Aspects of meson properties in dense nuclear matter

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We investigate the modification of meson spectral densities in dense nuclear matter at zero temperature. These effects are studied in a fully relativistic mean field model which goes beyond the linear density approximation and also includes baryon resonances. In particular, the role of $N^*(1520)$ and $N^*(1720)$ on the $\rho$ meson spectral density is highlighted. Even though the nucleon-nucleon loop and the nucleon-resonance loop contribute with the opposite sign, an overall reduction of $\rho$ meson mass is still observed at high density. Importantly, it is shown that the resonances cause substantial broadening of the $\rho$ meson spectral density in matter and also induces non-trivial momentum dependence. The spectral density of the $a_0$ meson is also shown. We study the dispersion relations and collective oscillations induced by the $\rho$ meson propagation in nuclear matter together with the influence of the mixing of $\rho$ with the $a_0$ meson. The relevant expression for the plasma frequency is also recovered analytically in the appropriate limit.

I. INTRODUCTION

Electromagnetic radiation constitutes a privileged probe of the properties of matter under extreme conditions. This owns partly to the fact that it decouples from the strongly interacting system without significant rescattering and also because the virtual photons enjoy a direct coupling to vector mesons. Lepton pairs carry thus valuable information about the in-medium properties.

Among the light vector mesons, the $\rho$ acquires a special importance because of its large decay width. Therefore, this might serve as a chronometer and thermometer to probe temporarily produced hot and dense hadronic matter. Even though $\omega$ or $\phi$ mesons do not have this desired short lifetime, in the medium they might undergo sufficient broadening leading to a shorter lifetime [1]. For the present purpose, however, we first mostly concentrate on the $\rho$ meson and we discuss the scalar-isovector sector later on.

The in-medium properties of the $\rho$ meson have been estimated in variety of models ranging from QCD sum-rules [2] to chiral models like Nambu-Jona Lasinio, effective hadronic Lagrangian approaches [3] and mean-field models. It is fair to say that, at this point, a clear consensus is still lacking but that important progress has been realized over the past few years. For a review see [4]. The angle of this work consists of uniting several physical aspects we find important but that had not been treated together in a unique approach. So here, the $\rho$ spectral density in dense nuclear matter is studied and the importance of the $N^*(1520)$ and $N^*(1720)$ is reiterated, but a relativistic calculation is performed and we go beyond the linear density approximation (LDA). A relativistic calculation has recently been presented in [5] in the LDA. We show that LDA is a good approximation for densities below nuclear matter density, but for higher densities multiple scattering becomes important. We report a quantitative comparison of the results obtained in the linear approximation with a resummed one loop calculation. We also incorporate the effect of interacting nuclear matter through the scalar and vector meson mean fields, motivated by the Walecka model. The role of resonances and that of the nucleon loops are examined separately. Finally, we include the recently discussed mixing effects [6,7], and report on the spectral density of the $a_0$ for the first time.

The paper is organized as follows. First the formalism is outlined followed by a discussion of the $\rho$ meson properties involving nucleons. Then we consider the effect of the resonances on the in-medium spectral densities. Later we discuss the effect of mixing. We also present the spectral density of the $a_0$ meson which supplements our understanding of the mixed propagator of the $\rho$ in nuclear matter. The calculations are done in a fully relativistic formalism including the effect of the mean field. At places the mathematical details are relegated to the appendix. Finally we discuss the results and conclude.

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II. FORMALISM

It is well known that the spectral density is actually the imaginary part of the propagator which in turn is related with the polarization functions of the meson. Therefore, we first discuss the properties of the ρ meson polarization function in dense nuclear matter.

Essentially, the spectral density is related with the collective excitation induced by the ρ meson by its propagation in nuclear matter. This is analogous to the photon propagation in QED plasma where the propagating particle picks up the collective modes from the system arising out of the density fluctuation. This is commonly known as plasma oscillation. Even though in the present work the main focus is not to recover the characteristic features of the plasma oscillation induced by the ρ meson, nevertheless, we outline the formalism à la Chin in order to be able to discuss the spectral density in terms of the dielectric response function of the nuclear matter. This enables us to incorporate the effect of meson mixing in a straightforward manner.

The ρ meson, being a massive spin one particle, can have both longitudinal and transverse excitations depending upon whether its momenta is perpendicular or parallel to the spin. Furthermore, in matter the Lorentz symmetry is broken and these two modes will have different characteristic features. These states are designated as \( \Pi_L(q_0, |q|) \) and \( \Pi_T(q_0, |q|) \) with \( L \) and \( T \) denoting longitudinal and transverse modes. As already mentioned, we consider the coupling of ρ meson with n-n, n-R and π-π states and therefore what we have is the following

\[
\Pi^{L(T)} = \Pi^{L(T)}_{nn} + \Pi^{L(T)}_{Rn} + \Pi^{L(T)}_{\pi\pi},
\]

with \( R = N^*(1520), N^*(1720) \). First we present a general formalism without the effect of mixing and later we shall address the issue of the possible mixing and of the corresponding modifications.

To describe the nuclear matter ground state we invoke the mean field approach of quantum hadrodynamics and consequently the effective nucleon mass is generated through the \( \sigma \) meson tadpole of scalar mean field potential. The nucleon mass is determined by solving the following equation self-consistently.

\[
m^*_n = m_n - 4(\sigma/m_\sigma)^2 \int_0^{k_F} \frac{d^3k}{(2\pi)^3} \frac{m^*_n}{\sqrt{k^2 + m^*_n^2}}.
\]

To study the collective excitation of the system, the relevant quantity is the dielectric function which actually characterizes the eigenvalue condition for the collective modes. In the language of field theory this is equivalent to solving the Schwinger-Dyson equation to determine the dressed propagator. The relevance of summing over the ring diagrams for the study of vector meson propagation is discussed at length in Ref. [8].

The vector meson propagation is calculated by summing over ring diagrams, a diagrammatic equivalent of the random phase approximation (RPA), which consists of repeated insertions of the lowest order polarization, as illustrated in Fig. 1 below.

