Implications of Solar and Atmospheric Neutrinos

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The importance of non-zero neutrino mass as a probe of particle physics, astrophysics, and cosmology is emphasized. The present status and future prospects for the solar and atmospheric neutrinos are reviewed, and the implications for neutrino mass and mixing in 2, 3, and 4-neutrino schemes are discussed. The possibilities for significant mixing between ordinary and light sterile neutrinos are described.

1. NEUTRINO MASS

Neutrino mass and properties are superb simultaneous probes of particle and astrophysics:

- Decays and scattering processes involving neutrinos have been powerful probes of the existence and properties of quarks, tests of QCD, of the standard electroweak model and its parameters, and of possible TeV-scale physics.
- Fermion masses in general are one of the major mysteries/problems of the standard model. Observation or nonobservation of the neutrino masses introduces a useful new perspective on the subject.
- Nonzero \( \nu \) masses are predicted in most extensions of the standard model. They therefore constitute a powerful window on new physics at the TeV scale, intermediate scales (e.g., \( 10^{12} \) GeV), or the Planck scale.
- There may be a hot dark matter component to the universe. If so, neutrinos would be (one of) the most important things in the universe.
- The neutrino masses must be understood to fully exploit neutrinos as a probe of the Solar core, of supernova dynamics, and of nucleosynthesis in the big bang, in stars, and in supernovae.

2. THEORY OF NEUTRINO MASS

There are a confusing variety of models of neutrino mass. Here, I give a brief survey of the principle classes and of some of the terminology. For more detail, see [1].

A Weyl two-component spinor is a left (\( L \))-handed particle state, \( \psi_L \), which is necessarily associated by CPT with a right (\( R \))-handed antiparticle state \( \psi_R^c \). One refers to active (or ordinary) neutrinos as left-handed neutrinos which transform as \( SU(2) \) doublets with a charged lepton partner. They therefore have normal weak interactions, as do their right-handed anti-lepton partners,

\[
\left( \begin{array}{c} \nu_e \\ e^- \end{array} \right)_L \xrightarrow{\text{CPT}} \left( \begin{array}{c} e^+ \\ \nu_e \end{array} \right)_R.
\]

(Sterile neutrinos will sometimes also be denoted \( \nu_s \).)

Mass terms describe transitions between right (\( R \)) and left (\( L \))-handed states. A Dirac mass

\[
N_R \xleftarrow{\text{CPT}} N_L^c.
\]
term, which conserves lepton number, involves transitions between two distinct Weyl neutrinos $\nu_L$ and $N_R$:

$$ L_{\text{Dirac}} = m_D(\bar{\nu}_L N_R + \bar{N}_R \nu_L) = m_D \bar{\nu} \nu, \quad (3) $$

where the Dirac field is defined as $\nu = \nu_L + N_R$. Thus a Dirac neutrino has four components $\nu_L$, $\nu_R$, $N_R$, $N_R^c$, and the mass term allows a conserved lepton number $L = L_\nu + L_N$. This and other types of mass terms can easily be generalized to three or more families, in which case the masses become matrices. The charged current transitions then involve a leptonic mixing matrix (analogous to the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix), which can lead to neutrino oscillations between the light neutrinos.

For an ordinary Dirac neutrino the $\nu_L$ is active and the $N_R$ is sterile. The transition is $\Delta I = \frac{1}{2}$, where $I$ is the weak isospin. The mass requires $SU(2)$ breaking and is generated by a Yukawa coupling

$$ L_{\text{Yukawa}} = \bar{\nu}_L(v_c \bar{e}_L) \left( \begin{array}{c} \nu^0 \\ \nu^- \end{array} \right) N_R + \text{H.C.} \quad (4) $$

One has $m_D = h_{\nu} v / \sqrt{2}$, where the vacuum expectation value (VEV) of the Higgs doublet is $v = \sqrt{2}(\phi^0) = (\sqrt{2} G_F)^{-1/2} = 246$ GeV, and $h_{\nu}$ is the Yukawa coupling. A Dirac mass is just like the quark and charged lepton masses, but that leads to the question of why it is so small: one requires $h_{\nu} < 10^{-10}$ to have $m_{\nu_L} < 10$ eV.

A Majorana mass, which violates lepton number by two units ($\Delta L = \pm 2$), makes use of the right-handed antineutrino, $\nu_R$, rather than a separate Weyl neutrino. It is a transition from an antineutrino into a neutrino. Equivalently, it can be viewed as the creation or annihilation of a neutrino and an antineutrino, and if present it can therefore lead to neutrinoless double beta decay. The form of a Majorana mass term is

$$ L_{\text{Majorana}} = \frac{1}{2} m_T(\bar{\nu}_L \nu_R^c + \bar{\nu}_R \nu_L) = \frac{1}{2} m_T \bar{\nu} \nu, \quad (5) $$

where $\nu = \nu_L + \nu_R^c$ is a self-conjugate two-component state satisfying $\nu = \nu^c = C \nu^T$, where $C$ is the charge conjugation matrix. If $\nu_L$ is active then $\Delta I = 1$ and $m_T$ must be generated by either an elementary Higgs triplet or by an effective operator involving two Higgs doublets arranged to transform as a triplet.

One can also have a Majorana mass term

$$ L_{\text{Majorana}} = \frac{1}{2} m_N(\bar{N}_L^c N_R + \bar{N}_R N_L^c) \quad (6) $$

for a sterile neutrino. This has $\Delta I = 0$ and thus can be generated by the VEV of a Higgs singlet.

Some of the principle classes of models for neutrino mass are:

- A triplet majorana mass $m_T$ can be generated by the VEV $v_T$ of a Higgs triplet field. Then, $m_T = h_T v_T$, where $h_T$ is the relevant Yukawa coupling. Small values of $m_T$ could be due to a small scale $v_T$, although that introduces a new hierarchy problem. The simplest implementation is the Gelmini-Roncadelli (GR) model \[2\], in which lepton number is spontaneously broken by $v_T$. The original GR model is now excluded by the LEP data on the $Z$ width.

