Abstract

We theoretically study the tunnelling current of a single electron transistor (SET) under optical pumping. It found that holes in the quantum dot (QD) created by optical pumping lead to new channels for the electrons tunnelling from emitter to collector. As a consequence, an electron can tunnel through the QD via additional channels, characterized by the exciton, trion and biexciton states. The binding energy of exciton complexes can be determined by the Coulomb oscillatory tunnelling current.

PACS numbers: 73.63.Kv, 73.23.Hk and 78.67.Hc
Recently, the spontaneous emission spectrum of a single quantum dot (QD) has been suggested as a single-photon source, which is important in the application of quantum cryptography\textsuperscript{1−4}. Experimentally, the spontaneous emission spectrum typically exhibits coexisting sharp emission peaks, which have been attributed to the electron-hole recombination in the exciton, trion, and biexciton formed in the QD\textsuperscript{5}. Nevertheless, it is difficult to experimentally determine the binding energy of exciton complexes due to lack of free electron-hole recombination in the QD. In this letter we propose that the tunnelling current of single electron transistor (SET) under optical pumping can be employed to determine the binding energy of exciton. Our calculation is based on the Keldysh Green’s function approach within the Anderson model for a two-level system. We find that the optical excitation creates holes in the QD, which provide new channels (via the electron-hole interaction) for the electron to tunnel from the emitter to the collector. Consequently, an electron can tunnel through the QD via four additional channels, characterized by the exciton, positive trion, negative trion, and biexciton states. Each addition channel can generate a new oscillatory peak in the tunnelling current characteristics in addition to the typical peaks caused by the electron-electron Coulomb interactions. The binding energy of exciton complexes as well as electron charging energy can be determined by using the tunnelling current as functions of gate voltage.

The system under the current study considers a single quantum dot (QD) sandwiched between two leads. Electrons are allowed to tunnel from the left lead (emitter) to the right lead (collector) under the influence of an optical pump. We start with the following Hamiltonian

\[ H = H_d + H_l + H_{d,l} + H_{d,e} + H_I, \]  

where the first term describes electrons in the InAs/GaAs QD. We assume that the quantum confinement effect is strong for the small QD considered here. Therefore, the energy spacings between the ground state and the first excited state for electrons and holes, $\Delta E_e$ and $\Delta E_h$, are much larger than thermal energy, $k_B T$, where $k_B$ and $T$ denote the Boltzmann constant and temperature. Only the ground state levels for electrons and holes, $E_e$ and $E_h$, are considered in $H_d$. The second term describes the kinetic energies of free electrons in the electrodes, where the correlation effects among electrons is ignored. Note that in the current setup, the gate electrode does not provide any electrons,
but merely controls the energy levels of the QD. The third term describes the coupling between the QD and the leads. The fourth term describes the interband optical pumping with a frequency of $\omega_0$, which is in resonance with the energy difference between an electron level in the wetting layer and the hole ground state level. Due to the large strain-induced splitting between the heavy-hole and light-hole band for typical QDs, we only have to consider the heavy hole band (with $J_z = \pm 3/2$) and ignore its coupling with light-hole band caused by the QD potential. Because the effect of inter-particle Coulomb interactions is significant in small semiconductor QDs, we take into account the electron Coulomb interactions and electron-hole Coulomb interactions in the last term.

Once the Hamiltonian is constructed, the tunnelling current of SET can be calculated via the Keldysh Green’s function method. We obtain the tunnelling current through a single dot

$$J = -\frac{2e}{h} \int \frac{d\epsilon}{2\pi} [f_L(\epsilon - \mu_L) - f_R(\epsilon - \mu_R)] \frac{\Gamma_L(\epsilon)\Gamma_R(\epsilon)}{\Gamma_L(\epsilon) + \Gamma_R(\epsilon)} \text{Im} G^r_{e,\sigma}(\epsilon).$$

(2)

Eq. (2) is still valid for the SET under optical pumping provided that the condition $\Gamma_L(\epsilon) > R_{eh}$ is satisfied, where $R_{eh}$ is the electron-hole recombination rate. $f_L(\epsilon)$ and $f_R(\epsilon)$ are the Fermi distribution function for the source and drain electrodes, respectively. The chemical potential difference between these two electrodes is related to the applied bias via $\mu_L - \mu_R = eV_a$. $\Gamma_L(\epsilon)$ and $\Gamma_R(\epsilon)$ denote the tunnelling rates from the QD to the left (source) and right (drain) electrodes, respectively. For simplicity, these tunnelling rates will be assumed energy and bias-independent. Therefore, the calculation of tunnelling current is entirely determined by the spectral function $A = \text{Im} G^r_{e,\sigma}(\epsilon)$, which is the imaginary part of the retarded Green’s function $G^r_{e,\sigma}(\epsilon)$.

