A Cut-Off Grade Optimization Model in Open Pit Mining Considering Reclamation Cost and Revenue

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Abstract. In open pit mining industry, cut-off grade has an important role in affecting the total profit that will be earned by the company. In this research, cut-off grade optimization model is developed for open pit mining industry to maximize the total profit. We consider the environmental aspect in this model which consists of reclamation cost and reclamation revenue. We also consider the revenue of sales and valuable wasted materials revenue, and also the cost of selling stage, processing cost, mining cost, waste removal/rehabilitation cost and fixed cost. The results show that the model is able to determine the optimal cut-off grade and total profit that will be earned by the company can be estimated. Besides that, we can also estimate the completion time of mining project and the value of Net Present Value (NPV) and Return on Investment (ROI). The application of the model can be illustrated using numerical example that given in this study.

Keywords: Open Pit Mining, Cut-Off Grade, Reclamation Process, Analytical Solution

1. Introduction

Essentially, mining is a multidisciplinary knowledge of three scientific fields consists of geology, mining engineering, and economics [1]. Mineral mining is a fairly complex process technique and can last for decades [2]. There are two kind of mining methods widely used, namely open pit mining and underground mining. One of the challenges faced by open pit mining industry is determining the optimal cut-off grade of the mine to be processed.

The metal fraction contained in a rock mass is used by mining engineers to describe the term of grade. Cut-off grade is the minimum grade required in a certain rock mass to be considered as ore [3]. Material that does not exceed this minimum criterion is defined as waste. Mining companies must be smart enough to decide the optimal cut-off grade since the cut-off grade value will affect the cost and revenue components of the company.

Several studies have been conducted to find the optimal cut-off grade in open pit mining. The first basic model of determining cut-off grade through the breakeven approach was introduced by Hening [4] and followed by an algorithmic/heuristic approach introduced by Lane [5]. In addition to using the two approaches above, several research have been conducted in open pit mining to determine the optimal cut-off grade using a mathematical approach through analytic solutions.

Exploration and exploitation will directly affect the depletion of natural resources and the surrounding environment such as air, water, land, flora and fauna which should continue to be preserved and utilized optimally. Therefore the company has an obligation to carry out the land reclamation. The purpose of reclamation according to [6] is to achieve stability, security for humans...
and animals, recovery of landscape aesthetics, eliminate risks, increase the economic value of final land formation and improve company image.

Nowadays, one of the important issues to be considered by the company is the use of the mining area after reclamation (post mining activities). In a study conducted by Narrei and Osanloo [7], post-mining land uses can be grouped into eight possibilities (forestry, construction, agriculture, intensive recreation, non-intensive recreation, lake or pool, conservation, and backfilling). The type of utilization above will certainly provide additional economic value to the area after the mining process is completed and provides a higher investment value in mining projects.

This research will develop a mathematical model that can be used to determine the optimal cut-off grade to maximize company’s profit by considering environmental factors in terms of reclamation cost and reclamation revenue. The model considers sales revenue, valuable wasted material revenue, processing cost, selling stage cost, mining cost, waste removal/rehabilitation cost and fixed cost.

2. System Description

In this research, we developed an optimal cut-off grade model to maximize total profit in open pit mining industry. We used mathematical approach through analytic method to solve the model. Gama [1] is the first one who introduced this model by considering sales revenue, processing cost, waste removal/rehabilitation cost, mining cost, dan fixed cost. Some researches already developed this model with adding more variables as to solve the complexity of the real system in open pit mining itself.

Muttaqin and Rosyidi [8] developed Gama’s model by considering selling stage/marketing cost. Further, Muttaqin et al. [9] developed the model of [8] by adding two variables namely reclamation cost and revenue of valuable wasted material. In the latest research, Muttaqin et al. [10] continue to develop the model by adding the project selection issue to select the best mining project/location based on the value of NPV and ROI. In this research, we develop the model conducted by Muttaqin et al. [9] by considering reclamation revenue and estimate the time completion of mining project followed by calculating the value of NPV and ROI in open pit mining project.

The objective function of this model is to maximize the total profit with the cut-off grade as the decision variable. There are number of costs and revenues components influenced the total profit. The revenue components include sales revenue and reclamation revenue. The sales revenues consist of revenue from final product which depends on sales price per ton \( V \), average grade of ore \( T \), and recovery rate \( U \). The next is revenue from valuable wasted material which influenced by stripping ratio \( R \), valuable waste ratio \( A \) and revenue waste material per ton \( I \). The last one is reclamation revenue which influenced by reclamation revenue per ton \( L \). For cost components, first there is selling stage cost which influenced by recovery rate \( U \), average grade of ore \( T \) and selling stage cost per ton \( S \). The next one is reclamation cost which influenced by reclamation cost per ton \( C \). After that, there are mining cost and processing cost which respectively influenced by mining cost per ton \( M \) and processing cost per ton \( B \). Also we have waste removal cost which influenced by stripping ratio \( R \) and waste rehabilitation per ton \( E \). And the last one is fixed cost which influenced by indirect and fixed cost \( F \) and production rate \( P \).

