Deterministic entanglement swapping in a superconducting circuit

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Entanglement swapping allows two particles that have never been coupled directly or indirectly to
be nonlocally correlated. Besides fundamental interest, this procedure has applications in complex
entanglement manipulation and quantum communication. Entanglement swapping for qubits has
been demonstrated in optical experiments, but where the process was conditional on detection of
preset photon coincidence events, which succeeded with only a small probability. Here we report an
unconditional entanglement swapping experiment with superconducting qubits. Using controllable
qubit-qubit couplings mediated by a resonator, we prepare two highly entangled qubit pairs and then
perform the Bell state measurement on two qubits coming from different entangled pairs, projecting
the remaining two qubits to one of four Bell states. The measured concurrences for these Bell states
are above 0.75, demonstrating the quantum nature of entanglement swapping. With this setup, we
further demonstrate delayed-choice entanglement swapping, confirming whether two qubits behaved
as in an entangled state or as in a separate state is determined by a later choice of the type of
measurement on their partners. This is the first demonstration of entanglement-separability duality
in a deterministic way, closing the detection loophole the previous experiments suffer from.

I. INTRODUCTION

Quantum entanglement, lying at the heart of the Einstein-Podolsky-Rosen (EPR) paradox [1], is one of
the most striking features of quantum mechanics. When two particles are put in an entangled state, they can
exhibit nonlocal correlation that cannot be interpreted in terms of any classical model as evidenced by violation
of Bell’s inequalities [2, 3]. In addition to fundamental tests of quantum mechanics, entanglement is an essential
resource for many quantum information tasks, such as quantum teleportation [4] and measurement-based quantum
computation [5]. The nonlocal characteristic of quantum-mechanical wavefunctions allows two particles that have never interacted to be put into an entangled state by means of entanglement swapping [6]. The process is illustrated in Fig. 1, where the two qubits (Q1 and Q4) to be entangled are first entangled with their respective partners (Q2 and Q3): Q1 and Q2 form the first entangled Bell pairs, while Q3 and Q4 form the second pair. Then a joint Bell state measurement applied to the partners Q2 and Q3 will project the remaining two qubits, Q1 and Q4, to one of four possible Bell states; which entangled state is produced depends on the outcome of the Bell state measurement. Entanglement swapping can be understood in terms of quantum teleportation [4]: The state of Q2 is teleported to Q4, which inherits the entanglement of Q2 with Q1, or the state of Q3 is teleported to Q1. On the contrary, if the partners are measured independently, the two remaining qubits will collapses to a separable state showing no entanglement. In other words, whether Q1 and Q4 are projected to an entangled state or to a separable state depends upon the choice of measurement on Q2 and Q3. Aside from fundamental interest, entanglement swapping has practical applications in quantum communication [7] and in multipartite entanglement manipulation necessary for construction of complex quantum networks [8].

Entanglement swapping has been experimentally demonstrated with photonic qubits [9–17]. However, in these optical experiments, entanglement was swapped conditional on the occurrence of preset photon coincidence events. These events were detected only in a small fraction of experimental runs due to the photon loss on optical components, lack of logic operations to completely distinguish all four Bell states, and restriction of photon detectors’ efficiency [17]. Experiments have realized heralded entanglement between two spatially separated atomic qubits, each entangled with its emitted photons before a partial Bell state analysis on these photons [18, 19]; the entanglement was swapped also with a small probability. With photonic continuous variables, unconditional entanglement swapping has been reported [20, 21], but where only a small portion of entan-

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glement was preserved after swapping due to the limitation of the degree of entanglement carried by the original entangled beams, which is the main source of the infidelity for teleportation of a continuous-variable state [22]. To our knowledge, high-fidelity entanglement swapping has not been realized in a deterministic manner, although unconditional teleportation has been demonstrated in experiments with nuclear magnetic resonance [23], trapped ions [24, 25], and superconducting qubits [26, 27], where the teleported states were preset and known to the experimenters. As pointed out in Ref. [10], entanglement swapping is the only known procedure that demonstrates the quantum nature of teleportation—the qubit whose state is to be teleported is entangled with another qubit, rendering it impossible to know this state; the teleportation of quantum entanglement manifests transfer of non-classicality.

We here implement a deterministic entanglement swapping experiment with superconducting qubits (labeled from $Q_1$ to $Q_4$). In our experiment, each of the qubit pairs $Q_1$-$Q_2$ and $Q_3$-$Q_4$ is first prepared in a Bell state through virtual photon exchange mediated by a resonator [28–32]. Then a complete Bell state measurement on $Q_2$ and $Q_3$, achieved via a two-qubit quantum logic gate, deterministically projects $Q_1$ and $Q_4$ to one Bell state, though they have never interacted with each other. We characterize the joint state of $Q_1$ and $Q_4$ after entanglement swapping by full quantum state tomography, achieved by individually manipulating and detecting them, finding the fidelities above 0.86 for all the four output Bell states. The measured $Q_1$-$Q_4$ concurrences after entanglement swapping are above 0.75, confirming genuinely quantum nature of the procedure. Unlike previous experiments with photonic qubits, the Bell states are produced deterministically and the measurement is single-shot, so that the entanglement is swapped unconditionally. We further realize a delayed-choice entanglement swapping experiment [33], where we choose to perform a Bell state measurement or a separable-state measurement on $Q_2$ and $Q_4$ after $Q_1$ and $Q_3$ have been detected. The results demonstrate this later choice decides the previous behavior of $Q_1$ and $Q_4$—whether they were entangled or separable. As far as we know, our experiment is the first one for deterministically implementing delayed-choice entanglement swapping, revealing two-party entanglement-separability duality without the detection loophole, to which the previous demonstrations with optical systems [15–17] are subject.

II. RESULTS

A. GENERATION OF BELL STATES

The device used to perform the experiment swapping is identical to that used in Ref. [34], where a resonator with a fixed frequency $\omega_r = 2\pi \times 5.588$ GHz is controllably coupled to five superconducting Xmon qubits, whose frequencies can be individually adjusted on nanosecond timescales using flux bias lines. The device is sketched in Fig. 2A, and the optical image shown in Fig. 2B. Throughout the experiment, $Q_5$ (unused) is tuned far off-resonance with the resonator and the other qubits, and will not be included in the description of the system. The parameters of the system are detailed in Supplementary Material. All the qubits and the resonator are initially in their ground states. The experiment starts with applying $\pi$ pulses to $Q_1$ and $Q_3$, transforming each of them from the ground state $|0\rangle$ to the excited state $|1\rangle$ at its idle frequency, with the experimental sequence shown in Fig. 2C. Then the qubit pairs $Q_1$-$Q_2$ and $Q_3$-$Q_4$ are redetuned from the resonator by $\Delta_1 = \Delta_2 = 2\pi \times 308$ MHz and $\Delta_3 = \Delta_4 = 2\pi \times 238$ MHz, respectively. With this setting, the resonator will not exchange photons with the qubits and remain in the ground state, but it can simultaneously mediate two entangling gates, each operating on one qubit pair, with the coupling between these two qubit pairs being negligible owing to their large detuning [28, 32]. In the interaction picture, the resonator-induced $Q_1$-$Q_2$ and $Q_3$-$Q_4$ couplings are described by the effective Hamiltonian

$$H_e = -\hbar \left( \lambda_{1,2} S_1^+ S_2^- + \lambda_{3,4} S_3^+ S_4^- + H.c. \right),$$

where $S_j^+ = |1_j\rangle \langle 0_j|$, $S_j^- = |0_j\rangle \langle 1_j|$, and $\lambda_{j,k} = g_j g_k / \Delta_j$, with $g_j$ being the on-resonance coupling strength between the $j$th qubit and the resonator.

