A Formal Definition of Stochastic Activity Networks Templates

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Abstract—Model-based evaluation has been extensively used to estimate performance and reliability metrics of computer systems, especially critical systems, for which experimental approaches are not always applicable. A significant challenge is constructing and maintaining the models for large-scale and possibly evolving systems. In a recent work we defined the Template Models Description Language (TMDL) framework, an approach to improve reuse in the specification of performability models. The approach is based on the concept of libraries of model templates that interact using well-defined interfaces. To apply the framework, some assumptions must be satisfied. In particular, a template-level version of the formalism that will be used for the analysis needs to be defined. A template-level formalism is essentially a parameterized abstracted version of a certain modeling specific formalism, from which concrete instances can be automatically derived. In this paper we give the formal definition of Stochastic Activity Networks Templates (SAN-T), a formalism based on SANs with the addition of variability aspects that can be used to define model templates. We then discuss its concrete application to a simple example in the telecom domain.

Index Terms—model-based evaluation, templates, stochastic Petri nets, reuse, parametric models.

I. INTRODUCTION

Formal methods have been extensively used to estimate performance and reliability metrics of computer systems. They are especially useful for assessing non-functional properties of critical systems, for which experimental approaches are not always applicable. In fact, model-based evaluation [1] has the advantage of not exercising the real system, which may be dangerous, costly, or not feasible.

Traditionally, critical systems have been mostly isolated and monolithic. The main challenge for model-based evaluation has always been the solution process, in term of efficiency of state-space generation and accuracy of results. More recently, the problem of specifying complex models in convenient ways has increasingly gained attention. In particular, in the dependability [2] and performability [3] domain many works have proposed approaches to automatically generate formal models from design models (e.g., UML models) enriched with information on the failure/repair process. The idea behind these works is that software and systems engineers can take advantage of formal models without being proficient in them, because model transformations embed the knowledge of experts [4], [5].

However, these approaches are good in providing an application-specific abstraction to users of a certain domain, but they are not flexible enough to relieve dependability experts from the effort of modeling complex systems. In fact, they have two main limitations: i) they are tailored to the needs of system designers and not to those of formal methods experts, and ii) they define different transformation algorithms for different problems or class of systems.

In our recent work [6] we defined an approach to improve reuse in the specification of performability models, in particular, for the specification of models based on Stochastic Petri Nets (SPN) [7]. The approach is based on the concept of libraries of model templates that interact using well-defined interfaces. Such interfaces and their composition rules are specified using a domain-specific language that we call the Template Models Description Language (TMDL). In our formulation, a “model template” is essentially a parameterized abstracted version of a model in a specific formalism, from which concrete instances can be automatically derived by specifying values for its parameters.

In [6] we defined the overall idea of the TMDL framework, we formalized its definition, and we introduced the TMDL language itself. However, as prerequisites for applying the approach, we assumed the existence of i) a template-level formalism, ii) an instance-level formalism, and iii) a concretize function, to generate an instance-level model from a template-level model. In this paper we take a step forward, defining how the TMDL approach can be actually applied with Stochastic Activity Networks (SANs) [8]. We do this by defining i) a template-level formalism based on SANs and, ii) the associated concretize function.

The rest of the paper is organized as follows. In Section II we introduce the background and we discuss the related work. In Section III we summarize the TMDL framework introduced in [6], detailing how it relates to the contribution in this paper. In Section IV we present the overall idea of SAN templates with a running example, and then in Section V we give their formal definition. In Section VI we define the algorithm that derives a concrete SAN model from a SAN template (i.e., the concretize function). Then, in Section VII we show how the models of the running example can be defined using the introduced formulation. Finally, concluding remarks are drawn in Section VIII.
II. BACKGROUND

A. Model-Based Evaluation

Model-based evaluation [1] is a well-known technique for the verification and validation of complex systems, and it consists in estimating system-level metrics through formal models, which typically include stochastic behavior. Model-based evaluation plays a key role in the assessment of critical systems and large-scale infrastructures, where exercising the real system is not feasible.

Various formal models are used for this task. Approaches are typically categorized in combinatorial models and state-space models [1], [9]. Combinatorial models include simple formalisms that are used to describe which combinations of component failures lead to system failure, e.g., Fault Trees (FTs) or Reliability Block Diagrams (RBDs) [10]. These models are very popular in the industry, as they are simple to understand and they can be evaluated with well-known formulas. However, they assume independent events and therefore they cannot represent complex interactions between components or dynamic behavior.

On the other hand, state-space models explicitly represent the different states of a system and the possible transitions between them. While being more powerful, this kind of models can quickly become very complex, leading to well-studied problems like state-space explosion and stiffness [1]. One of the most popular formalisms are the class of models known as Stochastic Petri Nets (SPNs) and their numerous extensions [7]. In particular, the work in this paper is based on Stochastic Activity Networks (SANs), which can be considered as a variant of SPNs [8], although they adopt a different terminology (e.g., activity instead of transition).

B. Stochastic Activity Networks (SAN)

A formal definition of Stochastic Activity Networks (SANs) was given by Sanders and Meyer in [8]. We recall here the basic definitions, on which we will base later for the definition of SAN templates.

An Activity Network (AN) is defined as an eight-tuple [8]:

\[ AN = (P, I, O, \gamma, \tau, o, \iota, o) \]

where \( P \) is a finite set of places; \( I \) is a finite set of activities; \( O \) is a finite set of input gates; and \( O \) is a finite set of output gates. The function \( \gamma : A \to \mathbb{N}_0 \) specifies the number of cases for each activity, that is, the number of possible choices upon execution of that activity. \( \tau : A \to \{\text{timed, instantaneous}\} \) specifies the type of each activity; \( \iota : I \to A \) maps input gates to activities; and \( o : O \to \{(a, c) \mid a \in A \land c \in \{1, 2, \ldots, \gamma(a)\}\} \) maps output gates to cases of activities.

