An Adaptive Subspace Self-Organizing Map (ASSOM) Imbalanced Learning and Its Applications in EEG

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Abstract—This paper presents a novel oversampling technique that addresses highly imbalanced benchmark and electroencephalogram (EEG) data distributions. Presently, conventional machine learning technologies do not adequately address imbalanced data with an anomalous class distribution and underrepresented data. To balance the class distributions, an adaptive subspace self-organizing map (ASSOM) that combines a local mapping scheme and the globally competitive rule is proposed to artificially generate synthetic samples that focus on minority class samples and its application in EEG. The ASSOM is configured with feature-invariant characteristics, including translation, scaling, and rotation, and it retains the independence of the basis vectors in each module. Specifically, basis vectors that are generated via each ASSOM module can avoid generating repeated representative features that only increase the computational load. Several benchmark experimental results demonstrate that the proposed ASSOM method incorporating a supervised learning approach could be superior to other existing oversampling techniques, and two EEG applications present the improvement of classification accuracy using the proposed ASSOM method.

Index Terms— Imbalanced Learning, Oversampling, Synthetic Sample Generation, Subspace, EEG, Classification

I. INTRODUCTION

Learning from imbalanced data has attracted growing attention in the research community in recent years because it is present in a variety of real-world application problems, including medical diagnosis, anomaly detection and financial fraud detection [1-4]. Under these circumstances, the use of computationally intelligent methods has the potential to play an essential role in solving these problems; however, there are still many challenges to this research topic.

Specifically, a classification task can be regarded as an imbalanced problem whenever some types of data distribution significantly dominate the others. In this paper, for simplicity, we focus on the two-class imbalanced classification problem, which is a topic of major interest in the research community. The underlying challenge manifests itself in two common forms: relative imbalance and absolute imbalance. Relative imbalance occurs when minority samples are well represented but severely outnumbered by the majority of samples, whereas absolute imbalance arises in datasets in which minority samples are scarce and underrepresented. Either form of imbalance poses a great challenge to conventional classification algorithms because it becomes extremely difficult to detect minority class samples. The reason arises from the fact that the algorithm tends to favor the majority class samples or simply omit the minority class samples in the training process, which thereby results in a biased classifier. This phenomenon becomes troublesome when the detection of minority class samples is crucially important, such as in cancer diagnosis.

Current solutions to the imbalanced problem can be divided into two categories: internal methods and external methods. Internal methods target the imbalanced problem by modifying the underlying classification algorithm. A popular approach in this category is cost-sensitive learning [5], which uses a cost matrix for different types of errors or instances to facilitate the learning directly from an imbalanced dataset. A higher cost of misclassifying a minority class sample compensates for the scarcity of the minority class. In [6], a cost-sensitive framework for applying the support vector machine is proposed. In [7], Zhou and Liu investigated the applicability of cost-sensitive neural networks on the imbalanced classification problem. In contrast, external methods aim to address the imbalanced problem by manipulating the input data to form a more balanced data set. External methods can further be divided into under-sampling and oversampling. Under-sampling methods compensate for the imbalanced problem by reducing the instances of the majority class. A cluster-based under-sampling approach is proposed in [8]. A study demonstrates class cover catch diagrams (CCCDs) capture the density of majority class as radii of the covering balls as to preserve the information during the under-sampling process [9]. In contrast to under-sampling methods that remove majority class samples, oversampling methods balance the data set by generating synthetic samples for the minority classes. The synthetic minority oversampling technique (SMOTE) [10] algorithm generates an arbitrary number of synthetic minority samples to eliminate the classifier learning bias. A collection of extension works based on the SMOTE algorithm has been proposed to address the imbalanced classification problem, e.g., the Borderline-SMOTE [11], SMOTE-Boost [12], majority weighted minority oversampling technique (MWMOTE) [13] and adaptive synthetic sampling (ADASYN) [14]. In [15], an enhanced structure preserving oversampling (ESPO) method...
that is based on a combination of the multivariate Gaussian distribution and an interpolation-based algorithm is developed. In 2017, we proposed a kernel adaptive subspace method to address nonlinear boundary problem [16]. Kernel method achieves remarkable performance on extreme imbalanced data but comes with a high computational cost. In this paper, we proposed an adaptive subspace self-organizing map (ASSOM) oversampling method to address the imbalanced problem.

In terms of electroencephalogram (EEG) applications, motor imagery (MI) signals recorded via electroencephalography (EEG), a mental process by which an individual rehearses or simulates a given action, is the most convenient basis for designing brain-computer interfaces [17, 18]. This helps motor disabled people to communicate with the device by performing sequence of MI tasks, but the motor imagery samples were usually found highly imbalanced. Furthermore, clinical applications of EEG have increasingly gained attention within the biomedical engineering community. It can be applied to end-users with prediction or classification of neurological diseases including migraine [19], depression [20] and sleep [21]. However, the minority of EEG samples may lead an negative influence on classification accuracy. This study tries to tackle the same problem with merely linear computation involved. The ASSOM is feature-invariant regarding translation, scaling, and rotation. By assuring the independence of the basis vectors of each module, we can generate representative synthetic EEG samples for the minority class.

The remainder of this paper is organized as follows. In Section 2, a concise survey of existing oversampling techniques is presented. The details of the proposed ASSOM algorithm are discussed in Section 3. Section 4 discusses the experimental results on various benchmark data and provides a comparison with other existing oversampling algorithms. Section 5 presents the experimental results on EEG data collected in this study. Finally, conclusions are drawn in Section 6.

