Study on the Vibration Testing Technology in Nondestructive Testing of Masonry Structures

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Abstract. This paper presents a method for the application of vibration testing techniques in the nondestructive testing of masonry structures, which gives the masonry compressive and shear resistance as well as vibration deformation performance by calculation, and achieves accurate nondestructive testing of the structure about the actual measured data. The practical example shows that the method is feasible and it provides a scientific basis for the testing of masonry structures.

Keywords: Concrete masonry structure, vibration testing, nondestructive testing, deformation properties, dynamic characteristics

1. Introduction
With the rapid development of China's economy and accelerated urbanization, some of the old building complexes are gradually becoming new urban centers, and the safety of the old building structures is being paid attention to by everyone, and most of the buildings in the complexes are masonry structures, so the testing of masonry structures has become a subject of considerable interest[1]. The masonry structure testing discussed in this paper provides a basis for testing masonry structures and safety and reliability assessment by obtaining the frequencies and vibration patterns of the structures through vibration testing techniques [2] without damaging or changing the masonry structure itself, and by analyzing the story stiffness and compressive strength of the masonry structures by combining the data of the materials themselves.

2. Mechanical Properties of Masonry

2.1. The Main Factors Affecting the Strength of Masonry
From a macroscopic point of view, masonry is a monolithic material made of blocks and mortar masonry, but from the internal point of view it is not continuous whole [3], nor is it a completely elastic material. Thus, the main factors affecting the strength of brick masonry are: the strength of the brick, the strength of the mortar, the thickness of the mortar layer laid, and the quality of masonry [4].
2.2. Calculation of compressive and shear strength of masonry

The maximum compressive stress that masonry can withstand is called masonry compressive strength, which is an important indicator to determine the compressive damage capacity of masonry and its members [5]. A large number of masonry compressive strength tests have been carried out for various types of masonry in China, laying the foundation for the establishment of various masonry compressive strength calculation formulas in line with the actual situation in China [6]. The following formula for calculating the average compressive strength applicable to various types of masonry is proposed.

\[ f_m = \beta_f f'_f (1 + 0.07 f'_f) \beta_f \]  

(1)

where:

- \( f_m \) = average value of masonry compressive strength
- \( f'_f \), \( f'_f \) = average value of blocks and mortar compressive strength
- \( \beta_f \) = the parameter that considers the influence of block shape, size, etc. of various masonry blocks
- \( \beta_f \) = a correction factor

Besides, many foreign researchers have established various empirical formulas based on experimental data. For example, Professor Oniszczyk LI [7] of the former Soviet Union proposed the following Eq. (2)

\[ f_m = A f'_f (1 - \frac{a}{b + f'_f / a f'_1}) \eta \]  

(2)

Where:

\[ A = \frac{100 + f'_1}{100m + nf'_1} \]; \( m, n, a, b \) are empirical coefficients related to the type of masonry and various influencing factors of masonry strength, which can be determined by the test data.

The average value of compressive strength of masonry is the average level that indicates its compressive strength taken. The standard value of compressive strength of masonry is the basic representative value of its compressive strength [8]. The shear strength of masonry is mainly determined by the mortar strength. The average value of shear strength is given by:

\[ f_{vm} = K s \sqrt{f_2} \]  

(3)

where:

- \( f_2 \) = compressive strength of mortar
- \( K_s \) = coefficient of different kinds of masonry

According to the strength design value taking method and the influence of strength variation and other factors, the masonry shear strength design value is:

\[ f_v = \frac{1}{r_f} f_{vm} (1 - 1.645 \delta) \]  

(4)

since \( r_f = 1.5 \), \( \delta = 0.2 \), then \( f_v = 0.447 f_{vm} \).
2.3. Deformation properties of masonry

2.3.1. Masonry Stress-strain Relationship and its Modulus of Elasticity. The test results show that the masonry is an elastoplastic material and the stress-strain relationship can be expressed by the following logarithmic function relationship.

