1 Introduction

It is common believed that the usual parton distributions (PDFs) can only give the longitudinal information of a hadron target in the deep inelastic scattering (DIS) processes, while the generalized parton distributions (GPDs) have the promising ability to shade light on the transverse information, which gives rise to the idea of “quark/gluon imaging” of hadrons [1]. Moreover, the impact parameter distributions (IPDs), obtained by the Fourier transform of GPDs with respect to the transverse momentum transfer, may show some information about the transverse impact space position of partons [2]. This impact parameter representation is useful in processes such as high-energy scattering and hard processes [3]. It is also argued that, in position space, IPDs play a similar role to the charge distributions, and are, thus, very promising for understanding the hadron internal structures.

As we know, $G_{C}(Q^2)$ is the form factor of the conserved local current, and is thus independent of the renormalization scale $\mu$. It can be obtained through the sum rules from GPDs, which by definition are probed in hard processes [3]. In the case of Fourier transforms of GPDs, Burkardt pointed out that, when $\xi = 0$, the Fourier transforms of GPDs have the interpretation of a density of partons with longitudinal momentum fraction $x$, localized at $b_\perp$ relative to the transverse center in the impact parameter space, which is allowed by the Heisenberg uncertainty principle [4, 5]. Due to the significance of the form factors in the impact parameter space, many theoretical works have been devoted to study the IPDs of pions, kaons and nucleons [5–15].

It should be mentioned that our recent work [16] gave a discussion of the $\rho$ meson unpolarized GPDs in momentum space with a Light-Cone Constituent Quark Model (LCCQM). The form factors and some other low-energy observables of the $\rho$ meson were calculated and our numerical results agreed with the previous publications and some experimental data [17]. In the literature, the constituent quark model is also used to describe the form factors of pions, nucleons, deuterons, etc. [18–20]. Moreover, the contributions from the valence and non-valence regimes to the form factors and generalized parton distributions were discussed and analyzed in detail. In addition, the reduced matrix elements, which are the moments of the DIS structure functions, were also estimated and the obtained values were compatible with the available lattice calculation at the same scale ratio [21]. In general, our numerical results for the unpolarized GPDs [16] were reasonable and satisfying. Therefore, in this work, we extend the phenomenological model to study the IPDs of the $\rho$ meson and to calculate the impact parameter dependent PDFs of $q(x,b_\perp)$ and $q(b_\perp)$ and the form factors of $q^{C,M,Q}(x,b_\perp)$ and $q^{C,M,Q}(b_\perp)$.

The paper is organized as follows. In Section 2, the framework of the impact parameter dependent PDFs is presented. In Section 3, we discuss the wave packets and the cutoff for the numerical calculation. The definitions of the impact parameter dependent FFs are given in Sec-

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Our numerical results for the PDFs and FFs in the impact parameter space are shown in Section 5, and Section 6 gives a short summary and conclusion.

2 Impact parameter dependent PDFs

When considering the nucleon GPDs without helicity flip, Burkardt [22] identifies the Fourier transform of its GPD \( H_q(x, \xi = 0, -\Delta^2_t) \) w.r.t. \(-\Delta^2_t\) as a distribution of partons in the transverse plane, i.e., the probability of finding a quark with longitudinal momentum fraction \( x \) and at transverse impact space position \( b_\perp \). The impact parameter dependent PDF for a nucleon (a spin-1/2 target), given by Ref. [22], reads

\[
q_N(x, b_\perp) = |N|^2 \int \frac{dp_+}{(2\pi)^2} \int \frac{dp'_+}{(2\pi)^2} \times \langle p^+, p'_+ | \lambda \rangle \left[ \int \frac{dz^-}{4\pi} \bar{q}(\frac{z^-}{2}, b_\perp) q(\frac{z^-}{2}, b_\perp) e^{iz^+ x} \right] [p^+, p'_+, \lambda] \\
= |N|^2 \int \frac{dp_+}{(2\pi)^2} \int \frac{dp'_+}{(2\pi)^2} H_q(x, \xi = 0, - (p_+ - p'_+)^2) e^{ib_\perp (p_+ - p'_+)} \\
= \int d^2 \Delta_\perp H_q(x, 0, - \Delta_\perp^2) e^{-ib_\perp \Delta_\perp} \\
= \int_0^\infty \Delta_\perp d\Delta_\perp \frac{1}{2\pi} J_0(b\Delta_\perp) H_q(x, 0, -\Delta_\perp^2) \\
= q_N(x, b),
\]

