Aspects of Accretion Processes On a Rotating Black Hole

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Abstract: We describe the most general nature of accretion and wind flows around a compact object and emphasize on the properties which are special to black hole accretion. The angular momentum distribution in the most general solution is far from Keplerian, and the non-Keplerian disks can include standing shock waves. We also present fully time dependent numerical simulation results to show that they agree with these analytical solutions. We describe the spectral properties of these accretion disks and show that the soft and hard states of the black hole candidates could be explained by the change of the accretion rate of the disk. We present fits of the observational data to demonstrate the presence of sub-Keplerian flows around black holes.

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Introduction

Centers of galaxies are believed to be the seats of massive black holes ($M_{BH} \sim 10^6-9 M_\odot$) and some evolved compact binary systems are believed to be the seats of small mass black holes ($M_{BH} \sim 3 - 10 M_\odot$). In the 1970s, the standard accretion disk models around black holes were constructed (Shakura & Sunyaev 1973; Novikov & Thorne 1973) assuming Keplerian angular momentum distribution in the orbiting matter. In the early 1980s, these disks became the favorite candidates for the explanation of the “big blue bump” seen in the UV region of the continuum spectra of active galaxies (Malkan 1982; Sun & Malkan 1989) with some success.

However, from X-ray and $\gamma$-ray spectra and line emissions from these objects (see, Chakrabarti, 1996a for a general review), it is becoming clear that the accretion disks cannot be simple Keplerian type nor are as simple as spherically symmetric Bondi flows (Bondi, 1952). To satisfy the inner boundary condition on the horizon, matter must cross the horizon supersonically (Chakrabarti 1990, hereafter C90) and therefore must pass through a sonic point where the radial velocity of the flow is the same as the sound velocity. This means that the angular momentum must deviate from a Keplerian flow. All these considerations require one to solve the most general set of equations which must include effects of rotation, flow pressure, viscosity, advection, heating and cooling processes. Below, we present these equations and discuss the solutions and their implications. For the purpose of the present review we emphasize those properties which are special to a black hole accretion.

We consider a perfect flow around a Kerr black hole with the metric (e.g., Novikov & Thorne, 1973; using $t, r, \theta & z$ as coordinates and the units which render $G = M_{BH} =$
\( c = 1 \).

\[
ds^2 = - \frac{r^2 \Delta}{A} dt^2 + \frac{A}{r^2} (d\phi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2,
\]

where,

\[
A = r^4 + r^2 a^2 + 2 a r^2,
\]

\[
\Delta = r^2 - 2 r + a^2,
\]

\[
\omega = \frac{2 a r}{A}.
\]

Here, \( a \) is the Kerr parameter.

The stress-energy tensor of a perfect fluid with pressure \( p \) and mass density \( \rho = \rho_0 (1 + \pi) \), \( \pi \) is the internal energy is given by,

\[
T_{\mu\nu} = \rho u_\mu u_\nu + p (g_{\mu\nu} + u_\mu u_\nu).
\]

We shall concentrate on the time independent solution of the governing equations (Chakrabarti 1996b, hereafter C96b). The equation for the balance of the radial momentum is obtained from \((u_\mu u_\nu + g_{\mu\nu}) T^{\mu\nu} = 0\). Here the advection term due to significant radial velocity is included. The baryon number conservation equation (continuity equation) is obtained from \((\rho_0 u^\mu)_\mu = 0\). The conservation of angular momentum is obtained from \((\delta^\mu_{\nu} T^{\mu\nu})_\nu = 0\). Here the angular momentum is allowed to be non-Keplerian. Entropy generation equation is obtained from the first law of thermodynamics along with baryon conservation equation: \((S^\mu)_{\mu} = \frac{1}{T}[2\eta \sigma_{\mu\nu} \sigma^{\mu\nu}]\), where, \( T \) is the temperature of the flow and \( \eta \) is the coefficient of viscosity. \( \sigma_{\mu\nu} \) is the shear tensor which is responsible for the viscous transport of angular momentum.

