Sequential Aggregate Signatures with Short Public Keys without Random Oracles*

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Abstract

The notion of aggregate signature has been motivated by applications and it enables any user to compress different signatures signed by different signers on different messages into a short signature. Sequential aggregate signature, in turn, is a special kind of aggregate signature that only allows a signer to add his signature into an aggregate signature in sequential order. This latter scheme has applications in diversified settings such as in reducing bandwidth of certificate chains and in secure routing protocols. Lu, Ostrovsky, Sahai, Shacham, and Waters (EUROCRYPT 2006) presented the first sequential aggregate signature scheme in the standard model. The size of their public key, however, is quite large (i.e., the number of group elements is proportional to the security parameter), and therefore, they suggested as an open problem the construction of such a scheme with short keys.

In this paper, we propose the first sequential aggregate signature schemes with short public keys (i.e., a constant number of group elements) in prime order (asymmetric) bilinear groups that are secure under static assumptions in the standard model. Furthermore, our schemes employ a constant number of pairing operations per message signing and message verification operation. Technically, we start with a public-key signature scheme based on the recent dual system encryption technique of Lewko and Waters (TCC 2010). This technique cannot directly provide an aggregate signature scheme since, as we observed, additional elements should be published in a public key to support aggregation. Thus, our constructions are careful augmentation techniques for the dual system technique to allow it to support sequential aggregate signature schemes. We also propose a multi-signature scheme with short public parameters in the standard model.

Keywords: Public-key signature, Aggregate signature, Sequential aggregate signature, Dual system encryption, Bilinear maps.

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1 Introduction

Aggregate signature is a relatively new type of public-key signature (PKS) that enables any user to combine \( n \) signatures signed by \( n \) different signers on \( n \) different messages into a short signature. The concept of public-key aggregate signature (PKAS) was introduced by Boneh, Gentry, Lynn, and Shacham [10], and they proposed an efficient PKAS scheme in the random oracle model using bilinear groups. After that, numerous PKAS schemes were proposed using bilinear groups [1, 7, 8, 15, 16, 27, 28] or using trapdoor permutations [3, 29, 31].

One application of aggregate signature is the certificate chains of the public-key infrastructure (PKI) [10]. The PKI system has a tree structure, and a certificate for a user consists of a certificate chain from a root node to a leaf node, each node in the chain signing its predecessor. If the signatures in the certificate chain are replaced with a single aggregate signature, then bandwidth for signature transfer can be significantly saved. Another application is to the secure routing protocol of the Internet protocol [10]. If each router that participates in the routing protocol uses a PKAS scheme instead of a PKS scheme, then the communication overload of signature transfer can be dramatically reduced. Furthermore, aggregate signatures have other applications such as reducing bandwidth in sensor networks or ad-hoc networks, as well as in software authentication in the presence of software update [1].

1.1 Previous Methods

Aggregate signature schemes are categorized as full aggregate signature, synchronized aggregate signature, and sequential aggregate signature depending on the type of signature aggregation. They have also been applied to regular signatures in the PKI model and to ID-based signatures (with a trusted key server).

The first type of aggregate signature is full aggregate signature, which enables any user to freely aggregate different signatures of different signers. This full aggregate signature is the most flexible aggregate signature since it does not require any restriction on the aggregation step (though restriction may be needed at times for certain applications). However, there is only one full aggregate signature scheme, proposed by Boneh et al. [10]. Since this scheme is based on the short signature scheme of Boneh et al. [11], the signature length it provides is also very short. However, the security of the scheme has just been proven in the idealized random oracle model and the number of pairing operations in the aggregate signature verification algorithm is proportional to the number of signers in the aggregate signature.

The second type of aggregate signature is synchronized aggregate signature, which enables any user to combine different signatures with the same synchronizing information into a single signature. The synchronized aggregate signature has one drawback: all signers should share the same synchronizing information (such as a time clock or another shared value). Gentry and Ramzan [15] introduced the concept of synchronized aggregate signature. They proposed an identity-based synchronized aggregate signature scheme using bilinear groups, and they proved its security in the random oracle model. We note that identity-based aggregate signature (IBAS) is an ID-based scheme and thus relies on a trusted server knowing all private keys (i.e., its trust structure is different from that in regular PKI). However, it also has a notable advantage in that it is not required to retrieve the public keys of signers in the verification algorithm since an identity string plays the role of a public key (this lack of public key is indicated in our comparison table as public key of no size!). Recently, Ahn et al. [11] presented a public-key synchronized aggregate signature scheme without relying on random oracles.

\(^1\)Subsequent to our work, Hohenberger et al. [19] proposed an identity-based aggregate signature scheme that supports full aggregation based on the recently introduced candidate multilinear maps of Garg et al. [13].
The third type of aggregate signature is sequential aggregate signature (SAS), which enables each signer to aggregate his signature to a previously aggregated signature in a sequential order. The sequential aggregate signature has the obvious limitation of signers being ordered to aggregate their signatures in contrast to the full aggregate signature and the synchronized aggregate signature. However, it has an advantage in that it is not required to share synchronized information among signers in contrast to the synchronized aggregate signature, and many natural applications lead themselves to this setting. The concept of sequential aggregate signature was introduced by Lysyanskaya, Micali, Reyzin, and Shacham [29], and they proposed a public-key sequential aggregate signature scheme using the certified trapdoor permutations in the random oracle model. Boldyreva et al. [7] presented an identity-based sequential aggregate signature scheme in the random oracle model using an interactive assumption, but it was shown by Hwang et al. [20] that their construction is not secure. After that, Boldyreva et al. [8] proposed a new identity-based sequential aggregate signature by modifying their previous construction and proved its security in the generic group model. Recently, Gerbush et al. [16] showed that the modified IBAS scheme of Boldyreva et al. [8] is secure under static assumptions using the dual form signatures framework.

The first sequential aggregate signature scheme without random oracle idealization was proposed by Lu et al. [27, 28]. They converted the PKS scheme of Waters [34] to the PKAS scheme and proved its security under the well known CDH assumption. However, their scheme has a drawback since the number of group elements in a public key is proportional to the security parameter (for a security of $2^{80}$ they need 160 elements, or about 80 elements in a larger group); so they left as an open question how to design a scheme with shorter public keys. Schröder proposed a PKAS scheme with short public keys relying on the Camenisch-Lysyanskaya signature scheme [33]; however the scheme's security is proven under an interactive assumption (which, typically, is a relaxation used when designs based on static assumptions are hard to find). Therefore, the construction of an SAS scheme with short public keys without relaxations such as random oracles or interactive assumptions was left as an open question.

1.2 Our Contributions

Challenged by the above question, the motivation of our research is to construct an efficient SAS scheme secure in the standard model (i.e., without employing assumptions such as random oracle or interactive assumptions as part of the proof) with short public keys (e.g., a constant number of group elements). To achieve this goal, we use the PKS scheme derived from the identity-based encryption (IBE) scheme, which adopts the innovative dual system encryption techniques of Waters [26, 35]. That is, an IBE scheme is first converted to a PKS scheme by the clever observation of Naor [9]. The PKS schemes that adopt the dual system encryption techniques are the scheme of Waters [35], which includes a random tag in a signature, and the scheme of Lewko and Waters [26], which does not include a random tag in a signature. The scheme of Waters is not appropriate to aggregate signatures since the random tags in signatures cannot be compressed into a single value. The scheme of Lewko and Waters in composite order groups is easily converted to an aggregate signature scheme if an element in $G_{p_3}$ is moved from a private key to a public key, but it is inefficient because of composite order groups. Gerbush et al. [16] showed that a modified Camenisch-Lysyanskaya signature scheme in composite order groups is secure under static assumptions. However, it is unclear whether the construction of Schröder can be directly applied to this modified Camenisch-Lysyanskaya signature scheme. The reason is that aggregating $G_{p_1}$ and $G_{p_3}$ subgroups is hard and a private key element $g_{2,3} \in G_{p_1 p_3}$ cannot be generated by the key generation algorithm of an aggregate signature scheme. Additionally, our work and findings are independent of the work of Gerbush et al.

We can safely move the element in $G_{p_1}$ from a private key to a public key since it is always given in assumptions. Lewko obtained a prime order IBE scheme by translating the Lewko-Waters composite order IBE scheme using the dual pairing vector
Table 1: Comparison of aggregate signature schemes

| Scheme | Type | ROM | KOSK | PK Size | AS Size | Sign Time | Verify Time | Assumption |
|--------|------|-----|------|---------|---------|------------|-------------|------------|
| BGLS [10] | Full | Yes | No | $1k_p$ | $1k_p$ | 1E | /P | CDH |
| GR [15] | IB, Sync | Yes | No | $2k_p + \lambda$ | 3E | 3P + /E | CDH |
| AGH [1] | Sync | Yes | Yes | $1k_p$ | 2$k_p + 32$ | 6E | 4P + /E | CDH |
| AGH [1] | Sync | No | Yes | $1k_p$ | $2k_p + 32$ | 10E | 8P + /E | CDH |
| LMRS [29] | Seq | Yes | No | $1k_f$ | $1k_f$ | /E | /E | cert TDP |
| Neven [31] | Seq | Yes | No | $1k_f$ | $1k_f + 2\lambda$ | 1E + 2/M | 2/M | uncert CFP |
| BGOY [8] | IB, Seq | Yes | No | $3k_p$ | 4P + /E | 4P + /E | Interactive |
| GLOW [16] | IB, Seq | Yes | No | $5k_f$ | 10P + 2/E | 10P + 2/E | Static |
| LOSSW [27] | Seq | No | Yes | $2\lambda k_p$ | $2k_p$ | 2P + 4\lambda /M | 2P + 2\lambda /M | CDH |
| Schröder [33] | Seq | No | Yes | $2k_p$ | $4k_p$ | /P + 2/E | /P + /E | Interactive |
| Ours | Seq | No | Yes | $11k_p$ | $8k_p$ | 8P + 5/E | 8P + 4/E | Static |
| Ours | Seq | No | Yes | $13k_p$ | $6k_p$ | 6P + 6/E | 6P + 3/E | Static |

ROM = random oracle model, KOSK = certified-key model, IB = identity based
\(\lambda\) = security parameter, \(k_p, k_f\) = the bit size of elements for pairing and factoring, \(l\) = the number of signers
P = pairing computation, E = exponentiation, M = multiplication

Therefore, we start the construction from the IBE scheme of Lewko and Waters (LW-IBE) [26] in the prime order (asymmetric) bilinear groups. However, this LW-PKS scheme, which is directly derived from the LW-IBE scheme, is not easily converted to an SAS scheme (as far as we see). The reason is that we need a PKS scheme that supports multi-users and public re-randomization to construct an SAS scheme by using the randomness reuse technique of Lu et al. [27], but the LW-PKS scheme does not support these two properties. Technically speaking, this directly converted LW-PKS scheme does not support multi-users and public re-randomization since group elements \(g, u, h \in G\) cannot be published in a public key. To resolve this problem, we devised two independent solutions. Our first solution for this problem is to randomize the verification algorithm of the LW-PKS scheme and publish \(g, u, h \in G\) in the public key. That is, the verification components are additionally multiplied by \(\hat{v}, \hat{v}_3, \hat{v}_\pi\) to prevent the verification of invalid signatures. Our second solution for this problem is to randomize the group elements of the public key. That is, we publish \(gw^c_1, uw^c_1, hw^c_1 \in G\) in the public key instead of \(g, u, h \in G\).

Here we first construct two PKS schemes in prime order (asymmetric) bilinear groups that support multi-users and public re-randomization by applying our two solutions to the LW-PKS scheme, and we prove their security by using the dual system encryption technique. Next, we convert the modified PKS schemes to SAS schemes with short public keys by using the randomness reuse technique, and then we prove their security based on the traditional static assumptions without random oracles. Additionally, we present an efficient multi-signature scheme based on our modified PKS scheme. Table 1 gives the comparison of past aggregate signature schemes with ours.
1.3 Additional Related Work

There are some works on aggregate signature schemes that allow signers to communicate with each other or schemes that compress only partial elements of a signature in the aggregate algorithm [2, 4, 12, 18]. Generally, communication resources of computer systems are very expensive compared with computation resources. Thus, it is preferred to perform several expensive computational operations rather than one single communication exchange. Additionally, a signature scheme with added communications does not correspond to a pure PKS scheme, but corresponds more to a multi-party protocol. In addition, PKS schemes that compress just partial elements of signatures cannot be considered aggregate signature schemes since the total size of signatures is still proportional to the number of signers.

Another research area related to aggregate signature is multi-signature [6, 21, 27]. Multi-signature is a special type of aggregate signature in which all signers generate signatures on the same message, and then any user can combine these signatures into a single signature. Aggregate message authentication code (AMAC) is the symmetric key analogue of aggregate signature: Katz and Lindell [22] introduced the concept of AMAC and showed that it is possible to construct an AMAC scheme based on any message authentication code scheme.

2 Preliminaries

In this section, we define asymmetric bilinear groups and introduce the complexity assumptions for our schemes. The description of LW-IBE and LW-PKS schemes is given in Appendix A.

2.1 Asymmetric Bilinear Groups

Let $G$, $\hat{G}$ and $G_T$ be multiplicative cyclic groups of prime order $p$. Let $g$ and $\hat{g}$ be generators of $G$ and $\hat{G}$, respectively. The bilinear map $e: G \times \hat{G} \rightarrow G_T$ has the following properties:

1. Bilinearity: $\forall u \in G, \forall \hat{v} \in \hat{G}$ and $\forall a, b \in \mathbb{Z}_p$, $e(u^a, \hat{v}^b) = e(u, \hat{v})^{ab}$.

2. Non-degeneracy: $e(g, \hat{g}) \neq 1$, that is, $e(g, \hat{g})$ is a generator of $G_T$.

We say that $G$, $\hat{G}$, $G_T$ are bilinear groups with no efficiently computable isomorphisms if the group operations in $G$, $\hat{G}$, and $G_T$ as well as the bilinear map $e$ are all efficiently computable, but there are no efficiently computable isomorphisms between $G$ and $\hat{G}$.

2.2 Complexity Assumptions

We employ four assumptions in prime order bilinear groups. The SXDH and DBDH assumptions have been used extensively, while the LW1 and LW2 assumptions were introduced by Lewko and Waters [26].

Assumption 2.1 (Symmetric eXternal Diffie-Hellman, SXDH). Let $(p, G, \hat{G}, G_T, e)$ be a description of the asymmetric bilinear group of prime order $p$. Let $g, \hat{g}$ be generators of $G, \hat{G}$ respectively. The assumption is that if the challenge values

$$D = ((p, G, \hat{G}, G_T, e), g, \hat{g}, g^a, \hat{g}^b)$$

and $T$, are given, no PPT algorithm $B$ can distinguish $T = T_0 = \hat{g}^{ab}$ from $T = T_1 = g^c$ with more than a negligible advantage. The advantage of $B$ is defined as

$$\text{Adv}_{\text{SXDH}}^B(\lambda) = \left| \Pr[B(D, T_0) = 0] - \Pr[B(D, T_1) = 0] \right|$$

where the probability is taken over the random choice of $a, b, c \in \mathbb{Z}_p$. 

6
Assumption 2.2 (LW1). Let \((p, G, G_T, e)\) be a description of the asymmetric bilinear group of prime order \(p\) with the security parameter \(\lambda\). Let \(g, \hat{g}\) be generators of \(G, \hat{G}\) respectively. The assumption is that if the challenge values

\[
D = ((p, G, \hat{G}, G_T, e), g, g^a, g^b, g^c, \hat{g}, \hat{g}^a, \hat{g}^b, \hat{g}^c, g^{ab}, g^{ac}, g^{bc}, \hat{g}^{ab}, \hat{g}^{ac}, \hat{g}^{bc}) \text{ and } T
\]

are given, no PPT algorithm \(B\) can distinguish \(T = T_0 = \hat{g}^{ab} \hat{c}\) from \(T = T_1 = g^d\) with more than a negligible advantage. The advantage of \(B\) is defined as \(\text{Adv}_B^{\text{LW1}}(\lambda) = |\Pr[B(D, T_0) = 0] − \Pr[B(D, T_1) = 0]|\) where the probability is taken over the random choice of \(a, b, c, d \in \mathbb{Z}_p\).

Assumption 2.3 (LW2). Let \((p, G, G_T, e)\) be a description of the asymmetric bilinear group of prime order \(p\). Let \(g, \hat{g}\) be generators of \(G, \hat{G}\) respectively. The assumption is that if the challenge values

\[
D = ((p, G, \hat{G}, G_T, e), g, g^a, g^b, g^c, \hat{g}, \hat{g}^a, \hat{g}^b, \hat{g}^c, g^{ab}, g^{ac}, g^{bc}, \hat{g}^{ab}, \hat{g}^{ac}, \hat{g}^{bc}) \text{ and } T
\]

are given, no PPT algorithm \(B\) can distinguish \(T = T_0 = g^{bc}\) from \(T = T_1 = \hat{g}^d\) with more than a negligible advantage. The advantage of \(B\) is defined as \(\text{Adv}_B^{\text{LW2}}(\lambda) = |\Pr[B(D, T_0) = 0] − \Pr[B(D, T_1) = 0]|\) where the probability is taken over the random choice of \(a, b, c, x, d \in \mathbb{Z}_p\).

Assumption 2.4 (Decisional Bilinear Diffie-Hellman, DBDH). Let \((p, G, G_T, e)\) be a description of the asymmetric bilinear group of prime order \(p\). Let \(g, \hat{g}\) be generators of \(G, \hat{G}\) respectively. The assumption is that if the challenge values

\[
D = ((p, G, \hat{G}, G_T, e), g, g^a, g^b, g^c, \hat{g}, \hat{g}^a, \hat{g}^b, \hat{g}^c, g^{ab}, g^{ac}, g^{bc}, \hat{g}^{ab}, \hat{g}^{ac}, \hat{g}^{bc}) \text{ and } T
\]

are given, no PPT algorithm \(B\) can distinguish \(T = T_0 = e(g, \hat{g})^{abc}\) from \(T = T_1 = e(g, \hat{g})^d\) with more than a negligible advantage. The advantage of \(B\) is defined as \(\text{Adv}_B^{\text{DBDH}}(\lambda) = |\Pr[B(D, T_0) = 0] − \Pr[B(D, T_1) = 0]|\) where the probability is taken over the random choice of \(a, b, c, d \in \mathbb{Z}_p\).

The LW1 and LW2 assumptions are falsifiable since they are not interactive (or even \(q\)-type) assumptions and they obviously hold in the generic bilinear group model since the target polynomial in \(T\) is independent of given polynomials in \(D\).

3 Public-Key Signature

In this section, we propose two PKS schemes with short public keys and prove their security under static assumptions.

3.1 Definitions

The concept of PKS was introduced by Diffie and Hellman [13]. In PKS, a signer first generates a public key and a private key, and then he publishes the public key. The signer generates a signature on a message by using his private key. A verifier can check the validity of the signer’s signature on the message by using the signer’s public key. A PKS scheme is formally defined as follows:

Definition 3.1 (Public-Key Signature). A public key signature (PKS) scheme consists of three PPT algorithms \(\text{KeyGen}, \text{Sign}, \text{and Verify}\), which are defined as follows:
**KeyGen**$(1^\lambda)$. The key generation algorithm takes as input the security parameters $1^\lambda$ and outputs a public key $PK$ and a private key $SK$.

**Sign**$(M,SK)$. The signing algorithm takes as input a message $M$ and a private key $SK$ and outputs a signature $\sigma$.

**Verify**$(\sigma,M,PK)$. The verification algorithm takes as input a signature $\sigma$, a message $M$, and a public key $PK$ and outputs either 1 or 0, depending on the validity of the signature.

