3D Smith charts scattering parameters frequency-dependent orientation analysis and complex-scalar multi-parameter characterization applied to Peano reconfigurable vanadium dioxide inductors

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Recently, the field of Metal-Insulator-Transition (MIT) materials has emerged as an unconventional solution for novel energy efficient electronic functions, such as steep slope subthermionic switches, neuromorphic hardware, reconfigurable radiofrequency functions, new types of sensors, terahertz and optoelectronic devices. Employing radiofrequency (RF) electronic circuits with a MIT material like vanadium Dioxide, VO2, requires appropriate characterization tools and fabrication processes. In this work, we develop and use 3D Smith charts for devices and circuits having complex frequency dependences, like the ones resulting using MIT materials. The novel foundation of a 3D Smith chart involves here the geometrical fundamental notions of oriented curvature and variable homothety in order to clarify first theoretical inconsistencies in Foster and Non Foster circuits, where the driving point impedances exhibit mixed clockwise and counter-clockwise frequency dependent (oriented) paths on the Smith chart as frequency increases. We show here the unique visualization capability of a 3D Smith chart, which allows to quantify orientation over variable frequency. The new 3D Smith chart is applied as a joint complex-scalar 3D multi-parameter modelling and characterization environment for reconfigurable RF design exploiting Metal-Insulator-Transition (MIT) materials. We report fabricated inductors with record quality factors using VO2 phase transition to program multiple tuning states, operating in the range 4 GHz to 10 GHz.

The Smith chart, invented in 19391, is a graphical tool widely used in various fields of electrical engineering and applied physics when dealing with frequency dependent reflection coefficients or impedances. The Smith chart is extensively employed in the design/measurement stage of a large variety of circuits, from metasurfaces2 to coils3 (Supplementary Fig. 3 in3) or scanning microwave microscopy4, while being mostly present in microwave-terahertz frequency region in the design and characterization of antennas5, transmission lines6,7.
The 3D Smith chart proposed in\(^1\) generalizes the Smith chart (which is limited within the unit circle to circuits with reflection coefficients \(\Gamma = \Gamma_r + j\Gamma_i\) magnitudes smaller than unity Fig. 1a) onto the Riemann sphere in order to make it usable for all circuits (Fig. 1b,c) further conveying the theoretical support and advance for an intuitive spherical drawing for which the first insights have been presented in\(^1\). The next developments of the 3D Smith chart\(^13,14\) propose a Java tool with the 3D Smith chart which additionally displays the group delay and amplifier stability circles too.

An essential drawback of the Smith chart and previous 3D Smith chart representations\(^1\)–\(^14\) is the lack of visualization of the variable parameter (frequency), thus the orientation changes and dynamics of the scattering parameters (S) frequency dependency being impossible to be pictured. Although many circuits exhibit a clockwise orientation of their driving point impedances and reflection parameters curves as frequency increases\(^2\)–\(^5,9\), \((–\text{ unspecified})^6,10\), the absence of a clockwise motion (i.e. discontinuity points or counter-clockwise motion) was often reported leading to diverse interpretations. In active devices (transistors), as for example in\(^15,16\) it is referred as “kink-phenomenon”, in lossless (purely reactive) non-Foster circuits (such as negative capacitors and inductors)\(^17\)–\(^20\), as an intrinsic phenomenon. Unfortunately, this counter-clockwise dynamics phenomenon recurrently exists in lossy circuits with non-Foster elements\(^23\)–\(^24\). Led to some misleading conclusions: in\(^21\)–\(^24\) the authors assume that the existence of this phenomenon proves the presence of a non-Foster element, while in\(^25\) it is stated that passive linear devices cannot exhibit driving point immittances with counter-clockwise frequency dependency on the Smith chart.

Figure 1. Smith chart limitations and clockwise and counter-clockwise frequency dependency of Foster and non-Foster elements on a newly introduced frequency dependent 3D Smith chart. (a) Smith chart. (b) Extended Smith chart (c), 3D Smith chart as in 2018 (without frequency dependency). (d) 3D Smith chart representation of the two port reflection coefficient and newly implemented frequency dependent 3D Smith chart representation for it. For a capacitor with purely negative capacitance and an inductor with positive inductance their reflection coefficient frequency representation overlaps on the Smith chart for a wide frequency range, their orientation changes cannot be distinguished. On the newly implemented frequency dependent 3D Smith chart (when compared to our previous works in\(^1,13,14\) one can clearly see their clockwise motion with increasing frequency for the inductor while the counter-clockwise motion for the capacitor with negative capacitance. (e) 3D Smith chart representation of the reflection coefficient and newly implemented frequency dependent 3D Smith chart for a negative valued inductor and capacitor with positive capacitance. Again, their trace is identical on a Smith chart, their intrinsic opposed frequency dependency cannot be seen. On the 3D Smith chart one can directly see the clockwise motion of the reflection coefficient of the capacitor and the counter-clockwise motion of the reflection coefficient of the negative valued inductor.
Because of their lack in zooming capacity, 2D Smith chart representations may oversee also the changes of orientation occurring for the input impedances and reflection coefficients (while increasingly sweeping the frequency), in simple passive networks too. These reversals occur in networks with only Foster elements too as empirically observed in [17]. Otherwise orientation reversal phenomenon which occurs in lossy networks with Foster elements is often overlooked or seen as an interpolation error and its presence is often ignored. Since the paths of the reflection coefficients of distinct circuits may coincide within a specific frequency on the Smith chart (as for negative inductors and positive capacitors and vice versa [17, 19, 20, 26]), a frequency dependency visualization is missing in order to get an insight beyond the reflection coefficients paths, regarding their dynamics within it.

