Investigation of chaotic and regular dynamics of the asteroids – Venus companions

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Abstract. The paper deals with the investigation of regular and chaotic dynamics of near-Earth asteroids (NEAs) in the vicinity of resonance 1:1 with Venus. The degree of chaoticity of NEA orbits is determined by analyzing the behavior of parameters MEGNO and OMEGNO. It is shown that the asteroid motion in the resonance vicinity and/or multiple close approaches of NEAs with planets lead to the appearance of dynamical chaoticity.

The investigation of regular and chaotic dynamics of near-Earth asteroids (NEA) is very topical currently. One of the main reasons of particular attention to the NEA dynamics is the asteroid hazard problem. The mean motion resonances with major planets are of great importance in NEA motion because stable resonances can be a protection mechanism from asteroid close encounters. However, the motion in the vicinity of the resonances can lead to significant changes of asteroid orbital parameters and to possible close approaches with the planets, particularly with the Earth. The investigation of NEA chaotic dynamics allows one to determine a time interval of asteroid motion predictability and the time after that the motion may be considered as unpredictable.

This paper presents some results of investigating regular and chaotic dynamics of the asteroids being Venus companions which are revealed at the beginning of 2015. There were five such objects at the time of the revealing – 322756 2001 CK32, 2002 LT24, 2002 VE68, 2007 AG, 2013 ND15 [1].

For all the investigated NEAs the orbital evolution has been simulated over a time interval about 3000 years. The behavior of resonant characteristics and chaoticity parameters has been studied over the same span. The investigation has been carried out by numerical integration of motion differential equations by Everhart method [2]. The initial data have been taken from Bowell catalog dated February of 2015 [3]. The force model includes the gravitational influence of the planets, Pluto, the Moon, Ceres, Pallas and Vesta. For all the computations has been used a specially developed software [4] which enables one to perform high-precision prediction of asteroid motion.

We used the following resonant characteristics: a critical (resonant) argument $\beta$ which determines the longitude of the connection for an asteroid and Venus [5]:

$$\beta = \lambda_1 - \lambda_2,$$  \hspace{1cm} (1)

and its time derivative $\alpha$ (called resonant “band”) [6]:

$$\alpha \approx n_1 - n_2,$$  \hspace{1cm} (2)

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where \( n_1, n_2 \) are the mean motions, \( \lambda_1, \lambda_2 \) are the mean longitudes of the asteroid and Venus respectively. At \( \alpha = 0 \) the asteroid is in the exact resonance with the planet due to the mean motion commensurability 1:1. The asteroid moves in the vicinity of the resonance if \( \alpha \) and \( \beta \) librate around the value of the exact commensurability so that \( |\beta - \tilde{\beta}| \leq 180^\circ \) and \( |\alpha| \leq \alpha_{\max} \) (\( \tilde{\beta} \) is the libration center for the critical argument). The value \( \alpha_{\max} \) characterizes boundaries of the resonant motion and is determined by the maximum amplitude of the critical argument librations.

We used the indicators MEGNO (Mean Exponential Growth factor of Nearby Orbits) [7] and OMEGNO (Orthogonal MEGNO) [8] to study regularity or chaoticity of orbital motion in the vicinities of resonances. Analysis of these characteristics allows separating regular and chaotic regimes of motion surely.

For example, the averaged parameter MEGNO \( \tilde{Y}(t) \) has the reference value 2 relative to which the chaoticity and regularity are determined. For chaotic orbits with exponential divergence of close trajectories the averaged parameter MEGNO is greater than 2 and it grows almost linearly. For stable quasi-periodical (regular) orbits with linear divergence of close trajectories \( \tilde{Y}(t) \) always tends to 2 and for stable orbits described by equations for a harmonic oscillator \( \tilde{Y}(t)=0 \) [9].

The orthogonal parameter MEGNO, i.e. the parameter OMEGNO \( Y_\perp(t) \), allows not only distinguishing chaotic and regular orbits but also revealing and separating periodic trajectories in the domains of regular motion of continuous dynamic systems [10]. This is an advantage of the given parameter.

The parameter OMEGNO has two reference values \( Y_\perp(t)=2 \) and \( Y_\perp(t)=0 \). When \( Y_\perp(t)>2 \) and the parameter OMEGNO grows almost linearly, a motion is considered as chaotic. For quasi-periodic (regular) orbits \( Y_\perp(t)\rightarrow2 \) and the deviations of \( Y_\perp(t) \) from 2 allows determining whether the trajectory under investigation is close to a stable or unstable periodic orbit. That is to say, if \( Y_\perp(t)\leq2 \) then the trajectory is close to a stable periodic orbit and if \( Y_\perp(t)\geq2 \) then the trajectory is close to an unstable periodic orbit. At last, for any periodic orbit \( Y_\perp(t)=0 \).

