PARADOXES IN STRANGE QUARK CONTRIBUTIONS TO HYPERON SPINS

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ABSTRACT

Data for magnetic moments and semileptonic decays show disagreements between experimental values and theoretical predictions not easily explained by simple models for Λ and Σ hyperons.

1. Introduction - A Strong Disagreement with SU(6)

Baryon magnetic moment data agree with the naive SU(6) quark model at the 15% level but are now good to a few per cent[1] and may provide clues to new physics. Detailed models have too many free parameters for significant tests against 8 or 9 experimental numbers. Instead we suggest investigating:
1) General features like the roles of the strange quarks in the Λ, Σ and proton;
2) New kinds of data like hyperon beam investigations of Primakoff excitation of Σ∗ and Ξ∗ resonances[2, 3]; 3) Lattice QCD results for spin structure of baryons [4] or exotic multiquark states like H dibaryon and Pentaquark[5].

This talk focuses on the contributions ∆s(Λ) and ∆s(Σ) of the strange quark in the Λ and Σ to the hyperon spin.

The SU(6) values ∆s(Λ)SU(6) = 1 and ∆s(Σ)SU(6) = −1/3 lead to predictions in disagreement with experiment[6] in opposite directions for weak decays and magnetic moments. Semileptonic decays give too large a value for the ratio ∆s(Σ)/∆s(Λ); magnetic moments give too low.

\[-1/3 = \frac{\Delta s(\Sigma)_{SU(6)}}{\Delta s(\Lambda)_{SU(6)}} = \frac{(G_A/G_V)_{\Sigma^{-}\rightarrow n}}{(G_A/G_V)_{\Lambda^{-}\rightarrow p}} = -0.473 \pm 0.026 \quad (YY1a)\]

\[-1/3 = \frac{\Delta s(\Sigma)_{SU(6)}}{\Delta s(\Lambda)_{SU(6)}} = \frac{\mu_{\Sigma^+} + 2\mu_{\Sigma^-}}{3\mu_{\Lambda}} = -0.06 \pm 0.02 \quad (YY1b)\]
This enormous discrepancy by a factor of $8 \pm 2$ is not easily overcome. The excellent agreement obtained\cite{7} for $\mu_\Lambda$ by assuming $\Delta s(\Lambda) = 1$ supports naive SU(6) for the $\Lambda$; $\mu_{\Sigma^\pm}$ disagree with SU(6). The excellent agreement with SU(6) for $(G_A/G_V)_{\Sigma^- \rightarrow n}$ supports naive SU(6) for the $\Sigma$; the $\Lambda$ decay disagrees with SU(6). These $\Lambda - \Sigma$ disagreements are very general; they do not assume flavor SU(3) symmetry and only consider states having nearly the same masses. We now examine the underlying physics.

2. A General Treatment of $\Sigma$ Magnetic Moments

We define $\mu_f$ as the most general one-body operator acting only on quarks and antiquarks of flavor $f$, including orbital and sea contributions. We write the $\Sigma$ moments as the expectation value of the sum of three terms, one for each flavor, each proportional to the quark charge, in the most general wave functions satisfying isospin invariance.

\[
\mu_{\Sigma^\pm} = \frac{2}{3} \mu_u - \frac{1}{3} \mu_d - \frac{1}{3} \mu_s |\Sigma^\pm\rangle = \frac{2}{3} \mu_d - \frac{1}{3} \mu_u - \frac{1}{3} \mu_s |\Sigma^\mp\rangle \quad (YY3)
\]

\[
\frac{\mu_{\Sigma^+} + 2\mu_{\Sigma^-}}{3\mu_{\Sigma^+}} = \frac{\langle \Sigma^+ | \mu^V_s + \mu^S_s - \mu^S_d |\Sigma^+\rangle}{3\mu_{\Sigma^+}} = 0.015 \pm 0.005 \quad (QQ4)
\]

where we have separated valence and sea contributions denoted by $\mu^V_f$ and $\mu^S_f$. Thus the contribution to $\mu_{\Sigma}$ of the valence strange quark is either anomalously low, only $1.5\%$ of $\mu_{\Sigma^+}$ as previously noted\cite{8}, or mysteriously canceled by an appreciable contribution from SU(3) breaking in the sea. This result is more general than previous similar treatments\cite{9}.

3. A General Treatment of Hyperon Semileptonic Decays

A previous analysis of semileptonic $n$, $\Lambda$, $\Sigma$ and $\Xi$ decay data\cite{10} showed the contrast between neutron and $\Lambda$ decays in strong disagreement with simple SU(6) predictions but both smaller by the same factor, and $\Sigma$ decays in striking agreement with SU(6), while large errors left $\Xi$ data within two standard deviations of both SU(6) predictions and consistency with $n$ and $\Lambda$.

