Analytical modelling of a novel test for determination of porosity and permeability of porous materials

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Abstract. Porosity and permeability are important properties of porous materials, such as rocks and concrete. This paper presents the physical-mathematical modelling of a novel test, based on one previously developed by one of the authors (standardized in Switzerland, Japan and China) for measuring the air-permeability of concrete structures. In the present case, a cylindrical specimen is placed inside an air-tight cell, subjected to an initial vacuum pressure \(P_0\), which is afterwards isolated from the pump. The rate of pressure increase (due to the extraction of air originally at atmospheric pressure \(P_a\)) is related to the coefficient of permeability of the material whilst the final pressure attained is a function of the porosity (total amount of air extracted). The analysis assumes a unidirectional radial flow of air, which can be achieved by a special serial three-chamber vacuum cell (with pressure regulation of the external chambers) or by an air-tight sealing of the extreme faces of the cylinder. The analysis is developed under the assumption of viscous laminar flow. To account for the molecular diffusion flow, the test can be performed under vacuum \((P_0 \ll P_a)\) and under overpressure \((P_0 \gg P_a)\), enabling the application of the Klinkenberg correction to get the intrinsic coefficient of permeability.

1. Introduction
Porosity \(\phi\) and permeability \(k\) are two important properties of porous materials. Regarding water, oil and gas exploitation, the porosity of the rocks indicates their potential storage capacity, whilst their permeability indicates the easiness with which the fluids can be extracted from the reservoirs. When we consider stone and concrete as building materials, several engineering properties are closely associated to the pore structure of the material, namely strength, stiffness, thermal conductivity, frost resistance and durability, the latter being a function of the resistance of the material to the penetration of aggressive species, reflected in its permeability.

Given their importance, several test methods have been developed to measure porosity \(\phi\) and permeability \(k\) of rock (for different applications) and concrete, some of them included in local and international standards.

Total porosity of concrete can be measured by petrographic techniques, e.g. ASTM C457. The effective porosity of concrete can be measured following ASTM C642, based on the water saturation (including boiling) of a previously oven-dried sample. In the case of rocks, the effective porosity can be measured by mercury intrusion porosimetry (ASTM D4404) or by He pycnometry (ASTM D5550).

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The determination of the coefficient of gas-permeability under steady state conditions is covered by ASTM C4525 for rocks and, for concrete, by several standards in Europe: Portugal (LNEC E 392), Spain (UNE 83981) and in France (XP P18-463), all based on the ‘Cembureau’ method [1]. The above-mentioned test methods are applicable to specimens, either cast in the case of concrete or drilled and saw-cut from concrete structures or rock cores (small specimens called ‘plugs’) and are based on Darcy-Hagen-Poiseuille law for gases, assuming viscous laminar flow, equation (1).

\[
Q = \frac{dV}{dt} = k \cdot \frac{A}{2\mu L} \frac{p_i^2 - p_e^2}{p_i}
\]

where:
- \( k \): coefficient of gas-permeability (m²)
- \( Q \): gas flow rate (m³/s), measured at pressure \( P_i \)
- \( dV \): differential volume of gas (m³) traversing the sample in time differential \( dt \) (s)
- \( L \): length of the cylindric sample (m)
- \( A \): cross-section area of the cylinder (m²)
- \( \mu \): dynamic viscosity of the gas (Pa.s)
- \( P_i \): outlet gas pressure (Pa), usually atmospheric pressure
- \( P_e \): inlet gas pressure (Pa)

2. Background

For concrete, the quality of specimens cast and matured in the laboratory (under quasi-ideal conditions), called ‘Labcrete’, is not representative of the true quality of the concrete in the structure (cast and matured under widely different conditions), called ‘Realcrete’ [2]. On the other hand, drilling cores from a concrete structure is damaging and expensive, which limits the availability of samples.

With the above principles in mind, one of the authors [3] developed a non-destructive testing technique to measure, under non-steady conditions, the coefficient of air-permeability on site, sketched in figure 1, which is briefly described.

![Figure 1](image-url)  
**Figure 1.** Sketch (l.) of NDT to measure the coefficient of air-permeability \( kT \); vacuum cell (r.)

