Vanishing Higgs Quadratic Divergence in the Scotogenic Model and Beyond

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Abstract

It is shown that the inherent quadratic divergence of the Higgs mass renormalization of the standard model may be avoided in the well-studied scotogenic model of radiative neutrino mass as well as other analogous extensions.
In quantum field theory, the additive renormalization of $m^2$ for a scalar field of mass $m$ is a quadratic function of the cutoff scale $\Lambda$. The elegant removal of this quadratic divergence is a powerful theoretical argument for the existence of supersymmetric particles. However, given the recent discovery of the 126 GeV particle [1, 2] at the Large Hadron Collider (LHC), presumably the long sought Higgs boson of the standard model, and the nonobservation of any hint of supersymmetry, it may be a good time to reexamine an alternative solution of the quadratic divergence problem.

It was suggested a long time ago [3] that in the standard model of quarks and leptons, the condition

$$\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{3}{4}m_H^2 = \sum_f N_f m_f^2,$$  

(1)

where $N_f = 3$ for quarks and $N_f = 1$ for leptons, would make the coefficient of the $\Lambda^2$ contribution to $m_H^2$ vanish. This would predict $m_H = 316$ GeV, which we now know to be incorrect.

The same idea may be extended to the case of two Higgs doublets [4, 5, 6, 7] where $\langle \phi_{1,2} \rangle$ = $v_1$, $v_2$, with $v = \sqrt{v_1^2 + v_2^2} = 174$ GeV. In that case, the vanishing of quadratic divergences would also depend on how $\Phi_{1,2}$ couple to the quarks and leptons. In the scotogenic model of radiative neutrino mass [8], there are two scalar doublets ($\phi^+, \phi^0$) and ($\eta^+, \eta^0$), distinguished from each other by a discrete $Z_2$ symmetry, under which $\Phi$ is even and $\eta$ odd. Thus only $\phi^0$ acquires a nonzero vacuum expectation value $v$. This same discrete symmetry also prevents $\eta$ from coupling to the usual quarks and leptons, except for the Yukawa terms

$$L_Y = h_{ij}(\nu_i \eta^0 - l_i \eta^+) N_j + H.c.,$$  

(2)

where $N_j$ are three neutral singlet Majorana fermions odd under $Z_2$. As a result, neutrinos obtain one-loop finite radiative Majorana masses as shown in Fig. 1. This is a well-studied model which also offers $\sqrt{2}Re(\eta^0)$ as a good dark-matter candidate [9]. The lightest $N$
Figure 1: One-loop generation of neutrino mass with $Z_2$ symmetry.

may also be a dark-matter candidate [10] but is more suitable if the dark-matter discrete symmetry $Z_2$ is extended to $U(1)_D$ as proposed recently [11].

The scalar potential of the scotogenic $Z_2$ model is given by [8]

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2$$

$$+ \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \frac{1}{2} \lambda_5 [(\Phi^\dagger \eta)^2 + (\eta^\dagger \Phi)^2].$$

(3)

Let $\phi^0 = v + H/\sqrt{2}$ and $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, then

$$m^2(H) = 2\lambda_1 v^2,$$

(4)

$$m^2(\eta^\pm) = m_2^2 + \lambda_3 v^2,$$

(5)

$$m^2(\eta_R) = m_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)v^2,$$

(6)

$$m^2(\eta_I) = m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)v^2.$$  

(7)

The corresponding two conditions for the vanishing of quadratic divergences are

$$\frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \frac{3}{4} m_H^2 + (\lambda_3 + \frac{1}{2} \lambda_4)v^2 = 3m_t^2,$$

(8)

$$\frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \left(\frac{3}{2} \lambda_2 + \lambda_3 + \frac{1}{2} \lambda_4\right)v^2 = \sum_{i,j} h_{ij}^2 v^2.$$  

(9)

Consequently, the following two sum rules are obtained:

$$\lambda_3 + \frac{1}{2} \lambda_4 = \frac{3}{v^2} \left(m_t^2 - \frac{1}{2} M_W^2 - \frac{1}{4} M_Z^2 - \frac{1}{4} m_H^2\right) = 2.063,$$

(10)
where \(3h^2 = \sum_{i,j} h_{ij}^2\). Since \(\lambda_2\) must be positive, Eq. (11) cannot be satisfied without the Yukawa couplings of Eq. (2). In other words, the existence of \(N\), hence the radiative generation of neutrino mass, is necessary for this scenario. In a model with simply a second “inert” scalar doublet \([12, 13]\), vanishing quadratic divergence will not be possible. To test Eq. (10), Eqs. (5) to (7) may be used, i.e.

\[
2\lambda_4 v^2 = m^2_R + m^2_I - 2m^2_+.
\] (12)

As for \(\lambda_3\), it may be extracted \([14, 15]\) from \(H \rightarrow \gamma\gamma\) using also \(m_+\). However Eq. (11) is very difficult to test, because \(h^2\) and \(\lambda_2\) are not easily measurable.

Analogous extensions of the scotogenic model may also accommodate vanishing quadratic divergences. As an example, consider the addition of a charged scalar \(\chi^+\) odd under \(Z_2\), then the electron may acquire a radiative mass by assigning \(e_R\) to be odd with the Yukawa couplings \(f \bar{e}_R N_L \chi^-\) as shown in Fig. 2, where \(N_L\) is even under \(Z_2\), but the soft Dirac mass term \(\bar{N}_L N_R\) breaks \(Z_2\) explicitly. With the addition of \(\chi^+\), the scalar potential has the extra terms

\[
\begin{align*}
V' &= m_3^2 \chi^+ \chi^- + \frac{1}{2} \lambda_6 (\chi^+ \chi^-)^2 + \lambda_7 (\chi^+ \chi^-)(\Phi^\dagger \Phi) + \lambda_8 (\chi^+ \chi^-)(\eta^\dagger \eta) \\
&+ [\mu (\eta^+ \phi^0 - \eta^0 \phi^+)] \chi^- + H.c.].
\end{align*}
\] (13)
The conditions for vanishing quadratic divergence in this model are then:

\[
\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \frac{3}{4}m_H^2 + (\lambda_3 + \frac{1}{2}\lambda_4 + \frac{1}{2}\lambda_7)v^2 = 3m_t^2, \tag{14}
\]

\[
\frac{3}{2}M_W^2 + \frac{3}{4}M_Z^2 + \left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4 + \frac{1}{2}\lambda_8\right)v^2 = \sum_{i,j} h_{ij}^2 v^2, \tag{15}
\]

\[
3(M_Z^2 - M_W^2) + (\lambda_6 + \lambda_7 + \lambda_8)v^2 = f^2 v^2. \tag{16}
\]

Again, verification is possible, at least in principle. Other more involved scenarios such as the scotogenic $U(1)_D$ model \[11\] or that of a recent proposal \[16\], where all quark and lepton masses are radiative with either $Z_2$ or $U(1)_D$ dark matter, may also have similar viable solutions.

I thank Maria Krawczyk for discussions at Scalars 2013. This work is supported in part by the U. S. Department of Energy under Grant No. de-sc0008541.

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