The saga of bottom production in $p\bar{p}$ collisions

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Abstract. I review here the history of bottom quark cross section measurements and theoretical predictions. Starting from the early days of UA1, and going through the sequence of the large excesses reported during run 0 and I at the Tevatron by CDF and D0, I summarize how both data and theory have evolved in time, thanks to improved experimental techniques, more data, and improved control over the main ingredients of the theoretical calculations. I conclude with the discussion of the preliminary data from run II, which appear to finally give a satisfactory picture of the data vs theory comparison.

INTRODUCTION

The study of events with bottom quarks has led in the past 10 years to some of the most important Tevatron results: the discovery and study of the top quark, the appreciation of the colour-octet-mediated quarkonium production mechanisms, as well as general results in $b$-hadron physics (spectroscopy, lifetimes, mixing, $\sin^2 2\beta$). These results have been obtained while both CDF and D0 were reporting factor-of-3 discrepancies between observed and predicted $b$-hadron cross-sections. To claim that we need to understand $b$ production in order to make new discoveries is therefore a bit exaggerated: important discoveries should be able to stand on their feet without appealing to the prediction of a QCD calculation. Nevertheless, lack of confidence in the ability to describe the properties of events containing $b$ quarks, in addition to raising doubts over the general applicability of perturbative QCD in hadronic collisions, does limit our potential for the observation of new dynamical regimes (e.g. small-$x$ physics [1]-[4]) or for the discovery of new phenomena (e.g. Supersymmetry [5]). In some cases, the existing measurements challenge the theory in ways which go beyond simple overall normalization issues, pointing at effects which are apparently well beyond reasonable theoretical systematics: this is the case of recent CDF studies, which detected anomalies in both rates and properties of events with secondary vertices and soft leptons [6]. It cannot be contested, therefore, that the study of $b$ production properties should be one of the main priorities for Run II at the Tevatron, with implications which could go beyond the simple study of QCD.

Starting from the situation as it developed during the early Tevatron runs, I will review here the progress in the theoretical predictions. More details on the historical evolution
of the cross section measurements can be found in [7], as well as in [8, 9], which also review the status of fixed-target heavy quark studies. For a recent review including γγ and ep data as well, see [10]. I will then present the implications of the preliminary results from Run II. Their complete theoretical analysis is contained in [11].

**REVIEW OF RUN 0 AND RUN I RESULTS**

The prehistory of b cross-section measurements in hadronic collisions start with UA1 at the S ¯ppS (\( \sqrt{S} = 630 \) GeV) collider [12]. The data were compared with theoretical predictions [13, 14], showing good agreement, within the rather large (±40%) theoretical uncertainty. “Theory”, in those days, already meant a full NLO QCD calculation [13, 14], including all mass effects, state-of-art NLO PDF fits [15], and b → B non-perturbative fragmentation functions parameterized according to [16], with a parameter \( \epsilon = 0.006 \) extrapolated from fits [17] to charm fragmentation data in \( e^+e^- \), using the relation \( \epsilon_b = \epsilon_c \times (m_c/m_b)^2 \). At the beginning only predictions for total cross-sections and inclusive pb mass spectra were available. Later on, more exclusive calculations were performed, allowing for the application of general cuts to the final states, as well as for the study of correlations between the b and \( \bar{b} \) [18].

After such a good start in UA1, the first published data from CDF [19] appeared as a big surprise. CDF collected a sample of 14 ± 4 fully reconstructed \( B^± \rightarrow \psi K^± \) decays, leading to:

\[
\sigma(p\bar{p} \rightarrow bX; p_T^b > 11.5 \text{GeV}, |y| < 1) = \begin{array}{cc}
\text{CDF:} & 6.1 \pm 1.9_{\text{stat}} \pm 2.4_{\text{syst}} \mu b \\
\text{theory:} & 1.1 \pm 0.5 \mu b
\end{array} (1)
\]

