The effective action of $W_3$-gravity.

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Abstract

A new method for integrating anomalous Ward identities and finding the effective action is proposed. Two-dimensional supergravity and $W_3$-gravity are used as examples to demonstrate its potential. An operator is introduced that associates each physical quantity with a Ward identity, i.e., a quantity that is transformed without anomalous terms and can be nullified in a consistent manner. A covariant form of the action for matter field interacting with a gravitational and $W_3$-gravitational background is proposed.

1 INTRODUCTION

The tremendous upsurge of interest in $W_N$-algebras [1], that followed their discovery by A.B. Zamolodchikov can be explained by the fact that the basic relationships in $W_N$-algebras, in contrast to those of ordinary Lie algebras, are multilinear and that the mathematical aspects had not been systematically studied. A big achievement in this area of research was the use of the Drinfel’d-Sokolov reduction scheme [2], which reduces $W$-algebras to Lie algebras and relates them to the second hamiltonian structure of the generalized Korteweg-de Vries hierarchies; $W_N$-algebras contain Virasoro algebra as a subalgebra. In the context of string theory, the appearance of the latter is a reflection of invariance under reparametrization of the string world surface. The extension of this symmetry to invariance under $W$-gravity transformations leads to the theory of $W$-strings in Polyakov approach, i.e., to the theory of interaction of matter fields with ordinary (spin-2) and $W$-gravitational (spin-$N$) background fields. Thus, symmetry under transformations of $W$-gravity is the leading principle that makes it possible to write the interaction for the fields with spin $\geq 2$, at least in two dimensions.

However, progress in this area of research was fraught with considerable difficulties. First the chiral theory of the interaction of matter and $W$-gravity was formulated by Hull [3]. Then Schoutens et al. [4] generalized the theory to the non-chiral case but encountered significant technical difficulties: the action in the theory proved to be infinitely nonlinear in the matter fields and non-local, so any further analysis is extremely complicated. By calculating the functional integral over the matter fields with a central charge $c$ interacting with $W_3$-gravity Schotens et. al. also found the induced action of $W_3$-gravity in the form of a $1/c$ expansion [5]. The same researchers (see Ref. [6]) found the induced action of
chiral $W_3$-gravity, a direct analog of the Polyakov’s action \[7\] for ordinary gravity, by integrating the anomaly in the limit $c \to \infty$.

Clarification of the geometrical meaning of $W$-transformations would help $W$-gravity studies considerably. This aspect was studied by Figueroa-O’Farrill et al. and Hull \[12\].

At present it is generally hoped that $W$-gravity studies will help to overcome the strong coupling barrier $c = 1$ for a system consisting of conformal matter and two-dimensional grivation, which will probably make it possible to avoid the fractional dimensionality established by Knizhnik and et. al. \[8\] for quantum gravity in the weak-coupling mode. Direct generalization of the results of Ref. \[9\] to $W$-gravity in the absence of matter fields done by Matsuo \[13\].

The present investigation develops a method for integrating two-dimensional anomalous Ward identities. Its application is illustrated by examples of two-dimensional gravity, supergravity, and $W_3$-gravity. The essence of the method consists in the following. By expressing anomalous currents in terms of free fields via bosonization formulae, we can lower the order of these differential equations and integrate them. The resulting effective action reproduces the anomaly correctly. When the regularization scheme changes, local counterterms are added to the non-local effective action, and the emergence of these counterterms changes the form and symmetry of the Ward identities. The bosonized fields, being free in one regularization scheme, in another scheme are related by the fact that they satisfy certain Ward identities. When the chiral Weyl-invariant regularization scheme is replaced by the diffeomorphism-invariant scheme, local counterterms are added in such a way that the kinetic part of the effective action becomes invariant both under diffeomorphism and under Weyl transformations. The remaining (topological) part of the effective action is fixed by requirement that the total action, being diffeomorphism-invariant, under Weyl transformations, be symmetric in the quantum or projective sense, i.e., is transformed as a 1-cocycle.

In Sect. 2 and 3 the application of this method is demonstrated using the well-known examples of ordinary and (N=1)-supergravity, and a differential operator $R$ is introduced, which with each physical quantity associates its Ward identity. The operator is actually a Slavnov operator, which was studied by Zucchini \[9\] in connection with two-dimensional gravity in conjunction with an additional inhomogeneous term that destroys the anomalous contribution in the transformation law.

In Sect. 4 these calculations are generalized to the case of chiral $W_3$-gravity. It was found that the result is in full agreement with that of Ooguri et. al. \[10\].

Finally, in Sect. 5 deals with the covariant action of matter interacting with nonchiral $W_3$-gravity. In addition to exhibiting parametrization symmetry and $W$-diffeomorphism symmetry, this action is $W$-Weyl invariant and can serve as the kinetic part of the effective action of $W_3$-gravity calculated in the diffeomorphism-invariant scheme.
2 Two-dimensional gravity

The Polyakov action, which was derived in Ref. [7] as the effective action induced by chiral matter interacting with two-dimensional gravity, is the determinant of the two-dimensional Laplasian calculated in a regularization scheme that conserves Weyl symmetry and half the reparametrization symmetry. The presence of a conformal anomaly manifests induces an explicit dependence of the Polyakov action on one of the reparametrization functions. In other words, this effective action can be calculated by integrating the appropriate variational equation, the Ward identity.

The Ward identity of two-dimensional gravitation theory in the light-cone gauge is well known:

\[ R_T = (\bar{\partial} - h\partial - 2\partial h)T - \partial^3 h = 0. \]  

It expresses the anomalous conservation of the system’s energy-momentum tensor \( T \). The field \( h \) in this expression denotes the nonvanishing metric component that remains after light-cone gauge is specified. It is covariant under the transformations

\[ \delta h = (\bar{\partial} - h\partial + \partial h)\epsilon, \]
\[ \delta T = (\partial^3 + 2T\partial + \partial T)\epsilon, \]  

i.e

\[ \delta \epsilon R_T = (\epsilon\partial + 2\partial\epsilon)R_T. \]  

Equation (2.3) expresses the Wess-Zumino self-consistency condition. If we use the bosonization formula and parametrize the energy-momentum tensor via a scalar field,

\[ T = \partial^2 \varphi - \frac{1}{2} (\partial \varphi)^2, \]  

the order of the anomalous term in (2.1) can be reduced:

\[ R_\varphi = (\bar{\partial} - h\partial)\varphi - \partial h = 0, \]
\[ \delta_\varphi R_\varphi = \epsilon \partial R_\varphi. \]  

