The role of a higher derivative bulk scalar in stabilizing a warped spacetime

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Abstract
The back-reaction on the Randall–Sundrum warped spacetime is determined in the presence of a scalar field in the bulk. A general condition for the stability of such a model is derived for a bulk scalar field action with non-canonical higher derivative terms. It is further shown that the gauge hierarchy problem can be resolved in such a stabilized scenario by appropriate choice of various parameters of the theory.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction
Braneworld models have gained considerable attention in recent years. A wide number of applications of such models provide new ways to encounter some of the unsolved problems in physics. From the particle physics point of view one of the intriguing problems is the well-known ‘gauge hierarchy’, which is related to the mass renormalization of the standard model Higgs boson due to radiative corrections. The Higgs boson appears naturally in the standard model (SM) so as to give appropriate masses to the other fundamental bosons and fermions. While the mass of the Higgs boson cannot be stabilized due to the lack of symmetry, it is phenomenologically suggested to be in the range of TeV scale. The radiative corrections draw this TeV scale mass to the Planck scale, leading to the well-known fine tuning problem of the Higgs mass. One possible solution of such a problem is supersymmetry, which is the symmetry between fermions and bosons. However, such a new symmetry leads to a large number of superpartner particles corresponding to all of the SM fundamental particles. Lack of experimental signatures of these superpartners to date puts serious constraints on supersymmetry. A possible alternative way of resolving the hierarchy problem is provided...
by the large or warped compact extra-dimensional models, proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) and Randall–Sundrum (RS) [1, 2]. The main essence of such models is a geometric realization of the different scales in a theory. One important ingredient in this approach is the localization of all SM fields on a three-dimensional hypersurface, namely a ‘3-brane’ [1–7], while only the fields coming from the gravity sector can propagate through the bulk. In the 2-brane RS setup, the brane with negative tension, i.e., the (visible) TeV brane on which all the SM fields live, is at one end, while the brane with positive tension, the so-called (hidden) Planck brane, is situated at the other end of the extra dimension. The geometry of the extra dimension is a circle modded out by the $Z_2$ symmetry, namely, $S^1/Z_2$ orbifold. The two ends of the extra dimension correspond to the two orbifold fixed points. The model contains various parameters, the bulk cosmological constant $\Lambda$, the brane tensions ($V_{\text{vis}}, V_{\text{hid}}$) corresponding to the visible and hidden branes respectively, and the brane separation $r_c$. Now, the static solution for the bulk Einstein equations [2] leads to a relation between the bulk and brane cosmological constants $V_{\text{hid}} = -V_{\text{vis}} = \sqrt{-24\Lambda M^3}$ ($M$ being the five-dimensional Planck mass) preserving the four-dimensional Poincaré invariance. For the desired warping one should have $kr_c \sim 12(k = \sqrt{-\Lambda/(24M^3)}$, where the bulk spacetime is an anti-de Sitter spacetime with a negative cosmological constant. If $k \sim$ Planck mass, then $r_c$ should have a value near the Planck length. To stabilize the value of this extra dimension, Goldberger and Wise (GW) [8] proposed a simple mechanism by introducing a massive scalar field with the usual (canonical) kinetic term in the bulk. The aim had been to obtain an effective four-dimensional potential for the modulus $r_c$ by plugging back the classical solution for the scalar field in the bulk action and integrating out along the extra compact dimension. The minimum of this effective potential gives us the stabilized value of the compactification radius, $r_c$. Several other works have been done in this direction [9–15]. However, in the calculation of GW [8], the back-reaction of the scalar field on the background metric was ignored. Such a back-reaction was included later in [16] and exact solutions for the background metric and the scalar field have been given for some specific class of potentials, motivated by five-dimensional gauged supergravity analysis [17]. Assigning appropriate values of the scalar field on the two branes, a stable value of the modulus $r$ was estimated from the solution of the scalar field. It should however be noted that the scalar action in [13, 16] is inspired by a supergravity-based model and has no correspondence with that in [8] even in a limiting sense. These two models correspond to two different classes of bulk scalar actions such that for [13, 16] one can calculate the exact warp factor with the full scalar back-reaction, while Goldberger and Wise [8] examine stability of the modulus by ignoring the back-reaction of the bulk scalar.

