Crossover from Antiferromagnetic Phase to Fermiology Regime in a Weakly Coupled Half-Filled Chain System

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Effects of the electron-electron umklapp process on dimensional crossovers caused by the interchain one-particle hopping, $t_\perp$, in a weakly-coupled half-filled chain system have been studied, based on the perturbative renormalization-group approach. The variance of $t_\perp$ and the umklapp process under scaling is taken into account. We found that the intrachain umklapp process causes a finite crossover value of $t_\perp$, $t_\perp^{\text{cr}}$. For $t_\perp < t_\perp^{\text{cr}}$, as the temperature decreases, the system undergoes a crossover from the Tomonaga-Luttinger (TL) liquid to an incipient one-dimensional Mott insulator and finally makes a phase transition into an antiferromagnetic long-range-ordered phase at a transition temperature $T_N$ which increases with the increasing $t_\perp$. For $t_\perp > t_\perp^{\text{cr}}$, the system undergoes a crossover from the TL liquid to the Fermiology regime where the interchain propagation of a quasi-particle is coherent. We discuss relevance of the present result to the experimentally suggested, pressure-induced crossover phenomena in the quasi-one-dimensional organic systems.

KEYWORDS: umklapp scattering, dimensional crossover, Tomonaga-Luttinger liquid, Mott insulator, SDW, Fermiology, perturbative renormalization-group, quasi-one-dimensional organic system

In one-dimensional (1D) systems, quantum fluctuations prevent the system undergo a phase transition toward a long-range ordered phase and consequently only incipient instability can exist. [1] When 1D systems are weakly coupled via a one-particle hopping, $t_\perp$, they behave as isolated 1D systems at high temperature, $T \gtrsim t_\perp$. As the temperature decreases, they undergo either of two dimensional crossovers. [2] A “two-particle crossover” converts the incipient instability to the instability toward a long-range-order, while a “one-particle crossover” converts the transverse incoherent propagation of a quasi-particle to a

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coherent one. In the latter case, below the crossover temperature, the Fermi surface effects become important. The scaling theory of dimensional crossovers based on the perturbative renormalization-group (PRG) approach \[2\] tells us which crossover the system undergoes essentially depends on the geometry (chain, ladder...) and the universality (weak-coupling or strong-coupling) of the corresponding isolated system. \[3\]

In quasi-one-dimensional organic systems, \((\text{TMTTF})_2\)X (X=Br, PF\(_6\), SCN...), a dimerization along the conducting stack makes the band effectively half-filled instead of three-quarter filled as expected from the 2:1 stoichiometry. \[4\] At ambient pressure, \((\text{TMTTF})_2\)Br is semiconducting with a shallow minimum in resistivity around \(T_\rho \sim 100\)K and undergoes a phase transition at \(T_N \sim 13\)K to a commensurate spin-density wave (CSDW) phase characterized by the SDW amplitude \(0.14\mu_B/\text{molecule}\) and the wave-number \((1/2,1/4,0)\). \[5\] Recently the irrelevance of the Fermi surface to a CSDW transition was reported in \((\text{TMTTF})_2\)SCN \[6\] which undergoes a clear metal-insulator transition accompanying ordering of the non-centrosymmetric anions (SCN\(^-\)) at \(T = 160\)K \[7\] without any anomaly of magnetic susceptibility and a phase transition to a CSDW phase occurs at \(T_N \sim 7\)K with the SDW amplitude and the wave-number being identical to those of \((\text{TMTTF})_2\)Br. \[6\] This fact strongly suggests that the Fermi surface effects are not responsible for the phase transition to the ICSDW phase in these TMTTF-salts. \[6\]

On the other hand, \((\text{TMTSF})_2\)PF\(_6\) shows metal-like behavior down to a phase transition at \(T_N \sim 12\) K to an incommensurate SDW (ICSDW) phase characterized by the SDW amplitude \(0.08\mu_B/\text{molecule}\) and the wave-vector \((0.5, 0.24 \pm 0.03, -0.06 \pm 0.20)\) \[8\] which is quite close to the calculated optimal nesting vector. \[9\] The incommensurate character is also supported through \(\mu\)SR studies, \[10\] observations of motional narrowing of the NMR line shape and non-Ohmic electrical transport caused by a pinning of the ICSDW. \[11\] It is well established that the Fermi surface nesting is responsible for the ICSDW phase of \((\text{TMTSF})_2\)PF\(_6\). \[12\] The relevance of the Fermi surface effects is also supported by the success of Fermiology arguments on the field-induced SDW phenomena in the TMTSF-salts. \[13,14\]

