Construction and analysis of assembly 3D deviation model considering part tolerance and deformation

Yunlong Wang¹, Bo Yuan¹, Chao Zhang¹, Xiaokai Mu*, Qingchao Sun¹, Wei Sun¹ and Chong Liu¹

¹ School of Mechanical Engineering, Dalian University of Technology, Dalian, Liaoning Province, 116023, China
*Corresponding author’s e-mail: muxiaokai@dlut.edu.cn

Abstract. The existing assembly precision design mainly uses the tolerance and fit to control the geometric accuracy, and does not accurately grasp the influence of part deformation on assembly performance. In this study, a design theory and method for flexible body assembly is proposed, which considers the factors of tolerance and deformation. Firstly, based on the spatial multi-body kinematics, a reasonable modeling method of 3D tolerance of part surface suitable for product design process are given; Secondly, the deformation geometric features are calculated, and the 3D tolerance mathematical model including the part deformation is obtained by combining with the 3D tolerance modeling method; Finally, taking an engineering example as the research object, a 3D assembly deviation mathematical model including part deformation is constructed. The analysis results show that the cumulative deviation at the end of the system has a great influence on the z-direction precision in the system space, and the error is -0.028 mm. Therefore, the influence of part deformation deviation on assembly performance should be fully considered in the process of mechanical system assembly design. This research provides a theoretical basis for further research on precision prediction, simulation and control methods of mechanical system.

1. Introduction
From the control perspective of machine precision, the design tolerance, manufacturing deviation and assembly deformation deviation between parts’ mating surfaces run through the whole assembly process, and then affect the final mating accuracy of the product. Therefore, it can be considered that the mechanical system is a multi-deviation coupling complex [1]. For example, the reliability of the assembly accuracy of the machine tool guide rail not only depends on the dimensional tolerance and geometric tolerance of the guide rail installation surface, but also depends on the deformation in the working process. Due to the existence of contact deformation on the surface of the guide rail, the contact stiffness and internal stress distribution of the actual guide rail have great uncertainty, which leads to the decline of the overall performance of the guide rail assembly. Therefore, in the trend of increasing precision of mechanical equipment, the construction of 3D tolerance model including the tolerance and deformation deviation of parts has become a key point for engineering designers.

Tolerance modeling is an effective expression of the tolerance information of the part surface in digital form, which mainly describes the mathematical expression of the variation of the geometric elements of the part in its tolerance domain [2]. Hillyard R. C. [3] first proposed the concept that the dimension and geometric tolerance of the part surface can be expressed in the form of computer-aided. At present, many scholars have studied the mathematical modeling method of part tolerance, but mainly based on the dimension and geometric tolerance elements. Roy U. and Li B. [4,5] have carried out
tolerance mathematical modeling for plane geometry containing dimensional tolerance and geometric tolerance. Vignat and Villeneuve [6] used the small displacement torsor (SDT) theory to model the machining error of the part surface, but mainly the part surface obtained by turning. Homri et al. [7] constructed the tolerance mathematical expression model in the feasible region based on the variable region of variable geometric entity. Luo et al. [8] elaborated the relevant principles of creating geometric tolerance model by space envelope method, and then realized the effective construction of a variety of geometric feature tolerance mathematical models. Ametage et al. [9] studied the tolerance model of plane geometric features through t-map model, and analyzed the model. Mansuy et al. [10] put forward the concept of T-MAP according to the relationship between geometric variable elements and tolerance region, and established the tolerance model of geometric parts with regular section shape. Zhang et al. [11] proposed a unified tolerance integrated modeling method based on key features (KFs) and graph theory, which can be used to deal with multiple tolerance types.

In recent years, many scholars have explored and analyzed the 3D tolerance model which considers the deformation factors of parts, but there is no unified modeling method and standard. Sun et al. [12] analyzed the relationship between surface processing deviation and assembly deviation between parts, and elaborated the deviation transfer law of parts under load. Chun et al. [13] used statistical theory and numerical analysis method to study the relationship between input and output tolerances of components, and then put forward the tolerance analysis theory of components under load. Benichou and Anselmetti [14] took the non-rigid body assembly as the object of tolerance analysis, and considered the thermal deformation and other factors in the process of assembly and work. Korbi et al. [15] analyzed the translation and rotation of the mating surface in the flexible body assembly process, and constructed the covariance model of the geometry through the mixed form of multiple function polynomials. Mu et al. [16] considered various deformation factors in the process of part assembly, and constructed the tolerance model of part assembly process by using Jacobian screw theory.

To sum up, the construction of tolerance model mostly considers the factors of part size and geometric tolerance. Although some scholars take part deformation into account in the modeling process, they have not formed a unified 3D tolerance modeling theory. Therefore, in order to effectively analyze the influence of part deformation on assembly accuracy, it is very important to build a 3D tolerance model including part deformation factors.

