INTERPLAY OF INTERFERENCE EFFECTS IN PRODUCTION OF $J/\Psi$ AND $\Psi'$ OFF NUCLEI

Jörg Hufner
Institut für Theor. Physik der Universität
Philosophenweg 19, 69120 Heidelberg, Germany

Boris Kopeliovich
Max-Planck-Institut für Kernphysik, 69029 Heidelberg, Germany, and
Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia

Abstract

Our main observations are:

(i) The effective path length of the $c\bar{c}$ wave packet, which is produced in the nucleus, grows with the energy of the produced charmonium. The variation is controlled by the coherence length $l_c = 2E_\Psi/M_\Psi^2$.

(ii) A colorless $c\bar{c}$ wave packet produced in $pp$-interaction is a specific linear combination of the $J/\Psi$ and $\Psi'$ states, and interacts substantially weaker that any of its components. The time evolution of this wave packet is controlled by the formation length, $l_f = 2E_\Psi/(M_{\Psi'}^2 - M_\Psi^2)$.

Exact formulas incorporating with these effects are derived. The interplay of the two phenomena results in a nontrivial energy- and $x_F$-dependence of the nuclear suppression of the charmonium production in proton-nucleus and nucleus-nucleus collisions and explains some of the experimentally observed effects.

*Invited talks presented by B.K. at the XXXI st Rencontres de Moriond, "QCD and High-Energy Hadronic Interactions" March 23-30, 1996, Les Arcs, France and at PANIC'96, May 22-28, 1996, Williamsburg, USA*
1. Introduction

The experiments of producing a $\Psi$ (we use the symbol $\Psi$ instead of $J/\Psi$) or a $\Psi'$ meson in a collision of a hadron $h$ with a nucleus $A$, at energies $E_h$ of several hundreds of GeV have yielded a number of unexpected results, of which we will recall the most significant ones. Since we want to limit ourselves to nuclear effects and not absolute cross sections, it is convenient to introduce the nuclear suppression function

$$S^A_h(E, x_F) = \frac{1}{A} \frac{d\sigma^{hA\rightarrow\Psi A}(E_h, x_F)}{d\sigma^{hN\rightarrow\Psi A}(E_h, x_F)} dF,$$

which depends on the energy $E_h$ of the hadron, and the Feynman variable $x_F$ of the $\Psi$.

The experimental results under discussion here are:

(i) The value for $S^{pA}_\Psi(E_h, x_F)$ in the interval $0 < x_F < x_0$ seems to depend on $E_h$, the suppression factor being smaller for $E = 800$ GeV $^1$ than for $E = 200$ GeV $^2$.

(ii) For $x_F > 0$ in $pA$ collisions one has nearly the same nuclear suppression for the $\Psi$ and $\Psi'$ mesons$^1$.

(iii) $\Psi'$ turns out to be more suppressed than $J/\Psi$ in nucleus-nucleus collisions, $S^{AB}_\Psi < S^{AB}_\Psi$ $^3$.

In the present paper we discuss quantum interference effects, which are not ad hoc mechanisms (and do not need any unknown parameters), but can explain, at least partially, the above effects.

2. Glauber theory. The coherence length.

Nuclear suppression of charmonium depends on the production mechanism even in the simplest case of eikonal approximation. Charmonium production on a nucleon at high energy may be seen in the lab. frame as interaction of a fluctuation of the projectile hadron, containing charm quarks, which frees the charmonium. We single out two types of interaction:

a) Direct interaction of the $c\bar{c}$ projectile fluctuation with the target. This interaction must be sufficiently hard to resolve the size of the $c\bar{c}$ pair in order to make it colorless.

b) Interaction of the light spectator partons accompanying the $c\bar{c}$ pair, freeing the charmonium. This interaction can be soft i.e. have a large cross section at $x_1 \rightarrow 1$ $^4$.

We assume hereafter the dominance of the direct mechanism a), which restricts our consideration to small values of $x_F$.