Another interesting work is in the context of a scalar field potential with a supersymmetric form, where it has been shown that the resulting model is stable [18].

In a previous work [19], starting from the scalar bulk and boundary terms in the action, we re-examined the stability issue by generalizing the work of Goldberger and Wise [8]. Exploring into the region of the parameter space beyond the approximations adopted by Goldberger and Wise, we bring out some interesting features of the modulus potential and the corresponding stability conditions.

In this work, we examine the modulus stabilization by resorting to the usual modulus potential calculation and its subsequent minimization for a very general class of bulk scalar actions. Keeping the non-canonical as well as higher derivative term in the bulk scalar action we find the general condition for the modulus stabilization for the back-reacted RS model [2]. We exhibit the role of higher power of the derivative terms in stabilizing the modulus as well as resolving the gauge hierarchy problem.

The plan of this paper is as follows: in section 2, we begin with a general bulk scalar action and find the condition on the back-reacted warp factor to achieve modulus
stabilization by estimating the derivative of the modulus potential without resorting to any specific choice for the bulk scalar action. In section 3, we find the solutions for the bulk scalar and the warp factor for some specific choices for the bulk scalar action along the line of [13, 14, 16]. In section 4, following the GW approach [8], we perform a complete stability analysis of scalar back-reacted RS models [2] when the higher derivative and non-canonical terms are present. Here, we determine the correlations among the various parameters in the scalar potential along with the parameters associated with the non-canonical and higher derivative terms and the corresponding stabilized value of the brane separation modulus $r$ which resolves the electroweak gauge hierarchy problem.

2. General issues

We start with a general action similar to that in our earlier work [20]. In recent years, there have been many models where the presence of a bulk scalar field is shown to have an important role in the context of stability issue of braneworld scenario, bulk-brane cosmological dynamics, higher-dimensional black-hole solutions and also in many other phenomenological issues in particle physics [8, 16, 21–24]. Here, we resort to a somewhat general type of self-interacting scalar field along with the gravity in the bulk in order to analyze the stability of the RS-type two-brane model. We consider the following five-dimensional bulk action:

$$S = \int d^5x \sqrt{-g}[-M^3 R + F(\phi, X) - V(\phi)] - \int d^4x dy \sqrt{-g} \delta(y - y_a) \lambda_a(\phi),$$

(1)

where $X = \delta_\phi \delta^A \phi$, with ‘$A$’ spanning the whole five-dimensional bulk spacetime. The index ‘$a$’ runs over the brane locations and the corresponding brane potentials are denoted by $\lambda_a$. The scalar field is assumed to be only the function of the extra spatial coordinate $y$. We emphasize at this point that we have taken a generalized scalar action only in terms of higher powers of $x$, i.e. have not included the dependence on the second derivative of $\phi$. Inclusion of such a term in the scalar Lagrangian density will imply a modified equation of motion with generalized Cauchy initial conditions. We do not address this issue here. Moreover, string-inspired models like tachyon-like action yield a dependence on higher powers $X$ only as chosen by us. Taking the metric as

$$ds^2 = e^{-2A(y)} \eta_{\mu \nu} dx^\mu dx^\nu - dy^2,$$

(2)

where $\{y\}$ is the extra compact coordinate with radius $r_c$ such that $dy^2 = r_c^2 d\theta^2, \theta$ being the angular coordinate. The field equations turn out to be

$$F_X \phi'' - 2F_X \phi' \phi'' = 4F_X \phi' A' - \frac{\partial F_X}{\partial \phi} \phi'^2 - \frac{1}{2} \left( \frac{\partial F}{\partial \phi} - \frac{\partial V}{\partial \phi} \right) + \frac{1}{2} \sum_a \frac{\partial \lambda_a(\phi)}{\partial \phi} \delta(y - y_a),$$

