Consistency conditions and trace anomalies in six dimensions

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Abstract

Conformally invariant quantum field theories develop trace anomalies when defined on curved backgrounds. We study again the problem of identifying all possible trace anomalies in $d = 6$ by studying the consistency conditions to derive their 10 independent solutions. It is known that only 4 of these solutions represent true anomalies, classified as one type A anomaly, given by the topological Euler density, and three type B anomalies, made up by three independent Weyl invariants. However, we also present the explicit expressions of the remaining 6 trivial anomalies, namely those that can be obtained by the Weyl variation of local functionals. The knowledge of the latter is in general necessary to disentangle the universal coefficients of the type A and B anomalies from calculations performed on concrete models.
1 Introduction

Six-dimensional conformal field theories (CFT\textsubscript{6}) have attracted some interest in view of recent advances in string/M-theory. In particular, the low energy dynamics of the collective coordinates of \(N\) coinciding M5 branes of M-theory realize a quite interesting class of non-trivial CFT\textsubscript{6} with maximal supersymmetry, the so-called \(\mathcal{N} = (0,2)\) interacting theories originally described in \([1,2]\). While they still lack a lagrangian formulation, the AdS/CFT conjecture \([3,4,5,6]\) has provided concrete tools to extract some informations on these rather mysterious theories in their large \(N\) limit, such as their trace anomalies \([7]\), the spectrum of their operators \([8]\) and some of their 2- and 3-point correlation functions \([9,10,11]\). To understand better the structure of these interacting theories, in refs. \([12,13]\) some of their more accessible properties were compared with those of another maximally supersymmetric CFT\textsubscript{6}: the non-interacting one made up by \(N\) copies of the free \(\mathcal{N} = (0,2)\) tensor multiplet containing 5 scalars, 1 two-form with selfdual field strength and 2 Weyl fermions. In particular, in \([13]\) the trace anomalies for the free theory were computed and compared with the corresponding ones for the interacting theory obtained through the AdS/CFT conjecture \([4]\). In this comparison it is crucial to disentangle the coefficients of the universal part of the anomalies by separating out a trivial sector. The latter can always be cancelled by the variation of a local counterterm which can be added to the effective action. This disentanglement will be the subject of the present paper and we will start addressing it after a brief introduction to the topic of trace anomalies.

Trace anomalies can be characterized by the anomalous Weyl variation of a general coordinate invariant effective action depending on a background metric. Discovered originally in \([14]\) (and reviewed in \([15]\)) they can be computed using Feynman graphs, as in the original papers, or more efficiently using the heat kernel methods of De Witt \([16]\) (as employed e.g. in \([17,18]\)) or by a quantum mechanical representation as proposed in \([19]\). Their structure has been analyzed in \([20,21,22]\) by cohomological methods which encode the information on the Wess–Zumino consistency conditions \([23]\) specialized to the Weyl symmetry. Finally, a useful classification was described in \([24]\) where trace anomalies are divided into three classes: type A (always proportional to the topological Euler density), type B (made up by independent Weyl invariants) and trivial anomalies (obtainable as Weyl variations of local functionals and in general expressible as total derivatives). The number of type B and trivial anomalies grows quite rapidly with the number of dimensions. Already in six dimensions there are 1 type A, 3 type B and 6 trivial anomalies satisfying the consistency conditions on a set of 17 independent terms with the property of being cubic in the curvature (and thus with the correct dimensions to constitute possible trace anomalies). Usually a concrete calculation
on a specific model delivers the anomaly as a linear combination of these 17 terms, and one is left with the problem of disentangling the correct universal part of the anomaly (type A and B).

This problem was solved pragmatically in [13] by expressing the 6d trace anomalies in a special basis for the curvature invariants (a basis which employs the Weyl tensor and traceless Ricci tensor instead of the Riemann and Ricci tensors). This special basis makes it easier to cast the anomaly into a form with the expected type A and B contributions plus a combination of total derivatives. The latter part was interpreted as a trivial anomaly since on general grounds one expects trivial anomalies to be total derivatives. This procedure worked for the four cases of free scalars, fermions, two-forms and interacting (0, 2) theory considered in [13]. Now, one may object that the calculations made there used a basis of 7 independent total derivatives while the cohomological analysis of [21] predicts only 6 trivial anomalies. Thus, to make sure of the correct identification of the various anomalies, we have decided to perform again a cohomological analysis to derive a basis for the trivial anomalies. This allows us to check that indeed there are only 6 trivial anomalies, for which we find the explicit expressions and identify the local counterterms that can cancel them. Then we verify that one specific linear combination of the 7 total derivative terms used in [13] doesn’t solve the consistency conditions, but never appears in the results for the trace anomalies of the various cases treated there, thus confirming the correctness of those results.

