The Human Group Optimizer (HGO): Mimicking the collective intelligence of human groups as an optimization tool for combinatorial problems.

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Abstract

A large number of optimization algorithms have been developed by researchers to solve a variety of complex problems in operations management area. We present a novel optimization algorithm belonging to the class of swarm intelligence optimization methods. The algorithm mimics the decision making process of human groups and exploits the dynamics of this process as an optimization tool for combinatorial problems. In order to achieve this aim, a continuous-time Markov process is proposed to describe the behavior of a population of socially interacting agents, modelling how humans in a group modify their opinions driven by self-interest and consensus seeking. As in the case of a collection of spins, the dynamics of such a system is characterized by a phase transition from low to high values of the overall consensus (magnetization). We recognize this phase transition as being associated with the emergence of a collective superior intelligence of the population. While this state being active, a cooling schedule is applied to make agents closer and closer to the optimal solution, while performing their random walk on the fitness landscape. A comparison with simulated annealing as well as with a multi-agent version of the simulated annealing is presented in terms of efficacy in finding good solution on a NK - Kauffman landscape. In all cases our method outperforms the others, particularly in presence of limited knowledge of the agent.

Keywords: Optimization algorithm, Artificial Intelligence, Collaborative Decisions, Decision Making, Group Decision, Social interactions, Complexity, Markov chains.
I. INTRODUCTION

Researchers have developed a large number of meta-heuristic algorithms inspired by nature with the aim of solving combinatorial optimization problems. A common way to classify them is to distinguish between trajectory and population-based algorithms. Trajectory algorithms, such as Simulated Annealing (SA) and Quantum Annealing [2, 3], describe a trajectory (usually a random walk) in the search space to reach the solution. Population-based algorithms perform multiple search processes, each of them carried out by a different agent. Population-based algorithms can be further distinguished in two classes: i) Evolutionary algorithms and ii) Swarm-based algorithms [4]. The evolutionary algorithms mimic the processes of natural evolution, such as mutation, selection, and inheritance, to identify the best solution. An example of such a class of algorithms is the genetic algorithm [5]. The swarm algorithms exploit the collective intelligence of the social groups, such as flock of birds, ant colonies, and schools of fish, in accomplishing different tasks. They include the Ant Colony Optimization (ACO) [6–8], the Particle Swarm Optimization [9], the Differential Evolution [10], the Artificial Bee Colony [11, 12], the Glowworm Swarm Optimization [13, 14], the Cuckoo Search Algorithm [15], and very recently the Grey Wolf Optimizer [16] and the Ant Lion Optimizer [17].

All these algorithms have been successfully applied to solve production and operation management problems. For example, Simulated Annealing has been mainly employed to solve the traveling salesman problem [18], scheduling problems [19, 20], facility location and supply chain design problems [21, 22]. The Genetic Algorithms count a larger number of applications compared to Simulated Annealing, even though the wideness of the areas to which they have been applied is quite narrow [23]. Aytug et al. [24] provide an interesting review of the use of genetic algorithms for solving different types of operations problems including production control, facility layout design, line balancing, production planning, and supply chain management.

These last years have seen a huge growth of the applications of swarm-based algorithms (in particular, ACO, bee colony, and swarm particle algorithms) in operations management context [25–28]. They share remarkable features, such as decentralization, self-organization, autonomy, flexibility, and robustness, which have been proven very useful to solve complex operational tasks [29, 30]. Applications of ACO algorithm mainly concern the traveling salesman problem, scheduling, vehicle routing, and sequential ordering [31]. More recently, they have been also employed in supply chain contexts to solve production-inventory problems [32, 33] and network design [34].

In particular, these algorithms reproduce the collective decision making that makes social groups superior in solving tasks compared to single individuals. Agents (ants, bees, termites, fishes) make choices, pursuing their individual goals (forage, survive, etc.) on the basis of their own knowledge and amount of information (position, sight, etc.), and adapting their behavior to the actions of the other agents. The group-living enables social interactions to take place as a mechanism for knowledge and information sharing [35–45]. Even though the single agents may possess a limited knowledge, and their actions are usually very simple, the collective behavior, enabled by the social interactions, leads to the emergence of a superior intelligence of the group.