These set of equations are solved simultaneously along with the possibility that the shock waves may form in the flow, where, the following Rankine-Hugoniot conditions must be fulfilled:

\[
W_- n^\nu + (W_- + \Sigma_-) (u_-^\mu n_\mu) u_-^\nu = W_+ n^\nu + (W_+ + \Sigma_+) (u_+^\mu n_\mu) u_+^\nu.
\]

Here, \( n_\mu \) is the four normal vector component across the shock, and \( W \) and \( \Sigma \) are vertically integrated pressure and density on the shock surface.

**Solution Topologies:**

First note that the above mentioned equations are valid for any compact object whose external spacetime is similar to that of a Kerr black hole. However, on a neutron star surface matter has to stop and corotate with the surface velocity. The inner boundary condition is therefore sub-sonic. On a black hole, on the other hand, the flow must enter through the horizon with the velocity of light, and therefore must be supersonic. Stationary shock waves may form when matter starts piling up behind the centrifugal barrier (which arises due to centrifugal force \( \propto \lambda^2 / r^3 \)). The supersonic flow becomes subsonic at the shock and again becomes supersonic before entering through the horizon. Clearly, the flow has to become supersonic, before forming the shock as well, and therefore pass through another sonic point at a large distance away from the black hole. Thus, as a whole, the flow may deviate from a Keplerian disk and (a) enter through the inner sonic point only, or, (b) enter through the outer sonic point only, or, (c) pass through the outer sonic point, then a shock, and finally through an inner sonic
point if the shock conditions are satisfied. If the angular momentum is too small, then the flow has only one sonic point and shocks cannot be formed as in a Bondi flow. For more details, see Chakrabarti (1989, hereafter C89), C90 and C96a.

Three flow parameters govern the topology of the flow: the $\alpha$ parameter (Shakura & Sunyaev, 1973) which determines the viscosity, the location of the inner sonic point $r_{in}$ through which matter must pass through, and the specific angular momentum $\lambda_{in}$ of the matter at the horizon (or, that at $r_{in}$). It so happens that these parameters are sufficient to completely determine the solution when a cooling process is provided. To illustrate the flow topologies, we first choose the viscosity of the flow to be negligible (C96b). In the inviscid case, the angular momentum and energy remain constant $hu_o = l$ and $hu_t = E$. Hereafter, we use $\lambda = l/E$ to be the specific angular momentum. Flow entropy remains constant unless it passes through a shock wave where it goes up within a thin layer. In Fig. 1 we show all possible solutions of weakly viscous accretion flows around a Kerr black hole of rotation parameter $a = 0.5$. (For flows in Schwarzschild geometry, see, C89, C90.) The adiabatic index $\gamma = 4/3$ has been chosen. Vertical equilibrium flow model with the vertical height prescription of NT73 is used. In the central box, we divide the parameter space spanned by $(\lambda, E)$ into nine regions marked by $N$, $O$, $NSA$, $SA$, $SW$, $NSW$, $I$, $O^*$, $I^*$. The horizontal line at $E = 1$ corresponds to the rest mass of the flow. Surrounding this parameter space, we plot various solution topologies (Mach number $M = v_r/a_o$ vs. logarithmic radial distance where $v_r$ is the radial velocity and $a_o$ is the sound speed) marked with the same notations (except $N$). Each of these solution topologies has been drawn using flow parameters from the respective region of the central box. Though each contour of each of the boxes represents individual solutions (differing only by specific entropy), the relevant solutions are the ones which are self-crossing, as they are transonic. The crossing points are 'X' type or saddle type sonic points and the contours of circular topology surround 'O' type sonic points. If there are two 'X' type sonic points, the inner one is called the inner sonic point, and the outer one is called the outer sonic point. If there is only one 'X' type sonic point in the entire solution, then the terminology of inner or outer is used according to whether the sonic point is close to or away from the black hole. The solutions from the region 'O' has only the outer sonic point. The solutions from the regions $NSA$ and $SA$ have two 'X' type sonic points with the entropy density $S_o$ at the outer sonic point less than the entropy density $S_i$ at the inner sonic point. However, flows from $SA$ pass through a standing shock (See Fig. 2) as the Rankine-Hugoniot condition is satisfied. The entropy generated at the shock is exactly $S_i - S_o$, which is advected towards the hole to enable the flow to pass through the inner sonic point. Rankine-Hugoniot condition is not satisfied for flows from the region $NSA$. Numerical simulation indicates (Ryu, Chakrabarti & Molteni, 1996) that the solution is very unstable and show periodic changes in emission properties as the flow constantly try to form the shock wave, but fail to do so. The solutions from the region $SW$ and $NSW$ are very similar to those from $SA$ and $NSA$. However, $S_o \geq S_i$ in these cases. Shocks can form only in winds from the region $SW$. The shock condition is not satisfied in winds from the region $NSW$. This may make the $NSW$ flow unstable as well. A flow from region $I$ only has the inner sonic point and thus can form shocks (which require the presence of two saddle type sonic points) only if the inflow is already supersonic due to some other physical processes.

