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Polar Motion of the Triaxial Nonrigid Earth and Atmospheric Excitation

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Abstract The present study aims to extend the traditional rotation theory of the rotational-symmetric Earth to the triaxial Earth. We re-formulate the Liouville equations and their general solutions for the triaxial nonrigid Earth and find that the traditional theory introduces some theoretical errors in modeling the excitation functions. Furthermore, we apply that theory to the atmospheric excitation and find that theoretical errors should not be neglected given the present measurement accuracy. Thus we conclude that the traditional theory of the rotation of the rotational-symmetric Earth should be revised and upgraded to include the effects of the Earth's triaxiality.

Keywords Earth rotation; triaxiality of the Earth; atmospheric excitation

CLC number P223

Introduction

In a previous study by Chen et al. (2010),¹ the atmospheric excitation of polar motion for a rotational-symmetric (i.e. \( A = B \)), single-layer and elastic Earth model was discussed. The authors demonstrated that atmospheric excitation is a significant factor in the seasonal wobble (e.g., the 18-month wobble), the Chandler Wobble (CW), as well as some distinct polar wobbles corresponding to some great diurnal and semi-diurnal atmospheric tides.

However, recent measurements have confirmed that all the Earth's principal moments of inertia, \( A, B \) and \( C \), are different from each other.²⁻⁵ The Earth is triaxial rather than rotational-symmetric. Consequently, the Earth's rotation state, closely related to the principal moments of inertia, should be more or less different from that predicted by the traditional theory.

Recently, more and more studies have paid attention to the rotation of the triaxial Earth. Van Hoolst & Dehant pointed out that the triaxiality of the Earth could reduce the frequency values of the CW and the Free Core Nutation (FCN).⁶ Wang suggested that the triaxial nature might be responsible for the decadal polar motion.⁷ Folgueira & Souchay discussed...
the free polar motion of the triaxial and elastic Earth in Hamiltonian formalism, and found that both the longitude and latitude of the pole oscillate with the semi-Chandlern period.\cite{Shen} Shen et al. did an elementary study on the free Euler motion of a triaxial rigid Earth and found that the triaxial nature could give rise to a small fluctuation with a semi-Euler period in the Length Of Day (LOD).\cite{Chen} By extending and developing the study of\cite{Shen}, Chen et al.\cite{Chen} and Chen et al.\cite{Chen} calculated a new set of elliptic function solutions to the Euler-Liouville dynamic equations as well as a theoretical model for the Frequency-Amplitude Modulation (FAM) of the free wobble. The FAM might be a candidate to explain the positive correlation between the amplitude and period of the Chandlern wobble, which has been demonstrated by long time observations.\cite{Chen-1,Chen-2,Chen-3,Chen-4,Chen-5} Paper\cite{Chen} also found that the triaxial wobble, which has been demonstrated by long time observations.\cite{Chen-1,Chen-2,Chen-3,Chen-4,Chen-5} Based on the above discussion, we expected the atmospheric excitation for a triaxial Earth should be somewhat different from that for a rotationally-symmetric one. In addition, the Liouville equations and their general solutions should be developed to incorporate the effect of the Earth’s fluid core and the triaxialities of the whole Earth and the fluid core. Rather than single-layer, the Earth can be roughly divided into three layers: the mantle, the fluid outer core and the solid inner core. The effects of the inner core on Earth’s rotation are negligible in most cases with the present measurement accuracy.\cite{Chen-6}

1 Liouville equations for the triaxial Earth: General expressions

When considering triaxiality, the reference frame of the Earth should be defined as one whose coordinate axes coincide with the mean principal axes of the mantle, namely the Tisserand axial system of the mantle, to ensure that the products of inertia $c_{ij} \ (i, j = 1, 2, 3)$ are sufficiently small and their products with other small quantities can be neglected. The reference frame uniformly rotates around the mean rotational axis of the mantle with the mean sidereal rate $\Omega$. In this frame, the vector Liouville equations could be written as\cite{Chen-6,Chen-7,Chen-8}

$$ L = \frac{\partial}{\partial t} H + \omega \times H, \quad H = I \cdot \omega + h $$

