Evolution of pomeran and odderon at all angular momenta

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Abstract

In the QCD the small $x$ evolution of the interacting pomerons and odderons is studied with all angular momenta $l$ taken into account. The resulting system of coupled nonlinear evolution equations is formulated in the momentum space and solved numerically. Excellent convergence in $l$ is observed. Also it is found that states with $l > 1$ play an important role and substantially reduce the basic pomeron state at large rapidities.

1 Introduction

Since long ago one of the main features of the strong interaction has been the dominance at high energies of the $C$ even exchange over the $C$ odd one ("the Pomeranchuk theorem"). In the Regge language the large energy asymptotic of the $C = +1$ amplitude is due to pomeron exchanges and that of the $C = -1$ amplitude by the odderon exchanges, both pomerons and odderon corresponding to the leading singularities of the relevant amplitudes in the complex angular momentum plane.

Whereas the behavior of the $C = +1$ transitions is more or less confirmed by experiments, which show the growing cross-sections, the $C = -1$ behavior has been somewhat elusive up to this date. Since the pioneering work [1] this behavior has been attributed to the odderon (whether a pole or not in the complex angular momentum plane) with the intercept close or exactly equal to one.

However the experimental evidence of its existence remains inconclusive in spite of many assertions [2, 3, 4, 5, 6]. Remarkably on the theoretical level the existence of the odderon has been well predicted within the QCD paradigm. In this picture the odderon appears as an object made of three reggeized gluons ("reggeons") in the $d$-color state, as opposed to the pomeron made of two reggeons in the colorless state. Moreover within the Regge kinematics (fixed $t$, $s/t \to \infty$) and in the leading approximation in $\alpha_s \ln s$ both the pomeron and odderon leading intercepts has been found to be $1 + \Delta$ [7] and exactly unity [8]. Here $\Delta > 1$ is the well-known BFKL intercept, which predicted the growth of cross-sections in the strong interaction at high energies.
This picture hints to some reasons for the weakness of the odderon exchange. The flatness of the corresponding cross-sections as compared to the rising ones for the pomeron exchange already make its observation very difficult. Also an extra power of $\alpha_s$ related to its three reggeon components instead of two in the pomeron presumably make its coupling to the hadrons weaker.

Many theoretical estimates of the cross sections for various odderon mediated processes [9, 10, 11, 12, 13, 14, 15] confirm this weakness and predict small cross sections, below the sensitivity of current experiments. The only exception, for which some evidence of the odderon contribution was probably measured, is the elastic $pp$ and $p\bar{p}$ scattering at non-zero momentum transfer [2, 3, 4, 5, 6]. However the final conclusions from these experimental observations remain not too convincing up to now.

As is well known, in the QCD the perturbative small $x$ evolution equation of the pomeron amplitudes, taking into account non-linear unitary corrections was derived by Balitski [16] and Kovchegov [17]. In the diagrammatic language, the Balitski-Kovchegov (BK) equation resums BFKL pomeron fan diagrams in the large $N_c$ limit. The BK equation may be also obtained as the mean-field limit of the effective theory of small $x$ gluons in the hadron wave function (the Color Glass Condensate approach [18]).

As mentioned, in the QCD the odderon consists of three $t$-channel reggeons in the color singlet $d$-state. Generally these three gluons may occupy three different spatial points. The small $x$ evolution equation of this odderon (the BKP equation) was derived long time ago [19, 20]. Its leading intercept was found to lie below unity [21] meaning that $C$-odd cross-sections should decrease with energy. However later a new odderon solution was discovered [8] with two of the three reggeons located at the same spatial point This "degenerated" BLV odderon has its intercept equal to exactly unity, so that its contribution to the high-energy $C$-odd cross-sections is dominating. With the two reggeons fused into one the wave function effectively coincides with the pomeron wave function with the negative spatial symmetry. So its small-$x$ evolution is described by the equation analogous to the BK equation for the pomerons [22]. Under some approximations this equation has recently been solved [23] where fast decrease of the odderon amplitude at large rapidity has been found.

