Advances in high-dimensional quantum entanglement

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Abstract | Since its discovery, quantum entanglement has challenged some of the best established views of the world: locality and reality. Quantum technologies promise to revolutionize computation, communication, metrology and imaging. Here we review conceptual and experimental advances in complex entangled systems involving many multilevel quantum particles. We provide an overview of the latest technological developments in the generation and manipulation of high-dimensionally entangled photonic systems encoded in various discrete degrees of freedom such as path, transverse spatial modes or time–frequency bins. This overview should help to transfer various physical principles for the generation and manipulation from one degree of freedom to another and thus inspire new technical developments. We also show how purely academic questions and curiosity led to new technological applications. Fundamental research provides the necessary knowledge for upcoming technologies, such as a prospective quantum internet or the quantum teleportation of all information stored in a quantum system. Finally, we discuss some important problems in the area of high-dimensional entanglement and give a brief outlook on possible future developments.

The basic unit of quantum information is the qubit, which is a two-state system. Nature, however, uses a four-letter alphabet for arguably the most important information storage system, DNA. Although it is still unclear why evolution developed a four-letter system1, its existence may give us an indication that we should also look for more complex systems than qubits.

In fact, in quantum communication, protocols based on larger alphabets offer certain advantages: a higher information capacity and the increased resistance to noise, which is important for applications2–4. In fundamental tests of nature, such as the violation of local realistic theories, higher-dimensional systems are advantageous as these allow for lower detection efficiency than qubits1.

Several physical systems allow for the encoding of higher-dimensional quantum information. These systems span Rydberg atoms, trapped ions5, polar molecules, cold atomic ensembles6, artificial atoms formed by superconducting phase qudits7, and defects in solid states8 or photonic systems.

In photonic systems, there are two fundamentally different approaches to encoding information. Whereas the continuous-variable1,2 quantum information processing approach is based on coherent or squeezed states, the discrete-variable approach is based on single-photon Fock states. The continuous-variable and discrete-variable approaches differ in the photon-number statistics they obey. Experimentally, this leads to considerable differences in the generation and detection of the respective states. These differences, in turn, lead to advantages and disadvantages in different application scenarios.

Continuous-variable quantum information is mostly encoded in the quantized electric field quadratures (amplitude and phase), which are continuous degrees of freedom (DoFs). Measurements of these field quadratures can be performed using beamsplitters and positive–intrinsic–negative (PIN) diodes. The squeezing of the two quadratures leads to entanglement and enables teleportation protocols. Whereas the teleportation protocol is in principle deterministic, the fidelity is fundamentally limited by the squeezing parameter r. Note, only r→∞ would yield a fidelity of one, but would also require an infinite amount of energy. Experimentally, fidelities up to 85% have been reported. This also holds true for more complex graph states9–11. In quantum metrology, the use of the continuous-variable approach has a real advantage, as demonstrated in REF 11, which reported using the squeezing of the amplitude and phase states to beat the standard quantum limit and thereby enhance the sensitivity of the LIGO interferometer.

In stark contrast with continuous-variable systems, discrete-variable systems are based on the creation and detection of single quanta of light (photons). In discrete-variable approaches, quantum information...
More particles and dimensions

Interestingly, high-dimensional entangled quantum systems have been considered theoretically since the beginning of quantum mechanics, first by Albert Einstein, Boris Podolsky and Nathan Rosen, and later by Erwin Schrödinger\(^{25,26}\). They considered the external and continuous parameters of quantum systems, namely the position and momentum of two strongly correlated (entangled) particles. Only later did David Bohm\(^{27}\) come up with the idea to investigate two entangled spin-1/2 particles, that is, two-state systems now called qubits. The work\(^{28}\) of John Stewart Bell moved the formerly purely philosophical questions about entanglement, reality and locality to an experimentally testable theorem. These developments aroused the interest of many experimental physicists, who in the following years attempted to generate entangled quantum systems in the laboratory\(^{29-34}\). These experiments unexpectedly led to actual technological applications (FIG. 1).

Historically, it seems that going from complex systems to simpler ones enabled several important advances in quantum mechanics. However, we would like to challenge this chronological perspective and consider the system size in terms of numbers of particles and their local dimensionality (complexity) as the driving factors behind the development of both fundamental insight and technological applications. To better illustrate our view, we use FIG. 1 and introduce two categories: fundamental insights and technological applications. These two categories emphasize that all potential applications of quantum technology today have their origin in fundamental insights and have been driven initially by curiosity only.

The double-slit experiment is a prime example of the superposition principle that contains the essential features of quantum mechanics\(^ {35} \). Consider a single electron incident on a double slit; after passing through the double slit, the quantum state is in a superposition of the two possible ways the electron could have gone. The answer to whether one can simply copy this unknown quantum state is ‘no’\(^ {36} \). At first glance, this no-cloning theorem seems like a strong limitation. This is true especially when compared with classical information science, in which the ability to copy information is essential for long-distance communication or error-correcting schemes. Nonetheless, the superposition principle and the no-cloning theorem lead to a useful application\(^ {37} \), namely QKD. In QKD schemes, a key is distributed between two parties in a secure way such that if an eavesdropper gains too much information about the key, that can be detected.

Single quantum particles in higher dimensions lead to questions about possible representations using hidden variable models\(^ {38} \). In such systems, the contextuality of quantum measurements and its corresponding hidden value can be investigated. Interestingly, there is already a strong contradiction between quantum and classical physics, without even considering entanglement. It has been found in quantum mechanics that the measurement outcome depends on the measurement performed, meaning the context of the measurement itself\(^ {39} \). On the technological side, increasing the dimensionality of
a single particle leads not only to higher information capacities but also, more importantly, to unprecedented levels of noise resistance in QKD schemes\(^\text{2,3}\).

Going from single to two locally separated quantum systems, questions regarding locality have had an enormous influence on our worldview. Schrödinger coined the term entanglement to describe non-classically correlated quantum systems and considered it as “not one but rather the characteristic trait of quantum mechanics”\(^\text{45}\). Philosophically, and of fundamental importance, is whether quantum mechanics is compatible with local realistic hidden-variable theories, as considered by Einstein, Podolsky and Rosen (EPR)\(^\text{46}\) and Bell\(^\text{47}\). A series of experiments published in 2015 that closed all essential experimental loopholes denied this hypothesis\(^\text{48–51}\). Historically, the first theoretical investigation into generalized Bell-like violations in higher-dimensional bipartite systems showed classical behaviour for high spin values\(^\text{52}\). It was unclear at that time whether this result was a natural consequence of a quantum-to-classical transition or a weak version of Bell's inequality. Later studies found stronger Bell violations than in the qubit case, and also unveiled a higher noise resistance as dimensionality increased\(^{53–55}\), which triggered the interest of many experimental physicists.

It is not obvious that entanglement can be used for technological applications as entanglement itself does not allow information to be transmitted. But it can assist the construction of the ultimate cryptographic channel. Using entanglement and its Bell-type violations in combination with classical communication enables ultimately secure and device-independent key distribution concepts for cryptography\(^{56,57}\).

Multiparticle systems with particle numbers \(N \geq 2\) not only increase the Hilbert space exponentially but also lead to qualitatively new insights into the relationship between classical and quantum physics. One such example is the Greenberger–Horne–Zeilinger (GHZ) theorem\(^\text{58}\). The essential difference between the Bell theorem and the GHZ theorem is that in the case of GHZ, the predicted outcomes by the quantum theory are deterministic. Thus, the assignment of a local hidden variable is more direct from a conceptual point of view. On the application side, the use of many qubits results in the possibility of universal quantum computation. Also, because physical calculators are never perfect and classical error-correcting schemes cannot be applied to quantum computing because of the no-cloning theorem, multiparticle entanglement in the form of GHZ states can be used for quantum error correction schemes\(^{59,60}\).

