THE IMPORTANCE OF ANISOTROPIC INTERSTELLAR TURBULENCE AND MOLECULAR-CLOUD MAGNETIC MIRRORS FOR GALACTIC COSMIC-RAY PROPAGATION

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ABSTRACT

Recent studies suggest that when magnetohydrodynamic (MHD) turbulence is excited by stirring a plasma at large scales, the cascade of energy from large to small scales is anisotropic, in the sense that small-scale fluctuations satisfy the inequality \( k_\parallel \ll k_\perp \), where \( k_\parallel \) and \( k_\perp \) are, respectively, the components of a fluctuation’s wave vector \( \parallel \) and \( \perp \) to the background magnetic field. Such anisotropic fluctuations are very inefficient at scattering cosmic rays. Results based on the quasilinear approximation for scattering of cosmic rays by anisotropic MHD turbulence are presented and explained. The important role played by molecular-cloud magnetic mirrors in confining and isotropizing cosmic rays when scattering is weak is also discussed.

1. INTRODUCTION

In diffusion models of Galactic cosmic-ray propagation, cosmic rays are scattered by small-scale fluctuations in the interstellar magnetic field. For cosmic-ray energies below \( 10^2 - 10^3 \) GeV, these small-scale fluctuations can arise from resonant waves that the cosmic rays generate themselves. At higher energies, it is believed that self-confinement is not possible, because the growth rates of the resonant modes become too small in comparison to the rates at which the modes are damped (Cesarsky 1980, Berezinskii et al. 1990). For cosmic rays with energies above \( 10^5 - 10^5 \) GeV, scattering can result from turbulence that is generated by large-scale stirring of the interstellar medium (ISM), which results in a cascade of magnetic energy from large to small scales. Recent studies of magnetohydrodynamic (MHD) turbulence, however, find that the small-scale fluctuations resulting from a turbulent cascade satisfy the inequality \( k_\parallel \ll k_\perp \), where \( k_\parallel \) and \( k_\perp \) are, respectively, the components of a fluctuation’s wave vector \( \parallel \) and \( \perp \) to the background magnetic field \( \mathbf{B}_0 \) (e.g., Goldreich & Sridhar 1995). This paper explains why fluctuations with this anisotropy are very inefficient at scattering cosmic rays, and presents results from quasi-linear theory for the scattering mean free paths resulting from anisotropic MHD turbulence. The role of molecular clouds in confining and isotropizing cosmic rays when scattering is weak is also discussed.

2. ANISOTROPIC MHD TURBULENCE AND THE GOLDFREICH-SRIDHAR SPECTRUM

Early studies of MHD turbulence assumed that when a plasma is stirred on some large scale \( l \), the cascade of energy from large scales to small scales proceeds isotropically in \( k \)-space (Kraichnan 1965). More recent studies, however, find that energy cascades efficiently to large values of \( k_\perp \), but not very efficiently to large values of \( k_\parallel \) (e.g., Goldreich & Sridhar 1995, Shebalin et al. 1983). (Note: if the mean-magnetic field is weaker than the fluctuating magnetic field, then within any stirring-scale cell of volume \( l^3 \) there is a preferential field direction which can be thought of as the background field \( \mathbf{B}_0 \) for all of the small-scale fluctuations within that cell. Local anisotropy is then determined relative to the direction of \( \mathbf{B}_0 \) within each stirring-scale cell.) Goldreich & Sridhar (1995) have proposed an inertial-range power spectrum for strong anisotropic MHD turbulence in which the magnetic field fluctuations are comparable to \( \mathbf{B}_0 \):

\[
E_B(k_\perp, k_\parallel) \propto k_\parallel^{-10/3} l^{-1/3} g \left( \frac{k_\parallel}{k_\perp^{2/3} l^{-1/3}} \right),
\]

where the dimensionless function \( g(x) \) is \( \sim 1 \) for \( |x| \lesssim 1 \) and rapidly approaches 0 for \( |x| \gg 1 \). Evidence in support of equation (1) has been found in direct numerical simulations of MHD turbulence (Maron 2000, Cho & Vishniac 2000). In this spectrum, there is only power at small scales when \( k_\parallel \sim k_\perp^{2/3} l^{-1/3} \). In this region of \( k \)-space, in which turbulence is excited, the linear incompressible Alfvén-wave period \( (k_\parallel v_A)^{-1} \) is greater than the nonlinear energy-transfer time \( (k_\perp v_k)^{-1} \), where

\[
\mathbf{v}_A = \mathbf{B}_0 / \sqrt{4 \pi \rho}
\]

is the Alfvén speed, \( \rho \) is the mass density of the medium, and

\[
\mathbf{v}_k \sim v_A (k_\perp l)^{-1/3}
\]

is the rms velocity fluctuation on a perpendicular scale of \( k_\perp^{-1} \). Because of this inequality, the Iroshnikov-Kraichnan mechanism for slowing energy-transfer does not apply (Kraichnan 1965).

