Application of Sliding Smoothing Method Denoising in Model Updating Damage Identification

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Abstract. The accuracy of finite element model updating for structural damage identification is easily affected by the noise in the measured modal, so the sliding smoothing method is introduced to reduce the impact of noise. In each iteration step, the sliding smoothing method denoising is applied to the difference between measured modal and simulated modal from the finite element model because the time-frequency characteristic of actual modal is different from noise modal. The modal parameters after denoising are used to construct objective function for modal updating, and the optimal question is solved to determine the stiffness in finite element model. Finally, the stiffness change can indicate the damage position and magnitude. The numerical analysis shows the proposed method can improve the robustness of finite element model updating for the structural damage identification.

1. Introduction
In recent years, structural damage detection based on modal parameter identification has attracted the attention of researchers. The measured modal parameters of the structure are used to update the finite element model, so that the calculated modal parameters of the updated finite element model tend to coincide with the measured modal parameters. This process is the updating of the structural finite element model. According to the change of the element stiffness in the model updating result, the damage location and damage degree evaluation of the structure can be carried out [1].

Structural model updating is a kind of inverse problem, and the error of input parameters has a great influence on the updating result [2]. Due to the influence of environmental factors and human factors, the measured modal parameters as input parameters will inevitably introduce noise, and its accuracy is difficult to meet the requirements of model updating, and it is easy to cause misjudgment of structural damage identification.

The method in this paper takes the structural finite element model as the benchmark, considers the difference between the real modal and the modal noise in the time-frequency domain, and takes the difference between the measured modal and the calculated modal of the finite element model as the object to perform three-point linear sliding smoothing denoising processing, Simultaneously updates the measured modal and finite element model, and the damage of the structure can be identified after the iterative updating calculation. This paper takes a three-span continuous beam with multiple damages as an example, and uses this method to identify the location and degree of structural damage. Numerical examples show that the method in this paper can improve the robustness of the finite element model updating method to structural damage identification.

2. Sliding smooth denoising method for measured modal
The sliding smoothing method [3,4] is divided into linear sliding smoothing method and nonlinear
sliding smoothing method. Suppose the observation object is Y, and the corresponding independent variable is X. Taking the interval $\Delta x$ as the independent variable, the equally spaced independent variable is $x, x_1, x_2, \cdots, x_{i-1}, x_i, x_{i+1}, \cdots, x_n$, at these value positions, observe the value of the dependent variable Y, and the obtained observation data sequence is $y_1, y_2, \cdots, y_i, \cdots, y_n$. The working principle of the linear sliding smoothing method is: take the data of the point i and several nearby points, determine a fitted straight line according to the principle of least squares, and then calculate the dependent variable of the point i from the straight line equation as the smoothed data value. As the number of points participating in the fitting changes, there will be different linear sliding smoothing methods.

2.1. Three-point linear sliding smooth

For any point i, take itself and a total of three data points as:

\[(x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1})\]

According to the principle of least squares, a straight line is used to fit these three data points, and the fitted linear equation is:

\[u = a_0 + a_1 x\]  \hspace{1cm} (1)

Calculate the value of $\mu$ at $x = x_i$ from the above formula: $u_i = a_0 + a_1 x_i$, this value is the smooth value of $y_i$. According to the principle of least squares, the residual sum of squares should be:

\[Q = \sum_{k=1}^{n} (y_{i+k} - a_0 - a_1 x_{i+k})^2\]  \hspace{1cm} (2)

is the smallest, according to the principle of extreme value, the partial derivatives of $a_0$ and $a_1$ should be calculated separately for $Q$ and the conditions that should be met when $Q$ is the minimum are:

\[
\frac{\partial Q}{\partial a_0} = \sum_{k=1}^{n} 2(y_{i+k} - a_0 - a_1 x_{i+k}) = 0 \\
\frac{\partial Q}{\partial a_1} = \sum_{k=1}^{n} 2(y_{i+k} - a_0 - a_1 x_{i+k}) \cdot x_{i+k} = 0
\]  \hspace{1cm} (3)

Without loss of generality, with $x_i = 0, \Delta x = 1$, and substituting $x_i = 0, x_{i-1} = -1, x_{i+1} = 1$ in the above formula, we get

\[
\begin{align*}
y_{i-1} + y_i + y_{i+1} - 3a_0 &= 0 \\
y_{i-1} + y_i + y_{i+1} - 2a_1 &= 0
\end{align*}
\]  \hspace{1cm} (4)

Solutions have to

\[
\begin{align*}
a_0 &= (y_{i-1} + y_i + y_{i+1}) / 3 \\
a_1 &= y_{i+1} - y_{i-1} / 2
\end{align*}
\]

Because of $x_i = 0$, $u_i = a_0$, at $x = x_i$, the three-point linear sliding smoothing formula for the dependent variable is

\[u_i = (y_{i-1} + y_i + y_{i+1}) / 3\]  \hspace{1cm} (5)

Where $u_i$ is the smoothed value of $y_i$.

