Asteroseismology of KIC 7107778: a binary comprising almost identical subgiants

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ABSTRACT
We analyze an asteroseismic binary system: KIC 7107778, a non-eclipsing, unresolved target, with solar-like oscillations in both components. We used Kepler short cadence time series spanning nearly two years to obtain the power spectrum. Oscillation mode parameters were determined using Bayesian inference and a nested sampling Monte Carlo algorithm with the DIAMONDS package. The power profiles of the two components fully overlap, indicating their close similarity. We modelled the two stars with MESA and calculated oscillation frequencies with GYRE. Stellar fundamental parameters (mass, radius and age) were estimated by grid modelling with atmospheric parameters and the oscillation frequencies of \( l = 0 \) modes as constraints. Most \( l = 1 \) mixed modes were identified with models searched using a bisection method. Stellar parameters for the two sub-giant stars are: \( M_A = 1.42 \pm 0.06 M_{\odot}, \) \( M_B = 1.39 \pm 0.03 M_{\odot}, \) \( R_A = 2.93 \pm 0.05 R_{\odot}, R_B = 2.76 \pm 0.04 R_{\odot}, t_A = 3.32 \pm 0.54 \) Gyr and \( t_B = 3.51 \pm 0.33 \) Gyr. The mass difference of the system is \( \sim 1\% \). The results confirm their simultaneous birth and evolution, as is expected from binary formation. KIC 7107778 comprises almost identical twins, and is the first asteroseismic sub-giant binary to be detected.

Key words: binaries: general – stars: oscillations – stars: evolution

1 INTRODUCTION

Binary star systems provide an ideal astronomical laboratory to study stellar structure and evolution. The fact that two components share same metal abundance and age provides powerful constraints on models. Eclipsing binaries are especially useful, since mass and radius can be directly measured from orbits and eclipses through spectroscopic observations and precise light curves analysis (Andersen 1991).

Asteroseismology ushers in a new way to study binaries (Huber 2014), even for unresolved ones (Miglio et al. 2014), by analyzing the two components separately. Determining stellar mass and radius is feasible for single stars with solar-like oscillations (e.g. Kallinger et al. 2010; Chaplin et al. 2014). Sub-giant and red-giant oscillating stars displaying \( p \) (pressure dominated in the envelope) and \( g \) (gravity dominated in the core) mixed modes (Chaplin & Miglio 2013), are good indicators of evolutionary stages. Their great sensitivity to mass and age can produce precise estimates of stellar parameters (e.g. Metcalfe et al. 2010; Benomar et al. 2012). Binary systems with pulsators can be analyzed from modulation of the pulsation frequencies (Murphy et al. 2014; Murphy & Shibahashi 2015; Shibahashi & Kurtz 2012), so that system parameters can be determined with radial velocity curves simply derived through photometry.

White et al. (2016) pointed out that, until now, only five binary star systems have been detected with solar-like oscillations from both components. They are \( \alpha \) Cen A and B (Kjeldsen et al. 1999; Bedding et al. 1998; Bouchy & Carrier 2001; Carrier & Bourban 2003), 16 Cyg A and B (KIC 12069424 and KIC 12069449; Metcalfe et al. 2012; Metcalfe et al. 2015), KIC 9139163 and KIC 9139151 (Appourchaux et al. 2014; Appourchaux et al. 2012), HD 177412 (KIC 7510397; Appourchaux et al. 2015), and HD 176465 (KIC 10124866, also known as Luke & Leia; White et al. 2016). The latter two systems are not resolved by Kepler, such that their light variations are mixed in a...
single time series. Both systems were analyzed with one power spectrum, from which two sets of oscillation profiles were measured. Specifically, HD 177412 contains two main-sequence stars with mass difference ~ 7.5%, with separable oscillation frequency ranges, and HD 176465 contains two extremely similar main-sequence stars with mass difference ~ 3%, leading to two significantly overlapping oscillation ranges. In addition, KIC 10080943 is another unresolved binary system, comprising two δ Sct/γ Dor hybrid pulsators on the main-sequence (Keen et al. 2015; Schmid et al. 2015).

In this context, these binaries are not strictly ‘twins’ in that the masses are not equal to within 2 percent (Lucy 2006; Simon & Obbie 2009). Twins are almost always found at small separations (P ≲ 25 d, Lucy & Ricco 1979), though not all close binaries are twins. Further, they are more common among lower-mass systems, both in observational data (e.g. Tokovinin 2000; Simon & Obbie 2009) and in binary star-formation simulations (e.g. Bate 2009). Their importance lies in their ability to discriminate between dominant physical processes operating in pre-main-sequence binaries, as those authors have discussed.

Here we report on the widely-separated twin binary system KIC 7107778, with mass difference ~ 1% based on our findings, an analog to Luke & Leia (White et al. 2016). The system was not resolved by Kepler but was detected with oscillations from two stars in the mixed time series. It proves to be the first asteroseismic sub-giant binary system to be detected, with completely overlapping power spectra of two component stars. The paper will be structured as follows. Section 2 provides observations from previous literature and describes the processing of Kepler data. Section 3 illustrates the oscillation mode parameters. Section 4 details our asteroseismic analysis with stellar models of two stars, and is followed by discussions and conclusions in Section 5.

