Proper acceleration, the geometric tachyon and the
dynamics of a fundamental string near Dp branes

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Abstract
We present a detailed analysis of our recent observation that the origin of the
geometric tachyon, which arises when a Dp brane propagates in the vicinity of
a stack of coincident NS5 branes, is due to the proper acceleration generated
by the background dilaton field. We show that when a fundamental string
(F-string), described by the Nambu–Goto action, is moving in the background
of a stack of coincident Dp branes, the geometric tachyon mode can also appear
since the overall conformal mode of the induced metric for the string can act as
a source for proper acceleration. We also studied the detailed dynamics of the
F-string as well as the instability by mapping the Nambu–Goto action of the F-
string to the tachyon effective action of the non-BPS D-string. We qualitatively
argue that the condensation of the geometric tachyon is responsible for the
(F,Dp) bound state formation.

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1. Introduction

Type II string theories are known to admit two types of Dp branes as classical solutions. The
BPS Dp branes [1–3] are supersymmetric and stable while the non-BPS Dp branes [4, 5] are
neither supersymmetric nor stable. The instability of the non-BPS Dp branes arises because
the lowest lying state of the open string theory with both ends attached to the brane corresponds
to a tachyon. The dynamics of both the BPS as well as the non-BPS Dp branes are described
by Dirac–Born–Infeld (DBI) actions involving the appropriate background fields. The DBI
action for the non-BPS Dp brane [6–9] is a functional of the tachyon field and has a prefactor
in the Lagrangian density that corresponds to the potential for the tachyon field. The tachyon
potential has the interesting property that it has a finite maximum at the origin (in the field
space) while it vanishes asymptotically leading to a vanishing pressure for late times [10]. The DBI action for the BPS $D_p$ brane [11–13], on the other hand, does not have any functional dependence on a tachyon degree of freedom and the multiplicative factor in the Lagrangian density depends on the dilaton background of the theory.

In spite of these differences, it has been noted recently that in the vicinity of a stack of NS5 branes, the dynamics of a BPS $D_p$ brane resembles that of a non-BPS $D_p$ brane [14]. In fact, it has been shown that the DBI action for the radial mode living on the BPS $D_p$ brane can be mapped to the tachyon field on the non-BPS $D_p$ brane. In this case the exponential dilaton prefactor in the Lagrangian density of the BPS $D_p$ brane can be thought of as playing the role of the effective potential for the tachyon. This potential falls off exponentially to zero at short distances while it behaves like a gravitational potential for large separations so that as the BPS $D_p$ brane approaches the NS5 branes, this potential resembles the tachyon potential in the DBI action for the non-BPS $D_p$ brane. Furthermore, the pressure for this system has also been observed to fall off exponentially to zero at late times, much like that in the case of the rolling tachyon in the decay of a non-BPS $D_p$ brane to tachyon matter whose equation of state corresponds to that of a pressureless fluid [15–17]. As a result, the radial mode of the BPS $D_p$ brane near a stack of NS5 branes has been called a ‘geometric tachyon’. Such an instability can be physically understood because the configuration of the BPS $D_p$ brane in the background of the stack of NS5 branes breaks all of the spacetime supersymmetry of the system.

This instability has been further studied by compactifying one of the transverse directions of the NS5 branes on a circle and placing the BPS $D_p$ brane at a point diametrically opposite to the NS5 branes [18]. Such a configuration corresponds to a saddle point in the potential energy of the BPS $D_p$ brane leading to a tachyonic mode for translations along the compactified direction (circle). More recently a close relationship between the geometric tachyon and the universal open string tachyon has been extensively discussed by Sen [19] (see also [20, 21]). The geometric tachyon has also played an interesting role in the inflationary models in cosmology [22–27].

In a recent letter [28], we studied the origin of the tachyonic instability in the motion of a BPS $D_p$ brane in the vicinity of the stack of coincident NS5 branes in terms of the motion of a point particle in the transverse space of the five branes. We observed that in this case the particle experiences a proper acceleration due to the background dilaton field and as a result deviates from its geodesic. We argued that it is this dilaton-dependent acceleration which is responsible for the motion of the radial mode to be that of an inverted simple harmonic oscillator at small separations, leading to the tachyonic instability. In this paper we give a detailed and systematic analysis of our results.

We extend our observation to other dynamical systems involving different backgrounds. For example, instead of the BPS $D_p$ brane, we consider the motion of a fundamental string (F-string) described by the Nambu–Goto action where there is no exponential dilaton prefactor. If the F-string is in the background of the NS5 branes, we know that the system is supersymmetric (it is S-dual to the D1–D5 system) and stable so that the F-string is nondynamical. On the other hand when the fundamental string is in the background of a stack of coincident $D_p$ branes, the dynamics of the F-string would still be described by the Nambu–Goto action without the dilaton prefactor. Naively, therefore, in this background one would not expect a proper acceleration associated with the motion of the F-string. However, we know that the F-D5 system is S-dual to the D1-NS5 system and for the latter, as we have discussed the dynamics of the radial mode becomes tachyonic near the NS5 branes. For this to be compatible with our proposal, therefore, it is necessary that there should be a source of proper acceleration in the system. That this is indeed the case can be seen from the fact that,
even though the Nambu–Goto action is free from the dilaton prefactor, it inherits one from the induced metric when the overall conformal factor in the background metric of the D5 brane is taken out. This conformal factor gives rise to the proper acceleration which is responsible for the deviation from geodesic motion leading to the tachyonic instability. As before, this conformal factor can be thought of as the tachyon potential in the open string effective action for the tachyon. The same conclusion also follows for other $D_p$-brane backgrounds with $p < 5$.

We study in detail the dynamics of the F-string in various $D_p$-brane backgrounds with $p \leq 5$. We map the Nambu–Goto action to the effective action for the tachyon in a non-BPS D-string. We show that when the F-string is far away from the $D_p$ brane, the potential $V(T)$ (where $T$ denotes the tachyon field to be defined later in terms of the radial mode $R$) reduces to the expected attractive gravitational potential, but as it approaches the $D_p$ brane the potential vanishes exponentially for $p = 5$ but as a power of $T$ for $p < 5$. This fall off is the expected behavior of the effective potential for the tachyon [10]. We solve the dynamical equation for the F-string and determine its trajectory in various cases of interest. We obtain the expression for pressure when the separation between the F-string and the $D_p$ brane approaches zero and we show that at late times, the pressure vanishes exponentially for $p = 5$ and as a power when $p < 5$, resulting in a pressureless dust (or tachyon matter) [16]. This is indeed expected of an unstable $D_p$ brane with the tachyon rolling down to the minimum of the potential. We thus recover the analog of a rolling tachyon on an unstable D-string as the dynamics of the F-string propagating in the $D_p$ brane background. Of course, it would be interesting to know what would correspond to the end-point state of this tachyon condensation. From the tension formula following from [29, 30], we conjecture that as the F-string approaches the $D_p$ branes, the F-string loses almost all of its energy density and thus the F-string really gets melted down into the $D_p$ branes forming an $(F, D_p)$ bound state. This would correspond to a non-threshold bound state and would preserve half of the spacetime supersymmetry.

This paper is organized as follows. In the following section, we briefly discuss the motion of a point particle in a gravitational field subjected in addition to a proper acceleration. We point out how a Lagrangian formulation can be used to obtain the geodesic motion, modified by the acceleration term, as Euler–Lagrange equation of motion. In section 3, we investigate the dynamics of a $D_p$ brane in the background of a stack of NS5 branes as a point particle motion in the transverse space. This analysis is then extended to the motion of a fundamental string in the background of a stack of coincident $D_p$ branes in section 4. We discuss the case of $D5$ branes in subsection 4.1 and other $D_p$ ($p < 5$) branes in subsection 4.2. Here we also briefly discuss the limitations of the supergravity description as well as the end-point of the geometric tachyon condensation. We present our conclusions in section 5.

