The properties of the ordered bilinear form semigroup related to their regular elements

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Abstract. A semigroup is an algebraic structure which only involve one binary operation that is associative. Research on a semigroup has grown rapidly, included an ordered semigroup. An ordered semigroup is a semigroup that is added an ordered operation such that it meets some axioms. A bilinear forms semigroup is a semigroup whose the elements are adjoin ordered pairs relative to a bilinear form. In this study, we have investigated the properties of an ordered bilinear form semigroup based on its regular elements. We have obtained some properties associated with the biggest inverse element and the biggest idempotent element. The other results we investigated the properties of Right (Left) Green equivalence classes on the ordered bilinear form semigroup related to their biggest inverse element and the biggest idempotent element.

1. Introduction
An algebraic structure is a set that is added with one or more binary operations, which fulfill one or more specific axioms. A semigroup is an algebraic structure that only involves one binary operation and fulfills an axiom that is associative. Research related to semigroups theories have grown rapidly. It is followed by the development of an ordered semigroup theories. An ordered semigroup is a semigroup which is added a binary operation, which is called an ordered operation. In studies by previous researchers [1-3], obtained several properties related to an ordered semigroup. Karyati and Abadi, AM [4], has investigated the properties of fuzzy ordered semigroups, fuzzy ordered bilinear form semigroups. Several researchers [5-7] have also worked and produce several properties of the ordered semigroup based on their fuzzy subsets, their fuzzy ideals and their fuzzy bi-ideals. In a study conducted by Karyati and Dhoriva [8], several theories have been obtained related to the characterization of ordered bilinear form semigroups based on their fuzzy subsets and their fuzzy (right / left) ideals. Another study has been done by Karyati & Abadi, AM [9], which is related to the characteristics of ordered bilinear form semigroups which are weakly regular, intra-regular and semisimple, associated with their fuzzy subsets.

A semigroup \((S, \cdot)\) in which is added an order operation \(\leq\), so that \((S, \leq)\) forms a poset and it meets for each \(x, y, z \in S\) with \(x \leq y\) implies \(xz \leq yz\) and \(xz \leq yz\), then \((S, \cdot)\) is called ordered semigroup, [10]. Some studies related to ordered semigroups have been developed by many researchers [11-13]. By defining this partial sequence is very influential on the ideal definitions (left / right), ideal quasi (left / right), relations, ideal (left / right) fuzzy, ideal quasi (left / right) fuzzy which will later give rise to new nature and theories.
2. Bilinear Form Semigroups

A bilinear form semigroup is a semigroup that has special characteristics. The special element has a unique character. They are ordered pairs which are adjoin each other related to a bilinear form [11-12]. The sets $L(X)$ and $L(Y)$ are sets of all linear operators $X$ and $Y$, respectively. If $f \in L(X)$, then we can construct sets of Kernel and Range that are denoted by $N(f) = \{ u \in X | f(u) = 0 \}$ and $R(f) = \{ v \in X | f(x) = v, \text{ for any } x \in X \}$, respectively. An element $f \in L(X)$ is called adjoin to $g \in L(Y)$ related to a bilinear form $B$ and vice versa, if $B(x, g(y)) = B(f(x), y)$ for all $x \in X$ and $y \in Y$. Furthermore, we construct the following sets:

\[
L(X) = \{ f \in L(X) | N(B) \subseteq N(f), R(f) \cap N(B) = \{ 0 \} \},
\]

\[
L(Y) = \{ g \in L(Y) | N(B^*) \subseteq N(g), R(g) \cap N(B^*) = \{ 0 \} \}.
\]

Karyati et al. proved that the set forms a semigroup of the following binary operations: $(f, g) \circ (f', g') = (f \circ f', g' \circ g)$ [14]. This semigroup is called a bilinear form semigroup and denoted by $S(B)$ is hereinafter called semigroup bilinear form. Some properties associated with this bilinear form semigroup have been investigated by Nambooripad et al. [12] and followed by Karyati et al. [15].

