Enhanced $2\pi$-periodic Aharonov-Bohm effect as a signature of Majorana bound states probed by nonlocal measurements

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We present the $2\pi$-periodic Aharonov-Bohm (AB) effect as a nonlocal probe of Majorana zero modes (MZMs) without the restriction of fermion parity. We demonstrate the enhancement of the AB effect, where the topological protection of MZMs yields amplified and robust Andreev reflection mediated by MZMs at multiple superconductor-normal metal junctions. We investigate the influence of trivial bound states and show that a nonlocal index enables a more explicit distinction between the trivial and topological bound states than local probes.

I. INTRODUCTION

Owing to exotic properties such as topological protection and non-Abelian braidings, Majorana quasiparticles have been one of the central topics in condensed matter physics. The potential application for fault-tolerant quantum computation [1, 2] has motivated the pursuit of platforms to host MZMs is topological superconductors. Several physical systems have been proposed to realize the topological bound states, including Rashba spin-orbit coupled nanowires with Zeeman energy in the proximity of $s$-wave superconductors [3–14]. This system is particularly favorable to experimentalists thanks to the emergence of the exotic property from the combination of conventional ingredients.

Despite the substantial effort to prove the existence of MZMs, the consensus is yet to be established. The main focus of previous experimental studies has been local conductance measurements [15–22]. The topologically protected Andreev reflection mediated by MZMs causes quantized zero-bias conductance peaks (ZBCPs) [10, 11], which are one of the most well-known signatures of MZMs. For example, the experimentally observed near-quantized ZBCPs [23] had been interpreted as evidence of MZMs. However, theoretical studies have pointed out that the trivial bound states called partially-separated Andreev bound states (ps-ABSs) can mimic the signature [11] 24–42. The indistinguishability of the signatures emerging from MZMs and ps-ABSs questions the validity of the previous studies based on local conductance measurements, motivating exploration for alternative experimental schemes to distinguish the trivial and topological bound states [43–53].

The methods to probe the nonlocality peculiar to MZMs, among nonlocal transport measurements [54–60], include the “teleportation” interferometry [61–65]. The process is the phase-coherent single electron transmission through MZMs, which reflects the nonlocal nature as the independence of the distance between MZMs. In this process, the phase of the transmission amplitude is proportional to the occupation number of the complex fermion composed of the MZMs. The interference measurement not only provides a smoking-gun signature of MZMs but also serves as readouts of topological qubits [66, 67].

While the interferometry scheme attracts attention from both theoretical and experimental researchers [68–74], it requires parity conservation [69]. The condition demands the fabrication of mesoscopic systems with floating superconductors, which may lead to experimental intricacy. Therefore, it is desirable to develop a new approach for detecting the nonlocality of MZMs without the condition of parity restrictions.

In this paper, we present the $2\pi$-periodic Aharonov-Bohm (AB) effect as a nonlocal probe of Majorana bound states without the requirement of parity restrictions. We introduce two indices that characterize the AB effect and discuss their capability for distinguishing MZMs from ps-ABSs. We discuss how the robustness of the indices demonstrates the difference between the topological and trivial bound states. We show that the effect can exhibit a more conspicuous contrast between topological and trivial bound states than local conductance measurements.

II. 2$\pi$-PERIODIC AB EFFECT

Here we consider the AB effect in the setup expressed in Fig. 1(a) by introducing Peierls phase factors to the hopping of the metallic lead in the tight-binding model. Under the condition of parity restriction, MZMs at the ends of the wire mediate teleportation in the topological phase. The interference between the two paths of electron conduction, namely the metallic lead and the superconducting wire, yields the $4\pi$-periodic AB effect [65]. In the trivial phase, the absence of transmission channels inside the energy gap eliminates the effect (see Fig. 1(b)).