From the (differential) geometrical point of view the input impedance and reflection coefficients are particular cases of parametric curves of the frequency variable for each single RF circuit. Their parametric curves equations describe more than a simple geometrical image (their path on the Smith chart), they also contain the information about their motion on it (as for example: orientation and speed)

For this purpose, we first introduce here the notion of oriented curvature $k^\star$ (in our case, frequency dependent $k(\omega)$ where $\omega$ represents the angular frequency and apply it to the reflection coefficients analysis. We prove the mixed clockwise-counter-clockwise orientation phenomenon in lossy circuits with Foster elements and propose a frequency orientation quantification (while increasing the sweeping range) with a new implementation within the 3D Smith chart tool by using the topology of the Riemann sphere [29]. Thus, while (increasingly) sweeping the frequency, a new vision to detect this orientation reversal for both Foster and Non-Foster [27–29, 31–33] elements-based networks is first presented. This phenomenon was impossible to quantify for a 2D parametrical plot such as the Smith chart or basic 3D Smith chart (where only the Riemann sphere surface is used).

Further the 3D Smith chart is additionally exploited and developed here in order to display simultaneously parameters needed in reconfigurable frequency designs while dealing with inductors. Frequency dependent inductances based on the $Y_{\pi}$ [31–36] admittance parameter (shunt models $Lshunt$), series models of inductances $Lseries$ based on the $Y_{\pi}$ [37, 38] parameter, quality factors [24–39] ($Q$), self-resonant frequencies, Smith chart information [39] ($I$) need ideally all to be simultaneously modeled and analyzed over a wide frequency range during the designs and characterizing stages of inductors. This leads [34–39] to a larger number of graphics or to different scaling in order to grasp all parameters of interest. Here all these factors are implemented and analyzed on a single combined mode of display using the 3D Smith chart topology and different perspectives.

The newly developed capabilities are particularly suited to explore reconfigurable microwave characteristics, here with reconfigurable CMOS-compatible inductors equivalent circuit modelling for microwaves frequencies using a phase change (PC) material like vanadium Dioxide (VO$_2$) [37, 38, 40–49] for tuning the values of inductance. Indeed, it is known that VO$_2$ behaves like an insulator under its phase transition temperature $T_c = 68^\circ C$ (or higher when doped) [43] with monoclinic crystal structure [40] while deposited on SiO$_2$/Si substrates. Because of its ease of integration, reversible insulator to metal transition (IMT), low transition temperature and fast switching time, the employment of VO$_2$ as a reconfigurable radio frequency (RF) material has been just recently investigated for a variety of RF-reconfigurable devices [37, 38, 40, 44, 46, 48]. Still, much of existing studies are carried out in the frequency range of terahertz or far-infrared [41, 43, 46, 48], leaving (RF) VO$_2$ a largely uncharted area for exploration in development. The conductivity levels of VO$_2$ in its insulating (off) state and in its conductive (on) state vary over a wide range depending on the substrate [37, 38, 40–46] causing limitations in the RF devices performances (being below 50,000 S/m for SiO$_2$/Si depositions in the on state).

The on state (limited) conductivity levels of VO$_2$ restricted the maximum quality factors ($Q_{\text{max, on}}$) of the reconfigurable inductors fabricated with this PC material to sub-unitary values [38], or to values below three in [37] for CMOS compatible processes on SiO$_2$/Si substrates.

Here, after evaluating the VO$_2$ conductive/insulating properties, using the new implemented multi-parameter displays, we design and fabricate a new type of SiO$_2$/Si CMOS compatible reconfigurable inductors with VO$_2$ switches based on Peano curves [50] and extract their equivalent circuit while analyzing their behavior in a complex-scalar 3D Smith chart-based environment. The aim was to improve the performances obtained in [37] on SiO$_2$/Si substrates in terms of: $Q_{\text{max, on}}$ tuning range and $Q_{\text{max, on}}/Q_{\text{max, off}}$ ratio (where $Q_{\text{max, off}}$ denotes the maximum quality factor in the insulating phase of the VO$_2$).

The experimentally fabricated and measured reconfigurable inductors improve by 2.33 times the $Q_{\text{max, on}}$ values previously reported in [37] for the VO$_2$ based inductors fabricated within the same CMOS technology on SiO$_2$/Si substrate (and by orders of magnitude in respect to [36]), while also increasing the number of reconfiguring states too (from two in [37] to three here). Further due to their original geometry the inductors exhibit a tuning range of 77% (improving the 55% in [37] or 32% in [36]) and have a $Q_{\text{max, on}}/Q_{\text{max, off}}$ ratio of 0.87 unlike 0.27 in our previous work [37]. It is worth pointing out that the inductor while facing the limited conductivity levels of VO$_2$ on SiO$_2$/Si substrates, reveals a 2.33 better $Q_{\text{max, on}}$ value in respect to other VO$_2$ based reconfigurable inductors such as the SiO$_2$/Sapphire inductors reported in [36] where the conductivity levels of VO$_2$ exceed 300,000 S/m.