- A very different class of models are those in which the neutrino masses are zero at the tree level (typically because no sterile neutrino or elementary Higgs triplets are introduced), but only generated by loops \[3\], i.e., radiative generation. Such models generally require the ad hoc introduction of new scalar particles at the TeV scale with nonstandard electroweak quantum numbers and lepton number-violating couplings. They have also been introduced in an attempt to generate large electric or magnetic dipole moments. They also occur in some supersymmetric models with cubic $R$ parity violating terms in the superpotential \[3\].

- In the seesaw models \[3\], a small Majorana mass is induced by mixing between an active neutrino and a very heavy Majorana sterile neutrino $M_N$. The light (essentially active) state has a naturally small mass

$$ m_\nu \sim \frac{m_D^2}{M_N} \ll m_D. \quad (7) $$

4In principle this could also be generated by a bare mass, but this is usually forbidden by higher symmetries in extensions of the standard model.
There are literally hundreds of seesaw models, which differ in the scale $M_X$ for the heavy neutrino (ranging from the TeV scale to grand unification scale), the Dirac mass $m_D$ which connects the ordinary and sterile states and induces the mixing (e.g., $m_D \sim m_\nu$ in most grand unified theory (GUT) models, or $\sim m_e$ in left-right symmetric models), the patterns of $m_D$ and $M_X$ in three family generalizations, etc. One can also have mixings with heavy neutrinos in supersymmetric models with right symmetric models), the patterns of $m_D$ and $M_X$ in three family generalizations, etc. One can also have mixings with heavy neutrinos in supersymmetric models with $R$ parity breaking, induced either by bilinears connecting Higgs and lepton doublets in the superpotential or by the expectation values of scalar neutrino potential. Similarly, one may obtain triplet and singlet Majorana neutrino masses, which differ in the scale $M_X$, the Dirac mass $m_D$ for the heavy neutrino (ranging from the TeV scale to grand unification scale), the Dirac mass $m_D$ which connects the ordinary and sterile states and induces the mixing (e.g., $m_D \sim m_\nu$ in most grand unified theory (GUT) models, or $\sim m_e$ in left-right symmetric models), the patterns of $m_D$ and $M_X$ in three family generalizations, etc. One can also have mixings with heavy neutrinos in supersymmetric models with $R$ parity breaking, induced either by bilinears connecting Higgs and lepton doublets in the superpotential or by the expectation values of scalar neutrino potential. Depending on the dimensions $P$ of the various operators and on the scale $\langle S \rangle$, it may be possible to generate an interesting hierarchy for the quark and charged lepton masses and to obtain naturally small Dirac neutrino masses. Similarly, one may obtain triplet and singlet Majorana neutrino masses, $m_T$ and $m_N$ by analogous higher-dimensional operators. The former are small. Depending on the operators, the latter may be either small, leading to the possibility of significant mixing between ordinary and sterile neutrinos, or large, allowing a conventional seesaw.

• Mixed models, in which both Majorana and Dirac mass terms are present, will be further discussed in the section on sterile neutrinos.

3. SOLAR NEUTRINOS

Tremendous progress has been made recently in solar neutrinos. For many years there was only one experiment, while now there are a number that are running or finished, and more are coming on line soon. The original goal of using the solar neutrinos to study the properties of the solar core underwent a 30 year digression on the study of the properties of the neutrino itself. The quality of the experiments themselves and of related efforts on helioseismology, nuclear cross sections, and solar modeling is such that the revised goal of simultaneously studying the properties of the Sun and of the neutrinos is feasible.

3.1. Experiments

The experimental situation is very promising. We now have available the results of five experiments, Homestake (chlorine), Kamiokande, GALLEX, SAGE, and Superkamiokande. Especially impressive are the successful $^{51}$Cr source experiments for SAGE and GALLEX (which probe a combination of the extraction efficiencies and the neutrino absorption cross section, yielding $0.95 \pm 0.07^{+0.04}_{-0.03}$ of the expected rate), and the successful $^{71}$As spiking experiment completed at the end of the GALLEX run to test the extraction efficiency (yielding $R = 1.00 \pm 0.01$ for the ratio of actual to expected extractions).

Coming soon, there should be results from SNO, Borexino, The Gallium Neutrino Observatory (GNO), and the next phase of SAGE, which will yield much more detailed, precise, or model independent information on the $^8B$ (SNO), $^7Be$ (Borexino), and $pp$ (GNO, SAGE) neutrinos. Future generations of even more precise experiments should especially be sensitive to the $^7Be$ and $pp$ neutrinos. The overall goal of the program should be very ambitious, i.e., to measure the arriving flux of $\nu_e$, $\nu_{\mu+\tau}$, and $\nu_\alpha$ (sterile neutrinos), and even possible antineutrinos, for each of the initial flux components, as well as to measure or constrain possible spectral distortions, day-night (earth) effects, seasonal and solar cycle variations, and mixed (e.g., simultaneous spectral and day-night) effects.
3.2. Interpretation

The observed fluxes are in strong disagreement with the predictions of the standard solar model (SSM). The overall rates are compared with the predictions of the new Bahcall-Pinsonneault 1998 (BP 98) model \[16\] in Table \[1\], where it is seen that all of the fluxes are much lower than the expectations. BP 98 contains a number of refinements compared to earlier theoretical calculations, but the most important changes are a 20% (1.3 $\sigma$) lower $^8B$ flux, as described below, and 1.1 $\sigma$ decreases in the $^{37}Cl$ and $^{71}Ga$ capture rates.

Recent results in helioseismology \[14,20\] leave little room for deviations from the standard solar model. The eigen-frequencies effectively measure the sound speed $T/\mu$, where $T$ and $\mu$ are respectively the temperature and density, as a function of radial position, down to 5% of the solar radius. The results agree with the predictions of BP 98 to $\sim 10^{-3}$, even though $T$ and $\mu$ individually vary by large values over the radius of the Sun. This leaves very little room for non-standard solar models (NSSM), which would typically have to deviate by several percent to have much impact on the neutrino flux predictions. The only aspect of the SSM relevant to the neutrino fluxes that is not severely constrained are nuclear cross sections, especially $S_{17}$ and $S_{34}$, which are respectively proportional to the cross sections for $^8B$ and $^7Be$ production, and to the absorption cross sections for the radiochemical experiments.

