Confining phase in SUSY SO(12) gauge theory with one spinor and several vectors

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Abstract
We study the confining phase structure of \( \mathcal{N}=1 \) supersymmetric \( SO(12) \) gauge theory with \( N_f \leq 7 \) vectors and one spinor. The explicit form of low energy superpotentials for \( N_f \leq 7 \) are derived after gauge invariant operators relevant in the effective theory are identified via gauge symmetry breaking pattern. The resulting confining phase structure is analogous to \( N_f \leq N_c + 1 \) SUSY QCD. Finally, we conclude with some comments on the search for duals to \( N_f \geq 8 \) \( SO(12) \) theory.

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1 Introduction

Our understanding of non-perturbative nature in $\mathcal{N} = 1$ supersymmetric gauge theories has much progressed since the pioneering works of Seiberg and his collaborators [1, 2]. Especially, physics of confining phase in $\mathcal{N} = 1$ supersymmetric gauge theories was enriched (quantum deformed moduli space, “s-confinement”). These works have been also extended to the theories with various types of gauge groups and matter contents [3, 4, 5, 6]. Furthermore, these theories have recently been applied to construction of models with dynamical supersymmetry breaking or SUSY composite models.

In this paper, we study the confining phase in $\mathcal{N} = 1$ supersymmetric $SO(12)$ gauge theory with $N_f \leq 7$ vectors and one spinor. There are two motivations we are interested in this particular model. First, from the theoretical point of view, it will provide useful informations for finding the dual to $SO(N_c)(N_c > 10)$ with an arbitrary number of vectors and spinors. Although the duality of this class of models has only been generalized to $SO(10)$ [7], the known dualities have the following remarkable properties which are not contained in Seiberg’s duality [13]: 1. Chiral-Nonchiral duality. 2. Reducibility to the exceptional group ($G_2$) duality. 3. Simple and Semi-simple group duality (without “deconfinement”). 4. Identification of massive spinors and $Z_2$ monopoles under duality [4, 5, 6, 10, 11, 12, 14]. Therefore, it is natural to ask whether these properties exist in the duality for $SO(N_c)(N_c > 10)$ theory. However, looking for this dual seems to be highly non-trivial from the result of Ref. [7]. Cho [4] has already investigated in detail the confining phase of $SO(11)$ gauge theory with $N_f \leq 6$ vectors and a spinor and extracted some clues in search for duals. It is interesting enough to pursue further following the line of his argument in order to clarify the dual to $SO(N_c)(N_c > 10)$ theory. Second, as mentioned in the above paragraph, the theory under consideration may provide phenomenologically viable models with dynamical supersymmetry breaking or SUSY composite models.

This paper is organized as follows. In section 2, gauge invariant operators relevant to the low energy physics are identified. We derive the explicit form of low energy superpotentials for $N_f \leq 7$ in section 3. In the last section, summary and some comments on the search for duals to $N_f \geq 8$ $SO(12)$ theory are given.
2 The $SO(12)$ model

The model we consider has following symmetry groups

$$G = SO(12)_{\text{gauge}} \times [SU(N_f)_V \times U(1)_V \times U(1)_Q \times U(1)_R]_{\text{global}}$$

(1)

under which the superfields transform as

$$V^i_\mu \sim (12, \Box, 1, 0, 0),$$

(2)

$$Q_\alpha \sim (32, 1, 0, 1, 0)$$

(3)

and no tree level superpotential. Note that since each of the $U(1)$ symmetries in Eq.(1) are anomalous, the action is transformed as

$$S \rightarrow S - iC\alpha \int d^4x \frac{g^2}{32\pi^2} F \tilde{F},$$

(4)

where $C$ denotes the anomaly coefficient of the corresponding $U(1)_V SO(12)^2$, $U(1)_Q SO(12)^2$ or $U(1)_R SO(12)^2$ anomalies and $\alpha$ is a transformation parameter. If the theta parameter in the Lagrangian is shifted under these anomalous $U(1)$’s as $\theta \rightarrow \theta + C\alpha$, then anomalies can be cancelled. Recalling the relation

$$\left( \frac{\Lambda}{\mu} \right)^{b_0} = \exp \left( -\frac{8\pi^2}{g^2(\mu)} + i\theta \right),$$

