The Kinematic Modeling and Simulation of an Adjustable Pivot Point of Six Degree-of-Freedom (DOF) Parallel Robot

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Abstract. For the pose requirements of secondary mirror in the large aperture space optical remote sensor, the pose adjustment is applied to the pivot point of secondary mirror. So the adjustable pivot point kinematic model must be set up. Based on the characteristics of the pivot point fixed with the moving platform, the corresponding coordinate system is built, and the adjustable pivot point kinematic model is established on the Euler rotation. Then virtual prototype model and the pivot point are built in ADAMS, and the kinematic simulation method is given. Finally, a 6-DOF parallel robot as secondary mirror pose adjustment device of an optical remote sensing system is taken as an example, and the adjustable pivot point kinematic model and ADAMS simulation method are carried out. The results show that the model and simulation method are correct and practical, which can be used in any field of a 6 DOF parallel robot with adjustable pivot point.

Keywords: pivot point, kinematic, parallel robot, ADAMS

1. Introduction
In a space optical system, large aperture optical cell is more sensitive to the thermal field and gravity field change, so the pose of large aperture optical cell needs to be more precise to detection performance and image quality [1, 2]. In this case, the traditional passive technology means such as lightweight and athermalization have been unable to meet the demand of large diameter optical system of aerospace engineering. In view of the difficulties in this project, active optics and adaptive optics have been adopted abroad with precise active adjustment devices added to the optical system as the secondary mirror positioner [3, 4]. It ensures the detection performance and imaging quality of the large-aperture high-performance optical system while in orbit. In engineering implementation, six degree-of-freedom (DOF) parallel robot is used to adjust the secondary mirror pose. It has been proved very effective in the field of space optical system abroad [5-7]. The six DOF parallel robot has high precision, high rigidity, less weight and so on [8].

In the application of 6 DOF parallel robot to adjust the secondary mirror pose, it is essential to change the specific pivot point of a secondary mirror according to the mission and index of the satellite system. Generally, the pivot point should be located at the center of the optical surface of the secondary mirror. However, at present, the reported six DOF parallel robot kinematics models are all based on the origin of the moving platform. If this model is directly applied to adjust the secondary mirror pose, the secondary mirror center will deviate from the optical axis, which will affect the imaging quality seriously. Therefore, when the pivot point is adjustable, the kinematic model must be established to apply the six DOF parallel robot to the secondary mirror adjustment.
2. The Kinematic Modeling

2.1. The Coordinates

Stewart platform is a typical six DOF parallel robot. In this paper, 6-UCU as shown in figure 1 is studied: The robot is connected to the moving and fixed platforms by six struts, with two hooker hinges at each end and a cylindrical pair in the middle.

![6-UCU parallel robot](image)

**Figure 1.** 6-UCU parallel robot.

When the moving platform’s pose in space are determined, the length of each strut, namely the displacement of each moving pair, is obtained, which is the inverse kinematic of Stewart platform. Inverse kinematic is a basis of workspace, error analysis and precision compensation.

Bi and bi (i=1,…, 6) present the centers of joints on the fixed platform and on the moving platform respectively, as shown in figure 2. R and r are circumradii of fixed and moving platform. Triangle B1 B2 B3 , B4 B5 B6 , b1 b2 b3 and b4 b5 b6 are regular triangles. βis central angle of arc B1B2. α is central angle of arc b1b6.

![Points distribution](image)

(1) points of fixed platform. (2) points of moving platform.

**Figure 2.** The distribution of points.

Figure 3 shows the P-xyz coordinates is fixed on the moving platform. P is both P-xyz coordinates’ origin and the center of circumscribed hexagon b1b2b3 b4b5b6. Pz-axis is perpendicular to the moving platform upward. Px-axis is perpendicular to line b1b6 and Py-axis is defined by right-hand rule. The O-XYZ coordinates is fixed on the fixed platform. O is both the O-XYZ coordinates’ origin and the center of circumscribed hexagon B1B2B3B4B5B6. OZ-axis is upward perpendicular to the fixed platform. OX-
axis is perpendicular to line \( B_1B_6 \) and OY-axis is defined by right-hand rule. In the general kinematic model, P is the pivot point.

Suppose the pivot point of the secondary mirror is point C, and the coordinate system is C-XcYcZc. The pivot virtual platform coincides with C-XcYc. When the pivot platform rotates, CXc-axis is always parallel to Px-axis, CYc-axis is parallel to Py-axis, and CZc-axis is parallel to Pz-axis. There is no orientation change between coordinate system C-XcYcZc and coordinate system P-xyz, only position movement. For any vector \( \mathbf{v} \), its expression in the P-xyz is denoted by \( \mathbf{p} \), in the O-XYZ is denoted by \( \mathbf{v} \), and in the C-XcYcZc is denoted by \( \mathbf{c} \). There is the transformation relationship: \( \mathbf{p} \mathbf{c} = \mathbf{c} \mathbf{v} \).