II. RELATED WORK

The critical issue in imbalanced data is that learning algorithms tend to be biased toward majority classes or less important negative classes that consist of a large number of samples. For a two-class imbalanced problem, various oversampling approaches have been proposed to balance the distribution of different classes, including SMOTE [10], ADASYN [14], ESPO [15], MWMOTE [13] and ADG [22]. These methods alter the imbalanced ratios by augmenting the minority class (or positive class) with synthetically generated samples. Then, a classifier is trained according to the balanced dataset. This mechanism of oversampling has proven to efficiently improve the performance in the recognition task.

A. Synthetic Minority Over-sampling Technique

SMOTE [10] utilizes minority (positive) samples as seed samples and evenly generates synthetic samples from each selected seed. The minority class is oversampled by introducing synthetic samples along with the k-nearest neighbor (KNN) of the minority class. Synthetic samples are generated using the following steps: 1) taking the difference between a sample under consideration and its nearest neighbor, 2) multiplying this difference by a random number between 0 and 1, and 3) adding this weighted difference to the sample under consideration. This process causes the selection of a random point along with the line segment between two specific samples and effectively determines the decision boundary of the minority class, which causes the classifier to become more general in a classification task. In addition, in [10], the authors indicate that a combination of under-sampling the majority class and oversampling the minority class can provide superior system performance compared with either an under-sampling or an oversampling approach. The SMOTE that combines under-sampling of the majority class introduces a bias toward the minority class. Therefore, SMOTE provides more related minority class samples that a classifier can learn from, and the broader areas can be carefully carved, which results in a better approximation of the minority class. However, over-generalization in the SMOTE severely influences the system performance. The over-generalization in the SMOTE algorithm is mainly caused by its generated synthetic samples. Specifically, SMOTE gives the same number of synthetic data samples for each primitive minority sample and does not carefully consider the distribution of the neighboring samples, which leads to the occurrence of overlap between different classes.

B. Adaptive Synthetic Sampling

Various adaptive oversampling techniques have been proposed in the recent past to overcome the limitation of the SMOTE. The ADASYN [14] algorithm employs an adaptive mechanism in which the number of synthetic samples generated by each minority (positive) seed is determined by the ratio of majority (negative) samples in its neighborhood. The nucleus of ADASYN is to evaluate the level of learning difficulty for each minority class sample. A weighted distribution approach is used to allocate specific weights to different minority samples, where more synthetic data would be generated for a minority sample that is more difficult to learn compared with minority samples that are easier to learn. As a result, the ADASYN approach improves the distribution of different classes in two phases, including 1) reducing the bias introduced by the class imbalance, and 2) adaptively adjusting the decision boundary with respect to the difficult regions. These two objectives are accomplished by a dynamic adjustment of the weights and an adaptive learning procedure according to the density distribution.
C. Enhanced Structure Preserving Oversampling

ESPO [15] was proposed by Cao et al. to handle highly imbalanced time series classification. ESPO uses a multivariate Gaussian distribution to estimate the covariance matrix of the minority samples and regularize the unreliable eigen spectrum. The main portion of synthetic samples is generated by ESPO in the eigen decomposed subspace and is regularized by the eigen spectrum. To aim to protect the seed samples that are difficult to classify in the minority class, an interpolation-based technique is employed to augment a smaller portion of the synthetic population. By preserving the main covariance architecture and creating protective variances in the trivial eigen dimensions, the ESPO can successively generate synthetic samples that still have partial dissimilarity with respect to the existing minority class samples.

D. Majority Weighted Minority Oversampling Technique

Barua et al. proposed the MWMOTE [13], which identifies the most informative minority class samples that are more difficult to classify. To address this issue, the clustering approach is applied to adaptively assign appropriate weights to each of the minority samples according to their importance in the learning procedure. The samples that are closer to the decision boundary are given higher weights than others. Similarly, the samples of the small-sized clusters are given higher weights to reduce the within-class imbalance. As a result, the MWMOTE generates synthetic samples using those weighted seed samples. This architecture of the MWMOTE guarantees that each generated sample resides inside a certain minority class cluster, which prevents noisy synthetic sample generation. The MWMOTE is the first attempt at identifying the difficult-to-learn minority class samples and assigns them proper weights according to their Euclidean distance from the nearest majority class sample. The essences of the MWMOTE include 1) selecting the most informative subset from the primitive minority samples, 2) calculating the weights to the selected samples according to their importance (Euclidian distance) in the dataset, and 3) exploiting a clustering approach to augment the synthetic minority class samples.

E. Absent Data Generator

Pourhabib et al. proposed the ADG [22] to tackle the imbalanced problem by oversampling minority class. ADG employs kernel Fisher discriminant analysis to generate synthetic data near the discriminative boundary of minority and majority class as data close to boundary carry more information for hyperplane separation in the feature space.

F. Support Vector Machines for Class Imbalance

Support vector machines (SVMs) is a popular machine learning technique, which works effectively with balanced datasets. However, when it comes to imbalanced datasets, SVMs produce suboptimal classification models. Joachims proposed a support vector method, called SVM-light [23], for optimizing multivariate based on the sparse approximation algorithm for the structural SVM. Instead of learning a univariate rule that predicts the label of a single example, the SVM-light exploited a multivariate prediction of all examples in the dataset during the learning phase. The SVM-light can effectively address a large imbalance between positive and negative examples by directly optimizing the measure of interest (e.g., recall, precision, F-value). In addition, the “balanced” mode of SVM is also applied in this study to separate the hyperplane for imbalanced classes [24]. The “balanced” mode automatically adjusts weights inversely proportional to class frequencies in the input data.