![FIG. 1. Ring diagrams relevant for the random phase approximation.](image-url)

We make use of the Dyson equation to carry out the summation

\[
D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\alpha}(q) \Pi^{\alpha\beta}(q) D_{\beta\nu}(q).
\]

The poles are found from the equation

\[
\det [\delta_{\mu\nu} - D^0_{\mu\alpha}(q) \Pi^{\alpha\nu}] = 0.
\]
\[ \epsilon_{\mu} = \delta_{\mu} - D_{\mu}^0 \Pi^{\nu} \]  
the determinant of which, denoted later by \( \epsilon(q) \), is the dielectric function. The eigenconditions for collective modes can now be expressed as \( \epsilon(q) = 0 \). The relevance of the set of ring diagrams and the origin of such an eigencondition can be understood from linear response theory where the fluctuation of the current density, the source term for the meson field in nuclear matter, is "picked up" by the vector field.

For later convenience we define longitudinal and transverse dielectric functions as

\[ \epsilon_L(q) = (1 + D^0 \Pi_{00})(1 - D^0 \Pi_{33}) + D^0 \Pi_{03} D^0 \Pi_{30} \]  
\[ \epsilon_T(q) = 1 - D^0 \Pi_{11} = 1 - D^0 \Pi_{22} = 1 - D^0 \Pi_T. \]  

Here, the modified \( \Pi \) functions are used and the Dirac part has also been incorporated. \( D_0 = 1/(q^2 - m_v^2) \) is the free vector meson propagator of mass \( m_v \).

The eigenmodes of the collective oscillations are given by

\[ \epsilon(q) = \epsilon_T^2(q) \epsilon_L(q) = 0, \]  
\[ \epsilon_L(q) = 0, \]  
\[ \epsilon_T(q) = 0 \]  
which yield the relevant dispersion curves.

A. Pion-pion loop

It is well-known that the free space decay width of the \( \rho \) meson is dominantly determined by the two-pion channel. In other words its coupling to \( \pi - \pi \) loops determines the shape of the free space \( \rho \) spectral density.

The interaction between a neutral vector meson and the pions are given by

\[ \mathcal{L} = g_{\rho\pi\pi}(\pi \times \partial_{\mu} \pi) \cdot \rho^\mu. \]  

\[ \text{FIG. 2. } \rho - \pi\pi \text{ Loop.} \]

The real and imaginary part of the pion-pion loop have been discussed at length in many places \[11,12\], we just quote the results here:

\[ \text{Re} \Pi^{L(T)} = \frac{g_{\rho\pi\pi}^2 M^2}{48\pi^2} [(1 - \frac{4m_v^2}{M^2})^{3/2} \ln |1 + \frac{\frac{1}{1 - \frac{4m_v^2}{M^2}}}{1 - \frac{1}{1 - \frac{4m_v^2}{M^2}}}| + 8m_v^2 \left( \frac{1}{M^2} - \frac{1}{m_\rho^2} \right) - 2 \left( \frac{4m_v^2}{\omega_0} \right)^3 \ln \left( \frac{\omega_0 + p_0}{m_\pi} \right)], \]  

\[ \text{Im} \Pi^{L(T)} = -\frac{g_{\rho\pi\pi}^2 M^2}{48\pi^2} (1 - \frac{4m_\pi^2}{M^2})^{3/2}. \]

The free-space spectral density of \( \rho \) meson is given by the following expression:

\[ S_\rho(q^2) = \frac{1}{\pi} \left( \frac{\text{Im} \Sigma_\rho(q^2)}{q^2 - m_\rho^2 - \Sigma_\rho^2} + \text{Im} \Sigma_\rho^2 \right). \]

In vacuum, the above is a Lorentz invariant quantity and a function of \( q^2 \). In matter, however, this breaks down and we shall have non-degenerate spectral densities for the longitudinal and transverse mode of the \( \rho \) meson.
B. Nucleon-nucleon loop

The $\rho$-nucleon interaction Lagrangian may be written as

$$\mathcal{L}_{\text{int}} = g_{\rho}[\bar{N}_\gamma \gamma_\mu \tau N + \frac{\kappa_\rho}{2M} \bar{N}_\sigma \sigma_\mu \tau^a \sigma^\alpha N \sigma^\nu] \rho^a$$ \hspace{1cm} (15)

where

$$G_{\mu} = \gamma_\mu - \frac{\kappa_\rho}{2m_n} \sigma_\mu q^\nu.$$ \hspace{1cm} (17)

In Eq. (16), $G(k)$ is the in-medium nucleon propagator given by

$$G(k, |\vec{k}|) = G_F(k) + G_D(k_0, \vec{k})$$ \hspace{1cm} (18)

with

$$G_F(k) = \frac{(\vec{k} + m_n^*)}{\vec{k}^2 - m_n^2 + i\epsilon}$$ \hspace{1cm} (19)

and

$$G_D(k_0, |k|) = (\vec{k} + m_n^*) \frac{i\pi}{E_k} \delta(k_0 - E_k) \theta(k_F - |k|).$$ \hspace{1cm} (20)

The first term in $G_F^0(k)$, namely $G_F^0(k)$, is the same as the free propagator of a spin $\frac{1}{2}$ particle while the second part, $G_D^0(k)$, involving $\theta(k_F - |k|)$, arises from Pauli blocking and describes the modifications brought about in nuclear matter at zero temperature. It deletes the on mass-shell propagation of the nucleon in nuclear matter with momenta below the Fermi momentum. It should be noted that here, unlike the situation obtained in usual field theory in the absence of extraneous matter, the standard destruction operator operating on the ground state does not give zero as long as the momentum lies below the Fermi momenta simply because there are particles with such momenta in the ground state. On the other hand, particles can only be created with a momentum higher than $k_F$ (because of Pauli blocking). This clearly is the origin of the second term in the propagator which has a non-covariant form. Actually, the in-medium nucleon propagator is defined in a preferred frame, the rest frame for nuclear matter, and the vertex for $\rho$-nn is

$$\Gamma_\mu = \gamma_\mu - \frac{\kappa_\rho}{2m_n} \sigma_\mu q^\nu.$$ \hspace{1cm} (21)

