More on nucleon-nucleon cross sections in symmetric and asymmetric matter

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Following a recent work, we present numerical results for total two-nucleon effective cross sections in isospin symmetric and asymmetric matter. The present calculations include the additional effect of Pauli blocking of the final state.

I. BRIEF DESCRIPTION OF THE UPDATED CALCULATION

In a recent paper [1], we discussed in-medium nucleon-nucleon (NN) cross sections in a dense hadronic environment with and without isospin asymmetry. This note is a continuation of our previous work and the present purpose is to provide a large set of numerical results for possible applications in transport equations.

The cross sections we display here differ from those of Ref. [1] due to the inclusion of Pauli blocking of the final state. The need for such correction may be dependent on the details of the chosen Boltzmann-Uehling-Uhlenbeck (BUU) equation, as this effect is sometimes incorporated at the level of the transport calculation. In such case, though, a Pauli blocking function is usually applied together with in-medium differential cross sections. However, total effective cross sections with properly Pauli-blocked final states should be a better indication of the degree of suppression the interaction actually undergoes in the medium. Furthermore, this correction must be included for a realistic calculation of the nucleon mean free path in nuclear matter.

We calculate the total effective cross section as

$$\sigma(q_0, P_{tot}, \rho) = \int \frac{d\sigma}{d\Omega} Q(q_0, P_{tot}, \theta, \rho) d\Omega,$$

where \( \frac{d\sigma}{d\Omega} \) is given by the usual sum of amplitudes squared and phase space factors and \( Q \) is the Pauli operator. Ignoring Pauli blocking on the final momenta amounts to setting \( Q=1 \) in the integrand above, as done in Ref. [1]. (Note that the intermediate states are always Pauli-blocked in the microscopic calculation of the amplitudes.) All definitions as in Ref. [1], that is, \( q_0 \) is the nucleon momentum in the two-body c.m. frame (or the relative momentum of the two nucleons), and \( P_{tot} \) is the total momentum of the two-nucleon system with respect to the nuclear matter rest frame. In free space, the momentum \( q_0 \) and the incident laboratory kinetic energy are related through the well-known formula \( T_{lab} = 2q_0^2/m \), with \( m \) the free nucleon mass.

The presence of the Pauli operator in Eq.(1) restricts the integration domain to [2]

$$\frac{k_F^2 - P^2 - q_0^2}{2Pq_0} \leq \cos \theta \leq \frac{P^2 + q_0^2 - k_F^2}{2Pq_0},$$

where \( k_F \) is the Fermi momentum.
where $k_F$ is the Fermi momentum. The integral becomes zero if the upper limit is negative, whereas the full angular range is used if the upper limit is greater than one. (Following a previously used convention [2], $P$ in Eq.(2) denotes one-half the total momentum.) Notice that the angle $\theta$ in Eq.(2), namely the angle between the directions of $q_0$ and $\vec{P}$, is also the colatitude of $q_0$ in a system where the $z$-axis is along $\vec{P}$ and, thus, in such system it coincides with the scattering angle to be integrated over in Eq.(1).

Except for the inclusion of $Q$ in Eq.(1), the present microscopic calculation is the same as in Ref. [1]. We adopt the Bonn-B potential and the Dirac-Brueckner-Hartree-Fock (DBHF) approach to nuclear matter, unless otherwise stated. For simplicity, we use in-vacuum kinematics to define the total two-nucleon momentum in the nuclear matter rest frame (that is, the target nucleon is, on the average, at rest).

II. NUMERICAL RESULTS FOR PP AND NP EFFECTIVE CROSS SECTIONS IN SYMMETRIC MATTER

We begin with discussing our predictions in symmetric matter. These are displayed in Table I and II for $pp$ and $np$ scattering, respectively. The given values of $q_0$ cover a range between approximately 20 MeV and 350 MeV in terms of in-vacuum laboratory kinetic energy. The range of densities goes from zero to about twice saturation density. Due to the presence of the Pauli operator in Eq.(1), it’s clear that the cross sections will drop to zero at certain densities depending on the value of the momentum.

$pp$ cross sections:

At very low energy, we observe strong sensitivity to variations of the Fermi momentum when approaching the density where the cross section becomes identically zero.

At very low (fixed) density, the cross section decreases with energy, a behavior similar to what happens in free space. At higher densities, though, the cross section grows with energy. This is due to the fact that the Pauli operator in Eq.(1) becomes less effective at the higher energies.