This paper is devoted to the case without parity restriction, where the AB effect is $2\pi$-periodic [75–77]. The halved periodicity is attributed to the two processes with Andreev reflections at both ends of the superconducting wire (see Fig. 1). The Andreev reflections at both superconductor-normal metal (SN) junctions cause electrons and holes to gain the opposite-signed phase, resulting in the $2\pi$ periodicity. The halved periodicity is universal whether the system is in the topological or trivial phase. However, the amplitude is more conspicuous...
by the orders of magnitude in the topological phase because the topological bound states mediate the perfect Andreev reflection \[78, 79\]. Due to the spatial separation and the topological protection of MZMs, the processes of multiple Andreev reflections via MZMs are more robust than those via independent unprotected states at SN junctions. Although the effect solely relies on local Andreev reflections, it can provide a more striking difference between topological and trivial bound states because the processes depend on the conditions of multiple SN junctions. That is, in the case of the topological superconducting state, there are topologically protected MZMs at both two ends, and hence, the sequential Andreev reflections mediated via these two MZM can maintain the interference effect robustly. On the other hand, in the trivial case, for preserving the interference effect due to the sequential scatterings at two junctions, one needs to tune system parameters precisely to make the energy levels of two ps-ABSs at two ends nearly equal. In the following sections, we confirm these predictions by performing precise model calculations.

### III. MODEL

We model a topological superconductor as a Rashba spin-orbit coupled nanowire with Zeeman energy and s-wave superconducting proximity effect. The Hamiltonian is expressed as \[7\]

\[
\mathcal{H} = \mathcal{H}_{\text{wire}} + \mathcal{H}_{\text{lead}} + \mathcal{H}_{\text{hop-LW}},
\]

(1)

\[
\mathcal{H}_{\text{wire}} = \sum_{j,\sigma} \left[ -t_{\text{wire}} c_{j+1,\sigma} c_{j,\sigma} + \mu_{\text{wire}} c_{j,\sigma} c_{j,\sigma} + \hbar \sum_{j} \left[ c_{j,\uparrow} \lambda c_{j,\uparrow} - c_{j,\downarrow} \lambda c_{j,\downarrow} \right] + \sum_{j} \left[ -\lambda c_{j-1,\downarrow} c_{j,\downarrow} + \lambda c_{j+1,\downarrow} c_{j,\downarrow} + h.c. \right] + \sum_{j} \left[ \Delta c_{j,\uparrow} c_{j,\downarrow} + h.c. \right] \right],
\]

(2)

\[
\mathcal{H}_{\text{lead}} = \sum_{j,\sigma} \left[ -t_{\text{lead}} \psi_{j+1,\sigma} \psi_{j,\sigma} + h.c. \right] - \mu_{\text{lead}} \sum_{j,\sigma} \psi_{j,\sigma} \psi_{j,\sigma},
\]

(3)

\[
\mathcal{H}_{\text{hop-LW}} = \sum_{\sigma} \left[ -t_{\text{wire}} c_{1,\sigma} \psi_{1,\sigma} - t_{\text{wire}} c_{N,\sigma} \psi_{N,\sigma} + h.c. \right].
\]

(4)

Throughout this paper, we set parameters \( \lambda = 0.3 t_{\text{wire}}, \Delta = 0.1 t_{\text{wire}}, t_{\text{lead}} = t_{\text{wire}}, \mu_{\text{lead}} = 0 \). The length of the nanowire is \( L_{\text{wire}} = 500a \) unless stated otherwise, where \( a \) is the lattice constant of the tight-binding model. The nonlocal conductance \( G = dI_R/dV_L \) (see Fig. 1(a)) is calculated as,

\[
G = \frac{e^2}{h} \int_{-\infty}^{\infty} dE \left( \frac{\partial f(E - eV)}{\partial E} \right) [T_{ee}(E) - T_{eh}(E)].
\]

(5)

Here \( T_{ee} \) (\( T_{eh} \)) is the transmission rate from electron sectors of the left lead to electron (hole) sectors of the right lead. The transmission rates are calculated with the help of Kwant, which is the Python package for the scattering...
problem in tight-binding models [80].

IV. RESULTS

A. Pristine nanowire

Figure 2(b) shows the magnetic field dependence of the conductance of the SN junction system. The system undergoes topological phase transition by changing magnetic field $h$, with the critical value of $h_c$ (see Fig. 2(a)). The trivial system ($h < h_c$) yields a nonzero and constant conductance inside the energy gap. The conductance reflects the gap-closing behavior with increasing $h$. After the phase transition and the emergence of MZMs, the conductance shows the valley structure at zero energy. The emergence of the structure can be interpreted as a signature of the topological phase.

