Research of friction loss for setting slurry flow in inclined pipe

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Abstract: By analyzing the momentum transfer and velocity both of solid particles and water over the acceleration time of solid particles, and interplay mechanism of water and solid particles, a new model is proposed to predict friction loss for setting slurry flow in inclined pipe. Based on the experimental data of some scholars, the hydraulic gradient formulas of the author and other inclined pipelines are verified. The results show that the deviation between the theoretical and measured values of the model proposed by the author is the smallest, not more than 13.33%. At last the causes of error is analysed.

1. Introduction

With the increasing demand for mineral resources in remote areas and increasingly stringent environmental requirements, economic and environmental slurry pipeline transportation is more and more widely used. In long distance slurry pipeline transport project, it often needs to calculate hydraulic gradient of settling Slurry in inclined pipeline, but up to now, the research about this aspect is not much and scholars such as Rose[1], Kawashima[2] and Worster[3] have done research in this field. Rose has only studied a number of slurry transport parameters in inclined pipelines (hydraulic gradient is not involved), the study Kawashima and Worster for inclined slurry flow is either not perfect in form or inaccurate in prediction.

The movement status of solid particles directly decides the pressure loss of setting slurry. Based on a new theoretical analysis method[4], a new model that can open out the relationship between the movement status of solid particles and the pressure loss of setting slurry. Using this model, the hydraulic gradient of setting slurry in inclined pipe can be predicted which is transported in suspension, by salination, or in partly suspension and by salination.

2. The velocity change of the water during solid particles’ accelerating

Using a new theoretical analysis method thought out by author own, a balance equation of momentum and continuous equation between the solid particles with mass $M_s$ and water with mass $M_w$ discharged from the section $A$ of the outlet of the pipe with the time $\Delta t$ may be set up and the following equation are obtained[4]:

$$V - V_s = \frac{k^2}{k_1(1 - \bar{q})\rho} \frac{\rho_s V_s^2}{V_m}$$

(1)

$$V_m = V_w(1 - \bar{q}) + V_s \bar{q}$$

(2)

where and $V$ are the average rate of the water before momentum transference, $\rho$ and $\rho_s$ are the density of water and solid particles, $q$, $V_w$, $V_s$, $V_m$ are separately in-situ average concentration (by volume) of
solid particles, the average velocity of the water, the solid particles and settling slurry in the section $A$. $k_1$ and $k_2$ are coefficient. And the equation to calculate $k_1$, $k_2$ are:

$$k_1 = \frac{1}{1 - 0.56\psi(1) \phi(\theta)} \tag{3}$$
$$k_2 = 1 + k_4 (33\lambda \cos \theta \pm \sin \theta) \frac{k_1 \sqrt{L_a g}}{V_m} \left(1 - \frac{1}{\delta} \right) \tag{4}$$

Rose’s experimental study shows that the value of $\psi(1)$ is related to the average velocity $V_m$ of two-phase flow in the pipeline, the particle diameter $d$ and the density ratio $\delta$ of solid particles to fluid. After analyzing the experimental data of several scholars, the fitting curve between $\psi(1)$ and $\log_{10} \left( \frac{V_m^2}{g d\delta^2} \right)$ is shown in Figure 1 [5]. $\phi(\theta)$ is a function of $\theta$ as shown in Figure 2 for up-inclined pipe according to Rose’s research [1]. For down-inclined pipe, $\phi(\theta)$ is smaller than that of upward inclination. Here, we assume that $\phi(\theta)$ for up-inclined pipe and down-inclined pipe are symmetrica, so $\phi(\theta)$ for down-inclined pipe can be determined from Figure 3.

$$\delta = \rho_s / \rho, \ k_3 = 3.742, \ \lambda \ is \ friction \ factor, \ when \ solid \ particles \ are \ transported \ in \ suspension, \ k_4 \ represents \ the \ proportion \ coefficient \ of \ solid \ particles \ in \ the \ slip \ state \ to \ the \ total \ weight \ of \ solid \ particles. \ When \ all \ solid \ particles \ are \ suspended, \ k_4 = 0; \ when \ solid \ particles \ are \ transported \ by \ saltation \ \ k_4 = 1; \ when \ some \ solid \ particles \ are \ transported \ in \ suspension \ and \ others \ are \ by \ saltation, \ k_4 = \frac{V_t}{V_m}, \ V_m \ is \ the \ average \ velocity \ of \ setting \ slurry, \ V_t \ is \ terminal \ velocity \ of \ a \ particle, \ L_a \ is \ length \ of \ pipe \ required \ for \ full \ acceleration \ of \ the \ solid. \ And \ the \ equation \ to \ calculate \ L_a \ is \ [1]:}
where $M_s$ is mass flow rate of solid, $D$ is pipe diameter, $d$ is solid particle diameter, $g$ is gravitational acceleration.

