Clustering in N-Body gravitating systems

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Abstract 
Self-gravitating systems have acquired growing interest in statistical mechanics, due to the peculiarities of the 1/r potential. Indeed, the usual approach of statistical mechanics cannot be applied to a system of many point particles interacting with the Newtonian potential, because of (i) the long range nature of the 1/r potential and of (ii) the divergence at the origin. We study numerically the evolutionary behavior of self-gravitating systems with periodical boundary conditions, starting from simple initial conditions. We do not consider in the simulations additional effects as the (cosmological) metric expansion and/or sophisticated initial conditions, since we are interested whether and how gravity by itself can produce clustered structures. We are able to identify well defined correlation properties during the evolution of the system, which seem to show a well defined thermodynamic limit, as opposed to the properties of the “equilibrium state”. Gravity-induced clustering also shows interesting self-similar characteristics. 

\textbf{Key words:} Classical statistical mechanics, Few- and many-body systems, Non-extensive thermodynamics 
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1 Introduction

Many-body systems with long range interactions are widespread in nature. In particular, gravitating systems dominate in an impressive range of spatial...
scales ($10^{-1}pc - 10^8pc$). It is very difficult to trace the specific role of gravity in the evolution and the statistical properties of such systems, since usually many different physical processes should be considered. For this reason, many theoretical and numerical studies have focussed on the specific physical problem, rather than on the general statistical features of an infinite gravitating system. On the other hand, the usual statistical and thermodynamical approaches to the understanding of an infinite system with an interaction of some kind fail, or have to be taken very carefully, in the case of long range interactions. One of the properties of such interactions is that they make the system non extensive, that is, the system cannot be considered as made of smaller independent parts. Therefore one cannot study the typical properties of finite sample to extrapolate safely the properties of the whole infinite sample, as one does with short range systems. Many simple idealized examples have been considered to analyse the effect of non extensivity (e.g. [1–4]). They have given interesting clues on the behavior of non extensive system. On the other hand, it would be more satisfying to deal with three dimensional systems with a real interaction potential, as gravity, and a full treatment of the dynamics of the system. Our approach has been to deal with a system as realistic as possible, i.e. three dimensional N-body gravitating systems.

2 The simulations

The model we investigate is usually a system with particles placed randomly in a cube with periodical boundary conditions. The particles have the same mass and are initially at rest. Since we want to understand the behaviour of an infinite system, we run a series of simulations with different number of particles, but with the same density. It is important to stress that, due to long range nature of gravitational interaction, one has to use subtle analytic tools to evaluate the force on a particle due to an infinite system. We use Ewald formula for this purpose [5]. It is also important to observe that the gravitational potential in such systems is not well defined, since the potential energy of a particle would diverge. Nevertheless, due to isotropy of the system on large scales, the force is well defined, since contributions from far away are balanced. We use a numerical code [6] which solves the $N$ coupled equations of motions in discrete time steps. A well known algorithm (tree code) is used to evaluate the forces in an efficient approximated way. The time integrator is a leapfrog (Verlet) algorithm, with individual time steps. For numerical reasons, the potential is smoothed at very small scales (much smaller than the mean interparticle distance), that is the divergence in the origin is removed and replaced with a smoother behavior. We have checked by changing the smoothing length that such procedure does not affect our results.

We consider in the following the results of a series of simulations, with the
same number density but different number of particles, \((8^3, 16^3, 32^3)\). In fig.1 we show a picture of the evolution of particle positions in time. Note that the systems develop clusters on small scale first, then on larger and larger scales. When the typical size of the clusters is of the order of the box size, the evolution is influenced by finite size effect. Actually, even if the periodic system is infinite, the maximum number of particles in the simulation volume is fixed.

In fig.2 we plot the kinetic energy per particle versus time. This figure shows that the evolution of the kinetic energy per particle is independent of the number of particles in the system, in spite of the non extensivity, up to a time which increases approximately with \(lnN\). After this time, the kinetic energy becomes constant. This “equilibrium state”, though, is a finite size effect. Therefore we argue that in the “thermodynamic limit” (defined as the limit behavior when \(N, V \to \infty\) keeping the density constant) the behavior of the kinetic energy will be the same. It is also interesting to look at the distribution of velocities along an axis at a fixed time, shown in fig.3. The picture refers to a given time before the finite size effects become relevant. A gaussian fit is shown for the reference. The distribution is non gaussian, but this is not surprising, since the system is not in equilibrium. It can be interesting to try to apply
Fig. 3. Normalized distribution of velocities along an axis for a $32^3$ particle simulation. The inner continuous line is the gaussian fit, the outer one is a stretched exponential fit.