The expression of retarded Green’s function, $G^r_{e,\sigma}(\epsilon)$, can be obtained by the equation of motion of $G^r_{e,\sigma}(t) = -i\theta(t)\langle\{d_{e,\sigma}(t), a^\dagger_{e,\sigma}(0)\}\rangle$, where $\theta(t)$ is a step function, the curly brackets denote the anti-commutator, and the bracket $\langle\ldots\rangle$ represents the thermal average. After some algebras, the retarded Green’s function of Eq. (2)

$$G^r_{e,\sigma}(\epsilon) = (1 - N_{e,-\sigma})\left\{\frac{1}{\epsilon - E_e + i\Gamma_e} \left[ \frac{1}{\epsilon - E_e + U_{eh} + \frac{i\Gamma_e}{2}} - \frac{1}{\epsilon - E_e - U_{eh} + \frac{i\Gamma_e}{2}} \right] \right\} + \frac{n_{h,\sigma} + n_{h,-\sigma} - 2n_{h,\sigma}n_{h,-\sigma}}{\epsilon - E_e + U_{eh} + \frac{i\Gamma_e}{2}} + \frac{n_{h,\sigma}n_{h,-\sigma}}{\epsilon - E_e + 2U_{eh} + \frac{i\Gamma_e}{2}} + N_{e,-\sigma}\left\{\frac{1}{\epsilon - E_e - U_e + \frac{i\Gamma_e}{2}} \right\}$$

(3)
\[ + \frac{n_{h,\sigma} + n_{h,-\sigma} - 2n_{h,\sigma}n_{h,-\sigma}}{\epsilon - E_e - U_e + U_{eh} + i\frac{\Gamma_e}{2}} + \frac{n_{h,\sigma}n_{h,-\sigma}}{\epsilon - E_e - U_e + 2U_{eh} + i\frac{\Gamma_e}{2}} \}. \]

In Eq. (3), \( \Gamma_e \) is the electron tunnelling rate \( \Gamma_e \equiv \Gamma_L + \Gamma_R \). It is worth noting that the electron-hole recombination effect have not been directly included into \( G^r_{e,\sigma}(\epsilon) \), because we assumed that \( \Gamma_e >> \Gamma_{eh} \). The typical value of \( \Gamma_{eh} \) for InAs/GaAs QDs is \( \sim 1/\text{ns} \). The electron occupation number of the QD can be solved in a self-consistently via the relation

\[ N_{e,\sigma} = -\int \frac{d\epsilon}{\pi} \frac{\Gamma_L f_L(\epsilon) + \Gamma_R f_R(\epsilon)}{\Gamma_L + \Gamma_R} \text{Im} G^r_{e,\sigma}(\epsilon). \] (4)

\( N_{e,\sigma} \) is limited to the region \( 0 \leq N_{e,\sigma} \leq 1 \). Eq. (4) indicates that the electron occupation numbers of the QD, \( N_{e,-\sigma} \) and \( N_{e,\sigma} \), are primarily determined by the tunnelling process. To obtain the electron and hole occupation numbers \( (n_{e,-\sigma} = n_{e,\sigma} \) and \( n_{h,-\sigma} = n_{h,\sigma} \) arises from the optical pumping, we solve the rate equations and obtain

\[ n_e = n_{e,-\sigma} = n_{e,\sigma} = \frac{\gamma_{e,c} N_{e,k}}{\gamma_{e,c} N_{e,k} + \Gamma_{eh} n_h + \Gamma_e}, \] (5)

and

\[ n_h = n_{h,-\sigma} = n_{h,\sigma} = \frac{\gamma_{h,c} N_{h,k}}{\gamma_{h,c} N_{h,k} + \Gamma_{eh} (n_e + N_e) + \Gamma_h}, \] (6)

where \( \gamma_{e(h),c} \) and \( N_{e(h),k} \) denote the captured rate for electrons (holes) from the wetting layer to the QD and the occupation number of electrons (holes) in the wetting layer. Here, we assume that \( N_{e(h),k} \) is in proportion to the intensity of excitation power, \( p_{\text{exc}} \). \( \Gamma_h \) denotes the nonradiative recombination rate for holes in the QD.

