Finite element modeling of the processes of elastic-plastic deformation of reclamation objects of the agro-industrial complex

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Annotation. To study the processes of nonlinear deformation of reclamation objects and engineering systems of the agro-industrial complex, taking into account the plastic stage of the used structural material, a finite element model was created based on a volumetric prismatic discretization element with quadrangular bases. The plastic stage of deformation of the applied structural material of the object is taken into account on the basis of the provisions of the deformation theory of plasticity. The plasticity matrix at the \((j+1)\)-th stage of sequential loading was compiled as a result of applying the operation of differentiating the stress tensor components accumulated over the \(j\) previous stages of sequential loading with respect to the strain tensor components. The stiffness matrix and the column of nodal forces of a prismatic discretization element with quadrangular bases at the \((j+1)\)-th stage of loading were obtained by minimizing the Lagrange functional. Numerical experiments on the calculation of a tubular outlet proved the adequacy of the developed finite element model and the accuracy of calculating the required strength parameters of the studied reclamation objects and other engineering systems of the agro-industrial complex, sufficient for engineering practice.

1. Introduction
Sustainable development of agricultural production in the Volgograd region and in the Lower Volga region as a whole is impossible without the revival of reclamation irrigation systems and the corresponding infrastructure. Achievement of this goal provides for the construction of new and reconstruction of existing reclamation systems and facilities.

To reduce the payback period of such systems, the optimal design of objects and systems for reclamation purposes becomes very important, which is unthinkable without the use of modern numerical methods of calculation [1, 2, 3, 4, 5], which, in particular, includes the finite element method (FEM) [6, 7, 8, 9]. Moreover, the application of this method at the present stage is based on the use as discretization elements, mainly, three-dimensional finite elements [10, 11, 12, 13, 14], as the most versatile, adapted for modeling both thin-walled fragments of reclamation structures (pipelines, tanks,
tanks) and thick-walled massive bodies (dams, retaining walls, joints and others). The most relevant computational algorithms based on the FEM are currently finite element models that take into account the plastic stage of the used structural material [15, 16, 17, 18]. The presented work presents a computational algorithm for studying the strength parameters of reclamation systems and objects, based on the modeling of the latter by three-dimensional prismatic finite elements with quadrangular bases, taking into account the plastic stage of the applied structural material.

2. Materials and methods

2.1. Object geometry

Suppose that the reference surface of the investigated reclamation object can be specified by the radius vector in a rectangular cartesian coordinate system with unit vectors \( \vec{e}_k \) \( (k = 1, 2, 3) \)

\[
\vec{R}^0 = \chi^p(\theta^p)\vec{e}_k,
\]

where \( x^k \) are cartesian coordinates, which are functions of generalized curvilinear coordinates \( \theta^p \) \( (p = 1, 2) \).

The differentiation procedure (1) by \( \theta^p \) can determine the local basis of point \( M^0 \) of the reference surface

\[
\vec{a}_p = \hat{R}_{,\theta^p}^0; \quad \vec{a}_s = \hat{\alpha}_1^0 \times \hat{\alpha}_2^0 / |\hat{\alpha}_1^0 \times \hat{\alpha}_2^0|.
\]

For point \( M^0_t \), spaced from point \( M \) at a distance of \( t \) along the perpendicular to the reference surface, the radius vector will have the form

\[
\vec{R}^{0_t} = \vec{R}^0 + t\vec{a}^0.
\]

To linearize the set task, in the overwhelming majority of cases, the method of successive loading of the studied reclamation object is used. In the process of implementing this method, point \( M^0_t \) will sequentially occupy new positions: after a series of \( j \) consecutive loads is \( M^t \) and after the \( (j+1) \)-th loading is \( M^{*t} \). The specified positions can be defined by radius vectors

\[
\vec{R}^t = \vec{R}^{0_t}; \quad \vec{R}^{*t} = \vec{R}^{0_t} + \vec{V}; \quad \vec{R}^{*t} = \vec{R} + \vec{W},
\]

where \( \vec{V} \) and \( \vec{W} \) are the displacement vectors of point \( M^0_t \) after \( j \) consecutive loads and after the \( (j+1) \)-th loading, respectively.