(3a)

$$A'^2 = 4C F_X \phi'^2 + 2 |F(X, \phi) - V(\phi)|,$$

(3b)

$$A'' = 8C F_X \phi'^2 + 4C \sum_a \lambda_a(\phi) \delta(y - y_a),$$

(3c)

where

$$C = \frac{1}{24M^3}, \quad F_X = \frac{\partial F(X, \phi)}{\partial X}, \quad F_{XX} = \frac{\partial^2 F(X, \phi)}{\partial X^2}.$$
and prime [''] denotes partial differentiation with respect to $y$. Two of the above equations (3) are independent and the other one automatically follows from the energy conservation in the bulk.

The boundary conditions are

$$2 (F_X \phi') |_{y=0} = \frac{1}{2} \frac{\partial \lambda_0 (\phi_0)}{\partial \phi},$$
$$-2 (F_X \phi') |_{y=\pi r_c} = \frac{1}{2} \frac{\partial \lambda_\pi (\phi_\pi)}{\partial \phi}. \tag{4}$$

$$2 A' (y) |_{y=0} = 4 C \lambda_0 (\phi_0),$$
$$-2 A' (y) |_{y=\pi r_c} = 4 C \lambda_\pi (\phi_\pi). \tag{5}$$

Now, without knowing the solutions to the above equations explicitly, we may analyze the stability of the modulus $r_c$, following the mechanism developed by Goldberger and Wise [8]. The brane separation $r_c$, in general, is a dynamical variable associated with the metric component $g_{55}$. Integrating out the scalar field action over the extra coordinate $y$ in the background of the back-reacted five-dimensional metric, the four-dimensional effective potential for the modulus $r_c$ is obtained as

$$V_{\text{eff}} (r_c) = -2 \int_0^{r_c \pi} dy \ e^{-4 A(y)} [-M^3 R + F(X, \phi) - V(\phi)]$$
$$+ e^{-4 A(0)} \lambda_0 (\phi_0) + e^{-4 A(\pi)} \lambda_\pi (\phi_\pi). \tag{6}$$

It may be noted that the effective potential is calculated with the warp factor $A(y)$, which takes care of the full back-reaction of the scalar field on the 5D metric through the equations of motion. Therefore, the effective potential for the modulus $r_c$ is calculated by integrating out the full action in the five-dimensional background back-reacted metric. The modulus $r_c$ in general can be a dynamical variable and the minimum of its effective potential determines the corresponding stable value. The role of the bulk scalar field here is to stabilize the modulus associated with $g_{55}$ in the bulk five-dimensional background spacetime.

Now, using the above two boundary conditions and expression for the potential from the above equation of motion

$$V (\phi) = -\frac{1}{2C} A'^2 + 2F_X \phi'^2 + F(X, \phi), \tag{7}$$

and also the expression for the Ricci scalar $R = 20A'(y)^2 - 8A''(y)$, one gets the expression for the effective potential as

$$V_{\text{eff}} = -16 M^3 [A'(0) + A'(r_c \pi) e^{-4 A(r_c \pi)}]. \tag{8}$$

Now, taking derivative with respect to $y = \pi r_c$, one finds, by the use of the equations of motion (3), the following algebraic equation:

$$\frac{\partial V_{\text{eff}} (r_c)}{\partial (\pi r_c)} = 16 M^2 e^{-4 A(y)} [A''(y) - 4A'(y)^2]_{y=r_c}, \tag{9}$$

where $A(y)$ are specific solutions of equations (3).