Since the charmonium production is a hard process, a soft initial/final state interaction, which cannot resolve the $c\bar{c}$ fluctuation, does not produce any shadowing. Charmonia produced on different nucleons add up incoherently, since the longitudinal momentum transfer is large, $q_L = E_q(1 - x_1)$, where $x_1 = (x_F + \sqrt{x_F^2 + 4M_{\Psi}^2/s})/2$.

There is also a possibility of an additional hard scattering which frees the $c\bar{c}$ pair "elastically" in advance of the inelastic interaction. Namely, the projectile hadron can experience a hard diffractive excitation with a colorless exchange (Pomeron) in $t$-channel, which puts the $c\bar{c}$ fluctuation on mass shell. The longitudinal momentum transfer to the target nucleon may be small, provided that the energy is high,

$$q_c \approx \frac{M_{\Psi}^2}{2E_{\Psi}},$$

(2)
If \( q_c \ll 1/R_A \), different nucleons contribute coherently.

The excited hadron propagates through the nucleus and produces the final charmonium with energy \( x_1 E_h \) in another interaction. Although the hard diffractive cross section is quite small, such a correction turns out to be very important, since it substantially increases the attenuation of the charmonium at high energy.

We skip the full expression for \( S_{\gamma A}^{\Psi} \), which is too lengthy and can be found in \(^5,^6\), where it was derived for the first time. That expression can be simplified using smallness of \( \sigma_{\Psi N}^{\Psi N} \langle T \rangle \ll 1 \) (compare with \(^7\)).

\[
S_{\Psi}^{hA}(E_{\Psi}) \approx 1 - \frac{1}{2} \sigma_{\Psi N}^{\Psi N} \langle T \rangle \left[ 1 + F_A^2(q_c) \right],
\]

where \( \langle T \rangle \) is the mean nuclear thickness and the nuclear "longitudinal formfactor”;

\[
F_A^2(q_c) = \frac{1}{A(T)} \int d^2 b \left| \int_{-\infty}^{\infty} dz \rho_A(b, z) e^{i q_c z} \right|^2
\]

(4)

takes into account the phase shifts between the waves produced at different points.

We conclude from \(^\|\) that the nuclear shadowing correction at high energy \( (F_A^2(q_c) \to 1) \) is twice as big as at low energy \( (F_A^2(q_c) \to 0) \). This important result has a natural space-time interpretation: at high energy the lifetime of the \( c \bar{c} \) fluctuation of the photon, \( t_c = 1/q_c \) (called coherence time or length), is long and the mean path of the \( c \bar{c} \) pair in the nucleus is doubled compared to that at low energy \(^8\). Thus, \( S_{\Psi}^{hA} \) decreases with \( x_F \).

3. **Beyond the Glauber model. The formation length**

In order to improve the eikonal Glauber approximation one should take into account the off diagonal diffractive rescatterings of the charmonium in the nucleus. We restrict our consideration to a two-coupled-channel problem, including \( \Psi = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \) and \( \Psi' = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \). The initially produced \( c \bar{c} \) state \( \Psi_0 \) has its representation

\[
|\Psi_0\rangle = \frac{1}{\sqrt{1 + R^2}} \left( \begin{array}{c} 1 \\ R \end{array} \right),
\]

(5)

The evolution of this state through the nucleus can written in matrix representation,

\[
i \frac{d}{dz} \begin{pmatrix} \alpha(z) \\ \beta(z) \end{pmatrix} = \hat{U}(b, z) \begin{pmatrix} \alpha(z) \\ \beta(z) \end{pmatrix},
\]

(6)

where

\[
\hat{U} = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} - \frac{i}{2} \sigma_{\Psi N}^{\Psi N} \rho_A(b, z) \begin{pmatrix} 1 & \epsilon \\ \epsilon & r \end{pmatrix},
\]

(7)

with

\[
\epsilon = \frac{\langle \Psi' | \hat{f} | \Psi \rangle}{\langle \Psi | \hat{f} | \Psi \rangle}
\]

