A systematic study of finite BRST-BFV transformations in $Sp(2)$-extended generalized Hamiltonian formalism

Igor A. Batalin\textsuperscript{(a)}, Peter M. Lavrov\textsuperscript{(b,c)}, Igor V. Tyutin\textsuperscript{(a)}

\textsuperscript{(a)} P.N. Lebedev Physical Institute, Leninsky Prospect 53, 119 991 Moscow, Russia
\textsuperscript{(b)} Tomsk State Pedagogical University, Kievskaya St. 60, 634061 Tomsk, Russia
\textsuperscript{(c)} National Research Tomsk State University, Lenin Av. 36, 634050 Tomsk, Russia

Abstract

We study systematically finite BRST-BFV transformations in $Sp(2)$-extended generalized Hamiltonian formalism. We present explicitly their Jacobians and the form of a solution to the compensation equation determining the functional field dependence of finite Fermionic parameters, necessary to generate arbitrary finite change of gauge-fixing functions in the path integral.

Keywords: Generalized Hamiltonian formalism, field-dependent BRST-BFV transformation

PACS numbers: 11.10.Ef, 11.15.Bt

\textsuperscript{1}E-mail: batalin@lpi.ru
\textsuperscript{2}E-mail: lavrov@tspu.edu.ru
\textsuperscript{3}E-mail: tyutin@lpi.ru
1 INTRODUCTION

In our recent articles [1, 2], we have studied finite BRST transformations in the framework of the generalized Hamiltonian (BFV) formalism [3, 4, 5], as well as of the field-antifield (BV) formalism [6, 7]. An important result was obtained that a finite BRST transformation was capable of generating an arbitrary finite change of gauge-fixing conditions in these quantization methods. Both these quantization schemes are essentially based on the BRST symmetry principle [8, 9, 10]. In addition to usual BRST symmetry, the anti-BRST symmetry was also known [11, 12] for Yang-Mills theories in special gauges. For a long time, an opinion was dominating that Hamiltonian quantization of dynamical systems with constraints, preserving the BRST-anti-BRST (or extended BRST) symmetry, could be performed only if structure coefficients of a gauge algebra were independent of phase variables [13], and in turn, the Lagrangian quantization respecting the extended BRST symmetry was possible only for gauge theories with closed algebra [14]. Later, quantization methods were discovered based essentially on the extended BRST symmetry principle in the Hamiltonian formalism of arbitrary dynamical systems with constraints [15, 16, 17], as well as in the Lagrangian formalism of general gauge theories [18, 19, 20]. As the variables of extended phase space in the Hamiltonian formalism, as well as the variables of extended configuration space in the Lagrangian formalism, form irreducible representations of the $Sp(2)$ group, these methods are labeled with abbreviation "Sp(2)".

In this paper, we will extend the results of our previous paper [1] to the case of $Sp(2)$ generalized Hamiltonian formalism [15, 16, 17], with two Fermionic parameters, so that the global $Sp(2)$ symmetry is included by construction. Physically, that $Sp(2)$ symmetry establishes the "democracy" between ghosts and anti-ghosts. Notice that in the $Sp(2)$ symmetric formalism, a gauge-fixing function is a Boson, in contrast to the standard case [3, 4, 5]. At the same time, the $Sp(2)$ vector-valued BRST-BFV generators enter the unitarizing Hamiltonian quadratically. We define effectively the $Sp(2)$-extended BRST-BFV transformations by the corresponding Lie equations in the "plane" of the two Fermionic parameters. In principle, our new construction follows the same general logic as we did in our previous article [1]. The main new feature is that the Jacobian of the $Sp(2)$-extended BRST-BFV transformation is expressed in terms of a determinant of a $2 \times 2$ matrix, so that the compensation equation becomes $2 \times 2$ matrix-valued as well.

