A GEOMETRIC APPROACH TO THE QUANTUM MECHANICS OF DE BROGLIE AND VIGIER

W.R. WOOD
Faculty of Natural and Applied Sciences
Trinity Western University, 7600 Glover Road
Langley, British Columbia V2Y 1Y1, Canada

AND

G. PAPINI
Department of Physics, University of Regina
Regina, Saskatchewan S4S 0A2, Canada

Abstract. Following de Broglie and Vigier, a fully relativistic causal interpretation of quantum mechanics is given within the context of a geometric theory of gravitation and electromagnetism. While the geometric model shares the essential principles of the causal interpretation initiated by de Broglie and advanced by Vigier, the particle and wave components of the theory are derived from the Einstein equations rather than a nonlinear wave equation. This geometric approach leads to several new features, including a solution to the de Broglie variable mass problem.

1. Introduction

It is a pleasure to acknowledge the role that Professor Vigier has played in the development of the casual interpretation of quantum mechanics [1]. His demonstration of an explicit solitonic solution [2] has made de Broglie’s conception of a double solution [3] a reality. As well, his extensive work [4] on issues relating to relativistic causal or stochastic models has been very helpful in our own efforts to formulate the principles of the causal interpretation within a geometrical framework.

In the geometric theory discussed here, a particle is represented by a thin shell or bubble solution to the Einstein equations rather than a solitonic solution to a nonlinear wave equation. The Gauss-Mainardi-Codazzi
(GMC) formalism (a familiar tool in general relativity) is used to facilitate the analysis of the dynamics of the bubble. The junction conditions in the GMC formalism provide a tensorial description of the balance of energy and momentum across the thin shell. As a consequence, the geometric model provides a framework by which the influence of external fields, such as the wave field $\psi(x)$, on the motion of the particle can be rigorously analyzed.

In the classical theory of general relativity, the guidance mechanism is well-known: the geometry, which is determined by the distribution of matter, in turn, governs the motion of the matter itself. However, this classical guidance mechanism becomes insignificant when applied to particles at the microscopic scale where de Broglie’s guidance principle is required to explain quantum effects such as the interference pattern in the two-slit experiment. It appears that a theory of gravitation whose domain of validity encompasses the microscopic scale is required if the desired guidance mechanism is to be given a geometric interpretation. A natural candidate is Weyl’s conformally invariant theory of gravitation and electromagnetism [5]. Weyl generalized the Riemannian geometry of general relativity by supposing that a vector parallel transported around a closed circuit would also experience a change in length according to the formula $\delta \ell = \ell \kappa_{\mu} \delta x^\mu$. The vector field $\kappa^\mu$, together with the metric tensor $g_{\mu\nu}$ that is defined modulo an equivalence class, comprise the fundamental fields of the new geometry.

Apart from providing a means to investigate the self-consistency of the dynamical aspects of the causal interpretation, the geometric model also offers several new interesting features. For example, by formulating the theory in the context of curved spacetime, new opportunities arise for considering the role that nonlocal interactions may play in a relativistic causal theory. As well, the geometric model provides a resolution to the problem of de Broglie’s variable mass.

2. The GMC Formalism

In Weyl geometry, one introduces a gauge-covariant calculus [8] based on the gauge-covariant derivative $\Box$ and a semimetric connection $\Gamma_{\alpha\mu\nu}^\rho$, where an overbar is used to distinguish objects from their Riemannian counterparts. In the GMC formalism, a timelike hypersurface $\Sigma$, which represents the history of the thin shell, divides spacetime into two four-dimensional
regions \((V^I\) and \(V^E)\), both of which have \(\Sigma\) as their boundary. The intrinsic metric on \(\Sigma\) is given by \(g_{\mu\nu} = n_{\mu}n_{\nu}\), where \(n^\mu\) is a unit spacelike \((n_{\mu}n^\mu = 1)\) vector field normal to \(\Sigma\). The extrinsic curvature tensor in Weyl geometry is defined by \(K_{\mu\nu} = h_{\mu\nu} + h_{\mu\rho}n^{\rho}\kappa_{\nu}\). The development of the GMC formalism in Weyl geometry ultimately yields the equations \([9]\)

