Cooperative vehicle platoon control considering longitudinal and lane-changing dynamics

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\textbf{ABSTRACT}
This paper presents a distributed cascade Proportional Integral Derivate (DCPID) control algorithm for the connected and automated vehicle (CAV) platoon considering the heterogeneity of CAVs. Furthermore, a real-time cooperative lane-changing model for CAVs, which can seamlessly combine the DCPID algorithm and sine function is developed. The DCPID algorithm determines the appropriate longitudinal acceleration and speed of the lane-changing vehicle considering the speed fluctuations of the front vehicle on the target lane (TFV). In the meantime, the sine function plans a reference trajectory which is further updated in real time using the model predictive control (MPC) to avoid potential collisions. Simulation results indicate that the DCPID algorithm can provide robust control for tracking and adjusting the desired spacing and velocity for all 400 scenarios. Besides, the proposed cooperative lane-changing model can guarantee effective and safe lane changing with different speeds and even in emergency situations.

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\section{Introduction

With the rapid development of connected and automated vehicle (CAV) technologies, the transition from the traditional human-driven traffic to the CAV traffic is undergoing. CAVs can make full use of connectivity, especially fast V2V and V2I communication, to improve average traffic speed (Pan et al. 2021), traffic capacity (Zhou, Qu, and Qi 2021), safety (Zheng et al. 2021) and stability (Mena-Oreja, Gozalvez, and Sepulcre 2018; Pasquale et al. 2018; Talebpour and Mahmassani 2016). Among the applications of CAV technologies, the vehicle platooning has gained a lot of attention in recent years. Vehicle platooning is a coordinated movement mechanism where vehicles travel with small headways without any mechanical
linkage (Maiti, Winter, and Kulik 2017). The main benefits of vehicle platooning include energy consumption savings due to less aerodynamic drag (Lee et al. 2020), increased road capacity and enhanced traffic safety by providing lower reaction time (Wang, Almeida Correia, and Lin 2020; Bhoopalam, Agatz, and Zuidwijk 2018). Therefore, it is of great significance to investigate the automatic control of vehicles for platooning.

Basically, the vehicle platooning is mainly composed of two aspects: the longitudinal tracking control and lateral lane-changing control. In the literature, considerable research has been carried out to develop longitudinal control models, among which the cooperative adaptive cruise control (CACC) model developed by the PATH laboratory, the model predictive control (MPC) models and the linear control models have gained substantial attention in both traffic and control engineering. The CACC technologies enable a vehicle to follow the preceding vehicle smoothly while maintaining a safety distance based on information obtained from the preceding vehicles, thus to improve the traffic efficiency and reduce oscillations (Milanés et al. 2014; van Arem, van Driel, and Visser 2006; Wang et al. 2014). Milanés et al. (2014) first develop the CACC system by introducing V2V communications to the commercially available ACC systems. The field road tests show that the CACC system could improve highway capacity and string stability. They further compare the CACC with the Intelligent Driver Model (IDM) and a commercial ACC through field experiments, and the test results indicate that the CACC performs best in terms of providing smooth and stable car following response (Milanés and Shladover 2014). Later, Liu et al. (2018) extend the CACC model to consider both car following and lane-changing behaviour of CAVs in mixed traffic. Dong et al. (2021) integrate the CACC model and machine learning algorithms to evaluate road efficiency and traffic safety at an off-ramp bottleneck using NGSIM dataset. However, the control performance of the CACC model is highly dependent on scenarios and parameter settings.

The MPC models aim at optimising the control decisions (e.g. acceleration/deceleration) of the following vehicles in the platoon for a certain prediction horizon based on the vehicles’ prevailing state information Wang, Gong, et al. (2019). Gong, Shen, and Du (2016) model the platoon as a multi-agent interconnected dynamic system and propose a one-step MPC control algorithm based on constrained optimisation and distributed computation. Gong and Du (2018) further extend their model to the cooperative MPC-based platoon control in mixed traffic flow with CAVs and human-driven vehicles (HDVs) where system optimisers are developed to consider multiple objectives including traffic efficiency and driving/riding comfort. Liu, Kurt, and Ozguner (2019) formulate the platoon as a dynamically decoupled system and propose a two-step distributed MPC algorithm to solve the dynamic formation problem. The aforementioned platoon control strategies require the optimal control problem be solved instantaneously at each sampling time instant without considering the control delay. To address the problem of control delay, Wang, Gong, et al. (2019) develop two real-time deployable approaches based on the MPC which can provide efficient cooperative control for all following vehicles in the platoon to damp traffic oscillations. One significant advantage of the MPC model is that it can deal with multiple criteria and constraints on vehicles’ state and control variables. However, the MPC approach, especially the centralised MPC which involves vehicles in a platoon, requires large amount of computation time and the stability of the controlled platoon through mathematical proofs is sometimes quite challenging (Zhou, Wang, and Ahn 2019). In addition, a plausible car-following model should satisfy the rational driving constraints (RDC) (Wilson and Ward
which, however, is always ignored since the mathematical derivation of the RDC for the MPC model is rather difficult.

The linear control models usually deal with designing feedback control laws (feedback gain matrices) for the state variables of the platoon system under different information flow topologies. The control values of the following vehicles are then calculated ensuring the stability of the closed-loop platoon system (Guo and Yue 2012; Zheng et al. 2016a). Guo and Yue (2012) propose a linear controller by acquiring the information of the preceding and leading vehicles of the platoon. Their approach can stabilise the platoon with robustness for a given level of disturbance attenuation; while (Ghasemi, Kazemi, and Azadi 2013) design a decentralised linear controller using the information of the preceding and following vehicles as inputs. Zheng et al. (2016b) summarise six kinds of commonly used information flow topologies and establish linear controller gains for vehicle platoons. They further investigate the scalability and stability margin of vehicle platoons under the undirected and bidirectional topologies (Zheng et al. 2016a). However, there always exists oscillation around the steady-state (with zero spacing error) for the linear controllers discussed above.

Besides the aforementioned linear controllers, the PID controller is also widely used for longitudinal tracking control and lateral lane-changing control to attain a steady-state of the vehicle platoon (Dasgupta et al. 2017; Ying et al. 2014; Karoui et al. 2017). These PDs are generally centralised controllers, which need to collect all vehicle information and perform large-scale optimisation calculations. To reduce computation costs, it is preferable to consider the vehicle platoon as a multi-agent system (MAS) where the distributed framework can be applied to design controllers. In this case, each distributed controller only performs local optimal control for one vehicle, e.g. see (Fiengo et al. 2019), where a distributed robust PID control approach is proposed to ensure good leader-tracking performance for a platoon of CAVs. It is worth noting that the PID controllers developed in (Zheng et al. 2016b; Ghasemi, Kazemi, and Azadi 2013; Zheng et al. 2016a; Dasgupta et al. 2017) are all single-layer PID controllers which couldn’t deal with problems involving constraints and multi-objective optimisation. To deal with these problems, (Lui, Petrillo, and Santini 2020) proposed a fully distributed PID control strategy which can be used to solve both the leader-tracking and containment control problems without requiring the fulfilment of additional constraints on the control input matrix.