**Oriented Curvature of Input Impedances, Reflection Coefficients, Slope of Reactance and 3D Smith Chart Implementation of Frequency Dependency Orientation**

Based on $k(\omega)$ [38] described in detail within the Supplementary Material, we show that the changes in sign of the reactance frequency derivative do not always imply changes in orientation neither for the input impedance nor for the reflection coefficient of 1-port networks (when losses occur). We provide the conditions and equations under which one may have the same orientation (more details in Supplementary Section 1) for both reflection coefficient and input impedances. The lossless (reactive) cases (purely Foster [17, 19, 27] and non-Foster [17–26]) become particular cases where the reflection coefficients are direct inverse (Mobius) and indirect inverse transformations [30] of the oriented imaginary axes of the impedance plane. The clockwise and counter-clockwise motions on circles are a consequence of the reactance slope and sign in the lossless cases.
By introducing the geometrical notion of oriented curvature in this field we prove that the assumptions made by other authors may not apply (see Supplementary Section 1). Further, as seen also for the input impedance of an antenna in, the negative frequency derivative of the reactance of a lossy 1-port network does not imply counter-clockwise motions in the case of lossy 1-port networks. The paths of both 1-port and 2-port networks (such as in Fig. 1) become simple consequences of the magnitude and sign changes of $k(\omega)$ (\text{sgn}(k(\omega))):

For any parametric curve $C(j\omega)$ (all reflection coefficients and input impedances are particular cases of frequency dependent parametric curves) its corresponding $k(\omega)$ is given by (1) (b).

$$C(j\omega) = a(\omega) + jb(\omega)$$

(a) $k(\omega) = \frac{-b'(\omega)a''(\omega) + a'(\omega)b''(\omega)}{(a'(\omega)^2 + b'(\omega)^2)^{3/2}} = \frac{a'(\omega)b'(\omega) - b'(\omega)a'(\omega)}{(a'(\omega)^2 + b'(\omega)^2)^{3/2}}$ (b)

(1)

Figure 2 illustrates the concept of oriented curvature along several frequency dependent curves.

Consider now a 1-port network terminated on a resistive load $r$. The input impedance is given by (2), where $r_m(\omega)$ denotes its resistive part and $x_m(\omega)$ its reactive part, while its reflection coefficient is given by (3). Computing the oriented curvature values for both of them (Supplementary Section 1) we get $k_m(\omega)$ (the oriented curvature of the input impedance) and $k_{zm}(\omega)$ (the oriented curvature of the 1-port reflection coefficient) as (4) and (5).

$$z_m(j\omega) = r_m(\omega) + jx_m(\omega)$$

$$\Gamma_{zm}(j\omega) = \frac{z_m(j\omega)}{r} - 1$$

$$k_m(\omega) = \frac{r_m'(\omega)}{(r_m''(\omega)^2 + x_m''(\omega)^2)^{3/2}}$$

$$k_{zm}(\omega) = \frac{r_m'(\omega)^2[(r + r_m(\omega))^2 + x_m(\omega)^2]^{3/2}}{2(r_m'(\omega)^2 + x_m''(\omega)^2)^{3/2}}$$

(4)

(5)
Figure 3. Reflection coefficient orientation changes and the sign of the oriented curvature for different circuits. (a) Smith chart representation of the reflection coefficient for a 1-port negative capacitance (purely non-Foster circuit) and a positive inductance (purely Foster). For a capacitor with purely negative capacitance and an inductor with positive inductance their reflection coefficients \( \Gamma_{12m}(j\omega) \) overlap on the Smith chart on a wide frequency range. Their opposite orientation is given by the different sign of their oriented curvature \( k_{zmNF}(\omega) \). Their same path is given by the same absolute value of their oriented curvature. (b) On the newly implemented frequency dependent 3D Smith chart one can see the clockwise motion with increasing frequency for the inductor with positive inductance and the counter-clockwise motion for the negative valued capacitor, the motion is on the contour of the equatorial plane (lossless circuits). (c) Mixed motion for a fabricated circuit containing non-Foster (lossy elements). (d) Mixed clockwise and counter-clockwise motion of the reflection coefficient of a passive lossy network described by the positive real function \( z_m(s) \) with the 1 port reflection coefficient \( \Gamma_{12m}(j\omega) \). The reflection coefficient has a clockwise orientation from \(-2 < \omega < -0.28\) and for \(0.28 < \omega < 2\), while counter-clockwise orientation for \(-0.28 < \omega < 0\). The sign changes of its 1-port reflection coefficient curvature \( k_{zmNF}(\omega) \) generates the changes of orientation of its path on the Smith chart. It is interesting to notice that mixed motion can exist on limited bandwidth also for lossy circuits with only Foster elements and thus that the counter-clockwise motion is by no means a prove of an existence of a non-Foster element in the network. A more detailed description on oriented curvature and 1-port and two port networks is given in Supplementary Section 1.

Denoting with \( x_{mF}(\omega) \) and \( B_{mF}(\omega) \) the reactance and susceptance of purely Foster elements and with \( x_{mNF}(\omega) \) and \( B_{mNF}(\omega) \) the ones for Non Foster elements we obtain: for Foster networks \( r_m(\omega) = 0 \) while \( \frac{dx_{mF}(\omega)}{d\omega} > 0 \) and \( \frac{dx_{mNF}(\omega)}{d\omega} < 0 \) and using (3)-(4) we get the input impedance and 1–port reflection coefficient curvatures for them as: \( k_{zmF}(\omega) = 0 \) and \( k_{zmNF}(\omega) = -1 \) (Supplementary Section 1). For Non-Foster networks \( r_m(\omega) = 0 \) too, while \( \frac{dx_{mNF}(\omega)}{d\omega} < 0 \) and \( \frac{dx_{mNF}(\omega)}{d\omega} > 0 \) hold thus via (4) and (5) we get the input impedance and 1–port reflection coefficient curvatures for them as \( k_{NFmF}(\omega) = 0 \) and \( k_{NFmNF}(\omega) = 1 \).

