Numerical Analysis of 2d Rectangular Plate

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Abstract. In this article two dimensional heat equation is solved using finite difference method for the metals such as gold, zinc, tin, marble and bronze. Dirichlet conditions are applied on the boundaries of the rectangular plate. The problem is solved for different metals and the comparison is made for reaching the steady state.

The first section in your paper

Keywords

Laplace equation, Finite Difference Method, Dirichlet condition, Steady State, Temperature.

1. Introduction

The problem of conduction of heat in a square plate has been studied by many researchers. Reddy J.N. and Gera R [1] determined a finite difference analysis of thin rectangular elastic plates. Inversion of thermal conductivity of heat transfer in two dimensional unsteady state was presented by Shoubin et al [2]. By boundary element and sequential function specification method, S. Wang et.al [3] solved two dimensional unsteady inverse heat conduction problem. Temperature distribution in a rectangular plate by a moving heat source was discussed by Kidawa – Kukla [4] and conduction of heat by Beck.J.V [5]. By apply FDM to the study of thin plates by Dolicanin. C.B et.al [6] and 2D varying thickness and thermal effect was discussed by sharma [7]. S. Wang et al [8,9] determined solution of two dimensional steady inverse heat transfer problems with conjugate gradient method and SPSO algorithm. P. Duda [10] solved multidimensional heat transfer problems and Li [11] created an algorithm for geometry boundary of heat conduction problem.Thermal Performance of Heat Conduction in a Square Plate at different angle is discussed by [12] and Numerical analysis of MATLAB code in a square plate by [13].

In engineering numerous physical issues can be numerically modelled by either ordinary differential equation or partial differential equations. In many cases, finding the logical arrangements of PDE is awkward. In such cases numerical strategies plays a significant job. The heat equation is a partial differential equation that describes how the dispersion of heat evolves over time in a solid medium. It spontaneously flows from higher towards lower.

In practice, widely used numerical methods are FDM, FVM, FEM etc. Compared to other numerical methods Finite difference method is easy to implement and can be extended to higher dimensions. The purpose of our study is that the problem is solved for different metals and the comparison is made for reaching the steady state.

2. Methodology

In this article a thin rectangular plate is considered which is of conducting material. The plate is heated along the boundary of the plate. The plate considered in this study have uniform specific heat, uniform density, internally there is no heat sources and perfectly insulated on top and bottom of the plate. Under these conditions the two dimensional heat equation is given by
\[ \rho C_p \frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \] 

(1)

Where,
- \( \rho \) is the material density.
- \( C_p \) is the specific heat.
- \( k \) is the thermal conductivity.

Here \( T(t,x,y) \) denotes the temperature of the plate at time \( t \) and \( (x,y) \) is the position of the plate.

Boundary Conditions are,
- \( T(x,0,t) = 100^\circ \)
- \( T(x,1,t) = 50^\circ \)
- \( T(0,y,t) = 150^\circ \)
- \( T(1,y,t) = 0^\circ \)

Initial condition is \( T(x,y,0)=0 \)

\( T(0,y,t) = 150^\circ \quad T(1,y,t) = 0^\circ \)

Fig. 1 Two dimensional square plate

The derivatives in the governing differential equations are replaced by difference quantities at some selected points of the plate while applying the FDM. These points are situated at the joints of a rectangular, square, triangular called a finite difference mesh. Each mesh point can replace by an appropriate finite difference equation. A comparable methodology is taken with the expressions describing the boundary conditions.

Thus equation (1) solved using FDM to an algebraic equation as

\[
\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = \frac{k}{\rho C_P} \left( \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} \right)
\]

In this manner, the FDM applies a mathematical discretization of the plate continuum yielding a lot of algebraic equations, from which the plate deflections at the mesh points can be acquired.

Governing equations were solved in MATLAB which use finite difference method.

3. Result

Using finite difference method the temperature distribution is performed numerically. By applying material properties like conductivity, specific heat and density of five metals Table (1), detailed study of temperature and steady state is made.

| Material | Conductivity \( K \, J/m.c.sec \) | Specific Heat \( C_p \, J/kg \, c \) | Density \( \rho \, kg/m^3 \) |
|----------|---------------------------------|---------------------------------|-------------------|
| Bronze   | 109                             | 380                             | 8730              |
| Gold     | 314                             | 386                             | 8960              |
| Zinc     | 122                             | 388                             | 7144              |
| Tin      | 68.2                            | 228                             | 7304              |
| Marble   | 294                             | 880                             | 2770              |

Temperature distribution of five metals inside a square plate is shown in Fig. 2 to Fig. 6.
By considering five interior points inside the square plate, convergence and the temperature distribution at five positions are calculated shown in Fig. 2 to Fig. 6.

**ZINC**

**Fig. 2** Temperature distribution inside the Zinc plate and temperature at 5 interior points

**MARBLE**

**Fig. 3** Temperature distribution inside the Marble plate and temperature at 5 interior points

**BRONZE**
Fig. 4 Temperature distribution inside the Bronze plate and temperature at 5 interior points.

TIN

Fig. 5 Temperature distribution inside the Tin plate and temperature at 5 interior points.