Research related to bilinear form semigroup introduced by Rajendran and Nambooripad [12]. Based on this research, a theory of fuzzy semigroup bilinear forms has been developed by Karyati et al. [14,15]. Karyati and Dhoriva [8,9] learned about the characteristics of an ordered bilinear form semigroup based on their fuzzy subsets. Moreover, Karyati and Abadi [16] have examined the characteristics of an ordered bilinear form semigroup that is weakly regular and semisimple.

2.1. Ordered Semigroup (po_semigroup)

Let $S$ be a non empty set. The set $S$ is added by an operation $\cdot$ is called a semigroup if it is closed under the operation, i.e.: $(\forall x, y \in S) x \cdot y \in S$ and it is associative, i.e.: $(\forall x, y, z \in S) (x \cdot y) \cdot z = x \cdot (y \cdot z)$. A non empty set $P$ is called ordered operation $\leq$ if it is reflexive, i.e. $(\forall x \in P) x \leq x$, antisymmetric, i.e. $(\forall x, y \in P) x \leq y \land y \leq x \Rightarrow x = y$, and transitive, i.e. $(\forall x, y, z \in P) x \leq y \land y \leq z \Rightarrow z \leq z$.

The partial ordered set is also called a posset. Suppose that $S$ is a non-empty set. The set $S$ with binary operations $\cdot$ and $\leq$ is called partial ordered semigroup if: $(S, \cdot)$ is a semigroup, $(S, \leq)$ is an ordered set, $(\forall a, b, x \in S) a \leq b \Rightarrow xa \leq xb$ and $ax \leq bx$. Based on the definition of an ordered semigroup, the definition of an of an ordered semigroup need to be added several axioms. The definition of an ideal of an ordered semigroup is given as follow: suppose $(S, \cdot, \leq)$ is an ordered semigroup, the non-empty subset $I$ is called an ideal of $S$ if $(\forall a \in S)(\forall b \in I) a \cdot b \Rightarrow a \in I$, $IS \subseteq I$ and $SI \subseteq I$.

2.2. Green’s Relation

Let $S$ be a semigroup. A non empty subset $I \subseteq S$ is called a right ideal if $IS \subseteq I$ (a left ideal if $SI \subseteq I$). The subset $I$ is called an ideal (two sided) if $I$ is both a right ideal and a left ideal (Karyati, []). The right (left) ideal generated by $x \in S$ is denoted by $\langle x \rangle_R$ ($\langle x \rangle_L$) and an ideal generated by $x \in S$ is denoted by $\langle x \rangle$. The Green relation on a semigroup has been introduced by Howie (1976). They are right Green relation ($R$), the left Green relation ($L$) and the Green relation ($J$). The Green relation $R, L, J$ are equivalence relations, denoted by: $R = \{ (x, y) \in S \times S | \langle x \rangle_R = \langle y \rangle_R \}$, $L = \{ (x, y) \in S \times S | \langle x \rangle_L = \langle y \rangle_L \}$, $J = \{ (x, y) \in S \times S | \langle x \rangle = \langle y \rangle \}$. Some studies related to the fuzzy ideal of semigroups, the fuzzy ideal of semigroups generated by a fuzzy singleton and their properties have been introduced by Karyati.

3. Results and Discussion

An ordered bilinear form semigroup is called primary ordered if for every $(f, g) = \hat{a} \in S(B)$ there is element $\hat{a}^* = \max\{ \hat{b} \in S(B) | \hat{a} \hat{b} \hat{a} \leq \hat{a} \}$. This element has properties as follow:

M1. For all $\hat{a} \in S(B)$, $\hat{a} = \hat{a}^* \hat{a}$.
M2. For all class of left Green’s relations $\mathcal{L}$, denoted by $\bar{[a]}_\mathcal{L}$, consist a biggest idempotent element. It is denoted by $\bar{a}^*\bar{a}$.

