Markov-Switching Vector Autoregressive Modelling (Intercept Adjusted); Application to International Trade and Macroeconomic Stability in Nigeria (2000M1–2019M6)

Tuaneh, Godwin Lebari1* and Essi, Isaac Didi2

1Department of Agricultural and Applied Economics, Rivers State University, P.M.B. 5080, Port Harcourt, Nigeria.
2Department of Mathematics, Rivers State University, P.M.B. 5080, Port Harcourt, Nigeria.

Authors’ contributions

This work was carried out in collaboration between the two authors. Author TGL designed the study, wrote the first draft of the manuscript including the literature searches, performed the statistical analysis and results interpretation author EID supervised the work and the protocol. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2021/v12i430294

Editors:
(1) Dr. Dariusz Jacek Jakóbczak, Koszalin University of Technology, Poland.
(2) Kristijan Krstic, University of Kragujevac, Serbia.

Reviewers:
(1) Magda Mohamed Mohamed Haggag, Damanhour University, Egypt.

Complete Peer review History: http://www.sdiarticle4.com/review-history/68062

Received 27 February 2021
Accepted 04 May 2021
Published 10 May 2021

Abstract

Economic relationships are often modelled without consideration of a possible regime switch, the transmission from one regime to another and the duration of stay in a particular regime which are not captured by linear models. This study aimed to model and estimate the interdependence existing among Nigeria’s International Trade and Macroeconomic Stability. Specifically, this study sought to estimate and compare the estimated Models, select the best Model and determine the probabilities of stay, the expected duration of stay in a particular regime. The study adopted a quasi-experimental design. Time series data on the study variables from January 2000 to June 2019 were obtained from the Statistical Bulletin of the Central Bank of Nigeria. Models were specified accordingly, the statistical analyses were carried out using the Markov Switching Intercept Vector Autoregressive Models, the pre and post-diagnostic tests were also conducted. The unit root test results showed I (1). VAR lag length selection criteria choose lag 2. The MS-VAR analysis identified two regimes (expansion and contraction), the information criteria selected the
Markov-Switching Intercept Autoregressive Heteroschedastic 2 Variance Auto-regression 2 [MSIARH (2) - VAR (2)]. The MS-VAR results in regime 1 showed that lags 1 and 2 of total export significantly affected total export and total import, Lags 1 and 2 of total import had significant effects on exchange rate while lags 1 of exchange rate and lags 1 and 2 of exchange rate had significant effects on inflation rate. In Regime 2, lag 1 of total export and lag 2 of exchange rate had significant effects on total export. Only lag 2 of inflation rate had significant effects on exchange rate while lag 2 of total export and lags 1 and 2 of exchange rate had significant effects on the inflation rate. The results also showed an 89% probability of staying in regime 1 for a duration of 8 months 8 days and 57% probability of staying in regime 2 for 2 months 10 days. It was concluded that the MSVARH (2) - VAR (2). It was recommended that the right-hand side variables should be tested for endogeneity before concluding on single or system equation. It was also recommended that the possibility of regimes should be verified before concluding on linear or nonlinear models.

Keywords: Markov-switching Vector autoregressive model (MS-Var); MSI-Var; international trade; macroeconomic stability; Nigeria.

1 Introduction

Macroeconomic relationship are often modelled with linear econometric methods, these suggest that the economy is always in appreciation or a depreciation state, and consequently ruling out periods of shocks and the resultant effects. This excludes periods of normalcy where the exchange rate may respond to the economic fundamentals like import and export, [1]. The dynamics and interdependence among variables are in consequence modelled without recourse to the existence of regimes. Consequently, the models do not only ignore the unobservable state, structural breaks, regime switches and duration of stay in a state but are also inappropriate with incomplete structural inference. To bridge this gap in the literature, this study adopts a multivariate Markov-Switching VAR (MS-VAR) to explore the regime-dependent dynamic relationships among International Trade and Macroeconomic Stability in Nigeria.

Macro-economic variables typically and persistently fluctuate around high and low levels, hence an unobservable egordic Markov process and the possibility of a regime shift. One appropriate method which captures the unobservable state, the transmission from one regime to another and the duration of stay in a particular regime often ignored by the linear methods is the Markov-Switching Variance Autoregressive model. The MS-VAR model can provide a systematic ability to implementing statistical methods and the model can also estimate an efficient and consistent parameters, detect recent changes and correct the VAR model when the regimes change [2].

The Markov-Switching Vector Autoregressive (MS-VAR) model is a non-linear model as it is characterized by a data generation mechanism that is non-linear. This is done by limiting the method to be linear in a particular unobservable and discrete regime. It was introduced by [3] following the Hamilton concept. The MS-VAR model is also a generalization of the simple finite order of the VAR model. The key concept of the model is that the observables time series vectors are reliant on an unobserved state.

The Markov Switching model is set up to achieve an unobservable state through a discrete-time and state of Markov stochastic mechanism with the transition probabilities. More so, the MS-VAR model estimates and forecast time-varying problems when a change in parameters occurs. In empirical studies, certain parameters are conditioned on the state of the Markov chain, whereas the other parameters are allowed to be regime invariant. Consequently, there are; Markov-Switching Intercept Term, Markov-Switching Mean, Markov-Switching Autoregressive Parameters and Markov-Switching Heteroskedasticity. This Study focused on the Markov-Switching Intercept.

1.1 International trade and macroeconomic stability in Nigeria

The study used export and import as proxy for international trade while the exchange rate and inflation rate were used as measures of macroeconomic stability. This study investigates the maintenance or alteration in macroeconomic stability resulting from the relationship between international trade (export and import) and the macroeconomic stability variables described using the MSI-VAR model.
[4] reported that Nigeria has historically witnessed macroeconomic turmoil, like other developing countries. He further clarified that macroeconomic instability refers to volatile and dynamic macroeconomic environments. It is a phenomenon that makes the domestic macroeconomic system less stable and unpredictable. This is a concern because unpredictability hampers decisions, investment, and growth in resource allocation.

