Signal-Locality and Subquantum Information in Deterministic Hidden-Variables Theories

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It is proven that any deterministic hidden-variables theory, that reproduces quantum theory for a ‘quantum equilibrium’ distribution of hidden variables, must predict the existence of instantaneous signals at the statistical level for hypothetical ‘nonequilibrium ensembles’. This ‘signal-locality theorem’ generalises yet another feature of the pilot-wave theory of de Broglie and Bohm, for which it is already known that signal-locality is true only in equilibrium. Assuming certain symmetries, lower bounds are derived on the ‘degree of nonlocality’ of the singlet state, defined as the (equilibrium) fraction of outcomes at one wing of an EPR-experiment that change in response to a shift in the distant angular setting. It is shown by explicit calculation that these bounds are satisfied by pilot-wave theory. The degree of nonlocality is interpreted as the average number of bits of ‘subquantum information’ transmitted superluminally, for an equilibrium ensemble. It is proposed that this quantity might provide a novel measure of the entanglement of a quantum state, and that the field of quantum information would benefit from a more explicit hidden-variables approach. It is argued that the signal-locality theorem supports the hypothesis, made elsewhere, that in the remote past the universe relaxed to a state of statistical equilibrium at the hidden-variable level, a state in which nonlocality happens to be masked by quantum noise.
1 Introduction

Bell’s theorem shows that, with reasonable assumptions, any deterministic hidden-variables theory behind quantum mechanics has to be nonlocal [1]. Specifically, for pairs of spin-1/2 particles in the singlet state, the outcomes of spin measurements at one wing must depend instantaneously on the axis of measurement at the other, distant wing. Historically, Bell’s theorem was inspired by a specific nonlocal hidden-variables theory: the pilot-wave theory of de Broglie and Bohm [2–10]. In his famous review article [11], Bell asked if all hidden-variables theories that reproduce the quantum distribution of outcomes have to be nonlocal like pilot-wave theory. He subsequently proved that this is indeed the case.

A further property of pilot-wave theory was also proved to be a general feature, namely ‘contextuality’. In general, so-called quantum measurements are not faithful: they do not reveal the value of an attribute of the system existing prior to the ‘measurement’. This property of pilot-wave theory was discovered by Bohm [3], and the Kochen-Specker theorem [12] tells us that any hidden-variables interpretation of quantum mechanics must share this property.

The question naturally arises: are there any other properties of pilot-wave theory that are actually universal, in the sense of necessarily being properties of any viable hidden-variables theory? In this paper we shall prove that, indeed, yet another feature of pilot-wave theory – the ‘signal-locality theorem’ – is in fact generally true in any deterministic hidden-variables interpretation: there are instantaneous signals at the statistical level for hypothetical ‘nonequilibrium’ ensembles whose distribution differs from that of quantum theory.

We shall also obtain – assuming certain symmetries – lower bounds on the nonlocal flow of ‘subquantum information’ between entangled systems, and we shall check in detail that these bounds are satisfied (indeed saturated) by pilot-wave theory. It will be suggested that the ‘degree of nonlocality’, which quantifies nonlocal information flow, be explored as a new measure of entanglement, and that the field of quantum information generally would benefit from a more explicit hidden-variables perspective.

Finally, it will be urged that the results of this paper be viewed in a cosmological context, as supporting the hypothesis that quantum theory is merely a theory of an equilibrium state, to which the universe relaxed in the remote past, perhaps soon after the big bang.

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5 It is assumed in particular that there is no ‘conspiracy’ or common cause between the hidden variables and the measurement settings, and that there is no backwards causation (so that the hidden variables are unaffected by the future outcomes). Bell’s original paper addressed only the deterministic case. The later generalisations to stochastic theories are of no concern here.

6 Note that, contrary to a widespread misunderstanding, at the 1927 Fifth Solvay Congress de Broglie proposed the full pilot-wave dynamics in configuration space for a many-body system, and not just the one-body theory. See ref. [2].

7 Even if the original paper by Kochen and Specker erroneously claimed to prove the nonexistence of hidden variables.
2 Signal-Locality in Pilot-Wave Theory

In pilot-wave theory, an individual system with wavefunction $\psi(x,t)$ is assumed to have a definite configuration $x(t)$ at all times, whose velocity is determined by the de Broglie guidance equation $\dot{x}(t) = j(x,t)/|\psi(x,t)|^2$ where $j$ is the usual quantum probability current. Given $\psi(x,t)$, this formula determines the velocity $\dot{x}(t)$. The wavefunction $\psi$ is interpreted as an objective ‘guiding field’ in configuration space, and satisfies the usual Schrödinger equation.

To recover quantum theory, it must also be assumed that an ensemble of systems with wavefunction $\psi_0(x)$ at $t = 0$ begins with a distribution of configurations given by $\rho_0(x) = |\psi_0(x)|^2$ (which guarantees that $\rho(x,t) = |\psi(x,t)|^2$ at all future times). In other words, the Born probability distribution is assumed as an initial condition. In principle, however, the theory – considered as a theory of dynamics – allows one to consider arbitrary initial distributions $\rho_0(x) \neq |\psi_0(x)|^2$, which violate quantum theory. The ‘quantum equilibrium’ distribution $\rho = |\psi|^2$ is analogous to thermal equilibrium in classical mechanics: in both cases, the underlying dynamical theory allows one to consider nonequilibrium; and in both cases equilibrium may be accounted for on the basis of an appropriate $H$-theorem [13, 14]. Thus, pilot-wave theory is richer than quantum theory, containing the latter as a special case. (It has in fact been argued that nonequilibrium $\rho \neq |\psi|^2$ existed in the early universe, and may still exist today for some relic cosmological particles [5, 10, 13–17].)

Now at the fundamental hidden-variable level, pilot-wave theory is nonlocal. For example, for two entangled particles $A$ and $B$ the wavefunction $\psi(x_A, x_B, t)$ has a non-separable phase $S(x_A, x_B, t)$ and the velocity $dx_A/dt = \nabla_A S(x_A, x_B, t)/m$ of particle $A$ depends instantaneously on the position $x_B$ of particle $B$. And in general, operations performed at $B$ (such as switching on an external potential) have an instantaneous effect on the motion of particle $A$ no matter how distant it may be.

However, at the quantum level, where one considers an ensemble with the equilibrium distribution $\rho(x_A, x_B, t) = |\psi(x_A, x_B, t)|^2$, operations at $B$ have no statistical effect at $A$: as is well known, quantum entanglement cannot be used for signalling at a distance.

On the other hand, this masking of nonlocality by statistical noise is peculiar to the distribution $\rho = |\psi|^2$. If one considers an ensemble of entangled particles at $t = 0$ with distribution $\rho_0(x_A, x_B) \neq |\psi_0(x_A, x_B)|^2$, it may be shown by explicit calculation that changing the Hamiltonian at $B$ induces an instantaneous change in the marginal distribution $\rho_A(x_A, t) \equiv \int dx_B \rho(x_A, x_B, t)$ at $A$. For a specific example it was found that a sudden change $\hat{H}_B \to \hat{H}'_B$ at $B$ – say a change in potential – leads after a short time $\epsilon$ to a change $\Delta \rho_A \equiv \rho_A(x_A, \epsilon) - \rho_A(x_A, 0)$ at $A$ given by [15]

$$\Delta \rho_A = -\frac{\epsilon^2}{4m} \frac{\partial}{\partial x_A} \left( a(x_A) \int dx_B \frac{\rho_0(x_A, x_B) |\psi_0(x_A, x_B)|^2}{|\psi_0(x_A, x_B)|^2} b(x_B) \right)$$

(Here $a(x_A)$ is a factor depending on $\psi_0$; the factor $b(x_B)$ also depends on $\hat{H}'_B$...
and vanishes if $\hat{H}_B' = \hat{H}_B$. In equilibrium $\rho_0 = |\psi_0|^2$ the signal vanishes; while in general, for $\rho_0 \neq |\psi_0|^2$ there are instantaneous signals at the statistical level.\footnote{Of course, the signal may vanish for some special $\rho_0 \neq |\psi_0|^2$, but not in general.}

This is the signal-locality theorem of pilot-wave theory: in general, there are instantaneous signals at the statistical level if and only if the ensemble is in quantum nonequilibrium $\rho_0 \neq |\psi_0|^2$ \cite{15}. We wish to show that the same is true in any deterministic hidden-variables theory.

## 3 Bell Nonlocality

It is convenient first of all to review Bell’s theorem in its original formulation \cite{1}.

Consider two spin-$1/2$ particles lying on the $y$-axis at $A$ and $B$ and separated by a large distance. If the pair is in the singlet state $|\Psi\rangle = (|z+,z-⟩ - |z-,z+⟩)/\sqrt{2}$, spin measurements along the $z$-axis at each wing always yield opposite results. But we are of course free to measure spin components along arbitrary axes at each wing. For simplicity we take the measurement axes to lie in the $x-z$ plane, so that their orientations may be specified by the (positive or negative) angles $\theta_A, \theta_B$ made with the $z$-axis. In units of $\hbar/2$, the possible values of outcomes of spin measurements along $\theta_A, \theta_B$ at $A, B$ – that is, the possible values of the quantum ‘observables’ $\hat{\sigma}_A, \hat{\sigma}_B$ at $A, B$ – are $±1$. Quantum theory predicts that for an ensemble of such pairs, the outcomes at $A$ and $B$ are correlated: $\langle \Psi | \hat{\sigma}_A \hat{\sigma}_B | \Psi \rangle = -\cos(\theta_A - \theta_B)$.

One now assumes the existence of hidden variables $\lambda$ that determine the outcomes $\sigma_A, \sigma_B = ±1$ along $\theta_A, \theta_B$. It is further assumed that there exists a ‘quantum equilibrium ensemble’ of $\lambda$ – that is, a distribution $\rho_{eq}(\lambda)$ that reproduces the quantum statistics (where $\int d\lambda \rho_{eq}(\lambda) = 1$). Each value of $\lambda$ determines a pair of outcomes $\sigma_A, \sigma_B$ (for given $\theta_A, \theta_B$); for an ensemble of similar experiments – in which the values of $\lambda$ generally differ from one run to the next – one obtains a distribution of $\sigma_A, \sigma_B$, which is assumed to agree with quantum theory. In particular, the expectation value

$$\sigma_A \sigma_B = \int d\lambda \rho_{eq}(\lambda) \sigma_A(\theta_A, \theta_B, \lambda) \sigma_B(\theta_A, \theta_B, \lambda)$$

must reproduce the quantum result $\langle \hat{\sigma}_A \hat{\sigma}_B \rangle = -\cos(\theta_A - \theta_B)$. Bell showed that this is possible only if one has nonlocal equations

$$\sigma_A = \sigma_A(\theta_A, \theta_B, \lambda), \quad \sigma_B = \sigma_B(\theta_A, \theta_B, \lambda)$$

in which the outcomes depend on the distant angular settings \cite{1}.

