Abstract

The $\pi_0$ neutral meson pole mass is calculated in a strongly magnetized medium using the SU(2) Nambu-Jona-Lasinio model within the random phase approximation (RPA) at zero temperature and zero baryonic density. We employ a magnetic field dependent coupling, $G(eB)$, fitted to reproduce lattice QCD results for the quark condensates. Divergent quantities are handled with a magnetic field independent regularization scheme in order to avoid unphysical oscillations. A comparison between the running and the fixed couplings reveals that the former produces results much closer to the predictions from recent lattice calculations. In particular, we find that the $\pi_0$ meson mass systematically decreases when the magnetic field increases while the scalar mass remains almost constant. We also investigate how the magnetic background influences other mesonic properties such as $f_{\pi}$ and $g_{\piqq}$.

Keywords: neutral meson mass, RPA approximation, NJL model in a strong magnetic field, magnetized medium, effective models of QCD

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1. Introduction

Very recently [1] the $\pi_0$ pole mass was calculated in a strongly magnetized medium using the two flavor Nambu-Jona-Lasinio model [2][3][4][5][6]. With this aim, a generalization of the standard $B=0$ evaluation, within the RPA (or ladder approximation) [4], has been carried out to take into account the presence of a strong magnetic field. The formalism employed in Ref.[1] is based on the use of field dependent Feynman propagators [7][8] and a field independent regularization scheme (MFIR) [9][10] which separates divergent vacuum contributions from finite thermo-magnetic ones. The advantages of using the MFIR, specially in connection with high density effects, have been recently discussed in detail [11][12]. In those references it has been discussed that calculations which employ $B$-dependent regularizations display, in general, a spurious behavior specially at high magnetic field values which constitute a serious drawback for their use in many physical situations of interest as discussed in the literature [13][14][15]. Other powerful tools which have been employed in the evaluation of pionic observables include chiral perturbation theory (ChPT) [16][17] and effective quark-antiquark lagrangians [18][19].

The magnetic catalysis phenomenon (MC), i.e., the enhancement of the quark condensate when the magnetic field increases, is a common characteristic of all mean field calculations. However, accurate lattice calculations at zero chemical potential and finite temperature predict exactly the opposite behavior close to the (pseudo) critical temperature. This effect has been called inverse magnetic catalysis (IMC) or magnetic inhibition.
In some recent calculations [23] thermo-magnetic effects were included in the standard two-flavor version of the NJL model, where the coupling constant $G$ has been allowed to become temperature and magnetic field dependent, i.e., $G \rightarrow G(eB, T)$, and fitted according to recent lattice results for the quark condensates, emulating the running of the QCD coupling constant with magnetic field and temperature, and thus incorporating IMC. It has been shown in Ref. [23] that the thermodynamic quantities calculated with the SU(2) NJL model using $G(eB, T)$ behave in accordance with lattice QCD simulations [26, 27]. At pion mass, which represents the soft mode, is in excellent numerical agreement with lattice predictions, some-
tice fields. The Letter is organized as follows. In the next section we present the model and the formalism. The numerical results are discussed in Sec. 3 and our concluding remarks are presented in Sec. 4.

2. General formalism

Let us start by reviewing the main steps related to the evaluation of the mesonic properties using the RPA formalism within the MFIR framework as done in Ref. [1]. We also present the ansatz for the magnetic coupling at vanishing temperatures.

2.1. Meson properties under strong magnetic field

In the presence of a magnetic field the standard two-flavor NJL model is described by

\[ \mathcal{L} = \bar{\psi}_f (i \slashed{D} - m) \psi_f + G \left[ (\bar{\psi}_f i \gamma_5 \vec{\tau} \psi_f) \right]^2 + \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \]

(1)

where a sum over repeated $f$ is implied. The electromagnetic gauge field is represented by $A^\mu$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\vec{\tau}$ is the isospin matrix, the coupling constant by $G$ while $Q = \text{diag}(q_u = 2e/3, q_d = -e/3)$ represents the charge matrix, $D^\mu = (i \partial^\mu - QA^\mu)$ is the covariant derivative, $\vec{\tau}^\dagger \psi_u (\bar{\psi}_d)$ is the quark fermion field and $m = \text{diag}(m_u, m_d)$ represents the bare quark mass matrix.

