Peristaltic modes of single vortex in the Abelian Higgs model

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Using the Abelian Higgs model, we study the radial excitations of single vortex and their propagation modes along the vortex line. We call such beyond-stringy modes peristaltic modes of single vortex. With the perspective of the static vortex, we derive the vortex-induced potential, i.e., single-particle potential for the Higgs and the photon field fluctuations around the static vortex, and investigate the coherently propagating excitations which corresponds to the vibration of the vortex. We derive, analyze and numerically solve the field equations of the Higgs and the photon field fluctuations around the static vortex with various Ginzburg-Landau parameter and topological charge n. Around the BPS value or critical coupling $\gamma = 1/2$, there appears a significant correlation between the Higgs and the photon field fluctuations mediated by the static vortex. As a result, for $\gamma = 1/2$, we find the characteristic new-type discrete pole of the peristaltic mode corresponding to the quasibound-state of coherently fluctuating elds and the static vortex. We investigate its excitation energy, correlation energy of coherent fluctuations, spatial distributions, and the resulting magnetic flux behavior in detail. Our investigation covers not only usual Type-II vortices with \( n = 1 \) but also Type-I and Type-II vortices with \( n = 2 \) for the application to various general systems where the vortex-like objects behave as the essential degrees of freedom.

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I. INTRODUCTION

There have been wide interests in the study of the topological objects such as the vortices \([1] \), monopoles \([2,3] \), and instantons \([4] \) because of their importance for understanding some crucial aspects of the non-linear field theories \([5,6] \). They are not included in the original action manifestly, but appear as the localized field solutions of the non-linear eld equations. These solutions can be classified with the topological number which comes from the topological property of the system. Since the topological objects have spatially localized, finite energy configurations and some topologically conserved charges, it is fascinating to regard them as dynamical degrees of freedom in the system.

One of such objects is the Abrigosov vortex \([1] \) in Ginzburg-Landau (GL) theory, which appears as the magnetic ux squeezed by the Cooper-pair condensate. When the external magnetic eld applied to the superconductor exceeds some critical strength, the Cooper-pair is dissociated and the node of the Cooper-pair forms a vortex line. The single-valued property of the Cooper-pair around vortex lines leads to the quantization of the total magnetic flux, as $\Phi = 2\pi n ( n \geq 2) \pi$, where $\Phi$ represent the effective gauge coupling of the Cooper-pair, the total magnetic flux, and the topological number, respectively. The topological number $n$ characterizes the topology of the system and we can classify vortices in terms of $n$.

There exist many applications playing the concept of the Abrikosov vortex and its relativistic version, the Nielsen-Olesen vortex \([2] \). One example is the squeezed color-electric ux between the quark and anti-quark in QCD. Nambu \([3] \), 't Hooft \([4] \) and Mandelstam \([5] \) discussed that there exist the vortices which appear as the color-electric flux tubes squeezed by the condensation of magnetic monopoles. As a result of the color-electric flux squeezing, we can only observe colorless monopoles and baryons, which are color singlet bound states of the colored quarks with the bond of the color-electric flux. This scenario is a possible explanation for "color con renмент", which is the experimental fact in hadron physics \([6] \). The lattice QCD Monte Carlo calculations \([7] \) for QQ and QQQ potentials strongly support the flux tube picture, and show the universality of the string tension of the flux-tube, $\sigma = 0.89 \text{ GeV}$ in both QQ and QQQ cases.

An another example is the cosmic string in astrophysics as a seed of the galaxy formation and baryogenesis \([8] \). Kobayashi \([9] \) and Zucker \([10] \) discussed the distribution of the topological defects and cosmo logical evolution after the quench of the early Universe and the resulting phase transition. They argued that the cosmic strings as topological defects would produce a characteristic signature in the cosmic microwave background. Following their scenarios, the phase transition dynamics is closely investigated in the numerical simulations using the phenomenological time-dependent GL (TDGL) theory \([11] \) in both cases with global symmetries and local gauge symmetries. In particular, in the case of the gauge symmetry, it is demonstrated that the initial thermal magnetic fluctuations just after the quench play an essential role in the formation of the vortices such as the clusters of vortices \([12,13] \). These investigations are closely correlated with the condensed matter physics in a viewpoint to understand the phase transition dynamics, and provide the interesting subject to link different elds of research.

Now we turn to the dynamical aspects of single vortex as a "pseudoparticle". For the treatment of the vortex motion, the vortex is usually regarded as one dimensional (1-D) thin object like a string, neglecting the extension...
perpendicular to the vortex line. Such a treatment is expected to be sufficient when the length of the string is much larger than the length scale in the radial direction. This condition comes from the fact that the excitation energy of string as a 1-D object is typically \( E \approx 1/L \) (where \( L \) is the string length). Then, for the vortex with large length, the stringy excitation is more important than radial one with changing its thickness. Even if the above condition is not well satisfied, we have only to consider extensive modes of the magnetic \( \kappa \) when the Type-II nature is very strong, i.e., the \( S \) = \( 1/2 \). Such modes are characterized by the \( \kappa \) and \( \kappa \) excitations. In particular, the time-dependent photon excitations, which are sometimes neglected, play important roles in the radial direction of the vortex edge, considering the vortex with the infinite length. The bending of the vortex is also neglected. We investigate both Type-I and Type-II vortices with not only usual \( n = 1 \) but also \( n = 2 \) for the future applications to the non-equilibrium or multi-vortices systems where the giant vortices can appear as excited states. In the Type-I case, if the system contains only the magnetic \( \kappa \) in contrast to the equilibrium case usually re-

Putting applications to the above examples in perspective, we will discuss the axial-symmetric radial excitation modes and their propagation along the vortex line. Hereafter, we will call this excitation mode peristaltic mode. The schematic picture of the peristaltic mode is shown in Fig. 1. To discuss the peristaltic modes, we utilize the Abelian Higgs action which includes the Higgs and the photon fields. In terms of this action, the peristaltic modes are characterized by the Higgs and photon excitations. In particular, the time-dependent photon excitations, which are sometimes neglected, play important roles in the radial direction of the vortex edge, considering the vortex with the infinite length. The bending of the vortex is also neglected. We investigate both Type-I and Type-II vortices with not only usual \( n = 1 \) but also \( n = 2 \) for the future applications to the non-equilibrium or multi-vortices systems where the giant vortices can appear as excited states. In the Type-I case, if the system contains only the magnetic \( \kappa \) in contrast to the equilibrium case usually re-

![Diagram](attachment:diagram.png)

**FIG. 1:** The illustration for the peristaltic mode of the vortex. The axial-symmetric oscillation perpendicular to the vortex line propagates with the excitation of the Higgs and photon fields. The wave solid line in the left side represents the surface where the amplitude of the Higgs field equals some constant value. The right hand side represents the amplitude of the Higgs and the magnetic fields in the cross section perpendicular to the vortex line. Through this process, the conserved total magnetic \( \kappa \) repeatedly contracts and relaxes.
alized in the laboratory, these ux gather and become giant vortex because Type-I vortices attract each other. On the other hand, in the Type-II case, the giant vortex is unstable and splits into the small vortices. In spite of this, when \( n^2 \) is not too large, it is worth considering the vibration of the giant vortex because the gradient of the vortex-vortex interaction is small for the small separation of vortices and we can expect that the annihilation process develops slowly.

