Validation of AE tomography based on model tests of soil specimen with heterogeneous elastic wave velocity distribution

K Nakamura1*, Y Kobayashi1, K Oda1, S Shigemura1 and K Ikebata1

1 Department of Civil Engineering, College of Science and Technology, Nihon University, Tokyo, Japan

* nakamura.katsuya@nihon-u.ac.jp

Abstract. In order to investigate the progressive failure of soil-structures, non-destructive inspections have been applied in recent. Use of Acoustic Emission (AE), the elastic wave which is generated by the occurrence of cracks, is attractive in this field to detail the progressive failure if the measurement is conducted with sufficiently small time interval. Although this technique potentially detects the cracks clearly, conventional methods of AE source localization consider that elastic wave velocity distribution is homogenous, and there are the limitations of its application due to the diffraction and the refraction of elastic wave if the heterogeneity of the elastic wave velocity distribution is not negligible. In this study, AE-Tomography (AET), which performs the identification elastic wave velocity distribution and the localization of AE sources with consideration of the diffraction and the refraction of elastic wave, is validated about the performance of the identification of the elastic wave velocity distribution and the AE sources localizations on numerical tests and model tests. Artificial AE events are generated in the soil specimen that have some parts of different relative density and validation of AET is conducted by using the arrival times of the artificial AE. According to the results of the validation, it is suggested that AET identifies elastic wave velocity distributions and AE source locations adequately on the heterogeneous elastic wave velocity distributions.

1. Introduction
The failure of soil-structures is well known as the progressive behavior of granular materials. However, the consideration of the progressive failure of soil-structures is difficult for the design, because the mechanism of soil particles slide and crush is unclear.

In the studies about the mechanism of the progressive failure, experimental and numerical approaches have been attempted. For instance, Digital image correlation (DIC) method is quoted as a technique that measures the deformation of a specimen from a series of images which are continuously taken. This technique has been applied to detect the evolution of shear bands [1, 2]. Although the results visualize the motion of the surface of the soil specimens, it is difficult to reveal the inside failures. DIC also requires sufficient framerate of cameras to detail motions of particles. To observe the process of inside failure of soil, X-ray CT, it reveals difference of densities and atomic compositions by X-ray absorptions, have been applied [3, 4]. Although, it detects and visualizes the progressive failure inside soil specimens, a larger space in comparison with the specimen for triaxial tests is required in the X-ray apparatus, and the apparatus should be installed in a secure area which is a barrier surrounded by the walls. Even though the technique that combines X-ray CT and DIC was proposed and it observed the
correlation of surface and inside behavior of soil specimens [5], the difficulties on the use of X-Ray still remains.

Use of AE, the elastic wave which is emitted at the occurrence of the cracks and/or the frictions is attractive to reveal the mechanism of failures [6]. AE is measured by AE sensors located on the surface of targets. The measurement system is portable, and the special facilities to stop X-Ray is not required. Further, it is able to apply any specimens as far as the wave can be detected. Due to the above characteristic, AE tests have been applied to geomaterials and observe the failure of soil-structures by AE parameters [7, 8, 9]. However, the conventional methods of AE source localization, it detects the failure locations, assume that elastic waves are transmitted in homogenous materials, and it is limitedly applied on heterogeneous elastic wave velocity distributions due to the diffraction and the refraction of the elastic waves.

AET simultaneously identifies the elastic wave velocity distribution and AE source locations with the consideration of the diffraction and the refraction of elastic waves [10]. In this paper, the performance of AET is validated on the heterogenous elastic wave velocity distribution to develop a method to visualize the progressive failure in soil-structures. It is expected that AET reveals the correlation between local density and failure locations by identifying elastic wave velocity distributions and AE source locations. For the validation, numerical investigations respected for the heterogeneous soil-structure are conducted. In addition, the method is validated by model tests in which the model has similar elastic wave distributions with the numerical models for the numerical investigations to check the performance of AET on heterogeneous dried sands.

