Simple Rules for Evanescent Operators in One-Loop Basis Transformations

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Abstract

Basis transformations often involve Fierz and other relations which are only valid in $D = 4$ dimensions. In general $D$ space-time dimensions however, evanescent operators have to be introduced, in order to preserve such identities. Such evanescent operators contribute to one-loop basis transformations as well as to two-loop renormalization group running. We present a simple procedure on how to systematically change basis at the one-loop level by obtaining shifts due to evanescent operators. As an example we apply this method to derive the one-loop basis transformation from the BMU (Buras, Misiak and Urban) basis useful for NLO QCD calculations, to the JMS (Jenkins, Manohar and Stoffer) basis used in the matching to the SMEFT.
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1 Introduction

In recent years a lot of progress has been made concerning Next-to-Leading Order (NLO) analyses, which involve one-loop matching calculations as well as solving two-loop Renormalization Group Equations (RGEs) for the Wilson coefficients of Effective Field Theories. For an up to date review see [1]. For instance, concerning the Standard Model Effective Theory (SMEFT), the full matching from the SMEFT onto the Weak Effective Theory (WET) valid below the electroweak (EW) scale is known at tree-level [2] and since recently also at the one-loop level [3,4]. Furthermore, the one-loop RGEs in the SMEFT [5,7] and in the WET [8,9] are known.

In the process of performing a NLO analysis, it is often necessary to perform one-loop transformations between different operator bases, since for instance anomalous dimension matrices (ADMs) are known only in a particular basis, whereas the matching conditions are given in a different one. This is for example the case for the WET, where the two-loop ADMs are known in the BMU (Buras, Misiak and Urban) basis [10] as elaborated on recently in [11]. On the other hand the one-loop matching from SMEFT onto WET is given in the JMS (Jenkins, Manohar and Stoffer) basis defined in [2].

Generally, in order to translate the results from one basis to another one at NLO, one-loop basis transformations have to be taken into account. In this respect particular care has to be taken, if the tree-level (Leading Order (LO)) transformations involve Dirac space identities, which are only valid in four space-time dimensions. Examples include Fierz identities or identities involving gamma matrices. When using dimensional regularization, where the divergent loop integrals are continued to $D$ dimensions, such identities cannot be used directly, but need to be generalized by introducing evanescent (EV) operators [10]. Such evanescent operators vanish in four dimensions to conserve the original identities but are non-zero in $D$ dimensions. They are therefore formally speaking proportional to $\epsilon = (4 - D)/2$, which implies that they give non-zero contributions when inserted into divergent loop diagrams. These are exactly the contributions which enter basis transformations at the one-loop order. In this article we discuss a simple procedure on how to obtain these contributions by computing one-loop corrections resulting from the presence of the evanescent operators. To this end the evanescent operators are simply defined as the difference between operators and their transformed versions using $D=4$ identities. The resulting contributions will manifest themselves in shifts in the corresponding Wilson coefficients of the initial operators.

Having a simple algorithm at hand to perform NLO basis changes is important when performing one-loop matching calculations or in the computation of two-loop ADMs. In this work we explain the underlying formal framework and provide such an algorithm, based on Greek projections [13], which facilitates the aforementioned calculations. In this manner this procedure is an important ingredient in the pursuit of a complete NLO SMEFT analysis.

The rest of the article is organized as follows: In Section 2 we outline the general procedure on how to compute one-loop basis transformations between two operator bases. In Section 3 we show an explicit example by performing a NLO change from the BMU to

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1We follow here the WCxf convention defined in [12].

2A more formal procedure on how to perform NLO basis changes can be found in [10].
In this section we discuss the full NLO basis transformation between general operator bases. However, in order to make the subsequent sections more transparent we will dub the two operator bases JMS (see Table. 1 for the list of operators considered in this work) and BMU. We start with the simple LO basis transformation which will also set the notation used. Then, in the second subsection we discuss the issue of evanescent operators which become relevant when performing basis changes at the one-loop level.

## 2 Procedure

Table 1: Non-leptonic $\Delta F = 1$ operators (baryon and lepton number conserving) in the JMS basis. Note that $Q_{1,LR}$ and $Q_{1,LR}$ have Hermitian conjugates. The same holds for the operators $(LR)(LR)$. This choice of basis eliminates all operators with Dirac structures $\sigma^{\mu\nu}$. The class of operators $(LR)(LR) + \text{h.c.}$ does not contain non-leptonic operators, but only semi-leptonic ones.

In Section 3 we define and calculate the EV operators which in turn gives us the transformation matrix between BMU and JMS bases at the one-loop level. Finally we conclude in Section 5. Additional material used in the calculations is collected in the appendices.
2.1 Tree-level transformation

Let us consider the two bases

\[ \mathcal{O}_{\text{BMU}} = \{Q_1, Q_2, \ldots, Q_N\}, \quad \mathcal{O}_{\text{JMS}} = \{O_1, O_2, \ldots, O_N\}, \]

(1)

containing \( N \) operators each. At tree-level each operator is given by a linear combination of operators from the other basis:

\[ \mathcal{O}_{\text{JMS}} = \hat{R}^{(0)} \mathcal{O}_{\text{BMU}}, \]

(2)

where the \( N \times N \) matrix \( \hat{R}^{(0)} \) denotes the linear transformation between the two bases. The superindex '(0)' denotes the tree-level transformation in anticipation of the one-loop transformation discussed in the next subsection. The matrix \( \hat{R}^{(0)} \) is obtained by applying identities such as Fierz relations and gamma matrix identities to the operators on the LHS of eq. (2). It is independent of the renormalization scale and only contains numerical factors.\(^3\)

As seen in eq. (2) the BMU operator basis is transformed via \( \hat{R}^{(0)} \) into the JMS basis. This transformation is useful because the hadronic matrix elements of operators are usually calculated in the BMU basis and this transformation allows to obtain them in the JMS basis. However, the Wilson coefficients are nowadays calculated in the JMS basis, since the SMEFT matching results are only available in that particular basis. Therefore for Wilson coefficients the transformation from JMS to BMU is more useful, which reads

\[ \tilde{C}^{(0)}_{\text{BMU}} = \left( \hat{R}^{(0)} \right)^T \tilde{C}^{(0)}_{\text{JMS}}, \]

(3)

that is \( (\hat{R}^{(0)})^T \) is involved.

Since \( \hat{R}^{(0)} \) is invertible by definition, one can easily express the BMU operators in terms of JMS ones and the JMS Wilson coefficients in terms of BMU ones. For our purposes however, the transformations in eqs. (2) and (3) are more convenient.

It should be stressed that although the procedure outlined below is general and can be used for any basis transformation the explicit values of various coefficients are given in our paper in the NDR-\( \overline{\text{MS}} \) scheme as defined in \( [14] \) with evanescent operators entering two-loop calculations defined by the so-called Greek method. The details in the context of WET and SMEFT are discussed in Appendix E of \( [15] \).

2.2 One-loop transformation

At the one-loop level special care has to be taken when identities have been used in the LO transformation, that are only valid in \( D = 4 \) space-time dimensions. Therefore, for all the lines in \( R^{(0)} \) which were obtained using \( D = 4 \) relations, evanescent operators have to be introduced to generalize these identities. In the case of the BMU→JMS transformation we proceed as follows:

**Step 1:**

\(^3\)This is true up to possible normalization factors, which can contain coupling constants or other parameters that depend on the renormalization scale.
We perform a Fierz transformation on every operator in the BMU basis \( Q_i \) and denote the result of this transformation by \( \tilde{Q}_i \) given generally by

\[
\tilde{Q}_i = \sum_k \omega_{ik} Q_k ,
\]

with operators \( Q_k \) belonging to the BMU basis and coefficients \( \omega_{ik} \) determined through the Fierz identities collected in the Appendix [B]. It should be emphasized that the relation above and analogous relations below should be interpreted as effective contributions from this operator in one loop diagrams. This means that the calculations performed in two bases, in our case BMU and JMS, will give the same results for physical observables when this transformation is taken into account at all stages, therefore when calculating also anomalous dimensions. This relations and analogous relations involving evanescent operators can also be used for the calculations of two-loop anomalous dimensions of operators because there the evanescent operators contributing to two-loop anomalous dimensions appear in one-loop sub-diagrams [14]. In order to go to the next order in perturbation theory the shifts should include corrections of order \( \mathcal{O}(\alpha_s^2) \) that can be obtained through insertions of the operators into two-loop diagrams.