3. Analysis of the forces that the solid particles exerted

When slurry flow in inclined pipe, there are four forces acting on solid particles, that is dragging force of fluid on particles $F_D$, sliding component of gravity of solid particle $W_b\sin(\theta)$, interference force of other particles $F_h$ and friction on particles $K_{df}$, as shown in Figure 4.

Figure 4. Diagram of force acting on particles in inclined pipeline

The movement equation of the accelerating solid particle can be given by following equation for slurry flow in inclined pipe

$$
\frac{\pi}{6} d_s^3 \left( \rho_s + \frac{\rho}{2} \right) \frac{dV_s}{dt} = F_D - F_h - k_s f_f + W_b \sin \theta
$$

where

$$F_D = \frac{\pi}{4} d_s^3 C_d \frac{(V_s - V_w)^2}{2} \rho$$

$$F_h = \left\{ 1 - (1 - \bar{q})^{2(n-1)} \left( \sqrt{\text{Rep} \alpha} + \sqrt{\text{Rep} \alpha^2 + 4\sqrt{48 \alpha \beta} (1 - \bar{q})^{n-1}} \right)^2 \right\} W_b$$

$$f_f = \frac{\pi}{6} d_s^3 \lambda (\rho_s - \rho) g \cos \theta$$

where $F_D$ is the drag force from the water, $F_h$ is the interference force from other solid particles, $f_f$ is the friction that acts on single solid particle, $d$ is the equal ball diameter of solid particles $w_b$ is the valid gravity of solid particles in water and $W_b = \pi d_s^3 (\rho_s - \rho) g / 6$, $C_d$ is drag coefficient according to $V_s - V_w$, $\text{Rep}$ is Reynolds factor of solid particle, $\alpha, \beta$ is Swanson shape factor; $n$ is a prime number (calculated with Sato Hiroshi equation), $CD$ is the drag coefficient of particles. After finishing the accelerating course of the solid particles, the setting slurry comes to a steady flowing status, and the velocities of the water, the solid particles and the setting slurry come to be constant too. Then the acceleration of solid particles $dV_s / dt = 0$, according to equation (6), the following equation can be obtained:

$$F_D - F_h - k_s f_f + W_b \sin \theta = 0$$
Where for flow in inclined upward pipeline, $W_b \sin(\theta)$ is negative, and for flow in inclined downward pipeline, $W_b \sin(\theta)$ is positive.

Substituting equation (7), (8) into equation (10) with equation (9) gives:

$$V_s = V_v - \sqrt{\frac{8(F_b + k_i f_i) \mp W_b \sin(\theta)}{\pi d_e^2 C_D \rho}}$$

Substituting equation (3) into equation (1) with equation (4) gives:

$$V = V_v + \left[1 - 0.56 \phi(1) \phi(\theta)\right] \frac{\delta \eta}{(1 - \theta) V_v} V_v^2 \left[1 + k_i(33 \lambda \cos(\theta) \pm \sin(\theta)) \frac{k_i \sqrt{L_s \delta}}{V_v} \left(1 - \frac{1}{\delta}ight)\right]^2$$

4. The hydraulic gradient of setting slurry

When water is in turbulence status the hydraulic gradient can be given out by the following equation:

$$i = \frac{\lambda}{2gD} V^2$$

where $D$ is the inner radius of the pipe, $V$ is the average velocity of the water. Obviously, as to the setting slurry when the solid particles are transported in suspension or by saltation or some are in suspension while others are by saltation, equation (13) can’t be used to calculate hydraulic gradient, however, from (12), we can easily see that: Actually the setting slurry’s flowing with average velocity $V_m$ can be regarded as the water’s flowing with the average velocity $V$. To say it more clearly, if the pressure’s difference existing in the part of pipe can make water move forward with the velocity $V$, then after being intervened by group of solid particles with the average volume concentration $\bar{Q}$, this pressure difference can only make the mixture (setting slurry) move forward with the velocity $V_m$. Therefore, the hydraulic gradient produced when setting slurry with the velocity $V_m$ is flowing in the pipe can be regarded as that of the water with the velocity $V$ flowing in the pipe. The above discussion can be illustrated by the equivalent Resistance model in Figure 5.

So, $V$ can be used to calculate the hydraulic gradient of the setting slurry flowing in the above state only when $V$ is calculated under the given transporting condition.