When calculating the polarization function (16) with the nucleon propagator (18), there will be terms containing "$G_F G_F$", "$G_F G_D + G_D G_F$" and "$G_D G_D$". The first term accounts for the free part i.e. the contributions of the Dirac vacuum ($\Pi_{\mu\nu}^D$), while the rest provides the density dependent part of the polarization ($\Pi_{\mu\nu}^F$), and we can write

$$\Pi_{\mu\nu}(q) = \Pi_{\mu\nu}^F(q) + \Pi_{\mu\nu}^D(q)$$ \hspace{1cm} (22)

$$\Pi_{\mu\nu}^F(q) = -\frac{i}{(2\pi)^4} g_\nu^2 S I \int d^4k \text{Tr}[\Gamma_\rho G_F(k + q) \tilde{\Gamma}_\nu G_F(k)]$$ \hspace{1cm} (23)

and

$$\Pi_{\mu\nu}^D(q) = -\frac{i}{(2\pi)^4} g_\nu^2 S I \int d^4k \text{Tr}[\Gamma_\rho G_D(k + q) \tilde{\Gamma}_\nu G_D(k) + \Gamma_\rho G_D(k + q) \tilde{\Gamma}_\nu G_D(k)] + \Gamma_\rho G_D(k + q) \tilde{\Gamma}_\nu G_D(k))$$ \hspace{1cm} (24)
The free part of the ρ self-energy denoted by $\Pi_{\mu\nu}^F$ is divergent and therefore needs to be regularized. We used the dimensional regularization scheme with the following condition:

$$\partial^n \Pi^F(q^2)/\partial (q^2)^n|_{M^*_q \to M, q^2 = m^2} = 0 (n = 0, 1, 2, \ldots, \infty).$$  \hfill (26)

The real part of the density dependent piece of the polarization is given by

$$\Pi^D_{\mu\nu} = \frac{g^2_\rho S_I}{(2\pi)^4} \int \frac{d^4 k}{E^+(k)} \delta(k^0 - E^+(k)) \theta(k_F - |\vec{k}|)$$

$$\times \left[ \frac{T_{\mu\nu}(k - q, k)}{(k - q)^2 - M^*^2} + \frac{T_{\mu\nu}(k, k + q)}{(k + q)^2 - M^*^2} \right].$$  \hfill (27)

Beside this there is another term which involves two $\theta(k_F - |\vec{k}|)$ function. That becomes operative beyond 2 times the Fermi energy $E_F$.

The trace involved in the calculation of the loop diagram has three parts corresponding to vector-vector, vector-tensor and tensor-tensor terms in the vertex function $(37)$. These can be cast into the following form

$$T^v_{\mu\nu}(k, k + q) = 4[k_\mu(k + q)\nu + (k + q)\mu k_\nu - k_\nu(k + q)g_{\mu\nu} + M^*^2 g_{\mu\nu}]$$  \hfill (28)

$$T^{vt+tv}_{\mu\nu}(k, k + q) = 4M^* \frac{k_\mu}{M} q^2 Q_{\mu\nu}$$  \hfill (29)

$$T^{tt}_{\mu\nu}(k, k + q) = 16\left(\frac{k_\mu}{4M} q^2 \right)^2 \left(2(k \cdot q)^2 - q^2 k^2 + q^2(k \cdot q) - q^2 M^*^2\right) - 2q^2 K_{\mu\nu}. $$  \hfill (30)

Hence the self energy can be written as

$$\Pi^D_{\mu\nu}(q) = \Pi^{v}_{\mu\nu}(q) + \Pi^{vt+tv}_{\mu\nu}(q) + \Pi^{tt}_{\mu\nu}(q)$$  \hfill (31)

The $\Pi^D_{\mu\nu}(q)$ functions in this case are as follows

$$\Pi^{v}_{\mu\nu} = \frac{g^2_\rho}{\pi^3} S_I \int^k_0 \frac{d^3 k}{E^+(k)} \frac{K_{\mu\nu} q^2 - Q_{\mu\nu}(k \cdot q)^2}{q^4 - 4(k \cdot q)^2}$$  \hfill (32)

$$\Pi^{vt+tv}_{\mu\nu} = \frac{g^2_\rho}{\pi^3} S_I \left(\frac{\kappa M^*}{4M}\right)^2 2q^2 Q_{\mu\nu} \int^k_0 \frac{d^3 k}{E^+(k)} \frac{1}{q^4 - 4(k \cdot q)^2}$$  \hfill (33)

$$\Pi^{tt}_{\mu\nu} = -\frac{g^2_\rho}{\pi^3} S_I \left(\frac{\kappa}{4M}\right)^2 4q^4 \int^k_0 \frac{d^3 k}{E^+(k)} \frac{K_{\mu\nu} + Q_{\mu\nu} M^*^2}{q^4 - 4(k \cdot q)^2}$$  \hfill (34)

where $K_{\mu\nu} = (k_\mu - \frac{k^2}{q^2} q_\mu)(k_\nu - \frac{k^2}{q^2} q_\nu)$, $Q_{\mu\nu} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})$ and $E_k^* = \sqrt{k^2 + M^*^2}$. To include the overall degeneracy factor, the above expressions are multiplied by a factor of two coming from the nucleon and proton loop. It is clear that the form for the polarization tensor conforms to the requirement of current conservation, i.e.

$$q^\nu \Pi^D_{\mu\nu} = 0 = \Pi^D_{\mu\nu} q^\nu.$$  \hfill (35)

In the present case we observe that the free part and the dense part of the polarization tensor individually satisfies the above condition.