2. Proper acceleration

In this section, we review briefly the concept of proper acceleration in both flat spacetime as well as in a curved background. To begin with let us consider a relativistic point particle moving along a straight trajectory (linear motion) in flat spacetime with the line element

$$ds^2 = -dt^2 = \eta_{\mu\nu} dx^\mu dx^\nu,$$

where we are using the metric with signature $(-, +, \ldots, +)$. If the particle is moving along the $x$-axis, the Lorentz factor follows from (1) to be (we are setting $c = 1$)

$$\gamma = \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - v^2}},$$

(2)
where
\[ v = \frac{dx}{dt}, \]  
(3)
represents the instantaneous velocity of the particle along the \( x \)-axis. It follows from the definition in (2) that
\[ \gamma^2 v^2 + 1 = \gamma^2. \]  
(4)
The motion of the particle in this case is effectively two dimensional and neglecting the other spatial coordinates we note that the proper velocity of the particle can be represented as the two-dimensional vector
\[ u^\mu = \frac{dx^\mu}{d\tau} = (\gamma, \gamma v), \]  
(5)
which can be seen using (2) to satisfy
\[ \eta_{\mu\nu} u^\mu u^\nu = -1. \]  
(6)
If \( v \) is not constant (namely, if there is an acceleration along the \( x \)-axis), the motion of the particle is nonuniform and we can define the proper acceleration of the particle as
\[ a^\mu = \frac{du^\mu}{d\tau} = \gamma \left( \frac{dy}{dt}, \frac{d(\gamma v)}{dt} \right). \]  
(7)
Using the identity in (4) we can easily show that
\[ \frac{dy}{dt} = v \frac{d(\gamma v)}{dt} = \alpha v, \]  
(8)
so that we can write
\[ a^\mu = \frac{du^\mu}{d\tau} = \alpha (\gamma v, \gamma) = \alpha \epsilon^{\mu\nu} u_\nu, \]  
(9)
where we have identified
\[ \alpha = \frac{d(\gamma v)}{dt}, \]  
(10)
and it is clear from (9) that \( \alpha \) is simply the instantaneous acceleration of the particle. Furthermore, \( \epsilon^{\mu\nu} \) denotes the two-dimensional Levi-Civita tensor with \( \epsilon^{01} = 1 \) and it now follows from (5) and (9) that
\[ \eta_{\mu\nu} u^\mu a^\nu = 0, \quad \eta_{\mu\nu} d^\mu a^\nu = \alpha^2, \]  
(11)
so that the proper acceleration is space like and is orthogonal to the proper velocity. As a result, (6) continues to hold even when the particle is subjected to a proper acceleration.

If we assume that the instantaneous acceleration \( \alpha \) is a constant, the dynamical equations (7) can be solved leading to the fact that a constant proper acceleration leads to a hyperbolic motion for the particle (as opposed to parabolic motion in the nonrelativistic case). This can be seen easily as follows. From (9), we note that for constant \( \alpha \), we have
\[ \frac{d^2 u^\mu}{d\tau^2} = \alpha^2 u^\mu, \]  
(12)
which already reflects the hyperbolic nature of particle motion. Explicitly, for constant acceleration, we see from (9) that
\[ u^\mu - \alpha \epsilon^{\mu\nu} x_\nu = \text{constant}, \]  
(13)
and the solutions of (12) are given by
\begin{align}
t(\tau) &= \frac{1}{\alpha} \sinh(\alpha \tau), \\
x(\tau) &= \frac{1}{\alpha} (\cosh(\alpha \tau) - 1 + \alpha x_0),
\end{align}
where \(x_0 = x(\tau = 0)\) and we have chosen the initial conditions \(t(\tau = 0) = 0, v(\tau = 0) = 0\).

The particle follows a hyperbolic space-like trajectory
\begin{align}
t^2 + \left( x - x_0 + \frac{1}{\alpha} \right)^2 &= \frac{1}{\alpha^2},
\end{align}
with the asymptotes of the hyperbola determined by the value of \(\alpha\).

The notion of a proper acceleration can be carried over to a curved background with metric \(G_{\mu\nu}\) in a straightforward manner. Here, of course, coordinates are not four vectors, but the proper velocity is a four vector which satisfies the condition
\begin{align}
G_{\mu\nu} u^\mu u^\nu = -1,
\end{align}
The definition of the proper acceleration (7) in this case can be generalized covariantly as
\begin{align}
a^\mu = D_\tau u^\mu = \frac{du^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda,
\end{align}
where \(\Gamma^\mu_{\nu\lambda}\) denotes the Christoffel symbol constructed from the metric \(G_{\mu\nu}\) of the curved background. Using the metric compatibility condition \(D_\tau G_{\mu\nu} = 0\), it follows easily from (17) that
\begin{align}
G_{\mu\nu} u^\mu a^\nu = 0,
\end{align}
and the normalization condition (16) continues to hold even in the presence of an acceleration.

Let us next see how the geodesic motion of a particle is modified due to the presence of an acceleration. There are several ways of incorporating acceleration into an action formalism. From the point of view of our discussions in the following sections, let us consider the action given by
\begin{align}
S = -\int dt f(x) \sqrt{-G_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} = \int dt L = -\int d\tau f(x) \sqrt{-G_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}},
\end{align}
where \(f(x)\) is a given function of the coordinates and \(G_{\mu\nu}\) is the metric of the curved spacetime. The line element in this case is given by
\begin{align}
-d\tau^2 = G_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau},
\end{align}
so that the Lorentz factor takes the form,
\begin{align}
\gamma = \frac{dt}{d\tau} = \frac{1}{\sqrt{-G_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}},
\end{align}
where an ‘overdot’ represents a derivative with respect to \(t\). It now follows from the action (19) that
\begin{align}
\frac{\partial L}{\partial x^\mu} &= f(x)G_{\mu\nu} \frac{dx^\nu}{d\tau}, \\
\frac{\partial L}{\partial \dot{x}^\mu} &= \frac{1}{2} f(x) \partial_{\mu} G_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - \frac{d\nu}{d\tau} \partial_{\mu} f(x).
\end{align}
Therefore, the Euler–Lagrange equation for the system takes the form
\begin{align}
\frac{d}{d\tau} \left( f(x)G_{\mu\nu} \frac{dx^\nu}{d\tau} \right) = \frac{1}{2} f(x) \partial_{\mu} G_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \partial_{\mu} f(x) = 0.
\end{align}
The above equation can be simplified leading to the geodesic equation with an acceleration term of the form

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = a^\mu,$$

where

$$a^\mu = -\left(G^{\mu\nu} + \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}\right) \partial_\nu(\ln f(x)), \quad (26)$$

and

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\rho}(\partial_\lambda G_{\rho\nu} + \partial_\nu G_{\rho\lambda} - \partial_\rho G_{\nu\lambda}). \quad (27)$$

We see from (26) that by construction we have

$$G_{\mu\nu}(\frac{dx^\mu}{d\tau})u^\nu = G_{\mu\nu}u^\mu u^\nu = 0,$$

where we have used $G_{\mu\nu}u^\mu u^\nu = -1$, which shows that the proper velocity and the proper acceleration are orthogonal to each other. We see from this simple example that a coordinate-dependent overall multiplicative factor in front of the Lagrangian for a point particle motion leads to a proper acceleration which modifies the trajectory of the particle from that of a geodesic. In the following sections, where we consider the motion of a BPS Dp brane in the background of a stack of coincident NS5 branes and the motion of a fundamental string in the background of a stack of coincident Dp branes, we will show how the motion can be regarded as the motion of a relativistic point particle in the transverse space of NS5 branes in the first case and the Dp branes in the second case, respectively. We will make use of our discussion in this section there and show that the motion in those cases also will be subjected to a proper acceleration orthogonal to the proper velocity and thereby leading to a deviation from the geodetic motion of the particle. This in turn will be shown to give rise to the tachyonic instability when the branes come close to each other.

### 3. Dp-brane motion in NS5-brane background

We now present a detailed analysis of the dynamics of a Dp brane propagating in the background of a stack of N number of coincident NS5 branes by formulating this as the motion of a relativistic point particle in the transverse space of the five branes. We take the five branes to be stretched in the directions $(x^1, x^2, \ldots, x^5)$ and denote the world volume directions to be $x^{\bar{\mu}}, \bar{\mu} = 0, 1, \ldots, 5$. The transverse directions are labeled by $x^m, m = 6, 7, 8, 9, \ldots$.