The flows from regions $I^*$ and $O^*$ are interesting in the sense that each of them has two sonic points (one 'X' and one 'O') only and neither produces complete and global solution. The region $I^*$ has an inner sonic point but the solution does not extend subsonically to a large distance. The region $O^*$ has an outer sonic point, but the solution does not extend supersonically to the horizon! In both the cases a weakly viscous flow is expected to be unstable. When a significant viscosity is added, the closed topology
of $I^*$ is expected to open up (Fig. 3 below; Chakrabarti, 1996c; hereafter C96c) and then the flow can join with a Keplerian disk.

Examples of ‘Discontinuous’ Solutions:

In Fig. 1 we presented continuous solutions and mentioned the possibility of shock formation. In Fig. 2(a-b) we give examples of solutions which include shock wave discontinuities. Mach number is plotted along Y-axis and logarithmic radial distance is plotted along X-axis. Fig. 2a is drawn with parameters from the region $SA$ and Fig. 2b is drawn with parameters from the region $SW$.

In Fig. 2a, the single arrowed curve represents a solution coming subsonically from a large distance and becoming supersonic at $O$, the outer sonic point. Subsequently, the flow jumps onto the subsonic branch (at the place where Rankine-Hugoniot condition is satisfied) along the vertical dashed line and subsequently the flow enters the black hole through the inner sonic point at $I$ (double arrowed curve). The flow chooses to have a shock since the inner sonic point has a higher entropy. The parameters of the flow are $a = 0.5$, $\mathcal{E} = 1.003$ and $\lambda = 3$. In Fig. 2b, on the other hand, the accretion flow can straightaway pass through the inner sonic point (single arrowed curve) and will have no shocks. However, outgoing winds, which may be originated closer to the black hole can have a shock discontinuity (double arrowed curves). Here the flow first passes through $I$, the shock (vertical dashed line) and the outer sonic point ‘$O$’. The parameters in the case are $a = 0.5$, $\mathcal{E} = 1.007$ and $\lambda = 3$.

Above mentioned discussions are valid for flows around a black hole only. For flows around a neutron star, the inner boundary condition must be subsonic, and therefore, the shock transition (in Fig. 2a, for example) must take the flow to a branch which will remain subsonic thereafter, instead of a branch which is likely to become supersonic again.

