Updated Combined Fit of Low Energy Constraints
to Minimal Supersymmetry

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Abstract

The new precise LEP measurements of $\alpha_S$ and $\sin^2 \theta_W$ as well as the
new LEP II mass limits for supersymmetric particles and new calculations
for the radiative (penguin) decay of the $b$-quark into $s\gamma$ allow a further
restriction in the parameter space of the Constrained Minimal Supersym-
metric Standard Model (CMSSM).

1 Introduction

In this paper we repeat our previous fit of the supergravity parameters[1] with the
new input data from LEP concerning the coupling constants and new higher order
calculations for the $b \rightarrow s\gamma$ decay rate branching ratio[2]. The latter indicate that
next to leading log (QCD) corrections increase the SM value by about 10%. This
can be simulated in the lowest level calculation by choosing a renormalization
scale $\mu = 0.57m_b$, which is done in the following. In addition, a new preliminary
measurement of the $b \rightarrow s\gamma$ rate has been given by the ALEPH Coll.[3]: $BR = (3.38 \pm 0.74 \pm 0.85) \times 10^{-4}$, which is slightly higher than the previously measured
value by the CLEO Coll.[4]: $BR = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$. Both are consistent
with the SM expectation $BR = (3.52 \pm 0.33) \times 10^{-4}$. This value was calculated
for $\alpha_s = 0.122$, which is the best fit value to the electroweak data from LEP, thus
excluding the SLC value of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, which is inconsistent with the present Higgs
limit of 79 GeV[5], as shown in fig. 1. It remains to be seen, if the SLC value
is a statistical fluctuation or due to a systematic uncertainty. For the moment
we have taken the LEP values for the coupling constants ($\alpha_s = 0.122 \pm 0.003$
and $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.2319 \pm 0.00029$), which are slightly higher than the fit values
from the combined LEP and SLC data ($\alpha_s = 0.120 \pm 0.003$ and $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.2315 \pm 0.0004$) and lead to different unification conditions, as shown in fig. 2.
Figure 1: The electroweak mixing angle versus Higgs mass. The diagonal band corresponds to the SM prediction; the width of the band is determined by the uncertainty in the top mass. The point at $m_h = 14.6^{+28.6}_{-14.6}$ GeV corresponds to the SLC value of the mixing angle, while the point labeled LEP corresponds to the LEP value and the intermediate point corresponds to the combined data.

In addition to the statistical errors on $\sin^2 \Theta_{\text{eff}}$, we added in quadrature the error from the uncertainty in the QED coupling constant at $M_Z$, which is 0.0026 and arises mainly from the uncertainty of the hadronic vacuum polarization. We did not use the world average of $\alpha_s = 0.118$, but used only the LEP value, for which the 3rd order QCD corrections have been calculated and found to be small, so the uncertainties from the higher order corrections are small.

In addition LEP has provided a new chargino mass limit around 90 GeV in case of a gaugino-like chargino and a heavy sneutrino [6], which is exactly the MSSM case considered here: ($\mu > m_{1/2}$ and $m_{\tilde{l}} > 100$ GeV). The Higgs mass limit is now 79 GeV at 95% C.L. for the SM Higgs [5], which corresponds to the CMSSM case too, since all the other Higgses are heavy and decouple in the CMSSM limit considered here.

2 Results

The fitted supergravity parameters are mainly sensitive to the following input data: The GUT scale $M_{\text{GUT}}$ and coupling constant $\alpha_{\text{GUT}}$ are determined from gauge coupling unification, the Yukawa couplings $Y_t$, $Y_b$, $Y_{\tau}$ at the GUT scale from
Figure 2: The values of $\alpha_s$ and $\sin^2 \Theta$ yielding unification for $m_0 = 200$, $m_{1/2}=400$ and $m_0 = 1000, m_{1/2} = 1000$ GeV. The upper curve fits better the LEP couplings, while the lower curve fits better the combined data of LEP and SLC.

The masses of the third generation, $\mu$ from electroweak symmetry breaking and $\tan \beta$ from $b\tau$-unification. Of course, in a $\chi^2$-fit all parameters are determined simultaneously, thus taking all correlations into account. Since $m_0$ and $m_{1/2}$ enter in all observables, and are strongly correlated, we perform the fit for all combinations of $m_0$ and $m_{1/2}$ between 100 GeV and 1 TeV in steps of 100 GeV.

The most restrictive constraints are the gauge coupling constant unification and the requirement that the unification scale has to be above $10^{15}$ GeV from the proton lifetime limits, assuming decay via s-channel exchange of heavy gauge bosons. They exclude the SM as well as many other models.