The establishment of an effective 3D tolerance model is an effective measure to ensure the product performance from the design point of view. In order to build 3D tolerance model, firstly, based on the spatial multi-body kinematics, a reasonable 3D tolerance modeling method is proposed to adapt to the product design process; Secondly, the 3D tolerance mathematical model of geometric features including part deformation is constructed by combining the 3D tolerance modeling method; Finally, taking an engineering example as the research object, a 3D assembly deviation mathematical model including part deformation is constructed, and then the product spatial error distribution law under load is obtained through the analysis of 3D tolerance.

2. Construction of 3D tolerance model of mating surface
Tolerance model is the expression of the limited range of different geometric features, which reflects the relationship between the ideal geometric features and the actual geometric features of parts. At present, the product design mainly takes the early geometric technical specification standard system as reference, which will cause some deviation between the design results and the actual situation, and ultimately lead to the final quality not meeting the design requirements. Therefore, in the process of product design, it is very important to establish the tolerance model for improving the assembly accuracy of mechanical system and the success rate of one-time assembly.

The construction of 3D tolerance model mainly consists of two parts [17]: (1) the determination of the change direction and position of geometric features; (2) The determination and expression of geometric change domain. 3D tolerance modeling is the process of determining the variation range of geometric features, and then using mathematical methods to effectively express.
This research focuses on the construction method of 3D tolerance mathematical model of plane geometric features.

![Diagram showing dimensional tolerance of plane geometric features.](image)

Figure 1. Dimensional tolerance of plane geometric features.

As shown in Fig.1, the dimension between the datum plane and the center point of the change plane is taken as the ideal dimension \( L \), \( TP \) and \( TF \) are the upper and lower limits of the change domain, respectively. According to the size change limit, in the coordinate system \( o-xyz \), the mathematical expression of the change plane within the allowable range is as follows:

\[
-T_p \leq z(x, y) \leq +T_p
\]

(1)

where, \( z(x, y) \) is the change plane in the coordinate system \( o-xyz \).

Thus, the spinor of the distance \( \Delta z(x, y) \) between any two points on the variable plane within the boundary is expressed as follows,

\[
-(T_p+T_f) \leq \Delta z(x, y) \leq (T_p+T_f)
\]

(2)

After calculation,

\[
\frac{-(T_p+T_f)}{2} \leq \beta d\theta_x + \alpha d\theta_y \leq \frac{(T_p+T_f)}{2}
\]

(3)

Then, the expression of the distance between two points on the variable plane is,

\[
\frac{(T_p+T_f)}{2} \leq yd\theta_x + xd\theta_y \leq \frac{(T_p+T_f)}{2}
\]

(4)

The relationship between the spinor parameters is as follows,

\[
\begin{cases}
-T_r \leq yd\theta_x + xd\theta_y + dz \leq T_r \\
\frac{-(T_p+T_f)}{2} \leq yd\theta_x + xd\theta_y \leq \frac{(T_p+T_f)}{2}
\end{cases}
\]

(5)

According to the relationship between the screw parameters of the plane dimension tolerance, the feature matrix expressed by the screw parameters is obtained as follows,

\[
S = \begin{bmatrix}
1 & 0 & d\theta_y & 0 \\
0 & 1 & -d\theta_x & 0 \\
-d\theta_y & d\theta_x & 1 & dz \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(6)

where, \( dz \), \( d\theta_x \), and \( d\theta_y \) are the pose deviation of the plane in the coordinate system.
3. 3D coupling deviation mathematical modeling considering part deformation

At present, the construction method of tolerance model does not consider the influence of part deformation on tolerance design, which has a big deviation from the actual situation[16]. Therefore, it is very important to study the mapping mechanism of the flexible body to the assembly accuracy, and build a 3D deviation mathematical model which is close to the actual working condition.

In this study, the numerical analysis method is used to analyze the assembly, and the deformation point cloud data at each node of the mating surface is extracted. According to the fitting operation method of geometric features[18], the plane geometric features are fitted, and the pose spinor parameters of the fitting deformation plane are obtained, which provides effective data support for the construction of 3D coupling deviation mathematical model. Fig.2 shows the solution process of 3D deviation model of plane geometric features.

\[ A \cdot x + B \cdot y + C \cdot z + D = 0, \quad (C \neq 0) \]  

where, \( a = A/C; \quad b = B/C; \quad c = D/C. \)

According to the operation method of least square fitting plane, the sum of squares of the differences between the ideal plane elements and the actual deformation elements is as follows,

\[
\sum_{i=1}^{n} \left( ax_i + by_i + cz_i - d_i \right)^2
\]

The unknown parameters \((a, b, c)\) of the plane equation are obtained by Eq. (10), and the fitting deformation plane of the part is obtained by taking them into the plane equation.

4. Case analysis

In this study, a double column horizontal machining center is taken as the analysis object, which is mainly composed of ram, slider, slipway, bed, column and other key parts, as shown in Fig.3.
This part mainly analyzes the influence of the deformation of the matching surface (bed guide (1) - slide (2) - slipway guide (3) - slide (4) - ram (5)) on the mechanical assembly accuracy. Therefore, in order to improve the assembly accuracy of mechanical system, it is particularly important to build a 3D mathematical model considering the dimensional tolerance, geometric tolerance and deformation deviation.

