Properties of Quarkonia at $T_c$

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Abstract. We discuss how the spectral changes of quarkonia at $T_c$ can reflect the “critical” behaviour of QCD phase transition. Starting from the temperature dependencies of the energy density and pressure from lattice QCD calculation, we extract the temperature dependencies of the scalar and spin 2 gluon condensates near $T_c$. We also parameterize these changes into the electric and magnetic condensate near $T_c$. While the magnetic condensate hardly changes across $T_c$, we find that the electric condensate increases abruptly above $T_c$. Similar abrupt change is also seen in the scalar condensate. Using the QCD second-order Stark effect and QCD sum rules, we show that these sudden changes induce equally abrupt changes in the mass and width of $J/\psi$, both of which are larger than 100 MeV at slightly above $T_c$.

1. Introduction

$J/\psi$ emanating from heavy ion collision is considered to be an intriguing tool to investigate the properties of quark gluon plasma (QGP) expected to form in the early stages of a heavy ion collision [1, 2]. Indeed measurements at past SPS and recent RHIC data show nontrivial suppression patterns that could be consistent with the original prediction that the $J/\psi$ will dissolve in QGP. However, the subject has gained a new turn recently as lattice calculations show that $J/\psi$ will survive past the critical temperature $T_c$ up to about $1.6T_c$, while $\chi_c$ and $\psi'$ will dissolve just above $T_c$ [3, 4]. The notion that $J/\psi$ will not dissolve immediately above $T_c$ was also considered before [5], as it was known that non-perturbative aspects of QCD persist in QGP [6, 7]. These results suggest that the sudden disappearance of $J/\psi$ at $T_c$ is not the order parameter of QCD phase transition. On the other hand, the phase transition is accompanied by sudden changes in the chiral condensate, the heavy quark potential and the energy density, and should therefore have some effect on the properties of quarkonia at $T_c$.

Unfortunately, the lattice results based on the maximum entropy method have large error bars, and one can not determine the detailed properties of quarkonium above $T_c$, except its existence. Solving the Schrödinger equation with potential extracted from the lattice is one possibility [8]. However, here there are some controversies on how to extract the correct potential from the free energy calculations. Even with a chosen prescription, one has to solve the Schrödinger equation at discrete temperatures steps, and it becomes not clear how the sudden critical behaviour is translated into the discrete temperature
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steps. Thermal perturbation alone is also not sufficient to probe the temperature region from $T_c$ to $2.5T_c$, which is now known to be strongly interacting.

Therefore, a systematic non-perturbative method is essential to treat the phase transition region. Here, we will summarize the recent developments [10, 11] to attach the problem using QCD sum rules and the QCD second order Stark effect. The inputs are the temperature dependencies of local gluonic operators, which undergoes abrupt changes across the phase transition as does the energy density.

2. The gluon matter

The order parameter for QCD phase transition is the thermal Wilson line for pure gauge theory, and the quark condensates for QCD with massless quarks. For realistic QCD none of them are order parameters in the strict sense. But simulation shows that the sudden changes in both parameters take places at the same critical temperature $T_c$, which is determined by the susceptibilities. At the same $T_c$, the energy density also makes a drastic change, which is somehow universal for any quark flavours. The sudden changes apparent in the energy density and pressure can be translated to temperature dependencies of local gluonic operators, which are expected to embody the non-perturbative nature of QCD. The link is obtained through the energy momentum tensor, which can be written in terms of symmetric traceless part (twist-2 gluon) and the trace part (gluon condensate).

\[ T^{\alpha\beta} = -S T (G^{\alpha\lambda} G^{\beta}_\lambda) + \frac{g^{\alpha\beta} \beta(g)}{4g} G^{\alpha}_{\mu\nu} G^{\beta}_{\mu\nu}, \]  

where we will use the LO beta function $\beta(g) = -\frac{g^3}{(4\pi)^2} (11 - \frac{2}{3}N_f)$. The thermal expectation value of this operator can be related to the lattice measurements of energy-momentum tensor through the following relation.

