Holographic Thermalization in Quark Confining Background

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Abstract: We study holographic thermalization of a strongly coupled theory inspired by two colliding shock waves in a vacuum confining background. Holographic thermalization means a black hole formation, in fact a trapped surface formation. As a vacuum confining background we considered a well know bottom-up AdS/QCD model that provides the Cornell potential as well as reproduces QCD β-function. We perturb vacuum background by colliding domain shock waves, that are assumed to be holographically dual to heavy ions collisions. Our main physical assumption is that we can make a restriction on the time of a trapped surface formation that makes a natural limitation on the size of the domain where the trapped surface is produced. This limits the intermediate domain where the main part of the entropy is produced. In this domain one can use an intermediate vacuum background as an approximation to the full confining background. We have found that the dependence of multiplicity on energy for the intermediate background has an asymptotic expansion, which first term depends on energy as $E^{1/3}$, that is very similar to the experimental dependence of particles multiplicities on colliding ions energy obtained from RHIC and LHC. However, this first term, at the energies where the approximation of the confining metric by the intermediate works, does not saturate the exact answer and one has to take into account the non-leading terms.

Keywords: Holography and quark-gluon plasma, gauge-gravity correspondence
1 Introduction

QCD, which is the currently accepted theory of strong interactions, still has the well-known problems with description of a strong coupling phenomena. The physics of heavy ion collisions, in particular QGP formation, involves real-time strong coupled phenomena, that makes difficult to study these phenomena within standard QCD methods. In the recent years a powerful approach to QGP is explored. This method is based on a holographic duality between the strong coupling quantum field in $d$-dimensional Minkowski space and classical gravity in $d + 1$-dimensional anti-de Sitter space (AdS) \[1–3\]. In particular, there is a considerable progress in the holographic description of equilibrium QGP \[4\]. The holographic approach is also applied to non-equilibrium QGP. Within this holographic approach thermalization is described as a process of formation of a black hole in AdS.

The AdS/CFT correspondence is based on string theory and perfectly works for $\mathcal{N}=4$ SUSY Yang Mills theory, while the dual description of real QCD is unknown. A lot of efforts have been made in searching for holographic QCD from string theory, in particular, \[5–7\]. This approach is known as the "top-down" approach. Other approach, known as the "bottom-up" approach, is supposed to propose a suitable holographic QCD models from experimental data and lattice results \[8–15\]. Main idea of
this approach is using natural prescriptions of the general AdS/CFT correspondence try to recover non-perturbative QCD phenomena, in particular non-perturbative vacuum phenomena, finite temperature, high-dense and non-zero chemical potential phenomena.

The 5-dim metrics that reproduce the Cornell potential [16], as well as $\rho$-meson spectrum etc., have been proposed [10, 13, 14]. A so-called improved QCD (IHQCD) that is able to reproduce the QCD $\beta$-function has been constructed [15]. Thermal deformation of these backgrounds are intensively studied in the last years, see for review [4].

The problem of QGP formation is the subject of intensive study within holographic approach in last years, see [17, 18] and references therein. There is a considerable progress in understanding of the thermalization process from the gravity side as BH formation. Initially this process has been considered starting from the AdS background [19–25]. However the pure AdS background is unable to describe the vacuum QCD with quark confinement, as well it is not able to reproduce the QCD $\beta$-function. There are backgrounds that solve one, or even two of these problems. The first one has been solved in [10] (see also [13, 14]), where a special version of soft-wall has been proposed, and the $\beta$-function has been reproduced from IHQCD [12, 15].

To describe the thermalization it is natural to study deformations of these backgrounds. Suitable deformations of IHQCD by shock waves have been studied in [26, 27] and it has been shown that without additional assumptions IHQCD metric does not reproduce the experimental multiplicity dependence on energy. In [28] it has been noticed that holographic realization of the experimental multiplicity requires an unstable background.

The goal of this paper is to cover this gap and to show that the model that reproduces the Cornell potential at the same time can be used as a gravity background to give a correct energy dependence of multiplicities produced during a finite time. As a bonus of our approach we get a reasonable estimation for the thermalization time.

