The Type-II Singular See-Saw Mechanism

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The singular see-saw mechanism is a variation of the see-saw mechanism whereby the right-chiral neutrino Majorana mass matrix is singular. Previous works employing the singular see-saw mechanism have assumed a vanishing left-chiral Majorana mass matrix. We study the neutrino spectrum obtained under a singular see-saw mechanism when the left-chiral neutrinos possess a non-zero Majorana mass matrix. We refer to this as the type-II singular see-saw mechanism. The resulting neutrino spectrum is found to be sensitive to the hierarchy of the Dirac and Majorana mass scales used and we explore the phenomenological consequences of the candidate hierarchies. The compatibility of the resulting spectra with the body of neutrino oscillation data is discussed. It is found that neutrino mass matrices with this structure result in 3 + 1 or 2 + 2 neutrino spectra, making it unlikely that this mass matrix structure is realized in nature. If the left-chiral Majorana mass matrix is also singular we show that a type-II singular see-saw mechanism can realize a spectrum of one active-sterile pseudo-Dirac neutrino in conjunction with two active Majorana neutrinos effectively decoupled from the sterile sector. This realizes a scheme discussed in the literature in relation to astrophysical neutrino fluxes.

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I. INTRODUCTION

Over recent years our understanding of neutrino physics, in particular their masses and mixings, has dramatically increased (for a review, see for example [1, 2, 3]). The solar [4] and atmospheric [5] data, together with the terrestrial experiments KamLand [6] and K2K [7], can be adequately explained by active neutrino oscillations. The bounds on $|U_{e3}|$, obtained by the reactor experiments CHOOZ [8] and Palo Verde [9] mean that the solar and atmospheric neutrino oscillations are practically decoupled [10], whilst the reconciliation of the LSND [11] result with the other data remains puzzling. Currently favoured fits imply bounds on the sterile component of active flavours produced in the sun and the atmosphere that are increasingly stringent (though questions regarding atmospheric $\nu_\mu \rightarrow \nu_\tau$ versus $\nu_\mu \rightarrow \nu_\tau$ distinction [12] and the use of the 2 + 2 sum rule [13] have been asked). Nevertheless there exist regions of the $(\sin^2 2\theta, \delta m^2)$ plane not yet experimentally probed and the possibility remains that sterile neutrinos play a role in these regions, independent of any role played in current experiments.

The singular see-saw mechanism is a variation of the see-saw mechanism whereby the right-chiral neutrino Majorana mass matrix is singular. In previous works, where the singular see-saw mechanism was introduced to accommodate the LSND result, the left-chiral neutrino Majorana mass matrix was assumed to vanish [14]. In this paper we extend the singular see-saw mechanism by including a left-chiral Majorana mass matrix. This is analogous to the type-II see-saw mechanism whereby the see-saw mechanism is extended to include a left-chiral Majorana mass matrix. The resulting neutrino spectrum is found to be sensitive to the hierarchy of the Dirac and Majorana mass scales used and we explore the phenomenological consequences of the candidate hierarchies. We find that neutrino mass matrices with this structure can produce both 3 + 1 and 2 + 2 spectra, but as these spectra have difficulties accommodating all the oscillation data it is unlikely that this mass matrix structure is realized in nature.

We also show that if the left-chiral Majorana mass matrix is singular the type-II singular see-saw mechanism can give rise to an active-sterile pseudo-Dirac pair of Majorana neutrinos. This realizes a scheme recently discussed in the literature where one or more of the mass eigenstates that participate in the solar and atmospheric neutrino oscillations forms a pseudo-Dirac neutrino with a near degenerate sterile neutrino [15, 16].

The structure of this paper is as follows. In Section II we review the see-saw mechanism in both its standard and type-II (non-canonical) form. Section III contains a discussion of the singular see-saw mechanism. The equivalent of a type-II form for the singular see-saw mechanism is introduced in Section IV and the phenomenological consequences of the relevant scale hierarchies are compared with the oscillation data. In Section V we discuss some of the phenomenology resulting from one active-sterile pseudo-Dirac pair with very small mass splitting.