The experimental and theoretical status of the nuclear cross sections were critically examined at a workshop at the Institute for Nuclear Theory in 1997 (INT 97) \[21,22\]. The participants recommended a lower $S_{17}$, by relying on the best documented individual measurements rather than an average, and also a larger uncertainty in $S_{34}$, both of which were incorporated in BP 98. Haxton has recently argued \[23\] that there are still considerable uncertainties in the $Ga$ absorption cross sections, but this possibility is strongly disfavored by the $^{51}Cr$ source and $^{71}As$ spiking experiments.

Even the relatively large shift in $S_{17}$ advocated by INT 97 and used by BP 98 does little to change the basic disagreement between the observations and the standard solar model. Even if a particular NSSM could be consistent with helioseismology, it would be difficult to account for the observations. The Kamiokande and Superkamiokande results can be regarded (in the absence of neutrino oscillations) as a measurement of the $^8B$ flux. Subtracting this “experimental” $^8B$ flux from either the gallium or chlorine predictions, the observed fluxes are still inconsistent with the observed solar luminosity.

This line of reasoning is developed in the “model-independent” analyses of the neutrino flux components \[22,24\], which can be viewed as a measurement of “global” spectral distortions. The idea is that all plausible astrophysical or nuclear physics modifications of the standard solar model do not significantly distort the spectral shape of the $pp$ or $^8B$ neutrinos: all that they can do is modify the overall magnitude of the $pp$, $^7Be$, $^8B$, and minor flux components. Furthermore, the observed solar luminosity places a linear constraint on the $pp$, $^7Be$, and CNO fluxes (provided that the time scale for changes in the solar core is long compared to the $10^4$ yr required for a photon to diffuse to the surface).

By combining the different experiments, each class of which has a different spectral sensitivity, one concludes that

$$\frac{\phi(^7Be)}{\phi(^7Be)_{SSM}} \ll \frac{\phi(^8B)}{\phi(^8B)_{SSM}}.$$  \hspace{1cm} (10)

where SSM refers to the standard solar model predictions. The same result holds even if one discards any one of the three types of experiment (chlorine, gallium, water), or ignores the luminosity constraint. No plausible astrophysical model has succeeded in suppressing $^7Be$ neutrinos significantly more than $^8B$ neutrinos, mainly because $^8B$ is made from $^7Be$. Models with a lower core temperature or with a lower $S_{17}$ do not come anywhere near the data. The Cumming-Haxton model \[26\] with large $^3He$ diffusion comes closest, but even that is far from the data. That model is probably also excluded by helioseismology, but Haxton has argued \[22\] that final judgment should wait until a self-consistent model with $^3He$ diffusion is constructed to be compared with the helioseismology data.
Table 1
Results of Solar neutrino experiments, compared with the predictions of BP 98. The chlorine and gallium results are in units of SNU \((10^{-36} \text{s}^{-1} \text{captures per target atom})\), and the water Cerenkov results are in units of \(10^6 / \text{cm}^2 \text{s}\).

| experiment          | BP-98   |
|---------------------|---------|
| Homestake (chlorine)| 2.56 ± 0.23 |
| GALLEX, SAGE (gallium)| 72.2 ± 5.6 |
| Kamiokande, SuperK \((\nu e \rightarrow \nu e)\)| 2.44 ± 0.10 |

Figure 1. Allowed regions for the \(^7\text{Be}\) and \(^8\text{B}\) fluxes (normalized by BP 98), compared with the predictions and uncertainties in the SSM and various non-standard solar models. Courtesy of N. Hata.

3.3. Possible Solutions

As discussed in the previous section, an astrophysical/nuclear explanation of the solar neutrinos experiments is unlikely. The most likely particle physics explanations include:

- A matter enhanced (MSW) transition of \(\nu_e\) into \(\nu_\mu\) or \(\nu_\tau\). There are the familiar small (SMA) and large (LMA) mixing angle solutions \([23]\) with \(\Delta m^2 \sim 10^{-5} \text{eV}^2\), as well as the low mass (LOW) solution with \(\Delta m^2 \sim 10^{-7} \text{eV}^2\) and near maximal mixing. The latter is a very poor fit, but sometimes shows up in fits at the 99% cl.

- There is also a small mixing angle MSW solution for \(\nu_e\) into a sterile neutrino \(\nu_s\). The major difference between \(\nu_\mu, \tau\) and \(\nu_s\), and the reason there is no LMA solution, is that in the first case the \(\nu_\mu, \tau\) can scattering elastically from electrons in the water Cerenkov experiments, with about \(1/6\) the \(\nu_e\) cross section, leading to a lower survival probability for \(\nu_e\) than for astrophysical or sterile neutrino solutions. There is also a small difference for the MSW conversion rate for sterile neutrinos in the Sun, but that is proportional to the neutron density, and is much less important.

- The vacuum (“just so” \([27]\)) oscillation solutions \([23]\), with near maximal mixing and \(\Delta m^2 \sim 10^{-10} \text{eV}^2\) are another possibility. These are somewhat fine-tuned, with \(\Delta m^2\) such that the Earth-Sun distance is at roughly half an oscillation length, \(L_{\text{osc}}\), or an odd multiple. Since \(L_{\text{osc}} = 4\pi E/\Delta m^2\), one expects a significant variation of the \(\nu_e\) survival probability with neutrino energy.

- The above solutions are such that only two neutrinos are important for the Solar neutrinos. However, it is possible that transitions between all three neutrinos are important. There could be generalized MSW solutions involving more than one value of \(\Delta m^2\), or mixed MSW and vacuum solutions \([28]\). In both cases, there could be considerably different spectral distortions than in the two-neutrino case.

Other possibilities include:
Figure 2. Allowed MSW solutions, not including Superkamiokande spectral data. Courtesy of N. Hata.

- Maximal mixing [29] (i.e., vacuum oscillations with $\Delta m^2 \gg 10^{-10} \text{eV}^2$), combined with a low $S_{17}$. Such solutions lead to an energy independent suppression of the $\nu_e$ survival probability. Even allowing a suppressed $^8B$ production rate, this possibility is viable only if one ignores (or greatly expands the uncertainties in) the Homestake Chlorine experiment.

