Einstein-aether theory as an alternative to dark energy model?

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\textbf{A B S T R A C T}

In the generalized Einstein-aether theories by taking a special form of the Lagrangian density of aether field, the possibility of Einstein-aether theory as an alternative to dark energy model is discussed in detail, that is, taking a special aether field as a dark energy candidate. We compute the joint statistic constraints on this special model's parameters by using the recent type Ia supernovae (SNe Ia) data, the Cosmic Microwave Background (CMB) shift parameter data, and the Baryonic Acoustic Oscillations (BAOs) data traced by the Sloan Digital Sky Survey (SDSS). Furthermore, we analyze other constrains from the Observational Hubble parameter Data (OHD). The comparison with the standard cosmological model (cosmological constant $\Lambda$ Cold Dark Matter (LCDM) model) is clearly shown with new features; also we comment on an interesting relation between the coupling constant $M$ in this model and the possible existence of a special accelerating scale in the MOdified Newtonian Dynamics (MOND) model initially given by Milgrom with the hope for interpreting the galaxy rotation curves without introducing mysterious dark matter.

\section{1. Introduction}

The independent discovery respectively in 1998 and 1999 that the current universe expansion is actually speeding up rather than previously thought slowing-down due to the evolution dynamics dominated by then cosmic matter component (mainly the speculated mysterious cold dark matter) [1] has been an amazing result. To account for that cosmic accelerating expansion, together with other astrophysics observations, such as CMB, large scale structure survey, like the SDSS, and the universe age or Hubble constant measurements, a so coined dark-energy component (even more puzzling than the dark matter concept) with enough negative pressure has been hypothesized. According to the Wilkinson Microwave Anisotropy Probe (WMAP) 7-year data-set analysis [2] it makes up about 72.8\% of the universe contents. However, dark energy may be the most mysterious component of the universe hitherto as envisioned, we know little about what it is and its nature. Over the past decade there have been many theoretical models for mimicking the dark energy behaviors, such as the simplest (just) cosmological constant and the popular quintessence models [3]. An alternatively instructive idea is that the general theory of relativity may fail to describe the universe evolution on very large cosmic scales. Some attempts have been made at modifying the standard general relativity such as the $f(R)$ extended gravity models [4], string theory inspired cosmology models, brane cosmology and the holographic principle applicable to the universe evolution modelings. In the present work we concentrate on the generalized Einstein-aether theories as proposed by T.G. Zlosnik, P.G. Ferreira and G.D. Starkman [5,6], which is a generalization of the Einstein-aether theory developed by Ted Jacobson and David Mattingly [7,8].

These years a lot of work has been done in generalized Einstein-aether theories on macroscopic scales, from the Solar System [9], clusters [10], to large scale structure and CMB [6,11,12]. It seems that this theory could provide a consistent way to explain dark matter and dark energy (still with some problems on dark matter [11]). We discuss a special case of generalized Einstein-aether theories and hope to get more constraints from other observations.

Arrangement for this Letter is as follows. In the next section, Section 2, we briefly review the framework of generalized Einstein-aether theory by providing its basic equations for our later use. In Section 3, we take a special form of the Lagrangian density of aether field and discuss the corresponding modified Friedmann equations. In Section 4 followed, we describe how to employ the observational data sets used for joint statistics analysis in details with the hope that this clear development can be useful to the related astrophysics and cosmology community. In Section 5 by...
figures and tables, we show our results compared with the currently standard cosmology ΛCDM model. The last section, Section 6 contains our conclusions and discussions.

2. Einstein-aether theory

In the early history for modern physics, the concept aether is considered to be a physical medium homogeneously occupying every point in our universe. It determines a special rest reference frame, in which everything has absolute relative velocity respect to it. That suits for Newtonian dynamics very well. Later, this puzzling concept is rejected by Einstein’s relativity with mainly optics experiments. By saying “aether” framework, a term in the present letter, we do not mean a mechanical medium naively, but rather a locally preferred state resting for each point of the spacetime in our physical evolutionary universe, determined by some hitherto unknown physics or its physical state is to be specified by some physical conditions with its environment. Some people even argue that the smoothly distributed CMB everywhere may be regarded as a modern version of aether. Einstein-aether theories were popularized by Gasperini in a series of papers [13]. A vector–tensor theory is suggested by Ted Jacobson and David Mattingly [7,8], where in a modern version of aether. Einstein-aether theories were popularized by T.G. Zlosnik, P.G. Ferreira and G.D. Starkman [5,6].