We should also observe that Eq. $(34)$ is proportional to $Q_{\mu\nu}$ and therefore contribute equally to the longitudinal and transverse mode. In fact, it is $K_{\mu\nu}$ which in matter, induces the splitting of these two modes as we shall discuss later. Evidently, the Dirac part is also proportional to $Q_{\mu\nu}$ and therefore the modes remain degenerate on account of Lorentz symmetry. It might be mentioned that at $|q| = 0$ they are degenerate in matter because of the rotational symmetry.
Also, it is worth to point out that we could describe these effects in the linear density approximation for low baryonic densities. In this approximation, the term $K_{\mu\nu}$ cannot contribute and therefore the longitudinal and transverse part become degenerate and Eqs. (33-35) give

$$\Pi_{vv}^{(L)} = -4g_v^2 \frac{M^* q_0^2}{q^4 - 4M^* q_0^2} \rho_B$$ \hspace{1cm} (37)

$$\Pi_{tt}^{(L)} = 4g_v^2 (\frac{\kappa M^*}{2M})^2 \frac{q^4}{q^4 - 4M^* q_0^2} \rho_B (38)$$

$$\Pi_{tt}^{(L)} = 4g_v^2 (\frac{\kappa M^*}{2M})^2 \frac{q^4}{q^4 - 4M^* q_0^2} \rho_B (39)$$

These results could also directly be obtained by multiplying the forward scattering amplitude with the density. In the present case we implemented the same by expanding the integrand up to second order or in other words schematically we retained terms like $f_0^{kF} d^3k f_{\mu\nu}(q_0, |q|)(1 + O(k))$.

To provide further insight, one can make a long wavelength approximation and recover the familiar results of the plasma oscillations. In this limit, when $q_0 < E_F$ and $|q| < k_F$, Eq. (38) reduces to $\Pi_{vv}^{(L)} = g_v^2 / M^* \rho_B$. In this limit, with $\kappa = 0$ the dispersion relation of the density dependent part alone becomes

$$q_0^2 = |q|^2 + m_v^2 + \Omega^2,$$ \hspace{1cm} (41)

where the plasma frequency $\Omega^2 = g_v^2 / M^* \rho_B$. This is the non-relativistic form of the results presented by Chin [8] for the case of $\omega$ meson propagation in nuclear matter. Furthermore, replacing $g_v$ by the electronic charge “$e$” and putting $m_v = 0$ what one obtains is nothing but the familiar plasma frequency encountered in condensed matter physics [14].

In Fig. (4) we compare resummed one loop results of the $\rho$-meson self-energy with the ones calculated in the linear density approximation. The ratio of the self-energies are shown as a function of density.

![Graph showing the ratio of self-energies](image)

FIG. 4. Comparison of the $\rho$-meson self-energy calculated at the one-loop level with the one calculated in the linear density approximation.

It is now apparent that the linear approximation can be justified only up to normal nuclear matter density and we notice a rather strong dependence on energy, the approximation becoming worse for high values.
C. Nucleon-Resonance Loop

In the present work, among the resonances, we consider only \( N^*(1520) \) and \( N^*(1720) \) which couples strongly with the \( \rho \) meson as indicated in [13]. The corresponding relativistic interactions are given by

\[
\mathcal{L}_{\text{int}} = \begin{cases} 
\frac{f_{\rho N N}}{m_\rho} \bar{\psi} \gamma^\mu \psi F_{\mu\nu} & \text{for } I(J^P) = \frac{1}{2}(-) \\
\frac{f_{\rho N N}}{m_\rho} \bar{\psi} \gamma^5 \gamma^\mu \psi F_{\mu\nu} & \text{for } I(J^P) = \frac{1}{2}(+) 
\end{cases}
\]  

(42)

Here \( \psi^\mu \) denotes the resonance spinor and \( \psi \) the nucleon spinor, \( \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \) and \( F^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu \). It might be mentioned that \( N^*(1520) \) and \( N^*(1720) \) have total widths of \( \sim 120 \text{ MeV} \) and \( 150 \text{ MeV} \) respectively with corresponding branching ratios into the \( \rho-N \) channel of \( \sim 24 \text{ MeV} \) and 115 MeV.

![FIG. 5. \( \rho \) self-energy.](image)

From the Fig.5 it is evident that the polarization tensor has two parts designated as direct and exchange term. We present only the direct term and the exchange term can be calculated in a similar fashion.

\[
-i\Pi^{\pm(\text{dir})}_{\mu\nu} = S_I \int \frac{d^4k}{(2\pi)^4} \frac{T^\pm_{\mu\nu}(k, k-q)}{(k-q)^2-m_R^2} \left[ \frac{1}{k^2-m_n^2} + \frac{i\pi}{E_k} \delta(k^2-m_n^2)\theta(k_F-|k|) \right],
\]

(43)

where \( S_I \) is the isospin factor. The vertex factors for \( R^{3/2} N\rho \) could be written as

\[
\Gamma^{\pm}_\mu = \frac{f_\rho}{m_\rho} (\gamma_\alpha) \frac{i\gamma_5}{2} (\gamma_\mu q_\alpha - \gamma_\alpha q_\mu)
\]

(44)

It is evident that like the N-N loop, \( \Pi^{\pm(\text{dir})}_{\mu\nu} \) also contains a “free” and a density dependent part. The detailed expression for the free part is given in the appendix and has a form \( \Pi_{\mu\nu} = Q_{\mu\nu}(q^2) \). Note also that

\[
T^\pm_{\mu\nu}(k+q, k) = Tr[i(k+m_\rho)i\Gamma^\pm_\mu i\mathcal{R}^{\alpha\beta}_{3/2}(k-q)i\Gamma^\pm_\nu] \\
= Tr[(k+m_\rho)(\gamma_\mu q_\alpha - \gamma_\alpha q_\mu)(k-q+m_R)] \\
\times P^{\alpha\beta}_{3/2}(k-q)(q_\beta q_\nu - \gamma_\nu q_\beta).
\]

(45)

In the above equation \( \mathcal{R}^{\mu\nu}_{3/2}(p) \) is the Rarita-Schwinger propagator, given by

\[
\mathcal{R}^{\mu\nu}_{3/2}(p) = (\not{p} + m_R)P^{\mu\nu}_{3/2}(p) \\
= (\not{p} + m_R)[-g^{\mu\nu} + \frac{1}{3} \gamma^{\mu} \gamma^{\nu} + \frac{2}{3} \frac{p^\mu p^\nu}{m_R^2} - \frac{1}{3} \frac{p^\mu \gamma^{\nu} - \gamma^\mu p^\nu}{m_R}]
\]

(46)

(47)

There is an overall sign ambiguity with spin 3/2 particles which arises from the special choice of the point transformation properties of the spin 3/2 Lagrangian [14,15]. This has also been discussed in Ref. [16]. However, we do not adopt here the prescription suggested in Ref. [16] to deal with the spin 3/2 propagator.