(1)

where $L$ is the external torques, $\omega = (m_1, m_2, 1+m_3)\Omega$ is the angular velocity vector, $H$ is the angular moment vector, and

$$ I = \begin{bmatrix} A + c_{11} & c_{12} & c_{13} \\ c_{12} & B + c_{22} & c_{23} \\ c_{13} & c_{23} & C + c_{33} \end{bmatrix} $$

(2)

is the Earth’s inertia moment tensor where the principal moments $A, B$ and $C$ are assumed to be constants and $c_{ij} \ (i, j = 1, 2, 3)$ are the time-dependent components of that tensor ($c_{ij}$ corresponds to deformations in the Earth). Wherein, $m_1 \ (i = 1, 2, 3), \ c_{ij}, \ \alpha = (C-A)/B$ and $\beta = (C-B)/A$ are all small quantities. Accurate to their first order, one gets:

$$ H = \Omega \begin{bmatrix} A m_1 + c_{13} \\ B m_2 + c_{23} \\ C (1 + m_3) + c_{33} \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} $$

(3)

and

$$ \omega \times H = \omega \times (I\omega) + \omega \times h $$

$$ = \Omega^2 \begin{bmatrix} (C-B) m_2 - c_{23} \\ -(C-A) m_1 - c_{13} \\ 0 \end{bmatrix} + \Omega \begin{bmatrix} -h_2 \\ h_1 \\ 0 \end{bmatrix} $$

(4)

then Eq.(1) changes to the scalar form:

$$ A \Omega \dot{m}_1 + \Omega^2 (C-B) m_2 = \Omega^2 c_{23} - \Omega c_{13} + \Omega h_2 - \dot{h}_1 + L_1 $$

(5)

$$ B \Omega \dot{m}_2 - \Omega^2 (C-A) m_1 = -\Omega^2 c_{13} - \Omega c_{23} - \Omega h_1 - \dot{h}_2 + L_2 $$

(6)

$$ C \Omega \dot{m}_3 = -\Omega c_{33} - \dot{h}_3 + L_3 $$

(7)

Obviously, Eq. (7) has the same form as it has in the rotational-symmetric case and so does its solution, while Eqs. (5) and (6) differ from the traditional ones. Thus we limit our attention on the solutions to Eqs. (5) and (6).

Now setting $M_1 = m_1 (\beta / \alpha)^{1/4}$ and $M_2 = m_2 (\alpha / \beta)^{1/4}$, then $m_1 = M_1 (\alpha / \beta)^{1/4}$ and $m_2 = M_2 (\beta / \alpha)^{1/4}$, and Eqs.(5) and (6) change to:

$$ \sigma E^{-1} M_1 + M_2 $$
\[
\sigma_e = \Omega \sqrt{q_B} = \Omega \sqrt{\frac{(C - A)(B - C)}{AB}}
\]

is the Euler frequency for the triaxial Earth. Obviously, Eq. (10) will reduce to the traditional form \( \sigma_e = \frac{C}{A} \Omega \) once \( A = B \) is assumed.

By introducing the modifying coefficients:

\[
\begin{align*}
K_1 &= \left[ \frac{A(C - A)}{B(C - B)} \right]^{1/4} \\
K_2 &= \left[ \frac{B(C - A)}{(C - A)^3} \right]^{1/4}
\end{align*}
\]

and the modified excitation functions:

\[
\begin{align*}
\psi_1 &= K_1 \frac{\Omega^2 c_{13} + \Omega \dot{c}_{13} + \Omega \dot{h}_2 - \dot{h}_1 + \frac{L}{2}}{\Omega^2 (C - A)} \\
\psi_2 &= K_2 \frac{\Omega^2 c_{23} - \Omega \dot{c}_{23} + \Omega h_2 - \dot{h}_1 + \frac{L}{2}}{\Omega^2 (C - A)}
\end{align*}
\]

Eqs. (8) and (9) can be rewritten as

\[
\begin{align*}
\sigma_e^{1/4} M_1 + i M_2 &= \psi_2 \\\n\sigma_e^{1/4} M_2 - M_1 &= -\psi_1
\end{align*}
\]

Setting

\[
\begin{align*}
M &= M_1 + i M_2 \\
\psi &= \psi_1 + i \psi_2
\end{align*}
\]

the two equations of Eq. (13) combine to

\[
\frac{i}{\sigma_e} M + M = \psi
\]

As is well known, the solution to Eq. (15) is

\[
M = e^{i \sigma_e \tau} \left[ M_0 - i \sigma_e \int \psi(\tau) e^{-i \sigma_e \tau} d\tau \right]
\]

and finally, one gets

\[
\begin{align*}
m_1 &= M_1 (\alpha / \beta)^{1/4} \\
m_2 &= M_2 (\beta / \alpha)^{1/4}
\end{align*}
\]

which describe the motion of the rotational pole.