Similarly to Petri nets, places can hold tokens. The number of tokens in each places determines the state of the network, also called its marking. More formally, if \( S \) is a set of places \( S \subseteq P \), a marking of \( S \) is a mapping \( \mu : S \to \mathbb{N} \). The value \( \mu(p) \) is the marking of place \( p \), i.e., the number of tokens it holds. Similarly, the set of possible markings of \( S \) is the set of functions \( MS = \{\mu \mid \mu : S \to \mathbb{N}\} \).

An input gate is defined as a triple \((G, e, f)\), where \( G \subseteq P \) is the set of input places associated with the gate, \( e : M_G \to \{0, 1\} \) is the enabling predicate of the gate, and \( f : M_G \to M_G \) is the input function of the gate. An output gate is a pair \((G, f)\), where \( G \subseteq P \) is the set of output places associated with the gate, and \( f : M_G \to M_G \) is the output function of the gate.

An input gate \( g = (G, e, f) \) holds in a marking \( \mu \) if \( e(\mu_G) = 1 \). We say that an activity \( a \) is enabled in a marking \( \mu \) if all the input gates associated with it hold. Intuitively, the behavior of the network is regulated by the following rules: i) when an activity is enabled it can fire; ii) instantaneous activities have priority over timed activities; and iii) when an activity fires, one of its cases is selected. When an activity \( a \) fires in marking \( \mu \), the new marking is given by \( \mu' = f_{G_a}(\ldots f_{O_a}(f_{I_a}(\ldots f_{I_0}(\mu)))) \), where \( g_i = (G_{I_i}, e_i, f_{I_i}) \) is the \( i \)-th input gate of the activity, and \( o_j = (G_{O_j}, f_{O_j}) \) is the \( j \)-th output gate of the selected case. That is, all the functions of all the input gates are computed first, and then all the functions of the output gates are computed. The complete formal definitions that characterize the behavior of a SAN can be found in [8].

A marking in which no instantaneous activities are enabled is a stable marking. An activity network is stabilizing if, essentially, there is no marking from which it is possible to fire an infinite sequence of instantaneous activities, i.e., a stable marking is always reached.

Given an activity network that is stabilizing in some initial marking \( \mu_0 \in M_P \), a Stochastic Activity Network (SAN) is formed by adding functions \( C_a, F_a, \) and \( G_a \) for each activity \( a \), where \( C_a \subseteq C \) is a function specifying the probability distribution of its cases; \( F_a \subseteq F \) is a function specifying the probability distribution of its firing delay; and \( G_a \subseteq G \) is a function that describes its reactivation markings [8].

That is:

\[ SAN = ((P, A, I, O, \gamma, \tau, o, \iota, o), \mu_0, C, F, G) \].

SANs have an intuitive graphical notation that is illustrated in Figure 1. Places are represented as circles, instantaneous activities as thin bars and timed activities as thick bars. Input gates are represented as left-oriented triangles, while output gates as right-oriented triangles. Cases are represented as small circles next to the activity; when there is a single case it is omitted from the diagram. Input arcs and output arcs are considered special cases of input and output gates, respectively, in which the function \( f \) simply removes or adds one token to the connected place.

Metrics are defined using reward functions, and under certain conditions the stochastic process underlying a SAN has an exact solution. In general, they can be evaluated by discrete-event simulation, as the one provided by Möbius [11].

C. Related Work

The problem of simplifying the construction of performability models has been approached in different ways in the literature.
Different variants of the original Petri Nets formalism have been defined, some of them enabling more compact and reusable specifications. For example, Colored Petri Nets (CPNs) \cite{12} allow tokens to be distinguished, by attaching data to them. Tokens can be of different data types, called colors. Hierarchical CPNs support modularization by means of substitution transitions, i.e., a transition is replaced by a whole subnet in a more detailed model. Stochastic Reward Nets (SRNs) \cite{13} also contain features that allow for a compact specification of SPNs, e.g., marking dependency, variable-cardinality arcs, priorities, etc. Furthermore, they embed extensions to define rewards. These kind of models fold a complex Petri net in a compact specification. Conversely, our approach focuses on specifying “template” models, from which multiple Petri net in a compact specification. Conversely, our approach focuses on specifying “template” models, from which multiple different instances can be automatically derived.

As mentioned earlier, SANs can also be considered a variant of SPNs \cite{8}. In their Möbius implementation \cite{11}, they support tokens having different datatypes, including structured datatypes. The input gate and output gate primitives can be used to specify arbitrary complex functions for the enabling of transitions (called activities) and for their effects. SANs models can be composed using the Rep/Join state sharing formalism \cite{14}. However, which state variables are composed, and how, must be specified manually for each composition. The Möbius implementation of SANs permits using variables, however they only impact the behavior of the model, and not its structure. In this paper we define parametric (“template”) SAN models, whose structure and behavior can depend on parameters.

A well-established research line focuses on applying Model-Driven Engineering (MDE) \cite{15} techniques for the automatic derivation of dependability models from UML models or similar representations, e.g., see \cite{16, 17, 18}. However, such approaches typically provide an application-specific abstraction to users of a certain domain, and then they automatically derive formal models defined by an expert. Instead, our approach is targeted at dependability modeling experts, and it focuses on constructing models that could be reused across different domains or systems.

Finally, it should be noted that existing modeling frameworks, e.g., Möbius \cite{11} or CPNTools \cite{12}, provide some means for reducing the effort in the specification of complex models. For example, they both allow multiple instances of a subnet to be reused. However, instances have identical structure, and each of them still needs to be manually connected to the rest of the model. The objective of our proposal is to facilitate the selection, parametrization, and composition of templates from model libraries, without knowledge of their internal implementation.

