Superstring Dualities

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Abstract

This talk is divided into two parts. The first part reviews some of the duality relationships between superstring theories. These relationships are interpreted as providing evidence for the existence of a unique underlying fundamental theory. The second part describes my recent work on the $SL(2,\mathbb{Z})$ duality group of the type IIB superstring theory in ten dimensions and its interpretation in terms of a possible theory of supermembranes in eleven dimensions.

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1 Relationships among superstring theories

As of ten years ago we knew five different supersymmetric string “theories” in ten dimensions (nine space and one time): Types I, IIA, IIB, and two heterotic theories. The word “theories” is placed in quotation marks, because we only had rules for characterizing a classical vacuum and for computing quantum corrections in perturbation theory. Still, this was a major advance over anything else on the market. Non-perturbative physics was completely out of reach. At the time, my colloquium lectures [1] stressed that we really didn’t want five consistent quantum theories containing gravity. It would be much nicer if there was just one, assuming of course that it successfully describes nature. I suggested that some of them might turn out to be inconsistent or some might be equivalent or a combination of the two. Over the past year a great deal has been learned about non-perturbative properties of string theory, which makes it now possible to reassess the situation.

The first two “unifications” to emerge were between the two type II theories [2] and between the two heterotic theories [3]. While the IIA theory and IIB theory in ten dimensions are certainly different – indeed the former is non-chiral and the latter is chiral – they become equivalent after compactification of one spatial dimension on a circle. However, this happens in a surprising way: they are T dual. This means that the compactification radius of the IIA theory (call it $R_A$) is inversely proportional to the compactification radius of the IIB theory (call it $R_B$). The radius corresponds to the classical value of a scalar field in nine dimensions, and the two 10D theories correspond to two different limits of this value. The scalar field in question has a flat potential, so the IIA and IIB theories are smoothly connected as boundary points of the classical moduli space of the common 9D theory. This relationship, like most T dualities, is valid order-by-order in string perturbation theory (though it is non-perturbative on the string world-sheet), and is, therefore, understood very well.

The two 10D heterotic theories have gauge groups $E_8 \times E_8$ and $O(32)$, the only possibilities allowed by anomaly cancellation. They are related in much the same way as the two type II theories. Compactification to nine dimensions gives a theory with a 17-dimensional moduli space. One of these moduli describes the compactification radius in the $E_8 \times E_8$ picture and another one gives the radius in the $O(32)$ picture. Thus, the two 10D theories again correspond to two distinct boundary points of the classical moduli space for nine dimensions.

We have now reduced the number of “theories” to three – type I, type II, and heterotic. To do better than this requires going beyond string perturbation theory, and is necessarily more
speculative. After reviewing the evidence, I’ll present my interpretation of its significance.

The type II theory is related to the heterotic theory by “string-string duality” \(\text{[4, 5, 6, 7, 8]}\). It has various manifestations, but the basic one is described in six dimensions. The conjecture asserts that the type IIA theory compactified on the K3 manifold is non-perturbatively equivalent to the heterotic theory compactified on a four-torus. Both theories have the same massless sectors and the same \(O(4, 20; \mathbb{Z})\) discrete group of symmetries. The situation is much the same as before. Namely, the 10D theories correspond to different boundary points in the 80-dimensional moduli space of the common 6D theory. Moreover, the common low-energy effective supergravity theory gives rise to both kinds of strings as soliton-like solutions. These arguments, as well as a variety of others, make a compelling case. This identification is remarkable, because weak coupling is mapped to strong coupling, and therefore perturbative results in one description correspond to non-perturbative results in the other.

