Small-\( x \) dynamics in forward-central dijet decorrelations at the LHC

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Abstract

We provide a description, within the High Energy Factorization formalism, of central-forward dijet decorrelation data measured by the CMS experiment and the predictions for nuclear modification ratio \( R_{pA} \) in \( p+Pb \) collisions. In our study, we use the unintegrated gluon density derived from the BFKL and BK equations supplemented with subleading corrections and a hard scale dependence. The latter is introduced at the final step of the calculation by reweighting the Monte Carlo generated events using suitable Sudakov form factors, without changing the total cross section. We achieve a good description of data in the whole region of the azimuthal angle.

Introduction

In Quantum Chromodynamics at high energies, there is a continuous search for definite signatures of small-\( x \) dynamics. Examples of expected signatures are: high energy-enhanced rate of minijets between two hard jets that are far away in rapidity [1], suppression of rates of hadron production in “dilute-dense” scattering as compared to “dilute-dilute” scattering [2] or diffractive processes at high energies [3]. Indeed, there have already been studies reporting evidence of small-\( x \) effects for some observables in \( p+p \) and \( d+Au \) collisions [4, 5, 6].

One of the best observables to study the dynamics at small-\( x \) is the azimuthal decorrelation, i.e. the differential distribution in the difference of the azimuthal angle between two leading jets [7, 8, 9, 10, 11, 12, 13]. One of the reasons is that small-\( x \) effects are inseparably related to the notion of internal transverse momenta of (off-shell) gluons inside a hadron, which, according to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) formalism [14, 15, 16], can be large but cannot be arbitrarily small because of the importance of nonlinear effects absent in the BFKL equation [17, 18, 19]. The internal transverse momentum of a gluon can be viewed as a direct source of azimuthal decorrelations, since it creates a jet momentum imbalance on the transverse plane. On the other hand the decorrelations can be also made by parton showers by means of explicit additional emissions. For the observables considered here, general purpose Monte Carlo generators like herwig++ [20] and pythia [21] were the only alternatives to describe the data [22].
With the present experimental program of CMS and ATLAS, one can test these effects by investigating the pattern of radiation in a wide kinematical domain. Of particular interest are jet observables, since in the factorization picture of a collision, the jets momenta are kinematically linked to momenta of initial state partons. Therefore, scanning over a wide domain of jet $p_T$ allows one to test various physics assumptions on the properties of gluon distribution.

The effects that turn out to play a crucial role are the kinematical effects enforcing momenta of the gluons to be dominated by the transverse components \cite{23, 24} and the effects related to soft gluon radiation of Sudakov type \cite{25}. The need for such effects in gluon cascades has been already recognized in \cite{26, 27, 28} and has been formalized as the Catani-Ciafaloni-Fiorani-Marchesini (CCFM) evolution equation or its nonlinear extension \cite{29, 30}. The latter takes into account saturation effects. At present, the most commonly used framework providing a hard scale dependence (i.e. Sudakov effects) in the parton density function is the Kimber-Martin-Ryskin (KMR) evolution \cite{28, 31}. In its final form, it is based on small-$x$ dynamics, angular ordering, and it incorporates DGLAP effects. Recently, it has been shown in \cite{32}, for the case of color neutral particle production, that the double logarithms of the type $\ln^2(\mu^2/k_T^2)$, where $\mu$ is the hard scale of the process and $k_T$ is the transverse momentum of the initial state gluon, can be conveniently resummed in the dipole approach, on top of small-$x$ resummation.

Theoretical framework

In the present work, we study central-forward dijet decorrelations using the High Energy Factorization (HEF) approach \cite{2, 33, 34, 35}. In this framework, similarly to the standard collinear factorization, the cross section is calculated as a convolution of a ‘hard sub-process’ \cite{36, 37} and nonperturbative parton densities, which take into account longitudinal and transverse degrees of freedom. In practice, at low $x$, the gluons dominate over the quarks and therefore one usually deals with the unintegrated gluon densities (UGDs) only. We shall describe different UGDs later in this section.