The $\Lambda$ and $\Sigma$ decays are very simply described as a valence $s \rightarrow u$ quark transition with all remaining degrees of freedom including any combination of valence $d$ quarks, $q\bar{q}$ pairs, gluons and orbital angular momenta remaining inert spectators. Any $s\bar{s}$ pairs in the nucleon have a completely different momentum spectrum from the valence quarks and can play no active role in weak decays at zero momentum transfer. In the baryon rest frame the spectator total angular momentum has only two allowed values, zero and one and the $\Sigma$, $\Lambda$
and neutron wave functions can be written

\[ |B\rangle = \cos \theta^B |S^B_1; f\rangle + \sin \theta^B |S^B_o; f\rangle \]  \hspace{1cm} (QQ5)

where \( S^B_o \) and \( S^B_1 \), denote the spectator states in the baryon \( B \) wave function with angular momentum zero and one respectively and \( |S^B_k; f\rangle \) denotes a state in baryon \( B \) in which the angular momenta of the spin-\( k \) spectator and an active quark of flavor \( f \) are coupled to total angular momentum 1/2 and \( \theta^B \) denotes a “mixing angle” parameter defining the relative contribution of the two terms.

In both current and constituent quark models the decay is an \( s \to u \) transition with or without spin flip while leaving the spectators unchanged. The transition matrix element factorizes into the product of a weak \( s \to u \) matrix element and an overlap factor between the final baryon state and the state produced by the flavor and spin change of the active quark. The value of \( G_A/G_V \) for the transition is measured experimentally by the ratio of the flip to nonflip amplitudes.

\[
(G_A/G_V)_{B_i \to B_f} = \frac{\langle u \downarrow | H_{\text{weak}} | s \uparrow \rangle}{\langle u \uparrow | H_{\text{weak}} | s \uparrow \rangle}
\]

\[
\sin \theta^i \langle B_f \downarrow | u \downarrow | S^0_0 \rangle - (1/3) \cos \theta^i \langle B_f \downarrow | (S^1_1; u) \downarrow \rangle
\]

\[
\sin \theta^i \langle B_f \uparrow | u \uparrow | S^0_0 \rangle + \cos \theta^i \langle B_f \uparrow | (S^1_1; u) \uparrow \rangle
\]

The two overlap factors \( \langle B_f \downarrow | u \downarrow | S^0_0 \rangle \) and \( \langle B_f \uparrow | u \uparrow | S^0_0 \rangle \) are equal by rotational invariance, completely independent of the structure of the final state baryon \( B_f \), and similarly for \( \langle B_f \downarrow | (S^1_1; u) \downarrow \rangle \) and \( \langle B_f \uparrow | (S^1_1; u) \uparrow \rangle \). The ratio of the matrix elements of \( H_{\text{weak}} \) can be interpreted as the value of \( (G_A/G_V) \) at the quark level. Thus

\[
(G_A/G_V)_{B_i \to B_f} = -\frac{1}{3} \cdot (g_A/g_V)_{s \to u} \cdot \frac{1 - 3\xi}{1 + \xi} \]  \hspace{1cm} (QQ7a)

where

\[
\xi = \tan \theta^i \cdot \frac{\langle B_f \downarrow | u \downarrow | S^0_0 \rangle}{\langle B_f \downarrow | (S^1_1; u) \downarrow \rangle} \]  \hspace{1cm} (QQ7b)

The SU(6) results for \( (G_A/G_V)_{\Sigma^{-} \to n} \) and \( (G_A/G_V)_{\Lambda \to p} \) are seen to follow only from assuming that the spectator degrees of freedom in the hyperons are coupled to the SU(6) values 1 and 0 respectively, to give \( (\theta^{\Sigma} = 0; \theta^{\Lambda} \neq 0) \).
\( \theta^\Lambda = 90^\circ \) with no assumptions about the structures of the spectators nor the final state nucleon.

\[
(G_A/G_V)_{B_i \rightarrow B_f} = -(1/3)(g_A/g_V)_{s \rightarrow u} \quad \text{if } \sin \theta^i = 0 \quad (QQ8a)
\]

\[
(G_A/G_V)_{B_i \rightarrow B_f} = (g_A/g_V)_{s \rightarrow u} \quad \text{if } \cos \theta^i = 0 \quad (QQ8b)
\]

This highlights the difference between the agreement of \((G_A/G_V)(\Sigma^- \rightarrow n)\) with SU(6) and the disagreement of \((G_A/G_V)\Lambda \rightarrow p\).

The \( \Lambda \rightarrow p \) decay is particularly simple in any model where the strange quark carries the spin of the baryon and the other degrees of freedom including the nonstrange quarks are coupled to angular momentum zero. Then \( \cos \theta^\Lambda = 0 \), eq. (10b) is valid and comparison with experiment [6] gives

\[
(G_A/G_V)_{\Lambda \rightarrow p} = (g_A/g_V)_{s \rightarrow u} = 0.718 \pm 0.015 \quad (QQ9)
\]

Thus either the the value of \((G_A/G_V)\) at the quark level is reduced to the experimental value 0.72 or the \( \Lambda \) wave function with all the spin carried by the strange quark is not valid.

