Braneworld inflation driven by dynamics of a bulk scalar field

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We review a viable alternative scenario of the inflationary universe in the context of the Randall-Sundrum (RS) braneworld. In this scenario, the dynamics of a 5-dimensional scalar field, which we call a bulk scalar field, plays the central role. Focusing on the second (single-brane) RS model, we discuss braneworld inflation driven by a bulk scalar field without introducing an inflaton on the brane. As a toy model, for the bulk scalar field, we consider a minimally coupled massive scalar field in the 5-dimensional spacetime, and look for a perturbative solution of the field equation in the anti-de Sitter background with an inflating brane. For a suitable range of the model parameters, we find a solution that realizes slow-roll inflation on the brane. When the Hubble parameter on the brane and the mass of a bulk scalar field are much smaller than a typical 5-dimensional mass scale, it is found that this proposed inflation scenario reproduces the standard inflation scenario in the 4-dimensional theory.

§1. Introduction

It is very likely that our four-dimensional universe is a subspace of a higher-dimensional spacetime. In fact, string theory, which is a candidate for the unified theory, is a higher-dimensional theory. Therefore it is of great interest to develop a higher dimensional cosmological scenario which is consistent with existing observational data and which predicts new phenomena that can be experimentally or observationally tested. In particular, the braneworld scenario has attracted much attention and a model proposed by Randall and Sundrum (RS2) ignited active research of braneworld cosmology.

As an alternative to the standard 4-dimensional theory of inflation, we recently proposed a brane cosmological model in which slow-roll inflation is driven not by an inflaton on the brane but solely by dynamics of a scalar field living in the 5-dimensional bulk. The scalar field we introduced in Ref. 4) should probably be a dilaton-like gravitational field from the unified theoretical point of view. In fact, it is natural that the 5-dimensional theory is itself an effective theory which originates from a yet higher-dimensional theory, and the 5-dimensional effective action includes some scalar fields of gravitational origin. Provided there is a bulk scalar field with a suitable potential, it was shown in Ref. 4) that there exists a field configuration in the bulk that indeed realizes inflation on the brane. In particular, we found that the standard inflationary cosmology is reproduced, when $|m^2|\ell^2 \ll 1$ and $H^2\ell^2 \ll 1$ are satisfied, where $\ell$ is the curvature radius of AdS$_5$, $m$ is the mass of the bulk scalar field, and $H$ is the Hubble parameter on the brane.

This paper is organized as follows. In §2, we review the basic picture of the

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inflation scenario driven by a bulk scalar field. In §3, we investigate the behavior of the bulk scalar field on the inflating braneworld and discuss the effective description of the dynamics from the 4-dimensional point of view. Section 4 is devoted to conclusion.

§2. Braneworld inflation without inflaton on the brane

Based on the braneworld scenario of the RS2 type, we consider a 5-dimensional bulk with a single positive tension brane which is located at the fixed point of the \( Z_2 \) symmetry. We assume that the 5-dimensional gravitational equations are

\[ R_{ab} - \frac{1}{2} g_{ab} R + \Lambda_5 g_{ab} = \kappa_5^2 (T_{ab} + S_{ab} \delta(r - r_0)) , \]  

(2.1)

where \( r \) is the coordinate normal to the brane and the brane is assumed to be located at \( r = r_0 \). As for \( S_{ab} \) and \( T_{ab} \), we neglect the contribution to \( S_{ab} \) from matter fields confined on the brane and consider a minimally coupled bulk scalar field with the potential \( V(\phi) \). Thus we have

\[ S_{ab} = -\sigma q_{ab} , \quad \text{and} \quad T_{ab} = \phi, a \phi, b - g_{ab} \left( \frac{1}{2} g^{cd} \phi, c \phi, d + V(\phi) \right) , \]  