(5)

where $b_0$ represents 1-loop beta function coefficient

$$b_0 = \frac{1}{2}[3\mu(\text{Adj}) - \sum_{\text{matter}} \mu(R)] = 26 - N_f,$$

(6)

and $\Lambda$ is the strong coupling scale of the theory, the spurion superfield $\Lambda^{b_0}$ is transformed as

$$\Lambda^{b_0} \sim (1, 1, 2N_f, 8, 12 - 2N_f).$$

(7)

Using these symmetries and holomorphy, we can easily fix the form of the dynamically generated superpotential $W_{\text{dyn}}$ for the small value of $N_f$ so that $U(1)_R$ charge of $W_{\text{dyn}}$ be 2 and $U(1)_V, U(1)_Q$ charges vanish. The results are summarized in Table 1. $N_f = 6$ case is special because R charge of $\Lambda^{b_0}$ vanishes, therefore we cannot construct

\footnote{We implicitly regard the 32 dimensional $SO(12)$ spinor as the projection $Q = P_- Q_{64}$ where $Q_{64}$ means the 64 dimensional spinor of $SO(13)$ and $P_- = \frac{1}{2}(1 - \Gamma_{13})$.}

\footnote{$\mu$ denotes quadratic Dynkin index defined as $\text{Tr} T^a(R)T^b(R) = \mu(R)\delta^{ab}$ ($T^a$: the generators of the group, $R$: representation which the superfield belongs to). We use the following values : $\mu(12) = 2$, $\mu(32) = 8$, $\mu(66) = 20$.}
Table 1: The form of dynamically generated superpotentials

| $N_f$ | $R(Λ_{b0})$ | $W_{dyn}$ |
|-------|-------------|-----------|
| 0     | 12          | $(Λ^{20}/Q^{8})^{1/6}$ |
| 1     | 10          | $(Λ^{25}/Q^{8}V^2)^{1/5}$ |
| 2     | 8           | $(Λ^{24}/Q^{8}V^4)^{1/4}$ |
| 3     | 6           | $(Λ^{23}/Q^{8}V^6)^{1/3}$ |
| 4     | 4           | $(Λ^{22}/Q^{8}V^8)^{1/2}$ |
| 5     | 2           | $Λ^{21}/Q^{8}V^{10}$ |
| 6     | 0           | $X(Q^{8}V^{12} - Λ^{20})$ |
| 7     | -2          | $Q^{8}V^{14}/Λ^{19}$ |

the dynamically generated superpotential. However, the classical constraint among matter superfields is modified by non-perturbative effects and this quantum constraint can be included in the superpotential by using the Lagrange multiplier superfield $X$ [1]. Therefore $N_f = 6$ case is analogous to $N_f = N_c$ SUSY QCD (quantum deformation of moduli space). Furthermore, $N_f = 7$ case is analogous to $N_f = N_c + 1$ SUSY QCD (“s-confinement”) [1] and $N_f \leq 5$ case is analogous to $N_f \leq N_c - 1$ SUSY QCD (runaway superpotential) [15].

In order to describe the low energy effective theory, we need to find gauge invariant operators which behave as the moduli space coordinate [1]. It is in general troublesome to do this task. However, if the gauge symmetry breaking pattern is known at generic points in the moduli space, one can easily identify these gauge invariant operators. We illustrate below how it works in the present model. The gauge symmetry breaking pattern we utilize is [17]

$$SO(12) \xrightarrow{<32>^*} SU(6) \xrightarrow{<12>^*} SU(5) \xrightarrow{<12>^*} SU(4) \xrightarrow{<12>^*} SU(3) \xrightarrow{<12>^*} SU(2) \xrightarrow{<12>^*} 1.$$ (8)

With this information in hand, counting degrees of freedom of gauge invariant operators is nothing but a group theoretical exercise. We display in Table 2 parton degrees of freedom, unbroken subgroups, eaten degrees of freedom by Higgs mechanism and hadron degrees of freedom. We are now in a position to construct gauge invariant operators explicitly. Before doing this, we need to notice that $SO(12)$ spinor product

