An SO(10) GUT Model With Lopsided Mass Matrix and Neutrino Mixing Angle $\theta_{13}$

Xiangdong Ji,$^1$ Yingchuan Li,$^1$ and R. N. Mohapatra$^1$

$^1$Department of Physics, University of Maryland, College Park, Maryland 20742

(Dated: October 15, 2018)

Abstract

An alternative supersymmetric SO(10) grand unification model with lopsided fermion mass matrices is introduced. It generates a large solar-neutrino-mixing angle through the neutrinos’ Dirac mass matrix constrained by the SO(10) group structure, avoiding the fine-tuning required in the Majorana mass matrix of right-handed neutrinos. The model fits well the known data on masses and mixings of quarks and leptons, and predicts a sizable lepton mixing $\sin^2 2\theta_{13} \simeq 0.074$, which is significantly larger than that of the original lopsided model.
The discovery of neutrino oscillation has opened up a fascinating window for physics beyond the standard model. Experimental data on neutrino mass differences and mixings help to constrain various theoretical models of new physics. Assuming three light flavors, the lepton Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix is characterized by three mixing angles \( \theta_{12}, \theta_{13}, \theta_{23} \) and three CP-violating phases when neutrinos are Majorana fermions. The atmospheric and accelerator neutrino data have determined \( \theta_{23} \) to a good accuracy, and the solar neutrino and reactor neutrino experiments have measured \( \theta_{12} \) with an even better precision \([1]\). These results have already helped to eliminate a large class of neutrino mass matrix models in the literature. The CHOOZ reactor experiment has found that \( \sin^2 2\theta_{13} \), if non-zero, should be smaller than 0.1 \([2]\). The next generation of neutrino experiments under proposal aims to push the limit to \( \sin^2 2\theta_{13} \approx 0.01 \) \([3, 4]\), which undoubtedly will teach us a great deal about the mechanism of neutrino mass generation.

If small neutrino masses are assumed to arise from the seesaw mechanism \([5]\), the first thing one learns from present data is that the seesaw scale (the mass of the right-handed neutrinos) must be very high (close to \( 10^{15} \) GeV or so). This strongly suggests that the seesaw scale may be connected with one of the leading ideas for new physics beyond the standard model, i.e., supersymmetric grand unification theory (GUT) according to which all forces and matter unify at very short distances corresponding to energies of order \( 10^{16} \) GeV. Since the GUT models unify the quarks and leptons they build in more constraints and have better predictive power \([6]\) which can connect neutrino parameters to the well-determined quark parameters.

The most minimal GUT models that incorporate the seesaw mechanism are based on SUSY SO(10) since its \( 16 \) dimensional spinor representation contains all fermions of the standard model along with the right-handed neutrino needed for this purpose as well as the fact that it has \( B - L \) as a subgroup whose breaking gives rise to masses to the right-handed neutrinos. Depending upon which set of Higgs multiplets is chosen to break the \( B - L \) subgroup of SO(10), there emerge two classes of SO(10) models: one that uses \( 10_H, 126_H, \bar{126}_H \) and \( 120_H \) \([7, 8]\), and the other that uses \( 10_H, 16_H, \bar{16}_H \) and \( 45_H \) \([9, 10, 11]\). While most of these models are quite successful in fitting and predicting the known experimental masses and mixing angles of leptons and quarks, they predict very different values for the poorly-known neutrino mixing angle \( \theta_{13} \)—majority of models with high-dimensional Higgses tend to yield \( \theta_{13} \) close to the current experimental upper bound and majority of those with low-dimensional Higgses generally result in a small \( \theta_{13} \), hence, a small CP violation in the lepton sector. Thus it appears that \( \theta_{13} \) might be an excellent observable to differentiate the two classes of SO(10) models.

