Design of precision finishing method for cold extruded sun gear with internal-external tooth shapes

Zuofa Liu 1 · Qiuyun Wang 1 · Jie Zhou 1 · Wenjie Feng 2 · Qiang Liang 3

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Abstract

The dimensional accuracy of cold extruded sun gear is poor since the material flow is considerably complex in the forming process. A novel precision finishing method was proposed in this study to improve the dimension accuracy of cold extruded sun gear, and a new finite element (FE) prediction strategy was developed to obtain an in-depth understanding of the deviation distribution laws of the finished sun gear. Then, the forming laws of internal-external gears such as material flow, tooth deformation, forming load, and tooth accuracy were analyzed using FE simulation and verified experiment. The investigation results show that the single deviations in profile and helix both first diminished remarkably, and then rose gradually, while the single \( M \) value deviation decreased remarkably with increasing the tooth width. The simulation results of teeth deviations are well agreed with the experimental ones, which verifies the reliability of the FE prediction strategy. Moreover, the profile accuracy of external gear can be improved from ninth to seventh class, lead accuracy can be enhanced from tenth to eighth class, and total \( M \) value deviation of internal spline is reduced to 72.3 \( \mu \text{m} \), which proves the feasibility and effectiveness of the precision finishing method.

Keywords
Sun gear · Precision finishing · Cold extrusion · FE prediction · Gear accuracy · Internal-external tooth shapes

1 Introduction

In recent years, the market demand for sun gear in engineering production has been remarkably increased with the rapid development of the automobile industry. It is difficult to satisfy the requirements of mass production by using conventional cutting method because of the shortages of high production costs, high equipment occupation rate, low processing efficiency, and large material consumption [1–3]. Forging or extrusion processes have been extensively introduced in the manufacturing of gear-shaped parts since having the significant superiorities of low material wastage, high production efficiency, and high mechanical properties. Tierman et al. [4] developed a cold extrusion process forming high-grade aluminum and designed the extrusion dies for the experimental confirmation. A relief-cavity approach in the hot forging of cylindrical gear was introduced by Zuo et al. [5], and the influence laws of process variables and die designs on billet and die stress, corner filling effect, and forming force were investigated. Kanani and Lalwani [6] developed a strategy of providing relief-hole and avoiding tooth corner filling to reduce forming force in the hot forging of pure aluminum gear. Song and Im [7] adopted the computer system of process design in the forward extrusion of hollow spur gear. The results show that the presented computer system can be applied to design the process and die set for cylindrical gear forming. Silveira and Schaeffer [8] evaluated the effects of two different shrink rings on the pre-stressing grade of the cold forging dies by applying the Simufact Forming software.
Since the surface quality and dimension accuracy of gears formed by extrusion or forging methods cannot meet the specification requirements of precision products, there has been an increased interest in precision finishing technology as the post-operation for manufacturing gears. Chang et al. [9] introduced the ironing process as a post-operation for the warm forged spur gears and studied the influences of initial interference values on dimension accuracy, geometrical profile, and surface quality. A cold sizing process of thick-walled spur gear as a useful post-operation was proposed by Stone et al. [10], and the effects of various process conditions on surface finish and dimensions were examined. Behrens and Doege [11] employed a post sizing operation for the hot- or cold-formed gears to obtain the desired productions. Liu et al. [12] developed a compound process of warm forging and cold finishing in the production of cylindrical gear with a large module and discussed the effects of different billet specifications on material flow and load. A comparison between the compressing sizing and ironing sizing processes as the post-forging operation for manufacturing spur gears was undertaken by Li et al. [13]. Franulovic et al. [14] investigated the effects of various pitch deviations on the forming load during the cold finishing of cylindrical gears. Li et al. [15] presented the compound extrusion-finishing operation for the manufacturing of the conjunction gears. The significant advantages of this process were confirmed by the numerical and experimental results.

