Multi-objective optimal sliding mode control design of active suspension system with MOPSO algorithm

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Abstract. In order to improve the ride comfort and driving stability of the vehicle, a design method of sliding mode controller based on multi-objective particle swarm optimization algorithm is proposed for the vehicle active suspension. Based on the model of 1/4 vehicle active suspension system, the sliding mode controller is designed by using sliding mode control method. The stability of the sliding mode surface is verified by using Lyapunov stability theory. The stability conditions of the sliding mode surface are obtained by combining Hurwitz stability criterion. The particle swarm optimization algorithm is used to optimize the multi-objective optimization of the active suspension and the numerical simulation is carried out. The results show that after multi-objective parameter optimization, the ride comfort and driving stability of the vehicle are significantly improved, which provides a theoretical basis for the research of the vehicle active suspension system.

1. Introduction
Active suspension is the main development direction of future automobile suspension and one of the main symbols of modern automobile. Compared with traditional passive suspension, active suspension can be in the best state of vibration reduction and plays an important role in the further improvement of automobile performance [1-3]. In recent years, classical control methods have been widely used in automotive active suspension control, such as LQR control, PID control, fuzzy control, adaptive control and sliding mode control [4-7]. Among them, the sliding mode control is used to deal with the nonlinearity of the system, the uncertainty of the model and the external disturbance, and has strong robustness [8-9].

At present, some random search methods are widely used in the optimization design of automotive suspensions, such as particle swarm optimization, genetic algorithm, simulated annealing algorithm, et al [11-12]. Particle swarm optimization is a random optimization algorithm based on swarm intelligence. Compared with the widely used genetic algorithm, it has the advantages of easy realization, high precision, fast convergence, etc. The algorithm has many advantages, such as high search efficiency, multi-variable, non-linear and discontinuous problems [13-16].

The main means to improve the ride comfort of the vehicle is to control the vertical acceleration of the vehicle body. However, the vertical acceleration of the vehicle conflicts with the suspension travel and the wheel dynamic load. In order to coordinate the contradiction between the three targets, the multi-objective optimization control method has been extensively and deeply studied [17-19].

In order to further improve the vibration damping performance of active suspension, based on the
previous work, this paper takes a two-degree-of-freedom 1/4 automotive nonlinear active suspension system as the research model, and derives the sliding mode based on sliding mode control and Lyapunov stability conditions. The controller selects the controller coefficients according to the Hurwitz criterion of the stability of the equation of motion, and uses the particle swarm optimization algorithm to optimize the parameters of the sliding mode controller.

2. Nonlinear vehicle active suspension system model

This paper uses a 1/4 vehicle nonlinear suspension model of the body and wheel two-degree-of-freedom vibration system, as shown in Figure 1.

The tire in the figure is simplified as a constant coefficient spring $k$ and a constant coefficient damping $c$. The suspension consists of spring $k$, damping $c$ and active controller $u$, $m_1$ and $m_2$ are sprung mass and unsprung mass, $x_s$, $x_u$ and $q$ respectively indicates the body displacement, tire displacement and random road roughness input when the suspension spring is not deformed, and $x_1$ and $x_2$ are the displacements of the suspension from the equilibrium position.

According to Newton's second law, the differential equation of the suspension system can be established. The dynamic model of the suspension system is as follows

$$\begin{align*}
\dot{m}_1 \ddot{x}_1 &= -k_{11} (x_1 - x_u) - k_{12} (x_1 - x_s) - c (\dot{x}_1 - \dot{x}_s) - m_1 g + u \\
\dot{m}_2 \ddot{x}_2 &= k_{11} (x_1 - x_u) + k_{12} (x_1 - x_s) + c (\dot{x}_1 - \dot{x}_s) - k_2 (x_2 - q) - c (\dot{x}_2 - \dot{q}) - m_2 g - u
\end{align*}$$

(1)