For the lane-changing trajectory planning, most existing research applies the curve interpolation method or artificial potential field method. The curve interpolation method is to select an appropriate curve to fit the path, such as the polynomial curve, Bessel curve, B-spline curve, Sine curve, etc. The time-dependent polynomial curve trajectory planning method (Li et al. 2019; Xu et al. 2019; Heil, Lange, and Cramer 2016) is to design polynomial functions of the longitudinal and lateral positions with respect to time, where a large number of parameters need to be calibrated according to the state assumptions at the beginning and end of the lane-changing process. To relax these assumptions (e.g. the acceleration at the beginning and end of lane changing is 0), Yang et al. (2018) propose a time-independent polynomial curve method using the moving coordinate system, wherein the initial coordinate of the lane-changing trajectory for each time step is set as (0, 0). McNaughton et al. (2011) propose a curvature-dependent cubic polynomial spiral method. In their proposed approach, a method evaluating the Jacobian of the abridged endpoint state vector with respect to the unknown parameter vector is adopted in order to select the parameters which could satisfy the endpoint constraints more flexibly. Werling et al. (2010)
design a cost function to make the best compromise between the ease and comfort and the efficiency in order to find the optimal quintic polynomial curve-based lane-changing trajectory. Nevertheless, the above polynomial-based model parameters still lack physical significance and are difficult to compute to achieve a deterministic motion state (González et al. 2016). Maekawa et al. (2010) apply the cubic B-spline curve to generate the lane-changing trajectory. Fan et al. (2018) propose a spline-based curve method to generate a one-dimension-lane-changing trajectory in a Frenet frame. Their approach couldn’t guarantee the optimal solution because a lot of attention is paid to the continuity of the curvature, while the maximum lateral acceleration generated by the lane-changing trajectory is difficult to control. Chen et al. (2013) develop a quadratic Bessel curve-based lane-changing trajectory planning algorithm considering the safe distance of lane-changing and the riding comfort. This method could provide flexibility in selecting control points and modifying the lane-changing distance once determined. Recently, Lee et al. (2020) develop a vehicle trajectory generation approach for truck platooning by combing Kalman filter-based vehicle state estimation, front-rear trajectory buffering and the 3rd polynomial curve fitting method. The simulation results show that their proposed approach could maintain string stability for truck platooning in different conditions.

The artificial potential field method plans the vehicle path by assigning different potential functions to different types of obstacles and road structures (Rasekhipour et al. 2017). Rasekhipour et al. (2017) develop a potential field-based model predictive path-planning controller by combining the artificial potential field method with vehicle dynamics. Their method can guarantee vehicle stability during the path-tracking process. Huang et al. (2020) use the velocity information in the path planning and regulate the velocity within the predicted horizon of the current state to satisfy a feasible local trajectory. However, the artificial potential field method usually has the problems of target unreachability and falling into local minimum. In addition, the performance of this method becomes worse with the increase of the speed resulting in poor robustness Wang, Zhang, et al. (2019). By far, most of the lane-changing trajectory planning algorithms are static planning method, wherein the vehicle follows the one-time planned trajectory during the entire lane-changing process or the states of the surrounding vehicles of the lane-changing vehicle are assumed to keep unchanged in the whole lane-changing process. Thus, the static planning model cannot capture the intermediate interruptions caused by the abnormal behaviours of the surrounding vehicles during the lane-changing process (Venthuruthiyil and Chunchu 2022). In order to address the limitation, the dynamic trajectory planning approaches are considered, wherein the vehicle trajectory is replanned at a certain frequency according to the prevailing condition during the process of lane changing (Liu et al. 2021). Though Yang et al. (2018) propose a dynamic lane-changing trajectory planning model without considering the cooperative manoeuvrability of vehicles during lane changing, which could be inconsistent with the real environment.

In this paper, we propose a cooperative vehicle platooning control approach which is composed of a distributed cascade PID (DCPID)-based longitudinal control model and a cooperative lane-changing model. The main contributions of this paper are as follows.

1. We propose a distributed double-layer DCPID control algorithm by combining an inner loop controlling the velocity error and an outer loop controlling the spacing error with objectives that both errors are zero. A parameter tuning optimisation algorithm based
on the genetic algorithm is developed to guarantee the rational driving constraints (RDC), stability and good performance of the control system, overcoming the limitation of ignoring the RDC for many existing MPC controllers.

(2) The proposed DCPID algorithm has good robustness with respect to the uncertainty of the inertial lag of vehicle dynamics. The performance of the DCPID controller is insensitive to the control parameters, which is a significant improvement compared with the CACC model, whose control performance is highly dependent on scenarios and parameter settings.

(3) We develop a cooperative lane-changing model which consists of the DCPID control algorithm and a sine curve trajectory planning method. The first component ensures safe lane-changing by instructing the speed of the lane-changing vehicle. The second component plans a continuous path during which the passenger discomfort caused by lateral acceleration is considered. The improved sine function is simpler with fewer model parameters (all are explicable with physical significance), thus requiring fewer deterministic motion state assumptions (mostly unreasonable).

(4) The proposed model explicitly considers the heterogeneous behaviour of the CAVs in terms of the inertial lag and desired time headway. The uncertainty of lane-changing caused by the speed change of the front vehicle on the target lane, and the cooperative manoeuvre between the vehicle and the platoon on the target lane are both considered.

The rest of this paper is organised as follows: Section 2 presents the detailed description of the proposed cooperative vehicle platoon control model, and Section 3 provides the detailed mathematical analysis of the stability conditions for the DCPID and parameters tuning algorithm. In Section 4, the feasible domain of the parameter $a_p$ for trajectory planning is discussed. Section 5 gives the numerical examples both for the longitudinal control and cooperative lane-changing, and the simulation results are analysed and discussed. Section 6 concludes the paper, and some discussion and the prospect of future work are provided. Throughout the rest of the paper, all vehicles are referred to as CAVs.

2. Cooperative vehicle platoon control model

2.1. Description of the cooperative control framework

We consider a vehicle platoon travelling on a two-lane freeway as shown in Figure 1. A vehicle on the adjacent lane plans to change lanes and merge into the platoon. $SV$ denotes the vehicle which desires to change lanes and join the platoon in the target lane; $TFV$ and $TRV$ represent the vehicles in front and in rear of the desired position of the $SV$ in the target lane, respectively. Both the $TFV$ and the $TRV$ are the following vehicles in the original platoon and the $LV$ is the leading vehicle of the platoon.

Figure 2 illustrates the real-time cooperative vehicle platoon control framework which includes the longitudinal control and cooperative lane-changing models. The longitudinal control model is composed of a cascade PID control algorithm and a kinematic model, ensuring safe and efficient operation of the vehicle platoon. In case any vehicle in the platoon is disturbed, the longitudinal control model can respond quickly to make the platoon return to the steady state. The cooperative lane-changing model consists of three
2.2. DCPID algorithm for longitudinal control

Considering a platoon of $N$ vehicles, we define any two adjacent vehicles on the same lane as a subsystem. Let $P_{i-1}$ denote the subsystem consisting of the preceding vehicle $i - 1$ and following vehicle $i$. The DCPID algorithm controls the operation of the following vehicle of each subsystem. The longitudinal dynamics of the following vehicle in the subsystem is described by the linearised third-order state space equations as:

$$\begin{align*}
\dot{x}_i(t) &= v_i(t) \\
\dot{v}_i(t) &= a_i(t) \\
\dot{a}_i(t) &= -\frac{1}{\tau_i} a_i(t) + \frac{1}{\tau_i} u_i(t)
\end{align*}$$ (1)

We consider the heterogeneity of vehicles where different vehicles have different values of the inertial lag $\tau_i$. 

Figure 1. Illustration of vehicle lane changing.

Figure 2. The framework for real-time cooperative control.