The vectors of bases of points \( M^0_t \), \( M^t \), and \( M^{*t} \) can be obtained by calculating the derivatives of (3) and (4) with respect to the generalized curvilinear coordinates \( \theta^p \) and \( t \)

\[
\vec{g}^{0_k} = \hat{R}_{,\theta^p}^{0_t}; \quad \vec{g}^{tk} = \hat{R}_{,\theta^p}^{0_t}; \quad \vec{g}^{*tk} = \hat{R}_{,\theta^p}^{0_t} \quad (k = 1, 2, 3).
\]

Subscripts 1, 2, 3 after the decimal point indicate that differentiation is performed along curvilinear coordinates \( \theta^1 \), \( \theta^2 \), and \( \theta^3 \).

Applying the scalar product operation to (5), one can obtain the covariant components of the metric tensors at points \( M^0_t \), \( M^t \) and \( M^{*t} \), respectively

\[
g^{0_{mn}} = \vec{g}^{0_m} \cdot \vec{g}^{0_n}; \quad g^{m_n} = \vec{g}^{m} \cdot \vec{g}^{n}; \quad g^{*_{mn}} = \vec{g}^{*m} \cdot \vec{g}^{*n}.
\]

The deformations of the investigated reclamation object at point \( M^0_t \) after a series of \( j \) successive loads and their increments after the \( (j+1) \)-th loading can be calculated by the formulas proposed by academician Sedov [19]

\[
\varepsilon_{mn} = (g^{0_{mn}} - g^{*_{mn}}) / 2; \quad \Delta \varepsilon_{mn} = (g^{*_{mn}} - g_{mn}) / 2.
\]

2.2. Physical relationships
When taking into account the plastic stage of deformation of the applied structural material, the deformation theory of plasticity or the theory of plastic flow is usually used. In this work, the deformation theory of plasticity was used, according to the hypotheses of which [20], we can write the following relation

$$\sigma_{mn} = \frac{2}{3} \sigma_i \varepsilon_{mn} - \frac{1}{3} I_1(\varepsilon) g_{mn} \left( \frac{2}{3} \sigma_i - K \right),$$

(8)

where $K = E/(1-2\nu)$; $\sigma_i$, $\varepsilon_i$ are stress and strain intensities determined by formulas

$$\sigma_i = \sqrt[3]{\frac{1}{2} S_{mn} S_{mn}}; \quad \varepsilon_i = \sqrt[3]{\frac{2}{3} E_{mn} E_{mn}}.$$

(9)

The co- and contravariant components of the stress and strain deviators included in (9) are calculated from the following dependences

$$S_{mn} = \sigma_{mn} - I_1(\sigma) g_{mn}/3; \quad S'^{mn} = \sigma'^{mn} - I_1(\sigma') g_{mn}/3;$$

$$E_{mn} = \varepsilon_{mn} - I_1(\varepsilon) g_{mn}/3; \quad E'^{mn} = \varepsilon'^{mn} - I_1(\varepsilon') g_{mn}/3,$$

(10)

where $I_1(\sigma) = \sigma_{mn} g_{mn} = \sigma_{mn} g_{mn}$ is the first invariant of the stress tensor; $I_1(\varepsilon) = \varepsilon_{mn} g_{mn} = \varepsilon_{mn} g_{mn}$ is the first invariant of the strain tensor.

The increments in stresses at the $(j+1)$-th stage of sequential loading can be expressed in terms of the increments in deformations at the same stage as follows

$$\Delta\sigma_{mn} = \frac{\partial \sigma_{mn}}{\partial \varepsilon_{i1}} \Delta\varepsilon_{i1} + \frac{\partial \sigma_{mn}}{\partial \varepsilon_{i2}} \Delta\varepsilon_{i2} + \frac{\partial \sigma_{mn}}{\partial \varepsilon_{i3}} \Delta\varepsilon_{i3} + \frac{\partial \sigma_{mn}}{\partial \varepsilon_{i2}} \Delta\varepsilon_{i2} + \frac{\partial \sigma_{mn}}{\partial \varepsilon_{i3}} \Delta\varepsilon_{i3} + \frac{\partial \sigma_{mn}}{\partial \varepsilon_{i3}} \Delta\varepsilon_{i3},$$

(11)

or in matrix form

$$\begin{bmatrix} \Delta\sigma_{mn} \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_{mn} \end{bmatrix},$$