Long ago the theory predicted that the pomeron may split into two odderons [24]. Therefore in the course of small-$x$ evolution the pomeron fan diagrams may generate pairs of odderons, so that the pomeron and odderon evolutions are interrelated. The system of coupled non-linear equations involving both the $C$-even and $C$-odd amplitudes was derived in [25]. This system was studied in [26] under some important approximations: the translational invariance and the lowest angular momenta $l = 0$ for the pomeron and $l = 1$ for the odderon. The first approximation compelled to substitute the odderon contribution to the pomeron to its average over the angle.

In this study we retain the first approximation (translational invariance) but give up the second to study evolution at all angular momenta.

### 2 Formalism.

Let $N(x, y; \tau)$ be the pomeron density in the transverse position plane for the collision off a large nucleus and $O(x, z; \tau)$ be the similar density of the BLV odderon. Let also the rapidity be $\tau = \log(1/x)$. The $\tau$-evolution of the $C$-even amplitude $N(x, y; \tau)$ and $C$-odd amplitude $O(x, z; \tau)$ in
the leading order approximation in powers of $\alpha_s \tau$ is described by a system of equations [25]:

\begin{align}
\frac{\partial N(x, y; \tau)}{\partial \tau} &= \frac{\bar{\alpha}_s}{2\pi} \int d^2z \frac{(x - y)^2}{(x - z)^2(z - y)^2} [N(x, z; \tau) + N(z, y; \tau) - N(x, y; \tau)] - N(x, z; \tau)N(z, y; \tau) + O(x, z; \tau)O(z, y; \tau), \\
\frac{\partial O(x, y; \tau)}{\partial \tau} &= \frac{\bar{\alpha}_s}{2\pi} \int d^2z \frac{(x - y)^2}{(x - z)^2(z - y)^2} [O(x, z; \tau) + O(z, y; \tau) - O(x, y; \tau)] - O(x, z; \tau)N(z, y; \tau) - N(x, z; \tau)O(z, y; \tau),
\end{align}

(1)

In fact $x$, $y$ and $z$ represent positions of the end points in the transverse plane of color dipoles, which interact with a large target. The pomeron and the odderon amplitudes have definite parities with respect to exchange of the gluon positions, that is:

\[ N(y, x; \tau) = N(x, y; \tau), \quad O(y, x; \tau) = -O(x, y; \tau). \]

(3)

If we separate the central-of-mass (c.m) coordinate $b = (x + y)/2$ the amplitudes become $N(b, y - x; \tau)$ and $O(b, y - x; \tau)$. In the large nucleus at rest the individual nucleons interact with a very small transverse momentum transfer, of the order of $1/R_A$ where $R_A$ is the nuclear radius. So all transverse momentum transfers along the pomeron fan diagram result to be of the same small order and can be taken as zero. In this case the $C$-even amplitude $N$ can be taken in the forward direction, which means that the impact parameter $b$ is not changed in the evolution and enters only as an external parameter. In particular at the start of the evolution with a symmetric nuclear target one can take $N(b, y - x, \tau = 0) = N_b((y - x)^2, \tau = 0)$. Of course this form satisfies condition [3].

Inclusion of the odderon radically changes the situation. The requirement of antisymmetry in $x$ and $y$ implies that the amplitude has to depend on $(b, x - y)$ and be antisymmetric in this argument. Integration over $b$ then gives zero, which means that the amplitude vanishes at zero momentum transfer. In principle this implies that one has to consider the amplitude at finite momentum transfers. Apart from difficulties for application to collisions with a large nucleus this leads to the necessity to study evolution equations in the whole space of two independent variables $x$ and $y$, both changing in the course of evolution. Having mostly in mind to study the influence of the odderon on the evolution we shall try to simplify the problem following the idea of [26].

