Deng, Hang; Zhang, Cun-Hui
Isotonic regression in multi-dimensional spaces and graphs. (English) Zbl 1462.62208
Ann. Stat. 48, No. 6, 3672-3698 (2020).

This work considers the minimax and adaptation rates in the multivariate isotonic regression setting. The minimax theorem is used as the theoretical foundation to build a case for developing estimators which lie in-between the max-min and min-max estimators over possibly smaller classes of upper and lower sets, including a subclass of block estimators. Further, based on appropriate moment conditions on the noise, the research proceeds by providing corresponding risk bounds for such general estimators for isotonic regression on graphs. The asymptotic properties of these estimates are studied in detail and in particular the block estimate is shown to possess the so-called oracle property in variable selection.

Reviewer: Dimitrios Bagkavos (Ioannina)

MSC:
62G05 Nonparametric estimation
62G08 Nonparametric regression and quantile regression
62H22 Probabilistic graphical models
62K20 Response surface designs
62C20 Minimax procedures in statistical decision theory

Keywords:
isotonic regression; multiple isotonic regression; isotonic regression on graphs; max-min estimator; min-max estimator; block estimator; lattice design; random design; minimax rate; adaptive estimation; variable selection; oracle property

Full Text: DOI arXiv Euclid

References:
[1] Ayer, M., Brunk, H. D., Ewing, G. M., Reid, W. T. and Silverman, E. (1955). An empirical distribution function for sampling
with incomplete information. Ann. Math. Stat. 26 641-647. · Zbl 0066.38502 · doi:10.1214/aoms/1177728423
[2] Bellec, P. C. (2018). Sharp oracle inequalities for least squares estimators in shape restricted regression. Ann. Statist. 46
745-780. · Zbl 1408.62066 · doi:10.1214/17-AOS1566
[3] Birgé, L. and Massart, P. (1993). Rates of convergence for minimum contrast estimators. Probab. Theory Related Fields 97
113-150. · Zbl 0805.62037 · doi:10.1007/BF01199316
[4] Brunk, H. D. (1955). Maximum likelihood estimates of monotone parameters. Ann. Math. Stat. 26 607-616. · Zbl 0066.38503
· doi:10.1214/aoms/1177728420
[5] Chatterjee, S., Guntuboyina, A. and Sen, B. (2015). On risk bounds in isotonic and other shape restricted regression problems.
Ann. Statist. 43 1774-1800. · Zbl 1317.62032 · doi:10.1214/15-AOS1324
[6] Chatterjee, S., Guntuboyina, A. and Sen, B. (2018). On matrix estimation under monotonicity constraints. Bernoulli 24
1072-1100. · Zbl 1419.62135 · doi:10.3150/16-BEJ865
[7] Deng, H. and Zhang, C.-H. (2020). Supplement to “Isotonic regression in multi-dimensional spaces and graphs.” https://doi.org/10.1214/20-AOS1947SUPP
[8] Donoho, D. L. (1990). Gelfand n-widths and the method of least squares. Preprint.
[9] Durot, C. (2007). On the \((\mathbb{L}_{\infty})_{p}\)-error of monotonicity constrained estimators. Ann. Statist. 35 1080-1104. · Zbl 1129.62024 · doi:10.1214/0090536060000001497
[10] Durot, C. (2008). Monotone nonparametric regression with random design. Math. Methods Statist. 17 327-341. · Zbl 1231.62066 · doi:10.3103/S1066530708040042
[11] Dykstra, R. L. (1983). An algorithm for restricted least squares regression. J. Amer. Statist. Assoc. 78 837-842. · Zbl 0535.62063 · doi:10.1080/01621459.1983.10477029
[12] Fokianos, K., Leucht, A. and Neumann, M. H. (2017). On integrated \((\mathbb{L}_{\infty})_{1}\)-convergence rate of an isotonic regression esti-
mator for multivariate observations. Preprint. Available at arXiv:1710.04813. · Zbl 1452.62282 · doi:10.1109/TIT.2020.3013390
[13] Gao, C., Han, F. and Zhang, C.-H. (2017). Minimax risk bounds for piecewise constant models. Preprint. Available at
arXiv:1705.06386.

[14] Grenander, U. (1956). On the theory of mortality measurement. II. Skand. Aktuarietidskr. 39 125-153 (1957). - Zbl 0077.33715

[15] Groeneboom, P. (1985). Estimating a monotone density. In Proceedings of the Berkeley Conference in Honor of Jerzy Neyman and Jack Kiefer, Vol. II (Berkeley, Calif., 1983). Wadsworth Statist./Probab. Ser. 539-555. Wadsworth, Belmont, CA. - Zbl 1373.62144

[16] Han, Q., Wang, T., Chatterjee, S. and Samworth, R. J. (2019). Isotonic regression in general dimensions. Ann. Statist. 47 2440-2471. - Zbl 1437.62124 · doi:10.1214/18-AOS1753

[17] Kyng, R., Rao, A. and Sachdeva, S. (2015). Fast, provable algorithms for isotonic regression in all \((l_p\))-norms. In Advances in Neural Information Processing Systems 2719-2727.

[18] Meyer, M. and Woodroofe, M. (2000). On the degrees of freedom in shape-restricted regression. Ann. Statist. 28 1083-1104. - Zbl 1105.62340 · doi:10.1214/aos/1015956708

[19] Prakasa Rao, B. L. S. (1969). Estimation of a unimodal density. Sankhyā Ser. A 31 23-36. - Zbl 0181.45901

[20] Robertson, T., Wright, F. T. and Dykstra, R. L. (1988). Order Restricted Statistical Inference. Wiley Series in Probability and Mathematical Statistics: Probability and Mathematical Statistics. Wiley, Chichester. - Zbl 0645.62028

[21] Stout, Q. F. (2015). Isotonic regression for multiple independent variables. Algorithmica 71 450-470. - Zbl 1312.62084 · doi:10.1007/s00453-013-9814-z

[22] van de Geer, S. (1990). Estimating a regression function. Ann. Statist. 18 907-924. - Zbl 0709.62040 · doi:10.1214/aos/1176347632

[23] van de Geer, S. (1993). Hellinger-consistency of certain nonparametric maximum likelihood estimators. Ann. Statist. 21 14-44. - Zbl 0779.62033 · doi:10.1214/aos/1176349013

[24] Wang, Y. (1996). The \(\langle L_2 \rangle\) risk of an isotonic estimate. Comm. Statist. Theory Methods 25 281-294. - Zbl 0875.62143

[25] Woodroofe, M. and Sun, J. (1993). A penalized maximum likelihood estimate of \(\langle f(0+)\rangle\) when \(\langle f\rangle\) is nonincreasing. Statist. Sinica 3 501-515. - Zbl 0822.62020

[26] Yang, F. and Barber, R. F. (2019). Contraction and uniform convergence of isotonic regression. Electron. J. Stat. 13 646-677. - Zbl 1478.62098 · doi:10.1214/18-EJS1520

[27] Zhang, C. - Zbl 1012.62045 · doi:10.1214/aos/1021379864

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.