Correlation of strontium anharmonicity with charge-lattice dynamics of the apical oxygens and their coupling to cuprate superconductivity

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Received 20 September 2023, revised 28 November 2023
Accepted for publication 20 December 2023
Published 8 January 2024

Abstract
By means of Cu K edge x-ray absorption spectra of overdoped high-pressure oxygenated superconducting YSr2Cu2.75Mo0.25O7.54 and Sr2CuO3.3, we demonstrate a remarkably strong three way correlation between the superconductivity and the local dynamics of their highly anharmonic Cu–Sr and Cu-apical O pairs. We model the latter as aspects of the Internal Quantum Tunneling Polaron (IQTP) that give the two-site distributions in extended x-ray absorption fine structure and inelastic pair distribution function measurements. This finding obviates the common assumption that the universal Ba/Sr-apical O dielectric layer, far from only maintaining the separation of charges between the charge reservoir and the CuO2 conducting domains, plays an unexpectedly active role in the unusual electronic properties of cuprate superconductors. Furthermore, we investigate the effects of the dynamic structure associated with these pairs by means of the exact diagonalization of a prototype Hamiltonian based on a six-atom cluster, with two neighboring Cu-apical O pairs bridged by an anharmonically coupled

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Sr atom and a planar O atom. In terms of the Kuramoto model for synchronization, these calculations show a first order phase transition, driven by anharmonicity, to a synchronized state of the IQTPs, in which a fraction of the charge originally confined to the apical O sites is transferred onto the planar O in the superconducting plane. This combination of experimental results and theory demonstrates that the Ba/Sr-apical O layer of cuprates most likely plays an important role in high temperature superconductivity via its collective charge dynamics.

Keywords: superconductivity, cuprates, polarons, synchronization, EXAFS

1. Introduction

Since the discovery of high-temperature superconductivity (HTSC) in cuprates and related materials [1], decades of intense study have revolutionized branches of condensed matter physics, but have yet to yield a consensus on the origins of the superconductivity. This has led the community to call for new ideas [2–5]. Notably in the search for a universal characteristic, two-site distributions in the dynamic structures of certain Cu–O pairs (figures 1(a) and (b)), coupled to HTSC have been observed in virtually all hole-doped cuprates via extended x-ray absorption fine structure (EXAFS) measurements that probe the instantaneous structure factor, \( S(Q, \omega = 0) \) [6–20]. Concomitant with the original EXAFS report [6], differences between the elastic, \( S(Q, \omega = 0) \), and inelastic, \( S(Q, \omega > 0) \), structure factors from neutron scattering measurements corroborated those findings [21, 22]. Analogous behavior has also been found in non-cuprate HTSC compounds [23, 24].

These structural anomalies were originally designated tunneling polarons [9, 10, 25]. A more precise term differentiating them from Holstein-type and related small polarons is ‘Internal Quantum Tunneling Polarons’ (IQTPs). IQTPs occur when a fraction of the excess charge of a Cu-centered small polaron is localized on one of its neighboring oxygen ions via hybridization to the Zhang–Rice singlet state [26–29]. This decrease in its charge, \( \delta \), causes a displacement of the oxygen from its crystallographic position to give a second, distinct Cu–O distance. The IQTP occurs when this localized charge and associated displacement exchange with an identical oxygen ion bonded to this same or an adjacent copper ion via quantum (non-thermal) tunneling at a rate much faster than the host polaron hops to another copper site, confining them within this local cluster (figure 1(c)). Their essential characteristic is this fundamental charge–lattice dynamics that is internal within a small number of neighboring Cu–O pairs. IQTPs are therefore observed as two Cu–O distances in the dynamic structure measured by EXAFS or inelastic neutron scattering, whereas diffraction finds only one [10, 14]. We note that the transformation between static and dynamic structure is not an averaging process because it is not related to static disorder.

The potential significance of IQTPs is that at a critical value of an anharmonic coupling between the two oxygen ions the pairwise exchange of the charge and displacement between them synchronizes, merging the wave functions to encompass the entire cluster built to model their aspects [30, 31]. This correlated motion of charge and atoms makes IQTPs especially interesting from the perspective that the HTSC driver in cuprates is electron–electron interactions [32–36] that, as found by angle-resolved photoemission spectroscopy (ARPES), are augmented by strongly coupled phonons [37–43]. Other types of measurements [44–51] and calculations [52–54] show the same trend. As a form of direct electron–phonon coupling in which intrinsic quantum oscillations of the atoms between their two sites constitute lattice-assisted dynamical charge transfer, the electron-electron interactions and entanglement that are a consequence of the in-phase oscillations of the Cu–O pairs of the IQTPs provide a conceptual basis for a much more significant role for the lattice. Whether the lattice is involved in the mechanism beyond simply providing a platform for the carriers has been one of the principal issues in HTSC since shortly after its discovery.