For fixed momentum, the cross section typically decreases with increasing density. In Ref. [1] we discussed the tendency (of the $pp$ cross section, in particular) to rise again at high density and for the higher momenta. This typical “Dirac effect”, already observed in previous DBHF calculations, is of course still present in the interaction, but is less noticeable in these cross sections due to the additional Pauli suppression in Eq.(1). To better demonstrate its origin, we also show results obtained with the conventional Brueckner-Hartree-Fock approach (BHF) (Table III and IV for $pp$ and $np$, respectively). The differences are most apparent when moving down to the higher densities in the last few columns of Table I and comparing with the corresponding entries in Table III.

$np$ cross sections:

As in the $pp$ case, very strong $k_F$-sensitivity can be seen at the lowest momenta just before the cross section is totally suppressed.
For fixed momentum, the cross section typically decreases with increasing density. An interesting difference between \( pp \) and \( np \) cross sections is the fact that the latter rises with density at very low density and for the lowest momenta. We traced this effect to an enhancement of the \( ^3S_1 \) partial wave and did not observe a similar phenomenon in isospin-1 partial waves. The presence of low-density “peaks” (in the \( np \) channel in particular), was discussed before as a possible bound-state signature [3,4].

Finally, the differences between the values in Table II and Table IV for high momentum and high density are much less pronounced than in the \( pp \) case.

III. NUMERICAL RESULTS FOR \( PP \) AND \( NN \) EFFECTIVE CROSS SECTIONS IN ASYMMETRIC MATTER

Predictions for \( pp \) and \( nn \) total cross sections in isospin-asymmetric matter are displayed in Table V for different degrees of asymmetry. These are obtained with our DBHF model. The asymmetry parameter, \( \alpha \), is defined in the usual way in terms of neutron and proton densities, namely, \( \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \). We ignore the \( \alpha \)-dependence in the \( np \) channel as we previously found it to be very weak (due to the “competing” roles of protons and neutrons).

The \( nn \) cross section is (almost always) smaller than the \( pp \) cross section. This is due to the additional Pauli blocking included in Eq.(1), together with the fact that the neutron’s Fermi momentum is larger than the proton’s. In fact, as a consequence of Eq.(1), the \( nn \) effective cross section drops to zero more quickly (for the same average density). This is apparent from the Table. Although at low density the \( nn \) cross section tends to start larger, Pauli blocking soon takes over. Particularly for large values of \( \alpha \), the \( pp \) cross section “survives” much larger densities than the \( nn \). This is to be expected, because the proton Fermi momentum, \( k_F^p = k_F(1 - \alpha)^{1/3} \), tends to remain small for large \( \alpha \).

All of the above suggests the following observation: the region of the density/momentum phase space where \( \sigma_{nn} \) is nearly or entirely suppressed whereas \( \sigma_{pp} \) is still considerably different than zero should be a suitable ground to look for a clear experimental signature of their difference.

Some comments are in place concerning the relative sizes of \( \sigma_{pp} \) and \( \sigma_{nn} \). Predictions based on scaling the free-space cross section through the use of effective masses in the phase-space factor will generally have the trend:

- \( \sigma_{nn} > \sigma_{pp} \), if \( m_n^* > m_p^* \) [7];
- \( \sigma_{pp} > \sigma_{nn} \), if \( m_p^* > m_n^* \) [8].

Notice that our predictions in Ref. [1] are qualitatively consistent with the first case above, due to the dominant role of the effective masses in that calculation (we predict \( m_n^* > m_p^* \) [5]).

Obviously, empirical constraints that might shed light on these issues can only be indirect, for instance through analyses of carefully selected heavy-ion observables. Such efforts are presently under way [6,7]. In fact, a recent study indicates that the transverse flow of neutrons and protons may be a reliable probe of in-medium cross section [7]. As the present discussion suggests, when comparing theoretical predictions with experimental constraints it is important to be clear about the nature of the extracted effective cross section, namely
what medium effects are included in the quantity that is being constrained by the analysis.

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TABLE I. \(pp\) total effective cross sections in symmetric matter calculated with the DBHF model and according to Eq.(1). Kinematics and definitions of variables are explained in the text.

