Design of narrowband multiplierless comb compensator with low passband deviation

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Abstract. The simplest decimation filter is a comb filter which has all coefficients equal to unity, and consequently does not require multipliers for its implementation. However, comb filter does not have the flat passband characteristic, which is increases with the increase of the number of cascaded comb filters. The passband droop must be compensated by a filter called comb compensator. In this paper we present novel narrowband multiplierless compensator which results in a low absolute value of the passband deviation of the compensated comb, which is less than 0.01dB. The compensator has a magnitude response synthesized as sine wave functions. The parameters of design are the amplitudes of sine wave functions, which depend on the order of the comb filter and practically do not depend on the decimation factors. The designs for two values of the passband edge frequencies are elaborated and illustrated with examples.

1. Introduction
Decimation is the process of decreasing sampling rate in a digital domain, by an integer, called the decimation factor [1]. This process introduces replicas of the main spectrum, called aliasing. Aliasing may deteriorate the decimated signal if the signal is not filtered before decimation, by a low pass filter known as a decimation filter. Comb filter is the most popular decimation filter due to its simplicity. Comb filter has all coefficients equal to unity, and consequently does not require multipliers for its implementation. Its transfer function is given as:

\[ H(z) = \left[ \frac{1}{M} \frac{1-z^{-M}}{1-z^{-1}} \right]^K, \]

where \( M \) is decimation factor and \( K \) is the number of the cascaded combs, also known as order of the comb.

The magnitude characteristic of comb filters is in \( \sin x/x \) form and is given as:

\[ |H(e^{j\omega})| = \left[ \frac{1}{M} \frac{\sin(\omega M/2)}{\sin(\omega/2)} \right]^K. \]

Figure 2 illustrates the comb magnitude characteristic for \( M=9 \) and three values of \( K \), \( K=1, 3, \) and \( 5 \). The figure shows that the comb magnitude characteristic is not flat in the passband and there is a passband droop which increases with the increase of comb parameter \( K \). The passband zoom is also shown to see better the values of the passband droop.
The passband droop may deteriorate the decimated signal and must be compensated by the filter called compensation filter. Different methods have been proposed for design of comb compensators [2-6]. The review of the methods can be found in [7].

We consider here design of narrowband comb compensators, taking the passband edge frequency \( \omega_p \leq \pi/4M \). More precisely we elaborated two cases:

- Case 1: Very narrow passband with the passband edge frequency \( \omega_p = \pi/8M \)
- Case 2: Narrow passband with the passband edge frequency \( \omega_p = \pi/4M \)

Some desirable characteristics of comb compensator are multiplierless design, and a low absolute value of the passband deviation of the compensated comb.

The magnitude response of comb compensator has to approximate the inverse comb magnitude characteristic. In that way the cascade of comb and compensator has the passband characteristic approximately equal to 1.

In this paper we present novel narrowband multiplierless compensator with a low absolute value of the passband deviation.

The rest of the paper is organized in the following way. Next section describes the proposed compensator and its principal features. Section 3 describes the designs for two passband edge frequencies: \( \omega_{p1} = \pi/8M \) and \( \omega_{p2} = \pi/4M \), and it is illustrated with examples.

2. Proposed compensator

In [5] is introduced a wideband comb compensator with magnitude characteristic based on sine wave functions:

\[
G(e^{j\omega}) = \left[1 + A\sin^4(\omega M / 2)\right] \times \left[1 + B\sin^2(\omega M / 2)\right],
\]

where \( A \) and \( B \) are amplitude of sine functions.

From (3) we get:

\[
|G(e^{j\omega})| = 1 + A\sin^4(\omega M / 2) + B\sin^2(\omega M / 2) + AB\sin^6(\omega M / 2).
\]
Taking into account that we consider here the design of narrowband compensator with the passband edges frequency \( \omega_{p1} = \pi/8M \) and \( \omega_{p2} = \pi/4M \), and that the values of \( A \) and \( B \) are less than 1, last term in (4) for \( \omega_{p1} \) and \( \omega_{p2} \) becomes:

\[
AB\sin^6(\omega_{p1}M/2) = AB\sin^6(\pi/16) = AB \times 5.5133 \times 10^{-5}
\]

\[
AB\sin^6(\omega_{p1}M/2) = AB\sin^6(\pi/8) = AB \times 3.31 \times 10^{-3}.
\]

Consequently, we propose here the magnitude characteristic of narrowband compensator in the form:

\[
|G_p(e^{j\omega})| = 1 + A\sin^4(\omega M/2) + B\sin^2(\omega M/2).
\]

(5)

