Estimates of W-exchange contributions to $\Xi_{cc}$ decays

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Abstract

Encouraged by the recent discovery of the $\Xi_{cc}$ baryon, we investigate two-body nonleptonic weak decays of doubly charmed, $\Xi_{cc}$, baryons. We calculate the branching ratios for Cabibbo-Kobayashi-Maskawafavored and -suppressed modes in factorization and pole model approaches. The preliminary estimates of nonfactorizable W-exchange contributions are obtained using the pole model. We find that the W-exchange contributions to $\Xi_{cc}$ decays, being sizable, cannot be ignored.

Keywords: Doubly-heavy baryons; Charm baryons; Electro-weak decays; Symmetry breaking; W-exchange process; Branching Ratios.

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I. INTRODUCTION

The resolution of the longstanding puzzle involving the Ξ_{cc} states has been much awaited since their first observations were reported by the SELEX collaboration [1, 2]. Most recently, the LHCb collaboration announced the observation of a doubly charmed Ξ^{++}_{cc} baryon found in the Λ^{+}K−π+π+ mass spectrum. The mass of the observed state is determined as \( M_{\Xi^{++}_{cc}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14 \) MeV, while the mass difference \( M_{\Xi^{++}_{cc}} - M_{\Lambda^{+}} = 1334.94 \pm 0.72 \pm 0.27 \) MeV. The new observation of a doubly heavy charm baryon has revamped the interest of heavy flavor physicists as being a good candidate with which to study the heavy-quark dynamics. Although, the life time, \( \tau_{\Xi_{cc}} \), has not yet been given experimentally, ample theoretical estimates exist in literature that range from \( \sim 50 - 670 \) fs [4–9]. Another interesting aspect of doubly heavy baryons is their spectroscopy [8–11].

In addition to the three quark dynamics, the doubly heavy baryons can be identified by the set of quantum numbers \((J^P, S_d)\) in diquark picture, where \( S_d \) is the spin of the heavy diquark. Thus, spins of the two heavy-quarks are coupled to form the \((S_d = 1)\) symmetric spin configuration of a diquark \( \{Q_1Q_2\} \) and the \((S_d = 0)\) antisymmetric spin configuration of a diquark \( [Q_1Q_2] \). The general convention is to denote the antisymmetric state as a primed one i.e. \( |B'\rangle \) and symmetric heavy-diquark state as an unprimed, \(|B\rangle\) state. Also, the wave functions of the \(|B\rangle\) and \(|B'\rangle\) states are expected to mix [12–18]. However, in the present work we consider three quarks as an independent dynamical entities.

Theoretically, the mass spectra, magnetic moments, and radiative and semi-leptonic decays of the doubly charmed baryons has been the center of interest for the last decade [4–36]. On the contrary, the progress in the heavy-baryon nonleptonic weak decays has been very slow [37–46], although, the recent experimental observations have revived the activities in nonleptonic decays of heavy baryons in last few years [47–61]. Thus, we put our focus on the two-body nonleptonic weak decays of doubly charmed baryons. Very recently, weak decays of doubly heavy baryons were analyzed in SU(3) symmetry and in the quark-diquark picture using factorization and the light front approach [57, 58]. In another interesting work, the analysis of factorizable \( \Xi^{++} \rightarrow \Sigma^{++}_{c} \bar{K}^{(*)0} \) decays was carried out using the covariant confined quark model (CCQM) [59]. The theoretical interpretation of the experimentally favored decay chain \( \Xi^{++} \rightarrow \Sigma^{++}_{c} (\rightarrow \Lambda^{+}_{c}\pi^{+}) + \bar{K}^{(*)0} (\rightarrow K^{-}\pi^{+}) \) due to the dominant branching ratios of the daughter decays is first presented in [60]. In addition, the short-distance and long-distance (W-exchange) contributions to the decay channels of Ξ_{cc} baryons are calculated more systematically using factorization and final-state interaction (FSI) rescattering, respectively [60]. The branching ratios of nonleptonic decays of the doubly heavy baryons are predicted in the perturbative QCD (pQCD) [61].

It may be emphasized that in heavy baryon decays, unlike meson decays, the W-exchange contributions could be as important as factorizable for being free from helicity and color suppression [62–71]. In fact, many of the observed charm baryon decays receive contributions solely from W-exchange diagrams. Therefore, in the present work, we give preliminary estimates of W-exchange (pole) contributions using the pole model. To obtain the factorization contributions, we use the nonrelativistic quark model (NRQM) [72] and...
The charm changing two-body nonleptonic decays of (doubly heavy) baryons, emitting pseudoscalar ($P$) meson, proceed through usual current $\otimes$ current effective weak Hamiltonian,

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ud}V_{us}^* \left[ c_1(\bar{d}u)_{V-A}(\bar{c}s)_{V-A} + c_2(\bar{s}d)_{V-A}(\bar{u}c)_{V-A} \right]_{(\Delta C=\Delta S=-1)} + \right. $$

$$\left. V_{ud}V_{cd}^* \left[ c_1\{(\bar{s}c)_{V-A}(\bar{u}s)_{V-A} - (\bar{d}c)_{V-A}(\bar{u}d)_{V-A}\} + \right. $$

$$\left. c_2\{(\bar{c}v)_{V-A}(\bar{s}s)_{V-A} - (\bar{u}c)_{V-A}(\bar{d}d)_{V-A}\} \right]_{(\Delta C=-1, \; \Delta S=0)} - \right. $$

$$\left. V_{us}V_{cd}^* \left[ c_1(\bar{d}c)_{V-A}(\bar{u}s)_{V-A} + c_2(\bar{u}c)_{V-A}(\bar{d}s)_{V-A}\right]_{(\Delta C=\Delta S=-1)} \right\};$$

(1)

where $V_{ij}$ denote the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and $(\bar{q}_i q_j)_{V-A} \equiv \bar{q}_i \gamma_\mu (1 - \gamma_5) q_j$, the weak $V-A$ current. The Hamiltonian consists of CKM-favored ($\Delta C = \Delta S = -1$), CKM-suppressed ($\Delta C = -1, \Delta S = 0$) and CKM-doubly-suppressed ($\Delta C = -1, \Delta S = -1$) decay modes. The QCD (Wilson) coefficients $c_1(\mu) = 1.2$, $c_2(\mu) = -0.51$ at $\mu \approx m_c^2$ in the large $N_c$ limit are used in analysis (for a review see [77]). The coefficients $c_1$ and $c_2$ may be treated as free parameters for being affected by nonfactorizable contributions. In general, the transition amplitude can be expressed in terms of reduced matrix element for $B_i(\frac{1}{2}^+, p_i) \to B_f(\frac{1}{2}^+, p_f) + P_k(0^-, q)$ decay process:

$$A(B_i \to B_f P) \equiv \langle B_f(p_f)P_k(q) | \mathcal{H}_{\text{eff}}^f | B_i(p_i) \rangle = i\bar{u}_{B_f}(p_f)(A + B \gamma_5)u_{B_i}(p_i),$$

(2)

where $u_{B_i}$ represent Dirac spinors for initial and final ($\frac{1}{2}^+$) baryons $B_i$ and $B_f$. $A$ and $B$ denotes the parity-violating (PV) $s$-wave and parity-conserving (PC) $p$-wave amplitudes,
respectively.

The decay rate formula for \( B_i \rightarrow B_f P \) process is given by

\[
\Gamma(B_i \rightarrow B_f P) = \frac{p_c}{8\pi} \frac{E_f + m_f}{m_i} \left[ |A|^2 + \frac{E_f - m_f}{E_f + m_f} |B|^2 \right].
\] (3)

Here \( m_i \) and \( m_f \) are the masses of the initial and final state baryons. The magnitude of the three-momentum \( p_c \) of the final-state particles in the rest frame of \( B_i \) is

\[
p_c = \frac{1}{2m_i} \sqrt{[m_i^2 - (m_f - m_P)^2][m_i^2 - (m_f + m_P)^2]},
\]

where \( m_P \) is the mass of emitted pseudoscalar meson, and

\[
E_f \pm m_f = \frac{(m_i \pm m_f)^2 - m_P^2}{2m_i}.
\]

The corresponding asymmetry parameter is given by

\[
\alpha = \frac{2 \frac{p_c}{E_f + m_f} \text{Re}[A \ast B]}{(|A|^2 + \frac{p_c^2}{(E_f + m_f)^2} |B|^2)}.
\] (4)

To estimate the decay rate and asymmetry parameters we require to calculate numerically the amplitudes, \( A \) and \( B \).

