Geometric model of persecution by a group of one goal

A A Dubanov¹, T V Ausheev²

¹Buryat State University, 24 A, Smolin-street, Ulan-Ude, Russia
²East Siberian State University of Technology and Management, 40 V, Klyuchevskaya-street, Ulan-Ude, Russia

alandubanov@mail.ru, atv62@mail.ru

Abstract. This article describes a geometric model where a group of four objects pursues a single goal. The movement occurs on a plane, but if necessary, this model can be applied to an explicitly specified surface. The movement speed of all participants, both pursuers and targets, is constantly modulo. The goals and strategies of each of the pursuers are different. The goal and strategy of the target is determined by the behavior of one of the pursuers.

The key words: pursuit, evasion, escape, simulation

1. Introduction
Quasi-discrete models in the pursuit problem are a possibility of approximate calculation of dynamic processes with further visualization. In our problem, we introduce a time sampling period, during which the pursuer takes a step and changes the movement direction. This article considers a quasi-discrete geometric model of group pursuit of a single goal. Since the system of computer mathematics is used for testing the geometric model, the goal and strategy of each participant in the pursuit task are formed separately in the form of a procedure-function. A separate goal and strategy is formed for each participant. Goals and strategies include setting points at which the task is completed. Enter vectors for these points are set. You also set a set of conditions for forming trajectories should be adhered to at any given time.

In our work, we relied on the theoretical results presented in the sources [1-7]. When compiling the algorithms, the result, presented in [10-15], were taken into account.

2. Formulation of the problem
Consider a group of pursuers that will catch up with a single target. In this group of pursuers, everyone will have their own goal and their own strategy for achieving the goal.
Let the group of pursuers consist of four participants. Let's analyze the goals and strategies for each participant in this group. Let the target at some point in time have a position T on the plane with the speed $V_T$.

3. Goal and strategy of the first object-pursuer
Figure 1. First pursuer’s strategy

The pursuer \( P_1 \) with the speed \( V_1 \) aims simply to catch up with the object \( T \), which means that the points \( P_1 \) and \( T \) overlap with some degree of accuracy \( |P_1 - T| \leq \varepsilon \). As an indicator of accuracy, we can suggest \( \varepsilon = |V_1| \cdot \Delta T \), where \( \Delta T \) is the sampling period in time. In addition, the object \( P_1 \) has a maximum angular velocity \( \omega_1 \), which limits the radius of curvature of the trajectory \( R_1 = \frac{|V_1|}{\omega_1} \).

The strategy of the pursuer \( P_1 \) is that the coordinates of the point \( T \) are converted into a coordinate system \((v_1, v_2)\) with the origin at the point \( P_1 \) (Figure 1):

\[
\begin{align*}
v_1 &= \frac{V_P}{|V_P|} \\
v_2 &= \begin{bmatrix} -v_{1y} \\ v_{1x} \end{bmatrix}
\end{align*}
\]

There the coordinates of the point \( T \) will look like this:

\[
T_v = \begin{bmatrix} (T - P_1) \cdot v_1 \\ (T - P_1) \cdot v_2 \end{bmatrix}
\]

Next, we analyze the coordinates of the point \( T_v \) for belonging to the upper or lower half-plane in the coordinate system \((v_1, v_2)\) with the origin at the point \( P_1 \):

\[
P_{1,v} = \begin{cases} \frac{|V_1| \cdot \Delta T \cdot \cos(\omega_1 \cdot \Delta T)}{|V_1| \cdot \Delta T \cdot \sin(\omega_1 \cdot \Delta T)} & \text{if } T_{vy} \geq 0 \\
\frac{|V_1| \cdot \Delta T \cdot \cos(\omega_1 \cdot \Delta T)}{-|V_1| \cdot \Delta T \cdot \sin(\omega_1 \cdot \Delta T)} & \text{if } T_{vy} < 0 \end{cases}
\]

It is necessary to constantly compare the values of the angles \( \omega_1 \cdot \Delta T \) and \( \alpha \), where \( \alpha \) is the angle between the vectors \((P_1, T)\) and \( V_1 \).

If the angle \( \alpha \) is less than the angle \( \omega_p \cdot \Delta T \), then the coordinates of the point \( P_{1,v} \) will look different:
\[ P_{L,v} = \begin{cases} \frac{V_P}{|V_P|} \cdot \Delta T \cdot \cos(\alpha) & \text{if } T_{v,y} \geq 0 \\ -\frac{V_P}{|V_P|} \cdot \Delta T \cdot \sin(\alpha) & \text{if } T_{v,y} < 0 \end{cases} \]

4. Second and third pursuers’ strategies

The pursuers \( P_2 \) and \( P_3 \) move at speeds \( V_2 \) and \( V_3 \), respectively. For the objects \( P_2 \) and \( P_3 \), the goal is to combine with a certain degree of accuracy \( \varepsilon \) not with the point \( T \), but with the points \( T_2 \) and \( T_3 \), respectively (Figure 2).