The absence of excessive volatility in the main macroeconomic variables refers to economic stability. An economy with a reasonably steady rate of growth, whose inflation rate is low and fairly stable, the interest rate is low and fairly stable and has an acceptable and stable exchange rate. The World Bank defines the macroeconomic system as stable when "real interest rate is reasonable, the inflation rate is low and predictable, the real exchange rate is efficient and accurate and the balance of payments condition is viewed as realistic " [5]. International trade between countries or foreign trade is the trade across international boundaries or territories, of goods and services. In general, foreign trade and the associated financial transactions are carried out to ensure that a nation imports the goods it needs and exports those it produces in abundance. International trade between citizens of the reporting economy and the rest of the world takes place (ROW). Therefore, Foreign Trade Statistics (ITS) tests the amounts and values of products flowing into (importing) or out of (exporting) countries [6]. Accordingly, this analysis takes total exports and total imports as the dimensions for foreign trade.

Exchange rate is defined as a rate for which a country’s currency exchange for the other country’s currency. It tells us how much foreign currency your currency is worth. The management of foreign exchange and exchange rates in Nigeria has grown over the years. After 1986, it has changed from an officially pegged exchange rate regime from 1970 to 1985 to a market-determined system. The Naira exchange rate is now calculated based on demand and supply through the foreign exchange market. The dollar is the market intervention currency, while other currencies exchange rates are based on a cross-reference to the naira-dollar exchange rate [6].

Inflation is the percentage change in the general level of price during a particular time. This reduces the purchasing power of each unit of the currency. When the exchange rate suffers much like the Nigerian Naira has become less valuable relative to foreign currencies like the American dollar, European euro, or the English pounds, it makes goods and services from these countries (imports) more expensive to the Nigerian consumers and simultaneously making Nigerian goods and services (exports) cheaper to consumers overseas. The above analogy indicates that inter-relationship exists among inflation rates, exchange rates, export and import. This study is to espouse this inter-relationship and determine any existing interdependence among them and the significance of the interdependence.

1.2 Aim and objectives

The main thrust of the study was to apply the MSIVAR in modelling and estimating within the context of the Nigerian economy, the interdependence between international trade (export and import) and the macroeconomic stability (Exchange rate and inflation rate). Specifically, the study; (i) Estimated the Markov Switching Intercept Vector Autoregressive Models and select the best one based on information criteria, and (ii) estimated the interdependence existing among total export, total import, exchange rate, and inflation rate. (iii) Determined the probabilities of transition, and the duration of stay in a regime, from the selected MS-VAR models (iv) Forecasted total export, total import, exchange rate, and inflation rate for 12 months.

2 Literature

Markov Switching Vector Autoregressive (MS-VAR) models and their application to complex multivariate systems have been studied. In a series of papers, [7-10] used the MS-VAR model to discuss the characterization and the test business cycle asymmetries. Series of authors, [11-14] have used MS-VAR in macroeconomic research.

[15] studied Inflation Targeting for Turkey by analyzing the behaviour of the inflation rate in Turkey. They used monthly data spanning from January 2003 to August 2014. The study used the Markov Switching Intercept Autoregression (MSI-AR). The study identified two regimes and the results the regime changes were slow in Turkey.
[1] studied Intercept Adjusted Markov Switching Vector Autoregressive Model in Macro-economic Time Series Data. The study ascertained the asymmetry and state-switching behaviour of the data using the Markov Switching Vector Autoregressive models particularly the MSI-VAR model with adjusted Intercept. The study identified two regimes and the result showed a smooth transition of the stock index changes from recession state to growth state. The study showed that the gold price and oil price affected stock exchange. It, however, concluded that the MSI-VAR model provides significant, valid and reliable results.

[16] carried out a study on global capital flows, time-varying fundamentals and transitional exchange rate dynamics. The purpose of the investigation was to examine the dynamic relationship between the economic and financial fundamentals, and exchange rate, it also sought to investigate if the relationship depends on overvaluation and undervaluation of the exchange rate. The study applied the Markov Switching Vector Auto Regression (MSVAR) model on annual data from 1972-2009. The study identified 2 states (overvaluation and undervaluation). The results showed a varying relationship among the variables in both regimes.

[17] carried out a study on the Economic regimes and stock market performance in Nigeria: Evidence from the regime-switching model. The study analyzed volatility spillover between stock market in bull and bear periods and exchange rate in the Nigerian stock market. The study applied a regime heteroskedastic Markov switching model to daily data spanning from 1st January 2010 to 31st December 2017. The results from preliminary investigations showed evidence of two regimes (bear and bull markets) and showed that both stock returns and exchange rate series were characterized with non-normal distribution, presence of unit root and ARCH effects. The result showed high transition probabilities, (0.9455 and 0.8686), for bear and bull respectively. However, the duration of stay in the regime is higher in the bull market (regime 2) than it was in the bear market (regime 1) at 5958.12 days and 18.406 days, respectively.

[18] analyzed the New Monetarist Phillips curve. The study aimed to ascertain the cointegration and causality relationships between inflation, GDP and unemployment in the USA. The Markov Switching –VAR was applied on quarterly data from 1957 second quarter to 2014 third quarter. The study identified 3 regimes and estimated different MS-VAR models and selected the best model based on the AIC and LR test. The result showed important asymmetries in inflation, GDP and unemployment and the changes in the behaviour of the variables were detected with the MS-VAR models.

[19] studied the effects of fiscal policy on economic activity over the business cycle in Algeria. They applied the Markov Switching Vector Autoregressive (MSVAR) mode. Annual data which spanned from 1970 to 2011 were converted to quarterly data using the cubic spine interpolation. The results showed that the effects of government spending and revenue multipliers were direct in the short run in both regimes. The result also showed that the effect of public revenue was weaker than the effect of government spending during the recession. Also, shocks to fiscal policy had a stronger impact during economic stress than during expansion.

[20] studied money growth and inflation using the Markov switching Bayesian VAR. The study analyzed the relationship between inflation and money growth in the Euro area, the US, Japan and UK, over an estimated period stretching from 1960 to 2012. The study described multiple inflation states showing simple and diversified features. The study showed that monetary changes marginally strengthened the signal of moving to a high inflation state or regime. The study also found that inflation tended to be relatively weak during times of low and steady inflation.

3 Methodology

In this section, we present the research design, sources and types of data, methods used in data collection, data analysis techniques and model specification.