The paper is organized as follows. In Sect. 2.1 we remind confining metrics, that reproduce the Cornell potential. In Sect. 2.2 we remind the previous results concerning the multiplicities dependence on energy. In Sect. 2.3 we present the main formula for the size of trapped surfaces formed in collision of the domain walls. In Sect. 3 we consider a special metric that is faraway from the confining metrics, but gives a suitable entropy. We also notice that a restriction of the size of the trapped surface permits to determine the thermalization time. In Sect. 4 we show that the confining metric [14] can be approximated at intermediate values of the holographic coordinate $z$ by the metric considered in Sect. 3. As a result this gives for the entropy produced during a short time, $\tau_{\text{term}} \sim 0.25 fm$, an asymptotic expansion with the leading term which has
dependence on energy as $\sim E^{1/3}$. The same is true for metric [10].

2 Setup

2.1 Confining Backgrounds

It is well known that the AdS space does not reproduce the quark confinement. To reproduce quark confinement, in particular the appropriate glueball spectrum, Polchinski-Strassler [8] imposed the cut-off in the AdS space, "hard wall model". Another modifications of the AdS space, "soft wall models" [9], are related with the dilaton. In the bottom-up approach, the metric is usually taken to be

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2), \quad (2.1)$$

where $b^2(z)$ is some function usually taken to be the AdS in the UV zone (this leads to the Coulomb potential in the UV) and is deformed AdS in the IR. The deformation in the IR should be taken in such a way, that the quark-antiquark potential exhibits confinement.

The experimental model of potential which is used to fit lattice and experimental data [16] is usually taken to be the Cornell potential. In principle this potential should reproduce quarkonia spectrum, interpolating between one-gluon exchange in the UV and linear confinement in the IR.

The model proposed in [10] uses the following warp factor:

$$b^2(z) = \frac{L^2 h(z)}{z^2}, \quad h_{AZ} = e^{a z^2}, \quad a = 0.42 GeV^2. \quad (2.2)$$

In [11] it has been shown that this factor reproduces the static interquark potential obtained from $SU(3)$ lattice calculations [16].

In [14] the following modification

$$h(z) = \exp \left( -\frac{\sigma z^2}{2} \right) \left( \frac{z_{IR} - z}{z_{IR}} \right)^{c_0}, \quad \sigma = 0.34 GeV^2, \quad c_0 = 1, \quad z_{IR} = 2.54 GeV^{-1} \quad (2.3)$$

has been considered. This modification is in fact very close to the model [13] for $0 < z < 2 \text{ fm}$ and reproduces the Cornell potential and $\beta$-function.

In this paper we consider the modification (4.1), see below, of factor (2.2), that also fits the Cornell potential well.
2.2 Multiplicities

The experimental data for multiplicities in heavy-ion collisions at RHIC and LHC indicate [29]

$$M_{\text{exp}} \sim E^{0.3} + \ldots$$  \hspace{1cm} (2.4)

Multiplicities obtained for the simplest holographic calculation in conformal background with the $AdS_5$ metric [19–25],

$$M_{AdS_5}(E) \sim E^{2/3}$$  \hspace{1cm} (2.5)

are in fact worse than the Landau bound

$$M_{\text{Landau}}(E) \sim E^{1/2}.$$  \hspace{1cm} (2.6)

To improve the energy dependence of multiplicities Kiritsis and Taliotis [26] have proposed to use modifications of the $b$-factor. They have considered $b$-factors corresponding to conformal and non-conformal backgrounds. More precisely, they have considered the holographic point-like sources collision in dilaton models and got estimations for a variety of models (depending on the dilaton potential)

$$M_{a>1/3} \sim E^{\frac{2a+3}{3a+2}},$$  \hspace{1cm} (2.7)

$$M_{a\leq-1/3} \sim E^{\frac{2a+1}{3a}}.$$  \hspace{1cm} (2.8)

Note that they also used a perturbative QCD inspired UV cut-off. This modification provides log-corrections. Following [24], where the energy-dependent cut-off in the high-energy limit has been proposed, Kiritsis and Taliotis [26] have shown that this cut-off reduces powers in (2.7) and (2.8) as

$$M_{a>1/3} \sim E^{\frac{2a+3}{3a+1}},$$  \hspace{1cm} (2.9)

$$M_{a\leq-1/3} \sim E^{\frac{2a+1}{3a-1}}.$$  \hspace{1cm} (2.10)

