Retraction

Retraction: Simulation and analysis of the shaft strength of a centrifugal pump in an in-situ leaching method of uranium (IOP Conf. Ser.: Mater. Sci. Eng. 665 012010)

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Simulation and analysis of the shaft strength of a centrifugal pump in an in-situ leaching method of uranium

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Abstract. This research shows the performance of the strength analysis and dynamic analysis of the shaft strength of a centrifugal pump in an integrated NASTRAN / PATRAN system. There is conducted the calculation of a mechanical scheme for determining the stress-strain state of the rotor shaft of a centrifugal pump and constructed a 3D-model of the rotor of a seven-step centrifugal pump. Furthermore, there are determined voltage levels for zero and maximum flow. On the basis of these dynamic calculations, a damping element has been proposed that ensures a smooth entrance of CP into an operating mode - a magnetic coupling.

1 Introduction
An in-situ leaching technique (ISL), usually used in the extraction of uranium. A solution of the sulfuric acid of low concentration, or another oxidizing agent (for example, formation water saturated with oxygen in the air) is pumped through an injection well into a formation and filtered through a uranium-containing rock, while gradually dissolving it. Then, a metal-enriched solution is pumped to a surface through a pumping well, leaving mostly an intact underground mountain system, i.e. the metal mining is carried out at the place of the occurrence of ores. Subsequently, uranium is extracted on special technological equipment (sorption on ion-exchange resin), and a leach solution is recycled. Before re-supplying the leach solution to a formation, it is enriched with leaching reagents.

In-situ leaching accounts for about 20% of the world uranium industry. A leadership yet retain underground mines (40%) and open careers (30%). However, the fact says that such advanced uranium mining countries as the United States, Uzbekistan and the undisputed leader of the industry, Kazakhstan, give preference to ISL [1].

Pumps are one of the key elements of an in-situ leaching technology: they are involved in all stages of production. That is why its quality and durability take the first place in the selection of equipment - breaks in work are unacceptable, as well as too few hours of work. Indeed, in the latter case, the pumps have to be changed frequently, and these are additional financial and time costs [2].

As part of the research work, there has been created an optimized design of a central pump on the basis of the model of a pump brand ‘ODDESSE zentralasien’ - UPP 13-7 / 6. The task of the work is to increase the efficiency of the central pump from an existing value of 40% to a reference value of 60%. To achieve this goal, one of the first stages of research is to conduct an automated strength calculation in PATRAN.

In the practice of pump design, centrifugal sizes are most often set not from a strength condition, but from design considerations and depending on technological capabilities. In addition, new pumps are usually designed on the basis of tested prototypes. Therefore, when designing, the main importance is the verification calculation of the strength of the main elements of a pump: shaft, impeller, housing, keyway or pin joint, coupling [3].

The task of the verification calculation is to determine the values of parameters with which in each case the loss of the strength (destruction) of a particular element (normal or tangential stresses,
deformations, rotational speed, etc.) is associated with their subsequent comparison with some limit permissible values.

The automated strength analysis of the rotor shaft of a pump has been carried out in a NASTRAN / PATRAN program.

For a calculation, there has been simulated a calculated mechanical scheme according to a method:

![Figure 1. Design diagram a) pump stages b) pressure distribution in pump stages](image)

Axial and radial forces act on a pump stage. The axial force acting on the impeller of a pump is a resultant force acting on an impeller and it is directed in a direction corresponding to the direction of the action of the larger of these forces.

### 2. Calculation of forces acting on impellers

Testing for the static strength of a shaft is carried out on the greatest loads that may occur in the various modes of pump operation - this is a mode with zero flow (or close to zero) and a mode with a maximum flow allowed during operation. The fixed scheme of a vertical submersible pump is shown in Figure 2.

![Figure 2. Circuit diagram of a centrifugal pump](image)

The lower end of a shaft is worn in a nylon sleeve and it is free until the start of a suction. Later, when modeling in PATRAN, this boundary condition has been modeled as a degree of freedom allowing the movement of ∆ according to an assembly’s working drawing ∆ = 4 mm.