### III. DECAY AMPLITUDES

The hadronic matrix element for the \( B_i \rightarrow B_f + P_k \) process can receive dominant contributions from factorization and pole processes, thus can be given as follows:

\[
\langle B_f P_k |H_W|B_i \rangle \equiv A_{\text{Pole}} + A_{\text{Fac}},
\] (5)

where \( A_{\text{Pole}} \) and \( A_{\text{Fac.}} \) denotes pole and factorization amplitudes, respectively. The pole diagrams mainly involves the W-exchange process contributions that are evaluated using the pole model framework \([63]\). In the pole model, the weak and strong vertices are separated by introduction of a set of intermediate states into the decay process. It may also be noted that factorization may be considered as a correction to pole contributions where \( t \)–channel pole process is equivalent to the tree-level diagram i.e. factorizable process. The contribution of both pole and factorization processes can be summed up in terms of \( s \)-wave (PV) and \( p \)-wave (PC) amplitudes. We wish to point out that we have ignored the relative strong phases involved in the decay amplitudes in our calculation for being difficult to estimate in the present scenario, however, such phases can contribute to some of the \( CP \)-violating asymmetries.
A. Pole amplitudes

The decay amplitude, $A_{\text{Pole}}$, can be calculated from the reduced matrix element

$$\langle B_f | H | B_i \rangle = \bar{u}_{B_i} (A + \gamma_5 B) u_{B_f}, \quad (6)$$

between two $\frac{1}{2}^+$ baryon states expressed in terms of PV and PC amplitudes, $A$ and $B$, receptively. The baryonic decay in pole model involves hadronic intermediate state which first is produced in the strong process and then go through a weak transition to the final baryon. Thus, $A$ and $B$ can simply be expressed in terms of masses, strong couplings and weak matrix elements. The pole amplitude consisting of contributions of $s$ and $u$ channels for positive-parity intermediate baryon ($J^P = \frac{1}{2}^+$) poles are denoted by $A_{\text{pole}}$ and $B_{\text{pole}}$,

$$A_{\text{pole}} = \sum_n \left[ \frac{g_{P_k} B_n b_{n_i}}{m_i + m_n} + \frac{g_{P_k} B_i b_{f_n}}{m_f + m_n} \right], \quad (7)$$

$$B_{\text{pole}} = -\sum_n \left[ \frac{g_{P_k} B_n a_{n_i}}{m_i - m_n} + \frac{g_{P_k} B_i a_{f_n}}{m_f - m_n} \right], \quad (8)$$

where $g_{ij}^k$ is the strong meson-baryon coupling constants. The weak baryon-baryon matrix elements $a_{ij}$ and $b_{ij}$ are defined as

$$\langle B_i | H_W | B_j \rangle = \bar{u}_{B_i} (a_{ij} + \gamma_5 b_{ij}) u_{B_j}. \quad (9)$$

As a preliminary study, we will restrict ourself to the contributions from parity-conserving amplitudes for the following reasons:

1. It is well known that the PV matrix element $b_{ij}$ vanishes in SU(3) flavor symmetry limit i.e. $\langle B_f P_k | H_{W}^{PV} | B_i \rangle = 0$. Since for charmed baryon decays $b_{ij} \ll a_{ij}$, the contributions of $\frac{1}{2}^- -$ poles are expected to be suppressed in $s$-wave amplitudes and dominant in $p$-wave amplitudes. Moreover, presence of sum of the baryon masses in the denominator further suppresses their contributions. Thus, consideration of PC terms only turns out to be a good approximation for heavy baryon decays.

2. Estimation of $\frac{1}{2}^- -$pole terms is nontrivial task in the present scenario as it involves knowledge of strong coupling constants and weak metrics elements of $\frac{1}{2}^- -$ baryons.

3. Furthermore, it has been argued by Fayyazuddin and Riazuddin [37] that, in the leading nonrelativistic approximation, one can ignore $J^P = \frac{1}{2}^-, \frac{3}{2}^-,...$ and higher (orbital) resonances in order to connect them to relevant the ground-state ($s$-wave) wave function in the overlap integral to satisfy the normalization condition: thus, only the PC amplitude survives.

B. Weak transitions

The flavor symmetric and quark model weak Hamiltonian [41, 67] involved in weak transitions for the quark-level process $q_i + q_j \rightarrow q_l + q_m$ is given by

$$H_W \simeq V_{ij} V_{jm}^* c_{m_c} (m_c) [B_{[l,m]} B_{i,j} H_{l,m}] , \quad (10)$$
here \( c_- = c_1 + c_2 \), and the antisymmetrization among the indices is represented by the brackets, \([\ ,\ ]\). The spurion transforms like \( H^{[1,3]}_{[2,4]} \). Equation (10) can be written in terms of the weak amplitude, \( a_W \), for CKM-favored and CKM-suppressed modes:

\[
H_W \simeq a_W [\bar{B}^{[i,j]}k B_{[i,m]k} H^{[q,m]}_{[i,j]}].
\] (11)

As discussed in the literature \([42, 70, 71]\), a rough estimate of \( a_W \) can be made based on symmetry arguments. However, SU(4) symmetry (being badly broken) ignores QCD enhancements due to hard gluon exchanges, contributing through \( c_- \), at corresponding mass scales, that will affect the weak transition.

To calculate numerical values of pole terms, the weak matrix element \( \langle \bar{B}_f | H^{PC}_W | B_i \rangle \) can be treated in leading nonrelativistic approximation \([37]\). Moreover, decays of doubly heavy baryons involve heavy-to-heavy transitions: thus, the use of nonrelativistic approximation suits the present analysis. Following the analysis of Riazuddin and Fayyazuddin \([37]\), we obtained the weak transition amplitudes for the charm baryons as a first approximation,

\[
\mathcal{M}^{PC} = \frac{G_F}{\sqrt{2}} V_{du} V_{cs} \sum_{i\neq j} (\gamma_i^- \alpha_j^+ + \alpha_i^+ \gamma_j^-) (1 - \sigma_i \cdot \sigma_j),
\] (12)

where \( S_i = \sigma_i / 2 \) are Pauli spinors representing the spin of \( i \)th quark. The operators \( \alpha_i^+ \) and \( \gamma_j^- \) convert \( d \rightarrow u \) and \( c \rightarrow s \), respectively \([76]\). The weak Hamiltonian can be obtained by using Fourier transformation of (12),

\[
H_W^{PC} = \frac{G_F}{\sqrt{2}} V_{du} V_{cs} \sum_{i \neq j} \alpha_i^+ \gamma_j^- (1 - \sigma_i \cdot \sigma_j) \delta^3(r),
\] (13)

which gives the first estimate of the pole terms. The spatial baryon wave function overlap, \( \delta^3(r) \equiv \langle \psi_f | \delta^3(r) | \psi_i \rangle \), is usually assumed to be flavor invariant such that

\[
\langle \psi_f | \delta^3(r) | \psi_i \rangle_c \approx \langle \psi_f | \delta^3(r) | \psi_i \rangle_s.
\] (14)

The relation (14) connects nonleptonic charmed baryon decays with hyperon decays in SU(4) symmetry. However, the SU(4) being badly broken due to the large mass difference between \( s \) and \( c \) quarks should yield a larger mismatch between strange and charm baryon wave function overlaps. Several methods have been proposed in the literature to address this issue by the introduction of a correction factor based on different arguments (for a summary, see Ref. \([71]\)). In the present analysis, we follow our previous work \([76]\) by treating \( |\psi(0)|^2 \) (based of dimensionality argument) as a flavor-dependent quantity. It may be noted that a reliable estimate of baryon ground-state wave function at the origin (at charm mass scale) can be obtained from, precisely known, experimental masses of baryons using hyperfine splitting, which in turn yields

\[
\frac{m_{\Sigma_c} - m_{\Lambda_c}}{m_{\Sigma} - m_{\Lambda}} = \frac{\alpha_s(m_c) m_s(m_c - m_u)|\psi(0)|^2_c}{\alpha_s(m_s) m_c(m_s - m_u)|\psi(0)|^2_s}.
\] (15)

Thus, we get

\[
\frac{|\psi(0)|^2_c}{|\psi(0)|^2_s} \approx 2.1,
\] (16)
for \( \frac{\alpha_s(m_c)}{\alpha_s(m_s)} \approx 0.53 \). Thus, the variation of flavor-dependent baryon spatial wave function overlap would lead to a substantial correction in branching ratios of doubly heavy baryons. The numerical results are discussed in Sec. IV.