The relevant trace can be written in the following suggestive form

\[
T^\pm_{\mu\nu}(k-q) = \left( \frac{f_\rho}{m_\rho} \right)^2 \alpha^\pm Q_{\mu\nu} + \beta^\pm K_{\mu\nu}
\]

(48)

where,
\[ \alpha^\pm = \frac{8}{3} k^2 q^2 + m_R m_R q^2 + k^2 q^4 - 2 \frac{k^2 q^2 (k \cdot q)}{m_R^2} - \frac{q^2 (k \cdot q)}{m_R} + (k \cdot q)^2 + \frac{k^2 (k \cdot q)^2}{m_R^2} + 2 \frac{q^2 k \cdot q^2}{m_R^2} - \frac{(k \cdot q)^3}{m_R^2} \]

\[ \beta^\pm = \frac{8}{3} (k \cdot q - k^2 - m_R^2) \frac{q^2}{m_R}. \]

(49)

It is evident that this structure is similar to what we had for the nucleon loop and therefore satisfies the condition of current conservation \((q^\mu \Pi_{\mu\nu} = 0 = \Pi_{\mu\nu} q^\nu\) in the momentum space). \(T_{\mu\nu}^\pm\) involves the same gauge invariant forms \(K_{\mu\nu}\) and \(Q_{\mu\nu}\).

In order to evaluate \(\Pi_{\mu\nu}^D\) conveniently, we choose \(\vec{q}\) to be along the \(z\) axis i.e. \(q = (q_0, 0, 0, |\vec{q}|)\), and \(k \cdot q = E(k) q_0 - |\vec{k}| |\vec{q}| \chi\), where \(\chi\) is the cosine of the angle between \(\vec{k}\) and \(\vec{q}\). After \(\phi\) integration the non-vanishing components \(\Pi_{\mu\nu}^D\) are

\[ \begin{pmatrix} \Pi_{00} & 0 & 0 & \Pi_{03} \\ 0 & \Pi_{11} & 0 & 0 \\ 0 & 0 & \Pi_{22} & 0 \\ \Pi_{30} & 0 & 0 & \Pi_{33} \end{pmatrix}. \]

(50)

Moreover for isotropic nuclear matter we have \(\Pi_{22}^D = \Pi_{33}^D\) and \(\Pi_{00}^D = \Pi_{33}^D\), and hence, taking all this into account, we have only two non-vanishing independent components of \(\Pi_{\mu\nu}^D\), linear combinations of which gives us the longitudinal and transverse components of \(\Pi_{\mu\nu}^D\), in other words: \(\Pi_{L}^D(q) = -\Pi_{00}^D + \Pi_{33}^D\) and \(\Pi_{T}^D(q) = \Pi_{11}^D = \Pi_{22}^D\).

We can now estimate the dispersion curves of the \(\rho\) meson in nuclear matter. As mentioned, they appear as poles in the propagator and therefore zeros of the dielectric functions shown in Eqs. (9) and (10). In Fig. 6 we show the dispersion curves for \(\rho\) meson for \(\rho = 1.5\rho_0\) baryonic density. It is clear that the \(\rho\) meson physical mass (when \(q_z = 0\)) drops in nuclear matter from its free space value. The dashed curve shows the results with n-n loop only and the solid one corresponds to the case where the direct coupling of the \(\rho\) with \(N^*(1520)\) and \(N^*(1720)\) is also considered. It should be noted that the resonance-particle excitations contribute with opposite sign to that of the n-n loop. This partially offsets the dropping of \(\rho\) meson mass in nuclear matter.

Fig. 6 shows the variation of the invariant mass of the \(\rho\) meson mass as a function of nuclear density. The Dirac vacuum and the density dependent part of the self-energy contribute with opposite sign to the invariant mass.
FIG. 7. The invariant mass of the $\rho$ meson as a function of density showing explicitly the effect of the baryonic resonances.

At lower densities the free part is responsible for the dropping of the $\rho$ mass while at higher densities the masses again tend to increase when the contribution from the density dependent part increases. This behaviour was also observed in the case of the $\omega$ and $\sigma$ mesons in Ref. [9].

D. $\rho$-$\sigma_0$ Mixing via N-N Loop

Before we start the discussion on the $\rho$-$\sigma_0$ mixing involving n-n polarization in nuclear matter, we should say a few words on the $\sigma_0$ coupling to the nucleon. A more detailed study on the $\sigma_0$ propagation in a dense medium could be found in [6]. The interaction can be described by the following Lagrangian:

$$L_{int} = g_{\sigma_0} \bar{\psi} \phi_{\sigma_0} a_0^{\tau^a} \psi$$

(51)

where $\psi$ and $\phi_{\sigma_0}$ correspond to the nucleon and $\sigma_0$ fields, and $\tau^a$ is a Pauli matrix. The values used for the coupling parameters are obtained from Ref. [10]. We do not involve the coupling of $\sigma_0$ to the baryonic resonances since currently this is not precisely known.

