Realistic Inflation Models and Primordial Gravity Waves

Qaisar Shafi
Bartol Research Institute, Department of Physics and Astronomy, University of Delaware,
Newark, DE 19716, USA
E-mail: shafi@bartol.udel.edu

Abstract. We consider a variety of realistic inflationary potentials and discuss their
predictions for the tensor to scalar ratio $r$, a canonical measure of gravity waves generated
during inflation. For a Standard Model gauge singlet inflaton with a Higgs-like potential,
for example, we find that $r \gtrsim 0.02$, which may be observed by the Planck satellite. We also identify
supersymmetric hybrid inflation models where $r$ can be as high as 0.03, again within the reach
of Planck. The scalar spectral index in these models lies within the WMAP one sigma bounds.

Introduction

Successful primordial inflation should:

- Explain the observed flatness and high degree of isotropy;
- Provide origin of $\delta T / T$;
- Offer testable predictions for $n_s$, $r$, $dn_s / d \ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Need physics beyond the Standard Model (SM)?

Inflation is usually driven by some potential $V(\phi)$ and the relevant slow-roll parameters are
defined as

$$\epsilon = \frac{m_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{m_p^2}{2} \left( \frac{V'}{V} \right).$$

Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index $n_s$ and the tensor to
scalar ratio $r$ are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \quad r \simeq 16\epsilon.$$
The tensor to scalar ratio \( r \) is related to the energy scale of inflation by
\[
V(\phi_0)^{1/4} = 3.3 \times 10^{16} r^{1/4} \text{ GeV}.
\]
The amplitude of the curvature perturbation is given by
\[
\Delta^2_R = \frac{1}{24\pi^2} \left( \frac{V}{m^4} \right)_{\phi = \phi_0} \approx 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.
\]
The number of e-folds after the comoving scale \( l_0 = 2\pi/k_0 \) has crossed the horizon is given by
\[
N_0 = \frac{1}{m^2} \int_{\phi_0}^{\phi_e} \left( \frac{V}{V'} \right) d\phi.
\]
Inflation ends when \( \text{max}[\epsilon(\phi_e), |\eta(\phi_e)|] = 1 \).

**Higgs Inflation**

Consider the following Higgs Potential:
\[
V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{M} \right)^2 \right]^2 \quad \leftarrow \text{ (tree level)}
\]
Here \( \phi \) is a gauge singlet field. As discussed in Refs. [1, 2], slow roll inflation may occur from either above or below the VEV \( M \). For shorthand, we henceforth denote these regimes as BV (below VEV) and AV (above VEV) solutions. As shown in Fig. 2, WMAP data favors BV inflation, whereas, the AV branch is found to lie outside of the 1-\( \sigma \) bounds in Ref. [2].

Consider next the following interaction of inflaton \( \phi \) with some GUT symmetry breaking scalar boson \( \Phi \):
\[
\mathcal{L}_{\text{int}} = \frac{\lambda^2}{2} \phi^2 \Phi^2.
\]
Including radiative corrections (quantum smearing) we have
\[
V(\phi) \approx \left( \frac{m^2 M^2}{4} \right) \left[ 1 - \left( \frac{\phi}{M} \right)^2 \right]^2 + A \phi^4 \left[ \ln \left( \frac{\phi}{M} \right) - \frac{1}{4} \right] + AM^4 \frac{4}{4},
\]
where \( V(\phi = 0) \equiv V_0 = \frac{m^2 M^2}{4} + \frac{AM^4}{4} \) and \( A = \frac{\lambda^2}{32\pi^2} \). Note that we can use ‘Minkowski space’ CW corrections provided the propagating fields have masses \( \gg H \) (Hubble constant).
**Figure 2.** $r$ vs. $n_s$ for tree level Higgs inflation, shown together with the WMAP 1-σ (68% confidence level) bounds [3]. The blue dotted and blue dashed curves correspond to number of e-foldings $N_0 = 50$, $N_0 = 60$, respectively. Small (big) green circles correspond to the quadratic potential (QP) with $N_0 = 50$ ($N_0 = 60$).

**Figure 3.** $r$ vs. $n_s$ for the tree level Higgs potential ($A = 0$) and the radiatively corrected Higgs potential ($A = 10^{-14.0}$, $10^{-13.6}$, $10^{-13.3}$). Blue and red curves represent the predictions of Coleman-Weinberg Potential (CWP) with the number of e-foldings $N_0 = 55$ and $N_0 = 60$, respectively. Small (big) green circles correspond to quadratic potential (QP) with $N_0 = 50$ ($N_0 = 60$).