Mediated by the resonator, each qubit independently swaps energy with its partner. The qubit pair, $Q_j$-$Q_k$ ($j = 1$, $k = 2$ or $j = 3$, $k = 4$), evolves to the Bell state $|\Psi_{j,k}^+\rangle = (|1_j\rangle |0_k\rangle + i |0_j\rangle |1_k\rangle) / \sqrt{2}$ after the corresponding $\sqrt{iSWAP}$ gate with the duration $\tau_{j,k} = \pi / 4 \lambda_{j,k}$. In our experiment, $\lambda_{1,2} = 2\pi \times 0.82$ MHz and $\lambda_{3,4} = 2\pi \times 1.1$ MHz, corresponding to $\tau_{1,2} = 153$ ns and $\tau_{3,4} = 114$ ns. As soon as $|\Psi_{j,k}^+\rangle$ is generated, $Q_j$ and $Q_k$ are detuned from each other to stop their coupling. The measured density matrices for these two produced entangled pairs are displayed in Supplementary Material. Their fidelities to the ideal Bell states are respectively $F_{1,2} = 0.982 \pm 0.006$ and $F_{3,4} = 0.978 \pm 0.007$, where $F_{j,k}$...
is defined as \( F_{\tau,k} = \langle \Psi_{j,k}^+ | \rho_{j,k} | \Psi_{j,k}^+ \rangle \), with \( \rho_{j,k} \) being the density operator of the produced joint state for \( Q_j \) and \( Q_k \).

The infidelities are mainly due to the imperfect large detuning conditions and the qubits’ decoherence effects, including energy relaxation and dephasing. The concurrences for these two produced Bell states are 0.966 ± 0.011 and 0.957 ± 0.013, respectively.

**B. ENTANGLEMENT SWAPPING**

The product of the two Bell states \( |\Psi_{1,2}^+\rangle \) and \( |\Psi_{3,4}^+\rangle \) can be expanded as

\[
|\psi\rangle = \frac{1}{2} \left( |\Psi_{1,4}^+\rangle + i |\Psi_{1,4}^-\rangle + |\Psi_{2,3}^+\rangle + i |\Psi_{2,3}^-\rangle \right) \tag{2}
\]

where \( |\Psi_{j,k}^\pm\rangle = (|0_j\rangle |0_k\rangle \pm |1_j\rangle |1_k\rangle) / \sqrt{2} \) and \( |\Phi_{j,k}^\pm\rangle = \frac{1}{\sqrt{2}} (|0_j\rangle |0_k\rangle \pm |1_j\rangle |1_k\rangle) \). The key element of realizing entanglement swapping is the measurement of \( Q_2 \) and \( Q_3 \) in the Bell basis \( \{ |\Psi_{2,3}^+\rangle, |\Psi_{2,3}^-\rangle, |\Phi_{2,3}^+\rangle, |\Phi_{2,3}^-\rangle \} \), which will project \( Q_1 \) and \( Q_4 \) to one Bell state. A complete Bell state measurement can be implemented by mapping the Bell basis onto the computational basis \( \{ |0_2\rangle |0_3\rangle, |0_2\rangle |1_3\rangle, |1_2\rangle |0_3\rangle, |1_2\rangle |1_3\rangle \} \) through a dressed-state phase gate [35, 36]. To do so, we tune \( Q_1 \) and \( Q_4 \) back to their idle frequencies, so that neither of them can interact with other qubits, and then red-detune \( Q_2 \) and \( Q_3 \) from the resonator by the same amount \( \Delta_2 = \Delta_3 = 2\pi \times 308 \text{MHz} \), switching on their interaction via the resonator-induced virtual photon exchange, with the coupling strength \( \lambda_{2,3} = g_{23} g_3 / \Delta_2 \). At the same time, we apply a resonant continuous drive to each of these two qubits, whose phase is inverted in the middle of the two-qubit interaction with a duration \( \tau_{2,3} = \pi / 2\lambda_{2,3} \). When the difference of the Rabi frequencies of these two drives is much larger than \( \lambda_{2,3} \), a dressed-state phase gate between \( Q_2 \) and \( Q_3 \) is achieved. As a result, the four Bell states of \( Q_2 \) and \( Q_3 \) evolve as (see Supplementary Mate-
FIG. 4: (Color online) Measured $Q_1$-$Q_4$ density matrices conditional on outcomes of delayed-choice $Q_2$-$Q_3$ measurement. (A), (B), (C), (D) Results obtained from the four subsets of data correlated with the outcomes $\{0_2, 0_3, 1_2, 0_3, 1_2, 1_3\}\}$ of $Q_2$-$Q_3$ measurement performed after the dressed-state phase gate. Compared with Fig. 2B, the temporal orders of $Q_2$-$Q_3$ Bell measurement and $Q_1$-$Q_4$ joint state tomography are inverted, with the experimental pulse sequence shown in Supplementary Material. (E), (F), (G), (H) Results obtained from the four subsets of data correlated with the outcomes of the later $Q_2$-$Q_3$ measurement without the dressed-state phase gate.

The combination of this transformation and the subsequent detection of $Q_2$ and $Q_3$ in the computational basis $\{0_2, 0_3, 1_2, 0_3, 1_2, 1_3\}\}$ effectively realizes the complete Bell state analysis, enabling us to distinguish all the four Bell states. Consequently, $Q_1$ and $Q_4$ are randomly projected onto one of the four Bell states $\{\Phi_{1,4}^+, \Psi_{1,4}^+, \Psi_{1,4}^+, \Phi_{1,4}^-\}$ depending on the $Q_2$-$Q_3$ measurement outcome.