All that now remains is to relate the type I superstring theory to the others. This appears very challenging, because type I superstrings are unoriented open and closed strings, whereas type II and heterotic strings are oriented closed strings. However, these strings are used to define perturbation expansions, and the identifications are supposed to be non-perturbative, so this may not be as big an obstacle as we used to believe. One could imagine that heterotic or type II strings would arise as solitons of the type I theory and vice-versa. Indeed, the low-energy supergravity theories obtained from the 10D \(O(32)\) type I and heterotic strings are the same, but the correspondence implies that the dilaton field obtained from the one string corresponds to the negative of the dilaton obtained from the other, which means that the coupling constants are inversely proportional \([9]\). This motivated the conjecture that the string theories are really the same, and that the perturbative expansion of one corresponds to the strong coupling expansion of the other (just as in the case of the 6D string-string duality described above). While it is understood how to obtain the heterotic string as a soliton solution of the low-energy supergravity theory \([9, 10]\), a type I string solution would involve additional subtleties and has not yet been constructed. Curiously, there is no candidate for an analogous dual string description of the \(E_8 \times E_8\) heterotic string in ten dimensions.

We have now connected all five of the 10D superstring theories, but there is one more “theory” that should be mentioned, namely 11D supergravity. Unlike the string theories, it is not perturbatively renormalizable, but it might exist as a quantum theory nonperturbatively. Townsend \([11]\) and Witten \([6]\) have pointed out that the 10D type IIA theory is actually
eleven dimensional! The IIA theory has a hidden circular eleventh dimension, whose radius scales as the two-thirds power of the string coupling constant. Thus, in perturbation theory the IIA theory looks ten dimensional, but in the strong coupling limit one obtains full 11D Poincaré symmetry described at low energy by an effective 11D supergravity theory.

Having reviewed the relevant facts, let us now contemplate their implications. It seems to me that these duality relations imply that the various different string “theories” should be viewed as recipes for finding classical solutions (and their perturbative quantum improvements) of a single underlying theory. What this underlying theory should be is quite uncertain at this time, but it may be unlike anything we’ve seen before. It needs to be able to give rise to all the solutions obtainable by any of the known recipes and account for all of their duality symmetries and relationships. It might also have additional solutions that are not obtainable by any of the currently known recipes. The remarkable role of duality symmetries and their geometrically non-intuitive implications suggest to me that the theory might look very algebraic in structure without evident geometric properties, so that no space-time manifold is evident in its formulation. In this case, the existence of space-time would have to emerge as a property of a class of solutions. Other solutions might not have any such interpretation. Even stringy one-dimensional structures might not play a more prominent role than other $p$-branes in the underlying theory [12, 13], in which case the subject will require a new name. One line of inquiry that may be helpful for inventing the theory is to focus on determining the complete group of duality symmetries. Assuming this is a well-defined notion (so that the group that appears is not just an artifact of formalism), the duality group could turn out to be some very large discrete subgroup of a hyperbolic Lie algebra such as $E_{10}$ or the monster Lie algebra. In the past, symmetries have been a useful guide to dynamics. Maybe that will turn out to be the case once again.

On the other hand, a radically different type of description, which is much more geometrical and less algebraic, could turn out to be correct. One such possibility is a theory based on fundamental supermembranes in eleven dimensions [14, 15]. This proposal used to look very unattractive to me, because of its bad perturbative quantum behavior and its lack of chirality. Now, I am beginning to take it much more seriously, because there are remarkable heuristic arguments for how various string theories could be deduced from such a starting point. The next section describes how it can lead to the chiral IIB theory in ten dimensions. Most of the results that follow have been presented previously in ref. [16], but one new result
an interpretation of type IIB superstrings as wrapped supermembranes – is given at the end of the section.

2 Type IIB Superstrings

Among the various conjectured duality symmetries of superstring theories, the proposed $SL(2, \mathbb{Z})$ symmetry of the type IIB superstring theory in ten dimensions is especially interesting \[4, 7\]. Like the $SL(2, \mathbb{Z})$ S duality of the $N = 4$ 4D heterotic string \[18, 19\], it relates weak and strong coupling. However, unlike the heterotic example in which the symmetry relates particles carrying electric and magnetic charges of the same gauge field, the IIB duality relates strings carrying electric charges of two different gauge fields. In this respect it is more like a T duality \[20\]. Combined with ordinary T dualities, the IIB $SL(2, \mathbb{Z})$ duality implies the complete U duality symmetry of toroidally compactified type II strings in dimensions $D < 10$ \[4, 9\].