In principle, the nonlocality might be just ‘one-way’, with only one of $\sigma_A, \sigma_B$ depending on the distant setting. For instance, one might have $\sigma_A = \sigma_A(\theta_A, \theta_B, \lambda)$ and $\sigma_B = \sigma_B(\theta_A, \theta_B, \lambda)$ (where $\lambda$ are initial values) allows for this. See ref. \cite{4}, chapter 8.

\footnote{Here $\lambda$ are the initial values of the hidden variables, for example just after the source has produced the singlet pair. Their later values may be affected by changes in $\theta_A, \theta_B$, and writing $\sigma_A = \sigma_A(\theta_A, \theta_B, \lambda), \sigma_B = \sigma_B(\theta_A, \theta_B, \lambda)$ (where $\lambda$ are initial values) allows for this. See ref. \cite{4}, chapter 8.}
where \( S \) of \( S \), equilibrium distribution (Bell’s theorem requires some nonlocal dependence in at least one direction.)

\( \rho \) respect to the equilibrium measure \( \rho \), mathematically – a ‘nonequilibrium’ distribution \( \rho \). (Such theories certainly exist, and pilot-wave theory provides an example.)

Given a distribution \( \rho_{eq}(\lambda) \), one can always contemplate – purely theoretically – a ‘nonequilibrium’ \( \rho \) distribution \( \rho(\lambda) \neq \rho_{eq}(\lambda) \), even if one cannot prepare such a distribution in practice. For example, given an ensemble of values of \( \lambda \) with distribution \( \rho_{eq}(\lambda) \), mathematically one could pick a subensemble such that \( \rho(\lambda) \neq \rho_{eq}(\lambda) \).

The theorem to be proved is then the following: in general, there are instantaneous signals at the statistical level if and only if the ensemble is in quantum nonequilibrium \( \rho(\lambda) \neq \rho_{eq}(\lambda) \).

Proof: Assume first that \( \sigma_A \) has some dependence on the distant setting \( \theta_B \). (Bell’s theorem requires some nonlocal dependence in at least one direction.)

Now consider an ensemble of experiments with fixed settings \( \theta_A, \theta_B \) and an equilibrium distribution \( \rho_{eq}(\lambda) \) of hidden variables \( \lambda \). In each experiment, a particular value of \( \lambda \) determines an outcome \( \sigma_A = \sigma_A(\theta_A, \theta_B, \lambda) \) at \( A \). Some values of \( \lambda \) yield \( \sigma_A = +1 \), some yield \( \sigma_A = -1 \). What happens if the setting \( \theta_B \) at \( B \) is changed to \( \theta_B' \)?

The set \( S = \{ \lambda \} \) of possible values of \( \lambda \) may be partitioned in two ways:

\[
S_{A+} = \{ \lambda | \sigma_A(\theta_A, \theta_B, \lambda) = +1 \}, \quad S_{A-} = \{ \lambda | \sigma_A(\theta_A, \theta_B, \lambda) = -1 \}
\]

where \( S = S_{A+} \cup S_{A-} \), \( S_{A+} \cap S_{A-} = \emptyset \), and

\[
S'_{A+} = \{ \lambda | \sigma_A(\theta_A, \theta_B', \lambda) = +1 \}, \quad S'_{A-} = \{ \lambda | \sigma_A(\theta_A, \theta_B', \lambda) = -1 \}
\]

where \( S = S'_{A+} \cup S'_{A-} \), \( S'_{A+} \cap S'_{A-} = \emptyset \). (There could exist a pathological subset of \( S \) that gives neither outcome \( \sigma_A = \pm 1 \), but this must have measure zero with respect to the equilibrium measure \( \rho_{eq}(\lambda) \), and so may be ignored.) It cannot be the case that \( S_{A+} = S'_{A+} \) and \( S_{A-} = S'_{A-} \) for arbitrary \( \theta_B' \), for otherwise the outcomes at \( A \) would not depend at all on the distant setting at \( B \). Thus in general

\[
T_A(+, -) \equiv S_{A+} \cap S'_{A-} \neq \emptyset, \quad T_A(-, +) \equiv S_{A-} \cap S'_{A+} \neq \emptyset
\]

In other words: under a shift \( \theta_B \rightarrow \theta_B' \) in the setting at \( B \), some values of \( \lambda \) that would have yielded the outcome \( \sigma_A = +1 \) at \( A \) now yield \( \sigma_A = -1 \); and some \( \lambda \) that would have yielded \( \sigma_A = -1 \) now yield \( \sigma_A = +1 \).

Of the equilibrium ensemble with distribution \( \rho_{eq}(\lambda) \), a fraction

\[
\nu_A^{eq}(+, -) = \int_{T_A(+, -)} d\lambda \rho_{eq}(\lambda)
\]
make the nonlocal ‘transition’ \(\sigma_A = +1 \rightarrow \sigma_A = -1\) under the distant shift \(\theta_B \rightarrow \theta_B'\). Similarly, a fraction

\[
\nu_A^\text{eq}(-, +) = \int_{T_A(-,+)} d\lambda \rho_{\text{eq}}(\lambda)
\]

make the ‘transition’ \(\sigma_A = -1 \rightarrow \sigma_A = +1\) under \(\theta_B \rightarrow \theta_B'\).

Now with the initial setting \(\theta_A, \theta_B\), quantum theory tells us that one half of the equilibrium ensemble of values of \(\lambda\) yield \(\sigma_A = +1\) and the other half yield \(\sigma_A = -1\). (That is, the equilibrium measures of \(S_{A+}\) and \(S_{A-}\) are both 1/2.) With the new setting \(\theta_A, \theta_B'\), quantum theory again tells us that one half yield \(\sigma_A = +1\) and the other half yield \(\sigma_A = -1\) (the equilibrium measures of \(S_{A+}'\) and \(S_{A-}'\), again being 1/2). The 1:1 ratio of outcomes \(\sigma_A = \pm 1\) is preserved under the shift \(\theta_B \rightarrow \theta_B'\), from which we deduce the condition of ‘detailed balancing’

\[
\nu_A^\text{eq}(+,-) = \nu_A^\text{eq}(-,+)
\]

The fraction of the equilibrium ensemble that makes the transition \(\sigma_A = +1 \rightarrow \sigma_A = -1\) must equal the fraction that makes the reverse transition \(\sigma_A = -1 \rightarrow \sigma_A = +1\).

But for an arbitrary nonequilibrium ensemble with distribution \(\rho(\lambda) \neq \rho_{\text{eq}}(\lambda)\), the ‘transition sets’ \(T_A(+,-)\) and \(T_A(-,+)\) will generally have different measures

\[
\int_{T_A(+,-)} d\lambda \rho(\lambda) \neq \int_{T_A(-,+)} d\lambda \rho(\lambda)
\]

and the nonequilibrium transition fractions will generally be unequal,

\[
\nu_A(+,-) \neq \nu_A(-,+)
\]

(Note that \(T_A(+,-)\) and \(T_A(-,+)\) are fixed by the underlying deterministic theory, and are therefore independent of \(\rho(\lambda)\).) Thus, if with the initial setting \(\theta_A, \theta_B\) we would have obtained a certain nonequilibrium ratio of outcomes \(\sigma_A = \pm 1\) at \(A\), with the new setting \(\theta_A, \theta_B'\) we will obtain a different ratio at \(A\). Under a shift \(\theta_B \rightarrow \theta_B'\), the number of systems that change from \(\sigma_A = +1\) to \(\sigma_A = -1\) is unequal to the number that change from \(\sigma_A = -1\) to \(\sigma_A = +1\), causing an imbalance that changes the outcome ratios at \(A\). In other words, in general the statistical distribution of outcomes at \(A\) is altered by the distant shift \(\theta_B \rightarrow \theta_B'\), and there is a statistical signal from \(B\) to \(A\). (Of course, the signal vanishes for special \(\rho(\lambda) \neq \rho_{\text{eq}}(\lambda)\) that happen to have equal measures for \(T_A(+,-)\) and \(T_A(-,+)\), but not in general.)

Similarly, if \(\sigma_B\) depends on the distant setting \(\theta_A\), one may define non-zero transition sets \(T_B(+,-)\) and \(T_B(-,+)\) ‘from \(A\) to \(B\)’; and in nonequilibrium there will generally be statistical signals from \(A\) to \(B\).

In the special case of ‘one-way’ nonlocality, only one of the pairs \(T_A(+,-)\), \(T_A(-,+)\) or \(T_B(+,-)\), \(T_B(-,+)\) has non-zero measure, and nonequilibrium signalling occurs in one direction only.
As an illustrative example of signalling from $B$ to $A$, one might have a theory in which the variables $\lambda$ consist of pairs of real numbers $(p, q)$ confined to the area of a unit circle centred on the origin ($p^2 + q^2 \leq 1$). Imagine that, for the initial setting $\theta_A, \theta_B$, the right half of the circle yields $\sigma_A = +1$ while the left half yields $\sigma_A = -1$ (that is, $S_{A+} = \{(p, q) | p > 0\}$, $S_{A-} = \{(p, q) | p < 0\}$). Imagine further that under a small shift $\theta_B \to \theta_B'$ the vertical chord dividing the circle into $S_{A+}$ and $S_{A-}$ rotates slightly about the origin, with the area to the right of the rotated chord yielding $\sigma_A = +1$ and the area to the left yielding $\sigma_A = -1$. If we take $\rho_{eq}(\lambda)$ to be uniformly distributed over the area of the circle, we obtain the quantum 1:1 ratio of outcomes $\sigma_A = \pm 1$, both before and after the shift $\theta_B \to \theta_B'$. But in general, for a nonuniform distribution $\rho(\lambda) \neq \rho_{eq}(\lambda)$, not only will the outcome ratio at $A$ with the initial setting $\theta_A, \theta_B$ be different from 1:1, the ratio at $A$ will change as the chord dividing the circle is rotated by the shift $\theta_B \to \theta_B'$.