The material flow is remarkably complex during the precision finishing process since the sun gear has inner-outer teeth shapes, resulting in difficulty to control the dimensional accuracy of teeth. Thus, the evolution of tooth deviations is still an important engineering issue in the precision finishing of spur gears. Qin and Balendra [16] established an elastic-plastic model to predict the influences of die-elasticity deformation during the forward extrusion. Ou and Balendra [17] investigated the elastic deflections of the forging dies using numerical simulation and developed a FE strategy to modify the die profiles based on the nominal dimensions of forging dies. A reasonable prediction method was presented by Lee et al. [18] for the machined and shrink-fitted die dimensions in cold forging. Kang et al. [19] introduced a new manufacturing approach for gear forging tools and studied the influences of die elastic deformation in the spur gear forging process. Zuo et al. [20] proposed a theoretical model to predict the profile and lead deviations of spur gears in the hot forging process. Zhuang et al. [21] presented a novel finite element analysis strategy for predicting the tooth deviations during the forging process of spur gears.

From the above analysis, it can be concluded that substantial progress has been achieved in producing cylindrical gear-like components with either external or internal teeth. However, there is a lack of investigation on the dimensions and surface quality of precision finished sun gear with external-internal tooth shapes. In this study, a novel precision finishing method was proposed to improve the dimensional accuracy of sun gears formed by the cold extrusion process. Also, a new FE prediction strategy was developed to obtain an in-depth understanding of the deviation distribution laws of the finished sun gear. The forming laws of internal-external gears such as material flow, tooth deformation, forming load, and tooth accuracy were analyzed using FE simulation. Finally, the experiment was conducted to confirm the feasibility of the precision finishing method and the reliability of the FE prediction strategy.

2 Cold extrusion process and defects

2.1 Cold extrusion process

Figure 1 shows the main parameters and geometry of the sun gear. It has two kinds of tooth shapes, including internal spline and external gear. The number of external and internal teeth is 23 and 26, respectively. The modules of external and internal gears are 4 and 2, respectively. And, the pressure angles of external and internal teeth are 20° and 30°, respectively.

The cold extrusion process of sun gear, developed in the previous work, is displayed in Fig. 2. Firstly, a hot-forged ring-shaped workpiece was put into the cavity of cylindrical die and spline mandrel. When the downward-moving punch contacted with the upper end of the billet, the material of billet started to flow downward, leading to the formed internal spline. When the punch reached the set stroke, another billet was inserted into the tools. Two billets moved down under the press of punch and the lower workpiece was extruded out of the die cavity through the upper billet, thereby a continuous extrusion of internal spline was achieved, as illustrated in Fig. 2(a). Secondly, the extrusion principle of outer tooth shapes was similar to that of inner spline, as shown in Fig. 2(b). The blank with inner tooth shapes was inserted into the toothed die cavity, and the punch and spline mandrel moved down together during the forming process. Then, the spline mandrel was first passed through the inner tooth shapes under a hydraulic press, and the outer gear forming was completed as the lower blank was extruded out of the die cavity.

2.2 Defect analysis

Figure 3 shows the sun gears formed by the developed cold extrusion process, where the upper end is defined as the large tooth width and the lower end represents small tooth width. The total profile deviation (Fα) and total helix deviation (Fβ) can be considered as two important indexes of external gear accuracy, and the total M value deviation (FM) indicates the
dimensional accuracy of internal spline. A formed sun gear was randomly selected for inspecting the dimensional accuracy of internal-external teeth by using the gear measuring center denoted in Fig. 4. The total deviations in profile, helix, and $M$ value of the sun gear were inspected for four teeth numbered 1, 5, 10, and 14, which were evenly distributed on the circumference of gears. Table 1 lists the inspected results. The tolerances and accuracy grade ($Q$) are for the cutting spur gears according to the International Organization for Standardization (ISO). It can be seen that the profile accuracy of external gear was ninth class and the lead accuracy was tenth class, which failed to meet the accuracy requirement of eight class specified in the International Organization for Standardization. For the dimensional accuracy of internal spline, the $M$ value deviation gradually reduced with the increase of tooth width, and the total deviation of $M$ value was 276.4 $\mu$m, which was much larger than the final product requirement of 100 $\mu$m. The inspection results show that the dimensional accuracy of sun gear formed by cold extrusion process is poor and fails to satisfy the requirements of precision production. Thus, it is valuable to introduce a precision finishing process as the post-extrusion operation for enhancing the dimensional accuracy of the cold extruded sun gear.
3 Precision finishing method design

