Physics of the Insulating Phase in the Dilute Two-Dimensional Electron Gas

Victor M. Yakovenko*1, Victor A. Khodel++

*Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA
+Russian Research Centre Kurchatov Institute, Moscow, 123182, Russia

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We propose to use the radio-frequency single-electron transistor as an extremely sensitive probe to detect the time-periodic ac signal generated by sliding electron lattice in the insulating state of the dilute two-dimensional electron gas. We also propose to use the optically-pumped NMR technique to probe the electron spin structure of the insulating state. We show that the electron effective mass and spin susceptibility are strongly enhanced by critical fluctuations of electron lattice in the vicinity of the metal-insulator transition, as observed in experiment.

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Detecting electron lattice with the single-electron transistor. The metal-insulator transition (MIT) in the two-dimensional electron gas (2DEG) attracts considerable interest [1, 2]. In this paper, we focus on physics of the insulating phase. The great majority of experiments are transport measurements, and only few are thermodynamic. Dultz and Jiang [3] measured compressibility $\kappa$ of the 2DEG as a function of carrier concentration $n$ and found that it tends to vanish in the insulating phase, i.e. the phase is incompressible. The experimental dependence $\kappa(n)$ was semi-quantitatively reproduced within both the scenario of electron localization (E-LOC) [5] and the scenario of electron lattice (E-LAT) formation [6, 7]. We use the term E-LAT to denote any state with local periodic modulation of electron density. The Wigner crystal (WC) and the charge-density wave (CDW) are the limiting cases of E-LAT, where the modulation amplitude is comparable to $n$ and is much less than $n$, respectively. For simplicity, we call the carriers electrons, even though they may actually be holes.

Ilani et al. [6, 7] measured compressibility locally, using the single-electron transistor (SET) as a microscopic probe. They found that $\mu(n)$ has a series of quasirandom jumps, which become very strong in the insulating phase. These jumps were interpreted as single-electron charging events [6, 7] within the E-LOC scenario. Alternatively, the jumps can be interpreted as a manifestation of E-LAT [7]. When the average carrier concentration $n$ is changed by the back gate, the period $l$ of E-LAT must adjust, because it is proportional to the average distance between electrons $a = 1/\sqrt{n}$. However, because E-LAT is pinned by impurities, it cannot adjust its period continuously. Instead, E-LAT accumulates stress until it overcomes the pinning force and then makes a sudden local rearrangement of the lattice, which results in a jump of the local potential. Both the E-LOC and E-LAT scenarios are plausible, and it is difficult to decide between them on the basis of the known experimental data. Here we propose a modification of the experiments [6, 7], which may help to distinguish between the two scenarios.

In Ref. [8], Pudalov et al. observed a very nonlinear current-voltage ($I-V$) relation in the insulating phase of the 2DEG in Si-MOSFET. Almost no current $I$ flows until electric field reaches the threshold field $E_t$, and then $I$ sharply surges at $E > E_t$, accompanied by the broad-band noise. Pudalov et al. interpreted their findings in terms of collective sliding of E-LAT depinned by the strong electric field $E > E_t$, which produces the large current $I$ and generates the broad-band noise due to the local slip-stick motion. The $I-V$ nonlinearity was found to be extremely sharp, with the differential conductivity increasing by the factor of $10^6$, in the samples with the highest mobility and rounded in the samples with poor mobility [9]. These results suggest that the transition to the insulating state is not driven by disorder, as assumed by the E-LOC theories, but by E-LAT formation. The $I-V$ nonlinearity was also observed in GaAs samples [10]. It was shown that the MIT deduced from the temperature dependence of resistivity is the same one as deduced from the $I-V$ nonlinearity [11].