2 $Sp(2)$-EXTENDED FINITE BRST-BFV TRANSFORMATIONS AND THEIR JACOBIANS

Let

$$z^i = (q; p), \quad \varepsilon(z^i) = \varepsilon_i$$

be a complete set of canonical variables specific to the phase space of $Sp(2)$-extended generalized Hamiltonian formalism. We proceed with the following path integral representation for the
partition function $Z_F$,
\[
Z_F = \int Dz \exp \left[ \left( \frac{i}{\hbar} \right) W_F \right],
\]
where the action $W_F$ is defined as (all the $Sp(2)$ indices such as ”$a, b, ...$” take the two values $a = 1, 2$; by $F$ we denote a gauge-fixing Boson in the $Sp(2)$-extended formalism),
\[
W_F = \int \left[ \left( \frac{1}{2} \right) z^i(t) \omega_{ik} \dot{z}^k(t) - H_F(t) \right] dt,
\]
\[
H_F(t) = \mathcal{H}(t) + \left( \frac{1}{2} \right) \varepsilon_{ab} \{\{F, \Omega^a\}, \Omega^b\}_t,
\]
\[
\{\Omega^a, \mathcal{H}\} = 0, \quad \{\Omega^a, \Omega^b\} = 0,
\]
\[
\varepsilon(\mathcal{H}) = \varepsilon(F) = 0, \quad \varepsilon(\Omega^a) = 1,
\]
\[
\mathcal{H} = H_0 + \ldots, \quad \Omega^a = \varepsilon^a T + \ldots.
\]
Here in (2.3), $z^i(t)$ are functions of time (trajectories), $\dot{z}^k(t) = dz^k(t)/dt$, $H_F(t)$, $\mathcal{H}(t)$, $\Omega(t)$, $F(t)$ are local functions of time: $H_F(t) = H_F(z)|_{z \rightarrow z(t)}$ and so on, $\{, \}_t$ means the Poisson superbracket for fixed time $t$: $\{F, \Omega^a\}_t = \{F(z), \Omega^a(z)\}|_{z \rightarrow z(t)}$ and so on, $\{z^i, z^k\} = \omega_{ik} = \text{const} = -\omega_{ki}(-1)^{\varepsilon_i \varepsilon_k}$ is an invertible even matrix; $\omega_{ik} = \omega_{ki}(-1)^{\varepsilon_i \varepsilon_k}(\varepsilon_{i+1} \varepsilon_{k+1})$ stands for an inverse to $\omega^{ik}$; $\varepsilon_{ab}$ is the constant $Sp(2)$-invariant antisymmetric tensor, while $\varepsilon_{ab}$ stands for an inverse to $\varepsilon^{ab}$, $\varepsilon^{ab} \varepsilon_{bc} = \delta^a_c$, $\mathcal{H}$ and $F$ are Bosons while $\Omega^a$ are Fermions. It follows from (2.4) that
\[
\{\Omega^a, H_F\} = 0.
\]
We define finite BRST-BFV transformations in their differential form, by the following Lie equations in the ”plane” of their Fermionic parameters $\mu_a$,
\[
\bar{z}^i(z, \mu) \partial^a = \{\bar{z}^i, \Omega^a\}_\tau, \quad \bar{\Omega}^a = \frac{\partial}{\partial \mu_a},
\]
\[
\bar{\Omega}^a = \Omega^a(z), \quad \bar{z}^i|_{\mu=0} = z^i.
\]
It follows from (2.9), (2.10) that
\[
\bar{z}^i(z, \mu) \partial^a \partial^b = \{\{\bar{z}^i, \Omega^a\}_\tau, \Omega^b\}_\tau,
\]
and then
\[
\bar{z}^i = z^i + \{z^i, \Omega^a\}_{\mu_a} + \frac{1}{2}\{\{z^i, \Omega^a\}, \Omega^b\}_{\mu_b \mu_a} = z^i \exp[\{\ldots, \Omega^a\}_{\mu_a}].
\]
The last equality in (2.12) does confirm explicitly the canonicity of that transformation with \( \mu_a = \text{const.} \).

