Understanding Modified Two-Slit Experiments Using Path Markers

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Received: 8 December 2022 / Accepted: 8 March 2023 / Published online: 18 March 2023
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Abstract
Some modified two-slit interference experiments were carried out showing an apparent paradox in wave–particle duality. In a typical such experiment, the screen, where the interference pattern is supposed to be formed, is replaced by a converging lens. The converging lens forms the images of the two slits at two spatially separated detectors. It was claimed that each of these two detectors give information about which slit a photon came from, even though they come from the region of interference. These experiments generated a lot of debate. The various refutations pointed out that the controversial claims involved some questionable assumptions. However the refutations were largely philosophical in nature, and one may like to substantiate those with arguments which are testable, at least in principle. Here such an experiment is theoretically analyzed by introducing path markers which are two orthogonal polarization states of the photon. Analyzing the polarization at the two detectors shows that the photons which give rise to interference, and reach a particular detector, always come from both the slits. This provides clarity in understanding such experiments by making use of testable quantum correlations.

Keywords Wave–particle duality · Complementarity · Two-slit interference

1 Introduction

The two-slit experiment with massive particles, or single photons is probably the simplest experiment which captures the most intriguing features of quantum mechanics, especially in a situation where one wants to probe which of the two slits the particle passed through. If the particles show interference, it is not possible to know which slit each of them passed through. The moment one acquires the knowledge about which slit the particle passed through, the interference is lost. Notable is

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the fact that no real experimenter or classical apparatus is needed here—even if the
path information gets encoded in another quantum system, by way of entanglement,
that is enough to destroy interference. Niels Bohr elevated this concept to the sta-
tus of a separate principle, the principle of complementarity [1]. Feynman believed
that this experiment captures the only mystery in quantum mechanics [2]. It is obvi-
ous that quantum superposition plays an important role in the experiment. The new
understanding that entanglement is at the heart of the principle of complementarity
[3–5], reinforces Feynman’s belief.

The principle of complementarity, or wave–particle duality, as it is more com-
monly referred to, has stood its ground in the face of various theoretical and experi-
mental investigations. Some experiments were carried out early this century which
appeared to show a violation of this principle [6, 7], and attracted lot of popular
attention [8]. A schematic diagram of a typical such experiment is shown in Fig. 1.
It consists of a standard two-slit experiment, with a converging lens just behind the
location where one would put a conventional screen for obtaining the interference
pattern. The experiment uses pinholes instead of conventional slits, but that does
not make any difference. The single photons passing through the two holes A and B,
form a sharp interference pattern on the screen. If the screen is removed, the light
passes through the lens and produces two images of the holes, which are captured
on two detectors $D_A$ and $D_B$ respectively. Opening only the hole A results in only
detector $D_A$ clicking, and opening only hole B leads to only $D_B$ clicking. The authors
argue that the detectors $D_A$ and $D_B$ yield information about which hole, A or B, the
photon initially passed through.

Now without a screen one cannot know if the photons interfere and would
yield an interference pattern. The authors devise a clever scheme for establish-
ing the existence of the interference pattern without actually observing it. The
exact location of the dark fringes is found out by observing the interference pat-
tern on a screen. Then the screen is removed and thin wires are placed in the
exact locations of the dark fringes. Their argument was that if the interference
pattern exists, sliding in wires through the dark fringes will not affect the inten-
sity of light on the two detectors. If the interference pattern is not there, the wires
would surely scatter some photons, thus diminishing the photon count at the two

![Fig. 1](image-url)
detectors. In this manner, one can establish the existence of the interference pattern, without actually disturbing the photons in any way. The reader would notice the similarity of this scheme with the “interaction-free measurements” where the non-observation of a particle along one path establishes that it followed the other possible path, without actually measuring it [9]. The results of the experiments can be summarized as follows.

1. If only hole \( A \) (\( B \)) is opened, only the detector \( D_A \) (\( D_B \)) detects photons.
2. If wires are introduced when only one hole is open, the intensity at the single detector, which received the photons, is reduced.
3. If both holes are opened, both the detector \( D_A \) and \( D_B \) detect photons.
4. If wires are introduced when both the holes are open, the intensity at the two detectors remains unaffected, for all practical purposes.

