A single-photon router based on a modulated cavity optomechanical system

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We investigate the routing of a single-photon in a modulated cavity optomechanical system, in which the cavity is driven by a strong coupling field, and the mechanical resonator (MR) is modulated with a weak coherent field. We show that, when there is no a weak coherent field modulating the MR, the system cannot act as a single-photon router, since the signal will be completely covered by the quantum and thermal noises. By introducing the weak coherent field, we can achieve the routing of the single-photon by adjusting the frequency of the weak coherent field, and the system can be immune to the quantum and thermal noises.

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I. INTRODUCTION

It is well known that quantum routers are important ingredients of quantum networks. In the past few decades, scientists have demonstrated that many physical effects and physical systems, such as quantum interference [1], electromagnetically induced transparency [2], coupled waveguide array [3-5], can be used to realize the routing of photons. Recently, many theoretical and experimental researches aiming at achieve the quantum router in the single-photon level have been reported [6-10]. Hall et al. demonstrated the routing of single-photon without disturbing the photons’ quantum states with the help of strong cross-phase modulation [11]. Hoi et al. achieved a single-photon router in the microwave regime by using a superconducting transmon qubit [12]. Zhou et al. proposed an experimentally accessible single-photon routing scheme using a three-level atom embedded in a coupled-resonator waveguide [13].

We also notice that the realization of a single-photon router has been researched in cavity optomechanical system. In Ref. [14], the authors have shown how nanomechanical mirrors in an optical cavity can be used to build single-photon routers. However, their analysis is inadequate. We find that, their scheme actually cannot achieve the routing of a single-photon, since the signal will be completely covered by the quantum and thermal noises. In the present paper, we propose a scheme, based on a modulated cavity optomechanical system, to realize the single-photon router. In our system, the cavity is driven by a strong coupling field, and the mechanical resonator (MR) is modulated with a weak coherent field. We can achieve the routing of the single-photon by changing the frequency of the weak coherent field. Moreover, our system can be immune to the quantum andthermal noises when the MR is cooled to its quantum ground state.

This paper is organized as follows. In Section II, we introduce the theoretical model. In Section III, we consider the case in which there is no a weak coherent field modulating the MR. We show and explain why in this case the system cannot act as a single-photon router. Next in Section IV, we consider the case in which the MR is modulated by a weak coherent field. We exhibit how the single-photon router works in this situation. We also discuss the effects of the quantum and thermal noises on the single-photon router. Finally in Section V, we provide a brief summary.

II. MODEL

Our proposed scheme is shown in Fig. 1. We consider a cavity optomechanical system with a mechanical resonator (MR) of partial reflection inserted between two fixed mirrors. The cavity is driven by a strong coupling field with amplitude \( \varepsilon_p = \sqrt{2 \kappa P/\hbar \omega_p} \) and frequency \( \omega_p \). The MR is modulated by a weak coherent field with amplitude \( \varepsilon_d \) and frequency \( \omega_d \), this modulation can be realized by, e.g., parametrically modulating the spring constant of the MR at twice of the MR’s resonance frequency [12, 16]. The Hamiltonian of the system in the rotating frame at the frequency \( \omega_p \) of the coupling field

\[ H = \hbar \omega_p a^\dagger a + \hbar \omega_d d^\dagger d + \hbar \kappa (a^\dagger + a) (d^\dagger + d) + \hbar \varepsilon_p P (a^\dagger d + ad) \]

where \( a \) and \( a^\dagger \) are the cavity annihilation and creation operators, \( d \) and \( d^\dagger \) are the mechanical annihilation and creation operators, \( \kappa \) is the cavity decay rate, and \( P \) is the strong coupling field.
is given by ($\hbar = 1$)

$$
H = \Delta \hat{a}^\dagger \hat{a} + g_0 \hat{a}^\dagger \hat{b} + \hat{b} \hat{b} + \epsilon_p (\hat{a}^\dagger - \hat{a})
+ \omega_m \hat{b}^\dagger \hat{b} + i \epsilon_d [i \hat{b}^\dagger e^{-i\omega_d t} - \hat{b} e^{i\omega_d t}],
$$

where $\Delta = \omega_c - \omega_p$ is the frequency detuning between the cavity field and the coupling field. $\hat{a}$ and $\hat{b}$ are the annihilation operators of the cavity mode and the mechanical mode with frequency $\omega_c$ and $\omega_m$, respectively, $g_0$ is the single-photon optomechanical coupling strength between the cavity mode and the mechanical mode.

