Abstract

When the dimensions decrease to nano-scale, many essential phenomena appear which are not important at macro scales. In this paper, two effects of these sub-micron phenomena are demonstrated and considered for simulation of pull-in instability of beam-type nano-actuator. The first phenomenon that becomes important at nano-distances is the presence of dispersion forces such as van der Waals attraction. The second issue that must be considered in modeling nano-structures is characterization of real boundary condition. The obtained results agree well with numerical solution and other models in the literature.

Keywords: NEMS NEMS; MAD; Real boundry conditions; van der Waals force

1. Introduction

Beam-type NEMS are increasingly used in many applications such as electrical, medical, optical and microscopy devices [1,2]. Consider a beam-type actuator constructed from two conductive electrodes which one is fixed and the other is movable. Applying voltage difference between these electrodes causes the movable electrode to deflect towards the fixed electrode (ground electrode), because of the electrostatic forces. At a critical voltage, which is known as pull-in voltage, the electrode becomes
unstable and pulls-in onto the ground electrode. The pull-in behavior of micro-electromechanical systems (MEMS) has been studied for over two decades without considering nano-scale effects [3-5]. With decrease in dimensions to nano-scale, many essential phenomena appear which are not important at macro scales. In this paper, two effects of these sub-micron phenomena are demonstrated and considered for simulation of pull-in instability of beam-type nano-actuator.

The first phenomenon that becomes important at nano-distances is the presence of dispersion forces such as van der Waals attraction. This attraction can significantly influence the NEMS performance when the initial gap between the components of nanoactuator is typically below several ten nanometers [2]. In this case, the attraction between two surfaces is proportional to the inverse cube of the separation and is affected by material properties [6]. Batra et al. [7] studied the pull-in behavior of micro-plates considering vdW attraction. Spengen et al. [8] studied the stiction in micro-electromechanical systems (MEMS) due to the van der Waals force. Dequesnes et al. [9] calculated the effect of van der Waals attraction on the instability voltage of carbon-nanotube-based NEMS switches. Rotkin [10] obtained an analytical relation to express the effect of van der Waals force on the pull-in gap and voltage of a nano-actuator. The dynamic behavior of a nanoscale electrostatic actuator was investigated by Lin and Zhao [11] considering the effect of van der Waals force on the instability of NEMS using a two-parameter lumped model. Ramezani et al. [12] applied distributed parameter model to study the pull-in instability of nano-cantilevers.

The second issue that must be considered in modeling nano-structures is characterization of real boundary condition. Supported boundary characterization is important in many applications such as flexible optical waveguides [13] atomic force microscope probes [14] micro-bridges [15] and MEMS/NEMS switches [16]. It is obvious that the boundary conditions of real nano-structures are not always homogeneous and may be flexible by rotation [16]. The static and dynamic responses of structure members such as beam element become varied under different boundary conditions. Rinaldi et al. [18] characterized the non-classical B.C. of micro-cantilevers through electromechanical test. Yunqiang et al. [19] studied the B.C. effect on the static and dynamic responses of micro-plates. According to limitations of manufacturing techniques in nano-scale productions, any ideal boundary condition (B.C.), for instance, clamped or simple supported condition might not be acceptable and the boundary support conditions need to be theoretically quantified and experimentally validated [19].

To the knowledge of the authors, none of the two mentioned phenomena have contributed together in any of the pull-in models proposed by previous researchers. In this study, the effects of van der Waals attraction and elastic B.C. demonstrated together on the pull-in behavior of beam-type NEMS for the first time. The Euler beam model is applied as a time-saving continuum approach to obtain constitutive governing equations [20-25, 12]. The rotational artificial springs are used at the supported ends to model the B.C. of double supported nano-beams. In order to solve the constitutive equation of nano-structures, modified Adomian decomposition (MAD) is utilized. The MAD results are compared with the numerical data as well as other results reported in the literature.

2. Governing equation

Figure 1 show NEMS actuators. The actuators are modelled by a beam of length $L$ with a uniform rectangular cross section of width $B$ and thickness $H$ which are suspended over a conductive substrate. Two artificial angular springs with spring stiffness of $K_{1\theta}$ and $K_{2\theta}$ is used to model the real B.C. in nano-actuator. Note that considering axial tractions and forces along the beam is out of the scope of this work and these effects will be considered in further publications. In order to develop the governing equation of
the beams, we apply the minimum energy principle which implies equilibrium when the free energy reaches a minimum value. The final governing equilibrium equation written as

\[
\left( EI \right) \frac{d^4 w}{dx^4} = \frac{\varepsilon_0 B V^2}{2 \left( g - w \right)} \left( 1 + 0.65 \frac{g - w}{B} \right) + \frac{\bar{AB}}{6\pi \left( g - w \right)^3}.
\]

