Nonlinear Detection of Voltage Source Converter Control Systems in Wind Farms Based on Higher-Order Spectral Analysis

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Abstract—In recent years, the sub-synchronous oscillation (SSO) accidents caused by wind power have received extensive attention. A method is needed to detect the nonlinearity of the collected equal-amplitude accident waveform record. The theory of higher-order statistics (HOS) has become a powerful nonlinear detection tool since 1960s. However, HOS analysis was most applied in condition monitoring and fault diagnosis of mechanical equipment, even in the power system and wind farms. This paper focuses on the VSC control systems in wind farms and classifies the nonlinearity based on HOS analysis. First, the traditional describing function is extended to obtain more frequency domain information, and hereby the harmonic characteristics of bilateral and the unilateral saturation hard limit are studied. Then the bispectrum and trispectrum are introduced as HOS, which are extended into bicoherence and tricoherence spectrums to eliminate the effects from linear parts in the VSC control system. The effectiveness of nonlinear detection and classification based on HOS is strictly proved and its detailed calculation and estimation process is listed. Finally, the proposed method is demonstrated and further discussed through simulation results.

Index Terms—Bicoherence spectrum, higher-order statistics (HOS), nonlinear detection, tricoherence spectrum, voltage source converter (VSC), wind farms.

I. INTRODUCTION

With the implementation of the energy development strategy in China, the installation capacity of wind power increases in high speed. However, the stability problems brought about by wind integration are also becoming more and more serious. Among them, the sub-synchronous oscillation (SSO) is the most prominent. After the SSO accidents, the collected accident waveform record tends to be equal-amplitude oscillation because the transition process is very fast. Among current literatures, most scholars mainly analyze SSO with the complex torque coefficient method [6], [7] and so on. However, there are nonlinear parts such as hard limits in the control system of wind power, and the induced self-sustained oscillation is also of constant amplitude, so it is unreasonable to straightforwardly analyze the amplitude and frequency of SSO with a linearized method, and adopt corresponding measures to suppress the oscillation. Therefore, it is necessary to perform nonlinearity detection on the waveform records.

Since its emergence in the early 1960s, the theory of higher-order statistics (HOS) has become a powerful analysis tool in condition monitoring and fault diagnosis of mechanical equipment. Its applications mainly include three aspects at the very beginning: harmonic retrieval [8], system identification [9], and feature extraction [10]. Later, researchers applied HOS to nonlinear detection [11]. On the basis of bispectrum, [12] and [13] proposed the definitions of bicoherence spectrum and inverted bispectrum, respectively. Since then, scholars have proposed different statistical indicators of nonlinear detection based on HOS for nonlinear detection.

In the field of the power system, inspired by the above ideas, as early as 1995, researchers have proved that bispectral analysis can be introduced into fault identification and condition monitoring of three-phase induction motors to analyze and identify motor asymmetric faults and stator winding failure. [14] Since then, the research on fault diagnosis using HOS has achieved fruitful results. Reference [15] detected and identified asymmetric faults in induction motors by measuring vibration data and analyzing motor nonlinearity using the bicoherence spectrum. In [16], considering the Gaussian noise and non-Gaussian noise of mechanical signals, a new rolling bearing detection method was proposed, which integrated bispectral analysis and improved ensemble empirical mode decomposition.

In wind farms, [17] used a modulated signal bispectrum detector to diagnose bearing faults of wind turbines for doubly-fed induction generator wind turbines. [18] used bispectral analysis to identify single-point defects in rolling bearings. [19] proposed an improved signal separation method based on the Vold Kalman filter and the HOS analysis for rotating mechanical systems under strong background noise.

However, the current application of bispectral analysis mainly focuses on the mechanical defects of the power system, and there are few studies analyze the nonlinearity of the control system in the power system or specifically in the wind farm.
This paper attempts to give an analytical proof of the effectiveness of HOS applied in the voltage source converter (VSC) control system, which is an essential component of wind turbines and static var generators (SVGs), and to detect nonlinear oscillations in wind power systems. In this paper, the studied nonlinearity is considered as induced by hard limits.

The remainder of this paper is organized as follows. Section II extends the traditional describing function and analyze two types of hard limits. In Section III, HOS is introduced in definition. In Section IV, the VSC control system is modeled and nonlinear detection based on HOS is studied and strictly proved. The detailed calculation process for nonlinear detection of the VSC control system is described in Section V, and its effectiveness is proved by the case studies in Section VI. Finally, conclusions derived from this paper is presented in Section VII.

II. ANALYSIS OF HARMONIC CHARACTERISTICS OF HARD LIMIT

A. The Extended Describing Function (DF)

As shown in Fig. 1, assume that the input signal of the studied nonlinear part is a sinusoidal signal which is described as

\[ x(t) = A \sin \omega t \]  

(1)

where A and \( \omega \) are the amplitude and the frequency of the input sinusoidal signal, respectively. The output \( y(t) \) of the hard limit is a periodic non-sinusoidal signal, which can be expanded into a Fourier series as

\[ y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t) \]  

(2)

where \( A_0 \) is the magnitude of the DC component; \( A_n \) and \( B_n \) are the cosine part and sine part of the magnitude of the n-th Fourier harmonics, respectively.

In the traditional DF-based analysis method [20] used in analyzing the characteristics of sustained oscillations caused by nonlinearity, the nonlinear part is considered oddly symmetrical and the linear part of the system is considered to be low-pass. Then \( A_0 = 0 \) is derived and \( y(t) \) can be approximated as

\[ y(t) \approx y_1(t) = A_1 \cos \omega t + B_1 \sin \omega t = Y_1 e^{j\varphi_1} \]  

(3)

where

\[ A_1 = \frac{1}{\pi} \int_0^{2\pi} y(t) \cos \omega t \, dt \quad Y_1 = \sqrt{A_1^2 + B_1^2} \]  

(4)

\[ B_1 = \frac{1}{\pi} \int_0^{2\pi} y(t) \sin \omega t \, dt \quad \varphi_1 = \arctan \frac{B_1}{A_1} \]

The ratio of the first-order Fourier series of \( y(t) \) and the magnitude of the input signal is defined as the DF of the nonlinear part:

\[ N(A) = \frac{Y_1}{A} e^{j\varphi_1} \]  

(5)

In the self-sustained oscillations, the fundamental frequency and the corresponding amplitude of the oscillation are the key factors deciding the characteristics of the oscillation, so only the first-order Fourier series of the oscillation is reserved in the traditional DF. In nonlinear detection, however, the characteristics at the fundamental frequency are not enough. Otherwise, it will be not possible to tell apart the sustained oscillations induced by hard limits and negatively damped oscillations induced by incorrectly configured control parameters.

