Robust Graph Representation Learning for Local Corruption Recovery

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Abstract
Real-world graph observations may contain local corruptions by abnormal behaviors. While existing research usually pursues global smoothness in graph embedding, these rarely observed anomalies are harmful to an accurate prediction. This work establishes a graph learning scheme that automatically detects corrupted node attributes and recovers robust embedding for prediction tasks. The detection operation does not make any assumptions about the distribution of the local corruptions. It pinpoints the positions of the anomalous node attributes in an unbiased mask matrix, where robust estimations are recovered with an $\ell_{p,q}$ regularizer. We alleviate an inertial alternating direction method of multipliers to approach a new embedding that is sparse in the framelet domain and conditionally close to input observations. Extensive experiments validate the model recovers robust graph representations from black-box poisoning and achieves excellent performance.

1. Introduction
Graph neural networks (GNNs; Bronstein et al. (2017); Wu et al. (2020b); Zhou et al. (2020); Zhang et al. (2020); Atz et al. (2021)) have received tremendous success in the past few years. Graphs, as the input of GNNs, record useful features and structural information, and they exist widely in many fields such as biomedical science (Ahmedt-Aristizabal et al., 2021), social networks (Fan et al., 2019), and recommendation systems (Wu et al., 2019), and recommendation systems (Wu et al., 2020a).

Similar to other real-world observations, inaccurate observations are ubiquitous in graphs with a noticeable side-effect for graph representation learning. For instance, fraudulent users in social media tend to fake user avatars or online activities. A recommender might be dysfunctional by mislabeled items or users. Such disruptive observations hinder the model fitting and prediction. The feature aggregation in graph representation learning accentuates the negative influence of irregular entities to their neighborhoods, resulting in a misleading latent feature representation for the predictor.

To this end, this research proposes to protect the graph prediction performance by recovering a robust representation for the irregular entities.

Existing works are aware of the harmful graph anomalies. Graph anomaly detection (Ding et al., 2019; Peng et al., 2020; Zhu et al., 2020; Ma et al., 2021) identifies the small portion of problematic nodes; graph defense refines the learning manner of a classifier to provide promising predictions against potential threats (Dai et al., 2018; Zügner & Günnemann, 2019; Xu et al., 2020); optimization-based graph convolutions smooth out global noise by special regularization designs (Liu et al., 2021; Zhou et al., 2021; Zhu et al., 2021; Chen et al., 2021). However, the first candidate detects irregular nodes rather than node attributes, and it does not amend the flawed representation of the identified outlier. The second approach provides an adequate solution to amend the prediction performance, but most researches focus on graph rewiring, as edges are believed more vulnerable to adversarial attacks. Last but not least, the third choice usually makes assumptions on the distribution of feature corruptions that they are normally observed in all inputs.

Instead, this paper develops a ‘detect-and-then-recovery’ strategy to save graph representation learning from a small portion of hidden corruptions in input node attributes. In particular, an unsupervised encoder module first exposes suspicious attributes and assigns a mask matrix to record their positions. The detector requires no prior on the distribution of the anomalous attributes. The constructed mask is submitted to a sparsity regularizer with graph framelet transforms (Dong, 2017; Zheng et al., 2021) to find a robust approximation of the initial input, where the iterative optimizing scheme acts similar to graph convolutional layers that constantly smooth the hidden feature representation.
for GNN prediction tasks. With the transforms of the input signal, the posterior mask guarantees that the denoising is predominantly conducted at the essential spots, and the representation pumps up the robustness significantly.

Combining the three key ingredients of Mask, ADMM, and Graph, we name our model as MAGNET. Figure 1 illustrates the three components for unsupervised mask construction, localized robust optimization, and graph representation learning. The proposed MAGNET detects local corruptions in an unsupervised manner and approximates a robust representation for graph network training. We also develop an efficient inertial alternating direction method of multipliers (ADMM) algorithm, which provides a faster convergence than the plain ADMM with time complexity of $O(k^{-1})$.

2. Problem Formulation

An undirected attributed $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{X})$ has $n = |\mathcal{V}|$ nodes with the interactions described by an adjacency matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$. The observed $d$-dimensional node features $\mathbf{X} \in \mathbb{R}^{n \times d}$ is a noised version of the ground truth signal $\mathbf{U}$, i.e.,

$$\mathbf{X} = \mathbf{U} + \mathbf{E}_1 + \mathbf{E}_2,$$

where we call $\mathbf{E}_1$ global noise and $\mathbf{E}_2$ outliers or local corruptions. The main difference between $\mathbf{E}_1$ and $\mathbf{E}_2$ is that the former might universally exist on the entire graph, and the latter generally take a small chance of existence so it cannot be observed from a large set of nodes. We hereby make three assumptions on the properties of $\mathbf{U}$, $\mathbf{E}_1$, and $\mathbf{E}_2$: (a) $\mathbf{E}_1 \sim \mathcal{D}(0, \sigma)$ where $\sigma$ is considerably small; (b) $\mathbf{E}_2$ take a small portion of the entire graph; and (c) $\mathbf{X}$ is close to $\mathbf{U}$ besides the anomalous locations. The associated objective function to estimate $\mathbf{U}$ reads

$$\min_{\mathbf{U}} \alpha \text{Reg}(\mathbf{U}) + \beta \text{Loss}(\mathbf{E}_2)$$

such that $\mathbf{X}|_M = (\mathbf{U} + \mathbf{E}_1 + \mathbf{E}_2)|_M,$

where $\alpha, \beta$ are tuning parameters, and $M$ is a mask matrix of outliers indices. The $\text{Reg}(\cdot), \text{Loss}(\cdot)$ denote regularizers and loss functions satisfying assumptions (a) and (b).