The system dynamics is fully described by the set of the quantum Langevin equations (QLEs)

$$
\frac{d\hat{a}}{dt} = -(2\kappa + i\Delta)\hat{a} - ig_0 \hat{b}(\hat{b}^\dagger + \hat{b}) + \epsilon_p + \sqrt{2\kappa} \hat{c}_{in} + \sqrt{2\kappa} \hat{d}_{in},
\tag{2}
$$

$$
\frac{d\hat{b}}{dt} = -(\gamma + i\omega_m)\hat{b} - ig_0 \hat{a}^\dagger \hat{a} + 2\epsilon_d e^{-i\omega_d t} \hat{b}^\dagger
+ \sqrt{2\gamma} \hat{b}_{in},
\tag{3}
$$

where $2\kappa$ is the total damping rate of the cavity and $\gamma$ is the mechanical damping rate. $\epsilon_{in}, \epsilon_{in}^*, \hat{c}_{in}$, and $\hat{b}_{in}$ are the input quantum fields with zero mean values.

We assume that the cavity field is driving by a strong coupling field $\epsilon_p$ and the MR is modulated by a weak coherent field $\epsilon_d$. The steady-state mean values of the operators can be obtained from the QLEs (2)-(3) by making a transformations $\hat{a} \rightarrow \alpha + \delta \hat{a}$, and $\hat{b} \rightarrow \beta + \delta \hat{b}$, where $\alpha(\beta)$ and $\delta \hat{a}(\delta \hat{b})$ are the steady state mean value and quantum fluctuation of the cavity mode (mechanical mode), respectively, then we have

$$
\alpha = \frac{\epsilon_p}{2\kappa + i\Delta + ig_0 (\beta + \beta^*)},
\tag{4}
$$

$$
\beta = \frac{-ig_0 |\alpha|^2}{\gamma + i\omega_m}.
\tag{5}
$$

And for simplicity in symbols, we rewrite $\delta \hat{a}(\delta \hat{b})$ as $a(\hat{b})$ in the following sections.

### III. WITHOUT THE WEAK COHERENT FIELD

In this section, we consider the case in which there is no a weak coherent field modulating the MR. We would like to point out that this situation has been discussed in Ref. [14]. However, the analysis there is inadequate and the conclusion is incorrect, so we make a re-calculation and a re-discussion about this situation. When $\epsilon_d = 0$, the linearized Hamiltonian of the system can be expressed as

$$
H_1 = \Delta' \hat{a}^\dagger \hat{a} + \omega_m \hat{b} \hat{b} + G(\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger)
+ G^* (\hat{b}^\dagger \hat{b} + \hat{b} \hat{b}^\dagger),
\tag{6}
$$

where $G = g_0 \alpha$, $\Delta' = \Delta + g_0 (\beta + \beta^*) \simeq \Delta$.

We define a vector $v(t) = (\hat{a}(t), \hat{b}(t), \hat{a}^\dagger(t), \hat{b}^\dagger(t))^T$ in terms of the operators of the system. By substituting $v(t)$ and $H_1$ into the quantum Langevin equation, we can obtain

$$
\frac{dv(t)}{dt} = M v(t) + \sqrt{2\kappa} \nu_{v,in}(\omega) + \sqrt{2\gamma} \nu_{b,in},
\tag{7}
$$

where $\nu_{v,in} = (\nu_{v,in}(0), 0, \nu_{v,in}(t), 0)^T (x = c, d)$, $v_{b,in} = (0, \nu_{b,in}(0), 0, \nu_{b,in}(t))^T$, and

$$
M = \begin{pmatrix}
-2\kappa - i\Delta & -iG & 0 & -iG \\
-iG^* & -\gamma - i\omega_m & -iG & 0 \\
0 & iG^* & i\Delta - 2\kappa & iG^* \\
iG^* & 0 & iG & i\omega_m - \gamma
\end{pmatrix}.
\tag{8}
$$

The system is stable only when the real parts of all the eigenvalues of matrix $M$ are negative. The stability conditions can be explicitly given by using the Routh-Hurwitz criterion [14–19], and the stability conditions are fulfilled in the system with our used parameters. Moreover, for simplicity, we take $G$ as a real number in the following calculations.