In this study, the finite element method is used for mechanical analysis of the machining center, and the deformation data of each mating surface node is extracted, and then the fitting deformation plane under the load is obtained by using the fitting algorithm. Because the guide rail and the slider are in the same space plane, rather than assembly in sequence, the fitting operation of the matching surface of the guide rail and the slider is performed again, and then the total matching plane in the assembly plane is formed, as shown in Fig.4.

By calculating the mutual position relationship between the total fitting deformation surface and the ideal mating surface, the total pose spinor parameters of the mating surface are obtained,

$$\begin{bmatrix}
\sum_{i=1}^{n} x_i \\
\sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} z_i 
\end{bmatrix}
=\begin{bmatrix}
\sum_{i=1}^{n} x_i x_i \\
\sum_{i=1}^{n} x_i y_i \\
\sum_{i=1}^{n} x_i z_i 
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c 
\end{bmatrix}
=\begin{bmatrix}
\sum_{i=1}^{n} y_i y_i \\
\sum_{i=1}^{n} y_i z_i \\
\sum_{i=1}^{n} z_i z_i 
\end{bmatrix}
$$

(11)

Finally, the 3D mathematical model of the mating surface between the guide rail and the slider is obtained,
With the assembly process, the same method can be used to obtain the 3D deviation mathematical model of the slider and slipway,

\[
S = \begin{bmatrix}
1 & 0 & -1.675e-08 & 0 \\
0 & 1 & 4.283e-07 & 0 \\
1.675e-08 & -4.283e-07 & 1 & -0.00144 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (13)

The 3D deviation mathematical model of the guide rail (with the slipway) and the sliding is:

\[
S = \begin{bmatrix}
1 & 0 & 1.693e-07 & 0 \\
0 & 1 & 3.250e-06 & 0 \\
-1.693e-07 & -3.250e-06 & 1 & -0.00366 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (14)

The 3D mathematical model of the mating surface between ram and slider is:

\[
S = \begin{bmatrix}
1 & 0 & 1.480e-07 & 0 \\
0 & 1 & 3.243e-06 & 0 \\
-1.480e-07 & -3.243e-06 & 1 & -0.00367 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (15)

The 3D deviation mathematical model of the ram hole at the end of the system is as follows,

\[
S = \begin{bmatrix}
1 & -7.216e-07 & 0 & -1.971e-04 \\
7.216e-07 & 1 & 4.459e-06 & 0 \\
0 & -4.459e-06 & 1 & -0.0144 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (16)

In order to explain the influence of part deformation on the assembly accuracy of the system, 3000 samples are randomly selected within the variation range of each mating surface. Through the analysis of the end cumulative deviation of the rigid and flexible system, the deviation distribution of the system end relative to the datum is obtained, as shown in Fig.5.

Figure 5. Deviation distribution of system end relative to datum under different working conditions.
It can be seen from Fig. 5 that the accumulated amount of deviation under load moves down greatly along the z direction, which indicates that the deformation between the mating surfaces has a great influence on the assembly accuracy of the whole system.

In order to intuitively see the accumulated deviation at the end of the system, the homogeneous coordinate transformation method is used to transfer and accumulate the deviation screw parameter matrix of each mating surface, and finally the accumulated deviation of each assembly process and the terminal system in the whole assembly system is obtained, as shown in Fig. 6.

It can be seen from Fig. 6 that the fitting surface deviation at each assembly stage and the accumulated deviation at the end of the system have little influence on the assembly accuracy of the system in x and y directions, but the accumulated deviation in z direction is -0.028 mm, which indicates that the deformation deviation under load should be the main factor affecting the assembly accuracy of the system. Therefore, the influence of the dimension tolerance, geometric tolerance and deformation deviation on the assembly performance of the product should be fully considered in the process of parts assembly design.

5. Conclusions

The existing assembly accuracy design mainly uses the tolerance and fit theory to control the geometric accuracy to ensure the final assembly accuracy, but it does not accurately grasp the influence of part deformation on assembly performance. In this study, a design theory and method for flexible body assembly is proposed, which considers the factors of tolerance and deformation.

(1) A reasonable modeling method of 3D tolerance for product design process is given, and the 3D tolerance mathematical model of plane dimension tolerance is established, which provides the basis for the follow-up research on the 3D tolerance mathematical model of assembly;

(2) According to the Geometrical Production Specification, the fitting algorithm of plane geometric features and the solution process of deviation model are given, and then the 3D deviation mathematical model of plane geometric features including part deformation is constructed;

(3) Taking an engineering example as the research object, a 3D assembly deviation mathematical model including part deformation is constructed, and the 3D deviation is analyzed. The results show that the cumulative deviation in z direction of the system is -0.028 mm, so the influence of the deformation deviation of parts on the assembly performance should be fully considered in the process of assembly design.

This research content can fully consider the influence of part deformation on assembly accuracy, and the construction of assembly 3D deviation mathematical model including part deformation can provide theoretical basis for the research of mechanical product accuracy prediction, simulation and control methods.
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