Adiabatic hydrodynamization in the rapidly-expanding quark-gluon plasma

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Based on:
JB, Li Yan, and Yi Yin [arXiv:1910.00021]
and ongoing work
Expected applicability of hydrodynamics

Non-equilibrium initial conditions

Many modes are important

Hydrodynamic modes dominate

“Gradients are large so hydro doesn’t work”

“Gradients are small so hydro works”
Observation of attractor behavior

Heller and Spalinski [1503.07514], many follow-ups

Suggestive of simplified bulk description before $\tau_{\text{Hydro}}$

Fig: Romatschke [1704.08699]
Bjorken-expanding kinetic theory in the relaxation time approximation
Bjorken-expanding kinetic theory in the relaxation time approximation
Many modes are important

“Pre-hydrodynamic” modes dominate

Hydrodynamic modes dominate

\( \frac{p_L}{\epsilon} \)

\( \tau / \tau_C \)

\( \tau \)

\( \tau_{Redu} \)

\( \tau_{Hydro} \)
Adiabatic theorem:

“A system prepared in its (instantaneous) ground state will remain in its (instantaneous) ground state under adiabatic evolution of the Hamiltonian”
Adiabatic interpretation of far-from-equilibrium behavior

Pre-hydrodynamic mode is instantaneous ground state of an effective Hamiltonian
Adiabatic interpretation of far-from-equilibrium behavior

Dominance of ground state at early times driven by rapid longitudinal expansion
Hamiltonian formulation from kinetic theory

Bjorken-expanding kinetic theory

\[
\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f]
\]

longitudinal expansion

collisions
Hamiltonian formulation from kinetic theory

**Bjorken-expanding kinetic theory**

\[
\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f]
\]

longitudinal expansion collisions

Moments of kinetic equation give evolution of more macroscopic quantities

\[
\int_{|\mathbf{p}|} |\mathbf{p}| f(p_z, p_\perp; \tau) = \epsilon(\tau)
\]

Energy density

\[
\int_{|\mathbf{p}|} |\mathbf{p}| f(p_z, p_\perp; \tau) = F_\epsilon(\cos \theta; \tau)
\]

Angular distribution contributing to energy density
Hamiltonian formulation from kinetic theory

\[ \int_{|p|} \left( \frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f] \right) \]

\[ \frac{\partial}{\partial \tau} F_\varepsilon \quad - \frac{1}{\tau} (...) F_\varepsilon \]
Hamiltonian formulation from kinetic theory

\[ \int_{|p|} \left( \frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f] \right) \]

\[ \frac{\partial}{\partial \tau} F_\epsilon - \frac{1}{\tau} (...) F_\epsilon \]

For some $\hat{C}[f]$

\[ \tau \frac{\partial}{\partial \tau} F_\epsilon = (...) F_\epsilon \quad \leftrightarrow \quad \partial_y \psi = -\mathcal{H}(y)\psi \quad y = \log \left( \frac{\tau}{\tau_I} \right) \]

Schrodinger-type evolution of generalized Hamiltonian $\mathcal{H}$
\[ F_\epsilon(\cos \theta; \tau) = \epsilon(\tau) + \sum_{n=1}^{4n + 1} \frac{1}{2} \mathcal{L}_n(\tau) P_{2n}(\cos \theta) \]

\[ \psi = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, \ldots) \]
\[ F_\varepsilon(\cos \theta; \tau) = \epsilon(\tau) + \sum_{n=1}^{4n+1} \frac{4n + 1}{2} \mathcal{L}_n(\tau) P_{2n}(\cos \theta) \quad \overset{\longleftrightarrow}{\psi} = (\epsilon, \mathcal{L}_1, \mathcal{L}_2, \ldots) \]

Fig adapted from KMPST [1805.00961]
Is the system prepared in the ground state?