In experiments, the fluctuations of the electromagnetic field are more convenient to be measured in the frequency domain than in the time domain. Therefore, we introduce the Fourier transform of the operators

$$
\hat{\delta}(\omega) = \int_{-\infty}^{+\infty} \hat{\delta}(t) e^{i\omega t} dt,
\tag{9}
$$

$$
\hat{\delta}^\dagger(\omega) = \int_{-\infty}^{+\infty} \hat{\delta}^\dagger(t) e^{i\omega t} dt,
\tag{10}
$$

where $\hat{\delta} = \hat{a}, \hat{b}$, then we can solve the linearized QLEs (7) in the frequency domain

$$
v(\omega) = -(M + i\omega I)^{-1} \left[ \sqrt{2\kappa} \nu_{v,in}(\omega) + \sqrt{2\gamma} \nu_{b,in}(\omega) \right],
\tag{11}
$$

where $v(\omega) = (\hat{a}(\omega), \hat{b}(\omega), \hat{a}^\dagger(\omega), \hat{b}^\dagger(\omega))^T$, $v_{x,in}(\omega) = (\hat{x}_{in}(\omega), 0, \hat{a}^\dagger_{in}(\omega), 0)^T (x = c, d)$, and $v_{b,in}(\omega) = (0, \hat{b}_{in}(\omega), 0, \hat{b}^\dagger_{in}(\omega))^T$, then we can obtain

$$
\hat{a}(\omega) = f(\omega) v_{x,in}(\omega),
\tag{12}
$$

where $f(\omega) = (f_1(\omega), f_2(\omega), f_3(\omega), f_4(\omega), f_5(\omega), f_6(\omega))$, and $v_{x,in}(\omega) = (\hat{c}_{in}(\omega), \hat{d}_{in}(\omega), \hat{b}_{in}(\omega), \hat{c}_{in}^\dagger(\omega), \hat{d}_{in}^\dagger(\omega), \hat{b}^\dagger_{in}(\omega))^T$. The concrete forms of the coefficients $f_1(\omega), \cdots, f_6(\omega)$ are tediously long, we will not write them out here.

In this paper, we consider that the input field $\hat{c}_{in}$ is in a single-photon Fock state, and the correlation functions are $\langle \hat{c}_{in}^\dagger(\Omega) \hat{c}_{in}(\omega) \rangle = S_{in}(\omega) \delta(\omega - \Omega)$, $\langle \hat{c}_{in}(\Omega) \hat{c}_{in}^\dagger(\omega) \rangle = [S_{in}(\Omega) + 1] \delta(\omega + \Omega)$. It should be point out that, when we use such a single-photon state as the input state to the cavity, we also assume that its center frequency is resonant with the cavity [20, 21]. Its spectrum is given by the Lorentzian lineshape $S_{in}(\omega) = \frac{\Gamma/2}{(\omega - \omega_0)^2 + \Gamma^2/4}$, in which
The thermal phonon occupation number at a finite temperature is characterized by \( \langle \hat{a}_{in}(\Omega)\hat{a}_{in}^\dagger(\omega) \rangle = \delta(\Omega + \omega) \). The mechanical input operator \( \hat{b}_{in} \) satisfies \( \langle \hat{b}_{in}(\Omega)\hat{b}_{in}^\dagger(\omega) \rangle = n_{th}\delta(\Omega + \omega) \), where \( n_{th} \) is the thermal phonon occupation number at a finite temperature.

The relation among the input, internal, and output fields is given as \[ \hat{x}_{out}(\omega) = -\hat{x}_{in}(\omega) + \sqrt{2\kappa}\hat{a}(\omega), \quad x = c, d. \tag{13} \]

Then we can write the operators of the output fields as
\[
\hat{c}_{out}(\omega) = f^c(\omega)\hat{v}_{in}(\omega), \quad \hat{d}_{out}(\omega) = f^d(\omega)\hat{v}_{in}(\omega),
\]
where \( f^c(\omega) = (f_1^c(\omega) - 1, f_2^c(\omega), f_3^c(\omega), f_4^c(\omega), f_5^c(\omega), f_6^c(\omega)) \), \( f^d(\omega) = (f_1^d(\omega), f_2^d(\omega) - 1, f_3^d(\omega), f_4^d(\omega), f_5^d(\omega), f_6^d(\omega)) \), and \( f_j^c(\omega) \equiv \sqrt{2\kappa}f_j(\omega) \) (\( j = 1, 2, 3, 4, 5, 6 \)).