2 OBSERVATION AND DATA PROCESSING

KIC 7107778 is a binary system observed as a single target, with Kepler magnitude Kp = 11.38. The realization of its binary identity was from speckle interferometry (Horch et al. 2012). The angular separation of the two stars is $P = 0.0288$″, measured with filters whose central wavelengths are 692 nm and 880 nm. The physical separation can be determined by combining luminosities from models with the apparent magnitude; however, such estimation is only approximate because the apparent magnitude is a blended contribution from both stars. We took advantage of the parallax determined from Gaia (Gaia Collaboration et al. 2016). 1.74 ± 0.43 mas, to infer the distance to Earth and physical projected separation between the two stars, which are $d = 574.180$ pc, and $s = 16.52 ± 3.54$ AU. Note that this estimation is very approximate because Gaia parallaxes are not yet fully reliable at the present mission stage (see Fabricius et al. 2016). Assuming the orbit is circular, and adopting Kepler’s third law, it suggests the orbital period of this system is about 39 yr. Therefore, we would not expect to analyze its orbit using radial velocity curves. On the other hand, such a wide separation means fully independent evolution without tidal effects or mass transfer. In the following analysis, we treat them as two single stars that have not interacted.

| Parameter | Value | Reference |
|-----------|-------|-----------|
| $T_{\text{eff}}$ [K] | 5129 ± 82 | Drimmel et al. (2003) |
| 5045 ± 105 | Casagrande et al. (2010) |
| 5149 ± 150 | Xiang et al. (2015) |

| [Fe/H] [dex] | 0.11 ± 0.10 | Xiang et al. (2015) |
| 0.05 ± 0.08 | Buchhave & Latham (2015) |

Several works have measured astrophysical parameters of this system. Effective temperatures measured by SDSS and IRFM are $T_{\text{eff}} = 5129 ± 82$ K and $T_{\text{eff}} = 5045 ± 105$ K, respectively. The binary system was also covered by the LAMOST-Kepler project, which used LAMOST low-resolution ($R \approx 1800$A) optical spectra in the waveband of 3800–9000 Å and observed Kepler-field targets in September 2014. The LAMOST Stellar Parameter Pipeline (LSP3, Xiang et al. 2015) gives $T_{\text{eff}} = 5149 ± 150$ K, and metallicity [Fe/H] = 0.11 ± 0.10 dex. Buchhave & Latham (2015) used the Tillinghast Reflector Echelle Spectrograph (TRES) to obtain medium resolution ($R \approx 44000$A) spectra in the waveband of 3800–9100 Å at Fred Lawrence Whipple Observatory. They measured the metallicity [Fe/H] = 0.05 ± 0.08 dex. However, all these observations treated two stars as one single target. We do not know to what extent the mixed value deviates from the real ones. Table 1 summarizes the observations in the literature.

The Kepler mission observed the target in long-cadence mode (LC; 29.43 min sampling) over the whole mission and in short-cadence mode (SC; 58.84 s sampling) during Q2.1, Q5 and Q7 – Q12 (Q represents three-month-long quarters). The pulsation frequency range is centered at 550 μHz, which is well above the Nyquist frequency of long cadence data (~ 283 μHz). Therefore, we only considered the short-cadence time series. We concatenated the data and processed it following Garcia et al. (2011), correcting outliers, jumps and drifts. Then it passed through a high-pass filter which was based on a Gaussian smooth function with a width of one day. This largely minimized instrumental side-effects and only affected frequencies lower than ~ 12 μHz, far below the frequency range we intended to analyze. We obtained the power spectrum by applying a Lomb-Scargle Periodogram (Lomb 1976; Scargle 1982) to the time series with a frequency resolution ~ 0.012 μHz. The power spectrum is shown in Fig. 1 and 2 in both logarithmic and linear scales.

The signature of solar-like oscillation is a Gaussian-like envelope located at $\nu_{\text{max}}$, the so-called frequency of maximum power, and is comprised of numerous oscillation modes. In main-sequence stars, p-mode oscillations are approximately equally spaced in frequency, as described by the asymptotic equation with radial orders n and spherical degrees l (Tassoul 1980):

$$\nu_n^l = \Delta \nu (n + \frac{l}{2} + \varepsilon) - \delta \nu_{\text{wd}}.$$  \hspace{1cm} (1)$$

where $\Delta \nu$ is the large separation, which measures the spacing of adjacent modes with the same l, $\delta \nu_{\text{wd}}$ is the small separation, and $\varepsilon$ is an offset. In more evolved stars, the core will have g-mode oscillations, which are approximately equally spaced in period. The analogous asymptotic equa-
tion is specified by order $n_\ell$:
\[
\Pi_{\ell d} = v_{\ell d}^{-1} = \Delta \Pi_l (n_\ell + \xi_\ell),
\]
where $\Delta \Pi_l$ is the period spacing, and $\xi_\ell$ is an offset.

As the star evolves off the main-sequence, central hydrogen depletes, and $p$ and $g$ mixed modes of $l \geq 1$ appear to have “avoided crossings” (Aizenman et al. 1977), whereby oscillation frequencies are no longer equally spaced in either frequency or period. The $l = 0$ $p$ modes are unaffected, which assisted us in determining the mean large separation $\langle \Delta \nu \rangle$ in an échelle diagram. The best value for $\langle \Delta \nu \rangle$ is the one that makes the $l = 0$ ridge vertical. Fig. 3 displays échelle diagrams for the two stars. To avoid ambiguity, the star with smaller $\langle \Delta \nu \rangle$ is named as KIC 7107778 A, and the larger one KIC 7107778 B. The best values are: $\langle \Delta \nu \rangle_A = 31.83 \pm 0.02 \mu$Hz and $\langle \Delta \nu \rangle_B = 34.55 \pm 0.01 \mu$Hz.