Table 1 summarises some commonly used notations which will be mentioned in the model formulation.
Table 1. Notation list.

| Notation | Description |
|----------|-------------|
| $\rho_{i-1}$ | Subsystem consisting of the preceding vehicle $i-1$ and following vehicle $i$ |
| $i$ | Sequence number of a vehicle |
| $t$ | Time |
| $k$ | Number of time steps |
| $T_S$ | Sampling time interval |
| $x_i$ | Position of vehicle $i$ |
| $v_i$ | Velocity of vehicle $i$ |
| $a_i$ | Acceleration of vehicle $i$ |
| $u_i$ | Control input (which is the desired acceleration for vehicle $i$) |
| $\tau_i$ | Inertial lag of the longitudinal dynamics of vehicle $i$ |
| $d_i$ | Distance between the rear bumper of the leading vehicle $i-1$ and the front bumper of the following vehicle $i$ |
| $S_i$ | Desired distance between the rear bumper of the predecessor $i-1$ and the front bumper of the follower $i$ |
| $d_0$ | Minimum safe distance |
| $h_i$ | Desired time headway of vehicle $i$ |
| $e_{di}$ | Spacing error between the desired spacing and actual spacing of vehicle $i$ |
| $e_{vi}$ | Velocity error between the desired velocity and actual velocity of vehicle $i$ |
| $K_{pL_i}, K_{iL}, K_{dL_i}$ | Parameters of the outer loop PID of vehicle $i$ |
| $K_{pI_i}, K_{iI}, K_{dI_i}$ | Parameters of the inner loop PID of vehicle $i$ |
| $SV$ | Vehicle that desires to change lanes and join the platoon in the target lane |
| $LV$ | Leading vehicle of the platoon |
| $TFV$ | Vehicle in front of the desired position of $SV$ in the target lane |
| $TRV$ | Vehicle in rear of the desired position of $SV$ in the target lane |
| $d_{SV}$ | Longitudinal spacing between $SV$ and $TFV$ |
| $d_{TRV}$ | Longitudinal spacing between $TRV$ and $SV$ |
| $y_0$ | Lateral distance at the beginning of the lane-changing manoeuvre between $TFV$ and $SV$ |
| $v_{SV}$ | Velocity of $SV$ |
| $a_p$ | Planned acceleration considering the comfort of lane changing |
| $\psi_r$ | Desired yaw angle |
| $\delta_{fr}$ | Desired front wheel angle |
| $\psi$ | Yaw angle |
| $\delta_f$ | Front wheel angle |
| $L$ | Distance between the front and the rear axles of the vehicle |
| $N_p$ | Prediction horizon |
| $N_c$ | Control horizon |

An effective platoon controller should be able to provide good control ability for tracking and adjusting the desired spacing and velocity. Since a single-stage PID cannot handle control problems involving two objectives, we propose a DCPID control algorithm combined with vehicle dynamics. Figure 3 shows the proposed DCPID control structure. It is worth noting that the DCPID is a double control structure with the inner and outer loops. The outer loop PID controls the distance between adjacent vehicles, and the inner loop PID controls the vehicle speed. The way two loops are “stringed” in the cascade PID is to take the output of the outer loop as the target value of the inner loop. Specifically, in the outer loop PID, the target and feedback values are the desired and actual spacings, respectively. The output value of the outer loop PID is considered as the target velocity error for the inner loop PID at the current time. The actual relative velocity is a feedback value of the inner loop PID. And then a control variable $u$ (acceleration) is generated to control the dynamics of the rear-vehicle.

Let $d_i$ denote the distance between the rear bumper of the leading vehicle $i-1$ and the front bumper of the following vehicle $i$, we can obtain:

$$d_i(t) = x_{i-1}(t) - x_i(t) - l_{i-1}$$ (2)
where $l_{i-1}$ is the length of vehicle $i - 1$. The desired spacing control strategy we adopt here is the constant time headway spacing strategy, wherein the desired distance $S_i$ is defined as:

$$S_i(t) = d_0 + v_i(t)h_i$$  \hspace{1em} (3)

To simplify the problem, we assume all subsystems have the same $d_0$.

The goal of the DCPID control algorithm is to help each subsystem maintain the desired spacing and consistent speed, and respond to any vehicle disturbance quickly such that the stability of the whole platoon can be maintained. Two errors, which are used to measure the control target of the subsystem $P_{i,j-1}$, are defined as:

$$e_{xi}(t) = d_i(t) - S_i(t)$$  \hspace{1em} (4)

$$e_{vi}(t) = v_{i-1}(t) - v_i(t)$$  \hspace{1em} (5)

where $e_{xi}$ is the spacing error between the desired spacing and actual spacing of vehicle $i$. $e_{vi}$ is the velocity error between the desired velocity and actual velocity of vehicle $i$, while the desired velocity is equal to the velocity of $i$’s leading vehicle $i - 1$.

The outer loop PID control equation of the subsystem $P_{i,j-1}$ is given by:

$$o_i(t) = K_{px}e_{xi}(t) + K_{ix} \sum_{j=0}^{k} e_{xi}(jT_s) + K_{dx}^i[e_{xi}(t) - e_{xi}(t - T_s)]$$  \hspace{1em} (6)

where $o_i$ is the output of the outer loop PID; $j$ represents the time step between 0 and $k$ with $k = \frac{t}{T_c}$.

The inner loop PID control equation of the subsystem $P_{i,j-1}$ is given by:

$$u_i(t) = K_{pv}e_i(t) + K_{iv} \sum_{j=0}^{k} e_i(jT_s) + K_{dv}^i[e_i(t) - e_i(t - T_s)]$$  \hspace{1em} (7)

$$e_i(t) = o_i(t) - e_{vi}(t)$$  \hspace{1em} (8)

where $e_i$ and $u_i$ represent the input and output of the inner loop PID, respectively.

The delay of executing the desired acceleration commanded by the controller can occur in the vehicle system. Thus, we consider the inertial lag $\tau_i$ in vehicle dynamics as shown in

![Figure 3. The DCPID control structure.](image-url)
Equation (1), instead of assuming instantaneous realisation of acceleration. Equation (1) can be discretised by using the method of difference approximation, and the discrete model of the acceleration is obtained as:

\[ a_i(t + T_s) = \left( 1 - \frac{T_s}{\tau_i} \right) a_i(t) + \frac{T_s}{\tau_i} u_i(t) \]  

(9)

with constraints:

\[ u_{\text{min}} \leq u_i(t) \leq u_{\text{max}} \]
\[ a_{\text{min}} \leq a_i(t) \leq a_{\text{max}} \]
\[ v_{\text{min}} \leq v_i(t) \leq v_{\text{max}} \]  

(10)

where \( u_{\text{min}} \) and \( u_{\text{max}} \) are the minimum and maximum desired accelerations, respectively; \( a_{\text{min}} \) and \( a_{\text{max}} \) are the minimum and maximum actual accelerations, respectively; \( v_{\text{min}} \) and \( v_{\text{max}} \) are the minimum and maximum velocities, respectively. The first constraint considers the vehicle performance limitation. The second constraint relates to passenger comfort, and the third constraint reflects the road condition.

### 2.3. Cooperative lane-changing model

Figure 4 shows the flow chart of the proposed cooperative lane-changing model which includes three parts: (1) lane-changing decision; (2) dynamic trajectory planning; (3) trajectory tracking control.

![Figure 4](image-url)
2.3.1. Lane-changing decision

The status of the platoon vehicles can be divided into three types: (1) following the platoon, (2) joining the platoon and (3) leaving the platoon. This section discusses the second type of behaviour. If a vehicle wants to merge into the platoon on the target lane, the vehicle and platoon make cooperative decisions to complete the manoeuvre as shown in Figure 5. FV1 and FV2 are following vehicles in the original platoon in the target lane.