M3. For all class of right Green’s relations $\mathcal{R}$, denoted by $\bar{[a]}_\mathcal{R}$, consist a biggest idempotent element. It is denoted by $\bar{a}\bar{a}^*$.

M4. For all $\bar{a} \in S(B)$, have consequence $\bar{a}^{**} = \bar{a}^*$.

M5. For all $\bar{a} \in S(B)$ have biggest inverse, i.e. $\bar{a}^\Theta = \bar{a}^*\bar{a}\bar{a}^*$.

M6. For all $\bar{a} \in S(B)$, $\bar{a}^\Theta \leq \bar{a}^*$.

M7. For all $\bar{a} \in S(B)$, $\bar{a} \leq \bar{a}^{**} = \bar{a}^\Theta = \bar{a}^\infty$

Theorem 3.1. If the bilinear form semigroup $S(B)$ is primary ordered, then the following statements are equivalence:

(i) For every class left Green’s relation $\mathcal{L}$ has a biggest element which is idempotent,

(ii) For all $\bar{a} \in S(B)$, $\bar{a}^*\bar{a} = \max[\bar{a}]_\mathcal{L}$

(iii) For every class right Green’s relation $\mathcal{R}$ has a biggest element which is idempotent,

(iv) For all $\bar{a} \in S(B)$, $\bar{a}\bar{a}^* = \max[\bar{a}]_\mathcal{R}$

(v) For $\bar{a} \in S(B)$, $\bar{a}^2 \leq \bar{a}$

(vi) For $\bar{a} \in S(B)$, $\bar{a}^* \in E(S(B))$

Proof.

(i) $\iff$ (ii).

Assume that (i) be hold and $\bar{e} = \bar{e}^2 = \max[\bar{e}]_\mathcal{L}$. Refer to M2., so we have $\bar{e} = \bar{e}^*\bar{e} = \bar{a}^*\bar{a}$. So the condition (ii) is fulfilled. By inverted the proofing, we can prove (ii)$\implies$(i).

(iii)$\iff$(iv).

We assume that (iii) be hold and $\bar{e} = \bar{e}^2 = \max[\bar{e}]_\mathcal{R}$. Refer to M3., so we have $\bar{e} = \bar{e}\bar{e}^* = \bar{a}\bar{a}^*$. So the condition (iv) is fulfilled. By inverted the proofing, we can prove (iv)$\implies$(iii).

(ii)$\iff$(v).

If for all $\bar{a} \in S(B)$, then $\bar{a}^*\bar{a} = \max[\bar{a}]_\mathcal{L}$ so we get $\bar{a} \leq \bar{a}^*\bar{a}$. Refer to (i) we have $\bar{a}^2 \leq \bar{a}^*\bar{a} = \bar{a}$. Otherwise, if for all $\bar{a} \in S(B), \bar{a}^2 \leq \bar{a}$, then $\bar{a}^3 \leq \bar{a}^2 = \bar{a}$. There for we get $\bar{a} \leq \bar{a}^*$, and also we have $\bar{a}\bar{a}^* \leq \bar{a}^* \leq \bar{a}^*$. The consequence, $\bar{a} = \bar{a}\bar{a}^* \leq \bar{a}^* \bar{a}$. Finally, we obtain $\bar{a}^* = \max[\bar{a}]_\mathcal{L}$.

(iv)$\iff$(v) This is the dual of (ii)$\iff$(v).

(v)$\iff$(vi).

We have for $\bar{a} \in S(B), \bar{a}^2 \leq \bar{a}$. Similarly with the proofing statement (ii)$\iff$(iv). Furthermore, $\bar{b} \mathcal{R} \bar{a}$, so refer to the (iv) we get $\bar{b} \leq \bar{a}\bar{a}^*$. As the consequence, $\bar{b}\bar{a} \leq \bar{a}\bar{a}^* = \bar{a}$. According (v), it is hold $\bar{a}\bar{a}^* \leq \bar{a}^2 \leq \bar{a}$ and also we get $\bar{b} \leq \bar{a}^*$. Especially for the case $\bar{b} = \bar{a}\bar{a}^*$ implies $\bar{a}\bar{a}^* \leq \bar{a}^*$ for all $\bar{a} \in S(B)$. If we replace $\bar{a}$ with $\bar{a}^*$, then we have $\bar{a}^* \leq \bar{a}^*$. Refer to (ii), then we obtain $\bar{a}^* = \bar{a}^*\bar{a}^* \in E(S(B))$.