- RSFP [30] (resonant spin flavor precession), involving rotations of left handed neutrinos into sterile right handed neutrinos, combined with MSW flavor transitions. These were motivated by possible hints (not confirmed by other experiments) of time dependence correlated with the Sunspot activity in the chlorine experiment. This could only occur if there are extremely large neutrino electric or magnetic dipole moments or transition moments, which would present a considerable challenge to the model builder. Although such effects have not been reported by other groups, there is still a somewhat surprising difference in rates observed by the GALLEX collaboration in their third and fourth data taking intervals. However, this could also be a statistical fluctuation. In any case, such RSFP effects could be probed experimentally by studying the $\bar{\nu}_e$ and $\bar{\nu}_\mu$ spectra [31].

- Flavor changing neutral current effects [32], possibly generated by $R$-parity violating terms in supersymmetry, could be an alternative means of generating enhanced neutrino flavor conversions in the Sun.

- The possible violation of Lorentz invariance [33] could affect not only the Solar neutrinos, but could also be relevant to the observed ultra high energy cosmic rays.

- There could be a lepton flavor dependent violation of the equivalence principle [34].

Perhaps the most important possibility or complication is that more than one thing could be going on simultaneously. There could be any of the above effects in conjunction with non-standard properties of the Sun or nuclear cross sections. Many but not all such NSSM possibilities are excluded by helioseismology and neutrino source experiments. While it is very unlikely than such effects could by themselves account for the data, their combination with new neutrino properties could considerably confuse the interpretation of future experimental results. This is one or the reasons that it is important to have as many independent precise experimental results as possible.

3.4. Needs

To distinguish the many possibilities we need as much precise data as possible. Especially useful are observables that are independent of or insensitive to the initial $\nu_e$ fluxes, and therefore to the astrophysical and nuclear cross section uncertainties. Such observables include:

- The neutral to charged current interaction ratio (NC/CC), which will be measured by SNO.
for deuteron dissociation. Since the NC cross section is the same for all active neutrinos, the NC rate measures the sum of the $\nu_e$, $\nu_\mu$, and $\nu_\tau$ fluxes, while the CC only measures $\nu_e$. An anomalous NC/CC ratio would provide definitive evidence for transitions of $\nu_e$ into $\nu_\mu$ or $\nu_\tau$, either by MSW or vacuum oscillations. Although the NC measurement is difficult, SNO should have the requisite sensitivity. A confirmation could be obtained by comparing the SNO CC rate with the fluxes determined in $\nu e \rightarrow \nu e$ measurements, since $\nu_{\mu, \tau}$ also contribute to the latter, with about 1/6th $\nu_e$ (by the SMA hypothesis).

Transitions of $\nu_e$ into a sterile neutrino $\nu_s$ would not lead to an anomalous NC/CC ratio. This would make it much harder to verify $\nu_s$ transitions, but would serve as evidence for sterile neutrinos if MSW or vacuum oscillations are established by other means.

- There is no known astrophysical mechanism that can significantly distort the $^8B$ neutrino spectrum from the expected $\beta$ decay shape. Not only would a spectral distortion establish a non-astrophysical solution to the solar neutrinos, but it would be a powerful probe of the mechanism. Study of the $^8B$ spectrum can be viewed as a cleaner extension (by individual experiments) of the “global” spectral distortion inferred from the combined experiments.

One expects significant spectral distortions for the MSW SMA solution, for vacuum oscillations, and for hybrid solutions, but not for the LMA solution. The ratio of observed to expected spectrum can be conveniently parametrized by the first two moments, i.e., a linear approximation, for the SMA case, while the other cases can exhibit more complicated shapes. Measurement of the spectral distortion is very difficult, and requires excellent energy calibrations and extending the measurement to as low an energy as possible. Both SuperKamiokande and SNO have the capability to measure a spectral distortion. SuperK has the advantage of higher statistics. However, the $\nu$ energy is shared between the final electron and neutrino, so any spectral distortion is partially washed out in the observed $e^-$ spectrum. SNO, on the other hand, has the advantage that the electron in the CC reaction carries all of the neutrino energy (plus the known binding energy), leading to a harder electron spectrum and an essentially direct measurement of the $\nu$ spectrum.

One of the highlights of this conference was the preliminary new statistics-limited SuperKamiokande spectrum, from around 6.7 to 14.5 MeV, obtained after a series of careful calibrations of their detector using an electron Linac [34]. The lower energy data are consistent with no distortions, but there is evidence for a significant excess of events in the three energy bins above 13 MeV. These data, for the first time, give a statistically significant indication of a spectral distortion: the no oscillation hypothesis (and also LMA solution) is disfavored at the 95-99% CL level. The SMA MSW solution is also a very poor fit, although it is allowed at 95% CL. The best fit favors vacuum oscillations. The favored $\Delta m^2 \sim 4 \times 10^{-2} \text{ eV}^2$ gives a much better fit to the data than for the lower range $\Delta m^2$ around $10^{-2} \text{ eV}^2$ found in recent global analyses of the total event rates. However, new studies based on BP 98 with its larger $S_{17}$ allow a larger $\Delta m^2$, consistent with the spectral distortions.

An alternate interpretation of the high energy excess is that the flux of hep neutrinos ($^3He + p \rightarrow ^4He + e^+ + \nu_e$) has been seriously underestimated. Their flux would have to be larger by a factor of twenty or so from the usual estimates for them to contribute significantly to the excess, but it has been emphasized that there is no direct experimental measure of or rigorous theoretical bound on the cross section [35]. The issue can be resolved by a careful study of the energy range 14-18.8 MeV, above the endpoint of the $^8B$ spectrum. (The highest energy SuperK bin is centered above this endpoint, but there is a significant energy un-
The SuperK spectrum has important implications, but it is still preliminary. In addition to finalizing the analysis, additional lower energy points are expected that should help clarify the situation.