3.1 Research design

This study adopted a quasi-experimental design. This was used because the study sought to find out the causes or effects relationship of the variables.
3.2 Types and sources of data

The research used monthly time series data on total export, total import, exchange rate, and inflation rate spanning from January 2000 to June 2019. The data were obtained from the Central Bank of Nigeria (CBN) Statistical Bulletin 2019.

3.3 Methods of data analysis

The study used the Markov Switching Vector Autoregressive Model (MSVAR) particularly the Markov Switching Intercept Vector Autoregressive Model (MSI-VAR) a non-linear modelling technique. However, pre-test particularly the unit root test were conducted to ascertain the stationarity status of the study variables.

3.4 Model specification

The models stated in this section will be used to evaluate the interdependence between variables that served as proxies for international trade and macroeconomic stability indicators.

The general form of a standard VAR model is;

\[ Y_t = \psi_0 + \sum_{i=1}^{p} \psi_i Y_{t-i} + \epsilon_t \]  

(3.0)

Following Krolzig 1997, the above is modified to allow for regime change so that \( Y_t \) follows a VAR procedure that is dependent on an unobservable discrete regime variable \( s_t \)

MSI:

\[ Y_t = \psi_0(s_t) + \sum_{i=1}^{p} \psi_i(s_t) Y_{t-i} + \epsilon_t \]  

where: \( \epsilon_t \sim iid, N(0, \Sigma(s_t)) \)  

(3.1)

where; \( m = 1, 2, \ldots, M \) possible regime and in period \( T \) when \( s_t = m \)

Shift in intercept according to Krolzig 1997 results to smooth adjustment of the time series.

The eight (8) basic classes of Markov Switching Intercept Vector Autoregressive (MSI-VAR) models noted earlier are presented in equation 3.2-3.9 and tabulated in Table 1 below.

- MSI(m)-VAR(p): \( Y_t = \psi_0(s_t) + \sum_{i=1}^{p} \psi_i Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim iid, N(0, \Sigma) \)  

(3.2)

- MSIH(m)-VAR(p): \( Y_t = \psi_0(s_t) + \sum_{i=1}^{p} \psi_i Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim iid, N(0, \Sigma(s_t)) \)  

(3.3)

- MSIAR(m)-VAR(p): \( Y_t = \psi_0(s_t) + \sum_{i=1}^{p} \psi_i(s_t) Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim iid, N(0, \Sigma) \)  

(3.4)

- MSIARH(m)-VAR(p): \( Y_t = \psi_0(s_t) + \sum_{i=1}^{p} (s_t) \psi_i Y_{t-i} + \epsilon_t \) where: \( \sim N(0, \Sigma(s_t)) \)  

(3.5)

- MSH(m)-VAR(p): \( Y_t = \psi_0 + \sum_{i=1}^{p} \psi_i Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim iid, N(0, \Sigma(s_t)) \)  

(3.6)

- MSAR(m)-VAR(p): \( Y_t = \psi_0 + \sum_{i=1}^{p} \psi(s_t) Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim iid, N(0, \Sigma) \)  

(3.7)
MSARH(m)-VAR(p): \( Y_t = \psi_0(s_t) + \sum_{i=1}^{p} (s_t)^i \psi_i Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim N(0, \Sigma) \) (3.8)

Linear-(m)-VAR: \( Y_t = \psi_0 + \sum_{i=1}^{p} \psi_i Y_{t-i} + \epsilon_t \) where: \( \epsilon_t \sim \text{iid, } N(0, \Sigma) \) (3.9)

### Table 1. Special cases of Markov Switching Intercept Variance Autoregressive (MSIVAR) models

| S/n | Markov Switching Intercept (MSI-VAR) models | \( \lambda_t \) | \( \Lambda_t \) | \( \Sigma \) |
|-----|------------------------------------------|----------------|-------------|------------|
| 1   | MSI(m)-VAR(p)                            | V             | NV          | NV         |
| 2   | MSIAR(m)-VAR(p)                          | V             | V           | NV         |
| 3   | MSIH(m)-VAR(p)                           | V             | NV          | V          |
| 4   | MSIARH(m)-VAR(p)                         | V             | V           | V          |
| 5   | MSAR(m)-VAR(p)                           | NV            | V           | NV         |
| 6   | MSH(m)-VAR(p)                            | NV            | NV          | V          |
| 7   | MSARH(m)-VAR(p)                          | NV            | V           | V          |
| 8   | LINEAR-(m)-VAR(p)                        | NV            | NV          | NV         |

Source: Krulzig, (1998), Guidoli (2012)

Where: (m) = Number of regime, (p) = number of lags, AR = autoregressive parameter, \( H(\Sigma) \) = Variance (Heteroschedastic parameter), I = Intercept, V = Varying, NV = Not Varying

### 3.5 Unobservable state/regime switching

Fig. 1 showed that the study identified 2 regimes, regime 1 (expansion) and regime 2 (contraction). As noted earlier, macroeconomic variables are not always increasing or always decreasing, hence linear models may not always be the best. Fig. 1 indicates that the variables fluctuate between high and low hence the high is modeled as regime 1 and the low is modeled as regime 2. The lag order selection criteria as shown earlier chose a lag length of 2.
Markov Switching Variance Autoregressive Model with 2 Regimes and 2 Lags

\[
\begin{align*}
\text{Y}_t & \sim \text{st}(1|s_{t-1}) \begin{pmatrix} \text{DTXR} \\ \text{DTIM} \\ \text{DEXR} \\ \text{DINR} \end{pmatrix} = \begin{pmatrix} \psi_{1,1} & \psi_{1,2} & \psi_{1,3} & \psi_{1,4} \\ \psi_{2,1} & \psi_{2,2} & \psi_{2,3} & \psi_{2,4} \\ \psi_{3,1} & \psi_{3,2} & \psi_{3,3} & \psi_{3,4} \\ \psi_{4,1} & \psi_{4,2} & \psi_{4,3} & \psi_{4,4} \end{pmatrix} \begin{pmatrix} \text{DTXR}_{t-1} \\ \text{DTIM}_{t-1} \\ \text{DEXR}_{t-1} \\ \text{DINR}_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \\ \epsilon_{t,4} \end{pmatrix} + \text{st}(s_t \xi_t) \\
\text{Y}_t & \sim \text{st}(2|s_{t-1}) \begin{pmatrix} \text{DTXR} \\ \text{DTIM} \\ \text{DEXR} \\ \text{DINR} \end{pmatrix} = \begin{pmatrix} \psi_{1,1} & \psi_{1,2} & \psi_{1,3} & \psi_{1,4} \\ \psi_{2,1} & \psi_{2,2} & \psi_{2,3} & \psi_{2,4} \\ \psi_{3,1} & \psi_{3,2} & \psi_{3,3} & \psi_{3,4} \\ \psi_{4,1} & \psi_{4,2} & \psi_{4,3} & \psi_{4,4} \end{pmatrix} \begin{pmatrix} \text{DTXR}_{t-2} \\ \text{DTIM}_{t-2} \\ \text{DEXR}_{t-2} \\ \text{DINR}_{t-2} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \\ \epsilon_{t,4} \end{pmatrix} + \text{st}(s_t \xi_t) 
\end{align*}
\]

