Computing Microwave Background Polarization Power Spectra from Cosmic Birefringence

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We present a new publicly available code, \texttt{class.rot}, which modifies \texttt{class} to enable fast non-perturbative calculation of cosmic microwave background polarization power spectra due to both isotropic and anisotropic polarization rotation from cosmic birefringence. Cosmic birefringence can arise from new parity-violating physics such as axion dark matter with a Chern-Simons coupling to photons or Faraday rotation due to a primordial magnetic field. Constraints on these effects can be obtained by comparing measurements to precise numerical calculations of the polarization power spectra. We describe the implementation of \texttt{class.rot} in terms of both mathematical formalism and coding architecture. We also provide usage examples and demonstrate the accuracy of the code by comparing with simulations.

\section{1. INTRODUCTION}

Parity-violating physics in the early universe may cause an effect known as cosmic birefringence, in which photons with different polarizations travel differently along their propagation paths, resulting in a net rotation on the polarization directions of cosmic microwave background (CMB) photons. Such an effect can arise from many types of beyond-the-Standard-Model physics, such as from the coupling between axion-like particles and photons through a Chern-Simons interaction (see, e.g., [1]), from pseudoscalar fields introduced in early dark energy models to resolve the Hubble tension [2], or from primordial magnetic fields through Faraday rotation (see, e.g., [3]).

Cosmic birefringence can cause both isotropic and anisotropic rotation of the microwave background polarization. Since the polarization field is dominated by an E-mode signal from primordial density perturbations, small rotations of polarization effectively turn E-mode into B-mode polarization, leaving observable imprints in the polarization power spectra. Isotropic birefringence, in particular, leads to non-zero parity-odd power spectra in the CMB including TB and EB (see, e.g., [1, 4]). Various experiments have placed constraints on isotropic rotation angle, such as Planck [5], WMAP [6], and ACT [7]. The observational challenge in constraining isotropic birefringence is that its effect is highly degenerate to that of a calibration error in the orientation of polarized detectors (see, e.g., [8, 9]).

Anisotropic birefringence, on the other hand, leads only to parity-even spectra and contributes negligibly to the B-mode power spectrum. Anisotropic rotation also induces off-diagonal correlations in the microwave background multipoles, which allows reconstruction of the anisotropic rotation field using a quadratic estimator approach similar to lensing reconstruction of the deflection field (see, e.g., [10–12]). Such an effect has been used to derive observational constraints on anisotropic rotation; for examples, Planck [13], BICEP2 / Keck [14], ACT [15], and SPT [16] have all derived upper bounds on anisotropic rotation field with a scale-invariant power spectrum.

Despite the physical importance of a possible rotation field, to our knowledge no publicly available codes exist that compute CMB power spectra from cosmic birefringence. Here we present a modified version of \texttt{class} [17]\textsuperscript{a}, named \texttt{class.rot}\textsuperscript{b}, which implements this calculation and allows for fast computation of the rotated EB, TB, EE, and BB power spectra due to both isotropic and anisotropic rotation from cosmic birefringence. In particular, we implement a non-perturbative calculation based on the angular correlation function of the rotation field [1, 18]. Our code has an accuracy better than 1\% at all multipoles from $l = 2$ to $l = 4000$, which we verify through comparison with power spectra of simulated sky maps including random rotation fields.

This paper is structured as follows. In Sec. 2, we describe the basics of cosmic birefringence. In Sec. 3 we show the non-perturbative calculation method that is implemented in \texttt{class.rot}, focusing on the effect of cosmic birefringence on the CMB power spectra. In Sec. 4, we demonstrate the code implementation and give usage examples, and we present comparisons between the results from \texttt{class.rot} and numerical simulations. Sec. 5 provides a brief concluding discussion about the uses of this code in the context of current and upcoming experiments.

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\textsuperscript{\textcopyright} https://github.com/lesgourg/class_public
\textsuperscript{\textcopyright} https://github.com/catketchup/class_rot
2. COSMIC ROTATION FIELD

The rotation effect from cosmic birefringence can be effectively expressed as a rotation field $\alpha(\mathbf{n})$, which can have both an isotropic part and an anisotropic part [4], given by

$$\alpha(\mathbf{n}) = \bar{\alpha} + \delta\alpha(\mathbf{n}),$$

with $\bar{\alpha}$ the isotropic part, and $\delta\alpha(\mathbf{n})$ the anisotropic part with a zero mean,

$$\langle \delta\alpha(\mathbf{n}) \rangle = 0. \quad (2)$$

As a result of rotation, Stokes parameter $Q$ and $U$ transform as

$$(\hat{Q} \pm i\hat{U})(\mathbf{n}) = \exp(\pm 2i\alpha(\mathbf{n}))(Q \pm iU)(\mathbf{n}), \quad (3)$$

where we have used tildes to denote rotated quantities.