The final output of our analysis is a systematic classification of the type A, type B and trivial anomalies for trace anomalies in six dimensions. This knowledge can be useful to put new calculations of such anomalies in a preferred basis and extract unambiguously the universal coefficients of the type A and B parts. It is presumably the difficulties related to the proper factorization of trivial anomalies that has caused a miscalculation of the trace anomalies for a scalar field in [25].

Thus in sect. 2 we review and solve the consistency conditions. In sect. 3 we cast those solutions into a more useful basis by taking into account their character as anomalies of type A, B and trivial. In sect. 4 we present our conclusions. Finally, we leave appendices A, B, C and D for more technical parts where we list useful results of our calculations.

## 2 Consistency conditions

Let’s consider the effective action $W[g]$ for a CFT coupled to a background metric $g_{ab}$. We will use an euclidean signature. For simplicity we assume absence of chiral gravitational anomalies and thus can consider $W[g]$ to be general coordinate invariant. This assumption is not necessary [21]. Under an infinitesimal Weyl transformation depending on an infinitesimal
arbitrary function $\sigma(x)$

$$\delta_\sigma g_{ab}(x) = 2\sigma(x)g_{ab}(x),$$

(1)

the effective action generically suffers an anomalous variation

$$\delta_\sigma W[g] = \int d^6 x \; \sqrt{g} \sigma(x) A(x).$$

(2)

It is well known that by functional differentiation with respect to $\frac{2}{\sqrt{g}} \frac{\delta}{\delta g_{ab}}$ the effective action $W[g]$ generates correlation functions of the stress tensor $T^{ab}$. Thus eq. (2) produces an anomalous trace to the stress tensor

$$\langle T^a_a(x) \rangle = A(x)$$

(3)

which depends on the background curvature. General coordinate invariance guarantees that the anomaly $A(x)$ is a scalar and dimensional considerations in $d = 6$ fix it to be cubic in the curvature (two covariant derivatives count as one curvature). However, those particular anomalies that can be obtained also from the Weyl variation of local functionals of the metric are considered trivial since they can be cancelled by subtracting the same local functionals from the original non-local effective action.

Since the anomaly $A(x)$ is obtained by varying a functional, there are integrability conditions that can be identified by applying the commutator algebra of the Weyl symmetry, $[\delta_\sigma_1, \delta_\sigma_2] = 0$, to the effective action. Such integrability conditions are generically known as Wess–Zumino consistency conditions, originally derived for chiral anomalies in [23].

Now, we follow the work of Bonora et al. [21] as a guideline to study the consistency conditions

$$[\delta_\sigma_1, \delta_\sigma_2] W[g] = 0$$

(4)

and derive all of their solutions in $d = 6$. We use the following conventions for the curvature tensors

$$[\nabla_a, \nabla_b] V^c = R^c_{ab} V^d, \quad R_{ab} = R^{c}_a b, \quad R = R^a_a,$$

(5)

so that the scalar curvature of a sphere is positive, and use the same basis of 17 independent
curvature invariants as in [21]

\[ K_1 = R^3 \]
\[ K_2 = RR_{ab}^2 \]
\[ K_3 = RR_{abcd}^2 \]
\[ K_4 = R_a^m R_m^i R_i^a \]
\[ K_5 = R_{ab} R_{mn} R_{mah} \]
\[ K_6 = R_{ab} R_{amnl} R_{bml} \]
\[ K_7 = R_{ab}^m R_{mn} R_{ij} R_{ij}^a b \]
\[ K_8 = R_{amnl} R_{mij} R_i^a j \]
\[ K_9 = R \nabla^2 R \]
\[ K_{10} = R_{ab} \nabla^2 R_{ab} \]
\[ K_{11} = R_{abmn} \nabla^2 R_{abmn} \]
\[ K_{12} = R_{ab} \nabla_a \nabla_b R \]
\[ K_{13} = (\nabla_a R_{mn})^2 \]
\[ K_{14} = \nabla_a R_{bm} \nabla_b R_{am} \]
\[ K_{15} = (\nabla_i R_{abmn})^2 \]
\[ K_{16} = \nabla^2 R^2 \]
\[ K_{17} = \nabla^4 R. \]

All other terms cubic in the curvature are linear combinations of the above invariants after taking into account the symmetry properties and the Bianchi identities of the Riemann tensor.