The action of this theory with the normal Einstein–Hilbert part action can be written in the form below

\[ S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} + L_A + L_M \right), \]

where \( L_A \) is the vector field Lagrangian density while \( L_M \) denotes the Lagrangian density for all other matter fields. The Lagrangian density for the vector part consists of terms quadratic in the field and its derivatives [5]:

\[ L_A = \frac{M^2}{16\pi G} F(K) + \frac{1}{16\pi G} \lambda (A^\alpha A_\alpha) + 1, \]

\[ \mathcal{K} = M^{-2} F^\alpha, \gamma_\sigma \nabla_A A^\gamma \nabla_\beta A^\sigma, \]

\[ K_{\alpha\beta}^{\gamma\sigma} = c_1 g^{\alpha\beta} g_{\gamma\sigma} + 2 c_2 g^{\alpha\sigma} g_{\beta\gamma} + 2 c_3 g^{\alpha\gamma} g_{\beta\sigma}, \tag{2} \]

where \( c_1 \) are dimensionless constants and the coupling constant \( M \)
has the dimension of mass. The \( F(K) \) is a free function that we do not know a priori, and the \( \lambda \) is a Lagrange multiplier that enforces the unit constraint for the time-like unit vector field which picks out a preferred frame at each point in the spacetime. Then it is generalized by T.G. Zlosnik, P.G. Ferreira and G.D. Starkman [5,6].

We choose the inverse metric tensor \( g^{\alpha\beta} \) and the contravariant vector field \( A^\beta \) to be our dynamic degrees of freedom. Field equations from varying the action (1) with respect to \( g^{\alpha\beta} \) and \( A^\beta \) respectively are given by

\[ G_{\alpha\beta} = \tilde{T}_{\alpha\beta} + 8\pi G T_{\alpha\beta}^{\text{matter}}, \]

\[ \nabla_\alpha (F^\alpha, \gamma_\sigma) = 2\lambda A_\beta, \]

where \( \tilde{T}_{\alpha\beta} \) is the stress–energy tensor for the vector field and

\[ F = \frac{df}{d\kappa}, \quad J^\alpha_\sigma = 2\kappa g^{\alpha\sigma} \gamma_\sigma \nabla_\beta A^\gamma. \]

For the choice (2), \( \tilde{T}_{\alpha\beta} \) is given by [5]

\[ \tilde{T}_{\alpha\beta} = \frac{1}{2} \nabla_\sigma \left[ F (J^\sigma (A^\beta) - J^\beta (A^\sigma) - J_\alpha (A)) \right] \]

\[ - F_J (A^\alpha) + \frac{1}{2} g_{\alpha\beta} M^2 F + \lambda A_\alpha A_\beta, \]

where the subscript \( (\alpha\beta) \) means symmetric with respect to the indices involved and

\[ Y_{\alpha\beta} = -c_1 \left[ (\nabla_\alpha A_\beta) (\nabla_\gamma A^\gamma) - (\nabla_\alpha A_\gamma) (\nabla_\gamma A^\beta) \right]. \tag{7} \]

In addition, the constraint that \( A \) is a time-like unit vector field gives \( A^\alpha A_\alpha = -1 \).