In this paper we propose a novel swarm intelligence optimization algorithm to solve complex combinatorial problems. The proposed algorithm is inspired by the behavior of human groups and their ability to solve a very large variety of complex problems, even when the individuals may be characterized by cognitive limitations. Although it is widely recognized that human groups, such as organizational teams, outperform single individuals in solving many different tasks including new product development, R&D activities, production and marketing issues, literature is still lacking of optimization algorithms inspired by the problem solving process of human groups. Similarly to other social groups, human groups are collectively able, by exploiting the potential of social interactions, to achieve much better performance than single individuals can do. This specific ability of human groups has been defined as group collective intelligence [46, 47] that recently is receiving a growing attention in the literature as to its antecedents and proper measures [46, 47].

The proposed algorithm, hereafter referred to as Human Group Optimization (HGO), is developed within the methodological framework recently proposed by Carbone and Giannoccaro [48] to model the collective decision making of human groups. This model captures the main drivers of the individual behavior in groups, i.e., self-interest and consensus seeking, leading to the emergence of collective intelligence. The group is conceived as a set of individuals making choices based on rational calculation and self-interested motivations. However, any decision made by the individual is also influenced by the social relationships he/she has with the other group members. This social influence pushes the individual to modify the choice he/she made, for the natural tendency of humans to seek consensus and avoid conflict with people they interact with [49]. As a consequence, effective group decisions spontaneously emerge as the result of the choices of multiple interacting individuals.

To test the ability of HGO algorithm, we compare its performance with those of some benchmarks chosen among trajectory-based and population-based algorithms. In particular, the HGO is compared with the Simulated Annealing (SA) and a Multi Agent version of the Simulated Annealing (MASA). We also compare HGO with a well-established swarm algorithm such as ACO in solving the traveling salesman problem.

The paper is organized as follows. In Sec. III we briefly present the decision making model of human groups, which
the HGO algorithm relies on. In Sec. III we discuss the conditions which lead to the emergence of the group collective intelligence. Then, in Sec. IV we present the Human Group Optimization (HGO) algorithm and its main features. Sec. V tests the HGO algorithm in solving N\textsuperscript{P} problems of increasing complexity and compares it with the Simulated Annealing and a Multi-Agent version of Simulated Annealing. In Sec. VI we draw the main conclusion and discuss future perspectives.

II. THE DECISION MAKING MODEL OF HUMAN GROUPS

Here we briefly summarize the decision making model presented in Ref. [48]. We consider a human group made of \( M \) socially interacting members, which is assigned to accomplish a complex task. The task is modelled in terms of \( N \) binary decisions and the problem consists in solving a combinational decision making problem by identifying the set of choices (configuration) with the highest fitness, out of \( 2^N \) configurations.

As an example of application of the method, the fitness landscape, i.e., the map of all configurations and associated fitness values, is generated following the classical NK procedure, where \( N \) are the decisions and \( K \) the interactions among them. Each decision \( d_i \) of the vector \( \mathbf{d} \) is a binary variable \( d_i = \pm 1, i = 1, 2, ..., N \). Each vector \( \mathbf{d} \) is associated with a certain fitness value \( V(\mathbf{d}) \) computed as the weighted sum of \( N \) stochastic contributions \( W_j\left(d_j, d_{j1}, d_{j2}, ..., d_{jK}\right) \) that each decision leads to the total fitness. The contributions \( W_j\left(d_j, d_{j1}, d_{j2}, ..., d_{jK}\right) \) depend on the value of the decision \( d_j \) itself and the values of other \( K \) decisions \( d_{ij}, i = 1, 2, ..., K \), and are determined following the classical NK procedure [50–52]. The fitness function is then defined as

\[
V(\mathbf{d}) = \frac{1}{N} \sum_{j=1}^{N} W_j\left(d_j, d_{j1}, d_{j2}, ..., d_{jK}\right)
\]

The integer index \( K = 0, 1, 2, ..., N - 1 \) corresponds to the number of interacting decision variables, and tunes the complexity of the problem: increasing \( K \) increases the complexity of the problem.

