On new conservative aspects of a derived $d$-KdV equation in transonic plasma

P. K. Karmakar

Department of Physics, Tezpur University, Napaam, Tezpur-784028, Assam, India.

E-mail: pkk@tezu.ernet.in

Abstract. A linear source driven non-linear dynamical evolution system governed by the $d$-KdV equation of our present concern arises in the following form with all usual notations in the theoretical description of the non-linear normal mode behaviour of plasma ion acoustic mode through transonic plasma of an assumed finite extension. The ion acoustic wave-induced lowest order perturbed potential $\Phi_m = f(\xi,\tau)$ in transonic plasma equilibrium configuration can be written in a normalized form as the following

$$K_0 \frac{\partial \Phi_m}{\partial \tau} + M \frac{\partial \Phi_m}{\partial \xi} + \frac{1}{2} \frac{\partial^4 \Phi_m}{\partial \xi^4} = \gamma K_0 \Phi_m.$$

Motivated by the search for the explicit forms of all the possible conservation laws realistically applicable under a defined transonic plasma equilibrium, it is mathematically modelled into the repeated hydrodynamically standard form of fluid flux conservation construct $\partial_j \rho + \partial_j J = 0$, where associated conserved density $\rho = f(\Phi_m, (\Phi_m)_{\xi}, (\Phi_m)_{\xi\xi}, ...)$ and associated conserved flux $J = g(\Phi_m, (\Phi_m)_{\xi}, (\Phi_m)_{\xi\xi}, ...)$ vanish asymptotically at $\xi \to \pm\infty$. In the present contribution a large number of conservation laws of applied mathematical significance associated with the $d$-KdV flow dynamics followed by realistic physical discussions are presented in transonic plasma domain. A detailed numerical analysis for our $d$-KdV profiles and corresponding phase space portraits methodologically carried out for the first time to investigate its geometries in the defined transonic plasma equilibrium configuration.

1. Introduction

A non-linear dynamical system governed by KdV equation is found to be a conservative system possessing an infinite number of conserved quantities [1-4]. The conservation laws associated with KdV equation can be obtained by applying different mathematical techniques [3]. Such a KdV equation with a linear driving source, called driven KdV equation ($d$-KdV equation), arises in the theoretical description of the non-linear normal mode behavior of plasma ion acoustic wave with inertia-induced acoustic excitation theory [5-10] in a transonic plasma of an assumed finite extension.

To whom any correspondence should be addressed.
under transient time action where the free energy source is the transonic flow itself. The ‘transient
time’ means the time taken by an ion acoustic soliton to traverse over its own width through transonic
plasma zone [8].

Numerically, however, the $d$-KdV equation predicts two distinct classes of non-linear eigen-modes:
oscillatory shock-like solution and usual KdV soliton depending on source perturbation scaling [8]. The $d$-KdV equation is indeed found to be analytically integrable through methodological integrability
tests [9]. But it is well known that a nonlinear dynamical evolution system is integrable if and only if it
possesses a sufficient number of conserved quantities [9]. Motivated by the search for all the possible
conservation laws as in the case of a usual KdV dynamical system [1-3], all the new possible
conserved quantities associated with our $d$-KdV equation under a quasi-hydrostatic type of equilibrium
are derived by the method of flux conservation. Detailed numerical analyses of the $d$-KdV dynamics
are carried out for investigating its dynamical features in coordination space as well as in phase space
geometry in which $d$-KdV equation dynamically evolves.

Contents of this research article are organized as follows. Section 2 includes general physical
discussions on our plasma model under which the $d$-KdV equation is analytically derived. Section 3
highlights the methodology of analytical derivation of almost all the approximate explicit forms of the
$d$-KdV conservation laws by applying various methods of mathematical interest. Section 4 includes
the physical results and discussions, in brief, obtained thereof. Last, section 5 contains a compact
summary of the conclusions drawn from this present contribution followed by its future scopes in
realistic physical situations of scientific interest.

2. Physical model

A simple fluid background model of two-component non-isothermal, field-free and collisionless
plasma system is considered to study $d$-KdV flow dynamical evolution [8-10] with a uniform bulk
flow. Inertial particles are naturally drifting with uniform supersonic speeds. Such situations, of
course, exist in bi-polar positive ion rich plasma sheaths [8] formed around a mesh grid in a double
plasma device (DPD) where acoustic solitons are launched in the source chamber and detected in the
target chamber through transonic zone [6]. Unstable conditions are equally likely to exist in such
transonic zones. A hydrodynamic equilibrium configuration with a uniform bulk plasma flow is
assumed to pre-exist satisfying global quasi-neutrality. Thus a self-similar plasma flow is considered.
The basic physical mechanism responsible for the phenomenon of the resonant excitation of the ion
acoustic wave-induced electrostatic potential is the plasma electron fluid-compressibility, which
otherwise, is totally absent for the usual Boltzmannian plasma electrons.

It will, in addition, be pertinent to comment that the plasma ions are self-consistently drifting or
streaming through a negative neutralizing background of hot electrons having relatively zero inertia.
As such the time response of the plasma electron fluid for ion sound wave description is normally
ignored. As a result, the unique role of weak but finite electron inertia having a unique ability to
resonantly destabilize the ion sound wave in transonic plasma equilibrium even within fluid model
approach of normal mode description is masked.

3. Mathematical analyses

All the previous calculations of plasma ion acoustic wave analysis [4-10] provide a clear-cut idea of
the possible existence of ion acoustic wave turbulence in transonic zone of plasma flows within a
hydrodynamic fluid model approach. The derived $d$-KdV equation [8-9] describes the nonlinear
normal mode excitation of plasma ion acoustic wave dynamics under transient limit in such a
transonic equilibrium condition in terms of the lowest order perturbed acoustic potential $\Phi_m = f(\xi, \tau)$
in a normalized form as follows

$$K_0 \frac{\partial}{\partial \tau} (\Phi_m) + M_0 (\Phi_m) \frac{\partial}{\partial \xi} (\Phi_m) + \frac{1}{2} \frac{\partial^2}{\partial \xi^2} (\Phi_m) = \gamma K_0 \Phi_m. \quad (1)$$
This an interesting point to be noted that in the limiting cases of the Global Phase Transformation Method (GPTM) [8-9], the compact form of the linear source driven KdV (d-KdV) equation (1) is well verified and physically justified as well. Here the usual notations $K_0$ and $M_0$ termed as complex response coefficients [8-9] and the linear resonant growth rate $\gamma$ of the ion acoustic wave propagating through transonic plasma zone appearing in equation (1) are defined mathematically as follows