3.1 Finishing method design of external gear

The cold extruded sun gear was evenly divided into five section planes from the lower to the upper ends, and then the M values of internal-external gears were measured by using the pins. Then, the conversion formulas were applied for calculation, so as to obtain the distribution laws of both the tooth thickness of external gear and the space width of internal spline. Figure 5 shows the schematic diagram of external gear thickness measurement. Given the external gear with module \( m_e \), teeth \( z_e \), pressure angle \( \alpha_e \), diameter of measuring pin \( d_{pe} \), and the \( M \) value of external gear \( M_{Re} \) can be calculated as

\[
M_{Re} = \frac{m_e z_e \cos \alpha_e \cos(90^\circ/z_e)}{\cos \alpha_{Me}} \times \cos(90^\circ/z_e) + d_{pe} \tag{1}
\]

Using Eq. (1), the pressure angle of contact point of measuring pin \( \alpha_{Me} \) can be described as

\[
\alpha_{Me} = \arccos \frac{m_e z_e \cos \alpha_e \cos(90^\circ/z_e)}{M_{Re} - d_{pe}} \tag{2}
\]

The involute profile is described by the function

\[
\text{inv}_{\alpha_{Me}} = \frac{d_{pe}}{m_e z_e \cos \alpha_e} + \frac{S}{m_e z_e} + \text{inv}_{\alpha_e} - \frac{\pi}{z_e} \tag{3}
\]

\[
\text{inv}_{\alpha_{Me}} = \tan \alpha_{Me} - \alpha_{Me} \tag{4}
\]

Using Eqs. (3) and (4), the tooth thickness of external gear \( S \) can be written as

\[
S = m_e \pi + m_e z_e (\text{inv}_{\alpha_{Me}} - \text{inv}_{\alpha_e}) \frac{d_{pe}}{\cos \alpha_e} \tag{5}
\]

Using formulas (1)–(5), the measured \( M \) value of external tooth \( M_{Re} \) can be converted into tooth thickness \( S \). Figure 6 illustrates the tooth thickness distribution of external gear. With the rise of tooth width, the tooth thickness decreased firstly, and then grew remarkably, indicating the shape of external gear was anti-crown. It is well-known that the anti-crown gear with big two ends and small middle has the disadvantages of poor load-bearing capacity and high noise. Therefore, it is necessary to modify the external gear obtained by cold extrusion process, so as to change the tooth thickness distribution and improve the dimensional accuracy. According to the tooth accuracy requirements of the target gear, the precision finishing method of external gear was designed and is displayed in Fig. 7. The tooth profile of finishing tool was 0.2 mm less than that of the target gear, which means that the tip, flank, and root of external gear will be all finished.

3.2 Finishing method design of internal spline

The schematic diagram of internal spline space width measurement through pins is shown in Fig. 8. Given the internal spline with module \( m_i \), teeth \( z_i \), pressure angle \( \alpha_i \), and diameter of measuring pin \( d_{pi} \), the \( M \) value of internal spline \( M_{Ri} \) can be represented as

\[
M_{Ri} = \frac{m_i z_i \cos \alpha_i}{\cos \alpha_{Mi}} - d_{pi} \tag{6}
\]
Using Eq. (6), the pressure angle of contact point of measuring pin $\alpha_{Mi}$ can be expressed as

$$\alpha_{Mi} = \arccos \frac{m_i z_i \cos \alpha_i}{M_{Ri} + d_{pi}}$$

(7)

