A PROCEDURE FOR GENERATING INFINITE SERIES IDENTITIES

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A procedure for generating infinite series identities makes use of the generalized method of exhaustion by analytically evaluating the inner series of the resulting double summation. Identities are generated involving both elementary and special functions. Infinite sums of special functions include those of the gamma and polygamma functions, the Hurwitz Zeta function, the polygamma function, the Gauss hypergeometric function, and the Lerch transcendent. The procedure can be automated with Mathematica (or equivalent software).

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1. Introduction. The generalized method of exhaustion [5] provides series expansions for Riemann integrals of the form

\[
\int_a^b f(x) \, dx = (b - a) \lim_{N \to \infty} \sum_{n=1}^{N} \sum_{m=1}^{2^n-1} (-1)^{m+1} 2^{-n} f\left( a + \frac{m(b-a)}{2^n} \right)
\]

(1.1)

The first expression is identical to the limit of the Riemann sum of order \(2^N\) for \(f(x)\) over \([a,b]\), except for the missing endpoints \(f(a)\) or \(f(b)\), which can be neglected when \(N \to \infty\). Thus, convergence of the expansion is guaranteed for \(N \to \infty\) when the integral exists.

The double summation expression can be reduced to that of a single summation when the inner finite series can be evaluated analytically. Analytical sums are occasionally expressible in terms of elementary functions or (more often) in terms of special functions. Some examples of special functions are the gamma and polygamma functions, the Gauss hypergeometric function, the Hurwitz Zeta function, and the Lerch transcendent. Identities involving these and other functions can stimulate solutions to engineering or physics problems [6], or lead to new insights into existing solutions.

The analytical evaluation of the inner series can often be automated with the use of software such as Mathematica. A few of the more important results are presented in this paper, along with a table of integrals permitting analytical evaluations of the inner series.

2. Generation of identities. The simplest analytic expressions resulting from the use of (1.1) involve geometric series, one form of which is as follows:

\[
\sum_{m=1}^{2^n-1} (-1)^{m+1} e^{m x/2^n} = 1 + \frac{e^x - 1}{e^{x/2^n} + 1}.
\]

(2.1)
This result can then be applied to the following definite integral:

\[
\int_0^x e^z \, dz = e^x - 1 = x \sum_{n=1}^{2^n-1} \sum_{m=1}^{m+1} (-1)^{m+1} 2^{-n} e^{mx/2^n} = x \sum_{n=1}^{\infty} 2^{-n} \left( 1 + \frac{e^x - 1}{e^{x/2^n} + 1} \right) = x + (e^x - 1) x \sum_{n=1}^{\infty} \frac{2^{-n}}{e^{x/2^n} + 1},
\]

or

\[
\frac{1}{x} = \frac{1}{e^x - 1} + \sum_{n=1}^{\infty} \frac{2^{-n}}{e^{x/2^n} + 1}.
\]

Integration (and some algebra) leads to

\[
\ln x = (x - 1) \prod_{n=1}^{\infty} \frac{2}{1 + x^{2^{-n}}}.
\]

Setting \( x = i \theta \) in (2.2) and following a similar procedure leads to the well-known result [4]

\[
\sin \theta = \theta \prod_{n=1}^{\infty} \cos \left( \frac{\theta}{2^n} \right).
\]

A second expression for the logarithm can be derived from (1.1) as follows:

\[
\int_1^x \frac{dz}{z} = \ln x = \sum_{n=1}^{\infty} \sum_{m=1}^{2^n-1} (-1)^{m+1} (x - 1) \frac{1}{2^n + m(x - 1)}.
\]

