Research on Dispersion Performance of Harmonic Diffractive Lenses

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Abstract. Harmonic diffractive lens, which is also called multiorder diffractive lens, was proposed to decrease the chromatic aberration by Dean Faklis, G. Michael Morris, Donald W. Sweeney and Gary E. Sommargren in 1995. Based on the Kirchhoff’s scalar diffraction theory and the empirical formula of refractive index with the change of wavelength, the designing formulas for harmonic diffractive lenses with continuous relief and multilevel relief are given, and their chromatic dispersion performances are studied synthetically. The preconditions, which the formulas for designing harmonic diffractive lenses with low numerical apertures come into existence, are educed, and then the more exact design formulas are given. In the same time, the relationships among the intensity distributions in focal plane and axial direction, diffraction efficiency, focal distance, wavelength, structural parameters, material factor are analyzed in detail by using the self-programming emulating package. The analytical and simulated results achieved in this paper have an important meaning for designing achromatic only-diffractive visual optical system in the future.

1. Introduction

Diffractive lens is a kind of typical diffractive optical element, which has been proved to be very useful in many practical applications [1,2], for instance wavefront shaping, wavefront sensing, optical communication, optical interconnection, optical data storage, optical scanner, infrared focal plane array, micro-opto-electro-mechanical system, etc. When a diffractive optical lens is illuminated by white light, its focal distance has relations not only with material and wavelength, but also with numerical aperture. In order to decrease the chromatic aberration, harmonic diffractive lens, which is also called multiorder diffractive lens, was proposed by Dean Faklis, G. Michael Morris, Donald W. Sweeney and Gary E. Sommargren in 1995 [3,4]. The optical path difference between its adjacent rings is \( p \) times of the designing wavelength, and positive integer \( p \) is called harmonic order. In this paper, the chromatic dispersion performances of harmonic diffractive lenses are studied completely by using the self-programming emulating package.

2. Designing formulas for lenses with low numerical aperture

Based on the Kirchhoff’s scalar diffraction theory and paraxial approximation condition, the phase modulation function of a refractive lens with lower numerical aperture can be obtained below

\[
\phi(r) = -\frac{\pi r^2}{\lambda_0 f_0}
\]
where, \( r \) is the radius, \( \lambda_0 \) is the designing wavelength, \( f_0 \) is the designing focal length.

If the designing wavelength \( \lambda_0 \) is given, the phase modulation function of corresponding harmonic diffractive lens can be written

\[
\phi_0(r) = 2\pi rp - \frac{(2\pi r)^2}{\lambda_0 f_0}, \quad r_k \leq r < r_{k+1}
\]

and the designing formulas for harmonic diffractive lenses with continuous relief and multilevel relief are given respectively as follows

\[
r_k = (2kp\lambda_0 f_0)^{1/2}, \quad 0 \leq k \leq K
\]

\[
r_{k,l} = \left[2\left(k + \frac{l}{L}\right)p\lambda_0 f_0\right]^{1/2}, \quad 0 \leq k < K, \quad 0 \leq l < L
\]

in the above equations, \( r_k \) is the radius of No. \( k \) ring for harmonic diffractive lens with continuous relief, \( r_{k,l} \) is the radius of corresponding step for harmonic diffractive lens with multilevel relief, \( K \) is the total number of ring zone, \( L \) is the total number of step.

The above formulas are only used to design harmonic diffractive lenses with lower numerical aperture, namely the paraxial approximation condition must be satisfied. The precondition, which equation (3) comes into existence, is \( kp\lambda_0 << 2f_0 \). For the outmost ring, we can obtain

\[
Kp\lambda_0 << 2f_0
\]

and \( K \) can be expressed as

\[
K = \frac{R^2}{2p\lambda_0 f_0}
\]

where, \( R \) is the radius of harmonic diffractive lens.

Substitute equation (6) into equation (5), the approximate condition can be educed below

\[
\frac{2R}{f_0} << 4
\]

that is to say, for the harmonic diffractive lenses with smaller relative aperture, the above approximate condition come into existence. But for the harmonic diffractive lenses with higher numerical apertures, equation (7) isn’t satisfied. Therefore the harmonic diffractive lenses designed using equation (3) will have bigger aberration, especially chromatic aberration.

Similarly, for the harmonic diffractive lenses with multilevel relief, the equation (7) must be satisfied. In addition, the following equation must be also fulfilled

\[
\frac{2R}{f_0} < \frac{2p\lambda_0}{vL}
\]

where, \( v \) is the minimum feature size of diffractive lenses with multilevel relief, which is decided by the adopted manufacturing technics.

