Unruh effect and condensate in and out of an accelerated vacuum

Sanjin Benić\textsuperscript{1,2} and Kenji Fukushima\textsuperscript{2}

\textsuperscript{1}Physics Department, Faculty of Science, University of Zagreb, Zagreb 10000, Croatia
\textsuperscript{2}Department of Physics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

We address a physical interpretation of the Hawking-Unruh effect with our emphasis put on judicious consideration of observables and vacua with and without acceleration. In particular we discuss thermal-like corrections using explicit computation of correlation functions. We identify the correspondence between thermo-field dynamics and accelerated systems. Then we make it clear that a genuine thermal state corresponds to a non-accelerated frame, while vacuum states correspond to Rindler wedges, which accounts for a negative contribution of the thermal-like correction measured in the accelerated vacuum. We apply our results to investigate how the acceleration effect would affect a condensate of fields. Our conclusion is that a larger acceleration should enhance a condensate as compared to those in a non-accelerated vacuum.

I. INTRODUCTION

Quantum field theory in non-trivial spacetime associates an event horizon with the Hawking-Unruh temperature \( T_H = \kappa/(2\pi) \) for the black-hole case. Here \( \kappa = 1/4M \) is the surface gravity at the horizon and \( M \) is the black-hole mass. For \( M \sim M_\odot \) (solar mass), this temperature is of the order of \( 10^{-8} \text{K} \). It is, therefore, difficult to detect any signal for the Hawking radiation directly from astrophysical observations. Nevertheless, it is still a fascinating idea to seek for an analogous and more controllable system having an event horizon. One example is found in a moving fluid that could be expressed in terms of an acoustic metric. The particle production from quantum fluctuations may happen at the horizon where the fluid velocity exceeds the sound velocity, and its spectrum is supposed to be thermal, which awaits experimental tests in systems of cold atoms \cite{3}.

Recent developments in the high-intensity laser physics provides us with another analogue to the Hawking-Unruh temperature \cite{4,5}: strong fields realized by the laser \cite{6,7} may open an opportunity to test the Unruh radiation \cite{2,8,11} (see also Refs. \cite{12,13} for recent calculations). With a pettawatt laser (i.e., \( 10^{20} \text{W/cm}^2 \)) of the Extreme Light Infrastructure project, for example, electrons can reach an acceleration of the order \( 10^{25} \text{cm/s}^2 \) so that the Hawking-Unruh temperature can be of the order of \( 10^9 \text{K} \). Moreover, the Hawking-Unruh effect might have a phenomenological impact in high-energy collision experiments where the ratio of particle species agrees with thermal distributions \cite{16,17}. In the relativistic heavy-ion collision a microscopic mechanism of thermal evolution toward a quark-gluon plasma is still under active debates, and the theoretical estimate of particle production is strongly demanded. Such calculations involve quantum fluctuations under strong chromo-fields in a non-trivial (expanding) geometry \cite{18}. For the concrete evaluation of physical observables, we should carefully specify how we should define the vacuum with and without acceleration, and distinguish the observers (i.e., detectors) set in each vacuum.

Observers measure the physical observables that include a condensate or an order parameter for a global symmetry of a theory. We know that the ground state in Quantum Chromodynamics, that is commonly called the QCD vacuum, is endowed with condensates in Minkowski (flat) spacetime; the chiral condensate makes fermionic excitations gapped and the gluon condensate arises from the trace anomaly. In the electroweak sector the Minkowski vacuum accommodates the Higgs condensate and the Higgs phenomena are ubiquitous in condensed matter experiments. In the contexts of the heavy-ion collision as well as in the early universe, investigating a possible modification of condensates under gravity and acceleration has a wide variety of potential applications \cite{19,20}. In fact, there may be an even better chance that strong laser fields could access a non-trivial vacuum structure via a strong acceleration effect, i.e., the Unruh effect. Instead of treating the electric field and its acceleration as they appear, we will adopt an idealized setup where the acceleration is encoded in a specific geometry called the Rindler spacetime. We emphasize that this particular way to explore the strong external field effect should lead us to an evident link between the Hawking and the Unruh effects.

In this work we consider a real scalar field \( \phi \) in Rindler spacetime. We explicitly compute the Wightman two-point functions in the Rindler (accelerated) vacuum and in the Minkowski (inertial) vacuum using the point-splitting regularization. Our calculations conclude that two-point functions are intact regardless of in which of the Rindler and the Minkowski vacua \( \phi \) is quantized, as long as the state that takes the expectation value is common. The explicit calculations clearly show that this non-trivial situation is a consequence from taking different basis functions in Rindler and Minkowski spacetimes. As a matter of fact, isolating the UV divergent terms, we see that the acceleration-induced corrections have an opposite sign to what is expected from the thermal interpretation. We make a comparison to a formalism of thermo-field dynamics to clarify the the origin of this opposite sign.

To quantify a modification on the condensates, we as-
sume to have a finite and homogeneous scalar condensate in Minkowski spacetime without acceleration. We wish to make it clear what happens to the condensate when a finite acceleration is applied if seen in a co-accelerated frame. The possible response of the condensate to the Unruh effect is especially intriguing from the point of view of analogies to the temperature; for example, in ordinary thermal field theories, on the one hand, temperature tends to destroy the condensate. The effect of finite acceleration, on the other hand, may not necessarily lead to the same conclusion. Thus, the interpretation of the Unruh effect is far less clear than the interpretation in the case of temperature; e.g. see Refs. [21–28] for several scenarios.

As we already mentioned, the acceleration-induced corrections may be accompanied by an opposite sign, and such a reversal of roles for thermal-like effects brings an exotic possibility that the condensate could grow with increasing acceleration. This is the case when the condensate is measured in the co-accelerated frame that is realized if the inertial vacuum is itself accelerated together with the observer. By utilizing the first-order perturbation theory, we will solve the equation of motion and calculate the condensate numerically and analytically under a scaling Ansatz. We employ a boundary condition that parametrizes this trajectory corresponding to a fixed value of $\alpha$, that is:

$$z = -\bar{\rho} \cosh \bar{\eta}, \quad t = \bar{\rho} \sinh \bar{\eta},$$

(3)

and the associated metric is found to be

$$ds^2 = \bar{\rho}^2 d\bar{\eta}^2 - d\bar{\rho}^2 - d\mathbf{x}_\perp^2.$$  

(4)

It is important to note that these new coordinates, $\rho$ and $\eta$, can cover only a part of the Minkowski spacetime as long as $\rho$ is non-negative and $\eta$ is real. Because the region of $z > |t|$ is spanned with $\rho$ and $\eta$ in Eq. (1), we call $\rho$ and $\eta$ the right-wedge Rindler coordinates. We can also introduce the coordinates, $\bar{\rho}$ and $\bar{\eta}$ for the left-wedge in a similar fashion, that is:

$$z = -\bar{\rho} \cosh \bar{\eta}, \quad t = \bar{\rho} \sinh \bar{\eta},$$

and the associated metric is found to be

$$ds^2 = \bar{\rho}^2 d\bar{\eta}^2 - d\bar{\rho}^2 - d\mathbf{x}_\perp^2.$$  

(4)

In what follows we will focus on the right-wedge Rindler coordinates only. We can always setup the trajectory of an accelerated particle in this region with $z > 0$ without loss of generality.