![Figure 5. Equivalent Resistance model](image)

According to the analyzing results obtained before, obviously the equation to calculate hydraulic gradient of the setting slurry can be obtained as the following:

$$i = \frac{\lambda}{2gD} \left(V_v + \left[1 - 0.56 \phi(1) \phi(\theta)\right] \frac{\delta \eta}{(1 - \theta) V_v} V_v^2 \left[1 + k_i(33 \lambda \cos(\theta) \pm \sin(\theta)) \frac{k_i \sqrt{L_s \delta}}{V_v} \left(1 - \frac{1}{\delta}\right)\right]^2\right)$$

where $C_D$ is drag coefficient according to $V_m - V_v$. 

4
5. The criterion of the solid particles’ moving status

In formula (12), \( k_i \) represents the proportion coefficient of solid particles in the slip state to the total weight of solid particles, and it is related to the state of particle movement[6]. When average velocity of setting slurry is bigger than the homogeneous limit velocity \( V_h \), solid particles can be considered as being transported by saltation. When average velocity of setting slurry is between the deposit limit velocity \( V_b \) and the homogeneous limit velocity \( V_h \), solid particles can be considered as being transported by suspension partly and by saltation partly. Obviously, it’s easy to define the moving status of the solid particles with the single diameter, but it’s hard to define as to the setting slurry when the distribution of the solid particles’ diameter is in a certain scope and the industry slurry when the fine solid particles divine a certain ratio. As to the setting slurry when the diameter of the solid particles are rather single, the floating limit velocity \( V_f \) and the homogeneous limit velocity \( V_h \) can be calculated with the following Newitt and Lazarus’ equations[7]:

\[
\begin{align*}
V_f &= 17 \nu_i \quad (15) \\
V_h &= 4.4 \sqrt{gD} C_D^{-0.25} (\delta - 1)^{\frac{3}{2}} \quad (16)
\end{align*}
\]

6. The comparison of the theoretical results with experimental results

Up to date, several experts have put forward their model of calculating slurry friction loss in inclined pipe. In paper, Kawashima’s model[2] and Worster’s model[2] are used to compare with the new model.

Kawashima’s model can be expressed in following:

\[
\frac{i - i_w}{i_w C_v} = \pm \frac{3}{2} \left( \frac{d}{D} \right) \frac{(1 - \zeta)^2}{\zeta} \quad (17)
\]

Where \( i_w \) is the hydraulic gradient of water, \( C_v \) is slurry volume concentration, \( \zeta \) is a variable related to particle size \( d \), specific gravity \( \delta \), and pipe inclination \( \theta \)[2].

When \( \mu_s \cos \theta + \sin \theta > 0 \), the right sign of equation (17) is positive, and the expression of \( \zeta \) is as follows:

\[
\zeta = 1 + \frac{1}{V_w} \sqrt{\frac{4(\rho - 1)gd}{3C_b}} \left( \mu_s \cos \theta + \sin \theta \right) \quad (18)
\]

When \( \mu_s \cos \theta + \sin \theta < 0 \), the right sign of equation (17) is negative, and the expression of \( \zeta \) is as follows:

\[
\zeta = 1 - \frac{1}{V_w} \sqrt{\frac{4(\rho - 1)gd}{3C_b}} \left( \mu_s \cos \theta + \sin \theta \right) \quad (19)
\]

When \( \mu_s \cos \theta + \sin \theta = 0 \), \( \zeta = 1 \), \( i = i_w \).

Where \( u_i \) is the friction coefficient.

Worster’s model can be expressed in following[2]:

\[
i_w = i_0 + (i_b - i_0) \cos(\theta) \pm C_v (\delta - 1) \sin(\theta) \quad (20)
\]

Where \( i_b \) is the hydraulic gradient of slurry in horizontal pipe with the same concentration.

In Figure 6, Figure 7 and Figure 8, authors use Huang zhao-lin’s experiment data(down inclined) to verify theoretical model[8]. In Huang zhao-lin’s experiment, solid particle diameter are 0.2042mm, and relative density are 1.552. The slurry volume concentration is 21.74%. In Figure 6, pipe inclination is 15 degree, pipe inside diameter is 20 mm. In Fig.7, pipe inclination is 30 degree, pipe inside diameter is 15 mm. In Figure 8, pipe inclination is 40 degree, pipe inside diameter is 15 mm.
In figure 6, the maximum deviation between the author formula calculating value and measured values is 9.71%, and 11.09%, 52.91% for Worster and Kawashima formula respectively.

It can be seen from Figure 7 that the maximum deviation between the author formula calculating value and measured values is 10.33%, and 63.44%, 7.07% for Worster and Kawashima formula respectively. Generally speaking, the calculation deviation of the author's formula is small.

