Non-Gaussianity from Inflation

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Abstract

Correlated adiabatic and isocurvature perturbation modes are produced during inflation through an oscillation mechanism when extra scalar degrees of freedom -other than the inflaton field are present. We show that this correlation generically leads to sizeable non-Gaussian features both in the adiabatic and isocurvature perturbations. The non-Gaussianity is first generated by large non-linearities in some scalar sector and then efficiently transferred to the inflaton sector by the oscillation process. We compute the cosmic microwave background angular bispectrum, providing a characteristic feature of such inflationary non-Gaussianity, which might be detected by upcoming satellite experiments.

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1 Introduction

It is generally believed that inflation provides the causal mechanism to seed structure formation in the Universe. One of the most interesting aspects of these primordial perturbations is their statistical nature. The simplest and most generally accepted idea is that these primordial perturbations were Gaussian distributed. However, this issue is far from being settled: there is still ample room for some level of non-Gaussianity in the initial conditions.

One way of parametrizing the possible presence of non-Gaussianity in the primordial gravitational potential $\Phi$ is to expand it in the following way \cite{1, 2, 3, 4}

$$\Phi = \phi + f_{\text{NL}}(\phi^2 - \langle \phi^2 \rangle) + \mathcal{O}(f_{\text{NL}}^2),$$

where $\phi$ is a zero-mean Gaussian random field and $f_{\text{NL}}$ is an expansion parameter which can be observationally constrained.

It is commonly believed that primordial perturbations generated during inflation are necessarily adiabatic and Gaussian. Although this is essentially the case for the simplest model, where a single inflaton field undergoes a slow-roll transition, the range of possibilities is actually much wider and more interesting than such a standard lore may tell. Even in the case of a single, slowly rolling inflaton field, it has been shown that the effect of field non-linearities and their backreaction on the underlying geometry is to generate a small, but calculable, non-Gaussianity \cite{2, 3, 4, 5}. The non-Gaussianity, or non-linearity, parameter $f_{\text{NL}}$ can be expressed in terms of the standard slow-roll parameters $\epsilon$ and $\eta$ as $f_{\text{NL}} \sim 3\epsilon - 2\eta \ \cite{5, 6, 7}$. Since the slow-roll parameters have to be much smaller than unity for inflation to occur, the typical value of $f_{\text{NL}}$ in single-field inflationary models is inevitably tiny. These constraints can be partially relaxed if the inflaton potential contains ‘features’ in that part corresponding to the last $\sim 60$ e-foldings \cite{8, 9, 10}.

The common belief that non-Gaussianity of inflation generated perturbations is small
comes from this theoretical argument applied to single-field models of inflation. On the other hand the presence of non-Gaussianity is only mildly constrained by observations. Let us focus on the evidence for Gaussian primordial fluctuations coming from the analysis of primary anisotropies of the Cosmic Microwave Background (CMB), as these certainly provide the most direct probe of initial conditions and the most efficient way to look for non-Gaussianity of the type described by Eq. (1) [3]. Recent analyses of the angular bispectrum from 4-year COBE data [10] yield a weak upper limit, $|f_{NL}| < 1.5 \times 10^3$. The analysis of the diagonal angular bispectrum of the Maxima dataset [11] also provides a very weak constraint: $|f_{NL}| < 2330$. According to Komatsu and Spergel [4], the minimum value of $|f_{NL}|$ that will become detectable from the analysis of MAP and Planck data, after properly subtracting detector noise and foreground contamination, is as large as $\sim 20$, and $5$ respectively.

In this paper we show that sizeable and detectable non-Gaussian perturbations both in the adiabatic and the isocurvature modes naturally arise during inflation when extra scalar degrees of freedom are present other than the inflaton field. In such a case, the adiabatic and isocurvature perturbations are correlated [12, 13, 14, 16] as a result of an oscillation mechanism similar to the phenomenon leading to neutrino oscillations [14]. This may happen, for instance, if the inflaton field is coupled to the other scalar degrees of freedom, as expected on particle physics grounds. If these scalar degrees of freedom have large self-interactions, their quantum fluctuations are intrinsically non-Gaussian. This non-Gaussianity is transferred to the inflaton sector through the oscillation mechanism and is left imprinted in the adiabatic and isocurvature modes. We show that the CMB angular bispectrum is sourced not only by the intrinsic adiabatic and isocurvature bispectrum but also by cross-correlation terms, providing a characteristic and detectable signature of these non-Gaussian inflationary perturbations.

The idea that an isocurvature perturbation mode produced during inflation could

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1This phenomenon was first described in Ref. [15] where it was pointed out that dangerous relics may be generated as coherent states through the oscillation mechanism with the inflaton field in the preheating phase after inflation, leading to tight constraint on the reheating temperature.
be non-Gaussian is certainly not new [17, 18, 19]. These scenarios, however, have the
disadvantage that it is generally difficult to fit the observed pattern of CMB anisotropies
in terms of isocurvature perturbations alone. The possibility of generically cross-
correlating the adiabatic and isocurvature modes is attractive [20, 21, 22] both because
of its wider capability of reproducing the observed CMB angular power-spectrum and
because of the possibility of introducing non-Gaussianity in the adiabatic mode too.
Moreover the characteristic signatures of these non-Gaussian inflationary perturbations
could be also a way to break some degeneracies between the cosmological parameters
which usually arise in scenarios where correlated adiabatic and isocurvature perturba-
tions are present [21, 22]. Indeed there exist other mechanisms to produce non-Gaussian
primordial perturbations, such as single-field models allowing for an initial state which
is not the ground state [23] or, outside the inflationary paradigm, cosmic defects [24].
The scenario we propose differs from the other mechanisms since it can generate non-
Gaussian perturbations which are adiabatic through the oscillation mechanism men-
tioned above.

