

**Unitary limit and linear scaling of neutrons in harmonic trap with tuned CD-Bonn and square-well interactions**

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We study systems of finite-number neutrons in a harmonic trap at the unitary limit. Two very different types of neutron-neutron interactions are applied, namely, the meson-theoretic CD-Bonn potential and hard-core square-well interactions, all tuned to possess infinite scattering lengths, and with effective ranges comparable to or larger than the trap size. The potentials are renormalized to equivalent, scattering-length preserving low-momentum potentials, $V_{\text{low-}k}$, with which the particle-particle hole-hole ring diagrams are summed to all orders to yield the ground-state energy $E_0$ of the finite neutron system. We find the ratio $E_0/E_{0\text{free}}$ (where $E_{0\text{free}}$ denotes the ground-state energy of the corresponding non-interacting system) to be remarkably independent from variations of the harmonic trap parameter, the number of neutrons, the decimation momentum of $V_{\text{low-}k}$, and the type and effective range of the unitarity potential. Our results support a special virial linear scaling relation of $E_0$. Certain properties of Landau’s quasi-particles for trapped neutrons at the unitary limit are also discussed.

**Introduction.** The scenario of “unitary limit” was originally formulated by Bertsch in 1999, asking what will be the ground-state properties of a spin-1/2 fermion system with an interaction of infinite scattering length. With impressive advances of cold-atom experimental techniques, such unitary Fermi systems became experimentally accessible at the atomic level, and have attracted intensive attention. This limit is also of great interest to nuclear systems, because the $S_0$ scattering lengths of realistic nucleon-nucleon interactions are all fairly large, such as $-18.97$ fm in the CD-Bonn potential, in comparison with other length scales in the nuclear system.

For a Fermi gas at the unitary limit, the ground-state energy $E_0$ is expected to be proportional to the energy of the corresponding free gas $E_{0\text{free}}$, i.e., $E_0 = \xi E_{0\text{free}}$ with $\xi$ an universal constant shared by all unitary Fermi gases. There have been huge efforts devoted to describing universal behaviors of unitary Fermi gases in regard to the ground-state properties, as well as collective excitations, thermodynamic properties, and non-equilibrium aspects, see e.g. Refs. [3–22]. Our previous results of unitary neutron matter [11, 12] by summing the low-momentum particle-particle hole-hole ring diagrams to all orders, with very different unitarity potentials, give $\xi$ values all closely equal to 0.44. Very recently, extensive studies on few-nucleon systems [23, 24] and neutron stars [21] from perspective of the unitary limit have been carried out.

A unitary system of finite-number fermions confined in a harmonic trap is receiving increasing attention [25–30]. In previous works, see e.g. [27, 28, 30], unitary systems confined in a harmonic trap, whose size $a_{\text{h.o.}} = \sqrt{\hbar/(m\omega)}$ is much larger than the effective range $r_e$ of the inter-fermion interaction, were studied. The trapped unitary systems with $r_e$ comparable to or larger than $a_{\text{h.o.}}$, however, remain largely unknown. In particular, of great interest and importance is the question whether or not such trapped systems at the unitary limit are universally related to corresponding non-interacting ones. In this work we find the ratio $E_0/E_{0\text{free}}$ for a trapped unitary system with $r_e$ comparable to or larger than $a_{\text{h.o.}}$, is remarkably invariant from variations of the harmonic trap parameter, the number of fermions, the decimation momentum of the low-momentum interaction $V_{\text{low-}k}$ [31–37], and the type and effective range of the inter-fermion potential, suggesting a universal nature of these systems. Our results support a special linear scaling relation of $E_0$, which is shown to be analytically consistent with the unitary-limit virial theorems of Refs. [26, 29]. We also study regularities of trapped unitary neutrons from perspective of Landau’s Fermi liquid theory [38].