C. Strong coupling constants

In general, meson-baryon strong couplings are obtained from the SU(4)-invariant strong Hamiltonian. In the present work, we follow a relatively accurate method used by Khanna and Verma \([73]\) to calculate the baryon-baryon-pseudoscalar (BB‘P) couplings. We extend their analysis to include SU(4)-breaking effects by employing the null result of Coleman and Glashow for the tadpole-type symmetry-breaking. The SU(4)-broken (SB) baryon-meson strong couplings are calculated by

\[
g_{BB'P}^{SB}(\text{SB}) = \frac{M_B + M'B'}{2M_N} \left( \sqrt{\frac{8}{3}} \frac{m_s - m_u}{m_c - m_u} \right) g_{BB'P}^{SB}(\text{Sym}),
\]

where \( g_{BB'P}^{SB}(\text{Sym}) \) is the value of SU(4) symmetric couplings \([74, 76]\). Effects of symmetry-breaking are such that it should yield larger values of strong couplings as compared to symmetric ones due to mass dependence, consequently, leading to larger pole contributions for heavy-baryon decays. The obtained absolute numerical values and expressions of relevant SB strong meson-baryon coupling constants are presented in Table I. The \( g_{BB'P}^{SB}(\text{SB}) \) are expressed in terms of \( g_D(=8.4) \) and \( g_F(=5.6) \) \([41, 78]\).

D. Factorization

The factorizable decay amplitudes (ignoring the scale factors) can be expanded in terms of the following reduced matrix elements:

\[
A^{Fac}(B_i \rightarrow B_f + P_k) \equiv \langle P_k(q)|A_\mu|0 > < B_f(p_f)|V^\mu + A^\mu|B_i(p_i) >.
\]

The baryon-baryon matrix elements of the weak currents can be expressed in terms of form factors \( f_i \) and \( g_i \) (as functions of \( q^2 \)) \([62, 63]\) as

\[
\langle B_f(p_f)|V_\mu|B_i(p_i) > = \bar{u}_f(p_f) \left[ f_1 \gamma_\mu - \frac{f_2}{m_i} i\sigma_{\mu\nu}q^\nu + \frac{f_3}{m_i} q^\mu \right] u_i(p_i),
\]

and

\[
\langle B_f(p_f)|A_\mu|B_i(p_i) > = \bar{u}_f(p_f) \left[ g_1 \gamma_\mu \gamma_5 - \frac{g_2}{m_i} i\sigma_{\mu\nu}q^\nu \gamma_5 + \frac{g_3}{m_i} q^\mu \gamma_5 \right] u_i(p_i).
\]

The decay constant \( f_P \) of the emitted pseudoscalar meson, \( P_k \), is defined as

\[
\langle P_k(q)|A_\mu|0 > = i f_P m_P.
\]
TABLE I: Expressions of strong-coupling constants and their absolute numerical values.

| Strong Couplings | Absolute values | Strong Couplings | Absolute values |
|------------------|-----------------|------------------|-----------------|
| $g_{D}^\Xi\Lambda$ | $(\frac{g_{D}}{\sqrt{2}} + \frac{g_{F}}{3\sqrt{2}})$ | 3.30 | $g_{D}^\Xi\Xi_{c}$ | $\sqrt{2}g_{D}$ | 32.60 |
| $g_{D}^{\Xi}\Lambda_{c}$ | $\frac{\sqrt{3}}{\sqrt{2}}(g_{D} - g_{F})$ | 1.60 | $g_{K}^{\Xi}\Omega_{c}^{0}$ | $-\frac{2g_{F}}{\sqrt{3}}$ | 17.80 |
| $g_{D}^{\Xi}\Sigma_{c}$ | $-(g_{D} + g_{F})$ | 11.00 | $g_{K}^{\Xi}\Xi_{c}$ | $2g_{D}$ | 47.20 |
| $g_{D}^{\Xi}\Xi_{c}$ | $-(g_{D} + g_{F})$ | 1.40 | $g_{D}^{\Xi}\Xi_{c}$ | $-(\sqrt{3}g_{D} + \frac{g_{F}}{\sqrt{3}})$ | 8.86 |
| $g_{D}^{\Xi}\Xi_{c}$ | $-(\frac{g_{D}}{\sqrt{2}} + \frac{g_{F}}{\sqrt{3}})$ | 0.01 | $g_{D}^{\Xi}\Xi_{c}$ | $-(g_{D}+g_{F})$ | 11.16 |
| $g_{D}^{\Xi}\Xi_{c}$ | $-\sqrt{3}g_{D} - \frac{g_{F}}{\sqrt{3}}$ | 8.60 | $g_{D}^{\Xi}\Xi_{c}$ | $\sqrt{2}(g_{D} - g_{F})$ | 15.46 |
| $g_{D}^{\Xi}_{c}\Sigma_{c}$ | $-(g_{D} + g_{F})$ | 1.40 | $g_{D}^{\Xi}_{c}\Sigma_{c}$ | $\sqrt{2}(g_{D} - g_{F})$ | 1.72 |
| $g_{K}^{\Xi}_{c}\Xi_{c}$ | $(\sqrt{2}g_{D} - 2\sqrt{2}g_{F})$ | 16.70 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $-\sqrt{2}g_{D}$ | 16.04 |
| $g_{K}^{\Xi}_{c}\Xi_{c}$ | $(g_{D} - 2g_{F})$ | 12.30 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $\sqrt{2}(g_{D} - g_{F})$ | 16.96 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $-\frac{2g_{F}}{\sqrt{3}}$ | 8.70 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $\frac{2g_{F}}{\sqrt{3}}$ | 16.33 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $g_{D}^{\Xi}_{c}\Xi_{c}$ | 10.80 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $2g_{D}$ | 43.91 |
| $g^{\Xi}_{c}\Xi_{c}$ | $0.12g_{D} + 0.08g_{F}$ | 1.50 | $g^{\Xi}_{c}\Xi_{c}$ | $(g_{D} - g_{F})$ | 10.82 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $-0.96g_{F}$ | 14.50 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $1.55g_{D}$ | 33.98 |
| $g^{\Xi}_{c}\Xi_{c}$ | $0.80(g_{D} - g_{F})$ | 8.40 | $g^{\Xi}_{c}\Xi_{c}$ | $0.77(g_{D} - g_{F})$ | 8.38 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $1.70g_{D} - 1.1g_{F}$ | 21.20 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $1.27g_{D}$ | 27.81 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $0.27g_{F}$ | 4.00 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $0.63(g_{D} - g_{F})$ | 6.66 |
| $g^{\Xi}_{c}\Xi_{c}$ | $0.60(g_{D} - g_{F})$ | 6.90 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $2g_{D}$ | 45.02 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $-\sqrt{2}g_{D}$ | 12.30 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $\Lambda^{p}(g_{D} + \frac{2g_{F}}{\sqrt{3}})$ | 7.37 |
| $g^{\Xi}_{c}\Xi_{c}$ | $(\sqrt{2}g_{D} - 2\sqrt{2}g_{F})$ | 17.40 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $\Sigma^{p}(g_{D} + \frac{2g_{F}}{\sqrt{3}})$ | 1.22 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $2g_{D}$ | 17.00 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $-\sqrt{2}(g_{D} + g_{F})$ | 15.47 |
| $g^{\Xi}_{c}\Xi_{c}$ | $g_{D}$ | 23.10 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $\sqrt{2}(g_{D} + \frac{2g_{F}}{\sqrt{3}})$ | 6.35 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $0.12g_{D}$ | 2.80 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $(\sqrt{3}g_{D} + \frac{g_{F}}{\sqrt{3}})$ | 8.87 |
| $g^{\Xi}_{c}\Xi_{c}$ | $0.77(g_{D} - g_{F})$ | 8.40 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $\sqrt{2}(g_{D} - g_{F})$ | 1.86 |
| $g_{D}^{\Xi}_{c}\Xi_{c}$ | $1.7g_{D}$ | 39.80 | $g_{D}^{\Xi}_{c}\Xi_{c}$ | $1.55g_{D} - 1.03g_{F}$ | 17.60 |
| $g^{\Xi}_{c}\Xi_{c}$ | $-\frac{2g_{F}}{\sqrt{3}}$ | 12.30 | $g^{\Xi}_{c}\Xi_{c}$ | 0 | 0 |
| $g^{\Xi}_{c}\Xi_{c}$ | $-\frac{2g_{F}}{\sqrt{3}}$ | 16.33 | $g^{\Xi}_{c}\Xi_{c}$ | 0 | 0 |
| $g^{\Xi}_{c}\Xi_{c}$ | $-\frac{2g_{F}}{\sqrt{3}}$ | 16.33 | $g^{\Xi}_{c}\Xi_{c}$ | 0 | 0 |
The factorizable amplitudes could be simplified to

\[ A_{1}^{fac} = -\frac{G_{F}}{\sqrt{2}}F_{C}f_p c_k [(m_i - m_f) f_1^{B_i,B_f}(m_P^2)], \]

\[ B_{1}^{fac} = \frac{G_{F}}{\sqrt{2}}F_{C}f_p c_k [(m_i + m_f) g_1^{B_i,B_f}(m_P^2)], \]

where the factor \( F_{C} \) is a product of appropriate CKM factors and Clebsch-Gordan (CG) coefficients and \( c_k \) are corresponding QCD coefficients.