Recently pressure-induced crossover phenomena in \((\text{TMTTF})_2\)Br were reported, based on transport and NMR experiments. \[14,15\] \(dT_N/dP > 0\) and a commensurate SDW character are observed at low pressure \(P < P_{\text{opt}} = 5\)kbar, whereas \(dT_N/dP < 0\) and an incommensurate SDW character at low pressure \(P > P_{\text{opt}}\). The crossover between the CSDW and ICSDW regimes is coincident with the vanishing of a charge localization gap \(\Delta_\rho\). \[13\] The features of the NMR rate \(\left(1/T_1^{-1}\right)\) in the ICSDW phase at \(P > P_{\text{opt}}\) are common to \((\text{TMTSF})_2\)PF\(_6\) under ambient pressure. \[16\]

From theoretical viewpoints, the importance of the umklapp process in \((\text{TMTTF})_2\)X and \((\text{TMTSF})_2\)X was first addressed by Emery \textit{et al.} \[17\] with emphasis on the relation between the dimerization and the umklapp process. They treated the dimensionality effects in a qualitative manner by controlling the interference between the lowest-order particle-particle and particle-hole fluctuations.

On the other hand, based on the PRG approach, Bourbonnais \[2,18\] suggested that the two-particle crossover becomes possible when the one-particle crossover temperature, \(T_{x_1}\), is
strongly suppressed by the development of a charge localization gap $\Delta_p$. Then the CSDW phase transition is induced by the interchain coherence of an electron-hole pair (a $2k_F$ spin correlation). Based on the Stoner criteria (mean field treatment of the interchain coupling), Bourbonnais obtained $T_N \sim t_\perp^2 / \Delta_p$ (where $t_\perp$ denotes a renormalized $t_\perp$) which increases as $\Delta_p$ decreases under pressure. In Bourbonnais’s arguments, however, the variance of $t_\perp$ and the umklapp process under scaling is not explicitly taken into account.

It is also to be noted that based on a mean-field calculation for a one-dimensional conductor at quarter filling, Seo and Fukuyama recently found that the CSDW phase is accompanied by $4k_F$ charge ordering when the nearest neighbor Coulomb interaction is strong enough.

In this paper, we develop a scaling theory of the dimensional crossovers in the weakly-coupled half-filled chain system by taking account of the variance of $t_\perp$ and the umklapp process under scaling. It is pointed out in our previous paper that the asymptotic behavior of $t_\perp$ plays an essential role in determining which crossover the system undergoes. We start with the path integral representation of the partition function, $Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{S}$, where the action consists of four parts, $S = S_{\parallel}^{(1)} + S_{\perp}^{(1)} + S_{\parallel}^{(2)} + S_{\perp}^{(2)}$, with $S_{\parallel}^{(1)}$, $S_{\perp}^{(1)}$, $S_{\parallel}^{(2)}$, and $S_{\perp}^{(2)}$ being the actions for the intrachain one-particle hopping, interchain one-particle hopping, intrachain two-particle scattering and interchain two-particle scattering processes, respectively, which we shortly describe below. $\mathcal{D}$ symbolizes the measure of the path integral over the fermionic Grassmann variables. In Figs. 1(a), 1(b) and 1(c), we show fundamental processes included in $S_{\perp}^{(1)}$, $S_{\parallel}^{(2)}$, and $S_{\perp}^{(2)}$, respectively. The interchain two-particle processes included in $S_{\perp}^{(2)}$ are generated in the course of the scaling.

The intrachain one-particle dispersion is linearized at the Fermi points $\pm k_F$ and is given by $\varepsilon_{RR}(k_\parallel) = v_F(k_\parallel - k_F)$ and $\varepsilon_{RL}(k_\parallel) = v_F(-k_\parallel - k_F)$ with $v_F$ being the Fermi velocity. In both branches ($L$ and $R$) of bands, the energy variables, $\varepsilon_\nu (\nu = R, L)$, run over the region, $-E/2 < \varepsilon_\nu < E/2$, with $E$ denoting the bandwidth cutoff. The interchain one-particle hopping process [Fig. 1(a)] causes a dispersion, $\varepsilon(k_\perp) = -2t_\perp \cos k_\perp$. Momenta along and perpendicular to the chain are denoted by $k_\parallel$ and $k_\perp$, respectively.