Point P in the C-XcYcZc coordinate system are set as \( T_{cPC} = \begin{bmatrix} x_{pc} & y_{pc} & z_{pc} \end{bmatrix}^T \). The vectors of the six struts are denoted as \( \mathbf{l}_i \), and the struts length are denoted as \( l_i (i=1,2,...,6) \).

The expressions of \( b_i \) in P-xyz and B_i O-XYZ are respectively are shown in the equation (1) and equation (2):

\[
p_i = r \begin{bmatrix} \cos \alpha_i & \sin \alpha_i & 0 \end{bmatrix}^T
\]

\[
B_i = R \begin{bmatrix} \cos \beta_i & \sin \beta_i & 0 \end{bmatrix}^T
\]

Where \( \alpha_i \) is the angle of \( \mathbf{P}_i \) to Px-axis, and \( \beta_i \) is the angle of \( \mathbf{O}_i \) to OX-axis.

The expressions of \( \mathbf{b}_i \) in C-XcYcZc is shown in the equation (3):

\[
\mathbf{b}_i = \left[ r \cos \alpha_i + x_{pc} \right] \mathbf{c} \begin{bmatrix} \cos \alpha_i & \sin \alpha_i & z_{pc} \end{bmatrix}^T
\]

2.2. The Orientation of the Pivot Platform

When the coordinate system of the secondary mirror does not duplicate the origin P, it is necessary to use the coordinate system C-XcYcZc to calculate the kinematics and other models. Any vector \( \mathbf{c} \) in C-XcYcZc can be transformed into \( \mathbf{v} \) in fixed coordinates O-XYZ by coordinate transformation. The pivot virtual platform position can be expressed by point C in the fixed coordinate system \( C = [x_c, y_c, z_c]^T \), and the pivot platform orientation can be expressed by the orientation of C-XcYcZc coordinates relative to O-XYZ coordinates. The rotation transformation between the coordinate systems adopts the Euler Angle transformation. For convenience, the origin of the rotating coordinate system C-XcYcZc and the
fixed coordinate system O-XYZ coincide, and the three Euler angles \( \theta_x, \theta_y \) and \( \theta_z \) are respectively defined as: first, rotate the coordinate system C-XcYcZc about the OZ-axis by \( \theta_z \) to obtain the intermediate coordinate system O-X'Y'Z; then rotate O-X'Y'Z about the OY'-axis by \( \theta_y \) to get the intermediate coordinate system O-XcYcZc; finally, rotate O-XcYcZc about the OXc-axis by \( \theta_x \) to obtain the final coordinate system C-XcYcZc, as shown in figure 4.

![Figure 4. The orientation of the pivot platform.](image)

The C-XcYcZc is projected to O-XYZ coordinates, the direction cosine matrix \( R \) representing the orientation of the pivot virtual platform is shown in equation (4):

\[
R = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} \\
    r_{21} & r_{22} & r_{23} \\
    r_{31} & r_{32} & r_{33}
\end{bmatrix} =
\begin{bmatrix}
    c\theta_x c\theta_y & c\theta_z s\theta_y s\theta_x - s\theta_z c\theta_x & c\theta_z s\theta_x c\theta_y + s\theta_z s\theta_x \\
    s\theta_x c\theta_y & s\theta_z s\theta_y s\theta_x - c\theta_z c\theta_x & s\theta_z c\theta_x c\theta_y - c\theta_z s\theta_x \\
    -s\theta_y & c\theta_y s\theta_x & c\theta_y c\theta_x
\end{bmatrix}
\]

(4)

Where

\[
\begin{align*}
    c\theta_z &= \cos \theta_z, s\theta_z = \sin \theta_z \\
    c\theta_y &= \cos \theta_y, s\theta_y = \sin \theta_y \\
    c\theta_x &= \cos \theta_x, s\theta_x = \sin \theta_x
\end{align*}
\]

2.3. The Adjustable Pivot Point Kinematic Model

The expression of \( b_i \) in O-XYZ coordinates is \( q_{b_i} \), as shown in equation (5), which is obtained through the transformation of the rotation matrix:

\[
q_{b_i} = R \cdot C + C
\]

(5)

Then formula (6) is the vector of strut \( i \):

\[
l_i = q_{b_i} - B_i = R \cdot C + C - B_i
\]

(6)

So the adjustable pivot point kinematic model is shown in equation (7):

\[
l_i = \sqrt{(R \cdot C + C - B_i)^T (R \cdot C + C - B_i)}
\]

(7)
3. The Adjustable Pivot Point Kinematic Simulation in ADAMS

The parameterized model of the Stewart virtual prototype in the analysis process is established in ADAMS/View. The fixed pivot point on the moving platform is shown in Fig 5.

Adjustable pivot point model must be built in ADASMS for simulation. The steps are as follows:

1) Click "Construction Geometry: Point" in "Bodies" and select "Add to Part" and "Don't Attach" in the left column. Then click on any Point of the upper platform and the space in sequence.

2) Find the new dot built in step1 in the "Model Browse" on the left and rename it "Pivot_Point".

