High-energy few-cycle pulses: post-compression techniques

Tamas Nagy\textsuperscript{a}, Peter Simon\textsuperscript{b} and Laszlo Veisz\textsuperscript{c}

\textsuperscript{a}Max Born Institute for Nonlinear Optics and Short Pulse Spectroscopy, Berlin, Germany; \textsuperscript{b}Institute for Nanophotonics Göttingen e.V.*, Göttingen, Germany; \textsuperscript{c}Department of Physics, Umeå University, SE-901 87, Umeå, Sweden

\textbf{ABSTRACT}

Contemporary ultrafast science requires reliable sources of high-energy few-cycle light pulses. Currently two methods are capable of generating such pulses: post compression of short laser pulses and optical parametric chirped-pulse amplification (OPCPA). Here we give a comprehensive overview on the post-compression technology based on optical Kerr-effect or ionization, with particular emphasis on energy and power scaling. Relevant types of post compression techniques are discussed including free propagation in bulk materials, multiple-plate continuum generation, multi-pass cells, filaments, photonic-crystal fibers, hollow-core fibers and self-compression techniques. We provide a short theoretical overview of the physics as well as an in-depth description of existing experimental realizations of post compression, especially those that can provide few-cycle pulse duration with mJ-scale pulse energy. The achieved experimental performances of these methods are compared in terms of important figures of merit such as pulse energy, pulse duration, peak power and average power. We give some perspectives at the end to emphasize the expected future trends of this technology.

\textbf{ARTICLE HISTORY}

Received 31 July 2020
Accepted 29 October 2020

\textbf{KEYWORDS}

Nonlinear optics; post compression; ultrafast lasers; few-cycle pulses

\section*{CONTACT}

Tamas Nagy \(\text{\texttt{nagy@mbi-berlin.de}}\) Max Born Institute for Nonlinear Optics and Short Pulse Spectroscopy, Max-Born-Str. 2A, 12489 Berlin, Germany

*formally Laser-Laboratorium Göttingen e.V.

\(\copyright\) 2020 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
1. Introduction

The shortest events the mankind could ever generate are light pulses. In the visible to near-infrared spectral range, they have a duration of a few femtoseconds [1], which paved the way to the emergence of attosecond science by enabling the generation of XUV pulses of several tens of attoseconds duration via high-harmonic generation [2,3]. They are the tools of choice [4] for exploring electron dynamics inside atoms, molecules and solids [5–10] or in nanostructures [11]. They are necessary for understanding fundamental phenomena like magnetism [12] or charge migration inside molecules [13]. Few-fs laser pulses can also be utilized for the generation of relativistic electron bunches [14] and for the imaging of the plasma waves that accelerate these electrons [15]. Such pulses are at the limit posed by fundamental natural laws in several aspects. As light is an oscillating electromagnetic field, an ultimate limit of its shortest possible duration is half an oscillation period. As this ‘optical cycle’ is the reciprocal of the carrier frequency (the first moment of the spectrum), it is strongly wavelength dependent e.g. 0.67 fs, 2.67 fs and 6.67 fs for pulses with a central wavelength of 200 nm, 800 nm and 2000 nm, respectively. This also means that in the quest for ever shorter pulses one migrates towards shorter central wavelengths, this is why the overwhelming majority of attosecond pulses are generated in the extreme ultraviolet (XUV) spectral range. As the pulses getting shorter approaching the ultimate limit, the number of optical cycles inside the full width at half maximum (FWHM) of the pulse envelope will decrease, as shown in Figure 1, causing growing jumps between the consecutive electric field maxima.

The carrier-envelope phase of such pulses (see Figure 1(a)), i.e. the phase shift between the sinusoidal carrier wave and the peak of the intensity or field envelope, starts to play an important role in light-matter interaction. In

![Figure 1](image-url)
In this paper we focus on the generation of high-energy light pulses containing only a few optical cycles in the FWHM of the pulse envelope.

In order to see what is needed for the generation of few-cycle pulses we have to understand a fundamental behavior of light pulses (electromagnetic wave packets): a pulse can be fully described either in time or in frequency domain which are connected by the Fourier transformation. This is analogous to the representation of particles in quantum mechanics where the two domains are space and momentum. According to the bandwidth theorem [16] – which describes an inherent property of the Fourier transform – the product of the widths in both representations cannot be arbitrary small but has a minimum value. In quantum mechanics this is known as Heisenberg’s uncertainty relation, in ultrafast optics it reads as

$$\Delta t \cdot \Delta \omega \geq \text{const.}$$

where $\Delta t$ is the pulse duration and $\Delta \omega$ denotes the spectral bandwidth. According to (1) for a given spectrum the pulse duration has a minimum value where (1) becomes equality, which is called Fourier-transform limit. This happens if all components of the spectrum are in phase giving rise to constructive superposition. Eq. 1 has direct consequences for the generation of few-cycle pulses: (i) the spectral width should be sufficiently broad to support the pulse duration, e.g. about an octave for single-cycle duration and (ii) the spectral phase should be nearly constant over the entire spectrum (transform-limit). Both conditions are challenging to fulfill but the first one has larger influence on the architecture of the light source. Most of the frequently used lasers have too narrow gain bandwidth due to the finite widths of the energy levels of the amplifying medium, therefore the spectrum of their pulses needs to be broadened first before they can be compressed to few-cycle duration. This technique is called post compression (compression after amplification) which is overviewed in this paper. Another appealing approach is to find media, which unlike most lasers have enough gain bandwidth to directly generate or amplify few-cycle pulses. Optical parametric processes involving virtual states can support arbitrary broad spectral range where only the phase-matching condition together with the spectral transmission range of the amplifying medium pose limitation on the practically achievable bandwidth. In many practical cases optical parametric chirped-pulse amplifiers (OPCPA) can indeed have enough bandwidth required for direct few-cycle generation or amplification. This approach is the subject of a separate publication.

The rest of this paper concentrates on the post-compression techniques. Spectral broadening can be achieved by inducing nonlinear interaction between intense light pulses and the propagation medium. In the course of such interactions the pulses suffer nonlinear phase distortion leading to the emergence of new spectral components. Although the spectrum of the
pulses broadens, the pulse shape does not change much during the interaction. As a result a phase-modulated (chirped) pulse is obtained which has a much shorter transform-limited duration. This potential can be exploited by a proper chirp management which brings all spectral components in phase making the spectral phase of the pulse constant. Therefore, in most cases post compression encompasses two distinct steps: spectral broadening followed by chirp compensation. This strategy was introduced already in the late 60s. In the first approach an active frequency shifter was utilized for introducing new spectral components [17] which was able to achieve a factor of two compression of 500 ps pulses of a mode-locked helium-neon laser [18]. Shortly later, R.A. Fisher et al. proposed a scheme [19] which forms the basis of almost all pulse compressors till today: it uses self-phase modulation driven by the optical Kerr-effect for spectral broadening and a grating pair for compression [20,21]. In the first experimental demonstration ten times compression of 20 ps pulses was achieved [22].

Dependent on the pulse duration, bandwidth and central wavelength there are different options for the chirp management. The vast majority of post compressors incorporate a set of chirped mirrors [23–25] and an additional glass wedge pair for continuous tuning of the chirp. For very precise chirp compensation needed for sub-1.5-cycle pulses additionally to the chirped mirrors one can incorporate third-order dispersion (TOD) compensation using a combination of bulk materials with different ratios of the group-delay dispersion (GDD) to the TOD [26,27]. Other techniques include pulse shapers [28,29] or pulse synthesizers, where the ultra-broadband spectrum is first split into spectral branches compressed separately by different sets of chirped mirrors and finally recombined [30,31]. In the infrared spectral range, where the material dispersion changes sign, simple propagation through bulk glass can replace the chirped mirrors [32], making the arrangement simpler.

In some special cases such as filamentation or soliton dynamics, the pulses might self-compress at some distinct positions in the course of the nonlinear propagation without the need of extra chirp management.

The typical nonlinear interaction that is used for spectral broadening is self-phase modulation due to the optical Kerr-effect. However, self-phase modulation can also be induced by other nonlinear effects, such as photoionization. These will be discussed in the following section.

2. Basics of spectral broadening

In order to understand the potential and limits of post-compression techniques, we need to briefly overview the fundamentals of spectral broadening. Here we focus on self-phase modulation, as it is applied in the overwhelming majority of post compressors, and give a heuristic view on the creation of new spectral components.
The electric field of a short light pulse propagating along the z-axis can be described by \( E(t, z) = a(t, z) \cos(\phi(t, z)) \) with an envelope \( a(t, z) \) and a phase term \( \phi(t, z) = \omega_0 t - k_0 z = \omega_0 t - z \omega_0 n / c \), where \( \omega_0 \) denotes the angular carrier frequency, \( n \) the refractive index of the medium and \( c \) the speed of light in vacuum. It is useful to introduce the instantaneous frequency, which is defined as the time-derivative of the phase term [33]:

\[
\omega_{\text{inst}}(t, z) = \frac{\partial \phi}{\partial t} = \omega_0 - \frac{\omega_0}{c} \frac{\partial n(t, z)}{\partial t} z. \tag{2}
\]

Eq. 2 suggests that the instantaneous frequency shifts if the refractive index of the medium changes during the passage of the light pulse leading to the creation of new spectral components and thus to spectral broadening. If the refractive index change is induced by the pulse itself, then we call it self-phase modulation (SPM), while if the change is caused by another pulse (e.g. due to four-wave mixing or stimulated Raman scattering [34]) then it is called cross-phase modulation (XPM). Most of the post compressors utilize SPM because it is much simpler than using two well-synchronized pulses. In the followings, we introduce two physical mechanisms that can result in such fast phase modulation: the optical Kerr-effect and photoionization.

2.1. Spectral broadening based on the optical Kerr-effect

If the intensity of a light pulse propagating in a medium is high enough to significantly distort the atomic potential of the medium constituents, then the valence electrons will oscillate anharmonically changing the macroscopic properties of the medium. In particular, the refractive index becomes intensity dependent which is known as the optical Kerr effect: \( n = n_0 + n_2 I(t, z) \), where \( n_0 \) is the linear refractive index and \( n_2 \) is the nonlinear refractive index with units of \( \text{cm}^2 / \text{W} \) and \( I \) denotes the light intensity. In gases, the nonlinear refractive index is linearly proportional to the gas pressure \( (p) \): \( n_2 = p \cdot \kappa_2 \). In the presence of the optical Kerr-effect the instantaneous frequency takes the following form:

\[
\omega_{\text{inst}}(t, z) = \omega_0 - \frac{\omega_0}{c} n_2 \frac{\partial I(t, z)}{\partial t} z. \tag{3}
\]

On the slope of the pulse envelope, new spectral components are created, and as the slope changes the instantaneous frequency shifts, as illustrated in Figure 2.

Figure 2 helps in understanding the characteristic features of the spectrum formed by Kerr-induced SPM: (i) The leading edge of the pulse broadens the red side while the trailing edge the blue side of the spectrum. (ii) The maximal spectral shifts at both sides of the spectrum and thus the total amount of broadening are determined by the highest slopes of the
Figure 2. Self-phase modulation due to the optical Kerr-effect. (a) input pulse shape, (b) normalized instantaneous angular frequency shift, (c) SPM-broadened spectrum at a B-integral of $9\pi$ (the input spectrum is marked by a shaded area).
pulse (blue line). Furthermore, the pulse energy is proportional to the integral of both the pulse shape and the spectrum. Therefore, the steeper the slope is, the less energy it contains which is distributed in a broader spectral region. Consequently, the spectral amplitude corresponding to a steeper part of the pulse decreases with the steepness rapidly, as it is seen in Figure 2. This can be often observed in spectra measured after filamentation exhibiting a very broad blue spectral tail with an amplitude at the percent level. These wings are generated by extreme self-steepening of the pulse in the course of filamentary propagation [35]. (iii) Besides these extrema on each side of the pulse with the maximum slope at the inflection points, there are always two positions with the same slope. These parts of the pulse generate the same spectral component at different times (red lines) leading to spectral interference. As the time difference continuously increases with decreasing slope (smaller spectral shift), we can observe spectral fringes with decreasing period towards the central wavelength. In fact, the SPM-broadened spectra exhibit a number of $B/\pi$ peaks (e.g. 9 in Figure 2). (iv) Near the peak of the pulse and at the low-intensity pedestals no spectral shift occurs, these regions all contribute to the spectrum around the center wavelength (green lines). However, if the pulse has considerable pedestals then the low slope occurs at several largely separated temporal moments leading to fast oscillations with large amplitude near the center wavelength (well observable in most experimentally recorded SPM spectra). We note that if a temporal contrast enhancement method (e.g. nonlinear ellipse rotation) is applied to an uncompressed SPM-broadened pulse which cuts the low-intensity parts of the pulse, then the spectral modulations can be effectively suppressed leading to a bell-shaped spectrum [36].