### 3 MODE PARAMETERS

To model the power spectrum, we used high-Dimensional And multi-Modal NestD Sampling code (DIAMONDS; Corsaro & De Ridder 2014). The DIAMONDS code utilizes Bayes’s theorem:
\[
p(\theta | D, M) = \frac{f(\theta | D, M) \pi(\theta | M)}{p(D | M)},
\]
where $\theta = (\theta_1, \theta_2, ..., \theta_k)$, $f(\theta | D, M)$, $\pi(\theta | M)$, $p(\theta | D, M)$ are the parameter vector, likelihood function for a given model $M$ and dataset $D$, prior probability density function, and the posterior probability density function, respectively. DIAMONDS uses a sampling algorithm, Nested Sampling Monte Carlo (NSMC), to tackle the high-dimensional problem.

We simultaneously fitted the two stars to separate their overlapping oscillations with the following steps. First, we modelled the power spectrum, $P(\nu)$, with:
\[
P(\nu) = W + R(\nu) \left\{ \sum_{i=1}^{k} \frac{2 \sqrt{2}}{\pi} \left( \frac{a_i^2}{b_i} \right) + H_0 \exp \left[ \frac{(\nu - v_{\text{max}})^2}{2 \sigma^2} \right] \right\},
\]

similar to the background models presented in Kallinger et al. (2014) and Corsaro et al. (2015). The right-hand side of the equation comprises a flat white noise $W$, a sum of Harvey power profiles (Harvey 1985) with parameters $(a_i, b_i)$, and a Gaussian envelope with $(H_0, \nu_{\text{max}}, \sigma)$. The Harvey profile models stellar background caused by granulation. In the case of KIC 7107778, the background was well-fitted using three Harvey profiles, i.e., $k = 3$. The Gaussian envelope denotes the range of oscillation for two overlapping stars. All these components except the noise are modulated by the response function,
\[
R(\nu) = \sin^2 \left( \frac{\nu \pi}{2 \nu_{\text{Nyq}}} \right),
\]
where $\nu_{\text{Nyq}} = 8496.36 \mu$Hz denotes the Nyquist frequency. The total number of free variables is $1 + 2 \times 3 + 3 = 10$. The results are shown in Table 2. As argued by Kallinger et al. (2014), the granulation frequencies $b_i$ should scale with $v_{\text{max}}$. They provided empirical relations $b_2 = 0.317 (v_{\text{max}} / \mu$Hz$)^{0.970} \mu$Hz and $b_3 = 0.948 (v_{\text{max}} / \mu$Hz$)^{0.992} \mu$Hz, which result in $b_2 = 148.87 \mu$Hz and $b_3 = 511.87 \mu$Hz in our case. $b_2$ is similar to our fit but not $b_3$. This should be expected because the power spectrum is a superposition of the twins. $b_1$ should be treated carefully because the light curve was processed by a high-pass filter, which affected the low-frequency spectrum.

Then, each mode was fitted with a Lorentzian profile with three free parameters: frequency centroid $\nu_0$, amplitude $A$, and linewidth $\Gamma$. The Bayesian evidence balances the goodness of fit and the need to fit, as it can be written as a product of the maximum of the likelihood and an Occam factor (Knuth et al. 2014). We adopted a splitting model of $l = 1$ modes
\[
L(\nu) = R(\nu) \left[ \frac{A^2 / \Gamma}{1 + (\nu - \nu_0)^2 / \Gamma^2} \right].
\]

The power spectrum was fitted with a sum of 32 Lorentzian profiles. The total number of free variables is $3 \times 32 = 96$. It is possible that the star rotates and lifts degeneracy of $m$-degree of the modes (Gizon & Solanki 2003). Thus we performed a hypothesis test with Bayesian evidence denoted by $p(D | M)$ in Equation 3. The Bayesian evidence balances the goodness of fit and the need to fit, as it can be written as a product of the maximum of the likelihood and an Occam factor (Knuth et al. 2014). We adopted a splitting model of $l = 1$ modes
\[
L(\nu) = R(\nu) \left[ \sum_{i=-1}^{l} \frac{\xi_i A^2 / \Gamma}{1 + (\nu - \nu_0 - m \delta \nu_\text{split})^2 / \Gamma^2} \right].
\]

Table 2. Granulation background parameters.

| Parameter | Value | 68.3% credible |
|-----------|-------|----------------|
| $W$ [ppm$^2$/\mu Hz] | 11.927 | 0.100 |
| $a_1$ [ppm] | 58.874 | 8.314 |
| $a_2$ [\mu Hz] | 5.203 | 1.460 |
| $a_3$ [ppm] | 76.134 | 7.947 |
| $b_1$ [\mu Hz] | 400.646 | 64.318 |
| $H_0$ [ppm$^2$/\mu Hz] | 17.977 | 2.852 |
| $v_{\text{max}}$ [\mu Hz] | 568.051 | 7.935 |
| $\sigma$ [\mu Hz] | 55.811 | 8.690 |

Notice that only $l = 0$ and $l = 2$ modes can be identified straightforwardly and allocated to one of the two stars, since they are regularly spaced. The $l = 1$ mixed modes are strongly bumped and no clear patterns could be followed, so we used stellar models to help the identification, as discussed in Section 4. We also measured the frequency of maximum power, $v_{\text{max}}$, by fitting a Gaussian profile in power density.
to the $l = 0$ mode peaks. The results are $v_{\text{max}, A} = 523 \pm 16 \ \mu \text{Hz}$ and $v_{\text{max}, B} = 570 \pm 18 \ \mu \text{Hz}$, where the uncertainties are half of $\langle \Delta v \rangle$.

Fig. 3 shows the échelle diagram of both stars. The circular and square open symbols represent identified $l = 0$ and $l = 2$ modes. Red and blue colors denote star A and star B, respectively. The mode parameters are shown in Tables 6 and 7. The $l = 1$ modes in the tables are further discussed in Section 4.3.