We have said that, even if $\sigma_A$ depends on $\theta_B$, the signal from $B$ to $A$ will vanish for special $\rho(\lambda) \neq \rho_{eq}(\lambda)$ such that the transition sets $T_A(+, -), T_A(-, +)$ have equal measure. This hardly affects our argument: the point remains that for a general nonequilibrium distribution the measures will be unequal and there will be a signal. At the same time, it would be interesting to know if the signal can vanish for some special $\rho(\lambda) \neq \rho_{eq}(\lambda)$ for all angular settings or only for some. (Clearly, for specific angles $\theta_A, \theta_B, \theta_B'$ and associated sets $T_A(+, -), T_A(-, +)$, one may trivially choose a special $\rho(\lambda) \neq \rho_{eq}(\lambda)$ such that $T_A(+, -), T_A(-, +)$ have the same measure – for example, $\rho(\lambda) \propto \rho_{eq}(\lambda)$ for $\lambda \in T_A(+, -) \cup T_A(-, +)$ and $\rho(\lambda) = 0$ otherwise. But it is not known whether there can exist a $\rho(\lambda) \neq \rho_{eq}(\lambda)$ that, like $\rho_{eq}(\lambda)$, has equal measures for $T_A(+, -), T_A(-, +)$ for all $\theta_A, \theta_B, \theta_B'$. One suspects not, but the matter should be examined further.)

5 Degree of Nonlocality

The possibility of nonlocal signalling from $B$ to $A$ (or from $A$ to $B$) depends on the existence of finite transition sets $T_A(+, -), T_A(-, +)$ (or $T_B(+, -), T_B(-, +)$). We have shown by a detailed-balancing argument that $T_A(+, -), T_A(-, +)$ (or $T_B(+, -), T_B(-, +)$) have equal equilibrium measure, so that the signal vanishes in equilibrium $\rho(\lambda) = \rho_{eq}(\lambda)$. On the other hand, a general nonequilibrium distribution $\rho(\lambda) \neq \rho_{eq}(\lambda)$ will imply different measures for $T_A(+, -), T_A(-, +)$ (or $T_B(+, -), T_B(-, +)$), resulting in signalling from $B$ to $A$ (or from $A$ to $B$).

But how large can the signal be? There is no signal at all in equilibrium $\rho(\lambda) = \rho_{eq}(\lambda)$; while if $\rho(\lambda)$ is concentrated on just one of $T_A(+, -), T_A(-, +)$ (or on just one of $T_B(+, -), T_B(-, +)$), then all the outcomes are changed by the distant shift. Thus the size of the signal – measured by the fraction of outcomes that change at a distance – can range from 0% to 100%.

Now Bell’s theorem guarantees that at least one of the pairs $T_A(+, -), T_A(-, +)$ or $T_B(+, -), T_B(-, +)$ has non-zero equilibrium measure: otherwise
we would have a local theory. But so far we have no idea how large these sets have to be; we know only that $T_A(\pm, -)$ and $T_A(\pm, +)$ must have equal equilibrium measure, as must $T_B(\pm, -)$ and $T_B(\pm, +)$. (In this sense, Bell’s theorem tells us there must be some nonlocality hidden behind the equilibrium distribution, but not how much.) The size of the transition sets is important because if they have very tiny equilibrium measure, then to obtain an appreciable signal the nonequilibrium distribution $\rho(\lambda) \neq \rho_{eq}(\lambda)$ would have to be very far from equilibrium – that is, concentrated on a very tiny (with respect to the equilibrium measure) set. We shall therefore try to deduce the equilibrium measure of the transition sets.

In other words, we now ask the following quantitative question: for an equilibrium distribution $\rho_{eq}(\lambda)$ of hidden variables, what fraction of outcomes at $A$ are changed by the distant shift $\theta_B \to \theta'_B$, and what fraction at $B$ are changed by $\theta_A \to \theta'_A$?

The quantity
\[ \alpha \equiv \nu_{eq}^A(\pm, -) + \nu_{eq}^A(\pm, +) \]
(the sum of the equilibrium measures of $T_A(\pm, -)$ and $T_A(\pm, +)$) is the fraction of the equilibrium ensemble for which the outcomes at $A$ are changed under $\theta_B \to \theta'_B$ (irrespective of whether they change from $+1$ to $-1$ or vice versa, the fractions doing each being $\alpha/2$). There is a ‘degree of nonlocality from $B$ to $A$’, quantified by $\alpha = \alpha(\theta_A, \theta_B, \theta'_B)$. Similarly, one may define a ‘degree of nonlocality from $A$ to $B$’, quantified by the fraction $\beta = \beta(\theta_A, \theta_B, \theta'_A)$ of outcomes at $B$ that change in response to a shift $\theta_A \to \theta'_A$ at $A$. Bell’s theorem tells us that, in general, the ‘total degree of nonlocality’ $\alpha + \beta > 0$.

Positive lower bounds on $\alpha + \beta$, and on $\alpha$ or $\beta$ alone, may be obtained if one assumes certain symmetries.

6 A General Lower Bound

First, we derive a general lower bound for the quantity $\alpha + \beta$, where
\[ \beta \equiv \nu_{eq}^B(-, +) + \nu_{eq}^B(+, -) \]
is the equilibrium fraction of outcomes that change at $B$, under the local shift $\theta_B \to \theta'_B$ (with $\nu_{eq}^B(-, +)$ and $\nu_{eq}^B(+, -)$ defined similarly to $\nu_{eq}^A(-, +)$ and $\nu_{eq}^A(+, -)$ above). In other words, we obtain a lower bound on the sum of the nonlocal and local effects of $\theta_B \to \theta'_B$.

The quantity $\frac{1}{2} |\sigma_A(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda)|$ equals 1 if the outcome $\sigma_A$ changes under $\theta_B \to \theta'_B$, and vanishes otherwise. Since $\rho_{eq}(\lambda) d\lambda$ is by definition the fraction of the equilibrium ensemble for which $\lambda$ lies in the interval $(\lambda, \lambda+d\lambda)$, the fraction for which $\sigma_A$ changes is
\[ \alpha = \frac{1}{2} \int d\lambda \rho_{eq}(\lambda) |\sigma_A(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda)| \]
\[ \int d\lambda \rho_{eq}(\lambda) \]

\[ ^{10} \text{Of course, } \alpha + \beta \text{ could vanish for specific angles, but not in general.} \]
Similarly, the fraction for which $\sigma_B$ changes is

$$\tilde{\beta} = \frac{1}{2} \int d\lambda \rho_{eq}(\lambda) |\sigma_B(\theta_A, \theta'_B, \lambda) - \sigma_B(\theta_A, \theta_B, \lambda)|$$

Now

$$-\cos(\theta_A - \theta_B) = \int d\lambda \rho_{eq}(\lambda) \sigma_A(\theta_A, \theta_B, \lambda) \sigma_B(\theta_A, \theta_B, \lambda)$$

and so

$$|\cos(\theta_A - \theta'_B) - \cos(\theta_A - \theta_B)| \leq \int d\lambda \rho_{eq}(\lambda) |\sigma_A(\theta_A, \theta'_B, \lambda) \sigma_B(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda) \sigma_B(\theta_A, \theta_B, \lambda)|$$

$$= \int d\lambda \rho_{eq}(\lambda) \left| \sigma_A(\theta_A, \theta'_B, \lambda) \sigma_B(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda) \sigma_B(\theta_A, \theta_B, \lambda) \right|$$

$$\leq \int d\lambda \rho_{eq}(\lambda) |\sigma_A(\theta_A, \theta'_B, \lambda) - \sigma_A(\theta_A, \theta_B, \lambda)|$$

$$+ \int d\lambda \rho_{eq}(\lambda) |\sigma_B(\theta_A, \theta'_B, \lambda) - \sigma_B(\theta_A, \theta_B, \lambda)|$$

$$= 2\alpha + 2\tilde{\beta}$$

Thus we have the lower bound

$$\alpha(\theta_A, \theta_B, \theta'_B) + \tilde{\beta}(\theta_A, \theta_B, \theta'_B) \geq \frac{1}{2} |\cos(\theta_A - \theta'_B) - \cos(\theta_A - \theta_B)| \quad (1)$$

The maximum value of the right hand side is 1. From this inequality alone, then, one could have $\alpha$ arbitrarily close to zero, with $\tilde{\beta} \to 1$ – that is, an arbitrarily small fraction could change at $A$ in response to $\theta_B \to \theta'_B$, provided virtually all the outcomes change at $B$. So far, then, we have no lower bound on the nonlocal effect from $B$ to $A$, as quantified by $\alpha$: it is only the sum $\alpha + \tilde{\beta}$ of the nonlocal and local effects that is bounded.

### 7 Symmetric Cases

A lower bound on $\alpha + \beta$ – the sum of the nonlocal effects from $B$ to $A$ and from $A$ to $B$ – may be obtained if we assume an appropriate rotational symmetry at the hidden-variable level. A lower bound on $\alpha$ or $\beta$ alone may be obtained if we also assume that the measurement operations at $A$ and $B$ are identical – that
is, use identical equipment and coupling – so that there is symmetry between the two wings.

**Rotational Symmetry**: Consider the effect of a shift \( \theta_B \to \theta_B' = \theta_B + \delta \) at B. This changes certain fractions \( \alpha(\theta_A, \theta_B, \theta_B + \delta) \) and \( \beta(\theta_A, \theta_B, \theta_B + \delta) \) of the (equilibrium) outcomes at A and B respectively. Let us assume that the same changes are effected by the shift \( \theta_A \to \theta_A' = \theta_A - \delta \) at A. This means that \( \beta(\theta_A, \theta_B, \theta_A - \delta) \) – the fraction of outcomes at B that change (nonlocally) in response to a shift \( \theta_A \to \theta_A' = \theta_A - \delta \) at A – is equal to \( \beta(\theta_A, \theta_B, \theta_B + \delta) \), and so from (1)

\[
\alpha(\theta_A, \theta_B, \theta_B + \delta) + \beta(\theta_A, \theta_B, \theta_A - \delta) \geq \frac{1}{2} |\cos(\theta_A - \theta_B - \delta) - \cos(\theta_A - \theta_B)|
\]

(2)

For \( \theta_A = \theta_B = 0 \),

\[
\alpha(0, 0, \delta) + \beta(0, 0, -\delta) \geq \frac{1}{2}(1 - \cos \delta)
\]

(3)

Thus, for example, the fraction \( \alpha \) that changes at A due to a shift \(+\pi/2\) at B plus the fraction \( \beta \) that changes at B due to a shift \(-\pi/2\) at A must be at least 50%.

