Performance Improvement of OFDM System Using New Scrambling Technique with Precoded Methods

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Abstract. One of the challenging issues for Orthogonal Frequency Division Multiplexing (OFDM) system is its high Peak to Average Power Ratio (PAPR). This degrades performance of the system. In this paper, a new scrambling algorithm to reduce PAPR is presented. The new algorithm uses chaos system to generate several Chaos Binary Sequences (CBS) that scrambled with the data symbols to get several sequences. Only one sequence is chosen with smallest PAPR value for transmission. To reduce the PAPR more and improve Bit Error Rate (BER) performance of the new scrambling algorithm, precoded methods are used. Two precoded methods are presented in this work: Discrete Cosine Transform (DCT) and Walsh Hadamard Transform (WHT). Simulation results using MATLAB show that when number of subcarriers is 1024 and number of CBS is 8, the PAPR is approximately equal to 7 dB by using new scrambling technique only. While the PAPR is approximately reduced by 1 dB when using WHT with new technique. Finally, when using DCT with new technique, the PAPR is approximately reduced by 3.8 dB. Also DCT with new technique gives better BER performance than WHT with new technique.

Keywords: OFDM, Chaos, PAPR, BER, DCT and WHT.

1. Introduction
Multicarrier techniques in wireless communication systems are widely used in the recent years. Several types of Multicarrier techniques are presented, the most attractive technique is OFDM [1]. OFDM provides multiple advantages such as high transmission data rate, immunity against selective fading channel, high spectral efficiency, require easy channel equalization in receiver and etc. [2]. In OFDM, system bandwidth (BW) is divided into N orthogonal subcarriers \((f_i=i\Delta f)\), where \((\Delta f=\frac{BW}{N})\) is the band of each subcarrier and \(i=0,1,\ldots,N-1\) [3]. Data are spread over these orthogonal subcarriers. Therefore, Inverse Fast Fourier Transform (IFFT) is used in transmitter as modulation technique. Thus, the transmitted discrete time OFDM signal is \(x(k)\):

\[
x(k) = \sum_{i=0}^{N-1} X(i) e^{j\frac{2\pi i k}{N}} \quad \text{for} \ 0 \leq k \leq N - 1
\]

In receiver, Fast Fourier Transform (FFT) is used to demodulate OFDM signal [4]. With all these advantages, OFDM system has major disadvantage so that the transmitted signal has large PAPR. This makes OFDM signal more likely to be clipped when passing through a power amplifier in transmitter. Clipping the signal causes reduction in the signal power efficiency, degrading BER performance, and spectral spreading which degrades the spectral efficiency [5]. These disadvantages give us motivation...
to reduce PAPR. There are multiple methods that proposed to decrease PAPR. So that, they are classifying into two groups: distortion techniques and distortionless techniques. Distortion techniques distort the shaping of transmitted signal and they include clipping and filtering [6], companding[7], etc.. While distortionless techniques do not distort the shaping of transmitted signal and they include Selective Mapping scrambling techniques (SLM) [8], Partial Transmit Sequence scrambling techniques (PTS) [9], precoded techniques [10], etc.. In this paper, proposed system is presented to reduce PAPR which combines new algorithm of scrambling technique with precoded techniques. The proposed system gives good PAPR reduction performance without producing any signal distortion and improved BER performance. Also, it has simple hardware compared with another PAPR reduction techniques.

2. PAPR Characteristics of OFDM Signal

PAPR is ratio of peak instantaneous power to average power. From (1), mathematically, the PAPR can be given by [4]:

\[
PAPR = \frac{\max_{0 \leq k \leq N-1} |x(k)|^2}{P_{av}}
\]  

(2)

Where \(P_{av}\) is average power of \(x(k)\) in (1). PAPR can be characterize into two different aspects [11], as shown in the following sections below.