At early times \( \hat{C}[f] = 0 \quad \rightarrow \quad \partial_y \psi = -\mathcal{H}_F \psi \)

\[
\psi(\tau) \sim \sum_n \beta_n(\tau) \phi_n^F \quad \psi(\tau) \sim \phi_0^F
\]

\[
\beta_n(\tau) \sim \beta_n(\tau_I) e^{-\mathcal{E}_n^F \tau} \quad \text{gives time scale for decay of initial conditions}
\]
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At early times  \( \hat{C}[f] = 0 \)  \( \rightarrow \)  \( \partial_y \psi = -\mathcal{H}_F \psi \)

Since \( \mathcal{H}_F \) is gapped, \( \psi \) decays toward the ground state.

\[ \psi(\tau) \sim \sum_n \beta_n(\tau) \phi_n^F \]

\( \beta_n(\tau) \sim \beta_n(\tau_I) e^{-\varepsilon_n^F \gamma} \)  gives time scale for decay of initial conditions.

\( \frac{pL}{\epsilon} \quad 0.4 \]

\( 0.2 \quad \psi(\tau) \sim \phi_0^F \]

\( 0.0 \quad \tau / \tau_C \quad 10^{-2} \quad 10^{-1} \)
Is the system prepared in the ground state?

At early times $\hat{C}[f] = 0$ \quad $\partial_y \psi = -\mathcal{H}_F \psi$

Since $\mathcal{H}_F$ is gapped, $\psi$ decays toward the ground state.

$\beta_n(\tau) \sim \beta_n(\tau_I) e^{-\varepsilon_n^F} \tau$ gives time scale for decay of initial conditions.

Attractor behavior doesn’t imply collectivity.
System prepared in ground state

Is the subsequent evolution adiabatic?
Definition of adiabatic hydrodynamization:

System evolution determined by the instantaneous ground state

\[ \psi(y) \sim \phi_0(y) \]
Definition of adiabatic hydrodynamization:

System evolution determined by the instantaneous ground state \( \psi(y) \sim \phi_0(y) \)

Predicts non-trivial relations between components of \( \psi \)

\[
\partial_y \psi = -\mathcal{H}(y)\psi \quad \implies \quad \partial_y \phi_0(y) = -\mathcal{E}_0(y)\phi_0(y)
\]

e.g. \( g(y) \equiv 1 + p_L/\epsilon = \mathcal{E}_0(y) \)

Test extent to which these relations are satisfied!
Hydrodynamization in RTA kinetic theory is adiabatic!

Far-from-equilibrium evolution dominated just by evolution of instantaneous ground state ("pre-hydrodynamic") mode!
Implies presence of a small “adiabatic” parameter that suppresses contributions from other modes.

\[ \frac{|g-E_0|}{g} \]

$\tau/\tau_C$

> 95% of $g$ described by instantaneous ground state mode
Why would the rapidly-expanding QGP be adiabatic?

Suppression of excited states: \[ \delta_A \sim \frac{\partial \tau \log \lambda}{\Delta E_n} \langle 0(\tau) | H | n(\tau) \rangle \]
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Hamiltonian evolves slowly compared to energy gap

\[ \delta_A \text{ small near hydro limit} \]
Why would the rapidly-expanding QGP be adiabatic?

Suppression of excited states:

$$\delta_A \sim \frac{\partial_{\tau} \log \lambda}{\Delta E_n} \braket{0(\tau)|H|n(\tau)}$$

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Small overlap between ground and excited states

$\delta_A$ small near hydro limit

$\delta_A$ small at early times
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\( \delta_A \) small near hydro limit

\( \delta_A \) small at early times

Bottom-up thermalization: parametrically narrow window when \( \delta_A \) can be large
A new perturbative expansion

Including contributions from the excited states to the evolution at $\mathcal{O}(\delta_A)$ shows explicitly that they are a small correction.
Many modes are important

“Pre-hydrodynamic” modes dominate

Hydrodynamic modes dominate

Single “pre-hydrodynamic” mode