Here, we adopt the Landau gauge, i.e., $A^\mu = \delta_{\mu2} x_1 B$, thus $\vec{B} = B \vec{\epsilon}_3$. Then, in the mean field approximation the NJL lagrangian is given by [6]

\[ \mathcal{L} = \bar{\psi}_f \left( i \slashed{D} - M_f \right) \psi_f + G \left( \bar{\psi}_f \gamma_5 \psi_f \right)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \]

(2)

where $(\bar{\psi}_f \gamma_5 \psi_f)$ represents the quark condensates. The effective quark mass for a given flavor is

\[ M_f = m_f - 2G(\langle \bar{\psi}_f \psi_f \rangle + \langle \bar{\psi}_d \psi_d \rangle), \]

(3)

with $i, j = u, d$ and $i \neq j$. Note that by taking $m_u = m_d$, as we do here, one may set $M_u = M_d = M$ since the different condensates enter in a symmetric manner. It has been shown in Ref. [1] that in the RPA approximation the $\pi_0$ meson mass in a magnetized medium can be calculated selecting the quantum numbers associated to the neutral pion. From the Bethe-Salpeter equation one obtains:

\[ (ig_{\text{qq}})^2 iD_{\pi 0}(k^2) = \frac{2iG}{1 - 2G\Pi_{\pi 0}(k^2)}, \]

(4)

As usual in the last equation the left hand side of the equality is calculated by representing the quark-pion interaction with the following Lagrangian density [4]:

\[ \mathcal{L}_{\pi \text{qq}} = ig_{\pi \text{qq}} \bar{\psi} \gamma_5 \vec{\tau} \vec{\pi} \psi, \]

(5)

where $\vec{\pi}$ stands for the pion field while $g_{\pi \text{qq}}$ represents the coupling strength between pions and quarks. Both sides of eq. (4) can be calculated using the standard meson propagator [28],

\[ D_{\pi 0}(k^2) = \frac{1}{k^2 - m_\pi^2}, \]

(6)

as well as the quark (dressed) propagator in a magnetic medium [7] [8],

\[ S_q(x, x') = e^{ig_{\text{qq}}(x, x')} \sum_{n=0}^{\infty} S_{q,n}(x - x'), q = u, d. \]

(7)

The quark propagator in a strong magnetic field is given by the product of a gauge dependent factor, $g_{\text{qq}}(x, x')$, called Schwinger phase, times a translational invariant term and its explicit expression can be found in Ref. [8]. In the present calculation, which involves only neutral particles, the Schwinger phase cancels out. Through the use of standard Feynman rules the pseudo-scalar polarization loop reads (see Ref. [1] for further technical details):

\[ \frac{1}{i} \Pi_{\pi 5}(k^2) = - \sum_{q=u,d} \int \frac{d^3p}{(2\pi)^3} \text{Tr} \left[ i\gamma_5 \mathcal{L}_q \left( p + \frac{k}{2} \right) i\gamma_5 \right] \times iS_q \left( p - \frac{k}{2} \right). \]

(8)
As shown in Ref. [1] an analogous expression can be obtained for the scalar channel. Then, from Eq. (4), one can obtain the $\pi_0$ mass pole as:

$$1 - 2G \Pi_{PS}(k^2)|_{\omega=m_{\pi_0}} = 0.$$  \hspace{1cm} (9)