### III. The TMDL Framework

As mentioned before, we defined the TMDL framework in \cite{6}, with the objective to improve the reuse of performability models, in particular SPN-based models. In this section we briefly recall it, and discuss how this paper relates to it.

The TMDL framework is based on the following simple idea, organized in three steps: i) there exist a library of parametric reusable submodels, defined with a template-level formalism, and called model templates; ii) based on the scenario to be modeled, a set of templates is selected and proper parameters are assigned; and iii) models in the instance-level formalism are automatically generated and assembled to obtain the overall system model. The corresponding workflow is detailed in Figure 2.

In Step #1, a library of reusable model templates is created by an expert. In Step #2, the different system configurations that should be analyzed are defined in terms of “scenarios”. Scenarios are composed of model variants, that is, a selection of model templates with their parameter assignment. In Step #3 all the needed model instances are automatically created and assembled, thus generating the complete system model for each scenario. Note that the steps in the workflow are not strictly sequential. In particular, the creation of the model library is performed once, and the library is stored for future access.

What makes the model templates reusable is that they have well-defined interfaces and parameters. Briefly summarizing, interfaces specify how they can be connected to other templates, while parameters make it possible to derive different concrete models from the same template. A model template has a specification (of its parameters and interfaces), and an implementation.

The specification of a template is provided with TMDL. The implementation of an atomic template is given using a template-level formalism, that is, a modeling formalism that defines partially specified models in the concrete formalism of
choice. With “partially specified”, we mean that some aspects of the structure and behavior of the model are controlled by parameters, e.g., the number of cases of an activity. Conversely, we call instance-level formalism the modeling formalism concretely used for the analysis (e.g., “normal” SANs), which is the language of the models generated in Step #3.

In [6] we provided several contributions to the realization of such approach:

- definition of the general idea of the workflow (reproduced in Figure 2).
- formalization of the concepts of model template, model variant, and model instance;
- definition of the TMDL, a domain-specific language that is used to define specifications of model templates and to select and parameterize them for a given scenario; and
- definition of the instantiation and composition algorithm that determines how templates should be instantiated and connected together (via state sharing).

However, we also introduced some assumptions, both for simplicity but also to keep the approach independent of a specific modeling formalism. In particular, we assumed that for a certain instance-level formalism it was possible to define:

1) a corresponding template-level formalism, to represent model templates;

2) a notion of compatibility between the TMDL specification of a template (i.e., interfaces and parameters) and its implementation using the template-level formalism; and

3) a concretize function that, given a model in the template-level formalism and an assignment of values to its parameters, generates a model in the instance-level formalism.

In this paper we provide the above required definitions, considering SANs as the instance-level formalism. That is, we provide: i) a definition of a template-level version of SANs (Section V), and ii) the corresponding concretize function that generates instance-level models (Section VI). We then apply the proposed formulation to a simple example in the telecom domain. These definitions complete the formal definition of the framework for its concrete realization using SANs.

IV. RUNNING EXAMPLE

The idea of a template-level formalism based on SANs is that of a formalism to define partially specified SAN models, in which some elements of their structure and behavior depend on parameters. We call this new formalism Stochastic Activity Networks Templates (SAN-T).

SAN-Ts are employed to provide the internal implementation of atomic templates. The idea of a SAN-T model is to provide an abstract representation of multiple SAN models with a similar structure and behavior, having some systematic differences that can be parameterized. Then, from such base skeleton, different variants can be generated, based on the values assigned to its parameters.

This idea is visualized in Figure 3, which will be used as running example. The SANs models in Figures 3(a)-(b) are derived from the models adopted in [17]. We chose this example for its simplicity and compatibility with page limitations. A more recent work in which we build a template-based model is for example [18].

The system we modeled in [17] was a cellular network in a mobile setting, and the objective was to evaluate different metrics of performance (e.g., congestion) and reliability (e.g., probability of disconnection). The model had to take into account for different service kinds (e.g., file transfer or videoconference), having different characteristics but a similar behavior. Also, different kinds of user models were needed, having access to different subsets of these services, and requiring services with different probabilities.

The behavior represented by the two SAN models in Figures 3(a)-(b) is the following. Each user is initially in idle state, and may then request a network service. With a certain probability they can request one of the services that are available to them, by adding a token in the corresponding place. While the service is being delivered, a token stays in the place with the corresponding identifier (e.g., Req1 or Req5). The request can fail or be dropped; in these cases a token is received in the corresponding place, and the user goes back to idle.

It is clear that the two models have a similar structure. In fact, they differ only by: i) the number of services available to the user, ii) their identifiers, and iii) the probabilities of requesting each service. The structure of these two models can be generalized by establishing the following informal rule: “Create one place ReqX for each of the services that are available to the user, and name them according to the identifier of these services. The activity Request should have the same amount of cases as the number of ReqX places, and each of them should have an output arc connecting the case to the corresponding ReqX place.” This would result in a SAN “template”, depicted in Figure 3(c) which abstracts the common structure among different user models.

Having to maintain similar models that only differ from some details is a common issue in the modeling of complex systems. A formal specification of SAN-T aims to simplify their management. The actual formal definition of the User SAN-T model discussed above is provided later in Section VII.

V. TEMPLATE-LEVEL FORMALISM BASED ON SANs

A. Preliminary Definitions

We first introduce some basic notations that will be used in the rest of the paper. In particular, we need the following definitions to understand what is a parameter of a template model, and how it connects to the rest of the formalism.

As done in [6], we adopt the definitions of sort, operator, term and assignment from the ISO/IEC 15909 standard [19], which apply to a wide range of PN-based formalisms, including SANs. According to that formalization, the set of possible values of a place is defined by its associated sort (i.e., type).