III. THE ADAPTIVE-SUBSPACE SELF-ORGANIZING MAP

The idea of using subspaces that are subsets of the largest principal components for data generation is an emerging technology. The eigenvectors of the input correlation matrix are called the principal components, which are composed on the corresponding linear subspaces. As shown in Figure 1, the proposed ASSOM model, which is extended by the concept of the self-organizing map (SOM), is used to artificially evolve useful samples according to the above mechanism. Therefore, in this section, we describe the ASSOM algorithm in the presence of a structure and a learning scheme.

An invariant feature of the input vector \( x \) represents the signal subspaces. A linear subspace \( L \) of dimensionality \( H \) is defined given the linearly independent basis vectors \( b_1, \ldots, b_H \), and the reconstructed signal is obtained as shown in Eq. (1); however, there exist infinitely many equivalents and non-unique combinations of the \( b_i \) for the same \( L \). In the parameter learning phase, this study utilizes a gradient descent (GD) algorithm to achieve updating. The detailed functions of each layer are described below.

A. ASSOM Structure

ASSOM presents as module structure as in Figure 1. Three layers are included, layer 1 takes input data, layer 2 maps the input data to feature space(subspace) and layer 3 manages to reconstruct original data. Each node in layer 2 can be represented as a linear-subspace neural unit. Each node is a linearly independent basis vector. The output function of layer 3 is written as

\[
\hat{x} = \sum_{i=1}^{H} x^T b_i \cdot b_i
\]  

(1)

where \( x \) denotes input data. \( b_i \) denotes the orthonormal form and \( H \) denotes the number of hidden nodes.

Here, a set of equivalent orthonormal basis vectors for \( L \) can be computed by the familiar Gram-Schmidt process. The reconstructed signal relies on the orthonormal basis; in other words, the reconstructed signal \( \hat{x} \) that belongs to \( L \) is the...
orthogonal projection of $x$ onto $L$.

We expect that the reconstructed signal is approximately similar to the original signal; thus, the criterion using the Euclidean distance as $\|\tilde{x}\| = \|x - \tilde{x}\|$ is present to determine whether they are similar or even identical. Finally, a projection operator matrix $P$ is defined as Eq. (2), and the following properties hold: $P^2 = P$ and $P^T = P$.

$$P = \sum_{i=1}^{H} b_i b_i^T$$

where $\tilde{x} = Px$ and $\tilde{x} = (I - P)x$, in which $I$ represent the identity matrix.

### B. Learning Scheme

Because the learning mechanism of the SOM is inherited, an ASSOM also possesses the abilities of competitive learning for parameter learning, which are vital contributions to the effectiveness and robustness of the system. First, we would like to describe the procedure of competitive learning as in Figure 2. The different modules are generated to compete on the input signal subspaces to find the minimum distance as the winner, which represents important information in that a given signal subspace that is best wins; The number of competing module is defined by the expression

$$N = \text{round}\left(\frac{\text{# of majority class}}{\text{# of minority class}}\right) - 1$$

$$= \text{round}(\text{imbalance ratio}) - 1.$$  

where $N$ denote the number of competing modules.

Consequently, the updated weight vectors in each module follow the representative winner. As the modules in the neighbourhood of the winner are adapted to represent the input better, the neighbouring modules gradually approximate the winner of the inputs. The representative winner can be defined by the expression

$$c = \arg\min_{n \in N} \left\{ \sum_{t \in S} \|x - \tilde{x}^n(t)\|^2 \right\}$$

$$= \arg\min_{n \in N} \left\{ \sum_{t \in S} \|\tilde{x}^n(t)\|^2 \right\}$$

where $S$ is denoted as the total number of input samples, and $c$ as the index of the winning module.

After obtaining the winning module via competitive learning, the free parameters of the other modules must be adjusted dependently by a factor in terms of the distance between their input subspace and the subspace of the winning module, to effectively achieve the phase of learning. Therefore, we define the objective as minimizing the error function. The error function is considered to be two factors that correspond to the neighborhood factors, $g^c(t)$, as follows:

$$g^c(t) = \exp\left[\frac{-\|x^c(t) - \bar{x}^c(t)\|^2}{2\sigma^2}\right]$$

where $\sigma$ is a constant.

Cost function $E$ is defined as the summation of projection error for all modules and data.

$$E = \sum_{n} \sum_{t \in S} g^c(t) \|\tilde{x}^n(t)\|^2$$

Consequently, this GD algorithm is performed for each piece of incoming data. By using the GD algorithm for the updated basis vectors of each module, we have

$$b_i^n(t + 1) = b_i^n(t) - \eta \frac{\partial E}{\partial b_i^n(t)}$$

where $b_i^n$ is the i-th basis vector of module $n$ and the factor $\eta$ is a learning rate, and the derivation is computed as

$$\frac{\partial E}{\partial b_i^n(t)} = -2 \sum_{t \in S} g^c(t) x(t)x(t)^T b_i^n(t)$$