Using the identity [1]

\[
\psi_0(z) = \int_0^{\infty} \left[ \frac{e^{-t}}{t} - \frac{e^{-zt}}{1 - e^{-t}} \right] dt,
\]

where \( \psi_0(x) = (d/dx) \ln \Gamma(x) \) is the digamma function, it can be shown that

\[
\psi_0 \left( 2^n + \frac{2^n}{(x - 1)} \right) - \psi_0 \left( 1 + \frac{2^n}{(x - 1)} \right) = \sum_{m=1}^{2^n-1} \frac{(x - 1)}{2^n + m(x - 1)},
\]

\[
\psi_0 \left( \frac{2^n}{1 + \frac{2^n}{(x - 1)} \right) - \psi_0 \left( \frac{1}{2} + \frac{2^n}{(x - 1)} \right) = \sum_{m=1}^{2^n-1} \frac{2(x - 1)}{2^n + 1 + (2m - 1)(x - 1)},
\]
and thus,
\[
\sum_{m=1}^{2n-1} \frac{(-1)^{m+1}(x-1)}{2^n + m(x-1)} = \frac{1}{2} \psi_0 \left(1 + \frac{2^{n-1}}{x-1}\right) - \frac{1}{2} \psi_0 \left(\frac{1}{2} + \frac{2^{n-1}}{x-1}\right) \\
+ \frac{1}{2} \psi_0 \left(\frac{1}{2} + \frac{2^{n-1}x}{x-1}\right) - \frac{1}{2} \psi_0 \left(\frac{2^{n-1}x}{x-1}\right).
\] (2.11)

This leads to the result
\[
\ln x = \frac{1}{2} \sum_{n=0}^{\infty} \left[ \psi_0 \left(1 + \frac{2^{n-1}}{x-1}\right) - \psi_0 \left(\frac{1}{2} + \frac{2^{n-1}}{x-1}\right) \right] \\
+ \frac{1}{2} \sum_{n=0}^{\infty} \left[ \psi_0 \left(\frac{1}{2} + \frac{2^{n-1}x}{x-1}\right) - \psi_0 \left(\frac{2^{n-1}x}{x-1}\right) \right].
\] (2.12)

The next example involves the exponential integral, that is,
\[
\int_0^x \frac{e^z - 1}{z} \, dz = \text{Ei}(x) - \ln x - \gamma = \sum_{n=1}^{\infty} \sum_{m=1}^{2^{n-1}-1} (-1)^m \frac{e^{m\frac{x}{2^n}} - 1}{m}.
\] (2.13)

Here, \(\gamma\) is the Euler-Mascheroni constant. From (2.13), Mathematica yields the result
\[
\sum_{m=1}^{2^{n-1}} (-1)^{m+1} \frac{e^{m\frac{x}{2^n}} - 1}{m} = \ln \left(\frac{1}{2} + \frac{1}{2} e^{x/2^n}\right) + 2^{-n} e^{x/2^n} \binom{2, 1, 1 + 2^n, -e^{x/2^n}}{2^n, 1, 1, -e^{x/2^n}} \\
+ \frac{1}{2} \psi_0 (2^{n-1}) - \frac{1}{2} \psi_0 \left(\frac{1}{2} + 2^{n-1}\right),
\] (2.14)

where \(\binom{a}{b, c, d}\) is the Gauss hypergeometric function. Noting that [2, 3, 7]
\[
\gamma = \frac{1}{2} \sum_{n=1}^{\infty} \left[ \psi_0 \left(\frac{1}{2} + 2^{n-1}\right) - \psi_0 (2^{n-1}) \right] = \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^k}{2^n + k},
\] (2.15)

an identity for the exponential integral becomes
\[
\text{Ei}(x) = \ln x + \sum_{n=1}^{\infty} \ln \left(\frac{1}{2} + \frac{1}{2} e^{x/2^n}\right) + e^x \sum_{n=1}^{\infty} 2^{-n} \binom{2^n, 1, 1 + 2^n, -e^{x/2^n}}{2^n, 1, 1, -e^{x/2^n}}.
\] (2.16)