Consider the first term on the right-hand side of Eq. [1] in variables with the extracted c.m. coordinate. It has the form

\[
\frac{\bar{\alpha}_s}{2\pi} \int d^2z \frac{(x - y)^2}{(x - z)^2(z - y)^2} N(b', x - z; \tau)
\]

where $b' = (x + z)/2$ is the evolved c.m. coordinate. Using $x = b + (x - y)/2$ we have $b' - b = (x + z)/2 - (x + y)/2 = (z - y)/2$. So in the course of evolution the impact parameter changes by the order of the average dipole dimension. If we take the impact parameter very large as compared to the average dipole dimension then one can neglect this change so that $b$ also becomes a fixed external parameter for the evolution. However in this case it is a fixed vector parameter with not only the magnitude of $b$ but also its direction as evolution parameter. Then both $N$ and $O$ will depend on the vector $x - y$ in the presence of the external direction given by the fixed $b$: Thus, we assume

\[ N(x, y; \tau) = N_b(y - x, \tau), \quad O(x, y; \tau) = O_b(y - x, \tau). \]

(4)
having in mind to find the influence of the odderon in Eqs. (1) and (2) and leaving aside its relevance for the actual physical case of the collision with a large nucleus.

To simplify the non-linear terms we pass to the momentum space, and define the momentum dependent functions $\Phi(k, \tau)$ and $\Psi(k, \tau)$ describing the pomeron and the odderon dipole densities respectively

$$\Phi(k, \tau) = \int \frac{d^2r}{2\pi r^2} N(r, \tau) \exp(-ikr), \quad \Psi(k, \tau) = \int \frac{d^2r}{2\pi r^2} O(r, \tau) \exp(-ikr).$$

(5)

From Eqs. (1) and (2) with the amplitudes having the forms (4) one obtains a system of equations

$$\frac{\partial \Phi(k, \tau)}{\partial \tau} = \tilde{\alpha}_s (K \otimes \Phi)(k, \tau) - \tilde{\alpha}_s \Phi^2(k, \tau) + \tilde{\alpha}_s \Psi^2(k, \tau),$$

(6)

$$\frac{\partial \Psi(k, \tau)}{\partial \tau} = \tilde{\alpha}_s (K \otimes \Psi)(k, \tau) - 2\tilde{\alpha}_s \Phi(k, \tau) \Psi(k, \tau),$$

(7)

where the linear terms describe the standard BFKL evolution in the forward direction

$$(K \otimes \Phi)(k, \tau) = \frac{1}{\pi} \int \frac{d^2k'}{(k - k')^2} \left[ \Phi(k', \tau) - \frac{k^2 \Phi(k', \tau)}{k^2 + (k - k')^2} \right]$$

(8)

and similar for $\Psi$.

In the presence of the external direction we develop both $\Phi$ and $\Psi$ in angular momenta $l$

$$\Phi(k, \tau) = \sum_{l, even} \Phi_l(k, \tau) e^{il\varphi}, \quad \Psi(k, \tau) = \sum_{l, odd} \Psi_l(k, \tau) e^{il\varphi},$$

(9)

Here $\varphi$ is the angle between $k$ and the fixed direction in the transverse plane. The angular momentum $l$ goes from $\infty$ to $+\infty$ but the parity condition requires even angular momenta for $\Phi$ and odd ones for $\Psi$.

We obtain the following system of coupled equations for $\Phi_l$ and $\Psi_l$

$$\frac{\partial \Phi_l(k, \tau)}{\partial w} = \int_0^\infty dk'^2 \left\{ \Phi_l(k', \tau) \left( \frac{k'^2}{k^2} \right)^{|l|} \frac{1}{k'^2 - k^2} - \Phi_l(k, \tau) \frac{k^2}{k'^2} \left( \frac{1}{k'^2 - k^2} - \frac{1}{\sqrt{4k'^4 + k^4}} \right) \right\}$$

$$- \sum_m \Phi_m(k, \tau) \Phi_{l-m}(k, \tau) + \tilde{\alpha}_s \sum_m \Psi_m(k, \tau) \Psi_{l-m}(k, \tau)$$

