Modulated and Rotating Turbulent Plane Couette Flow: Direct numerical simulations

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Direct numerical simulations of turbulent Rotating Plane Couette Flow (RPCF) with a periodically modulated plate velocity are conducted to study the effect of modulated forcing on turbulent shear flows. The time averaged shear Reynolds number is fixed to \( Re_S = 3 \cdot 10^4 \), which results in a frictional Reynolds number of approximately \( Re_\tau \approx 400 \). The modulating frequency is varied in the range \( Wo \in (20, 200) \), while the modulating amplitude is kept fixed at 10% of the shear velocity except to demonstrate that varying this parameter changes little. The resulting shear at the plates are found to be independent of the forcing frequency, and equal to the non-modulated baseline. For the non-rotating simulations, two clear flow regions can be seen: a near wall region that follows Stokes’ theoretical solution, and a bulk region that behaves similar to Stokes’ solutions but with an increased effective viscosity. For high driving frequencies, the amplitude response follows the scaling laws for modulated turbulence of von der Heydt et al. (Physical Review E 67, 046308 (2003)). Cyclonic rotation is not found to modify the system’s behaviour in a substantial way, but anti-cyclonic rotation significantly changes the system’s response to periodic forcing. We find that the persistent axial inhomogeneities introduced by mild anti-cyclonic rotation make it impossible to measure the propagation of the modulation adequately, while stronger anti-cyclonic rotation creates regions where the modulation travels instantaneously.

Key words: xxx

1. Introduction

Turbulent flows subjected to periodic modulation appear at multiple scales and in many disparate contexts: pulsatile blood flow through arteries (Ku 1997), the flow of fuel, air, and other combustion products in internal combustion engines (Shelkin 1947; Dent & Salama 1975; Baumann et al. 2014), and tidal currents and weather patterns in geophysical flows (Bouchet & Venaille 2012; Jackson 1976; Turner 1986). A common feature in all such flows is that the turbulence field adjusts to the the degree of modulation, so while ordinary turbulence is often thought to have a continuum of relevant, fluctuating timescales, there is evidence that at high modulation frequencies, a dominant scale emerges that is correlated to the forcing frequency (von der Heydt et al. 2003a,b; Kuczaj et al. 2006; Bos et al. 2007; Kuczaj et al. 2008). This effect can lead to phenomena such as resonances or couplings between the forcing and the existing turbulent structures that results in heavily amplified energy injection and dissipation (Cekli et al. 2010, 2015).

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Experimental studies of modulated turbulence have generally been performed in wind tunnels through the use of static grids to inject energy into a flow by air streams (Comte-Bellot & Corrsin 1966) or through “active” grids that use a grid of rods articulated by servo motors (Makita 1991). This has allowed researchers to tune the turbulence’s properties and to study the details of the dissipation rates and other features of turbulence (Mydlarski & Warhaft 1996; Poorte & Biesheuvel 2002). Among other things, these studies have found that the largest energy input was reached when the time scale of the active grid forcing matched that of the largest eddies of the wind-tunnel turbulence (Cekli et al. 2010, 2015). On the other hand, studies of modulated turbulence through direct numerical simulation (DNS) have been more scarce, as they require resolving all the length- and time-scales of a fully developed turbulent flow, as well as running the simulation for a sufficiently long time to capture reliable statistics. This results in high computational costs, limiting such runs to only a few studies as listed in Yu & Girimaji (2006) and Kuczaj et al. (2006, 2008), which simulate randomly forced turbulence; in consonance with experiments, these groups have found resonance enhancement of mean turbulence dissipation when the forcing and flow scales match.

Compared to turbulence generated by wind-tunnels or random numerical forcing, modulated wall-bounded flows and boundary layers have received much less attention. Modulated wall-bounded flows, which consist of oscillatory flows superimposed on nearly steady currents are ubiquitous. In the laminar regime, the oscillatory solution can often be superimposed to the steady solution through the use of linearity. Even if this eliminates the possibility of resonances, it is often possible to exactly solve the Navier-Stokes equations to determine the phase and amplitude of the oscillation, and obtain exact results for both the steady and the oscillatory components. This provides some insights on the physics, such as the time-scales in which modulation travels through a wall-bounded flow. The two canonical examples of this are Stokes’ second problem, i.e. the flow in a semi-infinite domain driven by an oscillating wall, and pulsatile pipe flow, also known as Womersley flow (Womersley 1955), i.e. the flow in a pipe driven by an oscillatory pressure gradient. Both of these problems have exact solutions often available in textbooks (Landau & Lifshitz 1987), which reveal the relevant non-dimensional groups for analyzing modulated flows such as the Womersley number (which we will use below). It is also worth noting that both problems differ in the way the oscillatory component is imposed: Womersley pipe flow is driven by an oscillatory pressure gradient, which drives the bulk flow pulsations that perturb the pipe boundary layer, while in Stokes’ second problem, the flow is driven by oscillating walls and hence the modulation of momentum is transported from the boundary layer towards the bulk.

The superposition principle no longer holds up in a turbulent flow. Therefore, a full simulation or experimental study is required to study the interaction of modulation with a constant flow, which may include the forementioned resonant interactions. Moreover, wall-bounded flows could present a better case to study modulation resonance as the largest scales in the flow are naturally set by boundaries. Our interest here is in flows which are driven from the boundary, which has seen less attention than modulation introduced through pressure driving (Scotti & Piomelli 2001; Zamir & Budwig 2002; Ku & Giddens 1983; Ling & Atabek 1972). We focus on the Plane Couette Flow (PCF) problem, the flow between two plates with a differential velocity. PCF is similar to the Stokes’ second problem with an additional wall that closes the system, and is ideal for studying the way a perturbation is transmitted from the wall through the boundary layer in a confined geometry.