Before making lane-changing decision, the position of the SV in the platoon and the pair of vehicles TFV and TRV involved in the lane-changing process need to be determined. We assume that the location information of each vehicle where it leaves the freeway is available beforehand. It is worth noting that the manoeuvre of vehicles leaving the platoon (also called ‘splitting’) could cause disturbances, and thus affect platoon stability. In particular, if the vehicle, which is in the middle of the platoon, makes lane changes, its following vehicles need to adjust their spacings and velocities to restore the equilibrium state. This adjustment process could cause instability of the platoon. To reduce the impact of the splitting process on platoon stability, the position sequence of each vehicle in the platoon is determined according to the order in which it leaves the platoon. Specifically, the distance between the departure position and current position is used to determine the relative position of each vehicle in the platoon: the vehicle with the smallest distance is at the end of the platoon, while the vehicle with the largest distance is at the head of the platoon. In this way, the last vehicle in the platoon always leaves the platoon first.

When the position of the SV in the platoon is determined, the vehicles in front and rear of the SV in the target lane become the TFV and TRV, respectively, as shown in Figure 6. The SV sends the request of joining the platoon to the TFV and TRV.

Secondly, the subsystem composed of the TRV and TFV is disconnected. The TFV and the vehicles in front of the TFV continue to move with their current status. We define the position of the SV at the moment when Equation (11) is satisfied as the starting point of lane changing. The TRV and SV manoeuvre cooperatively to meet the conditions of the lane-changing: (1) The SV accelerates appropriately when the SV is behind the starting point of lane changing, or decelerates appropriately when the SV is in front of the starting point of lane changing; (2) The TRV decelerates to provide a safe lane-changing spacing for the

![Figure 5. Illustration of cooperative lane changing.](image)

![Figure 6. Lane-change spacing for vehicles.](image)
Lane changing is performed when the two spacings \(d_{SV}, d_{TRV}\) satisfy the following conditions:

\[
\begin{align*}
    d_{SV} &= S_{SV} \\
    d_{TRV} &\geq d_0
\end{align*}
\]  

where \(S_{SV}\) represents the desired spacing between the \(SV\) and \(TFV\).

Thirdly, when the lane changing is triggered, the \(SV\) and \(TFV\) constitute a new subsystem. Meanwhile, the \(TRV\) and \(SV\) constitute another subsystem to ensure safe longitudinal driving. It is noteworthy that we assume these two subsystems do not interfere with each other when the lane changing mechanism is triggered. Then, the lane-changing manoeuvre is carried out according to the model proposed in subsection 2.3.2 until the whole lane-changing process is completed.

### 2.3.2. Dynamic trajectory planning

To model the lateral motion of vehicles during the platoon formation process, the sine function curve is adopted to describe the lane-changing curve because the structure of the sine function is simpler with fewer model parameters, thus requiring fewer deterministic motion state assumptions (mostly unreasonable), and it can be combined with the DCPID more easily compared with other function curves. In addition, the sine function has good smoothness with the continuous second derivative. Therefore, we propose a novel dynamic lane-changing trajectory planning method, which integrates the DCPID longitudinal control algorithm with the sine function curve. The DCPID model described by Equations (1–9) is applied to update the acceleration of the \(SV\) during the lane-changing process, where we define a function \(F_{DCPID}\) as shown in Equation (12b). As shown in Figure 7, \(SV'\) and \(TFV'\) represent the final \(SV\) and final \(TFV\) when the \(SV\) completes the lane-changing process, respectively. Then, the \(SV'\) keeps the desired spacing \(S_{SV}\) from its preceding vehicle \(TFV'\).

Let \(x_{SV}^0, y_{SV}^0\) and \(y_{TFV}^0\) denote the longitudinal and lateral position of the \(SV\) and the position of the \(TFV\) at the beginning of the lane-changing process, respectively. The dynamic lane-changing trajectory planning model including dynamic update of the lane-changing speed and lane-changing trajectory generation is given by:

\[
\begin{align*}
    y_d^0 &= y_{TFV}^0 - y_{SV}^0 \\
    a_{SV}(t) &= F_{DCPID}(x_{SV}(t - T_s), x_{TFV}(t - T_s), v_{SV}(t - T_s), v_{TFV}(t - T_s)) \\
    v_{SV}(t) &= v_{SV}(t - T_s) + a_{SV}(t) \ast T_s, \quad t \in [t_0, t_{end}] \\
    y_r(x_r(t)) &= y_{SV}^0 + \frac{y_{SV}^0}{2\pi} \left\{ \frac{2\pi}{M(t)} (x_r(t) - x_{SV}^0) - \sin \left[ \frac{2\pi}{M(t)} (x_r(t) - x_{SV}^0) \right] \right\} \\
    M(t) &= v_{SV}(t) \sqrt{\frac{2y_{SV}^0}{a_p}} \\
    x_r(t) &\in [x_{SV}^0, x_{SV}^0 + M(t)]
\end{align*}
\]  

(12)

Figure 7. The lane-changing trajectory using the sine function.
where \((x_r(t), y_r(t))\) is the desired lane-changing trajectory of SV at time \(t\); \(t_0\) and \(t_e\) represent the start time and end time of lane changing, respectively; \(v_{SV}\) is the velocity of the SV; \(M(t)\) is the planned longitudinal lane-changing distance, which is determined by \(v_{SV}(t)\) and \(a_p\).

As shown in Equation (12), the dynamic trajectory planning model is basically a type of path-speed decoupled methods. This approach could provide more flexibility in both path and speed planning. The speed profile is calculated using Equations (12b) and (12c) derived based on the DCPID longitudinal control algorithm, which guarantees the safety of lane changing. Equation (12e) is used to determine the longitudinal length of the lane-changing trajectory according to the current speed of the SV.

In the general sine function \((y = A \sin(wx + \Theta) + b)\), the parameters \(A, w, \Theta\) and \(b\) have no physical meaning. By combining the sine function with the DCPID controller, \(A, w, \Theta\) and \(b\) are replaced by \(y_d^0, 2\pi/v_{SV}(t)\sqrt{\frac{2y_d^0}{a_p}}, x_{SV}^0\), and \(y_{SV}^0\) as shown in Equation (12d). The improved sine function has the following properties: (1) All the parameters of the sine function are given specific physical meaning involved in lane-changing; (2) The ride comfort can be explicitly considered in the lane-changing trajectory planning process by determining the feasible domain of the parameter \(a_p\) using the yaw-rate of the vehicle (The sensitivity analysis of \(a_p\) will be presented in section 4); (3) Notably, the dynamic trajectory planning can be realised with the proposed improved sine function using the \(v_{SV}\) in Equation (12d), which plans a series of trajectories with a certain frequency in the process of lane changing. That is, the planned reference trajectory can be updated at each time step, so that the safety of lane changing both in the steady state and extreme situations (such as when the TFV decelerates suddenly) can be guaranteed.