Properties of Viscous Transonic Flows

The topologies shown above become more complicated once various cooling and viscous heating effects are included (C90, C96c). We present the case of ‘isothermal gas’ where the flow adjusts heating and cooling in such a way that matter remains at a constant temperature. Figs. 3(a-d) show the ‘phase space’ of the accretion flow. We assume Schwarzschild black hole $a = 0$ for simplicity and consider Shakura-Sunyaev (1973) viscosity prescription where the viscous stress at a given location depends on the local thermal pressure $p$: $t_{r\phi} = -\alpha p$. We note the general change in topology of the flow. First of all, the circular topology of the inviscid flow (‘$O$’ type sonic point) is converted into spiral topology as in a damped harmonic oscillator (C90). Secondly, the closed topology has opened up, partially or completely, depending upon flow parameters. In Fig. 3a, the flow parameters are $\alpha = 0.05$, $\lambda_{in} = 1.8$, $r_{in} = 2.8$. The spiral is ‘half closed’. This topology is still good for shock formation as shown in the vertical curve (C90). In Fig. 3b, we use, $\alpha = 0.1$, $\lambda_{in} = 1.8$, $r_{in} = 2.8$. Here the spiral is complete with branches from both sonic points. In Fig. 3c, we use, $\alpha = 0.05$, $\lambda_{in} = 1.77$, $r_{in} = 2.8$ and in Fig. 3d, we use, $\alpha = 0.05$, $\lambda_{in} = 1.8$, $r_{in} = 2.65$. The topology remains similar to Fig. 3b, but whereas Fig. 3b is attained by increasing viscosity, Fig. 3c and Fig. 3d are obtained by decreasing angular momentum and decreasing the inner sonic point respectively. What is common in Fig. 3(b-d), is that both the sonic points allow flows to become Keplerian (where, $\mathcal{M} \sim 0$) but since the dissipation in the flow passing through $I$ is higher, we believe that the disk will choose this branch. More importantly, the flow cannot have a standing shock wave with this topology. All
these solutions, except where $M \sim 0$, are sub-Keplerian as illustrated in Figs. 3(e-h) where the flow angular momentum (solid curve) is compared with Keplerian angular momentum (dotted curve). The location where the flow joins a Keplerian disk may be somewhat turbulent (description of which is not within the scope of our solution) so as to adjust pressures of the subsonic Keplerian disk and the sub-Keplerian transonic viscous flow. These figures illustrate the existence of critical viscosity parameter $\alpha_c$, critical angular momentum $\lambda_c$, and critical inner sonic point $r_c$ at which the flow topologies are changed (C90).

The above results obtained for isothermal flow can be generalized easily using the most general set of equations with heating and cooling processes (C96c). If $Q^+$ and $Q^-$ denote the heating and cooling rates, and if, for illustration purpose, one assumes that $f = (Q^+ - Q^-)/Q^- = constant$, then one can easily solve the general equations to find the degree at which the flow deviates from a Keplerian disk. In Fig. 4a we show the ratio $\lambda_{disk}/\lambda_{Kep}$ for various viscosity and cooling parameters $\alpha$ and $f$. Clearly as the flow starts deviating from a Keplerian disk $\lambda_{disk}/\lambda_{Kep} = 1$, it becomes about 20-30% of Keplerian. As it approaches the black hole, it becomes close to Keplerian again (and sometimes super-Keplerian also) before plunging in to the black hole in a very sub-Keplerian manner. In Fig. 4b, we show the ratio $v_r/v_\phi$ for the same disks. Note that near the outer edge, as the flow deviates from a Keplerian disk, $v_r << v_\phi$, i.e., the flow is rotation dominated. Around $r \sim 100$ the radial velocity becomes dominant, and subsequently, even closer to the black hole, the flow becomes rotation dominated (due to the centrifugal barrier). Near the horizon the radial (advection) velocity dominates once more. The case $f = 0.5$, $\alpha = 0.05$ showing a sudden jump in velocity ratio represents a solution which includes a standing shock. The corresponding angular momentum distribution in the upper panel does not show discontinuity since shear is chosen to be continuous across a shock wave. The region where the disk deviates from a Keplerian disk could be geometrically thick and would produce thick accretion disks. The post-shock region, where the flow suddenly becomes hot and puffed up would also form another thick disk of smaller size.

The solutions presented so far is of most general nature encompassing the entire parameter space. Any other solutions of black hole accretion or winds are special cases of these solutions.