(8)

and

\[
r = \frac{\langle \Psi' | \hat{f} | \Psi \rangle}{\langle \Psi | \hat{f} | \Psi \rangle} = \frac{\sigma_{\Psi N}^{\Psi N}}{\sigma_{\Psi N}^{\Psi N}},
\]

(9)
\( f \) is the operator of the \( c\bar{c} \)-nucleon diffractive scattering amplitude. \( q_1 \) and \( q_2 \) are the transferred longitudinal momenta in photoproduction of \( \Psi \) and \( \Psi' \) respectively, as it is defined in eq. (2).

\[
q_f = q_2 - q_1 = \frac{M_{\Psi'}^2 - M_{\Psi}^2}{2E_{\Psi}},
\]

(10)

Exact solution is presented in 5), however, it is instructive to solve the equation (6) to first order in \( \sigma_{\text{tot}}^{\Psi} \), neglecting the effect of coherence length considered above. Then we find for the nuclear suppression of the \( \Psi \) and \( \Psi' \) states

\[
S_{\Psi}^{pA}(E_{\Psi}) \approx 1 - \frac{1}{2}\sigma_{\text{tot}}^{\Psi} \langle T \rangle \left[ 1 + \epsilon R F_A^2(q_f) \right],
\]

(11)

\[
S_{\Psi'}^{pA}(E_{\Psi}) \approx 1 - \frac{1}{2}\sigma_{\text{tot}}^{\Psi'} \langle T \rangle \left[ 1 + (\epsilon/rR) F_A^2(q_f) \right],
\]

(12)

We estimate matrix elements (8) - (9) at \( \epsilon = -\sqrt{2/3} \) and \( r = 7/3 \) and from experimental data \( |R_{ex}| = 0.48\pm0.06 \). As opposite to the effect of the coherence length discussed in the previous section, the growth of the formation length leads to the increase of \( S_{\Psi}^{pA} \). This is because the produced initial state \( |\Psi_0\rangle \) turns out to be nearly an eigenstate of interaction, provided that the parameters \( \epsilon, r \) and \( R \) have values we estimated. Such an eigenstate has the absorption cross section smaller than any of its components, \( \Psi \) or \( \Psi' \). This explains the growth of \( S_{\Psi}^{pA} \) with \( E_{\Psi} \), predicted by (11).

As soon as the produced \( c\bar{c} \) state \( |\Psi_0\rangle \) is the eigen state, it does not change its \( \Psi-\Psi' \) content during propagation through the nucleus, i.e. the relative yields of \( \Psi' \) to \( \Psi \) has no \( A \)-dependence at high \( E_{\Psi} \) as was observed in 1). This is demonstrated in Fig. 1 versus \( x_F \) 5) in comparison with data 1,3).

![Figure 1: The relative nuclear suppression \( S_{\Psi'/\Psi}^{pA} = S_{\Psi'}^{pA}/S_{\Psi}^{pA} \) in \( p-W \) collision at 800 and 200 GeV, calculated in the two-coupled channel approach 5) at in comparison with data 1,3).](image)
Now we can turn on the coherence length and combine the two effects. In the approximation of small $\sigma_{\text{tot}}^{\Psi N}(T) \ll 1$ we get

$$S_{\Psi}^A(E_{\Psi}) \approx 1 - \frac{1}{2} \sigma_{\text{in}}^{\Psi}(T) \left[ 1 + F_A^2(q_c) \right] \left[ 1 + \epsilon R F_A^2(q_f) \right]$$

(13)

$$S_{\Psi'}^A(E_{\Psi'}) \approx 1 - \frac{1}{2} \sigma_{\text{in}}^{\Psi'}(T) \left[ 1 + F_A^2(q_c) \right] \left[ 1 + \frac{\epsilon}{r R} F_A^2(q_f) \right]$$

(14)

Since the effects of the coherence and formation lengths act in opposite directions, their interplay leads to a nontrivial $x_F$-dependence of the nuclear transparency as is shown in fig. 2. We present the results of exact solution of the two-channel problem, described in 9).