By applying the operators \( \overleftarrow{\partial^a} \) to \( \Omega^a \) and \( \mathcal{H} \), and using the Lie equations (2.9) together with (2.5), we get

\[
\overleftarrow{\Omega}^a = \Omega^a, \quad \overleftarrow{\mathcal{H}} = \mathcal{H}.
\]  

(2.13)

In the same way, we get for the unitarizing Hamiltonian \( H_F \) (2.4)

\[
\overleftarrow{H}_F = H_F.
\]  

(2.14)

The finite BRST-BFV transformations of trajectories \( z^i(t) \) have the form

\[
\tilde{z}^i(i) = z^i(t) + \{ z^i, \Omega^a \}_t \mu_a + \frac{1}{2} \{ \{ z^i, \Omega^a \}, \Omega^b \}_t \mu_b \mu_a.
\]  

(2.15)

In general, the two Fermionic parameters in rep. (2.15) are allowed to be integral functionals \( \mu_a = \mu_a[z] \) of the whole trajectory \( z(t), -\infty < t < \infty \). However, by themselves, \( \mu_a \) are independent of the current time \( t \) and local position \( z \),

\[
d_t \mu_a[z] = 0, \quad \partial_i \mu_a[z] = 0,
\]  

(2.16)

where we have denoted

\[
d_t = \frac{d}{dt}, \quad \partial_i = \frac{\partial}{\partial z^i}.
\]  

(2.17)

Thus, only a functional derivative such as \( \delta/\delta z \) is capable to differentiate the \( \mu_a[z] \) nontrivially.

By applying the operators \( \overleftarrow{\partial^a} \) to the kinetic part of the action (2.3) one and two times, and using the Lie equation (2.9), we get for the total action (2.3)

\[
\overleftarrow{W}_F = W_F + \frac{1}{2} \left[ (N - 2)\Omega^a \mu_a + \frac{1}{2} \{ (N\Omega^a), \Omega^b \}_t \mu_b \mu_a \right]_t^{+\infty} - \left. \right|_{-\infty} - \infty,
\]  

(2.18)

where \( N = z^i \partial_i \), and we have used (2.14). For the class of trajectories whose asymptotic is such that the boundary term in the square bracket in the r. h. s. in (2.18) is zero, the total action (2.3) is invariant,

\[
\overleftarrow{W}_F = W_F.
\]  

(2.19)

Now, let us consider the functional Jacobian,

\[
J = \text{sDet} \left[ \left( \overleftarrow{z}^i(t) \overleftarrow{\delta z}^i(t') \right) \right] = \text{sDet} \left[ \left( \overleftarrow{z}^i(z, \mu) \overleftarrow{\delta z}^i(t') \right) \right] = \text{sDet} \left[ \left( \overleftarrow{z}^i(z, \mu) \overleftarrow{\delta z}^i(t') \right) \right] \left( \mu_a[z] \overleftarrow{\delta} \right) \left( \frac{\delta}{\delta z^i(t')} \right).
\]  

(2.20)
We factorize the Jacobian (2.20) in the form
\[ J = J_1 J_2 , \] (2.21)
where
\[ J_1 = \text{sDet} \left[ (G^{-1})^i_k (t, t'; \lambda = 1) \right] , \] (2.22)
\[ J_2 = \text{sDet} \left[ \left( \frac{\delta}{\delta z^i} \right)_t \delta (t - t') \right] , \] (2.23)
\[ G^i_k (t, t' ; \lambda) = \delta^i_k \delta (t - t') - \lambda \{ z^i, \Omega^a \} t A_{ak} (t'') , \] (2.24)
\[ A_{ak} (t'') = \int dt' \mu_a [z] \frac{\delta}{\delta z^i (t')} \{ z^i, \Omega^a \} t G^i_k (t', t''; \lambda) = \] \[ = [(1 + \lambda \kappa)^{-1}]^b_a \left( \mu_b [z] \frac{\delta}{\delta z^k (t'')} \right) , \] (2.25)
\[ \kappa^b_a = \mu_a [z] \int dt \frac{\delta}{\delta z^i (t)} \{ z^i, \Omega^b \} t . \] (2.26)

Notice that the original explicit form of the integral equation for the Green function \( G \) is given by (2.24) with the first expression in (2.25) standing for \( A_{ak} (t'') \). Then, by multiplying by \( \mu_a [z] [\delta / \delta z^i (t)] \) from the left and taking the \( t \)-integral, it follows a simple linear algebraic equation whose solution is given by the second expression for \( A_{ak} (t'') \) in (2.25).