Based on Eqs. (7) and (8), the basis vectors are updated as follows:

$$b_i^n(t + 1) = [I + \eta g^c(t) x(t)x(t)^T]b_i^n(t)$$

In the case of the rotation operation, the learning rate $\eta$ should be such that it guarantees a monotonically increasing function of $\|\tilde{x}^n\|$ or a monotonically decreasing function of $\|\tilde{x}^n\|$. For the monotonic correction, we must be proportional to $x^Tb_i^n$, and thus, one of the simplest ways is to divide the learning rate $\eta$ by the crisp value $\|\tilde{x}^n\|/\|x\|$. Let us denote the learning rate as $\lambda$; then, Eq. (9) is rewritten as
\[ b_i^n(t + 1) = \left[ I + \lambda^i(t) \frac{x(t)x(t)^T}{\|x(t)\|^2} \right] b_i^n(t) \]  \hspace{1cm} (10)

where \( \lambda^i(t) = \eta g_i^n(t) \).

During the learning process, we set the magnitude of the small components of the basis vectors \( b_i^n \) to zero to reduce those degrees of freedom; thus, \( b_i^n \) is forced to approximate the dominant frequency components. \( \tilde{b}_i^n \) by a dissipation effect can be described by

\[ \tilde{b}_i^n = sgn(b_i^n) \cdot \max \left[ 0, \text{abs}(b_i^n) - \varepsilon \right] \]  \hspace{1cm} (11)

where \( \varepsilon \) is a small fraction of magnitude vector that can modeling as the following equation:

\[ \varepsilon = \alpha \cdot \text{abs}[b_i^n(t) - b_i^n(t - 1)] \]  \hspace{1cm} (12)

where \( \alpha \) is a small constant. Based on the dissipation effect, it must be applied after the GD algorithm is performed and prior to normalization. To ensure basis vectors of all competing modules are orthonormal, Gram-Schmidt process is applied.

To sum up, the learning steps of an ASSOM are as follows. Once we receive each piece of training data, the procedure will be divided into the following steps:

- **Step 1:** Generate the competing modules by Equ (3).
- **Step 2:** Find the winning module by Equ. (4).
- **Step 3:** Update the basis vectors of each module via a gradient descent algorithm by Equ. (10) and (11).
- **Step 4:** Orthonormalize the basis vectors of each module via the Gram-Schmidt process.

### IV. EXPERIMENTS AND RESULTS

To effectively display the performance from using different oversampling methods, we apply two fundamental classifiers, artificial neural networks (ANNs) and SVMs. ANNs and SVMs both play important roles in solving classification problems. In particular, ANNs [25] possess the special characteristics of self-organization, adaptive learning capability, and robustness. ANNs are inspired by biological neural networks, and the structures of the ANNs are commonly composed of interconnected neurons (non-linearity mapping) that deliver information to each other. The SVM [26] is a popular machine learning method for various learning tasks or classification applications. SVMs belong to the non-probabilistic model that classifies unknown data using hyper-planes. The SVM implicitly maps samples to a high-dimensional space through a kernel trick instead of increasing a non-linear operation.

#### A. Benchmark Data

Eight benchmark datasets from the UCI machine learning repository [27] and KEEL dataset [28] are employed to test the proposed method compared with other existing oversampling or synthetic data generation technologies. Four assessment metrics, including the recall, precision, G-mean, and F1-value, are considered to reveal the significant advantage of the proposed method. Finally, the results demonstrate that we must account for the oversampling techniques to avoid having the classification of the minority party dominated by the majority party. Specifically, the information contributions from the minority party are more important than those from the majority party.

#### B. Assessment Metric

Four assessment metrics, the recall, precision, G-mean, and F-value, are considered to determine the benefits of the ASSOM for imbalanced class distributions. Four metrics reply by counting the number of true positive (TP), true negative (TN), false positive (FP) and false negative (FN) samples. These metrics are shown in Eqs. (28) - (31).

\[ \text{Precision} = \frac{TP}{TP + FP} \]  \hspace{1cm} (28)

\[ \text{Recall} = \frac{TP}{TP + FN} \]  \hspace{1cm} (29)

\[ G – \text{mean} = \sqrt{\frac{TP}{TP + FN} \times \frac{TN}{FP + TN}} \]  \hspace{1cm} (30)

\[ F1 – \text{value} = 2 \frac{\text{Recall} \times \text{Precision}}{\text{Recall} + \text{Precision}} \]  \hspace{1cm} (31)

The function G-mean is to evaluate the overall performance

| Dataset Name | # of Total Examples | # of Attributes | Minority Class | Majority Class | # of Minority Examples | # of Majority Examples | Imbalanced Ratio |
|--------------|-------------------|----------------|----------------|---------------|-----------------------|-----------------------|-----------------|
| Abalone      | 731               | 7              | Class of ‘18’  | Class of ‘9’  | 42                    | 689                   | 16.40           |
| Breast cancer| 683               | 9              | Class of ‘malignant’ | Class of ‘benign’ | 239                  | 444                   | 1.86            |
| E. coli      | 336               | 7              | Class of ‘im’  | All other classes | 77                   | 259                   | 3.36            |
| Glass        | 214               | 9              | Class of ‘5,6,7’ | All other classes | 51                   | 163                   | 3.20            |
| Pima         | 768               | 8              | Class of ‘1’   | Class of ‘0’  | 268                   | 500                   | 1.87            |
| Vehicle      | 846               | 18             | Class of ‘van’ | All other classes | 199                  | 647                   | 3.25            |
| Yeast        | 1484              | 8              | Class of ‘ME3’, ‘ME2’, ‘EXC’, ‘VAC’, ‘POX’, ‘ERL’ | All other classes | 304                   | 1180                  | 3.88            |
| Ozone        | 1848              | 72             | Class of ‘1’   | Class of ‘0’  | 57                    | 1791                  | 31.42           |

Table 1. Information on the imbalanced data sets
of a classifier associated with the accuracies on both the positive and negative class samples. Unlike the G-mean, which concerns both classes, the F1-value measures the effectiveness of the classification in terms of a ratio of the weighted importance on either the recall or precision for a single class.

