A New View of the Size–Mass Distribution of Galaxies: Using $r_{20}$ and $r_{80}$ Instead of $r_{50}$

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Abstract

When investigating the sizes of galaxies it is standard practice to use the half-light radius, $r_{50}$. Here we explore the effects of the size definition on the distribution of galaxies in the size–stellar mass plane. Specifically, we consider $r_{20}$ and $r_{80}$, the radii that contain 20% and 80% of the total luminosity, along with the canonical measure of $r_{50}$. We will use Somerville et al. (2018) to define the size of a galaxy. We will use Somerville et al. 2018 as the standard measurement of the size of a galaxy.

1. Introduction

The sizes of galaxies hold clues about the physical processes that shape them. They can be predicted by galaxy formation models (Mo et al. 1998; Dutton et al. 2011; Kravtsov 2013; Somerville et al. 2018) and can help distinguish between different evolutionary models (Carollo et al. 2013; van Dokkum et al. 2015; Matharu et al. 2018). However, the sizes of galaxies are difficult to define, as their surface brightness profiles decrease smoothly with radius and have no well-defined edge. A common method is to use the half-light, also known as the effective radius, $r_{50}$, which contains 50% of a galaxy’s total luminosity. It is generally applicable to all galaxies and does not trivially correlate with other properties such as a galaxy’s luminosity. Due to these properties, $r_{50}$ has become the standard measurement of the size of a galaxy. Studies of $r_{50}$ over the past decades have shown that it correlates with stellar mass, the so-called size–mass distribution, which in turn varies with galaxy color, type, and redshift (Shen et al. 2003; Ferguson et al. 2003; Trujillo et al. 2006; Williams et al. 2010; Ono et al. 2013; van der Wel et al. 2014; Lange et al. 2015; Mowla et al. 2018).

When investigating the size–mass distribution it is important to assess the effect of the choice of the size parameter, as a single number fails to capture information about the distribution of light within a galaxy. In practice, a second parameter is typically introduced to separately study the form of the light profile. The Sérsic index $n$ ( Sérsic 1968) has become the parameter of choice, derived from 1D or 2D fits of the form $\log I(r) \propto (r/r_0)^{n}$ to the surface brightness profile.

In this Letter we explore an alternative approach to studying the structure of galaxies. We compare and contrast the size–mass distribution that arises from using different measures for the size of a galaxy. We will use $r_{20}$ and $r_{80}$, the radii that contain 20% and 80% of the total luminosity, along with the canonical measure of $r_{50}$. This Letter will focus on the difference between star-forming and quiescent galaxies at a fixed stellar mass to investigate the different evolutionary processes that shape them. In an accompanying Letter, Mowla et al. (2019), we investigate the relation between $r_{50}$ and a galaxy’s dark matter halo.

2. Data

2.1. Galaxy Sample

In this Letter we employ two different galaxy surveys: 3D-HST/CANDELS ( Koekemoer et al. 2011; Brammer et al. 2012) and COSMOS-DASH (Momcheva et al. 2016; Mowla et al. 2018). The CANDELS survey covers 0.22 degree$^2$ with extensive ground- and space-based photometry ranging from 0.3 to 8 $\mu$m, which is supplemented by WFC3 grism spectroscopy spanning three-quarters of that area. Galaxy sizes are measured in van der Wel et al. (2014) from the $H_{160}$ and $I_{814}$ bands for ~30,000 galaxies above $M_\ast > 10^9 M_\odot$ with $0 < z < 3$. Galaxy properties such as stellar mass, redshift, and rest-frame colors for this sample are taken from the 3D-HST catalog (Skelton et al. 2014). We supplement this sample with the COSMOS-DASH survey, which covers 0.66 degree$^2$ with $H_{160}$ imaging. The larger survey area affords proper sampling of the bright end of the luminosity function for 1.5 $< z < 3$, which is not possible in the smaller CANDELS survey. Combined with 1.7 degree$^2$ of ACS-COSMOS imaging ( Koekemoer et al. 2007), Mowla et al. (2018) measured the sizes of 910 galaxies with $M_\ast > 2 \times 10^{11} M_\odot$ at $0 < z < 3$. Masses and redshifts for the COSMOS-DASH sample are taken from the UltraVISTA catalog (Muzzin et al. 2013a), as described in Mowla et al. (2018).

