Elementary explanation of the inexistence of decoherence at zero temperature for systems with purely elastic scattering

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This note has no new results and is therefore not intended to be submitted to a "research" journal in the foreseeable future, but to be available to the numerous individuals who are interested in this issue. Several of those have approached the author for his opinion, which is summarized here in a hopefully pedagogical manner, for convenience. It is demonstrated, using essentially only energy conservation and elementary quantum mechanics, that true decoherence by a normal environment approaching the zero-temperature limit is impossible for a test particle which can not give or lose energy. Prime examples are: Bragg scattering, the Mössbauer effect and related phenomena at zero temperature, as well as quantum corrections for the transport of conduction electrons in solids. The last example is valid within the scattering formulation for the transport. Similar statements apply also to interference properties in equilibrium.

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I. INTRODUCTION

What diminishes the interference of, say, two waves (see Eq. 3 below) is an interesting fundamental question, some aspects of which are, surprisingly, still being debated. This process is called "dephasing" or "decoherence" 1. Decoherence occurs due to an interaction of the interfering entity (henceforth referred to as "the particle") with the degrees of freedom (dubbed "the environment", examples for which are lattice vibrations, E-M fields, interactions with other particles, etc.) which are not measured in the interference experiment. More specifically, this interaction will eliminate the interference if and only if the two (in the simplest case) partial waves leave the environment in orthogonal states. The diminishing of the interference can then be described as due to either tracing over the environment’s states or to the random fields exerted by the fluctuating environment on the particle. These two descriptions are exactly equivalent 2, 3. After almost twenty years of research on Mesoscopic Physics 4, it is now fully and universally accepted that purely elastic scattering, for example by static defects in solids, can modify the interference terms but does not cause decoherence. An extreme case occurs if one of the two interference paths is blocked. The interference obviously vanishes (as will a half of the classical contribution). But this is not decoherence. Likewise, one can diminish the interference by various averaging processes (For example, via a broad enough distribution of incident particle energies or wavelengths) but, again, this is not proper decoherence.

The issue of whether the zero point modes of an environment can dephase 4 a test particle which can not lose energy 5 is still current in modern literature. An answer to the affirmative has been suggested theoretically in the case of the coupling of a conduction-electron in a solid to lattice vibrations (phonons) 14 years ago 6 and immediately refuted vigorously 7. Interest in this problem has resurfaced due to experiments by Mohanty et al 8 which determine the dephasing rate of conduction electrons by weak-localization magnetoconductance 8. According to these rather careful experiments, the dephasing rate of a conduction electron does not vanish when the temperature $T \to 0$, but rather goes to a finite limit which is then interpreted as due to the coupling with the zero-point electromagnetic fluctuations in the solid. For conductors, these fluctuations are mainly due to the fields produced by the other conduction electrons, hence this decoherence can also be described as due to electron-electron interactions. This dephasing rate has been calculated before 4 and found to vanish at the zero temperature limit. An apparent contradiction between theory and experiment results.

This contradiction is not just with a specific model calculation. As mentioned above, decoherence is very generally understood as what may be called a "which path" detection by the "environment", effected by the exchange of an excitation (i.e. an inelastic process) 1 between the particle and the environment. Such excitation exchange processes can not occur when both the environment and the test particle are at zero temperature and in the linear transport regime (where the conduction electron has an excess energy which tends to zero) 10. No excitation can be exchanged if neither the particle nor the environment can lose energy. It is assumed that, as is usually the case, the environment does not have a large degeneracy of the ground state. The above argument that no excitation can be exchanged between the test-particle and the environment can actually be formulated in terms of an integral over products of certain correlation functions,
The only exceptions being an environment having a pathologically increasing density of states at low energies and the above-mentioned case where the environment has a large ground state degeneracy. A viable model for the latter are uncompensated and isolated magnetic impurities at zero magnetic field.

