Application of the Harmonic Balance method for regime change prediction using Francis-99 test case

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Abstract. An efficient method for predicting the turbine start-up and shut-down flow parameters is presented in this paper. Due to start-up or shut-down usually lasting a large number of periods, simulations of such processes are highly demanding and long lasting. To alleviate this, a modified Harmonic Balance method is used, thus casting a problem into spectral space and solving start-up and shut-down simultaneously, as a complete period. By doing this, the results are obtained in a fraction of time required by time-accurate simulation. Furthermore, the same approach could be used for any change of regime from operating point OP1 to operating point OP2 and back to OP1. The method is implemented in foam-extend and suitable for solving incompressible turbulent Navier–Stokes equations. Validation is performed on two test cases: a simple 2D pump case for initial validation, followed by a Francis turbine. Francis turbine test case and experimental data are provided by the Francis-99 Workshop. For Francis turbine the power curve is assembled and compared with experiment.

1. Introduction

Turbomachinery CFD simulations have become a standard in industry, although still presenting quite expensive and time consuming process. In order to alleviate this, a number of tools and methods have been developed, being an approximation or simplification of the ongoing process within turbomachinery. Some of these methods include the steady state Multiple Reference Frame approach (MRF) [1], taking into account the rotation even though a steady state simulation is run. Another more recent method is the Harmonic Balance method, a quasi-steady state method due to a number of time instants being solved via coupled steady state equations. Finally, the approach with least approximations is the transient simulation where a large number of successive time steps are solved, thus obtaining the detailed insight into flow development, wakes propagation, etc. In order for transient simulation to present valid results, a periodic steady state has to be reached: simulating a single period is not enough. It should be made sure that the simulation start instabilities do not affect the solution and that resulting flow features are no longer within the domain, therefore reaching periodic steady state is what makes transient simulation expensive. In certain cases this means simulating 5-10 periods, but if a high level of unsteadiness is present, the number of needed periods can even go up to 50 [2]. From the perspective of CPU time consumption, the transient simulation is the most expensive one, followed by Harmonic Balance and then by MRF as the shortest [3].

The focus of this work is on the Harmonic Balance method, which was originally developed as a periodic boundary condition by He [4]. He and Ning [5] extended its application for solving the two-dimensional Navier-Stokes equations, which finally led to its current form and its extensive
use in turbomachinery. More complex turbomachinery simulations using the Harmonic Balance were done by Hall et al. [6]. Shape design optimisations [7] and optimisation studies [8] were performed as well. A recent Harmonic Balance development unrelated to turbomachinery was presented by Gatin et al. [9], who used the Harmonic Balance method for naval hydrodynamics applications: running regular wave propagation and sea-keeping simulations.

Compared to conventional transient methods, the benefit of the Harmonic Balance method is the ability to capture transient flow features at significantly lower CPU time cost, while still being sufficiently accurate. Cvijetić et al. [3] presented the comparison of transient approach, steady state MRF method and Harmonic Balance, demonstrating the speed-up of at least an order of magnitude. Depending on the number of harmonics used, the size of the system of equations changes, as well as the accuracy. $n$ number of harmonics yield a system of $2n + 1$ coupled equations, where larger number of harmonics will take more time to converge, offering greater accuracy, as higher order effects do not get neglected. As the method is based on the Fourier series expansion, the frequency of the motion should be known in advance, suggesting that problems with imposed periodic motion are the most suitable for Harmonic Balance. In this work, the Harmonic Balance method is implemented in the Finite Volume framework within the open source software OpenFOAM, using a segregated pressure-based algorithm. In order to account for rotation, moving mesh has to be used, thus always solving for the equation set for the corresponding rotor position.

Turbomachinery start-up and shut-down present a challenging problem for CFD investigation. Furthermore, change of operating points requires special attention as well. Depending on the type of the machine considered, the change of regime can take from several periods up to several dozen periods, making the simulations of such process highly expensive. The change of regime is a transient process during which the flow can change significantly and mass flow through the machine changes according to the newly reached regime. Rotor angular velocity can change as well in this process, making steady state simulations unusable.