- For some regions of MSW parameters, one expects an asymmetry between day and night event rates due to regeneration of $\nu_e$ at night as the converted neutrinos travel through the Earth [37]. Superkamionde has binned their data for daytime and for a number of different nighttime zenith angles (i.e., different paths through the earth). They see no evidence for a zenith angle dependence, and their overall day-night asymmetry is

$$\frac{D - N}{D + N} = -0.023 \pm 0.020 \pm 0.014,$$

where $D$ ($N$) refers to day (night) rates and the first (second) error is statistical (systematic). The absence of an effect excludes a significant region of MSW parameter space independent of the details of the solar model (and with only a small uncertainty from the Earth’s density profile). This excludes the lower $\Delta m^2$ part of the LMA solution, but has little impact on the SMA solution. (The part of the SMA solution with the smallest $\sin^2 2\theta$ was expected to have a barely observable day-night asymmetry, but the effect is predicted to be smaller with the new BP 98 fluxes, which shift the SMA region to slightly smaller $\sin^2 2\theta$.)

Several authors have emphasized recently that the Earth effect is significantly enhanced for neutrinos passing through the core of the Earth [38]. (There is an analogous effect for atmospheric neutrinos.) This parametric (or oscillation length) resonance, in which the oscillation length is comparable to the diameter of the core, was included automatically in previous numerical studies, but not explicitly commented on. It is larger for transitions into $\nu_{\mu,\tau}$ than for $\nu_e$. Since relatively few of the solar neutrinos pass through the core for the existing high latitude detectors, it has been suggested that there should be a dedicated experiment at low latitude [39].

- For vacuum oscillations [23], the Earth-Sun distance is typically at a node of the oscillations. This is somewhat fine-tuned, leading to the name “just so”. Since the oscillation length is $4\pi E/\Delta m^2$ there is a strong energy dependence to the survival probability. One also expects a strong seasonal variation, due to the eccentricity of the Earth’s orbit. However, the seasonal variation can be partially washed out as one averages over energies, so one should ideally measure the spectral shape binned with respect to the time of the year [40].

- RSFP could lead to long term variations in the neutrino flux, e.g., correlated with Sunspots or Solar magnetic fields. Other changing magnetic effects could conceivably alter the solar neutrinos in other ways, e.g., by changing the local density. Only the Homestake experiment has seen any significant hint of a time variation, and that hint has been considerably weakened by more recent Homestake data. Nevertheless, it is conceivable that time dependent effects are energy dependent, and therefore different experiments have different sensitivity. They could also have been somewhat hidden in the water Cerenkov experiments because of the neutral current. It would be useful to run all of the experiments simultaneously through a solar cycle.

- RSFP [30] could also lead to the production of $\bar{\nu}_e$ which can be observed in the SNO detector by the delayed coincidence of the $\gamma$ ray emitted by the capture of the neutron from $\bar{\nu}_e p \rightarrow e^+ n$.  

3.5. Outlook

The model independent observables that can be measured by SuperK and SNO for the $^8B$ neutrinos should go far towards distinguishing the different possibilities. However, it will be especially difficult to establish transitions into sterile neutrinos. It will also be very difficult to sort out what is happening in a three-flavor or hybrid scenario, such as MSW transitions combined with non-standard solar physics. For these reasons, we would like to have accurate information on
spectral distortions, day-night effects (especially for neutrinos passing through the Earth core), NC/CC ratios, and absolute fluxes arriving at the Earth for the $^7$Be and $pp$ neutrinos as well. There is a strong need for the next generations of experiments.

A challenging but realistic goal is to simultaneously establish the neutrino mechanism(s) (e.g., MSW SMA solution), determine the neutrino parameters, and study the Sun [11]. Even with existing data, if one assumed two-flavor MSW but allowed an arbitrary solar core temperature $T_C$, it was possible to simultaneously determine the MSW parameters (with larger uncertainties than when the SSM is assumed) and $T_C$, with the result that $T_C = 0.99^{+0.62}_{-0.63}$ with respect to the SSM prediction of $1.57 \times 10^7$ K [24]. In the future, it should be possible to determine the neutrino parameters and simultaneously the $^7B$ and $^7Be$ fluxes, for comparison with the SSM predictions. It will also be possible to constrain density fluctuations in the Sun [12], which can smear out the MSW effects. However, recent estimates suggest that such effects are negligible [31].

To fully exploit the future data, it will be important to carry out global analyses of all of the observables in all of the experiments (possibly incorporating helioseismology data as well). Global analyses are difficult because of difficulties with systematic errors. However, they often contain more information than the individual experiments, and allow uniform treatment of theoretical uncertainties. For this purpose, it is important that each experiment publish all of their data, such as double binning the data with respect to energy and zenith angle, including full systematics and correlations.

4. ATMOSPHERIC NEUTRINOS

Although the prediction for the absolute number of $\mu$ or $e$ produced by the interactions of neutrinos produced in cosmic ray interactions in the atmosphere has a theoretical uncertainty of around 20%, it is believe that the ratio $N(\mu)/N(e)$ can be predicted to within 5% [14]. To zeroth approximation, the ratio is just two, independent of the details of the cosmic ray flux or interactions, because each produced pion decays into two $\nu_\mu$ and one $\nu_e$ (I am not distinguishing $\nu$ from $\bar{\nu}$), and for energies large compared to $m_\mu$ the interaction cross sections are the same. Of course, the actual ratio depends on the neutrino energies, and therefore on the details of the hadronic energies, polarization of the intermediate muons from $\pi$ decay, etc.

For years the ratio $R$ of observed $N(\mu)/N(e)$, normalized by the predicted value, found in the water Cerenkov experiments (Kamiokande, IMB, SuperKamiokande) has been around 0.6 [25]. This has recently been confirmed by the higher SuperK [16] statistics ($R = 0.63(3)(5)$ for sub-GeV events and 0.65(5)(8) for multi-GeV), and independently by the iron calorimeter experiment at Soudan [27] (0.58(11)(5)) and by Macro [18] (0.53(15) for upward events and 0.71(21) for stopping or downgoing events). This depletion of $\mu$ events suggests the possibility of $\nu_\mu$ oscillations into $\nu_e$, $\nu_\tau$, or $\nu_s$, with near-maximal mixing ($\sin^2 2\theta > 0.8$) and $\Delta m^2 \sim 10^{-3} - 10^{-2}$ eV$^2$.

To confirm oscillations, more detailed information is needed. Already, the CHOOZ [19] (France) reactor $\bar{\nu}_e$ disappearance experiment excludes the $\nu_\mu \rightarrow \nu_e$ interpretation of the atmospheric neutrino anomaly for $\Delta m^2 > 10^{-3}$ eV$^2$. This should be extended by the coming Palo Verde experiment [50], and the planned Kamland [51] experiment at Kamiokande (sensitive to many nearby reactors) should extend the sensitivity down to the MSW LA solar neutrino range.