Problem of Angular Momentum Transport

A curious property of a black hole accretion flow is that the shear stress $\sigma_{r\phi}$ is not a monotonic function of distance, and it can be negative close to the black hole. In the Newtonian geometry, it has the form $-r^{2/3}$ and its magnitude is monotonically increasing inward. For a cold radial flow below the marginally bound orbit this was pointed out by Anderson & Lemos (1988). In Fig. 5(a-b) we show this for the complete and exact global solutions of the accretion flow presented in the earlier Section (C96b). In Fig. 5a, we present the variation of shear and angular velocity gradient for a typical prograde flow ($a = 0.95$, $\lambda = 2.3$ and $E = 1.001$) and in Fig. 5b we show the results of a retrograde flow ($a = 0.95$, $\lambda = -4.0$ and $E = 1.0025$). The solid curves are computed using the most general definition of shear tensor: $\sigma_{\alpha\beta} = 1/2(u_{\alpha\mu}P^\mu_{\beta\gamma} + u_{\beta\mu}P^\mu_{\alpha\gamma}) - \Theta P_{\alpha\beta}/3$ where $P_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$ is the projection tensor and $\Theta = u^\alpha_{\alpha\gamma}$ is the expansion. The velocity components have been taken from the exact solution which passes through the outer sonic point. The subscript $+$ sign under $\sigma_{\phi}$ refers to the supersonic branch of the solution. We also present the same quantity (long dashed curve) for the subsonic branch ($\sigma_{\phi-\phi}$) which passes through the outer sonic point. For comparison, we present
the component of shear tensor $\sigma_{r\phi}^r|_{\text{rot}}$ (short dashed curve) where we ignore the radial velocity completely $v_r \ll v_\phi$. We also present the variation of $d\Omega/dr$ (dotted curve) to indicate the relation between the angular velocity gradient and shear.

Some interesting observations emerge from this peculiar shear distribution. First, for prograde flows, the shear reverses its sign and becomes negative just outside the horizon. This shows that very close to the horizon, the angular momentum transport takes place ‘towards the black hole’ rather than away from it. Second, $\sigma_{\phi+}^r|_{\text{rot}}$ and $\sigma_{\phi-}^r$ which have negligible radial velocities, vanish on the horizon whereas $\sigma_{\phi+}$ does not. Existence of the negative shear component shows the $\alpha$ viscosity prescription (SS73) is invalid close to a horizon, since pressure cannot be negative. However, the error by choosing a positive (e.g., $-\alpha p$) shear may not be significant, since one requires a (unphysically) large viscosity so as to transport significant angular momentum inwards. In any case, the inward angular momentum transport may change angular momentum distribution near the horizon (from that of an almost constant to something perhaps increasing inwards, similar to the Keplerian distribution below marginally stable orbit). Third, in retrograde flows, the $\sigma_{\phi+}^r$ reverses twice but $\sigma_{\phi-}^r$ and $\sigma_{\phi-}^r|_{\text{rot}}$ reverse once since they vanish on the horizon as in the prograde flows.

The physical mechanism underlying the reversal of the shear component is simple. In general relativity, all the energies couple one another. It is well known that the ‘pit in the potential’ of a black hole is due to coupling between the rotational energy and and the gravitational energy (Chakrabarti, 1993). As matter approaches the black hole, the rotational energy, and therefore ‘mass’ due to the energy increases which is also attracted by the black hole. This makes gravity much stronger than that of a Newtonian star. When the black hole rotates, there are more coupling terms (such as thse arise out of spin of the black hole and the orbital angular momentum of the matter) which either favour gravity or go against it depending on whether the flow is retrograde or prograde (Chakrabarti & Khanna, 1992) respectively. This is the basic reason why retrograde and prograde flows display different reversal properties.

**Numerical Simulations of Black Hole Accretion**

The steady state solution topologies described in the previous Sections need not be the final solution of a set of time dependent equations. Depending on the stability of the solutions, the flow may or may not settle on the steady state solution. However, it so happens that except for the solutions in regions $NSA$ and $NSW$, numerical results actually match with analytical results. The accuracy of the matching primarily depends on the ability of the numerical code to conserve angular momentum and energy. In Chakrabarti & Molteni (1993) one dimensional simulations and in Molteni, Lanzafame & Chakrabarti (1994) and Molteni, Ryu & Chakrabarti (1996) two dimensional simulations have been presented using parameters from $SA$ and $SW$ regions. Simulation by diverse codes produced similar results. These suggest that these analytical solutions could be used for rigorous tests of the codes in curved spacetime. The parameters from regions $NSA$ and $NSW$ show unstable behaviours in a multidimensional flow (Chakrabarti et al, 1996, in preparation) though in strictly one dimension they match with the analytical work. Preliminary solutions of cold flows ($E = 1$) have already been reported in the literature (Ryu, Chakrabarti & Molteni, 1996).