Comparing (2.1) with (2.4) and (2.5), we obtain

\[ R_T = \partial^2 R_\varphi - \partial \varphi \partial R_\varphi. \]  

The transformation law for the scalar field \( \varphi \) is also anomalous:

\[ \delta \varphi = \partial \epsilon + \epsilon \partial \varphi. \]  

If for the field \( \varphi \) we postulate the free-field Poisson bracket,

\[ \{ \varphi(x); \varphi(x') \} = \delta'(x - x'), \]  

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the $\epsilon$-variation of any quantity $A$ can be determined by its Poisson bracket with energy-momentum tensor:

$$\delta_\epsilon A() = \int d^2x' \epsilon(x') \{T(x'); A(x)\}$$  \hspace{1cm} (2.9)

Clearly, this definition for the field $\varphi$ coincides with (2.7). The bracket of tensor $T$ with itself is

$$- \left\{ T(x); T(x') \right\} = \delta'''(x-x') + (T(x) + T(x')) \delta'(x-x').$$  \hspace{1cm} (2.10)

Although the energy-momentum tensor can be expressed in terms of $\varphi$, there is no way in which we can express the gauge field $h$ in terms $\varphi$ in (2.5) in a local manner. To do this we must introduce a quantity that satisfies the regular Ward identity, i.e., a quantity that transforms as a scalar. The anomaly can be removed from the Ward identity by introducing a scalar field $f$ in the following way:

$$\varphi = \log \partial f.$$  \hspace{1cm} (2.11)

The transformation law for $f$ and the corresponding Ward identity have the form

$$\delta_\epsilon f = \epsilon \partial f,$$

$$R_f = (\bar{\partial} - h\partial)f = 0,$$

$$\delta_\epsilon R_f = \epsilon \partial R_f.$$  \hspace{1cm} (2.12)

Now, when all of the quantities are expressed in terms of the function $f$ locally, we can integrate the variational equation for the effective action of the theory, which can also be expressed in terms of $f$ locally and is given by the Polyakov formula [7]. Detailed calculations are given in Sect. 3 for the more interesting case of supergravity.

Comparing Eqs. (2.3), (2.6) and (2.12), we can see that the gauge variation $\delta$ and the Ward identities $R$ are commutative operations on the fields $T$, $\varphi$ and $f$.

The relationship between $R_f$, $R_\varphi$ and $R_T$ is specified by the following formulae:

$$R_\varphi = \frac{\partial R_f}{\partial f},$$

$$R_T = (\partial^3 + 2T\partial + \partial T)(\frac{R_f}{\partial f}).$$  \hspace{1cm} (2.13)

We see that the operator $R$ associates with each physical quantity $X$ a covariant expression $R_X$, its Ward identity, which in view of its covariance under gauge transformations can be consistently made to vanish. But since the theory lacks quantities of the required dimensionality, this expression must to be set to zero. Comparing (2.13) with (2.11) and (2.6) with (2.4), we can see that $R$ obeys the Newton-Leibniz rule. This property of $R$ makes it possible to write the Ward identities for the correlation functions of the fields $T$, $\varphi$ etc. immediately.
If we apply the Legendre transformation

\[ Z[h] = \Gamma[h] + \int d^2 x T h, \]  

(2.14)

Eq. (2.1) can be written as

\[ (\partial^3 + 2 T \partial + \partial T) \frac{\delta \Gamma}{\delta T} = -\bar{\partial} T, \]  

(2.15)

where the Bol operator \[\Gamma[10]\] on the left-hand side is the covariant form of \(\partial^3\) on an arbitrary Riemann surface, and contains a projective connection, for which we may take \(T\). This notation expresses the covariance of the Ward identity (2.1) just as the Wess-Zumino self-consistency condition (2.3) does.

3 Simple supergravity

In this section we generalize all the ideas of Sect. 2 to the case of simple supergravity and calculate the effective action of the theory.

Polyakov’s result was generalized by Polyakov and Zamolodchikov [14] to the case of (1.0)-supergravity. The corresponding generalization of the Polyakov action represents the effective action obtained in a regularization scheme that conserves the Weyl and super-Weyl symmetries and half the supercoordinate symmetry [15]. The nontrivial dependence of the action on the other coordinate functions (odd- and even-parity) is determined by a superconformal anomaly.

The Ward identities of two-dimensional supergravity in the light-cone gauge can be written as [4]

\[ R_T = (\bar{\partial} - h \partial - 2 \partial h) T - \left( \frac{1}{2} \chi \partial + \frac{3}{2} \partial \chi \right) S - \partial^3 h = 0, \]

\[ R_S = (\bar{\partial} - h \partial - \frac{3}{2} \partial h) S - \frac{1}{2} \chi T - \partial^2 \chi = 0. \]  

(3.1)

They are covariant under the transformations

\[ \delta h = (\bar{\partial} - h \partial + \partial h) \varepsilon + \frac{1}{2} \kappa \chi, \]

\[ \delta \chi = (\bar{\partial} - h \partial + \frac{1}{2} \partial h) \kappa + (\varepsilon \partial - \frac{1}{2} \partial \varepsilon) \chi, \]

\[ \delta T = (\bar{\partial}^3 + 2 T \partial + \partial T) \varepsilon + \left( \frac{1}{2} \kappa \partial + \frac{3}{2} \partial \kappa \right) S, \]  

(3.2)

\[ \delta S = (\varepsilon \partial + \frac{3}{2} \partial \varepsilon) S + (\partial^2 + \frac{1}{2} T) \kappa, \]

i.e.,

\[ \delta R_T = (\varepsilon \partial + 2 \partial \varepsilon) R_T + \left( \frac{1}{2} \kappa \partial + \frac{3}{2} \partial \kappa \right) R_S, \]

\[ \delta R_S = (\varepsilon \partial + \frac{3}{2} \partial \varepsilon) R_S + \frac{1}{2} \kappa R_T, \]  

(3.3)
which means that the Ward identity $R_A$ transforms in the same way as the quantity $A$ but without anomalous terms.