3. Model Development

The notations of the proposed model are shown in Table 1. The total profit earned by the company is expressed in Equation (1).

\[
Y = Q \left[ \left( IAR + TUV + L \right) - \left( \frac{R}{P} + TUS + C + B + ER + M \right) \right] \tag{1}
\]
There are two important criteria before a company determines whether a mining project can provide benefits or not. The two criteria are maximum allowable stripping ratio \((R_{\text{max}})\) and minimum allowable cut-off grade \((T_{\text{min}})\). The profit can be generated if mining project has the lower stripping ratio \((R)\) compared to the maximum allowable stripping ratio \((R_{\text{max}})\) and the higher average grade of ore \((T)\) compared to the minimum allowable cut-off grade \((T_{\text{min}})\). Both of stripping ratio \((R)\) and average grade of ore \((T)\) can be determined using their extreme point which causes the function of total profit in Equation (1) to be zero.

The determination of minimum allowable cut-off grade is shown in Equation (2)

\[
0 = IAR + TUV + L - \left( \frac{F}{P} + TUS + C + B + ER + M \right)
\]

\[
T(UV - US) = \left( \frac{F}{P} + C + B + ER + M - IAR - L \right)
\]

\[
T_{\text{min}} = \frac{F + C + B + ER + M - IAR - L}{U(Y-S)}
\]  

The determination of maximum allowable stripping ratio is shown in Equation (3)

\[
0 = IAR + TUV + L - \left( \frac{F}{P} + TUS + C + B + ER + M \right)
\]
The next step after determining maximum allowable stripping ratio and minimum allowable cut-off grade is determining the optimal cut-off grade. According to [1], the relationship between cut-off grade \( (T_c) \) to average grade of ore \( (T) \), stripping ratio \( (R) \), and ore tonnage \( (Q) \) is linear and can be explained through mathematical equations, as shown in the Equation (4) through Equation (6).

\[
T = c_0 + c_1 T_c
\]  

(4)

\[
R = b_0 + b_1 T_c
\]  

(5)

\[
Q = a_0 + a_1 T_c
\]  

(6)

The three equations (4-6) are then substituted into the total profit function in Equation (1) which then forms the new total profit function shown in Equation (7).

\[
Y = (a_0 + a_1 T_c) \left( IA(b_0 + b_1 T_c) + UV(c_0 + c_1 T_c) + L - \left( \frac{E}{P} + SU(c_0 + c_1 T_c) + C + B + \right) (b_0 + b_1 T_c) E + M \right)
\]  

(7)

Then Equation (7) is derived towards \( T_c \) to get the optimal cut-off grade formula. Wolfram Mathematica 12.0 is used to solve the equation. The optimal cut-off grade formula is shown in equation (8). After determining the optimal cut-off grade, we can calculate the total profit using Equation (7).

\[
T_c^* = \frac{a_0 P \left( (E + AI) b_1 + (V - S) c_1 U - a_1 \left( P(B + C \cdot L + b_0 E - b_0 AI + M + c_0 SU - c_0 UV) + F \right) \right) \left( 2 a_1 P \left( c_1 U (S - V) + b_1 (E - AI) \right) \right)}{2 a_1 P \left( c_1 U (S - V) + b_1 (E - AI) \right)}
\]  

(8)

4. Numerical Example

In this paper, a numerical example is given to illustrate the implementation of the optimal cut-off grade model. The datas of iron ore grade distribution are shown in Table 2. In addition, economic parameters and some datas are also given in Table 3. These datas are obtained from latest research conducted by Benazir et al. [10]
The first step to determine the optimal cut-off grade is to calculate the minimum allowable cut-off grade. Using Equation (2), the value of minimum allowable cut-off grade is found to be 30.64%. Based on that value we can conclude that the mining activity is profitable since the average grade of iron ore for the minimum stripping ratio (R=0) is higher than minimum allowable cut-off grade (33.24%>30.64%).

After concluding that the mining project is profitable, the next step is to determine the regression coefficient values of variables $T$, $Q$ and $R$ using Equation (4), (5), (6) respectively. We analyze it using a statistical software. The regression formula for those variables shown in Table 4 are well fit by looking at the $R^2$ values which are mostly quite close to 100%.

The coefficient value obtained is then substituted into Equation (8) to get the optimal cut-off grade and then calculate the maximum profit using Equation (7). The results obtained show that the optimal cut-off grade is 22.67% with a maximum total profit is $3,838,153,287.
In case of the company wants to know the values of NPV and ROI of their mining project, we can calculate it by determining the time completion of mining project/locations. We can obtain the time completion by comparing the tonnage of mined material \( Q_m \) and tonnage of iron ore \( Q_p \) to the capacity limits for each activities (mining, processing and marketing). Those processes are sequential in mining production system and we can use each limiting stage to estimate the completion time of the mining. The data of each capacity process is given in Table 5.