The organization of this paper is as follows. In Sec. II we briefly review the treatment of the static profile of single vortex. In Sec. III we derive the effective action and the resulting equation of motion for the fluctuations around the vortex. The effective action for the static vortex background on the fluctuations are emphasized. In Sec. IV we show the numerical results for the excitation modes of the \( n=1 \) vortex with 0.1 \( n^2 \)-1.0. Based on the classification with \( \frac{1}{2} \), we closely discuss their excitation energy, correlation energy, and magnetic ux behavior. It will be shown that, in the case of \( \frac{1}{2} \neq \frac{1}{2} \), the interplay of the Higgs and the photon vortices play an essential role for the existence of the new type discrete pole. We also discuss the excitation modes around \( j \frac{1}{2} \) vortices. In Sec. V we summarize this paper with perspective for the future direction.

II. ABELIAN HIGGS MODEL AND THE STATIC VORTEX

In this section, we briefly review the treatment of the static vortex in terms of Abelian Higgs model. This type of action is very popular and has been used to describe chiral strings in the astrophysics, the color-ux tube in QCD, and so on. We use this Lorentz-invariant action to investigate the properties of the vortex as a "pseudo-particle".

For the finite temperature cases, one usually uses the phenomenological model with replacing the 2nd time-derivative in Abelian Higgs model with the 1st derivative although the microscopic foundation of this treatment is still not conclusive. The noise term is sometimes further added. These replacement change the equation of motion from the wave equation to the dissipative equation which respects the dissipation effects due to the thermal fluctuations. We will discuss the finite temperature case elsewhere.

A. The action and the \( \phi \) equation

Let us consider the ideal system which contains only one vortex with finite length along the \( z \)-axis. Our purpose is to investigate the excitation mode of the vortex in such a system. We start with the following action with natural units \((c=\hbar =1)\)

\[
S = \int d^4x \ j @ \ iA \ f \frac{1}{4} \ (A @ A)^2
\]

where \( A \) and \( \phi \) represent the Higgs field and photon field respectively. Here \( \tau \) are the effective gauge coupling constant, the strength of self-coupling, and the vacuum expectation value of the Higgs field, respectively. \( x \) denote space-time coordinates and \( \partial / \partial t \). Through this paper, we describe the variables in physical unit with bar. For later convenience, we rescale the \( \phi \) and the coordinates to make them dimensionless quantities,

\[
\phi = \phi / \phi_c; A = A / A_c; x = e x v : \quad (2)
\]

We also rewrite the action

\[
S = e^2 S; \quad (3)
\]

The energy relation between the physical and our rescaled unit can be obtained with noting the following relation,

\[
S = dt L \ f \frac{1}{2} \ (v e L)^2; \quad (4)
\]

Here \( L \) and \( L \) are Lagrangians in physical and rescaled unit, respectively, and they are related as \( L = \frac{L^2}{L} \). Similarly, we can obtain the energy and momentum in the physical unit with multiplying \( v \) to our rescaled energy and momentum.

After these rewriting, the action reads

\[
S = \int d^4x \ j @ \ iA \ f \frac{1}{2} \ (A @ A)^2
\]

\[
\frac{1}{4} \ (j @ A)^2 \ : \quad (5)
\]

Here we have introduced the GL parameter \( \tau = \frac{\phi}{\phi_c} = \frac{1}{A} \), which is used to distinguish the Type-I (\( < 1 = \frac{1}{2} \)) and Type-II (\( > 1 = \frac{1}{2} \)) superconductors. The critical case of \( \tau = 1 = \frac{1}{A} \) is called the BPS case, which lies between Type-I and Type-II. As a remarkable fact in the BPS case, the photon mass and Higgs mass coincide as \( m_\phi = m_\chi \), because of \( m_\chi = \phi \), which can be derived from the original action.

In this work, we will consider the system which is translation invariant in the \( z \)-direction and contains single vortex with finite length along the \( z \)-axis. We also impose the axial-symmetry around the \( z \)-axis on the static solution. With cylindrical coordinates \((x; \ z)\), we denote the complex field \( \chi \) in the polar decomposition form

\[
(x) = (x; \ z)e^{i(\varphi; \ z)} \quad (0 < \varphi < 2\pi); \quad (6)
\]
where \( n \) is the topological number (\( n = 0, 1, \ldots \)) related to the phase of the Higgs field, i.e., wave function around the z-axis. For single vortex with the topological number \( n = 2 \), the sign of \( n \) is not relevant because of charge conjugation symmetry of the Abelian Higgs model, and therefore we have only to consider \( n = 0, 1 \), \ldots\) cases \( \) without loss of generality. Since the Abelian Higgs model has the U(1) local gauge symmetry, we have the degrees of freedom to choose the gauge. For the treatment of the vortex solution, the familiar gauge is the \( A_0 = 0 \) gauge. Instead, we adopt the gauge which eliminates the small amplitude of the phase,

\[
\theta = 0; \quad (7)
\]
to simplify the equations of motion in the later analysis. For \( n = 0 \), this gauge \( \theta \) corresponds to the unitary gauge.

From the rotational symmetry and translational invariance along the z-direction, we adopt the following Ansatz for the static solutions

\[
\begin{align*}
\phi (r) &= \phi (r) e^{i n}; \\
A (r) &= (0; \ A (r) \sin ; \ A (r) \cos ; \ 0); \quad (8)
\end{align*}
\]

and the static electric and magnetic fields are obtained as

\[
E (r) = 0; \ H (r) = (0; \ 0; \ \frac{1}{r} \frac{d}{dr} (rA )); \quad (9)
\]

As seen soon later in Eq. (13) and (14), our gauge \( \theta \) and Ansatz for the static fields reproduce the same results for the static solutions as those in usual \( A_0 = 0 \) gauge.

The total static energy of the system (in the rescaled unit) is obtained as

\[
E_{\text{static}} = \int_0^Z dr dr A (r); \quad (10)
\]

where

\[
E [A] = \frac{1}{2} \left( \frac{n^2}{r^2} + \frac{1}{r^2} \right) \phi^2 \\
+ \frac{1}{2} \left( \frac{\partial \phi}{\partial A} \right)^2 + \frac{1}{2} \left( \frac{\partial A}{\partial A} \right)^2 \\
+ \frac{1}{2} \left( \frac{n \rho A}{r^2} \right)^2 + 2 (\frac{\partial \phi}{\partial A} \frac{\partial A}{\partial A})^2; \quad (11)
\]

(The full action and the energy functional are given in Appendix A.)