Going over to the scalar multiplet of matter fields $(\phi, \lambda)$, with

$$\delta \phi = (\partial + \partial \phi) \epsilon + \frac{1}{2} \kappa \lambda,$$  
$$\delta \lambda = (\epsilon \partial + \frac{1}{2} \partial \epsilon) \lambda + (\partial + \frac{1}{2} \partial \phi) \kappa,$$  

which are related to the current fields by the rule

$$T = \partial^2 \phi - \frac{1}{2} \partial (\partial \phi)^2 + \frac{1}{2} \lambda \partial \lambda,$$  
$$S = \partial \lambda - \frac{1}{2} \lambda \partial \phi.$$

we can reduce the order of the derivatives in (3.1). The operator $R$ acts on the fields $\phi$ and $\lambda$ in the following manner:

$$R\phi = (\bar{\partial} - h \partial) \phi - \frac{1}{2} \chi \lambda - \partial h,$$  
$$R\lambda = (\bar{\partial} - h \partial - \frac{1}{2} \partial h) \lambda - \frac{1}{2} \chi \partial \phi - \partial \chi.$$

We see that the gauge fields $h$ and $\chi$ cannot be expressed in terms of $\phi$ and $\lambda$ locally, a situation resembling that of Sect. 2.

The fields $\phi$ and $\lambda$ form an algebra of free fields in Poisson bracket:

$$\{\partial \phi(x); \partial \phi(x')\} = \delta'(x - x'),$$  
$$\{\lambda(x); \lambda(x')\} = -\delta(x - x').$$

Then, with respect to this bracket, (3.5) suggests the existence of the following algebra for the current fields:

$$-\{T(x); T(x')\} = \delta''(x - x') + (T(x) + T(x')) \delta'(x - x'),$$  
$$-\{T(x); S(x')\} = (S(x) + \frac{1}{2} S(x')) \delta'(x - x'),$$  
$$-\{S(x); S(x')\} = \delta''(x - x') - \frac{1}{2} T(x) \delta(x - x').$$

To parametrize the gauge fields in a convenient manner, we must introduce a scalar multiplet $(f, \psi)$ without anomalous dimensionality:

$$\delta f = \epsilon \partial f + \frac{1}{2} \kappa \psi,$$  
$$\delta \psi = \epsilon \partial \psi + \frac{1}{2} \partial \epsilon \psi + \frac{1}{2} \kappa \partial f,$$  

$$R_f = (\bar{\partial} - h \partial) f - \frac{1}{2} \chi \psi,$$  
$$R_\psi = (\bar{\partial} - h \partial - \frac{1}{2} \partial h) \psi - \frac{1}{2} \chi \partial f.$$
This multiplet is related to the matter fields as follows:

\[
\varphi = \log \partial f + \frac{\psi \partial \psi}{(\partial f)^2}, \quad (3.10)
\]
\[
\lambda = 2 \frac{\partial \psi}{\partial f} - \psi \frac{\partial^2 f}{(\partial f)^2}. \quad (3.11)
\]

There is no simple way in which we can deduce such a complicated relationship from the condition that the appropriate terms appear in the transformation laws. However, the problem can be simplified if superfields are introduced.

Since the superfield formulation of chiral supergravity contains no auxiliary fields, the meaning of all previous expressions is not altered when we go over to superfields:

\[
R_U = (\bar{\partial} - H \partial - \frac{1}{2} DHD - \frac{3}{2} \partial H)U - \partial^2 DH,
\]
\[
\delta U = (D\partial^2 + \frac{3}{2} U \partial - \frac{1}{2} DUD + \partial U)E, \quad (3.11)
\]
\[
\delta H = (\bar{\partial} - H \partial - \frac{1}{2} DHD + \partial H)E,
\]

where \( U = S + \theta T, H = h + \theta \chi, D = \partial \theta + \theta \partial, \) and \( E = \epsilon + \theta k \) with \( \theta \) the anticommuting coordinate. Then the current superfield \( U \) is related to the matter superfield \( \Phi = \varphi + \theta \lambda \) as follows:

\[
U = D\partial \Phi - \frac{1}{2} D\Phi \partial \Phi, \quad (3.12)
\]

and accordingly,

\[
\delta \Phi = \partial E + E \partial \Phi + \frac{1}{2} DED \Phi, \quad (3.13)
\]
\[
R_U = D\partial R_\Phi - \frac{1}{2} D\Phi \partial R_\Phi - \frac{1}{2} \partial \Phi DR_\Phi.
\]

The scalar multiplet \( F = f + \theta \psi, \) with

\[
\delta F = E \partial F + \frac{1}{2} DEDF, \quad (3.14)
\]

is related to the superfield \( \Phi \) by:

\[
\Phi = \log \partial F + \frac{DF}{\partial F} D \log \partial F, \quad (3.15)
\]

while the relationship between corresponding Ward identities is

\[
R_\Phi = \left( \frac{DF}{(\partial F)^2} \partial D + \frac{1}{\partial F} - 2 \frac{DFD \partial F}{(\partial F)^3} \right) \partial - \frac{D \partial F}{(\partial F)^2} D \right) R_F. \quad (3.16)
\]
The formulae that link the current Ward identities with \( R_f \) and \( R_\psi \) are

\[
R_T = (\partial^3 + 2T\partial + \partial T) \left( \frac{R_f}{\partial f} - \frac{\psi}{(\partial f)^2} \right) + \frac{\psi R_\psi}{(\partial f)^2} + \frac{\psi R_f}{(\partial f)^2} \]  
\[
- \left( \frac{3}{2} S\partial + \frac{1}{2} \partial S \right) \left( \frac{2R_\psi}{\partial f} - \frac{\psi}{\partial f} \partial \left( \frac{R_f}{\partial f} \right) - 2 \frac{\partial \psi R_f}{(\partial f)^2} \right), \tag{3.17}
\]

\[
R_S = (\partial^2 + \frac{1}{2} T) \left( \frac{2R_\psi}{\partial f} - \frac{\psi}{\partial f} \partial \left( \frac{R_f}{\partial f} \right) - 2 \frac{\partial \psi R_f}{(\partial f)^2} \right) \]
\[
- \left( \frac{3}{2} S\partial + \partial S \right) \left( \frac{R_f}{\partial f} - \frac{\psi}{(\partial f)^2} \frac{\partial \psi R_\psi}{R_f} + \frac{\psi R_f}{(\partial f)^2} \right). \]

If we now use the Legendre transformations to proceed from the partition function to the effective action,

\[
\Gamma[T, S] = Z[h, \chi] - \int d^2x(hT + \chi S), \tag{3.18}
\]

the Ward identity becomes

\[
(\partial^2 D + 3U\partial + DU D + 2\partial U) \frac{\delta \Gamma}{\delta U} = 0 \tag{3.19}
\]

or, in components,

\[
(\partial^3 + 2T\partial + \partial T) \frac{\delta \Gamma}{\delta T} + \left( \frac{3}{2} S\partial + \frac{1}{2} \partial S \right) \frac{\delta \Gamma}{\delta S} = -\partial T, \tag{3.20}
\]

\[
(\partial^2 + \frac{1}{2} T) \frac{\delta \Gamma}{\delta T} + \left( \frac{3}{2} S\partial + \partial S \right) \frac{\delta \Gamma}{\delta S} = -\partial S,
\]

i.e., there emerges a supersymmetric Bol operator, which has also been described in Ref. 12. In this form the covariance of Ward identities under gauge transformations (3.2), which is equivalent to the Wess-Zumino conditions for an anomaly, becomes explicit.