The $SL(2, \mathbb{Z})$ duality of the IIB theory will be explored here by considering string-like (or ‘one-brane’) solutions of the 10D IIB supergravity theory. It will be argued that there is an infinite family of such solutions forming an $SL(2, \mathbb{Z})$ multiplet. (This possibility was hinted at in section 5 of Ref. \[8\].) Once these string solutions have been constructed, we will consider compactification on a circle and compare the resulting 9D spectrum with that of 11D supergravity compactified on a two-torus. The conclusion will be that the $SL(2, \mathbb{Z})$ duality group of the IIB theory in ten dimensions corresponds precisely to the modular group of the torus\[^3\] and that all type II superstrings can be interpreted as wrapped supermembranes of 11D supergravity.

All 10D supergravity theories contain the following terms in common

\[ S_0 = \frac{1}{2\kappa^2} \int d^{10} x \, \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{12} e^{-\phi} H^2 \right), \]  

(1)

where $H$ is a three-form field strength ($H = dB$), and $\phi$ is the dilaton. Moreover, in each case, a solution to the classical equations of motion derived from $S_0$ can be regarded as a solution of the complete supergravity theory with all other fields set equal to zero. A macroscopic string-like solution, which was identified with the heterotic string, was constructed by Dabholkar

\[^3\]This conclusion has been reached independently by Aspinwall \[21\]. The 11D origin of the $SL(11 - d, \mathbb{Z})$ subgroup of the U duality group of type II string theory in $d$ dimensions is explained in Witten’s paper \[6\]. See also Appendix B of Ref. \[22\].
et al. [23] (This was generalized to $p$-branes in Ref. [24].) Restricted to ten dimensions, it is given by

$$ds^2 = A^{-3/4}[-dt^2 + (dx^1)^2] + A^{1/4}d\mathbf{x} \cdot d\mathbf{x},$$

$$B_{01} = e^{2\phi} = A^{-1},$$

where

$$A = 1 + \frac{Q}{3r^6},$$

$$\mathbf{x} = (x^2, x^3, \ldots, x^9), \quad \mathbf{x} \cdot \mathbf{x} = r^2 = \delta_{ij}x^ix^j, \quad \text{and} \quad Q \text{ is the } H \text{ electric charge carried by the string.}$$

(Recall that the electric charge of a $(p + 2)$-form field strength is carried by a $p$-brane.) Strictly speaking, the $S_0$ equations are not satisfied at $r = 0$, the string location, because $\nabla^2 A$ has a delta-function singularity there. In Ref. [23] it was proposed that this could be fixed by coupling to a string source, which means considering $S = S_0 + S_\sigma$ instead, where

$$S_\sigma = -\frac{T}{2} \int d^2\sigma (\partial^\alpha X^\mu \partial_\alpha X^\nu G_{\mu\nu} + \ldots),$$

$$T \text{ is the string tension, and } G_{\mu\nu} = e^{\phi/2}g_{\mu\nu} \text{ is the string metric.}$$

From the coefficient of the delta function one can deduce the relation $Q = \kappa^2 T/\omega_7$, where $\omega_7 = \frac{1}{3}\pi^4$ is the volume of $S^7$.

The type IIB theory has two three-form field strengths $H^{(i)} = dB^{(i)}, \ i = 1, 2$ [24]. $H^{(1)}$ belongs to the NS–NS sector and can be identified with $H$ in the preceding discussion. $H^{(2)}$ belongs to the R–R sector and does not couple to the (usual) string world sheet. In addition, the type IIB theory has two scalar fields, which can be combined into a complex field $\lambda = \chi + ie^{-\phi}$. The dilaton $\phi$ is in the NS–NS sector and can be identified with $\phi$ in the preceding discussion, while $\chi$ belongs to the R–R sector. The other bose fields are the metric $g_{\mu\nu}$ and a self-dual five-form field strength $F_5$. The five-form field strength will be set to zero, since the corresponding charges are carried by a self-dual three-brane, whereas the focus here is on charges carried by strings. Once we set $F_5 = 0$, it is possible to write down a covariant action that gives the desired equations of motion [10]:

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} (R + \frac{1}{4} tr(\partial M \partial M^{-1}) - \frac{1}{12} H^T M H).$$

Here we have combined $H^{(1)}$ and $H^{(2)}$ into a two-component vector $H = dB$, and introduced the symmetric $SL(2, \mathbb{R})$ matrix

$$M = e^\phi \begin{pmatrix} |\lambda|^2 & \chi \\ \chi & 1 \end{pmatrix}.$$
This action has manifest invariance under the global $SL(2, \mathbb{R})$ transformation

$$M \rightarrow \Lambda M \Lambda^T, \quad B \rightarrow (\Lambda^T)^{-1} B.$$  \hfill (8)

The choice $\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ corresponds to

$$\lambda \rightarrow \frac{a \lambda + b}{c \lambda + d}, \quad B^{(1)} \rightarrow dB^{(1)} - cB^{(2)}, \quad B^{(2)} \rightarrow aB^{(2)} - bB^{(1)}.$$  \hfill (9)

Given the symmetry of this system, it is clearly artificial to only consider solutions carrying $H^{(1)}$ electric charge and not $H^{(2)}$ electric charge. Measured in units of $Q$, we will consider solutions carrying charges $(q_1, q_2)$. Since there exist five-brane solutions carrying magnetic $H$ charge \cite{27}, the generalized Dirac quantization condition \cite{26} implies that $q_1$ and $q_2$ must be integers. Moreover, $q_1$ and $q_2$ should be relatively prime, since otherwise the solution is neutrally stable against decomposing into a multiple string solution — the number of strings being given by the common divisor.\footnote{This is the same counting rule that was required in a different context in Ref. \cite{27}. We will show that it leads to sensible degeneracies after compactification on a circle. As was pointed out in Ref. \cite{27}, a different rule is sometimes appropriate in other situations.} Also, the $(q_1, q_2)$ string and the $(-q_1, -q_2)$ string are related by orientation reversal ($x^1 \rightarrow -x^1$). A complete description of string solutions requires specifying the vacuum in which they reside. In the IIB theory this means choosing the asymptotic value of $\lambda$ as $r \rightarrow \infty$, denoted by $\lambda_0$. The simplest choice is $\lambda_0 = i$, which corresponds to $\chi_0 = \phi_0 = 0$. The $(1, 0)$ string in this background is given by the solution in eqs. (2) and (3). By applying an appropriate $SL(2, \mathbb{R})$ transformation to that solution, we can obtain the solution describing the $(q_1, q_2)$ string for arbitrary $\lambda_0$.

The general solution describing a $(q_1, q_2)$ string in the $\lambda_0$ vacuum found in ref. \cite{16} is

$$ds^2 = A_q^{-3/4}[-dt^2 + (dx^1)^2] + A_q^{1/4}d\mathbf{x} \cdot d\mathbf{x},$$  \hfill (10)

$$B_{01}^{(i)} = q_i \Delta_q^{-1/2} A_q^{-1},$$  \hfill (11)

$$\lambda = \frac{i(q_2 \chi_0 + q_1 |\lambda_0|^2) A_q^{1/2} - q_2 e^{-\phi_0}}{i(q_1 \chi_0 + q_2) A_q^{1/2} + q_1 e^{-\phi_0}},$$  \hfill (12)

where\footnote{The original preprint version of this paper, and also of Ref. \cite{16}, had an error in eq. (13).}

$$\Delta_q = (q_1, q_2) \mathcal{M}_0^{-1} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = e^{\phi_0}(q_2 \chi_0 - q_1)^2 + e^{-\phi_0} q_2^2.$$  \hfill (13)
\[ A_q(r) = 1 + \Delta_q^{1/2} \frac{Q}{3r^6}. \]  

