CPhD Tracking Algorithm Based on BPF for Multiple Group Targets

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Abstract. Aim to solve the tracking of multiple group targets in strong clutter environment, a rectangular box is used to deal with extension state of group targets. Combining the box particle filter algorithm and the cardinalized probability hypothesis density (CPHD) algorithm, a CPHD algorithm for multiple group targets tracking based on box particle filter (BPF), which is called BPF-CPHD, is proposed in this paper. Compared with the existed algorithm of multiple group targets tracking, the proposed algorithm can improve the tracking accuracy and efficiency greatly. Simulation results confirm the effectiveness of the BPF-CPHD.

1. Introduction

According to the number of targets in the group, the group targets are divided into two categories: large group targets and small group targets [1]. Large group target refers to a group composed of a large number of targets, which is suitable for tracking the overall state of the group and unable to track all individual targets within the group. Small group target refers to a group composed of a small number of targets, which is suitable for tracking individual targets within the group and estimating the group structure.

Koch [2], a German scholar, used kinematics model and symmetric positive definite matrix to describe the change of group centroid and group extension, respectively. On the basis of ignoring the influence of measurement error on the expansion state, the idea of group tracking as a whole was proposed based on Bayesian theory. In literature [2], a single Singer model was used to describe the group target tracking algorithm of the group centroid motion. When the group target was maneuvering or strongly maneuvering, the tracking accuracy of the algorithm would be significantly descend [3]. Under the framework of interactive multi-model algorithm, the literature [4] considered the influence of measurement error on the extended state to track single group target. On the basis of literature [4], literature [5] updated the measurement formula based on the variable decibel Bayesian iterative method, but the model conversion probability needs a prior setting. The methods in literature [4] and [5] could improve the accuracy of group target tracking to a certain extent, but there are some problems such as the need to set the model conversion probability matrix in advance and the large maneuvering tracking error.

In the single group target tracking problem, the single-model and multi-model tracking methods are usually used to solve the problem of tracking model setup and filtering estimation of corresponding state. Single-model tracking algorithm generally adaptively adjusts the state model and filtering algorithm.
In terms of group shape modeling, the literature [6] combined the random matrix method with Gaussian mixture PHD filtering algorithm to process the tracking target shape approximately as an ellipse. When the number of clutter is large and the multiple group targets are moving cross each other, the target measurement is difficult to achieve accurate division and the tracking effect is poor. The group target is a target occupying a certain area around the group centroid in the state space. Multiple group target measurements are distributed around its equivalent centroid, and the range of distribution can be equivalent to the extended range of the group targets. Therefore, the extended range of the group targets can be described by the rectangular box. Based on the BPF algorithm, the stable tracking of the group targets can be achieved. The principle of measurement and inclusion box for group targets are shown in Figure 1.

**Figure 1. Group measurement and inclusion box**

In terms of multiple group targets tracking filtering algorithm, PHD filtering algorithm in literature [7] can effectively track multiple group targets. However, PHD algorithm assumes that the number of targets obeys Poisson distribution, and the estimation of target number becomes imprecise under the influence of clutter and false alarm. In the CPHD filtering algorithm, the recursion of the target number cardinalized distribution is added, so the filtering accuracy is higher.

Based on above analysis, through combining the interval analysis method with the box particle filter, the CPHD tracking algorithm based on BPF is proposed for multiple group targets. The proposed algorithm use a controllable non-zero size rectangular area instead of the traditional multiple description of measurement point measurement, and exploits the probability hypothesis density filter algorithm to deal with the cardinalized balance multi-group state. Compared with the existed algorithm of multiple group targets tracking, the proposed algorithm can improve the tracking accuracy and efficiency greatly.

2. Main steps of the BPF-CPHD algorithm

The main steps of the proposed BPF-CPHD algorithm are as follows:

1. Initialization

According to the measurement division results at the last moment, \( N^j_k \) rectangular regions subject to normal \( N((z)_j^{-1}, \Sigma) \) distribution is generated, where \( N^j_k = N / m_{k-1}, j = 1, \cdots, m_{k-1} \). \( N \) is the total number of box particles and \( m_{k-1} \) is the number measured at the last moment. The initialization of this moment is done by the state transition function.

2. Measurement division

The measurement is divided by distance division method. According to the inclusion function \( [f] \), the partition result is contained into a rectangular box with a certain size, and the measured distribution range in the partition unit is described by the size of the rectangular box.

\[
[z_k] = ([f] W_x), \lambda = 1, \cdots, m
\]  

where \( m \) is the number of partition units and \( W \) is the subset of partition.

3. Generation of newborn particles
Newborn targets are more likely to be generated in the area where the measurement appears. The measurement at the previous moment is used to supplement newborn box particles, and $N_b$ new box particles are generated. Each measurement generates $N_b^j$ box particles

$$N_b^j = N_b \cdot m_{k-1}, j = 1, \ldots, m_{k-1}$$ (2)

where $m_{k-1}$ is the number of measurements at the previous moment.