\[ \varepsilon = -\frac{1}{\xi \sqrt{f_m}} \cdot \ln(1 - \frac{\sigma}{f_m}) \]  

where: \( \xi \) = elasticity eigenvalue

Ours is obtained by the ordinary least squares \( \xi = 463 \), then the masonry is subjected to the compressive stress-strain relationship as:

\[ \varepsilon = -\frac{1}{463 \sqrt{f_m}} \cdot \ln(1 - \frac{\sigma}{f_m}) \]  

In the stress-strain relationship curve, the slope of the tangent line at any point is the tangent line modulus of elasticity at that point.

\[ E' = \frac{d\sigma}{d\varepsilon} = \xi \cdot f_m \sqrt{f_m} (1 - \frac{\sigma}{f_m}) \]  

When \( \sigma = 0 \), this is the initial modulus of elasticity through the origin.

\[ E_0 = \xi \cdot f_m \sqrt{f_m} \]  

When \( \sigma = 0.43 f_m \)

\[ E = \frac{0.43 f_m}{\xi \sqrt{f_m}} = \frac{0.8 \xi \cdot f_m \sqrt{f_m} = 370 f_m \sqrt{f_m}}{\xi \sqrt{f_m}} \]  

2.3.2. Shear Modulus of Masonry. The shear modulus of masonry is related to the modulus of elasticity, Poisson's ratio, and according to the mechanics of materials, it is known that the shear modulus of masonry is:

\[ G = E / (2(1 + \nu)) \]  

Since the Poisson's ratio of brick masonry is generally taken as 0.15, while block masonry is generally taken as 0.3, substitution into the above equation yields.

\[ G = (0.43 \cdot 0.38)E \]  

Therefore, in general, the shear modulus of all types of masonry can be taken as 0.4E.

3. Characteristics of the Measured Dynamic Characteristics of Building Projects

Researchers and designers at home and abroad have carried out a large number of actual measurements of building projects, the main purpose of which is twofold: one is to verify whether the
inherent frequencies and vibration patterns calculated by the design are correct; the other is to establish statistical and empirical formulas for design reference or seismic performance, wind performance verification and earthquake damage prediction of housing buildings.

3.1. Examples of Actual Measurements in Building Projects
In this paper, the measured dynamic characteristics of 15 buildings are counted (Table 1), and the purpose of this paper is to illustrate the inherent vibration characteristics of building engineering structures to serve as a basis for simplifying the calculation model of building structures. With the number of layers as the horizontal coordinate and the frequency ratio as the vertical coordinate, the distribution of frequency ratio versus the number of layers is plotted as shown in Figs.1 and 2. The general bending type vibration and shear-type vibration frequency ratios are shown in Table 2.

![Figure 1. Distribution of f2/f1 versus the number of layers.](image1)

![Figure 2. Distribution of f3/f1 versus number of layers.](image2)
### Table 1. Measured dynamic properties of buildings.