Figure 3 illustrates the AB effect. In the trivial phase, the conductance shows almost the constant value inside the gap, with the slight Peierls phase dependence of $2\pi$-periodicity. In the case of the topological phase, the periodicity is identical to the trivial one. However, the amplitude is more prominent by a few orders of magnitude. The result demonstrates the enhancement of $2\pi$-periodic AB effect mediated by topologically protected MZMs, as mentioned in the previous section.

B. Trivial bound states

Now let us consider the effects of low-energy trivial states referred to as ps-ABSs, which appear at SN junctions with smooth gate potentials. In the previous section, we assumed a clean superconductor, where MZMs mediate Andreev reflection and amplify the $2\pi$-periodic AB effect. In this case, the absence of MZMs in the trivial phase strongly suppresses the amplitude. Meanwhile, ps-ABSs, a possible cause of the experimentally observed ZBCPs [23], can also contribute to Andreev reflection. Here we investigate the effect of ps-ABSs on nonlocal conductance. We compare the topological and trivial bound states in terms of the periodicity, amplitude, and stability of the AB effect.

The trivial ABSs are implemented by introducing smooth gate potentials at the ends of the wire. The spatial variation of the potential and the superconducting gap is

$$V(x) = \frac{V_L}{2} \left( 1 - \tanh \left( \frac{x - \Delta x_L}{\delta x_V} \right) \right) + \frac{V_R}{2} \left( 1 + \tanh \left( \frac{x - L_{wire} + \Delta x_R}{\delta x_V} \right) \right) ,$$

$$\Delta(x) = \frac{\Delta_0}{2} \left( 1 + \tanh \left( \frac{x - \Delta x_{SC}}{\delta x_{\Delta}} \right) \right) \times \frac{1}{2} \left( 1 - \tanh \left( \frac{x - L_{wire} + \Delta x_{SC}}{\delta x_{\Delta}} \right) \right) .$$

Parameters in the following calculations are $\delta x_V = 5a$, $\delta x_{\Delta} = 3a$, $\Delta x_{SC} = 17.8a$, $k_B T = 0.0015 \Delta$, $t_{lead} = 0.25t_{lead}$, $\mu_{wire} = -1.975t_{wire}$, unless stated otherwise.

The trivial and topological bound states show contrasting spatial distributions. First, we introduce the gate potential at the left end (see Fig. 3(b)). By diagonalizing the superconducting wire Hamiltonian, we find the eigenfunctions $\phi^{e,\sigma}_{\tau}(x)$ with eigenenergy $\varepsilon$, labeled by electron-hole degrees of freedom $\tau$ and spin $\sigma$. Figure 4(c, d) show the distributions of the squared wave functions of ps-ABSs and MZMs in the trivial and topological regime, respectively. The distributions are calculated by

$$|\psi_{\pm}(x)|^2 = \sum_{\tau, e, h, \sigma = \uparrow, \downarrow} \frac{|\phi^{e,\sigma}_{\tau}(x) \pm \phi^{h,\sigma}_{\tau}(x)|^2}{2}$$

These are consistent with the previous research [31].

Now we introduce the gate potentials to both ends of the wire and calculate the conductance, as illustrated in Fig. 5. The MZMs and the corresponding zero energy structure are observed in the topological regime of
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off, and sequential Andreev reflection at both junctions is unnecessary. Since the index relies on local Andreev reflection at a single SN junction, it corresponds to signatures of local conductance, which only depends on states at either end. Although the index is defined for nonlocal conductance, its value is determined primarily by the local condition. In this sense, the index encodes local information. On the other hand, Fig. 7(b) shows that “amplitude” survives only when both hoppings are finite. This dependence arises because the index relies on sequential Andreev reflection at both junctions. Therefore, “amplitude” depends on the condition of the spatially separated ends of the wire, potentially encoding the nonlocal information. In this sense, “amplitude” is a nonlocal index.