The Dp brane lies parallel to the NS5 branes along the directions $x^{\bar{\mu}}, \bar{\mu} = 0, 1, 2, \ldots, p$ with $p \leq 5$. Thus the Dp brane is taken to be point like in the directions $x^{\bar{\mu}}$. Note that if we place the Dp brane at a finitely large distance $r = (x^m x^m)^{1/2}$, from the coincident five branes, it experiences an attractive force due to gravitational and dilatonic interactions between them. Since at weak string coupling ($g_s$), the Dp brane is lighter than the stack of NS5 branes, their mass being proportional to $1/g_s$ and $N/g_s^2$ respectively, it will move toward the NS5 branes. This motion depends upon the details of the background fields created by the NS5 branes. So we give below the background metric, the dilaton and the field strength for the NSNS gauge field of a stack of coincident NS5 branes as $[37, 38],$

$$ds^2 = -dr^2 + \eta_{\bar{\mu}\bar{\nu}} dx^{\bar{\mu}} dx^{\bar{\nu}} + G_{mn} dx^m dx^n,$$

$$G_{mn} = H(r) \delta_{mn},$$

$$e^{2(\phi - \phi_0)} = H(r) = 1 + \frac{N \ell_s^2}{r^2},$$

$$H_{mpq} = -e^{\phi(\delta_{mp} \partial_q \phi + \text{terms}),}$$

where $[31–33]$ for some related earlier works and $[34–36]$ for the (NS5,Dp) supergravity configurations.
where \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1, \ldots, +1) \), \( H \) is the harmonic function in the transverse space describing the \( N \) coincident NS5 branes, \( t_s \) denotes the string length and the string coupling is \( g_s = e^{\phi_0} \).

We denote the world volume coordinates of the Dp brane by \( \xi^\mu, \mu = 0, 1, \ldots, p \). However, we can use the reparametrization invariance on the world volume and identify \( \xi^\mu = x^\mu \). Since our interest lies in the dynamics of the Dp brane in the transverse space of the five branes, denoted by \( (x^6, x^7, x^8, x^9) \), they will give rise to the scalar fields, \( X^m(\xi^\mu), m = 6, 7, 8, 9 \) on the world volume of the Dp branes. The dynamics of the Dp brane is described by the dynamics of these scalar fields governed by the DBI action

\[
S_p = -\tau_p \int d^{p+1}\xi \, e^{-\phi(\xi)} \sqrt{-\det (G_{\mu\nu} + B_{\mu\nu})},
\]

where \( \tau_p \) is the tension of the Dp brane. \( G_{\mu\nu} \) and \( B_{\mu\nu} \) are the induced metric and the B-field on the world volume, given by \( (X^\mu = x^\mu) \)

\[
G_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu} \frac{\partial X^B}{\partial \xi^\nu} G_{AB}(X),
\]

\[
B_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu} \frac{\partial X^B}{\partial \xi^\nu} B_{AB}(X).
\]

Here the indices \( A, B = 0, 1, \ldots, 9 \) and \( G_{AB} \) and \( B_{AB} \) are the metric and the B-field in ten dimensions, given in (28). We are interested in the case when the fields representing the position of the Dp brane \( X^m, m = 6, \ldots, 9 \), depend only on time, \( X^m = X^m(t) \). In this case, the action (29) simplifies considerably as we get contribution from the metric only (the antisymmetric field does not contribute) and takes the form,

\[
S_p = -\tau_p V_p \int dt \, e^{-(\phi-\phi_0)} \sqrt{1 - G_{mn} X^n X^m},
\]

where \( V_p \) is the volume of the \( p \)-dimensional space in which the Dp brane is stretched out. The dilaton field and the metric \( G_{mn} \) are related to the harmonic function as noted in (28).

We rewrite the above action in a suggestive form as,

\[
S_p = -\tau_p V_p \int dt \, e^{-\phi} \sqrt{-G_{\tilde{m}\tilde{n}} X^{\tilde{m}} X^{\tilde{n}}}
\]

\[
= -\tau_p V_p \int dt \, e^{-\phi} \sqrt{-G_{\tilde{m}\tilde{n}} \frac{dX^{\tilde{m}}}{dt} \frac{dX^{\tilde{n}}}{dt}},
\]

where \( \phi = \phi - \phi_0, G_{\tilde{m}\tilde{n}} = (-1, G_{mn}) \) with \( \tilde{m} \) and \( \tilde{n} \) taking values \( \tilde{m} = (0, m) = 0, 6, 7, 8, 9; X^0 = t \) and \( t \) is the proper time. The action (32) can be thought of as describing the dynamics of a relativistic point particle in gravitational as well as dilatonic backgrounds. We remark that the action (29) reduces to that of a relativistic point particle because we assumed \( X^{\tilde{m}} \) to depend only on time. In principle it could depend on other world-volume coordinates of the Dp brane and then the action cannot be reduced to the point particle action. Note that \( \phi \) does not depend on \( X^0 \) and, therefore, on time explicitly. However, since it depends upon \( X^m \), which is a function of time, it has an implicit time dependence. We note the similarity of the action (32) with (19) when \( e^{-\phi} \) is identified with \( f \). The Lorentz factor here takes the form,

\[
\gamma = \frac{dt}{d\tau} = \frac{1}{\sqrt{-G_{\tilde{m}\tilde{n}} X^{\tilde{m}} X^{\tilde{n}}}}.
\]

Denoting the proper velocity of the particle as \( u^\tilde{m} = dX^{\tilde{m}}/d\tau \), it can be checked easily that \( G_{\tilde{m}\tilde{n}} u^\tilde{m} u^{\tilde{n}} = -1 \) as in (16). As we mentioned in the introduction, in the vicinity of the NS5 branes, the dynamics of a BPS Dp brane resembles that of a non-BPS Dp brane and...
there is a tachyonic instability for the BPS D_p brane. However, we note that the momentum of the particle as obtained from (32) is given as \( P_\bar{m} = \tau_p V_p e^{-\bar{\phi}} G_{mn} u^m \), which leads to \( P^2 = G^{\bar{m}\bar{n}} P_{\bar{m}} P_{\bar{n}} = - (\tau_p V_p)^2 e^{-2\bar{\phi}} \) and this makes it clear that such a particle is not tachyonic in the conventional sense unless the dilaton field becomes complex.

Therefore, to understand the origin of the tachyonic instability, let us analyze the equations of motion following from (32) which correspond to the motion of a particle in a curved background subject to an acceleration, namely,

\[
\frac{d^2 X^\mu}{d\tau^2} + \Gamma^\mu_{\bar{p} \bar{n}} \frac{dX^\bar{p}}{d\tau} \frac{dX^\bar{n}}{d\tau} = a^\mu,
\]

(34)

where \( \Gamma^\mu_{\bar{p} \bar{n}} \) is the Christoffel symbol constructed from the metric \( G_{mn} \) and the proper acceleration \( a^\mu \) can be computed from equation (26) with \( f = e^{-\bar{\phi}} \) as,

\[
a^\mu = \left( G^{\bar{m}\bar{n}} + \frac{dX^\bar{m}}{d\tau} \frac{dX^\bar{n}}{d\tau} \right) \partial_\bar{n}\bar{\phi}.
\]

(35)

Thus we note that the dilaton background is responsible for a proper acceleration leading to a deviation of the trajectory of the particle from its geodesic. It can be checked easily that \( G_{\bar{m} \bar{n} \bar{p}} a^\bar{p} = 0 \) so that the proper acceleration is orthogonal to the proper velocity as would be expected for a relativistic system. This is reminiscent of a Rindler particle executing hyperbolic motion [39] and clarifies the origin of the hyperbolic solution obtained in [14].

We can compute the energy–momentum tensor by using the general formula

\[
T^{\mu\nu} = -\frac{\partial L}{\partial (\partial_\mu X^m)} \partial^\nu X^m + \eta^{\mu\nu} L,
\]

(36)

where \( L \) is the Lagrangian of the action (32) and the non-vanishing components take the explicit forms,

\[
T^{00} = \tau_p V_p \gamma e^{-\bar{\phi}} \equiv E, \quad T^{ij} = -\tau_p V_p \gamma^{-1} e^{-\bar{\phi}} g^{ij} \equiv p \delta^{ij},
\]

(37)

where \( \gamma \) is the Lorentz factor defined in (33) and \( E, p \) denote the energy and the pressure of the system, respectively. From time translation invariance we expect energy to be conserved and similarly rotational invariance in the transverse space leads to the conservation of angular momentum in the system.

The time component \((\bar{m} = 0)\) of the equation of motion (34) yields

\[
\frac{d\gamma}{d\tau} = \gamma \frac{d\bar{\phi}}{d\tau},
\]

(38)

which we recognize from (37) to lead to conservation of energy. On the other hand, using (38) the dynamical equation (34) for \( \bar{m} = m \) takes the form

\[
X^m + \Gamma^m_{\bar{p} \bar{n}} X^\bar{p} X^\bar{n} + G_{pq} X^p X^q G^{mn} \partial_n \bar{\phi} = 0.
\]

(39)

We note that for large separations the leading behavior of this equation is the free particle motion described by \( \ddot{X}^m = 0 \). This corresponds to the vanishing of the gravitational force as well as the acceleration \( a^m \) for large separations in the leading order. We study below the dynamics of the system in the next to leading order.

