The paper proposes a correction method of the oblique-angle vibration for laser doppler vibrometry. It briefly discusses the key mathematical approach considering the surface of the analysed object to be a reference plane and gives a practical example of the method proper application. The proposed correction method is practically verified by laboratory measurement of natural frequencies and mode shapes for vibrations of high voltage transformer housing. The results are further compared to equivalent accelerometer measurement.

Keywords: laser, doppler, vibrometry, measurement, prototype, oblique-angle, correction

1 Introduction

Laser Doppler Vibrometry (LDV) has been primarily used in medicine [1-3] but offers a wide range of technical capabilities also in engineering applications involving contactless vibration measurement [4-8]. It has been introduced as a very sensitive method applicable for measurements that assess movements of high-dynamic components. This method is particularly important for applications where physical contact with analyzed device is not technically possible, typically rotating machines in operation, measurement of hot surfaces or surfaces under high voltage.

Commercially available LDV are typically constructed with a single beam to measure radial and axial vibrations or with parallel beams to measure pitch and torsional vibrations. Therefore, they can only collect data at one point at a time. In such a case, performing a complex modal analysis to obtain an operational deformation shape is usually a relatively long process. This problem is even more difficult for larger structures or for low natural frequency structures, such as aircraft, space structures or civil structures [9-16].

As stated in [17], the market offers several types of laser-scanning vibrometers allowing multiple points to be measured simultaneously, but they are usually very expensive and complicated. The authors of the same work [17] presented their own design of a three-dimensional measuring system visible on Figure 1 and consisting of a single point vibrometer that can redirect the laser beam using computer-controlled mirrors. The only disadvantage of the proposed system was the lack of an observation angle correction algorithm when measuring large surface from one position.

This study extends the authors’ original work [17] with presenting a simple method of correcting observation angles that can be used generally for any other similar application.

2 Oblique-angle correction

In engineering, most projects require vibration measurements of relatively large technical surfaces, such as a transformer housing, building walls or similar structures. Often, there is insufficient space to install the measuring system at a suitable distance. Therefore, the laser beam senses the vibrations at the specific measuring point under a large angle. This angle changes according to measured point coordinates and the consequent correction is not an easy task.

A practical example is shown in Figure 2. The aim is to investigate natural frequencies and mode shapes for vibrations of high voltage transformer housing. The matrix of investigated points is marked with red dots. The position of the laser vibrometer is considered to be at coordinates $[x_0, y_0, z_0]$. The arms $r_1 - r_4$ represent the distances measured (with a laser) between the vibrometer lens and all of the transformer housing corners.

As obvious, the oscillations scanned at different positions must be accordingly recalculated prior the final evaluation. The triangle seen in Figure 3 proposes the method to determinate necessary correction coefficient for any evaluated point.

Considering the surface of the transformer housing to be the reference plane, it is possible to determine the observing position by solving the set of Equations (1).
Assuming $x_2 = x_3$, Equation (2) changes into Equation (3).

$\begin{align*}
(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2 - r_1^2 &= 0 \\
(x_0 - x_2)^2 + (y_0 - y_2)^2 + (z_0 - z_2)^2 - r_2^2 &= 0 \\
(x_0 - x_3)^2 + (y_0 - y_3)^2 + (z_0 - z_3)^2 - r_3^2 &= 0
\end{align*}$

In this case, Equation (1) can be rewritten according to Figure 2 into a simpler Equation (2).

$\begin{align*}
(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2 - r_1^2 &= 0 \\
(x_0 - x_2)^2 + (y_0 - y_2)^2 + (z_0 - z_2)^2 - r_2^2 &= 0 \\
(x_0 - x_3)^2 + (y_0 - y_3)^2 + (z_0 - z_3)^2 - r_3^2 &= 0
\end{align*}$

Assuming $x_2 = x_3$, Equation (2) changes into Equation (3).

$\begin{align*}
x_0^2 + y_0^2 + z_0^2 - r_1^2 &= 0 \\
(x_0 - x_2)^2 + (y_0 - y_2)^2 + (z_0 - z_2)^2 - r_2^2 &= 0 \\
(x_0 - x_3)^2 + (y_0 - y_3)^2 + (z_0 - z_3)^2 - r_3^2 &= 0
\end{align*}$

By solving the Equation (3) we get the coordinates (4)-(6).

$\begin{align*}
x_0 &= \frac{x_2^2 + r_1^2 - r_2^2}{2x_2}
\end{align*}$
The method can be used for any complex shape by dividing the whole surface of the analyzed object into smaller but simpler regions and applying it repeatedly. It is similar to application of the finite element method.

### 3 Experimental Measurement

The validity of the proposed correction method is practically demonstrated by laboratory measurement of natural frequencies and mode shapes for vibrations of high voltage transformer housing shown in Figure 2 right. A block diagram of the measurement system is shown in Figure 4.