$$K_0 = \left[A^2 + B^2\right]^{1/2},$$

where

$$A = \left(\frac{M_0^2 + M_0^2 - 3M_0^2M_i^2}{M_0^2 + M_i^2}\right)$$

and

$$B = \left(\frac{M_0^2 + M_0^2 - 3M_0^2M_i^2}{M_0^2 + M_i^2}\right);$$

$$M_n = \left[C^2 + D^2\right]^{1/2},$$

where

$$C = \frac{1}{2}\left[\frac{3\left(M_0^2 - M_i^2\right)^2 - 4M_0^2M_i}{M_0^2 + M_i^2}\right] - \frac{M_i^2}{\epsilon_m}\right] - 1\right]$$

and

$$D = -\frac{1}{2}\left[\frac{12\left(M_0^2 - M_i^2\right)M_0M_i}{M_0^2 + M_i^2} + \frac{4\left(M_0^2 - M_i^2\right)M_0M_i}{\epsilon_m}\right];$$

$$\gamma = \sqrt{\frac{2}{\epsilon_m}}k\hat{\lambda}_{De}\left|\left(1 - v_{eo}\right)^{1/2}\right|.$$

Also the notation $M_{Dr}$ represents the real part of the Doppler shifted ion sound wave Mach number and $M_i$ is the imaginary counterpart correlated to the linear growth rate as in earlier case [8]. The other conventional notations like $k\hat{\lambda}_{De}$ and $v_{eo}$ represent the normalized ion acoustic wave number and equilibrium ion fluid flow, respectively. Likewise, as in usual case, $\epsilon_m = m_i/m_e$ denotes the ion to electron mass ratio.

3.1. Application of flux conservation method
Motivated by the search for the explicit forms of all the possible conservation laws, our d-KdV equation (1) is mathematically modeled repeatedly into the standard form of flux conservation construct [1-4] of a hydrodynamic flow into the form of equation of continuity (in absence of any source or sink) by taking various order moments with $\Phi_m = f(\xi, \tau)$ as follows

$$\partial_\tau \rho + \partial_\xi J = 0,$$

where d-KdV associated conserved density $\rho = f(\Phi_m, (\Phi_m)_\xi, (\Phi_m)_{\xi\xi}, ...)$ and conserved flux $J = g(\Phi_m, (\Phi_m)_\xi, (\Phi_m)_{\xi\xi}, ...)$ vanish asymptotically at $\xi \to \pm\infty$. The d-KdV equation (1) is now directly put into the following form
\[
\left[ K_0(\Phi_m') - \gamma K_0 \int (\Phi_m') d\tau \right] + \left[ M_0 \frac{I}{2} (\Phi_m')^2 + \frac{I}{2} (\Phi_m')_{\xi_{\xi}} \right] = 0.
\]  \hspace{1cm} (3)

Now comparing the equations (2) and (3) in the laboratory frame of reference, the explicit forms of the lowest order or zeroth order conserved density and flux respectively are given as

\[
\rho_0 = K_0(\Phi_m') - \gamma K_0 \int (\Phi_m') d\tau, \quad \text{and} \quad
\]

\[
J_0 = M_0 \frac{I}{2} (\Phi_m')^2 + \frac{I}{2} (\Phi_m')_{\xi_{\xi}}.
\]  \hspace{1cm} (4)

Now multiplying both sides of equation (1) by \(\Phi_m'\) and re-arranging, one gets

\[
K_0(\Phi_m') \cdot (\Phi_m') + M_0 (\Phi_m')^2 + \frac{I}{2} (\Phi_m') \cdot (\Phi_m')_{\xi_{\xi}} = \gamma K_0 (\Phi_m')^2.
\]  \hspace{1cm} (5)

Equation (6) can further be simplified and re-written as follows

\[
\left[ K_0 \frac{I}{2} (\Phi_m')^2 - \gamma K_0 \int (\Phi_m')^2 d\tau \right] + \left[ M_0 \frac{I}{3} (\Phi_m')^2 + \frac{I}{2} (\Phi_m') \cdot (\Phi_m')_{\xi_{\xi}} - \frac{(\Phi_m')^2}{2} \right] = 0.
\]  \hspace{1cm} (7)

Again comparing equation (7) with equation (2), one gets the conserved density and flux associated with \(d\)-KdV flow dynamics governed by equation (1) respectively as follows

\[
\rho_I = K_0 \frac{I}{2} (\Phi_m')^2 - \gamma K_0 \int (\Phi_m')^2 d\tau, \quad \text{and} \quad
\]

\[
J_I = M_0 \frac{I}{3} (\Phi_m')^2 + \frac{I}{2} (\Phi_m') \cdot (\Phi_m')_{\xi_{\xi}} - \frac{(\Phi_m')^2}{2}.
\]  \hspace{1cm} (8)

Now multiplying both sides of equation (1) by \(\Phi_m'\), one gets the following

\[
K_0 (\Phi_m') \cdot (\Phi_m') + M_0 (\Phi_m') \cdot (\Phi_m')_{\xi_{\xi}} + \frac{I}{2} (\Phi_m') (\Phi_m')_{\xi_{\xi}} = \gamma K_0 (\Phi_m')^2.
\]  \hspace{1cm} (9)

After integration and simplification, now one can re-write equation (10) as follows

\[
\left[ K_0 \frac{I}{3} (\Phi_m') - \gamma K_0 \int (\Phi_m')^2 d\tau \right]_{\xi} + \left[ M_0 \frac{I}{4} (\Phi_m')^2 + \frac{I}{2} \left( (\Phi_m')^2 (\Phi_m')_{\xi_{\xi}} - \frac{2}{3} (\Phi_m') \cdot (\Phi_m')_{\xi_{\xi}} d\xi \right) \right]_{\xi} = 0.
\]  \hspace{1cm} (10)
Similarly comparing equation (11) with equation (2), the next higher order conserved density and flux associated with $d$-KdV flow dynamics respectively can be obtained as

$$\rho_2 = K_0 \frac{I}{3} (\Phi_m)^i - \gamma K_0 \int (\Phi_m)^i d\tau, \text{ and}$$

$$J_2 = M_0 \frac{I}{4} (\Phi_m)^i + \frac{I}{2} \left\{ (\Phi_m)^i (\Phi_m)_{\xi\xi} - \int 2(\Phi_m)_\xi \cdot (\Phi_m)_{\xi\xi} d\xi \right\}. \quad (12)$$