We propose to combine the SET experiment with the nonlinear $I-V$ experiments. Suppose a strong pulling
electric field $\mathcal{E} > \mathcal{E}_c$ is applied, and E-LAT slides. Then
the SET would register a time-periodic ac signal with the
frequency $\nu = v/l$ produced by E-LAT of the spatial
period $l$, which slides with the velocity $v$. This effect is
nothing but the narrow-band noise (NBN), well-known
for CDW in the quasi-one-dimensional (Q1D) conductors [12].
Unlike in the Q1D conductors, attempts to observe the NBN in regular transport measurements in
the 2DEG failed thus far [13]. We propose that the SET is a better tool for detecting the NBN, because of
its very high sensitivity and because it is a local, micro-
scopic probe, unlike the macroscopic current leads.
In the experiment [7], the SET was situated at the dis-
cance $d = 400$ nm from the 2DEG. This distance is com-
parable to the average distance between the carriers
$a = 1/\sqrt{n} = 100$ nm in the experiment [7] performed
on $p$-GaAs with the typical hole concentration in the
insulating phase $n = 10^{10}$ cm$^{-2}$. Because $d$ and $l \sim a$
are comparable, the SET should experience a noticeable
time-dependent signal when the periodically-modulated
electron charge density slides past the SET. Reducing $d$
and bringing the SET closer to the 2DEG would further
increase sensitivity.

Let us estimate the frequency $\nu = v/l$ of the ac signal.
The E-LAT period $l$ is of the order of the average
distance between the carriers $l \sim a = 1/\sqrt{n}$.
The sliding velocity $v$ is related to the current density
$j = I/W = env$, where $I$ is the total current, and $W$ is
the transverse width of the sample. Thus we find

$$\nu \approx \frac{I}{e} \frac{1}{\sqrt{n}W} \approx \frac{6}{\sqrt{n}W} \frac{\text{MHz}}{\text{pA}} \approx \frac{I}{\sqrt{n}W}, \quad (1)$$

For a crude estimate of the current density in the sliding
regime, we use the data from Ref. [10]. $j = I/W \approx
0.4$ nA/0.4 mm = 1 nA/mm. (The data from Ref. [8]
give a similar estimate.) Substituting these numbers
into Eq. (1), we find $\nu \approx 600$ kHz. The frequency scale
is similar to that of the Q1D CDW [12]. Unfortunately,
the frequency range of a typical SET is limited to less
than 1 kHz. Thus, it is necessary to use the radio-
frequency SET (RF-SET) [14], which can operate from
dc to 100 MHz. With this experimental setup, it should be
desirable to detect the ac signal at the frequency $\nu$.

Eq. (1) shows that $\nu$ is proportional to the current $I$
carried by the sliding E-LAT, and the slope of that
dependence is proportional to $1/\sqrt{n}$. An experimental
observation of this effect would be the definitive proof
of the existence of E-LAT in the dilute 2DEG. Peri-
docity in time is the direct consequence of periodicity
in space, and the E-LOC scenarios cannot produce a
periodic ac signal from the dc current. Although dis-
order destroys the long-range order of E-LAT [15, 16],
the local periodicity is preserved and would produce
the NBN peak in the Fourier spectrum. On the other hand,
even if the RF-SET will not find a time-periodic signal,
the measured time series would provide important mi-
roscopic information about electron conduction, such as
the variable-range hopping. For example, uncorre-
lated single-electron hops would generate the Poisson
stochastic process in the simplest case.

**Probing spin order with the optically pumped NMR.** Besides the question of charge ordering in the
insulating state of the MIT, there is a question of spin
ordering in that state. One of the great tools for obtaining
information about electron spins is the nuclear magnetic
resonance (NMR). In the quantum Hall regime, the optically-pumped NMR measurements on the $^{71}$Ga
nuclei in $n$-GaAs detected formation of skyrmions in the
electron spin configuration for small deviations from the
filling factor $\nu = 1$ [17]. In the $\nu = 1$ state, electrons are
spontaneously spin-polarized and produce a significant
effective magnetic field on the nuclei via the hyperfine
interaction. Thus, the NMR line of the nuclei in contact
with the 2DEG experiences the measurable Knight
shift proportional to the spontaneous spin polarization
of electrons [17, 18].

We propose to use a suitable modification of the
same method to study the spin properties of the 2DEG
in the insulating state in zero effective magnetic field.
A magnetic field is needed for NMR, but we want to
eliminate its effect on electrons. This can be achieved
by engineering a situation where the electron $g$-factor is
zero. For example, this is the case for a magnetic field
parallel to the [100] surface of $p$-GaAs [19, 20]. It is
also possible to achieve $g = 0$ by applying hydrostatic
pressure [21].