How much the Kerr effect modifies the pulse, can be described by the nonlinear part of the phase shift $\phi$ accumulated at the peak of the pulse during propagation, which is expressed by the so-called B-integral:

$$B = \frac{\omega_0}{c} \int_0^L n_2(z)I(z)dz = \frac{2\pi}{\lambda} n_2 I_0 L_{\text{eff}},$$

(4)

where $\lambda = 2\pi c/\omega_0$ is the central wavelength in vacuum and $I_0 = I(t = 0, z = 0)$ the peak intensity of the input pulse, and $L_{\text{eff}} = \int_0^L n_2(z)/n_2 \cdot I(z)/I_0 dz$ denotes the effective length of the nonlinear interaction. The effective length takes into account intensity changes during propagation, e.g. due to losses or gain and also if the nonlinearity of the medium changes, e.g. due to pressure variations in gases. In the particular case of Gaussian pulses the link between the spectral broadening and the B-integral takes an analytical form [37]:
\[ F = \frac{\Delta \omega_{out}}{\Delta \omega_{in}} = \sqrt{1 + \frac{4}{3\sqrt{3}} B^2}, \quad (5) \]

where \( F \) denotes the spectral broadening factor, \( \Delta \omega_{in} \) and \( \Delta \omega_{out} \) are the rms spectral bandwidths of the pulse before and after the nonlinear interaction. If the B-integral exceeds 2.5 then the spectral broadening factor is approximately linearly proportional to the B-integral: \( F \approx 0.88B \). Although this analytical expression is only valid for Gaussian pulses, it gives a surprisingly good approximation for more complex cases: e.g. in case of Figure 2(c) the simple approximation results in 24.9, which agrees with the numerical value of 25.3 within an error of 2%.

The self-phase modulation can also be interpreted in a way that due to the Kerr effect the phase velocity \( v = \omega/k = c/n \) of the light becomes intensity dependent, which leads to spectral broadening. However, if the refractive index is intensity dependent then the group velocity \( v_g = \frac{d\omega}{dk} = \frac{c}{n + \omega n_2} \) will also be intensity dependent. As the pulse envelope propagates with the group velocity, the peak of the pulse will propagate slightly slower than its flanks. Therefore, the slope of its leading edge decreases while the falling edge will be steeper (similarly to Figure 2(a)). This effect is called self-steepening and plays a significant role for the propagation of short pulses (<50 fs) or by longer pulses with very large B-integral (>40).

The Kerr-effect can modify intensive light pulses not only in the time (or frequency) domain, but it can also influence their spatial properties. Specifically, intense pulses having a bell-shaped spatial distribution (e.g. Gaussian) collect maximum phase shift in the middle of the beam profile during propagation in Kerr-media and the phase shift gradually decreases as the intensity drops along the beam radius. This acts analogously to the propagation through a convex lens, therefore this effect is called self-focusing. Dependent on the strength of the nonlinearity the Kerr lensing can overcome the natural divergence of the free propagating beam. As both effects induce a spherical wavefront curvature which scales inversely proportional with the square of the beam size, it is not the intensity but the peak power of the pulse which governs the propagation [38]. If the peak power of the pulse reaches the critical power of self-focusing \( (P_{cr}) \) then the two effects balance each other and the beam size remains constant during propagation, beyond that threshold the beam will be focused. There are numerous definitions of \( P_{cr} \) slightly differing from each other by a constant factor dependent on the defining mechanism [38] and on the beam profile [39]. In order to gain simple analytical results we will use the following definition:

\[ P_{cr} = \frac{1}{2\pi n_0 n_2} \lambda^2. \quad (6) \]
2.2. Photoionization-induced spectral broadening in gases

Another effect which can lead to SPM is photoionization [40]. The free-electron plasma emerging from the ionization process has a refractive index less than unity:

\[ n_p = \sqrt{1 - \frac{\omega_p^2}{\omega_0^2}} \approx 1 - \frac{\omega_p^2}{2\omega_0^2} , \]

where \( \omega_p \) denotes the plasma frequency and \( \rho \) is the free-electron number density. As the refractive index of gases is very close to unity, its change due to the free-electrons is

\[ \Delta n \approx - \frac{\omega_p^2}{2\omega_0^2} = - \frac{e^2}{2m_e\varepsilon_0\omega_0^2} \rho(t,z). \]

The recombination time of the ions is at least in the order of several picoseconds, much longer than the laser pulses, therefore, the electron density during the pulse is a monotonically increasing step-shaped function. Furthermore, the evolution of the electron density is well described by a rate equation, which can be further simplified in case of weak ionization:

\[ \frac{d\rho}{dt} = w(\rho_0 - \rho) \approx w\rho_0, \]

where \( \rho_0 \) is the number density of the gas and \( w \) is the ionization rate, which is a strongly nonlinear function of the light intensity [41–44]. Combining Eqs. 2, 8 and 9 we obtain the instantaneous frequency in case of ionization as:

\[ \omega_{\text{inst}}(t,z) \approx \omega_0 + \frac{\rho_0 e^2}{2m_e\varepsilon_0 c\omega_0} w(I(t))z. \]

It shows that in this case the newly created spectral components have always higher frequency than the carrier. Therefore, the center of gravity of ionization-broadened spectra is at lower wavelength compared to the input, which is often referred to as a blue shift. Furthermore, as the ionization rate and thus the instantaneous frequency is a sharp bell-shaped function of time, the induced chirp will have significant higher-order content, which makes the chirp management more difficult.

3. Spectral broadening during free propagation

Using free propagation in homogeneous nonlinear media is conceptually the simplest spectral broadening method. However, there are inherent problems connected to free-space propagation: (i) as the spectral broadening is
proportional to the B-integral it can vary across the beam profile if the intensity is not uniform. Furthermore, (ii) if the peak power of the pulse is higher than the critical power then self-focusing sets in, not only for the full beam [45] but also on the small scale, which leads to beam break-up in case of the B-integral exceeds ~3 [46]. This can only be mitigated by splitting the nonlinear propagation into separate steps each with a moderate B-integral and applying careful beam filtering between the subsequent stages.

3.1. Free propagation in bulk

In the first report on high-energy fs pulse compression by C. Rolland and P. B. Corkum on quasi uniform spectral broadening in a bulk medium [47] by filtering out the low-intensity wings of the beam 19–24 fs pulses with 7 to 110 µJ were obtained representing an energy record on that time scale. However, there was a trade-off between the spectral uniformity and thus temporal purity and the transmission, which varied between ~4 and 20%. In a modified arrangement 12 fs pulses with 70 µJ or 14 fs pulses with 220 µJ could be achieved [48].

This concept experienced a boost in the context of the compression of multi-J, multi-TW to PW lasers having well-engineered flat-top beam profiles. Using the so-called Thin Film Compression (TFC) G. Mourou et al. envisioned the generation of single-cycle 100 J-level pulses focused to ultra-relativistic intensities of $10^{24}$ W/cm$^2$ on the basis of simulations [49]. Currently, the reach of these goals is still far away, nevertheless substantial steps in energy scaling are achieved at moderate compression factors [50]. Especially, the fivefold compression of a part of a 17 J, 250 TW laser beam down to 14 fs (4.6 optical cycles) is promising [51]. In a recent publication the group demonstrated the generation of 3.25 J pulses at an average duration of 13 fs whereas the pulse duration strongly varied between 6.4 and 29 fs along the beam profile [52].

3.2. Multiple-plate continuum generation

In the followings, we overview a spectral broadening method, which at first glance might be very similar to the above TFC technique; however, in fact it is different.

The idea of A. H. Kung et al. in [53] is using very thin (0.05–0.5 mm) glass plates, which are placed in an intense converging beam at ~20 TW/cm$^2$ intensity, as shown in Figure 3(a). As the peak power is several hundred times larger than the critical power in the plate the beam starts self-focusing. However, the plate is so thin that the beam exits the plate before collapse and gets focused behind the plate in air. Behind the focus another plate is placed into the divergent beam which will be refocused again by the self-focusing
effect inside the next plate. As each plate can introduce a B-integral of 4–5 \[54,55\], a sequence of such refocusing thin plates can result in supercontinuum (Figure 3(b)) and very large compression factors down to the single-cycle limit \[56\]. The downside of this technique is the poor energy scalability (sub-mJ regime), as it inherently relies on condensed materials with high nonlinearity, and a small amount of remaining spatial chirp of the output beam \[57\].

### 3.3. Multi-pass cells

The idea of multi-pass cell compressors is similar to the previous method in the sense that the nonlinear interaction takes place in the vicinity of a sequence of foci. However, unlike the multi-plate arrangement, here the refocusing is obtained by conventional optics instead of self-action inside the nonlinear media. In this technique, the nonlinear medium is placed inside a Herriott-type cavity which preserves the beam parameters \[58–60\] and acts as a ‘discrete’ waveguide (Figure 4(a)). In such a cavity it is possible to achieve several tens of passes through the nonlinear medium which supports large spectral broadening, as shown in Figure 4(b), at low amount of nonlinearity per pass \(B << \pi\), as was pointed out in the original publication of J. Schulte et al. \[61\]. The low single-pass B-integral is essential to keep the beam parameters constant, matching the cavity. Furthermore, finely dosed Kerr-nonlinearity separated by free-space propagation steps results in a homogeneous spectral distribution across the beam profile, as was shown by numerical simulations \[62\] and also experimentally \[63\]. Originally, the multi-pass cell compressor incorporated glass material as nonlinear medium for low-energy (few \(\mu\)J) lasers \[61,63,64\]. Later, in order to scale up the pulse energy to the mJ level, the Herriott-cells were placed in a sealed container and used noble gas as nonlinear medium instead of solids \[65–67\].

The multi-pass cell compression was developed for high-repetition-rate, high-power ytterbium-based slab and thin-disk lasers delivering pulses of

---

**Figure 3.** Schematics of multiple-plate continuum generation (a) and the input and broadened spectra after indicated number of plates as well as just in air (b). Reprinted with permission from \[53\] © The Optical Society.
several 100 fs to ps duration. They exhibit excellent throughput in the range of 85–95%, dependent on the reflectivity of the cavity mirrors and on the number of roundtrips. With a single-stage gas-filled multi-pass cell, 1.3 ps pulses with the so far highest energy of 18 mJ were compressed to 41 fs duration at an average power of 90 W in a 45-pass cell filled with 600 mbar argon (Figure 4) [67]. This results in a record compression factor of ~32 at an excellent overall transmission of 95.7%. With this technique, the highest average power of 530 W was achieved compressing 590 fs 1.1 mJ pulses to 30 fs (20 times compression) at 500 kHz repetition rate in a 44-pass cell with a throughput of 96% [68].

Most of the achievements till today report on the compression to a duration not shorter than ~20 fs. The first successful attempt to reach the few-cycle regime with this technique utilized a cascade of two krypton-filled cells [69]. The first stage compressed the 2 mJ 1.2 ps pulses to 32 fs with 80% efficiency in 44 passes while the second stage could only be used at a reduced input energy of 0.8 mJ which then compressed the pulses to 13 fs in 12 passes at a throughput of only 46%. While the first stage performed optimally, the second one was clearly suffering from the high peak power of the pulses and from the low performance of the silver cavity mirrors in terms of both reflection and damage threshold.

3.4. Filaments

If the peak power of an ultrashort pulse propagating in a transparent medium exceeds the critical power of self-focusing several times (5–10 times) then a self-guided propagation can occur, which is called filamentation. It is a result of a very complex dynamical interplay mainly between the Kerr-effect (self-focusing, SPM and self-steepening), photoionization (absorption and defocusing) and linear effects (diffraction and dispersion) [70,71]. In the

Figure 4. Schematics of continuum generation in a multi-pass cell with 45 passes (a) and input (orange) and broadened output spectrum (blue) with retrieved spectrum (black) and phase (green) from the SHG-FROG measurement showing 41 fs compressed duration (b). Reprinted with permission from [67] © The Optical Society.
course of filamentary propagation the spectrum of the pulses undergoes an asymmetric spectral broadening resulting in a long weak tail on the blue side of the spectrum formed primarily by the optical Kerr-effect involving strong self-steepening [72] (see also Sect. 2).

Filaments can occur both in condensed materials (such as solids and liquids) and gases in a similar way, however, at very different intensities according to the different linear and nonlinear properties of the media. Filamentation in solids is induced by low-energy fs to ps pulses of the order of a few µJ. This process leads to coherent white light or supercontinuum generation spanning over several octaves with a spectral density of ~10 pJ/nm. This simple and robust technique finds numerous applications in spectroscopy or in light sources, e.g. by seeding OPCPA systems. Although filamentation in bulk solid media is a very well-developed and broad research field, it is less relevant from the energy scaling perspective; therefore, we refer to an in-depth review of [73].

At higher pulse energies of the order of a mJ, filaments can be generated in gases what we will briefly summarize here. U. Keller’s group was the first to utilize filamentation in a pulse compression scheme for Ti:sapphire laser pulses [74]. C.P. Hauri et al. achieved 5.7 fs CEP-stable pulses with 0.38 mJ energy by spectral broadening in two cascaded argon filaments. Later, the group could gradually improve the results to 5.1 fs and finally to 4.9 fs [75,76]. Using 2-cycle pulses as input for filamentation, very broad spectra spanning from below 300 nm to beyond 1 µm were obtained in argon [77] and in helium [78] with an energy of up to 200 µJ. M. Kovacev’s group obtained 7 fs pulses [79] and subsequently sub-4 fs pulses [80] by inducing a single filament in a semi-infinite gas cell filled with argon using 35 fs long input pulses. By varying the chirp introduced by chirped mirrors after the filament, they observed compressed pulses at several chirp settings suggesting a rather complex pulse structure.