Again multiplying equation (1) by $(\Phi_m)^i$, one gets the following

$$K_0 (\Phi_m)^i \cdot (\Phi_m)_{\xi} + M_0 (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi} + \frac{I}{2} (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi\xi} = \gamma K_0 (\Phi_m)^i. \quad (13)$$

Equation (14) on further integration and simplification yields,

$$\left\{ K_0 \frac{I}{4} (\Phi_m)^i - \gamma K_0 \int (\Phi_m)^i d\tau \right\}_\tau + \left\{ M_0 \frac{I}{5} (\Phi_m)^i + \frac{I}{2} \left\{ (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi} - \int 3(\Phi_m)^i \cdot (\Phi_m)_\xi \cdot (\Phi_m)_{\xi\xi} d\xi \right\}_\xi \right\} = 0. \quad (15)$$

Again comparing equation (15) with equation (2), the next higher order conserved density and flux associated with $d$-KdV flow dynamics respectively can be given by

$$\rho_3 = K_0 \frac{I}{5} (\Phi_m)^i - \gamma K_0 \int (\Phi_m)^i d\tau, \text{ and}$$

$$J_3 = M_0 \frac{I}{6} (\Phi_m)^i + \frac{I}{2} \left\{ (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi} - \int 4(\Phi_m)^i \cdot (\Phi_m)_\xi \cdot (\Phi_m)_{\xi\xi} d\xi \right\}. \quad (17)$$

Now, again multiplying equation (1) by $(\Phi_m)^i$, one gets

$$K_0 (\Phi_m)^i \cdot (\Phi_m)_{\xi} + M_0 (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi} + \frac{I}{2} (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi\xi} = \gamma K_0 (\Phi_m)^i. \quad (18)$$

After integration by parts and simplification, equation (18) gets transformed into

$$\left\{ K_0 \frac{I}{5} (\Phi_m)^i - \gamma K_0 \int (\Phi_m)^i d\tau \right\}_\tau + \left\{ M_0 \frac{I}{6} (\Phi_m)^i + \frac{I}{2} \left\{ (\Phi_m)^i \cdot (\Phi_m)_{\xi\xi} - \int 4(\Phi_m)^i \cdot (\Phi_m)_\xi \cdot (\Phi_m)_{\xi\xi} d\xi \right\}_\xi \right\} = 0. \quad (19)$$

Applying the same previous methodology, the next higher order conserved density and flux associated with $d$-KdV flow in the form of equation (19) respectively are given by
\[ \rho_s = K_0 \frac{I}{3} (\Phi_m)^{\gamma} - \gamma K_0 \int (\Phi_m)^{\gamma} d\tau, \quad \text{and} \]  

\[ J_s = M_0 \frac{I}{6} (\Phi_m)^{\gamma} + \frac{I}{2} \left\{ (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} - \int \xi (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} d\xi \right\}. \]  

Again multiplying both sides of equation (1) by \((\Phi_m)^{\gamma}\), one gets the following form

\[ K_0 (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\tau} + M_0 (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi} + \frac{I}{2} (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi\xi} = \gamma K_0 (\Phi_m)^{\gamma}. \]  

After integration and simplification, equation (22) can now be put into the following form of flux conservation

\[ \left\{ K_0 \frac{I}{6} (\Phi_m)^{\gamma} - \gamma K_0 \int (\Phi_m)^{\gamma} d\tau \right\} + \left[ M_0 \frac{I}{7} (\Phi_m)^{\gamma} + \frac{I}{2} \left\{ (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} - \int \xi (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} d\xi \right\} \right] = 0. \]  

Again comparing equation (23) with equation (2), the next higher order conserved density and flux associated with equation (23) respectively can be given by

\[ \rho_s = K_0 \frac{I}{6} (\Phi_m)^{\gamma} - \gamma K_0 \int (\Phi_m)^{\gamma} d\tau, \quad \text{and} \]  

\[ J_s = M_0 \frac{I}{7} (\Phi_m)^{\gamma} + \frac{I}{2} \left\{ (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} - \int \xi (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} d\xi \right\}. \]  

Similarly multiplying both sides of equation (1) by \((\Phi_m)^{\gamma}\), one gets

\[ K_0 (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\tau} + M_0 (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi} + \frac{I}{2} (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi\xi} = \gamma K_0 (\Phi_m)^{\gamma}. \]  

Now integration and simplification of equation (26) will give the following form of flux conservation governed by equation of continuity (2) as follows

\[ \left\{ K_0 \frac{I}{7} (\Phi_m)^{\gamma} - \gamma K_0 \int (\Phi_m)^{\gamma} d\tau \right\} + \left[ M_0 \frac{I}{8} (\Phi_m)^{\gamma} + \frac{I}{2} \left\{ (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} - \int \xi (\Phi_m)^{\gamma} \cdot (\Phi_m)_{\xi\xi} d\xi \right\} \right] = 0. \]  

Correspondingly the next higher order conserved density and flux associated with \(d\)-KdV flow in the form of equation (27) respectively can be given by
\[ \rho_0 = K_0 \frac{1}{7} \langle \Phi_m \rangle^7 - \gamma K_0 \int \langle \Phi_m \rangle^7 d\tau, \quad \text{and} \]  
(28)

\[ J_0 = M_0 \frac{1}{8} \langle \Phi_m \rangle^8 + \frac{1}{2} \left\{ \langle \Phi_m \rangle^8 \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(29)