**Choice of Loss($\mathbf{E}_2$).** The loss function plays the role of finding an object $\mathbf{U}$ that approximates the input feature $\mathbf{X}$. Assumption (b) requires a measure with respect to the sparsity of $\mathbf{E}_2$ has to be optimized. The best choice for sparsity, in theory, is $\ell_0$-norm that counts the number of nonzero entries in $\mathbf{E}_2$. While solving a $\ell_0$ constrained problem is NP-hard, $\ell_1$-norm allows feasible solution for (2) that promotes good sparsity measure of $\mathbf{E}_2$ (Chen et al., 2015). Such a design has been practiced in residual analysis-based anomaly detection methods to label suspicious small outliers (Li et al., 2017; Peng et al., 2018).

**Choice of Reg($\mathbf{U}$).** The regularization term is a penalty complementary to the loss, which controls the noise level of $\mathbf{X}$ by the smoothness of $\mathbf{U}$, and it is usually quantified by some type of energy. For instance, GCN (Kipf & Welling, 2017) utilizes normalized Dirichlet energy of $\mathbf{U}$ by $\text{tr}(\mathbf{U}^\top \mathbf{L} \mathbf{U})$, where $\mathbf{L}$ denotes the normalized graph Laplacian from $\tilde{\mathbf{L}} = \mathbf{I} - \mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}}$ with the adjacency matrix $\mathbf{A}$ and its degree matrix $\mathbf{D}$. Minimizing such energy encourages message transmissions among connected nodes. By extracting the summary of neighborhood space, unnoticeable noise is likely to be smoothed out. In addition, minimizing the Dirichlet energy implies a low-rank solution to $\mathbf{U}$ (Monti et al., 2017). Such a graph smoothing effect...
(Zhu et al., 2021; Liu et al., 2021) that minimizes Dirichlet energy can be found in many spatial-based graph convolutions (Klicpera et al., 2018; Xu et al., 2018; Wu et al., 2019). Alternatively, spectral-based methods regularizes the graph signal in a transformed domain by $L$. For example, Dong (2017) and Zhou et al. (2021) minimize the $\ell_1$-norm total variation of framelet coefficients, and Mahmood et al. (2018) take a similar regularization in the wavelet domain. As the total variation reflects redundant local fluctuations, a solution to minimize it is believed to remove noisy details while simultaneously preserving essential patterns.

**Restriction on $E_1 + E_2$.** Compared to the main ingredients of the objective function, the treatment to the fidelity constraint is rather trivial if the outlier $E_2$ does not exist, where a regularizer can be adopted to minimize the difference between $X$ and $U + E_1$. When $E_2$ is assumed with a small percentage, it becomes irrational to force $U$ to approximate $X$ especially on anomalous locations. Instead, a conditional approximation should be placed with the index matrix $M$, in which case only attributes at regular positions are required aligned, i.e., $\text{Reg} (M \odot (X - U))$. The regularization can be appended to the main objective by

$$\min_{U} \alpha \text{Reg}(U) + \beta \text{Loss} (M \odot (X - U)).$$

The above optimization for graph signal recovery is well-defined. Nevertheless, it has three issues in application. First, a direct minimization of the Dirichlet energy in spatial domain usually falls into the pitfall of over-smoothing, where the recovered graph loss expressivity drastically (Balciar et al., 2020). On the other hand, spectral transforms can be sophisticated and time-consuming, which is generally circumvented by the majority of studies. Second, restricting a $\ell_1$ or $\ell_2$ norm does not adapt to a specific dataset or application, which could give rise to the precision loss of the recovered graph representation. Last but not least, attaining the mask matrix $M$ in (3) can be nasty in practice, as the prior knowledge of $M$ is generally inaccessible.

We hereby formulate a new objective function

$$\min_{U} \|
u \mathcal{W} U\|_{p,G} + \frac{1}{2} \|M \odot (U - X)\|_{q,G}^2,$$  \tag{4}

where $\mathcal{W}$ is a set of multi-scale and multi-level framelet decomposition operators. See Appendix A for a brief explanation on the fast approximation of $\mathcal{W}$. These decomposition operators transform the input feature $U$ to low-pass and high-pass framelet coefficients in the framelet (spectral) domain. The two hyperparameters $\alpha, \beta$ in (3) are deduced to $\nu$, which is a set of tunable parameters adaptive to high-frequency framelet coefficients in different scales. We also replace the ordinary Euclidean $\ell_p$-norm with a graph $\ell_k$-norm, denoted as $\ell_{k,G}$, to assign higher penalties to influential nodes. For an arbitrary node $v_i$ of degree $D_{ii}$, $\|v_i\|_{k,G} := \left(\|v_i\|^k \cdot D_{ii}\right)^{\frac{1}{k}}$.

Compared to the initial design (2), (4) made three adjustments to tackle the identified issues. First, minimizing the first component smooths out $E_1$ from high-pass framelet coefficients, which avoids information loss by spatial Dirichlet energy minimization. In other words, the global noise $E_1$ is removed without sacrificing small-energy features. Secondly, we adopt $\ell_{p,q}$-norm to adaptively restrict the sparsity of recovered graph with tunable $p \in [1, 2], q \in [0, 1]$. As introduced in Section 4, the optimization can be solved by an inertial version of the alternating direction method of multipliers with promising convergence.

A potential solution to the unreachable mask matrix $M$ is to add a sub-problem of optimization to (4), which introduces a two-stage optimization problem. However, this approach blows up the difficulty of solving the existing problem and the mask could be a very complicated region to reveal. Instead, we consider an unsupervised GNN scheme to automate the anomalous positions discovery. We approximate the anomaly score of the raw feature matrix with a classic graph anomaly detection scheme that looks for the reconstruction error between the raw feature matrix and the reconstructed matrix from neural networks.

### 3. Graph Anomaly Detection for Mask Matrix

Graph anomaly detection is an important subproblem of the general graph signal recovery where outliers are assumed to exist in the input. A detector identifies nodes with $E_2$ as community anomalies (Ma et al., 2021), which is defined as nodes that have distinct attribute values compared to their neighbors of the same community. The underlying assumption here is that the outlier is sparse and their coefficients are antipathetic from the smooth graph signal $U$. We hereby consider GNN-based algorithms to investigate the difference of each node from the representation of its local community.

