α-induced reaction cross sections in the mass range
A ≈ 20 – 50: a critical review

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Abstract. In a recent review it was shown that the cross sections of α-induced reactions in the A ≈ 20 – 50 mass range follow a general and smooth trend in most cases. For comparison of cross sections of different targets at various energies the method of reduced cross sections σ_red and reduced energies E_red was used. Four outliers were identified: 36Ar and 40Ar with unusual small cross sections and 23Na and 33S with unusual huge cross sections. New data for 23Na were presented at this NPA-7 conference; contrary to the previous data, these new data fit into the general systematics. In addition, a relation between the most effective energy E_0 for astrophysical reaction rates (the so-called Gamow window) and the reduced energy E_red is presented.

1. Introduction
In a recent review [1] the cross sections of α-induced reactions for target nuclei in the mass range A ≈ 20 – 50 were studied. For comparison of cross sections of different targets at various energies the method of reduced cross sections σ_red and reduced energies E_red [2] was used:

\[ E_{\text{red}} = \frac{(A_P^{1/3} + A_T^{1/3})E_{\text{c.m.}}}{Z_PZ_T} \] (1)

\[ \sigma_{\text{red}} = \frac{\sigma_{\text{reac}}}{(A_P^{1/3} + A_T^{1/3})^2} \] (2)

The reduced energy E_red takes into account the different heights of the Coulomb barrier in the systems under consideration, whereas the reduced reaction cross section σ_red scales the measured total reaction cross section σ_reac according to the geometrical size of the projectile-plus-target system. (All energies are given as E_{c.m.} except explicitly noted.) In the A ≈ 20 – 50 mass range under study the total reaction cross section σ_reac can often be taken from the dominating (α,n) or (α,p) reaction channel. It was found in [1] that most of the experimental cross sections show a general and smooth trend for σ_red vs. E_red. The two outliers 36Ar and 40Ar show significantly smaller cross sections; however, this finding is based only on one very old experiment [3]. Surprisingly, two recent experiments on 23Na [4] and 33S [5] show unexpected huge cross sections. In addition, the data for 23Na [4] show a steeper energy dependence, and the data for 33S show a flatter energy dependence compared to most of the other nuclei in this mass region.

The present study presents two extensions to the recent review [1]. First, a relation between the reduced energy E_red and the most effective energy E_0 for astrophysical reaction rates (the
so-called Gamow window) is derived. Second, the review [1] is updated with two data sets for the previous outlier $^{23}$Na which were presented at this NPA-7 conference [6, 7] and have been published now [8, 9]. These new data for $^{23}$Na show that $^{23}$Na should not be considered as outlier any more. Some further information on the other outlier $^{33}$S was already given in [10].

2. Gamow window and reduced energy $E_{\text{red}}$

The astrophysical reaction rate $N_{A}\langle \sigma v \rangle$ is defined by the Maxwellian-averaged cross section $\langle \sigma v \rangle$

$$\langle \sigma v \rangle = \int_{0}^{\infty} \Phi(v) \sigma(v) v \, dv = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_{0}^{\infty} E \, \sigma(E) \exp \left( \frac{-E}{kT} \right) \, dE$$  \hspace{1cm} (3)

For an energy-independent astrophysical $S$-factor the integrand in Eq. (3) shows a maximum at the energy $E_0$ which is given by

$$E_0 \approx 0.122 \text{ MeV} \left( \frac{Z_P^2 Z_T^2 A_{\text{red}} T_9^2}{3} \right)^{1/3}$$  \hspace{1cm} (4)

with the charge numbers $Z_P$ and $Z_T$ of projectile and target, the reduced mass number $A_{\text{red}} = \frac{A_P A_T}{A_P + A_T}$, and the plasma temperature $T_9$ in $10^9$ Kelvin. Although it has been shown that the underlying simple approximation $S(E) \approx \text{const.}$ does not hold exactly [11], the energy $E_0$ is still a reasonable estimate for the most effective energy, and the energy window around $E_0$ is usually called Gamow window.