The Harmonic Balance method is deployed here as a quasi-steady method in order to reduce the simulation time and capture the behavior during regime change. Non–periodic process such as start-up or shut-down is made periodic by considering both start-up and shut-down as a complete process. The period then consists of two complementary regime changes, with $2n + 1$ simulations throughout the period of start-up and shut-down. Due to two distinctive time-scales of rotor period (inner) and complete start-up/shut-down period (outer), a nested Harmonic Balance structure is deployed. Therefore, the $2n + 1$ Harmonic Balance simulations for $2n + 1$ time instants are interconnected with additional Harmonic Balance source term for the outer coupling.

The paper is organised as follows. The mathematical model is presented in section 2 giving the overview of the Harmonic Balance formulation and its application to the Navier-Stokes equations. The 2D pump test case for initial validation is presented in section 3, followed by the Francis test case and comparison with experimental data. Finally the results are summarized in the conclusion.

2. Mathematical Model

An overview of the Harmonic Balance (HB) method is presented in this section. HB treatment, transforming the time-derivative term into a source term is briefly explained, while complete derivation can be found in [3]. The equations presented are valid for an arbitrary number of harmonics, with remarks on how the time derivative term is transformed into additional source terms, thus changing the transient equation into a set of coupled steady state equations. Greater attention is given to expanding the general HB for use in start-up/shut-down simulations. For this, two harmonic balance loops are formed, creating a single outer loop with a number of inner loops.
2.1. **Navier-Stokes Equations**

A general transport equation for variable $\mathcal{Q}$ can be written in a condensed form:

$$\frac{\partial \mathcal{Q}}{\partial t} + \mathcal{R} = 0, \quad (1)$$

Under the assumption that fields of interest change periodically in time, one can expand a continuous field variable $\mathcal{Q}$ and residual $\mathcal{R}$ into a Fourier series with a finite number of harmonics:

$$\mathcal{Q}(t) = Q_0 + \sum_{i=1}^{n} Q_S i \sin(i\omega t) + Q_C i \cos(i\omega t), \quad (2)$$

$$\mathcal{R}(t) = R_0 + \sum_{i=1}^{n} R_S i \sin(i\omega t) + R_C i \cos(i\omega t), \quad (3)$$

where $\mathcal{Q}$ represents a variable in time domain, while $Q$ stands for the same variable in the frequency domain. The same analogy is valid for $\mathcal{R}$ and $R$. $\omega$ is a base radian frequency and $Q_S / R_S$ and $Q_C / R_C$ represent sine and cosine Fourier coefficients.

The Equation 2 and Equation 3 are inserted into Equation 1 in order to obtain the Harmonic Balance form and organized using the trigonometric identities and summation of appropriate terms. In order to obtain the momentum equation, our solving variable $\mathcal{Q}$ will denote velocity $u$. This is done for all the used equations that contain the temporal term. Therefore, the Navier–Stokes equation set along with continuity equation becomes:

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

$$\nabla \cdot (\mathbf{u} \nabla \mathbf{u}) - \nabla (\nu \nabla \mathbf{u}) = -\frac{\nabla p_{i,j}}{\rho} - \frac{2\omega}{2n + 1} \left( \sum_{l=1}^{2n} P_{l,j} \mathbf{u}_l \right), \quad \text{for } j = 1...2n + 1, \quad (5)$$

where $P_{l,j}$ is defined as:

$$P_{l,j} = P_{l} = \sum_{k=1}^{n} k \sin(lk\omega \Delta t), \quad \text{for } l = -2n...2n, \quad (6)$$

and:

$$\Delta t = \frac{T}{2n + 1}. \quad (7)$$

Equation (4) represents the incompressible continuity equation in Harmonic Balance form. It can be seen that the equation has not changed as it has no time derivative term, but it is valid for time instant $t_j$ only. The continuity equation must hold for each of $2n + 1$ time instants. Equation (5) represents the momentum equation which consists of convection and diffusion term, pressure gradient term and additional HB source term. The equations are incompressible as they are implemented in this form, however, there are no limitations of the method if compressibility should be included.
Observing (5), the following can be concluded, which is explained in greater detail in [3):

- The initially continuous variable in time $u$ is now a discrete variable, calculated only in $2n + 1$ equally spaced time instants within the period.

