Deuteron stripping and pickup involving halo nuclei $^{11}$Be and $^{15}$C

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Three-body calculations of $(d, p)$ and $(p, d)$ reactions involving one-neutron halo nuclei $^{11}$Be and $^{15}$C are performed using the framework of Faddeev-type scattering equations. Important effects of the optical potential nonlocality are found improving the description of the experimental data. The obtained values for the neutron spectroscopic factor are consistent with estimations from other approaches.

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I. INTRODUCTION

Deuteron stripping and pickup reactions $(d, p)$ and $(p, d)$ constitute an important tool for extracting nuclear structure information such as spectroscopic factors or spin/parity. Of special interest are cases involving exotic weakly bound systems such as one-neutron halo nuclei assumed to consist of a core with a mass number $A$ and a neutron $n$; well known examples are $^{11}$Be, $^{15}$C, and $^{19}$C. The reaction $d + A \rightarrow p + (An)$ and its time reverse $p + (An) \rightarrow d + A$ may therefore be described in a three-body model $(p, n, A)$. These reactions have been analyzed by several approximate methods such as distorted-wave Born approximation (DWBA) and various DWBA-type adiabatic approaches [1, 2, 3, 4, 5] or continuum-discretized coupled-channels (CDCC) method [6, 7, 8, 9]. Recently, also the application of exact three-body Faddeev/Alt, Grassberger, and Sandhas (AGS) scattering theory [10, 11] to three-body nuclear reactions has become possible [12, 13] due to a novel implementation [14] of the screening and renormalization method [15, 16, 17] for including the long-range Coulomb force between charged particles. Faddeev/AGS framework, though being technically and computationally most complicated and expensive, has an advantage that, once numerically well-converged results are obtained, all discrepancies with the experimental data can be attributed solely to the shortcomings of the used optical potentials (OP) or to the inadequacy of the three-body model. Recent comparison [12] of Faddeev/AGS and CDCC results revealed that CDCC is indeed a reliable method to calculate $d + A$ elastic and breakup cross sections but may lack accuracy for transfer reactions such as $p + ^{11}$Be $\rightarrow d + ^{10}$Be. No benchmark with adiabatic approaches is performed yet. Furthermore, the nonlocal optical potential (NLOP), so far, has been included only in the Faddeev/AGS framework [18] where an important effect of the OP nonlocality was found in deuteron stripping on $^{12}$C and $^{16}$O, improving the description of the experimental data. Therefore we expect the OP nonlocality to be significant also in transfer reactions on one-neutron halo nuclei which we aim to study in the present work using momentum-space AGS equations.

The results for $(d, p)$ and $(p, d)$ reactions involving $^{11}$Be and $^{15}$C nuclei for which the experimental data is available are presented in Sec. III. The summary is given in Sec. IV.

II. AGS EQUATIONS

The AGS equations [11]

$$U_{\beta\alpha}(Z) = \delta_{\beta\alpha} G_0^{-1}(Z) + \sum_{\gamma} \delta_{\beta\gamma} T_{\gamma}(Z) G_0(Z) U_{\gamma\alpha}(Z),$$

(1)

are a system of Faddeev-like coupled integral equations for the transition operators $U_{\beta\alpha}(Z)$; their on-shell matrix elements $\langle \psi_\beta | U_{\beta\alpha}(E+i0) | \psi_\alpha \rangle$ at the available three-particle energy $E$ are amplitudes for all scattering processes (elastic, inelastic, transfer, breakup) allowed by the chosen Hamiltonian $H = H_0 + \sum_{\gamma} v_\gamma$ where $H_0$ is the free Hamiltonian and $v_\gamma$ the potential for the pair $\gamma$ in odd-man-out notation. In Eq. (1) $\delta_{\beta\alpha} = 1 - \delta_{\beta\alpha}$, $G_0(Z) = (Z - H_0)^{-1}$ is the free resolvent, and

$$T_{\gamma}(Z) = v_\gamma + v_\gamma G_0(Z) T_{\gamma}(Z)$$

(2)

is the two-particle transition matrix. The channel states $|\psi_\gamma\rangle$ for $\gamma = 1, 2, 3$ are the eigenstates of the corresponding channel Hamiltonian $H_\gamma = H_0 + v_\gamma$ with the energy eigenvalue $E$; thus, $|\psi_\gamma\rangle$ is a product of the bound state wave function for pair $\gamma$ and a plane wave with fixed on-shell momentum $q_\gamma$ corresponding to the relative motion of particle $\gamma$ and pair $\gamma$ in the initial or final state.