where \( I \) is the second moment of cross-sectional area around \( Z \)-axis, \( A \) is the cross-sectional area of the beam, in Eq. (1) first and second right hand terms are the electrostatic force and van der Waals forces per unit length of the beam, respectively. \( \varepsilon_0 = 8.854 \times 10^{-12} \text{C}^2\text{N}^{-1}\text{m}^{-2} \) is the permittivity of vacuum, \( V \) is the applied voltage, \( g \) is the initial gap between the movable and the ground electrode and \( \bar{A} \) is the Hamaker constant. In this equation, the constitutive material of the nano-actuator is assumed linear elastic and only the static deflection of the nano-beam is considered.

\[
\begin{align*}
\left( EI \right) \frac{d^4 w}{dx^4} &= \frac{\varepsilon_0 B V^2}{2 \left( g - w \right)} \left( 1 + 0.65 \frac{g - w}{B} \right) + \frac{\bar{AB}}{6\pi \left( g - w \right)^3} \\
\frac{d^2 w}{dx^2} &= K_w \frac{dw}{dx} \quad \left( EI \right) \frac{d^2 w}{dx^2} = -K_w \frac{dw}{dx}
\end{align*}
\]

In above equations, the non-dimensional parameters, \( \alpha, \beta \) and \( \gamma \) are defined as

\[
\alpha = \frac{\bar{AB} L^4}{6\pi g^4 E I}, \quad \beta = \frac{\varepsilon_0 B V^2 L^4}{2g^3 E I}, \quad \gamma = 0.65 \frac{g}{B}
\]

Relations (2-3) present the governing equation of beam-type nanostructures. In order to study pull-in behavior of nanostructures, Eq. (2) solved numerically using MAPLE commercial software. Furthermore, MAD is applied to the boundary value problem and the analytical results are compared with those of numerical solution in the following section (for detail of MAD see [25]). Geometry and constitutive material of the beams are identified in Table 1.

Table 1. parameters and material properties of the nano-beam

| Material Properties | Geometrical Dimensions |
|---------------------|------------------------|
| \( E \) (GPa)       | \( v \)                | \( L \) (\( \mu \text{m} \)) | \( W \) (\( \mu \text{m} \)) | \( H \) (\( \mu \text{m} \)) | \( g \) (\( \mu \text{m} \)) |
| 77                  | 0.33                   | 300                          | 0.5                           | 1                             | 2.5                            |
The comparison between pull-in voltages obtained by MAD and those of the literature is presented in Table 2. It reveals that the difference between MAD and numerical value is within the range of those of other methods presented in the literature.

Table 2. pull-in voltage ($V$) comparison for cantilever beam, vdW force is neglected

| Ref. [3] | Ref. [28] | Ref. [29] | Ref. [30] | Ref. [12] | Numerical | MAD  |
|----------|-----------|-----------|-----------|-----------|-----------|------|
| 1.23     | 1.20      | 1.21      | 1.21      | 1.29      | 1.24      | 1.27 |

3. Results and discussion

Figure 2 depicted the effect of elastic B.C. on the pull-in voltage of clamp-clamp beam for $\alpha=0$. Neglecting van der Waals force is a common practice in MEMS literature. As seen, stiffening the B.C. increases the pull-in voltage of the beam.

Fig. 2. Effects of rotational spring coefficients ($K_1$ and $K_2$) on pull-in voltage ($\alpha_{\text{PI}}$) for $\alpha=0$

Figure 3 shows the variation of $\alpha_C$ for freestanding NEMS. Despite MEMS, intermolecular force cannot be neglected in NEMS. When the gap between the ground and movable electrode is small enough, the movable nano-beam might collapse onto the ground without applying voltage due to the van der Waals force ($\alpha>>\alpha_C$). This phenomenon must be considered in designing the minimum gap of NEMS to ensure that the NEMS does not adhere to the substrate as a result of intermolecular force. Figure 3 also reveals that stiffening the B.C. increases the $\alpha_C$ of the beam.

Fig. 3. Effects of rotational spring coefficients ($K_1$ and $K_2$) on critical van der Waals force ($\alpha_C$) for $\beta=0$. 
4. Conclusions

In this article, MAD solving method has been introduced to investigate the effect of van der Waals attraction and real boundary conditions on the nonlinear pull-in behavior of supported NEMS. Results reveal that van der Waals force decreases the pull-in voltage and increase deflection of NEMS in submicron scales. Also, the instability of NEMS strongly depends on the type of applied B.C.. This emphasizes the importance of characterizing real B.C. in design and analyze of NEMS. The proposed MAD solutions avoid time-consuming numerical iterations and make parametric studies possible.

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