The order of numbering is model, regime, lag, and variable for the coefficients and Model, regime for the intercept term.

MSIARH(2)-VAR(2) model requires the estimation of a 4x4 matrix for each regime which gives 64 autoregressive parameters, a column of 4 intercept terms for each of the 2 regimes, a matrix of 4 variances and 6 co-variances for each regime, and 2 independent transition probabilities, given a total of 94 parameters as also shown in the Eviews specification.

It is often assumed that the state variable is governed by the Markov chain:

\[
\begin{align*}
P(s_t = 1|s_{t-1} = 1) &= P^{11} \\
P(s_t = 2|s_{t-1} = 1) &= P^{12} \\
P(s_t = 1|s_{t-1} = 2) &= P^{21} \\
P(s_t = 1|s_{t-1} = 2) &= P^{22} 
\end{align*}
\]

Which is often presented as:

\[
P_{ij} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}
\]

These transition probabilities are restricted so that

\[
P_{11} + P_{12} = 1
\]

and

\[
P_{21} + P_{22} = 1
\]
MSI-VAR Models of 2 Regimes and lag 2 for Eviews Analysis

Regime 1

\[
\begin{align*}
DTEX &= C(1,1)DTEX(-1) + C(1,2)DTEX(-2) + C(1,3)DTIM(-1) + C(1,4)DTIM(-2) + \\
&\quad C(1,5)DEXR(-1) + C(1,6)DEXR(-2) + C(1,7)DINR(-1) + C(1,8)DINR(-2) + C(1,9) \\
DTIM &= C(2,1)DTEX(-1) + C(2,2)DTEX(-2) + C(2,3)DTIM(-1) + C(2,4)DTIM(-2) + \\
&\quad C(2,5)DEXR(-1) + C(2,6)DEXR(-2) + C(2,7)DINR(-1) + C(2,8)DINR(-2) + C(2,9) \\
DEXR &= C(3,1)DTEX(-1) + C(3,2)DTEX(-2) + C(3,3)DTIM(-1) + C(3,4)DTIM(-2) + \\
&\quad C(3,5)DEXR(-1) + C(3,6)DEXR(-2) + C(3,7)DINR(-1) + C(3,8)DINR(-2) + C(3,9) \\
DINR &= C(4,1)DTEX(-1) + C(4,2)DTEX(-2) + C(4,3)DTIM(-1) + C(4,4)DTIM(-2) + \\
&\quad C(4,5)DEXR(-1) + C(4,6)DEXR(-2) + C(4,7)DINR(-1) + C(4,8)DINR(-2) + C(4,9) \\
SIGMA(DTEX, DTEX) &= C(1, 10) \\
SIGMA(DTEX, DTIM) &= C(1, 11) \\
SIGMA(DTEX, DEXR) &= C(1, 12) \\
SIGMA(DTEX, DINR) &= C(1, 13) \\
SIGMA(DTIM, DTEX) &= C(2, 11) \\
SIGMA(DTIM, DTIM) &= C(2, 12) \\
SIGMA(DTIM, DEXR) &= C(2, 13) \\
SIGMA(DTIM, DINR) &= C(2, 14) \\
SIGMA(DEXR, DTEX) &= C(3, 12) \\
SIGMA(DEXR, DTIM) &= C(3, 13) \\
SIGMA(DEXR, DINR) &= C(3, 14) \\
SIGMA(DINR, DTEX) &= C(4, 13) \\
SIGMA(DINR, DTIM) &= C(4, 14) \\
SIGMA(DINR, DEXR) &= C(4, 15) \\
SIGMA(DINR, DINR) &= C(4, 16)
\end{align*}
\]

Regime 2

\[
\begin{align*}
DTEX &= C(1,20)DTEX(-1) + C(1,21)DTEX(-2) + C(1,22)DTIM(-1) + C(1,23)DTIM(-2) + \\
&\quad C(1,24)DEXR(-1) + C(1,25)DEXR(-2) + C(1,26)DINR(-1) + C(1,27)DINR(-2) + C(1,28) \\
DTIM &= C(2,20)DTEX(-1) + C(2,21)DTEX(-2) + C(2,22)DTIM(-1) + C(2,23)DTIM(-2) + \\
&\quad C(2,24)DEXR(-1) + C(2,25)DEXR(-2) + C(2,26)DINR(-1) + C(2,27)DINR(-2) + C(2,28) \\
DEXR &= C(3,20)DTEX(-1) + C(3,21)DTEX(-2) + C(3,22)DTIM(-1) + C(3,23)DTIM(-2) + \\
&\quad C(3,24)DEXR(-1) + C(3,25)DEXR(-2) + C(3,26)DINR(-1) + C(3,27)DINR(-2) + C(3,28) \\
DINR &= C(4,20)DTEX(-1) + C(4,21)DTEX(-2) + C(4,22)DTIM(-1) + C(4,23)DTIM(-2) + \\
&\quad C(4,24)DEXR(-1) + C(4,25)DEXR(-2) + C(4,26)DINR(-1) + C(4,27)DINR(-2) + C(4,28) \\
SIGMA(DTEX, DTEX) &= C(1, 29) \\
SIGMA(DTEX, DTIM) &= C(1, 30) \\
SIGMA(DTEX, DEXR) &= C(1, 31) \\
SIGMA(DTEX, DINR) &= C(1, 32)
\end{align*}
\]
SIGMA(DTIM, DTEX) = C(2, 30)  
SIGMA(DTIM, DTIM) = C(2, 31)  
SIGMA(DTIM, DEXR) = C(2, 32)  
SIGMA(DEXR, DTEX) = C(3, 31)  
SIGMA(DEXR, DTIM) = C(3, 32)  
SIGMA(DINR, DTEX) = C(4, 32)  