(2.2)

where \( \sigma \) is the tension of the brane and \( q_{ab} \) is the induced metric on the brane. In order to recover the Randall-Sundrum flat braneworld when \( T_{ab} \) vanishes, we choose \( \Lambda_5 \) as

\[ \Lambda_5 = -\frac{\kappa_5^4}{\kappa_4^2} \frac{\sigma}{6} . \]  

(2.3)

Then, the effective 4-dimensional Einstein equations on the brane become

\[ G_{\mu\nu} = \kappa_4^2 T_{\mu\nu}^{(s)} - E_{\mu\nu} , \]  

(2.4)

where

\[ T_{\mu\nu}^{(s)} = \frac{\ell_0}{6} \left( 4 \phi,_{\mu} \phi,_{\nu} + \left( \frac{3}{2} (\phi, r)^2 - \frac{5}{2} q^{\alpha\beta} \phi,_{\alpha} \phi,_{\beta} - 3V(\phi) \right) q_{\mu\nu} \right) , \]  

(2.5)

\[ E_{\mu\nu} = (5) C_{\mu\nu} q_{\rho\sigma} q^{\rho\sigma} . \]  

(2.6)

Here \( \ell_0 = 6/(\kappa_5^2 \sigma) \) is the AdS\(_5\) curvature radius and \((5) C_{\mu\nu}\) is the 5-dimensional Weyl tensor with its two indices projected in the \( r \)-direction.

Focusing on the zeroth order description of the cosmological model, we consider the case in which the metric induced on the brane is isotropic and homogeneous. Because of the assumed \( Z_2 \) symmetry, the boundary condition for the bulk scalar field at the position of the brane is given by

\[ \partial_r \phi |_{r=r_0} = 0 . \]  

(2.7)

*) For simplicity, we do not consider a possible coupling of \( \phi \) to the metric on the brane, though an extension to such a case may be worth investigating in the future.
Then, the 4-dimensional effective Friedmann equation is given by

\[ 3 \left( \frac{\dot{a}}{a} \right)^2 + \frac{K}{a^2} = 3H^2 = \kappa_4^2 \rho_{\text{eff}}, \tag{2.8} \]

with

\[ \rho_{\text{eff}} = \frac{\ell_0}{2} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right) - \frac{\ell_0}{\kappa_5^2} E_{tt}, \tag{2.9} \]

where \( a(t) \) is the cosmological scale factor of the brane and \( K = \pm 1, 0 \). The equations for \( \phi \) and \( E_{tt} \), are basically 5-dimensional. However, in the present spatially homogeneous case, the Bianchi identities supply the evolution equation of \( E_{tt} \) on the brane as \(^4\)

\[ E_{tt} = \frac{\kappa_5^2}{2a^4} \int^t a^4 \dot{\phi} (\partial_r^2 \phi + \frac{\dot{a}}{a} \dot{\phi}) dt. \tag{2.10} \]

We then see that \( E_{tt} \) can be neglected if both \( \dot{\phi} \) and \( \partial_r^2 \phi \) are sufficiently small on the brane. Thus a sufficient condition for inflation to occur on the brane is that \( \phi \) is a slowly varying function with respect to both \( t \) and \( r \) in the vicinity of the brane.

### 2.1. The field equation in the de Sitter brane background

Our purpose is to find a solution of the field equations that has nontrivial dynamics in the bulk and gives rise to inflation on the brane. For this purpose, we investigate the general behavior of the solution of 5-dimensional scalar field equation in AdS\(_5\) background with an inflating brane.

