Reconstructing generalized ghost condensate model with dynamical dark energy parametrizations and observational datasets

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Observations of high-redshift supernovae indicate that the universe is accelerating at the present stage, and we refer to the cause for this cosmic acceleration as “dark energy”. In particular, the analysis of current data of type Ia supernovae (SNIa), cosmic large-scale structure (LSS), and the cosmic microwave background (CMB) anisotropy implies that, with some possibility, the equation-of-state parameter of dark energy may cross the cosmological-constant boundary (\( w = -1 \)) during the recent evolution stage. The model of “quintom” has been proposed to describe this \( w = -1 \) crossing behavior for dark energy. As a single-real-scalar-field model of dark energy, the generalized ghost condensate model provides us with a successful mechanism for realizing the quintom-like behavior. In this paper, we reconstruct the generalized ghost condensate model in the light of three forms of parametrization for dynamical dark energy, with the best-fit results of up-to-date observational data.

I. INTRODUCTION

It has been confirmed admittedly that our universe is experiencing an accelerating expansion at the present time, by many cosmological experiments, such as observations of large scale structure (LSS) [1], searches for type Ia supernovae (SNIa) [2], and measurements of the cosmic microwave background (CMB) anisotropy [3]. This cosmic acceleration observed strongly supports the existence of a mysterious exotic matter, dark energy, with large enough negative pressure, whose energy density has been a dominative power of the universe. The astrophysical feature of dark energy is that it remains unclustered at all scales where gravitational clustering of baryons and nonbaryonic cold dark matter can be seen. Its gravity effect is shown as a repulsive force so as to make the expansion of the universe accelerate when its energy density becomes dominative power of the universe. The combined analysis of cosmological observations suggests that the universe is spatially flat, and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although we can affirm that the ultimate fate of the universe is determined by the feature of dark energy, the nature of dark energy as well as its cosmological origin remain enigmatic at present. However, we still can propose some candidates to interpret or describe the properties of dark energy. The most obvious theoretical candidate of dark energy is the cosmological constant \( \Lambda \) (vacuum energy) [4,5], which has the equation of state \( w = -1 \). However, as is well known, there are two difficulties arise from the cosmological constant scenario, namely the two famous cosmological constant problems — the “fine-tuning” problem and the “cosmic coincidence” problem [6]. The fine-tuning problem asks why the vacuum energy density today is so small compared to typical particle scales. The vacuum energy density is of order \( 10^{-123} \text{GeV}^4 \), which appears to require the introduction of a new mass scale 14 or so orders of magnitude smaller than the electroweak scale. The second difficulty, the cosmic coincidence problem, says: Since the energy densities of vacuum energy and dark matter scale so differently during the expansion history of the universe, why are they nearly equal today? To get this coincidence, it appears that their ratio must be set to a specific, infinitesimal value in the very early universe.

Theorists have made lots of efforts to try to resolve the cosmological constant problem, but all these efforts were turned out to be unsuccessful.1 However, there remain other candidates to explaining dark energy. An alternative proposal for dark energy is the dynamical dark energy scenario. The cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is canceled to exactly zero by some unknown mechanism and introducing a dark energy component with a dynamically variable equation of state. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. Actually, this mechanism is enlightened to a great extent by the inflationary cosmology. As