\[
n_{\mu}n^\nu G^\mu_\nu = -\frac{1}{2} \left( 3R + K_{\mu\nu}K^{\mu\nu} - K^2 \right) - D_\mu \kappa^\mu + 2h_\mu^\nu \kappa^\mu \kappa_\nu + 2Kn^{\mu}\kappa_\mu. \tag{1}
\]

\[
n_{\mu}h_\alpha^\nu G^\mu_\nu = D_\alpha K - D_\mu K^\mu_\alpha, \tag{2}
\]

\[
h_\mu^\alpha h_\beta^\nu G^\mu_\nu = 3G^\alpha_\beta \quad + \quad (K^\alpha_\beta - h^\alpha_\beta K)_n - KK^\alpha_\beta + \frac{1}{2}h^\alpha_\beta(K_{\mu\nu}K^{\mu\nu} + K^2)
\quad - \quad 2(K^\alpha_\beta - h^\alpha_\beta K)n^\lambda \kappa_\lambda + 2h^\alpha_\beta h_\nu^\mu \kappa^\mu \kappa_\nu. \tag{3}
\]

The intrinsic stress-energy tensor on \(\Sigma\), which is defined by

\[
S^\mu_\nu \equiv \lim_{\varepsilon \to 0} \int_{-\varepsilon}^\varepsilon T^\mu_\nu d\kappa,
\]

corresponds to the distributional part of \(T_{\mu\nu}\). The junction conditions for the gravitational field are given by \(h_\mu^\alpha h_\beta^\nu S^\mu_\nu = \gamma^\alpha_\beta - h^\alpha_\beta \gamma\) and \(n_\mu S^\mu_\nu = 0\), where the jump in the extrinsic curvature is denoted by \(\gamma^\mu_\nu \equiv [K^\mu_\nu]\), \(\gamma \equiv \gamma^\mu_\mu\), and \(g_{\mu\nu}\) and \(\kappa^\mu\) are assumed to be continuous across \(\Sigma\), but their normal derivatives discontinuous. It is also assumed that \(\kappa^\mu = 0\) in the interior geometry \(V^I\) so that length integrability is established in the spacetime region occupied by the particle. Using (1) and (2), the jump in the equations \(n_\mu G^\mu_\nu = n_\mu T^\mu_\nu\) yields the intrinsic tensor equations

\[
D_\mu(h_\alpha^\mu h_\beta^\nu S^\alpha_\beta) + [n_\alpha h_\nu^\beta T^\alpha_\beta] = 0, \tag{5}
\]

\[
\{K^\nu_\nu\} S_\mu^\nu + [n_\mu n^\nu T^\mu_\nu] = 0, \tag{6}
\]

where \(\{K^\mu_\nu\}\) denotes the average of \(K^\mu_\nu\) across \(\Sigma\). Equations (5) and (6) describe the balance of stress-energy-momentum between neighboring external fields and the thin shell. It is this balance that governs the dynamical behavior of the thin shell. Indeed, the requirement that the fields in the exterior Weyl space join at \(\Sigma\) in accordance with the junction conditions places constraints on the motion of the bubble since \(\Sigma\) represents the history of the thin shell. Within the context of the causal interpretation of quantum mechanics, it is particularly significant that the interplay between the particle and wave aspects of the problem is an inherent feature of the present geometric formulation.
3. The Geometric Model