Although the original GHZ argument regarding three or more particles in 2D was formulated three decades ago\(^{50,52}\), its generalization to arbitrary local dimensions was only found in the past decade\(^{53–56}\). A key difference to the 2D qubit case is that the \(N\)-partite observables do not form a commuting set of observables and are not Hermitian. However, all observables have the multipartite and high-dimensional GHZ state as a common eigenstate, and thus still predict an outcome with certainty. Most interestingly, so far, no local realistic violation of the GHZ type could be constructed using local Hermitian operators. All attempts to generalize the GHZ argument use local unitary observables and thus have complex eigenvalues. This is in sharp contrast to the prevailing view in physics that physical observables must have real eigenvalues, as famously stated by Paul Dirac\(^\text{61}\).

This fact calls for a more general definition of what properties a physical observable should obey. A more general class of operators that fulfill the requirement of orthogonal eigenstates are normal operators\(^\text{62,63}\).

A further increase in system size in terms of number of particles and dimensionality leads to quantum chemistry and quantum biology. Accumulations of many atoms and their complex interaction are of interest, not only to understand why nature stores information in four-dimensional DNA but also to use and understand biological processes such as nitrogen fixation by the enzyme nitrogenase\(^\text{64}\), which produces ammonia in an energy-efficient way\(^{65}\). (Ammonia is important for synthetic fertilizer and is usually produced in the

| Fundamental insight | System size | Technological application |
|---------------------|-------------|--------------------------|
| Superposition and no-cloning | \(N=1 \mid d=2\) | QKD                      |
| Contextuality       | \(N=1 \mid d\geq3\) | Noise-resistant QKD       |
| Entanglement (Bell’s inequality) | \(N=2 \mid d\geq2\) | Device-independent QKD    |
| Teleportation and GHZ theorem | \(N\geq3 \mid d\geq2\) | Quantum computation       |
| New refutations of local realistic world views | \(N\geq3 \mid d\geq3\) | Complete teleportation of a single photon |

Fig. 1 | Curiosity-driven questions and technological applications of quantum mechanics, for varying system sizes.
Left: the curiosity-driven fundamental questions and theorems with an increasing number of involved particles (\(N\)), with different local dimensionalities (\(d\)). Right: these findings have inspired actual technological applications that will power the second quantum revolution\(^{63}\), that is, the application of quantum mechanical rules to measurement, information and communication technology. QKD, quantum key distribution; GHZ, Greenberger–Horne–Zeilinger.
energy-intensive Haber–Bosch process.) Understanding such chemical and biological processes is highly relevant and could potentially be accomplished using advanced quantum computers or simulators61.

**Photonic high-dimensional entanglement**

In this section, we discuss different physical and technical methods to generate and manipulate higher-dimensional entangled pairs of photons. We also survey the experimental ability to detect high-dimensional entanglement. We focus on local measurements as they are interesting from both a fundamental and an application point of view. Because photons are the ideal carriers of quantum information over long distances, we also present present-day approaches to distribute the quantum information stored in different DoFs.

Despite the important theoretical and experimental effort to deterministically generate single and multiple photons, a discussion of these methods would go beyond the scope of this Review Article. Moreover, many of the schemes to create entangled photon pairs in two or higher dimensions actually rely on the inherently probabilistic nature of the photon source. Hence, in the following, we focus on probabilistic photon-pair sources such as spontaneous parametric down-conversion (SPDC) and spontaneous four-wave mixing (SFWM).

SPDC uses a material with a $\chi^{(2)}$ nonlinearity to convert a single higher-energy pump photon into a pair of photons (a signal photon and an idler photon). The phase-matching conditions are given by

$$\omega_p = \omega_i + \omega_s,$$

$$k_p = k_i + k_s$$

(1)

with $\omega$ describing the photon frequency, $k$ the linear momentum, and $p$ refers to pump, $i$ refers to idler and $s$ refers to signal. The first equation in equation (1) stems from energy conservation (because the energy $E$ of a photon is given by $E = h\omega$) and the second equation from conservation of linear momentum. Satisfying the phase-matching condition results in the spontaneous (that is, completely probabilistic) down-conversion of a pump photon into two photons according to equation (1). In the low-pump-power regime, the quantum state of the down-converted photon pair can be written as a Taylor expansion

$$|\psi\rangle \approx |0,0\rangle + \alpha |1,1\rangle + \alpha^2 / 2 |2,2\rangle + \cdots$$

(2)

where the photon-number basis (Fock basis) is used and normalization has been omitted for the sake of legibility. (Taking only the vacuum and two-photon terms into account, the properly normalized equation (2) reads $|\psi\rangle = \sqrt{1-x} |0,0\rangle + \sqrt{x} |1,1\rangle$ for small $x$ and hence the vacuum term usually dominates.) Owing to the inherently probabilistic emission of photon pairs with probability amplitude $\alpha$, there is also the probability that two or more photon pairs are emitted simultaneously. Thus $\alpha$ is usually set to $\alpha \ll 1$. In the case of multiphoton experiments using SPDC, each crystal should only produce one pair within a given time frame, which would suggest that one should choose a small $\alpha$. However, doing so results in a low probability to achieve the outcome that $N$ SPDC processes create a photon-pair simultaneously — the probability of this scales with $\alpha^N$. Possible solutions are to use photon-number resolving detectors to detect and exclude unwanted multipair emissions or time-multiplexing of SPDC sources to reduce multipair emissions of a single source while enhancing the simultaneous emission of multiple sources.

In contrast, SFWM is based on the $\chi^{(3)}$ nonlinearity and the corresponding phase-matching conditions (using the same notation as for equation (1)) are

$$2\omega_p = \omega_i + \omega_s,$$

$$2k_p = k_i + k_s$$

(3)

In this case, there are two pump fields. Thus, in SFWM, two pump photons are converted into two output photons, which can also be described by equation (2).

Several specific advantages arise using either SPDC or SFWM for different implementations and applications, as we discuss in the following sections. A thorough review of single-photon sources, as well as single-photon detectors, can be found in Ref.64.

**Path degree of freedom**

Encoding quantum information in the path DoF is appealing because there exist arbitrary single-photon transformations in any dimension65, even in a highly symmetrical and loss-tolerable manner66. These schemes use only beam splitters and phase shifters and allow implementation of any single-photon qudit transformation [FIG. 2]. In addition, experimental realization is possible using bulk optical elements or integrated optics, such as silicon chips with extreme interferometric stability and high indistinguishability67–70 [FIG. 2c]. Conceptually, one method of creating high-dimensionally entangled photon pairs relies on the intrinsic momentum conservation within the SPDC process62. In this scheme, the intrinsic linear momentum conservation leads to the coherent emission of a single photon pair on a cone (FIG. 2a). The momentum conservation is responsible for the diametrically opposite positions on the emission cone of the two photons. Collecting these photon pairs with $d$ single-mode fibre pairs arranged evenly spaced on the emission cone leads to the existence of a $d$-dimensionally entangled quantum state of two photons in their respective fibres or paths72.

Instead of using single-mode fibres, the high-dimensional entanglement stored in the linear momentum can be directly harnessed using a deformable mirror device in combination with a single-photon detector. Doing so leads to pixel entanglement73. In this scheme, the basis transformation for certifying entanglement can be performed using a single lens only, because a lens effectively performs a Fourier transform and thus transforms between the position and momentum bases. Efficient detection methods using sparse-matrix techniques make it possible to certify a channel capacity of 8.4 bits per photon63.

A conceptually different method to create high-dimensionally entangled photon pairs is to utilize $d$ indistinguishable photon-pair sources74–77 [FIG. 2b],
The physical principle of this method is based on coherently pumping $d$ nonlinear crystals (NLCs), resulting in a coherent superposition of a single-photon pair emitted in one of the paths. The crucial ingredient is to have $d$ indistinguishable photon-pair sources in all DoFs except the path, where the quantum information is stored. Experimentally, this scheme can either be realized using bulk optical elements, integrated fibre optical elements, or on-chip techniques (Fig. 2c). Using bulk optical elements guarantees higher efficiencies ($>99.9\%$) due to special anti-reflection coatings, but this approach is difficult to stabilize and scale to very high dimensions.

