Dependence of linear polarization of radiation in accretion disks on the spin of central black hole

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Abstract

We suppose that linear optical polarization is due to multiple scattering in optically thick magnetized accretion disk around central black hole. The polarization degree is very sensitive to the spin of black hole - for Kerr rotating hole the polarization is higher than for Schwarzschild non-rotating one if both holes have the same luminosities and masses. The reason of this effect is that the radius of the first stable orbit for non-rotating hole is equal to three gravitational radiiuses, and for fast rotating Kerr hole is approximately 6 times lesser. Magnetic field, decreasing from first stable orbits, is much larger in the region of escaping of optical radiation for the case of Schwarzschild hole than for Kerr one. Large magnetic field gives rise to large depolarization of radiation due to Faraday rotation effect. This explains the mentioned result. It seems that the ensemble of objects with observed polarization mostly consists of Kerr black holes.

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1 Introduction

The estimation of the black hole spin, i.e. the dimensionless parameter $a_*=J/J_{max}=Jc/GM^2$, which determines the angular momentum, is very difficult from observational point of view. To some extent the observation of luminosity $L_{bol}=\varepsilon\dot{M}c^2$ of accretion disk allow us qualitatively estimate this parameter, if we compare this value with theoretical models predictions. Here $\dot{M}$ is the mass accretion velocity and $c$ being light velocity. Dimensionless parameter $\varepsilon$ demonstrates what part of mass transforms to the radiation energy. For Schwarzschild’s black hole ($a_*=0$) the theoretical estimation gives $\varepsilon=0.057$, and for fast rotating Kerr hole ($a_*=1$) the value $\varepsilon=0.42$ (See Blandford (1990), Krolik (2007) and Shapiro (2007)). So, one can qualitatively consider that the bright objects most probably correspond to the Kerr black holes, and the objects with weak luminosity correspond to Schwarzschild ones. Evidently, any other technique to estimate the parameter $a_*$, may be only qualitatively, will be of interest in this situation.

In this paper we demonstrate that the degree of linear polarization from magnetized accretion disk strongly depends on the parameter $a_*$.

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consequently, the linear polarization describes by known Milne’s problem for conservative magnetized atmosphere. Note that for non-magnetized electron atmosphere the maximal degree of polarization is equal to $p(\mu = 0) = 11.71\%$ and is observed in direction $\mathbf{n}$ perpendicular to surface normal $\mathbf{N}$ (see Chandrasekhar 1950).

Here and what follow $\mu = \cos i$ is cosine of disk’s inclination angle.

For estimations of linear polarization from magnetized disks we use the approximate analytical formulae (see Silan’ev 2002 and Silant’ev et al. 2009). The far located disks we observe as a point sources. Therefore, we use azimuthally averaged values of polarization. In this case the integral polarization is described by simple formulae:

$$p(\mu, \mathbf{B}) = \frac{p(\mu)}{[1 + C]^4 + 2(1 + C)(a^2 + b^2) + (a^2 - b^2)^2]^{1/4}};$$

$$\tan 2\chi = \frac{2(1 + C)a}{(p(\mu)/p(\mu, \mathbf{B}))^2 + (1 + C)^2 + b^2 - a^2}.$$  

(1)

Here $p(\mu)$ is the degree of polarization for non-magnetized atmosphere (Chandrasekhar’s value), $\mathbf{B} = \mathbf{B}_\parallel + \mathbf{B}_\perp$ is magnetic field inside the disk, the component $\mathbf{B}_\parallel$ is directed parallel to the disk normal $\mathbf{N}$, and $\mathbf{B}_\perp = \sqrt{B^2\rho + B^2\phi}$ is magnetic field perpendicular to $\mathbf{N}$. Dimensionless parameters $a$ and $b$ describe the Faraday depolarization of radiation:

$$a = 0.8\lambda^2 B_\parallel \mu \equiv \delta_\parallel \mu, \quad b = 0.8\lambda^2 B_\perp \sqrt{1 - \mu^2} \equiv \delta_\perp \sqrt{1 - \mu^2}$$

(3)

In all the paper we take the magnetic field in Gauss, the wavelengths - in micrometers ($\mu$m), and the distances - in centimeters. Physically the dimensionless parameter $\delta = 0.8\lambda^2 B$ corresponds to Faraday rotation of polarization plane along the Thomson optical length $\tau = 2$, if the magnetic field directed along line of sight $\mathbf{n}$. Positional angle $\chi = 0$ corresponds to wave electric field oscillations in the direction perpendicular to the plane ($\mathbf{n}\mathbf{N}$), i.e. parallel to disk’s plane.