4 ASTEROSEISMIC ANALYSIS

As discussed in section 2, KIC 7107778 has a long orbital period and no evidence of eclipses from which stellar fundamental parameters could be estimated. Therefore, estimating them through stellar models with asteroseismology is necessary.

4.1 Stellar Models

We constructed stellar models using Modules for Experiments in Stellar Astrophysics (MESA; Paxton et al. 2011; Paxton et al. 2013; Paxton et al. 2015). Paxton et al. (2011) have discussed the input physics of MESA. Here we list them for completeness. The equation of state was delivered by OPAL EOS tables (Rogers & Nayfonov 2002), SVCH tables (Saumon et al. 1995) at low temperatures and densities, and HELM (Timmes & Swesty 2000) and PC (Potekhin & Chabrier 2010) tables under other circumstances. Opacities were taken from Iglesias & Rogers (1996) at high temperature and Ferguson et al. (2005) at low temperature. Nuclear reaction rates were based on NACRE (Angulo et al. 1999) and CF88 (Caughlan & Fowler 1988) when NACRE was unavailable. The convection was implemented with Mixing Length Theory (MLT) illustrated in Cox & Giuli (1968). The mixing length parameter $\alpha_{\text{MLT}} = 1.917$ was employed according to the MESA standard solar model (Paxton et al. 2011). Overshoot mixing was set according to Herwig (2000), with overshooting parameter $f_{\text{ov}} = 0.16$. The initial helium abundance was estimated through

$$Y_{\text{ini}} = Y_0 + \frac{\Delta Y}{\Delta Z} Z_{\text{ini}},$$

where $Y_0 = 0.249$ (Planck Collaboration et al. 2016) and $\Delta Y/\Delta Z = 1.33$, calculated using the initial abundances of helium and heavy elements of the calibrated solar model (Paxton et al. 2011). The relation of metallicity and element abundance ratio we adopted here was

$$[\text{Fe/H}] = \log(Z/X) - \log(Z/X)_\odot,$$

where the solar value is $(Z/X)_\odot = 0.02293$ (Grevesse & Sauval 1998).

The varying input parameters for grid modelling are mass $M$ and metallicity [Fe/H]. They were set as follows. First, the mass was set according to asteroseismic scaling relations. The mean large separation $\Delta v$ and frequency of maximum power $v_{\text{max}}$ are related to mean density $\rho$, surface gravity $g$ and effective temperature $T_{\text{eff}}$: $\Delta v \propto \sqrt{\rho}$, $v_{\text{max}} \propto g/\sqrt{T_{\text{eff}}}$ (Kjeldsen & Bedding 1995), i.e.

$$\frac{\Delta v}{\Delta v_\odot} \approx \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{R_\odot}\right)^{-3/2},$$

$$\frac{v_{\text{max}}}{v_{\text{max}, \odot}} \approx \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{R_\odot}\right)^{-3/2},$$
**Figure 2.** Power spectrum around the oscillation range on a linear scale. The gray line denotes the original power spectrum superimposed with fitted $l = 1$ modes in black, $l = 0$ and $l = 2$ modes of star A in red, and those of star B in blue.
and (11) summarize the input parameters of grid modelling.

Table 3. Input parameters of grid modelling.

| Parameter | Value | Step size |
|-----------|-------|-----------|
| M/M⊙ | 1.34±1.60 | 0.1 |
| [Fe/H] | 0.01±0.21 | 0.1 |
| a_{ALT} | 1.917 | fixed |
| f_0 | 0.016 | fixed |

\[
\frac{\nu_{\text{max}}}{\nu_{\text{max}, \odot}} \approx \left( \frac{M}{M_{\odot}} \right)^{1/3} \left( \frac{R}{R_{\odot}} \right)^{-2} \left( \frac{T_{\text{eff}}}{T_{\text{eff}, \odot}} \right)^{-1/2},
\]

where \( \Delta \nu_{\odot} = 135.1 \) µHz, \( \nu_{\text{max}, \odot} = 3050 \) µHz (Chaplin & Miglio 2013) and \( T_{\text{eff}, \odot} = 5777 \) K. Thus, the mass prescription can be deduced from equation 10 and 11:

\[
\frac{M}{M_{\odot}} \approx \left( \frac{\Delta \nu}{\nu_{\text{max}, \odot}} \right)^{-4} \left( \frac{\nu_{\text{max}}}{\nu_{\text{max}, \odot}} \right)^{3} \left( \frac{T_{\text{eff}}}{T_{\text{eff}, \odot}} \right)^{3/2}.
\]

Considering the uncertainties, the mass range of the grid should at least cover 1.34 to 1.60 \( M_{\odot} \). Second, we adopted metallicity \([\text{Fe/H}]=0.01, 0.11 \) and 0.21 according to spectral observations from LAMOST. Table 3 summarizes the input parameters of our grid modelling.

4.2 Modelling the \( l = 0 \) and \( l = 2 \) modes

We calculated oscillation frequencies for models that met the requirements of \( \Delta \nu \) and \( \nu_{\text{max}} \) with GYRE (Townsend & Teitler 2013), which solves the adiabatic pulsation equations with stellar structure data.

The calculated frequencies deviated from the observations due to the improper simulation of the stellar surface. Therefore, we followed the method described by Ball & Gizon (2014) to correct them. The correction formula we adopted here was:

\[
\delta \nu = (a_{-1} \nu^{-1} + a_{3} \nu^{3})/I,
\]

where \( a_{-1} \) and \( a_{3} \) are coefficients determined through least-squares fit, and \( I \) is the mode inertia.