An active role for the apical oxygen (Oap) atoms in HTSC has been demonstrated by numerous experiments and calculations. The collective behavior of the charges in the CuO₂ planes that is manifested as the numerous ordered phases poses the same question as the calculations: what are the ramifications of coupling the Oap atoms, which would be linked through the Ba/Sr atoms, as in figure 1? This possibility has been mostly neglected as unlikely because of the absence of any experimental data and the inert charge of the alkaline earth cations [55]. However, it could result in functionality for the entire Ba/Sr–Oap plane in the YBa₂Cu₃O₇₋ₓ-type (YBCO) or Bi/Hg/Tl-based compounds and the La₁₋ₓBa/Srₓ–O apical layer in the La₂₋ₓCuO₄₋ₓ-type (LCO) compounds analogous to the other discrete sections of the structures. This is often referred to as the dielectric domain whose function has been presumed to be passively maintaining the separation of the excess holes injected into the CuO₂ conducting layers from the charge reservoir layer where they originate [56] (figure 1). However, our recent reports on the EXAFS of high-pressure oxygenated (HPO) YSr₂Cu₂₋₄M₀₋₂₋₅O₇₋₄₋₅ (YSnO–Mo, \( T_\text{c} = 84 \text{ K} \), figure 1) and Sr₂CuO₃ (SCO, \( T_\text{c} = 95 \text{ K} \)) samples, while focused on the Cu–O pairs and their IQTPs, also showed that \( \chi(R) \) from the Cu–Sr pair in YSCO–Mo along the \( c \) axis began to decrease at \( T_\text{c} \). This is an anomaly since EXAFS amplitudes are expected to increase with decreasing temperature and thermal motion. The unusual behavior of the Cu–Sr pair raises the question of the possibility of the structural dynamics associated with the dielectric layer possessing a role in the superconducting physics of these systems. Another aspect of the dielectric layer is that, in contrast to the relatively rigid CuO₂ planes and the structures of the charge reservoirs being
Figure 1. Structures of (a) YSCO–Mo and (b) SCO derived from crystallography and EXAFS, using the standard atom labels from the former. The directions of alignment in a magnetic field are shown at the bottom. The functional domains are: violet = conducting CuO planes; green = charge reservoir layer for YSCO–Mo; beige = dielectric layer that for HPO SCO contains Sr and Oap, but in conventional divalent cation-doped La2CuO4-type compounds is composed of La and the Ca/Sr/Ba. The Mo environment becomes octahedral by incorporating the excess O into neighboring vacancies, introducing a fifth O around adjacent Cu ions. For SCO, half of the O sites between Cu ions along the a direction and many of the Oap sites are vacant. In its dynamic structure we have postulated that the long Cu–O distances are Oap tilting towards adjacent Oap vacancies (yellow), others in red have the short, 1.9 Å Cu–O distance. Cu–O pairs that will form the IQTPs are outlined in purple. (c) The symmetry-allowed atom displacements of the X2 = (a, 0, 0) zone-boundary phonon for YSCO–Mo described below. The displacement magnitudes are arbitrary. The exact diagonalization calculations were performed on the six atom cluster within the red rectangle in which the expanded Cupl-Oap IQTP exchanges its displacement and δ+ charge with the adjacent contracted Cupl-Oap IQTP.

those of the parent oxides, the local geometries of the Ba and Sr are highly disordered relative to the rock salt lattices of BaO and SrO.

Although the positive identification of IQTPs is performed by inelastic neutron pair distribution function (pdf) analysis, [21, 60] their coupling to, and role in, HTSC are elucidated by EXAFS [6, 19]. Here, we exploit the capabilities of EXAFS to probe the Cu–Sr pairs and extract changes in their distributions with high precision to advance our findings on the IQTPs in YSr2Cu2.75Mo0.25O7.54 and Sr2CuO3.3 [57–59]. These data are conjugate to vibrational spectroscopy that probes vibrational states via the energies of the collective displacements of atoms in a lattice. Alternatively, the EXAFS signal contains a snapshot of the real space distribution of specific pairs averaged over the probed volume. Anharmonic deviations from Gaussian-like distributions that soften and distort IR and Raman peaks [61] are manifested in the EXAFS as anomalously shaped increases in the normal exponential damping of its pairwise amplitudes in χ(k) that substantially diminish the magnitude of their Fourier transforms, χ(R) [11]. Since the quantitative analysis of EXAFS can only be performed if the pair distributions are known, which these obviously are not, we have adopted an approach that we have used on other systems utilizing direct comparisons of the isolated Cu–Sr components of the spectra. This technique was not performed in our previous reports [57–59].

We find that the Cu–Sr pair distributions along the c axis are not only unusually soft, extending previous reports on their disorder [62, 63] but, surprisingly, they are also highly correlated with both the HTSC and the IQTPs. These results indicate that the entire dielectric domain, both the cations and the Oap atoms, is highly active in HTSC via the local dynamical behavior of its constituent atoms. In addition to these experimental findings, we have performed exact quantum diagonalization calculations on a small cluster (see figure 1(c)) of the structure containing the two Cu–Oap pairs forming the IQTPs bridged by an electronic active Opl site. This introduces the conducting plane in the system, and also a structurally active Sr site coupling the Oap atoms, forming a triatomic Oap–Sr–Oap ‘molecule’. By analyzing the numerical results in the context of the Kuramoto model for synchronization [64, 65], we point to a possible mechanism for the active role of the dielectric domain in the interplay of electronic and structural degrees of freedom of these systems. Moreover, these calculations elucidate the functionality of IQTPs that would underlie an important role for the dynamic properties of these entities in HTSC.

This paper is divided as following: in section 2 we describe the materials and methods used in the experiments. In section 3 we show the experimental results of EXAFS performed in the HPO compounds studied in this work, namely YSCO–Mo and SCO. In section 4 we discuss and show the numerical calculations that supports the interpretation given to the experimental results. Finally, in section 5 we summarize our findings and discuss the implications of our results within the framework of the IQTPs in HPO cuprates.