**Step 2:**

We insert \( Q_i \) and \( \tilde{Q}_i \) defined by (4) into current-current and QCD penguin diagrams of Figs. 1 and 2, respectively. Due to the presence of evanescent operators which are simply defined by

\[
Q_i = \tilde{Q}_i + EV_i ,
\]

the insertion of \( Q_i \) and \( \tilde{Q}_i \) into one-loop diagrams will generally differ at \( \mathcal{O}(\alpha_s) \), leading to the result

\[
Q_i = \tilde{Q}_i + \frac{\alpha_s}{4\pi} \sum_r \tilde{\omega}_{ir} Q_r ,
\]

where the operators in the sum can again be written in the BMU basis but are generally different from the ones in the definition of \( \tilde{Q}_i \) in (4) and also \( \omega_{ik} \neq \tilde{\omega}_{ir} \). Note that this can easily be done at the one-loop level because in transforming the operators entering these corrections \( D = 4 \) Fierz identities can be used as will be explained below. Expressing the shift in terms of BMU operators allows to use these results for transformations of the BMU basis to any basis, not just the JMS one.

To compute the \( \alpha_s \)-corrections resulting from the evanescent operator \( EV_i \) in (5), one simply inserts the difference \( Q_i - \tilde{Q}_i \) into the relevant one-loop diagrams. Since the evanescent operator is formally \( \mathcal{O}(\epsilon) \), only the divergent pieces of the loop integrals will contribute and consequently the finite pieces can be discarded in the calculation.

The relation between operators in JMS basis and the Fierz transformed operators in BMU are further discussed in Sec. 3.2. In particular the tree-level relations are given in eqs. (46)-(49), and the one-loop relations in eqs. (50)-(52).

**Step 3:**

Having the relations in (6) and inspecting which Fierz transformations had to be performed to find the LO matrix \( \hat{R}^{(0)} \) we can generalize the basis change matrix \( \hat{R}^{(0)} \) to
the one-loop order, as follows\footnote{In the case JMS→BMU we will focus on QCD corrections. The relation in eq. (7) can easily be generalized to include other one-loop corrections.}

\[ \hat{O}_{\text{JMS}} = \hat{R} \hat{O}_{\text{BMU}}, \quad \hat{R} = \hat{R}^{(0)} + \frac{\alpha_s}{4\pi} \hat{R}^{(1)}, \]

(7)

where \( \hat{R}^{(1)} \) is the one-loop basis transformation matrix resulting exclusively from the evanescent operators.

When computing the one-loop corrections to the difference \( Q_i - \tilde{Q}_i \), new Dirac structures can appear, which are not present in the original basis, in our case the BMU basis. To reduce these structures to the ones in \( \hat{O}_{\text{BMU}} \) it is essential to use the same projections as in the calculations of two-loop anomalous dimensions of the operators. Only then are the evanescent operators entering the two-loop calculations the same as the ones used in the one-loop matching between WET and SMEFT and in particular they correspond to the ones in the basis change. This prescription guarantees that the renormalization scheme dependence of two-loop matrix elements can be canceled by the one present in one-loop contributions so that the physical amplitudes are renormalization scheme independent.

These issues have been discussed in the context of the NLO QCD calculations of Wilson coefficients at length in [14, 16, 17] and the summary can be found in Section 5.2.9 of [1]. There the so-called Greek projection [13], properly generalized to include evanescent operators in [14], has been discussed in detail. As pointed out in [17] this is not the only way to include evanescent operators but in fact the simplest one. It has been used in all two-loop calculations performed by the second author and will be used in the following but this time in the context of basis change. It should be kept in mind that this procedure defines the evanescent operators in the NDR-MS scheme combined with the Greek projections as used in [14]. While giving the same results it is much simpler than the formal method presented in [10] and used recently in [11].

The NLO transformation in terms of the Wilson coefficients is given as follows.

\[ \hat{\bar{C}}_{\text{BMU}} = \hat{R}^T \hat{\bar{C}}_{\text{JMS}}, \]

(8)

Writing

\[ \hat{\bar{C}}_{\text{JMS}} = \tilde{\bar{C}}^{(0)}_{\text{JMS}} + \frac{\alpha_s}{4\pi} \tilde{\bar{C}}^{(1)}_{\text{JMS}}, \quad \hat{\bar{C}}_{\text{BMU}} = \tilde{\bar{C}}^{(0)}_{\text{BMU}} + \frac{\alpha_s}{4\pi} \tilde{\bar{C}}^{(1)}_{\text{BMU}}, \]

(9)

the transformation for Wilson coefficients reads

\[ \tilde{\bar{C}}^{(0)}_{\text{BMU}} = (\hat{R}^{(0)})^T \tilde{\bar{C}}^{(0)}_{\text{JMS}}, \quad \tilde{\bar{C}}^{(1)}_{\text{BMU}} = (\hat{R}^{(0)})^T \tilde{\bar{C}}^{(1)}_{\text{JMS}} + (\hat{R}^{(1)})^T \tilde{\bar{C}}^{(0)}_{\text{JMS}}. \]

(10)

A few remarks concerning the generality of our results are in order: The procedure laid out in this article is not limited to the BMU and JMS basis, but is valid for any pair of operator bases, that are related via Fierz transformations. The full set of relevant evanescent operators is generated by applying Greek identities, or any other identities to reduce the resulting Dirac structures from the one-loop calculation. This choice of Dirac reduction, together with the chosen finite counterterms and the treatment of \( \gamma_5 \) fixes the renormalization scheme [17]. In this article we have chosen the NDR-MS scheme in combination with the Greek identities. Choosing a different renormalization scheme would
change the entries of the rotation matrix $R^{(1)}$ in eq. (7), but is related via a trivial change of scheme to our results [18,19]. Furthermore, the procedure is independent of possible group theory relations between the two bases, since these are still valid in $D \neq 4$ space-time dimensions, and therefore do not obtain any one-loop shifts from evanescent structures. Finally, we note that the operator shifts presented in this paper can be interpreted as one-loop corrections to the original Fierz identities. This has been shown in two recent publications [20,21], in which the shifts for all possible four-fermi operators together with the contributions from dipole operators have been taken into account. A similar procedure in the SMEFT has been employed in [22].

2.3 How to use this procedure

Having the results in (6) to be presented in the next section, our goal will be to find the matrix $\hat{R}$. To this end comparing the BMU and JMS bases one has to find those operators or groups of them for which a Fierz transformation on operators in the BMU basis has to be performed in order to obtain the operators in the JMS basis with order $\alpha_s$ corrections taken into account.

Generally the matrix $\hat{R}^{(0)}$ will have a block structure so that operators in a given block can be separately considered from other blocks. In this context the following three cases arise.

- If no Fierz transformations are required in a given block the corresponding matrix $\hat{R}^{(1)}$ will vanish and tree-level results will be valid also at one-loop. One can use the corresponding block in $\hat{R}^{(0)}$ in that case.

- If Fierz transformations in a given block are required but the contributions of evanescent operators will vanish the corresponding block in the tree-level matrix $\hat{R}^{(0)}$ will again represent the corresponding block in the full $\hat{R}$.

- Finally, in certain blocks the necessity of performing Fierz transformations will introduce evanescent operators which will contribute to $\hat{R}^{(1)}$.

In the next section we will present the three step procedure outlined in this section in explicit terms.

3 BMU to JMS translation at one-loop

3.1 Basic method

As outlined above the transformation of the BMU basis to the JMS basis requires Fierz transformations on some of the BMU operators. This generates then additional contributions to the one-loop matching performed within the JMS basis. In order to find these one loop contributions one has to insert the difference $Q_i - \tilde{Q}_i$ into current-current and penguin diagrams of Figs. 1 and 2 respectively. The details of the calculation for the 1-loop insertions are given in Appendix C.
With all BMU operators listed in Appendix A it turns out that

• For $Q_k$ with $k = 1 - 18$ these additional contributions come only from penguin insertions and moreover only for a few among these operators listed below.

• Fierz transformations on $Q_{11}^{SLR,Q}$ and $Q_{21}^{SLR,Q}$ do not generate any evanescent contributions and consequently in this case $D = 4$ identities can be used.

• For $Q_{k}^{SRR,Q}$ with $k = 1 - 4$ and $Q_{l}^{SRR,D}$ with $l = 1, 2$ the contributions come only from current-current operators and involve all operators considered. However, all these operators do not contribute to $K$ and $B$ decays being forbidden within SMEFT. For completeness we list these contributions below because they could be useful for charm physics.