Fig. 6 shows the Mach number and density variations in an one dimensional flow around a Schwarzschild black hole (Molteni, Ryu and Chakrabarti, 1996). The solution is chosen from region $SA$ so that analytically a stable shock is expected in a thin flow
of polytropic index $\gamma = 4/3$. The solid curve shows the analytical solution: at the shock the density goes up as matter virtually stops behind the shock and the Mach number goes down from supersonic to subsonic. The long-dashed and the short-dashed curves are the results of simulations using Total Variation Diminishing (TVD) method (see, Harten 1983; Ryu et al, 1995) and the Smoothed Particle Hydrodynamics (SPH) method (see, Monaghan, 1985; Molteni & Sponholz, 1994). The results show that the shocks form in an accretion flow, and they are stable. These simulations also verify the shock free solutions.

Spectral Properties of Generalized Accretion Disks

How does an accretion disk radiate? What are the observational signatures of a black hole? How to distinguish a black hole and a neutron star from observations? In order to answer these questions one clearly needs to solve the hydrodynamic equations in conjunction with radiative transfer. In the first approximation one could construct a realistic accretion disk model based on analytical solutions mentioned above, and then compute the radiations emitted out of this model flow.

In Fig. 3, we showed that above a critical viscosity or below a critical angular momentum or inner sonic point location, the flow does not have a shock wave after deviating from a Keplerian disk. Since with height the sonic point is expected to be closer to a black hole, and a Keplerian disk above a sub-Keplerian flow may be Rayleigh-Taylor unstable, it may be worthwhile to consider the important alternative that the flow viscosity decreases with height. This makes a solution topology of the kind as in Fig. 3b to be on the equatorial plane with a Keplerian disk close to a black hole, while a solution with a shock (Fig. 3a) at a higher elevation. Fig. 7 shows a typical disk model where Keplerian disk is flanked by sub-Keplerian ‘halo’ which undergoes a shock wave at around $r \sim 10 - 30$ in Schwarzschild geometry and roughly at half as distant in extreme Kerr geometry (Chakrabarti & Titarchuk, 1995). Photons from the Keplerian disks are in the optical-UV range in the case of massive black holes at galactic centers, and in the soft X-ray range in the case of small black holes in X-ray binaries. These photons are intercepted by the post-shock flow and are energized through inverse Compton process, and are eventually re-radiated at higher energies (as soft X-rays in massive black holes, or, in hard X-rays in smaller black holes). First consider a galactic black hole candidate of mass $\sim 5 M_\odot$. When accretion rate of the Keplerian component is very low, the soft X-ray from the Keplerian component is weak, but the hard X-ray intensity from the post-shock region is very strong. This is known as the hard state. As the accretion rate increases, electrons in the post-shock region become cooler and the object eventually goes to the soft state where only the soft-X rays from the Keplerian disk dominates. Occasionally, one finds a weaker hard component tail of slope $\alpha_{\nu} \sim 1.5$ even in the soft state. This is interpreted (Chakrabarti & Titarchuk, 1995) as due to the ‘bulk motion Comptonization’ where the converging flow energizes soft photons not due to thermal Comptonization (i.e., by thermal motion of the hot electrons) but due to direct transfer of bulk motion momentum of the cool electrons to cooler photons (Blandford & Payne, 1981; Titarchuk, Mastichiadis & Kylafis, 1996).