Proceeding in the same way repeatedly into the standard form of flux conservation construct [1-3] of a hydrodynamic flow, one gets the next higher order conserved density and flux associated with \( d \)-KdV flow dynamics governed by equation (1) respectively as

\[ \rho_j = K_0 \frac{1}{8} \langle \Phi_m \rangle^8 - \gamma K_0 \int \langle \Phi_m \rangle^8 d\tau, \]  
(30)

\[ J_j = M_0 \frac{1}{9} \langle \Phi_m \rangle^9 + \frac{1}{2} \left\{ \langle \Phi_m \rangle^9 \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(31)

\[ \rho_8 = K_0 \frac{1}{9} \langle \Phi_m \rangle^9 - \gamma K_0 \int \langle \Phi_m \rangle^9 d\tau, \]  
(32)

\[ J_8 = M_0 \frac{1}{10} \langle \Phi_m \rangle^{10} + \frac{1}{2} \left\{ \langle \Phi_m \rangle^{10} \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(33)

\[ \rho_9 = K_0 \frac{1}{10} \langle \Phi_m \rangle^{10} - \gamma K_0 \int \langle \Phi_m \rangle^{10} d\tau, \]  
(34)

\[ J_9 = M_0 \frac{1}{11} \langle \Phi_m \rangle^{11} + \frac{1}{2} \left\{ \langle \Phi_m \rangle^{11} \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(35)

\[ \rho_{10} = K_0 \frac{1}{11} \langle \Phi_m \rangle^{11} - \gamma K_0 \int \langle \Phi_m \rangle^{11} d\tau, \]  
(36)

\[ J_{10} = M_0 \frac{1}{12} \langle \Phi_m \rangle^{12} + \frac{1}{2} \left\{ \langle \Phi_m \rangle^{12} \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(37)

\[ \rho_{11} = K_0 \frac{1}{12} \langle \Phi_m \rangle^{12} - \gamma K_0 \int \langle \Phi_m \rangle^{12} d\tau, \]  
(38)

\[ J_{11} = M_0 \frac{1}{13} \langle \Phi_m \rangle^{13} + \frac{1}{2} \left\{ \langle \Phi_m \rangle^{13} \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(39)

\[ \rho_{12} = K_0 \frac{1}{13} \langle \Phi_m \rangle^{13} - \gamma K_0 \int \langle \Phi_m \rangle^{13} d\tau, \]  
(40)

\[ J_{12} = M_0 \frac{1}{14} \langle \Phi_m \rangle^{14} + \frac{1}{2} \left\{ \langle \Phi_m \rangle^{14} \cdot (\Phi_m)_\xi \cdot (\Phi_m)_\xi d\xi \right\}. \]  
(41)
\[
\rho_{13} = K_0 \frac{I}{I_{14}} (\Phi_m)^{14} - \gamma K_0 \int (\Phi_m)^{14} d\tau.
\]

\[
J_{13} = M_0 \frac{I}{I_{15}} (\Phi_m)^{15} + \frac{I}{2} \left\{ (\Phi_m)^{13} \cdot (\Phi_m)_{\xi} - \int 13 (\Phi_m)^{12} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]

\[
\rho_{14} = K_0 \frac{I}{I_{15}} (\Phi_m)^{15} - \gamma K_0 \int (\Phi_m)^{15} d\tau.
\]

\[
J_{14} = M_0 \frac{I}{I_{16}} (\Phi_m)^{16} + \frac{I}{2} \left\{ (\Phi_m)^{14} \cdot (\Phi_m)_{\xi} - \int 14 (\Phi_m)^{13} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]

\[
\rho_{15} = K_0 \frac{I}{I_{16}} (\Phi_m)^{16} - \gamma K_0 \int (\Phi_m)^{16} d\tau.
\]

\[
J_{15} = M_0 \frac{I}{I_{17}} (\Phi_m)^{17} + \frac{I}{2} \left\{ (\Phi_m)^{15} \cdot (\Phi_m)_{\xi} - \int 15 (\Phi_m)^{14} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]

\[
\rho_{16} = K_0 \frac{I}{I_{17}} (\Phi_m)^{17} - \gamma K_0 \int (\Phi_m)^{17} d\tau.
\]

\[
J_{16} = M_0 \frac{I}{I_{18}} (\Phi_m)^{18} + \frac{I}{2} \left\{ (\Phi_m)^{16} \cdot (\Phi_m)_{\xi} - \int 16 (\Phi_m)^{15} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]

\[
\rho_{17} = K_0 \frac{I}{I_{18}} (\Phi_m)^{18} - \gamma K_0 \int (\Phi_m)^{18} d\tau.
\]

\[
J_{17} = M_0 \frac{I}{I_{19}} (\Phi_m)^{19} + \frac{I}{2} \left\{ (\Phi_m)^{17} \cdot (\Phi_m)_{\xi} - \int 17 (\Phi_m)^{16} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]

\[
\rho_{18} = K_0 \frac{I}{I_{19}} (\Phi_m)^{19} - \gamma K_0 \int (\Phi_m)^{19} d\tau.
\]

\[
J_{18} = M_0 \frac{I}{I_{20}} (\Phi_m)^{20} + \frac{I}{2} \left\{ (\Phi_m)^{18} \cdot (\Phi_m)_{\xi} - \int 18 (\Phi_m)^{17} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]

\[
\rho_{19} = K_0 \frac{I}{I_{20}} (\Phi_m)^{20} - \gamma K_0 \int (\Phi_m)^{20} d\tau.
\]

\[
J_{19} = M_0 \frac{I}{I_{21}} (\Phi_m)^{21} + \frac{I}{2} \left\{ (\Phi_m)^{19} \cdot (\Phi_m)_{\xi} - \int 19 (\Phi_m)^{18} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi} d\xi \right\}
\]
\[ \rho_{20} = K_0 I_{\frac{1}{21}} (\Phi_m)_{\xi}^{1/2} \gamma K_0 \int (\Phi_m)_{\xi}^{1/2} d\tau, \quad (56) \]

\[ J_{20} = M_0 I_{\frac{1}{22}} (\Phi_m)_{\xi}^{22} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{20} \cdot (\Phi_m)_{\xi \xi}^{22} - \int 20 (\Phi_m)_{\xi}^{20} \cdot (\Phi_m)_{\xi \xi}^{22} d\xi \right\}, \quad (57) \]

\[ \rho_{21} = K_0 I_{\frac{1}{22}} (\Phi_m)_{\xi}^{22} - \gamma K_0 \int (\Phi_m)_{\xi}^{22} d\tau, \quad (58) \]

\[ J_{21} = M_0 I_{\frac{1}{23}} (\Phi_m)_{\xi}^{23} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{21} \cdot (\Phi_m)_{\xi \xi}^{23} - \int 21 (\Phi_m)_{\xi}^{21} \cdot (\Phi_m)_{\xi \xi}^{23} d\xi \right\}, \quad (59) \]