Note that our assumption of ‘rotational symmetry’ refers to the hidden-variable level\(^{11}\). How it relates to fundamental rotational invariance is not clear, for in the absence of a definite theory one does not know how the hidden variables transform under a rotation. In any case, the assumption seems stronger than fundamental rotational invariance: it might be violated if the measuring apparatus happens to pick out an effectively preferred direction in space.

**Rotational and Exchange Symmetry**: Assuming further an exchange symmetry between A and B – specifically, that the effect at A of a shift \( \theta_B \to \theta_B' = \theta_B + \delta \) at B equals the effect at B of a shift \( \theta_A \to \theta_A' = \theta_A - \delta \) at A – we also have \( \alpha(\theta_A, \theta_B, \theta_B + \delta) = \beta(\theta_A, \theta_B, \theta_A - \delta) \).\(^{12}\) Thus we obtain a lower bound

\[
\alpha(\theta_A, \theta_B, \theta_B + \delta) \geq \frac{1}{4} |\cos(\theta_A - \theta_B - \delta) - \cos(\theta_A - \theta_B)|
\]

(4)

on \( \alpha \) (or \( \beta \)) alone.

For \( \theta_A = \theta_B = 0 \),

\[
\alpha(0, 0, \delta) \geq \frac{1}{4}(1 - \cos \delta)
\]

(5)

If the measurement angle at B is shifted by \( \pi/2 \), at least 25% of the outcomes change at A (and similarly from A to B).

Clearly, in these symmetric cases, the transition sets are necessarily very large, and even a mild disequilibrium \( \rho(\lambda) \neq \rho_{eq}(\lambda) \) will entail a significant signal.

\(^{11}\)Quantum theory alone says nothing at all about \( \alpha, \beta, \tilde{\alpha}, \tilde{\beta} \). And at the quantum level, the singlet state is of course rotationally invariant.

\(^{12}\)This assumption excludes ‘one-way’ nonlocality.
8 Comparison with Pilot-Wave Theory

It behoves us to check that the bounds obtained above are satisfied for the specific hidden-variables model provided by pilot-wave theory. To do so, we use the theory of spin measurements due to Bell.\footnote{See ref. [4], chapter 15.}

**Pilot-Wave Spin Measurements:** At each wing $A$ and $B$ let there be an apparatus that performs a quantum measurement of spin. The pointer positions $r_A$ and $r_B$ begin in ‘neutral’ states centred at the origin. More precisely, we assume that at $t=0$ the pointers have identical localised wavepackets $\phi(r_A)$, $\phi(r_B)$ centred at $r_A$, $r_B = 0$. During the measurement, each packet will move ‘up’ or ‘down’ (that is, towards positive or negative values of $r_A$, $r_B$), thereby indicating an outcome of spin up or down.

In the case of a Stern-Gerlach apparatus, the ‘pointer position’ would in fact be the position of the particle itself as it is deflected upon passing through the magnetic field. But here, $r_A$, $r_B$ are simply abstract pointer positions for measuring equipment that is brought into interaction with the particles.\footnote{There is in fact an important difference between the measurement process described here and the Stern-Gerlach method, in the case of angles differing by $\pi$. This is discussed in section 9 below.}

Consider, then, the initial total wavefunction $\psi_{ij}(r_A, r_B, 0) = \phi(r_A)\phi(r_B)a_{ij}$, where the indices $i, j = \pm$ denote spin up or down at $A$, $B$ and the $a_{ij}$ are spin amplitudes for the singlet state. The pointers are initially independent, but the spins are entangled $a_{ij} \neq b_{i}c_{j}$. The total Hamiltonian

$$\hat{H} = g_A(t)\hat{\sigma}_A (-i\partial/\partial r_A) + g_B(t)\hat{\sigma}_B (-i\partial/\partial r_B)$$

(where the couplings $g_A(t)$, $g_B(t)$ are switched on at $t=0$) describes ideal von Neumann measurements of spin at $A$ and $B$.\footnote{The Hamiltonians of the particles and pointers themselves are assumed to be negligible compared to the interaction part (so that the free spreading of the pointer packets is negligible). This may be justified for a realistic model by considering strong couplings over short times; or, one can simply accept the above as an illustrative model of measurement.}

The Schrödinger equation then reads

$$i\frac{\partial \psi_{ij}}{\partial t} = g_A(t)(\sigma_A)_{ik} \left(-i\frac{\partial}{\partial r_A}\right) \psi_{kj} + g_B(t)(\sigma_B)_{jk} \left(-i\frac{\partial}{\partial r_B}\right) \psi_{ik}$$

where repeated indices are summed over, and $\sigma_{A,B}$ is the Pauli spin matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(where in the spin subspace at $A$ the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ denotes spin up along the axis $\theta_A$ while in subspace $B$ the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ denotes spin up along $\theta_B$).

The quantum equilibrium distribution for the pointer positions is

$$\rho_{eq}(r_A, r_B, t) = |\psi_{++}|^2 + |\psi_{+-}|^2 + |\psi_{-+}|^2 + |\psi_{--}|^2$$
From the Schrödinger equation one readily derives a continuity equation for $\rho_{eq}$ and associated probability currents

$$j_A = g_A \psi^*_k (\sigma_A)_{kl} \psi_l, \quad j_B = g_B \psi^*_k (\sigma_B)_{kl} \psi_l$$

The hidden-variable pointer positions $r_A(t), r_B(t)$ have velocities given by the de Broglie guidance formulas $v_{A,B} = j_{A,B}/\rho_{eq}$. Explicitly

$$v_A = g_A \left( |\psi_{++}|^2 + |\psi_{+-}|^2 - |\psi_{+--}|^2 - |\psi_{--+}|^2 \right)/\rho_{eq}$$

$$v_B = g_B \left( |\psi_{++}|^2 - |\psi_{+-}|^2 + |\psi_{+--}|^2 - |\psi_{--+}|^2 \right)/\rho_{eq}$$

If $\psi_{ij}(r_A, r_B, 0) = \phi(r_A)\phi(r_B)a_{ij}$ then with

$$h_A(t) \equiv \int_0^t dt' \, g_A(t'), \quad h_B(t) \equiv \int_0^t dt' \, g_B(t')$$

the Schrödinger equation implies that at $t > 0$

$$\psi_{++} = a_{++} \phi(r_A - h_A)\phi(r_B - h_B), \quad \psi_{+-} = a_{+-} \phi(r_A - h_A)\phi(r_B + h_B)$$

$$\psi_{-+} = a_{-+} \phi(r_A + h_A)\phi(r_B - h_B), \quad \psi_{--} = a_{--} \phi(r_A + h_A)\phi(r_B + h_B)$$

The branches $\psi_{ij}$ eventually separate in configuration space, and the actual configuration $(r_A(t), r_B(t))$ can end up occupying only one of them – leading to a definite outcome. For example, if at large $t$ the actual $(r_A, r_B)$ lies in $\psi_{+-}$ alone, then the guidance formulas imply $v_A = g_A, v_B = -g_B$ and the pointer positions will be $r_A(t) \approx h_A(t), r_B(t) \approx -h_B(t)$, corresponding to $\sigma_A = +1, \sigma_B = -1$.

The outcomes $\sigma_A, \sigma_B$ depend on the hidden variables $r_A(0), r_B(0)$ and on the settings $\theta_A, \theta_B$. The question now is: what fraction of the outcomes change under $\theta_B \to \theta'_B$ (or $\theta_A \to \theta'_A$)?

More precisely, for given settings $\theta_A, \theta_B$ the outcomes $\sigma_A, \sigma_B$ are determined by the initial wavefunction $\psi_{ij}(r_A, r_B, 0)$ and the initial pointer positions $r_A(0), r_B(0)$, so strictly speaking the ‘hidden variables’ are $\lambda = (\psi_{ij}(r_A, r_B, 0), r_A(0), r_B(0))$.

However, because $\psi_{ij}(r_A, r_B, 0)$ is assumed to be the same in every run of the experiment, there is no need to consider it, and effectively $\lambda = (r_A(0), r_B(0))$.

We wish to calculate the fraction of the equilibrium ensemble of $r_A(0), r_B(0)$ for which the outcome $\sigma_A$ (or $\sigma_B$) changes under $\theta_B \to \theta'_B$ (or $\theta_A \to \theta'_A$).

If we take square initial pointer packets $|\phi(r_A)|^2, |\phi(r_B)|^2$, each equal to $1/\Delta$ for $-\Delta/2 < r_A, r_B < \Delta/2$ and zero elsewhere, then in the $r_A - r_B$ plane the initial distribution $\rho_{eq}(r_A, r_B, 0) = |\phi(r_A)|^2 |\phi(r_B)|^2$ is uniform within a square of side $\Delta$ centred on the origin (with no support outside the square). Fractional areas within the square then represent statistical fractions of the

\[\text{Note that the hidden-variable velocities are independent of the spin basis used.}\]
initial ensemble. Thus, we need to calculate the fractional area of initial points within the square for which $\sigma_A$ (or $\sigma_B$) changes under $\theta_B \rightarrow \theta_B'$ (or $\theta_A \rightarrow \theta_A'$).

**Symmetric Case:** First we look at a symmetric case with equal couplings at each wing: $g_A(t) = g_B(t) = a\theta(t)$, where $a$ is a positive constant, and $h_A(t) = h_B(t) = at$ (for $t \geq 0$). The packets move with equal speeds along the $r_A$- and $r_B$-axes.