C. Evaluation Results

This section presents the performance of the ASSOM and compares it with other state-of-the-art methods. The proposed ASSOM in this paper has been successfully validated on eight real-world imbalanced problems from the UCI machine learning repository and KEEL dataset repository, including Abalone, Breast cancer, E. coli, Glass, Pima, Vehicle, and Yeast. These sets are chosen such that they have different characteristics in terms of their samples, features, classes, and imbalanced ratios. Some of these datasets have samples of more than two classes. For simplicity, these datasets are transformed into a two-class problem in this study. Table 1 describes the relevant items that are associated with the data attributes and properties. There exist highly imbalanced ratios in the presence of the problems that have two categories.

Extensive experiments with two well-known supervised learning methods, ANNs and SVMs, demonstrate the performance of each dataset on the classification task after employing different oversampling approaches. The proposed method is evaluated by the before-and-after test to show the improvement compared to the classifiers that were constructed based on primitive datasets, for which the datasets are not oversampled. After the before-and-after test, the ASSOM is further compared to existing state-of-the-art approaches, namely, SMOTE, ADASYN, ESPO, MWMOTE, SVM-light, and SVM-balanced. To show the improvement realized by the proposed method.

For each comparative model in the validation process, 70% of the data are randomly selected to build the training data set, whereas the remaining data serve as test data. To maintain the imbalanced ratio in each dataset, the selection of majority and minority samples are processed from the original dataset, respectively. Furthermore, the classification task was

Table 2. Average ANN performance comparison for different comparative methods

| Dataset    | Measure | Original | SMOTE | ADASYN | MWMOTE | ESPO | ASSOM |
|------------|---------|----------|-------|--------|--------|------|-------|
| Abalone    | Recall  | 0.401    | 0.765 | 0.511  | 0.683  | 0.634| 0.622 |
|            | Precision | 0.414 | 0.355 | 0.345  | 0.426  | 0.299| 0.446 |
|            | F1 value | 0.394 | 0.483 | 0.407  | **0.518** | 0.404| 0.513 |
|            | G mean   | 0.606 | **0.832** | 0.687 | 0.797  | 0.753| 0.766 |
| Breast cancer | Recall | 0.862 | 0.940 | 0.902  | 0.9494 | 0.969| 0.958 |
|            | Precision | 0.937 | 0.934 | 0.936  | 0.929  | 0.936| 0.947 |
|            | F1 value | 0.896 | 0.937 | 0.918  | 0.938  | 0.952| 0.952 |
|            | G mean   | 0.913 | 0.952 | 0.933  | 0.954  | **0.966** | 0.964 |
| E. coli    | Recall  | 0.714 | 0.864 | 0.734  | 0.818  | 0.863| **0.887** |
|            | Precision | 0.645 | 0.664 | 0.655  | **0.716** | 0.608| 0.681 |
|            | F1 value | 0.674 | 0.746 | 0.689  | 0.761  | 0.710| **0.766** |
|            | G mean   | 0.791 | 0.863 | 0.803  | 0.858  | 0.846| 0.879 |
| Glass      | Recall  | 0.817 | 0.863 | 0.790  | 0.871  | 0.859| **0.880** |
|            | Precision | 0.800 | 0.842 | 0.857  | 0.843  | 0.889| 0.836 |
|            | F1 value | 0.800 | 0.849 | 0.817  | 0.850  | 0.867| 0.852 |
|            | G mean   | 0.870 | 0.903 | 0.857  | 0.906  | 0.908| 0.908 |
| Pima       | Recall  | 0.556 | 0.739 | 0.634  | 0.708  | 0.677| 0.655 |
|            | Precision | 0.604 | 0.596 | 0.551  | 0.603  | 0.568| **0.626** |
|            | F1 value | 0.577 | 0.657 | 0.589  | 0.649  | 0.617| 0.637 |
|            | G mean   | 0.667 | **0.730** | 0.677 | 0.726  | 0.700| 0.716 |
| Vehicle    | Recall  | 0.889 | 0.949 | 0.935  | 0.962  | 0.967| **0.969** |
|            | Precision | 0.904 | 0.907 | 0.899  | **0.920** | 0.894| 0.884 |
|            | F1 value | 0.900 | 0.926 | 0.917  | **0.940** | 0.903| 0.924 |
|            | G mean   | 0.933 | 0.959 | 0.951  | **0.968** | 0.956| 0.964 |
| Yeast      | Recall  | 0.674 | 0.806 | 0.782  | **0.809** | 0.775| 0.758 |
|            | Precision | **0.730** | 0.636 | 0.558  | 0.641  | 0.649| 0.721 |
|            | F1 value | 0.700 | 0.710 | 0.650  | 0.714  | 0.706| **0.736** |
|            | G mean   | 0.793 | 0.842 | 0.809  | **0.845** | 0.831| 0.835 |
| Ozone      | Recall  | 0.035 | 0.360 | 0.358  | 0.362  | 0.499| 0.280 |
|            | Precision | 0.089 | 0.173 | 0.172  | 0.172  | 0.139| **0.252** |
|            | F1 value | 0.047 | 0.228 | 0.228  | 0.228  | 0.214| 0.256 |
|            | G mean   | 0.094 | 0.572 | 0.569  | 0.566  | **0.659** | 0.513 |
| Average    | Recall  | 0.620 | **0.786** | 0.706 | 0.770  | 0.780| 0.751 |
|            | Precision | 0.640 | 0.638 | 0.622  | 0.656  | 0.617| **0.674** |
|            | F1 value | 0.624 | 0.692 | 0.652  | 0.700  | 0.672| **0.705** |
|            | G mean   | 0.708 | **0.832** | 0.787 | 0.828  | 0.827| 0.818 |
| Average Rank | Recall | 1.13 | 4.50 | 2.25  | **4.63** | 4.38| 4.13 |
|            | Precision | 3.50 | 3.38 | 2.63  | 4.00  | 2.50| **4.75** |
|            | F1 value | 1.13 | 4.00 | 2.25  | 4.75  | 3.25| **5.13** |
|            | G mean   | 1.13 | **4.63** | 2.13 | **4.63** | 4.00| 4.38 |
| Average Overall Rank | 1.72 | 4.13 | 2.31  | 4.50  | 3.53| **4.59** |
conducted 50 times to evaluate each comparative classifier to prevent the bias in the initial state parameters during the supervised learning procedure. This overall process of validation is repeated 5 times; hence, the average of 250 runs is compared against other methods.

