Proton Decay in a 6D $SO(10)$ model

L. COVI

Theory Division, CERN Department of Physics, Geneva, Switzerland

We present a study of proton decay in a supersymmetric $SO(10)$ gauge theory in six dimensions compactified on an orbifold. The dimension 5 proton decay operators are absent, but the dimension 6 are enhanced due to the presence of KK towers. We resum the KK modes up to the cut-off of the theory and find the rate for the dominant mode $p \rightarrow \pi^0 e^+$. We explore also the flavour dependence, due to the different localization of states in the extra dimensions and find that it is possible to distinguish the model from the usual 4D $SU(5)/SO(10)$ models.

Proton decay is still the main signature of Grand Unified Theories as it was realized more than 30 years ago\textsuperscript{1}. Unfortunately for GUT model-builders, it is still escaping observation..., but it is perhaps not surprising since the lifetime scales as the fourth power of the GUT scale and the experimental limits can be improved only at the cost of huge detectors. At present the most stringent experimental bounds come from the SuperKamiokande collaboration, that at this conference announced some new limits, e.g. $\tau(p \rightarrow e^+\pi^0) \geq 6.9 \times 10^{33}$ years and $\tau(p \rightarrow K^+\bar{\nu}) \geq 1.6 \times 10^{33}$ year\textsuperscript{2}. Such values exclude simple non-supersymmetric $SU(5)$ models\textsuperscript{a} and start to reduce the parameter space also of the supersymmetric version, depending on the flavour structure assumed (see e.g.\textsuperscript{3}).

In this talk I will describe the results for proton decay in a particular orbifold model computed in\textsuperscript{4}. I will try to argue both that the signal could really be around the corner in this case and also that if the signal is seen, we could perhaps have a chance to distinguish if the GUT model is of the 4 dimensional type or presents some extra-dimensional features.

\textsuperscript{a}Actually such models do not fare well with the unification of the gauge couplings either, as we all know, so probably excluding them is superfluous...
Our starting point is an $SO(10)$ gauge theory in 6D with $N = 1$ supersymmetry compactified on the orbifold $T^2/(Z_2 \times Z_2^{PS} \times Z_2^{GG})$. The theory has four fixed points, $O_I$, $O_{PS}$, $O_{GG}$ and $O_{fl}$, located at the four corners of a ‘pillow’ corresponding to the two compact dimensions. At $O_I$ only supersymmetry is broken whereas at the other fixed points, $O_{PS}$, $O_{GG}$ and $O_{fl}$, also the gauge group $SO(10)$ is broken to its three GUT subgroups $G_{PS} = SU(4) \times SU(2) \times SU(2)$, $G_{GG} = SU(5) \times U(1)$ and flipped $SU(5)$, $G_{fl} = SU(5)' \times U(1)'$, respectively. The intersection of all these GUT groups yields the standard model group with an additional $U(1)$ factor, $G_{SM} = SU(3) \times SU(2) \times U(1)_Y \times U(1)'_Y$, as unbroken gauge symmetry below the compactification scale. The field content of the theory is strongly constrained by imposing the cancellation of irreducible bulk and brane anomalies. We study the model proposed in [8], containing 3 16-plets $\psi_1$, $i = 1 \ldots 3$, as brane fields and 6 10-plets, $H_1, \ldots, H_6$, and 4 16-plets, $\Phi, \Phi^c, \phi, \phi^c$, as bulk hypermultiplets. Vacuum expectation values of $\Phi$ and $\Phi^c$ break the surviving $U(1)_B-L$. The electroweak gauge group is broken by expectation values of the anti-doublet and doublet $H_u/d$ contained in $H_1$ and $H_2$.

We choose the parities of $H_5, H_6$ and $\phi, \phi^c$ such that their zero modes

$$L = \begin{pmatrix} \nu_4 \\ e_4 \end{pmatrix}, \quad L^c = \begin{pmatrix} \nu_4^c \\ e_4^c \end{pmatrix}, \quad D^c = d_4^c, \quad D = d_4$$

act as a (vectorial) fourth generation of $d$-quarks and leptons and mix with the three generations of brane fields, located on the three branes where $SO(10)$ is broken to its three GUT subgroups. This leads to a characteristic pattern of mass matrices of the lopsided type as described in [8]. In particular the hierarchy and mixing angles for the quark sector can be accounted for and GUT relations do not hold for all the generations due to the presence of split multiplets. See [8] for the details and [4] for the explicit form of the mixing matrices.