To extend, the higher-order harmonics are calculated as

\[ A_n = \frac{1}{\pi} \int_0^{2\pi} x(t) \cos n\omega t \, dt \]  

\[ B_n = \frac{1}{\pi} \int_0^{2\pi} x(t) \sin n\omega t \, dt \]  

(6)

B. Bilateral Saturation Hard Limit

As shown in Fig. 2(a), when a sine wave goes through a bilateral saturation hard limit in the time domain, its upper and lower part exceeding the limit value is ignored, which can be described as

\[ x(t) = A \sin \omega t \]  

(7)

where \( A \) and \( \omega \) are the amplitude and the frequency of the input sinusoidal signal, and \( a (> 0) \) is the upper limit of the hard limit.

The bilateral hard limit is oddly symmetrical, and the output periodic signal is an odd function, so the coefficients of the DC component and the cosine components in the Fourier series are 0, i.e., \( A_n = 0 \) (n = 0,1,2, ...). According to (4), the sine fundamental component of the output can be derived as

\[ B_1 = \frac{1}{\pi} \int_0^{2\pi} x(t) \sin \omega t \, dt \]  

\[ = \frac{2}{\pi} \left( \int_0^a \sin \omega t \, dt + \int_{\pi/2}^{\pi/2} \sin \omega t \, dt \right) \]  

\[ = \frac{2}{a} \left( \arcsin \frac{a}{A} + \frac{a}{A} \sqrt{1 - \left( \frac{a}{A} \right)^2} \right) \]  

\[ x(t) \rightarrow N(A) \rightarrow y(t) \]

Fig. 1. A typical hard limit with a sinusoidal input

\[ y(t) \rightarrow x(t) \rightarrow A_1 + a \]

\[ x(t) \rightarrow A_1 + a \]

(a) bilateral-saturation (b) unilateral-saturation

Fig. 2. Time-domain characteristics of bilateral-saturation and unilateral-saturation hard limits
The n-th components are calculated similarly and the results show that only
\[ B_{2n+1} \neq 0 \quad (n = 0, 1, 2, \cdots) \] (9)
The detailed calculation results can be referred to Table A in the appendix.

Define the limit saturation level \( \eta_{sat} \) as the ratio of the input sine wave amplitude to the limit value, i.e.,
\[ \eta_{sat} = \frac{A}{a} \] (10)
then the third-order harmonic distortion \( HD_{3b}(\eta_{sat} \geq 1) \) of the bilateral saturation hard limit is derived as
\[
HD_{3b} = \frac{B_3}{B_1} = \frac{\sqrt{\frac{1 - \eta_{sat}^2}{\eta_{sat}^2} + \frac{2}{3} \cos\left(\frac{1}{\eta_{sat}}\right)}}{\eta_{sat} \arcsin\left(\frac{1}{\eta_{sat}}\right)}
\] (11)
\( HD_{3b} \) is monotonically increasing in the domain of definition \( \{\eta_{sat} \mid \eta_{sat} \geq 1\} \). Therefore, if \( HD_{3b} \) of the time series of the hard limit’s output can be calculated, the limit saturation level \( \eta_{sat} \) in the operating system can be uniquely determined.

C. Unilateral Saturation Hard Limit
As shown in Fig. 2 (b), when a sine wave goes through a unilateral saturation hard limit in the time domain, its upper (or lower, depending on the actual situation) part exceeding the limit value is ignored, which can be described as
\[
x(t) = A \sin \omega t
\]
y(t) = \begin{cases} x(t) & x(t) \leq A_0 + a \\ A_0 + a & x(t) > A_0 + a \end{cases}
(12)
where \( A \), \( A_0 \) and \( \omega \) are the amplitude, offset and the frequency of the input sinusoidal signal, and \( A_0 + a \) is the upper limit of the hard limit.

Similar to II.B, the n-th components can be calculated and the results show that
\[
A_{2n} \neq 0 \quad (n = 0, 1, 2, \cdots) \\
B_{2n+1} \neq 0 \quad (n = 0, 1, 2, \cdots)
\] (13)
The detailed calculation results can be referred to Table B in the appendix.

To determine the limit saturation level \( \eta_{sat} \), the second-order harmonic distortion \( HD_{2u}(\eta_{sat} \geq 1) \) of the unilateral saturation hard limit is derived as
\[
HD_{2u} = \frac{A_2}{B_1} = \frac{4 \sqrt{1 - \eta_{sat}^2} (-1 + \eta_{sat}^2)}{3 \eta_{sat} \left(\frac{1}{\eta_{sat}^2} + \frac{2}{3} \eta_{sat} \arcsin\left(\frac{1}{\eta_{sat}}\right)\right)}
\] (14)
\( HD_{2u} \) is also monotonically increasing in the domain of definition and can uniquely determine \( \eta_{sat} \).

III. HOS ANALYSIS
The eigenfunction method is one of the important tools of statistical analysis, which can easily lead to the definition of higher-order moments and higher-order cumulants. The first joint eigenfunction of \( k \) continuous random variables is defined as
\[
\Phi(\omega_1, \cdots, \omega_k) \triangleq \mathbb{E} e^{j \sum_{i=1}^{k} \omega_i x_i} - \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1, \cdots, x_k) e^{j \sum_{i=1}^{k} \omega_i x_i} dx_1 \cdots dx_k
\] (15)
where \( f(\cdot) \) is the probability density function.

