Constellation Design for Quadrature Spatial Modulation

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Abstract. Quadrature space modulation (QSM) was proposed recently as a viable multiple-input multiple-output (MIMO) communication technique for next generation wireless system. QSM achieves the more spatial gain than spatial modulation (SM) due to transmitting more additional information bits by the freedom of transmit antennas. In this paper, without increasing the complexity of detection at the receiver, an improved \(M\)-ary quadrature amplitude modulation (I-\(M\)QAM) signal constellation for the QSM system is proposed to further increase the squared Minimum Euclidean distance (MED) between the transmitted spatial vectors (TSVs). Then an upper bound on the average bit error probability (ABEP) is elaborately analyzed. Simulation results using Monte Carlo demonstrate that the QSM system with the proposed I-\(M\)QAM has better bit error ratio (BER) performance than the QSM system with \(M\)-QAM/PSK in wireless network.

1. Introduction

Multiple-input multiple-output (MIMO) technologies, which meet high data rate in next generation communications, significantly improve the spectral efficiency and reliability in wireless communications, thanks to the achievable spatial multiplexing and diversity gains. How to more effectively improve the spectral efficiency through the limited transceiver antennas and to reduce bit error ratio (BER) have been attracted much interest in the field of wireless communications.

The Vertical-Bell Labs Layered Space-Time (V-BLAST) reported in [1], one of MIMO technologies, greatly increases the spectral efficiency and achieves the multiplexing gains without increasing transmit power and system bandwidth. However, V-BLAST causes inter-channel interference and synchronization problems. Due to this problem, spatial modulation (SM) reported in [2], which is proposed to transmit one constellation symbol by only one index antenna, relieves the shortcomings of the V-BLAST scheme, the other transmit antennas are idle. Such that the inter-channel interference and synchronization are avoided. Furthermore, a lot of works have been done [3-4], Generalized Spatial Modulation (GSM) scheme that transmits one or more than one modulated symbol by more than one antenna at one time solves the problem that the number of transmit antennas does not need to be proportional to the power of 2. Quadrature Spatial Modulation (QSM) [5] is further exploited to achieve the spatial gain by transmitting more additional spatial antenna index bits without increasing the detective complexity at the receiver. In the QSM system, through two antenna indexes, the real and imaginary part of the traditional \(M\)-ary quadrature amplitude modulation or phase shift keying (\(M\)-QAM/PSK) are respectively modulated on the real and imaginary part of the transmitted spatial vector (TSV). Note that the minimum Euclidean distance (MED) between the TSVs has changed and is not...
equivalent to the MED between the conventional constellation points when the maximum likelihood detection is used to retrieve the origin information bits.

Based on this background, an improved M-QAM modulation (I-MQAM), in which the Euclidean distance between the adjacent constellation points is narrowed for reducing the average power of each constellation point, is proposed to further increase the squared MED between the TSVs for the QSM system. Simulation results demonstrate the QSM system is capable of achieving a lower bit error ratio (BER) performance than the QSM system with M-QAM/PSK, SM/GSM schemes.

2. System model
2.1. Transmitter

In this paper, considering the QSM system with transmit antennas $N_t$, as shown in Fig.1.

\[ B_1 = \log_2(N_t) \text{ bits} \]
\[ B_2 = \log_2(M) \text{ bits} \]
\[ B_3 = \log_2(N) \text{ bits} \]

The first group $B_1$ bits are modulated into a constellation point $s = \text{Re} + js_{\text{im}}$ in $M$-QAM/PSK constellation. The second group, containing $B_2$ $N_t$ bits, is mapped into a antenna index number for selecting an antenna index vector $\mathbf{A}_i$ ($i = 1, 2, \cdots, N_t$) that activates one antenna to transmit the real part $s_{\text{re}}$ of the modulated symbol. Similarly, the third group, containing $B_3$ $N_t$ bits, is mapped into the antennas index number for selecting an antenna index vector $\mathbf{B}_i'$ ($i' = 1, 2, \cdots, N_t$) that activates one antenna to transmit the imaginary part $s_{\text{im}}$ of the modulated symbol. $\mathbf{A}_i$ and $\mathbf{B}_i'$ are respectively $i$-th and $i'$-th column vector of the identity matrix $\mathbf{I}_{N_t}$. Specifically, the real $s_{\text{re}}$ and imaginary $s_{\text{im}}$ part are respectively multiplied with the vector $\mathbf{A}_i$ and $\mathbf{B}_i'$. Then the resulting in $s_{\text{re}} \cdot \mathbf{A}_i$ and $s_{\text{im}} \cdot \mathbf{B}_i'$ are considered as the real and imaginary part of the TSV $\mathbf{S}$, respectively. Finally, by adding the real part and the imaginary part, the TSV is given by

\[ \mathbf{S} = s_{\text{re}} \cdot \mathbf{A}_i + j \cdot s_{\text{im}} \cdot \mathbf{B}_i' \]  

(1)

2.2. Receiver

The TSV symbols are transmitted over the flat Rayleigh distribution channel and the additive white Gaussian noise (AWGN) in MIMO channel. Thus, the received signal may be described as.