It is a characteristic feature of the factor (2.22) that the operator therein is nontrivial only for \( \mu_a \) depending actually on fields. On the other hand, in the factor (2.23), the corresponding operator has a nontrivial part proportional to undifferentiated \( \mu_a \). Let us consider the factors (2.22), (2.23) in more detail. For the \( J_1 \) factor, we have
\[ \ln J_1 = \int_0^1 d\lambda \int dt dt' G^i_j (t, t'; \lambda) \{ z^j, \Omega^a \}_t \left( \mu_a [z] \frac{\delta}{\delta z^i (t)} \right) (-1)^{\xi i} = \]
\[ = - \int_0^1 d\lambda \int dt A_{aj} (t') \{ z^j, \Omega^a \}_t = \]
\[ = - \int_0^1 d\lambda \int dt' [(1 + \lambda \kappa)^{-1}]^b_a \left( \mu_b [z] \frac{\delta}{\delta z^k (t')} \right) \{ z^i, \Omega^a \}_t = \]
\[ = - \int_0^1 \text{tr} [(1 + \lambda \kappa)^{-1}] d\lambda = - \text{tr} \left[ \ln (1 + \kappa) \right] . \] (2.27)

Thus, we have finally for \( J_1 \) (2.22)
\[ J_1 = [\det (1 + \kappa)]^{-1} . \] (2.28)
Now, let us consider the ultra-local Jacobian $J_2$ (2.23). As that Jacobian involves only undifferentiated $\mu_a$, one is allowed to consider the $\mu_a$ as a constant. On the other hand, in the latter case the transformation is canonical, so that the Liouville theorem tells us that

$$J_2 = 1.$$ (2.29)

Indeed, by applying the operator $\hat{\partial}_a$ to $J_2$ and using the Lie equation (2.9), one can confirm the equality (2.29) explicitly,

$$\left[ \text{str ln} \left( \Xi'(z, \mu) \hat{\partial}_j \right) \right] \hat{\partial}_a = \left( \Xi'(z, \mu) \frac{\hat{\partial}}{\hat{\partial}z^i} \right) \left( \Xi'(z, \mu) \hat{\partial}_i \hat{\partial}_a \right) (-1)^{\epsilon_i} =$$

$$= \left\{ \Xi^k, \Omega^a \right\} \frac{\hat{\partial}}{\hat{\partial}z^k} = \omega^{kj} \frac{\partial}{\partial z^j} \frac{\partial}{\partial z^k} \Xi^a = 0.$$ (2.30)

Now, we have: $\ln J_2 = \delta(0) \int dt \text{str ln} [\Xi'(z, \mu) \hat{\partial}_j]_t$; as usual, we assume that the zero in (2.30) is the principal factor. In that case, we arrive at (2.29).

So, from (2.21), (2.28), (2.29) we conclude finally

$$J = J_1 = [\det (1 + \kappa)]^{-1},$$ (2.31)

with $\kappa$ given by (2.26).

### 3 MATRIX-VALUED COMPENSATION EQUATION AND ITS EXPLICIT SOLUTION

Now, we would like to use the Jacobian (2.31) to generate arbitrary finite change $\delta F$ of the gauge Boson $F$ in the action (2.3),

$$F \rightarrow F_1 = F + \delta F$$ (3.1)

Due to the invariance (2.19) of the action (2.3), we have for the partition function (2.2) in the new variables

$$Z_F = \int D\Xi \exp \left[ \left( \frac{i}{\hbar} \right) W_F \right] = \int Dz J \exp \left[ \left( \frac{i}{\hbar} \right) W_F \right].$$ (3.2)

Let us require the following condition to hold

$$J = \exp \left[ - \left( \frac{i}{\hbar} \right) \int dt \left( \frac{1}{2} \right) \{\delta F, \Omega^c\}, \Omega^a \right]_t \right].$$ (3.3)

It follows then the gauge-independence property of the partition function,

$$Z_{F_1} = Z_F,$$ (3.4)
for arbitrary finite $\delta F$. We call the condition (3.3) "a compensation equation". Due to the formula (2.31), it follows that (3.3) is rewritten as

$$\text{tr} \left\{ \left[ \ln (1 + \kappa) \right]^b_a \right\} = \text{tr} \left( x^b_a \right),$$