Consider, for example, the SO(10) model with low-dimensional Higgses and the so-called lop-sided fermion mass matrices proposed by Albright, Babu, and Barr \([10, 12, 13]\). The lop-sidedness built within the Yukawa couplings between the second and third families generates, among other interesting physical consequences, the large atmospheric-neutrino mixing angle \( \theta_{23} \) while keeping \( V_{cb} \) in the Cabbibo-Kobayashi-Moskawa (CKM) matrix small. The large solar-neutrino mixing angle \( \theta_{12} \), however, is generated less elegantly. It is obtained through a fine-tuning which not only requires the 2-3 and 3-2 entries in the Majorana mass matrix \( M_R \) of the right-handed neutrinos to be of order of parameter \( \epsilon \) appearing in Dirac mass matrices of quarks and leptons, but also requires them to be exactly \(-\epsilon \) \([12]\). By varying the four parameters in the \( M_R \) \([13]\), the predication of \( \theta_{13} \) from this model was found to lie in the range of \( 10^{-5} \leq \sin^2 \theta_{13} \leq 10^{-2} \). A narrower range of \( 0.002 \leq \sin^2 \theta_{13} \leq 0.003 \) is obtained when constraints are imposed on the parameter space. If \( \bar{\nu}_e \) disappearance is observed in
the next generation of short baseline reactor experiments \[4\], the original lopsided model would be ruled out.

Given that the lopsided fermion matrix model is one of the most successful GUT theories incorporating all the known experimental facts, two obvious questions arise immediately. First, is there a more natural way to realize the large solar-neutrino mixing angle without fine tuning? And second, if such an alternative model exists, is $\theta_{13}$ consistently small? In this paper, we present a modified lopsided model which uses an alternative mechanism to generate the solar-neutrino mixing angle. We assume that the right-handed neutrino Majorana mass matrix $M_R$ has a simple diagonal structure, and introduce additional off-diagonal couplings in the upper-type-quark and neutrino Dirac mass matrices to generate 1-2 rotation. We found that all the fermion masses and mixing angles can be fitted well in the new model. The mixing angle $\theta_{13}$, however, is close to the upper limit from the CHOOZ experiment and therefore definitely within the reach of next generation reactor experiments.

Before we present our model for fermion mass matrices, it is instructive to review some of the salient features of the SUSY SO(10) model with lopsided fermion mass matrices \[10, 12\]. Through couplings with a set of Higgs multiplets $10_H$, $16_H$, $\overline{16}_H$ and $45_H$ and the constraint from the flavor $U(1) \times Z_2 \times Z_2$ symmetry, the fermion mass matrices have the following forms,

\[
\begin{align*}
U &= \begin{pmatrix}
\eta & 0 & 0 \\
0 & \epsilon/3 & 1 \\
0 & -\epsilon/3 & 1
\end{pmatrix} M_U , \\
N &= \begin{pmatrix}
\eta & 0 & 0 \\
0 & -\epsilon & 1 \\
0 & \epsilon & 1
\end{pmatrix} M_U , \\
D &= \begin{pmatrix}
\eta & \delta & \delta' \epsilon^i \phi \\
\delta & \sigma + \epsilon/3 & 1 \\
\delta' \epsilon^i \phi & -\epsilon/3 & 1
\end{pmatrix} M_D , \\
L &= \begin{pmatrix}
\eta & \delta & \delta' \epsilon^i \phi \\
\delta & \sigma + \epsilon & 1 \\
\delta' \epsilon^i \phi & -\epsilon & 1
\end{pmatrix} M_D , \\
M_R &= \begin{pmatrix}
e^2 \eta^2 & -b \epsilon \eta & a \eta \\
-b \epsilon \eta & \epsilon^2 & -\epsilon \\
a \eta & -\epsilon & 1
\end{pmatrix} \Lambda_R,
\end{align*}
\]