Now let us turn to the problem of finding the partition function of the theory:

\[
\delta Z = \int d^2x(T\delta h + S\delta \chi) = \int d^2x d\theta \delta H \tag{3.21}
\]

\[
= - \int d^2x d\theta E(\bar{\partial} - H\bar{\partial} - \frac{1}{2}DHD + \frac{1}{2}\partial H)U.
\]

We see that the integrand is the Ward identity \( R_U \) without anomalous term. In the chiral scheme, i.e., a regularization scheme that conserves half the reparametrization symmetry and the Weyl symmetry as well as their superpartners, the fields \( \varphi \) and \( \lambda \) are related by (3.10), and the corresponding Ward identities \( R_\varphi \) and \( R_\lambda \) vanish. On the other hand, in a regularization scheme that preserves supercoordinate symmetry the group parameters \( f \) and \( \psi \) vanish and the fields \( \varphi \) and \( \lambda \)
are unconstrained, but Ward identities $R_\varphi$ and $R_\lambda$ still play an important role. Multiplaying $R_U$ by $E$ and integrating by parts, we obtain

$$\int d^2x \theta R_U E = \int E (D\partial - \frac{1}{2} D\Phi \partial - \frac{1}{2} \partial \Phi D) R_\Phi = \int d^2x \theta R_\Phi (D\partial E + \frac{1}{2} \partial (ED\Phi) + \frac{1}{2} D(E\partial \Phi)) (3.22)$$

$$= \int d^2x \theta R_\Phi D\delta \Phi.$$

If we "err" twice, i.e., take (3.22) for $\delta Z$ rather than (3.21) and ignore the relationship between $H$ and $\Phi$, expressed by the fact that $R_\Phi$ is zero, we can integrate this variational equation and arrive at the following expression for $Z[H]$: $Z[H] = -\frac{1}{2} \int d^2x \theta (\bar{\partial} \Phi - H \partial \Phi - 2\partial H) D\Phi$. (3.23)

The fact that (3.17) reproduces the anomaly correctly can easily be verified. Thus, assuming that the superfield $\Phi$ is independent, we can reproduce the anomaly by directly adding the appropriate term to the action.

This conclusion agrees with our ideas about the "transfer" of the anomaly from one regularization scheme to another. A detailed description of the process in which a conformal anomaly is transformed into a gravitational anomaly in the two-dimensional gravitation theory can be found in [1] and [11].

4 $W_3$-gravity

The difference between the theory of $W_3$-gravity and above cases in that the chiral formulation of this theory is not only more convenient but is also the only one amenable to quantum analysis. The nonchiral version formulated in [4], contains an infinite number of derivatives of matter fields and is too complicated even at the classical level.

The Ward identities of chiral $W_3$-gravity are

$$R_T = (\bar{\partial} - h\partial - 2\partial h)T - (2b\partial + 3\partial b)W - \partial^3 h,$$

$$R_W = (\bar{\partial} - h\partial - 3\partial h)W + (2b\partial^3 + 9\partial B\partial^2$$

$$+ 15\partial^2 b\partial + 10\partial^3 b + 16bT\partial + 16\partial bT)T - \partial^5 h.$$ (4.1)

Here $b$ denotes the single nonvanishing component of a third rank symmetric tensor - the gauge field of $W$-gravity, the partner of the metric in the multiplet - and $W$ denotes the corresponding spin-3 current, the partner of the energy-momentum tensor. The chiral general-coordinate and $W$-transformations have the form

$$\delta T = (\partial^3 + 2T\partial + \partial T)\epsilon + 3W\partial \lambda + 2W\lambda,$$
\[\delta W = \partial W \epsilon + 3W \partial \epsilon + (\partial^2 + 10T \partial^3 + 15\partial T \partial^2) + 9\partial^2 T \partial + 2\partial^3 T + 16T^2 \partial + 16T \partial T)\lambda, \tag{4.2}\]
\[\delta h = (\partial - h \partial + \partial h) \epsilon + 2\lambda \partial^3 b - 3\partial \lambda \partial^2 b + 3\partial^2 \lambda \partial b - 2\partial^3 b + 16T(\lambda \partial b - b \partial \lambda), \]
\[\delta b = \epsilon \partial b - 2\partial \epsilon b + (\partial - h \partial + 2\partial h) \lambda.\]

The quantities \(R_T\) and \(R_W\) are covariant under the transformations (4.2), i.e.,
\[\begin{align*}
\delta R_T &= (\epsilon \partial + 2\partial \epsilon)R_T + (2\lambda \partial + 3\partial \lambda)R_W, \\
\delta R_W &= (\epsilon \partial + 3\epsilon \partial)R_W + (2\lambda \partial^3 + 9\partial \lambda \partial^2 + 15\partial^2 \lambda \partial \\
&\quad + 10\partial^3 \lambda + 32T \partial \lambda + 16T \lambda \partial + 16\lambda \partial T)R_T.
\end{align*}\tag{4.3}\]

Thus, taking \(R\) as the differential operator, we conclude that (4.3) yields a universal relation
\[\left[\delta, R\right] = 0 \tag{4.4}\]
on the current fields \(T\) and \(W\).