(1)

where the normalization factor \( N \) satisfies \( |N|^2 \int \frac{dp_+}{(2\pi)^2} = 1 \), and \( \Delta_\perp = |\Delta_\perp| = \sqrt{\Delta_x^2 + \Delta_y^2} \) and \( b = |b_\perp| = \sqrt{b_x^2 + b_y^2} \). Cylindrical symmetry is applied in the last but one step and \( J_0 \) is the Bessel function of the first kind \( J_0(z) \) with \( \nu = 0 \). The parton distribution depends on transverse impact space position \( b_\perp \) only through its norm \( b \) being the consequence of the longitudinal polarization. In the third step the integral turns to the total and transverse momentum transfer, i.e., \( d^2 p_+ d^2 p'_+ = d^2 \Delta_\perp d^2 P_\perp \), with \( \Delta_\perp = p'_\perp - p_\perp \) and \( P_\perp = (p'_\perp + p_\perp)/2 \), and using the fact that GPD \( H \) is independent of total transverse momentum \( P_\perp \). Ignoring the helicity flip, the spin projection \( \lambda \) can be dropped. In the forward limit, namely \( \xi = 0 \), we have \( t = (p' - p)^2 = -\Delta_1^2 \).

Note that Hoodbhoy [23] has already pointed out the DIS structure function \( F_1, F_2, g_1, \) and \( g_2 \) of spin-1 targets can be precisely measured in the same way as that of spin-1/2 targets. Analogous to the fact that the structure function \( F_1 \) connects to GPD \( H_q \) for spin-1/2 targets, we simply assume \( F_1 \) connects to the GPD \( H^I_1 \) for spin-1 targets as well. As shown by Eqs. (37~39) in Ref. [16], the isospin combination implies that

\[
\int_{-1}^1 dx H_i^+(x, \xi, t) = \int_{-1}^1 dx H_i^{=1}(x, \xi, t).
\]

(2)

Hereafter we omit the label of quark flavor \( u \) and isospin \( I = 1 \) for simplicity. Due to the similar roles of \( H_q \) and \( H^I_1 \), we introduce the impact parameter dependent PDF for spin-1 targets (for the \( u \) quark),

\[
q(x, b) = \int_{-1}^1 \frac{\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H_U(x, 0, -\Delta_\perp^2),
\]

(3)

One can further define the total parton distribution in the impact parameter space as

\[
q(b) = \int_0^1 dx q(x, b).
\]

(4)

Notice that \( \int d^2 b_\perp q(x, b) = H_1(x, 0, 0) \), which is equal to the usual PDF \( q(x) \) in the forward limit \( t = \Delta^2 \to 0 \). Therefore, \( q(x, b) \), the Fourier transform of the GPD \( H_1(x, \xi = 0, -\Delta^2_t) \) w.r.t. \(-\Delta^2_t\), can be identified, in analogy to the nucleon case, with the probability of finding a quark with longitudinal momentum fraction \( x \) and transverse impact space position \( b_\perp \) in the p meson.

It should be emphasized that in Ref. [2], the nucleon impact parameter dependent PDF \( q_N \) was proved to satisfy the positive constraints for the so-called “good” quark field. In our model calculation, the phenomenological vertexes (see Eq. (24) in Ref. [16]) involve the loop momentum \( k \), and the form of the vertexes is fixed according to the constraints from isospin symmetry. Our sophisticated model cannot simply reproduce the procedure of Ref. [2] to fold the correlation function into a norm of a quantity (see Eq. (23) of Ref. [2]). Therefore, the positive constraint for \( q(x, b) \) with a realistic model calculation needs to be proven further.

