Hybrid Quantile Regression Neural Network Model for Forecasting Currency Inflow and Outflow in Indonesia

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Abstract: Regression analysis which can explain the relationship between variables on various quantiles has been developed using quantile regression. Moreover, quantile regression can be applied in forecasting analysis. The aim of this study was to find the best model for forecasting inflow and outflow in Indonesia which contains heteroscedasticity and nonlinearity problems. In order to improve the forecast accuracy, quantile regression will be combined with neural network method, known as quantile regression neural network (QRNN). Then, the forecast accuracy of QRNN will be compared with ARIMAX method based on some forecast accuracy criteria, i.e. RMSE, MAE, MdAE, MAPE, and MdAPE. Two types of data are used as case studies in this research, i.e. simulation and real data about 6 currencies of inflow and outflow in Indonesia. The result of simulation study shows that QRNN is the best method to solve heteroscedasticity and nonlinearity problem. Furthermore, the comparison results on real data shows that QRNN yield better result than ARIMAX for four currencies.

1. Introduction
Quantile regression was developed from Ordinary Least Square (OLS) regression. When OLS regression only explains the relationship between independent variables and the mean of dependent variable, quantile regression can explain this relationship on various quantiles. Furthermore, quantile regression can solve the problem of non-homogeneous data distribution and obtain an interval forecasting. Quantile regression will be applied in modeling currency inflow and outflow due to heteroscedasticity problem of the data. Study about forecasting of money circulation shows that there is an ARCH effect on the data. In order to solve heteroscedasticity problem, modeling of money circulation has been done by using GARCH. Forecasting inflow and outflow is necessary to achieve the suitable currency in circulation for the needs of the society. Thus, forecasting inflow and outflow should be done accurately. On average, combination of two methods will result better forecast than using single method. Previous study also shows that there is a nonlinear pattern of currency in circulation data, so in this study, quantile regression will be combined with neural network, known as Quantile Regression Neural Network (QRNN).
QRNN has been used to forecast multiperiod returns in various holding periods. The result is that QRNN performs better than GARCH with empirical distributions, and tends to have the same accuracy as GARCH with Gaussian distribution. Money circulation data has been widely forecasted using ARIMA. Thus, the forecast value of inflow and outflow will be compared with ARIMA with exogenous variables (ARIMAX). The criteria used in comparing forecast accuracy are RMSE, MAPE, MdAPE, MdAE, and MdAE criteria.

2. Experimental Design
In this study, there are two studies conducted, i.e. simulation studies and applications on inflow and outflow data. The monthly inflow and outflow data are secondary data obtained from Bank Indonesia for 2003 to 2016. The data will be divided into training and testing data. The first 13 years data are used as training data, while the remaining data as testing data. The research variables in this study can be shown in Table 1.

| Inflow Variable | Description     | Outflow Variable | Description   |
|-----------------|-----------------|------------------|--------------|
| $Y_{1,t}$       | Rp5,000 currency| $Y_{4,t}$        | Rp5,000 currency|
| $Y_{2,t}$       | Rp10,000 currency| $Y_{5,t}$        | Rp10,000 currency|
| $Y_{3,t}$       | Rp20,000 currency| $Y_{6,t}$        | Rp20,000 currency|

Dummy variables in this study are trend, seasonal, and calendar variations. Completely, dummy variables in this study can be shown in Table 2.

| Dummy Variables | Description                                      |
|-----------------|--------------------------------------------------|
| Trend           | $t$, with $t=1,2,\ldots,n$                       |
| $M_{t,t}$       | $\begin{cases} 1, & \text{January in the } t\text{-th period} \\ 0, & \text{otherwise} \end{cases}$ |
| Seasonal        | $M_{2,t}$                                        |
| $M_{2,t}$       | $\begin{cases} 1, & \text{December in the } t\text{-th period} \\ 0, & \text{otherwise} \end{cases}$ |
| $V_{i,t}$       | $\begin{cases} 1, & \text{month when Eid Fitr (period } t) \text{ occurs in week-}i, \text{ with } i=1,2,3,4 \\ 0, & \text{otherwise} \end{cases}$ |
| Calendar Variations | $V_{i,t+1}$                                      |
| $V_{i,t+1}$     | $\begin{cases} 1, & \text{month before Eid Fitr (period } t) \text{ in week-}i, \text{ with } i=1,2,3,4 \\ 0, & \text{otherwise} \end{cases}$ |
| $V_{i,t+1}$     | $\begin{cases} 1, & \text{month after Eid Fitr (period } t) \text{ in week-}i, \text{ with } i=1,2,3,4 \\ 0, & \text{otherwise} \end{cases}$ |