In the case of two port networks with equal port impedances similar computations can be done for purely Foster and non-Foster elements resulting in the corresponding reflection coefficients \( \Gamma_{21mF} \) and \( \Gamma_{21mNF} \) (6) with their corresponding oriented curvatures (computed in Supplementary Section 1) \( k_{zmF}(\omega) = -2 \) and \( k_{zmNF}(\omega) = 2 \).

\[
\Gamma_{21mF}(j\omega) = \frac{j x_{mF}(\omega)}{x_{mF}(\omega) + 1} \quad \Gamma_{21mNF}(j\omega) = \frac{-j x_{mNF}(\omega)}{-j x_{mNF}(\omega) + 2}
\]

Their oriented curvature magnitudes explain the reflection coefficients paths on 0.5 radius circles in Fig. 1d,e on the Smith chart and 3D Smith chart (see Supplementary Section 1). Their oriented curvature opposite signs in (6) determines their reversed orientation.

The reflection coefficients of purely reactive 1-port elements are given in Fig. 3a–d (purely reactive Foster and non-Foster circuits in Fig. 3a, lossy circuits with non-Foster elements in Fig. 3c and lossy circuits with Foster...
The results in Fig. 3d show that reflection coefficients and input impedance orientation reversal can occur at lossy 1-port networks containing only Foster elements too.

The results plotted in Figs. 1,e and 3b,c show the new 3D Smith chart implementations capable of detecting orientation changes phenomenon.

The main new insight is given by the representation of the frequency parameter over the 3D Smith chart representation of the reflection coefficient $\omega S_{11}$ via a variable homothety with its center in the center of the 3D Smith chart: each frequency that corresponds to a point of the 3D Smith chart reflection point of $\omega S_{11}$ will be displayed as a segment on the line that passes from the center of the 3D sphere and the point of the 3D Smith chart surface curve of $\omega S_{11}$. The length of the segment will be given by the normalized frequency and the direction will be outwards of the surface of the 3D sphere. Figure 1d displays in the 3D Smith chart surrounding space the counter-clockwise dynamics of the two port negative capacitor reflection coefficient while the clockwise dynamics of the two port reflection coefficient of the positive inductor. In Fig. 1e one may see the clockwise frequency increasing orientation of the reflection coefficient of the positive capacitor and the counter-clockwise orientation of the negative inductor. (Supplementary Section 2 describes the 3D implementation in detail).

The Smith chart plot can detect the magnitude of the curvature $|k_{F,2D}(\omega)|$ (which gives the path of the reflection coefficient-curve shape) but cannot perceive its sign which determines its direction; the new 3D Smith chart frequency orientation quantification and visualization implementation (the frequency sweeping is always increasing in our modelling) detects its sign (see additional video) and thus its orientation. In the cases presented in Fig. 1d,e, $|k_{F,2D}(\omega)|$ is constant but not zero thus the shape of the curves is a circle. The same happens in Fig. 3a,b. In more complex circuits one does not deal with reflection coefficients curves with constant curvatures anymore, $k_{F,2D}(\omega)$, $k_{F,2D}(\omega)$ alternate in sign and magnitude values exhibiting orientation changes for both Foster and non-Foster circuits as seen in Fig. 3c,d. In Fig. 3d it can be seen that even a network characterized by a positive real function can generate mixed oriented curvature in its input impedance and reflection coefficient. These reversals of...
orientation may be easy overlooked on the Smith chart if the zooming scales are not properly chosen but using (4) and (5) this is clearly discovered in Fig. 3d within the sign changes of $\omega_k$.$^{1,3}$

New 3D visualization insights of frequency dependent series and shunt inductances and quality factors. The S parameters of the inductors are directly converted by the new implementations in the 3D Smith conceptual software tool into the series inductance model $L_{\text{series}}(\omega)$ and shunt inductance model $L_{\text{shunt}}(\omega)$ using classical conversion techniques of two port parameters (see Supplementary Section 2). The series and shunt inductances values are then normalized to their maximum value over the frequency range of interest and we get the corresponding normalized values $L_{\text{series}}(\omega)$ and $L_{\text{shunt}}(\omega)$. The reflection parameter $S_{11}(j\omega)$ of the inductor is then plotted first on the surface of the 3D Smith chart as $\omega_{S_{11}}$. Then the 3D space surrounding the 3D Smith chart is used by means of a variable homothety with the homothetic center in the center of the sphere through the $\omega_{S_{11}}$. The $\omega_{S_{11}}$ parameter is sent now here to another point in 3D at a distance corresponding to $L_{\text{series}}$ or $L_{\text{shunt}}$.
and enters the capacitive hemisphere too. It is interesting to notice that the display contains already more information than \( \text{d} \), \( S \) instances (in some) identical (no resistive losses in their equivalent Pi model), thus a change of hemisphere of the zeros of the \( S \) inductances and \( Q \) of the inductor with 2 switches in on state, for the 600 nm long switches, 1.5 \( \mu \)m and 2.3 \( \mu \)m. \( \text{f} \), Simultaneous 3D Smith chart representation for the 4 GHz-10 GHz frequency of the \( L_{\text{series}} \), \( Q \) and \( S_{11} \) parameters for the three different inductors with 600 nm long switches, 1.5 \( \mu \)m and 2.3 \( \mu \)m long switches.

\[ L_{\text{series}} \cdot shunt_{3d}(\omega) = (L_{\text{seriesN}} \cdot shunt_{3d}(\omega) + 1) \ast S_{11s}(\omega) \]  

(7)

\( Q(\omega) \) of the inductors is also computed by the new 3D Smith chart tool implementation using classical conversion formulas (from the \( S \) parameters) (see Supplementary Section 2) and normalized to its maximum value over the frequency range of interest obtaining \( Q_{3}(\omega) \). Using the 3D representation of the \( L_{\text{series}}, shunt_{3d}(\omega) \) curves we then use the normal plane of the curves to associate to each (frequency) point of the curves the \( Q(\omega) \) as a cylinder \( Q_{3d}(\omega) \) of variable radius associated to its normalized value \( Q_{3}(\omega) \). In Fig. 4 one may see the new representations done for the novel fabricated Peano inductor whose performances are compared with the spiral inductor in\(^37\) (the description on the new inductor design will be presented in the following section).