### 4 Results and Discussion

#### 4.1 Comparing and selecting the markov switching intercept vector autoregressive Models

Sixteen models were estimated, eight each for the Markov switching mean model and the Markov switching intercept model. The results are summarized in Table 2 below.

| S/n | Estimated Models | Log-likelihood | Akaike info criterion | Schwarz criterion | Number of coefficients |
|-----|-----------------|----------------|-----------------------|------------------|----------------------|
|     | Markov Switching Intercept Models |                  |                       |                  |                     |
| 1   | MSI(2)-VAR(2)   | 540.9196        | -4.233                | -3.458           | 52                   |
| 2   | MSIAR(2)-VAR(2) | 802.1219        | -6.218                | -4.966           | 84                   |
| 3   | MSIH(2)-VAR(2)  | 1011.418        | -8.220                | -6.796           | 62                   |
| 4   | **MSIARH(2)-VAR(2)** | **1049.340**     | **-8.271**            | **-6.871**       | **94**               |
| 5   | MSAR(2)-VAR(2)  | 847.4278        | -6.644                | -5.452           | 80                   |
| 6   | MSH(2)-VAR(2)   | 1006.639        | -8.213                | -6.349           | 58                   |
| 7   | MSARH(2)-VAR(2) | 1016.639        | -8.023                | -6.682           | 90                   |
| 8   | LINEAR(2)-VAR(2) | 698.4901        | -5.606                | -4.846           | 51                   |

*Source: Researchers’ Computation with E-views 11.0*

Table 2 revealed that the Markov switching intercept autoregressive heteroskedastic (2) VAR(2) [MSIARH(2)-VAR(2)] model of the Markov switching intercept model had the highest log likelihood (1049.34) and the least information criteria (Akaike Information Criteria = -8.27, Schwarz Information Criteria = -6.871), consequently, the MSIARH(2)-VAR(2) of the Markov switching intercept model with 94 parameters was chosen.

#### 4.2 The interdependence existing among total export, total import, exchange rate, and inflation rate using the selected Ms-Var model (MSIARH(2)-VAR(2))

| Parameter estimates of MSIARH (2) – VAR (2) Model |
|--------------------------------------------------|

|                  | DTEX | DTIM | DEXR | DINR | DTEX | DTIM | DEXR | DINR | DINR |
|------------------|------|------|------|------|------|------|------|------|------|
| **Intercept**    |      |      |      |      |      |      |      |      |      |
| C                | -0.352 | 0.046 | -0.001 | 0.008 | -0.495 | 0.464 | -0.023 | -0.750 |
| [ -4.595 ]       | [ 0.420 ] | [ -1.121 ] | [ 0.102 ] | [ -4.289 ] | [ 1.781 ] | [ -0.465 ] | [ -1.559 ] |
| **Regime dependent autoregressive parameter** |      |      |      |      |      |      |      |      |      |
| DTEX(-1)         | 0.095* | -0.629* | 0.001 | 0.041 | -0.074 | -0.191 | 0.030 | 0.006 |
| [ 1.977 ]        | [ -9.343 ] | [ 0.635 ] | [ 0.962 ] | [ -0.948 ] | [ -1.069 ] | [ 0.877 ] | [ 0.018 ] |
### Representation of Regime 1 (MSIARH (2) - VAR (2))

\[
\begin{bmatrix}
\text{DTEX} \\
\text{DTIM} \\
\text{DEXR} \\
\text{DINR}
\end{bmatrix}
= \begin{bmatrix}
-0.352 \\
0.046 \\
0.001 \\
0.008
\end{bmatrix} + \begin{bmatrix}
0.095 \\
0.629 \\
0.005 \\
0.041
\end{bmatrix} \text{SIGMA} + \begin{bmatrix}
0.108 \\
0.029 \\
0.002 \\
0.004
\end{bmatrix} \text{SIGMA}^{-1}
\]

\[
\begin{bmatrix}
\text{DTIM} \\
\text{DEXR} \\
\text{DINR}
\end{bmatrix} = \begin{bmatrix}
-0.495 \\
-0.023 \\
0.750
\end{bmatrix} + \begin{bmatrix}
0.276 \\
-0.202 \\
0.018
\end{bmatrix} \text{SIGMA} + \begin{bmatrix}
0.464 \\
0.031 \\
0.006
\end{bmatrix} \text{SIGMA}^{-1}
\]

### Representation of Regime 2 (MSIARH (2) - VAR (2))

\[
\begin{bmatrix}
\text{DTEX} \\
\text{DTIM} \\
\text{DEXR} \\
\text{DINR}
\end{bmatrix} = \begin{bmatrix}
0.295 \\
0.277 \\
0.217 \\
0.231
\end{bmatrix} + \begin{bmatrix}
2.766 \\
0.216 \\
0.200 \\
0.088
\end{bmatrix} \text{SIGMA} + \begin{bmatrix}
-0.267 \\
-0.746 \\
-0.906 \\
-0.167
\end{bmatrix} \text{SIGMA}^{-1}
\]

### Representation of Regime 2 (MSIARH (2) - VAR (2))

\[
\begin{bmatrix}
\text{DTEX} \\
\text{DTIM} \\
\text{DEXR} \\
\text{DINR}
\end{bmatrix} = \begin{bmatrix}
0.276 \\
0.111 \\
0.003 \\
0.000
\end{bmatrix} + \begin{bmatrix}
0.002 \\
0.080 \\
0.209 \\
0.080
\end{bmatrix} \text{SIGMA} + \begin{bmatrix}
0.495 \\
0.231 \\
0.001 \\
0.102
\end{bmatrix} \text{SIGMA}^{-1}
\]