For this purpose it is simpler to work in the spherical-polar coordinates. Using the fact that angular momentum is conserved, we can restrict the motion of the particle to a plane with the radial mode \( R \) and the angular mode \( \Theta \). In this case the line element in (28) takes the form

\[
-d\tau^2 = -d\tau^2 + H dR^2 + H R^2 d\Theta^2,
\]

(40)

and correspondingly the Lorentz factor (33) becomes,

\[
\gamma = \frac{1}{\sqrt{1 - H R^2 - H R^2 \Theta^2}}.
\]

(41)
The nonvanishing components of $\Gamma$ are given by,

$$\Gamma^{R}_{RR} = \partial_{R} \ln \sqrt{H}, \quad \Gamma^{\alpha}_{R\alpha} = -R^{2} \partial_{R} \ln \sqrt{H R^{2}}.$$

In the spherical coordinates (39) has the form

$$\dot{R} - R \ddot{\Theta}^{2} + \frac{1}{2 H^{2}} \partial_{R} H (2 H \dot{R}^{2} - 1) = 0,$$

$$\dot{\Theta} + \frac{1}{H R^{2}} \partial_{R} (H R^{2}) \dot{R} = 0.$$

Since $\Theta$ is an angular coordinate, its conjugate gives the angular momentum of the form

$$L = \tau_{p} V_{p} \gamma e^{-\delta} H R^{2} \dot{\Theta} = E H R^{2} \dot{\Theta}.$$

Defining the quantity

$$H R^{2} \dot{\Theta} = \frac{L}{E} = \ell,$$

we note that the second equation in (43) leads to the conservation condition

$$\frac{d}{dt} (H R^{2} \dot{\Theta}) = \frac{d}{dt} \left( \frac{L}{E} \right) = \frac{d\ell}{dt} = 0,$$

namely, the angular momentum associated with the motion of the particle is conserved.

The true dynamics of the system is contained in the $R$-equation in (43). Using (44) as well as the fact that energy in (37) is conserved, we obtain from (41)

$$R^{2} = \frac{1}{H} \left( 1 - \frac{1}{H} \left( \left( \frac{\tau_{p} V_{p}}{E} \right)^{2} + \ell^{2} \right) \right) \geq 0,$$

which determines (using the form of $H$ in (28)) that for $\frac{\tau_{p} V_{p}}{E} \geq 1$, we must have $(N \ell_{s}^{2} - \ell^{2}) \geq 0$ and $R^{2} \leq R_{0}^{2}$, while for $\frac{\tau_{p} V_{p}}{E} \leq 1$, we can have either $(N \ell_{s}^{2} - \ell^{2}) \geq 0$ without any restriction on $R$, or $(N \ell_{s}^{2} - \ell^{2}) \leq 0$ with $R^{2} \geq R_{0}^{2}$ where $R_{0}^{2} = \frac{1}{(2 \tau_{p} V_{p})^{-1}}$. Since we are interested in the behavior of the system close to the origin $R \approx 0$, it is clear that we must have $(N \ell_{s}^{2} - \ell^{2}) \geq 0$ independent of the value of the ratio $\frac{\tau_{p} V_{p}}{E}$. As it is natural to assume that the D$p$ brane starts out infinitely far away, we would assume $\frac{\tau_{p} V_{p}}{E} \leq 1$, although what is really important for the analysis of the behavior near the NS5 branes is that $N \ell_{s}^{2} - \ell^{2} \geq 0$.

Using the angular–momentum conservation relation (46), the radial equation in (43) can be simplified to have the form,

$$\ddot{R} - \frac{(N \ell_{s}^{2} - \ell^{2}) N \ell_{s}^{4}}{(R^{2} + N \ell_{s}^{2})^{3}} \left( R + \frac{2}{R_{0}^{2}} - \frac{1}{N \ell_{s}^{2}} \right) R^{3} = 0.$$

This is an exact expression and we will see the behavior of the radial mode at the two extremes when $R \gg \sqrt{N \ell_{s}}$ as well as when $R \ll \sqrt{N \ell_{s}}$ by expanding $(R^{2} + N \ell_{s}^{2})^{3}$. In the first case, when $R \gg \sqrt{N \ell_{s}}$, we have

$$\ddot{R} + (N \ell_{s}^{2} - \ell^{2}) N \ell_{s}^{2} \left( \frac{1}{N \ell_{s}^{2}} - \frac{2}{R_{0}^{2}} \right) \frac{1}{R^{3}} + O \left( \frac{1}{R^{5}} \right) = 0.$$

On the other hand for $R \ll \sqrt{N \ell_{s}}$, we have,

$$\ddot{R} - \frac{(N \ell_{s}^{2} - \ell^{2})}{(N \ell_{s}^{2})^{2}} R + \frac{2}{(N \ell_{s}^{2})^{3}} \left( 2 - \frac{N \ell_{s}^{2}}{R_{0}^{2}} \right) R^{3} \frac{3(N \ell_{s}^{2} - \ell^{2})}{(N \ell_{s}^{2})^{4}} \left( 3 - \frac{2 N \ell_{s}^{2}}{R_{0}^{2}} \right) R^{3} + O(R^{7}) = 0.$$
Thus we note from (49) that when the Dp brane is far away from the stack of NS5 branes, the associated particle moves in an attractive $-1/R^2$ potential (since $(N\ell_s^2 - \ell^2) > 0$) in four spatial dimensions (transverse directions to the NS5 branes) as expected. On the other hand, when the Dp brane is at a distance $\sqrt{N\ell_s}$ away from the stack of NS5 branes, the behavior of the radial mode becomes much more transparent if we make a coordinate transformation $Z = 1/R$. In this coordinate we can rewrite equation (50) as,

$$\ddot{Z} - \frac{(N\ell_s^2 - \ell^2)}{(N\ell_s^2)^2} \left( \sqrt{N\ell_s} Z - \left( 3 - \frac{2(N\ell_s^2)}{R_0^2} \right) \frac{1}{(\sqrt{N\ell_s} Z)^3} + O \left( \frac{1}{(\sqrt{N\ell_s} Z)^5} \right) \right) = 0. \quad (51)$$

We thus find that for $R \ll \sqrt{N\ell_s}$, i.e., $\sqrt{N\ell_s} Z \gg 1$ the above equation reduces to the leading order,

$$\ddot{Z} - \left( N\ell_s^2 - \ell^2 \right) Z = 0. \quad (52)$$

For $(N\ell_s^2 - \ell^2) > 0$, which is the case of our interest, we recognize the above equation to correspond to the inverted simple harmonic oscillator. In the absence of the acceleration ($aR = 0$), it can be easily checked that the radial equation for small $R$ reduces to $\ddot{Z} = 0$. The origin of the tachyonic instability is now clear, namely, it is the acceleration due to the dilatonic background which is the source of the instability. Even though $P^2 < 0$ indicating that the particle is not tachyonic in the conventional sense, the dilatonic force that it experiences in the background of the NS5 branes leads to hyperbolic motion and the tachyonic instability in the system. We can solve equation (52) and the solution is given as (for $Z \sqrt{N\ell_s} \gg 1$)

$$Z = \frac{1}{R} = \frac{\tau_p V_p}{E(N\ell_s^2 - \ell^2)} \cosh \left( \frac{\sqrt{N\ell_s^2 - \ell^2}}{N\ell_s^2} t \right). \quad (53)$$

Using this solution, we can simplify the expression for the pressure given in (37) as

$$p = -\tau_p V_p \gamma^{-1} e^{-\bar{\phi}} = -E(N\ell_s^2 - \ell^2) \frac{(N\ell_s^2 - \ell^2)}{N\ell_s^2} \exp \left( -2 \frac{\sqrt{N\ell_s^2 - \ell^2}}{N\ell_s^2} t \right). \quad (54)$$

So, the pressure falls off exponentially to zero at late times, indicating that the system evolves into the tachyon matter state at late times as for the case of rolling tachyon on non-BPS branes. We note here that although (52) exhibits a tachyonic instability, in order to see where the instability occurs we look at the complete radial equation (48). From here we identify the potential $V(R)$ in which the particle moves by equating $\ddot{R} = -dV(R)/dR$. It is now easy to see from (48) that the potential has a maximum at $R = 0$ (note that for $R_0 < \sqrt{N\ell_s}$, the case in which we are interested, this is the only extremum) and so this is the point of instability. Now we can also calculate the effective mass squared of the particle and it has the form,

$$m^2 \equiv \frac{d^2V(R)}{dR^2} \bigg|_{R=0} = -\frac{(N\ell_s^2 - \ell^2)}{(N\ell_s^2)^2}. \quad (55)$$

As the mass squared is negative, the particle effectively behaves as a tachyon in the vicinity of NS5 branes. However, it should be noted that since at $R = 0$, the effective string coupling $e^{\phi} = H^{1/2}$ blows up, a full quantum string theoretic treatment is necessary to identify the true position of the instability.