After the Bell state analysis, we perform joint 2-qubit state tomography to reconstruct the density matrix for $Q_1$ and $Q_4$. The measured density matrices of $Q_1$ and $Q_4$ conditional on the measurement outcomes $\{0_2, 0_3, 1_2, 0_3, 1_2, 1_3\}$ of $Q_2$ and $Q_3$ are displayed in Fig. 3A, B, C, and D, respectively, where the off-diagonal elements $\rho_{01,01}, \rho_{01,10}$ or $\rho_{11,00}, \rho_{00,11}$ manifest the entanglement. These four results are obtained with probabilities of $0.249 \pm 0.004, 0.250 \pm 0.006, 0.255 \pm 0.005$, and $0.246 \pm 0.004$, each approximate to $1/4$. Ideally, for these four outcomes $Q_1$ and $Q_4$ are projected onto $\{\Phi_{1,4}, \Psi_{1,4}^+, \Psi_{1,4}^+, \Phi_{1,4}^-\}$, respectively. The fidelities for the four obtained Bell states to the ideal ones are $F_{\Phi^+} = 0.893 \pm 0.010, F_{\Psi^+} = 0.879 \pm 0.010, F_{\Psi^-} = 0.872 \pm 0.011$, and $F_{\Phi^-} = 0.884 \pm 0.010$, with the concurrences $C_{\Phi^+} = 0.794 \pm 0.020, C_{\Psi^+} = 0.779 \pm 0.020, C_{\Psi^-} = 0.758 \pm 0.024$, and $C_{\Phi^-} = 0.785 \pm 0.021$, respectively. These results are in well agreement with numerical simulations based on master equation (see Table S2 of Supplementary Material), and show that $Q_4$ ($Q_1$) inherits most of the entanglement of $Q_2$ with $Q_1$ ($Q_3$ with $Q_4$) after the swapping, which is in stark contrast with experiments with photonic continuous variables [20, 21], where only a small portion of entanglement is inherited (e.g., about 29% in Ref. [20]). The error sources include imperfect preparation of the $Q_1$-$Q_2$ and $Q_3$-$Q_4$ Bell states, infidelity of the $Q_2$-$Q_3$ dressed-state phase gate, and decoherence effects of $Q_1$ and $Q_4$ during this gate. Without discriminating the four $Q_2$-$Q_3$ measurement outcomes, the $Q_1$-$Q_4$ density matrix, given by the weighted average of the results associated with the four measurement outcomes of $Q_2$ and $Q_3$, is displayed in Fig. 3E. As expected, this corresponds to the classical mixture of the four Bell states, which is equivalent to a statistical mixture of the four computational states. These results unambiguously demonstrated that the entanglement between $Q_1$ and $Q_4$ is produced by Bell state measurement on their partners, other than by direct interaction.

C. DELAYED-CHOICE ENTANGLEMENT SWAPPING

Going one step further, we delay the $Q_2$-$Q_3$ Bell state measurement until the joint $Q_1$-$Q_4$ state has been de-
tected. The detailed pulse sequence is shown in Fig. S4B of Supplementary Material, where the Q2-Q3 readout pulse is applied about 219 ns after the end of Q1-Q4 readout pulse. Since the correlation between the outcomes of Q2-Q3 measurement and Q1-Q4 measurement is independent of their temporal order, this arrangement will result in entanglement swapping in a delayed manner [33]. The probabilities for Q2-Q3 measurement outcomes \{\{0\}_2\{0\}_3,\{0\}_2\{1\}_3,\{1\}_2\{0\}_3,\{1\}_2\{1\}_3\} after the dressed-state phase gate are \{0.248\pm 0.004, 0.256\pm 0.005, 0.248 \pm 0.006, 0.248 \pm 0.006\}. According to these outcomes, the data of Q1-Q4 joint state measurement are sorted into four subsets, from which four density matrices are reconstructed, and shown in Fig. 4A, B, C, and D. As in the non-delayed case, these four density matrices correspond to four Bell states, with the respective fidelities \(F_{914}=0.891 \pm 0.012\), \(F_{926}=0.891 \pm 0.012\), \(F\)\(_{241}=0.896 \pm 0.010\), and \(F\)\(_{248}=0.897 \pm 0.010\), and concurrences \(C_{248}=0.815 \pm 0.026\), \(C\)\(_{241}=0.816 \pm 0.024\), \(C\)\(_{926}=0.806 \pm 0.022\), and \(C\)\(_{914}=0.807 \pm 0.019\). The fidelities and concurrences are slightly higher than those in the non-delayed case due to the fact that Q1-Q4 joint state is detected earlier so that the measured data is less affected by decoherence effects.

We also perform another experiment, where we choose to measure Q2 and Q4 in the computational basis (without performing the dressed-state phase gate before detection of their states). Again, this measurement is performed after Q1-Q4 joint state detection, with the pulse sequence shown in Fig. S3C of Supplementary Material. The density matrices reconstructed from the four subsets of Q1-Q4 measurement data, each associated with one of Q2-Q3 measurement outcomes \{\{0\}_2\{0\}_3,\{0\}_2\{1\}_3,\{1\}_2\{0\}_3,\{1\}_2\{1\}_3\}, are presented in Fig. 4E, F, G, and H, respectively. The probabilities for these four outcomes are \{0.258 \pm 0.005, 0.250 \pm 0.005, 0.251 \pm 0.005, 0.241 \pm 0.008\}. As expected, these matrices correspond to product states \{\{1\}_1\{1\}_4,\{1\}_1\{0\}_4,\{0\}_1\{1\}_4,\{0\}_1\{0\}_4\} with the fidelities \{0.907 \pm 0.011, 0.914 \pm 0.009, 0.930 \pm 0.009, 0.949 \pm 0.008\}. The concurrence associated with each of these reconstructed matrices is approximate to 0 (see Table S4 of Supplementary Material). These results demonstrate that whether or not the already measured qubits Q1 and Q4 previously behaved as an entangled pair depends on the later choice of the type of measurement on Q2 and Q3. As a generalization of Wheeler’s delayed-choice experiment proposed for illustrating the wave-particle duality of a single particle [37], the delayed-choice entanglement swapping experiment reveals the entanglement-separability duality of two particles [38]. A realization of this gedanken experiment was previously reported with photonic qubits [17], but where only two out of four basis states could be distinguished in each of the two mutually exclusive measurements, so that the entanglement-separability duality was only partially demonstrated: Whether the Q1-Q4 states associated with the two distinguishable Q2-Q3 basis states manifested a quantum or a classical correlation could not be confirmed. Another problem is only a small fraction of events coinciding with the distinguishable basis states was detected owing to the photon loss on optical components (only 4.4% photons left) and nonunity photon detection efficiency.

The present experiment represents the first deterministic demonstration of the entanglement-separability duality, where the state of each qubit is read out in a single-shot manner and the results are obtained using the outcomes of every experimental run.