Using the notion of the proper time $\tau$ and the velocity four-vector $u^\mu = dx^\mu/d\tau$, we can define the acceleration four-vector as $a^\mu = du^\mu/d\tau$. Then, the proper acceleration $\alpha$ is given by

$$\alpha^2 = -a_\mu a^\mu.$$  

(5)

A trajectory of a point particle with a proper acceleration $\alpha$ in terms of the Minkowski coordinates can be parametrized as

$$z(\tau) = \frac{1}{\alpha} \cosh(\alpha \tau), \quad t(\tau) = \frac{1}{\alpha} \sinh(\alpha \tau).$$  

(6)

Therefore, in terms of the right-wedge Rindler coordinates, this trajectory corresponds to $\tau$ dependent $\eta$ with a fixed value of $\rho$, that is:

$$\rho = \frac{1}{\alpha}, \quad \eta = \alpha \tau.$$  

(7)

It is important to stress here that $\rho$ and $\eta$ have dual roles as coordinates and as parameters characterizing a trajectory with acceleration. As sketched in Fig. 1, the constant-$\rho$ trajectories move away from the light-cone, and their shape straightens, as $\rho$ is increased. This clearly means that a larger $\rho$ represents a smaller acceleration, which agrees with the identification of Eq. (7).

**B. Scalar field theory**

The action for a real scalar field theory in a general coordinate system (apart from the curvature) [30] is

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right),$$  

(8)

where $V(\phi)$ is a potential term. The wave equation, or the equation of motion, takes the following form:

$$\Box \phi - V'(\phi) = 0, \quad \Box = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu).$$  

(9)
In this section we limit ourselves to a special case with a quadratic potential, i.e., we consider a mass term as

\[ V(\phi) = \frac{m^2}{2} \phi^2 . \quad (10) \]

The Unruh effect in Rindler spacetime is in some sense similar to the physics of the Schwinger mechanism that is responsible for the particle production under strong electric fields \[21\]. However, while the source of the non-trivial Bogolyubov transformation in the Schwinger effect is the difference between the in- and out-states, in the Unruh case the non-trivial Bogolyubov transformation (to be discussed in the following) is introduced by an inequivalence of states or observers throughout the whole spacetime history \[8\], which may or may not be connected adiabatically. This sometimes causes confusion in conceptual understanding of the Unruh effect.

Let us now introduce two distinct observers; a Minkowski observer (without acceleration) and a Rindler observer (with acceleration). The Minkowski (and Rindler) observer quantizes the fields in terms of the Minkowski (and Rindler, respectively) coordinates. We will indicate observables quantized in these ways by denoting $\phi_M$ and $\phi_R$ for the Rindler and the Minkowski observers for the moment. As we will discuss in great details later, after all, it is irrelevant to distinguish $\phi_M$ and $\phi_R$.

In the Minkowski coordinate system we have a flat metric and the quantum field is expanded in terms of a complete set of basis functions (denoted by $f's$) with the creation and annihilation operators as

\[ \phi_M = \int_{-\infty}^{\infty} dk_z \int d^2k_\perp \left[ a(k_\perp, k_z) f(k_\perp, k_z, x) + a^\dagger(k_\perp, k_z) f^*(k_\perp, k_z, x) \right] . \quad (11) \]

When we impose no acceleration, it would be the most convenient to take the plane waves as the basis functions:

\[ f(k_\perp, k_z, x) = \frac{1}{(2\pi)^{3/2}(2k_0)^{1/2}} e^{ik_\perp \cdot x + ik_z z - ik_0 t} . \quad (12) \]

We can thus define the Minkowski vacuum $|M\rangle$ as a solution of $a(k_\perp, k_z)|M\rangle = 0$.

With the Rindler metric \[2\] we should replace the basis functions as $f(k_\perp, k_z, x) \rightarrow f_R(k_\perp, \omega, x) \quad (see, \ e.g. \ [11, 26, 32] for technical details)$, where

\[ f_R(k_\perp, \omega, x) = \frac{\sqrt{1 - e^{-2i\omega}}} {2(2\pi)^{1/2}} K \left( \frac{\omega}{2}, \frac{\kappa \rho}{\sqrt{2}} \right) e^{ik_\perp \cdot x - i\omega \eta} , \quad (13) \]

and we introduced $\kappa \equiv \sqrt{k^2_\perp + m^2}$ in the above. We note that $\omega$ is a dimensionless conjugate of $\eta$. From the trajectory \[7\] we should understand that $\alpha \omega$ corresponds to a Rindler energy. The longitudinal part of the above basis functions is explicitly given by

\[ K(\omega, \alpha, \beta) = \int_0^\infty ds s^{i\omega} e^{-is \alpha + is^2/s} . \quad (14) \]

We should consider this function in the physical region of $\omega > 0$ where we can check the following:

\[ K(\omega, \alpha, \alpha) = 2e^{\pi \omega / 2} K_{\alpha}(2\alpha) , \quad (15) \]

where $K_{\alpha}(x)$ is the modified Bessel function of imaginary order. We can then expand the quantized field for the Rindler observer in terms of $f_R(k_\perp, \omega, x)$, which leads to

\[ \phi_R = \int_0^\infty d\omega \int d^2k_\perp \left[ a_R(k_\perp, \omega) f_R(k_\perp, \omega, x) + a_R^\dagger(k_\perp, \omega) f_R^*(k_\perp, \omega, x) \right] . \quad (16) \]

In the same way as for the Minkowski spacetime, we can define the vacuum for the observer in the right-wedge Rindler coordinates by solving $a_R(k_\perp, \omega)|R\rangle = 0$. It is also possible to perform an expansion of the field in the left-wedge and construct the left-wedge vacuum from $a_L(k_\perp, \omega)|L\rangle = 0$.

C. Bogolyubov transformation

We now establish the relations between $a(k_\perp, k_z)$ and $a_{R/L}(k_\perp, \omega)$, which is formulated conveniently in terms of the Bogolyubov transformations. Even though the transformation \[1\] is just a change of variables, the existence of a causal horizon at $t = \pm z$ for the accelerated observer introduces a non-trivial structure through the Bogolyubov transformation and this is at the heart of the Unruh effect.

FIG. 1. Schematic picture of the Rindler coordinates. In the right-wedge Rindler coordinates several curves with different values of $\rho$ are shown; A larger $\rho$ makes the constant-$\rho$ trajectory straightened corresponding to a less acceleration.
The Bogolyubov transformation connects the creation and annihilation operators in Minkowski and Rindler spacetimes through a sequence of transformations. Following Ref. [32] we first define an operator \( a_1(k_\perp, k^+) \) in the light-cone coordinates from \( \sqrt{k^+} a_1(k_\perp, k^+) = \sqrt{k_\perp} a_1(k_\perp, k_\perp) \) with \( k^\pm = (k_0 \pm k^z)/\sqrt{2} \). Then we find another operator by a variable change from \( k^+ \) to \( \omega \):

\[
a_2(k_\perp, \omega) = \int_0^\infty \frac{dk^+}{(2\pi k^+)^{1/2}} a_1(k_\perp, k^+) e^{i\omega \log[(k^+\sqrt{2})/\kappa]},
\]

where, as we have already defined before, \( \kappa = \sqrt{k_\perp^2 + m^2} \). For \( \omega > 0 \) these new operators \( a_2(k_\perp, \pm \omega) \) are related to the Rindler operators \( a_{R/L}(k_\perp, \omega) \) through a linear transformation, i.e.,

\[
\begin{pmatrix}
a_R(k_\perp, \omega) \\
a_L(k_\perp, \omega) \\
a_R(-k_\perp, \omega) \\
a_L(-k_\perp, \omega)
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & -\beta_\omega & \alpha_\omega \\
0 & \alpha_\omega & -\beta_\omega & 0 \\
0 & \beta_\omega & 0 & \alpha_\omega \\
0 & \alpha_\omega & \beta_\omega & 0
\end{pmatrix}
\begin{pmatrix}
a_2(k_\perp, \omega) \\
a_2(k_\perp, -\omega) \\
a_2(-k_\perp, \omega) \\
a_2(-k_\perp, -\omega)
\end{pmatrix}.
\]

The Bogolyubov coefficients are given as

\[
\alpha_\omega = \frac{1}{\sqrt{1 - e^{-2\pi \omega}}} \quad \beta_\omega = \frac{e^{-\pi \omega}}{\sqrt{1 - e^{-2\pi \omega}}}.
\]

The thermal interpretation of the accelerated vacuum follows immediately from these coefficients.

