Monoenergetic neutrinos from WIMP annihilations in Jupiter

George French
William & Mary

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Monoenergetic neutrinos from WIMP annihilations in Jupiter

A thesis submitted in partial fulfillment of the requirement for the degree of Bachelor of Science with Honors in Physics from the College of William and Mary in Virginia,

by

George M. French

Accepted for

Honors

(Honors or no-Honors)

Advisor: Prof. Marc Sher

Prof. Keith Griffioen

Prof. Alexander Angelov

Williamsburg, Virginia

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Monoenergetic neutrinos from WIMP annihilations in Jupiter

by

G. M. French

Dr. M. Sher (Research Advisor)

May 2023
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Abstract

Weakly interacting massive particles (WIMPs) can be captured by the Sun and can annihilate in the core, which may result in production of kaons that can decay at rest into monoenergetic 236 MeV neutrinos. Several studies of detection of these neutrinos at DUNE have been carried out. It has been shown that if the WIMP mass is below 4 GeV, then they will evaporate prior to annihilation, suppressing the signal. Since Jupiter has a cooler core, WIMPs with masses in the 1-4 GeV range will not evaporate and can thus annihilate into monoenergetic neutrinos. We calculate the flux of these neutrinos near the surface of Jupiter and find that it is comparable to the flux at DUNE for masses above 4 GeV and substantially greater in the 1-4 GeV range. Of course, detecting these neutrinos would require a neutrino detector near Jupiter. Obviously, it will be many decades before such a detector can be built, but should direct detection experiments find a WIMP with a mass in the 1-4 GeV range, it may be one of the few ways to learn about the annihilation process. A liquid hydrogen time projection chamber might be able to get precise directional information and energy of these neutrinos (and hydrogen is plentiful in the vicinity of Jupiter). We speculate that such a detector could be placed on the far side of one of the tidally locked Amalthean moons; the moon itself would provide substantial background shielding and the surface would allow easier deployment of solar panels for power generation.
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Chapter 1

Introduction

1.1 Dark matter

Various lines of evidence - such as galaxy rotation curves, gravitational lensing, and anisotropies in the cosmic microwave background - point to the existence of a form of matter which interacts with gravity but does not interact with the electromagnetic field, earning it the title “dark matter” (DM). Cosmology indicates that DM is highly abundant, and it is estimated that it comprises about 85% of all matter in the universe. While DM has not yet been directly detected, there is plenty of indirect evidence to support its existence.

There are many proposed candidates for DM (e.g., axions and sterile neutrinos) which cover a wide mass range. One of the most highly motivated DM candidates is the weakly interacting massive particle (WIMP), a hypothetical particle with a mass on the GeV scale which resides in a DM halo around a galaxy. A WIMP only interacts with gravity and weak-scale nuclear forces making it very difficult to detect in the laboratory. If they exist, WIMPs were produced in the early Universe, and the properties necessary to result in the abundance of DM we observe today accords well with the properties of a hypothetical new particle predicted by extensions of the Standard Model of particle physics. This is known as the “WIMP Miracle,” and it is the reason that WIMPs are such an appealing candidate for DM.

A variety of methods exists to detect WIMPs both directly and indirectly, but, as of yet, no experiment has conclusively done so. That said, the experiments have not been fruitless. Each one serves to tighten the bounds on key DM parameters. Those parameters are the mass ($m_\chi$) and cross section ($\sigma_{SD/SI}$). The two possible subscripts
on the cross section signify that WIMPs can interact either spin-dependently or spin-independently. This work considers only the former case, but more will be said on this later.

Throughout this work, WIMPs are assumed to be the only type of DM. In reality, this may not be the case as others may exist as well in other regions of the DM mass spectrum. However, they are not of much concern for the limited scope of this inquiry. The Boltzmann distribution of DM, $f_X(u)$, used here is taken to be entirely comprised of WIMPs.

## 1.2 WIMP detection

Three primary detection methods exist in the search for WIMPs: those involving particle colliders, those that search for WIMPs directly, and those that search indirectly for signals of annihilations between WIMPs and anti-WIMPs. The first method involves searching for missing energy in proton beam collisions. It is not relevant to this work, so no more will be said of it. The second method relies on detecting collisions between WIMPs and atomic nuclei. We will look at its limits in the next section. The third method looks for Standard Model (SM) particles produced when WIMPs annihilate with their antimatter counterparts (or when those particles decay into other particles). The efficacy of each of these strategies is very dependent on the mass and interactions of the WIMPs, and thus all three must be deployed. However, the relevant method for this work is indirect detection, so we ought to outline the basics of how it works. This will be the subject of the next chapter.

## 1.3 Previous searches for WIMPs

### 1.3.1 Direct experimental searches

Direct searches for WIMPs are limited on Earth by the neutrino background due mostly to atmospheric and solar neutrinos. It used to be thought that there was a hard line (the “neutrino floor”) below which direct detection experiments could not probe, but certain statistical methods allow for exceptions to this. Thus, the term “neutrino fog” has been adopted to describe the boundary at which the probability of observing a WIMP drastically decreases [1]. This “fog” is dependent on the material used in the WIMP detector, thus why Figure 1.1 has separate regions for fluorine and xenon. Existing and future experiments are quickly approaching the fog, so there will be more of a need for indirect detection methods with time.
Indirect detection does not face the same difficulties, especially if one moves the detector into space, as will be proposed later. This is because the most important kind of indirect detection method searches for neutrinos with a very specific energy (MeV-scale) that tends to be distinguishable from the atmospheric neutrino background (GeV-scale). This will be the focus of the following section.