We used \( \chi^2 = \Sigma (\nu_{\text{obs}} - \nu_{\text{mod}})^2 / \sigma^2 \) as the quality of the fit. Frequencies used in the \( \chi^2 \) calculation were all \( l = 0 \) and \( l = 2 \) modes. The smaller the \( \chi^2 \) of the model, the better the match. We took the best 10% of the models, as measured by \( \chi^2 \), for further considerations. The choice of this criterion was a trade-off. Small values could be biased by fluctuations in limited samples. The problem was tackled by sorting samples according to the values of \( \chi^2 \), performing difference between two adjacent quantities. We found the trend around 10% went smoothly, which ruled out the fluctuation effect. Including more models would make the selection less useful. This point was addressed by inspecting the value of \( \chi^2 \) around the 10% cut-off and assuring that it was not too large.

With the lowest 10% \( \chi^2 \) models, we calculated the mean value of the stellar parameters as the centroid. Table 4 lists them with standard deviations. Additionally, Fig. 7 and 8 display histograms of each stellar parameter, which reflect the distribution in the lowest 10% \( \chi^2 \) models with \([\text{Fe/H}]=0.11\). Red and blue indicate star A and star B, respectively.

Considering that \([\text{Fe/H}]=0\) was estimated with uncertainties, we combined stellar parameters derived based on different \([\text{Fe/H}]=0\) together, as the ultimate results. Here we list mass, radius and age: \( M_A = 1.42\pm0.06 \ M_{\odot}, \ M_B = 1.39\pm0.03 \ M_{\odot}, \ R_A = 2.93\pm0.05 \ R_{\odot}, \ R_B = 2.76\pm0.04 \ R_{\odot}, \ t_A = 3.32\pm0.54 \ \text{Gyr} \) and \( t_B = 3.51\pm0.33 \ \text{Gyr} \).

4.3 Modelling the \( l = 1 \) mixed modes

We next searched for the best models which could also fit the frequencies of \( l = 1 \) modes from both stars. KIC 7107778...
contains two sub-giant stars. Tiny changes to the mass of the models influence oscillations greatly. Only extremely fine grids produce satisfying results, which makes the task demanding. Since our purpose was to find a pair of stellar models for both stars that fit observations best, we used the bisection method to search.

The main idea of the bisection method is summarized as follows, similar to finding solutions for equation $f(x) = 0$. First, we started with two masses, $M_1$ and $M_2$, which lied on opposite sides of the best model. This choice was realized by visually inspecting oscillation frequencies on the échelle diagram. Second, we bisected this range, i.e. calculated frequencies of $M_3 = (M_1 + M_2)/2$. Third, we evaluated the result of $M_3$ and determined $M_3$ and whichever of $M_1$ or $M_2$ yield the best model. Fourth, we kept bisecting the mass range until the difference became sufficiently small.

We followed this search scheme under three different metallicities [Fe/H]: 0.01, 0.11 and 0.21. The maximum precision in calculation reaches 0.00001 $M_\odot$. The results revealed three combinations of best fitting models for the two stars. Although they do still deviate from observed peaks, they give a reasonable fit to most $l = 1$ modes. Most model masses reach 0.001 $M_\odot$ precision. The reason of such small precision is that the mixed modes are very sensitive to the change of internal structure, and subgiants evolve very fast. Each combination has less than 0.2 Gyr difference in age, consistent with the idea that two components formed at the same time. Table 5 presents the fundamental parameters of the three model pairs, and Fig. 6 displays them on échelle diagrams. Open and filled symbols represent observational and theoretical frequencies. Red and blue indicate modes of star A and star B, respectively. Circles, squares and triangles denote $l = 0$, $l = 1$, and $l = 2$ modes.

Based on theoretical models, we found that most observed peaks could be matched with $l = 1$ modes. Tables 6 and 7 display the mode parameters in each Lorentzian profile we fitted to each peak, with associated $l$ degrees. Modes which share the same peak on the power spectrum are labeled with an asterisk mark. Some modes from the two stars stand too close and cause ambiguity; they are labeled with a question mark. Here we remind readers that this solution is not unique, considering that the models still differ from the observation. In Fig. 4, we show the mode linewidth as a function of mode frequency. The circles and triangles represent the $l = 0$ and $l = 1$ modes respectively. The $l = 1$ modes shown here are only those which have relative certain identity (not associated with "?" or "?" in Table 6 and 7). The $l = 2$ modes are instead not presented because each $l = 2$ mode region is fitted with a Lorentzian profile, the linewidth of which is not necessarily representing the real mode lifetime.

5. Conclusion

We applied asteroseismology to a binary target KIC 7107778, and confirmed that the two stars are in the subgiant phase. We successfully identified their $l = 0$ and $l = 2$ oscillation modes and distinguished $l = 1$ modes to the greatest extent. We derived stellar fundamental parameters for the two stars: $M_A = 1.43^{+0.08}_{-0.08}$ $M_\odot$, $M_B = 1.40^{+0.05}_{-0.06}$ $M_\odot$, $R_A = 2.94^{+0.06}_{-0.06}$ $R_\odot$, $R_B = 2.77^{+0.04}_{-0.04}$ $R_\odot$, $t_A = 3.19^{+0.60}_{-0.64}$ Gyr, and $t_B = 3.26^{+0.40}_{-0.40}$ Gyr. All the evidence suggests that they formed at the same time and possess nearly equal masses.

The results yield the similarity of masses for two stars, and the best models, derived through bisection modelling, determined the mass difference as 1.42%, 0.68% and 0.70%, from which we conclude it is ~1%. The H-R diagram verifies this, with extremely close tracks.