The experiment is quite simple and straightforward. The authors argue that even when both the holes are open, the detectors \( D_A \) and \( D_B \) continue to tell us which hole each photon came from. Since the introduction of the wires does not affect the intensity, it is natural to conclude that there are dark fringes, and hence an interference pattern, in the region of spatial overlap of the two photon amplitudes. This leads one to an apparent paradox in wave–particle duality: the existence of interference should prohibit any knowledge of which hole a photon passed through, yet the two detectors at the end appear to provide this very information for every photon.

As expected the experiment started a heated debate, with people trying to find flaws in the experiment [10–21]. However, the various criticisms do not agree among themselves regarding the perceived flaw in the experiment. The criticism which is in the right direction is that although the two detectors give which-way information when only one hole is open, they do not give which-way information when both the holes are open [17, 19, 20]. However, some of the arguments use the assertion that since the state of the particle is a superposition of the two paths, there is no which-way information to start with [19, 20]. While this assertion may be correct in principle, the assumption that which-way information is carried by the particle in a Mach–Zehnder interferometer is widely used today. For instance, in the setup shown in Fig. 2, it is widely believed that if the second beam-splitter is removed (Fig. 2b) the detectors \( D_1 \) and \( D_2 \) tell us which path the particle followed, but if BS2 is present

![Fig. 2](image-url) Fig. 2 In a Mach–Zehnder interferometer, it is widely believed that if the second beam-splitter BS2 is removed, the detectors \( D_1 \) and \( D_2 \) give information on which of the two paths a photon followed.
(Fig. 2a), they do not. Although this belief cannot be defended using standard quantum mechanics, in most experiments its fallacy does not become apparent.

So the situation is that the controversial claims of these modified two-slit experiments have been refuted, but mostly on philosophical grounds. The refuting argument is that it is incorrect to assume that when both the holes are open, the two detectors give which-way information about every photon. However, there is no way to prove that the two detectors do not give which-way information, and a student is bound to be unconvinced because it is common to accept the same very assumption while dealing with Mach–Zehnder interferometer (see Fig. 2). It is also unavoidable to have the feeling, mostly stemming from classical prejudice, how can the detector (say) \( D_A \), in Fig. 1, receive photons from hole \( B \)?

In the following analysis we introduce quantum path markers in the two paths of the photons to unambiguously tell which path the photon followed. If a path marker can distinguish whether a photon emerged from hole A or B (i.e., by being detected at \( D_A \) or \( D_B \)), we would like to show that in such a situation, interference cannot be detected at the lens plane. In addition, we wish to demonstrate that to get interference at the lens plane, distinguishability by the path markers must be erased. This is because a photon that can self-interfere at the lens plane necessarily has wave components from both holes and may therefore be detected by either \( D_A \) or \( D_B \). Using this strategy we analyze these modified two-slit experiments, and show that the photons which show interference (via introduction of wires) and reach a particular detector, always follow both the paths.

2 Two-slit experiment with path markers

Let us assume there is a quantum path-detector which interacts with the photons as it passes through the two holes. The two orthogonal states of the path-detector get correlated with the two paths of photons. In the experiment shown in Fig. 1, this can be achieved by having a photon source which produces linearly polarized photons, and then putting behind the two holes, two quarter-wave plates, which convert the passing linearly polarized photons to left-circular and right-circular polarization,

![Fig. 3](image-url)

An experiment which is a modification of the one shown in Fig. 1, by introducing quarter-wave plates in front of the two slits/holes, and putting horizontal polarizers in front of the two detectors.
respectively (see Fig. 3). The combined state of the photon with its polarization, as it comes out of the two holes, is given by

\[ |\Psi_1\rangle = \frac{1}{\sqrt{2}}(|\psi'_A|L\rangle + |\psi'_B|R\rangle), \] (1)

where \( |\psi'_A\rangle, |\psi'_B\rangle \) are the states corresponding to the photon coming out of the hole \( A \) and \( B \), respectively, and \( |L\rangle, |R\rangle \) are the left- and right-circular polarization states, respectively. As the photon travels and reaches the lens, or the location of a potential screen, the states \( |\psi'_A\rangle, |\psi'_B\rangle \) broaden and overlap with each other. The state at this time can be written as

\[ |\Psi_2\rangle = \frac{1}{\sqrt{2}}(|\psi_A|L\rangle + |\psi_B|R\rangle), \] (2)

where \( |\psi_A\rangle, |\psi_B\rangle \) are the states corresponding to the photon coming from the hole \( A \) and \( B \), respectively. This is an entangled state, and has an important implication. If the photon is found in the polarization state \( |L\rangle \) (\( |R\rangle \)), it implies that it came from the hole \( A \) (\( B \)). So the polarization of the photon can be used any time, to determine the path it took, as long as this entangled state is intact. In the absence of the quarter wave plates, the state would simply have been

\[ |\Psi_2'\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle + |\psi_B\rangle). \] (3)

