Searching spin-mass interaction using a diamagnetic levitated magnetic resonance force sensor

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Axion-like particles (ALPs) are predicted to mediate exotic interactions between spin and mass. We propose an ALP-searching experiment based on the levitated micro-mechanical oscillator, which is one of the most sensitive sensors for spin-mass forces at a short distance. The experiment tests the spin-mass resonant interaction between the polarized electron spins and a diamagnetically levitated microsphere. By periodically flipping the electron spins, the contamination from non-resonant background forces can be eliminated. The levitated micro-oscillator can prospectively enhance the sensitivity by nearly 3 orders over current experiments for ALPs with mass of 7 meV.

I. INTRODUCTION

Light pseudoscalars exist in a number of Beyond the Standard Model theories. One well-motivated example is the axion [1, 2], which is introduced via spontaneously broken the Peccei-Quinn (PQ) $U(1)$ symmetry [3, 4] to solve the strong CP problem, and also a low-mass candidate for the dark matter in the universe [5]. Generalized axion-like particles (ALPs) rise from dimensional compactification in string theory, which share similar interaction with electromagnetic fields, and share a similar phenomenological role with the axions [6–8]. Motivated by axion and ALP’s potential role in particle physics and cosmology, a number of experimental methods and techniques have been developed over the past few decades, such as the method proposed by Moody and Wilczek to detect cosmic axion [9], the photon-axion-photconverion light shining through wall experiments [10, 11], the axion emission from the Sun [12, 13], the dichroism and birefringence effects in external fields [14, 15], and the light pseudoscalar mediated macroscopic mass-mass [16], spin-mass [17–24] and spin-spin [25, 26] forces.

The pseudoscalar exchange between fermions results in spin-dependent forces [27]. Most prior works detecting exotic spin-dependent forces [19–22, 28, 29] are focused on the so-called axion window [30], where the interaction range is 200 µm – 20 cm. Due to the interest in non-zero mass, it is desired to find experimental techniques to search for such anomalous spin-dependent interactions at even shorter distances [31].

The levitated micro-mechanical and nano-mechanical oscillators have been demonstrated as one of the ultra-sensitive force sensors [32–38] due to its ultra-low dissipation and small size. It is one of the ideal methods to measure short-range force [39–44] with high precision. However, in short-ranged force measurements, surface noises from the electric static force fluctuation, the Casimir force and magnetic force, limit the final sensitivity.

Here we propose a new method to investigate the spin-mass interaction using an ensemble of electron spins and a levitated diamagnetic microsphere mechanical oscillator. By periodically flip the electron spin at the resonant frequency with the mechanical oscillator, the postulated force between electron spins and the microsphere mass is preserved while the spin-independent force noise from the surface is eliminated.