\[ \langle T^{\alpha\beta} \rangle_T = (\varepsilon + p) \left( u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta} \right) + \frac{1}{4} (\varepsilon - 3p) g^{\alpha\beta}. \]  

(2)

Here, $u^\alpha$ is the four velocity of the heat bath. With the following definitions,

\[ \langle \frac{\alpha_s}{\pi} G^{\alpha}_{\mu\nu} G_{\lambda}^{\mu\nu} \rangle_T = G_0(T) \]  

(3)

\[ \langle ST (\frac{\alpha_s}{\pi} G^{\alpha\lambda} G_{\lambda}^{\alpha\beta}) \rangle_T = \left( u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta} \right) G_2(T), \]  

(4)

we find for pure SU(3) gauge theory,

\[ G_0(T) = G_0^{\text{vac}} - \frac{8}{11} (\varepsilon - 3p), \quad G_2(T) = -\frac{\alpha_s(T)}{\pi} (\varepsilon + p), \]  

(5)

where $G_0^{\text{vac}}$ is the value of the scalar gluon condensate in vacuum.

At low temperature, the scalar part is dominated and characterized by the non-perturbative contribution of the QCD vacuum. At high temperature, its behaviour will be dominated by the quasi-particle contribution. Just above $T_c$, it is still dominated
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by the non-perturbative contribution. For the heat bath at rest, one can rewrite the thermal expectation values in terms of electric and magnetic condensate.

$$\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_T = -\frac{1}{4} G_0(T) - \frac{3}{4} G_2(T)$$  \hspace{1cm} (6)

$$\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T = \frac{1}{4} G_0(T) - \frac{3}{4} G_2(T).$$  \hspace{1cm} (7)

Figure 1 shows the temperature dependence of $G_0$, $G_2$ or $E^2$ and $B^2$. One should first note that the sudden increase of energy density is translated to the anomalously large and sudden decrease in $G_0$, which deviates largely from the asymptotic $T^4$ behaviour. In contrast, $G_2$ reaches the asymptotic value quickly. Similarly, one finds that there is a sudden increase in the electric condensate $E^2$, while the magnetic condensate $B^2$ hardly changes above $T_c$. This can be related to the fact that the area law behaviour of the space time Wilson loop changes to the perimeter law above $T_c$, while that of the space-space Wilson loop does not. The connection comes in through the operator product expansion (OPE) of rectangular Wilson loop, which was found to be expressible in terms of the electric condensates and the magnetic condensates for the space-time and space-space Wilson loops respectively. To investigate the consequences of the abrupt change of condensates to the properties of $J/\psi$, we will use perturbative QCD and QCD sum rules.

3. Results from perturbative QCD

The perturbative QCD formalism for calculating the interaction between heavy quarkonium and partons was first developed by Peskin. The counting scheme in this formalism is obtained in the non-relativistic limit. Because of this, the formula for the mass shift reduces to the second-order Stark effect in QCD, which was used to calculate the mass shift of charmonium in nuclear matter. The information needed from the medium is the electric field square. Noting that the dominant change across the phase transition is that of the electric condensate, one finds that the QCD second-order Stark effect is the most natural formula to be used across the phase transition.
Figure 2. Mass shift from the second-order Stark effect (solid line) and the maximal mass shift obtained from QCD sum rules from Ref. [10] (points).

The QCD second-order Stark effect for the ground state charmonium with momentum space wave function normalized as \( \int \frac{d^3p}{(2\pi)^3} |\psi(p)|^2 = 1 \) is as follows,

\[
\delta m_{J/\psi} = -\frac{1}{18} \int_0^\infty dk^2 \left( \frac{\partial \psi(k)}{\partial k} \right)^2 \frac{k}{k^2/m_c + \epsilon} \left< \frac{\alpha_s}{\pi} \Delta E^2 \right>_T,
\]