The transformations (4.2) of an arbitrary quantity \(A\) are generated by the currents \(T\) and \(W\) via Poisson brackets:
\[\delta A = \int d^3x \left(\epsilon(x) \{T(x); A\} + \lambda(x) \{W(x); A\}\right). \tag{4.5}\]

In terms of these brackets, the transformations (4.2) themselves become
\[\begin{align*}
-\{T(x); T(x')\} &= \delta''(x - x') + (T(x) + T(x'))\delta'(x - x'), \\
-\{T(x); W(x')\} &= (W(x) + 2W(x'))\delta'(x - x'), \\
-\{W(x); T(x')\} &= (2W(x) + W(x'))\delta'(x - x'), \\
-\{W(x); W(x')\} &= \delta^V + 5(T(x) + T(x'))\delta'''(x - x') \\
&+ 8(T^2(x) + T^2(x'))\delta'(x - x') - 3(T''(x) + T''(x'))\delta'(x - x').
\end{align*}\tag{4.6}\]

This algebra can be reproduced by expressing the current fields in terms of the matter fields \(\varphi\) and \(\psi\), which obey the algebra of free fields:
\[\begin{align*}
\{\varphi'(x); \varphi'(x')\} &= \delta'(x - x'), \\
\{\varphi'(x); \psi'(x')\} &= 0, \\
\{\psi'(x); \psi'(x')\} &= -\delta'(x - x').
\end{align*}\tag{4.7}\]

if we define \(T(x)\) and \(W(x)\) in the following manner:
\[\begin{align*}
T(x) &= \partial^2 \varphi - \frac{1}{2}(\partial \varphi)^2 + \frac{1}{2}(\partial \psi)^2, \\
W(x) &= \partial^2 \psi - 3\partial \varphi \partial^2 \psi - \partial^2 \varphi \partial \psi \\
&\quad + 2(\partial \varphi)^2 \partial \psi + \frac{2}{3}(\partial \psi)^3. \tag{4.8}\end{align*}\]
This corresponds to the following transformation law for the matter fields:
\[
\delta \varphi = \partial \epsilon + \partial \varphi \epsilon - 4\lambda \partial \varphi \partial \psi + 2\lambda \partial^2 \psi - \partial \lambda \partial \psi, \tag{4.9}
\]
\[
\delta \psi = \epsilon \partial \psi + \partial^2 \lambda + 3\partial \lambda \partial \varphi + 2\lambda (\partial^2 \varphi + (\partial \varphi)^2 + (\partial \psi)^2).
\]

The anomalous equation of motion of the matter multiplet have the form
\[
R_\varphi = \bar{\partial} \varphi - h \partial \varphi + \partial b \partial \psi - 2b(\partial^2 \psi - 2\partial \varphi \partial \psi) - \partial h, \tag{4.10}
\]
\[
R_\psi = \bar{\partial} \psi - h \partial \psi - 3\partial b \partial \varphi - 2b(\partial^2 \varphi + (\partial \varphi)^2 + (\partial \psi)^2) - \partial^2 b.
\]

With respect to \(\epsilon\)- and \(\lambda\)-diffeomorphisms, these relationships are also covariant
\[
\delta R_\varphi = (\epsilon \partial - 4\lambda \partial \psi \partial)R_\varphi + (2\lambda \partial^2 - \partial \lambda - 4\lambda \partial \varphi \partial)R_\psi, \tag{4.11}
\]
\[
\delta R_\psi = (\epsilon \partial + 4\lambda \partial \psi)R_\psi + (3\partial \lambda \partial + 2\lambda \partial^2 + 4\lambda \partial \varphi \partial)R_\varphi.
\]

This also establishes the validity of Eq. (4.4) when acts on matter multiplet.

The Ward identities (4.1) can easily be transformed into
\[
R_T = (\partial^2 - \partial \varphi)R_\varphi + \partial \psi \partial R_\psi, \tag{4.12}
\]
\[
R_W = (4\partial \varphi \partial \psi \partial - \partial \psi \partial^2 - 3\partial^2 \psi \partial)R_\varphi + (\partial^3 - 3\partial \varphi \partial^2 + 2(\partial \varphi)^2 \partial + 2(\partial \psi)^2 \partial)R_\psi.
\]

The \(\epsilon\) - and \(\lambda\)-transformations constitute a closed algebra on the multiplets of currents \(\{T, W\}\), matter fields \(\{\varphi, \psi\}\) and gauge fields \(\{h, b\}\):
\[
[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta(\epsilon_3 = \epsilon_2 \partial \epsilon_1 - \epsilon_1 \partial \epsilon_2),
\]
\[
[\delta(\epsilon_1), \delta(\lambda_2)] = \delta(\lambda_3 = 2\lambda_2 \partial \epsilon_1 - \epsilon_2 \partial \lambda_1), \tag{4.13}
\]
\[
[\delta(\lambda_1), \delta(\lambda_2)] = \delta(\epsilon_3 = 16T(\lambda_2 \partial \lambda_1 - \lambda_1 \partial \lambda_2),
\]
\[+ 2\lambda_2 \partial^2 \lambda_1 - 3\partial \lambda_2 \partial^2 \lambda_1 + 3\partial^2 \lambda_2 \partial \lambda_1 - 2\partial^3 \lambda_2 \lambda_1).
\]

The partition function of the theory is calculated in the same way as in supergravitation theory: by multiplaying Eqs. (4.12) respectively, by \(\epsilon\) and \(\lambda\), we obtain
\[
\int d^2x(\epsilon R_T + \lambda R_W) = \int d^2x(R_\varphi \delta \partial \varphi - R_\psi \delta \partial \psi). \tag{4.14}
\]

If the Ward identities (4.1) were to have no anomalous terms, the left-hand side of Eq. (4.14) would be the variation of the partition function with opposite sign. If, in addition, the fields \(\varphi\) and \(\psi\) were to be free and Eqs. (4.11) were not link them with gauge fields \(h\) and \(b\), the right-hand side of Eq. (4.14) would be the total variation of the following expression:
\[
Z[h, b] = \int d^2x \left( \frac{1}{2} \partial \varphi (\bar{\partial} \varphi - h \partial \varphi) - \frac{1}{2} \partial \psi (\bar{\partial} \psi - h \partial \psi) - \partial h \partial \varphi + \right.
\]
\[+ \partial^2 b \partial \psi + b(2(\partial \varphi)^2 \partial \psi - \partial^2 \varphi \partial \psi - 3\partial \varphi \partial^2 \psi + \frac{2}{3}(\partial \psi)^3) \right)
\[= \int d^2x[\frac{1}{2}(\bar{\partial} \varphi \partial \varphi - \bar{\partial} \psi \partial \psi) + hT + bW].
\]
Equation (4.15) is the action of the matter fields interacting with two-dimensional chiral gravity and $W_3$-gravity. The variation of (4.15) with respect to (4.2) and (4.9) is

$$
\delta Z = \int d^2 x \left( h \partial^3 \epsilon + b \partial^5 \lambda + 16 \lambda (T^2 \partial b + b T \partial T) \right).
$$

This expression differs from that for the quantum anomaly of the minimal type by the presence of terms quadratic in $T$. This discrepancy is due to the differences in defining the transformation law for the field $h$ under $\lambda$-diffeomorphisms. The transformation law (4.2) is motivated by the closure of the algebra (4.13) on the fields $h$ and $b$ and by the Wess-Zumino self-consistency condition, with Eqs. (4.3) being valid.