In the case of the SM operators $Q_k$ with $k = 1 - 10$ all the contributions from Fierz transformations for LL (left-left) operators can be obtained from two properties:

$$Q_1 = \bar{Q}_1, \quad Q_2 = \bar{Q}_2 + \frac{1}{3} \frac{\alpha_s}{4\pi} P,$$

with

$$P = Q_4 + Q_6 - \frac{1}{3} (Q_3 + Q_5).$$

$$Q_{21}^{SLR,Q}$$

$$Q_{22}^{SLR,Q}$$

Figure 1: The QCD current-current insertions at one-loop.

Figure 2: The QCD-penguin insertions with open–type [left] and closed–type [right] fermion loops.
We find
\[ Q_3 = \tilde{Q}_3 + \frac{2\alpha_s}{34\pi} P, \quad Q_4 = \tilde{Q}_4 - \frac{N_f\alpha_s}{34\pi} P, \quad Q_5 = \tilde{Q}_5, \quad Q_6 = \tilde{Q}_6, \]
\[ Q_7 = \tilde{Q}_7, \quad Q_8 = \tilde{Q}_8, \quad Q_9 = \tilde{Q}_9 - \frac{1}{3\pi} \frac{N_f\alpha_s}{3} P, \quad Q_{10} = \tilde{Q}_{10} - \frac{1}{3} (N_u - \frac{N_d}{2}) \frac{N_f\alpha_s}{3} P. \] (13)

We observe that the Fierz transformations on the VLR operators \( Q_k \) with \( k = 5 - 8 \) do not bring any contributions from evanescent operators.

In the case of the NP operators \( Q_k \) with \( k = 11 - 18 \) only the Fierz transformation on \( Q_{11} \) brings a contribution from evanescent operators so that
\[ Q_{11} = \tilde{Q}_{11} + \frac{2\alpha_s}{34\pi} P, \quad Q_k = \tilde{Q}_k, \quad k = 12 - 18. \] (15)

The corresponding results for \( Q_{1,2,3,4}^{SRR,D} \) with \( D = d_i \) or \( d_j \) operators can be obtained by using the results of [23], in particular the results in equations (29)-(32) of that paper. In this case the \( Q_k^{SRR,D} \) operators with \( k = 1 - 4 \) are given as follows:
\[ \tilde{Q}_1^{SRR,D} = -\frac{1}{2} Q_2^{SRR,D} + \frac{1}{8} Q_4^{SRR,D}, \quad \tilde{Q}_2^{SRR,D} = -\frac{1}{2} Q_1^{SRR,D} + \frac{1}{8} Q_3^{SRR,D}, \]
\[ \tilde{Q}_3^{SRR,D} = 6Q_2^{SRR,D} + \frac{1}{2} Q_4^{SRR,D}, \quad \tilde{Q}_4^{SRR,D} = 6Q_1^{SRR,D} + \frac{1}{2} Q_3^{SRR,D}. \] (16) (17)

As this time only current-current diagrams are involved the flavour structure relative to the one considered in [23] does not matter and the full calculation of the matrix elements of \( \tilde{Q}_i \) operators can be readily performed in no time using results of [23]. The shifts caused by evanescent operators involve four operators but those with \( k = 1,3 \) can be eliminated\(^5\) using
\[ Q_3^{SRR,D} = 6Q_2^{SRR,D} + \frac{1}{2} Q_4^{SRR,D} + \text{Fierz ev.} \] (18)
\[ Q_1^{SRR,D} = -\frac{1}{2} Q_2^{SRR,D} + \frac{1}{8} Q_4^{SRR,D} + \text{Fierz ev.} \] (19)
so that the shifts depend only on \( Q_{2,4}^{SRR,D} \). Note that in the BMU basis \( Q_{2,4}^{SRR,D} \) are denoted as \( Q_{1,2}^{SRR,i} \) or \( Q_{1,2}^{SRR,j} \) in Ref. [11], see Appendix [A] for definitions.

We find then
\[ Q_1^{SRR,D} = \tilde{Q}_1^{SRR,D} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} A_k Q_k^{SRR,D}, \] (20)
\[ Q_2^{SRR,D} = \tilde{Q}_2^{SRR,D} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} B_k Q_k^{SRR,D}, \] (21)
\[ Q_3^{SRR,D} = \tilde{Q}_3^{SRR,D} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} C_k Q_k^{SRR,D}, \] (22)
\[ Q_4^{SRR,D} = \tilde{Q}_4^{SRR,D} + \frac{\alpha_s}{4\pi} \sum_{k=2,4} D_k Q_k^{SRR,D}, \] (23)

\(^5\)Equivalently one could also eliminate any other two operators but here we follow the conventions used in Ref. [11].
with the coefficients $A_k, B_k, C_k, D_k$ given as follows:\footnote{$N_c$ is the number of colours with $N_c = 3$ in the final results.}

\begin{align}
A_2 &= \frac{1}{2} + \frac{5}{N_c} - \frac{7N_c}{4} = \frac{37}{12}, \quad A_4 = -\frac{1}{2} + \frac{1}{4N_c} - \frac{N_c}{16} = -\frac{29}{48}, \quad (24) \\
B_2 &= -\frac{17}{4} + \frac{1}{N_c} = -\frac{55}{12}, \quad B_4 = -\frac{3}{16} + \frac{3}{4N_c} - \frac{N_c}{8} = -\frac{5}{16}, \quad (25) \\
C_2 &= 36 + \frac{28}{N_c} - 7N_c = \frac{73}{3}, \quad C_4 = -\frac{1}{2} - \frac{5}{N_c} + \frac{3N_c}{4} = \frac{1}{12}, \quad (26) \\
D_2 &= -21 - \frac{44}{N_c} + 14N_c = \frac{19}{3}, \quad D_4 = \frac{13}{4} + \frac{1}{N_c} = \frac{43}{12}. \quad (27)
\end{align}

The same procedure can be applied to $Q_{1,2,3,4}^{\text{SRR},Q}$ with $Q = u_k, d_k \neq d_i, d_j$ defined in eq. (100). The rules for the shifts read

\begin{align}
Q_{1}^{\text{SRR},Q} &= \bar{Q}_1^{\text{SRR},Q} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} a_k Q_k^{\text{SRR},Q}, \quad (28) \\
Q_{2}^{\text{SRR},Q} &= \bar{Q}_2^{\text{SRR},Q} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} b_k Q_k^{\text{SRR},Q}, \quad (29) \\
Q_{3}^{\text{SRR},Q} &= \bar{Q}_3^{\text{SRR},Q} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} c_k Q_k^{\text{SRR},Q}, \quad (30) \\
Q_{4}^{\text{SRR},Q} &= \bar{Q}_4^{\text{SRR},Q} + \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} d_k Q_k^{\text{SRR},Q}. \quad (31)
\end{align}

Here the flavour structure of the tilde operators $\bar{Q}_{1,2,3,4}^{\text{SRR},Q}$ (see (16),(17) for the definition with $D$ replaced by $Q$) is $(d_j \Gamma_Q)(Q \Gamma d_i)$ and the BMU operators $Q_{1,2,3,4}^{\text{SRR},Q}$ have the form $(d_j \Gamma d_i)(Q \Gamma Q)$.

The coefficients $a_k, b_k, c_k, d_k$ are given as follows

\begin{align}
a_1 &= \frac{N_c}{2} - \frac{1}{N_c} = \frac{7}{6}, \quad a_2 = \frac{1}{2}, \quad (32) \\
a_3 &= -\frac{N_c}{4} + \frac{3}{4N_c} = -\frac{1}{2}, \quad a_4 = -\frac{1}{2}. \quad (33) \\
b_1 &= 1, \quad b_2 = -\frac{1}{N_c} = -\frac{1}{3}, \quad (34) \\
b_3 &= -\frac{5}{8}, \quad b_4 = -\frac{N_c}{8} + \frac{3}{4N_c} = -\frac{1}{8}. \quad (35)
\end{align}
\[ c_1 = 8N_c - \frac{44}{N_c} = \frac{28}{3}, \quad c_2 = 36, \]  
\[ c_3 = -\frac{N_c}{2} + \frac{1}{N_c} = -\frac{7}{6}, \quad c_4 = -\frac{1}{2}, \]  
\[ d_1 = 30, \quad d_2 = 14N_c - \frac{44}{N_c} = \frac{82}{3}, \]  
\[ d_3 = -1, \quad d_4 = \frac{1}{N_c} = \frac{1}{3}. \]  

With these rules we can find the matrix \( \hat{R} \) as defined in (36).