However, the experimenters have taken great pains to eliminate the effect of magnetic impurities and have presented serious arguments as to why that was not an important effect in their samples. An assumed high enough density of other low-energy modes, such as two-level systems (TLS) was also shown to be able to account, in principle, for the observed anomalies. Such low-energy modes might depend on the metallurgy of a given sample. Other experiments have reported a vanishing dephasing rate as $T \to 0$ in a different material. Later experiments have in fact demonstrated that the $T \to 0$ anomaly seems to depend on sample preparation. Thus some lattice defects such as the TLS might be relevant. Uncompensated magnetic impurities were also implicated in Ref. but that possibility was argued to have been negated for the experiments of Ref. Here we certainly do not attempt to pass judgement on the experimental controversy. Furthermore, as indicated by some of the early experiments one must use extremely small measurement currents to be in the linear transport regime. Ovadyahu studied this issue very carefully. His findings can be summarized as showing that it may be possible to eliminate the anomalous decoherence by using extra-small probing currents. However, there is a large range of currents that do not heat the electrons, as found in Ref., but appear nevertheless not to be small enough for the transport to be in the truly linear regime. Finding the mechanism leading to this, which probably also involves some special low-frequency modes, still presents an important unsolved problem in low-temperature metal Physics. Therefore, we again refrain from offering a verdict on what the final answer provided by all the experiments will be and we trust that that will be cleared up soon. From now on we confine ourselves to the, still hotly debated, theoretical question only, which ought to be decidable by applying known principles correctly.

That question has become controversial as well, due to reports of calculations which claimed to have produced a finite dephasing rate as $T \to 0$, in contradiction with refs. and with the general arguments summarized above. Refs. and have disagreed strongly with these calculations. To which criticism the authors have rebutted. The theoretical controversy is still going on (e.g., and). Unlike the experimental situation, experience has shown that questions such as "who has calculated the correct diagrams correctly" tend to linger and produce unnecessary controversy. This is compounded by further reports of the weakening of quantum interference, even truly at equilibrium, at $T \to 0$, by a coupling to the environment. It has been claimed that "Vacuum fluctuations are a source of irreversibility and can decohere an otherwise coherent process". That statement is of course trivially valid (if certain matrix-elements do not vanish) for a system that can lose energy to the vacuum fluctuations. However, that does not include an electron just on the Fermi-level. The persistent current in a mesoscopic ring was found to be weakened at $T \to 0$ by coupling to harmonic oscillators in their ground state. Models invoking such oscillators with a specific coupling and density of states (DOS), chosen to mimic the dissipation can be used to describe a viscous resistance to the electron’s motion (although, at best, important modifications, such as a spatial distribution of these oscillators might be necessary).

Thus, the theoretical situation needs clarification, independently of the final experimental verdict. We shall attempt in this very informal note to clarify the possible misunderstandings that can lead to the belief in the $T \to 0$ decoherence. We shall show that some of the confusion may be due to interpreting a reduction of interference as due to dephasing, when it really is something else. To explain that, we shall first treat in section II an elementary model, really a rehash of the arguments of Ref., which illustrates the whole issue very simply.

## II. A SIMPLE MODEL: TWO-WAVE INTERFERENCE WITH VIBRATING SCATTERERS

Let us first imagine two elastic, rigid, point scatterers placed at points $\vec{x}_1$ and $\vec{x}_2$, separated from each other by a vector $\vec{d}$ along the x axis. A particle wave with wave vector $\vec{k}_0$ impinges upon these scatterers. We look at the (elastic) scattering into the state with a wavevector $\vec{k}$. We denote the momentum-transfer vector by $\vec{K} = \vec{k}_0 - \vec{k}$. If the scattering amplitude from each of the scatterers is $A_K$, the (lowest order) scattering probability from the system will be proportional to

$$S_K = 2|A_K|^2 + 2\text{Re}|A_K|^2 \exp(i\vec{K} \cdot \vec{d}) = S_K^{\text{cl}} + S_K^{\text{nu}},$$

(1)

where the first and second terms are, in obvious notation, the classical and the interference contributions, bearing a strong similarity to the case of diffraction by a double slit. $\vec{K} \cdot \vec{d}$ is the phase shift due to the difference in the optical paths between the waves scattered by the two scatterers. Somewhat similarly to the Bohr discussion of diffraction from two fluctuating slits, we now let each scatterer be bound by a parabolic potential, so that the frequency of the motion in the x-direction is $\omega_0$. The case of interest to us here is when the scatterers are at zero temperature, but the generalization is obvious. We write $\vec{x}_i = X_i + \vec{u}_i,$
\( i = 1, 2 \). Each \( \hat{x}_i \) performs zero-point fluctuations (whose coordinate operator is \( \hat{u}_i \)) around its equilibrium position \( X_i = \langle x_i \rangle \). To obtain the new scattering pattern we must average the scattered intensity over the wavefunction of the scatterers. That wavefunction is a product of two gaussians, of \( u_1 \) and \( u_2 \). The standard deviation of each is \( \sigma \), given by \( \sigma^2 = \frac{h^2}{2Mw} \), where \( M \) is the mass of the scatterer. Using this fact that for a gaussian distribution of \( v \) with an average \( \langle v \rangle \) and a variance \( \langle \Delta v^2 \rangle \),