GOLD
Fig. 6. Temperature distribution inside the Gold plate and temperature at 5 interior points.

Table 2 explains time taken to attain steady state for Bronze, Gold, Zinc, Tin and Marble. From the table 2, it is clear that marble takes less time whereas Tin takes more time to reach steady state. In Table 3 steady state temperature at five specified points of the plate for the metals are tabulated. In steady state, temperature distribution for Zinc, Tin and Marble are same at all points of the plate. When compared to other four metals, Bronze has less temperature distribution in steady state and Gold has higher temperature distribution in steady state.

| Material | Steady State | Conductivity K J/m.c.sec | Specific Heat C J/kg c | Density ρ kg/m³ |
|----------|--------------|--------------------------|------------------------|-----------------|
| Bronze   | 3537 seconds | 109                      | 380                    | 8730            |
| Gold     | 1963 seconds | 314                      | 386                    | 8960            |
| Zinc     | 3554 seconds | 122                      | 388                    | 7144            |
| Tin      | 3733 seconds | 68.2                     | 228                    | 7304            |
| Marble   | 1297 seconds | 294                      | 880                    | 2770            |

Table 3 Steady state temperature at five specified points on the plate for five metals

| Material | RED Lx/4, Ly/4 | GREEN Lx/2, Ly/2 | BLUE 3*Lx/4, 3*Ly/4 | YELLOW 3*Lx/4, Ly/4 | BLACK Lx/4, 3*Ly/4 |
|----------|----------------|------------------|---------------------|---------------------|--------------------|
| Bronze   | 109.84         | 70.52            | 36.47               | 57.48               | 93.90              |
| Gold     | 113.82         | 78.84            | 41.22               | 63.03               | 99.55              |
| Zinc     | 112.38         | 73.30            | 39.02               | 61.97               | 98.53              |
| Tin      | 112.38         | 73.30            | 39.02               | 61.97               | 98.53              |
| Marble   | 112.38         | 73.30            | 39.02               | 61.97               | 98.53              |

4. Conclusion

Numerical investigation of temperature distribution in a square plate was proposed in this paper. Two-dimensional differential heat equation is solved using finite difference method. The five metals gold, zinc, tin, marble and bronze attains steady state at different time seconds. The interior values of zinc, tin and marble is converges to similar points on account of their material properties. The heat
transmission inside the plate is solved numerically and further investigation is needed to investigate experimentally the thermal performance of the plate.

References

[1] Reddy JN and Gera R. An improved finite difference analysis of bending of thin rectangular elastic plates. Computers and Structures, vol 10, pp 431-438.

[2] Shoubin Wang, Li Zhang, Xiaogang Sun, and Huangchao Jia. Inversion of thermal conductivity in two dimensional unsteady state heat transfer system based on boundary element method and decentralized fuzzy inference. Comp. vol 9, Article ID 8783946.

[3] Shoubin Wang, Deng Y, and Sun X. Solving of two dimensional unsteady inverse heat conduction problems based on boundary element method and sequential function specification method. Comp. Article ID 6741632, 11 pages.

[4] Kidawa-kukla J. Temperature distribution in a rectangular plate heated by a moving heat source. International Journal of Heat and Mass Transfer, Vol 51, pp 865-72.

[5] Beck J V, Neil T Wright, Haji-Sheikh A, Kevin D Cole, and Donald E Amos. Conduction in rectangular plates with boundary temperatures specified. International Journal of Heat and Mass Transfer, Vol 51, pp 4676-4679.

[6] Dolicanin C B, Nikolic VB, and Dolicanin D C. Application of finite difference method to study the phenomenon in the theory of thin plates. Applied Mathematics and Informatics and Mechanics, Vol 2, pp 29-43.

[7] Sharma, Subodh Kumar, and Ashish Kumar Sharma. Mechanical vibration of orthotropic rectangular plate with 2D linearly varying thickness and thermal effect. International Journal of research in advent Technology, Vol 2, pp 184-190.

[8] Wang S, Zhang L, X Sun et al. Solution to two-dimensional steady inverse heat transfer problems with interior heat source based on the conjugate gradient method. Mathematical Problems in Engineering, vol 2017, Article ID 2861342, 9 pages.

[9] S Wang, H Jia, X, Sun et al. Two-dimensional steadystate boundary shape inversion of CGM-SPSO algorithm on temperature information. Advances in Materials Science and Engineering, vol 2017, Article ID 2461498, 12 pages.

[10] P Duda A. A general method for solving transient multidimensional inverse heat transfer problems. International Journal of Heat and Mass Transfer, vol. 93, pp 665–673.

[11] B Li and L Liu. An algorithm for geometry boundary identification of heat conduction problem based on boundary element discretization. ResearchGate Proceedings of the CSEE, vol. 28, pp 38-43.

[12] Mohammad A Saraireh. Thermal Performance of Heat Conduction in a Square Plate. DOI 10.5013/IJSSST.a.18.04.14 ISSN: 1473-804x online, 1473-8031 print.

[13] Heat Diffusion in 2D Square Plate Using Finite Difference Method with Steady-State Solution. Numerical Analysis- Course project.