Table 4 indicates the sensitivity of metallicity to age, and the two stars’ ages are equal within the error. We could conclude that they formed at the same time, as is expec-
Figure 5. Hertzsprung-Russell diagram. Grid model tracks are displayed as black dotted lines. The best models, derived through bisection, are drawn with star symbols located on evolutionary tracks (dashed lines) in red for star A and in blue for star B. The solid lines indicate the standard deviation of effective temperature $T_{\text{eff}}$ and luminosity $L$, which are shown in Table 4. The left, middle and right panel represent models for metallicity $[\text{Fe/H}]=0.01$, 0.11 and 0.21 respectively.

Table 4. Fundamental properties of models from grid modelling.

| Star | $[\text{Fe/H}]$ | $M$ | Age | $T_{\text{eff}}$ | $L$ | $R$ | $\log g$ | $\Delta \nu$ | $v_{\text{max}}$ |
|------|-----------------|-----|-----|-----------------|-----|-----|-----------|-----------|-----------|
|      | [dex]           | $[M_{\odot}]$ | [Gyr] | [K] | $[L_{\odot}]$ | $[R_{\odot}]$ | [dex] | [$\mu$Hz] | [$\mu$Hz] |
| A    | 0.01            | 1.41 ± 0.06 | 3.178 ± 0.440 | 5216 ± 258 | 5.813 ± 1.508 | 2.925 ± 0.047 | 3.654 ± 0.005 | 32.01 ± 0.18 | 534 ± 8   |
| B    | 0.01            | 1.38 ± 0.03 | 3.289 ± 0.263 | 5233 ± 132 | 5.140 ± 0.692 | 2.754 ± 0.025 | 3.699 ± 0.003 | 34.77 ± 0.13 | 592 ± 5   |
| A    | 0.11            | 1.43 ± 0.07 | 3.266 ± 0.583 | 5184 ± 290 | 5.787 ± 1.663 | 2.945 ± 0.055 | 3.656 ± 0.007 | 31.98 ± 0.17 | 539 ± 9   |
| B    | 0.11            | 1.38 ± 0.03 | 3.606 ± 0.313 | 5092 ± 105 | 4.594 ± 0.549 | 2.751 ± 0.025 | 3.700 ± 0.004 | 34.81 ± 0.12 | 601 ± 4   |
| A    | 0.21            | 1.41 ± 0.07 | 3.643 ± 0.551 | 5037 ± 199 | 5.032 ± 1.160 | 2.926 ± 0.054 | 3.655 ± 0.006 | 32.06 ± 0.15 | 546 ± 6   |
| B    | 0.21            | 1.40 ± 0.04 | 3.657 ± 0.317 | 5045 ± 106 | 4.473 ± 0.535 | 2.765 ± 0.028 | 3.701 ± 0.003 | 34.78 ± 0.13 | 606 ± 4   |

Note: the models are selected with the lowest 10% $\chi^2$. Column 2 is the input parameter for grid modelling.

Table 5. Fundamental properties of best models from bisection method.

| NO. | Star | $M$ | $[\text{Fe/H}]$ | Age | $T_{\text{eff}}$ | $L$ | $R$ | $\log g$ | $M_{\text{H,core}}$ | $\Delta \nu$ | $v_{\text{max}}$ |
|-----|------|-----|-----------------|-----|-----------------|-----|-----|-----------|---------------------|-----------|-----------|
| #   |      | $[M_{\odot}]$ | [dex] | [Gyr] | [K] | $[L_{\odot}]$ | $[R_{\odot}]$ | [dex] | [M$_{\odot}$] | [$\mu$Hz] | [$\mu$Hz] |
| 1   | A    | 1.41 | 0.01            | 3.063 | 5130 | 5.342 | 2.929 | 3.655 | 0.165 | 32.04 | 540   |
| 2   | B    | 1.39 | 0.01            | 3.229 | 5195 | 4.982 | 2.759 | 3.699 | 0.157 | 34.74 | 594   |
| 3   | A    | 1.48 | 0.11            | 2.873 | 5120 | 5.464 | 2.975 | 3.661 | 0.170 | 32.00 | 548   |
| 4   | B    | 1.47 | 0.11            | 2.929 | 5267 | 5.478 | 2.814 | 3.705 | 0.162 | 34.64 | 599   |
| 5   | A    | 1.42 | 0.21            | 3.533 | 4987 | 4.763 | 2.927 | 3.656 | 0.165 | 32.10 | 549   |
| 6   | B    | 1.41 | 0.21            | 3.548 | 5038 | 4.443 | 2.770 | 3.703 | 0.160 | 34.80 | 608   |

Note: column 3 and 4 are the input parameters for grid modelling.

Acknowledged by binary formation that they originate from the same protostellar cloud.

The KIC 7107778 system contains two extremely similar components, with fully overlapping power spectra. This is one of the few identical twin systems to be found, proving the full potential of asteroseismology.

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1 http://www.cosmos.esa.int/gaia
2 http://www.cosmos.esa.int/web/gaia/dpac/consortium
References