As a toy model, we assume the potential of the form,

\[ V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2, \tag{2.11} \]

and consider the region \( |m^2| \phi^2 / 2 \ll V_0 \). Here we do not specify the signature of \( m^2 \), though \( m^2 \) should be negative for inflation to end if the model should describe the braneworld inflation self-consistently. Then, the effective 5-dimensional cosmological constant becomes

\[ \Lambda_{5, \text{eff}} = \Lambda_5 + \kappa_5^2 V_0. \tag{2.12} \]

Here we assume that \( |\Lambda_5| > \kappa_5^2 V_0 \) so that the background spacetime is still effectively AdS\(_5\) with the effective curvature radius \( \ell = \sqrt{6/\Lambda_{5, \text{eff}}} \). Note that \( \ell > \ell_0 \) where \( \ell_0 = \sqrt{6/\Lambda_5} \). Then the bulk metric may be written as \(^9\)

\[ ds^2 = dr^2 + (H\ell)^2 \sinh^2 (r/\ell) [-dt^2 + H^{-2}e^{2Ht} dx^2]. \tag{2.13} \]

Here we have adopted the spatially flat slicing \( (K = 0) \) of de Sitter space for simplicity. Note that the Friedmann equation (2.8) at the lowest order determines the Hubble parameter \( H \) as \( H^2 = \kappa_5^2 V_0/6 \).

At the first order in the amplitude of \( \phi \), the background metric is unaffected. Hence we look for a perturbative solution for \( \phi \) on this effective AdS\(_5\) background.
Then the field equation in the bulk becomes
\[
(-\Box_5 + m^2)\phi = \left[ -\hat{L}_r + \frac{1}{(\ell H)^2 \sinh^2(r/\ell)} \left( \hat{L}_t - H^2 e^{-2Ht} \partial_x^2 \right) \right] \phi = 0 ,
\]
where
\[
\hat{L}_t = \frac{\partial^2}{\partial t^2} + 3H \frac{\partial}{\partial t} \quad \text{and} \quad \hat{L}_r = \frac{1}{\sinh^2(r/\ell)} \frac{\partial}{\partial r} \sinh^4(r/\ell) \frac{\partial}{\partial r} - m^2 ,
\]
with the boundary condition (2.7). The equation of motion for the $x$-independent bulk scalar field can be separated by setting $\phi = u(r)\psi(t)$ as
\[
\left[ \hat{L}_r + \frac{\lambda^2}{\ell^2 \sinh^2(r/\ell)} \right] u(r) = 0 ,
\]
\[
\left[ \hat{L}_t + \lambda^2 H^2 \right] \psi(t) = 0 .
\]
Eq. (2.16) determines the spectrum of $\lambda^2$, while Eq. (2.17) is the equation of motion for a 4-dimensional scalar field with effective mass-squared $\lambda^2 H^2$ in a de Sitter space of radius $H^{-1}$.

To see the structure of the mass spectrum, it is convenient to rewrite Eq. (2.16) in the standard Schrödinger form. To do so, we introduce the conformal radial coordinate $y$ through $dr/R(r) = dy$, where $R(r) = \ell \sinh(r/\ell)$. Then the metric (2.13) is expressed as
\[
ds^2 = R^2 \left( dy^2 - H^2 dt^2 + e^{2Ht} dx^2 \right) , \quad (-\infty < y < +\infty)
\]
where $R(y) = \ell \sinh^{-1}(|y| + y_0)$ and $y_0$ is defined by $\sinh(y_0) = \sinh^{-1}(r_0/\ell) = H\ell$. Then putting $u = R^{-3/2} f(y)$, Eq. (2.16) becomes
\[
-f'' + \tilde{V} f = \lambda^2 f ,
\]
where the prime denotes the $y$-derivative and $\tilde{V}$ takes the form of a “volcano potential”,
\[
\tilde{V} = \frac{9}{4} + \frac{15 + 4m^2\ell^2}{4 \sinh^2(|y| + y_0)} - 3 \coth(|y| + y_0) \delta(y) .
\]
It is clear that the volcano potential $\tilde{V}$ approaches the constant $9/4$ at $y \to \pm \infty$. We find that Eq. (2.19) has a normalizable bound-state solution in the region
\[
\lambda^2 < 9/4 ,
\]
for $m^2$ smaller than a critical value ($\sim 9H^2/2$) and the continuous spectrum starts at
\[
\lambda = 3/2 .
\]
Here we note that this bound-state solution corresponds to the zero-mode solution $u = $ const in the case $m^2 = 0$. We also note that the continuous spectrum is
independent of the 5-dimensional mass of the field. These continuous modes are called the Kaluza-Klein modes and the existence of them is the main signature of the braneworld.