According to Eq. (1), the corresponding reduced energy $E_{\text{red},0}$ is given by

$$E_{\text{red},0} = \frac{\left( \frac{A_P^{1/3} + A_T^{1/3}}{Z_P Z_T} \right)^{1/3}}{E_0} \approx 0.122 \text{ MeV} \times T_9^{2/3} \times f(Z_P, Z_T, A_P, A_T)$$  \hspace{1cm} (5)

where the function $f(Z_P, Z_T, A_P, A_T)$ is given by

$$f(Z_P, Z_T, A_P, A_T) = \left( \frac{A_P A_T}{Z_P Z_T} \right)^{1/3} \times \frac{1 + (A_P/A_T)^{1/3}}{[1 + (A_P/A_T)]^{1/3}}$$  \hspace{1cm} (6)

From the exponents of 1/3 in Eq. (6) already a relatively smooth dependence of $E_{\text{red},0}$ on the mass and charge numbers of projectile and target can be expected. A closer inspection of Eq. (6) shows that there is a further compensation between the first and second factor in Eq. (6) which leads to a reduced Gamow energy $E_{\text{red},0}$ which is almost independent of target charge and mass for $\alpha$-induced reactions ($Z_P = 2, A_P = 4$). The first factor increases from light nuclei (with $N_T \approx Z_T$ and thus $A_T/Z_T \approx 2$) towards heavier nuclei (with $N_T \approx 1.5 Z_T$ or $A_T/Z_T \approx 2.5$). The second factor decreases with increasing target mass number $A_T$. Some numerical examples for $f(Z_P, Z_T, A_P, A_T)$ are $f(^{20}\text{Ne}) = 2.367$, $f(^{51}\text{V}) = 2.288$, and $f(^{208}\text{Pb}) = 2.165$. In the mass range under study $f$ varies only by a few per cent, leading to

$$E_{\text{red},0} \approx 0.284 \text{ MeV} \times T_9^{2/3}$$  \hspace{1cm} (7)

for the mass range under study. Explicitly, the numbers are $E_{\text{red},0} \approx 0.284 \text{ MeV}$ for $T_9 = 1$, $E_{\text{red},0} \approx 0.451 \text{ MeV}$ for $T_9 = 2$, and $E_{\text{red},0} \approx 0.591 \text{ MeV}$ for $T_9 = 3$.

3. Update for $^{23}\text{Na} + \alpha$

The total reaction cross section of $\alpha + ^{23}\text{Na}$ is dominated by the $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$ reaction at low energies. The $^{23}\text{Na}(\alpha,n)^{26}\text{Al}$ reaction has a negative $Q$-value of about $-3 \text{ MeV}$.
Two new experimental data sets for the $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$ reaction have been presented at this NPA-7 conference \[6, 8, 7, 9\]. The results of both experiments agree with each other, but are in disagreement with the previous result of \[4\].

At the Aarhus accelerator forward kinematics was applied \[6, 8\]. The target composition of the NaCl target was carefully monitored using Rutherford scattering on $^{23}\text{Na}$ and $^{nat}\text{Cl}$. Because of the relatively low beam current, no significant deterioration of the target was observed. Angular distributions for the $p_0$ and $p_1$ protons were measured over a broad angular range. Thus, the derived absolute cross sections of the $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$ reaction should be reliable within the given uncertainties in \[8\] which are of the order of $15-25\text{ per cent}$.

At the ISAC facility at TRIUMF a helium-filled gas cell was mounted in the TUDA scattering chamber for a measurement in inverse kinematics \[7, 9\]. The number of target nuclei was controlled via the gas pressure, and the number of beam particles was measured by Rutherford backscattering on the gas cell entrance window and an additionally mounted thin gold foil. Also this procedure should lead to a reliable absolute normalization of these experimental data. Absolute uncertainties of slightly below $20\%$ at higher energies and about $30-40\%$ at the lowest energies are reported in \[9\].