Figure 6. Comparison of Several Calculating Formulas for D=20mm, θ=15°

Figure 7. Comparison of Several Calculating Formulas for D=15mm, θ=30°
In figure 8, the maximum deviation between the author formula calculating value and measured values is 13.33%, 84.29%, 20.93% for Worster’s and Kawashima’s formula respectively. Generally speaking, the calculation deviation of the author’s formula is small.

Seen from Figure 6, Figure 7 and Figure 8, it can be seen that except for individual data points, the deviation calculated by the author's formula is the smallest.

In Figure 9 and Figure 10, author use Diniz and Coiado[9] inclined experiment results(up inclined). In his experiment, solids diameter are 0.20mm, pipe inside diameter is 75mm and relative density is 2.68. The slurry concentration in volume is 5%. In Figure 9 and Figure 10, pipe inclination is 11 and 34 degree, respectively.

In Figure 9, the maximum deviation between the author formula calculating value and measured values is 8.71%, 14.21% and 42.36% for Worster’s model and Kawashima’s formula respectively. In Figure 10, the maximum deviation between the author formula calculating value and measured values is 5.79%, 7.47% and 22.84% for Worster’s model and Kawashima’s formula respectively. So from Figure 9 and Figure 10, the deviation of the author’s formula is the smallest.
Figure 10. Comparison of Several Calculating Formulas

From above fig.6-10, the author’s model agrees with measured value and error is not exceed 13.33%, while the other two models have larger error. It is obvious that when the slurry velocity is low, all the three model’s calculating error is large. Seen from the deduction of Kawashima’s model, it is not used for solid in suspending state. In low velocity, the error of Kawashima’s model is mainly that value of coefficient of pressure loss is not suitable for medium particle. For Worster’s model, reason of error is probably that its factors considered is few.

In Figure 6, only experiment data for down inclined flow is given out, in order to facilitate comparison and analysis, the upward inclined flow data and the horizontal flow data under the condition of Fig. 6 are also given, as shown in Table 1.

Table 1. Comparison and Analysis of Slurry Inclined Flow and Horizontal Flow

| Slurry average $V_m$ /m/s | hydraulic gradient $i$ / (m/m) |
|---------------------------|-------------------------------|
|                           | measured value                | Author formula | Kawashima     | Worster       | measured value |
|                           | up-inclined pipe( down-inclined pipe) |                   |               |               |               |
| 0.5                       | 0.1375(0.0875)                | 0.1375(0.079)   | 0.0412        | 0.1340(0.0778) | 0.0145        |
| 1.1875                    | 0.1938(0.1625)                | 0.1874(0.1639)  | 0.1295        | 0.2027(0.1405) | 0.0820        |
| 1.6875                    | 0.2375(0.2125)                | 0.2312(0.2032)  | 0.2038        | 0.2535(0.1913) | 0.1657        |
| 1.9688                    | 0.2813(0.2688)                | 0.2988(0.2501)  | 0.2685        | 0.3101(0.2479) | 0.2255        |
| 2.2813                    | 0.3375(0.325)                 | 0.3434(0.3358)  | 0.3289        | 0.3544(0.2922) | 0.3028        |

It can be seen that under Huang zhao-lin’s experiment, when pipe inclination is 15 degree and -15 degree, hydraulic gradient values are completely different. The maximum deviation between the author formula calculating value and measured values is 6.22%, and 70%, 10.24% for Kawashima and Worster formula for up-inclined flow, respectively.

According to Kawashima formula calculated result, $\mu_\ast \cos \theta + \sin \theta < 0$, hydraulic gradient values of up-inclined and down-inclined are same, and this is unreasonable. So it can be seen that from Figure 6–Figure 10 and table 1 that there is a big deviation between the calculated value of Kawashima formula and the measured value.

For Worster’s formula (20), solid diameter and friction factor have not appeared, but they influence the hydraulic gradient in inclined pipe. So Worster’s formula is imperfect, especially for the slurry flow at low speed.
As for author’s model, possible reason is that the parameter $k$ is not properly determined when flow velocity is low. It is obvious that when slurry velocity is very low, the flow of solid is very complex. So next work is to discuss the parameter $k$ when slurry velocity is very low.

7. Conclusion
By analyzing the change in momentum, velocity, both of solid particles and water over the acceleration time of solid particles, and interplay mechanism of water and solid particles, a new model that can calculate the hydraulic gradient of settling slurry in inclined pipe is put forward in this paper. The proposed new model was verified by experimental data from several experts, and the calculated values were not much different from the experimental values. So when d is about 0.2mm, pipe diameter is 15-75mm, and inclination angle is 10-40 degree, the author’s model is superior to Kawashima and Worster’s model. As to other case, more research has to be done in this aspect.

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