Any trace anomaly can be expanded in the above basis

\[ \delta_\sigma W[g] = \int d^6 x \sqrt{g} \sigma(x) \sum_{i=1}^{17} a^i K_i \]  

and after computing a second Weyl variation one obtains

\[ [\delta_\sigma_2, \delta_\sigma_1] W[g] = \int d^6 x \sqrt{g} \sum_{i=1}^{17} \sum_{\alpha=1}^{9} f^\alpha_i a^i H_\alpha \]  

where the rectangular matrix of coefficients \( f^\alpha_i \) can be constructed using the variations of the terms entering eq. (2) and reported in appendix A, and where the 9 independent (unintegrated) 2-cochains \( H_\alpha \) are given by

\[ H_1 = R^2 \sigma_1 \nabla^2 \sigma_2 \]
\[ H_2 = R_{ab}^2 \sigma_1 \nabla^2 \sigma_2 \]
\[ H_3 = RR_{ab} \sigma_1 \nabla^a \nabla^b \sigma_2 \]
\[ H_4 = R_{mn} \sigma_1 \nabla^m \nabla^l \sigma_2 \]
\[ H_5 = R_{abmn}^2 \sigma_1 \nabla^2 \sigma_2 \]
\[ H_6 = (\nabla^2 R) \sigma_1 \nabla^2 \sigma_2 \]
\[ H_7 = R \sigma_1 \nabla^4 \sigma_2 \]
\[ H_8 = R_{ab} \sigma_1 \nabla^a \nabla^b \nabla^2 \sigma_2 \]
\[ H_9 = R_{mn} R_{amnl} \sigma_1 \nabla^a \nabla^b \nabla^2 \sigma_2. \]

Now the consistency condition eq. (4) applied to eq. (8) requires that

\[ \sum_{i=1}^{17} f^\alpha_i a^i = 0, \quad \alpha = 1, \ldots, 9. \]  

1 However we differ in the definitions of the various curvature tensors, so there are some sign differences with respect to ref. [21]. In particular, we agree on the sign of the Riemann tensor \( R_{abcd} \) but have opposite sign for the Ricci tensor \( R_{ab} \) and scalar curvature \( R \).

2 The symbol \( \cdot \cdot \) denotes antisymmetrization, namely \( a_1 b_2 = a_1 b_2 - a_2 b_1. \)
The $9 \times 17$ matrix $f^\alpha_i$ has rank 7, so there are 7 independent constraints for the coefficients $a^i$ to form a consistent anomaly. The resulting 10 independent anomalies can be presented as

$$M_I(x) = \sum_{i=1}^{17} a^i_I K_i(x)$$

(11)

where the 10 vectors $a^i_I$, $I = 1, \ldots, 10$, form a basis for the solutions of eq. (10). These vectors are constructed in appendices A (where one can read off the matrix $f^\alpha_i$) and C. We list them later on in eqs. (13–22).

Trivial anomalies are those that can be obtained by varying a local functional. To recognize them we compute the Weyl variation of the most general local functional obtained as a linear combination with coefficients $c^i$ of the integrated curvature invariants $K_i$ (note that we can restrict the index $i \leq 10$ since by partial integration the remaining terms are not linearly independent)

$$\delta_\sigma \int d^6x \sqrt{g} \sum_{i=1}^{10} c^i K_i = \int d^6x \sqrt{g} \sigma(x) \sum_{i=1}^{10} \sum_{j=1}^{17} g^i j^j K_j.$$  

(12)

The matrix of coefficients $g^i j^j$ has rank 6 and therefore identifies 6 trivial anomalies. These are constructed in appendices B (where one can read off the matrix $g^i j^j$) and C, and reported here below in eqs. (17–22). In appendix C one may also find the local functionals which generate the trivial anomalies (see eq. (41)).

Now we present the solutions of the consistency conditions just described. A basis for the non-trivial anomalies is given by

$$M_1 = \frac{19}{800} K_1 - \frac{57}{160} K_2 + \frac{3}{40} K_3 + \frac{7}{16} K_4 - \frac{9}{8} K_5 - \frac{3}{4} K_6 + K_8$$

(13)

$$M_2 = \frac{9}{200} K_1 - \frac{27}{40} K_2 + \frac{3}{10} K_3 + \frac{5}{4} K_4 - \frac{3}{2} K_5 - 3 K_6 + K_7$$

(14)

$$M_3 = -K_1 + 8 K_2 + 2 K_3 - 10 K_4 + 10 K_5 - \frac{1}{2} K_9 + 5 K_10 - 5 K_{11}$$

(15)

$$M_4 = -K_1 + 12 K_2 - 3 K_3 - 16 K_4 + 24 K_5 + 24 K_6 - 4 K_7 - 8 K_8$$

(16)

where we have chosen to agree with the ones reported in ref. [21]. Instead a suitable basis for the remaining trivial anomalies is given by

$$M_5 = 6 K_6 - 3 K_7 + 12 K_8 + K_{10} - 7 K_{11} - 11 K_{13} + 12 K_{14} - 4 K_{15}$$

(17)

$$M_6 = -\frac{1}{5} K_9 + K_{10} + \frac{2}{5} K_{12} + K_{13}$$

(18)
\[ M_7 = K_4 + K_5 - \frac{3}{20}K_9 + \frac{4}{5}K_{12} + K_{14} \]  
\[ M_8 = -\frac{1}{5}K_9 + K_{11} + \frac{2}{5}K_{12} + K_{15} \]  
\[ M_9 = K_{16} \]  
\[ M_{10} = K_{17}. \]

This is the main result we were searching for.