\[ \rho_{22} = K_0 I_{\frac{1}{23}} (\Phi_m)_{\xi}^{23} - \gamma K_0 \int (\Phi_m)_{\xi}^{23} d\tau, \quad (60) \]

\[ J_{22} = M_0 I_{\frac{1}{24}} (\Phi_m)_{\xi}^{24} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{22} \cdot (\Phi_m)_{\xi \xi}^{24} - \int 22 (\Phi_m)_{\xi}^{22} \cdot (\Phi_m)_{\xi \xi}^{24} d\xi \right\}, \quad (61) \]

\[ \rho_{23} = K_0 I_{\frac{1}{24}} (\Phi_m)_{\xi}^{24} - \gamma K_0 \int (\Phi_m)_{\xi}^{24} d\tau, \quad (62) \]

\[ J_{23} = M_0 I_{\frac{1}{25}} (\Phi_m)_{\xi}^{25} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{23} \cdot (\Phi_m)_{\xi \xi}^{25} - \int 23 (\Phi_m)_{\xi}^{23} \cdot (\Phi_m)_{\xi \xi}^{25} d\xi \right\}, \quad (63) \]

\[ \rho_{24} = K_0 I_{\frac{1}{25}} (\Phi_m)_{\xi}^{25} - \gamma K_0 \int (\Phi_m)_{\xi}^{25} d\tau, \quad (64) \]

\[ J_{24} = M_0 I_{\frac{1}{26}} (\Phi_m)_{\xi}^{26} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{24} \cdot (\Phi_m)_{\xi \xi}^{26} - \int 24 (\Phi_m)_{\xi}^{24} \cdot (\Phi_m)_{\xi \xi}^{26} d\xi \right\}, \quad (65) \]

\[ \rho_{25} = K_0 I_{\frac{1}{26}} (\Phi_m)_{\xi}^{26} - \gamma K_0 \int (\Phi_m)_{\xi}^{26} d\tau, \quad (66) \]

\[ J_{25} = M_0 I_{\frac{1}{27}} (\Phi_m)_{\xi}^{27} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{25} \cdot (\Phi_m)_{\xi \xi}^{27} - \int 25 (\Phi_m)_{\xi}^{25} \cdot (\Phi_m)_{\xi \xi}^{27} d\xi \right\}, \quad (67) \]

\[ \rho_{26} = K_0 I_{\frac{1}{27}} (\Phi_m)_{\xi}^{27} - \gamma K_0 \int (\Phi_m)_{\xi}^{27} d\tau, \quad (68) \]

\[ J_{26} = M_0 I_{\frac{1}{28}} (\Phi_m)_{\xi}^{28} + \frac{1}{2} \left\{ (\Phi_m)_{\xi}^{26} \cdot (\Phi_m)_{\xi \xi}^{28} - \int 26 (\Phi_m)_{\xi}^{26} \cdot (\Phi_m)_{\xi \xi}^{28} d\xi \right\}, \quad (69) \]

\[ \rho_{27} = K_0 I_{\frac{1}{28}} (\Phi_m)_{\xi}^{28} - \gamma K_0 \int (\Phi_m)_{\xi}^{28} d\tau, \quad (70) \]
\[ J_{27} = M_0 \frac{1}{29} (\Phi_m)^{29} + \frac{1}{2} \left\{ (\Phi_m)^{27} \cdot (\Phi_m)_{\xi \xi} - \int 27 (\Phi_m)^{26} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi \xi} d\xi \right\} \] (71)

\[ \rho_{28} = K_0 \frac{1}{29} (\Phi_m)^{29} - \eta K_0 \int (\Phi_m)^{29} d\tau, \] (72)

\[ J_{28} = M_0 \frac{1}{30} (\Phi_m)^{30} + \frac{1}{2} \left\{ (\Phi_m)^{28} \cdot (\Phi_m)_{\xi \xi} - \int 28 (\Phi_m)^{27} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi \xi} d\xi \right\} \] (73)

\[ \rho_{29} = K_0 \frac{1}{30} (\Phi_m)^{30} - K_0 \int (\Phi_m)^{30} d\tau, \] (74)

\[ J_{29} = M_0 \frac{1}{31} (\Phi_m)^{31} + \frac{1}{2} \left\{ (\Phi_m)^{29} \cdot (\Phi_m)_{\xi \xi} - \int 29 (\Phi_m)^{28} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi \xi} d\xi \right\} \] (75)

\[ \rho_{30} = K_0 \frac{1}{31} (\Phi_m)^{31} - K_0 \int (\Phi_m)^{31} d\tau, \] (76)

\[ J_{30} = M_0 \frac{1}{32} (\Phi_m)^{32} + \frac{1}{2} \left\{ (\Phi_m)^{30} \cdot (\Phi_m)_{\xi \xi} - \int 30 (\Phi_m)^{29} \cdot (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi \xi} d\xi \right\} \] (77)

and this way the sequential series of explicit forms of d-KdV conservation laws goes on.

It is now clear that our d-KdV equation (1) can mathematically be transformed into a generalizing standard form of equation of continuity in hydrodynamic flow dynamics by taking its \( n^{th} \) order moment with \( \Phi_m = f(\xi, \tau) \) as follows

\[ \partial_\xi \rho_n + \partial_\xi J_n = 0, \text{ where } n \in \mathbb{Z}. \] (78)

Accordingly the \( n^{th} \) order conserved density \( \rho_n \) and conserved flux \( J_n \) associated with the d-KdV hydrodynamic flow dynamics governed by equation (1) are generalized as follows

\[ \rho_n = \frac{K_0}{(n+1)} (\Phi_m)^{n+1} - \eta K_0 \int (\Phi_m)^{n+1} d\tau, \] (79)

\[ J_n = \frac{M_0}{(n+2)} (\Phi_m)^{n+2} + \frac{1}{2} \left\{ (\Phi_m)^n \cdot (\Phi_m)_{\xi \xi} - \int n(\Phi_m)^{n-1} (\Phi_m)_{\xi} \cdot (\Phi_m)_{\xi \xi} d\xi \right\}. \] (80)

It is now interestingly conjectured that our d-KdV equation (1) drives the electron inertia-induced ion acoustic wave instability through transonic plasma under a hydrodynamic type of equilibrium configuration in such a way that its associated density \( \rho_n \) and flux \( J_n \) remain conserved with the plasma flow dynamics.