As a matter of fact, using the Bogolyubov transformation [18] and the operator definition [17], we can readily see that the vacuum expectation value of the number operator is finite

\[
\langle M| a_R^\dagger(k_\perp, \omega) a_R(k_\perp', \omega') |M \rangle = \beta_\omega \alpha_\omega \delta(2) \int_0^\infty \frac{dk^+}{2\pi k^+} e^{-i(\omega - \omega') \log[(k^+\sqrt{2})/\kappa]} = \beta_\omega^2 \alpha_\omega^2 \delta(2) \delta(\omega - \omega').
\]

One may be tempted to interpret the above as \( \langle M \langle \rangle \) being a thermal bath if seen from the operators quantized in the Rindler vacuum with its temperature deduced to be \( a_\omega/(2\pi) \); from Eq. [19] \( \beta_\omega^2 \) is nothing but a Bose-Einstein distribution function with a dimensionless temperature \( 1/(2\pi) \) and the mass dimension should be recovered from the identification of \( \alpha_\omega \) as an energy. Such an argument for the thermal interpretation could be found in some literature, but it is sometimes applied too easily. We will elucidate this point by means of explicit calculations in the next section.

III. IRRELEVANCE OF THE FIELD INTERPRETATION IN THE WIGHTMAN FUNCTIONS

In this section we shall illustrate how the following equalities for the two-point Wightman functions follow.

That is, for the Minkowski vacuum,

\[
\langle M| \phi(x) \phi(x') |M \rangle = \langle M| \phi_R(x) \phi_R(x') |M \rangle = \langle M| \phi_M(x) \phi_M(x') |M \rangle.
\]

and for the Rindler vacuum,

\[
\langle R| \phi(x) \phi(x') |R \rangle = \langle R| \phi_M(x) \phi_M(x') |R \rangle = \langle R| \phi_R(x) \phi_R(x') |R \rangle.
\]

Thanks to them, as is already used in the above expressions, we can drop \( M \) and \( R \) from \( \phi(x) \) regardless of the vacuum on which \( \phi(x) \) is quantized. These relations may be surprising at first, for we know \( \langle M| a_R^\dagger a_R |M \rangle = \langle M| a_R^\dagger a_R |M \rangle = \langle M| a_L^\dagger a_L |M \rangle = \langle M| a_L^\dagger a_L |M \rangle = 0 \) and \( \langle R| a_R^\dagger a_R |R \rangle = \langle R| a_R^\dagger a_R |R \rangle = 0 \).

The relation [21] has been addressed in the literature [33] where a proof of Eq. [21] was given based on the functional integral technique, while Refs. [21, 22] utilized the Schrödinger functional formalism with an explicit point-splitting regularization to prove Eq. [21]. We note that, in contrast, a general proof of [22] is not really established.

We must emphasize that the irrelevance of the difference between \( \phi_M(x) \) and \( \phi_R(x) \) is quite natural. The field \( \phi(x) \) is to be expanded in any complete set of basis functions and the creation and annihilation operators are specified accordingly. The Bogolyubov transformation actually guarantees the equivalence of \( \phi(x) \) in different descriptions. In this sense, therefore, we should be able to compute the expectation value of any operator \( \mathcal{O}[\phi] \) uniquely without referring to the quantization procedures. Thus, it is just a matter of convenience which we use, \( \phi_M(x) \) of Eq. [11] or \( \phi_R(x) \) of Eq. [16], to arrive finally at the same physical answer. Nevertheless, our claim is not trivial on the level of mathematical details. As we will discuss below, \( \phi_M(x) \) and \( \phi_R(x) \) are not identical (and they should not be because \( \phi_R(x) \) and \( \phi_L(x) \) decouple by definition).

We will articulate a step-by-step demonstration of Eqs. [21 and 22] based on the expansion in Sec. [11] It is useful to take this exercise in such an explicit way because it provides us with a deeper insight into the analogy to thermo-field dynamics for the two-point Wightman functions. We comment that our calculation will not rely on any specific regularization scheme.

A. Minkowski vacuum

We can readily evaluate the right side of Eq. [21] to find,

\[
\langle M| \phi_M(x) \phi_M(x') |M \rangle = \int_{-\infty}^{\infty} dk^z \int \frac{d^2k_\perp}{(2\pi)^2 k_0} e^{-ik_0(t-t')} e^{ik^z(z-z')} e^{ik_\perp(x-x')}.
\]
Using the expanded form (21) for the left-hand side of Eq. (21) we have,
\[ \langle M | \phi_R(x) \phi_R(x') | M \rangle \]
\[ = \int_0^\infty d\omega \int_0^\infty d\omega' \int d^2 k_\perp \int d^2 k'_\perp \left( f_{R} f'_{R} \langle M | a_R a_R^\dagger | M \rangle \\
+ f_{R} f'_{R} \langle M | a_R a_R^\dagger | M \rangle + f_{R} f'_{R} \langle M | a_R a_R^\dagger | M \rangle \right) , \] \[
(24) \]
where we introduced a compact notation with the prime for quantities with \( \omega' \) and \( k'_\perp \).

In order to prove the equivalence between Eqs. 23 and 24, we plug Eq. (20) and similar expressions for other combinations of the creation/annihilation operators into Eq. (24). We further replace \( \omega \) with \( -\omega \) to obtain,
\[ \langle M | \phi_R(x) \phi_R(x') | M \rangle \]
\[ = \frac{1}{2(2\pi)^4} \int \frac{dk^+}{2\pi k^+} \int d^2 k_\perp e^{ik_\perp \cdot (x-x')_\perp} \int d^0 \omega F(\omega, k^+, k_\perp) \]
\[ \times \left[ \int_0^\infty d\omega F(\omega, k^+, k_\perp) \int_0^\infty d\omega' G(\omega', k^+, k_\perp) + \int_{-\infty}^0 d\omega F(\omega, k^+, k_\perp) \int_0^\infty d\omega' G(\omega', k^+, k_\perp) \right] , \] \[
(25) \]
with the following functions that we define by
\[ F(\omega, k^+, k_\perp) = e^{-i\omega \eta} e^{-i\omega \log(k^+ \sqrt{2/\kappa})} K\left( \omega, \frac{\kappa \rho}{2}, \frac{\kappa \rho}{2} \right) , \]
\[ G(\omega', k^+, k_\perp) = e^{i\omega' \eta'} e^{-i\omega' \log(k^+ \sqrt{2/\kappa})} K\left( \omega', \frac{\kappa \rho}{2}, \frac{\kappa \rho}{2} \right) . \] \[
(26) \]
These are, of course, functions of \( t \) and \( z \) through \( \eta \) and \( \rho \), which is not indicated in the argument of \( F \) and \( G \) for notational clarity. Also, in deriving Eq. (24), we employed the analyticity property of the function \( K(\omega, \alpha, \beta) \). Now we can deform the quantity in the angle parentheses of Eq. (25) as
\[ \int_0^\infty d\omega F \int_0^\infty d\omega' G + \int_0^\infty d\omega F \int_{-\infty}^0 d\omega' G \]
\[ + \int_{-\infty}^0 d\omega F \int_{-\infty}^0 d\omega' G + \int_0^\infty d\omega F \int_0^\infty d\omega' G \] \[
(28) \]
without changing its value; two latter terms among four in the left-hand side of the above expression give no contribution under the \( k^+ \) integration. After the \( k^+ \) integration, in fact, we can easily show that the resulting contribution is proportional to \( \delta(\omega - \omega') \). It is obvious that such a singularity at \( \omega = \omega' \) in Dirac’s delta function cannot be picked up in the integrals like \( \int_0^\infty d\omega \int_{-\infty}^0 d\omega' \) and \( \int_{-\infty}^0 d\omega \int_0^\infty d\omega' \).