3. WHY SCATTERING IS WEAK IN ANISOTROPIC TURBULENCE

If the magnetic power spectrum is isotropic in \( k \)-space or possesses slab symmetry (in which the wave vectors of fluctuations are \( \parallel \mathbf{B}_0 \)), then cosmic-ray scattering is dominated by magnetostatic gyroresonant interactions, in which the cosmic ray and fluctuation satisfy the resonance relation

\[
k_\parallel \mathbf{v}_l = n \Omega,
\]

where \( \mathbf{v}_l \) is the component of a cosmic ray’s velocity along \( \mathbf{B}_0 \), \( n \) is a non-zero integer (\( n = \pm 1 \) for slab symmetry), and \( \Omega \) is the cosmic ray’s gyrofrequency. In the general resonance relation, the linear wave frequency \( \omega \) appears on the left-hand side of equation (2). It is neglected here since for incompressible Alfvén waves \( \omega \simeq k_\parallel v_A \), which is \( \ll k_\parallel \mathbf{v}_l \) unless the angle between a cosmic-ray’s velocity vector \( \mathbf{v} \) and \( \mathbf{B}_0 \), the pitch angle, is very close to 90°. If a cosmic ray’s pitch angle isn’t too close to 0°, 90°, or 180°, then equation (2) implies that the fluctuations that dominate scattering satisfy \( k_\parallel \sim \rho^{-1} \), where \( \rho = v_\perp / \Omega \) is a cosmic
ray’s gyroradius and $v_\perp$ is the component of $v \perp$ to $B_0$. However, if $p^{-1} \gg l^{-1}$, where $l$ is the scale at which the turbulence is stirred, and if the turbulence has the type of anisotropy described by equation (1), then the only fluctuations with $k_\parallel \sim p^{-1}$ have $k_\perp \gg p^{-1}$, as depicted in figure 1. But if $k_\perp p \gg 1$, then during a single gyro orbit a cosmic ray traverses many uncorrelated turbulent fluctuations of the required $k_\parallel$. The contributions from these different fluctuations tend to cancel, resulting in highly inefficient scattering.

Scattering rates have been calculated for turbulence with a power spectrum described by equation (1) in the quasilinear approximation (Chandran 2000a). If
\[
\delta = \frac{v_\perp}{v} \ll 1,
\]
and
\[
\varepsilon = \frac{v_\perp}{v_\parallel} \ll 1,
\]
then when $\varepsilon^{3/2} \ll (-\ln \varepsilon)\delta$ the coefficient of spatial diffusion along the magnetic field resulting from the quasilinear scattering rates is given by (Chandran 2000a)
\[
\kappa_\parallel = v\left(-\delta \ln \varepsilon\right)^{-1} \left(\frac{5}{2} - \frac{3\pi}{4}\right).
\]

This value is far too large to explain confinement of cosmic rays to the Galaxy. Thus, some mechanism besides turbulence described by equation (1) must be invoked to explain cosmic-ray confinement. For cosmic rays with energies less than $10^9$–$10^{10}$ GeV, waves excited by the cosmic rays may provide the confinement. For higher energy cosmic rays for which self confinement does not appear possible (Cesarsky 1980, Berezinskii et al. 1990), molecular-cloud magnetic mirrors may play an important role. It should be noted that equation (5) applies to both cosmic-ray nuclei and electrons.

4. WHY MOLECULAR CLOUDS CAN HELP CONFINING COSMIC RAYS WHEN SCATTERING IS WEAK

Molecular-clouds are characterized by a range of sizes, masses, and densities. Cloud mass spectra obey power-law scalings over several decades of cloud masses, and a power-law mass-size relation also holds over a range of scales (Elmegreen & Falgarone 1996, Heithausen et al. 1998, Blitz & Williams 1997). Elmegreen (1997) has proposed a useful quasi-fractal model for this hierarchy of structure, with smaller, denser objects nested within structures that are larger and more diffuse. Using straightforward rules to generate fractal structures, Elmegreen estimates through numerical modeling that a line-of-sight through a fractal molecular cloud complex has a $50 \pm 10$% chance of entering dense molecular material, and a $50 \pm 10$% chance of passing through a hole in the fractal complex filled with diffuse matter. Elmegreen interprets the standard 8 large absorption lines per kpc (Blaauw 1952) as evidence for an average of 3 fractal cloud complexes per kpc along a typical line of sight. Because of the 50% see-through probability, however, photons can travel $\sim 600$ pc without entering molecular material. The 8 absorption lines per kpc are then not spaced at even distances along a line of sight, but rather are clustered in groups of $\sim 5$ per cloud complex.