Aizenman M., Smeyers P., Weigert A., 1977, A&A, 58, 41
Andersen J., 1991, A&ARv, 3, 91
Angulo C., et al., 1999, Nuclear Physics A, 656, 3
Appourchaux T., et al., 2012, A&A, 543, A54
Appourchaux T., et al., 2014, A&A, 566, A20
Appourchaux T., et al., 2015, A&A, 582, A25
Ball W. H., Gizon L., 2014, A&A, 568, A123
Bate M. R., 2000, MNRAS, 392, 590
Bedding T. R., Kjeldsen H., Frandsen S., dall T. H., 1998, in Deubner F.-L., Christensen-Dalsgaard J., Kurtz D., eds, IAU Symposium Vol. 185, New Eyes to See Inside the Sun and Stars, p. 285 (arXiv:astro-ph/9710359)
Benomar O., Bedding T. R., Stello D., Deheuvels S., White T. R., Christensen-Dalsgaard J., 2012, ApJ, 745, L33
Benomar O., et al., 2013, ApJ, 767, 158
Bouchy F., Carrier F., 2001, A&A, 374, L5
Buchhave L. A., Latham D. W., 2015, ApJ, 808, 187
Carrier F., Bourban G., 2003, A&A, 406, L23
Casagrande L., Ramírez I., Méhéndez J., Bessell M., Asplund M., 2010, A&A, 512, A54
Cauglain G. R., Fowler W. A., 1988, Atomic Data and Nuclear Data Tables, 40, 283
Chaplin W. J., Miglio A., 2013, ARA&A, 51, 353
Chaplin W. J., et al., 2014, ApJS, 210, 1
Corsaro E., De Ridder J., 2014, A&A, 571, A71
Corsaro E., De Ridder J., García R. A., 2015, A&A, 579, A83
Cox J. P., Giuli R. T., 1968, Principles of stellar structure
Drimmel R., Cabrera-Lavers A., López-Corredoira M., 2003, A&A, 409, 205
Dupret M.-A., et al., 2009, A&A, 506, 57
Fabricius C., et al., 2016, A&A, 595, A3
Ferguson J. W., Alexander D. R., Allard F., Barman T., Bodnarik J. G., Hauschildt P. H., Heffner-Wong A., Tamanai A., 2005, ApJ, 623, 585
Gaia Collaboration Brown A. G. A., Vallenari A., Prusti T., de Bruijne J., Mignard F., Drimmel R., co-authors ., 2016, preprint, (arXiv:1609.04172)
García R. A., et al., 2011, MNRAS, 414, L6
Gizon L., Solanki S. K., 2003, ApJ, 589, 1009
Grevesse N., Sauval A. J., 1998, Space Sci. Rev., 85, 161
Harvey J., 1985, in Rolfe E., Battrick B., eds, ESA Special Publication Vol. 235, Future Missions in Solar, Heliospheric & Space Plasma Physics.
Herwig F., 2000, A&A, 360, 952
Horch E. P., Howell S. B., Everett M. E., Ciardi D. R., 2012, AJ, 144, 165
Huber D., 2014, preprint, (arXiv:1404.7501)
Iglesias C. A., Rogers F. J., 1996, ApJ, 464, 943
Jeffreys H., 1961, Theory of Probability. International series of monographs on physics, Oxford University Press,
https://books.google.co.jp/books?id=tXp5AQAACAAJ
Kallinger T., et al., 2010, A&A, 522, A1
Kallinger T., et al., 2014, A&A, 570, A41
Kee M. A., Bedding T. R., Murphy S. J., Schmid V. S., Aerts C., Tkachenko A., Ouazzani R.-M., Kurtz D. W., 2015, MNRAS, 454, 1792
Kjeldsen H., Bedding T. R., 1995, A&A, 293
Knuth K. H., Habeck M., Malakar N. K., Mubeen A. M., Placek B., 2014, preprint, (arXiv:1411.3013)
Lomb N. R., 1976, Ap&SS, 39, 447
Lucy L. B., 2006, A&A, 457, 629
Lucy L. B., Ricco E., 1979, AJ, 84, 401
Metcalfe T. S., et al., 2010, ApJ, 723, 1583
Metcalfe T. S., et al., 2012, ApJ, 748, L10
Metcalfe T. S., Creevey O. L., Davies G. R., 2015, ApJ, 811, L37
Miglio A., Chaplin W. J., Farmer R., Kolb U., Girardi L., Elsworth Y., Appourchaux T., Handberg R., 2014, ApJ, 784, L3
Murphy S. J., Shibahashi H., 2015, MNRAS, 450, 4475
Murphy S. J., Bedding T. R., Shibahashi H., Kurtz D. W., Kjeldsen H., 2014, MNRAS, 441, 2515
Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F., 2011, ApJS, 192, 3
Paxton B., et al., 2013, ApJS, 208, 4
Paxton B., et al., 2015, ApJS, 220, 15
Planck Collaboration et al., 2015, A&A, 594, A13
Potechkin A. Y., Chabrier G., 2010, Contributions to Plasma Physics, 50, 82
Rogers F. J., Nayfonov A., 2002, ApJ, 576, 1064
Saumon D., Chabrier G., van Horn H. M., 1995, ApJS, 99, 713
Scargle J. D., 1982, ApJ, 263, 835
Schmid V. S., et al., 2015, A&A, 584, A35
Shibahashi H., Kurtz D. W., 2012, MNRAS, 422, 738
Simon M., Ollibe R. C., 2009, AJ, 137, 3442
Tassoul M., 1980, ApJS, 43, 469
Timmes F. X., Swesty F. D., 2000, ApJS, 126, 501
Tokovinin A. A., 2000, A&A, 360, 997
Townsend B. H. D., Teitler S. A., 2013, MNRAS, 435, 3406
White T. R., et al., 2016, preprint, (arXiv:1609.09581)
Xiang M. S., et al., 2015, MNRAS, 448, 822

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Table 6. Mode parameters of KIC 7107778 A.

