Open Cosmic Strings in Black Hole Space-Times

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We construct open cosmic string solutions in Schwarzschild black hole and non-dilatonic black p-brane backgrounds. These strings can be thought to stretch between two D-branes or between a D-brane and the horizon in curved space-time. We study small fluctuations around these solutions and discuss their basic properties.

I. INTRODUCTION

General relativity offers a very good description of gravity in terms of a curved space-time metric. However, this description breaks down when a sufficiently large distribution of matter collapses due to gravitational attraction. At the end of the collapse, a singularity is inevitable provided that general relativity is correct and that the strong energy condition holds for matter. According to the cosmic censor conjecture, no singularity is ever visible to any observer. The endpoint of a collapse will be a black hole where the singularity is located in a region of space-time causally disconnected from the asymptotic observer. Even if the cosmic censor conjecture is true, one may still insist on a well defined physical picture, rather than having a secret region where physics is no longer operational. It is natural to expect that quantum gravitational effects to be important in the very strong fields near a singularity. More precisely, we expect that at the Planck scale the classical description of space-time becomes completely inadequate.

Classically, nothing can escape from a black hole. However, Hawking showed that black holes radiate energy as if they were perfect blackbody emitters with the temperature $T = \frac{\kappa}{2\pi}$, where $\kappa$ is the surface gravity at the horizon $\Delta$. Qualitatively, the radiation can be thought to arise due to vacuum fluctuations of matter fields near the horizon. The pairs of particles and anti-particles are produced, and as one member with negative energy falls into the hole, the other member can escape to infinity. Moreover, as the black hole emits radiation its temperature increases. Therefore, one expects that a black hole radiates all of it’s energy in a finite proper time and disappears. In Hawking’s derivation, it turns out that the radiation coming from the hole is insensitive to the details of matter that makes up the hole. If the semiclassical reasoning of Hawking is completely reliable at all times during the evaporation, then this implies an important modification for quantum mechanics applied to general relativity. Namely, in quantum gravity it is possible to have a process where an initial pure state (matter before the collapse) may evolve to a mixed state (thermal radiation).

To avoid such a possibility, one should assume that Hawking radiation can carry out information about the collapsed matter. Since the pairs are created near the horizon, one would suggest to include quantum gravitational effects in describing near horizon physics. However, curvature invariants near the horizon scales as $1/M^2$, where $M$ is the mass of the hole. Thus for a sufficiently heavy black hole the near horizon geometry is locally close to flat space. Moreover, the distance between the collapsed matter concentrated near the singularity and the horizon is proportional to $M$. Thus, this distance can in principle be much greater than the Planck length, which is the scale where quantum gravitational effects become important. Therefore, it seems that the problem should be addressed and solved in a semiclassical context.

Recent progress in string theory strongly suggests that black hole evaporation is a unitary process. However, it is hard (if it is indeed possible) to quantize string theory exactly in a black hole background. In this situation, one can implement a semiclassical approximation near a suitable classical solution. As pointed out above, such an approximation might also offer a solution to the information puzzle. A number of closed string solutions in classical backgrounds have been found and studied before (see for instance [3]-[10]). For example, in [10], the center of mass trajectories were classified in Schwarzschild black hole and semiclassical quantization were studied to the first order in fluctuations. In this letter, we will consider open strings (ending on D-branes) since in string theory black holes can be realized as black p-brane solutions corresponding to D-branes and it is for these objects one can provide a statistical foundation for black hole thermodynamics.