The polarization vector through which the $\sigma_0$ couples to $\rho$ via the n-n loop is obtained by evaluating the Feynman diagram depicted in Fig. 8 and is given by

$$\Pi_\mu(q_0, |\vec{q}|) = 2i g_{\rho a_0} g_\rho \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)\Gamma_\mu G(k + q)]$$

(52)

where 2 is an isospin factor. With the evaluation of the trace and after a little algebra Eq. (52) can be put into a suggestive form:
In Eq. (55) the noninteracting propagator for \( a \)

\[
\Pi_{\mu}(q_0, |q|) = \frac{g_0 g_{a0}}{\pi^3} \frac{2q^2(2m_n^*)}{2m_n^*} \int_{0}^{k_F} d^3k \frac{k_\mu - \frac{2a_0}{q} (k \cdot q)}{E^*(k) q^4 - 4(k \cdot q)^2}.
\]

This immediately leads to two conclusions. First, it respects the current conservation condition, \( q^\mu \Pi_{\mu} = 0 = \Pi_\nu q^\nu \).

Secondly, there are only two components which survive the integration over azimuthal angle. This guarantees that it is only the longitudinal component of the \( \rho \) meson which couples to the scalar meson while the transverse mode remains unaltered. Furthermore, current conservation implies that out of the two non-zero components of \( \Pi_{\mu} \), only one is independent.

In presence of mixing the combined meson propagator can be written in a matrix form where the dressed propagator would no longer be a diagonal matrix:

\[
D = D^0 + D^0 \Pi D.
\]

It is to be noted that the free propagator is diagonal having the following form:

\[
D^0 = \begin{pmatrix}
D^0_{00} & 0 \\
0 & \Delta_0
\end{pmatrix}
\]

In Eq. (53) the noninteracting propagator for \( a_0 \) and \( \rho \) are given respectively by

\[
\Delta_0(q) = \frac{1}{q^2 - m_0^2 + i\epsilon},
\]

\[
D^0_{\mu\nu}(q) = \frac{-g_{\mu\nu} + 2a_0 q_\mu q_\nu}{q^2 - m_0^2 + i\epsilon}.
\]

In fact, it is the polarization matrix which involves non-diagonal elements as shown below characterizing the mixing

\[
\Pi = \begin{pmatrix}
\Pi_{\mu\nu}(q) & \Pi_\nu(q) & \Pi^{a0}(q) \\
\Pi_\mu(q) & \Pi_\nu(q) & \Pi^{a0}(q) \\
\Pi^{a0}(q) & \Pi_\nu(q) & \Pi^{a0}(q)
\end{pmatrix}
\]

After \( \phi \) integration the non-vanishing components \( \Pi \) are as shown below

\[
\begin{pmatrix}
\Pi_{00} & 0 & 0 & \Pi_{03} & \Pi_0 \\
0 & \Pi_{11} & 0 & 0 & 0 \\
0 & 0 & \Pi_{22} & 0 & 0 \\
\Pi_{30} & 0 & 0 & \Pi_{33} & \Pi_3 \\
\Pi_0 & 0 & 0 & \Pi_3 & \Pi^{a0}
\end{pmatrix}
\]

For \( a_0 \) meson the free part of the self-energy is given by:

\[
\Pi^{ao}(q^2) = \frac{3g_{a0}^2}{2\pi^2} |3(m^* - m^2) - 4(m^* - m) m - (m^* - m^2)| \int_0^1 dx \ln \left[ \frac{m^* - x(1 - x)q^2}{m^2 - x(1 - x)q^2} \right] \]

\[
- \int_0^1 dx \left( m^2 - x(1 - x)q^2 \right) \ln \left[ \frac{m^2 - x(1 - x)q^2}{m^2 - x(1 - x)q^2} \right].
\]

To determine the collective modes, one defines the dielectric function in presence of the mixed terms [8]:

\[
\epsilon(q_0, |q|) = \det(1 - D^0 \Pi) = \epsilon_T^2 \times \epsilon_{mix}
\]

where \( \epsilon_T \) corresponds to two identical transverse (T) modes and \( \epsilon_{mix} \) correspond to the longitudinal mode with the mixing. The latter also characterizes the mode relevant for the \( a_0 \) propagation

\[
\epsilon_T = 1 - d_0 \Pi_T, \quad d_0 = \frac{1}{q^2 - m_{a0}^2 + i\epsilon}
\]

\[
\epsilon_{mix} = (1 - d_0 \Pi_L)(1 - \Delta_0 \Pi_s) - \frac{q^2}{|q|^2} \Delta_0 d_0 (\Pi_0)^2.
\]

It is evident that only the longitudinal component gets modified because of the mixing, and when \( \Pi_0 = 0 \) we recover the same expression as Eq. (50).
III. $\rho$ SPECTRAL DENSITY IN NUCLEAR MATTER

Unlike in free space, the longitudinal and transverse $\rho$ spectral densities are non-degenerate. This is because of the fact that, in the nuclear matter rest frame, Lorentz symmetry is broken and they are functions of $q_0$ and $|\mathbf{q}|$, independently. Furthermore, in matter the scalar and vector mesons can mix. This also modifies the longitudinal $\rho$ spectral density through the off-diagonal mixing terms in Eq. (58).

Now in the presence of mixing, the spectral densities can be defined in terms of the dielectric function as

$$S_{L}(q_0, |\mathbf{q}|, \rho_B) = -\frac{1}{\pi} \text{Im} \left[ \frac{d_0 (1 - \Delta_0 \Pi_s)}{\epsilon_{SL}} \right].$$

On the other hand the transverse spectral density is unaffected by the mixing and has the following form

$$S_{T}(q_0, |\mathbf{q}|, \rho_B) = -\frac{1}{\pi} \text{Im} \left[ \frac{d_0}{1 - d_0 \Pi_T} \right].$$

Fig. 9 shows the longitudinal spectral density ($S_L$) at a density $\rho = 1.5\rho_0$ as a function of three momenta ($|\mathbf{q}|=q_z$) and invariant mass ($M_i$). This includes the effect of N-N loop and the direct coupling of the $\rho$ meson with $N^*(1520)$ and $N^*(1720)$. The three-peak structure of the spectral density is similar to what has already been observed in non-relativistic calculations in Ref. [18]. Such a characteristic behaviour of the spectral density in presence of resonance has been also observed in pion-nucleon dynamics. The collective modes induced by the density fluctuations can be identified as the $\rho$ meson mode, N-hole and N-resonance mode similar to what we observe in case of pion propagation in nuclear matter.
Fig. 10 is as Fig. 9 but at higher density. We note that at higher density the spectral density gets even broader. Furthermore, with increasing momenta all the peaks merge into one broad peak indicating the fact that at high momenta the collective behaviour dies down and the meson shows a behaviour of a free propagation in matter.