The plan of the paper is as follows. In Section 2 we derive a general formula for
the CMB angular bispectrum in models where correlated adiabatic and isocurvature
perturbation modes are present. The physical mechanism by which these modes can
be produced during inflation is summarized in Section 3. The resulting form for the
angular bispectrum is obtained in Section 4 in the case of adiabatic plus cold dark
matter isocurvature perturbations. Section 5 contains our conclusions.

2 The CMB Angular Bispectrum

In order to investigate possible non-Gaussian features of the CMB one can consider the
angular three point correlation function

$$
\left\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \frac{\Delta T}{T}(\hat{n}_3) \right\rangle = \sum_{l_1,m_1} \sum_{l_2,m_2} \sum_{l_3,m_3} \langle a_{l_1m_1} a_{l_2m_2} a_{l_3m_3} \rangle Y_{l_1m_1}(\hat{n}_1) Y_{l_2m_2}(\hat{n}_2) Y_{l_3m_3}(\hat{n}_3), \quad (2)
$$

where hats denote unit vectors and we have used the usual expansion of the CMB temperature anisotropy in spherical harmonics $Y_{lm}(\hat{\mathbf{n}})$ with coefficients

$$a_{lm} = \int d\hat{\mathbf{n}} Y_{lm}^*(\hat{\mathbf{n}}) \frac{\Delta T}{T} (\hat{\mathbf{n}}).$$

(3)

The angular CMB bispectrum is the harmonic conjugate of the three-point correlation function and is given by

$$\langle a_{l_1m_1} a_{l_2m_2} a_{l_3m_3} \rangle = \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1l_2l_3},$$

(4)

where the first term is the Wigner 3j symbol and $B_{l_1l_2l_3}$ is the angle-averaged bispectrum, which is the observational quantity.

To calculate the bispectrum one has to properly take into account the initial conditions in the radiation dominated epoch after the end of inflation. Such initial conditions may reflect either the adiabatic or isocurvature nature of the cosmological perturbations. In general, however, one expects a mixture of adiabatic and isocurvature perturbations with a nonvanishing cross-correlation \[12, 13, 14, 19, 20, 21, 22\].

For pure adiabatic perturbations the harmonic coefficients $a_{lm}$ are given by \[25\]

$$a_{lm} = 4\pi (-i)^l \int d^3k \hat{\Phi}(k) \Delta_i(k) Y_{lm}^*(\hat{k}),$$

(5)

where $\hat{\Phi}(k)$ indicates the primordial gravitational potential perturbation and $\Delta_i(k)$ is the radiation, or CMB, transfer function. In the large scale limit, one recovers the Sachs-Wolfe effect

$$\frac{\Delta T}{T} = \frac{1}{3} \Phi$$

(6)

where $\Phi$ is the gravitational potential at recombination, by choosing $\Delta_i(k) = 1/3 j_i[k(\tau_0 - \tau_{rec})]$, $\tau_0$ being the conformal time at present and $\tau_{rec}$ the conformal time at recombination.

In the case of pure isocurvature perturbations one simply inserts the initial entropic perturbation $S(k)$ in Eq. (7) in place of the gravitational potential perturbation (see, for example, \[26\]). Of course, this corresponds to a different radiation transfer function.
which can be called $\Delta^S_l(k)$.

Having the expression for $a_{lm}$ it is possible to calculate the bispectrum. Following the formalism of Ref. [6], one finds for pure adiabatic perturbations the following expression

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = (4\pi)^3 (-i)^{l_1 + l_2 + l_3} \int d^3 k_1 d^3 k_2 d^3 k_3$$

$$\times Y_{l_1 m_1}^* (\hat{k}_1) Y_{l_2 m_2}^* (\hat{k}_2) Y_{l_3 m_3}^* (\hat{k}_3)$$

$$\times \delta^3 ({k_1 + k_2 + k_3}) P^{(3)}_\Phi (k_1, k_2, k_3)$$

$$\times \Delta_{l_1} (k_1) \Delta_{l_2} (k_2) \Delta_{l_3} (k_3)$$

where

$$\langle \Phi (k_1) \Phi (k_2) \Phi (k_3) \rangle = \delta^3 (k_1 + k_2 + k_3) P^{(3)}_\Phi (k_1, k_2, k_3)$$

is the three-dimensional bispectrum of the gravitational potential. A similar expression holds for pure isocurvature perturbations. We now analyze what happens in the most general case in which both adiabatic and isocurvature modes are present and are correlated.

