SUSY GUTs with Yukawa unification:
a go/no-go study using FCNC processes

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We address the viability of exact Yukawa unification in the context of general SUSY GUTs with universal soft-breaking sfermion and gaugino mass terms at the GUT scale. We find that this possibility is challenged, unless the squark spectrum is pushed well above the limits allowed by naturalness. This conclusion is assessed through a global fit using electroweak observables and quark flavour-changing neutral current (FCNC) processes. The problem is mostly the impossibility of accommodating simultaneously the bottom mass and the BR$(B \to X_s \gamma)$, after the stringent CDF upper bound on the decay $B_s \to \mu^+ \mu^-$ is taken into account, and under the basic assumption that the $b \to s \gamma$ amplitude have like sign with respect to the Standard Model one, as indicated by the $B \to X_s \ell^+ \ell^-$ data.

With the same strategy, we also consider the possibility of relaxing Yukawa unification to $b - \tau$ Yukawa unification. We find that with small departures from the condition $\tan \beta \simeq 50$, holding when Yukawa unification is exact, the mentioned tension is substantially relieved. We emphasize that in the region where fits are successful the lightest part of the SUSY spectrum is basically fixed by the requirements of $b - \tau$ unification and the applied FCNC constraints. As such, it is easily falsifiable once the LHC turns on.

I. INTRODUCTION

The hypothesis of grand unification is able to address many of the unanswered questions of the Standard Model (SM), like charge quantization or the quantum number assignments of the SM fermions. Moreover, augmenting grand unified theories (GUTs) with supersymmetry (SUSY) not only stabilizes the large mass hierarchy between the electroweak (EW) and the GUT scale but also leads to the possibility of exact gauge coupling unification.

Although this unification works remarkably well in the minimal supersymmetric Standard Model (MSSM), in order to test the idea of grand unification one needs other, independent observables. A first candidate in this respect would be proton decay. However, the absence of a signal at proton decay experiments constrains mostly non-supersymmetric GUTs, whereas the strong model-dependence of the proton decay rate contributions from dimension-five operators makes it difficult to draw general conclusions on the viability of SUSY GUTs.

A different way to test SUSY GUTs is by their predictions for the masses and mixings of the SM fermions. In this respect, SO(10) is especially attractive because it unifies all quarks and leptons of one generation into a 16 representation of the gauge group, leading to the opportunity of a unified Yukawa coupling for the fermions of that generation. Though this is not phenomenologically viable for the two light generations, unification of the top, bottom, tau, and tau-neutrino Yukawa couplings might be possible if $\tan \beta$, the ratio of the vacuum expectation values of the two Higgs doublets, is close to 50.

It must be taken into account, however, that the success of this Yukawa unification sensitively depends not only on $\tan \beta$, but also on the SUSY spectrum and parameters, because the Yukawa couplings are much more sensitive to weak scale threshold corrections than are the gauge couplings. In the absence of a clear signal in favour of supersymmetry or an a priori knowledge of the SUSY spectrum, one would need a set of additional observables to sufficiently constrain the allowed ranges for the SUSY spectrum itself, in order to test GUT predictions for fermion masses. It turns out that flavour-changing neutral current (FCNC) processes – loop-suppressed observables that are highly sensitive to SUSY particle contributions – are especially suited for this purpose.

In [9], an SO(10) SUSY GUT model proposed by Dermišek and Raby (DR) [10], and featuring Yukawa unification, has been reconsidered in a global analysis in light of all the most precise data on FCNCs in the quark sector. While the model successfully describes EW observables as well as quark and lepton masses and mixings [14, 11], in [9] it was found that the simultaneous description of these observables and all the FCNC processes considered is impossible unless the squark masses are pushed well above the limits allowed by naturalness and within reach of the Large Hadron Collider (LHC).

The aim of this Letter is twofold. First, we show that the problem pointed out in [9] and mentioned above is a general feature of SUSY GUT models with Yukawa unification and universal sfermion and gaugino mass terms at the GUT scale, thus challenging the viability of these hypotheses, when considered together. Our second aim is then to explore a possible
remedy, namely relaxing the hypothesis of Yukawa unification in favour of the less restrictive $t-\nu$ and $b-\tau$ Yukawa unifications. The departure from exact Yukawa unification can be quantified by the parameteric departure from the condition $\tan \beta \approx 50$. As clarified below, this case will be relevant not only for SU(5), but also for SO(10). This study will allow us to address the question whether a range of large $\tan \beta \lesssim 50$ exists, where a successful prediction for the bottom mass and full compatibility with quark FCNCs are possible at the same time.