The new implementation allows the concurrent view of complex valued-scalar parameters: \( S(\omega), L_{\text{series}}, shunt_{3d}(\omega), Q_{3d}(\omega) \) and frequency (not plotted in Fig. 4 since the dynamics of \( S_{11}(\omega) \) is clockwise anyway in this case, unlike the cases presented in Fig. 3). The use of different perspectives and the topology of the 3D Smith chart permits one thus to simultaneously analyze complex parameters (Smith chart) and visualize series and shunt inductances and \( Qs \) all on the same interactive display. This plays an insightful role in investigation for directly understanding multiple phenomenon on a single view. The information contained in Fig. 4a–c can be visualized together using three scaling on a common 2D plot, however still without having any information on \( S_{11}(\omega) \) of the inductor. In Fig. 4d we may see just \( S_{11s}(\omega) \) and \( L_{\text{series}}, shunt_{3d}(\omega) \), the display contains already more information than in the 2D Fig. 4a, allowing us to understand that the series inductance model is linear for both analyzed inductors and that \( S_{11s}(\omega) \) is still in the East hemisphere (in the 4 GHz-10 GHz frequency range) (inductive). Additionally, the zeros of the \( S_{11}(\omega) \) are strongly related to the zeros of the \( Y_{11}(\omega) \) of an inductor and under certain circumstances (in some) identical (no resistive losses in their equivalent Pi model), thus a change of hemisphere of the \( S_{11}(\omega) \) is strongly related to the self-resonances of the inductors model implying in most cases that the \( Q \) fails to be positive anymore (see Supplementary Section 3). In Fig. 4e we can see \( S_{11s}(\omega), L_{\text{series}}, shunt_{3d}(\omega) \) and \( Q_{3d}(\omega) \) for the 4 GHz-10 GHz frequency range, again the information contained offers an insight on three parameters impossible to visualize together in 2D. In Fig. 4f we may see how by increasing the analysis range up to 15 GHz the \( Q \) for both inductors become zero, while the \( S_{11s}(\omega) \) enters the capacitive hemisphere too. It is interesting to notice that the series inductance model stays linear. Using the 3D Smith chart tool (see Supplementary Video) one may see the exact frequencies at which \( S_{11s}(\omega) \) changes hemisphere and the frequency for which \( Q \) becomes zero (the 3D gen-
Generalized cylinders become curves—their radius becomes zero (at 14.44 GHz for both inductors). In Fig. 4g, the 3D Smith chart enters the interior of the 3D Smith chart from that frequency point. In Fig. 4h, we may see the parameters $\omega S_1$, $\omega L_{\text{shunt}}$, and $\omega Q_D$, clearly $\omega Q_D$ becomes zero once $\omega L_{\text{shunt}}$ enters the 3D Smith chart. In Fig. 4i, all these parameters are shown together.

Peano reconfigurable inductors using VO$_2$ switches. As a case study of the 3D Smith chart and its usefulness for radiofrequency characterization with VO$_2$, and with the aim to improve our reported results (in terms of $Q_{\text{max}}$, $Q_{\text{max,off}}$, tuning states and tuning range) we have first designed, fabricated and then extracted its equivalent circuit a reconfigurable inductor based on the Peano curve of order 2 by means of VO$_2$ switches (Figs. 5–8).

The design of the inductor was done using the similar procedures as in $^{37}$, targeting an extracted $L_{\text{series}}$ of around 1.5 nH in the off state of the both switches and of 0.9 nH in the on state of both while reaching around 1.3 nH if only one switch is used, while targeting to maximize the $Q$ within 4–10 GHz frequency in the on state of the VO$_2$ where conductivity levels prove to be below 50,000 S/m (please see the Fig. 9 in the Methods).

Their Peano geometry is presented in Fig. 5. In Fig. 5a one may see the cross-sectional view of the technology used. The inductors were fabricated using standard microelectronic processes starting with a high-resistivity (10000 Ω·cm) 525 μm thick silicon substrate. A 300 nm thick amorphous silicon layer was first deposited to improve radiofrequency performances $^{52,53}$. The substrate was then passivated with 500 nm SiO$_2$ deposited by sputtering. 140 nm-thick VO$_2$ and films a Pulsed Laser Deposition (PLD). The film was then patterned using photolithography followed by dry etching and Cr (20 nm)/Al (400 nm) bi-layer was deposited to contact the patterned VO$_2$ film. This thin contact layer allowed here the realization of smaller than 0.6 μm gaps between the contact pads (unlike 2 μm in our previous work$^{57}$ or 1 μm in$^{54}$). Additionally, a 2.4 μm-thick Al layer was deposited on top of these contact pads by conventional lift-off methods to provide low RF losses (to create the final CPW elements), the photo of the fabricated inductor being shown in Fig. 5b.

VO$_2$ limited on state conductivity levels on Si/SiO$_2$ substrates constitute a challenge for the reconfigurable VO$_2$ RF design$^{54}$, while to reduce its impact, 1μm switches were employed in the fabricated designs$^{37,54}$. 