### 2.1 Mixture of adiabatic and entropy perturbations

In the case of initial adiabatic plus entropy perturbations, we write the coefficient $a_{lm}$ as

$$a_{lm} = 4\pi (-i)^l \int d^3 k \left[ \Phi (k) \Delta^A_l (k) + S(k) \Delta^S_l (k) \right] Y_{lm}^* (\hat{k}) ,$$

where $\Delta^A_l (k)$ and $\Delta^S_l (k)$ are the transfer functions for the adiabatic and the entropy perturbation modes, respectively. This expression is consistent with the fact that the equations for the evolution of cosmological perturbations are linear. As a check, one can consider the Sachs-Wolfe effect for adiabatic ($\Phi$) plus cold dark matter isocurvature ($S_c$) perturbations [20]:

$$\left( \frac{\Delta T}{T} \right)_{SW} = \left( \frac{\Delta T}{T} \right)_{AD} + \left( \frac{\Delta T}{T} \right)_{ISOC}$$

$$= \frac{1}{3} \Phi_A + 2 \Phi_S .$$
The first term on the r.h.s. of Eq. (10), containing the gravitational potential at large scales \( k \ll aH \) at the time of recombination, corresponds to the case of pure adiabatic perturbations. The second term corresponds to the case of pure isocurvature perturbations. The two potentials are given by

\[
\Phi_A = \frac{3}{10} \left( 3 + \frac{4}{5} \Omega_{\nu RD}^\prime \right) \hat{\Phi}, \quad \Phi_S = -\frac{1}{5} \Omega_c^{MD} S_c, \tag{11}
\]

where \( \Omega_c^{MD} \) is the density parameter for the cold dark matter during the matter era and \( \Omega_{\nu RD}^\prime \) the one for neutrinos during the radiation era. One can recover this result from Eq. (9) with the transfer functions \( \Delta^A_l(k) = 1/3 j_l(k\chi) \) and \( \Delta^S_l(k) = -2/5 j_l(k\chi)\Omega_c^{MD} \).

Note that the full transfer functions take into account all the other effects playing a role in the generation of the temperature anisotropies \( \Delta T/T \) (such as the integrated Sachs-Wolfe effect emerging – for example – in the presence of a cosmological constant, and various small scale effects [4]).

Given the expression (9), if adiabatic and entropy perturbations are correlated we find for the bispectrum a result similar to Eq. (7), but with a more complicated structure

\[
\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3} \rangle = (4\pi)^3(-i)^{l_1+l_2+l_3} \int d^3k_1d^3k_2d^3k_3 \\
\times Y^*_{l_1m_1}(\hat{k}_1)Y^*_{l_2m_2}(\hat{k}_2)Y^*_{l_3m_3}(\hat{k}_3) \times \delta^3(k_1+k_2+k_3) \\
\times [P^A_{\Phi}(k_1,k_2,k_3) \Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^A_{l_3}(k_3)] \\
+ P^S_{A}(k_1,k_2,k_3) \Delta^S_{l_1}(k_1)\Delta^S_{l_2}(k_2)\Delta^S_{l_3}(k_3) \\
+ P^A_{AS}(k_1,k_2,k_3) \Delta^A_{l_1}(k_1)\Delta^A_{l_2}(k_2)\Delta^S_{l_3}(k_3) \\
+ (A, S, A) + (S, A, A) + (S, S, A) + (S, A, S) + (A, S, S)]. \tag{12}
\]

As expected, the bispectrum gets contributions from the adiabatic modes (3), from the isocurvature modes

\[
\langle S(k_1)S(k_2)S(k_3) \rangle = \delta^3(k_1+k_2+k_3)P^S_{A}(k_1,k_2,k_3) \tag{13}
\]

and from the terms parametrizing the cross-correlation between adiabatic and isocur-
vature modes, for example

\[
\langle \hat{\Phi}(k_1)S(k_2)\hat{\Phi}(k_3) \rangle = \delta^3(k_1 + k_2 + k_3) P^{(3)}_{A \cdot S \cdot A}(k_1, k_2, k_3)
\]  

(14)

where we have adopted the notation

\[
(A, S, A) \equiv P^{(3)}_{A \cdot S \cdot A}(k_1, k_2, k_3) \Delta^A_{l_1}(k_1) \Delta^S_{l_2}(k_2) \Delta^A_{l_3}(k_3).
\]  

(15)

Performing the angular integration following Ref. [6], we obtain Eq. (16), where

\[
B_{l_1 l_2 l_3} = (8\pi)^3 \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right)
\]

\[
\times \int dk_1 k_1^2 dk_2 k_2^2 dk_3 k_3^2 J_{l_1 l_2 l_3}(k_1, k_2, k_3) \times
\]

\[
[P^{(3)}_{\Phi}(k_1, k_2, k_3) \Delta^A_{l_1}(k_1) \Delta^A_{l_2}(k_2) \Delta^A_{l_3}(k_3) +
\]

\[
P^{(3)}_{S}(k_1, k_2, k_3) \Delta^S_{l_1}(k_1) \Delta^S_{l_2}(k_2) \Delta^S_{l_3}(k_3) +
\]

\[
P^{(3)}_{A \cdot A \cdot S}(k_1, k_2, k_3) \Delta^A_{l_1}(k_1) \Delta^A_{l_2}(k_2) \Delta^S_{l_3}(k_3) +
\]

\[
(A, S, A) + (S, A, A) + (S, S, A) + (A, A, S) + (A, S, S)
\]

Note that the integral in Eq. (16) is proportional to the reduced bispectrum defined in Ref. [4]. Indeed it contains all the physical information on the bispectrum.