#### 3.1. Graph Autoencoder

Autoencoder is a powerful tool for reconstructing corrupted objects (Aggarwal, 2017; Kovenko & Bogach, 2020). GAE (Kipf & Welling, 2016) is a GCN-based classic graph autoencoder network, which revisions have drawn substantial interest for graph anomaly detection (Ding et al., 2019; Peng et al., 2020; Zhu et al., 2020).

**Network Training.** A GAE layer takes the information of $G$ to obtain an embedding $Z \in \mathbb{R}^{n \times h}$ of a hidden size $h$ by $Z = \text{GCN}(X, A)$. Alternatively, one trains a feed-forward network to recover graph attributes by $X' = \text{FFN}(Z)$. The $X'$ is a smoothed version of $X$, which is believed removing essential noise or minor corruptions of the raw input. In the design by Ding et al. (2019) and Peng et al. (2018) for graph anomaly detection, the learning objective of the neural network is to best reconstruct the graph data.
(X′, A′), which loss function is the weighted average of the squared reconstruction errors
\[ \mathcal{L} = \alpha \| X - X' \|_2. \]

**Anomaly Score.** Once the graph is reconstructed, it is straightforward to find outliers by comparing the input and output representation. For node entities, a corrupted object has different patterns from its local community, so its initial representation should be distinct from the smoothed representation. The reconstruction error is thus an appropriate indicator for diagnosing such divergence. The anomaly score for a particular node vi reads
\[ \text{score}(v_i) = \| x_i - x_i' \|_2. \]

3.2. Mask Matrix Generation

Unlike conventional anomaly detection tasks, we aim at generating a mask matrix of size n × d that identifies the anomalous attributes, instead of corrupted node entities. In addition, our primary focus is on graph recovery which does not explicitly consider the disordered node connections or the adjacency matrix. We thus define the mask matrix by tweaking the anomaly score function
\[ M = 1 - \text{threshold}(\| X - X' \|_1, \tau) \]  

By differentiating the raw feature matrix and the reconstructed matrix from a graph autoencoder, we establish a mask matrix that deduces element-wise reconstruction errors. It is then binarized with the threshold value \( \tau \in (0, 1) \). For the jth feature of node i, \( M_{ij} = 0 \) indicates the value is trustworthy and \( M_{ij} = 1 \) suggests assigning a new value to avoid potential corruption.

Under assumption (b) that \( E_2 \) rarely exist, the vast majority of the mask matrix should be 1, i.e., \( 1 - M \) should be a sparse matrix. This requirement is double-confirmed by the autoencoder algorithm and the thresholding function. As the autoencoder does simply blurs X to reach the smooth U and the magnitude of \( E_1 \) is assumed smaller than \( E_2 \)'s, the difference of X and X' mainly comes from \( E_2 \). On top of that, the tunable \( \tau \) makes further adjustments on the sparsity of M. Figure 2 supplements empirical evidence for the effect of selecting different \( \tau \) on the sparsity of \( 1 - M \) with perturbed citation networks. An increasing \( \tau \) is illustrated to reduce the number of non-zero elements in \( 1 - M \) from a considerably sparse level to a lower degree.

4. An Inertia Alternating Direction Method of Multipliers (ADMM)

This section introduces the numerical scheme to solve (4).
4.2. Subproblems

The solutions to the subproblems of $Z_{k+1}$ and $U_{k+1}$ depend on $p$ and $q$. Here we enumerate the special cases of $p = \{0, 1\}$, $q = \{1, 2\}$ which covers the maximum and minimum sparsity in regularization.

**Choices of $p$.** Different values of $p$ affects the thresholding operator in the update of $Z_{k+1}$. For the case of $p = 1$, $\|\nu Z\|_{p,G}$ becomes the $\ell_1$-norm and its update requires a soft-thresholding, that is,

$$S_\alpha(x) = \text{sign}(x) \circ \max \{ |x| - \alpha, 0 \}.$$  

When $p = 0$, a hard-thresholding defines $H_\alpha(x) = x$ when $|x| > \alpha$, and $H_\alpha(x) = 0$ otherwise.

**Choices of $q$.** Compared to $Z_{k+1}$, the update of $U_{k+1}$ in (8) is more complicated as it involves more regularization terms. When $q = 2$, the objective is a quadratic problem.

With $\mathbf{W}^T \mathbf{W} = \text{Id}$, differentiating the function gives

$$U_{k+1} = \left( M \odot X - \mathbf{W}^T \tilde{V}_{k+1} \right) / (M + \gamma),$$  

which requires an element-wise division. Since the fast approximation of $\mathbf{W}$ is implemented by the Chebyshev approximation, it happens when the approximation degree is considerably small that the approximation has noticeable error, i.e., $\mathbf{W}^T \mathbf{W} \neq \text{Id}$. Alternatively, the descent type methods such as gradient descent, conjugate gradient can be applied to inexacty solve the sub-problem with $I$ steps of iteration. At the $i$th ($i \leq I$) iteration,

$$U^{(i+1)} = U^{(i)} - \alpha \left( M \odot (U^{(i)} - X) + \gamma \mathbf{W}^T (\mathbf{W} U^{(i)} + \tilde{V}_{k+1}/\gamma) \right),$$  

where $\alpha$ is the step-size. The solution is then $U_{k+1} = U^{(I)}$.

In the case of $q = 1$, let $Q = U - X$ and consider

$$\frac{1}{2} \| M \odot Q \|_{1,G} + \frac{1}{2} \| Q + X + \mathbf{W}^T \tilde{V}_{k+1}/\gamma \|_2^2,$$

under the optimality condition. The $Q_{k+1}$ is the soft-thresholding of $-X - \mathbf{W}^T \tilde{V}_{k+1}/\gamma$ and

$$U_{k+1} = Q_{k+1} + X.$$  

5. Numerical Experiments

5.1. Experimental Protocol

**Benchmark Preparation.** We examine MAGNET on five benchmark datasets: Cora, Citeseer and PubMed of the citation networks (Yang et al., 2016), Wiki-CS (Mernyei & Cangea, 2020) that classifies articles from Wikipedia database, and Coauthor-CS (Shchur et al., 2018) that labels the most active field of authors. As the given datasets do not provide ground truth of anomalies, we conduct two types of black-box poisoning methods that have been used in graph anomaly detection and graph defense. A **attribute injection** method (Song et al., 2007; Ding et al., 2019) perturbs attribute through swapping attributes of the most distant samples in a random subgraph. A set of anomalies can be obtained from different random subgraphs. We also adopt a **graph adversarial attack** (Zügner & Günnemann, 2019) that leverages meta-learning to pollute node attributes with the meta-gradient of the loss function.