![Figure 1.1: The solid portions are the neutrino-fog limits for fluorine and xenon (two common materials for WIMP detectors) for direct detection experiments. The lines indicate the current limits of existing detectors.](image)

### 1.3.2 Indirect experimental searches

Searches for high energy neutrinos from WIMP annihilation in the Sun have been carried out [2–4]. Furthermore, it was later shown [5, 6] that in models in which the WIMPs annihilate into light quarks (or heavy quarks which then decay into light quarks) there will be a large number of low-energy (sub-GeV) neutrinos produced. These papers focused on decays of muons and pions. However, in a series of papers by Rott, In, Kumar and Yaylali (RIKY) [7–9], it was argued that the pions and kaons would come to rest before decaying and thus would decay into monoenergetic neutrinos. Pions yield 32 MeV neutrinos and kaons yield (64% of the time) a 236 MeV neutrino. RIKY noted that WIMPs with masses below 3-4 GeV would evaporate, but that masses in the 4 – 10 GeV range would cover a region of parameter-space which could be detected at DUNE and would not be excluded by direct detection experiments. A flux of 236 MeV neutrinos coming from the Sun would be a smoking gun for dark matter annihilation. Recently,
DUNE [10] has analyzed this possibility and shown that spin-dependent cross-sections as low as $10^{-38} \text{ cm}^2$ can be reached.

## 1.4 Motivation

Detection of a monoenergetic flux of neutrinos from the Sun would certainly tell us a great deal about WIMP dark matter, but unless one also had direct detection or collider evidence, there would remain many unanswered questions. Are there other celestial bodies that could provide information about dark matter annihilation? WIMP capture in the Earth would be very rare, since Earth has a much smaller size and a much smaller escape velocity. In early papers, Kawasaki et al. [11] and Adler [12] discussed strongly interacting dark matter as source for heating of gas giant planets; Leane et al. [13] looked at the possibility that dark matter could be focused by celestial bodies, increasing the rate of annihilation; and Leane and Linden [14] studied gamma ray emission from dark matter annihilation in Jupiter. Very recently, Li and Fan [15] discussed WIMP capture in Jupiter. They also pointed out that Jupiter is a particularly promising celestial object because it is the largest gas giant and its core is relatively cool, reducing the evaporation rate. As a result, WIMP annihilation for masses well below the 4 GeV evaporation limit from the Sun could collect in the core. Li and Fan studied the possibility that the WIMPs could annihilate into long-lived dark mediators which would convert to electrons and positrons after leaving Jupiter. Current data from the Galileo and Juno orbiters gave interesting constraints on dark matter models.

These works all considered WIMPs that eventually decay into charged particles. Could one detect a monoenergetic flux of neutrinos from Jupiter? Obviously, there is no current detector in orbit that could detect neutrinos, nor is there likely to be one for many decades. But such a detector could encounter a huge flux of neutrinos. The inverse square law alone would give an enhancement of (roughly) the square of 1 A.U./$R_{\text{Jupiter}}$, which is a factor of five million relative to DUNE. This could far exceed the reduction due to the smaller size (relative to the Sun) of Jupiter and the smaller escape velocity. Hopefully by the end of this century robust exploration of the jovian system will be underway and the idea of orbiting a neutrino detector will not be unthinkable. Obviously if dark matter is detected and annihilation into light quarks is possible, then this type of detector could be helpful. Even if the annihilation into light quarks is detected at an earlier stage, such a detector could give us direct information about the Jovian interior. Thus, there is reason to think that it is valuable to study the question of WIMP annihilation into kaons in Jupiter and the detection of the neutrinos, acknowledging that such a detection would be decades away.
Chapter 2

Theory

In this chapter we will see how the flux of monoenergetic neutrinos arriving at a detector can be calculated based on the rate capture, evaporation, and annihilation rates of WIMPs. Two cases are considered. The first is the flux of neutrinos coming from the solar core and arriving at a detector on Earth (e.g., DUNE). The second is the flux of neutrinos coming from the jovian core and arriving at a detector in low-Jupiter orbit. The two are compared for a range of WIMP masses and for three possible values of the spin-dependent, WIMP-proton cross section.