2.1 Autocorrelation Characteristic of IFFT Input

The relationship between autocorrelation of input to IFFT and PAPR is discussed briefly in this section. In IFFT, the operation can be considered as multiplying sinusoidal functions to the input sequence, then summing the result. Thus, high correlation of input to IFFT makes the arrangement of the sinusoidal functions in-phase form [11]. When these in-phase sinusoidal functions are summing, this might result large amplitude in the output of IFFT.

The discussion above can be formulated in terms of the autocorrelation and PAPR. Autocorrelation \(\rho_k\) for discrete input sequence to IFFT, \(X(i)\), can be given by: [12]

\[
\rho_k = \sum_{i=1}^{N-k} X_{i+k}^* X_i, k = 0, 1, 2, ..., N - 1
\]

(3)

Where \(^*\) is the conjugate operation. Then the maximum (\(\gamma\)) of PAPR is

\[
\gamma \leq 1 + \frac{2}{N} \sum_{k=0}^{N-1} |\rho_k|
\]

(4)

2.2 Probability Distribution of IFFT Output

The second aspect of PAPR is probability distribution of IFFT output instantaneous power [13]. When each input to IFFT is independent and identical distributed, real and imaginary parts of the IFFT output have approximately Gaussian distribution for large number of N based on central limit theorem [14]. Then power of IFFT output signal follows Chi-square distribution. So that the instantaneous power \((\Psi)\) of \(x(k)\) can be given by [11]:

\[
\Psi = \text{Re}\{x(k)\}^2 + \text{Im}\{x(k)\}^2
\]

(5)

Where \(\text{Re}\{\cdot\}\) and \(\text{Im}\{\cdot\}\) are taking real and imaginary parts of a complex value. Chi-square distribution \((\tilde{f}(\Psi))\) of (5) can be given by:

\[
\tilde{f}(\Psi) = \frac{1}{\sigma^2} e^{-\frac{\Psi}{\sigma^2}}, \Psi \geq 0
\]

(6)

Where \((\sigma^2 = E(|x(k)|^2)/N=\tilde{P}_{av})\), where \(E(.)\) is the expectation of \(x(k)\). Cumulative Distribution Function (CDF) of (6) is:

\[
\text{Prob}[\Psi < \Psi_o] = \int_0^{\Psi_o} f(u) du
\]
Prob[Ψ < Ψ₀] = [1 - e⁻¹]⁻¹ (7)

Where Ψ₀ is a specified threshold level of power. For N subcarriers and the Complementary CDF (CCDF) of the instantaneous power that exceeds the certain power level Ψ₀ is given as:

Prob[Ψ > Ψ₀] = [1 - e⁻¹]⁻¹ N (8)

Equation (8) can be given in terms of PAPR as shown below [4]

Prob[PAPR > δ] = [1 - e⁻¹]⁻¹ (9)

Equation (9) gives the probability that PAPR exceeds a certain threshold level δ.

3. Proposed System

From section 2, it can conclude that PAPR value can be reduced by reducing two mainly factors. The first factor is the autocorrelation of the input to IFFT as in (4) and the second factor is the probability that PAPR higher than δ as in (9). To achieve that, the proposed system in this paper combines precoded methods with new algorithm of scrambling technique.

3.1 Precoded Methods

In these methods, the modulated data symbols will multiply with shaping matrix before IFFT operation as shown in figure 1 below [15].

![OFDM system with precoding method](image)

Figure 1. OFDM system with precoding method.

There are different methods to generate the precoding matrix [P] such as Discrete Cosine Transform (DCT), Walsh-Hadamard Transform, pulse shaping, Zadoff-Chu sequence, etc.. In this paper, DCT and WHT are used. In fact, the purpose of using these methods is to reduce the autocorrelation of input sequence to IFFT, also improve BER performance of the system.