According to Eq. (3), particle Coulomb interactions will significantly affect the tunnelling current of SET. To illustrate this effect, we apply our theory to a self-assembled InAs/GaAs QD with pyramidal shape. First, we calculate the inter-particle Coulomb interactions using a simple but realistic effective-mass model. The electron (hole) in the QD is described by the equation

\[ \left[ -\nabla^2 + \frac{\hbar^2}{2m_{e(h)}^*} + V_{QD}^{e(h)}(\rho, z) \mp eFz \right] \psi_{e(h)}(\mathbf{r}) = E_{e(h)} \psi_{e(h)}(\mathbf{r}), \] (7)
where \(m^*_e(\rho, z)\) (a scalar) denotes the position-dependent electron effective mass, which has \(m^*_{eG} = 0.067m_e\) for GaAs and \(m^*_{eI} = 0.04m_e\) for InAs QD. \(m^*_h(\rho, z)\) denotes the position-dependent effective mass tensor for the hole. It is a fairly good approximation to describe \(m^*_h(\rho, z)\) in InAs/GaAs QD as a diagonal tensor with the \(x\) and \(y\) components given by \(m^*_h^{-1} = (\gamma_1 + \gamma_2)/m_e\) and the \(z\) component given by \(m^*_h^{-1} = (\gamma_1 - 2\gamma_2)/m_e\). \(\gamma_1\) and \(\gamma_2\) are the Luttinger parameters. \(V_{QD}^e(\rho, z)\) (\(V_{QD}^h(\rho, z)\)) is approximated by a constant potential in the InAs region with value determined by the conduction-band (valence-band) offset and the deformation potential shift caused by the biaxial strain in the QD. These values have been determined by comparison with results obtained from a microscopic model calculation\(^7\) and we have \(V_{QD}^e = -0.5eV\) and \(V_{QD}^h = -0.32eV\). The \(eFz\) term in Eq.(7) arises from the applied voltage, where \(F\) denotes the strength of the electric field. Using the eigenfunctions of Eq. (7), we calculate the inter-particle Coulomb interactions via

\[
U_{i,j} = \int dr_1 \int dr_2 \frac{e^2 |n_i(r_1)n_j(r_2)|}{\epsilon_0 |r_1 - r_2|},
\]

where \(i(j) = e, h\). \(n_i(r_1)\) denotes the charge density. \(\epsilon_0\) is the static dielectric constant of InAs. The Coulomb energies are different in different exciton complexes, but the difference is small.\(^5\) Therefore, only the direct Coulomb interactions have been taken into account in this study.

For the purpose of constructing the approximate wave functions, we place the system in a large confining cubic box with length \(L\). Here we adopt \(L = 40nm\). The wave functions are expanded in a set of basis functions, which are chosen as sine waves

\[
\psi_{nlm}(\rho, \phi, z) = \frac{\sqrt{8}}{V^{3/2}} \sin(k_n \rho) \sin(k_m \phi) \sin(k_l \phi),
\]

where \(k_n = n\pi/L, k_m = m\pi/L, k_\ell = \ell\pi/L\). \(n, m\) and \(\ell\) are positive integers. The expression of the matrix elements of the Hamiltonian of Eq. (7) can be readily obtained. In our calculation \(n = 20, m = 10\) and \(\ell = 10\) are used to diagonalize the Hamiltonian of Eq. (7). Fig. 1 shows the inter-particle interactions as functions of QD size. The ratio of height and base length is \(h/b = 1/4\), while \(h\) varies from 2.5nm to 6.5nm. The strengths of Coulomb interactions are inversely proportional to the QD size. However as the QD size decreases below a threshold value (around \(b = 12nm\)), \(U_e\) is significantly reduced due to the leak out of electron density for small QDs. These Coulomb
interactions approach approximately the same value in the large QD limit. This indicates similar degree of localization for electron and hole in large QDs. We also note that $U_{eh}$ is smaller than $U_e$ in the large QD. This is due to the fact that in large QDs the degree of localization for the hole becomes similar to that for electron, while the anisotropic nature of hole wave function reduces $U_{eh}$. The repulsive Coulomb interactions, $U_e$ and $U_h$, are the origin of Coulomb blockade for electrons and holes, respectively. The attractive Coulomb interaction $U_{eh}$ gives rise to the binding of the exciton.