| Projects                                             | Number of floors | Direction   | f1   | f2   | f3   | f2/ f1 | f3/ f1 |
|------------------------------------------------------|------------------|-------------|------|------|------|--------|--------|
| Dalian Personnel Bureau Talent Exchange Center Building | 14               | Horizontal  | 1.06 | 3.34 | 8.84 | 3.15   | 8.34   |
|                                                      |                  | Longitudinal| 1.59 | 4.63 | 9.13 | 2.91   | 5.74   |
| Dalian Li Yuan Building                              | 28               | Horizontal  | 0.890| 3.125| 6.688| 3.51   | 7.51   |
|                                                      |                  | Longitudinal| 0.890| 2.828| 5.531| 3.18   | 6.21   |
| Shenyang Railway Branch Office Building Apartments    | 10               | Horizontal  | 1.61 | 5.22 | 8.44 | 3.24   | 5.24   |
|                                                      |                  | Longitudinal| 1.95 | 5.56 | 9.22 | 2.85   | 4.72   |
| Shenyang Northeast Light Industry United Company Business Building | 12               | Horizontal  | 1.46 | 5.47 | 10.25| 3.74   | 7.00   |
|                                                      |                  | Longitudinal| 1.66 | 5.47 | 10.54| 3.30   | 6.35   |
| Shenyang Construction Road Slab Type Residential Building | 16               | Horizontal  | 0.977| 3.221| 5.466| 3.30   | 6.34   |
|                                                      |                  | Longitudinal| 1.854| 5.563| 9.174| 3.00   | 4.95   |
| Shenyang High Floor Hotel                            | 17               | Horizontal  | 0.976| 3.611| 5.27 | 3.70   | 5.60   |
|                                                      |                  | Longitudinal| 1.464| 4.490| 7.125| 3.07   | 4.87   |
| Shenyang Zhong Xing Commercial Building              | 19               | Horizontal  | 0.878| 2.830| 5.026| 3.34   | 6.34   |
|                                                      |                  | Longitudinal| 0.878| 2.928| 5.563| 3.22   | 5.72   |
| Shenyang Auto and Parts Trade Building                | 19               | Horizontal  | 0.976| 3.465| 6.881| 3.55   | 7.05   |
|                                                      |                  | Longitudinal| 0.976| 3.855| 7.222| 3.95   | 7.40   |
| Shandong Yan Tai Port Complex                        | 16               | Horizontal  | 1.20 | 2.97 | 4.76 | 2.48   | 3.97   |
| Zhengzhou University of Light Industry 1# Teaching Building | 6               | Horizontal  | 2.23 | 5.70 | 9.80 | 2.56   | 4.40   |
| Tower Residential Building in Tai Po, Hong Kong       | 32               | Horizontal  | 0.73 | 3.53 | 7.28 | 4.81   | 9.92   |
|                                                      |                  | Longitudinal| 0.81 | 3.90 | 7.35 | 4.84   | 9.34   |
| Hong Kong Dabao Court Residential Building           | 35               | Horizontal  | 0.52 | 2.42 | —    | 4.63   | —      |
|                                                      |                  | Longitudinal| 0.76 | 2.56 | —    | 3.39   | 7.29   |
| Kwun Tong Tower Residential Building, Hong Kong       | 36               | Horizontal  | 0.82 | 3.30 | 6.83 | 4.03   | 8.35   |
|                                                      |                  | Longitudinal| 0.92 | 3.69 | 7.22 | 4.01   | 7.83   |
| Hong Kong China Resources Building                    | 50               | Horizontal  | 0.43 | 1.56 | 2.95 | 3.63   | 6.86   |
|                                                      |                  | Longitudinal| 0.63 | 1.93 | 3.90 | 3.06   | 6.19   |
| Hong Kong Hopewell Centre                            | 65               | Horizontal  | 0.45 | 1.62 | 2.95 | 3.60   | 6.56   |
|                                                      |                  | Longitudinal| 0.45 | 1.61 | 2.85 | 3.58   | 6.33   |

1 f1 is the First-order frequency, f2 is the Second-order frequency, f3 is the Third-order frequency

### Table 2. Frequency ratio of bending type vibration and shear type vibration.

| Frequency Ratio | f1/f1 | f2/f1 | f3/f1 |
|-----------------|-------|-------|-------|
| Bending Type    | 1     | 6.27  | 17.55 |
| Shear Type      | 1     | 3     | 5     |

3.2. Structure Mechanical Model Discrimination

From the experimental data, it can be seen that the measured frequency of the general building structure is relatively close to the shear vibration. The vibration model can be simplified as a shear multi-mass system model, that is, the stiffness array for three diagonal types, the mass array for the main diagonal type, each layer can be placed at the floor as a concentrated mass, the story stiffness as a spring.
4. Principles and Methods of Nondestructive Testing of Masonry Structures

The NDT method in this paper differs from the traditional method in that it uses dynamic measurement techniques to obtain the necessary mechanical parameters, and in the structural dynamic analysis to obtain the stiffness of the structure, and then determine the relationship between stiffness and strength for the overall assessment of structural strength, to achieve the overall structural NDT.