The k-order moments and k-order cumulants of \( k \) random variables are derived respectively as
\[
E(x_1, \cdots, x_k) = (-j)^k \frac{\partial^k \Phi(\omega_1, \cdots, \omega_k)}{\partial \omega_1 \cdots \partial \omega_k} \bigg|_{\omega_1=\cdots=\omega_k=0}
\]
(16)
\[
cum(x_1, \cdots, x_k) = (-j)^k \frac{\partial^k \ln \Phi(\omega_1, \cdots, \omega_k)}{\partial \omega_1 \cdots \partial \omega_k} \bigg|_{\omega_1=\cdots=\omega_k=0}
\] (17)

For a stationary continuous random signal \( x(t) \), set \( x_1 = x(t), x_2 = x(t + \tau_1), \cdots, x_k = x(t + \tau_{k-1}) \) in (17), then the k-order cumulant of the random signal \( x(t) \) is represented as
\[
cum_{\omega_1}(\tau_1, \cdots, \tau_{k-1}) = \sum_{\omega_k} \cum_{\omega_1}(\tau_1, \cdots, \tau_{k-1}) e^{j \omega_k x(t + \tau_{k-1})}
\] (18)

The k-order cumulant spectrum is defined as the (k-1)-dimensional discrete Fourier transform of the k-order cumulant, which is calculated as
\[
S_{\omega_k}(\omega_1, \cdots, \omega_{k-1}) = \sum_{\tau_k} \cum_{\omega_k}(\tau_1, \cdots, \tau_{k-1}) e^{-j \omega_k \tau_k}
\] (19)

Generally, the higher-order cumulant spectrum is simply referred to as the higher-order spectrum. In particular, the third-order spectrum \( S_3(\omega_1, \omega_2) \) is called bispectrum because it is an energy spectrum of two frequencies, and is represented by \( B(\omega_1, \omega_2) \) in this paper. Likewise, the fourth-order spectrum \( S_4(\omega_1, \omega_2, \omega_3) \) is referred to as the trispectrum, herein denoted by \( T(\omega_1, \omega_2, \omega_3) \).

IV. NONLINEAR DETECTION OF VSC CONTROL SYSTEM
A. Modeling of VSC Control System
A typical direct-drive wind farm has 30–60 generators. Groups of several direct-drive permanent magnet synchronous generators (PMSGs) are connected to form a string structure. Then several strings are attached to PCC and finally to the main grid through a series of boosting transformers and transmission lines (which is equivalent to a set of impedances).[21] Fig. 3 is the schematic of a typical direct-drive wind generator, which consists of a wind turbine, a PMSG, and a full-power converter (including machine- and grid-side converters).

![Fig. 3. Structure of a typical direct-drive wind generator.](image-url)

As the machine-side converter applies maximum power tracking, its interaction with the grid is small. As noted by [22] and [23], the grid-related oscillation dynamics strongly depend
on the DC capacitor and grid-side converter but are weakly affected by the wind turbine, PMSG, and machine-side converter. Therefore, the machine-side component (including the wind turbine, PMSG, and machine-side converter) is equivalent to a power source that outputs wind power received by the wind turbine and the grid-side converter is modeled as a VSC and its corresponding control system.

Meanwhile, the static reactive power compensation equipment such as static VAR generator (SVG) is usually installed in wind farms. The SVG model in this paper adopts a double closed-loop control strategy and terminal voltage orientation. The d-axis control loop stabilizes the DC bus voltage, and the q-axis control loop varies according to the control mode. When a SVG runs in a constant-voltage control mode, the control target of this loop is the terminal voltage; when it runs in constant reactive power control mode, the q-axis control loop varies according to the control targets in the d-axis and the q-axis control loop, as is shown in Fig. 4. In this paper, 4 hard limits are hereby modeled as a VSC and its corresponding control system.

Therefore, we obtain a unified VSC control system for both PMSGs and SVGs in wind farms. The only difference is the choice of the control targets in the d-axis and the q-axis control loop, as is shown in Fig. 4. In this paper, 4 hard limits are installed in wind farms. The SVG model in this paper adopts a VSC and its corresponding control system.

Therefore, we obtain a unified VSC control system for both PMSGs and SVGs in wind farms. The only difference is the choice of the control targets in the d-axis and the q-axis control loop, as is shown in Fig. 4. In this paper, 4 hard limits are considered as the nonlinear parts: 2 in the inner control loop of current and 2 in the outer control loop of voltage.

![Fig. 4. Structure of VSC control systems.](image)

The meanings of the symbols in Fig. 4 are as follows: $v_d(P)$ and $v_{dq}(P^*)$ are the measurement and the reference of the DC bus voltage (DC power), respectively; $v_d(Q)$ and $v_{dq}(Q^*)$ are the measurement and the reference of the d-axis terminal voltage (the output reactive power), respectively; $i_d$ and $i_q$ are the $d$ axis component and $q$ axis component of the output current of the grid-side converter, respectively; $e_d$ and $e_q$ are the $d$ axis component and $q$ axis component of the fundamental output voltage of the grid-side converter, respectively; $V_d$, $V_q$, and $V_g$ are the phase voltage amplitude of the terminal, the PCC and the grid, respectively; $\delta$ is the output angle of the PLL; $R_1$ and $L_1$ ($R_2$ and $L_2$) are the equivalent resistance and inductance of the transformer and the transmission line between the terminal and PCC, respectively; $R_g$ and $L_g$ are the equivalent resistance and inductance of the transmission line between PCC and the grid, respectively.

### B. Elimination of the Effects from Linear Parts

While the nonlinear parts take effect inside the control system, the accident waveform record collected from the phasor measurement unit (PMU) offers only the voltage and current information at the terminal of the VSC. Therefore, it’s necessary to derive the relationship between the HOS of the hard limit output and of the terminal electrical quantities.