Now if the photons are to show interference, there should be some parts of the amplitudes \( |\psi_A\rangle, |\psi_B\rangle \) which cancel with each other, to give destructive interference. This is an essential requirement of an interference pattern. So we assume the two states to have the following form

\[ |\psi_A\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle + |\phi_-\rangle), |\psi_B\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle - |\phi_-\rangle), \] (4)

where we make no assumption on the form of \( |\phi_+\rangle, |\phi_-\rangle \), except that they are orthonormal. The part \( |\phi_+\rangle \) will constitute the bright fringes, and the part \( |\phi_-\rangle \), rather its absence, will constitute the dark fringes. Note that \( |\psi_A\rangle \) and \( |\psi_B\rangle \) are orthogonal as they result from the same unitary evolution from the orthogonal initial states \( |\psi'_A\rangle, |\psi'_B\rangle \).

One may wonder if introduction of thin wires would lead to photons scattering off the wires. Indeed it will, and for that reason we assume that the wires are fully absorptive, and do not reflect (scatter) photons. However, even if the wires are fully absorptive, introduction of these would be like introducing a grating, and diffraction effects would be there. This point has been discussed in detail by Jacques et al. [16]. However, the experiments in question did acknowledge that if only one hole is open, the photons will be scattered/absorbed, and that will reduce the photon count at the detector. The central argument of these experiments uses just the fact that when only one hole is open, only one detector receives photons, in the absence of any wires. In our analysis, we will not include the effect of thin wires explicitly, but just use the fact that their presence may affect the photon count, and that can be used to infer the presence or absence of interference. Our path distinguishability analysis will be in
the absence of the wires, as the experiments in question claim that in the absence of wires, with the both the holes open, the two detectors give which-hole information.

Next we need to incorporate the effect of the lens on the photon. Without going into the details of how a lens works, here it suffices to consider its effect on \( |\psi_A \rangle \) and \( |\psi_B \rangle \). We know that the lens takes the state \( |\psi_A \rangle \) to the detector \( D_A \) and \( |\psi_B \rangle \) to the detector \( D_B \). So, the effect of the lens can be treated as a unitary operator with the effect

\[
U_L|\psi_A \rangle = \frac{1}{\sqrt{2}} U_L(|\phi_+ \rangle + |\phi_- \rangle) = |D_A \rangle \\
U_L|\psi_B \rangle = \frac{1}{\sqrt{2}} U_L(|\phi_+ \rangle - |\phi_- \rangle) = |D_B \rangle,
\]

where \( |D_A \rangle (|D_B \rangle) \) is the state of the photon if it registers at the detector \( D_A \) (\( D_B \)). Now we wish to see the effect of the lens on the state (2), which has photon polarization included. The state of the photon when it arrives at the two detectors, is given by

\[
|\Psi_3 \rangle = U_L|\Psi_2 \rangle = \frac{1}{\sqrt{2}} U_L(|\psi_A \rangle |L \rangle + |\psi_B \rangle |R \rangle) \\
= \frac{1}{\sqrt{2}} (|D_A \rangle |L \rangle + |D_B \rangle |R \rangle).
\]

It is clear that the photons that arrive at \( D_A \), have the polarization \( |L \rangle \), and those that arrive at \( D_B \), have the polarization \( |R \rangle \), which means those that landed at \( D_A \) came from hole \( A \), and those that arrived at \( D_B \) came from hole \( B \). However, these photons will not show any interference. This can be inferred from the fact that the photons that landed at \( D_A \), with polarization \( |L \rangle \), had the state \( \frac{1}{\sqrt{2}} (|\phi_+ \rangle + |\phi_- \rangle) \) before the lens. This state has the part \( |\phi_- \rangle \) present, which means the photons pass through the regions which would be dark if there were interference.