**Figure 7.** Proposed equivalent circuit of the Peano reconfigurable inductors and VO$_2$ switches in the off and on state for the 4 GHz–10 GHz frequency range. (a) Off state modelling of the VO$_2$ switch: $R_{\text{VO2-off}}$ is modelling the undesired losses in between the adjacent turns where the VO$_2$ switches are present. Ideally $R_{\text{VO2-off}} = \infty$, however the non-zero conductivity of VO$_2$ (whose measurement in DC is present in Fig. 9-Methods) will generate undesired conductive losses between turns. Similarly, the frequency dependent dielectric constant of the VO$_2$ can in theory generate undesired couplings between the adjacent turns where the VO$_2$ is present: $C_{\text{VO2-ground1}}$, $C_{\text{VO2-ground2}}$, and $C_{\text{VO2-ground3}}$. $R_{\text{sub1}}$, $R_{\text{sub2}}$ describe the RF losses within the various substrates layers. $R_p$ is modelling the undesired conductive losses between the adjacent turns due to the VO$_2$ conductivity. In the off state we would like $R_p$ to be $\infty$, however the conductive losses in the VO$_2$ switches will make this term being un-unneglectable. $R_s$ is modelling the series resistance-dependent on the Al deposition and trace width, counterintuitive, the on/off resistances corresponding to the current paths through the turns $R_{\text{turns-off}}$, but including too the VO$_2$ resistive losses $R_{\text{VO2-off}}$ in the on state of the switches. (c) On state modelling of the VO$_2$ switch: $R_{\text{VO2-on}}$ is modelling the undesired resistive losses due to the limited conductivity of the VO$_2$ in the on state. This resistance will contribute to a higher value of the $R_s$ in the on state since the current will pass through the switches.
The small length of the VO$_2$ switches obtained here, below 0.6 $\mu$m, at the limit of photolithography, minimizes the losses while in the on state (measured as 48,000 S/m), while their increased width (120 $\mu$m) contributes too to this effort (a tilted photo of the switch in Fig. 5b, is in detail in (Supplementary Section 3). Switch photos and current distributions are further shown in Fig. 5c–f.

In order to validate this claim we additionally fabricated inductors with 1.5 $\mu$m and 2.3 switches lengths (Fig. 6a–c and visualized their underwhelming performances in Fig. 6c,d. The inductor has been simulated in the Ansys HFSS commercial software tool while visualizing the extracted inductances and $Q$ on the new 3D Smith chart tool. The position of the VO$_2$ switches was optimized in order to maximize $Q_{\text{max,on}}/Q_{\text{max,off}}$, tuning states and tuning range: the final current distribution at 5 GHz being shown as simulated in Fig. 5c (when both switches are off), Fig. 5e when one switch is off and one on and Fig. 5f with both switches on.

The measured inductances and $Q$s plotted in Fig. 4a,b are compared to our previous work$^{37}$ for the on state of the inductor with two switches show (“on” when measured at 100 °C): Peano inductors more than double the $Q$s while also dealing with a smaller inductance (0.9 nH unlike 1.35 nH in$^{37}$, while usually the $Q_{\text{max}}$ decrease a lot while using lower inductances$^{34}$) for the 5GHz-10 GHz frequency range. Further the series inductance is stable with an average value of 0.95 nH within the 4GHz-10 GHz frequency range and thus over-performing our previous reported results$^{37}$. In terms of shunt inductance (untreated in$^{37,38}$) the inductor is stable within the 3 GHz-6 GHz frequency range with an average of 1 nH as seen in Fig. 4c.

Figure 8. Fabricated VO$_2$ reconfigurable Peano inductor equivalent circuit modelling within 4 GHz-10 GHz for the inductor with 2 switches, in on state and off state compared to measured ones. (a) The resulted fitting of the equivalent circuit extracted Lseries_circuit and Lseries from the measured S parameters. The values are extracted optimizing simultaneously the Lseries_circuit, Q_circuit, module and phase of the S$_{ij,circuit}(j\omega)$ (2 scalar parameters and 3 complex values due to reciprocity). Off state values: $L_s = 1.48$ nH $R_s = 2.79$ Ohms, $R_p = 1341$Ohms, $C_s = 0.17$fF, $C_{ox1} = C_{ox2} = 721$fF, $C_{sub1} = C_{sub2} = 201$fF, $R_{sub1} = R_{sub2} > 10^9$Ohm. On state values: $L_s = 0.90$ nH, $R_s = 4.01$ Ohms, $R_p = 10^7$ Ohm, $C_s = 0.15$fF, $C_{ox1} = C_{ox2} = 724$fF, $C_{sub1} = C_{sub2} = 201$fF, $R_{sub1} = R_{sub2} > 10^9$Ohm. (b) Scalar Q and extracted Q_circuit resulted fitting (c), Simultaneous 3D Smith chart representation for Lseries_circuit, Q_circuit, S$_{ij,circuit}(j\omega)$, and Lseries, Q, S parameters of the measured inductors. The fitting of the S$_{ij}$ parameters is almost perfect, the fitting of the S$_{11}$ parameters (whose values are extremely small in magnitude) is not perfect—but it overlaps on a specific frequency range too. S$_{11}$ is more sensitive to the frequency dependency of the elements neglected in the simplified equivalent circuit in Fig. 7b. (d) Simultaneous 3D Smith chart representation for all the S$_{ij}$ (S$_{ij,circuit}(j\omega)$ and measured S$_{ij}(j\omega)$) parameters and their frequency dependency. It is interesting to notice their change of hemisphere just below 10 GHz for the off state modeled and measured inductors (this happening before their Q becomes 0—the radius of the cylinders in c is not yet zero, or in b one may see still the positive values of Q). This means that the imaginary part of S$_{11}$ becomes “capacitive” before its Y$_{11}$ imaginary part becomes 0—which determines the sign changes in Q. Referring to$^{34,57}$ the classical quality factor$^{34–39}$ analysed and implemented here too is a measure of the inductor performances while short-cutting the second port. If the inductor is connected in shunt in the final circuit the change of hemisphere of S$_{11}$ below 10 GHz can be neglected, else depending on the configuration in which this is finally used- the change of hemisphere warns that under 50 Ohm load the inductor starts behaving capacitive before 10 GHz.
The overall performances of the inductor with two switches on, two switches off and of a fabricated inductor with one switch on are all plotted together in Fig. 5g (series inductances), Fig. 5h Qs and Fig. 5i both Qs and normalized 3D series inductances and Qs. The results show 77% tuning range and $Q_{\text{max,off}}/Q_{\text{max,off}} = 0.88$ and thus 3.26 times higher than $Q_{\text{max,off}}/Q_{\text{max,off}}$ reported in37. The $Q_{\text{max,off}}$ exceeds 7 being comparable with the off state even though the inductance is tuned with $77\%$ down to 0.95 nH.