### 3.2 Transformation matrices at one-loop

In this section we present our final result for the transformation matrices between the BMU and JMS bases at the 1-loop level. The details of the calculation are given in Section 4. The calculation can be split into the three disconnected sectors VLL (Vector Left-Left), VLR (Vector Left-Right) and SRR (Scalar Right-Right), which denote the \( \gamma_\mu P_L \otimes \gamma^\mu P_L \), \( \gamma_\mu P_L \otimes \gamma_\mu P_R \) and \( P_R \otimes P_R \) Dirac structures of the involved four-fermi operators, respectively.

For the BMU operator we use the following reference ordering

\[
\text{VLL: \{Q}_1, Q_2, Q_3, Q_4, Q_9, Q_{10}, Q_{11}, Q_{14}\},
\]

\[
\text{VLR: \{Q}_5, Q_6, Q_7, Q_8, Q_{12}, Q_{13}, Q_{15}, \ldots, Q_{24}\},
\]

\[
\text{SRR: \{Q}_{25}, \ldots, Q_{40}\}.
\]

For the JMS basis we use the ordering

\[
\text{VLL: \{[O}^1_{11}, [O}^8_{11}, [O}^1_{22}, [O}^8_{22}, [O}^1_{33}, [O}^8_{33}\}.
\]

\[
\text{VLR: \{[O}^1_{11}, [O}^8_{11}, [O}^1_{22}, [O}^8_{22}, [O}^1_{33}, [O}^8_{33}\}.
\]

\[
\text{SRR: \{[O}^1_{11}, [O}^8_{11}, [O}^1_{22}, [O}^8_{22}, [O}^1_{33}, [O}^8_{33}\}.
\]
At tree-level the transformation matrix $\hat{R}^{(0)}$ reads

$$
\hat{R}^{(0)} = \begin{pmatrix}
\hat{R}_{\text{VLL}}^{(0)} & 0_{8\times16} & 0_{8\times16} \\
0_{16\times8} & \hat{R}_{\text{VLR}}^{(0)} & 0_{16\times16} \\
0_{16\times8} & 0_{16\times16} & \hat{R}_{\text{SRR}}^{(0)}
\end{pmatrix},
$$

(46)

$$
\hat{R}_{\text{VLL}}^{(0)} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\
\frac{1}{6} & -\frac{1}{2} & -\frac{1}{12} & \frac{1}{6} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\
0 & 0 & \frac{2}{3} & 0 & -\frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2}
\end{pmatrix},
$$

(47)

$$
\hat{A}_{\text{SRR}}^{(0)} = \begin{pmatrix}
1 & 0 \\
-\frac{5}{12} & \frac{1}{16}
\end{pmatrix},
$$

(48)

$$
\hat{B}_{\text{SRR}}^{(0)} = \begin{pmatrix}
0 & 1 & 0 & 0 \\
\frac{1}{2} & -\frac{1}{5} & 0 & 0 \\
-\frac{1}{2} & 0 & \frac{1}{8} & 0 \\
\frac{1}{12} & -\frac{1}{3} & -\frac{1}{16}
\end{pmatrix},
$$

(49)
At 1-loop level the corrections due to the EV operators are given by the matrix $\hat{R}^{(1)}$

$$\hat{R}^{(1)} = \begin{pmatrix} \hat{R}_{\text{VLL}}^{(1)} & \hat{R}_{8\times16}^{(1)} & 0_{8\times16} \\ 0_{16\times8} & \hat{R}_{VLR}^{(1)} & 0_{16\times16} \\ 0_{16\times8} & 0_{16\times16} & \hat{R}_{SRR}^{(1)} \end{pmatrix},$$  

(50)

$$\hat{R}_{\text{VLL}}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{18} & -\frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{18} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \ 
$$

$$\hat{R}_{8\times16}^{(1)} = \begin{pmatrix} 0 & 0 & \ldots & 0 \\ \frac{1}{18} & -\frac{1}{6} & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ -\frac{1}{18} & \frac{1}{6} & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ -\frac{1}{3} & 1 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \end{pmatrix}, \ 
$$

(51)

Here, we have used $N_f = 5, N_d = 3$ and $N_u = 2$.

## 4 Evanescent operators

In this section, we define and calculate the EV operators which in turn gives us the matrix $\hat{R}^{(1)}$.

### 4.1 Definition of EV operators in the VLL sector

At the 1-loop level, we need to add an EV contribution to a tree-level basis transformation rule if it involves a Fierz transformation. On the other hand, we do not need an EV contribution for the cases in which no Fierz is required for the tree-level basis change. In the VLL sector, there are total eight independent operators where only five of them involve Fierz relation (105) for the change of basis. The corresponding EV operators

$$\hat{A}_{SRR}^{(1)} = \begin{pmatrix} A_{SRR}^{(1)} & 0_{2\times2} & 0_{2\times4} & 0_{2\times4} & 0_{2\times4} \\ 0_{2\times2} & A_{SRR}^{(1)} & 0_{2\times4} & 0_{2\times4} & 0_{2\times4} \\ 0_{4\times2} & 0_{4\times2} & B_{SRR}^{(1)} & 0_{4\times4} & 0_{4\times4} \\ 0_{4\times2} & 0_{4\times2} & 0_{4\times4} & B_{SRR}^{(1)} & 0_{4\times4} \\ 0_{4\times2} & 0_{4\times2} & 0_{4\times4} & 0_{4\times4} & B_{SRR}^{(1)} \end{pmatrix}, \ 
$$

$$\hat{A}_{SRR}^{(1)} = \begin{pmatrix} 0 & 0 \\ -\frac{37}{24} & -\frac{29}{96} \end{pmatrix}, \ 
$$

$$\hat{B}_{SRR}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{7}{12} & -\frac{17}{36} & -\frac{5}{144} & -\frac{3}{144} \\ -\frac{55}{36} & -\frac{13}{12} & -\frac{11}{144} & -\frac{1}{48} \end{pmatrix}. \ 
$$

(52)
$E_{I}^{VLL}$, $I = 1 - 5$ are defined by the relations:

$$[O_{ud}^{V_{1},LL}]_{11ji} \equiv \mathcal{F} Q_1 + E_1^{VLL},$$

$$[O_{ud}^{V_{8},LL}]_{11ji} \equiv -\frac{1}{6} Q_1 + \frac{1}{2} Q_2 + E_2^{VLL},$$

$$[O_{ud}^{V_{1},LL}]_{22ji} \equiv -Q_1 + \frac{1}{3} Q_3 + \frac{2}{3} Q_9 + E_3^{VLL},$$

$$[O_{ud}^{V_{8},LL}]_{22ji} \equiv \frac{1}{6} Q_1 - \frac{1}{2} Q_2 - \frac{1}{18} Q_3 + \frac{1}{6} Q_4 - \frac{1}{9} Q_9 + \frac{1}{3} Q_{10} + E_4^{VLL},$$

$$[O_{dd}^{V_{1},LL}]_{jikk} = \frac{2}{3} Q_3 - \frac{2}{3} Q_9 - Q_{11},$$

$$[O_{dd}^{V_{1},LL}]_{jikk} = \frac{2}{3} Q_4 - \frac{2}{3} Q_{10} - Q_{11} + E_5^{VLL},$$

$$[O_{dd}^{V_{1},LL}]_{jiu} = \frac{1}{2} Q_{11} + \frac{1}{2} Q_{14},$$

$$[O_{dd}^{V_{1},LL}]_{jijj} = \frac{1}{2} Q_{11} - \frac{1}{2} Q_{14}.$$

Here $\mathcal{F}$ indicates that the Fierz identity (105) is needed for the change of basis.

### 4.2 Calculation of the EV operators in the VLL sectors

Now we are in position to use the rules presented in Sec. 3.1 to obtain $E_{I}^{VLL} - E_{5}^{VLL}$ which contribute to $\hat{R}^{(1)}$. In order to use the rules of Sec. 3.1 first we need to express the JMS operators on the l.h.s. in terms of the $\tilde{Q}_I$ operators. In general, there are three categories of operators as discussed in Sec. 2.3.

#### 4.2.1 Operators requiring no Fierz

Since the following set of operators in the VLL sector do not require Fierz transformations for the basis change

$$[O_{dd}^{V_{1},LL}]_{jikk}, [O_{dd}^{V_{1},LL}]_{jiu}, [O_{dd}^{V_{1},LL}]_{jijj},$$

there are no EV operator contributions at the 1-loop level basis transformation given by (57), (59) and (60). Hence, the corresponding entries in the matrix $\hat{R}^{(1)}$ vanish.