Under the above circumstances, we consider the static vortex solutions \( (r) \) and \( A (r) \) which minimize the static energy. From the variational principle

\[
\frac{E_{\text{static}} (r)}{E_{\text{static}} (A) (r)} = 0; \quad (12)
\]

and then the static field equations read

\[
\begin{align*}
\frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \phi \right) + \frac{n^2}{r^2} \phi + 2 (\frac{\partial \phi}{\partial A} \frac{\partial A}{\partial A})^2 &= 0; \quad (13) \\
\frac{d}{dr} \left( A \frac{dA}{dr} \frac{d}{dr} A \right) + A \frac{dA}{dr} + 2 (n \rho A)^2 &= 0; \quad (14)
\end{align*}
\]

Since we are interested in the vortex solution, we consider the boundary condition (B.C.) which is appropriate to describe the vortex at the center. For \( r = 0 \),

\[
(r) \mid_{r=0} \text{ const.} \quad E [A] \mid_{r=0} \text{ const.} \quad (15)
\]

and for \( r = \infty \),

\[
(r) \mid_{r=\infty} = 1 + \text{const.} \quad r \frac{d}{dr} \phi = 0; \\
A \mid_{r=\infty} \text{ const.} \quad r \frac{dA}{dr} = 0; \quad (16)
\]

These boundary conditions are obtained by the asymptotic analysis of the static field equations which satisfy the following physical requirements to describe the vortex at the center: (I) at \( r = 0 \) vacuum expectation value of Higgs field takes zero. (II) the static energy \( U \) asymptotically approaches to the vacuum energy when \( r \to \infty \). Note that the first condition represents the existence of the vortex at \( r = 0 \) and the second one represents the Higgs field is completely screened in the region far from the center of the vortex. The boundary conditions (15) and (16) also lead to the quantization of the total magnetic flux

\[
\int_0^Z dr \phi = n \phi \quad (17)
\]

(0 corresponds to \( e \) in the physical unit.)

We can numerically solve the static field equations (13) and (14) with the boundary conditions (15) and (16) and obtain the static solutions \( \phi (r) \) and \( A (r) \) for the Higgs field and the photon field. Figure 2 shows the static profile of the Higgs field \( \phi (r) \), the magnetic field \( H (r) = \frac{1}{2 \pi} \frac{\partial}{\partial r} (r A) \), and the resulting static energy distribution \( E (r) \) in the vortex with various values.

**B. The vortex mass and the classification of Type-I and Type-II superconductors**

Using the distribution of the static field energy, we can calculate the vortex mass (with quantum number \( n \)) per unit length in the z-direction, i.e.,

\[
M_n \equiv \int_0^Z dr \frac{dE [A]}{dr} = \frac{1}{n} A = A^{(c)}; \quad (18)
\]

Note that by definition of the action, the origin of energy is zero when the system has no vortex. The vortex mass is classified in the following way,

\[
M_n < 2 \pi \quad (n = \text{odd}; \text{Type I}); \quad (19)
\]

\[
M_n = 2 \pi \quad (n = \text{even}; \text{BPS}); \quad (20)
\]

\[
M_n > 2 \pi \quad (n = \text{odd}; \text{Type II}); \quad (21)
\]

These relations are analytically [27,34] and numerically investigated. Figure 3 and 4 showed the numerically
solved $2$-dependence of $M_n = n$, the vortex mass divided by the topological number $n$, and we can see

$$M_n < nM_1 \quad (2 < 1=2);$$
$$M_n > nM_1 \quad (2 > 1=2);$$

(22)

These relations indicate that the interaction between vortices is attractive for $2 < 1/2$, and repulsive for $2 > 1/2$. These were confirmed by the studies of the vortex-vortex interaction analytically in large separation case [32] and numerically in arbitrary distance case [33].

The qualitative understanding for the potential property is as follows. The attractive part of the potential comes from the reduction of the topological defect of the Higgs field condensate. This is because the total energy of the system decreases when the domain of the normal conducting state decreases. On the other hand, the repulsive part comes from the magnetic field interaction of the vortices. Then, in the case of $2 < 1/2$ where the coherence length of the Higgs field exceeds the penetration depth of the magnetic field, the attractive part becomes larger than the repulsive part in the whole region, and the vortex-vortex potential becomes attractive. On the other hand, in the case of $2 > 1/2$, the relation is reversed and the vortex-vortex potential becomes repulsive.

In the case of the vortex-anti-vortex case, the potential becomes attractive independently from $2$ since the magnetic field interaction between the vortex and the anti-vortex becomes attractive in contrast to the vortex-vortex case, and there is no repulsive part in the potential.

Although we consider the isolated vortex under the idealized situation from the beginning, it is worth mentioning under which situations single vortex appears referring to the potential properties explained above. In Type-I superconductor, the multi-vortices with total topological number $N$ transfer to the single, giant vortex with topological number $N$ due to their attractive interaction. After the successive fusion processes, the multi-vortex system transfers to the normal conducting state. For the finite size superconducting materials under the homogeneous magnetic field and them at equilibrium, such fusion processes lead the difficulty to observe the Type-I vortices because the magnetic fluxes successively invade from the interface between the normal and superconducting phases. Nevertheless, under the non-equilibrium situations or inhomogeneous magnetic fields concentrated on the small area, it is expected that the Type-I vortex or, more generally, the domain of the normal state in the superconductor can appear as a transient state in the
process such as the dynamical evolution of the thermodynamic unstable state into the equilibrium state, i.e., the nucleation or spinodal decomposition processes \[27,28\]. From the viewpoint of particle physics, another interesting situation where Type-I vortex appears is the case where magnetic monopoles are immersed in superconductors and the isolated vortex emerges between the monopole and antimonopole \[29\]. In this case, no collapse with the other vortices occurs and Type-I vortex behaves as the stable object due to the topological charge conservation.

On the other hand, in Type-II superconductor, the giant vortex with topological charge N vortices to the N vortices with charge n = 1 due to their repulsive nature \[30\]. This repulsive interaction prevents the vortices to coalesce and, under the thermodynamic equilibrium, the n = 1 vortices form some stable structure like the Abrikosov triangle lattice \[30\]. Then, in Type-II superconductor, the n = 1 case is realistic in the thermodynamic equilibrium in contrast to Type-I case. However, when the system experiences a certain quenching \[30\] such as the rapid change of the temperature or density, the n = 2 vortex may be generated as the excited states. This is the reason why we also consider the Type-II giant vortex in the following. Although they are unstable against the sion into small vortices, when \( n > 2 \) is not too large, it may be interesting to consider the vibrations of the giant vortex because such vibrations can occur during the slow development of the sion process due to the small gradient of the vortex-vortex potential for the small separation of vortices \[30\]. Moreover, the study of the ucation modes of the n = 2 vortex may give some insight for the early stage of the evolution of the giant vortex into the multi-vortices.

III. AXIAL-SYMMETRIC F LUCTUATIONS

In this section, using the results of the previous section, we analyze axial-symmetric ucations in the system which has only one vortex in the background. In subsection A, using the static vortex model, we derive the vortex-induced potential for the Higgs and photon excitations, and the equation of motion. In subsection B, we closely discuss the potential feature to classify the type of the excitation modes.

A. Field equation for the ucations

We investigate the axial-symmetric ucations of the Higgs excitations, and the related photon excitations around the static vortex. We expand the excitations with small amplitude ucations around the static excitations,

\[
\begin{align*}
(\psi; \zeta; r) &= \phi(\zeta) + \phi'(\zeta; r), \\
(\psi; \zeta; r) &= A^{\ast} \phi(\zeta) + a(\zeta; r), \\
A &= 0.2 \text{ (23)}
\end{align*}
\]

Here we only consider the axial-symmetric ucations and we include \( t = 0 \)-dependence at this stage. It is important to notice that, with the Ansatz for the static solution, all the other type of ucations such as ucations of \( A_{0}^{\ast} A_{x} \) and \( A_{r} \) and the \( t \)-dependent ucations are decoupled from the axial-symmetric ucations which we consider. Hence we can analyze the axial-symmetric ucations \[23\] independently of the other type of ucations and we drop such ucation terms from the following Lagrangian. The complete form of the Lagrangian is given in the Appendix \[A\].