In spite of the large uncertainties, which led to a mere 1.5σ discrepancy, attention focused on the large data/theory=5.5 excess. Theoretical work to explain the apparent contradiction between the success of the NLO theory at 630 GeV and the disaster at 1.8 TeV concentrated at the beginning on possible effects induced by the different x range probed at the two energies: PDF uncertainties [20] and large small-x effects [1]-[3], where \( x \sim m_b/\sqrt{S} \). In the first case marginal fits to both data sets could be obtained at the cost of strongly modifying the gluon density, in a way which however would not survive the later accurate determinations of \( g(x) \) from HERA. In the second case, conflicting conclusions were reached: on one side the first paper of [3] obtained increases by factors of 3-5 due to small-x effects; on the other, the analysis of [1] proved that the resummation of small-x logarithms could only augment the total rate by 30% (or less, in the case of \( g(x) \) more singular than \( 1/x^3 \)).

The ball was therefore back on the experimentalists’ court. CDF expanded the set of measurements, including final states with inclusive \( \psi \) and \( \psi' \) [22] and inclusive leptons [23], summarised in fig. 1 The measurement of the b cross section from the

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2 For lack of time, I will however focus my attention in this presentation on inclusive \( p_T \) spectra.

3 The option of very large small-x effects being manifest only at 1.8 TeV will be definitely ruled out several years later, when CDF measured [21] the b cross section at \( \sqrt{S} = 630 \) GeV and showed that the scaling from 630 to 1.8 TeV was consistent with the predictions of pure NLO QCD.
inclusive charmonium decays turned out later to be incorrect. In run 0, in fact, CDF could not measure secondary vertices, so that charmonium states from direct production and from $B$ decays could not be separated. The extraction of a $b$ rate from these final states was based on theoretical prejudice about the prompt production rates, prejudice which in run I, when the secondary vertices started being measured by CDF, turned out to be terribly wrong \cite{24}. The data on inclusive leptons, while high compared to the central value of the theoretical prediction, were nevertheless consistent with its upper value, and in any case within $1\sigma$.

Increased statistics in run I allowed CDF to improve its measurement of fully reconstructed exclusive decay modes, leading to the measurements in fig. 2. For this measurement CDF used 19pb$^{-1}$ of data, leading to approximately 55 $B^0 \rightarrow \psi K^*$ and 125 $B^\pm \rightarrow \psi K^\pm$ decays. The cross section was still high compared to the central value of the theoretical prediction (data/theory=1.9±0.3), but this was already a marked improvement over the first measurement from run 0, when this ratio was equal to 6.1! More explicitly, the 1995 measurement gave $\langle \sigma(p_T(B) > 6\text{GeV}, |y| < 1) \rangle = 2.39 \pm 0.54 \mu$b, compared to the 1992 measurement of $\langle \sigma(p_T(B) > 9\text{GeV}, |y| < 1) \rangle = 2.8 \pm 1.4 \mu$b (where $\langle \sigma(B) \rangle \equiv \langle \sigma(B^+) + \sigma(B^0) \rangle/2$). Taking into account that the $b$ rate is expected to increase by 2.7 when going from a 9 GeV to a 6 GeV threshold, the 1992 measurement appears to be a factor of 3.2 higher than the 1995 result, consistent with the 6.1/1.9 ratio. This drop in the experimental cross-section was not inconsistent with the large statistical and systematic uncertainties of the 1992 measurement, but somehow the common belief that theory was way off had already stuck. It is also worth noting that the same data, when compared to theoretical predictions obtained a couple of years later using the same QCD calculations, but up-to-date sets of input PDFs (MRST \cite{27} with $\alpha_s(m_Z) = 0.1175$, and CTEQ5M \cite{28} with $\alpha_s(m_Z) = 0.118$), gave very good agreement. This is shown in the

