Effects of $f(R)$ Dark Energy on Dissipative Anisotropic Collapsing Fluid

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Abstract

The purpose of this paper is to study the effects of dark energy on dynamics of the collapsing fluid within the framework of metric $f(R)$ gravity. The fluid distribution is assumed to be locally anisotropic and undergoing dissipation in the form of heat flow, null radiations and shear viscosity. For this purpose, we take general spherical symmetric spacetime. Dynamical equations are obtained and also some special solutions are found by considering shearing expansionfree evolution of the fluid. It is found that dark energy affects the mass of the collapsing matter and rate of collapse but does not affect the hydrostatic equilibrium.

Keywords: $f(R)$ theory; Dissipative anisotropic fluid; Dynamical equations.

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1 Introduction

Dark energy (DE) and gravitational collapse are the two noteworthy issues of cosmology and gravitational physics. Recent observational data [1]-[11]

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indicate that our universe is expanding. This acceleration is explained in terms of DE, which may be explained in modified gravity models. On the other hand, gravitational collapse is the basic process driving evolution within galaxies, assembling giant molecular clouds and producing stars.

When the Einstein-Hilbert (EH) gravitational action in General Relativity (GR),
\[ S_{EH} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R, \]  
(1.1)
is re-written in the modified form as follows
\[ S_{\text{modif}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R), \]  
(1.2)
the presence of \( f(R) \) function may be understood as the introduction of an effective fluid which is not restricted to hold the usual energy conditions. An important feature of this theory is that the modified field equations can be written in the form of Einstein tensor which makes it easy to compare with GR. This is done by taking all the higher order corrections to the curvature on the right hand side of the field equations and defining it as a ”Dark source” term or ”curvature fluid”. In the following, the field equations in the metric approach are obtained as
\[ F(R)R_{\alpha\beta} - \frac{1}{2} f(R)g_{\alpha\beta} - \nabla_\alpha \nabla_\beta F(R) + g_{\alpha\beta} \Box F(R) = \kappa T_{\alpha\beta}, \quad (\alpha, \beta = 0, 1, 2, 3), \]  
(1.3)
where \( F(R) \equiv df(R)/dR. \) When we re-write this equation in the above mentioned form, it follows that
\[ G_{\alpha\beta} = \frac{\kappa}{F}(T_{\alpha\beta}^m + T_{\alpha\beta}^D), \]  
(1.4)
where
\[ T_{\alpha\beta}^D = \frac{1}{\kappa} \left[ \frac{f(R) + RF(R)}{2} g_{\alpha\beta} + \nabla_\alpha \nabla_\beta F(R) - g_{\alpha\beta} \Box F(R) \right]. \]  
(1.5)
In this way, DE can be thought of as having the geometrical origin rather than some additional scalar fields which are added by hand to the matter part. Therefore, \( f(R) \) theory of gravity may be used to explain the present accelerating expansion of the universe.

This theory has many applications in cosmology and gravity such as inflation, local gravity constraints, cosmological perturbations and spherically
symmetric solutions in weak and strong gravitational backgrounds. In the last few years, a considerable amount of analysis and theoretical observations have been made in order to compare the preliminary successes of $f(R)$ models with the great achievements of GR. For example, Capozziello et al. [12] analyzed the relation between spherical symmetry and the weak field limit of $f(R)$ theory and compared the results with GR. In strong gravitational background such as neutron star and white dwarfs, one needs to take into account the backcreation of gravitational potentials to the field equations. The structure of the relativistic stars in $f(R)$ theory has been discussed by many authors [13]-[17].

Cai et al. [18] derived the generalized Misner-Sharp energy in $f(R)$ gravity for spherically symmetric spacetime. They found that unlike GR, the existence of the generalized Misner-Sharp energy depends on a constraint condition. Erickcek et al. [19] found unique exterior solution for a stellar object by matching it with interior solution in the presence of matter source. Kainulainen et al. [20] studied the interior spacetime of stars in Palatini $f(R)$ gravity. de la Cruz-Dombriz et al. [21] discussed the problem of finding static spherically symmetric black hole solutions in $f(R)$ theory. They explored several aspects of constant curvature solutions and thermodynamical properties. In a recent paper [22], we have investigated perfect fluid gravitational collapse in this theory and found that constant scalar curvature term $f(R_0)$ acts as a source of repulsive force and thus slows down the collapse of matter.