Substituting Eq. (23) into Eq. (5) and using the static solution, the following expression for the axial-ucation part of Lagrangian can be derived as

\[
L_{\text{axial}}[ A, \psi; \zeta] = L_{\text{axial}}^{(2)} + L_{\text{axial}}^{(3)} + L_{\text{axial}}^{(4)}: \text{(24)}
\]

The first term \( L_{\text{axial}}^{(2)} \) represents \( L_{\text{axial}}^{(2)} = 0 \) \( c_{1}; A, c_{1} \). The 1st order terms of ucations \( a \) do not appear because we expand around the static solution. The 2nd, 3rd, and 4th order ucation terms take the following form:

\[
L_{\text{axial}}^{(2)} = \frac{1}{2} \left( \zeta' \right)^{2} \left( \frac{z_{1}}{2} \right)^{2} \left( \frac{\zeta}{2} \right)^{2} \left( \frac{1}{2} \zeta + a \right) \equiv V_{a} + \psi; \zeta. \quad \text{(25)}
\]

\[
L_{\text{axial}}^{(3)} = 4 \zeta' + 2 A_{\zeta} \zeta + V_{a} \psi; \zeta. \quad \text{(26)}
\]

\[
L_{\text{axial}}^{(4)} = 2 \psi; \zeta \psi; \zeta. \quad \text{(27)}
\]

Here \( V_{a}, V_{a}, V_{a} \) denote the vortex-induced potential, i.e., these interactions are induced by the static vortex configuration which can be regarded as the large amplitude background external field \( V_{a} \) and \( V_{a} \) play roles as the single particle potential for \( a \) and \( a \) respectively, and \( V_{a} \) is the interaction which makes \( a \) and \( a \). Hereafter, we will simply call these interactions \( \psi; \zeta \) potentials \( a \).

The concrete form of the potentials are

\[
\begin{align*}
V_{a}(r) &= \frac{n}{r} \frac{2}{2} A_{1}^{2} + 2 \left( 3 \right) \left( 2 \right) \left( 1 \right); \\
V_{a}(r) &= 4 \frac{n}{r} A_{1}^{2} c_{1}; \\
V_{a}(r) &= 2 c_{1}. \text{(28)}
\end{align*}
\]

To investigate the ucation spectrum, we solve the Euler-Lagrange equation for \( L_{\text{axial}}^{(2)} \) neglecting the higher order ucations. The quantum loop corrections can be calculated using the Ginzburg's functions constructed of bases which diagonalize \( L_{\text{axial}}^{(2)} \). However, because of the inhomogeneous vortex-background, the Ginzburg's functions are not functions of the distance between arbitrary two points, i.e., \( G(x; x) \neq G(x; x) \), and the calculations of the loop corrections are not straightforward. In addition, higher order terms of the axial-symmetric ucations couple with the terms of the non-axial-symmetric ucations.
We reserve the calculation of the loop corrections for the future work and, in this work, we safely neglect the quantum corrections constraining ourselves to the case where the perturbative expansion parameters \( e^2 \) and \( e^2 \) are sufficiently small. (Recall that the perturbative corrections are calculated with expansion of \( e^2 = e^{-a} \). See also Eq. (28).) These constraints are not so strict because the Higgs-photon coupling \( e \) is usually small and we mainly consider \( e^2 \) cases, where \( e^2 \) is also sufficiently small.

For later convenience to discuss the effect of the potential, we change the variables

\[
\Lambda^2(t; z) = e^{1/2} \quad a = \frac{p_z}{r} \tag{29}
\]

These changes of variables enable us to considerably simplify the equation of motion as will be seen. The Euler-Lagrange equation for 2nd order Lagrangian is obtained through the variation,

\[
\frac{Z}{\epsilon} \frac{d}{dt} \frac{d}{dz} rL(2) = \frac{Z}{\epsilon} \frac{d}{dt} \frac{d}{dz} rL(2) = 0 \tag{30}
\]

Then we obtain the following coupled equation

\[
\begin{align*}
(\theta^2, \theta_z^2) & \quad \Lambda^2(t; z) = e^{1/2} \\
\theta_t^2 + \frac{4m_A}{2r^2} + \frac{1}{2} & \quad + 2 \frac{2}{(3c + 1)} \quad \frac{p_z}{r} \quad \Lambda^2(t; z) \\
\theta_t^2 + \frac{1}{4r^2} + 2 \frac{2}{c(1)} & \quad \Lambda^2(t; z) \\
\theta_t^2 + V(r) & \quad V(r) \quad \Lambda^2(t; z) \\
\theta_t^2 + V(r) & \quad V(r) \quad \Lambda^2(t; z) \\
\end{align*} \tag{31}
\]

Here \( V(r) = V(r) \) (This is the consequence of the change of the variables in Eq. (28)). Note that the \( t; z \) dependence of \( \Lambda \) and \( a \) is trivial because the background static vortex configuration is \( t; z \)-independent and \( t; z \) dependence does not appear in the potential. Then the partial differential equation can be decoupled into the total differential equations for \( t; z; r \), respectively.

The system has the translation invariance in both \( z \)-direction and \( t; z \)-direction, and then the solution takes the following plane wave solution,

\[
\Lambda^2(t; z) = e^{i(t - k; z)} \quad (r) \tag{33}
\]

Then the total differential equation in the radial direction for \( r \) and \( r \) is obtained as

\[
\begin{align*}
\Lambda^2(t; z) & \quad \Lambda^2(t; z) \\
\theta_t^2 + V(r) & \quad V(r) \quad \Lambda^2(t; z) \\
\theta_t^2 + V(r) & \quad V(r) \quad \Lambda^2(t; z) \\
\theta_t^2 + V(r) & \quad V(r) \quad \Lambda^2(t; z) \\
\end{align*} \tag{34}
\]

where \( m^2 = k^2 + k^2 \). Note that the Hermiticity of the matrix in the RHS in Eq. (34) ensures the real value of \( m^2 \) and our later analysis show \( m^2 > 0 \); i.e., the stability of the static potential against the fluctuations. The root of the eigenvalues for the Eq. (34), \( m^2 \), plays a role of them as the propagating mode in the \( z \)-direction. The Hermiticity of Eq. (34) also enables us to take the functions \( (r) \) and \( (r) \) real and the orthonormal condition for the \( (r) \) and \( (r) \) is expressed as

\[
Z \int_0^1 \frac{d}{dz} \frac{d}{dt} (r) (k; z) b = \int_0^1 \frac{d}{dz} \frac{d}{dt} (r; k) \tag{35}
\]

Here we give a brief explanation for the qualitative picture for the excitation modes. The shape of the radial direction is given by \( j(r) \) and \( j(r) \). This wave function propagates in the \( z \)-direction with the frequency \( j(k) \) and the wave vector \( k \). Therefore the root of the eigenvalue \( m^2 \) of the radial wave functions \( j(r) \) and \( j(r) \) play roles of the mass for the propagation in the \( z \)-direction. Then we call this type of new mode as the peritalic mode of the vortex as illustrated in Fig. [6].

B. The feature of the potential

We show in Fig. [5] the single particle potentials \( V \), \( V \) and \( V \) around the vortex for various values of \( \epsilon \). We can deduce the considerable information of the excitation modes from the examination of these potentials. To get the qualitative picture about the wave functions of fluctuations, it is convenient to divide the region of the coordinate space \( r \) into three regions, central (0.05), inner (0.55), and asymptotic (4) region in the renormalized Abelian Higgs model.