We use the NRQM [72] and the HQET [73] to calculate the baryon-baryon transition form factors \( f_i \) and \( g_i \). In the NRQM calculations, the form factors are calculated in the Breit frame and include several corrections like the hard-gluon QCD contributions, the \( q^2 \) dependence of the form factors, and the wave-function mismatch. Later, in the heavy-quark sector, a \( 1/m_Q \) correction to the baryon-baryon transition form factors was introduced within the heavy-quark symmetry constraints using HQET. The obtained transition form factors are given in Table II.

We use the mixing scheme for \( \eta \) and \( \eta' \) mesons:

\[ \eta'(0.958) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \cos \phi_P + (s\bar{s}) \sin \phi_P, \]
\[ \eta(0.547) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \sin \phi_P - (s\bar{s}) \cos \phi_P, \]  \hspace{1cm} (22)

where \( \phi_P = \theta_{ideal} - \theta_{phy}^p \) and \( \theta_{phy}^p = -15.4^\circ \) [79]. The decay constants [79, 80] relevant for the present analysis are given as

\[ f_\pi = 131 \text{ MeV}, \ f_\eta = 133 \text{ MeV}, \ f_{\eta'} = 126 \text{ MeV}, \ f_K = 160 \text{ MeV}, \]
\[ f_D = 207.4 \text{ MeV} \text{ and } f_{D_s} = 255 \text{ MeV}. \]

IV. NUMERICAL RESULTS AND DISCUSSIONS

The preliminary results for the various decay channels of \( \Xi_{cc} \) are obtained as a sum of the factorization and the pole contributions to different PV and PC amplitudes. As mentioned before, SU(4) symmetry-breaking could be substantially large thus: the use of exact SU(4) symmetry could be questioned. Therefore, we include SU(4)-breaking effects in evaluating strong coupling constants as well weak transitions. First, we evaluate the factorizable amplitudes using NRQM-and HQET-based form factors for CKM-favored, CKM-suppressed and CKM-doubly-suppressed modes as listed in columns 3 and 4 of Tables I-II-VII. The flavor-independent pole amplitudes are calculated by using SU(4) broken strong coupling constants as shown in column 5 of Tables III-VII. The flavor-dependent effects in weak transition amplitudes through hyperfine splitting. The variation of the spatial baryon wave function overlap, \( |\psi(0)|^2 \), with flavor results in larger pole contributions. The numerical values flavor-dependent pole amplitudes of \( \Xi_{cc} \) decays in CKM-favored, CKM-suppressed and CKM-doubly-suppressed modes
are given in column 6 of Tables III–VII. It can be clearly seen that the pole contributions are enhanced by a factor of \( \sim 2 \) due to flavor-dependent effects caused by SU(4) breaking. Moreover, the increment in pole amplitudes could be viewed as variation of scale (charm to strange) by 2.

We wish to remark that a significant contribution to the parity-violating amplitudes may come from, \( \frac{1}{2}^- \), the lowest-lying negative-parity excited baryons, however, the estimation of such terms is far from simple, as discussed in [63–66, 71]. In addition, symmetry-based attempts have also been made to estimate their contributions for singly charmed baryons. Such attempts required sufficient experimental information on decays which is not available at present for doubly heavy \( \Xi_{cc} \) baryons. Therefore, we have only considered ground-state \( \frac{1}{2}^+ \) intermediate baryon pole terms as a first estimate of pole contributions. It may be noted that a large theoretical uncertainty in the lifetime of \( \Xi_{cc} \) states could be seen as another

| Transitions | Models | Form Factors |
|-------------|--------|--------------|
|             | [72][73] | \( f_1 \) | \( g_1 \) |
| \( \Xi_{cc}^{++} \rightarrow \Lambda_c^+ \) | NRQM | -0.35 | -0.19 |
|             | HQET  | -0.59 | -0.27 |
| \( \Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \) | NRQM | -0.39 | -0.96 |
|             | HQET  | -0.54 | -1.35 |
| \( \Xi_{cc}^{++} \rightarrow \Sigma_c^+ \) | NRQM | -0.27 | -0.68 |
|             | HQET  | -0.38 | -0.95 |
| \( \Xi_{cc}^{++} \rightarrow \Xi_c^+ \) | NRQM | -0.57 | -0.24 |
|             | HQET  | -0.74 | -0.29 |
| \( \Xi_{cc}^{++} \rightarrow \Xi_c^0 \) | NRQM | -0.37 | -0.78 |
|             | HQET  | -0.43 | -0.91 |
| \( \Xi_{cc}^{+} \rightarrow \Lambda_c^+ \) | NRQM | 0.35  | 0.19  |
|             | HQET  | 0.59  | 0.27  |
| \( \Xi_{cc}^+ \rightarrow \Sigma_c^{+} \) | NRQM | -0.27 | -0.68 |
|             | HQET  | -0.38 | -0.95 |
| \( \Xi_{cc}^+ \rightarrow \Sigma_c^0 \) | NRQM | -0.39 | -0.96 |
|             | HQET  | -0.54 | -1.35 |
| \( \Xi_{cc}^+ \rightarrow \Xi_c^0 \) | NRQM | -0.57 | -0.24 |
|             | HQET  | -0.74 | -0.29 |
| \( \Xi_{cc}^+ \rightarrow \Xi_c^0 \) | NRQM | -0.37 | -0.78 |
|             | HQET  | -0.43 | -0.91 |
TABLE III: Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}} V_{uq} V_{cq}^*$) for CKM-favored ($\Delta C = \Delta S = -1$) mode.

| Decays | Models | $A^{fac}$ | $B^{fac}$ | Flavor independent | Flavor dependent |
|--------|--------|-----------|-----------|-------------------|-----------------|
| $\Xi^{++}_{cc} \rightarrow \Sigma^+ D^+$ | NRQM | 0 | 0 | 0.101 | 0.212 |
| | HQET | 0 | 0 | 0.101 | 0.212 |
| $\Xi^{++}_{cc} \rightarrow \Xi^+_c \pi^+$ | NRQM | 0.110 | -0.250 | 0.372 | 0.782 |
| | HQET | 0.142 | -0.290 | 0.372 | 0.782 |
| $\Xi^{++}_{cc} \rightarrow \Sigma^{++}_c \bar{K}^0$ | NRQM | -0.042 | 0.520 | 0 | 0 |
| | HQET | -0.060 | 0.730 | 0 | 0 |
| $\Xi^{++}_{cc} \rightarrow \Xi^{'+}_c \pi^+$ | NRQM | 0.064 | -0.800 | 0 | 0 |
| | HQET | 0.076 | -0.930 | 0 | 0 |

$A$ and $B$ represent PV and PC amplitudes, respectively.

source of uncertainty in the results. We use $\tau_{\Xi^{++}_{cc}} = 300$ fs and $\tau_{\Xi^{+}_{cc}} = 100$ fs $^{58}$ to obtain the branching ratios in the present work.

After adding factorizable and pole contributions, we calculate the branching ratios and asymmetry parameters for two-body weak decays of doubly heavy $\Xi_{cc}$ baryons for the flavor-independent and flavor-dependent cases. To emphasize the importance of the W-exchange contribution to $\Xi_{cc}$ decays, we present our predictions for the branching ratios of $\Xi_{cc}$ decays receiving contributions only from pole amplitudes in Tables VIII and IX. The prediction for branching ratios receiving contributions from both the factorization and pole or factorization-only are given in Tables X-XII for CKM-favored, CKM-suppressed and CKM-doubly suppressed, respectively. We draw the following observations:

1. As expected, a large number of the $\Xi_{cc}$ decay channels receive contributions from the W-exchange process. Upon comparison with factorizable contributions, we find that the pole amplitudes are not only equipollent but also are dominant in several decays.
2. In the CKM-favored ($\Delta C = \Delta S = -1$) decay mode, most of the decays come from the pole diagrams alone, and only two of the decay channels come from factorization. The rest of the decays receive dominant pole contributions except for $\Xi^{+}_{cc} \rightarrow \Xi^{0}_c \pi^+$. The order of the branching ratios for all the decays range from $10^{-2}$ to $10^{-5}$ for flavor-independent case. While the inclusion of flavor-dependent effects enhances the pole contributions, consequently the branching ratios of dominant modes become $O(10^{-1}) \sim O(10^{-3})$.
3. The pole and factorizable amplitudes can interfere constructively or destructively in decay modes with both, factorizable and pole, contributions. The pole and factorization amplitudes interfere constructively, in $\Xi^{++}_{cc} \rightarrow \Xi^+_c \pi^+$, $\Xi^{+}_{cc} \rightarrow \Xi^{0}_c \pi^+$ and $\Xi^{+}_{cc} \rightarrow \Sigma^+_c \bar{K}^0$.
TABLE IV: Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}} V_{uq} V_{cq}^*$) for the CKM-favored $(\Delta C = \Delta S = -1)$ mode.