**Training Setup.** For a fast recovering from the corrupted graph, an inertial ADMM is iterated for 15 times. The GAE to approximate binary reconstruction errors for the mask matrix consists of two GCN encoding layers followed with two GCN attribute decoding layers. The hidden representation is sent to GAE attribute reconstruction module for the last case does not participate in the performance comparison. Instead, it indicates the upper limit of MAGNET with the perfect mask matrix. We compare our model to three popular baseline models for graph smoothing but with different design philosophy: APPNP (Klicpera et al., 2018) that avoids global smoothness with residual connections; GNNguard (Zhang & Zitnik, 2020) that modifies the neighbor relevance in message passing to mitigate local corruption; and ELASTICGNN (Liu et al., 2021) that pursues local smoothness with a mix of $\ell_1$ and $\ell_2$ regularizers.
Table 2. Average performance with targeted adversarial attack

| Module          | CORA         | CITESER     | PUBMED      |
|-----------------|--------------|-------------|-------------|
| clean           | $81.26\pm0.65$ | $71.77\pm0.29$ | $79.01\pm0.44$ |
| corrupted       | $75.07\pm0.64$ | $55.32\pm2.22$ | $72.88\pm0.30$ |
| APPNP           | $73.49\pm0.59$ | $55.67\pm0.28$ | $70.63\pm1.97$ |
| GNNGuard        | $72.02\pm0.61$ | $57.64\pm1.31$ | $71.10\pm0.32$ |
| ELASTICGNN      | $79.25\pm0.50$ | $67.29\pm1.17$ | $71.95\pm0.52$ |
| MAGNET-gae (ours) | $77.11\pm0.45$ | $62.49\pm1.70$ | $75.83\pm2.05$ |
| MAGNET-gae (ours) | $79.04\pm0.50$ | $67.40\pm0.73$ | $78.63\pm0.32$ |
| MAGNET-true     | $80.88\pm0.37$ | $67.16\pm0.95$ | $79.16\pm0.41$ |

5.2. Node Classification with Graph Recovery

Table 1-2 compare model performance under two types of local corruptions, where Table 2 excludes Wiki-CS and Coauthor-CS as the attribute poison of meta-attack is ineffective on them. We highlight each score in red (green) by their relative improvement (regression) over the non-smoothed corrupted baseline (row 2).

It can be observed that MAGNET-gae outperforms its competitors and recovers at most 94% prediction accuracy from the perturbed attributes. Furthermore, a more accurate mask approximation could push the prediction performance of graph representation up to MAGNET-true’s scores. On the other hand, the three baseline graph smoothing methods fail to denoise local corruption within the input. In some cases they are fooled to make worse predictions, as they are not able to distinguish outliers from regular patterns. APPNP leverages residual connections to avoid global smoothness by reinforcing local anomalies; GNNGuard makes modifications on the edge connection to indirectly influence node representations; ELASTICGNN, although realizing the importance of local smoothness, designs the optimization with low-pass filters and restricts stringent consistency of the new representation even on anomalous positions.

5.3. Further Investigation

We next visualize the effect of the GAE-oriented mask approximation and ADMM optimization.

Mask Approximation. We first verify the quality of GAE through the recall of the approximation. Figure 3 pictures the conditional mask matrix from model reconstruction error on Cora with attribute injection. A $200 \times 200$ sub-region is amplified for both the ground truth matrix and the approximated mask matrix at the middle, which indicates clearly the sparsity of both matrices. The approximated mask matrix succeeds in seizing 60% of anomalies, which provides a reliable foundation to the subsequent recovering work. Other visualizations of different datasets are in Appendix C.

ADMM Optimization. Figure 4 visually exhibits the effect of ADMM optimization on local graph in inpainting with

6. Conclusion

We develop MAGNET, a graph neural network for recovering a robust graph data representation from locally corrupted node attributes. The key computational unit for coping with regional outliers is based on a sparse and multi-scale regularizer with a mask of the anomalous positions in graph node attributes, which is approximated with an unsupervised graph autoencoder that requires no prior knowledge on the distribution of anomalies. The optimization problem is decorated with an $l_{p,q}$ regularization with an efficient inertial ADMM, where the tunable $p$, $q$ stimulate the maximum ability of the optimizer. The multi-level framelet coefficients
removes global noises and regularizes local anomalous attributes simultaneously. Our proposed model achieves satisfying performance to recovering a robust graph embedding from local corruptions. In contrast, graph smoothing and defense baseline methods fail to provide a decent solution.

Acknowledgements

YW, JL and XZ acknowledge support from the Shanghai Municipal Science and Technology Major Project (2021SHZDZX0102). YW and JL also acknowledges support from SJTU and Huawei ExploreX Funding (SD6040004/034). We are also grateful to the anonymous reviewers for their feedback.

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A. Graph Framelet Transform

This section briefs the fast computation of undecimated framelet transform on graphs. The initial idea was proposed by Dong (2017), which is then developed to a graph convolution by Zheng et al. (2021). The key to the implementation is to make fast approximation on the framelet decomposition and reconstruction operators through Chebyshev approximation.