3) Click the "Design Variable" of "Design Exploration" to establish a new Variable, named "Pivot_x", as the position of adjustable pivot point relative to the origin of coordinate system P-xyz in the X direction;

4) Similarly, the variables "Pivot_y" and "Pivot_z" are established as the positions of adjustable pivot point relative to the origin of P-xyz coordinates in the Y and Z directions.

5) Click "Table Editor" in "Tools" to modify the values of "Pivot_x", "Pivot_y" and "Pivot_z", and the robot with adjustable pivot point has been established, shown as in figure 6.

Then the pose driving function is added to the Pivot_Point, and the reference point selects the center point O of the fixed platform, thus ensuring that the reference system of the given motion index is O-XYZ coordinates. At the same time, the displacement measurement of upper and lower hinge points is established, and the distance between two points is measured to obtain the real-time struts length curve.

In general, the sinusoidal displacement/Angle driver function is loaded at the adjustable pivot point of the robot in the six degrees of freedom direction at the same time, and the motion under this drive is simulated in ADAMS. Then the length curve of the six struts can be obtained through post-processing, which is the adjustable pivot point kinematic solution.
4. An Example of Modeling and Simulation

In a large aperture space optical remote sensor, the structural parameters of a 6-DOF parallel robot for secondary mirror pose adjustment are shown in Table 1.

| Parameters | r/mm | R/mm | H/mm | α/rad | β/rad |
|------------|------|------|------|-------|-------|
| Data       | 290  | 295  | 175  | 0.36  | 0.44  |

The pivot point of the secondary mirror component is C, and the moving distance relative to the original point P in the moving platform is 20, 30, 100 mm respectively. Then the point P in the C-XcYcZc is set as $C = \begin{bmatrix} x_{pc} \\ y_{pc} \\ z_{pc} \end{bmatrix}^T = \begin{bmatrix} -20 \\ 30 \\ -100 \end{bmatrix}^T$.

4.1. The Necessary of Adjustable Pivot Point Kinematic

In practical engineering application, if the change of pivot point is ignored, the given secondary mirror’s pose is applied to the origin of the upper platform. Based on that, the kinematics solution and control are carried out. Then the position and orientation of the secondary mirror cannot reach the expected pose.

The simulation rule to the pivot platform is composite motion of low-frequency translation and high-frequency rotation, as shown in Table 2. The total simulation time is 50S and the simulation step length is 0.5s.

| Directions | Rule | Units |
|------------|------|-------|
| Translation | x $2\sin(2\pi/12*\text{time})$ | mm |
|            | y $2\sin(2\pi/24* \text{time})$ | mm |
|            | z $2\sin(2\pi/48* \text{time})$ | mm |
| Rotation   | x $1\sin(2\pi/2* \text{time})$ | ° |
|            | y $1\sin(2\pi/6* \text{time})$ | ° |
|            | z $1\sin(2\pi/3* \text{time})$ | ° |

In ADAMS, the adjustable pivot point was set. The motion was applied to the origin of the upper platform as shown in Table 2 to obtain the length increment of the six struts, and then the increments were used to drive the motion of the six struts. At this time, the position and orientation of the origin of the upper platform and the adjustable pivot point of the secondary mirror were measured, as shown in figure 7 and figure 8. The figures showed that the position of the secondary mirror was inconsistent with the expected position, and the orientation is consistent with the expected orientation. If the adjustable pivot point was not taken into account, the position of the secondary mirror would be seriously deviated and could not meet the adjustment requirements.

![Figure 7. Positions of upper platform and secondary mirror.](image-url)
Figure 8. Orientations of upper platform and secondary mirror.

4.2. The Consistency of Model and Simulation

The adjustable pivot point kinematic model was established in MATLAB, and the mathematical calculation was carried out to obtain the variation diagram of the length of six struts over time, as shown in the figure 9. The adjustable pivot point was set in ADAMS to carry out kinematic simulation, and the variation diagram of the length of six struts with time were obtained, as shown in the figure 10.

Figure 9. The length of six struts of adjustable pivot point kinematic model.

Figure 10. The length of six struts of adjustable pivot point ADAMS model.

Then the theoretical model calculation curve was subtracted from simulation curve of ADAMS model on each moment to get error curve, which was shown in figure 11. The error quantity of six struts length of two models stayed at $10^{-5}$ mm level, which means struts length of theory model and ADAMS virtual prototype model at the same time is consistent. So adjustable pivot point kinematic theoretical model and ADAMS model are correct.
Figure 11. The error of the length of six struts of two models.

5. Conclusion
The kinematics model is the basis of the control algorithm of 6-DOF parallel robot. The adjustable pivot point kinematic model and ADAMS simulation analysis method of a 6 DOF parallel robot are established to satisfy the demand of the secondary mirror adjustment pose. The modeling process is relatively simple, and the kinematics simulation analysis in ADAMS is clearly. The results of kinematic calculation and simulation method are consistent, which verifies the accuracy of this mathematical kinematic model. In the optical remote sensing system, the adjustable pivot point kinematic model can ensure the image quality by adjusting the pose of the secondary mirror as the real requirements. This method can be applied to any field of a 6 DOF parallel robot with adjustable pivot point.

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