The correctness requirement is that for any $(PK,SK)$ output by KeyGen and any $M \in M$, we have Verify$(\text{Sign}(M,SK),M,PK) = 1$. We can relax this notion to require that the verification is correct with overwhelming probability over all the randomness of the experiment.

The security model of PKS is defined as existential unforgeability under a chosen message attack (EUF-CMA), and this was formally defined by Goldwasser et al. [17]. In this security model, an adversary adaptively requests a polynomial number of signatures on messages through the signing oracle, and he finally outputs a forged signature on a message $M^*$. If the message $M^*$ was not queried to the signing oracle and the forged signature is valid, then the adversary wins this game. The security of PKS is formally defined as follows:

**Definition 3.2 (Security).** The security notion of existential unforgeability under a chosen message attack is defined in terms of the following experiment between a challenger $C$ and a PPT adversary $A$:

1. **Setup:** $C$ first generates a key pair $(PK,SK)$ by running KeyGen, and gives $PK$ to $A$.

2. **Signature Query:** Then $A$, adaptively and polynomially many times, requests a signature query on a message $M$ under the challenge public key $PK$, and receives a signature $\sigma$ generated by running Sign.

3. **Output:** Finally, $A$ outputs a forged signature $\sigma^*$ on a message $M^*$. $C$ then outputs 1 if the forged signature satisfies the following two conditions, or outputs 0 otherwise: 1) Verify$(\sigma^*,M^*,PK) = 1$ and 2) $M^*$ was not queried by $A$ to the signing oracle.

The advantage of $A$ is defined as $\text{Adv}_{A}^{\text{PKS}}(\lambda) = \Pr[C = 1]$ where the probability is taken over all the randomness of the experiment. A PKS scheme is existentially unforgeable under a chosen message attack if all PPT adversaries have at most a negligible advantage in the above experiment (for a large enough security parameter).

### 3.2 Construction

We construct PKS schemes with a short public key that will be augmented to support multi-users and public re-randomization. To construct a PKS scheme with a short public key, we may convert the LW-IBE scheme [26] in prime order groups to the LW-PKS scheme in prime order groups by using the transformation of Naor [9]. However, this directly converted LW-PKS scheme does not support multi-users and public re-randomization since it is necessary to publish additional public key components: Specifically, we need to publish an element $g$ for multi-users and elements $g,u,h$ for public re-randomization. Note that $\hat{g},\hat{u},\hat{h}$ are already in the public key, but $g,u,h$ are not. One may try to publish $g,u,h$ in the public key, but a technical difficulty arises in this case in that the simulator of the security proof can easily distinguish from the normal
verification algorithm to the semi-functional one, without using an adversary. Thus the simulator of Lewko and Waters sets the CDH value into the elements \( g, u, h \) to prevent the simulator from creating these elements.

To solve this problem, we devise two independent solutions. The first solution allows a PKS scheme to safely publish elements \( g, u, h \) in the public key for multi-users and public re-randomization. The main idea is to additionally randomize the verification components using \( \hat{\nu}, \hat{\nu}^3, \hat{\nu}^{-\pi} \) in the verification algorithm. If a valid signature is given in the verification algorithm, then the additionally added randomization elements \( \hat{\nu}, \hat{\nu}^3, \hat{\nu}^{-\pi} \) are canceled. Otherwise, the added randomization components prevent the verification of an invalid signature. Therefore, the simulator of the security proof cannot detect the changes of the verification algorithm even if \( g, u, h \) are published, since the additional elements \( \hat{\nu}, \hat{\nu}^3, \hat{\nu}^{-\pi} \) prevent the signature verification.

Our second solution for this problem is to publish randomized components \( gw_1^{c_x}, uw_1^{c_y}, hw_1^{c_h} \) that are additionally multiplied with random elements rather than directly publishing \( g, u, h \). In this case, the simulator can create these elements since the random exponents \( c_x, c_y, c_h \) can be used to cancel out the CDH value embedded in the elements \( g, u, h \). Additionally, the simulator cannot detect the changes of verification components for the forged signature because of the added elements \( w_1^{c_x}, w_1^{c_y}, w_1^{c_h} \). This solution does not increase the number of group elements in the signatures, rather it increases the number of public keys since additional elements \( w_2^{c_x}, w_2^{c_y}, w_2^{c_h}, w_3^{c_h} \) should be published.

### 3.2.1 Our PKS1 Scheme

Our first PKS scheme in prime order bilinear groups is described as follows:

**PKS1.KeyGen(1^k):** This algorithm first generates the asymmetric bilinear groups \( \mathbb{G}, \hat{\mathbb{G}} \) of prime order \( p \) of bit size \( \Theta(\lambda) \). It chooses random elements \( g, w \in \mathbb{G} \) and \( \hat{g}, \hat{\nu} \in \hat{\mathbb{G}} \). Next, it chooses random exponents \( v_1, v_2, v_3, \phi_2, \phi_3 \in \mathbb{Z}_p \) and sets \( \tau = \phi_1 + v_1 \phi_2 + v_2 \phi_3, \pi = \phi_2 + v_3 \phi_3 \). It selects random exponents \( a, x, y \in \mathbb{Z}_p \) and sets \( u = g^x, h = g^y, \hat{u} = \hat{g}^x, \hat{h} = \hat{g}^y, w_1 = w^\phi_1, w_2 = w^\phi_2, w_3 = w^\phi_3 \). It outputs a private key \( SK = \alpha \) and a public key as

\[
PK = \left( (p, \mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_T, e), g, u, h, w_1, w_2, w_3, w, \hat{g}, \hat{\nu}, \hat{\nu}^2, \hat{\nu}^{-\tau}, \\
\hat{u}, \hat{u}^2, \hat{u}^{\tau}, \hat{h}, \hat{h}^2, \hat{h}^{-\tau}, \hat{\nu}, \hat{\nu}^3, \hat{\nu}^{-\pi}, \Omega = e(g, \hat{g})^\alpha \right).
\]

**PKS1.Sign(M, SK):** This algorithm takes as input a message \( M \in \{0, 1\}^k \) where \( k < \lambda \) and a private key \( SK = \alpha \). It selects random exponents \( r, c_1, c_2 \in \mathbb{Z}_p \) and outputs a signature as

\[
\sigma = \left( W_{1,1} = g^a (u^M)^r w_1^{c_1}, W_{1,2} = w_2^{c_1}, W_{1,3} = w_3^{c_1}, W_{1,4} = w_1^{c_1}, \\
W_{2,1} = g^r w_1^{c_2}, W_{2,2} = w_2^{c_2}, W_{2,3} = w_3^{c_2}, W_{2,4} = w_2^{c_2} \right).
\]

**PKS1.Verify(\sigma, M, PK):** This algorithm takes as input a signature \( \sigma \) on a message \( M \in \{0, 1\}^k \) under a public key \( PK \). It first chooses random exponents \( t, s_1, s_2 \in \mathbb{Z}_p \) and computes verification components as

\[
V_{1,1} = \hat{g}^t, V_{1,2} = (g^{\nu_1})^t \hat{\nu}_1^{s_1}, V_{1,3} = (g^{\nu_2})^t \hat{\nu}_1^{s_1}, V_{1,4} = (g^{-\tau})^t \hat{\nu}_1^{s_1}, \\
V_{2,1} = (\hat{u}^M)^t, V_{2,2} = ((\hat{\nu}_2)^M \hat{h}^{\nu_1})^t \hat{\nu}_2^{s_2}, V_{2,3} = ((\hat{\nu}_1)^M \hat{h}^{\nu_1})^t \hat{\nu}_2^{s_2}, V_{2,4} = ((\hat{\nu}^{-\tau})^M \hat{h}^{-\tau})^t \hat{\nu}_2^{s_2}.
\]

Next, it verifies that \( \prod_{i=1}^{4} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{4} e(W_{2,i}, V_{2,i})^{-1} = \Omega \). If this equation holds, then it outputs 1. Otherwise, it outputs 0.
We note that the inner product of \((\phi_1, \phi_2, 1)\) and \((1, v_1, v_2, -\tau)\) is zero since \(\tau = \phi_1 + v_1\phi_2 + v_2\phi_3\), and the inner product of \((\phi_1, \phi_2, 1)\) and \((0, 1, v_3, -\pi)\) is zero since \(\pi = \phi_2 + v_3\phi_3\). Using these facts, the correctness of PKS is easily obtained from the equation

\[
\prod_{i=1}^{4} e(W_{1,i}, V_{1,i}) \prod_{i=1}^{4} e(W_{2,i}, V_{2,i})^{-1} = e(g^\alpha (u^M h)^\tau, \hat{g}^\tau \cdot e(g', (\hat{u}^M \hat{h})^\tau)^{-1} = \Omega'.
\]

3.2.2 Our PKS2 Scheme

Our second PKS scheme in prime order bilinear groups is described as follows:

**PKS2.KeyGen(\(\lambda\)):** This algorithm first generates the asymmetric bilinear groups \(G, \hat{G}\) of order \(p\) of bit size \(\Theta(\lambda)\). It chooses random elements \(g, w \in G\) and \(\hat{g} \in \hat{G}\). Next, it selects random exponents \(v, \phi_1, \phi_2 \in \mathbb{Z}_p\) and sets \(\tau = \phi_1 + v\phi_2\). It also selects random exponents \(x, y \in \mathbb{Z}_p\) and sets \(u = g^x, h = g^y\). It chooses a random exponent \(\nu \in \mathbb{Z}_p\), and outputs a private key \(SK = (\alpha, g, u, h)\) and a public key by selecting random values \(c_g, c_u, c_h \in \mathbb{Z}_p\) as

\[
PK = \left(\left(p, G, \hat{G}, G_T, e\right), gw^\ell, w_2^\ell, w^\ell, uw_1^\ell, w_2^\ell, w^\ell, h \nu_1^\ell, w_2^\ell, w^\ell, w_1, w_2, w, \hat{g}, \hat{g}^\nu, \hat{g}^{-\tau}, u, \hat{u}, \hat{u}^{-\tau}, \hat{h}, \hat{h}^\nu, \hat{h}^{-\tau}, \Omega = e(g, \hat{g})^\alpha\right).
\]

**PKS2.Sign(\(M, SK\)):** This algorithm takes as input a message \(M \in \mathbb{Z}_p\) and a private key \(SK = (\alpha, g, u, h)\) with \(PK\). It selects random exponents \(r, c_1, c_2 \in \mathbb{Z}_p\) and outputs a signature as

\[
\sigma = \left(W_{1,1} = g^\alpha (u^M h)^{\tau}, W_{1,2} = w_2^\ell, W_{1,3} = w^\ell, W_{2,1} = g^\nu w_1^\ell, W_{2,2} = w_2^\ell, W_{2,3} = w^\ell\right).
\]

**PKS2.Verify(\(\sigma, M, PK\)):** This algorithm takes as input a signature \(\sigma\) on a message \(M \in \mathbb{Z}_p\) under a public key \(PK\). It chooses a random exponent \(t \in \mathbb{Z}_p\) and computes verification components as

\[
V_{1,1} = \hat{g}^\tau, V_{1,2} = (\hat{g}^\nu)^t, V_{1,3} = (\hat{g}^{-\tau})^t, V_{2,1} = (\hat{u}^M \hat{h})^t, V_{2,2} = ((\hat{u}^v)^M \hat{h}^\nu)^t, V_{2,3} = ((\hat{u}^{-\tau})^M \hat{h}^{-\tau})^t.
\]

Next, it verifies that \(\prod_{i=1}^{3} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i}, V_{2,i})^{-1} = \Omega'.\) If this equation holds, then it outputs 1. Otherwise, it outputs 0.

We note that the inner product of \((\phi_1, \phi_2, 1)\) and \((1, v, -\tau)\) is zero since \(\tau = \phi_1 + v\phi_2\). Using this fact, the correctness of PKS is easily obtained from the following equation

\[
\prod_{i=1}^{3} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i}, V_{2,i})^{-1} = e(g^\alpha (u^M h)^\tau, \hat{g}^\tau \cdot e(g', (\hat{u}^M \hat{h})^\tau)^{-1} = \Omega'.
\]
3.3 Security Analysis

We prove the security of our PKS schemes without random oracles under static assumptions. To prove the security, we use the dual system encryption technique of Lewko and Waters [26]. The dual system encryption technique was originally developed to prove the full-model security of IBE and its extensions, but it also can be used to prove the security of PKS by using the transformation of Naor [9]. Note that Gerbush et al. [16] developed the dual form signature technique that is a variation of the dual system encryption technique to prove the security of their PKS schemes.

3.3.1 Analysis of PKS1

Theorem 3.3. The above PKS1 scheme is existentially unforgeable under a chosen message attack if the SXDH, LW2, DBDH assumptions hold. That is, for any PPT adversary A, there exist PPT algorithms \( B_1, B_2, B_3 \) such that \( \text{Adv}^\text{PKS}_A(\lambda) \leq \text{Adv}^\text{SXDH}_B(\lambda) + q\text{Adv}^\text{LW2}_B(\lambda) + \text{Adv}^\text{DBDH}_B(\lambda) \) where \( q \) is the maximum number of signature queries of A.

Proof. To use the dual system encryption technique of Lewko and Waters [26], we first describe a semi-functional signing algorithm and a semi-functional verification algorithm. They are not used in a real system; rather, they are used in the security proof. When comparing our proof to that of Lewko and Waters, we employ a different assumption since we have published additional elements \( g, u, h \) used in aggregation (in fact, direct adaptation of the earlier technique will break the assumption and thus the proof). A crucial idea in our proof is that we have added elements \( \hat{\nu}, \nu^3, \hat{\nu}^\pi \) in the public key that are used in randomization of the verification algorithm. In the security proof when moving from normal to semi-functional verification, it is the randomization elements \( \hat{\nu}, \nu^3, \hat{\nu}^\pi \) that are expanded to the semi-functional space; this enables deriving semi-functional verification as part of the security proof under our assumption, without being affected by the publication of the additional public key elements used for aggregation.

For the semi-functional signing and verification, we set \( f = g^y, \hat{f} = \hat{g}^y \) where \( y_f \) is a random exponent in \( \mathbb{Z}_p \).

PKS1.SignSF. The semi-functional signing algorithm first creates a normal signature using the private key. Let \( (W_{1,1}', \ldots, W_{2,4}') \) be the normal signature of a message \( M \) with random exponents \( r, c_1, c_2 \in \mathbb{Z}_p \). It selects random exponents \( s_\delta, z_\delta \in \mathbb{Z}_p \) and outputs a semi-functional signature as

\[
\sigma = \left( W_{1,1} = W_{1,1}'(f^{y_1-v_1-v_2})^{s_\delta z_\delta}, W_{1,2} = W_{1,2}'(f^{-v_1})^{s_\delta z_\delta}, W_{1,3} = W_{1,3}'(f^{v_1 z_\delta})^{s_\delta z_\delta}, W_{1,4} = W_{1,4}', \\
W_{2,1} = W_{2,1}'(f^{v_1 v_2})^{s_\delta}, W_{2,2} = W_{2,2}'(f^{-v_1})^{s_\delta}, W_{2,3} = W_{2,3}'(f^{v_1 z_\delta})^{s_\delta}, W_{2,4} = W_{2,4}' \right).
\]

PKS1.VerifySF. The semi-functional verification algorithm first creates normal verification components using the public key. Let \( (V_{1,1}', \ldots, V_{2,4}') \) be the normal verification components with random exponents \( l, s_1, s_2 \in \mathbb{Z}_p \). It chooses random exponents \( s_c, z_c \in \mathbb{Z}_p \) and computes semi-functional verification components as

\[
V_{1,1} = V_{1,1}', V_{1,2} = V_{1,2}', V_{1,3} = V_{1,3}'(f^{-\phi_3})^{s_c}, V_{1,4} = V_{1,4}'(f^{-\phi_3})^{s_c}, \\
V_{2,1} = V_{2,1}', V_{2,2} = V_{2,2}', V_{2,3} = V_{2,3}'(f^{-\phi_3})^{s_c}, V_{2,4} = V_{2,4}'(f^{-\phi_3})^{s_c}.
\]

Next, it verifies that \( \prod_{i=1}^{4} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{4} e(W_{2,i}, V_{2,i})^{-1} \equiv \Omega' \). If this equation holds, then it outputs 1. Otherwise, it outputs 0.
Note that if the semi-functional verification algorithm verifies a semi-functional signature, then the left part of the above verification equation contains an additional random element $e(f, \hat{f})^k s_k z_k - z_c)$. If $z_k = z_c$, then the semi-functional verification algorithm succeeds. In this case, we say that the signature is nominally semi-functional.

The security proof uses a sequence of games $G_0, G_1, G_2, \text{ and } G_3$: The first game $G_0$ will be the original security game and the last game $G_3$ will be a game such that an adversary $A$ has no advantage. Formally, the hybrid games are defined as follows:

**Game $G_0$.** This game is the original security game. In this game, the signatures that are given to $A$ are normal and the challenger use the normal verification algorithm Verify to check the validity of the forged signature of $A$.

**Game $G_1$.** We first modify the original game to a new game $G_1$. This game is almost identical to $G_0$ except that the challenger uses the semi-functional verification algorithm VerifySF to check the validity of the forged signature of $A$.

**Game $G_2$.** Next, we change $G_1$ to a new game $G_2$. This game is the same as the $G_1$ except that the signatures that are given to $A$ will be semi-functional. At this moment, the signatures are semi-functional and the challenger uses the semi-functional verification algorithm VerifySF to check the validity of the forged signature. Suppose that $A$ makes at most $q$ signature queries. For the security proof, we define a sequence of hybrid games $G_{1,0}, \ldots, G_{1,k}, \ldots, G_{1,q}$ where $G_{1,0} = G_1$. In $G_{1,k}$, a normal signature is given to $A$ for all $j$-th signature queries such that $j > k$, and a semi-functional signature is given to $A$ for all $j$-th signature queries such that $j \leq k$. It is obvious that $G_{1,q}$ is equal to $G_2$.

**Game $G_3$.** Finally, we define a new game $G_3$. This game differs from $G_2$ in that the challenger always rejects the forged signature of $A$. Therefore, the advantage of this game is zero since $A$ cannot win this game.

For the security proof, we show the indistinguishability of each hybrid game. We informally describe the meaning of each indistinguishability as follows:

- **Indistinguishability of $G_0$ and $G_1$:** This property shows that $A$ cannot forge a semi-functional signature if it is only given normal signatures. That is, if $A$ forges a semi-functional signature, then it can distinguish $G_0$ from $G_1$.

- **Indistinguishability of $G_1$ and $G_2$:** This property shows that the probability of $A$ forging a normal signature is almost the same when the signatures given to the adversary are changed from a normal type to a semi-functional type. That is, if the probability of $A$ forging a normal signature is different in $G_1$ and $G_2$, then $A$ can distinguish the two games.

- **Indistinguishability of $G_2$ and $G_3$:** This property shows that $A$ cannot forge a normal signature if it is only given semi-functional signatures. That is, if $A$ forges a normal signature, then it can distinguish $G_2$ from $G_3$.

The security (unforgeability) of our PKS scheme follows from a hybrid argument. We first consider an adversary $A$ attacking our PKS scheme in the original security game $G_0$. By the indistinguishability of $G_0$ and $G_1$, we have that $A$ can forge a normal signature with a non-negligible $\epsilon$ probability, but it can forge
a semi-functional signature with only a negligible probability. Now we should show that the \( \varepsilon \) probability of \( A \) forging a normal signature is also negligible. By the indistinguishability of \( G_1 \) and \( G_2 \), we have that the \( \varepsilon \) probability of \( A \) forging a normal signature is almost the same when the signatures given to \( A \) are changed from a normal type to a semi-functional type. Finally, by the indistinguishability of \( G_2 \) and \( G_3 \), we have that \( A \) can forge a normal signature with only a negligible probability. Summing up, we obtain that the probability of \( A \) forging a semi-functional signature is negligible (from the indistinguishability of \( G_0 \) and \( G_1 \)) and the probability of \( A \) forging a normal signature is also negligible (from the indistinguishability of \( G_2 \) and \( G_3 \)).