As PCF can be hard to construct experimentally, the two plates are often substituted by two cylinders, resulting in cylindrical Couette Flow, also known as Taylor-Couette Flow.
flow (TCF). It is worth noting that for certain low values of the Reynolds numbers, TCF produces a modulated response even when the driving is steady (Barenghi & Jones 1989). We must distinguish this case, usually denoted as the modulated wavy Taylor vortex regime, from the case that interests us, i.e. fully turbulent TCF with modulated forcing. Fully turbulent TCF with modulated forcing was recently studied by Verschoof et al. (2018), who found that the system response follows the forcing signal well for lower frequencies but falls out of phase at higher frequencies. However, they did not identify a proper time scale where the behaviour of the flow transitions from the low frequency regime to the high frequency regime. They also held the amplitude of modulation constant, and could not measure torques due to the nature of their setup. Furthermore, the effects of solid-body rotation on the response were not investigated, as the study was limited to pure inner cylinder rotation. A proper treatment of this parameter is critical, as solid body rotation is responsible for the presence or absence of certain types of large-scale structures in the turbulent regime of rotating PCF and TCF (Sacco et al. 2020). From the previous discussion we expect the role of large-scale structures and their interaction with the modulation to be of paramount importance in determining the response of the system.

Another flow where large-scale dynamics heavily impact the system response is Rayleigh-Bénard convection (RBC), the flow in a fluid layer heated from below and cooled from above. RBC has been shown to be in close analog to TCF (Busse 2012), with the analog to the angular momentum transport between cylinders being the heat transfer between plates. Modulated RBC convection has been studied experimentally in Jin & Xia (2008), who found no increase in the mean heat transfer at the plates as if a sinusoidal modulation of the bottom temperature was applied. However, if the modulation was introduced through pulses or “kicks”, a maximum heat transport enhancement of 7% could be achieved when the pulse was synchronized to the existing energy scales, showing resonant enhancement in this system. Jin & Xia (2008) rationalized this as “spikier” pulses being better for heat transfer enhancement than “flatter” ones. Jin & Xia (2008) also found that amplitude of the fluctuations in the heat transfer and temperature were found to depended on both amplitude and frequency of the modulation in the case of pulsatile modulation. Yang et al. (2020) extend these results through simulations, finding a modification of the mean heat transfer of a maximum of 25% in two- and three-dimensional RBC when the boundary temperature was modulated at frequencies close to the frequencies of the existing flow structures. The main difference between both cases is the amplitude of the modulation: the perturbations in the first study were much smaller than those in the second study, which were of equal size to the fixed temperature.

The RBC results give us a guideline to what we can expect when large-scale structures are present. To restrict the scope of this work, we have decided to only study sinusoidal modulations which can be directly benchmarked against Verschoof et al. (2018). Hence, we will use Rotating Plane Couette Flow (RPCF) to study the effect of flow modulation induced by a sinusoidally oscillating wall at different frequencies and for different rotation ratios in wall turbulence. We will also consider solid body rotation ratios that are both anticyclonic and cyclonic, which will allow us to better compare against the experiment and against RBC; thus we can study the interaction of large-scale structures with modulation, as well as what happens when these structures are not present. We will analyze how the flow responds to different modulation frequencies by looking at torque and velocity statistics. We will also study the effect of the modulating amplitude on the flow behaviour. The larger quantity and in-depth examination of available statistics will extend the findings of Verschoof et al. (2018), allowing us to include dissipation and spectral analysis data that was previously not available.
The paper is organized as follows: in §2 we describe the numerical setup (mathematical formulation, non-dimensional parameters, domain size, resolution study). In §3, we detail the results obtained for the non-rotating case, while in §4 we add rotation and highlight the different features that arise. A brief summary and conclusions is provided in §5, which includes an outlook for future investigations.

2. Numerical setup

To simulate RPCF we use the three-dimensional Cartesian domain shown in Fig. 1. The top and bottom plates have length $L_x$ (streamwise) and $L_z$ (spanwise) and are separated by gap width $d$. Periodic conditions are imposed at the $x$ and $z$ domain boundaries. The top and bottom plates are prescribed to have opposite streamwise velocities $\pm U/2$. In addition to this, a modulation is superimposed onto the bottom plate’s velocity with a perturbation frequency $\omega = 2\pi/T$ and magnitude $A_0$. Solid body rotation in the $z$-direction is added through a Coriolis force. This represents the differential motion of the cylinders in a Taylor-Couette system as the curvature vanishes (Brauckmann et al. 2016). With this, the incompressible non-dimensional Navier-Stokes equations which govern this problem are:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + R\Omega (\mathbf{e}_z \times \mathbf{u}) = -\nabla p + Re_s^{-1} \nabla^2 \mathbf{u},$$

which alongside the incompressibility condition defines the flow field,

$$\nabla \cdot \mathbf{u} = 0.$$

Here $\mathbf{u}$ is the non-dimensional velocity, $\Omega$ is the background spanwise rotation, $p$ the pressure, $t$ is time and $\mathbf{e}_z$ the unit vector in the $z$ direction. Equations 2.1-2.2 have been non-dimensionalized using the gap-width $d$ and the plate velocity $U$, which gives two non-dimensional control parameters: a shear Reynolds number, $Re_s = Ud/\nu$ and the Coriolis parameter (sometimes known as the Rossby or Rotation number), $R\Omega = 2\Omega d/U$. Two more control parameters are provided by the modulated boundary condition: the non-dimensional modulation amplitude $\alpha = A_0/U$ and the non-dimensionalized modulation frequency, which is written as the Womersley number ($Wo$) defined as $d\sqrt{\Omega/\nu}$ following Verschoof et al. (2018).