The following line of reasoning can motivate another definition of $\delta \lambda h$: if we perform the Legendre transformation

$$
Z[h, b] = \Gamma[T, W] + \int d^2 x (hT + bW),
$$

the expressions for $R_T$ and $R_W$ acquire the Bol operators $L_3$ and $L_5$:

$$
R_T = \bar{\partial} T + \left( \partial^3 + 2 T \partial + \partial T \right) \frac{\delta \Gamma}{\delta T} + (3 W \partial + 2 \partial W) \frac{\delta \Gamma}{\delta W},
$$

$$
R_W = \bar{\partial} W + (3 W \partial + \partial W) \frac{\delta \Gamma}{\delta T} + \left( \partial^5 + 10 T \partial^3 + 15 T \partial^2 + \right.
$$
$$
\left. + (9 \partial^2 T + 16 T^2) \partial + (2 \partial^3 T + 16 T \partial T) \right) \frac{\delta \Gamma}{\delta W}.
$$

Thus,

$$
\delta Z = \int d^2 x (T \delta h + W \delta b)
$$

$$
= - \int d^2 x \left( \epsilon \partial^3 h + \lambda \partial^5 b - 16 \lambda (T^2 + b T \partial T) \right).
$$

To obtain an anomaly of the minimum type it appears reasonable to define $\delta \lambda h$ with $\delta_{extra} h = -8 T (\lambda \partial b - b \partial \lambda)$.

The anomaly can be completely removed from the transformation laws and the Ward identities if we transform to variables $(f, g)$, which form a scalar multiplet:

$$
\varphi = \log \partial f + \frac{1}{2} \log (1 + \varphi^2 g),
$$

$$
\psi = \gamma^{-1} \log (1 + \varphi^2 g),
$$

where $\varphi \equiv \frac{1}{\partial r} \partial$ and $\gamma^2 = -12$.

The transformation law for the fields $f$ and $g$ is

$$
\delta f = \epsilon \partial f - \gamma (\lambda \partial^2 f + \frac{1}{2} \partial \lambda \partial f + \frac{2}{3} \lambda \partial f \partial \log (1 + \varphi^2 g)),
$$

$$
\delta g = \epsilon \partial g - \frac{1}{2} \gamma \partial \lambda \partial g - \gamma \lambda [ (\partial f)^2 - \partial^2 g + \frac{2}{3} \partial g \partial \log (1 + \varphi^2 g) + 2 \partial g \frac{\partial^2 f}{\partial f}].
$$
Accordingly, the Ward identities are

\[ R_f = \partial f - h \partial f + \gamma \frac{\partial}{\partial f} \partial b \partial f + \gamma b \left( \partial^2 f + \frac{2}{3} \partial f \partial \log(1 + \varphi^2 g) \right) \]  
\[ R_g = \partial g - h \partial g + \gamma \frac{\partial}{\partial g} \partial b \partial f + \gamma b \left[ 2 \partial g \frac{\partial^2 f}{\partial f} + \frac{2}{3} \partial f \partial \log(1 + \varphi^2 g) - (\partial f)^2 - \partial^2 g \right]. \]

Setting \( R_f \) and \( R_g \) to zero, we can express the gauge multiplet in terms of \( f \) and \( g \):

\[ b = \gamma^{-1} \frac{(\partial g - \frac{\partial f}{\partial f} \partial g)}{(\partial f)^2 (1 + \varphi^2 g)}, \]  
\[ h = \frac{\partial f}{\partial f} + \frac{\gamma}{2} \partial b + \gamma b \left( \frac{\partial^2 f}{\partial f} + \frac{2}{3} \partial f \partial \log(1 + \varphi^2 g) \right). \]

These formulae coincide, up to renormalization of \( \gamma \) and \( \lambda \), with the solution found by Ooguri et. al., who interpreted \( W_3 \)-gravity as a constrained Wess-Zumino \( SL(3,R) \)-theory. Plugging (4.20) and (4.21) into (4.15), we reproduce the chiral action of Ooguri et. al., which must be interpreted as the effective action induced by quantum fluctuations of the matter fields \( \varphi \) and \( \psi \), which interact with the multiplet of chiral \( W_3 \)-gravity via (4.15). The anomalous dependence of this action on the ”coordinate” functions \( f \) and \( g \) is due to the \( W \)-gravity anomaly.

Continuing the analogy with the cases of two-dimensional gravity and supergravity, we can assume that this action is a result of choosing a regularization scheme that conversed the Weyl and \( W \)-Weyl symmetries, as well as half the coordinate symmetries of the covariant action that describes the interaction of matter fields and ordinary gravity and \( W \)-gravity.

5 Conclusion

By generalizing some of the laws governing ordinary gravitation we were able to find the effective action of chiral \( W_3 \)-gravity. It would be more interesting, however, to continue the analogy and find the \( W \)-analog of the Liouville action; namely the covariant action that describes the interaction of matter fields with a gravitational and \( W \)-gravitational background.

To understand the nature of the symmetry properties of \( W \)-gravity theory it is advisable to first turn to classical theory.

As a classical gauge theory, \( W \)-gravity was first examined by Hull. The nonchiral formulation of this theory was later performed by Schoutens et. al. They started with the action

\[ S = \frac{1}{2} \int d^2x \partial^+ \varphi \partial^- \varphi. \]  

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With respect to the ordinary diffeomorphisms \( \delta \varphi = \epsilon^\alpha \partial_\alpha \varphi \), the variation of (5.1) is:

\[
\delta S = \int d^2 x [\partial_\alpha (\epsilon^\alpha \partial_\alpha \varphi \partial_\alpha \varphi) + \partial_\alpha \epsilon^- (\partial_\alpha \varphi)^2 + \partial_\alpha^+ (\partial_\alpha \varphi)^2],
\]

vanishes if \( \partial_\alpha \epsilon^- = \partial_\alpha \epsilon^+ = 0 \). To generalize this symmetry to a local one, we must, in accordance with Noether’s theorem, add to (5.1) the currents \( t_{++} = \frac{1}{2} (\partial_+ \varphi)^2 \) and \( t_{--} = \frac{1}{2} (\partial_- \varphi)^2 \) multiplied by the corresponding gauge fields \( kh_{++} \) and \( kh_{--} \)