Let \( \text{Adv}^G_A \) be the advantage of \( A \) in \( G_j \) for \( j = 0, \ldots, 3 \). Let \( \text{Adv}^{G_{1,k}}_A \) be the advantage of \( A \) in \( G_{1,k} \) for \( k = 0, \ldots, q \). It is clear that \( \text{Adv}^{G_0}_A = \text{Adv}^{PKS}_A(\lambda) \), \( \text{Adv}^{G_{1,q}}_A = \text{Adv}^{G_1}_A \), \( \text{Adv}^{G_{1, q}}_A = \text{Adv}^{G_2}_A \), and \( \text{Adv}^{G_3}_A = 0 \). From the following three Lemmas, we prove that it is hard for \( A \) to distinguish a semi-functional signature with only a negligible probability. Now we should show that the probability of \( A \) forging a normal signature is also negligible. By the indistinguishability of \( G_0 \) and \( G_1 \)

\[
\begin{align*}
\text{Adv}^{PKS}_A(\lambda) & = \text{Adv}^{G_0}_A + \sum_{i=1}^{3} (\text{Adv}^{G_i}_A - \text{Adv}^{G_i}_A) - \text{Adv}^{G_i}_A \\
& = \text{Adv}^{\text{SXDH}}_{B_1}(\lambda) + \sum_{k=1}^{q} \text{Adv}^{\text{DW2}}_{B_2}(\lambda) + \text{Adv}^{\text{DBDH}}_{B_3}(\lambda).
\end{align*}
\]

This completes our proof.

\textbf{Lemma 3.4.} If the SXDH assumption holds, then no polynomial-time adversary can distinguish between \( G_0 \) and \( G_1 \) with non-negligible advantage. That is, for any adversary \( A \), there exists a PPT algorithm \( B_1 \) such that \( |\text{Adv}^{G_0}_A - \text{Adv}^{G_1}_A| = \text{Adv}^{\text{SXDH}}_{B_1}(\lambda) \).

\textbf{Proof.} Before proving this lemma, we introduce the parallel-SXDH assumption as follows: Let \( (p, G, \hat{G}, \hat{G}_T, e) \) be a description of the asymmetric bilinear group of prime order \( p \). Let \( k, \hat{k} \) be generators of \( G, \hat{G} \) respectively. The assumption is stated as follows: given a challenge tuple \( D = ((p, G, \hat{G}, \hat{G}_T, e), k, \hat{k}, \hat{k}^a, \hat{k}^d, \hat{k}^{ld}) \) and \( T = (A_1, A_2) \), it is hard to decide whether \( T = (\hat{k}^a, \hat{k}^{ld}) \) or \( T = (\hat{k}^d, \hat{k}^{ld}) \) with random choices of \( a, d, l_1, d_2, d_3, d_4 \in \mathbb{Z}_p \). It is easy to prove by simple hybrid arguments that if there exists an adversary that breaks the parallel-SXDH assumption, then it can break the SXDH assumption. Alternatively, we can tightly prove the reduction using the random self-reducibility of the Decisional Diffie-Hellman assumption.

Suppose there exists an adversary \( A \) that distinguishes between \( G_0 \) and \( G_1 \) with non-negligible advantage. Simulator \( B_1 \) that solves the parallel-SXDH assumption using \( A \) is given: a challenge tuple \( D = ((p, G, \hat{G}, \hat{G}_T, e), k, \hat{k}, \hat{k}^a, \hat{k}^d, \hat{k}^{ld}) \) and \( T = (A_1, A_2) \) where \( T_0 = (A_0^0, A_0^2) = (\hat{k}^a, \hat{k}^d) \) or \( T_1 = (A_1^1, A_1^2) = (\hat{k}^a, \hat{k}^d) \). Then \( B_1 \) that interacts with \( A \) is described as follows: \( B_1 \) first chooses random exponents \( v_1, v_2, \phi_1, \phi_2 \in \mathbb{Z}_p \), then it sets \( T = \phi_1 + v_1 \phi_2 + v_2 \phi_3 \). It selects random exponents \( \alpha, x, y, r, s, y, w \in \mathbb{Z}_p \) and sets \( g = k^\alpha, u = g^x, h = g^y, w_1 = k^\alpha \phi_1, w_2 = k^\alpha \phi_2, w_3 = k^\alpha \phi_3, w = k^\alpha \phi_3, \hat{u} = g^x, \hat{h} = g^y \). It implicitly sets \( v_3 = a, \pi = \phi_2 + a \phi_3 \) and publishes a public key \( PK \) as

\[
\begin{align*}
g, u, h, w_1, w_2, w_3, y, g, g^y, \hat{g}^y, \hat{g}^{-y}, \hat{u}^y, \hat{u}^{-y}, \hat{h}, \hat{h}^y, \hat{u}^y, \hat{u}^{-y}, \hat{v} = \hat{k}^y, \hat{v}^y = (\hat{k}^a)^y, \hat{v}^{-y} = \hat{k}^a, \hat{v}^{-y} \phi_2 (\hat{k}^a)^{-y} \phi_1, \Omega = e(g, \hat{g})^a.
\end{align*}
\]

It sets a private key \( SK = \alpha \). Additionally, it sets \( f = k, \hat{f} = \hat{k} \) for the semi-functional signature and verification. \( A \) adaptively requests a signature for a message \( M \). To response this sign query, \( B_1 \) creates a normal signature by calling \( \text{PKS1.Sign} \) since it knows the private key. Note that it cannot create a semi-functional
signature since it does not know $k^a$. Finally, $A$ outputs a forged signature $\sigma^* = (W_{1,1}^*, \ldots, W_{2,4}^*)$ on a message $M^*$ from $A$. To verify the forged signature, $B_1$ first chooses a random exponent $t \in \mathbb{Z}_p$ and computes verification components by implicitly setting $s_1 = d_1$, $s_2 = d_2$ as

\begin{align*}
V_{1,1} &= g^\tau, V_{1,2} = (g^{\nu_1})^{(k^{d_1})^{\nu_1}}, V_{1,3} = (g^{\nu_2})^{(k^{d_1})^{\nu_2}}, V_{1,4} = (g^{\nu_3})^{(k^{d_1})^{\nu_3}}(A_1)^{\nu_4},
V_{2,1} &= (\hat{u}^{\nu_1} h) \hat{y}, V_{2,2} = (\hat{u}^{\nu_2} h \hat{y})^{(k^{d_2})^{\nu_1}}, V_{2,3} = (\hat{u}^{\nu_2} h \hat{y})^{(k^{d_2})^{\nu_2}} (A_2)^{\nu_3},
V_{2,4} &= (\hat{u}^{\nu_3} h \hat{y})^{(k^{d_2})^{\nu_3}}(A_2)^{\nu_4}.
\end{align*}

Next, it verifies that $\prod_{i=1}^4 e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^4 e(W_{2,i}, V_{2,i})^{-1} = \Omega'$. If this equation holds, then it outputs 0. Otherwise, it outputs 1.

To finish this proof, we show that the distribution of the simulation is correct. We first show that the distribution using $D, T_0 = (A_1^0, A_2^0) = (\hat{k}^{ad_1, d_2}, \hat{k}^{ad_2})$ is the same as $G_0$. The public key is correctly distributed since the random blinding values $y_a, y_\pi$ are used. The signatures is correctly distributed since it uses the signing algorithm. The verification components are correctly distributed as

\begin{align*}
V_{1,3} &= (g^{\nu_2})^{(\hat{g}^{\nu_1})^{(k^{d_1})^{\nu_1}}} = (g^{\nu_2})^{(k^{d_1})^{\nu_1}},
V_{1,4} &= (g^{\nu_3})^{(\hat{g}^{\nu_1})^{(k^{d_1})^{\nu_1}} (A_1)^{\nu_4}},
V_{2,3} &= (\hat{u}^{\nu_1} h \hat{y})^{(\hat{g}^{\nu_1})^{(k^{d_1})^{\nu_1}} (A_1)^{\nu_4}},
V_{2,4} &= (\hat{u}^{\nu_3} h \hat{y})^{(\hat{g}^{\nu_1})^{(k^{d_1})^{\nu_1}} (A_1)^{\nu_4}}.
\end{align*}

We next show that the distribution of the simulation using $D, T_1 = (A_1^1, A_2^1) = (\hat{k}^{ad_1+d_2, d_2}, \hat{k}^{ad_2+d_2})$ is the same as $G_1$. We only consider the distribution of the verification components since $T$ is only used in the verification components. The difference between $T_0 = (A_1^0, A_2^0)$ and $T_1 = (A_1^1, A_2^1)$ is that $T_1 = (A_1^1, A_2^1)$ additionally has $(\hat{k}^{d_1}, \hat{k}^{d_2})$. Thus $V_{1,3}, V_{1,4}, V_{2,3}, V_{2,4}$ that have $T = (A_1, A_2)$ in the simulation additionally have $(\hat{k}^{d_1})^{\nu_1}, (\hat{k}^{d_2})^{\nu_1}, (\hat{k}^{d_1})^{\nu_2}, (\hat{k}^{d_2})^{\nu_2}$ respectively. If we implicitly set $s_c = y_a d_3, z_c = d_4/d_3$, then the verification components for the forged signature are semi-functional since $d_3, d_4$ are randomly chosen. This completes our proof.

**Lemma 3.5.** If the LW2 assumption holds, then no polynomial-time adversary can distinguish between $G_1$ and $G_2$ with non-negligible advantage. That is, for any adversary $A$, there exists a PPT algorithm $B_2$ such that $|Adv_{A,1}^{G_1} - Adv_{A,1}^{G_2}| = Adv_{B_2, LW2}^{\lambda}.$

**Proof.** Suppose there exists an adversary $A$ that distinguishes between $G_{1,k-1}$ and $G_{1,k}$ with non-negligible advantage. A simulator $B_2$ that solves the LW2 assumption using $A$ is given: a challenge tuple $D = ((p, G, G_T, e), k, a, b, c, \hat{a}, \hat{b}, \hat{c}, \hat{k}, \hat{\alpha}, \hat{\beta}, \hat{v})$ and $T$ where $T = T_0 = k^{bc}$ or $T = T_1 = k^{bc+d}$. Then $B_2$ that interacts with $A$ as described is follows: $B_2$ first selects random exponents $v_1, v_2, v_3, \pi, \alpha, \beta, a, y_a, y_b, y_c, y_d, y_e, z, y_t \in \mathbb{Z}_p$ and sets $g = k^a, u = (k^a)^{k^b} v, h = (k^a)^{k^b} v^2, w = k^b, s = k^a, \hat{u} = (k^a)^{\hat{a}} \hat{k}^{\hat{a}}, \hat{v} = (k^a)^{\hat{a}} \hat{k}^{\hat{a}}$. It implicitly sets $\phi_1 = (v_1 v_3 - v_2) b - v_1 \pi + (a + y_t), \phi_2 = -v_3 b + \pi, \phi_3 = b, \tau = a + y_t$ and publishes a public key $PK$ as

\begin{align*}
g, u, h, w_1 &= ((k^a)^{v_1} v_3 - v_2 k^{-v_1} (k^a)^{k^a})^v, w_2 = ((k^a)^{-v_1} k^a)^v, w_3 = (k^b)^y, w, 
\hat{g}, \hat{g} v_1, \hat{g} v_2, \hat{g} \tau &= (k^a)^{(k^a)^{v_1}(k^a)^{v_2} (k^a)^{v_3}} , \hat{u}, \hat{u} v_1, \hat{u} v_2, \hat{u} \tau = ((k^a)^{k^b} (k^a)^{k^b})^{-1}, 
\hat{v}, \hat{v} v_1, \hat{v} v_2, \hat{v} \tau &= ((k^a)^{k^b} (k^a)^{k^b})^{-1}, \hat{v}, \hat{v} v_3, \tau - \pi, \Omega = e(k^a, \hat{k}^a)^{\alpha}.
\end{align*}
Additionally, it sets \( f = k, \hat{f} = \hat{k} \) for the semi-functional signature and verification. \( A \) adaptively requests a signature for a message \( M \). If this is a \( j \)-th signature query, then \( B_2 \) handles this query as follows:

- **Case \( j < k \)**: It creates a semi-functional signature by calling \( \text{PKS1.SignSF} \) since it knows the tuple \((f^{v_1 v_2}, f^{v_3}, f, 1)\) for the semi-functional signature.

- **Case \( j = k \)**: It selects random exponents \( r', c'_1, c'_2 \in \mathbb{Z}_p \) and creates a signature by implicitly setting \( r = -c + r', c_1 = c(AM + B)/\nu + c'_1, c_2 = c/\nu + c'_2 \) as

  \[
  \begin{align*}
  W_{1,1} &= g^\alpha (k^e)^{-\nu\nu(M+y_0)}(u^M h)^r (T)^{(v_1 v_2 - v_3)(AM+B)} (k^e)^{-v_1 \pi + v_2}(AM+B)w_1^{c'_1}, \\
  W_{1,2} &= (T)^{v_3(AM+B)} (k^e)^{\pi(AM+B)}w_2^{c'_2}, \\
  W_{1,3} &= (T)^{(AM+B)} w_3^{c'_3}, \\
  W_{1,4} &= (k^e)^{(AM+B)}w_4^{c'_4}, \\
  W_{2,1} &= g^\nu (T)^{(v_1 v_2 - v_3)} (k^e)^{-v_1 \pi + v_2}w_1^{c'_1}, \\
  W_{2,2} &= (T)^{-v_3(AM+B)} (k^e)^{\nu(AM+B)}w_2^{c'_2}, \quad W_{2,3} = (T)^{v_3}(k^e)^{\nu(AM+B)}w_3^{c'_3}, \\
  W_{2,4} &= (k^e)^\nu w_4^{c'_4}.
  \end{align*}
  \]

- **Case \( j > k \)**: It creates a normal signature by calling \( \text{PKS1.Sign} \) since it knows \( \alpha \) of the private key. Note that \( x, y \) are not required.

Finally, \( A \) outputs a forged signature \( \sigma^* = (W_{1,1}^*, \ldots, W_{2,4}^*) \) on a message \( M^* \). To verify the forged signature, \( B_2 \) first chooses random exponents \( t', s_1, s_2 \in \mathbb{Z}_p \) and computes semi-functional verification components by implicitly setting \( t = bx + t', s_c = -a^2x, z_c = AM^* + B \) as

\[
\begin{align*}
V_{1,1} &= \hat{k}^{abx} (\hat{k} a t') \nu, \\
V_{1,2} &= (\hat{k}^{abx})^{v_1} (\hat{k} a t')^{v_2} \nu^{v_3} (\hat{k} a x)^{-1}, \\
V_{1,3} &= (\hat{k}^{abx})^{v_2} (\hat{k} a t')^{v_3} \nu^{v_4} (\hat{k} a x)^{-1}, \\
V_{1,4} &= (\hat{k}^{abx})^{-\nu t} (\hat{k} a t')^{-t'} (\hat{k} a)^{-v_1 \pi + v_2}(AM+B)w_1^{c'_1}, \\
V_{2,1} &= (\hat{k}^{abx})^{\nu(AM+B)} (\hat{k} a t')^{v_1} \nu^{v_2} (\hat{k} a x)^{-1}, \\
V_{2,2} &= (\hat{k}^{abx})^{\nu(AM+B)} (\hat{k} a t')^{v_2} \nu^{v_3} (\hat{k} a x)^{-1}, \\
V_{2,3} &= (\hat{k}^{abx})^{\nu(AM+B)} (\hat{k} a t')^{v_3} \nu^{v_4} (\hat{k} a x)^{-1}, \\
V_{2,4} &= (\hat{k}^{abx})^{-v_3(AM+B)} (\hat{k} a t')^{-v_3(AM+B)} (\hat{k} a)^{-v_3(AM+B)}(AM+B)w_2^{c'_2}.
\end{align*}
\]

Next, it verifies that \( \prod_{i=1}^{t} e(W_{1,i}^*, V_{1,i}) \cdot \prod_{i=1}^{t} e(W_{2,i}^*, V_{2,i})^{-1} = e(k^a, \hat{k}^{abx})^\alpha \cdot e(k^a, \hat{k} a)^{\alpha t'} \). If this equation holds, then it outputs 0. Otherwise, it outputs 1.

To finish the proof, we should show that the distribution of the simulation is correct. We first show that the distribution of the simulation using \( D, T_0 = \hat{k}bc \) is the same as \( G_{1,k-1} \). The public key is correctly distributed since the random blinding values \( y_\nu, y_\nu^2, y_\nu, y_\nu \) are used. The \( k \)-th signature is correctly distributed as

\[
\begin{align*}
W_{1,1} &= g^\alpha(u^M h)^r w_1^{c'_1} = g^\alpha (k^{abx})^{\nu(AM+B)}(\hat{k} a t')^{v_1 v_2 - v_3}(k^a)^{(v_1 v_2 - v_3)(AM+B)}(k^e)^{-v_1 \pi + v_2}(AM+B)w_1^{c'_1}, \\
W_{1,2} &= w_2^{c'_2}, \\
W_{1,3} &= w_3^{c'_3}, \\
W_{1,4} &= w_4^{c'_4}.
\end{align*}
\]

The semi-functional verification components are correctly distributed as

\[
\begin{align*}
V_{2,1} &= (\hat{k} a t')^{v_1} \nu^{v_2} = (\hat{k} a t')^{v_1 v_2 - v_3}(k^e)^{-v_1 \pi + v_2}(AM+B)w_2^{c'_2}, \\
V_{2,2} &= (\hat{k} a t')^{v_2} \nu^{v_3} = (\hat{k} a t')^{v_1 v_2 - v_3}(k^e)^{-v_1 \pi + v_2}(AM+B)w_2^{c'_2}, \\
V_{2,3} &= (\hat{k} a t')^{v_3} \nu^{v_4} = (\hat{k} a t')^{v_1 v_2 - v_3}(k^e)^{-v_1 \pi + v_2}(AM+B)w_2^{c'_2}.
\end{align*}
\]
The simulator can create the semi-functional verification components with only fixed $z_c = AM^* + B$ since $s_c, s_c$ enable the cancellation of $\hat{k}aBx$. Even though the simulator uses the fixed $z_c$, the distribution of $z_c$ is correct since $A, B$ are information theoretically hidden to $A$. We next show that the distribution of the simulation using $D, T_1 = k^{b+c+d}$ is the same as $G_{1,k}$. We only consider the distribution of the $k$-th signature since $T$ is only used in the $k$-th signature. The only difference between $T_0$ and $T_1$ is that $T_1$ additionally has $k^d$. The signature components $W_{1,1}, W_{1,2}, W_{1,3}, W_{2,1}, W_{2,2}, W_{2,3}$ that have $T$ in the simulation additionally have $(k^d)\nu T - \nu T = (\bar{\nu})T - \nu T$ such that $\bar{\nu} = \nu T = \nu T$ that have $T$ in the simulation additionally have $(k^d)\nu T - \nu T = (\bar{\nu})T - \nu T$ such that $\bar{\nu} = \nu T = \nu T$ and $s_c, s_c$ that have $T$ in the simulation additionally have $(k^d)\nu T - \nu T = (\bar{\nu})T - \nu T$ such that $\bar{\nu} = \nu T = \nu T$ and $s_c, s_c$ look random to the unbounded adversary since $\bar{\nu} = \nu T = \nu T$ and $s_c, s_c$ look random to the unbounded adversary since $\bar{\nu} = \nu T = \nu T$.