II. SCHEME

We use a levitated diamagnetic microsphere mechanical oscillator to investigate the spin-mass interaction (Fig. 1(a)). The microsphere is trapped in the magnetogravitational trap and levitated stably in high vacuum. The diamagnetic-levitated micro-mechanical oscillator achieves the best sensitivity in micro- and nano-mechanical systems to date, orders of magnitude improvement over the reported state-of-the-art systems based on different principles. The cryogenic diamagnetic-levitated oscillator described here is applicable to a wide range of mass, making it a good candidate for measuring force with ultra-high sensitivity [45]. The position of the microsphere is mainly determined by the equilibrium between the gravity force and the main magnetic force of the trap. A uniform magnetic field is applied to tune the
The electrons spins are initially polarized along the magnetic field under high field and low temperature, so that \( \rho_e (0) = \rho_{e0} \), where \( \rho_{e0} \) is the electron density of the spin-source. Then they are flipped periodically in resonance with the microsphere mechanical oscillator (see Fig. 1(b)). On one hand, the spin-independent interactions, such as the Casimir force, will be off-resonance and become eliminated (Fig. 1(c)). On the other hand, the spin-mass interaction is preserved on the resonance condition. The spin autocorrelation function is defined as 
\[
\langle \rho_e (t) | \rho_e (0) \rangle = \rho_{e0} (0)^2 P(t) = \rho_{e0} (0)^2 e^{-1/T_1 \xi(t)},
\]
where \( T_1 \) is the electron spin-lattice relaxation time and \( \xi(t) \) is the modulation function (see Appendix B). The microwave \( \pi \) pulses flip the electron spin periodically with frequency \( 2\omega_z \). \( \xi(t) \) jumps between -1 and +1 every time the electron spins are flipped. The corresponding power spectral density (PSD) of the spin-related force is proportional to \( G(\omega) \), which is the Fourier transform of \( P(t) \). The PSD of spin-mass force is then:
\[
S_{F_{sm}}(\omega) = \left( \frac{\hbar^2 g_s^N g_p^e \rho_m}{8\pi m_e} \zeta_{sm}(R, d, \lambda) \right)^2 \rho_{e0}^2 \tilde{G}(\omega). \tag{3}
\]
If spin-mass interaction signal is observed on resonance \( (\omega = \omega_z) \), the coupling \( g_s^N g_p^e \) can be derived as
\[
g_s^N g_p^e = \sqrt{S_{F_{sm}} (\omega_z) / G(\omega_z)} \frac{8\pi m_e}{\zeta_{sm} \hbar^2 \rho_m \rho_{e0}}. \tag{4}
\]
Apart from the spin-mass force, spin-induced magnetic force \( F_s \) between electron spins and the diamagnetic microsphere is recorded during the measurement. Fortunately, well designed spin-source geometry can eliminate most of the force (see Appendix C). Then the residual spin-induced magnetic force is
\[
F_s(t) = \rho_e(t) \frac{\mu_B}{2} \frac{\partial B_{0z}}{\partial z} \zeta_s(R, d), \tag{5}
\]
where $\zeta(R,d)$ is the effective volume for spin-induced force. Similarly, the PSD of $F_1$ is

$$S_{\|}^m(\omega) = \left( \frac{1}{2} \mu_B \chi_m \frac{\partial B_{\|}}{\partial \omega} \zeta_m(R,d) \right)^2 \rho_{e_0}^2 \tilde{g}(\omega).$$  \hspace{1cm} (6)

Considering the fluctuating noise, the equation of motion for the system center of mass is

$$m\ddot{z} + m\gamma \dot{z} + m\omega^2 z = F_{\|}(t) + F_e(t) + F_{\text{sm}}(t),$$  \hspace{1cm} (7)

where $m$ is the mass of the microsphere, $\omega^2/2\pi$ is the resonance frequency, $\gamma/2\pi$ is the intrinsic damping rate and $F_{\|}(t)$ is the fluctuating noise force that includes the thermal Langevin force $F_{\text{th}}(t)$ and the radiation pressure fluctuations $F_{\text{ba}}(t)$. \[46\]

The total detected displacement PSD is given by:

$$S_{zz}^{\text{tot}}(\omega) = S_{zz}^{\text{imp}}(\omega) + \frac{|\chi(\omega)|^2}{m^2} \left( S_{\|\|}^{\text{ba}} + S_{\|\|}^{\text{th}} + S_{\|\|}^{\text{th}} + S_{\|\|}^{\text{sm}} \right)$$  \hspace{1cm} (8)

where $\chi(\omega)$ is the mechanical susceptibility given by $|\chi(\omega)|^2 = 1/(\omega^2 - \omega_0^2 + \gamma^2\omega^2)$; $S_{zz}^{\text{imp}}(\omega)$ denotes the PSD of the detector imprecision noise; $S_{\|\|}^{\text{ba}}$, $S_{\|\|}^{\text{th}}$, $S_{\|\|}^{\text{sm}}$ and $S_{\|\|}^{\text{th}}$ are the PSDs of $F_{\text{ba}}(t)$, $F_{\text{th}}(t)$, $F_e(t)$ and $F_{\text{sm}}(t)$, respectively. The total fluctuation noise $S_{\|\|}^{\text{tot}}(\omega_z) = S_{\|\|}^{\text{th}} + S_{\|\|}^{\text{ba}} + m^2 S_{zz}^{\text{imp}}|\chi(\omega_z)|^{-2}$. Due to these noises, the detection limit of spin-mass coupling strength $g_N^N_g^e$ is thus:

$$(g_N^N_g^e)^{\text{limit}} = \frac{S_{\|\|}^{\text{th}}(\omega_z) + S_{\|\|}^{\text{ba}}(\omega_z)}{G(\omega_z)} \frac{8\pi m_e}{\zeta_{\text{sm}} h^2 \rho_m \rho_{e_0}}.$$  \hspace{1cm} (9)

### III. RESULTS

As a reasonable example we consider a microsphere with mass $m = 1.5 \times 10^{-13}$ kg and radius $R = 3.2$ $\mu$m of density $1.1 \times 10^3$ kg/m$^3$. Thus, the corresponding nucleon density is $\rho_m = 6.7 \times 10^{29}$ m$^{-3}$. The magnetic susceptibility of the microsphere is $-9.1 \times 10^{-6}$. The whole system is placed in a cryostat with temperature $T = 20$ mK and external uniform magnetic field $B_{\text{ext}} = 1.85$ T. A permanent magnet provides 0.15 T magnetic field and correspondingly the $z$ direction magnetic gradient $\partial B_{\|}/\partial z = 750$ T/m. The microsphere is then levitated with a surface distance $d = 1.46$ $\mu$m above the spin source. The whole mechanical oscillator system have a typical frequency of 24 Hz [47] and the electron density of the spin-source is $\rho_{e_0} = 2.3 \times 10^{27}$ m$^{-3}$. The direction of the electron spins is initially prepared along the external magnetic field, which in our design is approximately along the z axis, with a maximum tilted angle of 4$^\circ$.

The total measurement time is set as 1s. We take the experimental sensitivity limited by the total fluctuation noise as $S_{\|\|}^{\text{th}}(\omega_z) = S_{\|\|}^{\text{th}} + S_{\|\|}^{\text{ba}} + m^2 S_{zz}^{\text{imp}}|\chi(\omega_z)|^{-2}$. Here $S_{\|\|}^{\text{th}}$ is estimated to be $5.14 \times 10^{-41}$ N$^2$/Hz according to $S_{\|\|}^{\text{th}} = 4m\gamma k_B T$, with $\gamma/2\pi = 10^{-6}$ Hz [45].

Imprecision noise and backaction noise are related, when they contribute equally, the sum has a minimum $S_{\|\|}^{\text{sum}}(\omega_z) = S_{\|\|}^{\text{ba}} + m^2 S_{zz}^{\text{imp}}|\chi(\omega_z)|^{-2} = 2m|\chi(\omega_z)|^{-1}\hbar/\eta^2$. In a practical condition, the measurement efficiency $\eta \geq 0.001$ [48], which imply $S_{\|\|}^{\text{sum}}(\omega_z) = 9.36 \times 10^{-49}$ N$^2$/Hz. Thus, the total fluctuation noise is dominated by the thermal noise, with $S_{\|\|}^{\text{th}}(\omega_z) \approx 5.14 \times 10^{-41}$ N$^2$/Hz. Under such an experimental sensitivity, $(g_N^N_g^e)^{\text{limit}} = \frac{8\pi m_e (S_{\|\|}^{\text{th}}(\omega_z)/G(\omega_z))^{1/2}}{\zeta_{\text{sm}} h^2 \rho_m \rho_{e_0}}$. As $G(\omega_z)$ is proportional to the electron spin-lattice relaxation time, $(g_N^N_g^e)^{\text{limit}}$ decreases as $T_1$ increases, which is shown in green in Fig. 2.