Periodic aspect ratios of $L_x/d = 2\pi$ and $L_z/d = \pi$ are used. The Reynolds number
Table 1: Summary of Wormersley numbers used for all simulations, and their corresponding forcing periods $T = 1/(2\pi \omega)$ in $d/U$ and $d/u_\tau$ time units.

| $Wo$ | $TU/d$ | $Tu_\tau/d$ | $Tu_\tau/[d/2]$ |
|------|--------|-------------|-----------------|
| 26   | 266.4  | 7.35        | 14.7            |
| 44   | 96.1   | 2.65        | 5.30            |
| 77   | 31.8   | 0.88        | 1.76            |
| 114  | 14.4   | 0.40        | 0.79            |
| 200  | 4.71   | 0.13        | 0.26            |
| 300  | 2.09   | 0.055       | 0.11            |
| 400  | 1.18   | 0.032       | 0.065           |

$Re_s$ is fixed at $3 \times 10^4$, resulting in a frictional Reynolds number $Re_\tau \approx 770$ for the non-rotating case, where $u_\tau$ is the shear velocity defined as $u_\tau = \sqrt{\tau_w/\rho}$, where $\tau_w$ is the average wall shear stress and $\rho$ is the fluid density. The rotation parameter $R_\Omega$ is varied in the range $[-0.1, 0.3]$, with positive values denoting anti-cyclonic rotation, such that the spanwise rotation vector is anti-parallel to the vorticity of base flow, whereas negative values of the Coriolis signify cyclonic behavior, i.e. the spanwise rotation is parallel to the vorticity vector of the base flow. The perturbation amplitude $\alpha$ is kept constant at $\alpha = 0.1$ unless stated otherwise. The Wormersley number $Wo$ is varied in the range $Wo \in [26, 200]$, with selected cases at higher $Wo$. Table 1 shows how these values of $Wo$ correspond to the different time-scales in the flow.

The equations are discretized using finite differences: second-order accurate energy conserving in space, third-order accurate in time using Runge-Kutta for the explicit terms. The viscous term is discretized in the wall normal direction using a second-order Adams-Moulton scheme. The discretized equations are solved using the parallel FORTRAN based code, AFID (www.afid.eu). This code has been used in previous studies to study turbulent Rayleigh-Bénard convection and Taylor-Couette flow (Van Der Poel et al. 2015) and has been thoroughly validated. Details of the code algorithms are documented in Verzicco & Orlandi (1996) and Van Der Poel et al. (2015). Spatial resolution of the simulations are selected as $N_x \times N_y \times N_z = 512 \times 384 \times 512$ in the streamwise, wall-normal and spanwise directions, respectively. This is in accordance with the spatial resolution selected for $Re_s = 3.61 \times 10^4$ in the study of turbulent Taylor rolls in Sacco et al. (2020). A variable time-stepping scheme is defined such that the maximum CFL condition does not exceed 1.2. To exclude the start-up transients, the first few periods of the simulation are not included while evaluating turbulence statistics. The duration of the simulation to evaluate turbulence statistics is in the range of 5 to 10 periods for a given $Wo$ number, or one thousand simulation time units, whichever is larger. Temporal convergence is also checked by monitoring that the deviations of the average shear stress $\tau_y = \nu \partial_y \langle u_x \rangle_{t,z,x} + \langle u_x u_y \rangle_{t,z,x}$ do not exceed 1% from the average value, where $\langle \cdot \rangle_{t,z,x}$ denotes averaging with respect to time and to the streamwise and spanwise directions.
3. Results for non-rotating Plane Couette Flow

The first case we analyze is Plane Couette flow without rotation, i.e. $R_\Omega = 0$. Non-rotating Plane Couette flow contains large-scale structures that extend significantly in the streamwise direction (Tsukahara et al. 2006). However, these structures do not dominate the transport of momentum in the same way that the Taylor rolls present for $R_\Omega = 0.1$ do (Brauckmann et al. 2016; Sacco et al. 2020). Instead momentum is transferred through a hierarchy of eddies which spans many length- and time-scales (Townsend 1980), so in principle we do not expect that there are natural time-scales in the flow with which the modulation could couple to produce resonances.

We first look at the volumetrically averaged instantaneous dissipation $\varepsilon$, which is a quantity of interest in studies of modulated turbulence. We note that the temporal average of $\varepsilon$ is equal to the shear force at the plates (modulo scaling factors) due to the exact balances of energy: in the statistically stationary state on average the energy input through the walls must be balanced out by the viscous dissipation. Following Brauckmann et al. (2016) and Eckhardt et al. (2020), we show the viscous dissipation as a non-dimensional Nusselt number $\text{Nu}$, where $\text{Nu} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$ with $\text{Nu} = 1$ corresponding to the dissipation for the purely streamwise flow.

In the left panel of Figure 2, we show the instantaneous values of $\text{Nu}$ for two different values of $Wo$ (44, 200) and for the unmodulated flow. In the left panel, time is non-dimensionalized using the flow time-scale $d/(2u_\tau)$. In the center panel we use the modulation period $T$ to non-dimensionalize time, and show only the instantaneous values of $\text{Nu}$ for the modulated cases. This choice rescales the horizontal axis differently for both lines, and reveals how there are two clear time-scales in the flow, one given by the modulated forcing from the wall, and of the order $O(T)$, and one given by the turbulent flow itself, of the order of $O(d/u_\tau)$. The fluctuations due to natural turbulence appear to be much smaller than those introduced by modulation (left panel), which are of the order of $20 - 30\%$ around the mean value of $\text{Nu}$ even when the wall-modulation amplitude is only $10\%$ of the average wall velocity. We also note that the fluctuation size does not appear to change appreciably with the modulation frequency, unlike what was observed in Jin & Xia (2008).

