RELATIVISTIC LENGTH AGONY CONTINUED

Dragan V Redžić
Faculty of Physics, University of Belgrade, PO Box 368, 11001 Beograd, Serbia, Yugoslavia
E-mail address: redzic@ff.bg.ac.rs

Abstract
An attempt is made to remedy confusing treatments of some basic relativistic concepts and results in recent papers by Franklin (2010 Eur. J. Phys. 31 291-8) and by McGlynn and van Kampen (2008 Eur. J. Phys. 29 N63-N67). The authors’ misconceptions are recurrent points in the literature.
1. Introduction

Recently, Franklin [1] published a thought-provoking and curious paper on Lorentz contraction and related issues. While his intention was ‘to correct students’ misconceptions due to conflicting earlier treatments’, I believe that the paper could be confusing reading for the student. It is a hardly extricable and certainly challenging mixture of truths, half-truths and erroneous statements about some basic relativistic concepts and results.¹ Thus Franklin’s argument may sound correct to the unexperienced ear. The situation is aggravated by the circumstance that some of his contentions are in conformity with interpretations found in authoritative books on the subject.

In another recent paper, McGlynn and van Kampen [2] contend that the phenomenon of different values of charge densities in a current-carrying wire as measured by observers in different inertial frames, due to relativistic length contractions, is an effect ‘which perfectly demonstrates “the pole in a barn” paradox’ at room velocities. I think, however, that the authors are wrong here, namely, two distinct aspects of relativistic length contraction are exemplified in the two situations.

Neither paper is an exception. It is a notorious fact that understanding relativity is a painful, nay agonizing process.² We remind the reader of the old duel between Dingle and Born about the reciprocity of time dilation [5,6]

¹I have a vague feeling that perhaps just the challenge is the concealed essence of [1]. That is, as if Franklin’s true intention was to push to extremes some problematic points in special relativity and in this way to stir up the student to brood over them. Considered as a spur, Ref. [1] seems to be an excellent reading.

²With relativity, we enter the zone of evolution of the human race in a dramatic way. At a first step towards the conquest of relativistic mentality, one strikes the hard wall of everyday language. As Schrödinger pointed out ‘... everyday language is prejudicial in that it is so thoroughly imbued with the notion of time - you cannot use a verb (verbum, “the” word, Germ. Zeitwort) without using it in one or the other tense. ... [Special Relativity] ... meant the dethronement of time as a rigid tyrant imposed on us from outside, a liberation from the unbreakable rule of “before and after”’. [3] More recently, Mermin argued that ‘... to deal with relativity one must either critically reexamine ordinary language, or abandon it altogether.’ [4] The present author believes that alas it is impossible to abandon altogether the metalanguage of everyday speech. Physical meaning is unavoidably blurred by linguistic meaning and vice versa.

On the other hand, the word ‘agony’ need not necessarily have a painful connotation; choosing the title of this paper, I had also in mind its Greek sense (‘a contest for victory’).
which aroused a prolonged controversy in the Nature. A natural inference that the final outcomes of events must be the same with respect to two inertial observers cannot be generalized to two infinite continuous sequences of inertial observers, a lesson the present author has fully learned only very recently [7,8]. Even the meaning of the Lorentz contraction, which is generally accepted to be the simplest relativistic phenomenon, is hard to grasp and becomes the stumbling block in various contexts [9 - 11].

This is small wonder. Recall that, as the first physical consequence of the Lorentz transformations, the student learns that the length of a rod which is uniformly moving along itself with velocity $v$ is reduced by a factor $(1 - v^2/c^2)^{1/2}$ as compared to the rod’s length measured in its rest frame. While the phenomenon has been dubbed ‘relativistic length contraction’, the student is immediately warned ‘but of course nothing at all has happened to the rod itself’. However, the term ‘contraction’ connotes shrinking (‘cold contracts metals’), shrinking connotes change, and ‘change’ in physics involves some happening; what happens before and after is measured in one and the same reference frame. Thus, learning that the rod contracts, yet nothing has happened to it, the student strikes the hard wall of everyday language. Moreover, in the usual textbook presentations of special relativity it is explained that ‘... the different measures of length are intimately connected with the lack of absolute simultaneity’ [16], or, in the same vein, 

---

3In my opinion, Born won. Dingle has made the same kind of error the student usually makes: two different quantities are denoted by one and the same symbol and thus confused. (A compound event that takes place at various spatial points of an inertial frame $K$ (corresponding to the motion of a clock at constant velocity $v$), and has a duration of $1/\sqrt{(1 - v^2/c^2)}$ $K$-seconds, and a compound event that takes place at one spatial point of the same K-frame and has the same duration of $1/\sqrt{(1 - v^2/c^2)}$ $K$-seconds must not be identified; those events are two distinct straight lines in Minkowski space.)

4The most recent example of this (related to the fact that traps of language always lurk in special relativity) seems to be the recognition that what is a well defined wire segment in one inertial frame is no more a wire segment as measured in another frame in the framework of an elementary model of a current-carrying wire [12 - 14].

5A consequence of a similar relativistic terminological muddle was described some time ago in Adler’s paper ‘Does mass really depend on velocity, dad?’ [15]. It appears that in the meantime velocity-dependent mass disappeared from special relativity curricula, as it should.
‘... the contraction, when we observe it, is not a property of matter but something inherent in the measuring process’ [17].

In a recent paper [18], I pointed out that such interpretations confuse derivation of the phenomenon and its root, thus adding a conceptual problem to the terminological one. I argued that there is also a fundamental active aspect of relativistic length contraction: a rod initially at rest in an inertial frame, after a constant velocity \(v\) is imparted to it so that it moves freely and uniformly along itself, is contracted (its length is reduced), all with respect to that frame; the phenomenon is due to acceleration of the rod relative to that frame, and is described by the well-known formula, under the proviso that the acceleration was rest length-preserving in the final outcome. I inferred that without the active aspect of length contraction, i.e. without the rest length-preserving accelerations, there is no special relativity. The Lorentz transformation, even the formulation of the principle of relativity, is built on the active aspect. Thus, there is a dynamical content of the Lorentz transformation.

It appears that the authors of Refs. [1] and [2] were unaware of [18] or they chose to ignore it. Probably my argument was not perspicuous enough. In the present paper I will point out what I find to be weak points in Refs. [1] and [2], and elsewhere. That would perhaps sharpen my argument of [18] and, hopefully, save the student of relativity some time and effort.

2. The length of a moving rod revisited

In this section, for the convenience of the reader I will briefly summarize main conclusions of [18], slightly improving terminology.

Consider two inertial reference frames \(S\) and \(S'\) in standard configuration, \(S'\) is uniformly moving at speed \(v\) along the common positive \(x, x'\)-axes, and the \(y\)- and \(z\)-axis of \(S\) are parallel to the \(y'\)- and \(z'\)-axis of \(S'\), respectively.