\section{A useful basis for six dimensional trace anomalies}

In the previous section we have derived the solutions to the consistency conditions. We now put those solutions into a more useful basis by taking into account their character as type A, B or trivial anomalies, as classified in \[24\].

The type A anomaly is unique and proportional to the six dimensional topological Euler density and can be written as

\[ E_6 = -\epsilon_{m_1n_1m_2n_2m_3n_3}^{\ a_1b_1a_2b_2a_3b_3} R^{m_1n_1}_{\ a_1b_1} R^{m_2n_2}_{\ a_2b_2} R^{m_3n_3}_{\ a_3b_3} = 8M_4. \]  

The anomalies of type B are given instead by the three following Weyl invariants

\[ I_1 = C_{amnb} C^{mijn} C_{ij}^{\ ab} = M_1 \]  
\[ I_2 = C_{ab}^{\ mn} C_{mn}^{\ ij} C_{ij}^{\ ab} = M_2 \]  
\[ I_3 = C_{mabc} (\nabla^2 \delta^{m}_{n} + 4R^{m}_{n} - \frac{6}{5}R \delta^{m}_{n}) C^{mabc} + \nabla_i J^i = \frac{16}{3}M_1 + \frac{8}{3}M_2 - \frac{1}{5}M_3 + \frac{2}{3}M_4 + \nabla_i J^i \]

where

\[ C_{abcd} = R_{abcd} - \frac{1}{4}(g_{ac}R_{bd} + g_{bd}R_{ac} - g_{ad}R_{bc} - g_{bc}R_{ad}) + \frac{1}{20}(g_{ac}g_{bd} - g_{ad}g_{bc})R \]

is the Weyl tensor in 6 dimensions and

\[ \nabla_i J^i = -\frac{2}{3}M_5 - \frac{13}{3}M_6 + 2M_7 + \frac{1}{3}M_8 \]

is a trivial anomaly that make \( I_3 \) locally Weyl invariant once multiplied by the measure \( \sqrt{g} \) \[26\]. Finally the independent six trivial anomalies can be identified by \( M_5, M_6, M_7, M_8, M_9, M_{10} \) as listed in eqs. (17–22).

To summarize, a preferred basis for the trace anomalies which takes into account the classification of ref. \[24\] is given by \((E_6; I_1, I_2, I_3; M_5, M_6, M_7, M_8, M_9, M_{10})\). The first four
elements make up a basis for the true trace anomalies and in that form have been used in the calculations of ref. [13].

It may be useful to recall that in the basis of Bonora et al. [21] $M_4$ gives the type A anomaly, while $M_1$ and $M_2$ are type B anomalies. On the other hand, $M_3$ contains a spurious contribution from the Euler density and it is not classifiable as the remaining type B anomaly: it is preferable to use $I_3$ instead.

Anselmi has introduced in ref. [27] the notion of pondered Euler density $\tilde{E}_6$ by adding a suitable trivial anomaly to $E_6$ to make it linear in the conformal factor once evaluated on conformally flat metrics

$$\tilde{E}_6 = E_6 + \left( \frac{288}{5} - 20\zeta \right) M_6 + \left( 20\zeta - \frac{408}{5} \right) M_7 + \left( \frac{\zeta}{2} - \frac{9}{25} \right) M_9 - \frac{24}{5} M_{10}. \quad (29)$$

This is an equivalent way of presenting the type A anomaly which may be useful for various applications. Note that $\zeta$ labels a 1-parameter family of trivial anomalies. It can be chosen at will showing that the definition of a pondered density is not unique. This construction can be extended to any even dimension [27].

A further characterization of type A anomalies has been proposed in [28] by studying the AdS/CFT holographic correspondence, while their systematic computation for free models in arbitrary dimensions has been carried out recently in [29]. Finally, it is worth mentioning that CFTs with a special linear relation between the type A and B anomalies, the so-called $c = a$ theories, have been identified in [30] as an interesting subclass of conformal theories with special properties.

