Ocean Acoustic Tomography with Moving Node Based on Tikhonov Regularized Kalman Filter

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Abstract. Large scale and long period observation of ocean state can be a really tough task using direct measurement. Ocean acoustic tomography (OAT) is an impactful method to monitor large scale ocean state (sound speed, current etc.). This paper studies a sequential tracking approach for the tomography problem from travel-time data in deep water. Tomographic networks combined both fixed mooring receivers and a moving source are considered, which provide a sufficient information to estimate the three-dimensional (3D) sound speed field (SSF). Empirical orthogonal function (EOF) coefficients are used to parameterize the SSF, and a first order autoregressive (AR(1)) model is formed to describe the evolution of sound speed changes in short interval. So that linear state-space model can be established combining ray travel time inversion which means the inversion could be solved under data assimilation framework. Furthermore, in order to overcome the illness of observation matrix in the tomography tracking problem, Tikhonov regularization is integrated into the basic Kalman filter, which induced a Tikhonov regularized Kalman filter (TRKF). Simulations are conducted to test the feasibility of proposed tracking approach in deep water.

1. Introduction

Ocean acoustic tomography was firstly proposed by Munk and Munsch in 1979 [1]. The basic idea of OAT is calculating the ocean properties (sound speed or current velocity), given the properties of transmitted and received signals. It is a typical inverse problem and many linear methods have been put forward to solve the problem, such as ray travel time inversion and modal travel time inversion. Also match field tomography is a brute-force searching method for tomography with same idea as match field processing. But all these methods just consider the static ocean state, doesn’t use the dynamic properties of ocean state.

Oceanographic variability includes the temporal and spatial variability of sound speed. So the tomography problem can be reformed as a data assimilation problem if a plausible model to describe the evolution of ocean, and under this framework the oceanic state and observation at different times were connected. Olivier Carrière [2] et al. have used the Gauss-Markov process to model the dynamic evolution of the range-dependent SSF in time-varying shallow water. Obviously, Kalman filter (KF) is
a well-known and effective method to uses observations sequentially in time in order to make a better estimation combining both measurements and prior model under linear and Gaussian assumptions. But the ill-condition property of the observation matrix may affect the performance of Kalman filter, and a spatial smooth constraint are considered as a regularization term to release the effect.

For monitoring a 3D environment filed, in [3], moving ship tomography technique were proposed to solve the undersampling problem in fixed mooring tomography system. Comparing the traditional fixed mooring node, one or more additional moving node will sample the ocean from the different angles in the horizontal slice which like computerized tomography. For example, N fixed transceivers have $N(N-1)/2$ ray paths while a moving system with S ship stations will have $N(N-1)/2 + N*S$ paths. And we can adjust the position of the ship station to get different topological structure for better sample. Moving ship tomography were used here for providing sufficient information to reconstruct the 3D SSF in an area about $100^*100$km.

In this paper, first-order autoregressive model was used to model the SSF evolution during short interval. TRKF was proposed to track 3D SSF with the help of an additional moving node. In section 2, a brief introduction of ray time inversion method and TRKF are presented. Simulation environment and results are described in section 3, conclusion and discussion are shown in section 4.

2. Theory and Method

2.1. Empirical orthogonal function

Empirical orthogonal function is used to reduce the dimension of unknown parameters in many ocean problem. EOF could acquire from the history sound speed data. Supposed there are P sound speed profiles $c_1(r), c_2(r), ... c_P(r)$, $r = (x, y, z)$ means the space vector, and the mean sound speed profile is obvious

$$c_0(r) = \frac{1}{P} \sum_{i=1}^{P} c_i(r)$$

The sound speed is considered as a random perturbation adding to mean sound speed. The perturbation covariance matrix can be formed with the expression

$$R = \frac{1}{P} [c_1(r) - c_0(r) \cdots c_P(r) - c_0(r)] [c_1(r) - c_0(r) \cdots c_P(r) - c_0(r)]^T$$

Eigen-decomposition to the covariance matrix, the eigenvector is so-called EOF, and corresponding eigenvalue denotes the contribution to the speed variability. It is easily to determine the orders of EOFs by the a compromise of parameterization error and model parameter dimension, always, three to six orders is sufficient to describe the variability.