where $U$, $D$, $L$, and $N$ denote up-type-quark, down-type-quark, charged lepton, and neutrino Dirac mass matrices, respectively, and $M_R$ is the Majorana mass matrix of the right-handed neutrinos. As explained in \[10, 12\], the various entries in the mass matrices come from different SO(10) invariants in the superpotential, e.g., $\eta$ from $16, 16, 10_H$; $\epsilon$ from $16_2 16_3 10_H 45_H$, $\delta, \delta'$ from $16_1 16_2 3 16_H 16_H'$; and $\sigma$ from $16_2 16_H 16_3 16_H'$. The parameter $\sigma$ is of order one, signaling the lopsidedness between the second and third families in $D$ and $L$. This feature leads to a large left-handed neutrino mixing in the PMNS matrix and a small left-handed quark mixing shown in the CKM matrix. The parameter $\epsilon$ is one order-of-magnitude smaller than $\sigma$ and generates the hierarchy between the second and third families. In extending to the first family, $\delta$ and $\delta'$ were introduced into the $D$ and $L$. The large solar-neutrino mixing angle is from the left-handed neutrino seesaw mass matrix which in turn depends on a very specific structure in $M_R$.

Since the lepton mixing matrix is defined as

\[
U_{\text{PMNS}} = U_L^\dagger U_{\nu},
\]

the large solar mixing angle can either be generated from $U_L^\dagger$ or $U_{\nu}$ or a combination of both. If there is a non-vanishing 1-2 rotation from $U_{\nu}$, it can either be generated from the Dirac mass matrix of the left-handed neutrinos or from the Majorana mass matrix of the right-handed neutrinos or a combination of both. In the following, we focus on the possibilities...
in which one of the matrices generates a large solar-neutrino mixing angle, keeping in mind though that a general situation might involve a mixture of the extreme cases. In the fermion mass model in Eq. (1), the large solar-neutrino mixing is induced mainly by the right-handed neutrino mass matrix.

Thus, an alternative possibility is to produce the large solar-neutrino mixing from the charged lepton matrix. In fact, in Ref. [14], a model was proposed in which both large solar and atmospheric neutrino mixings are generated from the lopsided charged-lepton mass matrix. The value of $\sin^2 2\theta_{13}$ is again found to be small, 0.01 or less.

Here we study yet a third possibility of generating a large size 1-2 rotation in the lepton mixing from the neutrinos’ Dirac mass matrix $N$. The easiest way to achieve this might be to use a lopsided structure in the 1-2 entries of $N$. However, this is impossible in group theory of SO(10). A large rotation, however, can be generated through 1-3 and 2-3 entries without affecting, for example, the quark mass hierarchy between the first and second generations. Thus we introduce the following modifications of the up-type quark and neutrino mass matrices in Eq. (1).

$$U = \begin{pmatrix} \eta & 0 & \kappa + \rho/3 \\ 0 & 0 & \omega \\ \kappa - \rho/3 & \omega & 1 \end{pmatrix} M_U, \quad N = \begin{pmatrix} \eta & 0 & \kappa - \rho \\ 0 & 0 & \omega \\ \kappa + \rho & \omega & 1 \end{pmatrix} M_U,$$

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R,$$

(3)

The symmetric entries $\omega$ and $\kappa$ in $U$ and $N$ can be generated from the dimension-5 operator $16,16,[16_H,16_H^\prime]_{10}$, and the antisymmetric $\rho$ entries in $U$ and $N$ are from dimension-6 operator $16,16,[16_H,16_H^\prime]_{10}45_H$, where the subscript 10 indicate that the spinor Higgses are coupled to 10 of SO(10). Because of the modification, the $\epsilon$ entries in $D$ and $L$ now must be generated from dimension-6 operator $16,16,[16_H,16_H^\prime]_{10}45_H$. We assume as in the past that $45_H$ Higgs develops a vacuum expectation value (VEV) in the $B - L$ direction. $16_H$ and $16_H^\prime$ are the Higgs spinors which break the SO(10) to SU(5) by taking the VEV in the singlet direction of SU(5). The second pair of $16_H^\prime$ and $16_H^\prime$ develop VEV in the $\mathbf{5}$ and $\mathbf{5}$ of SU(5), respectively, and therefore the operators involving $16_H^\prime$ and $16_H^\prime$ contribute to up and down sectors as weak doublets, respectively.