The AGS equations [11] are applicable only to short-range interactions. Nevertheless, the long-range Coulomb force between charged particles can be included in this framework using the method of screening and renormalization [12, 16, 17] where one has to solve AGS equations with nuclear plus screened Coulomb potential to obtain the Coulomb-distorted short-range part of the transition amplitude; the convergence of the results with the screening radius has to be established. The method has been successfully applied to proton-deuteron elastic scattering and breakup [12, 13] and to direct nuclear reactions dominated by three-body degrees of freedom [12, 13].

The results for $(d, p)$ and $(p, d)$ reactions involving $^{11}$Be and $^{15}$C nuclei for which the experimental data is available are presented in Sec. III. The summary is given in Sec. IV.
We use momentum-space partial-wave basis. The screened Coulomb potential and most of the nuclear potentials are given in coordinate space; they have to be transformed to the momentum space where local and nonlocal potentials are treated in the same manner. The AGS equations then become a system of integral equations with two continuous variables which are the absolute values of Jacobi momenta. The technique of numerical solution is described in Refs. 12, 20, 24.

III. RESULTS

The dynamic input to AGS equations are the potentials \( v_\gamma \) for the three pairs of particles. For the np interaction we take the realistic CD Bonn potential \[22\]. In order to describe reactions involving one-neutron halo nuclei we need a real \( nA \) potential that reproduces the bound state spectrum of nucleus \((AN)\). This is a standard choice in \( p + (AN) \) reactions where the \( pA \) potential is complex and is taken at the proton lab energy. As discussed in Ref. 15, \( d + A \rightarrow p + (AN) \) and \( p + (AN) \rightarrow d + A \) reactions are related by time reversal provided the energy is complex and is taken at the proton lab energy. As discussed in Ref. 15, \( d + A \rightarrow p + (AN) \) and \( p + (AN) \rightarrow d + A \) reactions are related by time reversal provided the energy is complex and is taken at the proton lab energy.

\[
V_\gamma = V_c f(r, R, a) + \sigma \cdot L V_{so} \frac{2}{J} \frac{d}{dr} f(r, R, a),
\]

with \( f(r, R, a) = [1 + \exp((r - R)/a)]^{-1} \) and \( R = r_0 A^{1/3} \). The \( n^{10}Be \) potential is taken from Ref. 23 while the one for \( n^{14}C \) is constructed in the present work assuming standard values for \( r_0 = 1.25 \text{ fm} \), \( a = 0.65 \text{ fm} \), and \( V_{so} = 5.5 \text{ MeV} \) and adjusting \( V_c \) to the binding energy of \( ^{15}C \) ground and excited state and to the neutron separation energy of \( ^{14}C \). For completeness the parameters of both potentials are given in Table I while the resulting values for binding energies are listed in Table II. Pauli forbidden bound states are projected out as described in Ref. 12.

For the hadronic part of the \( pA \) interaction we take the NLOP of Giannini et al. \[24\] which is based on Watson’s multiple scattering theory; the nonlocality arises mainly due to the fully off-shell two-nucleon transition matrix \[25, 26\]. In the configuration space the NLOP is parametrized as

\[
v_\gamma (r', r) = H_c(x) U_c(y) + H_s(x) U_s(y) \frac{A - 2Z}{A} + H_s(x) V_s(y) \sigma \cdot L
\]

with \( x = |r' - r| \), \( y = |r' + r|/2 \), \( H_j(x) = (\pi \beta_j^2)^{-3/2} \exp(-x^2/\beta_j^2) \), and \( Z \) being the number of protons in the nucleus \( A \). The values of the nonlocality parameters are \( \beta_c = 1.015 \text{ fm} \), \( \beta_s = 1.633 \text{ fm} \), \( \beta_s = 0.789 \text{ fm} \). The nonlocality of the OP mainly Both \( U_c(y) \) and \( U_s(y) \) contain real volume and imaginary surface parts, while the spin-orbit part \( V_s(y) \) is real; all of them are parametrized in the standard way using Woods-Saxon functions and their derivatives \[24\], i.e.,

\[
U_c(y) = -V_c f(y, a_R) - 4i W_c f(y, a_I)[1 - f(y, a_I)],
\]

\[
V_s(y) = V_s \frac{2}{y} \frac{\partial f(y, a_R)}{\partial y},
\]

\[
f(y, a) = [1 + \exp((y - R)/a)]^{-1}
\]