Individuals are characterized by cognitive limits, i.e. they possess a limited knowledge. The level of knowledge of the \( k \)-th member of the group is identified by the parameter \( p \in [0, 1] \), which is the probability that each single member knows the contribution of the decision to the total fitness.

Based on the level of knowledge, each member \( k \) computes his/her own perceived fitness (self-interest) as follows:

\[
V_k(\mathbf{d}) = \frac{\sum_{j=1}^{K} D_{kj}W_j\left(d_j, d_{j1}, d_{j2}, ..., d_{jK}\right)}{\sum_{j=1}^{K} D_{kj}}.
\]

where \( \mathbf{D} \) is the matrix whose elements \( D_{kj} \) take the value 1 with probability \( p \) and 0 probability \( 1-p \).

During the decision making process, each member of the group makes his/her choices to improve the perceived fitness (self-interest) and to seek consensus within the group. The dynamics is modelled by means of a continuous-time Markov process where the state vector \( \mathbf{s} \) of the system has \( M \times N \) components \( s = (s_1, s_2, ..., s_n) = (\sigma_1^1, \sigma_1^2, ..., \sigma_1^N, \sigma_2^1, ..., \sigma_2^N, ..., \sigma_M^1, ..., \sigma_M^N) \). The variable \( \sigma_k^l = \pm 1 \) is a binary variable representing the opinion of the member \( k \) on the decision \( j \). The probability \( P(s, t) \) that at time \( t \), the state vector takes the value \( s \) out of \( 2^N \) possible states, satisfies the master equation

\[
\frac{dP(s, t)}{dt} = -\sum_{l} w(s_l \to s'_l) P(s_l, t) + \sum_{l} w(s'_l \to s_l) P(s'_l, t)
\]

where \( s_l = (s_1, s_2, ..., s_l, s_n) \) and \( s'_l = (s_1, s_2, ..., -s_l, ..., s_n) \). The transition rate of the Markov chain (i.e. the probability per unit time that the opinion \( s_l \) flips to \(-s_l\) while the others remain temporarily fixed) is defined so as to be the product of the transition rate of the Ising-Glauber dynamics [52], which models the process of consensus seeking to minimize the conflict level, and the Weibull exponential rate [53], which models the self-interest behavior of the agents:

\[
w(s_l \to s'_l) = \frac{1}{2} \left[ 1 - s_l \tanh \left( \frac{\beta J}{\langle \kappa \rangle} \sum_{h} A_{lh} s_h \right) \right] \exp \left\{ \beta' [\Delta V(s'_l, s_l)] \right\}
\]

In Eq. (4) \( A_{ih} \) are the elements of the adjacency matrix, \( J/\langle \kappa \rangle \) is the social interaction strength and \( \langle \kappa \rangle \) the mean degree of the network of social interactions. The quantity \( \beta \) is the inverse of the social temperature that is a measure
FIG. 1: The stationary values of the normalized averaged fitness $\eta_\infty$ as a function of $\beta J$, (a); and of the statistically averaged consensus $\chi_\infty$ as a function of $\beta J$, (b). Results are presented for $p = 0.5$, $K = 5$ and for three different team sizes: $M = 6, 12, 24$.

of the degree of confidence the members have in the other judgement/opinion. Similarly, the quantity $\beta'$ is related to the level of confidence the members have about their perceived fitness (the higher $\beta'$, the higher the confidence).