van der Wel et al. (2014) and Mowla et al. (2018) used very similar methods to measure the size of galaxies. GALFIT ( Peng et al. 2010) is used to fit 2D single-component Sérsic profiles to each galaxy and extract a best-fit Sérsic index and
effective radius. This forward modeling approach allows the measurement of galaxy sizes that are comparable to the instrumental point-spread function (PSF). The ACS/F814W filter is used for galaxies with \( z < 1.5 \), and the WFC3/F160W filter is used at higher redshift. Redshift- and mass-dependent color gradients are taken into account to ensure that the sizes of all galaxies are measured at the same rest-frame wavelength (5000 Å). Throughout this Letter we will separate galaxies into two populations: star-forming and quiescent. This is done using their rest-frame \( UBV \) colors according to the prescription in Muzzin et al. (2013b).

### 2.2. Calculating \( r_{20} \) and \( r_{80} \)

Given that the sizes of galaxies at high redshift are comparable to the PSF, one cannot simply measure \( r_{20} \) and \( r_{80} \) directly from the surface brightness profile. Thus we choose to calculate \( r_{20} \) and \( r_{80} \) from the Sérsic profile derived by GALFIT (Peng et al. 2010). For a single-component Sérsic profile it is straightforward to convert between \( r_{50} \), \( r_{20} \) and \( r_{80} \). The fraction of light contained within a projected radius \( r \) is

\[
\frac{L(<r)}{L_{tot}} = \frac{\gamma(2n, b_n(r/r_{eff})^{1/n})}{\Gamma(2n)}.
\]

Here, \( \gamma \) is the incomplete gamma function, \( \Gamma \) is the complete gamma function, and \( b_n \) is the solution to the equation \( \Gamma(2n) = 2\gamma(2n, b_n) \), which we approximate as \( b_n = 1.9992n - 0.3271 \) (Capaccioli 1989). Comparing \( L(<r_{20}) \) to \( L(<r_{50}) \) we derive the following.

\[
\frac{L(<r_{20})}{L(<r_{50})} = \frac{0.2}{0.5} = \frac{\gamma(2n, b_n(r_{20}/r_{50})^{1/n})}{\gamma(2n, b_n)}.
\]

For a given value of \( n \), we numerically solve Equation (2) for the value of \( r_{20}/r_{50} \). A similar procedure is used to calculate \( r_{80}/r_{50} \). We perform this calculation for a range of Sérsic indices, with results shown in Figure 1. For higher Sérsic indices \( r_{20}/r_{50} \) decreases, corresponding to the steeper central profile, and \( r_{80}/r_{50} \) increases, corresponding to the extended wings at large radius. We present fitting formulas for \( r_{20}/r_{50} \) and \( r_{80}/r_{50} \) as a function of Sérsic index, shown below in Equation (3). These fitting functions are accurate to within 5% for \( n = 0.25-10 \):

\[
\frac{r_{20}}{r_{50}}(n) = -0.0008n^3 + 0.0178n^2 - 0.1471n + 0.6294
\]

\[
\frac{r_{80}}{r_{50}}(n) = 0.0012n^3 - 0.0123n^2 + 0.5092n + 1.2646.
\]