| Table 1 | Deviations of cold extruded sun gear |
|---------|-------------------------------------|
| **Profile deviation** | ![Image](left|right) |
| $F_\alpha (Q)$ | $30.2 \mu$m (9) |
| Tolerances ($Q$ in ISO) | $19 \mu$m (7), $27 \mu$m (8), $38 \mu$m (9), $54 \mu$m (10) |
| **Helix deviation** | ![Image](left|right) |
| $F_\beta (Q)$ | $55.9 \mu$m (10) |
| Tolerances ($Q$ in ISO) | $20 \mu$m (7), $28 \mu$m (8), $39 \mu$m (9), $56 \mu$m (10) |
| **M value deviation** | ![Image](left|right) |
| $F_M$ | $276.4 \mu$m |
| Tolerance | $100 \mu$m |
For the internal spline, the involute profile is described by the function $\text{inv}_{\alpha_i}^M_i$ calculated as

$$\text{inv}_{\alpha_i}^M_i = \frac{E}{m_i z_i} + \text{inv}_{\alpha_i} - \frac{d_{pi}}{m_i z_i \cos \alpha_i}$$  \hspace{1cm} (8)

Using Eq. (8), the space width of internal spline $E$ can be represented as

$$E = (\text{inv}_{\alpha_i}^M_i - \text{inv}_{\alpha_i})d + \frac{d_{pi}}{\cos \alpha_i}$$  \hspace{1cm} (9)

The measured $M$ value of internal spline $M_{Ri}$ can be converted into space width $E$ by using formulas (6)~(9). Figure 9 shows the space width distributions of internal spline obtained by the cold extrusion process. It can be seen that the space width gradually reduced with the increase of tooth width. To improve the dimensional accuracy of internal spline, the precision finishing method was designed and is shown in Fig. 10. The tooth profile of the finishing spline mandrel was 0.1 mm larger than that of the target gear, indicating the tooth profile of internal spline will be reshaped by the entire finishing method. Besides, the material radial flow can be promoted by the reshaping of external gear, resulting in the internal tooth fits the spline mandrel more closely. Thus, the tooth accuracy of internal spline can be significantly improved while the external gear accuracy is enhanced. Figure 11 shows the diagram of the designed precision sizing process. The spline mandrel was first passed through the internal tooth by a press, and then the external gear was reshaped during the precision finishing process.

4 FE prediction strategy and verified experiment procedure

4.1 Finite element model

The four parts for the finite element model of precision finishing operation displayed in Fig. 12 were constructed with Unigraphics-NX for geometric structure and Deform for numerical simulation, where the error billet was established by using the measurement data, including space width of inner spline and tooth thickness of outer gear. Due to the rotational symmetry of billet and tools, 1/26 of the FE model was performed. The tool sets (punch, spline mandrel, and toothed die) were considered as rigid bodies while the error workpiece was modeled as an elastic-plastic one. A total of 100000 tetrahedron meshes were generated on the workpiece and the elements on the internal and external teeth regions were refined with the ratio of 0.1. An automatic re-meshing strategy was applied for the numerical simulation. The punch and spline mandrel moved down together at the speed of 20 mm/s. A shear friction coefficient $f=0.12$ was widely adopted for
simulation of cold forming process. The materials of error billet and tools were AISI-4120 and AISI-D2, respectively. The main chemical composition and flow stress curves of billet material are given in Table 2 and Fig. 13, respectively. The details of precision finishing simulation are listed in Table 3.

### 4.2 FE prediction strategy

Most of the present works are mainly concentrated on the total deviations of forged or extruded gears, such as cumulative pitch error, total profile deviation, and total helix error. However, the material flow is remarkably complex during the precision finishing process since the sun gear has inner-outer teeth shapes, resulting in difficulty to control the dimensional accuracy of teeth. To improve the understanding of error distribution laws on the precision finished gears, a novel finite element prediction strategy shown in Fig. 14 was developed in this study. First, the finished sun gear was evenly divided into nine section planes from the lower to the upper ends, thereby obtaining nine tooth profiles. Then, the simulated tooth profiles were imported into UG 10.0 software for the measurement of the teeth deviations. The descriptions of the FE prediction strategy are presented as follows:

(I) The maximum of the single profile deviations ($f_{\alpha_j}$) is assumed to the total profile deviation ($F_{\alpha}$), which can be formulated as:
\[ F_\alpha = \max \{ f_{\alpha j} \} \quad (j = 1, 2, \cdots, 9) \]  

(10)

where \( f_{\alpha j} \) is the single profile deviation of the \( j \)-th section plane.