Eventually we can rewrite Eq. (22) into a form of Eq. (21) as confirmed from
\[ \langle M | \phi_R(x) \phi_R(x') | M \rangle \]
\[ = \frac{1}{2(2\pi)^4} \int \frac{dk^+}{2\pi k^+} \int d^2 k_\perp e^{ik_\perp \cdot (x-x')_\perp} \]
\[ \times \int_{-\infty}^\infty d\omega F(\omega, k^+, k_\perp) \int_{-\infty}^\infty d\omega' G(\omega', k^+, k_\perp) \]
\[ = \frac{1}{2k^+} \int d^2 k_\perp e^{ik_\perp \cdot (x-x')_\perp} e^{i(k^+ - k^-)(x^- - x'^-)} e^{i(k^- - k^+)(x^+ - x'^+)} . \] \[
(29) \]
In this concrete process of calculations we note that we used,
\[ e^{i(k^+ z^- + k^- z^+)} \]
\[ = \int_{-\infty}^\infty d\omega e^{-i\omega \eta} e^{i\omega \log(k^+ \sqrt{2/\kappa})} K\left( \omega, \frac{\kappa \rho}{2}, \frac{\kappa \rho}{2} \right) , \] \[
(30) \]
from the second line to the third line of Eq. (29).

### B. Rindler vacuum

Now we want to check Eq. (22). The right-hand side is readily evaluated as
\[ \langle R | \phi_R(x) \phi_R(x') | R \rangle \]
\[ = \frac{1}{4\pi^4} \int_0^\infty d\omega \int d^2 k_\perp e^{ik_\perp \cdot (x-x')_\perp} e^{-i\omega (\eta - \eta')} \]
\[ \times \sinh(\pi \omega) K_{\omega}(\kappa \rho) K_{\omega}(\kappa \rho') . \] \[
(31) \]
In the left-hand side, on the other hand, we have,
\[ \langle R | \phi_M(x) \phi_M(x') | R \rangle \]
\[ = \int_0^\infty d\omega \int_0^\infty d\omega' \int d^2 k_\perp \int d^2 k'_\perp \]
\[ \times \left( f f'(R | a_1 a_1^\dagger | R) + f' f(R | a_1 a_1^\dagger | R) \right) \]
\[ + f' f'(R | a_1 a_1^\dagger | R) + f^* f'(R | a_1 a_1^\dagger | R) \] \[
(32) \]
where we introduced a compact notation, \( f' \) and \( a_1' \) for quantities with \( x' \). Using the definition (17) and the Bogolyubov transformation (18) we have,
\[ \langle R | a_1(k_\perp, k^+) a_1(k'_\perp, k'^+) | R \rangle \]
\[ = -\delta(2)(k_\perp + k'_\perp) \frac{1}{k^+ k'^+} \]
\[ \times \int_0^\infty d\omega \alpha_\omega \beta_\omega e^{-i\omega \log(k^+ \sqrt{2/\kappa})} e^{i\omega \log(k'^+ \sqrt{2/\kappa})} , \] \[
(33) \]
and similar expressions for the expectation values involving other combinations of the creation and annihilation operators. Now we plug these terms into Eq. (32) and use Eq. (14) to define the \( K \) function in a form of the \( k^+ \) and \( k'^+ \) integrations. For example, we can show that,
\[ \int_0^\infty \frac{dk^+}{k^+} e^{ik^+ (x^- - x'^-)} e^{-i\omega \log(k^+ \sqrt{2/\kappa})} \]
\[ = K\left( \omega, \frac{\kappa \rho}{2}, \frac{\kappa \rho}{2} \right) . \] \[
(34) \]
Therefore, the integrations over \( k_\perp \) and \( \omega \) remain. Collecting all four terms, we finally find,

\[
\langle R|\phi_M(x)\phi_M(x')|R\rangle = \frac{1}{2(2\pi)^2} \int_0^\infty d\omega \int \frac{d^2k_\perp}{[2(2\pi)^4]^{1/2}} \left[ a_2(k_\perp,\omega)K\left(\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2}\right)e^{ik_\perp\cdot(x-x')_\perp}e^{-i\omega(x-x')_\parallel} \right. \\
\times \left( a^*_2(-k_\perp,-\omega)K^*(-\omega,-\frac{\kappa\rho}{2},\frac{\rho\beta}{2})e^{i(k_\perp\cdot(x-x')_\perp-i\omega\pi)} \right. \\
+ \int_0^\infty d\omega \int \frac{d^2k_\perp}{[2(2\pi)^4]^{1/2}} \left[ a_2(k_\perp,\omega)K\left(\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2}\right)e^{i(k_\perp\cdot(x-x')_\perp-i\omega\pi)} \right. \\
\times \left. \left( a^*_2(-k_\perp,-\omega)K^*(-\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2})e^{-i(k_\perp\cdot(x-x')_\perp+i\omega\pi)} \right) \right].
\]

Using \( \alpha_2^2 - 2\alpha_2\beta_\omega e^{-\omega} + \beta_\omega^2 e^{-2\omega} = 1 - e^{-2\omega} \), together with Eq. (15), we easily see that Eq. (35) is reduced to Eq. (31).

C. Generalization to arbitrary operators

In Ref. [33], the equality in the two-point Wightman functions evaluated for the Minkowski vacuum has been generalized to \( n \)-point functions, so that

\[
\langle M|\phi(x_1)\phi(x_2)\ldots\phi(x_n)|M\rangle = \langle M|\phi_R(x_1)\phi_R(x_2)\ldots\phi_R(x_n)|M\rangle = \langle M|\phi_M(x_1)\phi_M(x_2)\ldots\phi_M(x_n)|M\rangle,
\]

holds for general interacting theories. The proof relies on the fact that the Minkowski vacuum \( |M\rangle \) is a mixed state of \( |R\rangle \). This allows one to define the path integral for the right-wedge through a density matrix by tracing out the states in the left-wedge. The crucial point is that the Rindler Hamiltonian itself is a function of the coordinate \( \rho \) which exactly compensates the thermal-like effect brought in by the density matrix. These details will play an important role in later discussions.