Framelet transform divides an input signal to multiple channels by a set of low-pass and high-passes framelet operators. For a specific nodes $p$ of a signal $x$ at scale $j \in \mathbb{Z}$, we define the undecimated framelets $\varphi_{j,p}(g), \psi_{j,p}(g)$ by

$$
\begin{align*}
\varphi_{j,p}(g) & := \sum_{\ell=1}^{N} \tilde{\alpha} \left( \frac{2^j}{\pi} \right) u_{\ell}(p) u_{\ell}(v), \\
\psi_{j,p}(g) & := \sum_{\ell=1}^{N} \tilde{\beta}(r) \left( \frac{2^j}{\pi} \right) u_{\ell}(p) u_{\ell}(v), \quad r = 1, \ldots, n.
\end{align*}
$$

(12)

For two integers $J, J_1$ ($J > J_1$), an undecimated framelet system from a scale $J_1$ UFS$_J^J$ $(\Psi, \eta)$ is an undecimated tight frame for $l_2(\mathcal{G})$, which is a non-homogeneous, stationary affine system:

$$
\begin{align*}
\text{UFS}_{J_1}^J (\Psi, \eta) & := \text{UFS}_{J_1}^J (\Psi, \eta; \mathcal{G}) \\
& := \{ \varphi_{J_1,p} : p \in V \} \cup \{ \psi_{J_1,p} : p \in V, j = J_1, \ldots, J \}_{r=1}^n.
\end{align*}
$$

(13)

The elements in UFS$_J^J$ $(\Psi, \eta)$ are the undecimated tight framelets on $\mathcal{G}$.

We call $\Psi = \{ \alpha; \beta^{(1)}, \ldots, \beta^{(K)} \}$ a set of scaling functions, and it is determined by a filter bank $\eta := \{ a; b^{(1)}, \ldots, b^{(K)} \}$.

In particular, we consider the Haar-type filter with one high pass. For $x \in \mathbb{R}$, it defines

$$
\tilde{\alpha}(x) = \cos(x/2) \quad \text{and} \quad \tilde{\beta}^{(1)}(x) = \sin(x/2).
$$

The other component that plays a key role for embedding graph topology is the eigenpairs $\{ (\lambda, u) \}_{j=1}^n$ of the graph Laplacian $\mathcal{L}$. The set of framelet decomposition operator projects input signals to a transformed domain as framelet coefficients. To allow fast approximation of the filter spectral functions, $m$-order Chebyshev polynomials is considered. Denote the $m$-order approximation of $\alpha$ and $\{ \beta^{(1)}, \ldots, \beta^{(K)} \}$ by $\mathcal{T}_m^0$ and $\{ \mathcal{T}_k^m \}_{k=1}^K$, the framelet decomposition operators at $(r,j) \in \{(1,1), \ldots, (J_1, \ldots, (n,1), \ldots, (n,J) \} \cup \{(0,J)\}$ is defined by

$$
W_{k,1} = \begin{cases} 
\mathcal{T}_r (2^{K} \mathcal{L}) , & j = 1, \\
\mathcal{T}_r (2^{K+j-1} \mathcal{L}) \mathcal{T}_0 (2^{K+j-2} \mathcal{L}) \ldots \mathcal{T}_0 (2^{K} \mathcal{L}) , & \text{otherwise},
\end{cases}
$$

where the dilation scale $K$ satisfies $\lambda_{\max} \leq 2^K \pi$. In this definition, the finest scale is $1/2^{K+J}$ that guarantees $\lambda_{\ell}/2^{K+J-\ell} \in (0, \pi)$ for $j = 1, 2, \ldots, n$.

The approximated $W$ is used in the penalty term of the objective function (4) formulated in Section 2. To learn more facts about the undecimated framelet transform on graph, we refer the readers to the work by Dong (2017); Zheng et al. (2021).
We denote $Z$ (Bot & Csetnek, 2014; Bot et al., 2015). This section provides essential details for the inertial ADMM to understand the update rules defined in Section 4.

B. More details on the ADMM algorithm

This section provides essential details for the inertial ADMM to understand the update rules defined in Section 4.

B.1. Inertial ADMM

We denote $Z = \mathcal{W}U$ and rewrite (4) as

$$\min_{U, Z} \|\nu Z\|_{p,G} + \frac{1}{2} \|M \circ (U - X)\|_{q,G}^{q}, \text{ such that } Z = \mathcal{W}U.$$ 

This forms a standard formulation of problems that can be solved by Alternating Direction Method of Multipliers (ADMM; Gabay & Mercier (1976)). The associated augmented Lagrangian reads

$$L(U, Z; Y) := \|\nu Z\|_{p,G} + \frac{1}{2} \|M \circ (U - X)\|_{q,G}^{q} + \langle Y, \mathcal{W}U - Z \rangle + \frac{1}{2} \|\mathcal{W}U - Z\|^{2}.$$ 

To find a saddle-point of $L(U, Z; Y)$, ADMM applies the following iteration

$$Z_{k+1} = \arg \min_{Z} \|\nu Z\|_{p,G} + \frac{\gamma}{2} \|\mathcal{W}U - Z\|^{2} + \langle Y_{k}, \mathcal{W}U - Z \rangle,$$

$$U_{k+1} = \arg \min_{U} \frac{1}{2} \|M \circ (U - X)\|_{q,G}^{q} + \frac{\gamma}{2} \|\mathcal{W}U - Z_{k+1}\|^{2} + \langle Y_{k}, \mathcal{W}U - Z_{k+1} \rangle,$$

$$Y_{k+1} = Y_{k} + \gamma (\mathcal{W}U_{k+1} - Z_{k+1}).$$

The above iteration can be equivalently written as

$$Z_{k+1} = \arg \min_{Z} \|\nu Z\|_{p,G} + \frac{\gamma}{2} \|\mathcal{W}U - Z + Y_{k}/\gamma\|^{2},$$

$$U_{k+1} = \arg \min_{U} \frac{1}{2} \|M \circ (U - X)\|_{q,G}^{q} + \frac{\gamma}{2} \|\mathcal{W}U - Z_{k+1} + Y_{k}/\gamma\|^{2},$$