Usually a rotation is connected with the mass spectrum. However, in our case the 1-2 rotation angle from $U$ will be combined with the 1-2 rotation from $D$ to obtain the Cabibbo angle $\theta^c$, and a constraint from the up-type quark spectrum must be avoided. Thus, the first two families in the $U$ and $N$ cannot be coupled to each other directly, but can be coupled indirectly through the third family. The 1-2 rotations in $U$ and $N$ generated from this way are proportional to the ratios $\gamma \equiv (\kappa - \rho/3)/\omega$ and $\gamma' \equiv (\kappa + \rho)/\omega$, respectively.

Taking the approximation $\eta = 0$, the dependence of various mass ratios and CKM elements on parameters can be seen roughly from the following approximate expressions (the
superscript 0 indicates the relevant quantity is at GUT scale)

\[
m_0^0/m_\tau^0 \approx 1 - \frac{2}{3} \frac{\sigma}{\sigma^2 + 1} \epsilon ,
m_u^0/m_t^0 \approx 0 ,
m_e^0/m_t^0 \approx (1 + \gamma^2)\omega^2 ,
m_\mu^0/m_t^0 \approx \frac{\epsilon}{\sigma^2 + 1} ,
m_s^0/m_b^0 \approx \frac{1}{3} \frac{\epsilon}{\sigma^2 + 1} ,
m_e^0/m_\mu^0 \approx \frac{1}{9} t_L t_R ,
m_d^0/m_s^0 \approx t_L t_R ,
\]

\[
\begin{align*}
V_{cb}^0 & \approx -\sqrt{1 + \gamma^2} \omega - \frac{1}{\sqrt{1 + \gamma^2} 3(1 + \sigma^2)} \epsilon , \\
V_{us}^0 & \approx \frac{1}{\sqrt{1 + \gamma^2}} (-\gamma + t_L e^{i\theta}) , \\
V_{ub}^0 & \approx \frac{1}{\sqrt{1 + \gamma^2} 3(\sigma^2 + 1)} (\gamma - t_L e^{i\theta} + \sqrt{1 + \sigma^2} t_R) ,
\end{align*}
\]  

(4)

where \( t_L, t_R \) and \( \theta \) are defined as \( t_L e^{i\theta} \equiv 3(\delta - \sigma \delta^* e^{i\phi})/(\sigma \epsilon) \) and \( t_R \equiv 3\delta \sqrt{\sigma^2 + 1}/(\sigma \epsilon) \). The expressions for mass ratios in down-type quark and charged lepton sectors are the same as those in the original lopsided model. The expressions for \( m_e^0/m_\mu^0 \) and elements in CKM matrix are new. These approximations allow us to design strategies to fit various parameters to experimental data.

First, we use the up-type quark and lepton spectra and the parameters in the CKM matrix to determine 10 parameters \( \sigma, \epsilon, \delta, \delta', \phi, \omega, \gamma, \eta, M_U \) and \( M_D \). Our best fit yields \( \sigma \) and \( \epsilon \) approximately the same as those in the original lopsided model, and thus the successful prediction for the mass ratios \( m_u^0/m_t^0 \) and \( m_e^0/m_\mu^0 \) are kept. The two CKM elements \( |V_{us}| \) and \( |V_{ub}| \), together with the CP violation phase \( \delta_{CP} \) and the constraint on the product \( t_L t_R \) from mass ratio \( m_e^0/m_\mu^0 \), can fix the \( t_L, t_R, \gamma \) and \( \theta \). Then \( \omega \) and \( \eta \) can be fixed from \( m_u^0/m_t^0 \) and \( m_e^0 \), respectively. The down-type quark mass spectrum come out as predictions.