When the hard limit in the d-axis inner control loop of current takes effect, the output $\dot{v}_d$ is produced by the nonlinear part after PI. Assume the phase-locked loop (PLL) performs well and the keeps synchronization stability, then the d-axis frame model of the main circuit is written as

$$\Delta v_d' - \Delta v_d = (sL + R) \Delta i_d - \alpha_b L \Delta i_q$$

Ignoring the dynamics of the PWM, assume $v_d' = e_d$, then the relationship between $\dot{v}_d$ and $v_d'$ is derived as

$$\Delta v_d' = \Delta \bar{v}_d - \alpha_b L \Delta i_q + \Delta v_d$$

Combine (20) and (21), we get

$$\Delta i_d = \frac{1}{sL + R} \Delta \bar{v}_d$$

Similarly, when the hard limit in the q-axis inner control loop of current takes effect, the equation stands as

$$\Delta i_q = \frac{1}{sL + R} \Delta \bar{v}_q$$

When the hard limit in the d-axis outer control loop of voltage takes effect, the output $\dot{v}_d$ is produced by the nonlinear part after PI. (21) turns into

$$\Delta v_d' = G(s) \cdot (\Delta \bar{i}_d - \Delta i_d) - \alpha_b \bar{L} \Delta i_q + \Delta v_d$$

Combine (20) and(24), we get

$$\Delta i_d = \frac{G(s)}{sL + R + G} \Delta \bar{i}_d$$

Similarly, when the hard limit in the q-axis outer control loop of voltage takes effect, the equation stands as

$$\Delta i_q = \frac{G(s)}{sL + R + G} \Delta \bar{i}_q$$

Therefore, combining (22), (23), (25) and (26), the output of the nonlinear part in the control system can always be obtained from the current measured at the terminal after going through a linear part. To eliminate the effects from linear parts using HOS, consider the system in Fig. 5, with the input $e(n)$ and the output $y(n)$. $H(z)$ represents a linear time-invariant part.

![Fig. 5. A typical linear system.](image)
\begin{equation}
\begin{aligned}
B_i(\omega_1,\omega_2) &= B_i(\omega_1,\omega_2)\frac{H(\omega_1)\cdot H(\omega_2)}{H'\omega_1+\omega_2} \quad (29)
\end{aligned}
\end{equation}

Define \( bic_i(\omega_1,\omega_2) \) as the bicoherence at the frequency pair \((\omega_1, \omega_2)\), which is calculated as

\begin{equation}
\begin{aligned}
bic_i(\omega_1,\omega_2) &= \frac{b_i(\omega_1,\omega_2)}{P(\omega_1)P(\omega_2)P(\omega_1+\omega_2)} \quad (30)
\end{aligned}
\end{equation}

Combining (28) and (29), it can be proved that \( bic_x(\omega_1, \omega_2) = bic_i(\omega_1, \omega_2) \).

Define tricoherence \( tric(\omega_1,\omega_2,\omega_3) \) as

\begin{equation}
\begin{aligned}
tric(\omega_1,\omega_2,\omega_3) &= \frac{b_i(\omega_1,\omega_2,\omega_3)}{P(\omega_1+\omega_2+\omega_3)P(\omega_1)P(\omega_2)P(\omega_3)} \quad (32)
\end{aligned}
\end{equation}

Similarly, it can be proved that \( tric_i(\omega_1,\omega_2,\omega_3) = tric_i(\omega_1,\omega_2,\omega_3) \).

Therefore, the linear part does not change the bicoherence and the tricoherence of the system. Combining (22), (23), (25) and (26), by measuring the waveform of \( i_d \) and \( i_q \) at the terminal and performing HOS analysis on them, the nonlinearity of the VSC system can be detected.

C. Bicoherence Spectrum and Unilateral Saturation Hard Limit Detection

From the results in II.C, when the self-sustained oscillation occurs, the output \( y \) of the unilateral saturation hard limit contains the second harmonic of the oscillation frequency with the same phase as the fundamental frequency. Without loss of generality, let its initial phase be 0, i.e., let

\begin{equation}
\begin{aligned}
y(t) = B_1 \sin 2\pi f t + B_2 \sin 2\pi f t \quad (33)
\end{aligned}
\end{equation}

where \( f \) is the oscillation frequency. \( B_1 \) and \( B_2 \) are the cosine parts of the magnitude of the fundamental and second Fourier harmonics, respectively.

The Fourier transform is performed twice on the second-order autocorrelation function of \( y(t) \) and then the bispectrum of \( y(t) \) can be derived as

\begin{equation}
\begin{aligned}
B_i(\omega_1,\omega_2) &= \int \int y(t)y(t+\tau_1)y(t+\tau_2)e^{-j2\omega_1\tau_1}e^{-j2\omega_2\tau_2}d\tau_1d\tau_2 \quad (34)
\end{aligned}
\end{equation}

A bispectrum has 12 symmetry regions [24], as is shown in Fig. 6. Therefore, it is possible to take a symmetrical area in the result of \( B_y(\omega_1,\omega_2) \) for analysis, to completely describe the whole bispectrum. Considering \((\omega_1,\omega_2)|0 \leq \omega_1 \leq \omega_2 \) , \( B_y(\omega_1,\omega_2) \) is calculated as

\begin{equation}
\begin{aligned}
B_y(\omega_1,\omega_2) &= \pi B_1^2 B_2^2 \delta(\omega_1 - 2\pi f) \delta(\omega_2 - 2\pi f) \quad (35)
\end{aligned}
\end{equation}

where \( \delta() \) is the Dirac Delta function, which is described as

\begin{equation}
\begin{aligned}
\delta(x) &= \left\{ \begin{array}{ll}
0 & (x \neq 0) \\
1 & (x = 0)
\end{array} \right. \\
\int_{-\infty}^{\infty} \delta(x)dx &= 1 \quad (36)
\end{aligned}
\end{equation}

\( \delta(x) \) is a finite maximum at \( x = 0 \) when the input signal is discretized. So in (35), if and only if \( \omega_1 = \omega_2 = 2\pi f \), \( B_y(\omega_1,\omega_2) \) is a finite maximum, otherwise it is zero. Therefore, a peak can be observed at the x-y coordinate \((2\pi f, 2\pi f)\) in the 3-dimensional graph of \( \{\omega_1, \omega_2, B_y(\omega_1, \omega_2)\} | 0 \leq \omega_1 \leq \omega_2 \). Furthermore, considering symmetry, because \((2\pi f, 2\pi f)\) is on the symmetry axis \( \omega_1 = \omega_2 \), when the area is extended to \((\omega_1, \omega_2)|0 \leq \omega_1 \leq \omega_2 \) (Area 1 and 2 in Fig. 6), there is also only one peak at \((2\pi f, 2\pi f)\).