II. THE SEE-SAW MECHANISM

It is known [17] that the relative lightness of the neutrinos can be explained by employing the so called see-saw mechanism. In its standard form, the neutrino mass matrix in the Majorana basis is given by:

$$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix},$$

where the Dirac mass matrix $M_D$ and the Majorana mass matrix $M_R$ are $n \times n$ matrices for $n$ generations. If the ele-
ments of $M_R$ are of order $M$ and the elements of $M_D$ are of order $m$, with $M \gg m$, then diagonalization produces $n$ light Majorana neutrinos and $n$ heavy Majorana neutrinos with masses of order $m^2/M$ and $m_N \sim M$ respectively. To first order the heavy eigenvalues are found by diagonalizing $M_R$ whilst diagonalizing $M_{Light} = -M_D^T M_R^{-1} M_D$ gives the light eigenvalues. Typically one identifies the light Majorana neutrinos with the active neutrinos observed in nature, whilst the scale $M$ is set by new physics. When the heavy Majorana neutrinos are integrated out the light neutrinos are described by an effective dimension-five operator [18]:

$$\mathcal{L}_{eff} = \frac{1}{\Lambda} \phi^0 (\nu_i)^c \nu_j$$  \hspace{1cm} (1)$$

where $\phi^0$ is the standard model scalar and $\Lambda^{-1} = g_i g_j / 2M$. $g_i$ is the Yukawa coupling constant for the Dirac mass. The number of light neutrinos that the see-saw mechanism can naturally accommodate is determined by the rank of the matrix $M_R$ [19] and in particular if $M_R$ is rank $n$ one may obtain $n$ naturally light neutrinos. Hence at most one can naturally obtain three generations of light neutrinos if three heavy sterile neutrinos exist [20].

If a non-zero active Majorana mass matrix $M_L$ is present, the see-saw formula is modified. The effective mass matrix in the light-sector takes the form:

$$M_{Light} \approx M_L - M_D^T M_R^{-1} M_D.$$  \hspace{1cm} (2)$$

This form has been referred to as the type-II see-saw formula. In general the scale of the entries in $M_L$ are completely independent from those in $M_R$. When $M_L \neq 0$ the question of which term is larger in eq. (2) arises. If the see-saw term $-M_D^T M_R^{-1} M_D$ dominates, the original motivation for the see-saw mechanism remains, with the scale of the light neutrinos set by the suppressing large scale of $M_R$. Though the presence of $M_L$ in the type-II see-saw means the question ‘Why are the neutrinos so light?’ is effectively rephrased as ‘Why is $M_L$ so light?’. Alternatively, if $M_L$ dominates the see-saw mechanism no longer plays any part in setting the light sector mass scale (for a possible connection between large $\nu_\mu - \nu_e$ mixing and a dominant $M_L$ in a type-II see-saw scenario within SO(10) see [21]). The type-II see-saw with dominant $M_L$ allows the neutrinos to possess large Dirac mass terms which do not significantly affect the light eigenvector structure. The existence of the relatively light active neutrinos is attributed to the light scale of $M_L$.

III. THE SINGULAR SEE-SAW MECHANISM

It should be noted that the approximations used to develop the see-saw mechanism rely on the existence of $M_R^{-1}$. There is in fact a history of study into the neutrino spectrum when $M_R$ is singular. Singular Majorana mass matrices were of interest in SO(10) when it was realized that combining [25] the Stech [23] and Fritsch [24] ansatz for the quark mass matrices in an SO(10) framework [27] could lead to singular behaviour in the Majorana mass sector [22].

The apparent observation of a small admixture of a $\sim 17$ keV neutrino state in $\nu_e$ [29] motivated studies that could produce such a state. The size of the claimed mass meant the properties of the neutrino were constrained by both neutrino-less double $\beta$-decay experiments and cosmological arguments and a (pseudo-)Dirac particle seemed necessary to explain the data. The singular see-saw mechanism [30, 31] received interest in this context as it could produce a pseudo-Dirac particle with the required properties. Specific models that realized a singular $M_R$ amended the standard model to include an extra Abelian symmetry [32], a non-Abelian symmetry [33] and horizontal symmetries [34, 35], whilst a supersymmetric model was also developed [36].