The predictions of the radiatively corrected Higgs potential are shown in Fig. 3 and have been discussed in Ref. [4]. The parameter $A$ here quantifies the radiative corrections. The CWP prediction represents the maximal smearing of the radiatively corrected Higgs inflation results.

Note that $r \gtrsim 0.02$ if $n_s \gtrsim 0.96$. Thus, Planck will test Higgs inflation soon!
Susy Hybrid Inflation

Some attractive features of supersymmetry (susy) are:

- Resolution of the gauge hierarchy problem
- Unification of the SM gauge couplings at 
  \[ M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV} \] (see Fig. 4)
- Cold dark matter candidate (LSP), typically a neutralino.

Other good reasons:

- Radiative electroweak breaking
- String theory requires susy

Leading candidate is the MSSM (Minimal Supersymmetric Standard Model).

The susy hybrid model [5, 6, 7, 8] is an attractive scenario in which inflation can be associated with symmetry breaking \( G \rightarrow H \)

- Simplest inflation model is based on

\[ W = \kappa S (\Phi \bar{\Phi} - M^2). \]

\( S = \) gauge singlet superfield, \((\Phi, \bar{\Phi})\) belong to suitable representation of \( G \).

- We need \( \Phi, \bar{\Phi} \) pair in order to preserve susy while breaking \( G \rightarrow H \) at scale \( M \gg \text{TeV} \), susy breaking scale.
- R-symmetry

\[ \Phi \bar{\Phi} \rightarrow \Phi \bar{\Phi}, \quad S \rightarrow e^{i\alpha} S, \quad W \rightarrow e^{i\alpha} W \]
$\Rightarrow$ $W$ is a unique renormalizable superpotential.

- Some examples of gauge groups:

\[ G = U(1)_{B-L}, \text{ (Supersymmetric superconductor)} \]

\[ G = SU(5) \times U(1), \text{ (}\Phi = 10\text{-plet of } SU(5)\text{), (Flipped } SU(5)\text{)} \]

\[ G = 3_c \times 2_L \times 2_R \times 1_{B-L}, \text{ (}\Phi = (1, 1, 2, +1)\text{)} \]

\[ G = 4_c \times 2_L \times 2_R, \text{ (}\Phi = (1, 1, 2)\text{),} \]

\[ G = SO(10), \text{ (}\Phi = 16\text{)} \]

- The susy hybrid tree level potential is

\[ V_F = \kappa^2 (M^2 - |\Phi|^2)^2 + 2\kappa^2 |S|^2 |\Phi|^2, \]

with susy vacua

\[ |\langle \Phi \rangle| = |\langle \Phi \rangle| = M, \langle S \rangle = 0. \]

Take into account radiative corrections (because during inflation $V \neq 0$ and susy is broken by $F_S = -\kappa M^2$):

- Mass splitting in $\Phi - \overline{\Phi}$

\[ m^2_{\pm} = \kappa^2 S^2 \pm \kappa^2 M^2, \quad m^2_F = \kappa^2 S^2 \]

- One-loop radiative corrections

\[ \Delta V_{\text{1loop}} = \frac{1}{16\pi^2} \text{Str}[\mathcal{M}^4(S)(\ln \frac{M^2(S)}{q^2} - \frac{3}{2})] \]
• In the inflationary valley (Φ = 0)

\[ V \simeq \kappa^2 M^4 \left( 1 + \frac{\kappa^2 N}{8\pi^2} F(x) \right) \]

where \( x = |S|/M \) and

\[ F(x) = \frac{1}{4} \left( (x^4 + 1) \ln \left( \frac{x^4 - 1}{x^4} \right) + 2x^2 \ln \left( \frac{2x^2 - 1}{x^2} \right) + 2 \ln \frac{2M^2x^2}{\kappa^2} - 3 \right). \]

Tree Level plus radiative corrections:

\[ \delta T/T \propto (M/M_P)^2 \sim 10^{-5} \longrightarrow \text{attractive scenario (} M \sim M_G \text{)} \]

\( n_s \approx 1 - \frac{1}{\alpha_0} \approx 0.98 \)

• The minimal Kähler potential can be expanded as

\[ K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2. \]

• The sugra scalar potential is given by

\[ V_F = e^{K/m_p^2} \left( K_{ij}^{-1} D_{z_i} W D_{z_j} W^* - 3m_p^{-2} |W|^2 \right), \]

where we have defined

\[ D_{z_i} W = \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W; \quad K_{ij} = \frac{\partial^2 K}{\partial z_i \partial z_j}, \]

and \( z_i \in \{ \Phi, \overline{\Phi}, S, \ldots \} \).