---

6Professor Franklin has kindly informed me that he had not known of my paper [18], but he does not think ‘it would be more appropriate to refer to it than to any of a great number of papers written about the Lorentz transformation over a span of 120 years.’

7The ubiquitous worm of doubt reminds me of the possibility that my analysis of the length contraction phenomenon was wrong. However, brooding over the issue and brooding again, I was unable to find any flaws in my argument [18].
Take a solid rod parallel to the $x, x'$-axes, at a permanent rest with respect to the $S'$ frame, and let $l'_0$ be the length of the rod as measured in $S'$ by a given measuring rod also at rest in $S'$. What is the length of the rod as measured in $S$ employing the same measuring rod which is now at rest in $S$?

Following Einstein's prescription for ascertaining ‘the length of a uniformly moving rod ... in the ‘resting’ frame $S'$’ [19], by using the Lorentz transformation, one deduces in the well-known way that the length of our uniformly moving rod as measured in $S$, $l_v$, is given by

$$l_v = l'_0 \sqrt{1 - v^2/c^2}. \tag{1}$$

The phenomenon expressed by equation (1), that one and the same rod has different lengths $l_v$ and $l'_0$ as measured in the $S$ and $S'$ frames, respectively, where $S'$ is the rest frame of the rod, in what follows I will call the relativistic length reduction.\(^9\)

Now in the Relativity Paper [19], Einstein stated that if the same rod to be measured is at rest in $S$, then ‘according to the principle of relativity’ its length as measured in $S$, $l_0$, must be equal $l'_0$.\(^8\)

\(^8\)In [18], I called the prescription Einstein’s ‘very natural operational definition’ of the length of a uniformly moving rod. However, the term ‘definition’ can be misleading. It could imply that perhaps some other definition, leading to a different value of length of a uniformly moving rod, could be legitimately introduced. (As Dieks pointed out in his article “The ‘reality’ of the Lorentz contraction” the term ‘definition’ ‘... possesses the connotations of arbitrariness and conventionality.’) As far as I understand special relativity, the uniformly moving rod has no other length than the part of a (stationary) straight line taken up instantaneously by the rod, all relative to $S$.

\(^9\)Equation (1) is usually called ‘relativistic length contraction’ or ‘conventional length contraction’ [20]. The term ‘reduction’ seems to be more proper here, as being perhaps more neutral, than ‘contraction’ (which connotes shrinking, as was pointed out above). I am grateful to Professor Giuliano Boella for stimulating correspondence concerning this terminological point.

Of course, the content of equation (1) is not exhausted by the length of a moving rod, cf footnote 6 of [18]. It should be noted that the term ‘one and the same rod’ above has a peculiar, special relativistic meaning: it connotes that no action was exerted upon the rod by a mere different choice of inertial reference frame (observer), and yet the rod does not have the same length in the various frames (corresponding to various cross-sections of the world-strip of the rod, cf Appendix A). Again, everyday language is the problem in special relativity.
Equations (1) and (2) imply

\[ l_v = l_0 \sqrt{1 - v^2/c^2}. \]

Equation (3) relates the length \( l_0 \) of the rod at rest to its length \( l_v \) when it is in uniform motion along itself at the speed \( v \), all with respect to the inertial frame \( S \). The phenomenon described by equation (3) in what follows I will call the relativistic FitzGerald-Lorentz contraction, or shortly the FitzGerald-Lorentz contraction, as I did in [18]. Obviously, in this case there is a change of the rod relative to \( S \) (its length has changed); the change is due to acceleration of the rod from rest to the state of uniform motion.

---

10 One should make a clear distinction between the rest length Lorentz-invariance, which is a truism, and the rest length frame-independence, which is a fundamental physical assumption. Namely, if a rod is uniformly moving along itself with respect to an inertial frame, making a Lorentz boost to its rest frame \( S^* \), one can measure its rest length \( l_0^* \). The quantity \( l_0^* \) is a fortiori Lorentz invariant. This of course means, on the basis of equation (1), that \( l_0^* = l_{v_1}/\sqrt{1 - v_1^2/c^2} = l_{v_2}/\sqrt{1 - v_2^2/c^2}, \) etc, where \( v_1, v_2, \ldots \), are speeds of the rod relative to inertial frames \( S_1, S_2, \ldots \), respectively (all the frames being in standard configuration with \( S^* \)). On the other hand, Einstein’s assumption (2) has quite a different meaning, which was perhaps most clearly expressed by Resnick ([21], p 93): ‘The rest length of a rod is an absolute quantity, the same for all inertial observers: If a given rod is measured by different inertial observers by bringing the rod to rest in their respective frames, each will measure the same length.’ This rest length frame-independence could be also termed the absolute Lorentz-invariance of rest length.

11 While equation (3) is an obvious consequence of eqs. (1) and (2), one point should be stressed. Namely, according to Einstein, a rod at rest with its axis lying along the \( x \)-axis, having the length \( l_0 \), after ‘a uniform motion of parallel translation (with velocity \( v \)) along the \( x \)-axis ... is imparted to the rod’ will have the length \( l_v \) given by equation (3), all with respect to the same inertial frame \( S \) [19]. This is so regardless of the way the speed \( v \) was imparted to the rod (Einstein made no restrictions). Thus, according to Einstein, an arbitrary acceleration of an arbitrary rod, starting from rest, with the only proviso that the acceleration leads essentially to a uniform (unconstrained) motion of the rod along its length, in a persistent internal state, does not (eventually) change the rest length of the rod.

12 It should be pointed out that the FitzGerald-Lorentz contraction does not have the same content as the original idea put forth by FitzGerald and Lorentz long time ago, as will be explained in subsection 3.4 below.
This is contrary to the situation described by equation (1), in which case there is no change of the rod in the standard physicists’ sense of the word (involving alterations in the rod with time in one and the same inertial reference frame); only the frame $S$ world-map is substituted for the frame $S'$ world-map, without exerting any action upon the rod itself. It seems natural to call the content of eqs. (1) and (3) a passive and an active interpretation of relativistic length contraction, respectively.\footnote{One important point should be stressed here in relation to the passive interpretation of length contraction. Throughout the present article I insisted that no action was exerted upon the rod by a mere different choice of inertial reference frame (and that consequently there is no change of the rod in the standard physicists’ sense of the word). This is of course true. It should be noted, however, that even in the case of the relativistic length reduction there is a change of the rod in a certain physically reasonable sense. This important point, whose neglect makes the content of equation (1) hard to grasp, will be discussed in some detail in Appendix B. The essence is that something has happened to the rod even in the passive interpretation of length contraction.}