The singular see-saw mechanism also received interest in light of LSND [11] and the three distinct $\Delta m^2$ scales required to simultaneously explain the solar data [3], the atmospheric data [5] and the LSND results in terms of neutrino oscillations. A general analysis was carried out in [14] and the possibility of a hierarchy in the Dirac mass matrix was considered in [37]. A model using Abelian symmetries that simultaneously produced singularities in the Dirac mass matrix and the sterile Majorana mass matrix was also constructed [39]. General discussions regarding the coexistence of large active-active and large active-sterile mixing in the presence of a singular Majorana mass matrix can be found in [40].

The singular see-saw mechanism is a variation of the original see-saw mechanism in which an $n$-dimensional matrix $M_R$ has rank $(n-1)$ (or less). Depending on the model, it may be possible to obtain four relatively light Majorana neutrinos, for three generations, via the singular see-saw mechanism for a rank 2 matrix $M_R$. Only two of the light neutrinos have a naturally light mass, whilst the lightness of the other two requires a light Dirac mass matrix. To see this we consider the example of three generations of active and sterile neutrinos with a matrix $M_R$ of rank 2 [14, 31]. We first proceed by diagonalizing the matrix $M_R$ and placing the zero eigenvalue in the one-one entry so that the full mass matrix now reads:

$$
\begin{pmatrix}
0 & M_D' & M_D'' \\
M_D'^T & M_R' & M_{\omega} \\
M_D'' & M_{\omega}' & M_\gamma
\end{pmatrix}
\equiv M'
$$

where $M_R' = R M_R R^T = \text{diag}(0, M_1, M_2, M_3)$ and $M_D' = M_D R^T$ and $R$ is the rotation matrix used to diagonalize $M_R$. Now define new matrices:

$$
\begin{pmatrix}
0 & M_D' & M_D'' \\
M_D'^T & M_R' & M_\omega \\
M_D'' & M_\omega' & M_\gamma
\end{pmatrix} \equiv M''
$$

where $M_D = \text{diag}(M_1, M_2)$ is a $2 \times 2$ matrix, $M_\omega$ is a $4 \times 2$ matrix and $M_\omega'$ is a $4 \times 4$ matrix with the zero eigenvalue of the matrix $M_R$ in the entry $M_\omega_{44}$. The general form of $M_\omega$ is

$$M_\omega = \begin{pmatrix}
0 & 0 & 0 & a_1 \\
0 & 0 & a_2 & 0 \\
a_3 & 0 & 0 & 0
\end{pmatrix}.$$  \hspace{1cm} (3)$$

The eigenvalue equation may now be solved. It is given by:

$$M' \Omega = \lambda \Omega$$  \hspace{1cm} (4)$$
where $\Omega^T = (\psi^T_{\text{Light}}, \psi^T_{\text{Heavy}})$ and $\psi_{\text{Light}}$ ($\psi_{\text{Heavy}}$) is a four (two) dimensional column vector. Equation (4) is equivalent to the two coupled equations

$$M_{\omega}\psi_{\text{Light}} + M_{\gamma}\psi_{\text{Heavy}} = \lambda\psi_{\text{Light}}$$  \hspace{1cm} (5)$$

$$M_{\gamma}^T\psi_{\text{Light}} + M_{d}\psi_{\text{Heavy}} = \lambda\psi_{\text{Heavy}}.$$  \hspace{1cm} (6)

Solving (6) for $\psi_{\text{Heavy}}$ and substituting into (5) gives

$$(M_{\omega} - M_{\gamma}(M_{d} - \lambda)^{-1}M_{\gamma}^T)\psi_{\text{Light}} = \lambda\psi_{\text{Light}}$$  \hspace{1cm} (7)

which has the form $f(\lambda)\psi_{\text{Light}} = \lambda\psi_{\text{Light}}$. This equation may be solved to obtain the four lighter eigenvalues. As we expect $\lambda \ll M$ for the light eigenvalues we can use $(M_{d} - \lambda)^{-1} \approx M_{d}^{-1}$, giving:

$$(M_{\omega} - M_{\gamma}M_{d}^{-1}M_{\gamma}^T)\psi_{\text{Light}} = \lambda\psi_{\text{Light}}$$  \hspace{1cm} (8)

as the light sector eigenvalue equation to first order. The zeroth order eigenvalues are obtained by solving