Because magnetic field lines are focused into strong-field regions, and because the magnetic field is stronger within molecular clouds than in the ICM, a magnetic field line passing through a cloud complex has a higher probability of entering molecular material than a straight line of sight (figure 2). If the mean field strength within the molecular material is $m$ times the typical field strength in the ICM, and if 50% of the line-of-sight through the complex intersect molecular material, then one would expect a fraction $P$ on the order of $m/(m+1)$ of the magnetic flux (and, therefore, field lines) through the complex to pass through molecular material. The line-of-sight average of a cloud’s magnetic field can be obtained through Zeeman splitting. Troland & Heiles (1996) report Zeeman measurements of molecular-cloud field strengths ranging from 9 $\mu$G to 120 $\mu$G. The typical field strength in the ICM is $\sim 4$–$5$ $\mu$G (Zweibel & Heiles 1997). The ratio $m$ is thus fairly large, and $P$ is close to 1. Moreover, due to magnetic focusing, the spacing of cloud complexes along field lines may be smaller than the $\sim 300$-pc spacing of complexes along straight lines. On the other hand, tangling of field lines in the ICM may tend to increase the spacing of successive cloud complexes along a given field line as measured along that field line. It seems reasonable, however, to take the typical distance between cloud complexes as measured along a field line to be $l_{\text{intercloud}} \approx 300$ pc.

Once inside a molecular cloud, field lines tend to be focused into dense clumps where the field strength is larger than the average field strength in the cloud. A range of observations suggests that at particle densities $n$ above $10^2$ cm$^{-3}$, the magnetic field strength $B$ scales as (Vallée 1997)
\[
B \sim n^{0.5}.
\]

According to Heithausen et al. (1998),
\[
M \sim r^{2.3},
\]
\[
n \sim r^{-0.7},
\]
\[
dN/dr \sim r^{-1.0},
\]
where $M$ is the mass of a clump of linear dimension $r$, and $(dN/dr)\Delta r$ is the number of clumps with linear dimension $r$ in the interval $(r, r + \Delta r)$. Since the flux through a clump is $\sim B r^2$, equations (7) through (10) imply that the total flux $\Phi$ through clumps with linear dimension between $r$ and $2r$ scales as
\[
\Phi \sim r^{-0.35}.
\]

That is, there is more flux through the smaller, denser clumps than through the cloud complex as a whole. This means that as a single field line passes through a complex, it must on average pass through several of the densest clumps described by the power-law scalings, clumps in which the magnetic field strength is large.

If $B_{\text{ICM}}$ is the field strength in the ICM and $B_{\text{max}}$ is the maximum field strength encountered by a cosmic ray in a molecular-cloud complex, then the cosmic ray will be magnetically reflected by the cloud complex provided that its pitch-angle cosine $\xi_{\text{ICM}} = v_\parallel/v$ in the ICM as it approaches the complex satisfies the inequality
\[
\left|\xi_{\text{ICM}}\right| < \sqrt{1 - \chi^{-1}},
\]
where
\[
\chi \equiv \frac{B_{\text{max}}}{B_{\text{ICM}}}.
\]

The above discussion suggests that $\chi \gg 1$, and thus molecular clouds can magnetically reflect a large fraction of cosmic rays.

When the scattering mean-free path $l_\parallel/v_\parallel$ is much greater than $l_{\text{intercloud}}$, cosmic rays travel between molecular clouds without significant scattering. Phase-space orbits of cosmic
Once inside molecular material, field lines tend to be focused into dense clumps where the field is strongest.

Fig. 2.—
rays in this weak-scattering limit are depicted in figure 3, where $\xi = v_\perp / v$. Particles with $|\xi|$ close to 1 are not magnetically reflected and can pass through molecular clouds. Trapped cosmic rays move on closed orbits in the $x$-$\xi$ plane, where $x$ is distance along a field line.

The way in which molecular-cloud magnetic mirrors affect cosmic-ray transport depends upon cosmic-ray energy and also the efficiency of cosmic-ray scattering. Approximate coefficients of diffusion perpendicular to the Galactic plane have been calculated for several different propagation regimes (Chandran 2000b). Broadly speaking, cosmic rays can escape from magnetic traps in one of two ways, by scattering into the passing region of phase space and then traveling along the magnetic field through a molecular cloud, or by drifting perpendicular to the magnetic field. The second case is depicted in figure 4: a cosmic ray initially trapped between clouds A and B can drift perpendicular to the magnetic field and end up trapped between clouds A and D.