Denoting the bound state mode simply by $\psi(t)u(r)$ and a Kaluza-Klein mode by $\psi_p(t)u_p(r)$ where $p^2 = \lambda^2 - 9/4$, the general solution for $\phi$ is given by

$$\phi(t,u) = \psi(t)u(r) + \int_{-\infty}^{\infty} dp \psi_p(t)u_p(r). \quad (2.23)$$

It should be noted that the mode functions $u(r)$ and $u_p(r)$, which satisfy the boundary condition (2.7), are square-integrable in the usual sense but are singular in the limit $r \to 0$ except for $u(r)$ in the case of negative $m^2$.

§3. Bulk scalar field mimicking 4d inflaton dynamics

In Ref. 4, we assumed $m^2 < 0$ and that the scalar field is described by the zero-mode solution $\phi = \psi(t)u(r)$. Then it was shown that the dynamics of this system may be well described by the effective 4-dimensional scalar field and the standard slow-roll inflation is realized by the bulk scalar field when $|m^2|\ell^2 \ll 1$ and $H^2\ell^2 \ll 1$.

However, as given by Eq. (2.23), the general solution will contain the contribution from the Kaluza-Klein modes. Furthermore, if the backreaction of the scalar field to the geometry is taken into account, which appears at the second order in $\phi$, the inclusion of the Kaluza-Klein modes is indispensable to make the geometry at $r = 0$ (for fixed $t$) regular even in the case $m^2 < 0$.

Therefore, for the scenario to be viable, it is necessary to show that the inclusion of the Kaluza-Klein modes does not affect the dynamics of the brane too much. In this section, we analyze the general behavior of the scalar field for a much more general class of initial conditions for which there is no need to assume the separable form nor the case $m^2 < 0$. Then we show how the description in terms of the effective 4-dimensional (zero mode) field is recovered on the brane.

3.1. Dynamics induced on the inflating brane

We consider arbitrary, regular initial data for this scalar field and investigate the generic behavior of the scalar field at sufficiently late times by analyzing the properties of the retarded Green function.

We start with the construction of the Green function. The retarded Green function satisfies

$$(-\Box_5 + m^2)G(x,x') = \frac{\delta^5(x-x')}{\sqrt{-g}}, \quad (3.1)$$

with the causal condition that $G(x,x') = 0$ for $x'$ not in the causal past of $x$. For given initial data on the hypersurface $t = t_i$, the time evolution of a scalar field is given by

$$\phi(x) = \int_{t' = t_i} \left[ (N^a\partial'_a G(x,x'))\phi(x') - G(x,x')N^a\partial'_a \phi(x') \right] \sqrt{\gamma(x')} \ d^4x', \quad (3.2)$$

* Although $u(r)$ vanishes at $r = 0$ for $m^2 < 0$, its derivative diverges for $|m^2| \lesssim H^2$.  


where \( N^a \) is the time-like unit vector normal to the initial hypersurface, and \( \gamma \) is the determinant of the metric induced on this initial hypersurface. Since we are interested in the spatially homogeneous brane, we focus on the \( x \)-independent scalar field configurations. Namely, we consider the spatially averaged Green function defined by

\[
G(t, r; t', r') := \int dx' G(x, x').
\]  

(3.3)

Since \( G(x, x') \) depends on the spatial coordinates \( x \) and \( x' \) though the form \( |x - x'| \), the \( x \)-dependence also disappears after taking the average over \( x' \). The equation for \( G(t, r; t', r') \) follows from Eq. (3.1) as

\[
\left[ \frac{\hat{L}_t}{(Ht)^2 \sinh^2(r/\ell)} - \hat{L}_r \right] G(t, r; t', r') = \frac{\delta(t - t')\delta(r - r')}{Ht^4 \sinh^4(r/\ell)e^{3Ht}},
\]

(3.4)

where \( \hat{L}_t \) and \( \hat{L}_r \) are the operators defined in Eq. (2.15).