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\footnote{Incidentally, this fact puts into question the UA1 results, which heavily relied on the $\psi$ final states and on explicit assumptions about the prompt charmonium rates!}
right panel of fig. 2, taken from an update of [9]. The crucial change between the two predictions was the change in the value of the QCD coupling strength $\alpha_s$ extracted from global PDF fits. The fits used in the CDF 1995 publication, MRSD0 [26], did not include HERA data and had $\alpha_s(m_Z)=0.111$, significantly lower than what we were getting from LEP, namely $\alpha_s(m_Z) \sim 0.120$. This 10% difference, when evolved to the low scales of relevance to $b$ production, becomes much more significant, especially because $b$ rates grow like $\alpha_s^2$. This is shown more explicitly in fig. 3. The left panel shows the ratio of
the rates obtained by using post-HERA PDF sets, normalized to the MRSD0 set used in the CDF 1995 comparison. The right panel shows the integrated total cross section (for \(|y| < 1\)) for several PDF sets, ordered versus the date of release. One can notice a constant increase, with the most recent sets being almost a factor of 2 higher than the older ones. Notice that this increase is due by and large to the increased value of \(\alpha_s\) returned by the PDF fits. Forcing \(\Lambda_{QCD}\) to take the value consistent with LEP’s \(\alpha_s(m_Z)\), one would have obtained for each PDF set the values corresponding to the crosses in the plots. There the increase relative to the pre-HERA fit MRSD0 is significantly smaller.

While the improvements in the PDF fits were reducing the difference between data and theory, as shown fig. [2], a new CDF measurement from the full sample of run I exclusive \(B\) decays in the range \(6 \text{ GeV} < p_T < 20 \text{ GeV}\) appeared in 2001 [31], and is shown here in Fig. 4. The total rate turned out to be 50\% larger than in the previous 1995 publication [25]: \(\sigma(p_T(B^+) > 6 \text{ GeV}, |y| < 1) = 3.6 \pm 0.6 \mu b\), compared to the previous \(2.4 \pm 0.5 \mu b\), a change in excess of 2\(\sigma\). The ratio between data and the central value of the theory prediction was quoted as \(2.9 \pm 0.5\): a serious disagreement was back!

On the other side of the Tevatron ring, the D0 experiment started presenting the first \(b\) cross section measurements in 1994. The first preliminary results [32] were in perfect agreement with QCD, as shown in the left panel of Fig. 5. They were eventually published, after significant changes, in [33]. The results from a larger dataset of \(6.6 \text{ pb}^{-1}\) appeared in [34], where \(\psi\) dimuons were added. They are shown in the central panel of the figure, and they show a clear increase over the preliminary analysis, but are still consistent with the QCD expectations. The same data set underwent further analysis, and eventually appeared few years later in [35]. They are shown in the right panel of the figure. Now the data are significantly higher than QCD, and certainly higher than in 1996, especially in view of the fact that in the meantime the theory predictions had increased by almost a factor of 2 as a result of the use of new PDF sets (this is clearly visible by the shift of the theory curves between the central and right panels).
As in the case of the CDF exclusive analysis, this evolution underscores the difficulty in performing these measurements, and indicates that it was not just the theory that was having difficulties in coming to grips with the problem!

An additional element was added to the puzzle when D0 reported [36] the measurement of $b$ production at large rapidity, using inclusive forward muons ($2.4 < |y_\mu| < 3.2$). The results, shown in fig. 6, indicated an excess over NLO QCD by a factor larger than what observed in the central region. This anomaly could not be explained away by assuming some extra systematics related to PDFs. From the point of view of perturbation theory, furthermore, there was no reason to expect a significant deterioration of the predictive power when going to large rapidity. So when this result first appeared in its preliminary form I was led [38] to review our assumptions about the non-perturbative part of the calculation, in particular the impact of the fragmentation function. A crucial observation is that in hadronic collisions the fragmentation function is probed in different ranges of $z$ as we change rapidity. This is easily seen as follows. Let us assume that the $b$ $p_T$ spectrum takes the simplified form:

\[
\frac{d\sigma(b)}{dp_T} \sim \frac{1}{p_T^N},
\]

where the slope $N$ will typically depend on rapidity, becoming larger at higher $y_b$. The meson spectrum is then obtained via convolution with the fragmentation function $f(z)$,
leading to the simple result:

\[
\frac{d\sigma(B)}{dp_T} \equiv \int \frac{dz}{z} \frac{d\sigma(b)}{dp_T}(p_T = P_T/z) = \int \frac{dz}{z} \left( \frac{z}{P_T} \right)^N f(z) = f_N \frac{d\sigma(b)}{dp_T},
\]