In Fig. 8 we compare the robustness of the two indices for the topological or trivial bound states. In the case of MZMs, both indices are robust against variations in potential, as expected from the topological protection. The trivial bound states can also exhibit similar behavior in certain parameter regimes for the local index “dip”, posing potential difficulty in differentiating ps-ABSs from MZMs. These regimes correspond to the case where both junctions independently satisfy the optimal condition for Andreev reflection. The dependence suggests an absence of nonlocal information in the index. On the other hand, “amplitude” of ps-ABSs mimics that of MZMs only in a limited region compared to “dip.” Even in the parameter regime where the distinction between trivial and topological bound states by the local index “dip” is difficult, “amplitude” for ps-ABSs is highly sensitive to variations of the potential profile. This result reflects the nonlocal nature of “amplitude”, which requires sequential Andreev reflection at both ends of the superconductor. In the topological case, MZMs are topologically guaranteed to be localized at the ends and remain at zero energy. On the other hand, ps-ABSs are local, and their energies are less stable than those of MZMs. Since sequential Andreev reflection and enhanced 2π-periodic AB effect require optimal conditions of multiple SN junctions, the instability of ps-ABSs is more likely to manifest in the nonlocal index than in the local one.

Furthermore, to confirm that our results do not depend on specific model systems, we performed numerical calculations for other system parameters. The results are shown in Appendices A and B. As clearly shown in Figs. 10 and 11 in Appendix, the point addressed above holds even for the parameter regime where, as shown in Fig. 7(b) the trivial ps-ABSs appear in the field range much wider than the case discussed in this section. The instability of the trivial bound states in the nonlocal index is confirmed, which implies the generality of the results and conclusions.

V. CONCLUSION

In this paper, we discussed the 2π-periodic AB effect in the context of the Majorana signature in nonlocal transport. This effect is attributed to Andreev reflection at multiple SN junctions and does not require parity restriction. We demonstrated the enhancement of the effect, where Andreev reflection mediated by MZMs strongly enhances the oscillation amplitude. The processes are robust thanks to the topological protection of MZMs, which guarantees the existence of MZMs at both ends of the superconducting wire. By introducing the two characteristic indices, we investigated the influence of trivial bound states. The trivial and topological bound states exhibit similar phase dependence in the case with the parameters set ideally for ps-ABSs to mimic MZMs. However, the robustness against variation of potential profile demonstrated striking contrast between the trivial and topological bound states. Even in a parameter regime where the local index for ps-ABSs is indistinguishable from the topological one, their nonlocal index, i.e. “amplitude”, strongly depends on the junction profile, making a sharp contrast to MZMs. The difference can enable a more explicit distinction between the trivial and topological bound states than local conductance measurements. Therefore, the observation of the effect will advance the exploration of their existence.

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Appendix A: Numerical results for other system parameters

Here we show results for other system parameters regimes. We set parameters such that the trivial bound states exist in a wider magnetic field range compared to the case considered in the main text (see Fig. 7). We set the chemical potential $\mu_{\text{wire}} = -2\mu_{\text{wire}} + 4\Delta$, Rashba spin-orbit coupling $\lambda = 0.18\mu_{\text{wire}}$, and the induced superconducting gap $\Delta = 0.026\mu_{\text{wire}}$. We set the parameters in $V(x)$ (Eq. (6)) and $\Delta(x)$ (Eq. (7)) as $\Delta_{\text{L}} = \Delta_{\text{R}} = 40a$, $\delta x_V = \delta x_{\text{L}} = 6.5a$, $\Delta_{\text{SC}} = 55a$.

