Spin-filtering by field dependent resonant tunneling

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We consider theoretically transport in a spinfull one-channel interacting quantum wire placed in an external magnetic field. For the case of two point-like impurities embedded in the wire, under a small voltage bias the spin-polarized current occurs at special points in the parameter space, tunable by a single parameter. At sufficiently low temperatures complete spin-polarization may be achieved, provided repulsive interaction between electrons is not too strong.

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Introduction.— Control and manipulation of spin degree of freedom in nanoscale electronic devices is an active new field of research [1, 2]. In quantum wires spin selective transmission of electrons was considered in the past in a number of publications [3–7]. In [8] a strong asymmetry of the spin dependent conductances in a Luttinger liquid (LL) with a magnetic impurity was observed, which is related to the Zeeman energy splitting Δ of the impurity states. In [4] the authors consider the effect of the magnetic field is to polarize the electrons. Contrary to weak fields, the backscattering of electrons having spin parallel to the field may be suppressed making the impurity transparent, whereas electrons antiparallel to the field are still reflected.

In the present paper we report on spin selective transmission of electrons in a quantum wire through a quantum dot formed by two impurities. The mechanism consists in lifting the degeneracy of the condition for resonant tunneling of up and down electrons through the quantum dot [8, 9] by an external magnetic field \(H\). Whereas the transmission for the spin direction which fulfills the resonance condition is finite for repulsive interaction, it vanishes for the other spin direction due to the Coulomb blockade in the quantum dot. The mechanism requires sufficiently low temperatures such that the Zeeman splitting \(Δ = gμBH\) and the Coulomb energy of the quantum dot \(\approx E_F/\rho\) are large compared to \(T\). Here \(n\) denotes the number of electrons in the quantum dot.

For weak impurities we find a resonance in the region of repulsive electron interaction where the transmission for one spin direction is perfect, provided the impurity is weaker than a critical value, whereas the other spin direction is completely blocked. For strong impurities transmission is found to change smoothly from perfect to zero when the interaction strength is increased. As a difference to the case \(H = 0\) considered in [8], we find that the resonance condition for \(H \neq 0\) is not same in the two limiting cases of strong and weak impurities and leads to two scenarios shown in Fig. 4.

A similar setup, but under very different conditions, has been considered recently in [6]. There, the Coulomb blockade effect was ignored and the magnetic field was assumed to be unrealistically strong, \(Δ = O(E_F)\).

Model.— We consider electrons in a one dimensional wire along \(x\) axis exposed to an external magnetic field. Since electrons are confined in the directions transverse to \(x\), orbital effects are suppressed and the only field effect of the magnetic field is to polarize the electrons. In the noninteracting case the Zeeman energy splits the Fermi momentum \(k_{F,s}(s = ↑, ↓)\) of the up and down spin electrons by \(|k_{F↑} - k_{F↓}|/(k_{F↑} + k_{F↓}) \approx Δ/E_F ≪ 1\). The Hamiltonian for electrons in the external impurity potential \(V(x)\) can be described by the Tomonaga-Luttinger model

\[
H = \sum_s \int dx \left\{ -i\hbar v_F \left[ \psi_{Rs}^\dagger \partial_x \psi_{Rs} - \psi_{Ls}^\dagger \partial_x \psi_{Ls} \right] \right\} + V(x)\rho_s (x) + \frac{1}{2} \sum s, s' \int dx dx' W(x - x')\rho_s (x)\rho_{s'} (x'),
\]

where \(\psi_{Rs}(x), \psi_{Ls}(x)\) are the annihilation operators for right- and left-moving spin-s electrons, \(\psi = \psi_{Rs} + \psi_{Ls}\) is the annihilation operator for spin-s electrons, \(\rho_s = \psi_{Rs}^\dagger \psi_{Rs}\) is the spin-s electron density, and \(W(x - x')\) is the screened Coulomb interaction between electrons [11].