The validation results of ANNs and SVMs with different oversampling approaches on the eight datasets are shown in Table 2 and Table 3, respectively. The best performance is shown in bold face. The results of the ANN and SVM show that our proposed ASSOM outperforms than existing methods for the majority of the real-world problems.

To better show the improvement of the proposed ASSOM, all of the comparative approaches are ranked based on the results of each assessment metric. Under each assessment metric, the method with the best performance is scored as the highest points (ANN: 6 and SVM: 8), and the worst is scored as 1 point. Consequently, we compute the average rank of the four-assessment metrics across the eight datasets to quantify the relative performances. By further averaging these four-assessment metrics, an overall assessment matrix is integrated to evaluate the comparative approaches. The ANN and SVM with the best performance, which possesses the highest number of points, are shown in the last row of Table 2 and Table 3. The average overall rank of the ASSOM is 4.59 for the ANN and 5.66 for the SVM, which is higher than any of the other state-of-the-art approaches. These experimental results suggest that our proposed ASSOM model can yield a significant improvement in the performance of the imbalanced correction.

V. EEG EXPERIMENT

After presenting the results of these experiments, two practical problems of electroencephalography (EEG) classification related to oversampling implementation are
addressed in this section. The collection of a considerable amount of valid EEG data typically has a high cost; however, the performance of the modeling is nonstationary if the data collection is insufficient.

With this motivation, this section applies the proposed ASSOM model to solve the problem mentioned above because it can automatically generate synthetic samples according to the distribution of the original data.

EEG data collection and analysis has attracted a substantial amount of attention for many years. A significant advantage of EEG over other extraction methodologies is that it provides convenient real-time measurements. Therefore, EEG signals are commonly used in real-world applications [29-31]. In the EEG-based brain-computer interface (BCI) design, well-recorded data are difficult to collect because most of the subjects are affected by interior and exterior disturbances. These disturbances greatly reduce the quality of the collected data; therefore, the collection of substantial EEG data is always a challenge for constructing BCIs. In contrast, for recognition problems, if the data collection is insufficient, then the performance of the modeling is often nonstationary. The oversampling approach is intuitively considered to be an effective method to compensate for insufficient information by generating synthetic samples. In this study, the EEG signals collected from the motor imagery (MI) task and migraine patients with different phases are used to validate the effect of the use of an ASSOM in the classification task.

A. Motor Imagery (MI) Task

1) Experiment and subjects

Four healthy young adults participated in the MI experiment in this study. For each subject, the EEG data were collected across the sensorimotor area, as shown in Figure 3. Each subject was instructed to sit on a standard chair and keep his/her hands placed on a table statically. During the MI task, the screen was kept blank for 2 seconds at the beginning of each trial. A cross mark was then displayed on the centre of the screen for 2 seconds as an alarm prior to a left/right imagery event. Subsequently, a left/right arrow mark was randomly introduced on the screen for 10 seconds. During this 10 seconds imagery period, the participants were required to imagine the left/right hand movement in accordance with the direction of the arrow that appears. After the end of the imagination, the inter-trial intervals of random imagery events were set to 7-10 seconds. The MI paradigm consisted of four sessions, in which each session consisted of 40 trials. This study was approved by the Institutional Review Board at Taipei Veterans General Hospital. Informed consent was obtained from all subjects before they joined the study.