\[ \mathbf{Y} = \mathbf{HS} + \mathbf{N} \]
\[ = \sqrt{E_s} (\mathbf{h}_i s_{\text{re}} + j \mathbf{h}_i s_{\text{im}}) + \mathbf{N}, i, i' \in (1, 2, \cdots, N_t) \]  

(2)
Where $Y \in \mathbb{C}^{N_t \times 1}$, $H = [h_1, \ldots, h_{N_t}] \in \mathbb{C}^{N_t \times N_r}$, whose each entry is independent and identically distributed complex-valued Gaussian random variables with $CN(0, 1)$. $N \in \mathbb{C}^{N_t \times 1}$ denotes the AWGN vector with $CN(0, \sigma^2 I_{N_t})$.

Assuming that the channel state information (CSI) is perfectly known at the receiver, maximum likelihood (ML) detection algorithm is used to recover the original bits.

$$\hat{i}, \hat{i'}, \hat{s} = \arg \min_{i, i', s} \left\| Y - \sqrt{E_s} \cdot (h_s s_{\text{Re}} + j \cdot h_s s_{\text{Im}}) \right\|^2$$ (3)

Where $\hat{i}, \hat{i'}$ are the detected antenna indexes, $E_s$ denotes the transmitted energy.

It can be seen from (3) that the information bits are recovered by jointly decoding the antenna indexes and the modulated symbols.

3. Proposed improved MQAM

When the traditional $M$-QAM/PSK is extensively used in single-input single-output (SISO), single-input multiple-output (SIMO), multiple-input multiple-output (MIMO) wireless communication systems, the squared MED between the transmitted symbols, which is equivalent to the squared MED between the constellation points, can be calculated by

$$d^2_{s, \text{min}} = \frac{4}{E_{av}} \cdot \min \left\{ |s_{\text{Re}}|^2, |s_{\text{Im}}|^2 \right\}$$ (4)

Where $E_{av}$ denotes the average power of each constellation point in $M$-QAM/PSK constellation.

However, when $M$-QAM/PSK is used for the QSM system in MIMO channel, the squared MED $d^2_{s, \text{min}}$ between TSVs is not equivalent to the squared MED $d^2_{s, \text{min}}$. The squared MED $d^2_{s, \text{min}}$ may be calculated by

$$d^2_{s, \text{min}} = \left\| S - \hat{S} \right\|^2 = \frac{2}{E_{av}} \cdot \min \left\{ |s_{\text{Re}}|^2, |s_{\text{Im}}|^2 \right\}.$$ (5)

Compared with (4), $d^2_{s, \text{min}} \neq d^2_{s, \text{min}}$.

Based on the above analysis, the design of the improved MQAM (I-MQAM) constellation suitable for the QSM system is introduced in following.

In order to further increase MED $d^2_{s, \text{min}}$, we make some improving on the Euclidean distance between the adjacent MQAM constellation points. The design method is that the intermost four constellation points remain unchanged due to the real and imaginary part of these points are unit one, namely $|s_{\text{Re}}| = |s_{\text{Im}}| = 1$, the other constellation points are narrowed inward in a unit for reducing the $E_{av}$, such that a new I-MQAM symbol $\mathcal{Z}$ can be obtained as
\[ \mathcal{S} = \pm \frac{A}{\sqrt{E_{av}^{1}}} \pm j \frac{B}{\sqrt{E_{av}^{1}}}, \]
\[ A \in \{1, 2, \cdots, \alpha\}, \quad B \in \{1, 2, \cdots, \beta\}, \]
\[ \alpha = \beta = \left\lceil \sqrt{M / 4} \right\rceil \]  

Where \( \lceil \cdot \rceil \) denotes the ceil operation. \( E_{av}^{1} \) denotes the average power of each constellation point in I-MQAM constellation.

Such that the useful power is concentrated to each constellation point for maximizing the \( d_{S,\text{min}}^{2} \).

Fig.2 shows that an example of I-8QAM constellation. Obviously, the MED \( d_{S,\text{min}}^{2} \) of I-8QAM is larger than that of 8QAM, as follows:

\[ d_{S,\text{min}}^{2} |_{8\text{QAM}} = \frac{2}{E_{av}^{1}} \cdot \min \left\{ |s_{\text{Re}}^{1}|, |s_{\text{Im}}^{1}| \right\} = \frac{2}{10}, \]
\[ d_{S,\text{min}}^{2} |_{1-8\text{QAM}} = \frac{2}{E_{av}^{1}} \cdot \min \left\{ |s_{\text{Re}}^{1}|, |s_{\text{Im}}^{1}| \right\} = \frac{2}{5}. \]

\[ \text{Fig 2.} \quad 8\text{QAM converted into I-8QAM} \]