| Decays          | Models       | Factorization | $A^{fac}$ | $B^{fac}$ | Flavor independent | Flavor dependent |
|-----------------|--------------|---------------|-----------|-----------|--------------------|-----------------|
| $\Xi^{+}_{cc} \rightarrow \Lambda^{0} D^{+}$ | NRQM         | 0             | 0         | 0         | 0.082              | 0.172           |
|                 | HQET         | 0             | 0         | 0         |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Sigma^{+} D^{0}$ | NRQM         | 0             | 0         | 0         | 0.119              | 0.249           |
|                 | HQET         | 0             | 0         | 0         |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Sigma^{0} D^{+}$ | NRQM         | 0             | 0         | 0         | 0.156              | 0.327           |
|                 | HQET         | 0             | 0         | 0         |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{0} D^{+}$  | NRQM         | 0             | 0         | 0         | -0.114             | -0.239          |
|                 | HQET         | 0             | 0         | 0         |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \pi^{0}$ | NRQM         | 0.043         | -0.102    | -0.407    | -0.854            |
|                 | HQET         | 0.072         | -0.144    |            |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \pi^{0}$ | NRQM         | 0             | 0         | -0.562    | -1.179            |
|                 | HQET         | 0             | 0         | -1.179    |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \eta$  | NRQM         | 0.211         | 0.444     | 0         | 0.240              | 0.504           |
|                 | HQET         | 0             | 0         | 0.240     |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \eta$  | NRQM         | 0             | 0         | 0.353     | 0.741             |
|                 | HQET         | 0             | 0         | 0.353     |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \eta'$ | NRQM         | 0             | 0         | -0.349    | -0.733            |
|                 | HQET         | 0             | 0         | -0.349    |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \eta'$ | NRQM         | 0             | 0         | -0.097    | -0.205            |
|                 | HQET         | 0             | 0         | -0.097    |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \eta'$ | NRQM         | 0.110         | -0.250    | -0.422    | -0.887            |
|                 | HQET         | 0.143         | -0.290    | -0.887    |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Xi^{+} \eta'$ | NRQM         | 0.064         | -0.802    | 0.299     | 0.628             |
|                 | HQET         | 0.080         | -0.940    | 0.628     |                    |                 |
| $\Xi^{+}_{cc} \rightarrow \Omega^{0} K^{+}$ | NRQM         | -0.030        | 0.370     | -0.291    | -0.612            |
|                 | HQET         | -0.042        | 0.515     | -0.612    |                    |                 |
TABLE V: Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}} V_{uv} V_{cq}^*$) for the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) mode.

| Decays | Models | Factorization | Pole Amplitude |
|--------|--------|---------------|----------------|
|        |        | $A^{fac}$ | $B^{fac}$ | Flavor independent | Flavor dependent |
| $\Xi_{cc}^{++} \rightarrow pD^+$ | NRQM | 0 | 0 | 0.087 | 0.182 |
| | HQET | 0 | 0 | | |
| $\Xi_{cc}^{++} \rightarrow \Sigma^+ D_s^+$ | NRQM | 0 | 0 | 0.099 | 0.207 |
| | HQET | 0 | 0 | | |
| $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ \pi^+$ | NRQM | 0.078 | -0.190 | 0.322 | 0.676 |
| | HQET | 0.131 | -0.270 | | |
| $\Xi_{cc}^{++} \rightarrow \Xi_c^+ K^+$ | NRQM | 0.150 | -0.320 | 0.354 | 0.743 |
| | HQET | 0.190 | -0.380 | | |
| $\Xi_{cc}^{++} \rightarrow \Xi_c^+ K^+$ | NRQM | 0.090 | -1.060 | 0 | 0 |
| | HQET | 0.100 | -1.230 | | |
| $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ \pi^+$ | NRQM | 0.022 | -0.280 | 0 | 0 |
| | HQET | 0.030 | -0.400 | | |
| $\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \eta$ | NRQM | 0.042 | -0.530 | 0 | 0 |
| | HQET | 0.062 | -0.730 | | |
| $\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \eta'$ | NRQM | -0.017 | 0.170 | 0 | 0 |
| | HQET | -0.023 | 0.230 | | |
| $\Xi_{cc}^{++} \rightarrow \Sigma_c^+ \pi^+$ | NRQM | 0.050 | -0.690 | 0 | 0 |
| | HQET | 0.080 | -0.960 | | |

decay channels, however, these interfere destructively in $\Xi_{cc}^{+} \rightarrow \Lambda_c^+ K^0$ and $\Xi_{cc}^{+} \rightarrow \Xi_{c}^{0} \pi^+$ decays. Because of flavor dependence, branching ratios of the most dominant modes: $B(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+} \pi^+)$, $B(\Xi_{cc}^{+} \rightarrow \Xi_{c}^{0} \pi^+)$, and $B(\Xi_{cc}^{+} \rightarrow \Xi_{c}^{+} \pi^0)$ are enhanced by an order of magnitude, and the last decay comes from W-exchange diagrams only. The large decay width of $\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+} \pi^+$ decay makes it the best candidate to look out for in experimental searches.

4. The factorization contributions obtained from NRQM and HQET differ owing to the difference in form factors. The results based on HQET, in general, have larger values.

5. In the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) decay mode, the most of the dominant decays receive contributions from both pole and decay amplitudes via their constructive inference. The flavor-dependent branching ratios of such decay channels are $\mathcal{O}(10^{-2}) \sim \mathcal{O}(10^{-3})$ with a few exceptions. However, the pole-only decays have branching ratios of $\mathcal{O}(10^{-3}) \sim \mathcal{O}(10^{-5})$. The most dominant decays in this mode are: $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ \pi^+$,
TABLE VI: Decay amplitudes (in units of $\frac{\sqrt{2}}{\sqrt{3}} V_{uq} V_{cq}^*$) for the CKM-suppressed $(\Delta C = -1, \Delta S = 0)$ mode.

| Decays                  | Models  | Factorization | Pole Amplitude |
|-------------------------|---------|---------------|----------------|
|                         | [72][73] | $A^{fac}$ | $B^{fac}$ | Flavor independent | Flavor dependent |
| $\Xi_{cc}^{+} \rightarrow pD^0$ | NRQM    | 0          | 0          | $-0.111$          | $-0.234$        |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow nD^+$ | NRQM    | 0          | 0          | 0.198             | 0.416          |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Lambda^0 D_s^+$ | NRQM    | 0          | 0          | 0.056             | 0.117          |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Sigma^0 D_s^+$ | NRQM    | 0          | 0          | 0.070             | 0.147          |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Lambda^0 \pi^0$ | NRQM    | $-0.022$   | 0.054      | $-0.228$          | $-0.478$        |
|                         | HQET    | $-0.037$   | 0.077      |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Lambda^+ \eta$ | NRQM    | $-0.044$   | 0.010      | 0.194             | 0.407          |
|                         | HQET    | $-0.074$   | 0.144      |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Lambda^+ \eta'$ | NRQM    | $-0.018$   | 0.034      | $-0.159$          | $-0.333$        |
|                         | HQET    | $-0.028$   | 0.046      |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Xi^+ K^0$ | NRQM    | 0          | 0          | $-0.710$          | $-1.49$         |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Xi^+ K^0'$ | NRQM    | 0          | 0          | $-0.249$          | $-0.523$        |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Xi^0 K^+$ | NRQM    | 0.150      | $-0.330$   | $-0.352$          | $-0.739$        |
|                         | HQET    | 0.190      | $-0.380$   |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Xi^0 K^+$ | NRQM    | 0.087      | $-1.060$   | $-0.249$          | $-0.523$        |
|                         | HQET    | 0.103      | $-1.230$   |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Sigma^{++} \pi^-$ | NRQM    | 0          | 0          | $-0.343$          | $-0.721$        |
|                         | HQET    | 0          | 0          |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Sigma^{+} \pi^0$ | NRQM    | 0.015      | $-0.200$   | 0.343             | 0.721           |
|                         | HQET    | 0.022      | $-0.275$   |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Sigma^+ \eta$ | NRQM    | 0.030      | $-0.370$   | 0                 | 0               |
|                         | HQET    | 0.043      | $-0.520$   |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Sigma^+ \eta'$ | NRQM    | $-0.011$   | 0.122      | 0                 | 0               |
|                         | HQET    | $-0.016$   | 0.171      |                   |                |
| $\Xi_{cc}^{+} \rightarrow \Sigma^0 \pi^+$ | NRQM    | 0.076      | $-0.971$   | 0.343             | 0.721           |
|                         | HQET    | 0.110      | $-1.360$   |                   |                |
TABLE VII: Decay amplitudes (in units of $\frac{G_F}{\sqrt{2}}V_{uq}V_{cq}^*$) for the CKM-doubly suppressed $(\Delta C = -\Delta S = -1)$ mode.