There is a severe energy limitation for 800 nm driver pulses, which can be mitigated by up-scaling the laser wavelength and matching the geometry of the propagation [81]. Thus higher pulse energies can be obtained in the mid-infrared spectral range mostly due to the advantageous wavelength scaling of the critical power [82] (see Sect. 2.1). Recently, at 1.8 µm central wavelength 11.8 fs, sub-2-cycle pulses with 2.1 mJ were obtained at a peak power of 0.2 TW by a two-stage argon filament compressor [83]. In this work the short pulses were successfully used to generate high-harmonics in a helium-filled gas cell covering the entire water window. At even longer wavelength of 3.9 µm, 15 mJ, 100 fs output of a mid-infrared OPCPA was spectrally broadened in a single filament in nitrogen and subsequently compressed to 2.5-cycle duration at 31.5 fs with 13 mJ energy at a peak power of 0.3 TW [84].

However, post-compression in the visible/near-infrared spectral range utilizing filamentation has been abandoned by most of the groups mainly
due to the following reason: in the filament the temporal pulse profile varies considerably with the lateral position in the beam profile having optimal shape only in the center, as shown in [76,85]. Therefore, only the center region of the beam, containing a small fraction of the full energy well below 1 mJ, can be used as a source of short pulses.

Besides spectral broadening, the rich dynamics during filamentation can lead to self-compression which will be treated later in Sect. 5.

4. Spectral broadening in waveguides

Using waveguides for spectral broadening can optimally solve the two fundamental problems of free-space propagation: (i) In a waveguide the pulses can maintain their beam size and thus high intensity on an extended length defined solely by the waveguide geometry, which can exceed the Rayleigh length of the focusing by orders of magnitude. This allows the accumulation of very large B-integrals and therefore can result in extended spectral broadening. At the same time (ii) in the course of propagation in the waveguide the beam undergoes permanently reflections leading to a coherent mixing of the different parts of the beam and to the formation of eigenmodes. The eigenmodes exhibit the same spectral and temporal behavior over the entire beam profile [86], allowing in principle perfect pulse compression. Furthermore, taking the ideal (diffraction-limited) beam properties of the eigenmodes into account, the pulse energy can also be ideally focused leading to the highest possible intensity. However, this ideal case is only valid, if only one eigenmode is involved in the propagation, since the different modes propagate at different speeds leading to non-compensable temporal smearing (modal dispersion) and deteriorated focusing properties.

The most straightforward way of mitigating modal dispersion is to use a waveguide where only one mode can propagate. The first controlled large spectral broadening was achieved in a single-mode silica fiber in the late seventies [87]. A few years later, combined with chirp compensation by a grating pair, generation of the first few-cycle pulses with gradually decreasing durations was demonstrated [88–90]. In less than a decade, the technique culminated in a long-standing record of the generation of 6 fs pulses [91], containing only 3 optical cycles inside the FWHM. It was reached thanks to more sophisticated dispersion compensation up to the third order by a sequence of a grating compressor and a prism compressor [92,93].

Although these results were a revolution for ultrafast science achieving unprecedented temporal resolution, the compressed pulse energy could not exceed a few nJ resulting in peak powers of up to the MW level and focused intensities up to $10^{11}–10^{12}$ W/cm², thus limiting the field of applications mainly to time-resolved spectroscopy.
In the followings, we concentrate on basic scaling laws that elucidate fundamental limitations of this technique and give hints and guidelines for the design of high-energy pulse compressors.

4.1. Energy and power scaling of propagation in waveguides

There are two main criteria of well-controlled pulse propagation in waveguides [94]: (i) the peak power of the input pulse should be below the critical power of self-focusing \( P_0 < P_{cr} \) and (ii) the peak intensity of the pulse should stay below the ionization threshold of the nonlinear medium \( I_0 < I_{th} \). These are necessary for avoiding damage of solid-core waveguides. Furthermore, in multimode fibers both self-focusing and the free electrons emerging from photoionization change the divergence of the beam and therefore cause coupling between eigenmodes (mode mixing) leading to modal dispersion and increased losses.

In a waveguide each eigenmode has its own effective area \( A_{eff} \) which is only dependent on the mode profile [86]. The effective mode area establishes a constant relationship between the peak power and peak intensity of the pulses during the propagation:

\[
I_0 = \frac{P_0}{A_{eff}}. \tag{11}
\]

According to (i), while increasing the input peak power self-focusing can only be avoided if the critical power is also increased accordingly. Taking Eq. 6 into account it requires the reduction of the nonlinear refractive index, which can be expressed by the following inequality:

\[
n_2 < \frac{\lambda^2}{2\pi n_0 P_0}. \tag{12}
\]

Furthermore, substituting Eqs. 11 and 12 into Eq. 4 we obtain [95]:

\[
B = \frac{2\pi}{\lambda} n_2 I_0 L_{eff} < \frac{\lambda}{n_0 A_{eff}} L_{eff}. \tag{13}
\]

Remarkably, in order to avoid self-focusing in the waveguide, the achievable spectral broadening for a given central wavelength in the medium is only limited by the fiber geometry and does not depend on other parameters of the pulse or on the properties of the medium.

Taking Eq. 11 into account, in order to prevent ionization by up-scaling the peak power the effective mode area needs to be increased according to the following inequality:
\[ A_{\text{eff}} > \frac{P_0}{I_{\text{th}}}. \]  \hfill (14)

The physical content of Eqs. 13 and 14 can be synthesized into a single, combined relationship showing the limits of spectral broadening during a well-controlled propagation in a waveguide:

\[ B < \frac{\lambda}{n_0 P_0 L_{\text{eff}}}. \]  \hfill (15)

According to Eq. 15, in order to avoid mode mixing due to self-focusing or ionization in a waveguide the maximal achievable spectral broadening is dependent on the input pulse \((\lambda, P_0)\), on the nonlinear medium \((n_0, I_{\text{th}})\) and also on the waveguide geometry \(L_{\text{eff}}\). In general, for a given central wavelength, by up-scaling the peak power of the input pulse (and correspondingly matching the effective area of the waveguide and the nonlinear index of refraction) the achievable spectral broadening decreases which can only be compensated by increasing the length of the nonlinear interaction. For a given central wavelength all the above considerations can be condensed into a single scaling law:

\[ P_0 \propto \frac{1}{n_2} \propto A_{\text{eff}} \propto L_{\text{eff}}. \]  \hfill (16)

In the light of the above findings, we can identify two practical routes of energy scaling: (i) using single-mode fibers with large mode area, or (ii) use large-aperture multi-mode waveguides, both with gas as nonlinear medium. Furthermore, migration to longer wavelengths e.g. to the mid infrared range offers a significant energy up-scaling potential due to the beneficial wavelength scaling of the critical power (see Eq. 6 and the first criterion (i)) and due to the also advantageous wavelength dependence of the ionization rate [41].

### 4.2. Photonic-crystal fibers

A significant step forward in energy scaling was made possible by the invention of microstructured fibers or photonic-crystal fibers (PCF) [96] by P.St.J. Russell’s group. Instead of relying on total internal reflection between a high-index core and a low-index bulk cladding, here the wave-guiding is achieved either by photonic band-gap effect or by other novel mechanisms arising from the periodic structure of the waveguide cladding. In this way, low-loss and effectively single-mode operation can be obtained where the fundamental mode is confined by the bandgap formed by the surrounding cladding while the higher-order modes are not blocked by the cladding structure letting them leaking out. This kind of guiding mechanism can form single-mode waveguides with much larger core sizes than standard single-mode silica fibers.
have and works also for structures with a hollow core (HC-PCF) [97] which makes them especially attractive for power scaling. Furthermore, the dispersion properties of such fibers can also be designed to match a predefined spectral dispersion function. However, the photonic bandgap can only guide light with narrow spectrum which is less suited for pulse compression. The first broad-band HC-PCF is demonstrated by F. Benabid et al. [98] exhibiting low loss of a few dB/m for almost two octave bandwidth using a novel cladding structure called Kagome-lattice. The very effective guiding mechanism was interpreted in terms of inhibited coupling and also by antiresonance coupling (ARROW) models. Later, similar performance could also be achieved by negative curvature HCFs having a substantially simpler cladding structure, such as the single-ring tubular lattice fiber introduced in 2011 [99]. The losses of both types could be reduced to the \(~10\) dB/km level keeping broadband operation [100,101]. Nowadays the core size of HC-PCF-based single-mode fibers can reach several tens of \(\mu\)m with a record exceeding 100 \(\mu\)m [102] making them suitable for spectrally broaden laser pulses with several \(\mu\)J energy at an average power of up to 100 W [103,104]. Therefore, these waveguides find numerous applications in sub-mJ, high-power, high-repetition rate systems such as oscillators [103,105] and fiber lasers [106–108]. For an in-depth review on this very large research field, we refer to [109].

### 4.3. Capillaries or hollow-core fibers

In 1996 a revolutionary paper appeared from O. Svelto’s group reporting on the generation of 10 fs pulses with 240 \(\mu\)J energy which exceeded the energy of previously generated few-cycle pulses by more than three orders of magnitude [110]. This spectacular boost of energy was achieved by the combination of two deciding factors: the nonlinearity was decreased by a factor of thousand relative to those of the silica core of former single-mode fibers by using a gas medium; and at the same time the mode area was also increased by a large factor, in accordance with the guidelines of the scaling law (Eq. 16). For this purpose M. Nisoli and coworkers gave up the single-mode fiber approach and used a gas-filled capillary or hollow-core fiber (HCF) with a core diameter of 140 \(\mu\)m, instead. This was a radically new and risky idea, as hollow capillaries are weakly guiding multi-mode waveguides. Since the core of the capillary (gas) has smaller refractive index than the cladding (glass), there is no total internal reflection on the interface between the two as in usual silica fibers. Therefore, the guiding relies on grazing-incidence reflection which makes it inherently lossy and sensitive to bending [111]. Furthermore, the price of increased mode area is multi-mode guiding. In order to fulfill single-mode propagation in such a waveguide, which is essential for pulse compression, two conditions need to be fulfilled: (i) the incoming pulse is coupled into a single eigenmode. This can be
achieved by proper choice of the focusing geometry leading to a beam waist radius matched to the HCF: \( w_0 \approx 0.6436 \cdot a \), where \( a \) denotes the bore radius of the HCF [112]. Furthermore, (ii) the full pulse energy is kept in this mode throughout the propagation (prevention of mode mixing). It turned out that both criteria can be fulfilled. The details of this technique can be found in an in-depth review of the inventors [113]. Shortly after its introduction this technique allowed the generation of 4.5 fs pulses [114]. The hollow-core fiber compressor together with chirped mirrors for dispersion compensation [23,24] made a major impact, as they are two key technologies essential for providing sufficiently intense few-cycle drivers [1,115,116] for efficient isolated attosecond pulse generation via high-harmonic process in noble gases [117–119].

By using sufficiently short input pulses of \( \approx 23 \) fs and taking much care of keeping the nonlinearities in the laser compressor low, sub-2-cycle pulses could be obtained with 0.5 mJ [120] and later with 1 mJ [121]. Further energy scaling can be achieved by using circularly polarized light [122], as it reduces the nonlinearity: the nonlinear refractive index for circular polarization is \( 2/3 \) of the value for the linear case and also the ionization threshold is higher. By using circularly polarized light and neon as the nonlinear medium 4.3 fs pulses with 1 mJ energy [123] and later 1.9 mJ pulses with longer duration of 5.7 fs [124] could be obtained.

Due to their similar geometry, HCF compressors are especially well matched to fiber lasers. They serve as a natural extension of ytterbium-doped fiber lasers, emitting pulses at 1030 nm with durations of a few hundred fs, in order to shorten their pulses [125] down to the few-cycle regime [126]. As the average power of the fiber lasers exceeded the 50 W level, J. Limbert’s group developed a water-cooled HCF compressor for their sources capable of handling high average powers up to the kW level [127]. In a first study 700 fs long 0.2 mJ pulses at 1 MHz repetition rate were compressed to 81 fs duration at record-breaking average power of 93 W [128]. Later, the group achieved 0.54 mJ, 26 fs pulses at an average power of 135 W [129]. In a cascaded two-stage arrangement J. Rothhardt et al. compressed 1 mJ, 210 fs pulses at 150 kHz repetition rate to few-cycle duration of 7.8 fs with an energy of 0.35 mJ at 52.5 W [130]. Subsequently, S. Hädrich et al., achieved few-cycle 6.3 fs pulses at an unprecedented average power of 216 W by a cascaded HCF setup [131]. The first stage of that compressor operated at an even higher power of 406 W.