Now, there is a well-known tradition, originated by Einstein [19], to present relativistic length contraction as a purely kinematical effect. Thus, in the usual textbook presentations of special relativity, the active interpretation of length contraction is either neglected or introduced tacitly.\footnote{For example, after a brief discussion of the relativistic length reduction, equation (1), as a kind of a velocity perspective effect (‘but of course nothing at all has happened to the rod itself’), Rindler [22] stated that the phenomenon ‘is no “illusion”: it is real and, in principle, usable.’ However, giving an argument for the last statement, as is clear from the context, he was tacitly assuming equation (2), i. e. he had tacitly passed from the passive to the active interpretation of length contraction. Rindler is no exception. For some mysterious reason, the pride of place has been given to the passive interpretation by various authors including Einstein, Born, Pauli. The heuristic level of special relativity, ‘helping us to recognize a great miracle of the world’ [18], seems to be usually kept sub rosa.} However, without the active interpretation there is no special relativity as a physical theory which ‘asserts definite properties of real bodies’. This is clear from Einstein’s definition of two inertial frames in standard configuration (which conceptually precedes the formulation of the principle of (special) relativity and a derivation of the Lorentz transformation and which, as far as I am aware, cannot be replaced by another definition), and from a related Einstein’s assumption of ‘the boostability of rulers and clocks’ [23], made...
explicit by Born [24], cf footnotes 4 and 12 of [18].\textsuperscript{15} As was pointed out above, the active interpretation involves changes.

Thus, if there were no change in the (macroscopic) object in special relativity, then special relativity would not exist as a valid physical theory (it could not even be formulated). However, changes which appear in special relativity may have curious properties, requiring a thorough reexamination of everyday language. As was pointed out in [18], the FitzGerald-Lorentz contraction described by equation (3) possesses the following peculiarity: a free rod in uniform motion along its length is contracted (shrunk) with respect to the $S$ frame and yet it is perfectly relaxed (with no stress relative to both $S$ and $S'$ frames), the contraction being its natural state when it is in that state of motion (all this under the proviso that the rod was unstressed when initially it was at rest in $S$). Also, contrary to what was sometimes stated in the literature, the contraction is not due to the relative motion of a body; it is due to acceleration (or deceleration, in the reciprocal case of elongation) of the body relative to an inertial frame.

One last point. My key inference in [18] seems to be that a weaker assumption than Einstein’s original ‘boostability of rulers and clocks’ is sufficient for foundation of special relativity. The weaker assumption, which I called in [18] ‘the universal boostability assumption’, states that it is possible to boost a measuring rod or clock in a way which leaves their measuring capacity untouched.\textsuperscript{16} As far as I am aware, this implies that the rest length of a rod need not be preserved under arbitrary boosts. There is no guarantee of the absolute Lorentz-invariance of rest length.

3. Some weak points in Refs. [1] and [2]

3.1. There is no change in the object in special relativity

Franklin based his discussion of length contraction in special relativity on the following premise: In special relativity, there is no change in the object.

\textsuperscript{15}Current textbook literature seems to imply that two inertial frames in standard configuration can be introduced simply by fiat, which is in my opinion incorrect.

\textsuperscript{16}Einstein took it for granted that the measuring capacity of a measuring rod or clock would remain untouched under arbitrary boosts, cf footnote 12 of [18].
It is only the reference frame that is changed from $S$ to $S'$. Now since that premise runs as a common thread through various authoritative discussions of the topic, it perhaps deserves some clarification.

As a representative example, I choose the famous book *Einstein’s Theory of Relativity* by Max Born [24]. In a section under catchy title *Appearance and Reality*, Born pointed out that some opponents of special relativity assert that Einstein’s theory implies ‘... a violation of the causal law. For if one and the same measuring rod, as judged from the system $S$, has a different length according to its being at rest in $S$ or moving relative to $S$, then, so these people say, there must be a cause for this change. But Einstein’s theory gives no cause; rather it states that the contraction occurs by itself, that it is an accompanying circumstance of the fact of motion.’ Born defended special relativity by arguing that the opponents have ‘... a too limited view of the concept “change”’. He explained that ‘the standpoint of Einstein’s theory about the nature of the contraction is as follows: A material rod is physically not a spatial thing but a space-time configuration. Every point of the rod exists at this moment, at the next, and still at the next, and so on, at every moment of time. The adequate picture of the rod under consideration (one-dimensional in space) is thus not a section of the $x$-axis but rather a strip of the $x, ct$-plane [parallel to the $ct$-axis] ... The “contraction” does not affect the strip at all but rather a section cut out of the [corresponding] $x$-axis. It is, however, only the strip as a manifold of world points (events) which has physical reality, and not the cross-section. Thus the contraction is only a consequence of our way of regarding things and is not a change of physical reality. Hence it does not come within the scope of the concepts of cause and effect.’

It is clear that Franklin’s premise concurs with Born’s explanation: there is no change in the object in special relativity. It is also clear that the authors would be right if their arguments referred only to the relativistic length reduction, described by equation (1). However, if equation (1) were the whole contents of length contraction, i.e. if there were no change in the object in special relativity, then special relativity would not exist as a
physical theory, as is pointed out in the preceding section (cf also [18]).

To do justice to Born, it should be noted that in the first part of section Appearance and Reality of [24], introducing his ‘principle of the physical identity of the units of measure’, he essentially argued for a change in the object in special relativity (namely, that equation (1) and assumption (2) imply the physical validity of equation (3)).

Unfortunately, in the sequel he confused the (relativistic) FitzGerald-Lorentz contraction (where change is obvious) with the relativistic length reduction (where there is no change in the usual physicists’ sense of the word), ascribing properties of the second phenomenon to the first one. Thus his defense of special relativity failed.

3.2. There is only one length: the ‘rest frame length’

As was pointed out above, Franklin’s premise in [1] is that there is no change in the object in special relativity. However, since properties like length undergo changes, the author cut the Gordian knot as follows: the Einstein length of a moving object is not a physical attribute of the object! Only its ‘rest frame length’ is a physically reasonable attribute - length of the object.

Moreover, discussing the relativistic length reduction, equation (1), as a velocity perspective effect, he inferred that ‘the “shortening” of a stick that is rotated in four dimensions by a Lorentz transformation is ... illusory.’