(3)

where \( f_N \) is the \( N \)-th moment of \( f(z) \). This means that a steeper partonic spectrum selects higher moments. Since the index \( N \) is larger for forward production, a relative difference in \( B \) production rates in the forward/central regions could be explained by making the fragmentation function harder, enhancing the larger moments (which measure the large-\( z \) behaviour of \( f(z) \)). A related observation is that \( f(z) \) fits to \( e^+e^- \) data are mostly driven by the value of the first moment \( f_1 \), which measures the average of the fragmentation variable \( z \). It is therefore possible that different choices of \( f(z) \), giving equivalent overall fits to \( e^+e^- \), could make very different predictions for the higher moments of relevance to hadronic production (in this case \( N \) is in the range 4-6).

One way to understand whether indeed the inaccurate description of the fragmentation process could affect the theoretical predictions was therefore to think of measurements not affected by this systematics. The most obvious observable of this kind is the \( E_T \) spectrum of jets containing a \( b \) quark \[40\]. Since the tagging of a \( b \) inside the jet is only marginally affected by the details of the \( b \to B \) fragmentation, measuring the rate of \( b \) jets is a direct measurement of the \( b \) production rate with negligible fragmentation systematics. In addition, this measurement is also insensitive to higher-order large-\( p_T \) logarithms which are present in the \( p_T^b \) spectrum, therefore improving in principle the perturbative accuracy. D0 carried out the measurement, publishing \[40\] the results shown in Fig. 7. The agreement with NLO QCD \[39\] is better than in the case of the \( p_T^b \) spectrum, as was hoped. We took this as strong evidence that a reappraisal of the fragmentation function systematics may have led to a better description of the \( p_T^b \) and \( y_\mu \) distributions.

The necessary ingredients to carry out this programme are perturbative calculations of matching accuracy for \( b \) spectra in both \( e^+e^- \) and \( p\bar{p} \) collisions, in addition of course to
accurate $e^+e^-$ data to be used in the fits. These tools had just become available towards the end of the 90’s. The resummation of the logarithms of $p_T/m_b$, with next-to-leading logarithmic accuracy (NLL), and the matching with the fixed-order (FO), exact NLO calculation for massive quarks, had been performed in [41] (Fixed-Order with Next-to-Leading-Log resummation: FONLL) and a calculation with this level of accuracy for $e^+e^-$ collisions was presented in [42]. Here it had been used for the extraction of the non-perturbative fragmentation function $f(z)$ from LEP and SLC data [43], with the main result that the Peterson functional form is strongly disfavoured over other alternatives [44]. The equivalence of the perturbative inputs allows one to consistently apply this fit to the FONLL $b$-quark spectra in hadronic collisions, leading to FONLL predictions for the $b$ hadron ($H_b$) spectrum. A comparison of these predictions with the final CDF data at 1.8 TeV for $B^\pm$-meson production in the range 6 GeV < $p_T$ < 20 GeV has been presented in [45]. The results are shown in Fig. 8: the left panel compares the CDF data from [31] with the theory curve evaluated using CTEQ5M PDF, FONLL, and fragmentation functions fitted to LEP and SLC data. The right panel shows a comparison [46] with the D0 forward muon data. In both cases the agreement with data is much improved. In the case of the CDF central cross section, the ratio between data and theory improves from 2.9 ± 0.5 to 1.7 ± 0.7. As discussed in detail in [45], the improvement is due to the sum of three independent 20% effects ($1.2^3 \sim 2.9/1.7$), all going in the same direction: the resummation of $p_T$ logarithms, the change in functional form of the fragmentation function, and the use of the LEP/SLC $b$ fragmentation data.

The heritage of run I was therefore a set of measurements, more or less consistent with each other, normalized with a factor of about 1.5 to 2 higher than the central theoretical prediction, but still compatible with the upper end of the theoretical systematics band.
The final phase of this history deals with the new run II data from CDF [48]. A great improvement took place in the ability to trigger on very low $p_T$ events, allowing for a measurement down to $p_T \sim 0$, although still in the limited rapidity range $|y_b| \lesssim 0.6$. This is also accompanied by very large statistics, allowing a fine binning in $p_T$. The measurement down to very small $p_T$ is important because the total rate has a much reduced dependence on the fragmentation systematics, and because it is particularly sensitive to possible small-$x$ phenomena.