3 Wave packets

The Fourier transform of a plane wave is not well defined, thus, one can start with the wave packets instead of the plane wave. In the non-relativistic limit, the Fourier transform of the charge form factor \( G_C(Q^2) \) can be interpreted as the charge distribution in the transverse direction. In other words, as long as the wave packets peak sharply at some point in position space, by taking the non-relativistic limit, the Fourier transform of the
charge distribution equals the form factor. By the way, a Gaussian weighting factor was also adopted in a recent lattice QCD calculation [24], in order to suppress the unphysical oscillatory behaviour. The oscillation is due to the finite lattice size and nucleon momentum. The result in the small Bjorken $x(<0.3)$ region is different from that in Ref. [5]. This action will add two Gaussian functions in the expression, where $G$ charge distribution equals the form factor. By the way, the definition of the impact parameter dependent PDF

$$G(x,|b|) = \frac{1}{2\pi} \int \frac{d^2p}{(2\pi)^2} e^{-i\mathbf{p}\cdot \mathbf{x}} G(p,|b|)$$

was also adopted in a recent lattice QCD calculation [24], in order to suppress the unphysical oscillatory behaviour. The oscillation is due to the finite lattice size and nucleon momentum. The result in the small Bjorken $x(<0.3)$ region is different from that in Ref. [5].

Moreover, as pointed out by Burkardt [2, 4], the interpretation of the Fourier transform of the form factor as the charge distribution may receive relativistic corrections in the rest frame. However, such a problem may disappear in either Breit frame or infinite momentum frame (IMF). In the relativistic case, the transform receives relativistic corrections when the wave packet is localized with a size smaller than the Compton wavelength of the system. In the IMF, the relativistic correction can be managed to be very small, and therefore, the wave packet does not change the interpretation, as long as the wave packets are set slowly varying w.r.t. $\Delta$. To be specific, the width of the wave packets must be much larger than a typical QCD scale $\Lambda_{QCD} \sim 0.23$ GeV. For a Gaussian form wave packet, one gets $\sigma \ll 1/\Lambda_{QCD} \sim 3/M$, with $M$ being the $p$ meson mass.

On the other hand, as Diehl [5] has discussed, a real hadron is an extended object and is smeared out by an amount $\sigma$. From the experimental viewpoint, there is a largest measured value $|t|_{\text{max}}$ and thus there is the accuracy of the measurement $\sigma \sim (|t|_{\text{max}})^{-1/2}$. According to the observations and to the limit of the effect from unmeasured values of $t$, a Gaussian form wave packet can also be reasonably introduced. Thus we have

$$q_\sigma(x,b) = \int_0^{\Delta_0} \frac{d\Delta}{2\pi} J_0(b\Delta) G(\mathbf{p}_\perp, \sqrt{2} \sigma) H_1(x,0,-\Delta^2)$$

and

$$q_\sigma(b) = \int_0^1 dx q_\sigma(x,b)$$

Reference [5] also argued that in order to give a well-defined (positive, or without sign flip) longitudinal momentum $p^z$, $|p_\perp| \ll p^z$ is required. However, as one can see in Eq. (5), $p_\perp$ and $p^z$ are separated in the wave packet and this requirement actually does not affect the result of the integrals. This can also be seen from the property of GPDs. In the forward limit, $H(x,0,-\Delta^2)$ is not affected by this requirement either.

According to the above discussions, the relation $\sigma \sim (|t|_{\text{max}})^{-1/2}$ inspires us to introduce a cutoff $(\Delta_0)$ of the momentum transfer in the integral as well

$$q(x,b,\Delta_0) = \int_0^{\Delta_0} \frac{d\Delta}{2\pi} J_0(b\Delta) H_1(x,0,-\Delta^2)$$

and

$$q(b,\Delta_0) = \int_0^1 dx q(x,b,\Delta_0)$$

This assumption is supported by a comparison between the results of the integrals with a wave packet, $q_\sigma(b)$ (width $\sigma \sim 1/\Delta_0$) and the one with a cutoff $q(b,\Delta_0)$. This will be shown in Section 5.

## 4 Impact parameter dependent FFs

We emphasize that the unpolarized impact parameter dependent PDFs are proposed to describe the transverse distribution of unpolarized partons in an unpolarized target. As shown in previous sections, the IPDs can be obtained through Fourier transform of the unpolarized GPD $H_1$. We notice that the conventional charge, magnetic dipole and quadrupole FFs are the integrals of the linear combination of $H_i$. This motivates us to explore the possibility of obtain the IPDs with respect to the three FFs. The sum rules relating to the GPDs and the FFs $G_i$ are [27]

$$\int_{-1}^{1} dx H_i(x,\xi, t) = G_i(t) \quad (i=1,2,3)$$

$$\int_{-1}^{1} dx H_i(x,\xi, t) = 0 \quad (i=4,5)$$

These are the sum rules of $x$ and $\xi$ and are satisfied by all the FFs with $G_i(0) = 0$.