(10)

with $l$ even and

$$\frac{\partial \Psi_l(k, \tau)}{\partial w} = \int_0^\infty dk'^2 \int_0^\infty dk'^2 \left\{ \Psi_l(k', \tau) \left( \frac{k'^2}{k^2} \right)^{|l|} \frac{1}{k'^2 - k^2} - \Psi_l(k, \tau) \frac{k^2}{k'^2} \left( \frac{1}{k'^2 - k^2} - \frac{1}{\sqrt{4k'^4 + k^4}} \right) \right\}$$

$$- 2 \sum_m \Phi_m(k, \tau) \Psi_{l-m}(k, \tau)$$

(11)

with $l$ odd. Here $k_\leq = \min(k, k')$ and $k_\geq = \max(k, k')$. We also introduce the rescaled rapidity $w = \tilde{\alpha}_s \tau$. Of course $\Phi_n = 0$ for $l$ odd and $\Psi_n = 0$ for $n$ even. Also $\Phi_l = \Phi_{-l}$ and $\Psi_l = \Psi_{-l}$. So one can rewrite

$$\sum_m \Phi_m \Phi_{l-m} = \Phi_0 \Phi_{|l|} + \sum_{m=1}^\infty \Phi_m (\Phi_{|l-m|} + \Phi_{|l+m|}),$$

(12)

$$\sum_m \Psi_m \Psi_{l-m} = \Psi_0 \Psi_{|l|} + \sum_{m=1}^\infty \Psi_m (\Psi_{|l-m|} + \Psi_{|l+m|})$$

(13)
and
\[ \sum_m \Phi_m \Psi_{l-m} = \Phi_0 \Psi_{|l|} + \sum_{m=1}^{\infty} \Phi_m (\Psi_{|l-m|} + \Psi_{|l+m|}) = \Psi_0 \Phi_{|l|} + \sum_{m=1}^{\infty} \Psi_m (\Phi_{|l-m|} + \Phi_{|l+m|}). \] (14)

These equations form the basis of our numerical calculations.

Since \( \Phi_l = 0 \) for \( l \) odd and \( \Psi_l = 0 \) for \( l \) even it is convenient to introduce
\[ \phi_l \equiv \Phi_2^l, \quad \psi_l \equiv \Psi_2^{l+1}. \] (15)

Then both \( \phi_l \) and \( \psi_l \) are different from zero for all \( l = 0, 1, \ldots \). In terms of \( \phi_l \) and \( \psi_l \)
\[ \Phi = \phi_0 + 2 \sum_{l=1}^{\infty} \phi_l \cos 2l \phi, \quad \Psi = 2 \sum_{l=0}^{\infty} \psi_l \cos(2l + 1) \phi. \] (16)

In the following we denote the angular momentum of the pomeron as \( L = 2l \) and of the odderon as \( L = 2l + 1 \).

The sums in our equations are then transformed as follows
\[ C_{l}^{PP} = \sum_{m=-\infty, even}^{+\infty} \Phi_m \Psi_{2l-m} = \phi_0 \phi_l + \sum_{m=1}^{\infty} \phi_m (\phi_{|l-m|} + \phi_{|l+m|}), \] (17)
\[ C_{l}^{OO} = \sum_{m=-\infty, odd}^{+\infty} \Psi_m \Psi_{2l-m} = \sum_{m=0}^{\infty} \psi_m (\psi_{m'} + \psi_{l+m}) \] (18)
where \( m' = (|2l - 2m - 1| - 1)/2 \). Finally
\[ C_{l}^{PO} = \sum_{m=-\infty, odd}^{+\infty} \Psi_m \Phi_{2l+1-m} = \sum_{m=0}^{\infty} \psi_m (\phi_{|l-m|} + \phi_{l+m+1}). \] (19)