The pay-off function $\Delta V(s_l, s_l)$ is simply the change of fitness perceived by the agent when its opinion on the decision $j$ changes from $s_l$ to $-s_l$. The group fitness value Eq. (1) is used as a measure of the performance of the collective-decision making process. To calculate the group fitness value, the vector $d = (d_1, d_2, ..., d_N)$ needs to be determined. To this end, consider the set of opinions $(\sigma_{j1}, \sigma_{j2}, ..., \sigma_{jM})$ that the members of the group have about the decision $j$, at time $t$. The decision $d_j$ is obtained by employing the majority rule, i.e. we set:

$$d_j = \text{sgn} \left( M^{-1} \sum_k \sigma_{jk}^j \right), \quad j = 1, 2, ..., N \quad (5)$$

If $M$ is even and in the case of a parity condition, $d_j$ is, instead, uniformly chosen at random between the two possible values $\pm 1$. The group fitness is then calculated as $V[d(t)]$ and the ensemble average $\langle V(t) \rangle$ is then evaluated. The efficacy of the group in optimizing $\langle V(t) \rangle$ is then calculated as

$$\eta(t) = \frac{\langle V(t) \rangle - V_{\text{min}}}{V_{\text{max}} - V_{\text{min}}} \quad (6)$$

where $V_{\text{max}}$ and $V_{\text{min}}$ are the maximal and minimal payoffs of the fitness landscape. Note that $0 \leq \eta(t) \leq 1$.

The degree of consensus among the members is also computed. Following Ref. [48] this is defined as:

$$\chi(t) = \frac{1}{M^2 N} \sum_{j=1}^{N} \sum_{kh=1}^{M} R_{hk}^j(t) \quad (7)$$

where $R_{hk}^j(t) = \langle \sigma_{jk}^j(t) \sigma_{kh}^j(t) \rangle$. Observe that $0 \leq \chi(t) \leq 1$.

III. CRITICALITY AND SWARM INTELLIGENCE

We refer to the case of a fully connected network of $M$ agents. We simulate the Markov process by using the well-know stochastic simulation algorithm proposed by Gillespie [46, 57], see also Ref. [48].

Results are shown in Fig. 1 where the stationary values of efficacy $\eta_\infty = \eta(t \to \infty)$ and the degree of consensus $\chi_\infty = \chi(t \to \infty)$ are reported as a function of the quantity $\beta J$ for different group sizes $M = 6, 12, 24$, $N = 12$, $K = 5$, $\beta' = 10$ and for an average level of knowledge $p = 0.5$. Results clearly show that a critical threshold value of $\beta J$ exists at which both consensus and payoff have a sharp and concurrent increase. Notably, the transition from low to high payoff, accompanied by an analogous transition from low to high consensus, becomes sharper as the group size $M$ is increased. However, in all cases, given $\beta' = 10$, the transition occurs for $(\beta J)_C \approx 1$. Interestingly this threshold value actually corresponds to the critical ordering transition of the Ising model on a complete graph, in
FIG. 2: The time-evolution of the efficacy \( \eta(t) \) and degree of consensus \( \chi(t) \) for \( p = 0.1, 0.3, 0.5, 0.8, 1.0 \), and \( K = 5, 9, 11 \).

the thermodynamic limit of large \( M \). This result can be obtained by using the findings by Vespignani and Mendes \[58, 59\], who independently demonstrated that for general graphs the critical transition of the Ising model occurs at

\[
\left( \beta J \langle \kappa \rangle \right)_C = -\frac{1}{2} \log \left( 1 - 2 \frac{\langle \kappa \rangle}{\langle \kappa^2 \rangle} \right)
\]

Thus, considering that for complete graph \( \langle \kappa \rangle = M - 1 \), \( \langle \kappa^2 \rangle = (M - 1)^2 \) and that \( M \) is large, expanding Eq. 8 at first order in \( \langle \kappa \rangle / \langle \kappa^2 \rangle \) gives \( (\beta J)_C = 1 \). However, calculations shows that increasing \( \beta' \) above 10 makes the transition occur at values of \( \beta J \) smaller than one.