**Formalism.** We consider a system of neutrons trapped in a harmonic trap, with Hamiltonian $H = T + U_{\text{osc}} + V$, where $T$ represents the kinetic energy, $U_{\text{osc}}$ the oscillator potential $\sum_i \frac{1}{2} m \omega^2 r_i^2$, and $V$ a uni-
tary neutron-neutron interaction which will be described later. Let us consider a closed-shell system first. The ground-state (g.s.) energy $E_0$ of this system will be calculated using a linked-diagram expansion \cite{39, 41} of the form

$$E_0 = E_0^\text{free} + \Delta E_0 ,$$

$$\Delta E_0 = \langle \Phi_0 | W(t, t') | \Phi_0 \rangle \bigg|_{t' \to -\infty} ,$$

$$E_0^\text{free} = \langle \Phi_0 | U_\text{cor} | \Phi_0 \rangle + \langle \Phi_0 | T | \Phi_0 \rangle = U_0 + T_0 ,$$

where $U(t, t')$ is the time evolution operator. $E_0^\text{free}$ is the non-interacting g.s. energy, and $\Delta E_0$ is given by the sum of all the linked diagrams generated by the interaction $V$. $\Phi_0$ is a closed-core wave function, such as a shell-model closed-core state. The ratio $\sigma$ for the CD-Bonn potential is carried out by slightly adjusting its scattering lengths approaching infinity. The tuning of the $\text{HCSW}$ potential \cite{12}, both tuned to have their scattering length and the effective range. For most cases, we shall use the decimation momentum $\Lambda=2.0$ fm$^{-1}$ as used in a number of nuclear structure studies \cite{33}. We shall also check whether the ratios are invariant with respect to the type of the unitarity potential and with the variation of the effective range.

Next we obtain a low-momentum interaction $V_{\text{low-}k}$ from the above unitarity potential. A renormalization procedure \cite{11, 31, 37} where the high-momentum components of such a potential are integrated out is enacted. Note that the resulting $V_{\text{low-}k}$ preserves both the scattering length and the effective range. For most cases, we shall use the decimation momentum $\Lambda=2.0$ fm$^{-1}$ as used in a number of nuclear structure studies \cite{33}. We shall also check whether the ratios are invariant with the variation of $\Lambda$.

We calculate the g.s. energy shift $\Delta E_0$ using a ring-diagram method (see Refs. \cite{11, 42} for details), where the particle-particle hole-hole ($\text{pphh}$) ring diagrams are summed to all orders. The low-order $\text{pphh}$ ring diagrams can be readily calculated. The 1st- and 2nd-order diagrams are respectively

$$D^{(1)} = \frac{1}{2} \sum_{ab} \langle ab | V_{\text{low-}k} | ab \rangle n_a n_b ,$$

$$D^{(2)} = \sum_{abcd} \langle ab | V_{\text{low-}k} | cd \rangle^2 n_a n_b (1 - n_a) (1 - n_b) \frac{1}{4 (c - e - e_c + e_d) .}$$

Here $n_a$ is the occupation number of the orbit $a$, given by $n_a=1$ if $a \leq k_F$ and $=0$ if otherwise, and $\langle ab | V_{\text{low-}k} | cd \rangle$ denotes the anti-symmetrized and normalized shell-model matrix element. $\bar{c}$ is a shell-model Hartree-Fock (smHF) s.p. energy. We dress the single-particle and single-hole lines with self-energies, or HF insertions, and include such insertions to all orders. Sample diagrams illustrating the general structure of the $\text{pphh}$ ring diagrams and their self-energy insertions have been given in e.g. Refs. \cite{11, 42, 43}. The net effect of such all-order insertions is the replacement of the oscillator s.p. energy $\epsilon_a^{(0)}$ of the undressed ring diagrams by the smHF s.p. energy

$$\bar{\epsilon}_a = \epsilon_a^{(0)} + \sum_b \langle ab | V_{\text{low-}k} | ab \rangle n_b .$$

Unless specified otherwise, we shall use $\bar{\epsilon}$ in all the calculations reported in this work.