To illustrate how such a rotation field can arise from parity-violating physics in the early universe, consider for example a Chern-Simons-type interaction of photons and axions with a Lagrangian given by

$$\mathcal{L}_{cs} = \frac{\beta \phi}{2M} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (4)$$

where $\beta$ is a dimensionless coupling constant, $\phi$ is the axion field, $M$ is its mass scale, and $F_{\mu\nu}$ is the electromagnetic tensor with $\tilde{F}_{\mu\nu}$ being its dual. This term modifies the Euler-Lagrange equations for electromagnetic field and induces a rotation in the polarization direction of a photon if $\phi$ varies along its propagation path [19–21], with the rotation angle given by

$$\alpha = \frac{\beta}{M} \Delta \phi, \quad (5)$$

where $\Delta \phi$ is the change of $\phi$ along the photon path. In the case that the axion field $\phi$ is spatially homogeneous, Eq. (5) introduces an isotropic rotation field to the CMB; an inhomogeneous axion field gives an anisotropic rotation field in the CMB.

A convenient way to express an anisotropic rotation field, $\alpha(\mathbf{n})$, is to expand it in the basis of spherical harmonics as

$$\delta\alpha(\mathbf{n}) = \sum_{LM} \alpha_{LM} Y_{LM}(\mathbf{n}). \quad (6)$$

We assume that $\alpha(\mathbf{n})$ follows Gaussian random statistics, in which case the statistical information of the rotation field $\alpha(\mathbf{n})$ can be completely specified by its power spectrum $C_{L}^{\alpha\alpha}$, given by

$$\langle a_{LM} a_{LM'} \rangle = \delta_{LL'} \delta_{MM'} C_{L}^{\alpha\alpha}. \quad (7)$$

In this paper we only consider a scale-invariant power spectrum of the anisotropic rotation field, which is physically well-motivated [22], though the formalism presented here is broadly applicable to an arbitrary rotation field power spectrum. Following the convention in [23], we parametrize a scale-invariant power spectrum as

$$\frac{L(L+1)}{2\pi} C_{L}^{\alpha\alpha} = A_{CB}, \quad (8)$$

with $A_{CB}$ the amplitude of the cosmic birefringence power spectrum.

3. IMPACTS ON MICROWAVE BACKGROUND POLARIZATION POWER SPECTRA

In this section, we briefly review the rotated CMB power spectra calculation implemented in `class_rot`. We consider a rotation field with both an isotropic contribution and a Gaussian random anisotropic contribution

\[\text{Note that } A_{CB} \text{ defined in this paper is } 10^{-4} \text{ times of that in [15] and } 10^{-5} \text{ of that in [12].}\]
as described in Eq. (1). We adopt the non-perturbative method introduced in [1, 18], which is similar to the calculation method of lensed CMB power spectra in [24]. Here we briefly review the non-perturbative calculations relevant to the implementation of class_rot; we refer interested readers to [1, 18] for more calculation details.

In this method, the starting point is to connect the real-space correlation functions of rotated quantities, such as $\hat{T}(\mathbf{n})$, $\hat{Q}(\mathbf{n})$, and $\hat{U}(\mathbf{n})$, to the rotated power spectra, e.g., $\tilde{C}_\ell^{EE}$, $\tilde{C}_\ell^{BB}$, with

$$\tilde{\xi}_+(\beta) \equiv \left\langle (\hat{Q} + i\hat{U})^* (\mathbf{n}) (\hat{Q} + i\hat{U}) (\mathbf{n}') \right\rangle = \sum_{\ell'} \frac{2\ell' + 1}{4\pi} \left( \tilde{C}_\ell^{EE} + \tilde{C}_\ell^{BB} \right) d_{22}(\beta),$$

$$\tilde{\xi}_-(\beta) \equiv \left\langle (\hat{Q} + i\hat{U}) (\mathbf{n}) (\hat{Q} + i\hat{U}) (\mathbf{n}') \right\rangle = \sum_{\ell'} \frac{2\ell' + 1}{4\pi} \left( \tilde{C}_\ell^{EE} - \tilde{C}_\ell^{BB} + 2i\tilde{C}_\ell^{EB} \right) d_{22}(\beta),$$