To elucidate how the average value of dissipation (and wall-shear) depends on the parameters of the unsteady forcing, we show the temporally averaged values of $\text{Nu}$ in the third panel of Figure 2. We cannot observe any definite patterns in the resulting values for total dissipation: they deviate from the value of $\langle \text{Nu} \rangle = 20.6 \pm 0.2$ obtained...
with no modulation, but this baseline value is generally contained within the error bars of the simulation. This provides a first indication that the modulation does not significantly couple with any existing structures in the flow, even for \( Wo = 77 \) when the modulation roughly matches the time-scale of the flow, i.e. \( T \approx d/[2u_\tau] \). To assess the possible effects of box-size dependence on these results, we simulated two additional cases at twice the domain size for \( Wo = 77 \) and \( Wo = 200 \), shown with orange markers in the panel. These values of \( Nu \) also show some dispersion around the baseline value. We note that the \( Wo = 77 \) case is 4% below the previous value, which would result in a smaller friction at the walls and is consistent with similar studies of PCF (Tsukahara et al. 2006). However, this is not seen for \( Wo = 200 \), where the resulting \( Nu \) is instead larger. We conclude by stating that the error introduced by the small domain size is comparable to or larger than any variation due to \( Wo \), which means we cannot make any definite statements on the \( Nu(Wo) \) dependence; even if following Jin & Xia (2008), we do not expect there to be an effect.

We first check the effect of modulation on the flow structures by showing the streamwise and spanwise spectra of the streamwise velocity in Figure 3. The periodic modulation does not introduce significant modifications of the energy spectra at the mid-gap. Only a small degree of variation between the cases, especially at the low wavenumber end can be seen, and among these there is no discernible pattern of behavior as the curves are not ordered by \( Wo \). This is similar as to what was seen for \( Nu \), where no discernible pattern could be seen as \( Wo \) was changed. We also note that for the streamwise spectra \( \Phi_{xx} \), the unmodulated case is closest of all to the lowest \( Wo \) curve (\( Wo = 44 \)), something which is unexpected. Due to the absence of obvious patterns, we may attribute these differences to lack of statistics.

To understand how the modulation is transferred through the flow, we turn towards the streamwise velocity field itself. To isolate the effect of the modulation from the turbulent background fluctuations, we first average the field in the span- and stream-wise directions. A space-time visualization of the result in shown in the left panel of Figure 4. The modulation imposed by the unsteady boundary condition can be clearly seen. Due to the turbulent fluctuations, the average velocities are not periodic. To separate the effect introduced by the periodic motion of the wall, we conduct a phase average over several of the periods simulated. This reduces the temporal domain to \( 0 \leq t/T < 1 \), resulting in a phase-averaged velocity field that we denote as \( \bar{u}(y,t/T) \). We show a space-time
Figure 4: Space-time pseudocolor plots of the averaged streamwise velocity up to the mid-gap for \( Wo = 44 \), non-rotating case. Left: Instantaneous stream- and span-wise averaged velocity. Right: Spanwise, streamwise and phase averaged velocity.

visualization of \( \bar{u} \) in the right panel of Figure 4, where we can now clearly see how the modulation wave travels from the wall into the rest of the fluid, observing that as the distance from the wall increases, the phase lag becomes larger.

We can decompose \( \bar{u} \) in the following manner:

\[
\bar{u}(y,t/T) = \bar{u}_0(y) + f(y,t/T),
\]

where \( \bar{u}_0 \) is the temporally averaged velocity, and \( f(y,t/T) \) denotes the periodic effect introduced by the modulation. There is no a priori reason to think that \( f \) cannot contain any harmonics of the fundamental modulation of period \( T \), i.e. \( T/2, T/3, \) etc. However, the visualization shown in the right panel of Figure 4 indicates that the dominant temporal scale is that associated with the modulation, and not to higher harmonics. This is further confirmed by Fourier analysis, which shows that the first harmonic has an amplitude that is a factor 10\(^{-300} \) times smaller than the first fundamental, depending on the distance to the wall.

Therefore, we use the ansatz that \( f \) has the following functional dependence:

\[
f(y,t/T) = A_u(y) \sin[2\pi t/T + \phi_d(y)],
\]

where \( A_u \) is the amplitude response, and \( \phi_d \) the phase lag, both of which are dependent on the distance to the wall. To determine the values of these quantities we use two methods. The first is to simply take a Fourier transform of \( f \), and determine \( \phi_d \) and \( A_u \) from the results of this transform. We truncate \( \phi_d \) when the amplitude of the Fourier mode is smaller than 10\(^{-3} \) for reasons that will become apparent below. For comparison purposes, we also follow Verschoof et al. (2018), and determine the phase-delay using the peak of the cross-correlation between the wall-velocity and \( f \). We show the results obtained from both methods in the top panels of Figure 5 for values of \( Wo \) in the (26,200) range. We have also added dashed lines which represent the exact solution for Stokes’ oscillating laminar boundary layer, \( \phi_d = (y Wo)/\sqrt{2} \).

We first notice that the phase delay results for the cross-correlation are limited to the range \([0, \pi]\), while those obtained from the Fourier transform have a larger range of \( \phi \) which is only limited by truncation. The results are also qualitatively similar to each other, with a few minor differences. In the cross-correlation method, the phase delay is effectively zero for the lowest \( Wo \) very close to the wall, and is slightly smaller than the one obtained through the Fourier transform. This method also shows some “graininess” due to the numerical inadequacies of using the maximum operator. Hence, we use the
Figure 5: Top panels: Phase delay $\phi_d$ against wall distance for the non-rotating cases measured through FFT (left) and using the cross-correlation method of Verschoof et al. (2018) (right). Bottom left: Perturbation time lag with respect to wall against wall distance for non-rotating cases. Bottom right: Amplitude response $A_u$ measured through the FFT method against wall distance for the non-rotating cases. Symbols: $Wo = 26, 44, 77, 114, 200$ from light red to dark red. Dashed lines in select panels are the theoretical results from Stokes’ problem.