\[
\langle \exp(iv) \rangle = \exp(i\langle v \rangle - \langle \Delta v^2 \rangle / 2)
\]  
(a similar result is valid also in the quantum case \[30, 31\]), we obtain

\[
S_K = \langle S_{K}^{cl} \rangle + 2\text{Re}|A_K|^2 \exp(i\vec{K} \cdot \vec{d}) \exp(-2W),
\]
where \( 2W \) is the well-known Debye-Waller factor

\[
2W = K^2 \sigma^2,
\]

obtained by writing \( \langle (u_1 - u_2)^2 \rangle = 2\sigma^2 \), since the \( u \)'s are independent random variables with zero mean value each.

Thus, the quantum interference term is indeed reduced by the zero-point motion. One may (as several researchers have done) now jump to the wrong conclusion that the zero-point motion indeed reduces the quantum interference term without apparently affecting the classical terms. In other words, one would write:

\[
\langle S_{K}^{cl} \rangle = 2|A_K|^2.
\]

This would imply decoherence by the zero-point motion! The \( \text{(superficially valid only)} \) reason for the above mistaken conclusion is that in the classical terms one has \( \langle e^{iK \cdot u_e^{-iK \cdot w}} \rangle = 1. \)

The fallacy in the above naive thinking is in the application of the last seemingly innocuous equality, Eq. \[3\]. Because it automatically includes all the inelastic scattering \[32\], which should not appear. This fact has been known for at least 45 years \[29, 33\] and has been reinforced when the M"ossbauer effect was discovered and understood (see, for example, \[30, 31\]). Next, we will explain this issue.

Let us review the derivation of the inelastic nature \[29\] of part of the above expressions. The exact elastic scattering amplitude \( A_K \) of the isolated static scatterer is first parameterized in terms of a suitably defined pseudopotential \( V(r) \), whose Fourier transform is \( V_K \). As if anticipating the current sophisticated red-herring type discussions of the perturbative vs the "nonperturbative" nature of the theory, Fermi in 1936 \[55\] defined a pseudopotential \( V(r) \), so that the lowest-order Born approximation scattering amplitude due to it (which is proportional to \( V_K \)) equals the actual, correct, \( A_K \). Therefore, using the Born approximation with the pseudopotential, already implies taking into account all the multiple scattering by the same \[55\] scatterer. Within this picture, van Hove considered in 1954 the (in general) inelastic Born approximation scattering, which for our system is proportional to

\[
S(K, \omega) = |A_K|^2 \sum_f |\langle f \rangle| \exp(iK \hat{x}_1) + \exp(iK \hat{x}_2) ||g||^2 \delta(h\omega + E_f - E_g)
\]

where \( h\omega \) is the energy, \( (\hbar^2/2m)(k_0^2 - k^2) \), lost by the scattered particle and \( E_f \) and \( E_g \) are the energies of the final state \( |\langle f \rangle| \) and initial one \( |\langle g \rangle| \) of the oscillators. The \( \delta \) function simply gives total energy conservation in the process. Since the scatterers are at \( T \to 0 \), the only possible initial state is the ground state, \( |g\rangle \), of the two-oscillator system.

A slightly more advanced treatment of Eq. \[3\] will be given in the next section. Right now, we simply enforce the condition of no energy exchange in the scattering. We remind the reader that for electrons in metals, this most important condition follows from the fact that neither the oscillators (which are at their ground states) nor the conduction electron at the Fermi level have any states to go down to. This is valid for an arbitrary, interacting, system because there are no states below the ground state. In the noninteracting fermion picture, this would follow formally, from the Fermi-Pauli factors \( \langle n_f(1 - n_f) \rangle \) that should then be included in Eq. \[3\]. This enforces the choice of only the term with \( f = g \) in the sum. As a result, the \( \delta(\omega) \) part of \( S(K, \omega) \), which is the elastic scattering cross section and the only surviving part of \( S(K, \omega) \), is given here by

\[
S_K \equiv \int d\omega S^{cl}(K, \omega) = |A_K|^2 \langle g \rangle| \exp(iK \hat{x}_1) + \exp(iK \hat{x}_2) ||g||^2
\]