| \(k_F (fm^{-1})\) | \(\sigma_{pp} (mb)\) \(q_0 = 100MeV\) | \(\sigma_{pp} (mb)\) \(q_0 = 150MeV\) | \(\sigma_{pp} (mb)\) \(q_0 = 200MeV\) | \(\sigma_{pp} (mb)\) \(q_0 = 250MeV\) | \(\sigma_{pp} (mb)\) \(q_0 = 300MeV\) | \(\sigma_{pp} (mb)\) \(q_0 = 350MeV\) | \(\sigma_{pp} (mb)\) \(q_0 = 400MeV\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.0             | 171.2           | 69.33           | 39.86           | 28.93           | 23.96           | 21.47           | 20.27           |
| 0.2             | 157.1           | 61.71           | 34.97           | 25.56           | 21.70           | 20.05           | 19.47           |
| 0.4             | 133.4           | 52.91           | 29.90           | 22.32           | 19.63           | 18.80           | 18.82           |
| 0.6             | 64.34           | 42.58           | 24.97           | 19.39           | 17.85           | 17.78           | 18.31           |
| 0.8             | 0.00            | 29.06           | 20.11           | 16.71           | 16.30           | 16.92           | 17.91           |
| 0.9             | 0.00            | 19.86           | 17.61           | 15.42           | 15.57           | 16.52           | 17.71           |
| 1.0             | 0.00            | 8.980           | 14.99           | 14.86           | 16.13           | 17.51           | 17.31           |
| 1.1             | 0.00            | 0.00            | 11.98           | 12.70           | 14.07           | 15.72           | 17.31           |
| 1.2             | 0.00            | 0.00            | 8.602           | 11.14           | 13.24           | 15.29           | 17.13           |
| 1.3             | 0.00            | 0.00            | 5.025           | 9.478           | 12.37           | 14.89           | 17.01           |
| 1.4             | 0.00            | 0.00            | 1.583           | 7.793           | 11.54           | 14.56           | 16.98           |
| 1.5             | 0.00            | 0.00            | 0.00            | 6.123           | 10.05           | 14.20           | 17.25           |
| 1.6             | 0.00            | 0.00            | 0.00            | 4.466           | 10.05           | 14.20           | 17.25           |
| 1.7             | 0.00            | 0.00            | 0.00            | 2.698           | 9.286           | 14.04           | 17.40           |

TABLE II. \(np\) total effective cross sections in symmetric matter calculated with the DBHF model and according to Eq.(1). Kinematics and definitions of variables as explained in the text.

| \(k_F (fm^{-1})\) | \(\sigma_{np} (mb)\) \(q_0 = 100MeV\) | \(\sigma_{np} (mb)\) \(q_0 = 150MeV\) | \(\sigma_{np} (mb)\) \(q_0 = 200MeV\) | \(\sigma_{np} (mb)\) \(q_0 = 250MeV\) | \(\sigma_{np} (mb)\) \(q_0 = 300MeV\) | \(\sigma_{np} (mb)\) \(q_0 = 350MeV\) | \(\sigma_{np} (mb)\) \(q_0 = 400MeV\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.0             | 453.3           | 174.1           | 86.41           | 55.02           | 41.65           | 34.60           | 30.15           |
| 0.2             | 459.7           | 159.6           | 75.48           | 47.35           | 36.23           | 30.78           | 27.52           |
| 0.4             | 508.3           | 144.7           | 63.55           | 39.26           | 30.68           | 26.85           | 24.72           |
| 0.6             | 421.6           | 128.8           | 52.49           | 31.98           | 25.73           | 23.40           | 22.26           |
| 0.8             | 0.00            | 101.3           | 42.49           | 25.71           | 21.51           | 20.45           | 20.18           |
| 0.9             | 0.00            | 72.45           | 39.58           | 22.91           | 19.61           | 19.14           | 19.23           |
| 1.0             | 0.00            | 31.84           | 32.37           | 20.28           | 17.85           | 17.89           | 18.35           |
| 1.1             | 0.00            | 0.00            | 25.82           | 17.45           | 15.96           | 16.56           | 17.39           |
| 1.2             | 0.00            | 0.00            | 17.91           | 14.50           | 14.06           | 15.19           | 16.39           |
| 1.3             | 0.00            | 0.00            | 9.578           | 11.44           | 12.14           | 13.81           | 15.39           |
| 1.4             | 0.00            | 0.00            | 2.628           | 8.512           | 10.35           | 12.52           | 14.48           |
| 1.5             | 0.00            | 0.00            | 0.00            | 5.899           | 8.763           | 11.41           | 13.69           |
| 1.6             | 0.00            | 0.00            | 0.00            | 3.761           | 7.454           | 10.52           | 13.08           |
| 1.7             | 0.00            | 0.00            | 0.00            | 2.018           | 6.386           | 9.810           | 12.60           |
TABLE III. As in Table I, but with the BHF model.