| ℓ | Frequency [µHz] | 68.3% credible [µHz] | Amplitude [ppm] | 68.3% credible [ppm] | Linewidth [µHz] | 68.3% credible [µHz] |
|---|-----------------|----------------------|-----------------|----------------------|-----------------|----------------------|
| 0 | 460.294         | −0.022/ + 0.021      | 15.66           | −0.80/ + 1.19        | 0.36            | −0.09/ + 0.10        |
| 0 | 491.947         | −0.013/ + 0.014      | 24.17           | −1.29/ + 1.66        | 0.36            | −0.10/ + 0.13        |
| 0 | 523.603         | −0.012/ + 0.018      | 25.84           | −1.85/ + 1.35        | 0.19            | −0.07/ + 0.06        |
| 0 | 555.473         | −0.133/ + 0.090      | 22.65           | −5.44/ + 2.56        | 0.33            | −0.16/ + 0.18        |
| 0 | 587.314         | −0.420/ + 1.462      | 9.65            | −2.24/ + 3.40        | 0.41            | −0.17/ + 0.18        |
| 1 | 477.158         | −0.031/ + 0.042      | 15.47           | −0.18/ + 0.25        | 0.18            | −0.02/ + 0.02        |
| 1 | 488.837         | −0.022/ + 0.019      | 10.70           | −0.48/ + 0.64        | 0.26            | −0.09/ + 0.07        |
| 1 | 507.928         | −0.014/ + 0.013      | 25.92           | −0.83/ + 0.99        | 0.37            | −0.06/ + 0.06        |
| 1 | 522.897         | −0.002/ + 0.002      | 21.55           | −0.75/ + 1.00        | 0.04            | −0.01/ + 0.01        |
| 1 | 539.794         | −0.011/ + 0.008      | 32.01           | −1.36/ + 2.02        | 0.13            | −0.04/ + 0.05        |
| 1 | 558.466         | −0.172/ + 0.072      | 9.08            | −3.06/ + 3.49        | 0.06            | −0.02/ + 0.04        |
| 1 | 572.522         | −0.827/ + 0.844      | 9.03            | −2.80/ + 4.84        | 0.24            | −0.13/ + 0.09        |
| 1 | 595.812         | −0.040/ + 0.042      | 12.35           | −1.44/ + 2.36        | 0.17            | −0.11/ + 0.08        |

Note: Modes which share the same peak on the power spectrum are labeled with ** marks. Modes from two stars standing too close and causing ambiguity are denoted with * marks.

Table 7. Mode parameters of KIC 7107778 B.

| ℓ | Frequency [µHz] | 68.3% credible [µHz] | Amplitude [ppm] | 68.3% credible [ppm] | Linewidth [µHz] | 68.3% credible [µHz] |
|---|-----------------|----------------------|-----------------|----------------------|-----------------|----------------------|
| 0 | 501.006         | −0.019/ + 0.020      | 14.08           | −0.55/ + 1.06        | 0.24            | −0.07/ + 0.05        |
| 0 | 535.615         | −0.016/ + 0.017      | 19.47           | −0.66/ + 0.94        | 0.16            | −0.03/ + 0.04        |
| 0 | 570.025         | −0.146/ + 0.423      | 27.22           | −6.39/ + 8.01        | 0.24            | −0.11/ + 0.10        |
| 0 | 604.609         | −0.026/ + 0.022      | 15.92           | −2.44/ + 1.51        | 0.23            | −0.08/ + 0.11        |
| 0 | 639.344         | −0.076/ + 0.068      | 13.11           | −0.74/ + 0.99        | 0.38            | −0.11/ + 0.13        |
| 1 | 512.368         | −0.008/ + 0.012      | 11.26           | −0.93/ + 1.19        | 0.09            | −0.06/ + 0.04        |
| 1 | 520.851         | −0.020/ + 0.024      | 19.66           | −0.31/ + 0.64        | 0.19            | −0.05/ + 0.04        |
| 1 | 547.122         | −0.735/ + 2.480      | 11.15           | −2.42/ + 4.08        | 0.40            | −0.16/ + 0.17        |
| 1 | 559.789         | −0.177/ + 0.168      | 15.37           | −5.30/ + 5.61        | 0.39            | −0.19/ + 0.17        |
| 1 | 584.175         | −0.393/ + 0.370      | 15.78           | −2.33/ + 2.80        | 0.38            | −0.13/ + 0.14        |
| 1 | 596.595         | −0.001/ + 0.001      | 18.12           | −1.00/ + 1.37        | 0.02            | −0.00/ + 0.01        |
| 1 | 620.979         | −0.049/ + 0.040      | 16.97           | −1.01/ + 1.17        | 0.33            | −0.07/ + 0.10        |
| 2 | 497.849         | −0.096/ + 0.091      | 10.21           | −0.74/ + 0.89        | 0.38            | −0.11/ + 0.12        |
| 2 | 532.691         | −0.342/ + 0.315      | 11.42           | −1.28/ + 2.08        | 0.41            | −0.14/ + 0.14        |
| 2 | 566.534         | −0.731/ + 0.830      | 10.72           | −2.78/ + 7.32        | 0.44            | −0.17/ + 0.30        |
| 2 | 601.946         | −0.106/ + 0.109      | 14.16           | −0.93/ + 1.22        | 0.36            | −0.10/ + 0.10        |
| 2 | 636.541         | −0.380/ + 0.493      | 9.04            | −1.04/ + 1.20        | 0.44            | −0.14/ + 0.13        |

Note: Modes which share the same peak on the power spectrum are labeled with *** marks. Modes from two stars standing too close and causing ambiguity are denoted with ** marks.
Figure 6. Échelle diagrams of KIC 7107778. Number series (1)∼(6) on the upper left corner of each panel corresponds to the series of models in Table 5, whose oscillation frequencies are displayed in the corresponding échelle. These models are without surface corrections. Open and filled symbols represent observational frequencies and theoretical ones. Red and blue represent modes of star A and star B respectively. Circles, triangles, and squares denote $l = 0$, $l = 1$ and $l = 2$ modes.
Figure 7. Parameter distributions for the lowest 10% $\chi^2$ of star A grid models with [Fe/H]=0.11 dex.
Figure 8. Parameter distributions for the lowest 10% $\chi^2$ of star B grid models with $[\text{Fe/H}]=0.11$ dex.