where \( k = |k| \) and \( \left< \frac{\alpha_s}{\pi} \Delta E^2 \right>_T \) denotes the value of change of the electric condensate from its vacuum value so that \( \delta m_{J/\psi} = 0 \) in vacuum. The second line is obtained for the Coulomb wave function. Here, \( \epsilon \) is the binding energy and \( m_c \) the charm quark mass. These parameters are fit to the size of the wave function obtained in the potential model [16], and to the mass of \( J/\psi \) assuming it to be a Coulombic bound state in the heavy quark limit [13]. The fit gives \( m_c = 1704 \text{ MeV}, \ a = 0.271 \text{ fm} \) and \( \alpha_s = 0.57 \). Few comments are in order. The minus sign in (8) is a model independent result and follows from the fact that the second-order Stark effect is negative for the ground state. Second, the important parts of the formula are the Bohr radius \( a \), the binding energy \( \epsilon \) and the temperature dependence of the electric condensate squared. In the formula, the factor of \( a^2 \) follows from the dipole nature of the interaction, and the binding energy from the inverse propagator, characterizing the separation scale [13, 17]. Therefore, the actual value of the mass shift does not depend much on the form of the wave function as long as the size of the wave function is fixed.

The solid line in Fig. 2 shows the mass shift obtained from the second-order Stark effect with the extracted lattice input for the electric condensate shown in Fig. 1-b. As the formula is based on multipole expansion, it will break down if the higher dimensional condensates become large. At this moment, there is no lattice calculation for the temperature dependence of higher dimensional gluonic operators. However, as discussed before, up to \( 1.1 T_c \) the temperature dependence is dominated by the sudden increase in the electric condensate with little change in the magnetic counterpart. Moreover, the
OPE in the QCD sum rules were found to be reliable up to $1.05T_c$ [10]. Therefore our result should at least be valid up to the same temperature. Above that temperature, the change of the electric condensate amounts to more than 100% of its vacuum value and higher order corrections should be important. As can be seen in Fig. 1, the mass reduces abruptly above $T_c$ and becomes smaller by about 100 MeV at $1.05T_c$, reflecting the critical behaviour of the QCD phase transition.

4. QCD sum rule result

The QCD sum rule for quarkonium was found to be very reliable at zero temperature. This is because the expansion parameter of the OPE for the correlation function for the heavy quark current-current correlation function is $\Lambda_{QCD}^2/(Q^2 + 4m_c^2)$. Therefore, the OPE for the correlation function can be reliably estimated even at $Q^2 = 0$. Moreover, for the heavy quark system, only gluon operators appear, for which reliable estimates can be made at least for the lowest dimension 4 operator.

How to generalize the sum rule method to finite temperature depends on the magnitude of the temperature. If the temperature is larger than the separation scale $Q^2 + 4m_c^2$, we have to calculate the temperature effect into the Wilson coefficient. If the temperature is low, then all the temperature effects can be put into the temperature dependent operators. In this case, the new expansion parameter in the OPE will be $(\Lambda_{QCD} + cT)^2/(Q^2 + 4m_c^2)$, where $c$ is some constant. Whether such approximation is valid or not can be directly checked by looking at the convergence of the OPE at finite temperature. An additional ingredient at finite temperature is that unlike at zero temperature, where only scalar gluon operators appear, operators with Lorentz indexes have to be added. Therefore, up to dimension 4, both the scalar gluon ($G_0$) and twist-2 gluon ($G_2$) operators contribute. Since we have extracted the temperature dependence of these operators from the lattice, no ambiguities exist for the OPE side of the sum rule for a consistent analysis.