- The time derivative term has been replaced by a coupled source term, thus transforming a single transient equation to a set of coupled steady state equations. Instead of solving one equation, a system of $2n + 1$ coupled equations is solved for a single transport equation, assuming time periodicity. These equations are coupled through a source term that has resulted from the Fourier expansion of the time derivative term.

- Each time step has its own set of variables $p$ and $u$ defining the flow field at that time step, therefore denoted with index $t_j$: $p_{t_j}$, $u_{t_j}$. Furthermore, variables with index $t_j$ are calculated by the corresponding continuity, pressure and momentum equations valid for the time instant $t_j$ only. $P_{t-j}$ term takes into account the temporal distance between solution time instants for coupling between equations.

- By calculating the Fourier coefficients and doing a reconstruction, the solution can be obtained at any time instant within a period (as a post-processing step).

- The solution is always perfectly time-periodic with a given number of frequencies, which would not be the case in the equivalent transient simulation.

If the exact procedure is performed again on the obtained equation set, equation (5), with additional frequency, an additional source term is obtained. This is of major importance as for simulation of start-up/shut-down the two distinct time scales have to be captured. Smaller time scale depends on the rotor period (inner period), while the larger time scale reflects the complete start-up/shut-down process (outer period). By doing this, the outer period is split into $2n + 1$ time instants. These $2n + 1$ time instants are simulated as a regular HB simulation, with additional source coupling term responsible for outer-period coupling.

2.2. Start-up and shut-down

In order to use HB for aperiodic problems, such as start-up and shut-down, the problem is made periodic first and then solved. In terms of turbine regime change, the periodicity is created by adding complementary regime change so to return to initial operating point. Therefore, if first regime change is from operating point OP1 to operating point OP2, the complete considered period is OP1-OP2-OP1. This period can further be deconstructed to following parts, Figure 1:

- a Steady-state condition at OP1,
- b followed by change from OP1 to OP2,
- c OP2 maintained for a number of periods to reach steady-state,
- d change from OP2 to OP1,
- e reaching OP1 steady-state.
In terms of HB, it is necessary to determine a sufficient number of harmonics $m$ so that a HB curve follows the original profile closely. Figure 2 shows approximation of original profile using different number of harmonics. For $m$ harmonics period consists of $M = 2m + 1$ time instants, while for accuracy in each of $2m + 1$ time instants additional single-harmonic HB simulation is run. Therefore, the outer (single) HB loop accounts for changes related to regime change, while inner $2m + 1$ HB loops account for rotor-stator interaction and rotor rotation. The two loops are interconnected over mutual source term.

3. Pump test case
In this section the initial validation of the approach presented in section 2 is demonstrated. A 2D pump test case is used, consisting of rotor and stator, performing periodic change of regime. Start-up and shut-down curves are assembled using a time-accurate simulation and compared with HB results. Start-up and shut-down is performed at constant rpm by regulating the inlet flow rate. The flow rate function is defined arbitrarily.

3.1. Geometry and setup
Test case geometry consists of two regions, rotor with 4 blade and stator with 3 blades. The complete domain is discretized with 12,580 hexahedral cells, as shown in Figure 3. Rotor speed is set to 60 rpm.

Figure 1: Stages of complete period.

Figure 2: HB approximation functions.