The power spectrum of \( y(t) \) is derived as

\begin{equation}
\begin{aligned}
P_y(\omega) &= \int_{-\infty}^{\infty} y(t)y(t+\tau)e^{-j2\omega\tau}d\tau \\
&= \frac{1}{2}B_1^2 \frac{\pi}{\sqrt{2}} \delta(\omega - 2\pi f) + \frac{1}{2}B_2^2 \frac{\pi}{\sqrt{2}} \delta(\omega - 4\pi f) \quad (37)
\end{aligned}
\end{equation}

\( HD_{21} \) can be obtained from \( B_1^2 \) and \( B_2^2 \), then the limit saturation level \( \eta_{sat} \) under the current operating state of the system can be calculated according to II.C.
To make it easier to generate intuitive graphs, when the area is frequency with the same phase as the fundamental frequency. Therefore, two peak s can be observed at the x-y-z coordinate \( (i \cdot 2\pi f, j \cdot 2\pi f) \), \( (i, j = 1, 2, 3, \ldots) \), and their corresponding bicoherence value also equals to 1. Furthermore, it can be proved that when \( y(t) = \sum_{n=1}^{\infty} A_{2n} \cos 2\pi \cdot 2nft + \sum_{n=0}^{\infty} B_{2n+1} \sin 2\pi \cdot (2n + 1)f t \) according to (13), which accurately represents the output of the unilateral saturation hard limit, the conclusion remains the same.

**D. Tricoherence Spectrum and Bilateral Saturation Hard Limit Detection**

From the results in II.B, when the self-sustained oscillation occurs, the output \( y \) of the bilateral saturation hard limit contains the 3-rd and 5-th harmonics of the oscillation frequency with the same phase as the fundamental frequency. Without loss of generality, let its initial phase be 0, i.e., let

\[
y(t) = B_1 \sin 2\pi ft + B_2 \sin 2\pi f \cdot 3t + B_3 \sin 2\pi f \cdot 5t
\]

where \( f \) is the oscillation frequency. \( B_1, B_2, B_3 \) are the cosine parts of the magnitude of the fundamental, 3-rd and 5-th Fourier harmonics, respectively.

The Fourier transform is performed three times on the third-order autocorrelation function of \( y(t) \) and then the trispectrum of \( y(t) \) can be derived as

\[
R_i(t, t, t_j) = \int_{-\infty}^{\infty} y^2(t) y^2(t + \tau) d\tau
\]

A trispectrum has 96 symmetry regions [24]. Therefore, it is possible to take a symmetrical area in the result of \( T_i(y(t)) \) for analysis, to completely describe the whole trispectrum. Considering \( \{ (\omega_1, \omega_2, \omega_3) | 0 \leq \omega_1 \leq \omega_2 \leq \omega_3 \} \), \( T_i(y(t)) \) is calculated as

\[
T_i(\omega_1, \omega_2, \omega_3) = \int \int \int \int R_i(t, t, t) e^{i \omega_1 t_1 + i \omega_2 t_2 + i \omega_3 t_3} dt_1 dt_2 dt_3
\]

If only if \( \omega_1 = \omega_2 = 2\pi f \), \( \omega_1 = \omega_3 = 2\pi f \), \( \omega_3 = 6\pi f \), \( \omega_1 = 2 \pi f \), \( \omega_2 = 3\pi f \), \( \omega_3 = 6\pi f \), the tricoherence spectrum of \( y(t) \) reaches its peak.

When \( \omega_1 = \omega_2 = 2\pi f \), it is calculated as

\[
tric_i(2\pi f, 2\pi f, 2\pi f) = \frac{B_1^2 B_2^2}{4\pi^2} \frac{\delta(\omega_1 - 2\pi f)\delta(\omega_1 - 2\pi f)}{2\pi^2}
\]

When \( \omega_1 = 2 \pi f, \omega_2 = 3\pi f, \omega_3 = 6\pi f \), \( \omega_1 = 3\pi f, \omega_2 = 2\pi f, \omega_3 = 6\pi f \), it is calculated as

\[
tric_i(2\pi f, 2\pi f, 6\pi f) = \frac{B_1^2 B_2^2 B_3^2}{4\pi^2} \frac{\delta(\omega_1 - 2\pi f)\delta(\omega_1 - 2\pi f)\delta(\omega_1 - 6\pi f)}{2\pi^2}
\]

Obviously, \( tric_i(\omega_1, \omega_2, \omega_3) \geq 0 \) in (44). Therefore, the range of the corresponding tricoherence value of each x-y-z coordinate in the tricoherence spectrum is [0,1]. The larger the tricoherence value, the stronger the nonlinear phase coupling among the three frequencies corresponding to the coordinate, that is, the stronger the nonlinearity.

In (40), when \( y(t) \) extends to \( y(t) = \sum_{n=1}^{\infty} B_{2n+1} \sin 2\pi \cdot (2n + 1)f t \) according to (9), which accurately represents the output of the bilateral saturation hard limit, similarly, it can be proved that in the 4-dimensional graph of \( \{ (\omega_1, \omega_2, \omega_3, \omega_4) | \omega_4 \geq 0, \omega_2 \geq 0, \omega_3 \geq 0 \} \), peaks exist at the x-y-z coordinate \( (2i + 1) \cdot 2\pi f, (2j + 1) \cdot 2\pi f, (2k + 1) \cdot 2\pi f) \), \( (i, j, k = 0, 1, 2, 3, \ldots) \).