The test method consists in applying a vacuum cell on the concrete surface, which presents two concentric chambers separated by elastomeric soft rings (see figure 1). By means of a pump, a vacuum of ca. 30 mbar is created inside the cell, adhering it to the tested surface. After 60 s, the internal chamber (test chamber) is isolated from the pump, moment at which its pressure \( P_i \) starts to grow due to air in the pores of the concrete (originally at atmospheric pressure) flowing towards the evacuated cell. The rate of pressure rise in the internal chamber is higher the higher the permeability of the tested concrete. The key feature of the test method is the role of the external chamber, the pressure of which \( (P_e) \) is regulated
to match that of the internal chamber, i.e. at all times is \( P_e = P_i \). As sketched in figure 1, the flow of air into the internal chamber can be considered as unidirectional (a cylinder of gas). Under this assumption, it was possible to model the air-flow and develop a formula to calculate the coefficient of air-permeability \( kT \), as described in [4,5].

This test method has gained acceptance and has been standardized in Switzerland (SIA 262/1), China’s Jiangsu Province (DGJ32/TJ 206), Japan (NDIS 3436-2) and shortly in Argentina (IRAM 1892). Two commercial instruments are on the market, the Torrent Permeability Tester (Proceq SA) and the PermeaTORR family (Materials Advanced Services), the latter produced by the authors.

It is important to remark that the test method has been successfully applied also on rocks, both in the laboratory and in the field. The applications in the laboratory investigated the potential of the test method on rocks for oil exploitation [6] and on construction stones [7]. The field application investigated the tightness of rocks in connection with nuclear waste disposal sites [8].

3. Objective
The objective of this paper is to present the physical and analytical model of a novel test method aimed at measuring simultaneously the coefficient of air-permeability \( k \) and the porosity \( \phi \) of a cylindrical sample of a porous material, of special application to rocks and concrete-like materials. It is based on the concept of creating a vacuum on the curved walls of a cylinder, under conditions that generate a uniaxial radial flow of air from the sample.

4. Theoretical Background and Physical Model
Let us assume that a cylindrical sample of a porous material like rock or concrete, with its free pores containing air at atmospheric pressure \( P_a \), is enclosed in a chamber, which is evacuated by means of a vacuum pump bringing it instantaneously to a pressure \( P_0 \), lower than \( P_a \). If all air flow along the longitudinal axis is impeded (as described in Section 6), the flow of air from the sample to the surrounding chamber can be assumed as radial (uniaxial), see arrows in figure 2 (left). The air that is extracted from the sample will cause an increase in the pressure of the surrounding chamber until the air in the pores of the sample is in pressure equilibrium with the chamber. Intuitively, the rate of pressure rise will be ‘proportional’ to the permeability \( k \) of the sample and the final pressure reached to the amount of air extracted, i.e. to the porosity \( \phi \) of the sample, as sketched in figure 2 (right).

![Diagram of the test method and expected response](image)

**Figure 2.** Sketch of the proposed test method (l.) and expected response (r.)

Figure 3 presents a diagram with the assumed pressure profiles in the material’s pores along the radius of the cylinder, assumed of radius \( \rho \), for four different instants of the test. The free volume of the surrounding chamber, i.e. that not occupied by the sample, is \( V_c \) (is the volume of the gap in figure 2).
The following assumptions are made:

i. At $t = 0$, initiation of the test, all the pores in the sample are at atmospheric pressure $P_a$ (assumed = 1000 mbar) and the vacuum chamber pressure is $P << P_a$.

ii. The vacuum chamber is completely airtight

iii. The porosity $\phi$ and permeability $k$ are constant across the sample affected by the test (as well as the temperature $T$ and the saturation degree)

iv. The flow of air is unidirectional (radial) and is viscous laminar

v. Although the conditions are non-steady, the distribution of pressure is regarded as linear between the atmospheric pressure front (vacuum front) and the surface of the sample