Figure 3: Full mesh of a pump test case.
In order to obtain accurate transient results for regime change from operating point OP1 to operating point OP2, a number of periods have to be run between regime changes. These periods should lead to reaching the periodic steady state of OP1 or OP2. Therefore, prior to change from OP1 to OP2, the simulation is run at OP1 in order to reach periodic steady state. Then, the flow rate is reduced gradually until reaching OP2. The flow rate is held constant at OP2 to reach periodic steady state and gradually increased again to OP1 from where it is run to periodic steady state. OP1 is characterised by inlet velocity of 8 m/s, while for OP2 the inlet is considered closed, i.e. inlet velocity is 0 m/s.

The change from OP1 to OP2 is achieved by reducing the inlet velocity, where the velocity change is a sine function. Period of the complete simulation (shut-down and start-up) is 130 s, divided into 5 stages, as explained in subsection 2.2. During stage a, the inlet velocity is kept constant for 15 s, followed by inlet velocity reduction during stage b, taking further 50 s. During stage c inlet velocity is held 0 m/s for 10 s and increased to 8 m/s during next 50 s. Stage e is 5 s long.

![Figure 4: Inlet velocity profile and time instants of the HB simulation.](image1)

![Figure 5: Location of probes.](image2)

### 3.2. Results

HB simulation is performed with six harmonics in outer loop, yielding 13 time instants within a period. For comparison, velocity and pressure profiles are acquired in four points throughout the domain. The comparison of probe data for transient and HB simulation is shown in Figure 6. The pressure and velocity data agrees very well, which also holds true for visual comparison of velocity field through the domain. This concludes that the proposed method gives expected results and can be used on Francis turbine.
Figure 6: Velocity and pressure ($p/\rho$) over time in different points for time-accurate (red) and HB (black) simulation.
Figure 7: Velocity fields over time, HB (left) and time-accurate (right).
3.3. CPU Time Comparison

The main idea behind the HB-approach for turbine shut-down and start-up was the reduction of CPU time for assessment of flow features. In this subsection the CPU time comparison is presented. Numbers presented here were obtained on a same desktop computer for all simulations, running Intel Core i5-3570K CPU @ 3.40 GHz. Transient simulation was run with a variable time step, varying between $4 - 8 \cdot 10^{-4}$s, which was limited by the maximum Courant number of 0.5. All the other settings were the same for HB and time-accurate simulation. Table 1 presents the calculation time for time-accurate and HB simulations, showing the reduction obtained using the HB approach. With time-accurate simulation lasting almost 19 hours and HB simulation taking 40 minutes and being more than 28 times faster, this approach shows beneficial for future use.

|                  | Full HB simulation | Transient simulation | Reduction (Transient/HB) |
|------------------|--------------------|---------------------|--------------------------|
| Time             | 0.66 h             | 18.7 h              | 28.3333                  |

4. Francis turbine

Francis test case and experiment, based on the Francis-99 Workshop [10] is presented in this section. Organizers of the workshops provided a large amount of technical documentation concerning the geometry and working regime of the turbine, which is what this work is based on and the authors are grateful for provided data. The experimental work is performed by the NTNU - Norwegian University of Science and Technology and publicly released under the Francis-99 Workshop series. Francis turbine is a water turbine consisting of spiral inlet, followed by 14 stay vanes, 28 guide vanes and 30 runner blades, ending with the draft tube outlet, Figure 8. Within the workshop, data for best efficiency point (BEP), part load (PL), high load (HL), start-up and shut-down are provided. In this work only the BEP, start-up and shut-down are considered. Simulations are run using the HB approach presented in section 2 and compared with experimental data. Start-up and shut-down are regulated by changing the inlet mass flow rate achieved by changing the guide vanes angle, i.e. by opening and closing the blades. During start-up and shut-down the rotational velocity is held constant. However, as noted by the experimentalists from the Workshop, certain inconsistencies related to experimental data exist, which are explained in the following subsection.