We again have a classical term which is a sum of a term due to \( x_1 \) and one due to \( x_2 \), and the interference term which is the ground-state average of \( 2\text{Re}|A_K|^2 \exp(iK \hat{x}_1) \exp(-iK \hat{x}_2) \), yielding the Debye-Waller factor as before. The whole point is now, that each classical term is given by \( |A_K| \langle g \rangle \exp(iK \hat{x}_1) ||g||^2 \), \( i = 1, 2 \). Therefore, the classical terms too are reduced by the same Debye-Waller factors as the interference term. Thus, again, it is not that the classical terms stay unchanged and the interference is decreased, as would be the case for a true decoherence, but both the non-interfering classical part and the interference, "quantum
correction” part are renormalized in the same fashion. Therefore, the reduction of the interference part by the coupling to the environment has nothing to do with decoherence. Looking at the average of the classical terms, it is instructive to point out that the reduction of the scattering in a distributed scatterer is due to the addition of the scattering amplitudes from the various positions occupied by that scatterer. These amplitudes add coherently because the scattering is elastic. This amplitude addition is familiar from diffraction theory in optics or from antenna theory.

### III. A MORE ADVANCED TREATMENT

For a fuller treatment, van Hove [29] started by writing Eq. 3 for N scatterers. The term in the square brackets is just \( \sum_{i=1,N} \exp(i \vec{K} \cdot \vec{x}_i) \), which is just the Fourier transform, \( n_K \), of the particle density operator \( \hat{n}(\vec{x}) \equiv \sum_{i=1,N} \delta(\vec{x} - \vec{x}_i) \). He then used a series of simple but ingenious manipulations (Fourier-representing the energy \( \delta \) function, inserting a complete set of states and using the Heisenberg representation for the \( n_K \) to write \( S(K, \omega) \) as the Fourier transform of the temporal correlation function, \( \langle n_{-K}(0)n_K(t) \rangle \). For phonon systems, where \( \hat{u}_i \) (the fluctuation of \( \vec{x}_i \) from its average position in the lattice) is a linear superposition of harmonic oscillators, the correlators relevant for inelastic neutron scattering and treated extensively in that literature in the late 50’s (of the last century) [30], are therefore

\[
\exp(-J_{ij}(t)) = \langle \exp(i \vec{K} \cdot \vec{x}_i(t)) \exp(-i \vec{K} \cdot \vec{x}_j(0)) \rangle \tag{8}
\]

The elastic part of the scattering is given by the \( t \to \infty \) limit (or time-independent) part of this, since the Fourier transform of a constant is a \( \delta \) function. In the modern so-called \( P(E) \) [31] theory of phase fluctuations, similar calculations have been done and similar functions appear with harmonic oscillators taken to represent the lossy environment. Looking at the interference between two paths, one again sees immediately that the two-path interference term is affected, but that at the same time the single-path terms are affected in the same fashion. This completes our demonstration that zero-point fluctuations do not decohere. Some details will still be presented below.

It is perhaps useful to point out that the mistake in neglecting the Debye-Waller factors in the first terms of Eqs. 1 and 3 occurs because there one looks at the equal-time correlators. These are related to the integrals of \( S(K, \omega) \) over all frequencies [32]. Now, the inelastic parts of \( S(K, \omega) \) have to be discarded, as discussed above. The \( t = 0 \) correlator (leading to Eq. 3), obtained by neglecting this important constraint, is very different from the relevant \( t \to \infty \) limit of the correlator (leading to Eq. 7), which is related to the elastic scattering. We are now ready to discuss the question of what happens when the \( t \to \infty \) limit of the above correlators vanishes. The answer depends on the physical situation considered. In the scattering problem, the vanishing of the Debye-Waller factor means that there is no strict Bragg-type scattering or Mössbauer effect (but interesting things can still happen, see for example Refs. [33, 34]). If the inelastic scattering is blocked, no scattering remains and the particle can only go forward "unscathed" through the system (remaining unscattered). However, other situations can exist. One can have a vibrating double-slit system that will stop transmitting altogether. An electron in a solid with phonons can also localize. A sizable increase of the electron’s effective mass was found by Holstein [10] in the small-polaron problem (which is also related to the issues discussed here). Later, Schmid [41] found that for the phonon problem similar to the one discussed here, with a suitable "ohmic" spectrum, this mass renormalization is so severe that the mass diverges for a strong enough coupling, and the particle becomes localized. This effect appeared later, in the context of what has been called "Macroscopic Quantum Coherence" [42], for mesoscopic SQUIDS. It should perhaps be reemphasized that this mass renormalization and possible ensuing localization are due to an elastic "orthogonalization catastrophe" and not to decoherence [43]. We shall discuss this further in the next section.