Our goal is now to show that large contributions to the bispectrum (12) may naturally arise when adiabatic and isocurvature modes are correlated.

### 3 Adiabatic and entropy perturbations from inflation

Correlated adiabatic and isocurvature modes can be generated during a period of inflation in which several scalar fields are present [12, 13, 14, 16]. Indeed, adiabatic (curvature) perturbations are produced during a period of cosmological inflation that is driven by a single scalar field, the inflaton. On particle physics grounds – though – it is natural to expect that this scalar field is coupled to other scalar degrees of freedom. This gives rise to oscillations between the perturbation of the inflaton field and
the perturbations of the other scalar degrees of freedom, similar to the phenomenon of neutrino oscillations. The crucial observation is that – since the degree of mixing is governed by the squared mass matrix of the scalar fields – the oscillations can occur even if the energy density of the extra scalar fields is much smaller than the energy density of the inflaton. The probability of oscillation is resonantly amplified when perturbations cross the horizon and the perturbations in the inflaton field may disappear at horizon crossing giving rise to perturbations in scalar fields other than the inflaton. Adiabatic and isocurvature perturbations are inevitably correlated at the end of inflation \[14, 16].

It is exactly this strong correlation which may give rise to large non-Gaussian features in the CMB anisotropy spectrum. This is a simple, but important point. Gaussian features in the CMB anisotropies are usually expected in inflationary models because the inflaton potential is required to be very flat. This amounts to saying that the interaction terms in the inflaton potential are present, but small and non-Gaussian features are suppressed since the non-linearities in the inflaton potential are suppressed too. On the other hand, nothing prevents the inflaton field from being coupled to another scalar degree of freedom whose energy density is much smaller than the one stored in the inflaton field. It is natural to expect that the self-interactions of such extra field or the interaction terms with the inflaton field are sizeable and they represent potential non-linear sources for non-Gaussianity. If during the inflationary epoch, oscillations between the perturbation of the inflaton field and the perturbations of the other scalar degrees of freedom occur, the non-Gaussian features generated in the system of the extra field are efficiently communicated to the inflaton sector and may be left imprinted in the CMB anisotropies.

Let us consider for simplicity the case of two scalar fields \( \phi \) and \( \chi \) interacting through a generic potential \( V(\phi, \chi) \). The study of the field fluctuations \( \delta \phi \) and \( \delta \chi \) can be done
using the Sasaki-Mukhanov variables\textsuperscript{1, 24}

\[ Q_I \equiv \delta \phi_I + \frac{\dot{\phi}_I}{H} \psi \]  

(17)

where \( I = 1, 2 \) with \( \delta \phi_1 = \delta \phi, \delta \phi_2 = \delta \chi \) and \( \psi \) is the linear perturbation in the line element of the metric

\[ ds^2 = -(1 + 2A)dt^2 + 2aB_i dx^i dt + a^2[(1 - 2\psi)\delta_{ij} + 2E_{ij}]dx^i dx^j. \]  

(18)

Using such variables it is possible to define the adiabatic and entropy fields \( Q_A \) and \( \delta s \) in terms of the original field perturbations \( Q_\phi \) and \( Q_\chi \) \textsuperscript{13}

\[ Q_A = (\cos \beta) Q_\phi + (\sin \beta) Q_\chi, \]  

(19)

\[ \delta s = (\cos \beta) Q_\chi - (\sin \beta) Q_\phi, \]  

(20)

where

\[ \cos \beta \equiv c_\beta = \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}, \quad \sin \beta \equiv s_\beta = \frac{\dot{\chi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}, \]  

(21)

and the dots stand for the derivatives with respect to the cosmic time \( t \).

The curvature perturbation \textsuperscript{[25]}

\[ \mathcal{R} = H \sum_I \left( \frac{\dot{\phi}_I}{\sum_{J=1}^N \dot{\phi}_J} \right) Q_I \]  

(22)

deep in the radiation era can be written in terms of the adiabatic field \( Q_A \)

\[ \mathcal{R}_{\text{rad}} = \frac{H}{c_\beta \phi + s_\beta \dot{\chi}} Q_A \]  

(23)

where the r.h.s of this equation is evaluated at the end of inflation.

Let us now introduce the slow-roll parameters for the two scalar fields \( \phi \) and \( \chi \)

\[ \epsilon_I = \frac{M_{Pl}^2}{16\pi} \left( \frac{V_{\phi_I}}{V} \right)^2 \quad \text{and} \quad \eta_{IJ} = \frac{M_{Pl}^2 V_{\phi_I \phi_J}}{8\pi V}, \]  

(24)

where \( M_{Pl} \) is the Planck mass, \( V_{\phi_I} = \partial V/\partial \phi_I \), and \( \phi_I = \phi, \chi \).