High Energy Factorization is obviously applicable only in a certain domain and, in particular, one should be cautious about the following points: (i) although we use tree-level off-shell matrix elements and thus we do not have explicit problems with factorization breaking by soft emissions, we have to accept that the UGDs might not be universal, (ii) below the saturation scale, we should use more than just one UGD, but for the proton-proton collision studied here, the nonlinear effects are rather weak. For more details concerning various factorization issues we refer to \cite{42, 43, 42, 44, 45, 46}.

In situations where the final state populates forward rapidity regions, one of the longitudinal fractions of the hadron momenta is much smaller then the other, $x_A \ll x_B$, and the following ‘hybrid’ HEF formula is used \cite{9}

\[
d\sigma_{AB \to X} = \int \frac{d^2k_{TA}}{\pi} \int \frac{dx_A}{x_A} \int dx_B \sum_b F_{g^*/A} (x_A, k_{TA}, \mu) f_{b/B} (x_B, \mu) d\hat{\sigma}_{g^*b \to X} (x_A, x_B, k_{TA}, \mu), \tag{1}
\]

where $F_{g^*/A}$ is a UGD, $f_{b/B}$ are the collinear PDFs, and $b$ runs over the gluon and all the quarks that can contribute to the production of a multi-particle state $X$. Note that both $f_{b/B}$ and $F_{g^*/A}$ depend on the hard scale $\mu$. As we explain below, it is important to incorporate the hard scale dependence also in UGD. The off-shell gauge-invariant matrix elements for multiple final states reside in $d\hat{\sigma}_{g^*b \to X}$. The condition $x_B \gg x_A$ is imposed by proper cuts on the phase

\footnote{For recent developments in hard matrix elements within HEF we refer the Reader to \cite{38, 39, 40, 41}.}
space of $X$. It was shown in [47] that the phase space cuts for central-forward jets do imply the aforementioned asymmetry condition.

In our computations, we used several different unintegrated gluon densities $F_{g^*/A}(x,k_T,\mu)$:

- The nonlinear KS (Kutak-Sapeta) unintegrated gluon density [11], which comes from the extension of the BK (Balitsky-Kovchegov) equation [48] following the prescription of KMS (Kwiecinski-Martin-Stasto) [24] to include kinematic constraint on the gluons in the chain, non-singular pieces of the splitting functions as well as contributions from sea quarks. The parameters of the gluon were set by the fit to $F_2$ data from HERA. This gluon can be determined for an arbitrary nucleus and, in the following, we shall use the KS densities for the proton and for lead.

- The linear KS gluon [11], determined from the linearized version of the equation described above.

- The KMR hard scale dependent unintegrated gluon density [28, 31]. It is obtained from the standard, collinear PDFs supplemented by the Sudakov form factor and small-$x$ resummation of the BFKL type. The Sudakov form factor ensures no emissions between the scale of the gluon transverse momentum, $k_T$, and the scale of the hard process, $\mu$. The upper cutoff in the Sudakov form factor is chosen such that it imposes angular ordering in the last step of the evolution. The KMR gluon used in our study is based on MSTW 2008 LO [49].

- The standard collinear distribution $F_{g^*}(x,k_T,\mu^2) = xg(x,\mu^2_2)\delta(k_T^2)$, which, when used in Eq. (1), reduces it to the collinear factorization formula. In this study we used the CTEQ10 NLO PDF set [50].

In addition, we supplement the KS linear and nonlinear UGDs with the Sudakov resummation, which, as we shall see, turns out to be a necessary ingredient needed to describe the data at moderate $\Delta \phi$. The resummation is made on top of the Monte Carlo generated events and it is motivated by the KMR prescription of the Sudakov form factor. It effectively incorporates the dependence on a hard scale $\mu$ into the KS gluons, which by themselves do not exhibit such dependence. A short description of the resummation model is presented in the appendix.