Based on these outcomes, the condition that leads to the emergence of the collective intelligence (i.e. high value of efficacy) is simply identified by the critical transition point at which consensus sets in. At this value of consensus, a fully exploitation of the potential of social interactions is obtained. In fact, as soon as the critical threshold value of \( (\beta J)_C \) is reached, the agent with limited knowledge, driven by the social interactions, will make good choices following those group members, who have higher knowledge about the problem.
IV. THE HUMAN GROUP OPTIMIZATION ALGORITHM

In this section we design the HGO algorithm exploiting the collective intelligence property of the decision making process to solve combinatorial problems. To this aim, we emulate the process followed to design the Simulated Annealing algorithm \[1\]. We first observe that the Markov process defined in Eq. (3) with transitions rates Eq. (4) converges to the stationary probability distribution \[48\]

$$P_0(s_l) = \frac{\exp[-\beta E(s_l) + 2\beta' \bar{V}(s_l)]}{\sum_k \exp[-\beta E(s_k) + 2\beta' \bar{V}(s_k)]}$$  (9)

where the total level of conflict is $E(s) = -0.5 (\kappa)^{-1} J \sum_{ij} A_{ij} s_i s_j$. Eq. (9) is a Boltzmann distribution with effective energy

$$E_{\text{eff}}(s_l) = -\bar{V}(s_l) + \alpha E(s_l)$$  (10)

where $\alpha = \beta / (2\beta')$. We then make the parameters $\beta J$ and $\beta'$ change during the process as follows:

$$\beta' = \beta_0' \log (i + 1)$$

$$\beta J = \min \{\mu (i - 1), (\beta J)_{C}\}$$  (11)

where $i$ is the time iterator, $\mu$ is chosen by the user, and $\beta_0'$ is set according to Ref. \[60\]. These requirements assure that the critical transition to the collective intelligence state is completed during the process, and that $\alpha$ vanishes...
FIG. 5: A comparison between the proposed HGO, SA, and MASA, in terms of steady-state efficacy \( \eta_\infty \) as a function of the knowledge level \( p \), for \( N = 12, K = 11 \), (a) and \( N = 18, K = 17 \), (b).

in the long term limit so as to allow \( E_{\text{eff}}(s_t) \to -\bar{V}(s_t) \). Note that, when individuals possess complete knowledge \( (p = 1) \), the latter condition, akin the Simulated Annealing, makes the proposed algorithm converge in probability to the optimum of \( V(d) \) [61, 62].

Also observe that the choice \( \beta J = 0 \) identifies an optimization algorithm very closely related to the Simulated Annealing, except that the fitness landscape is explored by \( M \) non-interacting agents. Hereafter, this algorithm will be referred to as Multi Agent Simulated Annealing (MASA). Observe that MASA is characterized by the absence of social interactions among the agents, and, as such, it is unable to exploit the swarm intelligence of the group.

V. SIMULATION AND RESULTS

In this section we first analyze the performance of the HGO algorithm for the case of a \( NK \) landscape with \( N = 12 \) and \( K \) ranging from 5 to 11. A much more complex case is also analyzed with \( N = 18 \) and \( K = 17 \). We also investigate the effect of the size of the group on the performance of the HGO, by making \( M \) range from 3 to 15.

In all simulations each stochastic process is simulated by generating 50 different realizations and the ensemble average of the results is then calculated. The simulation is stopped at steady-state, i.e., when changes in the time-averages of consensus and pay-off over consecutive time intervals of a given length is sufficiently small.

HGO performance in solving complex problems

In Fig. 2 the time-evolution \( (i \) is the time iterator) of the HGO performance are reported for \( N = 12, K = 5, 9, 11 \), and different levels of knowledge \( p \) ranging from 0.1 to 1. We observe that independently of the complexity level \( K \) and level of knowledge \( p \), the increase of \( \eta(t) \) is always accompanied by simultaneously increase of \( \chi(t) \). This confirms that the transition to swarm intelligence always occurs, and that this condition is necessary to guarantee high performance of the HGO algorithm. The complexity parameter \( K \) only marginally affects the performance of the method. The level of knowledge \( p \) of the agents, instead, strongly affects the performance of the optimization algorithm. However, high efficacy can be achieved already at moderate levels of knowledge: \( p = 0.5 \) determines a final efficacy close to 0.9. This result is also shown in Fig. 3 where the steady-state values of the efficacy \( \eta_\infty \) [Fig. 3(a)], and consensus \( \chi_\infty \) [Fig. 3(b)] are plotted as a function of the level of knowledge \( p \), for \( K = 5, 9, 11 \).