It has become customary to describe the finite resistance of an electron in a real metal as due to coupling with the Nyquist–Feynmann–Vernon–Zwanzig–Caldeira-Leggett oscillators [14]. We put aside, for the time being, the serious question of how well can a resistance given physically when \( T \to 0 \) by (mainly) elastic impurity scattering, be described by inelastic (and therefore phase-breaking) processes. Within this model the phase decay for an electron coupled to a dissipative bath is described by the temporal correlator \( \langle \exp(i \phi(t)) \exp(-i \phi(0)) \rangle \). It is governed [15] when \( T \to 0 \) by

\[
\exp[-\text{const} \int_0^\infty d\omega \eta(\omega) \left( \frac{1 - \cos \omega t}{\omega} \right)], \tag{9}
\]

where \( \eta(\omega) \) is the dissipation constant, assumed to have a finite \( \omega \to 0 \) limit. We emphasize that this is the naive result, obtained by neglecting the fact that inelastic processes are absent. To set up an analogy with a real lattice dynamics problem we note that the "self" correlator for the single-particle density \( \langle \exp(i \vec{K} \cdot \vec{x}_i(t)) \exp(-i \vec{K} \cdot \vec{x}_i(0)) \rangle = \langle -J_{ii}(t) \rangle \) at \( T = 0 \) for a 1D Debye lattice is also (see e.g. Ref. [33]) decaying (see [43]) via exp(−const \( \int_0^\infty d\omega \frac{1 - \cos \omega t}{\omega} \)). Thus, these two problems are mathematically similar. Both have, when \( T \to 0 \), the same logarithmic long-time behavior of the exponent, which becomes a power-law decay of the correlator. However, in the latter case,
these zero point-effects are very physical – for example, an atom emitting radiation can produce phonons (in the non-Mössbauer channels) in its recoil. However, as we explained, the electron on the Fermi level can not decay and produce an excitation. Its inelastic channels are blocked. The amplitude for the particle to go between two points is the sum of the amplitudes of a huge number, $N$, of paths. The probability has two types of terms: $N$ classical ones (the absolute value squared of the amplitude of a single path) and $N^2/2$ interference terms (twice the real part of the product of a path amplitude with the complex conjugate of the amplitude of a different path). Once the constraint of no energy transfer is imposed, there is again no major difference between the (Debye-Waller type) reduction factors for the classical and the interference terms. Hence there is no intrinsic decoherence in the Physics of a particle coupled to a normal bath at $T \to 0$. Unusual low-energy properties of the bath might of course still give anomalous decoherence at low temperatures. Such anomalies do not exist in the theoretical models on which the present controversy ranges [18, 19, 21, 22, 23].

IV. QUANTUM EFFECTS IN CLOSED RINGS, "ANTIDECOHERENCE"???

Besides real decoherence, there are other ways to reduce quantum interference effects. Averaging over the energy of the incoming particle is often confused with decoherence, but it should not [6]. Using a variable energy filter one may recover the many interference patterns belonging to different energies in the beam. This is impossible for decoherence via "which path" identification. A rather aggressive way to kill quantum interference is, for example, to simply block (at least) one of the two interfering paths. Alternatively, if the Fermi-surface electron interacts with the $T \to 0$ bath only when it goes via one of the interfering paths, the amplitude of that path will be reduced by a Debye-Waller type factor similar to Eq. (9) (see [17]),

$$\exp[-\text{const} \int_0^\infty d\omega \lambda(\omega)(\frac{1 - \cos \omega t}{\omega})], \quad (10)$$

where $\lambda(\omega)$ contains the coupling constants and the DOS of the oscillators at frequency $\omega$. The limit of effectively "cutting" the path is achieved when this factor becomes very small. A similar reduction will be achieved if only one of the scatterers in the simple model of section II, is allowed to fluctuate. If the two paths are cut or severely weakened, a decrease of both the interference and the classical terms is obtained. It is hoped that the reader already understands very well that this reduction is distinct from decoherence.