At longer wavelengths in the short-wavelength infrared (SWIR) sub-3-cycle pulses of 0.4 mJ energy at 1425 nm [132] and sub-2-cycle pulses at 1.8 \( \mu \)m [133,134] were generated using standard HCFs. Compressing pulses of a thulium-doped fiber laser emitting at 2 \( \mu \)m in a HCF 7-cycle pulses were obtained at an average power of over 15 W [135]. HCFs were also utilized for the generation of ultrashort pulses in the deep ultraviolet (DUV). In the first
experiment at 270 nm the gas-filled HCF served both for sum-frequency generation and for spectral broadening of the pulses [136]. In this way 8 fs pulses were obtained with 1 μJ energy. Later, 100 fs pulses of a KrF excimer laser emitting at 248 nm were compressed by our group in a HCF to sub-20 fs at 20 μJ energy [137].

After the great initial success of the HCF compressors the technique saturated at the sub-mJ energy level. This can be understood by analyzing the scaling law (Eq. 16) for case of gas-filled HCFs:

$$P_0 \propto \frac{1}{p \cdot \kappa_2} \propto a^2 \propto L_{\text{eff}}.$$  (17)

The reason for the stagnation had a pure technical nature: the length of the HCFs were practically limited to ~1 meter due to the following reasons: The HCFs are very sensitive to bending (see Sect. 4.4) which leads to rapid mode mixing and increased guiding losses. Therefore, the capillaries need to be very straight; in fact the straightness is the main figure of merit of the hollow fibers. The capillaries used at that time for the compression were thick-walled rigid fused silica tubes with bore diameters of 200 to 300 μm and with an outer diameter of 1–2 mm. In order to keep them straight they were placed into a mechanically machined holder with a V-groove, as shown in Figure 5(a). It turned out that in this construction proper guiding is practically limited to about 1 meter length by the quality of the capillary and the V-groove. In this case the playroom is restricted to the application of materials with reduced nonlinearity, such as neon or helium [138]. This limitation is clearly observable in [139], where 6 mJ, 200 fs long pulses at 1030 nm were compressed by using neon in a standard HCF. The obtained compressed pulse energy of 4 mJ is a good achievement; however, due to the short length the HCF did not support pulses shorter than 35 fs.

Figure 5. Hollow fibers. a. standard capillary; b. stretched flexible hollow-core fiber.
Much larger power scaling can be achieved by overcoming the length limitation of HCFs what we discuss in the next section.

4.4. *Stretched flexible hollow-core fibers*

In Sect. 4.1 we showed that for energy scaling one needs to increase both the effective mode area, which is proportional to the bore radius \(a\), and the waveguide length \(L\). The transmission of a hollow waveguide can be described by Beer’s law: \(T = \exp(-\alpha L)\), where \(\alpha\) denotes the waveguide losses. According to [111] it can be factorized in the following way:

\[
\alpha = \alpha_0 + \alpha_R, \quad \text{where} \quad \alpha_0 \propto \frac{1}{a^2} \quad \text{and} \quad \alpha_R \propto \frac{1}{a_0 R^2}.
\]

Here \(\alpha_0\) denotes the loss coefficient of a straight HCF and \(\alpha_R\) describes an extra loss due to bending of the waveguide with a constant radius of curvature of \(R\). Since the linear loss coefficient is inversely proportional to the third power of the inner radius, the up-scaling of the fiber geometry with the peak power results in a net increase of the transmission of an ideal HCF. However, the last part of Eq. 18 shows that at the same time the bending losses scale inversely proportional with \(\alpha_0\) making the waveguide very sensitive to bending. Therefore, for efficient power scaling it is essential to obtain very straight hollow waveguides with large dimensions which is clearly beyond the possibilities of common rigid capillaries.

In 2008 a new idea was published by our group at Laser-Laboratorium Göttingen: instead of using common rigid capillaries, one can also utilize capillaries having much thinner walls in the order of a few tens of microns, which makes them flexible. Instead of putting them into a machined V-groove one can pull both ends and span them to be straight [140]. In principle, this is equivalent to the catenary problem which describes a chain fixed at both ends sagging under the force of gravity (see Figure 6).

The solution is well known: the chain with a length of \(l\) takes a form described by the following function:

\[
y = R_{\min} \cosh(x/R_{\min}),
\]

![Figure 6. The catenary curve.](image)
where \( x \) and \( y \) denote the horizontal and vertical coordinates, respectively, and \( R_{\text{min}} \) is the minimum radius of the curvature situated in the middle of the curve. It can be shown, that \( R_{\text{min}} = T/w \) is the ratio of the stretching force to the weight per unit length of the chain \( (w = W/l) \). Therefore, the minimal radius of curvature of a stretched flexible hollow-core fiber (SF-HCF) can be optimized solely by increasing the stretching force independent on the length of the waveguide which makes this kind of capillary virtually freely scalable. Thanks to the small wall thickness, the specific weight of a capillary is usually less than 1 gram/m. Therefore, it is easy to achieve a minimal radius of curvature of several kilometers by applying a moderate stretching force.

In order to determine the wave-guiding properties of the new type of hollow fiber (see Figure 5(b)), a virtually perfect diffraction-limited beam of a helium-neon laser was launched into a 2.5 m long SF-HCF. The transmission of the waveguide was measured by a photodiode showing that the transmission of the waveguide approaches the theoretical value within 1%. Furthermore, the beam emerging from the fiber was recorded at a set of distances from the fiber end exhibiting perfectly round beam profiles with a divergence matching the numerically calculated value, as displayed in Figure 7 [140]. This proves that the output beam of the SF-HCF is practically diffraction-limited.

Thanks to their free length scalability and at the same time near ideal wave-guiding properties, the SF-HCFs are now well established as

---

**Figure 7.** Wave-guiding properties of a SF-HCF. Adapted with permission from [140] © The Optical Society.
the second generation of hollow fibers in demanding compression tasks. In fact, most of the records in post-compression techniques to date were achieved by SF-HCFs (see Sect. 6).

4.5. Techniques for further scaling: pressure gradient, ionization, planar waveguides, multiplexing, and molecular gases

The scaling laws described in Sect. 4.1 are based on conservative premises which ensure pure Kerr-based spectral broadening. They are very useful as general guidelines for the design of compressors, but at the same time, they are also very restrictive prescribing very long geometries which are often hard to build in a limited lab space. A number of methods were developed to further scale waveguide-based post compressors by ‘overdriving’ the spectral broadening process. These techniques always make a careful trade-off between achievable spectral broadening, peak power and multimode operation.

One of the most powerful techniques was introduced by K. Midorikawa’s group. In order to mitigate severe incoupling losses in the high-intensity regime where filamentation can occur in the gas medium, A. Suda and coworkers separated the gas volumes at both ends of the HCF, evacuated the beam path in front of the waveguide and applied gas at the output side of the fiber [141]. In this way optimal incoupling can be maintained at high intensities preventing filamentation-induced beam reshaping in front of the fiber. Due to the pressure gradient inside the fiber the local pressure can be calculated as:

\[ p(z) = \sqrt{p_0^2 + \frac{z}{L} (p_L^2 - p_0^2)}, \]

where \( p_0 \) and \( p_L \) are the gas pressures in front of and behind the capillary of length \( L \). With this scheme, the nonlinear interaction builds up gradually which delays self-focusing and ionization inside the fiber and thus suppresses the population of higher-order eigenmodes [142]. The down side of the method is the reduced effective interaction length \( L_{\text{eff}} \approx 2/3L \). This issue can be mitigated by using long waveguides, as demonstrated by our study comparing the performance of standard HCFs with long SF-HCFs [143], whose construction naturally supports pressure-gradient operation.

In a 1 m long 300 \( \mu \)m inner diameter HCF differentially pumped with neon gas C.H. Nam’s group generated 5.5 fs pulses at 0.2 TW [144] and later 3.7 fs pulses at 0.3 TW peak power [145]. In these studies the compressed pulse energy up to 1.2 mJ was limited by the 1 m length of the available hollow fiber.

Using pressure gradient in a He-filled 2.2 m long HCF of 500 \( \mu \)m inner diameter the K. Midorikawa’s group could compress the pulses of a Ti:
sapphire laser first to sub-10 fs [141, 146], later to 2-cycle duration [147] with multi-mJ energy which culminated in a long-standing record of the generation of 5 mJ, 5 fs pulses [148]. For few-cycle pulses it is very important that their carrier envelope phase (CEP) can be stabilized and controlled. Therefore, it was important to show that the laminar gas flow inside a HCF used with pressure gradient does not deteriorate the CEP stability of the pulses [149, 150]. Furthermore, the well-controlled incoupling using pressure gradient in a HCF allowed easy pressure tuning of the Fourier-limited duration of high energy pulses [151].

Combining all above-mentioned methods for energy scaling, e.g. (i) using helium with (ii) circularly polarized light (iii) in a large-aperture and long capillary with (iv) pressure gradient, we achieved CEP stabilized 3 mJ, 4 fs pulses in a 2 m long SF-HCF with 450 µm bore diameter by compressing 8 mJ 23 fs pulses in close collaboration with R. Lopez-Martens’ group [95]. In this study we found out that excessive nonlinearity ($B > 3$ rad) inside the input window of the HCF assembly deteriorates the incoupling and thus the transmission of the waveguide which cannot be recovered by longitudinally translating the fiber entrance. Therefore, as the overall length available for the compressor was strictly limited to 5.5 m, further increasing the input energy of the system to 10 mJ required the construction of a vacuum beamline with integrated HCF. In the current setup all focusing/recollimating optics are situated in the vacuum/gas chambers and only a large collimated beam at low intensity passes the windows without inducing unwanted nonlinearities. In this way well-controlled CEP-stable 3.5 fs pulses with 3.5 mJ are routinely generated [152]. Recently, an up-scaled version of the system has been commissioned at the Max Born Institute with an overall length of 8.2 m where 14 mJ 50 fs pulses are compressed to 1.5 optical cycles at 3.8 fs duration with an output energy of 6.1 mJ with a peak power of 1.2 TW [153] clearly breaking the TW barrier.

At even higher input peak powers and pulse energies (TW-scale input pulses) one can utilize phase modulation induced by photoionization instead of the Kerr-effect, as described in Sect. 2.2. Based on this phase modulation mechanism French groups from Bordeaux and Saclay compressed ~70 mJ, 40 fs pulses to 11 fs with 13.7 mJ energy in low-pressure helium-filled HCF [154–156] at 10 Hz repetition rate. In a later work, they achieved 9 fs pulses at 8.7 mJ energy [157]. However, the high losses and the long compressed pulse durations do not make this technique attractive.

An alternative solution can be to drastically increase the mode area by utilizing waveguides with more dimensions. Instead of capillaries, one can use hollow planar waveguides, as first proposed by K. Midorikawa’s group [158, 159], obtaining 2 mJ, 12 fs pulses. A. Mysyrowicz’s group advanced the technology to reach about 10 mJ pulse energy at a duration of 10–12 fs [160–162]. Later, C. Arnold’s group reached the terawatt level at sub-15 fs
duration [163], but since then this method has not found further applications. The main obstacle of this technique is connected to the incomplete wave-guiding properties of the arrangement as there is no guiding parallel to the plates. Therefore, the beam profile cannot be controlled along this direction leading to inhomogeneous spectral broadening and at higher intensities to multi-filamentation.

According to a substantially different energy scaling technique independently introduced by J. Limpert’s and P. Georges’ groups the input pulses are divided to a sequence of $N$ identical pulses with $N$ times lower energy before they enter the HCF. In the course of the nonlinear propagation each pulse experiences spectral broadening independently. In a subsequent step the pulses are recombined to form a high-energy pulse with a broadened spectrum [164,165]. This multiplexing method performs well with longer pulses with narrow spectra; however, it is non-trivial to apply it to few-cycle pulses due to the lack of broadband dispersion-free polarizing beam splitters and combiners. An elegant adaptation to few-cycle regime is explored by the R. Lopez-Martens’ group utilizing birefringence in crystals for both the division and for the recombination [166,167]. Analogously to the temporal multiplexing scheme one can also use spatial multiplexing where the divided pulses are spectrally broadened parallel in separate HCFs before they are recombined [168].

Some sources require HCF scaling to opposite direction: those where either due to low pulse energy and/or long pulse duration the peak power is insufficient for using noble gases-filled HCFs for spectral broadening. In this case molecular gases offer a viable solution where molecular rotation (Raman scattering) cause a non-instantaneous nonlinear response [169] leading to a pronounced enhancement of the red side of the spectrum [140,170]. On the one hand this slow nonlinearity can be much larger than the instantaneous Kerr-effect which makes it suitable to the compression of low peak-power pulses, on the other hand the input pulse duration needs to be sufficiently long to experience the Raman-effect (>100 fs). A further advantage of many molecular gases against atomic species is that the proportion between nonlinearity and ionization potential is often better than in atomic gases which results in larger spectral broadening capacity [171,172]. Furthermore, the spectral broadening and chirp can be greatly optimized by proper matching of the input pulse duration to the time constant of the delayed nonlinearity (or find a proper molecule for the input pulses), as it was shown in a recent publication of J.E. Betar et al. [173] achieving 45-times compression and two-octave spanning spectrum. This technique fits well to the compression of several 100 fs long pulses of Yb-based lasers. However, the heat generated by the friction of the molecular motion may limit the average-power-scaling using molecular gases
thus questioning their suitability for high repetition-rate systems with several 10s or 100s of watts of power.