Fourier transform for measuring $A_u$ and $\phi_d$, as it appears superior, even if it is evident that the cross-correlation calculation used in Verschoof et al. (2018) also gives reasonable results.

Turning to the results themselves, we can observe there are two distinct regions where the phase and the wall distance are related in an approximately linear manner, albeit with different slopes. The linear relationship between phase and wall distance can be understood as the modulation travelling into the flow with a constant speed, with steeper slopes signal slower travel. The transition between them occurs at $y \approx 0.05$, which corresponds to $y^+ \approx 40$, i.e. the buffer sub-region of the boundary layer. The near-wall region ($y \leq 0.05$) approximately corresponds to the viscid sub-region. The modulation travels slower, mainly through viscosity. However, because the slopes obtained are shallower than the purely viscid solutions (denoted as dashed lines), other mechanisms that accelerate the transport are at play. We also highlight that the distance between the purely viscous solution and the actual solution decreases as the frequency increases, meaning that these corrections become more important. The second region ($y \geq 0.05$) has a shallower slope, which means that the modulation travels faster, as it is essentially transported by turbulent fluctuations.

We can also observe that the phase delay is larger with increasing $Wo$ at a given wall distance. This does not mean that the perturbation itself travels slower. Instead, this means that increasing the frequency of the perturbation does not increase the travel
speed of the perturbation in a sufficient amount to make the phase delay constant at a
given distance. To emphasize this point, we show the actual delay time $t_d$ against wall
distance for all values of $Wo$ in the bottom left panel of Figure 5. Again, we can see
two regions with different behavior: the viscous subregion at $y \leq 0.05$ and the turbulent
region for $y > 0.05$. In the near wall region, there are two clear behaviors: if $Wo$ is
large (dark curves), the perturbation travel speed is fast and depends on $Wo$. If $Wo$ is
small (light curves), the travel speed of the perturbation is slower and approximately
$Wo$-independent. This is consonant with the fact that high-$Wo$ curves follow the Stokes’
solution better than their low-$Wo$ counterparts in the viscid region.

We can observe similar behaviour in the turbulent region, where all lines reported
(except for $Wo = 26$) have a similar slope indicating that the actual velocity at which
the perturbation travels is approximately $Wo$-independent in the bulk. The transition
between high-$Wo$ and low-$Wo$ behavior is not easy to delimit, as some curves show
different characteristic behavior depending on the wall distance. For $Wo = 26$, due to
the slowness of travel we do not expect a simple picture. Once the delay time grows
beyond $d/(2u_{\tau})$, the information from more than one modulated cycle will be affecting
the flow. Finally, we notice that for $Wo = 77$, the phase delay is approximately $2\pi$
at the center; in contrast for $Wo = 44$, it takes approximately one flow timescale measured
in $u_{\tau}$ units for the perturbation to reach the center. This confirms the fact that we are
forcing close to the natural time-scale of the flow as suggested by Table 1. However, we
do not see any sort of resonant behaviour in the dissipation—neither for $Wo = 77$ nor for
$Wo = 44$.

We now turn to the amplitude response $A_u$. We only show results here obtained through
the Fourier transform method, which presents much smaller oscillations than applying
the method used in Verschoof et al. (2018) on our data. In in the bottom right panel of
5, we show $A_u$ as a function of wall-distance for the same five $Wo$. We also include the
solutions of Stokes’ second problem for the three largest values of $Wo$, which is given by
$A_u = \exp(-\sqrt{2}Wo y)$. These are represented as straight lines in our semi-logarithmic
plot. The same two regions as in the phase plot can be seen: There is an inner viscid
region where the perturbation amplitude decays rapidly, and which closely tracks the
viscous solution. There is also a turbulent region where the perturbation decay is slower
down. And again, the transition between both regions happens at $y \approx 0.05$. In this
region, the perturbations are transported through turbulence, which can be understood
as an effective increase of the viscosity which in turns facilitates the propagation of the
perturbation making it decay at a slower rate. For the cases of $Wo = 26$ we can observe
a third region that starts at around $y = 0.3$ where the slope further decreases to the
point that the amplitude is almost constant. The origin of this third region is unclear,
as similar changes did not clearly appear in the phase delay but were present when
looking at the delay time. In principle, we can rule out averaging errors due to the rather
large magnitude of $A_u$. Finite-averaging and other numerical errors are expressed in the
manner seen for the two highest values of $Wo$: through oscillations which only start to
dominate once $A_u < 10^{-3}$, i.e. as the perturbation amplitude has decreased by a factor of
a hundred. A possible source of this could be that this is the region where the slow travel
of the perturbation causes the results from more than one modulated cycle to interact.

To allow a more direct comparison to the results from Verschoof et al. (2018), we
plot the amplitude and phase lag as a function of $Wo$ for different $y$ locations in Figure
6. We have also indicated on both figures the line at which $T = d/[2u_{\tau}]$. Unlike the
corresponding figures in Verschoof et al. (2018), this plot shows that the amplitude and
phase delay of the perturbation is a strong function of the distance to the wall. There
are two main possible sources for this discrepancy. First, we show values of $y$ which are
Figure 6: Left panel: Amplitude response ($A_u$) against $Wo$ for non-rotating cases. The dashed line shows the scaling $A_u \sim Wo^{-2}$, and the inset shows the compensated amplitude $A_u Wo^2$ against $Wo$ plot to emphasize the scaling. Right panel: Phase delay ($\phi_d$) using FFT against $Wo$ for non-rotating cases. Cases shown are $y = 0.01, 0.05, 0.1, 0.5$ from light blue to dark blue.

much closer to the wall, down to $y = 0.01$, while Verschoof et al. (2018) use distances which correspond to the range $0.2 < y < 0.8$. Second, not only the Reynolds number is different, but also the rotation rate. Taylor-Couette with a pure inner cylinder rotation corresponds to a rotation parameter of $R\Omega = 1 - \eta$, which means that the effective value of $R\Omega$ in the experiments is $R\Omega = 0.29$. We will visit that value of $R\Omega$ in a later section, and show that the effective solid-body rotation is indeed the main source of our discrepancy.