In GR, Oppenheimer and Snyder [23] innovated the first mathematical model for the description of gravitational collapse of stars. After that many approaches are adopted for the physical description of the fluid in order to form self-gravitating objects. During fluid evolution, self-gravitating objects may pass through phases of intense dynamical activities for which quasi-static approximation is not reliable. For instance, the collapse of very massive stars [24], the quick collapse phase yielding neutron star formation [25] and the peculiar stars. The peculiar stars are very dense, strongly magnetic and are created when massive stars die by collapse. Many of the cooler chemically peculiar stars are the result of the mixing of nuclear fusion products from the interior of the star to its surface.

Misner and Sharp [26] discussed the gravitational collapse by taking spherically symmetric ideal fluid. They provided a full account of the dynamical equations governing the adiabatic relativistic collapse. Dissipative process plays dominant role in the formation and evolution of stars. Vaidya
introduced the idea of outgoing radiations in collapse giving non-vacuum exterior outside the stars. It was physically a quite reasonable assumption as radiation is a confirmation that dissipative processes are occurring, causing loss of thermal energy of the system which is an effective way of decreasing internal pressure. Cai and Wang [28] studied the formation of black holes in the background of DE. Herrera et al. [29] investigated the dynamics of gravitational collapse which undergoes dissipation in the form of heat flow and radiation. The same authors [30] also provided detailed discussion on the physical meaning of expansionfree fluid evolution. Di Prisco et al. [31] explored gravitational collapse by adding charge and dissipation in the form of shear viscosity. The dynamical and transport equations were coupled to observe the effects of dissipation over collapsing process. Recently, Sharif et al. [32]–[39] explored different aspects of gravitational collapse by using all the three types of symmetry.

In this paper, we discuss how DE generated by curvature fluid affects the dynamics of dissipative gravitational collapse. The format of the paper is as follows. In next section 2, we present spacetimes and energy-momentum tensor for dissipative fluid. Section 3 is devoted to formulate the modified field equations and dynamical equations in $f(R)$ gravity. In section 4, special solutions are discussed. The last section 5 concludes the main results of the paper.

### 2 Spacetimes and Collapsing Matter

We take spherical symmetry about an origin $O$ which is divided into two regions, interior and exterior by 3D hypersurface $\Sigma$ centered at $O$. The interior spacetime to $\Sigma$ can be represented by the line element

$$ds^2 = A^2(t, r)dt^2 - B^2(t, r)dr^2 - C^2(t, r)(d\theta^2 + \sin^2 \theta d\phi^2).$$  

For the exterior spacetime to $\Sigma$, we take the Vaidya spacetime given by the line element

$$ds^2 = [1 - \frac{2m(\nu)}{r}]d\nu^2 + 2drd\nu - r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$  

where $m(\nu)$ represents the total mass and $\nu$ is the retarded time. In the interior region, we assume a distribution of anisotropic collapsing fluid which
undergoes a dissipation in the form of heat flow, null radiations and shearing viscosity. The energy-momentum tensor with such properties is given by

\[ T_{\alpha\beta} = (\rho + p_\perp)u_\alpha u_\beta - p_\perp g_{\alpha\beta} + (p_r - p_\perp)\chi_\alpha \chi_\beta + u_\alpha q_\beta + q_\alpha u_\beta + \epsilon l_\alpha l_\beta - 2\eta \sigma_{\alpha\beta}. \]  

Here, we have \( \rho \) as the energy density, \( p_\perp \) the tangential pressure, \( p_r \) the radial pressure, \( q_\alpha \) the heat flux, \( \eta \) the coefficient of shear viscosity, \( u_\alpha \) the four-velocity of the fluid, \( \chi_\alpha \) the unit four-vector along the radial direction, \( l_\alpha \) a radial null four-vector and \( \epsilon \) the energy density of the null fluid describing dissipation in the free streaming approximation. These quantities satisfy the relations

\[ u_\alpha u_\alpha = 1, \quad \chi_\alpha \chi_\alpha = -1, \quad l_\alpha l_\alpha = 0, \quad l_\alpha u_\alpha = 0, \quad l_\alpha = 0 \]  

which are obtained from the following definitions in co-moving coordinates

\[ u^\alpha = A^{-1} \delta_0^\alpha, \quad \chi^\alpha = B^{-1} \delta_1^\alpha, \quad q^\alpha = q B^{-1} \delta_1^\alpha, \quad l^\alpha = A^{-1} \delta_0^\alpha + B^{-1} \delta_1^\alpha. \]  