Later, in [28] we have confirmed results (2.7), (2.8) considering the domain wall collision models that generalized the Lin-Shuryak model [30, 31] to non-conformal cases. In [28] we have also noticed that the model with the $b$-factor $b(z) = \frac{L_{\text{eff}}}{z}$ gives a more realistic bound

$$M_{p_{\text{ph-dilaton}}}(E) \sim E^{1/3}$$  \hspace{1cm} (2.11)

that is closer to (2.4). But the price for this modification is the phantom kinetic term for the dilaton. Note that we have not performed any UV cut-off in this model to get estimation (2.11).
2.3 Trapped Surface for Domain-wall Shock Waves

The equation for the domain-wall wave profile $\phi^w(z)$ in the space with the $b$-factor is

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^w(z) = -C\frac{\delta(z - z_*)}{b^3(z)}, \quad (2.12)$$

where $C$ is a dimensionless variable

$$C = \frac{16\pi G_5 E}{L^2}. \quad (2.13)$$

The solution of (2.12) is given as

$$\phi^w(z) = \phi_a \Theta(z_0 - z) + \phi_b \Theta(z - z_0), \quad (2.14)$$

where

$$\phi_a = C_a \int_{z_a}^{z_0} b^{-3}dz, \quad \phi_b = C_b \int_{z_b}^{z_0} b^{-3}dz. \quad (2.15)$$

The constants $C_a$ and $C_b$ can be represented in the form, see [28, 30]:

$$C_a = \frac{\int_{z_a}^{z_0} b^{-3}dz}{\int_{z_a}^{z_0} b^{-3}dz}, \quad C_b = \frac{\int_{z_a}^{z_0} b^{-3}dz}{\int_{z_a}^{z_0} b^{-3}dz}. \quad (2.16)$$

As has been mentioned in Introduction we consider the collision of two shock domain walls in 5-dimensional space time as a holographical model of heavy ion collisions in real 4-dimensional space time. The shock wave profile $\phi^w$ satisfies equation (2.12). The trapped surface formed in the wall-on-wall collision obeys equation (2.12) and special boundary conditions. From these boundary conditions we can find that the trapped surface is located in z-direction from some point $z_a$ to point $z_b$, such that $z_a < z_0 < z_b$.

The points $z_a$ and $z_b$ can be found from the following relations

$$\frac{C b^{-3}(z_a) \int_{z_a}^{z_0} b^{-3}dz}{\int_{z_a}^{z_0} b^{-3}dz} = 1, \quad \frac{C b^{-3}(z_b) \int_{z_a}^{z_0} b^{-3}dz}{\int_{z_a}^{z_0} b^{-3}dz} = -1. \quad (2.17)$$

Relations (2.17) guaranty that the trapped surface forms and the trapped surface is located between $z_a$ and $z_b$ and the collision point $z_0$ is located between $z_a$ and $z_b$, $z_a < z_0 < z_b$ (see details in [28, 30]).

From (2.17) we get

$$\frac{C}{2} = b^3(z_a) + b^3(z_b), \quad (2.18)$$
\[ F(z_a) = \frac{b^{-3}(z_a)F(z_b) + b^{-3}(z_b)F(z_a)}{b^{-3}(z_a) + b^{-3}(z_b)}, \quad (2.19) \]

where
\[ \int_{z_i}^{z_j} b^{-3}dz = F(z_j) - F(z_i) \quad (2.20) \]

and
\[ z_a < z_* < z_b. \quad (2.21) \]

There is the following formula for the entropy density \([28]\) of the trapped surface
\[ s = \frac{S_{\text{trap}}}{\int d^2x_\perp} = \frac{1}{2G_5} \int_{z_a}^{z_b} b^3 dz. \quad (2.22) \]

## 3 Intermediate Background

### 3.1 Entropy

In this section we consider the metric (2.1) with \(b\)-factor
\[ b = b_1 \equiv \left( \frac{L_{\text{eff}}}{z} \right)^{1/2}. \quad (3.1) \]

The entropy dependence on the energy can be read from the formula
\[ s_1(z_a, z_b) = \frac{L_{\text{eff}}}{G_5} \left( \left( \frac{L_{\text{eff}}}{z_a} \right)^{1/2} - \left( \frac{L_{\text{eff}}}{z_b} \right)^{1/2} \right), \quad (3.2) \]

where
\[ \frac{z_a}{z_b} = \left( \frac{C}{2} \frac{z_b}{L_{\text{eff}}} \right)^{3/2} - 1 \quad (3.3) \]

Substituting (3.3) into (3.2) we get
\[ s_1(C, z_b) = \frac{L_{\text{eff}}}{G_5} \left( \frac{L_{\text{eff}}}{z_b} \right)^{1/2} \left[ \left( \frac{C}{2} \left( \frac{z_b}{L_{\text{eff}}} \right)^{3/2} - 1 \right)^{1/3} - 1 \right]. \quad (3.4) \]