$$M_{\omega}\psi_{\text{Light}} = \lambda\psi_{\text{Light}}$$

and we see that the scale of the mass eigenvalues is set by the eigenvalues of $M_{\omega}$, which are $\lambda^{(0)} = \{0, 0, m_{\omega}, -m_{\omega}\}$ with $m_{\omega} = (a_2^2 + a_2^2 + a_3^2)^{1/2}$. We expect the $a_i$'s to be $O(m)$, where $m$ is the scale of the neutrino Dirac mass matrix, so that two of the eigenvalues are ultra light whilst the other two have a mass set by the Dirac scale. It is worth noting that $M_{\omega}$ possesses a lepton number symmetry $\tilde{L} = L_e + L_\mu + L_\tau + L_s$, as the only non-zero entries are Dirac mass terms coupling the active neutrinos to the massless eigenvalue of $M_{\omega}$. Consequently the lowest order eigenstates must be Dirac particles or massless Majorana (Weyl) states \[41\]. The term $M_{\omega}M_{d}^{-1}M_{\gamma}^T$ provides corrections to the eigenvalues of order $m^2/M$. It will break $\tilde{L}$ and split the Dirac particle to a pseudo-Dirac pair. The degeneracy of the massless particle will in general be lifted and a pair of massive Majorana neutrinos results. The two intermediate scale eigenvectors are:

$$\nu_{3,4} = \frac{1}{\sqrt{2}} \left( a_1\nu_e + a_2\nu_\mu + a_3\nu_\tau \pm \nu_s \right)$$  \hspace{1cm} (9)

and the light sterile field is seen to reside predominantly in the pseudo-Dirac pair. The emergence of the pseudo-Dirac pair justified the interest in the singular see-saw scenario in view of Simpson's 17 keV neutrino results. In the more modern context, the pseudo-Dirac pair produced near maximal mixing between an active linear combination and the light sterile state. If the active component of the pseudo-Dirac pair is taken as mostly $\nu_e$ ($\nu_s$) then near maximal oscillations between $\nu_e$ ($\nu_s$) and $\nu_s$ occur.

Eq. (8) looks similar to the type-II see-saw formula \[2\] but an important difference exists. In the type-II formula the scales of $M_L$ and the see-saw term $M_D^2M_R^{-1}M_D$ are independent and the question of relative size arises. In the singular see-saw mechanism the first term $M_L$ and the see-saw type term $M_sM_d^{-1}M_\gamma^T$ are related and the first term always dominates. Thus the gross structure of the eigenstates obtained via the singular see-saw mechanism is set by $M_{\omega}$, giving a pseudo-Dirac pair which contain a large sterile component and a pair of lighter Majorana neutrinos with a mass suppressed by the large sterile Majorana scale.

**IV. NON-ZERO ACTIVE MAJORANA MASS**

We now consider the singular see-saw mechanism with a non-zero Majorana mass matrix for the active neutrinos. The equivalent of a type-II form for the singular see-saw mechanism follows immediately. The mass matrix is

$$M = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} = O^T \begin{pmatrix} U M_L U^T & U M_D R^T \\ (U M_D R^T)^T & R M_R R^T \end{pmatrix} O,$$

where $O = \text{diag}(U, R)$ is a $6 \times 6$ orthogonal matrix such that $U (R)$ is the $3 \times 3$ sub-matrix that diagonalizes $M_{L(R)}$. As we are taking $M_R$ to be singular we choose $R$ such that $R M_R R^T = \text{diag}(0, M_1, M_2)$ and we have $U M_L U^T = \text{diag}(m_1, m_2, m_3)$. Defining:

$$\tilde{M}_L \equiv \begin{pmatrix} U M_L U^T & 0_{3 \times 1} \\ 0_{3 \times 1} & 0 \end{pmatrix} = \text{diag}(m_1, m_2, m_3, 0),$$

with $0_{3 \times 1} = (0, 0, 0)$, we repartition the mass matrix as:

$$M = O^T \begin{pmatrix} \tilde{M}_L + M_{\omega} & M_{\gamma} \\ M_{\gamma}^T & M_R \end{pmatrix} O.$$