Finally, we show that the adversary cannot distinguish the nominally semi-functional signature from the semi-functional signature. The main idea of this is that the adversary cannot request a signature for the forgery message $M^*$ in the security model. Suppose there exists an unbounded adversary, then the adversary can gather $z_k = AM + B$ from the $k$-th signature and $z_c = AM^* + B$ from the forged signature. It is easy to show that $z_k$ and $z_c$ look random to the unbounded adversary since $f(M) = AM + B$ is a pair-wise independent function and $A, B$ are information theoretically hidden to the adversary. This completes our proof.

**Lemma 3.6.** If the DBDH assumption holds, then no polynomial-time adversary can distinguish between $G_2$ and $G_3$ with non-negligible advantage. That is, for any adversary $A$, there exists a PPT algorithm $B_3$ such that $|\text{Adv}_{G_2}^A - \text{Adv}_{G_3}^A| = \text{Adv}_{B_3}^{\text{DBDH}}(\hat{\lambda})$.

**Proof.** Suppose there exists an adversary $A$ that distinguishes $G_2$ from $G_3$ with non-negligible advantage. A simulator $B_3$ that solves the DBDH assumption using $A$ is given: a challenge tuple $D = ((p, G, \hat{G}, G_T, e), k, k^a, k^b, k^c, \hat{k}, \bar{k}^b, \bar{k}, \hat{k}^b)$ and $T$ where $T_0 = e(k, \hat{k})^{ab}$ or $T_1 = e(k, \hat{k})^d$. Then $B_3$ that interacts with $A$ is described as follows: $B_3$ first chooses random exponents $v_1, v_3, \phi_1, \phi_2, \phi_3 \in \mathbb{Z}_p$ and sets $\pi = \phi_2 + v_3 \phi_1$. It selects random exponents $y_1, x, y_1, y_2, y_3, y_1 \in \mathbb{Z}_p$ and sets $g = k^\nu, u = g^\tau, h = g^\nu, w_1 = k^\nu \phi_1, w_2 = k^\nu \phi_2, w_3 = k^\nu \phi_3, w = k^\nu, \hat{g} = \hat{k}^s, \hat{u} = \hat{g}^\tau, \hat{h} = \hat{g}^\nu, \hat{v} = \hat{k}^\nu$. It implicitly sets $v_2 = a, \tau = \phi_1 + v_1 \phi_2 + a \phi_3, \alpha = ab$ and publishes a public key $PK$ as

$$g, u, h, w_1, w_2, w_3, w, \hat{g}, \hat{u}, \hat{v} = (k^a)^{y_1} h^\nu, (k^a)^{y_2} h^\nu, (k^a)^{y_3} h^\nu, (k^a)^{y_4} h^\nu, (k^a)^{y_5} h^\nu, (k^a)^{y_6} h^\nu, (k^a)^{y_7} h^\nu, (k^a)^{y_8} h^\nu,$$

Additionally, it sets $f = k, \hat{f} = \hat{k}$ for the semi-functional signature and semi-functional verification. $A$ adaptively requests a signature for a message $M$. To respond to this query, $B_3$ selects random exponents.
Let $r, c_1, c_2, s_k, z_k \in \mathbb{Z}_p$ and create a semi-functional signature by implicitly setting $z_k = by_g/s_k + z'_k$ as

$$W_{1,1} = (u^M h)^r w_1^{c_1} (k^b v_1 v_2)^k v_1 v_2 s_k (a^a - s_k z'_k),$$

$$W_{1,2} = w_2^{c_1} (k^b)^{-v_2 s_k z'_2}, \quad W_{1,3} = w_3^{c_1} (k^b)^{y_g v_2 s_k z'_2}, \quad W_{1,4} = w_{c_1},$$

$$W_{2,1} = g^r w_1^{c_1} v_1 v_2 s_k (k^a)^{-s_k z'_k}, \quad W_{2,2} = w_2^{c_1} k^{-v_2 s_k}, \quad W_{2,3} = w_3^{c_1} k^{-a}, \quad W_{2,4} = w_{c_2}.$$

The simulator can only create a semi-functional signature since $s_k, z_k$ enables the cancellation of $k^{ab}$. Finally, $\mathcal{A}$ outputs a forged signature $\sigma^* = (W_{1,1}, \ldots, W_{2,4})$ on a message $M^*$. To verify the forged signature, $\mathcal{B}_3$ first chooses random exponents $s_1, s_2, s'_1, z'_c \in \mathbb{Z}_p$ and computes semi-functional verification components by implicitly setting $t = c, s_c = -acy_g + s'_c, z_c = -acy_g (xM^* + y)/s_c + z'_c/s_c$ as

$$V_{1,1} = (\hat{k}^c)^{s_1}, \quad V_{1,2} = (\hat{k}^c)^{s_2} (k^{v_1 v_2} s_k)^{z'_c}, \quad V_{1,3} = \hat{g}^{v_3 s_k z'_k}, \quad V_{1,4} = (\hat{k}^c)^{-y_g (s_1 v_1 + v_2 + a g) c} \hat{g}^{-s_k} \hat{k}^{-a c y_g + s'_c},$$

$$V_{2,1} = (\hat{k}^c)^{s_1} (k^{v_1 v_2} s_k)^{z'_c}, \quad V_{2,2} = (\hat{k}^c)^{s_1} (k^{v_1 v_2} s_k)^{z'_c}, \quad V_{2,3} = \hat{g}^{v_3 s_k z'_k}, \quad V_{2,4} = (\hat{k}^c)^{-y_g (s_1 v_1 + v_2 + a g) c} \hat{g}^{-s_k} \hat{k}^{-a c y_g + s'_c}.$$

Next, it verifies that $\prod_{i=1}^3 e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^3 e(W_{2,i}, V_{2,i})^{-1} = (T)^2$. If this equation holds, then it outputs 0. Otherwise, it outputs 1.

To finish the proof, we first show that the distribution of the simulation using $D, T = e(k, \hat{k})^{abc}$ is the same as $G_2$. The public key is correctly distributed since the random blinding values $y_g, y_w, y_v$ are used. The semi-functional signature is correctly distributed as

$$W_{1,1} = g^r (u^M h)^r w_1^{c_1} (k^{v_1 v_2} s_k)^{z_k} = k^{v_1 v_2} (u^M h)^r w_1^{c_1} (k^{v_1 v_2} s_k)^{z_k}.$$

The semi-functional verification components are correctly distributed as

$$V_{1,3} = (\hat{g}^{v_3})^{s_1} (\hat{g}^{v_3})^{s_2} (\hat{g}^{v_3})^{z'_k}, \quad V_{1,4} = (\hat{g}^{v_3})^{s_1} (\hat{g}^{v_3})^{s_2} (\hat{g}^{v_3})^{z'_k},$$

$$V_{2,3} = (\hat{g}^{v_3})^{s_1} (\hat{g}^{v_3})^{s_2} (\hat{g}^{v_3})^{z'_k}, \quad V_{2,4} = (\hat{g}^{v_3})^{s_1} (\hat{g}^{v_3})^{s_2} (\hat{g}^{v_3})^{z'_k},$$

$$\Omega' = e(g, \hat{g})^a = e(k, \hat{k})^{2abc} = (T_0)^2.$$

We next show that the distribution of the simulation using $D, T_1 = e(k, \hat{k})^d$ is almost the same as $G_3$. It is obvious that the signature verification for the forged signature always fails if $T_1 = e(k, \hat{k})^d$ is used except with $1/p$ probability since $d$ is a random value in $\mathbb{Z}_p$. This completes our proof.

### 3.3.2 Analysis of PKS2

**Theorem 3.7.** The above PKS2 scheme is existentially unforgeable under a chosen message attack if the LW1, LW2, DBDH assumptions hold. That is, for any PPT adversary $\mathcal{A}$, there exist PPT algorithms $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3$ such that $\text{Adv}_{\mathcal{B}_1}^{\text{PKS}}(\lambda) \leq q \text{Adv}_{\mathcal{B}_2}^{\text{LW1}}(\lambda) + q \text{Adv}_{\mathcal{B}_3}^{\text{LW2}}(\lambda) + \text{Adv}_{\mathcal{B}_3}^{\text{DBDH}}(\lambda)$ where $q$ is the maximum number of signature queries of $\mathcal{A}$. 


Proof. Before proving the security, we first define two additional algorithms for semi-functional types. For the semi-functionality, we set $f = g^{y_f}, \hat{f} = \hat{g}^{y_{\hat{f}}}$ where $y_f$ is a random exponent in $\mathbb{Z}_p$.

**PKS2.SignSF.** The semi-functional signing algorithm first creates a normal signature using the private key. Let $(W_{1,1}, \ldots, W_{2,3})$ be the normal signature of a message $M$ with random exponents $r, c_1, c_2 \in \mathbb{Z}_p$. It selects random exponents $s_k, z_k \in \mathbb{Z}_p$ and outputs a semi-functional signature as

$$
\sigma = \left( W_{1,1} = W_{1,1}' \cdot (f^{-v})^{z_k}, W_{1,2} = W_{1,2}' \cdot f^{v z_k}, W_{1,3} = W_{1,3}' ,
W_{2,1} = W_{2,1}' \cdot (f^{-v})^s, W_{2,2} = W_{2,2}' \cdot f^{v s}, W_{2,3} = W_{2,3}' \right).
$$

**PKS2.VerifySF.** The semi-functional verification algorithm first creates normal verification components using the public key. Let $(V_{1,1}', \ldots, V_{2,3}')$ be the normal verification components with a random exponent $t \in \mathbb{Z}_p$. It chooses random exponents $s_c, z_c \in \mathbb{Z}_p$ and computes semi-functional verification components as

$$
V_{1,1} = V_{1,1}', V_{1,2} = V_{1,2}' \cdot \hat{f}^{s_c}, V_{1,3} = V_{1,3}' \cdot (\hat{f}^{-\phi_2})^{s_c},
V_{2,1} = V_{2,1}', V_{2,2} = V_{2,2}' \cdot \hat{f}^{z_c}, V_{2,3} = V_{2,3}' \cdot (\hat{f}^{-\phi_2})^{z_c}.
$$

Next, it verifies that $\prod_{i=1}^{3} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i}, V_{2,i})^{-1} \equiv 0'$. If this equation holds, then it outputs 1. Otherwise, it outputs 0.

If the semi-functional verification algorithm is used to verify a semi-functional signature, then an additional random element $e(f, \hat{f})^{s_k (2-\omega)}$ is left in the left part of the above verification equation. If $z_k = z_c$, then the semi-functional verification algorithm succeeds. In this case, we say that the signature is nominally semi-functional.

The security proof uses a sequence of games $G_0, G_1, G_2$, and $G_3$. The definition of these games is the same as that of Theorem 3.3. From the following three lemmas, we prove that it is hard for $A$ to distinguish $G_{i-1}$ from $G_i$ under the given assumptions. Therefore, we have that

$$
\text{Adv}_{PKS}(\lambda) = \text{Adv}_{A}^{G_0} + \sum_{i=1}^{2} (\text{Adv}_{A}^{G_i} - \text{Adv}_{A}^{G_{i-1}}) - \text{Adv}_{A}^{G_3} \leq \sum_{i=1}^{3} |\text{Adv}_{A}^{G_{i-1}} - \text{Adv}_{A}^{G_{i}}|
$$

$$
= \text{Adv}_{B_1}^{LW}(\lambda) + \sum_{k=1}^{q} \text{Adv}_{B_2}^{LW}(\lambda) + \text{Adv}_{B_3}^{DBDH}(\lambda).
$$

This completes our proof. \qed

**Lemma 3.8.** If the LW1 assumption holds, then no polynomial-time adversary can distinguish between $G_0$ and $G_1$ with non-negligible advantage. That is, for any adversary $A$, there exists a PPT algorithm $B_1$ such that $|\text{Adv}_{A}^{G_0} - \text{Adv}_{A}^{G_1}| = \text{Adv}_{B_1}^{LW}(\lambda)$.

Proof. Suppose there exists an adversary $A$ that distinguishes between $G_0$ and $G_1$ with non-negligible advantage. A simulator $B_1$ that solves the LW1 assumption using $A$ is given: a challenge tuple $D = (\{p, G, G_T, e\}, k, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b, k^b)$ and $T$ where $T = T_0 = k^{ab^2c}$ or $T = T_1 = k^{ab^2\alpha+d}$. Then $B_1$ that interacts with $A$ is described as follows: $B_1$ first chooses random exponents $\phi_2, A, B, \alpha \in \mathbb{Z}_p$, random values $y_8, y_9, y_{10}, y_w \in \mathbb{Z}_p$. It computes $w_1 = w^\phi_1 = (k^b)^{y_w}, w_2 = w^\phi_2 = k^w \phi_1, w = k^w$ by implicitly
setting $\phi_1 = b$. It implicitly sets $c_k = -b/y_w + c'_k$, $c_u = -bA/y_w + c'_u$, $c_h = -bB/y_w + c'_h$, $v = a$, $\tau = b + a\phi_2$ and publishes a public key $PK$ by selecting random values $c'_g, c'_u, c'_h \in \mathbb{Z}_p$ as

$$
gw_1^{c_g} = k^{b_c}w_1^{c_g}, \quad w_2^{c_g} = (k^{b_c})^{-b_c}w_2^{c_g}, \quad w_3^{c_g} = (k^{b_c})^{-1}w_3^{c_g},
$$

$$
u w_1^{c_u} = k^{b_u}w_1^{c_u}, \quad w_2^{c_u} = (k^{b_u})^{-b_u}w_2^{c_u}, \quad w_3^{c_u} = (k^{b_u})^{-1}w_3^{c_u},
$$

$$
hw_1^{c_h} = k^{b_h}w_1^{c_h}, \quad w_2^{c_h} = (k^{b_h})^{-b_h}w_2^{c_h}, \quad w_3^{c_h} = (k^{b_h})^{-1}w_3^{c_h},
$$

$$
\hat{g} = \hat{k}^b, \hat{y}_d, \hat{g}^v = \hat{k}^{ab^2} \hat{k}^{a}_{y_v}, \quad \hat{g}^{-1} = \hat{k}^{b} \hat{k}^{a}_{y_v} \hat{k}^{b_2} \hat{k}^{a}_{y_v} \hat{k}^{b_2} \hat{k}^{a}_{y_v},
$$

$$
\hat{u} = (\hat{k}^{b_2}) A \hat{y}_w, \quad \hat{u}^v = (\hat{k}^{ab^2}) A \hat{k}^a \hat{y}_v, \quad \hat{u}^{-1} = ((\hat{k}^{b}) A (\hat{k}^{b}) \hat{k}^a \hat{y}_v (\hat{k}^{ab^2}) A \hat{y}_v),
$$

$$
\hat{h} = (\hat{k}^{b}) A \hat{k}^a \hat{y}_v, \quad \hat{h}^v = (\hat{k}^{b}) A \hat{k}^a \hat{y}_v, \quad \hat{h}^{-1} = ((\hat{k}^{b}) A (\hat{k}^{b}) \hat{k}^a \hat{y}_v (\hat{k}^{ab^2}) A \hat{y}_v),
$$

$$
\Omega = (e(k^{b}, k^{b}) \cdot e(k^{b^2}, k^{2b_2}) \cdot e(k, k)^{\alpha}).
$$

It implicitly sets $g = k^{b_2} k^{y_v}, u = (k^{b_2})^A k^{y_v}, h = (k^{b_2})^B k^{y_v}$, but it cannot create these elements since $k^{b_2}$ is not given. Additionally, it sets $f = k, \hat{f} = \hat{k}$ for the semi-functional signature and verification. $A$ adaptively requests a signature for a message $M$. To respond to this sign query, $B_1$ first selects random exponents $r, c'_1, c'_2 \in \mathbb{Z}_p$. It implicitly sets $c_1 = -b(\alpha + (AM + B)r)/y_w + c'_1, c_2 = -br_1/y_w + c'_2$ and creates a normal signature as

$$
W_{1,1} = k^{y_v \alpha + (y_w + y_v)} r (w_1^{c_1}), \quad W_{1,2} = (W_{1,3})^{\phi_2}, \quad W_{1,3} = (k^{b})^{-\alpha - (AM + B)r} w_3^{c_1},
$$

$$
W_{2,1} = k^{y_v \alpha} (w_1^{c_2}), \quad W_{2,2} = (W_{2,3})^{\phi_2}, \quad W_{2,3} = (k^{b})^{-r} w_3^{c_2}.
$$

Finally, $A$ outputs a forged signature $\sigma^* = (W_{1,1}, \ldots, W_{2,3})$ on a message $M^*$ from $A$. To verify the forged signature, $B_1$ first chooses a random exponent $t \in \mathbb{Z}_p$ and computes verification components by implicitly setting $t = c$ as

$$
V_{1,1} = \hat{k}^{b_2} \hat{k}^c, \quad V_{1,2} = T(\hat{k}^c)^{y_v}, \quad V_{1,3} = ((\hat{k}^{b_2} c) \hat{k}^{c} \hat{k}^{c}) (T) \Phi_2 (\hat{k}^c c) \hat{k}^c \hat{k}^c - 1,
$$

$$
V_{2,1} = (\hat{k}^{b_2} c)^{AM + B} (\hat{k}^c)^{y_v M + y_v}, \quad V_{2,2} = (T)^{AM + B} (\hat{k}^c)^{y_v M + y_v},
$$

$$
V_{2,3} = ((\hat{k}^{b_2} c)^{AM + B} (\hat{k}^c)^{y_v M + y_v} (T) \Phi_2 (AM + B) (\hat{k}^c c) \hat{k}^c (y_v M + y_v)) - 1.
$$

Next, it verifies that $\prod_{i=1}^{3} e(W_{1,i}^{\ast}, V_{1,i}^{\ast}) \cdot \prod_{i=1}^{3} e(W_{2,i}^{\ast}, V_{2,i}^{\ast})^{-1} \equiv \Omega'$. If this equation holds, then it outputs 0. Otherwise, it outputs 1.

To finish this proof, we show that the distribution of the simulation is correct. We first show that the distribution using $D, T_0 = \hat{k}^{ab^2} c$ is the same as $G_0$. The public key is correctly distributed as

$$
gw_1^{c_g} = (k^{b_2} k^{y_v}) (k^{b_2} y_w) = k^{b_2} w_1^{c_g}.
$$

The simulator cannot create $g, u, h$ since $k^{b_2}$ is not given in the assumption, but it can create $g w_1^{c_g}, u w_1^{c_g}, h w_1^{c_g}$ since $c_g, c_u, c_h$ can be used to cancel out $k^{b_2}$. The signature is correctly distributed as

$$
W_{1,1} = g^r (u^{y_v} h) w_1 = (k^{b_2} y_w)^{\alpha} (k^{b_2 A + y_u} (k^{b_2 B + y_u})^{-b(\alpha + (AM + B)r)/y_u + c_2})^r k^{b_2} w_1^{c_2} = k^{y_v} w_1^{c_2},
$$

$$
W_{2,1} = g^r (w^{b_1}) c_2 = (k^{b_2 + y_w})^{-b r + c_2} = k^{y_v} w_1^{c_2}.
$$
It can create a normal signature since $c_1, c_2$ enable the cancellation of $k^2$, but it cannot create a semi-functional signature since $k^a$ is not given. The verification components are correctly distributed as

$$V_{1,1} = \hat{g}^c = (\hat{k}^{b_2+y_3})^c = k^{b_2} (\hat{k}^c)^{y_3}, \quad V_{1,2} = (\hat{g}^v)^t = \hat{k}^{(b_2+y_3)c} = T_0(\hat{k}^{ac})^{y_b},$$

$$V_{1,3} = (\hat{g}^{-\tau} f) = \hat{k}^{(b_2+y_3)(b+a\phi_2)c} = (\hat{k}^{b_2} k^{b_3-B+y_3})^c = T_0(k_0)_{\phi_2}(\hat{k}^{ac}) y_2 \phi_2(-1),$$

$$V_{2,1} = (u^{M^+} h)^t = k^{(2bA+y_0)M^+} k^{b_2B+y_3})^c = (k^{b_2} k^{M^+})^c, \quad V_{2,2} = ((u^v)^t M^+ h^v) = (k^{(2bA+y_0)M^+})(b+a\phi_2)^c = (T_0)^{AM^+} + B(k^{ac}) y_2 M^+ + y_3,$$

$$V_{2,3} = ((u^{-\tau} M^+ - \tau) f) = (\hat{k}^{b_2} k^{M^+ + B}(b+a\phi_2)^c)^{y_3} = (T_0)^{(AM^+ + B)}(k^{ac}) y_2 M^+ + y_3.$$

We next show that the distribution of the simulation using $D, T_1 = \hat{k}^{aBc+d}$ is the same as $G_1$. We only consider the distribution of the verification components since $T$ is only used in the verification components. The difference between $T_0$ and $T_1$ is that $T_1$ additionally has $\hat{k}^d$. Thus $V_{1,2}, V_{1,3}, V_{2,2}, V_{2,3}$ that have $T$ in the simulation additionally have $\hat{k}^d, (\hat{k}^d)_{\phi_2}, (\hat{k}^d)^{AM^+ + B}, (\hat{k}^d)_{\phi_2 (AM^+ + B)}$ respectively. If we implicitly set $x_c = d, z_c = AM^+ + B$, then the verification components of the forged signature are semi-functional since $A$ and $B$ are information-theoretically hidden to the adversary. This completes our proof.