The spectrums of the output fields are defined by
\[
S_{x, out}(\omega) = \int d\Omega \langle \hat{x}_{out}^\dagger(\Omega)\hat{x}_{out}(\omega) \rangle, \quad x = c, d. \tag{15} \]

By substituting the expressions of \( \hat{c}_{out}(\omega) \) and \( \hat{d}_{out}(\omega) \) into Eq. (15), and using the correlation functions, one can obtain
\[
S^I_{c, out}(\omega) = F^c_1S_{in}(\omega) + F_{3th} + F_4S_{vac}(-\omega) + F_5 + F_6, \tag{16} \]
\[
S^I_{d, out}(\omega) = F^d_1S_{in}(\omega) + F_{3th} + F_4S_{vac}(-\omega) + F_5 + F_6. \tag{17} \]

We can see that both the spectrums \( S^I_{c, out}(\omega) \) and \( S^I_{d, out}(\omega) \) of the output fields contain five components. For \( S^I_{c, out}(\omega) \), \( F^c_1 \) and \( F_4 \) represent, respectively, the scattering probabilities of the input field \( \hat{c}_{in}(\omega) \) and its fluctuation. \( F_3 \) is the scattering probability of the fluctuation of the vacuum field \( \hat{d}_{in}(\omega) \). \( F_5 \) and \( F_6 \) denote, respectively, the scattering probabilities of the thermal noise and quantum noise input to the mechanical mode. It can be seen that even if there is no input signal photon, the output fields will also be generated by the vacuum fluctuations and thermal noises. A good single photon router should not be influenced by these quantum and thermal noises.

Next we numerically calculate the reflection spectrum \( S^I_{c, out}(\omega) \) and the transmission spectrum \( S^I_{d, out}(\omega) \). The parameters we used are the same as that in Ref. \[ \text{[14]} \]: \( \lambda = 1054 \text{ nm}, L = 6.7 \text{ cm}, m = 40 \text{ ng}, \omega_m = 2\pi \times 134 \text{ kHz}, \gamma_m = 0.76 \text{ Hz}, \kappa = 0.1\omega_m, \Delta = \omega_m \).

First we do not consider the effects of the noises. In Fig. 2(a) and Fig. 2(b), we plot the spectrums of the scattering probabilities \( F^c_1 \) and \( F^d_1 \) for different effective optomechanical coupling strength \( G \). It can be seen that, when \( G \) is small \( (G = 10^{-4}\omega_m) \), the point \( \omega - \omega_c = \omega_m \) is a valley, while the point \( \omega - \omega_c = -\omega_m \), the spectrum \( F^c_1 \) exhibits a valley, while the spectrum \( F^d_1 \) exhibits a peak, and at \( \omega - \omega_c = \omega_m \), we have \( F^c_1 \approx 0 \) and \( F^d_1 \approx 1 \), this means that the single-photon will completely transmit through the cavity and exit from the right output port. For a larger \( G \), e.g., for
$G = 0.1\omega_m$ or $0.2\omega_m$, at $\omega - \omega_c = \omega_m$, we have $F^c_1 \approx 1$ and $F^d_1 \approx 0$, this means that the single-photon will be completely reflected from the cavity and exit from the left output port. These results are similar with that in Ref. [14], and indicate that one can realize a single-photon router by adjusting the effective optomechanical coupling strength $G$. However, it should be pointed out that one obtains these results when one ignores the noises. Things will be different if the noises are taken into account, and this will be discussed in the following. Here we would like to point out another phenomenon: the spectrums will exhibit a split at $\omega - \omega_c = \pm G$, and this is associated with the normal mode splitting [24, 25].

Now we estimate the order of magnitude of the signal. It can be seen that, in this case, the operating frequency of the system is at $\omega - \omega_c = \omega_m$, hence the signal can be expressed as $S_{\text{in}}(\omega_c + \omega_m) = \frac{F^c_1}{\omega_m^2 + \omega_c^2 + 1}$. Its maximum value is about $10^{-7}$, which appears at $\Gamma = \omega_m$.

Then we consider the effects of the quantum and thermal noises. In Fig. 2(c)-(f), we plot the spectrums of the scattering probabilities $F_3, \cdots, F_6$, respectively. We find that, at $\omega - \omega_c = \omega_m$, $F_3, \cdots, F_6$ have the following order of magnitudes: when $G = 10^{-4}\omega_m$, one has $F_3 \sim 10^{-2}$, $F_4 \sim F_5 \sim 10^{-5}$, and $F_6 \sim 10^{-14}$; when $G = 0.1\omega_m$ or $0.2\omega_m$, one has $F_3 \sim 10^{-5}$, $F_4 \sim F_5 \sim 10^{-2}$, and $F_6 \sim 10^{-17}$. That is, with the increase of $G$, the noises deriving from the input fields $c_{\text{in}}(\omega)$ and $d_{\text{in}}(\omega)$ have been strongly amplified, while the noises deriving from the input field $b_{\text{in}}(\omega)$ has been effectively suppressed.