### Regime dependent heteroschedastic parameter

| Parameter | Regime 1 | Regime 2 |
|-----------|----------|----------|
| SIGMA-DTEX | 0.022* | 0.007* |
| SIGMA-DTIM | 0.007* | 0.043* |
| SIGMA-DEXR | 0.00005 | 0.00004 |
| SIGMA-DINR | -0.004* | 0.002 |

Source: Researcher’s computation with Eviews 11.0
Regime 2 Variance Covariance Matrix (MSIARH (2)- VAR (2))

\[
\begin{pmatrix}
\text{SIGMA} \\
\text{DTEX} \\
\text{DTIM} \\
\text{DEXR} \\
\text{DINR}
\end{pmatrix}
\begin{pmatrix}
(DTEX) & (DTIM) & (DEXR) & (DINR) \\
0.015 & 0.007 & 0.001 & 0.015 \\
0.007 & 0.078 & 0.001 & -0.019 \\
0.001 & 0.001 & 0.003 & 0.001 \\
0.015 & -0.019 & -0.001 & -0.258
\end{pmatrix}
\]

The regime 1 results of the Markov Switching Intercept Autoregressive Heteroscedastic Variance Autoregressive Model above and presented in Table 3 shows that; the first and the second lag of total export significantly affected total export, lags 1 and 2 of total export had significant effects on total import, Lag 1 and lag 2 of total import had significant effects on exchange rate while the first leg of the exchange rate and lags 1 and 2 of exchange rate significantly affected inflation rate.

The regime 2 results of the Markov Switching Intercept Autoregressive Heteroscedastic Variance Autoregressive Model in Table 3 shows that; lag 1 of total export and lag 2 of exchange rate significantly affected total export, only lag 2 of inflation rate significantly affected exchange rate while lag 2 of total export and lags 1 &2 of exchange rate significantly affected on the inflation rate.

The Transition Probabilities and Expected Duration of Stay

\[
P_{ij} = \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix} = \begin{bmatrix}
0.879 & 0.121 \\
0.430 & 0.570
\end{bmatrix}
\]

Where;

\[P_{11}+P_{12}=1,\]
\[P_{21}+P_{22}=1\]

The result implied that; given that the current state is in expansion, the probability of transitioning to expansion in the next period is 0.879, and the probability of transitioning to a contraction in the next period is 0.121. Also, given that the current state is in contraction, the probability of transitioning to expansion in the next period is 0.430 while the probability of transitioning to a contraction in the next period is 0.570. The expected time spent in each state (expansion or contraction) is referred to as the expected duration. The closer \(P_{ij}\) is to 1 the higher is the expected duration of stay in the state. The expected duration is derived by \[\frac{1}{1-P_{ij}}\] is 8.28 in regime 1 and 2.33 in regime 2. The result implied that there is an 88% probability of staying in regime 1 for a duration of 8 months 8 days. Also, there is a 57% probability of staying in regime 2 for a duration of 2 months 10 days.

Post Estimation Analysis

On the post estimation analysis conducted, the AR root graph of regime 1 and 2 had all points in the unit root circle, indicating that the estimated parameters were stable. The Residual Normality test conducted showed that only one component was multivariate Normal.

4.3 Impulse response

Impulse Response of the Markov Switching VAR Model

The impulse response is often used not only to show the reaction of a variable to shocks in other variables but also to identify the direction of response resulting from the dynamic behaviour among the variables or shock to other variables. A shock to an endogenous variable causes own shocks and shocks to other variables in the system. The impulse response traces the effects of a one-time shock to one of the innovations on current and future values of the particular endogenous variable and other endogenous variables.
The impulse response of the Markov-switching VAR summarized in Table 5 showed four-unit structural innovations: (i) total export shock; (ii) total import shock; (iii) exchange rate shock, and (iv) inflation rate shock.

Table 4. Impulse Response from MSIARH (2)-VAR (2)

| Period | DTEX  | DTIM | DEXR | DINR | DTEX  | DTIM | DEXR | DINR |
|--------|-------|------|------|------|-------|------|------|------|
|        |       |      |      |      |       |      |      |      |
|        |       |      |      |      | Regime 1 |     |     |     |
| Response of DTEX: |       |      |      |      |       |     |     |     |
| 1      | 0.149 | 0.000| 0.000| 0.000| 0.121 | 0.000| 0.000| 0.000|
| 2      | 0.051 | 0.154| -0.001| -0.019| 0.000 | 0.060| 0.001| -0.015|
| 3      | -0.006| 0.229| 0.000| -0.002| -0.055| -0.086| -0.005| 0.010|
| . . . | . . . |      |      |      | . . . |     |     |     |
| 12     | 0.073 | -0.419| 0.001| 0.012| 0.006| 0.005| 0.003| -0.013|
| Response of DTIM: |       |      |      |      |       |     |     |     |
| 1      | 0.044 | 0.203| 0.000| 0.000| 0.061 | 0.273| 0.000| 0.000|
| 2      | -0.082| 0.050| 0.000| -0.003| -0.023| -0.035| -0.007| 0.030|
| 3      | -0.090| -0.110| 0.001| 0.014| -0.063| -0.250| 0.000| 0.011|
| . . . | . . . |      |      |      | . . . |     |     |     |
| 12     | 0.195 | 0.273| -0.001| -0.026| 0.039| 0.116| 0.006| -0.026|
| Response of DEXR: |       |      |      |      |       |     |     |     |
| 1      | 0.000 | 0.000| 0.002| 0.000| 0.010 | -0.006| 0.053| 0.000|
| 2      | 0.010 | 0.047| 0.000| 0.000| 0.007 | 0.063| 0.002| -0.038|
| . . . | . . . |      |      |      | . . . |     |     |     |
| 12     | 0.037641| -0.020| 0.000| -0.002| -0.004| -0.017| 9.28E-05| -0.003|
| Response of DINR: |       |      |      |      |       |     |     |     |
| 1      | -0.025| 0.015| 0.007| 0.126| 0.121| -0.096| -0.019| 0.483|
| 2      | 0.000 | -0.034| 0.000| -0.005| -0.065| 0.126| -0.040| -0.280|
| . . . | . . . |      |      |      | . . . |     |     |     |
| 12     | -0.040| -0.030| 0.000| 0.004| 0.048| 0.155| 0.004| -0.036|

Cholesky Ordering: DTEX DTIM DEXR DINR

Source: Researcher’s computation with Eviews 11.0.