At half filling, the intrachain interaction generates the “normal” [Fig. 1(b-1)] and “umklapp” [Fig. 1(b-2)] scattering processes with dimensionless scattering strengths $g_1$, $g_2$, and $g_3$ of the backward, forward and umklapp scatterings, respectively. The usual scattering strengths with a dimension of the interaction energy are $\pi v_F g_i$. The intrachain scattering strengths are related to the on-site and nearest-neighbor Coulomb repulsion, $U$ and $V$, as $\pi v_F g_1 = \pi v_F g_3 = U - 2V$ and $\pi v_F g_2 = U + 2V$ at half filling only.

The 3rd order scaling equations for the intrachain two-particle processes [Fig. 1(b)] are given by

\begin{align}
\frac{dg_1}{dl} &= -g_1^2 - g_1^3/2, \\
\frac{dG}{dl} &= -g_3^2 (1 + G/2), \\
\frac{dg_3}{dl} &= -g_3 G (1 + G/4) - g_3^3/4,
\end{align}

where $G = g_1 - 2g_2$. The scaling parameter is defined by $l = \ln[E_0/T]$ with $E_0$ and $T$.
being the initial bandwidth cutoff and the absolute temperature, respectively. The charge
degrees of freedom in the intrachain system are governed by the combination \((G, g_3)\) with
flow lines \(G^2 - g_3^2 = \text{const.}\) When the initial values, \(G(0)\) and \(g_3(0)\) satisfy the condition,
\(G(0) < |g_3(0)|\), the umklapp process becomes relevant and \((1) \sim (3)\) give the non-trivial
fixed point, \(g_1^* = 0\) and \(|g_3^*| = -G^* = 2\). In this region, a Mott gap opens in the charge
excitation spectrum and the antiferromagnetic (AF) correlation is the most dominant one
(incipient instability) in the intrachain system.

The scaling equation for \(t_\perp\) [Fig. 1(a)] is given by \([20, 21]\)
\[
d\ln t_\perp/dl = 1 - \left( g_1^2 + g_2^2 - g_1 g_2 + g_3^2/2 \right)/4. \tag{4}
\]
In Fig. 2, we show the diagrams which contribute to the r.h.s of \((4)\). The renormalization of
\(t_\perp\) comes only from the intrachain self-energy processes. The non-trivial fixed point gives
\[
d\ln t_\perp(l)/dl \xrightarrow{l \to \infty} 1/4. \tag{5}
\]
Thus for large \(l\), \(t_\perp\) grows as \(t_\perp = t_\perp(0)e^{l/4}\). Consequently, in the present system, \(t_\perp\) is
a relevant perturbation and always attains an order of the initial bandwidth, \(E_0\), at some
crossover value of the scaling parameter, \(l_{cr} = \ln[E_0/T_{cr}]\), qualitatively defined by
\[
t_\perp(l_{cr}) = E_0. \tag{6}
\]
\(T_{cr}\) corresponds to \(T_{x1}\) which was introduced by Bourbonnais and Caron. \([2]\) The system
undergoes the one-particle crossover around \(T_{cr}\).

Here we note that the similar scaling arguments give \(d\ln t_\perp/dl \xrightarrow{l \to \infty} 1 - U^2/16\pi^2v_F^2\) for the
case of a coupled non-half-filled Hubbard chain system, \([2]\) and \(d\ln t_\perp/dl \xrightarrow{l \to \infty} -U^2/32\pi^2v_F^2 - 7/8\) for the case of a coupled non-half-filled Hubbard ladder system. \([3]\) In the former case,
\(t_\perp\) rapidly grows for small \(U\) and consequently the one-particle crossover always dominants
the two-particle crossover as far as \(U\) is not extremely large. \([2]\) On the contrary, in the
latter case, a spin gap opening in the intraladder system so strongly reduces the growth of
\(t_\perp\) that there appears a region where the two-particle crossover dominates the one-particle
crossover. \([3]\) We see that, in the case of the weakly-coupled half-filled chain system, \(t_\perp\) is
relevant but the growth of the intrachain umklapp process (Mott gap opening) strongly reduces
the growth of \(t_\perp\) and consequently \(t_\perp\) grows much more slowly than in the weakly-coupled
non-half-filled chain system. Consequently, as we shall show below, there appears a region
where the two-particle crossover dominates the one-particle crossover as in the coupled ladder
system.