These equations describe an \( SL(2,\mathbb{Z}) \) family of type IIB macroscopic strings carrying \( H \) charges \((q_1, q_2)\) and an \( SL(2,\mathbb{Z}) \) covariant spectrum of tensions given by

\[ T_q = \Delta_q^{1/2} T. \]  

For generic values of \( \lambda_0 \) one of these tensions is smallest. However, for special values of \( \lambda_0 \) there are degeneracies. For example, \( T_{1,0} = T_{0,1} \) whenever \( |\lambda_0| = 1 \). (More generally, \( T_{q_1,q_2} = T_{q_2,q_1} \) in this case.) Also, \( T_{0,1} = T_{1,1} \) whenever \( \chi_0 = \frac{1}{2} \). (More generally, \( T_{q_1,q_2} = T_{q_2-q_1,q_2} \) in this case.) Combining these, we find a three-fold degeneracy \( T_{1,0} = T_{0,1} = T_{1,1} \) for the special choice \( \lambda_0 = e^{\pi i/3} \).

Although we have only constructed infinite straight macroscopic strings, there must be an infinite family of little loopy strings whose spectrum of excitations can be analyzed in the usual way. Thus, in ten dimensions each of the \((q_1, q_2)\) strings has a perturbative spectrum given by

\[ M^2 = 4\pi T_q(N_L + N_R), \]

where \( N_L \) and \( N_R \) are made from oscillators in the usual way. Each of the strings has the same massless sector — the IIB supergravity multiplet — in common. The excited states are presumably distinct, with the excited levels of one string representing states that are non-perturbative from the viewpoint of any of the other strings. Of course, the formula for \( M^2 \) gives the free-particle spectrum only, which is not meaningful for two different strings at the same time, so comparisons of massive levels are only qualitative. In ten dimensions, the only states in short supersymmetry multiplets, for which we have good control of the corrections, are those of the supergravity multiplet itself.

We now turn to the theory compactified to nine dimensions, because much more of the spectrum is under precise control in that case.

Consider the \((q_1, q_2)\) IIB string compactified on a circle of radius \( R_B \). Then the resulting perturbative spectrum of this string has 9D masses given by

\[ M^2_B = \left( \frac{m}{R_B} \right)^2 + (2\pi R_B n T_q)^2 + 4\pi T_q(N_L + N_R), \]  

where \( m \) is the Kaluza–Klein excitation number (discrete momentum) and \( n \) is the winding number, as usual. Level-matching gives the condition \( N_R - N_L = mn \). Short multiplets (which saturate a BPS bound) have \( N_R = 0 \) or \( N_L = 0 \). (Ones with \( N_L = N_R = 0 \) are ‘ultrashort’.) Taking \( N_L = 0 \) gives \( M^2_B = (2\pi R_B n T_q + m/R_B)^2 \) and a rich spectrum controlled by \( N_R = mn \). The masses of these states should be exact, and they should be
stable in the exact theory. Note that

\[ n^2 T_q^2 = [\ell_2^2 + e^{2\phi_0}(\ell_2 \chi_0 - \ell_1)^2]e^{-\phi_0} T^2, \]  

(17)

where \( \ell_1 = nq_1 \) and \( \ell_2 = nq_2 \). Any pair of integers \((\ell_1, \ell_2)\) uniquely determines \( n \) and \((q_1, q_2)\) up to an irrelevant sign ambiguity. Winding a \((-q_1, -q_2)\) string \(-n\) times is the same thing as winding a \((q_1, q_2)\) string \( n \) times. Thus the pair of integers \((\ell_1, \ell_2)\) occurs exactly once, with the tension of the string determined by the corresponding pair \((q_1, q_2)\).

Since the IIB theory compactified on a circle is equivalent to the IIA theory compactified on a circle of reciprocal radius, and the IIA theory corresponds to 11D supergravity compactified on a circle, there should be a correspondence between the IIB theory compactified on a circle and 11D supergravity compactified on a torus. Therefore, let us consider compactification of 11D supergravity on a torus with modular parameter \( \tau = \tau_1 + i \tau_2 \).