In the central region, when \( r \) is 0, the asymptotic-be-
behavior of the potential is

\[ V = V \tau^n \quad \text{(negative const.)} \quad r^n - 1 \quad : \quad (36) \]

From these asymptotic behavior, it is expected that the fluctuations \( \tau \) and \( \eta \) around the vortex center are suppressed by the strong repulsive interaction proportional to \( r^2 \) and their distributions are shifted to the outside.

On the other hand, in the intermediate region, the strong repulsive part of \( V \) and \( V \) become weak and the attractive pocket emerges. The mixing term \( V \) also plays an important role to enhance the mixing between the Higgs and photon fields fluctuations, or and . Since the \( V \) always takes the negative value, the excitation energy is reduced when the relative sign of the and fluctuations is positive. To avoid the confusion, we stress that the minus sign of \( V \) does not mean attractive interaction. Even if \( V \) takes the positive value, the energy is reduced when the relative sign of and fluctuations is negative. This is also recognized from the fact that the replacement \( \tau \rightarrow \eta \) changes only the sign of the off-diagonal part of Eq. (38). In treatment up to the second order of the fluctuations, only the absolute value of \( V \) is relevant to the mixing of and . All these features of the potentials \( V \) and \( V \) make low energy modes of the fluctuations localized in the intermediate region.

Finally, we discuss the asymptotic region. When \( r \rightarrow 1 \), these potentials asymptotically behave as

\[ V \rightarrow 4^{2/3} r^2; \quad V \rightarrow 2 r; \quad V = V \rightarrow 0: \quad (37) \]

Note that the asymptotic behaviors do not depend on the topological charge \( n \). Since \( V \) goes to zero, the correlation between and vanishes and these fluctuations behave as independent modes. Then we have only to consider the one-particle problem in this region, i.e.,

we treat the asymptotic forms of Eq. (34) as

\[ \left( \frac{d^2}{dr^2} \right) \phi (r) = m^2 \phi (r); \quad (38) \]

\[ \left( \frac{d^2}{dr^2} + 2 \right) \phi (r) = m^2 \phi (r); \quad (39) \]

The values 4 and 2 in the LHS play roles of the (square of) continuum thresholds of the and respectively. These thresholds are equivalent to the masses of the fluctuations in the system with no vortex, i.e., corresponds to the mass of the Higgs field and 2 is the photon mass generated through the Anderson-Higgs mechanism [33, 40]. Then, the threshold energy for the continuum states is expressed as

\[ m_{\text{th}} = m \ln (2; \frac{P}{2}) : \quad (40) \]

When \( m \) is smaller (larger) than the thresholds, the fluctuations behave like bound (continuum) states.

It is also worth mentioning that \( V \) and \( V \) take the same value in the asymptotic region and the continuum thresholds are the same both for \( \tau \) and \( \eta \) for the case of \( \tau = 1/2 \), where BPS saturation occurs. This note is somewhat useful to roughly classify the type of the continuum spectrum in the low-energy region in terms of the type of superconductor, as is explained below. It is expected that, for Type-I case \( \tau < 1/2 \), the threshold for the Higgs field is lower than that for the photon field, and the Higgs field fluctuation dominates over the low energy physics. For Type-II case \( \tau > 1/2 \), the relation is reversed, the photon field fluctuation dominates.

We also expect that the collective properties induced by the mixing of the Higgs and photon fields fluctuations are most pronounced when the BPS saturation occurs. This is because the strength of both fluctuations are the same order and they are each other. In the next section, we will see the above mechanism is crucial to emerge the characteristic discrete pole around \( \tau = 1/2 \).
In this subsection, we show the numerical results for n=1 case in both Type-I and Type-II superconductors. In Fig. 6, the $^2$-dependence of $m_j^2$ for n=1 case and the thresholds for and are plotted. Shown in Fig. 7 are the same quantities as Fig. 6, but without channel coupling, which is artificially dropped off.

With smaller $^2$, the threshold for is shifted lower than threshold for , and can excite with smaller energy and the amplitude of is enhanced compared to that of . Then the lower excitation modes are mainly characterized by the eld variations. Under the normalization condition [35], there appears a large excess of the amplitude of compared to that of , and this mismatch between and leads to the suppression of the mixing effect. Then the properties of the lower excitations asymptotically approach those calculated without channel coupling. The property of the discrete pole can be interpreted as the bound state of the static vortex and the fluctuating Higgs eld.

On the other hand, with larger $^2$ ($> 1=2$), the enhancement of the threshold energy for leads to the suppression of the eld variations, and simultaneously enhances the eld ratio in the lower excitation modes. Then the increasing of $^2$ value leads to the reduction of the mixing correlation energy which is indispensable for the appearance of the discrete pole. Eventually, at $^2 = 0$, the discrete pole is buried in the continuum state spectrum of the fluctuating photon eld and no discrete pole appears for $^2 > 0$. This indicates that such a pole is difficult to be identified in most Type-II superconductors.

Finally, we consider the most interesting case, $^2 = 1=2$ case, i.e., near BPS saturation. In this case, the mixing effect becomes larger, and, as a result, the correlation between and considerably lowers the excitation energy. This is clearly seen from the comparison of Fig. 6 with Fig. 7. In Table I, we summarize the results for the lowest excitation energy $m_{th} = m_{inf} = 2g$, the lowest excitation energy without channel coupling, $m_0^2$, and the energy differences among them. The quantity $m_{th} = m_0$ represents the binding energy and specifies the spatial extension of the wave functions. The quantity $m_0^2$ is, roughly speaking, the correlation energy induced by the mixing effect between Higgs and photon eld excitations. As expected, the mixing $m_0$ mixing correlation is realized in the BPS case ($^2 = 1/2$).
in fact, $\phi(r)$ and the magnetic excitations $h_i(r)$ in Eq. (23).

The physical value can be obtained by multiplying $v_e$. $m_{\text{th}}, m_0, m^2_0$ represent the threshold energy, the lowest excitation energy and the lowest excitation energy without the channel coupling, respectively. The negative sign of $m_0$ indicates the bound state. The quantity $m_0, m^2_0$ roughly characterizes the correlation energy of the Higgs-photon excitations.

| $^2$ | $m_{\text{th}}$ | $m_0$ | $m_0^2$ | $m_{\text{th}}$ | $m_0$ | $m_0^2$ |
|-----|---------------|-------|--------|---------------|-------|--------|
| 0.3 | 1.095         | 1.003 | 0.092  | 1.068         | 0.065 |
| 0.4 | 1.265         | 1.138 | 0.127  | 1.229         | 0.091 |
| 0.5 | 1.414         | 1.247 | 0.167  | 1.371         | 0.124 |
| 0.6 | 1.414         | 1.333 | 0.081  | 1.414         | 0.081 |
| 0.7 | 1.414         | 1.394 | 0.020  | 1.414         | 0.020 |

The spatial behavior of the excitations and the character change of the excitation modes with

The features explained above can be explicitly seen from the behavior of the radial wave function of the Higgs and the photon excitations, i.e.,

\begin{align}
' (r) &= r^{1/2} (r); \quad a (r) = P_0 r^{1/2} (r); \quad (41)
\end{align}

For the argument of the radial part, we hereafter drop the trivial factor $e^{i l \phi z}$ appearing in the relations (20) and (33). In fact, $' (r)$ and $a (r)$ denote the radial part of the Higgs and the photon excitations in Eq. (23).