3.2. Application of numerical method

The partial differential equation (1) is conveniently transformed into a nonlinear stationary ordinary differential equation with a single independent variable \( X \) only in a new frame of reference moving
with a sonic phase speed. The corresponding transformation is defined by \( \xi=X-\tau \) with \( \partial_\xi=\partial_x \) and \( \partial_{\tau}=\partial_\chi \). For the theoretical investigations of all the possible changes in the steady-state ion acoustic dynamics in coordination space as well as in phase space, equation (1) is solved numerically (by usual Runge-Kutta IV method \([8]\)) over a large number of finite values of wave number \( (k\lambda_{Le}) \) and deviation \( (\delta=M_{Le}-1) \) from sonic flow point. This is because only these two parameters are responsible to influence the type of acoustic interaction – either of resonant or non-resonant types.

Figure 1 shows the profile of (a) ion acoustic potential \( \Phi_m \) with normalized space variable \( (\chi) \), and (b) phase space geometry of ion acoustic potential in a phase space described by \( \Phi_m \) and \( (\phi_\chi) \) with \( k\lambda_{Le}=1.0\times10^{-1} \) (fixed) for Case (1): \( \delta=0 \), Case (2): \( \delta=1.0\times10^{-9} \), Case (3): \( \delta=1.0\times10^{-8} \), and Case (4): \( \delta=1.0\times10^{-7} \).

Figure 1 shows the profile of (a) ion acoustic potential \( \Phi_m \) with normalized space variable \( (\chi) \), and (b) phase space geometry of ion acoustic potential in a phase space described by \( \Phi_m \) and \( (\phi_\chi) \) with \( k\lambda_{Le}=1.0\times10^{-1} \) (fixed) for Case (1): \( \delta=0 \), Case (2): \( \delta=1.0\times10^{-9} \), Case (3): \( \delta=1.0\times10^{-8} \), and Case (4): \( \delta=1.0\times10^{-7} \). The finite nonzero initial values arbitrarily chosen under a step-size \( (\chi_i-\chi_{i-1})=0.01 \) for running our programme are \( (\phi_\chi,\theta,\phi_\chi)/X_{\chi}=10^{-3} \), \( (\partial\phi_\chi/\partial X_{\chi})_{\theta}=10^{-14} \), and \( (\partial_2\phi_\chi/\partial X_{\chi})_{\theta}=10^{-20} \). Now corresponding to numerically obtained \( \Phi_m \)-evolution (figure 1a) in the form of oscillatory shock-like structures, almost closed or quasi-closed phase portraits (figure 1b) are obtained to show nature of \textit{d}-KdV dynamics. Figure 2 shows the same as figure 1, but with \( k\lambda_{Le}=1.5\times10^{-3} \) (fixed) for Case (1): \( \delta=0 \), Case (2): \( \delta=1.5\times10^{-10} \), Case (3): \( \delta=3.5\times10^{-7} \), and Case (4): \( \delta=5.0\times10^{-6} \). Figure 3 also shows the same as figure 1, but with \( k\lambda_{Le}=2.5\times10^{-6} \) (fixed) for Case (1): \( \delta=0 \), Case (2): \( \delta=2.5\times10^{-7} \), Case (3): \( \delta=3.5\times10^{-6} \), and Case (4): \( \delta=2.5\times10^{-6} \). Figure 4 depicts the same as figure 1, but with \( k\lambda_{Le}=2.5\times10^{-8} \) (fixed) for Case (1): \( \delta=1.0\times10^{-7} \), Case (2): \( \delta=2.5\times10^{-7} \), Case (3): \( \delta=5.0\times10^{-7} \), and Case (4): \( \delta=7.5\times10^{-7} \).
Figure 2. Same as figure 1, but with $k \lambda_{ac} = 1.5 \times 10^{-3}$ (fixed) for Case (1): $\delta = 0$, Case (2): $\delta = 1.5 \times 10^{-10}$, Case (3): $\delta = 3.5 \times 10^{-7}$, and Case (4): $\delta = 5.0 \times 10^{-6}$.

Figure 3. Same as figure 1, but with $k \lambda_{ac} = 2.5 \times 10^{-6}$ (fixed) for Case (1): $\delta = 0$, Case (2): $\delta = 2.5 \times 10^{-7}$, Case (3): $\delta = 3.5 \times 10^{-6}$, and Case (4): $\delta = 2.5 \times 10^{-6}$.

Figure 5 displays the same as figure 1, but with $k \lambda_{ac} = 1.0 \times 10^{-7}$ (fixed) for Case (1): $\delta = 1.0 \times 10^{-5}$, Case (2): $\delta = 2.0 \times 10^{-5}$, Case (3): $\delta = 3.0 \times 10^{-5}$, and Case (4): $\delta = 5.0 \times 10^{-5}$. Figure 6 again shows the same as figure 1, but with $k \lambda_{ac} = 1.0 \times 10^{-7}$ (fixed) for Case (1): $\delta = 0$, Case (2): $\delta = 5.0 \times 10^{-6}$, Case (3): $\delta = 5.5 \times 10^{-7}$, and Case (4): $\delta = 2.5 \times 10^{-6}$. The finite nonzero initial values arbitrarily chosen under $(X_i - X_{i-1}) = 0.01$ for running our programme now are $(\phi_{ac})_i = 10^{-10}$, $(\partial \phi_{ac}/\partial \mathbf{x})_i = 10^{-15}$, and $(\partial^2 \phi_{ac}/\partial \mathbf{x}^2)_i = 10^{-32}$. Figure 7 gives the same as figure 1, but with $k \lambda_{ac} = 1.0 \times 10^{-7}$ (fixed) for Case (1): $\delta = 1.5 \times 10^{-6}$, Case (2): $\delta = 3.5 \times 10^{-6}$, Case (3): $\delta = 6.0 \times 10^{-6}$, and Case (4): $\delta = 8.5 \times 10^{-6}$. The finite nonzero initial values arbitrarily chosen under $(X_i - X_{i-1}) = 0.1$ for running our programme here are $(\phi_{ac})_i = 10^{-5}$, $(\partial \phi_{ac}/\partial \mathbf{x})_i = 10^{-8}$, and $(\partial^2 \phi_{ac}/\partial \mathbf{x}^2)_i = 10^{-17}$. Figure 8 shows the same as figure 1, but with $k \lambda_{ac} = 1.0 \times 10^{-7}$ (fixed) for Case
The finite nonzero initial values now arbitrarily chosen for running our programme are $(\Phi_i)_0 = 10^{-2}$, $(\partial_i \Phi_i / \partial X)_0 = 10^{-4}$, and $[\partial^2_i \Phi_i / \partial X^2]_0 = 10^{-5}$ with a step-size $(x_i - x_{i+1}) = 0.01$.