#### 4.2.2 Operators requiring Fierz but no EV shifts

There are two operators in the JMS basis which require Fierz transformation but the EV contribution still vanish. The tree-level transformations for these operators are given by (53) and (55). To see this, one has to express the JMS operators on the LHS in terms of the $\tilde{Q}_I$ operators, doing so we obtain

$$E_{1}^{VLL} = [O_{ad}^{V_{1},LL}]_{11ji} - Q_1 = \tilde{Q}_1 - Q_1 = 0,$$

$$E_{3}^{VLL} = [O_{ad}^{V_{1},LL}]_{22ji} - \left(-Q_1 + \frac{1}{3} Q_3 + \frac{2}{3} Q_9\right) = \tilde{Q}_1 - Q_1 = 0.$$
Here the shift in $\tilde{Q}_1 - Q_1$ is given by the rule (11).

### 4.2.3 Operators requiring Fierz and EV shifts

Finally, we turn to the cases for which Fierz transformation at the tree-level as well as the EV contributions at the 1-loop level are necessary for the basis transformation. The tree-level transformations can be read from Eqs. (54), (56) and (58). The EV operators are then given by

\[
E_{\text{VLL}}^{VLL} = \left[ O_{ud}^{VLL} \right]_{11ji} - \left( -\frac{1}{6}Q_1 + \frac{1}{2}Q_2 \right) = \frac{1}{6}(\tilde{Q}_1 - Q_1) - \frac{1}{2}(Q_2 - \tilde{Q}_2) = -\frac{1}{6} \frac{\alpha_s}{4\pi} P, \tag{64}
\]

\[
E_{\text{VLL}}^{VLL} = \left[ O_{ud}^{VLL} \right]_{22ji} - \left( \frac{1}{6}Q_1 - \frac{1}{2}Q_2 - \frac{1}{18}Q_3 + \frac{1}{6}Q_4 - \frac{1}{9}Q_9 + \frac{1}{3}Q_{10} \right) = \frac{1}{6}(\tilde{Q}_1 - Q_1) + \frac{1}{2}(Q_2 - \tilde{Q}_2) = \frac{1}{6} \frac{\alpha_s}{4\pi} P, \tag{65}
\]

\[
E_{\text{VLL}}^{VLL} = \left[ O_{dd}^{VLL} \right]_{jkki} - \left( \frac{2}{3}Q_4 - \frac{2}{3}Q_{10} - Q_{11} \right) = \frac{2}{3}(\tilde{Q}_4 - Q_4) - \frac{2}{3}(\tilde{Q}_{10} - Q_{10}) = \frac{2N_f + N_d - 2N_u}{9} \frac{\alpha_s}{4\pi} P. \tag{66}
\]

Here the 1-loop shifts are given by the rules in (11), (13), (14) and (15), respectively.

### 4.3 Definition of EV operators in the SRR sector

In this case, in addition to the color identity (109), we need the Fierz relations given in (103) and (107)\(^7\). The 1-loop basis transformations including the EV operators $E_{i}^{\text{SRR}}$ read

\[
\left[ O_{dd}^{SL,RR} \right]_{jiij} = Q_{2}^{\text{SRR},i}, \tag{67}
\]

\[
\left[ O_{dd}^{SS,RR} \right]_{jiiij} \equiv -\frac{5}{12}Q_{2}^{\text{SRR},i} + \frac{1}{16}Q_{4}^{\text{SRR},j} + E_{\text{SRR},j}. \tag{68}
\]

Similar relations hold for $jiij \rightarrow jijj$. We note that a separate evanescent operator $E_{\text{SRR},j}$ is needed for this relation.

For the SRR,Q operators we find:

\(^7\)Note that here we have used the definition $\sigma_{\mu\nu} = \frac{i}{2}[^{\mu}\gamma,^{\nu}\gamma]$. Also we define the operators $Q_{2}^{\text{SRR},i}$ and $Q_{3}^{\text{SRR},Q}$, $Q_{4}^{\text{SRR},Q}$ with an additional negative sign as compared to Ref. [11].
\[ [O_{\text{ud}}^{S1,\text{RR}}]_{1ji} = \frac{1}{2} Q_{2}^{\text{SRR},u} - \frac{1}{6} Q_{2}^{\text{SRR},u}, \quad (69) \]
\[ [O_{\text{ud}}^{S8,\text{RR}}]_{1ji} = \frac{1}{12} Q_{1}^{\text{SRR},u} - \frac{1}{4} Q_{2}^{\text{SRR},u} - \frac{1}{48} Q_{3}^{\text{SRR},u} + \frac{1}{16} Q_{4}^{\text{SRR},u} + E_{1}^{\text{SRR},u}, \quad (70) \]
\[ [O_{\text{uddu}}^{S1,\text{RR}}]_{ij1} = -\frac{1}{2} Q_{1}^{\text{SRR},u} + \frac{1}{8} Q_{3}^{\text{SRR},u} + E_{1}^{\text{SRR},u}, \quad (71) \]
\[ [O_{\text{uddu}}^{S8,\text{RR}}]_{ij1} = \frac{1}{12} Q_{1}^{\text{SRR},u} - \frac{1}{4} Q_{2}^{\text{SRR},u} - \frac{1}{48} Q_{3}^{\text{SRR},u} + \frac{1}{16} Q_{4}^{\text{SRR},u} + E_{1}^{\text{SRR},u}. \quad (72) \]

Similar relations hold for $1 \to 2$ and $u \to c$ on the l.h.s. and r.h.s., respectively. Finally,
\[ [O_{\text{dd}}^{S1,\text{RR}}]_{jikk} = \frac{1}{2} Q_{2}^{\text{SRR},d_{k}}, \quad (73) \]
\[ [O_{\text{dd}}^{S8,\text{RR}}]_{jikk} = \frac{1}{12} Q_{1}^{\text{SRR},d_{k}} - \frac{1}{4} Q_{2}^{\text{SRR},d_{k}} - \frac{1}{48} Q_{3}^{\text{SRR},d_{k}} + \frac{1}{16} Q_{4}^{\text{SRR},d_{k}} + E_{1}^{\text{SRR},d_{k}}. \quad (74) \]
\[ [O_{\text{dd}}^{S1,\text{RR}}]_{jkki} = -\frac{1}{2} Q_{1}^{\text{SRR},d_{k}} + \frac{1}{8} Q_{3}^{\text{SRR},d_{k}} + E_{1}^{\text{SRR},d_{k}}, \quad (75) \]
\[ [O_{\text{dd}}^{S8,\text{RR}}]_{jkki} = \frac{1}{12} Q_{1}^{\text{SRR},d_{k}} - \frac{1}{4} Q_{2}^{\text{SRR},d_{k}} - \frac{1}{48} Q_{3}^{\text{SRR},d_{k}} + \frac{1}{16} Q_{4}^{\text{SRR},d_{k}} + E_{1}^{\text{SRR},d_{k}}. \quad (76) \]

### 4.4 Calculation of EV operators in the SRR sector

#### 4.4.1 Operators requiring no Fierz

In the SRR sector, the following operators do not require Fierz transformations for the basis change at the tree-level
\[ [O_{\text{ud}}^{S1,\text{RR}}]_{iiii}, [O_{\text{ud}}^{S8,\text{RR}}]_{iiii}, [O_{\text{ud}}^{S1,\text{RR}}]_{11ji}, [O_{\text{ud}}^{S8,\text{RR}}]_{11ji}, [O_{\text{ud}}^{S1,\text{RR}}]_{22ji}, \]
\[ [O_{\text{ud}}^{S8,\text{RR}}]_{22ji}, [O_{\text{dd}}^{S1,\text{RR}}]_{jikk}, [O_{\text{dd}}^{S8,\text{RR}}]_{jikk}. \quad (77) \]

Therefore, for the basis change at the 1-loop level no EV contributions are required.

#### 4.4.2 Operators requiring Fierz but no EV shifts

In this sector there are no such operators which require Fierz without having non vanishing EV shifts at the 1-loop level.