Parameter $C$ arises in turbulent magnetized plasma (see Silant’ev 2005) and characterizes new effect - an additional extinction of Stokes parameters $Q$ and $U$ due to incoherent Faraday rotation of the polarization plane at small scale turbulent curls.

$$C \approx 0.64\tau\lambda^4 \langle B^2 \rangle f_B / 3$$

(4)

where $\tau$ is mean Thomson optical depth of a turbulent curl , $\langle B^2 \rangle$ characterizes mean value of magnetic field fluctuations , $f_B \approx 1$ - dimensionless parameter, integrally describing correlation of $B'$ taking at two near places of an atmosphere.

Below we will use the case $\mathbf{B}_\perp = 0$. In this case formulae (1) and (2) are most simple:

$$p(\mu, \mathbf{B}_\parallel) = \frac{p(\mu)}{\sqrt{(1 + C)^2 + a^2}}, \quad \tan 2\chi = \frac{a}{1 + C}.$$  

(5)

If we take that the minimum observed polarization is equal to 0.1%, then we can observe the linear polarization in plasma without turbulent magnetic field ($C = 0$) in optical range ($\lambda \approx 0.55\mu$m) at $\mu < 0.7$ and magnetic fields in the emitting region in the interval from zero up $B_\parallel \mu \approx 400$. This is rather wide range of magnetic field variations. Thus, for the object NGC4258 authors (see Silant’ev 2009) estimated the magnetic field in the region of optical emission as $B_\parallel \approx 160$ Gauss.
2 Magnetic field in accretion disk

Usually one uses that near the black hole horizon the strong magnetic field $\mathbf{B}_H$ arises, which spreads into accretion disk by power law, $\sim r^{-n}$. Remember, that the horizon radius $R_H$ is determined by the formula:

$$R_H = \frac{1}{2} r_g (1 + \sqrt{1 - a_*^2}), \quad r_g = 2 \frac{GM_{BH}}{c^2} = 2.95 \times 10^5 \left( \frac{M_{BH}}{M_\odot} \right) [\text{cm}]$$

(6)

where $r_g$ is the gravitational radius of the central mass $M_{BH}$, dimensionless parameter $a_*$ determines the spin of a black hole, $G$ is gravitational constant, $c$ - the light velocity. For Schwarzschild’s black hole ($a_* = 0$) the value $R_H = r_g$, and for the fast rotating Kerr’s black hole with $a_* = 1$ the value $R_H \approx r_g/2$.

Regular power law decreasing of magnetic field is accepted from the radius $r_{ms}$ of first from the center stable orbit. Analytical expression for $r_{ms}$ has the form (see Zhang 2005, Murphy 2009):

$$r_{ms} \equiv q(a_*) r_g = \frac{1}{2} (3 + A_2 - [(3 - A_1)(3 + A_1 + 2A_2)]^{1/2}) r_g,$$

$$A_1 = 1 + (1 - a_*^2)^{1/3}[(1 + a_*^{1/3}) + (1 - a_*^{1/3})], \quad A_2 = (3a_*^2 + A_*^2)^{1/2}.$$

(7)

According to this formula, Schwarzschild’s hole has $r_{ms} = 3r_g$, and fast rotating Kerr’s hole ($a_* \approx 1$) has $r_{ms} = r_g/2$, therewith the decreasing of $r_{ms}$ near $a_* \approx 1$ is very fast: $r_{ms} \approx r_g(1 + (4\delta)^{1/3})/2$ at $a_* \approx (1 - \delta) (\delta \ll 1)$ (see Chandrasekhar 1983). The values $q(a_*)$ for a number of $a_*$ are presented in Table 1.

Usually in accretion disks models one considers that at horizon $R_H$ the magnetic field $\mathbf{B}_H$ is perpendicular to the spherical surface, but further it enters to accretion disk perpendicular to disk’s surface, i.e. in disk exists the component $B_{\parallel}(r)$ (see Wang 2002, 2003; Zhang 2005).

