Cardassian Universe Constrained by Latest Observations

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ABSTRACT: Several Cardassian universe models including the original, modified polytropic and exponential Cardassian models are constrained by the latest Constitution Type Ia supernova data, the position of the first acoustic peak of CMB from the five years WMAP data and the size of baryonic acoustic oscillation peak from the SDSS data. Both the spatial flat and curved universe are studied, and we also take account of the possible bulk viscosity of the matter fluid in the flat universe case.

KEYWORDS: Cosmology, Viscosity, Cardassian universe
1. Introduction

Nowadays, there are many dark energy models and modified gravity theories proposed to explain the current accelerating expansion of the universe, which has been confirmed by the observations like Type Ia supernovae (SNe Ia), CMB and SDSS et al. The dark energy models assume the existence of an energy component with negative pressure in the universe, and it dominates and accelerates the universe at late times. The cosmological constant seems the best candidate of dark energy, but it suffers the fine tuning problem and coincidence problem, and it may even have the age problem [1]. To alleviate these problems, many dynamic dark energy models were proposed. However, people still do not know what is dark energy.

Since the Einstein general gravity theory has not been checked in a very large scale, then one does not know whether this gravity theory is suitable or not for studying the observational data like SNe Ia, and maybe the accelerating expansion of universe is due to the gravity theory that differs from the general gravity. Thus, many modified gravity theories like \( f(R) \), DGP et al. are proposed to explain the accelerating phenomenology. The Cardassian model is a kind of model in which the Friedemann equation is modified by the introduction of an additional nonlinear term of energy density, and we will briefly review on this model in the next section.

Dissipative processes in the universe including bulk viscosity, shear viscosity and heat transport have been conscientiously studied[2]. The general theory of dissipation in relativistic imperfect fluid was put on a firm foundation by Eckart[3], and, in a somewhat different formulation, by Landau and Lifshitz[4]. This is only the first order deviation from equilibrium and may have a causality problem, the full causal theory was developed by Isreal and Stewart[5], and has also been studied in the evolution of the early universe[f]. However, the character of the evolution equation is very complicated in the full causal theory. Fortunately, once the phenomena are quasi-stationary, namely slowly varying on space and time scale characterized by the mean free path and the mean collision time of the fluid particles, the conventional theory is still valid. In the case of isotropic and homogeneous universe, the dissipative process can be modeled as a bulk viscosity \( \xi \) within a thermodynamical approach, while the shear viscosity \( \eta \) can be neglected, which is consistent with the usual practice[8]. For works on viscous dark energy models, see ref.[9].

The bulk viscosity introduces dissipation by only redefining the effective pressure, \( p_{\text{eff}} \), according to \( p_{\text{eff}} = p - 3\xi H \) where \( \xi \) is the bulk viscosity coefficient and \( H \) is the Hubble parameter. The condition \( \xi > 0 \) guaranties a positive entropy production, consequently, no violation of the second law of the thermodynamics[10]. The case \( \xi = \tau H \), implying the bulk viscosity is proportional to the fluid’s velocity vector, is physically natural, and has been considered earlier in a astrophysical context, see the review article of Grøn[11].
In this paper, we will focus on several Cardassian models including the original, modified polytropic and exponential Cardassian models and constrain their parameters by the latest Constitution Type Ia supernova data (SNeIa), the position of the first acoustic peak of the cosmic microwave background (CMB) from the five years WMAP data and the size of baryonic acoustic oscillation (BAO) peak from the SDSS data. We have consider the case of spatial flat and curved universe and the case of flat universe with the bulk viscosity. After a lengthy numerical calculation, we obtain the best fit values of the parameters in each Cardassian model.

This paper is organized as follows: In Section 2, we present a brief review of Cardassian models, and derive the Hubble parameter in terms of the redshift and some parameters for several models. In Section 3, we analysis each model with statistical method and constrain their parameters with the observational data. In the last section, we give some conclusions and discussions.