The mining and processing stages use the amount of mined material \( Q_m \) as in Equation (9), while the marketing stage uses the amount of ore material \( Q_p \) as in Equation (10). From Equation (9) and (10), we get the value of \( Q_m \) and \( Q_p \) respectively are 42,769,374 tons and 10,811,684 tons. By dividing \( Q_m \) and \( Q_p \) to each capacity process we can estimate the time completion of mining project as shown in Table 5. From the table, the project will be done within 14.26 years or rounded up into 15 years.

\[
Q_m = a_0 + a_1 T_c \tag{9}
\]
\[
Q_p = Q_m \times (c_0 + c_1 T_c) \times U \tag{10}
\]

### Table 4. Regression Coefficient Calculation for T, Q and R

| Cut-off grade (%Fe) | Ore Reserves (ton) | Stripping Ratio | Average Grade of Ore |
|---------------------|-------------------|-----------------|----------------------|
| 20%                 | 45,233,750        | 0.000           | 33.24%               |
| 25%                 | 43,125,000        | 0.049           | 34.16%               |
| 30%                 | 23,843,250        | 0.897           | 35.76%               |
| 35%                 | 6,267,250         | -               | 37.87%               |
| 1st coefficient    | 104,517,000       | -1.927          | 0.267                |
| 2nd coefficient    | -272,362,500      | 8.971           | 0.309                |
| R2                  | 92.30%            | 79.10%          | 97.10%               |

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After determining the time completion of mining project, we can calculate the NPV and ROI. First of all it is assumed that the profit is distributed equally at each year. With the total profit of $3,838,153,287 then the project will generate profit of $255,876,886 per year. It is also assumed that the investment cost for this mining project is $300,000,000 with the discount rate of 12% per year. The NPV and ROI values can be calculated using Equation (11) and equation (12). From the result shown in Table 6, the value of NPV is $1,442,742,795 with ROI of 481%.

\[
NPV = -C_0 + \frac{c_1}{1+r^t} + \frac{c_2}{(1+r)^t} + \ldots + \frac{c_t}{(1+r)^t} \tag{11}
\]

Where: \( r \): discount/interest rate
\( t \): time of the cash flow (year)

\[
ROI = \frac{NPV}{Investment} \tag{12}
\]

### Table 5. Process Capacity and Estimated Years of Completion Time

| Process Stage | Capacity | Units                        | Estimated Years |
|---------------|----------|------------------------------|-----------------|
| Mining        | 3,000,000| ton ore/year                 | 14.26           |
| Processing    | 3,500,000| ton ore/year                 | 12.22           |
| Marketing     | 2,400,000| ton iron (Fe) ore concentrate /year | 4.50       |
Table 6. NPV Calculation

| Year | Profit ($) | Investment Cost ($) | PV ($)      | PV Cum. ($) |
|------|------------|---------------------|-------------|-------------|
| 0    | -300,000,000 | -300,000,000        | -300,000,000|             |
| 1    | 255,876,886  | 228,461,505         | -71,538,495 |             |
| 2    | 255,876,886  | 203,983,487         | 132,444,992 |             |
| 3    | 255,876,886  | 182,128,113         | 314,573,105 |             |
| 4    | 255,876,886  | 162,614,387         | 477,187,492 |             |
| 5    | 255,876,886  | 145,191,417         | 622,378,909 |             |
| 6    | 255,876,886  | 129,635,194         | 752,014,102 |             |
| 7    | 255,876,886  | 115,745,709         | 867,759,811 |             |
| 8    | 255,876,886  | 103,344,383         | 971,104,193 |             |
| 9    | 255,876,886  | 92,271,770          | 1,063,375,963 |         |
| 10   | 255,876,886  | 82,385,509          | 1,145,761,473 |         |
| 11   | 255,876,886  | 73,558,490          | 1,219,319,963 |         |
| 12   | 255,876,886  | 65,677,223          | 1,284,997,186 |         |
| 13   | 255,876,886  | 58,640,378          | 1,343,637,564 |         |
| 14   | 255,876,886  | 52,357,480          | 1,395,995,045 |         |
| 15   | 255,876,886  | 46,747,750          | 1,442,742,795 |         |

5. Conclusion
In this study, we developed a mathematical model to determine optimal cut-off grade considering reclamation process which consists of reclamation revenue and cost compared to previous studies that have been conducted. The mining company can make decisions accurately, quickly and easily can be adapted into the real system condition by using the analytic method developed in this study. Mining company should consider these kind of environmental aspects in determining cut-off grade. Because mining is not about exploration and exploitation, but also considers environmental responsibility to protect environmental sustainability. Based on the results, it shows that cut-off grade has a crucial role decision in open pit mining project. Besides affecting the profit, it will also affecting the value of NPV and ROI. The model in this paper can be further developed by considering price uncertainty to make the model closer to the real problem.

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