Here \( q \) is a function of \( t \) and \( r \). The shear tensor \( \sigma_{ab} \) is defined by

\[ \sigma_{\alpha\beta} = u_{(\alpha;\beta)} - u_{(a} u_{\beta)} - \frac{1}{3} \Theta (g_{\alpha\beta} - u_\alpha u_\beta), \]  

where the acceleration \( a_\alpha \) and the expansion \( \Theta \) are given by

\[ a_\alpha = u_{\alpha;\beta} u^\beta, \quad \Theta = u_\alpha. \]  

The bulk viscosity does not appear explicitly as it has been absorbed in the form of radial and tangential pressures of the collapsing fluid. From Eqs. (2.5) and (2.6), the non-zero components of the shear tensor are

\[ \sigma_{11} = \frac{-2}{3} B^2 \sigma, \quad \sigma_{22} = \frac{1}{3} C^2 \sigma, \quad \sigma_{33} = \sigma_{22} \sin^2 \theta. \]  

The shear scalar \( \sigma \) is given by

\[ \sigma = \frac{1}{A} \left( \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right). \]  

Using Eqs. (2.5) and (2.7), it follows that

\[ a_1 = -\frac{A'}{A}, \quad a^2 = a^\alpha a_\alpha = \left( \frac{A'}{AB} \right)^2, \quad \Theta = \frac{1}{A} \left( \frac{\dot{B}}{B} + \frac{2}{C} \dot{C} \right), \]  

where dot and prime represent derivative with respect to \( t \) and \( r \) respectively.
3 The Field Equations and the Dynamical Equations in f(R) Gravity

The field equations (1.3) for the interior metric take the following form:

\[
\begin{align*}
\frac{AA''}{B^2} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{AA'B'}{B^3} - \frac{2\dot{C}}{C} + \frac{2\dot{A}}{AC} + \frac{2AA'C''}{2B^2C} - \frac{A^2 f(R)}{2F} \\
- \frac{A^2 F''}{B^2F} - \frac{\dot{F}^2}{B} + \frac{2\dot{C}}{C} - \frac{FB^2(\ddot{B}' - B'_B)}{FB^2} = 8\pi(\rho + \epsilon)A^2, \\
(3.1) \\
-2\left(\frac{\dot{C}''C}{C} - \frac{\dot{C}AC}{CA} - \frac{\dot{C}BC}{BC}\right) - \frac{\dot{F}'}{F} + \frac{\dot{A}'F}{AF} + \frac{\dot{B}'F}{BF} = -8\pi(q + \epsilon)AB, \\
(3.2) \\
- \frac{A''}{A} + \frac{\ddot{B}B}{A^2} - \frac{\dot{A}\dot{B}B}{AB} + \frac{A'B'}{AB} - \frac{2C''}{C} + \frac{2\dot{C}'B}{A^2C} + \frac{2B'C'}{BC} + \frac{B^2 f(R)}{2F} \\
+ \frac{\ddot{F}B^2}{A^2F} + \frac{\dot{F}'B^2}{A^2F} + \frac{\dot{F}A^2}{F} = 8\pi(p_r + \epsilon + \frac{4}{3}\eta\sigma)B^2, \\
(3.3) \\
\frac{\dot{C}}{CA^2} - \frac{\dot{A}C''}{CA^3} - \frac{A'C'}{B^2AC} - \frac{C''}{B^2C} + \frac{\dot{C}B}{A^2BC} + \frac{B'C'}{B^2C} + \frac{1}{C^2} \\
+ \frac{\dot{C}^2}{A^2C^2} + \frac{C^2 f(R)}{B^2C^2} - \frac{1}{2F} + \frac{\ddot{F}}{B^2} + \frac{\dot{F}'(\dot{A} - B')}{B^2} + \frac{\dot{C}'}{B^2} = 8\pi(p_\perp - \frac{2}{3}\eta\sigma). \\
(3.4)
\end{align*}
\]

In the most general case, we have nine variables with four equations. One cannot solve this system of equations unless some assumptions are imposed. An expansionfree motion of the system, i.e., \(\Theta = 0\), will be used in section 4 which may lead to some interesting results.