2. The Cardassian model

Assuming the universe is homogeneous and isotropic, i.e.

\[ ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \]  

(2.1)

where \( k \) is the spatial curvature, the modified Friedmann equation for Cardassian model is given by

\[ H^2 + \frac{k}{a^2} = \frac{g(\rho)}{3}, \]  

(2.2)

where \( \rho \) is the total energy density of matter and radiation and we will neglect the contribution of radiation for the late-time evolution of the universe. In eq. (2.2), the function \( g(\rho_m) \) reduces to \( \rho_m \) in the early universe, then eq. (2.2) reduces to the ordinary Friedmann equation during early epochs such as primordial nucleosynthesis. However, it differs from the FRW universe at the redshift \( z < O(1) \), during which it will gives rise to accelerated expansion. Different forms of the function \( g(\rho_m) \) corresponds to different Cardassian models, and we will focus on the original Cardassian model (OC) [2], the modified polytropic Cardassian model (MPC) [3], the exponential model (EC) [4], their flat versions ( FOC, FMPC, FEC ), in which the spatial curvature is neglected, and their viscous versions ( VOC, VMPC, VEC ) [7], in which the bulk viscosity of the matter is taken account and the spatial curvature is also neglected. We summaries these models in Table 1. Recent works on constraining the Cardassian universe, see ref. [7].

| \( g(\rho_m) \) | Model | \( E^2 = H^2/H_0^2 \) |
|----------------|----------------|----------------|
| \( \rho_m \left[ 1 + \left( \frac{\rho_m}{\rho_{card}} \right)^{n-1} \right] \) | FOC | \( \Omega_m(1+z)^3 + (1 - \Omega_m)(1+z)^3n \) |
| \( \rho_m \left[ 1 + \left( \frac{\rho_m}{\rho_{card}} \right)^{q(n-1)} \right] \) | OC | \( \Omega_m(1+z)^3 + (1 - \Omega_m - \Omega_k)(1+z)^3n + \Omega_k(1+z)^2 \) |
| \( \rho_m \left[ 1 + \left( \frac{\rho_m}{\rho_{card}} \right)^{q(n-1)} \right] \) | VOC | \( \Omega_m(1+z)^3(1-n)^{1-q(1-n)^{-1/4}} \left[ 1 + F_{voc}(z)(\Omega_m^{-1} - 1) \right] \) |
| \( \rho_m \exp \left[ \frac{\rho_m}{\rho_{card}} \right] \) | FMPC | \( \Omega_m(1+z)^3[1 + (1 - \Omega_k)^{3\Omega_k - 1}(1+z)^{3\Omega_k - n}]^{1/q} + \Omega_k(1+z)^2 \) |
| \( \rho_m \exp \left[ \frac{\rho_m}{\rho_{card}} \right] \) | MPC | \( \Omega_m(1+z)^3(1-n)^{1-q(1-n)^{-1/4}} \left[ 1 + F_{mpc}(z)(\Omega_m^{-1} - 1) \right] \) |
| \( \rho_m \exp \left[ \frac{\rho_m}{\rho_{card}} \right] \) | VMPC | \( \Omega_m(1+z)^3[1 + (1 - \Omega_k)^{3\Omega_k - 1}(1+z)^{3\Omega_k - n}]^{1/q} + \Omega_k(1+z)^2 \) |
| \( \rho_m \exp \left[ \frac{\rho_m}{\rho_{card}} \right] \) | FEC | \( \Omega_m(1+z)^3 \exp \left[ -(1+z)^{-3n} \ln(1 - \Omega_k) \right] \) |
| \( \rho_m \exp \left[ \frac{\rho_m}{\rho_{card}} \right] \) | EC | \( \Omega_m(1+z)^3 \exp \left[ -(1+z)^{-3n}(\ln(1 - \Omega_k + 1/4)) \right] + \Omega_k(1+z)^2 \) |
| \( \rho_m \exp \left[ \frac{\rho_m}{\rho_{card}} \right] \) | VEC | \( \Omega_m(1+z)^3 \exp \left[ -(1+z)^{-3n} \ln(1 - \Omega_k) \right] \) |

Table 1: Summary of Cardassian models with different functions of \( g(\rho_m) \). Here, \( \rho_{card} \) is a character energy density.
Energy conservation of pressureless matter is given by

\[ \dot{\rho}_m + 3H(\rho_m - 3\xi_mH) = 0, \quad (2.3) \]

where \( \xi_m \) is the bulk viscosity for the matter \( \rho_m \). Following [17], the function \( g \) could be written as \( g = \rho_m + \rho_k \), where \( \rho_k \) is so called Cardassian term, which may indicate that our observable universe as 3 + 1 dimensional brane in extra dimensions. Thus, the total energy density can be written as

\[ \dot{g} + 3H(g + p_T - 3\xi H) = 0, \quad (2.4) \]

where \( \xi \) is the bulk viscosity for the total energy density \( g(\rho_m) \). Here, \( p_T \) is defined as the effective pressure of total fluid without bulk viscosity, and the first law of thermodynamics in an adiabatic expanding universe gives

\[ p_T = \rho_m \frac{\partial g}{\partial \rho_m} - g. \quad (2.5) \]