Taking initial measurement angles $\theta_A = \theta_B = 0$, the singlet state has spin amplitudes

$$a_{++} = 0, \quad a_{+-} = 1/\sqrt{2}, \quad a_{-+} = -1/\sqrt{2}, \quad a_{--} = 0$$

We then have $\psi_{++} = \psi_{--} = 0$ and the pointer velocities are

$$v_A = a \left( \frac{|\psi_{+-}|^2 - |\psi_{-+}|^2}{|\psi_{+-}|^2 + |\psi_{+ -}|^2} \right), \quad v_B = -v_A$$

It is then straightforward to deduce that in the $r_A-r_B$ plane all initial points $(r_A(0), r_B(0))$ above the line $r_B = r_A$ (plotting $r_B$ as ordinate and $r_A$ as abscissa) end up in the branch $\psi_{+-}$, yielding outcomes $\sigma_A = -1$, $\sigma_B = +1$; while those below that line end up in $\psi_{-+}$, yielding $\sigma_A = +1$, $\sigma_B = -1$. [Where $\psi_{+-}$ and $\psi_{-+}$ overlap, $v_A = v_B = 0$; while if $\psi_{+-} \neq 0$ and $\psi_{-+} = 0$ then $v_A = +a$, $v_B = -a$; and if $\psi_{+-} \neq 0$ and $\psi_{-+} = 0$ then $v_A = -a$, $v_B = +a$.

It follows that as the branches separate — with $\psi_{+-}$ and $\psi_{-+}$ moving perpendicular to the line $r_B = r_A$, into the bottom-right and top-left quadrants respectively — a point $(r_A, r_B)$ initially above the line $r_B = r_A$ will begin at rest but will eventually be left behind by the branch $\psi_{+-}$, whereupon it will acquire the velocity $(v_A, v_B) = (-a, +a)$ due to guidance by $\psi_{+-}$. At large times $r_A \approx -at$, $r_B \approx +at$ yielding the outcomes $\sigma_A = -1$, $\sigma_B = +1$. Similar reasoning shows that a point $(r_A, r_B)$ initially below the line $r_B = r_A$ yields the outcomes $\sigma_A = +1$, $\sigma_B = -1$.]

Now, what happens if we change the axis of measurement at $B$ — say to $\theta_B' = \pi/2$? For the settings $\theta_A = 0$, $\theta_B' = \pi/2$, the singlet state has spin amplitudes

$$a'_{++} = 1/2, \quad a'_{+-} = 1/2, \quad a'_{-+} = -1/2, \quad a'_{--} = 1/2$$

and now all four branches $\psi_{ij}'$ contribute to the velocities, each branch moving into a different quadrant of the $r_A-r_B$ plane. It is not difficult to see that a point $(r_A, r_B)$ initially in the top-right quadrant ends up in $\psi_{++}'$, yielding $\sigma_A = +1$, $\sigma_B = +1$; while points in the bottom-right quadrant yield $\sigma_A = +1$, $\sigma_B = -1$; those in the top-left yield $\sigma_A = -1$, $\sigma_B = +1$; and those in the bottom-left yield $\sigma_A = -1$, $\sigma_B = -1$. [Where all four branches overlap, both velocity components vanish, $v_A' = v_B' = 0$; while just two overlap, one component vanishes (for example if only $\psi_{++}'$ and $\psi_{-+}'$ are non-zero then $v_A' = +a$, $v_B' = 0$); and where none overlap neither component vanishes (for example if only $\psi_{++}'$ is non-zero then $v_A' = +a$, $v_B' = +a$). Thus, for example, consider an initial point $(r_A, r_B)$
in the top-right quadrant and below the line \( r_B = r_A \). For as long as all four branches overlap at \((r_A, r_B)\), the point will remain at rest. But after a while only \( \psi_{++}^i \) and \( \psi_{+-}^i \) will overlap there, and for an interim period the point will move along \( r_A \), remaining in the same quadrant, having acquired velocity components \( v'_A = +a, \ v'_B = 0 \). Later, once \( \psi_{++}^i \) and \( \psi_{+-}^i \) have separated, the point is guided by \( \psi_{++}^i \) alone, acquiring velocity components \( v'_A = +a, \ v'_B = +a \), and at large times \( r_A \approx +at, \ r_B \approx +at \), yielding the outcomes \( \sigma_A = +1, \ \sigma_B = +1 \). For an initial point \((r_A, r_B)\) in the top-right quadrant but above the line \( r_B = r_A \), there is an interim period of motion along \( r_B \), again remaining in the same quadrant, after which the point is again carried by \( \psi_{++}^i \) to \( r_A \approx +at, \ r_B \approx +at \) at large times, again yielding \( \sigma_A = +1, \ \sigma_B = +1 \). Thus, all initial points in the top-right quadrant yield \( \sigma_A = +1, \ \sigma_B = +1 \). Similarly, points in the bottom-right quadrant yield \( \sigma_A = +1, \ \sigma_B = -1 \), while those in the top-left yield \( \sigma_A = -1, \ \sigma_B = +1 \) and those in the bottom-left yield \( \sigma_A = -1, \ \sigma_B = -1 \).

Clearly, some initial points \((r_A, r_B)\) that would have yielded \( \sigma_A = +1, \ \sigma_B = -1 \) with the settings \( \theta_A = 0, \ \theta_B = 0 \) still yield \( \sigma_A = +1, \ \sigma_B = -1 \) with \( \theta_A = 0, \ \theta_B = \pi/2 \) – namely, all points in the bottom-right quadrant. Similarly, all points in the top-left quadrant yield \( \sigma_A = -1, \ \sigma_B = +1 \) with both the old and new settings. But in the other two quadrants, the outcomes are changed by the shift from \( \theta_B = 0 \) to \( \theta'_B = \pi/2 \). In the top-right, whereas before half gave \( \sigma_A = +1, \ \sigma_B = -1 \) and half \( \sigma_A = -1, \ \sigma_B = +1 \), now all give \( \sigma_A = +1, \ \sigma_B = +1 \) in the bottom-left, before half gave \( \sigma_A = +1, \ \sigma_B = -1 \) and half \( \sigma_A = -1, \ \sigma_B = +1 \), whereas now all give \( \sigma_A = -1, \ \sigma_B = -1 \).

A simple count shows that 25% of the outcomes have changed at \( A \) and 25% have changed at \( B \). Thus, under \( \theta_B = 0 \rightarrow \theta'_B = \pi/2 \), the fraction \( \alpha(0,0,\pi/2) = 1/4 \) of outcomes that change at \( A \) is indeed equal to the fraction \( \beta(0,0,\pi/2) = 1/4 \) of outcomes that change at \( B \), and our inequality (5) is exactly saturated.

For a shift to an arbitrary angle \( \theta'_B = \delta \) at \( B \), we find that \( \alpha(0,0,\delta) = \beta(0,0,\delta) = 1/4(1 - \cos \delta) \); again, the fractional changes at \( A \) and \( B \) are equal, and (5) is exactly saturated. With the new settings the singlet state has spin amplitudes

\[
a'_{++} = \frac{1}{\sqrt{2}} \sin \frac{\delta}{2}, \quad a'_{+-} = \frac{1}{\sqrt{2}} \cos \frac{\delta}{2},
\]

\[
a'_{-+} = -\frac{1}{\sqrt{2}} \cos \frac{\delta}{2}, \quad a'_{--} = \frac{1}{\sqrt{2}} \sin \frac{\delta}{2}
\]

and again all four branches \( \psi'_{ij} \) contribute to the velocities and move into different quadrants of the \( r_A - r_B \) plane. As in the case \( \delta = \pi/2 \), where all four branches overlap both velocity components vanish, \( v'_A = v'_B = 0 \); but now, where just two overlap, neither component vanishes (for example, if only \( \psi'_{++} \) and \( \psi'_{+-} \) are non-zero then \( v'_A = +a, \ v'_B = -a \cos \delta \); and where none overlap the velocities are as for \( \delta = \pi/2 \) (for example if only \( \psi'_{++} \) is non-zero then \( v'_A = +a, \ v'_B = +a \)). Thus, again considering an initial point \((r_A, r_B)\) in the top-right quadrant and below the line \( r_B = r_A \), for as long as all four branches
overlap at \((r_A, r_B)\) the point will remain at rest. As before, after a while only \(\psi'_{++} \) and \(\psi'_{+-}\) will overlap there, but now there is an interim period during which the point not only moves along \(r_A\) with velocity \(v'_A = +a\), it also moves along \(r_B\) with velocity \(v'_B = -a \cos \delta\) and need not remain in the same quadrant. If the initial point is sufficiently close to the \(r_A\) axis, below the line \(r_B = (\cos \delta)r_A\), in fact it may cross that axis into the bottom-right quadrant before the four branches separate altogether. If that happens, once the branches separate completely the point will be guided by \(\psi'_{+-}\), yielding \(\sigma_A = +1, \sigma_B = -1\). Similarly, an initial point in the top-right quadrant and above the line \(r_B = r_A\) will cross into the top-left quadrant -- yielding \(\sigma_A = -1, \sigma_B = +1\) -- if it begins sufficiently close to the \(r_B\) axis (above the line \(r_B = r_A/\cos \delta\)). Thus, unlike for \(\delta = \pi/2\), not every initial point in the top-right quadrant yields \(\sigma_A = +1, \sigma_B = +1\), but only those between the lines \(r_B = (\cos \delta)r_A\) and \(r_B = r_A/\cos \delta\); the rest cross over into neighboring quadrants and yield different outcomes. Similar results are found for the opposite (bottom-left) quadrant, the fate of initial points being reflection-symmetric about the line \(r_B = -r_A\). Results for the other two quadrants are as for \(\delta = \pi/2\). Elementary geometry then shows that the regions of the initial square in which outcomes at \(A\) change under \(\theta_B = 0 \rightarrow \theta'_B = \delta\) have a total fractional area \(\frac{1}{4}(1 - \cos \delta)\), as do regions in which outcomes at \(B\) change.

The maximum change is obtained for \(\delta = \pi\), for which 50\% change at \(A\) and \(B\) \((\alpha(0, 0, \pi) = \tilde{\beta}(0, 0, \pi) = 0.5)\). Because our inequality (5) is exactly saturated for all \(\delta\), in a precisely defined sense it may be said that pilot-wave theory is 'minimally nonlocal'. Though whether (5) is saturated for non-square pointer packets is not known.