| $k_F (fm^{-1})$ | $\sigma_{pp}(mb)$ | $\sigma_{pp}(mb)$ | $\sigma_{pp}(mb)$ | $\sigma_{pp}(mb)$ | $\sigma_{pp}(mb)$ | $\sigma_{pp}(mb)$ | $\sigma_{pp}(mb)$ |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                 | $q_0 = 100 MeV$  | $q_0 = 150 MeV$  | $q_0 = 200 MeV$  | $q_0 = 250 MeV$  | $q_0 = 300 MeV$  | $q_0 = 350 MeV$  | $q_0 = 400 MeV$  |
| 0.0             | 171.2            | 69.33            | 39.86            | 28.93            | 23.96            | 21.47            | 20.27            |
| 0.2             | 156.8            | 62.36            | 35.68            | 26.06            | 21.90            | 19.95            | 19.12            |
| 0.4             | 131.4            | 53.62            | 30.85            | 22.91            | 19.69            | 18.33            | 17.86            |
| 0.6             | 61.28            | 42.74            | 25.72            | 19.71            | 17.49            | 16.72            | 16.61            |
| 0.8             | 0.00             | 28.39            | 20.36            | 16.53            | 15.33            | 15.15            | 15.39            |
| 0.9             | 0.00             | 19.02            | 17.57            | 14.95            | 14.27            | 14.37            | 14.78            |
| 1.0             | 0.00             | 8.431            | 14.68            | 13.36            | 13.21            | 13.60            | 14.17            |
| 1.1             | 0.00             | 0.00             | 11.72            | 11.83            | 12.21            | 12.87            | 13.00            |
| 1.2             | 0.00             | 0.00             | 8.404            | 10.15            | 11.08            | 12.04            | 12.93            |
| 1.3             | 0.00             | 0.00             | 4.900            | 8.353            | 9.831            | 11.08            | 12.15            |
| 1.4             | 0.00             | 0.00             | 1.526            | 6.526            | 8.526            | 10.06            | 11.30            |
| 1.5             | 0.00             | 0.00             | 0.00             | 4.714            | 7.175            | 8.973            | 10.37            |
| 1.6             | 0.00             | 0.00             | 0.00             | 3.011            | 5.823            | 7.836            | 9.380            |
| 1.7             | 0.00             | 0.00             | 0.00             | 1.493            | 4.499            | 6.665            | 8.332            |

TABLE IV. As in Table II, but with the BHF model.

| $k_F (fm^{-1})$ | $\sigma_{np}(mb)$ | $\sigma_{np}(mb)$ | $\sigma_{np}(mb)$ | $\sigma_{np}(mb)$ | $\sigma_{np}(mb)$ | $\sigma_{np}(mb)$ | $\sigma_{np}(mb)$ |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                 | $q_0 = 100 MeV$  | $q_0 = 150 MeV$  | $q_0 = 200 MeV$  | $q_0 = 250 MeV$  | $q_0 = 300 MeV$  | $q_0 = 350 MeV$  | $q_0 = 400 MeV$  |
| 0.0             | 453.3            | 174.1            | 86.41            | 55.02            | 41.65            | 34.60            | 30.15            |
| 0.2             | 462.3            | 162.0            | 77.26            | 48.79            | 37.38            | 31.61            | 28.04            |
| 0.4             | 520.4            | 149.4            | 66.39            | 41.42            | 32.38            | 28.06            | 25.39            |
| 0.6             | 464.9            | 135.6            | 55.71            | 34.23            | 27.46            | 24.57            | 22.78            |
| 0.8             | 0.00             | 109.2            | 45.62            | 27.64            | 22.91            | 21.30            | 20.34            |
| 0.9             | 0.00             | 78.78            | 40.53            | 24.59            | 20.77            | 19.76            | 19.18            |
| 1.0             | 0.00             | 34.49            | 35.03            | 21.68            | 18.72            | 18.26            | 18.05            |
| 1.1             | 0.00             | 0.00             | 28.91            | 19.00            | 16.85            | 16.89            | 17.02            |
| 1.2             | 0.00             | 0.00             | 20.99            | 16.15            | 14.89            | 15.43            | 15.89            |
| 1.3             | 0.00             | 0.00             | 11.91            | 13.16            | 12.88            | 13.88            | 14.66            |
| 1.4             | 0.00             | 0.00             | 3.466            | 10.13            | 10.90            | 12.33            | 13.42            |
| 1.5             | 0.00             | 0.00             | 0.00             | 7.161            | 8.971            | 10.78            | 12.14            |
| 1.6             | 0.00             | 0.00             | 0.00             | 4.442            | 7.137            | 9.263            | 10.85            |
| 1.7             | 0.00             | 0.00             | 0.00             | 2.129            | 5.432            | 7.790            | 9.562            |
TABLE V. \(pp\) and \(nn\) total cross sections in asymmetric nuclear matter calculated with the DBHF model and according to Eq.(1). Kinematics and definition of variables are given in the text.