4. Dynamics of F-string in the background of Dp branes

We next investigate the motion of a fundamental string in type II string theories in the background of a stack of N coincident Dp branes (for $p \leq 5$). However, since the behavior
of the F-string is quite different when the background is D5 branes from those of Dp branes, with \( p < 5 \) we will treat these two cases separately. First we will consider the D5-brane background and then discuss about the other Dp-brane backgrounds.

4.1. F-string in D5-brane background

The analysis of this system will follow closely that in the previous section. We recall that a well-separated system of a fundamental string and a stack of parallel Dp branes leads to a complete breakdown of the spacetime supersymmetries. Furthermore, as the F-string mass (\( \sim \) constant) is much less than the Dp-brane mass (\( \sim N/g_s \)), where \( N \) is the number of Dp branes, the F-string will move toward the Dp branes at small string coupling because of their gravitational and dilatonic interactions.

The background fields describing the supergravity solutions of the coincident D5 branes have the forms [2]

\[
d s^2 = H^{-1/2} \left( -d t^2 + \sum_{i=1}^{5} (d x^i)^2 \right) + H^{1/2} (d r^2 + r^2 d \Omega_3^2),
\]

\[
e^{2(\phi - \phi_0)} = H^{-1},
\]

\[
H(r) = 1 + \frac{N g_s \ell_s^2}{r^2},
\]

\[
F_7 = d H^{-1} \wedge d t \wedge d x^1 \wedge \cdots \wedge d x^5.
\]

Here we have assumed that \((t, x^1, \ldots, x^5)\) denote the coordinates along which the D5 branes are spread while \((x^6, \ldots, x^9)\) represent the transverse coordinates and \(r = \sqrt{x^m x^m} \). \( H(r) \) defines the harmonic function associated with the D5 branes and \( F_7 \) represents the 7-form field strength.

The Nambu–Goto action describing the dynamics of the bosonic sector of a fundamental string in the background of the D5 branes is given by

\[
S_F = -\frac{1}{2 \pi \ell_s^2} \int d^2 \xi \sqrt{-\det G_{\mu\nu}},
\]

with \( \mu, \nu = 0, 1 \) correspond to the string world sheet indices while \( \xi^0, \xi^1 \) denote the coordinates on the string world sheet. The induced metric on the world sheet of the fundamental string due to the background D5 branes defined in (56) is given by

\[
G_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu} \frac{\partial X^B}{\partial \xi^\nu} G_{AB},
\]

with \( A, B = 0, 1, \ldots, 9 \). As before, \( X^s \) denote the scalar fields associated with the coordinates \( x^s \). We can use the reparameterization invariance of the string world sheet to choose \( \xi^0 = x^0 = t \) and \( \xi^1 = x^1 \) so that the string is extended along \( x^1 \).

Let us note that there is no dilaton factor in the Nambu–Goto action (57) as opposed to the DBI action for the Dp brane in (29). However, using (56) and (58) we can rewrite

\[
\sqrt{-\det G_{\mu\nu}} = \frac{1}{\sqrt{H}} \sqrt{1 + \eta^{\mu\nu} G_{mn} \partial_\mu X^m \partial_\nu X^n},
\]

where \( \eta_{\mu\nu} = \text{diag}(-1, 1) \), and we have defined \( G_{mn} = H(r) \delta_{mn} \) and \( m, n = 6, 7, 8, 9 \) describe the transverse space of the D5 branes. Using (59), the Nambu–Goto action (57) can be written as

\[
S_F = -\frac{1}{2 \pi \ell_s^2} \int d t d x^1 \frac{1}{\sqrt{H}} \sqrt{1 + \eta^{\mu\nu} G_{mn} \partial_\mu X^m \partial_\nu X^n}.
\]
As in the previous section, we will be interested in the case when the scalar fields on the F-string depend only on time, namely, \( X^m = X^m(t) \). In this case, (60) simplifies and can be written as
\[
S_F = -\tau_F V_F \int dt \frac{1}{\sqrt{H}} \sqrt{-G_{\bar{a}\bar{b}}} \dot{X}^\bar{a} \dot{X}^\bar{b}
\]
\[
= -\tau_F V_F \int d\tau \frac{1}{\sqrt{H}} \sqrt{-G_{\bar{a}\bar{b}}} \frac{dX^\bar{a}}{d\tau} \frac{dX^\bar{b}}{d\tau},
\]
where we have identified \( \tau_F = \frac{1}{2\pi \ell^2 s} \), \( V_F = \int dx^1 \) and \( G_{\bar{a}\bar{b}} \) is defined in the last section.

Comparing (61) with (32) we observe that the overall multiplicative factor \( 1/\sqrt{H} \) in the Lagrangian in (61) can be thought of as playing the role of \( e^{-\phi} \) in the present case. Namely, this factor can be thought of effectively as producing the acceleration in the present case much like the exponential dilaton prefactor was the source of acceleration for particle motion in the last section (see (19)).

The dynamics of the fundamental string in the background of a stack of \( N \) coincident D5 branes now follows from (61) (see also (25) and (26))
\[
\frac{d^2 X^\bar{m}}{d\tau^2} + \Gamma^\bar{m}_{\bar{a}\bar{b}} \frac{dX^\bar{a}}{d\tau} \frac{dX^\bar{b}}{d\tau} = a^\bar{m},
\]
with the acceleration in the present case given by
\[
a^\bar{m} = \left( G_{\bar{m}\bar{a}} + \frac{dX^\bar{a}}{d\tau} \frac{dX^\bar{b}}{d\tau} \right) \frac{\partial_b H}{2H}.
\]

The energy and the pressure follows from the general form of the energy–momentum tensor given in (36) with the Lagrangian following from the action (61) and have the forms,
\[
T^{00} = \tau_F V_F \gamma H^{-1} = E
\]
\[
T^{ij} = -\tau_F V_F \gamma^{-1} \dot{g}^{ij} = p \delta^{ij},
\]
where the Lorentz factor ‘\( \gamma \)’ has the form given in (33). The time component (\( \bar{m} = 0 \)) of the equation of motion (62) gives the energy conservation relation \( d \ln (\gamma H^{-1/2})/dt = 0 \) whereas \( \bar{m} = m \) gives,
\[
\dot{X}^\bar{m} + \Gamma^m_{\bar{a}\bar{b}} \dot{X}^\bar{a} \dot{X}^\bar{b} + G_{\bar{p}\bar{q}} \dot{X}^\bar{p} \dot{X}^\bar{q} \frac{\partial_m H}{2H} = 0.
\]

The above equation being a second-order differential equation, the motion of the particle can be restricted to a plane. As before, introducing the spherical-polar coordinates, the radial and the angular modes can be shown, from (65), to satisfy exactly the same equations as given in (43). The \( \Theta \) equation yields the conservation of angular momentum as given in (46) and the radial equation has the form
\[
\ddot{R} - \left( \frac{N g_s \ell_s^2 - \ell_s^2}{R^2 + N g_s \ell_s^2} \right) \left( R + \frac{2}{R_0} - \frac{1}{N g_s \ell_s^2} \right) R^3 = 0.
\]

This is exactly the same equation we obtained for the motion of the relativistic point particle corresponding to the motion of Dp branes in NS5-brane background with \( \ell_s^2 \) replaced by \( g_s \ell_s^2 \). The reason for this is that the configuration of F-string in D5-brane background is S-dual to the configuration of D-string in NS5-brane background and under S-duality \( \ell_s^2 \) changes to \( g_s \ell_s^2 \). It is now obvious that the dynamics in this case is analogous to that of D-string in NS5-brane background we studied in the previous section. The radial equations in the two limits \( R \gg \sqrt{N g_s} \ell_s \) and \( R \ll \sqrt{N g_s} \ell_s \) have the same forms as in (49) and (50) with \( \ell_s^2 \).
replaced by $g_s \ell_s^2$. Thus the whole analysis of the previous section goes through in a completely parallel manner leading to the fact that in the vicinity of the stack of D5 branes, the motion of F-string, described by a relativistic point particle, is given by that of an inverted simple harmonic oscillator leading to the tachyonic instability.