![Fig. 2](g_N^N_g^e)^{\text{limit}} for the force range of $\lambda = 2\mu$m as an example. The green line denotes $(g_N^N_g^e)^{\text{limit}}$ calculated from the total fluctuation noise, which decreases as $T_1$ increases; the red line denotes the correction of $(g_N^N_g^e)^{\text{limit}}$ by taking the residual spin-induced magnetic force into account, which is independent of $T_1$; the blue curve denotes their sum. The correction from spin-induced magnetic force (red curve) is dominant when $T_1 > 1$ ms.

Practically, it is not feasible to completely eliminate the spin-induced magnetic force due to fabrication imperfection of the spin-source geometry (see Appendix C). A correction for $(g_N^N_g^e)^{\text{limit}}$ is introduced as follows. Since the spin-induced magnetic noise is spin-dependent while the $G(\omega_z)$ has the same scaling, its contribution to $(g_N^N_g^e)^{\text{limit}}$ is constant (blue curve in Fig. 2). For $T_1 > 1$ ms, $(g_N^N_g^e)^{\text{limit}}$ is dominated by the spin-induced magnetic force and approaches to the minimum $8\pi m_e (S_{\|\|}^{\text{th}}(\omega_z)/G(\omega_z))^{1/2}/\zeta_{\text{sm}} h^2 \rho_m \rho_{e_0}$.

Finally, Fig. 3 shows the calculated $(g_N^N_g^e)^{\text{limit}}$ (see Appendix E) set by this work at $\lambda = 0.5\mu$m - 50 $\mu$m together with reported experimental results for the constraints of spin-mass coupling. Here our result is estimated through supposing $T_1 = 1$ s, for spin-lattice relaxation time can be longer than the scale of seconds at low temperature [49, 50]. The limitation for our pro-
The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$. The estimated bound of our method is plotted in red for the spin-mass force range $55$.
spin source when the microsphere locates in equilibrium (Fig. 4), \( n_c = 0.059 \) characterizes the reduction in the Casimir force, depending on the dielectric functions of the microsphere and the spin-source. The value of \( E_p \) versus the displacement of the microsphere is shown in Fig. 4.

Thus our mechanical system can be described as a damping harmonic oscillator subject to \( F_{sm}, F_s \) and \( F_{flu} \), i.e.,

\[
m\ddot{z} + m\gamma \dot{z} + m\omega_z^2z = F_{sm} + F_s + F_{flu},
\]

where \( \omega_z \) is the resonant frequency of the microsphere,

\[
\omega_z = \sqrt{\frac{1}{m}\frac{\partial E_p}{\partial z}}.
\]

The equilibrium position of the microsphere can be derived by \( \partial E_p / \partial z = 0 \). The spin-induced magnetic field and \( V_{cas} \) are so weak that they have negligible influence on this trap, so that the equilibrium position is mainly determined by the gravity field and the main magnetic field \( B_{ext} \). Thus we can indirectly tune it by the uniform external magnetic field \( B_{ext} \).

**Appendix B: AUTOCORRELATION FUNCTION OF NET ELECTRON-SPINS DENSITY**

The autocorrelation function of electron polarization is \( \langle \rho_e(t)\rho_e(0) \rangle \). Suppose these electron spins are independent of each other, we have

\[
\langle \rho_e(t)\rho_e(0) \rangle = \rho_{e0}^2 P(t),
\]

where \( P(t) \) is the autocorrelation function of a single spin, i.e.,

\[
P(t) = p_\uparrow(t) - p_\downarrow(t). \tag{B2}
\]

Here \( p_\uparrow(t) \) represents the spin population on \( |\uparrow\rangle \) or \( |\downarrow\rangle \). Every time when a \( \pi \) pulse is applied to flip the electron spin,

\[
p_\uparrow(t) = p_\uparrow(t) + p_\downarrow(t)p_\downarrow(\tau)
\]

\[
p_\downarrow(t) = p_\uparrow(t)(1 - p_\downarrow(\tau)),
\]

where \( \tau \in (0, \tau_0) \), \( \tau_0 \) corresponds to the period between two adjacent \( \pi \) pulses (Fig. 5), \( p_1(\tau) = 1 - e^{-\tau/T_1} \) is the spin flip probability during \( \tau_0 \), \( T_1 \) is spin-lattice relaxation time.