Having a sucessfull period of inflation requires that the potential is flat enough, that

\[ \text{To simplify the calculation of the three-point correlation functions one can reduce to a particular gauge, such as the spatially flat gauge } (\psi = 0) \text{ in which the } Q_I \text{ variables coincide with } \delta \phi_I. \]
is $\epsilon_I$ and $|\eta_{IJ}| \ll 1$. Now, making an expansion in the slow roll parameters to lowest order, it is possible to write the gravitational potential $\dot{\Phi}$ as

$$\dot{\Phi} = \frac{2}{3} R_{rad} = \frac{2}{3} \frac{\sqrt{4\pi}}{M_{Pl}} \frac{1}{\sqrt{\epsilon_{\text{tot}}}} Q_A$$

(25)

where $\epsilon_{\text{tot}} = \epsilon_{\phi} + \epsilon_{\chi}$.

Under the hypothesis that the scalar field $\phi$ decays into “ordinary” matter (the present day photons, neutrinos and baryons), while the scalar field $\chi$ decays only into cold dark matter (or it does not decay at all, like in the case of the superheavy dark matter [30]), an adiabatic ($\dot{\Phi}$) and a cold dark matter isocurvature mode ($S_c$) will be generated in the post-inflationary epoch. To lowest order in the slow roll parameters, they are given by expression (25) and [14, 16]

$$S_c = -3 \frac{\sqrt{4\pi}}{m_{Pl}} \frac{\sqrt{\epsilon_{\text{tot}}}}{(\pm \sqrt{\epsilon_{\phi}})(\pm \sqrt{\epsilon_{\chi}})} \delta s$$

(26)

where the r.h.s. is evaluated at the end of inflation as a matching condition. In a short-hand notation we can write

$$\dot{\Phi} = A_0 Q_A, \quad S = S_0 \delta s,$$

(27)

where $A_0$ and $S_0$ are just the “amplitudes” of $\dot{\Phi}$ and $S_c$.

As we will show in the next section, since it is quite natural to expect a nonzero correlation between the adiabatic field $Q_A$ and the entropy field $\delta s$ generated during inflation [14], non-Gaussian features in the CMB anisotropies may be left imprinted.

4 Primordial non-Gaussianity from inflation

We are now in the position of relating the bispectrum in Eq. (12) with the expressions for $\dot{\Phi}$ and $S_c$ originated during a period of inflation.

Consider, for example, the $\langle \dot{\Phi}(k_1)\dot{\Phi}(k_2)\dot{\Phi}(k_3) \rangle$ term. One finds

$$\langle \dot{\Phi}(k_1)\dot{\Phi}(k_2)\dot{\Phi}(k_3) \rangle = A_0^3 Q_A(k_1)Q_A(k_2)Q_A(k_3)$$
\[ A_0^3 ((c_\beta Q_{\phi_1} + s_\beta Q_{\chi_1})(c_\beta Q_{\phi_2} + s_\beta Q_{\chi_2})(c_\beta Q_{\phi_3} + s_\beta Q_{\chi_3})) \] (28)
\[ = A_0^3 \left[ c_\beta^3 \langle Q_{\phi_1} Q_{\phi_2} Q_{\phi_3} \rangle + c_\beta^2 s_\beta \langle Q_{\phi_1} Q_{\phi_2} Q_{\chi_3} \rangle + c_\beta s_\beta^2 \langle Q_{\phi_1} Q_{\chi_2} Q_{\phi_3} \rangle \right. \\
\left. + c_\beta^2 s_\beta^2 \langle Q_{\phi_1} Q_{\chi_2} Q_{\chi_3} \rangle + s_\beta c_\beta^2 \langle Q_{\chi_1} Q_{\phi_2} Q_{\phi_3} \rangle + s_\beta^3 c_\beta \langle Q_{\chi_1} Q_{\chi_2} Q_{\phi_3} \rangle + s_\beta^3 \langle Q_{\chi_1} Q_{\chi_2} Q_{\chi_3} \rangle \right] 
\]

where, for example, \( Q_{\phi_1} \) stands for \( Q_\phi(k_1) \) and we have used Eq. (19). Analogous expressions hold for the remaining terms.

Our goal is now to show that a large amount of non-Gaussianity can be generated in the presence of correlated adiabatic and entropy perturbations. First of all, we note that the bispectrum is a sum of different three-point correlation functions. The coefficients in front of each correlation function involve mixing angles which parametrize the amount of mixing between the adiabatic and the isocurvature modes. If such mixing is sizeable, all coefficients are of order unity and one expects that nonlinearities in the perturbation of the scalar field \( \chi \) may be efficiently transferred to the inflaton sector, thus generating large non-Gaussian features.

Because expressions are quite lengthy and might obscure our point, from now on and just for illustrative purposes we make some simplifying hypothesis and assume that the dominant nonlinear terms are those sourced by the three-point correlation function \( \langle Q_{\chi_1} Q_{\chi_2} Q_{\chi_3} \rangle \). This could be the case for a Langrangian of scalar fields \( \phi \) and \( \chi \) in which the largest coupling is for the \( \frac{\xi}{3} \chi^3 \)-term. Let us also assume that the field \( \chi \) is lighter than the Hubble rate during inflation.