Here \( k \) is an expansion parameter, which is set to unity in the final result. After an infinite number of steps the action can be summed as a geometric progression to produce

\[
S = \frac{1}{2} \int d^2 x \frac{(\partial_+ \varphi - kh_{--} \partial_- \varphi)(\partial_- \varphi - kh_{++} \partial_+ \varphi)}{1 - k^2 h_{++} h_{--}}.
\]

Schoutens et. al. \[4\] then stated that the action (5.1) for \( n \) real scalar fields is invariant under holomorphic \( W \)-diffeomorphisms. Indeed, under the transformations

\[
\delta \varphi^i = d^{ijk} (\lambda^{++} \partial_+ \varphi^j \partial_+ \varphi^k + \lambda^{--} \partial_- \varphi^j \partial_- \varphi^k)
\]

the action transforms as

\[
\delta S = \frac{1}{3} \int d^2 x d_{ijk} (\partial_+ \varphi^j \partial_+ \varphi^k \partial_- \varphi \partial_- \lambda^{++} + \partial_- \varphi^j \partial_- \varphi^k \partial_+ \varphi \partial_+ \lambda^{--}).
\]

The fact that the algebra of holomorphic \( \epsilon \)- and \( \lambda \)-diffeomorphisms is closed imposes the following constraint on the symmetric constants \( d \) see \[3\]:

\[
d^{k(ij) l(mk)} = \delta^{(ij) \delta (l)},
\]

The action (5.1) can be made invariant under local \( \epsilon \)- and \( \lambda \)-transformations via the Noether procedure by introducing the appropriate gauge fields \( h_{++} \), \( h_{--} \), \( b^{+++} \) and \( b^{---} \). Unfortunately, in the given case the invariant action can be summed only by using auxiliary fields \( F^i_{\pm} \):

\[
S = \int d^2 x [\partial_+ \varphi^i \partial_- \varphi^i + F^i_+ F^i_- + F^i_+ (\partial_- \varphi^i - \frac{1}{3} d_{ijk} b^{+++} F^j_+ F^k_+) + (5.7)
\]

\[
F^i_- (\partial_+ \varphi^i - \frac{1}{3} d_{ijk} b^{---} F^j_- F^k_-)]
\]

After the auxiliary fields are eliminated, they replaced by "nested" covariant derivatives,

\[
F^i_{\pm} \rightarrow \hat{\partial} \varphi^i_{\pm} = e^\alpha_{\pm} \partial_\alpha \varphi - b^{+++} d_{ijk} \hat{\partial}_+ \varphi^j \hat{\partial}_+ \varphi^k,
\]

and the action becomes infinitely nonlinear. To avoid difficulties and operate from the start in a covariant setting, we must introduce more gauge fields than required by the Noether approach. Specifically, in the case of pure gravitation, we must introduce the full tensor \( h_{\alpha \beta} \) instead of the two components \( h_{++} \) \( h_{--} \).
Then the Noether procedure terminates after the first step, and the invariant action has the form

\[ S = \int d^2 x h^{\alpha \beta} \partial_\alpha \varphi \partial_\beta \varphi. \]

Here we expect a new symmetry to appear, a symmetry that would balance the superfluous degree of freedom related to the \( h^{+---} \)-component of the metric. By requiring that the energy-momentum tensor of the theory be traceless,

\[ T^\alpha_\alpha = T^{\alpha \beta} h^{\alpha \beta} = 0, \quad T^{\alpha \beta} \equiv \frac{\delta S}{\delta h^{\alpha \beta}}, \quad (5.9) \]

we require that the theory be Weyl-invariant, so that the equation \( h^{\alpha \beta} \frac{\delta S}{\delta h^{\alpha \beta}} = 0 \) has a functional of the type \( S = S(\sqrt{h^{\alpha \beta}}) \) as a solution, which is equivalent to invariance under \( h^{\alpha \beta} \rightarrow e^{\alpha} h^{\alpha \beta} \). In this way the final expression for the invariant action is

\[ S = \int \sqrt{h} d^2 x h^{\alpha \beta} \partial_\alpha \varphi \partial_\beta \varphi. \quad (5.10) \]

In the case of \( W_3 \)-gravity we propose introducing the \( h^{+---}, b^{++--} \) and \( b^{-++} \) components of the gauge fields, in order to produce the total tensors \( h^{\alpha \beta}, b^{\alpha \beta \gamma} \). The Noether procedure terminates after the first step, and the invariant action has the form

\[ S = \int d^2 x (h^{\alpha \beta} t^{\alpha \beta} + b^{\alpha \beta \gamma} \omega^{\alpha \beta \gamma}), \quad (5.11) \]

where \( t^{\alpha \beta} = \frac{1}{2} (\partial_\alpha \varphi \partial_\beta \varphi - \partial_\alpha \psi \partial_\beta \varphi) \) and

\[ \omega^{\alpha \beta \gamma} = \frac{2}{3} (\partial_\alpha \varphi \partial_\beta \varphi \partial_\gamma \psi + \partial_\alpha \varphi \partial_\beta \psi \partial_\gamma \varphi + \partial_\alpha \psi \partial_\beta \varphi \partial_\gamma \varphi + \partial_\alpha \psi \partial_\beta \psi \partial_\gamma \psi) \]

The action (5.11) is assumed to be invariant under the \( W \)-diffeomorphisms

\[ \delta \lambda \varphi = -4 \lambda^{\alpha \beta} \partial_\alpha \varphi \partial_\beta \psi, \quad (5.12) \]

\[ \delta \lambda \psi = 2 \lambda^{\alpha \beta} (\partial_\alpha \varphi \partial_\beta \varphi + \partial_\alpha \psi \partial_\beta \psi), \]

defined with a traceless parameter \( \lambda \), i.e., \( \lambda^{\alpha \beta} h_{\alpha \beta} = 0 \). The variation of (5.11) under the transformations (5.12) can be written in the form

\[ \delta S = \int d^2 x \left[ \delta \lambda h^{\alpha \beta \gamma} \omega^{\alpha \beta \gamma} - \omega^{\alpha \beta \gamma}(h^{\alpha \mu} \nabla_\mu \lambda^{\beta \gamma} + h^{\beta \mu} \nabla_\mu \lambda^{\alpha \gamma} + h^{\gamma \mu} \nabla_\mu \lambda^{\alpha \beta}) \\
- (h^{\alpha \mu} \lambda^{\beta \gamma} + h^{\beta \gamma} \lambda^{\alpha \mu}) \nabla_\mu \omega^{\alpha \beta \gamma} + \delta \lambda h^{\alpha \beta} t^{\alpha \beta} + 16 b^{\alpha \beta \gamma} \lambda^{\mu \nu} \right. \\
\times \left. (2 t^{\gamma \mu} \nabla_\mu t^{\alpha \beta} - t^{\beta \gamma} \nabla_\alpha t^{\mu \nu}) + 16 b^{\alpha \beta \gamma} \nabla_\alpha \lambda^{\mu \nu}(2 t^{\beta \mu} t^{\gamma \nu} - t^{\beta \gamma} t^{\mu \nu}) \right]. \quad (5.13) \]