One can perform the large \(C\) expansion in formula (3.4) to get
\[ s_1(C, z_b) = \frac{L_{\text{eff}}}{G_5} \left( \frac{C}{2} \right)^{1/3} - \left( \frac{L_{\text{eff}}}{z_b} \right)^{1/2} - \frac{1}{3} \left( \frac{2}{C} \right)^{2/3} \left( \frac{L_{\text{eff}}}{z_b} \right)^{3/2} + \ldots \quad (3.5) \]
i.e. the zeroth, first and second approximations are given by

\[
s_1^{(0)}(C) = \frac{L_{\text{eff}}}{G_5} \left( \frac{C}{2} \right)^{1/3},
\]

\[
s_1^{(1)}(C, z_b) = \frac{L_{\text{eff}}}{G_5} \left( \left( \frac{C}{2} \right)^{1/3} - \left( \frac{L_{\text{eff}}}{z_b} \right)^{1/2} \right),
\]

\[
s_1^{(2)}(C, z_b) = \frac{L_{\text{eff}}}{G_5} \left( \left( \frac{C}{2} \right)^{1/3} - \left( \frac{L_{\text{eff}}}{z_b} \right)^{1/2} - \frac{1}{3} \left( \frac{2}{C} \right)^{2/3} \left( \frac{L_{\text{eff}}}{z_b} \right)^{3/2} \right). \tag{3.7}
\]

The dependence of entropy \( s_1(C, z_b) \) on \( C \) at fixed \( z_b \) is presented by the green line in Fig.1.A. The approximation \( s_1^{(0)}(C) \) at large \( C \) has behavior \( C^{1/3} \) and it is shown by the black line in Fig.1.A. Note that due to relation (2.13) \( C \sim E \). More precisely, taking \( G_5 = 44.83 \text{ fm}^3 \) and \( L_{\text{eff}} = 4.4 \text{ fm} \) we get \( C = 580 E/\text{GeV} \) and the range of variation \( C \) at Fig.1.A corresponds to the energy around 0.1 TeV. Therefore, one can say that at large energies \( s_1(E, z_b) \sim E^{1/3} \) that in fact is rather close to the experimental dependence \( \sim E^{0.3} \).

However relation (3.3) written in the form

\[
\left( \frac{L_{\text{eff}}}{z_a} \right)^{3/2} + \left( \frac{L_{\text{eff}}}{z_b} \right)^{3/2} = \frac{C}{2},
\]

may give a restriction on possible variation range of energy. Indeed, by the construction \( z_a < z_b \) and to get a large value of \( C \) at fixed \( z_b \) one has to take a small \( z_a \). In the case of an additional restriction on \( z_a \), say \( z_a > z_{a,\text{min}} \), we get a restriction \( C < C_{\text{max}} \). It may happen that in this area of \( C \) we cannot restrict ourself by the zeroth approximation to \( s_1(C, z_b) \). Few examples of such behavior are presented in Fig.1.B. In Fig.1.B the energy dependencies of the exact entropy \( s_1(C, z_b) \) and approximated entropies \( s_1^{(1)}(C, z_b) \) and \( s_1^{(2)}(C, z_b) \) for different \( z_b \) are shown for relatively small values of \( C \). Fig.1.B shows that in the considered regions of \( C \) we have to take into account the first three terms of the approximation (3.5). The choice of \( L_{\text{eff}} = 20.7 \text{ fm} \) in Fig.1.B will be claire in Sect.4.

### 3.2 Thermalization Times

We estimate the thermalization time by a characteristic size of the trapped surface, i.e.

\[
\tau_{\text{therm}} \sim \frac{z_b - z_a}{2.4}. \tag{3.9}
\]