The numerical analyses (figures 1-8) carried out in detail over a wide range spectrum of initial inputs obviously show two distinct classes of phase portraits: (a) closed structures and (b) quasi-closed structures in phase space (figures 1b–8b) corresponding to (a) soliton-like and (b) oscillatory shock-like structures in d-KdV dynamics in coordination space (figures 1a–8a), respectively. The sets of closed and quasi-closed phase space trajectories clearly show the reality of conservative dynamics governed by d-KdV equation in general through an adiabatic energy exchange process between the driven acoustic instability [8-10] and background acoustic spectral components in transonic plasma equilibrium. Any distorting deviation from closed-packed forms of the obtained phase space

Figure 4. Same as figure 1, but with $k\lambda_{ac} = 2.5 \times 10^{-5}$ (fixed) for Case (1): $\delta = 1.0 \times 10^{-5}$, Case (2): $\delta = 2.5 \times 10^{-5}$, Case (3): $\delta = 5.0 \times 10^{-5}$, and Case (4): $\delta = 7.5 \times 10^{-5}$.

Figure 5. Same as figure 1, but with $k\lambda_{ac} = 1.0 \times 10^{-4}$ (fixed) for Case (1): $\delta = 1.0 \times 10^{-5}$, Case (2): $\delta = 2.0 \times 10^{-5}$, Case (3): $\delta = 3.0 \times 10^{-5}$, and Case (4): $\delta = 5.0 \times 10^{-5}$.
geometries for our $d$-KdV equation in either bounded or unbounded form (due to release or absorption of ion acoustic wave induced electrostatic potential energy) might essentially be attributed to non-conservative aspects of $d$-KdV dynamics.

**Figure 6.** Same as figure 1, but with $k_{i\text{De}} = 1.0 \times 10^{-1}$ (fixed) for Case (1): $\delta = 0$, Case (2): $\delta = 5.0 \times 10^{-6}$, Case (3): $\delta = 5.5 \times 10^{-7}$, and Case (4): $\delta = 2.5 \times 10^{-6}$.

**Figure 7.** Same as figure 1, but with $k_{i\text{De}} = 1.0 \times 10^{-1}$ (fixed) for Case (1): $\delta = 1.5 \times 10^{-6}$, Case (2): $\delta = 3.5 \times 10^{-6}$, Case (3): $\delta = 6.0 \times 10^{-6}$, and Case (4): $\delta = 8.5 \times 10^{-6}$.

4. Results and discussions
The detailed methodological analyses carried out in the present contribution clearly show that there are indeed an infinitely large number of explicit analytical forms of conserved quantities associated with $d$-KdV flow dynamics in transonic plasma domain. Interestingly it is observed that $d$-KdV equation drives electron inertia-induced ion acoustic wave instability [5] through the transonic zone of a two-component plasma system under the transient time action in such a way that all the derived explicit functional forms of the derived quantities remain conserved (through a standard form of equation of continuity). The basic physical mechanism through which they remain conserved is an adiabatic energy exchange process [8] between the driven acoustic instability and the background acoustic
spectral components. Therefore, since our \(d\)-KdV equation (1) is analytically integrable [9], accordingly it is found to be a conservative non-linear dynamical evolution system possessing a very large number of conserved quantities. Additional treatment with Lagrangian-Hamiltonian [3-4] formalism of the \(d\)-KdV dynamics might yield some new more associated conserved quantities. A comprehensive analysis of Lagrangian and Hamiltonian formalism of the \(d\)-KdV flow dynamics will, of course, be another interesting problem of future research of more applied mathematical importance.

\[
\begin{align*}
\text{Figure 8 (a)} & \quad \text{Figure 8 (b)} \\
\end{align*}
\]

Figure 8 shows the same as figure 1, but with \(kD_0 = 1.0 \times 10^{-3}\) (fixed) for Case (1): \(\delta = 1.5 \times 10^{-5}\), Case (2): \(\delta = 3.5 \times 10^{-5}\), Case (3): \(\delta = 6.5 \times 10^{-5}\), and Case (4): \(\delta = 8.5 \times 10^{-5}\).

In order to get numerical flavor of the conservative flow dynamics governed by equation (1), it is solved numerically as an initial value problem in coordination space as well as phase space over a wide range spectrum of initial values of relevant variables. The numerical results obtained are shown in figures 1-8. Two distinct classes of \(\Phi_m\)-profiles are obtained: (a) soliton-like structures, and (b) oscillatory shock-like structures (not exactly shock which is purely a mechanical blast wave of compressional type the thickness of the front of which is a consequence of balancing between nonlinear compressibility and dissipative mechanisms like viscosity, thermal conduction, etc.) as shown in figures 1a-8a. The presence of the linear driving source term in \(d\)-KdV dynamical equation (1) introduces an asymmetry into ion acoustic soliton structures perturbatively. Oscillatory shock-like structures are formed due to resonant (crest formation) and non-resonant (trough formation) interaction between the propagating ion acoustic wave through transonic plasma and background ion acoustic spectral components in an adiabatic energy sharing process. Accordingly, two distinct classes of phase portraits are observed: (a) closed phase portraits (for long scale perturbation), and (b) quasi-closed (for short scale perturbation) phase portraits as shown in figures 1b-8b. The set of closed or quasi-closed forms of phase space trajectories of \(\Phi_m\)-evolution shows that \(d\)-KdV equation really represents a conservative non-linear dynamical evolution system.

This, of course, is still an experimental challenge to devise a set up to produce an extended length of the transonic plasma zone to a sufficient extent to resolve the desired unstable wave spectral components and test the associated conservation principles of the driven ion acoustic instability. In case of sheath edge boundary, transonic layer could be probed by a high-resolving diagnosis of the Debye scale length order. The desired experiments of spectral analysis of the unstable ion acoustic waves in transonic plasma condition may be quite useful to resolve the mystery of sheath edge singularity [8]. Applying the basic principle of de-Laval nozzle [8-9] of hydrodynamic flow, experiments could be designed to produce transonic transition layer of a desired length. It is based on a...
gas-dynamic analog of plasma fluid hydrodynamically flowing through a cylindrical chamber of longitudinally varying cross-sectional area.