Initially, at time $t = 0$, all the pores in the sample contain air at atmospheric pressure $P_a$ ($\approx$1000 mbar), situation a) in figure 3, whilst the surrounding evacuated chamber is assumed to be at a much lower pressure $P_0$ (e.g. 30 mbar). Due to the pressure gradient, air will flow from the material’s pores into the vacuum chamber while the atmospheric pressure front (vacuum front) recedes towards the centre of the specimen. Let us assume that, at time $t$, the atmospheric pressure front has receded a distance $y$, as sketched in diagram b) of figure 3. During a successive differential time $dt$, a volume of air $dV$ will have flown into the vacuum chamber, with a further penetration $dy$ of the atmospheric pressure front, as illustrated in diagram c) of figure 3. In turn, this volume of air $dV$ will generate an increase in the pressure of the airtight vacuum chamber of magnitude $dP$, expressed as:

$$dP = \frac{P \cdot dV}{V_c}$$  \hspace{1cm} (2)

where $P$ and $V_c$ are the pressure and volume of the vacuum chamber, respectively.

Applying equation (1) to the situation c) of figure 3, the volume of air $dV$ that flows into the vacuum chamber during the interval $dt$ can be calculated as:

$$dV = \frac{k \cdot A}{2 \cdot \mu \cdot y} \cdot \frac{P_a^2 - P^2}{P} \cdot dt$$  \hspace{1cm} (3)

where:

$dV$: differential volume of air that flows into the vacuum chamber in the interval $dt$

$k$: coefficient of air-permeability (m$^2$)

$A$: mean area of sample traversed by the air flow (m$^2$)

$\mu$: dynamic viscosity of air (Pa.s)
\[ y : \text{penetration depth of vacuum front (m)} \]
\[ P : \text{pressure in the inner chamber at time } t \text{ (Pa)} \]
\[ P_a : \text{atmospheric pressure (Pa)} \]

The area \( A \) is, see figure 3 b):
\[ A = 2.\pi \left( r + \frac{y}{2} \right) L \quad (4) \]
where:
\[ r = \text{radius to the vacuum front (m), see figure 3 b)} \]
\[ L = \text{length of the cylindrical tested sample (m)} \]

with
\[ y = q - r \quad (5) \]

Replacing (4) and (5) into (3):
\[ dV = \frac{\pi k L}{2\mu} \cdot \frac{(q + r)}{(q - r)} \cdot \frac{p_a^2 - p^2}{p} \cdot dt \quad (6) \]

Applying the general gas equation, the number of air moles \( dn \) that correspond to a volume \( dV \) at a pressure \( P \) and absolute temperature \( T \) is:
\[ dn = \frac{P}{RT} \cdot dV \quad (7) \]

where \( R \) is the universal gas constant.

Substituting \( dV \) in (7) by its value in (6):
\[ dn = \frac{\pi k L}{2\mu} \cdot \frac{(q + r)}{(q - r)} \cdot \frac{p_a^2 - p^2}{R T} \cdot dt \quad (8) \]

By mass conservation principle, the number of moles of air \( dn \) entering the vacuum chamber must correspond to the same number affected by a differential deepening \( dy (= -dr) \) of the vacuum front \( y \), dotted lines in situation (c) of figure 3. The affected volume of air is:
\[ dV = -\phi \cdot L \cdot 2 \cdot \pi \cdot r \cdot dr \quad (9) \]

where:
\[ \phi : \text{open porosity of the material (-)} \]

This volume can be assumed to be at the mean pressure \((P_a + P) / 2\), resulting in the following number of moles, applying equations (7) and (9):
\[ dn = -\frac{P_a + P}{2RT} \phi \cdot L \cdot 2 \cdot \pi \cdot r \cdot dr = -\frac{P_a + P}{RT} \phi \cdot L \cdot \pi \cdot r \cdot dr \quad (10) \]