We put the factor 2.4 taking into account the relation between the interquark distance \( x \) and the string maximum holographic coordinate \( z_m \). The dependence of the interquark
Figure 1. A. The black line represents $s_1^{(0)}(C)$ and the green one represents $s_1(C, z_b)$ at $z_b = 1.7 \text{ fm}$. Here we take $L_{eff} = 4.4 \text{ fm}$ and $G_5 = 44.83 \text{ fm}^3$. B. The dependence of the entropy $s_1(C, z_b)$ on $C$ for $z_b = 1.7 \text{ fm}$ (blue thick line) and $z_b = 1.3 \text{ fm}$ (blue thin line). Approximations $s_1^{(1)}(C, z_b)$ (khaki lines) and $s_1^{(2)}(C, z_b)$ (green lines) for $z_b = 1.7 \text{ fm}$ (thick lines) and $z_b = 1.3 \text{ fm}$ (thin lines). Here $L_{eff} = 20.7 \text{ fm}$. For thick lines $z_a$ varies from $1 \text{ fm}$ to $1.7 \text{ fm}$, for thin lines $z_a$ varies from $0.6 \text{ fm}$ to $1.3 \text{ fm}$.

distance $x$ on the maximum of string $z$-coordinate, $z_m$, is given by

$$x = \int_0^{z_m} \frac{2}{\sqrt{b_4(z)} - 1} dz,$$

(3.10)

see for details for example [13]. For metric (2.1) with $b_1$ given by (3.1) this dependence is presented in Fig.2 by the solid magenta line and we see that $x = z_m/2.4$.

Note that the formula (3.9) is written using general causal arguments. We assume that the time of formation of an object extended along the holographic direction from $z_a$ to $z_b$ is the same as the time formation of an extended object along x-direction with a characteristic scale $\Delta x = (z_b - z_a)/2.4$. This is in accordance with (3.10). Formation of such object can be performed no faster as $\Delta x$.

The dependence of the thermalization time on $C$ for a given value of $z_b$ can be estimated substituting $z_a$ from (3.3) into the right hand site of (3.9). In Fig.3 the dependence of the thermalization time on $C$ for different values of $z_b$ is presented. To vary $C$ we vary $z_a$. In the plot different domains of $z_a$ are shown by lines with different thickness. Small values of $z_a$ correspond to large values of the energy.
Figure 2. Dependence of the interquark distance $x$ on the string maximum holographic coordinate $z_m$ for metric with the factor $b_1(z)$ (solid magenta line) and for the metric $b_2(z)$ (dashed blue line).

Figure 3. The dependence of the thermalization time on $C$ for different values of $z_b$ ($z_b = 1.7\, fm$ – coral lines, $z_b = 1.3\, fm$ – cyan lines). Different range of variation of $z_a$ are shown by lines with different thickness.
4 Intermediate Background as a Part of Confining Background

In this section we consider the metric (2.1) with confining factor $b(z)$

$$b(z) = b_2(z) \equiv \frac{Le^{\frac{z^2}{4}}\sqrt{1+g^2}}{z}, \quad g = -0.02 \text{GeV}. \quad (4.1)$$

A schematic picture of the bulk scales is presented in Fig.4.

![Figure 4. Schematic picture of the bulk scales.](image)

Fig.5 shows that for $L = 4.4\text{fm}$ at the region of intermediate holographic coordinate $z$, $1.3 \text{fm} = z_{UV} < z < z_{IR} = 1.8 \text{fm}$ the factor $b_1$ with $L_{eff} = 20.86 \text{fm}$ coincides with $b_2$ up to 3% and for $1.4 \text{fm} < z < 1.7 \text{fm}$ these factors are almost coincided. Instead of (4.1) we can use the $b$-factor (2.2) from [10]. This leads to a slight variation of parameters $L_{eff}$, $z_{IR}$ and $z_{UV}$, see Fig.6.

Note that the metric with $b$-factor $b_1$ leads to the potential of interquark interaction of the form $V(r) \sim A \log \left( \frac{r}{r_0} \right)$, where $A$ and $r_0$ are some constants. This form of the potential was suggested as the simple model to fit identical spin-averaged charmonia and bottomonia level splitting, see [16] and refs therein.

The dependence of the interquark distance $x$ on the string maximum $z$-coordinate $z_m$ for metric (2.1) with confining factor $b_2$ is presented in Fig.2 by dashed blue line. Note that at $z \approx 2.2 \text{fm}$ there is the string breaking, that is in accordance with [32]. This point is out of our intermediate zone.

In formula for entropy (2.22) we take the usually accepted value $G_5 \approx 44.83 \text{fm}^3$ [19].
Figure 5. A. $b$-factors $b_1(z)$ and $b_2(z)$. Here $L = 4.4 \text{ fm}$ and $L_{\text{eff}} = 20.86 \text{ fm}$. B. The same $b$-factors as in A in the intermediate region $1.2 \text{ fm} < z < 1.8 \text{ fm}$. Solid magenta lines correspond to $b_1 = b_1(z)$, dashed blue lines correspond to $b_2 = b_2(z)$.