In order to develop dynamical equations that help to study the properties of collapsing process, we shall use Misner and Sharp [26] formalism. The mass function is defined by

\[
M = \frac{C}{2} (1 + g^{\mu\nu}C_{\mu\nu}) = \frac{C}{2} \left(1 + \frac{\dot{C}^2}{A^2} - \frac{C^2}{B^2}\right). \\
(3.5)
\]

The proper time and radial derivatives are given by

\[
D_T = \frac{1}{A} \frac{\partial}{\partial t}, \quad D_C = \frac{1}{C} \frac{\partial}{\partial r}, \\
(3.6)
\]
where $C$ is the areal radius of a spherical surface inside the boundary. The velocity of the collapsing fluid is defined by the proper time derivative of $C$, i.e.,

$$U = D_T C = \frac{\dot{C}}{A},$$

(3.7)

which is always negative. Using this expression, Eq. (3.5) implies that

$$E \equiv \frac{C'}{B} = \left[1 + U^2 + \frac{2M}{C}\right]^{1/2}.$$

(3.8)

When we make use of Eqs. (2.9), (2.10) and (3.6) in Eq. (3.2), we obtain

$$E \left[1 \frac{1}{3} D_C \left(\Theta - \sigma\right) - \frac{\sigma}{C}\right] = 4\pi \left[\frac{q + \epsilon}{E} C' + D_T F' - \frac{UC'}{C} D_C A D_T F - \frac{UEF'}{C} D_C B\right].$$

(3.9)

The rate of change of mass in Eq. (3.5) with respect to proper time, with the use of Eqs. (3.1)-(3.4), is given by

$$D_T M = \frac{C^2}{F} \left[ -4\pi \left\{ (\rho + 2e + p_r - 2p_\perp + \frac{8}{3} \eta \sigma) U + E(q + \epsilon) \right\} - U \left\{ \frac{F}{2C^2} - \frac{FE^2}{C^2} + \frac{FD_T U}{C} + \frac{E^2}{C^2} \left\{ \frac{\dot{F} C'}{C} + \frac{3F''}{2} - E \dot{F} D_C B \right\}\right\} + \frac{ED_A D_T F}{2C} + \frac{f(R)}{2} \right\} - U^3 \left\{ \frac{F}{C^2} + \frac{\dot{F}}{C^2} + \frac{\dot{F}}{C} \left( \frac{D_T A}{2C} - \frac{1}{C} \right) \right\} + U^2 \left\{ \left( \frac{F}{C} - \frac{F'}{2C'} \right) D_C A + \frac{3\dot{F} D_T B}{2EC'} \right\} \frac{E^2}{C} - \frac{E^3}{2C'} \left[ D_T F' + ED_C F D_T B \right].$$

(3.10)

This represents variation of total energy inside a collapsing surface of radius $C$. The first two terms inside the square brackets have negative signs which show that the total energy is being dissipated during collapse. However, in the case of collapse $U < 0$, the first term $(\rho + 2e + p_r - 2p_\perp + \frac{8}{3} \eta \sigma)$ increases the energy density through the rate of work being done by the effective anisotropic pressure and radiation density of the null fluid. Here we may use
equation of state to change energy density into pressure. The second term $E(q + \epsilon)$ has negative sign which describes that energy is leaving the system due to heat flux and radiations. All other terms show the contribution of the DE in the form of function $f(R)$ and its derivatives. We know that DE exerts a repulsive force on its surrounding thus we may conclude from the above expression that DE reduces the mass of the collapsing matter due to its negative pressure. It is mentioned here that in GR, only the first two terms excluding energy density $\rho$ and tangential pressure appear.

Similarly, we can calculate

$$D_CM = \frac{C^2}{2F} \left[ 8\pi \{ \rho + 2\epsilon + p_r + \frac{4}{3} \eta \sigma + \frac{U}{E}(q + \epsilon) \} ight. + U^2 \left( \frac{F}{C} - D_T F D_C A \right) - U \left\{ E D_C F \left( \frac{D_T B}{C'} - \frac{E D_C A}{C} \right) \right. \left. + \frac{E F D_C A}{C C'} \right\} + D_{TT} F - \frac{F'' E^2}{C''} - \frac{E D_T F'}{C'} - \frac{E}{C'} (D_T F D_T B) + \left. E^2 D_C F D_C B + \frac{F E^2}{C^2} + \frac{F}{C^2} + \frac{2 F D_T U}{C} \right] \right]. \quad (3.11)$$