Equating \( dn \) in equations (8) and (10):
\[ \frac{\pi k L}{2\mu} \cdot \frac{(q + r)}{(q - r)} \cdot \frac{p_a^2 - p^2}{RT} \cdot dt = -\frac{P_a + P}{RT} \phi \cdot L \cdot \pi \cdot r \cdot dr \quad (11) \]

or, rearranging terms:
\[ \frac{k}{2\mu \phi} \cdot (P_a - P) \cdot dt = -\frac{(q - r)}{(q + r)} \cdot r \cdot dr \quad (12) \]

or, by introducing variable \( x \) and its differential \( dx \) into equation (12) and operating:
\[ x = \frac{r}{q} ; \quad dx = \frac{dr}{q} \quad (13) \]
\[
\frac{k}{2\mu}\phi \cdot (P_a - P) \cdot dt = -q^2 \cdot \frac{x-x^2}{1+x} \cdot dx
\] (14)

Remembering equations (2) and (6) and introducing variable \(x\) from equation (13):

\[
dP = \frac{\pi kL}{2\mu V_c} \cdot \frac{(1+x)}{(1-x)} \cdot (P_a^2 - P^2) \cdot dt
\] (15)

where \(x\) (relative distance of the vacuum front from the centre of the cylinder) is a function of \(t\).

Now, from equation (14):

\[
\frac{k}{2\mu} \cdot dt = -\frac{\phi}{(P_a - P)} \cdot q^2 \cdot \frac{x-x^2}{1+x} \cdot dx
\] (16)

which, substituting into equation (15) yields:

\[
dP = -\frac{\pi \phi L}{V_c} \cdot \frac{(1+x)}{(1-x)} \cdot q^2 \cdot \frac{x-x^2}{1+x} \cdot (P_a + P) \cdot dx
\] (17)

or:

\[
\frac{dP}{P_a + P} = -\frac{\pi \phi L}{V_c} \cdot q^2 \cdot x \cdot dx
\] (18)

and, integrating equation (18) between the initial state \((t = 0; P = P_0; x = 1)\) and any intermediate state \((t; P; x)\):

\[
\ln(P_a + P) \bigg|^{P_f}_{P_0} = -\frac{\pi \phi L}{V_c} \cdot q^2 \cdot \left[ \frac{x^2}{2} \right]_1^x
\] (19)

\[
P = -P_a + (P_a + P_0) \cdot \exp \left[ \frac{\pi \phi L q^2}{2V_c} (1-x^2) \right]
\] (20)

Similarly, integrating equation (16):

\[
\frac{k}{2\mu} \cdot \int_0^t dt = -\frac{\phi}{(P_a - P)} \cdot q^2 \cdot \int_1^x \frac{x-x^2}{1+x} \cdot dx
\] (21)

and neglecting \(P\) in comparison to \(P_a\):

\[
t = \frac{\phi \mu q^2}{k P_a} \cdot \left[ 4 \ln(x+1) + (x-4) x + 3 - 4 \cdot \ln 2 \right]
\] (22)

Equations (20) and (22) solve the problem, as they relate the two measurable variables \((t \text{ and } P)\) through the unmeasurable intermediate variable \(x\), as function the properties of the material \((k \text{ and } \phi)\) and geometrical features of the test \((\rho, L \text{ and } V_c)\).

The test ends when the vacuum front reaches the centre of the specimen \((x=0)\), which happens at a certain time \(t_f\), with a pressure \(P_f\) in the vacuum chamber. Applying these conditions to both equations (20) and (22) we get:

\[
\phi = \frac{2V_c}{\pi q^2 L} \cdot \ln \left( \frac{P_a + P_f}{P_a + P_0} \right)
\] (23)
\[ k = (3 - 4 \cdot \ln 2) \cdot \frac{\mu \phi \varphi^2}{t_f \cdot P_a} \]  

or, introducing Eq. (23) into (24):

\[ k = (6 - 8 \cdot \ln 2) \cdot \frac{\mu V_c}{\pi P_a L} \cdot \ln \left( \frac{P_a + P_f}{P_a + P_0} \right) \cdot \frac{1}{t_f} \]  

Equations (23) and (25) allow the calculation of the open porosity \( \phi \) and the coefficient of air-permeability \( k \) of the sample, respectively, as function of the final and initial pressures \( P_f \) and \( P_0 \) in the vacuum chamber and of the time \( t_f \) taken to reach the final condition d) in figure 3. As predicted intuitively (see figure 2), the porosity value \( \phi \) depends exclusively on the pressure rise whilst the permeability depends also on the duration of the test, i.e. a form of rate of pressure increase in the chamber.