A many-sorted signature is a pair \((S, O)\), where \(S\) is a set of sorts and \(O\) is a set of operators, together with their arity.

In [6] we defined atomic and composite templates. The implementation of composite templates simply includes composition rules, and it is given with TMDL. Composite templates are therefore out of the scope of this paper.
Arity is a function from the set of operators to $S^* \times S$, where $S^*$ is the set of finite sequences over $S$, including the empty string $\varepsilon$. An operator is thus denoted as $o_{(\sigma, s)}$, where $\sigma \in S^*$ are the input sorts, and $s \in S$ is the output sort. Constants are operators with empty input sorts, and are denoted as $o_{(\varepsilon, s)}$ or simply $o_s$.

We denote with $\Delta$ a set of parameters; an element of $\Delta$ of sort $s \in S$ is denoted by $\delta_s$. $\Delta_s \subseteq \Delta$ is the set of parameters of sort $s$.

**Terms** of sort $s \in S$ may be built from a signature $(S, O)$ and a set of parameters $\Delta$. The set of terms of sort $s$ is denoted by $\text{TERM}(O \cup \Delta)_s$. Intuitively, it is the set of all the possible expressions of sort $s$ made of any legit combination of operators in $O$ and parameters in $\Delta$ [19]. To simplify the notation, in the rest of the paper we will use $\text{TERM}_s$, unless there are ambiguities.

A many-sorted algebra $H = (S_H, O_H)$ provides an interpretation of a signature $(S, O)$. For every sort $s \in S$ there is a corresponding set of values $H_s \in S_H$, and for every operator $o_{s_1...s_n, s} \in O$ there is a corresponding function in $o_H \in O_H$, such that $o_H: H_{s_1} \times \ldots \times H_{s_n} \rightarrow H_s$.

Given a many-sorted algebra $H$, and many-sorted parameters in $\Delta$, an assignment for $H$ and $\Delta$ is a family of functions $\xi$, comprising a function $\xi_s: \Delta_s \rightarrow H_s$ for each sort $s \in S$. The concept of assignment may be extended to terms, thus obtaining the family of functions $Val_\xi$ comprising the function $Val_{s, \xi}: \text{TERM}_s \rightarrow H_s$ for each sort $s \in S$ [19].

To support the subsequent definitions, we require the existence of at least the “integer”, “real”, “boolean”, “set of integers”, and “set of reals” sorts. Formally, we assume a signature $(S, O)$, such that\{Int, Real, Bool, Set[Int], Set[Real]\} $\subseteq S$, and $O$ contains the common operators applicable on such sorts. The corresponding many-sorted algebra is $(S_H, O_H)$, with\{\mathbb{N}, \mathbb{R}, \{0,1\}, \mathcal{P}(\mathbb{N}), \mathcal{P}(\mathbb{R})\} $\subseteq S_H$, and $O_H$ containing the set of functions corresponding to operators in $O$.

**B. Stochastic Activity Network Templates**

Based on the previous definitions, we can now introduce the formal definition of SAN-T. Formally, a Stochastic Activity Network Template (SAN-T) is a tuple:

$$SAN-T = (\Delta, \tilde{P}, \tilde{A}, \tilde{I}, \tilde{O}, \tilde{\gamma}, \tilde{i}, \tilde{o}, \tilde{\mu}_0, \tilde{C}, \tilde{F}, \tilde{G})$$

where $\Delta$ is a set of parameters, and elements marked with a tilde accent, $\tilde{}$, are modified versions of elements existing in plain SANs (see Section II-B), reformulated to take parameters into account. In more details:

- $\Delta$ is a sorted set of parameters of the template.
- $\tilde{P}$ is a finite set of place templates. A place template can be seen as a placeholder for multiple places that, in a regular SAN model, would be strongly related to each other. Based on parameters’ values, a template place will be expanded to a precise set of concrete places. Place $\text{Req}$ in Figure 3(c) is an example of place template.

Formally, a place template is defined as a pair $(\tau, k)$, where $\tau$ is the name of the place, and $k \in \text{TERM}_{\text{set}(\text{Int})}$ is its multiplicity. Evaluating the term $k$ with respect to an assignment $\xi$ identifies a set of integer indices $K \subseteq \mathbb{N}$. Such indices determine the set of places to which, with the given assignment of parameters, the place template is expanded. Normal places (i.e., those always expanding to a single place of ordinary SANs) are those for which $Val_\xi(k) = \{1\}$ for any assignment $\xi$.

- $\tilde{A}$ is a finite set of activity templates.
- $\tilde{I}$ is a finite set of input gate templates.
- $\tilde{O}$ is a finite set of output gate templates.
- $\tilde{\gamma}: A \rightarrow \text{TERM}_{\text{Int}}$ specifies the number of cases for each activity template. For any activity template $\tilde{a} \in \tilde{A}$, evaluating $\tilde{\gamma}(\tilde{a})$ with respect to an assignment $\xi$ returns an integer number, which determines the number of cases of $\tilde{a}$ under that assignment, i.e., $Val_\xi(\tilde{\gamma}(\tilde{a})) \in \mathbb{N}$.

- $\tilde{i}: \tilde{I} \rightarrow \tilde{A}$ maps input gate templates to activity templates.
- $\tilde{o}: \tilde{O} \rightarrow \tilde{A}$ maps output gate templates to activity templates.