2.2. Five, seven, m point linear sliding smooth

Applying the same derivation method as the three-point linear sliding smoothing method, the calculation formula of the linear sliding smoothing method for adjacent five, seven, and m points can be obtained.
Figure 1 shows the effect of the linear sliding smoothing method. Among them, the curve Y is artificially synthesized by three different frequency curves of low, medium and high. It can be seen from the figure that the linear sliding smoothing method is actually a low-pass filter. As the number of points increases, the stronger the suppression of high-frequency noise, the more obvious the low-frequency signal changes in the smooth result. The nonlinear sliding smoothing method uses a nonlinear function as the fitting function, and the simplest and common one is the quadratic polynomial sliding smoothing method. For example, suppose the fitting equation is \( u = a_0 + a_1x + a_2x^2 \). In various polynomial smoothing methods, for polynomials of the same degree, the more points are taken, the smoother the result obtained, that is, the stronger the suppression of high-frequency components. If the number of points taken is the same, when a higher degree polynomial is used, the smooth curve obtained will be closer to the real situation of the signal, or the smooth result will be more precise.

![Figure 1. The effect of the linear sliding smoothing method.](image)

In addition, for the serious frequency leakage problem, this paper uses the difference between the measured modal \( \hat{\phi} \) of the structure and the reference modal \( \phi_0 \) provided by the modified finite element model as the signal for noise reduction. That is, sliding smooth denoising method is performed on \( \hat{\phi} - \phi_0 = \Delta \phi + s_n \) (\( \phi \) is the true modal, \( s_n \) is random noise, \( \Delta \phi = \phi - \phi_0 \)). At this time, the reference modal is removed from the measured modal, which greatly reduces the value of the low-frequency component in the signal. Therefore, after sliding smooth denoising, the error generated is only the high-frequency component of a small value \( \Delta \phi \), the impact of the frequency leakage is significantly reduced. Moreover, with the continuous iterative updating of the finite element model, \( \Delta \phi \) will tend to zero, and the error caused by the corresponding denoising will also be reduced.

### 3. The damage identification of simultaneous updating of the measured modal and structure model

In the damage identification method based on the finite element model, the damage degree of the structure is usually characterized by the reduction of the element Young’s modulus. Therefore, the dimensionless damage parameter \( \alpha_i^e \) is used to describe the damage of the i-th element, that is, \( E_i^e = E_{i0}(1 - \alpha_i^e) \), where \( E_{i0}^e \) and \( E_i^e \) are the Young’s modulus of the i-th unit material before and after structural damage. Then the corresponding element stiffness matrix can be expressed as:

\[
K_i^e = K_{i0}^e(1 - \alpha_i^e)
\]

\( K_{i0}^e \) and \( K_i^e \) are the element stiffness matrices of the i-th element before and after structural damage.
When using finite element model updating for damage identification, the problem can be attributed to the solution of a constrained optimization problem. In order to minimize the residuals between the calculated and measured modal of the finite element model, the objective function of the constrained optimization problem can be expressed as a nonlinear least squares problem [5,6]:

$$\min_{\alpha} f(\alpha) = \frac{1}{2} \left\| r(\alpha) \right\|^2 = \frac{1}{2} \left\| r_j(\alpha) \right\|^2$$  \hspace{1cm} (7)

The natural frequency residual function $r_j(\alpha)$ and the modal residual function $r_s(\alpha)$ are expressed as

$$r_j(\alpha) = \left\{ \frac{\lambda_i(\alpha) - \tilde{\lambda}_i}{\tilde{\lambda}_i} \right\}$$  \hspace{1cm} (8)

$$r_s(\alpha) = \left\{ \frac{\phi_i(\alpha)}{\phi_i'(\alpha)} - \frac{\tilde{\phi}_i' - \phi_i'}{\phi_i'} \right\}$$  \hspace{1cm} (9)

Here $\tilde{\lambda}_i$ and $\tilde{\phi}_i$ are the $i$-th natural frequency calculated and measured by the structural finite element respectively; $\phi_i'(\alpha)$ and $\tilde{\phi}_i'$ are the calculated and measured modal components on any degree of freedom, respectively, Where the modal component on the reference degree of freedom is the modal maximum value.