5. Self-compression techniques

Besides post-compression techniques using spectral broadening and subsequent compression steps, there is another slightly different approach, which utilizes self-compression dynamics during nonlinear propagation. These techniques have a great promise to achieve the shortest optical pulses (not counting high-harmonic generation, which is a secondary process) down to the sub-cycle regime with multi-octave-spanning spectra well beyond the capabilities of today’s chirp compensation techniques. On the down side, we note that due to the highly nonlinear dynamics, which is very sensitive to the parameters of the input pulse, it is very difficult to scale the pulse energy and peak power in these schemes. Today few-cycle pulses generated by self-compression techniques can exceed the mJ level only for driver pulses in the mid-infrared spectral range. Furthermore, the self-compressed pulses usually exhibit a rather long pedestal on the leading edge of the pulse.

In a pioneering work self-compression in the course of ionization-dominated propagation in a short hollow waveguide filled with low-pressure argon was observed [174]. The group of H.C. Kapteyn and M.M. Murnane obtained 13 fs pulses by coupling 2.2 mJ 30 fs Ti:sapphire pulses into a 2.5 cm long HCF with 150 μm inner diameter.

Self-compression can also be obtained in filamentation as a result of the complex interplay between self-focusing, ionization-induced defocusing, SPM and self-steepening as was uncovered by numerical simulations [175,176]. In the visible/near-infrared range it was observed experimentally generating sub-10 fs [177] and 8 fs [85,178,179] pulses. Later, self-compression down to 11 fs was observed directly inside the filament after terminating it by abruptly removing the nonlinear medium in a differential vacuum stage [180]. With a similar technique, the pulse evolution along the filament was tracked first by a spectrometer [181] and later by a FROG device placed in the vacuum behind the filament revealing the rich spatio-temporal dynamics leading to the formation of even shorter pulses of 5 fs duration [182]. At a longer wavelength of 2 μm, CEP-stable 2.5-cycle pulses of 17 fs duration with an energy of 1.2 mJ were generated in a xenon filament [183].

In the mid infrared most of the materials exhibit anomalous dispersion which, combined with the Kerr-effect, can also lead to compression due to soliton propagation. This is a well-known effect in silica fibers [86] which was recently also demonstrated using a multi-pass cell incorporating a fused silica plate as the nonlinear medium at 1550 nm [184]. In case of filaments in this spectral range, it is not easy to clearly identify whether filamentation dynamics or soliton propagation is dominant in self-compression. In an
experiment at 3.9 μm wavelength the observed self-compression of 21 mJ 94 fs pulses to 30 fs in a bulk YAG plate [185] and later, positively chirped 18 mJ, 100 fs pulses to 39 fs at 16 mJ energy in nitrogen was interpreted in terms of soliton propagation [186]. In another experiment with the same light source self-compression of positively chirped 150 fs pulses with 29.5 mJ energy, having similar peak power as in the former case, to sub-3-cycle with a duration of 28 fs at a peak power of 0.34 TW was interpreted as light bullet formation in air filament [187].

Well-controlled self-compression at μJ energy level can be achieved by high-order soliton formation in hollow waveguides having negative dispersion. It was shown theoretically by J. Herrmann’s group that hollow-core Kagome-lattice PCFs exhibit the necessary transmission bandwidth and negative dispersion over the entire visible and UV spectral range in order to support soliton propagation [188]. Furthermore, they predicted high-energy supercontinuum generation and sub-10 fs UV-VUV pulse generation via resonant dispersive-wave (RDW) emission, which is tunable by the gas pressure in the waveguide [189]. Both effects were then experimentally demonstrated by P.St.J. Russell’s group [190]. The generated RDW pulses were tunable in the range of 240–350 nm which was later extended to 120–200 nm [191] containing a few tens of nJ energy. A broad supercontinuum spanning more than three octaves was also generated [192]. Later, by using a large-core single-ring HC-PCF the group successfully demonstrated self-compression to the single-cycle regime [108], scaled up the UV pulse energy to the μJ-level at MHz repetition rate [104] and demonstrated CEP stability of the self-compression [193]. Using longer wavelengths can help in energy scaling also in case of soliton compression. T. Balciunas et al. compressed 35 μJ, 80 fs pulses at 1.8 μm wavelength in an argon-filled Kagome-fiber to sub-cycle duration of 4.5 fs with 25 μJ energy [194]. At even longer wavelength of 3.25 μm, near-single-cycle pulses of 14.5 fs duration were generated at a high average power of 9.6 W [195]. Further details on nonlinear optics in HC-PCFs can be found in the reviews [109,196].

Much larger energy was achieved by J.C. Travers et al. as they recognized that gas-filled capillaries can also exhibit negative dispersion at proper choice of parameters enabling soliton dynamics [197]. Since the energy scaling requires several-meter-long high-quality capillaries, their approach relies on the SF-HCF technology. In their seminal paper, J.C. Travers’ group compressed 10 fs near-infrared pulses from a HCF compressor to sub-fs duration at several 100 μJ energy in a helium-filled 3 m long SF-HCF with 250 μm inner diameter. Furthermore, they generated few-cycle UV pulses up to 16 μJ pulse energy tunable between 120 and 350 nm by changing the gas pressure, as shown in Figure 8. In a recent work, the group solved the chirping problem of the output window using the waveguide with inverse pressure gradient: applying the gas at the entrance of the waveguide and putting the output side into vacuum [198].
6. State-of-the-art

In this section we briefly summarize the best compression results and compare the different approaches. There are many aspects which are important for post-compression arrangements and thus serve as figures of merit: compressed pulse duration, pulse energy, peak power, compression factor and average power. The improvement of these parameters invokes major technical challenges and has crucial impact on the applications. Most of them are limited by basic physical laws, others such as the compression factor and the average power behind a compressor is more of technical nature. Figures 9 and 10 display the most relevant compression results for the near infrared spectral range, where the vast majority of the post-compression results were obtained. The free propagation in bulk and the filament results are not included in the figures, because the spatial inhomogeneity of their spectrum is either too large or not well defined in the reports. The plotted results are achieved using two different class of lasers: Ti:sapphire lasers, which have been the work horse of ultrafast science in the last decades, emitting pulses with a duration of 20–100 fs at around 800 nm operating up to a few tens of watts average power at up to a few kHz repetition rate (marked by solid symbols in Figures 9 and 10). The other results are obtained with neodymium or ytterbium-doped high-power (up to kW), high-repetition rate (from 100 kHz to tens of MHz) lasers based on fiber, slab or thin-disk architectures [199]. These lasers emit considerably longer pulses at durations typically between 250 fs and 1.5 ps (marked by hollow symbols in Figures 9 and 10).

The choice of the laser gain medium clearly manifests in the tendencies observable in Figures 9 and 10: the shortest pulses were mostly generated by
using shorter input pulses of Ti:sapphire lasers while the highest average powers were achieved with ytterbium-based systems. Comparing the post-compression methods one can observe that the multi-pass cell technique is the only one, which has not demonstrated sub-3-cycle operation, yet. The HC-PCFs have the least capacity in terms of pulse energy and peak power, not exceeding 20 µJ and 1–2 GW. Hollow fibers and multi-pass cells perform best in terms of pulse energy and average power; however, only hollow fibers could reach the terawatt peak power level so far (red-shaded area in Figure 9). The highest average power of post-compressed systems exceeds 100 W. Here the most remarkable results are 530 W at 30 fs duration [68] and 375 W at 170 fs [61], both obtained by multi-pass cells, and 406 W/30 fs and 216 W/6.3 fs [131] in HCF as well as 318 W/10 fs in SF-HCF [200].

The HCF technology is the only post-compression technique which reaches the high-energy few-cycle regime with above-mJ, sub-3-cycle pulses (red-shaded area in Figure 9), clearly dominating this field at present. Two of the three best results in this regime were obtained with SF-HCFs (6.1 mJ/3.8 fs [153] and 3.5 mJ/3.4 fs [152]) and the third with a long HCF (5 mJ/5 fs [148]). In the terawatt regime the picture is similar: here in addition to the

Figure 9. Peak versus average power achieved by post compression in the near-infrared range. The red-shaded area represents the terawatt range.
previously mentioned works there is a preliminary result achieved with SF-HCF technology (40 mJ/25 fs [201]).

A further important figure of merit is the compression ratio. We distinguish between results achieved by a single-stage compressor and that obtained in a cascaded setup [202,203]. The former ones are much simpler and more robust; however, the cascaded solutions offer larger compression capacity. The best single-stage results are based on multi-pass cells with many roundtrips or very long SF-HCFs. Multi-pass cells reach 37.5 (the first stage of [69]) and 31.7 times temporal compression [67] with 44 and 45 passes, respectively. Using 6 m long SF-HCFs, 30 [200] and 33 times compression [204] were achieved. Using molecular gases in a SF-HCF a record compression factor of 45 was recently achieved [173]. For comparison, the largest compression factor obtained by a standard 1 m long HCF is 13.1 [129]. The largest compression factors obtained by cascaded setups are 92.3 with multi-pass cells [69], 84.2 by HC-PCFs [108], 53 by multi-plate arrangements [56], 48.5 by a combination of a multi-pass cell and a subsequent HCF [205] and 38.1 by HCFs [131] and 33 by cascaded second-order nonlinearity followed by an SF-HCF [206].

The above results were achieved in the near infrared spectral range. However, other ranges such as deep ultraviolet or mid to far infrared play increasingly
prominent role for a broad range of applications. In the DUV range, the highest-energy pulse compression was carried out by using the very first SF-HCF to obtain 24 fs duration with ten-times higher pulse energy than previous works at a level of 200 µJ [207]. Much shorter few-cycle pulses tunable across the deep-UV were obtained by RDW emission during soliton propagation of 800 nm pulses in HC-PCFs and later in SF-HCFs (see Sect. 5). The highest-energy few-cycle pulses of up to 16 µJ in the UV was achieved by utilizing SF-HCFs [197].

In the short-wavelength infrared (SWIR) range at a wavelength of 1.8 µm the highest-energy compressed pulses were achieved by using a 3 m long SF-HCF obtaining 12 fs long 2-cycle pulses with 5 mJ energy at a peak power in excess of 0.4 TW [208]. The shortest pulses with sub-cycle duration of 4.5 fs were generated by compression in a HC-PCF [194]. In the mid-wave infrared (MWIR) at 3.2 µm 22 fs long 2.1-cycle pulses with 2.5 mJ energy were achieved by compression in a SF-HCF [209], while another group obtained 21.5 fs long 1.6 cycle pulses with 2.6 mJ energy with a very similar arrangement at a slightly longer wavelength of 4 µm [210]. Multi-mJ pulses of an OPCPA operating at 3.9 µm were compressed during filamentation to sub-3-cycle duration with up to 16 mJ pulse energy [84,186,187].

7. Future perspectives

Currently the most established and widely used technologies are HCFs for mJ-level and HC-PCFs for µJ-level pulse compression as well as solid filaments for white-light generation. It is foreseeable that for mJ systems SF-HCFs and MPCs will gradually spread in labs taking over the place of standard HCFs. In the µJ-level sources molecular gas-filled HCFs or SF-HCFs may challenge HC-PCFs at moderate average power levels due to their easier usage. Altogether; there are several fast developing techniques with great potential. Here we give our vision on the most important trends in post compression.

A fascinating development which can have a big impact on ultrafast science and applications is the RDW generation during soliton self-compression in SF-HCFs. With its unprecedented combination of easy tunability, intrinsic short pulse duration (few-cycle) and sufficiently high pulse energy (several tens of µJ) this method is an ideal light source for a broad range of applications including ultrafast spectroscopy. This technology covers not only the full ultraviolet spectral range but also the entire visible spectrum, when using 1.8 µm pumping pulses, as was very recently demonstrated by J.C. Travers’ group [211]. This brings a very versatile ultrafast source in the labs.

The performance of multi-pass cells rose in the last years quickly, seriously challenging the HCF technology. Especially for compression of sub-ps pulses at high-average-powers, they offer an excellent solution. However, an exciting question remains: can they cope with the few-cycle regime? The difficulty comes from the combination of requirements on the cavity mirrors, as they
should (i) support near octave-spanning spectral range necessary for few-cycle pulses; (ii) have high reflectivity, well above 99%, to keep the throughput high; and (iii) exhibit a very high laser-induced damage threshold, to keep the size reasonable. All three requirements have to be fulfilled at the same time which greatly challenges the coating technology, thus influencing the success of multi-pass cells on their way towards the few-cycle regime.

At a given wavelength further energy scaling with the currently available techniques can only be achieved by drastically increasing the physical length of the compressor, as one can recognize in the newest results: both multi-pass cells [212] and SF-HCFs [201] become inconveniently long (6–8 m) at several tens of mJ pulse energy. Following the scaling law (Eq. 16 or 17), the key parameter posing physical limitations to the Kerr-driven spectral broadening process is the input peak power. Therefore, targeting 50–100 mJ few-cycle pulses in the near-infrared spectral range requires starting from longer pulses of 100–150 fs duration in order to keep the peak power at a reasonable level. Moreover, it is necessary to apply a technique with large compression capabilities ($F > 30$) to directly reach the few-cycle regime. Currently the only approach which is capable of providing sufficiently large spectral broadening at sufficiently large (octave-spanning) spectral widths is the SF-HCF technology. Therefore, a viable route to 100 mJ-scale few-cycle pulses compresses sub-ps pulses of a high-energy slab or thin-disk laser in a cascaded setup: The first stage, which can be either a HCF, SF-HCF or multi-pass cell, brings the pulses to a duration from which a subsequent very long (6–10 m), large-aperture SF-HCF can further compress the pulses down to the few-cycle regime. We anticipate that further development of post-compression techniques will pave the way of the future of ultrafast science and technology.