Figure 1 displays the direct measurements of \( r_{80}/r_{50} \) and \( r_{20}/r_{50} \) ratios for 127 isolated galaxies in the GOODS-South field. We select these galaxies as being isolated if there is not another sources within \( \sim 10 \) \( r_{50} \). Their size and magnitude distributions matches those of the overall sample. We use the \( H_{160} \) images to directly measure the different radii using the residual-corrected surface brightness profile as described above. We find that the direct measurements of \( r_{80}/r_{50} \) and \( r_{20}/r_{50} \) match the simple calculation based on the Sérsic profile well. This is consistent with studies that have shown that high-redshift galaxies are generally well fit by a single-component Sérsic profile (Szomoru et al. 2012). The scatter of the observed points around the Sérsic relation does not correlate with Sérsic index, redshift, or galaxy type, but it does increase for galaxies with \( m_{F160W} > 23 \). Given the success in reproducing \( r_{80}/r_{50} \) and \( r_{20}/r_{50} \) based on the Sérsic index alone, we apply this simple calculation to the rest of our sample with the caveat that the values can be uncertain for individual galaxies.

### 3. The Distributions of \( r_{20} \) and \( r_{80} \)

#### 3.1. The Size–Mass Plane

In Figure 2 we show the distribution of galaxies in the size–mass plane using three different measures of galaxy size: \( r_{20} \), \( r_{50} \) and \( r_{80} \). The size distributions are offset toward larger sizes when going from \( r_{20} \) to \( r_{50} \) and \( r_{80} \), as follows from their definitions. However, we also find that the distributions of star-forming and quiescent galaxies are very different depending on which radius is used. Using \( r_{20} \) the two populations occupy separate regions of the size–mass plane with very little overlap. The quiescent galaxies are consistently smaller at a given stellar mass across the entire sample. The \( r_{80} \) mass plane affords a different view. The star-forming and quiescent populations appear to follow the same distribution, with little difference between the two types of galaxies. The canonical size–mass distribution, using \( r_{50} \), lies between these two extremes. The

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**Figure 1.** Ratios \( r_{20}/r_{50} \) and \( r_{80}/r_{50} \) are shown as a function of Sérsic index. The blue lines display the calculation for a Sérsic function based on Equation (2). Gray crosses display measurements of isolated galaxies in the GOODS-South field using direct integration of the residual-corrected surface brightness profile. We find that the observations match the calculation based on the Sérsic index very well.
distribution of galaxies in this plane is often interpreted in the context of the distinct relations that star-forming and quiescent galaxies follow (see van der Wel et al. 2014; Mowla et al. 2018, and references therein), but as we show in Figure 2 this conclusion depends sensitively on the definition of size.

The distribution of $r_{20}$ and $r_{80}$ for star-forming and quiescent galaxies across a range of stellar masses and redshifts is shown in Figure 3. We observe that the two galaxy populations represent two distinct distributions of $r_{20}$, while they appear to follow the same distribution in $r_{80}$. The bimodality in the distribution of $r_{20}$ is most clear for intermediate stellar mass ($10 < \log M_*/M_\odot < 11$) and high redshift ($z > 1$). Here the peaks of the distributions for star-forming and quiescent galaxies are clearly separated and a valley between the two distributions is apparent. By contrast, the distributions of $r_{80}$ for the two populations are nearly identical. Across the entire range of stellar mass and redshift the peaks and widths of the $r_{80}$ distribution appear at nearly the same location for star-forming and quiescent galaxies.

3.2. Bimodality in the Distribution of $r_{20}$

To highlight and quantify these trends, we focus on the distribution of radii in a single stellar mass and redshift bin in Figure 4. We investigate the overall distribution of sizes, without separating star-forming and quiescent galaxies. The distribution of $r_{20}$ appears to be bimodal. To test this hypothesis, we employ Hartigan’s dip test (Hartigan & Hartigan 1985), which tests the null hypothesis that the sample is drawn from a unimodal distribution. When analyzing the logspace distribution of $r_{20}$ in this mass and redshift bin we find $p = 0.043$, which means that the null hypothesis of a unimodal distribution can be rejected with >95% confidence. As an additional test, we fit one- and two-component Gaussian mixture models to the logspace distributions of $r_{20}$, $r_{50}$, and $r_{80}$ and compare the Bayesian information criterion (BIC) of each.