4 Conclusions

We have presented a systematic derivation and classification of trace anomalies in six dimensions. We have solved the consistency conditions and listed the 10 independent solutions as type A ($E_6$), type B ($I_1$, $I_2$, $I_3$) and trivial ($M_5$, $M_6$, $M_7$, $M_8$, $M_9$, $M_{10}$) trace anomalies. We summarize them by using the $K_i$ basis in Table 1.

Our motivation to perform this analysis was to make sure that the identifications of type A and B anomalies made in [13] for various models was correct. As we show explicitly in appendix D that is the case: a spurious term, which doesn’t solve the consistency conditions but enters the basis of total derivatives used to identify trivial anomalies, always drops out in the relevant cases. On the other hand, the calculation of the trace anomalies for a scalar field performed in [25] didn’t produce the same result as in [13] because trivial anomalies were not properly factorized: in [25] the coefficients of $K_4$, $K_5$, $K_7$ and $K_8$ were interpreted as the coefficients of true trace anomalies but those structures appear in our list of trivial
\[ E_6 = -8K_1 + 96K_2 - 24K_3 - 128K_4 + 192K_5 + 192K_6 - 32K_7 - 64K_8 \]

\[ I_1 = \frac{19}{800}K_1 - \frac{57}{100}K_2 + \frac{3}{40}K_3 + \frac{7}{10}K_4 - \frac{2}{5}K_5 - \frac{3}{4}K_6 + K_8 \]

\[ I_2 = \frac{9}{200}K_1 - \frac{27}{40}K_2 + \frac{3}{10}K_3 + \frac{5}{2}K_4 - \frac{3}{2}K_5 - 3K_6 + K_7 \]

\[ I_3 = -\frac{11}{50}K_1 + \frac{27}{10}K_2 - \frac{6}{5}K_3 - K_4 + 6K_5 + 2K_7 - 8K_8 + \frac{3}{5}K_9 - 6K_{10} + 6K_{11} + 3K_{13} - 6K_{14} + 3K_{15} \]

\[ M_5 = 6K_6 - 3K_7 + 12K_8 + K_{10} - 7K_{11} - 11K_{13} + 12K_{14} - 4K_{15} \]

\[ M_6 = -\frac{1}{2}K_9 + K_{10} + \frac{2}{5}K_{12} + K_{13} \]

\[ M_7 = K_4 + K_5 - \frac{3}{20}K_9 + \frac{4}{5}K_{12} + K_{14} \]

\[ M_8 = -\frac{1}{5}K_9 + K_{11} + \frac{2}{5}K_{12} + K_{15} \]

\[ M_9 = K_{16} \]

\[ M_{10} = K_{17} \]

Table 1: Trace anomalies in six dimensions: type A, type B and trivial anomalies

anomalies, so their coefficients do not have a universal meaning since they can be modified by adding local counterterms to the effective action. Nevertheless, the method of [25] can still be used to compute trace anomalies. By inspecting Table 1 one can recognize the terms that are not corrupted by trivial anomalies: a simple set of 4 independent structures is given by \( K_1, K_2, K_3, K_4 - K_5 \).

With the present knowledge of trivial anomalies at hand, an interesting investigation could be to study their flows in CFT\(_6\) deformed by relevant operators. This would provide a test in six dimensions of some of the properties studied in ref. [31].
Appendix A

We define the 1-cochains

$$W_i^{(a)} = \int d^6x \sqrt{\sigma_a}(x)K_i, \quad i = 1, \ldots, 17$$

(30)

where $\sigma_a$ denote infinitesimal parameters of Weyl transformations and $K_i$ belong to the list of curvature invariants given in eq. (6), and the 2-cochains

$$H_\alpha = \int d^6x \sqrt{\sigma}H_\alpha, \quad \alpha = 1, \ldots, 9$$

(31)

with the list of $H_\alpha$ reported in eq. (9). Now, we compute the variations

$$\Delta W_i \equiv \delta_{\sigma_2}W_i^{(1)} - \delta_{\sigma_1}W_i^{(2)}$$

(32)

which can be expanded in the basis of the functionals $H_\alpha$. We find:

$$\begin{align*}
\Delta W_1 &= -30H_1 \\
\Delta W_2 &= -2H_1 - 10H_2 - 8H_3 \\
\Delta W_3 &= -8H_3 - 10H_5 \\
\Delta W_4 &= -3H_2 - 12H_4 \\
\Delta W_5 &= 2H_2 + 2H_3 - 2H_4 - 8H_9 \\
\Delta W_6 &= -H_1 + 4H_2 + 4H_3 - 12H_4 - 2H_5 + 12H_9 \\
\Delta W_7 &= -3H_1 + 12H_2 + 12H_3 - 24H_4 - 3H_5 + 24H_9 \\
\Delta W_8 &= -\frac{2}{3}H_1 + 3H_2 + 3H_3 - 6H_4 - \frac{3}{4}H_5 \\
\Delta W_9 &= -2H_1 - 10H_6 - 10H_7 \\
\Delta W_{10} &= H_1 - 6H_2 - 4H_3 - 4H_4 - 3H_6 - H_7 - 4H_8 - 16H_9 \\
\Delta W_{11} &= 4H_1 - 12H_2 - 16H_3 + 16H_4 - 4H_5 - 2H_6 - 4H_8 - 32H_9 \\
\Delta W_{12} &= -H_1 - 5H_6 - 10H_8 \\
\Delta W_{13} &= -H_1 + 6H_2 + 4H_3 + 4H_4 + 3H_6 - H_7 + 8H_8 + 16H_9 \\
\Delta W_{14} &= \frac{1}{2}H_1 + H_2 - 2H_3 + 14H_4 + 5H_6 - \frac{3}{2}H_7 + 8H_8 + 8H_9 \\
\Delta W_{15} &= -4H_1 + 12H_2 + 16H_3 - 16H_4 + 4H_5 + 2H_6 - 2H_7 + 8H_8 + 32H_9 \\
\Delta W_{16} &= 0 \\
\Delta W_{17} &= 0.
\end{align*}$$

(33)

From this list one can read off the matrix $f^\alpha_i$ of eq. (10).
Appendix B

To find all trivial anomalies, we compute the Weyl variation of the most general dimensionless local functional of the metric

$$K = \sum_{i=1}^{10} c^i K_i \quad \text{where} \quad K_i = \int d^6 x \sqrt{g} K_i.$$  \hspace{1cm} (34)

Defining as before $\mathcal{W}_i = \int d^6 x \sqrt{g}(x) K_i$ we have

$$\delta_\sigma K_1 = -30 \mathcal{W}_6$$
$$\delta_\sigma K_2 = 4\mathcal{W}_9 - 20\mathcal{W}_{10} - 8\mathcal{W}_{12} - 20\mathcal{W}_{13} - 6\mathcal{W}_{16}$$
$$\delta_\sigma K_3 = 4\mathcal{W}_9 - 20\mathcal{W}_{11} - 8\mathcal{W}_{12} - 20\mathcal{W}_{15} - 4\mathcal{W}_{16}$$
$$\delta_\sigma K_4 = -12\mathcal{W}_4 - 12\mathcal{W}_5 + 3\mathcal{W}_9 - 6\mathcal{W}_{10} - 12\mathcal{W}_{12} - 6\mathcal{W}_{13} - 12\mathcal{W}_{14} - \frac{3}{2} \mathcal{W}_{16}$$
$$\delta_\sigma K_5 = -10\mathcal{W}_4 - 10\mathcal{W}_5 - 4\mathcal{W}_6 + 2\mathcal{W}_7 - 8\mathcal{W}_8 - \frac{1}{2} \mathcal{W}_9 + 12\mathcal{W}_{10} + 2\mathcal{W}_{11} - 4\mathcal{W}_{12} + 20\mathcal{W}_{13} - 18\mathcal{W}_{14} + \frac{3}{4} \mathcal{W}_{16}$$
$$\delta_\sigma K_6 = 6\mathcal{W}_6 - 3\mathcal{W}_7 + 12\mathcal{W}_8 + \mathcal{W}_9 - 4\mathcal{W}_{10} - 7\mathcal{W}_{11} - 2\mathcal{W}_{12} - 16\mathcal{W}_{13} + 12\mathcal{W}_{14}$$
$$-4\mathcal{W}_{15} - \frac{1}{2} \mathcal{W}_{16}$$
$$\delta_\sigma K_7 = 12\mathcal{W}_6 - 6\mathcal{W}_7 + 24\mathcal{W}_8 - 12\mathcal{W}_{10} - 12\mathcal{W}_{11} - 24\mathcal{W}_{13} + 24\mathcal{W}_{14} - 6\mathcal{W}_{15}$$
$$\delta_\sigma K_8 = -6\mathcal{W}_4 - 6\mathcal{W}_5 + 6\mathcal{W}_{10} - \frac{3}{2} \mathcal{W}_{11} - 3\mathcal{W}_{12} + 6\mathcal{W}_{13} - 6\mathcal{W}_{14} - \frac{3}{2} \mathcal{W}_{15}$$
$$\delta_\sigma K_9 = -2\mathcal{W}_{16} - 20\mathcal{W}_{17}$$
$$\delta_\sigma K_{10} = -20\mathcal{W}_4 - 20\mathcal{W}_5 - 8\mathcal{W}_6 + 4\mathcal{W}_7 - 16\mathcal{W}_8 + 3\mathcal{W}_9 + 4\mathcal{W}_{10} + 4\mathcal{W}_{11} - 16\mathcal{W}_{12} + 20\mathcal{W}_{13} - 36\mathcal{W}_{14} - \frac{3}{2} \mathcal{W}_{16} - 6\mathcal{W}_{17}.$$