II. YUKAWA UNIFICATION AND FCNCs

It is well-known [12, 13] that, under the assumptions of a universal sfermion mass $m_{16}$ and a universal gaugino mass $m_{1/2}$ at the GUT scale, and with a positive $\mu$ parameter, Yukawa unification prefers the region in MSSM parameter space characterized by the relations

$$-A_0 \approx 2 \, m_{16}, \quad \mu, m_{1/2} < m_{16},$$  

(1)

because they ensure a cancellation of potentially large $\tan \beta$-enhanced SUSY threshold corrections to the bottom quark mass [14], which could otherwise spoil the Yukawa unification. Through renormalization group effects, these relations lead to an inverted scalar mass hierarchy (ISMH) [15], i.e., light third generation and heavy first and second generation sfermions.

Relations [11], together with the large value of $\tan \beta \approx 50$ required for Yukawa unification, have an important impact on the SUSY spectrum and on the predictions for FCNCs. In particular, the branching ratio of the decay $B_s \to \mu^+\mu^-$ receives large $\tan \beta$-enhanced contributions from Higgs-mediated neutral currents that are proportional to $A_t^2 (\tan \beta)^6 / M_A^4$ [16, 17]. With large $\tan \beta$ and a large trilinear coupling $A_t$ following from relations [11], the stringent most recent experimental bound [18]

$$\text{BR}(B_s \to \mu^+\mu^-)_{\exp} < 5.8 \times 10^{-8} \quad (95\% \text{ C.L.})$$  

(2)

can only be met with quite heavy $A^0$, $H^0$, and $H^+$ Higgs bosons.

Another important process in this respect is the tree-level decay $B^+ \to \tau^+\nu$. Using the SM fit value for the CKM matrix element $V_{us}$ [19, 20] one obtains a SM prediction for the branching ratio

$$\text{BR}(B^+ \to \tau^+\nu)_{\text{SM}} = (0.82 \pm 0.11) \times 10^{-4}$$  

(3)

that is quite low compared to the experimental value

$$\text{BR}(B^+ \to \tau^+\nu)_{\exp} = (1.41 \pm 0.43) \times 10^{-4}.$$  

(4)

In the MSSM with large $\tan \beta$, the dominant additional contribution to this process comes from charged Higgs bosons and is found to interfere always destructively [21] with the SM contribution, thus further reducing the theory prediction. Hence, similarly to $B_s \to \mu^+\mu^-$, the $B^+ \to \tau^+\nu$ decay requires a heavy Higgs spectrum to be in agreement with the experimental data. However, given the large experimental uncertainty in [11], the $B_s \to \mu^+\mu^-$ constraint turns out usually to be more stringent.

Finally, a very important constraint is the inclusive decay $B \to X_s\gamma$, which receives the dominant SUSY contributions from a chargino–stop loop and a top–charged Higgs loop. The chargino contribution is $\tan \beta$-enhanced and, with the large negative trilinear parameters implied by relations [11], adds destructively to the SM branching ratio. The charged Higgs contribution, on the other hand, adds constructively to the branching ratio, but is suppressed by the heavy Higgs masses required to be consistent with $B_s \to \mu^+\mu^-$. Thus a near cancellation between the two contributions, which is necessary in view of the good agreement between the experimental determination [22]

$$\text{BR}(B \to X_s\gamma)_{\exp} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$$  

(5)

and the SM prediction [23]

$$\text{BR}(B \to X_s\gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4},$$  

(6)

is difficult to achieve.

Note that the solution with the chargino contribution so large that the sign of the $b \to s\gamma$ amplitude is reversed [24] is challenged in our framework by the experimental data on $\text{BR}(B \to X_s\ell^+\ell^-)$ [30, 31, 32]. In fact, it would lead to a $3 \sigma$ discrepancy between the prediction and the experimental figure for this branching ratio.