---

17 This point will be discussed in some detail in Appendix A.
18 In Appearance and Reality, Born switched several times, tacitly and obviously unconsciously, between the active and the passive interpretation of length contraction, using the same term ‘contraction’ for both phenomena, and confusing their meanings. This section of Born’s book ([24], pp 251-62) is perhaps a perfect example of how unavoidably terminological confusion leads to conceptual confusion. This confusion seems to be commonplace in the literature. Thus Pauli in his discussion of the Lorentz contraction [25] confused eqs. (1) and (3), obviously assuming tacitly equation (2). While dealing only with the relativistic length reduction, he ended the discussion with the following query which clearly refers to the active aspect of the phenomenon: ‘Should one then ... completely abandon any attempt to explain the Lorentz contraction atomistically? We think that the answer to this question should be No.’ ([25], pp 11-15) Needless to say, it would be much simpler to explain equation (3) atomistically, than equation (1).
19 It seems that Franklin thus introduced a novel and radical interpretation of special relativity which, I think, can be summarized by the following manifesto: If some consequences of special relativity are surprising and hard to understand, they should be proclaimed devoid of physical sense.
However, stating that the relativistic length reduction is an illusion would represent a falsification of special relativity.

Now one of Franklin’s starting assertions, that ‘the measured length of a moving object depends on the “particular way” in which it is measured’, is perfectly correct. Indeed, one and the same moving object may have various measured lengths, depending on which definition (i. e. which procedure of measurement) of the length of a moving object is being used, all with respect to one and the same reference frame. However, for some reason Franklin ignored the fundamental fact that according to Einstein’s special relativity the moving object has only one length in the (stationary) frame $S$, that obeying equation (1); it is the only physical reality (world-map) for the $S$-observer. A photograph of a (small) moving object would indeed be identical to a photograph of an object that is somewhat rotated, but of the same shape and dimensions as compared with the moving object in its rest frame, under the proviso that the rotated object is at rest, as Franklin recalled. However, as is well known, that inference is reached assuming special relativity which means, inter alia, that the moving object has only one length with respect to $S$, i. e. that the relativistic length reduction had taken place (cf [17], pp 150-2, [16], pp 163-8, [26], [27]). What Franklin characterizes as ‘the belief that a moving object has a different length’, is the only physical reality for the $S$-observer; the ‘belief’ is obviously built into the standard Lorentz transformations as $x'\sqrt{1-v^2/c^2} = x- vt$, i. e. in the more familiar form

\[ x'\sqrt{1-v^2/c^2} = x- vt, \]

Recall that at the end of his paper [28] Terrell stressed that none of his statements there ‘should be construed as casting any doubt on either the observability or the reality of the Lorentz contraction [i. e. the relativistic length reduction], as all the results given are derived from the special theory of relativity.’ It is perhaps worthwhile to mention here that, analyzing in 1905 how the shape of a body depends on the reference frame in which it is measured, Einstein occasionally used the verb ‘betrachten’. This German verb has two meanings: first, to observe, to see, and second, to consider, depending on the context. Various English translators of the Relativity Paper seem to be unanimous that Einstein used ‘betrachten’ in the first sense. (The present author shares this point of view.) Wind hindsight, we know today that Einstein (and translators) should have been using to consider, or perhaps better to measure (‘messen’, in German), instead of to observe, to view. (Here of course to measure is used in the sense of Einstein’s ‘operational definition’.) The moral of the story seems to have been known to Democritus: things are not found where their picture is.
\[ x' = \gamma_v (x - vt). \]

On the other hand, it is clear that Franklin in \[1\] does assume the validity of the Lorentz transformations. Thus his argument appears to be self-contradictory. Contrary to Franklin’s statement, one and the same moving rod has infinitely many lengths in infinitely many inertial frames in the standard configuration with its rest frame \(S'\), respectively. (Note, however, that the different lengths are, in a certain sense, due to changes of the rod, cf Appendix B.) According to special relativity, none of the lengths is less or more physically real than the rod’s rest length \(l_0'\); each inertial observer possesses her or his perfectly legitimate physical reality.\(^{21}\)

Another Franklin’s basic assertion is that two different inertial frames are required ‘in order to compare the measured length of a moving object to its measured length in a system in which it is not moving’. This is of course true in the case of the relativistic length reduction, described by equation (1). However, if that were the whole content of special relativity, then it would not exist as a physical theory. As was pointed out in \[18\] and also above, the foundation of special relativity requires the rest length-preserving accelerations (and also, more generally, it requires the rest properties-preserving accelerations (cf \[29, 30\])). In the case of such gentle accelerations (which are sine qua non for special relativity) one inertial frame would be enough for a comparison of the two lengths.

3.3. Who contends stresses can be induced by Lorentz contraction

\(^{21}\)It could be perhaps somewhat misleading to state, as French does, that Einstein’s length of a moving rod in the stationary frame \(S\), obeying equation (1), ‘does refer to measurements of a particular kind ...’ ([17], p 152). Einstein’s ‘operational definition’ is not so much a measurement of a particular kind but rather a perfectly classical explanation of what is the length of a moving rod (regardless of how it moves), whose only peculiarity is that one instant of the \(S\)-time should be understood according to special relativity. Namely, the Einstein length is the length of a segment of a stationary line taken up instantaneously by the moving rod, all with respect to \(S\). What else on earth could be the length of a moving object? Measuring that stationary length would hardly be a measurement of a particular kind. However, to ascertain the stationary line segment would require, e. g., taking a photograph of the moving rod and of course a clever interpretation of the photograph [27].
Franklin stated in [1] that Lorentz contraction (by which he obviously meant the relativistic length reduction, equation (1)) could not induce strains and stresses. He illustrated this with a simple example of a brittle wine glass at rest on a table, pointing out that moving past the wine glass at constant velocity (and looking at it) could not shatter the wine glass. This is of course true: an object at permanent rest and perfectly relaxed in the $S'$ frame, is perfectly relaxed also relative to the $S$ frame (no action was exerted upon the object in the change of the inertial reference frame from $S'$ to $S$).\[22\]

Now Franklin also stated that Refs. [31-34] contended that stresses and strains could be induced by Lorentz contraction. However, as far as I can see, there is no hint of such a contention in FitzGerald’s five-sentence letter to Science, where he had suggested a hypothesis that ‘the length of material bodies changes, according as they are moving through the ether or across it, by an amount depending on the square of the ratio of their velocity to that of light’ ([31], cf also [35]).\[23\] Also, despite appearances, no such contention is the essence of Refs. [33], [34]. Namely, Bell clearly stated that ‘... the artificial prevention of the natural contraction imposes intolerable stress’ [34], where by ‘the natural contraction’ he obviously meant the relativistic FitzGerald-Lorentz contraction, described by equation (3). (Nowhere in [34] Bell stated that ‘the natural contraction’ itself induces stresses.) On the other hand, it is true that Dewan and Beran [33] described their Gedankenexperiment as a demonstration ‘that relativistic contraction can introduce stress effects in a moving body’. However, the authors were somewhat sloppy in their wording\[24\], as is often the case in discussions dealing with special relativ-

\[22\] Moreover, as was pointed out in [18], with the rest length-preserving accelerations the object is perfectly relaxed relative to $S$, while being contracted (shrunk) relative to $S$.