The values $B_{\parallel}(r_{ms}) \equiv kB_H$ are slightly different for holes with different rotation velocity. Thus, for $a_* = 1$ the factor $k = 0.5$, and for $a_* = 0$ $k = 1/3$ takes place (see Wang 2002). So, we can take

$$B_{\parallel}(r) = B_{\parallel}(r_{ms}) \left( \frac{r_{ms}}{r} \right)^n \equiv k q^n(a_*) B_H \left( \frac{r_g}{r} \right)^n$$

(8)

For power index one usually accepts $1 \leq n < 3$ (see Pariev 2003).

The value $B_H$, according to Li 2002, Wang 2002, and Ma 2007, is presented by the expression:

$$B_H = \frac{\sqrt{2k_m c M}}{R_H} \approx \frac{1.66 \sqrt{k_m}}{1 + \sqrt{1 - a_*^2}} \left( \frac{M_{BH}}{M_\odot} \right)^{-1} \dot{M}^{1/2} \approx$$

$$\approx \sqrt{k_m} \left( \frac{M_{BH}}{M_\odot} \right)^{-1/2} \left( \frac{L_{bol}}{L_{edd}} \right)^{1/2} \frac{10^{8.8}}{1 + \sqrt{1 - a_*^2}}$$

(9)

Remember that the Eddington luminosity $L_{edd} = 1.3 \times 10^{38} (M_{BH}/M_\odot)$, and the bolometric luminosity of a disk $L_{bol} = \varepsilon \dot{M} c^2$. The parameter $k_m = P_{magn}/P_{gas}$ relates magnetic pressure with the gas pressure into disk. At equilibrium usually one takes $k_m \approx 1$. The parameter $\varepsilon$, as it was mentioned, characterizes the effectiveness of transition of accretion energy (the accretion velocity $\dot{M}$) to form of radiation. This parameter depends on particular mechanisms of transformation and also on the spin $a_*$. So, for $a_* = 0$ is found $\varepsilon = 0.057$, and for $a_* = 1$ one takes $\varepsilon = 0.42$ (see Blandford 1990, Krolik 2007, and Shapiro 2007).
As a result, we derive for $B$ the expressions:

$$B_{\parallel}(R_{\lambda}) = 10^{8.8-4.52k}q(a_*)^{\sqrt{k_m}}\frac{kq(a_*)^{\sqrt{k_m}}}{1+\sqrt{1-a_*^2}}\lambda^{-4n/3} \left(\frac{M_{BH}}{M_\odot}\right)^{n/3-1/2} \left(\frac{L_{bol}}{\epsilon L_{edd}}\right)^{1/2-n/3}.$$

(12)

$$B_{\parallel}(R_{\lambda}) = 10^{0.22+1.2k}q(a_*)^{\sqrt{k_m}}\frac{kq(a_*)^{\sqrt{k_m}}}{1+\sqrt{1-a_*^2}}\lambda^{-4n/3} \left(\frac{M_{BH}}{M_\odot}\right)^{2n-1} \left(\dot{M}\right)^{1/2-n/3}.$$

(13)

The first expression is convenient for calculation of $B_{\parallel}(R_{\lambda})$, if is known the bolometric disk luminosity and the black hole mass. The second formula relates directly $B_{\parallel}(R_{\lambda})$ with the accretion velocity. Equations (12) and (13) contain the function $f_n(a_*) = q^n(a_*)/(1 + \sqrt{1-a_*^2})$, which basically determine the dependence of magnetic field ($B \sim f_n$) in the radiation region from the black hole spin.

The values of this function are presented in Table 1.

As it was spoken, the factor $k$ depends also on the spin $a_*$, but this dependence is weak: $k = 1/3$ for $a_* = 0$ and $k = 1/2$ for $a_* = 1$. This dependence does not break the monotonic increase of $B_{\parallel}(R_{\lambda})$ with the decrease the spin. For this reason, further we shall neglect this weak effect. Thus, the Faraday depolarization, which is proportional to $B_{\parallel}(R_{\lambda})$, increases with the spin decrease. Therefore, the observed degree of polarization $p_{obs}$ will decrease.