Particularly interesting are the technical advances in recent years in on-chip photonics. This technology is a promising platform for quantum optics experiments that encode the quantum information in the path DoF. High-quality interferometers with extinction ratios up to $66\,\text{dB}$ reported in the literature $^{78}$ and scalability in the number of optical components allow a variety of applications. Probabilistic processes based on either SFWM $^{89}$ or SPDC $^{90}$ are used as photon-pair sources. For SPDC, lithium-niobate waveguides are used, for example. SFWM has the advantage that all materials used (such as silicon or Hydex) are CMOS compatible. A common source of photon pairs is the SFWM-based spiral waveguide source, in which the phase-matching conditions can be achieved by appropriate design of the spiral waveguides through anomalous dispersion.

Sources of this type have recently been used to certify $3\text{D}$ and $14\text{D}$ entanglement on-chip $^{82}$. In the latter, a total of 16 identical spiral waveguide sources based on SFWM with a total of more than 550 optical components were implemented on-chip. The observed fidelities range from $96\%$ for $d=4$ to $81\%$ for $d=12$. Typically, each of the 16 sources produces photon pairs at a rate of $2\,\text{kHz}$.

Another experiment recently demonstrated an optical general two-qubit quantum processor implemented on-chip $^{82}$. In it, two pre-entangled guquarts are used to probabilistically perform any two-qubit unitary operation, such as CNOT gates, between two qubits encoded in the path DoF. Also, quantum Hamiltonian learning algorithms $^{91}$ have been performed using higher-dimensionally encoded path qudits on-chip $^{94}$. In that work, a negatively charged nitrogen-vacancy centre was coupled to a trusted quantum simulator, in this case, the photonic silicon chip. This chip was then used to learn the Hamiltonian of the nitrogen-vacancy centre.

Distributing the classical information using the path DoF is conveniently possible using multicore fibres (MCFs). MCFs find applications in classical communication networks, specifically where multiplexing schemes for higher information capacities are necessary. However, transferring quantum information using MCFs is more involved, because the phase between different cores must be interferometrically stable to avoid dephasing $^{41}$.
Random dephasing results in incoherent and thus classical mixtures that yield non-entangled quantum states. However, recent results achieved 4D QKD using MCFs of up to 0.3 km in length\cite{86,87}. Also, high-dimensional entanglement has been transmitted through two MCFs recently\cite{88}. An interesting approach is to send pixel entanglement via a conventional graded-index multimode fibre. Using entanglement itself to measure the complex transmission matrix, the scrambling of the optical modes within the optical fibre can be compensated by adjusting the measurement bases accordingly. To date, 6D entanglement over a length of 2 m can be sent\cite{89}. These proof-of-principle demonstrations show the possibilities of using MCFs for high-dimensional quantum information to connect optical chips\cite{90}.

**Transverse spatial modes**

The transverse spatial modes of single photons can perfectly encode high-dimensional quantum information. Different mode families have been studied in the context of entanglement, including Laguerre–Gauss\cite{91}, Hermite–Gauss\cite{92}, Bessel–Gauss\cite{93} and Ince–Gauss\cite{94} modes. We discuss only the Laguerre–Gaussian (LG) modes\cite{95}, because these are relevant for photons with orbital angular momentum (OAM), the subject of most experimental work regarding high-dimensional entanglement\cite{96}. The OAM describes the transverse wavefront of photons. LG modes describe photons with OAM because the main feature of the LG modes is the existence of singularities within the transverse phase\cite{97,98}. The amount of OAM (in units of $\hbar$) corresponds to the direction and number of windings $\ell$ of the phase around these singularities. At the singularity, the phase is not defined, resulting in the typical doughnut-shaped intensity distributions\cite{99}. The reason why OAM is interesting for quantum experiments is twofold. First, OAM entanglement can readily be created in a single NLC, because the OAM is conserved within the SPDC process\cite{100} and thus directly yields high-dimensionally entangled photon pairs\cite{101}[Fig. 5a], except in specific experimental situations (type II non-colinear SPDC) in which the rotational symmetry of the Hamiltonian is broken and thus total OAM is not conserved\cite{102}. Second, there are several known experimentally feasible techniques for manipulating and measuring OAM states of single photons\cite{91,102,103}.

A quantum state of two photons generated in an NLC can be described according to $\sum_{\ell} c_{\ell} \langle \ell | \ell \rangle$, where the dimensionality is given by $d = 2(\ell_{\max} + 1)$. The exact distribution of the complex coefficients $c_{\ell}$ is called the spiral spectrum, and mainly depends on the length of the crystal, the beam waist of the pump laser and the collection beam waist\cite{105}. It is even possible to optimize the entanglement dimensionality using the phase-matching conditions\cite{106}. Using OAM-entangled photon pairs created...
in a single NLC, high-dimensional generalized Bell inequalities have been violated in up to 12 dimensions. LG modes are represented by two indices: \( \ell \) denotes the azimuthal phase of the topological charge in terms of OAM and \( p \) represents the radial quantum number. Including the complete LG modes — namely, \( \ell, p \)-modes — of the 2D transverse spatial field of the photons into the entangled two-photon quantum states can lead to >100D entanglement in the laboratory. Maximally entangled, that is, \(|\psi\rangle = 1/\sqrt{d} \sum_p |\ell_p\rangle\), states can be created either by using procrustean filtering techniques or by taking the natural spiral spectrum of the SPDC into account and counter-acting with a corresponding superposition of different OAM quanta in the pump beam.

In the past few years, a conceptually new method for creating OAM-entangled quantum states has been introduced, relying on indistinguishability and path identity. In analogy to the creation of path entanglement explained above, \( d \) NLCs were pumped coherently. However, here the NLCs are aligned in series and their respective paths are identically aligned, such that the resulting quantum state is in a coherent superposition of one photon pair being emitted in one of the \( d \) NLCs (FIG. 5b). To create an OAM-entangled quantum state, a spiral phase plate (SPP) is inserted after each NLC. An SPP is a device that adds \( m \) quanta of OAM to the incoming photons, thus realizing the operation \(|\ell\rangle \rightarrow |\ell + m\rangle\). Q-plates are ideal for this task, especially in the collinear regime. Depending on the relative pump powers and phases between the NLCs, an arbitrary \( d \)-dimensionally entangled two-photon quantum state is created. Currently, fidelity for 3D entanglement of approximately \( F \approx 90\% \) is achieved. Future efforts in integrating the NLCs directly with q-plates could pave the way to create tens of dimensions with even higher fidelities.

Manipulation of OAM in higher dimensions has proved difficult. Although projective measurements using phase plates or fork holograms in combination with single-mode fibres are well established and provide a powerful tool to detect high-dimensional entanglement in only two non-orthonormal bases, even seemingly simple transformations such as cyclic transformations were not known until recently and have only been discovered using the computer algorithm MELVIN. At the heart of these efforts lies a simple but elegant device that is capable of sorting OAM quanta according to their parity (even or odd). Based on a Mach–Zehnder interferometer with a Dove prism inserted that introduces an OAM-dependent phase, different parity OAM values can be sorted. Cascading the parity sorter leads to the capability of sorting arbitrarily many different OAM modes, thus enabling multi-outcome measurements. Interestingly, to construct \( d \)-level cyclic transformations only \( \log(d) \) parity sorters are necessary (FIG. 5c). Analogously, this concept has been used to sort different \( p \)-modes and thus yield access to the complete 2D transverse spatial wavefront of single photons. Parity sorters for OAM also make it possible to route high-dimensionally entangled states, and play a crucial role in creating genuinely high-dimensional and multiphoton entangled quantum states, as discussed below.