To see the dependence of the lepton mixing PMNS matrix on various parameters, we construct the Majorana mass matrix of left-handed neutrino from the see-saw mechanism, \( m_\nu = -N M_R^{-1} N \),

\[
m_\nu = - \begin{pmatrix} 
\frac{\eta^2}{a} + (\kappa + \rho)^2 & (\kappa + \rho) \omega & \eta(\kappa - \rho)/a + (\kappa + \rho) \\
(\kappa + \rho) \omega & \omega^2 & \omega \\
\eta(\kappa - \rho)/a + (\kappa + \rho) & \omega & 1 + (\kappa - \rho)^2/a + \omega^2/b 
\end{pmatrix} M_U^2/\Lambda_R ,
\]  

(5)

which depends on the four unknown parameters, \( \gamma', \Lambda_R, a \) and \( b \). With parameter \( a \) taking a reasonably large value, say, order of 0.001 or larger, the \( \eta \) dependent terms can be neglected. Then one readily sees that the \( m_\nu \) matrix can be diagonalized by a 1-2 rotation of angle \( \theta_{12}^\nu \) with \( \tan \theta_{12}^\nu = \gamma' \), and followed by a 2-3 rotation by angle \( \theta_{23}^\nu \), with

\[
\tan 2\theta_{23}^\nu = \frac{2 \sqrt{1 + \gamma^2} \omega}{1 + (\kappa - \rho)^2/a + \omega^2/b - (1 + \gamma^2)\omega^2} .
\]  

(6)
The neutrino Majorana masses of the second and the third families are

\[
m_{\nu2} = - \left[ (1 + \gamma^2)\omega^2 + \sqrt{1 + \gamma^2} \omega (\cot 2\theta^\nu_{23} - \csc 2\theta^\nu_{23}) \right] M_R^2 / \Lambda_R,
\]

\[
m_{\nu3} = - \left[ (1 + \gamma^2)\omega^2 + \sqrt{1 + \gamma^2} \omega (\cot 2\theta^\nu_{23} + \csc 2\theta^\nu_{23}) \right] M_R^2 / \Lambda_R,
\]

with \( m_{\nu1} = 0 \) as the result of the approximation \( \eta = 0 \). Therefore, the present model constrains the neutrino mass spectrum as hierarchial, which means that the parameters in the light-neutrino mass matrix, the mass eigenvalues and mixings, do not run significantly from GUT to low-energy scales. The mass difference \( \Delta m^2_{\nu12} \) can be used to fix the right-handed neutrino mass scale \( \Lambda_R \).

Taking into account rotations from matrices \( m_{\nu} \) and \( L \), we arrive at the elements in the PMNS matrix,

\[
U_{e2} = \left( \frac{-\gamma'}{\sqrt{1 + \gamma^2}} - \frac{t_R}{3} \frac{1}{\sqrt{1 + \gamma^2}} \right) \cos \theta^\nu_{23} - \frac{t_R}{3} \frac{\sigma}{\sqrt{1 + \sigma^2}} \sin \theta^\nu_{23},
\]

\[
U_{\mu3} = - \frac{\sigma}{\sqrt{1 + \sigma^2}} \cos \theta^\nu_{23} + \frac{1}{\sqrt{1 + \gamma^2}} \left( \gamma' \frac{t_R}{3} + \frac{1}{\sqrt{1 + \sigma^2}} \right) \sin \theta^\nu_{23},
\]

\[
U_{e3} = \frac{t_R}{3} \frac{\sigma}{\sqrt{1 + \sigma^2}} \cos \theta^\nu_{23} + \left( \frac{-\gamma'}{\sqrt{1 + \gamma^2}} - \frac{t_R}{3} \frac{1}{\sqrt{1 + \sigma^2}} \right) \sin \theta^\nu_{23}.
\]

The data on the solar-neutrino mixing \( U_{e2} \), together with the ratio of mass differences, \( \Delta m^2_{\nu12}/\Delta m^2_{\nu23} = m^2_{\nu2}/(m^2_{\nu3} - m^2_{\nu2}) \), can fix \( \gamma' \) and \( \theta^\nu_{23} \), where the latter depends on a combination of \( a \) and \( b \). Having fixed \( \gamma' \) and parameters in \( M_R \), the atmospheric-neutrino mixing \( U_{\mu3} \) and \( U_{e3} \) are obtained as predictions.