In Fig. 10 we compare the robustness of the two indices with the topological or trivial bound states. As mentioned in the main text, the result shares the same characteristics as Fig. 7, the nonlocal index “amp” is more sensitive to the variation of the potential profile than the local index. The similarity of the result implies
FIG. 8. The top and bottom rows show the stabilities of the indices “dip” and “amplitude,” respectively, for changes in potential parameters. (a)–(d) demonstrate the robustness against changes in the potential height at the left and right ends. In these cases, the width of the potential profile is \( \Delta L = \Delta R = 20.5a \). (e)–(h) show the robustness against changes in the width, with the height set to be \( V_L = V_R = 0.35\Delta \). The bound states are trivial for (a), (b), (e), (f) with \( h/h_c = 0.75 \) and topological for (c), (d), (g), (h) with \( h/h_c = 1.25 \). The plots are normalized by the maximum values (dip\(_{\text{max}}\) and amp\(_{\text{max}}\)) in each dataset, which are displayed at the bottom left corners of the plots.

FIG. 9. Magnetic field dependence of the energy eigenvalues of the isolated superconductor. We set \( \mu_{\text{wire}} = -2t_{\text{wire}} + 4\Delta \), \( \lambda = 0.182t_{\text{wire}} \), \( \Delta = 0.026t_{\text{wire}} \), \( \Delta x_L = \Delta x_R = 40a \), \( \delta x_L = \delta x_R = 6.5a \), \( \Delta x_{\text{SC}} = 55a \).

that our conclusions do not rely on the fine-tuning of the system parameters.

Appendix B: Plateau in the nonlocal index and hybridization of MZMs

In Fig. 11, we show the “dip” and “amplitude” indices as functions of \( V_{\text{symm}} \equiv V_L = V_R \), with different values of the SN junction coupling \( t_{\text{LW}} \). We use the same system parameters as Appendix A. For relatively small values of \( t_{\text{LW}} \), there exists a parameter regime where both “dip” and “amplitude” for MZMs are stable. For ps-ABSs, the local index “dip” appears relatively robust against variation of \( V_L = V_R \equiv V_{\text{symm}} \). However, the nonlocal index “amplitude” is more sensitive to the potential profile, and there is no plateau in “amplitude” for ps-ABSs inside the parameter regime where “dip” is stable. In fact, the nonlocal index for ps-ABSs decays more rapidly than that of MZMs, presenting a qualitative difference. In the case of MZMs, the nonlocal index “amplitude” is robust against the changes of system parameters provided that \( t_{\text{LW}} \) is sufficiently small. However, the robustness is spoiled for the larger values of \( t_{\text{LW}} \). The probable cause of this fragility is the hybridization of the two MZMs at each end mediated via the metallic lead. The larger the value of \( t_{\text{LW}} \), the longer the hybridization becomes, leading to the level-splitting. If the energy splitting is larger than the energy scale of the temperature, the AB effect loses the benefit of the topological protection of MZMs, causing the fluctuation. Conversely, the robustness of the nonlocal index can be recovered by adjusting the temperature.
FIG. 10. The top and bottom rows show the color maps of the indices “dip” and “amplitude,” respectively, as functions of potential parameters. (a)–(d) demonstrate the robustness against changes in the potential height at the left and right ends. In these cases, the width of the potential profile is \( \Delta x_L = \Delta x_R = 40 \Delta \). \( k_B T = 0.003 \Delta \). (e)–(h) show the robustness against changes in the width, with the height set to be \( V_L = V_R = 5.5 \Delta \). \( k_B T = 0.0015 \Delta \). The bound states are trivial for (a), (b), (e), (f) with \( h/h_c = 0.75 \) and topological for (c), (d), (g), (h) with \( h/h_c = 1.25 \). The plots are normalized by the maximum values (\( \text{dip}_{\text{max}} \) and \( \text{amp}_{\text{max}} \)) in each dataset, which are displayed at the bottom left corners of the plots.

FIG. 11. Potential height dependence of dip and amplitude. The potential heights \( V_L \) and \( V_R \) are set to be symmetric, denoted as \( V_{\text{symm}} \). The vertical axes are normalized by the maximum values in each plot, which are displayed on the top right sides. (a)–(d) are for the trivial case with \( h/h_c = 0.75 \) and (e)–(h) are for the topological case with \( h/h_c = 1.25 \). The values of the hopping amplitude at the SN junctions are \( t_{LW}/t_{lead} = 0.5, 0.4, 0.3, 0.25 \) for the first to fourth row, respectively. \( k_B T = 0.003 \Delta \).
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