SYNTHETIC SPECTRA FROM PIC SIMULATIONS OF RELATIVISTIC COLLISIONLESS SHOCKS

LORENZO SIRONI AND ANATOLY SPITKOVSKY
Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544-1001

ABSTRACT

We extract synthetic photon spectra from first-principles particle-in-cell simulations of relativistic shocks propagating in unmagnetized pair plasmas. The two basic ingredients for the radiation, namely accelerated particles and magnetic fields, are produced self-consistently as part of the shock evolution. We use the method of Hededal & Nordlund (2005) and compute the photon spectrum via Fourier transform of the electric far-field from a large number of particles, sampled directly from the simulation. We find that the spectrum from relativistic collisionless shocks is entirely consistent with synchrotron radiation in the magnetic fields generated by Weibel instability. We can recover the so-called “jitter” regime only if we artificially reduce the strength of the electromagnetic fields, such that the wiggler parameter \( K \equiv qB\lambda/mc^2 \) becomes much smaller than unity (\( B \) and \( \lambda \) are the strength and scale of the magnetic turbulence, respectively). These findings may place constraints on the origin of non-thermal emission in astrophysics, especially for the interpretation of the hard (harder than synchrotron) low-frequency spectrum of Gamma-Ray Bursts.

Subject headings: acceleration of particles — gamma rays: bursts — radiation mechanisms: non-thermal — shock waves

1. INTRODUCTION

Non-thermal photon spectra from Pulsar Wind Nebulae, jets from Active Galactic Nuclei, Gamma-Ray Bursts and Supernova Remnants are usually explained as synchrotron radiation from a power-law population of particles, presumably accelerated in collisionless shocks. The microphysical details of shock acceleration are still poorly known, however, and are the subject of active research.

Particle-in-cell (PIC) simulations of colliding plasma shells have shown that Weibel instability (Weibel 1959, Medvedev & Loeb 1999, Gruzinov & Waxman 1999) converts the free energy of counter-streaming flows into small scale (skin-depth) magnetic fields (Nishikawa et al. 2003, 2005, Silva et al. 2003, Frederiksen et al. 2004, Hededal et al. 2004). The fields grow to sub-equilibrium levels and deflect and randomize the bulk flow, thus creating a shock (Spitkovsky 2005, 2008a, Keshet et al. 2009). A few percent of the incoming particles repeatedly scatter off the magnetic turbulence created by Weibel instability, and eventually populate a power-law high-energy tail in the particle spectrum behind the shock (Spitkovsky 2008b, Martins et al. 2009b, Sironi & Spitkovsky 2009).

Since most of the magnetic power generated by Weibel instability is concentrated on scales as small as a few plasma skin depths, it has been speculated that the emission mechanism in unmagnetized collisionless shocks may be the so-called “jitter” radiation. Whereas the standard synchrotron emission applies to large-scale fields, the jitter regime is realized if the scale \( \lambda \) of the turbulence is such that the wiggler parameter \( K \equiv qB\lambda/mc^2 \ll 1 \) (Medvedev 2000, 2006, Fleishman 2006a, b). Jitter radiation has been proposed as a solution for the so-called “line of death” puzzle in Gamma-Ray Burst (GRB) spectra, which below the peak frequency are sometimes harder than expected from synchrotron radiation (e.g., Preece 2009). Since PIC simulations self-consistently produce both the strength and the spatial structure of electromagnetic fields, as well as the particle distribution, it is possible to calculate the photon spectrum from first principles, thus determining whether the emission is more synchrotron-like or jitter-like.

In this work, we present synthetic spectra extracted from PIC simulations of relativistic unmagnetized collisionless shocks. In §2 we show the simulated shock structure and particle energy spectrum. The numerical technique that we employ to compute the photon spectrum is described in §3.1, and in §3.2 we show our results. In §4 we discuss the implications of our findings for the interpretation of the non-thermal radiation from astrophysical sources.

2. SHOCK STRUCTURE AND PARTICLE ENERGY SPECTRUM

We use the three-dimensional (3D) electromagnetic PIC code TRISTAN-MP (Buneman 1993, Spitkovsky 2005) to simulate a relativistic shock propagating into an unmagnetized pair plasma. The shock is triggered by reflecting an incoming cold “upstream” flow off a conducting wall at \( x = 0 \) (e.g., Sironi & Spitkovsky 2009). The simulation is performed in the “wall” or “downstream” frame. The incoming flow propagates along \( -\hat{z} \) with Lorentz factor \( \gamma_0 = 15 \), and the shock moves along \( +\hat{z} \).

To follow the shock evolution for longer times with fixed computational resources, we use a 2D simulation domain in the \( xy \) plane. In the case of an unmagnetized 2D shock, only the in-plane components of the velocity, current and electric field, and only the out-of-plane component of the magnetic field are present. Each computational cell is initialized with 16 particles per species. The relativistic plasma skin depth for the upstream flow \( (e/\omega_p) \) is resolved with 10 cells and the simulation timestep is \( \Delta t = 0.045 \omega_p^{-1} \). The simulation
~ 10% of the upstream kinetic energy density (Fig. [1]). The magnetic field decays farther downstream, where the field is confined within islands of typical scale ~ 20 c/ωp (Fig. [1]).