This equation describes how different quantities influence the mass between neighboring surfaces of radius $C$ in the fluid distribution. The first two terms and their description in the above expression are almost the same as in GR except for the factor $(p_r + \frac{4}{3} \eta)$. The appearance of this factor is due to the complicated field equations. The remaining terms represent contribution of DE due to curvature fluid. Taking integral of Eq.(3.11) over $C$, we have

$$M = \frac{1}{2} \int_0^C \left[ 8\pi \{ \rho + 2\epsilon + p_r + \frac{4}{3} \eta \sigma + \frac{U}{E}(q + \epsilon) \} ight. + U^2 \left( \frac{F}{C} - D_T F D_C A \right) - U \left\{ E D_C F \left( \frac{D_T B}{C'} - \frac{E D_C A}{C} \right) \right. \left. + \frac{E F D_C A}{C C'} \right\} + D_{TT} F - \frac{F'' E^2}{C''} - \frac{E D_T F'}{C'} - \frac{E}{C'} (D_T F D_T B) + \left. E^2 D_C F D_C B + \frac{F E^2}{C^2} + \frac{F}{C^2} + \frac{2 F D_T U}{C} \right] \right] dC. \quad (3.12)$$

The dynamical equations can be obtained from the contracted Bianchi identities. Consider the following two equations

$$T^\alpha_\beta u_\alpha = 0, \quad T^\alpha_\beta \chi_\alpha = 0 \quad (3.13)$$
which yield
\[
\frac{1}{A} \left[ (\rho + \epsilon)' + (\rho + 2\epsilon + p_r + \frac{4}{3} \eta F) \frac{\dot{B}}{B} + 2(\rho + \epsilon + p_\perp - 2\frac{3}{3} \eta \sigma) \frac{\dot{C}}{C} \right] \\
+ \frac{1}{B} \left[ (q + \epsilon)' + 2(q + \epsilon) \frac{(AC)'}{AC} \right] = 0, \tag{3.14}
\]
\[
- \frac{1}{A} \left[ (q + \epsilon)' + 2(q + \epsilon) \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] - \frac{1}{B} \left[ (p_r + \epsilon + \frac{4}{3} \eta \sigma)' \right. \\
+ (\rho + p_r + 2\epsilon + \frac{4}{3} \eta \sigma) \frac{A'}{A} + 2(p_r - p_\perp + \epsilon + 2\eta \sigma) \frac{C'}{C} = 0. \tag{3.15}
\]

Using Eqs. (2.9), (2.10), (3.6) and (3.8), it follows that
\[
D_T (\rho + \epsilon) + \frac{1}{3} (3\rho + 4\epsilon + p_r + 2p_\perp) \Theta + \frac{2}{3} (\epsilon + p_r - p_\perp - 2\eta \sigma) \sigma \\
+ E D_C (q + \epsilon) + 2(q + \epsilon) (a + \frac{E}{C}) = 0, \tag{3.16}
\]
\[
D_T (q + \epsilon) + \frac{2}{3} (q + \epsilon) (2\Theta + \sigma) + E D_C (p_r + \epsilon + \frac{4}{3} \eta \sigma) + (\rho + p_r) \\
+ 2\epsilon + \frac{4}{3} \eta \sigma)a + 2(p_r - p_\perp + \epsilon + 2\eta \sigma) \frac{E}{C} = 0. \tag{3.17}
\]

The acceleration \( D_T U \) of the collapsing matter inside the hypersurface is obtained by using Eqs. (3.4)-(3.6) and (3.8)
\[
D_T U = \frac{2M}{C^2} + \frac{8\pi C}{F} (p_\perp - \frac{2}{3} \eta \sigma) + Ea + E D_C E - \frac{U E D_T B}{C'} \\
- \frac{C f(R)}{2F} - \frac{C}{F} \left\{ -D_{TT} F + E D_C (\frac{F'}{B}) - \sigma D_T F \right\} \\
+ E^2 (\frac{UD_C A}{C} + \frac{1}{C}) D_C F. \tag{3.18}
\]
Substituting \(a\) from Eq. (3.18) into (3.17), it follows that

\[
(\rho + p_r + 2\epsilon + \frac{4}{3}\eta\sigma)D_T U = (\rho + p_r + 2\epsilon + \frac{4}{3}\eta\sigma) \left[ \frac{8\pi C}{F}(p_\perp - \frac{2}{3}\eta\sigma) + 2MC^2 - CF(R) + 2\epsilon(2UC + \sigma) + (p_r + 2\epsilon + \frac{4}{3}\eta\sigma) \right]
\]