**Lemma 3.9.** If the LW2 assumption holds, then no polynomial-time adversary can distinguish between $G_1$ and $G_2$ with non-negligible advantage. That is, for any adversary $A$, there exists a PPT algorithm $B_2$ such that $|Adv_{G_1}^A - Adv_{G_2}^A| = Adv_{LW2}^A (\lambda)$.

**Proof.** Suppose there exists an adversary $A$ that distinguishes between $G_{1,k-1}$ and $G_{1,k}$ with non-negligible advantage. A simulator $B_2$ that solves the LW2 assumption using $A$ is given: a challenge tuple $D = ((G, \hat{G}, \hat{G}, G_T, e), k, k^a, k^b, k^c, k^{b_2}, k^{b_3}, k^{b_4}, k^{b_5}, k^{b_6}, k^{b_7}, k^{b_8}, k^{b_9})$ and $T$ where $T = T_0 = k^{bc}$ or $T = T_1 = k^{bc + d}$. Then $B_2$ that interacts with $A$ is described as follows: $B_2$ first selects random exponents $v, y_\tau, a, b, \alpha, y_w, y_h, y_w, y_w \in \mathbb{Z}_p$. It computes $w_1 = w^{\phi_1} = (k^{(b')^w} k^{(b')^{w'}}), w_2 = w^{\phi_2} = (k^{(b')^w} w = k^{w'}$ by implicitly setting $\phi_1 = -vb + (a + y_{\tau}), \phi_2 = b$. It implicitly sets $\tau = a + y_{\tau}$ and publishes a public key $PK$ by selecting random values $c_8, c_9, c_11 \in \mathbb{Z}_p$ as

$$g^{c_8} = k^{a w_1^c_8} w_1^{c_9}, g^{c_9} = k^{w_2^c_8} w_2^{c_9}, u^{w_1^c_9} = (k^{a})^{k^{w_1^c_9}}, h^{w_1^c_9} = (k^{a})^{k^{w_1^c_9}}, u^{w_2^c_9} = (k^{a})^{k^{w_2^c_9}}, w_1^{w_2^c_9}, w_1^{w_2^c_9}, w_1 w_2 w_3, w_1 w_2 w_3,$$

$\hat{g} = \hat{k}^{a}, \hat{g}^t = \hat{g}^t, \hat{g}^{-\tau} = (\hat{k}^{a} (\hat{k}^c)^{\tau}(-1)), \hat{u} = (\hat{k}^{a})^{\hat{k}^{a}}, \hat{u}^{t} = (\hat{k}^{a} (\hat{k}^c)^{AM^+ + B} (\hat{k}^c) \phi_2 (AM^+ + B)^{-1},\hat{h} = (\hat{k}^{a})^{\hat{k}^{a}}, \hat{h}^t = (\hat{k}^{a})^{B (\hat{k}^c)^{AM^+ + B} (\hat{k}^c)^{AM^+ + B})^{-1}, \Omega = e(\hat{k}^{a}, \hat{k}^{a}).$

Additionally, it sets $f = k, \hat{f} = \hat{k}$ for the semi-functional signature and verification. $A$ adaptively requests a signature for a message $M$. If this is a j-th signature query, then $B_2$ handles this query as follows:

- **Case j < k**: It creates a semi-functional signature by calling PKS2.SignSF since it knows the tuple $(f^{-v}, f, 1)$ for the semi-functional signature.

- **Case j = k**: It selects random exponents $r', c_1', c_2' \in \mathbb{Z}_p$ and creates a signature by implicitly setting $r = -c + r', c_1 = c(AM + B)/y_w + c_1', c_2 = c/y_w + c_2'$ as

$$W_{1,1} = g^{c_1} (k^c)^{(b+y_0)w_1^c} (u^{M^+} h)^r(T)^{v(AM+B)} (k^{c})^{y_2(AM+B)} w_1^{c_1'}, W_{1,2} = (T)^{(AM+B)} w_2^{c_1'}, W_{1,3} = (k^c)^{(AM+B)} w_2^{c_1'}, W_{2,1} = g^{c_2'} (T)^{-v}(k^c)^{y_2} w_1^{c_2'}, W_{2,2} = T w_2^{c_2'}, W_{2,3} = k^c w_2^{c_2'}.$$
• Case $j > k$: It creates a normal signature by calling \textbf{PKS2.Sign} since it knows the private key.

Finally, $A$ outputs a forged signature $\sigma' = (W_{1,1}, \ldots, W_{2,3})$ on a message $M^*$. To verify the forged signature, $B_2$ chooses a random exponent $t' \in \mathbb{Z}_p$ and computes semi-functional verification components by implicitly setting $t = bx + t'$, $s_c = -a^2x$, $z_c = AM^* + B$ as

\[
\begin{align*}
V_{1,1} &= \tilde{k}^{abx}(\tilde{k}^{ax})^{t'}, \quad V_{1,2} = (\tilde{k}^{abx})^v(\tilde{k}^{ax})^{t'}, \quad V_{1,3} = (\tilde{k}^{abx})^{-\gamma_t}(\tilde{g}^{-\gamma_t})^{t'}, \\
V_{2,1} &= (\tilde{k}^{abx})^{AM^*+B}(\tilde{k}^{hx})^{y_M+y_h}(\tilde{u}^M)^{t'}, \quad V_{2,2} = (\tilde{k}^{abx})^{(AM^*+B)v}(\tilde{k}^{hx})^{(y_M+y_h)v}(\tilde{u}^{AM^*})^{t'}, \\
V_{2,3} &= (\tilde{k}^{abx})^{-(AM^*+B)y_c}(\tilde{k}^{hx})^{-(y_M+y_h)}(\tilde{k}^{hx})^{-(y_M+y_h)}y_c((\tilde{u}^{AM^*})^\tau\tilde{h}^{t'-\tau})^{t'}.
\end{align*}
\]

Next, it verifies that $\prod_{i=1}^{3} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i}, V_{2,i})^{-1} = e(k^a, \tilde{k}^{abx})^v \cdot e(k^a, \tilde{k}^{ax})^{t'}$. If this equation holds, then it outputs 0. Otherwise, it outputs 1.

To finish the proof, we should show that the distribution of the simulation is correct. We first show that the distribution of the simulation using $D, T_0 = k^{bc}$ is the same as $G_{1,k}$. The public key is correctly distributed since the random blinding values $y_u, y_h, y_w$ are used. The $k$-th signature is correctly distributed as

\[
\begin{align*}
W_{1,1} &= g^a (u^M)^t \tilde{w}_1^c = g^a (k^{(aA+y_u)}k^{ab+y_h})^{t'+t''} (k^{y_u}\gamma_{v(b+a+y_c)}c)^{(AM^*+B)/y_u}, \\
&= g^a (k^{c})^{-(y_M+y_h)}(u^M)^t (T)^{-v(AM^*)} (k^{c})^{y_c(AM^*)} w_1^c, \\
W_{1,2} &= w_2^c = (k^{b})^{c(AM^*)/y_u+c'} (T)^{(AM^*)} w_2^c, \\
W_{1,3} &= w_3^c = (k^{w})^{c(AM^*)/y_u+c'} (k^{c})^{(AM^*)} w_3^c.
\end{align*}
\]

The semi-functional verification components are correctly distributed as

\[
\begin{align*}
V_{2,1} &= (\tilde{u}^{AM^*})^{t'} (\tilde{k}^{(aA+y_u)}k^{ab+B})^{bx+t''} (\tilde{k}^{abx})^{(AM^*+B)v}(\tilde{k}^{hx})^{(y_M+y_h)v}(\tilde{u}^{AM^*})^{t'}, \\
V_{2,2} &= ((\tilde{u}^v)^{(AM^*)})^{t'} \tilde{x}_2 = (\tilde{k}^{(aA+y_u)v})^{(AM^*)} (\tilde{k}^{ab+y_h})^{bx+t''} (\tilde{u}^{AM^*})^{t'}, \\
V_{2,3} &= ((\tilde{u}^{AM^*})^{t'})^{(AM^*)} (\tilde{k}^{hx})^{-(y_M+y_h)}((\tilde{u}^{AM^*})^{t'})^{(AM^*)} (\tilde{u}^{AM^*})^{t'}.
\end{align*}
\]

The simulator can create the semi-functional verification components with only fixed $z_c = AM^* + B$ since $s_c, s_c$ enable the cancellation of $\tilde{k}^{abx}$. Even though it uses the fixed $z_c$, the distribution of $z_c$ is correct since $A, B$ are information theoretically hidden to $A$. We next show that the distribution of the simulation using $D, T_1 = k^{bc+d}$ is the same as $G_{1,k}$. We only consider the distribution of the $k$-th signature since $T$ is only used in the $k$-th signature. The only difference between $T_0$ and $T_1$ is that $T_1$ additionally has $k^{d}$. The signature components $W_{1,1}, W_{1,2}, W_{2,1}, W_{2,2}$ that have $T$ in the simulation additionally have $(k^{d})^{-\nu(AM^*)}, (k^{d})^{(AM^*)}$, $(k^{d})^{-(AM^*)}$, respectively. If we implicitly set $s_k = d, z_k = AM^* + B$, then the distribution of the $k$-th signature is the same as $G_{1,k}$ except that the $k$-th signature is nominally semi-functional.

Finally, we show that $A$ cannot distinguish the nominally semi-functional signature from the semi-functional signature. The main idea of this is that $A$ cannot request a signature for the forgery message $M^*$ in the security model. Suppose there exists an unbounded adversary, then he can gather $z_k = AM^* + B$ from the $k$-th signature and $z_c = AM^* + B$ from the forged signature. It is easy to show that $z_k, z_c$ look random to the unbounded adversary since $f(M) = AM^* + B$ is a pair-wise independent function and $A, B$ are information theoretically hidden to the adversary. This completes our proof. \qed
Lemma 3.10. If the DBDH assumption holds, then no polynomial-time adversary can distinguish between $G_2$ and $G_3$ with non-negligible advantage. That is, for any adversary $A$, there exists a PPT algorithm $B_3$ such that $|\text{Adv}^{G_2}_{\hat{A}} - \text{Adv}^{G_3}_{\hat{A}}| = \text{Adv}^{\text{DBDH}}_{B_3}(\hat{\lambda})$.

Proof. Suppose there exists an adversary $\hat{A}$ that distinguish $G_2$ from $G_3$ with non-negligible advantage. A simulator $B_3$ that solves the DBDH assumption using $\hat{A}$ is given: a challenge tuple $D = ((p, \mathbb{G}, \hat{\mathbb{G}}, G_T, e), k, k^a, k^b, k^c, \hat{k}, k^a, \hat{k}, k^b, \hat{k}^c)$ and $T$ where $T = T_0 = e(k, \hat{k})$. Then $B_3$ that interacts with $\hat{A}$ is described as follows: $B_3$ first chooses random exponents $\phi_1, \phi_2, y, x, y \in \mathbb{Z}_p$ and a random element $w \in \mathbb{G}$. It computes $g = k^{x}, u = g^y, h = g^{x}, \hat{g} = \hat{k}^{x}, \hat{u} = \hat{g}^{x}, \hat{h} = \hat{g}^{x}, w_1 = w^\phi_1, w_2 = w^\phi_2$. It implicitly sets $\nu = a, \tau = \phi_1 + a\phi_2, \alpha = ab$ and publishes a public key $PK$ by selecting random values $c_\nu, c_\tau, c_\alpha \in \mathbb{Z}_p$ as

$$g w_1^{\nu} w_2^{\tau} w_1^{\alpha} w_2^{\alpha} w_3^{\alpha}, h w_1^{\nu} w_2^{\tau} w_1^{\alpha} w_2^{\alpha} w_3^{\alpha}, w_1, w_2, w_3.$$  

Additionally, it sets $f = k, \hat{f} = \hat{k}$ for the semi-functional signature and semi-functional verification. $\hat{A}$ adaptively requests a signature for a message $M$. To respond to this query, $B_3$ selects random exponents $r, c_1, c_2, s_k, z_k \in \mathbb{Z}_p$ and creates a semi-functional signature by implicitly setting $z_k = by_{\nu}/s_k + z'_k$ as

$$W_{1,1} = (u^M h)^r w_1^{c_1} (k^a)^{-x_s}, W_{1,2} = w_2^{c_1} (k^b)^{x_y} k^{x_{z_k}}, W_{1,3} = w^{c_1},$$

$$W_{2,1} = g^r w_1^{c_2} (k^a)^{-x_s}, W_{2,2} = w_2^{c_2} k^{x_s}, W_{2,3} = w^{c_2}.$$  

It can only create a semi-functional signature since $s_k, z_k$ enables the cancellation of $k^{ab}$. Finally, $\hat{A}$ outputs a forged signature $\sigma' = (W_{1,1}', \ldots, W_{2,3}')$ on a message $M'$. To verify the forged signature, $B_3$ first chooses random exponents $s_1, s_2, s'_c, z_c \in \mathbb{Z}_p$ and computes semi-functional verification components by implicitly setting $t = c, s_c = -acy_{\nu} + s'_c, z_c = -acy_{\nu}(xM^r + y)/s_c + z'_c/s_c$ as

$$V_{1,1} = (\hat{k}^c)^{y_c}, V_{1,2} = \hat{k}^{s_c}, V_{1,3} = (\hat{k}^c)^{-x_y} \hat{k}^{-s_c},$$

$$V_{2,1} = (\hat{k}^c)^{y_c}(xM^r + y), V_{2,2} = \hat{k}^{s_c}, V_{2,3} = (\hat{k}^c)^{-x_y} \hat{k}^{-s_c}.$$  

Next, it verifies that $\prod_{i=1}^3 e(W_{1,i}', V_{1,i}) \cdot \prod_{i=1}^3 e(W_{2,i}', V_{2,i})^{-1} \equiv (T)^{y_{\nu}}$. If this equation holds, then it outputs 0. Otherwise, it outputs 1.

To finish the proof, we first show that the distribution of the simulation using $D, T = e(k, \hat{k})$ is $\hat{A}$, the same as $G_2$. The public key is correctly distributed since the random values $y_{\nu}, y, x, c_\nu, c_\tau, c_\alpha$ are used. The semi-functional signature is correctly distributed as

$$W_{1,1} = g^a (u^M h)^r w_1^{c_1} (f^a)^{s_k} = k^{x_s} (u^M h)^r w_1^{c_1} (k^{-a})^{s_k} (by_{\nu}/s_k + z'_k) = (u^M h)^r w_1^{c_1} (k^{-a})^{-x_s}.$$  

The simulator can only create a semi-functional signature since $z_k = by_{\nu}/s_k + z'_k$ enables the cancellation of $k^{ab}$. The semi-functional verification components are correctly distributed as

$$V_{1,1} = \hat{g}^{c_1} = (\hat{k}^c)^y, V_{1,2} = (\hat{g}^{c_1})^{y_c} \hat{k}^{s_c} = (\hat{k}^c)^{y_c} \hat{k}^{-s_c},$$

$$V_{1,3} = (\hat{g}^{c_1})^{y_c} (\hat{k}^c)^{x_y} \hat{k}^{-s_c} = (\hat{k}^c)^{-x_y} \hat{k}^{-s_c},$$

$$V_{2,1} = (\hat{k}^c)^{y_c}(xM^r + y), V_{2,2} = \hat{k}^{s_c}, V_{2,3} = (\hat{k}^c)^{-x_y} \hat{k}^{-s_c}.$$  

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\[ V_{2,3} = (u^{-T\theta} h^{-T} (f^{-\phi_2})^{\lambda z_c} = (k^{-y_s(\phi_1 + a\phi_2) + y} e (k^{-\phi_2} - acy_s(aM^r + y) + z_c, 
\Omega' = e(g, \hat{g})^{at} = e(k, \hat{k}) s^{abc} = (T_0)^{a}. \]

We next show that the distribution of the simulation using \( D, T_1 = e(k, \hat{k})^d \) is almost the same as \( G_3 \). It is obvious that the signature verification for the forged signature always fails if \( T_1 = e(k, \hat{k})^d \) is used except with \( 1/p \) probability since \( d \) is a random value in \( \mathbb{Z}_p \). This completes our proof. \( \square \)

4 Sequential Aggregate Signature

In this section, we propose two SAS schemes with short public keys and prove their security based on that of our PKS schemes.

4.1 Definitions

The concept of SAS was introduced by Lysyanskaya et al. \cite{29}. In SAS, all signers first generate public keys and private keys, and then publishes their public keys. To generate a sequential aggregate signature, a signer may receive an aggregate-so-far from a previous signer, and creates a new aggregate signature by adding his signature to the aggregate-so-far in sequential order. After that, the signer may send the aggregate signature to a next signer. A verifier can check the validity of the aggregate signature by using the pubic keys of all signers in the aggregate signature. An SAS scheme is formally defined as follows:

Definition 4.1 (Sequential Aggregate Signature). A sequential aggregate signature (SAS) scheme consists of four PPT algorithms Setup, KeyGen, AggSign, and AggVerify, which are defined as follows:

Setup(\( 1^\lambda \)). The setup algorithm takes as input a security parameter \( 1^\lambda \) and outputs public parameters PP.

KeyGen(PP). The key generation algorithm takes as input the public parameters PP, and outputs a public key PK and a private key SK.

AggSign(\( AS', M, PK, M, SK \)). The aggregate signing algorithm takes as input an aggregate-so-far \( AS' \) on messages \( M = (M_1, \ldots, M_l) \) under public keys \( PK = (PK_1, \ldots, PK_l) \), a message \( M \), and a private key SK, and outputs a new aggregate signature AS.

AggVerify(AS, PK). The aggregate verification algorithm takes as input an aggregate signature AS on messages \( M = (M_1, \ldots, M_l) \) under public keys \( PK = (PK_1, \ldots, PK_l) \), and outputs either 1 or 0 depending on the validity of the sequential aggregate signature.

The correctness requirement is that for each PP output by Setup, for all (PK, SK) output by KeyGen, any M, we have that AggVerify(AggSign(\( AS', M, PK, M, SK \), M'), M', ||M, PK|| PK) = 1 where \( AS' \) is a valid aggregate-so-far signature on messages \( M' \) under public keys \( PK' \).