Figure 8: Francis turbine geometry.
4.1. Data manipulation
Data provided for turbine start-up and shut-down consists of pressure and velocity probes measurements, head measurements, and flow rate and blade angle measurements with relation to time. However, due to incompressible fluid used (water), the relation between guide vane angle and mass flow rate should be linear, which was not the case. It was stated on the Workshop website that the used flowmeter had too slow response time and was causing a significant discrepancy between blade angle and measured mass flow rate. Experimentalists suggest using linearized mass flow rate based on guide vane angle (openness). The linearized data was used by Minakov et al. [11] who obtained good agreement, therefore the same approach was adopted here. Flow rate was then considered to change from 0.202 m$^3$/s at 9.84° blade angle to 0.018 m$^3$/s at 0.8° blade angle. In order to assess a complete experimental curve, values from reached steady-state points are used, as flow meter response time is $\approx 2$ s. Figure 9 shows normalized curves for start-up and shut-down for flow rate (measured and linearized) and blade angle against time.

![Figure 9: Normalized flow rate as measured (Q), linearized (Q$_{lin}$) and blade angle (α).](image_url)

4.2. Setup
The available geometry was reduced to using only two blade passages instead of a full annulus, which was possible due to applied periodic boundary conditions. It should be noted that with 14 stay vanes, 28 guide vanes and 30 runner blades, the rotor blade passage was reduced slightly in angular direction to be able to fully use periodic boundary conditions. Therefore, periodic boundary conditions allowed computational domain reduction 14 times. For connecting stationary and rotating domains, the overlapGgi is used [12]. However, the best efficiency point (BEP) is simulated using the complete geometry, consisting of 574 431 hexahedral cells. The turbulence is modelled using the $k – \varepsilon$ model with the wall functions and turbine operating regime is changed by opening or closing the guide vanes at constant rotor angular velocity of 333 rpm.

4.3. Best efficiency point
Best efficiency point was simulated to assess the accuracy of a single harmonic HB simulation in case of periodic steady state problem, with non-changing operating conditions. The values measured are power, head and efficiency and compared with experimental data, Table 2. All three values are predicted well by the simulation, with errors below 4%.
Table 2: Comparison of integral quantities.

|          | P [W] | H [m] | η [%] |
|----------|-------|-------|-------|
| Experiment | 21,617 | 11.94 | 92.39 |
| Simulation | 22,457 | 11.53 | 94.40 |
| Error     | 3.74%  | 3.43% | 2.13% |

Within the experiment the measurements in several probing points within the turbine were provided. In this work probes VL2, DT5 and DT6 were compared against experiment, Table 3. Again, the predicted values agree well with the experiment, with the highest error being 4.12%.

Pressure and velocity fields through the runner and inlet are presented in Figures 10 and 11, respectively.

Table 3: Comparison of pressure values in three different points.

| Measurement locations | VL2  | DT5  | DT6  |
|-----------------------|------|------|------|
| Experimental pressure, [kPa] | 173.60 | 105.01 | 104.80 |
| Simulation pressure, [kPa]    | 170.43 | 109.53 | 109.15 |
| Error                   | 1.80% | 4.12% | 3.98% |

Figure 10: Gauge pressure p [kPa] in the guide vane and runner.
4.4. Start-up and shut-down

Main object of shut-down and start-up comparison of HB simulation with the experimental data was assessment of power curve and pressure reading at one of the probes. The HB simulation was run using six harmonics, yielding $2n+1 = 13$ time instants with single-harmonic simulations throughout the complete shut-down and start-up period. Due to possibility to arbitrary change the steady-state stages of the curve (stages a, c, e in Figure 1), this was used to assemble a symmetric function. As regime change is dictated by guide vane angle, 6 harmonics would require meshes with 13 blade positions for each time instant. By making the function symmetrical, only 7 blade positions were needed, further yielding additional savings in terms of man-hour.