Where $k_{11}$ and $k_{12}$ are the linear stiffness and nonlinear stiffness of the suspension spring, $g$ is acceleration due to gravity, the total pressure received by the tire during static is $(m_1 + m_2)g$, denoted by $x_{uo}$ is given by

$$x_{uo} = \frac{(m_1 + m_u)g}{k_2}$$

(2)

Considering the reference position $x_u$ of the unsprung mass, the reference position $x_v$ of the sprung mass is defined as

$$x_v = x_{uo} + \delta_0$$

(3)

Where $\delta_0 < 0$ is the static spring deflection. Define the state variable of the suspension system model as

$$\mathbf{x} = [x_1, x_2, x_u, x_v]$$

(4)

$$\dot{x}_1 = x_1 - x_u, \quad \dot{x}_2 = \dot{x}_1, \quad \dot{x}_u = \dot{x}_u - x_{uo}, \quad \dot{x}_v = \dot{x}_u$$

(5)

The equation of state of the system is given by
\[
\begin{align*}
\dot{x}_1 &= x_1 \\
\dot{x}_2 &= f_2(x) - b_2 f(x) + b u \\
\dot{x}_3 &= x_3 \\
\dot{x}_4 &= f_4(x) + b_4 f(x) + k_b b q + c_4 b \dot{q} - b u
\end{align*}
\]

Where

\[
f_2(x) = b_2 \left[ -k_1 (x - x_1 + \delta) - c_1 (x_1 - x_2) - m_2 g, \right] \quad f_4(x) = b_4 \left[ k_3 (x_3 - x_4 + \delta_3) + c_3 (x_3 - x_2) - k_3 (x_3 + x_4) - c_3 x_2 - m_3 g \right]
\]

3. Sliding mode control design based on Lyapunov theory

Define the tracking error of the two states as

\[
\begin{align*}
\epsilon_1(t) &= x_1(t) - x_{1r}(t) \\
\epsilon_2(t) &= x_2(t) - x_{2r}(t)
\end{align*}
\]

Where \( x_{1r}(t) \) and \( x_{2r}(t) \) are the desired reference trajectories of \( x_1(t) \) and \( x_2(t) \), and the switching function of the sliding mode control is expressed as follows

\[
s(t) = \dot{s}(t) = \dot{\lambda}_1 \epsilon_1(t) + \dot{\lambda}_2 \epsilon_2(t) + \mu_1 \dot{\epsilon}_1(t) + \mu_2 \dot{\epsilon}_2(t)
\]

Where \( \dot{\lambda}_1, \dot{\lambda}_2, \mu_1 \) and \( \mu_2 \) are slip surface parameters. These parameters will be determined later in the paper.

Satisfying \( s(t) = 0 \) and \( \dot{s}(t) = 0 \) on the sliding surface, then

\[
\dot{s}(t) = \ddot{\lambda}_1 \epsilon_1(t) + \ddot{\lambda}_2 \epsilon_2(t) + \mu_1 \dot{\epsilon}_1(t) + \mu_2 \dot{\epsilon}_2(t) = 0
\]

Where \( \sigma = \frac{1}{\mu_1 b_1 - \mu_2 b_2} \), \( \rho_2 = \dot{\lambda}_2 \dot{\epsilon}_2(t) + \mu_2 \dot{\epsilon}_2(t) \).

Introducing a robust control rate to the handover control

\[
u = u_a + u_m = -\sigma [\dot{\lambda}_1 x_1 + \dot{\lambda}_2 x_2 + \mu_1 f_1(x) + \mu_2 f_2(x) - b_2 \dot{f}(x) + b_1 \dot{f}(x) + k_b b \dot{q} + c_4 b \dot{q} - \rho_s + K \text{sign}(s)]
\]

Where \( \dot{f}(x), \dot{q} \) and \( \dot{\dot{q}} \) are the estimated values of \( f(x), q \) and \( \dot{q} \).