Now we want to sketch how Eq. (22) can be generalized to \( n \)-point Wightman functions. First, we must understand that \( \phi_R \) is simply a restriction of \( \phi_M \) on the Rindler right-wedge. We can explicitly see this from Eq. (30) that transposes the \( (t,z) \) component of the plane wave into the Rindler basis. Then, \( \phi_M \) becomes

\[
\phi_M = \int_0^\infty d\omega \int \frac{d^2k_\perp}{[2(2\pi)^4]^{1/2}} \left[ a_2(k_\perp,\omega)K\left(\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2}\right) \\
+ a^*_2(-k_\perp,-\omega)K^*(-\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2})e^{i(k_\perp\cdot(x-x')_\perp-i\omega\pi)} \right. \\
+ \int_0^\infty d\omega \int \frac{d^2k_\perp}{[2(2\pi)^4]^{1/2}} \left[ a_2(k_\perp,\omega)K\left(\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2}\right)e^{i(k_\perp\cdot(x-x')_\perp-i\omega\pi)} \right. \\
\times \left. \left( a^*_2(-k_\perp,-\omega)K^*(-\omega,\frac{\kappa\rho}{2},\frac{\rho\beta}{2})e^{-i(k_\perp\cdot(x-x')_\perp+i\omega\pi)} \right) \right].
\]

By using the Bogolyubov transformations (18), the restriction to the right-wedge \( (\rho > 0) \) gives \( \phi_R \), while the restriction to the left-wedge \( (\bar{\rho} = -\rho > 0) \) gives \( \phi_L \). Overall, we can write

\[
\phi_M = \theta(\rho)\phi_R + \theta(-\rho)\phi_L.
\]

Since the creation and annihilation operators from the left-wedge and the right-wedge mutually commute, and since left-wedge operators do not act on \( |R\rangle \) by definition, it follows that

\[
\langle R|\phi_M(x_1)\phi_M(x_2)\ldots\phi_M(x_n)|R\rangle = \langle R|\phi_R(x_1)\phi_R(x_2)\ldots\phi_R(x_n)|R\rangle.
\]

For an odd value of \( n \) the correlation function is simply vanishing if \( Z_2 \) symmetry is preserved.

IV. THERMAL-LIKE CORRECTIONS

In this section we explicitly evaluate the two-point Wightman functions for \( m = 0 \) and discuss the correspondence to thermo-field dynamics (see Ref. [34] also). We will introduce several situations in Sec. V where totally different conclusions are, as shown in this section, a reflection of the relations between the Wightman functions.

A. Negative thermal-like corrections

For the calculation we use the point-splitting regularization. In the case \( m = 0 \) we have

\[
\langle M|\phi(x)\phi(x')|M\rangle = -\frac{1}{4\pi} \frac{1}{(t-t' - i\epsilon)^2 - |x_\perp - x'_\perp|^2 - |z - z'|^2}.
\]

In our scheme we subtract the UV divergence in Eq. (40) so that in the \( x' \to x \) limit we have \( \langle M|\phi^2|M\rangle_{\text{reg}} = 0 \) for the regularized expectation value. Unlike Eq. (40), the Wightman function \( \langle R|\phi(x)\phi(x')|R\rangle \) has both a UV divergent part and a finite part. The divergent part is intact and equal to Eq. (40) (see also Refs. [21, 22]). We can then obtain the finite part after a subtraction as

\[
\langle R|\phi^2|R\rangle_{\text{reg}} = \lim_{x' \to x} \left( \langle R|\phi(x)\phi(x')|R\rangle - \langle M|\phi(x)\phi(x')|M\rangle \right) = -\frac{1}{4\pi} \int_0^\infty d\omega e^{-\pi\omega} \int d^2k_\perp K^2_\perp(k_\perp\rho) \\
= -\frac{1}{2\pi^2\rho^2} \int_0^\infty d\omega e^{2\pi\omega} = -\frac{1}{48\pi^2\rho^2},
\]

for \( m = 0 \). Recall that the trajectory of constant proper acceleration \( \alpha \) is defined as \( \rho = 1/\alpha \). By defining the local Unruh temperature \( T_{\text{loc}} = \alpha/(2\pi) \) we can interpret this result as

\[
\langle R|\phi^2|R\rangle_{\text{reg}} = \frac{T_{\text{loc}}^2}{12}.
\]

Notice that this expression [42] has a thermal-like character reminiscent of finite-temperature calculations, but comes with an opposite sign. We will account for the origin of this opposite sign in what follows below (see Refs. [21, 22, 23, 35, 36] for related discussions).
Table I. Correspondence between different vacua in thermo-field dynamics and in Rindler spacetime.

| | \( |\beta\rangle \) | \( |M\rangle \) |
|---|---|---|
| \( |0\rangle \) | \( |R\rangle \) |
| \( |0\rangle \) | \( |L\rangle \) |

B. Analogue to thermo-field dynamics

The comparison with thermo-field dynamics will provide us with an intuitive understanding of the results \([41]\) and \([42]\). In thermo-field dynamics one deals with the thermal vacuum \( |\beta\rangle \) which is represented by the so-called non-tilde \( |0\rangle \) and tilda \( |0\rangle \) vacua \([37]\). Therefore, \( |\beta\rangle \) is excited relative to \( |0\rangle \).

For a Rindler observer \( |M\rangle \) is a mixed state of \( |R\rangle \). Then, \( |M\rangle \) in Rindler spacetime is analogous to \( |\beta\rangle \) in thermo-field dynamics, while \( |R\rangle \) (and \( |L\rangle \)) should correspond to \( |0\rangle \) and \( |0\rangle \) (or \( |0\rangle \)). We summarize these relations between vacua in Table I. We can also see a clear correspondence from expectation values of the number operator in Rindler spacetime, which is non-zero when applied to the Minkowski vacuum:

\[
\langle M|a_R^\dagger(k_\perp,\omega)a_R(k_\perp',\omega')\rangle_M = \frac{\delta(\omega - \omega')\delta^2(k_\perp - k_\perp')}{e^{\omega/T_\text{U}} - 1}, \tag{43}
\]

which is nothing but the Unruh effect itself \([2]\), where \( T_\text{U} = 1/(2\pi) \) is the dimensionless Unruh temperature.

A striking similarity in thermo-field dynamics is found, which takes the form of

\[
\langle \beta|a_0^\dagger(k_\perp,k^z)a_0(k_\perp',k^z')\rangle_\beta = \frac{\delta(k^2 - k'^2)\delta^2(k_\perp - k_\perp')}{e^{\sqrt{k^2 + k'^2 + m^2}} - 1}. \tag{44}
\]

Here \( \beta = 1/T \) with \( T \) being the temperature. These expressions leave an impression that the Unruh bath or the Minkowski vacuum \( |M\rangle \) definitely has a thermal character like \( |\beta\rangle \).

Finally, we can also appreciate the minus sign in Eqs. \([41]\) and \([42]\) as a consequence of the fact that \( |M\rangle \) is excited as compared to \( |R\rangle \), just like \( |\beta\rangle \) is excited as compared to \( |0\rangle \). From the correspondence as listed in Table I, we can retrieve the following well-known relation for a free massless scalar field:

\[
\langle \beta|\phi^2|\beta\rangle_{\text{reg}} = \lim_{x \to x'} (\langle \beta|\phi(x)\phi(x')|\beta\rangle - \langle 0|\phi(x)\phi(x')|0\rangle) = \frac{T^2}{12}, \tag{45}
\]

directly from Eq. \([41]\). \( \bowtie \)

V. THOUGHT EXPERIMENTS

It would be useful to put our understanding from the previous sections into a less abstract context by considering some thought experiments involving acceleration. In each situation we will find different behavior attributed to various choices of vacua and observers.