In Fig. 8 we show the wave functions $' (r)$ and $a (r)$ of the low-lying modes and the corresponding magnetic excitations

\begin{align}
h_z (r) &= \frac{1}{r} \frac{d}{dr} (r a (r)); \quad (42)
\end{align}

for $^2 = 0.1, 0.5, 1.0$. For $^2 = 0.1$ and 0.5, we plot the lowest eigenfunction with radial mass smaller than the threshold $m_{\text{th}}$. On the other hand, for $^2 = 1.0$, there is no discrete pole because the radial mass is larger than the threshold $m_{\text{th}} = 2$, i.e., $m_{\text{th}}^2 > 2$. Then we plot the typical wave function of the low-lying mode whose radial mass satisfying $2 < m^2_{\text{th}} < 4$.  

**FIG. 8:** The behavior of the low-lying mode of Higgs excitations $' (r)$ and $a (r)$, the photon excitation $h_z (r)$ and the magnetic excitation $h_i (r)$ for $^2 = 0.1, 0.5, 1.0$. For $^2 = 0.1$ and 0.5, we plot the lowest eigenfunction with radial mass smaller than the threshold $m_{\text{th}}$. On the other hand, for $^2 = 1.0$, there is no discrete pole because the radial mass is larger than the threshold $m_{\text{th}} = 2$, i.e., $m_{\text{th}}^2 > 2$. Then we plot the typical wave function of the low-lying mode whose radial mass satisfying $2 < m^2_{\text{th}} < 4$.  

**TABLE I:** $^2$ dependence of the various energy in the rescaled unit. The physical value can be obtained by multiplying $v_e$. $m_{\text{th}}, m_0, m^2_0$ represent the threshold energy, the lowest excitation energy and the lowest excitation energy without the channel coupling, respectively. The negative sign of $m_0$ indicates the bound state. The quantity $m_0, m^2_0$ roughly characterizes the correlation energy of the Higgs-photon excitations.
The behavior of the utication elds coincides with our previous expectations. For \( z = 0.1 \), the Higgs eld utication considerably exceeds the photon utication. Although both of utication elds have the smaller excitation energy than the threshold \( m_\phi \), and this leads localization of them around the vortex core, the distribution of the Higgs eld utication spreads broader than that of the photon eld utication. This is because the lowest excitation energy \( m_0 \) is close to the threshold for the Higgs eld utications, \( 2 \), and the lowest mode is characterized as the shallow bound-state between the static vortex and the Higgs eld utication.

When we change to the \( z = 0.5 \), the mixing correlation induces the competitive behavior of the Higgs and photon eld utications and enlarges the binding energy \( m_\chi \). As a result, the lowest excitation mode becomes the deeper bound-state, and then, the Higgs eld localizes closer to the core, which aects the magnetic eld vibration more strongly.

For \( z = 1.0 \), because of \( 2 = m_0^2 < 4 \) for low-lying modes, only Higgs eld utication is localized and the photon eld a \((r)\) oscillates asymmetrically \((r < 1)\) as

\[
a(r) = e^{ik_{r}r} = r^{1/2};
\]

leading to the asymptotic behavior of the magnetic eld \( h_z(r) \) like

\[
h_z(r) = \frac{1}{r} \frac{d}{dr} (a(r)) e^{ik_{r}r} = r^{1/2}
\]

with \( k_{r}^2 = m_0^2 / 2 \). More precisely, \( a(r) \) and \( h_z(r) \) are expressed as the linear combinations of the \( \cos(k_{r}r) = r^{1/2} \) and \( \sin(k_{r}r) = r^{-1/2} \) because \( \cos \) \((r)\) and \( a(r) \) are real functions.

Here we comment on the total magnetic utx. In the static case, the total magnetic utx takes the quantized value \( 2 \eta \), as seen in Eq. (17). On the other hand, the photon utication generally induces a time-dependent magnetic utx, whose proper time-average vanishes due to the factor \( e^{it} \). The contribution from the radial part of the magnetic eld utication is

\[
Z_1^{1/2} \left( \frac{dr}{2} \right) \phi \left( \frac{dr}{2} \right) \phi \left( \frac{dr}{2} \right) = 2 \eta \left( \frac{r}{r_{0}} \right)_{r=1}^{r=0}:
\]

For \( r < 0 \), \( a(r) \) asymptotically approaches to zero. On the other hand, for \( r > 1 \),

\[
ra(r) = C_{r=2} \left( \frac{m_{0}}{r} \right) \left( \frac{r}{r_{0}} \right)_{r=1}^{r=0} \left( \frac{r}{r_{0}} \right)_{r=0}(\text{discrete pole});
\]

\[
D_{r=2} \left( \frac{r}{r_{0}} \right)_{r=0}(\text{continuum states});
\]

where \( C, D \) represent some constants. Then, for the discrete pole, becomes zero and thus the total magnetic utx is conserved in the arbitrarily cross-section perpendicular to the vortex line. In contrast, for the continuum states, the total magnetic utx becomes sensitive to the situation far from the vortex core because of the long tail behavior of \( h_z(r) \). For the continuum state, the radial photon utication \( a(r) \) asymptotically behaves as a two-dimensional spherical wave in the homogenous no-vortex vacuum, and hence its asymptotic part can be regarded as a non-vortex-origin utication. In other words, the continuum states can be regarded as the vortex plus a \"scattering wave\".

To discuss the character change of the excitation modes in wider range of \( z \), we define the functions

\[
A_{<z}^{1/2} \left( \frac{dr}{2} \right)_{r=0}^{r=1} B_{>z}^{1/2} \left( \frac{dr}{2} \right)_{r=0}^{r=1} (47)
\]
Notice that \( R_i \) depends on the potential \( V \). When the oscillation is symmetrical, the integral takes the large value. In Fig. 3, we plot the behavior of the functions \( A^2, B^2 \) and \( A \) and \( B \) for various \( n \). As expected, the ratio of \( (r) \) or \( H \) increases as \( n \) increases. Photon energy density also shifts outward. The physical meaning of the change in potential \( V \) is to increase the size of the attractive pocket in \( V \). When \( n \) increases, the resulting changes in the static conditions and the vortex-induced potential lead to the change in the excitation energy of the peristaltic modes. Now we see the excitation spectra of the giant vortex \( n = 2; 3 \) and their dependence on the \( \mu \) value. The numerical results for \( n = 2; 3 \) are shown in Fig. 5, respectively. For larger \( n \), the excitation energy for the lowest mode becomes much smaller. In this case, the attractive potential \( V \) becomes very small, as seen in the lower panel in Fig. 9.

### C. The peristaltic modes for \( n = 2 \) cases

In this subsection, we show the results for the peristaltic modes of the \( n = 2 \) vortex, which is realistic in the case of \( n = 2 \). We emphasize the difference of the static vortex between \( n = 1 \) vortex and \( n = 2 \) vortex, and then discuss their effects on the potential behavior and the resulting excitation spectrum. In this subsection, when we argue \( n \)-dependence of the classical potential \( V \) and the potential, \( \mu \) value is fixed to the BPS value, i.e., \( \mu = 1/2 \).