The coordinates of points \( T_2 \) and \( T_3 \) are formed as follows:

\[ T_{2,3} = T + n_{2,3} \]

The normal vector \( n_{2,3} = \pm \frac{1}{|V_T|} \begin{bmatrix} -V_T^y \\ V_T^x \end{bmatrix} \cdot \Delta S_{2,3} \), where \( \Delta S_{2,3} \) is the distance from the points \( T_2 \) and \( T_3 \) to the point \( T \), respectively.

For the trajectories of objects \( P_2 \) and \( P_3 \), the following conditions are defined: they came to the points \( T_2 \) and \( T_3 \) with the velocity directions \( V'_2 \) and \( V'_3 \). The radius of curvature of the trajectories must not be less than \( R_{2,3} = \frac{|V_{2,3}|}{\omega_{2,3}} \), where \( \omega_{2,3} \) are the maximum angular rotation speeds of the pursuers \( P_2 \) and \( P_3 \).

The simulated trajectory at some point in time consists of a straight section \([P_{2,3}, P_{\tan,2,3}]\) and the arc segment \((P_{\tan,2,3}, T_{2,3})\).

At each iteration stage, the \( P_2 \) and \( P_3 \) objects perform discrete rotation and discrete translational movement to reach the simulated trajectories. This was described in detail in section 5.1.

In our test program, written based on the materials of this paragraph, objects \( P_2 \) and \( P_3 \), as soon as they enter a course parallel to the course \( T \), begin to move at speeds equal to \( V_T \).
5. Goal and strategy of the fourth pursuer

Consider the fourth participant from the group of pursuers. If the behavior of the first participant can be qualified as the main "hunter". The behavior of the second and third pursuers can be qualified as assistants that do not allow the goal to escape, the fourth pursuer’s role can be interpreted as a player from the "ambush".

Figure 3 shows two cases of forming the paths of the fourth pursuer. In the first case the trajectory of the pursuer is included in the position directly to the target $T$ in a direction perpendicular to its velocity $V_T$. In the second case, the trajectory of the pursuer enters a point $Q$ at a speed opposite the target velocity $V_T$. The point $Q$ located on a straight line from point $T$ forming $V_T$.

The point $Q$ can be placed at any point in the plane, nothing prevents us from doing this. It’s just that the goal may not be achieved.

When you reach the $Q$ point, you can change the strategy of the pursuer. Let's say we reset the speed to 0 and wait for the target to approach a distance less than $\varepsilon$. You can change the strategy when you reach the $Q$ point to the strategy of the first pursuer.

6. Target’s strategy and goal
Consider the behavior of the object of persecution. In our model under consideration, the target’s goal is to evade the first pursuer.
Figure 4. Target’s strategy

Figure 4 illustrates the strategy of the pursued object \( T \). In this figure, the object \( T \) with the speed \( V_T \) and the angular rotation speed \( \omega_T \) during the sampling period \( T \) rotates by the angle \( \omega_T \cdot \Delta T \) and moves a distance \( |T_i - T| = |V_T| \cdot \Delta T \). The direction of rotation of the point \( T \) depends on which half-plane the pursuer \( P_1 \) is located in. In addition, we can suggest an alternative strategy illustrated in Figure 5.

Figure 5. Additional target’s strategy

7. Conclusion

Based on the materials presented in this Chapter, a test program is written in the MathCAD system that calculates the trajectories of a group of four pursuers and a target that evades them. Each participant in the geometric model has its own goal and strategy. Fig. 6 shows a screenshot from the video, where you can see how one pursuer implements the chase on the trail. Two pursuers take and accompany the target along parallel trajectories. One pursuer goes perpendicular to the projected trajectory of the target. In the program, we deliberately changed the goal and strategy of the fourth pursuer to show that within our program, it is quite simple to set the coordinates of the entry points and the entry vectors to the points.
Figure 6. The simulation result of group pursuit of single target

When writing this article, the theoretical provisions set out in [1-6] are used as a basis. Description of the algorithm for following predicted trajectories is located on the resource [7]. Fig. 6 is supplemented with a link to the resource [8], where the video on the results of the program is posted. The source code of the program is available on the resource [9]

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