We have introduced $\tilde{M}_R \equiv \text{diag}(M_1, M_2)$ and the $4 \times 2$ matrix $M_{\gamma}$. The matrix $M_{\omega}$ is a $4 \times 4$ matrix with the same form as eq. (5) and contains the zero eigenvalue of $M_{\omega}$ as its $(4,4)$ element. We denote the scale of the non-zero elements in the Majorana and Dirac mass matrices as $O(M_{L_R}) \sim \tilde{m}$, $O(M_{R_R}) \sim M$ and $O(M_{D}) \sim m$. Taking $\tilde{m} \ll M$ and $m \ll M$ allows us to block diagonalize the mass matrix in (IV) up to $O(m^2/M)$ as:

$$\begin{pmatrix} \tilde{M}_L + M_{\omega} & M_{\gamma} \\ M_{\gamma}^T & \tilde{M}_R \end{pmatrix} = \begin{pmatrix} 1 & S \\ -S^T & 1 \end{pmatrix} \begin{pmatrix} M_{\gamma}^{\text{eff}} & 0 \\ 0 & \tilde{M}_R \end{pmatrix} \begin{pmatrix} 1 & S \\ -S^T & 1 \end{pmatrix}$$

with $S = M_{\gamma} \tilde{M}_R^{-1}$ and:

$$M_{\gamma}^{\text{eff}} = \tilde{M}_L + M_{\omega} - M_{\gamma} \tilde{M}_R^{-1} M_{\gamma}^T.$$  \hspace{1cm} (11)

We refer to (11) as the type-II singular see-saw formula. The distinction between the matrices $\tilde{M}_L$ and $M_{\omega}$ has been maintained as the scale of their non-zero entries will in general be independent. As with the type-II see-saw formula, the question of which matrix sets the scale for the light neutrino sector arises. Though now it is a comparison of the Dirac scale and the active Majorana scale that is relevant. We consider the two cases.
A. Dirac Scale Dominance

If we take \( m \gg \tilde{m} \) then \( M_L \) will determine the structure of the four lightest eigenstates. This case is essentially the same as the singular see-saw case. To lowest order we obtain two massless states and a Dirac particle with mass \( m_{\omega} \) as defined in Section III. The active Majorana mass terms will now contribute to the splitting of the massless states and the Dirac particle. If the the active mass scale is larger than the see-saw correction \( M_L M_R^{-1} M_L^T \), the size of the splittings will differ from the canonical singular see-saw case. Otherwise the neutrino spectrum of the singular see-saw case is reproduced.

B. Non-Negligible Active Majorana Mass

The eigenvector structure is quite different when the effects of the active Majorana mass matrix must be included. The lack of knowledge of the overall neutrino mass scales necessitates a discussion of the different viable alternatives. If the eigenvalues of \( M_L \) are all of order \( \tilde{m} \) and \( \tilde{m} \gg m \), the gross structure of the eigenstates is set by the active Majorana mass matrix. To lowest order the light mass eigenstates will be three purely active Majorana neutrinos with masses \( m_i, i = 1, 2, 3 \), corresponding to the mass eigenstates of \( M_L \), and one massless sterile state. The non-zero elements in \( M_\omega \) will mix these states. In general each of the purely active states will develop a small sterile component, whilst the sterile state will acquire a corresponding active component. Denoting the eigenvalues of \( M_L \) as \( \nu_i^{(0)} \), \( i = 1, 2, 3 \), and the massless eigenvector of \( M_R \) as \( \nu_4^{(0)} \), we consider \( M_\omega \) as a perturbation on \( M_L \). The perturbed eigenvectors are

\[
\nu_i^{(1)} = \nu_i^{(0)} + \frac{a_i}{m_i} \nu_4^{(0)},
\]

\[
\nu_s^{(1)} = \nu_4^{(0)} - \sum_{i=1}^{3} \frac{a_i}{m_i} \nu_i^{(0)},
\]

where we have labelled \( M_\omega \) as in eq. (12). The eigenvalues do not shift to first order. The predominantly sterile state develops a non-zero mass at second order, given by

\[
m_4 = \sum_{i=1}^{3} \frac{a_i^2}{m_i^2}.
\]