$$\tilde{\xi}_X(\beta) \equiv \left\langle \hat{T}(\mathbf{n}) (\hat{Q} + i\hat{U}) (\mathbf{n}) \right\rangle = -\sum_{\ell'} \frac{2\ell' + 1}{4\pi} \left( \tilde{C}_\ell^{EE} + i\tilde{C}_\ell^{TB} \right) d_{02}(\beta),$$

where $\mathbf{n}$ and $\mathbf{n}'$ are two directions in the spherical coordinate system, $\cos \beta = \mathbf{n} \cdot \mathbf{n}'$, and $d_{mm'}$ is the Wigner d-function. Taking advantages of the orthogonality relations of Wigner d-functions,

$$\int_{-1}^{1} d\cos \beta d_{mk}(\beta)d_{m'k'}(\beta) = \frac{2}{2\ell + 1} \delta_{mm'}\delta_{kk'}\delta_{\ell\ell'},$$

one can invert Eq. (9) to express rotated power spectra in terms of correlation functions, such as

$$\tilde{C}_\ell^{EE} + \tilde{C}_\ell^{BB} = 2\pi \int_{-1}^{1} d\cos \beta \tilde{\xi}_+(\beta)d_{22}(\beta).$$

Applying Eq. (3), $\tilde{\xi}_+(\beta)$ can be expressed by un-rotated quantities as

$$\tilde{\xi}_+(\beta) = e^{-4C^{(0)}(0)+4C^{(0)}(\beta)} \sum_{\ell'} (2\ell' + 1)(C_{\ell'}^{EE} + C_{\ell'}^{BB})d_{22}(\beta).$$

Here $C^{(0)}(\beta)$ is the correlation function of rotation angles in the two directions separated by $\beta$ and can be expressed as

$$C^{(0)}(\beta) = \left\langle \delta(\alpha_1) \delta(\alpha_2) \right\rangle = \sum_{L} \frac{2L + 1}{4\pi} C_{L}^{\alpha\alpha} P_{L}(\cos \beta) = \sum_{L} \frac{2L + 1}{4\pi} C_{L}^{\alpha\alpha} d_{00}(\beta),$$

where $C_{L}^{\alpha\alpha}$ is a generic rotation field power spectrum introduced in Eq. (7), $P_{L}(\cos \beta)$ is the Legendre Polynomial, and we have applied $P_{L}(\cos \beta) = d_{00}(\beta)$.

Equipped with Eq. (12), Eq. (11) can be written as

$$\tilde{C}_\ell^{EE} + \tilde{C}_\ell^{BB} = \frac{1}{2} e^{-4C^{(0)}(0)} \int d\cos \beta e^{4C^{(0)}(\beta)} d_{22}(\beta) \left[ \sum_{\ell'} (2\ell' + 1)(C_{\ell'}^{EE} + C_{\ell'}^{BB})d_{22}(\beta) \right].$$

Similarly, one can also obtain

$$\tilde{C}_\ell^{TE} - \tilde{C}_\ell^{TB} = \frac{1}{2} e^{-4C^{(0)}(0)} \cos 4\alpha \int d\cos \beta e^{-4C^{(0)}(\beta)} d_{{22}}(\beta) \left[ \sum_{\ell'} (2\ell' + 1)(C_{\ell'}^{EE} - C_{\ell'}^{BB})d_{{22}}(\beta) \right],$$

$$\tilde{C}_\ell^{EB} = \frac{1}{2} e^{-4C^{(0)}(0)} \sin 4\alpha \int d\cos \beta e^{-4C^{(0)}(\beta)} d_{{22}}(\beta) \left[ \sum_{\ell'} (2\ell' + 1)(C_{\ell'}^{EE} - C_{\ell'}^{BB})d_{{22}}(\beta) \right].$$

Note that the rotated CMB EE, BB and EB power spectra in Eq. (14) and Eq. (15) are given by real-space integrals, which avoids convolution in the $\ell m$ space which is computationally expensive. A similar strategy that uses real-space integral instead of convolution in $\ell m$ space can be found in delensing calculation [25] which significantly reduces computational cost. Also note that we have ignored the correlations between the rotation field and both CMB temperature and (unrotated) E-polarization fields, which may arise in certain axion-like models, such as models with nonzero potential under adiabatic initial conditions [22]. A similar calculation that takes account of these correlations can be found in [4].

