Inhomogeneous longitudinal electric field-induced anomalous Hall conductivity in a ferromagnetic two-dimensional electron gas

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It is known that the anomalous Hall conductivity (AHC) in a disordered two dimensional electron system with Rashba spin-orbit interaction and finite ferromagnetic spin-exchange energy is zero in the metallic weak-scattering regime because of the exact cancellation of the bare-bubble contribution by the vertex correction. We study the effect of inhomogeneous longitudinal electric field on the AHC in such a system. We predict that AHC increases from zero (at zero wavenumber), forms a peak, and then decreases as the wavenumber for the variation of electric field increases. The peak-value of AHC is as high as the bare-bubble contribution. We find that the wave number, $q$, at which the peaks occur is the inverse of the geometric mean of the mean free path of an electron and the spin-exchange length scale. Although the Rashba energy is responsible for the peak-value of AHC, the peak position is independent of it.

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\section{I. INTRODUCTION}

Hall resistance in a ferromagnetic material is known to have two contributions: 1) proportional to magnetic field, 2) proportional to magnetization. While the former is responsible for conventional Hall effect, the latter is due to anomalous Hall effect (AHE). The anomalous Hall coefficient is the zero field Hall resistance which is estimated by extrapolating Hall resistance up to zero magnetic field. The physical mechanisms which contribute to the anomalous Hall conductivity (AHC), are broadly categorized into two classes: intrinsic and extrinsic. While the properties of the bands are responsible for the former, the spin dependent scattering of electrons belong to the extrinsic mechanism. Karplaus and Luttinger\textsuperscript{1} proposed intrinsic mechanism due to the spin-orbit interaction (SOI) for the polarized electrons in ferromagnetic materials. This mechanism was later realized as the outcome of the Berry phase\textsuperscript{2} from band topology. Smit\textsuperscript{3} proposed an extrinsic mechanism which is known as ‘skew-scattering’ (SS) in which electrons scatter from impurities with spin asymmetry. In addition, contribution due to side-jump (SJ) scattering was put forward by Berger\textsuperscript{4} through a semiclassical theory.

The debate on the true mechanism for AHE grew stronger with the contradicting results\textsuperscript{5} for various systems. The linear transport theories, which are now mostly used to study AHE, seem to have settled some of the confusing issues in the recent years. Although, different approaches of this theory often tend to conflict with each other, there are some successful attempts\textsuperscript{6, 7} to establish links between them. To this end, the two dimensional ferromagnetic-Rashba gas, \textit{i.e.}, electrons with Rashba SOI and ferromagnetic exchange coupling has proved to be a simple and elegant model to work with because it captures almost all the important and fundamental features relevant to AHE. Due to the Rashba SOI, characterized by the Rashba energy $\Delta_{R}$, degeneracy is lifted by the formation of two chiral subbands with opposite helicity. Further, ferromagnetic spin-exchange energy produces a gap between these subbands.

A number of studies on AHC have been performed within the model of two-dimensional ferromagnetic-Rashba gas, using the Kubo formalism\textsuperscript{6, 8-14}, the Keldysh technique\textsuperscript{6, 15, 16}, and the semiclassical treatment of Boltzmann transport equation\textsuperscript{17, 18}. According to the studies in Kubo formalism, the magnitude of AHC is sensitive to whether only one or both the subbands are occupied by the electrons, \textit{i.e.}, whether or not the Fermi level is situated below the band-gap created by exchange energy, $\Delta_{ex}$. In the presence of non-magnetic impurities, both the Fermi sea and Fermi surface contributions in AHC vanish\textsuperscript{11} when $\epsilon_{F} > \Delta_{ex}$, where $\epsilon_{F}$ is the Fermi energy. The bare-bubble contribution is exactly canceled by the vertex correction\textsuperscript{10, 11} due to SJ mechanism, and the SS term is absent\textsuperscript{12}. On the contrary, all these terms are finite for $\epsilon_{F} < \Delta_{ex}$. A comprehensive analysis of these works can be found in the detailed study of Nun- ner \textit{et al}\textsuperscript{11}. Using the Keldysh technique in a T-matrix approximation, Onoda \textit{et al}\textsuperscript{5, 19} however, find a strong SS contribution that leads to non-zero AHC even when $\epsilon_{F} > \Delta_{ex}$. They also observe that the AHC changes sign when the Fermi level enters into the band gap. The Keldysh technique automatically incorporates the higher order Born scatterings and enables them to arrive at such striking conclusions when the Fermi energy is located around the anticrossing of band dispersions in momentum space. Kovalev \textit{et al}\textsuperscript{16} later included the higher order scatterings explicitly in the Kubo formalism to match the results of Onoda \textit{et al}\textsuperscript{5, 19} and were able to unify the these approaches\textsuperscript{7} at least for a certain range of parameters.