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3Vacuum expectation values (VEVs) of these gauge invariant operators are in one to one correspondence to the solutions of D-flatness conditions [16]
Table 2: Degrees of freedom of independent gauge invariants

| \( N_f \) | Parton | Unbroken | Eaten | Hadron |
|---|---|---|---|---|
| 0 | 32 | SU(6) | 66 - 35 = 31 | 1 |
| 1 | 44 | SU(5) | 66 - 24 = 42 | 2 |
| 2 | 56 | SU(4) | 66 - 15 = 51 | 5 |
| 3 | 68 | SU(3) | 66 - 8 = 58 | 10 |
| 4 | 80 | SU(2) | 66 - 3 = 63 | 17 |
| 5 | 92 | 1 | 66 | 26 |
| 6 | 104 | 1 | 66 | 38 |
| 7 | 116 | 1 | 66 | 50 |

decomposes into the following irreducible representations

\[
32 \times 32 = [0]_A + [2]_S + [4]_A + [\bar{6}]_S,
\]

where \([n]\) represents rank-\(n\) antisymmetric tensor, subscripts “A” and “S” mean antisymmetry and symmetry under spinor exchange, and the tilde of the last term implies that the rank-6 tensor is self-dual. Since our model has only one spinor, gauge invariant operators can include \([2]_S\) and \([\bar{6}]_S\) in Eq.(9).

Taking this into account, we can construct gauge invariant composites as follows\(^4\):

\[
\begin{align*}
L &= \frac{1}{2!^2!} (Q^T \Gamma_{[\mu} \Gamma_{\nu]} CQ) (Q^T \Gamma_{[\mu} \Gamma_{\nu]} CQ) \sim (1; 1; 4N_f; 4R), \\
M^{(ij)} &= (V^\mu)^T V_\mu^j \sim (1; \quad ; -8; 2R), \\
N^{[ij]} &= \frac{1}{2} Q^T \Gamma^i \Gamma^j CQ \sim (1; \quad ; 2N_f - 8; 4R), \\
P^{[ijklmn]} &= \frac{1}{6!} Q^T \Gamma^i \Gamma^j \Gamma^k \Gamma^l \Gamma^m \Gamma^n CQ \sim (1; \quad ; 2N_f - 24; 8R), \\
R^{[ijklmn]} &= \frac{1}{6!} \epsilon^{\mu_1 \cdots \mu_2} (Q^T \Gamma_{\mu_1} \Gamma^\nu CQ) (Q^T \Gamma_{\mu_2} \Gamma_{\mu_3} \cdots \Gamma_{\mu_6} CQ) V_\mu^i V_\nu^j V_\kappa^k V_\lambda^l V_\mu^m V_\nu^n \sim (1; \quad ; 4N_f - 24; 10R),
\end{align*}
\]

where the square bracket means the antisymmetrization of the corresponding indices and \(\bar{\Psi}^i \equiv V^i_\mu \Gamma^\mu\) and we use here the following SO(12) Gamma matrices,

\(^4\)The representations and the charge in Eq.\((\mathbb{1})\) are those under Eq.\((\mathbb{1})\)
\[\Gamma_1 = \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_2 = -\sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_3 = 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_4 = -1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_5 = 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_6 = -1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_7 = 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_8 = -1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3\]
\[\Gamma_9 = 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3\]
\[\Gamma_{10} = -1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3\]
\[\Gamma_{11} = 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2\]
\[\Gamma_{12} = -1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1\]
\[\Gamma_{13} = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3\]

and \(C\) is a charge conjugation matrix.

In order to see that these gauge invariant operators are in fact the coordinates of the moduli space, one has to check whether the total degrees of freedom of these gauge invariant operators in eq (10) coincide with hadronic degrees of freedom. In Table 3,

| \(N_f\) | Hadron DOF | L | M | N | P | R | Constraints |
|--------|------------|---|---|---|---|---|-------------|
| 0      | 1          | 1 | 0 | 0 | 0 | 0 |             |
| 1      | 2          | 1 | 1 | 0 | 0 | 0 |             |
| 2      | 5          | 1 | 3 | 1 | 0 | 0 |             |
| 3      | 10         | 1 | 6 | 3 | 0 | 0 |             |
| 4      | 17         | 1 | 10| 6 | 0 | 0 |             |
| 5      | 26         | 1 | 15| 10| 0 | 0 |             |
| 6      | 38         | 1 | 21| 15| 1 | 1 | -1          |
| 7      | 50         | 1 | 28| 21| 7 | 7 | -14         |