![Graph 1](image1.png)

**Figure 1.** Cross sections $\sigma$ of the $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$ and $^{23}\text{Na}(\alpha,n)^{26}\text{Al}$ reactions vs. laboratory energy $E_{\text{lab}}$. The experimental data have been taken from \[4, 8, 9, 14, 15\]. The new data by Howard et al. \[8\] and Tomlinson et al. \[9\] are significantly lower than the earlier data by Almarez-Calderon et al. \[4\]. The calculations are taken from \[1\]. Further discussion see text.

![Graph 2](image2.png)

**Figure 2.** Reduced cross section $\sigma_{\text{red}}$ vs. reduced energy $E_{\text{red}}$ for the $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$ reaction. The reduced cross sections $\sigma_{\text{red}}$ slightly increase towards lower target masses; the expected range is indicated by three calculations for $^{21}\text{Ne}$, $^{36}\text{Ar}$, and $^{51}\text{V}$ (taken from \[1\]; for details see there). The new data \[8, 9\] fit into the general systematics whereas the earlier data \[4\] are much higher.

In both experiments the target can be considered as relatively thick ($\approx 80-100\text{ keV}$ in the Aarhus experiment and $\approx 140\text{ keV}$ in the TRIUMF experiment in the c.m. system). Thus, the data show a relatively smooth energy dependence because of the averaging of the cross section in a broad energy interval. The high data point of the Aarhus experiment at $E_{\text{lab}} \approx 2.5\text{ MeV}$ corresponds to a strong resonance which has been seen also in earlier experiments \[12, 13\]. The new data are shown together with previous data for the $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$ \[4\] and $^{23}\text{Na}(\alpha,n)^{26}\text{Al}$...
reactions [14, 15] in Fig. 1 as cross sections vs. \( E_{\text{lab}} \) and in Fig. 2 as \( \sigma_{\text{red}} \) vs. \( E_{\text{red}} \). These figures are updates of the corresponding Figs. 56 and 57 of the review [1].

It is obvious from Fig. 1 that the new data from Howard et al. [8] and Tomlinson et al. [9] are in good agreement with each other and with the prediction from the statistical model (taken from [1]). Contrary, the previous data by Almarez-Calderon et al. [4] are more than one order of magnitude higher. Fig. 2 shows the same data as reduced cross sections \( \sigma_{\text{red}} \) vs. reduced energies \( E_{\text{red}} \). Here it has been found in [1] that there is a smooth trend for increased \( \sigma_{\text{red}} \) towards lower target masses. The typical range of \( \sigma_{\text{red}} \) for the \( A \approx 20 - 50 \) mass range is indicated by three calculations for the total reaction cross sections of \( \alpha + ^{21}\text{Ne} \), \( \alpha + ^{36}\text{Ar} \), and \( \alpha + ^{51}\text{V} \) (full lines from left to right in Fig. 2; for details see [1]). The new data of [8, 9] are located very close to the expectation from systematics.

4. Summary and Conclusions

It has been shown in [1] that \( \alpha \)-induced reaction cross sections for targets in the \( A \approx 20 - 50 \) mass range show a relatively smooth and systematic behavior. This behavior can be nicely visualized using so-called reduced cross sections \( \sigma_{\text{red}} \) and reduced energies \( E_{\text{red}} \). The present work provides a simple relation between the reduced energy \( E_{\text{red}} \) and the most effective energy \( E_0 \) for astrophysical reaction rates (the so-called Gamow window).

Four exceptions from the general systematics of \( \alpha \)-induced reaction cross sections have been identified in [1]: \( ^{36}\text{Ar} \) and \( ^{40}\text{Ar} \) with smaller cross sections [3], \( ^{23}\text{Na} \) [4] and \( ^{33}\text{S} \) [5] with larger cross sections. New data [8, 9] for \( ^{23}\text{Na} \) which have been presented first at this NPA-7 conference [6, 7] supersede the previous data [4]. Contrary to the previous data by [4], these new data [8, 9] indicate a regular behavior of \( ^{23}\text{Na} \). New data for the remaining outliers \( ^{36}\text{Ar}, ^{40}\text{Ar} \) (and \( ^{38}\text{Ar} \) where no data are available) and \( ^{33}\text{S} \) would be helpful to confirm or reject the irregular behavior of these nuclei.

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