Leaving aside, for the moment, the possibility of a sign flip in the $b \to s\gamma$ amplitude (we will return to this issue in section III), the above discussion implies that a tension between the prediction for $B \to X_s\gamma$ and the bound on $B_s \to \mu^+\mu^-$ should generally be expected in models with Yukawa unification, as a direct consequence of relations [11] and the large value of $\tan \beta$. By the nature of the argument, this tension should be completely independent of the mechanism (flavour symmetries or other) embedded in the SUSY GUT to explain the light quark masses and mixings. In section IV we will come back to this issue, showing that indeed this tension occurs generally in SUSY

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1 This prediction is obtained by normalizing the branching ratio to $\Delta M_d$ [21]. The value in eq. 3 agrees well with those reported in [4, 19, 22].

2 In the actual numerical analysis we include all the relevant contributions, in particular gluino-down squark loops as well. The latter are found to play a negligible role.

3 This statement holds, barring non-negligible new physics contributions to the Wilson coefficients $C_{9,10}^H$ (see Ref. [31]), which is impossible in our case.
GUT models with Yukawa unification and quantifying the tension numerically with a \( \chi^2 \)-procedure.

Technically, the most immediate potential remedy to the above mentioned problem seems to be to lower \( \tan \beta \). This in fact alleviates the pressure from \( B_3 \rightarrow \mu^+ \mu^- \), permitting in turn larger Higgs and smaller chargino contributions to \( B \rightarrow X_s \gamma \) and thereby making possible that those two contributions indeed cancel to a large extent.

Lowering \( \tan \beta \) means breaking the unification of the top and bottom Yukawa couplings, so that the full Yukawa unification is relaxed to \( b - \tau \) unification, occurring, e.g., in SU(5). Such breaking in \( t - b \) unification is actually also a general possibility in SO(10) SUSY GUTs once all the representations needed for a realistic GUT-breaking sector are taken into account. For example, the “minimal breaking scheme” introduced by Barr and Raby \[33\] requires the presence of a \( 16'_{H} \) spinor. In this framework, the MSSM Higgs doublet \( H_d \) can naturally be a mixture between a doublet contained in the same \( 10_{H} \) representation as the doublet \( H_u \) and one doublet contained in this \( 16'_{H} \) spinor, since the two have the same quantum numbers. One then has \[3, 4, 34\]

\[
\begin{align*}
H_u &= H(10_{H}), \\
H_d &= \overline{H}(10_{H}) \cos \gamma + \overline{16'_{H}} \sin \gamma.
\end{align*}
\]

(7)

Consequently, the Yukawa unification relation \( \lambda_t = \lambda_b \) is effectively broken to

\[
\frac{\lambda_b}{\lambda_t} = \cos \gamma.
\]

(8)

At the EW scale, this relation leads to a value of \( \tan \beta \lesssim 50 \) parametrically smaller than in the exact unification case, depending on the amount of mixing in the second of eqs. (7).

We would like to emphasize that the two cases of SU(5) and SO(10) with minimal breaking scheme mentioned above are just intended as examples. Our analysis will be completely general in SUSY GUTs with \( b - \tau \) unification.

It should be stressed as well that, even without \( t - b \) unification, SUSY GUT models with \( b - \tau \) unification maintain in fact most of their predictivity, since the relation between the \( b \) and \( \tau \) Yukawa couplings requires the ISMH relations, eq. (1), to be satisfied in order to obtain a correct prediction for \( m_b \). In addition, a crucial observation is that \( b - \tau \) unification requires \( \tan \beta \) either close to unity (which is however excluded by the Higgs mass bound \[32\] or O(50), because otherwise the predicted bottom quark mass is in general too large \[32, 37\]. Although the case \( \tan \beta = O(50) \) can be significantly modified by the \( \tan \beta \)-enhanced threshold corrections to \( m_b \) mentioned above, \( b - \tau \) unification is difficult to achieve for \( \tan \beta \lesssim 35 \). Therefore the strategy to lower \( \tan \beta \) is not a trivial one in our context, since \( b - \tau \) unification pushes by itself lower values.

With the above arguments, departure from third generation Yukawa unification and restriction to \( b - \tau \) unification seems to be a promising approach to retain the predictivity of GUT models, while at the same time possibly removing tensions in FCNC observables, thanks to \( \tan \beta < 50 \). The rest of our Letter is thus an attempt to address the following two questions:

- Is the tension between FCNC observables a general feature of GUT models with third generation Yukawa unification and universal masses for sfermions and gauginos at the GUT scale?
- Is this tension relieved when \( \tan \beta \) is (slightly) below 50, i.e., if one moves from exact Yukawa unification but retains \( b - \tau \) unification.