The design compensator parameters are the values \( A \) and \( B \). The compensator (6) has to approximate the inverse comb magnitude characteristic (2) in the narrow passband defined with the passband edge frequencies, \( \omega_{p1} \) and \( \omega_{p2} \).

\[
|G_p(e^{j\omega})| = H^{-1}(e^{j\omega}) \quad \text{for} \quad \omega \in [0, \omega_p), \omega_p = \pi/4M, \text{and} \quad \pi/8M.
\]

(6)

For each value of \( K, K=1,\ldots,5 \), we find the parameters \( A \) and \( B \), imposing the following condition:

\[
\min_{\omega \in [0, \omega_p]} \max_{j \in [\omega_p, \pi]} |H^{-1}(e^{j\omega}) - G_p(e^{j\omega})| \leq \delta,
\]

(7)

where \( \delta \) is the desired absolute value of the maximum passband deviation.

The useful imposed features of the proposed narrowband compensators are:

- The compensator should not deteriorate the alias attenuation in the comb folding bands (the bands around the comb zeros).
- The design parameters \( A \) and \( B \) are defined only by the comb parameter \( K \), and generally do not depend on the decimation factor \( M \).
- Like comb, the compensator structure does not require multipliers. To this end the parameters \( A \) and \( B \) are chosen in form of sum of powers of two, and thus can be implemented using adders and shifts.

Next section presents design of compensator using (7).

### 3. Design of proposed compensator

We consider two cases, taking the maximum absolute value of the passband deviation equal to \( \delta = 0.01 \text{dB} \).

#### 3.1. Case 1: \( \omega_{p1} = \pi/8M \)

The values of the parameters \( A \) and \( B \) for each value of the comb parameter \( K, K=1,\ldots,5 \), are obtained by MATLAB simulations and are presented in Table 1.

| \( K \) | \( A \) | \( B \) |
|---|---|---|
| 1 | 2^3 | 2^3+2^4 |
| 2 | 2^3 | 2^2+2^4 |
| 3 | 1 | 2^1 |
| 4 | 1 | 2^1+2^3 |
| 5 | 1 | 2^1+2^2+2^4 |
The method is illustrated in the following example.

Example 1:
In order to show that the design does not depend on the comb parameter $M$, we consider two values of $M$: $M=11$ and $M=18$ and the same value of $K=4$. From Table 1 the parameters $A$ and $B$ are equal to $1$ and $2^{-1}+2^{-3}$, respectively. Figure 2 contrasts the overall magnitude responses of comb and compensated comb in order to show that the alias rejection is not deteriorated. The passband zooms are also shown to prove that the passband deviation is less than $0.01\text{dB}$ in both cases.

![Figure 2](image)

**Figure 2.** Magnitude responses of comb and compensated comb in Example 1.
3.2. Case 2: $\omega_p^2=\pi/4M$.
The values of the parameters $A$ and $B$ for each value of the comb parameter $K$, $K=1,\ldots,5$, are obtained by MATLAB simulations and are presented in Table 2.

**Table 2.** Case 2: Parameters $A$ and $B$ for different values of $K$.

| $K$ | $A$     | $B$                        |
|-----|---------|----------------------------|
| 1   | $2^3$   | $2^3+2^5+2^6$              |
| 2   | $2^4$   | $2^2+2^5+2^6$              |
| 3   | $2^1$   | $2^4+2^5$                  |
| 4   | $2^1$   | $2^1+2^3+2^5$              |
| 5   | 1       | $1-2^3-2^4-2^6$            |

The method is illustrated in the following example.
Example 2:
Similarly as in Example 1 we chose here two values of $M$: $M=16$ and $M=21$, and the value of $K=5$ for both value of the comb parameter $M$. From Table 2 we get the parameters $A$ and $B$ for the comb parameter $K=5$: $A=1$ and $B=1-2^3-2^4-2^6$. Figure 3 contrasts the overall magnitude responses of comb and compensated comb in order to show that the alias rejection is not deteriorated. Additionally, the passband zoom shows that the maximum absolute value of the passband deviation of the compensated comb is less than 0.01 for both values of $M$.

![Graph](image.png)

**a.** $M=16$. 
4. Conclusion
This paper presents a novel method for design of comb narrowband compensator. The magnitude characteristic of compensator is synthesized as sine wave forms. The design for two passband edge frequencies are presented and illustrated with examples. In both cases the maximum value, of the absolute value of the passband deviation of the compensated comb is less than 0.01dB. The parameters of the design are the amplitudes of sine functions which are presented in form of sums of powers of two. In that way the compensator can be considered as a multiplierless filter.

References
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