### III. DISCUSSION

We have demonstrated deterministic entanglement swapping with superconducting qubits controllably coupled to a resonator. The qubit-qubit couplings mediated by the resonator allows for both the controlled generation of the Bell states and complete Bell state analysis. After entanglement swapping, two qubits that have never interacted with each other are entangled. We have further deterministically realized delayed-choice entanglement swapping, demonstrating whether two qubits were in an entangled state or in a separable state can be a posteriori decided after they have been measured. We note that this does not mean that future actions can affect already recorded events. Instead, it indicates quantum entanglement of two quantum systems, like the wave-like or particle-like behavior for a single quantum system, is not a reality, but is a manifestation of the statistical correlations of the measured data. Whether an entangled state or a separable state can be assigned to two systems sometimes depends on how one looks at their measurement data. Our results reveal that the same set of data may show different kinds of correlations and have different interpretations when grouped in different manners.

The deterministic feature distinguishes our experiment from those with photonic qubits [9–17], where the data used for statistics were acquired from only a small fraction of experimental runs, and consequently, the detection loophole was left open: The success probability is not high enough to exclude possibility that the subensemble consisting of these successful experimental runs coincides with quantum-mechanical predictions, but the entire ensemble does not. This loophole is closed in our experiment, where the qubits’ state readout is single-shot and all four Bell states are distinguished in each experimental run.

The procedure demonstrated in this work can be used for complex entanglement manipulation [8, 39, 40]. Consider a superconducting circuit composed of two sets of qubits, which are coupled to two resonators, respectively. In addition, the two resonators are commonly coupled to a qubit. All the qubits coupled to the first resonator can be prepared in an entangled state by the qubit-qubit couplings mediated by this resonator, while the other qubits can be entangled via the interactions mediated by second resonator. Then a Controlled-NOT gate between
the common qubit and one qubit from the second set followed by a measurement on the common qubit will merge these two initially independent sets into a larger entangled state, entangling all the qubits that are not measured.

IV. MATERIALS AND METHODS

The experiment is performed with a quantum circuit involving five Xmon qubits coupled to a resonator with a fixed frequency $\omega_r$. The system Hamiltonian, in the interaction picture, is

$$H = \hbar \sum_{j=1}^{5} g_j \left( e^{-i\Delta_j t} S^+_j a + e^{i\Delta_j t} S^-_j a^\dagger \right),$$

where $S^+_j$ and $S^-_j$ are the flip operators for $Q_j$, $a$ and $a^\dagger$ denote the annihilation and creation operators of the field in the resonator, and $\Delta_j = \omega_r - \omega_j$ with $\omega_j$ being the frequency of $Q_j$. The frequency of each qubit is fast tunable through a flux bias line, which enables full control over each qubit’s interactions with the resonator as well as with the other qubits. When $|\Delta_j| \gg g_j$, $|\Delta_k| \gg g_k$, and $|\Delta_j - \Delta_k| \gg g_j g_k \left( \frac{1}{|\Delta_j|} + \frac{1}{|\Delta_k|} \right)$, $Q_j$ and $Q_k$ are effectively decoupled from the resonator and from each other due to large detunings. In our experiment, $Q_3$ is highly detuned from the resonator and all other qubits, so that it does not affect the dynamics of the other qubits throughout the experiment. When $Q_1$ and $Q_2$ are detuned from the resonator by the same amount $\Delta_1$, and $Q_3$ and $Q_4$ detuned from the resonator by $\Delta_3$, with $|\Delta_1| \gg g_1, g_2, |\Delta_3| \gg g_3, g_4$, and $|\Delta_1 - \Delta_3| \gg g_1 g_2 / |\Delta_1|, g_3 g_4 / |\Delta_3|$, the resonator mediates $Q_1$-$Q_2$ and $Q_3$-$Q_4$ couplings in parallel, allowing simultaneous generation of two independent Bell entangled pairs [32]. During the Bell state measurement performed on $Q_2$ and $Q_3$, $Q_1$ and $Q_4$ are each highly detuned from the resonator and all other qubits, so that they are decoupled from each other and do not intervene in the $Q_2$-$Q_3$ dressed-state phase gate.

Each qubit is dispersively coupled to its own readout resonator for individual state detection. All the readout resonators are connected to a common transmission line for multiplexed readout of all qubits. The high-fidelity and quantum non-demolition single-shot readout of the state for each qubit is enabled by an impedance-transformed Josephson parametric amplifier. The readout fidelities for the ground states of $Q_1$, $Q_2$, $Q_3$, and $Q_4$ are about 0.975, 0.975, 0.961, and 0.979, while those for their excited states are about 0.927, 0.925, 0.919, and 0.822, respectively.

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[1] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 777–780 (1935).
[2] J. S. Bell, On the Einstein Podolsky Rosen paradox. Physics 1, 195–200 (1965).
[3] J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt, Proposed experiment to test local hidden-variable theories. Phys. Rev. Lett. 23, 880–884 (1969).
[4] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W. K. Wootters, Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 70, 1895–1899 (1993).
[5] R. Raussendorf, H. J. Briegel, A one-way quantum computer. Phys. Rev. Lett. 86, 5188–5191 (2001).
[6] M. Żukowski, A. Zeilinger, M. A. Horne, A. K. Ekert, “Event-ready-detectors” Bell experiment via entanglement swapping. Phys. Rev. Lett. 71, 4287–4290 (1993).
[7] W. Dür, H.-J. Briegel, J. I. Cirac, P. Zoller, Quantum repeaters based on entanglement purification. Phys. Rev. A 59, 169–181 (1999).
[8] S. Bose, V. Vedral, P. L. Knight, Multiparticle generalization of entanglement swapping. Phys. Rev. A 57, 822–829 (1998).
[9] J.-W. Pan, D. Bouwmeester, H. Weinfurter, A. Zeilinger, Experimental entanglement swapping: entangling photons that never interacted, Phys. Rev. Lett. 80, 3891–3894 (1998).
[10] J.-W. Pan, M. Danielli, S. Gasparoni, G. Weihs, A. Zeilinger, Experimental demonstration of four-photon entanglement and high-fidelity teleportation. Phys. Rev. Lett. 86, 4435–4438 (2001).
[11] H. de Riedmatten, I. Marcikic, W. Tittel, H. Zbinden, N. Gisin, Long-distance entanglement swapping with photons from separated sources. Phys. Rev. A 71, 050302 (2005).
[12] M. Hald, A. Beveratos, N. Gisin, V. Scarani, C. Simon, H. Zbinden, Entangling independent photons by time-measurement, Nat. Phys. 3, 692–695 (2007).
[13] C.-Y. Lu, T. Yang, J.-W. Pan, Experimental multiparticle entanglement swapping for quantum networking.
C. Schmid, N. Kiesel, U. K. Weber, R. Ursin, A. Zeilinger, H. Weinfurter, Quantum teleportation and entanglement swapping with linear optics logic gates. New J. Phys. 11, 033008 (2009).

F. Sciarrino, E. Lombardi, G. Milani, F. De Martini, Delayed-choice entanglement swapping with vacuum–one-photon quantum states. Phys. Rev. A 66, 024309 (2002).