In order to define input and output gate templates, the concept of marking needs to be extended, making it applicable to place templates. The idea is to let the marking function anticipate that the place template will be mapped to a set
of places, and thus allow the marking for each of them to be specified, through an index. Formally, if $S \subseteq P$ is a set of place templates, a marking of $S$ is a mapping $\tilde{\mu}: S \times N \rightarrow N$. For example, $\tilde{\mu}(\tilde{p}, 2) = 10$ means that the place generated from $\tilde{p}$ having index 2 contains 10 tokens. The set of possible markings of $S$ is the set of functions $M_S = \{\tilde{\mu} \mid \tilde{\mu}: S \times N \rightarrow N\}$.

As in ordinary SANs, an input gate template defines an enabling condition for an activity template and an input function, that is, how the marking is altered upon the firing of the activity. An input gate template will always result in a single input gate in the concrete SAN model. Still, the projected output gate may depend on the assignment of parameters. Formally, an input gate template is still defined as a triple $(\tilde{G}, \tilde{e}, \tilde{f})$, where $\tilde{G} \subseteq \tilde{P}$ is the set of input places associated with the gate, $\tilde{e}: \tilde{M}_{\tilde{G}} \rightarrow \text{TERM}_{\text{Bool}}$ is the enabling predicate, and $\tilde{f}: \tilde{M}_{\tilde{G}} \times \Xi \rightarrow \tilde{M}_{\tilde{G}}$ is the input function, where $\Xi$ is the set of all the possible assignments. In SANs, an output gate defines an output function that is executed upon the firing of an activity. Differently from an input gate, it can be associated to individual cases of an activity. An output gate template has a similar purpose. However, since the number of cases of an activity template is not known, the gate is connected directly to the activity. When a regular SAN is generated from the template, an output gate template will be expanded to multiple concrete output gates, depending on the number of cases of the activity to which it is connected.

Formally, an output gate template is a pair $(\tilde{G}, \tilde{f})$, where $\tilde{G} \subseteq \tilde{P}$ is the set of output places associated with the gate, and $\tilde{f}: \tilde{M}_{\tilde{G}} \times \Xi \rightarrow \tilde{M}_{\tilde{G}}$ is the output function of the gate. It should be noted that the output function $\tilde{f}$ depends on the index of the case of the associated activity template $(\Xi)$, as well as on the assignment of values to parameters $(\Xi)$. The probability of cases of an activity template is given by the case distribution assignment $C$, which defines a function $\tilde{C}_a \in \tilde{C}$ for each activity template $\tilde{a} \in \tilde{A}$. Such functions also depend on parameters, thus $\tilde{C}_a: \tilde{M}_{\tilde{P}(\tilde{a})} \times \Xi^{+} \times \Xi \rightarrow [0, 1]$, where $\tilde{P}(\tilde{a})$ is the set of input and output places of the activity. For the model to be well-formed, $\tilde{C}_a(\mu, i, \xi) = 0$ should hold $\forall i > \text{Val}_{\xi}(\gamma(\tilde{a}))$, i.e., the probability of cases beyond those generated with the given assignment $\xi$ should be zero.

Similarly, the firing time of activities is given by the activity time distribution assignment $F$, which defines a function $\tilde{F}_a \in \tilde{F}$ for any timed activity template $a$, with $\tilde{F}_a: \mathbb{R} \times \tilde{M}_{\tilde{P}} \times \Xi \rightarrow [0, 1]$. That is, the probability of a certain firing time $(\mathbb{R})$ depends on the marking $(\tilde{M}_{\tilde{P}})$ and on the parameters assignment $(\Xi)$. The reactivation function of activity templates is given by the reactivation function assignment $\tilde{G}$, such that for any timed activity $a$, function $\tilde{G}_a \in \tilde{G}$ defines the reactivation markings, with $\tilde{G}_a: \tilde{M}_{\tilde{P}} \times \Xi \rightarrow \gamma(\tilde{M}_{\tilde{P}})$ and $\gamma(\tilde{M}_{\tilde{P}})$ denoting the power set of $\tilde{M}_{\tilde{P}}$.

Finally, the initial marking of a SAN-T should also depend on the assignment of values to parameters. For this reason, it is defined by the function $\tilde{\mu}_0: \Xi \rightarrow \tilde{M}_{\tilde{P}}$. It should be noted that the original definition of SANs requires the initial marking $\mu_0(\xi) \in \tilde{M}_{\tilde{P}}$ to be a stable marking in which the network is stabilizing (see Section II-B). However, since the actual structure of the model is not completely specified until a value is assigned to all the SAN-T parameters, we relax this constraint. Well-formedness checks on the structure of the resulting SAN models can be performed at the time of instantiation, based on techniques available for ordinary SAN models (e.g., [20]).

VI. CONCRETIZATION OF SAN-T MODELS

In this section we defined the concretize function that generates an ordinary SAN model from a from a pair $(\text{SAN-T}, \xi)$, that is, from a SAN-T model and an assignment of values to its parameters. Together with the definition of the template-level formalism in the previous section, this is a requirement for applying the TMDL framework.

A. Overview

Given a SAN-T $S_\Delta$:

$$S_\Delta = (\Delta, \tilde{P}, \tilde{A}, \tilde{I}, \tilde{O}, \tilde{\gamma}, \tilde{\tau}, \tilde{\delta}, \tilde{\mu}_0, \tilde{C}, \tilde{F}, \tilde{G}),$$

and a parameter assignment function $\xi$, this section defines how the corresponding concrete SAN model $S^\xi$ (also called SAN-T instance) is obtained, that is, how to obtain:

$$S^\xi = (P^\xi, A^\xi, I^\xi, O^\xi, \gamma^\xi, \tau^\xi, \delta^\xi, \mu^\xi_0, C^\xi, F^\xi, G^\xi).$$

We separate the presentation of the algorithm in two parts: i) concretization of the individual places, marking, and gates; and ii) concretization of the overall model structure.