Therefore, one can get \( \xi = \frac{\partial g}{\partial \rho_m} \xi_m \) from eqs. (2.3) and (2.4). In the following, we will choose \( \xi = \tau H \), in which the cosmological dynamics can be analytically solvable [15] and \( \tau \) is a constant. Then, the conservation law (2.4) becomes

\[ \left( f + \frac{\partial f}{\partial y} \right) y' + 3 \left[ \frac{\partial f}{\partial y} + (1 - \tau)f \right] = 0, \quad (2.6) \]

where \( f = g/\rho_m \), \( y = \ln(\rho_m/\rho_{\text{card}}) \) the prime denotes the derivative with respect to \( x \equiv \ln(a) = -\ln(1 + z) \), and \( z \) is the redshift.

For the VOC model, \( f = 1 + e^{(n-1)y} \), and the solution is

\[ \rho_m = \rho_{m0}(1 + z)^{3(1 - \tau)} \left[ \frac{1 - \tau + (n - \tau)(\rho_{m0}/\rho_{\text{card}})^{n-1}}{1 - \tau + (n - \tau)(\rho_m/\rho_{\text{card}})^{n-1}} \right]^{\frac{\tau}{1 - n}}, \quad (2.7) \]

where \( \rho_{m0} \) is the present value of the matter’s energy density, and the Hubble parameter \( E^2 = H^2/H_0^2 \), is given by

\[ E^2 = \Omega_{m0}(1 + z)^3 \left[ \frac{1 - \tau + (n - \tau)(\rho_{m0}/\rho_{\text{card}})^{n-1}}{1 - \tau + (n - \tau)(\rho_m/\rho_{\text{card}})^{n-1}} \right]^{\frac{\tau}{1 - n}}, \quad (2.8) \]

where \( \Omega_{m0} = \rho_{m0}/(3H_0^2) \), \( H_0 \) is the present value of the Hubble parameter and

\[ \tilde{\tau}_1 = \left( \frac{\Omega_{m0}}{1 - \Omega_{m0}} \right) \left( \frac{1 - \tau}{n - \tau} \right). \quad (2.9) \]

Here the function \( F_{\text{voc}}(z) = (\rho_m/\rho_{m0})^{n-1} \) satisfies

\[ F_{\text{voc}} = (1 + z)^3(1 - \tau)(n-1) \left[ \frac{1 - \tau + (n - \tau)(\rho_{m0}/\rho_{\text{card}})^{n-1}}{1 - \tau + (n - \tau)(\rho_m/\rho_{\text{card}})^{n-1}} \right]^{\frac{\tau(n-1)}{1 - n}}, \quad (2.10) \]

from which one can get the solution for \( F_{\text{voc}} \) and substitute it into eq. (2.8), then one obtains the Hubble parameter in terms of \( z \) and parameters \( \Omega_{m0}, n, \tau \). When \( \tau = 0 \), the solution is rather simple, and the Hubble parameter (2.8) becomes

\[ E^2 = \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^3n. \quad (2.11) \]