The identity (2.6) leads to
\[
\sum_{n=1}^{\infty} \ln \left(\frac{1}{2} + \frac{1}{2} e^{x/2^n}\right) = \ln (e^x - 1) - \ln x
\] (2.17)
Table 2.1 lists some integrals permitting analytical evaluation of the inner series when expanded with (1.1), and the form of the resulting infinite series.
3. Method of exhaustion variants. By using an approach similar to that in [5], a family of expansions similar to (1.1) can be generated for the definite integral, that is,

\[
\int_{a}^{b} f(x) \, dx = (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{3^{n-1}} 3^{-n} f \left( a + \frac{m(b-a)}{3^n} \right) \\
- (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{3^{n-1}} 3^{-n+1} f \left( a + \frac{m(b-a)}{3^{n-1}} \right)
\]

\[= (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{4^{n-1}} 4^{-n} f \left( a + \frac{m(b-a)}{4^n} \right) \]

\[- (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{4^{n-1}} 4^{-n+1} f \left( a + \frac{m(b-a)}{4^{n-1}} \right) \]

\[= (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{5^{n-1}} 5^{-n} f \left( a + \frac{m(b-a)}{5^n} \right) \]

\[- (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{5^{n-1}} 5^{-n+1} f \left( a + \frac{m(b-a)}{5^{n-1}} \right), \]

and so forth.

Or, in general,

\[
\int_{a}^{b} f(x) \, dx = (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{p^n-1} p^{-n} f \left( a + \frac{m(b-a)}{p^n} \right) \\
- (b - a) \sum_{n=1}^{\infty} \sum_{m=1}^{p^n-1} p^{-n+1} f \left( a + \frac{m(b-a)}{p^{n-1}} \right), \quad p = 2, 3, 4, \ldots.
\]

Note that

\[ (b - a) \sum_{n=1}^{N} \sum_{m=1}^{p^n-1} p^{-n} f \left( a + \frac{m(b-a)}{p^n} \right) \]

\[- (b - a) \sum_{n=1}^{N} \sum_{m=1}^{p^n-1} p^{-n+1} f \left( a + \frac{m(b-a)}{p^{n-1}} \right), \quad p = 2, 3, 4, \ldots,
\]

is precisely the Riemann sum of order \(p^N\) for \(f(x)\) over \([a, b]\), except for the missing terms \(f(a)\) or \(f(b)\), which can be neglected in the limit \(N \to \infty\). Thus, as \(N \to \infty\), (3.2) is valid for all Riemann integrable functions.

Each of these expressions leads to distinct identities, once the inner finite series is summed. A family of identities can thus be generated similar to each presented in the last section. For example, a family of identities for the logarithm similar to (2.6) is as
follows:

\[
\ln x = (x - 1) \prod_{n=1}^{\infty} \frac{2}{1 + x^{1/2^n}} \\
= (x - 1) \prod_{n=1}^{\infty} \frac{3}{1 + x^{1/3^n} + x^{2/3^n}} \\
= (x - 1) \prod_{n=1}^{\infty} \frac{4}{1 + x^{1/4^n} + x^{2/4^n} + x^{3/4^n}} \\
= (x - 1) \prod_{n=1}^{\infty} \frac{5}{1 + x^{1/5^n} + x^{2/5^n} + x^{3/5^n} + x^{4/5^n}} \\
= (x - 1) \prod_{n=1}^{\infty} \frac{p}{\sum_{q=0}^{p-1} x^{q/p^n}}, \quad p = 2, 3, 4, \ldots
\]  

(3.4)

A family of identities similar to (2.7) can also be generated

\[
\sin x = x \prod_{n=1}^{\infty} \frac{1}{2} \left( e^{-ix/2^n} + e^{ix/2^n} \right) = x \prod_{n=1}^{\infty} \cos \left( \frac{x}{2^n} \right) \\
= x \prod_{n=1}^{\infty} \frac{1}{3} \left( e^{-ix/3^n} + 1 + e^{ix/3^n} \right) = x \prod_{n=1}^{\infty} \left( 1 - \frac{4}{3} \sin^2 \left( \frac{x}{3^n} \right) \right) \\
= x \prod_{n=1}^{\infty} \frac{1}{4} \left( e^{-ix/4^n} + e^{-ix/4^n} + e^{ix/4^n} + e^{3ix/4^n} \right) \\
= x \prod_{n=1}^{\infty} \frac{1}{5} \left( e^{-ix/5^n} + e^{-2ix/5^n} + 1 + e^{2ix/5^n} + e^{4ix/5^n} \right) \\
= x \prod_{n=1}^{\infty} \sum_{p=2, 3, 4, \ldots}^{q=1} \frac{1}{p} e^{(2q-p-1)ix/p^n}, \quad p = 2, 3, 4, \ldots
\]  