We first consider the system without impurities. Then the model [11] describes an interacting quantum wire with four Fermi points [12]. In that situation it is useful to split terms arising from the interaction into inter-subband and intra-subband terms [13]. For repulsive and spin independent interaction electrons stay in their bands during scattering processes and the only allowed inter-subband process is the forward scattering [14]. While mutually noninteracting subsystems consisting of spin up and spin down electrons are described in the bosonized representation by the standard LL Euclidean action[15] in terms of bosonic fields \(\varphi^s, \bar{\varphi}^s\) with the Luttinger parameter (LP) \(K = \left( 1 + \frac{\bar{W}(0) - \bar{W}(2k_F)}{\pi \hbar v_F} \right)^{-1/2}\), the inter-
subband interaction is diagonalized in symmetric \( \varphi_0 = (\varphi_1 + \varphi_2)/\sqrt{2} \) and antisymmetric \( \varphi_\sigma = (\varphi_1 - \varphi_2)/\sqrt{2} \) combinations. \( \varphi_0 \) describe charge and \( \varphi_\sigma \) spin degrees of freedom. The action of the system in the absence of impurities is then given by (for details see [16, 18])

\[
\frac{S_0}{\hbar} = \sum_{\ell=0,\sigma} \frac{1}{2\pi K_\ell} \int dx \, r \left[ \frac{1}{V_\ell} (\partial_x \varphi_\ell)^2 + v_\ell (\partial_z \varphi_\ell)^2 \right],
\]

where \( K_\ell = K \left( 1 \pm \frac{K^2 V(0)}{\pi \hbar} \right)^{-1/2} \) with the convention that the upper (lower) sign corresponds to \( \ell = \rho, \sigma \). The velocities of excitations are \( v_\ell = v_F/K_\ell \), where \( v_F \) is the Fermi velocity.

Non-trivial effects come from impurities. We consider two point-like impurities, modeled as \( \delta \)-functions of the strength \( V \) and placed at \( \pm a/2 \). Introducing the displacement fields at the impurity positions \( \phi_{s1}(\tau) = \varphi_s(-a/2, \tau) \) and \( \phi_{s2}(\tau) = \varphi_s(a/2, \tau) \), the bosonized form of electron-impurity interaction reads [8, 10] \( S_1 = \sum_{s} (V k_{F,s}/\pi) \int d\tau \cos (2\phi_s + k_{F,s} a) \). To analyze the full action \( S_0 + S_1 \) it is useful to integrate out degrees of freedom outside the impurities. In that way one gets an action in terms of four fluctuating fields in imaginary time. For low frequencies, \(|\omega| \ll v_\ell/a\), the effective action reads

\[
S_{\text{eff}} = \sum_{\ell, \rho, \sigma} \sum_{k=\pm} \int \frac{d\omega}{16\pi^2} \frac{\hbar |\omega|}{K_\ell} |\Phi_{\ell k}(\omega)|^2 + \int d\tau V_{\text{eff}},
\]

where effective potential energy \( V_{\text{eff}} \) reads

\[
V_{\text{eff}}(\phi_{1\uparrow}, \phi_{2\uparrow}, \phi_{1\downarrow}, \phi_{2\downarrow}) = \sum_{\ell} \frac{1}{2} U_\ell (\phi_{s\ell}(\tau))^2 + \sum_{s} V_s \left[ \cos (2\phi_{1s} + k_{F,s} a) + \cos (2\phi_{2s} - k_{F,s} a) \right].
\]

Here we have introduced \( U_\ell = \frac{\hbar v_{\ell}}{2ak_{F,\ell}} \), \( V_s = V k_{F,s}/\pi \) and the fields \( \Phi_{\ell k} = \phi_{s\ell} + k_{F,s} a/2 \pm k_{F,s} a \) where \( k = \pm \) and our sign convention for \( \ell = \rho, \sigma \) applies. \( \phi_{s\ell} \) and \( \Phi_{\ell s} \) determine the total charge and spin, respectively, between the impurities.