An alternative method to sort different OAM states is based on log-polar transforms. This method essentially converts the circularly varying phase pattern to a linear grating, resulting in the conversion of OAM to linear momentum, and thus different OAM modes appearing in different positions (paths) after propagation. Mode sorters for mode numbers up to 50 have been demonstrated, with next-mode overlap of about 2%.

In the past decade, a new technique based on multiplane light conversion has been developed and demonstrated. Using several consecutive phase planes with propagation in between allows unitary transformation of arbitrary 2D transverse light fields. With this approach, sorting of 210 modes defined in the LG modes into a spatial pattern of Gaussian spots has been demonstrated. In terms of efficiency, the device has a theoretical insertion loss of 2.5 dB and a measured insertion loss of 6 dB. The difference arises mainly from reflection inefficiencies of the spatial light modulator (SLM). In this demonstration, in total seven reflections on an SLM with 0.5 dB per reflection were used.) The average cross-talk per mode is measured to yield ~30 dB, resulting in a measured channel capacity of 6.25 bits per photon (the theoretical expectation is \( \log_2(210) = 7.71 \)). Multiplane light conversion is not limited to sorting LG modes but can also be applied to arbitrary unitary transformations, such as cycling operations or controlled operations on a single-photon level. This promising route towards complete control of qudits encoded in the LG modes showed a high process purity of 99% for 3D cyclic gates using three-phase planes at the SLM.

There are several demonstrations of distributing classical and quantum information using the OAM DoF. Free-space links allowed the transmission of classical information in high-speed terabit configurations and turbulent intracity links, underwater channels or over large distances, up to 143 km between two islands. Also, specifically designed optical fibres have been employed to transmit classical and quantum information with OAM. High-dimensional quantum key distribution (HD-QKD) using OAM has been demonstrated in turbulent environments such as intracity free-space links, fibre-based systems and even entanglement distribution.

In addition to all the properties mentioned above, OAM is also perfectly suited for fundamental tests in quantum mechanics regarding 2D entanglement incorporating a very high angular momenta quanta of up to 10.010. In these experiments, the question is whether a macroscopic bound in terms of the amount of electromagnetic action \( h \) exists. Experiments up to 10.010 show that there only seems to exist a technical, not a fundamental, limitation. Also, the appealing patterns formed by different OAM superpositions of entangled photons or remotely prepared quantum states have been imaged live.

**Discretized time and frequency modes**

Quantum information stored in time bins or the frequency DoF of single photons is perfectly suited for transmission over large distances using free-space links or optical fibres. The idea of using time bins
as the physical carrier of quantum information was first discussed theoretically in 1989, in terms of violating Bell inequalities. A series of experiments later demonstrated 2D time-bin entanglement. One possibility to create entanglement between two photons encoding quantum information in the time-bin domain is to utilize two indistinguishable laser pulses separated by a fixed time difference $\Delta$ (REF 162). Each laser

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**Entanglement creation**

- **a** Conservation of energy

  ![Diagram](image1)

  $|\psi\rangle_{ab} = \frac{1}{\sqrt{N}}(|\tau_0, \tau_0⟩ + |\tau_0, \tau_1⟩ + \cdots + |\tau_N, \tau_0⟩)$

- **b** Mode-locked pulses (time bin)

  ![Diagram](image2)

  $|\psi\rangle_{ab} = \int d\omega_1 d\omega_2 f(\omega_1, \omega_2) |\omega_1, \omega_2⟩_{ab}$

- **c** Microring cavity (frequency bin)

  ![Diagram](image3)

  $|\psi\rangle_{ab} = |\omega - \delta, \omega + \delta⟩ + |\omega - 2\delta, \omega + 2\delta⟩ + \cdots + |\omega - N\delta, \omega + N\delta⟩$ for arbitrary unitary transformations (frequency bin)

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**Manipulation**

- **d** Franson interferometers (time bin)

  ![Diagram](image4)

  - Phase modulators
  - $\tau_0$, $\tau_1$, $\tau_2$, $\cdots$
  - $\sum i |\tau_i⟩$

- **e** Ultrafast pulse shaping (time bin)

  ![Diagram](image5)

  - Input pulse
  - Output waveform

- **f** Arbitrary unitary transformations (frequency bin)

  ![Diagram](image6)

  - Phase modulator
  - Phase shaper
  - $\phi_0$, $\phi_1$, $\phi_2$, $\cdots$

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**Fig. 4 | Entanglement creation and manipulation concepts for time and frequency bin encoding.**

- **a** In principle, conservation of energy within the spontaneous parametric down-conversion process in a nonlinear crystal (NLC) can yield very high-dimensionally entangled two-photon quantum states $|\psi⟩_{ab}$. The joint spectral amplitude $f(\omega_1, \omega_2)$ with $\omega_1, \omega_2$ describing the frequency of photon $i$ determines the dimensionality. **b** An alternative concept relies on N indistinguishable mode-locked laser pulses. Each laser pulse creates probabilistically one photon pair at time $\tau_i$. Considering N such creation events yields a high-dimensionally entangled two-photon state $|\psi⟩_{ab}$. **c** A microring cavity can be exploited to create high-dimensionally entangled photon pairs from spontaneous four-wave mixing (SFWM). The microring cavity allows for an efficient SFWM process due to the high Q-value cavity environment. $\delta$ refers to the free spectral range (FSR) of the microring resonator. Additionally, the free spectral range of this cavity only allows for certain frequency modes to be hosted in this frequency comb (graph). **d** The concept introduced by James Franson (REF 157) to coherently manipulate time-bin quantum information relies on several unbalanced Mach–Zehnder-type interferometers. In this configuration, it allows production of an equal superposition of all time bins at the centre outgoing bin. **e** A 4-f non-dispersive pulse shaper with a programmable spatial light modulator can be utilized to arbitrarily transform the quantum information stored in time bins. **f** A new method for arbitrary unitary transformations of frequency bins. An electro-optic modulator (EOM) coherently populates many neighbouring frequency bins. A consecutive pulse shaper operating in the complete output space of the EOM introduces arbitrary phase shifts to the separate frequency bins. A final EOM coherently combines all frequency modes and interference between them yields the desired transformation. The phase modulators are driven by phase patterns given by the functions of time $\phi_i(t)$ and $\phi_j(t)$. The pulse shaper applies certain phase shifts $\phi_i$ to each bin. The graph in panel **c** reprinted with permission from REF 158, AAAS. Panel **e** adapted with permission from REF 159, OSA. Panel **f** reprinted from REF 160, Springer Nature Limited.
pulse can create a photon pair within the NLC [FIG. 4c]. If there is in principle no information available about which of the two pulses the photon pair \( ab \) has been created in, the resulting state can be written as a superposition of the two creation times \( \tau_1, \tau_2 \) with emission amplitudes \( a, b \), such that the quantum state in the Fock basis can be written as \( |0, 0\rangle \alpha + |1, 1\rangle \alpha + \alpha^2/2 |2, 2\rangle \alpha + \cdots \) and numbers within the ket vectors refer to the photon occupation number of the respective path mode \( a \) or \( b \). The probability of emitting \( n \) pairs simultaneously is \( \alpha^n \), because the \( n \) events are independent and thus the probability is given by the product of the individual pair probabilities. The probability of emitting more than one pair simultaneously over the time \( T_{\text{coh}} \) is kept well below 0.1 by keeping \( \alpha \) small enough. In principle, this scheme can be scaled up to \( d \) dimensions by employing a sequence of \( d \) indistinguishable and coherent mode-locked pump pulses (FIG. 4b), as has been demonstrated in REF.165 for 11 dimensions.

An alternative method to create high-dimensional time-bin photon pairs relies on the intrinsic energy conservation in the SPDC process. By choosing the nonlinear optical material, the length of the NLC, pump-pulse duration, wavelength and bandwidth of the pump, signal and idler beams, the joint spectral amplitude yields the joint temporal amplitude168. The conjugate variable of frequency is time, and Fourier transformation of the joint spectral amplitude yields the joint temporal amplitude. Thus, by appropriately choosing the parameters mentioned above, a high-dimensional entangled photon pair can be generated. In principle, it is possible to generate a very large number of modes within a given time frame. However, as discussed below, in this case the restrictions are imposed by the detection system rather than in the creation part.