An initial step to calculate the energy shift given by the all-order sum of the $\text{pphh}$ ring diagrams (denoted as...
\[ \Delta E_0^{pp} = \int_0^1 d\lambda \sum_{m} Y_m(ab, \lambda) \times Y_m^*(cd, \lambda) \times \langle ab|V_{\text{low}-k}|cd \rangle. \]

Here \( \lambda \) is a strength parameter, to be varied from 0 to 1. The indices \( (a, b) \) can be either both particles \( (p, p') \) or both holes \( (h, h') \) and so are \( (c, d) \). The subscript \( P \) refers to the ranges for particles and holes used in the calculation. As an example, for the 40-neutron case we have \( (h, h') \) in the core composed of the \( (0s, 0p, 0d1s, 0f1p) \) shells and \( (p, p') \) in the two major shells above the core. The above equation has two sets of solutions \( (\omega^+_m, Y_n) \) and \( (\omega^-_m, Y_m) \), with \( Y_n \) and \( Y_m \) dominated respectively by their \( (p, p') \) and \( (h, h') \) components. With the wave functions of the latter set, the energy shift is then given by \[ \Delta E_0^{pp} = \int_0^1 d\lambda \sum_{m} Y_m(ab, \lambda) \times Y_m^*(cd, \lambda) \times \langle ab|V_{\text{low}-k}|cd \rangle. \] The energy shift can also be calculated using a different and considerably simpler method \[ \Delta E_0^{pp} = -\sum_m \omega^-_m + \sum_{ab} (\bar{\epsilon}_p n_p + \bar{\epsilon}_h n_h), \]

where \( \omega^-_m \) is obtained by solving Eq. (8) with \( \lambda = 1 \). We shall carry out our calculations using both methods to cross check our numerical results.

**Results and discussions.** In this section we present and discuss the calculated g.s. properties for finite neutron systems confined in a harmonic trap interacting with unitary interactions. In Table I we present the trap ratio \( R_t \) of Eq. (2) for two closed-shell systems with \( A=40 \) and 70, as well as the intrinsic ratio \( R_i \) of Eq. (3) for \( A=70 \). In fact the ratios \( R_t \) and \( R_i \) are simply related by \( 2R_t = (1 + R_i) \), if we neglect \( K_{cm} \) in Eq. (4) which is obviously small compared with \( T_0 \) for our cases. The ratios with the superscripts “all” are given by the g.s. energy shifts calculated by summing the pp hh ring diagrams to all orders, using both the method of Eq. (9) and that of Eq. (10). The results given by the two methods are identical (to fourth decimal), providing a check of our numerical calculations. The ratios with the superscripts “1” and “2” are given by the 1st- and 2nd-order approximations, whose energy shifts are respectively \( D^{(1)} \) and \( D^{(1)} + D^{(2)} \) of Eqs. (5) (6).

In Table I one sees that, starting from the 1st-order approximation, the ratios decrease slightly by including the 2nd-order diagram. The ratios given by the 2nd-order approximation are further decreased, only very slightly, by summing the diagrams to all orders. These very small differences indicate that our linked-diagram expansion provides a rapidly converging framework for calculating the g.s. energies of trapped closed-shell unitary systems. As also shown in Table I of great interest is that the \( R_{ti}^{(all)} \) (and \( R_{ti}^{(all)} \)) values are remarkably invariant in regard to variations of the trap parameter \( \omega \), the decimation momentum \( \Lambda \) of \( V_{\text{low}-k} \), and the type (either the meson-exchange CD-Bonn potential or the HCSW ones) and effective range \( r_e \) of the unitarity potential.

Below we study a connection between our results and the unitary-limit virial theorems \[ 26, 29 \]. Let us begin with the theorem of Werner and Castin \[ 26, 29 \]. According to this theorem, the g.s. energies of unitary fermion systems in a harmonic trap satisfy the relation

\[ (\Psi_0|T + U_{\text{osc}} + V)|\Psi_0) = E_0 = 2\langle \Psi_0|U_{\text{osc}}|\Psi_0 \rangle, \]

where \( E_0 \) and \( \Psi_0 \) are respectively the g.s. energy and wave function of the system.