2 Short review of 4D proton decay in supersymmetry

Proton decay arises from effective 4-fermion operators joining three quarks and a lepton, which are of dimension 6. It can therefore be a bit puzzling to hear about “dimension 4” or “dimension
5” operators, as happens in supersymmetric models. So I will stop a little and review the terminology before discussing the dominant contribution in our case.

In supersymmetric models, there are contributions to proton decay from superpotential terms, either renormalizable or obtained integrating out heavy states, from kinetic terms and also from supersymmetry breaking terms. The first type of contributions, are usually classified according to the dimension of the superpotential terms that break the baryon or lepton number. In general we have

- **dimension four operators:**

\[ W = \lambda L E^c + \lambda' L Q D^c + \lambda'' U^c D^c D^c; \]  \( \text{(2)} \)

they are renormalizable and give very rapid proton decay via an intermediate scalar squark (therefore the effective 4-fermion operator is just suppressed by \( \frac{\lambda \lambda''}{m_{susy}^2} \), where \( m_{susy}^2 \) is the typical squark mass) and have to be excluded by a discrete symmetry, usually R-parity.

- **dimension five operators:**

\[ W = \frac{1}{M_{H_C}} \left[ \frac{1}{2} Y_{qq} Y_{qf} QQQL + Y_{ue} Y_{ud} E^c U^c U^c D^c \right]; \]  \( \text{(3)} \)

they arise e.g. from integrating out the heavy colored Higgs triplets with mass \( M_{H_C} \) and allow the decay via a loop of scalar superpartners. They produce effective 4-fermion operators that scale as \( \frac{1}{M_{H_C} m_{susy}} \), are color antisymmetric and therefore must be also flavour non-diagonal, so that the dominant channel results in \( p \to K^+ \bar{\nu} \). Such operators give the dominant contribution in the simple supersymmetric SU(5) case, and they can give proton decay even above the present limit.

- **“real” dimension 6 operators, arising from the fermion kinetic terms and mediated by the gauge multiplet; they are therefore not of the chiral type and cannot be written as superpotential terms.** They do not involve sparticles and are therefore independent of \( m_{susy} \), apart for the weak dependence coming from determining the GUT scale by RGEs. As an example, in \( SU(5) \) we have the exchange of the \( X \) leptoquark gauge bosons with masses \( M_X \). The effective vertex is give by the Fermi-type coupling:

\[ \mathcal{L}_{eff} = - \frac{g_5^2}{2M_X^2} \epsilon_{\alpha\beta\gamma} u_{\alpha,i}^c \gamma^\mu Q_{\beta,i} \left[ \bar{e}_{\beta,j} \gamma_\mu Q_{\gamma,j} - \bar{d}_{\beta,k} \gamma_\mu L_k \right] + \text{h.c.}, \]  \( \text{(4)} \)

where \( i,j \) and \( k \) count the generations. With Fierz reordering, one can write the operators in the usual form as

\[ \mathcal{L}_{eff} = - \frac{g_5^2}{M_X^2} \epsilon_{\alpha\beta\gamma} \left[ \bar{e}_{\beta,j} u_{\alpha,i}^c Q_{\gamma,j} - \bar{d}_{\beta,k} \bar{d}_{\beta,i} Q_{\gamma,i} L_k \right] + \text{h.c.}. \]  \( \text{(5)} \)

Note that these operators scale as \( M_X^{-2} \) and are proportional to a gauge coupling and not a Yukawa; still some flavour dependence arises from the quark and lepton mixing matrices. Assuming quarks and leptons to be embedded into multiplets according to their hierarchy, the dominant decay channel is the one involving only first generation fermions, i.e. \( p \to \pi^0 e^+ \).

- **dimension 6 operators coming from supersymmetry breaking, e.g. the one mediated by intermediate Higgs scalars mixing via the soft SUSY breaking mass terms;** they are usually much more suppressed compared to the previous ones and are usually neglected.
3 6D proton decay

In our 6D orbifold model we have a residual 4D N=1 supersymmetry and we could have in principle dimension 4 and dimension 5 proton decay. Luckily in extra-dimensional models, we can exclude them both with appropriate choice of R-symmetry, forbidding the $\lambda', \lambda''$ couplings and also the $\mu$ term. Note also that in extra-dimensional models, the two heavy triplet Higgs bosons become massive together with their N=2 superpartners, not directly with each other, so the mixing between them is generated only by supersymmetry breaking. So in general the dominant contribution to proton decay in orbifold models comes from dimension 6 operators. Moreover in our model, since the first generation of $u$ quarks are confined to live on the fixed point where $SU(5)$ is unbroken, we can use $SU(5)$ language to describe the operators, even if the bulk symmetry is $SO(10)$.