We briefly explain how to construct the Green function \( G(t, r; t', r') \) (see Ref. 10 for details). We begin by considering a set of eigenfunctions \( \psi_p(t) \) of the operator \( \hat{L}_t \). The equation is given by Eq. (2.17), with the change of labeling from \( \lambda \) to \( p \) \((\lambda^2 = p^2 + 9/4)\). We find

\[
\psi_p(t) = (2\pi)^{-1/2} e^{(-ip-3/2)Ht}.
\]

(3.5)

Recall that \((p^2 + 9/4)H^2\) is the four-dimensional effective mass squared for each mode. The functions \( \psi_p(t) \) satisfy the orthonormality and the completeness conditions for real \( p \).

Next we consider the eigenfunctions \( u_p(r) \) for \( \hat{L}_r \). The equation for \( u_p(r) \) is given by Eq. (2.16). We denote an eigenfunction which is regular on the upper half complex \( p \) plane by \( u_p^{(\text{out})}(r) \). It is given by

\[
u
\]

(3.6)

where \( P_{\nu-1/2}^p(z) \) is the associated Legendre function of the first kind and \( \nu = \sqrt{m^2 \ell^2 + 4} \). The reason for assigning the superscript ‘(out)’ to this eigenfunction is that \( \psi(t)u_p^{(\text{out})}(r) \propto e^{-ip(Ht + \ln r)} \) describes a wave propagating out to the Cauchy horizon given by \( r \to 0 \) with \( Ht + \ln r \) = const. On the other hand, we denote the eigenfunction which satisfies the Neumann boundary condition at the position of the brane (2.7) by \( u_p^{(Z_2)}(r) \). Here the superscript ‘(Z2)’ is assigned because of its \( Z_2 \)-symmetric property. We can describe \( u_p^{(Z_2)}(r) \) by a linear combination of two independent solutions as \( u_p^{(Z_2)}(r) = u_p^{(\text{out})}(r) - \gamma_p u_p^{(\text{in})}(r) \) and where \( \gamma_p \) is determined by the Neumann boundary condition at the brane.

With these eigenfunctions, we can express the Green function as

\[
G(t, r; t', r') = \int_C dp G_p(r, r') \psi_p(t) \bar{\psi}_p(t'),
\]

(3.7)
where $C$ is a path extending from $p = -\infty$ to $p = \infty$ on the complex $p$-plane with the property that the integrand contains no pole nor branch cut above the path (see Fig. 1), and $G_p$ is constructed from $u_p^{(out)}$ and $u_p^{(Z_2)}(r)$ as

$$G_p(r, r') = \frac{1}{W_p} \left( u_p^{(out)}(r) u_p^{(Z_2)}(r') \theta(r' - r) + u_p^{(out)}(r') u_p^{(Z_2)}(r) \theta(r - r') \right), \quad (3.8)$$

with $W_p$ being the Wronskian given by

$$W_p = \ell^4 \sinh^4 \left( \frac{r}{\ell} \right) \left[ (\partial_r u_p^{(out)}(r)) u_p^{(Z_2)}(r) - u_p^{(out)}(r) (\partial_r u_p^{(Z_2)}(r)) \right]$$

$$= 2ip\ell^3 \gamma_p. \quad (3.9)$$

Fig. 1. Contour of integration for the retarded Green function.