#### 4.4.3 Operators requiring Fierz and EV shifts

The SRR operators which require the Fierz relation and non vanishing EV operators for the basis transformation are given by (68), (71), (72), (75), and (76). The EV operators are then given by
E_{SRR,i} = [O_{dd_{SD}}]_{iii} - \left( -\frac{5}{12} Q_{SRR,i}^2 + \frac{1}{16} Q_{SRR,i}^4 \right) = -\frac{1}{2}(\tilde{Q}_{SRR,i}^1 - Q_{SRR,i}^1)
= -\frac{\alpha_s}{4\pi} \left( \frac{37}{24} Q_{SRR,i}^2 + \frac{29}{36} Q_{SRR,i}^4 \right), \quad (78)

E_{SRR,u} = [O_{uddu}^{S1,RR}]_{ijj} - \left( -\frac{1}{2} Q_{SRR,u}^1 + \frac{1}{8} Q_{SRR,u}^3 \right)
= -\frac{1}{2}(\tilde{Q}_{SRR,u}^1 - Q_{SRR,u}^1) + \frac{1}{8}(\tilde{Q}_{SRR,u}^3 - Q_{SRR,u}^3)
= \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} p_k Q_{SRR,u}^k, \quad (79)

E_{SRR,u} = [O_{uddu}^{S8,RR}]_{ijj} - \left( \frac{1}{12} Q_{SRR,u}^1 - \frac{1}{4} Q_{SRR,u}^2 - \frac{1}{48} Q_{SRR,u}^3 + \frac{1}{16} Q_{SRR,u}^4 \right)
= \frac{1}{12}(\tilde{Q}_{SRR,u}^1 - Q_{SRR,u}^1) - \frac{1}{4}(\tilde{Q}_{SRR,u}^2 - Q_{SRR,u}^2)
- \frac{1}{48}(\tilde{Q}_{SRR,u}^3 - Q_{SRR,u}^3) + \frac{1}{16}(\tilde{Q}_{SRR,u}^4 - Q_{SRR,u}^4)
= \frac{\alpha_s}{4\pi} \sum_{k=1,2,3,4} q_k Q_{SRR,u}^k. \quad (80)

The coefficients $p_k$ and $q_k$ are found to be

\begin{align*}
p_1 &= -\frac{7}{12}, & p_2 &= -\frac{17}{4}, & p_3 &= -\frac{5}{48}, & p_4 &= -\frac{3}{16}, \quad (81)\\
q_1 &= -\frac{55}{36}, & q_2 &= -\frac{13}{12}, & q_3 &= -\frac{11}{144}, & q_4 &= -\frac{1}{48}. \quad (82)
\end{align*}

The evanescent operators $E_{SRR,i}$ and $E_{SRR,u}^{1,2,3}$ follow from the corresponding $E_{SRR,i}$ and $E_{SRR,u}^{1,2,3}$ by the corresponding colour replacements.

In the above calculation, the shifts $Q_{SRR,D}^1 - \tilde{Q}_{SRR,D}^1$, and $Q_{SRR,Q}^1 - \tilde{Q}_{SRR,Q}^1$ for $Q = u$ or $d_k$ are obtained using the rules (20) and (28) - (31), respectively. It is worth noting at the 1-loop QCD only current-current insertions are involved for the SRR operators in obtaining these rules. Therefore flavour structure of the operators is immaterial. For instance, the operators $[O_{uddu}^{S1,RR}]_{ijj}$ and $Q_{SRR,u}^2$ having same color and Lorentz structures can be treated on the same footing even though they have different flavour structures.

4.5 Definition of EV operators in the VLR sector

In this subsection we turn our attention to the VLR sector and define the corresponding evanescent operators. Using the Fierz relation in eq. (106) as well as colour relations one
finds:

\[
\begin{align*}
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{j1i} &= \frac{1}{6} Q_5 + \frac{1}{3} Q_7 + \frac{1}{2} Q_{18} + E_1^{\text{VLR}}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]_{j1i} &= -\frac{1}{36} Q_5 + \frac{1}{12} Q_6 - \frac{1}{18} Q_7 + \frac{1}{6} Q_8 + \frac{1}{4} Q_{17} - \frac{1}{12} Q_{18} + E_2^{\text{VLR}}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{ji2} &= \frac{1}{6} Q_5 + \frac{1}{3} Q_7 - \frac{1}{2} Q_{18} + E_3^{\text{VLR}}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]_{ji2} &= -\frac{1}{36} Q_5 + \frac{1}{12} Q_6 - \frac{1}{18} Q_7 + \frac{1}{6} Q_8 - \frac{1}{4} Q_{17} + \frac{1}{12} Q_{18} + E_4^{\text{VLR}}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{jikk} &= \frac{2}{3} Q_5 - \frac{2}{3} Q_7 - Q_{13}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]_{jikk} &= -\frac{1}{9} Q_5 + \frac{1}{3} Q_6 + \frac{1}{9} Q_7 - \frac{1}{3} Q_8 - \frac{1}{2} Q_{12} + \frac{1}{6} Q_{13} + E_5^{\text{VLR}}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{jiii} &= \frac{1}{2} Q_{13} + \frac{1}{2} Q_{16}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]_{jiii} &= \frac{1}{4} Q_{12} - \frac{1}{12} Q_{13} + \frac{1}{4} Q_{15} - \frac{1}{12} Q_{16}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{jjjj} &= \frac{1}{2} Q_{13} - \frac{1}{2} Q_{16}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]_{jjjj} &= \frac{1}{4} Q_{12} - \frac{1}{12} Q_{13} - \frac{1}{4} Q_{15} + \frac{1}{12} Q_{16}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]^{\dagger}_{1ij1} &= -2 Q_{19}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]^{\dagger}_{1ij1} &= \frac{1}{3} Q_{19} - Q_{20}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]^{\dagger}_{2ij2} &= -2 Q_{21}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]^{\dagger}_{2ij2} &= \frac{1}{3} Q_{21} - Q_{22}, \\
\left[ O_{\text{V1,LR}}^{V1,LR} \right]^{\dagger}_{jkki} &= -2 Q_{23}, \\
\left[ O_{\text{V8,LR}}^{V8,LR} \right]^{\dagger}_{jkki} &= \frac{1}{3} Q_{23} - Q_{24}.
\end{align*}
\]

In the VLR sector, the following set operators do not require Fierz transformations for the basis change at the tree-level

\[
\begin{align*}
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{jikk}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]_{ji3}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]_{ji3}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]_{ji3}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]_{ji3}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]^{\dagger}_{1ij1}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]^{\dagger}_{1ij1}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]^{\dagger}_{2ij2}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]^{\dagger}_{2ij2}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]^{\dagger}_{jkki}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]^{\dagger}_{jkki}.
\end{align*}
\]

Therefore, for the basis change at the 1-loop level no EV contributions are required. The rest of the operators in the VLR sector requiring Fierz are

\[
\begin{align*}
\left[ O_{\text{V1,LR}}^{V1,LR} \right]_{ji11}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]_{ji11}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]_{ji22}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]_{ji22}, \left[ O_{\text{V1,LR}}^{V1,LR} \right]_{jikk}, \left[ O_{\text{V8,LR}}^{V8,LR} \right]_{jikk}.
\end{align*}
\]
A $\Delta F = 1$ BMU basis for $N_f = 5$

However, as discussed in Subsec. 3.1 the corresponding EV vanish:

$$E_1^{\text{VLR}} = E_2^{\text{VLR}} = E_3^{\text{VLR}} = E_4^{\text{VLR}} = E_5^{\text{VLR}} = 0.$$  \hfill (86)

## 5 Conclusions

We have presented a simple recipe to perform one-loop basis transformations involving evanescent operators. The procedure consists of computing the commutator of a one-loop $(L)$ correction using dimensional regularisation and a Fierz ($\mathcal{F}$) transformation of a given operator $Q$, which in all generality is non-vanishing:

$$[L, \mathcal{F}]Q \neq 0.$$  \hfill (87)

The presented method has already been used successfully in several contexts such as NLO basis transformations \cite{11, 15, 24}, one-loop matching calculations \cite{25} as well as in several two-loop calculations \cite{10}. But it has not been presented in any detail and in particular in this generality in the literature so far. The present paper should help to clarify possible issues involving evanescent operators. In the coming years one-loop matching and two-loop running effects will become more important in NP analyses than they are now.

We illustrated the outlined procedure by computing explicitly the complete one-loop basis change from the BMU to the JMS basis at $O(\alpha_s)$ and this example should allow the reader to perform the transformation between different bases. In this context our method will serve as a simple tool to perform one-loop basis transformations. One particular example would be the basis change to the CMM (Chetyrkin, Misiak and Munz) basis \cite{18}, which is most suited for multi-loop computations.

Since the one-loop basis change consists of a series of simple algebraic manipulations, it would be interesting to automate this procedure. After having computed all one-loop corrections to the operators in question a simple algorithm might be included in codes such as for example $\text{abc-eft}$ \cite{26}.