In all the following definitions, $\xi$ is the assignment of values to parameters (see Section V-A) that is used to generate the concrete SAN model.

B. Places, Marking, and Gates

Places. Given a place template of the SAN-T model, $\tilde{p} = (\tau, k) \in \tilde{P}$, a set of places $\{p_1^\xi, \ldots, p_{\text{Val}_{\xi}(k)}^\xi\}$ is created in the concrete SAN model. Note that the cardinality of this set is given by the cardinality of the set of indices that defines the multiplicity, obtained by applying the assignment function. We denote with $\Pi(\tilde{p}, i) \in P^\xi$ the $i$-th concrete place originating from place template $\tilde{p}$.

Marking. Given a marking of the SAN-T model, $\tilde{\mu} \in \tilde{M}_{\tilde{P}}$, we denote the corresponding marking of the concrete SAN model as $\Gamma(\tilde{\mu}) = \mu^\xi \in M_P$. The marking $\mu^\xi$ is defined as:

$$\mu^\xi(\Pi(\tilde{p}, i)) = \tilde{\mu}(\tilde{p}, i), \quad \forall \tilde{p} \in \tilde{P},$$

that is, the marking of the $i$-th generated place is the marking specified in the SAN-T for index $i$. Given a marking $\mu^\xi$ of the concrete (generated) SAN model we denote as $\Gamma^{-1}(\mu^\xi)$ the corresponding marking $\tilde{\mu}$ of the originating SAN-T.

Input Gates. Given an input gate template $\tilde{g} = (\tilde{G}, \tilde{e}, \tilde{f}) \in \tilde{I}$, we denote with $\alpha(\tilde{g})$ the corresponding input gate $g^\xi =$
(G^ξ, e^ξ, f^ξ) ∈ I^ξ in the concrete SAN model, which is obtained as:

\[ G^ξ = \{ \Pi(\tilde{\nu}, j) | j ∈ Val_k(\tilde{k}), \tilde{\nu} = (\tau, k) ∈ G \}, \]

\[ e^ξ(\Gamma(\tilde{\nu})) = Val_k(\tilde{\nu}, \tilde{k}), \]

\[ f^ξ(\Gamma(\tilde{\nu})) = \tilde{f}(\tilde{\nu}, i, ξ). \] (7)

That is, the input places G^ξ are all the places generated from input place templates in G, the input predicate applied to a marking Γ(μi) is the result of applying the assignment function to the predicate of the gate template, and the output function applied to marking Γ(μi) is the input function of the gate template applied on marking μ and assignment ξ.

**Output Gates.** Given an output gate template (G, f) ∈ O, we denote with β(γ, i) the i-th output gate (G^ξ, f^ξ) ∈ O generated from it in the SAN model, which is obtained as:

\[ G^ξ = \{ \Pi(\tilde{\nu}, j) | j ∈ Val_k(\tilde{k}), \tilde{\nu} = (\tau, k) ∈ G \}, \]

\[ f^ξ(\Gamma(\tilde{\nu})) = \tilde{f}(\tilde{\nu}, i, ξ). \] (8)

That is, the output places G^ξ are all the places generated from output place templates in G, and the output function applied to marking Γ(μi) is the input function of the gate template applied on marking μ, index i, and assignment ξ.

**C. Concretization of SAN Structure**

We can now provide the complete specification of the SAN derived from a SAN-T and an assignment ξ. That is, defining all the elements in Equation 3 as follows:

\[ P^ξ = \bigcup_{\tilde{\nu} = (\tau, k) ∈ P} \{ \Pi(\tilde{\nu}, i) | i ∈ Val_k(\tilde{k}) \}; \]

\[ A^ξ = \tilde{A}; \]

\[ \gamma^ξ(a) = Val_k(\tilde{\gamma}(\tilde{a})); \]

\[ I^ξ = \{ \gamma(\tilde{g}) | \tilde{g} ∈ \tilde{I} \}; \]

\[ O^ξ = \bigcup_{\tilde{g} ∈ \tilde{O}} \{ β(\tilde{g}, 1), ..., β(\tilde{g}, Val_k(\tilde{\gamma}(\tilde{a}))) | \tilde{a} = \tilde{\gamma}(\tilde{g}) \}; \]

\[ τ^ξ = \tilde{τ}; \]

\[ \nu^ξ(\gamma(\tilde{g}, i)) = \tilde{o}(\tilde{g}), \quad \forall g ∈ \tilde{I}; \]

\[ \nu^ξ(β(\tilde{g}, i)) = \tilde{ao}(\tilde{g}), \quad \forall g ∈ \tilde{O}, \forall i ∈ \{1, ..., Val_k(\tilde{\gamma}(\tilde{a}))\}; \]

\[ μ^ξ_0(\tilde{μ}) = \tilde{μ}_0(\tilde{ξ}). \]

The rationale behind the above derivation can be summarized as follows: i) the set of places P^ξ is given by all the places derived from all place templates in P; ii) the set of activities remains unchanged; iii) the function γ that specifies the number of cases of an activity is the result of applying the assignment function to the γ function; iv) there is an input gate in I^ξ for each input gate template in I; v) each output gate template in O is expanded to a number of output gates, given by the number of cases of the activity to which it is connected; vi) the function τ that determines if an activity is timed or instantaneous remains unchanged; vii) if an input gate template is connected to an activity template, then its concrete projection is connected to the projection of the activity template; viii) if an output gate template is connected to an activity template, then all its concrete projections are connected to the projection of the activity template; and ix) the initial marking μ^ξ_0 is given by the initial marking of the SAN template applied to the assignment ξ.