4.1. Derivation of Formulas for the Vibration Inverse Problem

From the vibration characteristics of the building structure, it can be simplified to a multi-mass system model. The undamped free equation of motion of the system is:

\[ [M] \ddot{U} + [K]U = 0 \]  

where: \([M]\) and \([K]\) are the mass and stiffness array. The mass array can be expressed as:

\[
[M] = \begin{bmatrix}
m_1 & 0 \\
0 & m_2 \\
& \ddots \\
& & 0 & m_n
\end{bmatrix}
\]  

The stiffness array is of the tridiagonal matrices.

\[
[K] = \begin{bmatrix}
k_1 + k_2 & -k_2 & \cdots & 0 \\
-k_2 & k_2 + k_3 & \ddots & \ddots \\
& \ddots & \ddots & \ddots \\
& & -k_{n-1} & k_{n-1} + k_n & -k_n \\
& & & -k_n & k_n
\end{bmatrix}
\]  

The vectors to be used in the derivation of the formula are force vectors \(P = \{P_1, P_2, \ldots, P_n\}^T\); simplify the displacement vector of each node of the model \(U = \{u_1, u_2, \ldots, u_n\}^T\). They are related as follows:

\[ P = k[E]TU \]  

The equation of motion of the system is derived from Newton's second law as:

\[ [M] \ddot{U} = -[E]P \]  

where \(U = A \sin(\omega \cdot t + \phi)\), substituting into equation given:

\[ \omega^2[M] \{A\} = [E]k[E]^\top \{A\} \]  

Therefore:
\[
\begin{bmatrix}
  k_1 \\
  k_2 \\
  \vdots \\
  k_n \\
\end{bmatrix}
\begin{bmatrix}
  A_1 \\
  A_2 - A_1 \\
  \vdots \\
  A_n - A_{n-1} \\
\end{bmatrix}
= \omega^2
\begin{bmatrix}
  m_1 A_1 + m_2 A_2 + \cdots + m_n A_n \\
  m_2 A_2 + \cdots + m_n A_n \\
  \vdots \\
  m_n A_n \\
\end{bmatrix}
\tag{18}
\]

where:
\[
k_r = \omega^2 \frac{m_r A_r + m_{r+1} A_{r+1} + \cdots + m_n A_n}{A_r - A_{r-1}} \quad (r=2, 3, \ldots, n)
\tag{19}
\]

For programming purposes, the general formula can be written as:
\[
k_i = (2\pi \cdot f)^2 M \cdot \sum_{j<i} \alpha_j A_j / (1 + \sum_{l=2}^{n} \alpha_l) \cdot (A_j - A_{j-1})
\tag{20}
\]

where: \( M \)=total mass of the construction work
\( \alpha = \frac{m_j}{m_i} \), \( f \), and \( A_i , A_{i+1} , \ldots , A_i \)=measured data

4.2. Relationship Between Stiffness and Strength

From the experiments, it can be seen that the masonry compressive stress-strain curve is:
\[
\sigma = f_n \left(1 - e^{-\xi \sqrt{\varepsilon}}\right)
\tag{21}
\]

The compressive modulus of elasticity is:
\[
E = \alpha \cdot f_n \sqrt{f_m} = 370 f_n \sqrt{f_m}
\tag{22}
\]

The lateral force stiffness \( k_j \) of each wall piece of the load-bearing wall.
\[
k_j = \sum k_{ij}
\tag{23}
\]

The lateral transfer stiffness of the wall is:
\[
k = \frac{1}{\delta} = \frac{Et}{3\rho + \rho^3}
\tag{24}
\]