In this case the type-II singular see-saw gives rise to three predominantly active Majorana neutrinos and one lighter predominantly sterile Majorana neutrino. If one was to attempt to accommodate LSND with such a mechanism the resulting spectrum would be classified as a \( 3 + 1 \) scenario. If \( O(a_i/m_i) \sim 0.1 \) and \( \tilde{m} \sim 1 \text{ eV} \) the resulting \( 3 + 1 \) spectrum can accommodate the atmospheric and solar data in terms of oscillations amongst the active states \( \nu_i^{(1)} \), with large angle mixing built into the rotation \( U \), which diagonalizes \( M_L \). Such a spectrum may not be able to explain the solar, atmospheric and LSND data, with recent analysis suggesting a best overall goodness of fit for \( 3 + 1 \) spectra to be \( 5.6 \times 10^{-3} \) [42]. Should the future interpretation of the LSND result not require neutrino oscillations the scale of \( \tilde{m} \) can be much lower. The sterile component in \( \nu_i^{(1)} \) may be small enough to comply with all other experimental constraints. This component produces small amplitude oscillations into the sterile state with the relevant mass-squared differences for oscillations between the \( \nu_i^{(1)} \)'s and \( \nu_i^{(1)} \) set by the the the active mass scales, \( \Delta m_{i4}^2 = m_{i4}^2 - m_i^2 \approx m_i^2 \). A current bound of \( \sin^2 \theta < 0.52 \) at 3\sigma for \( \nu_e \to \cos \eta \nu_\alpha + \sin \eta \nu_s \), with \( \nu_s \) containing only active flavours, has been derived for sterile mixing with solar neutrinos [43]. An atmospheric bound of \( \sin^2 \theta < 0.19 \) at the 90\% C.L. for \( \nu_\mu \to \cos \xi \nu_\tau + \sin \xi \nu_s \) has also been obtained [44].

We have taken the eigenvalues of \( M_L \) to be at least of order \( \tilde{m} \). This can be the case for the quasi-degenerate hierarchy \( (m_1 \approx m_2 \approx m_3) \), the normal hierarchy \( (m_1 \ll m_2 \ll m_3) \) and the inverted hierarchy \( (m_2 \gg m_1 \gg m_3) \), though it need not be an accurate assumption. The depletion of solar and atmospheric neutrinos due to oscillations leads to lower bounds for two of the mass eigenstates:

\[
m_\iota \geq (\Delta m_{\text{atm}}^2)^{1/2} \approx 5 \times 10^{-2} \text{eV},
\]

\[
m_\j \geq (\Delta m_{\text{solar}}^2)^{1/2} \approx 7 \times 10^{-2} \text{eV},
\]

where the value attributed to \( i, j \) depends on the mass pattern. For the normal and inverted hierarchies the lightest mass is unconstrained and may have a vanishingly small value. Thus we can ask what happens to the type-II singular see-saw spectrum if \( M_L \) is also singular? This may be expected in, for example, a left-right symmetric model. In this case we have \( 0 \sim m_1 < a \sim m \ll m_2, m_3 \sim \tilde{m} \). The lowest order eigenvalues are obtained by diagonalizing the matrix \( M_L + M_\omega \), where now \( M_L = \text{diag}(0, m_2, m_3, 0) \). To order \( m/\tilde{m} \) the eigenvalues are \( \lambda^0 \) \( \{ \pm a, m_2, m_3 \} \) and the sterile state is seen to form a Dirac particle with the zero eigenvalue of \( M_L \), corresponding to the non-standard lepton number symmetry \( L_1 + L_s \) (where \( L_1 \) is a lepton number given to \( \nu_i \)) present in \( M_L + M_\omega \) when \( a_{2,3} \to 0 \). Higher order corrections to the mass matrix will split the Dirac particle to form a pseudo-Dirac pair of Majorana neutrinos. The interpretation of this spectrum depends on whether one attempts to accommodate the LSND result or not.