We summarize our input and detailed fits as follows. For CKM matrix elements, we take \( |V_{us}| = 0.224, |V_{ub}| = 0.0037, |V_{cb}| = 0.042 \) and \( \delta_{CP} = 60^\circ \) as inputs at electro-weak scale. With a running factor of 0.8853 for \( |V_{ub}| \), and \( |V_{cb}| \) taken into account, we have \( |V_{ub}^0| = 0.0033 \) and \( |V_{cb}^0| = 0.037 \) at GUT scale. For charged lepton masses and up-type quark masses, we take the values at GUT scale corresponding to \( \tan \beta = 10 \) from Ref. [15].

For neutrino oscillation data, we take the solar-neutrino angle to be \( \theta^\nu_{solar} = 32.5^\circ \) and mass square differences as \( \Delta m^2_{\nu12} = 7.9 \times 10^{-5}\text{eV}^2 \) and \( \Delta m^2_{\nu23} = 2.4 \times 10^{-3}\text{eV}^2 \). The result for the 12 fitted parameters is

\[
\sigma = 1.83, \quad \epsilon = 0.1446, \quad \delta = 0.01,
\]

\[
\delta' = 0.014, \quad \phi = 27.9^\circ, \quad \eta = 1.02 \times 10^{-5},
\]

\[
\omega = -0.0466, \quad \rho = 0.0092, \quad \kappa = 0.0191,
\]

\[
M_U = 82.2 \text{ GeV}, \quad M_D = 583.5 \text{ MeV}, \quad \Lambda_R = 1.85 \times 10^{13} \text{ GeV}
\]

There is a combined constraint on \( a \) and \( b \), and thus the right-handed Majorana mass spectrum is not well determined. As examples, if \( a = b \), \( a = -2.039 \times 10^{-3} \); and if \( a = 1, b = -1.951 \times 10^{-3} \).

We show the result for the down-type quark masses and right-handed Majorana neutrino masses (taking \( a = b \)) as follows,

\[
m_d^0 = 1.08 \text{ MeV}, \quad m_s^0 = 25.97 \text{ MeV}, \quad m_b^0 = 1.242 \text{ GeV},
\]

\[
M_1 = 3.77 \times 10^{10}\text{GeV}, \quad M_2 = 3.77 \times 10^{10}\text{GeV}, \quad M_3 = 1.85 \times 10^{13}\text{GeV}.
\]
The predictions for the mixing angles in the PMNS matrix are,

$$\sin^2 \theta_{atm} = 0.49, \quad \sin^2 2\theta_{13} = 0.074. \quad (11)$$

The result for $\theta_{atm}$ is particularly interesting: Although the lopsided mass model is built to produce a large atmospheric-neutrino mixing angle, the charged lepton mass alone produces a 2-3 rotation of $63^\circ$ instead of $45^\circ$ because of the constraint from the lepton mass spectrum. With an additional rotation $\theta_{\nu_{23}} \simeq 21^\circ$ fixed mainly from the ratio of mass differences $\Delta m^2_{\nu_{12}}/\Delta m^2_{\nu_{23}}$, the nearly maximal atmospheric mixing $44.6^\circ$ comes out as a prediction. If one releases the best-fit value of $\Delta m^2_{\nu_{12}}$ and $\Delta m^2_{\nu_{23}}$ and only imposes the $3\sigma$ constraint as $7.1 \times 10^{-5} \text{eV}^2 \leq \Delta m^2_{\nu_{12}} \leq 8.9 \times 10^{-5} \text{eV}^2$ and $1.4 \times 10^{-3} \text{eV}^2 \leq \Delta m^2_{\nu_{23}} \leq 3.3 \times 10^{-3} \text{eV}^2$, one would obtain, as shown in Fig. 1, $0.44 \leq \sin^2 \theta_{atm} \leq 0.52$ which is well within the $1\sigma$ limit, and $0.055 \leq \sin^2 2\theta_{13} \leq 0.110$ which, as a whole region, lies in the scope of next generation of reactor experiments.