## 2 Inclusion of the effects of rotational deformation and fluid core

The rotation of the Earth will give rise to a centrifugal force and that force would give rise to a deformation of the non-rigid Earth. That is the Earth's principal departure from the rigid body. Also, there are some other non-rotational deformations, such as the luni-solar tidal deformation. We can decompose the total deformation as:

\[
c_{13} = c_{13}^g + c_{13}^{NR}, \quad c_{23} = c_{23}^g + c_{23}^{NR}
\]

and correspondingly,

\[
\psi = \psi^g + \psi^{NR}
\]

where the superscripts \( R \) and \( NR \) correspond to the rotational and non-rotational deformations respectively.

The rotational-induced perturbation of tensor might be written as\(^{18-20}\)

\[
c_{13} = \frac{ka^2}{3G} m_1, \quad c_{23} = \frac{ka^2}{3G} m_2, \quad \frac{ka^2}{3G} = \frac{k}{k_S} (C - A)
\]

where \( k \) is the second degree Love number and \( k_S = \frac{3G(C - A)}{a^2 \Omega^2} \) is the secular Love number. One could easily verify that Eq. (20) also holds for the case of triaxial Earth rotation since the centrifugal potential is axial-symmetric in space with respect to the rotation axis.

Based on Eq. (20), the excitation functions due to the rotational deformation might be written as:

\[
\begin{align*}
\psi_1^g &= K_1 \frac{\Omega^2 c_{13}^g + \Omega \dot{c}_{13}^g}{(C - A) \Omega^2} = K_1 \frac{k}{k_S} \left( m_1 + \frac{\dot{m}_1}{\Omega} \right) \\
\psi_2^g &= K_2 \frac{\Omega^2 c_{23}^g - \Omega \dot{c}_{23}^g}{(C - A) \Omega^2} = K_2 \frac{k}{k_S} \left( m_2 + \frac{\dot{m}_2}{\Omega} \right)
\end{align*}
\]

Noting that the terms \( \frac{M_i}{\Omega^i} (i = 1, 2) \) are actually negligible (see the Appendix), also noting that \( \frac{A(C - A)}{B(C - B)} \sim [1 + O(10^{-3})] \) and \( \frac{B(C - A)}{A(C - B)} \sim [1 + O(10^{-3})] \) \((O(x) \text{ reads on the order of } x)\), Eq. (21) reduces to

\[
\begin{align*}
\psi_1^g &= \frac{k}{k_S} M_1 \\
\psi_2^g &= \frac{k}{k_S} M_2
\end{align*}
\]
when the small quantities like $O(10^{-5})$ are neglected. Adopting complex notations, Eq. (22) can be rewritten as

$$\psi^* = \psi^1 + i\psi^2 = \frac{k}{k_s} M \tag{23}$$

Substituting Eqs. (19) and (23) into Eq. (15), one gets

$$\frac{i}{\sigma_c} M + (1 - \frac{k}{k_s}) M = \psi^\text{NR} \tag{24}$$

Now, we use a simple method to add the effect of the fluid core. The presence of the core influences mainly the Chandler frequency and introduces a new mode, namely the Free Core Nutation (FCN). Only the former will be considered here while the FCN for a triaxial fluid core has been dealt with in a separate study \cite{10} and will not be discussed here.