It is assumed that the estimation of the nonlinear suspension stiffness and the random road surface roughness input estimation error function are bounded again, and the limit is as shown in the following equation.

\[
\left| f(x) - \hat{f}(x) \right| \leq F_1, \quad \left| q - \hat{q} \right| \leq F_2, \quad \left| \dot{q} - \hat{\dot{q}} \right| \leq F_3
\]

Where \( F_1 = a k_2 (x_2 - x_1 + \delta_3)^2, F_2 = a, \) and \( F_3 = b \) are the error limits of the estimated \( f(x), q \), and \( \dot{q} \).

The determination of the error limit of the nonlinear spring stiffness estimation is based on the linear part system model of the equation, \( a \) is the nonlinear stiffness control coefficients, and the estimated error limit of the random road roughness input is assumed by the pavement function, \( a \) and \( b \) respectively. The extremum of the pavement function and the pavement function derivative.

In order to make the design control law ensure that the system can reach the sliding surface in any initial state, that is, the arrival condition of the sliding mode control is satisfied. Define a Lyapunov function in terms of the sliding surface:

\[
V(x) = \frac{1}{2} s^2(t)
\]

Hence, we have

\[
\dot{V}(x) = s(t) s(t) \leq \left( (\mu_1 b_2 - \mu_2 b_1) F_{11} + k_b b b_{11} + c_4 b b_{33} - K \right) |s(t)|
\]

When the value of \( K \) in the above formula is as follows, \( V(x) \leq -\eta |s(t)| \) is established.
Besides, the sliding surface should meet the following reaching condition and sliding conditions

\[\sigma(b_2\mu c_1 - b_1\lambda_2 + b_1\lambda_2) > 0\]
\[\sigma(b_2\lambda_1 c_2 + b_2\mu k_1) > 0\]
\[\sigma(b_2\lambda_2 k_2) > 0\]
\[\sigma(b_2\mu c_1 - b_1\lambda_2 + b_1\lambda_2)(\lambda_2 c_2 + \mu k_2) > \lambda_2 k_2\]

The reader can find the details of these four inequations from the appendix. They will serve as the inequation conditions to constraint the design space in the following MOP design.

In actual control, in order to avoid high frequency vibration, the saturation function \(\text{sat}(s(t), \phi)\) is substituted for the symbol function \(\text{sign}(s)\) and the saturation function is as follows

\[\text{sat}(s(t), \phi) = \begin{cases} 1, & s(t) > \phi \\ \frac{s(t)}{\phi}, & -\phi < s(t) < \phi, \phi > 0 \\ -1, & s(t) < -\phi \end{cases}\]

Where \(0 < \phi < 1\) is the boundary layer thickness of the switch control term, and the stability control of the saturation function can be proved in the same way.

4. Multi-objective optimization design

4.1 Target function selection
The ride comfort indicators in the design of the vehicle suspension are respectively the vertical acceleration of the vehicle body representing the ride comfort and the suspension travel motion which affects the vehicle body posture and is related to the structural design and arrangement. The driving stability index is a tire dynamic load representing the grounding property of the tire. The objective function is shown in equation (18).

\[f_1 = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \ddot{x}_i(t)}\]
\[f_2 = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i(t) - x_i(t-1))^2}\]
\[f_3 = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (k_i(x_i(t) + x_i(t) - q(t)) + c_i(x_i(t) - \ddot{q}(t)))}\]

4.2 Proposal for multi-objective optimization problems
The control performance and stability are determined by the 6 parameters \(K = [\lambda_1, \mu_1, \lambda_2, \mu_2, \eta, \phi]\) in the sliding mode controller. The values of these parameters are selected to satisfy the 3 targets. At the same time, the multi-objective optimization design problem can be written as follows by the 6 parameters of the sliding mode control.

\[\min_{K \in Q} \{F(K)\} = \min_{K \in Q} \{f_1, f_2, f_3\}\]

\[Q = \{K \in R^6 | g(K) > 0\}\]

Where \(g(K) > 0\) denote inequality constraints in vector format.