Finally, we make some remarks on CP phases in the lepton mixing matrix in our model. Since we essentially treat all our parameters as real, the CP-violation in the lepton sector is essentially absent. One might wonder why the $\phi$ phase in $L$ and $D$, which generates the CP phase in the CKM matrix, does not give contribution to the CP phases in the PMNS matrix. The answer is the lopsided structure of the $L$ matrix. In fact, the unitary matrix diagonalizing $L^\dagger L$ is nearly real, whereas that diagonalizing $D^\dagger D$ is not. There is, of course, some trivial CP phases due to specific choices of flavor basis. For example, we find $\delta_{CP}^{PMNS} \sim \pi$, and some of the Majorana phases are close to $\pi/2$, all of which are believed to be artifacts of the model.

In summary, we have presented an SUSY SO(10) GUT model for the fermion masses and mixings, which is developed from the original lopsided model of Albright, Babu and Barr [10]. It contains 13 parameters. After fitting them to experimental data, it yields a number of predictions. Whenever the experimental data are available, they work well. Most interestingly, the model predicts a $\sin^2 2\theta_{13}$ around 0.074, which is significantly larger than that from any of previous lopsided models. It can surely be tested through the next generation of reactor neutrino experiments. It will also have its characteristic predictions for lepton flavor violation, leptogenesis as well as proton decay. These issues are currently under investigation.
X. Ji and Y. Li are partially supported by the U. S. Department of Energy via grant DE-FG02-93ER-40762 and by National Natural Science Foundation of China (NSFC). R. Mohapatra is supported by National Science Foundation (NSF) Grant No. PHY-0354401. We thank Carl H. Albright for communication about original lopsided model.

[1] For a recent review on data, see J. W. F. Valle, Nucl. Phys. Proc. Suppl. 145, 141 (2005) [arXiv:hep-ph/0508067]; G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, arXiv:hep-ph/0506307; R. D. McKeown and P. Vogel, Phys. Rept. 394, 315 (2004) [arXiv:hep-ph/0402025].

[2] M. Apollonio et al., nuclear Eur. Phys. J. C 27, 331 (2003) [arXiv:hep-ex/0301017].

[3] K. Anderson et al., arXiv:hep-ex/0402041.

[4] K. M. Heeger, S. J. Freedman and K. B. Luk, AIP Conf. Proc. 698, 303 (2004).

[5] P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, Supergravity (P. van Nieuwenhuizen et al. eds.), North Holland, Amsterdam, 1980, p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95; S. L. Glashow, The future of elementary particle physics, in Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons (M. Lévy et al. eds.), Plenum Press, New York, 1980, pp. 687; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980).

[6] For some recent reviews on neutrino mass models, see R. N. Mohapatra, Nucl. Phys. Proc. Suppl. 138, 257 (2005) [arXiv:hep-ph/0402035]; G. Altarelli and F. Feruglio, New J. Phys. 6, 106 (2004) [arXiv:hep-ph/0405048]; Z. z. Xing, Int. J. Mod. Phys. A 19, 1 (2004) [arXiv:hep-ph/0307359]; A. Y. Smirnov, Int. J. Mod. Phys. A 19, 1180 (2004) [arXiv:hep-ph/0311259].