We would like to point out here that the motion of a D3 brane, described by the DBI action, in the background of the D5 branes, is given by the present analysis as well. This is because, for the D3 brane, the dilaton field satisfies $e^{2\phi} = 1$ so that there is no dilaton prefactor in the DBI action. However, the overall multiplicative factor $H^{-\frac{1}{2}}$ would again arise from the induced metric as discussed in this section and this would be the source of proper acceleration. Therefore, a geometric tachyon would also arise for this system near the D5 branes. The dynamics of a $D_p$ brane in the background of a $D_{p'}$ brane has been studied in [40].

The tachyonic behavior of F-string in the vicinity of a stack of D5 branes can also be understood, following [14], by mapping the Nambu–Goto action (60) to the tachyon effective action of the non-BPS branes. For this purpose, let us assume that in (60) only the radial mode $R = \sqrt{X_m X^m}$ has been excited and the angular modes are kept fixed, then the action reduces to,

$$S_F = -\frac{1}{2\pi \ell_s^2} \int dt d^4x \sqrt{1 + \eta^{\mu \nu} H(R) \partial_\mu R \partial_\nu R}.$$  

(67)

Now comparing this with the tachyon effective action of the non-BPS D-string

$$S_{\text{nonBPS}} = -\int dt d^4x V(T) \sqrt{1 + \eta^{\mu \nu} \partial_\mu T \partial_\nu T},$$

(68)

where $T$ is the open string tachyon living on the non-BPS D-string and $V(T)$ is the tachyon potential. Now (67) can be mapped to (68) if we identify

$$\frac{dT}{dR} = \sqrt{H(R)} = \frac{1}{\sqrt{1 + \frac{N g_s \ell_s^2}{R^2}}}.$$  

(69)

and

$$V(T) = \frac{1}{2\pi \ell_s^2} \frac{1}{\sqrt{H(R)}} \frac{\tau_F}{\sqrt{H(R)}}.$$  

(70)

The solution of equation (69) has the form,

$$T(R) = \frac{1}{2} \sqrt{N g_s \ell_s} \ln \frac{\sqrt{R^2 + N g_s \ell_s^2} - \sqrt{N g_s \ell_s^2}}{\sqrt{R^2 + N g_s \ell_s^2} + \sqrt{N g_s \ell_s^2} + R^2}$$

(71)

up to an unimportant additive constant which we set to zero. From (71), we find that as $R \to 0$,

$$T(R \to 0) \to \sqrt{N g_s \ell_s} \ln \frac{R}{\sqrt{N g_s \ell_s}} \to -\infty$$

(72)

and as $R \to \infty$,

$$T(R \to \infty) \to R \to \infty.$$  

(73)

Using these relations (72) and (73), we can find out the behavior of the tachyon potential (70) at the two extremes which we give below.

As, $R \to \infty$, \quad $V(T(R)) \to \tau_F \left(1 - \frac{N g_s \ell_s^2}{2R^2}\right) \simeq \tau_F \left(1 - \frac{N g_s \ell_s^2}{2T^2}\right)$

(74)

as, $R \to 0$, \quad $V(T(R)) \simeq \tau_F R \exp \left( \frac{T}{\sqrt{N g_s \ell_s}} \right)$.

(75)
This shows that when the F-string is far away \( R \gg \sqrt{N g_s \ell_s} \) from the stack of D5 branes, the potential goes as \(-1/T^2 \approx -1/R^2\) which is the attractive gravitational potential in the transverse four-dimensional space as expected and is consistent with our previous observation \((50)\). On the other hand when the F-string comes closer \( R \ll \sqrt{N g_s \ell_s} \) to the stack of D5 branes, \( T \rightarrow -\infty \) and the potential vanishes exponentially. This is what is expected of a tachyon potential on the non-BPS branes. This behavior of the potential leads to the two crucial properties \([17]\) of a tachyon effective action, namely, the absence of the plane wave solution around the minimum of the potential (this is also related to our observation that near the D5 branes, the motion of the F-string, described by a relativistic point particle, is given by the inverted simple harmonic oscillator) and the exponential fall off of the pressure at late times. In order to understand the exponential fall off of pressure, we need to solve the radial equation of motion. We have already given the solution for D-string in NS5-brane background in \((53)\). The solution for the motion of F-string in D5-brane background can be obtained from there by replacing \( \tau_1 V_1 \) by \( \tau_F V_F \) and \( \ell_2^s \) by \( \ell_2^s g_s \). So, the solution in this case would be,

\[
Z = \frac{1}{R} = \frac{\tau_F V_F}{E \sqrt{N g_s \ell_F^2 - \ell^2}} \cosh \left( \frac{\sqrt{N g_s \ell_F^2 - \ell^2}}{N g_s \ell_2^s} t \right). \tag{76}
\]

Therefore, the pressure given in \((64)\) can be shown to behave as,

\[
p = -\frac{E (N g_s \ell_F^2 - \ell^2)}{N g_s \ell_2^s} \exp \left( -2 \frac{\sqrt{N g_s \ell_F^2 - \ell^2}}{N g_s \ell_2^s} t \right). \tag{77}
\]

This shows that the pressure indeed falls off exponentially to zero at late times. However, we would like to remark here that the trajectory we have given in \((76)\) for \( R \ll \sqrt{N g_s \ell_s} \) is not sensible at all times. The reason is that the supergravity configuration of a stack of D5 branes given in \((56)\) has a curvature singularity at \( R = 0 \) (or \( t \rightarrow \infty \)). In fact in order for the supergravity description to remain valid the curvature must be small compared to the string scale and so we have,

\[
R = \left( \sqrt{N g_s \ell_s} \right)^\frac{1}{2} \ll \frac{1}{\ell_s}. \tag{78}
\]

This implies that \( R \gg \ell_s / (\sqrt{N g_s}) \). So, the range in which the trajectory \((76)\) remains valid is

\[
\ell_s / \sqrt{N g_s} \ll R \ll \sqrt{N g_s \ell_s}. \tag{79}
\]

This can be satisfied for large \( N \) and then solution \((76)\) will remain valid in the above region \((79)\) only.

### 4.2. F-string in Dp-brane \((p < 5)\) backgrounds

Let us next consider the motion of an F-string in the background of a stack of coincident Dp branes (for \( 1 \leq p < 5 \)). The generalities of the dynamics will remain exactly the same as in the previous subsection and we will follow closely to that discussion. The dynamics of the F-string will again be given by the Nambu–Goto action \((57)\). However, since now the background is different, the induced metric \( G_{\mu \nu} \) will be different. The background fields describing the supergravity solution of coincident Dp branes have the forms \([2]\),

\[
d^2 = H_{p+2}^{-\frac{1}{2}} \left( -dt^2 + \sum_{i=1}^{p} (dx_i)^2 \right) + H_{p}^{\frac{1}{2}} (dr^2 + r^2 d\Omega_{8-p})
\]

\[
e^{2(\phi - \phi_0)} = H_{p+2}^{-\frac{1}{2}}
\]

\[
F_{[p+2]} = dH^{-1} \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p,
\]

\[
(80)
\]
where \( \mu, \nu = 0, 1, \ldots, p \) label the coordinates along the Dp branes and \( m, n = p + 1, \ldots, 9 \) label the coordinates transverse to the Dp branes. \( H \) is a harmonic function in the transverse space given by

\[
H = 1 + d_p N g_{ij} \frac{I^j}{r^{l-p}} = 1 + \frac{Q_p}{r^{l-p}}, \tag{81}
\]

where \( Q_p = d_p N g_{ij} I^j \) and \( d_p \) is a \( p \)-dependent constant given as \( d_p = 2^{5-p} \pi \Gamma((7-p)/2) \), \( N \) is the number of Dp branes and \( F_{[p+2]} \) is the field strength of an RR \((p+1)\)-form gauge field that couples to the Dp branes. When \( p = 3 \), the corresponding field strength would be self-dual.