Figure 4 shows \( \eta_\infty \) and \( \chi_\infty \) as a function of \( p \) for different group sizes \( M = 3, 7, 11, 15 \), \( N = 18 \) and \( K = 17 \). We note that, for \( p > 0.3 \), increasing \( M \) slightly improves the outcome of the process i.e. the efficacy of the optimization method [Fig. 3(a)]. In all cases we still notice that the best results are obtained for \( p > 0.5 \) which seems to be a threshold value that must be exceeded to guarantee a high degree of consensus \( \chi_\infty \) among the agents [Fig. 3(b)], and, in turn, high fitness values [Fig. 3(a)].

Fig. 5 compares the HGO with SA and MASA. Results are shown for \( N = 12, K = 11 \), [Fig. 5(a)] and \( N = 18, K = 17 \) [Fig. 5(b)], with \( p \) ranging from 0 to 1. In the case of HGO and MASA, we use \( M = 7 \). In all cases the HGO algorithm outperforms the other methods. However, the most significant differences are observed in the case of limited knowledge of the agents. In these situations HGO strongly outperforms the Simulated Annealing and the Multi-agent Simulated Annealing. In this case, the social interaction among the agents pushes individuals, who do
VI. CONCLUSIONS

In this paper we proposed a novel swarm-based optimization algorithm mimicking the collective decision-making behavior of human groups. This algorithm, which we termed Human Group Optimization (HGO), describes the decision process of the agents in terms of a time-continuous Markov chain, where the transition rates are defined so as to capture the effect of the self-interest, which pushes each single agent to increase the perceived fitness, and of social interactions, which stimulate member to seek consensus with the other members of the group. The Markov chain is, then, characterized by a couple of parameters that, likewise the Simulated Annealing, are subjected to a specific cooling schedule that in the long-time limit makes the system converge in probability to the optimal value. The choice of the parameters is made in order to guarantee the transition to a consensus state at which the group of agents shows a very high degree of collective intelligence. While being in this state, the agents explore the landscape by sharing information and knowledge through social interactions, so as to achieve very good solutions even in the case of a limited knowledge.

To test the proposed HGO algorithm, we considered the hard-NP problem of finding the optimum on NK fitness landscape and compared the methodology with other well established algorithms as the Simulated Annealing and a multi-agent version of it. In all cases the HGO has been shown to significantly outperform the other two algorithms, especially under limited knowledge conditions. Summarizing, our algorithm presents several advantages that make it very suitable to solve complex operation management problems. It is flexible because it can be applied to almost any combinatorial problem by identifying the number of decisions the agents should make. However, its most attractive feature relies in its ability to identify very good solutions, even in presence of partial knowledge of the agents. For this reason it appears very promising for applications in distributed decision making contexts such as supply chains. Furthermore, while the vast majority of swarm intelligent algorithms, mimicking the behavior of social groups like insects and animals, are based on the mechanism of the stigmergy, our algorithm introduces a mechanism based on the direct communication among individuals, which is a more powerful and effective way to achieve coordination. Under this perspective, the proposed code is novel and unique within the class of swarm intelligent optimization codes.

We recognize that this first version of the algorithm could be further improved in future research by identifying better cooling schedules. The algorithm could be also fine-tuned to solve specific operations management problems characterized by distributed decision making and information asymmetry, such as multi-stage production scheduling, location routing problem, supply chain inventory problem, just to name a few. Additional numerical tests and theoretical investigation, not in the scope of present study, are however needed to quantify pros and cons.

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