T. Jennewein, G. Weihs, J.-W. Pan, A. Zeilinger, Experimental nonlocality proof of quantum teleportation and entanglement swapping. Phys. Rev. Lett. 88, 017903 (2001).

X.-S. Ma, S. Zotter, J. Kofler, R. Ursin, T. Jennewein, D. N. Matsukevich, P. Maunz, D. L. Moehring, S. Olmstead, M. Riebe, H. Haffner, C. F. Roos, W. Hansel, J. Benhelm, M. A. Nielsen, E. Knill, R. Laflamme, Complete quantum teleportation using nuclear magnetic resonance. Nature 396, 52–55 (1998).

M. Riebe, H. Häffner, C. F. Roos, W. Hänsel, J. Benhelm, G. P. T. Lancaster, T. W. Kürber, C. Becher, F. Schmidt-Kaler, D. F. V. James, R. Blatt, Deterministic quantum teleportation with atoms. Nature 429, 734–737 (2004).

M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri, D. J. Wineland, Deterministic quantum teleportation of atomic qubits. Nature 429, 737–739 (2004).

M. Baur, A. Fedorov, L. Steffen, S. Filipp, M. P. da Silva, and A. Wallraff, Benchmarking a quantum teleportation protocol in superconducting circuits using tomography and an entanglement witness. Phys. Rev. Lett. 108, 040502 (2012).

L. Steffen, A. Fedorov, M. Oppliger, Y. Salathe, P. Kurpiers, M. Baur, G. Puebla-Hellmann, C. Eichler, A. Wallraff, Deterministic quantum teleportation with feed-forward in a solid state system. Nature 500, 319–322 (2013).

S.-B. Zheng, G.-C. Guo, Efficient scheme for two-atom entanglement and quantum information processing in cavity QED. Phys. Rev. Lett. 85, 2392–2395 (2000).

S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, S. Haroche, Coherent control of an atomic collision in a cavity. Phys. Rev. Lett. 87, 037902 (2001).

M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. Hansel, C. F. Roos, H. Häffner, M. A. Nielsen, E. Knill, R. Laflamme, Complete quantum teleportation using nuclear magnetic resonance. Nature 396, 52–55 (1998).

C. Song, K. Xu, W. Liu, C. Yang, S. Zheng, H. Deng, Q. Xie, K. Huang, Q. Guo, L. Zhang, P. Zhang, D. Xu, D. Zheng, X. Zhu, H. Wang, Y.-A. Chen, C.-Y. Lu, S. Han, J.-W. Pan, H. Wang, Emulating anyonic fractional statistical behavior in a superconducting quantum circuit. Phys. Rev. Lett. 119, 180511 (2017).

A. Peres, Delayed choice for entanglement swapping. J. Mod. Opt. 47, 139–143 (2000).

C. Song, S.-B. Zheng, P. Zhang, K. Xu, L. Zhang, Q. Guo, W. Liu, D. Xu, H. Deng, K. Huang, D. Zheng, X. Zhu, H. Wang, Continuous-variable geometric phase and its manipulation for quantum computation in a superconducting circuit. Nat. Commun. 8, 1061 (2017).

Q. Guo, S.-B. Zheng, J. Wang, C. Song, P. Zhang, K. Li, W. Liu, H. Deng, K. Huang, D. Zheng, X. Zhu, H. Wang, Deephasing-insensitive quantum information storage and processing with superconducting qubits. Phys. Rev. Lett. 121, 130501 (2018).

C. Song, D. Xu, P. Zhang, J. Wang, Q. Guo, W. Liu, K. Xu, H. Deng, K. Huang, D. Zheng, S.-B. Zheng, H. Wang, X. Zhu, C.-Y. Lu, J.-W. Pan, Demonstration of topological robustness of anyonic braiding statistics with a superconducting quantum circuit. Phys. Rev. Lett. 121, 030502 (2018).

J. A. Wheeler, in Quantum Theory and Measurement, J. A. Wheeler, W. H. Zurek, Eds. (Princeton Univ. Press, New Jersey, 1984), pp. 182–213.

C. Brukner, M. Aspelmeyer, A. Zeilinger, Complementarity and information in “delayed-choice for entanglement swapping”. Found. Phys. 35, 1909–1919 (2005).

A. Zeilinger, M. A. Horne, H. Weinfurter, M. Zukowski, Three-particle entanglements from two entangled pairs. Phys. Rev. Lett. 78, 3031–3034 (1997).

X. Su, C. Tian, X. Deng, Q. Li, C. Xie, K. Peng, Quantum entanglement swapping between two multipartite entangled states. Phys. Rev. Lett. 117, 240503 (2016).
Supplementary material for “Deterministic entanglement swapping in a superconducting circuit”

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Contents

1. Device parameters
2. Characterization of Bell states produced by √iSWAP gates
3. Complete Bell state measurement
4. Experimental pulse sequences
5. Numerical results
References

1. DEVICE PARAMETERS

The device used to perform the present experiment is the same as that reported in Ref. [1], where five frequency-tunable superconducting Xmon qubits, labeled from Q₁ to Q₅, are capacitively coupled to a common resonator. In our experiment, Q₅ is not used. The full Hamiltonian of the system can be described as

\[ H = \hbar \omega_r a^+ a + \hbar \sum_{j=1}^{5} \omega_j S_j^+ S_j^- + \hbar \sum_{j=1}^{5} g_j(S_j^+ a + S_j^- a^+) \]

\[ + \hbar \sum_{j,k} \lambda_{j,k}^r(S_j^+ S_k^- + S_k^+ S_j^-). \]  (S1)

Qubit frequencies \( \omega_j \) are individually tunable from 5 to 6 GHz while the resonator frequency \( \omega_r \) is fixed at about 5.588 GHz. \( g_j \) is the coupling strength between qubit \( Q_j \) and the resonator with the magnitude listed in Table S1.

By equally detuning the frequency of any two qubits far away from that of the resonator, we can realize the effective qubit-qubit interaction with the coupling strength of \( g_j g_k/\Delta (\Delta = \omega_j - \omega_r = \omega_k - \omega_r, |\Delta| \gg g_j, g_k) \), which enables realizations of the √iSWAP gate and dressed-state phase gate used in the experiment. Note that except for the dominant resonator-mediated interaction, there exists very small direct couplings \( \lambda_{j,k}^r \) in the system which have been reported elsewhere in a similar devices [2, 3].

Characterization of qubit performance is presented in Table S1. For technical details about the superconducting qubits, see Supplemental Material of Ref. [2], which shares similar control methods to our experiment.