with \( R_N = 1.16A^{1/3} \text{ fm} \), \( a_R = 0.57 \text{ fm} \), \( a_I = 0.54 + \text{0.0032} A \text{ fm} \), \( V_c = 85 \text{ MeV} \), \( V_I = 127 + 11ZA^{-1/3} \text{ MeV} \), \( V_s = 9.1 \text{ MeV}, \( W_t = 13 \text{ MeV} \), and \( W_c = w_N[1 - \exp(-0.05E)] \text{ MeV} \) where \( E = E_p^{cm} + 1.08 - 1.35ZA^{-1/3} \text{ MeV} \) and \( E_p^{cm} \) is the proton energy (MeV) in the c.m. frame. We adjust \( w_N \) to improve the description of the experimental \( pA \) scattering data in the energy regime relevant for the considered three-body reactions. In the case of \( p^{14}C \) we fit the NLOP to the data at proton lab energy \( E_p = 14.5 \text{ MeV} \[27\]. We need the \( p^{10}Be \) potential in a broader energy range than \( E_p = 12 - 16 \text{ MeV} \) where experimental data \[28\] is available. Since the local energy-dependent OP by Watson et al. \[25\] provides rather satisfactory description of that data as demonstrated in Ref. \[28\] and confirmed by our own calculations, we adjust the NLOP to the predictions of Watson OP. The obtained values for \( w_N \) are given in Table III and other parameters are taken from Ref. \[24\]. This time we do not use the local OP obtained from NLOP by equivalence transformation \[24\]; instead, the Watson OP that

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**TABLE I: Partial-wave dependent strengths of central and spin-orbit parts of \( n^{10}Be \) and \( n^{14}C \) potentials, all in units of MeV.**

| \( L \) | \( V_c(n^{10}Be) \) | \( V_c(n^{14}C) \) | \( V_s(n^{10}Be) \) | \( V_s(n^{14}C) \) |
|---|---|---|---|---|
| 0 | 56.413 | 50.29 |
| odd | 42.498 | 11.953 | 46.13 | 5.5 |
| even, \( \geq 2 \) | 56.413 | 5.38 | 49.14 | 5.5 |

**TABLE II: Binding energies (MeV) of the bound states corresponding to the potential parameters of Table I.**

| \( ^{11}Be \) | \( ^{15}C \) |
|---|---|
| \( 1s_{1/2} \) | 28.730* | 28.194* |
| \( 2s_{1/2} \) | 0.503 | 1.218 |
| \( 1p_{3/2} \) | 6.812* | 11.522* |
| \( 1p_{1/2} \) | 0.183 | 8.175* |
| \( 1d_{5/2} \) | 0.479 |
is often used also in DWBA-type and CDCC calculations serves as a local reference potential.

Although some parameters of those $pA$ potentials vary with energy, in the calculations of present paper based on the Hamiltonian theory for three-body scattering they are taken at a fixed energy, corresponding to the proton lab energy in the $p + (An)$ reaction. Therefore the chosen Hamiltonian with complex $pA$ potential prevents the calculation of $d + A \to n + (Ap)$ and $p + (An) \to n + (Ap)$ reactions since the nucleus ($Ap$) is not bound. These reactions can be described [30] allowing for an explicitly energy-dependent $pA$ potential which becomes real at negative relative $pA$ energies and reproduces bound states of the nucleus ($Ap$). However, such an approach is not free of theoretical complications due to the absence of the Hamiltonian theory as discussed in Ref. [30]; we therefore refrain from using it in the present work. Furthermore, the results of Ref. [30] indicate that the effect of the energy-dependence in the $pA$ potential on the $(d,p)$ and $(p,d)$ cross section is rather insignificant at small c.m. scattering angles up to $\Theta_{c.m.} \approx 40^\circ$, although it may become sizable for large angles where the cross section itself is small.

The interaction between $np$, $nA$, and $pA$ pairs is included in partial waves with pair orbital angular momentum $L \leq 3, 6$, and 12, respectively, and the total angular momentum is $J \leq 25$; the results are well converged. The $pA$ channel is more demanding than the $nA$ channel due to the screened Coulomb force, where the screening radius $R \approx 10$ to 12 fm for the short-range part of the scattering amplitude is sufficient for convergence.