There is no controversy, however, in the metallic weak-scattering regime ($\epsilon_{F} \tau > 1$), where $\tau$ is the mean scattering time of an electron. When the Fermi level goes far above the band gap ($\epsilon_{F} > \Delta_{ex}$), \textit{i.e.}, when both the
chiral subbands are occupied, the AHC vanishes\textsuperscript{5,7,11,20}, making this regime a somewhat uninteresting one. We however note that AHE is intimately related to spin hall effect\textsuperscript{21,22} and there are instances of enhancement of spin-Hall conductivity\textsuperscript{23} and occurrence of a induced transverse force\textsuperscript{24} due to inhomogeneity of the applied electric field. In this paper, we study the effect of inhomogeneous longitudinal electric field ($E_q \parallel q$) on the AHC in the metallic weak-scattering regime when both the subbands are occupied.

We find, within the Kubo formalism, that AHC, $\sigma^{AH}$, is zero at $q = 0$, increases with $q$ and forms a peak at certain $q$ before it decreases to zero at large $q$. $\sigma^{AH}$ depends on the parameters $\epsilon, \tau, \Delta_{ex}\tau$, and $\Delta_R\tau$. We find that the peak-value of AHC is proportional to $(\Delta_R\tau)^2(\epsilon/\tau)^{-1}$ and the peak occurs when $q\ell_{eff} \approx 1$, where $\ell_{eff}=(l_{ex})^{1/2}$, $l$ is the mean free path of an electron, and $l_{ex} = v_F/(4\Delta ex)$ is the length-scale corresponding to spin-exchange, with $v_F$ being the Fermi velocity. We note that the momentum at which the peaks occur does not depend on $\Delta_R$.

The paper is organized as follows. The following section contains a brief formalism for the evaluation of the anomalous Hall conductivity. In Section III, we present our results and discuss about the relevant scales and limits. Finally, we summarize our results in Section IV.

II. ANOMALOUS HALL CONDUCTIVITY

A. The model

A system of spin-polarized two dimensional electron gas with Rashba spin-orbit interaction in a disordered environment may be expressed by the Hamiltonian

$$H = \left(\frac{\hbar^2}{2m} + V(r)\right) \sigma_0 + \alpha(\hat{\sigma} \times \hat{\nabla}) \cdot \hat{z} - \Delta_{ex}\sigma_z,$$  

(1)

where $m$ is the effective mass of an electron, $\alpha$ is the Rashba spin-orbit coupling parameter, $\Delta_{ex}$ is the exchange energy which favors one kind of spin over the other, $\hat{\sigma}$ represents three Pauli matrices and $\sigma_0$ is a 2 x 2 unit matrix. Here $V(r)$ is the spin independent disorder potential for the randomly located $\delta$-function impurities and has the form $V(r) = \sum V_i \delta(r-R_i)$ satisfying $\langle V_i \rangle = 0$, $\langle V_i^2 \rangle = V_2 \neq 0$ and $\langle V_i^3 \rangle = V_3 \neq 0$. (We have set the unit $\hbar = 1$ and $c = 1$.)