Results

In this section, we present the results of our study of the azimuthal decorrelations in the forward-central dijet production. Our framework enables us to describe two scenarios considered in the CMS forward-central dijet measurement [22]:

- **Inclusive scenario**, which, in the experiment, corresponds to selecting events with the two leading jets satisfying the cuts: $p_T,_{1,2} > 35$ GeV, $|y_1| < 2.8, 3.2 < |y_2| < 4.9$ and with no extra requirement on further jets. These results are shown in Fig. 1.

- **Inside-jet-tag scenario**, with the same selection on the two hardest jets but, this time, a third jet with $p_T > 20$ GeV is required between the forward and the central region. The corresponding results are shown in Fig. 2.

In Fig. 1 we present our results for the case of the inclusive selection and compare them with the data from CMS. We show the results obtained with the nonlinear and linear KS gluon, supplemented with the Sudakov form factor (top left and top right, respectively), the KMR gluon (bottom left) and an unmodified KS gluon (bottom right). We see that the KS+Sudakov and KMR describe the data well. The error bands on the predictions were obtained by varying the hard scale appearing explicitly in the running coupling, UGDs, and the collinear PDFs by a factor $2^{\pm 1}$. The calculations were performed independently by three programs: LxJet [51], forward [52], and a program implementing the method of [38].
The results presented in Fig. 1 provide evidence in favour of the small-x (or BFKL-like) dynamics. This dynamics produces gluon emissions, unordered in $k_T$, which build up the non-vanishing $\Delta \phi$ distributions away from $\Delta \phi = \pi$. (A pure DGLAP based approach, without the use of a parton shower, could of course only produce a delta function at $\Delta \phi = \pi$.) Furthermore, combining the above result with the recent analysis performed in [12], we conclude that the effects of higher orders, like kinematical effects that allow for emissions at low $\Delta \phi$ are of crucial importance. This alone is however not enough, since, as shown in Fig. 1, one necessarily needs the Sudakov resummation to improve the moderate $\Delta \phi$ (or equivalently moderate $k_T \sim 50$ GeV) region. These Sudakov effects are needed to resum virtual emissions between the hard scale provided by the external probe and the scale of the emission from the gluonic ladder. In other words, one has to assure that the external scale is the largest scale in the scattering event. Finally, Fig. 1 indicates no need for saturation to describe the azimuthal decorrelations in proton-proton collisions with these particular experimental selection cuts, although we see that it decreases the last bin at $\Delta \phi \sim \pi$ towards the data point.

Fig. 2 shows the results obtained in the HEF approach with the $2 \rightarrow 3$ hard matrix elements. We also show the corresponding result from pure DGLAP, i.e. with the HEF formula (1) used in
Figure 2: Comparison of CMS data for the dijet inside-jet-tag scenario with model predictions. 

The collinear limit. We see that the linear and nonlinear KS results without (top left) and with (top right) the Sudakov form factor nicely follow the experimental data from the inside-jet-tag scenario. The description is also very good when the KMR gluon is used (bottom left). In the case of pure DGLAP calculation (bottom right), the 3rd parton produced in the final state allows for generation of the necessary transverse momentum imbalance between the two leading jets. This, in turn, leads to a good description of the experimental data, even without the use of a parton shower.

In Fig. 3, we compare the results obtained using the KS gluon, with and without the Sudakov form factor, and the KMR gluon. We see that when the Sudakov effects are included, the curves are almost on top of each other. This confirms the necessity of incorporating the hard scale dependence to the unintegrated gluon densities. Further studies with in CCFM-based approaches would be needed in order to get better understanding of these effects.

Finally, having described decorrelations data well, we are now ready for providing predictions for the nuclear modification ratios, $R_{pA}$, in p+Pb collisions. They are defined as the ratios of p+Pb to p+p cross sections normalized to the number of nucleons. In Fig. 4, we see that the suppression is more pronounced in the inclusive scenario and further enhanced by the Sudakov
Figure 3: Illustration of the effect of the Sudakov resummation model on the KS nonlinear gluon density for inclusive dijet (left) and inside-jet-tag scenario (right). For comparison we plot also the KMR result.