2.1 WIMP population

As WIMPs from the DM halo pass through Jupiter, a portion of them scatter off of atomic nuclei and enter into bound orbits. While some scatter back out after additional collisions (i.e., evaporate), the rest remain bound and thermalize in the planet’s core where they annihilate into Standard Model particles [7, 16]. The rate at which the total population of WIMPs changes with time inside of Jupiter is governed by the following differential equation:

\[
\frac{dN_\chi(t)}{dt} = C - EN_\chi(t) - AN^2_\chi(t)
\] (2.1)

where the coefficients \(C\), \(E\), \(A\) correspond to capture, evaporation, and annihilation, respectively and \(N_\chi\) is the number of WIMPS in Jupiter [17]. A schematic of this process can be seen in Figure 2.1. It is assumed throughout this work that the coefficients are time-independent and that WIMPs are their own anti-particle\(^1\). The solution to this

\(^1\)It has been pointed out [7] that if the WIMP is a Majorana fermion and if one assumes minimal flavor violation, then the annihilation rate is suppressed by light quark masses. Of course, the WIMP could be a scalar or minimal flavor violation might not be realized. In any event, this assumption will not affect our results substantially.
The derivations of the capture, annihilation, and evaporation rates will be given in the following sections. All of the steps will be given and explained in sufficient detail, but we direct curious readers to Refs. [15–17, 19–22] for discussions on some of the subtler parts of the calculations.

\begin{equation}
N_X(t) = \frac{4\tau}{\tau+1} \frac{C \tanh(t/\tau)}{\tanh(t/\tau)}
\end{equation}

where \( \tau = 1/\sqrt{CA + \epsilon^2/4} \) is the time it takes for the system to reach equilibrium [18]. For a 1 GeV WIMP, Li and Fan [15] find that \( t_J/\tau \sim 10 \) where \( t_J \approx 4 \) Gyr is a proxy for the age of the solar system. This is an important result because the annihilation rate, \( \Gamma_A = AN_X^2/2 \), is maximum when \( \tanh(t/\tau) \approx 1 \). Since the outgoing neutrino flux is proportional to the annihilation rate, it is maximized as well.

As a proof-of-concept, this work only aims for a rough comparison of the monoenergetic neutrino flux near Jupiter to that at a detector on Earth. For this reason, certain simplifying assumptions will be made throughout this analysis:

1. Both Jupiter and the Sun are treated as targets of uniform density.

2. Jupiter and the Sun are treated as purely hydrogen targets in order to focus on probing the spin-dependent (SD) WIMP-proton cross section \( \sigma_{SD} \). While the bounds will also apply to the spin-independent (SI) cross section, direct detection experiments will generally provide much tighter bounds on the SI cross section.

3. The SD cross section is small enough that Jupiter and the Sun can be treated as optically thin, so only the single-scattering case is considered.
While these assumptions certainly will decrease the accuracy of the numerical results in both cases, a consistent application of them should not alter the comparison between the cases significantly.

### 2.2 Capture

#### 2.2.1 Deriving the capture rate

There has been considerable work done to show that celestial bodies (e.g., Jupiter and the Sun) are capable of capturing and collecting passing WIMPs. [19, 20, 23–26] To illustrate how this occurs, consider a Maxwell-Boltzmann distribution of WIMPs,

\[
f(u) du = n_\chi \left( \frac{3}{2\pi \bar{v}^2} \right)^{3/2} \exp \left( -\frac{3u^2}{2\bar{v}} \right) 4\pi u^2 du
\]  

(2.3)

where \( \bar{v} = \sqrt{kT_\chi/m_\chi} \approx 270 \) km/s is a velocity dispersion and \( n_\chi = \rho_\chi/m_\chi \) with \( \rho_\chi \) being the local dark matter density. Following the literature, we will take \( n_\chi \) to be 0.4 GeV/cm\(^3\). [15, 16, 27] Next, we must consider the rate at which WIMPs with a given velocity \( u \) are captured by a celestial body.

Consider a unit surface element on a large shell that is centered on a celestial body. The flux of WIMPs passing inwardly through this surface at an angle \( 0 \leq \theta \leq \frac{\pi}{2} \) w.r.t. the radial direction is given by,

\[
dF = \frac{1}{4} f(u) u du \cos^2 \theta.
\]  

(2.4)

Changing variables, we can find the total number of WIMPs passing through the surface element per unit time in terms of the angular momentum per unit mass \( J = uR \sin \theta \),

\[
dF = 4\pi R^2 dF = \pi f(u) du dJ^2.
\]  

(2.5)

We now have the differential accretion rate of WIMPs. However, this is not the same as the capture rate; WIMPs must not only pass through the surface of the celestial object, they must interact with the material and scatter down to a velocity less than the escape velocity. We can find this scattering rate by using conservation of energy to find the velocity \( w \) of a WIMP near the shell,

\[
w^2 = u^2 + v_{esc}^2(r)
\]  

(2.6)

\[\text{After this work was completed, we learned of a recent paper by Leane and Smirnov [33] which does a very detailed analysis of the dark matter distribution in the Sun, Earth and Jupiter, disagreeing somewhat with previous analyses. This will not affect our qualitative results.}\]
then by defining a rate $\Omega_w(w)$ per unit time at which a WIMP with velocity $w$ scatters to a velocity below the escape velocity. We need to know how long the WIMP spends passing through a thin shell of material a radius $r$ with thickness $dr$. Again using energy conservation, we can write the interaction probability as

$$\Omega_w(w)dt = \frac{2\Omega_w(w)dr}{w\sqrt{1 - \left(\frac{J}{rw}\right)^2}} \Theta(rw - J)$$