3.1.1 Discrete Fourier Transform

DCT was proposed by Ahmed et al.[16]. This transform is like to discrete Fourier transform (DFT), but it uses real values. One dimensional DCT (1D-DCT) of length N can be defined as (A(k)) [3]:

A(k) = α(k)Σ⁻¹ₐ(i)cos(π(2i+1)k/2N) (10)

For k=0,1,2,…,N-1, and the inverse-DCT (IDCT) is:

a(i) = Σ⁻¹ₐ(k)A(k)cos(π(2i+1)k/2N) (11)

For i=0,1,2,…,N-1, and α(k) is given as:

α(k) = \begin{cases} 
\frac{1}{\sqrt{N}} & \text{for } k = 0 \\
\frac{2}{\sqrt{N}} & \text{for } k = 1, 2, 3, ..., N - 1 
\end{cases} (12)
1D-DCT basis sequences are real, discrete-time sinusoids are \( C_N(i,k) \) [17]:
\[
C_N(i,k) = \cos \left( \pi \frac{(2i+1)k}{2N} \right) \tag{13}
\]
Equation (10) in matrix form is:
\[
A = C_Na \tag{14}
\]
The columns (or rows) of the matrix \( C_N \) are orthogonal vectors. In fact, this property is very useful to decrease PAPR, so that it reduces autocorrelation of input signals to IFFT which is the main reason of PAPR problem.

### 3.1.2 Walsh-Hadamard Transform

WHT is unitary transform which is used in many ranges of applications. WHT is implemented using Sylvester matrix, also called Hadamard matrix \( W_N \), which can be built as follows [18]:
\[
W_N = \begin{bmatrix}
W_{N/2} & W_{N/2} \\
W_{N/2} & -W_{N/2}
\end{bmatrix} \tag{15}
\]
Where \( W \) has elements that belongs to the group \{+1,-1\}. The Walsh-Hadamard matrix is orthogonal matrix [18]. Notice that Walsh-Hadamard matrix is symmetric. The matrix in (15) can be used to implement WHT. Therefore, the WHT of the data \( d = [d_0, d_1, \ldots, d_{N-1}]^T \) is:
\[
D_W[i] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} [W_N]_{i,k} d(k), \quad i=0,1,2,\ldots,N-1 \tag{16}
\]
Also, it can be considered the WHT in matrix notation \( (D_W) \) as:
\[
D_W = \frac{1}{\sqrt{N}} W_N^* d \tag{17}
\]
while its inverse is given by:
\[
d = \frac{1}{\sqrt{N}} W_N^H * D_W \tag{18}
\]

### 3.2 The New algorithm of Scrambling Technique

In the conventional scrambling techniques such as SLM, several IFFT and FFT blocks are needed. So that, increasing the blocks lead to reduce PAPR, but this also will increase cost as well as computational complexity. PTS technique suffer from the difficulty of finding the optimum set of phase vectors when increasing the number of sub-blocks. In this paper, new and simple scrambling technique is presented that require only one IFFT block and one FFT block without affecting on the PAPR reduction performance. Also it generates set of optimum random sequences using simple method and select one sequence with lowest PAPR for transmission. Figure 2 below shows flowchart diagram of the new scrambling technique.
Initially, it generates several aperiodic pseudorandom sequences \((R^n_m)\) where \(0 \leq m \leq M-1\) and \(M\) is the number of the generated sequences and \(0 \leq n \leq N_m-1\) \((N_s\) is part of \(N\) that carry information data\). Then information block \(X_n\) is XOR with \(R^n_m\) to get \(W^n_m\), so that:

\[
W^n_m = X_n \oplus R^n_m
\]
\[ W_n^m = X_n \oplus R_n^m \]  

(19)

Where is XOR. The remaining of N subcarriers (\(N_{\text{SI}} = N - N_r\)) is specified for the indication (m) of sequence \(W_n^m\). This means that if \(W_n^0\) then \(m = 0\), if \(W_n^1\) then \(m = 1\), and so on. Therefore, each \(m\) value is converted into \([\log_2(M)]\) binary bits.