### 3. Designing formulas for lenses with high numerical aperture

When the harmonic diffractive lenses have high numerical aperture, the paraxial approximation condition isn’t satisfied. Therefore the phase modulation function of a refractive lens can be denoted by

\[
\phi(r) = -2\pi \left(\frac{(r^2 + \rho^2)^{1/2} - f_0}{\lambda_0}\right)^{-1}
\]

If the designing wavelength \( \lambda_0 \) is given, the phase modulation function of corresponding harmonic diffractive lens can be written

\[
\phi_0(r) = 2\pi rp - 2\pi \left(\frac{(r^2 + \rho^2)^{1/2} - f_0}{\lambda_0}\right)^{-1}, \quad r_k \leq r < r_{k+1}
\]

and the more exact designing formulas for harmonic diffractive lenses with continuous relief and multilevel relief are shown below respectively
4. Empirical formula of refractive index for optical material

If refractive indexes of some characteristic line spectrums are given, the empirical formula of refractive index with the change of wavelength for this optical material can be express as [5]

\[ n(\lambda) = n_A B_A + n_C B_C + n_F B_F + n_h B_h \]  

(13)

where, \( n_A, n_C, n_F \) and \( n_h \) are the refractive indexes of characteristic line spectrums \( \lambda_A=0.76820 \mu m, \lambda_C=0.65628 \mu m, \lambda_F=0.48613 \mu m \) and \( \lambda_h=0.40466 \mu m \) respectively,

\[ B_A = -9.74802711 + 12.81327634 \lambda^2 + \frac{2.005673172}{\lambda^2 - 0.035} - \frac{0.1314220822}{(\lambda^2 - 0.035)^2} \]  

(14)

\[ B_C = 17.41684396 - 18.92716898 \lambda^2 - \frac{3.958684180}{\lambda^2 - 0.035} + \frac{0.2723451558}{(\lambda^2 - 0.035)^2} \]  

(15)

\[ B_F = -9.051099716 + 8.120605102 \lambda^2 + \frac{2.777953720}{\lambda^2 - 0.035} - \frac{0.2296678272}{(\lambda^2 - 0.035)^2} \]  

(16)

\[ B_h = 2.382282858 - 2.006712462 \lambda^2 - \frac{0.824942713}{\lambda^2 - 0.035} + \frac{0.08874475383}{(\lambda^2 - 0.035)^2} \]  

(17)

in equations (13), (14), (15), (16) and (17), \( \lambda \) is arbitrary wavelength whose unit is \( \mu m \). In the range of 0.4~0.8\( \mu m \), the computing error using this empirical formula is less than 0.00003.

5. Simulation results

Based on the Kirchhoff’s diffraction theory and the empirical formula of refractive index with the change of wavelength, the chromatic dispersion performances of harmonic diffractive lenses are obtained by using the self-programming emulating package. The simulating results are shown in figure 1, and the parameters are assumed as follows: K9 glass is chosen as optical material, and \( \lambda_0=0.58929 \mu m, \theta_0=1.51630, \theta_A=1.511003, \theta_C=1.513895, \theta_F=1.521955, \theta_h=1.529820, R=5mm, f_0=100mm. \)

![Figure 1](image1.png)

Figure 1. The dependency curves of performance parameters with the change of wavelength \((p=10)\): (a)focal distance; (b)diffraction efficiency for different steps lens.

In the same time, the intensity distributions in focal plane and axial direction of harmonic diffractive lenses are shown in figure 2 and figure 3, and the corresponding parameters are assumed as follows: \( R=10/3mm, f_0=20mm, \lambda_0=0.6328 \mu m, 2R/f_0=1/3 \). In figure 2, \( r_0 \) is the Airy disk radius.
Figure 2. The intensity distributions in the focal plane: (a) 8-steps designed by equation (4); (b) continuous relief designed by equation (3); (c) 8-steps designed by equation (12); (d) continuous relief designed by equation (11).

Figure 3. The intensity distributions in the axial direction: (a) 8-steps designed by equation (4); (b) continuous relief designed by equation (3); (c) 8-steps designed by equation (12); (d) continuous relief designed by equation (11).
From the above figures, it can be known that the intensity distributions of harmonic diffractive lenses become small in the focal plane, and become wide in the axial direction, when the relative aperture is 1/3, which are designed by using equations (3) or equations (4), no matter continuous relief or multilevel relief. That is to say, when the relative aperture is bigger, the equation (7) can’t be satisfied, therefore the harmonic diffractive lenses, which are computed by using equation (3) or equation (4), will have bigger aberration, especially chromatic aberration. At this time, we must utilizing the more exact formulas, equation (11) and (12), to design harmonic diffractive lenses with higher numerical apertures.

6. Conclusion
Based on the Kirchhoff’s scalar diffraction theory, the chromatic dispersion performances of harmonic diffractive lenses are studied synthetically. The designing formulas for harmonic diffractive lenses with continuous relief or multilevel relief are given. The preconditions, which the formulas for designing harmonic diffractive lenses with low numerical apertures come into existence, are educed, and then the more exact design formulas are given. In the same time, the intensity distributions in focal plane and axial direction are analyzed in detail by using the self-programming emulating package. The analytical and simulated results achieved in this paper have an important meaning for designing achromatic only-diffractive visual optical system in the future.

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