In general, the detection of entangled quantum states requires the ability to measure at least two different bases locally. The most common method to perform such a basis transformation for time bins is to use an unbalanced interferometer162. The original proposal was inherently vulnerable to loss (50%) but this can be overcome by using active-switching techniques169. An unbalanced interferometer creates a superposition between two time bins \( i \) and \( i + \Delta \) with an arbitrary phase \( \phi \) as \( \left| \tau_i \right> + e^{i\phi} \left| \tau_{i+\Delta} \right> \) (FIG. 4d). To get a feeling for how large \( \Delta \) is in this unbalanced Franson interferometer, consider a mode-locker laser with a repetition rate of 1 GHz. This corresponds to roughly 30 cm of propagation distance, assuming that light travels at the same speed as in vacuum. Hence, for 2D superpositions of two time bins created with a 1 GHz repetition laser, a physical propagation difference of 30 cm is necessary. For a high-dimensional quantum state (such as a 10D state), it is not only necessary to connect ten such interferometers but also the necessary distances increase to several metres. To circumvent this requirement, a theoretical dimensionality bound has been developed that requires only nearest- and next-nearest-neighbour time-bin (that is, superpositions of \( j \in \{1, 2\} |i\rangle + |i + j\rangle \) superposition measurements. This technique is used to bound the entanglement dimensionality being created from below170. Using this approach, an 18-dimensionally entangled quantum state has been observed.

Another approach to performing transformations of the quantum information stored in the time-bin domain is inspired by techniques developed in the ultrafast optical pulse shaping community171–173. The basic idea is to utilize a 4f pulse shaper with a programmable SLM inserted (FIG. 4e). This technique is used to shape the temporal distribution of entangled photon pairs174,171.

To overcome timing constraints from detectors and counting electronics, and thus enabling ultrafast timing detection in the picosecond regime, nonlinear optical approaches based on coherent sum-frequency generation can be employed175,176. This technique has allowed the direct characterization of the spectral and temporal properties of energy–time-entangled photon pairs on the subpicosecond timescale177.

Recent developments in upcoming on-chip schemes enable the creation of high-dimensionally entangled photon pairs in the form of discrete frequency bins178–179. This new approach allows for versatile and high-quality sources of two-photon and multiphoton quantum states in higher dimensions180. The most commonly used techniques are based on integrated Kerr frequency combs181. Owing to energy conservation in the SFWM process inside the microring cavity, perfectly anti-correlated frequency-bin entangled qudits are created (FIG. 4f). The number of dimensions achievable is determined by two factors. First, the phase-matching range of the SFWM process sets the overall bound for the correlated frequency range. For currently used materials (such as Hydex182 or silicon nitride-based materials183), a bandwidth of roughly 100 nm at 1,550 nm telecom wavelength can be achieved. In this process, the discrete frequency bins are directly created within the microring cavity. Their spacing \( \delta \) is given by the free spectral range (FSR) of the microring resonator. Typical values of the FSR are 200 GHz with a linewidth of about 800 MHz (REF.179). These values allow for entanglement of up to several tens of frequency bins (FIG. 4f). Most importantly, the large FSR also enables the use of off-the-shelf telecom equipment for accessing and manipulating (individually or collectively) the quantum information encoded in frequency bins.

This promising source technology, in combination with advances in creating quantum gates for discrete frequency bins such as generalized splitters in higher dimensions180, makes frequency-encoded qudits a promising platform. A pulse shaper sandwiched between two electro-optical modulators (EOMs) is used as a frequency tritter, that is, a generalized splitter for three dimensions (FIG. 4f). The first EOM scatters any input mode coherently over its neighbouring modes. However, doing so introduces unwanted loss as the EOM scatters some of the modes outside the desired state space. To overcome this limitation, a pulse shaper operating in all possible output modes of the EOM can apply arbitrary phases to each frequency bin. A final EOM then coherently recombines all modes. This procedure allows for any arbitrary unitary transformation. An important example is a frequency tritter or quantum-Fourier
transform [FIG. 4b], which takes any of the three frequency bins as input and creates an equal and coherent output distribution of all three frequency bins, including phases of $\exp(\frac{2\pi}{3} im)$ for unitarity. The high-fidelity operation of up to 99.9% with standard telecom equipment is promising for technological applications. A remaining problem is the relatively large coupling loss of 3–4 dB per element. This loss might be overcome using on-chip techniques.

An equivalent device to the even/odd sorter for OAM also exists for frequency bins. It is called an optical frequency interleaver with two input and two output channels. This feature enables it to sort frequency bins with a spacing $\Delta$ into even frequencies with respect to some central frequency $f_0$ and odd frequencies with respect to some other central frequency $f_0 + 2m\Delta$ ($m \in \mathbb{N}^*$). This system, which has characteristic numbers of positive states, or also integer numbers $\mathbb{Z}^+$, is called an optical frequency sorter (ODS). Because the frequency interleaver is a multi-input/multi-output device, it can readily create special types of quantum gates using concepts originally found for OAM, ranging from high-dimensional quantum gates to genuine high-dimensional and multidimensional entanglement, even to high-dimensional quantum gates.

The time-bin DoF is ideally suited for HD-QKD. One possibility to implement an HD-QKD protocol is to use visibility measurements in Franson-type interferometers. Another experiment achieved a 2.7 Mbit s$^{-1}$ secure key rate over a transmission distance of 20 km in an optical fibre. In this experiment, the security of the HD-QKD protocol is ensured by measurements of the Franson visibility. The security analysis includes all Gaussian attacks (such as beamsplitter attacks). However, intercept-and-resend attacks are not considered. Unconditional secure HD-QKD schemes have been proposed using Franson interferometry in combination with conjugate Franson interferometry, for example. Dispersion cancellation systems are commercially available. Hence the long-distance distribution of discrete frequency bins via standard optical fibres up to 24 km is possible and has been demonstrated.

A recent study investigated the noise resistance of higher-dimensional entangled photon pairs. Using time bins, entanglement was certified with a noise fraction of 92% for time-bin entanglement in 80 dimensions. In the same study, a parallel experiment using OAM encoding achieved a noise robustness of 63% in a 7D space.

**Combining and converting different DoFs**

A powerful technique to achieve high-dimensional quantum states is to use several DoFs of single photons simultaneously. One possibility is to use the polarization ($p$), OAM ($\rho$) and time–frequency ($t$–$f$) DoFs entangled in 2D and 3D, yielding

$$\rho_{t} \times \rho_{p} \times \rho_{t-f} = (|H, H\rangle + |V, V\rangle) \otimes \frac{|0, 0\rangle + |1, -1\rangle + |1, 1\rangle}{\text{polarization}} \otimes \frac{|k, s\rangle + |l, l\rangle}{\text{OAM, time}}$$

with $H$ denoting horizontal polarization, $V$ vertical polarization, $s$ describing short creation time and $l$ describing length of creation time of the photon pair, and numbers within the ket vectors referring to quanta of OAM in units of $\hbar$. This state can be rewritten in terms of the 12 orthogonal states $|H, 0, s\rangle$, $|H, 0, l\rangle, \ldots, |V, 1, l\rangle$ and results in a $(2 \times 3 \times 2 = 12)$-dimensionally entangled quantum state in three DoFs simultaneously. Such states can be created by employing techniques described above or in Ref. 192. The advantage is that for hybrid DoFs there exist deterministic CNOT gates, for example, which is important for applications such as superdense coding or superdense quantum teleportation. In addition, deterministic purification of entanglement and a complete Bell-state analysis is possible using hyperentanglement. Even measurement-device-independent quantum steering and random number generation beyond qubits has recently been demonstrated using polarization and path DoFs. Experiments, combining OAM modes with $p$-modes have shown the potential for deterministic controlled gates on a single photon. In addition, using the time–frequency DoFs simultaneously demonstrated potential applications for one-way quantum processing using $d$-level cluster states.