Asymmetric Case: Next we look at an asymmetric case where the couplings at each wing are unequal: \(g_A = a_A \theta(t), g_B = a_B \theta(t)\) with \(a_A = 2a_B\) so that the packets move with different speeds along the \(r_A\)- and \(r_B\)-axes. With the settings \(\theta_A = 0, \theta_B = 0\) it is found that instead of the \(r_A - r_B\) plane being divided by the line \(r_B = r_A\) into points yielding \(\sigma_A = -1, \sigma_B = +1\) (above the line) and \(\sigma_A = +1, \sigma_B = -1\) (below it), as it was for the case \(a_A = a_B\), it is now divided by the line \(r_B = \frac{1}{2}r_A - \frac{1}{2}\) for \(-\frac{1}{2} \leq r_A \leq 0\), the vertical line \(r_A = 0\) for \(-\frac{1}{2} \leq r_B \leq +\frac{1}{2}\), and \(r_B = \frac{1}{2}r_A + \frac{1}{2}\) for \(0 \leq r_A \leq +\frac{1}{2}\). When the angle at \(B\) is reset to \(\theta'_B = \pi/2\), it is found that the four quadrants yield the same outcomes as above for \(a_A = a_B\) (the top-right yielding \(\sigma_A = +1, \sigma_B = +1\), the bottom-right \(\sigma_A = +1, \sigma_B = -1\), the top-left \(\sigma_A = -1, \sigma_B = +1\), and the bottom-left \(\sigma_A = -1, \sigma_B = -1\)), there being no crossing from one quadrant into another. Inspection shows that a fraction \(\alpha(0, 0, \pi/2) = 1/8\) of outcomes have changed at \(A\), while a fraction \(\tilde{\beta}(0, 0, \pi/2) = 3/8\) have changed at \(B\).

Thus, \(\alpha \neq \tilde{\beta}\) in this asymmetrical case: the nonlocal and local effects are unequal, as expected. Further, the general bound (1) is satisfied, and indeed exactly saturated.

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17 We have assumed that \(\cos \delta > 0\). If \(\cos \delta < 0\), instead of points leaving the top-right quadrant they can enter it from neighbouring quadrants. But the resulting fractional changes at \(A\) and \(B\) are the same.
In the limit \( a_A/a_B \to \infty \) – where the measurement at \( A \) takes place much more rapidly than at \( B \) (in the sense of rate of branch separation) – it is found that \( \alpha(0,0,\pi/2) \to 0, \beta(0,0,\pi/2) \to 1/2 \). The bound (1) is again saturated, the nonlocal effect being arbitrarily small and the local effect approaching 50%.

[With \( \theta_A = 0, \theta_B = 0 \), there are just two branches \( \psi_{+-} \) and \( \psi_{-+} \) which rapidly separate along \( r_A \) before hardly any motion has occurred along \( r_B \), where in the \( r_A - r_B \) plane \( \psi_{+-} \) moves to the right and \( \psi_{-+} \) moves to the left: thus the right half of the initial square yields \( \sigma_A = +1, \sigma_B = -1 \) and the left half yields \( \sigma_A = -1, \sigma_B = +1 \). When the setting at \( B \) is shifted to \( \theta'_B = \pi/2 \), the four quadrants again yield the same outcomes as for \( a_A = a_B \), there being still no crossing between quadrants. Thus the right half still yields \( \sigma_A = +1 \) and the left half still yields \( \sigma_A = -1 \), so there are no changes at \( A \); but the top-right quadrant now yields \( \sigma_B = +1 \) and the bottom-left \( \sigma_B = -1 \), so 50% change at \( B \).]

This example proves that it is impossible to deduce a general lower bound on \( \alpha \) alone without assuming an exchange symmetry between the two wings. If the measurement at \( A \) is completed long before the measurement at \( B \), the degree of nonlocality from \( B \) to \( A \) tends to zero.

On the other hand, again in the limit \( a_A/a_B \to \infty \), under a shift \( \theta_A = 0 \to \theta'_A = -\pi/2 \) at \( A \) (keeping \( \theta_B = 0 \) fixed), it is found that a fraction \( \beta(0,0,-\pi/2) \to 1/2 \) change at \( B \). Thus \( \beta(0,0,-\pi/2) = \tilde{\beta}(0,0,\pi/2) \), in accordance with rotational symmetry, and while the degree of nonlocality is zero from \( B \) to \( A \) it is large from \( A \) to \( B \), saturating (3). [We may calculate the degree of nonlocality \( \beta(0,0,-\pi/2) \) from \( A \) to \( B \) for \( a_A/a_B \to \infty \) by noting that it must be the same as the degree of nonlocality \( \alpha(0,0,\pi/2) \) from \( B \) to \( A \) for \( a_B/a_A \to \infty \). In the latter case, for \( \theta_A = 0, \theta_B = 0 \) the branches \( \psi_{+-} \) and \( \psi_{-+} \) rapidly separate along \( r_B \), so that the top half of the initial square yields \( \sigma_A = -1, \sigma_B = +1 \) and the bottom half \( \sigma_A = +1, \sigma_B = -1 \). With \( \theta_A = 0, \theta'_B = \pi/2 \), once again the four quadrants yield the same outcomes as for \( a_A = a_B \), and it is now seen that 50% have changed at \( A \), so that for \( a_B/a_A \to \infty \) we have \( \alpha(0,0,\pi/2) \to 1/2 \). Thus we deduce that \( \beta(0,0,-\pi/2) \to 1/2 \) for \( a_A/a_B \to \infty \).]

This last result is worth emphasising: if the coupling at \( A \) is made much larger than at \( B \), the degree of nonlocality \( \alpha \) from \( B \) to \( A \) becomes small and the degree of nonlocality \( \beta \) from \( A \) to \( B \) becomes large, while the total degree of nonlocality \( \alpha + \beta = 1/2 \) is unchanged (at least for angular shifts \( \pm \pi/2 \) and square pointer packets).

\section{Angular Settings Differing by \( \pi \)}

For measurements of spin, one must be careful to consider only those settings \( \theta_A, \theta_B \) that correspond to physically distinct experimental arrangements.\footnote{I am grateful to Lucien Hardy for raising this point.}

This can depend on how the measurements are carried out. For the interaction Hamiltonian considered above, settings that differ by \( \pi \) are physically distinct;
but in the case of Stern-Gerlach measurements, settings that differ by π (at one or both wings) must be identified with the same experiment.

To see this, it suffices to consider pilot-wave theory for a single spin. With the Hamiltonian \( \hat{H} = g(t)\hat{\sigma}_\theta(-i\partial/\partial r) \), if the state \( |z+\rangle \) is measured for \( \theta = 0 \) one finds that all initial pointer positions \( r(0) \) evolve towards \( +\infty \); that is, one always obtains \( \sigma_0 = +1 \). If on the other hand one takes \( \theta = \pi \), all \( r(0) \) evolve towards \(-\infty\), corresponding to \( \sigma_\pi = -1 \). Thus, for \( |z+\rangle \), a shift \( \theta = 0 \rightarrow \theta = \pi \) induces a change in every outcome. The interaction is of course constructed so that, assuming \( g \) induces a change in every outcome. The interaction is of course constructed so that, assuming \( g(\theta) \geq 0 \), the sign of the pointer position \( r(t) \) at large times tells us the value of the ‘observable’ \( \hat{\sigma}_\theta \). The Hamiltonian changes sign \( \hat{H} \rightarrow -\hat{H} \) under a shift \( \theta = 0 \rightarrow \theta = \pi \) (where \( \hat{\sigma}_\pi = -\hat{\sigma}_0 \)), corresponding to a genuinely distinct experiment. For \( |z+\rangle \) this induces a reversal of the actual motion of the pointer. Though of course, in every case the outcome corresponds to spin up along the (fixed) z-axis, as it must.

For \( |x+\rangle \), however, a shift \( \theta = 0 \rightarrow \theta = \pi \) leads to no change in outcome \( \sigma_\theta \), corresponding to a reversal of spin along the z-axis. To see this, simply note that for both \( \theta = 0 \) and \( \theta = \pi \) half of the initial values \( r(0) \) must evolve towards \( +\infty \) and half towards \(-\infty\) (to yield the correct outcome ratios); and since the trajectories \( r(t) \) cannot cross (the de Broglie-Bohm velocity field being single-valued), it follows that in both cases \( r(0) > 0 \) yields \( r(t) \rightarrow +\infty \) \( (\sigma_0 = +1, \ \sigma_\pi = +1) \) and \( r(0) < 0 \) yields \( r(t) \rightarrow -\infty \) \( (\sigma_0 = -1, \ \sigma_\pi = -1) \). Thus, while it may seem puzzling at first sight, in this particular model an initial hidden-variable state that yields spin up along z for \( \theta = 0 \) (that is, \( \sigma_0 = +1 \)) yields spin down along z for \( \theta = \pi \) (that is, \( \sigma_\pi = +1 \)) and similarly, spin down becomes spin up.

It is therefore clear that, for an interaction \( \hat{H} = g(t)\hat{\sigma}_\theta(-i\partial/\partial r) \), measurements along angles differing by \( \pi \) correspond to physically distinct experiments that can yield physically distinct results. And so, it is not surprising that in our pilot-wave calculations for the singlet state we found that a shift \( \theta_B = 0 \rightarrow \theta'_B = \pi \) at \( B \) can induce a large change in the outcomes at \( A \).

In contrast, for a Stern-Gerlach measurement on a single spin, the position of the particle itself serves as a ‘pointer’ [18]. And if the magnet is rotated by \( \pi \), this induces no change in (the relevant part of) the Hamiltonian, the experimental arrangement is exactly the same, and therefore the time evolution of the wavefunction and of the de Broglie-Bohm trajectories must also be the same. Thus, for the state \( |z+\rangle \), the motion of the ‘pointer’ is not affected by

\[ \text{This seems strange from a quantum viewpoint, because one tends to make the implicit and mistaken assumption that so-called quantum ‘measurements’ really do tell us the value of something that existed beforehand. But contextuality tells us that, in general, quantum ‘measurements’ are not faithful – that is, are not really measurements, but simply experiments of a particular kind. Thus, there is no reason why different ‘measurement’ setups (with different Hamiltonians) could not yield different values of spin along z for the same hidden-variable state, as in the example above.} \]

\[ \text{In an inhomogeneous magnetic field along the z-axis, we have a term \( \hat{H} = \mu\hat{\sigma}_z B_z \approx \mu\hat{\sigma}_z(B_z)_{z=0} + \mu\hat{\sigma}_z z(\partial B_z/\partial z)_{z=0}. \) Because of the second term, packets with opposite spins acquire opposite \( z \)-momenta and eventually separate [18]. If the magnet is rotated by \( \pi \), the sign of \( (\partial B_z/\partial z)_{z=0} \) is unchanged.} \]
\[ \theta = 0 \rightarrow \theta = \pi. \] There is no physical distinction between the measurements with \( \theta = 0 \) and \( \theta = \pi \), and in both cases the motion of the particle will indicate spin up the \( z \)-axis. Similarly, for \( \left| x^{+} \right> \) no detail of the evolution is affected by \( \theta = 0 \rightarrow \theta = \pi \): one obtains spin up or down, depending on the initial particle position within the initial packet, regardless of whether \( \theta = 0 \) or \( \theta = \pi \).