By comparing the order of magnitudes of the signal and the noises, we can see that, in the reflection spectrum $S_{\text{c, in}}^I(\omega)$ and the transmission spectrum $S_{\text{d, out}}^I(\omega)$, the contributions $F_3S_{\text{in}}(\omega_c + \omega_m)$ or $F_1S_{\text{in}}(\omega_c + \omega_m)$ from the signal is much less than the contributions $F_3S_{\text{in}} + F_4 + F_5$ from the noises, whether $G = 10^{-4}\omega_m$, $0.1\omega_m$, or $0.2\omega_m$. In other words, the signal will be completely covered by the quantum and thermal noises. Hence, we can conclude that, in this case, this system can not act as a single-photon router.

IV. WITH THE WEAK COHERENT FIELD

In this section, we consider the case in which there is a weak coherent field modulating the MR. In the rotation frame with $H' = \omega_d(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})$, the linearized Hamiltonian of the system can be expressed as

$$H_{11} = \delta\hat{a}^\dagger\hat{a} + \Delta_m\hat{b}^\dagger\hat{b} + G\hat{a}^\dagger\hat{b} + G^*\hat{a}\hat{b}^\dagger + i\varepsilon_d(\hat{b}^\dagger)^2 - (\hat{b})^2, \quad (18)$$

where $\delta = \Delta - \omega_d$, and $\Delta_m = \omega_m - \omega_d$. Here we have used the rotating-wave approximation to omit the high-frequency oscillation terms $\hat{a}\hat{b}^\dagger e^{i2\omega_d t}$ and $\hat{a}\hat{b} e^{-i2\omega_d t}$.

By substituting $v(t)$ and $H_{11}$ into the quantum Langevin equation, we can obtain

$$\frac{dv(t)}{dt} = M'v(t) + \sqrt{2\kappa_v}v_{\text{c, in}} + \sqrt{2\kappa_v}v_{\text{d, in}} + \sqrt{2\gamma}v_{\text{b, in}}, \quad (19)$$
where
\[ M' = \begin{pmatrix}
-2\kappa - i\delta & -iG & 0 & 0 \\
-iG^* & -\gamma - i\Delta_m & 0 & 2\varepsilon_d \\
0 & 0 & i\delta - 2\kappa & iG^* \\
0 & 2\varepsilon_d & iG & i\Delta_m - \gamma
\end{pmatrix}. \tag{20}\]

The stability conditions of the matrix \( M' \) have been verified by using the Routh-Hurwitz criterion with our used parameters. The subsequent calculations are similar with that in section III. The spectrums of the output fields are obtained as
\[ S_{c,\text{out}}(\omega) = L_1^c S_{in}(\omega) + L_{3\text{th}}, \quad S_{d,\text{out}}(\omega) = L_1^d S_{in}(\omega) + L_{3\text{th}}, \tag{21}\]
\[ + L_4 S_{\text{vac}}(-\omega) + L_5 + L_6, \tag{22}\]

in which \( L_1^c, L_1^d, L_3, \ldots, L_6 \) have the same physical meaning with \( F_1^c, F_1^d, F_3, \ldots, F_6 \), respectively. The concrete forms of \( L_1^c, L_1^d, L_3, \ldots, L_6 \) are too verbose to be given here.