![Figure 2: $x_F$-dependence of the nuclear suppression for production of $\Psi$ (a) and $\Psi'$ (b) in p-Fe collisions. The curves show predictions at the proton energies 200, 800 GeV and in the energy range of RHIC - LHC.](image)

Note that according to Fig. 2 we expect a decreasing energy-dependence of $S_{\Psi}^A$ as function of energy at fixed $x_F$. This may explain the observed $^{1,2}$ energy dependence of nuclear suppression. We remind that our calculations are restricted to small values of $x_F$.

4. Nucleus-nucleus collisions

One may expect new phenomena in heavy ion collisions. First of all, the multiparticle production becomes so intensive that it may cause an additional suppression of charmonium. There might be also an unusual phenomenon, a quark-gluon plasma formation, in such collisions. We still have no reliable calculations of those effects, but in any case one needs a solid theoretical base line to compare the measurements with. The so called standard absorption model, which corresponds to $l_c = l_f = 0$, is obviously oversimplified, because existence of the quantum interference effects, discussed above, is not negotiable, and they are very important.

We expect the nuclear effects for charmonium production in AA collisions to be quite different from what is known for pA collision. This is because the coherence and formation lengths depend on whether we are in the rest frame of the target or of the beam. Due to
the inverse kinematics the charmonium wave packet attenuates with different effective cross sections propagating through the two colliding nuclei. The nuclear suppression in $A_1A_2$ collision is simply related to that in $pA_1$ and $pA_2$ interactions,

$$S_{A_1A_2}(x_F) \approx S_{pA_2}(x_F)S_{pA_1}(-x_F)$$

Our previous conclusion about equal nuclear suppression of $\Psi$ and $\Psi'$ in $pA$ interaction is not valid for $AA$ collisions. The results of application of (15) to the ratio $S^{AB}_{\Psi'}/S^{AB}_{\Psi}$ are shown in Fig. 3.

![Figure 3](image)

Figure 3: $x_F$-dependence of relative nuclear suppression for $\Psi'$ to $\Psi$ in $S - Au$ collisions at 200 GeV. The solid curve shows prediction of the two-coupled channel approach $^5$). The dashed curves show nuclear suppression in $p - S$ and $p - Au$ collisions.

We see that this ratio, which is unity in $pA$ interactions is substantially below one in the case of nuclear collisions. However, this reduction explains only about a half of the observed effect, shown in Fig. 3 by the only available experimental point $^3$.

5. Conclusions

The quantum effects related to interference between amplitudes of charmonium production on different nucleons and to a composite structure of the produced charmonium wave packet lead to a substantial modification of the theoretical expectations. We predict quite an unusual $x_F$- and energy-dependence of the nuclear suppression for $\Psi$, which agree with available data. We are also able to explain why the nuclear suppression factors for $\Psi$ and $\Psi'$ are the same in proton-nucleus, but different in nucleus-nucleus collisions. Numerically, however, the observed effect seems to be larger. This invites one to take into account the interaction of the charmonium with other produced particles, which is the next step to be done.

References
1. D.M. Adle et al., Phys. Rev. Lett. 66 (1991) 133
2. The NA3 Collab., J. Badier et al., Z. Phys. C 20 (1983) 101
3. The NA38 Collaboration, C. Baglin et al., Phys. Lett. B345 (1995) 617 and M.C. Abreu et al., presented at QUARK MATTER’95
4. S.J. Brodsky, P. Hoyer, A.H. Müller and W.-K. Tang, Nucl. Phys. B 369 (1992) 519
5. J. Hufner and B.Z. Kopeliovich, Phys. Rev. Lett. 76 (1996) 192
6. J. Hufner, B.Z. Kopeliovich and J. Nemchik, DOE/ER/40561-260-INT96-19-03, nucl-th/9605007
7. O. Benhar, B.Z. Kopeliovich, Ch. Mariotti, N.N. Nikolaev and B.G. Zakharov, Phys.Rev.Lett. 69 (1992) 1156.
8. B.Z. Kopeliovich and B.G. Zakharov, Phys. Rev. D44 (1991) 3466
9. J. Hufner and B.Z. Kopeliovich, paper in preparation