The requirement of bottom-tau Yukawa coupling unification strongly restricts the possible solutions in the $m_t$ versus $\tan \beta$ plane. For $m_t = 175.6 \pm 5.5$ GeV only two regions of $\tan \beta$ give an acceptable $\chi^2$ fit, as shown in fig. 3. $Y_t$ is left free independent of $Y_b = Y_\tau$.

In fig. 4 the total $\chi^2$ distribution is shown as a function of $m_0$ and $m_{1/2}$ for the two values of $\tan \beta$ determined above. One observes minima at $m_0$, $m_{1/2}$ around (200,400) and (1000,1000), as indicated by the stars. The contours in the lower part show the regions excluded by the different constraints used in the analysis.

The requirement that the LSP is neutral excludes the regions with small $m_0$ and relatively large $m_{1/2}$, since in this case one of the scalar staus becomes the
Figure 3: The top quark mass as function of $\tan \beta$ (top) for values of $m_0, m_{1/2} \approx 1$ TeV. The curve is hardly changed for lower SUSY masses. The middle part shows the corresponding values of the Yukawa coupling at the GUT scale and the lower part the $\chi^2$ values. If the top constraint ($m_t = 175 \pm 6$, horizontal band) is not applied, all values of $\tan \beta$ between 1.2 and 50 are allowed (thin dotted lines at the bottom), but if the top mass is constrained to the experimental value, only the regions $1 \leq \tan \beta \leq 3$ and $20 \leq \tan \beta \leq 40$ are allowed.

LSP after mixing via the off-diagonal elements in the mass matrix. The LSP constraint is especially effective at the high $\tan \beta$ region, since the off-diagonal element in the stau mass matrix is proportional to $A_t m_0 - \mu \tan \beta$.

The $b \rightarrow s\gamma$ rate is too large in most of the parameter region for large $\tan \beta$, at least if one requires $b - \tau$ unification. Both $m_b$ and $b \rightarrow s\gamma$ have loop corrections proportional to $\mu \tan \beta$ and it is difficult to get both constraints fulfilled simultaneously unless $\mu$ can be chosen to be small, which can only be obtained for non-unified Higgs masses at the GUT scale. Alternatively, one has to give up $b\tau$ unification, in which case a much larger region for high $\tan \beta$ is allowed.

The long lifetime of the universe requires a mass density below the critical density, else the overclosed universe would have collapsed long ago. This requires that the selectron is sufficiently light for a fast annihilation through t-channel selectron exchange. For large $\tan \beta$ the Higgsino admixture becomes larger, which leads to an enhancement of $\tilde{\chi}^0 - \tilde{\chi}^0$ annihilation via the s-channel Z boson exchange, thus reducing the relic density. As a result, in the large $\tan \beta$ case the
Figure 4: The $\chi^2$-distribution for the low and high tan $\beta$ solutions. The different shades in the projections indicate steps of $\Delta \chi^2 = 1$, so basically only the light shaded region is allowed. The stars indicate the optimum solution. Contours enclose domains excluded by the particular constraints used in the analysis.

constraint $\Omega h_0^2 < 1$ is almost always satisfied unlike in the case of low tan $\beta$.

3 Summary

It is shown that the CMSSM can fit simultaneously the constraints from

- gauge couplings unification;
- $b - \tau$ Yukawa couplings unification;
- $b \to s\gamma$ rate;
- radiative EWSB;
- life time of the universe.

For the high tan $\beta$ scenario the present limits of LEP [8] on the chargino mass and the more precise NLO calculations of $b \to s\gamma$ decay rate together with the measurements from ALEPH and CLEO leave only the small upper right corner region in parameter space ($m_0 > 500, m_{1/2} > 500$ GeV) (see fig. [4]), where all squark masses are above 1 TeV.
Figure 5: Contours of the Higgs mass (solid lines) in the $m_0, m_{1/2}$ plane (above) and the Higgs masses (below) for both sign of $\mu$ for the low tan $\beta$ solution tan $\beta = 1.65$ and $m_t = 175$ GeV.

For the low tan $\beta$ scenario the Higgs mass is below 100 (90) GeV for $\mu > 0$ ($\mu < 0$) and $m_t \leq 181$ GeV. The excluded regions in the $(m_0, m_{1/2})$ plane for a given Higgs mass are shown in fig. 5 for two different signs of $\mu$. Clearly the $\mu < 0$ solution is largely excluded, since the present LEP run at 183 GeV did not observe a positive signal and preliminary limits above 85 GeV were quoted[7]. For $\mu > 0$ $m_{1/2} > 135$ GeV is required from the gaugino-like chargino limit $m_\chi \geq 90$ GeV and $m_0 < 350$ GeV from the relic density constraint (see fig. 4).

References

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