| \(\alpha\) | \(q_0 (\text{MeV/c})\) | \(k_F (\text{fm}^{-1})\) | \(\sigma_{pp} (\text{mb})\) | \(\sigma_{nn} (\text{mb})\) |
|---|---|---|---|---|
| 0.2 | 100 | 0.2 | 154.6 | 160.0 |
| 0.4 | 131.2 | 135.9 |
| 0.5 | 110.5 | 104.9 |
| 0.6 | 78.42 | 47.76 |
| 0.7 | 32.71 | 0.0 |
| 200 | 0.2 | 34.38 | 35.63 |
| 0.4 | 29.17 | 30.75 |
| 0.6 | 24.44 | 25.62 |
| 0.8 | 20.07 | 20.23 |
| 1.0 | 15.76 | 14.28 |
| 1.1 | 13.28 | 10.55 |
| 1.2 | 10.55 | 6.567 |
| 1.3 | 7.362 | 2.528 |
| 1.4 | 4.405 | 0.0 |
| 1.5 | 1.660 | 0.0 |
| 300 | 0.2 | 21.41 | 22.04 |
| 0.4 | 19.26 | 20.06 |
| 0.6 | 17.56 | 18.21 |
| 0.8 | 16.18 | 16.48 |
| 1.0 | 14.97 | 14.80 |
| 1.1 | 14.32 | 13.85 |
| 1.2 | 13.63 | 12.86 |
| 1.3 | 12.89 | 11.86 |
| 1.4 | 12.24 | 10.81 |
| 1.5 | 11.70 | 9.893 |
| 1.6 | 11.22 | 8.877 |
| 1.7 | 10.76 | 7.731 |
| 400 | 0.2 | 19.37 | 19.60 |
| 0.4 | 18.72 | 18.94 |
| 0.6 | 18.29 | 18.37 |
| 0.8 | 17.99 | 17.86 |
| 1.0 | 17.71 | 17.34 |
| 1.1 | 17.57 | 17.06 |
| 1.2 | 17.48 | 16.81 |
| 1.3 | 17.43 | 16.60 |
| 1.4 | 17.52 | 16.45 |
| 1.5 | 17.75 | 16.44 |
| 1.6 | 18.10 | 16.44 |
| 1.7 | 18.46 | 16.36 |
| 0.4 | 100 | 0.2 | 153.8 | 162.9 |
| 0.4 | 131.3 | 138.6 |
| 0.5 | 111.6 | 100.0 |
|   |   | 0.6 | 0.7 | 0.8 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|   |   | 200 |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   | 300 |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   | 400 |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   | 100 |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|   |   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 0.6|   |     |   |   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     
|   |     |     |     |
|---|-----|-----|-----|
|   | 200 | 300 | 400 |
| 0.2 | 33.21 | 20.82 | 19.15 |
| 0.4 | 27.77 | 18.57 | 18.57 |
| 0.6 | 23.45 | 17.04 | 18.33 |
| 0.8 | 20.01 | 16.00 | 18.26 |
| 1.0 | 17.14 | 15.24 | 18.21 |
| 1.1 | 15.53 | 14.84 | 18.21 |
| 1.2 | 13.70 | 14.42 | 18.58 |
| 1.3 | 11.71 | 14.02 | 18.26 |
| 1.4 | 9.672 | 13.70 | 18.26 |
| 1.5 | 7.618 | 13.51 | 18.57 |
| 1.6 | 5.727 | 13.54 | 18.33 |
| 1.7 | 4.124 | 13.77 | 18.26 |
|     |     |     |     |
| 0.2 | 37.04 | 22.76 | 19.87 |
| 0.4 | 32.59 | 21.03 | 19.27 |
| 0.6 | 27.08 | 19.05 | 18.58 |
| 0.8 | 20.50 | 16.94 | 17.86 |
| 1.0 | 12.35 | 14.72 | 17.08 |
| 1.1 | 7.168 | 13.46 | 16.65 |
| 1.2 | 2.156 | 12.20 | 16.26 |
| 1.3 | 0.00  | 10.82 | 15.85 |
| 1.4 | 0.00  | 9.389 | 15.48 |
| 1.5 | 0.00  | 8.046 | 15.48 |
| 1.6 | 0.00  | 6.488 | 15.22 |
| 1.7 | 0.00  | 4.664 | 14.91 |
|     |     |     |     |
| 0.2 | 20.91 | 14.50 |     |