For a 3D SSF, the horizontal slice is sliced into several grids, and assume that the sound speed is same in the grid. Provided the target region is sliced into total $NX*NY$ grids and each grid can be parameterized using L-order EOFs. So the 3D SSF can be represented by

$$c(x, y, z) = c_0(x, y, z) + \sum_{q=1}^{L} \sum_{i=1}^{NX} \sum_{j=1}^{NY} a_{ij} F_q(z) g_{ij}(x, y)$$

$$g_{ij}(x, y) = \begin{cases} 1; & X_{i-1} < x < X_i; Y_{j-1} < y < Y_j \\ 0; & \text{else} \end{cases}$$

$g_{ij}(x, y)$ is the gate function shown in equation (3), $a_{qij}$ donates the $q$ -order EOF coefficient in grid $(i, j), and F_q(z)$ denotes the $q$ -order EOF.
2.2. Ray travel time inversion
Ray travel time inversion was widely used in the OAT, which proposed by Munk. According to the eikonal equation, the travel time of signal in a 3D case, which propagates along ray path \( \Gamma_i \), can be expressed by integral of slowness(reciprocal of sound speed) along the path with a form as

\[
\tau_i = \int_{\Gamma_i} \frac{ds}{c(r)}
\]

where \( \tau_i \) is travel time of \( i \) th eigenray and \( \Gamma_i \) is \( i \) th eigenray path. \( c(r) \) is sound-speed on the spatial position \( r = (x, y, z) \) in Cartesian coordinate.

The ray travel time gave the necessary information for the estimate of the SSF. Provided there is reference sound speed, and the actual sound speed is the small perturbation on the reference. The difference travel time was between reference and actual sound speed are shown as equation (5), and under frozen-ray approximation, which means the ray path is not changed, we can get approximate linear relationship between delay difference and sound speed perturbation.

\[
\Delta \tau_i = \int_{\Gamma_i} \frac{ds}{c(r)} - \int_{\Gamma_i} \frac{ds}{c_0(r)} \approx -\int_{\Gamma_i} \frac{\Delta c(r)}{c_0} ds
\]

If that N ray arrivals could be identified in the receiver location, the sound speed along the ray path could be extracted by the N measurements. The problem is normally solved by discretization of the ray path and the use orthogonal functions to describe the sound speed perturbation.

2.3. State-space model of OAT
The ocean sound speed profiles (SSPs) does not vary too much in a short duration (one day here, actually the SSP changes during one day, here we only consider a daily average SSP. So the one day should be regard as a short interval here in most time). Hence the SSF evolution can be modeled as a time autoregressive process.

As shown above, SSPs can be parameterized by EOFs, usually the changes of EOF coefficients also indicates the ocean state natural evolution. So the time-evolving process of SSF is instead by evolution of the EOF coefficients, which is considered as an AR(1) model used most in practice. Firstly, considering the sound speed in a discrete grid, which can be represented by the vector \( \mathbf{v}_k(i, j) \), where \( \mathbf{v}_k(i, j) = [\alpha_{ij}, \alpha_{2j}, ..., \alpha_{lj}]^T \). The AR(1) model is formulated as

\[
\mathbf{v}_k(i, j) = \Phi_k \mathbf{v}_{k-1}(i, j) + \mathbf{w}_k(i, j), \Phi_k = \text{diag}([\lambda_1, ..., \lambda_n])
\]

where matrix \( \Phi_k \) denote the transfer matrix consisted of AR coefficient in time \( k \),and \( \mathbf{w}_k(i, j) \) is local state error. Extending to the all horizontal slice, we can get

\[
\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{u}_k, \mathbf{F}_k = \mathbf{I}_N \otimes \Phi_k
\]

\( \mathbf{I}_N \) is the N dimensional identity matrix, N is all grid numbers in the horizontal slice. \( \mathbf{x}_k \) is a column vector consist with all EOF coefficients, \( \mathbf{F}_k \) denotes the overall transfer matrix, which is time-varying matrix which can estimate from the previous \( N_x \) data using Yuler-Walker algorithm, \( \mathbf{u}_k \) is the state error.