We can see from Eq. (14) and Eq. (15) that both isotropic and anisotropic rotations contribute to BB power spectrum. In the upper panel of Fig. 1, we show the BB power spectrum contributed by an isotropic rotation field with $\alpha = 0.1^\circ$ and a scale-invariant anisotropic rotation field with $A_{CB} = 10^{-5}$, respectively. As a comparison, we also show the contributions from primordial tensor mode with $r = 0.004$ where $r$ is the tensor-to-scalar ratio, and the contribution from CMB lensing. One can see that the B-mode signal from rotation fields can be larger than that from the primordial tensor mode at $\ell \gtrsim 150$, which suggests that, apart from searching for parity-violating physics, rotation field is also an important systematic when searching for primordial tensor mode. We also note that rotation field generally contributes less than CMB lensing to B-mode polarization; this suggests that the ability to “de-lens” the CMB will help tighten the constraints on cosmic birefringence. From Eq. (15) we can also see that both $\tilde{C}_\ell^{TB}$ and $\tilde{C}_\ell^{EB}$ become non-zero when $\alpha$ is non-zero; this is consistent with the fact that an isotropic rotation field violates parity symmetry and induces odd-parity CMB power spectra.
4. THE SOFTWARE PACKAGE

In this section, we describe briefly the implementation of class_rot, give usage examples of its Python interface, and show comparisons to numerical simulations.

Code implementation: In class_rot, the calculations described in Sec. 3 are implemented as a new module to class, contained in rotation.c [8]. Internally, this rotation module takes the power spectra calculated from the harmonic module as inputs, by doing so we have implicitly neglected the effect of CMB lensing when calculating the rotated power spectra. This assumption significantly simplifies our code implementation and will only lead to sub-percent to percent level error due to the smallness of $C_{\ell}^{BB}$ relative to $C_{\ell}^{EE}$; to incorporate the effect of CMB lensing in the rotation module will be the subject of future work.

The rotation module can be turned on by specifying rotation = yes in the parameter file, and it can take two additional parameters that specify the rotation field, alpha and A_cb, which correspond to $\bar{\alpha}$, in unit of degrees, and $A_{CB}$, in radians as defined in Eq. (8), respectively. The rest of the parameters are identical to those in class. Note that by using $A_{CB}$ we implicitly assume that the rotation field follows a scale-invariant power spectrum – a choice of preference rather than necessity; other rotation power spectrum can be implemented by changing the rotation_claa_atl function defined in rotation.c [8]. We leave the support for taking in a generic rotational power spectrum as input to a future work.

The parameters can be specified in a parameter file and passed to the compiled class binary executable, in the same way as the original class. An example parameter file, explanatory_ROT.ini [8] is also provided as part of class_rot to illustrate the use of parameters. Note that this parameter file is only needed when calling class_rot from the command-line interface using its compiled binary executable. We have also provided Python bindings to the functions in the rotation module allowing them to be called in the Python interface, and we show some usage example below.

Usage example: Here we give an example of how to calculate the rotated CMB power spectra using the Python interface of class_rot:

```python
from classy import Class
params = {
    "output": "tCl,pCl,rCl",
    "l_max_scalars": 4000,
    "rotation": "yes",
    "alpha": 0.1,
    "A_cb": 1E-5,
}

cosmo = Class()
cosmo.set(params)
cosmo.compute(level="rotation")
cosmo.rotated_cl()
```

One can see that class_rot is meant to be used as a drop-in replacement to the original class as it is imported the same way and follows the same usage pattern. The parameters are specified in a Python dictionary, param, and passed to the cosmo object. Note that it is important to include rCl in the output option as it is required for computing the rotated power spectra. The option rotation turns on the rotation
module when its value is yes; alpha and A_cB specify the rotation parameters as can be used in a parameter file. Also note that when computing cosmological model with the function \texttt{cosmo.compute()}, one needs to include \texttt{level=["rotation"]} so that the rotation module and its dependencies are initialized properly. After running \texttt{cosmo.compute()}, the rotated power spectra can be obtained by the function call \texttt{cosmo.rotated_cl()}, in the form of a Python dictionary following the convention from \texttt{class}. This illustrates a basic usage of \texttt{class_rot}; we refer interested readers to the examples provided in the bundled Jupyter notebook in \texttt{class_rot} to find more detailed examples and explanations.