Similar to Jochmann (2009), \cite{21} we revise the Euler frequency as

$$\sigma_c = \Omega \sqrt{\frac{(C-A)(C-B)}{A_a B_m}} \tag{25}$$

and replace $\psi^\text{NR}$ with $\sqrt{\frac{AB}{A_a B_m}} \psi^\text{NR}$ in Eq. (24) to match the triaxial Earth model with a triaxial fluid core. In Eq. (25), $A_a$ and $B_m$ are the equatorial principal moments of inertia of the mantle, and $A_f = A - A_a$ and $B_f = B - B_m$ are those for the fluid core. Let

$$\sigma_c = (1 - k/k_s) \sigma_c = (1 - \frac{k}{k_s}) \sqrt{\frac{(C-A)(C-B)}{A_a B_m}} \Omega \tag{26}$$

Eq. (24) changes to

$$\frac{i}{\sigma_c} M + M = \frac{k_s}{k_s - k} \sqrt{\frac{AB}{A_a B_m}} \psi^\text{NR} \equiv \psi^\text{eff} = 1.61 \psi^\text{NR}$$

(27)

where $\sigma_c$ is the Chandler frequency for a triaxial two-layer Earth model and $\psi^\text{eff}$ might be called the effective excitation function. This process is inconsistent with \cite{21} since Eq. (27) would be equivalent to Eq. (39) on paper \cite{21} once we set $A = B$ and $A_a = B_m$.

Similar to Eq. (39), the solution to Eq. (27) is

$$M = e^{i\sigma_c t} \left[ M_0 + i\sigma_c \int_0^t \psi^\text{eff}(\tau) e^{-i\sigma_c \tau} d\tau \right] \tag{28}$$

Then the polar coordinate $(m_1, m_2)$ can be obtained from Eq. (17). In the special case that $\psi^\text{eff} = 0$, we get the Chandler wobble

$$\begin{align*}
m_1 &= M_0 \frac{(\alpha/\beta)^{1/4}}{1 - k/k_s} \cos(\sigma_c t + \chi) \\
m_2 &= M_0 \frac{(\beta/\alpha)^{1/4}}{1 - k/k_s} \sin(\sigma_c t + \chi) \tag{29}
\end{align*}$$

which is obviously an elliptic motion with its semi-major and minor axes parallel to the mean principal axes of the mantle (or, approximately of the whole Earth). These confirm some conclusions of Chen et al. (2009)\cite{10} stated in the introduction from a different aspect.

In fact, $c_{13}^\text{NR}$ and $c_{23}^\text{NR}$ denote the permanent rotational deformation of the Earth due to the rotation with a constant angular velocity $\Omega$. We can limit our interest in the non-rotational deformations since we can always adopt Eq. (27) to include this permanent deformation. Thus we will omit the superscript “NR” in the following text but one must keep in mind that the quantities such as $c_{13}^\text{NR}$, $c_{23}^\text{NR}$ and $\psi$ are actually $c_{13}$, $c_{23}$ and $\psi$ respectively.

3 Atmosphere-excited polar motion of the triaxial earth: An application

As to the influence of atmosphere on the rotation of the Earth, Barnes et al. (1983)\cite{22} introduced the so-called angular momentum functions, which makes the treatment of atmospheric effects much more convenient. The angular momentum functions are defined as:

$$\begin{align*}
X_1 &= \frac{\Omega c_{13} + h_1}{\Omega (C - A)} = \chi_{p1} + \chi_{w1} \\
X_2 &= \frac{\Omega c_{23} + h_2}{\Omega (C - A)} = \chi_{p2} + \chi_{w2} \tag{30}
\end{align*}$$

where the subscripts $p$ and $w$ denote the pressure term (relevant to $\Omega c_{13}$ and $\Omega c_{23}$) and the wind term (relevant to $h_1$ and $h_2$) respectively.

The atmospheric pressure will load the Earth, and the pressure term ($\chi_{p1}, \chi_{p2}$) will give rise to an additional term ($k' \chi_{p1}, k' \chi_{p2}$), which denotes the loading deformation of the Earth ($k'$ is the second order load Love number). Then, the total angular momentum function will be

$$\begin{align*}
X_1' &= (1 + k') \chi_{p1} + \chi_{w1} \\
X_2' &= (1 + k') \chi_{p2} + \chi_{w2} \tag{31}
\end{align*}$$

Considering Eqs. (30), (31), (12) and (27) (noting that only the non-rotational deformations are needed and concerned here), the effective angular momentum
might be expressed as:

\[
\psi_{eff} = K_i \frac{k_s}{k_s - k} \sqrt{\frac{A^2}{A_n B_n}} (\chi' + \frac{\dot{X}_t}{\Omega})
\]

\[
= K_i \frac{k_s}{k_s - k} \sqrt{\frac{A^2}{A_n B_n}} \left[ (1 + k') \chi_{p1} + \chi_{a1} + \frac{(1 + k') \dot{X}_{p1} + \dot{X}_{a1}}{\Omega} \right]
\]