1 Of course the theoretical consideration is still in process and has made some progresses. In recent years, many string theorists have devoted to understand and shed light on the cosmological constant or dark energy within the string framework. The famous Kachru-Kallosh-Linde-Trivedi (KKLT) model [6] is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Furthermore, string landscape idea [8] has been proposed for shedding light on the cosmological constant problem based upon the anthropic principle and multiverse speculation.
we have known, the occurrence of the current accelerating expansion of the universe is not the first time in the expansion history of the universe. There is significant observational evidence strongly supports that the universe underwent an early inflationary epoch, over sufficiently small time scales, during which its expansion rapidly accelerated under the driven of an “inflaton” field which had properties similar to those of a cosmological constant. The inflaton field, to some extent, can be viewed as a kind of dynamically evolving dark energy. Hence, the scalar field models involving a minimally coupled scalar field are proposed, inspired by inflationary cosmology, to construct dynamically evolving models of dark energy. The only difference between the dynamical scalar-field dark energy and the inflaton is the energy scale they possess. Familiar examples of scalar-field dark energy models include quintessence, K-essence, tachyon, ghost condensate and quintom, and so forth. Generically, there are two points of view on the scalar-field models of dynamical dark energy. One viewpoint regards the scalar field as a fundamental field of the nature. The nature of dark energy is, according to this viewpoint, completely attributed to some fundamental scalar field which is omnipresent in supersymmetric field theories and in string/M theory. The other viewpoint supports that the scalar field model is an effective description of an underlying theory of dark energy. In any case, the scalar field dark energy models must face the test of cosmological observations. A typical approach for this is to predict the cosmological evolution behavior of the models, such as the evolution of equation-of-state or Hubble parameter, by putting in the Lagrangian (in particular the potential) by hand or theoretically, and to make a consistency check of models by comparing it with observations. An alternative approach is to start from observational data and to reconstruct corresponding theoretical Lagrangian. It looks that the latter is more efficient to find out the best-fit models of dark energy from observations.

The accumulation of the current observational data has opened a robust window for probing the recent dynamical behavior of dark energy. An intriguing aspect in the study of dark energy is that the analysis of the observational data of type Ia supernova, mainly the 157 “gold” data listed in Riess et al. [16], including 14 high redshift data from the Hubble Space Telescope (HST)/Great Observatories Origins Deep Survey (GOODS) program and previous data, using the parametrization of Hubble parameter $H(z)$ or equation-of-state of dark energy $w(z)$, shows that the equation of state of dark energy $w$ is likely to cross the cosmological-constant boundary $-1$ (or phantom divide), i.e. $w$ is larger than $-1$ in the recent past and less than $-1$ today. The dynamical evolving behavior of dark energy with $w$ getting across $-1$ has brought forward great challenge to the model-building of scalar-field in the cosmology. The conventional scalar-field model, the quintessence with a canonical kinetic term, can only evolve in the region of $w \geq -1$, whereas the model of phantom with negative kinetic term can always lead to $w \leq -1$. Neither the quintessence nor the phantom alone can realize the transition of $w$ from $w > -1$ to $w < -1$ or vice versa. Although the $K$-essence can realize both $w > -1$ and $w < -1$, it has been shown that it is very difficult for $K$-essence to achieve $w$ crossing $-1$. Hence, the quintom model was proposed for describing the dynamical evolving behavior of $w$ crossing $-1$.

The nomenclature “quintom” is suggested in the sense that its behavior resembles the combined behavior of quintessence and phantom. Thus, a simple realization of quintom scenario is a model with the double fields of quintessence and phantom. The cosmological evolution of such model has been investigated in detail. It should be noted that such a quintom model would typically encounter the problem of quantum instability inherited from the phantom component. For the single real scalar field models, the transition of crossing $-1$ for $w$ can occur for the Lagrangian density $p(\phi, X)$, where $X$ is a kinematic term of a scalar-field $\phi$, in which $\partial p/\partial X$ changes sign from positive to negative, thus we require nonlinear terms in $X$ to realize the $w = -1$ crossing. When adding a high derivative term to the kinetic term $X$ in the single scalar field model, the energy-momentum tensor is proven to be equivalent to that of a two-field quintom model. In addition, it is remarkable that the generalized ghost condensate model of a single-real-scalar-field is a successful realization of the quintom-like dark energy. In Ref. [14], a dark energy model with a ghost scalar field has been explored in the context of the runaway dilaton scenario in low-energy effective string theory. The authors addressed for the dilatonic ghost condensate model the problem of vacuum stability by implementing higher-order derivative terms and showed that a cosmological model of quintom-like dark energy can be constructed without violating the stability of quantum fluctuations. Furthermore, a generalized ghost condensate model was investigated in Refs.[24, 25] by means of the cosmological reconstruction program. For another interesting single-field quintom model see Ref. [26], where the $w = -1$ crossing is implemented with the help of a fixed background vector field. Besides, there are also many other interesting models, such as holographic dark energy model and braneworld model, being able to realize the quintom-like behavior. In this paper we will focus on the generalized ghost condensate model and will reconstruct this model using various dark energy parametrizations and the up-to-date observational datasets.