2. CHARACTERIZATION OF BELL STATES PRODUCED BY √iSWAP GATES

The original \( Q_1-Q_2 \) and \( Q_3-Q_4 \) Bell states are produced simultaneously [2]. The parallel operations for generating these entangled states are realized by tuning the frequencies of these two qubit pairs to 5.28 GHz and 5.35 GHz, respectively. This frequency setting enables the resonator to mediate two independent qubit-qubit swapping interactions, one between \( Q_1 \) and \( Q_2 \) and the other between \( Q_3 \) and \( Q_4 \). We note that the magnitude of the measured coupling \( \lambda_{j,k}^r \) between qubits \( Q_j \) and \( Q_k \) is slightly smaller than the calculated resonator-induced coupling. This is due to the fact that direct coupling between these qubits with an opposite sign partly cancels out the resonator-induced coupling [2, 3].

We characterize the Bell states generated via the corresponding √iSWAP gates through joint state tomography, with the \( Q_1-Q_2 \) and \( Q_3-Q_4 \) density matrices displayed in Fig. S1 A and B, respectively. The entanglements in the Bell states are characterized by the magnitudes of the off-diagonal matrix elements \( \rho_{11,10} \) and \( \rho_{10,01} \), which are about 0.49 for both produced Bell pairs. The fidelities of these produced Bell states are respectively \( F_{1,2} = 0.982\pm0.006 \) and \( F_{3,4} = 0.978\pm0.007 \). The populations of \( |0_1\rangle |1_2\rangle |0_3\rangle |1_4\rangle, |0_1\rangle |1_2\rangle |1_3\rangle |0_4\rangle, \)

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Table S1: Qubits characteristics. \(\omega_j/2 \pi\) is the idle frequency of \(Q_j\) where single-qubit rotation pulses and tomographic pulses are applied. \(T_{1,j}\) and \(T_{2,j}\) are the energy relaxation time and Ramsey dephasing time of \(Q_j\) (Gaussian decay) respectively measured at the idle point. \(T_{2\text{SE}}^j\) is the dephasing time (Gaussian decay) with spin echo, while \(T_{2\text{DD}}^j\) denotes the dephasing time (exponential decay) under continuous driving for dynamical decoupling [3]. The continuous driving is used in our experiment as it protects the qubits from dephasing much more effectively compared with the spin-echo technique. \(g_j/2 \pi\) is the coupling strength between \(Q_j\) and resonator. \(F_{0,j}\) (\(F_{1,j}\)) is the probability of detecting \(Q_j\) in \(|0\rangle\) (\(|1\rangle\)) when it is prepared in \(|0\rangle\) (\(|1\rangle\)) state.

\[
|0\rangle\langle 0|, |0\rangle\langle 1|, |1\rangle\langle 0|, |1\rangle\langle 1|
\]

are red-detuned from the resonator by the same amount \(\Delta\) induced \(
Q_j\) corresponding qubit-resonator couplings, the resonator-pumping coupling and continuous resonant driving. When a combination of the resonator-mediated detunings between each pair of qubits and the resonator is enabled by a dressed-state phase gate realized by \(\chi\)-gate. The real and imaginary parts of each matrix element are separately characterized with two different colorbars. The black wire frames represent the matrix elements of the ideal Bell states.

3. COMPLETE BELL STATE MEASUREMENT

In our experiment, the \(Q_2-Q_3\) Bell state measurement is enabled by a dressed-state phase gate realized by a combination of the resonator-mediated \(Q_2-Q_3\) swapping coupling and continuous resonant driving. When \(Q_2\) and \(Q_3\) are red-detuned from the resonator by the same amount \(\Delta_2 = \Delta_3\) that is much larger than the corresponding qubit-resonator couplings, the resonator-induced \(Q_2-Q_3\) coupling strength is \(\lambda_{2,3} = g_2g_3/\Delta_2\) [2]. Under continuous driving, the dynamics of \(Q_2\) and \(Q_3\) is described by the effective Hamiltonian [3]

\[
H_{\text{eff}} = \hbar \left( -\lambda_{2,3} S_2^- S_3^- + \sum_{j=2,3} \Omega_j \sigma^j_3 \right) + H.c.,
\]

where \(\Omega_j\) and \(\varphi_j\) denote the Rabi frequency and phase of the drive applied to \(Q_j\). We here assume that \(\varphi_2 = \varphi_3 = \varphi\). Under the condition \(|\Omega_2 - \Omega_3| \gg |\lambda_{2,3}|\), the effective Hamiltonian approximates

\[
H_{\text{eff}}' = -\frac{1}{2} \hbar \lambda \sigma_{z,\varphi,3} S_{z,\varphi,3} + \hbar \sum_{j=2,3} \Omega_j S_{z,\varphi,j},
\]

where \(S_{z,\varphi,j} = |\pm_{\varphi,j}\rangle \langle \pm_{\varphi,j}| - |\mp_{\varphi,j}\rangle \langle \mp_{\varphi,j}|\), with \(|\pm_{\varphi,j}\rangle\) and \(|\mp_{\varphi,j}\rangle\) being the dressed states, defined as \(|\pm_{\varphi,j}\rangle = 1/\sqrt{2}(|0\rangle \pm e^{i\varphi} |1\rangle)\) and \(|\mp_{\varphi,j}\rangle = 1/\sqrt{2}(|0\rangle - e^{i\varphi} |1\rangle)\). When the phase of

\[
FIG. S1: Measured density matrices of Bell states produced via \(\sqrt{\text{SWAP}}\) gates. (A) \(Q_1-Q_2\) density matrix. (B) \(Q_2-Q_4\) density matrix. Each qubit pair is prepared in the state \(|1\rangle\langle 1|\) before the corresponding \(\sqrt{\text{SWAP}}\) gate. The real and imaginary parts of each matrix element are separately characterized with two different colorbars. The black wire frames represent the matrix elements of the ideal Bell states.

FIG. S2: Characterization of dressed-state phase gate. (A) Measured \(\chi\)-matrix for the dressed-state phase gate in the Pauli basis. The real and imaginary parts of each element are plotted separately with different colorbars. The black wire frames represent the elements of ideal \(\chi\). (B) Randomized benchmarking by inserting the dressed-state phase gate between random single-qubit Pauli gates. Plotted are the corrected probability of \(|00\rangle\)-state after a series of random Pauli gates with (red) and without (blue) the dressed-state phase gate inserted.

\[
|1\rangle\langle 1|, |0\rangle\langle 1|, |0\rangle\langle 0|, |1\rangle\langle 1|
\]
each drive is inverted in the middle of the pulse with the duration $\tau = \pi/2\lambda_{2,3}$, this effective Hamiltonian leads to the controlled $\pi$-phase gate in the dressed-state basis $\{|+\varphi,2\rangle, |+\varphi,3\rangle, |+\varphi,2\rangle, |\varphi,2\rangle, |\varphi,3\rangle, |\varphi,2\rangle, |\varphi,3\rangle, |\varphi,2\rangle, |\varphi,3\rangle\}$ up to single-qubit operations $\exp(i\pi S_{x,y,z}/4)$. For simplicity, we take $\varphi = 0$. Then the evolution operator in the computational basis $\{|0_2\rangle|0_3\rangle, |0_2\rangle|1_3\rangle, |1_2\rangle|0_3\rangle, |1_2\rangle|1_3\rangle\}$ is

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & i & 0 \\ 0 & i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}. \quad (S4)$$

After the application of this evolution operator, each of the four Bell states is transformed to a computational state, with the correspondence given by Eq. (3) of the main text.