The effective potential energy (4) consists of two types of terms: the charging energy \( E_C = \sum_{\ell} U_\ell \Phi_{\ell s}^2/2 \) suppresses the accumulation of charge and spin on the island between impurities, while the \( V_s \)-term tends to pin the displacement fields at the impurity positions. The part \(|\omega| |\Phi_{\ell s}|^2 \) of the action (3) is a fluctuation correction to \( E_C \) and is important at resonance points for strong impurities, when \( \Phi_{\ell s} \) are undetermined, see below.

In the following we will examine the system described by (3) in two limiting cases, for strong and weak impurity strengths. In the realistic case of repulsive interaction, we have \( K_\rho < 1, K_\sigma > 1 \) and \( U_\rho < U_\sigma \). We study the model at zero temperature, while influence of temperature is briefly considered at the end. Our strategy is to first determine the ground state from \( V_{\text{eff}} \) without fluctuations, see Eq. (3) and then to include fluctuations in order to check the stability of that ground state.

**Strong impurities.—**In the limit of very strong barriers, \( V_{\uparrow1}, V_{\downarrow1} \gg U_\rho, U_\sigma, E_F \), the ground state of the system is defined by subsequent minimization of the pinning and the charging energy, see Eq. (3). The pinning energy terms are minimal for \( 2\phi_{ps} = (1)^p k_{F,s} a + \pi (1 - 2n_{ps}) \), \( p = 1, 2 \) where \( n_{ps} \) are integers. The high degeneracy of the pinning energy is broken by the charging energy. Plugging \( \phi_{ps} \) into \( E_C \) and defining \( n_s = n_{2s} - n_{1s} \) one gets

\[
E_C(n_1, n_{\downarrow1}) = \frac{U_\rho}{2} ((k_{F,s} a + k_{F,\downarrow} a - \pi (n_1 + n_{\downarrow1}))^2
+ \frac{U_\sigma}{2} ((k_{F,s} a - k_{F,\downarrow} a - \pi (n_1 - n_{\downarrow1}))^2.
\]

To characterize different nonequivalent minima of (5) it is useful to restrict the Fermi momenta to satisfy \( n < k_{F,s} a/\pi, k_{F,\downarrow} a/\pi \leq n + 1 \), where \( n \geq 0 \) is an integer. This implies \( n \leq n_1, n_{\downarrow1} \leq n + 1 \). The particle number on the island is \( n_1 + n_{\downarrow1} \). The ground states resulting from the minimization of the charging energy (5) are shown in Fig. 1. For generic values of \( k_{F,s} a \), the ground state is uniquely determined. However, at special lines different ground states meet. These lines define the resonance conditions: while the number of particles on the island with one spin direction is fixed at the same value on both sides of the boundary, the number of electrons with the opposite spins changes by \( \pm 1 \). \( E_C(n_1, n_{\downarrow1}) = E_C(n_1 \pm 1, n_{\downarrow1}) \) and \( E_C(n_1, n_{\downarrow1}) = E_C(n_1 \pm 1, n_{\downarrow1} \pm 1) \) are the resonance conditions for the up and down spin electrons, respectively.

![Fig. 1](image-url)
As a result a particle having the degenerate spin can tunnel through the quantum dot in a sequential tunneling process without changing its energy. Hence we have a spin-selective barrier transparency.