(vi)$\iff$(ii).

It is always hold that $\bar{1} \in E(S(B))$. So we have $\bar{1} \leq \bar{1}^*$. So, if it is hold for every $\bar{a} \in S(B)$, $\bar{a}^* \in E(S(B))$ then $\bar{a}^* \leq \bar{a}^{**}$ and $\bar{a}^{**} \leq \bar{a}^{***} = \bar{a}^*$, then we get $\bar{a}^* = \bar{a}^{**}$. Furthermore, we will prove that if $\bar{b} \equiv \bar{a}\mathcal{R}$, then $\bar{b}^* = \bar{a}^*$. Assume that we have $\bar{b} \equiv \bar{a}\mathcal{R}$. Based on (vi), then we get $\bar{a}\bar{a}^*\bar{b}^* \bar{a}^* = \bar{b}^* \bar{a}^* \bar{a}^* = \bar{b}^* \bar{a}^* = \bar{a}^* \bar{a}^* = \bar{a}^*$, so $\bar{a}^* \bar{b}^* \leq \bar{a}^*$. Therefore we have $\bar{a}^* \bar{b}^* \bar{a}^* \leq \bar{a}^*$, and finally we obtain $\bar{b}^* \leq \bar{a}^{**} = \bar{a}^*$. We replace $\bar{a}$ with $\bar{b}$, it will obtain $\bar{b}^* = \bar{a}^*$. 
Especially if we take $\tilde{b} = \tilde{a} \tilde{a}^*$, then we have $\tilde{a} \tilde{a}^* \leq (\tilde{a} \tilde{a}^*)^* = \tilde{a}^*$ since $\tilde{a} = \tilde{a} \tilde{a}^* \tilde{a} \leq \tilde{a}^* \tilde{a}$ for all $\tilde{a} \in \tilde{S}(\tilde{B})$. If we have $\tilde{c} \in [\tilde{a}]_\tilde{L}$ then $\tilde{c} \leq \tilde{c}^* \tilde{a} = \tilde{a}^* \tilde{c}$ and we have $\tilde{a}^* \tilde{a} = \max[\tilde{a}]_\tilde{L}$. So, (ii) is hold.

**Theorem 3.2.** If the bilinear form semigroup $S(\mathcal{B})$ holds one of the statement of Theorem 3.1., then

- (vii) For all $\tilde{a} \in S(\mathcal{B})$, then $\max[\tilde{a}]_\tilde{R} = \tilde{a}^* = \tilde{a}^{**}$.
- (viii) $S(\mathcal{B})$ is a semiband and the Green’s relation $\mathcal{H}$ is equality
- (ix) $\tilde{a} \in S(\mathcal{B})$ is a complete regular element if and only if $\tilde{a} \in E(S(\mathcal{B}))$

**Proof:** We assume that the condition of Theorem 3.1 are hold.

(vii) As like the proof (vi) $\Rightarrow$ (ii), $\tilde{a}^* = \tilde{a}^{**} \in E(S(\mathcal{B}))$, and therefore $\tilde{a}^* = \tilde{a}^{**} \tilde{a}^*$. Based on statement (iv) and M4, then we have $\tilde{a}^* = \max[\tilde{a}^{**}]_\tilde{R} = \max[\tilde{a}]_\tilde{R}$. Similarly, It can be proven that $\tilde{a} = \tilde{a}^* \tilde{a} = \tilde{a}^{**} \tilde{a}^*$. Then, we have $\tilde{a} = \tilde{a}^* \tilde{a} = \tilde{a}^{**} \tilde{a}^* \tilde{a} = \tilde{a}^* \tilde{a}^* = \tilde{b}^* \tilde{b}^* \tilde{b} = \tilde{b} \tilde{b} \tilde{b} = \tilde{b}$. So, it can be said that the Green’s relation $\mathcal{H}$ changes the relation to become an equality.