(3.5)

The results (3.4) and (3.5) are proven below. Consider the expression

\[
\int_0^x e^t \, dt = e^x - 1 \\
= x \sum_{n=1}^{\infty} \frac{e^{mx/p}}{p} \sum_{m=1}^{p^n-1} p^{-n} e^{mx/p^n} - x \sum_{n=1}^{p^n-1} \frac{e^{mx/p^n} - 1}{p^n} - \sum_{n=1}^{p^n-1} \frac{e^{mx/p^n}}{p^n} - 1, \quad p = 2, 3, 4, \ldots
\]  

(3.6)

Each inner summation is a geometric series, leading to the result

\[
\sum_{m=1}^{p^n-1} p^{-n} e^{mx/p^n} - \sum_{m=1}^{p^n-1} p^{-n+1} e^{mx/p^{n-1}} = \frac{e^x - e^{x/p^n}}{p^n(e^{x/p^n} - 1)} - \frac{e^x - e^{x/p^{n-1}}}{p^{n-1}(e^{x/p^{n-1}} - 1)},
\]  

(3.7)
Next, making use of the identities

\[
\begin{align*}
(e^{x/pn-1} - 1) &= (e^{x/pn} - 1) \left( \sum_{q=0}^{p-1} e^{qx/pn} \right), \\
(e^{x/pn-1} - 1)(e^{x/pn} - 1) &= e^{(p+1)x/pn} - e^{x/pn-1} - e^{x/pn} + 1 \\
&= (e^{x/pn} - 1)^2 \left( \sum_{q=0}^{p-1} e^{qx/pn} \right),
\end{align*}
\]

(3.8)

it can be shown that

\[
\int_0^x e^t \, dt = e^x - 1
\]

\[
= x \sum_{n=1}^{\infty} \frac{e^x [e^{x/pn-1} - pe^{x/pn} + (p-1)]}{p^n (e^{x/pn} - 1)^2 \left( \sum_{q=0}^{p-1} e^{qx/pn} \right)} + x \sum_{n=1}^{\infty} \frac{(p-1)e^{(p+1)x/pn} - pe^{x/pn-1} + e^{x/pn}}{p^n (e^{x/pn} - 1)^2 \left( \sum_{q=0}^{p-1} e^{qx/pn} \right)}.
\]

(3.9)

Noting that

\[
(e^{x/pn} - 1)^2 \sum_{q=0}^{p-1} q e^{qx/pn} = e^{x/pn} - pe^{x/pn} + (p-1)e^{(p+1)x/pn},
\]

(3.10)

we again show that

\[
e^{x/pn-1} - pe^{x/pn} + (p-1)
\]

\[
= -(e^{x/pn} - 1)^2 \sum_{q=0}^{p-1} q e^{qx/pn} + (p-1) \left( e^{(p+1)x/pn} - e^{x/pn-1} - e^{x/pn} + 1 \right)
\]

(3.11)

\[
= (p-1)(e^{x/pn} - 1)^2 \left( \sum_{q=0}^{p-1} e^{qx/pn} \right) - (e^{x/pn} - 1)^2 \sum_{q=0}^{p-1} q e^{qx/pn},
\]

so that (3.9) becomes

\[
e^x - 1 = xe^x \sum_{n=1}^{\infty} \frac{p-1}{p^n} - x(e^x - 1) \sum_{n=1}^{\infty} \frac{(e^{x/pn} - 1)^2 \sum_{q=0}^{p-1} q e^{qx/pn}}{p^n (e^{x/pn} - 1)^2 \sum_{q=0}^{p-1} e^{qx/pn}}
\]

(3.12)

or

\[
\frac{1}{x} = \frac{e^x}{e^x - 1} - \sum_{n=1}^{\infty} \frac{\sum_{q=1}^{p-1} q e^{qx/pn}}{pn \sum_{q=0}^{p-1} e^{qx/pn}}.
\]