Other variations are not necessary since by partial integration we find

$$K_11 = -4K_4 - 4K_5 + 2K_6 - K_7 + 4K_8 - K_9 + 4K_{10}$$
$$K_12 = \frac{1}{2} K_9$$
$$K_13 = -K_{10}$$
$$K_14 = -K_4 - K_5 - \frac{1}{4} K_9$$
$$K_15 = -K_{11}$$
$$K_{16} = K_{17} = 0.$$  \hspace{1cm} (36)

From the set of Weyl variations written above one can easily constructs the matrix $g^j_i$ of eq. (12). This matrix has rank 6 and so there are 6 independent trivial anomalies, which can be identified as the variations of $K_1, K_2, K_3, K_5, K_6, K_9$. 

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Appendix C

Here we solve the consistency conditions in eq. (36). They consist in the following homogeneous system of linear equations (same as eq. (10) of the main text)

$$\sum_{i=1}^{17} f_{\alpha i} a^i = 0, \quad \alpha = 1, \ldots, 9. \quad (37)$$

The $9 \times 17$ matrix $f_{\alpha i}$ can be constructed form the calculations reported in appendix A. It has rank 7 and so one has 10 independent solutions for the $a^i$. A possible choice for the parameters of the solution is: $a^6, a^7, a^8, a^{10}, a^{11}, a^{13}, a^{14}, a^{15}, a^{16}, a^{17}$. Thus, one can straightforwardly derive a suitable basis for the anomalies:

\begin{align*}
A_1 &= -\frac{21}{200}K_1 + \frac{43}{20}K_2 - \frac{1}{5}K_3 - \frac{5}{4}K_4 + \frac{3}{2}K_5 + K_6 \\
A_2 &= -\frac{27}{100}K_1 + \frac{51}{20}K_2 - \frac{3}{10}K_3 - \frac{5}{2}K_4 + 3K_5 + K_7 \\
A_3 &= -\frac{11}{200}K_1 + \frac{9}{20}K_2 - \frac{3}{10}K_3 - \frac{1}{2}K_4 + K_8 \\
A_4 &= \frac{3}{25}K_1 - K_2 - 2K_5 - \frac{1}{10}K_9 + K_{10} - \frac{2}{5}K_{12} \\
A_5 &= \frac{8}{25}K_1 - \frac{13}{5}K_2 - \frac{2}{5}K_3 + 2K_4 - 4K_5 + K_{11} - \frac{2}{5}K_{12} \\
A_6 &= -\frac{3}{25}K_1 + K_2 + 2K_5 - \frac{1}{10}K_9 + \frac{4}{5}K_{12} + K_{13} \\
A_7 &= K_1 + K_5 - \frac{1}{10}K_9 + \frac{2}{5}K_{12} + K_{14} \\
A_8 &= -\frac{8}{25}K_1 + \frac{12}{5}K_2 + \frac{2}{5}K_3 - 2K_4 + 4K_5 - \frac{1}{5}K_9 + \frac{4}{5}K_{12} + K_{15} \\
A_9 &= K_{16} \\
A_{10} &= K_{17}.
\end{align*}

Now, in appendix B we have identified a basis of trivial anomalies. One can use that knowledge to make a change of basis and separate the 6 trivial anomalies from the non-trivial ones. We have chosen the latter to agree with those of ref. [21] obtaining

\begin{align*}
M_1 &= -\frac{3}{4}A_1 + A_3, \quad M_2 = -3A_1 + A_2, \quad M_3 = 5A_4 - 5A_5, \\
M_4 &= 24A_1 - 4A_2 - 8A_3, \\
M_5 &= 6A_1 - 3A_2 + 12A_3 + A_4 - 7A_5 - 11A_6 + 12A_7 - 4A_8, \\
M_6 &= A_4 + A_6, \quad M_7 = A_7, \quad M_8 = A_8 + A_5, \quad M_9 = A_9, \quad M_{10} = A_{10}.
\end{align*}