One may now consider a case where, in a sense, there are no classical terms. The sensitivity of the energy levels of an electron on an Aharonov-Bohm (AB) ring to the AB flux is a good example. There is no such sensitivity in the classical case. This sensitivity (which determines for example the persistent current through the ring, see, for example, [8]) follows from the phase shift experienced by the electron going around the ring enclosing the flux. This AB phase shift along such paths determines the flux sensitivity of, say, the energy levels and thence the persistent current. Obviously, if decoherence is strong enough, in the sense that the ring’s circumference $L$ is much larger than the electron’s coherence length $L_\Phi$, these purely quantum effects are exponentially reduced.

In the Holstein polaron model [10], the effective mass of the electron increases by the coupling with the zero-point environment. Imagine putting such an electron on an AB ring. The flux-sensitivity of the energy levels is proportional to the inverse mass. It will be reduced by the increase of the latter. In certain cases [11, 22] the effective mass diverges (see the previous section) and the flux sensitivity is altogether eliminated. However, describing this electron with the measured, renormalized, effective mass, will eliminate any need to even think about decoherence.

Likewise, the coupling of a Fermi-surface electron to a $T \to 0$ oscillator bath will reduce the amplitude of its flux-encircling paths by the Debye-Waller type factor, Eq. (10). Other paths, not encircling the ring, will experience the corresponding reductions as well, which will just renormalize the physics of the ring. The magnitude of the AB oscillations will be suppressed by such factors for the relevant paths. This is the effect found in Ref. [24]. Quantum interference is indeed reduced, but this is not decoherence.

We now prove the incorrectness of identifying the Debye-Waller reduction of interference as decoherence by reductio ad absurdum. It is easy to devise couplings to an external bath which enhance the effects of quantum interference. In a diffusive real mesoscopic ring, the persistent current is well known [11] to increase with the elastic mean free path $\ell$. The physics of the conduction electrons is governed by two Debye-Waller renormalizations, that of the scattering by the impurities [11], due to lattice vibrations and that of the electrons [11] due basically to the electron-electron interactions. Coming from different interactions, these two renormalizations are not strongly dependent (see below) on each other. Suppose, for simplicity, that all relevant atomic masses are light enough so that the former is more important for the renormalized conductivity than the latter. As found in section II, the elastic impurity scattering rate is proportional to the Debye-Waller factor (for further discussions of this, including the more complicated case of finite temperatures, see for example Refs. [28, 24]). Thus, the magnitude of the quantum interference for $T \to 0$ will increase with
the decreasing vibrational Debye-Waller factor of the impurities. This increase at $T = 0$ is real, can be huge and may be observable, in principle. Since the effects of the electron-electron interactions decrease with the elastic scattering strength, the increasing electron mean free path will actually decrease the effect of the electronic Debye-Waller factor and help the vibrational one dominate. The analogy with the identification of the decrease of quantum effects via the Debye-Waller factor as "decoherence" would now suggest to define this increase as "antidecoherence" by a suitable coupling to zero-point modes of a bath. It is hoped that by now the reader does not have to resist the urge to rush to the word processor in order to produce an announcement of this increase of quantum interference by the coupling to the bath, as a revolutionary new physical phenomenon, with a range of important applications.