The Nambu–Goto action \((57)\) in this background would take the same form as \((60)\) with \( G_{mn} = H(r) \delta_{mn} \) and \( H(r) \) as given in \((81)\). When the scalar fields on the F-string world sheet depend only on time, namely, \( X^m = X^m(t) \), the Nambu–Goto action would reduce to \((61)\) giving rise to the same equation of motion \((62)\) with the proper acceleration given in \((63)\). The time component of the equation of motion would give the energy conservation relation with the energy and the pressure having the same form as in \((64)\), whereas, the spatial component of the equation of motion would be the same as \((65)\). This being a second-order differential equation we can restrict the motion in a plane as before, then the radial and the angular mode would give rise to equations \((43)\) obtained earlier. The \( \Theta \) equations would give the conservation of angular momentum \((46)\) and the radial equation gives,

\[
\ddot{R} - \frac{1}{(R^{7-p} + Q_p)} \left( \frac{7-p}{2} Q_p R^{6-p} - (6-p) \frac{Q_p \ell^2 R^{2(6-p)-1}}{R_0} \right) = 0, \tag{82}
\]

where \( R_0 \) is given by the equation

\[
\left( \frac{\tau F}{E} \right)^2 = 1 - \frac{\ell^2}{R_0^2} + \frac{Q_p}{R_0^{4-p}}. \tag{83}
\]

Note that \((82)\) is an exact equation of the radial mode \( R \). When the F-string is far away \((R \gg Q_p^{-1/7-p})\) from the stack of Dp branes, equation \((82)\) gives,

\[
\ddot{R} - \frac{\ell^2}{R_0^3} + (7-p)Q_p \left( \frac{1}{2} + \frac{Q_p}{R_0^{7-p}} \right) \frac{1}{R^{8-p}} = 0. \tag{84}
\]

Thus we note that for the vanishing angular momentum \((\ell = 0)\) the particle motion is governed by an attractive \(-1/R^{7-p}\) gravitational potential as expected. On the other hand, when F-string is close \((R \ll Q_p^{-1/7-p})\) to the stack of Dp branes we use the full radial equation \((82)\) to identify the potential \( V(R) \) in which the particle moves by equating \( \ddot{R} = -dV(R)/dR \). From \((82)\) it is clear that the potential has an extremum at \( R = 0 \) and this is the only extremum when F-string is very close to the stack of Dp branes. Computing the second derivative at \( R = 0 \), we find that \( d^2V/dR^2|_{R=0} = 0 \). So, this is a point of inflexion, indicating that there is an instability at this point. However, we note that except for \( p = 3 \), in all other cases, either the effective string coupling \( e^\phi \) blows up (for \( p = 1, 2 \)) or the curvature blows up (for \( p = 4 \)) at \( R = 0 \) and so, we need full string theory to find the true position of the instability.
Now as in the previous subsection we can try to understand the instability of the motion of F-string in the vicinity of the stack of $D_p$ branes, by mapping the Nambu–Goto action (67) to the non-BPS D-string action (68). In the present case the mapping can be done if we identify,

\[
\frac{\text{d}T}{\text{d}R} = \sqrt{H(R)} = \sqrt{1 + \frac{Q_p}{R^{7-p}}}
\]

and

\[
V(T) = \frac{1}{2\pi \ell_s^2 \sqrt{H(R)}} = \frac{\tau_F}{\sqrt{H(R)}}.
\]

The solution of (85) is given as

\[
T(R) = -\frac{2R}{5-p} \left( 1 + \frac{Q_p}{R^{7-p}} \right)^{\frac{1}{2}} + \frac{2(7-p)}{(9-p)(5-p)} \frac{Q_p^{7-p}}{Q_p^{7-p}},
\]

(87)

where $F(a,b;c;-R^{7-p}/Q_p)$ is a hypergeometric function. Now from (87) we find that as $R \to 0$, $F(a,b;c;-R^{7-p}/Q_p) \to 1$ and so,

\[
T(R \to 0) \to -\frac{2Q_p^{7-p}}{5-p} \to -\infty.
\]

(88)

On the other hand as $R \to \infty$, the hypergeometric function behaves as

\[
F(a,b;c;-R^{7-p}/Q_p) \to \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} \left( \frac{Q_p^{7-p}}{R^{7-p}} \right)^a + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} \left( \frac{Q_p^{7-p}}{R^{7-p}} \right)^b
\]

(89)

provided $(a-b)$ is not an integer. Using (89) we find from (87) that in this case

\[
T(R \to \infty) \to R \to \infty.
\]

(90)

Using relations (88) and (90) we analyze the behavior of the potential given in (86) as the following:

For $R \to \infty$, $V(T(R)) = \tau_F \left( 1 - \frac{Q_p}{2R^{7-p}} \right) \simeq \tau_F \left( 1 - \frac{Q_p}{2T^{7-p}} \right)$

(91)

and for $R \to 0$, $V(T(R)) = \tau_F \left( \frac{R^{7-p}}{Q_p^{7-p}} \right) \simeq \tau_F Q_p^{\frac{1}{7-p}} \left( -\frac{5-p}{2} T \right)^{-\frac{1}{7-p}} \to 0.

(92)

We thus find that when F-string is far away ($R \gg Q_p^{\frac{1}{7-p}}$) from the stack of $D_p$ branes, the potential goes as $-1/T^{7-p} \simeq -1/R^{7-p}$ which is the attractive gravitational potential in the $(9-p)$-dimensional transverse space of the $D_p$ branes as expected. This is also consistent with our earlier observation (84). However, when the F-string comes closer ($R \ll Q_p^{\frac{1}{7-p}}$) to the stack of $D_p$ branes, the potential vanishes as power of the tachyon field.

In order to see how the pressure as given in (64) varies with time, we need to solve the equation of motion of the radial mode $R$. The equation of motion is given in (47) with the appropriate changes, namely,

\[
\ddot{R}^2 = \frac{1}{H} \left( 1 - \frac{1}{H} \left( \left( \frac{\tau_p V_F}{E} \right)^2 + \ell_s^2 \right) \right)
\]

(93)
with the harmonic function as given in (81). This equation can be solved only when \( R \ll Q^{\frac{1}{7-p}} \), i.e., for F-string very close to the stack of Dp branes. Even for this case, we have not been able to solve the equation in a closed form for the nonzero angular momentum. However, when the angular momentum is zero (\( \ell = 0 \)) equation (93) can be solved and we get,

\[
t = \frac{2Q^\frac{1}{7-p}}{5-p} R^{-\frac{1}{7-p}} \left( 1 - F \left( \frac{5-p}{2(7-p)}, 1; \frac{3}{2}; \frac{Q^\frac{1}{7-p}}{(\frac{5}{E})^2} \right) R^{-(7-p)} \right),
\]

where we have assumed the boundary condition that as \( R \to 0, t \to \infty \). Now as \( R \to 0 \), \( F \to 0 \) and so,

\[
t = \frac{2Q^\frac{1}{7-p}}{5-p} R^{-\frac{1}{7-p}}.
\]

Inverting this relation we get,

\[
R = \left( \frac{5-p}{2Q^\frac{1}{7-p}} \right)^{-\frac{1}{7-p}}.
\]

Substituting this into the expression of pressure given in (64) we get,

\[
p = -\frac{\tau V_2}{E} \left( \frac{5-p}{2} \right)^{-\frac{7-p}{2}} Q^\frac{1}{7-p} t^{-\frac{2(3-p)}{7-p}}.
\]

We, therefore, find that the pressure vanishes as a power of \( t \) at late times. This indicates that the system indeed evolves into the pressureless fluid or the tachyon matter state at late times as happens for the rolling tachyon solution of the non-BPS branes. We further note from (88) and (96), that as \( R \to 0 \),

\[
T(R \to 0) \to -t.
\]

So, the tachyon behaves as time, a typical of the rolling tachyon solution of the non-BPS branes [41].

It should be noted here that the trajectory we have given in (96) is not valid at all times. This is because, for \( p = 1, 2 \) the dilaton behaves as

\[
e^\phi = g_s H^{-\frac{1}{7-p}},
\]

and for \( R \ll Q^{\frac{1}{7-p}} \), the dilaton in general may not remain small for all times. In order for the supergravity description to remain valid we must ensure that \( e^\phi \ll 1 \). This implies a range of \( R \) for which the trajectory can be trusted and is given as,

\[
g_s Q_p^{-\frac{1}{7-p}} \ll R \ll Q_p^\frac{1}{7-p}.
\]

In terms of \( t \), the corresponding range is

\[
\frac{2}{5-p} g_s Q_p^{-\frac{1}{7-p}} \gg t \gg \frac{2}{5-p} Q_p^\frac{1}{7-p}.
\]

This is the range of time for which the trajectory can be trusted. For small string coupling \( g_s \), this gives a sufficiently long time for which the trajectory remains reliable. Note also that in these cases the curvature always remains small compared to the string scale.