2. Materials and methods

Descriptions of the materials, sample preparation, and experiments have previously been reported [57–59]. The YSCO–Mo was the same sample used before [66], rechecked to ensure no degradation. The SCO sample was made and characterized for these experiments and consisted solely of the Tc = 95 K variant [67, 68] confirmed by XRD and the susceptibility measurements. The grain morphology of HPO cuprates often results in relatively low superconducting fractions [69], but that for the 75 K phase of SCO was stated as ∼85% and muon-spin relaxation demonstrates that this is a surface effect, with the actual fraction being ∼80% for YSCO–Mo. On this matter, it has been shown both theoretically and experimentally in both cuprates and conventional superconductors that the measured Meissner fraction is highly dependent on the sizes and shapes of the grains, the internal arrangement of their superconducting domains, their packing in the sample, and their conductive connections, with polycrystalline samples usually giving values below ∼40% [70–72]. Post synthesis treatments such as a second annealing phase or others have been found to increase this value, leading to the conclusion that Meissner
fractions in this range demonstrate superconducting fractions that are identical to the values obtained for homogeneous superconductivity in single crystals.

The samples were oriented for the EXAFS measurements by suspending the powders in epoxy that was then placed in the bore of a 4–10 T magnet to set and cure. Orientation was confirmed according to [73]. Like all other cuprates, YSCO–Mo is oriented with the c axis parallel to H, while SCO was unique in orienting along an axis perpendicular to the crystallographic c axis that we labeled as a, with the perpendicular direction therefore being the bc plane. The transmission XAFS measurements were performed at the Cu K edge end station 2–2 of the Stanford Synchrotron Radiation Lightsource and its analysis have been described previously in our reports on the local structure at single temperatures [58, 59] and the near neighbor Cu–O pairs over a range of temperatures through the HTSC transition [57].

3. Experimental results

3.1. Data analysis

EXAFS measurements of the two HPO cuprates were performed over a range around $T_c$ to probe their dynamic structure and its coupling to HTSC. The previous reports described the behavior of the Cu–O pairs of the IQTPs, but only examined the temperature dependence of the Cu–Sr $\chi$ amplitude in YSCO–Mo without comparing it against any of the other components of the structure. The novel aspect of this paper is therefore the detailed comparative analysis of the IQTP and Sr contributions to the EXAFS over a wide temperature range through the superconducting transitions in both compounds and its ramifications in our calculations. This identifies the surprising strength of the correlation and corresponding coupling to the projection of the Cu–Sr dynamics along the c-axis that was not previously examined.

As stated in the introduction, the quantitative analysis of the spectra in terms of extracting the standard metrical parameters describing the local structure requires a priori knowledge of the exact distributions [11]. The difficulty in even assuming a distribution is evident in the complicated behavior of the spectra; the disappearance of the Cu–Oap signal just above $T_c$ in SCO does not lend itself to a determination of a unique pair distribution. An alternative approach for attaining our objective of finding correlations between the pairs and the superconducting transition is the direct comparison of the Cu–Oap and Cu–Sr components of the spectra. Changes in the distributions of the target pairs caused by either shifts of oxygen atoms between different components of their preexisting multisite distributions or originating in their local, pairwise dynamics are then easily identified in the amplitudes of the Fourier transforms, $\chi(R)$, of their isolated contributions to the EXAFS. We have used this for other systems as a method for visualizing trends or patterns in their local structure as a function of an extrinsic variable without having to know the actual shapes of their distributions [74–76].

This separation of the spectral contributions begins with curve-fitting with a harmonic Debye–Waller that validates the structural model by finding the static neighbor shells that conform to the crystal structure and the displaced and dynamic ones that may deviate from it [59]. The contribution of the target pair in $\chi(R)$ is then obtained by subtracting from the EXAFS these waves from those of the other shells of the structure. A critical requirement is the experimental resolution and the confidence that the signal from the target neighbor shell is not affected by and correlated with those of the neighbors that are being subtracted [75, 76]. This becomes especially important in isolating a smaller signal from a larger one with which it overlaps. The resolution of EXAFS is defined as $\Delta = \pi/2k_{max}$ where $k_{max}$ is the highest energy in the analysis [76]; the signature of two shells is the beat in the phase that differs in shape from simple damping from a broad anharmonic distribution. A not uncommon misconception is to relate the resolution to the width of the Fourier transform modulus peaks, in which case the degree of correlation in the signals between neighboring shells would follow from the overlap of their peaks in $\chi(R)$. However, the few to several tenths of an Å width of the $\chi(R)$ peaks is a function of the finite and relatively short total range of the data used in their calculation; the waves for 1.9 Å Cu–O distances complete only six full cycles by $k = 15$ Å$^{-1}$. That the FT widths are largely an artifact of their calculation is also demonstrated by the use of window functions applied to the $\chi(k)$ spectra that eliminate ripple while broadening the features. All $\chi(R)$ data presented here were calculated with a sine window. For these spectra extending to $k = 15$ Å$^{-1}$, 0.15 Å separations of the IQTP oxygen shells give a beat at $k = 11.5$ Å$^{-1}$ that is completely within their upper endpoint. Potential correlation between sets of shells from different elements, e.g. Sr and O, is further reduced by their amplitudes and phase shifts that vary with atomic number.

This process expands the interpretation from a table of metrical results from curve-fits to include a more qualitative visualization of correlations. Analogous to amplitude-ratioing analysis [77], it exploits the extraordinary sensitivity of EXAFS to changes in the pair distributions of closely related samples, e.g. temperature dependence. Not only are the errors in the calculated phases and amplitude and backgrounds canceled, but also the inability to precisely determine disordered structures and unknown pair distributions is rendered less important because the objective is to identify changes. Even if there are errors in the calculated waves of the initial curve-fit that render the metrical data incomplete or with large uncertainties, the essential aspect is the reduction of the contributions of these errors to a level below that where it interferes with the spectral component of interest.