The evolution of \( P(t) \) is shown in Fig. 6(a). \( P(t) \) presents a sawtooth-like wave of frequency \( 2\omega_z \) for \( t = k\tau_0 + \tau \gg T_1, \) (\( k = 0, 1, 2, \ldots \)),

\[
P(\tau + k\tau_0) = 1 - \frac{2e^{-\frac{\tau}{T_0}}}{1 + e^{-\frac{\tau}{T_0}}}, \quad \tau \in (0, \tau_0) \tag{B4}
\]

Only the signal with resonant frequency \( \omega_z \) needs to be collected. After dropping the sawtooth-like signal whose frequency is \( 2\omega_z \), the resonant signal is shown in Fig. 6(b). The resonant signal is a square wave with an exponential decay, i.e.,

\[
P(t) = e^{-t/T_1}\xi(t), \tag{B5}
\]

| PSD calculated at \( T_1 = 1s \) | Size \((N^2/Hz)\) | Contribution to \( \langle g_{\text{sp}}^2 \rangle_{\text{min}} \) at \( \lambda = 2\mu m \) |
|----------------------------------|-------------|----------------------------------|
| Of spin induced magnetic force \( S_{th}^m(\omega_z) \) | \( 2.59 \times 10^{-41} \) | 4.3 \times 10^{-22} |
| Of thermal noise \( S_{th}^t(\omega_z) \) | \( 5.14 \times 10^{-43} \) | 9.1 \times 10^{-24} |
| Of backaction noise plus imprecision noise \( S_{ff}^{\text{add}}(\omega_z) \) | \( 9.36 \times 10^{-49} \) | 1.2 \times 10^{-26} |
| **Total** | \( 2.59 \times 10^{-41} \) | 4.3 \times 10^{-22} |

**FIG. 5.** Microwave \( \pi \) pulses carried on with a frequency of \( 2\omega_z \). The time interval between two adjacent \( \pi \) pulses is \( \tau_0 = \pi/\omega_z \). Spin flips with a frequency of \( \omega_z \) and its amplitude varies slowly over time due to the spin-lattice relaxation.
FIG. 6. (a) Variation of the native $P(t)$ after equilibrium in the form of a sawtooth-like wave. The frequency of the sawtooth-like wave is $2\omega_z$, which is out of resonance with the microsphere oscillator and can be neglected. (b) Effective $P(t)$ after dropping the sawtooth-like wave. It is a square wave modulated signal that decays exponentially with time, which is determined by the spin-lattice relaxation time. The frequency of this signal is $\omega_z$.

where $\xi(t)$ is the modulation function of the following form

$$\xi(t) = \frac{2}{1 + e^{-\frac{\tau_0}{T_1}}} \csc(\omega_z t + \frac{\pi}{2}).$$  \hspace{1cm} (B6) 

Here $\csc(\omega_z t + \frac{\pi}{2})$ is a square wave of frequency $\omega_z$. According to the Wiener-Khinchine theorem, its single side PSD is:

$$\tilde{G}(\omega) = \frac{4}{1 + e^{-\tau_0/T_1}} \left( \frac{2T_1}{1 + T_1 \omega^2} - 4e^{-\tau_0/2T_1} T_1 \left( 1 + e^{-\tau_0/T_1} \right) \cos(\omega \tau_0/2) - \omega \left( 1 - e^{-\tau_0/T_1} \right) \sin(\omega \tau_0/2) \right) \left( 1 + \omega^2 \right).$$

Appendix C: PSD OF SPIN INDUCED MAGNETIC FORCE

Apart from the desired magnetic trap, the spin-source can induce a magnetic force $F_s$ on the microsphere as follows

$$F_s = \int_m dV \chi_m \left( B_{0z} \frac{\partial B_{sz}}{\partial z} + B_{sz} \frac{\partial B_{0z}}{\partial z} \right).$$

This force can be eliminated by deliberately designing the configuration of spin source (in Fig. 7).