Once the assumption is made that elementary particles follow trajectories that are influenced in part by a wave field \( \psi(x) \), it is only natural to consider the new field on equal footing with the gravitational and electromagnetic fields in a unified manner. In fact, the transfer of energy and momentum required in the guidance process suggests use of a tensorial formulation that would, hopefully, yield the Einstein-Maxwell theory in the classical limit. The fact that the hypothesized guidance mechanism is effective at the microscopic scale, while the corresponding mechanism in general relativity is significant only at large scales, suggests beginning with a conformally invariant theory to integrate quantum effects into a geometric theory. In this regard, it is of interest to note that under the local conformal transformation \( g_{\mu\nu} \rightarrow \rho^2 g_{\mu\nu} \), the scalar curvature transforms as \( R \rightarrow R + \frac{6}{\rho^2} \rho^2 \beta^\mu \beta^\nu \beta^\mu \beta^\nu + \frac{\lambda}{\rho^2} \beta^\mu \beta^\nu \beta^\mu \beta^\nu \), where the derivative term in \( \rho \) is the covariant generalization of the quantum potential in the causal interpretation of quantum mechanics. The conformally invariant geometry introduced by Weyl is particularly attractive because it also provides a geometric interpretation for the electromagnetic field.

For our purposes, the modified Weyl-Dirac theory \[8\]
\[
I_c = \int \left\{ -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + |\beta|^2 R + k |\beta| \beta^\mu \beta^\nu + \lambda |\beta|^4 + \rho \gamma^\mu (\bar{\rho} \rho - \varepsilon \rho \varphi, \mu) \right\} \sqrt{-g} d^4 x, \tag{7}
\]
where \( \varepsilon = \pm 1 \), \( k \) and \( \lambda \) are real arbitrary constants and \( \rho, \varphi \) and \( \kappa_\mu \) are real fields, is convenient because it gives the complex scalar field \( \beta = \rho e^{i\varphi} \) a geometrical status as well as maintaining a theory that is linear in the scalar curvature. This latter point is essential when the particle is associated with a region of Riemannian space where the conformal symmetry of the exterior Weyl space is broken and the Gauss-Mainardi-Codazzi (GMC) formalism \[9\] is used to join the interior and exterior regions. The constraint, \( \kappa^\mu = - (\ln \rho)_{,\mu} + \varepsilon \varphi_{,\mu} \), is introduced \[8\] to allow for quantization of flux and leads to a topologically nontrivial electrodynamics with \( \varepsilon f_{\mu\nu} = \varphi_{,\mu\nu} - \varphi_{,\mu \nu} \).

3.1. THE FIELD EQUATIONS

The field equations that follow from the action (7), given here in terms of the Riemannian fields, are \[8\]
\[
\Box^\nu f^{\mu\nu} = 4(k - 3\varepsilon) \rho^2 \varphi^\mu \equiv j^\mu, \tag{8}
\]
\[
G_{\mu\nu} = \frac{1}{2\rho^2} E_{\mu\nu} + I_{\mu\nu} + \frac{1}{2} \lambda g_{\mu\nu} \rho^2 + H_{\mu\nu} \equiv T_{\mu\nu}, \tag{9}
\]
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\[
\frac{1}{3}(\varepsilon k - 3)\varphi_{,\mu}\varphi^{\mu} = -\frac{1}{6}(R + 2\lambda\rho^2) + \frac{1}{\rho} \Box_{\mu} \Box^{\mu} \rho 
\]  

(10)

and

\[
(k - 3\varepsilon) \Box_{\mu} (\rho^2 \varphi^{\mu}) = 0,
\]

(11)

where \( E_{\mu\nu} \) is the usual Maxwell tensor,

\[
I_{\mu\nu} = \frac{2}{\rho}(\Box_{\nu} \Box_{\mu} \rho - g_{\mu\nu} \Box_{\alpha} \Box^{\alpha} \rho) - \frac{1}{\rho^2}(4\rho_{,\mu}\rho_{,\nu} - g_{\mu\nu}\rho_{,\alpha}\rho^{\alpha})
\]

(12)

and

\[
H_{\mu\nu} = -2(\varepsilon k - 3)(\varphi_{\mu}\varphi_{,\nu} - \frac{1}{2} g_{\mu\nu}\varphi_{,\alpha}\varphi^{\alpha}).
\]

(13)

Taking the trace of (9) one recovers (10), while (11) follows from the conservation equation associated with (8). The theory also contains the wave equation [10]

\[
(\Box^\lambda + i\kappa^\lambda)(\Box_{\lambda} + i\kappa_{\lambda})\psi - \frac{\lambda}{3} |\psi|^2 \psi - \frac{1}{6} R \psi = 0.
\]