2. Acoustic Emission Tomography (AET)

2.1. Ray-Tracing

In order to consider the diffraction and the refraction of elastic wave, Ray-Tracing which have been applied to seismic tomography [11] is adopted in AET for AE source locations. The outline of Ray-Tracing is illustrated in figure 1. It is assumed that the elastic wave velocity distribution consists of the cells in which the elastic wave velocity is constant, and elastic the wave velocity is identified in each cell to reproduce the heterogeneous velocity distribution. Thus, the ray-paths are approximated as straight line on each cell. In the first step of AET, initial velocity distributions are set to a homogeneous distribution of the elastic wave velocity in soundness conditions in the target. In the second step, travel times from AE sensors to candidates of AE sources are computed by using Ray-Tracing technique. The travel time $t_{ij}$ from the AE sensor $i$ to the candidate $j$ is defined as

$$t_{ij} = \sum_{k=1}^{n} s_{ik} \cdot t_{kj}$$  \hspace{1cm} (1)

![Figure 1. Outline of Ray-Tracing.](image-url)
where $S_k$ is slowness in cell $k$, and $l_{ijk}$ is length of the ray-path from the AE sensor $i$ to the candidate $j$ in the cell $k$. The ray-paths are approximated as polygonal lines which vertices are at the nodal points, and a path that gives the first travel time is sought from all of possible ray-paths on the heterogeneous velocity distribution. The possible pulse originated time is computed as subtraction of the arrival time of the first travel times. The possible pulse originated time $P_{ij}$ is defined as

$$P_{ij} = A_i - t_{ij}$$

(2)

where $A_i$ is the arrival time on the AE sensor $i$. The number of the estimated originated times at all candidates is same with the number of the sensors. AE source is selected as a nodal point where the variance of the originated time is minimum. The procedure is illustrated in figure 2. It should be noted that accuracies of trajectory of AE depended on the mesh in this algorithm.

![Figure 2. Localized AE sources and simulated trajectory of AE.](image)

### 2.2. Identification velocity distribution

The measured travel times is obtained as the sum of the first travel time and the average of the originated times at the localized points. The velocity distribution is updated to minimize the difference of observed and computed first travel times by inverse techniques. In this study, Simultaneous Iterative Reconstruction Technique (SIRT) is adopted as the identification technique because it is simple and solving matrix equations is not required, and consequently cost is lower. According to the figure 3, the velocity distributions of the cells where ray-paths cross, are only updated. Therefore, the identification technique requires sufficient number of AE events since all of the cells should be covered by the ray-paths for updating the velocity distribution accurately. SIRT update the velocity on the cells to minimize difference of the measured and computed first travel times. In order to prevent that velocity is identified negative values, upper limitation of the velocity is applied in this study.

![Figure 3. Targets of iteration for identification elastic wave velocity distributions.](image)
2.3. Minimizing mesh dependency

Accuracy of AE sources localization is related with the cell size if the nodal points is associated as the candidate of the source locations. However, shrinking the cell size increases the computational cost. In this study, Lagrange interpolation is applied to interpolate AE sources localization for minimizing the mesh dependency. Interpolation function $L(u)$ is defined as

$$L(u) = \sum_{j=1}^{n} v_j \left( \prod_{i=1}^{n} \frac{u - u_i}{u_j - u_i} \right)$$

(3)

where $u$ is coordinate of source location, and $v$ the variance of the originated time. $L(u)$ is applied around selected AE source in horizontal and vertical direction, respectively. According to the figure 4, AE sources is localized by the vertex of $L(u)$. In order to compute the clear vertex of $L(u)$, $L(u)$ should be consisted by 3 coordinates.

![Figure 4. Outline of AE source localization in the cell.](image)

3. Validation of AET by numerical tests

In order to validate performance of AET on heterogeneous elastic velocity distributions, a series of numerical tests is conducted.