These issues will be studied through a numerical procedure to be described below.

### III. PROCEDURE

We assume, at scales higher than the GUT scale \( M_G \), the existence of a grand unified group containing \( b - \tau \) unification. Beneath \( M_G \) the grand unified group is broken \[4\] to the SM group \( G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \) and the MSSM running is performed down to the EW scale. As for the GUT scale boundary conditions to this running, we include a unified gauge coupling \( \alpha_G \), allowing for a GUT scale threshold correction \( \epsilon_3 \) to the strong coupling constant, the Yukawa couplings for up- and down-type fermions of the third generation \( \lambda_t \) and \( \lambda_b \), and a right-handed neutrino mass \( M_R \). At \( M_G \) we also assume a soft SUSY-breaking sector, consisting of a universal trilinear coupling \( A_0 \), a universal sfermion mass \( m_{1/2} \), a universal gaugino mass \( m_{1/2} \), and as well as non-universal Higgs mass parameters \( m_{H_u}, m_{H_d} \).

We run all the parameters using one-loop RGEs for the soft sector and two-loop RGEs for the Yukawa and gauge couplings \[38\]. To take correctly into account the effects of right-handed neutrinos present in SO(10) and required for the see-saw mechanism, we include the contribution of a third-generation neutrino Yukawa coupling (with initial condition \( \lambda_{\nu_i} = \lambda_i \)) in all RGEs between \( M_G \) and \( M_R \) \[33, 40, 41\]. In our framework, there are thus no potentially large logarithmic GUT scale threshold corrections to either Yukawa unification or to Higgs splitting, which would be present if such contribution were neglected in the RGEs.\[6\] The remaining GUT scale threshold correction

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4 For our purposes, this can be assumed to happen in one single step.
5 On the Yukawa couplings of the lightest two generations we will comment later on in this section.
6 In contrast with statements made in the literature and in accord with the results of \[41\], we find that neutrino Yukawa
tions to the Yukawa couplings are expected to be small.

At the EW scale, we finally have the two additional free parameters $\tan \beta$ and $\mu$. The total number of free parameters is then 13 and they are collected in table I.

| Sector | # | Parameters |
|--------|---|------------|
| gauge | 3 | $\alpha_G$, $M_G$, $\epsilon_3$ |
| SUSY   | 5 | $m_{1/2}$, $A_0$, $m_{H_u}$, $m_{H_d}$ |
| Yukawas | 2 | $\lambda_t$, $\lambda_b$ |
| neutrino | 1 | $M_R$ |
| SUSY (EW scale) | 2 | $\tan \beta$, $\mu$ |

TABLE I: Model parameters. Unless explicitly stated, they are intended at the GUT scale.

After calculating the SUSY and Higgs spectra\(^7\) and the threshold corrections to third generation fermion masses\(^0\), we evaluate the flavour-changing observables using the effective Lagrangian approach of\(^{[47]}\). Thereafter, in order to have a quantitative test of the model, we construct a $\chi^2$ function defined as

$$
\chi^2(\vec{\theta}) = \sum_{i=1}^{N_{\text{obs}}} \frac{(f_i[\vec{\theta}] - \mathcal{O}_i)^2}{(\sigma_i^\text{exp})^2 + (\sigma_i^\text{theo})^2},
$$

composed of the quantities given in tables II and III.

| Observable | Value($\sigma_{\text{exp}}$) | Observable | Lower Bound |
|------------|-----------------------------|------------|-------------|
| $M_W$      | 80.403(29)                  | $M_{h_0}$  | 114.4       |
| $M_Z$      | 91.1876(21)                 | $M_{h+}$   | 104         |
| $1/\alpha_{em}$ | 1.166837(1)               | $M_t$      | 95.7        |
| $\alpha_s(M_Z)$ | 0.1176(20)               | $M_{\mu}$  | 170.9       |
| $m_t(m_t)$  | 4.20(7)                     | $M_{\tau}$ | 1.77(7)     |

TABLE II: Flavour conserving observables \(^{[48]}\) \(^{[49]}\) used in the fit. Dimensionful quantities are expressed in powers of GeV.

