Nuclear constraints on the core-crust transition density and pressure of neutron stars

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Abstract

Using the equation of state of asymmetric nuclear matter that has been recently constrained by the isospin diffusion data from intermediate-energy heavy ion collisions, we have studied the transition density and pressure at the inner edge of neutron star crusts, and they are found to be $0.040 \text{ fm}^{-3}$ and $0.065 \text{ fm}^{-3}$, respectively, in both the dynamical and thermodynamical approaches. We have further found that the widely used parabolic approximation to the equation of state of asymmetric nuclear matter significantly overestimates the transition density and pressure, especially for symmetric energies. With these newly determined transition density and pressure, we have obtained an improved relation between the mass and radius of neutron stars based on the observed minimum crustal fraction of the total moment of inertia for Vela pulsar.

1 Introduction

Exploring the properties of neutron stars, which are among the most mysterious objects in the universe, allows us to test our knowledge of matter under extreme conditions. Theoretical studies have shown that neutron stars are expected to have a liquid core surrounded by a solid inner crust, which extends outward to the neutron drip-out region. While the neutron drip-out density is relatively well determined...
to be about $4 \times 10^{31} \text{g/cm}^3$ or $0.00024 \text{fm}^{-3}$ [2], the transition density $\rho_c$ at the inner edge is still largely uncertain mainly because of our very limited knowledge on the nuclear equation of state (EOS), especially the density dependence of the symmetry energy ($E_{\text{sym}}$) of neutron-rich nuclear matter [3,4]. These uncertainties have hampered our understanding of many important properties of neutron stars [3,4,5] and related astrophysical observations [3,4,5,6,7,8,9,10,11,12,13,14]. Recently, significant progress has been made in constraining the EOS of neutron-rich nuclear matter using terrestrial laboratory experiments (See Ref. [15] for the most recent review). In particular, the analysis of the isospin-diffusion data [16,17,18] in heavy-ion collisions has constrained tightly the $E_{\text{sym}}$ in exactly the same sub-saturation density region around the expected inner edge of neutron star crust. The extracted slope parameter $L = 3.0 \frac{E_{\text{sym}}}{\rho}$ in the density dependence of the nuclear symmetry energy was found to be $86 \pm 25$ MeV [19], which has further been confirmed by a more recent analysis [20]. With this constrained nuclear symmetry energy and the corresponding EOS of asymmetric nuclear matter, we have obtained an improved determination of the values for the transition density and pressure at the inner edge of neutron star crusts. This has led us to obtain significantly different values for the radius of the Vela pulsar from those estimated previously. Also, we have found that the widely used parabolic approximation (PA) to the EOS of asymmetric nuclear matter gives much larger values for the transition density and pressure, especially for small symmetry energies.

2 Dynamic and Thermodynamical Approaches to the Stability of npe Matter

The inner edge of neutron star crusts corresponds to the phase transition from the homogeneous matter at high densities to the inhomogeneous matter at low densities. In principle, the inner edge can be located by comparing in detail relevant properties of the nonuniform solid crust and the uniform liquid core mainly consisting of neutrons, protons and electrons (npe matter). However, this is practically very difficult since the inner crust may contain nuclei having very complicated geometries, usually known as the ‘nuclear pasta’ [5,12,21,22,23]. Furthermore, the core-crust transition is thought to be a very weak first-order phase transition and model calculations lead to a very small density discontinuities at the transition [3,24,25,26]. In practice, therefore, a good approximation is to search for the density at which the uniform liquid first becomes unstable against small amplitude density fluctuations with clusterization. This approximation has been shown to produce very small error for the actual core-crust transition density and it would yield the exact transition density for a second-order phase transition [3,24,25,26]. Several such methods including the dynamical method [3,7,8,9,24,27,28], the thermodynamical method [4,29,30] and the random
phas e approx im at i on (RPA) [31,26] have been appl i e d exten si ve l y i n the lit er at ur e. O ur study here was bas ed on the dynam ical and ther modyn ical approaches.