The OPE for the correlation function of heavy quark currents is related to the phenomenological side via the dispersion relation. In the sum rule, one has to assume the form of the spectral density. Here, one notes that lattice calculations suggest that the peak structure in the spectral density persists above $T_c$, although the resolution is not good enough for a critical study. Therefore, we can assume a relativistic Breit-Wigner form for the spectral density. Using the moments for the correlation function, we can extract a constraint for the temperature dependence for the mass and width from the temperature dependence of the condensates. We find that if there is no change in the width, the mass critically decreases by few hundreds MeV slightly above $T_c$, equivalently if there is no change in mass, the width will increase by a similar amount [10]. The dots in Fig. 2 represent the maximum mass shift assuming no change in the width. This is a direct consequence of the critical change of the scalar condensate proportional to $\varepsilon - 3p$. As can be seen in the figure, the mass shift obtained from the second-order Stark effect is almost the same as the maximum mass shift obtained in the sum rule up to $T_c$ and
then becomes smaller. The mass shift at $T_c$ is about $-50$ MeV. In the QCD sum rules, only a constraint for the combined mass shift and thermal width could be obtained. This constraint can be crudely summarized as follows,

$$-\Delta m + \Gamma_T \simeq 80 + 17 \times (T - T_c) \ [\text{MeV}], \quad (9)$$

within the temperature range from $T_c$ to $1.05T_c$. Therefore, the difference between the Stark effect and the maximum mass shift obtained from QCD sum rules above $T_c$ in Fig. 2 could be attributed to the non-perturbative thermal width at finite temperature. In Fig. 3 we plot the thermal width obtained from combining the constraint in (9) with the mass shift obtained from the QCD second-order Stark effect. As can be seen in the figure, the thermal width at $1.05T_c$ becomes larger than $100$ MeV. Such width slightly above $T_c$ is larger than that estimated from a perturbative LO and NLO QCD matrix element calculation together with an assumption of weakly interacting quarks and gluons with thermal masses $[18, 19]$, but smaller than a recent phenomenological estimate $[20]$.

The mass of quarkonium at finite temperature was also investigated in the potential models $[21]$, where the mass was found to decrease at high temperature. However, the potential has to be extracted from the lattice at each temperature and hence a more detailed investigation are needed to identify the critical behaviour of $J/\psi$ in the temperature region near $T_c$.

5. Corrections from dynamical quarks

To consider the quark effects, first we consider the quark operators appearing in the OPE. We can neglect the light quark contribution to the OPE, because the light quark operators appear in the OPE at order $\alpha_s^2(q^2)$: This is why the light quark condensate can be neglected in the sum rules for heavy quark system. On the other hand, thermal heavy quarks that directly couple to the heavy quark current contribute...
to the OPE at leading order. This is different from the heavy quark condensates that are perturbatively generated from the gluon condensates, and contribute to the OPE through gluon condensates, whose Wilson coefficients are calculated in the momentum representation. The direct thermal quark contributions are called the scattering terms. However, similar terms also appear in the phenomenological spectral density, which also has free charm quark mode that is not coupled with a light quark in the form of a $D$ meson above $T_c$ as been recently studied in Ref. [22]. Therefore, the scattering term will cancel out between the OPE and the phenomenological spectral density in the QCD sum rule analysis for the deconfined medium.

Second, the gluon condensates itself can have a different temperature dependence in the presence of dynamical quarks. As discussed before, the important input for the mass and width change is the temperature dependence of gluon condensates in Fig. 1 in particular the dominant contribution comes from the temperature dependence of $G_0$. For that purpose, we note that the trace of the energy momentum tensor to leading order is given as,

$$T_\mu^\mu = - \left( \frac{11 - 2/3N_f}{8} \right) \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{a\mu\nu} + \sum_q m_q \langle \bar{q}q \rangle. \quad (10)$$

Therefore, we start from the lattice calculation of the trace of the energy momentum tensor for the full QCD with realistic quark masses as given in Ref. [23]. Then, we subtract the fermionic part of the trace anomaly, which was also shown in the literature, from the total. Then we divide the result for the relevant prefactor with $N_f = 3$ multiplying the gluon condensate as given in (10). Since the critical temperature $T_c$ differs, we compared it as a function of $T/T_c$ in which $T_c = 196$ MeV for the full QCD case [23]. As can be seen in Fig. 4, the magnitude of the resulting change near the critical temperature are remarkably similar between the full and pure gluon QCD; although the slope at $T_c$ is milder for full QCD as a consequence of rapid cross over transition instead of a first order phase transition. Since the change of the condensate sets in at a lower $T/T_c$ in the full QCD case, the mass and width of charmonia might start varying at a lower temperature in the realistic case than in the pure glue theory. Therefore we believe our main argument and the quantitative result will not be alter even in the realistic situation.

