In this Note, we make comments on the recent criticism of the description of highly excited hadrons in terms of fixed angular momentum \( L \) and quantify the relation \( M^2 \sim L + n + S/2 \) (\( n \) is the radial quantum number and \( S \) is the total quark spin) for masses of light mesons.

In the recent Comment \cite{1}, it was attempted to prove that a consistent holographic description of excited hadrons with fixed angular momentum \( L \) is not possible. This statement is not specific for the holographic description only, it has a generic character: As the highly excited hadrons composed of light quarks represent ultra-relativistic systems, the internal angular momentum \( L \) and the total quark spin \( S \) are not conserved separately, hence, any descriptions of excited hadrons based on fixed \( L \) and \( S \) (say, the potential models with valence quarks) are not consistent with the Lorentz invariance. Since this claim keeps regularly appearing in the literature and on the conferences, a clear-cut explanation is called for as to the physical reasons invalidating such a logic. In part, this question has been already addressed in the Response \cite{2} to the Comment above, in the first part of the present Note we will provide alternative arguments in favor of the description of excited hadrons at fixed \( L \) and discuss the physical sense of the relation

\[
M^2 \sim L + n, \tag{1}
\]

\( (n \) is the radial quantum number) for meson masses.

The excited hadron is not just a relativistic quantum system, it is observed as a complicated phenomenon that includes \( \text{in}^- \) and \( \text{out}^- \) states and the description of this phenomenon is inseparable from the \( \text{in}^- \) and \( \text{out}^- \) states, in other words, it must be clearly defined how it was produced and how it decayed. This point seems to be misunderstood when making the claim in question. Let us consider the simplest example — the creation of excited light meson and its decay into two pions in the center of mass system. The whole process can be divided into four stages which are displayed in FIG. 1. To create a highly excited meson one needs highly energetic quark \( q \) and antiquark \( \bar{q} \). Such a \( q\bar{q} \) pair can be produced, for instance, by a proton-antiproton collision. Due to the asymptotic freedom, the energetic quarks fly like almost free and well localized objects, hence, they have a well-defined relative angular momentum \( L \) and intrinsic spin. This first stage of the process represents the \( \text{in}^- \) state. Then the quark and antiquark interact forming a kind of correlated system (stage (2)) — the meson resonance. Strictly speaking, we can regard this effect as a resonance only if the correlation of the quark \( q \) with the given antiquark \( \bar{q} \) is stronger than with other surrounding quarks and antiquarks. At stage (3), the confinement interaction creates an additional quark-antiquark pair from the vacuum and finally (stage (4)) we observe two pions — the \( \text{out}^- \) state. Since the pions are \( S^- \) wave states (we know this from the fact that they are extensively produced in the point-like process like the \( e^+e^- \) annihilation) the relative angular momentum of pions is equal to \( L \) due to the angular momentum conservation. \( L \) is not a conserved quantum number at stages (2) and (3), but this is obviously not the case for the \( \text{in}^- \) and \( \text{out}^- \) states. Measuring experimentally the \( L \) of produced pions we find \( L \) of initial \( q\bar{q} \) pair. This very \( L \) enters the relation \( (1) \).

The relation \( (1) \) follows from the analysis of available experimental data \cite{3}. First of all, it tells us about the quantum nature of meson resonances — the \( q\bar{q} \) pair resonates only at integer values of relative \( L \) and at certain discrete values of \( q \) and \( \bar{q} \) momentum (dictated by the integer value of \( n \)). Second, a large degeneracy takes place due to the existence of principal quantum number \( N \sim L + n \) like in the hydrogen atom \cite{4}. Thus, a theoretical challenge is to explain the relation \( (1) \) and to find possible corrections. In this regard, the potential (or in some sense "quasipotential") models as well as other models describing the light mesons as bound states represent a possible line of research, one should only keep in mind that \( L \) and \( n \) should not be understood literally in this case (one deals with models, not with a theory), they are rather a convenient tool for the description and classification of resonances.