Under these assumptions, we can rewrite the bispectrum (12) as

\[
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = (4\pi)^3(-i)^{l_1 + l_2 + l_3} \int d^3k_1 d^3k_2 d^3k_3 \\
\times Y_{l_1 m_1}^* (\hat{k}_1) Y_{l_2 m_2}^* (\hat{k}_2) Y_{l_3 m_3}^* (\hat{k}_3) \times \delta^3(k_1 + k_2 + k_3) \\
\times \left\{ A_0^3 s_\beta^3 P_{Q_{\chi}}^{(3)}(k_1, k_2, k_3) \Delta^A_{l_1}(k_1) \Delta^A_{l_2}(k_2) \Delta^A_{l_3}(k_3) \
+ A_0^3 s_\beta^2 c_\beta P_{Q_{\chi}}^{(3)}(k_1, k_2, k_3) \Delta^S_{l_1}(k_1) \Delta^S_{l_2}(k_2) \Delta^S_{l_3}(k_3) \
+ A_0^2 s_\beta c_\beta^2 P_{Q_{\chi}}^{(3)}(k_1, k_2, k_3) [\Delta^A_{l_1}(k_1) \Delta^A_{l_2}(k_2) \Delta^S_{l_3}(k_3) \
+ \Delta^S_{l_1}(k_1) \Delta^S_{l_2}(k_2) \Delta^A_{l_3}(k_3)] \right\}
\]

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\[(k_1 \leftrightarrow k_3; l_1 \leftrightarrow l_3) + (k_2 \leftrightarrow k_3; l_2 \leftrightarrow l_3) \] 
\[+ S_0^2 A_0 \epsilon_\beta \epsilon_\beta P^{(3)}_{Qx}(k_1, k_2, k_3) \Delta^S_{l_1}(k_1) \Delta^S_{l_2}(k_2) \Delta^A_{l_3}(k_3) \] 
\[+ (k_1 \leftrightarrow k_3; l_1 \leftrightarrow l_3) + (k_2 \leftrightarrow k_3; l_2 \leftrightarrow l_3) \}\]

where

\[
\langle Q \chi_1 Q \chi_2 Q \chi_3 \rangle = \delta^3(k_1 + k_2 + k_3) P^{(3)}_{Qx}(k_1, k_2, k_3). \tag{30}
\]

The angular part of the integral can be calculated as done in subsection 2.1.

The next step is to further reduce the expression for the bispectrum by expanding \[\langle Q \chi_1 Q \chi_2 Q \chi_3 \rangle\]. This is necessary because, in the presence of large mixing, \(Q_\phi\) and \(Q_\chi\) are not "mass-eigenstates" of the system, but just interaction eigenstates. The situation here is analogous to what happens for light neutrinos where the three different flavors of neutrinos represent interaction eigenstates, but they do not represent mass-eigenstates because of the mixing among the flavors giving rise to the phenomenon of neutrino oscillations.

We first define the comoving fields \(\tilde{Q}_\phi = a Q_\phi\) and \(\tilde{Q}_\chi = a Q_\chi\) and then we introduce a basis for annihilation and creation operators \(a_i\) and \(a_i^\dagger\). We perform the decomposition (\(\tau\) is the conformal time):

\[
\begin{pmatrix}
\tilde{Q}_\phi \\
\tilde{Q}_\chi
\end{pmatrix} = U \int \frac{d^3k}{(2\pi)^{3/2}} \left[ e^{ik \cdot x} h(\tau) \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} + \text{h.c.} \right] ,
\]

\[
\begin{pmatrix}
\Pi \tilde{Q}_\phi \\
\Pi \tilde{Q}_\chi
\end{pmatrix} = U \int \frac{d^3k}{(2\pi)^{3/2}} \left[ e^{ik \cdot x} \bar{h}(\tau) \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix} + \text{h.c.} \right] ,
\tag{31}
\]

where \(\Pi \tilde{Q}_\phi\) and \(\Pi \tilde{Q}_\chi\) are the conjugate momenta of \(\tilde{Q}_\phi\) and \(\tilde{Q}_\chi\) respectively, and \(h\) and \(\bar{h}\) are two \(2 \times 2\) matrices satisfying the relation

\[
[h \bar{h}^* - \bar{h}^* \bar{h}]_{ij} = i \delta_{ij}, \tag{32}
\]

derived from the canonical quantization condition.

The matrix \(U\) is a rotation matrix

\[
U = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \tag{33}
\]
which diagonalizes the squared mass matrix of the two scalar field perturbations $Q_\phi$ and $Q_\chi$

$$\mathcal{M}_{IJ}^2 = V_{\phi I} V_{\phi J} - 8\pi M_{Pl}^2 a^3 \left( a^3 / H \right)^3 \left( \dot{\phi}_I \dot{\phi}_J \right) \simeq \frac{8\pi V}{M_{Pl}^4} \left[ \eta_{IJ} - 2 (\pm \sqrt{\epsilon_J})(\pm \sqrt{\epsilon_I}) \right],$$  \hspace{1cm} (34)

where the sign $\pm$ stands for the cases $\dot{\phi}_I(\dot{\phi}_J) > 0$ and $< 0$ respectively. The mixing angle $\theta$ is given by

$$\tan 2\theta = \frac{2 \mathcal{M}_{\chi \phi}^2}{\mathcal{M}_{\phi \phi}^2 - \mathcal{M}_{\chi \chi}^2}. \hspace{1cm} (35)$$

One can envisage different situations:

\textit{i}) Inflation is driven by the inflaton field $\phi$ and there is another scalar field $\chi$ with a simple polynomial potential $V(\chi) \propto \chi^n$ leading to a vacuum expectation value $\langle \chi \rangle = 0$. In such a case, $\sin \beta = \sin \theta = 0$ and there is no mixing between the inflaton field and the $\chi$-field as well as no cross-correlation between the adiabatic and isocurvature modes. Nonvanishing non-Gaussianity will be present in the isocurvature mode. This is indeed a known result [17, 19]. Non-Gaussian adiabatic perturbations may also arise if the $\chi$-field decays late after inflation [19, 29].