We further solve the model along the boundary line where $E_C(n + 1, n) = E_C(n + 1, n + 1)$. Similar results hold for other cases. The fields $\phi_{\sigma \uparrow}$, $p = 1, 2$ are locked by the strong impurity pinning and have fixed values of $n_\sigma$. Approximating the nonlinear cosine term by a quadratic term for the $V$ one can integrate them from the action $[8, 20, 21]$. The resulting effective action then reads

$$S'_{\text{eff}} = \int \frac{d\omega}{4\pi^2} \frac{h[\omega]}{K_{\text{eff}}} \left( |\phi_{1\downarrow}(\omega)|^2 + |\phi_{2\uparrow}(\omega)|^2 \right) + \int d\tau V_{\text{eff}} \left( -k_\uparrow \cos \phi_{1\downarrow} + \frac{k_\downarrow}{2} \phi_{1\downarrow}, \phi_{2\uparrow} \right),$$

with $K_{\text{eff}} = \frac{2K_{\rho} + K_{\sigma}}{K_{\rho} + K_{\sigma}}$. It describes the resonant tunneling of spin down electrons and is analogous to the case of spinless electrons $[8]$. The partition function is dominated by tunneling events connecting degenerate minima of the strong impurity potential. Using the Coulomb gas representation $[8, 20, 21]$ one can produce the renormalization group equations for the tunneling transparency $t_\downarrow$ of barriers for spin-$\downarrow$ electrons. For strong impurity potential $V_1$ it reads $dt_\downarrow = t_\downarrow \left( 1 - (2K_{\text{eff}}) \right)$, from which we get that for $K_{\text{eff}} > \frac{1}{2}$ the transparency $t_\downarrow$ increases, or equivalently, the strength of $V_1$ flows to smaller values at low energies. Outside the resonance lines, $t_\downarrow$ flows to zero for any repulsion, similar to the single impurity case.

Weak impurities.—In the limit of weak impurities, $V_1, V_\downarrow \ll U_\rho, U_\sigma$, the action $[3]$ is minimized for $\Phi_{\sigma \uparrow} = 0$. This corresponds to fixed charge and spin on the island. Integrating out the $\Phi_{\uparrow \downarrow}$ fluctuations from $[3]$, new scattering processes of the form $\sum_s 2V_\downarrow \cos(k_F s a) \cos(\phi_{1\downarrow} + \phi_{2\uparrow}) + V^{(2)} \sin(k_F s a) \sin(k_F a) \cos \Phi_{1\downarrow}$ are generated, where $V^{(2)} = V_1 V_\downarrow U_\rho U_\sigma / 2 d F_{\downarrow \uparrow}$. Other generated higher order processes are irrelevant for repulsive interaction. The resonance condition for the spin-$s$ particles is now given by $\cos(k_F s a) = 0$.

For the generic situation $\cos(k_F s a) \neq 0$, the single electron backscattering processes are the most important ones. To leading order in the impurity potential, the renormalization group (RG) flow equations is $d_t V_\downarrow = V_\downarrow \left[ 1 - (K_{\rho} + K_{\sigma}) / 2 \right]$, from which we conclude that backward scattering terms $V_\downarrow$ are relevant for $K_{\rho} + K_{\sigma} < 2$. Since the point impurity is a local quantity it can not renormalize bulk quantities such as $K_{\rho}$, $K_{\sigma}$, and the flow of $V_\downarrow$ is vertical $[8, 22]$. The flow diagram for $V_\downarrow$ is shown in Fig. 2a. Since the two limiting cases have opposite flow, it is plausible to expect a line of attractive fixed points somewhere in between, corresponding to a new phase, where spins of one direction (here down spins) have nonzero transmission at zero temperature, while the other spin direction is blocked $[23]$.  

In the resonance case $\cos(k_F s a) = 0$ and $|\cos(k_F s a)| \notin \{0, 1\}$, the two-particle scattering processes should be taken into account (only for spin-$\downarrow$ electrons since spin-$\uparrow$ already have backscattering in the lowest nonvanishing order). From the RG flow equation $d_t V^{(2)} = V^{(2)} \left( 1 - 2K_{\rho} \right)$, we conclude that spin-$\downarrow$ electrons are effectively free at low energies for $K_{\rho} > 1/2$. In Fig. 2b we show the flow of $V_1$. Again the flows of two limiting cases are opposite, resulting in a separatrix in between the two resulting phases: perfectly conducting for spin down for $K_{\rho} = K_{\sigma} = K_{\text{eff}} = 1$.