In fact, these bits are considered as Side Information (SI) that informs the receiver about the selected sequence which transmitted. SI is very important because any error occurs in its bits lead to lose the whole data block in receiver. Therefore, SI are protected very well using channel coding. In this paper, convolutional code with Viterbi decoding algorithm are used as channel coding [19][20]. The encoded SI bits (\(B_j^m\)) are added to \(W_n^m\) to get \(Z_n^m\) as follow:

\[ Z_n^m = B_n^m + W_n^m, \quad 0 \leq n \leq N_{\text{SI}} - 1 \]  

(20)

After that, \(Z_n^m\) are mapped onto Binary Phase Shift Keying (BPSK) constellation and the result input to IFFT block to produce the time domain OFDM signals \(d(t)\) that can be given as:

\[ d(t) = \sum_{i=0}^{N-1} s_i e^{-j2\pi f_i t}, \quad 0 \leq t \leq T \]  

(21)

Where \(s_i\) is BPSK mapping of \(Z_n^m\). Finally, PAPR is calculated for each \(d(t)\) and select one signal with lowest PAPR for transmission. Equation (21) shows that M independent random signals are getting at output of IFFT block. Therefore, M independent signals in (21) makes (9) as such:

\[ \text{Prob}(\text{PAPR}_\text{low} > \delta) = [\text{Prob}(\text{PAPR} > \delta)]^M = [1 - (1 - e^{-\delta})^N]^M \]  

(22)

Where PAPR\(_{\text{low}}\) is PAPR value when using the new algorithm. Equation (22) shows that CCDF is much less than CCDF in (9), i.e. if probability of high PAPR occurs is (Pb) when using (9), then using (22) probability of occurring high PAPR becomes (Pb\(^M\). In fact, it is not possible to obtain \(d^u(t)\)\((u=0,1,\ldots,M-1)\) and \(d^v(t)\)\((v=0,1,\ldots,M-1)\) that are mutually independent for all \(u \neq v\) because they contain the same information \(X_n\) [21]. So to solve this problem, the condition is to make the correlation between \(d^u(t)\) and \(d^v(t)\) as low as possible. To obtain this condition, M aperiodic pseudorandom sequences must be generated that have very low Variance of Correlation (VC) [21]. Chaos system is used to generate several CBS which have very low VC [21]. Finally, the proposed system combines the new algorithm in figure 2 with preceding methods. So that DCT or WHT are putting after the BPSK block and before the IFFT block in figure 2.

### 3.2.1 Chaotic Systems

Chaos is a aperiodic, random like, long-term non-predictive behavior which can be generated by using different nonlinear systems [22]. Chaos systems are very sensitive to the initial conditions. The chaos sequences can be generated using chaotic maps. Any chaotic map can be defined as [23]:

\[ y_{n+1} = f(y_n), \quad n=0,1,2,\ldots,N_r-1 \]  

(23)

Where \(y_{n+1}\) is the present value of \(y\) and \(y_n\) is the previous value. There are several types of chaotic maps such as logistic map, quadratic map, tent map, etc.. Quadratic map is used in this work to generate CBS. Quadratic map can be defined as:

\[ y_{n+1} = 1 - r \cdot y_n^2 \]  

(24)

Where \(r\) is bifurcation parameter lies within the range \([-0.25,2]\) and \(y_n \in [-1,1]\). figure 3 shows bifurcation diagram of quadratic chaotic map. In this figure, it is clear that chaotic behavior happens when bifurcation parameter \((r)\) lies in the range \([1.43,2]\) [23].
While figure 4 shows how small change in the initial condition leads to give different sequences.

After that, $y_n$ values in (24) can be converted into binary ($R_n$) as:

$$R_n = \begin{cases} 
1 & \text{when } y_n \geq 0.5 \\
0 & \text{when } y_n < 0.5 
\end{cases}$$ (25)

4. Simulation Results

Here, PAPR reduction performance and BER performance of the proposed system (the new algorithm with precoding methods) are evaluated using MATLAB. All parameters that used in simulation are shown in the table 1 below:

| Table 1. Simulation parameters. |
|---------------------------------|
| Simulation parameter          | Type/value                      |
| $M$                            | 2, 4, 6, 8                      |
| $N$                            | 64, 128, 256, 512, 1024         |
| Coding technique               | Convolutional code with code rate $\frac{1}{2}$ |
| Mod. scheme                    | BPSK                            |
| $N_{CP}$                        | $\frac{N}{4}$                    |
| Number of OFDM symbols         | 5000                            |
4.1 PAPR reduction performance

Firstly, figure 5 states the performance of the new algorithm only (without using preceded methods) with different values of $M$ and $N$ subcarriers.