This shows the role of different forces on the collapsing process. The term within the brackets on the left hand side stands for "effective" inertial mass and the remaining term is acceleration. The first term on the right hand side represents gravitational force (the passive gravitational mass by equivalence mass). The term within the first square brackets shows how dissipative terms and DE affect the passive gravitational mass. The first two terms in the second square brackets are gradient of the effective pressure and effect of local anisotropy of pressure with negative sign which increases the rate of collapse. While the other term shows the contribution of DE collective with effective pressure. Here the presence of \(U\) (with negative sign) indicates that this term slows down the rate of collapse. The last square brackets depicts the combine role of each type of matter component.

4 Some Special Solutions

4.1 Shearing Expansionfree Dissipative Fluid

Now we use the condition of expansionfree motion, i.e., \(\Theta = 0\), to find some solutions. Using this condition, Eq. (2.10) yields

\[
\frac{\dot{B}}{B} = -2\frac{\dot{C}}{C}.
\]
The physical meaning of this condition is discussed with detail in [30]. On integration, we get
\[ B = \frac{g_1(r)}{C^2}. \] (4.2)
where \( g_1(r) \) is an arbitrary function. Substituting Eq.(4.1) in (3.2), we get
\[ 2\left(\frac{\dot{C}'}{C} - \frac{A'}{A} - 2\frac{C'}{C}\right) + \left(\frac{\dot{F}'}{F} - \frac{A'\dot{F}}{AF}\right)C + 2\frac{F'}{F} = 8\pi(q + \epsilon)A^2 \frac{B}{C} \] (4.3)
which, by integrating, gives
\[ A = \frac{F^2\dot{C}C^2}{\tau(t)}e^{\int \left(8\pi(q + \epsilon)A\frac{g_1}{FC^2} + \frac{\dot{F}'}{F} - \frac{A'\dot{F}}{AF}\right)C dr}. \] (4.4)
Thus the interior metric becomes
\[ ds^2 = \left(\frac{F^2\dot{C}C^2}{\tau(t)}\exp\left[-\int \left(8\pi(q + \epsilon)A\frac{g_1}{FC^2} + \frac{\dot{F}'}{F} - \frac{A'\dot{F}}{AF}\right)C dr\right]\right)^2 dt^2 - \left(\frac{g_1}{C^2}\right)^2 dr^2 - C^2(t, r)(d\theta^2 + \sin^2 \theta d\phi^2). \] (4.5)
The above metric represents spherically symmetric anisotropic fluid which is going shearing expansionfree evolution. Assuming the condition of constant scalar curvature \( R = R_c \), according to which \( F(R_c) = constant \), the metric for nondissipative \( (q = \epsilon = 0) \) case is reduced to
\[ ds^2 = \left(\frac{F^2\dot{C}C^2}{\tau(t)}\right)^2 dt^2 - \frac{1}{C^4} dr^2 - C^2(t, r)(d\theta^2 + \sin^2 \theta d\phi^2). \] (4.6)
Here we take \( g_1(r) = 1 \). It is mentioned here that \( f(R) \) theory, in the case of constant scalar curvature, exhibits behavior just like solutions with cosmological constant in GR. This is one of the reason why the DE issue can be addressed using this theory. Moreover, in this case, Birkhoff theorem holds, i.e., stationary solutions are also static [12] which does not hold for every \( f(R) \).

4.2 Shearing Expansionfree Nondissipative Perfect Fluid

Here we discuss shearing nondissipative expansionfree perfect fluid with constant scalar curvature. For perfect fluid \( \eta = 0 \) and hence the metric reduces
to Eq. (4.6). Using values of $A$ and $B$ from Eq. (4.6) in the field equations Eqs. (3.1)-(3.4) along with Eq. (3.9), it follows that

$$-\frac{6\tau^2}{F_c^4 C^6} + \frac{C^4 \dot{C}''}{C} + C^3 (2C'' + \frac{8\dot{C}'' C'}{C}) + 10C'^2 C^2 - \frac{1}{2} \frac{f(R_c)}{F_c} = \frac{8\pi \rho}{F_c},$$