This is the basis reported in eqs. (16-22). The last six elements $M_5, \ldots, M_{10}$ are trivial and one can easily identify the local functionals generating them. Defining

$$\mathcal{M}_i = \int d^6x \sqrt{g} \sigma(x) M_i, \quad (40)$$

we have

\begin{align*}
\mathcal{M}_5 &= \delta_{\sigma} \left( \frac{1}{30} K_1 - \frac{1}{3} K_2 + K_6 \right), \quad \mathcal{M}_6 = \delta_{\sigma} \left( \frac{1}{100} K_1 - \frac{1}{20} K_2 \right), \\
\mathcal{M}_7 &= \delta_{\sigma} \left( \frac{37}{6000} K_1 - \frac{7}{150} K_2 + \frac{1}{75} K_3 - \frac{1}{10} K_5 - \frac{1}{15} K_6 \right), \quad \mathcal{M}_8 = \delta_{\sigma} \left( \frac{1}{150} K_1 - \frac{1}{20} K_3 \right), \\
\mathcal{M}_9 &= \delta_{\sigma} \left( -\frac{1}{30} K_1 \right), \quad \mathcal{M}_{10} = \delta_{\sigma} \left( \frac{1}{300} K_1 - \frac{1}{20} K_9 \right).
\end{align*}

\[12\]
Appendix D

In [13] (and in [32]) the following basis of invariants was also used

\[ B_1 = \nabla^4 R \]
\[ B_2 = (\nabla_a R)^2 \]
\[ B_3 = (\nabla_a B_{mn})^2 \]
\[ B_4 = \nabla_a B_{bn} \nabla^b B_{am} \]
\[ B_5 = (\nabla_i C_{abmn})^2 \]
\[ B_6 = R \nabla^2 R \]
\[ B_7 = B_{ab} \nabla^2 B^{ab} \]
\[ B_8 = B_{ab} \nabla_m \nabla^b B_{am} \]
\[ B_9 = C_{abmn} \nabla^2 C_{abmn} \]
\[ B_{10} = R^3 \]
\[ B_{11} = R B_{ab}^2 \]
\[ B_{12} = R C_{abmn}^2 \]
\[ B_{13} = B_{a}^{m} B_{m}^{i} B_{i}^{a} \]
\[ B_{14} = B_{ab} B_{mn} C_{ambn} \]
\[ B_{15} = B_{ab} C_{amnl} C_{b mnl} \]
\[ B_{16} = C_{ab}^{mn} C_{mn}^{ij} C_{ij}^{ab} \]
\[ B_{17} = C_{ambn} C_{aijb} C_{m n a} \]

where \( C_{abcd} \) is the Weyl tensor defined in (27) and \( B_{ab} \) is the traceless part of the Ricci tensor

\[ B_{ab} = R_{ab} - \frac{1}{6} R g_{ab}. \]  

The trivial anomalies in [13] are expressed using the following set of total derivatives [32]

\[ C_1 = B_1, \quad C_2 = B_2 + B_6, \quad C_3 = B_3 + B_7, \quad C_4 = B_4 + B_8, \quad C_5 = B_5 + B_9, \]
\[ C_6 = \frac{1}{5} B_2 - B_1 - \frac{1}{5} B_{11} - \frac{3}{2} B_{13} + B_{14}, \]
\[ C_7 = \frac{1}{60} B_2 - \frac{3}{4} B_3 + \frac{3}{4} B_4 + \frac{1}{4} B_5 + \frac{1}{12} B_{12} + \frac{1}{4} B_{15} - \frac{1}{4} B_{16} - B_{17}. \]  

However, these total derivatives do not form a subset of the space of trivial anomalies. In fact we find

\[ C_1 = M_{10}, \quad C_2 = \frac{1}{2} M_9, \quad C_3 = -\frac{1}{12} M_9 + (K_{10} + K_{13}), \]
\[ C_4 = -\frac{7}{6} M_6 + M_7 - \frac{5}{72} M_9 + \frac{7}{6} (K_{10} + K_{13}), \quad C_5 = -M_6 + M_8 + \frac{1}{20} M_9, \]
\[ C_6 = 2M_6 - M_7 + \frac{1}{8} M_9 - 2(K_{10} + K_{13}), \quad C_7 = \frac{1}{12} M_5 - \frac{1}{12} M_6 - \frac{1}{4} M_7 + \frac{7}{12} M_8 + \frac{1}{32} M_9. \]  

Because of the term \((K_{10} + K_{13})\) the space spanned by the \( C_i \) is larger than the space of trivial anomalies. The latter is expressed by

\[ \sum_{i=1}^{7} a_i^i C_i \quad \text{is a trivial anomaly iff} \quad 6a_3 + 7a_4 - 12a_6 = 0. \]  

In [13] all the linear combinations of \( C_i \) satisfy the above relation, i.e. are trivial anomalies. This is a good check on the correctness of the anomaly computations performed there.
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