For the VMPC model, \( f = (1 + e^{q(n-1)y})^{1/q} \), and the solution is

\[ \rho_m = \rho_{m0}(1 + z)^{3(1 - \tau)} \left[ \frac{1 - \tau + (n - \tau)(\rho_{m0}/\rho_{\text{card}})^{q(n-1)}}{1 - \tau + (n - \tau)(\rho_m/\rho_{\text{card}})^{q(n-1)}} \right]^{\frac{1 - \tau}{q(n-1)}} \quad (2.12) \]
and the Hubble parameter is given by
\[ E^2 = \Omega_m(1 + z)^3(1 - \tau) \left[ \frac{\tilde{\tau}_2 + 1}{\tilde{\tau}_2 + F_{vmpc}(z)} \right]^{\frac{\tau}{n - \tau}} [1 + F_{vmpc}(z)(\Omega_m - 1)]^{\frac{\tau}{n - \tau}} , \] (2.13)
where
\[ \tilde{\tau}_2 = \left( \frac{\Omega_m}{1 - \Omega_m} \right) \left( \frac{1 - \tau}{n - \tau} \right) . \] (2.14)
Here the function \( F_{vmpc}(z) = (\rho_m/\rho_m0)^{q(n-1)} \) satisfies
\[ F_{vmpc} = (1 + z)^{3q(1-\tau)(n-1)} \left[ \frac{\tilde{\tau}_2 + 1}{\tilde{\tau}_2 + F_{vmpc}} \right]^{\frac{\tau(n-1)}{n - \tau}} , \] (2.15)
and when \( \tau = 0 \), the Hubble parameter (2.18) becomes
\[ E^2 = \Omega_m(1 + z)^3 \left[ 1 + (\Omega_m - 1)(1 + z)^3(q(n-1)) \right]^{\frac{\tau}{n - \tau}} , \] (2.16)
For the VEC model, \( f = \exp\left( e^{-ny} \right) \), and the solution is
\[ \rho_m = \rho_m0(1 + z)^3 \left[ \frac{n - (\rho_m/\rho_{card})(1 - \tau)}{n - (\rho_m/\rho_{card})(1 - \tau)} \right]^{\frac{\tau(n-1)}{n - \tau}} , \] (2.17)
and the Hubble parameter is given by
\[ E^2 = \Omega_m(1 + z)^3 \left[ \frac{\tilde{\tau}_3 + 1}{\tilde{\tau}_3 + F_{vec}(z)} \right]^{\frac{\tau(n-1)}{n - \tau}} \exp \left( - F_{vec}^{-1} \ln \Omega_m \right) , \] (2.18)
where
\[ \tilde{\tau}_3 = \left( \frac{n}{1 - \tau} \right) \ln \Omega_m . \] (2.19)
Here the function \( F_{vec}(z) = (\rho_m/\rho_m0)^{n} \) satisfies
\[ F_{vec} = (1 + z)^{3n} \left[ \frac{\tilde{\tau}_3 + 1}{\tilde{\tau}_3 + F_{vec}} \right]^{\frac{\tau(n-1)}{n - \tau}} , \] (2.20)
and when \( \tau = 0 \), the Hubble parameter (2.18) becomes
\[ E^2 = \Omega_m0(1 + z)^3 \exp \left( - (1 + z)^{-3n} \ln \Omega_m0 \right) . \] (2.21)
We summarize all the solutions of Hubble parameters for each model in Table 1.

3. Statistical analysis with the observational data

In general, the expansion history of the universe \( H(z) \) or \( E(z) \) can be given by a specific cosmological model or by assuming an arbitrary ansatz, which may be not physically motivated but just designed to give a good fit to the data for the luminosity distance \( d_L \) or the 'Hubble-constant free' luminosity distance \( D_L \) defined by
\[ D_L = \frac{H_0 d_L}{c} , \] (3.1)
where the light speed \( c \) is recovered to show that \( D_L \) is dimensionless. In the following, we will take the first strategy that assuming the Hubble parameter \( H(z; a_1, \cdots, a_n) \) with some parameters
where \( \Omega_k = -k/(a_0^2 H_0^2) \) and \( \sinh(x) = \sin(x), x, \sinh(x) \) for respectively a spatially closed \( (\Omega_k < 0) \), flat \( (\Omega_k = 0) \) and open \( (\Omega_k > 0) \) universe.

On the other hand, the apparent magnitude of the supernova is related to the corresponding luminosity distance by

\[
\mu(z) = m(z) - M = 5 \log_{10} \left( \frac{d_L(z)}{\text{Mpc}} \right) + 25, \tag{3.3}
\]

where \( \mu(z) \) is the distance modulus and \( M \) is the absolute magnitude which is assumed to be constant for standard candles like Type Ia supernovae. One can also rewrite the distance modulus in terms of \( D_L \) as

\[
\mu(z) = 5 \log_{10} D_L(z) + \mu_0, \tag{3.4}
\]

where

\[
\mu_0 = 5 \log_{10} \left( \frac{c H_0^{-1}}{\text{Mpc}} \right) + 25 = -5 \log_{10} h + 42.38, \tag{3.5}
\]

is the zero point offset, which is an additional model independent parameter. Thus, we obtain the predicted value of \( \mu^{th} \) by using the value of \( D_L^{th} \) and the observational value of \( \mu^{obs} \) we used is the latest data called the constitution data \([18]\), which contains 397 data points including the the 307 Union data set \([19]\) and 90 CFAs data set \([9]\) and 90 CFAs data set.