Summarily, the linear standard state-space model of the ocean acoustic tomography can be formed by combining equation (5) and (7):

\[
\begin{align*}
\mathbf{x}_k &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{u}_k \\
\mathbf{y}_k &= \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k
\end{align*}
\]

\( \mathbf{y}_k \) means perturbative travel time of each ray path, \( \mathbf{H}_k \) is the observation matrix calculated by equation (3) and (5). \( \mathbf{u}_k \) and \( \mathbf{n}_k \) are respective state noise and observation noise and they are uncorrelated, which are assumed to be zero mean Gaussian white noise processes with covariance matrix \( \mathbf{Q}_k \) and \( \mathbf{R}_k \).
2.4. Tikhonov Regularized Kalman Filter

The basic Kalman filter [4] can be summarized as two procedures: forecast and correction.

Forecast stage gives the prior estimate by the state equation:

\[
\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} \\
P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k
\]

(9)

where \( P_{k|k-1} \) is the prior error covariance matrix in time \( k \), and \( P_{k-1|k-1} \) is the posterior error covariance matrix in last time index \( k-1 \).

The kernel of correction step is using the measurement to correct the predicted state by the innovation and Kalman gain \( K_k \).

\[
K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \\
\hat{x}_{k|k} = \hat{x}_{k|k-1} - K_k (z_k - H_k \hat{x}_{k|k-1}) \\
P_{k|k} = (I - K_k H_k) P_{k|k-1}
\]

(10)

where \( K_k \) is the optimum gain that minimizes the mean least square of the estimation error, which is equivalent to minimize the trace of the posterior error covariance matrix. But if the observation matrix \( H_k \) is ill-conditional, which will lead a numerical instability in the Kalman filter, a Tikhonov regularization should be considered to overcome the problem [5]. Tikhonov regularization minimizes a cost function extended by a regularization term to find \( x_{nk} \)

\[
x_{nk} = \text{arg min}_{x_{nk}} \| H x_{nk} - z_k \|_2^2 + \lambda \| L x_{nk} \|_2^2
\]

(11)

which is equivalent to solve the normal equation as follow

\[
( H^T H + \lambda L^T L ) x = H^T z
\]

(12)

To integrate Tikhonov regularization into the Kalman filter, so that a "fake" measurement equation (12) replaces the raw equation which deduce the Tikhonov regularization Kalman filter with new update stage shown in equation (13). The regularization matrix is second-order spatial smoothing operator, and the regularization parameter can be determined by the L curve.

\[
H_{\text{jpk}} = H_k^T H_k + \lambda L^T L \\
K_k = P_{k|k-1} H_k^T (H_{\text{jpk}} P_{k|k-1} H_k^T + H_k R_k H_k^T)^{-1} \\
\hat{x}_{k|k} = \hat{x}_{k|k-1} - K_k (H^T z_k - H_{\text{jpk}} \hat{x}_{k|k-1}) \\
P_{k|k} = (I - K_k H_{\text{jpk}}) P_{k|k-1}
\]

(13)

3. Simulation results

3.1. Simulation setup

In this paper, hybrid isopycnal-sigma-pressure (generalized) coordinate ocean model (HYCOM) data was used to generate the "real SSF". HYCOM consortium is a multi-institutional effort sponsored by the National Ocean Partnership Program (N OPP), as part of the U. S. Global Ocean Data Assimilation Experiment (GODAE). HYCOM can give three-dimensional depiction of the ocean state at fine resolution in real time. About 100 days data in the region covered from 21°N to 22°N and 118°E to 120°E are chosen as the real data with the time resolution 1day and spatial resolution 0.08°. The raw temperature, salinity and depth data were transformed to the sound speed used the equation in [6]. Both EOFs and the mean sound speed are calculated from the SSPs which are shown in Figure 1. For convenience the last input to ocean acoustic model is the SSF represented with only first-three EOFs which covers a 100km*100km ocean region.
Figure 1. Mean sound speed and the first-three order EOFs calculated from many sound speed profiles in the region