**E. Nonlinear Detection and Classification of VSC Control System**

To sum up, from IV.B, the nonlinearity inside the VSC control system can be detected by measuring the waveform of \( i_d \) and \( i_q \) at the terminal and performing HOS (i.e.,
bicoherence/tricoherence) analysis on them. The nonlinearity of \( i_d \) represents the nonlinearity in d-axis control loop of the VSC control system, while the nonlinearity of \( i_q \) represents the nonlinearity in q-axis.

Combining the conclusions in IV.C and IV.D, Table I is summarized. Essentially, the bicoherence spectrum is applied to detect “phase coupling” in the analyzed signal, which means there exist harmonics whose frequencies \( f_1 + f_2 = f_3 \) and phases \( \varphi_1 + \varphi_2 = \varphi_3 \) are satisfied at the same time. Meanwhile, the tricoherence spectrum is applied to detect if \( f_1 + f_2 + f_3 = f_4 \) and \( \varphi_1 + \varphi_2 + \varphi_3 = \varphi_4 \) are both satisfied. It is worth noting that the phase equation is actually a sufficient condition, which will be explained in detail in the case study.

As is shown in Table I, the output of the unilateral saturation hard limit contains each harmonic, so it satisfies both quadratic and cubic phase coupling, which means peaks exist both in its bicoherence and tricoherence spectrums. As a result, the bicoherence spectrum should be first examined, and then the tricoherence spectrum. The nonlinearity can be judged and classified as shown in Fig. 7.

### Table I

| Phase Coupling | Unilateral Saturation | Bilateral Saturation |
|----------------|------------------------|----------------------|
| Fourier Series | / \( y(t) = \sum_{n=m}^{\infty} A_{2n} \cos 2\pi n t \) \( + \sum_{n=m}^{\infty} B_{2n+1} \sin 2\pi n t \) / | \( y(t) = \sum_{n=m}^{\infty} A_{2n+1} \sin 2\pi n t \) \( + \sum_{n=m}^{\infty} B_{2n+1} \sin 2\pi n t \) / |
| Bicoherence     | \( f_1 + f_2 = f_3 \) \( \varphi_1 + \varphi_2 = \varphi_3 \) peaks no peaks | \( f_1 + f_2 + f_3 \) \( \varphi_1 + \varphi_2 + \varphi_3 = \varphi_4 \) peaks peaks |
| Tricoherence    | \( f_1 + f_2 + f_3 \) \( \varphi_1 + \varphi_2 + \varphi_3 = \varphi_4 \) peaks peaks |

### V. Calculation Process for Nonlinear Detection of VSC Control System

Collect the accident waveform record of current at the terminal of the VSC. The studied signals \( x_d(t) \), \( x_0(t) \) and \( x_b(t) \) are sampled from the three-phase currents \( i_d(t) \), \( i_b(t) \) and \( i_c(t) \) whose lengths are \( L \) and sampling interval are \( \Delta t \), i.e.,

\[
\begin{align*}
  x_d(t) &= i_d(t) \Delta t \\
  x_b(t) &= i_b(t) \Delta t \\
  x_c(t) &= i_c(t) \Delta t
\end{align*}
\]

(47)

Step 1: Perform dq transformation on the three-phase sampling signals \( x_d(t) \), \( x_b(t) \) and \( x_c(t) \) and the initial phase \( \theta_0 \) can be obtained by applying a PLL algorithm.

\[
\begin{bmatrix}
  x_d(t) \\
  x_c(t)
\end{bmatrix} = \begin{bmatrix}
  \cos(\omega_0 t + \theta_0) & \cos(\omega_0 t + \frac{2\pi}{3} + \theta_0) & \cos(\omega_0 t + \frac{4\pi}{3} + \theta_0) \\
  -\sin(\omega_0 t + \theta_0) & -\sin(\omega_0 t + \frac{2\pi}{3} + \theta_0) & -\sin(\omega_0 t + \frac{4\pi}{3} + \theta_0)
\end{bmatrix} \begin{bmatrix}
  x_d(t) \\
  x_c(t) \\
  x_b(t)
\end{bmatrix}
\]

(48)

where \( x_d(t) \), \( x_q(t) \) and \( x_b(t) \) are the transformation results in dq\(0\) coordinate system.

Step 2: Take the transformation result \( x_d(t) \) of the previous step as the subsequent signal processing object, i.e.,

\[
x(t) = x_d(t)
\]

(49)

Step 3: Divide \( x(t) \) into \( M \) segments, and each segment length is \( N \) \( (L = M \times N) \). Record each segment as \( x^{(i)}(t) \) \( (i = 1, \cdots, M; t = 1, \cdots, N) \).

Step 4: Select an appropriate window function, such as a Hanning window, which is described as

\[
w(l) = \frac{1}{2} \left[ 1 + \cos \left( 2\pi l \cdot \frac{l}{N-1} \right) \right]
\]

(50)

Multiply each segment of the signal by the window function, and use the obtained results \( x^{(i)}(t) \) for subsequent calculations to reduce leakage errors:

\[
x^{(i)}(l) = x^{(i)}(l) \cdot w(l)
\]

(51)

Step 5: For each segment \( x^{(i)}(l) \), subtract its mean:

\[
\hat{x}^{(i)}(l) = x^{(i)}(l) - \bar{x}^{(i)}(l)
\]

(52)

Step 6: Perform the Fast Fourier Transform (FFT) on each segment \( x^{(i)}(l) \):

\[
X_k^{(i)} = \frac{1}{N} \sum_{l=1}^{N} \hat{x}^{(i)}(l) e^{-j2\pi kl/N} \quad k = 1, \cdots, N/2, \quad i = 1, \cdots, M
\]

(53)

Step 7: Deal with the FFT results. Take a small parameter \( \sigma \) (such as \( \sigma = 0.001 \)), traverse \( i = 1, \cdots, M \), for any \( k \), if \( X_k^{(i)} < \sigma \), then \( X_k^{(i)} = \sigma \). This step can further increase the difference of the order of magnitude between the white noise and the peak value in the spectrum, so the judgment and analysis of the peak value will not be affected by the appearance of values close to zero in the area other than the peaks in the bicoherence spectrum.