Let us make a clear distinction of a state to see and an observable to be seen. In classical physics it would be sufficient to define a collection of fields in some coordinate system. This represents the observable and the coordinate system is related to the observer. In quantum physics, we should quantize fields or operators, so that the observable is obtained only after specifying the physical state in addition to the coordinates. Then, a matrix element of the operator measured in the physical state is an observable.
A. Negative thermal-like effect

First we discuss a setting where the role-reversal of thermal-like corrections becomes apparent. We start by considering a more familiar case in thermo-field dynamics [see the upper one of Fig. 2 (a)]. Imagine there is an observer standing at zero temperature where the observer quantizes an operator $\mathcal{O}$. Then, this observer makes a measurement with the zero-temperature vacuum state to find $\langle 0 | \mathcal{O} | 0 \rangle$. Now, suppose that there is a box representing a piece of material, heated to some temperature $1/\beta$. This box is in a thermal state $|\beta\rangle$. If the zero-temperature observer looks at $|\beta\rangle$ using the same operator $\mathcal{O}$, the observer should find a thermal expectation value, $\langle \beta | \mathcal{O} | \beta \rangle$. The relation between these observables in thermo-field dynamics depends on $\mathcal{O}$, but for a very simple choice of $\mathcal{O} = \phi^2$ in a scalar field theory, we typically have

$$\langle \beta | \mathcal{O} | \beta \rangle > \langle 0 | \mathcal{O} | 0 \rangle,$$  \hspace{1cm} (46)

where the left-hand side receives a thermal correction $\sim \beta^{-2}$. For the rest of this section we implicitly mean that $\mathcal{O}$ is positive definite like $\phi^2$.

Now imagine a similar experiment, where the vacuum state in the box is not a heated but an accelerated one as shown in the lower one of Fig. 2 (a). Some confusion might have arisen from a wrong analogy between $|\beta\rangle$ and $|R\rangle$. In the case with acceleration, however, the box is in the state $|R\rangle$ that corresponds to $|0\rangle$ in thermo-field dynamics, and the result should be rather opposite:

$$\langle R | \mathcal{O}_M | R \rangle < \langle M | \mathcal{O}_M | M \rangle.$$  \hspace{1cm} (47)

We note that we recover the subscripts $M$ and $R$ for the moment just to indicate where the observer quantizes the operator. The above is nothing but a generalization of Eq. (42).

B. Positive thermal-like effect

Imagine now an opposite situation; let us suppose that we accelerate the observer [see Fig. 2 (b)]. The observer then uses an operator $\mathcal{O}_R$ and make a measurement of the observable in the accelerated vacuum, which leads to $\langle R | \mathcal{O}_R | R \rangle$. If the observer performs another measurement of the same observable with $|M\rangle$, we have a difference as follows:

$$\langle M | \mathcal{O}_R | M \rangle > \langle R | \mathcal{O}_R | R \rangle.$$  \hspace{1cm} (48)

This is in fact equivalent to Eq. (47) on the mathematical level, but the physical environments look quite different as we see in the previous and the present subsections. We would emphasize that such opposite behavior between thermo-field dynamics in Eq. (46) and the Rindler spacetime in Eq. (48) results from the fact that the accelerated vacuum is less excited as compared to the non-accelerated (Minkowski) vacuum. This point is crucial in our discussion on the condensates in the next section.

C. Zero thermal-like effect

We shall imagine a kind of hybrid case. Suppose a Minkowski observer who measures $\langle M | \mathcal{O}_M | M \rangle$. Now at some point, this observer starts to accelerate adiabatically until a certain value. Then, the operator transforms from $\mathcal{O}_M$ to $\mathcal{O}_R$ and its expectation value seen with the non-accelerated vacuum is $\langle M | \mathcal{O}_R | M \rangle$ [see Fig. 2 (c)]. According to the calculations as discussed in this work, such a measurement would yield the identical result (modulo trivial coordinate transformation when $\mathcal{O}$ is not a scalar):

$$\langle M | \mathcal{O}_M | M \rangle = \langle M | \mathcal{O}_R | M \rangle.$$  \hspace{1cm} (49)

One might have an impression that this is almost trivial because there is no distinction between $\mathcal{O}_M$ and $\mathcal{O}_R$. We must stress, however, that this is quite surprising if we recall the usual interpretation based on Eq. (43) saying that the accelerated observer perceives $|M\rangle$ as a thermal bath. We believe that Eq. (49) might be related to the cancellation mechanism in Ref. [38].

VI. SPONTANEOUS SYMMETRY BREAKING

We are considering a real scalar field theory with $Z_2$ symmetry, which is assumed to be spontaneously broken in the Minkowski vacuum through the potential of the following form:

$$V(\phi) = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4.$$  \hspace{1cm} (50)

On the tree level, the state with a minimal energy favors a finite condensate given by

$$\langle M | \phi | M \rangle = \sqrt{\frac{\mu^2}{\lambda}}.$$  \hspace{1cm} (51)

We should note a trivial corollary of Eq. (21), which states that the Rindler observer cannot detect any changes in the condensate as long as the condensate is measured with $|M\rangle$. This situation is summarized in Sec. VA with Fig. 2 (c). The crucial relation, $\langle M | \phi_M | M \rangle = \langle M | \phi_R | M \rangle$ was recognized some time ago by Unruh and Weiss [31] and by Hill [21, 22].

Actually, the question we are addressing in this section is different from the one made by Unruh and Weiss and by Hill. As discussed in Sec. VA with Fig. 2 (a), let us consider that a box of some material is initially prepared in a state having a homogeneous condensate [51]. Then this box is accelerated, so that the condensate is changed due to acceleration with respect to the value measured in the Minkowski vacuum. In fact, the condensate can change with acceleration in this case, as was already described in Eq. (42).
A. One-loop effective action

We are calculating a condensate defined as
\[ \bar{\phi} \equiv \langle R | \phi | R \rangle . \] (52)

In the mean-field approximation, \( \bar{\phi} \) should extremize the effective action. The one-loop calculation leads to the following equation of motion from the effective action:
\[ \left( \Box - 3\lambda\langle \phi^2 \rangle \right) \bar{\phi} - V'(\bar{\phi}) = 0 . \] (53)

This is a non-linear equation for the background field \( \bar{\phi} \), which depends non-trivially on the quantum fluctuations encoded in \( \langle \phi^2 \rangle = \langle R | \phi^2 | R \rangle - \langle M | \phi^2 | M \rangle \). Our goal is to find \( \bar{\phi} \) as a function of \( \rho \), where \( \rho \) is the Rindler coordinate as introduced before. We should note that it is indispensable to keep the derivatives to find \( \bar{\phi} \) in the Rindler coordinates.

The quantum fluctuation \( \langle \phi^2 \rangle \) can be adopted from Sec. [11] To simplify the calculation we approximate the quantum fluctuation with its massless counterpart, which is given in Eq. (41). Assuming a non-trivial \( \rho \) dependence in the condensate, the problem boils down to solving the following equation:
\[ \frac{d^2 \bar{\phi}}{d\rho^2} + \frac{1}{\rho} \frac{d\bar{\phi}}{d\rho} - \nu^2 \frac{\bar{\phi}}{\rho^2} = V'(\bar{\phi}) , \quad \nu^2 = -\frac{\lambda}{16\pi^2} . \] (54)

B. Particle interpretation

Equation (54) allows for a particle-like interpretation; \( \bar{\phi} \) is to be interpreted as “position” of the “particle” and \( \rho \) as “time”. Then, this identification enables us to rewrite Eq. (54) in the “energy” form:
\[ \frac{d}{d\rho} \left[ \frac{1}{2} \left( \frac{d\bar{\phi}}{d\rho} \right)^2 - V(\bar{\phi}) \right] = -\frac{1}{\rho} \left( \frac{d\bar{\phi}}{d\rho} \right)^2 + \nu^2 \frac{d\bar{\phi}}{d\rho} \frac{\bar{\phi}}{\rho^2} , \] (55)

which gives us an interpretation that the particle is moving in a potential \(-V(\bar{\phi})\). It is crucial to point out that both terms in the right-hand side are negative, leading to an energy loss as a function of time.