Figure 2. Array configuration of the moving-mooring tomography system used for the simulation, red stars represent the fixed mooring and the black circles represent the moving ship stations

The spatial array configuration of the moving-fixed mooring system used to tomography is shown in Figure 2. There are 4 fixed mooring stations, with a 25 ship stations which provide totally 100 ray paths in the horizontal slice, and in the vertical slice multipath will provide more sample information of different layers. Each mooring station usually has a vertical line array receiver (only use one hydrophone here), and the moving source will transmit the signal in every station. In this simulation, we just consider a sound speed dependent case and simply assume a flat bathymetry with ocean depth 2500m.

The simulation of tomography can be summarized as follow:

a) generate the reference travel time $T$ based on the reference SSF;
b) generate the real travel time $T'$ based on the actual SSF;
c) match the ray and get the observation matrix and perturb travel time;
d) solve the inverse problem.

All the forward problem are dealt with a general ray-tracing program bellhop [7].

3.2. Simulation results
The interested region is sliced into 10*10 grids, with 3-order EOFs. There are totally 300 parameters to estimate. Both KF and TRKF were used to solve the problem. To evaluate the performance of the proposed algorithm, region-integrated root-mean-square error (RMSE) value is defined as

$$RMSE = \sqrt{\frac{1}{N_p} \sum_{p \in S} (c_p - \hat{c}_p)^2}$$

(14)

where $S$ is the sound speed vortex candidate set in the region whose depth over 1250m with vertical space 50m (the sound speed below the depth has a small changes which can be neglected). And $N_p$ is the numbers of vortex. $c_p$ is the real sound speed in the position $p$ while $\hat{c}_p$ is reconstructed sound speed.

The tracking results are shown in the Figure 3, we can find that both KF and TRKF are tracking well to first-order EOF, but with a low tracking performance about the second and third EOF coefficient, which attributes to that the travel time are more sensitive to the amplitude of EOF1. Comparing to the TRKF, KF may diverges in some time index which may cause estimation performance decreases.
Figure 3. EOF coefficients tracking results in a horizontal grid, (a) (b) (c) are respectively represents the first-three order EOF coefficients based on KF, (d) (e) (f) are respectively represents the first-three order EOF coefficients based on TRKF.

Figure 4 shows the horizontal slice reconstruction results in depth 300m in 20th day using TRKF. The figure indicates that the reconstruction is agree well with the real SSF, and the maximum error is less than 2m/s. And according to the evaluation criterion, the performance of KF and TRKF tomography results are shown in the Figure 5. It is clear that the TRKF is always superior to KF. TRKF using Tikhonov regularization with spatial smoothing operator has a more stable solution than KF which leads to a better performance.

Figure 4. Reconstructed result of sound speed on 20th day in horizontal slice with depth 300m. (a) real sound speed field, (b) reconstruction of the sound speed, (c) the error map of the reconstruction.

Also performance of two algorithms under different noise condition are considered. The noise standard deviation varies from 1ms to 10ms, and the RMSE (time-average results) are shown in the Figure 6. The colour line of TRKF has a less slope which means TRKF is more robust than KF if noise increases.
Figure 5. RMSE of KF tomography method and TRKF method in different time with the noise standard deviation 5ms

Figure 6. RMSE varies with different noise both KF tomography method and TRKF method

4. Conclusion
We have described a tracking scheme of 3D SSF reconstruction that using the TRKF to release the effects of ill conditional observation matrix in tomography problem. And spatial continuity is utilized as the regularization term in the TRKF which make sure a more robust solution than basic Kalman filter.

A moving source is a powerful auxiliary tools which provide densely horizontal samples like computerized tomography. Spatial temporal autoregressive model should be studied to model the evolving process, and optimal topological configuration of ship stations about the moving source will also be researched in the future.

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