(14)

In fact, if one writes \( \psi = \rho e^{i\chi} \) and defines \( \chi_{,\mu} \) according to \( \alpha \varphi_{,\mu} \equiv \chi_{,\mu} + \kappa_{\mu} \) with \( \alpha^2 \equiv (\varepsilon k - 3)/3 \), then the imaginary part of (14) yields (11), while the real part coincides with (10) which can be expressed as

\[
(\chi_{,\mu} + \kappa_{\mu})(\chi^{\mu} + \kappa^{\mu}) = -\frac{1}{6}(R + 2\lambda\rho^2) + \frac{1}{\rho} \Box_{\mu} \Box^{\mu} \rho.
\]

(15)

Since \( \varphi, \chi, \) and \( \kappa_{\mu} \) are real fields, \( \alpha \) must be a real constant. In the causal interpretation of quantum mechanics, equation (15) is identified as the Hamilton-Jacobi equation for a system of momentum \( \chi_{,\mu} + \kappa_{\mu} = Mu_{\mu} \), so that

\[
Mu_{\mu} = \alpha \varphi_{,\mu}.
\]

(16)

From (15) one finds

\[
M^2 = \frac{\lambda}{3} \rho^2 + \left( \frac{R}{6} - \frac{1}{\rho} \Box_{\mu} \Box^{\mu} \rho \right),
\]

(17)

which is the square of the de Broglie mass in the present model.

3.2. THE PARTICLE-WAVE SOLUTION

For \( \alpha \varphi_{,\mu} = Mu_{\mu} \), the tensor \( H_{\mu\nu} \) is seen to represent a perfect (irrotational) fluid with equal pressure and energy density: \( H_{\mu\nu} = -6M^2(u_{\mu}u_{\nu} + \frac{1}{2} g_{\mu\nu}) \).
In the present geometric model, $H_{\mu\nu}$ is identified with the Madelung fluid in the causal interpretation. The particle is represented by a static, spherically symmetric thin shell solution to the Einstein equations when the Madelung fluid tensor $H_{\mu\nu}$ is neglected.

Application of the GMC formalism requires the determination of the interior and exterior line elements

$$ds^2_{I,E} = -e^{\nu_{I,E}} dt^2_{I,E} + e^{\mu_{I,E}} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (18)$$

as well as the intrinsic stress-energy tensor $S_{\mu\nu}$ on the timelike hypersurface $\Sigma$. In the interior space it is assumed that $\kappa_\mu = 0$ and that the scalar field acquires a constant value $\rho = \rho_0$ which breaks the interior conformal invariance and fixes the scale of the particle. Under these conditions, the interior metric is given by [9]

$$e^{-\mu_I} = 1 + \frac{1}{6} \lambda \rho_0^2 r^2 = e^{\nu_I}, \quad (19)$$

so that the interior space is de Sitter ($\lambda < 0$), Minkowski ($\lambda = 0$) or anti-de Sitter ($\lambda > 0$). The exterior metric, expressed in terms of the arbitrary function $\rho(r)$, is given by [9]

$$e^{-\mu_E} = \left(1 + \frac{\rho'}{\rho}\right)^{-2} \left[1 - \frac{2m}{\rho r} + \frac{q^2}{4\rho^2 r^2} + \frac{1}{6} \lambda \rho^2 r^2 \right] \quad (20)$$

and

$$e^{\nu_E} = \ell_0 \rho^{-2} \left[1 - \frac{2m}{\rho r} + \frac{q^2}{4\rho^2 r^2} + \frac{1}{6} \lambda \rho^2 r^2 \right], \quad (21)$$

where $m$, $q$ and $\ell_0$ are integration constants and a prime denotes differentiation with respect to $r$. When it is assumed that $g_{\mu\nu}$ and $\kappa^\mu$ are continuous across $\Sigma$, but their normal derivatives discontinuous, the surface stress-energy tensor is found to take the form [9]

$$h^\alpha_{\mu} h_{\beta\nu} S^\mu_{\nu} = -2\sigma h^\alpha_{\beta}, \quad (22)$$