On the theoretical side, in addition to the calculations described above, a new tool has meanwhile become available, namely the MC@NLO code [49], which merges the full NLO matrix elements with the complete shower evolution and hadronization performed by the HERWIG Monte Carlo. As discussed in detail in [49], this comparison probes a few features where FONLL and MC@NLO differ by effects beyond NLO: the evaluation of subleading logarithms in higher-order emissions, in particular in the case of gluon emission from the $b$ quark, and the hadronization of the heavy quark, which in MC@NLO is performed through HERWIG’s cluster model, tuned on $Z^0 \rightarrow H_bX$ decays.

The comparison of the run II data with the theoretical calculations is given in Fig. 9 which shows the data with our prediction for the spectrum of $J/\psi$ from $B$ decays, obtained by convoluting the FONLL result with the $J/\psi$ momentum distribution in inclusive $B \rightarrow J/\psi + X$ decays. The theoretical error band is obtained by varying renormalization and factorization scales ($\mu_R, \mu_F = \xi_{R,F}\mu_0$, with $\mu_0^2 = p_T^2 + m_b^2$), the $b$-quark mass, and parton densities. The central values of our predictions are obtained with $\xi_{R,F} = 1$, $m_b = 4.75$ GeV and CTEQ6M. The mass uncertainty corresponds to the range $4.5 \text{ GeV} < m_b < 5 \text{ GeV}$. The scale uncertainty is obtained by varying $\mu_{R,F}$ over the range $0.5 < \xi_{R,F} < 2$, with the constraint $0.5 < \xi_R/\xi_F < 2$. The PDF uncertainty is calculated by using all the three sets of PDFs with errors given by the CTEQ, MRST and Alekhin
The data lie well within the uncertainty band, and are in very good agreement with the central FONLL prediction. I also show the two MC@NLO predictions corresponding to the two different choices of the $b$ hadronization parameters (see [11] for the details).

I stress that both FONLL and MC@NLO are based on the NLO result of [14] (henceforth referred to as NDE), and only marginally enhance the cross section predicted there, via some higher-order effects. The most relevant change in FONLL with respect to old predictions lies at the non-perturbative level, i.e. in the treatment of the $b \to H_b$ hadronization, which makes use [45] of the moment-space analysis of the most up-to-date data on $b$ fragmentation in $e^+e^-$ annihilation. The evolution of the NLO theoretical predictions over time is shown in Fig. 10. Here we plot the original central prediction of NDE for $\sqrt{S} = 1.8$ TeV (symbols), obtained using NLO QCD partonic cross sections convoluted with the PDF set available at the time, namely DFLM260 [15]. The same calculation, performed with the CTEQ6M PDF set (dotted curve), shows an increase of roughly 20% in rate in the region $p_T < 10$ GeV. The effect of the inclusion of the resummation of NLL logarithms is displayed by the dashed curve, and is seen to be modest in the range of interest. Finally, we compare the original NDE prediction after convolution with the Peterson fragmentation function ($\epsilon = 0.006$, dot-dashed curve), with the FONLL curve convoluted with the fragmentation function extracted in [45] (solid curve). Notice that the effect of the fragmentation obtained in [45] brings about a modest decrease of the cross section (the difference between the dashed and solid curves), while the traditional Peterson fragmentation with $\epsilon = 0.006$ has a rather pronounced effect (the difference between the symbols and the dot-dashed curve). Thus, the dominant change in the theoretical prediction for heavy flavour production from the original NDE calculation up to now appears to be the consequence of more precise experimental inputs to the bottom fragmentation function [43], that have shown that non-perturbative fragmentation effects in bottom production are much smaller than previously thought.