Having in mind that HB is based on Fourier expansion, the solution from obtained 13 time instants can easily be reconstructed to form a complete period. The reconstructed solution is presented in following figures. Figure 12 shows pressure comparison in probe VL2. It can be concluded that pressure from simulations shows good agreement with the experimental data as the trend in pressure drop matches between the two, suggesting the qualitative agreement between the simulated and real physical processes. It should be noted that results show the relative difference of around 4%, matching those in BEP simulation.
Figure 13 shows the power curve through time during shut-down and start-up. Good agreement between HB and experiment can be observed, with the largest discrepancy at ≈ 19s, at the end of the HB period. However, the value at the beginning of the HB period (t = 0s) is equal to the value at the end of the period (t = 19.5s), while experimental data differs significantly. In both cases the first and last point represent BEP, therefore similar measurements would be expected.

![Power curve plot]

Figure 13: Power over time during shut-down and start-up.

5. Conclusion
In this work the application of the Harmonic Balance method was expanded to simulating turbomachinery start-up and shut-down processes. As it was shown, the approach can be used for any other change of operating regime as well. Initial validation was performed on a simple 2D pump test case, comparing the HB with time-accurate simulation where good agreement was achieved. Values compared include pressure and velocity probes in four points during complete period. Additionally, CPU time was compared, proving HB ≈ 28 times faster than time-accurate simulation. For second test case the Francis turbine provided by Francis-99 Workshop was used. Power curve and single pressure probe were compared, yielding good agreement with experimental results. The obtained results present a good ground for future work and investigation in terms of capabilities of HB for problems presented here.
References

[1] Page, M., Beaudoin, M., and Giroux, A.-M., “Steady-state capabilities for hydroturbines with OpenFOAM,” International Journal of Fluid Machinery and Systems, Vol. 4, No. 1, 2011, pp. 161–171.

[2] Vukcevic, V., Jasak, H., Gatin, I., and Malenica, S., “Seakeeping Sensitivity Studies Using the Decomposition CFD Model Based on the Ghost Fluid Method,” 31st Symposium on Naval Hydrodynamics, Monterey, California, 2016.

[3] Cvijetic, G., Jasak, H., and Vukcevic, V., “Finite Volume Implementation of the Harmonic Balance Method for Periodic Non–Linear Flows,” 54th AIAA Aerospace Sciences Meeting, 2016, p. 0070.

[4] He, L., “Method of simulating unsteady turbomachinery flows with multiple perturbations,” AIAA Journal, Vol. 30, 11 1992.

[5] He, L. and Ning, W., “Efficient approach for analysis of unsteady viscous flows in turbomachines,” Aiaa Journal, Vol. 36, No. 11, 1998, pp. 2005–2012.

[6] Hall, K., Thomas, J., Ekici, K., and Voytovyc, D., “Frequency domain techniques for complex and nonlinear flows in turbomachinery,” Vol. 3998, 2003, p. 2003.

[7] Nadarajah, S. K. and Jameson, A., “Optimum shape design for unsteady three-dimensional viscous flows using a nonlinear frequency-domain method,” Journal of Aircraft, Vol. 44, No. 5, 2007, pp. 1513–1527.

[8] Huang, H. and Ekici, K., “A discrete adjoint harmonic balance method for turbomachinery shape optimization,” Aerospace Science and Technology, Vol. 39, 2014, pp. 481–490.

[9] Gatin, I., Cvijetić, G., Vukčević, V., and Jasak, H., “Wave Diffraction CFD Nonlinear Time–Spectral Simulations in foam–extend,” 19th Numerical Towing Tank Symposium, 2016.

[10] Trivedi, C., Cervantes, M. J., and Dahlhaug, O., Francis 99: A test-case on a high head Francis turbine, Master’s thesis, Waterpower laboratory, Department of Energy and Process Engineering, Faculty of Engineering, NTNU, Trondheim, Norway, 2019.

[11] Minakov, A., Sentyabov, A. and Platonov, D., “Numerical investigation of flow structure and pressure pulsation in the Francis-99 turbine during startup,” IOP Conf. Series: Journal of Physics: Conf. Series 782 (2017) 012004.

[12] Beaudoin, M. and Jasak, H., “Development of a generalized grid interface for turbomachinery simulations with OpenFOAM,” Open source CFD International conference, Vol. 2, 2008.