Transmission of hybrid-entangled polarization–OAM (2 × 2 dimensional) entangled quantum states has been demonstrated using a 5-m-long air-core fibre. A recent study investigated the noise resistance of higher-dimensional entangled photon pairs. Using time bins, entanglement was certified with a noise fraction of 92% for time-bin entanglement in 80 dimensions. In the same study, a parallel experiment using OAM encoding achieved a noise robustness of 63% in a 7D space.

**High-dimension multiphoton entanglement**

**Classes of entanglement**

Entanglement with multiple photons ($n > 2$) in higher dimensions ($d > 2$) can have complex structures even in the case of pure states. The Schmidt rank vector (SRV) enables classification of these structures and is defined as the collection of Schmidt ranks of all bipartitions such that

$$\text{SRV} = \{r_1, r_2, \ldots, r_k\}$$

holds, with $r_k$ denoting the rank of the reduced density matrix $\rho_k = \text{Tr}_k(|\psi\rangle \langle \psi|)$ and $k = 2^n - 1$ represents the number of possible bipartitions of a $N$-partite quantum system. Most of the experiments involving multiple photons in a high-dimensional DoF can be distinguished into three classes (FIG. 5).

- The first class (I) is entanglement involving many photons in a high-dimensional DoF. Such states involve more than two photons, but can be biseparable. An example is $|\psi\rangle = |\phi_A, \rho\rangle \otimes |\phi_C, l\rangle$. Their SRV has some entries being 1.
* The second class (II) is genuine multiphotonic entanglement in a high-dimensional DoF. These states are not separable, but their entanglement is not high-dimensional (even though encoded in a high-dimensional space). The state is not separable, and some or all parts are two-dimensionally entangled. An example would be a four-photon 2D GHZ state in the path DoF.

* The third class (III) is genuine multiphotonic high-dimensional entanglement, in which all photons are entangled in more than two dimensions. Examples of experimental implementations of the three classes are given in Table 1.

**Path degree of freedom.** Many multiphoton experiments in the path DoF have been motivated by Scott Aaronson and Alex Arkhipov’s boson sampling proposal, which shows that linear optics can perform transformations that are difficult to simulate on a classical computer. The purpose of these experiments has not been to produce well-defined entangled states, and they did not measure the full state but investigated the probability distribution between different modes. Because these experiments do not demonstrate coherent superpositions between different output distributions, they do not verify quantum entanglement of the final state. Therefore, although they are genuine multiphotonic experiments encoded in a high-dimensional DoF, they are not part of our entanglement classification.

Recently, a series of experiments demonstrated multiphoton capabilities of on-chip technologies using the path DoF. These are the first genuine multiphoton entangled states that have been generated and measured on programmable chips. Different types of graph state were demonstrated, among them the four-photon star graph $S_4$, which is locally equivalent to a 2D GHZ state. This is an example of a class II entangled state in the path DoF, and the first experimental demonstration of such a state. The photon pairs (with signal wavelength of 1.539 nm and idler wavelength of 1.549 nm) were created directly on the chip, which substantially improves the stability compared with bulk optical approaches, necessary for such complex experiments. The key component for producing genuine multiphoton entanglement is a reconfigurable, post-selected entangling gate that exploits Hong–Ou–Mandel interference to remove the ‘which-crystal information’ between the pairs from different origins. The chip contains four arbitrarily tunable single-qubit projections, allowing for the measurement of arbitrary multiphoton correlations in coincidences. The fidelity was 78% and count rates were about 5.7 mHz (20 per hour). The count rates are similar to those of the first photonic multiphoton experiments, and technical improvements could push four-photon rates from the mHz to the kHz regime. A main future objective is the reduction of photon loss, currently in the region of 19.3 dB for the presented device. On the basis of published values of record component efficiencies, an estimated four-photon count-rate improvement is of a factor of five million.

**Spatial modes of photons.** The first multiphoton entangled state with spatial modes of light was created in 2016 (REF.), and was a state in class II. The state was created using double-pair emissions of an SPDC crystal, which were then probabilistically split using three beamsplitters. The resulting state is a Dicke state in the form

$$|\Psi\rangle = C \left[ |0, 0, 1, 1\rangle + |0, 1, 0, 1\rangle + |1, 0, 0, 1\rangle + \frac{1}{\sqrt{6}} (|0, 1, 1, 0\rangle + |1, 0, 1, 0\rangle + |1, 1, 0, 0\rangle) \right]$$

where $|0\rangle$ and $|1\rangle$ stand for two different OAM modes. A fidelity of 62% with a count rate of about 0.2 Hz was attained.
Later in the same year, a different group demonstrated a three-photon entangled state, \(|\psi\rangle = \frac{1}{\sqrt{2}} (|0,0,0\rangle + |1,1,1\rangle + |2,2,2\rangle)\), in which part of the state was entangled in three dimensions\(^{2,9}\). Although not fully high-dimensionally entangled (and thus still a class II entangled state), this experiment was the first demonstration of a state in which part of a multiphoton entanglement is higher dimensional. The fidelity of the state was 80.1\% with a count rate of about 15 mHz.

In 2018, the first genuine high-dimensional multiparticle entanglement was created\(^{13}\), in form of a 3D GHZ state

\[
|\psi\rangle = \frac{1}{\sqrt{3}} (|0,0,0\rangle + |1,1,1\rangle + |2,2,2\rangle) \tag{7}
\]

The experimental setup, which was discovered using MELVIN\(^{11}\), consists of two sources of SPDC crystals that produce 3D photon pairs. The exploitation of a multiport that coherently manipulates several photons simultaneously in higher dimensions makes it possible to remove all cross-correlation terms. This experiment demonstrated for the first time class III entanglement. The state fidelity was 75.2\% with a count rate of 1.2 mHz (roughly 4 counts per hour).

It is important to note that all concepts are translatable between the different DoFs. It means, for example, that the multiport could be directly employed for path or time-bin encoding of high-dimensional quantum information.

**Discretized time and frequency modes.** In addition to efforts in boson sampling with time encoding\(^{15}\), only two experiments have demonstrated entanglement with more than two photons with discretized time bins\(^{17}\). The first made use of a quantum frequency comb, containing a large number of discrete, equally spaced frequency lines (about 100 bins within 100 nm) distributed symmetrically around the pump wavelength of 1,550 nm. Each symmetrical pair of frequency lines can be occupied by photon pairs. The pairs of bins were used for creating time-bin entanglement in the form

\[
|\phi\rangle = \frac{1}{\sqrt{N}} (|S_{0},S_{0},\ldots \rangle + e^{i\phi} |L_{0},L_{0},\ldots \rangle),
\]

for the nth frequency pair symmetrically arranged around the pump wavelength (S denotes short, L denotes long, s denotes signal and i idler).

It was then demonstrated that the quantum frequency comb allows for the occupation of two pairs at the same time in a coherent way. Specifically, the state

\[
|\Psi\rangle = |w_{1}\rangle \otimes |w_{2}\rangle
\]

\[= \frac{1}{2} \left( |S_{0},S_{0},S_{0},S_{0}\rangle + e^{i\phi} |L_{0},L_{0},L_{0},L_{0}\rangle \right) + e^{i\theta} |F_{1},F_{1},F_{1},F_{1}\rangle \right)
\]

was generated. Changing the phase \(\phi\) showed that the created state was indeed a coherent four-photon state. This state falls into class I, because it is separable.

Recent advances have made it possible to manipulate multiple degrees of freedom in a single photon. This has opened up new possibilities for quantum information processing, including the creation of high-fidelity entangled states with more than two photons. The ability to manipulate multiple degrees of freedom simultaneously in higher dimensions makes it possible to create states that are more difficult to prepare and verify experimentally. This has important implications for quantum computing and quantum communication, as it allows for the creation of more complex entangled states that can be used to perform more powerful quantum operations.
leads to an 18-qubit entangled GHZ state. The fidelity is measured to be 70.8% and the sixfold count rate is 55 mHz (about 200 counts per hour).