Furthermore:

- For each function C^ξ in the case distribution assignment C, a corresponding function C^ξ^μ is included in C^ξ, defined as C^ξ^μ(τ, k) = C^ξ(μ, k, ξ), ∀μ ∈ M^μ, ∀k ∈ N^+.
- For each function F^ξ in the activity time distribution assignment F, a corresponding function F^ξ^μ is included in F^ξ, defined as F^ξ^μ(τ, r, ξ) = F^ξ(μ, r, ξ), ∀μ ∈ M^μ, ∀r ∈ R.
- For each function G^ξ in the reactivation function assignment G, a corresponding function G^ξ^μ is added to G^ξ, defined as G^ξ^μ(τ, k) = G^ξ(μ, k, ξ)∀μ ∈ M^μ.

**VII. APPLICATION EXAMPLE**

In this section we apply the proposed formulation of SAN-Ts to the running example introduced in Section IV. A SAN-T that abstracts the behavior of a generic User model has been shown in Figure 3(c). Here we show how such template model can be specified in a formal way, following the SAN-T definition provided in Section V.

**A. User SAN-T Model**

Two parameters can be identified for the User model. The first, s, identifies the number and indices of services that the user can access, and it is therefore of type set of integers. The second, p, determines the probabilities of selection of each service, and it is of type set of reals.

The variable elements of the model are essentially the activity template Request, its associated output gate template OγRequest, and the place template Req (see Figure 3(c)). The Request activity has a variable number of cases, given by the cardinality of the array of integers assigned to parameter s, and each of these cases is selected with a probability given by parameter p. Place template Req is expanded to a number of concrete places that is again given by the cardinality of p. The selection of case i of the Request activity template results in the addition of a token in place Req.

The formal specification of the SAN-T User model, according to the definitions given in Section V, is provided in the following.

\[ \text{SAN-T}_{\text{User}} = (\Delta, \tilde{P}, \tilde{A}, \tilde{I}, \tilde{O}, \tilde{γ}, \tilde{τ}, \tilde{o}, \tilde{μ}_0, \tilde{C}, \tilde{F}, \tilde{G}), \]

\[ \Delta = \{ \text{Set}([\text{Int}), \text{Set}([\text{Real})] \}; \]

\[ \tilde{P} = \{ \text{Idle}, \text{Req}, \text{Dropped}, \text{Failed} \}; \]

\[ \tilde{A} = \{ \text{Request}, \text{Fail}, \text{Drop} \}; \]

\[ \tilde{I} = \{ \text{OγRequest}, \text{ArcInFail}, \text{ArcInDrop} \}; \]

\[ \tilde{O} = \{ \text{OγRequest}, \text{ArcOutFail}, \text{ArcOutDrop} \}; \]

\[ \tilde{γ} = \{ (\text{Request}, s), (\text{Fail}, 1), (\text{Drop}, 1) \}; \]

\[ \tilde{τ} = \{ (\text{Request, timed}), (\text{Fail, instantaneous}), \text{(Drop, instantaneous}) \}; \]

\[ \tilde{i} = \{ (\text{OγRequest, Request}), (\text{ArcInFail, Fail}) \}; \]
(ArcInDrop, Drop)}
\[ \tilde{\delta} = \{(\text{OGRequest}, \text{Request}),(\text{ArcOutFail}, \text{Fail}), (\text{ArcOutDrop}, \text{Drop})\} \]
\[ \tilde{\mu}(\xi) = \mu^\prime (\tilde{p}, i) = \begin{cases} 1 & \text{if } \tilde{p} = \text{Idle} \land i = 1, \\ 0 & \text{otherwise}. \end{cases} \]
\[ \tilde{C} = \{\tilde{C}_{\text{Request}}, \tilde{C}_{\text{Drop}}, \tilde{C}_{\text{Fail}}\} \]
\[ \tilde{C}_{\text{Request}}(\tilde{\mu}, i, \xi) = \begin{cases} \text{val}(\xi, p_k) & i \leq k \leq |p|, \\ 0 & \text{otherwise}. \end{cases} \]
\[ \tilde{C}_{\text{Drop}}(\tilde{\mu}, i, \xi) = \begin{cases} 1 & \text{if } i = 1, \\ 0 & \text{otherwise}. \end{cases} \]
\[ \tilde{F} = \{\tilde{F}_{\text{Request}}\} \]
\[ \tilde{G} = \{\tilde{G}_{\text{Request}}, \tilde{G}_{\text{Drop}}, \tilde{G}_{\text{Fail}}\} \]

In this case, the firing time of the Request activity is regulated by an uniform distribution, thus \( \tilde{F}_{\text{Request}} \) is set accordingly. None of the activities are reactivating, that is, \( \tilde{G}_{\text{Request}} = \tilde{G}_{\text{Fail}} = \tilde{G}_{\text{Drop}} \) map any marking to the empty set.

To fully specify the SAN-T, we need to complete the specification of input gates templates and output gates templates. We give here the specification of OGRequest only, which is the only one that includes variability in the User template:

\[ \tilde{G}_{\text{OGRequest}} = \{\text{Req}\}, \]
\[ \tilde{f}_{\text{OGRequest}}(\tilde{\mu}, i, \xi) = \mu'(\tilde{p}, k) = \begin{cases} 1 & \text{if } \tilde{p} = \text{Req} \land k = i, \\ \mu(\tilde{p}, i) & \text{otherwise}. \end{cases} \]

Summarizing, the above specification means that the function of the output gate associated with the i-th case should put one token into the i-th instance of the Req place, and leave the rest unchanged.

### B. Generation of Instances

Following the concretize algorithm described in Section VI, it is possible to derive multiple instances of the SAN-TUser template, that is, concrete SAN models.