One experimental marginal support to the conservation principles is through the observations made in terms of ion acoustic wave activities in transonic plasma in a Double Plasma Device (DPD) [8]. It has a source chamber and a target chamber through a mesh grid of 85% transparency kept electrically floated. Transformation of an ion acoustic signal launched at the source chamber into a soliton and oscillatory shock-like structure detected with an axially movable Langmuir probe in the target chamber through the unstable zone of transonic plasma in the system [8] is quite in accordance with the law of conservation of energy through an adiabatic process. Since there is no signature of irregular, intermittent and chaotic behavior observed in the detected signal counterpart (with the axially movable Langmuir probe and oscilloscope) in the target chamber of the DPD, it is conjectured to be a conservative non-linear dynamical evolution process.

In plasmas only nonlinear wave modes like- soliton, shock etc., in general, can travel for a long before suffering major modifications in shape and identity. A transonic plasma zone seems to act as an interesting site for rich varieties of nonlinear acoustic wave activities under a three-layer analysis of plasma-boundary interaction [5-7]. Like in MHD spectroscopy, the coining of the idea of acoustic spectroscopy [9] in unmagnetized plasma flows may have an interesting impact to study the unique property of the transonic plasmas of laboratory or space for basic and applied research. Similar situation of acoustic wave-turbulence activities governed by d-KdV dynamics in transonic zone may also arise in a multi-component colloidal plasma system [10] and astrophysical plasma treated with GES – model [11] in a Bernoulli kind of equilibrium configuration with or without magnetic field for future investigations.

5. Conclusions
It is, in brief, primarily conjectured that our d-KdV equation indeed possesses an infinitely large number of conservation laws. It is concluded that some of them do have physical significance like shape conservation, energy conservation, etc. under long scale perturbation, but some have only mathematical importance functionally. It, furthermore, gives an idea of how an ion acoustic wave travels through an acoustically random medium rich in turbulent degrees of freedom like in transonic plasma. The dynamical evolution profiles are elaborately obtained numerically in configuration space (figures 1a-8a) as well as in phase space (figures 1b-8b). The main conclusive points drawn from the detailed analyses entirely are summarily highlighted as follows:

1. The d-KdV system indeed is investigated to be exactly comparable to a driven fluid flow of hydrodynamic type (governed by the standard form of equation of continuity in absence of any external source or sink).

2. The d-KdV equation, in fact, possesses an infinitely large number of conserved quantities (expressible as explicit functional forms) under transonic plasma equilibrium configuration. This dynamical evolution system is, therefore, equivalent to a conservative process in a time scale at least greater than the plasma ion oscillation time scale within the transient time action limit [8-9].

3. This is informative to add that the nonlinear dynamical evolution system governed by our d-KdV system indeed behaves as a member of an infinite family of commuting flows (due to large conservation laws governed by the standard form of equation of continuity) under quasi-hydrostatic type of equilibrium configuration.

4. Lastly, two distinct classes of nonlinear profiles of the d-KdV equation are numerically obtained in conservational perspective – soliton-like structures and oscillatory shock-like structures (figures 1a-8a) over a wide range spectrum of input initial values of controlling parameters. Accordingly and more importantly, two distinct classes of phase portraits are observed – closed portraits and quasi-closed portraits (figures 1b-8b). These structures show the conservative features of d-KdV dynamics numerically through our defined phase space geometries.

The explicit analytical forms of the associated conserved quantities of d-KdV system may be used to study the hydrodynamic equilibrium states of plasma flows by a suitable analysis of the waves and
instabilities they realistically exhibit. In fact, the ambient turbulence-driven plasma flow is quite natural to occur in toroidal and poloidal directions of the magnetic confinement of a tokamak plasma device. This leaves a further scope of theoretical research in the subject of nonlinear acoustic wave dynamics in a transonic regime of plasma flow governed by a $d$-KdV system like ours'. It may be partially useful to understand the burning issue of plasma sheath-edge singularity problem under a three-scale theory of plasma-boundary interaction processes in particular and aerodynamical designing of supersonic/ hypersonic space vehicles as well in more particular. This variety of theoretical scenario of transonic plasmas offers a unique scope of acoustic spectroscopy to describe the internal state of transonic equilibrium of plasma flows, if it exists at all. This may also have potential applications in a thorough study of the ion acoustic wave turbulence related with aerodynamics, planetary nebulae, solar wind and space plasmas, fusion plasmas of future generation, industrial plasmas and plasma flows in astrophysical context in a quasi-hydrostatic type of equilibrium, etc.

6. References

[1] Miura R M, Gardner C S and Kruskal M D 1968 J. Math. Phys. 9 1204
[2] Kruskal M D, Miura R M and Gardner C S 1970 J. Math. Phys. 11 952
[3] Lakshamanan M and Rajasekhar S 2002 Nonlinear Dynamics: Integrability, Chaos, and Patterns Basic Theory of KdV Equation (URL: http://www.springer.de/phys/) (Berlin: Springer) chapter 13 pp 402-450.
[4] Newell A C 1985 Solitons in Mathematics and Physics Soliton Equation Families and Solution Methods (Society for Industrial and Applied Mathematics) (Philadelphia & Pennsylvania 19103) chapter 3 pp 61-111
[5] Dwivedi C B and Prakash R 2001 J. Appl. Phys. 90 3200
[6] Karmakar P K, Deka U and Dwivedi C B 2005 Phys. Plasmas. 12 032105
[7] Karmakar P K, Deka U and Dwivedi C B 2006 Phys. Plasmas. 13 104702 (1)
[8] Deka U, Sarma A, Prakash R, Karmakar P K and Dwivedi C B 2004 Phys. Scr. 69 303
[9] Karmakar P K and Dwivedi C B 2006 J. Math. Phys. 47 032901(1)
[10] Karmakar P K 2007 Pramana- J. Phys. 68 631
[11] Dwivedi C B, Karmakar P K and Tripathy S C 2007 Astrophys. J. 663 (2) 1340