In eq. (9) $\mathcal{O}_i$ indicates the experimental value of the observables listed in tables II and III and $f_i[\vec{\theta}]$ the corresponding theoretical prediction, which will be function of the model parameters listed in table I, collectively indicated with $\vec{\theta}$. The $\chi^2$ function is minimized upon variation of the model parameters, using the minimization algorithm MIGRAD, which is part of the CERNlib library\(^{[53]}\).

Some comments are in order on the determination of the errors. First, one can note that among the observables in table II, some have a negligible experimental error. In this case, we took as overall uncertainty 0.5% of the experimental value, which we consider a realistic estimate of the numerical error associated with the calculations. Concerning the theoretical errors on the flavour observables (table III), we note the following: the error on $\Delta M_s/\Delta M_d$ takes into account the SM contribution, dominated by $\xi^2$ and the new physics contributions, dominated by scalar operators; the error on $\text{BR}(B^+ \to \tau^+ \nu)$, after normalization by $\Delta M_d$\(^{[21]}\), is dominated by the lattice “bag” parameter $B_H$ and the relevant CKM entries; the error on $\text{BR}(B \to X_s \gamma)$ is taken as twice\(^8\) the total theoretical error associated with the SM calculation\(^{[28]}\); finally the error on $\text{BR}(B \to X_s \ell^+\ell^-)$ is taken as 25% of the experimental result, and is estimated from the spread of the theoretical predictions after variations of the scale of matching of the SUSY contributions.

In evaluating the $\chi^2$ function, we also included the bounds reported in tables II and III. These constraints are in the form of suitably smoothed step functions, which are added to the $\chi^2$-function of eq. (9). If any of the constraints is violated, the step functions add a large positive number to the $\chi^2$, while for respected constraints the returned value is zero, so that the $\chi^2$ is set back to its ‘unbiased’ definition (9).

A step function was also included in order to enforce the desired sign for the $b \to s \gamma$ calculated amplitude, thus permitting to systematically explore both cases of like sign or flipped sign with respect to the SM one. In the case of flipped sign, large SUSY contributions are necessary such that SUSY is not quite a correction to the SM result, but rather the opposite. As a consequence, one would need a theoretical control on the SUSY part at least as good as that on the pure SM calculation. In the absence of this knowledge, the amplitude in the flipped-sign case is generally very sensitive to variations of the matching scale, and the associated theoretical error hard to control. In order to be able to estimate as reliably as possible the

\(^7\) We calculate the Higgs VEVs and $M_A$ following\(^{[42]}\) and use FeynHiggs\(^{[43]}\) \(^{[44]}\) \(^{[45]}\) \(^{[46]}\) to obtain the light Higgs mass.

\(^8\) This choice is quite conservative, considering that in our case variations of the calculated $\text{BR}(B \to X_s \gamma)$ upon variation of the SUSY matching scale in the huge range [0.1, 1] TeV are typically around 4%. However, we feel it is justified in the case of large cancellations among new physics contributions.
\[ b \rightarrow s\gamma \] amplitude also in the flipped-sign case, we have taken advantage of the SusyBSG code \(^{54}\), which is directly called by the fitting procedure.

As already mentioned in section II, the scenario with flipped \( b \rightarrow s\gamma \) amplitude leads to \( \chi^2 \gtrsim 9 \) solely on account of the implied 3\sigma discrepancy in \( B \rightarrow X_s\ell^+\ell^- \). We calculated the BR(\( B \rightarrow X_s\ell^+\ell^- \)) using the results of Ref. \(^{52}\). We will address the scenario with flipped \( b \rightarrow s\gamma \) amplitude quantitatively in section IV.

An important observation is in order at this point, justifying why our analysis should be valid for any SUSY GUT model with \( b \rightarrow \tau \) unification and universal soft terms as in table I. As already mentioned in section II any such model prefers the region of parameter space leading to ISMH, i.e., third generation sfermion masses much lighter than first and second generation ones, the latter being of \( O(m_{16}) \). One should as well consider that, assuming hierarchical Yukawa matrices and large \( \tan \beta \), it is sufficient to include only the 33-elements of Yukawa matrices in the RGEs, that is, take GUT-scale boundary conditions for the Yukawa couplings as

\[ Y_{u,d} = \text{diag}(0, 0, \lambda_t, \lambda_b). \quad (10) \]

This will have a negligible effect on the determination of the first and second generation sfermion masses, given their heaviness. This observation makes it possible to separate the effects of specific Yukawa textures, which depend on the flavour model one embeds into the SUSY GUT, from those genuinely due to the unification of Yukawa couplings. The adoption of this strategy brings us to the following remarks:

- Given the approximation we adopt for the initial conditions on the Yukawas, we do not need to assume any particular flavour model. The low-energy input of the CKM matrix and of the fermion masses other than third generation ones, necessary for the calculation of many among the observables included in the fit, is then taken directly from experiment\(^9\).