(II) The differences between the target and finished tooth thickness (left and right surfaces) are assumed to single helix deviations, and the maximum of the single helix deviations \( f_{\beta Lj} \) and \( f_{\beta Rj} \) is defined as the total helix deviation \( F_\beta \), which can be expressed as:

\[ f_{\beta Lj} = S_{Lj} - S_t \quad (j = 1, 2, \cdots, 9) \]  

(11)

\[ f_{\beta Rj} = S_{Rj} - S_t \quad (j = 1, 2, \cdots, 9) \]  

(12)

\[ F_\beta = \max \{ f_{\beta Lj}, f_{\beta Rj} \} \quad (j = 1, 2, \cdots, 9) \]  

(13)

where

\( S_t \) is half tooth thickness of the target external gear;
\( S_{Lj} \) is the left tooth thickness of the \( j \)-th section plane;
\( S_{Rj} \) is the right tooth thickness of the \( j \)-th section plane;
\( f_{\beta Lj} \) is the single helix deviation of left flank of the \( j \)-th section plane;
\( f_{\beta Rj} \) is the single helix deviation of right flank of the \( j \)-th section plane.

(III) The differences of \( M \) values between the target and reshaped spline are defined as single \( M \) value deviations, and the maximum of the single \( M \) value deviations \( f_{Mj} \) is assumed to total \( M \) value deviation \( F_M \), which can be represented as:

\[ f_{Mj} = M_t - M_j \quad (j = 1, 2, \cdots, 9) \]  

(14)

\[ F_M = \max \{ f_{Mj} \} \quad (j = 1, 2, \cdots, 9) \]  

(15)

where

\( M_t \) is the \( M \) value of the target internal spline;
\( M_j \) is the \( M \) value of the \( j \)-th section plane;
\( f_{Mj} \) is the single \( M \) value deviation of the \( j \)-th section plane.

### 4.3 Verified experiment procedure

In order to prove the effectiveness of the precision finishing process and the reliability of the FE prediction strategy, the
finishing experiment was performed on a 4000-kN hydraulic press. The materials of tools (punch, spline mandrel, and toothed die) were AISI D2, and water-based graphite was adopted as the lubricant solution for the gear billet and tools. The equipment and tools for the experiments of precision finishing are shown in Fig. 15.

5 Results and discussion

5.1 Material flow

Figure 16 shows the image of material flow in the precision finishing process. The direction of material flow was mainly axial downward due to the push of punch, and the color indicated the value of the flow velocity. The material flow of the precision finishing process can be primarily divided into three stages, including initial stage, middle stage, and last stage.

The initial stage was before punch stroke of 40%, as depicted in Fig. 16(a). In the beginning, the material flow of the external gear in the lower end was faster than that of the internal spline. It can be attributed to the spline mandrel would have a constraint on the inner material, and the external tooth would be reshaped by the toothed die, which made the excess material of the outer teeth flow to the lower end at a faster velocity. Moreover, as the precision finishing process continued, more and more reshaping materials were accumulated at the external teeth, causing the outer gear at the lower end of the finished sun gear to bend inward. When the punch stroke was greater than 25%, the accumulated materials gradually flowed to the inner layer. The punch stroke of 40~70% was the middle stage. In this entire stage, the inner teeth and spline mandrel would move downward together under the press of punch, and the outer gear would be reshaped by the toothed die; thus, the material flow velocity of outer teeth was slower than that of the inner teeth, as illustrated in Fig. 16(b). Besides, the difference in velocities enhanced gradually with the increase of the punch stroke. When the stroke reached 70%, the external gear at the upper end had begun to bend inward. The last stage was larger than the stroke of 70%. Due to the upper end of external gear was not constrained by the punch, the accumulated finishing material would flow upward and be released gradually; thus, the velocity difference of inner and outer teeth was reduced. It can be seen from Fig. 16(c) that the outer gear at the upper end of the finished sun gear bent inward, which was similar to the lower end.