Transport.—Now we will consider the conductance of our system using the anticipated flow diagram, see Fig. 2 Assuming the applied voltage across the dot is $V_G$ and at the ends of the wire is $V_L$, an additional term should be included in the action $[4]$, which reads $-eV_G \int d\tau \Phi_{\rho \uparrow} / \pi - eV_L \int d\tau \Phi_{\rho \downarrow} / (2\pi)$. The voltage $V_L$ pushes the electrons to advance in one direction along the wire, while the gate voltage $V_G$ serves as a single tuning parameter. Due to nonzero $V_G$, the shifted Fermi momenta $k_{F\sigma} = k_{F\downarrow} - eV_G K_{\rho \uparrow}^2$ should be taken in the above results, e.g., for the resonance conditions. This means the latter can be achieved by adjusting $V_G$ for fixed both
magnetic field and distance between impurities.

Without impurities or for attractive interaction in the low energy limit the system is described by Eq. (2) and has the perfect non-spin-polarized conductance \( G_\uparrow = G_\downarrow = e^2/h \) \([\ref{footnote1}]\). The situation drastically changes when impurities are present. In the non-resonant case, our model translates into the single impurity problem with the LP \( K_{\text{eff}} \). Therefore, the conductance is suppressed at low \( V_L \) for repulsive interaction for both spin directions as \( \sim V_L^2/K_{\text{eff}}^{-2} \).

On the resonance that corresponds to Fig. 2b, i.e. for strong impurities when the charge state for spin-\( \downarrow \) electrons is degenerate on the island, one gets spin-polarized conductance. Inside the region where the new line of fixed points appears, different scattering is experienced by two spin orientations. While \( G_\uparrow \) is suppressed at low voltages as \( \sim V_L^2/K_{\text{eff}}^{-2} \) near the point \( K_{\text{eff}} = 1/2 \), and as \( \sim V_L^{(K_\rho + K_\sigma)^{-2}} \) for \( K_\rho + K_\sigma \to 2^- \), \( G_\downarrow \) is not suppressed even at very low voltages. It is controlled by the fixed point value \( V_L^{*}(K_{\text{eff}}) \) which determines the effective strength of impurity scattering for a given \( K_{\text{eff}} \). We can estimate the conductance as \( G_\downarrow(K_{\text{eff}}) \approx e^2/h \frac{1}{1+\left|\nu_1^{*}(K_{\text{eff}})/E_F\right|^2}. \) Within our approach we are not able to determine \( V_L^{*}(K_{\text{eff}}) \). We expect that the fermionic method used in Ref. [10], which is beyond the scope of the present paper, could give more results.

On the resonance that corresponds to weak impurities, Fig. 2a, the system again has spin-polarized conductance which is controlled by the fixed points. In the lowest non-trivial order we have \( G_\downarrow = e^2/h \) for \( K_\rho > 1/2 \) and \( G_\uparrow \sim V_L^{(K_\rho + K_\sigma)^{-2}} \), for not too big initial values of impurity strengths. Otherwise the spin polarization is destroyed and the conductance behaves as in the non-resonant case.

So far we considered zero temperatures. At finite temperature the picture will be qualitatively unchanged until the electron thermal energy is much smaller than the charging and Zeeman energy. In the opposite case, which is the high frequency limit \( |\omega| \gg v_F/a, \) or \( T \gg K_{\ell}/2U_{\ell} \), for the starting action one would get Eqs. (3) and (4) with the replacements \( K_\ell \to K_{\ell}/2, U_{\ell} \to 0 \). Then the coherent effects of impurities are missing and our system effectively has the single impurity behavior \([\ref{footnote1}, \ref{footnote2}]\).

Conclusions.— We have shown that a quantum wire with two impurities in an external magnetic field may have spin-filter properties for repulsive interaction. Our study is based on the resonance tunneling phenomenon which may be tuned by a single parameter for only one spin polarization.

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