From the above equation we readily have

\[ \omega \frac{d}{d\omega} E_0 = E_0. \]
TABLE I: The trap ratio $R_t$ for two closed-shell systems with $A=40$ and 70, as well as the intrinsic ratio $R_i$ for $A=70$. The ratios given by summing up the ppbh ring diagrams to all orders and the 1st- and 2nd-order approximations are denoted respectively as $R_t^{(1)}$, $R_t^{(2)}$ and $R_t^{(all)}$. The decimation momentum $\Lambda$ of the $V_{\text{low-k}}$ is in units of fm$^{-1}$, the effective range $r_e$ of the inter-neutron potential and the harmonic trap size $a_{\theta,\omega}=\sqrt{\hbar/(m\omega)}$ in fm, and $\hbar\omega$ in MeV.

| $\Lambda$ | $r_e$ | $a_{\theta,\omega}/\hbar\omega$ | 40 neutrons | 70 neutrons |
|-----------|-------|-------------------------------|------------|------------|
|           |       | $R_t^{(1)}$ | $R_t^{(2)}$ | $R_t^{(all)}$ | $R_i^{(1)}$ | $R_i^{(2)}$ | $R_i^{(all)}$ |
| CD-Bonn   | 2.0   | 2.54 | 2.35/7.5 | 0.763 | 0.758 | 0.756 | 0.754/0.505 | 0.752/0.502 | 0.752/0.501 |
| CD-Bonn   | 2.0   | 2.54 | 2.21/8.5 | 0.760 | 0.755 | 0.754 | 0.752/0.502 | 0.751/0.499 | 0.750/0.498 |
| CD-Bonn   | 2.0   | 2.54 | 2.09/9.5 | 0.759 | 0.754 | 0.753 | 0.752/0.502 | 0.751/0.499 | 0.750/0.498 |
| CD-Bonn   | 2.0   | 2.54 | 1.99/10.5 | 0.758 | 0.754 | 0.754 | 0.753/0.503 | 0.752/0.501 | 0.751/0.500 |
| CD-Bonn   | 2.0   | 2.54 | 1.86/12.0 | 0.759 | 0.756 | 0.755 | 0.755/0.503 | 0.752/0.501 | 0.751/0.500 |
| CD-Bonn   | 2.0   | 2.54 | 1.72/14.0 | 0.762 | 0.760 | 0.759 | 0.761/0.519 | 0.760/0.518 | 0.760/0.517 |
| CD-Bonn   | 2.3   | 2.54 | 1.99/10.5 | 0.758 | 0.754 | 0.754 | 0.753/0.503 | 0.752/0.501 | 0.751/0.500 |
| CD-Bonn   | 1.8   | 2.54 | 1.99/10.5 | 0.758 | 0.754 | 0.753 | 0.752/0.502 | 0.751/0.500 | 0.751/0.499 |
| HCSW-1    | 2.0   | 2.63 | 1.99/10.5 | 0.761 | 0.759 | 0.759 | 0.762/0.521 | 0.761/0.520 | 0.761/0.519 |
| HCSW-2    | 2.0   | 2.36 | 1.99/10.5 | 0.754 | 0.751 | 0.750 | 0.750/0.498 | 0.749/0.496 | 0.749/0.495 |
| HCSW-3    | 2.0   | 2.22 | 1.99/10.5 | 0.753 | 0.750 | 0.749 | 0.749/0.495 | 0.748/0.493 | 0.747/0.492 |
| HCSW-4    | 2.0   | 2.20 | 1.99/10.5 | 0.753 | 0.749 | 0.748 | 0.749/0.496 | 0.748/0.494 | 0.748/0.493 |
| HCSW-5    | 2.0   | 1.80 | 1.99/10.5 | 0.760 | 0.754 | 0.753 | 0.750/0.498 | 0.748/0.494 | 0.748/0.493 |

FIG. 2: Summation of $(e_{qp}(a) - \frac{1}{2}e_{qp}^{(0)})$ (equal to intrinsic quasi-particle energies $e_{qp}^{(int)}(a)$) up to the $n$-th orbit, for two closed-shell systems with $A=40$ and 70, respectively. The values over $\hbar\omega$ are presented here. See text for further explanations.