Because molecular clouds are confined to the Galactic disk, it is possible that they have no affect on cosmic-ray propagation in the halo. On the other hand, if the magnetic field lines in the halo possess numerous arcs that are anchored down to the disk on either end (analogous to closed field lines in the solar corona), then molecular cloud magnetic mirrors may affect propagation in the halo to some extent.

5. ISOTROPIZATION OF COSMIC RAYS BY MOLECULAR-CLOUD MAGNETIC MIRRORS

One of the sources of cosmic-ray anisotropy is the flow of cosmic rays along the magnetic field. Because molecular-cloud magnetic mirrors impede this flow, they reduce the level of anisotropy for any given level of weak scattering. Values of the harmonics of the cosmic-ray distribution function (as functions of the scattering rate, $\chi$, and $l_{\text{intercloud}}$) are given by Chandran (2000b) under the assumption that the pitch-angle scattering frequency is weak and independent of pitch angle.

6. IMPLICATIONS OF MOLECULAR-CLOUD MAGNETIC MIRRORS FOR DIFFUSE GAMMA RADIATION AND SECONDARY PRODUCTS

When scattering is weak, the density of cosmic rays within molecular clouds $n_{\text{cloud}}$ is determined by two competing effects. On the one hand, cosmic rays are reflected as they approach cloud complexes, which tends to reduce $n_{\text{cloud}}$. On the other hand, magnetic field lines are brought closer together in high-field regions, which acts to increase $n_{\text{cloud}}$. Since cosmic rays travel primarily along the magnetic field. It can be shown that when energy losses are neglected, these two effects cancel (Chandran 2000b). This point is important since if $n_{\text{cloud}}$ were in fact less than $n_{\text{ICM}}$, there would be a corresponding reduction in spallation and diffuse gamma radiation for a fixed average energy density of cosmic rays throughout the Galaxy. (For sufficient ionization losses at low cosmic-ray energies, it should be noted that the value of $n_{\text{cloud}}$ can be reduced below the cosmic-ray density in the intercloud medium $n_{\text{ICM}}$.)

The two competing effects described above are illustrated graphically in figure 5. Each of the two narrow flux tubes in figure 5 has a bounding surface that is everywhere parallel to the magnetic field. The cross-sectional area of each tube is proportional to $(1/B)$, where $B$ is the field strength. Since motion perpendicular to the magnetic field is suppressed, the cosmic rays within each flux tube to a good approximation remain within their respective flux tubes as they move along the field. Because of magnetic mirroring, the number of cosmic rays per unit length within a flux tube decreases in high-field regions. However, because the cross-sectional area of the flux tube also decreases, the number of cosmic rays per unit volume stays the same.

7. DOES THE MODEL FIT THE DATA?

At this stage, it is difficult to determine from observations whether molecular clouds play a role in cosmic ray confine-
ment. Although the observed energy dependence of the cosmic-ray path length at cosmic ray energies \( < 10^2 \) GeV provides important information on propagation at energies \( < 10^2 \) GeV, almost nothing is known about the path length at the energies \( 10^2 - 10^3 \) GeV at which self-confinement appears to break down and at which confinement may depend upon molecular clouds. There appear to be three main possibilities. First, molecular clouds may help confine cosmic rays at energies \( 10^2 - 10^3 \) GeV as described in this paper. Second, the arguments that self-confinement breaks down at energies \( > 10^2 - 10^3 \) GeV may be incorrect. Third, interstellar turbulence generated by large-scale stirring may possess features not described by the Goldreich-Sridhar theory that allow for stronger scattering.

8. CONCLUSION

Recent investigations into MHD turbulence are providing new and important results on the anisotropy of the small-scale fluctuations that result from a cascade of magnetic energy from large to small scales. As discussed in this paper, anisotropic small-scale fluctuations are inefficient at scattering cosmic rays. For cosmic rays with energies less than \( 10^2 - 10^3 \) GeV, resonant waves excited by streaming cosmic rays are believed to be sufficient to confine cosmic rays to the Galaxy regardless of the nature of the cascade in MHD turbulence. At higher energies, however, it is believed that such self-generated waves are insufficient. Thus, if scattering by the turbulence that is generated by large-scale stirring of the ISM is inefficient, then some additional mechanism is needed to confine and isotropize cosmic rays at energies above \( \sim 10^2 - 10^3 \) GeV. Such a mechanism may be provided by molecular-cloud magnetic mirrors.

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