We recall that \( \phi_0 \) is directly related to the non-integrated gluon density in the nucleus. In our normalization (see [27])
\[ \frac{\partial xG(x, k^2)}{\partial k^2} = \frac{N_c^2}{2\pi^3 \bar{\alpha}} k^2 \phi_0 \left( \ln \frac{1}{x}, k^2 \right). \] (20)

3 Calculations

3.1 Passing to a grid in \( \ln k^2 \)

We pass to a logarithmic variable \( t = \ln k^2 = \ln q = k^2 \) (the units in which \( q \) is measured is inferred from the initial functions for evolution). In variable \( t \) the non-integrated gluon density is
\[ \frac{\partial xG(x, k^2)}{\partial k^2} = \frac{N_c^2}{2\pi^3 \bar{\alpha}} \partial^2 \phi_0 (w, t) \] (21)
where \( w = \bar{\alpha}_s \ln \frac{1}{x} \).

We introduce a grid in \( t \).
\[ t_i = t_{\text{min}} + id, \quad i = 0, 1, \ldots, n, \quad d = \frac{t_{\text{max}} - t_{\text{min}}}{n}. \] (22)

At the grid points \( q_i = \exp(t_i) \) and the fields are \( \phi_{li} = \phi_l(q_i) \) and \( \psi_{li} = \psi_l(q_i) \). We approximate the integrals over \( t \) by finite sums
\[ \int_{-\infty}^{\infty} dt F(t) \approx \sum_{i=1}^{n} w_i F(t_i) \] (23)
with points $t_i$ and weights $w_i$ depending on the chosen approximation scheme.

Then Eqs. (10) and (11) pass into the system of finite matrix equations for evolution in the rescaled rapidity $w$

$$\frac{d\phi_i}{dw} = \sum_{j \neq i} q_j \left( B_{ij}^P \phi_j - A_{ij} \phi_i \right) + \frac{\phi_i}{\sqrt{5}} - C_{ii}^{PP} + C_{ii}^{OO}$$  \hspace{1cm} (24)$$

and

$$\frac{d\psi_i}{dw} = \sum_{j \neq i} q_j \left( B_{ij}^O \psi_j - A_{ij} \psi_i \right) + \frac{\psi_i}{\sqrt{5}} - 2C_{ii}^{PO}.$$  \hspace{1cm} (25)

In these equations

$$B_{ij}^P = \left( \frac{q_i}{q} \right)^{2i} \frac{1}{|q_i - q_j|}, \quad B_{ij}^O = \left( \frac{q_i}{q} \right)^{2i+1} \frac{1}{|q_i - q_j|}, \quad A_{ij} = \frac{q_i}{q_j} \frac{1}{\sqrt{q_i^2 + 4q_j^2}}$$  \hspace{1cm} (26)

and terms $C^{PP}$, $C^{OO}$ and $C^{PO}$ are the nonlinear terms (17), (18) and (19).

### 3.2 Initial conditions

We assume that the target initially interacts with the pomeron and odderon only at the lowest orbital momenta $L = 0$ for the pomeron and $L = 1$ for the odderon. Higher orbital momenta appear only as a result of evolution. The pomeron function is related to the forward scattering of a dipole on a large nucleus and its initial function can be taken in the standard manner, as in a numerous previous calculations. A popular choice of this initial function follows the form proposed in [29]. In the coordinate space

$$N(\tau = 0, x) = 1 - \exp \left( -\frac{1}{4} Q_A^2 x^2 \right)$$  \hspace{1cm} (27)

where $Q_A^2 = cA^{1/3}Q_1^2$ and $Q_1$ and $c$ are constants determined by the data. From [28] we have $Q_1^2 = 0.24 \text{ GeV}^2$ and $c \simeq 0.25$.