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## A $\Delta F = 1$ BMU basis for $N_f = 5$

In this Appendix we collect the full set of BMU operators.
We start the list with the vector operators, where the first ten operators are the well known SM operators $Q_1 - Q_{10}$:

\[
Q_1 = Q_1^{VLL,u} = (\bar{d}_j^\alpha \gamma_\mu P_L u^\beta)(\bar{u}^\beta \gamma_\mu P_L d^\alpha_i),
\]
\[
Q_2 = Q_2^{VLL,u} = (\bar{d}_j^\alpha \gamma_\mu P_L u^\alpha)(\bar{u}^\alpha \gamma_\mu P_L d^\beta_i),
\]
\[
Q_3 = (\bar{d}_j^\alpha \gamma_\mu P_L d^\alpha_i) \sum_q (\bar{q}^\beta \gamma_\mu P_L q^\beta),
\]
\[
Q_4 = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \sum_q (\bar{q}^\beta \gamma_\mu P_L q^\alpha),
\]
\[
Q_5 = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \sum_q (\bar{q}^\beta \gamma_\mu P_R q^\alpha),
\]
\[
Q_6 = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \sum_q (\bar{q}^\beta \gamma_\mu P_R q^\beta),
\]
\[
Q_7 = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d^\alpha_i) \sum_q (\bar{q}^\beta \gamma_\mu P_R q^\beta),
\]
\[
Q_8 = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \sum_q (\bar{q}^\beta \gamma_\mu P_R q^\beta),
\]
\[
Q_9 = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \sum_q (\bar{q}^\beta \gamma_\mu P_L q^\beta),
\]
\[
Q_{10} = \frac{3}{2} (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \sum_q (\bar{q}^\beta \gamma_\mu P_L q^\beta).
\]

The NP (New Physics) vector operators in the BMU basis are given by:

\[
Q_{11} = Q_{11}^{VLL,i+j} = (\bar{d}_j^\alpha \gamma_\mu P_L d^\alpha_i) \left[ (\bar{d}_j^\beta \gamma_\mu P_L d^\beta_i) + (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) \right],
\]
\[
Q_{12} = Q_{12}^{VLR,i+j} = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \left[ (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) + (\bar{d}_j^\beta \gamma_\mu P_R d^\alpha_i) \right],
\]
\[
Q_{13} = Q_{13}^{VLR,i+j} = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \left[ (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) + (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) \right],
\]
\[
Q_{14} = Q_{14}^{VLL,i-j} = (\bar{d}_j^\alpha \gamma_\mu P_L d^\alpha_i) \left[ (\bar{d}_j^\beta \gamma_\mu P_L d^\beta_i) - (\bar{d}_j^\beta \gamma_\mu P_L d^\beta_i) \right],
\]
\[
Q_{15} = Q_{15}^{VLR,i-j} = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \left[ (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) - (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) \right],
\]
\[
Q_{16} = Q_{16}^{VLR,i-j} = (\bar{d}_j^\alpha \gamma_\mu P_L d^\beta_i) \left[ (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) - (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i) \right],
\]
\[
Q_{17} = Q_{17}^{VLR,\mu - \bar{c}} = (\bar{d}_j^\alpha \gamma_\mu P_R d^\beta_i) \left[ (\bar{u}^\beta \gamma_\mu P_R u^\alpha) - (\bar{c}^\beta \gamma_\mu P_R c^\alpha) \right],
\]
\[
Q_{18} = Q_{18}^{VLR,\mu - \bar{c}} = (\bar{d}_j^\alpha \gamma_\mu P_R d^\beta_i) \left[ (\bar{u}^\beta \gamma_\mu P_R u^\beta) - (\bar{c}^\beta \gamma_\mu P_R c^\beta) \right].
\]

Finally, we introduce the scalar sector of the BMU basis. In the SRL sector we use the structures

\[
Q_1^{SRL,Q} = (\bar{d}_j^\alpha P_R d^\alpha_i) (\bar{Q}^\beta P_L Q^\alpha),
\]
\[
Q_2^{SRL,Q} = (\bar{d}_j^\alpha P_R d^\alpha_i) (\bar{Q}^\beta P_L Q^\beta),
\]

which define the operators

\[
(Q_{19}, Q_{20}) = (Q_1^{SRL,u}, Q_2^{SRL,u}),
\]
\[
(Q_{21}, Q_{22}) = (Q_1^{SRL,c}, Q_2^{SRL,c}),
\]
\[
(Q_{23}, Q_{24}) = (Q_1^{SRL,d_s}, Q_2^{SRL,d_s}).
\]

In the SRR sector with three equal quarks we introduce for completeness the redundant structures

\[
Q_1^{SRR,i} = (\bar{d}_j^\alpha P_R d^\alpha_i) (\bar{d}_j^\beta P_R d^\beta_i),
\]
\[
Q_2^{SRR,i} = -(\bar{d}_j^\alpha \gamma_\mu P_R d^\alpha_i) (\bar{d}_j^\beta \gamma_\mu P_R d^\beta_i),
\]
\[
(Q_{25}, Q_{26}) = (Q_1^{SRR,d_s}, Q_2^{SRR,d_s}).
\]
together with the operators

$$Q_{25} = Q_{2}^{SRR,i} = (\bar{d}_j^a P_R d_i^a)(\bar{d}_j^a P_R d_i^a), \quad Q_{26} = Q_{4}^{SRR,i} = -(\bar{d}_j^a \sigma_{\mu\nu} P_R d_i^a)(\bar{d}_j^a \sigma_{\mu\nu} P_R d_i^a),$$

and similar for the SRR, \(j\) sector

$$Q_{27} = Q_{2}^{SRR,j} = (\bar{d}_j^a P_R d_i^a)(\bar{d}_j^a P_R d_i^a), \quad Q_{28} = Q_{4}^{SRR,j} = -(\bar{d}_j^a \sigma_{\mu\nu} P_R d_i^a)(\bar{d}_j^a \sigma_{\mu\nu} P_R d_i^a),$$

together with

$$Q_{1}^{SRR,j} = (\bar{d}_j^a P_R d_i^a)(\bar{d}_j^a P_R d_i^a), \quad Q_{3}^{SRR,j} = -(\bar{d}_j^a \sigma_{\mu\nu} P_R d_i^a)(\bar{d}_j^a \sigma_{\mu\nu} P_R d_i^a).$$

Note that we choose the operators \(Q_{26}\) and \(Q_{28}\) with an opposite sign, compared to the basis in [11].

For the SRR sector with four different quarks we define the structures in [11]. With these definitions we define the operators where the tensor structures have again opposite sign compared to the convention adopted here the usual Fierz identities valid in 4 dimensions that we used in our analysis.

Finally, as far as the chirality-flipped operators are concerned, their numbering in the BMU basis is given by

$$Q_{40+i} = Q_{i}[P_L \leftrightarrow P_R],$$

i.e. they are found by interchanging \(P_L \leftrightarrow P_R\) in the “non-flipped” operators.

### B Fierz identities

In the process of transforming from one operator basis to another one requires the Fierz identities [27] that allow to transfer a given chain of spinors into another one. We list here the usual Fierz identities valid in \(D = 4\) dimensions that we used in our analysis.

All Fierz identities used are of the type (12)(34) \(\leftrightarrow\) (14)(32) in which the exchange of fermion fields \(2 \leftrightarrow 4\) (or equivalently \(1 \leftrightarrow 3\)) takes place. In the formulae below \(P_A\) and \(P_B\) stand for the usual projectors \(P_{L,R}\) but in a given relation \(P_A \neq P_B\). This means that if \(P_A = P_L\) then \(P_B = P_R\) and vice versa.

We have then

$$\begin{align*}
(\bar{\psi}_1 P_A \psi_2)(\bar{\psi}_3 P_A \psi_4) &= -\frac{1}{2}(\bar{\psi}_1 P_A \psi_4)(\bar{\psi}_3 P_A \psi_2) - \frac{1}{8}(\bar{\psi}_1 \gamma_{\mu} P_A \psi_4)(\bar{\psi}_3 \gamma^{\mu} P_A \psi_2), \\
(\bar{\psi}_1 P_A \psi_2)(\bar{\psi}_3 P_B \psi_4) &= -\frac{1}{2}(\bar{\psi}_1 \gamma_{\mu} P_B \psi_4)(\bar{\psi}_3 \gamma^{\mu} P_A \psi_2),
\end{align*}$$

(103)
Therefore the finite and infinite parts of

\[(\bar{\psi}_1 \gamma_\mu P A \psi_2)(\bar{\psi}_3 \gamma^\mu P A \psi_4) = (\bar{\psi}_1 \gamma_\mu P A \psi_4)(\bar{\psi}_3 \gamma^\mu P A \psi_2), \quad (105)\]

\[(\bar{\psi}_1 \gamma_\mu P A \psi_2)(\bar{\psi}_3 \gamma^\mu P B \psi_4) = -2(\bar{\psi}_1 P B \psi_4)(\bar{\psi}_3 P A \psi_2), \quad (106)\]

\[(\bar{\psi}_1 \sigma_{\mu\nu} P A \psi_2)(\bar{\psi}_3 \sigma^{\mu\nu} P A \psi_4) = -6(\bar{\psi}_1 P A \psi_4)(\bar{\psi}_3 P A \psi_2) + \frac{1}{2}(\bar{\psi}_1 \sigma_{\mu\nu} P A \psi_4)(\bar{\psi}_3 \sigma^{\mu\nu} P A \psi_2), \quad (107)\]

\[(\bar{\psi}_1 \sigma_{\mu\nu} P A \psi_2)(\bar{\psi}_3 \sigma^{\mu\nu} P B \psi_4) = 0. \quad (108)\]

For more Fierz identities involving charge conjugated fields see Appendix A.3 in [1].