The dressed-state phase gate is characterized by both quantum process tomography and randomized benchmarking in experiment. The $\chi$-matrix obtained by quantum process tomography is shown in Fig. S2, with a fidelity of 0.966±0.005, in agreement with that characterized by randomized benchmarking, which yields a fidelity of 0.971±0.002.

### 4. EXPERIMENTAL PULSE SEQUENCES

The pulse sequence for entanglement swapping in the normal temporal order is shown in Fig. S3A, where the time interval between $Q_2$-$Q_3$ readout pulse and $Q_1$-$Q_4$ readout pulse is about 40 ns. In our delayed-choice entanglement swapping experiment, we choose to measure $Q_2$ and $Q_3$ in the Bell basis or in the computational basis after joint $Q_1$-$Q_4$ measurement has been performed. The pulse sequence with the choice of measurement in the Bell basis is shown in Fig. S3B, where $Q_2$-$Q_3$ readout pulse is applied about 219 ns after the end of $Q_1$-$Q_4$ readout pulse. With this pulse sequence, the density matrices for $Q_1$ and $Q_4$ reconstructed from the four data subsets associated with the $Q_2$-$Q_3$ measurement outcomes $|0_2\rangle|0_3\rangle, |0_2\rangle|1_3\rangle, |1_2\rangle|0_3\rangle, |1_2\rangle|1_3\rangle$ are displayed in Fig. 4A, B, C, and D of the main text, respectively. The pulse sequence with the choice of measurement in the computational basis is shown in Fig. S3C, where the dressed-state phase gate is omitted before the detection of the states of $Q_2$ and $Q_3$, and $Q_2$-$Q_3$ readout pulse is applied about 40 ns after the end of $Q_1$-$Q_4$ readout pulse. In this case, the $Q_1$-$Q_4$ density matrices reconstructed from the four subsets associated with the $Q_2$-$Q_3$ measurement outcomes $|0_2\rangle|0_3\rangle, |0_2\rangle|1_3\rangle, |1_2\rangle|0_3\rangle, |1_2\rangle|1_3\rangle$ are displayed in Fig. 4E, F, G, and H of the main text, respectively.

As can be seen from Fig. S3A, $Q_2$-$Q_3$ Bell state measurement is enabled by the combination of a dressed-state phase gate operation and multiplexed readout pulse which totally lasts about 1000 ns. During this period of time $Q_1$ and $Q_4$ are idled and thus endure decoherence effects. To mitigate these effects, following the dephasing suppression scheme proposed in Ref. [3], we apply weak continuous and resonant drives to these two qubits respectively (dotted blue box) during their idle time. As the phase is reversed in the middle of the driving pulse, the dephasing effect is reduced significantly.

![Fig. S3: Experimental pulse sequence.](image-url)
thus the quantum state of $Q_1$ and $Q_4$ can be well protected. The same method is employed in delayed-choice entanglement swapping experiment shown in Fig. S3B, where the protection pulse is applied to $Q_2$ and $Q_3$.

In our experiment, $Q_1$-$Q_4$ joint density matrices associated with different $Q_2$-$Q_3$ measurement outcomes are reconstructed by postselection. For each of the three experiments shown in Fig. S3, we measure $2^4$ probabilities labelled as $P_k^j = \{P_{0,0,0,0,0}^k, P_{0,0,0,1,1}^k, P_{0,0,1,0,1}^k, P_{0,0,1,1,0}^k, \ldots, P_{1,1,1,1,1}^k\}$, where $k$ is the index of the $3^2$ tomographic operations applied to $Q_1$ and $Q_4$ before the joint readout. After readout correction, the probabilities are then sorted into four subsets, each of which is associated with one of $Q_2$-$Q_3$ measurement outcomes $\{|0\rangle|0\rangle, |0\rangle|1\rangle, |1\rangle|0\rangle, |1\rangle|1\rangle\}$. For example, probabilities $P_{0,0,0,0,0}^j = \{P_{0,0,0,0,0}^j, P_{0,0,0,1,1}^j, P_{0,0,1,0,1}^j, P_{0,0,1,1,0}^j, P_{1,0,0,0,1}^j, P_{1,0,0,1,1}^j\}$ are extracted to reconstruct the density matrix of $Q_1$ and $Q_4$ on condition that $Q_2$ and $Q_3$ are in $|0\rangle|0\rangle$ state. The same method applies to other cases.

5. NUMERICAL RESULTS

The imperfections of the experimental results mainly come from the decoherence during the process. Numerical simulations with decoherence considered are performed to confirm the experimental outcomes. We use Lindblad master equation to model the state evolution under the Hamiltonian in section 1, with $T_{1,2}$ measured in experiment. In simulation, we set the pure dephasing time of $Q_j$ to be $T_\phi^{DD}$ listed in Table S1 due to the fact that a resonant continuous drive is applied to each qubit to dynamically decouple it from dephasing noises when there is a significant interval between the Bell state preparation and its state readout [3]. We note that although no continuous drive is applied during the initial Bell state preparation, the dephasing is also greatly depressed due to the qubit-qubit interaction, which has been discussed elsewhere [4].

Numerical simulation shows the product of $Q_1$-$Q_2$ and $Q_3$-$Q_4$ Bell states produced by swapping interactions has a fidelity of 0.979 with respect to the ideal state, which is in well agreement with the measured value of 0.971±0.009. For the dressed-state phase gate, the numerical results yield a fidelity of 0.975, slightly higher than the experimental value 0.966±0.005.

The measured probabilities of different $Q_2$-$Q_3$ basis states, and the associated $Q_1$-$Q_4$ output state fidelities and concurrences, together with the simulated results, for the normal entanglement swapping, delayed-choice entanglement swapping, and delayed-choice separable-state projection are shown in Table S2, S3, and S4, respectively. The experimental results are in good agreement with the numerical simulation overall. Fidelities in delayed-choice entanglement swapping are higher than those in the non-delayed case due to the fact that $Q_1$-$Q_4$ joint state is detected earlier so that the measured data is less affected by decoherence effects. The slight differences between the experimental and numerical results are partly due to the fact that $T_\phi^{DD}$, which is taken as the effective dephasing time of $Q_j$ in our simulation, does not perfectly characterize the effect of the dephasing noises under continuous driving. As demonstrated in Ref. [3], the qubit’s dephasing time depends not only on intensity of the applied drive but also on its effective interaction strengths with the others, which is not taken into account in the simulation. Another reason for these differences is imperfect control of the parameters of the continuous drives, whose fluctuations affect the performance of the dressed-phase gate and introduce an extra error to the protected qubits.

