Study of Equilibrium Using Collision Dynamics

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Abstract

We discuss the possibility of equilibrium (and thermalization) in heavy-ion collisions at intermediate energies within a transport model. This was achieved by dividing the nuclear matter into different collision zones. We find that those nucleons which experience at least ten collisions are close to complete equilibrium whereas others never achieve any equilibrium.

1 Introduction

The intermediate energy heavy-ion collisions are very useful to study the non-equilibrium dynamics of finite size systems. In addition, one also has an opportunity to understand the properties of hot and dense nuclear matter that exists for a short span of time during the reaction. It is worth mentioning that no direct extraction can be made about these phenomena and therefore, one has to rely either on an indirect method of extraction or on a dynamical theoretical model that is capable of simulating the reaction from the start till the end where measurements are made. The question of equilibrium (and thermalisation) can be dealt within
those models which are not based on the assumption of any equilibrium. The transport models such as Boltzmann-Uehling-Uhlenbeck [BUU] \cite{1} or Quantum Molecular Dynamics [QMD] \cite{2, 3} are very helpful as these models can also handle the non-equilibrated nuclear matter formed during the early phase of the reaction. Naturally, this anisotropy in the momentum space (that reflects the non-equilibrium situation) should be taken into account while studying a heavy-ion collision.

We here plan to discuss the degree of equilibrium that can be reached in an intermediate energy heavy ion collision. This study is made within the framework of QMD model \cite{2, 3}. We shall show that the degree of equilibrium (studied via rapidity distribution) depends on the reaction geometry as well as on the number of collisions any individual nucleon suffers. The preservation of the initial memory of nucleons is directly related with the number of collisions one suffers. Therefore, the momentum space of those nucleons who suffer large number of collisions should be better thermalized.

The section 2 deals with details of the model. Section 3 depicts the results and discussion. Finally, we summarise the results in section 4.

2 The Model

The nucleons in a molecular dynamics picture interact via two- and three-body forces. The explicit two- and three-body interactions lead to the preservation of fluctuations and correlations that are important for N-body phenomena like multifragmentation. This is in contrast to the one-body dynamical models which are suitable for one-body observable only.

In QMD model \cite{2, 3}, each nucleon is represented by a Gaussian distribution whose centroid
propagates with classical equations of motion:

\[
\frac{d\mathbf{r}_i}{dt} = \frac{dH}{d\mathbf{p}_i},
\]

\[
\frac{d\mathbf{p}_i}{dt} = -\frac{dH}{d\mathbf{r}_i},
\]

where the Hamiltonian is given by:

\[
H = \sum_i \frac{p_i^2}{2m_i} + V_{\text{tot}},
\]

with

\[
V_{\text{tot}} = V_{\text{loc}} + V_{\text{Yuk}} + V_{\text{Coul}} + V_{\text{MDI}}
\]

Here \( V_{\text{loc}}, V_{\text{Yuk}}, V_{\text{Coul}} \) and \( V_{\text{MDI}} \), represent, respectively, the Skyrme, Yukawa, Coulomb and momentum dependent (MDI) parts of the interaction. The interaction without MDI part is called static interaction. The different values of the compressibility in the Skyrme force give possibility to look for the role of different equations of state termed as soft and hard equations of state. The inclusion of momentum dependent interactions are labelled as soft momentum dependent (SMD) and hard momentum dependent (HMD), respectively.

The G- matrix at higher excitation energies becomes complex in nature and its imaginary part acts like the collision term. We shall use here an energy dependent nucleon-nucleon cross-section. It is, however, worth mentioning that the reaction dynamics depends on the form and magnitude of the nucleon-nucleon cross-section \[3\].

3 Results and Discussion

The present study is made by simulating the reactions of Ca-Ca, Xe-Sn and Au-Au at different incident energies as well at different impact parameters. We here use a soft equation of state along with energy dependent nucleon-nucleon cross-section \[2\] through out the discussion.
There are several different ways to define the degree of equilibrium. The first quantity is the anisotropy ratio $\langle R_a \rangle$ which is defined as

$$\langle R_a \rangle = \frac{\sqrt{\langle p_x^2 \rangle} + \sqrt{\langle p_y^2 \rangle}}{2 \sqrt{\langle p_z^2 \rangle}}.$$  

(5)

This anisotropy ratio is an indicator of the global equilibrium of the system. The word *global* is due to the fact that this quantity does not depend on the local positions of nucleons and therefore, represents the equilibrium of the whole system. A full global equilibrium demands the anisotropy ratio to be close to unity. Another way to study the local equilibrium is to look for the relative momentum of two colliding Fermi spheres which indicates the deviation from a Fermi sphere. Note that the concept of local equilibrium is used by the hydrodynamical models to simulate the heavy-ion reactions. We shall, however, address the question of equilibrium with the help of rapidity distribution that also shows the stopping of nuclear matter in heavy-ion collisions.

The rapidity distribution can be defined as

$$Y(i) = \frac{1}{2} \ln \frac{E(i) - p_z(i)}{E(i) + p_z(i)},$$  

(6)

Where $E(i)$ and $p_z(i)$ are, respectively, the energy and the longitudinal momentum of the $i$-th particle. For a full equilibrium, one should get a Gaussian shape distribution peaked at the mid-rapidity region. In other words, both the anisotropy ratio and rapidity distribution are related and can give insight into the equilibrium process of a reaction.