3. Modified Friedmann equations

3.1. Background spacetime

Now we consider the case of a homogeneous and isotropic universe as preferred by the WMAP observations, which can be described by the Friedmann–Robertson–Walker metric

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{1}{1 - kr^2} dr^2 + r^2 d\Omega^2 \right). \tag{8} \]

where \( k \) is the curvature parameter. In such a case, the vector must respect the spatial homogeneity and isotropy of the universe at large. Thus the only component non-vanishing is the time-like component. Using the constraint \( A^\alpha A_\alpha = -1 \), we can get

\[ A^\alpha = (1, 0, 0, 0). \tag{9} \]

We take the matter component as a perfect fluid, so its energy–momentum tensor is of the form

\[ T_{\alpha\beta}^{\text{matter}} = \rho U_\alpha U_\beta + p (U_\alpha U_\beta + g_{\alpha\beta}), \]

where \( U_\alpha \) is the fluid four-velocity. By using (8) and (9), \( \mathcal{K} \) can be simplified as

\[ \mathcal{K} = M^{-2} (c_1 g^{\alpha\beta} g_{\gamma\sigma} + c_2 g^{\beta\gamma} g_{\alpha\sigma} + c_3 g^{\beta\sigma} g_{\alpha\gamma}), \]

\[ = 3\alpha H^2 \Omega_K \]}

where the coefficient \( \alpha = c_1 + 3c_2 + c_3 \) and Hubble parameter \( H \equiv \dot{a} / a \). It is easy to find by calculation that the stress–energy tensor for the vector field also takes the form of a perfect fluid, with an energy density given by (also see in [6])

\[ \rho_\lambda = 3\alpha H^2 \left( \frac{\dot{F}^\alpha}{2\kappa} - \frac{F}{2\kappa} \right) \]

and a pressure as

\[ p_\lambda = 3\alpha H^2 \left( -\frac{1}{3} F^\alpha + \frac{F}{2\kappa} - \frac{\dot{F}}{\kappa^2} \right) - \alpha \dot{F} H - \alpha F \ddot{\dot{a}}. \tag{13} \]

We can check that the vector field part contributions obey the cosmological energy conservation relation \( \dot{\rho}_\lambda + \dot{3}H(\rho_\lambda + p_\lambda) = 0 \). A simple case has been discussed by Sean M. Carroll and Eugene A. Lim in Ref. [14]. Now we show that this conservation relation is applicable to an arbitrary form of \( F(K) \) (also see in [12]).

Taking (8)–(11) into field equations (3) and (4), the modified Friedmann equations can be derived (see also in [6]):

\[ \left( 1 - \alpha F^\alpha + \frac{1}{2} \frac{\alpha F}{\kappa} \right) H^2 + \frac{8\pi G}{3} \frac{\dot{a}^2}{a^2} = \rho. \tag{14} \]

\[ \frac{d}{dt} \left( -2H + \alpha F^\alpha H + \frac{2k}{\alpha^2} \right) = 8\pi G (\rho + p). \tag{15} \]

In order to see what have been modified, we list the standard Friedmann equations in the ΛCDM model below for comparison:

\[ H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3} \rho. \tag{16} \]

\[ -\frac{dH}{dt} + \frac{2k}{a^2} = 8\pi G (\rho + p). \tag{17} \]
We can see that a few terms involving $\mathcal{F}(K)$ and its first order derivative are present, which may imply that an effective term involving cosmological “constant” or an effective cosmological “constant” (the effective vacuum energy for the universe) can be from the vector field’s contributions, the mysterious “aether”.

In the interesting works [6] and [11], a class of theories with $\mathcal{F}(K) = \gamma(-K)^n$ have been discussed. Noting that $\gamma < 0$ [15], $K$ is negative there. It has been shown that the scale $M \sim H_0$ and for appropriate parameters the generalized Einstein-aether theories can lead to a late-time acceleration of the universe expansion (for example, the $n = 0$ case is just corresponding to the $\Lambda\text{CDM}$ model). For that form of $\mathcal{F}(K)$, the first modified Friedmann equation (14) becomes [6]

$$
\left[1 + \epsilon \left(\frac{H}{M}\right)^{2(n-1)}\right] H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho, 
$$

where $\epsilon = -(2n - 1)\gamma(-2\alpha)^n/6$. We can see if $n = \frac{1}{2}$, $\epsilon = 0$ and the modified terms disappear. Thus, there will not be a modified term proportional to $H$. However, what about other forms of $\mathcal{F}(K)$?