Fig. 11. The longitudinal spectral density for the $\rho$ meson with mixing at density $\rho = 2.5\rho_0$ with and without considering resonances.

Next we show how individual resonant states modify the $\rho$ spectral density in matter. In Fig. 11, the dashed-dotted line represent the $\rho$ spectral density for $\rho$ coupled only to $\pi-\pi$ and N-N loop. This shows the $\rho$ meson peak shifts towards lower invariant mass.
This is because of the $\sigma$ meson mean field in nuclear matter which lowers the nucleon mass considerably in nuclear matter. The dashed curve has a double hump structure with the introduction of the $N^*(1720)$. For non-relativistic calculation, such a feature has already been observed in the static limit in Ref. [19]. Addition of $N^*(1520)$ also introduces a double peak structure but it moves the strength to the low invariant mass region. Hence we conclude that the resonant states are responsible for the broadening of the $\rho$ spectral density in nuclear matter. Furthermore, in presence of the resonant states the $\rho$ meson gets broadened so much so that the single particle interpretation of $\rho$ meson as quasi particle fails to carry any sense.

The transverse spectral density ($S_T$) also shows interesting features at low momenta. Again we observe that at higher momenta the $\rho$-spectral density gets flattened. The momentum dependence of $S_T$ is also observed to be different than that of $S_L$.

At higher densities we see even more pronounced effect as evident from Fig. [13].
The spectral density for the $a_0$ meson is defined as:

$$S_S(q_0, |q|, \rho_B) = \frac{1}{\pi} Im \left[ \frac{\Delta_0(1 - d_0 \Pi_s)}{\epsilon_{SL}} \right].$$  \hspace{1cm} (65)

FIG. 14. Spectral density for the $a_0$ meson with mixing at density $\rho = 1.5\rho_0$.

In Figs. 14 and 15 the spectral densities of the $a_0$ meson are presented at densities $\rho = 1.5\rho_0$ and $\rho = 2.5\rho_0$. It is evident that the $a_0$ spectral density also gets broadened in medium. A marked shift towards the lower invariant mass indicates that the $a_0$ mass also drops in nuclear matter.

FIG. 15. Spectral density for the $a_0$ meson with mixing at density $\rho = 2.5\rho_0$.

To illustrate the spectral density modification in matter, we present $S_L$ as a function of density. The solid line in Fig. 15 shows the free $\rho$ spectral density. We find that with increasing density it acquires more strength in the low
invariant mass region and also becomes flattened. Dashed, dotted and dashed-dotted lines represent results for 0.5, 1.5 and 2.5 normal nuclear matter densities. It might be mentioned that a small peak appears at the higher invariant mass region which indicates the effect of mixing.

FIG. 16. The longitudinal spectral density for the $\rho$ meson with mixing as a function of density.
In Fig. 17, the two dimensional projection of the $a_0$ spectral density at $|q| = 0.7$ shows that with increasing density $a_0$ gets narrower and moves towards low invariant mass region. This is understandable from the reduction of phase space. It might be recalled here that resonant states do not couple to $a_0$ and therefore we do not observe any broadening in the $a_0$ channel.

**IV. SUMMARY AND CONCLUSIONS**

In the present work we present quantitative results of the in-medium meson properties. The meson spectral densities are evaluated for the first time in a fully relativistic mean field model which goes beyond linear density approximation.
and we discuss the limitation of the applicability of LDA. We find that meson spectral densities in matter are quite different than in free space. The difference stems partly from the Lorentz symmetry breaking. While in free space the transverse and longitudinal component of the $\rho$ spectral density are degenerate, in matter, they show different qualitative behaviour. Furthermore, we include other purely in-medium effects, forbidden in vacuum on the account of Lorentz symmetry, like meson mixing. In matter the $\rho$ can also be modified because of its mixing with a scalar (isovector) $a_0$ meson. This effect, as we have seen, modifies only the longitudinal component of the spectral density.

In our model we observe that the $\rho$ meson spectral density gets flattened in nuclear matter with the incorporation of the resonant states like $N^*(1520)$ and $N^*(1720)$. In fact in nuclear matter the original free space peak of the $\rho$ meson become so broadened that it is no longer possible to interpret that as a quasi-particle excitation. This was also observed in Ref. [15,5]. It should also be noted that in presence of scalar mean field the $\rho$ meson mass goes down as a function of density.

This broadened of $\rho$ spectral density shows an accumulation of strength towards lower invariant mass region. This would definitely imply that in more production of dilepton pairs with low invariant mass in access to what we might expect from the free $\rho$ spectral density. We plan to extend this work to the finite temperature region in the near future. Furthermore, the broadened $\rho$ spectral density would also affect the width of the resonance states like $N^*(1520)$ or $N^*(1720)$ which probably would shed some light on the issue of mixing resonant states in the photoabsorption cross-sections. For the complete determination of the $\rho$ spectral density and the broadening of the resonant state, a self-consistent approach should be adopted [5]. Studies along such directions are in progress.