To exemplify the application of templates, we define two different assignment functions \( \xi_{\text{NormalUser}} \) and \( \xi_{\text{UserAmbulance}} \), which will result in the generation of SAN models of Figure 3(a) and Figure 3(b) respectively:

\[ \xi_{\text{NormalUser}} = \{(s, \{1, 6, 7\}), (p, \{0.7, 0.2, 0.1\})\} \]
\[ \xi_{\text{UserAmbulance}} = \{(s, \{3, 7\}), (p, \{0.6, 0.4\})\} \]

## VIII. CONCLUDING REMARKS

In this paper we gave the formal definition of Stochastic Activity Networks Templates (SAN-T), a formalism based on SANs, with the addition of variability aspects that can be used to define model templates, according to the TMDL approach of [6]. Such definitions will enable the concrete application of the approach with a widely used modeling formalism.

As current and future work, we are working on two parallel directions. The first is to apply the methodology to a real system; we are currently investigating an self-testing on-board system in the railway domain. As a parallel activity, and to facilitate the application of the approach by other researchers, we are creating a prototype editor for SAN-T models, based on the Eclipse Modeling Framework (EMF) \(^2\) and the Sirius\(^3\) modeling tools, to be released as open source software.

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### REFERENCES

[1] D. M. Nicol, W. H. Sanders, and K. S. Trivedi, “Model-based evaluation: from dependability to security,” IEEE Transactions on Dependable and Secure Computing, vol. 1, no. 1, pp. 48–65, 2004.

[2] A. Avižienis, J.-C. Laprie, B. Randell, and C. Landwehr, “Basic concepts and taxonomy of dependable and secure computing,” IEEE Transactions on Dependable and Secure Computing, vol. 1, no. 1, pp. 11–33, 2004.

[3] J. Meyer, “On evaluating the performance of degradable computing systems,” IEEE Transactions on Computers, vol. C-29, no. 8, pp. 720–731, 1980.

[4] L. Montecchi, P. Lollini, and A. Bondavalli, “Towards a MDE Transformation Framework for Dependability Analysis,” in 16th IEEE International Conference on Engineering of Complex Computer Systems (ICECCS), Las Vegas, USA, 2011, pp. 157–166.

[5] S. Bernardi, J. Merseguer, and D. C. Petriu, “Dependability modeling and analysis of software systems specified with UML,” ACM Computing Surveys, vol. 45, no. 1, 2012.

[6] L. Montecchi, P. Lollini, and A. Bondavalli, “A template-based methodology for the specification and automated composition of performability models,” IEEE Transactions on Reliability, vol. 69, pp. 293–309, 2020.

[7] G. Ciardo, R. German, and C. Lindemann, “A characterization of the stochastic process underlying a stochastic petri net,” Software Engineering, IEEE Transactions on, vol. 20, no. 7, pp. 506–515, 1994.

[8] W. Sanders and J. Meyer, “Stochastic activity networks: formal definitions and concepts,” in Lectures on formal methods and performance analysis, ser. LNCS. Springer, 2002, vol. 2000, pp. 315–343.

[9] A. Bondavalli, P. Lollini, I. Majzik, and L. Montecchi, “Modelling and model-based assessment,” in Resilience Assessment and Evaluation of Computing Systems. Springer, July 2012.

[10] M. Stamatelatos et al., “Fault Tree Handbook with Aerospace Applications,” NASA Office of Safety and Mission Assurance, August 2002.

[11] T. Courtney, S. Gaonkar, K. Keefe, E. W. D. Rozier, and W. H. Sanders, “Mobius 2.3: An extensible tool for dependability, security, and performance evaluation of large and complex system models,” in 39th IEEE/ACM International Conference on Dependable Systems Networks, Estoril, Portugal, 2009, pp. 353–358.

[12] K. Jensen and L. Kristensen, Coloured Petri Nets — Modelling and Validation of Concurrent Systems. Springer Berlin Heidelberg, 2009.

[13] J. K. Muppala, G. Ciardo, and K. S. Trivedi, “Stochastic reward nets for reliability prediction,” in Communications in Reliability, Maintainability and Serviceability, vol. 1, no. 2, 1994, pp. 9–20.

[14] W. H. Sanders and J. F. Meyer, “Reduced base model construction methods for stochastic activity networks,” IEEE Journal on Selected Areas in Communications, vol. 9, no. 1, pp. 25–36, 1991.

[15] D. C. Schmidt, “Guest editor’s introduction: Model-driven engineering,” Computer, vol. 39, no. 2, pp. 25–31, 2006.

[16] M. Cinque, D. Cotronero, and C. Di Martino, “Automated generation of performance and dependability models for the assessment of wireless sensor networks,” IEEE Transactions on Computers, vol. 61, no. 6, pp. 870–884, 2012.

[17] A. Bondavalli, P. Lollini, and L. Montecchi, “QoS Perceived by Users of Ubiquitous UMTS: Compositional Models and Thorough Analysis,” Journal of Software Engineering, vol. 4, no. 7, 2009.

[18] N. Veeraragavan et al., “Modeling QoS in Dependable Tele-Immersive Applications: A Case Study of World Opera,” IEEE Transactions on Parallel and Distributed Systems, vol. 27, no. 9, pp. 2667–2681, 2016.

[19] ISO/IEC 15909-1:2004, “Systems and software engineering – High-level Petri nets – Part 1: Concepts, definitions and graphical notation,” December 2004.

\(^2\) Siruis, https://www.eclipse.org/sirius/ accessed on April 9th, 2020.
[20] D. D. Deavours and W. H. Sanders, “An efficient well-specified check,” in *Proceedings of the 8th International Workshop on Petri Nets and Performance Models*, 1999, pp. 124–133.

[21] D. Steinberg, F. Budinsky, M. Paternostro, and E. Merks, *EMF: Eclipse Modeling Framework*, 2nd ed. Addison-Wesley Professional, 2008.