In the following part of this Letter we consider a special case, in which we take

$$
\mathcal{F}(K) = \beta \sqrt{-K} + \sqrt{\frac{3K}{\alpha}} \ln(-K), \tag{19}
$$

where $\beta$ is a constant. Taking Eq. (19) into (14), Eq. (14) then becomes

$$
H^2 - MH + \frac{k}{a^2} = \frac{8\pi G}{3} \rho, \tag{20}
$$

where we have used $K = 3\alpha H^2/M^2$ as in given Eq. (11).

It is easy to figure out that when $\rho \to 0$, $H \to M$ for the evolutionary universe geometry almost flat at the late stage as indicated from the WMAP observations now. That is to say at late time period of the universe evolution when $\rho \propto a^{-3} \to 0$, the universe will keep its accelerating expansion due to the existence of the aether field and an interesting scale appears $M \to H$ then. We shall return to discuss in more detail this point at the end of Section 6.

Furthermore, we can calculate the effective state parameter for the vector field part and the deceleration parameter for our choice of the function $\mathcal{F}(K)$:

$$
w_A = \frac{p_A}{\rho_A} = -\frac{\dot{H}}{H^2} - 1, \tag{21}
$$

$$
q = -\frac{\ddot{a}}{\dot{a}^2} = -\frac{\ddot{H}}{H^2} - 1 = 3(w_A + 1) - 1. \tag{22}
$$

From Eq. (22) we know directly that to explain the speeding up of the universe expansion as implied by lots of astrophysics observations, the effective state parameter for the vector field part contributions today must be smaller than $-\frac{1}{3}$ (instead of the $-\frac{1}{2}$ as given directly from the standard Friedmann equations for the $\Lambda\text{CDM}$ model).

3.2. Stability analysis

In [8,11,15–17], the stability of theories including “aether” fields has been considered. It is shown that there may be unstable propagating modes. So we analyze the stability of the special case we choose following the process in [11]. In that paper, Zuntz et al. calculated the linear perturbation of generalized Einstein-aether theories in FRW background. To avoid exponentially growing modes, they found the following constraints on parameters $c_2$ and $\mathcal{F}(K)$:

$$
1 + \mathcal{F}'(c_1 + c_3) > 0, \tag{23}
$$

$$
\dot{\alpha} + 2c_1(c_1 + c_3) > 0, \tag{24}
$$

$$
\mathcal{F}' \left(1 + \mathcal{F}' \frac{c_1^2 - c_3^2}{2c_1} \right) > 0. \tag{25}
$$

Primes denote derivatives with respect to $K$ and $\dot{\alpha} = (1 + 2\frac{\Omega_{\Lambda}}{A^2} - 2\alpha)\alpha$. In addition, to avoid violating energy conditions, they found two more constraints:

$$
1 - \frac{1}{2} \alpha^2 \mathcal{F}' > 0, \tag{26}
$$

$$
1 + \frac{1}{2} c_1 \mathcal{F}' > 0. \tag{27}
$$

For our choice of $\mathcal{F}(K)$,

$$
\mathcal{F}' = -\frac{1}{\sqrt{-K}} \left[\beta + \sqrt{-\alpha}(1 + \frac{1}{2} \ln(-K))\right], \tag{28}
$$

$$
\dot{\alpha} = \left(\frac{\beta}{\sqrt{-\alpha}}\right)^2 \left[\frac{1}{\sqrt{-K}} \left(1 + \frac{1}{2} \ln(-K)\right)\right]. \tag{29}
$$

From Eq. (20), it is easy to get that $H > M$ (we have taken $k = 0$), which leads to $K < 3\alpha$. We can immediately check that constraint (26) is satisfied. To make the problem more explicit, in the following part we consider a simple example: $c_1 < 0$ (this is often needed to be consistent with the MOND limit [5]), $\alpha \sim -1$, $\beta = -2\sqrt{-\alpha}$. Under this condition, we only need to consider the first three constraints since the last two are naturally satisfied. Thus we get the following constraints from (23), (24) and (25) respectively:

$$
c_1 + c_3 < \epsilon \sqrt{-\alpha}, \tag{30}
$$

$$
c_1 + c_3 < -\frac{\alpha}{\ln(-K_{lower})}, \tag{31}
$$

$$
0 < \frac{c_1}{c_1^2 - c_3^2} < \frac{\sqrt{3} \ln(-K_{lower})}{4\sqrt{\alpha} K_{lower}}. \tag{32}
$$

Here $K_{lower}$ denotes the lower bound of $K$. It is worth noting that if $|K| \gg 1$, as is the case of the early universe, constraint (32) would be rather strict. However, the above constraints still do not rule out the model we consider in this Letter.