Equation (12f) gives the range of \(x_r\) at time \(t\), wherein the starting point of the lane-changing trajectory is fixed, and the end point is updated according to \(M(t)\) in real time. It can be seen that \(v_{SV}\) and \(a_p\) jointly determine the safety, real-time capability, and ride comfort of the whole lane-changing process. Let \(y_r', y_r''\) and \(K\) denote the first derivative of \(y_r\), the second derivative of \(y_r\) and the curvature of \(y_r\) with respect to \(t\), respectively, as given by:

\[
y_r'(x_r(t)) = \frac{y_d^0}{M(t)} \left\{ 1 - \cos \left[ \frac{2\pi}{M(t)} (x_r(t) - x_{SV}^0) \right] \right\}
\]

\[
y_r''(x_r(t)) = \frac{\pi a_p}{v_{SV}^2(t)} \sin \left[ \frac{2\pi}{M(t)} (x_r(t) - x_{SV}^0) \right]
\]

\[
K(x_r(t)) = \frac{y_r'''(x_r(t))}{(1 + y_r'(x_r(t))^2)^{3/2}}
\]

Let \(\phi_r\) and \(\delta_r\) denote the desired yaw angle and desired front wheel angle of the SV, which can be calculated as:

\[
\phi_r(x_r(t)) = \tan^{-1}(y_r'(x_r(t)))
\]

\[
\delta_r(x_r(t)) = \tan^{-1}(L * K(x_r(t)))
\]

where \(\phi_r\) and \(\delta_r\) are in radians.

After the lane-changing decision, vehicle SV and vehicle TFV are regarded as a new sub-system controlled by the DCPID control algorithm. Under the premise of avoiding collision
between the two vehicles, the DCPID control algorithm can calculate the acceleration of the lane-changing vehicle and update the velocity of the vehicle in real time by considering the relative speed and distance between the two vehicles. Then, the speed is transferred to the sine function model instantly to plan a lane-changing path. The proposed model can update the lane-changing path in real time according to the speed change of the TFV, which not only ensures the safety of lane-changing, but also improves the flexibility of lane-changing.

### 2.3.3. Trajectory tracking based on MPC method

We use the MPC controller to perform the real-time tracking control of the planned trajectory which is generated in subsection 2.3.2. The state space equation is given by:

\[
\tilde{\Phi}(k+1) = A(k) \tilde{\Phi}(k) + B(k) \tilde{U}(k)
\]  

(18)

with

\[
A(k) = \begin{bmatrix}
1 & 0 & -v_r \sin \psi_r T_s \\
0 & 1 & v_r \cos \psi_r T_s \\
0 & 0 & 1
\end{bmatrix}
\]

(19)

\[
B(k) = \begin{bmatrix}
\cos \psi_r T_s & 0 \\
\sin \psi_r T_s & 0 \\
\tan \delta_f T_s L & v_r T_s / \cos^2 \delta_f
\end{bmatrix}
\]

(20)

where \( \tilde{\Phi}(k) = X(k) - X_r(k) \), \( \tilde{U}(k) = U(k) - U_r(k) \), among which, \( X = [x \ y \ \psi]^T \) and \( U = [v \ \delta_f]^T \) are the current state and current control variables, respectively; \( X_r = [x_r \ y_r \ \psi_r]^T \) and \( U_r = [v_r \ \delta_{fr}]^T \) are the desired state and control variables respectively obtained from the reference trajectory; \((x, y)\) is the coordinate of the rear axle centre of the vehicle.

We design the objective function to ensure that the vehicle can track the reference trajectory quickly and smoothly. The minimum cost function and constraints of the MPC are given by:

\[
\min J(k) = \sum_{i=1}^{N_p} \tilde{\Phi}^T(k+i|k) Q \tilde{\Phi}(k+i|k) + \sum_{i=1}^{N_c-1} \tilde{U}^T(k+i|k) R \tilde{U}(k+i|k)
\]  

(21)

with

\[
\begin{cases}
U_{min} \leq U(k+i|k) \leq U_{max} \\
\Delta U_{min} \leq \Delta U(k+i|k) \leq \Delta U_{max} \\
\Delta U(k+i|k) = U(k+i|k) - U(k+i-1|k) \\
i = 0, 1, \cdots, N_c - 1
\end{cases}
\]

(22)

\[
\Delta U(k) = \begin{bmatrix}
\Delta U(k|k) \\
\Delta U(k+1|k) \\
\vdots \\
\Delta U(k+N_c-1|k)
\end{bmatrix}
\]

(23)

where \( Q \) and \( R \) are the weight matrices; \( U(k+i|k) \) is the control variable at the prediction time step \( k+i \) given the time step \( k \). \( N_p \) and \( N_c \) are the prediction and control horizons,
respectively. \( \Delta U(k) \) is a series of the system control variable increment at time step \( k \) in the control time domain \( N_C \); \( U_{\text{min}}, U_{\text{max}} \) are the minimum and maximum of the control variable \( U \); \( \Delta U_{\text{min}}, \Delta U_{\text{max}} \) are minimum and maximum of the control variable increment \( \Delta U \).

Therefore, this problem is transformed into a standard Quadratic Programming (QP) problem under the MPC framework. The first part of Equation (21) reflects the tracking ability of the control system to the reference trajectory, while the second part represents the constraints of the control variables of the system.

3. Stability analysis of the DCPID algorithm

This section presents stability analysis of the proposed DCPID algorithm, which includes follower stability analysis for the single-follower system and string stability analysis for the homogeneous and heterogeneous platoon system. The vehicle can maintain an equilibrium state (for example, the desired speed and desired spacing) under disturbance, which is called follower stability (Montanino and Punzo 2021). The amplitude of the disturbance (for example, deviation from the desired speed and desired spacing) gradually attenuates as it propagates from the downstream to the upstream in the vehicle platoon, which is called string stability (Zhou, Wang, and Ahn 2019). Stability is the basic requirement for the vehicle and vehicle platoon control (Li et al. 2022). Thus, we use the stability analysis method proposed by (Ward 2009; Montanino, Monteil, and Punzo 2021; Montanino and Punzo 2021; Qin and Wang 2021) to derive sufficient conditions for the follower and string stabilities. These conditions show that stability can be achieved by proper parameter tuning of the proposed DCPID algorithm.

3.1. Derivation of partial differential equation

The general formation of the DCPID longitudinal control can be expressed as:

\[
\dot{v}_i(t) = f(v_i(t), e_{vi}(t), d_i(t)) \quad (24)
\]

where \( \dot{v}_i(t) \) is the acceleration of vehicle \( i \) at time \( t \).

The partial differential values \( f_{i,v}, f_{i,e_v} \) and \( f_{i,d} \) of Equation (24) for \( v_i(t), e_{vi}(t) \) and \( d_i(t) \) in the equilibrium state \((v_e, 0, d_e)\) can be obtained as:

\[
\begin{align*}
    f_{i,v} &= \left[ \frac{\partial f(v_i(t), e_{vi}(t), d_i(t))}{\partial v_i(t)} \right]_{(v_e, 0, d_e)} \\
    f_{i,e_v} &= \left[ \frac{\partial f(v_i(t), e_{vi}(t), d_i(t))}{\partial e_{vi}(t)} \right]_{(v_e, 0, d_e)} \\
    f_{i,d} &= \left[ \frac{\partial f(v_i(t), e_{vi}(t), d_i(t))}{\partial d_i(t)} \right]_{(v_e, 0, d_e)}
\end{align*}
\]  

(25)

By substituting Equations (1)–(9) into Equation (25), we can obtain:

\[
\begin{align*}
    f_{i,v} &= -\frac{T_s h_i}{\tau_i} (K^{i}_p + K^{i}_r + K^{i}_c) (K^{i}_{pv} + K^{i}_{rv} + K^{i}_{dv}) \\
    f_{i,e_v} &= \frac{T_s h_i}{\tau_i} (K^{i}_p + K^{i}_r + K^{i}_c) (K^{i}_{pv} + K^{i}_{rv} + K^{i}_{dv}) \\
    f_{i,d} &= -\frac{T_s h_i}{\tau_i} (K^{i}_{pv} + K^{i}_{rv} + K^{i}_{dv})
\end{align*}
\]  

(26)

3.2. Rational driving constraints (RDC)

A plausible car-following model should satisfy the RDC (Wilson and Ward 2011), i.e. \( f_{i,v} < 0, f_{i,e_v} > 0, f_{i,d} > 0 \) in Equation (26). Therefore, we can derive the parameter constraints of
Equation (26) as given by.

\[
\begin{align*}
K_{px}^i + K_{ix}^i + K_{dx}^i & < 0 \\
K_{pv}^i + K_{iv}^i + K_{dv}^i & < 0
\end{align*}
\]  

(27)

### 3.3. Sufficient condition of DCPID stability

The conditions of follower stability and string stability for the homogeneous platoon system of the DCPID are stated as follows.