| Decays                  | Models | Factorization $A^{fac}$ | Pole Amplitude |
|-------------------------|--------|------------------------|----------------|
|                         |        | $B^{fac}$              | Flavor independent | Flavor dependent |
| $\Xi^{++}_{cc} \rightarrow pD_{s}^+$ | NRQM   | 0                      | 0.085          | 0.178          |
|                         | HQET   | 0                      | 0              | 0              |
| $\Xi^{++}_{cc} \rightarrow \Lambda^+_c K^+$ | NRQM   | 0.110                 | 0.308          | 0.647          |
|                         | HQET   | 0.180                 | 0              | 0              |
| $\Xi^{++}_{cc} \rightarrow \Sigma^{++}_{c} K^0$ | NRQM   | -0.042                | 0              | 0              |
|                         | HQET   | -0.059                | 0              | 0              |
| $\Xi^{++}_{cc} \rightarrow \Sigma^+_c K^+$ | NRQM   | -0.004                | 0              | 0              |
|                         | HQET   | -0.005                | 0              | 0              |
| $\Xi^{+}_{cc} \rightarrow nD_{s}^+$ | NRQM   | 0                      | -0.085         | -0.178         |
|                         | HQET   | 0                      | 0              | 0              |
| $\Xi^{+}_{cc} \rightarrow \Lambda^+_c K^0$ | NRQM   | 0.043                 | 0.308          | 0.647          |
|                         | HQET   | 0.072                 | 0              | 0              |
| $\Xi^{+}_{cc} \rightarrow \Sigma^+_c K^0$ | NRQM   | -0.030                | 0              | 0              |
|                         | HQET   | -0.042                | 0              | 0              |
| $\Xi^{+}_{cc} \rightarrow \Sigma^0_{c} K^+$ | NRQM   | 0.104                 | 0              | 0              |
|                         | HQET   | 0.150                 | 0              | 0              |

$\Xi^{++} \rightarrow \Xi^{+}_{c} K^+$ and $\Xi^{+} \rightarrow \Sigma^0_{c} \pi^+$.

6. The decays, $\Xi^{+}_{cc} \rightarrow \Xi^0_{c} K^+$ and $\Xi^{+}_{cc} \rightarrow \Xi^0_{c} K^+$, present an interesting case of destructive interference between pole and factorization terms. It is worth noting that in $\Xi^{+}_{cc} \rightarrow \Xi^0_{c} K^+$ decay pole and factorization contributions to PC amplitudes are roughly comparable, while the PC factorization amplitude in $\Xi^{+}_{cc} \rightarrow \Xi^0_{c} K^+$ is predominant. Experimental searches for such decays will provide a useful test of the theory.

7. The decay channels in CKM-doubly suppressed $(\Delta C = \Delta S = -1)$ modes have branching ratios $O(10^{-4}) \sim O(10^{-6})$. Only two of the decays attain contributions from the pole alone. The decays having both pole and factorization contributions have larger branching ratios. It is interesting to note that decays with factorization-only contributions have branching ratios comparable to the decays with pole-only contributions.

8. We wish to point out that the flavor-dependent results enhance the contribution of pole terms roughly by a factor of 4, consequently, giving larger branching ratios. Thus, results based on flavor dependence and flavor-independent analyses provide a useful domain for experimental searches.
TABLE VIII: Branching ratios for the CKM-favored (ΔC = ΔS = −1) mode with only pole contributions. The branching ratios for an arbitrary lifetime can be obtained by using $(\frac{f_{π}^{0}}{300}) \times B(B_i \to B_f P)$ and $(\frac{f_{π}^{0}}{100}) \times B(B_i \to B_f P)$.

| Decays              | Flavor independent | Flavor dependent |
|---------------------|--------------------|------------------|
| $\Xi^{++}_{cc} \to \Sigma^{+}D^{+}$ | $2.0 \times 10^{-3}$ | $8.9 \times 10^{-3}$ |
| $\Xi^{0}_{cc} \to \Lambda^{0}D^{+}$ | $5.3 \times 10^{-4}$ | $2.4 \times 10^{-3}$ |
| $\Xi^{+}_{cc} \to \Sigma^{+}D^{0}$ | $9.4 \times 10^{-4}$ | $4.2 \times 10^{-3}$ |
| $\Xi^{+}_{cc} \to \Sigma^{0}D^{+}$ | $1.6 \times 10^{-3}$ | $7.0 \times 10^{-3}$ |
| $\Xi^{+}_{cc} \to \Xi^{0}_{c}D^{+}$ | $4.1 \times 10^{-4}$ | $1.8 \times 10^{-3}$ |
| $\Xi^{0}_{cc} \to \Xi^{+}_{c}\pi^{0}$ | $1.1 \times 10^{-2}$ | $5.0 \times 10^{-2}$ |
| $\Xi^{+}_{cc} \to \Xi^{+}_{c}\pi^{0}$ | $1.2 \times 10^{-3}$ | $5.4 \times 10^{-3}$ |
| $\Xi^{+}_{cc} \to \Xi^{+}_{c}\eta$ | $1.4 \times 10^{-3}$ | $6.4 \times 10^{-3}$ |
| $\Xi^{+}_{cc} \to \Xi^{+}_{c}\eta'$ | $2.2 \times 10^{-3}$ | $9.5 \times 10^{-3}$ |
| $\Xi^{+}_{cc} \to \Xi^{+}_{c}\eta''$ | $7.9 \times 10^{-4}$ | $3.5 \times 10^{-3}$ |
| $\Xi^{0}_{cc} \to \Omega^{0}_{c}K^{-}$ | $4.8 \times 10^{-3}$ | $2.1 \times 10^{-2}$ |
| $\Xi^{0}_{cc} \to \Sigma^{+}_{c}K^{-}$ | $2.2 \times 10^{-3}$ | $1.0 \times 10^{-2}$ |

To compare our results with other works, we present corresponding decay modes in Table XIII. We first compare our results with some of the very recent analyses of nonleptonic decays $\Xi_{cc}$ baryons based on the factorization scheme [58, 59]. W. Wang et al. [58] have given an analysis of weak decays of doubly heavy baryons in the quark-diquark picture using the light front approach. Their branching ratios for dominant CKM-favored modes $B(\Xi^{++}_{cc} \to \Xi^{(i)}_{c}\pi^{+})$ and $B(\Xi^{+}_{cc} \to \Xi^{(0)}_{c}\pi^{+})$ are of the order of a few percent. The $B(\Xi^{++}_{cc} \to \Xi^{+}_{c}\pi^{+})$ compares well with our result with no pole contribution (owing to a zero CG coefficient of baryon-baryon weak coupling for W-exchange pole terms). Despite the inclusion of dominant pole contributions and constructive interference between PC pole and factorization amplitudes, our result for the most dominant $B(\Xi^{++}_{cc} \to \Xi^{+}_{c}\pi^{+})$ is comparable to their result, i.e. 7.24%. Thus, the major difference in results is due to the different form factors used in both the works. As mentioned before, $\Xi^{+}_{cc} \to \Xi^{0}_{c}\pi^{+}$ and $\Xi^{+}_{cc} \to \Xi^{0}_{c}\pi^{+}$ represent peculiar cases of destructive and constructive interference between pole and factorization amplitudes, respectively. Therefore, the magnitude of the $B(\Xi^{+}_{cc} \to \Xi^{0}_{c}\pi^{+})$ in our case is smaller as compared to their branching 2.4% and vice versa for $B(\Xi^{+}_{cc} \to \Xi^{0}_{c}\pi^{+})$. Similarly, for CKM-suppressed and CKM-doubly suppressed modes, branching ratios are of same order when compared with Ref. [58], i.e. $O(10^{-3})$ and $O(10^{-4})$, respectively. In general, our results for

1 The weak coupling $a_{\pi^{+} \pi^{+} \pi^{+}}$ becomes zero following the operation of $(1 - \sigma_i \cdot \sigma_j)$ on the wave function using [37]: for details, see Ref. [37].
TABLE IX: Branching ratios for the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) and CKM-doubly suppressed ($\Delta C = -\Delta S = -1$) modes with only pole contributions.