(2.7)

where the factor of 2 arises because the WIMP will interact twice, depending on whether or not $rw > J$, a condition which is enforced by the step-function. In order to find the specific capture rate (WIMPs per unit volume per unit time), we need to integrate the product of Eqs. 2.7 and 2.5 over the all angular momenta,

$$\frac{dC}{dV} = \int_0^\infty du f_\chi(u) \frac{w\Omega_w(w)}{u} .$$

(2.8)

Thus far everything is known with the exception of $\Omega_w(w)$. We can derive this by considering the range of possible energies that a WIMP could have after colliding with a hydrogen nucleus. The kinematic solution to this scenario has a fractional energy loss within the following range,

$$0 \leq \left| \frac{\Delta E}{E} \right| \leq \frac{\mu}{\mu_+^2}$$

(2.9)

where $\mu = m_\chi/m_H$ and $\mu_\pm = (\mu \pm 1)/2$. However, if the WIMP is to be in a bound orbit after the collision, it must have a change in velocity of at least $w^2 - v_{esc}^2$. This changes the lower bound of Eq. 2.9 to

$$\frac{u^2}{w^2} \leq \left| \frac{\Delta E}{E} \right| \leq \frac{\mu}{\mu_+^2} .$$

(2.10)

Combining Eqs. 2.9 and 2.10 gives us the probability of scattering into a bound orbit:

$$\left(1 - \frac{\mu_+^2}{\mu} \frac{u^2}{w^2}\right) \Theta\left(\frac{\mu}{\mu_+^2} - \frac{u^2}{w^2}\right)$$

(2.11)

where the step-function is included to ensure that the energy loss is positive. The condition within the step-function is useful because it allows us to set an upper bound on the WIMP velocity at infinity,

$$u_{max} \equiv v_{esc} \sqrt{\frac{4m_Hm_\chi}{(m_H - m_\chi)^2}} .$$

(2.12)
Finally, to get the rate of scattering from $w$ to $u < v_{\text{esc}}$, $\Omega_w(w)$, we multiply by the total scattering rate, $\sigma_{SD} n_\chi w$, which results in the following expression,

$$w \Omega_+ (w) = \sigma_{SD} n_\chi v_{\text{esc}}^2 \left( 1 - \frac{\mu^2}{\mu^2 u_{\text{max}}^2} \right) \Theta(u_{\text{max}} - u), \quad (2.13)$$

or

$$w \Omega_+ (w) = \frac{\sigma_{SD} n_\chi v_{\text{esc}}^2}{A^2} \left( A^2 - x^2 \right) \Theta(A - x), \quad (2.14)$$

where $A^2 \equiv \frac{3}{2} \frac{v_{\text{esc}}^2}{v^2} \frac{u}{\mu}$ and $x^2 \equiv \frac{m_{\chi}}{2kT_\chi} u^2$. With this last piece, we can write the full integral in expression 2.8 as,

$$\frac{dC}{dV} = \frac{\sigma_{SD} \rho_\chi v_{\text{esc}}^2}{m_\chi} \int_0^{u_{\text{max}}} du \frac{f_\chi(u)}{u} \left( 1 - \frac{\mu^2}{\mu^2 u_{\text{max}}^2} \right), \quad (2.15)$$

where $n_\chi$ has been replaced by $\rho_\chi/m_\chi$. We have assumed that the shell temperature is zero and the (spin-dependent) cross section, $\sigma_{SD}$, is isotropic and velocity-independent.

The first approximation works because the effects of a finite temperature in the shell only reduce the capture rate by about 0.5% when the shell is at rest relative to the WIMP distribution [19]. The second could be accounted for by averaging the cross section over the velocities in all directions, but that is unnecessary for the illustrative purposes of this work. Finally, for completeness one should also perform a Galilean transformation into the shell’s reference frame to account for the motion of the shell relative to the WIMP distribution, but this is not particularly illuminating for the illustrative purposes of this work. The details of this added step are given by Baum et al [16]. They note that it is negligible.

The result is that the differential capture rate depends on two variables: radius and WIMP mass. The radial dependence comes from $v_{\text{esc}}^2$, which, in principle, is not constant throughout the celestial body. We can obtain the complete capture rate by summing over all shells to get

$$C(m_\chi) = \int_0^R dr (4\pi r^2) \frac{dC}{dV}(r, m_\chi). \quad (2.16)$$
Figure 2.2: Capture rate of WIMPs in both Jupiter and the Sun. The resonance peak at roughly 1 GeV corresponds to the WIMP mass being closely matched to the target. This effect is hardly noticeable in the Sun because of its larger escape velocity \( v_{\text{esc},S} \approx 10 v_{\text{esc},J} \) \([19, 27]\).