According to the numerical simulation, after the parallel Bell state preparations $Q_1$-$Q_4$ has a small probability of being populated in $|0\rangle|1\rangle$ when $Q_2$-$Q_3$ is projected to $|0\rangle|1\rangle$ due to the imperfect large detuning conditions, so that the elements associated with $|0\rangle|1\rangle\langle 1\rangle|0\rangle$ and $|1\rangle|0\rangle\langle 0\rangle|1\rangle$ of the corresponding projected $Q_1$-$Q_4$ density matrix have a magnitude of about 0.065, which accounts for the calculated $Q_1$-$Q_4$ concurrence (0.069) corresponding to the $Q_2$-$Q_3$ output $|0\rangle|1\rangle$ under the delayed-choice separable-state projection. However, in experiment the measured magnitude of these off-diagonal elements is only about 0.02 due to imperfect timing, to which these elements are extremely sensitive because they oscillate very fast during the parallel entangling gates. This magnitude is further reduced to about 0.007 by the delay of projection, during which extra noises are introduced. Consequently the corresponding concurrence is too small to detect in experiment. The difference between the calculated $Q_1$-$Q_4$ concurrence and the measured result corresponding to the $Q_2$-$Q_3$ output $|1\rangle|0\rangle$, shown in Table S4, is due to the same reason.
TABLE S2: Measured and calculated probabilities of different $Q_2$-$Q_3$ basis states, and the associated $Q_1$-$Q_4$ output state fidelities and concurrences for the normal entanglement swapping.

| $Q_2$-$Q_3$ probability distribution | $Q_1$-$Q_4$ Fidelity | $Q_1$-$Q_4$ Concurrence |
|-------------------------------------|---------------------|-------------------------|
| Experiment (Simulation)             | $|0\rangle\langle 0|$ | $0.249\pm0.004 (0.250)$ |
|                                     | $|1\rangle\langle 1|$ | $0.250\pm0.006 (0.257)$ |
| $|0\rangle\langle 0|$ | $\Phi_{1,4}^{++}$ | $0.893\pm0.010 (0.895)$ |
| $|1\rangle\langle 1|$ | $\Phi_{1,4}^{+}$  | $0.872\pm0.011 (0.885)$ |
| $|0\rangle\langle 0|$ | $\Phi_{1,4}^{+}$  | $0.884\pm0.010 (0.893)$ |
| $|1\rangle\langle 1|$ | $\Phi_{1,4}^{+}$  | $0.891\pm0.012 (0.908)$ |
| $|0\rangle\langle 0|$ | $\Phi_{1,4}^{+}$  | $0.890\pm0.012 (0.904)$ |
| $|1\rangle\langle 1|$ | $\Phi_{1,4}^{+}$  | $0.897\pm0.010 (0.908)$ |

TABLE S3: Measured and calculated probabilities of different $Q_2$-$Q_3$ basis states, and the associated $Q_1$-$Q_4$ output state fidelities and concurrences for the delayed-choice entanglement swapping.

| $Q_2$-$Q_3$ probability distribution | $Q_1$-$Q_4$ Fidelity | $Q_1$-$Q_4$ Concurrence |
|-------------------------------------|---------------------|-------------------------|
| Experiment (Simulation)             | $|0\rangle\langle 0|$ | $0.248\pm0.004 (0.251)$ |
|                                     | $|1\rangle\langle 1|$ | $0.256\pm0.005 (0.258)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{++}$ | $0.891\pm0.012 (0.908)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{+}$  | $0.896\pm0.010 (0.899)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{+}$  | $0.897\pm0.010 (0.908)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{+}$  | $0.815\pm0.026 (0.820)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{+}$  | $0.816\pm0.024 (0.827)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{+}$  | $0.806\pm0.022 (0.808)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{+}$  | $0.807\pm0.019 (0.821)$ |

TABLE S4: Measured and calculated probabilities of different $Q_2$-$Q_3$ basis states, and the associated $Q_1$-$Q_4$ output state fidelities and concurrences for the delayed-choice separable-state projection.

| $Q_2$-$Q_3$ probability distribution | $Q_1$-$Q_4$ Fidelity | $Q_1$-$Q_4$ Concurrence |
|-------------------------------------|---------------------|-------------------------|
| Experiment (Simulation)             | $|0\rangle\langle 0|$ | $0.258\pm0.005 (0.258)$ |
|                                     | $|1\rangle\langle 1|$ | $0.250\pm0.005 (0.257)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{0}$  | $0.930\pm0.009 (0.946)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{0}$  | $0.949\pm0.008 (0.958)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{0}$  | $0.907\pm0.011 (0.932)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{0}$  | $0.914\pm0.009 (0.934)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{0}$  | $0.016\pm0.018 (0.000)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{0}$  | $0.005\pm0.007 (0.006)$ |
| $|0\rangle\langle 0|$ | $\Psi_{1,4}^{0}$  | $0.004\pm0.005 (0.008)$ |
| $|1\rangle\langle 1|$ | $\Psi_{1,4}^{0}$  | $0.015\pm0.011 (0.000)$ |

[1] C. Song, S.-B. Zheng, P. Zhang, K. Xu, L. Zhang, Q. Guo, W. Liu, D. Xu, H. Deng, K. Huang, D. Zheng, X. Zhu, H. Wang, Continuous-variable geometric phase and its manipulation for quantum computation in a superconducting circuit. Nat. Commun. 8, 1061 (2017).

[2] C. Song, K. Xu, W. Liu, C. Yang, S. Zheng, H. Deng, Q. Xie, K. Huang, Q. Guo, L. Zhang, P. Zhang, D. Xu, D. Zheng, X. Zhu, H. Wang, Y.-A. Chen, C.-Y. Lu, S. Han, J.-W. Pan, 10-qubit entanglement and parallel logic operations with a superconducting circuit. Phys. Rev. Lett. 119, 180511 (2017).

[3] Q. Guo, S.-B. Zheng, J. Wang, C. Song, P. Zhang, K. Li, W. Liu, H. Deng, K. Huang, D. Zheng, X. Zhu, H. Wang, C.-Y. Lu, J.-W. Pan, Dephasing-insensitive quantum information storage and processing with superconducting qubits. Phys. Rev. Lett. 121, 130501 (2018).

[4] K. Xu, J.-J. Chen, Y. Zeng, Y.-R. Zhang, C. Song, W. Liu, Q. Guo, P. Zhang, D. Xu, H. Deng, K. Huang, H. Wang, X. Zhu, D. Zheng, H. Fan, Emulating many-body localization with a superconducting quantum processor. Phys. Rev. Lett. 120, 050507 (2018).