Figure 6. Factors $b_1(z)$ with $L_{\text{eff}} = 20.7 \text{ fm}$ (solid magenta line) and $b(z)$ given by (2.2) (dashed blue line) in the intermediate region $1.2 \text{ fm} < z < 1.8 \text{ fm}$.

In Fig.7 A, the entropy dependence on the energy is presented for the confining metric (dashed line) and $b_1^2 = \frac{L_{\text{eff}}}{z}$ (solid line). For both lines $z_b = 1.8 \text{ fm}$ and $z_a$ varies from $z_a = 1.2 \text{ fm}$ to $z_a = 1.8 \text{ fm}$. We see that the energy dependence of entropies are very close in this intermediate region $1.2 \text{ fm} < z < 1.8 \text{ fm}$. This intermediate region according (3.9) corresponds to the thermalization time $\tau_{\text{therm}} \approx 0.25 \text{ fm}$.

Let us note, that our assumption about restriction of area of the trapped surface formation and consideration here instead of the confining metric with $b$-factor (4.1) the non-confining one with $b$-factor (3.1), that in the asymptotic regime gives the desirable energy dependence of entropy, is similar in some sense to a proposal to use the energy-dependent cut-off in the high-energy limit [24]. We can also say that our estimations give an analytical realization of this proposal.

However, if we consider a wider region for possible values of $z_a$ and consider for example $z_a \to 0$, that corresponds to large energies, we get different entropy behavior in these two models. The model with factor $b_2$ has a typical behaviour $s(C) \sim C^{2/3}$.
[19, 31] and the model with factor $b_1$ has $s(C) \sim C^{1/3}$ at large $C$ [27]. This means that without change of asymptotics of the b-factor at the UV region we cannot change behavior of $s(C)$ at large energies. From other point of view the UV asymptotics of the b-factor is fixed by the Coulomb potential [1]. This lead us to a modification of the holographic scenario and consideration at small $z$ an anisotropic background, where we can expect that the most part of the entropy for large energy is produced [33].

![Figure 7](image)

**Figure 7.** A. The entropy dependence on the energy for the confining metric (dashed line) and $b_1^2 = \frac{L_{\text{eff}}}{z}$ (solid line). For both lines $z_b = 1.8 \text{ fm}$ and $z_a$ varies from $z_a = 1.2 \text{ fm}$ to $z_a = 1.8 \text{ fm}$. B. The thermalization time dependence on the energy. The dashed line corresponds to the confining metric, the solid line corresponds to $b_1^2 = \frac{L_{\text{eff}}}{z}$. $z_b$ and $z_a$ are the same as in A.

### 5 Conclusion

Our calculations show that, within the holographic model of heavy-ions collisions using the confining vacuum background and colliding domain shock waves, the produced entropy has an asymptotic expansion, the first term of which provides a suitable dependence on the energy $s_2^{(0)} \sim E^{1/3}$. However, the entropy produced during a time $\sim 0.25 \text{ fm}$ after colliding of two shock domain walls in the confining background cannot be saturated by the first term and contributions of non-leading terms have to be taken into account. This is related with the fact that to restrict our asymptotic expansion by the first term we have to consider the asymptotic expansion at large energies. Large energies correspond to small values of the holographic coordinate $z$, where our approximation of the metric with $b_2$ factor (4.1) by the metric with $b_1$ factor (3.1) fails. From the other hand, as mentioned in the text, we cannot change the asymptotic of $b_2$ since it is related with the Coulomb potential.

It seems that one of possible resolutions of the problem is a change of the scenario of the isotropic holographic thermalization to an anisotropic short time holographic thermalization scenario. This scenario assumes that the main part of multiplicity is...
produced in an anisotropic regime and this part of multiplicity can be estimated by the trapped surface produced under a collision of the two shock waves in an anisotropic background. This scenario is accepted in the recent paper [33], where collisions of shock waves in the Lifshitz-like background have been considered.

It would be interesting to compare our estimation of the thermalization time with thermalization time estimations given by the Vaidya confining bulk metric, as well the thermalization time obtained in holographic hard wall model using the homogeneous injection of the energy [34].

Note, that our consideration may have applications not only for heavy-ions collisions, but also in studies of thermalization process in a broader class of strongly correlated multi-particle systems.

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