2.2.2 Capture rate results

In the case of Jupiter (as well as the Sun) and given the approximations stated in Section 2.1, we estimate the capture rate to be\(^3\)

\[
C = \sqrt{\frac{6}{\pi}} \left( \frac{\rho_X}{m_X} \right) \left( \frac{M}{m_H} \right) \sigma_{\text{SD}} \frac{v_{\text{esc}}^2}{\bar{v}} (R) \left( 1 - \frac{1 - e^{-A^2}}{A^2} \right)
\]  

(2.17)

where \(v_{\text{esc}}(R)\) is the surface escape velocity which is taken to be constant throughout Jupiter (the Sun). The suppression factor \(A^2\), which depends on \(v_{\text{esc}}\), is evaluated at a constant \(v_{\text{esc}} = v_{\text{esc}}(R)\) throughout Jupiter (the Sun).

Our results for the capture rate are given in Figure 2.2. Not surprisingly, the capture rate for Jupiter is substantially lower than the Sun due to its smaller size and smaller escape velocity. The resonance peak at 1 GeV corresponds to the WIMP mass being closely matched to the nucleon mass. This effect is not noticeable in the Sun because its larger escape velocity smooths out the resonance peak \([19, 27]\).  

\(^3\)In the published version of this work, a factor of 0.28 was erroneously included. This factor arises when modeling the internal density using a polytropic model as seen in Ref. \([15]\). This approach was considered in earlier iterations of this work, but the constant-density model was eventually adopted for simplicity. The factor was overlooked in revision. That said, it only introduces a small change to the order-of-magnitude nature of the comparisons seen in Figure 2.2.
2.3 Annihilation

Figure 2.3: Annihilation of WIMPs into SM particles in Jupiter and the Sun. It should be noted that this is the total number of WIMP annihilations, but only a fraction of them produce kaons which decay into muon neutrinos. The annihilation rate is comparable to the capture rate above the evaporation mass because both Jupiter and the Sun are in equilibrium ($\tanh(t/\tau) \simeq 1$).

The rate at which WIMPs annihilate in the core is given by

$$\Gamma_A = \langle \sigma v \rangle_{\text{ann}} \int n^2(r, t) d^3r$$

where $\langle \sigma v \rangle_{\text{ann}} \simeq 3 \times 10^{-26} \text{ cm}^3/\text{s}$ [27, 28] is the velocity-averaged annihilation cross section\(^4\) and $\mathbf{n}(r, t)$ is the WIMP number density. Comparing this with the annihilation term in Eq. 2.2, we find that

$$\Gamma_A = \frac{1}{2} A N^2_\chi(t)$$

where the factor of one half is inserted to account for the fact that an annihilation involves two WIMPs.

If we assume that $\mathbf{n}(r, t)$ can be neatly separated out into its spatial and temporal components ($\mathbf{n}(r, t) = N(t) \tilde{\mathbf{n}}(r)$), then we can write the annihilation coefficient as [15–17]

$$A = \frac{\langle \sigma v \rangle_{\text{ann}}}{V_{\text{eff}}}$$

where the spatial component $\tilde{\mathbf{n}}(r)$ has been replaced by an effective volume given by $V_{\text{eff}} = 4/3\pi r^3_\chi$. The way to think about this scenario is as follows: As the bound WIMPs

\(^4\)In principle, $\langle \sigma v \rangle_{\text{ann}}$ will vary somewhat with $m_\chi$, but we neglect this for the sake of illustration.
continue to lose energy in further scatterings and settle in the core, they occupy an approximately isothermal region given by a scale radius $r_\chi$, temperature $T_c$, and density $\rho_c$. For both the Sun and Jupiter, this is well approximated by [15, 16, 28]

$$r_\chi = \sqrt{\frac{3T_c}{2\pi G\rho_c m_\chi}} \simeq 0.1R \sqrt{\frac{1\text{ GeV}}{m_\chi}}$$

using $T_c = 1.5 \times 10^4 \text{ K} \ (1.5 \times 10^7 \text{ K})$ and $\rho_c = 2 \times 10^4 \text{ kg m}^{-3} \ (1.5 \times 10^5 \text{ kg m}^{-3})$ for Jupiter (the Sun).

The results for the annihilation rate are given in Figure 2.3. Note that what is plotted is the full annihilation rate $\Gamma_A$, not the annihilation coefficient, so $A$ has been halved and multiplied by the square of the population of WIMPs. Thus, it includes the effects of evaporation, discussed in the next subsection. This explains the drop one sees in the solar annihilation rate at 4 GeV and in the Jovian annihilation rate at 1 GeV. It is also important to note that only a fraction of annihilations produce kaons. Above the evaporation mass, the annihilation rate is comparable to the capture rate since Jupiter and the Sun are in equilibrium.

### 2.4 Evaporation

![Figure 2.4: Evaporation of WIMPs inside Jupiter and the Sun. In both cases, we note that just above a certain mass $m_{\text{evap}}$, evaporation drops sharply to zero, whereas below $m_{\text{evap}}$ it is comparable to the rate of WIMPs being captured. We have estimated this mass to be about 1.3 GeV for Jupiter and 3.3 GeV for the Sun, in agreement with previous estimates [15–17, 20, 29]. This is not a great restriction on Jupiter as sub-GeV WIMPs are kinematically unable to annihilate into kaons.](image-url)
Like the capture rate, calculating the evaporation rate involves taking the probability of a flux of WIMPs scattering to $v > v_{esc}$ for a thin shell of material and integrating over the total volume [20]. The derivation of it is very similar with the exception that the conditional scattering rate $\Omega_w(w)$ is replaced by an analogous expression for kinematic scatterings out of bound orbits rather than into them. This process becomes more efficient for smaller masses, and it sets a minimum mass $m_{evap}$ that can remain bound. Below this, evaporation nearly equals capture.