4.3 Multi-objective particle swarm optimization algorithm
In the process of optimization, the particle swarm optimization algorithm uses the fitness of each particle as the criterion to distinguish the advantages and disadvantages of the particle. The appropriate fitness function can significantly increase the convergence speed of the algorithm. The objective
function determined by the active suspension optimization problem is to use the minimum value of the target as the optimization strategy. Therefore, the multi-objective function can be directly converted into the fitness function to obtain better results. The fitness function can be expressed as the following formula.

$$\text{Fit}(K) = \min \{ f_1, f_2, f_3 \}$$

(21)

The particle velocity and position update formula is as follows:

$$V_i(t+1) = \omega V_i(t) + c_1 r_1(P_i(t) - X_i(t)) + c_2 r_2(P_{opt}(t) - X_i(t))$$

$$X_i(t+1) = X_i(t) + V_i(t + 1)$$

(22)

Where $\omega$ is the inertia weight, $r_1$ and $r_2$ are the random numbers distributed in the interval $[0,1]$, $t$ is the current iteration number, $P_i$ is the individual optimal particle position, $P_{opt}$ is the global optimal particle position, and $c_1$, $c_2$ are the acceleration Factor, $V_i$ is the particle velocity, and $X_i$ is the particle position.

The initial population is set to 100, the maximum number of iterations is 200, the inertia weight $\omega_{max}=0.7$, $\omega_{min}=0.4$, the acceleration factor $c_1=c_2=1$, and the simulation program is built under Matlab for simulation. The simulation time is 15s. Figure 3 is the multi-objective optimal solution distribution map. Where $b_1, b_2, b_3, b_4$ are the positions corresponding to the targets $f_1$, $f_2$, and $f_3$ in the multi-objective optimization solution set under different design requirements are respectively shown in Table 1.

| Serial number | Design requirements |
|---------------|---------------------|
| SMC           | General Sliding Mode Control |
| PSOS$_1$      | Target $f_1$ optimal |
| PSOS$_2$      | Target $f_2$ optimal |
| PSOS$_3$      | Target $f_3$ optimal |
| PSOS$_4$      | 3-objective synthesis optimum |

5. Simulation results analysis

According to the established models and algorithms, the example simulation analysis is carried out. This paper uses the car suspension system and road conditions given in the literature [19] as an example, including the vehicle model parameters and the pavement function, to prove the correctness and feasibility of the active suspension sliding mode controller based on particle swarm optimization. Vehicle model parameters are as follows:

Parameters of plant: $m_i=290 \text{ kg}$, $m_s=59 \text{ kg}$, $k_i=14500 \text{ N/m}$, $k_{ii}=160000 \text{ N/m}$, $k_s=190000 \text{ N/m}$,
During the simulation, the road surface condition is determined by the road function (23).

\[\begin{align*}
&-0.0592r_i^2 + 0.1332r_i^2 + d(t), \quad 3.5 \leq t < 5 \\
&0.0592r_i^2 + 0.1332r_i^2 + d(t), \quad 5 \leq t < 6.5 \\
&0.0592r_i^2 - 0.1332r_i^2 + d(t), \quad 8.5 \leq t < 10 \\
&-0.0592r_i^2 + 0.1332r_i^2 + d(t), \quad 10 \leq t < 11.5 \\
&d(t), \quad \text{else}
\end{align*}\]

Based on the optimal controller and multi-objective optimization solution set obtained in the previous section, the simulation is performed in Matlab. In order to select a satisfactory solution, three targets need to be decided. Table 3 lists the comparison of controller parameters between the active suspension and the optimized active suspension under 4 different design requirements. Figure 3-6 shows the comparison of the vertical acceleration, suspension travel, tire dynamic load and controller control force between the active suspension and the optimized active suspension under 4 different design requirements. Figure 3 and Figure 5 are the Fourier contrast curves of the vertical acceleration of the vehicle body and the dynamic load of the tire.