| Decays | Branching ratios |
|--------|------------------|
|        | Flavor independent | Flavor dependent |
| ($\Delta C = -1, \Delta S = 0$) |                  |
| $\Xi^{++}_{cc} \to pD^+$   | $1.4 \times 10^{-4}$ | $6.0 \times 10^{-4}$ |
| $\Xi^{++}_{cc} \to \Sigma^+ D_s^+$ | $7.7 \times 10^{-5}$ | $3.4 \times 10^{-4}$ |
| $\Xi^+_{cc} \to pD^0$     | $7.6 \times 10^{-5}$ | $3.4 \times 10^{-4}$ |
| $\Xi^+_{cc} \to nD^+$     | $2.4 \times 10^{-4}$ | $1.1 \times 10^{-3}$ |
| $\Xi^+_{cc} \to \Lambda^0 D_s^+$ | $1.0 \times 10^{-5}$ | $4.5 \times 10^{-5}$ |
| $\Xi^+_{cc} \to \Sigma^0 D_s^+$ | $1.3 \times 10^{-5}$ | $5.6 \times 10^{-5}$ |
| $\Xi^+_{cc} \to \Xi^+ K^0$ | $7.0 \times 10^{-4}$ | $3.1 \times 10^{-3}$ |
| $\Xi^+_{cc} \to \Xi^+ K^0$ | $6.2 \times 10^{-3}$ | $2.7 \times 10^{-3}$ |
| $\Xi^+_{cc} \to \Sigma^+ \pi^-$ | $2.3 \times 10^{-4}$ | $1.0 \times 10^{-3}$ |
| ($\Delta C = -\Delta S = -1$) |                  |
| $\Xi^{++}_{cc} \to pD_s^+$ | $5.7 \times 10^{-6}$ | $2.5 \times 10^{-5}$ |
| $\Xi^{+}_{cc} \to nD_s^+$ | $1.9 \times 10^{-6}$ | $8.4 \times 10^{-6}$ |

branching ratios including both pole and factorization amplitudes are larger than their values as expected. The decay $\Xi^{++}_{cc} \to \Sigma^+_c \bar{K}^*0$ is first figured as a four-body process in Ref. [60], which is predicted to be one of the most dominant modes. Thomas Gutsche et al. [59] have analyzed weak decay of $\Xi^{++}_{cc}$ as decay chain $\Xi^{++}_{cc} \to \Sigma^+_c (\rightarrow \Lambda^+_c \pi^+) + \bar{K}^*0 (\rightarrow K^- \pi^+)$, which is expected to be experimentally favored due to the dominant branching ratios of the daughter decays. The $\Xi^{++}_{cc} \to \Sigma^+_c \bar{K}^{(*)0}$ decays are studied using the factorization scheme in CCQM. The obtained branching ratio: $\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^+_c \bar{K}^0) = 1.5\%$ at 300 fs, is of the same order when compared with our result. Other than the factorization scheme, the nonperturbative long-distance (W-exchange) contributions to $\Xi_{cc}$ decays have been calculated by Yu et al. [60]. The rescattering mechanism of FSIs, which has been ignored in the present work, is used to evaluate long-distance contributions. Authors have used the one-particle exchange method, where FSI is assumed to be dominated by rescattering of intermediate states [81]. Thus, the amplitude is expressed in terms of strong coupling (of particles on mass shell) and form factor (for exchanged baryons that are off mass shell). Here, also, the branching ratios in case of the CKM-favored and CKM-suppressed modes for factorizable decay channels (see Table XIII) are of the same order as compared to our results. However, their branching ratios for (pole-only) $\Xi^+_{cc} \to \Sigma^+_c K^-$ and $\Xi^{++}_{cc} \to pD^+$ decays are smaller by an order of magnitude as compared to our results for flavor-independent case. The difference in results may be attributed mainly to distinctive approaches. Although all the results compared here are based on different models/approaches, but they agree at least on the order of
### TABLE X: Branching ratios for the CKM-favored ($\Delta C = \Delta S = -1$) mode including factorization and pole contributions.

| Decays                  | Models | Flavor independent | Flavor dependent | Asymmetries ($\alpha$) |
|-------------------------|--------|--------------------|------------------|-------------------------|
| $\Xi^{++} \rightarrow \Xi^{++} \pi^+$ | NRQM   | $7.8 \times 10^{-2}$ | $15.1 \times 10^{-2}$ | $-0.997$ | $-0.856$ |
|                         | HQET   | $10.9 \times 10^{-2}$ | $18.5 \times 10^{-2}$ | $-0.991$ | $-0.942$ |
| $\Xi^{++} \rightarrow \Sigma^{++} \pi^0$ | NRQM   | $2.8 \times 10^{-2}$ | -                | $-0.760$ | -       |
|                         | HQET   | $5.5 \times 10^{-2}$ | -                | $-0.760$ | -       |
| $\Xi^{++} \rightarrow \Xi^{+} \pi^+$  | NRQM   | $6.4 \times 10^{-2}$ | -                | $-0.780$ | -       |
|                         | HQET   | $8.8 \times 10^{-2}$ | -                | $-0.780$ | -       |
| $\Xi^{+} \rightarrow \Lambda^{+} \pi^0$ | NRQM   | $6.0 \times 10^{-3}$ | $2.7 \times 10^{-2}$ | $0.927$ | $0.504$ |
|                         | HQET   | $8.3 \times 10^{-3}$ | $2.7 \times 10^{-2}$ | $0.964$ | $0.785$ |
| $\Xi^{+} \rightarrow \Xi^{0} \pi^+$ | NRQM   | $1.3 \times 10^{-2}$ | $2.7 \times 10^{-2}$ | $0.552$ | $0.996$ |
|                         | HQET   | $2.1 \times 10^{-2}$ | $3.3 \times 10^{-2}$ | $0.341$ | $0.972$ |
| $\Xi^{+} \rightarrow \Xi^{0} \pi^+$ | NRQM   | $3.3 \times 10^{-2}$ | $5.9 \times 10^{-2}$ | $-0.653$ | $-0.502$ |
|                         | HQET   | $4.7 \times 10^{-2}$ | $7.2 \times 10^{-2}$ | $-0.647$ | $-0.535$ |
| $\Xi^{+} \rightarrow \Sigma^{+} \pi^0$ | NRQM   | $1.3 \times 10^{-2}$ | $2.8 \times 10^{-2}$ | $-0.483$ | $-0.336$ |
|                         | HQET   | $2.0 \times 10^{-2}$ | $3.8 \times 10^{-2}$ | $-0.543$ | $-0.404$ |

The magnitude of the doubly charmed baryon decays. These results could be of great importance for experimentalists for future searches.

In the present work, we have ignored the $CP$ asymmetries as they have not yet been established in charmed baryon decays. However, like heavy-flavor mesons decays, the heavy-baryon decays are also prone to $CP$ violation. Even though it is well established that non-factorizable diagrams like $W$-exchange/annihilation have a sizable impact on baryon decays, it would be a difficult task to establish $CP$ violation in charmed baryon decays as the $CP$ asymmetries originating from the Standard Model (SM) are very small or even zero [82, 83]. Moreover, the production of three-body final states with relatively larger branching ratios and many $CP$ observables will require large amount of experimental data. On the other hand, $CP$ asymmetries has already been probed in two-body $\Lambda_b$ decays [78]. The theoretical investigation based on pQCD approach [52] indicates the dominance of nonfactorizable contributions in addition to penguin amplitudes. Similar conclusions were made by theoretical estimates based on generalized factorization and symmetries [47, 48, 51, 84]. Obviously, measurements of the $CP$ asymmetries provide a good tool to probe interference between the SM and new physics.
TABLE XI: Branching ratios for the CKM-suppressed ($\Delta C = -1, \Delta S = 0$) mode including factorization and pole contributions.