(ix) If $\tilde{a} \in S(\mathcal{B})$ is a complete regular element, there is $\tilde{c} \in V(\tilde{a})$ such that $\tilde{a} \tilde{c} = \tilde{c} \tilde{a}$. So that, according to (v), $\tilde{c} = \tilde{c} \tilde{a} \tilde{c} = \tilde{c}^2 \tilde{c} \leq \tilde{c} \tilde{a}$, and we have $\tilde{a} = \tilde{a} \tilde{a} \tilde{a} \tilde{c} \tilde{a} = \tilde{a}^2 \tilde{c}$. In the other word $\tilde{a} \in E(S(\mathcal{B}))$. Finally, we get the proof of the theorem.

Remember that the natural order $\leq_n$ on the idempotent of a regular semigroup is defined by $e \leq_n f$ if and only if $f = ef = fe$, and that an ordered regular semigroup $(S(\mathcal{B}); \leq)$ is said to be naturally ordered if the order $\leq$ extends the natural order, in the sense that if $e \leq_n \tilde{b}$ then $e \leq \tilde{b}$.

**Theorem 3.3.** If $S(\mathcal{B})$ is a naturally ordered bilinear form semigroup and the biggest idempotent $\tilde{b}$ then the semiband $(E(S(\mathcal{B})))$ is a pointed primary ordered bilinear form semigroup.

**Proof.** Let $\tilde{\phi} = \tilde{e}_1 \ldots \tilde{e}_n \in (E(S(\mathcal{B})))$ and $\tilde{\epsilon}$ is the biggest element of $(E(S(\mathcal{B})))$, then we get $\tilde{\phi} \tilde{\phi} = \tilde{\phi} \tilde{\phi}$ for every $\tilde{\phi} \in E(S(\mathcal{B}))$, and consequently $\tilde{\phi} \tilde{\phi} = \tilde{\phi}_1 \ldots \tilde{\phi}_n \tilde{\epsilon}_1 \ldots \tilde{\epsilon}_n \leq \tilde{\epsilon}_1 \tilde{\epsilon}_2 \ldots \tilde{\epsilon}_n = \tilde{\epsilon}_1 \ldots \tilde{\epsilon}_n = \tilde{\phi}$, since the subsemigroup $(E(S(\mathcal{B})))$ is primary ordered with $\tilde{\phi}^* = \tilde{\phi}$ for every $\tilde{\phi} \in (E(S(\mathcal{B})))$. Furthermore, $\tilde{\phi}^2 = \tilde{\phi} \tilde{\phi} \leq \tilde{\phi} \tilde{\phi} \leq \tilde{\phi}$, so according to Theorem 3.1 (v) $(E(S(\mathcal{B})))$ is a pointed.

For $\tilde{a}$, $\tilde{b}$ in the bilinear form semigroup $S(\mathcal{B})$, $\tilde{a}$ is related Green’s $\mathcal{D}$ with $\tilde{b}$ in $S(\mathcal{B})$, if $\tilde{a}^{\mathcal{D}} = \tilde{b}^{\mathcal{D}}$.

**Theorem 3.4.** For every class of Green’s relation $\mathcal{D}$ has a biggest element which is idempotent, $(\forall \tilde{a} \in S(\mathcal{B})) \tilde{a}^{\mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \max [\tilde{a}^{\mathcal{D} \mathcal{D}}]_\tilde{R} = \max[\tilde{a}]_\tilde{D} \in E(S(\mathcal{B}))$.