The steps of analysis used are performing descriptive statistical analysis, then modeling the data by ARIMAX and QRNN. In ARIMAX modeling, we will modeled the residual of time series regression (TSR) model using ARIMA. Predictors used in TSR modeling are dummy variables as in Table 2. In QRNN modeling there are 2 schemes to be used, i.e. one level modeling and using decomposition as preprocessing. In one level modeling, the process of estimating QRNN parameters will be performed directly, with dummy variables and lags of $Y_t$ from the stationary...
PACF as inputs and inflow and outflow data as output. Preprocessing using decomposition can improve the accuracy of forecasting data that contain trend and seasonal patterns. Thus, decomposition is done to eliminate the effects of trends, monthly seasonal, and calendar variations, by using time series regression. Residual of the time series regression will be modeled using QRNN which the input is residual lag and the output is residual value. After obtaining the forecast value for testing data, we will compare the methods using RMSE, MAE, MdAE, MAPE, and MdAPE, and select the best method using cross validation.

3. Results And Discussion

3.1. Simulation Study

A simulation study was conducted to find out the performance of the methods used in solve heteroscedasticity and nonlinearity. The model used in the simulation study is a decomposition pattern model that contains trend, seasonal, calendar variation, and noise as follows:

\[ Y_t = T_t + M_t + V_t + N_t \]  

(1)

where:
- \( T_t \) is a component for trend, where \( T_t = 0.1t \)
- \( M_t \) is the monthly seasonal, consist of homogeneous \((M_{t,1})\) and heterogeneous \((M_{t,2})\) pattern, where \( M_{t,1} = 20M_{t,1} + 23M_{t,1} + 25M_{t,1} + 23M_{t,1} + 20M_{t,1} + 15M_{t,1} + 10M_{t,1} + 7M_{t,1} + 5M_{t,1} + 7M_{t,1} + 10M_{t,1} + 15M_{t,1} \).
- \( V_t \) is the calendar variation, where \( V_t = 23V_{t,1} + 37V_{t,1} + 44V_{t,1} + 48V_{t,1} + 56V_{t,1} + 42V_{t,1} + 34V_{t,1} + 30V_{t,1} \).
- \( N_t \) is the noise that follows the linear \((N_{t,1})\) and nonlinear \((N_{t,2})\) pattern, where \( N_{t,1} = 0.7N_{t-1} + \epsilon \), with \( \epsilon - IIDN(0,1) \).
- \( N_{t,2} = 6.5N_{t-1} \exp(-0.25N_{t-1}) + \epsilon \), with \( \epsilon - IIDN(0,1) \).

Time series plot for trend, calendar variation, and seasonal components can be shown in Figure 1.

![Time series plot](image-url)
However, the identification of homogeneous and heterogeneous seasonal as well as linear and nonlinear noise cannot be done through time series plot. Thus, identification of seasonal components will be done using boxplot as in Figure 2,

Figure 2. Homogenous (a) and Heterogeneous (b) Seasonal

whereas identification of noise patterns is shown by the plot between \( N_t \) and \( N_{t-1} \), as in Figure 3.
In simulation study, the combination of monthly seasonal and noise patterns will result four scenarios i.e. homogeneous seasonal with linear noise, heterogeneous seasonal with linear noise, homogeneous seasonal with nonlinear noise, and heterogeneous seasonal with nonlinear noise. The decomposition pattern model will be obtained from the sum of the four components according to equation (1). In four scenarios, 5 replications will be performed. There are two schemes for QRNN method, as mentioned in previous chapter. Based on the analysis, it was found that preprocessing with decomposition has the least number of crossing with the smallest forecasting error. Thus, forecasting with QRNN will be done by preprocessing to eliminate the pattern of trend, monthly seasonal, and calendar variations effect. Further analysis will be done to find out the best method for each scenario and replication. Based on RMSE, MAE, MdAE, MdAPE, and MAPE criteria, the best model for each scenario and replication can be shown in Figure 4.

Figure 4 shows that in the 1st scenario, ARIMAX is the best method in one replication, while for the other four replications, the best model is obtained by using QRNN method, as well as the 2nd scenario. On contrary, in the 3rd scenario, QRNN method is the best method in one replication only. In 4th scenario, all replication indicate that QRNN is the best method for this scenario. This may be due to the presence of seasonal heteroscedasticity patterns and nonlinear noise in 4th scenario. Thus, the QRNN method can solve the problems of heteroscedasticity and nonlinearity, in accordance with the characteristics of quantile regression that can solve the problems of heteroscedasticity, and neural network that can solve nonlinearity problem.