**Theorem 3.1:** Follower stability of the vehicle controlled by the DCPID is assured if satisfying:

\[f_{i,v} - f_{i,e,v} < 0 \]  

(28)

**Theorem 3.2:** The homogeneous vehicle platoon under the DCPID is string stable if satisfying:

\[
\frac{1}{2}(f_{i,v})^2 - f_{i,v}f_{e,v} - f_{i,d} > 0
\]  

(29)

### 3.4. Stability of a heterogeneous platoon system

We consider the heterogeneity of the DCPID controller in terms of the inertial lag and desired time headway. The string stability conditions of a platoon system with a finite number of vehicles \(N\) \((N > 1)\) are derived by applying the approach proposed in (Montanino and Punzo 2021; Montanino, Monteil, and Punzo 2021).

**Theorem 3.3:** The \(L_2\) weak string stability is assured if satisfying:

\[
\prod_{\omega}^N \left[ (f_{i,e,v} - f_{i,v} \omega + (f_{i,d} - \omega^2)^2 \right] - \prod_{\omega}^N \left[ (f_{i,e,v} \omega)^2 + f_{i,d}^2 \right] \geq 0 \forall \omega \in \mathbb{R}_0^+
\]  

(30)

where \(\omega\) is the fluctuation frequency.

Equations (27)–(30) show that the RDC and stability of the DCPID is closely related to its parameters. Therefore, we can ensure stability of the DCPID through proper parameter tuning, which provides theoretical support for the parameter settings in the simulation experiment.

### 3.5. Parameter tuning algorithm to guarantee RDC and stability

In the DCPID model, six key control parameters need to be determined including 3 parameters \((K_{px}, K_{ix}, K_{dx})\) for the outer loop control and 3 parameters \((K_{pv}, K_{iv}, K_{dv})\) for the inner loop control as indicated in Equations (6) and (7). Different parameter values could have significant influence on the performance of platoon control. Thus, it is necessary to determine the parameters which could provide the highest control gains. In order to evaluate the performance of the proposed DCPID algorithm, three performance indexes, namely, the steady
state adjustment time $t_{\text{steady}}$, velocity overshoot $v_{\text{overshoot}}$ and integral square error $J_e$ are chosen, and the latter two are calculated as:

$$v_{\text{overshoot}} = 100 \cdot \left| \frac{\max(v(t)) - v_0}{v_d - v_0} - 1 \right|$$  \hspace{1cm} (31)

$$J_e = \int_0^{T_{\text{sim}}} e_x^2(t) dt$$  \hspace{1cm} (32)

where $v_d$ and $v_0$ are the steady state and initial velocities, respectively. $T_{\text{sim}}$ is the total simulation time.

Moreover, the cascade PID parameters must satisfy the RDC and stability constraints, which can be derived by substituting Equation (26) into Equations (27)–(30). The objective function and constraints are given by:

$$\min \{v_{\text{overshoot}}, t_{\text{steady}}, J_e\}$$  \hspace{1cm} (33)

Subject to

$$\begin{aligned}
K_i & + K_i + K_d^i < 0 \\
K_i & + K_i + K_d^i < 0 \\
1 + \frac{h_i}{2} + \frac{\tau_i}{h_i T_s (K_i + K_i + K_d)} & > 0
\end{aligned}$$  \hspace{1cm} (34)

Based on the previous analysis, the parameter tuning can be regarded as a multi-objective optimisation problem with nonlinear constraints, which can be solved using the genetic algorithm (GA).

4. Feasible domain of the parameter $a_p$ for trajectory planning

In the cooperative lane-changing model, the feasible domain of $a_p$ in Equation (12) is determined by considering vehicle ride comfort. We define the yaw-rate $\gamma$ of the vehicle as an indicator to measure the vehicle ride comfort given by:

$$\gamma(t) = \frac{\gamma(t)}{v_{SV}(t)}$$  \hspace{1cm} (35)

where $\gamma(t)$ and $a_y(t)$ are the yaw-rate and lateral acceleration of the SV at time $t$, respectively. The upper bound of the yaw-rate $\gamma_{\text{upper\_bound}}$ that can guarantee comfort during lane-changing process can be described by Chen et al. (2013) as:

$$\gamma_{\text{upper\_bound}} = 0.85 \frac{a_{y_{\text{max}}}}{v_{SV}} = 0.425 \frac{0.5}{v_{SV}} \text{(rad/s)}$$  \hspace{1cm} (36)

where $a_{y_{\text{max}}}$ is the maximum lateral acceleration and $a_{y_{\text{max}}} = 0.5 m/s^2$ (Chen et al. 2013). Ride comfort can be guaranteed during the lane-changing process when the following conditions are met:

$$|\gamma|_{\text{max}} \leq \gamma_{\text{upper\_bound}}$$

$$|\gamma|_{\text{max}} = \max_{t_0 \leq t \leq t_e} |\gamma(t)|$$  \hspace{1cm} (37)

Figure 8 shows the impact of $a_p$ on the yaw-rate at three different speeds. The three dashed lines correspond to $\gamma_{\text{upper\_bound}}$ of the SV at 20, 25 and 30 m/s from the top to the
Figure 8. Effect of $a_p$ on $|\gamma|_{\text{max}}$ at three different speeds.

Table 2. The values of $\gamma_{\text{upper bound}}$ and the feasible domains of $a_p$ at different velocities of the $SV$.

| $v_{SV}$ (m/s) | $\gamma_{\text{upper bound}}$ (rad/s) | Feasible domain of $a_p$ |
|---------------|------------------|-----------------|
| 20            | 0.0212           | (0, 0.122)      |
| 25            | 0.0170           | (0, 0.114)      |
| 30            | 0.0142           | (0, 0.106)      |

The part below the dashed line indicates that the corresponding $a_p$ value can ensure the requirement of the riding comfort, while the part above the dashed line indicates that the corresponding $a_p$ value cannot meet the requirement. It can also be seen from Figure 8 that the feasible domain of $a_p$ satisfying the ride comfort decreases as $v_{SV}$ increases. When $a_p$ is determined, the difference between $|\gamma|_{\text{max}}$ and $\gamma_{\text{upper bound}}$ decreases as $v_{SV}$ increases. This indicates that the requirements for the ride comfort become higher as the velocity increases during the lane-changing process. The $\gamma_{\text{upper bound}}$ values and the feasible domains of $a_p$ under three velocities of the $SV$ are shown in Table 2.

5. Simulations and results analysis

5.1. Longitudinal platooning without lane changing

To demonstrate the performance of the proposed DCPID model, a numerical simulation is conducted considering a heterogeneous platoon composed of one leading vehicle and seven following vehicles.