Defining the \( \lambda \)-variations of the gauge fields in such a way that the coefficients of the currents \( t^{\alpha \beta} \) and \( \omega^{\alpha \beta \gamma} \) vanish, we ensure that the action (5.11) is invariant. The transformations (5.12) represent a specific realization of the constants \( d^{ijk} \) in (5.4) for the case of two fields. Such a restriction is not accidental, the point
being that when there are three or more fields, the condition that the covariant algebra of the $W$-transformations

$$
\delta \varphi^i = d^{ijk} \lambda^{\alpha\beta} \partial_\alpha \varphi^j \partial_\beta \varphi^k)
$$

be closed imposes additional constraints on the $d^{ijk}$, restrictions which together with the conditions (5.6) have only a vanishing solution. In addition, the algebra becomes closed on the gauge fields only if the equations of motion are included, as happens in the simpler case of two-dimensional supergravitation (17).

Thus, to guarantee invariance under ordinary and $\lambda$-diffeomorphisms, we introduced seven gauge fields. Now we would like to impose constraints on the theory in such a way so as to obtain a three-parameter symmetry group that "balances" the three superfluous degrees of freedom.

By imposing the conditions that the energy-momentum tensor and $W$-current be traceless,

$$
\frac{\delta S}{\delta h^{\alpha\beta}} = 0, \quad \frac{\delta S}{\delta b^{\beta\gamma}} = 0
$$

conditions with a solution of the form $S = S(\hat{h}^{\alpha\beta}, \hat{b}^{\beta\gamma})$, with the quantities $\hat{h}^{\alpha\beta} \equiv \sqrt{h}h^{\alpha\beta}$ and $\hat{b}^{\beta\gamma} \equiv b^{\beta\gamma} - \frac{1}{4}h^{\mu\nu}(h^{\alpha\beta}b_{\gamma\mu\nu} + h^{\beta\gamma}b_{\alpha\mu\nu} + h^{\alpha\gamma}b_{\beta\mu\nu})$ invariant under Weyl and $W$-Weyl transformations, respectively, i.e.,

$$
\begin{align*}
W \text{eyl} : & \quad h^{\alpha\beta} \rightarrow e^{\sigma} h^{\alpha\beta}, \\
W - W \text{eyl} : & \quad b^{\beta\gamma} \rightarrow b^{\beta\gamma} + (\zeta^\alpha h^{\alpha\gamma} + \zeta^\beta h^{\beta\gamma} + \zeta^\gamma h^{\gamma\alpha})
\end{align*}
$$

we reduce the number of degrees of freedom. However, comparison of the variations $\delta S(\hat{h}, \hat{b})$ with (5.13) shows that the symmetry of the action under $\lambda$-diffeomorphisms is incompatible with Weyl invariance, since if the variation $\delta_{\lambda} t_{\alpha\beta}$ traceless, the variation $\delta_{\lambda} \omega_{\alpha\beta\gamma}$ is not, with the result that the latter cannot be made equal to the traceless variation $\delta \hat{h}$. Thus, Weyl invariance is incompatible with $W$- symmetry even at classical level, and to reduce the number of degrees of freedom we must replace the requirements that the energy-momentum tensor be traceless by a different one.

Schoutens et al. [16] proposed a covariant formulation of $W$-gravity. With each annihilation operator of the classical $W_3$-algebra they associated a gauge field and a local parameter, and with each creation operator they associated a field in the adjoint representation. Then they required that all corresponding curvatures vanish. The resulting theory has a finite number of degrees of freedom and, in addition, to being coordinate- and $W$-diffeomorphism invariant, it is locally Weyl- and Lorentz- and $W$-Weyl and $W$-Lorentz-invariant.

It would also be interesting to study this theory as a constrained Hamiltonian system.

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References

[1] A.B. Zamolodchikov, Teoret. Mat. Fiz. 65, (1985), 347.

[2] I.M. Gel’fand and L.A.Dikij Preprint IFM of the USSR Academy of Science, Moskow (1978); V.G. Drinfel’d and V.V.Sokolov, Current Problems of Mathematics [in Russian] Vol.24, VINITI, Moskow (1984).

[3] C.M.Hull Phys.Lett. B240, (1990), 110.

[4] K.Schoutens, A.Sevrin and P.van Nieuwenhuizen, Phys.Lett. B243, (1990), 248.

[5] K.Schoutens, A.Sevrin and P.van Nieuwenhuizen, Nucl.Phys. B364, (1991), 584; Nucl.Phys. B371, (1992),315.

[6] H.Ooguri, K.Schoutens, A.Sevrin and P.van Nieuwenhuizen Comm.Math.Phys. 145, (1992), 515.

[7] A.M.Polyakov, Mod.Phys.Lett.A2, (1987), 893.

[8] V.G.Knizhnik, A.M.Polyakov, A.B.Zamolodchikov, Mod.Phys.Lett.A3 (1988) 819.

[9] R.Zucchini, Phys.Lett. B260 (1991) 296.

[10] F.Gieres, Conformally covariant operators on Riemann Surfaces (with applications to conformal field theory and integrable models), CERN preprint 366/1991.

[11] D.R.Karakhanyan, R.P.Manvelyan, R.L.Mkrtchyan Phys.Lett. B329, (1994), 185.

[12] J.M.Figueroa-O’Farrill, S.Stanciu, E.Ramos, Phys.Lett. B297 (1992), 289. C.M.Hull, Phys.Lett. B269, (1991), 257.

[13] Y.Matsuo, Phys.Lett. B227, (1991), 117.

[14] A.M.Polyakov, and A.B.Zamolodchikov, Mod.Phys.Lett.A3 (1988) 819.

[15] D.R.Karakhanyan, Phys.Lett. B365, (1996), 56.
[16] K. Schoutens, A. Sevrin and P. van Nieuwenhuizen, Nucl. Phys. B349, (1991), 791.

[17] S. Deser, B. Zumino, Phys. Lett. B65, (1976), 369.