5. Feasibility Study

In order to analyse the feasibility of the proposed test method and optimize the design of specimen and cell, a simulation of the test has been performed to study the expected performance in terms of duration (neither too short not too long) and pressure increase (within the range and sensitivity of available pressure sensors).

A simulation is described assuming five concretes of widely different values of porosity \( \phi \) and air-permeability \( k \) (in concrete both properties are related [9]), as shown in Table 1. The specimen size is chosen of 50 mm diameter, with a gap between the curved wall of the cylinder and that of the cylindrical vacuum cell of 15 mm (figure 2 left), leading to a \( V_c / L \) ratio of 3060 mm². An initial pressure \( P_0 = 30 \) mbar in the vacuum cell has been assumed and the atmospheric pressure \( P_a \) is assumed = 1000 mbar.

| Case | \( k \) (\( 10^{-16} \) m²) | \( \phi \) (%) | Predicted outcome |
|------|-----------------|-------------|-----------------|
| VL   | 0.001           | 5           | 47              | 17              | 4 hours |
| L    | 0.01            | 10          | 64              | 34              | 47 min  |
| M    | 0.1             | 12          | 70              | 40              | 6 min   |
| H    | 1.0             | 16          | 84              | 54              | 45 sec  |
| VH   | 10              | 20          | 98              | 68              | 6 sec   |

Figures 4 and 5 show charts presenting, for the five cases, the relation between \( x \) and \( t \) and \( P \) and \( x \), obtained from equations (22) and (20), respectively. Figure 6 presents the predicted evolution of \( P \) during the test, derived from the data in figures 4 and 5. The predicted values of the final pressure reached in the test \( P_f \), of the pressure rise \( \Delta P = (P_f - P_0) \) and of the duration of the test \( t_f \) are presented in Table 1 for the five cases.

The predicted results in Table 1 indicate that the duration of the test is reasonable, perhaps too long for the VL concrete. The only way of shortening the time would be to reduce the specimen’s diameter, see equation (24), which may create problems of representativity of the specimen size, at least for concrete. The final pressure is below 100 mbar, i.e. \( P_a = 1000 \) mbar, fulfilling assumption i) of the model and the pressure increase is within the range 15-70 mbar, suitable for being accurately measured by the pressure sensors used in the *PermeaTORR* instruments (sensitivity = ± 0.1 mbar).
Figure 4. Predicted relative penetration of the vacuum front with log-time for five concretes

Figure 5. Predicted pressure rise with penetration of the vacuum front for five concretes

Figure 6. Predicted pressure increase with log-time for five concretes
6. Design details
One way of ensuring a unidirectional air-flow is to enclose the cylindrical specimen in a single airtight cylindrical vacuum cell, sealing the flat ends of the specimen with an air-tight material. In this case, the specimen must have a defined length.

Another way is described in figure 7, showing a sketch and a prototype of a serial three-chamber vacuum cell, that can be mounted on a long core and displaced along its axis for repeated measurements. The pressure of the external chambers (which are interconnected) is kept balanced with that of the measurement internal chamber, ensuring a unidirectional radial flow in the central part of the specimen (same principle as that described in figure 1).

![Figure 7](image_url)

**Figure 7.** Sketch (l.) and prototype (r.) of a serial three-chamber vacuum cell to measure $\phi$ and $k$

At the end of the test, the sample will have its pores at the final pressure $P_f$. At that moment, if the vacuum cell is rapidly brought to atmospheric pressure, a reversed test can be performed now intruding air in the sample (thus, two results can be obtained from the same sample). Moreover, in the case of the single chamber, the test could be performed under vacuum and under a sufficiently high overpressure ($> P_f$); the former should yield a higher air-permeability due to a relatively more intense molecular (slip) flow; with both permeability values, the Klinkenberg correction [10] can be applied to obtain the intrinsic coefficient of permeability of the material. If such high pressure is not feasible, the model has to be revised accordingly.

As soon as a prototype instrument is built and applied on a variety of materials, the results will be published validating (or not) the physical-mathematical model presented in this paper.

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