As it is seen from the derivation of formulae (12) and (13), the effect of polarization decreasing for slow rotating black holes as compared with the fast rotating ones, mainly is determined by the fact, that in the first case the orbit $r_{ms}$ is located far from the disk center than for the second case.
This effect became more strong with grow of index $n$ (for $n = 0.5, 1, 1.5, 2, 3$ the ratio $f_n(0)/f_n(1)$ acquires, the values $1.22, 3, 7.34, 18, 108$, respectively).

If one considers the objects with equal masses $M_{BH}$ and luminosities $L_{bol}$, then it is necessary to take into account that the parameter $\varepsilon$ also depends on the spin $a_*$. In this case the value of magnetic field $B_{\|}(R_\Lambda)$ (and also the polarization) is determined by $\Phi_n(a_*) = f_n(a_*)\varepsilon^{n/3-1/2}(a_*)$ ($B_{\|}(R_\Lambda) \sim \Phi_n$). The values $\Phi(a_*)$ for $a_* = 0$ and $1$ are given in Table 2.

For most probable values of index $n \approx 1.5 - 2$, magnetic field in the emission region for Schwarzschild black hole is approximately ten times higher than for fast rotating Kerr hole. Clearly, in the first case the Faraday depolarization is considerably higher than in second case. This means that the observed degree of polarization from active galactic nucleus (AGN) with the Kerr black hole will be higher than that for Schwarzschild’s hole, if we compare the systems with approximately equal masses and luminosities.

It is of interest to see what changes if to suppose that potential law (8) of magnetic field decreasing takes place directly from the black hole horizon, i.e. one exists the relation:

$$B_{\|}(r) = B_H \left( \frac{R_H}{r} \right)^n = 2^{-n}(1 + \sqrt{1 - a_*^2})^n B_H \left( \frac{r_g}{r} \right)^n. \quad (14)$$

Evidently, in this case we have to take $k = 1$ and $q = 1$. The magnetic field dependence from $a_*$ $B_{\|}(R_\Lambda)$ exists due to factor $\Psi_n(a_*) = (1 + \sqrt{1 - a_*^2})^{(n-1)}$ (this is analog to $f_n(a_*)$).

In this case at $n = 0.5, 1, 1.5, 2, 3$ the ratio $\Psi_n(0)/\Psi_n(1)$ acquires the values $0.71, 1, 1.41, 2, 4$. These values are considerably lower than those for $f_n(0)/f_n(1)$. For ratio $\Psi_n(a_*)\varepsilon^{n/3-1/2}$ (the analog $\Phi_n(a_*)$) at $a_* = 0$ to the value at $a_* = 1$ we obtain $\approx 1.4$ for all values of $n$. This is also is much higher than the corresponding values $\Phi_n(0)/\Phi_n(1)$. Thus, the strong difference between polarizations in Schwarzschild and Kerr cases really is due to large difference between the radiiuses of first stable orbits in these cases.

The results of Table 1 show that there exists the dependence of polarization degree grow with the grow of black hole spin. It seems for ensemble of AGNs with approximately close values of masses and luminosities we may to consider the spin $a_* \approx 1$ in systems with most high polarization, but with $p_{max} < 11.7\%$. For sufficiently large ensembles one has compare the observing polarization with the $\mu$-averaged values of formula (5). Such comparison might give the estimates for $\Phi_n(a_*) = f_n(a_*)\varepsilon^{n/3-1/2}(a_*)$ for intermediate between $a_* = 0$ and $a_* = 1$ values of spin, i.e. give the estimates for parameter $\varepsilon(a_*)$, and also for parameter $n$. It better for this to use the values of polarization in different wavelengths. This difficult problem is not considered in our paper.

4 The spin dependence of polarization from the region of broad lines

In this case the characteristic distance $R_{BLR}$ of region emitting broad lines is determined by the expression (see Shen 2009):

$$R_{BLR} = 2.1 \times 10^{13} \left( \frac{M_{BH}}{M_\odot} \right)^{1/2} \left( \frac{L_{bol}}{L_{edd}} \right)^{1/2} = 5.525 \times 10^4 \varepsilon^{1/2} \sqrt{M}. \quad (15)$$

Instead of formula (11) we obtain

$$\frac{r_g}{R_{BLR}} = 10^{-7.85} \left( \frac{M_{BH}}{M_\odot} \right)^{1/2} \left( \frac{L_{bol}}{L_{edd}} \right)^{-1/2} = 5.34 \left( \frac{M_{BH}}{M_\odot} \right)^{1/2} \frac{1}{\varepsilon^{1/2} \sqrt{M}}. \quad (16)$$
Table 2: Values of $\Phi_n(a_*)$ and $\bar{\Phi}_n(a_*)$.