The action for the interchain two-particle AF-interaction [Fig. 1(c)] is written as
\[
S^{(2)}_{\perp AF} = -\frac{\pi v_F}{4} \sum_Q V_{AF} O^{*}_{AF} O_{AF} - \frac{\pi v_F}{4} \sum_Q V_{um} [O^{*}_{AF} O^{*}_{AF} + O_{AF} O_{AF}]. \tag{7}
\]
The AF particle-hole field is defined as \(O_{AF}(Q) = T^{1/2} \sum_{K,\sigma} R_\sigma(K + Q)\vec{\sigma}_{\sigma\sigma'} L_{\sigma'}(K)\), with
\(K = (k_\|, k_\perp, i\varepsilon_l), Q = (q_\|, q_\perp, i\omega_n), q_\| = 2k_F = \pi, \varepsilon_l = (2l + 1)\pi T\) and \(\omega_n = 2n\pi T\) being
fermion and boson thermal frequencies, respectively. Of course, the interchain processes contain channels other than AF: charge-density-wave, singlet superconductivity and triplet superconductivity channels. In this work we consider only the region where growth of the AF correlation dominates the other channels.

Contributions to the lowest-order scaling equations for $V_{\text{AF}}$ [Fig. 1(c-1)] and $V_{\text{um}}$ [Fig. 1(c-2)] are depicted in Figs. 3(a) and 3(b), respectively, and are written as

$$
\frac{dV_{\text{AF}}}{dl} = \frac{1}{2} t_{\perp}^2 \left( g_2^2 + 4g_3^2 \right) \cos q_{\perp} + \frac{1}{2} \left( g_2 V_{\text{AF}} + 4g_3 V_{\text{um}} \right) - \frac{1}{4} \left( V_{\text{AF}}^2 + 4V_{\text{um}}^2 \right),
$$

(8)

$$
\frac{dV_{\text{um}}}{dl} = 2t_{\perp}^2 g_2g_3 \cos q_{\perp} + 2 \left( g_2 V_{\text{um}} + g_3 V_{\text{AF}} \right) - V_{\text{AF}} V_{\text{um}},
$$

(9)

where we put $q_{\perp} = \pi$. In Figs.3(a-1), 3(a-2) and 3(a-3), we show diagrams which contribute to the first, second and third terms on the r.h.s of eq.(8), which play separate roles. Although, $V_{\text{AF}} = V_{\text{um}} = 0$ at the initial step, the first term generates a finite magnitude of $V_{\text{AF}}$. At the intermediate step, the second term induces an exponential growth of $V_{\text{AF}}$. At the final step, the third term causes divergence of $V_{\text{AF}}$ at a critical scaling parameter $l_N = \ln[\frac{E_0}{T_N}]$ defined by

$$
V_{\text{AF}}(l_N) = -\infty.
$$

(10)

In Figs.3(b-1), 3(b-2) and 3(b-3), we show diagrams which contribute to the first, second and third terms on the r.h.s of eq.(9). It follows from the structure of eq.(9) that divergences of $V_{\text{AF}}$ and $V_{\text{um}}$ occur at the same $l_N$.

To see which of $T_{\text{cr}}$ and $T_N$ is larger, we solve the coupled scaling equations (1) ~ (4), (8) and (9). For the initial conditions, we take

$$
\begin{align*}
U &= 4V = 0.4\pi v_F, \\
t_{\perp}(0) &= t_{\perp0}, \\
V_{\text{AF}}(0) &= V_{\text{um}}(0) = 0.
\end{align*}
$$

(11)

In Figs. 4(a) and 4(b), we show the scaling flows of $t_{\perp}/E_0$, $V_{\text{AF}}$ and $V_{\text{um}}$ for $t_{\perp0} = 0.05E_0$ and $0.2E_0$, where the vertical lines show locations of $l_{\text{cr}}$ and $l_N$. We see that, for $t_{\perp0} = 0.05E_0$, $T_{\text{cr}} < T_N$, while, for $t_{\perp0} = 0.2E_0$, $T_{\text{cr}} > T_N$. In Fig. 4, we also show the scaling flows of the stiffness of the intrachain charge excitation, $K_\rho = \sqrt{(2 + G)/(2 - G)}$. Note again that the low-energy asymptotics of the isolated 1D electron system at half filling is characterized by $K_\rho = 0$, which corresponds to a fully opened Mott gap.