(3.13)

Integrating leads to

\[
\ln x = \ln(e^x - 1) - \sum_{n=1}^{\infty} \ln \left( \sum_{q=0}^{p-1} \frac{e^{qx/pn}}{p} \right),
\]

(3.14)
or

\[
\frac{e^x - 1}{x} = \prod_{n=1}^{\infty} \sum_{q=0}^{p-1} \frac{e^{q/p^n}}{p}.
\] (3.15)

When \( x = \ln z \),

\[
\ln z = (z - 1) \prod_{n=1}^{\infty} \left[ \frac{p}{\sum_{q=0}^{p-1} z^{q/p^n}} \right], \quad p = 2, 3, 4, \ldots.
\] (3.16)

The identities given by (3.5) can be proven if \( x = iz \) in (3.13),

\[
\frac{1}{iz} = \frac{e^{iz}}{e^{iz} - 1} - \sum_{n=1}^{\infty} \frac{p}{p^n \sum_{q=0}^{p-1} e^{iqz/p^n}}.
\] (3.17)

By evaluating the double geometric series, it can be shown that

\[
\sum_{q=0}^{p-1} \sum_{r=0}^{p-1} q e^{i(q-r)z/p^n} = \left[ \sum_{q=1}^{p} e^{i(2q-p-1)z/2p^n} \right]^{2},
\]

\[
\text{Im} \left\{ \sum_{q=1}^{p} \sum_{r=0}^{p-1} q e^{i(q-r)z/p^n} \right\} = \text{Im} \left\{ \left[ \sum_{q=1}^{p} e^{i(2q-p-1)z/2p^n} \right]^{p} \sum_{q=1}^{p} (q - p/2 - 1/2) e^{i(2q-p-1)z/2p^n} \right\}.
\] (3.18)

Taking the imaginary part of (3.17) and using (3.18) leads to

\[
-\frac{1}{z} = -\frac{-\sin z}{2 - 2\cos z} + \text{Im} \left\{ \sum_{n=1}^{\infty} \frac{\sum_{q=1}^{p} (q - p/2 - 1/2) e^{i(2q-p-1)z/2p^n}}{p^n \sum_{q=1}^{p} e^{i(2q-p-1)z/2p^n}} \right\}.
\] (3.19)

Integrating leads to

\[
-\ln z = -\ln \left[ \sin \left( \frac{z}{2} \right) \right] + \sum_{n=1}^{\infty} \ln \left[ \sum_{q=1}^{p} e^{i(2q-p-1)z/2p^n} \right] + C,
\] (3.20)

or

\[
\sin \left( \frac{z}{2} \right) = z e^{C} \prod_{n=1}^{\infty} \left[ \sum_{q=1}^{p} e^{i(2q-p-1)z/2p^n} \right].
\] (3.21)

Letting \( y = z/2 \),

\[
\sin y = 2y e^{C} \prod_{n=1}^{\infty} \left[ \sum_{q=1}^{p} e^{i(2q-p-1)y/p^n} \right].
\] (3.22)
Finally, the integration constant $C$ can be evaluated by setting $y = 0$, leading to

$$\sin y = y \prod_{n=1}^{\infty} \left[ \frac{1}{p} e^{i(2q-p-1)y/p^n} \right]. \quad (3.23)$$

The remaining identities presented in the previous section also appear in other forms below. Many of the results lead to limit representations. For example, following a procedure similar to that leading to (2.12) leads to

$$\ln x = \sum_{n=1}^{\infty} \left[ \psi_0 \left( \frac{p^n x}{x-1} \right) - \psi_0 \left( 1 + \frac{p^n}{x-1} \right) \right] + \sum_{n=0}^{\infty} \left[ \psi_0 \left( 1 + \frac{p^n}{x-1} \right) - \psi_0 \left( \frac{p^n x}{x-1} \right) \right], \quad p = 2, 3, 4, \ldots \quad (3.24)$$

or

$$\ln x = \lim_{n \to \infty} \left[ \psi_0 \left( \frac{p^n x}{x-1} \right) - \psi_0 \left( 1 + \frac{p^n}{x-1} \right) \right], \quad p = 2, 3, 4, \ldots \quad (3.25)$$