where $\sigma \equiv [\eta^\mu (\ln \rho)_{,\mu}]$; that is, the surface stress-energy tensor is induced when the normal derivative of $\ln \rho$ across $\Sigma$ is discontinuous. The intrinsic stress-energy tensor (22) is characteristic of a domain wall of surface energy $2\sigma$, where $h_{\mu\nu}$ is the intrinsic metric on $\Sigma$. For $\sigma > 0$, the bubble is under a surface tension that opposes the Coulomb repulsion due to the surface charge. In this way, the particle finds its origin in the field $\rho$ that (i) fixes the scale in $V^I$, (ii) ensures conformal invariance in $V^E$, and (iii) induces the surface tension needed for stability.
Taking $\varphi_{,\mu} = 0$ in $V^I$ and $h^{\nu}_{\mu'} \varphi_{,\nu}$ discontinuous across $\Sigma$ allows the bubble to be embedded in the Madelung fluid in accordance with (5) and (6), while the surface stress-energy tensor (22) remains unchanged. For $n^\mu \varphi_{,\mu} = 0$, the normal component of the exterior fluid momentum at $\Sigma$ takes the form $n_\mu H^{\mu}_{\nu} = -3M^2 n_\nu$. From this it follows that $n_\mu h^{\nu}_{\alpha'} H_{\alpha}^{\nu} = 0$ and $n_\mu n^\nu H^{\mu}_{\nu} = -3M^2$. As a consequence, the Madelung fluid tensor $H_{\mu\nu}$ does not contribute to (5), the time component of which, in the rest frame of the thin shell, governs the transfer of energy between the particle and its neighboring fields [12]. While the particle does not draw energy from $H_{\mu\nu}$ as one might expect [2], it can be shown that, if $[\rho, n]$ varies on $\Sigma$, then $I_{\mu\nu}$ will transfer energy to the thin shell such that $\sigma \neq \text{constant}$.

3.3. THE GUIDANCE CONDITION

Although the Madelung fluid doesn’t serve as an energy source for the particle, it does influence the motion of the thin shell through (6) which represents Newton’s second law [12]. In this manner, the bubble acquires a new dynamical nature as it is guided in its motion by the fluid in $V^E$. The realization of this guidance process can be seen by considering the dynamical behavior of a fluid element at a point $P$ on the exterior surface of the bubble, where the four-velocity of the fluid element is denoted $u^\mu_f(P) = dz^\mu(P)/ds_E$. By construction, the metric tensor is continuous across $\Sigma$ at $P$ and consequently, $ds_E^2(P) = ds_E^2(P)$. Hence, at any point $P$ on $\Sigma$, the four-velocity of the thin shell is given by

$$u^\mu_p(P) = \left. \frac{dz^\mu}{ds_\Sigma} \right|_P = \frac{dz^\mu}{ds_E} \bigg|_P = u^\mu_f(P) = \frac{\alpha}{M} \varphi^\mu(P)$$

(23)

which is recognized as the guidance formula advanced by de Broglie. In the present approach, the validity of the guidance condition can be extended beyond holding only at a given point by noting that the motion of the fluid along its worldline from $P$ to a subsequent point $P'$, at which the fluid element and the bubble are still in contact, can be viewed as the result of a conformal transformation induced by the factor [13]

$$\xi^2 = 1 - M^2 \left( \frac{\Box M}{ds} \right)^{-2} \left( \frac{u_\mu}{ds} \right)^2.$$

(24)

The metric tensor at $P'$ in $V^E$ can therefore be obtained from its corresponding value at $P$ by applying a conformal transformation with the same factor $\xi^2$ and, by continuity, the intrinsic metric at $P'$ is also determined. The resulting identity, $h_{\mu\nu}(P') = \xi^2 h_{\mu\nu}(P)$, leads to the conclusion
that the bubble and fluid must move in step. Consequently, the Hamilton-Jacobi equation (15) may be applied to the particle itself, as required in the causal interpretation of quantum mechanics.