The explicit expression for the pseudoscalar polarization loop, Eq. (8), is given by [1]:

$$\frac{1}{i} \Pi_{PS}(k^2) = -i \left( \frac{M - m}{2MG} \right) - \sum_{q=u,d} \beta_q N_c \frac{k_0^2}{(2\pi)^2} \sum_{n=0}^{\infty} g_n I_{q,n}(k_0^2),$$ \hspace{1cm} (10)

where

$$I_{q,n}(k_0^2) = \int d^2 p_1 \left[ \frac{1}{p_1^2 - M^2 - 2\beta_q n_1 (p + k_0^2 - M^2 - 2\beta_q n_1^2)} \right],$$ \hspace{1cm} (11)

where $\beta_q = |q| q_B$, $q_B = \sum q_B$, $g_1 = 2 - \delta_{u0}$, $p_1 = p_0 - p_1$, and $k_0 = k_0 - k_3$. Therefore, from Eq. (9), the $\pi_0$ mass can be written as:

$$m_{\pi_0}^2(B) = -\frac{m}{M(B)} \frac{M}{4\pi G N_c I(m_{\pi_0}^2, B)},$$ \hspace{1cm} (12)

where

$$I(m_{\pi_0}^2, B) = \frac{1}{4(2\pi)^2} \sum_{q=u,d} \beta_q \sum_{n=0}^{\infty} g_n I_{q,n}(k_0^2 = m_{\pi_0}^2).$$ \hspace{1cm} (13)

The $\sigma$-meson mass, $m_{\sigma}$, is readily evaluated in a completely analogous fashion by calculating the scalar polarization loop. This procedure yields [1]:

$$m_{\sigma}^2(B) = 4M^2(B) + m_{\pi_0}^2(B).$$ \hspace{1cm} (14)

Next, the pion decay constant is given by the expression:

$$f_{\pi_0}^2(B) = -i \sum_{q=u,d} \beta_q N_c M^2 \sum_{n=0}^{\infty} g_n I_{q,n}(0),$$ \hspace{1cm} (15)

where $I_{q,n}(0) \approx I_{q,n}(m_{\pi_0}^2)$. The following identity can be obtained from Eqs. (12, 15):

$$m_{\pi_0}^2(B) f_{\pi_0}^2(B) = \frac{m M(B)}{2G}.$$ \hspace{1cm} (16)

In the next section we perform an explicit numerical analysis concluding that the approximation $I_{q,n}(0) \approx I_{q,n}(m_{\pi_0}^2)$ provides results that differ from the exact one only by about 1% or less.

The gap equation, Eq. (3), can be used in order to eliminate the coupling constant $G$ so that the Gell-Mann-Oakes-Renner (GOR) relation in a magnetic medium is recovered.

$$m_{\pi_0}^2(B) f_{\pi_0}^2(B) = -m \left( \langle \bar{\psi} \gamma^\mu \gamma^\nu \psi \rangle \right)(B).$$ \hspace{1cm} (17)

In Ref. [1] the loop integral Eq. (11) was obtained as

$$I(k_0^2, B) = I_{vac}(k_0^2) + I(k_0^2, B),$$ \hspace{1cm} (18)

where

$$I_{vac}(k_0^2) = \frac{i}{8\pi^2} \int_0^1 dx \left[ \sinh^{-1} \left( \frac{\Lambda}{M} \right) - \frac{\Lambda}{\sqrt{\Lambda^2 + M^2}} \right],$$

and

$$I(k_0^2, B) = -\frac{\alpha}{(2\pi)^2} \sum_{q=u,d} \int_0^1 dx \left[ -\psi(x_q + 1) + \frac{1}{2x_q} + \ln x_q \right],$$ \hspace{1cm} (19)

with

$$x_q = \frac{M^2(k_0^2)}{2\beta_q}, \quad \frac{M^2(k_0^2)}{2\beta_q} = M^2 - x(1-x)(k_0^2).$$ \hspace{1cm} (20)

Following the MFIR prescription [10], we have disentangled overlapping divergences by dividing the polarization integral, Eq. (18), into two terms: the first takes into account divergent vacuum fluctuations and can be regularized through a non-covariant three-momentum cutoff, while the second, Eq. (19), represents the finite contribution due to magnetized medium. Note that using the MFIR scheme one recovers the usual vacuum term.