3.1 Effective operator in 6D

In our orbifold model there is an important difference compared to the 4D case, we have to take into account the presence of a Kaluza-Klein tower of X gauge bosons with masses given by

$$M_X^2(n,m) = \frac{(2n+1)^2}{R_5^2} + \frac{(2m)^2}{R_6^2}$$

for $n, m = 0$ to $\infty$. The lowest possible mass is $M_X(0,0) = 1/R_5$, as given by the $SU(5)$ breaking parity. Note that if we define the 4D gauge coupling as the effective coupling of the zero modes, the KK modes interact more strongly by a factor $\sqrt{2}$ due to their bulk normalization.

To obtain the low energy effective operator, we have then to sum over the Kaluza Klein modes. We can define

$$\frac{1}{(M_{\chi}^{eff})^2} = 2 \sum_{n,m=0} M_X^2(n,m) = 2 \sum_{n,m=0} \frac{R_5^2}{(2n+1)^2 + \frac{R_5^2}{R_6^2}(2m)^2};$$

taking the limit $R_6/R_5 \to 0$, we regain the finite 5D result,

$$2 \sum_{n=0}^{\infty} \frac{R_5^2}{(2n+1)^2} = \frac{\pi^2 R_5^2}{4}.$$  

But in 6D the summation shows a logarithmic divergence; since our theory is non-renormalizable and valid only below the scale $M_*$, where the theory becomes strongly coupled and 6D gravity corrections are no more negligible, we regulate the sum with the cut-off $M_*$, and obtain formally

$$\frac{1}{(M_{\chi}^{eff})^2} \simeq \frac{\pi}{4} R_5 R_6 \left[ \ln (M_* R_5) + C \left( \frac{R_5}{R_6} \right) + O \left( \frac{1}{R_5/R_6 M_*} \right) \right].$$

In the case $R_5 = R_6 = 1/M_c$ the expression can be approximated by

$$\frac{1}{(M_{\chi}^{eff})^2} \simeq \frac{\pi}{4M_c^2} \left[ \ln \left( \frac{M_*}{M_c} \right) + 2.3 \right],$$

which agrees within 1% with explicit discrete sum for $M_* = 10 \ldots 50$.

\footnote{A small effect from the other gauge bosons can arise from brane derivative operators.}
4 Flavour structure

Another important difference in 6D is the non-universal coupling of the $\mathcal{X}$ gauge bosons. In fact, due to the parities and the $SO(10)$ breaking pattern, their wavefunctions must vanish on the fixed points $O_{PS}$ and $O_{RT}$, and therefore no coupling arises with the charm and top quark or to the brane states $d_3^e, d_4$ and $l_4, l_1^e$. We have in principle couplings to the bulk states $d_3^s, d_4$ and $l_4, l_1^e$, but in this case, the charge current interaction always mixes the light states with the heavy KK modes and it is therefore irrelevant for the low energy proton decay\cite{10}. So the kinetic coupling in Eqn. \cite{5} arises only for the 1st flavour eigenstate, not for all flavours as in the usual 4D case.

Proton decay involves only the light quark states and the operators containing the combinations $uud$ and $udd$. Starting in the basis where the up-quark Yukawa is diagonal, we have to rotate the down-type quarks and the leptons from the weak into the mass eigenstates and single out the contributions for the lightest generation. We have then

$$d_L = U_L^d d_L', \quad e_L = U_L^e e_L', \quad \nu_L = U_L^{\nu} \nu'_L, \quad d_R = U_R^d d_R', \quad e_R = U_R^e e_R',$$

where the prime denotes mass eigenstates. Since the up quarks are diagonal, $U_L^d$ coincides with the CKM-matrix. We can write the proton decay operators of Eqn. \cite{5} in mass eigenstates as

$$\mathcal{L}_{eff} = \frac{g_5^2}{(M_N^f)^2} \epsilon_{\alpha\beta\gamma} \left[ 2 \bar{e}_k^\ell \left( U_R^{e\ell} \right)_{k1} \bar{u}_{\alpha,1} d_{\beta,m} \left( U_L^d \right)_{1m} u_{\gamma,1} ight. \right.$$  