3.4. NONLOCAL EFFECTS AND CURVED SPACETIME

A novel feature of the bubble model presented above is the manner in which the interior space \( V^I \) is made distinct from the exterior space \( V^E \). This property not only makes it possible to break the conformal invariance in the interior space, whereby standards of length can be introduced into the theory while Weyl’s geometric interpretation of the exterior electromagnetic field is preserved, but the local requirements of the exterior space need not be imposed in the interior space. In particular, nonlocal influences that have been observed\(^1\) in experiments employing correlated particles may simply be a consequence of the fact that, while the world tubes of the correlated particles diverge after the disintegration process, they actually share a common past geometry that affords nonlocal interactions. In this way, nonlocal effects could be explained without denying the objective reality of elementary particles or compromising the principles of relativity (in curved spacetime).

The suggestion that the separation of \( V^I \) and \( V^E \) plays an essential role in seeking to understand the intriguing nonlocal EPR-type correlations does not require any nonlocal effects to occur in the exterior Weyl space. This situation is clearly not in keeping with the idea that it is the quantum potential (that exists in \( V^E \) in the present model) that is responsible for nonlocal phenomena. Bohm et al.\(^{15}\) have argued that the invariance of the quantum potential under a scaling of \( \psi(x) \) by an arbitrary constant plays a fundamental role in the nonlocal nature of the theory. However, this invariance property is reminiscent of the global phase invariance of pre-gauge field theories that also “contradicts the letter and spirit of relativity”\(^{16}\), and as a consequence is replaced by local phase invariance. In the present geometric model, the generalized quantum potential in (17) is invariant under the conformal transformation

\[
\tilde{g}_{\mu\nu} = \sigma^2 g_{\mu\nu}, \quad \tilde{\rho} = \sigma^{-1} \rho
\]

for the arbitrary function \( \sigma^2(x) > 0 \). As mentioned earlier, by requiring invariance under the local scaling (25) one is naturally led to a conformally invariant theory. For the Weyl-Dirac theory considered above, information

\(^1\)It is interesting to note, however, that Squires\(^{14}\) has challenged the conclusion that the empirical evidence implies nonlocality by considering the time involved in the actual measuring process in an Aspect-like experiment.
regarding the particle’s environment is propagated in the exterior spacetime via the tensor field \( I_{\mu\nu} \) in a local manner.

### 3.5. A SOLUTION TO DE BROGLIE’S VARIABLE MASS PROBLEM

An outstanding issue in the de Broglie-Vigier causal interpretation of quantum mechanics [3] has been the problem associated with the reality of the variable mass \( M \). Within the context of second-order wave equations, this problem manifests itself in the mathematical existence of negative probability densities and negative energy solutions. When the usual probabilistic interpretation is applied, these solutions cannot, in general, be given a physically meaningful interpretation. In contrast, when a particle follows a timelike causal trajectory, the situation changes radically. In this case, positive energy solutions are necessarily correlated with positive values of \( M \) and positive probability densities and the sign of the energy remains fixed along the trajectory [17]. However, general solutions of the Klein-Gordon equation do not ensure the reality of \( M \).

This deficiency is overcome in the present geometric model [18] due to the existence of the timelike thin shell solution to Einstein’s equations which can be embedded in the Madelung fluid according to the junction conditions discussed above. While spacelike and timelike directions are distinguished in any relativistic theory, a geometric theory permits these directions in spacetime to be related to the motion of matter via Einstein’s field equations. That is, for a given foliation of spacetime, the GMC formalism requires the various timelike and spacelike components of \( G_{\mu\nu} \) to be equated to the corresponding components of \( T_{\mu\nu} \). In this way, a link is established between the properties of spacetime and matter that allows one to address the issue of whether or not a given four-vector that is associated with matter is timelike. It is due to the absence of this geometric structure that the possibility of spacelike four-momenta in de Broglie’s guidance formula \( P_\mu = M u_\mu \) cannot be excluded in previous formulations of the causal interpretation derived from a scalar wave equation. By basing the theory on the field equations (8)-(11) (from which the wave equation (14) is then identified), one is not bound to demonstrate that all possible generic solutions to the wave equation must be physically meaningful as is the case when the causal interpretation is based solely on a wave equation. It is the field equations (8)-(11) that determine the set of physically acceptable solutions in the geometric approach. In the geometric model discussed above, the timelike nature of \( M u_\mu \) can be demonstrated as follows.