V. CONCLUSIONS, REAL METALS

Obviously, the residual ($T \to 0$) resistivity of the metal is determined by the elastic scattering off the defects, renormalized by a Debye-Waller type factor similar to the one discussed above, including both the electron-phonon and electron-electron interactions. All the low-temperature physics of the metal is given in terms of this renormalized mean-free-path (and mass). Strictly speaking, we have basically repeated and explained the arguments of refs. These were devised to refute the "zero temperature decoherence" by the phonons. While we presented the detailed discussion for a single "Einstein model" oscillator (which is an excellent test-case for the theory), it is clear that the results are valid for an arbitrary set of harmonic oscillators to which the conduction electron is coupled, and for which the theorem about the gaussian distribution we used is of course valid as well. Nothing of principle will change in this argument if the phonon harmonic oscillators are replaced by another set of harmonic oscillators, such as those taken by Nyquist. Feynmann and Vernon, Zwanzig and Leggett and Caldeira to represent the dissipation in the solid. Moreover, there is no need to use just an "ohmic" model. An almost arbitrary set of oscillators can be used, including "subohmic", "superohmic" or whatever one pleases (see below why we used the word "almost"). These models are formally solvable as long as the coupling to the oscillators is linear, and the solution is elementary. In the same fashion as with the well-known fact that the observed physics of an electron in vacuum automatically includes the effect of the zero-point motion of the latter, the observed physics of the electron in a real piece of material automatically has all the renormalizations by whatever harmonic modes of that metal that are coupled to the electron. Special low energy excitations, such as TLS, magnetic modes, etc., can yield nontrivial low-temperature decoherence for linear response.

Of course, should there be special harmonic (or anharmonic) modes with an anomalously large DOS at low energies, the dephasing rate due to them would be anomalously large at the corresponding low temperatures. It, as well as that by the TLS, should however vanish at the strict $T \to 0$ limit, unless a massive ground-state degeneracy or a pathologically increasing low energy DOS would exist. We used here a scattering picture, as in the Landauer description: an electron comes from the outside and its transmission by the system determines the conductance of the latter. The Fermi-surface electron comes and exits at the same energy. Its transmission amplitude across the system is given by a sum of amplitudes, each of which multiplied by a Debye-Waller type factor similar to Eqs. Since no excitations can be left by this electron at the $T \to 0$ metal whose ground state is non-degenerate, no decoherence (in the sense of the interference terms becoming smaller than the "classical" terms) can follow. This also applies when the conventional low-temperature Fermi liquid picture is used for the electrons in the conductor. We found that unless further special ingredients are introduced, this picture is theoretically stable at low enough temperatures and energies. It should be emphasized that such a scattering or a quasiparticle picture is convenient in order to have a clear definition of interference and decoherence. We did not mention the subtleties associated with models in which the interaction with the same environment is always on. How to define the interfering particle becomes a nontrivial issue in that case. Such subtleties are theoretically interesting but are hardly of much physical relevance. When the electron moves in a real solid the interactions are of finite range and hence it interacts with different degrees of freedom when it is at different positions. Therefore the Physics is similar to the scattering situation and not to models where the same interactions are on all the time.

We repeat that we did not rule out, and have not discussed, special situations where some additional new ingredient will cause anomalous decoherence/relaxation for the low-energy electrons; nor have we discussed the nontrivial experimental situation in depth. Regardless of all that, the statement that decoherence is due to "inelastic" processes (change of state of the environment) is the valid guiding principle for this problem. No ingenious model or advanced technique can go around that. Delving deeper into models that are argued to violate that principle may only be useful in order to expose their limitations or as diagnostics for the calculation. We finally add that during the preparation of this note, the present interpretation was conveyed to F. Guinea and found to be completely consistent with his model calculations. These are therefore consistent with the notion that there is no decoherence in the $T \to 0$ limit.

The deep question of how good is the description of
a real metal via coupling to harmonic oscillators was avoided here. Besides not distinguishing between resistance due to elastic or inelastic scattering, that picture is for sure not exact and corrections may well exist and be relevant [28, 55]. For example, in the real system, there will be (hopefully small) corrections to the gaussian approximation for the fluctuations [24], which have not been seriously treated either here or elsewhere. Independently of that, the underlying statement that without transfer of excitation there will be no dephasing, should still be valid.

We briefly summarize the theoretical statements of this note. The coupling of a particle that can not lose energy to environmental degrees of freedom at $T \rightarrow 0$, can modify, sometimes very seriously, the quantum behavior. This can go in either direction and is not decoherence. On the other hand, the experiments still point out to some nontrivial effects that are not understood as yet, in the low-temperature Physics of disordered metals.

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[1] We shall use here the terms "dephasing" and "decoherence" interchangeably and will not even attempt to decide which of them is more appropriate. The clearest definition of such a process is provided by a two-wave interference situation where the intensity is a sum of a classical term and an interference term, as in Eq. [1] below. Decoherence is defined via the "contrast" of the observed picture, i.e. by the relative decrease of the latter compared to the former.