Substitution of Eq. (22) into the above equation yields the relationship between stiffness and compressive strength of masonry walls.
\[
k = \frac{\alpha \cdot f_n \sqrt{f_m} \cdot t}{3\rho + \rho^3}
\tag{25}\]
where: $k =$ lateral force stiffness of the wall 
$\alpha =$ masonry strength test coefficient and $\alpha = 370$
$f_m =$ average value of masonry compressive strength

$$f_m = \left[ k\left(3\rho + \rho^3\right)/(\alpha - t)\right]^{\frac{1}{3}} \quad (26)$$

From the above derivation, if the stiffness is obtained by the dynamic testing method, the strength value, the bearing capacity, can be obtained according to the relationship between stiffness and strength. Such a test method neither breaks the structure nor requires sampling.

### 4.3. Seismic Deformation Checking

The calculation equation for the elastic horizontal displacement of the frame is:

$$\Delta u_{ei} = \sum \frac{V_i}{D} \quad (27)$$

where: $\Delta u_{ei} =$ story elastic displacement of layer $i$ under the action of multiple encounter earthquake
$V_i =$ standard value of story seismic shear at layer $i$
$\sum D =$ sum of the values of column $D$ at level $i$

The elastic-plastic lateral shift of the frame weak layer is:

$$\Delta u_p = \eta_p \Delta u_e \quad (28)$$

$$\Delta u_e = \frac{V_e}{\sum D} \quad (29)$$

where: $\Delta u_p =$ elastic displacement between layers of the weak layer under the action of rare earthquakes
$\eta_p =$ elastic-plastic displacement increase factor
$\Delta u_e =$ story displacement analysed by elasticity under the effect of rare earthquakes
$V_e =$ standard value of story seismic shear force under the action of rare earthquakes

After calculating $V_e$ and using the dynamic measurement method to determine the weak layer $\sum D$, using Eq. (30) can be carried out to test the elastic-plastic lateral shift between layers.

$$\Delta u_p \leq [\theta_p]H \quad (30)$$

where: $[\theta_p] =$ elastic-plastic displacement angle limit between layers
$H =$ height of the weak layer

### 5. Engineering Application Examples

#### 5.1. 27# Commercial Residential Building Nondestructive Testing

Dalian a real estate development company 27# commercial residential building for six-story brick and concrete structure, its plan form is rectangular, 42.2m long, 9.6m wide, six total height 16.8m, floor height 2.8m, vertical and horizontal external wall thickness 37cm, internal wall thickness 24cm. With
M5 mortar, MU10 mechanism red-brick masonry, horizontal wall bearing. The test and analysis results are shown in the following table. Where $D_{fm}$ is the design story compressive strength. The vibration pattern, stiffness of each layer, compressive strength and strength reserve factor are shown in Figs. 3 to 6. The results are shown in the Table 3.

**Table 3.** 27# Building test and analysis results.

| N  | m(kg) | k(kN/m) | A | Sfm(MPa) | Dfm(MPa) | Str(MPa) | RA |
|----|-------|---------|---|----------|----------|----------|----|
| F1 | 57465 | 7704793.6 | 0. | 3.143 | 3.298 | 0.714 | 4.4 |
| F2 | 57465 | 7516205.4 | 0. | 3.092 | 3.298 | 0.590 | 5.2 |
| F3 | 57465 | 3642393.3 | 0. | 1.907 | 3.298 | 0.465 | 4.0 |
| F4 | 57465 | 2088585.5 | 0. | 1.314 | 3.298 | 0.341 | 3.8 |
| F5 | 56070 | 1434164.1 | 0. | 1.025 | 3.298 | 0.216 | 4.7 |
| F6 | 39953 | 1370903.1 | 1. | 0.994 | 3.298 | 0.096 | 10. |

![Figure 3. Building 27# vibration type.](image)

![Figure 4. 27# story stiffness (MN/m).](image)

![Figure 5. 27# compressive strength (MPa).](image)

![Figure 6. 27# compressive strength reserve factor.](image)
From the above analysis results, it can be seen that the sudden change in stiffness between the second and third floors of the 27# building is large. The original design masonry compressive strength is 3.298 MPa, only the first and second floors reach the design strength, and the rest of the floors are not reached. However, the compressive strength reserve factor of each layer is larger. Because of the compressive stress in the brick masonry is small, thus the compressive strength reserve is still sufficient.