\[23\] In my opinion, no distortion or deformation, and thus no stresses, are implied in the following FitzGerald’s sentence in [31]: ‘We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently.’ (It seems to me that the only way of finding stresses in the FitzGerald sentence would be to read Lorentz’s ideas into it.) However, I agree with Brown ([23], p 51) that the FitzGerald supposition was prompted by Heaviside’s result for the electromagnetic field of a point charge in uniform motion relative to the ether.

\[24\] For example, their formulation that the Lorentz transformation ‘... implies that a fast
ity; I think it is clear from the contents of [33] that their intended meaning is perfectly summarized by the above quotation from Bell. Thus it seems that only Lorentz ([36], pp 5-7, 21-23, 27-28) spoke explicitly about deformation (and thus about stresses) of a body in connection with his ‘by no means far-fetched’ hypothesis that if to a system $\Sigma'$ of particles in the equilibrium configuration, at rest relative to the ether, ‘the velocity $v = v\hat{x}$ is imparted, it will of itself change into the system $\Sigma$ [which is got from $\Sigma'$ by the deformation $(\frac{1}{\beta l}, \frac{1}{l}, \frac{1}{l})$, where $\beta = (1 - v^2/c^2)^{-1/2}$ and $l$ is a numerical factor allowing for a change in the $y$ and $z$ directions]. In other terms, the translation will produce the deformation $(\frac{1}{\beta l}, \frac{1}{l}, \frac{1}{l})$.^{25} (Note that Lorentz subsequently showed that $l = 1$ ([36], p 27).)

3.4. The original FitzGerald-Lorentz contraction and its relativistic counterpart

Franklin’s starting statement in [1] is that ‘Lorentz contraction [introduced by FitzGerald and Lorentz] is not what actually occurs for a moving body in special relativity’. While that statement is certainly correct, it seems that the author in the sequel ignored the fact that there is a perfectly legitimate FitzGerald-Lorentz contraction in special relativity. This point perhaps needs some clarification.

As is well known, FitzGerald and Lorentz introduced the contraction (shrinking) of bodies in motion relative to the ether. Thus a rod at rest moving object contracts in the direction of its velocity’ is in the general case incorrect. Namely, the relativistic length reduction involves no change in the object, as was pointed out above. Similarly, Bell [34] spoke about ‘systematic distortion of the field of fast particles’ (italics added by D. V. R.) as compared with the spherically symmetrical Coulomb field of a charge at rest. However, I am convinced it is simply a bad wording.

^{25}Perhaps the Lorentz formulation that bodies ‘have their dimensions changed by the effect of translation’ prompted Minkowski to characterize the hypothesis as sounding ‘extremely fantastical, for the contraction ... [is to be looked upon] simply as a gift from above, - as an accompanying circumstance of the circumstance of the motion.’ ([36], p 81). With the benefit of hindsight, and taking the liberty of rectifying FitzGerald and Lorentz, I believe that both eminent physicists were victims of the traps of ordinary language: concerning their statement that bodies are changed by their translational motion relative to the ether, I think that their intended meaning was that bodies are changed by their acceleration relative to the ether from rest until reaching a steady velocity.
on the earth may be contracted, depending on the direction of its motion through the ether, as compared (measured) with the same rod at rest in the ether. However, in the world-map of the FitzGerald and Lorentz, a rod at rest in the ether is not contracted in comparison with an identical rod which is brought to rest relative to the earth; rather, the former rod may be elongated as compared with the latter. In this sense, the contraction introduced by FitzGerald and Lorentz is absolute, there is no reciprocity in it.26 As Franklin pointed out, applying this originally introduced FitzGerald-Lorentz contraction to a variant of the Michelson-Morley apparatus would lead to a positive result (cf [37], p 236).27

Now a clear distinction should be made between the original FitzGerald-Lorentz contraction and its relativistic counterpart (which is also called - and justly so - the FitzGerald-Lorentz contraction). Namely, what the two conceptions have in common is shrinking (which I think is due to acceleration); however, contrary to the former, the latter actually occurs for an object to which a constant velocity is imparted in any inertial frame (under the proviso of the rest length-preserving accelerations). (Recall that there is no ether frame in special relativity simply by virtue of Ockham’s razor (for an interesting argument cf [38]).28)

3.5. The Bell spaceship paradox

Section 2 entitled ‘The Bell spaceship paradox’ seems to be the most mischievous part of [1]. While Franklin asserts that he presents ‘the nexus of the Bell spaceship paradox as originally presented by John Bell’ [34], actually this is not so. Namely, Bell took into account the Evett and Wangsness correction of the original Dewan-Beran formulation of the problem [41]. Thus, instead of connecting the tail of the front spaceship (R) and the nose of

---

26This is contrary to the relativistic length reduction, which is reciprocal. Note, however, that the relativistic FitzGerald-Lorentz contraction, which refers to one and the same reference frame, is not reciprocal too.

27Note, however, that that conclusion is based on the premise that the velocity of light is the same in all directions only in the ether frame, contrary to Franklin’s assertion.

28By the way, we remind the reader that the ether played a rather subtle part for the fin de siècle electrodynamicists (cf [39], and [40], pp 176-7).
the back spaceship (L) as is supposed in [1], [33], in the Bell formulation a thread connects the corresponding points of ships. Moreover, Franklin’s analysis of his version of Dewan-Beran-Bell’s problem (which obviously refers to the ‘tough’ variant of the problem) is basically incorrect: contrary to Franklin’s repeated statements, there is no common rest frame $S'$ for both ships (‘even for continually accelerated spaceships’), as is clear from the corresponding Minkowski diagram. (Events that are simultaneous in the $S$ frame are not simultaneous with respect to any other frame, and vice versa.) Consequently, there is no rest frame distance between ships (there is no frame in which both ships are simultaneously at rest, except of course the $S$ frame at $t = 0$) and equations (2)-(6) in [1] are meaningless, for continually accelerated ships. (They are not incorrect, they are meaningless, since there is no the $S'$ frame.)

Eventually, Franklin’s resolution of the Bell spaceship paradox ‘as no paradox’ is hard to fathom. It is of course true that special relativity allows no difference in any measurement of two equal lengths such as the distance

---

29 This point is specially clear in the ‘mild’ variant of the problem, in which at an instant of the $S$ time the ships’ acceleration ceases and they coast with the same constant velocity, as measured in the $S$ frame [11]. Namely, assuming the rest length-preserving accelerations of ships in the final outcome, ships will eventually FitzGerald-Lorentz contract according to equation (3), and thus the final distance between the tail of R and the nose of L will be greater than their initial distance, all with respect to $S$. It is perhaps worthwhile here to clarify the standard assumption that the thread connecting ships in no way affects the motion of ships. Namely, this does not mean that the thread does not affect ships (it does, cf [42]); instead that means that the work programmes of the ships’ motors are being constantly re-adjusted so as to provide ships having identical accelerations with respect to $S$.