In the Stern-Gerlach case, then, settings that differ by \( \pi \) correspond physically to the same experiment. And so, for an entangled state of two spins at \( A \) and \( B \), a shift \( \theta_B = 0 \rightarrow \theta_B' = \pi \) at \( B \) cannot induce any change at all at \( A \). (There is, of course, no contradiction with our pilot-wave calculations for the other case, which corresponds to a different interaction.)

In the lower bounds (2)–(5), then, one must take into account the possibility that some mathematically distinct settings may be physically identical. This can be done by restricting the rotational angle \( \delta \) to a range that does not overcount the number of distinct experiments. Thus, in a Stern-Gerlach case, where one must identify settings differing by \( \pi \), (3) and (5) will presumably be true – if the appropriate symmetries hold – for \( \delta \in (-\pi/2, +\pi/2) \), rotations larger than \( \pm \pi/2 \) being identified with \( \delta \pm \pi \). Then, in (5) for example, as the Stern-Gerlach magnet is rotated from \( \theta_B = 0 \) to \( \theta_B' = \pi/2 \), the effect at \( A \) will increase from \( \alpha = 0 \) to \( \alpha = 0.25 \); while further rotation – actually corresponding to smaller values of \( |\delta| \) – will decrease \( \alpha \), and \( \alpha = 0 \) when the magnet has rotated by \( \pi \).

This point should be studied further; and a detailed comparison with pilot-wave theory for the Stern-Gerlach case should be made.

### 10 Subquantum Information

We have defined the ‘total degree of nonlocality’ \( \alpha + \beta \) as the equilibrium fraction \( \alpha \) of outcomes at \( A \) that change under a shift \( \theta_B \rightarrow \theta_B' \) at \( B \), plus the fraction \( \beta \) of outcomes at \( B \) that change under a shift \( \theta_A \rightarrow \theta_A' \) at \( A \) (whether the changes are from \( +1 \) to \( -1 \) or vice versa). Clearly, \( \alpha \) may be interpreted as the average number of bits of information per singlet pair transmitted nonlocally (in equilibrium) from \( B \) to \( A \); and similarly for \( \beta \) from \( A \) to \( B \). Thus \( \alpha \) and \( \beta \) quantify the nonlocal flow of what might be termed ‘subquantum information’.

It has proved fruitful in recent years to look at quantum theory from an information-theoretic perspective. While some of that work makes use of insights from Bell’s theorem, in the author’s view the whole field of quantum information would benefit from a more explicit hidden-variables (including pilot-wave) perspective. Here are some examples.

**Universal Lower Bound on Nonlocal Information Flow:** The lower bound (2) on \( \alpha(\theta_A, \theta_B, \theta_B + \delta) + \beta(\theta_A, \theta_B, \theta_A - \delta) \) was derived assuming a rotational symmetry. Would it be possible to derive a general lower bound on the total nonlocal information flow in the singlet state, without this extra assumption?

The total degree of nonlocality \( \alpha(\theta_A, \theta_B, \theta_B') + \beta(\theta_A, \theta_B', \theta_A') \) depends on the initial and final angular settings. It quantifies the total nonlocal flow of information in the singlet state, generated by specific shifts \( \theta_B \rightarrow \theta_B' \) and \( \theta_A \rightarrow \theta_A' \) in the angular settings at \( B \) and \( A \). (That it depends on the settings has
been shown explicitly in the case of pilot-wave theory. Bell’s theorem tells us that \(\alpha + \beta\) cannot vanish for all settings. But for a general hidden-variables theory without any symmetries, there is no reason why \(\alpha + \beta\) could not be very small for some angles and very large for others. In other words: the underlying nonlocality required by Bell’s theorem could be ‘concentrated’ around certain ranges of settings. Thus it is not surprising that we were able to derive a lower bound on \(\alpha + \beta\) for specific settings only by assuming a rotational symmetry.

To circumvent this, one might consider the average total degree of nonlocality \(\bar{\alpha} + \bar{\beta}\) obtained by averaging \(\alpha + \beta\) over all possible initial and final settings \(\theta_A, \theta_B, \theta'_A, \theta'_B\). Alternatively, one might look at the maximum value \((\alpha + \beta)_{\text{max}}\). It should be possible to derive a general lower bound on \(\alpha + \beta\) or \((\alpha + \beta)_{\text{max}}\) for the singlet state, without any extra symmetry assumption. This would provide us with a universal lower bound on nonlocal information flow.

Degree of Nonlocality as a New Measure of Entanglement: For a general entangled pure state \(\Psi\) of two spin-1/2 systems – that is, for a pure bipartite state of two qubits – various measures of entanglement \(E(\Psi)\) have been proposed in the literature. A recent example is based on the decomposition \(\Psi = p |\Psi_e\rangle + \sqrt{1 - p^2} e^{i\phi} |\Psi_f\rangle\) into a maximally-entangled state \(|\Psi_e\rangle\) and an orthogonal factorisable state \(|\Psi_f\rangle\), where \(p\) and \(\phi\) are real; the measure \(E(\Psi) \equiv p^2\) has been shown to be closely related to a measure based on maximal violation of Bell’s inequality [19]. Since \(\alpha + \beta\) – the sum of the nonlocal effects from \(B\) to \(A\) and from \(A\) to \(B\) – quantifies nonlocality, it is natural to ask if \(\alpha + \beta\) can also be used as a measure of entanglement.

One approach would be to use pilot-wave theory to calculate the angular average \(\bar{\alpha} + \bar{\beta}\) or the maximum value \((\alpha + \beta)_{\text{max}}\) for a general entangled state \(\Psi\), and to take \(E(\Psi) = \bar{\alpha} + \bar{\beta}\) or \(E(\Psi) = (\alpha + \beta)_{\text{max}}\). Such calculations could easily be performed numerically, and the results compared with those derived from other measures. Another approach might be to try to derive theory-independent lower bounds on \(\alpha + \beta\) or \((\alpha + \beta)_{\text{max}}\) for general entangled states, and use the lower bounds as measures of entanglement.

Clearly, this proposal needs further study. Here, we restrict ourselves to pointing out that, in the symmetric pilot-wave case with \(a_A = a_B\) (and with the same square pointer packets at \(A\) and \(B\)), on the space of quantum states \(\alpha(0,0,\pi)\) is a local maximum for the maximally-entangled singlet state. It is straightforward to show that any infinitesimal perturbation of the singlet state decreases \(\alpha(0,0,\pi)\), recalling that for the singlet we found \(\alpha(0,0,\pi) = 1/2\).

\(^{21}\)We also saw that, individually, \(\alpha\) and \(\beta\) depend on the measurement couplings, but their sum satisfies the lower bound (3) obtained from rotational symmetry.

\(^{22}\)One can, for example, easily write down a local model that reproduces the quantum correlations for the specific settings \(\theta_A = 0, \theta_B = 0\) and \(\theta_A = 0, \theta'_B = \pi/2\) [1].

\(^{23}\)I am grateful to Guido Bacciagaluppi for this suggestion. (For a recent review of entanglement measures, see for example the article by Leah Henderson in this volume.)

\(^{24}\)Should the result turn out to depend on the details of the measurement process – such as the shape of the pointer packets (assumed square in the above) – one could simply take the minimum value. (For \(\alpha\) alone, of course, we have seen that even for the singlet state \(\alpha\) may be made arbitrarily small, in pilot-wave theory, by making the coupling at \(A\) arbitrarily large; but the sum \(\alpha + \beta\) is unaffected by this.)
First, for initial settings \(\theta_A = 0, \theta_B = 0\), if we write perturbed spin amplitudes

\[
\begin{align*}
a_{++} &= \epsilon_{++}, & a_{+-} &= \frac{1}{\sqrt{2}} + \epsilon_{+-}, \\
a_{-+} &= -\frac{1}{\sqrt{2}} + \epsilon_{-+}, & a_{--} &= \epsilon_{--}
\end{align*}
\]

with \(\epsilon \equiv \sqrt{2}(\epsilon_{++} + \epsilon_{+-}^*) = \sqrt{2}(\epsilon_{--} + \epsilon_{-+}^*)\) (the last equality following from normalisation), and work to lowest order in \(\epsilon\), the pointer velocities differ from the corresponding singlet case only where \(\psi_{++}\) and \(\psi_{+-}\) overlap: at such points \(v_A = -v_B = a\epsilon\) (instead of \(v_A = v_B = 0\)). As a result, instead of obtaining \(\sigma_A = -1, \sigma_B = +1\) and \(\sigma_A = +1, \sigma_B = -1\) for points respectively above and below the line \(r_B = r_A\), those values are obtained for points respectively above and below the line \(r_B = (1 + \epsilon)/(1 - \epsilon)\) for \(-\Delta/2 \leq r_A \leq -\epsilon\Delta/2\) and \(r_B = (1 - \epsilon)/(1 + \epsilon)\) for \(-\epsilon\Delta/2 \leq r_A \leq +\Delta/2\). Similarly, with the new settings \(\theta_A = 0, \theta_B = \pi\) the same (perturbed) state has the spin amplitudes

\[
\begin{align*}
a'_{++} &= \frac{1}{\sqrt{2}} + \epsilon_{++}, & a'_{+-} &= -\epsilon_{++}, \\
a'_{-+} &= \epsilon_{--}, & a'_{--} &= \frac{1}{\sqrt{2}} - \epsilon_{++}
\end{align*}
\]

and \(v'_A = v'_B = a\epsilon\) in the overlap of \(\psi'_{++}\) and \(\psi'_{+-}\), yielding \(\sigma_A = +1, \sigma_B = +1\) and \(\sigma_A = -1, \sigma_B = -1\) for points respectively above and below the line \(r_B = -(1 + \epsilon)/(1 - \epsilon)\) for \(-\Delta/2 \leq r_A \leq -\epsilon\Delta/2\) and \(r_B = -(1 - \epsilon)/(1 + \epsilon)\) for \(-\epsilon\Delta/2 \leq r_A \leq +\Delta/2\). Simple geometry then shows that the fractional area of points in the initial square for which outcomes have changed at \(A\), under \(\theta_B = 0 \rightarrow \theta'_B = \pi\), is now equal to

\[
\alpha(0, 0, \pi) = \frac{1}{2} - \frac{5}{4}\epsilon^2 + O(\epsilon^3)
\]

which is a local maximum at \(\epsilon = 0\). It remains to be seen if \(\alpha(0, 0, \delta)\) has a local maximum at \(\epsilon = 0\) for all angles \(\delta\).]