**Disclosure statement**

TN and PS are the inventors of the stretched flexible hollow fibers. The patent is owned by the Institute for Nanophotonics Göttingen e.V.

**Funding**

This work was supported by the Vetenskapsrådet [2016-05409]; Vetenskapsrådet [2019-02376].

**ORCID**

Tamas Nagy http://orcid.org/0000-0002-8158-4938
Laszlo Veisz http://orcid.org/0000-0002-7694-9066
References

[1] Brabec T, Krausz F. Intense few-cycle laser fields: frontiers of nonlinear optics. Rev Mod Phys. 2000;72:545–591.
[2] Chang Z, Corkum PB, Leone SR. Attosecond optics and technology: progress to date and future prospects [Invited]. J Opt Soc Am B. 2016;33:1081–1097.
[3] Li J, Lu J, Chew A, et al. Attosecond science based on high harmonic generation from gases and solids. Nat Commun. 2020;11:2748.
[4] Kobayashi T. Development of ultrashort pulse lasers for ultrafast spectroscopy. Photonics. 2018;5:11.
[5] Krausz F, Ivanov M. Attosecond physics. Rev Mod Phys. 2009;81:163–234.
[6] Corkum PB, Krausz F. Attosecond science. Nat Phys. 2007;3:381–387.
[7] Lépine F, Ivanov MY, Vrakking MJJ. Attosecond molecular dynamics: fact or fiction? Nat Photon. 2014;8:195–204.
[8] Nisoli M, Decleva P, Calegari F, et al. Attosecond electron dynamics in molecules. Chem Rev. 2017;117:10760–10825.
[9] Young L, Ueda K, Gühr M, et al. Roadmap of ultrafast x-ray atomic and molecular physics. J Phys B At Mol Opt Phys. 2018;51:032003.
[10] Leone SR, McCurdy CW, Burgdörfer J, et al. What will it take to observe processes in ‘real time’? Nat Photon. 2014;8:162–166.
[11] Dombi P, Papa Z, Vogelsang J, et al. Strong-field nano-optics. Rev Mod Phys. 2020;92:025003.
[12] Siegrist F, Gessner JA, Ossiander M, et al. Light-wave dynamic control of magnetism. Nature. 2019;571:240–244.
[13] Calegari F, Ayuso D, Trabattoni A, et al. Ultrafast electron dynamics in phenylalanine initiated by attosecond pulses. Science. 2014;346:336–339.
[14] Guénot D, Gustas D, Vernier A, et al. Relativistic electron beams driven by kHz single-cycle light pulses. Nat Photon. 2017;11:293–296.
[15] Schwab MB, Savert A, Jackel O, et al. Few-cycle optical probe-pulse for investigation of relativistic laser-plasma interactions. Appl Phys Lett. 2013;103:191118.
[16] Champeney DC. Fourier transforms and their physical applications. Academic Press London; 1973. Appendix F.
[17] Giordmaine J, Duguay M, Hansen J. Compression of optical pulses. IEEE J Quantum Electron. 1968;4:252–255.
[18] Duguay MA, Hansen JW. Compression of pulses from a mode-locked He–Ne laser. Appl Phys Lett. 1969;14:14–16.
[19] Fisher RA, Kelley PL, Gustafson TK. Subpicosecond pulse generation using optical Kerr effect. Appl Phys Lett. 1969;14:140–143.
[20] Treacy EB. Compression of picosecond light pulses. Phys Lett A. 1968;28:34–35.
[21] Treacy EB. Optical pulse compression with diffraction gratings. IEEE J Quantum Electron. 1969;QE 5:454–458.
[22] Laubereau A. External frequency modulation and compression of picosecond pulses. Phys Lett A. 1969;29:539–540.
[23] Szipocs R, Ferencz K, Spielmann C, et al. Chirped multilayer coatings for broad-band dispersion control in femtosecond lasers. Opt Lett. 1994;19:201–203.
[24] Szipocs R, Kohazi-Kis A. Theory and design of chirped dielectric laser mirrors. Appl Phys B. 1997;65:115–135.
[25] Matuschek N, Kartner FX, Keller U. Theory of double-chirped mirrors. IEEE J Sel Top Quantum Electron. 1998;4:197–208.
[26] Miranda M, Penedones J, Guo C, et al. Fast iterative retrieval algorithm for ultrashort pulse characterization using dispersion scans. J Opt Soc Am B. 2017;34:190–197.
[27] Timmers H, Kobayashi Y, Chang KF, et al. Generating high-contrast, near single-cycle waveforms with third-order dispersion compensation. Opt Lett. 2017;42:811–814.
[28] Yamane K, Zhang Z, Oka K, et al. Optical pulse compression to 3.4fs in the monocyte region by feedback phase compensation. Opt Lett. 2003;28:2258–2260.
[29] Matsubara E, Yamane K, Sekikawa T, et al. Generation of 2.6 fs optical pulses using induced-phase modulation in a gas-filled hollow fiber. J Opt Soc Am B. 2007;24:985–989.
[30] Wirth A, Hassan MT, Grguras I, et al. Synthesized light transients. Science. 2011;334:195–200.
[31] Hassan MT, Luu TT, Moulet A, et al. Optical attosecond pulses and tracking the nonlinear response of bound electrons. Nature. 2016;530:66–70.
[32] Bejot P, Schmidt BE, Kasparian J, et al. Mechanism of hollow-core-fiber infrared-supercontinuum compression with bulk material. Phys Rev A. 2010;81:63828.
[33] Mandel L. Interpretation of instantaneous frequencies. Am J Phys. 1974;42:840–846.
[34] Zhavoronkov N, Korn G. Generation of single intense short optical pulses by ultrafast molecular phase modulation. Phys Rev Lett. 2002;88:203901.
[35] Theberge F, Liu W, Luo Q, et al. Ultrabroadband continuum generated in air (down to 230 nm) using ultrashort and intense laser pulses. Appl Phys B. 2005;80:221–225.
[36] Khodakovskiy NG, Kalashnikov MP, Pajer V, et al. Generation of few-cycle laser pulses with high temporal contrast via nonlinear elliptical polarisation rotation in a hollow fibre compressor. Laser Phys Lett. 2019;16:095001.
[37] Pinault SC, Potasek MJ. Frequency broadening by self-phase modulation in optical fibers. J Opt Soc Am B. 1985;2:1318–1319.
[38] Diels J-C, Rudolph W. Ultrashort laser pulse phenomena. Academic Press; Amsterdam. 2006, pp. 205–207.
[39] Fibich G, Gaeta AL. Critical power for self-focusing in bulk media and in hollow waveguides. Opt Lett. 2000;25:335–337.
[40] Tempea G, Brabec T. Nonlinear source for the generation of high-energy few-cycle optical pulses. Opt Lett. 1998;23:1286–1288.
[41] Keldysh LV. Ionization in field of a strong electromagnetic wave. Sov Phys JETP. 1965;20:1307–1314.
[42] Perelomov AM, Popov VS, Terentev MV. Ionization of atoms in an alternating electric field. Sov Phys JETP. 1966;23:924–934.
[43] Ammosov MV, Delone NB, Krainov VP. Tunnel ionization of complex atoms and of atomic ions in an alternating electromagnetic field. Sov Phys JETP. 1986;64:1191–1194.
[44] Yudin G, Ivanov M. Nonadiabatic tunnel ionization: looking inside a laser cycle. Phys Rev A. 2001;64:013409.
[45] Kelley PL. Self-focusing of optical beams. Phys Rev Lett. 1965;15:1005–1008.
[46] Simmons WW, Hunt JT, Warren WE. Light-propagation through large laser systems. IEEE J Quantum Electron. 1981;17:1727–1744.
[47] Rolland C, Corkum PB. Compression of high-power optical pulses. J Opt Soc Am B. 1988;5:641–647.
[48] Mevel E, Tcherbakoff O, Salin E, et al. Extracavity compression technique for high-energy femtosecond pulses. J Opt Soc Am B. 2003;20:105–108.
[49] Mourou G, Mironov S, Khazanov E, et al. Single cycle thin film compressor opening the door to Zeptosecond-Exawatt physics. The European Physical Journal Special Topics. 2014;223:1181–1188.

[50] Khazanov EA, Mironov SY, Mourou GA. Nonlinear compression of high-power laser pulses: compression after compressor approach. Phys Usp. 2019;62:1096–1124.

[51] Ginzbarg V, Yakovlev I, Zuev A, et al. Fivefold compression of 250-TW laser pulses. Phys Rev A. 2020;101:013829.

[52] Mironov SY, Fourmaux S, Lassonde P, et al. “Thin plate compression of a sub-petawatt Ti: salaser pulses. Appl Phys Lett. 2020;116:241101.

[53] Lu C-H, Tsou Y-J, Chen H-Y, et al. Generation of intense supercontinuum in condensed media. Optica. 2014;1:400-406.

[54] Cheng Y-C, Lu C-H, Lin -Y-Y, et al. Supercontinuum generation in a multi-plate medium. Opt Express. 2016;24:7224–7231.

[55] He P, Liu Y, Zhao K, et al. High-efficiency supercontinuum generation in solid thin plates at 0.1TW level. Opt Lett. 2017;42:474–477.

[56] Lu C-H, Wu W-H, Kuo S-H, et al. Greater than 50 times compression of 1030 nm Yb: kW laser pulses to single-cycle duration. Opt Express. 2019;27:15638–15648.

[57] Lu C-H, Witting T, Husakou A, et al. Sub-4 fs laser pulses at high average power and high repetition rate from an all-solid-state setup. Opt Express. 2018;26:8941–8956.

[58] Herriott D, Kogelnik H, Kompfner R. Off-axis paths in spherical mirror interferometers. Appl Opt. 1964;3:523–526.

[59] Sennaroglu A, Fujimoto JG. Design criteria for Herriott-type multi-pass cavities for ultrashort pulse lasers. Opt Express. 2003;11:1106–1113.

[60] Kowalevicz AM, Sennaroglu A, Zare AT, et al. Design principles of q-preserving multipass-cavity femtosecond lasers. J Opt Soc Am B. 2006;23:760–770.

[61] Schulte J, Sartorius T, Weitenberg J, et al. Nonlinear pulse compression in a multi-pass cell. Opt Lett. 2016;41:4511–4514.

[62] Hanna M, Délen X, Lavenu L, et al. Nonlinear temporal compression in multipass cells: theory. Opt Soc Am B. 2017;34:1340–1347.

[63] Weitenberg J, Vernaleken A, Schulte J, et al. Multi-pass-cell-based nonlinear pulse compression to 115 fs at 7.5 μJ pulse energy and 300 W average power. Opt Express. 2017;25:20502–20510.

[64] Fritsch K, Poetzlberger M, Pervak V, et al. All-solid-state multipass spectral broadening to sub-20 fs. Opt Lett. 2018;43:4643–4646.

[65] Ueffing M, Reiger S, Kaumanns M, et al. Nonlinear pulse compression in a gas-filled multipass cell. Opt Lett. 2018;43:2070–2073.

[66] Lavenu L, Natile M, Guichard F, et al. Nonlinear pulse compression based on a gas-filled multipass cell. Opt Lett. 2018;43:2252–2255.

[67] Kaumanns M, Pervak V, Kormin D, et al. Multipass spectral broadening of 18mJ pulses compressible from 1.3ps to 41fs. Opt Lett. 2018;43:5877–5880.

[68] Russbueldt P, Weitenberg J, Schulte J, et al. Scalable 30fs laser source with 530W average power. Opt Lett. 2019;44:5222–5225.

[69] Balla P, Bin Wahid A, Sytcovich I, et al. Postcompression of picosecond pulses into the few-cycle regime. Opt Lett. 2020;45:2572–2575.

[70] Couairon A, Mysyrowicz A. Femtosecond filamentation in transparent media. Phys Rep. 2007;441:47–189.

[71] Berge L, Skupin S, Nuter R, et al. Ultrashort filaments of light in weakly ionized, optically transparent media. Rep Prog Phys. 2007;70:1633–1713.