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The fluctuation-dissipation theorem [47] relates [48] the coefficients are related by \( |C^2| = |c^2|^2 \omega^2 \). The usual explanation of this model goes by noting that the excitation of a quick and dirty derivation one may follow the Onsager-\( \delta \omega \) symmetry in the frequency domain [46]. As a result, a term in the cos \( \omega t \) to the cos \( \omega t \) in Eq. 5, which can be seen to eliminate the \( \omega < 0 \) portion of the power spectrum of \( \exp[i\omega t] \). This portion yields the energy transfer from the oscillators to the particle. Such a transfer is ruled out when the oscillators are at \( T = 0 \). At finite temperatures this imaginary part is what brings about the correct (detailed-balance type) ratio of the energy-gain and energy-loss branches of the power spectrum. It is not crucial, however, for the Debye-waller factor, which is the focus of our discussion here.

Actually, in the quantum regime the correlator in the quantum regime the correlator in the classical limit. This is however a technical detail which is irrelevant to our discussion here. It follows that the power spectrum of \( \hat{\phi} \) is proportional to \( \mu^2 \sum |\omega| \omega^2 (\omega + \lambda)^2 \), where \( (\omega + \lambda) \) is the power spectrum of \( \omega + \lambda \). We remember that the latter is given by \( \delta_\omega \). Here \( \nu \) is the average number of excited oscillator quanta \( \left[ \exp[h\omega/k_BT] - 1 \right]^{-1} \). The fluctuation-dissipation theorem relates \( \mu \) with the (negative) energy \( \omega \) at zero temperature 

\[
\sum |c^2|^2 \delta(\omega - \omega_x) \propto \frac{M}{\hbar \mu}.
\]

If the coupling is written as: \( \sum C \hat{x}_x \lambda \), then the coefficients are related by \( |C_x|^2 = |c_x|^2 \omega^2 \). The usual explanation of this model goes by noting that the excitation of a quick and dirty derivation one may follow the Onsager-\( \delta \omega \) symmetry in the frequency domain [46]. As a result, a term in the cos \( \omega t \) to the cos \( \omega t \) in Eq. 5, which can be seen to eliminate the \( \omega < 0 \) portion of the power spectrum of \( \exp[i\omega t] \). This portion yields the energy transfer from the oscillators to the particle. Such a transfer is ruled out when the oscillators are at \( T = 0 \). At finite temperatures this imaginary part is what brings about the correct (detailed-balance type) ratio of the energy-gain and energy-loss branches of the power spectrum. It is not crucial, however, for the Debye-waller factor, which is the focus of our discussion here.

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\[
\sum |c^2|^2 \delta(\omega - \omega_x) \propto \frac{M}{\hbar \mu}.
\]
It should perhaps be emphasized that in spite of claims otherwise, the Landauer (two-terminal) formulation is equivalent to the linear response theory for the same geometry, at least at $T \to 0$. We have in mind a finite sample, with arbitrary interactions, connected by ideal and noninteracting leads to particle reservoirs. The Landauer conductance is defined by the transmission of electrons across this sample. The linear response conductance can be evaluated for the same model. Their equivalence as $T \to 0$ for an interacting system was proven by Y. Meir and N. Wingreen, Phys. Rev. Lett. 68, 2512 (1992) [See also: T. K. Ng and P. A. Lee, Phys. Rev. Lett. 61, 1768 (1988)]. The case of finite temperatures where inelastic scattering may occur is more complicated, but it is not relevant to the present discussion, which concerns the $T \to 0$ (no inelastic scattering) limit only. This finite temperature case will hopefully be treated in a future publication.

In situations with such anomalous low-energy behavior, the Fermi liquid theory will break down at low temperatures. Two canonical examples for that are the two-channel Kondo model and the Luttinger liquid. It is perhaps helpful to point out, however, that in the real world, there may exist factors that can destabilize these deviations from the FLT at low enough temperatures (such as a small interchannel coupling or a finite separation between the two states [53] in the two-channel Kondo model, a finite length of the Luttinger liquid). Likewise, disordered systems that have theoretically a ground-state degeneracy (for example, the ice model [54] or glasses) may have a small real-life splitting of that degeneracy or never experience it within any finite measurement time. On the other hand, situations where truly robust deviations from the FLT as $T \to 0$ exist are obviously of genuine interest.

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It has been pointed out to the author by D. E. Khmel’nitskii that the treatments of Refs. 8 and 9 are valid only within the gaussian approximation for the fluctuations.