The setting of the main simulation parameters is shown in Table 3. Since the state of each subsystem is similar, we set the same DCPID parameters for each subsystem. The parameters are estimated by applying the proposed parameter tuning algorithm in section 3.5, and the cascaded PID with a $P$ inner loop controller and a $PD$ outer loop controller performed
Table 3. The main simulation parameters (Zheng et al. 2016a; Zheng et al. 2017).

| Parameter                          | Notation | Unit | Value          |
|------------------------------------|----------|------|----------------|
| Control parameters                 |          |      |                |
| sampling time                      | $T_s$    | s    | 0.02           |
| time headway                       | $h_{i}$  | m    | ∈ (2, 8)       |
| minimum safe distance              | $d_0$    | m    | 4              |
| minimum control value              | $u_{\text{min}}$ | m/s² | −3            |
| maximum control value              | $u_{\text{max}}$ | m/s² | 3             |
| minimum acceleration               | $a_{\text{min}}$ | m/s² | −3            |
| maximum acceleration               | $a_{\text{max}}$ | m/s² | 3             |
| Vehicle parameters                 |          |      |                |
| length of the vehicles             | $l_i$    | m    | 5              |
| inertial lag of longitudinal dynamics | $\tau_{i}$ | s    | ∈ (2, 8)       |

We best with the parameter values of $K_{px}^i = -9.7591$, $K_{ix}^i = 0$, $K_{dx}^i = -7.8351$, $K_{pv}^i = -3.59040$, $K_{iv}^i = 0$, $K_{dv}^i = 0$.

5.1.1. Numerical results

To demonstrate that the parameters of the DCPID obtained through stability analysis proposed in Section 3 can provide robust control for tracking and adjusting the desired spacing and speed, we use $e_x$ and $e_v$ to describe the state of the vehicle platoon. We increase $e_x$ linearly from $-10$ to $10$ m with a step length of $1$ m and increase $e_v$ linearly from $-5$ to $5$ m/s with a step of $0.5$ m/s. In total 400 scenarios are obtained. By simulating each scenario and calculating the steady-state adjustment time $t_{\text{steady}}$ and the velocity overshoot defined in Equation (31), we can evaluate whether the DCPID has good tracking and adjustment performance.

Figure 9 shows the response of the following vehicles to the desired speed and spacing under the extreme conditions where $e_x = -10$m and $e_v = -5$m/s. We can observe that all vehicles can adjust the velocity and spacing to the stable state despite that the initial state of the platoon is relatively extreme with large deviation from the stable state. This also reveals the robustness of the DCPID model. Figure 9(d) shows the trajectory in the longitudinal direction where no collision occurs during the entire process, which indicates that the safety of the DCPID controlled platoon can be assured.

We further analyse the simulation results for all 400 scenarios and conclude that the DCPID can guarantee string stability of the platoon for all scenarios. The velocity overshoot of each scenario is shown in Figure 10. From the figure, we can observe that the velocity overshoot rates of most scenarios are rather small with $v_{\text{overshoot}} < 5\%$ (the dark blue area).

To evaluate the impact of the inertial lag $\tau$ on the robustness of the DCPID control scheme, we simulate the leader tracking performance of the controller for different values of $\tau$, where $\tau \in [0.2s, 0.9s]$ using the Monte Carlo method. In the simulation, two vehicles (leader-follower) travel at initial velocities of 15 and 20 m/s, respectively, and the initial spacing error $e_x$ is set to be $-10$m. As can been seen from Figure 11, despite the inertial lag $\tau$ is uncertain, the following vehicle can adjust itself to the desired state (the spacing and velocity errors are 0) for different $\tau$ values. This demonstrates the robustness of the cascade PID controller.

5.1.2. Comparison analysis

In this section, we discuss the performance comparison of our proposed DCPID with the DMPC proposed by Zhou, Wang, and Ahn (2019) and the single PID. We consider a platoon
Figure 9. Simulation results of the DCPID Platoon under extreme conditions with $e_x = -10 \text{m}$ and $e_v = -5 \text{m/s}$: (a) velocity; (b) spacing error; (c) acceleration; (d) the longitudinal trajectory.

Figure 10. The velocity overshoot rate with $e_x \in [-10 \text{m}, 10 \text{m}]$, $e_v \in [-5 \text{m/s}, 5 \text{m/s}]$. 
Figure 11. Robustness analysis via the Monte Carlo method for the DCPID, where $\in [0.2s, 0.9s]$: (a) velocity; (b) spacing error; (c) velocity error.

Figure 12. Variation of spacing error under three methods: (a) DCPID; (b) single-PID; (c) DMPC in (Zhou, Wang, and Ahn 2019) of 8 vehicles using the same scenario setting as (Zhou, Wang, and Ahn 2019). The initial spacing errors are set to be 2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, and both the initial speed errors and acceleration errors are set to be 0. The parameters of the single-PID are set as $K_p = 8$, $K_i = 0$, $K_d = 10$, and the parameters of DMPC are set as the same as those provided in Zhou, Wang, and Ahn (2019). As can be seen from Figure 12, all algorithms can effectively adjust the spacing error to zero. Nevertheless, the proposed DCPID provides better tracking performance with the smallest spacing error variation. Compared with a single-loop PID, the parallel adjustment of the inner and outer loops of the DCPID achieves a faster response speed and a smaller overshoot.

We use stability performance (e.g. the response speed and the amplitudes of errors) to evaluate the anti-interference ability of the controller, that is, to observe whether the platoon system can quickly return to the steady state after the control variable value is subjected to external disturbance for a period. A disturbance $\epsilon_u$, which is considered as the external interference, is added to the control variable $u$. We consider a platoon of 8 vehicles in a steady state (the initial spacing errors and speed errors are all 0). During the time between $t = 6s$ and $t = 8s$, interference $\epsilon_u = 3$ is added to $u$ of the leading vehicle. As can be seen from Figure 13, all these three algorithms can well suppress the interference and restore the vehicle platoon to a stable state. We can also observe that with the proposed DCPID approach, the amplitudes of the spacing and speed errors caused by interference are significantly smaller than those with the DMPC and single-PID. This indicates that the DCPID has stronger anti-interference ability. The underlying reason is that the double-layer structure of the DCPID can adjust the spacing and speed parallelly to smooth the disturbance.
Figure 13. Comparison analysis of the DCPID with respect to the single-PID and DMPC in Zhou, Wang, and Ahn (2019).

Table 4. Parameter settings for different scenarios.

| Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
|------------|------------|------------|------------|------------|
| $\theta^0_1$ | $(75, -1.875, 20)$ | $(75, -1.875, 20)$ | $(116, -1.875, 25)$ | $(116, -1.875, 25)$ |
| $\theta^0_2$ | $(50, -1.875, 20)$ | $(50, -1.875, 20)$ | $(87, -1.875, 25)$ | $(87, -1.875, 25)$ |
| $\theta^0_3$ | $(25, -1.875, 20)$ | $(25, -1.875, 20)$ | $(35, -1.875, 25)$ | $(29, -1.875, 25)$ |
| $\theta^0_4$ | $(0, -1.875, 20)$ | $(0, -1.875, 20)$ | $(6, -1.875, 25)$ | $(0, -1.875, 25)$ |
| SV speed | $(10, 1.875, 20)$ | $(35, 1.875, 20)$ | $(50, 1.875, 25)$ | $(58, 1.875, 25)$ |
| TFV speed | Constant | Constant | Constant | Variable |

5.2. Cooperative lane changing

To evaluate the performance of the cooperative lane-changing model, we conducted several simulation tests listed in Table 4. The DCPID-controlled platoon is composed of four vehicles driving on the target lane and a SV driving on the adjacent lane. The initial state of each vehicle is defined as $\theta^0_i = (x_{i0}, y_{i0}, v_{i0})$, $i \in \{1, 2, 3, 4, SV\}$, where vehicles $i = \{2, 3\}$ represent the TFV and TRV in the platoon, respectively. We also consider different speed states of the TFV for different scenarios. The speed is set to be constant for scenarios 1–3 and 5 and to be variable for scenario 4. The planned acceleration $a_p$ in the planning process is set to be $0.1 m/s^2$ according to Table 2 to ensure ride comfort in the following scenarios. The distance $L$ between the front and rear axles is 2.9 m. The control parameters (here, $h_i = 0.8$) and vehicle parameters are the same as listed in Table 3.