(12)

$$\begin{bmatrix} \Delta\sigma_{mn} \end{bmatrix} = \begin{bmatrix} \Delta\sigma_{11} & \Delta\sigma_{12} & \Delta\sigma_{13} & \Delta\sigma_{22} & \Delta\sigma_{23} & \Delta\sigma_{33} \end{bmatrix}^T, \quad \begin{bmatrix} \Delta\varepsilon_{mn} \end{bmatrix} = \begin{bmatrix} \Delta\varepsilon_{11} & \Delta\varepsilon_{12} & \Delta\varepsilon_{13} & \Delta\varepsilon_{22} & \Delta\varepsilon_{23} & \Delta\varepsilon_{33} \end{bmatrix}^T,$$

$$\begin{bmatrix} \Delta\sigma_{mn} \end{bmatrix} = \begin{bmatrix} \Delta\varepsilon_{11} & 2\Delta\varepsilon_{12} & \Delta\varepsilon_{13} & 2\Delta\varepsilon_{22} & \Delta\varepsilon_{23} & 2\Delta\varepsilon_{33} \end{bmatrix}^T.$$

The plasticity matrix included in (12) is used to arrange the stiffness matrix of the prismatic discretization element at the $(j+1)$-th stage of loading.

3. Results

3.1. 3D prismatic bin with quadrangular bases

A universal element of discretization of a reclamation object, both thin-walled and in the form of a solid massive body, was a prism with quadrangular bases with nodes located at its vertices. The sought-for nodal unknowns of this bin are the components of the displacement vector of the $(j+1)$-th stage of sequential loading and the first-order partial derivatives of these components. In matrix form, the sought nodal unknowns in the global (superscript G) and local (L) coordinate systems can be represented as follows

$$\begin{bmatrix} W^G \end{bmatrix} = \begin{bmatrix} W_{11}^G \end{bmatrix}_{1=1}^{96},$$

(13)

$$\begin{bmatrix} W^L \end{bmatrix} = \begin{bmatrix} W_{11}^L \end{bmatrix}_{1=1}^{96},$$

(14)

where
\[
\begin{align*}
&\{Q^G_{1s32}\}^T = \{Q^i Q^j Q^k Q^l Q^m Q^n Q^p Q^q \cdots Q^s_{0i} Q^s_{0j} \cdots Q^s_{02} Q^s_{1i} \cdots Q^s_{1j} \cdots Q^s_{1s}\}; \\
&\{Q^L_{1s32}\}^T = \{Q^i Q^j Q^k Q^l Q^m Q^n Q^p Q^q \cdots Q^s_{\xi_{\xi}} Q^s_{\xi_{\eta}} \cdots Q^s_{\zeta_{\xi}} Q^s_{\zeta_{\eta}} \cdots Q^s_{\zeta_{\zeta}}\};
\end{align*}
\]

(15)

\[
Q = W^1, W^2, W^3; \quad -1 \leq \xi, \eta, \zeta \leq 1.
\]

Here, superscripts \(i, j, k, l\) denote nodes located at the vertices of the lower base, and subscripts \(m, n, p, s\) denote nodes located at the vertices of the upper base of the prismatic bin.

The matrix dependence can be established between (13) and (14)

\[
\{W^T\}_{1s96} = [T] \{W^G\}_{96s96},
\]

(16)

Local coordinate system \(-1 \leq \xi, \eta, \zeta \leq 1\) is used to organize the procedure for numerical integration over the volume of a prismatic sampling unit using Gaussian quadrature with a variable number of integration points.

The component of the displacement vector of the \((j + 1)\)th stage of the sequential loading of a point located in the inner region of the prismatic sampling unit can be expressed in terms of column (15) by means of the following interpolation dependence

\[
Q = \{\psi\}^T \{Q^L\}_{1s32}.
\]

(17)

where \(\{\psi\}^T_{1s32}\) is a row matrix whose elements are triadic products of third-order Hermitian polynomials.

Applying the differentiation operation to (17), we can obtain expressions for the derivatives of the components of the displacement vector of the \((j + 1)\)th stage of sequential loading

\[
\begin{align*}
&Q^n_{\psi_\theta} = \left(\{\psi_{\xi}\} [\psi_{\eta}] [\psi_{\zeta}][Q^L]\right); \\
&Q^n_{\eta} = \left(\{\psi_{\psi}\} [\psi_{\eta}][\psi_{\zeta}][Q^L]\right).
\end{align*}
\]

(18)