(3.5)

where the matrix-valued functional $x^b_a$ is defined by

$$x^b_a = \left( \frac{i}{\hbar} \right) \int dt \frac{1}{2} \varepsilon_{ac} \{ \delta F, \Omega^c \}_t.$$  

(3.6)

Now, let us require the matrix-valued counterpart to (3.5) to hold

$$\left[ \ln (1 + \kappa) \right]^b_a = x^b_a.$$  

(3.7)

Due to (2.26), Eq. (3.7) is rewritten in a more detail as

$$\mu_a[z] \int dt \frac{\delta}{\delta z^i(t)} \{ z^i, \Omega^b \}_t = [\exp(x) - 1]^b_a.$$  

(3.8)

That is a functional equation to determine $\mu_a$. There is an obvious explicit solution to that equation

$$\mu_a = \mu_a[z; \delta F] = \left( \frac{i}{\hbar} \right) [f(x)]^b_a \int dt \left( \frac{1}{2} \right) \varepsilon_{bc} \{ \delta F, \Omega^c \}_t,$$

(3.9)

where $x^b_a$ is defined in (3.6), and

$$[f(x)]^b_a = [(\exp(x) - 1)x^{-1}]^b_a.$$  

(3.10)

Functional operators in l. h. s. in (3.8), when applying to the rightmost factor in (3.9), yield the $x$ of (3.6) to cancel the factor $x^{-1}$ in (3.10). On the other hand, these functional operators do annihilate $x$ itself due to the Jacobi identity and the second in (2.5). Thus we have confirmed explicitly the compensation equation (3.8) to hold. In the first order in $\delta F$, explicit solution (3.9) takes the usual form

$$\mu_a[z; \delta F] = \left( \frac{i}{\hbar} \right) \int dt \left( \frac{1}{2} \right) \varepsilon_{ac} \{ \delta F, \Omega^c \}_t + O((\delta F)^2).$$  

(3.11)

4 Sp(2)-EXTENDED FUNCTIONAL BRST-BFV TRANSFORMATIONS FOR TRAJECTORIES

It appears quite natural to make our considerations above more transparent by introducing a concept of functional BRST-BFV transformations. Namely, let us define a functional operator of the form

$$\overset{\leftarrow}{\delta}^a = \int dt \frac{\overset{\leftarrow}{\delta}}{\delta z^i(t)} \{ z^i, \Omega^a \}_t.$$  

(4.1)
It follows from the second in (2.5) that the Fermionic operators (4.1) super-commute among themselves,
\[
\varepsilon(d^a, d^b) = 1, \quad [d^a, d^b] = d^a d^b + d^b d^a = 0, \quad d^a d^b d^c = 0.
\] (4.2)

The transformation (2.12) of a trajectory \(z^i(t)\) is rewritten in terms of the operators (4.1) as
\[
\zeta^i(t) = z^i(t) \left( 1 + \frac{\varepsilon(d^a \mu_a)}{2} + \frac{\varepsilon(d^a d^b \mu_b \mu_a)}{2} \right).
\] (4.3)

For trajectory-independent parameters \(\mu_a\), rep. (4.3) is transformed to the form
\[
\zeta^i(t) = z^i(t) \exp(d^a \mu_a).
\] (4.4)

Thus, the operators (4.1) are functional BRST-BFV generators at a trajectory.

Functional Jacobian (2.31) is rewritten in terms of the generator (4.1) as
\[
J = \left[ \det \delta_{ab} + (\mu_a[z] d^b) \right]^{-1}.
\] (4.5)

Compensation equation (3.8) takes the form
\[
\mu_a[z] d^b = [\exp(x) - 1]_a^b,
\] (4.6)

where
\[
x_a^b = \left( \frac{i}{\hbar} \right) \left( \frac{1}{2} \right) \varepsilon_{ac} \int dt (\delta F(t) d^c d^b).
\] (4.7)

Similarly to (4.7), the gauge-fixed unitarizing Hamiltonian at a trajectory \(z^i(t)\) is rewritten as
\[
H_F(t) = \mathcal{H}(t) + \frac{1}{2} \varepsilon_{ab} F(t) d^b d^a.
\] (4.8)

Thus, we conclude that all the main objects in our considerations can be expressed naturally in terms of the functional BRST-BFV generators (4.1).