2) EEG signal processing

In this study, EEG was used to measure each participant’s brain dynamics during the MI task. All of the EEG data were recorded using a portable 4-channel EEG recording device with dry spring-loaded sensors [32]. The EEG data were recorded at a sampling rate of 512 Hz by the hardware specifications. Then, down-sampling to 100 Hz was applied to reduce the computational complexity during the subsequent computational phase. The acquired EEG data were thereby processed and analyzed during the pre-processing stage for band-pass filtering, down-sampling, epoch extraction, and artificial removal. For each channel of interest associated with the cerebral cortex, the mean powers in the delta (1-3 Hz), theta (4-7 Hz), alpha (8-12 Hz) and beta (13-30 Hz) bands were collected for power spectrum analysis and feature extraction.

The main concept of using a common spatial pattern (CSP) [33] is to exploit a linear transformation to project the multi-channel EEG signals into low-dimensional spatial subspaces with a projection matrix, of which each row consists of weights for different channels. Significant channels were selected by searching the maximums of the spatial patterns in scalp mappings. Although the CSP is effective for manually finding the optimal band for each subject, it is also time-consuming, and the result is hardly repeated by other research groups.

Although it is effective to manually find a subject-specific frequency range for each subject, it is time consuming, and the result is nonstationary. Here, we employ a filter bank to decompose the EEG signals into 4 sub-bands: delta, theta, alpha, and beta bands as inputs to the CSP. This method refers to the sub-band CSP (SUBCSP) [34], which has proven to achieve a similar result to that of finding proper bands manually for each subject. In previous studies [34, 35], it is difficult for CSP to consider both the accuracy and efficiency simultaneously, but SUBCSP can achieve this feat. Figure 4 shows the comprehensive scheme regarding the filter bank, SUBCSP, ASSOM and neural networks. Delta, theta, alpha and beta bands are as features for ASSOM. ASSOM takes the band features of minority class to generate synthetic data for balancing the MI dataset. To be specific, following the Figure 4 and Table 4, the input data for ASSOM are the band features

![Figure 3. The measurement electrodes are placed across the sensorimotor area.](image)

![Figure 4. Scheme composed of the filter bank, SUBCSP, ASSOM and NN classifier.](image)
suggests that our proposed ASSOM model can offer improvements in the results by 6.6%, 10.6%, 9.4%, and 9.5% using the oversampling approach across three subjects. This finding illustrates the improvement rate (IR) of each channel of oversampling approach, the average performance of each subject using these oversampled data in the MI recognition task. In this study, we exploit the information from subject 1 to generate synthetic data as the training set, to test that of the remaining subjects. Experiments were performed in a quiet, dimly light room. During the first 2 minutes of the experiment, subjects were instructed to take several deep breaths while they adapted to the environment. Next, subjects were instructed to open their eyes for 30 seconds and close their eyes for 30 seconds and to repeat this sequence for a total of three times. Meanwhile, EEG signals were digitized and recorded at a sampling rate of 256 Hz via a Nicolet One EEG System (Natus Ltd., USA). Eighteen EEG leads (Fp1, Fp2, F7, F3, Fz, F4, F8, T3, C3, Cz, C4, T4, T5, P3, P4, T6, O1, and O2) were placed according to the International 10–20 system. Fz was used as the reference channel.

Table 4. Classification performance of each channel in subject 1 to test the remaining subjects in MI task.

| Training subject | Test subject | w/o oversampling | w oversampling | Training subject | Test subject | w/o oversampling | w oversampling |
|------------------|--------------|------------------|----------------|------------------|--------------|------------------|----------------|
| Subject 1 (1st channel) | Subject 2 | 75.0±15.6 | 82.5±11.7 | Subject 2 | 37.5±11.0 | 51.9±11.0 |
| | Subject 3 | 65.0±14.2 | 73.8±6.5 | Subject 3 | 62.5±11.4 | 66.3±8.9 |
| | Subject 4 | 60.0±6.7 | 63.6±5.1 | Subject 4 | 48.8±9.7 | 58.8±8.9 |
| Subject 1 (2nd channel) | Subject 2 | 55.0±15.3 | 70.6±14.1 | Subject 2 | 52.5±12.2 | 60.8±14.2 |
| | Subject 3 | 60.5±8.9 | 64.6±10.6 | Subject 3 | 46.3±11.9 | 59.4±14.2 |
| | Subject 4 | 51.8±10.5 | 63.75±14.4 | Subject 4 | 53.1±6.1 | 60.2±9.9 |

![Figure 5. Improvement rate (IR) results.](image)

of subject 1. Subjects 2-4 follow the EEG pre-processing procedure and feature extraction stage as Subject 1 to act as testing set.

3) Evaluation results

For the purpose of selecting the optimal channel used in the MI task, the channels in each subject were divided during the training phase in this study. For each subject, there were four input variables and 16 samples in each channel. To demonstrate the improvement of the use of the oversampling strategy, only one subject was employed as the representative of the sparse data for being oversampled. Subsequently, ANNs were trained using these oversampled data in the MI recognition task. In this study, we exploit the information from subject 1 to generate synthetic data as the training set, to test that of the remaining subjects.

The system performance is shown in Table 4, in which we compare the results with and without the ASSOM approach. To better show the improvement from the use of ASSOM, Figure 6 illustrates the improvement rate (IR) of each channel of subjects 2-4. Compared with the result without the oversampling approach, the average performance of each channel using the oversampling approach across three subjects boosts the results by 6.6%, 10.6%, 9.4%, and 9.5%. This finding suggests that our proposed ASSOM model can offer sufficient information based on regulating the amount of sparse data while addressing EEG identification problems.