Fourier transformation to the momentum space gives

$$\phi_0(\tau = 0, q) = -\frac{1}{2} \text{Ei} \left( -\frac{q}{Q_A} \right).$$  \hspace{1cm} (28)

To see this one can use the Fourier transform

$$\int \frac{d^2k}{2\pi} \text{Ei}(-k^2)e^{ikr} = \int_0^\infty kdk \text{Ei}(-k^2)J_0(kr)$$  \hspace{1cm} (29)

and the standard formula (2.12.47.8) from [30] at $n = 0$

$$\int_0^\infty kdk \text{Ei}(-bk^2)J_0(ck) = \frac{2}{e^c} \left[ 1 - \exp \left( -\frac{c^2}{4b} \right) \right].$$  \hspace{1cm} (30)

The inverse Fourier transform gives [28].

As to the odderon initial function, in absence of the underlying clear physical picture and motivated mostly by our desire to study the influence of the inclusion of the odderon in the evolution, we take its initial function $\psi_1(\tau = 0, q)$ in the same form (27) with a scaling factor $g_O$ which may take into account a possible weakness of the odderon coupling. In fact in our numerical calculations we take $g_O = 1$ to investigate qualitatively the odderon influence with a coupling of a similar strength. Diminishing of $g_O$ will inevitably make this influence weaker.
Figure 1: The Pomeron $\phi_0$ decoupled from the odderon (left panel) and the corresponding gluon density (right panel). Curves from top to bottom in the left panel and from left to right in the right panel correspond to $w = 0, 1, 3, 5$ and 7.

4 Numerical results

We present our numerical results for $\phi$ and $\psi$ at different scaled rapidities $w = \bar{\alpha}_s x$ rising from 0 to 7. With $\alpha_s \sim 0.2$ this corresponds to natural rapidities up to $\sim 35$. As the momentum variable we choose a dimensionless variable

$$x = \frac{t - t_{\min}}{t_{\max} - t_{\min}} = \frac{\ln(q/q_{\text{max}})}{\ln(q_{\text{max}}/q_{\text{min}})}, \quad 0 < x < 1$$

with $q_{\text{min,max}} = Q_A \exp(t_{\text{min,max}}/2)$. In our calculations we chose $t_{\min} = -20$ and $t_{\max} = 60$, and $n = 800$, which proved to be values sufficient for a reasonable precision ($\leq 0.0001$) at $t > -12$, that is at $Q^2 > 1.5e - 6$ GeV$^2$. With these $t_{\min}$ and $t_{\max}$ values $x = 0$ and $x = 1$ correspond to 4.95e-10 GeV$^2$ and 2.74e + 25 GeV$^2$ respectively. The typical momentum squared 10 (Gev)$^2$ corresponds to $x \approx 0.3$. For some cases apart from $\phi$ and $\psi$ we present the rescaled non-integrated gluon density

$$g(w, k^2) = k^2 \nabla_k k^2 \phi(w, k) = \partial_t^2 \phi(w, t)$$

and a similar function for $\psi$ (although the physical interpretation of the latter is somewhat obscure).

4.1 The Pomeron

For comparison we start with the well studied case of the pure pomeron evolution without coupling to the odderon. This corresponds to non-linear terms $C^{OO} = C^{PO} = 0$. Since we assume that only $\phi_0$ is initially coupled with the target, all $\phi_l$ with $l > 0$ remain zero after evolution and $\phi_0$ evolves according to the standard BK equation. The values of $\phi_0(w, x)$ and $g_0(w, x)$ for this case following from our calculations are shown in Fig. 1 for scaled rapidities $w = 0, 1, 3, 5, 7$. (with $\bar{\alpha} = 0.2$ this corresponds to natural rapidities 0, 5, 15, 25 and 35).

Now we take into account the coupling of the pomeron and odderon on the minimal level, introducing all nonlinear terms different from zero but restricting the partial waves for both $\phi$ and
ψ to \( l = 0 \) (that is taking the pomeron at \( L = 0 \) and odderon at \( L = 1 \)) Our results are shown in Fig. 2. As one observes the change due to coupling with the odderon is barely visible.