The particle energy spectrum behind the shock (xsh − 400 c/ωp < x < xsh; thin solid line in Fig. [1h]) consists of a relativistic Maxwellian and a high-energy tail, which can be fitted as a power-law of index α = 2.5 (dashed line in Fig. [1h]) with an exponential cutoff (Spitkovsky 2008b). In the upstream spectrum (xsh < x < xsh + 400 c/ωp; dotted line in Fig. [1h]), the unshocked beam populates the low-energy peak at p ≃ γβ ~ 15, whereas the shock-accelerated returning particles with γβ > 0 (see the hot, diffuse population in Fig. [1]) appear as a high-energy bump. Since at the highest energies the particles ahead of the shock account for nearly half of the total census (compare dotted and thin solid lines in Fig. [1h]), the high-energy tail in the total spectrum (xsh − 400 c/ωp < x < xsh + 400 c/ωp; thick solid line in Fig. [1h]) is significantly flatter than in the downstream spectrum.

3. SYNTHETIC PHOTON SPECTRUM

3.1. Numerical Technique

We summarize the method introduced by Hededal (2005) and Hededal & Nordlund (2005) (see also Nishikawa 2009; Martins et al. 2009a) to extract synthetic spectra from simulations of collisionless shocks. The electric far-field from a particle with charge q, velocity v = βc and acceleration v̇ = βc is (Jackson 1999)

\[ E(x, t) = \frac{q}{c} \left[ \frac{n \times \{(n - \beta) \times \beta\}}{(1 - \beta \cdot n)^3} R \right]_{ret} \]

where the unit vector n points toward the observer, at distance R from the emitting particle. Here, the quantity in square brackets is to be evaluated at the retarded time \( t' = t - R(t')/c \). The photon spectrum is then computed via the Fourier transform of Poynting flux associated with the field in eq. (1). The energy dW received per unit solid angle dΩ (around the direction n) and per unit frequency dω can be computed as (Jackson 1999)

\[ d^2W = \frac{q^2}{4\pi^2c} \left( \int_{-\infty}^{+\infty} n \times \{(n - \beta) \times \beta\} e^{i(\omega'(t') - n \cdot r(t')/c)} d\omega' \right)^2 \]

where \( r(t') \) is the particle trajectory. Here, we neglected the Tsyptovich-Razin effect due to the dispersive properties of the plasma (Rybicki & Lightman 1979).

In PIC simulations we know the positions, velocities and accelerations of simulation particles with time resolution \( \Delta t = 0.045 \omega^{-1} \). In order to accurately compute the integral in eq. (2), we interpolate the orbit of the selected particles so that to achieve an effective timestep of 0.1 \( \Delta t \). For a given choice of n, we can then integrate eq. (2) to obtain the photon spectrum from each particle. Assuming that the far-fields by different particles are phase-uncorrelated, the total spectrum will be the sum of the spectra of individual particles.

We have implemented eq. (2) and tested it for the cases of synchrotron, bremsstrahlung and wiggler/undulator radiation, finding good agreement with analytic solutions. Following Hededal (2005) and Hededal & Nord-
which we call "\( L \)". The particle population, but evolved in electromagnetic fields artificially reduced by a factor of 10 (100, respectively), is shown as dot-dashed (dashed, respectively) lines. Red lines are for head-on emission \((n = \hat{x})\), blue lines for edge-on emission \((n = \hat{y})\).

Since electric fields are negligible in the downstream medium (see Fig. 1e), the resulting photon spectrum will probe the strength and structure of the magnetic fields, and it will clarify which regime – synchrotron or jitter – is appropriate to describe the particle emission. The solid lines in Fig. 2 show that the spectrum can be well approximated by two power-law segments. Regardless of the observer’s direction \( n \) (red line for \( n_x = 1 \), blue line for \( n_y = 1 \)), the slope at the low frequencies is remarkably close to \( 2/3 \) (dotted line in Fig. 2), as expected for synchrotron emission from a 2D particle distribution (e.g., Jackson 1999). In \( N \) dimensions, the high-frequency slope should be \(-\alpha / (4 - N)/2\), which reduces to \(-\alpha / 2\) for \( N = 2 \) and \( \alpha = 2.5 \), in agreement with our spectra (see dotted line). The similarity between the cases \( n_x = 1 \) and \( n_y = 1 \) suggests, given the isotropy of the injected particle distribution, that the downstream magnetic fluctuations are spatially isotropic, as seen in Fig. 1e.