Figure 5 shows that PAPR is reduced when increase $M$ as in equation (22). If DCT is put before IFFT in the new algorithm, the PAPR reduction performance is shown in figure 6. In this figure, PAPR is much reduced comparing with the PAPR reduction in figure 5. This is because DCT reduces autocorrelation of input sequence before IFFT as in equation (4). On other hand, if WHT is replaced rather than DCT, the PAPR reduction performance is shown in figure 7.
Figure 6. PAPR reduction performance of proposed system (DCT with the new scrambling technique) with N = 64,128,256,512,1024 and (a) M=2 (b) M=4 (c) M=6 (d) M=8.

Figure 7. PAPR reduction performance of proposed system (WHT with the new scrambling technique) with N=64,128,256,512,1024 and (a) M=2 (b) M=4 (c) M=6 (d) M=8.
4.2 BER performance

Because SI is very important in the proposed system, (1/2 code rate) convolutional code with Viterbi decoding is used to protect it against channel impairments (without considering the effect of PAPR value on BER performance). Also BER performance of proposed system is improved due to use precoded methods. figure 8 shows BER performance of: conventional system, new algorithm only, new algorithm with DCT, and new algorithm with WHT, over AWGN channel. The new algorithm gives same performance of the conventional system. This indicates that SI does not effect on BER performance. While, the performance is improved when using precoded methods with the new algorithm.

![Figure 8. BER performance over AWGN channel.](image)

5. Conclusion and Discussion

This work presents simple and efficient PAPR reduction method that combines new scrambling algorithm with precoded methods. The new algorithm facilitates computational complexity of the scrambling techniques by reducing required number of IFFT and FFT blocks. In addition to that, it generates optimum M-phase sequences when using chaos system. Chaos system is simple method comparing with the method in PTS that needs more computational complexity. Increasing M leads to reduce PAPR and increase SI bits. This can be done by using one block of IFFT and one block of FFT. DCT and WHT precoded methods are used with this algorithm to reduce PAPR, because they reduce autocorrelation of input sequence to IFFT. Simulation results of PAPR reduction performance of new scrambling technique, WTH with new scrambling technique, and DCT with new scrambling technique are showing in table 2 below:

| N   | PAPR when using new scrambling technique | PAPR when using WHT with New scrambling technique | PAPR when using DCT New scrambling technique |
|-----|----------------------------------------|------------------------------------------------|---------------------------------------------|
| 64  | 4.5 db                                  | 3.25 db                                         | 2.5 db                                      |
| 128 | 5 db                                    | 3.9 db                                          | 2.75 db                                     |
| 256 | 5.9 db                                  | 4.65 db                                         | 2.88 db                                     |
| 512 | 6.4 db                                  | 5.4 db                                          | 3.09 db                                     |
| 1024| 7 db                                    | 6.17 db                                         | 3.24 b                                      |

PAPR reduction performance is better when using DCT with new scrambling technique. This is because DCT does not amplify amplitude of its coefficients while WHT amplify amplitude of its coefficients. This amplification increases PAPR value. Also the new scrambling algorithm with DCT give better BER performance by reducing energy per bit to noise power ratio \(\frac{E_b}{N_0}\) by 2db. So that BER\(=10^{-4}\) at \(\frac{E_b}{N_0}\) = 5.5db when using DCT while BER\(=10^{-4}\) at \(\frac{E_b}{N_0}\) = 7.5db when using WHT. This is because DCT has energy compaction property.
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