With the techniques described above, it is conceivable that this technique could be extended to time bins, which would allow for 24 qubits. Using a 12-photon entanglement source that was reported in 2018 (Ref. 211), 48-qubit entangled GHZ states seem feasible with current technology.

**Teleportation in high dimensions**

Quantum teleportation, the disembodied transmission of unknown quantum states, is one of the most fascinating processes allowed by quantum mechanics. Discovered in 1993 as a conceptual curiosity222, it has become a cornerstone in various quantum applications including quantum computation and long-distance QKD networks. To teleport a system from A to B, one has to share an entangled photon pair between the two locations222. A Bell-state measurement then projects the two particles — the system to be teleported and one of the entangled photon pair — at A into a joint state, thus removing their identities. The classical information of the joint measurement outcome is then sent to B, where an outcome-dependent local transformation recreates the initial quantum state.

![Concept for performing 3D teleportation](image)

**Applications in entanglement swapping**

Extending the teleportation scheme to a situation in which the teleported photon is itself entangled with another photon leads to entanglement swapping234. The intriguing fact in this scenario is that the two photons that become entangled never interacted before nor ever shared a common past. Entanglement swapping has

The first experimental demonstration of quantum teleportation transmitted the polarization information of a single photon, which resembles a two-state quantum system225. Stretching the idea to larger systems, researchers found ways to teleport the quantum information of multiple particles simultaneously226 and multiple properties of a single particle226.

The final obstacle, quantum teleportation of high-dimensional systems, has turned out to be conceptually more difficult. The major challenge in teleporting high-dimensional photonic quantum states was already identified by John Calsamiglia in 2002 (Ref. 227): high-dimensional Bell-state measurements, with linear optical components, require additional ancillary particles. The key insight is that with linear optics only, it is impossible to distinguish a single Bell state unambiguously from the other $d^2$ states, except for $d=2$. Several theoretical approaches have been developed to overcome Calsamiglia’s no-go theorem224–226. Additional challenges come from the fact that the teleportation fidelity needs to be $F \geq 2/3$ to demonstrate genuine 3D teleportation. Fidelities below 50% can be achieved with classical techniques, and fidelities between 50% and 66.6% can be achieved using qubit systems.

In 2019, two experiments were reported that demonstrate teleportation using 3D path encoding of a photon227. Reference 27 reports experimental demonstration of a 3D Bell-state measurement that exploits the quantum-Fourier transform and can be generalized to arbitrary dimensions, as depicted in Fig. 6. The approach is optimal in terms of required additional photons. The experiment required four photons: the teleporte, a three-dimensionally entangled photon pair and an ancillary photon to overcome Calsamiglia’s no-go theorem. The count rates were 110 mHz (about 400 counts per hour) and the teleportation fidelity was 75%, well beyond both the classical and qubit bound.

The experiment reported in Ref. 224 demonstrated teleportation using a Bell-state measurement that requires an entangled ancillary pair of photons — making the demonstration a six-photon experiment. The count rate was 2 mHz (about 10 counts per hour) and the teleportation fidelity was 63.8%. This value is well beyond the classical bound for teleportation.

Both experiments demonstrate high-quality, long-term stability of their (not integrated) experiments. The authors of Ref. 223 report that their interferometers retain remarkable interference visibilities beyond 98% for 48 h. The concepts and technologies can be transferred to and combined with technology in other high-dimensional DoFs, which would enable the deployment of these techniques over large distances (as has been achieved for qubit systems219), and follow the dream of teleporting the entire quantum information of a quantum system.
become an important fundamental concept, with applications such as overcoming long distances in quantum networks\(^{14,23-26}\) or in fundamental experiments regarding entanglement\(^{14,27,28}\). Generalizing this concept to more complex entanglement structures in higher dimensions has not yet been fully achieved, but some important intermediate results have been reported.

In 2017, the first entanglement swapping experiment with a high-dimensional DoF was reported\(^{28}\). In it, a Bell-state projection was performed onto a large number of 2D subspaces. Conditioned on twofold detections, the resulting state is an incoherent superposition of two-dimensionally entangled states, in the form

\[
\rho = c_1 \left| \psi_{-1,-1} \right\rangle \left\langle \psi_{-1,-1} \right| + c_2 \left| \psi_{-2,2} \right\rangle \left\langle \psi_{-2,2} \right| + c_3 \left| \psi_{-2,-2} \right\rangle \left\langle \psi_{-2,-2} \right| + c_4 \left| \psi_{-1,-1} \right\rangle \left\langle \psi_{-1,-1} \right| + c_5 \left| \psi_{-1,-2} \right\rangle \left\langle \psi_{-1,-2} \right| + c_6 \left| \psi_{-2,-1} \right\rangle \left\langle \psi_{-2,-1} \right| + c_7 \left| \psi_{-2,-2} \right\rangle \left\langle \psi_{-2,-2} \right| + c_8 \left| \psi_{-1,-2} \right\rangle \left\langle \psi_{-1,-2} \right|\]

(12)

where \(\left| \psi_{m,n} \right\rangle = 1 / \sqrt{2} \left( |m,n\rangle - |n,m\rangle \right)\) and \(m\) and \(n\) refer to different OAM states and \(c_i\) denotes the respective probability to observe a certain state. Average fidelities of the two-dimensionally entangled swapped subspaces were \(F = 80\%\) (background subtracted; raw, \(F = 57\%\)) for the six 2-dimensionally entangled systems. Notably, the system is not high-dimensionally entangled, because to achieve this one would need a 3D Bell-state measurement.

An interesting application of the method lies in ghost imaging\(^{29}\). In conventional ghost imaging\(^{30}\), the object never interacts with the photon used to image it, but only with its correlated partner photon. In the demonstrated scheme, the photon interacting with the object does not even share a common past with the photon used for imaging. Thus, it is conceptually interesting to consider whether quantum or classical correlations are required for this task.

**Future directions**

The proof-of-principle experiments detailed in this Review Article define the current technological and conceptual state of the art. Many essential challenges are awaiting in the next few years, to enhance conceptual understanding and practical applicability of high-dimensional multiphotonic entanglement.

On the technological side, highly efficient or deterministic single-photon\(^{21}3\) or photon-pair sources are necessary to increase count rates of multiphoton experiments\(^{24}\). The development of near-unity photon detectors with small timing jitter will be important for scaling up time-bin entangled states. The use of efficient multi-outcome detection schemes (for instance, with detector arrays) will be necessary to harness the high-dimensional information stored in single photons. Scaling integrated optics to more modes, and in particular, multiphoton generation on-chip, will be essential for path-encoding schemes\(^{214}\). The access to complex entangled states gives rise to interesting properties, such as absolutely maximally entangled quantum states\(^{214,215}\), and their peculiar features could be investigated in laboratories. A critical question is how to control and manipulate entangled multiphotonic states experimentally. It remains an open question whether some of the unconventional methods that have been demonstrated successfully for single qudits\(^{213,214,216,217}\) will still be feasible when many particles are concerned.

In basic research on high-dimensional many-body entanglement, several questions remain unanswered. For example, the generalization of high-dimensional multiphotonic all-versus-nothing violations of local realism is a way to extend our understanding of the severe difference between our classical worldview and predictions of quantum mechanics. Of particular importance are generalizations of the GHZ argument\(^{31-35,248}\) and its application to asymmetrically entangled states that appear only when both the number of particles and dimension is beyond two\(^{218}\). This application involves the understanding of whether non-Hermitian measurements are necessary to find the most extensive violations, as has appeared to be the case in recent work. The questions of what are the most efficient protocols for high-dimensional quantum teleportation, and how multiple (high-dimensional) DoFs can be experimentally teleported, remain open. Answering the latter will be essential for the philosophically appealing goal of teleporting the entire quantum information of a single photon.