At the next step we widen the set of partial waves to include values \( l = 0 \) and \( l = 1 \), This implies taking into account also the pomeron with \( L = 2 \) and odderon with \( L = 3 \). Calculation give the results shown in Fig. 3. As we observe inclusion of higher partial wave has a quite large influence on the evolution of the normal pomeron with \( L = 0 \). Already at \( w = 1 \) both its amplitude and gluon density become more than twice reduced, although the general behavior with the growth of rapidity remains the same.

Remarkably inclusion of more partial waves does not change the pomeron \( \phi_0 \). This is illustrated in Figs. 4 and 5 where we show \( \phi_0 \) and \( g_0 \) at \( w = 3 \) and \( w = 7 \) respectively for different sets of included waves: \((0),(0,1),(0-2)\) and \((0-3)\) together with the uncoupled case.

Inclusion of higher partial waves leads to appearance of amplitudes \( \phi_{1,2,3} \) which correspond to pomerons with \( L = 2,4,6 \), which result from the evolution in the presence of the external direction, although they are zero initially. These amplitudes are small and rapidly diminish with the rapidity in accordance with their behavior under the BFKL evolution. Starting from \( w = 3 \) they are all practically equal to zero. They are also practically independent from the inclusion of higher partial waves. So we illustrate them only at the earlier part of the evolution at \( w = 1 \) and \( w = 2 \) and for the minimal sets of partial waves. In Fig. 6 we show \( \phi_1 \) at \( w = 1 \) with included waves 0,1 and the corresponding "gluon density", that is its double derivative in \( t \). As one observes \( \partial_t^2 \phi(t) \) is not positive and can hardly be interpreted as "density".

In the next figure we illustrate \( \phi_1 \) at \( w = 2 \) and \( \phi_2 \) at \( w = 1 \) For \( \phi_1 \) waves with \( l = 0,1 \) are included, for \( \phi_2 \) waves from \( l = 0 \) to \( l = 2 \) are included.

4.2 The odderon

Again we start with the situation when the odderon and pomerons are decoupled. We assume that it is the "normal" odderon with \( L = 1 \), corresponding to \( \psi_0 \), which interacts with the target.
Figure 3: The Pomeron $\phi_0$ coupled to the pomeron $\phi_1$ and odderons $\psi_{0,1}$ (left panel) and the corresponding gluon density (right panel). Curves from top to bottom in the left panel and from left to right in the right panel correspond to $w = 0, 1, 3, 5$ and 7.

Figure 4: The Pomeron $\phi_0$ (left panel) and the corresponding gluon density (right panel) at $w = 3$ and for different sets of included waves. The upper curves in both panel correspond to uncoupling and waves restricted to $l = 0$ The lower curves corresponds to sets of waves $(0,1), (0,2)$ and $(0,3)$.
Figure 5: The Pomeron $\phi_0$ (left panel) and the corresponding gluon density (right panel) at $w = 7$ and for different sets of included waves. The upper curves in both panel correspond to uncoupling and waves restricted to $l = 0$ The lower curves corresponds to sets of waves $(0,1), (0-2)$ and $(0-3)$

Figure 6: The Pomeron $\phi_1$ (left panel) and the corresponding ”gluon density” (right panel) at $w = 1$ with included waves 0,1.
Then all odderons with higher angular momenta will be zero and $\psi_0$ will evolve according to the BFKL Hamiltonian. In this case our calculations give the results shown in Fig. 8 where apart from $\psi_0$ we illustrate also the corresponding "gluon density" $f_0 = \partial^2_t \psi_0$ (as we mentioned its physical interpretation is rather obscure). As for the pomeron we show both $\psi_0$ and $f_0$ as functions of $x$ for rising rapidities $w = 0, 1, 3, 5$ and 7. Since the odderon functions go to zero with the growth of momentum very fast we present these and the following results in the logarithmic scale.