During the experiment, the transformer housing was mechanically excited by the dynamic vibration generator (Type 4824) in the frequency sweep mode (1-100 Hz). It is a lightweight modal exciter capable of exciting oscillations

\[
y_0 = \frac{y_1^2 + y_2^2 - y_3^2}{2y_3}
\]

\[
z_{01} = \frac{\sqrt{x_1^2y_1^2 - y_1^2(r_1^4 - r_2^4)^2 - x_1^2[y_1^2 + (r_1^4 - r_2^4)^2] - 2y_1^3(r_1^4 + r_2^4)}}{2x_1y_3}
\]

\[
z_{02} = \frac{\sqrt{-x_1^2y_1^2 - y_1^2(r_1^4 - r_2^4)^2 - x_1^2[y_1^2 + (r_1^4 - r_2^4)^2] - 2y_1^3(r_1^4 + r_2^4)}}{2x_1y_3}
\]

The position of the observation point is then \([x, y, z]\). Further, if the length \(a\) seen in Figure 3 is calculated as Equation (7)

\[
\alpha = \sqrt{(x_0 - x_a)^2 + (y_0 - y_a)^2},
\]  

then the angle \(\alpha\) is given by Equation (8) and the angle \(\gamma\) is obtained from Equation (9).

\[
\alpha = \frac{z_{01}}{a}
\]  

\[
\gamma = \frac{\pi}{2} - \alpha
\]

Finally, the correction factor is according to Equation (10).

\[
k = \cos \gamma
\]
An additional equivalent accelerometer measurement was performed to validate the presented results as this method is usually considered to be a reference in a given scientific field. Comparison between LDV and accelerometer measurement is shown in Figure 7. For this purpose, a constant current line drive accelerometer (4533-B) was a perfect choice due to high sensitivity, wide frequency range and low noise.

Minor inaccuracies in the frequency domain are caused by the non-linear response of the transformer housing, whose frame is manufactured from a set of aluminum with a force of 100 N. Figure 5 shows the location of the exciter along with the transformer housing being analyzed. Measurements were repeated four times in a sequence and subsequently averaged. The transformer housing from the viewpoint of the laser vibrometer (laser targeting to the upper right corner) is shown in Figure 2.

The resulting correction factors calculated using Equations (1)-(10) are listed in the left side of Figure 6. The vibration spectrum measured at the measuring point “17” can be seen in the right side of the same figure.

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Minor inaccuracies in the frequency domain are caused by the non-linear response of the transformer housing, whose frame is manufactured from a set of aluminum.
structural beams interconnected with a plastic clutch assembly. Another slight difference manifested mainly at higher frequencies may be caused by the accelerometer mass. This additional weight can cause slight distortion in both amplitude and spectral regions by shifting individual antinodes.

The qualitative comparison between transformer housing mode shapes derived from results of accelerometer measurement and the LDV measurement is shown in Figure 8.

4 Conclusion

The paper has given a brief discussion on the technical background and the most important benefits of using laser Doppler vibrometry. It has proposed a correction method of the oblique-angle vibration for a vibration measurement with a laser beam. The study has brought a simple mathematical description of the analyzed geometrical problem and has shown a practical example of its proper usage. Moreover, it could be used for any complex shape by dividing the whole surface of the analyzed object into smaller but geometrically simpler regions and applying it repeatedly.

The method has been verified by a laboratory measurement performed on the high voltage transformer housing. The results have shown very good agreement between corrected LDV and the equivalent accelerometer measurement. Therefore, the proposed correction method should be considered as valid.

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References

[1] CHEN, M., O’SULLIVAN, J. A., SINGLA, N., SIREVAAG, E. J., KRISTJANSSON, D., LAI, P., KAPLAN A. D., ROHRBAUGH J. W. Laser Doppler vibrometry measures of physiological function: evaluation of biometric capabilities. IEEE Transactions on Information Forensics and Security [online]. 2010, 5(3), p. 449-460. ISSN 1556-6021. Available from: https://doi.org/10.1109/TIFS.2010.2051542

[2] SCALISE, L., ERCOLI, I., MARCHIONNI, P., TOMASINI, E. P. Measurement of respiration rate in preterm infants by laser Doppler vibrometry. IEEE International Symposium on Medical Measurements and Applications : proceedings [online]. IEEE, 2011. ISBN 978-1-4244-9338-8, p. 657-661. Available from: https://doi.org/10.1109/MeMeA.2011.5966740

[3] KAPLAN A. D., O’RUSULLIVAN, J. A., SIREVAAG E., J., LAI P., ROHRBAUGH, J. W. Hidden state models for noncontact measurements of the carotid pulse using a laser Doppler vibrometer. IEEE Transactions on Biomedical Engineering [online]. 2012, 59(9), p. 744-753. ISSN 0018-9294, eISSN 1558-2531. Available from: https://doi.org/10.1109/TBME.2011.2179297

[4] AMBROZINSKI, L., SPYTEK, J., DZIEDZIECH, K., PIECZONKA, L., STASZEWSKI W. J. Damage detection in plate-like structures based on mode-conversion sensing with 3D laser vibrometer. IEEE International Ultrasonics Symposium IUS : proceedings [online]. IEEE, 2017. ISSN 2076-3417, p. 1-4. Available from: https://doi.org/10.1109/ULTSYM.2017.8092957