Expand the objective function in formula (7) according to the Taylor series and cut off, plus the constraint condition of the damage parameter, the optimization problem of the model updating can be obtained:

$$\min_{\alpha} \Pi(z) = f_1 + [\nabla f_1]^T z + \frac{1}{2} z^T [\nabla^2 f_1] z$$  \hspace{1cm} (10)

Where $z = [\Delta \alpha_1' \cdots \Delta \alpha_n']^T$ is the increment of the current damage parameter $\alpha_1', f_1$, $\nabla f_1$ and $\nabla^2 f_1$ are the value, gradient and Hessian matrix of the residual $f$ in the current state respectively.

$$\nabla f_1 = \nabla f(\alpha) = \sum_{i=1}^{n} r_i(\alpha) \nabla r_i(\alpha) = J_s(\alpha)^T r(\alpha)$$  \hspace{1cm} (11)

$$\nabla^2 f_1 = \nabla^2 f(\alpha) = J_s(\alpha)^T J_s(\alpha) + \sum_{i=1}^{n} r_i(\alpha) \nabla^2 r_i(\alpha) = J_s(\alpha)^T J_s(\alpha)$$  \hspace{1cm} (12)

$J_s$ is the Jacobian matrix (or sensitivity matrix). Assuming that the change of the parameters to be identified does not cause the quality of the structure to change, the sensitivity of the natural frequency and modal to the damage parameters is:

$$\frac{\partial r_j}{\partial \alpha_i'} = \frac{1}{\lambda_j} \frac{\partial \lambda_j}{\partial \alpha_i'} = -\frac{\phi_i' \mathbf{K}_{ix} \tilde{\phi}_j}{\tilde{\lambda}_j}$$  \hspace{1cm} (13)

$$\frac{\partial r_s}{\partial \alpha_i'} = -\frac{\phi_i' \mathbf{K}_{ix} \tilde{\phi}_j}{\tilde{\lambda}_j} \frac{\phi_i}{\phi_i'} = \sum_{k=1, k \neq i}^{n} \frac{\phi_k' \mathbf{K}_{ix} \tilde{\phi}_j}{\phi_k}$$  \hspace{1cm} (14)

Among them, $\tilde{\phi}_i = \mathbf{G}_i^T \phi_i, \mathbf{G}_i'$ are the transformation matrices of the $i$-th element stiffness matrix in the group lumped stiffness.

This paper uses the following steps to simultaneously update the measured modal and finite element model to identify the damage of the structure:
Step 1. Establish the initial finite element model of the structure, construct the model updating objective function, perform sensitivity analysis, and modify the damage parameters of the finite element model;

Step 2. From the updating results, select the damage element that can be clearly judged, and modify the Young’s modulus of the finite element model element;

Step 3. Take the updated finite element model calculation modal as a benchmark, and perform sliding smoothing method noise reduction processing on the measured modal difference;

Step 4. Use the denoised modal parameters to construct the model updating objective function, perform sensitivity analysis, and modify the damage parameters of the finite element model;

Step 5. Compare the damage parameter vectors in the finite element model before and after the updating, the updating process ends when convergence; otherwise, select several larger damage parameters from the updating results obtained in Step 4 to update the finite element model, and transfer to Step 3 to enter the calculation of the next iteration step, until the final convergence. After iterative calculation, the damage of the structure can be identified more accurately.

4. Analysis of calculation example

The numerical calculation example uses a three-span continuous beam with multiple damages, as shown in figure 2. The beam has a total length of 80m and is divided into 32 beam elements (using the Beam44 element in ANSYS). The material density and elastic modulus are 2500m3 and 3.2E+10 N/m2, respectively, and the beam cross-sectional area and moment of inertia are 5.275m2 and 2.5755E-02m4, respectively. The structural damage is characterized by the stiffness reduction of the element. The Young’s modulus of the 3rd, 11th, 17th, and 28th elements are selected to reduce 15%, 30%, 20%, and 15% respectively to characterize the damage.

Figure 2. Three-span continuous beam.

In actual structures, only lower-order modal parameters can usually be measured. In this paper, the first four-order vertical bending modal parameters are used for structural damage identification. Considering the influence of noise on structural damage identification, the natural frequency and modal data are respectively applied with noise as follows:

\[
v_i = v_i^0 (1 + \varepsilon R_i) \quad (15)
\]

\[
\phi_{i,j} = \phi_{i,j}^0 (1 + \varepsilon R_{i,j}) \quad (16)
\]

Among them, \(v_i^0\) and \(v_i\) are the frequency values before and after noise addition, \(\phi_{i,j}^0\) and \(\phi_{i,j}\) are the i-th component of the j-th modal before and after noise addition; \(\varepsilon\) is the noise level; \(R_i\) and \(R_{i,j}\) are both \([-1, +1]\) normally distributed random variables, and the noise-added modal data is used as the measured modal for damage identification. The constrained optimization problem is solved by the standard confidence interval Newton method [7]. In order to improve the efficiency and accuracy of the optimization calculation, the optimization search step is limited to the confidence interval. The confidence interval of the damage parameter in the calculation example in this paper is \([-0.02, 0.4]\).