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4 This test is computed using the R package dip test (https://cran.r-project.org/web/packages/dipTest/).

5 We use the scikit-learn python package (Pedregosa et al. 2011).
Unsurprisingly, the distributions of $r_{20}$ and $r_{50}$ are better fit by the two-component model ($\Delta BIC = BIC_{1 \text{ comp.}} - BIC_{2 \text{ comp.}} = 49.5$ and 41.5, respectively). Interestingly, for $r_{80}$ we find that it is better fit by the single-component model ($\Delta BIC = -5.3$).

In the top panel of Figure 5 we broaden this analysis and quantify the separation of the distributions of star-forming and quiescent galaxies as a function of mass, redshift, and size definition. This is done through the Ashman’s $D$ parameter (Ashman et al. 1994), given by

$$D = \sqrt{2} \frac{|\mu(\log r_{SF}) - \mu(\log r_Q)|}{\sqrt{\sigma(\log r_{SF})^2 + \sigma(\log r_Q)^2}}. \quad (4)$$
is dominated by intrinsic scatter in this regime. 

Figure 5. Top panel: evidence for bimodality in size distributions as quantified using the Ashman D parameter (Equation (4)). Bottom panel: the observed scatter, estimated using the bi-weight scale, in $r_{20}$, $r_{50}$, and $r_{80}$, as a function of stellar mass and redshift. The reduced bimodality for $r_{80}$ leads to the smaller scatter.

Here, $\mu$ is the mean of each galaxy population and $\sigma$ is the standard deviation, which we estimate using the bi-weight location and scale, respectively (Beers et al. 1990). In the ideal case of a combination of two identical Gaussian distributions, the combined distribution shows two distinct peaks if $D > 2$ (Everitt & Hand 1981). This threshold of $D > 2$ is also used more broadly to indicate when a distribution is bimodal, regardless of the functional form. The Ashman D values for $r_{20}$, $r_{50}$, and $r_{80}$ are shown in Figure 5 as a function of stellar mass and redshift. At all masses and redshifts, the difference between star-forming and quiescent galaxies increases when going from $r_{20}$ to $r_{50}$ and from $r_{50}$ to $r_{80}$. For $r_{20}$, there is significant bimodality with $D > 2$ at all stellar masses in the range $1 \times 10^{10} < M_*/M_\odot < 5 \times 10^{10}$. The Ashman D value decreases at large stellar masses ($M_*/M_\odot > 10^{11}M_\odot$) for all size definitions, echoing the results of Mowla et al. (2018) for $r_{50}$. The Ashman D value for $r_{80}$ is $\ll 2$ at all stellar masses and redshifts, consistent with the GMM analysis.

3.3. Implications for the Observed Scatter in the Size–Mass Relation

The fact that the separation of star-forming and quiescent galaxies changes for different size definitions has implications for the scatter in the overall size–mass relation: it is significantly smaller for $r_{80}$ than for $r_{50}$ and (particularly) $r_{20}$. This is demonstrated in the bottom panel of Figure 5. The observed scatter, estimated using the bi-weight scale, is larger in $r_{20}$ than in $r_{50}$ by 0.08 dex, due to the fact that the distributions of star-forming and quiescent galaxies have a larger separation. The scatter in $r_{50}$ ($\approx 0.25$ dex, independent of mass and redshift) is generally smaller than in $r_{50}$. We note that the inverse scatter for the quiescent and star-forming galaxies as separate populations is also $\approx 0.25$ dex at all masses and redshifts, regardless of the choice of size indicator. This implies that the reduction of the scatter in $r_{50}$ with respect to $r_{20}$ and $r_{50}$ can be attributed to the fact that the size distributions of star-forming and quiescent galaxies overlap in $r_{80}$. We are showing the observed scatter in the sizes of galaxies, which is the combination of intrinsic scatter and observational uncertainty.6

To decouple these two quantities, we would require a careful analysis of the observational procedures and how they affect uncertainties in size measurements. Instead, our goal is to compare the relative scatter of different measures of the size.