[5] LI, R., WANG, T., ZHU, Z.; XIAO, W. Vibration characteristics of various surfaces using an ldv for long-range voice acquisition. IEEE Sensors Journal [online]. 2011, 11(6), p. 1415-1422. ISSN 1530-437X. Available from: https://doi.org/10.1109/TSEN.S.2010.2093125

[6] SPANIK, P., SEDO, J., DRGONA, P., FRIVALDSKY, M. Real time harmonic analysis of recuperative current through utilization of digital measuring equipment. Elektronika IR Elektrotechnika [online]. 2013, 19(5), p. 33-38. ISSN 1392-1215, eISSN 2029-5731. Available from: https://doi.org/10.5755/j01.eele.19.5.4304

[7] GAO, Ch., WANG, Q., WIE, G., LONG, X. A highly accurate calibration method for terrestrial laser Doppler velocimeter. IEEE Transactions on Instrumentation and Measurement [online]. 2017, 66(8), p. 1904-2003. ISSN 0018-9456, eISSN 1557-9662. Available from: https://doi.org/10.1109/TIM.2017.2885078

[8] BERNAT, P, KACOR, P. Utilisation of stray electromagnetic field for no-contact operational diagnostic of asynchronous machine. 10th International Conference ELEKTRO 2014 : proceedings [online]. University of Zilina, Faculty of Electrical Engineering. 2014. ISBN 978-1-4799-3721-9, p. 256-261. Available from: https://doi.org/10.1109/ELEKTRO.2014.6848898

[9] KIM, D., SONG, H., KHALIL, H.; J. LEE, L., WANG, S., PARK, K. 3-D vibration measurement using a single laser scanning vibrometer by moving to three different locations. IEEE Transactions on Instrumentation and Measurement [online]. 2014, 63(8), p. 2028-2033. ISSN 0018-9456, eISSN 1557-9662. Available from: https://doi.org/10.1109/TIM.2014.2302244

[10] KIM, M. G., JO, K., KWON, H. S., JANG, W., PARK, Y., Z., LEE J. Fiber-optic laser doppler vibrometer to dynamically measure MEMS actuator with in-plane motion. Journal of Microelectromechanical Systems [online]. 2009, 18(6), p. 1365-1370. ISSN 1057-7157, eISSN 1941-0158. Available from: https://doi.org/10.1109/JMEMS.2009.2031698
[11] PHENGPOM, T.; KAMADA, Y., MAEDA, T., MURATA J., KAGISAKI, Y., NISHIMURA, S. Experimental study on sectional performance of horizontal axis wind turbine at optimum operation by using LDV system. International Conference and Utility Exhibition on Green Energy for Sustainable Development (ICUE) : proceedings. IEEE, 2014. ISBN 978-1-4799-2627-5, p. 1-6.

[12] PAGAN, L., LAKE, K. Modal analysis: a comparison between finite element analysis (FEA) and practical laser Doppler vibrometer (LDV) Testing. 19th International Conference on Computer Modelling & Simulation UKSim-AMSS : proceedings [online]. IEEE, 2017. ISBN 978-1-5386-2736-5, p. 75-80. Available from: https://doi.org/10.1109/UKSim.2017.27

[13] REITEN, M. T., WRIGHT, R. G. Laser Doppler vibrometry use in detecting faulty printed circuit boards. IEEE Autotestcon : proceedings. 2008, p. 33-36.

[14] SUN, H., LIU, J., KENNEL, R. Improving the accuracy of laser self-mixing interferometry for velocity measurement. IEEE International Instrumentation and Measurement Technology Conference (I2MTC) : proceedings [online]. IEEE, 2017. ISBN 978-1-5090-3596-0, p. 1-5. Available from: https://doi.org/10.1109/I2MTC.2017.7969922

[15] OUAHABI, A., DEPOLLIER, E., SIMON, L., KOUME, D. Spectrum estimation from randomly sampled velocity data [LDV]. IEEE Transactions on Instrumentation and Measurement [online]. 1998, 47(4), p. 1005-1012. ISSN 0018-9456, eISSN 1557-9662. Available from: https://doi.org/10.1109/19.744659

[16] KWAPISZ, L., JAKUBOWSKI, P., RADZIENSKI, M. Problems with vibroacoustic LDV measurements for Windows. Joint Conference - Acoustics : proceedings [online]. 2018. ISBN 978-1-5386-7115-3, p. 1-4. Available from: https://doi.org/10.1109/ACOUSTICS.2018.8502350

[17] KAVALIR, T., KRIZEK, M., SIKA, J., KINDL, V. Upgrading of the single point laser vibrometer into a laser scanning vibrometer. Communications - Scientific Letters of the University of Zilina [online]. 2018, 20(1), p. 61-66. ISSN 1335-4205, eISSN 2385-7878. Available from: http://komunikacie.uniza.sk/index.php/communications/article/view/47