In the absence of disorder, the above Hamiltonian can be exactly solved with the eigen values

$$E_k^s = \frac{k^2}{2m} + s\zeta_k$$  

(2)

where $k$ is the momentum of an electron, $s = \pm$ is the label for two chiral subbands produced due to Rashba spin-orbit interaction and $\zeta_k = \sqrt{\Delta_{ex}^2 + \alpha^2 k^2}$. The corresponding Fermi momenta $K_s$ are given by

$$K_s^2 = 2m[\epsilon_F - s\zeta_s]$$  

(3)

where $\epsilon_F$ is the Fermi energy and

$$\zeta_s = \sqrt{\Delta_{ex}^2 + \Delta_R^2 + m^2\alpha^4 - s ma^2},$$  

(4)

with $\Delta_R = \alpha\sqrt{2m}\epsilon_F = \alpha\epsilon_F$. Here $\epsilon_F$ is the mean Fermi momentum, given by $\epsilon_F = \frac{k_F^2}{2m}$. The density of states at the Fermi level for the two subbands are

$$\nu_s = \nu_0 \left[1 - \frac{s m\alpha^2}{\sqrt{\Delta_{ex}^2 + \Delta_R^2 + m^2\alpha^4}}\right],$$  

(5)

where $\nu_0$ is the density of states for each spin in pure two dimensional electron gas. The free retarded (advanced) Green’s function of the system can then be expressed as

$$\tilde{G}_{R,A}(k) = \frac{1}{\nu_0} \sum_{s = \pm} \nu_0(\sigma \cdot k \cdot \hat{z} - \Delta_{ex}\sigma_z)/\zeta_k,$$  

(6)

where $\tilde{G}_{k}^s = E_k^s - \epsilon_F$. The self energy for the electrons due to scattering is then found to be

$$\Sigma^{R(A)}(\epsilon) = \frac{i}{4\tau\nu_0} \left[(\nu_+ + \nu_-)\sigma_0 - \Delta_{ex}\left(\nu_+\hat{\zeta}_+ - \nu_-\hat{\zeta}_-\right)\sigma_z\right],$$  

(7)

in Born approximation, where $\tau = (2\pi\nu_0\hbar V_2)^{-1}$ is the mean scattering time for an electron, and $n_i$ represents the concentration of impurity. We thus find the disorder-averaged Greens functions as

$$\tilde{G}_{k}^R(\epsilon) = \frac{1}{\nu_0} \sum_{s = \pm} \nu_0(\sigma \cdot k \cdot \hat{z} - \Delta_{ex}\sigma_z)/\tilde{\zeta}_k,$$  

(8)

where $\tilde{\zeta}_k = \epsilon + i(\nu_+ + \nu_-)/(4\tau\nu_0)$ and $\Delta_{ex} = \Delta_{ex}\left[1 + \frac{i}{4\tau\nu_0}(\nu_+\hat{\zeta}_+ - \nu_-\hat{\zeta}_-\right)\right]$. Here $\tilde{\zeta}_k$ and $\tilde{\zeta}_k$ get renormalized to $\tilde{\zeta}_k$ and $\tilde{\zeta}_k$ due to the presence of $\Delta_{ex}$ instead of $\Delta_{ex}$ in their respective expressions. When both the helicity subbands are occupied, $\nu_+ + \nu_- = 2\nu_0$ and $\nu_+\hat{\zeta}_+ + \nu_-\hat{\zeta}_- = \nu_0$ and therefore exchange energy does not get renormalized and $\tilde{\zeta}$ becomes $\epsilon + i/2\tau$.