With the above choice of the complex path $C$, the retarded boundary condition is satisfied, and the Green function vanishes for spatially separated $(t, r)$ and $(t', r')$. In particular, it vanishes in the limit $r \to 0$ for fixed values of $t$ and $t'$. This guarantees the regularity at $r = 0$ of the scalar field for arbitrary, regular initial data. As a result, the late-time behavior is understood by investigating the structure of singularities such as poles and branch cuts in the Green function $G_p$. The singularity on the complex $p$ plane with the largest imaginary part dominates the late-time behavior.

For $H^2 \ell^2 \ll 1$ and $m^2 \ell^2 \ll 1$, the equation that determines the location of a pole with the smallest imaginary part is given by

$$\frac{m^2 \ell^2}{2} - \left( p^2 + \frac{9}{4} \right) H^2 \ell^2 \approx 0. \quad (3.10)$$

This equation is always a good approximation irrespective of the value of $m^2/H^2$. 
We denote the two solutions by
\[ p_\pm = \pm i \sqrt{\frac{9}{4} - \frac{m^2_{\text{eff}}}{H^2}} + O \left( (H\ell)^2, (m\ell)^2 \right), \] (3.11)
where
\[ m^2_{\text{eff}} = \frac{m^2}{2}. \] (3.12)

In addition to these, there are an infinite sequence of poles that reduces to a branch cut in the limit \( H^2 \to 0 \). However, they have large imaginary parts and do not give a dominant contribution to the late-time behavior.

Now it is easy to see the late-time behavior of the Green function. When \( m^2_{\text{eff}}/H^2 < 9/4 \), the contribution from the pole \( p_+ \) dominates. The Green function after a sufficiently long lapse of time behaves as
\[ G(x,x') \propto e^{\sqrt{(9/4)H^2 - m^2_{\text{eff}}}} e^{-(3/2)Ht}. \] (3.13)

When \( m^2_{\text{eff}}/H^2 > 9/4 \), the contributions from both poles \( p_\pm \) are equally important. In this case, the asymptotic behavior of the Green function is given by
\[ G(x,x') \propto e^{-(3/2)Ht} \cos \left( \left[ m^2_{\text{eff}} - \frac{9}{4}H^2 \right]^{1/2} t + \eta \right), \] (3.14)
where \( \eta \) is a real constant phase. From the asymptotic behavior obtained in Eqs (3.13) and (3.14), we can conclude that the bulk scalar field evaluated on the brane behaves as an effective 4-dimensional field with the mass squared \( m^2_{\text{eff}} \) given by Eq. (3.12) after a sufficiently long period of de Sitter expansion.

### 3.2. Effective 4-dimensional scalar field

Let us now turn to the second order description of the system where the backreaction to the geometry appears. Since our solution guarantees the regularity of the geometry in the bulk, we focus on the dynamics of the brane. We consider the case when \( |m^2|\ell^2 \ll 1 \) and \( H^2\ell^2 \ll 1 \).

We note that from the late time behavior of the scalar field given by Eqs. (3.13) and (3.14), the bulk scalar field satisfies the equation,
\[ \ddot{\phi} + 3H\dot{\phi} + m^2_{\text{eff}}\phi = 0, \] (3.15)
on the brane at late times. On the other hand, the 5-dimensional field equation on the brane implies
\[ \ddot{\phi} + 3H\dot{\phi} - \partial^2_\ell \phi + m^2 \phi = 0, \] (3.16)
on the brane. Using the effective mass given by Eq.(3.12), we obtain
\[ \partial^2_\ell \phi = (m^2 - m^2_{\text{eff}})\phi = m^2_{\text{eff}}\phi = -\ddot{\phi} - 3H\dot{\phi}. \] (3.17)
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Inserting this to the integrand of Eq. (2.10), we find

\[ E_{tt} = -\frac{\kappa_5^2}{2a^4} \int a^4 \dot{\phi}(\ddot{\phi} + 2H \dot{\phi}) \, dt = -\frac{\kappa_5^2}{4a^4} \int \frac{d}{dt} \left( a^4 \dot{\phi}^2 \right) \, dt \]