FIG. 3: The quasi-particle energy $e_{qp}$ of the $n$-th orbit, for the closed-shell system with $A=70$. The values over $\hbar\omega$ are presented here. See text for further explanations.

In Fig. 1(a) we present our ring-diagram g.s. energies using the fine-tuned CD-Bonn potential and using $\hbar\omega=7.5, 8.5, 9.5, 10.5, 12.0, 14.0$ MeV, respectively. Three closed-shell systems with $A=20, 40,$ and 70 are considered. Plotted is the energy per neutron (dots), versus the $\hbar\omega$ value, as well as the corresponding linear-fitting result (line). One sees all lines fit the data nearly perfectly, and moreover they all converge to the origin $(E, \hbar\omega)=(0, 0)$ also near perfectly. Clearly our results are in highly satisfactory agreement with the linear scaling relation of Eq. (12). Returning to Table I the ratios $R_t$ which gives

$$E_0 = \alpha \hbar\omega$$

with $\alpha$ a constant independent of $\hbar\omega$. Eq. (12) is a special linear scaling relation for the g.s. energy of a unitary fermion system in a harmonic trap. Let us check if our calculated g.s. energies are consistent with this scaling relation.
presented there are practically invariant with the variation of $\hbar \omega$. This invariance is actually a consequence of the above scaling relation (recalling $R_t = E_0/E_{0\text{ free}}$ and $E_{0\text{ free}}$ is proportional to $\hbar \omega$).

We also generalize our study to open-shell systems with valence neutrons occupying the major shell just above the closed core. Based on the low-momentum unitary interactions the same as those used for closed-shell systems, we derive the one-body and two-body effective interactions for valence neutrons using a $Q$-box folded-diagram method \[32\] \[41\]. Here the $Q$-box vertex function is composed of 1st- and 2nd-order irreducible valence-linked diagrams, including the core-polarization ones. We then calculate and diagonalize the Hamiltonian matrix in the many-body model space of valence particles, using a standard shell-model code \[44\]. In Fig. 1(b) we exemplify the results of open-shell systems using those with $A=26$, 36, and 46. One sees the energy per neutron is also remarkably proportional to $\hbar \omega$, again in very good agreement with the scaling relation of Eq. (12). According to our calculation, the scaling relation also holds perfectly for odd-$A$ open-shell systems.

The g.s. energies $E_0$ of Fig. 1 are calculated with finite effective range $r_e \simeq a_{\text{h.o.}}$ \(\text{(see Table I)}\), and they are in good agreement with the linear scaling relation of Eq. (12) which is, however, based on the virial theorem \[20\] for $r_e \rightarrow 0$. There is a difference in $r_e$. In Ref. \[29\] a general virial theorem including the effect of finite $r_e$ was obtained. From this theorem, we have

$$\omega \frac{d}{d\omega} E_0 = E_0 + \frac{1}{2} (E_0 - \bar{E}_0) ,$$

(13)

where $E_0$ and $\bar{E}_0$ are the g.s. energies of systems with $r_e \neq 0$ and $=0$, respectively. As illustrated in Table I the ratios $R_t$ is invariant with the choice of the unitary interaction, and consequently $E_0$ is invariant with $r_e$ (recalling $R_t = E_0/E_{0\text{ free}}$ and $E_{0\text{ free}}$ is independent of $r_e$). Thus we have in general $E_0 = \bar{E}_0$, and the above differential equation becomes $\omega (d/d\omega) E_0 = \bar{E}_0$ which is identical to Eq. (11). This leads to an interesting conclusion, namely the scaling relation of Eq. (12) is applicable to not only trapped unitary systems with $r_e \rightarrow 0$ but also those with $r_e > 0$, which is supported by our results in Fig. 1.