B. Kubo Formula

Using Kubo formula, anomalous Hall conductivity may be expressed as

$$\sigma_{xy}^{AH} = \frac{1}{2\pi} \int \frac{dk}{(2\pi)^2} \text{Tr} \left[ j_y \left( k + \frac{\alpha}{2} \right) \tilde{G}_{k+q}^R(0) \right] \times \left\{ j_x \left( k + \frac{\alpha}{2} \right) + \hat{\Gamma}_x(q) \right\} \tilde{G}_{k}^A(0),$$  

(9)

where charge current operators $j_x(k) = \epsilon(k,\sigma_0 - \alpha\sigma_y)$ and $j_y(k) = \epsilon(k,\sigma_0 + \alpha\sigma_y)$, and $\hat{\Gamma}_x(q)$ represents the correction to the vertex of $j_x$ due to both side-jump and skew scattering contributions (see fig.1) which can be calculated by solving the self-consistent equation.
FIG. 1. (Color online) Diagrammatic representation of the self-consistent equation \[10\] including side-jump (ladder) and skew-scattering corrections. The small wave lines with a point vertex represent the bare charge current vertex \(j_x\), and the small wave lines with dressed vertex represent \(\hat{\Gamma}_x\). Retarded and advanced Green’s functions are represented by the solid lines with arrows in the opposite directions. Dashed lines refer to the impurity scattering. Lowest order side-jump (a) and skew-scattering (b,c) diagrams; diagrams (d,e,f) represent vertex correction to the diagrams (a,b,c) respectively. Here vertex correction due to both side-jump and skew-scattering.

\[
\hat{\Gamma}_x(q) = \gamma_1 \int \frac{dk'}{(2\pi)^2} \hat{G}^{R}_{k'q}(0) \left[ j_x \left( k' + \frac{q}{2} \right) + \hat{\Gamma}_x(q) \right] \hat{G}^{A}_{k}(0) \\
+ \gamma_2 \int \frac{dk'}{(2\pi)^2} \hat{G}^{R}_{k'q}(0) \int \frac{dk''}{(2\pi)^2} \hat{G}^{R}_{k''q} \\
\times \left[ j_x \left( k'' + \frac{q}{2} \right) + \hat{\Gamma}_x(q) \right] \hat{G}^{A}_{k''}(0) \\
+ \gamma_2 \int \frac{dk''}{(2\pi)^2} \hat{G}^{R}_{k''q}(0) \left[ j_x \left( k'' + \frac{q}{2} \right) + \hat{\Gamma}_x(q) \right] \\
\times \hat{G}^{A}_{k''}(0) \int \frac{dk'}{(2\pi)^2} \hat{G}^{A}_{k'}(0),
\]

where, \(\gamma_1 = n_1 V_2\) and \(\gamma_2 = n_1 V_3\). It is established from a number of studies \[6,11,12\] that the skew-scattering contribution to the homogeneous \((q = 0)\) anomalous Hall conductivity vanishes provided both the chiral subbands are occupied. This, however, is true for any \(q\) since the last two terms in Eq. (10) cancel each other because of the identity \(\int \frac{dk}{(2\pi)^2} \hat{G}^{R,A}_{k} = \mp i \nu_v \sigma_o\). Equation (9) therefore reduces to

\[
\sigma_{yx}^{AH} = \frac{1}{2\pi} \int \frac{dk}{(2\pi)^2} \Tr \left[ j_y \left( k + \frac{q}{2} \right) \hat{G}^{R}_{k+q}(0) \right. \\
\times \left. \left\{ j_x \left( k + \frac{q}{2} \right) + j_x(q) \right\} \hat{G}^{A}_{k}(0) \right]
\]

with \(j_x(q)\) containing side-jump types of scattering contribution of \(\hat{\Gamma}_x(q)\) only, i.e.,

\[
j_x(q) = \frac{1}{2\pi \nu_v \tau} \int \frac{dk'}{(2\pi)^2} \hat{G}^{R}_{k'+q}(0) \left\{ j_x \left( k + \frac{q}{2} \right) \\
+ \hat{\Gamma}_x(q) \right\} \hat{G}^{A}_{k}(0).
\]

We numerically evaluate \(\hat{J}_x(q)\) and hence \(\sigma_{yx}^{AH}\) below.