- Due to ISMH, the lighter stop is always the lightest sfermion and in fact its tree-level mass can be very small. Therefore, we include the one-loop corrections to the light stop mass to ensure our solutions are consistent with the lower bound in table II. In practice, due to its lightness, we calculate the stop pole mass with the same accuracy as the pole masses of the third generation fermions, the \( W, Z \), and the Higgs bosons.

- The validity of our approximation, eq. (10), was checked by performing the full analysis also with a SUSY GUT model with specific flavour textures, namely the DR model \(^{10}\) (with Yukawa unification relaxed as in eq. (8)). Our results were not significantly affected.

IV. RESULTS

In order to address the questions outlined in the introduction, we have explored in the \( m_{16} \) vs. \( \lambda_t/\lambda_b \) plane the class of GUT models defined in section III. Concretely, we fixed \( m_{16} \) to values \( \geq 4 \text{ TeV} \) and \( \lambda_t/\lambda_b \) to values \( \geq 1 \) (roughly equivalent to fixing \( \tan \beta \) to values \( \leq 50 \)) and minimized the \( \chi^2 \) function \(^{8}\) upon variation of the remaining model parameters. The minimum \( \chi^2 \) value provides then a quantitative test of the performance of the model.\(^{10}\) The results of our survey are reported in the four panels of Fig. I. In particular, panels (a) to (c) report the \( \chi^2 \) contours as solid lines in the \( m_{16} \) vs. \( \tan \beta \) plane. As reference, also the values of \( \lambda_t/\lambda_b \) are reported on a right-hand vertical scale. Superimposed to the \( \chi^2 \) contours are: in panels (a) and (b), the deviations of respectively BR(\( B \rightarrow X_s\gamma \)) and \( m_b \) from the central values in tables IIIII in units of the total error; in panel (c) the mass contours of the lightest stop. Finally, panel (d) shows the \( \chi^2 \) contributions from BR(\( B \rightarrow X_s\gamma \)), \( m_b \) and all the rest (as three stacked contributions, represented by solid lines) vs. \( m_{16} \) in the special case of \( \lambda_t/\lambda_b = 1 \), corresponding to exact Yukawa unification.

Various comments are in order on these plots.

1. Panel (d) shows that, for any \( m_{16} \lesssim 9 \text{ TeV} \), the \( \chi^2 \) contribution from \( B \rightarrow X_s\gamma \) alone is no less than roughly 4, corresponding to no less than 2\sigma deviation from the result of eq. \(^{55}\). Therefore, in the case of Yukawa unification, agreement among FCNCs is only achieved at the price of decoupling in the scalar sector.\(^{11}\) One should note in this respect the quite conservative choice of the \( B \rightarrow X_s\gamma \) error, already mentioned in footnote 8. The apparent non-monotonic behaviour of the \( B \rightarrow X_s\gamma \) \( \chi^2 \)-profile is due to the fact that, for \( m_{16} \lesssim 6 \text{ TeV} \), the model prediction becomes so bad that the minimization algorithm prefers to sacrifice the prediction for \( m_b \) (which in fact gets much worse).

2. For \( m_{16} \lesssim 4.7 \text{ TeV} \), fits usually prefer the flipped-sign solution for the \( b \rightarrow s\gamma \) amplitudes.

\(^9\) Specifically, the CKM input is taken from the new physics independent CKM fit \(^{12}\).

\(^{10}\) Of course such test cannot be attached a statistically rigorous meaning, since, e.g., the \( \chi^2 \)-entries are not all independently measured observables.