There are also some constraints from CMB and BAO observations. We will take the parameter \( R \) from the CMB data \([20]\) and the parameter \( A \) from the SDSS data \([21]\) as well as the supernova data to constrain parameters of the Cardassian models. The parameter \( R \) is defined as

\[
R = \sqrt{\Omega_{m0}} \sinh \left( \sqrt{\Omega_{k0}} \int_{0}^{z_{ls}} \frac{dz'}{E(z')} \right), \tag{3.6}
\]

where \( z_{ls} = 1090 \) is the redshift of the last scattering surface and the observational value is given by \( R_{obs} = 1.170 \pm 0.019 \). While the parameter \( A \) is defined as

\[
A = \sqrt{\Omega_{m0}} \frac{z_1}{E(z_1) |\Omega_{k0}|} \sinh^2 \left( \sqrt{\Omega_{k0}} \int_{0}^{z_1} \frac{dz'}{E(z')} \right)^{1/3}, \tag{3.7}
\]

where \( z_1 = 0.35 \) and the observational value is given by \( A_{obs} = 0.469(0.95/0.98)^{-0.35} \pm 0.017 \).

In order to determine the best value of parameters (with 1\( \sigma \) error at least ) in the Cardassian models, we will use the maximum likelihood method and need to minimize the following quantity

\[
\chi^2 = \chi^2_{SN} + \chi^2_{CMB} + \chi^2_{BAO}, \tag{3.8}
\]

where

\[
\chi^2_{CMB} = \left( \frac{R - 1.710}{0.019} \right)^2, \quad \chi^2_{BAO} = \left( \frac{A - 0.469(0.96/0.98)^{-0.35}}{0.017^2} \right)^2, \tag{3.9}
\]

and

\[
\hat{\chi}^2_{SN}(a_1, \ldots, a_n) = \sum_{i=1}^{397} \left[ \frac{\mu^{obs}(z_i) - 5 \log_{10} D_L^{th}(z_i; a_1, \cdots, a_n) - \mu_0}{\sigma_i^2} \right]^2, \tag{3.10}
\]
where $\sigma_i$ is the 1σ error of the observation value $\mu^{obs}(z_i)$. Since the nuisance parameter $\mu_0$ is model-independent, then we analytically marginalize it by using a flat prior $P(\mu_0) = 1$:

$$\chi^2_{SN} = -2 \ln \left( \int_{-\infty}^{+\infty} e^{-\chi^2/2} P(\mu_0) d\mu_0 \right) = a - \frac{b^2}{c} + \ln \left( \frac{c}{2\pi} \right), \quad (3.11)$$

where

$$a = \sum_{i=1}^{397} \left[ \frac{\mu^{obs}(z_i) - 5 \log_{10} D_L^th(z_i; a_1, \cdots, a_n)}{\sigma_i^2} \right]^2, \quad (3.12)$$

and

$$b = \sum_{i=1}^{397} \left[ \frac{\mu^{obs}(z_i) - 5 \log_{10} D_L^th(z_i; a_1, \cdots, a_n)}{\sigma_i^2} \right], \quad (3.13)$$

and

$$c = \sum_{i=1}^{397} \frac{1}{\sigma_i^2}, \quad (3.14)$$

then, from now on, we will work with $\chi^2_{SN}$ and to minimize $\chi^2$ in eq.(3.8). The best fit parameter values and the corresponding $\chi^2_{min}$ and $\chi^2_{min}/DOF$ will be summarized for each model in Table 2. Here DOF is the degree of freedom defined as

$$DOF = N - \nu, \quad (3.15)$$

where $N$ is the number of data points, and $\nu$ is the number of free parameters.

We now apply the maximum likelihood method for each model in Table 1, and we summarize the results in Table 2 including the minimum values of $\chi^2$ and the best fit parameters with 1σ confidence level for each model. Since both the FOC and FEC model contain two parameters, we also plot the contours from 1σ to 3σ confidence levels for them, see Fig. 1 and Fig. 2. For each model, the predicted dimensionless luminosity is plotted in Fig. 3, from which one can see that these Cardassian models predict almost the same luminosity distance with their best fit parameters.