In order to have an extremum for $V_{\text{eff}}$ at some value of $r_c$, the right-hand side of equation (9) must vanish at that (stable) value of $r_c$. This immediately implies that the value $A''(y)$ must be positive and equals the value of $4A'(y)^2$ at $y = r_c$. Thus for different classes of back-reacted solutions for the warp factor for different bulk scalar actions, the above condition determines the corresponding stable value of the modulus $r_c$.

3. General solution

Let us now resort to the general solutions of the full set of field equations (3).
We start with the case of a bulk scalar action with a simple non-canonical kinetic term, with the usual quadratic first derivative term of the scalar field:

$$F(X, \phi) = f(\phi) X,$$

(10)

where $f(\phi)$ is any well-behaved explicit function of the scalar field $\phi$. Let us assume that $f(\phi) = \partial g(\phi)/\partial \phi$, where $g(\phi)$ is another explicit function of $\phi$. Then for a specific form of the potential,

$$V(\phi) = \frac{1}{16} \left( \frac{\partial g}{\partial \phi} \right)^2 - 2C W(g(\phi))^2.$$

(11)

This generalizes the expression for the potential for the scalar field in the presence of non-canonical term. It is straightforward to verify, for some $W(g(\phi))$ and $g(\phi)$, that a solution to

$$\phi' = \frac{1}{4} \frac{\partial W}{\partial g} A' = 2C W(g(\phi))$$

(12)

is also a solution to equations (3), provided

$$[g(\phi) \phi]_a = \frac{1}{2} \frac{\partial \lambda_a}{\partial \phi_a}(\phi_a), \quad [A']_a = 4C \lambda_a(\phi_a).$$

(13)

It may be observed that the expression for the potential for the scalar field as obtained in equation (11) generalizes that obtained in [16] because of the presence of non-canonical kinetic term for the scalar field. For $g(\phi) = \phi$ the scalar kinetic term becomes canonical and we reproduce the expression for $V(\phi)$ as obtained by DeWolfe et al [16].

Now, let us consider a more general case to include higher derivative term such as

$$F(X, \phi) = K(\phi) X + L(\phi) X^2.$$

(14)

One of the motivations to consider this type of term in the Lagrangian originates from string theory [25]. The low-energy effective string action contains higher-order derivative terms coming from $\alpha'$ and loop corrections, where $\alpha'$ is related to the string length scale $\lambda_s$ via the relation $\alpha' = \lambda_s/2\pi$. The four-dimensional effective string action is generally given as

$$S = \int d^4x \sqrt{-g} [B_{\phi}(\phi) \tilde{R} + B_{\phi}^{(0)} X + \alpha'(c_1(1) B_{\phi}^{(1)} X^2 + \cdots) + O(\alpha'^2)],$$

(15)

where $\phi$ is the dilaton field that controls the strength of the string coupling $g_s^2$ via the relation $g_s^2 = e^\phi$. In the weak coupling regime, the coupling function has the dependence $B_{\phi} \simeq B_{\phi}^{(0)} \simeq B_{\phi}^{(1)} \simeq e^\phi$. If we make a conformal transformation $g_{\mu\nu} = B_{\phi}(\phi) \tilde{g}_{\mu\nu}$, the string-frame action (15) is transformed to the Einstein-frame action [25, 26] as

$$S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R + K(\phi) X + L(\phi) X^2 + \cdots \right].$$

(16)

where

$$K(\phi) = \frac{2}{3} \left( \frac{1}{B_{\phi}} \frac{dB_{\phi}}{d\phi} \right)^2 - B_{\phi}^{(0)} \frac{B_{\phi}^{(1)}}{B_{\phi}^{(0)}}, \quad L(\phi) = 2c_1(1) \alpha' B_{\phi}^{(1)}(\phi).$$

(17)

Thus the four-dimensional Lagrangian involves a non-canonical kinetic term for the scalar field, precisely in the same form as that which we are considering here in five dimensions (equation (14)). With an appropriate redefinition of the scalar field we can recast such a term in the Lagrangian as

$$F(X, \phi) = f(\phi)[X - \beta X^2].$$

(18)
where $\beta$ is a constant parameter (in this case it is equal to unity) and $\{X, \phi\}$ are new variables in terms of old variables. This type of action is common in the K-essence cosmological inflationary models [27]. In what follows, we will be using this K-essence-type scalar action in the five-dimensional bulk with a suitable potential function. The scalar field is, however, considered only a function of the extra (fifth) dimension $y$.