A trivial SAS scheme can be constructed from a PKS scheme by concatenating each signer’s signature in sequential order, but the size of aggregate signature is proportional to the size of signers. Therefore, a non-trivial SAS scheme should satisfy the signature compactness property that requires the size of aggregate signature to be independent of the size of signers.

The security model of SAS was defined by Lysyanskaya et al. \cite{29}, but we follow the security model of Lu et al. \cite{27} that requires for an adversary to register the key-pairs of other signers except the target
signer, namely the knowledge of secret key (KOSK) setting or the proof of knowledge (POK) setting. In this security model, an adversary first given the public key of a target signer. After that, the adversary adaptively requests a certification for a public key by registering the key-pair of other signer, and he adaptively requests a sequential aggregate signature by providing a previous aggregate signature to the signing oracle. Finally, the adversary outputs a forged sequential aggregate signature on messages under public keys. If the forged sequential signature satisfies the conditions of the security model, then the adversary wins the security game. The security model of SAS is formally defined as follows:

**Definition 4.2 (Security).** The security notion of existential unforgeability under a chosen message attack is defined in terms of the following experiment between a challenger $\mathcal{C}$ and a PPT adversary $\mathcal{A}$:

1. **Setup:** $\mathcal{C}$ first initializes a certification list $\mathcal{CL}$ as empty. Next, it runs Setup to obtain public parameters $PP$ and KeyGen to obtain a key pair $(PK, SK)$, and gives $PK$ to $\mathcal{A}$.

2. **Certification Query:** $\mathcal{A}$ adaptively requests the certification of a public key by providing a key pair $(PK, SK)$. Then $\mathcal{C}$ adds the key pair $(PK, SK)$ to $\mathcal{CL}$ if the key pair is a valid one.

3. **Signature Query:** $\mathcal{A}$ adaptively requests a sequential aggregate signature (by providing an aggregate-so-far $AS'$ on messages $M'$ under public keys $PK'$), on a message $M$ to sign under the challenge public key $PK$, and receives a sequential aggregate signature $AS$.

4. **Output:** Finally (after a sequence of the above queries), $\mathcal{A}$ outputs a forged sequential aggregate signature $AS^*$ on messages $M^*$ under public keys $PK^*$. $\mathcal{C}$ outputs 1 if the forged signature satisfies the following three conditions, or outputs 0 otherwise: 1) $\text{AggVerify}(AS^*, M^*, PK^*) = 1$, 2) The challenge public key $PK$ must exists in $PK^*$ and each public key in $PK^*$ except the challenge public key must be in $\mathcal{CL}$, and 3) The corresponding message $M$ in $M^*$ of the challenge public key $PK$ must not have been queried by $\mathcal{A}$ to the sequential aggregate signing oracle.

The advantage of $\mathcal{A}$ is defined as $\text{Adv}_{\mathcal{A}}^{\text{SAS}}(\lambda) = \Pr[\mathcal{C} = 1]$ where the probability is taken over all the randomness of the experiment. An SAS scheme is existentially unforgeable under a chosen message attack if all PPT adversaries have at most a negligible advantage in the above experiment.

### 4.2 Construction

To construct an SAS scheme from a PKS scheme, the PKS scheme should support multi-users by sharing some elements among all signers and the randomness of signatures should be sequentially aggregated to a single value. We can employ the randomness reuse technique of Lu et al. [27] to aggregate the randomness of signatures. To apply the randomness reuse technique, we should re-randomize the aggregate signature to prevent a forgery attack. Thus we build on the PKS schemes of the previous section that support multi-users and public re-randomization to construct SAS schemes.

#### 4.2.1 Our SAS1 Scheme

Our first SAS scheme in prime order bilinear groups is described as follows:

**SAS1.Setup($1^\lambda$):** This algorithm first generates the asymmetric bilinear groups $\mathbb{G}, \hat{\mathbb{G}}$ of prime order $p$ of bit size $\Theta(\lambda)$. It chooses random elements $g, w \in \mathbb{G}$ and $\hat{g}, \hat{v} \in \hat{\mathbb{G}}$. Next, it chooses random exponents
\( v_1, v_2, v_3, \phi_1, \phi_2, \phi_3 \in \mathbb{Z}_p \) and sets \( \tau = \phi_1 + v_1 \phi_2 + v_2 \phi_3, \pi = \phi_2 + v_3 \phi_3 \). It also sets \( w_1 = w^{\phi_1}, w_2 = w^{\phi_2}, w_3 = w^{\phi_3} \). It publishes public parameters as
\[
PP = \left( (p, \mathbb{G}, \mathbb{G}_T, e), g, w_1, w_2, w_3, w, \hat{g}, \hat{g}^{v_1}, \hat{g}^{v_2}, \hat{g}^{\tau}, \hat{\nu}, \hat{\nu}^{v_1}, \hat{\nu}^{\pi} \right).
\]

**SAS1.KeyGen**\((PP)\): This algorithm takes as input the public parameters \( PP \). It selects random exponents \( \alpha, x, y \in \mathbb{Z}_p \) and computes \( u = g^\alpha, h = g^x, \hat{u} = \hat{g}^x, \hat{u}^{v_1} = (\hat{g}^{v_1})^x, \hat{u}^{v_2} = (\hat{g}^{v_2})^x, \hat{h} = \hat{g}^y, \hat{h}^{v_1} = (\hat{g}^{v_1})^y, \hat{h}^{v_2} = (\hat{g}^{v_2})^y \). It outputs a private key \( SK = (\alpha, x, y) \) and a public key as
\[
PK = \left( u, h, \hat{u}, \hat{u}^{v_1}, \hat{u}^{v_2}, \hat{h}, \hat{h}^{v_1}, \hat{h}^{v_2}, \hat{h}^{\tau}, \Omega = e(g, \hat{g})^\alpha \right).
\]

**SAS1.AggSign**\((AS', M', PK', M, SK)\): This algorithm takes as input an aggregate-sofar \( AS' = (S'_{1,1}, \ldots, S'_{2,4}) \) on messages \( M' = (M_1, \ldots, M_{l-1}) \) under public keys \( PK' = (PK_1, \ldots, PK_{l-1}) \) where \( PK_i = (u_i, h_i, \Omega_i) \), a message \( M \in \{0, 1\}^k \) where \( k < \lambda \), a private key \( SK = (\alpha, x, y) \) with \( PK = (u, h, \ldots, PK_i) \) and \( PP \). It first checks the validation of \( AS' \) by calling \( \text{AggVerify}(AS', M', PK') \). If \( AS' \) is not valid, then it halts. If the public key \( PK \) of \( SK \) does already exist in \( PK' \), then it halts. Next, it creates temporal aggregate components by using the randomness of the previous aggregate-sofar as
\[
\begin{align*}
T_{1,1} &= S'_{1,1} \cdot g^\alpha (S'_{2,1})^{M+y}, \\
T_{1,2} &= S'_{1,2} \cdot (S'_{2,2})^{M+y}, \\
T_{1,3} &= S'_{1,3} \cdot (S'_{2,3})^{M+y}, \\
T_{1,4} &= S'_{1,4} \cdot (S'_{2,4})^{M+y}, \\
T_{2,1} &= S'_1 + S'_2, \\
T_{2,2} &= S'_2 + S'_3, \\
T_{2,3} &= S'_2 + S'_3, \\
T_{2,4} &= S'_2 + S'_3.
\end{align*}
\]
Finally, it selects random exponents \( r, c_1, c_2 \in \mathbb{Z}_p \) for re-randomization and outputs an aggregate signature as
\[
AS = \left( S_{1,1} = T_{1,1} \cdot \prod_{i=1}^{l-1} (u_i^{M_i} h_i)^{y} (u_i^{M_i} h_i)^{y}, S_{1,2} = T_{1,2} \cdot w^{c_1}, S_{1,3} = T_{1,3} \cdot w^{c_1}, S_{1,4} = T_{1,4} \cdot w^{c_1}, \\
S_{2,1} = T_{2,1} \cdot g^{c_1} w_1^{c_2}, S_{2,2} = T_{2,2} \cdot w_2^{c_2}, S_{2,3} = T_{2,3} \cdot w_3^{c_2}, S_{2,4} = T_{2,4} \cdot w_2^{c_2} \right).
\]

**SAS1.AggVerify**\((AS, M, PK)\): This algorithm takes as input a sequential aggregate signature \( AS \) on messages \( M = (M_1, \ldots, M_l) \) under public keys \( PK = (PK_1, \ldots, PK_l) \) where \( PK_i = (u_i, h_i, \Omega_i) \). It first checks that any public key does not appear twice in \( PK \) and that any public key in \( PK \) has been certified. If these checks fail, then it outputs 0. If \( l = 0 \), then it outputs 0 if \( S_1 = S_2 = 1 \), 0 otherwise. It chooses random exponents \( t, s_1, s_2 \in \mathbb{Z}_p \) and computes verification components as
\[
\begin{align*}
C_{1,1} &= \hat{g}^t, C_{1,2} = (\hat{g}^{v_1})^t (\hat{g}^{v_1})^t, C_{1,3} = (\hat{g}^{v_2})^t (\hat{g}^{v_2})^t, C_{1,4} = (\hat{g}^{\pi})^t (\hat{g}^{\pi})^t, \\
C_{2,1} &= \prod_{i=1}^{l} (u_i^{M_i} h_i)^{t}, C_{2,2} = \prod_{i=1}^{l} ((u_i^{v_1})^{M_i} h_i^{v_1})^{t}, C_{2,3} = \prod_{i=1}^{l} ((u_i^{v_2})^{M_i} h_i^{v_2})^{t} (\hat{g}^{v_2})^{t}, \\
C_{2,4} &= \prod_{i=1}^{l} ((u_i^{-t})^{M_i} h_i^{-t})^{t} (\hat{g}^{\pi})^{t}.
\end{align*}
\]
Next, it verifies that \( \prod_{i=1}^{l} e(S_{1,i}, C_{1,i}) \cdot \prod_{i=1}^{l} e(S_{2,i}, C_{2,i})^{-1} = \prod_{i=1}^{l} \Omega_i^{t} \). If this equation holds, then it outputs 1. Otherwise, it outputs 0.
The aggregate signature $AS$ is a valid sequential aggregate signature on messages $M' || M$ under public keys $PK' || PK$ with randomness $\vec{r} = r' + c_1' + c_1(xM + y) + c_2$, $\vec{c} = c_2 + c_2$ where $r', c_1', c_2'$ are random values in $AS'$. The sequential aggregate signature has the following form

$$S_{1,1} = \prod_{i=1}^{l} g^{c_{1'i}} \prod_{i=1}^{l} (u_i^M h_i)^{c_{1'i}} w_1^{c_{1'i}}, \quad S_{1,2} = w_2^{c_1}, \quad S_{1,3} = w_3^{c_1}, \quad S_{1,4} = w_4^{c_1},$$

$$S_{2,1} = g^{\vec{r}} w_1^{c_1}, \quad S_{2,2} = w_2^{c_1}, \quad S_{2,3} = w_3^{c_1}, \quad S_{2,4} = w_4^{c_2}.$$

### 4.2.2 Our SAS2 Scheme

Our second SAS scheme in prime order bilinear groups is described as follows:

**SAS2.Setup**$(\lambda)$: This algorithm first generates the asymmetric bilinear groups $\mathbb{G}, \hat{\mathbb{G}}$ of prime order $p$ of bit size $\Theta(\lambda)$. It chooses random elements $g, w \in \mathbb{G}$ and $\hat{g} \in \hat{\mathbb{G}}$. Next, it selects random exponents $\nu, \phi_1, \phi_2 \in \mathbb{Z}_p$ and sets $\tau = \phi_1 + \nu \phi_2, w_1 = w^{\phi_1}, w_2 = w^{\phi_2}$. It publishes public parameters by selecting a random value $c_2 \in \mathbb{Z}_p$ as

$$PP = \left( \left( p, G, \hat{G}, G_T, e, \right), g, w_1^{c_2}, w_2^{c_2}, w_3^{c_2}, w_4, w, \hat{g}, \hat{g}^\nu, \hat{g}^{-\tau}, \Lambda = e(g, \hat{g}) \right).$$

**SAS2.KeyGen**(PP): This algorithm takes as input the public parameters $PP$. It selects random exponents $\alpha, x, y \in \mathbb{Z}_p$ and sets $\hat{u} = g^x, \hat{h} = \hat{g}^y$. It outputs a private key $SK = (\alpha, x, y)$ and a public key by selecting random values $c'_u, c'_h \in \mathbb{Z}_p$ as

$$PK = \left( \left( u w_1^{c'_u} = (gw_1^{c'_u})^x w_2^{c'_u}, w_2^{c'_u} = (w_2^{c'_u})^x w_2^{c'_u}, \right), \right.$$

$$h w_1^{c'_h} = (gw_1^{c'_h})^y w_2^{c'_h}, w_2^{c'_h} = (w_2^{c'_h})^y w_2^{c'_h}, \quad \hat{u}, \hat{u}^\nu = (g^x)^y, \hat{h}, \hat{h}^\nu = (\hat{g}^y)^y, \hat{h}^{-\tau} = (\hat{g}^{-\tau})^y, \quad \Omega = \Lambda^\alpha \right).$$

**SAS2.AggSign**$(AS', M', PK', M, SK)$: This algorithm takes as input an aggregate-sofar $AS' = (S'_{1,1}, \ldots, S'_{2,3})$ on messages $M' = (M_1, \ldots, M_{l-1})$ under public keys $PK' = (PK_1, \ldots, PK_{l-1})$ where $PK_i = (u_i w_i^{c_{u,i}}), \ldots, \Omega_i)$, a message $M \in \mathbb{Z}_p$, a private key $SK = (\alpha, x, y)$ with $PK = (uw_1^{c_{u,u}}, \ldots, \Omega)$ and $PP$. It first checks the validity of $AS'$ by calling $\text{SAS.AggVerify}(AS', M', PK')$. If $AS'$ is not valid, then it halts. If the public key $PK$ of $SK$ does already exist in $PK'$, then it halts. Next, it creates temporal aggregate components by using the randomness of the previous aggregate-sofar as

$$T_{1,1} = S'_{1,1} (gw_1^{c_1})^{xM+y}, \quad T_{1,2} = S'_{1,2} (w_2^{c_1})^{xM+y}, \quad T_{1,3} = S'_{1,3} (w_3^{c_1})^{xM+y},$$

$$T_{2,1} = S'_{2,1}, \quad T_{2,2} = S'_{2,2}, \quad T_{2,3} = S'_{2,3}.$$

Finally it selects random exponents $r, c_1, c_2 \in \mathbb{Z}_p$ for re-randomization and outputs an aggregate signature as

$$AS = \left( S_{1,1} = T_{1,1}, \prod_{i=1}^{l} ((u_i w_i^{c_{u,i}})^M (h_i w_i^{c_{h,i}}))^r w_1^{c_1}, \right.$$

$$S_{1,2} = T_{1,2}, \prod_{i=1}^{l} ((w_i^{c_{h,i}})^M (w_i^{c_{h,i}}))^r w_2^{c_1}, S_{1,3} = T_{1,3}, \prod_{i=1}^{l} ((w_i^{c_{u,i}})^M (w_i^{c_{h,i}}))^r w_3^{c_1},$$

$$S_{2,1} = T_{2,1}, (gw_1^{c_1})^{r} w_1^{c_1}, S_{2,2} = T_{2,2}, (w_2^{c_1})^{r} w_2^{c_1}, S_{2,3} = T_{2,3} (w_3^{c_1})^{r} w_3^{c_1}.$$
**SAS2.AggregateVerify**(AS, M, PK): This algorithm takes as input a sequential aggregate signature AS on messages \( M = (M_1, \ldots, M_l) \) under public keys \( PK = (PK_1, \ldots, PK_l) \) where \( PK_i = (u_iw_i^c, \ldots, \Omega_i) \). It first checks that any public key does not appear twice in \( PK \) and that any public key in \( PK \) has been certified. If these checks fail, then it outputs 0. If \( l = 0 \), then it outputs 1 if \( S_1.1 = \cdots = S_2.3 = 1 \), 0 otherwise. It chooses a random exponent \( t \in \mathbb{Z}_p \) and computes verification components as

\[
C_{1,1} = g^t, \quad C_{1,2} = (\hat{g}^v)^t, \quad C_{1,3} = (\hat{g}^{-v})^t, \quad C_{2,1} = \prod_{i=1}^{l}(\hat{u}_i^M h_i^c)^t, \quad C_{2,2} = \prod_{i=1}^{l}(u_i^M h_i^c)^t, \quad C_{2,3} = \prod_{i=1}^{l}((\hat{u}_i^{-v})M h_i^{-v})^t.
\]

Next, it verifies that \( \prod_{i=1}^{3} e(S_{1,i}, C_{1,i}) \cdot \prod_{i=1}^{3} e(S_{2,i}, C_{2,i})^{-1} = \prod_{i=1}^{l} \Omega_i' \). If this equation holds, then it outputs 1. Otherwise, it outputs 0.

Let \( r', c_1', c_2' \) be the randomness of an aggregate-so-far. If we implicitly sets \( \bar{r} = r' + r \), \( \bar{c}_1 = c_1' + c_g a_l + \Sigma_{i=1}^{l} (c_u M_l + c_h i) r + c_1 \), \( \bar{c}_2 = c_2' + c_g r + c_2 \), then the aggregate signature is correctly distributed as

\[
S_{1.1} = \prod_{i=1}^{l} g^{\alpha_l} \prod_{i=1}^{l} (u_i^M h_i^c) \hat{w}_1^{-c_1}, \quad S_{1.2} = \hat{w}_2^{c_1}, \quad S_{1.3} = \hat{w}_3^{c_1}, \quad S_{2.1} = g^{\bar{r}} \hat{w}_1^{c_2}, \quad S_{2.2} = \hat{w}_2^{c_2}, \quad S_{2.3} = \hat{w}_3^{c_2}.
\]

### 4.3 Security Analysis

**Theorem 4.3.** The above SAS1 scheme is existentially unforgeable under a chosen message attack if the PKS1 scheme is existentially unforgeable under a chosen message attack. That is, for any PPT adversary \( A \) for the above SAS1 scheme, there exists a PPT algorithm \( B \) for the PKS1 scheme such that \( \text{Adv}^{\text{SAS}}_A(\lambda) \leq \text{Adv}^{\text{PKS}}_B(\lambda) \).

**Proof.** Our overall proof strategy for this part follows Lu et al. [27] and adapts it to our setting. The proof uses two properties: the fact that the aggregated signature result is independent of the order of aggregation, and the fact that the simulator of the SAS system possesses the private keys of all but the target PKS.