30 This means that ships have identical accelerations $a(t)$ in the positive $x$-direction starting simultaneously from rest, all with respect to $S$; in the general case, the ships’ acceleration never ceases.

31 Thus Franklin’s contention in section 1 of [1] that ‘it is only the rest frame length of an object that relates to strains and stresses on the object’ is in the general case wrong.

32 The correct distances between the corresponding points of ships R and L, when the points are performing identical hyperbolic motions relative to $S$, as measured in instantaneous rest frames of R and L at the same instant of their proper time $\tau$ are given, e.g., in [8]. Note that one and the same inertial frame first becomes the instantaneous rest frame of R and subsequently (with respect to that frame) it becomes the instantaneous rest frame of L [8].
between ships and the length of the thread between them. However, the
two distances are not of the same sort in the following sense. Consider,
for simplicity, the ‘mild’ variant of the problem when all transient effects
have died out and a steady velocity of ships is reached in $S$. Then to the
former distance the relativistic length reduction applies, whereas to the latter
(the length of the thread) both the length reduction and a stretching above
the natural relativistic FitzGerald-Lorentz contraction of the thread apply
(under the proviso that the thread remained unbroken and under the proviso
of course that special relativity is valid).33 While our Galilean instincts
would expect the thread never breaks in $S$ (why should it?), it must break
at a sufficiently high speed if special relativity is valid, and this is the core
of the paradox.

3.6. Rigid body motion in special relativity

Franklin starts the last section of [1] entitled ‘Rigid body motion in spe-
cial relativity’ by pointing out that in the motion described by Bell and by
himself, the acceleration of each spaceship is the same at equal times in $S$.
He then contends ‘this also corresponds to each [spaceship] having the same
acceleration $a'$ in their [mutual] instantaneous rest system ... if their rest

33It should be pointed out that the conclusion ‘a stretching above the natural FitzGerald-
Lorentz contraction of the thread applies’ in the mild variant of the problem is based on
the tacit assumption that releasing the thread ends from ships in the final rest frame $S'$
would lead to the thread’s shrinking in $S'$ to its initial rest length (the length it had
in $S$ before accelerations started), or in other words that the thread is perfectly elastic.
Without that simplifying assumption the analysis of Dewan-Beran-Bell’s problem becomes
tricky in $S$. Thus, without that simplifying assumption Bell’s resolution of the paradox
in $S$ is oversimplified. However, at a sufficiently high speed the thread would certainly
break regardless of its elasticity since, according to special relativity, its length in $S'$
would tend to infinity when $v \to c$. The same conclusion is reached in the $S$ frame, taking
into account that the thread’s natural (FitzGerald-Lorentz contracted) length when it is in
uniform motion would tend to zero when $v \to c$. (The appearance of [11] stimulated heated
discussions on some internet physics forums and several published [7,8,43] and unpublished
papers on the topic. An anonymous Russian author, a philosopher by profession, remarked
that ‘it would not be a big harm if a philosopher added into the barrel of professional
physicists’ honey a teaspoon of philosophical tar’. The present footnote is prompted by
the author’s comment that special relativity alone does not imply the thread would break
due to ‘the artificial prevention of the natural contraction.’)
system acceleration is constant in time.’ However, as was noted above, there is no common instantaneous rest frame for both ships; instead, ships’ accelerations are constant in their respective instantaneous rest frames. In the same way, Franklin’s next argument that ‘from the preceding paragraph we see that keeping lengths constant in the rest system requires different rest frame accelerations for different parts of a rigid body’ is inconclusive: there is no mutual instantaneous rest frame in Dewan-Beran-Bell’s problem.

It should be stressed that Franklin’s subsequent analysis of the motion of ships in the case of their constant but different rest frame accelerations, so as to keep the distance between ships constant in their mutual rest system, is exact and instructive. There is only one terminological point where I disagree with Franklin. Namely, his formulation ‘rigid body motion’, which reflects the concept originally introduced by Born [44], should be replaced by a more proper formulation ‘rigidly moving body’ (cf [25], pp 130-2, [40]).

3.7. Linking electrical current and the pole in a barn paradox

In a recent paper, McGlynn and van Kampen contend that the phenomenon of alterations in charge densities in a current-carrying wire as measured by different inertial observers ‘perfectly demonstrates “the pole in a barn” paradox’ [2]. However, this is wrong; the two phenomena exemplify two distinct aspects of relativistic length contraction, namely, the length reduction and the FitzGerald-Lorentz contraction, respectively. Since the confusion appears to be a recurrent point in various contexts [9, 10], and taking into account that it is closely connected with our preceding considerations, it is perhaps worthwhile to briefly discuss McGlynn and van Kampen’s contention.

The standard textbook derivations of the magnetic force that acts on a moving charge $q$ via special relativity consider the case of an infinite straight wire at rest in the ‘laboratory’ frame ([17], [45, 46]). The wire is modelled as consisting of two superposed lines of charge: one moving (that of free electrons moving at drift speed $v_d$) and the other, which has an equal but opposite charge density, at rest (that of fixed positive ions). Thus, the wire is taken to be electrically neutral in the laboratory frame ($S$), which implies that the distance between adjacent ions equals the (mean) distance between adjacent
electrons in \( S \). Then by applying the proper relativistic length reduction formulae (\textit{mutatis mutandis} in equation (1)) to those distances, the corresponding charge densities in the rest frame of the moving charge \( q \) are found. Eventually, following the well known relativistic path, making (tacitly) use of the happy circumstance that the Lorentz force is a pure relativistic force\footnote{This implies, \textit{inter alia}, that the Lorentz force transforms in the same way as \( \frac{d}{dt}(mu \gamma_u) \), where \( m \) is a \textit{time-independent Lorentz scalar}, and \( u \) is the instantaneous velocity of a particle.} ([22], [16], [47], cf also [48], p 129, [49]), the desired result for the magnetic force is obtained. Note that in the above scene-setting no contractions are involved in the \( S \) frame.\footnote{As Zapolsky [9] pointed out, Feynman \textit{et al} [45] and Purcell [46] introduce electrical neutrality of the current-carrying wire in the laboratory frame \( S \) by \textit{fiat}. On the other hand, Zapolsky offered an explanation for the neutrality arguing that a steady current is established in a conducting wire (which is electrically neutral in \( S \) when no current flows in it) turning on a constant electric field oriented along the wire \textit{simultaneously} (relative to \( S \)) at all points of the \textit{infinite} wire, etc. That explanation seems to be implicit in a related quotation from French ([17], p 259):

'It is important to note that \textit{no} contractions are involved from the standpoint of the laboratory frame, but only from the standpoint of a frame moving relative to the laboratory. The only difference between a wire carrying a current and a wire not carrying a current is the existence of a drift velocity for the electrons. The mean distance between the electrons remains unaffected as measured in the laboratory frame.'