| Model | $\chi^2_{min}$ | $\chi^2_{min}/DOF$ | Best Fit Parameters (1σ) |
|-------|----------------|---------------------|--------------------------|
| FOC   | 474.083        | 1.194               | $\Omega_{m0} = 0.270^{+0.024}_{-0.021}$, $n = 0.053^{+0.075}_{-0.075}$ |
| OC    | 473.084        | 1.195               | $\Omega_{m0} = 0.283^{+0.033}_{-0.033}$, $n = 0.023^{+0.098}_{-0.113}$, $\Omega_k = -0.010^{+0.015}_{-0.019}$ |
| VOC   | 473.101        | 1.195               | $\Omega_{m0} = 0.281^{+0.024}_{-0.031}$, $n = 0.010^{+0.010}_{-0.140}$, $\tau = 0.004^{+0.140}_{-0.150}$ |
| FMPC  | 473.746        | 1.196               | $\Omega_{m0} = 0.273^{+0.024}_{-0.023}$, $n = -0.600^{+0.380}_{-0.450}$, $q = 0.480^{+0.020}_{-0.080}$ |
| MPC   | 473.072        | 1.198               | $\Omega_{m0} = 0.285^{+0.030}_{-0.035}$, $n = 0.200^{+0.200}_{-3.100}$, $q = 1.48^{+1.320}_{-1.280}$, $\Omega_k = -0.015^{+0.010}_{-0.015}$ |
| VMPC  | 473.205        | 1.198               | $\Omega_{m0} = 0.279^{+0.026}_{-0.029}$, $n = -0.050^{+0.400}_{-2.900}$, $q = 0.900^{+2.000}_{-0.970}$, $\tau = 0.003^{+0.008}_{-0.007}$ |
| FEC   | 474.128        | 1.194               | $\Omega_{m0} = 0.277^{+0.024}_{-0.029}$, $n = 0.625^{+0.059}_{-0.051}$ |
| EC    | 474.127        | 1.197               | $\Omega_{m0} = 0.277^{+0.024}_{-0.027}$, $n = 0.626^{+0.076}_{-0.070}$, $\Omega_k = -0.0004^{+0.0213}_{-0.0196}$ |
| VEC   | 474.127        | 1.197               | $\Omega_{m0} = 0.276^{+0.024}_{-0.026}$, $n = 0.623^{+0.093}_{-0.089}$, $\tau = 0.0002^{+0.0082}_{-0.0082}$ |

**Table 2:** Result: The minimum value of $\chi^2$ and the best fit parameters with 1σ confidence level in each model.

### 4. Discussion

We have used the 397 SNe Ia, CMB and SDSS data to constrain several Cardassian models. We have summarized these model in Table 1, in which different forms of the function $g(\rho_m)$ have been
Figure 1: FOC: Constraints on $\Omega_{m0}$ and $n$ from 1$\sigma$ to 3$\sigma$ confidence level obtained by using 397 SNe Ia + CMB + BAO for the FOC model. The best fit point corresponds to $\Omega_{m0} = 0.270$, $n = 0.053$.

chosen and the corresponding Hubble parameters are also given. In particular, we discuss the viscous Cardassian models in Section 2., in which we rewrite the Hubble parameter in a continent way to do the statistical analysis.

The fitting results are presented in Table 2, in which we have shown the minimum value of $\chi^2$ and the minimum value $\chi^2_{\text{min}}$ per degree of freedom. The best fit parameters with 1$\sigma$ confidence level for each model are also presented in Table 2, from which one can see that, the latest observational data can not distinguish these models at this classical level. In other words, they predict almost the same evolution history of the universe and we need to take the perturbation of universe into account that will be studied in our further work.

In fact, the minimal of $\chi^2$ in eq.(3.8) is very sensitive to the observational error of the distance modulus. Once the error is smaller in the future data than that at present, not every model will fit the data well, then one can distinguish these models and even rule out some of them. Thus, more precise data are very needed.

Since in Cardassian universe, one can explain the accelerating expansion without introducing any dark energy component, it is very interesting and worth further studying. We also hope that future observation data could give more stringent constraints on the parameters in the Cardassian model.

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**Figure 2:** FEC: Constraints on $\Omega_{m0}$ and $n$ from $1\sigma$ to $3\sigma$ confidence level obtained by using 397 SNe Ia + CMB + BAO for the FEC model. The best fit point corresponds to $\Omega_{m0} = 0.277$, $n = 0.625$.

**References**

[1] C. J. Feng and X. Z. Li, Phys. Lett. B 680, 355 (2009), arXiv:0905.0527.