[72] Gaeta AL. Catastrophic collapse of ultrashort pulses. Phys Rev Lett. 2000;84:3582–3585.
[73] Dubietis A, Tamosauskas G, Suminas R, et al. Ultrafast supercontinuum generation in bulk condensed media. Lith J Phys. 2017;57:113–157.
[74] Hauri CP, Kornelis W, Helbing FW, et al. Generation of intense, carrier-envelope phase-locked few-cycle laser pulses through filamentation. Appl Phys B. 2004;79:673–677.
[75] Hauri CP, Guandalini A, Eckle P, et al. Generation of intense few-cycle laser pulses through filamentation - parameter dependence. Opt Express. 2005;13:7541–7547.
[76] Zaïr A, Guandalini A, Schapper F, et al. “patio-temporal characterization of few-cycle pulses obtained by filamentation. Opt Express. 2007;15:5394–5405.
[77] Akozbek N, Trushin SA, Baltuska A, et al. Extending the supercontinuum spectrum down to 200 nm with few-cycle pulses. New J Phys. 2006;8:177.
[78] Goulielmakis E, Koehler S, Reiter B, et al. Ultrabroadband, coherent light source based on self-channeling of few-cycle pulses in helium. Opt Lett. 2008;33:1407–1409.
[79] Steingrube DS, Schulz E, Binhammer T, et al. Generation of high-order harmonics with ultra-short pulses from filamentation. Opt Express. 2009;17:16177–16182.
[80] Steingrube DS, Kretschmar M, Hoff D, et al. Sub-1.5-cycle pulses from a single filament. Opt Express. 2012;20:24049–24058.
[81] Heyl CM, Couder-Alteirac H, Miranda M, et al. Scale-invariant nonlinear optics in gases. Optica. 2016;3:75–81.
[82] Zheltikov AM. Laser-induced filaments in the mid-infrared. J Phys B At Mol Opt Phys. 2017;50:092001.
[83] Schmidt C, Pertot Y, Balciunas T, et al. High-order harmonic source spanning up to the oxygen K-edge based on filamentation pulse compression. Opt Express. 2018;26:11834–11842.
[84] Mitrofanov AV, Voronin AA, Sidorov-Biryukov DA, et al. Subterawatt few-cycle mid-infrared pulses from a single filament. Optica. 2016;3:299–302.
[85] Skupin S, Stibenz G, Berge L, et al. Self-compression by femtosecond pulse filamentation: experiments versus numerical simulations. Phys Rev E. 2006;74:56604.
[86] Agrawal GP. Nonlinear fiber optics. Academic Press; Amsterdam; 2013 Chp. 5.
[87] Stolen RH, Lin C. Self-phase-modulation in silica optical fibers. Phys Rev A. 1978;17:1448–1453.
[88] Fujimoto JG, Weiner AM, Ippen EP. Generation and measurement of optical pulses as short as 16 fs. Appl Phys Lett. 1984;44:832–834.
[89] Halbout JM, Grischkowsky D. 12-fs ultrashort optical pulse compression at a high repetition rate. Appl Phys Lett. 1984;45:1281–1283.
[90] Knox WH, Fork RL, Downer MC, et al. Optical pulse-compression to 8 fs at A 5-kHz repetition rate. Appl Phys Lett. 1985;46:1120–1121.
[91] Fork RL, Cruz CHB, Becker PC, et al. Compression of optical pulses to 6 femtoseconds by using cubic phase compensation. Opt Lett. 1987;12:483–485.
[92] Fork RL, Martinez OE, Gordon JP. Negative dispersion using pairs of Prisms. Opt Lett. 1984;9:150–152.
[93] Bor Z, Racz B. Group-velocity dispersion in prisms and its application to pulse-compression and traveling-wave excitation. Opt Commun. 1985;54:165–170.
[94] Vozzi C, Nisoli M, Sansone G, et al. Optimal spectral broadening in hollow-fiber compressor systems. Appl Phys B. 2005;80:285–289.
[95] Böhle F, Kretschmar M, Jullien A, et al. Compression of CEP-stable multi-mJ laser pulses down to 4 fs in long hollow fibers. Laser Phys Lett. 2014;11:095401.
[96] Knight JC, Birks TA, Russell PS, et al. All-silica single-mode optical fiber with photonic crystal cladding. Opt Lett. 1996;21:1547–1549.
[97] Cregan RF, Mangan BJ, Knight JC, et al. Single-mode photonic band gap guidance of light in air. Science. 1999;285:1537–1539.

[98] Benabid F, Knight JC, Antonopoulos G, et al. Stimulated Raman scattering in hydrogen-filled hollow-core photonic crystal fiber. Science. 2002;298:399–402.

[99] Pryamikov AD, Biriukov AS, Kosolapov AF, et al. Demonstration of a waveguide regime for a silica hollow-core microstructured optical fiber with a negative curvature of the core boundary in the spectral region > 3.5 μm. Opt Express. 2011;19:1441–1448.

[100] Debord B, Amsanpally A, Chafer M, et al. Ultralow transmission loss in inhibiting-guiding hollow fibers. Optica. 2017;4:209–217.

[101] Maurel M, Chafer M, Amsanpally A, et al. Optimized inhibited-guiding Kagome fibers at Yb-Nd: YAG(8.5 dB/km) and Ti: sa(30 dB/km) ranges. Opt Lett. 2018;43:1598–1601.

[102] Debord B, Amsanpally A, Alharbi M, et al. Ultra-large core size hypocycloid-shape inhibited coupling Kagome fibers for high-energy laser beam handling. J Lightwave Technol. 2015;33:3630–3634.

[103] Emaury F, Saraceno CJ, Debord B, et al. Efficient spectral broadening in the 100-W average power regime using gas-filled kagome HC-PCF and pulse compression. Opt Lett. 2014;39:6843–6846.

[104] Köttig F, Tani F, Biersach CM, et al. Generation of microjoule pulses in the deep ultraviolet at megahertz repetition rates. Optica. 2017;4:1272–1276.

[105] Mak KF, Seidel M, Pronin O, et al. Compressing µJ-level pulses from 250fs to sub-10fs at 38-MHz repetition rate using two gas-filled hollow-core photonic crystal fiber stages. Opt Lett. 2015;40:1238–1241.

[106] Hadrich S, Krebs M, Hoffmann A, et al. Exploring new avenues in high repetition rate table-top coherent extreme ultraviolet sources. Light Sci Appl. 2015;4:e320–e320.

[107] Guichard F, Giree A, Zaouter Y, et al. Nonlinear compression of high energy fiber amplifier pulses in air-filled hypocycloid-core Kagome fiber. Opt Express. 2015;23:7416–7423.

[108] Köttig F, Schade D, Koehler JR, et al. Efficient single-cycle pulse compression of an ytterbium fiber laser at 10 MHz repetition rate. Opt Express. 2020;28:9099–9110.

[109] Markos C, Travers JC, Abdolvand A, et al. Hybrid photonic-crystal fiber. Rev Mod Phys. 2017;89:045003.

[110] Nisoli M, De Silvestri S, Svelto O. Generation of high energy 10 fs pulses by a new pulse compression technique. Appl Phys Lett. 1996;68:2793–2795.

[111] Marcatili EAJ, Schmelter RA. Hollow metallic and dielectric waveguides for long distance optical transmission and lasers. Bell Syst Tech J. 1964;43:1783–1809.

[112] Abrams RL. Coupling losses in hollow waveguide laser resonators. IEEE J Quantum Electron. 1972;QE 8:848–843.

[113] De Silvestri S, Nisoli M, Sansone G, et al. Few-cycle pulses by external compression. Topics Appl Phys. 2004;95:137–177.

[114] Nisoli M, De Silvestri S, Svelto O, et al. Compression of high-energy laser pulses below 5 fs. Opt Lett. 1997;22:522–524.

[115] Sartania S, Cheng Z, Lenzner M, et al. Generation of 0.1-TW 5-fs optical pulses at a 1-kHz repetition rate. Opt Lett. 1997;22:1562–1564.

[116] Cerullo G, Silvestri SD, Nisoli M, et al. Few-optical-cycle laser pulses: from high peak power to frequency tunability. IEEE J Sel Top Quantum Electron. 2000;6:948–958.

[117] Farkas G, Toth C. Proposal for attosecond light-pulse generation using laser-induced multiple-harmonic conversion processes in rare-gases. Phys Lett A. 1992;168:447–450.
[118] Corkum PB. Plasma perspective on strong-field multiphoton ionization. Phys Rev Lett. 1993;71:1994–1997.

[119] Hentschel M, Kienberger R, Spielmann C, et al. Attosecond metrology. Nature. 2001;414:509–513.

[120] Cavalieri AL, Goulielmakis E, Horvath B, et al. Intense 1.5-cycle near infrared laser waveforms and their use for the generation of ultra-broadband soft-x-ray harmonic continua. New J Phys. 2007;9:242.

[121] Schweinberger W, Sommer A, Bothschafter E, et al. Waveform-controlled near-single-cycle milli-joule laser pulses generate sub-10 nm extreme ultraviolet continua. Opt Lett. 2012;37:3573–3575.

[122] Ghimire S, Shan B, Wang C, et al. High-energy 6.2-fs pulses for attosecond pulse generation. Laser Phys. 2005;15:838–842.

[123] Chen XW, Jullien A, Malvache A, et al. Generation of 4.3 fs, 1 mJ laser pulses via compression of circularly polarized pulses in a gas-filled hollow-core fiber. Opt Lett. 2009;34:1588–1590.

[124] Anderson A, Lücking F, Prikoszovits T, et al. Multi-mJ carrier envelope phase stabilized few-cycle pulses generated by a tabletop laser system. Appl Phys B. 2011;103:531–536.

[125] Hädrich S, Rothhardt J, Eidam T, et al. High energy ultrashort pulses via hollow fiber compression of a fiber chirped pulse amplification system. Opt Express. 2009;17:3913–3922.

[126] Laven L, Natile M, Guichard F, et al. High-energy few-cycle Yb-doped fiber amplifier source based on a single nonlinear compression stage. Opt Express. 2017;25:7530–7537.

[127] Hädrich S, Rothhardt J, Demmler S, et al. Scalability of components for kW-level average power few-cycle lasers. Appl Opt. 2016;55:1636–1640.

[128] Rothhardt J, Hädrich S, Carstens H, et al. 1 MHz repetition rate hollow fiber pulse compression to sub-100-fs duration at 100 W average power. Opt Lett. 2011;36:4605–4607.

[129] Hädrich S, Klenke A, Hoffmann A, et al. Nonlinear compression to sub-30-fs, 0.5 mJ pulses at 135 W of average power. Opt Lett. 2013;38:3866–3869.

[130] Rothhardt J, Hädrich S, Klenke A, et al. 53 W average power few-cycle fiber laser system generating soft x rays up to the water window. Opt Lett. 2014;39:5224–5227.

[131] Hädrich S, Kienel M, Müller M, et al. Energetic sub-2-cycle laser with 216 W average power. Opt Lett. 2016;41:4332–4335.

[132] Giguere M, Schmidt BE, Shiner AD, et al. Pulse compression of submillijoule few-optical-cycle infrared laser pulses using chirped mirrors. Opt Lett. 2009;34:1894–1896.

[133] Schmidt BE, Bejot P, Giguere M, et al. Compression of 1.8 mu m laser pulses to sub two optical cycles with bulk material. Appl Phys Lett. 2010;96:121109.

[134] Schmidt BE, Shiner AD, Lassonde P, et al. CEP stable 1.6 cycle laser pulses at 1.8 μm. Opt Express. 2011;19:6858–6864.

[135] Gebhardt M, Gaida C, Stutzki F, et al. High average power nonlinear compression to 4 GW, sub-50 fs pulses at 2 μm wavelength. Opt Lett. 2017;42:747–750.

[136] Durfee CG, Backus S, Kapteyn HC, et al. Intense 8-fs pulse generation in the deep ultraviolet. Opt Lett. 1999;24:697–699.

[137] Klein-Wiele JH, Nagy T, Simon P. Hollow-fiber pulse compressor for KrF lasers. Appl Phys B. 2006;82:567–570.

[138] Nisoli M, Stagira S, De Silvestri S, et al. Toward a terawatt-scale sub-10-fs laser technology. IEEE J Sel Top Quantum Electron. 1998;4:414–420.
[139] Andriukaitis G, Kartashov D, Lorenc D, et al. Hollow-fiber compression of 6 mJ pulses from a continuous-wave diode-pumped single-stage Yb,Na: caF2chirped pulse amplifier. Opt Lett. 2011;36:1914-1916.

[140] Nagy T, Forster M, Simon P. Flexible hollow fiber for pulse compressors. Appl Opt. 2008;47:3264–3268.

[141] Suda A, Hatayama M, Nagasaka K, et al. Generation of sub-10-fs, 5-mJ-optical pulses using a hollow fiber with a pressure gradient. Appl Phys Lett. 2005;86:111116.

[142] Nurhuda M, Suda A, Midorikawa K, et al. Propagation dynamics of femtosecond laser pulses in a hollow fiber filled with argon: constant gas pressure versus differential gas pressure. J Opt Soc Am B. 2003;20:2002–2011.

[143] Nagy T, Pervak V, Simon P. Optimal pulse compression in long hollow fibers. Opt Lett. 2011;36:4422–4424.

[144] Sung JH, Park JY, Imran T, et al. Generation of 0.2-TW 5.5-fs optical pulses at 1 kHz using a differentially pumped hollow-fiber chirped-mirror compressor. Appl Phys B. 2006;82:5–8.

[145] Park J, Lee J-H, Nam CH. Generation of 1.5 cycle 0.3 TW laser pulses using a hollow-fiber pulse compressor. Opt Lett. 2009;34:2342–2344.