### 2.2. Field dependent coupling

Let us now obtain the magnetic dependence of the NJL model coupling by reproducing the lattice results of Ref. [29] for the quark condensate average at zero temperature, $(\Sigma + \Sigma_d)/2$. We remark that these precise LQCD results have been obtained for $N_f = 2 + 1$ whereas here we are considering the two flavor case. However, in general, translating LQCD predictions for the $N_f = 2 + 1$ case to $N_f = 2$ effective models can be quite safely done because the lattice results are often divided into results for the light ($u$ and $d$) and strange sectors. This is particularly true in the case of the condensates since only the ones related to light quarks (or rather, their average) represent the order parameter for the chiral transition.

In LQCD simulations, the condensates are normalized in a way which is reminiscent of Gell-Mann-Oakes-Renner (GOR), $-2m \langle \bar{\psi} \gamma_\mu \gamma_\nu \psi \rangle = m_\pi^2 f_\pi^2 + \ldots$, so that for a given flavor one has

$$\Sigma(B) = \frac{2m}{m_\pi^2 f_\pi^2} \left[ \langle \bar{\psi} \gamma_\mu \gamma_\nu \psi \rangle_B - \langle \bar{\psi} \gamma_\mu \gamma_\nu \psi \rangle_{00} \right] + 1,$$ \hspace{1cm} (21)

with $\langle \bar{\psi} \gamma_\mu \gamma_\nu \psi \rangle_{00}$ representing the quark condensate at $T = 0$ and $B = 0$. In order to fit the lattice results, the other physical quantities appearing in Eq. (21) should be those of Ref. [29], namely, $m_\pi = 135$ MeV, $f_\pi = 86$ MeV, and $m = 5.5$ MeV so that, by invoking the GOR relation, one can use the LQCD value $\langle \bar{\psi} \gamma_\mu \gamma_\nu \psi \rangle_{00} = -230.5$ MeV.

For selected values of $eB$ from zero to 1 GeV$^2$ and $T = 0$, we can fit the NJL coupling to the corresponding values resulting from lattice QCD calculations. Then we make an interpolation to generate a larger set, which, in turn, is fitted to a simple
\[ \frac{\Sigma u + \Sigma d}{2} = \frac{\Sigma u}{2} - \frac{\Sigma d}{2} \]

In principle, our results are rigorously valid for eB \leq 0.4 GeV\(^2\), which is the upper limit the cutoff scheme can account for. Hence, our results for large magnetic field strengths need to be taken as extrapolations as they give only a qualitative behavior in this limit.

To carry out numerical evaluations we need the four different sets of parameters displayed in Table 1. Notice that sets I and II are used when comparing with LQCD employing physical quark masses, as in Ref. [29], while sets III and IV are more appropriate for comparisons with simulations using heavy quark masses such as the ones performed in Refs. [26, 27]. Therefore, although the running of \( G(eB) \) has been determined from a simulation with physical quark masses [29] we can still compare with simulations which employ heavier quarks [26, 27] provided that we tune the NJL current quark masses in an appropriate way as our numerical results will demonstrate.

Note that the parameters of set I used in our calculations were determined by fitting the pion mass and its decay constant to their empirical values \( m_\pi = 138 \text{MeV} \) and \( f_\pi = 92.4 \text{MeV} \), respectively, and they are the same used in the literature (see, eg, Ref. [6]). Our set II was obtained fixing the NJL coupling constant at \( B = 0 \). The correct \( eB \to 0 \) limit of our ansatz requires that \( G_{III} = G(eB = 0) \).

Table 1: Parameter sets for the NJL model at \( T = B = 0 \). The correct \( eB \to 0 \) limit of our ansatz requires that \( G_{III} = G(eB = 0) \).

|       | \( m_\pi \) (MeV) | \( m_0 \) (MeV) | \( \Lambda \) (MeV) |
|-------|------------------|-----------------|-------------------|
| Set I | 135.62           | 5.0             | 664.3             |
| Set II| 143.31           | 5.5             | 650.0             |
| Set III| 417              | 48.41           | 664.3             |
| Set IV| 417              | 50.16           | 650.0             |

In Fig. 1 we show our numerical results for the average \((\Sigma u + \Sigma d)/2\) (upper panel) and the difference \((\Sigma u - \Sigma d)\) (lower panel) using the coupling constant \( G_{III} \) and the fitted coupling constant. This means a good fit to lattice simulations for the condensate average at \( T = B = 0 \). We remark that the present ansatz is different from the one obtained in Ref. [25], where the fit was performed at the high temperatures \( T > 110 \text{MeV} \). However, the interpolation procedure carried out to improve precision when finding the parameters for the ansatz is the same.