| Table 2 | Chemical composition (mass fraction, %) |
|---------|---------------------------------------|
| C       | 0.18 | Ti | 0.08 | Mn | 0.90 | Cr | 1.20 | Si | 0.26 | Cu | 0.22 | Ni | 0.18 | P  | 0.023 | S  | 0.02 | Fe | 0.00 | Balance |

Fig. 11 Schematic of precision finishing process: (a) assembly; (b) toothed die

Fig. 12 FE model of precision finishing process
5.2 Tooth deformation

Figure 17 displays the effective strain distribution of several important section planes of the finished sun gear and the material flow velocity in the X-direction. It can be concluded that the deformation zone of external gear was the entire tooth surface, and the deformation region gradually expanded to the inner layer with the increasing of the tooth width. While the deformation zone of internal spline was variable during the precision sizing process.

Section plane 1 represents the lower region. At the initial stage of the precision finishing process, the material of section plane 1 flowed to the center in the radial direction; thus, the external gear bent inward and the tooth tip of the internal spline occurred a certain amount of plastic deformation. With the continuous downward movement of the billet, the bending degree of the outer teeth gradually reduced and the material of internal spline flowed to outer teeth by the constraint of spline mandrel, leading to the inner teeth occurred little deformation or even only elastic deformation, as shown in section plane 2. Section plane 3 is the middle region. As the accumulated finishing material of the outer teeth constantly flowed to the inner teeth, it caused the material of internal spline to flow to the center. Therefore, a proper amount of plastic deformation was observed on both the external and internal teeth, indicating the reshaping effect of the gears in this region was optimum. In section plane 4, most of the material flowed to the center, and only a small amount of material at the inner zone flowed to the external gear due to the constraint of spline mandrel; thus, the internal spline occurred little deformation. Section plane 5 represents the upper zone. Due to the upper end of external gear was not constrained by the punch, the accumulated reshaping material would flow upward in the axial direction, which makes the outer gear at the upper end bent inward. In the radial direction, most of the material flowed to the center while part material in the outer zone flowed to outer circle, thereby a certain amount of deformation was observed in the inner teeth.

Table 3  Precision finishing simulation details

| Conditions                        | Values       |
|-----------------------------------|--------------|
| Friction coefficient             | 0.12         |
| Environment temperature (°C)     | 20           |
| Mesh number of error billet      | 100000       |
| Mesh size of error billet        | 1            |
| Mesh refining ratio of teeth regions | 0.1     |
| Punch velocity (mm/s)            | 20           |
| Material of error billet         | AISI-4120    |
| Material of tools                | AISI-D2      |

5.3 Forming load

The forming results of the precision finishing process obtained from numerical simulation and verified experiment are shown in Fig. 18. The surfaces of finished sun gear were smooth, and the rounded corners were fully filled, indicating the tooth shapes of internal-external gears were quite sound. It can be observed that the upper and lower ends of the experimental sun gear were both bent inward, which were well consistent with the simulation ones.

Figure 19 shows the forming load curves of the precision finishing process and the billet shape of several important stages. With the increase of punch stroke, the billet started to contact with the toothed die under the press of punch, and the forming load increased sharply and to a peak value. As the punch stroke continued to rise, partial sun gear had passed through the toothed die, and the load reduced remarkably. The peak loads obtained by FE simulation and verified experiment were 678 kN and 630 kN, respectively. The simulated results were well agreed with the experimental ones within a reasonable error of 7.08%, which verifies the feasibility of the proposed precision finishing process and the correctness of the numerical simulation.

5.4 Tooth accuracy

The profile deviations of external gear obtained by numerical simulation and experiment are illustrated in Fig. 20. With the increase of tooth width, the single profile deviations first diminished remarkably, then reduced slowly, and finally rose gradually, and the maximum value appeared at the lower end of sun gear, as large as 0.0158 mm. It can be explained as the flow velocity difference between the outer and inner teeth, and the upper and lower ends of external gear after finishing would bend inward; thus, larger profile deviations were observed at
Fig. 14 FE prediction strategy for dimensional deviations of finished sun gear: a simulation result; b tooth profiles; and c measurement method.