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where $G_i$ are the FFs in the decomposition of the local current. The FFs $G_{C,M,Q}$ can be expressed in terms of $G_{1,2,3}$ as [28]

$$
G_C(t) = G_1(t) + \frac{2}{3} \eta G_Q(t),
$$

$$
G_M(t) = G_2(t),
$$

$$
G_Q(t) = G_1(t) - G_2(t) + (1 + \eta) G_3(t),
$$

where $\eta = -t/4M^2$. Together with Eq. (11), one can obtain $G_{C,M,Q}$ directly from GPDs $H_{1,2,3}$. This allows us to bypass the well-known ambiguity of the angular condition [29]. With the above two equations, one can get the relations

$$
G_C(t) = \int_{-1}^{1} dx \left[ H_1(x, \xi, t) + \frac{2}{3} \eta \left[ H_1(x, \xi, t) - H_2(x, \xi, t) + (1 + \eta) H_3(x, \xi, t) \right] \right],
$$

$$
G_M(t) = \int_{-1}^{1} dx H_2(x, \xi, t),
$$

$$
G_Q(t) = \int_{-1}^{1} dx \left[ H_1(x, \xi, t) - H_2(x, \xi, t) + (1 + \eta) H_3(x, \xi, t) \right].
$$

Notice that by taking $\xi = 0$ and $\eta = -t/4M^2 = \Delta_0^2/4M^2$, one can get quantities similar to the integrands in Eq. (1). We have the impact parameter dependent FFs

$$
q_C^0(x, b) = \int_{0}^{\infty} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) e^{-\Delta_{\perp}^2 s^2/4}
$$

$$
\times \left[ H_1(x, 0, -\Delta_{\perp}^2) + \frac{1}{3} \Delta_{\perp}^2 [H_1(x, 0, -\Delta_{\perp}^2) - H_3(x, 0, -\Delta_{\perp}^2)] \right],
$$

$$
q_M^0(x, b) = \frac{1}{G_M(0)} \int_{0}^{\infty} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) e^{-\Delta_{\perp}^2 s^2/4}
$$

$$
\times H_2(x, 0, -\Delta_{\perp}^2),
$$

$$
q_Q^0(x, b) = \frac{1}{G_Q(0)} \int_{0}^{\infty} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) e^{-\Delta_{\perp}^2 s^2/4}
$$

$$
\times \left[ H_1(x, 0, -\Delta_{\perp}^2) - H_3(x, 0, -\Delta_{\perp}^2) + (1 + \frac{\Delta_{\perp}^2}{4M^2}) H_3(x, 0, -\Delta_{\perp}^2) \right],
$$

and

$$
q_{C,M,Q}^Q(b) = \int_{0}^{1} dx q_{C,M,Q}^Q(x, b).
$$

Comparing the impact parameter dependent FFs, Eq. (14), with the impact parameter dependent PDFs, Eq. (7), we introduce the “difference” quantities

$$
q_C^{QC}(x, b) = \int_{0}^{\infty} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) e^{-\Delta_{\perp}^2 s^2/4}
$$

$$
\times \left[ \frac{2}{3} \Delta_{\perp}^2 \left[ H_1(x, 0, -\Delta_{\perp}^2) - H_2(x, 0, -\Delta_{\perp}^2) \right] + (1 + \frac{\Delta_{\perp}^2}{4M^2}) H_3(x, 0, -\Delta_{\perp}^2) \right],
$$

$$
q_M^{QC}(b) = \int_{0}^{1} dx q_{C,M,Q}^{QC}(x, b),
$$

which receive the contribution from the quadrupole moment. The “difference” quantities satisfy

$$
q_C^{QC}(x, b) = q_C^0(x, b) - q_C(x, b),
$$

$$
q_M^{QC}(b) = q_M^0(b) - q_M(b).
$$

It is clear that the impact parameter dependent PDFs relate to the impact parameter dependent FFs

$$
\int_{0}^{1} dx \int_{-\infty}^{\infty} d^2b q_{C,M,Q}^{QC}(x, b) = 1.
$$

Thus, it is possible to interpret $q_C^0$, $q_M^0$ and $q_Q^0$ as the percentage of the contributions to the charge (normalized to 1), magnetic dipole $\mu_\rho$ and quadrupole moment $Q_\rho$ respectively, from the parton with the longitudinal momentum fraction $x$ and transverse impact space position $b_{\perp}$.