6. Summary

We have shown that the critical mass shift and width increase of quarkonia slightly above $T_c$ could effectively be an "order parameter" of QCD phase transition. The expected shift for $J/\psi$ is small and will be a challenge for LHC. However, larger shift is expected for $\chi_c$ at its formation temperature slightly above $T_c$. Therefore, direct measurements and confirmation is possible. The changes will also lead to changes in production ratios in the statistical model and $J/\psi$ suppression effects, which needs further detailed studies. This work was supported by BK21 Program of the Korean Ministry of Education, and by the Korean Research Foundation KRF-2006-C00011.
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Figure 4. Comparison of the scalar gluon condensate in the pure gauge theory with the one extracted from full lattice QCD. Horizontal errorbars in the full QCD case are drawn by assuming the 2% uncertainty in the conversion from the lattice units to physical temperature \cite{23}.

References

[1] Hashimoto T, Miyamura O, Hirose K, and Kanki T. Phys. Rev. Lett., 57:2123, 1986.
[2] Matsui T and Satz H. Phys. Lett. B, 178:416, 1986.
[3] Asakawa M and Hatsuda T. Phys. Rev. Lett., 92:012001, 2004.
[4] Datta S, Karsch F, Petreczky P, and Wetzorke I. Phys. Rev. D, 69:094507, 2004.
[5] Hansson T H, Lee S H, and Zahed I. Phys. Rev. D, 37:2672, 1988.
[6] Manousakis E and Polonyi J. Phys. Rev. Lett., 58:847, 1987.
[7] Lee S H. Phys. Rev. D, 40:2484, 1989.
[8] Wong C Y. Phys. Rev. C, 72:034906, 2005.
[9] Blaizot J P, Iancu E, and Rebhan, A. Phys. Rev. D, 68:025011, 2003
[10] Morita K and Lee S H. Phys. Rev. Lett., 100:022301, 2008; Critical behavior of charmonia across the phase transition: A QCD sum rule approach. Preprint \texttt{arXiv:0711.3998}.
[11] Lee S H and Morita K. Properties of $J/\psi$ at $T_c$: QCD second-order Stark effect. Preprint \texttt{arXiv:0802.4000}.
[12] Shifman M A. Nucl. Phys. B, 173:13, 1980.
[13] Peskin M E. Nucl. Phys. B, 156:365, 1979.
[14] Bhanot G and Peskin M E. Nucl. Phys. B, 156:391, 1979.
[15] Luke M, Manohar A V, and Savage M J. Phys. Lett. B, 288:355, 1992.
[16] Eichten E, Gottfried K, Kinoshita T, Lane K D, and Yan T M. Phys. Rev. D, 21:203, 1980.
[17] Oh Y, Kim S, and Lee S H. Phys. Rev. C, Phys. Rev. C(65):067901, 2002.
[18] Park Y, Kim K I, Song T, Lee S H, and Wong C Y. Phys. Rev. C, 76:044907, 2007.
[19] Song T, Park Y, Lee S H, and Wong C Y. Phys. Lett. B, 659:621, 2008.
[20] Mócsy Á. and Petreczky P. Phys. Rev. Lett., 99:211602, 2007.
[21] Alberico W M, Beraudo A, De Pace A, and Molinari A. Phys. Rev. D, 75:074009, 2007.
[22] Dominguez C A, Loewe M, and Rojas J C. Heavy-light quark pseudoscalar and vector mesons at finite temperature. Preprint \texttt{arXiv:0707.2844}.
[23] Cheng M, Christ N H, Datta S, van der Heide J, Jung C, Karsch F, Kaczmarek O, Laermann E, Mawhinney R D, Miao C, Petreczky P, Petrov K, Schmidt C, Soeldner W and Umeda T. Phys. Rev. D, 77:014511, 2008.