In the future [22], there will also be accelerator long baseline experiments for $\nu_\mu \rightarrow \nu_\mu, \tau$ appearance, or $\nu_\mu$ disappearance (into $\nu_e, \tau, s$). The KEK to Kamiokande (K2K) experiment will be sensitive to $\nu_\mu$ disappearance down to $\Delta m^2 \sim 5 \times 10^{-3}$ eV$^2$, while the Fermilab to Soudan (MINOS) experiment will probe both appearance and disappearance down to $10^{-3}$ eV$^2$. There are also proposals for a CERN to Gran Sasso experiment (ICARUS, OPERA), which would be sensitive to most of the parameter range suggested by Superkamiokande. These experiments should be able to confirm or refute the atmospheric neutrino oscillations, except possibly for the small-

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5The long baseline experiments were proposed when the
est $\Delta m^2 \sim 10^{-3}$ eV$^2$.

Much more detailed information can be derived from the atmospheric neutrino data itself, by searching for indications of the $\sin^2(2\Delta m^2 L/E)$ dependence of the transition probability characteristic of neutrino oscillations. ($L$ is the distance traveled and $E$ is the neutrino energy.) This can be studied by considering the zenith angle distribution for fixed neutrino energy (in practice, the data is divided into sub-GeV and mutli-GeV bins), or by up-down asymmetries $(U-D)/(U+D)$, where $U$ and $D$ are respectively the number of up and downgoing muons or electrons $^{[53]}$. The data can also be plotted as a function of $L/E$, but that is less direct since the full neutrino energy is not measured on an event by event basis in the water Cerenkov experiments.

The Kamiokande collaboration observed an indication of oscillations in their zenith angle distribution for contained events $^{[54]}$. However, the new Superkamiokande zenith angle distributions for contained events have much better statistics. They strongly indicate a zenith angle distribution in muon events consistent with oscillations, with an enhanced effect in the multi-GeV sample, consistent with expectations. There is no anomaly or excess in the electron events. This implies that $\nu_\mu$ is oscillating into $\nu_\tau$ or possibly a sterile neutrino $\nu_s$, and not into $\nu_e$. The latter result confirms the conclusions of CHOOZ. (Subdominant oscillations into $\nu_e$ in three-neutrino schemes are still possible.) The SuperK events virtually establish neutrino oscillations. Independent evidence is obtained by the zenith angle distributions for upward through-going muon events from SuperK, MACRO$^{[55]}$, and very preliminary results from SOUDAN.

Future atmospheric neutrino observations could possibly shed further light on the question of whether $\nu_\mu$ is oscillating into $\nu_\tau$ or into $\nu_s$, although they are all very difficult. These include (a) subtle (e.g., parametric resonance) effects on neutrinos propagating through the Earth’s core $^{[55]}$, which would affect $\nu_\mu \rightarrow \nu_s$, but not $\nu_\mu \rightarrow \nu_\tau$ (because $\nu_\mu$ and $\nu_\tau$ have the same neutral current interactions). In either scenario, secondary $\nu_\mu \rightarrow \nu_e$ oscillations would also be modified by Earth core effects. (b) The NC/CC ratio, including its zenith angle distribution and up-down asymmetry $^{[56]}$. The NC rate could in principle be measured in $\nu N \rightarrow \nu \pi^0 X$, although this is a very difficult measurement. The preliminary SuperK result $^{[10]}$ $R(\pi^0/e) = 0.93(7)(19)$ on the ratio or $\pi^0$ to $e$ events compared to expectations slightly favors $\nu_\mu \rightarrow \nu_\tau$ but does not exclude $\nu_\mu \rightarrow \nu_s$. (c) Direct observation of events in which $\nu_\tau$ produces a $\tau$ would establish $\nu_\mu \rightarrow \nu_\tau$ oscillations $^{[57]}$. However, this is extremely difficult.

There may also be significant three neutrino effects. For example, even if the dominant transition for the atmospheric neutrinos involves $\nu_\mu \rightarrow \nu_\tau$, there could be important subdominant $\nu_e$ effects.

There have been several careful phenomenological analyses of the atmospheric neutrino data in two neutrino and three neutrino mixing schemes $^{[58]}$. One important theoretical issue posed by the atmospheric neutrinos, is why is there nearly maximal mixing (i.e., $\sin^2 2\theta \sim 1$), when most theoretical schemes involving hierarchies of neutrino masses, as well as the analogs in the quark mixing sector, yield small mixings.

5. IMPLICATIONS FOR NEUTRINO MIXING

5.1. The Global Picture

Various scenarios for the neutrino spectrum are possible, depending on which of the experimental indications one accepts. The simplest scheme, which accounts for the Solar (S) and Atmospheric (A) neutrino results, is that there are just three light neutrinos, all active, and that the mass eigenstates $\nu_i$ have masses in a hierarchy, analogous to the quarks and charged leptons. In that case, the atmospheric and solar neutrino mass-squared differences are measures of the mass-squares of the two heavier states, so that $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 0.03 - 0.1$ eV; $m_2 \sim (\Delta m^2_{solar})^{1/2} \sim 0.003$ eV (for MSW) or $\sim 10^{-5}$ eV (vacuum oscillations), and $m_1 \ll m_2$.
The weak eigenstate neutrinos $\nu_a = (\nu_e, \nu_\mu, \nu_\tau)$ are related to the mass eigenstates $\nu_i$ by a unitary transformation $\nu_a = U_{ai} \nu_i$. If one makes the simplest assumption (from the Superkamiokande and CHOOZ data), that the $\nu_e$ decouples entirely from the atmospheric neutrino oscillations, $U_{e3} = 0$, (of course, one can relax this assumption somewhat) and ignores possible CP-violating phases [59], then

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 0 \\
0 & c_\alpha & -s_\alpha \\
0 & s_\alpha & c_\alpha
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

(12)

where $\alpha$ and $\theta$ are mixing angles associated with the atmospheric and solar neutrino oscillations, respectively, and where $c_\alpha \equiv \cos \alpha$, $s_\alpha \equiv \sin \alpha$, and similarly for $c_\theta$, $s_\theta$.