Finally, let us represent the equality in (4.3) in the form
\[
\zeta^i(t) = z^i(t) T(\mu),
\] (4.9)

where the T-operators are defined by
\[
T(\mu) = 1 + \frac{\varepsilon(d^a \mu_a)}{2} d^a d^b \mu_b \mu_a.
\] (4.10)

Their commutators have the form
\[
\left[ T(\mu), T(\mu') \right] = \varepsilon(d^a [\mu_a d^b] \mu_b') + (\mu_a d^b d^c \mu'_b \mu'_c) - (\mu \leftrightarrow \mu') +
\frac{\varepsilon(d^a d^b \mu_b \mu'_a \mu'_c)}{2} + (\mu_b \mu'_a d^c d^e \mu'_e \mu'_c) - (\mu \leftrightarrow \mu').
\] (4.11)

That nonlinear open algebra looks essentially more complicated as compared to the corresponding algebra in the standard case [1].
5 Sp(2) VECTOR-VALUED WARD IDENTITIES DEPENDENT OF BRST-BFV PARAMETERS/FUNCTIONALS

As we have defined finite BRST-BFV transformations, it appears quite natural to apply them immediately to deduce the corresponding modified version of the Ward identity. We will do that just in terms of functional BRST-BFV generators introduced in Sec. 4.

As usual for that matter, let us proceed with the external-source dependent generating functional,

\[ Z_F(\zeta, z^*, z^{**}) = \int Dz \exp \left[ \left( \frac{i}{\hbar} \right) W_F(\zeta, z^*, z^{**}) \right], \]  

(5.1)

\[ W_F(\zeta, z^*, z^{**}) = W_F + \int dt \left( \zeta_i z^i + z_{i a}^* z^i \hat{d}^a + z_i^{**} z^i \left( \frac{1}{2} \right) \hat{d}^a \hat{d}^b \varepsilon_{ba} \right), \]  

(5.2)

\[ \varepsilon(\zeta_i) = \varepsilon(z_{i a}^*) = \varepsilon_i, \quad \varepsilon(z_i^{**}) = \varepsilon_i + 1, \]  

(5.3)

were \( \zeta_i \) are arbitrary external sources to \( z^i \), while \( z_{i a}^* \), and \( z_i^{**} \) are the so-called antifields to \( z^i \), which are, in fact, external sources to BRST variations of \( z^i \) and to the composition of these BRST variations, respectively. Of course, in the presence of non-zero external sources, the path integral (5.1) is in general actually dependent of gauge-fixing Boson \( F \). However, due to the known equivalence theorem, the physical observables are gauge-independent [21]. It is just the Ward identity what measures the deviation of the path integral from being gauge-independent.

Let us perform in (5.1) the change \( z^i \to \bar{z}^i \) of integration variables, where \( \bar{z}^i \) is defined by (4.3) with arbitrary \( \mu_a[z] \). Then, by using the BRST-BFV invariance (2.19), as well as (4.5) for the Jacobian, we get what we call "a modified Ward identity",

\[ \left\langle \left[ 1 + \left( \frac{i}{\hbar} \right) \int dt_1 \zeta^i \left( \hat{d}^a \mu_a + \left( \frac{1}{2} \right) \hat{d}^a \hat{d}^b \mu_b \mu_a \right) + \left( \frac{i}{\hbar} \right) \int dt z_{i a}^* z^i \hat{d}^a \right] \hat{d}^b \mu_b \right\rangle_{F; \zeta, z^*, z^{**}} = 1, \]  

(5.4)

where we have denoted the source dependent mean value,

\[ <(...)>_{F; \zeta, z^*, z^{**}} = [Z_F(\zeta, z^*, z^{**})]^{-1} \int Dz (...) \exp \left[ \left( \frac{i}{\hbar} \right) W_F(\zeta, z^*, z^{**}) \right], \]  