Now we look for situations which can give us interference. The circular polarization states can also be written in terms of horizontal and vertical polarization states:

\[
|R \rangle = \frac{1}{\sqrt{2}} (|H \rangle + i|V \rangle), \quad |L \rangle = \frac{1}{\sqrt{2}} (|H \rangle - i|V \rangle),
\]

where \( |H \rangle, |V \rangle \) are the horizontal and vertical polarization states, respectively. Using (7), the state before the lens (2) can be written as

\[
|\Psi_2 \rangle = \frac{1}{2}(|\psi_A \rangle + |\psi_B \rangle) |H \rangle - \frac{i}{2}(|\psi_A \rangle - |\psi_B \rangle) |V \rangle).
\]

This entangled state implies that if the polarization state of the photon is found to be \( |H \rangle \), it implies that it came from both the holes. The same holds true for photons found in the polarization state \( |V \rangle \). The photon state, before the lens, can also be written as

\[
|\Psi_2 \rangle = \frac{1}{\sqrt{2}} (|\phi_+ \rangle |H \rangle - i|\phi_- \rangle |V \rangle).
\]
The photon state correlated to $|H\rangle$ polarization does not have the $|\phi_-\rangle$ part, which means it will show interference. So the conclusion is that photons which are found in the horizontal polarization state, will show interference. We can filter out such photons by putting a horizontal polarizer in front of the detectors $D_A$ and $D_B$ (see Fig. 3). However, first we would like to see the effect of the lens on these photons. From (5) we see that

$$U_L|\phi_+\rangle = \frac{1}{\sqrt{2}}(|D_A\rangle + |D_B\rangle),$$

which means that the lens will take the photons with horizontal polarization to both the detectors, and not just one of the two. The final state of the photon at the detectors will be

$$|\Psi_3\rangle = U_L|\Psi_2\rangle = \frac{1}{\sqrt{2}}U_L(|\phi_+\rangle|H\rangle - i|\phi_-\rangle|V\rangle)$$

$$= \frac{1}{2}(|D_A\rangle + |D_B\rangle)|H\rangle - i\frac{1}{2}(|D_A\rangle - |D_B\rangle)|V\rangle.$$  \hspace{1cm} (11)

The above state implies that the photons with horizontal polarization are equally likely to land at either of the two detectors. The photons that pass through the horizontal polarizer, will have the state

$$\langle H|\Psi_3\rangle = \frac{1}{2}(|D_A\rangle + |D_B\rangle),$$

meaning they can land at either of the two detectors. More importantly, their polarization state is $|H\rangle$ which, by looking at (8) implies that these photons, which show interference, came from both the holes. Thus all the photons landing at (say) $D_A$, with horizontal polarization, passed through both the holes. This goes against the conclusion of the modified two-slit interference experiments that each detectors tells us which hole the photon came from.

The reader might have guessed that an equivalent analysis can be done for photons with vertical polarization. Such photons will also show interference, but the interference pattern will be shifted such that the intensity minima will lie at the locations of the maxima of the photons with horizontal polarization. So, if one wants to test out the presence of interference in such a situation, the wires need to be inserted at different locations. The reader might also have recognized this phenomenon as the familiar quantum erasure [22].

### 3 Conclusion

We have analyzed a typical modified two-slit experiment by introducing quantum path markers. The polarization state of the photon is used as the path marker. The analysis shows that in this experiment, if the photons retain which-slit information, they do not show interference, but always land at a particular detector. The photons which do show interference, may land at any of the two detectors. However, their path marker shows that such photons always come from both the slits, and not one of...
the two, even though they eventually land at one of the two detectors at random. So for photons which show interference, their landing at a particular detector does not imply that they came from a particular slit. This shows that the assumptions in the much debated modified two-slit experiments, that the each detector gives information on which slit the photon came from, is incorrect. The conclusion of the present work is arrived at, not by any philosophical argument or assumptions, but by experimentally verifiable correlation between the photon paths and path marker states. Our modified experiment with path markers can be performed easily, since a quantum eraser using photon polarization was demonstrated long back [23].

Acknowledgements The authors wishes to thank the two anonymous referees for their various suggestions which led to much improved clarity and readability of the paper. The author is thankful to Alexandra Elbakyan for her support.

Declarations

Conflict of interest The author has no conflict of interest.

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