This paper is organized as follows: In section II we address the various dark energy parametrizations and
describe the analysis results of the current experimental data of various astronomical observations. In section III we perform a cosmological reconstruction for the generalized ghost condensate model from the dark energy parametrizations and the fitting results of the up-to-date observational data. Finally we give the concluding remarks in section IV.

II. DARK ENERGY PARAMETRIZATIONS AND RESULTS OF OBSERVATIONAL CONSTRAINTS

The distinctive feature of the cosmological constant or vacuum energy is that its equation of state is always exactly equal to $-1$. Whereas, the dynamical dark energy exhibits a dynamic feature that its equation-of-state as well as its energy density are evolutionary with time. An efficient approach to probing the dynamics of dark energy is to parameterize dark energy and then to determine the parameters using various observational data. One can explore the dynamical evolution behavior of dark energy efficiently by making use of this way, although the results obtained are dependent on the parametrizations of dark energy more or less. Among the various parametric forms of dark energy, the minimum complexity required to detect time variation in dark energy is to add a second parameter to measure a change in the equation-of-state parameter with redshift. This is the so-called linear expansion parametrization $w(z) = w_0 + w'z$, where $w' = dw/dz|_{z=0}$, which was first used by Di Pietro & Claeskens and later by Riess et al. However, when some “longer-armed” observations, e.g. CMB and LSS data, are taken into account, this form of $w(z)$ will be unsuitable due to the divergence at high redshift. A frequently used parametrization form of equation-of-state, $w(z) = w_0 + w_1z/(1 + z)$, suggested by Chevallier & Polarski and Linder, can avoid the divergence problem effectively. It should be noted that this parametrization form has been investigated enormously in exploring the dynamical property of dark energy in the light of observational data. We shall summarize some main constraint results for this parametrization in the follows.

A recently popular method of constraining the dark energy parametrization is the so-called “global fitting” which tries to make use of the most observational information including CMB, SNIa and LSS data as well as to consider the dark energy perturbation, employing the Markov chain Monte Carlo (MCMC) techniques. In the global fitting one usually should determine an eight-dimensional set of cosmological parameters, $P = \{\omega_b, \omega_c, \Theta_S, \tau, w_0, n_s, \log[10^{10} A_s]\}$, where $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$ are baryon and cold dark matter densities relative to the critical density, $\Theta_S$ is the ratio (multiplied by 100) of the sound horizon and the angular diameter distance, $\tau$ is the optical depth, $A_s$ is defined as the amplitude of the primordial scalar power spectra, and $n_s$ measures the spectral index. In Ref.32, using the first-year WMAP (Wilkinson Microwave Anisotropy Probe) temperature and polarization data and the “gold” dataset of 157 SNIa, the authors obtained the following fit results: for with dark energy perturbation, $\Omega_m = 0.319^{+0.030}_{-0.031}$, $w_0 = -1.167^{+0.190}_{-0.190}$ and $w_a = 0.597^{+0.657}_{-0.713}$; for without dark energy perturbation, $\Omega_m = 0.314^{+0.031}_{-0.031}$, $w_0 = -1.098^{+0.078}_{-0.080}$ and $w_a = 0.416^{+0.293}_{-0.153}$. Subsequently, with the announcement of the latest three-year WMAP data, Zhao et al. extended their previous results in Ref.32 and got the fit results as: for with dark energy perturbation, $w_0 = -1.146^{+0.170}_{-0.178}$ and $w_a = 0.600^{+0.622}_{-0.652}$; for without dark energy perturbation, $w_0 = -1.118^{+0.152}_{-0.147}$ and $w_a = 0.499^{+0.453}_{-0.498}$. Lately, Riess et al. released the new “gold” dataset of 182 SNIa, in which the full sample of 23 SNIa at $z \geq 1$ provides the highest-redshift sample known. In addition, it should be mentioned that the gamma ray bursts (GRBs) have been, in some studies, processed as “known candles” due to some intrinsic correlations between temporal or spectral properties of GRBs and their isotropic energies and luminosities. Such investigations have triggered studies on using GRBs as cosmological probes. The largest GRB sample compiled by Shafer was released lately. In a recent work, for including the most observational information in the analysis of probing the dynamical dark energy, the authors added the GRB data to the global fitting. Their fit results can be summarized as: for with GRB data, $\Omega_m = 0.296^{+0.023}_{-0.019}$, $w_0 = -1.09^{+0.22}_{-0.06}$ and $w_a = 0.89^{+0.11}_{-0.07}$; for without GRB data, $\Omega_m = 0.300^{+0.009}_{-0.033}$, $w_0 = -1.09^{+0.27}_{-0.06}$ and $w_a = 0.90^{+0.07}_{-1.01}$.