If the meson spin \( J \) is somehow established experimentally, we can restore the total quark spin \( S \) of initial \( q\bar{q} \) pair following the quantum mechanical rule \( J = L + S \), \( S = 0, 1 \). The Comment \cite{2} contains an interesting proposal that \( S \) determines the intercept in the relation \( (1) \), \( M^2 \sim L + n + S/2 \). We have performed a fit of experimental data used in \cite{4} exploring this suggestion, the result is (in GeV$^2$)

\[
S = 0 : \quad M^2 = 1.22(L + 1.04n + 0.22), \tag{2}
\]

\[
S = 1 : \quad M^2 = 1.12(L + 0.93n + 0.63). \tag{3}
\]

The estimated errors are within 10% for each constant. If we do not include the \( \pi \)-meson the fit \( (2) \) yields \( M^2 = 1.17(L + 1.04n + 0.36) \). The relations \( (2) \) and \( (3) \) are to be compared with the original fit \cite{3} that did not make

---

*Also at V. A. Fock Department of Theoretical Physics, St. Petersburg State University, 1 ul. Ulyanovskaya, 198504, Russia.
FIG. 1: The formation and decay of excited meson (see the text). The one-gluon exchange at stage (2) is the simplest interaction, under this plot we mean all possible interactions. The same is implied at stage (3).

separation between the $S = 0$ and $S = 1$ states,

$$M^2 = 1.10(L + n + 0.62).$$

Thus, the suggestion above seems to agree qualitatively with the available experimental data.

In the second part of the present Note, we raise an objection to a claim in [1] stating that the ultraviolet matching condition used in the holographic models is inconsistent with the chiral symmetry. The primary theoretical objects in the holographic approach are the correlation functions, therefore the situation here is in one-to-one correspondence with the QCD sum rules based on the Operator Product Expansion — it is well known that the manifest chiral invariance of the correlation functions in the ultraviolet does not imply, generally speaking, the asymptotic chiral invariance for the physical states saturating the correlation functions (unless some additional assumptions are used, see, e.g., [2]). This point can be easily seen qualitatively: Saturating, for instance, a two-point correlator of some quark operator by physical mesons with appropriate quantum numbers (we neglect the decay width and consider the Euclidean space),

$$\Pi(Q^2) \sim \sum_i \frac{Z_i}{Q^2 + m_i^2},$$

we have the residues $Z_i$ in the numerators. The physical sense of quantities $Z_i$ is the probability of creation of the corresponding meson at the origin. It is quite evident that, generally speaking, these probabilities strongly depend on $L$ of initial $q\bar{q}$ pair. Since the orbital momenta confronting the physical states related by the chiral transformations are different (because of opposite parities of chiral partners, $P = (-1)^{L+1}$) and not restricted by the chiral symmetry, the ultraviolet constraint for the correlators related by the chiral symmetry (let be $\Pi_+$ and $\Pi_-$),

$$\Pi_+(Q^2) - \Pi_-(Q^2) \to 0, \quad Q^2 \to \infty,$$

does not necessary imply for the masses of physical states to follow the pattern of degeneracy predicted by the chiral multiplets. The chiral basis seems to be useful for the classification of correlation functions, but its use for the physical resonances is not convincingly justified neither theoretically nor experimentally, instead the standard nonrelativistic $2S+1L_J$ basis turns out to be more natural and convenient for the spectroscopy of excited light mesons.

Finally, we would indicate the following delicate point. There exists an important theoretical case when the orbital momentum indeed cannot be confronted with the resonances in a straightforward manner — the planar limit of QCD, $N_c \to \infty$: The mesons do not decay in that limit, hence, $L$ cannot be deduced experimentally. On the other hand, it is rather obscure how to make an imaginary experiment on meson production in the planar world. The point is that the typical holographic models deal with precisely this limit, so the notion of $L$ in this case needs some additional clarification. A possible way out may consist also in effective account for finite $N_c$ by constructing bottom-up AdS/QCD models describing a final set of discrete resonances [3].

The author thanks Stan Brodsky and Guy de Téramond for their stimulation to express his insight in the written form. The research is supported by the Alexander von Humboldt Foundation and by RFBR, grant 09-02-00073-a.

[1] L. Ya. Glozman, [arXiv:0903.3923] [hep-ph].
[2] G. F. de Téramond and S. J. Brodsky, [arXiv:0903.4922] [hep-ph].
[3] S. S. Afonin, Mod. Phys. Lett. A 22, 1359 (2007).
[4] S. S. Afonin, Eur. Phys. J. A 29, 327 (2006); Phys. Lett. B 639, 258 (2006); Int. J. Mod. Phys. A 22, 4537 (2007); 23, 4205 (2008); Phys. Rev. C 76, 015202 (2007); P. Bicudo, Phys. Rev. D 76, 094005 (2007); M. Shifman and A. Vainshtein, Phys. Rev. D 77, 034002 (2008); E. Klempt and A. Zaitsev, Phys. Rept. 454, 1 (2007); El H. Mezoir and P. Gonzalez, Phys. Rev. Lett. 101, 232001 (2008).

[5] S. S. Afonin, A. A. Andrianov, V. A. Andrianov, and D. Espriu, JHEP 04, 039 (2004).

[6] S. S. Afonin, to be published in Phys. Lett. B; arXiv:0903.0322 [hep-ph].