For the Wigner crystal, different types of spin ordering
were proposed theoretically: ferromagnetic [22], anti-
ferrromagnetic [22], and various exotic orderings [13].
In the ferromagnetic state, the NMR line should experi-
ence a measurable Knight shift, detection of which
would be the proof of spontaneous spin polarization of
electrons. In the antiferromagnetic state, the NMR line
would broaden, because the nuclei experience a stag-
gered hyperfine field from the electrons. This method
is routinely used to detect formation of spin-density waves
(SDW) in Q1D conductors [12]. On the other hand,
when a strong electric field $\mathcal{E} > \mathcal{E}_c$ is applied, it forces
SDW or E-LAT to slide. Then the nuclei experience
the time-averaged hyperfine magnetic field produced by
electrons, and the NMR line becomes narrow again [23]
(the so-called motion narrowing). An observation of
these effects in NMR would provide a great deal of infor-
mation about spin ordering of electrons in the insulating state and would put the ongoing theoretical discussion of the subject on a firm experimental ground.

Enhancement of the effective mass and spin susceptibility. Experiments [2, 21, 22, 23, 24, 25, 26, 27, 28] consistently show that the electron effective mass \( m^* \) and the effective spin susceptibility \( \chi^* \) strongly increase when \( n \rightarrow n_c \) from the metallic side, where \( n_c \) is the critical density of the MIT. This phenomenon has a simple explanation within the E-LAT scenario. The theory was developed in Refs. 29, 30, 31, and here we only briefly summarize the main physical idea.

The experiments [2, 22] show that the threshold field \( \mathcal{E}_t \) and the thermal activation gap of resistivity continuously vanish at \( n \rightarrow n_c \). Thus, the phase transition between the metallic phase at \( n > n_c \) and the insulating phase at \( n < n_c \) is of the second order. More precisely, it was found to be slightly of the first order [32], as expected by the symmetry reasons for a triangular or hexagonal lattice [33]. These results are in qualitative agreement with the self-consistent Hartree-Fock calculations [4], which show that E-LAT continuously evolves from the CDW limit to the WC limit with the decrease of \( n \) from \( n_c \).

Assuming that the system has a tendency to form E-LAT with the wave vector \( q_c \sim 1/a \), we can write the charge response function \( S_0(q) = S(q, \omega = 0) \) in the following form [33] in the vicinity of the phase transition for \( n > n_c \):

\[
S_0(q) \simeq \frac{C_1}{n - n_c + (q - q_c)^2},
\]

where \( C_1 \) is a constant, and \( q \) is momentum transfer. Electrons can interact via exchange of the critical fluctuations [2]. This interaction manifests itself in the Landau interaction function \( f(\theta) \propto S_0(|p_1 - p_2|) \), where \( p_1 \) and \( p_2 \) are the momenta of the interacting electrons, and \( \theta \) is the angle between \( p_1 \) and \( p_2 \). Substituting this formula in the Landau equation for the effective mass \( m^* \) [34], we find

\[
\frac{1}{m^*} = \frac{1}{m} - C_2 \int \frac{\cos \theta \, d\theta}{n - n_c + (|p_1 - p_2| - q_c)^2},
\]

where \( C_2 \propto C_1 \) is another constant. Taking into account that \( |p_1| = |p_2| = p_F \), where \( p_F \) is the Fermi momentum, we obtain

\[
\frac{1}{m^*} = \frac{1}{m} - C_2 \int \frac{\cos \theta \, d\theta}{n - n_c + (2p_F \sin(\theta/2) - q_c)^2}.
\]

Assuming that \( q_c \sim 2p_F \), we see that the integral in Eq. (4) is peaked around \( \theta \sim \pi \), where \( \cos \theta < 0 \). Because of the Fermi statistics, the exchange interaction originating from the positive Coulomb repulsion is negative, so \( C_2 < 0 \). Thus, the interaction term in Eq. (4) causes an increase in the effective mass \( m^* \).