\textit{ii}) Inflation is driven by two scalar fields $\phi$ and $\chi$ with equal mass, $V = \frac{m^2}{2}(\phi^2 + \chi^2)$. In such a case the mixing is maximal, $\beta = \theta = \pi/4$. Nevertheless, the cross-correlation is again vanishing [13, 14, 16] and the bispectrum gets contributions from adiabatic and isocurvature modes independently, since in this case the terms parametrizing the cross-correlation in Eq. (12) vanish. A term $\frac{2}{3}\chi^3$ in the Lagrangian would be a source of non-Gaussianity and at the same time it would switch on a cross correlation between the adiabatic and the isocurvature modes, thus producing nonzero cross terms in Eq. (29). However, these non-Gaussianities would be small because of slow-roll conditions.

\textit{iii}) Inflation is driven by an inflaton field $\phi$ and there is another scalar field $\chi$ whose vacuum expectation value depends on the inflaton field and – eventually – on the Hubble parameter $H$ and some other mass scale $\mu$, $\langle \chi \rangle = f(\phi, H, \mu)$. Under these circumstances, $\langle \dot{\chi} \rangle = \partial f / \partial \phi \dot{\phi} + \partial f / \partial H \dot{H}$. As in illustrative case, let us restrict ourselves to the case in which $\partial f / \partial \phi \dot{\phi}$ is the dominant term and we can approximate $\langle \dot{\chi} \rangle = \partial f / \partial \phi \dot{\phi}$. We
have therefore $\tan \beta \simeq \partial f/\partial \phi$ and $\dot{\beta} \simeq (\partial f/\partial \phi)/[1 + (\partial f/\partial \phi)^2]$. In such a case, cross-correlation between the adiabatic and the isocurvature modes may be large and non-Gaussianity may be efficiently transferred from one mode to the other.

We can now reduce $\langle Q_{\chi_1} Q_{\chi_2} Q_{\chi_3} \rangle$ using the decomposition (34) and making some further approximations justified if slow-roll conditions are attained. In fact, using a perturbative method, it can be checked that the contributions to $\langle Q_{\chi_1} Q_{\chi_2} Q_{\chi_3} \rangle$ coming from terms proportional to the non-diagonal elements $h_{12}$ and $h_{21}$ can be neglected since $h_{12}$ and $h_{21}$ are $\mathcal{O}(\epsilon_{1}, \eta_{11})$, and on superhorizon scales $k \ll aH h_{11}$ and $h_{22}$ are Hankel functions [14]. Thus we can neglect the non diagonal terms of the $h$ matrix and write

$$
\bar{Q}_\chi(k) = [s_\theta h_{11} a_1(k) + c_\theta h_{22} a_2(k)] + [s_\theta h_{11}^* a_1^\dagger(-k) + c_\theta h_{22}^* a_2^\dagger(-k)] \quad (36)
$$

$$
= s_\theta [h_{11} a_1(k) + h_{11}^* a_1^\dagger(-k)] + c_\theta [h_{22} a_2(k) + h_{22}^* a_2^\dagger(-k)]
$$

$$
\equiv s_\theta h_{11} I_1 + c_\theta h_{22} I_2,
$$

where, for simplicity of notation, we have indicated the diagonal terms of the $h$ matrix as $h_{11} \equiv h_1$, $h_{22} \equiv h_2$ and we have defined two new fields $I_1$ and $I_2$ just by collecting the functions $h_{11}$ and $h_{22}$. After all these manipulations we arrive at three-point function

$$
\langle \bar{Q}_\chi Q_{\chi_2} \bar{Q}_{\chi_3} \rangle = \langle (s_\theta h_{11} I_{11} + c_\theta h_{21} I_{21})(s_\theta h_{12} I_{12} + c_\theta h_{22} I_{22})(s_\theta h_{13} I_{13} + c_\theta h_{23} I_{23}) \rangle
$$

$$
= s_\theta^3 (h_{11} h_{12} h_{13}) \langle I_{11} I_{12} I_{13} \rangle + s_\theta^2 c_\theta (h_{11} h_{12} h_{23}) \langle I_{11} I_{12} I_{23} \rangle + s_\theta^2 c_\theta (h_{11} h_{22} h_{13}) \langle I_{11} I_{22} I_{13} \rangle + s_\theta c_\theta^2 (h_{11} h_{22} h_{23}) \langle I_{11} I_{22} I_{23} \rangle
$$