$$Y_{k+1} = Y_{k} + \gamma (\mathcal{W}U_{k+1} - Z_{k+1}).$$

If we further define

$$V_{k+1} = Y_{k} - \gamma Z_{k+1}.$$ 

The above iteration can be reformulated as

$$Z_{k+1} = \arg \min_{Z} \|\nu Z\|_{p,G} + \frac{\gamma}{2} \|Z - \mathcal{W}U_{k} - Y_{k}/\gamma\|^{2},$$

$$= \arg \min_{Z} \|\nu Z\|_{p,G} + \frac{\gamma}{2} \|Z - (2Y_{k} - V_{k})/\gamma\|^{2},$$

$$V_{k+1} = Y_{k} - \gamma Z_{k+1},$$

$$U_{k+1} = \arg \min_{U} \frac{1}{2} \|M \circ (U - X)\|_{q,G}^{q} + \frac{\gamma}{2} \|\mathcal{W}U + V_{k+1}/\gamma\|^{2},$$

$$Y_{k+1} = Y_{k} + \gamma (\mathcal{W}U_{k+1} - Z_{k+1})$$

$$= V_{k+1} + \gamma \mathcal{W}U_{k+1}.$$ 

In this paper, we consider the inertial ADMM motivated by Alvarez & Attouch (2001), whose iteration is provided below:

$$Z_{k+1} = \arg \min_{Z} \|\nu Z\|_{p,G} + \frac{\gamma}{2} \|Z - (2Y_{k} - \bar{V}_{k})/\gamma\|^{2},$$

$$V_{k+1} = Y_{k} - \gamma Z_{k+1},$$

$$\bar{V}_{k+1} = V_{k+1} + a_{k}(V_{k+1} - V_{k}),$$

$$U_{k+1} = \arg \min_{U} \frac{1}{2} \|M \circ (U - X)\|_{q,G}^{q} + \frac{\gamma}{2} \|\mathcal{W}U + \bar{V}_{k+1}/\gamma\|^{2},$$

$$Y_{k+1} = Y_{k} + \gamma (\mathcal{W}U_{k+1} - Z_{k+1})$$

$$= \bar{V}_{k+1} + \gamma \mathcal{W}U_{k+1}.$$ 

In general, we have $a_{k} \in [0, 1]$ if $g_{k} \in [0, 1]$ (Boţ & Csetnek, 2014; Boţ et al., 2015).
B.2. Solution to Subproblem of $q = 1$

When $q = 1$, let $Q = U - X$, and consider

$$
\min_Y \frac{1}{2} \| M \odot Y \|_{1,G} + \frac{\gamma}{2} \| WY + WX + \tilde{V}_{k+1}/\gamma \|^2.
$$

The optimality condition yields

$$
\frac{1}{2} M \odot \partial \| M \odot Q \|_{1,G} + \gamma W^T (WQ + WX + \tilde{V}_{k+1}/\gamma)
\iff \frac{1}{2} M \odot \partial \| M \odot Q \|_{1,G} + \gamma (Q + X + W^T \tilde{V}_{k+1}/\gamma)
\iff \frac{1}{2} \| M \odot Q \|_{1,G} + \frac{\gamma}{2} \| Q + X + W^T \tilde{V}_{k+1}/\gamma \|^2.
$$

From above we have that $Q_{k+1}$ is the soft-thresholding of $-X - W^T \tilde{V}_{k+1}/\gamma$ and

$$
U_{k+1} = Q_{k+1} + X.
$$
C. Experiments

This section provides more details on the experiments conducted in this work.

C.1. Dataset

Table 3 documents key descriptive statistics of the five datasets for the node classification tasks. We make this particular selection to include the most classic citation network (Cora, Citeseer, and Pubmed (Yang et al., 2016)), a dataset with (relatively) dense edge connection (wiki-cs (Mernyei & Cangea, 2020)), and a dataset with (relatively) high dimension of feature attributes (coauthor-cs (Shchur et al., 2018)).

|                | Cora     | Citeseer | Pubmed   | wiki-cs  | coauthor-cs |
|----------------|----------|----------|----------|----------|-------------|
| # Nodes        | 2,708    | 3,327    | 19,717   | 11,701   | 18,333      |
| # Edges        | 5,429    | 4,732    | 44,338   | 216,123  | 100,227     |
| # Features     | 1,433    | 3,703    | 500      | 300      | 6,805       |
| # Classes      | 7        | 6        | 3        | 10       | 15          |
| # Training Nodes| 140      | 120      | 60       | 580      | 300         |
| # Validation Nodes | 500   | 500      | 500      | 1769     | 200         |
| # Test Nodes   | 1,000    | 1,000    | 1,000    | 5847     | 1000        |
| Label Rate     | 0.052    | 0.036    | 0.003    | 0.050    | 0.016       |

C.2. Poison Preparation

We detail here the pre-processing we conduct on the two types of graph poison methods.

Injection We add the injection noise following a similar strategy by Ding et al. (2019). A certain number of targeted nodes are randomly selected from the graph and ready to change their attributes without perturbing the edge connectivity. For each selected node $i$, we randomly pick another $k$ nodes from the graph and select the node $j$ whose attributes deviate the most from node $i$ among the $k$ nodes by maximizing the Euclidean distance $\|x_i - x_j\|_2$. Then, we substitute the attributes $x_i$ of node $i$ with $x_j$. In this work, we set the value of $k$ to 100 for small datasets such as CORA and CITeseer, and $k$ to 500 for relative larger datasets such as PUBMED, COAUTHOR-CS and WIKI-CS.

Meta-attack The method meta-attack in perturbing the attributes is achieved by (Zügner & Günnemann, 2019). Although meta-attack mainly perturbs the edge connectivity in (Zügner & Günnemann, 2019), we take the technique to create local noise on the graph by corrupting a small amount of attributes in the feature matrix of a graph. In this work, the amount of perturbation varies for different graphs to obtain a noticeable attack effect.

