Incorrect problem analyzing in the Finite Element Method

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Abstract. Nowadays, numerical methods are widely used in the field analysis of many electromagnetic devices, MEMS and more. Incorrect problem analysis using the Finite Element Method often leads to incorrect results and drawing wrong conclusions. Therefore, the role of the engineer who describes the model in the pre-processing stage cannot be forgotten. This paper presents an example of developing a computer model of a flat capacitor in FEMM program, using the 2-dimensional Finite Element Method. It has been shown, how simple can disturb the result after wrong input data implementing.

1. Introduction
The considerations presented below may concern many areas where electric field modelling is used. It is a very wide area of application. These can include:

- large high voltage equipment [1-3],
- power cables [4, 5] or windings in electrical machines [6],
- a large group of different types of sensors, e.g. a capacitive sensor for measuring the water content in the soil [7] or a capacitive sensor for measuring the oil-air flow [8],
- small MEMS devices [9, 10],
- modern dielectric materials using nanotechnology [11],
- industrial processes, e.g. electrospinning, i.e. a fibre production method using high potential differences [12] or Electrical Capacitance Tomography (ECT) used in many industrial processes for visualizations and determination of flow parameters [13],
- modern, thin, electrically conductive layers with low resistance used in wearable electronic devices [14],
- and during the design of many other devices and technological processes.

At the same time, it should be emphasized that the subject matter is of particular importance in the teaching of electric field theory to future engineers, because Finite Element Methods (FEM) are increasingly used to facilitate research, design and solving complex engineering problems [15, 16].

2. Numerical methods stages
Due to numerical methods, which give opportunity of preparing computer models, designing process of different kind of engineering devices can be improved end can be providing faster than in reality design. The advantages of using numerical modelling are: giving fast results, saving time, saving money. But the role of engineer / designer cannot be ignored. Her or his role takes place in the first stage in numerical methods - in pre-processor. We can distinguish three stages in preparing computer models in FEM: pre-processing, solving, post-processing.
2.1. Pre-processing
In this stage the engineer inputs the input data and design the device. In case of Finite Element Method the most important step in pre-processing is to generate mesh of finite elements and choose correct boundary conditions. Those parameters influence on results.

2.2. Solving
For solving specific equations is responsible the computer / program / system. The equations are solved in each node of finite element mesh. The time of solving depends on the number of mesh nodes.

2.3. Post-processing
The post-processing is the last stage in the numerical methods. Here cannot change any parameters. It is space to viewing results. The solver generates the file with a kind of results data. In case of error, the engineer must turn back to the pre-processor to improve the model.

3. Flat capacitor model - analytical errors
In this paper the authors present the tricky problem of input data implementation in the pre-processing stage. Below it is shown incorrect analysis of comparison two equations: analytical and numerical.

The first equation is for analytical counting the capacity of a flat capacitor:

\[ C_a = \varepsilon_0 \frac{S}{d} \]  \hfill (1)

The second equation is for numerical counting the capacity of a flat capacitor:

\[ C_n = \frac{2E}{U^2} \]  \hfill (2)

converted from the energy, loaded from the FEMM program:

\[ E = \frac{1}{2} C_n U^2 \]  \hfill (3)

The figure below presents initial model of a flat capacitor made of silicon with air surrounding. The surrounding was designed typically as a rectangle. The edges of the rectangle are fixed parallel to the capacitor edges (Fig.1).

![Figure 1](image1.png)

**Figure 1.** Model of a flat capacitor – 2-D analysis.

At the Figure 2 can observe automatically generated mesh of finite elements.
Next figure (Figure 3) presents the flux lines that run away from the surrounding. This way of propagation of the lines should give a thought to the engineer.

The energy was counted from the surface integral in FEMM program. The value of capacity was:
• according to the analytical method: \( C_a = 8.854 \times 10^{-15} \) F,
• according to the numerical method: \( C_n = 9.869 \times 10^{-15} \) F.

The difference / the error was 10.29%.

In search of an error, the authors thicken the mesh in places important from the point of view of numerical analysis. At the Figure 5 is presented improved mesh, generated by the engineer. In this approach the value of capacity was:
• according to the analytical method: \( C_a = 8.854 \times 10^{-15} \) F,
• according to the numerical method: \( C_n = 9.860 \times 10^{-15} \) F.

The difference / the error was 10.20%, which decreased slightly.

In the next steps, the authors changed boundary conditions. Thanks to this procedure the flux lines closed in the surrounding and can observed the potentials of the capacitor plates. The higher plate was set on 10 Volts and the lower plate was set on 0 Volts.

Despite the improvement of conditions, the error was still big. For the case in Figure 6, it was as high as 18.02%. In the next steps, the engineers increased the number of mesh elements.
First three times (Figure 7), and then even forty times (Figure 8). This way the error decreased to 13.41% and was still not satisfactory.
The above approach to the problem shows that the engineers misdiagnosed the problem. They did not demonstrate their knowledge of electrostatics. As a result, they made an elementary mistake. They misused the analytical equation and referred to the wrong surface (see Figure 4). In this case the energy should be calculated for the surfaces of the capacitor plates, what presents the Figure 9.

Figure 7. The model with five thousand of finite elements.

Figure 8. The model with two hundred thousand of finite elements.

Figure 9. The model with closed space between a capacitor plates.
After the correct diagnosis of the electrostatic problem and the changed of the model, the error was 0.079% and was close to satisfactory.

The authors of the article want to additionally present that the model could be simplified even more. Thanks to this optimization, the calculation time and error were reduced. This time it was 0.0000883% and it was close to zero.

Figure 10. The model of the space between a capacitor plates.

In the figure above (Figure 10), only the area between the capacitor plates is visible. The voltage and boundary conditions were applied to the edges of this rectangle. Upper edge was set on 10 Volts, bottom edge was set on 0 volts, and on the sides normal condition.

4. Conclusions

The authors present easy way to disturbing, spoiling or even manipulate with the result. The computer is like a sheet of paper – can absorb everything. In the future this analysis will be developing about comparison of wrong boundary conditions choosing. It will be enriched with literature items in which such errors have occurred.

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