\[ = -\frac{\kappa_5^2}{4} \dot{\phi}^2, \quad (3.18) \]

where we have neglected the integration constant term \((\propto a^{-4})\) that vanishes rapidly as time goes on. Then we find the effective energy density on the brane, given by Eq. (2.9), as

\[ \rho_{\text{eff}} = \frac{\ell_0}{2} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right) - \frac{E_{tt}}{\kappa_4^2} = \frac{1}{2} \dot{\Phi}^2 + V_{\text{eff}}(\Phi), \quad (3.19) \]

where

\[ \Phi = \sqrt{\ell_0} \phi \quad \text{and} \quad V_{\text{eff}}(\Phi) = \frac{\ell_0}{2} V(\Phi/\sqrt{\ell_0}). \quad (3.20) \]

Thus \(\rho_{\text{eff}}\) is given by an effective 4-dimensional scalar field \(\Phi\) with mass-squared \(m_{\text{eff}}^2 = m^2/2\). It is important to note that this is fully consistent with the first order solution for the system where the bulk scalar field is dominated by its zero mode. To conclude, provided \(H^2\ell^2 \ll 1\) and \(|m^2|\ell^2 \ll 1\), the effective dynamics of the Einstein-scalar system on the brane is indistinguishable from a 4-dimensional theory at the lowest order in \(H^2\ell^2\) and \(|m^2|\ell^2\).

§4. Conclusion

Based on the Randall-Sundrum type braneworld scenario, we proposed a new model of the braneworld inflation in which the inflation is caused solely by the dynamics of a 5-dimensional scalar field without introducing an inflaton on the brane universe. We noted that the scalar field we introduced in this paper should presumably be a scalar field originating in gravity. It is natural that the 5-dimensional action includes some scalar fields of gravitational origin from the viewpoint of unified theories in a yet higher dimensional spacetime.

Using the Green function given in Eq. (3.7), we derived the late time behavior of the bulk scalar field under the assumptions \(|m^2|\ell^2 \ll 1\) and \(H^2\ell^2 \ll 1\), and showed that the bulk scalar field seen on the brane behaves as a 4-dimensional effective scalar field with mass \(m_{\text{eff}} = m/\sqrt{2}\), irrespective of initial field configurations and of the value of \(|m^2|/H^2\). Moreover, we have examined the lowest order backreaction to the geometry which starts at the quadratic order in the amplitude of \(\phi\). We have found that the leading order backreaction to the geometry is consistently represented by a 4-dimensional effective scalar field \(\Phi\) with the effective 4-dimensional mass \(m_{\text{eff}}\) mentioned above, where \(\Phi\) is related to \(\phi\) by a simple scaling (3.20). Thus this inflation model turns out to be a viable alternative scenario of the early universe.

It is, however, the next order corrections in \(H^2\ell^2\) and \(m^2\ell^2\) that are of cosmological interest, since these corrections are expected to give genuine braneworld effects which can be used to test the scenario. Furthermore, if we recall that the bulk scalar
field is probably of gravitational origin, it may be natural to expect $m^2$ to be of the curvature scale of the rest of compactified extra dimensions which should be at least of the same order of $\ell_0^{-2}$. Since $|m^2|\ell_0^2 \lesssim |m^2|\ell^2$, this implies $|m^2|\ell^2 \gtrsim 1$ for realistic models (and which implies $H^2\ell^2 \gtrsim 1$ for inflation to occur). It was shown in Ref. 6) that the contribution of the Kaluza-Klein modes are non-negligible when $|m^2|\ell^2 \gg 1$ (though small; of the order of 10% at most) and affect the brane dynamics even at sufficiently late times. Thus it seems very important to study the case $|m^2|\ell^2 \gg 1$ in more details, or at least to clarify the effect of the next order corrections in $|m^2|\ell^2$.

Investigations in this direction is in progress.

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