In the dynam ical approach, the stability condition for a homoge neous npe m at ter against smal l peri odic density perturba tions can be we ll approx im ated by [6,7,8,9,27]

\[ V_{\text{dyn}}(k) = V_0 + \frac{4}{k^2 + k_{TF}^2} > 0; \]  

where \( k \) is the wavevector of the spatially peri odic density perturbations and

\[ V_0 = \frac{\Theta_p}{\Theta_p} \frac{(\Theta_n = \Theta_p)^2}{\Theta_n = \Theta_n}; k_{TF}^2 = \frac{4}{\Theta_e = \Theta_e}; \]  

\[ V_0 = D_{pp} + 2D_{np} + D_{nn}; \]  

with \( \Theta \) being the chem ical poten tial of particle type \( i \). The three term s in Eq. (1) repre sent, respec tively, the contributions from the bulk nuclear m at ter, the density-grad ient (surface) term s, and the Coulomb b interac tion. For the co e icients of density-grad ient term s, we use the em pirical values of \( D_{pp} = D_{nn} = D_{np} = 132 \text{ MeV fm}^6 \), which are consis tent with those from the Skyrme-Hartree-Fock calculations [27,32]. At \( k_{mn} = \left( (\frac{4}{e^2})^{1/2} k_{TF}^2 \right)^{1/2} \), \( V_{\text{dyn}}(k) \) has the mini mal value of \( V_{\text{dyn}}(k_{mn}) = V_0 + 2(4 e^2)^{1/2} k_{TF}^2 \) [6,7,8,9,27], and \( t \) is then det ermined from \( V_{\text{dyn}}(k_{mn}) = 0 \).

In the ther modynam ical approach, the stability conditions for the npe m at ter are [4,29,33]

\[ \frac{\Theta_P}{\Theta_P} > 0; \text{ and } \frac{\Theta}{\Theta_q} > 0; \]  

In the above, \( P = P_b + P_e \) is the total pres sure of the npe m at ter with \( P_b \) and \( P_e \) being the contributions from baryons and elec trons, respec tively; \( v \) and \( q \) are the vol um e and charge per baryon number; and \( \Theta = \frac{\Theta_p}{\Theta_p} \) is the chem ical poten tial of the npe m at ter. It can be sho wn that the first inequality in Eq. (4) is equi valent to requiring a positive bulk term \( V_0 \) in Eq. (1) [32], while the second inequality in Eq. (4) is almost always satis fied. Therefore, the ther modynam ical stability conditions are simply the lim it of the dynam ical one for \( k \rightarrow 0 \) when the surface term s and the Coulomb b interac tion are neglected.

3 Results for the transition density and pressure

We have used in our study a mom en tum-de penden t MDI inter ac tion that is based on the modi ed ni te-range Gogny e cti ve inter ac tion [34]. This inter ac tion, which has been exten sively studied in our previ ous work [15], is exactly the one used in analyzing the isospin diff usion data from heavy-ion reactions [17,18].
Shown in Fig. 1 is the transition density $\tau$ as a function of the slope parameter $L$ of the nuclear symmetry energy from the dynamic and thermodynamical approaches with and without the parabolic approximation in the MDI interaction. Taken from Ref. [32].

Figure 1: (Color online) The transition density $\tau$ as a function of the slope parameter $L$ of the nuclear symmetry energy from the dynamic and thermodynamical approaches with and without the parabolic approximation in the MDI interaction. Taken from Ref. [32].
is thus appreciable. This is especially the case for the stiffer symmetry energy which generally leads to a more neutron-rich npe matter at subsaturation densities. Furthermore, because of the energy curvatures involved in the stability conditions, larger factors are multiplied to the contributions from higher-order terms in the EOS than that multiplied to the quadratic term. These features agree with the early finding [35] that the transition density $\rho_t$ is very sensitive to the fine details of the nuclear EOS. According to results from the more complete and realistic dynamical approach, the constrained $L$ limit the transition density to $0.040$ fm$^{-3}$ to $0.065$ fm$^{-3}$ as shown in Fig. 1.