Figs. 8(a-b) show the variation of intensity $L_\nu$ of radiation as a function of frequency $\nu$ (Chakrabarti & Titarchuk, 1995). In Fig. 8a, we show contributions from various components of the disk. The dotted curve represents the black body radiation from the optically thick Keplerian flow. The long dashed curve represents the fraction of these photons which were intercepted by the postshock flow and were energized by hot electrons of the post-shock flow through thermal Comptonization processes. The dash-dotted curve is the reflection of these hot photons from the Keplerian disk. The
solid curve gives the overall spectra which has a bump in the soft X-ray and a power law in the 2-50 keV region. The above figure is for $M_{\text{BH}} = 5M_\odot$ and dimensionless disk accretion rate of $\dot{m}_{\text{disk}} = 0.1\dot{M}_{\text{Edd}}$. In Fig. 8b, we show the computed spectra as the accretion rate of the Keplerian component is varied. The solid, long-dashed, short-dashed and the dotted curves are for accretion rates $\dot{m}_{\text{disk}} = 0.001, 0.01, 0.1, 1$ respectively. We also plot the dash-dotted curve which includes the effect of the bulk-motion Comptonization and shows a weak hard tail. The bulk motion Comptonization is the process by which the cool electrons deposit their momentum (while rushing towards the black hole horizon) onto photons and energize them. The spectral index $\alpha_\nu (L_\nu \propto \nu^{-\alpha_\nu})$ of the weak hard tail computed from Chakrabarti & Titarchuk (1995) model is around 1.5, and is often observed in black hole candidates in their soft states. The conclusions regarding general spectral shape remain unchanged when computations are done for supermassive black hole.

**Comparison With Observations and Concluding Remarks**

As an example of the success of our understanding of black hole accretion processes, we use the aforementioned theoretical understanding to fit a spectrum of GRS 1009-45 (X-ray Nova in Vela) obtained by MIR-KVANT module experiments (Titarchuk et al. 1996) The source was discovered by WATCH/GRANAT all-sky monitor (Lapshov et al., 1993; IAUC 5864) and verified by BATSE/GRO (Harmon et al., IAUC 5864). Fig. 9 shows the TTM and HEXE data (the photon flux versus energy; Sunyaev et al. 1994) and the best theoretical fit given by the considerations of the converging inflow in the post-shock region. The parameters, namely, mass of the black hole ($M$), accretion rate in Keplerian disk ($\dot{m}_{\text{disk}}$), accretion rate in the sub-Keplerian disk ($\dot{m}_h$), the shock location ($R_{sh}$), the post-shock temperature ($T_{sh}$), fraction of the soft photons intercepted by the post-shock bulge ($H$) and the distance ($D$) of the object, which are derived from the best-fit of the data seem to be: $M = 3.7554M_\odot$, $\dot{M}_{\text{disk}}/\dot{M}_{\text{Edd}} = \dot{m}_{\text{disk}} = 1.94$, $\dot{M}_h/\dot{M}_{\text{Edd}} = \dot{m}_h = 1.1$, $R_{sh} = 11.74R_g$, $T_{sh} = 5.59(\text{KeV})$, $H = 0.015$, $D = 5.0(\text{Kpc})$ The $\chi^2 = 1.03$ for this fit. The general agreement clearly vindicates the claim that a black hole accretes a significant amount of sub-Keplerian matter. In a neutron star accretion, the weak hard tail is not expected.

Some of the black hole candidates show quasi-periodic oscillations of its spectra in some range of hard X-rays (Dotani, 1992). Moteni, Sponholz, & Chakrabarti (1996) and Ryu, Chakrabarti, Molteni (1996) show that these oscillations could be due to dynamic oscillations of the standing shock waves or even the sub-Keplerian region itself. The frequency and amplitude modulation of the radiation, as well as the time variation of the frequencies match well with observations.

In this review, we showed that understanding accretion processes on a black hole must necessarily include the study of sub-Keplerian flows and how they combine with Keplerian disks farther away. We showed that the so called viscous transonic flows are stable, and constitute the most general form of accretion flows. We showed that the accretion shock could play an important role in energetics of the radiated photons. Indeed we showed that hard and soft states of black hole candidates as well as Quasi-Periodic Oscillations could be explained if shock waves were assumed to be present. Both of these observations along with our model could be used to obtain the mass and the accretion rates of black holes. Success of these solutions clearly depend on more accurate observations of steady state and time varying spectra of the black hole candidates.
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Figure Captions

Fig. 1: Classification of the parameter space (central box) in the energy-angular momentum plane in terms of various topology of the black hole accretion. Eight surrounding boxes show the solutions from each of the independent regions of the parameter space. See text for details.