While the full expression that results is far from transparent, it simplifies a great deal in the limits of $m_\chi \sim m_p$, $E_c = m_\chi v_{esc}^2(0)/2 \gg T_c$, and $T_\chi \simeq 0.9T_c$ which hold for an order-of-magnitude estimate. Considering the isothermal region $V_{eff}$ from above, the evaporation coefficient can be approximated by [17, 20, 27]

$$\mathcal{E} = \sigma_H \frac{N_{0.95}}{V_{eff}} \left( \frac{8T_\chi}{\pi m_\chi} \right)^{1/2} \left( \frac{E_c}{T_\chi} \right) e^{-E_c/T_\chi}$$

with $N_{0.95}$ being the number of protons within the region where $T = 0.95T_\chi$. Only a portion of the interior of Jupiter (the Sun) is considered because it corresponds to a region where evaporation is significantly enhanced by the closely matched WIMP and nucleon temperatures. For our purposes, this provides a decent approximation for the overall evaporation rate. We take $N_{0.95} \sim 0.1M/m_p$ which is known to be a reasonable approximation for the Sun [17, 20, 29]. Here, we also use $v_{esc}^2(0) \simeq 1.5v_{esc}(R)$ for Jupiter and $v_{esc}^2(0) \simeq 5v_{esc}^2(R)$ for the Sun.

The results are plotted in Figure 2.4. One sees that the evaporation is negligible for WIMP masses above 3.3 GeV for the Sun and above 1.3 GeV for Jupiter, as expected. [15–17, 20, 29]

### 2.5 Neutrino flux

WIMP annihilations (which occur at a rate $\Gamma_A = AN_\chi^2/2$) can produce kaons for WIMPs with $m_\chi \gtrsim 1$ GeV. The kaons, upon coming to rest, decay into muon neutrinos via $K^+ \rightarrow \nu_\mu \mu^+$ with a branching ratio of about 64%. The outgoing flux is given by [7, 16, 27]

$$\frac{d^2 \Phi_{\nu_\mu}}{dE d\Omega} = \frac{\Gamma_A}{4\pi D^2} N_K B_{\nu_\mu} \delta(E - E_0) \delta(\Omega)$$

where $D$ is the core-detector distance, $N_K$ is the average number of $K^+$ produced per annihilation, and $B_{\nu_\mu}$ is the fraction of $K^+$ that decay into $\nu_\mu$. The two dirac delta terms enforce the conditions that the energy signal is monoenergetic ($E_0 \approx 236$ MeV) and that all neutrinos emanate from the jovian or solar core, respectively. We can express $N_K$ in
terms of the fraction \( r_K \) of the c.o.m. energy that is converted into \( K^+ \) as

\[
N_K = \frac{2m_X r_K}{m_K}.
\]

(2.24)

We take \( r_K \sim 1/50 \) for simplicity [7]. As mentioned above, the condition of equilibrium is important because it means that the flux is maximized for \( t \gg \tau \). However, in the region where evaporation dominates, the flux decreases drastically because annihilations occur far more infrequently.

![Annual neutrino flux](image)

**Figure 2.5:** The dot-dashed lines give the flux of 236 MeV neutrinos at the surface of the Earth from WIMP annihilation in the Sun for three different spin-dependent cross sections. The solid lines give the flux from WIMP annihilation in Jupiter, near the surface of Jupiter. Note that the flux near Jupiter is substantially higher in the 1-4 GeV region. We have included the phase space factor of \( \sqrt{1 - m_K^2/m_W^2} \) in the figure - this is negligible above 1 GeV.

From Figure 2.5, one can see that the flux of neutrinos in low-Jovian orbit is comparable to the flux from the Sun at 1 AU (i.e. at DUNE) in the mass range at or above 4 GeV. However, in the 1-4 GeV mass range, the flux at Earth orbit is negligible where as the flux in low-Jovian orbit is substantial (below 1 GeV, WIMP annihilation into kaons becomes negligible due to phase space). Thus we focus on the 1–4 GeV mass range.

If the cross section is spin-dependent, direct detection experiments currently can’t detect WIMPs below 2.3 GeV for any cross section. One reason for this is the energy thresholds of direct-detection experiments. For example, PICASSO reports a sensitivity to nucleus recoil energies as low as 1.7 keV [30]. For WIMPs in the halo, this gives a lower bound of 2.3 GeV. However, a proposal by CYGNUS could eventually lead to a sensitivity corresponding to WIMP masses as low as 1 GeV [31]. Thus, in the coming decade, the
range of $1 - 4$ GeV will be explored if the spin-dependent cross section is sufficiently large. If there is a WIMP in this range, 236 MeV neutrinos from the Sun will not be detectable, and one way to study the annihilation would be to look for neutrinos from Jupiter.
Chapter 3

Detection in Jovian orbit

Of course, to detect neutrinos in low-Jovian orbit, one would need to orbit a neutrino detector. This seems absurd, and for the next few decades is certainly utterly infeasible. One can imagine that later in this century, there will be human exploration of the Jovian system and a reasonably sized neutrino detector might be thinkable. In this chapter the nature of such a detector and its location will be discussed. We recognize that this is an extremely preliminary discussion, given the unknown nature of technological advances between now and then. But it is still interesting to speculate.