5.2. 29# Commercial Residential Building Nondestructive Testing
Dalian a real estate development company 29# commercial residential building for a six-story brick structure, its plan form is rectangular, 48.6m long, 9.6m wide, total height 16.9m, the first to five floors 2.8m high, the sixth-floor height 2.9m, vertical and horizontal external wall thickness 37cm, internal wall thickness 24cm. With M5 mortar, MU10 mechanism red-brick masonry, horizontal wall load-bearing. The test and analysis results are shown in the following table. The vibration pattern, stiffness of each layer, compressive strength and strength reserve factor are shown in Figs. 7 to 10.

Table 4. 27# Building test and analysis results.

|   | m(kg) | k(kN/m) | A | Sfm(MPa) | Dfm(MPa) | Str(MPa) | RA |
|---|-------|---------|---|----------|----------|----------|----|
| F1 | 59230 | 8207496.3 | 0. | 3.008    | 3.298    | 0.654    | 4.6 |
| F2 | 59123 | 6992091.8 | 0. | 2.703    | 3.298    | 0.539    | 5.0 |
| F3 | 59123 | 3733119.57 | 0. | 1.779    | 3.298    | 0.425    | 4.1 |
| F4 | 59123 | 2285763.5 | 0. | 1.283    | 3.298    | 0.311    | 4.1 |
| F5 | 59875 | 1660774.9 | 1. | 1.037    | 3.298    | 0.197    | 5.2 |
| F6 | 46772 | 1761081.6 | 1. | 1.105    | 3.298    | 0.081    | 13.|

Figure 7. Building 29# vibration type.

Figure 8. 29# story stiffness (MN/m).

Figure 9. 29# compressive strength (MPa).
From the above analysis results, it can be seen that the sudden change in stiffness between the second and third floors of the 29# building is large. The original design masonry compressive strength is 3.298 MPa, and only the first floor basically reaches the design strength, while the rest of the floors do not. However, the compressive strength reserve factor of each layer is larger. Due to the small compressive stress in the brick masonry, thus the compressive strength reserve is still sufficient.

6. Conclusion

Eq. (18) by the frequency and vibration type of masonry story stiffness, as long as the test data is accurate, story stiffness is accurate. Eq. (20) is the statistical results of a large number of test data, involving the geometry and material properties of the wall, reflecting the objective law of story stiffness and masonry strength. Eq. (1) is based on the specification, according to a large number of test data induction of the empirical formula. Thus, the actual detection of the frequency and vibration type of the average masonry strength is reliable and more objective. To make the method more perfect and accurate, there is a need to continuously accumulate the actual test data. In the case of conditions permitting, the method is best used in conjunction with the in-situ axial compression method for calibration. Also, accumulate the experience of field testing and improve the level of testing to make the test results better reflect the real situation. Strictly speaking, the strength of the materials tested in the laboratory or the design strength on the drawings are not a true representation of the actual on-site building masonry strength, only the strength data measured on the actual structure can objectively and truly represent the masonry strength condition of the on-site building.

The analysis of inter-story stiffness and compressive strength of masonry structure by vibration test data is more reliable and can find out more quickly and effectively in which adjacent layers large story stiffness mutations occur. Thus, effective reinforcement measures can be taken so that when the earthquake comes, these story stiffness mutations do not cause more serious damage, thus ensuring that the structure can function better to resist earthquake effects. This method can also be extended to other building structures that conform to shear-type vibration.

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