To study the behavior of tunnelling current, we consider a particular pyramidal InAs/GaAs QD with base length $b = 13\text{nm}$ and height $h = 3.5\text{nm}$. The other relevant parameters for this QD are $E_{e,0} = -0.14\text{eV}$, $E_{h,0} = -0.125\text{eV}$, $U_e = 16.1\text{meV}$, $U_{eh} = 16.7\text{meV}$ and $U_h = 18.5\text{meV}$.

Now, we perform detailed numerical calculation of the tunnelling current. For simplicity, we assume that the tunnelling rate $\Gamma_L = \Gamma_R = 0.5\text{meV}$ is bias-independent. We apply a bias voltage $V_a$ across the source-drain and $V_g$ across the gate-drain. The QD electron and hole energy levels, $E_e$ and $E_h$, will be changed to $E_e + \alpha eV_a - \beta eV_g$ and $E_h + \alpha eV_a - \beta eV_g$, where $\alpha$ and $\beta$ are the modulation factors. In our calculation, we assume $\alpha = 0.5$ and $\beta = 0.7$, which can also be determined by experiments. Meanwhile the chemical potentials of the electrodes with Fermi energy $E_F = 60\text{meV}$ (relative to the conduction band minimum in the leads) are assumed to be $70\text{meV}$ below the energy level of $E_e$ at zero bias. Parameters $\Gamma_h = 0.2\text{meV}$ and $R_{eh} = 10\mu\text{eV}$ are adopted.

Applying Eqs. (2), (4) and (6), we solve for the electron occupation number $N_e = N_{e,\sigma} = N_{e,-\sigma}$ and tunnelling current $J$. Fig. 2 shows the calculated results for $N_e$ and $J$ as functions of gate voltage with and without the photon-excitation power at zero temperature and $V_a = 2\text{mV}$. Solid line and dashed line correspond to $I = 0$ (no pump) and $I = 0.9$ (with pump), respectively. We have defined a dimensionless quantity, $I \equiv \gamma_{h,c}N_{h,k}/\Gamma_h$, which is proportional to the pump power. The electron occupation number displays several plateaus, while the tunnelling current displays an oscillatory behavior. We label four critical voltages (from $V_{g1}$ to $V_{g4}$) to indicate the resonance energies of retarded Green’s function. We see that the photon-excitation leads to additional two current peaks below the voltage $V_{g3}$, which is caused by the electron tunnelling assisted by the presence of a hole in the QD. This interesting phenomenon was observed by Fujiwara et al. in an SET composed of one.
silicon (Si) QD and three electrodes\textsuperscript{9}. The behavior of the photo-induced tunnelling current can be understood by the analysis of the poles of retarded Green’s function of Eq. (3); the first peak of dashed line corresponds to the tunnelling current through the energy level at $\epsilon = E_e - 2U_{eh}$ (corresponding to a positive trion state). The second peak is caused by a pair of poles at $\epsilon = E_e - U_{eh}$ (the exciton state) and $\epsilon = E_e + U_e - 2U_{eh}$ (the biexciton state). Since the magnitude of $U_e$ is very close to that of $U_{eh}$, these two poles almost merge together. The third peak is caused by another pair of poles at $\epsilon = E_e$ (the single-electron state) and $\epsilon = E_e + U_e - U_{eh}$ (the negative trion state). The last peak locating near $V_g = 123mV$ is due to the tunnelling current through the energy level at the pole $\epsilon = E_e + U_e$ (the two-electron state). The gate voltage difference $\Delta V_{g21} = V_{g2} - V_{g1}$ ($V_{g43} = V_{g4} - V_{g3}$) determine the strength of electron-hole interaction $\beta \Delta V_{g21} = U_{eh}$ ($\beta V_{g43} = U_e$). Once $U_{eh}$ is determined, we obtain the binding energy of exciton complexes.

In this study we have used the tunnelling current of an SET under optical pumping to determine the electron-hole interaction $U_{eh}$, which can be regarded as the binding energy of exciton. Although we used InAs/GaAs SET as an example, this idea can also be applied to Si/SiO\textsubscript{2} SET system\textsuperscript{9}.

**ACKNOWLEDGMENTS**

This work was supported by National Science Council of Republic of China under Contract Nos. NSC 93-2215-E-008-014 and NSC-93-2120-M-008-002
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Figure Captions

Fig. 1: Intralevel Coulomb interactions $U_e$ and $U_h$ and interlevel Coulomb interaction $U_{eh}$ as a function of the QD base length $b$.

Fig. 2: Electron occupation number $N_e$ and tunnelling current as functions of gate voltage at zero temperature for various strengths of optical excitation. Current density is in units of $J_0 = 2e \times meV/h$. 
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