### 3. Definition of axial force

Axial force can be approximately defined as the difference in pressure forces to right and left from \( R_1 \) to \( R_b \) [4].

\[
P_{axial} = \frac{\pi}{4 \left( R_1^2 - R_b^2 \right)} \cdot Y \cdot H
\]

where,
- \( P_{axial} \) - axial force in H;
- \( R_1 \) - outer radius of the inlet of an impeller m;
- \( R_b \) - shaft radius in m;
- \( H \) - pump head in m;
- \( Y \) - volume of liquid in kg / m³.
Considering that a centrifugal pump is designed for pumping sulfuric acid with a density of 1050 kg/m³, there has been found the value of axial force.

4. Definition of radial force
The main cause of shaft deflection is a radial force. For centrifugal pumps without a spiral outlet and when connecting sections with ties, the radial force arises from the dynamic unbalance of a rotor.

The dynamic imbalance of a rotor is a consequence of the inaccuracy of the manufacturing parts of the rotor (primarily with large radial dimensions, such as impellers). The presence of an imbalance of rotor parts during the rotation of a rotor leads to the appearance of dynamic loads.

It is possible to determine a strength at a known residual unbalance of an impeller by the following formula:

\[ R_D = m \omega^2 \]  \hspace{1cm} (2)

where, \( m \) – permissible sufficient imbalance of an impeller, kg · m (for example, an expression 'Permissible residual imbalance of 100 g · mm' means that for the impeller with a radius of 100 mm, balancing on an external radius should be carried out with an accuracy of 1 g, which is technologically achievable);

\( \omega \) – angular velocity of the rotation of a rotor pump, 1/s [5].

| Impeller outer diameter, mm | <300 | 300-500 | 500-1000 | 1000-2000 |
|----------------------------|------|---------|----------|-----------|
| Eccentricity, mm           | 0.075| 0.100   | 0.150    | 0.200     |

To calculate a radial force from imbalance, it is used SolidWorks capabilities to determine the mass of a 3D model, in our case for seven sections, \( m = 5.43 \text{ kg} \).

Considering the principle of the superposition of force, the radial force has been calculated as the sum of force from imbalance at each stage of a rotor.

Taking into account the pressure distribution scheme along with the steps of a centrifugal pump Figure 3, and using the above method of calculating radial and axial loads, there have been determined boundary conditions, input data for modeling the strength of a rotor shaft and impellers are given below:

| No | Parameter                                           | Value                                           |
|----|-----------------------------------------------------|-------------------------------------------------|
| 1  | Material                                            | Steel                                           |
| 2  | Modulus of elasticity E                             | \( 2 \times 10^{11} \) PA                      |
| 3  | Poisson's ratio                                     | 0.3                                             |
| 4  | Support 1                                           | Hinge with rotation around the axis of a model   |
| 5  | Support 2                                           | Hinge with rotation around the axis of a         |
5. Creating a model in PATRAN
A geometric model has been created directly in PATRAN, there has been generated from TET elements, force has been applied in radial and axial direction through a 'Force' command to a node, boundary conditions for fixings have been created with conditions Displacement <0,0,0>, Rotation <0,0,0> for upper and lower support, and for zero feed Displacement <0,0,0>, Rotation <0,0,0> for upper support, Displacement <0,0,0>, Rotation <0,1.4,0> for lower support.

6. Results analysis
The results of the calculation of equivalent stresses are shown in Figure 4.

Figure 3. Finite element model of a pump shaft
Figure 4. Voltage map for zero and maximum flow
An analysis of the results showed a sufficient safety factor at maximum flow, according to the diagram \( \sigma_{\text{max}} = 286 \) MPa, the allowable stress for steel 40X \( [\sigma] = 680 \) MPa, a safety factor was 2.37.

The results of displacements at zero flow showed the adequacy of a computer scheme created in PATRAN. \( y_{\text{max}} = 4 \times 10^{-3} \), which corresponds to boundary conditions.

The results of a strength analysis showed that the shaft of a newly designed centrifugal pump can be made of a less expensive material than 40X steel.

Increasing the pressure-energy performance of machines for pumping a liquid media is impossible without optimizing the dynamic parameters of a system at a design stage.

The reliability and service life of a centrifugal pump is largely determined by its vibration state. A technology for calculating the critical rotor speed of a pump is complex, and so far there is no possibility to accurately determine it due to the impossibility of the reliable prediction of coefficients that take into account an effect of all possible factors affecting the vibration state of a pump [4].