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APPENDIX A: FREE PART OF THE N-N LOOP

Polarization tensors arising out of the $n - \bar{n}$ excitation of the Dirac sea.

$$\Pi_{\mu\nu}^v = -\frac{1}{2}(\frac{g_v}{\pi})^2[\frac{1}{3}(\Delta + \ln\mu^2)q^2Q_{\mu\nu} - 2q^2Q_{\mu\nu}\int\limits_0^1 dx x(1-x)\ln D]$$

$$\Pi_{\nu\nu}^{\mu\tau} = \frac{g_v^2}{2\pi^2}\frac{M^*\kappa_v}{2M}Q_{\mu\nu}[\Delta + \ln\mu^2 - \int\limits_0^1 dx \ln D]$$

$$\Pi_{\mu\nu}^{tt} = -\frac{g_v^2}{(4\pi)^2}(\frac{\kappa}{M})^2Q_{\mu\nu}[\frac{1}{6}q^2(\Delta + m^2\ln(\mu^2) - m^2 + \frac{q^2}{2}) + q^2\int\limits_0^1 dx x^2\ln D - q^2\int\limits_0^1 dx x\ln D - m^2\int\limits_0^1 dx \ln D]$$

$$\Delta = \frac{1}{e} - \gamma + \ln 4\pi$$

$$D = M^*^2 - q^2 x(1-x)$$

All the terms containing $\Delta$ are infinite and need to be subtracted out.

APPENDIX B: FREE PART OF THE N-R LOOP

$$\Pi_{\mu\nu}^{RN(dir)} = Q_{\mu\nu}\Pi(q^2)$$

where,

$$\Pi(q^2, \mu) = (\Pi_1(q^2) + \Pi_2(q^2) + \Pi_3(q^2) + \Pi_4(q^2) + \Pi_5(q^2) + \Pi_6(q^2) + \Pi_7(q^2) + \Pi_8(q^2))$$
\[ \Pi_1(q^2) = \frac{[(D + 2x^2q^2)(\Delta + \ln m^2) + \int_0^1 dx(D - D \ln D - 2q^2x^2 \ln D)]q^2Q_{\mu\nu}}{32\pi^2} \]  
\[ \Pi_2(q^2) = \frac{2(3D^2 + 18Dq^2x^2 + 4q^4x^4)(\Delta + \ln \mu^2) + \int_0^1 dx(9D^2 + 36Dq^2x^2 + 6D^2 \ln D - 36Dq^2x^2 \ln D - 8q^4x^4 \ln D)]}{128m_R^2\pi^2} \]  
\[ \Pi_3(q^2) = -2q^2x(3D + 2q^2x^2)(\Delta + \ln \mu^2) + \int_0^1 dx(3q^2xD - 3q^2xD \ln D - 2q^4x^3 \ln D)]\frac{q^2Q_{\mu\nu}}{32m^2\pi^2} \]  
\[ \Pi_4(q^2) = -\int_0^1 dxDq^2x(\Delta + \ln \mu^2) + \int_0^1 dx\frac{D(\Delta - \ln \mu^2)}{16\pi^2} \]  
\[ \Pi_5(q^2) = -m_Nm_\alpha[(\Delta + \ln \mu^2) - \int_0^1 dx \ln D)]\frac{q^2Q_{\mu\nu}}{16\pi^2} \]  
\[ \Pi_6(q^2) = [D(\Delta + \ln \mu^2) + \int_0^1 dx(D - D \ln D)]\frac{q^2Q_{\mu\nu}}{32\pi^2} \]  
\[ \Pi_7(q^2) = [2D(3D + 2q^2x^2)(\Delta + \ln \mu^2) + \int_0^1 dx\frac{D(9D + 4q^2x^2 - 6D \ln D - 4q^2x^2 \ln D)]q^2Q_{\mu\nu}}{128\pi^2} \]  
\[ \Pi_8(q^2) = -\int_0^1 dxDq^2x(\Delta + \ln \mu^2) + \int_0^1 dx\frac{Dq^2x - q^2xD \ln D)]\frac{q^2Q_{\mu\nu}}{32\pi^2} \]

**APPENDIX C: IDENTITIES INVOLVING RARITA-SCHWINGER SPINORS**

\[ \Delta^{\mu\nu}(p) = \sum \Psi^\mu(p)\Psi^\nu(p) \]
\[ = (\phi + M_R)^{-1}[g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3}\frac{p^\mu p^\nu}{M_R^2} - \frac{1}{3}\frac{p^\mu \gamma^\nu - \gamma^\nu p^\mu}{M_R}] \]
\[ = (\phi + M_R)P^{\mu\nu}_{3/2}(p) \]  
\[ \gamma^\mu P^{\mu\nu}_{3/2}(p) = \frac{1}{3M_R}(M_R^2 - p^2)(\gamma^\mu - \frac{2p^\nu}{M_R}) \]  
\[ P^{\mu\nu}_{3/2}(p)\gamma^\nu = \frac{1}{3M_R}(M_R^2 - p^2)(\gamma^\mu - \frac{2p^\mu}{M_R}) \]  

Clearly the above and the following equations vanish when the spin 3/2 state is on-shell, so does Eq. (C3) also.

\[ P^{\mu\nu}_{3/2}(p)\gamma^\nu = \frac{1}{3M_R}(M_R^2 - p^2)(\gamma^\mu - \frac{2p^\mu}{M_R}) \]

Another useful relation in this respect is the identity

\[ (k - p)_\mu P^{\mu\nu}_{3/2}(p)(k - p)_\nu = \frac{2}{3}\frac{(p^2 - M_R^2)}{M_R^2}[p^2 - 2k \cdot p] + \frac{2}{3M_R^2}[(k \cdot p)^2 - M_R^2k^2] \]

It is to be noted that the above equation also takes much simpler form when we have on-shell spin 3/2 projection operator.

\[ p_\mu \Psi^\mu(p) = 0 = \Psi^\mu(p)p_\nu \]
\[ k_\mu P^{\mu\nu}_{3/2}(p)k_\nu = \frac{2}{3M_R^2}[(k \cdot p)^2 - M_R^2k^2] \]

It is evident that \( p_\mu P^{\mu\nu}_{3/2}(p)p_\nu = 0 \) when the spin 3/2 particle is on-shell *i.e.* \( p^2 = M_R^2 \). In the rest frame of the resonant state R, we, therefore, have \( k_\mu P^{\mu\nu}_{3/2}k_\nu = \frac{4}{3}k^2 \).
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