An alternative approach to constraining the dynamical dark energy parametrization is to use the measured value of the CMB shift parameter, together with the baryon acoustic oscillation (BAO) measurement from the SDSS, and the SNIa data, which provides a more economical scheme comparing to the global fitting method. The CMB shift parameter $R$ is perhaps the least model-dependent parameter that can be extracted from CMB data, since it is independent of $H_0$. The shift parameter $R$ is given by $R = \Omega_m^{1/2}\int z_{\text{CMB}} dz'/E(z')$, where $z_{\text{CMB}} = 1089$ is the redshift of recombination and $E(z) = H(z)/H_0$. The value of the shift parameter $R$ can be determined by three-year integrated WMAP analysis, and has been updated by Wand & Mukher-
jee [44] to be 1.70 ± 0.03 independent of the dark energy model. The measurement of the BAO peak in the distribution of SDSS luminous red galaxies (LRGs) [45] gives $A = 0.469(n_s/0.98)^{-0.35} ± 0.017$ (independent of a dark energy model) at $z_{BAO} = 0.35$, where $A$ is defined as $A \equiv \Omega_{m0}^{1/2}E(z_{BAO})^{-1/3}(1+z_{BAO})\int_0^{z_{BAO}} dz'/E(z')^{2/3}$. Here the scalar spectral index is taken to be $n_s = 0.95$ as measured by the three-year WMAP data [30]. Wang and Mukherjee [44] used this method $5^*$ and found the constraints on the Linder parametrization: for Riess04+WMAP3+SDSS, $w_0 = -0.813^{+0.293}_{-0.266}$ and $w_a = -0.510^{+1.269}_{-1.299}$; for Astier05+WMAP3+SDSS, $w_0 = -1.017^{+0.199}_{-0.200}$ and $w_a = -0.039^{+1.045}_{-1.052}$. Such results are qualitatively consistent with those of Zhao et al. [37], with significant differences that may be explained by the differences in the combination of data used, and perhaps by some data analysis details.

The aforementioned discussions are focussed on the Linder parametrization of dark energy for the equation of state. Besides the Linder parametrization, there are also some other parametrization forms for the dark energy equation-of-state parameter, for instance, the form $\Omega_{m0}(1+z)^3 + A_0 + A_1(1+z) + A_2(1+z)^2$, where $A_0 + A_1 + A_2 = 1 - \Omega_{m0}$. This form is an interpolating fit for $E^2(z)$ having the right behavior for both small and large redshifts. In this paper, for providing the base for reconstruction of the scalar-field dark energy model, we shall consider these three parametrization forms for dark energy: $w(z) = w_0 + w_a z/(1+z)^2$ [44]. In addition, the parametrization for the Hubble parameter $H(z)$ is also often considered. The form can be expressed as $6^*$

$$E(z) = \Omega_{m0} + A_0 + A_1(1+z) + A_2(1+z)^2,$$

where $A_0 + A_1 + A_2 = 1 - \Omega_{m0}$. This is an interpolating fit for $E^2(z)$ having the right behavior for both small and large redshifts. In this paper, for providing the base for reconstruction of the scalar-field dark energy model, we shall consider these three parametrization forms for dark energy: $w(z) = w_0 + w_a z/(1+z)$, marked as “parametrization 1”; $w(z) = w_0 + w_a z/(1+z)^2$, marked as “parametrization 2”; $E(z) = \Omega_{m0}(1+z)^3 + A_0 + A_1(1+z) + A_2(1+z)^2$, marked as “parametrization 3”.