4. Discussion

In this Letter, we have investigated the size–mass distribution of galaxies if $r_{20}$ or $r_{80}$, the radii containing 20% of 80% of the light, is used instead of the traditional measure of $r_{50}$. When using $r_{20}$, we find strong evidence of bimodality in the size distribution at fixed mass; to our knowledge, such a structural bimodality has not been observed before. The two peaks correspond to quiescent galaxies and star-forming galaxies. When using $r_{80}$, the size distribution is narrow, and star-forming and quiescent galaxies follow very similar size–mass relations at all redshifts. The results presented here have been anticipated from the well-known relations between quiescence, mass, size, and Sérsic index. Specifically, quiescent galaxies are observed to have a higher average Sérsic index, which means that $r_{20}/r_{50}$ ($r_{80}/r_{80}$) is lower (higher) when compared to star-forming galaxies. In this sense, the results presented here can be seen as a re-casting of these relations into a convenient form.

Understanding the distribution of light within galaxies aids our understanding of how they assembled (Hill et al. 2017; Huang et al. 2018), and the $r_{20}$ and $r_{80}$ distributions may highlight specific and distinct physical processes. Based on our results, it seems likely that $r_{20}$ is related to processes that affect star formation and quenching. Specifically, there appears to be a connection between the structural bimodality discussed in this study and the well-known color/sSFR bimodalities (Strateva et al. 2001; Baldry et al. 2004). It had already been recognized that these bimodalities are connected to the central density of galaxies (Barro et al. 2014; van Dokkum et al. 2015; Tacchella et al. 2017; Whitaker et al. 2017). These studies suggest a central density or velocity dispersion threshold above which galaxies quench. At fixed stellar mass, galaxies with a lower $r_{20}$ have a higher central density. Therefore, these quenching thresholds are qualitatively consistent with the clean separation of star-forming and quiescent galaxies in $r_{20}$.

Turning to $r_{80}$, this provides a reasonable proxy of the total baryonic extent. At the highest masses typical values of $r_{80}$ reach $\sim 20$ kpc, and given the similarity of the distributions of

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6 A similar analysis was done in van der Wel et al. (2012), who concluded that the observed scatter in $r_{50}$ is dominated by intrinsic scatter in this regime.
star-forming and quiescent galaxies in the $r_{80}$-mass plane, it is tempting to link this size to halo properties. Several studies have suggested a constant scaling between stellar and halo radius (Kravtsov 2013; Somerville et al. 2018). This connection between $r_{80}$ and the halos of galaxies is explored further in an accompanying Letter, Mowla et al. (2019). We note that the differences between $r_{20}$ and $r_{80}$ can also be interpreted in the context of dynamical timescales; for massive galaxies these are typically a factor of $\sim 20$ longer at $r_{80}$ than at $r_{20}$. $r_{20}$ is therefore sensitive to processes that can change rapidly, such as star formation rates or nuclear activity, whereas $r_{80}$ should be more or less immune to those.

The work presented here is an initial investigation into the differences in the galaxy size–mass distribution when using $r_{20}$–$r_{50}$ and $r_{80}$, with more detailed analyses to follow. We have not quantified the evolution of the slope or normalization of the size–mass relation of $r_{20}$. Describing these trends may give insight into galaxy quenching through cosmic time. For $r_{80}$, we refer the reader to Mowla et al. (2019), which details the evolution of the $r_{80}$-mass distribution and its connection to halo properties. Another improvement will be measuring the mass profile of galaxies. Recent studies have shown a relatively constant offset between mass-weighted and light-weighted $r_{50}$ (Szomoru et al. 2013; Mosleh et al. 2017), but it is not clear whether that would also apply to $r_{20}$ and $r_{80}$. Finally, it is important to continue developing non-parametric techniques for measuring the surface brightness profiles of high-redshift galaxies.

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