Figure 14 illustrates the initial positions of vehicles in different scenarios. Figure 14(a,c) shows scenarios 1 and 3 with $d_{SV} > S_{SV}$. According to Equation (11), we can tell that the SV needs to accelerate to satisfy lane-changing requirements; Figure 14(b) shows scenario 2 with $d_{SV} < S_{SV}$, indicating that the SV needs to decelerate to satisfy lane-changing requirements; Figure 14(e,d) shows scenarios 4 and 5 with $d_{SV} = S_{SV}$. In this case, the SV only needs to consider the longitudinal spacing between the TRV and itself ($d_{TRV}$) ensuring $d_{TRV} \geq d_0$, such that the SV can trigger the lane-changing manoeuvre.
Figure 15(a) shows the speed dynamics for each vehicle during the cooperative lane-changing process in scenario 1. After accelerating and decelerating, the SV finally reaches the same speed as the preceding vehicles 1 and TFV in the platooning state. As for the TRV and vehicle 4, they need to decelerate at first to make space for the SV to change lanes and then accelerate to catch up the SV. The whole cooperative lane-changing process lasts about 16.36 seconds. After that, all vehicles drive with the same speed in a steady state. The lane-changing process is illustrated by a three-dimensional plot of vehicle trajectories in Figure 15(b). The velocity error and spacing error dynamics are provided in Figure 15(c,d), respectively.

Figure 16 shows the performance of the SV during the lane-changing process in scenario 1. The comparison of the desired trajectory and the trajectory derived from the proposed model during the entire lane-changing process is illustrated in Figure 16(a). It can be observed that the model derived trajectory is quite smooth and perfectly fit with the desired trajectory. Figure 16(b) shows the lateral tracking error with the maximum error of 0.001m. This indicates that the proposed method can provide accurate trajectory tracking. The front wheel steering angle and yaw angle of the SV are shown in Figure 16(c,d), respectively, which illustrate that the lane changing is performed exactly as the desired.

Figure 17 shows the lateral tracking errors of the five scenarios. It can be seen that the lateral tracking errors of the SV reach 0, indicating that the SV can complete the lane changing
manoeuvre safely in all five scenarios. The maximum lateral tracking errors of scenarios 1, 2, and 3 are quite small about 0.001 m; the maximum errors of scenarios 4 and 5 are slightly larger about 0.018 and 0.012 m, respectively. The reasons behind are that the velocity of the TFV is changing during the lane-changing process in scenario 4, causing the SV to adjust its own velocity to avoid potential collisions. The fluctuations of the velocity also result in larger differences between the trajectories planned at different times, leading to the increased uncertainty of the lateral movement of the SV. In addition, the slight fluctuations may give rise to larger errors when the velocity is higher (e.g. scenario 5). This is also in line with the analysis in section 4 where the lateral error increases and the ride comfort decreases as the speed of the lane changing increases.

Figure 18 shows the longitudinal spacing errors of five scenarios. As can be seen the longitudinal spacing errors of all subsystems can return to the steady state without error in the five scenarios. This indicates that the SV can successfully join the platoon and form a new steady platoon in all five scenarios.

Table 5 shows the start time $t_0$, the end time $t_e$ and the duration of the lane-changing process in five scenarios. The start time $t_0$ is determined if the condition in Equation (11) is satisfied. While the end time of the lane-changing process $t_e$ can be obtained when the lateral tracking error reaches 0 as can be seen from Figure 17.
Figure 16. Performance of the SV during the lane-changing process in scenario 1: (a) trajectory; (b) lateral tracking error; (c) steering angle; (d) yaw angle.

Figure 17. The lateral tracking errors for five scenarios.
Figure 18. The longitudinal spacing errors for five scenarios: (a) scenario 1; (b) scenario 2; (c) scenario 3; (d) scenario 4; (e) scenario 5.

Table 5. The start and the end time of lane-changing in five scenarios.

| t₀ (s) | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
|-------|------------|------------|------------|------------|------------|
| t₀    | 7.16       | 5.72       | 5.34       | 0          | 0          |
| tₑ    | 16.36      | 14.7       | 14.36      | 8.52       | 8.42       |
| tₑ − t₀ | 9.2       | 8.98       | 9.02       | 8.52       | 8.42       |

The results of scenarios 1–3 show that reasonable parameter values can keep the lane-changing manoeuvre to be completed safely and efficiently. However, if the TFV’s speed changes, the trajectory planning part needs to re-plan a trajectory in line with the current state during the actual lane-changing process, otherwise the lane-changing could be unsafe. Figure 19(a) shows that the SV begins to change lanes at 0s, and the TFV begins to decelerate at 0s in scenario 4. The detailed process of the dynamic lane-changing trajectory planning of the SV in scenario 4 is illustrated in Figure 19(b). The proposed model updates the current optimal lane-changing trajectory at every sampling time, and the final lane-changing trajectory passes through the planned trajectory of all sampling times.

6. Concluding remarks

In this paper, we propose a cooperative vehicle platooning approach considering both the longitudinal and lateral control. The DCPID algorithm is developed to provide the longitudinal control for vehicle platooning. This algorithm can not only allow the platoon to run steadily with slight disturbance but can also make the platoon return to the steady state quickly even under the extremely unstable condition. Strong stability and good anti-interference performance make the proposed algorithm very promising for real-world implementation. Moreover, the simulation results show that the proposed DCPID approach is superior to the DMPC and single PID methods in terms of the anti-interference ability.

We further propose the cooperative lane-changing model by combining the DCPID algorithm with the sine function. The longitudinal acceleration and speed of the
lane-changing vehicle are determined by the DCPIID algorithm considering the speed variations of the vehicle in front on the target lane (TFV). The reference trajectory is planned using the improved sine function considering the ride comfortability at different speeds, and can be updated in real time in response to the state change of the TFV, such as emergency operations (sudden deceleration). Several numerical simulations were carried out to demonstrate the performance of the proposed method. The results show that the lane-changing trajectories generated by our proposed cooperative lane-changing algorithm are rather smooth, and highly consistent with the desired trajectories at different speeds.

Still, stability analysis of this study focuses on analysing the impact of small perturbations in the initial condition, e.g. some small disturbances are added to the first vehicle of the platoon. While the large perturbations, e.g. caused by lane changing, are not considered in this study. We would like to perform more comprehensive stability analysis in our future study. In addition, we would also like to continue our research on developing control strategies of vehicle platooning in more complex situations including on-ramps and off-ramps, as well as under different traffic flow conditions such as the free flow and transition from the free flow to the congested flow. One more interesting attempt is to validate our proposed algorithm using the field experimental datasets or carry out the real-world verification experiments on CAVs.

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