B. Migraine Phases

1) Experiment and subjects

Forty-three patients with migraine without aura, as having low-frequency migraine (1-5 days per month) were invited to join this study [36]. Each patient kept a headache diary and completed a structured questionnaire on demographics, headache profile, medical history, and medication use. On EEG study days, patients’ migraine phases were designated as inter-ictal or pre-ictal based on the patients’ headache diaries. Pre-ictal phase was coded when patients were within 36 hours before a migraine attack on the day of EEG study, and inter-ictal phase was scored for patients in a pain-free period between pre-ictal and after 36 hours of a migraine attack. This study was approved by the Institutional Review Board at Taipei Veterans General Hospital. Informed consent was obtained from all subjects before they joined the study.

Experiments were performed in a quiet, dimly light room. During the first 2 minutes of the experiment, subjects were instructed to take several deep breaths while they adapted to the environment. Next, subjects were instructed to open their eyes for 30 seconds and close their eyes for 30 seconds and to repeat this sequence for a total of three times. Meanwhile, EEG signals were digitized and recorded at a sampling rate of 256 Hz via a Nicolet One EEG System (Natus Ltd., USA). Eighteen EEG leads (Fp1, Fp2, F7, F3, Fz, F4, F8, T3, C3, Cz, C4, T4, T5, P3, P4, T6, O1, and O2) were placed according to the International 10–20 system. Fz was used as the reference channel.

2) EEG signal processing

During signal preprocessing, raw EEG signals were subjected to 1-Hz high-pass and 30-Hz low-pass finite impulse response filters. Filtered EEG data were inspected manually to remove artifacts and noisy channels (i.e., channels severely contaminated by eye movements, blinks, and muscle or heart activities). Eyes-open and eyes-closed resting-state signals of three blocks were extracted and concatenated for further analyses.
Coherence index, an indicator of synchronization between paired channels, was used to establish the connectivity structure in each migraine phase. Inter-channel coherence analysis was performed on pairs of EEG signals to determine the degree of synchronization between brain areas within particular frequency bands. The value of coherence between two EEG signals over frequency band was calculated by extension of Pearson’s correlation coefficient for complex number pairs.

In this paper, we proposed a promising and powerful method, ASSOM, which can effectively evolve useful samples using invariant features associated with rotation, translation, and scaling. To solve the imbalanced issues in the recognition problems and EEG applications, synthetic data are intuitively generated and inserted into the minority class, to reach the number of majority samples; therefore, classifiers trained via such an oversampling technology strategy can obtain superior performance compared with those that are trained with primitive imbalanced samples.

The magnitude of coherence varies between 0 (complete absence of synchronization) and 1 (perfect synchronization).

3) Evaluation results

In this study, we collected 30 inter-ictal patients and only 13 pre-ictal patients in total, because it is not easy to catch the short pre-ictal migraine phase. The feature dimensions of EEG coherence with 30 channels are 136. By ASSOM oversampling approach, we oversampled the number of pre-ictal EEG signals to 30, consistent with that in the inter-ictal phase. The classification performance of EEG coherence with and without oversampling approach in each cortical frequency band (delta, theta, alpha, and beta) are summarized in Table 5. Compared to the classification performance without oversampling, we noted that oversampled EEG coherence using ASSOM can achieve the higher overall accuracy (from 0.77 ± 0.10 to 0.81 ± 0.12 in delta band, from 0.92 ± 0.07 to 0.93 ± 0.06 in theta band, from 0.74 ± 0.10 to 0.78 ± 0.11 in alpha band, and from 0.62 ± 0.11 to 0.65 ± 0.15 in beta band), as well as improve accuracy for the pre-ictal phase, based on the RBF-SVM classifier.

### Table 5. Classification performance of migraine phases

| EEG Coherence | Migraine Phases | Accuracy by EEG frequency band |
|---------------|-----------------|--------------------------------|
|               | Delta           | Theta                         | Alpha                        | Beta                          |
| w/o oversampling | Overall         | 0.77 ± 0.10                   | 0.92 ± 0.07                  | 0.74 ± 0.10                   | 0.62 ± 0.11                   |
|               | Inter-ictal     | 0.90 ± 0.12                   | 0.96 ± 0.06                  | 0.86 ± 0.13                   | 0.77 ± 0.17                   |
|               | Pre-ictal       | 0.54 ± 0.23                   | 0.85 ± 0.15                  | 0.51 ± 0.23                   | 0.35 ± 0.20                   |
| w oversampling | Overall         | 0.81 ± 0.12                   | 0.93 ± 0.06                  | 0.78 ± 0.11                   | 0.66 ± 0.10                   |
|               | Inter-ictal     | 0.87 ± 0.10                   | 0.94 ± 0.07                  | 0.84 ± 0.12                   | 0.76 ± 0.15                   |
|               | Pre-ictal       | 0.65 ± 0.15                   | 0.88 ± 0.12                  | 0.60 ± 0.18                   | 0.44 ± 0.17                   |

In summary, the principal contributions of ASSOM are twofold. One is the learning ability, and the other is the use of the subspace concept. The learning procedure of an ASSOM is extended from SOM. As a result, the distinguished abilities of ASSOM still include the competitive learning and adaptive learning strategies to effectively adjust all of the free parameters. The purpose of competitive learning is to obtain the best module (winner) means that the smallest distance compared to the rest of the modules, and then, the updated weights have been followed by the winner. Subsequently, the ASSOM learning procedure adjusts the subspace of the winning module to make the free weights of each module approach the raw signal in the input subspace.

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