The $z$-direction component of magnetic field produced by a single spin is

$$B_{sz0} = \frac{\mu_0 \mu_B}{4\pi} \frac{3\cos^2 \theta - 1}{l^5},$$  \hspace{1cm} (C1) 

where $\theta$ is the polar angle and $l$ is the distance from the microsphere to the spin. The magnetic field of a spin-source cylinder at $z$ axis is then

$$B_{sz} = \int_{c_i} dV \rho_e(r_i, z_i, t) B_{sz0}.$$  \hspace{1cm} (C2) 

Here $i = 1$ and 2 correspond to cylinder1 and cylinder 2, $\int_{c_i} dV = \int_{-R_{a1}}^{R_{a1}} dz_1 \int_0^{R_{i1}^2-z_1^2} 2\pi r_1 dr_1$, and $\rho_e(r_i, z_i, t)$ is net spin density along the $z$ axis in the cylinder.

The microsphere is assumed to be right above the center of the cylinder, so that the magnetic field in the microsphere is approximately uniform in the transverse direction. Thus, the magnetic force produced by a cylinder

\[ \text{...} \]
FIG. 7. The spin source consisting of a large cylinder (cylinder1) with a small cylinder (cylinder2) removed. $R_{s1}$ and $R_{s2}$ are radius of the two cylinders, and $L_1$ and $L_2$ are their heights, respectively. The gray ball represents the microsphere. $d$ is the surface distance between the microsphere and the spin-source.

In the cylindrical coordinate system, we have $\int_m dV = \int_R^L dz \int_0^{\sqrt{R^2-z^2}} 2\pi r dr$, and $z' = z - L_2 - d - 3R$.

The geometry shape and the imperfections on fabrications are considered. The geometric parameters are optimized to make $F_s$ as small as possible. Table II lists the optimized geometric parameters and their standard deviations according to the practical condition. Here we exaggerate the $\rho_e(t)$ to be $\rho_0$. From the table, we can see that the value of optimized $F_s$ is $4.2 \times 10^{-22}$ N, while the total uncertainty of $\Delta F_s$ is $\Delta F_s = 5.03 \times 10^{-20}$ N. More generally, the variation of $\Delta F_s$ versus the standard deviations of geometric parameters is plotted in Fig. 8.

APPENDIX D: PSD OF SPIN-MASS FORCE

The spin-mass effective magnetic field generated by a polarized spin on an unpolarized nucleon is:

$$B_{sp}(r) = \frac{\hbar g_s^N g_p^e}{4\pi m_e \gamma} \left( \frac{1}{r \lambda} + \frac{1}{r^2} \right) e^{-r/\lambda} e_r.$$  \hspace{1cm} (D1)

The spin-mass effective magnetic field generated by the microsphere on a polarized spin is obtained by integrating the volume of the microsphere with Eq. (D1), i.e.,

$$B_m = \int_m dV \rho_m B_{sp}(r) = \rho_m \frac{\hbar g_s^N g_p^e}{4\pi m_e \gamma} g(R, \ell) e_\ell.$$  \hspace{1cm} (D2)
where

\[ g(R, \ell) = 2\pi \lambda^2 \left( (R - \lambda) e^{\frac{R}{\lambda}} + (R + \lambda) e^{-\frac{R}{\lambda}} \right) \left( \frac{1}{\Lambda^2} + \frac{1}{\ell^2} \right) e^{-\frac{R}{\lambda}}, \]  

(D3)

\( \rho_m \) is the nucleon density of microsphere, \( e_\ell \) and \( \ell \) are the unit vector and distance between the microsphere and the spin, respectively. From Eq. (D2) and Eq. (D3), we can find that in the calculation of spin-mass effective magnetic field, the microsphere is completely equivalent to a center mass. Therefore, Eq. (D2) is equivalent to the effective magnetic field produced by the CM of the microsphere.