$$\frac{D_C \sigma}{3} + \frac{\sigma}{C} = 0,$$

$$-\frac{\tau^2}{F_c^4 C^6} \left( \frac{6\dot{C}}{C} - \frac{\dot{\tau}}{\tau} \right) + C^4 \left( \frac{\dot{C}''}{C} + \frac{4C''}{C} + \frac{6\dot{C}' C'}{C^2} + \frac{10C'^2}{C^2} \right) + \frac{1}{2} \frac{f(R_c)}{F_c} = \frac{8\pi p_r}{F_c},$$

$$-\frac{-\tau^2}{F_c^4 C^6} \left( \frac{3\dot{C}}{C} - \frac{\dot{\tau}}{\tau} \right) - C^4 \left( \frac{\dot{C}'' C'}{CC} - \frac{3C'^2}{C^2} - \frac{C''}{C^2} \right) + \frac{1}{2} \frac{1}{C^2} + \frac{1}{2} \frac{f(R_c)}{F_c} = \frac{8\pi p_\perp}{F_c}.$$

In this case, the Bianchi identities are reduced to the same expression as in GR, i.e.,

$$\dot{\rho} + 2(p_\perp - p_r) \frac{\dot{C}}{C} = 0, \quad (4.7)$$

$$p'_r + (\rho + p_r) \frac{\dot{C}'}{C} + 2(\rho + 2p_r - p_\perp) \frac{C''}{C} = 0. \quad (4.8)$$

We would like to mention here that for isotropic fluid, i.e., $p_r = p_\perp$, Eq. (4.7) implies that energy density $\rho$ depends only on $r$.

### 4.3 Shearing Expansionfree Dust

In this case, we have $p_r = 0 = p_\perp$ and hence Eq. (4.8) becomes

$$\frac{\dot{C}'}{C} + 2\frac{C''}{C} = 0 \quad (4.9)$$

whose integration gives

$$\dot{C}' = \frac{g_2(t)}{C^2}, \quad C' = \frac{g_3(r)}{C^2}, \quad (4.10)$$

where $g_2(t)$ and $g_3(r)$ are arbitrary functions of $t$ and $r$ respectively. Further, integrating Eq. (4.10) with respect to the corresponding arguments of the arbitrary functions, we get

$$\frac{C^3}{3} = \int g_2(t) dt + \int g_3(r) dr \quad (4.11)$$
which can be written as
\[ C^3 = \psi(t) + \chi(r), \] (4.12)
where
\[ \psi(t) = 3 \int g_2(t) dt, \quad \chi(r) = 3 \int g_3(r) dr, \] (4.13)
In view of Eq.(4.10) and taking \( \tau(t) = g_2(t) \), the metric in Eq.(4.6) is reduced to
\[ ds^2 = F^4_c dt^2 - \frac{1}{C^4} dr^2 - C^2(t,r)(d\theta^2 + \sin^2 \theta d\phi^2) \] (4.14)
while Eq.(3.7) becomes
\[ U = \frac{\dot{C}}{F^2_c}. \] (4.15)
When we use the standard GR limit, i.e., \( F_c = 1 \), Eqs.(4.14) and (4.15) are reduced to the corresponding GR results.

5 Summary

This paper investigates the gravitational collapse of a spherically symmetric star in \( f(R) \) theory of gravity. The star is made up of viscous anisotropic fluid distribution which is dissipating energy in the form of heat flow, null radiations and shearing viscosity. The objective of this work is to explore the effects of DE which is generated by modifying EH action. We have discussed the consequences in view of existing GR results.

It is concluded that the contribution of DE terms decreases the mass of the collapsing fluid with the passage of time and hence prevents the fluid to collapse. In dynamical equations, such curvature terms appear oftentimes to affect the passive gravitational mass and rate of collapse.

We have found some solutions by assuming that fluid has no expansion, i.e., \( \Theta = 0 \). Herrera et al. \[30\] has made a comprehensive discussion on the physical meaning of expansionfree evolution of the fluid. According to them such an assumption causes the formation of a vacuum cavity inside the collapsing fluid. Using this condition, we have obtained general metric for dissipative fluid which is further reduced to nondissipative case. For nondissipative solution, a locally anisotropic perfect fluid solution is obtained with the assumption of constant scalar curvature. This is further reduced to the dust case.
It is noted here that implication of hydrostatic equilibrium limit yields the same expression as in GR [30], i.e.,

\[ D_C p_r + 2 \left( \frac{p_r - p_\perp}{C} \right) = - \frac{(\rho + p_r)}{C(C - 2M)}(M + 4\pi p_r C^3). \] (5.1)

The reason is that \( f(R) \) term does not affect contracted Bianchi identities which are used to obtain the above expression along with the use of Eq. (4.7) giving time independent \( \rho \) in an isotropic pressure case. This change in energy density can be interpreted as the rate of work being done by the force of locally anisotropic pressure. It would be worthwhile to investigate these issues using other symmetries in \( f(R) \) theory for the complete understanding of gravitational collapse.