A transition to the jitter regime should appear when the wiggler parameter \( K \) becomes significantly smaller than unity (Medvedev 2006; Fleishman 2006a, b). We have tried to artificially lower the value of \( K \) by decreasing the strength of the electromagnetic fields, by a factor of 10 (dashed lines in Fig. 2) and 100 (dashed lines in Fig. 2). The high-frequency power-law decreases in intensity and shifts to lower frequencies, proportionally to the average magnetic field. The low-frequency spectrum becomes softer for decreasing \( B \), approaching the flat slope expected in the jitter regime when the shock is viewed edge-on. When we impose a magnetic field spectrum of the form \( B(k) \propto \delta(k - k_0) \) (here, \( k_0 \) is a fixed direction in \( k \)-space) with wiggler parameter \( K \ll 1 \), we are able to recover the hard low-frequency slope (\( \propto \omega \)) discussed by Medvedev (2006) for head-on emission from shocks. However, we do not observe it for the downstream turbulence self-consistently generated in the simulation, suggesting that the magnetic field fluctuations are not sufficiently ordered.

In Fig. 3 we show the photon spectrum resulting from a sample of particles extracted directly from the PIC simulation, followed near \( \omega_p t = 2250 \) in the time-varying electromagnetic fields of the simulation. The selected particles start in the region \( x_{sh} - 400c/\omega_p < x < x_{sh} + 400 \). The photon spectrum (thick solid lines in Fig. 3) red for \( n_x = 1 \), blue for \( n_y = 1 \) confirms that the emission occurs in the synchrotron regime, as the 2/3 slope at the low frequencies suggests (black dashed line).

Most of the low-frequency emission is powered by the thermal particles behind the shock, with 4-velocity \( p \leq 50 \) (dot-dashed lines in Fig. 3a). At high frequencies \( \propto \omega^{1/3} \).  

For a 3D distribution the low-frequency spectrum is \( \propto \omega^{1/3} \).  

The slight difference at high frequencies between \( n_x = 1 \) and \( n_y = 1 \) is due to the residual electric fields at \( x \lesssim x_{sh} \) (see Fig. 1e), and it disappears if electric fields are neglected while computing the spectrum.
The evolution of the photon spectrum, from downstream (thin solid lines) and upstream (dotted lines) particles. c) Time-aticist beaming, an observer located along the solid lines, respectively red and blue). In fact, due to rel-

Fig. 3.— Photon spectrum (thick solid lines, in all panels) for particles extracted from the PIC simulation, evolved in time-varying electromagnetic fields around $\omega_{p}t = 2250$. Red lines are for head-on emission ($n = \hat{x}$), blue lines for edge-on emission ($n = \hat{y}$). a) Dot-dashed lines only include the contribution of thermal particles, with 4-velocity $p \leq 50$. b) Relative contribution of downstream (thin solid lines) and upstream (dotted lines) particles. c) Time evolution of the photon spectrum, from $\omega_{p}t = 2250$ (thick solid lines) to $\omega_{p}t = 4500$ (dot-dashed lines). Here, the case $n_{x} = 1$ has been shifted downward by a factor of 2 for clarity.

In retrospect, this is not surprising. The characteristic length scale of the magnetic turbulence is $\lambda \gtrsim 10c/\omega_{p}$, and in the shock region, where most of the emission is produced, the magnetic energy reaches a fraction $\varepsilon_{B} \approx 0.1$ of the upstream bulk kinetic energy. This implies $r_{L}/(c/\omega_{p}) = \varepsilon_{B}^{-1/2} \approx 3$, where $r_{L}$ is the relativistic Larmor radius of a particle moving with the upstream flow. It follows that $K = \lambda/(r_{L}/\gamma_{0}) \approx 3\gamma_{0}$, so that the condition $K \ll 1$ for jitter radiation is unlikely to be satisfied, even for moderately relativistic shocks. In deed, the photon spectrum we obtain from a $\gamma_{0} = 3$ shock is still consistent with synchrotron radiation. For electron-ion shocks, the wigglir parameter will be $m_{e}/m_{i}$ times larger, and we are even deeper in the synchrotron regime. Further downstream from the shock, although the magnetic field strength decays, the value of $K$ does not significantly change, since short-wavelength modes are progressively damped (Chang et al. 2008), and the characteristic scale of the turbulence increases.

Although the results presented here apply to a 2D particle distribution, the main conclusions should hold for 3D configurations as well. There, we expect the low-frequency slope to be 1/3, as in the standard synchrotron radiation. If the GRB emission results from high-energy particles accelerated in relativistic unmagnetized shocks, it seems that resorting to the jitter radiation is not a viable solution for the “line of death” puzzle (Preece 1998). At high frequencies, our results suggest that the contribution of upstream particles to the total emission, which is usually omitted in standard models, is not negligible. It causes the radiation spectrum to be flatter than the
corresponding downstream spectrum, thus partly masking the contribution of downstream thermal particles (Giannios & Spitkovsky[2009]). This could potentially explain the absence of clear signatures of downstream thermal emission in GRB shocks (Band[1993]). Simulations extending to longer times (and higher particle energies) will help to clarify these issues.

We remark that our calculations do not include radiative particle cooling, synchrotron self-absorption and inverse Compton radiation. Still, we have shown that the computation of synthetic spectra from self-consistent PIC simulations provides a powerful tool for studying the origin of astrophysical non-thermal emission.

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