Tsallis’ temptative velocity distribution (see e.g. [7]) on such data. We have also analysed clustering by means of the correlation functions. In particular we use the integrated conditional density $\Gamma^{*}(r,t)$ [8] and the integrated two point correlation function $\xi(r,t)$. Operatively they can be defined respectively as the mean average density and the mean average density fluctuation in a sphere of radius $r$ centered on a particle of the system. Therefore a system is clustered on a scale $r_0$ if $\Gamma^{*}(r_0,t) \gg n_0$, where $n_0$ is the mean number density. The evolution of $\Gamma^{*}(r,t)$ is shown in fig.4 for simulations with different number of particles. Two important conclusions can be drawn from fig.4. First of all, the growth of correlations is approximately the same for all the simulations, in the regime where clustering takes places on scales small compared to the box size. Together with fig.2, this result suggests that the dynamics in this regime is the same for any number of particles, and therefore also in the “thermodynamic limit”. We can also reasonably infer that the infinite system has no equilibrium state. Another important remark is that the functional form of the $\Gamma^{*}(r,t)$ on the largest scales on which clustering is taking place is the same at all times. So we suppose that the clustering process could be explained in terms of the same dynamical processes, acting as time goes on on larger and larger scales.

In two papers of ours [9,10] we have tried to explain the whole evolution of the correlations in such systems in terms of a discrete dynamics acting on larger and larger scales. At the beginning, the basic elements for the dynamics we consider are particles, which form clusters. At later times, the interactions between such clusters are mainly responsible for the further evolution of the system, and so on. This is a sort of renormalization of the dynamics. The justification behind such model lies on the fact that on scales much larger than the clustering scales the system is approximately isotropic, therefore the force due to far away objects almost balances. On the contrary, on the scale of clustering the system is anisotropic, therefore the force due to near clusters is large. In particular, it will be mostly in the direction of the nearest cluster.
Fig. 4. Evolution of $\Gamma^\ast(r, t)$ for simulations with $8^3$, $16^3$ (lines with pluses), $32^3$ (lines with filled circles) particles. Time flows from left to right.

Fig. 5. Comparison between $\xi$ evolution for a simulation with $32^3$ particles (filled circles lines) and our theoretical prediction (smooth line). The agreement for times smaller than the typical crossing time is quite good.

This model is successful in providing: (i) an analytical expression for early time evolution of correlations, with no free parameter. The agreement with numerical experiments is given in fig.5; (ii) a qualitative prediction for the rate of the clustering process at any time.

3 Conclusions

We analyse the problem of the evolution of an infinite system with long range (gravitational) interaction. We have studied it by numerical N-body simulations of periodical systems with different sizes. We identified some properties which can be inferred for the infinite system and we have given a model that could explain most of them. The relevant points of our approach are:
• We consider a thermodynamic approach to be highly problematic in the case of infinite self gravitating systems, since they presumably have no equilibrium state and they are non-extensive.

• We propose a fully dynamical model instead which explains the out-of-equilibrium evolution of the system. The fundamental elements for the dynamics in our approach are discrete objects. Discretisation turns out to be crucial to explain highly developed clustering, while standard (statistical mechanical or cosmological) approaches have mostly focussed on mean field or fluid like dynamics. In particular, the non-analiticality of the distribution of mass (that is, its discrete nature) is exported at larger and larger scale.

• The evolution of clustering on a scale depends on what has happened on smaller scales. This is an opposite view with respect to the hierarchical clustering model, often considered in astrophysics. In particular, in such model clustering on a scale depends on initial conditions on that scale.

References

[1] A. Torcini, M. Antoni, Phys. Rev. E 59, 2746 (1999)
[2] M. Antoni, S. Ruffo, Phys. Rev. E 52, 2361 (1995)
[3] V. Latora, A. Rapisarda, S. Ruffo, Phys. Rev. Lett. 80, 692 (1998)
[4] P. Hertel, W. Thirring, Ann. Phys. (N.Y.) 63, 520 (1971)
[5] L.A.Hernquist, F.R. Bouchet, Y. Suto, ApJS, 75, 231 (1991)
[6] R.Capuzzo Dolcetta, P. Miocchi, J. Comp. Phys. 143, 29 (1998)
[7] R. Silva Jr., A. R. Plastino, J.A.S. Lima, Phys. Lett. A, 249, 401 (1998)
[8] P. Coleman, L. Pietronero, Phys. Rep. 213, 311 (1992)
[9] M. Bottaccio, A. Amici, P. Miocchi, R. Capuzzo Dolcetta, M. Montuori, L. Pietronero, submitted to Europhysics Letters (2001)
[10] M. Bottaccio, A. Amici, P. Miocchi, R. Capuzzo Dolcetta, M. Montuori, L. Pietronero, in preparation