C. Numerical evaluation

The numerical evaluation of \(\sigma_{yx}^{AH}\) via Eqs. (11) and (12) proceeds with the same spirit of Refs. 14 and 23. Expanding \(\hat{J}_x(q)\) in the Pauli matrix basis: \(\hat{J}_x = \sum_{\alpha=0}^{3} J_\alpha(q) \sigma_{\alpha}\), Eq. (12) can be rewritten as

\[
\sum_{\alpha} J_\alpha(q) \sigma_{\alpha} = \frac{1}{2\pi \nu_v \tau} \int \frac{dk}{(2\pi)^2} \hat{G}^{R}_{k+q}(0) \left( \sum_{\alpha} J_\alpha(q) \sigma_{\alpha} \right)
\times \hat{G}^{A}_{k}(0) = \frac{1}{2\pi \nu_v \tau} \int \frac{dk}{(2\pi)^2} \hat{G}^{R}_{k+q}(0) \hat{J}_x(k + \frac{q}{2}) \hat{G}^{A}_{k}(0).
\]

Equating the coefficients of the four Pauli matrices in the above equation, we find a \(4 \times 4\) matrix equation:

\[
\begin{pmatrix}
1 - m_{00} & -m_{11} & -m_{22} & -m_{33} \\
m_{01} & 1 - m_{10} & -m_{12} & -m_{13} \\
m_{02} & -m_{13} & 1 - m_{20} & -m_{23} \\
m_{03} & im_{12} & -im_{21} & 1 - m_{30}
\end{pmatrix}
\begin{pmatrix}
J_0 \\
J_1 \\
J_2 \\
J_3
\end{pmatrix}
= \begin{pmatrix}
z_0 \\
z_1 \\
z_2 \\
z_3
\end{pmatrix}
\]

(14)

where \(m_{\alpha\beta}(q)\) and \(z_\beta(q)\) are given by \[25\]

\[
\sum_{\beta=0}^{3} m_{\alpha\beta} \sigma_\beta = \frac{1}{2\pi \nu_v \tau} \int \frac{dk}{(2\pi)^2} \hat{G}^{R}_{k+q} \sigma_{\alpha} \hat{G}^{A}_{k}(0)
\]

(15)

Inverting the matrix equation (14), we find \(J_\alpha\) and then substituting these in Eq.(11), we obtain \(\sigma_{yx}^{AH}\).

III. RESULTS AND DISCUSSIONS

In the metallic regime (\(\epsilon_x \tau \gg 1\)), there is no net \(\sigma_{yx}^{AH}\) when the applied electric field is uniform. When both subbands are partially occupied, the bare bubble contribution

\[
\sigma_{0y}^{AH} = \frac{\nu_{qy}}{2\pi} \frac{m_0^2 \Delta_{xx}}{2 \Delta_{R}^2 + \Delta_{xx}^2 + \frac{(w)^2}{2}}
\]

is exactly canceled by the ladder vertex correction, \(\sigma_{yx}^{AH}\), which is numerically equal to \(\sigma_{0y}^{AH}\) but opposite in sign. In the super clean limit (\(\tau \rightarrow \infty\)), the above value agrees with the previous result \[11\].