Step 8: The estimated values of the power spectrum, bispectrum and trispectrum of \( x(t) \) are calculated as

\[
\hat{P}(m) = \frac{1}{M} \sum_{i=1}^{M} X_m^{(i)} X_m^{(i)}
\]

(54)
\[
\hat{B}(m,n) = \frac{1}{M} \sum_{i=1}^{M} X_m^{(i)} X_n^{(i)} X_m^{*+n} \tag{55}
\]
\[
\hat{T}(m,n,o) = \frac{1}{M} \sum_{i=1}^{M} X_m^{(i)} X_n^{(i)} X_o^{(i)} X_m^{*+n+o} \tag{56}
\]

Step 9: In the power spectrum, note that \(B_1, A_2\) and \(B_3\) are the fundamental, the second and the third harmonic amplitude, respectively. Solve the equation in (11) or (14), then the limit saturation level \(\eta_{sat}\) can be obtained.

Step 10: Calculate the bicoherence spectrum:
\[
\hat{bic}(m,n) = \frac{\left| \hat{B}(m,n) \right|}{\sqrt{\hat{P}(m+n) \hat{P}(m) \hat{P}(n)}} \tag{57}
\]

Step 11: The obtained bicoherence spectrum is a 3-dimensional graph whose x-y coordinates are the frequencies \((m,n)\), and the z coordinate is the corresponding bicoherence value whose theoretical value range is \([0,1]\). Step 12: Calculate the tricoherence spectrum:
\[
\hat{tric}(m,n,o) = \frac{\left| \hat{T}(m,n,o) \right|}{\sqrt{\hat{P}(m+n+o) \hat{P}(m) \hat{P}(n) \hat{P}(o)}} \tag{58}
\]

Step 13: Define \(\sigma_b\) as the nonlinear threshold (preferably 0.3). A peak in the bicoherence or tricoherence spectrum whose value is greater than \(\sigma_b\) (generally close to 1) is considered to characterize the existence of a quadratic or cubic phase coupling, and the coordinates of the peak represent the corresponding frequencies. The judgement and classification of the nonlinearity can be completed following the process in Fig. 7.

Step 14: The steps above implement nonlinear detection on d-axis control loop of the VSC control system. To study the q-axis control loop, back to Step 2, let \(x(t) = x_q(t)\) and repeat Step 3-Step 13.

VI. CASE STUDY

A. Case I

Considering the following signal:
\[
x(n) = \cos\left[2\pi f_1 n / f_s + \phi_1 + w_1(n)\right] + \cos\left[2\pi f_2 n / f_s + \phi_2 + w_2(n)\right] + \cos\left[2\pi f_3 n / f_s + \phi_3 + w_3(n)\right] \tag{59}
\]

where \(f_1 = 0.6381\) Hz, \(f_2 = 0.8345\) Hz, \(f_3 = f_1 + f_2\) and \(w_i(n)(i = 1,2,3)\) is -20 dB Gaussian white noise.

In (59), let \(\phi_1 = \phi_2 = \phi_3 = 0\), recorded as \(x_1(n)\). Then, let \(\phi_1 = \phi_2 = 0, \phi_3 = \pi/2\), recorded as \(x_2(n)\).

Fig. 8(a) is the frequency spectrum of \(x_1(n)\), from which three frequency components \(f_1, f_2\) and \(f_3\) can be found, but the relationship among them can’t be determined. Fig. 8(b) is the bicoherence spectrum of \(x_1(n)\). Its peak appears at \((f_1, f_2)\) and the peak value is 1.0, which means the power of the frequency component \(f_3\) entirely comes from quadratic phase coupling of \(f_1\) and \(f_2\), which proves the conclusion in IV.C.

Fig. 8(c) and 8(d) are the frequency spectrum and the bicoherence spectrum of \(x_2(n)\), which are almost the same as Fig. 8(a) and 8(b), respectively. In our initial setting, however, \(\phi_1 + \phi_2 = \phi_3\) in \(x_1(n)\), while \(\phi_1 + \phi_2 \neq \phi_3\) in \(x_2(n)\).

Most of the previous researches [25]–[27] consider quadratic phase coupling equivalent to \(f_1 + f_2 = f_3\) and \(\phi_1 + \phi_2 = \phi_3\), which is actually a sufficient condition. As long as \(\phi_1 + \phi_2\) keeps a fixed difference with \(\phi_3\), i.e., \(\phi_1 + \phi_2 - \phi_3\) is constant, the phase coupling exists and peaks can be seen in the bicoherence spectrum. This conclusion is important because according to (13), the output of the unilateral saturation hard limit can be expressed as
\[
y(t) = \sum_{n=1}^{\infty} A_n \cos 2\pi \cdot 2n f t + \sum_{n=0}^{\infty} B_2 e i \sin 2\pi \cdot (2n + 1) f t \tag{60}
\]
\[
= \sum_{n=1}^{\infty} A_n \left( \sin \left( 2\pi \cdot 2n f t + \frac{\pi}{2} \right) + \sum_{n=0}^{\infty} B_2 e i \sin 2\pi \cdot (2n + 1) f t \right)
\]

Therefore, its peaks appear in the bicoherence spectrum like a “chessboard”, because any two integer multiples of the fundamental frequency have the property of quadratic phase coupling. Otherwise, if the bicoherence can only detect those satisfying \(f_1 + f_2 = f_3\) and \(\phi_1 + \phi_2 = \phi_3\) at the same time, there will be no peaks in the bicoherence spectrum at all, which does not match the actual situation.

B. Case II

Set up a detailed grid-connected PMSG model in PSCAD, the structure of the VSC control system is shown in Fig. 9. Adjust the parameters to make the hard limit of the PI in the d-axis outer control loop of voltage take effect. The setting of the parameters is listed in Table II.