Now, assuming $f(\phi) = \partial g(\phi)/\partial \phi$, for a specific form of the potential

$$V(\phi) = \frac{1}{16} \left( \frac{\partial g}{\partial \phi} \right)^2 \left[ \frac{\partial W}{\partial h} \right]^2 + \frac{3 \beta}{256} \left( \frac{\partial g}{\partial \phi} \right) \left[ \frac{\partial W}{\partial h} \right]^4 - 2CW^2. \quad (19)$$

This form of the potential is a further generalization of the scalar field potential in [16] in the presence of higher powers of the quadratic kinetic term in addition to the non-canonical kinetic term. For some arbitrary $W(h(g(\phi)))$ and $g(\phi)$, it is straightforward to verify that a solution to

$$\phi' = \frac{1}{4} \frac{\partial W}{\partial h}, \quad A' = 2CW(h(g(\phi))), \quad (20)$$

with the constraint relation

$$\frac{dh}{dg} = 1 + \frac{\beta}{8} \left( \frac{\partial W}{\partial h} \right)^2, \quad (21)$$

is also a solution to equations (3), provided we have

$$[g(\phi)(1 - 2\beta X)\phi']_u = \frac{1}{2} \frac{\partial \lambda_u}{\partial \phi_u}(\phi_u), \quad [A']_u = 4C\lambda_u(\phi_u). \quad (22)$$

In the $\beta \to 0$ limit we at once get back the system of equations dealing with only the simple non-canonical kinetic term (10) in the scalar action, as discussed in the first part of this section. If in addition, $f(\phi) \to 1/2$, one deals with the usual canonical kinetic term which has been discussed in detail in [16]. Thus, we have generalized the DeWolfe et al proposed form of the scalar potential [16] by $g(\phi)$ to include the non-canonical term and further defining $h(g(\phi))$, we have incorporated the effects of the higher-order derivative terms so that in both the cases an exact back-reacted solution of the warp factor can be obtained.

Considering now

$$W(h) = \frac{k}{2C} - uh^2(g(\phi)), \quad g(\phi) = \alpha \phi, \quad (23)$$

where $k \sim u$ and $\alpha$ are the initial constant parameters of our model with their appropriate dimensions, equation (21) gives the solution for $h(g(\phi))$:

$$h(g(\phi)) = \frac{1}{u\sqrt{\beta}/2} \tan(u\sqrt{\beta}/2\alpha \phi). \quad (24)$$

Clearly, in the limit $\beta \to 0$ we have $h = g$, which corresponds to what only the simple non-canonical kinetic term gives us (equations (11), (12)).

From equation (20) the solution for $\phi$ becomes

$$\sin(u\sqrt{\beta}/2\alpha \phi) = A_0 e^{-u\alpha y/2}, \quad (25)$$

where $A_0 = \sin(u\sqrt{\beta}/2\alpha \phi_0)$ with $\phi|_{y=0} = \phi_0$. The solution for the warp factor $A(y)$ takes the form

$$A(y) = ky - \frac{4C}{\beta u^2\alpha} \ln \left( 1 - A_0^2 e^{-u\alpha y} \right). \quad (26)$$

This is the exact form of the back-reacted warp factor where the first term on the right-hand side is the same as that obtained by RS in the absence of any scalar field and the second term originates from the effect of back-reaction due to the bulk scalar.
In the limit $\beta \to 0$ and $\alpha = \text{const.}$

\[ A(y) = ky + 2C\alpha \phi^2_0 e^{-u_\alpha y} \]  

(27)

which is exactly what has been discussed in [16]. It is now simple to determine the stable value for the modulus by using the minimization condition given by equation (9).