(5.5)

related to the source dependent action in (5.1). By construction, in (5.4) both \( (\zeta_i, z_{i a}^*, z_i^{**}) \) and \( \mu_a \) is arbitrary. The presence of arbitrary \( \mu_a \) in the integrand in (5.4) reveals the implicit dependence of the generating functional (5.1) on the gauge-fixing Boson \( F \) for nonzero external source \( \zeta_i \).
Let us denote by $R$ the expression in the first square bracket in the integrand in the left-hand side in (5.4),
\[
\left\langle R \left[ \det (\delta^b_a + \mu_a[z] \frac{\delta}{d^b}) \right]^{-1} \right\rangle_{F;\zeta,z^*,z^{**}} = 1. \tag{5.6}
\]
By identifying in (5.6) the $\mu_a[z] = \mu_a[z; -\delta F]$ with the solution to the compensation equation (4.6) with the inverse sign of $\delta F$, it follows from (5.6) the formula generalizing (3.4) to the presence of the external sources,
\[
Z_{F_1} = Z_F < R >_{F;\zeta,z^*,z^{**}}. \tag{5.7}
\]
For a field-independent $\mu_a = \text{const}$, the latter does contribute separately to each its order in (5.4), and we get from (5.4) to the linear order in $\mu_a$,
\[
\left\langle \int dt (\zeta_i z^i d^a + z^*_a z^{*i} d^b d^a) \right\rangle_{F;\zeta,z^*,z^{**}} = 0, \tag{5.8}
\]
which is exactly the standard $Sp(2)$-form of a Ward identity. In terms of the generating functional (5.1), the Ward identity (5.8) is rewritten in a variation-derivative form,
\[
\int dt \left[ \zeta_i \frac{\delta}{\delta z^{*a}} - \varepsilon^{ab} z^*_b \frac{\delta}{\delta z^{**a}} \right] Z_F(\zeta, z^*, z^{**}) = 0. \tag{5.9}
\]
Now, let $S(z, z^*, z^{**})$ be a functional Legendre transform to $(\hbar/i) \ln Z_F(\zeta, z^*, z^{**})$ with respect to the external source $\zeta_i$,
\[
z^k = \frac{\delta}{\delta \zeta_k} \left( \frac{\hbar}{i} \right) \ln Z_F(\zeta, z^*, z^{**}), \tag{5.10}
\]
\[
S(z, z^*, z^{**}) = \left( \frac{\hbar}{i} \right) \ln Z_F(\zeta, z^*, z^{**}) - \int dt \zeta_i z^i, \tag{5.11}
\]
\[
\frac{\delta}{\delta z^i} S = -\zeta_i. \tag{5.12}
\]
It follows then from (5.10)-(5.12) that the $Sp(2)$ master equation,
\[
\left( \frac{1}{2} \right) (S, S)^a + V^a S = 0, \tag{5.13}
\]
holds for $S$, where we have denoted the so-called "$Sp(2)$-antibracket",
\[
(F, G)^a = \int dt \left( F \left[ \frac{\delta}{\delta z^i} \frac{\delta}{\delta z^{*a}} - \frac{\delta}{\delta z^{*a}} \frac{\delta}{\delta z^i} \right] G \right) = -(G, F)^a (-1)^{(\varepsilon_F+1)(\varepsilon_G+1)}, \tag{5.14}
\]
and the "$V^a$-operators",
\[
V^a = \varepsilon^{ab} \int dt z^*_b \frac{\delta}{\delta z^{**a}} \tag{5.15}
\]
(for details and properties of operators used here, see [15]).
6 Discussions

In the framework of the $Sp(2)$-extended generalized Hamiltonian formalism \cite{15, 16, 17}, we have studied systematically finite BRST-BFV transformations with two parameters being odd functionals of phase variables. We have defined these transformation effectively by the corresponding Lie equations in the Fermionic "plane" of the two parameters. It was shown that the Jacobian of finite transformation can be represented explicitly in the form of a $2 \times 2$ determinant. We have formulated the $2 \times 2$ matrix-valued compensation equation which is sufficient to provide for generating an arbitrary finite change of a gauge-fixing function in the path integral. In this way, we have extended the proof of the gauge independence of the partition function under finite variations of gauge-fixing function. An efficient technique was developed based on the use of the $Sp(2)$ vector-valued functional differential as operating on the space of trajectories. It appears that all the main objects in our consideration can be represented in a natural way in terms of the functional differential proposed. By making use of that technique as applied to finite BRST-BFV transformation, we derive the $Sp(2)$ vector-valued modified Ward identities. As a particular case of the latter, we have derived the $Sp(2)$ vector-valued master equation for the effective action. As another particular case, we have derived the relation connecting the generating functionals for Green functions in two arbitrary admissible gauges.