C.3. Model Preparation

We disclose the full details of all the models examined in the experiments, including the access of model implementation, and their tuning space. All the experiments are conducted with PyTorch on NVIDIA® Tesla A100 GPU with 6,912 CUDA cores and 80GB HBM2 mounted on an HPC cluster. All benchmark datasets are publicly available in the PyTorch Geometry library.

C.3.1. Availability of Model Implementation

We have uploaded our model to https://github.com/bzh3923/MAGnet/tree/main/submission. In addition, we take the official implementation of the baseline models from the repository:

- GNNGuard: https://github.com/mims-harvard/GNNGuard
- ElasticGNN: https://github.com/lxiaorui/ElasticGNN
We now reveal the tuning space of the implemented models as the guarantee of reproducible results and fair comparison. Table 2. The precise relative scores are reported in Table 5 and Table 6, respectively. This section supplements the percentage improvement of node classification tasks mentioned in the main text and in Table 1 and Table 2. The relative score of improvements is calculated by

$$\text{Relative Score} = \frac{S - S_{\text{corrupted}}}{S_{\text{clean}} - S_{\text{corrupted}}}$$

where $S$ denotes the current score of the performed model, and $S_{\text{clean}}$ and $S_{\text{corrupted}}$ are the accuracy score of GCN on the clean dataset and corrupted dataset, respectively. We use the relative score to highlight the performance in Table 1 and Table 2. The precise relative scores are reported in Table 5 and Table 6, respectively.

### Table 4. Hyperparameter searching space for node classification.

| Hyperparameters | CORA absolute | CORA relative | CITISEER absolute | CITISEER relative | PUBMED absolute | PUBMED relative | COAUTHOR-CS absolute | COAUTHOR-CS relative | WIKI-CS absolute | WIKI-CS relative |
|-----------------|---------------|---------------|-------------------|-------------------|----------------|-----------------|----------------------|---------------------|----------------|-----------------|
| Learning rate   | 5e-3          | 5e-3          | 5e-3              | 5e-3              | 5e-3           | 5e-3           |                      |                     |                 |                 |
| Weight decay ($L_2$) | 5e-3          | 5e-3          | 1e-3              | 1e-3              | 1e-3           | 1e-3           |                      |                     |                 |                 |
| Hidden size     | 128           | 128           | 128               | 128               | 128            | 128            |                      |                     |                 |                 |
| Dropout ratio   | 0.5           | 0.5           | 0.5               | 0.5               | 0.5            | 0.5            |                      |                     |                 |                 |
| Epochs          | 100           | 100           | 100               | 100               | 100            | 100            |                      |                     |                 |                 |

### C.4. Percentage Performance Improvement in Node Classification

This section supplements the percentage improvement of node classification tasks mentioned in the main text and in Table 1 and Table 2. The relative score of improvements is calculated by

$$\text{Relative Score} = \frac{S - S_{\text{corrupted}}}{S_{\text{clean}} - S_{\text{corrupted}}}$$

where $S$ denotes the current score of the performed model, and $S_{\text{clean}}$ and $S_{\text{corrupted}}$ are the accuracy score of GCN on the clean dataset and corrupted dataset, respectively. We use the relative score to highlight the performance in Table 1 and Table 2. The precise relative scores are reported in Table 5 and Table 6, respectively.

### Table 5. Improvement percentage of average performance for node classification with non-targeted local corruption

| Module                  | CORA absolute | CORA relative | CITISEER absolute | CITISEER relative | PUBMED absolute | PUBMED relative | COAUTHOR-CS absolute | COAUTHOR-CS relative | WIKI-CS absolute | WIKI-CS relative |
|-------------------------|---------------|---------------|-------------------|-------------------|----------------|-----------------|----------------------|---------------------|----------------|-----------------|
| APPNP                   | -0.95%        | -5.44%        | 4.27%             | 17.34%            | 1.49%          | 8.92%           | -13.68%              | -148.29%            | -13.62%         | -73.15%         |
| GNNGUARD                | -10.36%       | -58.98%       | -4.58%            | -17.38%           | -1.20%         | 7.16%           | -2.11%               | -22.37%             | 0.00%           | 0.00%           |
| ELASTICGNN              | 12.47%        | 71.00%        | 12.21%            | 49.54%            | 5.23%          | 31.27%          | -3.03%               | -32.89%             | -1.93%          | -10.34%         |
| MAGNET-one (ours)       | 9.78%         | 55.68%        | 2.85%             | 11.31%            | 1.89%          | 10.80%          | 1.98%                | 20.95%              | 8.24%           | 44.25%          |
| MAGNET-gae (ours)       | 11.13%        | 63.34%        | 12.52%            | 50.81%            | 11.40%         | 68.20%          | 4.96%                | 52.57%              | 10.64%          | 57.14%          |
| MAGNET-true             | 13.50%        | 76.85%        | 19.05%            | 72.22%            | 11.73%         | 70.14%          | 8.28%                | 87.66%              | 15.37%          | 82.59%          |

### Table 6. Improvement percentage of average performance for node classification with targeted local corruption

| Module                  | CORA absolute | CORA relative | CITISEER absolute | CITISEER relative | PUBMED absolute | PUBMED relative |
|-------------------------|---------------|---------------|-------------------|-------------------|----------------|----------------|
| APPNP                   | -2.10%        | -25.53%       | 0.63%             | 2.13%             | -3.09%         | -36.70%        |
| GNNGUARD                | -4.06%        | -49.27%       | 4.19%             | 14.10%            | -2.44%         | -29.04%        |
| ELASTICGNN              | 5.57%         | 67.53%        | 21.64%            | 72.77%            | -1.28%         | -15.17%        |
| MAGNET-one (ours)       | 2.72%         | 32.96%        | 12.96%            | 43.59%            | 4.05%          | 48.12%         |
| MAGNET-gae (ours)       | 5.29%         | 64.14%        | 21.84%            | 73.43%            | 7.89%          | 93.80%         |
| MAGNET-true             | 7.74%         | 93.86%        | 21.95%            | 73.80%            | 8.62%          | 102.45%        |
We also report the absolute score of improvement in the two tables above for a direction comparison, which is

\[
\text{Absolute Score} = \frac{S - S_{\text{corrupted}}}{S_{\text{corrupted}}}
\]