3. Numerical results

In Fig. 1 we show our numerical results for the average \((\Sigma u + \Sigma d)/2\) (upper panel) and the difference \((\Sigma u - \Sigma d)\) (lower panel) using the coupling constant \( G_{III} \) and the fitted coupling constant. This means a good fit to lattice simulations for the condensate average at \( T = B = 0 \). We remark that the present ansatz is different from the one obtained in Ref. [25], where the fit was performed at the high temperatures \( T > 110 \text{MeV} \). However, the interpolation procedure carried out to improve precision when finding the parameters for the ansatz is the same.
$G(eB)$ of eq. (22) in accord with the recent LQCD data \[29\]. The top panel displays how the order parameter for the chiral transition represented by the scalar condensates increases with $B$ in a clear manifestation of the magnetic catalysis phenomenon. Fig. 2 shows the magnetized effective quark mass behavior changes drastically when one uses the running coupling. However, such a behavior could be anticipated by recalling that the initial motivation to adopt such coupling was to counterbalance the increase of the order parameter with $B$ so that the (non-observable) effective quark mass $M \sim G(\bar{q}/q_f)$ behaves differently from the case where $G$ is fixed. This was particularly important at finite temperatures since in general the (pseudo)temperature is proportional to the value of the effective mass value at zero temperature (see, e.g., Ref. \[30\]) and therefore IMC could be achieved by using $G(eB,T)$ in the evaluation of $M$.

In the upper panel of Fig. 3 we compare our results of the normalized neutral pion mass in the MFIR scheme for different coupling constants $G_I$, $G_{II}$ and $G(eB)$ for $eB$ up to 1.0 GeV$^2$. Although the curves qualitatively agree at very weak fields, the behavior of the neutral pion mass with $G_I$ and $G_{II}$ are opposite to the $G(eB)$ case at fields higher than $\approx 0.4$ GeV$^2$, when the decrease of the $\pi_0$ mass is stronger in the $G(eB)$ case when compared to the $G_I$ and $G_{II}$ cases which have a slight increase. We also compare our predictions for $m_{\pi_0}(B)$ with those presented in Ref. \[18\]. We predict values which are about 10% lower than those predicted in Ref. \[18\] when the $eB \lesssim 0.6$ GeV$^2$ while beyond this value our results indicate that $m_{\pi_0}(B)$ decreases in less dramatic way.

The lower panel of Fig. 3 shows the scalar meson mass where again the differences can be traced back to the fact that $m_\sigma \sim M$ as the figure again reveals. The results obtained with $G(eB)$ indicate that, just like $M$, the sigma meson mass is quite stable (varying less than 10% at intermediate field values) so that the correlation length, $\xi \sim 1/m_\sigma$ also remains almost constant. On the other hand the results obtained by using a fixed $G$ lead to the conclusion that the scalar mass increases so that this mode decouples while $\xi \to 0$.

In the upper panel of Fig. 4 our results for the neutral pion decay constant are shown. The same three sets of coupling constants of Fig. 3 have been considered. A systematic increase of
Notice that the validity of the approximation at fields greater than 0.6 GeV$^2$ is confirmed since one can hardly see the difference between the calculations using $I_{q,a}(m_{\pi})$ or $I_{q,a}(m_{\pi} = 0)$.