In regime 1, total export, the total import and inflation rate had a contemporaneous response to its own shocks. All variables had a contemporaneous response to shocks in total export and total import. There was no immediate response of other endogenous variables to shocks in the exchange rate and inflation rate.

In regime 2, total export, total import, exchange rate and inflation rate had a contemporaneous response to its own shocks. All variables had a contemporaneous response to shocks in total export, total import, and inflation rate. There was no immediate response of other endogenous variables to shocks in the exchange rate.

Fig. 2A and 2B shows the impulse response of the Markov switching VAR in regime 1 and 2 respectively. The zero values from the start at lag zero for the contemporaneous or immediate response to shocks are imposed by the Cholesky decomposition by the particular ordering. The first column of Fig. 2A and 2B represents the response of total export to own shocks and the response of other variables to shocks in total export, the second column represents variations in the endogenous variables resulting from shocks in total imports, the third column showed changes in exchange rates to own shocks and the response of other variables to shocks in
exchange rate while the fourth column showed changes in the endogenous variables resulting from shocks in the inflation rate.

4.4 Variance decomposition

Table 5. Variance decomposition of MSIARH (2)-VAR (2)

| Period | S.E. | DTEX | DT  | DEXR | DINR | S.E. | DTEX | DTIM | DEXR | DINR |
|--------|------|------|-----|------|------|------|------|------|------|------|
|        |      |      |     |      |      |      |      |      |      |      |
| Regime 1 | Variance Decomposition of DTEX: | | | | | | | | | |
| 1       | 0.149 | 100.0 | 0.000 | 0.000 | 0.000 | 0.121 | 100.0 | 0.000 | 0.000 | 0.000 |
| 2       | 0.221 | 50.77 | 48.53 | 0.002 | 0.701 | 0.136 | 79.46 | 0.009 | 1.187 | 0.009 |
| 3       | 0.318 | 24.61 | 75.05 | 0.011 | 0.844 | 0.171 | 60.87 | 0.080 | 1.089 | 0.080 |
| ...     |      |      |      |      |      |      |      |      |      |      |
| 12      | 0.970 | 19.85 | 79.83 | 0.011 | 0.320 | 0.289 | 26.047 | 0.088 | 1.574 | 0.157 |

| Regime 2 | Variance Decomposition of DTIM: | | | | | | | | | |
| 1       | 0.208 | 4.421 | 95.58 | 0.000 | 0.000 | 0.280 | 4.724 | 0.000 | 0.000 | 0.000 |
| 2       | 0.229 | 16.61 | 83.38 | 0.000 | 0.015 | 0.285 | 5.248 | 0.057 | 1.075 | 0.057 |
| 3       | 0.270 | 23.19 | 76.52 | 0.001 | 0.292 | 0.384 | 5.546 | 0.032 | 0.666 | 0.032 |
| ...     |      |      |      |      |      |      |      |      |      |      |
| 12      | 0.775 | 24.02 | 75.62 | 0.001 | 0.356 | 0.604 | 6.443 | 0.080 | 1.567 | 0.080 |

| Regime 1 | Variance Decomposition of DEXR: | | | | | | | | | |
| 1       | 0.002 | 2.089 | 19.85 | 0.000 | 0.000 | 0.092 | 1.806 | 0.000 | 0.000 | 0.000 |
| 2       | 0.048 | 4.510 | 95.24 | 0.000 | 0.000 | 0.092 | 1.806 | 0.000 | 0.000 | 0.000 |
| 3       | 0.053 | 18.20 | 81.58 | 0.000 | 0.000 | 0.107 | 1.334 | 0.000 | 0.000 | 0.000 |
| ...     |      |      |      |      |      |      |      |      |      |      |
| 12      | 0.155 | 22.35 | 77.32 | 0.000 | 0.000 | 0.129 | 2.538 | 0.000 | 0.000 | 0.000 |

| Regime 2 | Variance Decomposition of DINR: | | | | | | | | | |
| 1       | 0.130 | 3.612 | 1.391 | 0.258 | 94.74 | 0.507 | 5.688 | 3.595 | 0.145 | 90.57 |
| 2       | 0.134 | 3.370 | 7.858 | 0.241 | 88.53 | 0.597 | 5.281 | 7.002 | 0.557 | 87.16 |
| 3       | 0.139 | 7.081 | 9.832 | 0.226 | 82.86 | 0.703 | 6.010 | 23.863 | 0.814 | 69.313 |
| ...     |      |      |      |      |      |      |      |      |      |      |
| 12      | 0.19  | 15.6  | 40.42 | 0.120 | 43.84 | 0.88  | 6.674 | 44.19 | 0.705 | 48.43 |

Cholesky Ordering: DTEX DTIM DEXR DINR

Source: Researcher’s computation with Eviews 11.0

4.5 Variance decomposition results of MSIARH (2)-VAR (2)

In regime 1, the percentage of the forecast error variance as shown in Table 5 showed that in the short run, 100% forecast variance in total export was self-explained. Total import, exchange rate, and inflation rate, however, showed very weak influence in predicting total export hence they are strongly exogenous. Moving into the future, total export decreases while total import increased. The percentage forecast variance of total export was 19.85% in the long run while the percentage forecast variance of total import, in the long run, was 79.83%. In regime 2, the percentage of the forecast error variance as also shown in Table 5 revealed that in the short run, 100% forecast variance in total export was self-explained. Total import, exchange rate, and inflation rate were very weak in predicting total export hence they are strongly exogenous. Moving into the future, however, total export decreases while total import increased. In the long run, the percentage forecast variance of total export was 26% while total import was 72.3%.

4.5.1 Variance decomposition of total import [MSIARH (2)–VAR (2)]

In regime 1, 95.58% of forecast error variance of total import was explained by own shock, total export exchange rate and inflation rate, however, indicated very weak influence in predicting total import in the short run. The forecast error variance of total import decreases while that of total export, and inflation rate increases
as we move into the future but at a very slow rate and were not strongly exogenous. In the long run, however, the percentage forecast variance of total import was 75.62% while total export was 24.02%.

