Mathematical reflective thinking process based on cognitive style

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Abstract. Reflective thinking is the process of uniting information with real experiences obtained from data and facts to produce knowledge. Every individual has its own uniqueness in producing their knowledge, there are independent and some are dependent on the information they obtain. The purpose of this study was to analyze each stage in the reflective thinking process in students who have independent and field cognitive styles. The research method uses a qualitative approach. The subjects of this study were high school students with different cognitive styles chosen by purposive sampling. The instruments used were GEFT, Reflective Thinking Tests and interview guidelines. The results of data analysis indicate that students' reflective thinking processes in the cognitive style of Independent Field (FI) do not only depend on information obtained from available data, but by independently linking information obtained with real experience to form new knowledge in solving problems. While students with cognitive style Dependent Field (FD) depend on information obtained from available data only without using real experience so that the knowledge formed is incomplete and inaccurate in solving problems.

1. Introduction
Thinking is the process of linking information with other information obtained from data and facts to produce knowledge in solving problems. The process of thinking is an event of mixing, matching, combining, exchanging, and sorting concepts, perceptions, and previous experiences [9]. Real experience is needed as a tool to link the information obtained so as to produce a solution to the problem. The thought process that uses real experience to gain knowledge is also called reflective thinking.

The origins of the concept of reflective practice in education are often ascribed to John Dewey [1] which states that learning takes time to weave meaning between the lines of experience. According to [2] notes that reflection refers to an activity or process in which an experience is remembered, considered, and evaluated, usually in relation to broader goals. This is a way to respond to past experiences and involve awareness and experience as a basis for evaluation and decision making and as a source for planning and taking action. With experience, a person becomes smarter and can evaluate success and mistakes made.

According to [3] mentions that someone thinks reflective occurs when someone responds to information from outside, is forwarded to the mental activity in question and gets a problem or requires information that already has, the activity is to respond to things namely information/
knowledge or data used to respond, originating from within (internal), can explain what has been done, realize the mistake and fix it (if there is an error), and communicate ideas with symbols or images. Furthermore, responding to an issue that is external as an effect of reflective thinking, it continues to arrive at solving problems.

According to [4] there are 4 stages in the reflective thinking process, namely: (1) habitual action, is the stage when someone does a habit with a little awareness, such as writing data and facts that are available without understanding the meaning of the data., (2) understanding, is the level where someone can connect the data obtained so that it forms an information in the form of a concept without understanding the usefulness of the concept in the real world, (3) reflection, this stage besides being able to form a concept from the data obtained, but also have been able to use experience in the real world to obtain a knowledge, and (4) critical reflection, this stage is the highest stage of the reflective thinking process where one can explain the reasons for each step in connecting information that is associated with real experiences so that they can gain knowledge to solve problems.

In reflective thinking, it is very important to see how someone processes, remembers and uses the information obtained. Every individual has its own uniqueness. The way people think, the accuracy of perceptions, how they process and remember information or how they use information in problem solving is called cognitive style [5]. Cognitive style by [6] individual perceptions in different spatial orientations, which refer to the field of dependency and independence of the field. The difference from the two cognitive styles is in seeing a problem. According to [6] define the main characteristics of the cognitive style of the Dependent Field (FD), which are individuals who lack or cannot separate a part of a unit and tend to immediately accept the dominant part or context. While individuals with cognitive Field-Independent (FI) are individuals who can easily be 'free' from organized perceptions and can immediately separate a part of their unit.

2. Methodology

The purpose of this study was to look at the students' mathematical reflective thinking process in terms of their cognitive style, so that this study used qualitative research with descriptive methods. The subjects in the study were two Senior High School students in 12th grade in Indonesia. This study uses the Stratified Random Sampling and Purposive Sampling techniques to determine the research subjects. Subject characteristics based on cognitive style consist of field dependent and field independent where each cognitive style gives a different influence in mathematical reflective thinking ability. In classifying the cognitive style of students, psychiatric tests were carried out and representatives were chosen from each cognitive style group using purposive sampling technique.

In this case study, the researcher is the main instrument, but in the process of collecting data using supporting instruments namely cognitive style classification tests (Group Embedded Figure Test), tests of mathematical reflective thinking skills and interview guidelines. The research method is presented in the flow chart in Figure 1.

In Figure 1, it can be seen that the steps of the study began with a preliminary study then continued with identifying problems and formulating research problems. After the problem is formulated, the next step is to collect data. Data collection uses test instruments, namely the group Embedded Figure Test (GEFT) and the Mathematical Reflective Ability Test and also non-tests, namely structured and in-depth interviews. Interviews were carried out when the data obtained from the tests were not enough. After the data needed is felt sufficient, then proceed with processing data. After the data is processed, the data is analyzed so as to produce conclusions and suggestions.