Letting \( z = (x + iy)/2\pi R_{11} \), \( \psi_{\ell_1, \ell_2}(x, y) \) is evidently invariant under \( z \to z+1 \) and \( z \to z+\tau \). The contribution to the 9D mass-squared is given by the eigenvalue of \( p_x^2 + p_y^2 = -\partial_x^2 - \partial_y^2 \). Let us try to take the supermembrane idea [14, 15, 11] seriously, and suppose that it has tension (mass/unit area) \( T_{11} \). Wrapping it so that it covers the torus \( m \) times gives a contribution to the mass-squared of \((mA_{11} T_{11})^2\). (Different maps giving the same \( m \) are identified.)

The area of the torus in the 11D metric is \( A_{11} = (2\pi R_{11})^2 \tau_2 \). Therefore, states with wrapping number \( m \) and Kaluza–Klein excitations \((\ell_1, \ell_2)\) have 9D mass-squared (in the 11D metric)

\[ M_{11}^2 = \left( m(2\pi R_{11})^2 \tau_2 T_{11} \right)^2 + \frac{1}{R_{11}^2} \left( \ell_2^2 + \frac{1}{\tau_2} (\ell_2 \tau_1 - \ell_1)^2 \right) + \ldots, \]  

(19)

where the dots represent membrane excitations, which we do not know how to compute. This is to be compared to eq. (16) for \( M_B^2 \), allowing \( M_{11} = \beta M_B \), since they are measured in different metrics. Agreement of the formulas for the masses of BPS saturated states is only possible if the vacuum modulus \( \lambda_0 \) of the IIB theory is identified with the modular parameter \( \tau \) of the torus. Since \( SL(2, \mathbb{Z}) \) is the modular group of the torus, this provides strong evidence that it should also be the duality group of the IIB string. In addition, the identification \( M_{11} = \beta M_B \) gives

\[ R_B^{-2} = T T_{11} A_{11}^{3/2}, \]  

(20)

A detailed comparison of the 9D fields and dualities obtained by compactifying the 11D, IIA, and IIB supergravity theories is given in Ref. [28].

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\[ \beta^2 = 2\pi R_{11} e^{-\phi_A/2} T_{11}/T. \]  

(21)

These identifications imply predictions for the spectrum of membrane excitations – at least those that give short supermultiplets.

It is also interesting to explore how the type IIA string fits into the story. One can identify \( r_{11} = R_{11}\tau_2 \) as the radius of the circle that takes 11D supergravity to the 10D IIA theory, and \( R_{11} \) as the radius of circle taking the 10D IIA theory to nine dimensions. Type IIA and IIB superstrings in nine dimensions are usually considered to be related by an \( R_B \sim 1/R_A \) duality of two circles, as has already been mentioned. The identification of eqs. (16) and (19) can be interpreted to mean that wrappings of the supermembrane on the torus in the 11D theory correspond to Kaluza–Klein excitations of the circle of the IIB theory, and windings of an \( SL(2,\mathbb{Z}) \) family of type IIB strings on the circle correspond to Kaluza–Klein excitations of the torus. So the duality is really between a torus and a circle rather than between two circles. Equation (20) tells us that \( R_B \sim A_{11}^{-3/4} \).

In Ref. [6], Witten showed that \( r_{11} \sim e^{2\phi_A/3} \), where \( \phi_A \) is the vev of the dilaton field in the \( D = 10 \) IIA theory, by comparing the masses of IIA states in ten dimensions that saturate a BPS bound to those of Kaluza–Klein excitations of 11D supergravity compactified on a circle of radius \( r_{11} \). An alternative approach is to consider wrapping the 11D supermembrane on a circle of radius \( r_{11} \) to give a type IIA string with tension \( 2\pi r_{11} T_{11} \) [24]. This tension is measured in units of the 11D metric \( g^{(11)} \). It can be converted to the IIA string metric \( g_A^{(10)} \) using \( g^{(11)} = e^{-2\phi_A/3} g_A^{(10)} \). Denoting the IIA string tension in the IIA string metric by \( T_A \), we deduce that

\[ T_A = 2\pi r_{11} T_{11} e^{-2\phi_A/3}. \]  

(22)

Since \( T_A \) and \( T_{11} \) are constants independent of \( r_{11} \) and \( \phi_A \), we have confirmed that \( r_{11} \sim e^{2\phi_A/3} \) and even determined the constant of proportionality.