In regime 2, the percentage of the forecast error variance as also shown in Table 5 revealed that in the short run, 95.27% forecast variance in total import was self-explained. Total import, exchange rate, and inflation rate were very weak in predicting total export therefore they are strongly exogenous. However, total export decreased while total import increased at a very slow rate. In the long run, the percentage forecast variance of total import was 91.9% while total export was 6.44%.

4.5.2 Variance decomposition of exchange rate [MSIARH (2)–VAR (2)]

In regime 1, the percentage of the forecast error variance of the exchange rate as shown in Table 5 was 97.56% self-explained. Total export, total import, and inflation rate, however, showed very weak influence in predicting exchange rate hence they are strongly exogenous. Moving into the future, exchange rate decreases sharply while total export and total import increased sharply. The percentage forecast variance of the exchange rate was 0.025% in the long run while the percentage forecast variance of total export and total import, in the long run, were 22.355 and 77.32% respectively.

![Impulse response function graph of regime 1](image)

**Fig. 2A. Impulse response function graph of regime 1**

In regime 2, the percentage of the forecast error variance of the exchange rate was 95.35% self-explained. Total import, total export and inflation rate showed very weak influence in predicting exchange rate hence they are strongly exogenous. Moving into the future, however, the forecast error variance of exchange rate decreased while that of total import and inflation rate increased. In the long run, the percentage forecast variance of the exchange rate was 17.32% while total import was 59.09% and the inflation rate was 21.04%.
4.5.3 Variance decomposition of inflation rate [MSIARH (2) – VAR (2)]

In regime 1, the percentage of the forecast error variance of inflation rate was self-explained by 94.74% as shown in Table 5. Total export, total import, and exchange rate indicated a very weak influence in predicting the inflation rate. The forecast error variance of inflation rate decreased while that of total import increased. The percentage forecast error variance of the exchange rate was 43.84% in the long run while the percentage forecast variance of total import, in the long run, was 40.42% respectively.

In regime 2, the percentage of the forecast error variance of the exchange rate of 90.57% self-explained. Total import, total export and exchange rate indicated a very weak influence in predicting the inflation rate. The forecast error variance of inflation rate decreased while that of total import increased as we move into the future. In the long run, the percentage forecast variance of inflation rate was 48.43% while total import was 44.19%.

![Fig. 2B. Impulse response function graph of regime 2](image)

5 Conclusion and Recommendations

The MS-VAR analysis identified two regimes (expansion and contraction). The information criteria selected the Markov-Switching Intercept Autoregressive Heteroscedastic 2 Variance Auto-regression 2 Model [MSIARH (2) - VAR (2)]. The right-hand side variables (supposed exogenous variable) should be tested for endogeneity before the decision on a single equation or system equation estimation. The possibility of regimes should be
determined. All forms of the Markov-Switching Intercept Vector Autoregressive Models should be estimated and the best selected using the information criteria. Having seen that inflation was the most affected by the dynamic behaviour of all the variables in the system, policies to regulate the inflation rate must consider activities or dynamic behaviour of international trade.

**Competing Interests**

Authors have declared that no competing interests exist.

**References**

[1] Aliyu SUR. Exchange rate volatility and export trade in Nigeria: An empirical investigation. Munich Personal RePEc Archive; 2008.

[2] Wai SP, Ismail MT, Sek SK. A Study of intercept adjusted Markov switching vector autoregressive model in economic time series data. Information Management and Business Review 2003;5(8):379-384.

[3] Krolzig HM. Markov-switching vector auto-regression. Berlin: Springer; 1997.

[4] Tuaneh GL. On agricultural performance amidst macroeconomic instability in Nigeria; Autoregressive distributed lagged modelling (2010Q1-2017Q4). Asian Journal of Economics, Business and Accounting. 2019;10(2):1-13,

[5] World Bank; 1990.

[6] Central Bank of Nigeria CBNl; 2019.

[7] Krolzig HM. Modeling of Markov-switching vector auto-regression using MSVAR for Ox” Discussion Paper, Department of Economics, University of Oxford; 1998 . Available:http://www.economics.ox.ac.uk/hendry/krolzig

[8] Krolzig HM. Predicting Markov-switching vector autoregressive processes, Journal of Forecasting; 2000.

[9] Krolzig. Business cycle measurement in the presence of structural change: International evidence. International Journal of Forecasting. 2001;17(3):349-368.

[10] Clements MP, Krolzig HM. Can oil shocks explain asymmetries in the US business cycle? Empirical Economics. 2002;27(1):185-204.

[11] Krolzig HM. Markov-switching vector auto-regression. Berlin: Springer; 1997.

[12] Tillmann P. The regime-dependent determination of credibility: A new look at European interest rate differentials, German Economic Review. 2003;4(4):409–431.

[13] Chen SW, Shen CH. A sneeze in the US, a cough in Japan, but pneumonia in Taiwan? An application of the Markov-switching vector autoregressive model. Economic Modelling. 2007;24:1–14.

[14] Kiganda EO, Obange N, Adhiambo S. The relationship between exports and inflation in Kenya: An aggregated econometric analysis. Asian Journal of Economics, Business and Accounting. 2017;3(1):1-12.

[15] Tasar I, Bayat T. Inflation targeting for Turkey. Elsevier. 2015;32(1):861-869.

[16] Kal H, Gunduz I. Global capital flows, time varying fundamentals and transitional exchange rate dynamics: An MS-VAR Approach, Istanbul Journal of Economics. 2019;69(1):1-22.
[17] Aliyu SUR, Aminu AW. Economic regimes and stock market performance in Nigeria: Evidence from regime switching model Munich Personal RePEc Archive. 2018;21:1-25.

[18] Bildiricia M, Turkmen C. New monetarist Phillips curve. Istanbul Conference of Economics and Finance 2020;38(1):360-367.

[19] Chibi Benbouziane, Chekouri. The impact of fiscal policy on economic activity over the business cycle: an empirical investigation in the case of Algeria. Review of Middle East Economics and Finance. 2014;15(3).

[20] Amisano G, Colavecchio R. Money Growth and Inflation: Evidence from a Markov Switching Bayesian VAR. 2013;04.

© 2021 Tuaneh and Essi; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/68062