2.1. Classification of Cognitive Style

Group Embedded Figures Test (GEFT) is a test to classify cognitive styles that are valid and reliable. GEFT examines students' abilities through identifying simple forms that are in a more complex pattern. GEFT includes three parts. The first part is considered as an introduction consisting of seven questions. The other two parts (second and third) each consist of nine questions. Students can work on each part within the ten-minute time limit. The score for each student is the number of numbers in the
last two parts of the test. Each correct answer is given a score of 1. The maximum score is 18 points and a minimum of 0 points. Determination of the cognitive style FI and FD is based on the scores obtained by students. Scores are distributed into the FD category (score 0-10) and FI (score 11-18).

![Flow chart of the research method](image)

**Figure 1.** Flow chart of the research method

### 2.2. Mathematical Reflective Thinking Test

The mathematical reflective thinking test instrument was modified from [7] and [8] so that its validity and reliability were tested and feasible to use. This instrument consists of 2 items in the form of a description with indicators that are in accordance with the mathematical reflective thinking process (See Table 1).

**Table 1.** The test on mathematical reflective thinking

| No. | Items of the test                                                                                                                                                                                                 |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.  | A road with a slope of 0.005 starts from point O to point A. The height of point A from the plane is 5 meters. Analyze the following statement, then give your comment and write down the mathematical concept and/or the underlying formula/use. |
|     | a. Is the road relatively sloping or steep? Give explanations accompanied and concepts and or mathematical formulas used.                                                                                           |
|     | b. If Aline walks along the road starting at 07.00 with a speed of 2.5 km/hour, then he will reach the end of the road at around midday. Are these estimates really? Give an explanation accompanied by calculations and formulas used.   |
|     | (Modified from [7])                                                                                                                                                                                             |
| 2.  | Note the following description, then determine in which step the error lies. Explain why this is wrong. Then write down the concepts / rules / formulas used in the procedure!                                                   |
|     | If were known $a = b$                                                                                                                         |
|     | Then $a^2 = ab$(multiply the two segments by $a$)                                                                                                 |
|     | $a^2 - b^2 = ab - b^2$(subtract the two sections with $b^2$)                                                                                       |
|     | $(a + b)(a - b) = b(a - b)$ (invoice both segments)                                                                                               |
|     | $a + b = b$(divided the two segments by $a - b$)                                                                                            |
|     | $a + a = b$ (on the left side of substitution $b = a$)                                                                                        |
|     | $a = b$ (add the terms in the left section)                                                                                                       |
|     | $2a = a$ (on the right side of substitution $b = a$)                                                                                              |
|     | $2 = 1$ (devided the two segments by $a$) (Adopted by [7])                                                                                     |
3. Results and Discussion

3.1. Cognitive Style Grouping Results
The result of the GEFT test given to 42 senior high school students are shown in Table 2.

| Cognitive Styles    | The Number of Students |
|---------------------|------------------------|
| Field Independent   | 36                     |
| Field Dependent     | 6                      |

From Table 2, we can see that the students with independent field cognitive styles is 36 students and the students in the cognitive style of the Dependent Field is 6 students. Students who have been grouped are selected by purposive sampling taking into account the mathematical abilities classified as high ability are students who score more or equal to the class average score plus standard deviation above the recommendations of the mathematics teacher in the class and can express their opinions clearly, can understood and can communicate well. Based on this criterion, the research subjects were 2 students consisting of 1 student with the Field Dependent cognitive style and 1 student with the Field Independent cognitive style.

Students who have been grouped are selected by purposive sampling with consideration to seeing the criteria of cognitive style, also by considering mathematical abilities that are classified as high ability and can express opinions clearly. High ability is students who score more or equal to the average score plus standard deviation upon, the consideration of the mathematics teacher in the class and can express his opinion clearly, can be understood and can communicate well. Based on these criteria the research subjects obtained 2 students consisting of 1 student with a cognitive style of Field Dependent and 1 student with cognitive style Field Independent

3.2. Mathematical Reflective Thinking Test Results
The results of student tests of Field Independent are shown in Figure 2.

![Field Independent’s Answer on Item 1](image)

Based on Figure 2 and the results of the interview, we found that FI wrote data that was known in the problem by representing it in the form of an image with information. He understands the data available. In answering the question whether the road was classified as steep or sloping, he answered with the concept of a sloping field in the concept of physics. In other words, he was able to use the concepts in the calculation and at the same time use real experience with the statements he wrote, namely, the larger the angle formed, the steeper the road.

While the results of student tests of Dependent Field are shown in Figure 3.
Figure 3. Field Dependent’s Answer on Item 1

From Figure 3, we found that FD wrote data that was known to be in trouble by representing it in the form of images with inappropriate information. He wrote a description of the slope at the length of the track, the slope should be the ratio between the height and the plane. In answering the question whether the road was classified as steep or sloping, he answered gently. However, he did not use concepts related to the data available when solving problems. He answered that the road is categorized as sloping because the slope starts from point 0. Besides not understanding the concept of gradients, he also cannot associate with real experience. Based on the results of the interview, we found that he wrote the conclusion of the answer by guessing it because he forgot the concept of the gradient. The results of interviews that corroborate these findings are shown in the following transcript.