[146] Oishi Y, Suda A, Midorikawa K, et al. Sub-10 fs, multimillijoule laser system. Rev Sci Instrum. 2005;76:93114.

[147] Bohman S, Suda A, Kaku M, et al. Generation of 5 fs, 0.5 TW pulses focusable to relativistic intensities at 1 kHz. Opt Express. 2008;16:10684–10689.

[148] Bohman S, Suda A, Kanai T, et al. Generation of 5.0 fs, 5.0 mJ pulses at 1kHz using hollow-fiber pulse compression. Opt Lett. 2010;35:1887–1889.

[149] Okell WA, Witting T, Fabris D, et al. Carrier-envelope phase stability of hollow fibers used for high-energy few-cycle pulse generation. Opt Lett. 2013;38:3918–3921.

[150] Lücking F, Trabatoni A, Anumula S, et al. In situ measurement of nonlinear carrier-envelope phase changes in hollow fiber compression. Opt Lett. 2014;39:2302–2305.

[151] Robinson JS, Haworth CA, Teng H, et al. The generation of intense, transform-limited laser pulses with tunable duration from 6 to 30 fs in a differentially pumped hollow fibre. Appl Phys B. 2006;85:525–529.

[152] Ouilé M, Vernier A, Böhle F, et al. Relativistic-intensity near-single-cycle light waveforms at kHz repetition rate. Light Sci Appl. 2020;9:47.

[153] Nagy T, Kretschmar M, Vrakking MJJ, et al. Generation of above-terawatt 1.5-cycle visible pulses at 1 kHz by post-compression in a hollow fiber. Opt Lett. 2020; 45:3313–3316.

[154] Dutin CF, Dubrouil A, Petit S, et al. Post-compression of high-energy femtosecond pulses using gas ionization. Opt Lett. 2010;35:253–255.

[155] Auguste T, Gobert O, Dutin CF, et al. Application of optical-field-ionization-induced spectral broadening in helium gas to the postcompression of high-energy femtosecond laser pulses. J Opt Soc Am B. 2012;29:1277–1286.

[156] Auguste T, Fourcade Dutin C, Dubrouil A, et al. High-energy femtosecond laser pulse compression in single- and multi-ionization regime of rare gases: experiment versus theory. Appl Phys B. 2013;111:75–87.

[157] Hort O, Dubrouil A, Cabasse A, et al. Post-compression of high-energy terawatt-level femtosecond pulses and application to high-order harmonic generation. J Opt Soc Am B. 2015;32:1055–1062.

[158] Nurhuda M, Suda A, Bohman S, et al. Optical pulse compression of ultrashort laser pulses in an argon-filled planar waveguide. Phys Rev Lett. 2006;97:153902.
[159] Chen JF, Suda A, Takahashi EJ, et al. Compression of intense ultrashort laser pulses in a gas-filled planar waveguide. Opt Lett. 2008;33:2992–2994.

[160] Akturk S, Arnold CL, Zhou B, et al. High-energy ultrashort laser pulse compression in hollow planar waveguides. Opt Lett. 2009;34:1462–1464.

[161] Arnold CL, Zhou B, Akturk S, et al. Pulse compression with planar hollow waveguides: a pathway towards relativistic intensity with table-top lasers. New J Phys. 2010;12:73015.

[162] Chen S, Jarnac A, Houard A, et al. Compression of high-energy ultrashort laser pulses through an argon-filled tapered planar waveguide. J Opt Soc Am B. 2011;28:1009-1012.

[163] Jarnac A, Brizuela F, Heyl CM, et al. Compression of TW class laser pulses in a planar hollow waveguide for applications in strong-field physics. Eur Phys J D. 2014;68:373.

[164] Klenke A, Kienel M, Eidam T, et al. Divided-pulse nonlinear compression. Opt Lett. 2013;38:4593–4596.

[165] Guichard F, Zaouter Y, Hanna M, et al. Energy scaling of a nonlinear compression setup using passive coherent combining. Opt Lett. 2013;38:4437–4440.

[166] Jacqmin H, Jullien A, Mercier B, et al. Passive coherent combining of CEP-stable few-cycle pulses from a temporally divided hollow fiber compressor. Opt Lett. 2015;40:709–712.

[167] Jacqmin H, Jullien A, Mercier B, et al. Temporal pulse division in hollow fiber compressors. J Opt Soc Am B. 2015;32:1901–1909.

[168] Klenke A, Hädrich S, Kienel M, et al. Coherent combination of spectrally broadened femtosecond pulses for nonlinear compression. Opt Lett. 2014;39:3520–3522.

[169] Wahlströmd J, Cheng YH, Chen YH, et al. Optical Nonlinearity in Ar and N2 near the Ionization Threshold. Phys Rev Lett. 2011;107:103901.

[170] Li C, Rishad KPM, Horak P, et al. Spectral broadening and temporal compression of~ 100 fs pulses in air-filled hollow core capillary fibers. Opt Express. 2014;22:1143–1151.

[171] Haddad E, Safaei R, Leblanc A, et al. Molecular gases for pulse compression in hollow core fibers. Opt Express. 2018;26:25426–25436.

[172] Fan G, Safaei R, Kwon O, et al. High energy redshifted and enhanced spectral broadening by molecular alignment. Opt Lett. 2020;45:3013–3016.

[173] Beetar JE, Nrisimhamurty M, Truong T-C, et al. Multi-octave supercontinuum generation and frequency conversion based on rotational nonlinearity. Sci Adv. 2020;6:eabb5375.

[174] Wagner NL, Gibson EA, Popmintchev T, et al. Self-compression of ultrashort pulses through ionization-induced spatiotemporal reshaping. Phys Rev Lett. 2004;93:173902.

[175] Couairon A, Biegert J, Hauri CP, et al. Self-compression of ultra-short laser pulses down to one optical cycle by filamentation. J Mod Opt. 2006;53:75–85.

[176] Serebryannikov EE, Goulielmakis E, Zheltikov AM. Generation of supercontinuum compressible to single-cycle pulse widths in an ionizing gas. New J Phys. 2008;10:093001.

[177] Hauri CP, Trisorio A, Merano M, et al. Generation of high-fidelity, down-chirped sub-10 fs mJ pulses through filamentation for driving relativistic laser-matter interactions at 1 kHz. Appl Phys Lett. 2006;89:151125.

[178] Stibenz G, Zhavoronkov N, Steinmeyer G. Self-compression of millijoule pulses to 7.8 fs duration in a white-light filament. Opt Lett. 2006;31:274–276.

[179] Schulz E, Binhammer T, Steingrube DS, et al. “Intense few-cycle laser pulses from self-compression in a self-guiding filament. Appl Phys B. 2009;95:269–272.
[180] Trushin SA, Kosma K, Fuss W, et al. Sub-10-fs supercontinuum radiation generated by filamentation of few-cycle 800 nm pulses in argon. Opt Lett. 2007;32:2432–2434.

[181] Schulz E, Steingrube DS, Binhammer T, et al. Tracking spectral shapes and temporal dynamics along a femtosecond filament. Opt Express. 2011;19:19495–19507.

[182] Kretschmar M, Brée C, Nagy T, et al. Direct observation of pulse dynamics and self-compression along a femtosecond filament. Opt Express. 2014;22:22905–22916.

[183] Hauri CP, Lopez-Martens RB, Blaga CI, et al. Intense self-compressed, self-phase-stabilized few-cycle pulses at 2 μm from an optical filament. Opt Lett. 2007;32:868–870.

[184] Jargot G, Daher N, Lavenu L, et al. Self-compression in a multipass cell. Opt Lett. 2018;43:5643–5646.

[185] Shumakova V, Malevich P, Ališauskas S, et al. Multi-millijoule few-cycle mid-infrared pulses through nonlinear self-compression in bulk. Nat Commun. 2016;7:12877.

[186] Mitrofanov AV, Voronin AA, Rozhko MV, et al. Self-compression of high-peak-power mid-infrared pulses in anomalously dispersive air. Optica. 2017;4:1405–1408.

[187] Shumakova V, Ališauskas S, Malevich P, et al. Chirp-controlled filamentation and formation of light bullets in the mid-IR. Opt Lett. 2019;44:2173–2176.

[188] Im S-J, Husakou A, Herrmann J. Guiding properties and dispersion control of kagome lattice hollow-core photonic crystal fibers. Opt Express. 2009;17:13050–13058.

[189] Im SJ, Husakou A, Herrmann J. High-power soliton-induced supercontinuum generation and tunable sub-10-fs VUV pulses from kagome-lattice HC-PCFs. Opt Express. 2010;18:5367–5374.

[190] Joly NY, Nold J, Chang W, et al. Bright spatially coherent wavelength-tunable deep-UV laser source using an Ar-filled photonic crystal fiber. Phys Rev Lett. 2011;106:203901.

[191] Ermolov A, MakKF, Froz MH, et al. Supercontinuum generation in the vacuum ultraviolet through dispersive-wave and soliton-plasma interaction in a noble-gas-filled hollow-core photonic crystal fiber. Phys Rev A. 2015;92:033821.

[192] Belli F, Abdolvand A, Chang W, et al. Vacuum-ultraviolet to infrared supercontinuum in hydrogen-filled photonic crystal fiber. Optica. 2015;2:292–300.

[193] Ermolov A, Heide C, Dienstbier P, et al. Carrier-envelope-phase-stable soliton-based pulse compression to 4.4fs and ultraviolet generation at the 800kHz repetition rate. Opt Lett. 2019;44:5005–5008.

[194] Balciunas T, Fourcade-Dutin C, Fan G, et al. A strong-field driver in the single-cycle regime based on self-compression in a kagome fibre. Nat Commun. 2015;6:6117.

[195] Elu U, Baudisch M, Pires H, et al. High average power and single-cycle pulses from a mid-IR optical parametric chirped pulse amplifier. Optica. 2017;4:1024–1029.

[196] Russell PSJ, Hölzer P, Chang W, et al. Hollow-core photonic crystal fibres for gas-based nonlinear optics. Nat Photonics. 2014;8:278–286.

[197] Travers JC, Grigorova TF, Brahms C, et al. High-energy pulse self-compression and ultraviolet generation through soliton dynamics in hollow capillary fibres. Nat Photonics. 2019;13:547–554.

[198] Christian B, Federico B, John CT. Resonant dispersive wave emission in hollow capillary fibers filled with pressure gradients. Opt Lett. 2020;45:4456–4459.

[199] Rothhardt J, Hadrich S, Delagnes JC, et al. High average power near-infrared few-cycle lasers. Laser Photonics Rev. 2017;11:1700043.

[200] Nagy T, Hadrich S, Simon P, et al. Generation of three-cycle multi-millijoule laser pulses at 318W average power. Optica. 2019;6:1423–1424.
[201] Fan G, Carpeggiani PA, Tao Z, et al., “TW-peak-power post-compression of 70-mJ pulses from an Yb amplifier,” Conference on Lasers and Electro-Optics, SW4E.1 (San Jose, CA, 2019).

[202] Nisoli M, Sansone G, Stagira S, et al. Ultra-broadband continuum generation by hollow-fiber cascading. Appl Phys B. 2002;75:601–604.

[203] Schenkel B, Biegert J, Keller U, et al. Generation of 3.8-fs pulses from adaptive compression of a cascaded hollow fiber supercontinuum. Opt Lett. 2003;28:1987–1989.

[204] Jeong Y-G, Piccoli R, Ferachou D, et al. Direct compression of 170-fs 50-cycle pulses down to 1.5 cycles with 70% transmission. Sci Rep. 2018;8:11794.

[205] Lavenu L, Natile M, Guichard F, et al. High-power two-cycle ultrafast source based on hybrid nonlinear compression. Opt Express. 2019;27:1958–1967.

[206] Chen B-H, Kretschmar M, Ehberger D, et al. Compression of picosecond pulses from a hindisk laser to 30fs at 4W average power. Opt Express. 2018;26:3861–3869.

[207] Nagy T, Simon P. Generation of 200-mu J, sub-25-fs deep-UV pulses using a noble-gas-filled hollow fiber. Opt Lett. 2009;34:2300–2302.

[208] Cardin V, Thiré N, Beaulieu S, et al. 0.42 TW 2-cycle pulses at 1.8 μm via hollow-core fiber compression. Appl Phys Lett. 2015;107:181101.

[209] Fan G, Balčiūnas T, Kanai T, et al. Hollow-core-waveguide compression of multimillijoule CEP-stable 3.2 μm pulses. Optica. 2016;3:1308–1311.

[210] Wang P, Li Y, Li W, et al. 2.6mJ/100Hz CEP-stable near-single-cycle 4μm laser based on OPCPA and hollow-core fiber compression. Opt Lett. 2018;43:2197–2200.

[211] Brahms C, Belli F, Travers JC. Infrared attosecond field transients and UV to IR few-femtosecond pulses generated by high-energy soliton self-compression. Phys Rev Res. 2020;2:043037.

[212] Kaumanns M, Pervak V, Kormin D, et al., “Multipass spectral broadening with tens of Millijoule pulse energy,” in 2019 Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference (CLEO/Europe-EQEC) (Münich, Germany 2019), p. CD_9.2.