4. Stability analysis

Following the Goldberger–Wise mechanism [8], we now analyze the stability of our specific model, with $F(X, \phi)$ given by equation (18) in the preceding section. From equation (9), we have the extremality condition in a generic situation:

\[ 4A'^2 - A'' = 0. \]  

(28)

This is an algebraic equation given in terms of the parameter $r_c$, i.e., the compactification radius of extra dimension $y$. $A(y)$ are the classical solutions of the field equations (3). Now, putting the expressions for the various derivatives of the metric solution $A$ in equation (28) one gets the following quadratic equation:

\[ QY^2 + PY - \beta u_\alpha^2 k^2 C = 0, \]  

(29)

where various notations are:

\[ Q = u_\alpha^2 [1 - \frac{16C}{\beta u_\alpha^2}], \quad P = u_\alpha(8k + u_\alpha), \quad Y = \frac{A_0^2 e^{-u_\alpha y}}{1 - A_0^2 e^{-u_\alpha y}}. \]  

(30)

At this point it is worth noting that the numerical value of $Y$ should be positive.

From the above equation clearly, we can have two different cases.

Case (i). $Q > 0$, then the above equation has only one real solution considering the fact that $Y > 0$. The corresponding root is

\[ Y = \frac{1}{2Q} \left( \sqrt{P^2 + \frac{4Q\beta u_\alpha^2 k^2}{C}} - P \right), \]  

(31)

which gives us the maximum of the potential. So, the point we get is unstable.

Case (ii). $Q < 0$, which in turn says $\beta \ll 1$ provided $u \sim$ Planck scale, then we have two solutions

\[ \tilde{Y}_\pm = \frac{1}{2Q} \left( P \pm \sqrt{P^2 - \frac{4Q\beta u_\alpha^2 k^2}{C}} \right), \]  

(32)

where $|Q|$ is the absolute value of $Q$. As we have checked that the larger value of $Y = \tilde{Y}_+$ gives us the stable point for the modulus. So, naturally, the lower value $Y = \tilde{Y}_-$ gives the unstable point.

So, corresponding to these minimum $\tilde{Y}_+$ and maximum $\tilde{Y}_-$ of the effective radion potential, one gets respective distance moduli $y^\pm_\pi$ as

\[ y^\pm_\pi = \frac{1}{u_\alpha} \log \left[ \frac{(P + 2|Q|) \pm \sqrt{P^2 - \frac{4|Q|\beta u_\alpha^2 k^2}{C}}}{P \pm \sqrt{P^2 - \frac{4|Q|\beta u_\alpha^2 k^2}{C}}} A_0^2 \right]. \]  

(33)

Now, we are interested in studying our solution for the metric as well as the scalar field at this stable point. Before we start, it is useful to define some dimensionless parameters out of the various known dimensionful parameters as

\[ m = k/(u_\alpha), \quad n = 4C/(u_\alpha^2 \beta). \]  

(34)
From now on, we will read out the various expressions in terms of these new parameters. We are interested in studying the metric at the stable point $y^+ = k \pi r_s$, where $r_s$ is said to be the stable distance between the two branes, 

$$y^+ = \frac{m}{k} \ln \left[ \frac{(1 + \tilde{Y}^+_+)^2}{\tilde{Y}^+_+} - n \ln \left[ \frac{1}{1 + \tilde{Y}^+_+} \right] \right].$$

(36)

So, by using the above expressions, we get the expression for the stable value of $A(r_s)$ as 

$$A(r_s) = m \ln \left[ \frac{(1 + \tilde{Y}^+_+)^2}{\tilde{Y}^+_+} - n \ln \left[ \frac{1}{1 + \tilde{Y}^+_+} \right] \right].$$

(37)

So, from the above expression for the stable value of warp factor $A(r_s)$, we can calculate the values of the set of parameters $\alpha, \beta, u, k$ and $\phi_0$ of our model from various phenomenological constraints. For simplicity, we will do our analysis by setting the parameter $A_0 = \sin(\sqrt{\beta/2}u\phi_0) = 1$.