At last we study the unitary ratio guided by Landau’s Fermi liquid (FL) theory \[38\]. In this way, a simple relation between the ratio and the Fermi-liquid quasi-particles can be obtained. For a system of neutrons in a harmonic trap, the quasi-particle (q.p.) energy $\epsilon_{\text{qp}}$ and the g.s. energy $E_0$ are related as

$$e_{\text{qp}}(a) = \frac{\delta E_0}{\delta n_a} ,$$

$$E_0 = \frac{1}{2} \sum_a \epsilon_{\text{qp}}^{(0)} n_a + \sum_a e_{\text{qp}}(a) n_a .$$

(14)

The trap ratio of Eq. (2) is then given by $\delta E_0/\delta n_a = 2(R_t - 1/2)\hbar \omega$. To have $R_t \approx 0.75$, the FL quasi-particles should satisfy the sum rule

$$\sum_a [e_{\text{qp}}(a) - 1/2 \epsilon_{\text{qp}}^{(0)}] n_a = 0 .$$

(15)

We also define an intrinsic q.p. energy $\epsilon_{\text{qp}}^{\text{int}}(a) = e_{\text{qp}}(a) - \epsilon_{\text{qp}}^{(0)}$, and the intrinsic ratio of Eq. (3) is then given by $\sum_a \epsilon_{\text{qp}}^{\text{int}}(a) n_a = 2(R_t - 1/2)T_0$ if $K_{\text{cm}}$ is neglected. One easily sees the sum rule also gives $R_t \approx 0.5$.

As shown in Table 1 the g.s. energy given by the low-order approximation is closely equal to that including high-order diagrams. We have thus approximated $\epsilon_{\text{qp}}$ by $e_{\text{qp}}^{(2)}(a) = \epsilon_{\text{qp}}^{(0)} + \delta(D^{(1)} + D^{(2)})/\delta n_a$, and obtain $e_{\text{qp}}^{(2)}$ by solving this equation in a self-consistent manner. In Fig. 2 we exemplify the above sum rule, using the closed-shell systems with $A=40$ and 70. The results are based on the unitary CD-Bonn and HCSW-2 interactions, combined with the trap parameter $\hbar \omega = 8.5$ and 10.5 MeV, respectively. As shown in Fig. 2 the sum rule of Eq. (15) is indeed well satisfied by our results. Moreover, the scaled q.p. energies $\epsilon_{\text{qp}}/\hbar \omega$ are remarkably invariant with the choice of the unitary interaction and the choice of $\hbar \omega$, as shown in Fig. 2 and later in Fig. 3. In Fig. 3 we present the q.p. energies of the $A=70$ system, where one sees the $\epsilon_{\text{qp}}$ values for orbits within one harmonic oscillator shell are nearly degenerate. According to our calculation, the q.p. energies of $A=20$ and 40 also have this major-shell degeneracy. In ordinary nuclear systems the s.p. energy levels of each major shell are generally non-degenerate. It will be of much interest to check if this drastic change does take place experimentally.

**Summary.** We have studied systems of finite-number neutrons in a harmonic trap at the unitary limit. Two very different types of neutron-neutron interactions have been applied, namely, the meson-theoretic CD-Bonn potential and the hard-core square-well ones, all tuned to possess infinite scattering lengths, and with effective ranges comparable to or larger than the trap size. The potentials were renormalized to equivalent, scattering-length preserving low-momentum potentials, $V_{\text{low-k}}$, with which the particle-particle hole-hole ring diagrams are summed to all orders to yield the ground-state energy $E_0$ of the finite neutron system. Two different methods were employed for the above ring-diagram calcula-
tions, giving practically identical results. We find the ratio $R_t \equiv \frac{E_0}{E_0^{\text{free}}}$ to be remarkably invariant in regard to variations of the harmonic trap parameter, the number of neutrons, the decimation momentum of $V_{\text{low}} - \epsilon$, and the type and effective range of the unitarity potential. Our results support a special linear scaling relation between the virial theorems. Our results further suggest the scaling relation is applicable to not only trapped unitary systems with inter-fermion interactions of zero-range but also those with interactions of finite effective ranges. The $R_t$’s all flow to a specific value of 0.75, suggesting a sum rule of Landau’s quasi-particles for trapped unitary neutrons. The quasi-particle energies also exhibit a major-shell degeneracy behavior.

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