The stiffness matrix and the column of nodal forces of the discretization element in the form of a prism with quadrangular bases at the \((j + 1)\)th stage of sequential loading can be arranged using the procedure of minimizing the Lagrange functional

\[
\Phi_L = \int \left\{\Delta \varepsilon_{mn} \right\}^T \left(\left\{\sigma_{mn}\right\} + \left\{\Delta \sigma_{mn}\right\}\right) dV - \int \left\{\{W\}^T \left(\left\{P\right\} + \left\{\Delta P\right\}\right)\right\} dF,
\]

(19)

where \(\left\{\sigma_{mn}\right\}^T = \left\{\sigma_{11} \sigma_{12} \sigma_{13} \sigma_{22} \sigma_{23} \sigma_{33}\right\}\) are stresses accumulated over \(j\) stages of sequential loading; \(\{P\}, \{\Delta P\}\) is external surface load, also accumulated during \(j\) stages of loading and its increment at the \((j + 1)\)th stage of sequential loading.

The matrix-row of the components of the displacement vector in (19) at the \((j + 1)\)th stage of loading with the help of (17) can be represented by the matrix product

\[
\{W\}^T = \{W^L\}^T [A]^T,
\]

(20)

where

\[
[A] = \begin{bmatrix}
\{\psi\}^T & \{0\}^T & \{0\}^T \\
\{0\}^T & \{\psi\}^T & \{0\}^T \\
\{0\}^T & \{0\}^T & \{\psi\}^T
\end{bmatrix}_{1s32 1s32 1s32}.
\]

Expressing the increments of deformations at the \((j + 1)\)th stage of loading through the column of unknown unknowns

\[
\{\Delta \varepsilon_{mn}\} = [B]\{W^L\}^T,
\]

(21)
functional (19) can be represented in a form prepared for the minimization procedure

$$
\phi_l = \left\{ W^G \right\}_V \left[ T \right] \left[ B \right] \left[ C \right] [B] dV[T] W^G_1 + \left\{ W^G \right\}_F \left[ T \right] \left[ B \right] \left[ A \right] \left\{ \sigma_{mn} \right\}_d dV - \left\{ W^G \right\}_F \left[ T \right] \left[ A \right] \left\{ \Delta P \right\}_d dF - \left\{ W^G \right\}_F \left[ T \right] \left[ A \right] \left\{ P \right\} dF.
$$

(22)

Carrying out the procedure of minimization (22) by \( \left\{ W^G \right\}_F \), we can obtain the following matrix relation

$$
\left[ MG \right] \left\{ W^G \right\}_V = \left\{ f \right\} - \left\{ NR \right\},
$$

(23)

where \( \left[ MG \right] = \left[ T \right] \left[ B \right] \left[ C \right] [B] dV[T] \) is stiffness matrix of a prismatic sampling element with quadrangular bases at the \((j+1)\)-th stage of sequential loading; \( \left\{ f \right\} = \left[ T \right] \left[ A \right] \left\{ \Delta P \right\}_d dF \) is matrix-column of nodal forces of the discretization element at the \((j+1)\)-th stage of loading; \( \left\{ NR \right\} = \left[ T \right] \left[ B \right] \left\{ \sigma_{mn} \right\}_d dV - \left[ A \right] \left\{ P \right\} dF \) is column matrix of the Newton-Raphson correction.

4. Discussion

To assess the accuracy of calculating the required strength parameters of reclamation facilities and engineering systems of the agro-industrial complex, taking into account the plastic deformation of the applied structural material, a test problem was solved to study the stress-strain state of a tubular outlet made of carbon steel, loaded with various types of external surface load.

Various versions of the deformation diagram were used: with linear and nonlinear hardening. In the process of performing numerical experiments, the convergence of the computational process, the correspondence of the obtained values of strength parameters to the values corresponding to the equations of static equilibrium of the outlet were analyzed.

5. Conclusions

The analysis of the obtained numerical values of the sought parameters of the strength of the object under study made it possible to conclude that there is a convergence of the computational process when the grid of prismatic sampling elements with a quadrangular base increases and the number of stages of sequential loading increases.

At the controlled outlet points, the correspondence of the calculated values of the strength parameters to the values of these parameters obtained from the static equilibrium equations was recorded.

On the basis of the performed numerical experiments, the final conclusion was made about the adequacy of the developed finite element model and the accuracy of calculating the required strength parameters of the studied reclamation objects and engineering systems of the agro-industrial complex satisfactory for engineering practice.

6. Acknowledgments

The study was carried out with the financial support of the Russian Foundation for Basic Research and the Administration of the Volgograd Region in the framework of the research project No. 19-41-340005 r_a.

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