This type of analysis is typical for the calculation of false vacuum decay [59] when a potential energy has several inequivalent minima. One is then interested in finding an “instanton” solution that represents a trajectory from one to the other extrema of the potential \(-V(\bar{\phi})\). The solution of our current interest should satisfy a boundary condition:
\[ \bar{\phi}(\rho \rightarrow \infty) = \sqrt{\frac{\mu^2}{\lambda}} , \] (56)

so that the condensate is reduced to its vacuum value when the proper acceleration \( \alpha = 1/\rho \) vanishes. In the opposite limit, one might have been tempted to impose \( \bar{\phi}(\rho \rightarrow 0) = 0 \), leading to a picture of symmetry restoration induced by high acceleration. It is obvious, however, that such a boundary condition is incompatible, which is understood from Eq. (55) that cannot increase the energy of the particle. (The particle should climb up the potential consuming the initial kinetic energy.) One may then think that a boundary condition such as
\[ \left( \frac{d\bar{\phi}}{d\rho} \right)_{\rho=0} > 0 \] (57)
could work to lead to a consistent solution. We can easily check by linearizing Eq. (54) around \( \bar{\phi} = 0 \) to find that the resulting trajectories are
\[ \bar{\phi}(\rho) = C_1 \rho J_{\nu+\nu}(\rho \mu) + C_2 \rho N_{\nu+\nu}(\rho \mu) , \] (58)

which have zero gradient at \( \rho = 0 \). Therefore, using the boundary condition (57) would end up with an inconsistency.

C. Acceleration catalysis

For this problem, it would be a more reasonable choice to specify one more boundary conditions at \( \rho \rightarrow \infty \). Besides Eq. (56) we also require,
\[ \left( \frac{d\bar{\phi}}{d\rho} \right)_{\rho \rightarrow \infty} = 0 , \] (59)
so that in the limit of zero acceleration the solution smoothly approaches the value of the condensate in the Minkowski vacuum.

With the conditions (56) and (59) we can solve the equation of motion (55) numerically and we show our numerical results in Fig. 3 for \( \lambda = 0.1 \) and \( \lambda = 0.01 \). We find that the condensate grows as \( \rho \) decreases, or as the proper acceleration \( \alpha = 1/\rho \) increases. Within the numerical accuracy as \( \rho \rightarrow 0 \), the condensate exhibits diverging behavior. This increasing behavior of the condensate with
acceleration reminds us of the enhancement of the chiral condensate induced by the external magnetic field, which is sometimes referred to as the magnetic catalysis. So, it should be appropriate to name the acceleration-induced enhancement of the condensate acceleration catalysis.

Let us comment that not only $\bar{\phi}$ but also $d\bar{\phi}/d\rho$ diverge at $\rho = 0$, and this is how the particle can climb up the potential. That is, the initial kinetic energy is large enough for the particle to approach Eq. (56) while constantly losing energy in our particle interpretation.

D. Scaling solution

It is straightforward to see that Eq. (54) with $\mu = 0$ accommodates a “scaling” type of the solution:

$$\bar{\phi}(\rho) = \sqrt{\frac{1 + \nu^2}{\lambda}} \frac{1}{\rho},$$  \hspace{1cm} (60)

as recognized first by Stephens [36]. In fact, the scaling solution is qualitatively similar to the acceleration catalysis solution found numerically in Sec. VI C. The condensate blows up as the acceleration increases.

There is a useful consistency check related to Eq. (60). Since it is non-trivial even at the classical level, i.e., even when $\nu^2 = 0$, the correct quantum fluctuations $\langle \bar{\phi}^2 \rangle$ should be constructed on top of Eq. (60) and thereby $\bar{\phi}$ should be affected at the one-loop level. As discussed in Ref. [39], however, the quantum fluctuations around the scaling solution are limited from above by Eq. (12).

Indeed, we can easily convince ourselves that the scaling solution introduces a mass term in the dispersion relation in the Rindler coordinates. Therefore, we conclude that acceleration catalysis is robust to persist.

VII. DISCUSSION AND CONCLUSION

Strong laser intensities would introduce a strong acceleration effect so that the Unruh effect and the Unruh radiation could be probed. In addition, it has been conjectured that the Unruh effect might be relevant for heavy-ion collisions. There is also a chance to get a new insight into black-hole physics from these experiments.

In our setup the strong acceleration effect was represented by Rindler spacetime. As a first step we have studied the effect of acceleration on the two-point Wightman functions for a real scalar field, and found that the Wightman functions depend only on the vacuum but not on the operator quantization. This gives a natural explanation for the observation that the role of the temperature corrections in Rindler spacetime is reversed from analogous thermo-field dynamics. We showed that such relations of the Wightman functions have deep consequences when more familiar results from thermo-field dynamics are referred to as a comparison. We have classified several thought experiments where this leads to interesting and somewhat unexpected results.

Then, we have concentrated on a scalar field model with a spontaneously broken $Z_2$ symmetry. We studied the behavior of the scalar condensate as a function of acceleration. Based on our analysis we can conclude that the condensate will not change as long as the (Minkowski or Rindler) observer takes an expectation value with the non-accelerated (Minkowski) vacuum; the condensate remains intact and takes the vacuum value. This is quite non-trivial in view of the fact that the Rindler observer perceives thermal effects in the Minkowski vacuum according to the conventional interpretation. Even though the creation and annihilation operators and their combinations such as the number operator transform very non-trivially under the change in the quantization condition, any operator in terms of fields would not change as long as measured in the Minkowski vacuum.

In contrast to this situation, if the co-accelerated observer measures a condensate in the Rindler vacuum, this observer should see that the condensate changes depending on the acceleration. What we found was that the scalar condensate increases as a function of increasing acceleration. We have named this behavior of the condensate acceleration catalysis. In the limit of large acceleration the condensate appears to grow without upper bound, i.e., it appears to diverge.

It is well-known that Rindler spacetime is an approximation to Schwarzschild spacetime in the near-horizon region. Then, an observer at a fixed distance from the horizon would find a positive thermal-like effect according to the discussion in Sec. V B. Provided that the observer measures the condensate in the Minkowski vacuum (corresponding to a freely falling frame). Under such conditions it seems conceivable that a condensate would melt as we approach the black-hole horizon [40, 41]. If we observe physics in the Rindler vacuum, on the other hand, the acceleration catalysis would play a role near the horizon. In any case, when it comes to acceleration as opposed to well-established thermal physics, the richness of theoretically possible physical outcomes deserves further attention.