These three forms of parametrization have lately been considered by Gong & Wang [49]. In addition, it should be mentioned that similar results were also independently obtained in Ref. [50] for the “parametrization 1” and in Ref. [51] for the “parametrization 3”. In Ref. [49], the authors constrained the parameters using the new measurement of the CMB shift parameter [44], together with LSS data (the BAO measurement from the SDSS LRGs) [45] and SNIa data (182 “gold” data released recently) [39]. The fit results are summarized as follows: for “parametrization 1”, $\Omega_{m0} = 0.29^{+0.04}_{-0.06}$, $w_0 = -1.07^{+0.33}_{-0.28}$ and $w_a = 0.85^{+0.61}_{-1.38}$, for “parametrization 2”, $\Omega_{m0} = 0.28^{+0.04}_{-0.03}$, $w_0 = -1.37^{+0.58}_{-0.57}$ and $w_a = 3.39^{+0.53}_{-0.93}$; for “parametrization 3”, $\Omega_{m0} = 0.30^{+0.04}_{-0.02}$, $A_1 = -0.48^{+1.36}_{-1.47}$ and $A_2 = 0.25^{+0.52}_{-0.45}$. The three forms of parametrization are investigated uniformly in this work, so it is convenient to compare them with each other. The cases for evolution of $w(z)$ are plotted in Fig. 1 using the best-fit results. We shall use these fit results to reconstruct the generalized ghost condensate model in the next section.

![FIG. 1](image)

**III. GENERALIZED GHOST CONDENSATE MODEL AND ITS RECONSTRUCTION**

The reconstruction of scalar-field dark energy models has been widely studied. For a minimally coupled scalar field with a potential $V(\phi)$, the reconstruction is simple and straightforward [52]. Saini et al. [53] reconstructed the potential and the equation of state of the quintessence field by parameterizing the Hubble parameter $H(z)$ based on a versatile analytical form of the luminosity distance $d_L(z)$. This method can be generalized to a variety of models, such as scalar-tensor theories [54], $f(R)$ gravity [55], $K$-essence model [56], and also hessian model [57], etc. For the reconstruction of general scalar-field dark energy models, Tsujikawa has investigated in detail [24]. In this section, we shall focus on the reconstruction of the generalized ghost condensate model based on the aforementioned three forms of parametrization.

First, let us consider the Lagrangian density of a general scalar field $p(\phi, X)$, where $X = -g_{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$ is the kinetic energy term. Note that $p(\phi, X)$ is a general...
function of $\phi$ and $X$, and we have used a sign notation $(-, +, +, +)$. Identifying the energy momentum tensor of the scalar field with that of a perfect fluid, we can easily derive the energy density of dark energy, $\rho_{\text{de}} = 2Xp_X - p$, where $p_X = \partial p/\partial X$. Thus, in a spatially flat Friedmann-Robertson-Walker (FRW) universe involving dust matter (baryon plus dark matter) and dark energy, the dynamic equations for the scalar field are

\begin{align}
3H^2 &= \rho_m + 2Xp_X - p, \quad (1) \\
2\dot{H} &= -\rho_m - 2Xp_X, \quad (2)
\end{align}

where $X = \dot{\phi}^2/2$ in the cosmological context, and note that we have used the unit $M_P = 1$ for convenience. Introducing a dimensionless quantity

\begin{equation}
r = E^2 = H^2/H_0^2, \quad (3)
\end{equation}

we find from Eqs. (1) and (2) that

\begin{equation}
p = [(1 + z)r' - 3r]H_0^2, \quad (4)
\end{equation}

\begin{equation}
\dot{\phi}^2p_X = \frac{r' - 3\Omega_m(1 + z)^2}{r(1 + z)}, \quad (5)
\end{equation}

where prime denotes a derivative with respect to $z$. The equation of state for dark energy is given by

\begin{equation}
w = \frac{p}{\dot{\phi}^2p_X - p} = \frac{(1 + z)r' - 3r}{3r - 3\Omega_m(1 + z)^3}. \quad (6)
\end{equation}