**Classical Simulation of Entanglement:** We have said that \(\alpha (\beta)\) is equal to the average number of bits of subquantum information per singlet pair transmitted nonlocally, in equilibrium, from \(B\) to \(A\) (\(A\) to \(B\)). In other words, \(\alpha (\beta)\) is the average amount of information per pair that needs to be transmitted faster than light from \(B\) to \(A\) (\(A\) to \(B\)) in order to reproduce the EPR-correlations. It might be interesting to investigate how this is related to recent work on the simulation of quantum entanglement with classical communication [20].

\[25\] Averaging (5) over \(\delta \in (-\pi, \pi]\), we obtain a mean lower bound of 0.25 bits; if instead only \(\delta \in (-\pi/2, \pi/2]\) count as physically distinct settings, the mean lower bound is \(\frac{1}{4}(1 - \frac{1}{\pi}) \approx 0.09\) bits. One expects the results would be higher without the symmetry assumptions on which (5) is based.
Subquantum Computation: In pilot-wave theory it is straightforward to show that the hidden-variable trajectory of a particle can contain information corresponding to all the results of a parallel quantum computation – for example if the particle is guided by a superposition of overlapping energy eigenfunctions whose eigenvalues encode the results of the computations (where here the computations are ‘performed’ by the evolution of the pilot wave in configuration space, and the results are ‘read’ by the piloted particle). This information could be read by us if we had access to matter in a state of quantum nonequilibrium $\rho \neq |\psi|^2$, leading to a truly exponential speed-up in processing power not just for some problems, but quite generally [5, 10, 21].

11 Conclusion and Hypothesis

Summarising, we have demonstrated a general ‘signal-locality theorem’, which states that in any deterministic hidden-variables theory that reproduces quantum statistics for some ‘equilibrium’ distribution $\rho_{eq}(\lambda)$ of hidden variables $\lambda$, a generic ‘nonequilibrium’ distribution $\rho(\lambda) \neq \rho_{eq}(\lambda)$ would give rise to instantaneous signals at the statistical level (as occurs in pilot-wave theory). Further, for an equilibrium ensemble of EPR-experiments, assuming certain symmetries we have derived lower bounds on the fraction of systems whose outcomes change under a shift in the distant measurement setting, and we have verified that the bounds are satisfied by pilot-wave theory. We have also pointed out some potential benefits of a perspective based on ‘subquantum information’.

With the signal-locality theorem in hand, let us now consider what its physical implications might be.

Bell’s theorem is widely regarded as proving that if hidden variables exist then so do instantaneous influences. But there is no consensus on what to conclude from this. A widespread conclusion is that hidden variables do not exist, the ‘argument’ being that relativity would otherwise have to be violated. But this is rather like someone in 1901 arguing that atoms cannot exist because if they did Newtonian mechanics would have to be violated. There is no reason why known physical principles cannot be violated at some hitherto-unknown level.

It is important, then, to distinguish between Bell’s theorem and what various authors have concluded from it. Similarly, one must distinguish between the signal-locality theorem proved above and what this author proposes to conclude from it.

The author suggests that the signal-locality theorem has the following physical significance: it indicates that our universe is in a special ‘finely-tuned’ state in which statistical noise happens to precisely mask the effects of nonlocality – a

26 Though there is disagreement about the status of relativity even among those who do consider that hidden variables might exist: some try to construct nonlocal theories in which the symmetries of Minkowski spacetime are somehow preserved at the fundamental level, while others (including this author) propose that special relativity be abandoned, with Minkowski spacetime emerging only as an equilibrium phenomenology. See ref. [10].
state of statistical equilibrium which is not fundamental but merely contingent.

For it seems mysterious that nonlocality should be hidden by an all-pervading quantum noise, and that (as we have shown) any deviation from that noise would make nonlocality visible. It is as if there is some sort of ‘conspiracy’ in the laws of physics that prevents us from using nonlocality for signalling. But the apparent conspiracy evaporates if one recognises that our universe is in a state of statistical equilibrium at the hidden-variable level, a special state in which nonlocality happens to be hidden.

On this view, the physics we see is not fundamental; it is merely a phenomenological description of an equilibrium state [15]. Fundamentally, the universe is nonlocal, obeying laws that have yet to be uncovered (pilot-wave theory providing a possible example). Unfortunately our experience is confined to an equilibrium state that hides the true nature of things behind a veil of quantum noise. Since any small departure from equilibrium would reveal the underlying nonlocal physics, our present inability to observe nonlocality directly (as opposed to indirectly, via Bell’s theorem) is not enforced by any fundamental physical principle: it is merely a contingent feature of equilibrium.

Indeed, our general inability to control the hidden-variable level is a contingent feature of equilibrium. This is clear in pilot-wave theory, where the uncertainty principle holds if and only if $\rho = |\psi|^2$ [15]. And it may be shown that the same is true in any deterministic hidden-variables theory: the uncertainty principle is valid if and only if $\rho(\lambda) = \rho_{eq}(\lambda)$ [10]. From this perspective, immense practical resources – for communication, and also for computation – are hidden from us by a veil of uncertainty noise, because we happen to live in quantum equilibrium.

It is then natural to make the hypothesis that the universe began in a state of quantum nonequilibrium $\rho(\lambda) \neq \rho_{eq}(\lambda)$, where nonlocal signalling was possible and the uncertainty principle was violated, the relaxation $\rho(\lambda) \rightarrow \rho_{eq}(\lambda)$ taking place during the great violence of the big bang [5, 10, 14–16]. As this relaxation occurred, the possibility of nonlocal signalling faded away, and statistical uncertainty took over. The quantum equilibrium state $\rho(\lambda) = \rho_{eq}(\lambda)$ may then be seen, heuristically, as a kind of ‘quantum heat death’ – analogous to the classical thermodynamic heat death in which all systems in the universe have reached the same temperature. (In the classical heat death, thermal energy may no longer be used to do work; in the ‘quantum heat death’, the underlying nonlocality may no longer be used for signalling.) In effect we are suggesting that, some time in the remote past, a hidden-variables analogue of the classical heat death actually occurred in our universe.

One might also compare our present limitations with those of a Maxwell demon in thermal equilibrium with a gas, whose attempts to sort fast and slow molecules fail. On this view the common objection to hidden variables – that their detailed behaviour can never be observed – is seen to be misguided: for the theory can hardly be blamed if we happen to live in a state of statistical equilibrium that masks the underlying details. There is no reason why nonequilibrium $\rho(\lambda) \neq \rho_{eq}(\lambda)$ could not exist in the remote past or in distant regions of the universe [5, 10, 14, 15], in which case the details of the ‘hidden-variable
level’ would not be hidden at all.

It may seem gratuitous to draw such radical conclusions from what has been proven here. But our hypothesis may also be supported by arguments from other areas of physics.

The view that our universe is in an equilibrium state is arguably supported by quantum field theory in curved spacetime, where there is no clear distinction between quantum and thermal fluctuations [22]. On this basis it has in fact been argued by Smolin that quantum and thermal fluctuations are really the same thing [23]. This suggests that quantum theory is indeed just the theory of an equilibrium state, analogous to thermal equilibrium.

This view is strengthened by Jacobson’s demonstration that the Einstein field equations may be viewed as an ‘equation of state’, in the context of a ‘thermodynamics of spacetime’ [24]. According to Jacobson, quantum field fluctuations correspond to some sort of ‘equilibrium’ distribution that might be violated at high energies; here, we would interpret quantum fluctuations as corresponding to \( \rho(\lambda) = \rho_{eq}(\lambda) \), which may have been violated at early times.

Further support comes from cosmology. The notorious ‘horizon problem’ has been with us for decades: the cosmic microwave background is observed to be nearly isotropic, and yet, at the time when photons decoupled from matter, the observable universe supposedly consisted of a large number of causally disconnected regions [27]. Inflation was thought to avoid this, but more recent analysis shows that at least some inflationary models merely shift the problem to having to assume ‘acausal’ homogeneity as an initial condition in order to obtain inflation [25]. A number of workers have seen in the horizon problem a hint that some sort of superluminal causation is required at early times, whether by topological fluctuations that lead to an effective nonlocality [26], or by the more recent suggestion that the speed of light increases at high energies [27].

Our hypothesis offers another alternative: if the universe started in quantum nonequilibrium \( \rho(\lambda) \neq \rho_{eq}(\lambda) \), the resulting nonlocal effects may have played a role in homogenising the universe at early times [10].

But is there any prospect of testing these ideas experimentally? Investigations are proceeding on two fronts. First, if we accept the idea from inflation that the temperature fluctuations in the microwave background were ultimately seeded by quantum fluctuations at very early times, then precise measurements of the microwave background can be seen as probes of quantum theory in the very early universe: if \( \rho(\lambda) \neq \rho_{eq}(\lambda) \) during the inflationary era, this would leave an imprint on the microwave background that differs from the one predicted by standard quantum field theory [10, 17]. Second, certain exotic (perhaps supersymmetric) particles may have decoupled at very early times, before quantum equilibrium was reached; if so, they may still exist in a state of quantum disequilibrium today and they – or their decay products – would now violate quantum mechanics [10, 14, 16, 17].

\[27\] Though it should be pointed out that the calculation of the size of causal horizons assumes that the classical Friedmann expansion, with scale factor \( \propto t^{1/2} \) at early times, is valid all the way back to \( t = 0 \). It might equally be argued that the horizon problem is an artifact of this (rather naive) assumption.
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