5 Results

In our previous work [16] with a light-cone constituent quark model, we took the two model parameters of the constituent mass $m = 0.403$ GeV and regulator mass $m_R = 1.61$ GeV, and we calculated the GPDs of the $\rho$ meson. In our LCCQM, we introduced an effective Lagrangian for the $\rho-q\bar{q}$ interaction with a phenomenological vertex $\Gamma$ and a Bethe-Salpeter amplitude. By integrating the minus component of the quark momentum $k^-$ analytically and rest of the components numerically, we obtained the GPDs and FFs of the $\rho$ meson.

In this work, we simply extend the calculation to the impact parameter dependent PDFs $q(b)$ and impact parameter dependent PDFs $q_{C,M,Q}^{QC}(b)$. Figure 1 gives the $q(b)$ with a wave packet, $q_{\sigma}(b)$, and with a cutoff on the momentum transfer, $q(b, \Delta_0)$, respectively. The comparison shows that the cutoff ($\Delta_0$) has a similar effect as the wave packet with width $\sigma / 1 / \Delta_0$. Of course, we expect that the prediction of the constituent quark model is reasonable only in the region of $|t|^{1/2} \leq 2$ GeV and when the momentum transfer becomes larger the constituent quark model fails. The width of the wave packet is also constrained by the uncertainty principle: to have a valid probability interpretation of the initial and final states, the position dispersion ($\sim \sigma$) cannot be smaller than the
also becomes more transversely localized in the position
smaller, the wave functions of the initial and final states
get closer to a plane wave, and the parton distribution
2 GeV
dependent FFs
rotation. As we argued before, the cal results in Fig. 3(a)
agrees with this point of view.

The oscillation of the states, the position dispersion
wavelength in size will in general induce various relativistic
corrections [4]. With the help of Figs. 1 and 3(b), and Eq. (20),
the oscillation in \( q_{b}^{C}(b) \) can be explained as the
behaviour of \( q_{b}^{QC}(b) \) which is related to the quadrupole
moment. From the experimental aspect, since the \( \rho \) meson quadrupole
moment is small, this phenomenon is hard to determine.

Figures 5 and 6 show the numerical result of \( q_{a}(x,b) \)
and \( q_{a}^{C,M,Q}(x,b) \) with \( \sigma = 1 \text{ GeV}^{-1} \) and \( x =
1/10,3/10\text{ and }1/2 \) respectively. When \( x \leq 1/10, q_{a}^{C}(x,b) \)
has negative values as \( b<0.4 \text{ fm} \) (see Fig. 5(b)), due to the
oscillation of \( q_{a}^{QC}(x,b) \) (see Fig. 5(c)). In the small
region (like \( x < 1/10 \) in our case), it is believed that the
contribution of the gluon GPDs becomes more important,
which is beyond the scope of the present model.

Compton wavelength. In the later content, our numerical
results in Fig. 3(a) agree with this point of view.

Figure 2 gives the contour plots of the impact parameter
dependent PDF \( q_{a}(b) \) with \( \sigma = 1 \text{ GeV}^{-1} \) and
2 GeV\(^{-1}\). Since we choose the polarization in the
z direction, the parton distribution is invariant under rotation
around the z direction. We see that as \( \sigma \) becomes
smaller, the wave functions of the initial and final states
get closer to a plane wave, and the parton distribution
also becomes more transversely localized in the position
space, as shown in Fig. 1 and Fig. 2.