4. Current observational data

4.1. Type Ia supernovae

The observations of type Ia supernovae (SNe Ia) provide an excellent tool for probing the expansion history of the universe. Because all type Ia supernovae explode at about the same mass, their absolute magnitudes are considered to be all the same ($M \equiv -19.3 \pm 0.3$). This makes them very useful as standard candles. The observation of supernovae measure essentially the apparent magnitude $m$. The theoretical distance modulus is defined as

$$
m_{\text{th}} = m_{\text{th}} - M = 5 \log_{10} D_L(z) + \mu_0, \tag{33}
$$

where $D_L (z) \equiv H_0 \zeta_{DL} (z)$ is the dimensionless luminosity and $\mu_0 = 42.38 - 5 \log_{10} h$. Here $h$ is the dimensionless Hubble parameter today. $D_L (z)$ is given by

$$
D_L (z) = \frac{1 + z}{\sqrt{H_0 k}} \sinh \left[ H_0 \sqrt{\frac{k}{H(z)}} \int_0^z \frac{dz'}{H(z')}, \right]. \tag{34}
$$
where $\Omega_0$ is the fractional curvature density today. In this Letter we use the Union2 data set consisting of 557 supernovae. The corresponding $\chi^2_{SN}$ function to be minimized is

$$
\chi^2_{SN} = \sum_{i=1}^{557} \left[ \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \theta)}{\sigma_i} \right]^2,
$$

(35)

where $\theta$ denotes the model parameters. The minimization with respect to $\mu_0$ can be made trivially by expanding $\chi^2_{SN}$ with respect to $\mu_0$ as [18]

$$
\chi^2_{SN}(\theta) = A - 2\mu_0B + \mu_0^2C,
$$

(36)

where

$$
A(\theta) = \sum_{i=1}^{557} \left[ \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \theta)}{\sigma_i} \right]^2,
$$

(37)

$$
B(\theta) = \sum_{i=1}^{557} \left[ \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \theta)}{\sigma_i} \right],
$$

(38)

$$
C = \sum_{i=1}^{557} \frac{1}{\sigma_i^2}.
$$

(39)

Eq. (36) has a minimum for $\mu_0 = B/C$ at

$$
\chi^2_{SN}(\theta) = A(\theta) - \frac{B^2(\theta)}{C}.
$$

(40)

Thus instead of minimizing $\chi^2_{SN}$ we can minimize $\tilde{\chi}^2_{SN}$ which is independent of $\mu_0$.

4.2. Cosmic microwave background and baryonic acoustic oscillations

In addition to the type Ia supernovae data, we use the Cosmic Microwave Background (CMB) shift parameter and the baryonic acoustic oscillations to compute the joint constraints. The shift parameter $\tilde{\mathcal{R}}$ is defined in [19] as

$$
\tilde{\mathcal{R}} \equiv \sqrt{\Omega_mH_0^2(1+z_\ast)}D_A(z_\ast),
$$

(41)

where $z_\ast$ is the redshift of recombination and $D_A(z)$ is the proper angular diameter distance:

$$
D_A(z) = \frac{1}{H_0(1+z)\sqrt{\Omega_k}} \sinh \left[ H_0\sqrt{\Omega_k} \int_0^{z} \frac{dz'}{H(z')} \right].
$$

(42)

The seven-year WMAP results [20] have updated the redshift of recombination $z_\ast = 1091.3$ and the shift parameter $R = 1.725 \pm 0.018$. The $\chi^2$ for CMB shift is

$$
\chi^2_{CMB} = \frac{|\tilde{\mathcal{R}}(\theta) - 1.725|^2}{0.018^2}.
$$

(43)

Another constraint is from the Baryonic Acoustic Oscillations (BAOs) traced by the Sloan Digital Sky Survey (SDSS). In this Letter we use only one node at $z = 0.35$. The distance parameter $A$ is defined as [21]

$$
A \equiv D_V(0.35)\sqrt{\Omega_mH_0^2},
$$

(44)

where $D_V$ is the effective distance

$$
D_V(z) = \left[ (1+z)^2D_A^2(z) \frac{z}{H(z)} \right]^{\frac{1}{2}}.
$$

(45)

In that paper, Tong-Jie Zhang and Cong Ma summarize the up-to-date data of observational Hubble parameter date (OHD). See Table 1. The data points at $z = 0.24$ and $z = 0.43$ are derived from the "radial BAO size method", while the others are derived from the "differential age method" as named.