For maximal atmospheric neutrino mixing, $\sin^2 2\alpha \sim 1$, this implies $c_\alpha = s_\alpha = 1/\sqrt{2}$, so that

$$
U = \begin{pmatrix}
c_\theta & -s_\theta & 0 \\
s_\theta & c_\theta & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(13)

For small $\theta$, this implies that $\nu_{3,2} \sim \nu_{+,-} \equiv (\nu_\tau \pm \nu_\mu)/\sqrt{2}$ participate in atmospheric oscillations, while the solar neutrinos are associated with a small additional mixing between $\nu_e$ and $\nu_{\mu}$. Another limit, suggested by the possibility of vacuum oscillations for the solar neutrinos, is $\sin^2 2\theta \sim 1$, or $c_\theta = s_\theta = 1/\sqrt{2}$, yielding

$$
U = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
$$

(14)

which is referred to as bi-maximal mixing [60]. A number of authors have discussed this pattern and how it might be obtained from models, as well as how much freedom there is to relax the assumptions of maximal atmospheric and solar mixing (the data actually allow $\sin^2 2\alpha \gtrsim 0.8$ and $\sin^2 2\theta \gtrsim 0.6$) or the complete decoupling of $\nu_e$ from the atmospheric neutrinos. Another popular pattern,

$$
U = \begin{pmatrix}
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
$$

(15)

known as democratic mixing [61], yields maximal solar oscillations and near-maximal $(8/9)$ atmospheric oscillations.

In this hierarchical pattern, the masses are all too small to be relevant to mixed dark matter (in which one of the components of the dark matter is hot, i.e., massive neutrinos) or to neutrinoless double beta decay ($\beta\beta_{0v}$). However, the solar and atmospheric oscillations only determine the differences in mass squares, so a variant on this scenario is that the three mass eigenstates are nearly degenerate rather than hierarchical [62], with small splittings associated with $\Delta m_{\text{atm}}^2$ and $\Delta m_{\text{solar}}^2$. For the common mass $m_{\text{av}}$ in the 1-several eV range, the hot dark matter could account for the dark matter on large scales (with another, larger, component of cold dark matter accounting for smaller structures) [63]. If the neutrinos are Majorana they could also lead to $\beta\beta_{0v}$ [64]. Current limits imply an upper limit of

$$
\langle m_{\nu_x} \rangle = \sum_i \eta_i U_{xi}^2 |m_i| < 0.46 - 1 \text{ eV},
$$

(16)

on the effective mass for a mixture of light Majorana mass eigenstates, where $\eta_i$ is the CP-parity of $\nu_i$ and the uncertainty on the right is due to the nuclear matrix elements. (There is no constraint on Dirac neutrinos.) The combination of small $\langle m_{\nu_x} \rangle \ll m_{\text{av}}$, maximal atmospheric mixing, and $U_{e3} = 0$ would imply cancellations, so that $\eta_1 \eta_2 = -1$ and $c_\theta = s_\theta = 1/\sqrt{2}$, i.e., maximal solar mixing. Even the more stringent limit in (16) is large enough that there is room to relax all of these assumptions considerably. Nevertheless, there is strong motivation to try to improve the $\beta\beta_{0v}$ limits.

The LSND experiment [65] has reported evidence for $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations with $\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$ and small mixing $\sim 10^{-3} - 10^{-2}$, while the KARMEN experiment sees no candidates. KARMEN [66] is sensitive to most of the same parameter range as LSND, although
there is a small window of oscillation parameters for which both experiments are consistent. A resolution of the situation may have to wait for the mini-BOONE experiment at Fermilab. However, it is interesting to consider the implications if the LSND result is confirmed. In that case, there are three distinct mass-squared differences, \( \Delta m^2_{\text{LSND}} \approx 1 \text{ eV}^2 \), \( \Delta m^2_{\text{atm}} \approx 10^{-3} - 10^{-2} \text{ eV}^2 \), and \( \Delta m^2_{\text{solar}} \approx 10^{-5} \text{ eV}^2 \) (MSW) or \( 10^{-10} \text{ eV}^2 \) (vacuum), implying the need for a fourth neutrino. Since the Z lineshape measurements at LEP only allow \( 2.992 \pm 0.011 \) light, active neutrinos \(^7\), any light fourth neutrino would have to be sterile, \( \nu_s \).

Several mass patterns for the four neutrinos have been suggested \(^7\). (There course also be more than four light neutrinos \(^7\).) To be consistent with both LSND and CHOOZ, states containing \( \nu_\mu \) and \( \nu_e \) must be separated by about 1 eV. Assuming the atmospheric neutrinos involve \( \nu_\mu \rightarrow \nu_\tau \), one could have nearly degenerate \( \nu_{\pm,-} \equiv (\nu_\tau \pm \nu_\mu)/\sqrt{2} \) at around 1 eV, with the solar neutrinos described by a dominantly \( \nu_\nu \) state at \( \sim 0.003 \text{ eV} \) or \( \sim 10^{-5} \text{ eV} \) and a much lighter (dominantly) \( \nu_e \). (Solar neutrinos can be accounted for by a SMA MSW solution or possibly by vacuum oscillations, but not by a LMA MSW.) Alternatively, one could reverse the pairing, with a nearly degenerate \( \nu_s \) and \( \nu_e \) at \( \sim 1 \text{ eV} \), and \( \nu_{\pm,-} \) around 0.03-0.1 eV. The other models involve \( \nu_\mu \rightarrow \nu_e \) with near-maximal mixing for the atmospheric neutrinos, and \( \nu_e \rightarrow \nu_\tau \) for the solar neutrinos. Again, there are two possibilities, with the nearly degenerate \( \nu_s - \nu_e \) pair around 1 eV and a lighter \( \nu_\tau - \nu_e \), or the other way around.

All of these patterns involve two neutrinos in the eV range, and therefore the possibility of a significant hot dark matter component. The two which have the (dominantly) \( \nu_\tau \) state around 1 eV could contribute to \( \beta \beta_0 \) if the neutrinos are Majorana. A very small \( (m_\nu) \) due to cancellations would suggest near maximal mixing for the solar neutrinos, but this could again be relaxed significantly given all of the uncertainties.