Suppose there exists an adversary \( A \) that forges the above SAS1 scheme with non-negligible advantage \( \varepsilon \). A simulator \( B \) that forges the PKS1 scheme is first given: a challenge public key \( PK_{PKS} = ((p, G, \hat{G}, G_T, e), g, u, h, w_1, \ldots, w, \hat{g}, \ldots, \hat{u}, \ldots, \hat{\bar{u}}^{-T}, \hat{\bar{u}}^{-T}, \hat{h}, \ldots, \hat{\bar{h}}^{-T}, v, \hat{v}, \hat{\bar{v}}^{-T}, \hat{\bar{v}}^{-T}, \Omega) \). Then \( B \) interacts with \( A \) as described follows: \( B \) first constructs \( PP = ((p, G, \hat{G}, G_T, e), g, w_1, \ldots, w, \hat{g}, \ldots, \hat{\bar{u}}^{-T}, \hat{\bar{u}}^{-T}, \hat{h}, \ldots, \hat{\bar{h}}^{-T}, v, \hat{v}, \hat{\bar{v}}^{-T}, \hat{\bar{v}}^{-T}, \Omega = e(g, \hat{g})^a) \) from \( PK_{PKS} \). Next, it initializes a certification list \( CL \) as an empty one and gives \( PP \) and \( PK^* \) to \( A \). \( A \) may adaptively requests certification queries or sequential aggregate signature queries. If \( A \) requests the certification of a public key by providing a public key \( PK_i = (u_i h_i, \ldots, \Omega_i) \) and its private key \( SK_i = (a_i, x_i, y_i) \), then \( B \) checks the private key and adds the key pair \( (PK_i, SK_i) \) to \( CL \). If \( A \) requests a sequential aggregate signature by providing an aggregate-so-far \( AS' \) on messages \( M' = (M_1, \ldots, M_{l-1}) \) under public keys \( PK' = (PK_1, \ldots, PK_{l-1}) \), and a message \( M \) to sign under the challenge private key of \( PK^* \), then \( B \) proceeds the aggregate signature query as follows:

1. It first checks that the signature \( AS' \) is valid and that each public key in \( PK' \) exits in \( CL \).
2. It queries its signing oracle that simulates \( PKS1.Sign \) on the message \( M \) for the challenge public key \( PK^* \) and obtains a signature \( \sigma \).
3. For each $1 \leq i \leq l - 1$, it constructs an aggregate signature on message $M_i$ using $\text{SAS1.AggSign}$ since it knows the private key that corresponds to $PK_i$. The result signature is an aggregate signature for messages $M'_i || M$ under public keys $PK'_i || PK^*$ since this scheme does not check the order of aggregation. It gives the result signature $AS$ to $A$.

Finally, $A$ outputs a forged aggregate signature $AS^* = (S_{1,1}^*, \ldots, S_{2,4}^*)$ on messages $M^* = (M_1, \ldots, M_l)$ under public keys $PK^* = (PK_1, \ldots, PK_l)$ for some $l$. Without loss of generality, we assume that $PK_1 = PK^*$. $B$ proceeds as follows:

1. $B$ first checks the validity of $AS^*$ by calling $\text{SAS1.AggVerify}$. Additionally, the forged signature should not be trivial: the challenge public key $PK^*$ must be in $PK^*$, and the message $M_1$ must not be queried by $A$ to the signature query oracle.

2. For each $2 \leq i \leq l$, it parses $PK_i = (u_i, h_i, \ldots, \Omega_i)$ from $PK^*$, and it retrieves the private key $SK_i = (\alpha_i, x_i, y_i)$ of $PK_i$ from $CL$. It then computes

$$W_{1,1} = S_{1,1}^* \cdot \left( \prod_{i=2}^{l} \left( g^{\alpha_i} (S_{2,2}^*)^{y_{M_i+y_i}} \right)^{-1} \right), \quad W_{1,2} = S_{1,2}^* \cdot \left( \prod_{i=2}^{l} \left( (S_{2,2}^*)^{y_{M_i+y_i}} \right)^{-1} \right),$$

$$W_{1,3} = S_{1,3}^* \cdot \left( \prod_{i=2}^{l} \left( (S_{2,3}^*)^{y_{M_i+y_i}} \right)^{-1} \right), \quad W_{1,4} = S_{1,4}^* \cdot \left( \prod_{i=2}^{l} \left( (S_{2,4}^*)^{y_{M_i+y_i}} \right)^{-1} \right),$$

$$W_{2,1} = S_{2,1}^*, \quad W_{2,2} = S_{2,2}, \quad W_{2,3} = S_{2,3}, \quad W_{2,4} = S_{2,4}^*.$$

3. It outputs $\sigma = (W_{1,1}, \ldots, W_{2,4})$ as a non-trivial forgery of the PKS scheme since it did not make a signing query on $M_1$.

To finish the proof, we first show that the distribution of the simulation is correct. It is obvious that the public parameters and the public key are correctly distributed. The sequential aggregate signatures is correctly distributed since this scheme does not check the order of aggregation. Finally, we can show that the result signature $\sigma = (W_{1,1}, \ldots, W_{2,4})$ of the simulator is a valid signature for the PKS scheme on the message $M_1$ under the public key $PK^*$ since it satisfies the following equation:

$$\prod_{i=1}^{4} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{4} e(W_{2,i}, V_{2,i})^{-1}$$

$$= e(S_{1,1}^*, \tilde{g}^{\nu}) \cdot e(S_{1,2}^*, \tilde{g}^{\nu_1 \nu_1^{\nu_1}}) \cdot e(S_{1,3}^*, \tilde{g}^{\nu_2 + \nu_3^{\nu_2}}) \cdot e(S_{1,4}^*, \tilde{g}^{\nu_4 - \nu_4^{\nu_4}}) \cdot e(\prod_{i=2}^{l} g_1^{\alpha_i}, \tilde{g}^\nu)^{-1},$$

$$e(S_{2,1}^*, \tilde{g}^{\nu_1^{\nu_1^{\nu_1^{\nu_1^{\nu_1}}}}})^{-1} \cdot e(S_{2,2}^*, \tilde{g}^{\nu_2 + \nu_3^{\nu_2}}) \cdot e(S_{2,3}^*, \tilde{g}^{\nu_4 - \nu_4^{\nu_4}}) \cdot e(\prod_{i=2}^{l} g_1^{\alpha_i}, \tilde{g}^\nu)^{-1},$$

$$e(S_{2,4}^*, \tilde{g}^{\nu_1^{\nu_1^{\nu_1^{\nu_1^{\nu_1}}}}})^{-1} \cdot e(S_{2,2}^*, \tilde{g}^{\nu_2 + \nu_3^{\nu_2}}) \cdot e(S_{2,3}^*, \tilde{g}^{\nu_4 - \nu_4^{\nu_4}}) \cdot e(\prod_{i=2}^{l} g_1^{\alpha_i}, \tilde{g}^\nu)^{-1},$$

$$= e(S_{1,1}^*, C_{1,1}) \cdot e(S_{1,2}^*, C_{1,2}) \cdot e(S_{1,3}^*, C_{1,3}) \cdot e(S_{1,4}^*, C_{1,4}) \cdot e(\prod_{i=2}^{l} g_1^{\alpha_i}, \tilde{g}^{\nu})^{-1},$$

$$e(S_{2,1}^*, \tilde{g}^{\nu_1^{\nu_1^{\nu_1^{\nu_1^{\nu_1}}}}})^{-1} \cdot e(S_{2,2}^*, \tilde{g}^{\nu_2 + \nu_3^{\nu_2}}) \cdot e(S_{2,3}^*, \tilde{g}^{\nu_4 - \nu_4^{\nu_4}}) \cdot e(\prod_{i=2}^{l} g_1^{\alpha_i}, \tilde{g}^{\nu})^{-1}. $$
\[
e(S_{2,4}^*, \prod_{i=1}^{l} (\hat{\alpha}_i^M h_i)_{\pi} - \pi_{\delta_2})^{-1} = 4 \prod_{i=1}^{l} e(S_{1,1}^*, C_{1,1}) \prod_{i=1}^{l} e(S_{2,2}^*, C_{2,2})^{-1} \prod_{i=2}^{l} g^{\alpha_i} \cdot g_i \cdot \prod_{i=1}^{l} \Omega_i \cdot \prod_{i=2}^{l} \Omega_i^{-1} = \Omega_1^l
\]

where \( \delta_l = x_i M_i + y_i \) and \( \hat{\delta}_2 = \sum_{i=2}^{l}(x_i M_i + y_i) s_1 + s_2 \). This completes our proof. \( \square \)

**Theorem 4.4.** The above SAS2 scheme is existentially unforgeable under a chosen message attack if the PKS2 scheme is existentially unforgeable under a chosen message attack. That is, for any PPT adversary \( \mathcal{A} \) for the above SAS2 scheme, there exists a PPT algorithm \( \mathcal{B} \) for the PKS2 scheme such that \( \text{Adv}_{\mathcal{A}}^{\text{SAS}}(\lambda) \leq \text{Adv}_{\mathcal{B}}^{\text{PKS}}(\lambda) \).

**Proof.** Suppose there exists an adversary \( \mathcal{A} \) that forges the above SAS2 scheme with non-negligible advantage \( \varepsilon \). A simulator \( \mathcal{B} \) that forges the PKS2 scheme is first given: a challenge public key \( PK_{\text{PKS}} = ((p, \hat{G}, \hat{G}_T, e), gw_1^{\hat{c}_0}, w_2^{\hat{c}_2}, w_3^{\hat{c}_3}, \ldots, w_i^{\hat{c}_i}, \ldots, w_{\hat{s}_k}, w_{1,2}, w, \hat{g}, \hat{g}_y, \hat{g}^{-\tau}, \hat{u}, \ldots, \hat{h}^{-\tau}, \Omega) \). Then \( \mathcal{B} \) that interacts with \( \mathcal{A} \) is described as follows: \( \mathcal{B} \) first constructs \( PP = ((p, \hat{G}, \hat{G}_T, e), gw_1^{\hat{c}_0}, w_2^{\hat{c}_2}, w_3^{\hat{c}_3}, \ldots, w_i^{\hat{c}_i}, \ldots, w_{\hat{s}_k}, w_{1,2}, w, \hat{g}, \hat{g}_y, \hat{g}^{-\tau}, \hat{u}, \ldots, \hat{h}^{-\tau}, \Omega) \) by computing \( \Lambda = h = (gw_1^{\hat{c}_0}, \hat{g}, e(w_2^{\hat{c}_2}, \hat{g}_y), e(w_i^{\hat{c}_i}, \hat{g}^{-\tau}) = e(g, \hat{g}) \) and \( PK = (uw_1^{\hat{c}_0}, w_i^{\hat{c}_i}, w_{1,2}, w, \hat{g}, \hat{g}_y, \hat{g}^{-\tau}, \hat{u}, \ldots, \hat{h}^{-\tau}, \Omega) \) from \( PK_{\text{PKS}} \). Next, it initializes a certification list \( CL \) as an empty one and gives \( PP \) and \( PK \) to \( \mathcal{A} \). \( \mathcal{A} \) may adaptively requests certification queries or sequential aggregate signature queries. If \( \mathcal{A} \) requests the certification of a public key by providing a public key \( PK_i = (u_i w_i^{\hat{c}_i}, \ldots, \Omega_i) \) and its private key \( SK_i = (\hat{c}_i, x_i, y_i) \), then \( \mathcal{B} \) checks the private key and adds the key pair \( (PK_i, SK_i) \) to \( CL \). If \( \mathcal{A} \) requests a sequential aggregate signature by providing an aggregate-so-far \( AS' \) on messages \( M' = (M_1, \ldots, M_{l-1}) \) under public keys \( PK' = (PK_1, \ldots, PK_{l-1}) \), and a message \( M \) to sign under the challenge private key of \( PK' \), then \( \mathcal{B} \) proceeds the aggregate signature query as follows:

1. It first checks that the signature \( AS' \) is valid and that each public key in \( PK' \) exits in \( CL \).
2. It queries its signing oracle that simulates \( \text{SAS2.Sign} \) on the message \( M \) for the challenge public key \( PK' \) and obtains a signature \( \sigma \).
3. For each \( 1 \leq i \leq l - 1 \), it constructs an aggregate signature on message \( M_i \) using \( \text{SAS2.AggSign} \) since it knows the private key that corresponds to \( PK_i \). The result signature is an aggregate signature for messages \( M' || M \) under public keys \( PK' || PK' \) since this scheme does not check the order of aggregation. It gives the result signature \( AS \) to \( \mathcal{A} \).

Finally, \( \mathcal{A} \) outputs a forged aggregate signature \( AS^* = (S_{1,1}^*, \ldots, S_{2,3}^*) \) on messages \( M^* = (M_1, \ldots, M_l) \) under public keys \( PK^* = (PK_1, \ldots, PK_l) \) for some \( l \). Without loss of generality, we assume that \( PK_1 = PK^* \). \( \mathcal{B} \) proceeds as follows:

1. \( \mathcal{B} \) first checks the validity of \( AS^* \) by using \( \text{SAS2.AggVerify} \). Additionally, the forged signature should not be trivial: the challenge public key \( PK^* \) must be in \( PK' \), and the message \( M_i \) must not be queried by \( \mathcal{A} \) to the signature query oracle.
2. For each \( 2 \leq i \leq l \), it parses \( PK_i = (u_i w_i^{\hat{c}_i}, \ldots, \Omega_i) \) from \( PK' \), and it retrieves the private key \( SK_i = (\hat{c}_i, x_i, y_i) \) of \( PK_i \) from \( CL \). It then computes

\[
W_{1,1} = S_{1,1}^* \prod_{i=2}^{l} (g^{\alpha_i} (S_{2,2}^*)^{|x_i M_i + y_i}|)^{-1}, \quad W_{1,2} = S_{1,2}^* \prod_{i=2}^{l} ((S_{2,2}^*)^{|x_i M_i + y_i}|)^{-1}, \quad W_{1,3} = S_{1,3}^* \prod_{i=2}^{l} ((S_{2,3}^*)^{|x_i M_i + y_i}|)^{-1},
\]

\[
W_{2,1} = S_{2,1}^*, \quad W_{2,2} = S_{2,2}^*, \quad W_{2,3} = S_{2,3}^*.
\]
3. It outputs \( \sigma = (W_1, \ldots, W_2) \) as a non-trivial forgery of the PKS scheme since it did not make a signing query on \( M_1 \).

The public parameters and the public key are correctly distributed, and the sequential aggregate signatures are also correctly distributed since this scheme does not check the order of aggregation. The result signature \( \sigma = (W_1, \ldots, W_2) \) of the simulator is a valid PKS signature on the message \( M_1 \) under the public key \( PK^* \) since it satisfies the following equation:

\[
\prod_{i=1}^{3} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i}, V_{2,i})^{-1} = e(S_{1,1}, \hat{g}^v) \cdot e(S_{1,2}, \hat{g}^v) \cdot e(S_{1,4}, \hat{g}^{-v}) \cdot e(\prod_{i=2}^{l} g^{w_i}, \hat{g}^{v})^{-1}.
\]

\[
e(S_{2,1}, \prod_{i=2}^{l} (\hat{u}^{M_1} \hat{h}_i)^{w_i} \cdot e(S_{2,2}, \prod_{i=2}^{l} (\hat{u}^{M_1} \hat{h}_i)^{w_i} \cdot e(S_{2,3}, \prod_{i=2}^{l} (\hat{u}^{M_1} \hat{h}_i)^{w_i} \cdot e(S_{2,4}, (\hat{u}^{M_1} \hat{h}_i)^{-v})^{-1}.
\]

\[
e(S_{2,1}, (\hat{u}^{M_1} \hat{h}_i)^{-v})^{-1} \cdot e(S_{2,2}, (\hat{u}^{M_1} \hat{h}_i)^{-v})^{-1} \cdot e(S_{2,3}, (\hat{u}^{M_1} \hat{h}_i)^{-v})^{-1}
\]

\[
e(S_{2,1}, C_{1,1}) \cdot e(S_{2,2}, C_{1,2}) \cdot e(S_{2,3}, C_{1,3}) \cdot e(\prod_{i=2}^{l} g^{w_i}, \hat{g}^{v})^{-1}.
\]

\[
e(S_{2,1}, \prod_{i=1}^{l} (\hat{u}^{M_1} \hat{h}_i)^{w_i} \cdot e(S_{2,2}, \prod_{i=1}^{l} (\hat{u}^{M_1} \hat{h}_i)^{w_i} \cdot e(S_{2,3}, \prod_{i=1}^{l} (\hat{u}^{M_1} \hat{h}_i)^{w_i} \cdot e(S_{2,4}, (\hat{u}^{M_1} \hat{h}_i)^{-v})^{-1}
\]

\[
= \prod_{i=1}^{3} e(S_{1,i}, C_{1,i}) \cdot \prod_{i=1}^{3} e(S_{2,i}, C_{2,i}) \cdot e(\prod_{i=2}^{l} g^{w_i}, \hat{g}^{v})^{-1} = \prod_{i=1}^{l} \Omega_i \cdot \prod_{i=2}^{l} \Omega_i^{-1} = \Omega_1
\]

where \( \delta_i = x_i M_i + y_i \) and \( s_2 = \sum_{i=2}^{l} (x_i M_i + y_i) s_1 + s_2 \). This completes our proof.

4.4 Discussions

Multiple Messages. The SAS schemes of this paper only allow a signer to sign once in the aggregate algorithm. To support multiple signing per one signer, we can use the method of Lu et al. [27]. The basic idea of Lu et al. is to apply a collision resistant hash function \( H \) to a message \( M \) before performing the signing algorithm. If a signer wants to add a signature on a message \( M_2 \) into the aggregate signature, he first removes his previous signature on \( H(M_1) \) from the aggregate signature using his private key, and then he adds the new signature on the \( H(M_1 \|| M_2) \) to the aggregate signature.

5 Multi-Signature

In this section, we propose an efficient multi-signature (MS) scheme with short public parameters and prove its security without random oracles.

5.1 Definitions

Multi-Signature (MS) can be regarded as a special kind of PKAS in which different signatures generated by different signers on the same message are combined as a short multi-signature. Thus MS consists of four algorithms of PKS and additional two algorithms Combine and MultiVerify for combining a multi-signature and verifying a multi-signature. In MS, each signer generates a public key and a private key, and
he can generate an individual signature on a message by using his private key. To generate a multi-signature, anyone can combine individual signatures of different signers on the same message. A verifier can check the validity of the multi-signature by using the public keys of signers. An MS scheme is formally defined as follows:

**Definition 5.1** (Multi-Signature). A multi-signature (MS) scheme consists of six PPT algorithms *Setup*, *KeyGen*, *Sign*, *Verify*, *Combine*, and *MultVerify*, which are defined as follows:

- **Setup**($1^\lambda$): The setup algorithm takes as input a security parameter $\lambda$, and outputs public parameters PP.
- **KeyGen**(PP): The key generation algorithm takes as input the public parameters PP, and outputs a public key PK and a private key SK.
- **Sign**(M,SK): The signing algorithm takes as input a message M, and a private key SK. It outputs a signature $\sigma$.
- **Verify**(σ,M,PK): The verification algorithm takes as input a signature $\sigma$ on a message M under a public key PK, and outputs either 1 or 0 depending on the validity of the signature.
- **Combine**(σ,M,PK): The combining algorithm takes as input signatures $\sigma$ on a message M under public keys PK = (PK₁, ..., PKₗ), and outputs a multi-signature MS.
- **MultVerify**(MS,M,PK): The multi-verification algorithm takes as input a multi-signature MS on a message M under public keys PK = (PK₁, ..., PKₗ), and outputs either 1 or 0 depending on the validity of the multi-signature.

The correctness requirement is that for each PP output by *Setup*($1^\lambda$), for all (PK,SK) output by *KeyGen*(PP), and any M, we have that *Verify*(*Sign*(M,SK),M,PK) = 1 and for each $\sigma$ on message M under public keys PK, *MultVerify*(*Combine*(σ,M,PK),M,PK) = 1.

The security model of MS was defined by Micali et al. [30], but we follow the security model of Boldyreva [6] that requires for an adversary to register the key-pairs of other signers except the target signer, namely the knowledge of secret key (KOSK) setting or the proof of knowledge (POK) setting. In this security model, an adversary is first given the public key of a target signer. After that, the adversary adaptively requests the certification of a public key by registering the key-pair of other signer, and he adaptively requests a signature for the target signer on a message. Finally, the adversary outputs a forged multi-signature on a message $M^*$ under public keys. If the forged multi-signature satisfies the conditions of the security model, then the adversary wins the security game. The security model of MS is formally defined as follows:

**Definition 5.2** (Security). The security notion of existential unforgeability under a chosen message attack is defined in terms of the following experiment between a challenger $\mathcal{C}$ and a PPT adversary $\mathcal{A}$:

1. **Setup**: $\mathcal{C}$ first initialize the certification list CL as empty. Next, it runs *Setup* to obtain public parameters PP and *KeyGen* to obtain a key pair (PK,SK), and gives PP,PK to $\mathcal{A}$.