The simulation is implemented as follows:
\(t = 0.0\) s: use a voltage source to charge the DC capacitor; in the initial state, the PMSG is off-grid and the active power and reactive power references are both 0.
\(t = 0.2\) s: the DC capacitor side is switched to the power source, and the PMSG is connected to the grid.
\(t = 1.0\) s: the active power is set to 0.34 MW.
\(t = 2.0\) s: \(G_i(s)\) is set to 0.2 + 20/s.
\(t = 4.0\) s: \(G_i(s)\) is set to 0.012 + 12.5/s.
Fig. 9. The PMSG model for nonlinear detection.

### TABLE II

| Symbol | Description | Value |
|--------|-------------|-------|
| \( V_g \) | Grid line voltage | 0.69 kV |
| \( f_0 \) | Fundamental frequency | 50 Hz |
| \( P_N \) | Rated capacity of PMSG | 1.5 MW |
| \( H_{PLL}(s) \) | Phase-locked loop | \( 500 + 900/s \) |
| \( P \) | Active power | 0.34 MW |
| \( C \) | DC link capacitor | 200 mF |
| \( R \) | Connection resistance | 0.001 \( \Omega \) |
| \( L \) | Connection inductance | 0.35 mH |
| \( R_g \) | Grid-side resistance | 0.005 \( \Omega \) |
| \( L_g \) | Grid-side inductance | 0.4 mH |
| \( G_d(s) \) | DC-voltage controller | \( 9 + 500/s \) |
| \( G_q(s) \) | Reactive power controller | \( 0.3 + 50.28/s \) |
| \( G(s) \) | Inner-loop current controller | \( 0.012 + 12.5/s \) |

Fig. 11. Voltage and current at PCC in \( d \) axis.

Fig. 12. Bicoherence and power spectrum of \( i_{dref} \) and \( i_{dPCC} \).

Fig. 10 is the current reference \( i_{dref} \). With the parameters listed in Table II, after \( t = 4.0 \, s \), the system has a pair of characteristic roots on the right side of the imaginary axis, which induces a divergent oscillation. When the oscillation reaches the hard limit of the PI in the \( d \)-axis outer control loop of voltage, it becomes unilateral saturated. Meanwhile, at PCC, an equal-amplitude self-sustained oscillation of 33.8 Hz can be observed, as shown in Fig. 11. Considering the transient process is short, the collected accident waveform record may only contains the equal-amplitude part, which is of superficial resemblance with the linear weakly-damped oscillation.

Fig. 12 is the bicoherence and power spectrum of \( i_{dref} \) and \( i_{dPCC} \). The bicoherence spectra are figured as contour maps, whose x-y axis range is restricted in \( \{(f_1, f_2)\mid \omega_1 \geq 0, \omega_2 \geq 0\} \). As is shown in Fig. 12(a), the bicoherence spectrum of \( i_{dref} \) presents as a “chessboard”, which means quadratic phase coupling exists between any two integer multiples of the fundamental frequency. Normally, the transfer function of the VSC control system is lowpass, which can be seen by comparing Fig. 12(b) and (d). The bicoherence spectrum of \( i_{dPCC} \), however, still keeps the property of quadratic phase.
coupling, as is shown in Fig. 12(c). It is acceptable that the peaks of higher harmonics disappear because their amplitudes are too small to maintain distinction from background noise. Therefore, Fig. 12(c) proves that nonlinearity can be detected from collecting accident waveform records at the terminal of VSCs based on HOS analysis. Especially in this case, nonlinearity exists in the d-axis control loop of the VSC control system.

VII. CONCLUSION

This paper proposes a method based on HOS analysis for nonlinear detection of the VSC control system in wind farms, where PMSGs and VSGs are modeled as a unified VSC control model. The paper establishes a corresponding relationship for the bicoherence spectrum and the unilateral saturation hard limit output in the analyzed signal, which is further discussed in the case study part.

Further work includes extending the proposed method to hybrid systems that include VSC control systems. Besides, a nonlinear detection method and a nonlinear index suitable for online monitoring will be designed. In addition, combining the port-controlled Hamiltonian (PCH) model and energy structure with HOS analysis, a nonlinear oscillatory source localization method will be proposed.

**APPENDIX**

| N-TH COMPONENTS OF BILATERAL SATURATION HARD LIMIT OUTPUT |
|----------------|----------------|
| n  | A_n  | B_n  |
| 0  | 0    | 0    |
| 1  | 0    | \(2a \sqrt{1 - \frac{a_0^2}{A^2} + 2A \sin^{-1} \frac{a_0}{A}}\) \(\frac{\pi}{3}\) |
| 2  | 0    | 0    |
| 3  | 0    | \(4a(1 - \frac{a_0^2}{A^2})^{3/2}\) \(\frac{3\pi}{3}\) |
| 4  | 0    | 0    |
| 5  | 0    | \(4a(1 - \frac{a_0^2}{A^2}(8a_0^4 - 11a_0^2A^2 + 3A^4))\) \(\frac{15\pi A^4}{\pi}\) |
| 6  | 0    | 0    |
| 7  | 0    | \(\frac{38a \cos \left(\frac{7\pi}{2} \sin^{-1} \frac{a_0}{A} + 28A \sin \left(6 \sin^{-1} \frac{a_0}{A} \right) - 21A \sin \left(8 \sin^{-1} \frac{a_0}{A} \right)\right) - 38a_0}{\lambda^6}\) \(\frac{38\pi}{\lambda^6}\) |

| N-TH COMPONENTS OF UNILATERAL SATURATION HARD LIMIT OUTPUT |
|----------------|----------------|
| n  | A_n  | B_n  |
| 0  | \(a + 2A_0 \frac{2}{\pi} \sqrt{1 - \frac{a_0^2}{A^2} A - \frac{2a_0}{A \pi} \sin^{-1} \frac{a_0}{A}}\) |
| 1  | 0    | \(2a \sqrt{1 - \frac{a_0^2}{A^2} A + 2A \sin^{-1} \frac{a_0}{A}}\) \(\frac{\pi}{2}\) |

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