Next, let us consider the generalized ghost condensate model proposed in Ref. [24] (see also Ref. [25]), in which the behavior of crossing the cosmological-constant boundary can be realized, with the Lagrangian density

\begin{equation}
p = -X + h(\phi)X^2, \quad (7)
\end{equation}

where $h(\phi)$ is a function in terms of $\phi$. Dilatonic ghost condensate model [14] corresponds to a choice $h(\phi) = ce^{A\phi}$. From Eqs. (4) and (5) we obtain

\begin{equation}
\dot{\phi}^2 = \frac{12r - 3(1 + z)r' - 3\Omega_m(1 + z)^3}{r(1 + z)^2}, \quad (8)
\end{equation}

\begin{equation}
h(\phi) = \frac{6(2(1 + z)r' - 6r + (1 + z)^2\dot{\phi}^2)}{r^2(1 + z)^4\dot{\phi}^2} \rho_{\phi}^{-1}, \quad (9)
\end{equation}

where $\rho_{\phi} = 3H_0^2$ represents the present critical density of the universe. The evolution of the field $\phi$ can be derived by integrating $\phi'$ according to Eq. (5). Note that the field $\phi$ is determined up to an additive constant $\phi_0$, so it

\begin{figure}

**FIG. 2:** Reconstruction of the generalized ghost condensate model according to three forms of parametrization for dynamical dark energy with the best-fit values of the parameters. Parametrization 1 is $w(z) = w_0 + w_0z/(1 + z)$, parametrization 2 is $w(z) = w_0 + w_0z/(1 + z)^2$ and parametrization 3 is $E(z) = \Omega_m(1 + z)^3 + A_0 + A_1(1 + z) + A_2(1 + z)^2)^{1/2}$. In this plot, we show the cases of function $h(\phi)$, in unit of $\rho_{\phi}^{-1}$, corresponding to the best-fit results of the joint analysis of SNIa+CMB+LSS.

\begin{figure}

**FIG. 3:** Reconstruction of the generalized ghost condensate model according to three forms of parametrization for dynamical dark energy with the best-fit values of the parameters. Parametrization 1 is $w(z) = w_0 + w_0z/(1 + z)$, parametrization 2 is $w(z) = w_0 + w_0z/(1 + z)^2$ and parametrization 3 is $E(z) = \Omega_m(1 + z)^3 + A_0 + A_1(1 + z) + A_2(1 + z)^2)^{1/2}$. In this plot, we show the evolutions of the scalar field $\phi(z)$, in unit of the Planck mass $M_P$ (note that here the Planck normalization $M_P = 1$ has been used), corresponding to the best-fit results of the joint analysis of SNIa+CMB+LSS.

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7 This is the unit of Planck normalization, here $M_P \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass.
is convenient to take $\phi$ to be zero at the present epoch ($z = 0$). The function $h(\phi)$ can be reconstructed using Eq.\(^{(9)}\) when the information of observational data is obtained from the observational data. Generically, the Friedmann equation can be expressed as

$$r(z) = \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})f(z),$$

where $f(z)$ is some function encoding the information about the dynamical property of dark energy. For example, the “parametrization 1” corresponds to $f(z) = (1 + z)^{3(1 + w_a + w_p)} \exp[-3w_a z/(1 + z)]$, and the “parametrization 2” corresponds to $f(z) = (1 + z)^{3(1 + w_0)} \exp[3w_0 z^2/2(1 + z)^2]$. Whereas, for the “parametrization 3” one can straightforwardly write $r(z) = \Omega_{m0}(1 + z)^3 + A_0 + A_1(1 + z) + A_2(1 + z)^2$. Hence, we can reconstruct the function $h(\phi)$ for the generalized ghost condensate model in the light of these forms of parametrization and the corresponding fit results of the observational data. The reconstruction for $h(\phi)$ is plotted in Fig.\(^{2}\) using the three parametrizations and the corresponding best-fit values of parameters from the observational data analysis of SNIa+CMB+LSS. The differences between the shapes of $h(\phi)$ comes from the differences in these forms of parametrization because that uncertainties still remain large for “model-independent” observational constrains of dynamical dark energy. The crossing of the cosmological-constant boundary corresponds to $hX = 1/2$. The system can enter the phantom region ($hX < 1/2$) without discontinuous behavior of $h$ and $X$. In addition, the evolution of the scalar field $\phi(z)$ is also determined by the reconstruction program, see Fig.\(^{8}\). We see that the differences in the shapes of $\phi(z)$ are very little. It should be mentioned that the reconstruction of the generalized ghost condensate model has been carried out in Ref.\(^{24}\) in the light of “parametrization 3” from the best-fit results of the SNIa gold dataset \(^{16}\). However, the constraints from the 157 gold data of SNIa can only give very preliminary results \(^{55}\): $A_1 = -4.16 \pm 2.53$ and $A_2 = 1.67 \pm 1.03$, for the prior $\Omega_{m0} = 0.3$. It can be seen clearly that these results have significant differences from those derived from new observational data analysis. So our result of reconstruction significantly improves the previous result.