|   | 0.5  | 1   | 1.5 | 2   | 3   |
|---|------|-----|-----|-----|-----|
| $\Phi_n(0)$ | 2.25 | 2.42| 2.60| 2.79| 3.23|
| $\Phi_n(1)$ | 0.94 | 0.58| 0.35| 0.22| 0.08|
| $\Phi_n(0)/\Phi_n(1)$ | 2.38 | 4.18| 7.34| 12.9| 39.8|
| $\bar{\Phi}_n(0)$ | 3.64 | 6.28| 10.9| 18.8| 56.4|
| $\bar{\Phi}_n(1)$ | 1.09 | 0.77| 0.55| 0.38| 0.19|
| $\bar{\Phi}_n(0)/\bar{\Phi}_n(1)$ | 3.34 | 8.16| 20  | 50  | 297 |

Substituting this expression into general formula (8), we obtain the following expression for $B_{\parallel}(R_{BLR})$:

$$B_{\parallel}(R_{BLR}) = \frac{kq^n\sqrt{k_m}}{1 + \sqrt{1 - a_*^2}} 10^{8.8 - 7.85n} \left( \frac{M_{BH}}{M_\odot} \right)^{n/2 - 1/2} \left( \frac{L_{bol}}{L_{edd}} \right)^{-n/2 + 1/2} \frac{1}{\sqrt{\varepsilon}} =$$

$$= 10^{0.22 + 0.73n} \frac{k\sqrt{k_m}}{1 + \sqrt{1 - a_*^2}} q^n \left( \frac{M_{BH}}{M_\odot} \right)^{n-1} \left( \frac{\dot{M}}{L_{edd}} \right)^{1/2 - n/2} \left( \frac{\varepsilon}{1 - \varepsilon} \right)^{1/2}.$$  \hspace{1cm} (17)

So, in the case of known luminosity $L_{bol}$ and mass $M_{BH}$

$$B_{\parallel}(R_{BLR}) \sim \frac{kq^n\varepsilon^{-1/2}}{1 + \sqrt{1 - a_*^2}} = \Phi_n(a_*) = f_n(a_*)\varepsilon^{-1/2},$$

$$\varepsilon^{-1/2} = 4.188 \text{ for } a_* = 0, \quad \varepsilon^{-1/2} = 1.543 \text{ for } a_* = 1.$$ \hspace{1cm} (18)

Remember that for emission region in continuum the value $\varepsilon^{-1/2}$ is replaced by $\varepsilon^{n/3 - 1/2}$. In Table 2 we presented $\Phi_n(a_*)$ and $\bar{\Phi}_n(a_*)$ for $a_* = 0$ and $a_* = 1$. It is seen that magnetic field in broad line emission region is also considerably lower for Kerr holes than that for Schwarzschild hole. We stress once more, that this takes place mainly due to that radius $r_{ms}$ for Kerr black holes is located more far from the emission region, and consequently the magnetic field is lower than in the case of non-rotating Schwarzschild holes with its $r_{ms} = 3r_g$.

### 5 Conclusion

Using the model that the linear polarization of radiation from AGN is due to multiple scattering in magnetized accretion disks, we demonstrate that the polarization strongly depends on the spin of central black hole. For fast rotating Kerr-type black holes the linear polarization is considerably higher than that for non-rotating Schwarzschild’s holes. This is mainly occur due to fact that the radius of first stable orbit for Kerr holes is more far from the emission region $R_\lambda$ than that for Schwarzschild holes with This result is based on the supposition (seems very obvious) that the power law of magnetic field decrease takes place from the first stable orbit $r_{ms}$, which is greater ($r_{ms} = 3r_g$) for Schwarzschild hole. The far located the emission region from the first stable orbit the less is magnetic field in this region, and less the depolarization due to Faraday rotation effect. It is known that bolometric luminosity of accretion disk near fast rotating Kerr black hole is considerably higher than that for non-rotating hole with the same mass. It means that bright AGNs have to demonstrate higher polarization of optical radiation than weak AGNs.
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