In the present paper, I refrain from entering into tricky problems of how is a current established in a conducting wire, and in what frame is a current-carrying conductor neutral. For purposes of teaching the relationship between electricity and magnetism via special relativity, it seems reasonable to introduce the neutrality of the wire in \( S \) by \textit{fiat}. Thus \textit{a fortiori} no contractions are involved in the \( S \) frame.} The same scene was used in [2], except for the fact that McGlynn and van Kampen confined their attention to a segment of the wire in \( S \), which is irrelevant for the present discussion. (Note also that no contractions are involved (the thread apart) concerning the \textit{distance} between the corresponding points of ships in Dewan-Beran-Bell’s problem \textit{in the \( S \) frame}.)

The situation is different in the pole in a barn problem ([50], cf also [22]). Namely, a pole vaulter (who, according to Dewan’s original formulation, lives in ‘Tomkins’ Wonderland’ where the speed of light is low) must speed up his pole from rest in a rest length-preserving way, so that the FitzGerald-Lorentz
contraction formula (3) applies. In this case obviously there is contraction in the $S$ frame (which is now the barn frame). Thus, there is a basic distinction between the two phenomena described above, contrary to McGlynn and van Kampen’s claim in [2].

In more detail, the FitzGerald-Lorentz contraction (and thus also the corresponding preparatory stage in which the pole acquires its motion relative to the barn) is essential in the pole in a barn problem; the contraction makes it possible that the pole in motion enters (momentarily) the barn (while this was impossible when the pole was at rest with respect to the barn). Consequently, a change of the pole with respect to the (inertial) barn frame is essential, and thus the active aspect of length contraction, expressed by my eq. (3), is essential in the pole in a barn problem.

On the other hand, we have another story in the phenomenon of alterations in charge densities in a current carrying wire as measured by different inertial observers. First, there is no contraction wrt the wire (laboratory) frame, in the case of the steady state assumed by McGlynn and van Kampen [2], Zapolsky [9], French [17], Feynman et al [45], Purcell [46]. Second, what is essential for the phenomenon is the assumed steady state (the wire carrying the steady current is electrically neutral in the wire frame); the preparatory stage (starting from electrically neutral wire with no current) is irrelevant for the phenomenon. Third, the alterations in charge densities are found using the relativistic length reduction formula, my eq. (1). Therefore, the passive aspect of length contraction is exemplified in the phenomenon.

**Acknowledgments**

I have benefitted a great deal from stimulating and cordial correspondence with Brian Coleman, Giuliano Boella, Vladimir Hnizdo, Jerrold Franklin, Dmitry Peregoudov, Paul van Kampen and Jaykov Foukzon.

**Appendix A**

Assume a rod of unit length at rest in $S$, lying along the $x$-axis, taking up the segment between the origin and the point $x = 1$m. The adequate picture of the rod (one-dimensional in space) is a strip of the $x, ct$-plane, bounded
by the \(ct\)-axis \((x = 0)\) and the line \(x = 1\)m parallel to it. It is the strip as a manifold of world points which has \textit{objective reality}. At various instants of the \(S\)-time, the rod is represented by cross-sections of the strip parallel to the \(x\)-axis. In the \(S'\) frame, however, the same rod is represented by cross-sections of the strip parallel to the corresponding \(x'\)-axis, at various instants of the \(S'\)-time; the length of the rod is \(\sqrt{1 - v^2/c^2}\)m', as measured in \(S'\). (Recall that the lesser \(S'\)-length is a \textit{longer} line segment than the \(S\)-length on the corresponding Minkowski diagram, due to well-known properties of space calibration hyperbola \(x^2 - c^2t^2 = 1\).) Thus to one and the same objective reality (the strip) correspond various \textit{physical realities} (cross-sections of the strip parallel to the corresponding spatial axes), being the world-maps of the \textit{same} rod in various reference frames. In this sense, each inertial frame has its own physical reality.

Assume now that the velocity \(v = v\hat{x}\) is imparted to the rod so that it moves uniformly along its length (the \(x\)-axis) with respect to \(S\), and assume also that the acceleration was a rest length-preserving one. (This assumption is contained in Born’s ‘principle of the physical identity of the units of measure’.) In this case, the corresponding objective reality of the rod is depicted by a strip of the \(x, ct\)-plane inclining to the \(ct\)-axis, bounded by the \(ct'\)-axis \((x' = 0)\), and the line \(x' = 1\)m' parallel to it. Cross-sections of the inclined strip parallel to the \(x\)-axis are physical reality for the \(S\)-observer, their length being of course \(\sqrt{1 - v^2/c^2}\)m, whereas cross-sections parallel to the \(x'\)-axis are physical reality for the \(S'\)-observer, their length being \(1\)m' (cf footnote 8 of [18]).

It is clear that there is a change in the object due to acceleration with respect to the \(S\)-frame (or, equivalently, due to deceleration with respect to the \(S'\)-frame): objective reality (the strip) has changed; this is so, of course, for all inertial observers. (However, despite the physical change has happened, the object is still one and the same object in the sense that it is still a bound configuration consisting of the same material points.)

In the above argument, Born’s term ‘physical reality’ is replaced by ‘objective reality’; on the other hand, I used ‘physical reality’ of an inertial
observer as a synonym for Rindler’s world-map (cf [22], and also [18]). Note
that, despite appearances, my term ‘objective reality’ does not necessarily
imply a reality which would be independent of the realm of our perceptions.
Note also that my argument is in accord with that presented by Minkowski
in his famous address ‘Space and Time’ more than a hundred years ago ([36],
pp 74-91).

Appendix B

Even in the case of the relativistic length reduction, there is a change of
the rod in the following, physically reasonable sense. Namely, a change of
inertial frame from $S'$ to $S$, entails essentially the following procedure: it
entails accelerating a reference frame which is an exact copy of $S'$, that
was initially at rest with respect to $S'$, until reaching a steady velocity with
respect to the inertial $S'$, and this in a rest properties-preserving way. During
the acceleration, there is an inertial force acting on the rod with respect to
the accelerated frame; acceleration of the rod with respect to the accelerated
(non-inertial) frame is the cause of the rod’s shortening with respect to the
(eventually again inertial) frame $S$. This explains physically different lengths
that appear in the relativistic length reduction formula (1).