The spectrum may be of interest if the LSND result is not explained in terms of neutrino oscillations. The atmospheric and solar data would be accommodated by oscillations between \( \nu_2, \nu_3 \) and the active component of the light pseudo-Dirac pair, \( \nu_1 \). The mixing between members of the pseudo-Dirac pair is near maximal, with the oscillation length dependent on the size of their splitting. Provided the ratio \( m/\tilde{m} \) is small enough the oscillation length of the pseudo-Dirac pair may be larger than solar system length scales. At distances shorter than the oscillation length for the pseudo-Dirac pair the presence of the sterile partner will not be observable. Beyond the oscillation length a conversion of active flavours into the sterile state occurs. Regardless of the size of the role played by sterile neutrinos in solar and atmospheric neutrino phenomena (if any at all), they may still play a role in regions of the \( (\sin^2 2\theta, \delta m^2) \) plane not yet probed. The smallest mass-
squared difference thus far probed is about $10^{-11}\text{eV}^2$ with solar neutrinos. If we take $m_2 \sim 5 \times 10^{-3}$, $m_3 \sim 7 \times 10^{-2}$ (corresponding to the lowest bounds in a normal hierarchy) and $\alpha \lesssim 10^{-5}$, the splitting of the pseudo-Dirac pair is $\lesssim 10^{-11}$ and the sterile partner of the lightest mass eigenstate would thus far have escaped detection. With these values the mixing angle of the active states $\nu_1$ and $\nu_2$ with the sterile state are $\sim 10^{-7} - 10^{-8}$ and these states effectively decouple from the sterile sector. The scale of the Dirac mass matrix required here is small compared to the charged fermions, as is the case of the singular see-saw mechanism. Though it is not our intention to build models we note that mechanisms that induce small neutrino Dirac mass scales exist. These include methods of excluding Dirac masses at tree level [34], a Dirac see-saw [48] and mechanisms employing large extra dimensions [49].

V. DETECTING ONE PSEUDO-DIRAC PAIR

Observation of neutrino fluxes from astrophysical sources can reveal the existence of a pseudo-Dirac structure connecting active and sterile states. Over the huge path length from astrophysical neutrino sources the phases of the the relatively large atmospheric and solar mass-squared differences effectively decohere. The neutrino density matrix becomes mixed amongst the active flavours whilst remaining coherent amongst pseudo-Dirac partners. In the absence of pseudo-Dirac partners, the neutrino flux from astrophysical sources is expected to be flavour democratic [50]. Pseudo-Dirac splittings can lead to a flavour dependent oscillatory reduction in flux that is in principle detectable. The effects on neutrino fluxes from astrophysical sources with a sterile pseudo-Dirac partner for each active mass eigenstate (a similar pattern to that occurring, for example, in the mirror model [47]) are discussed in [13].

A scenario with one sterile neutrino forming a near degenerate mass pair with one active neutrino were considered in [16]. The flux ratio of ultra-high energy electron and muon neutrinos with the sterile state present, $R_{\mu\mu} \equiv \frac{\phi_{\nu_\mu}}{\phi_{\nu_s}}$, was compared with the value predicted by the standard model for a range of mixing angles between $\nu_1$ and $\nu_s$. Deviations between $R_{\mu\mu}$ and $R_{e\mu}^{SM}$ were found, permitting observation of a sterile pseudo-Dirac partner over astrophysical length scales. The deviations were most marked for near maximal mixing where there was no overlap between the range of $R_{\mu\mu}$ and $R_{e\mu}^{SM}$.

The recent improved determination of cosmological parameters provides strong constraints on the number of relativistic species present during Big Bang Nucleosynthesis. Sterile neutrino populations created and maintained by neutrino oscillations must have sufficiently small active-sterile mixing and/or mass-squared differences to avoid disturbing the standard nucleosynthesis of light elements. This leads to the bound $|\delta m^2| \sin^2 2\theta < 5 \times 10^{-8}\text{eV}^2$ for $\nu_e \leftrightarrow \nu_s$ mixing [48] whilst the bounds for the other flavours are less severe. The value of $\delta m^2 < 10^{-11}\text{eV}^2$, taken to avoid disrupting solar neutrino experiments, satisfies this bound. Thus neutrino oscillations involving one active-sterile pseudo-Dirac pair do not create a significant sterile population and the sterile pattern predicted by the type-II singular see-saw mechanism with a singular $M_L$ remains experimentally viable.

VI. CONCLUSION

In this paper the singular see-saw mechanism was extended to include a left-chiral Majorana mass matrix. It was found that, depending on the hierarchy between the Dirac and Majorana mass scales, the type-II singular see-saw predicted a $2+2$ or $3+1$ neutrino spectrum. As these spectra have difficulties in accommodating the body of neutrino oscillation data it is unlikely that this mass matrix structure is realized in nature. It was also shown that a type-II singular see-saw produces a spectrum containing one active-sterile pseudo-Dirac pair and two active Majorana neutrinos if the left-chiral Majorana mass matrix $M_L$ is also singular.

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