Table 3: Hadron degree of freedom count

deep degrees of freedom are listed as a function of \(N_f\). For \(N_f \leq 5\), hadronic degrees of freedom and that of \(L, M, N, P, R\) agree with each other. For \(N_f = 6\), degrees of freedom of \(L, M, N, P, R\) are larger than those of hadrons by one. This implies that \(L, M, N, P, R\) are not independent and a single constraint among them exists. This statement is also consistent with the previous argument for dynamically generated superpotentials. For \(N_f = 7\), 14 constraints are expected to come from the equations of motion as in \(N_f = N_c + 1\) SUSY QCD.

We can obtain more non-trivial support which convinces us that gauge invariant operators \(L, M, N, P, R\) are moduli. One of the powerful methods to study the low energy spectrum is ’t Hooft anomaly matching \([18]\). To see that anomalies between elementary fields and composite ones match, we take anomaly free symmetry group instead of Eq.(11)

\[G_{AF} = SO(12)_{gauge} \times [SU(N_f) \times U(1) \times U(1)]_{global} \]

(11)
where new $U(1)$ and $U(1)_R$ are linear combinations of the original $U(1)$’s in Eq. (1). Matter superfields transform under Eq. (11) as

$$V_i^j \sim (12, \Box, -4, R),$$

$$Q_{\alpha} \sim (32, 1, N_f, R)$$

where $R = N_f - 6 / N_f + 4$. We can calculate anomalies and see that anomalies match for $N_f = 7$; $SU(7)^3 : 12A(\Box), SU(7)^2U(1)_R : -\frac{120}{11} \mu(\Box), SU(7)^2U(1) : -48 \mu(\Box), U(1)_R : -\frac{434}{11}, U(1) : -112, U(1)^2 : 120, U(1)_R U(1)^2 : -\frac{29120}{11}, U(1)_R^2 : -\frac{28154}{1331}, U(1)^3 : 5600,$

where $A(\Box)$ and $\mu(\Box)$ are cubic and quadratic Dynkin indices for fundamental representation of $SU(N_f)$. Recalling that anomalies are saturated in $N_f = N_c + 1$ SUSY QCD, this coincidence for $N_f = 7$ is very natural and gives a strong support that the theory in this case is in “s-confinement” phase.

For $N_f \geq 8$, it is impossible to satisfy anomaly matching conditions without violating $N_f \leq 7$ result even if other gauge invariant operators are added. This implies that a confining phase of this model terminates at $N_f = 7$.

### 3 Low energy superpotentials

In this section, we determine explicitly low energy superpotentials in terms of $L, M, N, P$ and $R$. Since we know which gauge invariant operators are moduli in the previous section, it is straightforward to work out what should be in the superpotential. Following Ref. [4], we first determine the quantum deformed constraint in $N_f = 6$ theory. Dimensional analysis, symmetries and holomorphy restrict the superpotential as follows

$$W_{N_f=6} = X(R^2 + P^2L + 2PPfN + L^2 \det M$$

$$+ \frac{1}{2!4!} \epsilon_{i1i2i3i4i5i6} \epsilon_{j1j2j3j4j5j6} LN^{i1j1}N^{i2j2}M^{i3j3}M^{i4j4}M^{i5j5}M^{i6j6}$$

$$+ \frac{1}{4!2!} \epsilon_{i1i2i3i4i5i6} \epsilon_{j1j2j3j4j5j6} N^{i1j1}N^{i2j2}N^{i3j3}N^{i4j4}M^{i5j5}M^{i6j6} - \Lambda_6^{20})$$

where $\Lambda_6$ is the strong coupling scale of $SO(12)$ gauge theory with 6 vector flavors and a spinor. Coefficients of each terms are determined so that it reproduce the superpotential in $SO(11)$ gauge theory with 5 flavors [4]. (If VEV $< V_{12}^6 > \neq 0$ is given, $SO(12)$ gauge theory with 6 flavors under consideration here reduces to $SO(11)$ gauge theory with 5 flavors)

One may also determine these coefficients by using the symmetry breaking along the spinor flat direction $SO(12) \overset{<32>}{\rightarrow} SU(6)$, explicitly $<32>^T = (0, a, 0, \cdots, 0, a, 0)$. 