• Take into account sugra corrections, radiative corrections and soft susy breaking terms [8]:

\[ V \simeq \kappa^2 M^4 \left( 1 + \left( \frac{M}{m_p} \right)^4 \frac{x^4}{2} + \frac{\kappa^2 N}{8\pi^2} F(x) + a \frac{m_3/2x}{\kappa M} \right)^2 \]

where \( a = 2|2 - A| \cos[\arg S + \arg(2 - A)], \ x = |S|/M \) and \( S \ll m_p \).

Note: No ‘η problem’ with minimal (canonical) Kähler potential!
\[ r \lesssim 10^{-4} \] within 2-\( \sigma \) bounds of WMAP data (see [10])

Now consider flipped SU(5) \( \equiv SU(5) \times U(1)_X \)

- Chiral superfields are arranged as

\[ 10_1 = \begin{pmatrix} d^c & Q \\ \nu^c \end{pmatrix}, \quad 5_{-3} = \begin{pmatrix} u^c_L \end{pmatrix}, \quad 1_5 = e^c \]

- Compared to standard SU(5), these multiplets correspond to the interchange

\[ u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c \]

| Low scale susy | Flipped SU(5) (Minimal) | SU(5) (Minimal) |
|----------------|------------------------|-----------------|
| Doublet-triplet splitting | Yes | Yes |
| Dimension 5 proton decay | Yes | Eliminated! |
| \( \mu \) problem | No | No |
| Inflation | Easy | Difficult |
| Dimension 6 proton decay | \( \tau_p \sim 10^{34}-10^{36} \) yrs | \( \tau_p \sim 10^{35}-10^{36} \) yrs |
| Monopole problem | No | Yes |
| Seesaw mechanism | Automatic | No |
| Charge quantization | No | Yes |
| Unification of gauge couplings | Can be arranged | Yes |
| CDM | Yes | Yes |

- Consider susy hybrid inflation in flipped SU(5), where \( \Phi \) is a 10-plet
Allowing the soft mass squared to vary, the potential appears as

\[ V \simeq \kappa^2 M^4 \left( 1 + \left( \frac{M}{m_p} \right)^4 \frac{v^4}{2} + \frac{\kappa^2 \g' - \kappa^2}{8 \kappa} F(x) + \alpha \left( \frac{m_3^2}{\kappa M} \right) + \left( \frac{M_S^2}{\kappa M} \right)^2 \right) \]

- If \( M_S^2 < 0 \) the soft susy breaking mass squared term drives the spectral index toward red-tilted values
- The minimal model consistent with \( n_s = 0.96 - 0.97 \) leads to proton lifetime predictions of order \( 10^{34} - 10^{36} \) years (see Ref. [11])

![Figure 6](image)

**Figure 6.** \( \log_{10}(M/\text{GeV}) \) and \( \log_{10}(\kappa) \) vs. \( \log_{10}(|M_S|/\text{GeV}) \) in the flipped SU(5) model \((N = 10)\), with \( n_s \) fixed at the central value 0.967 (see Ref. [11]).

- The minimal susy hybrid inflation model yields \( r \) values \( \lesssim 10^{-4} \);
- A more general analysis with a non-minimal Kähler potential can lead to larger \( r \)-values (see Fig. 7 and Ref. [12]).

![Figure 7](image)

**Figure 7.** \( r \) vs \( n_s \) for susy hybrid model (see Ref. [12]).
Summary

- The predictions of $r$ (primordial gravity waves) for various models of inflation are as follows:
  - Gauge Singlet Higgs Inflation:
    $$ r \gtrsim 0.02 \text{ for } n_s \gtrsim 0.96 $$
  - Susy Higgs (Hybrid) Inflation:
    $$ r \lesssim 10^{-4} \text{ (minimal), } r \lesssim 0.03 \text{ (non-minimal)} $$

- Results from PLANCK are eagerly awaited!

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