![Figure 2: (Color online) The transition pressure $P_t$ as a function of $L$ and $\rho_t$ by using the dynamical approach with and without the parabolic approximation in the MDI interaction. Taken from Ref. [32].](image)

The transition pressure $P_t$ at the inner edge of the neutron star crust is also an important quantity that might be measurable indirectly from observations of pulsar glitches [4,11]. Shown in Fig. 2 is the $P_t$ as a function of $L$ and $\rho_t$ by using the dynamical approach with both the full MDI EOS and its PA. Again, it is seen that the PA leads to huge errors for large (small) $L$ ($\rho_t$) values. For the full MDI EOS, the $P_t$ decreases (increases) with increasing $L$ ($\rho_t$) while it displays a complex relation with $L$ or $\rho_t$ in PA. From the constrained $L$ values, the value of $P_t$ is limited between $0.01$ MeV/fm$^3$ and $0.26$ MeV/fm$^3$. 
4 Improved constraint on the radius-mass relation of neutron stars

The constrained values of $t$ and $P_t$ have important implications in many properties of neutron stars $[4,8,12,27]$. As an example, we have examined their effect on constraining the mass-radius ($M$-$R$) correlation of neutron stars. The crustal fraction of the total moment of inertia $I=I$ of a neutron star can be well approximated by $[3,4,11]$

$$\frac{I}{I} = \frac{28}{3M} R^3 \left(1 + 1.67 \times 10^{-2} \right) \left(1 + \frac{2P_t(1+5)}{5m_b c^2} \right)^{1/2} \left(1 + 2P_t(1+5) \times 10^{-2} \right)^{1/2}; \quad (5)$$

where $m_b$ is the baryon mass and $G = GM = R c^2$ with $G$ being the gravitational constant. As stressed in Ref. $[3]$, $I=I$ depends sensitively on the symmetry energy at subsaturation densities through $P_t$ and $t$, but there is no explicit dependence on the higher-density EOS. So far, the only known limit of $I=I > 0.014$ was extracted from studying the glitches of Vela pulsar $[11]$. This together with the upper bounds on $P_t$ and $t$ ($t = 0.065$ $\text{fm}^{-3}$ and $P_t = 0.26$ $\text{MeV/fm}^3$) sets approximately a minimum radius of $R = 4.7 + 4.0M = 8$ km for the Vela pulsar. The radius of Vela pulsar is predicted to exceed 10.5 km should it have a mass of $1.4M$. We notice that a constraint of $R = 3.6 + 3.9M = 8$ km for this pulsar has previously been derived in Ref. $[11]$ by using $t = 0.075$ $\text{fm}^{-3}$ and $P_t = 0.65$ $\text{MeV/fm}^3$. However, the constraint obtained in our study using for the first time data from both the terrestrial laboratory experiments and astrophysical observations is more stringent.

The above constraints are shown in Fig. $[2]$ together with the $M$-$R$ relation obtained by solving the Tolman-Oppenheimer-Volkov (TOV) equation. In the latter, we have used the well-known BPS EOS $[3]$ for the outer crust. In the inner crust with $t < 0$, the EOS is largely uncertain and following Ref. $[26]$, we use an EOS of the form $P = a + b$ with the constants $a$ and $b$ determined by the total pressure $P$ and total energy density at $t$. The full M D I EOS and its parabolic approximation with $x = 0$ and $x = 1$ are used for the uniform liquid core with $t$. Assuming that the core consists of only the npe matter without possible new degrees of freedom or phase transitions at high densities, the PA leads to a larger radius for a fixed mass compared to the full M D I EOS. Furthermore, using the full M D I EOS with $x = 0$ and $x = 1$ constrained by the heavy-ion reaction experiments, the radius of a canonical neutron star of $1.4M$ is tightly constrained within 11.9 km to 13.2 km.
Figure 3: (Color online) The mass-radius relation $M - R$ of static neutron stars from the full EOS and its parabolic approximation in the MDI interaction with $x = 0$ and $x = 1$. For the Vela pulsar, the constraint of $I = I > 0.014$ limits the allowed masses and radii. See text for details. Taken from Ref. [32] with small modifications.

5 Summary

Using the MDI EOS of neutron-rich nuclear matter constrained by recent isospin diffusion data from heavy-ion reactions in the same sub-saturation density range as the neutron star crust, we have determined the density and pressure at the inner edge, that separates the liquid core from the solid crust of neutron stars, to be $0.040 \text{ fm}^{-3}$, $0.065 \text{ fm}^{-3}$ and $0.01 \text{ MeV/fm}^3$, respectively. These constraints have allowed us to determine an improved mass-radius relation for neutron stars. Furthermore, we have found that the widely used parabolic approximation to the EOS of asymmetric nuclear matter leads to significantly higher core-crust transition densities and pressures, especially for stiff nuclear symmetry energies.

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