Coupling to the pomeron leads to strong reduction of odderon amplitudes. It is particularly strong if only waves 0 and 1 are taken into account. If the waves include $l = 0 - 3$ the reduction is weaker but still persists. This is illustrated in Fig. 9 in which $\psi_1$ is shown for both sets of waves.

As with the pomeron, further widening of the set of partial waves does not change the odderon function $\psi_1$. This is illustrated in Fig. 10 where we plot $\Psi_1$ at $w = 3$ and $w = 7$ for different sets of partial waves.

As to "gluon density" $f = \partial^2_t \psi$ it turns out to be very small, goes to zero very fast with rapidity.
Figure 9: The Odderon $\psi_0$ with waves $l = 0, 1$ included (left panel) waves $l = 0 - 3$ included (right panel). Curves from top to bottom in both panels correspond to $w = 0, 1, 3, 5$ and 7.

Figure 10: The Odderon $\psi_0$ at $w = 3$ (left panel) and $w = 7$ (right panel) with different sets of partial waves included.
Figure 11: The "gluon density" $f_0 = \partial_t^2 \psi$ at $w = 1$ (left panel) and $w = 0, 1, 3, 5$ (right panel, from top down) with partial waves included $l = 0, 1$.

Figure 12: The odderons with $L = 3$ (left panel) and $L = 5$ (right panel) at $w = 1, 3, 5, 7$ from top down

and changes sign. In Fig. 11 we show it in the logarithmic scale at all rapidities and also in the normal scale at $w = 1$. Use of the logarithmic scale leads to breaks in the curves at intervals where $f < 0$

We finally come to the odderons with higher $l$. They go down quickly with the growth of $l$ and their behavior with $x$ and rapidity is similar to the odderon with $L = 1$. In Fig. 12 we show the odderons with $L = 3$ and $L = 5$.

The corresponding "gluon densities" $f_1$ and $f_2$ at $w = 1$ are presented in Fig. 13. At larger $w$ all $f_l$ with $l \geq 1$ are extremely small.

5 Discussion

We have studied the system of coupled evolution equations for the pomeron and odderon, derived in \cite{22,25} in the translationally invariant approximation proposed in \cite{26} taking in account the
full angular dependence. Our numerical calculations on the whole confirm qualitative predictions made in these references about the strong damping of the odderon and its fast diminishing with the growing rapidity as a result of its interaction with the pomeron. In our calculations we discovered that the inverse influence on the pomeron of the interaction with the odderon also damps both the amplitude and the gluon density. as soon as one goes beyond the basic pomeron and odderon states with $L = 0$ and $l = 1$. With only the basic odderon state $L = 1$ the pomeron practically does not change. But inclusion of higher states, starting from the pomeron at $L = 2$ and odderon at $l = 3$ substantially reduces the basic pomeron state at $l = 0$ while preserving its qualitative dependence on rapidity and momentum. This reduction does not practically change with the the number of states with $L > 1$ included. This may be considered as the main and somewhat unexpected result of our calculations with possible physical consequences.

In physical applications for the collision of azimuthal symmetric projectile with the target nucleus the angular dependence is obviously average out, so that all partial waves go to zero expect at $L = 0$. So the only surviving state is precisely the basic pomeron with $l = 0$. Without the odderon it evolves according to the BK equation. Our calculations show that as soon as one takes into account states with $L > 1$, which appear in the course of the evolution, both pomeron amplitude and the corresponding gluon density turn out to be more than twice reduced. Of course this prediction has been made under the assumption that the interaction of the odderon with the nucleon is of the same magnitude as that of the pomeron. In perturbation theory the odderon interaction carries one extra $\alpha_s$ and so is significantly smaller. However this interaction is in fact non-perturbative and its magnitude is unknown apriori. One may hope that experimental observations may shed light on this question.

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Pomeron $l=0$ at $w=7$
Pomeron, $l=0$, $w=7$

Different sets of angular waves

$\phi(x)$

uncoupled to odderon and $l=0,1$

$l=0-3, 0-5, 0-7$