Of course, usually the most exciting advances are ‘unknown unknowns’, thus cannot be predicted. In light of the enormous progress in the experimental capabilities of high-dimensional multipartite quantum entanglement in just the past 5 years, we would like to encourage and invite theoretical and experimental quantum scientists to explore and exploit the hidden potential of complex, high-dimensional, many-body quantum entanglement.

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18. Lee, S.-W. & Jeong, H. Near-deterministic quantum teleportation and resource-efficient quantum computation using linear optics and hybrid qubits. Phys. Rev. A 87, 022326 (2013).
19. Amselem, U. & Nielsen, J. S., Van Loock, P. & Furusawa, A. Hybrid discrete- and continuous-variable quantum information. Nat. Phys. 11, 713–719 (2015).
20. Briegel, H. J. & Zeilinger, A. Quantum entanglement. Rev. Mod. Phys. 84, 423–4251 (2002).
21. Horodecki, R., Horodecki, P., Horodecki, M. & Horodecki, K. Quantum entanglement. Rev. Mod. Phys. 81, 865 (2009).
22. Gühne, O. & Tóth, G. Entanglement detection. Phys. Rep. 474, 1–75 (2009).
23. Plenio, M. B. & Disney, S. Quantum Information and Coherence 175–209 (Springer, 2014).
24. Fris, N., Vitagliano, G., Maik, M. & Huber, M. Entanglement and quantum information from theory to experiment. Nat. Rev. Phys. 1, 72–87 (2018).
25. Einstein, A., Podolsky, B. & Rosen, N. Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 777 (1935).
26. Schrödinger, E. Die gegenteilige Situation in der Quantenmechanik. Naturwissenschaften 25, 825–828 (1935).
27. Bohm, D. Quantum Theory (Prentice Hall, 1951).
28. Feynman, R. P., Leighton, R. B. & Sands, M. The Feynman Lectures on Physics (Addison-Wesley, 1965).
29. Kaszlikowski, D., Gwak, S., Kochen, S. & Specker, E. P. in Quantum Information and Computation. LNCS 18, 24–25 (1999).
30. Shalm, L. K. et al. Strong loophole-free test of local realism with quantum systems described by nonorthogonal states. Phys. Rev. Lett. 113, 103601 (2014).
31. Lu, D. L. et al. Three-dimensional entanglement on a silicon chip. NPJ Quantum Inf. 6, 30 (2020).
32. Xiang, Q. et al. Large-scale silicon quantum photonic systems implementing arbitrary two-qubit processing. Nat. Photon. 12, 534 (2018).
33. Wiebe, N., Granade, C., Ferrie, C. & Cory, D. G. Hamiltonian learning and certifying quantum resources. Phys. Rev. Lett. 104, 190401 (2010).
34. Wang, J. et al. Experimental quantum hamiltonian learning. Nat. Phys. 13, 551 (2017).
35. Xavier, G. B. & Lima, C. G. Quantum information processing with space–division multiplexing fibers. Commun. Phys. 1, 1–11 (2020).
36. Caves, C. R. et al. High-dimensional decoy-state quantum key distribution over multicore telecommunication fibers. Phys. Rev. A 96, 022317 (2017).
37. Ding, Y. et al. High-dimensional quantum key distribution based on multicore fiber using silicon photonic integrated circuits. npj Quantum Inf. 5, 25 (2019).
38. Lee, H.-J., Choi, S.-K. & Park, H. S. Experimental demonstration of four-dimensional spatial quantum entanglement between multi-core optical fibers. Optics Lett. 43, 40–42 (2018).
39. Valencia, N. H., Go, S., McCutcheon, W., Defienne, H. & Maik, M. Unscrambling entanglement through a complex medium. Preprint at https://arxiv.org/abs/1910.04490 (2019).
40. Ding, Y. et al. Demonstration of chip-to-chip quantum teleportation. Nat. Commun. 10, 3278 (2019).
41. Lee, J.-H., Choi, S.-K. & Park, H. S. Demonstration of four-dimensional spatial quantum entanglement between multi-core optical fibers. Optics Lett. 43, 40–42 (2018).
42. Howland, G. A. & Howell, J. C. Efficient high-dimensional quantum teleportation. Phys. Rev. A 85, 052329 (2012).
43. Page, D. N. & Turchette, R. C. Quantum imperfections in a programmable nanophotonic processor. Nat. Photon. 11, 447 (2017).
44. Sharping, J. E. et al. Generation of correlated photons in nanoscale silicon waveguides. Opt. Express 14, 12358–12359 (2006).
45. Jin, H. et al. On-chip generation and manipulation of entangled photons based on reconfigurable lithium-niobate waveguide circuits. Phys. Rev. Lett. 113, 103601 (2014).
46. Lu, D. L. et al. Three-dimensional entanglement on a silicon chip. NPJ Quantum Inf. 6, 30 (2020).
47. Xiang, Q. et al. Large-scale silicon quantum photonic systems implementing arbitrary two-qubit processing. Nat. Photon. 12, 534 (2018).
48. Wiebe, N., Granade, C., Ferrie, C. & Cory, D. G. Hamiltonian learning and certifying quantum resources. Phys. Rev. Lett. 104, 190401 (2010).
49. Wang, J. et al. Experimental quantum hamiltonian learning. Nat. Phys. 13, 551 (2017).
50. Xavier, G. B. & Lima, C. G. Quantum information processing with space–division multiplexing fibers. Commun. Phys. 1, 1–11 (2020).
51. Caves, C. R. et al. High-dimensional quantum key distribution based on multicore fiber using silicon photonic integrated circuits. npj Quantum Inf. 5, 25 (2019).
52. Lee, H.-J., Choi, S.-K. & Park, H. S. Experimental demonstration of four-dimensional spatial quantum entanglement between multi-core optical fibers. Optics Lett. 43, 40–42 (2018).
53. Valencia, N. H., Go, S., McCutcheon, W., Defienne, H. & Maik, M. Unscrambling entanglement through a complex medium. Preprint at https://arxiv.org/abs/1910.04490 (2019).
54. Ding, Y. et al. Demonstration of chip-to-chip quantum teleportation. Nat. Commun. 10, 3278 (2019).
55. Lee, J.-H., Choi, S.-K. & Park, H. S. Demonstration of four-dimensional spatial quantum entanglement between multi-core optical fibers. Optics Lett. 43, 40–42 (2018).
56. Howland, G. A. & Howell, J. C. Efficient high-dimensional quantum teleportation. Phys. Rev. A 85, 052329 (2012).
57. Page, D. N. & Turchette, R. C. Quantum imperfections in a programmable nanophotonic processor. Nat. Photon. 11, 447 (2017).
58. Sharping, J. E. et al. Generation of correlated photons in nanoscale silicon waveguides. Opt. Express 14, 12358–12359 (2006).
59. Jin, H. et al. On-chip generation and manipulation of entangled photons based on reconfigurable lithium-niobate waveguide circuits. Phys. Rev. Lett. 113, 103601 (2014).
60. Lu, D. L. et al. Three-dimensional entanglement on a silicon chip. NPJ Quantum Inf. 6, 30 (2020).
61. Xiang, Q. et al. Large-scale silicon quantum photonic systems implementing arbitrary two-qubit processing. Nat. Photon. 12, 534 (2018).
62. Wiebe, N., Granade, C., Ferrie, C. & Cory, D. G. Hamiltonian learning and certifying quantum resources. Phys. Rev. Lett. 104, 190401 (2010).
63. Wang, J. et al. Experimental quantum hamiltonian learning. Nat. Phys. 13, 551 (2017).
64. Xavier, G. B. & Lima, C. G. Quantum information processing with space–division multiplexing fibers. Commun. Phys. 1, 1–11 (2020).
65. Caves, C. R. et al. High-dimensional quantum key distribution based on multicore fiber using silicon photonic integrated circuits. npj Quantum Inf. 5, 25 (2019).
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