Fig. 15 Equipment and tools for finishing experiment: a 4000-kN hydraulic press; b punch; c spline mandrel; and d toothed die.
both ends of sun gear. While the middle region of the gear was well constrained by the spline mandrel and toothed die, and the reshaping effect of this region was sound, leading to minor deviations. The total profile deviation of experimental gear was 17.2 $\mu$m, being 8.14% greater than that of the simulation result. This might be mainly caused by the elastic deformation of tools and thermal deformation of billet, which were not considered in this study.

Figure 21(a) shows the single helix deviation curves of external gear obtained by FE simulation. With increasing tooth width, the single helix deviations of left and right surfaces first reduced significantly, and then rose constantly. Larger helix deviations were observed at both ends of the finished sun gear, and the maximum value appeared at the lower end, as large as 0.0182 mm. It was likewise caused by the difference in flow velocities of inner and outer gears, which was similar to the profile deviations. The inspection result of helix deviation is depicted in Fig. 21(b). It can be seen that the single $M$ value deviation decreased remarkably with the increase of tooth width, and the total $M$ value deviations of simulation and experiment were 0.0619 and 0.0723 mm, respectively, which was significantly smaller than that of the cold extruded sun gear. This is due to the tooth profile of spline mandrel is 0.1 mm larger than that of the target internal spline, thereby the billet can be preferably constrained by the spline mandrel. Moreover, the reshaping of external gear can promote the material radial flow in the precision finishing process, causing the internal tooth to fit the spline mandrel more closely. Thus, the tooth accuracy of internal spline can be significantly improved.

Table 4 provides a comparison of total deviations before and after finishing. The total deviations in profile, helix, and $M$ value after reshaping are significantly lower than those before finishing. The profile accuracy of external gear can be improved from ninth to seventh class, lead accuracy can be enhanced from tenth to eighth class, and total $M$ value deviation of internal spline is reduced to 72.3 $\mu$m, which satisfied the final product requirements. Therefore, it can be considered
Fig. 17 Effective strain distributions and X-velocities of finished sun gear

Fig. 18 Results of precision finishing process: a simulation; b experiment
that the precision finishing method is effective to improve the
tooth accuracy of cold extruded sun gears. After turning the
upper and lower end faces, turning the addendum circle of the
inner and outer teeth and chamfering, the final product of the
sun gear is shown in Fig. 23. It can be observed that the tooth
shapes of internal-external gears were quite sound. The results
found in this study can provide certain reference evidence to
the practical industrial production and feasible ideas for the
manufacture of such sun gear parts with internal-external
tooth shapes.
6 Conclusions

The FE simulation prediction and experimental confirmation of the precision finishing process have drawn the following conclusions.

(1) The direction of material flow was mainly axial downward under the push of punch. Due to the difference in material flow speed between the inner layer and the outer layer, the upper and lower ends of the finished sun gear bent inward.

(2) The deformation zone of external gear was the entire tooth surface, and gradually expanded to the inner layer with the increasing of the tooth width, while the deformation zone of internal spline was variable during the precision finishing process.

(3) The results of both simulation and experiment show that the tooth shapes of reshaped sun gear were quite sound, and the peak load obtained by FE simulation was well agreed with the experimental one within a reasonable error of 7.08%, which proves the feasibility of the proposed precision finishing process and the correctness of the numerical simulation.

| Deviations          | $F_A (Q)$ | $F_B (Q)$ | $F_M$ |
|---------------------|-----------|-----------|-------|
| Before finishing (μm)| 30.2 (9)  | 55.9 (10) | 276.4 |
| After finishing (μm)| 17.2 (7)  | 21.1 (8)  | 72.3  |

Fig. 21 Helix deviations of external gear: a FE prediction; b verified experiment

Fig. 22 $M$ value deviations of internal spline
With increasing the tooth width, the single deviations in profile and helix both first diminished remarkably, and then rose gradually, while the single \( M \) value deviation decreased remarkably. The simulation results of teeth deviations are well agreed with the experimental ones, which verifies the reliability of the FE prediction strategy.