3.2. Forecasting Inflow and Outflow with ARIMAX and QRNN
The growth of inflow and outflow for each currency can be shown at the time series plot in Figure 5.
These figures show that the highest inflow and outflow occurred around the Eid Fitr. The Eid Fitr that occurred on different week will affect the different impact on the increase of inflow and outflow. The first step in forecast using ARIMAX is modeling the data by time series regression. Then the diagnostic checking will be performed. When the residual has not fulfilled the white noise assumption, the residual needs to be modeled using ARIMA. This step is done for each currency.

Table 3. The Best ARIMA Model

| Data | Best ARIMA Model | Data | Best ARIMA Model |
|------|------------------|------|------------------|
| \( Y_1 \) | ARIMA \((1,2,12,14,0,23)\) | \( Y_4 \) | ARIMA \((11,12,23,0,0)\) |
| \( Y_2 \) | ARIMA \((0,0,1)(0,0,2)\) | \( Y_5 \) | ARIMA \((11,23,0,0)\) |
| \( Y_3 \) | ARIMA \((2,1,0)\) | \( Y_6 \) | ARIMA \((1,2,3,22,1,0)\) |
Table 3 shows the best residual ARIMA model for each currency.

In the QRNN modeling, the activation function used is hyperbolic tangent, with standardization used as preprocessing. The quantiles used are 0.025 as the lower bound, 0.5 as the forecasting result, and 0.975 as the upper bound. The combinations of neurons used are 1, 2, 3, 4, 5, 10, and 15. One of the weaknesses of the QRNN is the presence of crossing between quantiles. In order to minimize the occurrence of crossing, forecasting with QRNN will be done by using several schemes, as mentioned in previous chapter. Comparison the number of crossing can be shown in Figure 6.

Table 4 shows the best neuron number for each currency.

3.3. Comparison of ARIMAX and QRNN

The learning The comparison of forecast value with actual value for testing data can be visualized in Figure 7.
Figure 7. The Comparison of Forecasting Value with Actual Value for Testing Data

Figure 7 shows that for some currency, the forecast value of ARIMAX and QRNN are almost identical. Thus, it is also necessary to analyze the reduced forecasting error for QRNN methods compared with the ARIMAX. The comparison of these two methods can be shown by Table 5.

Table 5. The Comparison of ARIMAX and QRNN Methods

| Data  | Ratio of Forecast Accuracy ARIMAX to QRNN | Best Method |
|-------|------------------------------------------|-------------|
|       | RMSE  | MAE  | MdAE  | MdAPE | MAPE  |          |
| $Y_1$ | 0.96  | 1.28 | 2.42  | 2.43  | 1.60  | QRNN     |
| $Y_2$ | 0.65  | 0.71 | 0.86  | 1.03  | 0.80  | ARIMAX   |
| $Y_3$ | 1.08  | 1.27 | 1.37  | 1.21  | 1.39  | QRNN     |
| $Y_4$ | 0.79  | 0.91 | 1.08  | 1.52  | 1.03  | QRNN     |
| $Y_5$ | 0.84  | 0.89 | 1.27  | 1.24  | 0.99  | ARIMAX   |
| $Y_6$ | 1.00  | 1.08 | 1.15  | 1.55  | 1.01  | QRNN     |

The ratio value that greater than one indicates that QRNN were better than ARIMAX based on that criteria. Table 5 shows that QRNN were better than ARIMAX for four currencies, while ARIMAX is the best method for two currencies. Furthermore, forecasting inflow and outflow for 2017 period is done using the best method that has obtained. The results show that the highest inflow occurred in July, while the highest outflow occurred in June. This is appropriate with the Eid-Fitr which occurred in the fourth week of June.

4. Conclusion

Simulation study shows that quantile regression neural network is able to capture the pattern of heteroscedasticity and nonlinearity. The results for applications in inflow and outflow data shows that QRNN perform better compared to ARIMAX for four currencies based on the ratios value of some forecast accuracy. However, in QRNN modeling there is a crossing within quantile which can be caused by the estimates of each quantile that are calculated independently. Thus, to obtain upper and
lower limits that adhere to the cumulative distribution function (cdf) properties i.e. monotonic increasing function, it is necessary to conduct further research using bootstrap quantile regression, rearrangement, and regularization parameters.

Acknowledgments
This research was supported by DRPM under scheme of “Penelitian Berbasis Kompetensi”, No. 532/PKS/ITS/2017. The authors thank to the General Director of DIKTI for funding and to anonymous referees for their useful suggestions

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