For each asteroid, the graphs of evolution of the orbital elements (semi-major axis \( a \), eccentricity \( e \), and inclination \( i \)), the resonant argument \( \beta \) (1), the resonant band \( \alpha \) (2), the close encounters with planets, and also the graphs of evolution of the chaoticity parameters MEGNO and OMEGNO have been constructed and analysed.

The investigation results showed that all the five NEAs move in the vicinity of resonance 1:1 with Venus and have non-regular librations of the resonant band around the value of the exact commensurability of the mean motions of the asteroid and the planet. Only asteroid 322756 2001 CK32 moving on one side of resonance in past falls into the resonance in 2000 and regularly passes through the exact commensurability in future. This object has a quasi-periodic (regular) character of motion over the entire considerable time interval, as is followed from the evolution of the MEGNO and OMEGNO parameters. At the same time four from the five NEAs begin to show chaoticity features. The transition between resonant and nonresonant motions and close and/or multiple approaches of these asteroids with planets lead to the appearance of motion chaoticity.

As an example figure 1 and figure 2 present orbital evolution of asteroids 322756 2001 CK32 [11] and 2002 LT24 [12] respectively. Every figure shows evolution of the resonant band \( \alpha (a) \), evolution of the critical argument \( \beta (b) \), approaches with Venus (c), the Earth (d), Mercury (e), evolution of the OMEGNO (gray line) and averaged MEGNO (black line) parameters (f), evolution of the semi-major axis \( a (g) \), the eccentricity \( e (h) \), and the inclination \( i (i) \).

As is seen from Figure 1, the critical argument of asteroid 322756 2001 CK32 moving along one side of the commensurability circulates but does not librate. From 1250 the parameter MEGNO oscillates near 2, i.e. from that time the motion of 322756 2001 CK32 is regular. Moreover, evolution of the parameter OMEGNO shows that the asteroid orbit is close to a stable periodic orbit, \( Y_\perp(t)\rightarrow0 \). The transition to the resonant motion leads to a jump of both chaoticity parameters but, nevertheless, the object motion remains quasi-periodic. At the same time \( Y_\perp(t)\rightarrow2 \) from above, i.e. the orbit is close to unstable periodic.
Figure 1. Asteroid 322756 2001 CK32: evolution of the resonance band $\alpha$ (a), evolution of the critical argument $\beta$ (b), approaches with Venus (c), the Earth (d), Mercury (e), evolution of OMEGNO (gray line) and averaged MEGNO (black line) parameters (f), evolution of the semi-major axis $a$ (g), the eccentricity (h), and the inclination of the asteroid orbit $i$ (i)
Figure 2. The same as in Fig. 1 but for asteroid 2002 LT24

Figure 2 shows that the asteroid 2002 LT24 moves on one side of the exact commensurability most of the time. The resonant band changes its sign only several times and the critical argument circulates almost always. The parameters MEGNO and OMEGNO are close to zero from 1800 to 2500, i.e. its orbit is stable and periodic during about 700 years.
From the graphs of close encounters of 2002 LT24 with the Earth and Venus we can see that from 2500 the asteroid has a series of approaches with the planets, which leads to the development of chaoticty. During the orbit simulation towards the past the chaoticity reveals itself at the transition from the libration to the circulation of the critical argument. Also we can see that the orbital elements of both the asteroids change insignificantly over the considered time interval.

For asteroids 2002 VE68 [13] and 2013 ND15 [14] we carried out the orbit fitting by the least square method using all available optical observations and investigated their probabilistic orbital evolution [1]. The analysis of behavior of the chaoticty parameters shows that at $\tilde{Y}(t)>2$, $\bar{Y}(t)>2$, and their further growth the probabilistic domains increase significantly, which allows making the conclusion about exponential extension of the domains.

Thus the regular and chaotic dynamics of asteroids 322756 2001 CK32, 2002 LT24, 2002 VE68, 2007 AG, 2013 ND15, Venus companions, has been investigated. All these asteroids move in the vicinity of resonance 1:1 with Venus and have irregular librations of the resonance band near the value of the exact commensurability of the mean motions of the asteroids and the planet. The results of the evolution of the parameters MEGNO and OMEGNO are in a good agreement with each other but the parameter OMEGNO behaves more smoothly and makes it possible to reliably identify a stable periodic motion. The paper shows that the transition to a chaotic motion may occur at close and multiple encounters with planets and also at the entrance in a resonant domain and at the exit from it.

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