**Comparison with simulations:** To demonstrate the accuracy of \texttt{class_rot}, we compare the rotated CMB power spectra from \texttt{class_rot} with those from full-sky simulations. In particular, we first generate 100 realizations of un-rotated CMB maps in T, Q, and U based on a fiducial model given by the best-fit cosmology from Planck 2018 [26] with \( l_{\text{max}} = 6000 \). Additionally we set a non-zero tensor-to-scalar ratio \( r = 0.004 \). Next we generate 100 realizations of a full-sky rotation map with \( \bar{\alpha} = 0.1^\circ \) and \( A_{CB} = 10^{-5} \), which are then used to rotate each realization of unrotated CMB maps. These full-sky simulations are generated using \texttt{pixell} [27] in rectangular pixelization and CAR projection with a resolution of 1 arcminute. We apply each rotation field to rotate one realization of simulated CMB maps in pixel space using Eq. (3) and then calculate its power spectra after the rotations. We repeat this procedure for each realization to get 100 sets of rotated CMB power spectra. In Fig. 2, we show the average of the 100 realizations of rotated power spectra in comparison to the corresponding theory spectrum obtained from \texttt{class_rot}. One can clearly see that the output of \texttt{class_rot} is in an excellent agreement with simulations. For \( C_{iBB}^T \) we estimate an error of \( \lesssim 1\% \) at \( \ell \lesssim 4000 \); the accuracy noticeably degrades at larger \( \ell \) likely due to a combination of pixel effect, numerical precision, and the smallness of the signal of interests. Both \( C_{iEB}^T \) and \( C_{iEB}^T \) from \texttt{class_rot} agree with the simulations within the expected cosmic variance of the averaged power spectra up to \( \ell = 6000 \), which is the highest multipole we have tested.

5. DISCUSSION AND CONCLUSION

In this paper we present \texttt{class_rot}, a new publicly available modified \texttt{class} code, which calculates rotated CMB power spectra from cosmic birefringence using a non-perturbative method. \texttt{class_rot} supports both isotropic and anisotropic rotations, as can be specified by the isotropic rotation angle, \( \bar{\alpha} \), and the amplitude of scale-invariant rotation power spectrum, \( A_{CB} \), respectively. Hence, \texttt{class_rot} can be effectively used to search for cosmic birefringence signal that features a scale-invariant rotation power spectrum or an isotropic rotation in CMB polarization rotation, such as that from the coupling between axion-like particles and photons via Chern-Simons interaction. We leave the implementation of a more generic (i.e., not scale-invariant) rotation power spectrum in \texttt{class_rot} to a future work which will allow us to search for a broader range of rotation signal such as that caused by Faraday rotation from primordial magnetic field, which, depending on its generation mechanism, may induce a rotation field that is not scale-invariant (see [28] for a review).

In this paper we have also briefly reviewed the non-perturbative calculation implemented in \texttt{class_rot}, which makes use of the angular correlation function of the rotation field and does not require the rotation to be perturbatively small. Hence the calculation in \texttt{class_rot} offers a broader range of applicability. We leave the implementation of a perturbative calculation as well as a detailed comparison between the non-perturbative and perturbative methods, in terms of both speed and accuracy, to a future work.

We have briefly described the coding implementation and given an example of how to use \texttt{class_rot} with its Python interface. To demonstrate its accuracy we have compared the rotated CMB power spectra such as BB, TB, and EB obtained from \texttt{class_rot} to full-sky simulations and shown that they are in good agreements with \( \lesssim 1\% \) error. The upcoming experiments are expected to constrain cosmic birefringence with much higher precision. For example, while the current best limits lie around \( O(10^9) \) for isotropic rotation [5, 7] and around \( O(10^{-6}) \) for \( A_{CB} \) [15, 16], it has been forecasted that Simons Observatory [29] can improve the current limits by nearly an order of magnitude, achieving an uncertainty level of around 0.7° for isotropic rotation and around \( 10^{-7} \) for \( A_{CB} \) [30]. These limits will be further improved by the CMB-S4 experiment [31], reaching an uncertainty level of around 0.2° for isotropic rotation [23] and around \( 10^{-8} \) for \( A_{CB} \) [30]; this will allow for percent-level determinations of \( \bar{\alpha} \) and \( A_{CB} \) should there be a cosmic birefringence signal at our current observational limit. In light of these future prospects, it is important to have a robust code that computes the effect of cosmic birefringence in power spectra with better than percent-level accuracy. Hence, \texttt{class_rot} can be a powerful tool for searches of cosmic birefringence signal in the future.

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