3.2. EXAFS of YSCO–Mo

The Cu K edge $\chi(R)$ of the total structure, the isolated Cu2–Oap contribution, and c-oriented projection of the Cu–Sr pairs of YSCO–Mo from 53 to 110 K are shown in figure 2. These were obtained as described above. The $k^2$-weighted spectrum
Figure 2. $\chi(R)$ as a function of temperature and position. $R - \varphi$ is used here to indicate that the EXAFS phase shift displaces the modulus peaks from the actual Cu-neighbor positions. (a) Modulus of $\chi(R)$ of the total spectrum showing the neighbor atoms that are the origins of the spectral features. The arrow indicates $T_c = 84$ K. (b) Subtraction of the Cu1–Oap, Y, Sr and Cu–Cu waves from the spectra yields the $\chi(R)$ modulus of the two-site Cu2–Oap IQTP. The contributions of the two Oap sites to the broad or double peak are labeled Oap1 and Oap2. The Cu1–Oap* peak is interpreted as the spectral contribution of an anharmonic component of the Cu1–Oap distribution after subtraction of the Cu1–Oap fit with a Gaussian distribution. (c) Cu2–Sr $\chi(R)$ modulus after isolation by the same procedure.

The metrical data from the curve-fits that provide the additional details that make the essential connection are shown in figure 3. These were obtained by curve-fits of these residuals in which the sum of the numbers of oxygen atoms was fixed. The endpoint of $k = 14.7$ Å$^{-1}$ gives a resolution of 0.11 Å. Insofar as the separation between the two oxygen atoms is 0.17 Å, giving the beat in the composite wave in the EXAFS signifying the two Cu–O distances at $k = 9.3$ Å$^{-1}$, the Cu2–Oap $\chi(R)$ is not only asymmetric and flattened across its top, but shows two peaks when they are similar in amplitude. The Cu1–Oap distance that is the origin of the large oxygen peak in figure 2(a) is 0.19 Å shorter than the shorter of the Cu2–Oap pairs, and was curve-fit with the Cu1–Oap, a single Cu2–Oap, Cu2–Y, Cu2–Sr and Cu1–Cu2 neighbors. The curve-fits do not find the Cu1–Sr and Cu2–Cu2 waves, indicating that these pairs are so disordered that they meet the spectroscopically silent criteria. This disorder of the Cu2–Cu2 pair is most likely thermal because of the length and presumed poor correlation of the Cu–O–Y–O–Cu link. The absence of the Cu1–Sr contribution would be a more extreme extension of our findings for the Cu2–Sr pair that we discuss below, augmented by the disordered Sr site. All but the Cu2–Oap waves were then subtracted from the total spectra. The residuals were then Fourier transformed to give the results in figure 2(a), with the flatness of the regions $R > 2$ Å corroborating the negligible Cu1–Sr and Cu2–Cu2 contributions.

The temperature-dependent patterns and trends in the structure, i.e. the absence of any anomalies in the major spectral features and very small change on the high $R$ side of the oxygen peak are much more easily identified in these three-dimensional depictions and are much more easily evaluated than in the original presentation of these spectra as overlays of the moduli. This is especially true for the Cu2–Oap $\chi(R)$ (figure 2(b)). Above $T_c$, the peak at $R \approx 1.8$ Å that is the contribution of the Oap with the shorter Cu–O distance is highest. At $T_c$ there is an abrupt drop in its amplitude, followed at lower temperatures by a shift of spectral weight to the $R \approx 2.0$ Å peak that is assigned to the longer Oap. This plot highlights the extremely narrow width of the change at $T_c$ and the subsequent loss of distinct features over a range below it. A change of the Oap contribution from a double or structured modulus peak to a more symmetric one at the transition is similar to the behavior in other cuprates, although in those cases the cause was identified as a decrease in the separation between the two sites. YSCO–Mo differs from these other compounds in that, instead of recovering the original spectrum there is an obvious shift of spectral weight signifying a shift of some of the Oap from its position closer to Cu2 to the longer Cu2–Oap distance. Another peak at $R = 1.4$ Å exhibits a distinct increase at $T_c$. Since this is too short for an actual Cu–O pair, we interpret it as SC-coupled anharmonicity in the Cu1–Oap distribution that does not affect its main peak and is not captured by the fit of this pair with a Gaussian distribution. Similarly, the Cu2–Sr $\chi(R)$ is superior to the previous amplitude ratios in showing the onset of the slow amplitude reduction at $T_c$ followed by the accelerating decrease with decreasing temperature (figure 2(c)).
the Cu1–Sr is more than 1 Å longer. Being substantially larger than the resolution, the separation is validated. Similarly, although the difference between the Cu2–Y and Cu2–Sr distances is just under the limit and their atomic numbers differ by only one, comparison of their waves from the curve-fits shows that they begin in phase at low energy, as expected, but are π out of phase by the higher energy limit. Insofar as the crystal structure dictates two distinct shells and not a single wide one, the correlation between their signals is therefore negligible and the isolation procedure is effective for the Sr as well.