The constraint in (1) can be written in the gauge \( \kappa'_{\mu} = \kappa_{\mu} + (\ln \rho)_{,\mu} \) so that \( \kappa'_{\mu} = \varepsilon \varphi_{,\mu} \). This is permissible due to the gauge covariance of the theory and this particular gauge is viable since \( \rho \) must be greater than one in the
model in order for the thin shell to be under a surface tension that balances the Coulomb repulsion [9]. For the static solution of Section 3b, $\kappa'_\mu K^{\mu\nu} < 0$ and as a consequence $\varphi_\mu$ is timelike. It then follows from (16) that $M$ must be real since $\alpha$ is real. This result, obtained in the static case, also holds true in a frame comoving with the thin shell, and is therefore quite general. Indeed, due to the covariant nature of $M$ under conformal transformations that preserve the sign of the line element, $M^2$ must be positive in general.

The condition for $\varphi_\mu$ to be timelike can also be expressed within the context of the theorem of Frobenius. In terms of differential forms, equation (16) is given by

$$u = \frac{\alpha}{M} \, d\varphi \equiv h \, d\varphi.$$  \hspace{1cm} (26)

The condition for $u^\mu$ to be orthogonal to hypersurfaces of constant $\varphi$, and hence for $\varphi_\mu$ to be timelike, is given by $d\varphi \wedge u = 0$. Recognizing that, due to the multivalued nature of $\varphi$, $d\varphi$ is not closed even though it is an exact 1-form, the condition for timelike $\varphi_\mu$ becomes

$$d^2 \varphi \wedge d\varphi = 0.$$  \hspace{1cm} (27)

Equation (27) is satisfied in the static case considered above, where $d^2 \varphi \sim dx^0 \wedge dx^1$ and $d\varphi \sim dx^0$.

For timelike $\varphi_\mu$, it follows that the Maxwell current $j^\mu$ in (8) is also timelike without having to impose this as an auxiliary condition. In the present theory, the Maxwell current is proportional to the Klein-Gordon current [10] associated with the wave equation, $j^\mu_{KG} = \alpha \rho^2 \varphi_\mu$, and is therefore also timelike and as such does not suffer from the difficulties normally associated with the current for a second order wave equation. It should be noted, however, that $\psi$ in the present theory is a physical field and not immediately identifiable with a probabilistic wave function. In addition, the time component of the current can be made positive by choosing the positive sign of the radical in the definition of $\alpha$. It then follows that positive (negative) energy particles will correspond to positive (negative) values of $M$ and positive (negative) values of $j^\mu_{KG}$. In this regard, it is interesting to observe that the equation of motion

$$\frac{\varepsilon}{ds} (Mu_\mu) = -\varepsilon \alpha f_{\mu\nu} u^\nu - \Box_\mu M$$  \hspace{1cm} (28)

is invariant under charge conjugation and time reversal transformations as discussed by Dirac [19]. In addition, equation (28) is invariant under $M \rightarrow -M$ together with time reversal. This indicates that, in the present theory, negative energy particles may be interpreted as positive energy particles moving backward in time.
4. Summary

Although the geometric formulation of the causal theory presented above is in an early developmental stage, it nevertheless demonstrates that it is possible to inject the principles of the causal interpretation of quantum mechanics into a fully relativistic geometric theory in Weyl space. In the authors’ opinion, this is an essential step towards obtaining a satisfactory causal theory of quantum phenomena. By formulating the problem within the context of a theory of gravitation, whereby the description of the transfer of energy-momentum becomes an inherent feature, it becomes possible to demonstrate that the guidance principle is dictated by the physics rather than the physicist. As well, the geometric model provides a basis upon which issues such as the reality of the de Broglie variable mass and nonlocal interactions can be addressed.

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