3.1 Detection method

When a 236 MeV neutrino interacts with an oxygen or argon nucleus, the charged lepton that emerges is close to isotropic - the Fermi motion of the struck nucleon in the nucleus alone will tend to isotropize the charged lepton. The proton that emerges, however, will tend to be in the forward direction. Protons with a kinetic energy of a few hundred MeV will not emit Cerenkov radiation and thus water Cerenkov detectors will not be useful (this is unfortunate since the Jovian system has a substantial amount of water/ice). Liquid argon time projection chamber (TPC) detectors like DUNE would be able to detect these protons and can thus reconstruct the direction and energy of the incident neutrino. Of course, one would need a substantial number of events to determine the average incident neutrino detection - a precise energy and direction determination on an event-by-event basis would not be possible.

A more promising possibility\(^1\) is a liquid hydrogen TPC or bubble chamber detector. There is no Fermi motion and the charged lepton can be easily seen. The neutron

\(^1\)We thank Mike Kordosky for this suggestion.
emerging from the interaction will travel some distance and interact - that interaction can also be seen. Thus the entire event can be seen, leading to an event-by-event determination of the energy and direction of the initial neutrino. It should be noted that the source isn’t at the precise center of Jupiter but typically within 0.1R_J, thus the angle of approach will not be completely determined in advance. Neutral current neutrino interactions can also be studied, although the backgrounds might be substantial. One can also note that the lower energy neutrinos from pion decay (30 MeV) might be detectable as well, although the length of the nucleon track might be too short. While a large liquid hydrogen detector would be too dangerous to be built on Earth, this would not be a problem in Jovian orbit. And since Jupiter is almost entirely hydrogen, the liquid hydrogen needed for the detector would not have to be transported from Earth (unlike liquid argon or liquid scintillator).

3.2 Detector placement

What about the location? Even with good directional information, there will be backgrounds from cosmic ray interactions in the atmosphere. In addition, power generation would be a more serious problem in low-Jovian orbit - large solar panels attached to the orbiting detector could cause instabilities. One way to avoid these problems would be to place the detector on the back side of one of Jupiter’s tidally-locked moons. Solar panels could be spread on the surface fairly easily and the moon itself would provide shielding. Liquid hydrogen would need thermal isolation from sunlight but shielding should not pose great difficulties.

All four of Jupiter’s four innermost moons - Metis, Adrastea, Amalthea, and Thebe - are candidate locations on which to place a space-bound neutrino detector for close observations of Jupiter. Unfortunately, none of them have been the subject of thorough investigation, so only general statements can be made about them. The following is meant to be a brief survey of what relevant information is known thus far, especially the orbital periods and resultant observation times. General or prominent features of terrain and composition are mentioned insofar as they may affect a neutrino detector’s function.

Much of the information we have of these fours moons was gathered by the Voyager 1 and 2 and Galileo space probes. Certain general inferences can be made of all of them: They are expected to be rather porous with mean densities slightly less than water. They all lie within about 3R_J of Jupiter’s “surface”\(^2\), have synchronous orbits,

\(^2\)Tidal forces are not negligible at this range, but each of the moons are small enough that they are relatively immune to the effects. It does, however, contribute to their stretched, oblong shapes.
and deviate only slightly from Jupiter’s equator. Unfortunately, this puts them near the
strongest portions of Jupiter’s radiation belts - this is part of the motivation for placing
a neutrino detector onto one of their surfaces, perhaps within one of their many large
craters. A helpful summary of what is known is given by Thomas et al. [32]

Of the four candidates, Metis is the closest with an orbital radius of $1.8R_J$. It’s orbit is
synchronous with period of about 7 hours and lies roughly along Jupiter’s equator. With
a radius of less than 22 kilometers, it is not very large. The Galileo spacecraft found
Metis to have a large central crater with many smaller impact craters scattered across
its surface. The larger crater could serve to increase the total shielding if a detector
were placed inside of it. Additionally, Metis is is irregular in shape with its longest axis
radially aligned with Jupiter. Its surface may contain water ice.

Adrastea orbits only slightly further out from Jupiter compared to Metis with roughly
the same orbital period and radius. However, its rotational period is currently unknown.
Its size is also not well constrained, but it appears to be the smallest of the four. The
only existing images of Adrastea have significantly lower resolution than those of Metis,
so little to nothing is known of its surface; however, it is expected to feature hills and
craters as well as patches of water ice.

The largest in the group is Amalthea with a mean radius of nearly 84 km. It orbits
at $2.5R_J$ with an orbital period of nearly 12 hours. Unlike the other moons, Amalthea
has a reddish surface in the photographs taken by the Voyager (1 and 2) and Galileo
mission. Some of this brighter patches may be due to sulfur, but this is not known.
Like all of the innermost moons, it has a highly irregular shape and a heavily cratered
surface. One unique feature of this moon is that it radiates slightly more energy than
what it receives from the Sun, possibly due to tidal forces and other effects.