Our prior analysis of the Cu–Oap pairs found that, based on the two independent Cu–O distances and numbers of atoms, the IQTPs exhibit identical patterns of plateaus and discontinuities at the same temperatures. These delineate the normal phase, a fluctuation region at the transition, and two separate regions in the superconducting phase. Reproducing them here with the addition of the Sr χ(R) amplitude to the difference in the numbers of oxygen atoms accentuates the novel finding that the peak of χ(R) displays a remarkable correspondence with the shifts of the two oxygen atoms between their two sites, as shown in figure 3. This even includes the feature at 70 K that would be assigned to a noise spike if it only occurred in one of these results. We emphasize that nothing in the data analysis would bias this correlation between these three parameters and the superconducting transition. The c-oriented χ(R) peak for the Cu2–Sr (figure 2(c)) is relatively flat at higher temperature, and then begins a slow curve downward at Tc that subsequently drops more steeply, albeit with some additional features. This behavior is indicative of increasing disorder with decreasing temperature, the opposite to the conventional thermal broadening of the distribution. The correlation is with the difference in the populations of the two Cu2–Oap sites caused by changes in the relative energies of the two minima of the potential (figure 3). The high level of correlation demonstrates the direct coupling of the anharmonicity of the Cu2–Sr distribution to the renormalization of the Cu2–Oap potential and both of these to the SC, including the division into the four regions.

3.3. EXAFS of SCO

Although its composition and parent structure are simple, the EXAFS and local structure of our second material, SCO, are much more complicated than those of YSCO–Mo. A notable aspect is the large changes in the amplitudes with temperature. These are often opposite to the normal reduction with increasing temperature, and several occur over narrow ranges both in proximity to and away from the HTSC transition. Also remarkable is the broad peak at R = 2.0–2.8 Å that does not correspond to sites in the crystallographic structure (figure 1) that vanishes and then abruptly reappears at its maximum amplitude at Tc (figure 4(a)). The magnitude of this change is unprecedented. Our earlier study of SCO found that it is unique among cuprates in exhibiting oxygen vacancies not only in its apical positions but also in its CuO2 planes. That the planar vacancies occupy approximately half the sites on only one axis provides an explanation for why its alignment in a magnetic field is along that direction instead of its c axis. Because of this rotated magnetism, the Cu–O IQTP in SCO is observed in its crystallographic bc plane [58].

We originally postulated that the behavior of the spectra across the transition might include a strontium atom with a Cu–Sr distance less than 3 Å [57]. However, visualizing the affected spectral region by the isolation method now shows that the large amplitude features at R = 2.2–2.8 Å are present above as well as below the transition. The broad, flat top of this feature from 85–95 K (figure 4(b)) assigns it to the two-site Cu–O distribution characteristic of IQTPs. The remarkable result is its disappearance at Tc, signifying a radical change in the dynamic structure coupled to the superconducting transition, a finding not only outside of BCS superconductivity, but also for HTSC. The absence of a correlation between the amplitudes of the two contributions to the peak is also unique. The current best model for this behavior is that the IQTP oxygen atoms are most likely apical, next to an apical vacancy, displaced along the b axis to asymmetrically bridge two copper atoms at ∼2.66 and 2.91 Å. Both these distances and the 0.25 Å separation are much longer than any other cuprates. The radical difference in the copper geometries relative to other cuprates, including Opl–Cu–Oap angles lower than 90 degrees, could be a consequence of the much higher copper valence in this overdoped compound. The oxygen atom below them in the b-oriented Cu–O chain shifts away from this Oap to slightly expand its Cu–O bonds [57]. The IQTP could then
include tunneling of the oxygen atoms into equivalent sites across the ac plane that passes through the midpoint of the two coppers as well as between the bridging and vacancy sites. \( \chi(R) \) at 121 K in the normal phase has most of its amplitude in the peak for the longer Cu–O distance, with the shorter oxygen giving just a shoulder. The almost complete loss of amplitude at \( T_c \) could result from either the separation between the two sites decreasing to maximize the interference of their EXAFS waves and/or their distributions broadening in the fluctuation region of the transition. Below \( T_c \), the two contributions from the two sites are equal. They begin to fall off at \( \sim 80 \) K, with the higher \( R \) peak again being larger. Repeating, a possible explanation for the diminished overall amplitude of the IQTP at lower temperature is that in this postulated configuration for SCO the oxygen could shift to a normal Oap position directly above the copper along the c direction.

The Cu–Sr contributions were fit with two strontium neighbors at \( \sim 3.23 \) and 3.36 Å. This 0.13–0.14 Å separation is only marginally larger than the resolution for the \( k = 14.2 \text{Å}^{-1} \) upper limit of the curve-fit. In combination with the disorder that also drives the amplitude down at high \( k \), the increase in the amplitude after the beat will be minimal and the Cu–Sr waves in \( \chi(k) \) will be indistinguishable from a single, highly damped one. What is clearly observed now that was obscured in our prior report is that \( \chi(R) \) shows a single, round peak with minimal structure. The Cu–Sr \( \chi(R) \) amplitude rises as the temperature is lowered from \( \sim 78 \) K, also developing a more distinct shoulder on the high \( R \) side, where the second strontium at the longer distance needed for a complete curve-fit makes its contribution. This relatively normal trend is broken by the peak beginning near 106 K that falls immediately at \( T_c \), reflecting a narrowing and hardening of the Cu–Sr distribution in the fluctuation region that is eliminated on becoming superconducting. All of these changes can be presumed to originate in the shape and resultant ordering within the single peaked Cu–Sr distribution.