Moreover, the integral in Eq. (4) diverges at \( n \rightarrow n_c \). In the case \( q_c < 2p_F \), it diverges as \((n - n_c)^{-1/2} \) [29]:

\[
\frac{m}{m^*} = 1 - \frac{C_3}{\sqrt{n - n_c}},
\]

where \( C_3 > 0 \) is another constant. As \( n \) approaches \( n_c \), the effective mass \( m^* \) diverges in Eq. (5) even earlier, at \( n \rightarrow n_{\infty} = n_c + C_3^2 \), where the last term in Eq. (5) becomes equal to 1. Remember that we defined \( n_c \) in Eq. (2) as the electron density where E-LAT forms. The spin susceptibility is also enhanced via the standard relation \( \chi^* = g^* m^* \).

In Fig. 1 we compare \( m^*(n) \) given by Eq. (5) with the experimental data for Si-MOSFET from Fig. 27 of Ref. [2]. The experimental points were obtained from the Shubnikov-de Haas oscillation (SdH, squares) and from the spin-polarizing parallel magnetic field (\( B_{\parallel} \), circles). In order to determine the parameters in Eq. (5), we plot \((1 - m/m^*)^{-2}\) vs. \( n \) in Fig. 2. According to Eq. (5), the slope of this linear dependence is \( 1/C_3^2 \), and it crosses zero at \( n = n_c \). The effective mass \( m^* \) diverges at \( n = n_{\infty} \), where the straight line crosses level 1. The parameters of our fit are \( n_c = 0.7 \times 10^{-11} \text{ cm}^{-2} \), \( n_{\infty} = 0.9 \times 10^{-11} \text{ cm}^{-2} \), and \( C_3 = 0.46 \).

The purpose of Figs. 1 and 2 is not to produce a detailed quantitative fit of the experimental data, but only to demonstrate qualitative agreement between the theory and experiment. One should keep in mind that Eq. (5) is not applicable for \( n \gg n_c \) far away from the E-LAT transition. Very close to the transition point, the singularities predicted by Eq. (5) may be cut off by the weakly first-order character of the phase transition [32]. The divergence of \( m^* \) in Figs. 1 and 2 is obtained only as an extrapolation of the available experimental data. Taking into account frequency dependence of \( S(q, \omega) \) can modify the theory to make \( n_{\infty} = n_c \). Nevertheless, the qualitative agreement between the theory and experiment gives an argument in favor of the E-LAT scenario for the MIT in the 2DEG.

This theory is also applicable to other system experiencing transition for a liquid to a crystalline phase. Such a transition is observed in the 2D He-3, and the experiment [35] finds a very strong enhancement of \( m^* \) in the vicinity of the transition. Notice that there is no disorder in liquid He-3, so the E-LOC scenario is irrelevant in this case.

\footnote{In the case \( q_c = 2p_F \), the last term in Eq. (5) diverges as \((n - n_c)^{-3/4} \).}
Conclusions. We propose to use the radio-frequency single-electron transistor (RF-SET) \cite{11} as an extremely sensitive probe \cite{6, 7} to detect the time-periodic ac signal generated by sliding electron lattice (E-LAT) at $E > E_t$ in the insulating state of the 2DEG. An observation of this narrow-band-noise effect would be the definitive proof of E-LAT formation in the dilute 2DEG. We also propose to use the optically-pumped NMR technique \cite{17} to probe the electron spin structure of the insulating state, which may have ferromagnetic, antiferromagnetic, or exotic types of spin ordering. NMR can be performed in a magnetic field without disturbing electron spins in a situation where the electron $g$-factor is engineered to be zero \cite{19, 21}. Within the Landau theory of Fermi liquids, we show that critical fluctuations of E-LAT near the metal-insulator transition produce a strong enhancement of the effective mass $m^\ast$ and spin susceptibility $\chi^\ast$ \cite{29, 30, 31} in qualitative agreement with the experiments in the 2DEG \cite{24, 25, 26, 27, 28}, as well as in the 2D He-3 \cite{35}. This is an argument in favor of the E-LAT scenario.

Although we concentrated on physics of the 2DEG in zero magnetic field, the same ideas also apply to the Wigner crystal in a non-zero magnetic field perpendicular to the 2DEG \cite{36, 37}.

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