(4) Prediction

The prediction equation is as follows

$$[x_{k-1}^i] = [f_i(x_{k-1}^i)] + [w_{k-1}^i], \quad i = 1, \ldots, N$$ (3)

$$w_{k-1}^i = p_b((x_{k-1}^i))w_{k-1}^i, \quad i = 1, \ldots, N$$ (4)

where $N$ is the total number of box particles, $p_b$ is the probability of particle survival.

The prediction equation of the cardinalized distribution of the target number can be expressed as

$$p_{k|k-1}(n) = \sum_{n'=0}^{\min[n, n']} p_{k-1}(n')M(n, n')$$ (5)

$$M(n, n') = \sum_{i=0}^{\min[n, n']} p_{birth}(n-i)\binom{n'}{i}d^{n-i}(1-d)^i$$ (6)

where $p_{birth}(n)$ represents the probability of $n$ new target appearing through the target newborn model from $k-1$ moment to $k$ moment, $M$ is the Markov transition matrix.

(5) Update

$$\{w_{k|k-1}^i, [x_{k|k-1}^i]\}_{i=1}^N$$ is the box particle set of the assumed density of the cardinalized probability at time $k$. Particle weight, cardinalized distribution and likelihood function of group targets are calculated as follows

$$\tilde{w}_i^k = \left(1 - p_D\right)\frac{L([Z_k]|D)}{L([Z_k])] + p_D\frac{L([Z_k]|\bar{D})}{L([Z_k])]\right)w_{k-1}^i$$ (7)

$$p_k^i(n) = \frac{L([Z_k]|n)}{L([Z_k])} p_{k|k-1}(n)$$ (8)

$$L_k([Z_k]|[x_{k|k-1}^i]) = \frac{[h_{cp}((x_{k|k-1}^i), [z_j])]}{[x]}$$ (9)

where $D$ and $\bar{D}$ are the target detected and undetected shorthand symbols respectively, $h_{cp}$ are constraint propagation algorithm. The functions mentioned in the above equation are listed as follows

$$L([Z_k]|D) = \frac{1}{n_{k|k-1}}\sum_{j=0}^{n_{k|k-1}}\alpha_j \beta_j \sigma_j ([L_k])$$ (10)

$$L([Z_k]|\bar{D}) = \sum_{j=0}^{n_{k|k-1}}\beta_j \sigma_j ([L_k])$$ (11)

where $L([Z_k]|\alpha_j^k = s) = \frac{1}{n_{k|k-1}}\sum_{j=0}^{n_{k|k-1}}\beta_j \sigma_j ([L_k])$.

$$L([Z_k]|n) = \sum_{j=0}^{\min[n, n']}\beta_j \frac{n!(1-p_D)^{n-j}}{(n-j)!} \sigma_j ([L_k])$$ (12)

$$L([Z_k]|n) = \sum_{j=0}^{n_{k|k-1}}\alpha_j \beta_j \sigma_j ([L_k])$$ (13)
where \( \alpha^{(j)} = \sum_{i=0}^{n_j} \binom{n_j}{i} p_{k,j-1}(n)(1-p_D)^{n_j-i} \), \( \beta^{(j)} = p_c(m-j) \left( \frac{m-j}{m!} \right)^k \), \( \hat{p}_j \) represents the probability of the \( m \)th false alarm.

According to the measurement, the particle of the corresponding prediction box is shrunk, that is
\[
\hat{x}_k^{(j)} = [h_{cp}(z_k^{(j)}), [\hat{x}_k^{(j)}]] \tag{14}
\]
\( (6) \) Extraction of target state
Here we use Max A Posteriori (MAP) criteria to estimate the number of targets
\[
\hat{N}_k = \text{arg sup } p_k(n) \tag{15}
\]
where \( p_k(n) \) is the cardinalized distribution update of the target number, and \( \hat{N}_k \) is the number of estimated groups.

Click on the box particle to extract the target state
\[
\hat{x}_k = \sum_{i=1}^{N_k} \hat{w}_k \cdot \text{mid}([x_k]) \tag{16}
\]
where \( \text{mid}([\cdot]) \) is the center of the acquisition box. The obtained point particles are clustered, the number of clustering centers \( \hat{N}_k \) is the number of target estimation, and the state estimation of the target is the central state of clustering.

(7) Resampling
Set \( N_{k+1} \) as the number of resampled particles, calculated number of the expected target number \( n_k \) as the true measurement number. Dividing the box into multiple boxes to replace the traditional multiple copies, and finally normalize its weight
\[
\left\{ [x_k^{(j)}, n_k/N_{k+1}] \right\}_{j=1}^{N_{k+1}} \tag{17}
\]
In the resampling of group target tracking, since the size of the used box represents the extended range of the group, the velocity dimension of the target is chosen to be divided. Adding an increment \( \sigma \) to the initial velocity dimension, one can reduce the influence of prediction error on the extended range of the group, and the velocity interval can be expressed as \( [v+\sigma/2,v+\sigma/2] \).