[7] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 70, 2845 (1993) [arXiv:hep-ph/9209215]; M. C. Chen and K. T. Mahanthappa, Phys. Rev. D 62, 113007 (2000) [arXiv:hep-ph/0005292]; Chen:2000fp M. C. Chen and K. T. Mahanthappa, Phys. Rev. D 68, 017301 (2003) [arXiv:hep-ph/0212375]; T. Fukuyama and N. Okada, JHEP 0211, 011 (2002) [arXiv:hep-ph/0205066]; G. G. Ross and L. Velasco-Sevilla, Nucl. Phys. B 653, 3 (2003) [arXiv:hep-ph/0208218]; B. Bajc, G. Senjanovic and F. Vissani, Phys. Rev. Lett. 90, 051802 (2003) [arXiv:hep-ph/0210207]; M. Bando, S. Kaneko, M. Obara and M. Tanimoto, arXiv:hep-ph/0405071.

[8] H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B 570, 215 (2003) [arXiv:hep-ph/0303055]; H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Rev. D 68, 115008 (2003) [arXiv:hep-ph/0308197]; H. S. Goh, R. N. Mohapatra, S. Nasri and S. P. Ng, Phys. Lett. B 587, 105 (2004) [arXiv:hep-ph/0311330]; H. S. Goh, R. N. Mohapatra and S. Nasri, Phys. Rev. D 70, 075022 (2004) [arXiv:hep-ph/0408139]; B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Lett. B 603, 35 (2004) [arXiv:hep-ph/0406262]; Phys. Rev. Lett. 94, 091804 (2005) [arXiv:hep-ph/0412105]; Phys. Rev. D 70, 095002 (2004); W. M. Yang and Z. G. Wang, Nucl. Phys. B 707, 87 (2005) [arXiv:hep-ph/0406221].

[9] K. S. Babu, J. C. Pati and F. Wilczek, Nucl. Phys. B 566, 33 (2000) [arXiv:hep-ph/9812538]. K. S. Babu, J. C. Pati and P. Rastogi, Phys. Rev. D 71, 015005 (2005) [arXiv:hep-ph/0410200]. K. S. Babu, J. C. Pati and P. Rastogi, Phys. Lett. B 621, 160 (2005) [arXiv:hep-ph/0502152].
[10] C. H. Albright, K. S. Babu and S. M. Barr, Phys. Rev. Lett. 81, 1167 (1998) arXiv:hep-ph/9802314; C. H. Albright and S. M. Barr, Phys. Rev. D 58, 013002 (1998) arXiv:hep-ph/9712488. C. H. Albright and S. M. Barr, Phys. Rev. D 62, 093008 (2000) arXiv:hep-ph/0003251.

[11] T. Blazek, S. Raby and K. Tobe, Phys. Rev. D 62, 055001 (2000) arXiv:hep-ph/9912482; Z. Berezhiani and A. Rossi, Nucl. Phys. B 594, 113 (2001) arXiv:hep-ph/0003084; R. Kitano and Y. Mimura, Phys. Rev. D 63, 016008 (2001) arXiv:hep-ph/0008269; T. Asaka, Phys. Lett. B 562, 291 (2003) arXiv:hep-ph/0304124; R. Dermisek and S. Raby, Phys. Lett. B 622, 327 (2005) arXiv:hep-ph/0507045; Z. Berezhiani and F. Nesti, arXiv:hep-ph/0510011.

[12] C. H. Albright and S. M. Barr, Phys. Rev. D 65, 073004 (2002) arXiv:hep-ph/0108070. C. H. Albright and S. Geer, Phys. Rev. D 72, 013001, (2005) arXiv:hep-ph/0502161.

[13] K. S. Babu and S. M. Barr, Phys. Lett. B 525, 289 (2002) arXiv:hep-ph/0111215.

[14] C. R. Das and M. K. Parida, Eur. Phys. J. C 20, 121, (2001) arXiv:hep-ph/0010004.