(32)

\[
\psi_{eff} = K_2 \frac{k_s}{k_s - k} \sqrt{\frac{A^2}{A_n B_n}} (\chi' - \frac{\dot{X}_t}{\Omega})
\]

\[
= K_2 \frac{k_s}{k_s - k} \sqrt{\frac{A^2}{A_n B_n}} \left[ (1 + k') \chi_{p2} + \chi_{a2} - \frac{(1 + k') \dot{X}_{p2} + \dot{X}_{a2}}{\Omega} \right]
\]

By substituting the numerical values \( k = 0.30, \ k_s = 0.94 \) and \( k' = -0.30 \) to Eq.(32), one gets

\[
\psi_{eff} = \psi_{eff1} + iv_{eff2}
\]

where

\[
\psi_{eff1} = K_i \left[ 1.12 \chi_{p1} + 1.61 \chi_{a1} + \frac{1.12 \dot{X}_{p1} + 1.61 \dot{X}_{a1}}{\Omega} \right]
\]

\[
\psi_{eff2} = K_2 \left[ 1.12 \chi_{p2} + 1.61 \chi_{a2} - \frac{1.12 \dot{X}_{p2} + 1.61 \dot{X}_{a2}}{\Omega} \right]
\]

(33)

Then we can use Eq. (33) to calculate the atmospheric excitation function.

Here we adopt the National Centers for Environmental Prediction (NCEP) values for global Atmospheric Angular Momentum (AAM) as calculated from NCEP/NCAR (National Center for Atmospheric Research) re-analyses archived on pressure surfaces. \[23-26\] Data are given up to four times daily from 1948-1-1 to 2006-12-31, and are provided by the Global Geophysical Fluids Data (GGFD) Center of the International Earth Rotation and Reference Systems Service (IERS) (the AAM data are available at http://www.iers.org/MainDisp.csl?pid=43-25713). More information about the data can be accessed at the above mentioned website. A plot of the AAM data can be found in Chen et al. (2010). \[1\]

Based on Eq. (33) and the AAM data (with the Inverted Barometer (IB) correction added), the Atmospheric Excitation Function (AEF) for the triaxial two-layer Earth are obtained (denoted by \( \psi_{sym} = \psi_{sym}^i + iv_{sym}^i \)). We have also used the same AAM data to calculate the AEF for the triaxial two-layer Earth based on the traditional theory (denoted by \( \psi_{sym} = \psi_{sym}^i + iv_{sym}^i \)). The difference between \( \psi_{sym}^i \) and \( \psi_{sym}^i \) is given in Fig.2. One can see that the difference can reach \(-1\) mas in \( x \)-component and \(-0.2\) mas in \( y \)-component. These differences are so small that many might only consider the rotational-symmetric case at the times of Munk & MacDonald (1960), \[18\] Lambeck (1980), \[19\] and Moritz & Mueller (1987)\[20\]. However, we should not neglect these differences any longer since our measurement accuracy can now reach better than 0.1 mas for the pole coordinate. \[27\]

**Fig. 1** Difference between the excitation functions of the triaxial and rotational-symmetric Earth. The magnitudes of the differences \( \Delta \psi_x = \psi_{sym}^i - \psi_{sym}^i \) (top figure) and \( \Delta \psi_y = \psi_{sym}^i - \psi_{sym}^i \) (bottom figure) are about 0.2 mas and 1 mas respectively, where the subscripts “tri” and “sym” correspond to the triaxial and rotational-symmetric cases respectively.

It’s worthy noting that the details for atmospheric excitation of polar motion are referred to in the study of Chen et al. (2010), \[1\] except that all the parameters in the \( x \)- and \( y \)-directions should be timed by \( 1.12K_i \) and \( 1.12K_2 \) to include the effects of the fluid core and the Earth’s triaxiality.