3.1. Initial conditions

Figure 5 shows a model that is used for the numerical tests. Elastic velocity distributions on the model consist of high velocity area located in centre of the model and low velocity area on the left and the right side of the model. The elastic wave velocities in the areas are shown in table 1. The model simulates that dried sand with high relative density $D_r$ exists in the centre. It is assumed that the artificial AE events are generated here. The low velocity is determined by referring results of preliminary experiments in which AE were measured to compute the elastic wave velocity in dried sands $D_r$ of 88%. The preliminary experiments use Toyoura sand which is popular in experimental approaches [12, 13]. The high velocity is assumed as a velocity of Toyoura sand which is highly compacted by loads. 6 AE sensors are installed as illustrated in figure 5 around the model. The location of the sensors is related with the
accuracy of the source localization because of the localization based on difference of arrival times. If the sensors are located on side of models, the iteration of AE source localization is limitedly performed because of iteration reducing the errors of the AE source localization on the specific aspects. Thus, the error of iteration is remained on AE source locations. The simulated area is divided by square meshes with a size of $1.25 \times 10^{-2}$ m. Although increasing number of the cells is expected to identify detailed velocity distributions, it leads to increasing unknown quantities in SIRT and difficulty of obtaining accurate solutions, and further, calculation cost rises. In this study, 64 cells are adapted in consideration of the computation time. Initial conditions of the numerical tests are shown in table 2. In Case 1, AE source localization is conducted on the original distribution to validate the performance of AE source localization in which Lagrange interpolation is applied. In Case 2, the identification of AE source locations and elastic wave velocity distribution is performed. In order to validate the performance of Lagrange interpolation on the source localization, the interpolate function is applied in (a) and not applied in (b). Initial velocity distribution is homogenous and the velocity is the same with the upper limit of the velocity. The limit is set to 200 m/s which is the highest velocity in the original distribution. AE events is located randomly in the model. The arrival times are computed by performing Ray-trace from the generated source locations to the sensor locations on the original distribution.

![Figure 5. Original distribution.](image)

| Case               | One side of the square (m) | Initial velocity distribution(m/s) | Number of cells | Number of events | Upper limit of velocity(m/s) | Number of sensors |
|--------------------|---------------------------|------------------------------------|-----------------|-----------------|-----------------------------|-------------------|
| Case 1             | $1.25 \times 10^{-2}$     | -                                  | 64              | 1000            | -                           | 6                 |
| Case 2(a, b)       | $1.25 \times 10^{-2}$     | 200.0                              | 64              | 1000            | 200.0                       | 6                 |

**Table 1.** The elastic wave velocities in the areas.

| High velocity(m/s) | Low velocity(m/s) |
|--------------------|-------------------|
| 200.0              | 160.0             |

**Table 2.** Initial conditions.
3.2. Results of numerical tests
According to the figure 6, it is confirmed that the source locations are identified accurately. It is noteworthy that the source locations are identified inside of the cells and this is achieved by using the interpolation function with lower computational cost. These results suggest that AE source location are accurately without installing the other candidate points, if identified elastic wave velocity distributions is identical to the real distribution. According to the table 3, the average of the errors in Case 2(a) is smaller than (b). Furthermore, results of Case 2(a) shows that the localized sources are closer to the exact sources than the results of Case 2(b) according to figure 7. The results of case 2(b) shows the source locations are identified at the nodal points and it illustrates the source locations depend on the mesh. In results of identified elastic wave velocity distributions in Case 2, the high velocity area is identified more clear at the centre of the model in (a) in comparison with (b). This suggested that interpolation function contributes to raise the accuracy of AET results and minimize the influence of the mesh dependency. The elastic wave velocity is identified lower than the original velocity in the side of mode of (a). This is caused by the difficulty of localization in the vicinity of AE sensors. The localization is conducted with updating the velocity distributions. Hence, if the source around the sensor is localized, the cells in which wave pass is few and the influence of the path is stronger related to the other path. In order to prevent above problem, the localized source around the sensor would be excluded in the identification of the elastic wave velocity distribution.

| Case   | Average of errors (m) |
|--------|-----------------------|
| Case 1 | 9.22×10^{-4}         |
| Case 2(a) | 8.73×10^{-3}   |
| Case 2(b) | 1.14×10^{-2}     |

Table 3. Error of AE source localization.