The profile accuracy of external gear can be improved from ninth to seventh class, tooth lead accuracy is enhanced from tenth to eighth class, and total \( M \) value deviation of internal spline can be reduced to 72.3 μm, demonstrating the accuracy of extruded sun gear is effectively enhanced. The results found in this study can provide feasible ideas for the manufacture of such sun gear parts with internal-external tooth shapes.

**List of symbols**

- \( m_e \): Module of external gear; \( m_i \): Module of internal spline; \( z_e \): Number of teeth of external gear; \( z_i \): Number of teeth of internal spline; \( \alpha_e \): Pressure angle at pitch circle of external gear; \( \alpha_i \): Pressure angle at pitch circle of internal spline; \( \alpha_{Mi} \): Involute pressure angle of external gear; \( \alpha_{Mi} \): Involute pressure angle of internal spline; \( d_{pe} \): Diameter of measuring pin of external gear; \( d_{pi} \): Diameter of measuring pin of internal spline; \( M_{Re} \): \( M \) value of external gear; \( M_{Ri} \): \( M \) value of internal spline; \( S \): Tooth thickness of external gear; \( E \): Space width of internal spline; \( \alpha \): Involute function; \( D \): Pitch circle diameter of external gear; \( d_b \): Base diameter of external gear; \( D_b \): Base diameter of internal spline; \( d_{pk} \): Diameter of contact point between external gear and measuring pin; \( d_k \): Pitch circle diameter of internal spline; \( \alpha_{Me} \): Involute pressure angle of external gear; \( \alpha_{Mi} \): Involute pressure angle of internal spline; \( d_{pe} \): Diameter of measuring pin of external gear; \( d_{pi} \): Diameter of measuring pin of internal spline; \( M_{Re} \): \( M \) value of external gear; \( M_{Ri} \): \( M \) value of internal spline; \( S \): Tooth thickness of external gear; \( E \): Space width of internal spline; \( \alpha \): Involute function; \( D \): Pitch circle diameter of external gear; \( d_b \): Base diameter of external gear; \( D_b \): Base diameter of internal spline; \( d_{pk} \): Diameter of contact point between external gear and measuring pin; \( d_k \): Pitch circle diameter of internal spline; \( S \): Tooth thickness of external gear; \( E \): Space width of internal spline; \( \alpha \): Involute function; \( D \): Pitch circle diameter of external gear; \( d_b \): Base diameter of external gear; \( D_b \): Base diameter of internal spline; \( d_{pk} \): Diameter of contact point between external gear and measuring pin; \( d_k \): Pitch circle diameter of internal spline; \( W \): Tooth width; \( f_{\alpha j} \): Single profile deviation of finished external gear \((j=1\sim9)\); \( F_{\alpha} \): Total profile deviation of finished external gear; \( S \): Half tooth thickness of target external gear; \( S_{Lj} \): Half tooth thickness of left flank of finished external gear \((j=1\sim9)\); \( S_{Rj} \): Half tooth thickness of right flank of finished external gear \((j=1\sim9)\); \( f_{\beta Lj} \): Single helix deviation of left flank of finished external gear \((j=1\sim9)\); \( f_{\beta Rj} \): Single helix deviation of right flank of finished external gear \((j=1\sim9)\); \( S \): Accuracy grade of external gear; \( M_i \): \( M \) value of finished internal spline \((j=1\sim9)\); \( M_i \): \( M \) value of target internal spline; \( f_{Mj} \): Single \( M \) value deviation of finished internal spline \((j=1\sim9)\); \( F_M \): Total \( M \) value deviation of finished internal spline

**Author contribution** Zuofa Liu designed the experimental method. Zuofa Liu and Qiuyun Wang prepared experimental materials and performed the experiments. Wenjie Feng provided financial support for materials and equipment. Zuofa Liu and Jie Zhou wrote the paper. Wenjie Feng, Qiang Liang, and Qiuyun Wang reviewed and edited the manuscript. All authors read and approved the manuscript.

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**Data availability** The authors confirm that the data supporting the findings of this study are available within the article.

**Declarations**

**Ethical approval** Not applicable.

**Consent to participate** Not applicable.

**Consent to publish** All authors agree to publish.

**Competing interests** The authors declare no competing interests.

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