**Proof.** Analog with the proofing of Theorem 3.1, then we get $\tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^*$. Based on the statement of Theorem 3.1(iv), we have $\tilde{a} = \tilde{a} \tilde{a}^* = \tilde{a} \tilde{a}^* \tilde{a} = \tilde{a} \tilde{a}^* \tilde{a} = \tilde{a}^{\mathcal{D} \mathcal{D}}$. Similarly, it is hold for Left Green’s $\mathcal{L}$. According to (vii) and Theorem 3.1(vi), we have $\tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}}$. If $\tilde{a} \tilde{b} \tilde{b}$ then there exists $\tilde{c} \in S(\mathcal{B})$ such that $\tilde{a} \mathcal{L} \tilde{c} \mathcal{R} \tilde{b}$. As a consequence we obtain $\tilde{a}^* \tilde{a} = \tilde{c} \tilde{c}^* \tilde{c}$ and $\tilde{c} \tilde{c}^* = \tilde{b} \tilde{b}^*$. So, we have $\tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{b} \tilde{b}^*$. Consequently it is hold $\tilde{a} \mathcal{D} \tilde{b} \mathcal{D} \tilde{a} \mathcal{D} \tilde{b}$ and only if $\tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{b}^{\mathcal{D} \mathcal{D}}$. According to Theorem 3.1, (ii, iv), we get $\tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}}$. Finally we obtain $\tilde{a}^{\mathcal{D} \mathcal{D}} = \tilde{a}^{\mathcal{D} \mathcal{D}} \in E(S(\mathcal{B}))$.

**Theorem 3.5.** The element $\tilde{a} \in S(\mathcal{B})$ is a maximal idempotent element if and only if it is maximal element.
Proof. Let \( \bar{\xi} \) be a maximal element in \( S(B) \). If there is \( \bar{\alpha} \in S(B) \) such that \( \bar{\xi} \leq \bar{\alpha} \leq \bar{\alpha}^* \in E(S(B)) \), then we have \( \bar{\xi} \leq \bar{\alpha} \leq \bar{\alpha}^* \in E(S(B)) \). So, we claim that \( \bar{\xi} \) is the maximal element in \( E(S(B)) \), for \( \bar{\xi} = \bar{\alpha} \). Consequently, \( \bar{\xi} \) is idempotent in \( S(B) \). Otherwise, if \( \bar{\alpha} \in S(B) \) is a maximal element then \( \bar{\alpha} \leq \bar{\alpha}^* \) and we get \( \bar{\alpha} = \bar{\alpha}^* \). Refer to Theorem 3.1 (vi), it has consequence \( \bar{\alpha} \in E(S(B)) \).

Theorem 3.6. A bilinear form semigroup \( S(B) \) contains at most one maximum element.

Proof. Let \( \bar{\xi} \) and \( \bar{\delta} \) be maximal elements in \( S(B) \), respectively. According to Theorem 3.5, then both of \( \bar{\xi} \) and \( \bar{\delta} \) are idempotent elements. Based on Theorem 3.1(v), it is hold that \( \bar{\xi} \bar{\delta} \bar{\xi} \bar{\delta} = (\bar{\xi} \bar{\delta})^2 \leq \bar{\xi} \bar{\delta} \) and we obtain \( \bar{\xi} \leq (\bar{\xi} \bar{\delta})^* \). The consequence is \( \bar{\xi} = (\bar{\xi} \bar{\delta})^* \). In the analog way, we get \( \bar{\delta} = (\bar{\delta} \bar{\xi})^* \). Similarly, we can proof that \( \bar{\delta} = (\bar{\delta} \bar{\xi})^* = (\bar{\delta} \bar{\xi})^* \). Finally, we have proven that \( \bar{\xi} = \bar{\delta} \).

4. Conclusion

Based on the previous discussion, we have investigated the properties of an ordered bilinear form semigroup based on its regular elements. We have obtained some properties associated with the biggest inverse element and the biggest idempotent element. The other results we investigated the properties the class of Green’s \( \mathcal{H} \) and Green’s \( \mathcal{D} \) relations on the ordered bilinear form semigroup related to their biggest inverse element and the biggest idempotent element.

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