Shown in Fig. 5 are the \( n = 1, 2, 3 \) static vortex profiles and the corresponding cortex-induced potentials \( V \) for the excitations \( (r) \) and \( (r') \), respectively. Here we qualitatively explain the changes of the static vortex profile in terms of the penetration of the magnetic field from the vortex core. Since the penetration of the magnetic field is proportional to the topological number \( n \), the magnetic field around the vortex core increases with increasing \( n \). Then the enlarged magnetic field pushes the Higgs field outward or reduces the expected value of the Higgs field. Roughly, this process spreads the vortex phase region near the vortex core and leads to the outward shift of the interface between normal and superconducting phase. A consequence of the outward shift of the surface region, where the kinetic energy of the Higgs field is enhanced, is that the local density of energy density also shifts outward, as seen in the upper panel of Fig. 5.

Next we present the quantitative explanation for the single-particle potential for the excitations. First we give the explanation for the change of \( V \) for the Higgs excitations. Since the area of the magnetic core is increased with increasing \( n \), the interaction of the static magnetic field and the Higgs field moves outward. This leads to the outward shift of the potential \( V \).

The changes of \( V \) can be interpreted in some way through the following arguments. The fact that if \( n \) is increased, the depth enhancement of the attractive pocket in \( V \) with increasing \( n \). This enables the photon excitations to enhance and localize around the surface region more easily for larger \( n \) values. The localization of the photon excitations means some way that the spatial change of the wave function \( \psi (r) \), and the large enhancement of the magnetic field \( h_{\psi} (r) \). Then we can say that the depth enhancement of the attractive pocket in \( V \) for large \( n \) leads the large enhancement of the magnetic field. The large magnetic enhancement reaction the reduction of the Higgs field and the weakened magnetic field. In fact, the depth enhancement of the attractive pocket is closely related to the outward shift of the static Higgs field distribution.

Finally, we comment on the potential \( V \), which induces the magnetic correlation between the Higgs and the photon excitations. As seen in Fig. 5 with increasing \( n \), the maximum of \( V \) shifts outward. The physical meaning of the change in this potential is naturally explained as the outward shift of the interface between the static Higgs field and magnetic field because, at the interface, the fluctuations of the Higgs and photon excitations can appear.
V. SUMMARY AND CONCLUDING REMARKS

In this paper, as a new type of mode beyond the stringy one, we have studied the axial-symmetric peristaltic modes of the vortex, peculiar to the extension of the ux-tube. We have investigated the peristaltic modes both in Type-I and Type-II superconductors for $n = 1, 2, 3$ and $2 = 0; 1; 0$. We have treated single vortex in the situation where Type-I vortex can be realized without collapse with the other vortices and behaves as the stable object.

Here we summarize our main results about the excitation modes. With $2 < 1 = 2$, the lowest excitation modes are characterized by the Higgs excitations binding with the static vortex. On the other hand, with $2 > 1 = 2$, the mass of the Higgs excitations becomes heavier and the photons excitation modes are relevant. For $2 = 1 = 2$ (BPS cases), there appears the characteristic discrete pole. In this case, the Higgs and the photon excitations have the same masses, and then both excitations play important roles and their correlated behavior makes the lowest excitation softer one. This mechanism is the main origin of the characteristic discrete pole. Since the energy separation between this discrete pole and the continuum threshold is large around $2', 1 = 2$, the peristaltic mode may appear most clearly in the superconductor with $2', 1 = 2$.

We have also investigated the excitation modes of the giant vortex with the topological number $n = 2$. It is found that, with larger $n$, the photon excitation becomes more important for the lowest eigenmode because of the distribution of the static Higgs excitations is shifted outward and the photon excitation can excite easily around the core. We expect that, in $2 = 1 = 2$ cases, the axial-symmetric excitation mode with the bound-state-like pole plays an important role in the low-energy dynamics of the vortex. On the other hand, for $2 > 1 = 2$ cases, although the equations for axial-symmetric excitation modes are decoupled from those of the other modes, it is necessarily not only to investigate axial-symmetric modes but also to consider whether these excitation modes can be realized or not during the lifetime of the giant vortex. To estimate the lifetime, we have to treat the non-axial-symmetric modes and the excitations of $A_0; A_\tau$, which drive the motion of the single giant vortex to many $n = 1$ vortices. We expect that, for $2$ value not so far from the BPS value, $2 = 1/2$, the lifetime is long enough and we can explore the possibility of the appearance of the discrete pole.

Finally, we end with a future perspective toward the further investigation for the dynamics of the vortices (or color-electric uxtubes) regarding them as dynamical degrees of freedom.

Although we restrict ourselves to the axial-symmetric static vortex profile and excitations of $A_\theta$ and $A_\tau$, we will investigate the more general cases without using the axial-symmetric Ansatz for the static profile and classify the large class of the excitation modes, i.e., $\psi, A_\theta$, $A_\tau$. 

because these modes behave independently as far as the distortion is sufficiently small, as noticed in Sec.IIIA.

On the other hand, for $2 = 1 = 2$ cases, since the giant vortex is stable and the axial-symmetric modes are independent from the other modes not considered in this work, the strong bound-state-like modes may easily appear than the case for $2 > 1 = 2$ because of non-existence of the ssion processes. In addition, according to the negative surface tension, vortex with $2 = 1 = 2$ tends to reduce the surface area and then the axial-symmetric excitation becomes increasingly prominent with smaller $2$. Therefore, we expect that the discrete pole shown in this section shares the considerable relevance to the low-energy behavior of the giant vortex with $2 = 1 = 2$. 

---

\[ \text{FIG. 11: } \frac{1}{2}\text{-dependence of } m_\alpha^2 \text{ for } n = 2 \text{ case and the square of thresholds for and els, } 4^2 \text{ and } 2, \text{ respectively. The excitation energy is considerably lowered than that in } n = 1 \text{ case and the discrete pole appears up to considerably large } 2. \]

\[ \text{FIG. 12: The same plot as FIG. 11 except the replacement of the topological number } n \text{ from } n = 2 \text{ to } n = 3. \text{ The thresholds for and are unchanged.} \]
A, A_2) in = 0 gauge, or equivalently, ( , A , A_2) in A_0 = 0 gauge, which are not fully considered in this work. For this purposes, we will calculate the static vortex profile and excitation spectrum of the excitations in two-dimensional analysis in near future.

For Type-I case, we expect that, in low energy, the relevant excitation mode around single vortex is axial-symmetric since the Type-I vortex tends to reduce the surface area owing to the positive surface tension. As to from the point of view on kinetic energy, the axial-symmetric excitation is favorable. On the other hand, the argument for the Type-II case are more subtle. Because Type-II vortex tends to increase the surface area according to the negative surface tension, it may be important to compare the surface energy to the kinetic energy of the distortion.

For n = 2 giant vortices, interesting subjects are remained. One of them is the calculation of the lifetime of single giant vortex for \( 2 > 1 = 2 \) and the comparison to the excitation modes considered in this work. The lifetime of giant vortex is closely related to the dynamics of the multi-vortices in non-equilibrium systems. As well as the information for the vortex-vortex potential, the information for the lifetime of single giant vortex may provide the good building block to understand the interesting features of such vortex configuration such as the transition of the system from the thermodynamically unstable state to the stable one. In connection with them, we are also interested in the viations between multi-vortices and their evolution through the vortex-vortex scattering. As a successive study, we would like to investigate the results of the full two-dimensional analysis for excitation modes of the vortex in both single vortex and the multi-vortices system.