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**References**

[1] Perlmutter, S., et al.: Astrophys. J. 517(1999)565.
[2] Knop, R.A., et al.: Astrophys. J. 598(2003)102.
[3] Riess, A.G., et al.: Astrophys. J. 116(1998)1009.
[4] Tonry, J.L., et al.:Astrophys. J. 594(2003)1.
[5] de Bernardis, P., et al.: Nature 404(2000)955.
[6] Stompor, R., et al.: Astrophys. J. 561(2001)L7.
[7] Spergel, D.N. et al.: Astrophys. J. Suppl. 148(2003)175.
[8] Hinshaw, G., et al.: Astrophys. J. Suppl. 148(2003)135.
[9] Astier, P., et al.: Astron. Astrophys. 447(2006)31.
[10] Spergel, D.N., et al.: Astrophys. J. Suppl. 170(2007)377.
[11] Riess, A.G., et al.: Astrophys. J. 607(2004)665.
[12] Capozziello, S., Stabile, A. and Troisi, A.: Class. Quantum Grav. 25(2008)085004.
[13] Kobayashi, T. and Maeda K.I.: Phys. Rev. D78(2008)064091.
[14] Kobayashi, T. and Maeda K.I.: Phys. Rev. D79(2009)024009.
[15] Tsujikawa, S., Tamaki, T. and Tavakol, R.: JCAP 0905(2009)020.
[16] Babichev, E. and Langlois, D.: Phys. Rev. D80(2009)121501.
[17] Upadhye, A. and Hu, W.: Phys. Rev. D80(2009)064002.
[18] Cai, R-G., Cao, L-M., Hu, Y-P. and Ohta, N.: Generalized Misner-Sharp Energy in f(R) Gravity; arXiv/0910.2387v1.
[19] Erickcek, A.L., Smith, T.L. and Kamionkowski, M.: Phys. Rev. D74(2006)121501.
[20] Kainulainen, K., Reijonen, V. and Sunhede, D.: Phys. Rev. D76(2007)043503.
[21] de la Cruz-Dombrize, A., Dobado A. and Maroto, A.L.: Phys. Rev. D80(2009)124011.
[22] Sharif, M. and Kausar, H.R.: Astrophys. Space Sci. (to appear, 2010).
[23] Oppenheimer, J.R. and Snyder, H.: Phys. Rev. 56(1939)455.
[24] Iben, I.: Astrophys. J. 138(1963)1090.
[25] Myra, E. and Burrows, A: Astrophys. J. 364(1990)222.
[26] Misner, C.W. and Sharp, D.: Phys. Rev. 136(1964)B571.
[27] Vaidya, P.C.: Proc. Indian Acad. Sci. A33(1951)264.
[28] Cai, R-G.: JHEP 079(2006)0608.
[29] Herrera, L. and Santos, N.O.: Phys. Rev. D70(2004)084004.
[30] Herrera, L., Santos, N.O. and Wang, A.: Phys. Rev. D78(2008)080426.
[31] Di Prisco, A., Herrera, L., Denmat, G.Le., MacCallum, M.A.H. and Santos, N.O.: Phys. Rev. D76(2007)064017.

[32] Sharif, M. and Rehmat, Z.: Gen. Relativ. Grav. 42(2010)1795.

[33] Sharif, M. and Ahmad, Z.: Mod. Phys. Lett. A22(2007)1493; ibid 2947.

[34] Sharif, M. and Ahmad, Z.: Int. J. Mod. Phys. A23(2008)181.

[35] Sharif, M. and Ahmad, Z.: J. Korean Physical Society 52(2008)980.

[36] Sharif, M. and Ahmad, Z.: Acta Physica Polonica B39(2008)1337.

[37] Sharif, M. and Abbas, G.: Mod. Phys. Lett. A24(2009)2551.

[38] Sharif, M. and Fatima, S.: Gen. Relativ. Grav. (to appear, 2010).

[39] Sharif, M. and Siddiqua, A.: Gen. Relativ. Grav. (to appear, 2010), arXiv/1008.0695v1.