Acknowledgments

I. A. Batalin would like to thank Klaus Bering of Masaryk University for interesting discussions. The work of I. A. Batalin is supported in part by the RFBR grants 14-01-00489 and 14-02-01171. P. M. Lavrov thanks I. L. Buchbinder for useful discussions. The work of P. M. Lavrov is partially supported by the Ministry of Education and Science of Russian Federation, grant TSPU-122, by the Presidential grant 88.2014.2 for LRSS and by the RFBR grant 13-02-90430-Ukr. The work of I. V. Tyutin is partially supported by the RFBR grant 14-01-00489.

References

[1] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, A systematic study of finite BRST-BFV transformations in generalized Hamiltonian formalism, arXiv:1404.4154[hep-th].
[2] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, A systematic study of finite BRST-BV transformations in field-antifield formalism, arXiv:1405.2621[hep-th].
[3] E.S. Fradkin and G.A. Vilkovisky, Quantization of relativistic systems with constraints, Phys. Lett. B55 (1975) 224.
[4] I.A. Batalin and G.A. Vilkovisky, Relativistic $S$-matrix of dynamical systems with boson and fermion constraints, Phys. Lett. B69 (1977) 309.
[5] E.S. Fradkin and T.E. Fradkina, *Quantization of relativistic systems with Boson and Fermion first and second class constraints*, Phys. Lett. B72 (1978) 343.

[6] I.A. Batalin and G.A. Vilkovisky, *Gauge algebra and quantization*, Phys. Lett. B102 (1981) 27.

[7] I.A. Batalin and G.A. Vilkovisky, *Quantization of gauge theories with linearly dependent generators*, Phys. Rev. D28 (1983) 2567.

[8] C. Becchi, A. Rouet and R. Stora, *The abelian Higgs-Kibble, unitarity of the S-operator*, Phys. Lett. B52 (1974) 344.

[9] C. Becchi, A. Rouet and R. Stora, *Renormalization of Gauge Theories* Ann. Phys. (N. Y.) 98 (1976) 287.

[10] I. V. Tyutin, *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Institute preprint No. 39 (1975) [arXiv:0812.0580 [hep-th])].

[11] G. Curci and R. Ferrari, *Slavnov transformations and supersymmetry*, Phys. Lett. B63 (1976) 91.

[12] I. Ojima, *Another BRS transformation*, Prog. Theor. Phys. 64 (1979) 625.

[13] S. Hwang, *Properties of the anti-BRS symmetry in a general framework*, Nucl. Phys. B231 (1984) 386.

[14] V.P. Spiridonov, *Sp(2)-covariant ghost fields in gauge theories*, Nucl. Phys. B308 (1988) 527.

[15] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *Extended BRST quantization of gauge theories in generalized canonical formalism*, J. Math. Phys. 31 (1990) 6.

[16] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *An Sp(2) covariant version of generalized canonical quantization of dynamical system with linearly dependent constraints*, J. Math. Phys. 31 (1990) 2708.

[17] I.A. Batalin, P.M. Lavrov and I.V. Tyutin I.V. *An Sp(2) covariant formalism of generalized canonical quantization of systems with second-class constraints*, Int. J. Mod. Phys. 6 (1990) 3599.

[18] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *Covariant quantization of gauge theories in the framework of extended BRST symmetry*, J. Math. Phys. 31 (1990) 1487.

[19] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *An Sp(2)-covariant quantization of gauge theories with linearly dependent generators*, J. Math. Phys. 32 (1991) 532.

[20] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *Remarks on the Sp(2)-covariant quantization of gauge theories*, J. Math. Phys. 32 (1991) 2513.

[21] R.E. Kallosh and I.V. Tyutin, *The equivalence theorem and gauge invariance in renormalizable theories*, Sov. J. Nucl. Phys. 17 (1973) 98.