**4 Discussion and conclusion**

In this paper, we have introduced the assistant parameter \( M \) to facilitate combining the equatorial components of the Liouville equations into a simple complex equation similar to the traditional expression. We then develop a general theory to describe the rotation of the triaxial Earth (in fact, the theory can also be applied to other triaxial celestial bodies such as Mars). As a special case, we apply our theory to the atmospheric excitation of polar motion and find that adopting the traditional theory will introduce theoretical errors (~0.2...
mas in $x$-component and $\sim 1$ mas in $y$-component) to the atmospheric excitation function. These errors, though small, should not be neglected with the present measurement accuracy in some contexts.

The differences between the triaxial and rotational-symmetric cases are actually and mainly caused by two sets of coefficients, $\{K_i, K_j\}$ (see Eq.(11)) and $\{(\alpha / \beta)^{1/4},(\beta / \alpha)^{1/4}\}$, which are directly related to the dynamic figure of the Earth, or more precisely, the triaxiality of the Earth. These coefficients, appearing either in the excitation function (e.g., Eq. (12)) or in the final expression (e.g., Eq. (17)), would give rise to minor but perhaps not negligible corrections to all the factors (namely mass redistribution, relative motions, and torques) perturbing the Earth’s rotation, and the atmospheric excitation is just an exemplification. Thus we can expect that excitations from earthquakes, oceans, core motions and so on are also somewhat different from the predictions based on the traditional theory, and taking account the Earth’s triaxiality might be necessary for more accurate estimations.

However, the question remains, are the present theory and results reliable? One can easily verify that the coefficients $\{K_i, K_j\}$ and $\{(\alpha / \beta)^{1/4},(\beta / \alpha)^{1/4}\}$ will all equal to 1 when setting $A = B$, and thus our theory reduces to just the traditional one deduced.\cite{18-21}

In other words, the traditional Earth rotation theory is just a special case of the present theory.

Based on the above study and discussion, we can conclude that with the present measurement accuracy, the traditional theory of the rotation of the rotational-symmetric Earth should be revised and upgraded to include the effects of the Earth’s triaxiality. The theory presented in this study might be more appropriate to describe the rotation of the triaxial Earth though further studies might be needed to incorporate the effects of the ocean and the solid inner core on models of the Earth’s rotation.

### Appendix

In this appendix we will show that the terms $\frac{M_i}{\Omega} (i=1,2)$ are actually negligible.

We start with the traditional case of Earth rotation (namely setting $A = B$). In that case, the permanent rotational deformation $c^R$, due to the Earth rotation with a constant angular velocity $\Omega$, can be written as

$$c^R = c^R_{13} + ic^R_{23} = \frac{k}{k} (C - A)m$$

and the excitation function might be expressed as

$$\psi = \psi^R + \psi^{NR} = \frac{\Omega^2 c^R - i\Omega c^R}{\Omega^2 (C - A)} + \frac{\Omega^2 c^{NR} - i\Omega c^{NR}}{\Omega^2 (C - A) + \Omega h - ih + L}$$

In Eqs.(34) and (35), we have adopted the traditional complex notations

$$\begin{align*}
\{ m = m_i + im_j, \\
\{ c = c_{13} + ic_{23}, \\
h = h_i + ih_j, \\
L = L_i + iL_j
\end{align*}$$

Also, the superscript R and NR correspond to the rotational and non-rotational deformations respectively.

Substituting Eqs. (34) and (35) into the traditional Liouville equation

$$\frac{i}{\sigma_E} \dot{\psi} + m + m = \psi$$

one gets

$$\frac{i(k_x + k}{\Omega}) \dot{\psi} + (k_x - k)m = \frac{\Omega^2 c^{NR} - i\Omega c^{NR}}{\Omega^2 (C - A) + \Omega h - ih + L}$$

noting that $\frac{1}{\sigma_E} = 305 \frac{1}{\Omega}$ and $\frac{k_x}{k} = 0.94 \frac{1}{0.30} = 3.13$, thus $\frac{k}{\Omega} = 10 \frac{3}{3} k_x \sigma_E$ and is quite neglectable.

One can see easily that in the case of triaxial Earth, there will arise the additional coefficients $\frac{A(C - A)}{B(C - B)}$ and $\frac{B(C - B)}{A(C - A)}$ (see Eq.(21)), which are all quite close to 1, and that is the only significant difference between the rotational-symmetric and triaxial cases. Obviously, these coefficients will not alter the order of magnitude of the terms $\frac{M_i}{\Omega} (i=1,2)$, and thus we can still neglect them.

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