2. **Certification Query**: $\mathcal{A}$ adaptively requests the certification of a public key by providing a key pair (PK,SK). $\mathcal{C}$ adds the key pair (PK,SK) to CL if the private key is a valid one.

3. **Signature Query**: $\mathcal{A}$ adaptively requests a signature by providing a message M to sign under the challenge public key PK, and receives a signature $\sigma$. 

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4. Output: Finally, A outputs a forged multi-signature MS* on a message M* under public keys PK*. C outputs 1 if the forged signature satisfies the following three conditions, or outputs 0 otherwise: 1) MultiVerify(MS*, M*, PK*) = 1. 2) The challenge public key PK must exist in PK* and each public key in PK* except the challenge public key must be in CL, and 3) The message M* must not have been queried by A to the signing oracle.

The advantage of A is defined as $\text{Adv}^\text{MS}_A = \Pr[C=1]$ where the probability is taken over all the randomness of the experiment. An MS scheme is existentially unforgeable under a chosen message attack if all PPT adversaries have at most a negligible advantage in the above experiment.

5.2 Construction

To construct an MS scheme with short public parameters, we may use our PKS schemes that support multi-users and public re-randomization. To aggregate the randomness of signatures, we cannot use the technique of Lu et al. [27] since the randomness should be freely aggregated in MS. Instead we aggregate the randomness of signatures by using the fact that each signer generates a signature on the same message in MS. That is, if group elements $u, h$ that are related to message hashing are shared among all signers, then the randomness of each signer can be easily aggregated since the random exponent in a public key and the randomness of a signature are placed in different positions. Thus our two PKS schemes can be used to build MS schemes since $g, u, h$ in PKS1 or $gw_1^{c_2}, uw_1^{c_1}, hw_1^{c_1}$ in PKS2 are published in a public key. Note that it is not required for a signer to publicly re-randomize a multi-signature since each signer selects an independent random value.

To reduce the size of multi-signatures, we use our PKS2 scheme for this MS scheme. Our MS scheme based on the PKS2 scheme is described as follows:

**MS.Setup($1^\lambda$):** This algorithm first generates the asymmetric bilinear groups $G, \hat{G}$ of prime order $p$ of bit size $\Theta(\lambda)$. It chooses random elements $g, w \in G$ and $\hat{g} \in \hat{G}$. Next, it selects random exponents $\nu, \phi_1, \phi_2 \in \mathbb{Z}_p$ and sets $\tau = \phi_1 + \nu \phi_2$, $w_1 = w^\phi_1, w_2 = w^\phi_2$. It selects random exponents $x, y \in \mathbb{Z}_p$ and computes $u = g^x, h = g^\nu, \hat{u} = \hat{g}^x, \hat{h} = \hat{g}^\nu$. It publishes public parameters by selecting random values $c_g, c_x, c_h \in \mathbb{Z}_p$ as

$$PP = \left( (p, G, \hat{G}, G_T, e), gw_1^{c_2}, w_2^{c_1}, w^{c_2}, uw_1^{c_1}, w_2^{c_2}, w^{c_1}, w_1^{c_1}, w_2^{c_1}, w^{c_2}, c_g, c_x, c_h \right).$$

**MS.KeyGen($PP$):** This algorithm takes as input the public parameters $PP$. It selects a random exponent $\alpha \in \mathbb{Z}_p$ and computes $\Omega = \Lambda^\alpha$. Then it outputs a private key $SK = \alpha$ and a public key as $PK = \Omega$.

**MS.Sign($M, SK$):** This algorithm takes as input a message $M \in \mathbb{Z}_p$ and a private key $SK = \alpha$. It selects random exponents $r, c_1, c_2 \in \mathbb{Z}_p$ and outputs a signature as

$$\sigma = \left( \begin{array}{l}
W_{1,1} = (gw_1^{c_2})^\alpha (uw_1^{c_1}w_2^{c_2})^M(hw_1^{c_1})^r w_1^{c_1}, \\
W_{1,2} = (w_2^{c_2})^\alpha (w_2^{c_1})^r w_2^{c_2}, W_{1,3} = (w^{c_2})^\alpha ((w^{c_1})^M w^{c_2})^r w^{c_1}, \\
W_{2,1} = (gw_1^{c_2})^r w_1^{c_1}, W_{2,2} = (w_2^{c_2})^r w_2^{c_2}, W_{2,3} = (w^{c_2})^r w^{c_1} \end{array} \right).$$
MS.Verify($\sigma, M, PK$): This algorithm takes as input a signature $\sigma$ on a message $M$ under a public key $PK$. It chooses a random exponent $t \in \mathbb{Z}_p$ and computes verification components as

\[
V_{1,1} = \hat{g}^t, V_{1,2} = (\hat{g}^\nu)^t, V_{1,3} = (\hat{g}^{-\tau})^t, \\
V_{2,1} = (\hat{u}^M)^t, V_{2,2} = ((\hat{u}^\nu)^M)^t, V_{2,3} = ((\hat{u}^{-\tau})^M)^t.
\]

Next, it verifies that $\prod_{i=1}^{3} e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i}, V_{2,i})^{-1} \equiv \Omega^t$. If this equation holds, then it outputs 1. Otherwise, it outputs 0.

MS.Combine($\sigma, M, PK$): This algorithm takes as input signatures $\sigma = (\sigma_1, \ldots, \sigma_l)$ on a message $M$ under public keys $PK = (PK_1, \ldots, PK_l)$ where $PK_i = \Omega_i$. It first checks the validity of each signature $\sigma_i = (W_{1,1}^i, \ldots, W_{2,3}^i)$ by calling MS.Verify($\sigma_i, M, PK_i$). If any signature is invalid, then it halts. It then outputs a multi-signature for a message $M$ as

\[
MS = \left( S_{1,1} = \prod_{i=1}^{l} W_{1,1}^i, S_{1,2} = \prod_{i=1}^{l} W_{1,2}^i, S_{1,3} = \prod_{i=1}^{l} W_{1,3}^i, \\
S_{2,1} = \prod_{i=1}^{l} W_{2,1}^i, S_{2,2} = \prod_{i=1}^{l} W_{2,2}^i, S_{2,3} = \prod_{i=1}^{l} W_{2,3}^i \right).
\]

MS.MultVerify($MS, M, PK$): This algorithm takes as input a multi-signature $MS$ on a message $M$ under public keys $PK = (PK_1, \ldots, PK_l)$ where $PK_i = \Omega_i$. It chooses a random exponent $t \in \mathbb{Z}_p$ and computes verification components as

\[
V_{1,1} = \hat{g}^t, V_{1,2} = (\hat{g}^\nu)^t, V_{1,3} = (\hat{g}^{-\tau})^t, \\
V_{2,1} = (\hat{u}^M)^t, V_{2,2} = ((\hat{u}^\nu)^M)^t, V_{2,3} = ((\hat{u}^{-\tau})^M)^t.
\]

Next, it verifies that $\prod_{i=1}^{3} e(S_{1,i}, V_{1,i}) \cdot \prod_{i=1}^{3} e(S_{2,i}, V_{2,i})^{-1} \equiv \prod_{i=1}^{l} \Omega^t$. If this equation holds, then it outputs 1. Otherwise, it outputs 0.

5.3 Security Analysis

Theorem 5.3. The above MS scheme is existentially unforgeable under a chosen message attack if the PKS2 scheme is existentially unforgeable under a chosen message attack. That is, for any PPT adversary $A$ for the above MS scheme, there exists a PPT algorithm $B$ for the PKS2 scheme such that $Adv^A_{MS}(\lambda) \leq Adv^B_{PKS}(\lambda)$.

Proof: Suppose there exists an adversary $A$ that forges the above MS scheme with a non-negligible advantage $\varepsilon$. A simulator $B$ that forges the PKS2 scheme is given: a challenge public key $PK_{PKS} = (p, G, \hat{G}, G_T, e, gw_1^x, \ldots, \Lambda, \Omega)$. Then $B$ that interacts with $A$ is described as follows: $B$ first constructs $PP = (p, G, \hat{G}, G_T, e, gw_1^x, \ldots, \Lambda, \Omega)$ by computing $\Lambda = e(gw_1^x, \hat{g}) \cdot e(w_2^x, \hat{g}^\nu) \cdot e(w_3^x, \hat{g}^{-\tau}) = e(g, \hat{g})$ and $PK^* = \Omega$ from $PK_{PKS}$. Next, it initializes a certification list $CL$ as an empty one and gives $PP$ and $PK^*$ to $A$. $A$ may adaptively request certification queries or signature queries. If $A$ requests the certification of a public key by providing a public key $PK_i = \Omega_i$ and its private key $SK_i = \alpha_i$, then $B$ checks the key pair and adds $(PK_i, SK_i)$ to $CL$. If $A$ requests a signature by providing a message $M$ to sign under the challenge private key of $PK^*$, then $B$ queries its signing oracle that simulates PKS2.Sign on the message $M$ for the challenge public key $PK^*$, and gives the signature to $A$. Finally, $A$ outputs a forged multi-signature $MS^* = (S_{1,1}^*, \ldots, S_{2,3}^*)$ on a message $M^*$ under public keys $PK^* = (PK_1, \ldots, PK_l)$ for some $l$. Without loss of generality, we assume that $PK_1 = PK^*$. $B$ proceeds as follows:

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1. B first check the validity of $MS^*$ by calling $\text{MS.MultVerify}$. Additionally, the forged signature should not be trivial: the challenge public key $PK^*$ must be in $PK^*$, and the message $M$ must not be queried by $A$ to the signing oracle.

2. For each $2 \leq i \leq l$, it parses $PK_i = \Omega_i$ from $PK^*$, and it retrieves the private key $SK_i = g^{\alpha_i}$ of $PK_i$ from $CL$. It then computes

$$W_{1,1} = S_{1,1}^* \cdot \prod_{i=2}^{l} (g^{\alpha_i})^{-1}, \quad W_{1,2} = S_{1,2}^*, \quad W_{1,3} = S_{1,3}^*,$$

$$W_{2,1} = S_{2,1}^*, \quad W_{2,2} = S_{2,2}^*, \quad W_{2,3} = S_{2,3}^*.$$ 

3. It outputs $\sigma = (W_{1,1}, \ldots, W_{2,3})$ as a non-trivial forgery of the PKS scheme since it did not make a signing query on $M_1$.

To finish the proof, we first show that the distribution of the simulation is correct. It is obvious that the public parameters, the public key, and the signatures are correctly distributed. Next we show that the output signature $\sigma = (W_{1,1}, \ldots, W_{2,3})$ of the simulator is a valid signature for the PKS2 scheme on the message $M_1$ under the public key $PK^*$ since it satisfies the following equation

$$\prod_{i=1}^{3} e(W_{1,i},V_{1,i}) \cdot \prod_{i=1}^{3} e(W_{2,i},V_{2,i})^{-1}$$

$$= \prod_{i=1}^{3} e(S_{1,i}^*,V_{1,i}) \cdot \prod_{i=1}^{3} e(S_{2,i}^*,V_{2,i})^{-1} \cdot e(\prod_{i=2}^{l} g^{\alpha_i}, \hat{g})^{-1} = \prod_{i=1}^{l} \Omega_i \cdot \prod_{i=2}^{l} \Omega_i^{-\ell} = \Omega_i^\ell.$$ 

This completes our proof.

5.4 Discussions

Removing the Proof of Knowledge. In our MS scheme, an adversary should prove that he knows the private key of other signer by using a zero-knowledge proof system. Ristenpart and Yilek [32] showed that some MS schemes can be proven in the proof of possession (POP) setting instead of the POK setting. Our MS scheme also can be proven in the POP setting by using their technique. That is, if our MS scheme is incorporated with a POP scheme that uses a different hash function, and the adversary submits a signature on the private key of other signer as the proof of possession, then the security of our scheme is also achieved. In the security proof, a simulator cannot extract the private key element $g^\alpha$ from the signature of the POP scheme, but he can extract other values $g^{\alpha_1}w_1^{\ell_1}, g^{\alpha_2}w_2^{\ell_2}$ and these values are enough for the security proof.

6 Conclusion

In this paper, we first proposed two PKS schemes with short public keys that support multi-users and public re-randomization based on the LW-IBE scheme. Next, we proposed two SAS schemes with short public keys without random oracles and with no relaxation of assumptions (i.e., employing neither random oracles nor interactive assumptions) based on our two PKS schemes. The proposed SAS schemes are the first of this kind that have short (a constant number of group elements) size public keys and a constant number of pairing operations per message in the verification algorithm. We also proposed an MS scheme with short public parameters based on our PKS scheme and proved its security without random oracles.
There are many interesting open problems. The first one is to construct an SAS scheme with short public keys that is secure under standard assumptions without random oracles. A possible approach is to build an SAS scheme based on the practical PKS scheme of Böhl et al. [5] that is secure under the standard assumption. The second one is to build an SAS scheme with short public keys that supports lazy verification and has the constant size of aggregate signatures. Brogle et al. [12] proposed an SAS scheme with lazy verification, but the size of aggregate signatures in their SAS scheme is not constant.

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In this section, we describe the IBE scheme of Lewko and Waters (LW-IBE) \cite{LW-IBE} in prime order bilinear groups and the PKS scheme (LW-PKS) that is derived from the LW-IBE scheme.
A.1 The LW-IBE Scheme

The LW-IBE scheme in prime order bilinear groups is described as follows:

**IBE.Setup**($1^k$): This algorithm first generates the asymmetric bilinear groups $\mathbb{G}, \hat{\mathbb{G}}$ of prime order $p$ of bit size $\Theta(\lambda)$. It chooses random elements $g \in \mathbb{G}$ and $\hat{g}, \hat{w} \in \hat{\mathbb{G}}$. Next, it chooses random exponents $v, \phi_1, \phi_2 \in \mathbb{Z}_p$ and sets $\tau = \phi_1 + v\phi_2$. It selects random exponents $\alpha, x, y \in \mathbb{Z}_p$ and sets $u = g^x, \hat{u} = \hat{g}^y, \hat{h} = \hat{g}^\tau, \hat{w}_1 = \hat{w}^{\phi_1}, \hat{w}_2 = \hat{w}^{\phi_2}$. It outputs a master key $MK = (\alpha, g, \hat{u}, \hat{h}, \hat{w}_1, \hat{w}_2, \hat{w})$ and public parameters as

$$PP = \left( (p, \mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_T, e), g, g^v, g^{-\tau}, u, u^v, u^{-\tau}, h, h^v, h^{-\tau}, \Omega = e(g, \hat{g})^\alpha \right).$$

**IBE.GenKey**(ID, MK): This algorithm takes as input an identity ID $\in \{0, 1\}^k$ where $k < \lambda$ and the master key MK. It selects random exponents $r, c_1, c_2 \in \mathbb{Z}_p$ and outputs a private key as

$$SK_{ID} = \left( K_{1,1} = \hat{g}^r (d^{ID} h)^r \hat{w}_1^{c_1}, K_{1,2} = \hat{w}_2^{c_1}, K_{1,3} = \hat{w}_1^{c_1}, K_{2,1} = \hat{g}^r \hat{w}_2^{c_1}, K_{2,2} = \hat{w}_2^{c_2}, K_{2,3} = \hat{w}_2^{c_2} \right).$$

**IBE.Encrypt**(M, ID, PP): This algorithm takes as input a message $M \in \mathbb{G}_T$, an identity ID, and the public parameters PP. It first chooses a random exponent $t \in \mathbb{Z}_p$ and outputs a ciphertext as

$$CT = \left( C = e(g, \hat{g})^{\alpha t} M, C_{1,1} = g^t, C_{1,2} = (g^v)^t, C_{1,3} = (g^{-\tau})^t, C_{2,1} = (u^{ID} h)^t, C_{2,2} = ((u^v)^{ID} h^v)^t, C_{2,3} = ((u^{-\tau})^{ID} h^{-\tau})^t \right).$$

**IBE.Decrypt**(CT, $SK_{ID}$, PP): This algorithm takes as input a ciphertext CT, a private key $SK_{ID}$, and the public parameters PP. If the identities of the ciphertext and the private key are equal, then it computes

$$M = \left( C \cdot \prod_{i=1}^{3} e(C_{1,i}, K_{1,i})^{-1} \cdot \prod_{i=1}^{3} e(C_{2,i}, K_{2,i}) \right).$$

A.2 The LW-PKS Scheme

To derive a LW-PKS scheme from the LW-IBE scheme, we apply the transformation of Naor [9]. Additionally, we represent the signature in $\mathbb{G}$ instead of $\hat{\mathbb{G}}$ to reduce the size of signatures. The LW-PKS scheme in prime order bilinear groups is described as follows:

**PKS.KeyGen**($1^k$): This algorithm first generates the asymmetric bilinear groups $\mathbb{G}, \hat{\mathbb{G}}$ of prime order $p$ of bit size $\Theta(\lambda)$. It chooses random elements $g, w \in \mathbb{G}$ and $\hat{g} \in \hat{\mathbb{G}}$. Next, it chooses random exponents $v, \phi_1, \phi_2 \in \mathbb{Z}_p$ and sets $\tau = \phi_1 + v\phi_2$. It selects random exponents $\alpha, x, y \in \mathbb{Z}_p$ and sets $u = g^x, \hat{u} = \hat{g}^y, \hat{h} = \hat{g}^\tau, w_1 = w^{\phi_1}, w_2 = w^{\phi_2}$. It outputs a private key $SK = (\alpha, g, u, h)$ and a public key as

$$PK = \left( (p, \mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_T, e), w_1, w_2, w, \hat{g}, \hat{g}, g, g^v, g^{-\tau}, \hat{u}, \hat{u}^v, \hat{u}^{-\tau}, \hat{h}, \hat{h}^v, \hat{h}^{-\tau}, \Omega = e(g, \hat{g})^\alpha \right).$$

**PKS.Sign**(M, SK): This algorithm takes as input a message $M \in \{0, 1\}^k$ where $k < \lambda$ and a private key SK. It selects random exponents $r, c_1, c_2 \in \mathbb{Z}_p$ and outputs a signature as

$$\sigma = \left( W_{1,1} = g^x (u^t h)^t w_1^{c_1}, W_{1,2} = w_2^{c_1}, W_{1,3} = w^{c_1}, W_{2,1} = g^y w_2^{c_2}, W_{2,2} = w_2^{c_2}, W_{2,3} = w^{c_2} \right).$$
**PKS.Verify(σ,M,PK):** This algorithm takes as input a signature σ on a message \( M \in \{0, 1\}^k \) under a public key PK. It first chooses a random exponent \( t \in \mathbb{Z}_p \) and computes verification components as

\[
V_{1,1} = \hat{g}^t, V_{1,2} = (\hat{g}^\nu)^t, V_{1,3} = (\hat{g}^{-\tau})^t,
\]
\[
V_{2,1} = (\hat{u}^M \hat{h})^t, V_{2,2} = ((\hat{u}^\nu)^M \hat{h}^\nu)^t, V_{2,3} = ((\hat{u}^{-\tau})^M \hat{h}^{-\tau})^t.
\]

Next, it verifies that \( \prod_{i=1}^3 e(W_{1,i}, V_{1,i}) \cdot \prod_{i=1}^3 e(W_{2,i}, V_{2,i})^{-1} \overset{?}{=} \Omega^t \). If this equation holds, then it outputs 1. Otherwise, it outputs 0.

We can safely move the elements \( w_1, w_2, w \) from the private key to the public key since these elements are always constructed in the security proof of the LW-IBE scheme. However, this LW-PKS scheme does not support multi-user setting and public re-randomization since the elements \( g, u, h \) are not given in the public key.