The $\chi^2$ for OHD is

$$
\chi^2_{OHD} = \sum_{i=1}^{13} \frac{|H_0E(z_i) - H_{obs}(z_i)|^2}{\sigma_i^2}.
$$

(46)

where $E(z) \equiv H(z)/H_0$ is independent of $H_0$. Using the same trick mentioned before, the minimization with respect to $H_0$ can be made trivially by expanding $\chi^2_{OHD}$ with respect to $H_0$ as

$$
\chi^2_{OHD}(H_0) = H_0^2A - 2H_0B + C,
$$

(47)

where

$$
A = \sum_{i=1}^{13} \frac{E^2(z_i)}{\sigma_i^2},
$$

(48)

$$
B = \sum_{i=1}^{13} \frac{E(z_i)H_{obs}(z_i)}{\sigma_i^2},
$$

(49)

$$
C = \sum_{i=1}^{13} \frac{H_{obs}^2(z_i)}{\sigma_i^2}.
$$

(50)

Equation (49) has a minimum for $H_0 = B/A$ at

$$
\tilde{\chi}^2_{OHD} = -\frac{B^2}{A} + C.
$$

(51)

Thus, instead of minimizing $\chi^2_{OHD}$ we can minimize $\tilde{\chi}^2_{OHD}$ which is independent of $H_0$. From Table 1, we can see the errors of OHD
For the $\Lambda$CDM model, the general expression for the expansion relation can be directly written out as

$$H(z) = H_0 \sqrt{\Omega_m + 2 \Omega_k + \Omega_m (1+z)^3 + \Omega_k (1+z)^4},$$

(54)

where $\Omega_A$, $\Omega_k$, $\Omega_m$, $\Omega_R$ are the fractional density of vacuum, curvature, matter and radiation today, respectively.

$$\Omega_A \equiv \frac{\Lambda}{3H_0^2}, \quad \Omega_k \equiv -\frac{k}{a^2 H_0^2},$$

$$\Omega_m \equiv \frac{8\pi G \rho_m}{3H_0^2}, \quad \Omega_R \equiv \frac{8\pi G \rho_R}{3H_0^2}.$$  

(55)

In this Letter, we fix $\Omega_0 = \Omega_0 \{1 + 0.2271N_{\text{eff}}\}$ and take the present photon density parameter $\Omega_\gamma = 2.469 \times 10^{-5}h^{-2}$ (for $T_{\text{CMB}} = 2.725$ K) and the effective number of neutrino species at its standard value 3.04 [20]. We also use the prior $h = 74.2 \pm 3.6$ given in [26]. So there are only two independent parameters $\Omega_0$ and $\Omega_m$. The parameter $\Omega_k$ can be expressed by the others:

$$\Omega_k = 1 - \Omega_A - \Omega_m - \Omega_R.$$  

(56)

For the Einstein-aether theory with our choice of the function $\mathcal{F}(K)$, we can solve $H(z)$ from (20):

$$H(z) = H_0 \left[ \frac{\Omega_A}{2} + \sqrt{\frac{\Omega_A^2}{4} + \Omega_k (1+z)^2 + \Omega_m (1+z)^3 + \Omega_R (1+z)^4} \right].$$

(57)

where we define

$$\Omega_A \equiv \frac{M}{H_0}.$$  

(58)

There are also only two parameters $\Omega_A$ and $\Omega_m$. Similarly, $\Omega_k$ can be expressed as (56), in which $\Omega_A$ needs to be replaced by $\Omega_A$.