Figure 6. Result of Case 1.

Figure 7. Result of Case 2.
4. Validation of AET by the model test

4.1. Testing material and apparatus
AET is validated with model tests on dried sands. The soil specimen is made by air pluviation method and consists of dried Toyoura sands which $D_r$ is 88%. The size of the soil specimens is $4.90 \times 10^{-1} \text{ m}$ in length, $3.40 \times 10^{-1} \text{ m}$ in width, and $3.00 \times 10^{-1} \text{ m}$ in height. Figure 8 illustrates 6ch of the AE sensors that are located at $1.00 \times 10^{-1} \text{ m}$, $1.50 \times 10^{-1} \text{ m}$, and $2.00 \times 10^{-1} \text{ m}$ in depth. The location of the AE sensors is determined to make a sufficient margin from soil tank frame to avoid observing the waves that propagates the soil tank frame. Operating frequency of the AE sensors is 150-400 kHz. Measured AE is amplified 60 dB by the preamp and sampling frequency is 1 MHz. Therefore, arrival times include errors of minimum $10^{-6}$ s.

4.2. The method of generating AE and detecting arrival time
AE is generated artificially by penetrating a pile which is $5.0 \times 10^{-3} \text{ m}$ in length, $5.0 \times 10^{-3} \text{ m}$ in width, and $1.0 \text{ m}$ in height as illustrated in figure 9. The amount penetration is $1.80 \times 10^{-1} \text{ m}$ by hammering and centre of the specimen is compacted as consequence. The elastic velocity distribution of the specimen is supposed to be same as the original distribution in the previous section. Measured AE is shown in figure 10 as an example, and it is separated to a noise part and a signal part, respectively. Arrival time is determined as the start of the signal part, and it is detected by visual confirmation.

**Figure 8.** Dimension of the soil tank and AE sensors’ locations (the sensors are located centre of width $3.4\times10^{-1} \text{ m}$).

**Figure 9.** The method of generating AE occurred in heterogenous elastic wave velocity distributions.

**Figure 10.** Example of measured AE.
4.3. Results of the model tests and AET with measured AE
According to results of detection of arrival times by visual confirmation, 4 AE events are chosen from all AE for the validation. It should be noted that the noise part and the signal part is clearly separated in the events. Other Initial conditions are the same as Case 2 in the previous section. The localized AE sources and the identified elastic wave velocity distributions are shown in figure 11 and table 4 shows the identified depth and originated time of AE events, respectively. According to the results, AE sources is localized deeper with the pulse originated time. It implies that the progress of the pile penetration is adequately detected. Further, the identified distribution estimates high velocity distributions in centre of the specimen, and it may suggest that the compression of soil structures at the top of the pile is detected. However, the identified high velocity distribution is unclear than the result of Case 2(a) in figure 7. This would be caused because the number of events is smaller than Case 2(a) and the ray-paths did not cover all of cells. Thus, it is expected that more events would be necessary to rise accuracy of the identified velocity distribution.

![Image](image_url)

**Figure 11.** Results of AET with measured AE.

| Event No. | Pulse originated time (s) | Depth (m)   |
|-----------|---------------------------|-------------|
| No. 1     | 20.503508                 | 1.05×10⁻¹  |
| No. 2     | 21.980975                 | 1.09×10⁻¹  |
| No. 3     | 23.263172                 | 1.42×10⁻¹  |
| No. 4     | 24.731889                 | 1.69×10⁻¹  |

**Table 4.** Pulse originated time and depth of AE sources in the specimen.

5. Conclusions
In this paper, AET is validated by performing the numerical investigation and the model tests on the heterogenous elastic velocity distribution to develop a method for visualizing the progressive failure in the soil-structures. The conclusions that are obtained in this study are written below.