To summarize, there are two distinct regimes for $A_u(Wo)$: at low $Wo$, the amplitude is not a strong function of $Wo$. This region is especially pronounced for the data at $y = 0.01$. At high $Wo$, the amplitude rapidly decays as $Wo$ is increased. This decrease matches the prediction in von der Heydt et al. (2003a), who also predict that for high $Wo$ the amplitude should behave like $A_u \sim T$. This is shown in the figure as the dashed line $A_u \sim Wo^{-2}$ and in a compensated subplot. This theoretical scaling matches the data reasonably well. As we move away from the wall, the transition to the $A_u \sim Wo^{-2}$ dependence happens at lower values of $Wo$. We can attribute this to the flow finding it harder to adjust to the perturbation as the distance to the wall increases. This two region behaviour is also seen for the phase lag, shown in the right panel of Figure 6, even if no clear power law behaviour can be discerned, nor is it available from the theoretical derivations in von der Heydt et al. (2003a).

For completeness, we checked the effect of the amplitude $\alpha$ on the results by running all the cases shown above for $\alpha = 0.05$ and $\alpha = 0.2$. A short summary of the results is presented in Figure 7. In the left panels we show the amplitude response and the phase delay against wall distance for all values of $Wo$ and $\alpha = 0.2$. We can see the same qualitative phenomena we saw appear for $\alpha = 0.1$, which we have already discussed. We do not show these results for $\alpha = 0.05$ as they show the same patterns, but the numerical averaging errors appear for smaller values of $y$ due to the smaller amplitude of the perturbation. In the right panels of the figure, we show $A_u$ and $\phi_d$ against $y$ for different values of $\alpha$ and the same $Wo$. We can clearly see that the amplitude response of the system is simply offset by a factor, while the phase response is approximately independent of $\alpha$ except for some small discrepancies which we attribute to averaging effects.

Finally, in Figure 8, we show an analog to Figure 6 but for the two other values of $\alpha$ simulated (0.05 and 0.2). It shows the same $A_u \sim Wo^{-2}$ behaviour in the high $Wo$
regime. This gives us confidence in the fact that $\alpha$ is a physically unimportant parameter that is only relevant when considering the effect of numerical averaging errors as long as it is not too large. We can expect that for $\alpha \sim \mathcal{O}(1)$, more effects of the amplitude modulation will begin to be seen, similar to those in Yang et al. (2020).

4. Results for rotating Plane Couette flow

4.1. Modulation and Taylor rolls

Adding solid body rotation causes a drastic change in the flow behavior and modifies the underlying statistics such as dissipation and mean velocity (Brauckmann et al. 2016).
As mentioned earlier, it also triggers the formation of large-scale pinned structures known as Taylor rolls which become primarily responsible for the transport of shear. These rolls are in close analog with the large-scale structures in Rayleigh-Bénard flow that dominate heat transfer, and which were shown to couple to modulation introduced through the boundary driving (Jin & Xia 2008; Yang et al. 2020).

We start our discussion with $R_\Omega = 0.1$ which is the value of $R_\Omega$ for which the rolls are most energetic (Sacco et al. 2020). Therefore, we can expect this to be the most favourable case to observe resonant coupling between the modulation and the existing structures in the flow, as the structures have very well defined natural length- and time-scales. We first show the instantaneous $\langle Nu \rangle(t)$ in the left and center panels of Figure 9 for $R_\Omega = 0.1$ and $Wo = 44$, $Wo = 200$ and unmodulated flow (only left). The temporal fluctuations due to the inherent turbulence of the flow are even smaller than for the case with no rotation. The modulation introduces fluctuations in $\langle Nu \rangle(t)$ which are of the order of 20 – 30% of the mean value of $\langle Nu \rangle$, in the same order of magnitude as the values observed previously. We can also observe that the fluctuations in $\langle Nu \rangle$ are larger for smaller values of $Wo$, similar to what was observed in Jin & Xia (2008). We will will rationalize this below.

Similar to the non-rotating case, the mean value around which all curves fluctuate appears to be the same. To further quantify this, in the right panel of Figure 9 we show the time-averaged values of $\langle Nu \rangle$ as a function of $Wo$. As was seen for the non-rotating case, no strong dependence of $\langle Nu \rangle$ with $Wo$ is observed. This hints at the fact that no significant resonances happen between the modulated forcing and the existing structure. These results are consistent with those obtained in Jin & Xia (2008) for sinusoidally modulated RBC, who did not observe an enhancement in the time-averaged values of heat transport.

To further quantify this, we turn to the velocity spectra to assess how they are modified by the modulation. We show the energy spectrum of the streamwise velocity in Figure 10. The “sawtooth” pattern characteristic of Taylor rolls (c.f. Ostilla-Mónico et al. (2016a)) can be appreciated for the spanwise spectra $\Phi_{zz}$. Some modification of the pattern is observed for $Wo = 77$, which is the value of $Wo$ that more closely matches the natural time-scales of the flow and is the one which we expect to produce resonant effects. However, this modification does not correspond to a significant change in the value of $\langle Nu \rangle$. Instead what seems to be happening is that the modes which are normally dampened or eliminated by the Taylor rolls, i.e. the second, fourth and other even
Figure 10: Pre-multiplied streamwise (left) and spanwise (right) spectra of the streamwise velocity at the mid-gap ($y = 0.5$) for RPCF with $R\Omega = 0.1$. Symbols: $Wo = 44$ (black) $Wo = 77$ (dark red), $Wo = 200$ (light red), unmodulated (dark blue).