ACKNOWLEDGMENTS

This work was supported by JSPS KAKENHI Grant Number 24740169 (K.F.) and by NEWFELPRO Grant Number 48 (S.B.).
[1] S. W. Hawking, Particle Creation by Black Holes, Commun. Math. Phys. 43 (1975) 199 [Erratum-ibid. 46 (1976) 206].
[2] W. G. Unruh, Notes on black hole evaporation, Phys. Rev. D 14 (1976) 790.
[3] L. J. Garay, J. R. Anglin, J. I. Cirac and P. Zoller, Sonic black holes in dilute Bose-Einstein condensates, Phys. Rev. A 63 (2001) 023611 [gr-qc/0005131].
[4] J. Steinhauser, Observation of self-amplifying Hawking radiation in an analog black hole laser, Nature Phys. 10 (2014) 864 [arXiv:1409.6550 [cond-mat.quant-gas]].
[5] L. Labun and J. Rafelski, Strong Field Physics: Probing Critical Acceleration and Inertia with Laser Pulses and Quark-Gluon Plasma, Acta Phys. Polon. B 41 (2010) 2763 [arXiv:1010.1970 [hep-ph]].
[6] W. Greiner, B. Muller and J. Rafelski, Quantum Electrodynamics Of Strong Fields, Berlin, Germany: Springer (1985) 594 P. (Texts and Monographs in Physics)
[7] T. Tajima and G. Mourou, Zettawatt-exawatt lasers and their applications in ultrastrong-field physics, Rev. ST Accel. Beams 5 (2002) 033101.
[8] S. A. Fulling, Nonuniqueness of canonical field quantization in Riemannian space-time, Phys. Rev. D 7 (1973) 2850.
[9] W. G. Unruh and R. M. Wald, What happens when an accelerating observer detects a Rindler particle, Phys. Rev. D 29 (1984) 1047.
[10] S. Takagi, Vacuum noise and stress induced by uniform accelerator: Hawking-Unruh effect in Rindler manifold of arbitrary dimensions, Prog. Theor. Phys. Suppl. 88 (1986) 1.
[11] L. C. B. Crispino, A. Higuchi and G. E. A. Matsas, The Unruh effect and its applications, Rev. Mod. Phys. 80 (2008) 787 [arXiv:0710.5373 [gr-qc]].
[12] P. Chen and T. Tajima, Testing Unruh radiation with ultraintense lasers, Phys. Rev. Lett. 83 (1999) 256.
[13] R. Schutzhold, G. Schaller and D. Hals, Signatures of the Unruh effect from electrons accelerated by ultra-strong laser fields, Phys. Rev. Lett. 97 (2006) 121302 [Erratum-ibid. 97 (2006) 139902] [quant-ph/0604065].
[14] R. Schutzhold and C. Maia, Quantum radiation by electrons in lasers and the Unruh effect, Eur. Phys. J. D 55 (2009) 375 [arXiv:1004.2399 [hep-th]].
[15] S. Iso, Y. Yamamoto and S. Zhang, Stochastic Analysis of an Accelerated Charged Particle - Transverse Fluctuations-, Phys. Rev. D 84 (2011) 025005 [arXiv:1011.4191 [hep-th]].
[16] P. Castorina, D. Kharzeev and H. Satz, Thermal Hadronization and Hawking-Unruh Radiation in QCD, Eur. Phys. J. C 52 (2007) 187 [arXiv:0704.1426 [hep-ph]].
[17] T. S. Biro, M. Gyulassy and Z. Schram, Unruh gamma radiation at RHIC?, Phys. Lett. B 708 (2012) 276 [arXiv:1111.4871 [hep-ph]].
[18] K. Dusling, T. Epelbaum, F. Gelis and R. Venugopalan, Instability induced pressure isotropization in a longitudinally expanding system, Phys. Rev. D 86 (2012) 085040 [arXiv:1206.3336 [hep-ph]].
[19] T. Inagaki, T. Muta and S. D. Odintsov, Dynamical symmetry breaking in curved space-time: Four fermion interactions, Prog. Theor. Phys. Suppl. 127 (1997) 93 [hep-th/9711084].
[20] A. Flachi and K. Fukushima, Chiral Mass-Gap in Curved Space, Phys. Rev. Lett. 113 (2014) 9, 091102 [arXiv:1406.6548 [hep-th]]; A. Flachi, K. Fukushima and V. Vitagliano, Geometrically induced magnetic catalysis and critical dimensions, arXiv:1502.0690 [hep-th].
[21] C. T. Hill, Can the Hawking Effect Thaw a Broken Symmetry?, Phys. Lett. B 155 (1985) 343.
[22] C. T. Hill, One Loop Operator Matrix Elements in the Unruh Vacuum, Nucl. Phys. B 277 (1986) 547.
[23] T. Ohkatsu, Dynamical chiral symmetry breaking and its restoration for an accelerated observer, Phys. Lett. B 599 (2004) 102 [hep-th/0407067].
[24] D. Kharzeev and K. Tuchin, From color glass condensate to quark gluon plasma through the event horizon, Nucl. Phys. A 753 (2005) 316 [hep-ph/0501234].
[25] D. Ebert and V. C. Zhukovsky, Restoration of Dynamically Broken Chiral and Color Symmetries for an Accelerated Observer, Phys. Lett. B 645 (2007) 287 [hep-th/0612009].
[26] F. Lenz, K. Ohta and K. Yazaki, Static interactions and stability of matter in Rindler space, Phys. Rev. D 83 (2011) 064037 [arXiv:1012.3283 [hep-th]].
[27] P. Castorina and M. Finocchiaro, Symmetry Restoration By Acceleration, J. Mod. Phys. 3 (2012) 1703 [arXiv:1207.3677 [hep-th]].
[28] S. Takeuchi, Bose-Einstein condensation in the Rindler space-time, arXiv:1501.07471 [hep-th].
[29] K. G. Klimenko, Three-dimensional Gross-Neveu model at nonzero temperature and in an external magnetic field, Theor. Math. Phys. 90 (1992) 1 [Teor. Mat. Fiz. 90 (1992) 3]; Three-dimensional Gross-Neveu model at nonzero temperature and in an external magnetic field, Z. Phys. C 54 (1992) 323; V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Dimensional reduction and catalysis of dynamical symmetry breaking by a magnetic field, Nucl. Phys. B 462 (1996) 249 [hep-ph/9509320].
[30] L. Parker and D. Toms, Quantum Field Theory in Curved Space-time: Quantized Fields and Gravity, Cambridge University Press (2009).
[31] G. V. Dunne, Heisenberg-Euler effective Lagrangians: Basics and extensions, In "Shifman, M. (ed.) et al.: From fields to strings, vol. 1" 445-522 [hep-th/0406216].
[32] G. ’t Hooft, The Scattering matrix approach for the quantum black hole: An Overview, Int. J. Mod. Phys. A 11 (1996) 4623 [gr-qc/9607022].
[33] W. G. Unruh and N. Weiss, Acceleration Radiation in Interacting Field Theories, Phys. Rev. D 29 (1984) 1656.
[34] R. Laflamme, Entropy of a Rindler Wedge, Phys. Lett. B 196 (1987) 449.
[35] P. Candelas and D. J. Raine, Entropy of a Rindler Wedge, Phys. Lett. B 196 (1987) 449.
[36] C. R. Stephens, Symmetry Breaking In Inhomogeneous Space-times: A Tractable Example, Phys. Rev. D 33 (1986) 2813.
[37] H. Umezawa, H. Matsumoto and M. Tachiki, Thermo Field Dynamics And Condensed States, Amsterdam, Netherlands: North-Holland (1982) 591p.
[38] S. Iso, K. Yamamoto and S. Zhang, On the Cancellation Mechanism of Radiation from the Unruh detector, PTEP 2013 (2013) 6, 063B01 [arXiv:1301.7543 [hep-th]].
[39] S. Coleman, *Aspects of Symmetry*, ed. S. Coleman, Cambridge University Press, Cambridge, England, 1988

[40] S. W. Hawking, *Interacting Quantum Fields Around a Black Hole*, Commun. Math. Phys. 80 (1981) 421.

[41] A. Flachi and T. Tanaka, *Chiral Phase Transitions around Black Holes*, Phys. Rev. D 84 (2011) 061503 [arXiv:1106.3991 [hep-th]].