6
$V_\mu$ decomposes into $6 + \bar{6}$ under $SU(6)$, which is explicitly as

$$
V_\mu = \begin{pmatrix}
q_1 + \bar{q}_1 \\
i(q_1 - \bar{q}_1) \\
q_2 + \bar{q}_2 \\
i(q_2 - \bar{q}_2) \\
q_3 + \bar{q}_3 \\
i(q_3 - \bar{q}_3) \\
q_4 + \bar{q}_4 \\
i(q_4 - \bar{q}_4) \\
q_5 + \bar{q}_5 \\
i(q_5 - \bar{q}_5) \\
q_6 + \bar{q}_6 \\
i(q_6 - \bar{q}_6)
\end{pmatrix}
$$

(15)

where $q_i, \bar{q}_i (i = 1, \cdots, 6)$ mean $SU(6)$ quarks, antiquarks, respectively. The reduced theory is $SU(6)$ gauge theory with $6(\Box + \bar{\Box})$, therefore it has a single quantum constraint [3].

According to this decomposition rule, $SO(12)$ gauge invariant operators are decomposed into the following $SU(6)$ meson $m^{ij}$, baryon $b$ and anti-baryon $\bar{b}$;

$$
\begin{align*}
L &\to 12a^4 \\
M^{ij} &\to 2(m^{ij} + m^{ji}) \\
N^{[ij]} &\to -4ia^2(m^{ij} - m^{ji}) \\
P &\to 64ia^2(b + \bar{b}) + 2ia^2\epsilon_{ijklmn}m^{ij}m^{kl}m^{mn} \\
R &\to 64a^4(b - \bar{b})
\end{align*}
$$

(16)

Using this information, one can also determine coefficients so that $\det m - b\bar{b} = \Lambda^{12}$ be reproduced.

The superpotential of $N_f = 7$ case can be found in a similar way. In this case, the superpotential must have the following features. 1. It is smooth everywhere on the moduli space. 2. Equations of motion give classical constraints among vectors and a spinor. 3. Adding mass term for one vector flavor to this superpotential and integrating out this massive vector, the superpotential (14) must be reproduced. The result is

$$
W_{N_f=7} = \frac{1}{\Lambda^{19}}(M^{ij}R_iR_j - 2iN^{ij}P_iR_j + LP_iP_jM^{ij})
$$

5Although this superpotential has already been derived in Ref. [3] by using the index argument, a PMN$^3$ term was missing. Without this term, the result of Ref. [4] cannot be correctly recovered.
For loop matching of gauge coupling as follows one flavor vector field has non-vanishing VEV.

\[ N \] superpotentials for (14) and integrating out each massive vectors successively, we can readily derive the correctly the superpotentials for 

\[ \delta W \]

By adding the mass terms for vector fields \( \delta W = m_{ij} M^{ij} \) to the superpotential (14) and integrating out each massive vectors successively, we can readily derive the superpotentials for \( N_f \leq 5 \) systematically. As a matter of fact, we obtain

\[
W_{N_f=5} = \frac{\Lambda_5^{21}}{(L^2 \text{det} M + \frac{1}{2!} LN_{11}^1 N_{12}^2 M_{13}^3 M_{14}^4 M_{15}^5 \epsilon_{111214i5} \epsilon_{1j1j2j3j4j5})^{1/2},}
\]

\[
W_{N_f=4} = 2 \left( \frac{\Lambda_4^{22}}{(L^2 \text{det} M + \frac{1}{2!} LN_{11}^1 N_{12}^2 M_{13}^3 M_{14}^4 \epsilon_{111214i5} \epsilon_{1j1j2j3j4j5})^{1/2}} \right)^{1/2},
\]