The main improvement in the comparison between data and theory w.r.t. the final run I results discussed in [43] comes from the normalization of the run II CDF data, which tend to be lower than one would have extrapolated from the latest measurements at 1.8 TeV. To clarify this point, we collect in Fig. 11 the experimental results from the CDF measurements of the $B^\pm$ cross section in Run IA [25], in Run IB [31] and in Run II. The rate for $p_T(B^\pm) > 6$ GeV, evolved from $2.4 \pm 0.5 \, \mu b$ (Run IA) to $3.6 \pm 0.6 \, \mu b$ (Run IB), and decreased to $2.8 \pm 0.4 \, \mu b$ in Run II. The increase in the c.m. energy should have instead led to an increase by 10-15%. The Run II result is therefore lower than the extrapolation from Run IB by approximately 30%. By itself, this result alone would reduce the factor of 1.7 quoted in [45] to 1.2 at $\sqrt{S} = 1.96$ TeV. In addition, the results presented in [11] lead to an increase in rate relative to the calculation of [45] by approximately 10-15%, due to the change of PDF from CTEQ5M to CTEQ6M. We then conclude that the improved agreement between the Run II measurements and perturbative QCD is mostly a consequence of improved experimental inputs (which include up-to-date $\alpha_s$ and PDF determinations).
FIGURE 10. Evolution of the NLO QCD predictions over time, for $\sqrt{s} = 1800$ GeV. See the text for the meaning of the various curves.

FIGURE 11. Evolution of the CDF data for exclusive $B^\pm$ production: Run IA [25], Run IB [31] and Run II [48].

CONCLUSIONS

When I meet colleagues and discuss the latest $b$ results, and when I hear presentations or read conference proceedings, there is often a more or less explicit message that now things are OK because theorists kept beating on their calculations until they got them right. I hope that this note will dispel this prejudice. The history of the experimental measurements indicates that many things have also “strongly evolved” on the data side, often with changes well in excess of the standard $\pm 1\sigma$ variation. The “history” plot in fig. 10 shows on the other hand that not much has changed on the theory side, aside from data-driven modifications associated to the value of $\alpha_s(m_Z)$, to the low-$x$
behaviour of the gluon as determined by the HERA data, and to the improved data on $b \to B$ fragmentation. The theoretical improvements due to the resummation of the large-$p_T$ logarithms play a major role in allowing a consistent use of the fragmentation functions extracted from $e^+ e^-$ data, but have a very limited impact in the region of $p_T^b$ probed by the run II data. Their significance will only manifest itself directly at high $p_T^b$ ($p_T^b > 20 - 30 \text{GeV}$), where the resummation leads to a much reduced scale dependence, and to more accurate predictions, allowing more compelling quantitative tests of the theory. It is auspicious that the improved run II detectors and the higher statistics will make it possible to extend the range of the measurements to really large $p_T^b$ (in the range of 80-100 GeV). Tools are now available (MC@NLO) to compare data subject to complex experimental constraints directly with realistic NLO calculations, including a complete description of the hadronic final state. This will avoid the risky business of attempting to connect the observables to a $p_T$ spectrum of the $b$ quark, a practice which, although unavoidable in the past, has certainly contributed to the inflation of theoretical and experimental systematic uncertainties.

To this date, the recent CDF measurement of total $b$-hadron production rates in $p\bar{p}$ collisions at $\sqrt{s} = 1.96 \text{ TeV}$ is in good agreement with NLO QCD, the residual discrepancies being well within the uncertainties due to the choice of scales and, to a lesser extent, of mass and PDF. A similar conclusion is reached for the $p_T$ spectrum. The improvement in the quality of the agreement between data and theory relative to previous studies is the result of several small effects, ranging from a better knowledge of fragmentation and structure functions and of $\alpha_s$, which constantly increased in the DIS fits over the years, to the fact that these data appear to lead to cross sections slightly lower than one would have extrapolated from the measurements at 1.8 TeV. The currently still large uncertainties in data and theory leave room for new physics. However there is no evidence now that their presence is required for the description of the data, and furthermore the recent results of [52] rule out the existence of a scalar bottom quark in the range preferred by the mechanism proposed in [5]. The data disfavour the presence of small-$x$ effects of the size obtained with the approaches of refs. [3]. They are instead compatible with the estimates of [1].

While these results have no direct impact on other anomalies reported by CDF in the internal structure and correlations of heavy-flavoured jets [6], we do expect that the improvements relative to pure parton-level calculations present in the MC@NLO should provide a firmer benchmark for future studies of the global final-state structure of $b\bar{b}$ events.

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