Using the above description, we plan to relate the frequency of nucleon-nucleon collisions with the degree of equilibrium. This is achieved by dividing the rapidity distribution of the whole system into different collision zones. Among such zones, the spectator matter (SM) consists of all those nucleons who do not suffered any collision. We divide the participant region into three zones: (i) the low collision matter (LCM) which contains all nucleons who
suffer less than four collisions. (ii) the moderate collision matter (MCM) that has those nucleons with 5-9 collisions and (iii) high collision matter (HCM) which takes care of all those nucleons with more than ten collisions [6].

First of all, we study the evolution of rapidity distribution. In figure 1, we display the evolution of rapidity distribution $dN/dY$ for the reaction of Xe-Sn at 400 MeV/nucleon for different geometry. Here we display the evolution for central ($b=0$ fm), semi-central ($b=4$ fm) and peripheral ($b=8$ fm) impact parameters.

![Diagram](image)

Figure 1: Rapidity distribution ($dN/dY$) for the reaction of Xe-Sn at 400 MeV/nucleon. We display the outcome at three impact parameters $b = 0$ fm (dashed line), $b= 4$ fm (solid line) and $b = 8$ fm (dotted line).

The central reaction leads to a very high nucleon-nucleon collision rate and density whereas the peripheral reaction has a very small overlap. At the beginning, whole matter is grouped
into either projectile matter or into target matter. No nucleon-nucleon collisions occur till 9 fm/c, therefore, no change in the rapidity structure appears. Due to some nucleon-nucleon collision between 9 and 18 fm/c, few nucleons shifts to the mid-rapidity zone. On the other hand, drastic changes can be seen (for the central and semi-central reactions) between 18 and 28 fm/c. It is that time when the density and collision rate (not shown here) is maximal. The low frequency of the nucleon-nucleon collisions in peripheral geometry does not allow substantial changes in the shape of the rapidity distribution even at the end of the reaction. This also points towards the lack of degree of equilibrium in peripheral collisions. On the other hand, a nearly equilibrium can be seen in central collisions suggesting better equilibrated nuclear matter in these reactions. The study of anisotropy ratio (not shown here) also depicts similar picture. Its final values are close to 0.87, 0.67 and 0.37, respectively, for central, semicentral and peripheral reactions. As mentioned in the beginning, for a complete equilibrium, this ratio should be equal to one. Both the rapidity distribution and the anisotropy ratio suggest that central reactions should be better thermalised. It is worth mentioning that both these quantities depends on the mass of the system [4].

It will be of further interest to analyse the momentum space of nucleons of equilibrated matter. This is achieved by dividing the final state rapidity distribution into the above mentioned collision zones (see figure 2).

Naturally, the frequency of the nucleon- nucleon collisions increases with the incident energy, therefore, HCM percentage is much more at 1 GeV/nucleon than at 400 MeV/nucleon. The maximal collision rate for Xe-Sn reaction at 400 MeV/nucleon is 34 whereas it is 56 at 1 GeV/nucleon. We find that the gaussian shape is more prominent for those nucleons that suffer at least ten collisions. This is true at both incident energies. The nucleons with less than 10 collisions observe a partial equilibrium.
Figure 2: The final state rapidity distribution $dN/dY$ for the reaction of Xe-Sn at 400 MeV/nucleon (solid line) and 1000 MeV/nucleon (dashed line). Different graphs shows the break up of rapidity distribution into SM, LCM, MCM and HCM zones. For details, see the text.

In other words, a complete equilibrium is possible only for those nucleons that suffer at least 10 collisions in a reaction. The nucleon-nucleon collisions destroy the initial correlations, therefore, are dominant mode of achieving the thermalization in a reaction. From figure 2, it is also evident that the momentum distribution of nucleons with less than 4 collisions is still like that of projectile and target.

The mass dependence in heavy ion reactions has been found to affect the dynamics ranging from the fusion (at low incident energy) to the multifragmentation as well as collective flow and particle production (at intermediate and relativistic energy) [3,7]. To see the role of mass dependence in thermalization, we display in figure 3, the rapidity distribution for different col-
Figure 3: Same as figure 2, but for the semi-central reactions of Ca-Ca, Xe-Sn and Au-Au at 400 MeV/nucleon.

The conclusions drawn from figure 2 are also valid here. We find that independent of the mass of the system, at least ten collisions are needed to form nearly equilibrated nuclear matter. It is also evident that the heavier matter are better thermalized compared to lighter systems. Similar trends can also be seen from the evolution of the anisotropy ratio. In other words, the degree of equilibrium depends on the size of the interacting system.

In figure 4, we display the transverse momentum ($P_x/A$) as a function of rapidity distribution for the semi-central reactions of Ca-Ca, Xe-Sn and Au-Au at 400 MeV/nucleon. We see that the spectator matter does not have any transverse momentum whereas the intensity of transverse flow increases with collision rate. The HCM has maximum transverse flow. Further the transverse flow depends strongly on the mass of the system which is in agreement with all
previous observations and calculations [7]. If one extrapolates the above results to very low incident energy, one will find that the balance energy (i.e. the energy at which flow disappears) will be lower for heavier systems compared to lighter systems.

4 Conclusions

Summarizing, within a dynamical quantum molecular dynamics model, we present the study of equilibrium (i.e. thermalization) in heavy ion collisions at intermediate energies. This was achieved by simulating the reactions of Ca-Ca, Xe-Sn and Au-Au at different geometry as well as incident energies. We find that the momentum space of nucleons that suffer at least ten collisions is better thermalization whereas nucleons with less number of collisions exhibit a partial equilibrium. The nucleonic flow also shows strong collision dependence.
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