On the other hand, the frequency dependency of the off state $Q_f(\omega)$ is comparable with the one reported in37 (although dealing with a smaller inductance than in37). Overall exhibiting a better performance (in terms of $Q_f(\omega)$) in the low GHz frequency range and a more stable frequency dependency linearity (in terms of series inductance) the values are facing the same trend as in37. The maximum value is limited as in37 by the CMOS compatible CPW SiO2/Si technology used with Al metallization, (Supplementary Section 3).

**Peano reconfigurable inductors modelling and characterization.** The simplified proposed equivalent circuit of the inductor based on the Fig. 5 inductor’s layout, together with switches equivalent circuit models are presented in Fig. 7a–c. The simplified (frequency independent) Pi model proposed in Fig. 7 uses the classical elements present in inductor modelling8,35 as described in Fig. 7. The only additional element added is $R_s$ - which is modelling the conductive losses in the adjacent turns due to the presence of the VO2, and fabrication flow. In the off state the value of $R_p$ (instead of tending to $\infty$) will be affected by the conductive losses in the VO2 (as described in Fig. 7). VO2, whose DC conductivity is presented in Fig. 9 will have higher conductivity losses in RF ranges44, losses which in the off state will contribute to conductive losses between the turns. In the on state on the other hand, the presence of the VO2 (see Fig. 5f) on the main current path will influence the series resistive losses $R_s$ in increasing its value. $R_s$ will thus have an additional component in the on state (besides the turns resistance corresponding to the on state current path, component which will be generated by the VO2 resistive losses in the on state.

The extraction of the inductor equivalent circuit parameters is done using an original approach, based on common optimization of 3 (complex) S parameters ($S_{11}(\omega) = S_{21}(\omega)$, due to reciprocity) and extracted $L$series and $Q$. Thus we impose the measured $S_{11}(\omega) = S_{ij,\text{circuit}}(\omega)$ were the last denote the equivalent circuit S parameters ($i=1, 2, 3$) and the $L$series and $Q$ to be equal to the extracted $S_{ij,\text{circuit}}$ and $Q_{\text{circuit}}$ of the proposed equivalent circuit. It is worth mentioning that the extraction of the equivalent circuit is usually done using just fitting scalar parameters: series or shunt inductance and $Q$- as in34 (by us)35, (Supplementary File), or only series inductance38. These approaches are however incomplete, since the Q model classically used34–38, is neglecting the right arm of the Pi model, being based on $Y_{11}$, thus having the second port grounded56,57. These values (from both measured and equivalent circuit extracted models $Q_{\text{circuit}}$) are accurate only if the inductor is used connected to a ground load56,57. The inductance fitting on the other hand is modeling just an element of the equivalent circuit, model based on only one of the $Y_{11}$ or $Y_{31}$34–38 parameters, thus the overall $S$ parameters may not fit at all even though $Q$ and extracted inductance fitting is reached.

Visualizing the fitting of $S_{ij}(\omega)$ and $S_{ij,\text{circuit}}(\omega)$ only on the (2D) Smith chart, as in Figs. 1d,e and 3a may lead to possibly inaccurate solutions, since the paths on the Smith chart may coincide for two circuits on a frequency range even though their nature can be different (which determines their frequency parametrization).

Here we take benefit of the new 3D Smith chart implementations and while using the AWR Microwave Office optimization tool, we simultaneously optimize in the extraction process all $S_{ij,\text{circuit}}(\omega)$, $L_{\text{series, circuit}}$ and $Q_{\text{circuit}}$ while visualizing this complex-scalor process on the 3D Smith chart.

Figure 8a,b show the results on a classical 2D scalar plot, while Fig. 8c,d on the 3D Smith chart plot. The $S_{21,\text{circuit}}(\omega)$ parameter fitting is almost perfect (in both states) (all being in the West hemisphere), while the $S_{11,\text{circuit}}(\omega)$ starts diverging slightly from $S_{11}(\omega)$ in phase at the higher frequencies for both on and off states.

Finally, it can be observed in Fig. 8c or 8d that both off state $S_{11}(\omega)$ and $S_{11,\text{circuit}}(\omega)$ change the hemisphere before 10GHz (i.e. imag $S_{11}(\omega)$ and $S_{11,\text{circuit}}(\omega)$ becomes negative), while their $Q$ remains positive (the radius of the generalized cylinders being not yet a point), showing the good match (both change hemisphere) but also the limitations of the inductor when used in a different configuration57 than grounded. An inductor $S_{ij}(\omega)$ parameter should stay in the inductive part of the Smith chart (East hemisphere-3D Smith chart) for the entire frequency range of interest in order to assure its validity of use under any resistive loads conditions.