Figure 11: Temporal evolution of the energy of the fundamental mode associated to the Taylor roll in flow (left) and forcing (right) time units.

fundamentals, are more energetic than for the unmodulated values. However, because the first fundamental mode remains unaffected, which corresponds to the Taylor roll and hence transports most shear, the resulting transport of shear is also unaffected. Following Jin & Xia (2008), it could be that sinusoidal modulations are too smooth to actually affect the roll.

We can justify this by looking at the energy of the Taylor roll. Following Sacco et al. (2019), we measure this energy as the energy of the first $z$-fundamental (i.e. $k_x = 0$, $k_z = 2$) of the wall-normal velocity at the mid-gap. In Figure 11 we show the temporal behaviour of this quantity. The fluctuations in the energy of the roll increase as the period increases, as the inertia of the roll is large enough to absorb the modulations at $Wo = 200$. Only when the time-scale of the modulation is larger than $d/u_T$, the roll feels the fluctuation and its energy is changed. This explains the larger variations in $Nu(t)$ seen for small values of $Wo$.

Unfortunately, the presence of the roll prevents us from examining the way the propagation penetrates into the flow, as the introduced spanwise dependence of average quantities makes replicating the earlier analysis impossible. The decomposition introduced to calculate $f$ as in Eq. 3.1 produces large artifacts which we will discuss later, in context with the results from other values of $R\Omega$. 
4.2. Modulation and mean rotation

We start by analyzing the effect the remaining values of $R_\Omega$ have on the behaviour of the shear transport and the dissipation. In Figure 12 we show the time-averaged $Nu$ against $Wo$ for the two remaining values of $R_\Omega$ studied. As for the previous cases, no significant effect of $Wo$ can be appreciated on the average value of $Nu$. We may now conclude that there is no significant dependence of $Nu$ on $Wo$ across all our simulations, and that the modulation does not significantly modify the average shear transport.

We now turn to the way the modulation propagates into the flow. We use the decomposition from Eq. 3.1 to calculate the amplitude response and phase delay against wall distance. We show the results obtained, alongside those for the non-rotating case, in Figure 13. Significant differences between all curves can be seen. The case with cyclonic rotation ($R_\Omega = -0.1$) follows a similar pattern as the case with no rotation. Close to the wall in the viscous sub-layer, the modulation is transported at a slow and constant velocity (reflected as a linear behaviour of the phase delay), with an exponential decay of the amplitude response. As the distance is increased, a second region appears where the perturbation also travels at a constant but larger velocity, and decays exponentially but with a smaller exponent, equivalent to the flow gaining a larger effective viscosity. The transition between these regions is located at $y = 0.08$, slightly larger than the transition between the regions seen for the non-rotating case. We can attribute this to the lower levels of turbulence and to the lower frictional velocities present when cyclonic rotation is added, which means that the viscous sub-region extends further from the wall (c.f. Ostilla-Mónico et al. (2016b) for an analysis of Taylor-Couette with only outer cylinder rotation, which has an equivalent cyclonic solid body rotation of $R_\Omega = -0.1$). Indeed, for $R_\Omega = -0.1$, $y = 0.08$ corresponds to $y^+ \approx 40$, in the buffer region.

The results for anti-cyclonic rotation present more differences when compared to the non-rotating case. For $R_\Omega = 0.1$, the amplitude response drops very rapidly as we move away from the wall, and then remains constant from $y \geq 0.08$, while the phase delay also increases at first first, and then also remains constant from $y \geq 0.15$. The transition between one type of behaviour to the other happens at very different values of $y$, and the behaviour is very different from the constant velocity or constant exponential decay seen for no rotation or anti-cyclonic rotation, and also deviates strongly from the behaviour of Stokes’ solution, namely linear phase increases and exponential amplitude decay. As we will further justify below, we can attribute this strange behaviour to the spanwise
inhomogeneities introduced by the Taylor rolls, which introduce numerical artifacts and affect the calculation of $f$.

Turning to $R_\Omega = 0.3$, we can observe the two near-wall and bulk regions which have a linear behaviour for the phase delay and an exponential decaying behaviour for the amplitude response, as well as a third region for $y \geq 0.2$ which was previously unobserved and where both the phase delay and the amplitude response are constant. This indicates that rotation induces a region that is affected simultaneously and homogeneously by the flow modulations. Unlike for $R_\Omega = 0.1$, we believe that these results are not due to averaging artifacts, as they were also observed in the experiments in Verschoof et al. (2018), who observed no significant dependence of $A_u$ and $\phi_d$. As mentioned earlier, the setup in Verschoof et al. (2018) corresponds to an effective $R_\Omega = 0.27$, very close to our simulated value, and furthermore, the experiment only measured $A_u$ and $\phi_d$ at distances from the wall of $0.2 < y < 0.8$. According to our simulations, these distances would all correspond to the region where $A_u$ and $\phi_d$ are constant. This explains the discrepancy between our earlier results and the experiment: anti-cyclonic rotation adds a new physical phenomena where the perturbation coming from the wall modulation appears to be constant throughout the bulk.

To further justify that the presence of large-scale structures interferes with the measurement of the perturbation amplitude and phase delay, in figure 14 we show the effects of calculating $A_u$ and $\phi_d$ by only averaging over a fraction of the span-wise length. The non-rotating case is used as a baseline which shows that even taking an eighth of the spanwise domain length leads to reasonable results if large-scale structures are not present. Once the rotation is increased to $R_\Omega = 0.1$, it is impossible to obtain results which do not depend on the extent of the spanwise domain, and increasing the domain size leads to the amplitude response to drop rapidly for the same $y$ coordinate, as the effect of spanwise inhomogeneity contaminates the measurement of $f$. Finally, when $R_\Omega$ is further increased to 0.3, the large-scale structures disappear, and while neither an eighth of the domain nor a quarter is enough to produce accurate results, the results from half- and the full domain show very similar behaviour. We can thus use our results for $R_\Omega = 0.3$ with confidence.