Setting $q = p^n$ leads to

$$\ln x = \lim_{q \to \infty} \left[ \psi_0 \left( \frac{q x}{x-1} \right) - \psi_0 \left( 1 + \frac{q}{x-1} \right) \right]. \quad (3.26)$$

A procedure similar to that leading to (2.18) generates the following:

$$\text{Ei}(x) - \ln x - \gamma = e^{x} \sum_{n=1}^{\infty} p^{-n-1} {}_2F_1 \left( 1, p^{n-1}, 1 + p^{n-1}, e^{x/p^{n-1}} \right)$$

$$- e^{x} \sum_{n=1}^{\infty} p^{-n} {}_2F_1 \left( 1, p^{n}, 1 + p^{n}, e^{x/p^{n}} \right)$$

$$+ \sum_{n=1}^{\infty} \ln \left( \frac{1 - e^{x/p^{n-1}}}{1 - e^{x/p^{n}}} \right) + \sum_{n=1}^{\infty} [\psi_0(p^{n-1}) - \psi_0(p^{n})], \quad p = 2, 3, 4, \ldots \quad (3.27)$$

or

$$\text{Ei}(x) - \ln x - \gamma = \ln \left( 1 - e^{x} \right) + e^{x} {}_2F_1 \left( 1, 1, 2, e^{x} \right) + \psi_0(1)$$

$$- e^{x} \lim_{n \to \infty} p^{-n} {}_2F_1 \left( 1, p^{n}, 1 + p^{n}, e^{x/p^{n}} \right)$$

$$- \lim_{n \to \infty} \left[ \ln \left( 1 - e^{x/p^{n}} \right) + \psi_0(p^{n}) \right], \quad p = 2, 3, 4, \ldots \quad (3.28)$$

Noting that $[1] \psi_0(1) = -\gamma$ and $\sum_{n=1}^{\infty} F_1 (1, 1, 2, e^{x}) = -e^{-x} \ln(1 - e^{x})$, and setting $q = p^n$ leads to

$$\text{Ei}(x) = \ln x - \lim_{q \to \infty} \left[ \psi_0(q) + \frac{e^{x}}{q} {}_2F_1 \left( 1, q, 1 + q, e^{x/q} \right) + \ln \left( 1 - e^{x/q} \right) \right]. \quad (3.29)$$
Finally, a procedure similar to that leading to (2.21) generates the following:

\[
\Gamma(a+1) - \Gamma(a+1,x) = -x^{1+a} \lim_{n \to \infty} p^{-n(1+a)} \left[ e^{-x/p^n} \Phi(e^{-x/p^n}, -a, p^n) - e^{-x/p^n} \Phi(e^{-x/p^n}, -a, 1) \right].
\]

Setting \( q = p^n \) leads to

\[
\Gamma(a+1) - \Gamma(a+1,x) = -x^{1+a} \lim_{q \to \infty} q^{-(1+a)} \left[ e^{-x/q} \Phi(e^{-x/q}, -a, q) - e^{-x/q} \Phi(e^{-x/q}, -a, 1) \right].
\]

While (1.1) leads to expressions in the form of infinite series, (3.2) often leads to limit representations, unless further operations (e.g., integration) are first applied to the two individual series expressions.

4. Summary. The examples presented here illustrate a procedure for generating infinite series identities, consisting of evaluating an integral using the generalized method of exhaustion, and then analytically summing the inner finite series. *Mathematica* (or equivalent software) can automate the second step, allowing rapid investigation of integrals.

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**Call for Papers**

As a multidisciplinary field, financial engineering is becoming increasingly important in today’s economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems).

This special issue will include (but not be limited to) the following topics:

- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

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| Deadline                  | Date               |
|---------------------------|--------------------|
| Manuscript Due            | December 1, 2008   |
| First Round of Reviews    | March 1, 2009      |
| Publication Date          | June 1, 2009       |

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