Examples for the cross section in two-body $p + ^{10}\text{Be}$ and $p + ^{14}\text{C}$ and three-body $d + ^{10}\text{Be}$ and $d + ^{14}\text{C}$ elastic scattering are presented in Figs. 1 and 2 and compared with the available experimental data. Two-body results indicate that adjusting only one parameter $w_N$ of NLOP is sufficient to achieve the quality in fitting the $pA$ data comparable with the one of the Watson OP. However, the three-body $dA$ data seems to be reproduced better by the NLOP, although it also fails for $d + ^{10}\text{Be}$ at large angles. There is no $p + (An)$ elastic scattering data in the considered energy region; however, as demonstrated in Ref. [18], the theoretical predictions for $p + A$ and $p + (An)$ elastic scattering observables are strongly correlated and therefore also the nonlocality effect on $p + (An)$ elastic cross section is extremely small. Thus, the observed discrepancy in $p + ^{11}\text{Be}$ elastic cross section at higher energy [13, 32] cannot be resolved by the NLOP.

Results for $d + ^{10}\text{Be} \to p + ^{11}\text{Be}$ transfer at deuteron lab energy $E_d = 12$ and 25 MeV are presented in Fig. 3 for the $^{11}\text{Be}$ ground state $1/2^+$ and in Fig. 4 for the

| $E_p$     | $w_N$ |
|-----------|-------|
| $p + ^{10}\text{Be}$ | 9.0   |
| $p + ^{12}\text{Be}$ | 12.0  |
| $p + ^{14}\text{Be}$ | 20.9  |
| $p + ^{10}\text{Be}$ | 35.3  |
| $p + ^{14}\text{C}$  | 14.0  |

TABLE III: The parameter $w_N$ of NLOP adjusted to the $p + ^{14}\text{C}$ data and $p + ^{10}\text{Be}$ Watson OP predictions at given proton lab energies $E_p$ (all in units of MeV).
excited state $1/2^-$. The corresponding proton lab energy in the time-reverse reaction $p + ^{11}\text{Be} \rightarrow d + ^{10}\text{Be}$ is $E_p = 9.0$ (8.7) and 20.9 (20.5) MeV for the ground (excited) state, respectively. At least for the transfer to the ground state $1/2^+$ the NLOP reproduces the shape of the data clearly better than Watson OP. In all cases theory overpredicts the data and thereby yields neutron spectroscopic factors below 1. This is believed to be due to neglecting the $^{10}\text{Be}$ core excitation in the $^{11}\text{Be}$ bound state. For the ground state $1/2^+$, the spectroscopic factor value around 0.45 at 12 MeV is in agreement with the DWBA-type result of Ref. [5], while at 25 MeV the value around 0.75 is consistent with most other estimations collected in Ref. [34]. Slightly larger values of the neutron spectroscopic factor are obtained for the excited state $1/2^-$, around 0.6 and 0.85 at 12 and 25 MeV, respectively. In Fig. 4 results for $p + ^{11}\text{Be} \rightarrow d + ^{10}\text{Be}$ transfer at $E_p = 35.3$ MeV are presented. Again significant difference between NLOP and Watson OP predictions is found. The values for the $^{11}\text{Be}$ ground state neutron spectroscopic factor obtained using the NLOP from $(d,p)$ reaction at $E_d = 25$ MeV and $(p,d)$ reaction at $E_p = 35.3$ MeV are consistent with each other, in contrast to those obtained using the Watson OP.

Results for $d + ^{14}\text{C} \rightarrow p + ^{15}\text{C}$ transfer at $E_d = 14$, 16, and 17 MeV are presented in Fig. 6 for the $^{15}\text{C}$ ground state $1/2^+$ and in Fig. 7 for the excited state $5/2^+$. The corresponding proton lab energy in the time-reverse reaction $p + ^{15}\text{C} \rightarrow d + ^{14}\text{C}$ is $E_p = 12.0$ (11.2), 13.9 (13.1), and 14.8 (14.0) MeV for the ground (excited) state, respectively. The theoretical predictions vary smoothly with the energy while the data from Ref. [35] at 16 MeV are significantly lower than other two sets from Refs. [31, 50] at 14 and 17 MeV as already pointed out in Ref. [5]. This raises serious doubts on the normalization of the 16-MeV data. In all cases NLOP reproduces the shape of the data better than Watson OP, especially for the transfer to the ground state $1/2^+$ in the region of the first minimum and second maximum. At 14 MeV and at 17 MeV up to c.m. scattering angle $\Theta_{c.m.} \approx 60^\circ$ also the quantitative description of the data by the NLOP is quite satisfactory. The extracted neutron spectroscopic factor for the $^{15}\text{C}$ ground state $1/2^+$ is close to 1, in agreement with the DWBA-type result of Ref. [12], while for the excited state $5/2^+$ it is slightly below 1.
FIG. 6: (Color online) Differential cross section for $d + ^{14}$C $\rightarrow$ $p + ^{15}$C transfer to the $^{15}$C ground state $1/2^+$ at $E_d = 14$, 16, and 17 MeV. Curves as in Fig. 1 and the experimental data are from Refs. [31, 35, 36].