As far as I know, this important point was not emphasized in the litera-
ture.
References

[1] Franklin J 2010 Lorentz contraction, Bell’s spaceships and rigid body motion in special relativity Eur. J. Phys. 31 291-8
[2] McGlynn E and van Kampen P 2008 A note on linking electrical current, magnetic fields, charges and the pole in a barn paradox in special relativity Eur. J. Phys. 29 N63-N67
[3] Schrödinger E 1977 What is Life & Mind and Matter (Cambridge: Cambridge UP) pp 158-61
[4] Mermin N D 1999 Writing Physics
   online at http://www.lassp.cornell.edu/~cew2/KnightLecture.html
[5] Dingle H 1962 Special theory of relativity Nature 195 985-6
[6] Born M 1963 Special theory of relativity Nature 197 1287
[7] Peregoulov D V 2009 Comment on ‘Note on Dewan-Beran-Bell’s spaceship problem’ Eur. J. Phys. 30 L3-L5
[8] Redžić D V 2009 Reply to ‘Comment on “Note on Dewan-Beran-Bell’s spaceship problem”’ Eur. J. Phys. 30 L7-L9
[9] Zapolsky H S 1988 On electric fields produced by steady currents Am. J. Phys. 56 1137-41
[10] Cavalleri G and Tonni E 2000 Comment on “Čerenkov effect and the Lorentz contraction” Phys. Rev. A 61 026101-1-2
[11] Redžić D V 2008 Note on Dewan-Beran-Bell’s spaceship problem Eur. J. Phys. 29 N11-N19
[12] van Kampen P 2008 Lorentz contraction and current-carrying wires Eur. J. Phys. 29 879-83
[13] Redžić D V 2010 Comment on ‘Lorentz contraction and current-carrying wires’ Eur. J. Phys. 31 L25-L27
[14] van Kampen P 2010 Reply to ‘Comment on “Lorentz contraction and current-carrying wires”’ Eur. J. Phys. 31 L29-L30
[15] Adler C G 1987 Does mass really depend on velocity, dad? Am. J. Phys. 55 739-43
[16] Rosser W G V 1964 An Introduction to the Theory of Relativity (London: Butterworths)
[17] French A P 1968 *Special Relativity* (London: Nelson)
[18] Redžić D V 2008 Towards disentangling the meaning of relativistic length contraction *Eur. J. Phys.* **29** 191-201
[19] Einstein A 1905 Zur Elektrodynamik bewegter Körper *Ann. Phys.*, *Lpz.* **17** 891-921
[20] Styer D F 2007 How do two moving clocks fall out of sync? A tale of trucks, threads, and twins *Am. J. Phys.* **75** 805-14
[21] Resnick R 1968 *Introduction to Special Relativity* (New York: Wiley)
[22] Rindler W 1991 *Introduction to Special Relativity* 2nd edn (Oxford: Clarendon)
[23] Brown H R 2005 *Physical Relativity: Space-time Structure from a Dynamical Perspective* (Oxford: Clarendon)
[24] Born M 1965 *Einstein’s Theory of Relativity* (New York: Dover) (revised edition prepared in collaboration with Günther Leibfried and Walter Biem)
[25] Pauli W 1958 *Theory of Relativity* (London: Pergamon) (reprinted 1981 transl. G Field (New York: Dover))
[26] Gamow G 1961 Remarks on Lorentz contraction *Proc. Natl. Acad. Sci.* **47** 728-9
[27] Kraus U 2008 First-person visualizations of the special and general theory of relativity *Eur. J. Phys.* **29** 1-13
[28] Terrell J 1959 Invisibility of the Lorentz contraction *Phys. Rev.* **116** 1041-5
[29] Redžić D V 2005 Momentum conservation and Einstein’s 1905 Gedankenexperiment *Eur. J. Phys.* **26** 991-7
[30] Redžić D V 2006 Does $\Delta m = \Delta E_{\text{rest}}/c^2$? *Eur. J. Phys.* **27** 147-57
[31] FitzGerald G F 1889 The ether and the earth’s atmosphere *Science* **13** 390
[32] Lorentz H A 1892 De relatieve beweging van de aarde en den aether *Versl. Kon. Akad. Wetensch* **1** 74-9 (reprinted in translation: Lorentz H A 1937 The relative motion of the earth and the ether, in *Collected Papers* Vol 4 (The Hague: Nijhoff) pp. 219-223.)
[33] Dewan E and Beran M 1959 Note on stress effects due to relativistic
contraction *Am. J. Phys.* 27 517-8

[34] Bell J S 1976 How to teach special relativity *Prog. Sci. Cult.* 1 (2) 1-13 (reprinted in Bell J S 1987 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge UP) pp 67-80)

[35] Brush S G 1967 Note on the history of the FitzGerald-Lorentz contraction *Isis* 54 230-2

[36] Lorentz H A, Einstein A, Minkowski H and Weyl H 1952 *The Principle of Relativity* (New York: Dover)

[37] Panofsky W K H and Phillips M 1955 *Classical Electricity and Magnetism* 1st edn (Cambridge, MA: Addison-Wesley)

[38] Mirabelli A 1985 The ether just fades away *Am. J. Phys.* 53 493-4

[39] Torretti R 2006 Can science advance effectively through philosophical criticism and reflection? online at http://philsci-archive.pitt.edu/archive/00002875/

[40] Miller A I 1981 *Albert Einstein’s Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905-1911)* (Reading, MA: Addison-Wesley)

[41] Evett A A and Wangsness R K 1960 Note on the separation of relativistically moving rockets *Am. J. Phys.* 28 566

[42] Cornwell D T 2005 Forces due to contraction on a cord spanning between two spaceships *Europhys. Lett.* 71 699-704

[43] Podosenov S A, Foukzon J and Potapov A A 2009 J. Bell’s problem and research of the electronic clots in linear collider *Nonlinear World* 7 612-21 (in Russian)

[44] Born M 1909 Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips *Ann. Phys., Lpz.* 30 1-56

[45] Feynman R P, Leighton R B and Sands M 1975 *The Feynman Lectures on Physics* Vol 2 (Reading, MA: Addison-Wesley) Sec 13-6

[46] Purcell E M 1985 *Electricity and Magnetism* 2nd edn (New York: McGraw-Hill) pp 190, 192-9

[47] Jefimenko O D 1996 Derivation of relativistic force transformation equations from Lorentz force law *Am. J. Phys.* 64 618-20
[48] Redžić D V 2002 Electromagnetism of rotating conductors revisited *Eur. J. Phys.* **23** 127-134

[49] Redžić D V 2006 *Recurrent Topics in Special Relativity: Seven Essays on the Electrodynamics of Moving Bodies* (Belgrade: authorial edition) pp 1-2, 11-12, 36

[50] Dewan E M 1963 Stress effects due to Lorentz contraction *Am. J. Phys.* **31** 383-6