In this paper, we are considering the effect of inhomogeneous longitudinal electric field, i.e., \(q \parallel E_{qy}\) on the Anomalous Hall conductivity. We define three dimensionless parameters out of four energy scales \(\epsilon_x, \Delta_R, \Delta_{xx}\) and \(1/\tau\). They are \(\epsilon_x, \Delta_R \tau\) and \(\Delta_{xx} \tau\). We numerically calculate \(\sigma_{yx}^{AH}\) for different sets of these parameters given in Table-1, using Eqs. (11), (14)–(16) when the electric field is assumed to be applied along x-axis, i.e., \(E_x = E_0 e^{i \omega t}, E_y = 0\).
TABLE I. Five sets (i)–(v) of three parameters $\Delta$ and $l_{\text{eff}}$ corresponding to the exchange energy, the peak positions occur at $q_{x}l_{\text{eff}} \approx 1$, irrespective of the values of the parameters (see Fig. 2(b)). On the other hand, the magnitudes of the peaks depend on two parameters: $\sigma_{\text{eff}}^{AH} \sim (\Delta_{R}\tau)_{0}^{2}(\epsilon_{F}\tau)^{-1}$. Figure 3 shows a contour plot of $\sigma_{\text{eff}}^{AH} = \sigma_{\text{eff}}^{AH}(\epsilon_{F}\tau)(\Delta_{R}\tau)^{-2}$ as a function of $q_{x}$ and $(\Delta_{ex}\tau)^{1/2}(\epsilon_{F}\tau)^{-1}$, i.e., $(k_{F}l_{\text{eff}})^{-1}$. Note that for the nearly-linear darkest patch, in which the magnitude of $\sigma_{\text{eff}}^{AH}$ is maximum, corresponds to $q_{x}l_{\text{eff}} \approx 1$.

These results suggest that if an electric field of the form $E_{x} = E_{0}e^{iq_{x}x}$ is applied along the x-axis in the plane of a two dimensional electron gas, the AHC becomes finite. As the maximum magnitude of AHC occurs at $q_{x}/k_{F} = (\Delta_{ex}\tau)^{1/2}(\epsilon_{F}\tau)^{-1}$, the periodic variation of the applied field should be properly tuned in accordance with the relevant parameters of the system to increase the transverse charge current. Clearly, the nature of this periodicity depends on magnetization, disorder and carrier concentration (i.e., on $\Delta_{ex}\tau$, and $\epsilon_{F}$ respectively) and not on Rashba parameter $\alpha$. This is illustrated in Fig. 4. An increased variation of the electric field is necessary to produce maximum AHC in a system with higher $l_{\text{eff}}$, i.e., with increasing magnetization and disorder, and decreasing electron density.

IV. SUMMARY

As evident in Fig. 2(a), the magnitude of $\sigma_{\text{eff}}^{AH}$ increases, forms a peak, and then decreases with the increase of $q_{x}$. We note that the peak values for the different sets of parameters are close to the corresponding bare contributions $\sigma_{b}^{AH}$ listed in Table 1. This suggests that the effect of inhomogeneous electric field on the AHC is substantial. The peak positions and the maximum values of $\sigma_{\text{eff}}^{AH}$ depend on these three parameters. However, if we scale $q_{x}$ in the unit of inverse effective mean free path, $l_{\text{eff}} = \sqrt{l_{\text{ex}}}$, where $l = v_{F}\tau$ is the mean free path of an electron and $l_{\text{ex}} = v_{F}/(4\Delta_{ex})$ is the length scale corresponding to the exchange energy, the peak positions occur at $q_{x}l_{\text{eff}} \approx 1$, irrespective of the values of the parameters (see Fig. 2(b)). On the other hand, the magnitudes of the peaks depend on two parameters: $\sigma_{\text{eff}}^{AH} \sim (\Delta_{R}\tau)^{2}(\epsilon_{F}\tau)^{-1}$. Figure 3 shows a contour plot of $\sigma_{\text{eff}}^{AH} = \sigma_{\text{eff}}^{AH}(\epsilon_{F}\tau)(\Delta_{R}\tau)^{-2}$ as a function of $q_{x}$ and $(\Delta_{ex}\tau)^{1/2}(\epsilon_{F}\tau)^{-1}$, i.e., $(k_{F}l_{\text{eff}})^{-1}$. Note that for the nearly-linear darkest patch, in which the magnitude of $\sigma_{\text{eff}}^{AH}$ is maximum, corresponds to $q_{x}l_{\text{eff}} \approx 1$.

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FIG. 4. (Color online) Schematic variation of the inhomogeneous electric field $E_x$, causing resonance in AHC: The periodicity is illustrated by the color gradient. For increased magnetization and disorder and decreased Fermi energy, i.e., for increased $l_{\text{eff}}$, the spatial variation of the electric field should be increased to obtain this resonance.

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