We have also checked the results for the neutral pion-quark coupling in the lower panel of Fig. 4 predicting a initial decrease of its values up to 0.25 GeV$^2$, and then a steadily increase with higher fields for both $G_I$ and $G_{I\Pi}$ cases, while for $G(eB)$ case we obtain a prediction of a continuous decrease which again could be anticipated by recalling that $g_{\pi qq} \sim M/f_{\pi}$ and that $f_{\pi}$ increases with $B$. Note also that the curve has the same shape as the one showed in Fig. 3 for $M$. Finally, in Fig. 5 we show once again our results for the neutral pion mass but now, in mind a quantitative comparison with lattice QCD results, we use the parameter set IV of Table I. In this parametrization the current quark mass is set equal to 50.16 MeV in order to obtain for $B = 0$ the $m_0$ of 417 MeV, which is the value used in the lattice calculation [26][27]. Thus, we can compare the results using different coupling constants with the recent lattice results showing that the behavior of the masses as a function of $eB$ is qualitatively the same as found in the top panel of Fig. 5. That is, in accordance with LQCD predictions, our results indicate that the neutral pion remains a soft mode over a rather wide range of $B$ values. Note that Fig. 5 indicates that only when $G(eB)$ is used in conjunction with a heavy current quark mass a very good quantitative agreement with recent LQCD results within the Wilson Fermions Formulation [26][27] is obtained. In those investigations, the authors discuss how the LQCD results for the pion mass in external magnetic fields depend on the critical hopping parameters, in particular, they show that the impact of their results within the Wilson Fermions Formulation has been ignored in previous works. The use of constant bare quark masses in the LQCD calculations implies that the neutral pion mass consistently decreases when $eB$ grows. The agreement between our calculations and the LQCD results is also a good evidence that more sophisticated results can be achieved when one assumes that the NJL SU(2) coupling constant has a dependence on $eB$ as proposed in Refs. [23][25].

4. Conclusions

The properties of magnetized neutral mesons have been investigated using a fixed and a $B$-dependent coupling constant so that model predictions and LQCD results related to inverse magnetic catalysis agree. The evaluations have been performed using the two flavor NJL model following the RPA-MFIR framework presented in Ref. [1]. One of our main results shows that the $\pi_0$ remains a soft mode even at rather high field strengths ($\approx 1.5$ GeV$^2$) since its mass decreases by about 30%. The quantitative agreement between our results and recent LQCD predictions is remarkable. Another physically interesting result refers to the behavior of the scalar meson mass which is predicted to steadily increase when a fixed coupling is used reaching (at $eB \approx 1.0$ GeV$^2$) a value which is two and half times higher than its value at $B = 0$, also indicating a decrease of the correlation length, while our results predict that $m_{\pi}$ remains quite stable. The different predictions can be easily understood by recalling $m_{\pi} \propto M \propto G(0\psi_f \psi_f)$ and that, owing to the MC effect, the order parameter $\langle 0\psi_f \psi_f \rangle$ increases within both approaches. On the other hand, the effective quark mass naturally increases when one uses a constant $G_I$ (and $G_{I\Pi}$) and remains practically stable when $G(eB)$ is considered yielding the observed different type of behavior.

Although the quark mass does not necessarily represent a physical observable this is still an interesting result since the behavior of $M$ gets directly reflected in $m_{\pi} \propto 1/E$. When the different model prescriptions are used to evaluate the $m_0$ decay the one which employs $G(eB)$ predicts an increase which is sharper than the one predicted by using a constant coupling value and, together with our predictions for $m_{\pi_0}$ and quark condensates, observes the GOR relation. Finally, when comparing model predictions for the meson coupling constant $g_{\pi qq}$ we found that the use of $G(eB)$ and $G_{I\Pi}$ (and $G_{I\Pi}$) indicate an opposite behavior since the former predicts this quantity to decrease with $B$ while the latter predicts it to increase. Once again the differences are easily understood from the discussions above and by recalling that this coupling is proportional to $M/f_{\pi}$. The results obtained in this Letter seem to indicate that the use of a running coupling within a robust theoretical framework, such as the RPA-MFIR, turns the simple NJL into a useful tool to investigate magnetized quark matter.

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