4. **Performance analysis**

Assumed that \( \hat{S} \) is the estimated version of \( S \), the conditional pairwise error probability (CPEP) can be calculated as [6].
\[ P(S \rightarrow \hat{S} | \mathbf{H}) \]
\[ = P \left( \left| Y - \sqrt{E_s} \cdot \mathbf{H} \right|^2 > \left| Y - \sqrt{E_s} \cdot \mathbf{H} \hat{S} \right|^2 \right) \]
\[ = P \left( \sum_{i=1}^{N_r} \left| Y_i - \sqrt{E_s} \cdot \mathbf{H}_i \hat{S} \right|^2 > \sum_{i=1}^{N_r} \left| Y_i - \sqrt{E_s} \cdot \mathbf{H}_i \hat{S} \right|^2 \right) \]
\[ = P \left( \sum_{i=1}^{N_r} \left| Y_i - \sqrt{E_s} \cdot \mathbf{H}_i \hat{S} \right|^2 > \sum_{i=1}^{N_r} \left| Y_i - \sqrt{E_s} \cdot \mathbf{H}_i \hat{S} \right|^2 \right) \]
\[ = P \left( \sum_{i=1}^{N_r} \left[ \mathbf{H}_i \left( \mathbf{S} - \hat{S} \right) \right]^2 > \sum_{i=1}^{N_r} \left[ \mathbf{H}_i \left( \mathbf{S} - \hat{S} \right) \right]^2 \right) \]
\[ = Q \left( \sqrt{\frac{\sum_{i=1}^{N_r} \left[ \mathbf{H}_i \left( \mathbf{S} - \hat{S} \right) \right]^2}{2N_0}} \right) \]

(7)

Where \( R(\cdot) \) denotes the real operation, \( N_r^* \) denotes the conjugation of \( N_r \), \( Q(\cdot) \) denotes the Gaussian \( Q \) function.

After calculating the CPEP, the close form expression of the expectation of the CPEP in (7) can be given in [6] by.

\[ \overline{P}_c(S \rightarrow \hat{S}) \]
\[ = E_{\mathbf{H}} \left[ Q \left( \sqrt{\frac{\sum_{i=1}^{N_r} \left[ \mathbf{H}_i \left( \mathbf{S} - \hat{S} \right) \right]^2}{2N_0}} \right) \right] \]
\[ = \frac{1}{\pi} \int_{0}^{\pi} \frac{4N_0}{4N_0 \sin^2 \theta} d\theta \]
\[ = \frac{1}{\pi} \int_{0}^{\pi} \frac{\sin^2 \theta}{\sin^2 \theta + \frac{4N_0}{4N_0 \sin^2 \theta}} d\theta \]
\[ = \left( \frac{1 - \lambda}{2} \right)^{N_r} \sum_{k=0}^{N_r} \left( \frac{N_r - 1 + k}{k} \right) \left( \frac{1 + \lambda}{2} \right)^k \]

Where
\[ \lambda = \sqrt{\frac{\lambda}{1+\lambda}}, \quad \bar{\lambda} = \mathbb{E}\left( \sum_{i=1}^{N_r} \left| \mathbf{H} \left( \mathbf{S} - \mathbf{S}^\prime \right) \right|^2 \right) \]

Based on union bound technology [6], an upper union bound on the average bit error probability (ABEP) performance of the QSM system with the proposed QAM/PSK can be derived as.

\[ P_b = \frac{1}{m^2} \sum_s \sum_{s'} P_e(s \rightarrow \hat{s}) \cdot e(s \rightarrow \hat{s}) \]  

(9)

Where \( e(s \rightarrow \hat{s}) \) denotes the total number of erroneous bits occurring in the specific event \( (s \rightarrow \hat{s}) \).

5. Performance Results

Simulation results using Monte Carlo are provided to illustrate the BER performance of the QSM system with I-M/QAM and M/QAM/PSK modulation mode. Assuming that the channel information state is perfect and \([N_r, N_r] = [4, 4]\) is considered in the MIMO channel.

In Fig.3, it can be observed that the QSM system with I-8QAM has better BER performance than that of the QSM system with the square 8QAM (S8QAM) and the rectangle 8QAM (R8QAM), that of SM/GSM with 32QAM at the same spectral efficiency. From Fig.3, the QSM system with I-8QAM outperforms 3 dB gains over the system with S8QAM, 1.5 dB gains over the system with R8QAM, 0.8 dB gains over the system with 8PSK, 2.5 dB gains over SM/GSM with 32QAM at BER value of \(10^{-3}\). Furthermore, Fig.4 shows the BER performance of the QSM system with I-16QAM as compared with the QSM system with 16QAM, SM/GSM with 64QAM. It can be observed that the QSM system with I-16QAM can achieve 2 dB SNR gains over the QSM system with 16QAM, approximately 4 dB SNR gains over SM/GSM with 64QAM at BER value of \(10^{-3}\).

![Fig.3 BER comparison of QSM, SM schemes at 7 bit/s/Hz](image-url)
6. Conclusion

In this paper, an improved $M$-QAM scheme for the QSM system is proposed to further increase the squared MED between the TSVs. Such that the spatial gain from the signal constellation is achieved and the link transmission reliability is further enhanced. As a result, simulation results using Monte Carlo demonstrate that the QSM system with I-$M$QAM has better BER performance than the QSM system with the conventional $M$QAM/PSK and SM/GSM schemes at the same spectral efficiency.

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