7. Theoretical provisions of a numerical analysis of rotor dynamics

Free vibrations completely determine the dynamic properties of a mechanical system and have paramount importance in analyzing forced vibrations [5], therefore using a finite-element model, firstly, they determine the spectrum of the natural frequencies of the rotor of a centrifugal pump.

To describe a movement only under the action of a restoring (elastic) force without taking into account energy dissipation, the equation is used [5]:

\[
[M] \{q\} + [C] \{\dot{q}\} = 0
\]

where, \([M]\), \([C]\) – matrix of masses (inertia) and the rigidity of a system; \([q]\), \({\dot{q}}\) – generalized displacements of nodes and their derivatives.

The solution of equation (3) is sought in the form [7]:

\[
\{q\} = \{q_0\} \sin \omega_0 t (2-17)
\]

where, \(\omega_0\) – values of the natural frequencies of oscillations;

\([q]\) – full vector of the nodal displacements of a system;

\({\dot{q}_0}\) – amplitude column matrix.

A full vector \(q\) is a function of independent displacement components and rotation angles about corresponding axes. The full displacement vector is represented as:

\[
\{q\} = \left\{ q_1^{(1)}, q_2^{(2)}, \ldots, q_n^{(n)} \right\}^T
\]
In this case, a task is reduced to calculating the eigenvalues of frequencies \( \omega_0 \) and the eigenvalues of the vectors of a generalized displacement \( q \), hence, \( q \) determines the shape of natural oscillations at a corresponding frequency \( \omega_0 \). When implementing an automated finite element method for determining natural oscillations, a numerical solution of the system of algebraic equations in the algorithms of a PATRAN program \([91,92,103,104]\) is carried out using a Lanczos method \([6]\).

The forced oscillations of a rotor occur under an action of the harmonic centrifugal force of the inertia of the unbalanced masses of a rotor, which are represented as \( F_u = ma\omega^2 \cos(\omega t) \), then the equation of forced oscillations is written:

\[
[M] \cdot \{\ddot{q}\} + [B] \cdot \{\dot{q}\} + [C] \cdot \{q\} = [Me] \omega^2 \cos(\omega t)
\]  

(6)

where \([M], [B], [C]\) are a matrix of masses (inertia), damping and the rigidity of a system;
\( \{q\}, \{\dot{q}\}, \{\ddot{q}\} \) – generalized displacements of nodes and their derivatives;
\( \omega \) – angular velocity of a rotation;
\( e \) – specific imbalance.

The solution of an equation (6) is sought as:

\[
\{q\} = \{q_0\} \sin(\omega_0 t) + [Me] \omega^2 \cos(\omega t)
\]  

(7)

8. Automated calculation of the natural frequencies and oscillations of the rotor of a centrifugal pump

Initial data for a calculation are the physical properties of a shaft material (density \( \rho = 7850 \text{ kg} / \text{ m}^3 \) and the modulus of the elasticity of the 1-st kind \( E = 2.1 \cdot 10^{11} \text{ N} / \text{ m}^2 \), lengths \( L \), outer \( D \) and inner \( d \) diameters of sections, mass \( m \), and also stiffness from bearing support.

According to the design scheme (Figure 6), a beam model has been built in Patran, which includes 7 CBEAM elements (rotor shaft), 7 CONM2 elements (concentrated mass element simulating a rotor wheel).

The modeling of rigid supports is carried out by fixing rotor model nodes according to the corresponding degrees of freedom; in the design of a pump under study, the support was modeled taking into account a gap \( \Delta \) - mm (Figure 6).

The values of the natural frequencies of a rotor have been found using the NORMAL MODELS solver (modal analysis).

| Parameter | Value |
|-----------|-------|
|           |       |

Figure 6. Centrifugal pump rotor, the rod model of a rotor, built in Patran

Table 3. Found values of natural frequencies.
The coincidence of often disturbing oscillations with the frequencies of natural oscillations presented in a table and can lead to resonant phenomena.