R: What do you understand related to item 1?
FD: What I understand from the problem is determining the slope or steepness of the road.
R: Do you know what the concept / formula is?
FD: The Physics Concept, but I forget the formula.
R: Ok, do you still remember the gradient concept?
FD: I remember a little.
R: Does the problem have to do with gradients?
FD: Yes, there is. Slope right? But the gradient is a lot with numbers, so I forget the formula.
R: Why is slope connected to the slope starting at 0?
FD: I’m guessing.

In item 2, student test results from Independent Field are shown in Figure 4.

Figure 4. Field Independent’s Answer on Item 2

From Figure 4, we find that FI writes data that is known by representing it in the form of an image (Habitual Action Stage). At the stage of Understanding, he discovered the concept of material speed and writing it down. Before using the speed formula, he looks for information that he feels is necessary, namely looking for a distance from point O to the end of the road (point A). But there is a slight error in calculating, I should have obtained a distance of 1 KM, but he wrote 10 KM. So that in
the Critical Reflection stage, he doesn't get the right answer. The answer is correct, namely Aline will arrive before noon, but the correct answer is not at 11.00 WIB but at 07.24 WIB.

While the results of student tests of Dependent Field are shown in Figure 5 in item 2. From Figure 5, we find that FD writes known data (Habitual Action Stage). At the stage of Understanding, he discovered the concept of speed material and wrote down the formula. He uses the speed formula to find distance. But the time used is 6. So that the distance obtained is 15 KM (there is a unit error in the answer to FD) the trip starts at 07.00 WIB - 12.00 WIB. But he did not look for the distance that must be taken, so that in the Critical Reflection stage, he was out of sync in giving reasons for the answers he got.

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In this case, based on [6], Field Dependencethas difficulty in finding simple forms of more complex forms, in this case FDs have difficulty in finding concepts in problems. He also cannot use his real experience in solving problems. He was just fixated on the concept. So that when the concept is not mastered, the problem cannot be resolved completely.

Based on the results, we found that there were several types of similar errors made by FD on other items. The problem is almost the same, namely the discovery of concepts and associations with real experiences. So, in general, students in the cognitive style of the Field Dependent prefer to solve problems in groups so that someone guides them in solving the problem.

Based on the results of the analysis of the students' answers, the stages of the reflective thinking process in terms of the cognitive style of the Independent Field and Dependent fields can be seen in table 3.

From Table 3, we can see that the reflective thinking process of the Independent Field and Dependent Field students in the Habitual Action stage, Independent Field Students and Field Dependent students write down the known data. The difference is the accuracy in representing it. At the stage of Understanding, the two cognitive styles also find concepts related to the problem. While in the Reflective stage, Independent Field Students can find the relationship between the concepts used in solving problems with real experiences. While Field Dependent students, could not find a connection between the two. At the highest stage, namely the Critical Reflection Phase, Independent Field Students can provide reasons for the answers according to the procedures that have been done, so that they can get answers thoroughly. While the Field Dependent students, provide reasons that are not related to the procedures that have been done in solving the problem, so that the solutions obtained are not appropriate.

| Stages           | Field Independent                                                                 | Field Dependent                                                                 |
|------------------|----------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| Habitual Action  | Write down the available data according to the problem correctly.                | Write down the data available on the question but not complete.                  |
| Understanding    | Find and write concepts to solve problems.                                       | Find and write concepts to solve problem.                                       |
| Reflective       | Linking concepts with experiences in real life to solve problems.                | Not connecting concepts with experience in real life.                           |
| Critical Reflective | Write down the reasons for the answers given and explain according to the procedures that have been done. | Write down the reasons for the answers given but not according to the procedures that have been done. |

4. Conclusions
Based on the results described in the previous section, it can be concluded that in the mathematical reflective thinking process, each stage is related. How someone finds data, uses data to compile concepts, uses concepts and associates them with real experiences so they can gain knowledge to solve problems.

The fundamental difference seen in the reflective thinking process between students and the cognitive style of the Independent Field and Dependent Field is in finding the basic concepts of a problem and their independence. Independent Field Students can accurately find data that will be used in the problem solving process and have an open mind in applying the concept by connecting it with real life to solve problems, while Field Dependent students have difficulty finding accurate data for concept formation, besides Field students. Dependent only relies on available concepts without associating them with real life experiences. So that the solutions obtained are not complete and
accurate. They must be guided to the concept in question. This is in accordance with the cognitive style characteristics of the Dependent Field. [6,10]

Our study only applies to special cases in the type of public school. For further studies, it is expected to be able to analyze the process of reflective thinking in vocational students, because they are accustomed to applicative problems.

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