- According to results of Case 1, it suggests that if the identified elastic wave velocity distributions is identical to the real distributions, AE source locations are accurately without installing the other candidate points.
- According to results of Case 2, it is revealed that the interpolation function rises the accuracy of AET results and lead minimizing mesh dependency.
- If the source is localized around the sensor, the cells in which the waves crossed is few and updated of the velocity is focused on these cells.
- According to results of AET for the model test, AET successfully shows the progress of the penetration of the pile and the compression of the sand around the tip of the pile. These results suggest that the progressive failure and the change of local density in soil-structures would be able to visualize by the identified elastic wave velocity distributions and AE sources.
- Updating the clear elastic wave velocity distributions as Case 2(a) is required enough events more than 4 events.
References

[1] Rechenmacher A, Abedi S and Chupin O 2010 Evolution of force chains in shear bands in sands *Geotechnique* **60** pp 343–351 DOI: 10.1680/geot.2010.60.5.343

[2] Bhandari A R, Powrie W, and Harkness R M 2012 A Digital Image-Based Deformation Measurement System for Triaxial Tests *Geotechnical Testing Journal* **35** 2 Paper ID GTJ103821

[3] Hall S A, Bornert M, Desrues J, Pannier Y, Lenoir N, Viggiani G and Suelle P Be´ 2010 Discrete and continuum analysis of localised deformation in sand using X-ray CT and volumetric digital image correlation *Geotechnique* **60** 5 pp 315–322

[4] Ando´ E, Viggiani G, Hall S A and Desrues J 2013 Experimental micro-mechanics of granular media studied by x-ray tomography: recent results and challenges *Ge´otechnique Letters* **3** 3 pp 142–146 http://dx.doi.org/10.1680/geolett.13.00036

[5] Takanoa D, Lenoir N, Otanic J and Hall S A 2015 Localised deformation in a wide-grained sand under triaxial compression revealed by X-ray tomography and digital image correlation *Soils and Foundations* **55** 4 pp 906–915

[6] Leia X, Masuda K, Nishizawa O, Jouniaux L, Liu L, Ma W, Satoh T and Kusunose K 2004 Detailed analysis of acoustic emission activity during catastrophic fracture of faults in rock *Journal of Structural Geology* **26** pp 247–258

[7] Mao W, Aoyama S, Goto S and Towhata I 2015 Acoustic emission characteristics of subsoil subjected to vertical pile loading in sand *Journal of Applied Geophysics Journal of Applied Geophysics* **119** pp 119–127

[8] Mao W, Goto S and Towhata I 2020 A study on particle breakage behavior during pile penetration process using acoustic emission source location *Geosciences Frontier* **11** pp 413–427

[9] Wenli L, Liu A, Mao W and Koseki J 2020 Acoustic emission behavior of granular soils with various ground conditions in drained triaxial compression tests *Soils and Foundations*. **60** pp 929–943

[10] Kobayashi Y and Shiotani T 2016 Computerized AE Tomography: Innovative AE and NDT Techniques for On-Site Measurement of Concrete and Masonry Structures *State-of-the-Art Report of the RILEM Technical Committee 239-MCM* Springer pp 47-68

[11] Sassa K, Ashida Y, Kozawa T and Yamada M 1989 Improvement in the accuracy of seismic tomography by use of an effective ray-tracing algorithm *MMJ/IMM Joint Symposium Volume of Papers. Kyoto.* pp 129-136

[12] Chiaro G, Kiyota T and Koseki J 2013 Strain localization characteristics of loose saturated Toyoura sand in undrained cyclic torsional shear tests with initial static shear *Soils and Foundations* **53** 1 pp 23-34

[13] Shogaki T 2017 Mechanism of sample disturbance caused by tube penetration: Model tests on Toyoura sand *Soils and Foundations* **57** 4 pp 527-542