FIG. 7: (Color online) Same as Fig. 6, but for transfer to the $^{15}$C excited state $5/2^+$. In addition we performed the nearside-farside decomposition of the cross section [37]. Although at small angles where the cross section is largest the nearside usually dominates, the nonlocality effect may be equally important for both nearside and farside cross sections and therefore the observed nonlocality effect is a result of a complicated interference between nearside and farside.

The results of the present work together with those of Ref. [18] indicate that the OP nonlocality effect on transfer observables depends quite strongly on the reaction energy and internal orbital angular momentum of the ($An$) nucleus but far less on its binding energy and mass. The dependence on the internal orbital angular momentum is evident, and the dependence on the reaction energy is seen most clearly by comparing $d + ^{10}$Be $\rightarrow$ $p + ^{11}$Be and $p + ^{11}$Be $\rightarrow$ $d + ^{10}$Be results at $E_d = 12$ and 25 MeV and $E_p = 35.3$ MeV given in Figs. 3 and 5. However, as shown in Figs. 3 and 6, the OP nonlocality effect is very similar in ($d,p$) reactions at $E_d = 12$ and 14 MeV leading to the ground state $1/2^+$ of $^{11}$Be and $^{15}$C, respectively. Furthermore, the OP nonlocality effect in the $p + ^{13}$Be $\rightarrow$ $d + ^{10}$Be transfer at $E_p = 35.3$ MeV is very similar to the one shown in Figs. 5 and 7 of Ref. [18] for ($d,p$) reactions at comparable c.m. energies leading to the excited state $1/2^+$ of $^{13}$C and $^{17}$O nuclei with the binding energies of 1.857 and 3.272 MeV, respectively. There is also a mutual similarity between OP nonlocality effects given in Figs. 5 and 6 of Ref. [18] for ($d,p$) reactions around $E_d = 30$ MeV leading to $5/2^+$ state of $^{13}$C and $^{17}$O nuclei; the corresponding binding energies are 1.092 and 4.143 MeV.

As in Ref. [18] we checked that the OP nonlocality effect observed in transfer reactions is insensitive to small variations of the parameter $w_N$ and therefore is not a consequence of only approximate on-shell equivalence between NLOP and Watson OP. We also studied sensitivity of the results with respect to $np$ and $nA$ interactions. We performed additional test calculations using local AV18.
potential for \( np \); compared to mildly nonlocal CD Bonn potential the differences in the predictions were found to be entirely negligible. Varying parameters \( r_0 \) and \( a \) of the \( nA \) potential within reasonable limits, \( \pm 0.05 \) fm, or, in the case of \( n^{-10}\)Be, using \( L \)-independent \( V_{\text{so}} \), and adjusting the strengths \( V_\pi \) and \( V_{\text{so}} \) to reproduce the same binding energies, leads to the differences in transfer results that are still considerably smaller than the observed OP nonlocality effect. Thus, the cross section in the considered transfer reactions depends mostly on the \( pA \) interaction.

**IV. SUMMARY**

We performed calculations of \((d,p)\) and \((p,d)\) transfer reactions involving one-neutron halo nuclei \(^{11}\)Be and \(^{15}\)C. Exact three-body scattering equations in the AGS form were solved and the Coulomb interaction was included using the method of screening and renormalization; well converged results were obtained. With respect to dynamics, they are mostly sensitive to the proton-core interaction. For the first time in reactions with halo nuclei a nonlocal optical potential was used and important nonlocality effects were found. The OP nonlocality effect on transfer observables depends mostly on the reaction energy and internal orbital angular momentum of the halo nucleus and is rather insensitive to its binding energy and mass. The NLOP accounts for the experimental transfer data better than the local Watson OP and provides a more consistent description over a broader energy and angular range, especially for the \( 1/2^+ \) initial/final state of the halo nucleus. The values for the neutron spectroscopic factor obtained using the NLOP agree quite well with estimates based on other approaches, in contrast to those obtained using the nearly on-shell equivalent Watson OP.

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