Fig. 2(a-b): Mach number $M$ is plotted against logarithmic radial distance $r$. Contours are of constant entropy. $a = 0.5$, $\lambda = 3$. In (a), $\mathcal{E} = 1.003$ and in (b), $\mathcal{E} = 1.007$. $O$ and $I$ denote the outer and inner sonic points respectively. Single arrow shows the accretion flow path through $O$ while double arrow traces the path of more stable shocked flow which passes through both $I$ and $O$.

Fig. 3: Mach number variation (a-d) and angular momentum distribution (e-h) of
an isothermal viscous transonic flow. Only the topology (a) allows a shock formation.
Transition to open (no-shock) topology is initiated by higher viscosity or lower angular
momentum or inner sonic point location. In (e-h), flow angular momentum (solid) is
compared with Keplerian angular momentum (dotted).

**Fig. 4(a-b):** Ratio of (a) disk angular momentum to Keplerian angular momentum
($\lambda/\lambda_{Kep}$) and (b) radial to azimuthal velocities ($v_r/v_\phi$) are shown for a few solutions.
Parameters are marked on the curves. Note that $r_{Kep}$, where the flow joins a Keplerian
disk, depends inversely on the viscosity parameter.

**Fig. 5:** Comparison of rotational viscous stress $\sigma_{\phi+}^\text{rot}$ (short dashed curves) with
complete viscous stress $\sigma_{\phi+}^r$ (solid) along the supersonic branch (passing through outer
sonic point) for (a) a prograde flow (upper panel) and (b) a retrograde flow (lower
panel). Also shown is $d\Omega/dr$ (dotted curves). For comparison, results for the subsonic
branch is also shown (long dashed). Note the change in sign of the shear near the
horizon. $\sigma_{\phi+}^\text{rot}$ does not vanish on the horizon, but $\sigma_{\phi+}^r$ and $\sigma_{\phi+}^r$ do.

**Fig. 6:** Comparison of analytical and numerical results in a one-dimensional accretion
flow which allows a standing shock. The long and short dashed curves are the results
of the TVD and SPH simulations respectively. The solid curve is the analytical result
for the same parameters. Upper panel is the mass density in arbitrary units and the
lower panel is the Mach number of the flow.

**Fig. 7:** Schematic diagram of the accretion processes around a black hole. An optically
thick, Keplerian disk which produces the soft component is surrounded by an optically
thin sub-Keplerian halo which terminates in a standing shock close to the black hole.
The postshock flow Comptonizes soft photons from the Keplerian disk and radiates
them as the hard component. Iron line features may originate in the rotating winds.

**Fig. 8(a-b):** Analytical computation of the emitted radiation from a black hole. (a)
Contributions of various components to the net spectral shape (solid). Dotted, short
dashed, long dashed and dash-dotted curves are the contributions from the Shakura-
Sunyaev disk $r > r_s$, the reprocessed hard radiation by the Shakura-Sunyaev disk,
reprocessed soft-radiation by the postshock disk $r < r_s$ and the hard radiation reflected
from the Shakura-Sunyaev disk along the observer ($\mu = \cos\theta = 0.4$). Parameters are
$\dot{m}_{disk}=0.1$, $\dot{m}_h=1$, and $M=5 M_\odot$. (b) Variation of the spectral shape as the accretion
rate of the disk is varied. $\dot{m}_{disk}=0.001$ (solid line), 0.01 (long-dashed line), 0.1 (short-
dashed line), and 1 (dotted line). The dash-dotted curve represents the hard component
from convergent inflow near the black hole and has the characteristics of the slope $\sim 1.5$
in soft state.

**Fig. 9:** Spectral fits of broad band X-rays from GRS1009-45 using our composite disk
model. Data is obtained from MIR-KVANT experiments as reported in Sunyaev et al.
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