In addition, as has been pointed out by Tsujikawa \(^{22}\), it should be cautioned that the perturbation of the field $\phi$ is plagued by a quantum instability whenever it behaves as a phantom \(^{14}\). Even at the classical level the perturbation becomes unstable for $1/6 < hX < 1/2$, because that the speed of sound, $c_s^2 = p_X/(p_X + 2Xp_{XX})$, will become negative. This instability may be avoided if the phantom behavior is just transient. In fact the dilatonic ghost condensate model can realize a transient phantom behavior (see, e.g., Fig.\(^{4}\) in Ref.\(^{14}\)). In this case the cosmological-constant boundary crossing occurs again in the future, after which the perturbation will become stable. Nevertheless, one may argue that the field can be regarded as an effective one so as to evade problems such as stability. In particular, the present focus is that how to establish a dynamical scalar-field model on phenomenological level to describe the possible dynamics of dark energy observed, disregarding the field is fundamental or not.

**IV. CONCLUDING REMARKS**

In this paper, we have reconstructed the ghost condensate scalar-field model in the light of three forms of parametrization for dynamical dark energy with fit results of observational data for the parameters. The analysis of the data of astronomical observations suggests that dark energy may possess dynamical nature, i.e. the energy density as well as the equation of state are likely to exhibit dynamical evolution property during the expansion history of the universe. Furthermore, it is intriguing that recent analysis of observational data, especially the “gold” SNIa data, shows that the equation-of-state parameter of dark energy may, with some possibility, cross the cosmological-constant boundary $w = -1$ during the evolution of the universe. Although the scalar-field models of dark energy, such as quintessence and phantom, can provide us with dynamical mechanism for dark energy, the behavior of cosmological-constant crossing brings forward a great challenge to the model-building for dynamical dark energy, because neither quintessence nor phantom can realize this manner. The model of quintom was suggested to realize this behavior by means of the incorporation of the features of quintessence and phantom. The generalized ghost condensate model provides us with a successful single-real-scalar-field model for realizing the quintom-like behavior. For probing the dynamical nature of dark energy, one should parameterize dark energy first and then constrain the parameters using the observational data. In this paper, we reviewed the recent constraint results for various parametrization forms. Based upon three forms of parametrization for dynamical dark energy, $w(z) = w_0 + w_a z/(1 + z)$, and $E(z) = [\Omega_{m0}(1 + z)^3 + A_0 + A_1(1 + z) + A_2(1 + z)^2]^{1/2}$, with the best-fit values of parameters, we perform a reconstruction for the generalized ghost condensate model. The results of reconstruction show that there are some differences in the various forms of parametrization.

On the other hand, it should be noted that the cosmological constant will be more favored than a dynamical dark energy when the SNLS supernova dataset is considered instead of the “gold” one (see e.g. Ref.\(^{51}\) for details). This statement has become even stronger after the recent appearance of the ESSENCE dataset \(^{55}\). However, though the cosmological constant receives support from the SNLS+ESSENCE dataset, the dynamical dark energy can not be ruled out yet. Actually, it is difficult to reach firm conclusion on the property of dark energy from these data until strong model-independent analysis can be carried out. The increase of the quantity and quality of observational data in the future will un-
doubtedly provide a true model-independent manner for exploring the property of dark energy. We hope that the future high-precision observations (e.g. SNAP) may be capable of providing us with deep insight into the nature of dark energy driving the acceleration of the universe.

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