For \( p = 3 \), both the curvature and the string coupling can be made small. However, for \( p = 4 \), even though the effective string coupling can be made small, the curvature can become large and thereby invalidating the supergravity description. From the supergravity
configuration (80) we compute the curvature for \( p = 4 \) in the form \( R = (H^{1/2} R^2)^{-1/2} \) which must remain small compared to the string scale and so we have

\[
(H^{-1/2} R^2)^{-1/2} \ll \frac{1}{\ell_s}.
\]  

(102)

This implies

\[
R \gg \frac{\ell_s^4}{Q_4},
\]  

(103)

where \( Q_4 = \pi N g_s \ell_s^4 \). So, the supergravity description will remain valid for

\[
\frac{\ell_s^4}{Q_4} \ll R \ll Q_4^{1/3}.
\]  

(104)

In terms of \( t \), the corresponding range is

\[
\frac{2Q_4}{\ell_s^4} \gg t \gg 2Q_4^{1/3}.
\]  

(105)

For large \( N \) this gives a sufficiently long time for which the trajectory can be reliable.

Thus we observed that when a fundamental string moves in the vicinity of a stack of coincident Dp branes it behaves like a non-BPS D-string in the sense that it develops a tachyonic instability and eventually evolves into a pressureless fluid state much like the tachyon matter of the rolling tachyon solution of the non-BPS D branes. The tachyon here has a geometric interpretation in terms of the radial mode. At this point one would naturally be interested to know what is the end point state of this geometric tachyon condensation. From the spacetime point of view, it is known that when F-string and the stack of Dp branes are separated they break all the spacetime supersymmetry and so, it is not surprising that they develop a tachyonic instability. However, when the tachyon rolls down from the initial homogeneous configuration toward the minimum of the potential, F-string will evolve into the pressureless fluid or tachyon matter. Usually in this situation the non-BPS D branes evolving into pressureless fluid is interpreted as the decay of the unstable D branes into the closed string radiation \([42–45]\). Indeed, as argued for the case of motion of a Dp brane in the vicinity of a stack of coincident NS5 branes, F-string also will lose almost all of its energy (decay) when it merges with the stack of Dp branes to form a bound state. This can be understood from the tension or the energy per unit \( p \)-brane volume formula of an (F,Dp) bound state which has the form \([29, 30]\)

\[
E_{N,M} = T_0^p \sqrt{\frac{N^2}{g_s^2} + M^2},
\]  

(106)

where \( T_0^p = \left(2\pi\right)^p \sqrt{\frac{p+1}{p}} \). \( N \) is the number of Dp branes and \( M \) is the number of F-string per unit \((p-1)\)-dimensional volume (along Dp brane), with \( M \) and \( N \) relatively prime. It is clear from the energy formula that (F,Dp) is a non-threshold bound state and it is known to preserve half of the spacetime supersymmetry. Suppose we consider a single F-string (per unit \((p-1)\)-dimensional volume) moving toward a stack of \( N \) coincident Dp branes, where F-string lies parallel to the Dp branes. From the above formula (106), we find that when the F-string gets bound to the stack of Dp branes, the system has an energy density

\[
E_{N,1} = T_0^p \sqrt{\frac{N^2}{g_s^2} + 1}.
\]  

(107)

For the weak string coupling this can be expanded as

\[
E_{N,1} \simeq T_0^p \frac{N}{g_s} \left(1 + \frac{g_s^2}{2N^2}\right) = T_0^p \left(\frac{N}{g_s} + \frac{g_s}{2N}\right).
\]  

(108)
Note that the second term which is the F-string contribution is almost zero for weak string coupling and large $N$. Thus when the F-string approaches the stack of $D_p$ branes it loses almost all of its energy to form the bound state, much like the decay of unstable D branes to the pressureless tachyon matter. This is just a qualitative argument, but to understand the detail dynamics of the bound state formation, obviously, the supergravity description would not be sufficient and we need the full string theoretical treatment.

5. Conclusion

To summarize, in this paper after a brief review of proper acceleration in both flat as well as curved spacetime, we have given a systematic and detailed account of our observation that the origin of the geometric tachyon when a $D_p$ brane propagates in the close proximity of a stack of coincident NS5 branes is due to the proper acceleration generated by the background dilaton field. We have seen this by considering the motion of the $D_p$ brane, described by the DBI action, in terms of the motion of a relativistic point particle in the transverse space of the NS5 branes. The acceleration is orthogonal to the proper velocity and is responsible for changing the dynamics of the particle from that of a simple geodesic motion. Using this we have seen that the motion of the particle in an appropriate coordinate can be reduced to that of an inverted simple harmonic oscillator. Even though the particle is not tachyonic in the conventional sense (since $P^2 < 0$), effectively it behaves like a tachyon (as the motion is hyperbolic) and the potential in which it moves has a maximum. In this approach the maximum occurs at the location of the stack of NS5 branes. However, since at that specific point the effective string coupling blows up we cannot trust the supergravity description. We have computed the effective mass squared of the particle when it is close to NS5 branes and found it to be negative. This, therefore, suggests the occurrence of tachyonic instability in the motion of a $D_p$ brane in the vicinity of NS5 branes.

We have extended our observations to the other dynamical systems, namely, the motion of an F-string in the vicinity of a stack of coincident $D_p$ branes. The dynamics in this case is described by the Nambu–Goto action. The Nambu–Goto action does not contain a dilaton prefactor as the DBI action of a $D_p$ brane, however, the induced metric of the background $D_p$ brane gives rise to a conformal factor which acts as a source for the proper acceleration in this case. By using our previous observation we showed that even in this case the proper acceleration is orthogonal to the proper velocity and is responsible for the deviation of the dynamics of the particle from its geodesic motion. Since $p = 5$ case is quite different from other $p$'s we have separately discussed these two cases. The motion of the F-string in the background of a stack of D5 branes is S-dual to the motion of a D-string in the background of a stack of NS5 branes and so, we follow the discussion of $p = 5$ case in close parallel with the D-string in NS5-brane background. Here also, we showed that the acceleration is responsible for modifying the motion of the radial mode of the particle in the vicinity of $D_p$ branes to that of an inverted simple harmonic oscillator in an appropriate coordinate leading to the tachyonic instability. We have computed the mass squared of the particle and found it to be negative. We have further studied the tachyonic instability by mapping the Nambu–Goto action describing the dynamics of the F-string to the tachyon effective action of the non-BPS D-string. We showed how the tachyon potential behaves at various regimes by relating the radial mode to the tachyon field. In particular, near the D5 branes we found that the potential behaves like usual tachyon potential of a non-BPS brane. On the other hand when the F-string is far away from the $D_p$ branes the potential behaves like a usual attractive $-1/R^2$ gravitational potential. But in this approach we do not see the maximum of the potential where the instability begins to
occur. We also showed how the system evolves into pressureless fluid by solving the equation of motion and it indeed falls off exponentially at late times like a non-BPS brane.

Then we discussed the motion of an F-string in the vicinity of other Dp branes. All of our previous discussion goes through here, but the radial equation of the particle motion is a bit involved in this case. Here we have not been able to show (in an appropriate coordinate) that the motion of the radial mode in the vicinity of the Dp branes can be modified to that of an inverted simple harmonic oscillator. However, we showed that the potential in which the particle moves has an extremum at $R = 0$ and this extremum is a point of inflexion, indicating an instability in the system. We have studied this instability also by mapping the Nambu–Goto action to the tachyon effective action of a non-BPS D-string. By relating the radial mode to the tachyon field we have analyzed the behavior of the potential at various regimes. Here also we found that when the F-string is far away from the stack of Dp branes the potential behaves like $-1/R^{7-p}$ which is the attractive gravitational potential as expected. However when the F-string is close to the stack of Dp branes the potential vanishes as a power of the tachyon field unlike the D5-brane background where the potential falls off to zero exponentially. We have also solved the equation of motion of the radial mode when the F-string moves in the vicinity of the stack of Dp branes with the zero angular momentum (since for nonzero angular momentum the equation cannot be solved in a closed form) and related the radial mode with the time. Using this we have been able to show that the pressure of the system vanishes as power of $t$ (unlike the D5-brane background where pressure falls off to zero exponentially) at late times. This shows that the system indeed evolves into the pressureless fluid (tachyon matter) like the rolling tachyon in non-BPS branes. We have also shown that at late times the tachyon field behaves as time a property typical of the rolling tachyon solution in non-BPS branes. Further we have argued qualitatively that the (F, Dp) bound state can be understood as the result of this geometric tachyon condensation.

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