Let us show how the single-photon router works in our scheme. In our system, with the existence of the weak coherent field, the effective frequency of the mechanical mode becomes \( \Delta_m = \omega_m - \omega_d \). Figures 3(a) and 3(b) show the spectrums of the scattering probabilities \( L_3, \ldots, L_6 \) for different \( \omega_d \). We find that, when \( \omega_d = 0.8\omega_m \) (\( \Delta_m = 0.2\omega_m = 2\kappa \)), \( L_1^c \) exhibit a peak around the point \( \omega - \omega_c = \Delta_m + G = (0, 4\kappa) \), while \( L_1^d \) exhibits just the opposite. At \( \omega - \omega_c = 0 \), one has \( L_1^c \approx 0 \) and \( L_1^d \approx 1 \), this means that the single-photon will completely transmit through the cavity and exit from the right output port. With the increase of \( \omega_d \), the curves will integrally move to the left, for example, when \( \omega_d = \omega_m \) (\( \Delta_m = 0 \)), we have \( L_1^c \approx 1 \) and \( L_1^d \approx 0 \) at \( \omega - \omega_c = 0 \), this means that the single-photon will be completely reflected from the cavity and exit from the left output port. In this way, we can achieve the routing of the single-photon. If we consider that the single-photon has a Lorentzian lineshape with a narrower linewidth than the cavity (\( \Gamma = 0.01\kappa \)), we can estimate that the signal \( S_{in}(\omega_c) \) has the order of magnitude \( 10^{-4} \).

Now we consider the effects of the quantum and thermal noises in this case. In Fig. 3(c)-(f), we plot the spectrums of the scattering probabilities \( L_3, \ldots, L_6 \), respectively. We find that, in the range of the parameters we considered (\( \varepsilon_d/\kappa = 2.37 \times 10^{-4} \), at \( \omega - \omega_c = 0 \), \( L_3, \ldots, L_6 \) have the following order of magnitudes: when \( \omega_d = 0.8\omega_m \) or \( \omega_m \), one has \( L_3 \sim 10^{-5} \), \( L_4 \sim 10^{-8} \), and \( L_6 \sim 10^{-12} \).

By comparing with the case in which there is no a weak coherent field, we find that, in the present case, the signal is enhanced, and the noises are suppressed. By comparing the order of magnitudes of the signal and the noises, we can write the spectrum of the output fields as
\[ S_{c,\text{out}}^{\text{II}}(\omega) \approx L_1^c S_{in}(\omega) + L_{3\text{th}}, \quad S_{d,\text{out}}^{\text{II}}(\omega) \approx L_1^d S_{in}(\omega) + L_{3\text{th}}. \tag{23}\]
\[ S_{c,\text{out}}^{\text{II}}(\omega) \approx L_1^c S_{in}(\omega) + L_{3\text{th}}, \quad S_{d,\text{out}}^{\text{II}}(\omega) \approx L_1^d S_{in}(\omega) + L_{3\text{th}}. \tag{24}\]

It can be seen that if the thermal phonon occupation number \( n_{th} \lesssim 1 \), the signal cannot be covered by the noises.

In Fig. 4 we plot the spectrums of the output fields \( S_{d,\text{out}}^{\text{II}}(\omega) \) and \( S_{c,\text{out}}^{\text{II}}(\omega) \) for different \( \omega_d \). For \( \omega_d = 0.8\omega_m \) and at \( \omega - \omega_c = 0 \), we find \( S_{d,\text{out}}^{\text{II}}(\omega_c) \approx S_{in}(\omega_c) \), and \( S_{c,\text{out}}^{\text{II}}(\omega_c) \approx 0 \). If we increase \( \omega_d \), \( S_{d,\text{out}}^{\text{II}}(\omega) \) will gradually decrease, and \( S_{c,\text{out}}^{\text{II}}(\omega) \) will gradually increase near \( \omega - \omega_c = 0 \). When \( \omega_d = \omega_m \), we have \( S_{c,\text{out}}^{\text{II}}(\omega_c) \approx S_{in}(\omega_c) \), \( S_{d,\text{out}}^{\text{II}}(\omega_c) \approx 0 \). This shows that our system can act as a single-photon router.

V. CONCLUSION

In summary, we have investigated the routing of a single-photon in a modulated cavity optomechanical system, in which the cavity is driven by a strong coupling field, and the mechanical resonator is modulated by a weak coherent field. We have shown that, if there is no the weak coherent field, the signal will be completely covered by the quantum and thermal noises, and the single-photon router cannot be realized. By introducing a weak coherent field modulating the mechanical resonator, we can achieve the single-photon router by adjusting the frequency of the weak coherent field.

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[23] It should be point out that G cannot be zero, since G = g₀α, by definition g₀ ≠ 0, and the linearization of the Hamiltonian (from Eq. (1) to Eq. (6)) demands α ≠ 0 (and in turn ε₀ ≠ 0, P ≠ 0 ). In fact, the linearization demands |α| ≫ 1, i.e. G ≫ g₀. For the parameters here, g₀ is in the order of 10 Hz, so we take G = 10⁻²ω₁ ≈ 10² Hz.
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