Figure 4: Predictions for nuclear modification ratios for p+Pb collisions.
effects. In particular we see that inclusion of the Sudakov form factor changes the slope of the $R_{pA}$ ratio making the saturation effects visible in the wider range of the $\Delta \phi$.

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**Sudakov resummation model in Monte Carlo calculation**

The model is designed to supplement Monte Carlo generated events with Sudakov effects. It effectively incorporates an additional hard scale $\mu$ into the UGD and relies on two assumptions: (i) unitarity, i.e. the total cross section will not be affected, (ii) events with internal $k_T > \mu$ will be affected as little as possible.

Suppose we have a set of Monte Carlo generated events $(w_i, X_i)$, where $w_i$ is a weight and $X_i$ is a phase space point. An observable is calculated according to

$$O = \frac{\sigma}{W} \sum_i w_i F_i^O (X_i),$$

where $\sigma$ is the total cross section, $F_i^O$ is a function defining the observable $O$ (e.g. some combination of step functions) and $W = \sum_i w_i$ is the total weight. Let us rewrite the observable $O$ as follows

$$O = \frac{\sigma}{W} \left[ \sum_i w_i F_i^O (X_i) \Theta (\mu_i > k_{Ti}) + \sum_j w_j F_j^O (X_j) \Theta (k_{Tj} > \mu_j) \right].$$

For each of the events from the first sum we incorporate the effect of resummed unresolved emissions by including the Sudakov form factor $\Delta (\mu_i, k_{Ti})$ which gives the probability that a parton with transverse momentum $k_T$ will remain untouched while refining the scale up to $\mu$. This is done by redefining the observable as follows

$$\tilde{O} = \frac{\sigma}{\tilde{W}} \left[ \sum_i w_i \Delta (\mu_i, k_{Ti}) F_i^O (X_i) \Theta (\mu_i > k_{Ti}) + \sum_j w_j F_j^O (X_j) \Theta (k_{Tj} > \mu_j) \right]$$

with

$$\tilde{W} = \sum_i w_i \Delta (\mu_i, k_{Ti}) \Theta (\mu_i > k_{Ti}) + \sum_j w_j \Theta (k_{Tj} > \mu_j).$$

This changed the observable $O \to \tilde{O}$. Note, we do not change the total cross section, but we affect the events with $k_T > \mu$ contradicting our second assumption. This can be corrected by simply re-weighting them as follows

$$\overline{O} = \frac{\sigma}{\overline{W}} \left[ \sum_i w_i \Delta (\mu_i, k_{Ti}) F_i^O (X_i) \Theta (\mu_i > k_{Ti}) + \frac{\tilde{W}}{\overline{W}} \sum_j w_j F_j^O (X_j) \Theta (k_{Tj} > \mu_j) \right].$$
with $\bar{W}$ being the total weight as usual

$$\bar{W} = \sum_i w_i \Delta (\mu_i, k_{T_i}) \Theta (\mu_i > k_{T_i}) + \frac{\bar{W}}{W} \sum_j w_j \Theta (k_{T_j} > \mu_j). \tag{7}$$

As long as $\bar{W}/\bar{W} \sim 1$ our second assumption is satisfied. In case of the High Energy Factorization and hard scale taken to be e.g. average $p_T$ of jets this is satisfied easily. The events from the first sum will gain effectively the normalization $W/\bar{W} > 1$ relatively to the original observable $O$ and account for real emissions (a model of).

The Sudakov form factor appropriate for the above model can be constructed along the lines given in [31]

$$\Delta (\mu, k_T^2) = \exp \left( -\int_{k_T^2}^{\mu^2} \frac{dk_T'^2}{k_T^2} \frac{\alpha_s(k_T'^2)}{2\pi} \sum_i \int_0^{1-\epsilon(k_T', \mu)} dz P_{ij} (z) \right), \tag{8}$$

where $P_{ij}(z)$ are splitting functions, and $\epsilon (k_T, \mu) = \frac{k_T}{\mu + k_T}$.

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