The spin-mass potential between the microsphere and the spin-source is obtained by integrating the volume of spin-source with Eq. (D2):

\[ V_{sm}(t) = -\int_{cy_1} dV \rho_c(r_1, z_1, t) \mu_B \cdot B_m \]

\[ -\int_{cy_2} dV \rho_c(r_2, z_2, t) \mu_B \cdot B_m, \]

and \( \rho_c(r_1, z_1, t) \) represents the net electron spin density along the z axis in the spin-source.

Consequently, the spin-mass force between the micro-

\[ F_{sm}(t) = -\frac{\partial V_{sm}}{\partial z} \]

\[ = -\frac{\partial}{\partial z} \left( \int_{cy_1} dV \rho_c(r_1, z_1, t) \mu_B \cdot B_m \right. \]

\[ - \left. \int_{cy_2} dV \rho_c(r_2, z_2, t) \mu_B \cdot B_m \right), \]

where \( \zeta_{sm}(R, d, \lambda) \) is the effective volume for \( F_{sm}(t) \), reads:

\[ \zeta_{sm}(R, d, \lambda) = (2\pi) \left( (R - \lambda)e^{\frac{R}{\lambda}} + (R + \lambda)e^{-\frac{R}{\lambda}} \right)^2. \]  

(D4)

Accordingly, The PSD of spin-mass force reads:

\[ S_{ff}^\omega(Fs(t)) = \sqrt{\frac{S_{ff}^\omega(Fs(t))}{G(\omega)}} = \left( \frac{h^2 g_N^N g_p^p}{8\pi m_m} \zeta_{sm}(R, d, \lambda) \right)^2 \zeta_{sm}(R, d, \lambda) \rho_{sm}^2 \tilde{G}(\omega). \]  

(D5)

**Appendix E: CALCULATION OF \((g_N^N g_p^p)_{\text{limit}}\)**

To observe the spin-mass signal, \( g_N^N g_p^p \) needs to be no less than

\[ (g_N^N g_p^p)_{\text{limit}} = \frac{\sqrt{S_{ff}^\omega(\omega_s)}}{G(\omega_s)} \left( \frac{\zeta_{sm} h^2 \rho_m \rho_c}{8\pi m_m} \right) \]

For the worst situation, \( (g_N^N g_p^p)_{\text{limit}} \) takes its upper bound:

\[ \sup (g_N^N g_p^p)_{\text{limit}} = \frac{S_{ff}^\omega(\omega_s) \zeta_{sm} h^2 \rho_m \rho_c}{8\pi m_m} \]

(E1)

where \( \sup (\frac{\zeta_{sm} h^2 \rho_m \rho_c}{8\pi m_m} \zeta_{sm}(R, d, \gamma)) \) means the upper bound of \( \frac{\zeta_{sm} h^2 \rho_m \rho_c}{8\pi m_m} \zeta_{sm}(R, d, \gamma) \), and \( \min \left( \frac{h^2 \rho_m \rho_c}{8\pi m_m} \zeta_{sm}(R, d, \lambda) \right) \) is the minimum value of \( \frac{h^2 \rho_m \rho_c}{8\pi m_m} \zeta_{sm}(R, d, \lambda) \). We take

\[ \sup \left( \frac{\chi_{m p} B_{dz}}{2} \zeta_{sm}(d, R) \right) \]

\[ = \left( \frac{\chi_{m p}}{2} \frac{\partial B_{dz}}{\partial z} \zeta_{sm}(d, R) \right) \]

(E2)

and \( \min \left( \frac{h^2 \rho_m \rho_c}{8\pi m_m} \zeta_{sm}(R, d, \lambda) \right) \) is numerically calculated with parameters \( R \) and \( d \) taken within the uncertainty ranges (see Table II). Combined with Eq. (E1) and Eq. (E2), the estimated \((g_N^N g_p^p)_{\text{limit}}\) in the worst situation is shown in red in Fig. 3 in the main text.
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