\[
W_{N_f=3} = 3 \left( \frac{\Lambda_3^{23}}{(L^2 \text{det} M + \frac{1}{2!} LN_{11}^1 N_{12}^2 N_{13}^4 \epsilon_{111214i5} \epsilon_{1j1j2j3j4j5})^{1/2}} \right)^{1/3},
\]

\[
W_{N_f=2} = 4 \left( \frac{\Lambda_2^{24}}{(L^2 \text{det} M + LN^2)^{1/4}} \right),
\]

\[
W_{N_f=1} = 5 \left( \frac{\Lambda_1^{25}}{(L^2 M)^{1/5}} \right),
\]

\[
W_{N_f=0} = 6 \left( \frac{\Lambda_0^{26}}{L^2} \right)^{1/6},
\]

where the strong coupling scales for each flavor are related to each other through one-loop matching of gauge coupling as follows

\[
\Lambda_0^{26} = m_{11} \Lambda_1^{25} = m_{11} m_{22} \Lambda_2^{24} = m_{11} m_{22} m_{33} \Lambda_3^{23} = \frac{m_{11} m_{22} m_{33} m_{44} \Lambda_4^{22} = m_{11} m_{22} m_{33} m_{44} m_{55} \Lambda_5^{21} = m_{11} m_{22} m_{33} m_{44} m_{55} m_{66} \Lambda_6^{20} = m_{11} m_{22} m_{33} m_{44} m_{55} m_{66} m_{77} \Lambda_7^{19}}. \quad (19)
\]

It is worth to note that one can confirm the above superpotentials (18) to recover correctly the superpotentials for \( N_f \leq 4 \) in SO(11) theory [4], which is obtained when one flavor vector field has non-vanishing VEV.

Before closing this section, we briefly discuss dynamical supersymmetry breaking. For \( N_f = 6 \), if we take the tree level superpotential as

\[
W_{\text{tree}} = \lambda_1 S_1 V^2 + \lambda_2 S_2 Q^2 V^6 + \lambda_3 S_3 Q^4 V^6 \quad (20)
\]
where $S_{1,2,3}$ are singlet superfields, then equations of motion with respect to $S_{1,2,3}$ and the quantum constraint (14) are incompatible. Therefore, supersymmetry is dynamically broken \[19\]. For $N_f = 0$, since we cannot add terms which lift a classical flat direction (i.e., $L$) preserving $U(1)_R$ to the tree level superpotential, supersymmetry remains unbroken. The same argument seems to be applicable for $1 \leq N_f \leq 5$ \[4\].

4 Summary

In this paper, we have studied the confining phase in $\mathcal{N} = 1$ $SO(12)$ SUSY gauge theory with $N_f \leq 7$ vectors and a spinor. Utilizing the gauge symmetry breaking pattern at generic points on the moduli space which plays a crucial role in our study, we have identified gauge invariant operators which behave as the moduli coordinate. Then we have derived explicitly low energy superpotentials for $N_f \leq 7$.

Some clues in search for duals are obtained from the results of this work. Let us suppose that the dual with the gauge group $\tilde{G}$ exists for $N_f \geq 8$. The original $SO(12)$ theory breaks down to $SU(6) + N_f (\oplus \Box)$ along the spinor flat direction. On the other hand, the gauge group of the dual theory is usually unbroken since in the dual theory the gauge invariant operators develop VEV. Since $SU(6) + N_f (\oplus \Box) (N_f \geq 8)$ theory is dual to $SU(N_f - 6) + N_f (\Box + \Box)$ \[13\], we can guess that at least $\tilde{G}$ must include $SU(N_f - 6)$ as a subgroup to preserve the duality along this direction. Furthermore, the superpotential in the dual theory must recover $N_f = 7$ superpotential. We also note that $N_f = 8$ case in our model is known to be self-dual \[20\].

Although it seems to be quite difficult to find a dual which is compatible with the above requirements, we hope that this work will provide useful informations to search for the dual to $SO(N_c)(N_c \geq 11)$ theory.

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\footnote{In Ref. \[3\], the authors construct a model with dynamical supersymmetry breaking for $N_f = 1$ by promoting a global $U(1)$ to a local $U(1)$ and adding singlets to cancel $U(1)$ gauge anomaly.}
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