[2] B. Li and J. D. Barrow, Phys. Rev. D 79, 103521 (2009), arXiv:0902.3163 [gr-qc];
   J. D. Barrow, Phys. Lett. B 180, 335 (1987);
   I. Brevik, S. D. Odintsov, Phys. Rev. D 65, 067302 (2002) [arXiv:gr-qc/0110105];
   D. J. Liu and X. Z. Li, Phys. Lett. B 611, 8 (2005) [arXiv:astro-ph/0501506].

[3] C. Eckart, Phys. Rev. 58, 919 (1940).

[4] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Butterworth Heinemann, 1987).

[5] W. Israel, J. Ann. Phys. 100, 310 (1976).

[6] W. Israel and J. M. Stewart, Phys. Lett. A 58, 213 (1976);
   W. A. Miscock and J. Salmomson, Phys. Rev. D 43, 3249 (1991);
   R. Maartens, Class. Quantum Grav. 12, 1455 (1995).

[7] T. Harko and M. K. Mak, Class. Quantum Grav. 20, 407 (2003) [arXiv:gr-qc/0212075].

[8] I. H. Brevik and O. Gorbunova, Gen. Rel. Grav. 37, 2039 (2005) [arXiv:gr-qc/0504001].

[9] X. H. Zhai, Y. D. Xu and X. Z. Li, Int. J. Mod. Phys. D 15, 1151 (2006) [arXiv:astro-ph/0511814];
   J. Chen and Y. Wang, arXiv:0904.2808 [gr-qc];
   N. Cruz, S. Lepe and F. Pena, Phys. Lett. B 646, 177 (2007) [arXiv:gr-qc/0609013];
   X. H. Meng, J. Ren and M. G. Hu, Commun. Theor. Phys. 47, 379 (2007) [arXiv:astro-ph/0509250];
   J. Ren and X. H. Meng, Phys. Lett. B 633, 1 (2006) [arXiv:astro-ph/0511633];
   M. G. Hu and X. H. Meng, Phys. Lett. B 635, 186 (2006) [arXiv:astro-ph/0511615];
   A. Avelino and U. Nucamendi, JCAP 0904, 006 (2009) arXiv:0811.3253 [gr-qc].
   D. F. Mota, J. R. Kristiansen, T. Koivisto and N. E. Groeneboom, Mon. Not. Roy. Astron. Soc. 382, 793 (2007) arXiv:0708.0830 [astro-ph].
   T. Koivisto and D. F. Mota, Phys. Rev. D 73, 083502 (2006) [arXiv:astro-ph/0512135].
Figure 3: The observed 397 SNeIa distance modulus along with the theoretically predicted curves in the Cardassan models with best fit parameters, and we have taken a priori that current dimensionless Hubble parameter $h = 0.70$.

T. Padmanabhan and S.M. Chitre, Phys. Letts. A, 120, 433 (1987)

[10] W. Zimdahl and D. Pavón, Phys. Rev. D 61, 108301 (2000).
[11] Ø. Grøn, Astrophys. Space Sci. 173, 191 (1990).
[12] K. Freese and M. Lewis, Phys. Lett. B 540, (2002) 1.
[13] Y. Wang, K. Freese, P. Gondolo and M. Lewis, Astrophys. J. 594, (2003) 25.
[14] D. J. Liu, C. B. Sun and X. Z. Li, Phys. Lett. B 634, 442 (2006), arXiv:astro-ph/0512355.
[15] C. B. Sun, J. L. Wang and X. Z. Li, Int. J. Mod. Phys. D 18, 1303 (2009), arXiv:0903.3087 [gr-qc];
[16] T. S. Wang and N. Liang, arXiv:0910.5835 [astro-ph.CO].
T. S. Wang and P. Wu, Phys. Lett. B 678, 32 (2009), arXiv:0908.1438 [astro-ph.CO].
[17] P. Gondolo, K. Freese, Phys. Rev. D68, (2003) 063509.
[18] M. Hicken et al., Astrophys. J. 700, 1097 (2009), arXiv:0901.4804 [astro-ph.CO].
[19] M. Kowalski et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 686, 749 (2008),
arXiv:0804.4142 [astro-ph].
[20] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180, 330 (2009), arXiv:0803.0547
[astro-ph].
[21] D. J. Eisenstein et al. [SDSS Collaboration], Astrophys. J. 633, 560 (2005), arXiv:astro-ph/0501171.