The \( \Sigma^- \rightarrow n \) decay is uniquely simple in all models where a single “active” \( s \) quark in the \( \Sigma^- \) turns into a \( u \) quark in the neutron, with all the remaining degrees of freedom including all the \( d \) quarks remaining inert spectators. The value of \((G_A/G_V)(\Sigma^- \rightarrow n)\) is given by eq. (QQ7a) for the most general wave functions (QQ5) with \( \xi \) given by substituting (QQ5) into (QQ7b). Comparing this value with experiment [6] gives

\[
(G_A/G_V)(\Sigma^- \rightarrow n) = -(1/3) \cdot (g_A/g_V)_{s \rightarrow u} \cdot \frac{1 - 3\xi}{1 + \xi} = -0.340 \pm 0.017 \quad (QQ10a)
\]

\[
\xi = \tan \theta^\Sigma \tan \theta^n \cdot \frac{\langle S_n^o | S_\Sigma^o \rangle}{\langle S_n^o | S_\Sigma^o \rangle} \quad (QQ10b)
\]

In the SU(3) symmetry limit \( \xi > 0 \) and is expected to remain positive even when SU(3) is broken. Thus the result (QQ10a) requires \((g_A/g_V)_{s \rightarrow u} \geq 1\), while the experimental result [6] \( G_A/G_V = 1.259 \pm 0.004 \) for the neutron beta decay is in strong disagreement with the SU(6) prediction \( G_A/G_V = 5/3 \) implies \((g_A/g_V)_{d \rightarrow u} \approx 3/4\).

An analogous treatment is not possible for \((G_A/G_V)\Lambda \rightarrow p\), where there are two active \( u \) quarks in the proton, and the wave function \( B_f \) cannot be written in the form (QQ5) which has only one active quark. The neutron decay is even more complicated, since there are two active \( d \) quarks in the neutron as well as two active \( u \) quarks in the proton, either pair of active quark spins can be coupled to spin zero or spin one, and there are many more free parameters depending upon the wave functions in the expression for \((G_A/G_V)(n \rightarrow p)\).
Why is the Σ different from all other baryons? The large value in agreement of experiment with SU(6) \((G_A/G_V)_{\Sigma^-\to n} = -(1/3)\) for the simplest weak decay where the prediction is least dependent upon wave function structure implies that the spin projection of the strange valence quark in the Σ is antiparallel to the hyperon spin and has the largest possible value. Yet the contribution of this strange quark spin to the Σ magnetic moment seems to be mysteriously suppressed by a large factor or cancelled by some other unknown contribution. There is no such suppression observed in the contributions to nucleon magnetic moments of the \(d\) quark in the proton and the \(u\) quark in the neutron, which are directly related by \(SU(3)\) to the strange quark contribution in the Σ, and all other weak decays seem to have \((G_A/G_V)\) suppressed by a factor of the order of 3/4.

4. Additional Input Obtainable from Other Electromagnetic Properties

Additional insight may be obtained from experimental data on other electromagnetic properties; e.g. charge radii, polarizabilities and \(B \to B^*\) transitions[3]. Any difference between the electromagnetic properties of the Σ and proton can only arise from flavor \(SU(3)\) symmetry breaking. The two Σ states are isospin mirrors but have very different electrical quark structures. The Σ, like the nucleon and \(\Xi^0\), has valence quarks of two flavors having + and - electric charge. External fields act in opposite directions on the two flavors and rotate spins in opposite directions, thereby producing internal excitation. The Σ and \(\Xi^-\), have three valence quarks all with charge -1/3. External fields act in the same direction on all three and rotate spins in the same direction, producing no internal excitation in the \(SU(3)\) symmetry limit, as in the well-known \(SU(3)\) U-spin selection rule[2] \(\Gamma(\Sigma^- \to \Sigma^{*-}) = \Gamma(\Xi^- \to \Xi^{*-}) = 0\). Broken-SU(3) sum rules [3] have been derived under the assumption that the contributions of the two quarks of the same flavor in nucleons and Σ’s are not changed by \(SU(3)\) symmetry breaking; e.g.

\[
\sqrt{\Gamma(\Sigma^- \to \Sigma^{*-})} = \sqrt{\Gamma(\Sigma^+ \to \Sigma^{*+})} - \sqrt{\Gamma(N \to \Delta)} \tag{QQ11a}
\]

\[
\langle r_c^2 \rangle_{\Sigma^+} + \langle r_c^2 \rangle_{\Sigma^-} = 2(\langle r_c^2 \rangle_{\Sigma^+} - \langle r_c^2 \rangle_{p}) - \langle r_c^2 \rangle_{n} \tag{QQ11b}
\]

where \(\langle r_c^2 \rangle_B\) denotes the mean square charge radius of baryon B. The sum rule (QQ11a) should hold separately for the \(M1\) and \(E2\) octet-decuplet transitions.
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