Firstly, we compute the combined constraints from SNe Ia, CMB shift and BAO data sets. The results are shown in Table 2 and Fig. 1. The best-fit values for parameters of the $\Lambda$CDM model are consistent with the results given by WMAP 7 [2]. It gives a nearly flat universe geometry with a tiny $\Omega_k = 4 \times 10^{-3}$. For the Einstein-aether theory case as we choose, the best-fit $\Omega_m$ is a little bit smaller than that in the $\Lambda$CDM model, but it gives a larger $\Omega_k = 0.04$. The $\Lambda$CDM model fits better to the data sets as its $\chi^2_{\text{min}}$ is 26.182 smaller than that of Einstein-aether theory case as shown.

Then, we do similar analysis with the OHD (data sets). The results are shown in Table 2 and Fig. 2 as well. This time, we can see that the Einstein-aether theory case fits a little better to the data sets, and the difference of the $\chi^2_{\text{min}}$ is very small now.

Noting the similarity of our modified Friedmann equation to that in DGP brane world model [28–30] and data analysis being done to DGP model [31,32], these results are rather natural. Using the results of combined analysis we also plot the effective state parameter of the vector field part contribution $w_A(z)$

### Table 2

| Model                          | Best-fit parameters $\chi^2_{\text{min}}$ |
|--------------------------------|-------------------------------------------|
| SNe Ia+CMB shift+BAO (Union2)  | $\Omega_0 = 0.272$, $\Omega_A = 0.728$, $\Omega_m = 0.220$, $\Omega_k = 0.739$ | 542.693 |
| Einstein-aether theory         |                                           | 568.875 |

Fig. 1. Joint constraints from SNe Ia, CMB shift and BAO. The 1σ, 2σ, 3σ confidence interval contours of $\Omega_m$ and $\Omega_A$ or $\Omega_k$ for the Einstein-aether theory case in the $\Lambda$CDM model (left) and the Einstein-aether theory case (right).
Fig. 2. Constraints from OHD. The $1\sigma$, $2\sigma$, $3\sigma$ confidence interval contours of $\Omega_m$ and $\Omega_\Lambda$ (or $\Omega_A$ for Einstein-aether theory) in the $\Lambda$CDM model (left) and Einstein-aether theory (right).

Fig. 3. The effective state parameter of the vector field.

Fig. 4. The deceleration parameter. The thick solid line is the result of the Einstein-aether theory, while the thin dashed line represents the $\Lambda$CDM model.

6. Conclusions and discussions

In the present work we only consider in detail a special case of the Einstein-aether theory and compute the joint constraints from observations such as SNe Ia, CMB shift, BAO data sets, and OHD respectively. Even though we only investigate a special Einstein-aether model we already find that it has shown lots of interesting features, by well fitting to the combined data sets of the SNe Ia, CMB shift and BAO, as well as OHD respectively, and comparing with the $\Lambda$CDM model.

The observational Hubble parameter data we have possessed now are relatively few and not so accurate. However, with the improving quality of observational $H(z)$ data and more data points being measured (more sample compiled hopefully), it will be certainly a directly useful tool to test dark energy models and modified theories of general relativity, as well as corresponding cosmology models.

Moreover, it is clearly shown in this special model that $M = \Omega_A H_0 \sim H_0$, which is consistent with the requirements of the MOND limit [5,6,11]. It may give one possible explanation by the Einstein-aether model (other comments can be found in [11]) that why Milgrom [27] found the constant $q_0$, as introduced initially to explain rotation curves of galaxies, is likely to be associated with the Hubble constant $H_0$ globally describing the expansion of the universe (it might also relate to the Unruh temperature for the Hubble horizon).

For the last point of the present work we would like to make (but not the least importance), we should emphasize especially that theoretically we cannot take it for granted that the phenomenological MOND theory can reproduce all the systematics of Rotational Curves (RCs) observations, although the MOND model fits BETTER than the $\Lambda$CDM based mass models. It is still far away to reproduce the wide and far telling systematics of the spiral galaxies’ RCs (and of the mass distribution in the corresponding galaxies) [33], so there will be lots of detail work to be done with any modified gravity proposals at least on the galaxy scale.

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