### 9. Automated calculation of perturbed oscillation frequencies

To determine the frequency of disturbing oscillations, there has been used a COMPLEX EIGENVALUE solver (complex frequencies). A program can use the option of asynchronous precession (ASYNC) to determine the response of a system to external influences, which is independent of rotor speed. When using synchronous precession (SYNC) option, the system’s response to an imbalance or other excitation is determined, which depends on the rotor speed. With the help of the complex analysis of forms, it is possible to determine oscillation frequencies corresponding to direct and inverse precession, as well as critical rotational speed.

In the Spin Profile menu, a user sets individual rotor speed; for our rotor task of a centrifugal pump, an angular velocity \( \omega = 3000 \) rpm. It also takes the moments of the inertia of mounted elements that have been defined in a SolidWorks CAD system.

When choosing the type of calculation, the calculation of the complex eigenvalues of SOL 107 is a direct method, the frequency diagram is obtained by calculating complex eigenvalues by the direct method using an ASYNC option, at the rotor speed of 0, 900, 1800, 2700, 3000 rpm [7].

**Table 4.** Found values of perturbed oscillations.

| Parameter               | Value |
|-------------------------|-------|
| \( f_1 \) - first Hz    | 8     |
| \( F_2 \) - second Hz   | 434   |
| \( F_3 \) - third Hz    | 447   |
| \( F_4 \) - third Hz    | 696   |
| \( F_5 \) - third Hz    | 732   |
| \( F_6 \) - third Hz    | 892   |

Figure 7. The 3-rd form of perturbed oscillations at a frequency of 1800 rpm.
Critical speed is determined based on which eigenvalues are identical to the speed of the rotation of a rotor. For this purpose, a straight line is constructed on a diagram, corresponding to \( w = W \), i.e. (oscillation frequency = angular speed of the rotation of a rotor). The points of the intersection of a line with the curves of natural frequencies correspond to critical rotor speed [10].

The broken character of frequency lines indicates the unstable nature of vibrations associated with the design features of guide supports, in the form of a gap of a size \( \Delta \).

Automatically calculated eigenvalues (Figure 8), corresponding to the same oscillation forms, form a series of curves that are the functions of changing an oscillation frequency from the angular velocity of the rotation of a rotor. A Campbell diagram shows that all repeated critical rotational frequencies for the first modes of oscillation – 366, 488, 732 Hz are in the region of 47 Hz (2200 rpm) for reverse and direct precession, respectively. To compensate for vibrations during the start-up period, it is planned to replace a finger clutch with a magnetic clutch with better damping properties.

An operating mode has been modeled by a design scheme with support imposing restrictions on displacements in a plane perpendicular to the axis of a rotor shaft axis in reality – this layer of distilled liquid filling a pump cavity. The task has been calculated with the same input data as the circuit presented above.
Figure 9. Campbell diagram for a model with support without a gap.

Table 5. Frequency of disturbed oscillations in the second case of consolidation is summarized in a table

| Parameter       | Value |
|-----------------|-------|
| $f_1$ - first frequency Hz | 0     |
| $f_2$ - second frequency Hz  | 315   |
| $f_3$ - third frequency Hz   | 388   |
| $f_4$ - third frequency Hz   | 631   |
| $f_5$ - third frequency Hz   | 636   |

Despite the fact that the tones of the oscillations of the second model are 20% lower than those in guide bearings, it can be said that critical speed points (points of the intersection of speed multiplicity lines and frequency lines) are missing and a machine enters an operating mode relatively stable.

10. Conclusion
In an integrated NASTRAN / PATRAN system, strength and dynamic analysis have been performed and the following results have been obtained:

- calculated mechanical schemes for determining the stress-deformable state of the rotor shaft of a centrifugal pump;
- 3D model of a seven-stage centrifugal pump rotor has been built;
- there have been determined voltage levels for zero and maximum flow $\sigma_{max} = 286$ MPA for maximum flow, $\sigma_{max} = 158$ MPA for minimum flow.

The dynamic parameters of a rotor shaft have been analyzed in two operating modes in a COMPLEX EIGENVALUE solver of a ROTOR DYNAMICS NASTRAN module, the determination of the frequency of disturbing oscillations, critical rotational frequencies for the first oscillation forms – 366, 448, 732 Hz are in the region of 47 Hz (2200 rpm) for reverse and direct precession, respectively. On the basis of the dynamic calculations, a damping element has been proposed that ensures a smooth entrance of CP into an operating mode - a magnetic coupling.

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