In Fig. 5, we show a phase diagram spanned by $t_{\perp0}/E_0$ and $T/E_0$. Roughly speaking, we may regard increasing $t_{\perp0}$ as increasing applied pressure. As the temperature decreases, the effectively isolated chain systems are gradually scaled from the Tomonaga-Luttinger liquid (TL) regime to their low-energy asymptotics, the 1D Mott insulator phase characterized by $K_\rho = 0$. The gradual change of darkness in the incipient 1D Mott insulator phase in Fig. 5 schematically illustrates this situation.
There exists a crossover value of $t_{\perp}$: $t_{\perp \text{cr}} \sim 0.1E_0$. For $0 < t_{\perp 0} < t_{\perp \text{cr}}$, as the temperature decreases, the system undergoes the crossover from the TL liquid to the incipient 1D Mott insulator and finally the two-particle crossover leads the system to the phase transition into the AF state at $T_N$. Dependence of $T_N$ on $t_{\perp}$ obtained here supports the result obtained by Bourbonnais [18] based on the Stoner criteria for the interchain AF coupling and qualitatively reproduces the experimentally observed pressure dependence of $T_N$. [3] In the real quasi-one-dimensional (TMTTF)$_2$Br salts, there exists a small but finite $t_{\perp 0}$ even under ambient pressure. So it is appropriate to interpret that in our phase diagram, Fig. 5, the region where $t_{\perp}$ is very small is missing in reality. In this work, since we limited our attention only to a purely electronic system and neglected coupling of AF fluctuations to the $2k_F$ lattice distortions which becomes important in the case of strong dimerization, our view cannot cover the competition between the spin-Peierls and AF phases [23] which is actually observed in the low effective pressure region of this series of salts. [24] To reproduce the experimentally observed temperature dependence of $T_\rho$ is also beyond our scope, since the feedback effects of $t_{\perp}$ on the intrachain system is not taken into account in the present scheme.

For $t_{\perp \text{cr}} < t_{\perp 0}$, the system undergoes a crossover from the TL phase to the Fermiology (FL) regime, where the interchain propagation of a quasi-particle is coherent and the 1D incipient instability is completely lost. Based on the fact that, around the crossover scaling parameter, $l_{\text{cr}}$, the stiffness of the intrachain charge excitation is far from $K_\rho = 0$ corresponding to the fully opened Mott gap [see Fig. 4(b)], we interpret that the system undergoes the crossover from the TL phase directly to the FL regime around $T_\text{cr}$ without visible Mott gap opening.

In the FL phase, if the correlation is weak, the physical nature of the system would be well understood in terms of the topology of the anisotropic 2D Fermi surface, where the system adjusts the SDW vector to the best nesting of the Fermi surface. [12] In such a case the SDW transition temperature, $T_{\text{SDW}}$, decreases with the increasing pressure, since the pressure tends to decrease the degree of the nesting. [12]

In Fig. 6, we show how the crossover value, $t_{\perp \text{cr}}$, depends on the relative strength of the umklapp scattering, $g_3(0)/g_1(0)$, for fixed $g_1(0) = U - 2V$ and $g_2(0) = U + 2V$ with $U = 4V = 0.4\pi v_F$. We see that the crossover value, $t_{\perp \text{cr}}$, always exists for a finite umklapp scattering. When the umklapp scattering becomes less important, the AF phase shrinks. This situation supports the picture given by Emery et al. [17] that in the TMTSF-series the dimerization along the conducting stack is much weaker than that of the TMTTF-series and consequently the umklapp process becomes less important even at high-temperature region. Then $g_3(0)$ becomes much smaller and very small $t_{\perp 0}$ is enough to locate the ground state in the FL regime.