**Table 2. Sliding Mode Controller parameters**

| parameter | SMC  | PSOS, | PSOS, | PSOS, | PSOS, |
|-----------|------|-------|-------|-------|-------|
| $\lambda_1$ | 800  | 684.1 | 6499.4| 602.9 | 5796.2|
| $\mu_1$    | 15   | 7348.9| 2984.8| 5677.2| 7154.4|
| $\lambda_2$| 40   | 134.5 | 1060.1| 1128.6| 557.5 |
| $\mu_2$    | 0.5  | 34.1  | 349.1 | 1066.2| 360.4 |
| $\eta$     | 100  | 682.2 | 4366.7| 1960.7| 1373.3|
| $\phi$     | 30   | 29.8  | 883.3 | 215.2 | 763.5 |

**Figure 3. Vertical acceleration Fourier contrast curve**

As can be seen from Figure 4, comparing the five control strategies, the vertical acceleration of the vehicle body is most effectively controlled with PSOS control, while the worst effect is obtained with PSOS control. However, as can be seen from Figure 4 below, when using PSOS and PSOS control, the suspension travel is much larger than the other three control strategies. It can also be seen from Figure 4 -5 that when the PSOS control is used, the vertical acceleration of the vehicle body and the suspension travel are effectively controlled, and the control effect is better than the SMC control.

Similarly, it can be seen from Figure 5-6 that when the single-objective optimization method PSOS is adopted, the best effect on the dynamic load control of the tire is obtained. However, higher requirements are placed on the acceleration of the vehicle body and control force. Especially in terms...
of control force, the control force required by PSOS is as high as 2000N, which is a very high challenge for the active suspension. However, as can be seen from Figure 6 below, when using PSOS and PSOS control, the control force is about 1000, and control effects are better than SMC control, which is relatively easy to implement.

In summary, the multi-objective optimization control method has a certain degree of improvement in vertical acceleration, suspension travel and tire dynamic load, and the control force is significantly reduced. It can also be seen from Table 4 that compared with the active suspension before optimization, the rms value of the vertical acceleration of the vehicle body is reduced by 24.3%, the suspension travel is kept within the range of the suspension dynamic deflection. The rms value is reduced10.4%, which effectively improves the ride comfort of the car. The root mean square value of the dynamic load of the tire is reduced by 36.9%, which effectively improves the grip ability of the car tire and reduces the generation of lateral force, thereby improving the vertical Driving stability.

Table 4. Simulation results

| Target value | SMC | PSOS | PSOS | PSOS | PSOS |
|--------------|-----|------|------|------|------|
| $f_1$ / m    | 0.1097 | 0.0123 | 0.1747 | 0.2311 | 0.0830 |
| $f_2$ / m    | 0.0316 | 0.0401 | 0.0187 | 0.0451 | 0.0283 |
| $f_3$ / N    | 83.25 | 61.48 | 46.85 | 6.93 | 52.49 |

6. Conclusion

(1) This paper combines Lyapunov stability theory and Hurwitz stability criterion, and uses multi-objective particle swarm optimization to design and optimize the sliding mode controller of the non-linear active suspension, which provides a new optimization method for the selection of control parameters of the sliding mode controller of the active suspension.

(2) Through the analysis, the performance of optimized sliding mode control active suspension by multi-objective particle swarm optimization algorithm is better than that sliding mode control active suspension. The vertical acceleration of the vehicle body is significantly reduced, and the suspension travel is reduced to a certain extent while maintaining the range of the dynamic deflection, which improves the ride comfort of the automobile.

(3) Through the analysis, the dynamic load of the tire is significantly reduced, which enhances the ground adhesion of the automobile, reduces the possibility of the tire coming off the ground and the generation of the lateral force, and improves the running stability of the automobile.

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Appendix
In order for the system to slide to the origin, the sliding surface must satisfy the Hurwitz stability. On the sliding surface \( s(t) = 0 \), so we can ask for (24).

\[
\dot{\delta}_i(t) = \delta_i e_i(t) + \delta_i \dot{e}_i(t) + \delta_i \ddot{e}_i(t) \tag{24}
\]

Where \( \delta_i = -\frac{\dot{\lambda}_i}{\mu_i}, \delta_i = -\frac{\mu_i}{\mu_i}, \delta_i = -\frac{\dot{\lambda}_i}{\mu_i} \).