6. PARTICLE PHYSICS IMPLICATIONS: FROM THE TOP DOWN

Almost all extensions of the standard model predict non-zero neutrino mass at some level, often in the observable \( 10^{-5} - 10 \text{ eV} \) range. It is therefore difficult to infer the underlying physics from the observed neutrino masses. However, the neutrino mass spectrum should be extremely useful for top-down physics; i.e., the predicted neutrino masses and mixings should provide an important test, complementary to, e.g., the sparticle, Higgs, and ordinary fermion spectrum, of any concrete fundamental theory with serious predictive power.

Prior to the precision Z-pole measurements at LEP and SLC there were two promising paths for physics beyond the standard model: compositeness at the TeV scale (e.g., dynamical symmetry breaking, composite Higgs, or composite fermions), or unification, which most likely would have led to deviations from the standard model prediction at the few % or few tenths of a % level, respectively. The absence of large deviations strongly supports the unification route, which is the domain of supersymmetry, grand unification, and superstring theory. The implication is that non-zero neutrino masses are most likely not the result of unexpected new physics at the TeV scale, such as by loop effects associated with new ad hoc scalar fields. (They could, however, be due to neutrino-neutrino mixing or loop effects in supersymmetric models with \( R \) parity breaking.) Alternatively, they could be associated with new physics at very high energy scales, most likely either seesaw models or higher dimensional operators.
7. ORDINARY-STERILE NEUTRINO MIXING

As discussed in Section 5.1, the combination of solar neutrinos, atmospheric neutrino oscillations, and the LSND results, if confirmed, would most likely imply the mixing of ordinary active neutrinos with one (or more) light sterile neutrinos. One difficulty is that the sterile neutrinos could have been produced in the early universe by the mixings. For the range of mass differences and mixings relevant to LSND and the atmospheric neutrinos, the sterile neutrino would have been produced prior to nucleosynthesis, changing the freezeout temperature for $\nu_e n \leftrightarrow e^- p$ and leading to too much $^4\text{He}$ \[74\]. However, Foot and Volkas have recently \[75\] argued that MSW effects involving sterile neutrinos could amplify a small lepton asymmetry, leading to an excess of $\nu_e$ compared to $\bar{\nu}_e$, reducing the $^4\text{He}$. It has also been argued that ordinary-sterile neutrino mixing could facilitate heavy element synthesis by $r$-processes in the ejecta of neutrino-heated supernova explosions \[76,74\].

Most extensions of the standard model predict the existence of sterile neutrinos. For example, simple $SO(10)$ and $E_6$ grand unified theories predict one or two sterile neutrinos per family, respectively. The only real questions are whether the ordinary and sterile neutrinos of the same chirality mix significantly with each other, and whether the mass eigenstate neutrinos are sufficiently light. When there are only Dirac masses, the ordinary and sterile states do not mix because of the conserved lepton number. Pure Majorana masses do not mix the ordinary and sterile sectors either. In the seesaw model the mixing is negligibly small, and the (mainly) sterile eigenstates are too heavy to be relevant to oscillations. The only way to have significant mixing and small mass eigenstates is for the Dirac and Majorana neutrino mass terms to be extremely small and to also be comparable to each other. This appears to require two miracles in conventional models of neutrino mass.

One promising possibility involves the generation of neutrino masses from higher-dimensional operators in theories involving an intermediate scale \[7\] as described in Section 5. Depending on the intermediate scale and the dimensions of the operators naturally small Dirac and Majorana masses are possible, and in some cases they are automatically of the same order of magnitude \[7\]. Another interesting possibility \[7\] involves sterile neutrinos associated with a parallel hidden sector of nature as suggested in some superstring and supergravity theories. Other mechanisms in which one can obtain ordinary-sterile neutrino mixing are described in \[78\].

8. CONCLUSIONS

- Neutrino mass is an important probe of particle physics, astrophysics, and cosmology.
- There are several experimental indications or suggestions: (a) The Superkamiokande and other results on atmospheric neutrinos provide strong evidence for $\nu_\mu$ oscillations. (b) The combination of solar neutrino experiments implies a global spectral distortion, strongly supporting neutrino transitions or oscillations. The preliminary SuperK results on the $^8\text{B}$ spectrum suggests a spectral distortion, most consistent with vacuum oscillations but possibly with small angle MSW. (c) LSND has candidate events in both decay at rest and decay in flight. The non-observation of candidates by KARMEN is close to being an experimental contradiction, also there is still a small parameter space consistent with both. (d) Mixed dark matter is an interesting hint for eV scale masses, but is not established.
- In the future many solar neutrino experiments and (model independent) observables will be needed to identify the mechanism, determine the neutrino parameters, and simultaneously study the Sun. This program is complicated by possible three neutrino effects, possible sterile neutrinos, and the possibility that there are both neutrino mass effects and nonstandard solar physics (although the latter is constrained by helioseismology). Experiments that are sensitive to the $pp$ and $^7\text{Be}$ neutrinos are needed. Important observables include neutral to charged current ratios, spectral distortions,
day-night effects (possibly involving parametric core enhancement), and seasonal variations (especially for vacuum oscillations).

- For the atmospheric neutrinos, we need more detailed spectral and zenith angle information, and the neutral to charged current ratio as a function of the zenith angle. Independent information, including possible $\nu_\tau$ appearance, for the same parameter range should be forthcoming from long baseline experiments.

- The planned Mini-BOONE experiment at Fermilab should clarify the LSND-KARMEN situation.

- Future cosmic microwave anisotropy experiments and large scale sky surveys should be able to determine whether neutrinos contribute significantly to the dark matter.

- Significant improvements in $\beta\beta_0\nu$ would be very powerful probes of the Majorana nature of neutrinos in the mass ranges suggested by the LSND and atmospheric neutrino results.

- Most extensions of the standard model predict nonzero neutrino masses, so it is difficult to determine their origin in a “bottom-up” matter. However, the neutrino spectrum will be a powerful constraint on “top-down” calculations of fundamental models.

- The possibility of mixing between ordinary and light sterile neutrinos should be taken seriously.

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