We finalize this section by showing in Figure 15 the amplitude response as a function of $W_0$ for several wall distances at $R_\Omega = -0.1$ and 0.3. For $R_\Omega = -0.1$ we can observe very similar results to those seen for the non-rotating case, with a small region consistent
with $A_u \sim W_0^{-2}$ behaviour. This shows that for cyclonic rotation, the physics of the flow’s response to modulation remains largely unchanged.

For $R_\Omega = 0.3$, we cannot observe any behaviour consistent with power-laws. We can compare this case to the experiments of Verschoof et al. (2018), as we have some similarities. First, we can observe a similar collapse of $A_u$ for low $W_0$ at $0.05 < y < 0.5$, as well as a slow divergence of the curves as $W_0$ becomes larger. While in Verschoof et al. (2018) this collapse held until $W_0 > 100$, in our case the data can only really be seen to collapse for $W_0 = 26$. Furthermore, the experiments reported an exponential-like decay for $A_u$ as a function of $W_0$, but we do not observe behaviour consistent with exponential decay. The differences between the experiment and the simulations can probably be attributed to two sources: first, the experiments do not report data very close to the wall, so a fair comparison would not include the data points in the darkest blue. Second, the correct non-dimensional time-scale for the flow is not the viscosity based $W_0$ but $u_*/d$, as the flow is fully turbulent. Therefore, it does not make sense to compare cases at the same $W_0$, but instead we should compare cases and flow behaviour transitions for the same values of $Tu_*/d$. In our simulations $W_0 \approx 80$ corresponds to $Tu_*/d = 1$, while in their experiments we can expect this number to be closer to $W_0 \approx 220$. Therefore, the separation of data we see at $W_0 \approx 40$ approximately corresponds to the separation of curves observed at $W_0 \sim 110$ in the experiments, making the observation of $y$-independence for the amplitude response at large forcing periods consistent across simulations and experiments.

5. Summary and Outlook

We performed direct numerical simulation (DNS) of rotating plane Couette flow (RPCF) with a modulated plate at a fixed shear Reynolds number, $Re_s = 3 \times 10^4$, for Womersley numbers in the range $W_0 \in [26,200]$ while keeping the amplitude of the modulation constant at $\alpha = 0.1$. We also studied the effect of cyclonic and anti-cyclonic Coriolis forces in the system by varying the rotation parameter, $R_\Omega$ in the range $\in [-0.1,0.3]$.

The average shear at the walls and the instantaneous dissipation was found to be
Figure 15: Amplitude response ($A_u$) against $Wo$ for $\alpha = 0.1$ and $R_\Omega = -0.1$ (left) and $R_\Omega = 0.3$ (right). The dashed line shows the scaling $A_u \sim Wo^{-2}$, and the inset shows the compensated amplitude $A_u Wo^2$ against $Wo$ plot to emphasize the scaling.

independent of the modulation frequency regardless of the Coriolis force added, and no evidence of resonance between flow structures and modulation was found, consistent with the RBC results of Jin & Xia (2008).

The propagation of the modulation was measured using Fourier transforms of phase-averaged velocities to obtain the phase delay and amplitude response. For the non-rotating case ($R_\Omega = 0$), both the modulation response amplitude and the phase delay show a behaviour similar to the theoretical solution of Stokes’ boundary problem, i.e. a linear behaviour for the phase delay and an exponential decay for the amplitude response. There are two main regimes, a near-wall regime where the effective viscosity is close to the fluid viscosity, and a bulk regime where the effective viscosity appears to be much higher. The transition between slopes is observed at $y \approx 0.05$, which corresponds to $y^+ = 40$ in viscous units, i.e. the transition between the viscous- and log-law regions of the turbulent boundary layer. When plotting the amplitude response as a function of $Wo$, we found a high-$Wo$ regime with behaviour consistent with the $A_u \sim Wo^{-2}$ behaviour as is expected of modulated turbulence at high frequencies (von der Heydt et al. 2003a).

We also confirmed that the amplitude of the modulation is an unimportant parameter in determining the physics of the system in the range $\alpha < 0.2$.

The simulations with cyclonic rotation result in similar behaviour to that of the non-rotating case: the modulation amplitude falls off exponentially at two different slopes with the change of slope again occurring at $y \approx 0.08$, which again happens to be at $y^+ = 40$--the transition between the viscous sub-layer and the buffer layer. Results for anti-cyclonic conditions were presented for $R_\Omega = 0.1$ and 0.3. For both anti-cyclonic conditions the amplitude decay as a function of distance from the wall exhibits very different behavior than the non-rotating case or cycloic conditions and deviates from the Stokes’s solution. We attributed this marked difference at $R_\Omega = 0.1$ to the presence of Taylor rolls, which introduce spanwise inhomogeneities and prevent the adequate calculation of the amplitude response. Furthermore, for $R_\Omega = 0.3$, the amplitude decay and phase delay at $y \geq 0.2$ remains fairly constant and is consistent with the observations in Verschoof et al. (2018). We conclude by noting that the correct non-dimensional time-scale of modulated and rotating turbulent Couette flow is $u_\tau/d$, leading to good correspondence of amplitude behavior between current simulations and those observed in experiments of Verschoof et al. (2018).

The behaviour of the anti-cyclonic cases warrants further investigation, especially that seen at $R_\Omega = 0.3$ which corresponds physically to a bulk region which feels the modulation all at once. Other types of modulation, such as periodic pulses should also
be analyzed to find whether the average shear at the plates can be modified. To investigate these cases, more advanced ways of studying how the modulation propagates into the fluid must be developed, as the simple phase-averaged Fourier transform will not be able to produce adequate results.

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