C.5. Ablation Study

This section reports the performance of the ablation models on the node classification tasks. We investigate the influence of \( p, q \) in different datasets of different types of local corruption. The particular performance comparison under anomaly injection and mettack are listed in Table 7 and Table 8, respectively.

| Module | Dataset       | CORA   | CiteSeer | PUBMED | COAUTHOR-CR | WIKI-CR |
|--------|---------------|--------|----------|--------|-------------|---------|
| N/A    | \( L_2 \) L_2| 69.12±0.57 | 57.58±0.71 | 67.69±0.23 | 82.41±0.40 | 65.44±0.23 |
| ONE    | \( L_0 \) L_1| 58.36±0.89 | 57.30±0.71 | 65.77±1.03 | 82.17±0.41 | 64.58±0.21 |
|        | \( L_1 \) L_2| 68.74±0.22 | 54.07±1.23 | 58.52±1.02 | 80.63±0.59 | 63.63±0.27 |
|        | \( L_1 \) L_1| 69.12±0.57 | 57.58±0.71 | 67.69±0.20 | 82.17±0.41 | 64.63±0.18 |
|        | \( L_2 \) L_2| 75.88±0.42 | 59.22±0.34 | 68.97±0.21 | 84.04±0.56 | 70.83±0.29 |
| GAE    | \( L_0 \) L_1| 68.42±1.15 | 54.38±0.54 | 67.74±0.71 | 83.95±0.52 | 61.96±0.16 |
|        | \( L_1 \) L_2| 66.34±0.81 | 56.29±1.18 | 59.15±0.85 | 83.88±0.55 | 64.67±0.29 |
|        | \( L_1 \) L_1| 72.76±0.40 | 63.60±0.66 | 75.41±0.35 | 86.02±0.59 | 72.40±0.21 |
|        | \( L_2 \) L_2| 76.81±0.98 | 64.79±0.14 | 71.13±0.25 | 86.50±0.37 | 70.08±0.39 |
| TRUE   | \( L_0 \) L_1| 77.15±0.74 | 68.14±0.85 | 74.40±0.56 | 88.14±0.46 | 72.42±0.29 |
|        | \( L_2 \) L_2| 76.44±0.59 | 65.02±0.97 | 68.12±1.21 | 87.01±0.24 | 71.51±0.20 |
|        | \( L_1 \) L_1| 78.48±0.67 | 68.55±0.74 | 75.63±0.56 | 89.23±0.37 | 75.50±0.20 |
|        | \( L_2 \) L_2| 77.57±0.92 | 64.29±0.19 | 75.19±0.18 | 86.72±0.31 | 74.44±0.23 |

| Module | Dataset       | CORA   | CiteSeer | PUBMED |
|--------|---------------|--------|----------|--------|
| N/A    | \( L_2 \) L_2| 75.07±0.64 | 55.32±0.64 | 72.88±0.30 |
| ONE    | \( L_0 \) L_1| 71.21±1.12 | 55.38±2.12 | 71.52±0.43 |
|        | \( L_2 \) L_2| 74.46±0.60 | 57.75±2.23 | 59.76±1.91 |
|        | \( L_1 \) L_2| 75.07±0.64 | 55.32±2.22 | 72.88±0.65 |
|        | \( L_0 \) L_1| 77.11±0.45 | 62.49±1.70 | 75.83±0.35 |
| GAE    | \( L_0 \) L_1| 77.42±1.08 | 56.13±0.47 | 78.42±0.63 |
|        | \( L_2 \) L_2| 77.03±0.78 | 55.84±1.37 | 70.82±0.38 |
|        | \( L_1 \) L_2| 78.18±1.56 | 63.22±1.56 | 78.63±0.32 |
|        | \( L_0 \) L_2| 79.04±0.50 | 67.40±0.73 | 74.47±0.30 |
| TRUE   | \( L_0 \) L_1| 80.88±0.37 | 67.46±0.95 | 79.16±0.41 |
|        | \( L_2 \) L_2| 80.57±0.51 | 67.21±1.63 | 71.90±0.41 |
|        | \( L_1 \) L_2| 79.99±0.45 | 65.50±0.81 | 79.14±0.32 |
|        | \( L_2 \) L_2| 77.16±0.66 | 67.33±0.49 | 75.04±0.32 |

C.6. GAE visualization

Figure 5-Figure 7 visualize the sparse mask matrix of the five datasets with anomaly injection. We are interested in the recall of the mask matrix that exposes the quality of the mask matrix approximation. As a result, we print the conditional mask matrix that marks the positions of approximated index that are at the true anomalous spots. Note that we did not visualize
the mask matrix from the datasets under mettack perturbation, as such attack focuses major poisonings in a minority of node entities. When making visualizations on such matrices, they are nothing more than a few horizontal lines.
Figure 5. citeepr

Figure 6. first 3000 rows and columns only. The full matrix is too large to display (18000x7000)

Figure 7. pubmed: first 3000 nodes; 500 features; threshold=0.005

Figure 8. Wikics: first 3000 nodes; 300 features; threshold=0.05
C.7. Computational Speed

We show the inertial effect of ADMM by the counting the computational speed of our model on Cora. We specifically consider two settings with and without the boosting effect, i.e., $\gamma$ in (8) equals to 0 or not. Below we report the running time of a complete update process under certain convergent condition $\epsilon$, i.e., $\|U_{k+1} - U_k\| < \epsilon$. The speed is counted in seconds.

| Hyperparameter searching space for node classification. | w/ boost (sec) | w/o boost (sec) |
|--------------------------------------------------------|---------------|-----------------|
| $p = 1, q = 2$                                         | 49.38         | 54.16           |
| $p = 1, q = 1$                                         | 90.44         | 94.84           |
| $p = 0, q = 2$                                         | 12.43         | 13.05           |
| $p = 0, q = 1$                                         | 4.37          | 5.43            |