Figures 3 and 4 give the impact parameter
dependent FFs \( q_{a}^{C,M,Q}(b) \) and \( q_{a}^{QC}(b) \) with \( \sigma =
1/2 \text{ GeV}^{-1}, 1 \text{ GeV}^{-1}, 2 \text{ GeV}^{-1} \) respectively. Figure 4
shows that, as the wave packet becomes more sharply
localized (\( \sigma \) decreases), the contributions are
concentrated more in the small \( b_{\perp} \) region for both the
magnetic dipole \( \mu_{\perp} \) and quadrupole moment \( Q_{\perp} \). For
the impact parameter charge density, Fig. 3(a), the distributions with \( \sigma \) less than about 1 GeV\(^{-1}\) become obscure
due to the oscillation. As we argued before, the \( \rho \) meson is an extended object and its Compton wavelength
is \( 1/m_{\rho} = 1.3 \text{ GeV}^{-1} \). The position dispersion \( \langle \Delta x \rangle = \sigma \)
in the case of the Gaussian wave packet. The uncertainty
principle \( \langle \Delta x \Delta p \rangle \geq 1/2 \) in natural units gives the
constraint that, to maintain the probability interpretation
of the states, the position dispersion \( \langle \Delta x \rangle \) should
not be smaller than the Compton wavelength. Otherwise,
localizing a wave packet to less than its Compton
wavelength in size will in general induce various relativistic
corrections [4]. With the help of Figs. 1 and 3(b), and Eq. (20),
the oscillation in \( q_{a}^{C}(b) \) can be explained as the
behaviour of \( q_{a}^{QC}(b) \) which is related to the quadrupole
moment. From the experimental aspect, since the \( \rho \) meson quadrupole
moment is small, this phenomenon is hard to determine.

Figures 5 and 6 show the numerical result of \( q_{a}(x,b) \)
and \( q_{a}^{C,M,Q,Q}(x,b) \) with \( \sigma = 1 \text{ GeV}^{-1} \) and \( x =
1/10,3/10\text{ and }1/2 \) respectively. When \( x \leq 1/10, q_{a}^{C}(x,b) \)
has negative values as \( b<0.4 \text{ fm} \) (see Fig. 5(b)), due to the
oscillation of \( q_{a}^{QC}(x,b) \) (see Fig. 5(c)). In the small
region (like \( x < 1/10 \) in our case), it is believed that the
contribution of the gluon GPDs becomes more important,
which is beyond the scope of the present model.
Fig. 3. (color online) The impact parameter dependent FFs $q_0^C(b)$ and $q_0^{QC}(b)$ with $\sigma=1/2$ GeV$^{-1}$, 1 GeV$^{-1}$ and 2 GeV$^{-1}$.

Fig. 4. (color online) The impact parameter dependent FFs $q_0^M(b)$ with $\sigma=1/2$ GeV$^{-1}$, 1 GeV$^{-1}$, and 2 GeV$^{-1}$.

Fig. 5. (color online) The impact parameter dependent PDFs $q_\sigma(x,b)$ and FFs $q_\sigma^{QC}(x,b)$ with $\sigma=1$ GeV$^{-1}$ and $x=1/10$, 3/10 and 1/2.
The symmetry around $x \sim 1/2$ of the parton distributions, implied by the isospin symmetry, is not satisfied well due to this reason. In addition, we found, from Fig. 6, that the distributions approximately remain the same in $q_{\sigma}^Q(x,b)$ when $1/10 \leq x \leq 3/10$.

6 Summary and conclusions

In this work, analogous to the definition of the pion and nucleon impact parameter dependent PDFs, we introduce the p meson impact parameter dependent PDFs ($q(x,b)$ and $q(b)$) and impact parameter dependent FFs ($q^{C,M,Q}(x,b)$ and $q^{C,M,Q}(b)$). By employing the LC-CQM, as we have done previously, we carried out the numerical calculation of those quantities for the first time. We believe that $q^{C,M,Q}(x,b)$ may be interpreted as the percentages of the contributions to the charge (normalized to 1), magnetic dipole $\mu_\sigma$, and quadrupole moment $Q_\sigma$, respectively, from a parton with a longitudinal momentum fraction $x$ and a transverse impact space position $b$. Considering the facts that the p meson is an extended object and there exists a largest measured value of momentum transfer in realistic measurements, a Gaussian form wave packet is employed in our numerical calculation. Our numerical results show that the wave packet approach plays a similar effect to the cutoff in the integral, which is due to the validity of the constituent quark model. Our numerical results for impact parameter charge distributions also show that the width of the Gaussian wave packet should be larger than the Compton wavelength. We expect that this approach is needed in a phenomenological model calculation in order to remove the possible negative values of the impact parameter charge distributions $q_{\sigma}^Q(x,b)$, which cannot be understood by the density interpretation.

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