Because of the difficulty of curve-fitting to extract metrical parameters when the features have vanished from the spectra, the correlation with the strontium is derived from comparing the isolated \( \chi(R) \) peaks from the IQTP oxygen and strontium atoms at, respectively, \( R = 2.6 \) and 2.95 Å. The novel result is the high degree of correlation, or in this case anti-correlation, between the Cu–Sr and the IQTP \( \chi(R) \) (figure 5), most notably the dramatic response to the onset of the HTSC. As with the YSCO–Mo, the oxygen atoms of the IQTP and the strontium are strongly coupled both to each other and the HTSC. This new analysis therefore shows that the entire putatively inert dielectric layer is actually very strongly coupled to HTSC in SCO.

4. Model calculations

Our demonstration of an anomalous Cu–Sr distribution that is strongly correlated with both the dynamical structure of the Cu–Oap and HTSC poses the questions of the interaction of the strontium with the IQTPs and more generally the HTSC–lattice coupling. We have explored this by applying exact
quantum diagonalization to a six-atom cluster (figure 1(c)) that is an extension of the original, three atom O–Cu–O moiety previously used to elucidate the experimental signatures of IQTPs in the elastic and inelastic structure factors [10, 26]. Briefly summarizing the results of the calculations emphasizes the importance of the strong anharmonic coupling of the stron- tum to the IQTPs found in EXAFS and its possible or even likely role in the HTSC mechanism.

The minimal subunit encompassing the relevant constituents consists of a pair of neighboring Cu–Oap IQTPs bridged between their copper ions by an Opl and between their Oap by the strontium of the dielectric layer (figure 1) [30, 31]. The novel aspects enabled by this structure are the addition of the CuO₂ plane through the inclusion of Opl and the anharmonic structural dynamics of the Cu–Sr pair that is incorporated via the nontrivial Oap–Sr–Oap triatomic molecular structure indicated by these EXAFS results. In addition to the two localized charges in the Cu sites, a single extra hole initially localized on one of the two Oap atoms causes its displacement to give the two-site distribution. The starting point for the calculation is to consider this cluster as a clas- sic Holstein-type polaron in which a significant fraction of the excess charge is found on one Oap. The standard Hubbard-Holstein Hamiltonian used to describe the interplay between electronic and lattice degrees of freedom was augmented by an anharmonic phonon-phonon coupling describing the interaction between the triatomic O–Sr–O molecular vibration and the independent apical phonons located on the Oap sites. This extra term reads

\[ H_{Sr} = K \left( \beta L b_R b_L + \beta R b^*_R b^*_L \right) \],

(1)

where \( K \) is the strength of the anharmonic interaction between the independent Oap phonons, described by the creation (annihilation) operators \( b^i \) \((b^i)\), where \( i = L, R \) stands for left \((L)\) and right \((R)\) apical oxygens in the cluster, as shown in figure 1, and the triatomic molecular phonon modes \( \beta^L (\beta) \), associated with the locked vibration of the Oap–Sr–Oap chain in the dielectric layer. This anharmonic term is introduced in order to capture the dynamics associated with the Sr atom in the Cu–Oap IQTPs, as seen by the experimental results discussed above. The internal dynamics within this cluster defines the IQTP exchange as the oscillation of the excess charge and displacement on an Oap site via tunneling at a frequency higher than thermally activated hopping of the polaron to a neighboring site. This is accomplished by exchang- ing these characteristics with an adjacent, degenerate polaron atom.

In addition to the exact diagonalization calculations, the charge-lattice dynamics was evaluated via the Kuramoto approach for synchronization of coupled oscillators by equat- ing the anharmonic coupling \( K \) of the the O–Sr–O vibration to the coupling between phase oscillators in the Kuramoto model [64, 65]. In addition to its application here incorpo- rating the EXAFS results, this approach has also been applied to the emergence of superconductivity as a synchronization problem [78]. The calculations shown here were performed by varying the anharmonic interaction strength, \( K \), while fixing the electron–phonon coupling, \( \lambda \), in the non-adiabatic regime that defines the cluster. This local electron–phonon coupling is present only on the apical positions Oap in order to ensure polaron formation. Further details of the exact diagonalization method and the Kuramoto analysis can be found in [30, 31].

The diagonalization procedure, incorporated with the synchro- nization driven by anharmonicity via the Kuramoto model, showed that at a critical value of \( K, K_c \), the system undergoes a first order transition, where the order parameter is written as

\[ r = \sqrt{1 - \frac{K}{K_c}}. \]

(2)

The transition is also defined by discontinuities in the dens- ity of charges on the oxygen ions, namely the two apical oxygens, \( n_{el}(O_{ap}) \), the planar oxygen site, \( n_{el}(O_{pl}) \), and the tunneling frequency, \( \hbar \omega_T = E_1 - E_0 \), the difference in energy between the ground and first excited states, as summarized in figure 6. The Kuramoto analysis shows that this transition can be understood as a dynamic synchronization; when \( K > K_c \) correlated, anti-phase tunneling of the two Cu–Oap pairs is initiated between their opposite configurations of displacement and charge.