| Decays | Models | Branching ratios | Asymmetries ($\alpha$) |
|--------|--------|------------------|------------------------|
| $\Xi_{cc}^{*+} \rightarrow \Lambda_c^+ \pi^+$ | NRQM | $3.2 \times 10^{-3}$ | Flavor independent | $-0.930$ |
| | HQET | $5.8 \times 10^{-3}$ | Flavor dependent | $-0.900$ |
| $\Xi_{cc}^{*+} \rightarrow \Xi_{c}^{++} K^+$ | NRQM | $5.1 \times 10^{-3}$ | Flavor independent | $-0.970$ |
| | HQET | $7.6 \times 10^{-3}$ | Flavor dependent | $-0.920$ |
| $\Xi_{cc}^{*+} \rightarrow \Xi_{c}^{++} K^+$ | NRQM | $4.4 \times 10^{-3}$ | Flavor independent | $-0.850$ |
| | HQET | $6.0 \times 10^{-3}$ | Flavor dependent | $-0.850$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{++} \pi^0$ | NRQM | $5.3 \times 10^{-4}$ | Flavor independent | $-0.700$ |
| | HQET | $1.0 \times 10^{-3}$ | Flavor dependent | $-0.690$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{++} \eta$ | NRQM | $1.3 \times 10^{-3}$ | Flavor independent | $-0.780$ |
| | HQET | $2.7 \times 10^{-3}$ | Flavor dependent | $-0.780$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{++} \eta'$ | NRQM | $5.7 \times 10^{-5}$ | Flavor independent | $-0.980$ |
| | HQET | $1.1 \times 10^{-4}$ | Flavor dependent | $-0.980$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{-} \pi^+$ | NRQM | $3.2 \times 10^{-3}$ | Flavor independent | $-0.690$ |
| | HQET | $6.3 \times 10^{-3}$ | Flavor dependent | $-0.690$ |
| $\Xi_{cc}^{*+} \rightarrow \Lambda_c^+ \pi^0$ | NRQM | $2.5 \times 10^{-4}$ | Flavor independent | $-0.625$ |
| | HQET | $3.4 \times 10^{-4}$ | Flavor dependent | $-0.840$ |
| $\Xi_{cc}^{*+} \rightarrow \Lambda_c^+ \eta$ | NRQM | $1.2 \times 10^{-4}$ | Flavor independent | $0.734$ |
| | HQET | $2.8 \times 10^{-4}$ | Flavor dependent | $0.277$ |
| $\Xi_{cc}^{*+} \rightarrow \Lambda_{c}^{+} \eta'$ | NRQM | $4.6 \times 10^{-5}$ | Flavor independent | $-0.829$ |
| | HQET | $7.0 \times 10^{-5}$ | Flavor dependent | $-0.983$ |
| $\Xi_{cc}^{*+} \rightarrow \Xi_{c}^{0} K^+$ | NRQM | $1.1 \times 10^{-3}$ | Flavor independent | $0.061$ |
| | HQET | $1.8 \times 10^{-3}$ | Flavor dependent | $0.052$ |
| $\Xi_{cc}^{*+} \rightarrow \Xi_{c}^{0} K^+$ | NRQM | $1.0 \times 10^{-3}$ | Flavor independent | $-0.950$ |
| | HQET | $1.4 \times 10^{-3}$ | Flavor dependent | $-0.940$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{+} \pi^0$ | NRQM | $5.8 \times 10^{-4}$ | Flavor independent | $-0.290$ |
| | HQET | $7.7 \times 10^{-4}$ | Flavor dependent | $-0.351$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{+} \eta$ | NRQM | $2.3 \times 10^{-4}$ | Flavor independent | $-0.780$ |
| | HQET | $4.5 \times 10^{-4}$ | Flavor dependent | $-0.780$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{+} \eta'$ | NRQM | $1.0 \times 10^{-5}$ | Flavor independent | $-0.980$ |
| | HQET | $1.9 \times 10^{-5}$ | Flavor dependent | $-0.980$ |
| $\Xi_{cc}^{*+} \rightarrow \Sigma_{c}^{0} \pi^+$ | NRQM | $3.7 \times 10^{-3}$ | Flavor independent | $-0.552$ |
| | HQET | $6.3 \times 10^{-3}$ | Flavor dependent | $-0.589$ |
TABLE XII: Branching ratios for the CKM-doubly suppressed ($\Delta C = -\Delta S = -1$) mode including factorization and pole contributions.

| Decays                  | Models   | Branching ratios          | Asymmetries ($\alpha$) |
|-------------------------|----------|---------------------------|-------------------------|
|                         |          | Flavor independent        | Flavor dependent        | Flavor independent | Flavor dependent |
| $\Xi_{cc}^{++} \to \Lambda_{c}^{+} K^{+}$ | NRQM     | $2.1 \times 10^{-4}$     | $3.8 \times 10^{-4}$   | -1.000           | -0.860           |
|                         | HQET     | $4.2 \times 10^{-4}$     | $6.2 \times 10^{-4}$   | -0.970           | -1.000           |
| $\Xi_{cc}^{++} \to \Sigma_{c}^{++} K^{0}$ | NRQM     | $7.6 \times 10^{-5}$     | -                       | -0.760           | -                |
|                         | HQET     | $1.5 \times 10^{-4}$     | -                       | -0.760           | -                |
| $\Xi_{cc}^{+} \to \Sigma_{c}^{+} K^{+}$  | NRQM     | $2.3 \times 10^{-4}$     | -                       | -0.760           | -                |
|                         | HQET     | $4.6 \times 10^{-4}$     | -                       | -0.760           | -                |
| $\Xi_{cc}^{+} \to \Lambda_{c}^{+} K^{0}$  | NRQM     | $2.5 \times 10^{-5}$     | $7.2 \times 10^{-5}$   | -0.802           | -0.510           |
|                         | HQET     | $3.9 \times 10^{-5}$     | $8.9 \times 10^{-5}$   | -0.964           | -0.731           |
| $\Xi_{cc}^{+} \to \Sigma_{c}^{0} K^{+}$   | NRQM     | $1.3 \times 10^{-5}$     | -                       | -0.760           | -                |
|                         | HQET     | $2.5 \times 10^{-5}$     | -                       | -0.760           | -                |
| $\Xi_{cc}^{+} \to \Sigma_{c}^{0} K^{+}$   | NRQM     | $1.5 \times 10^{-4}$     | -                       | -0.760           | -                |
|                         | HQET     | $3.0 \times 10^{-4}$     | -                       | -0.760           | -                |

V. SUMMARY

The understanding of heavy-baryon decays is a long-standing problem as there does not exist a reliable approach for investigating the weak decays of heavy baryons as of yet. The dynamics of baryon decays, unlike meson decays, seems to get more complicated once they become heavier. Motivated by the recent observations, especially by LHCb, we have analyzed nonleptonic weak decays of doubly charmed baryons. The branching ratios of $\Xi_{cc}$ decays for CKM-favored and -suppressed modes are calculated using the factorization and pole model approaches. In the factorization scheme, we have obtained the form factors, $f_i$ and $g_i$, using nonrelativistic quark model [72] and heavy quark effective theory [73]. The nonfactorizable W-exchange diagrams, involving $\frac{1}{2}^+$ intermediate states, are calculated using the pole model approach. In the case of singly charmed baryon decays, it has been well established that the W-exchange contributions are comparable to factorization amplitudes. Therefore, the purpose of the present work is to give first estimates of W-exchange terms in doubly charmed $\Xi_{cc}$ decays to get a more comprehensive picture. As mentioned before, there has been some recent analysis involving doubly heavy baryons based mostly on factorization contributions only. However, the importance of W-exchange terms has also been emphasized in such works. Furthermore, we include SU(4)-breaking effects in meson-baryon strong couplings as well as in weak amplitudes. The results for the two scenarios, namely, flavor-independent and flavor dependent have been presented. We summarize our observations as follows:

1. We find that W-exchange amplitude contributes to the majority of the $\Xi_{cc}$ decays.
The branching ratios are compared for the lifetime $\tau = 3$, and thus, for Ref. [60], we have used $R_\tau = 0.3$.
In contrast to factorization contributions, the W-exchange contributions are not only comparable but also dominant in many decay channels. Thus, W-exchange contributions in $\Xi_{cc}$ decays cannot be ignored.

2. It is interesting to note that most of the CKM-favored decay channels receive contributions from W-exchange pole amplitudes only. The overall branching ratios in this mode range from $10^{-1} \sim 10^{-5}$, The $B(\Xi_{cc}^{+} \to \Xi_{c}^{+}\pi^{+})$ is as high as $\mathcal{O}(10^{-1})$ in flavor-dependent case. Several decays in this node have branching ratios of the order of a few percent, which could be of experimental interest.

3. We have shown that the pole and factorization amplitudes, depending on their signs, can interfere constructively and destructively. An experimental search of these decays could prove to be a useful test of theoretical models.

4. In CKM-suppressed and CKM-doubly suppressed modes, the dominant decays receive contributions from factorization as well as pole amplitudes indicating the importance of W-exchange processes. The branching ratios of dominant decay channels in the CKM-suppressed mode are $\mathcal{O}(10^{-2}) \sim \mathcal{O}(10^{-3})$.

5. The pole contributions are significantly enhanced due to the flavor-dependent factor. Thus, our results based on the NRQM and HQET picture alongside flavor-dependent W-exchange contributions provide a useful range to search for experimental evidence.

Experimental searches for heavy-baryon decays could help theorists understand the underlying dynamics of W-exchange processes in such decays. The importance of nonfactorizable contributions in CP asymmetries in heavy-baryon decays could prove to be a challenge to the theory as well as experiment. New measurements on of doubly heavy baryons are in future plans of several ongoing experiments at Fermilab and CERN. We hope that our results could prove to be useful in experimental searches for new modes.

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