$$
+ c_\theta^2 s_\theta^2 (h_{21} h_{12} h_{13}) \langle I_{21} I_{12} I_{13} \rangle + c_\theta^2 s_\theta (h_{21} h_{12} h_{23}) \langle I_{21} I_{12} I_{23} \rangle + c_\theta^2 s_\theta (h_{21} h_{22} h_{13}) \langle I_{21} I_{22} I_{13} \rangle + c_\theta^2 (h_{21} h_{22} h_{23}) \langle I_{21} I_{22} I_{23} \rangle,
$$

where the indices “1, 2, 3” , as usual, indicate that the quantities are evaluated at $k_1, k_2$ and $k_3$. If we indicate

$$
\langle I_{i1} I_{j2} I_{k3} \rangle = \delta^3(k_1 + k_2 + k_3) P_{ijk}^3(k_1, k_2, k_3), \ i, j, k = 1, 2 \quad (38)
$$

\(^3\text{In such a case the system is diagonalized by the matrix } \mathcal{U}.\)
because of rotation and translation invariance, it is easy to convince oneself that the 
terms $\langle I_1 I_2 I_3 \rangle$ are invariant under an exchange of $k_1, k_2, k_3$. Thus, taking into 
account that the operators $I_1$ and $I_2$ commute, one can check that, for example, 
$\langle I_{11} I_{12} I_{23} \rangle$ is equal to $\langle I_{11} I_{22} I_{13} \rangle$. The expression (37) is thus further simplified to 

$$
\langle \tilde{Q}_{\chi 1} \tilde{Q}_{\chi 2} \tilde{Q}_{\chi 3} \rangle = s_\delta^3 (h_{11} h_{12} h_{13}) \langle I_{11} I_{12} I_{13} \rangle + c_\delta^3 (h_{21} h_{22} h_{23}) \langle I_{21} I_{22} I_{23} \rangle 
$$

$$
+ s_\delta^2 c_\theta \langle I_{11} I_{12} I_{23} \rangle (h_{11} h_{12} h_{23} + h_{11} h_{22} h_{13} + h_{21} h_{12} h_{13}) 
$$

$$
+ c_\delta^2 s_\theta \langle I_{11} I_{22} I_{23} \rangle (h_{11} h_{22} h_{23} + h_{21} h_{12} h_{23} + h_{21} h_{22} h_{13}).
$$

(39)

In order to make a quantitative estimate of the three-point function (39), we first 
notice that the coefficients in front of the various terms are of order unity, provided the 
degree of mixing is large. We can borrow the expression for each three-point function 
appearing in Eq. (39) from the calculation of [2], which is done for an effectively massless 
scalar field $\chi$ with cubic self-interactions. In the de Sitter background it is given by 

$$
\langle (\chi (k_1) \chi (k_2) \chi (k_3)) = \frac{1}{6} \mu H^2 (k_1 k_2 k_3)^{-3} F(k_1, k_2, k_3) \delta^{(3)} (k_1 + k_2 + k_3),
$$

(40)

where 

$$
F(k_1, k_2, k_3) \simeq -\beta (k_1^3 + k_2^3 + k_3^3)
$$

(41)

and, for instance, $\beta \sim 60$ if one is interested in the scales relevant for large-angle CMB 
anisotropies. Notice that the functional form of the bispectrum is the same found by 
Gangui et al. [5], who made different assumptions and used the stochastic approach to 
inflation.

Plugging the above expressions into the CMB angular bispectrum one gets the standard relation [5] giving the angular bispectrum $B_{l_1 l_2 l_3}$ as a sum of products of two 
angular power-spectra, $C_{l_i} C_{l_j}$. The non-Gaussianity amplitude is monitored by the 
dimensionless strength $f_{NL} = \mathcal{O}(\mu/H)$.
5 Conclusions

In this paper we have studied inflationary models where extra scalar degrees of freedom other than the inflaton exist. This allows isocurvature perturbation modes to be switched on during the inflationary evolution besides the usual adiabatic one. As previously shown [13, 14, 16], a generic prediction of these models is that non-zero cross-correlations arise among adiabatic and isocurvature fluctuations. Here we exploited this physical process as an efficient tool to transfer non-Gaussian features from the isocurvature to the adiabatic mode. Sizeable non-Gaussianity can be easily produced in the non-inflatonic sector, by self-interactions leading to non-linearities in their evolution. This is because, unlike the inflaton case, the self-interaction strength in such an extra scalar sector does not suffer from the usual slow-roll conditions. In order to make use of our results for practical purposes, one might introduce a simple non-Gaussian model. For instance, one can parametrize the non-Gaussianity in the peculiar gravitational potential as

\[ \Phi = \varphi_1 + f_{NL} (\varphi_2^2 - \langle \varphi_2^2 \rangle) + \mathcal{O}(f_{NL}^2) \] (42)

(and a similar expression for the entropy mode), where \( \varphi_1 \) and \( \varphi_2 \) are zero-mean Gaussian fields with non-zero cross-correlation \( \langle \varphi_1 \varphi_2 \rangle \neq 0 \). The non-Gaussianity strength \( f_{NL} \), being sourced by the non-inflatonic scalar sector of the theory, is not generally constrained by the slow-roll conditions of inflationary dynamics. This may make the non-Gaussian signatures accessible by future CMB satellite experiments.

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