The dimensional crossovers in the present case are quite analogous to those in the weakly-coupled non-half-filled ladder system where a spin gap opening in the intraladder system strongly reduces the growth of $t_{\perp}$. [3] The 1D Mott insulator and the AF phases of the present case correspond to the spin-gap-metal (SGM) and the $d$-wave superconductivity (SCd) phase of the latter system, respectively.
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1) J. Sólyom: Adv. Phys. 28 (1979) 201.
2) C. Bourbonnais and L. G. Caron: Int. J. Mod. Phys. B 5 (1991) 1033.
3) J. Kishine and K. Yonemitsu: J. Phys. Soc. Jpn. 66 (1997) 3725.
4) D. Jérôme: Science 252 (1991) 1509.
5) T. Nakamura, T. Nobutoki, Y. Kobayashi, T. Takahashi and G. Saito: Synth. Met. 74 (1995) 1293.
6) T. Nakamura, R. Kinami, T. Takahashi and G. Saito: Synth. Met. 86 (1997) 2053.
7) C. Coulon et al.: Phys. Rev. B 26 (1982) 6322.
8) T. Takahashi, Y. Maniwa, H. Kawamura and G. Saito: J. Phys. Soc. Jpn. 55 (1986) 1364.
9) L. Ducasse, M. Abderrabba and B. Gallois: J. Phys. C, Solid State Phys. 18 (1985) L947.
10) Le, L.P. et al.: Europhys. Lett. 15 (1991) 547.
11) W. Kang, S. Tomić, J. R. Cooper and D. Jérôme: Phys. Rev. B 41 (1990) 4862.
12) K. Yamaji: J. Phys. Soc. Jpn. 52 (1983) 1361.
13) L. P. Gorkov and A. G. Lebed: J. Phys. Lett. 45 (1984) L440.
14) K. Yamaji: J. Phys. Soc. Jpn. 54 (1985) 1034.
15) B. J. Klemme, S. E. Brown, P. Wzietek, G. Kriza, P. Batail, D. Jerome and J. M. Fabre: Phys. Rev. Lett. 75 (1995) 2408.
16) B. J. Klemme, S. E. Brown, P. Wzietek, D. Jerome and J. M. Fabre: J. Phys. I France 6 (1996) 1745.
17) V. J. Emery, R. Bruinsma and S. Barisic: Phys. Rev. Lett. 48 (1982) 1039.
18) C. Bourbonnais: Synth. Met. 84 (1997) 19.
19) H. Seo and H. Fukuyama: J. Phys. Soc. Jpn. 66 (1997) 1249.
20) M. Kimura: Prog. Theor. Phys. 53 (1975) 955.
21) C. Bourbonnais: J. Phys. I France 3 (1993) 143.
22) H. J. Schulz: Phys. Rev. Lett. 64 (1990) 2831.
23) S. Inagaki and H. Fukuyama: J. Phys. Soc. Jpn. 52 (1983) 2504.
24) J. P. Pouget et al.: Mol. Cryst. Liq. Cryst. 79 (1982) 485.
Fig. 1: A zigzag line in (a) represents the *interchain* one-particle hopping amplitude, $t_{\perp}$. White and black circles represent the *intrachain* two-particle “normal” (b-1) and “umklapp” (b-2) scatterings, respectively. White and black squares represent the *interchain* tow-particle antiferromagnetic (AF) interaction (c-1) and the umklapp process between electrons on different chains (c-2). The solid and broken lines represent the propagators for the right-moving and left-moving electrons, respectively. $i$ and $j$ denote different chain indices.

Fig. 2: Diagrams which contribute to the scaling of $t_{\perp}$.

Fig. 3: Some typical diagrams which contribute to the scaling of $V_{\text{AF}}$ (a) and $V_{\text{um}}$ (b).

Fig. 4: Scaling flows of $t_{\perp}/E_0$, $V_{\text{AF}}$ and $V_{\text{um}}$ for $t_{\perp 0} = 0.05E_0$ (a) and $0.2E_0$ (b), where the vertical lines show locations of $l_{\text{cr}}$ and $l_N$. A broken line represents the scaling flow of the stiffness of the intrachain charge excitation, $K_{\rho}$.

Fig. 5: Phase diagram of a weakly coupled chain system at half filling. The abbreviations are as follows: **TL**= Tomonaga-Luttinger liquid, **1DMott**=incipient one-dimensional Mott insulator, **FL**= Fermiology regime. Thick broken lines denote the crossover boundaries.

Fig. 6: Dependence of the crossover value $t_{\perp \text{cr}}$ on the ratio $g_3(0)/g_1(0)$ for fixed $g_1(0) = U - 2V$ and $g_2(0) = U + 2V$ with $U = 4V = 0.4\pi v_F$. 
(a) $t_{\perp 0} = 0.05E_0$

(b) $t_{\perp 0} = 0.2E_0$