The main issue addressed in the RS two-brane model [2] is the large mass hierarchy from an extra dimension whose compactification radius is as small as of the order of Planck length ($l_p \sim 10^{-33}$ cm). In the present scenario, because of the scalar field in the bulk, the modified warp factor $A(y)$ is given by equation (26). Now, to get the acceptable hierarchy between the two fundamental scales, the modulus $r_c$ must be stabilized to a value $r_s$ such that the warp factor $A(r_s) \sim 36$. Figures 1 and 2 depict the variation of the value of $A(y)$ with the parameters $m$ (for fixed $n$) and $n$ (for fixed $m$), whereas in figure 3 we show the surface plot of the warp factor versus the two dimensionless variables $m = k/(u\alpha)$ and $n = 4C/(u^2\beta\alpha)$. Finally, a numerical plot between $m$ and $n$ corresponding to a fixed value $A(rs) = 36$ up to an error $\sim 2 \times 10^{-8}$ is shown in figure 4. It is evident that we can get an infinite number of values for the above parameters by which we can resolve the hierarchy problem in connection with the Higgs mass.
Figure 2. Variation of warp factor of the metric $A(r)$ with parameter $n = 4C/(u^2 \beta \alpha)$ setting fixed values ($= 1, 5, 10, 20$ and $30$) of the other parameter $m = k/(ua)$ and $A_0 = \sin(u \sqrt{\beta/2 \alpha \phi_0}) = 1$.

Figure 3. Variation of warp factor of the metric $A(r)$ with both the parameters $m = k/(ua)$ and $n = 4C/(u^2 \beta \alpha)$ with $A_0 = \sin(u \sqrt{\beta/2 \alpha \phi_0}) = 1$.

Now, we will try to estimate the values of the parameters $\alpha$, $\beta$, $u$, $k$ and $\phi_0$ for some values of $m$ and $n$. Since, we have a total of five free parameters in our model, we can specify any two of them by suitably defining a combination of all of them. Let us choose $k$ and $\alpha$ from the physical point of view, to be $\alpha = 1/2$ (making the scalar kinetic term canonical) and $k \sim M$. Then, we get the plot shown in figure 5. It is interesting to note that for values of $\beta \ll \alpha$ the stability requirement of the braneworld model is satisfied.
Figure 4. Variation of $n = 4C/(\alpha \beta u^2)$ with the variation of $m = k/(\alpha u)$ keeping fixed the value of warp factor $A(r_s) = 36$ which approximately gives the scaling down of the Planck scale to TeV scale.

Figure 5. Variation of $\beta$ with $u$ for which we get $A(r_s) \sim 36$ with fixed values for $\alpha = 1/2$ and $k \sim M$.

5. Conclusion

Starting from a general action with a higher derivative, non-canonical kinetic term for a bulk scalar in a RS-type two-brane model, we have derived the modulus stabilization condition on the scalar action in a back-reacted warped geometry. This result generalizes our earlier work [20] where a specific form of the higher derivative term namely tachyon-like action was shown to stabilize the model. Calculating the exact form of the warp factor with full back-reaction, the naturalness issue in the context of Higgs mass has been explored. A large set of values of
the parameters of the bulk scalar potential and the parameters responsible for the presence of non-canonical and higher derivative terms are shown to produce the desired warping from the Planck scale to the TeV scale as a resolution to the well-known gauge hierarchy problem. The correlation among these parameters is determined. This work thus determines the conditions on a very general bulk scalar action in a back-reacted Randall–Sundrum braneworld so that the gauge hierarchy problem as well as the problem of modulus stabilization can be resolved together.

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