\left. + \bar{d}_{\alpha,j} \left( U_R^{d\ell} \right)_{11} \bar{u}_{\beta,1} \left( U_L^e \right)_{1j} e'_j - d_{\gamma,m} \left( U_L^e \right)_{1m} \left( U_L^e \right)_{1j} \right] + \text{h.c.} \quad (12)$$

Note again that due the orbifold construction only the first weak eigenstates couple to the $\mathcal{X}$ bosons, instead of all of them. So the proton decay in this 6D model has naturally different branching ratios compared to a 4D model \emph{with the same mixing ratios}.

5 Results

5.1 Bound on $M_c$

Considering the dominant channel $p \rightarrow e^+ \pi^0$, a lower bound on the compactification scale can be derived from the SuperKamiokande limit on the lifetime. We have in fact

$$\Gamma \simeq K_{had}^{\pi^0} \frac{\pi^2}{16} M_4^4 \left( \ln \left( \frac{M_s}{M_c} \right) + 2.3 \right)^2 \left[ 4V_{ud}^4 + \frac{\tilde{M}_2^{d,e}}{M_1^{d,e} + M_2^{d,e}} \frac{\tilde{M}_3^{d,e}}{M_1^{d,e} + M_2^{d,e} + M_3^{d,e}} \right], \quad (14)$$

where $K_{had}^{\pi^0} = 1.87 \times 10^{-40}$ sec$^{-1}$ contains a factor $M_s^{-4}$, the hadronic matrix element, kinematical factors, gauge coupling and the running of the operator from the high to the proton scale\cite{11}. With $M_s = 10^{17}$ GeV and $\tilde{M}_2^{d,e} = 0$, the limit $\tau \geq 6.9 \times 10^{33}$ yields $M_c \geq 0.89 \times 10^{16}$ GeV, not far from the 4D GUT scale.

5.2 Rates and branching ratios

We calculate the branching ratios for dimension 6 proton decay in our model and in some lopsided 4D models with a similar flavour structure discussed in\cite{11}. As expected, we find sizable differences in many channels, most noticeably in $p \rightarrow \mu^+ K^0$ due to the absence of direct coupling of the second generation weak eigenstates to the $\mathcal{X}$ gauge bosons. Even changing the unknown high energy parameters does not modify the picture: if we vary the heavy masses $\tilde{M}_j\tilde{M} = 0, \ldots, 1$ and $\tilde{\mu}_3^{d,e}/\mu_3 = 1, \ldots, 5$, we still find $BR(p \rightarrow \mu^+ K^0) \leq 5\%$. 

decay channel | Branching Ratios [%] | 6D SO(10) | SU(5) × U(1)$_F$
| | case I | case II | models A & B |
$e^+\pi^0$ | 75 | 71 | 54 |
$\mu^+\pi^0$ | 4 | 5 | $\leq 1$ |
$\bar{\nu}\pi^+$ | 19 | 23 | 27 |
$e^+K^0$ | 1 | 1 | $\leq 1$ |
$\mu^+K^0$ | $\leq 1$ | $\leq 1$ | 18 |
$\bar{\nu}K^+$ | $\leq 1$ | $\leq 1$ | $\leq 1$ |
$e^+\eta$ | $\leq 1$ | $\leq 1$ | $\leq 1$ |
$\mu^+\eta$ | $\leq 1$ | $\leq 1$ | $\leq 1$ |

Table 1: Resulting branching ratios and comparison with SU(5) × U(1)$_F$. See 4 for the details of the computation.

6 Conclusions

We have studied dimension 6 proton decay in a particular orbifold model, where the flavour eigenstates are placed at different fixed points. We have found two very interesting results. First the predicted decay rate is enhanced compared to 4D, even if still compatible with the experimental bounds. In fact, for a cut-off scale $M^* = 10^{17}$ GeV, we set a strong lower bound on the compactification scale $M_c \geq 0.89 \times 10^{16}$ GeV, which means that the range of validity of our model is more restricted also than that of 5D orbifold models\(^{12}\). Secondly, the peculiar flavour structure can give striking signatures in the branching ratios for proton decay, suppressing strongly the decay into $\mu^+K^0$. This is due to the localization of states in the extra-dimension and the consequent non-universal coupling of the GUT bosons to the fermions.

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