3. Simulation experiment and analysis
Two group targets with different sizes are tracking in two-dimensional space under the strong clutter, and the proposed BPF-CPHD algorithm is compared with GIW-PHD in [6] and BPF-PHD in [7]. In the tracking region \([-200,600] \times [-1000,1000] \) , there are two different group targets with uniform linear motion, and the state of the group targets' center of mass is \( [x, \dot{x}, y, \dot{y}]^T \). In 0-40s, group 1 and group 2 move in a uniform and straight line, and they are cross in 11s. The initial state components of the group centroid are: \( \bar{m}_1 = [-190m, 19m/s, 220m, -22m/s]^T \), \( \bar{m}_2 = [-160m, 16m/s, -210m, 21m/s]^T \). The state equation and measurement equation of the target are as follow
\[
\begin{align*}
[x_k] &= F[x_{k-1}] + Gw \tag{18} \\
\tilde{z}_k &= Hx_k + e_k \tag{19}
\end{align*}
\]
where \( [x_k] = [x_k, y_k, [v_x^k], [v_y^k]]^T \), \( \tilde{z}_k = [x_k, y_k]^T \), \( w \) and \( e_k \) are process noise and measurement noise respectively. \( \hat{z}_{p_k} \) is the boxes of divided result formed by inclusion function, the measured numbers of group 1 and group 2 are Poisson distributions based on 5 and 10 respectively. The detection probability and survival probability of the group target are \( p_D = 0.99 \) and \( p_S = 0.95 \). The clutter
parameters are set as \( r = 25 \), and the number of box particles in the box particle PHD and box particle CPHD algorithm is 30, the number of newborn box particles is 10. The state transition matrix and the observation matrix of the target motion model are

\[
\begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
\]

and

\[
\begin{bmatrix}
\frac{T^2}{2} & T & 0 & 0 \\
0 & 0 & \frac{T^2}{2} & T \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix},
\]

respectively. The variance of measured noise \( R_k = \text{diag}[5m^2, 5m^2] \), the variance of state noise \( Q_k = \text{diag}[\sigma_x^2, \sigma_y^2] = \text{diag}[10m^2, 10m^2] \), the sampling period is \( T = 1s \).

The performance of the algorithm is evaluated by the mean value and the OSPA distance of the group targets number estimation.

The OSPA distance is

\[
\tilde{d}^{(c)}_{p}(X, Z) = \left( \frac{1}{n} \min_{\pi \in \Pi_n} \sum_{i=1}^{n} d^{(c)}(x_i, z_{\pi(i)})^p + c^p (n - m) \right)^{1/p}
\]

where \( p = 2 \), \( c = 100 \).

The PC platform used for simulation experiment in this paper is Intel (R) Core (TM) i3-2100 cpu@3.10ghz, with 2.00gb RAM. The experimental software is matlab (R2010b), and 100 Monte Carlo simulation experiments are run in the simulation.

Figure 2 is the measurement division of group targets tracking based on box.

As can be seen from Figure 2, in the tracking scenes of different size group targets, the extended state range of the group target is described by the box, and even in the environment of strong clutter, a better group measurement division can be obtained.

Figure 3 is a comparison diagram of the results of three algorithms for group targets tracking. As can be seen from Figure 3, since the non-cullable clutter measurement in the traditional measurement division, the GIW-PHD algorithm may overestimate or miss estimation due to the weight calculation problem. Because of the clutter participating in the weighted calculation of the target position, the position estimation of group targets is inaccurate. Whereas, for they are not necessary to strictly
distinguish the real measurement of the target from the clutter measurement in the likelihood calculation process, the BPF-CPHD and BPF-PHD algorithms propagate the extended size of the group targets, they can still obtain good tracking performance in the strong clutter scene.

![Figure 3. The comparison diagram of the results of three algorithms for group targets tracking](image)

Figure 3 and Figure 5 compare the group targets number estimation and group state OSPA distance of the three algorithms. It can be seen from figure 4 and figure 5 that BPF-CPHD has better performance than BPF-PHD and GIW-PHD algorithms in tracking multiple group targets. In GIW-PHD algorithm, clutter is also involved in the weighted calculation of target position, leading to overestimation of target number. But the proposed BPF-CPHD algorithm uses the ratio between the intersection area of the clutter measurement and the prediction box particle size and the size of the prediction box particle as the likelihood function. When the clutter measurement overlaps with the prediction box particle, the clutter measurement area only occupies a very small part of the predicted state box area of the group targets, and has little influence on the final targets state extraction. Therefore, the tracking accuracy of BPF-CPHD algorithm is higher.

![Figure 4. The estimation of group targets number of the three algorithms](image)
4. Conclusions
In order to improve the accuracy and efficiency of the multi-group target tracking algorithm in the strong clutter environment, a rectangular box is used to process the group targets’ extended state, and the BPF algorithm is combined with the cardinalized probability hypothesis density tracking algorithm. A CPHD tracking algorithm based on BPF is proposed in this paper. Because the proposed algorithm uses the ratio between the intersection area of the clutter measurement and the prediction box particle size and the size of the prediction box particle as the likelihood function, BPF-CPHD has higher filtering accuracy compared with BPF-PHD and GIW-PHD algorithms. Simulation results show that the BPF-CPHD algorithm can track multiple group targets stably and effectively.

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