**Conclusions**

We have first reported new theoretical foundations for a frequency-dependent 3D Smith with 3D visualization methods for the orientation of parametric curves and used them to quantify and understand curvature reversal, while sweeping the frequency, for driving point impedances and reflection coefficients of circuits in the RF frequency bands. Further we have additionally extended the capabilities of the 3D Smith chart tool to simultaneously visualize a variety of frequency dependent scalar-complex valued parameters required in the inductor modelling and thus proposed a unique multi-parameter display. We demonstrated by fabrication and measurements, original Peano reconfigurable inductors by employing the phase change VO2 materials in CPW/CMOS compatible technology on SiO2. The reported inductors improve the previously reported state of art in the incipient field of VO2 reconfigurable inductors design for the S, C and X bands of the radio frequency spectrum.

**Methods**

**Fabrication.** The devices were fabricated using a high resistivity (10000 Ω-cm) Si wafer (525 μm) as the starting substrate. A 300 nm-thick amorphous Si was deposited by low pressure chemical vapor deposition (LPCVD), to reduce the losses during measurement. A surface passivation using 500 nm sputtered SiO2 was then carried out, followed by deposition of 140 nm -thick VO2 using a Pulsed Laser Deposition (PLD) system. The film was deposited by pulsed lased deposition (PLD) using a Solmates SMP 800 system. The deposition was performed at 400 °C in oxygen ambient, with a chamber pressure of 0.01 mbar. The ablating V2O5 target was placed at 60 mm
distance from the wafer. Further the deposition, an annealing of 10 min at 475 °C was performed without breaking the vacuum in the chamber.

The electrical properties of the films were studied from room temperature up to 100 °C by determining their temperature dependent electrical resistivity converted than in conductivity. This was done by standard four-point probe measurements using a semiconductor parameter analyzer (HP 4156 C) and a control on the sample temperature up to 100 °C.

The fabrication process for the Peano inductors on the above substrate then commenced with a photolithography step to pattern the VO₂ followed dry etching to remove the VO₂ from the unwanted areas. A Cr-20 nm/Al-400 nm bi-layer metal stack was then deposited by evaporation after a subsequent photolithography and patterning on the VO₂. This thin metallization made it possible to realize sub-micron gaps (600 nm) which is critical for extracting a good Q in the conductive state of VO₂ for the inductors. This was followed by deposition of a 2.4 µm-thick Al layer on these contact pads by conventional lithography followed by metal lift-off procedure to form the CPW elements with low RF losses.

**Devices simulation and characterization.** The numerical simulations of the inductor and filters were done in HFSS ANSYS commercial software relying on the finite element method (FEM) to solve Maxwell equations. Considering the full wave electromagnetic simulation technique, we used the modal solution type for the inductors and the terminal solution type for the filter simulations. The conductivity of the Al was decreased to 3.1*10⁷ S/m and the VO₂ switches were modeled for the inductors as simple dielectrics with 20 S/m losses and a loss tangent of 0 in the off state. In the on state, the VO₂ switches were simulated as lossy metals of a conductivity of 48,000 S/m. Subsequently full inductors and filters models were built in the software according to the actual physical structure fabricated. The Peano shapes were implemented using the Equation Curve facility of the tool, their equations being written parametrically and where needed rotations and reflections were used for the very final shapes.
The devices were measured with the Anritsu Vector Star VNA in a Cascade Summit prober with controllable chuck temperature who was set to 20 °C in the ‘off state’ and 100 °C in the on state. For the 2D graphical interpretation the measured S parameters were converted using the Anritsu Star VNA installed Microwave Office too into the desired parameters analysed. In order to obtain the extracted equivalent circuit parameters circuit we used the Genetic Algorithm of the Anritsu-AWR MW Office installed tool and optimization of 3 complex parameters and \( S_{ij}(\omega) = S_{ij \_\text{circuit}}(\omega) \) and two scalars: \( L_{\text{series}} = L_{\text{series \_circuit}} \) and \( Q = Q_{\_\text{circuit}} \) – in the 4 GHz–10 GHz range using 15,000 iterations.

**3D smith chart implementation.** The 3D Smith Chart application is developed using the Java programming language and the following libraries and development environment are used:

- 3D rendering: OpenGL through the Java Binding for the Open GL API (JOGL2) library.
- Mathematical operations and complex data representation: The Apache Commons Mathematics Library.
- Development environment: The NetBeans IDE with Beans Binding Library for the implementation of the application GUI and JOGL2 usage.

Further implementation details about the new mode of visualization, new simulation parameters used in the paper and their 3D representation on the Riemann sphere can be found in Supplementary Section 2.

**Mathematical modelling of curvature.** The calculations for the oriented curvature was performed using Mathematica software tool by writing the frequency parametric equations of the curves analyzed.

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Author contributions

A.A.M. proposed the new quantification model of the orientation of frequency over the 3D Smith chart, use of oriented curvature in the mathematical modelling, 3D series and shunt inductor Q inductor modelling, designed, fabricated and measured the devices with VO2. A.M. supervised and guided the overall 3D Smith chart tool new capabilities implementations for this article. Y.A. contributed to the development of the new 3D Smith chart concepts, their actual Java implementation and revised and arranged the paper. E.S.-C. refined and contributed extensively to the oriented curvature mathematical computations (equations). R.A.K. contributed with the fabrication and process flow in different stages, especially in obtaining the sub-micronic gaps in the Al via photolithography. A.K. contributed with the conductivity measurements of the VO2. M.F.-B. contributed with constructive critical view in the preliminary design stages. M.C. made the PLD deposition of VO2. M.A.I. lead the work of A.A.M. and guided the implementation steps for each part of the work.

Competing interests

The authors declare no competing interests.

Additional information

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