In application to QCD, the dynamics of the color-electric ux-tubes, or gliconic excitations as nonperturbative modes are also important to discuss the hadrons, which are composed of quarks and color vortex between quarks. In particular, the analysis for the multi-color-ux-tube system can give an good insight to understand the color confinement and color transition in QCD, and properties of the hot and dense matter of hadrons or quarks. It is also interesting to consider heavy ion collisions and the successive occurring expansion of the hot quark matter in close analogy with the evolution of the early Universe. The distribution of the topological defects may affect collective properties of quark-gluon plasma and the resulting particle productions.

In this work, we have studied the dynamics of single vortex as an example of the dynamics of the topological objects. The treatments of topological objects as essential degrees of freedom are quite general in important to understand the crucial aspects of the nonlinear field theory, which som times can not be reached with the perturbative treatment for the nonlinear term s. We would like to extend our analysis to the other topological objects such as color-ux-tubes, instantons, magnetic monopoles in gauge theories, or branes in the string theory, and their many-body dynamical systems.

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Note added: After this work is completed, we notice a similar study using a different gauge where ghosts appear. Their numerical result on the physical modes is consistent with ours.

APPENDIX A: THE DECOUPLING OF THE AXIAL-SYMMETRIC FLUCTUATION MODES

We give the complete form of the action in the cylindrical coordinates (r, z). We use the polar-decom position of Eq. (3) for the Higgs field

\[
\phi(x) = (\phi_1; \phi_2) e^{i(\phi_1; \phi_2)} (\theta < ) ; \quad (A1)
\]

and the gauge to \( A = 0 \). Then we obtain the action as

\[
S = 0 = dr dz r L = 0 ; \quad (A2)
\]

where

\[
L = 0 = \left( \phi_1 \right)^2 \left( \phi_2 \right)^2 \frac{1}{r^2} \left( \phi_1 \right)^2 \left( \phi_2 \right)^2 \left( 2 \right)^2 \left( 1 \right)^2 \left( \frac{1}{r} A \right)^2 \left( 2 \right)^2 \left( \frac{1}{r^2} \right)^2 + \left( A_0^2 A_1^2 A_2^2 \right)^2
\]

\[
(A3)
\]
Here the electric and magnetic fields are described as
\[ E_r = \frac{1}{2} \frac{\partial A_r}{\partial t} + \frac{1}{2} \frac{\partial A_0}{\partial z} \]
\[ E_z = \frac{1}{2} \frac{\partial A_z}{\partial t} - \frac{1}{2} \frac{\partial A_0}{\partial r} \]
\[ B_r = \frac{1}{2} \frac{\partial A_r}{\partial z} + \frac{1}{2} \frac{\partial A_0}{\partial r} \]
\[ B_z = \frac{1}{2} \frac{\partial A_z}{\partial r} - \frac{1}{2} \frac{\partial A_0}{\partial z} \]
\[ (A4) \]
\[ (A5) \]
We decompose the fields into the static part and the fluctuation part,
\[ \phi(t; r, z) = \phi_1(t; r, z) + \epsilon(t; r, z) \]
\[ A_j(t; r, z) = A_{j1}(t; r, z) + a_j(t; r, z) \]
\[ (A6) \]
where \( j = (0; r; z) \). For the static part, we impose the Ansatz \([3]\), or equivalently,
\[ \phi_1(t; r, z) = \phi_1(t); \quad A_{j1}(t; r, z) = A_{j1}(t); \quad A_{j1} = A_{j1}^{(2)} = A_{j1}^{(3)} = 0 \]
\[ (A7) \]
Putting these forms into the Lagrangian \([3]\), we obtain
\[ L = 0 \left[ \frac{\partial}{\partial t} \right] = L^{(2)} + L^{(3)} + L^{(4)} \]
\[ (A8) \]
where
\[ L^{(2)} = \frac{1}{2} \frac{\partial A_{j1}}{\partial t} \frac{\partial A_{j1}}{\partial r} + \frac{1}{2} \frac{\partial A_{j1}}{\partial r} \frac{\partial A_{j1}}{\partial t} + \frac{1}{2} \frac{\partial A_{j1}}{\partial z} \frac{\partial A_{j1}}{\partial z} \]
\[ (A9) \]
\[ L^{(3)} = \frac{1}{2} \frac{\partial A_{j1}}{\partial t} \frac{\partial A_{j1}}{\partial r} \frac{\partial A_{j1}}{\partial z} \]
\[ (A10) \]
\[ L^{(4)} = \frac{1}{2} a_{j1}^{(2)} a_{j1}^{(2)} a_{j1}^{(2)} a_{j1}^{(2)} \]
\[ (A11) \]
\[ L^{(4)} = 2 \frac{\partial A_{j1}}{\partial r} \frac{\partial A_{j1}}{\partial z} \]
\[ (A12) \]
The Lagrangian for the static part, \( \phi_{j1}, L^{(2)} \) is related to static energy \([1]\) as \( E = \frac{\partial}{\partial t} \phi_{j1} \). The fluctuations of the electric and magnetic fields, \( \phi, \theta_0, \phi_0 \) and \( \theta_0, \phi_0 \) are expressed as the Eq. \([A4]\) with the substitution \( A_j \rightarrow a_j \). The linearized Euler-Lagrange equations for the fluctuations \( a_j, a_0, a_r, a_z \) are obtained as
\[ \frac{\partial^2}{\partial t^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} \frac{\partial}{\partial z} a = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \frac{\partial}{\partial z} a + \frac{1}{r^2} \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial t} a \]
\[ (A13) \]
\[ \frac{\partial^2}{\partial t^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} \frac{\partial}{\partial z} a = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \frac{\partial}{\partial z} a + \frac{1}{r^2} \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial t} a \]
\[ (A14) \]
\[ \frac{\partial^2}{\partial t^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} \frac{\partial}{\partial z} a = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \frac{\partial}{\partial z} a + \frac{1}{r^2} \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial t} a \]
\[ (A15) \]
\[ \frac{\partial^2}{\partial t^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} \frac{\partial}{\partial z} a = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \frac{\partial}{\partial z} a + \frac{1}{r^2} \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial t} a \]
\[ (A16) \]
\[ \frac{\partial^2}{\partial t^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} \frac{\partial}{\partial z} a = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \frac{\partial}{\partial z} a + \frac{1}{r^2} \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial t} a \]
\[ (A17) \]
It is important to notice that we can decouple the equations for the axial-symmetric fluctuations of \( \epsilon \) and the expanding the fluctuations with the Legendre polynomials. If we restrict ourselves to the \( l = 0 \) modes analysis, we can neglect angular-derivative terms in Eq. \([A13]-[A17]\). Therefore, we can treat the fluctuations \( \epsilon \) and \( a \) independently from fluctuations \( a_0, a_r, a_z \), because \( \epsilon \) and \( a \) are related to \( a_0, a_r, a_z \) only through the angular-derivative terms. As a result, we investigate the axial-symmetric fluctuations of \( \epsilon \) and \( a \), we have only to treat the Lagrangian \([23]\) as mentioned in Sec. IIIA.
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