For \( K < K_c \), the unsynchronized phase, the excess charge is localized on either the left or right Oap due to the strong local electron–phonon coupling, forming two separated potential wells (figure 7(a)), between which no tunneling occurs. When the system transforms to the synchronized, phase-locked state, the resulting delocalization of the charge transfers a fraction of
Figure 6. $K$ dependence of cluster characteristics. (a) Order parameter for synchronization of IQTPs within the Kuramoto model as in equation (2). (b) Excess hole density on each Oap. (c) Excess hole density on Opl. (d) Tunneling frequency associated with the IQTPs in units of the electronic hoping parameter.

the excess charge originally on the two Oap atoms to Opl. The synchronization extends the wave function to form this third local minimum on Opl (figure 7(a)) and collapses the energy levels so that the excitation energies are greatly reduced. This delocalization and dispersion throughout the cluster of the hole that started on a single Oap is a clear signature that the synchronization of the IQTPs through the anharmonic Oap–Sr–Oap phonon modifies the electronic properties of the entire system. The resulting triple-well functions as an anharmonic-structural-adiabatic-passage promoting charge delocalization, that once formed may further enhance the oscillator synchronization in a positive feedback loop. Using the example of the polaronic tunneling frequency, $\hbar \omega_T$ (figure 6(d)), it is seen that at $K$ values below the critical value $K_c$, $\hbar \omega_T = 0$, no tunneling occurs and the polarons are trapped on their original left or right sites. Above $K_c$ the abrupt increase in $\hbar \omega_T$ is the signature of the transition to the synchronized phase. Because it is essentially a spectroscopic parameter, an interesting aspect of $\omega_T$ is the structure in the synchronized phase above $K_c$ (figure 6(d)) that is caused by avoided crossings in the ground to excited state transition. Such structure does not occur in the other parameters.

5. Discussion and conclusion

Comparisons of the Cu–Sr and Cu–Oap contributions to the Cu K EXAFS of YSCO–Mo and SCO over their transition temperatures demonstrated very strong coupling of the local charge-lattice dynamics within the entire dielectric layer to their superconductivity. Since the two-site distributions of the Cu–Oap distributions constitute the IQTPs in these compounds, the high degree of correlation between the $\chi(R)$ moduli amplitudes for both neighbors over the full temper-
ature range and abrupt changes across the transition further established a strong, three-way coupling between the Cu–Sr anharmonic distributions, the IQTPs, and the superconductivity in both materials despite the very different structures of their Cu–O sublattices. This three-way correlation implies that even without the direct participation of the Sr$^{2+}$ ion forming the tri-atomic chain Oap–Sr–Oap of the cluster. Far from being passive [56], our calculations demonstrate that this coupling and consequent synchronized, anti-phase motions of the Oap atoms and their associated charges have substantial effects on the dynamics and charge distributions in the other domains of the materials, notably the CuO$_2$ planes where the charge on the oxygen is highly correlated with $T_c$ [79]. The extension of the localized wave function on Oap to multiple IQTPs and sites in the CuO$_2$ plane promotes electron–electron interactions that go far beyond the acknowledged special role of the Oap atom in controlling the carrier density [79–81] or, in the extreme case, the suggestion that pairing could occur through possible lateral vibrations [55] (although these are not observed in EXAFS). Furthermore, depending on the details of short- and long-range interactions between IQTPs, assemblies of finite densities of them will likely result in mesoscopic patterns, including clusters and stripes, and possibly dynamical charge density waves (CDW) [82]. However, given the importance of specific nonlinear and nonadiabatic degree-of-freedom that we have identified for our 6-atom single IQTP, conventional CDW descriptions are unlikely to be directly relevant. The synchronization proffered by the anharmonically coupled two-site distributions of the IQTPs therefore also entangles at least some of the relevant degrees of freedom between the conduction and dielectric layers and charge reservoirs.

A persisting problem with EXAFS of highly correlated systems including cuprates is integrating its information on the behavior of specific atoms pairs with other spectroscopic data that are ensemble averages. This is often exacerbated by its sensitivity to the dynamic structure that can cause certain results to contradict the more intuitive ones from crystallography. In contrast to the immediate utility of chemical speciation that is defined by local structure, the interpretation of many experiments in terms of reciprocal space quantities means that there is no simple transformation from, for example, phonon spectra to anomalies in the real space pair distributions identified by EXAFS. In their obvious demonstration of coupling between charge fluctuations, phonons, and pairing-HTSC our results here are complementary with recent reports on RIXS [82–85] where the characterization of several excitations came to the same conclusions about charge dynamics. However, the different dependence on temperature and locations within the structures show that the phenomena probed by these measurements are not the same. This problem with unifying data from diverse sources is not unique to EXAFS. The difficulty of incorporating disparate results from multiple experiments into a coherent picture is a major factor in the continued absence of an HTSC theory. In terms of overcoming instead of contributing to this dilemma, an often-overlooked fact from the EXAFS of IQTPs is relevant; the two-site distribution of the apical oxygen atoms applies to every one of those atoms in the entire crystal, a signal from a third oxygen that is not participating in an IQTP is not found. If the local charge fluctuations are in fact universal, then the different electronic phases including HTSC might originate in their organization and collective behavior.

These experimental and theoretical results provide a critical extension to our prior EXAFS studies that demonstrated that the two-site Cu–O distributions in the dynamic structure that constitute the IQTPs are strongly coupled to the HTSC [6, 8, 12, 13, 19, 30, 31] as well as these overdoped, superconducting SCO and YSCO–Mo [57]. It would be expected that a phenomenon that is coupled to the HTSC mechanism would exhibit a correlation with $T_c$. Such behavior has been reported for the Cu–O bond lengths, and the hole density on the oxygen atoms [79, 86] that is analogous to the charge transfer caused by the synchronization. The observed variability in the type and extent of the IQTPs in materials with similar critical temperatures could therefore seem to minimize their importance. This argument based on isolating microscopic components of the material is countered by the increased complexity of the IQTPs we have reported here. In contrast to our original description of an IR phonon acting on a Cu–Oap pair, the entity of interest consists of not only the phonon giving the two-site distribution, but now also the anharmonic one connecting the Oap to the strontium that, when the coupling exceeds a threshold strength, synchronizes them over all of the copper and Oap in an entire domain, including the Opl, to which hole density is transferred, all of which are entangled. A simple linear relationship between all of these parameters defining the IQTPs and the HTSC would only occur if the coupling between each of them was weak, which these results show is not the case.