Finally, Thebe orbits out at just over $3R_J$, completing its orbit every 16 hours or so.
Coming in at 49 km for its mean radius, it is the second largest of the four. Images
of Thebe indicate that it might be the least irregularly shaped of the bunch, but it is
still far from a perfect sphere. It’s defining feature is that it has one or two very large
craters that are comparable to the size of the moon itself; the largest of these faces away
from Jupiter. Thebe is thought to sit near Jupiter’s Roche limit which would lower the
escape velocity from the surface.

Given the above summaries, it is difficult to determine which is a better candidate
location for a neutrino detector. Little is known, and what is known well is characteristic
of all four. However, the least promising of the lineup may be Adrastea; it is the smallest
and has no known surface features that might serve to shield the detector from debris.
Amalthea is promising simply because of its size and variety of craters. However, if it exhibits tidal flexing then it may not be the ideal location to fix sensitive equipment.

One who wishes to continue this line of inquiry might consider the following questions about the moons: Which moon demonstrates the most stability on its surface? Do any of them lie within gaps in or weaker bands of Jupiter’s radiation belts? How often are they bombarded by material from the Gossamer Rings? Additionally, someone with more expertise in neutrino detectors might ask whether the orbital period of the moon will make a difference in neutrino detection. If the solar neutrino flux appreciably decreases as Jupiter eclipses the Sun, then the orbital period becomes an important consideration as it will determine the hourly variation in neutrino events. This, of course, will depend on how many neutrinos are expected to be detected per second.
Chapter 4

Conclusions

WIMPs in the galactic halo can interact in the Sun and their velocities can drop below the escape velocity. These captured WIMPs will gradually fall into the core and annihilate. While the annihilation products are very model-dependent, there are many models in which they decay into light quarks, leading to production of pions and kaons. The kaons will quickly slow down in the dense core and decay into (64% of the time) monoenergetic 236 MeV neutrinos. Studies have been done calculating the flux of these on Earth and if the WIMP spin-dependent cross section is sufficiently large then the flux will be large enough that these neutrinos can be detected by DUNE. However, if the WIMP mass is below 4 GeV, then the WIMPs will evaporate before annihilation. It has been pointed out that Jupiter has a colder core than the Sun and thus WIMPs in the 1-4 GeV range will annihilate.

In this work, we have calculated the flux of 236 MeV neutrinos from Jupiter near the Jovian “surface”. Comparing with DUNE, the flux near the surface gets a huge enhancement from the inverse-square law. For WIMP masses in the 4-10 GeV range, the flux is comparable to DUNE. However, in the 1-4 GeV range, the flux from the Sun drops off rapidly and the flux near the surface of Jupiter does not. We studied the possibility of a neutrino detector orbiting Jupiter. While obviously many decades away, we speculate on the type and location of such a detector. A liquid hydrogen TPC would possibly allow determination of the energy and direction of 236 MeV neutrinos on an event-by-event basis. Locating the detector on the far side of a tidally locked moon orbiting near Jupiter would allow the moon to act as shielding and provide more ease in producing power for the detector.

In the coming decade or two, direct detection searches will cover the entire 1-4 GeV region of WIMP masses down to some spin-dependent cross section level. Over the decades, this level will drop. Should a positive signal be found, we will only know the
cross-section and mass of the WIMPs. At that point, one might take the concept of a Jovian neutrino detector seriously as the only way to learn about the annihilation process. If so, a neutrino detector placed on the surface of a low-orbit Jovian moon is not a far-fetched proposition.
Appendix A

Fortran code

The original intent for this work was to perform all of the difficult calculations using DarkSUSY - a Fortran package which allows one to calculate a variety of observables for different DM models, including WIMPs. The DarkSUSY package contains interior models of both the Sun and the Earth, allowing one to calculate rates of capture, annihilation, etc. with relative ease. Unfortunately, it proved more difficult than expected to incorporate an interior model of Jupiter due to the structure of the DarkSUSY code. Instead, a custom Fortran-Python package was written, called $DMWIMPs$, which takes the approximations given in the sections above and plots them over a range of possible WIMP masses for three different values of the cross section. This is how all of the plots were generated.

Structurally, it is broken down into two main sections, one for calculating (Fortran) and the other for plotting (Python). The former is uses all the approximations given above to calculate the coefficients, the WIMP population, and finally the rates. The latter contains separate files for plotting each rate that can be accessed by a makefile in the Fortran section. This is prompted by a user selection.

What the code lacks, however, is the ability to perform the calculations for different models of the Jovian interior. Currently, it only implements the highly simplified interior model given earlier (See Section 2.1). Future iterations of this package would need to include more accurate interior models using Juno data, possibly with the option to select between competing models. That is, of course, if they make an appreciable difference. Additionally, the code does not allow for one to specify an observational distance - it assumes a detector place on the Jovian surface in every case (a decent approximation for Metis and Adrastea).
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