Bring the equivalent control \( u_{eq} \) into the third line of equation (6) and make a differential,

\[
\dot{\xi}_i = \dot{\lambda}_i = \frac{f(x) + f(x) - b \mu(x)}{\mu_i} \sigma \left[ \dot{\lambda}_i x_i + \mu_i f_1 + \lambda_i \dot{x}_i + \mu_i f_2 - \rho_i - b \mu x_i (x_i - x_i + \delta_i)^3 + b \mu x_i (x_i - x_i + \delta_i)^3 \right]
\]

\[
= \xi_i \dot{x}_i + \xi_i \dot{x}_i + \xi_i \dot{x}_i + \xi_i \dot{x}_i + \xi_i \dot{x}_i + \xi_i \dot{x}_i (x_i - x_i + \delta_i)^3
\]

Where \( \dot{\xi}_i = 0, \xi_i = \sigma \mu k_1 \), \( \xi_i = -\sigma \mu k_1 \), \( \xi_i = -\sigma \mu k_1 \), \( \xi_i = 0 \).

Also because of \( x_{eq} = 0, \dot{x}_{eq} = 0, \dot{x}_{eq} = 0, e_i = x_i, e_i = x_i, \xi_i = \lambda_i, \xi_i = \lambda_i, \), so equation (25) can also be written as

\[
\dot{\xi}_i = \xi_i \dot{e}_i(t) + \xi_i \dot{e}_i(t) + \xi_i \dot{e}_i(t) + \xi_i \dot{e}_i(t) + \xi_i \dot{e}_i(t) + \xi_i \dot{e}_i(t) (x_i - x_i + \delta_i)^3
\]

\[
= \beta_i e_i(t) + \beta_i \dot{e}_i(t) + \beta_i \ddot{e}_i(t) + \xi_i \dot{k}_i (x_i - x_i + \delta_i)^3
\]

Where \( \beta_i = -\sigma b_i \), \( \beta_i = -\sigma b_i \), \( \beta_i = -\sigma b_i \), \( \beta_i = -\sigma b_i \).

Define a new set of state variables

\[
y = \left[ y_1, y_2, y_3 \right] = \left[ e_i(t), \dot{e}_i(t), \ddot{e}_i(t) \right]
\]

From the new state variable, equation (26) can be written as,

\[
\dot{y}_1 = y_2
\]

\[
\dot{y}_2 = \beta_i y_1 + \beta_i y_2 + \beta_i y_3 + \xi_i \dot{k}_i (x_i - x_i + \delta_i)^3
\]

The equilibrium condition of equation (28) can be obtained as \( y_i = y_i = 0 \). Localize the equation locally at the origin is given by

\[
\dot{y}_i = \beta_i y_1 + \beta_i y_2 + \beta_i y_3, \dot{y}_i = \beta_i y_1 + \beta_i y_2 + \beta_i y_3
\]

The characteristic equation of the linearization system is given by

\[
y^\prime + \varepsilon_i y^\prime + \varepsilon_i y^\prime + \varepsilon_i = 0
\]

Where \( \varepsilon_i \) is a Laplace variable, and

\[
e_i = -\beta_i \delta_i, \quad e_i = \beta_i \delta_i, \quad e_i = \beta_i \delta_i, \quad e_i = \beta_i \delta_i
\]

Available from Hurwitz stability

\[
e_i > 0, \quad e_i > 0, \quad e_i > 0, \quad e_i, e_i
\]

We can derive the sliding surface stability condition

\[
\sigma b_i \beta_i k_i > 0
\]

\[
\sigma b_i \beta_i k_i > 0
\]

\[
\sigma b_i \beta_i k_i > 0
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\sigma b_i \beta_i k_i > 0
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\sigma b_i \beta_i k_i > 0
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\sigma b_i \beta_i k_i > 0
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\sigma b_i \beta_i k_i > 0
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