Apart from this we also need the color identity

\[T^A \otimes T^A = \frac{1}{2} (\gamma^0 - \frac{1}{N_c} \gamma^5). \quad (109)\]

### C Master Formulae for One-loop Operator Insertions

In this section, we present master formulae for the 1-loop operator insertions. These can be used to obtain the shifts given in Sec 3.1. Consider a 4-fermion operator

\[(\bar{q}_1 \hat{V}_1 \Gamma_1 q_2)(\bar{q}_3 \hat{V}_2 \Gamma_2 q_4), \quad (110)\]

here \(\hat{V}_{1,2}\) and \(\Gamma_{1,2}\) represent the color and Dirac structures. There are two types of penguin insertions, an open penguin, and the closed penguin. In the next two subsections we evaluate the corresponding amplitudes.

#### C.1 Open penguin insertion

The open penguin insertion of the operator \([110]\) gives

\[P_{op} = W_\lambda (-ig_s T^b \gamma_\lambda) \left( \frac{-ig^{\lambda\lambda'}}{q^2} \right), \quad (111)\]

\[W_\lambda = i^2 \hat{V}_1 (-ig_s T^a) \hat{V}_2 I^{\mu\nu} T^\lambda_{\mu\nu}, \quad (112)\]

where \(T^\lambda_{\mu\nu} = \Gamma_1 \gamma_\nu \gamma_\lambda \gamma_\mu \Gamma_2\) and

\[I^{\mu\nu} = \int \frac{d^D k}{(2\pi)^D} \frac{k^\nu(k-q)^\mu}{k^2(k-q)^2} = -\frac{i}{16 \pi^2} \epsilon \left( \frac{1}{6} \frac{q_\mu q_\nu}{q^2} + \frac{12}{q} g_{\mu\nu} \right) - \frac{i}{16 \pi^2} \frac{5}{12} \left( \frac{g_{\mu\nu} + 2 g_{q_\mu q_\nu}}{q^2} \right). \quad (113)\]

Therefore the finite and infinite parts of \(P_{op}\) can be written as

\[P_{op}(\text{infinite}) = -C_{op} \frac{\alpha_s}{4 \pi} \left( \frac{1}{6} \frac{q_\mu q_\nu}{q^2} + \frac{12}{q} g_{\mu\nu} \right) \Gamma_1 \gamma_\nu \gamma_\lambda \Gamma_2 \otimes \gamma^\lambda, \quad (114)\]

\[P_{op}(\text{finite}) = -C_{op} \frac{\alpha_s}{4 \pi} \left( \frac{5}{18} \frac{q_\mu q_\nu}{q^2} + \frac{2 g_{\mu\nu}}{9} \right) \Gamma_1 \gamma_\nu \gamma_\lambda \Gamma_2 \otimes \gamma^\lambda. \quad (115)\]

Here \(C_{op} = \hat{V}_1 T^b \hat{V}_2 \otimes T^b\).
C.2 Closed penguin insertion

The closed penguin insertion of the operator (110) gives

$$P_{cl} = W_\lambda (-i g_s T^b \gamma_\lambda) \left( -\frac{i g^{\lambda\lambda'}}{q^2} \right),$$

$$W_\lambda = (-1)(i^2) (-i g_s) \text{Tr}(\hat{V}_1 T^b) \hat{V}_2 \text{Tr}(\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu) \Gamma_2 I^{\mu\nu}. \quad (116)$$

Here the $I^{\mu\nu}$ is given by (113).

Therefore the finite and infinite parts of $P_{cl}$ can be written as

$$P_{cl}(\text{infinite}) = C_{cl} \frac{\alpha_s}{4\pi} \left( \frac{1}{6} g_\rho q^\rho \right) + \frac{1}{12} g_{\mu\nu} \text{Tr}(\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu) \Gamma_2 \otimes \gamma^\lambda, \quad (117)$$

$$P_{cl}(\text{finite}) = C_{cl} \frac{\alpha_s}{4\pi} \left( \frac{5}{18} g_\rho q^\rho \right) + \frac{2}{9} g_{\mu\nu} \text{Tr}(\Gamma_1 \gamma_\mu \gamma_\lambda \gamma_\nu) \Gamma_2 \otimes \gamma^\lambda. \quad (118)$$

Here $C_{cl} = \text{Tr}(\hat{V}_1 T^b) \hat{V}_2 \otimes T^b$.

C.3 Special Cases

In Tables 2 and 3 we give the finite and singular parts for the penguin operator insertions with various Dirac structures.

| Dirac Structure | $P_{op}(\text{infinite})$ | $P_{op}(\text{finite})$ |
|-----------------|--------------------------|------------------------|
| $\Gamma_1 = \gamma_\rho P_L, \Gamma_2 = \gamma_\rho P_L$ | $C_{op} \frac{\alpha_s}{4\pi} \gamma_\lambda P_L \otimes \gamma^\lambda$ | $-C_{op} \frac{\alpha_s}{4\pi} \frac{13}{9} \gamma_\lambda P_L \otimes \gamma^\lambda$ |
| $\Gamma_1 = \gamma_\rho P_L, \Gamma_2 = \gamma_\rho P_R$ | $0$ | $0$ |
| $\Gamma_1 = P_L, \Gamma_2 = P_L$ | $0$ | $0$ |
| $\Gamma_1 = P_L, \Gamma_2 = P_R$ | $-C_{op} \frac{\alpha_s}{4\pi} \frac{1}{6} \gamma_\lambda P_R \otimes \gamma^\lambda$ | $C_{op} \frac{\alpha_s}{4\pi} \frac{13}{8} \gamma_\lambda P_R \otimes \gamma^\lambda$ |
| $\Gamma_1 = \sigma_{\alpha\beta} P_L, \Gamma_2 = \sigma_{\alpha\beta} P_L$ | $0$ | $0$ |

Table 2: Finite and infinite parts of the open penguin insertion. $C_{op} = \hat{V}_1 T^b \hat{V}_2 \otimes T^b$.

| Dirac Structure | $P_{cl}(\text{finite})$ | $P_{cl}(\text{infinite})$ |
|-----------------|------------------------|------------------------|
| $\Gamma_1 = \gamma_\rho P_L, \Gamma_2 = \gamma_\rho P_L$ | $-C_{cl} \frac{\alpha_s}{4\pi} \frac{13}{9} \gamma_\lambda P_L \otimes \gamma^\lambda$ | $C_{cl} \frac{\alpha_s}{4\pi} \frac{13}{9} \gamma_\lambda P_L \otimes \gamma^\lambda$ |
| $\Gamma_1 = \gamma_\rho P_L, \Gamma_2 = \gamma_\rho P_R$ | $-C_{cl} \frac{\alpha_s}{4\pi} \frac{13}{9} \gamma_\lambda P_R \otimes \gamma^\lambda$ | $C_{cl} \frac{\alpha_s}{4\pi} \frac{13}{9} \gamma_\lambda P_R \otimes \gamma^\lambda$ |
| $\Gamma_1 = P_L, \Gamma_2 = P_L$ | $0$ | $0$ |
| $\Gamma_1 = P_L, \Gamma_2 = P_R$ | $0$ | $0$ |
| $\Gamma_1 = \sigma_{\alpha\beta} P_L, \Gamma_2 = \sigma_{\alpha\beta} P_L$ | $0$ | $0$ |

Table 3: Finite and infinite parts the of closed penguin insertion. $C_{cl} = \text{Tr}(\hat{V}_1 T^b) \hat{V}_2 \otimes T^b$. 
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