\(^{11}\) For similar findings in the context of Bayesian analyses of the CMSSM, see, e.g., Ref. \(^{54}\).
FIG. 1: Panels (a)-(c): $\chi^2$ contours (solid lines) in the $m_{16}$ vs. $\tan \beta$ plane. Superimposed as dashed lines are the pulls for $\text{BR}(B \to X_s \gamma)$ (panel (a)) and for $m_b$ (panel (b)) and the lightest stop mass contours (panel (c)). Panel (d): $\chi^2$ contributions vs. $m_{16}$ in the special case of exact Yukawa unification. All the plots assume a SM-like sign for the $b \to s\gamma$ amplitude, except for panel (d), where also the total $\chi^2$ for the flipped-sign case is shown as a dot-dashed line.

1. From panels (a)-(c) one can note a region of successful fits for $m_{16} \gtrsim 7$ TeV and $46 \lesssim \tan \beta \lesssim 48$, corresponding to a moderate breaking of $t - b$ unification, since it corresponds to $0.2 \gtrsim (\lambda_t/\lambda_b - 1) \gtrsim 0.1$.

4. The interesting region, characterized in point 3., emerges as a compromise between $B \to X_s \gamma$ and $m_b$, pushing $\tan \beta$ to respectively lower and larger values. In this region, the lightest stop mass is below order 1 TeV, but not less than roughly 800 GeV.

5. If $m_{16}$ is not too large, the interesting region
is clearly distinguished from the corresponding case with exact Yukawa unification, as far as the fit quality is concerned. By looking at panels (a)-(c), one can in fact recognize that the gradient of $\chi^2$ variation is close to vertical for, say, $m_{16} = 7$ TeV, when increasing $\tan \beta$ from around 48. On the other hand, increasing $m_{16}$ makes the case of breaking $t - b$ unification more and more indistinguishable, for the fit performance, from the decoupling regime of point 1.

An example of a fit in the interesting region is reported in table IV and shown in panels (a)-(c) as a black square. Note that the prediction for BR($B \rightarrow X_s \gamma$) still tends to be on the lower side of the experimental range from eq. (5), with a central value around $2.9 \times 10^{-4}$. As a comparison, a representative fit in the region with exact Yukawa unification, featuring a flipped $b \rightarrow s\gamma$ amplitude, is reported in table IV.

We note, point 1. above implies that, for any SUSY GUT with Yukawa unification, compatibility with FCNC observables and with the now precisely known value of $m_b$ selects the “partially decoupled solution”, advocated in Tobes and Wells [57], as the only phenomenologically viable. The low-$m_{16}$ solution originally found in [12, 13] is disfavoured when combining all the most recent data. This conclusion has been here quantitatively assessed with a $\chi^2$ procedure.  

Points 1 to 6 illustrate instead that compatibility among all the considered observables at values of $m_{16}$ of order 7 TeV can be recovered at the price of relaxing $t - b - \tau$ unification to just $b - \tau$ unification and without modifying universality in the soft terms at the GUT scale.

Note that, in our approach, exact Yukawa unification can be enforced, so that the lower bound on $m_{16}$ emerges transparently as a tension among observables. Instead, in, e.g., [58], it is low-energy observables (like $m_b$) to be fixed and a large value for $m_{16}$ is needed for Yukawa unification to occur within a given tolerance.
We conclude this section with a few additional remarks. A first interesting issue is whether the general tension involving FCNCs and $m_t$ studied in this Letter may be relieved if, instead or in addition to lowering $\tan \beta$, one allows for a complex phase in the trilinear coupling $A_t$. The latter in our analysis is real, since we take real $A_0$. A complex phase $\phi_t$ in $A_t$ would induce a $\cos \phi_t$ suppression factor in the leading chargino correction to $\text{BR}(B \to X_s \gamma)$. In the presence of a complex $A_t$, however, the $\tan \beta$-enhanced chargino contribution to the bottom mass would also become complex, so that a (chiral) redefinition of the down-quark fields would be necessary in order to end up with real and positive masses. This may impact non-negligibly the overall size of SUSY corrections to $m_b$ as well as the pattern of CP violation in B-physics observables. In addition, a complex phase in $A_t$ is also constrained by the electric dipole moments of the electron and neutron. Addressing these issues quantitatively goes beyond the scope of the present Letter, where we confine ourselves to real GUT-scale soft terms.

As a second remark, we note that, in our analysis of the parameter space, we restricted ourselves to positive values of the $\mu$ parameter. This is because, for negative $\mu$, the ISMH solution is lost, thus leading to much heavier third generation sfermions for a given value of $m_{16}$ (typically $m_{\ell} \gtrsim 2.5$ TeV), which is unattainable. This constraint can however be circumvented in many ways, such as a late-time entropy injection (see, e.g., [61]).