In addition to the challenges to existing theories posed by the HPO phase diagram, the differences between the structures of YSCO–Mo and SCO, the effects of the excess oxygen atoms on $T_c$, and the details of their IQTPs merit notice in their own right. In the search for a common factor in HTSC cuprates, the totality of the dynamic structure measurements indicate that strong coupling of the anharmonic, dynamic structure of Cu–Sr and IQTPs to each other and the HTSC is universal among all of the materials studied. However, the details in terms of the atoms involved, their locations in the lattice, and their displacements caused by the exchange can exhibit large variations, possibly even within different preparations of the same compound. These new EXAFS measurements greatly extend the range of the structure and behavior of IQTPs beyond the previously observed partial collapse of the double well potential near the HTSC transition. Not only are the two minima of the double well potential non-degenerate, but also the difference in their energies varies across the entire temperature range with a large shift at the superconducting transition even while their separation is unchanged. The essential factor in the HTSC-IQTP coupling is therefore not limited to the equal splitting in the dynamic structure of the Oap sites into
a two-site distribution along the $c$ axis with tunneling frequencies on the order of 100 cm$^{-1}$. The coupling can also manifest itself in the dynamic structure as double well potentials and their associated charge dynamics that display other values of the two energies, locations in the lattice, and responses to the HTSC transition.

A critical aspect for identifying and characterizing the IQTPs is the experimental signatures. As with HPO cuprates, dynamic structure is a topic that has not been thoroughly examined by the community, especially inelastic neutron PDF measurements. There has been much more continuity with EXAFS, but the absence of theoretical bases for both the origins of the dynamical multisite distributions and their effects on properties of interest that are the intersection of the local coordination chemistry with the physics have impeded the utilization of its findings. Recent improvements in the measurements that result in the much larger number of higher quality spectra provide the new evidence here, e.g. the contrast between the absence of any significant temperature dependence in the total spectra of figure 2(a) and the large change at the superconducting transition well above the noise level in the isolated Cu2–Oap contribution. We have already initiated a more thorough evaluation of IQTPs by applying these techniques to the more conventional HTSC materials.

The role of electron–phonon coupling in the HTSC has been mapped to that of certain phonons, for example the $B_{1g}$ phonon that is the antiphase buckling of the Cu–O–Cu moieties of the CuO$_2$ plane [52], was shown shortly after the discovery of HTSC [22, 87], although there is disagreement on the strength of its coupling to the HTSC [88]. Anomalies in phonon dispersions indicative of strong electron–phonon coupling that are correlated with both $T_c$ and the doping, e.g. the giant softening of the bond-stretching phonon frequencies off of the zone center caused by strong renormalization along [010] and [100] directions and avoided crossings, are a common characteristic of cuprates [45–48, 89]. Although these are still insufficient to cause HTSC, insofar as the doping-induced charge inhomogeneities are localized [90, 91], the widths of the phonon peaks reflect their resulting wide range of energies [92]. The comparison of the doping dependence of these ~70 meV phonon dispersion anomalies with the ‘kink’ in angle-resolved photoemission denotes different origins for these two spectral signatures. Whereas this feature in ARPES would indicate electron-anomalous phonon coupling, its presence in neutron scattering would result from collective charge excitations [48, 93] that could promote a novel HTSC mechanism. This would be a likely description of an important role for a synchronized IQTP phase that directly combines both lattice and charge dynamics sufficiently large to involve exchange among different sites. Although there are no zone center phonons with the antiphase displacements of the synchronized phase of the IQTPs, analogous to the neutron scattering, the correlated Cu–Oap motions could originate from a phonon not belonging to the Brillouin zone center. Utilizing the YBa$_2$Cu$_3$O$_7$ (YBCO)-type structure (figure 1(c)), the calculated atomic displacements are described by a symmetry-adapted coordinate belonging to the X2-irreducible representation at the X point of the tetragonal Brillouin zone, where only one branch of the $q = (0 1/2 0)$ phonon mode is involved. This off-zone-center phonon exhibits the same $d$-symmetry with respect to the centers of the CuO$_2$ squares as the extended synchronized regime for the IQTP’s, as shown in figure 7(b).

Summarizing, in a doped system with the appropriate combination of parameters and relative energies of the transition metal and oxygen states, the IQTPs are a natural outcome of coupling this phonon to the coordination chemistry of the transition metal ions and the preferred geometries associated with their valence. This arises from the necessary mix of ionic and covalent character in the Cu–O bond, that also has two species possessing metal-oxygen bonds separated by a barrier of requisite height and width. The partial localization of the charge causes the metal sites to adopt these geometries and bond lengths within the lattice. This would give the known sensitivity to the interatomic distances and angles [94, 95]. If the thermal motion of the Cu–O pair traverses this range, then instead of the potential being featureless it can evolve into the two-site distribution by developing distinct local minima at those locations corresponding to the preferred charge distributions and their bonding modes. Overall, these EXAFS results combined with the theoretical developments on the physics of synchronized IQTPs can provide a new route to the understanding of induced electron–electron interactions in high-temperature cuprate superconductors.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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