4.1. Beam Iterative Updating Damage Identification

In the case of multiple damages to the beam, 1% noise is applied to the natural frequency and 6% noise is applied to the modal, and four iterative updating are performed to identify the damage of the structure. The result of beam model updating damage identification is shown in figure 3. It can be seen from the figure that in the first iteration, due to the influence of random noise, the stiffness reduction of 6, 10, 12
and other elements is easy to make people misjudge the damage results. In the second iteration, select the significantly damaged elements 3 and 11 from the results of the first iteration and modify their Young’s modulus, recalculate the finite element model modal parameters, and use the updated finite element calculation modal as the benchmark. The measured modal difference is subjected to three-point smoothing method denoising. A total of four iterative updating are carried out. From the iterative results, the method in this paper can make a more accurate judgment on the location and extent of structural damage.

**Figure 3.** Results of iterative identification of multiple damages in beam.

### 4.2. Sliding smoothing denoising analysis of the measured modal

To analyze the denoising effect of the measured modal, figure 4 shows the difference between the fourth measured modal, the updating modal and the real modal, and table 1 shows the signal-to-noise ratio (SNR) and the Root Mean Squared Error (RMSE) of the measured modal initial, four iterative updating and directly updating the modal. It can be seen from the measured modal denoising results that after four iterative updating, the measured modal accuracy is significantly improved.

**Figure 4.** The difference between the fourth measured modal, the updating modal and the real modal.
### Table 1. SNR and RMSE of the measured modal initial, four iterative updating and directly updating the modal.

| Modal order | Initial SNR | SNR after four iterative updating | SNR after directly updating the modal | Initial RMSE | RMSE after four iterative updating | RMSE after directly updating the modal |
|-------------|-------------|-----------------------------------|--------------------------------------|--------------|-----------------------------------|--------------------------------------|
| 1           | 33.6984     | 41.1730                           | 25.9884                              | 2.0134e-005  | 8.5151e-006                       | 4.8913e-005                         |
| 2           | 35.5608     | 38.2068                           | 20.5249                              | 1.6367e-005  | 1.0891e-005                       | 9.2419e-005                         |
| 3           | 36.4982     | 39.0771                           | 16.3783                              | 1.4648e-005  | 1.0964e-005                       | 1.4852e-004                         |
| 4           | 33.3969     | 38.9854                           | 9.3124                               | 2.0969e-005  | 1.4852e-004                       | 3.3558e-004                         |

4.3. The Influence of Different Noise Levels on Model updating Damage Identification

In order to analyze the influence of different noise levels on the results of beam model updating damage identification, considering that the accuracy of the measured frequency is usually higher than the accuracy of the modal, the natural frequencies are all noised by 1%, and the modal noise is 2%, 4%, 6%, and 8%, iterate 3, 3, 4, 4 times respectively. Figure 5 shows the damage identification results under different noise levels, and Table 2 shows the identification errors of the damage unit stiffness under different noise levels. It can be seen from the numerical results that this method can make a more accurate judgment on the location and extent of structural damage under different noise levels.

**Figure 5.** Damage identification results under different noise levels.

**Table 2.** The identification errors of the damage unit stiffness under different noise levels.

| Unit stiffness identification error (%) | 2% noise | 4% noise | 6% noise | 8% noise |
|----------------------------------------|----------|----------|----------|----------|
| Unit 3                                 | 0.7508   | -3.6350  | -3.3680  | -3.3900  |
| Unit 11                                | 0.6117   | 6.2016   | 4.8679   | 5.7537   |
| Unit 17                                | 5.0467   | -0.2394  | 7.2040   | 7.2375   |
| Unit 28                                | 0.6467   | -5.8360  | -4.0783  | -4.1426  |
5. Conclusion
In this paper, the linear sliding smoothing method is used to reduce the noise of the measured modal difference data, and the finite element model is updated with the measured modal after noise reduction, and the measured modal and finite element model are updated synchronously. The iterative updating calculation can be more accurate to identify structural damage. At the same time, the influence of different noise levels on the model updating damage identification is analyzed. Numerical examples show that the method in this paper can make a more accurate judgment on the location and extent of structural damage, and at the same time has a certain degree of robustness.

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