In fact, in the region preferred by the $\chi^2$ function, the contributions to $a_\mu$ are much smaller than would be needed to explain the E821 result [59, 60]. This is a well-known fact [3, 11] and we decided not to include this observable in our $\chi^2$ function to obtain results unbiased by this issue, which still needs to be settled.

We also chose not to include the constraint from the 'standard' neutralino relic density calculation. The latter yields a posteriori a too large relic density in the interesting region of parameter space for well-known reasons, i.e., the heaviness of sfermions and that of the pseudoscalar Higgs, making the A-funnel region unattainable. This constraint can however be circumvented in many ways, such as a late-time entropy injection (see, e.g., [61]).

For the above reasons, we preferred to restrict the set of observables considered in the analysis to those listed in tables [11] and [11] on which consensus is broad.

V. CONCLUSIONS

In this Letter we have studied the viability of the hypothesis of $t - b - \tau$ Yukawa unification in SUSY GUTs under the assumption that soft-breaking terms for sfermions and gauginos be universal at the GUT scale. We found that this hypothesis is challenged, unless the squark spectrum is pushed well above 1 TeV. Our conclusion is assessed through a global fit including EW observables as well as quark FCNC processes. The origin of the difficulty is mostly in the specific parameter region chosen by Yukawa unification, which guarantees the correct value for the bottom mass and implies ISMH. In this region, it is impossible to accommodate simultaneously the experimental value for $\text{BR}(B \to X_s \gamma)$ and the severe upper bound on $\text{BR}(B_s \to \mu^+ \mu^-)$. This statement holds under the prior assumption that the sign of the $b \to s \gamma$ amplitude be the same as in the SM. For $m_{16} \lesssim 4.7$ TeV, fits prefer the flipped-sign solution for the $b \to s \gamma$ amplitude. In this instance it is however difficult to achieve agreement on the $b \to s \gamma$ prediction and, simultaneously, on those of EW observables and/or of the bottom mass. In addition, this possibility entails in our case a $3\sigma$ discrepancy in $B \to X_s \ell^+ \ell^-$ data.

We have shown that our above conclusions hold irrespectively of the flavour model embedded in the SUSY GUT, since, due to ISMH, first and second generation squark masses are much heavier than third generation ones.

We have also addressed the possibility of relaxing Yukawa unification to $b - \tau$ unification, which in fact allows to maintain most of the predictivity of SUSY GUTs. In this case, the tension underlined above is in fact largely relieved. The fit still prefers large values of $46 \lesssim \tan \beta \lesssim 48$, as a compromise between FCNCs and $m_{16}$, pushing $\tan \beta$ to respectively lower and larger values. The range for $\tan \beta$ corresponds to a moderate breaking of $t - b$ Yukawa unification, in the interval $\lambda_1/\lambda_b - 1 \in [0.1, 0.2]$.

In the interesting region, we find the lightest stop mass $\gtrsim 800$ GeV, a light gluino around 400 GeV and lightest Higgs, neutralino and chargino close to the lower bounds. This spectrum implies $\text{BR}(B_s \to \mu^+ \mu^-)$ in the range $2 \times 10^{-8}$ and $\text{BR}(B \to X_s \gamma)$, on the lower side of the "acceptable" range, $\approx 2.9 \times 10^{-4}$. We stress that the requirements of $b - \tau$ unification and the FCNC constraints are enough to make the above figures, exemplified in table [14] basically a firm prediction within the interesting region, hence easily falsifiable once the LHC turns on.

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13 The requirement that SO(10) SUSY GUTs with Yukawa unification do account in full for the standard CDM abundance has been recently reconsidered in [62].
cially useful in the assessment of the $b \rightarrow s \gamma$ theoretical error, and for providing us with code allowing a number of cross-checks. The work of W.A., D.G. and D.M.S. has been supported in part by the Cluster of Excellence “Origin and Structure of the Universe” and by the German Bundesministerium für Bildung und Forschung under contract 05HT6WOA. D.G. also warmly acknowledges the support of the A. von Humboldt Stiftung. S.R. acknowledges partial funding from DOE grant DOE/ER/01545-877.

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