Type IIB superstrings also have a simple supermembrane interpretation. To see this, let us begin with the 11D theory compactified on a two-torus and consider a toroidal supermembrane with one of its cycles mapped onto a \( (q_1, q_2) \) cycle of the spatial torus. The other membrane cycle describes the resulting 9D string. The integers \( q_1 \) and \( q_2 \) are taken to be relatively prime for the same reasons as before. Now suppose the membrane shrinks to the shortest cycle in the \( (q_1, q_2) \) homology class. Then it can be represented as a straight line from \( z = 0 \) to \( z = q_1 \tau + q_2 \), which has length \( L_q = 2\pi R_{11} |q_1 \tau + q_2| \). The resulting 9D string
has tension (in the 11D metric) $T_q^{(11)} = L q T_{11}$. Converting to the IIB metric by setting $T_q = \beta^{-2} T_q^{(11)}$ precisely reproduces the previous result $T_q = \Delta q^{1/2} T$. This means that in nine dimensions the entire $SL(2, \mathbb{Z})$ family of type II superstrings can be regarded as different wrappings of a unique supermembrane! Equation (21) implies that the 10D IIB superstring theory can be recovered by taking the limit in which the torus is shrunk to a point ($A_{11} \to 0$) while holding the modulus $\tau = \lambda_0$ fixed. Remarkably, the tensions $T_q$ are all independent of $A_{11}$, and therefore they are finite in the limit.

One can also identify the IIA theory in ten dimensions as a decompactification of the toroidally compactified supermembrane theory described previously. For this purpose, we let $R_{11} \to \infty$ while holding $r_{11} = R_{11} \tau_2$ fixed. In this limit the only $(q_1, q_2)$ string whose tension remains finite is the $(0, 1)$ string, whose tension in the 11D metric is $2 \pi r_{11} T_{11}$. (The other strings become zero-branes/black holes in the limit.) This limit implies that $\tau$ is outside the usual $SL(2, \mathbb{Z})$ fundamental region, but it can be mapped into the usual fundamental region by the $SL(2, \mathbb{Z})$ transformation $\tau \to -1/\tau$. This turns the $(0, 1)$ string into the $(1, 0)$ string, and thus the $(1, 0)$ string is identified as the (unique) fundamental IIA string. This resolves the puzzle of why a supermembrane can wrap around a two-torus any number of times, but it can wrap around a circle only once, at least if one accepts that $q_1$ and $q_2$ should be relatively prime.

To summarize, there is an infinite family of type IIB superstrings in ten dimensions labeled by a pair of relatively prime integers, which correspond to their $H$ charges. Any one of the strings can be regarded as fundamental with the rest describing non-perturbative aspects of the theory. This family of strings has tensions given by the $SL(2, \mathbb{Z})$ covariant expression in eqs. (13) and (15). Compactifying on a circle to nine dimensions and identifying with 11D supergravity compactified on a two-torus requires equating the modulus $\lambda_0$ of the IIB theory and the modular parameter $\tau$ of the torus, which is strong evidence for $SL(2, \mathbb{Z})$ duality. Other aspects of the spectrum are consistent with a supermembrane interpretation.

3 Conclusion

We have learned the following:

1. There are many conjectured non-perturbative dualities, all of which seem to be true.

2. Type IIB superstring theory in ten dimensions has an infinite multiplet of strings with
an $SL(2,\mathbb{Z})$ covariant spectrum of tensions. At least heuristically, the strings can be described as different wrappings of a supermembrane on a two-torus. This suggests that supermembranes might be more than just a heuristic tool.

3. There ought to be a completely unique underlying “superstring theory.” Whether it is based on something geometrical (like supermembranes) or something completely different is still not known. In any case, finding it would be a landmark in human intellectual history.

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