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Another motivation in searching open cosmic string solutions in black hole backgrounds originates from the following considerations. Consider in string theory a large number of coincident D-branes located at \( r = 0 \) in flat space and a single parallel D-brane located at \( r = d \), at weak string coupling \( g_s \). These branes interact via long open strings stretched between them and the low energy dynamics of the system can be described by a suitable gauge theory. There are also closed string excitations propagating in the bulk. Following [12], let us try to understand what happens as we increase \( g_s \). Specifically, we are interested in the fate of the long string stretched between \( r = 0 \) and \( r = d \). As \( g_s \) becomes larger, Newton’s constant \( G \) also increases since in string theory \( G \sim g_s^2 \) (in units where \( \alpha' = 1 \)). Thus the gravitational field produced by D-branes becomes stronger and at some point it would be more appropriate to switch to a curved space description since one can no longer ignore back reaction of the modes on the geometry [11]. For a sufficiently large number of coincident branes, one expects that there would form a black hole (or \( p \)-brane) which has an effective Schwarzschild radius proportional to \( GM \), where \( M \) is the mass of the coincident D-branes. On the other hand, the single D-brane can be treated as an hypersurface located outside the horizon where open stings can end. If we assume that everything goes smoothly as we increase \( g_s \) (which should be the case for an extreme or near-extreme configuration), then in curved space-time we should be able to find open strings stretched between heavy D-branes that make up the hole located inside the horizon and the single D-brane located outside. One natural framework of investigation is to study macroscopic cosmic string solutions.

In [12], an open macroscopic string extending radially from the Schwarzschild black hole horizon was studied. As we will discuss, the string of [12] cannot be interpreted as the open string mentioned above. In this paper, we construct different open cosmic string solutions in black hole and \( p \)-brane backgrounds which can be thought to stretch between two D-branes in curved space-time. Specifically, we also try to determine the fate of the long string extending between the D-branes that make up the hole and a test D-brane located outside the horizon. We find that the string splits into two semi-infinite parts on the horizon. In some sense the horizon acts like a D-brane and the strings can be thought to stretch between the D-branes and the horizon. However, the piece located inside the horizon turns out to be oriented along a time-like direction and gives rise to tachyonic excitations. This shows that this string is unstable and it will decay by emitting radiation. We argue that this radiation can escape from inside the horizon to outside and we interpret it as Hawking radiation. We also discuss possible implications of this picture of black hole radiance on information paradox (for some attempts to solve the information puzzle see, for instance, [13]-[19]).

The organization of the paper is as follows. In section II we construct open cosmic string solutions in Schwarzschild black hole. We discuss their classical properties and study small fluctuations to the linearized order. Fixing all world-sheet reparametrization invariance, one obtains free massless and massive fields corresponding to transverse fluctuations of the string. In section III, we consider non-dilatonic black \( p \)-brane backgrounds and find similar open cosmic string solutions. Unfortunately, for \( p \neq 0 \), we find that there is a generic stability problem for the perturbations along brane directions. In particular, one can no longer identify the long string stretched between D-branes that make up the hole and a test D-brane located outside the horizon. For \( p = 0 \), however, there is no stability problem. We conclude briefly in section IV.

II. OPEN COSMIC STRINGS IN SCHWARZSCHILD BLACK HOLE

Consider an open bosonic string in a vacuum space-time with the metric \( g_{\mu \nu} \). The motion of this string is determined by the Polyakov action

\[
S = \int dr d\sigma \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu g_{\mu\nu},
\]

where \( \sigma \in [\sigma_i, \sigma_f] \). The world-sheet metric \( \gamma^{\alpha\beta} \) is taken to be an independent field. The equations of motion read

\[
T_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x^\nu g_{\mu\nu} - \frac{1}{2} \gamma^{\alpha\beta} \gamma^{ab} \partial_\alpha x^a \partial_\beta x^b g_{\rho\sigma} = 0,
\]

\[
\nabla^2 x^\mu + \Gamma^\mu_{\rho\sigma} \partial_\rho x^a \partial_\sigma x^b \gamma^{\alpha\beta} = 0,
\]

where \( \nabla^2 \) is the world-sheet Laplacian of \( \gamma^{\alpha\beta} \) and \( \Gamma^\mu_{\rho\sigma} \) is the Levi-Civita connection of \( g_{\mu\nu} \). Since we consider open strings, these equations should be supplemented by boundary conditions to cancel unwanted surface terms in the variation of the action. We will discuss them in a moment. Using reparametrization invariance we impose the conformal gauge where

\[
\gamma^{\alpha\beta} = \eta^{\alpha\beta}.
\]
In this gauge, the momentum conjugate to the coordinate $x^\mu$ is equal

$$P_\mu = \int_{\sigma_i}^{\sigma_f} \partial_\tau x^\mu \ g_{\mu\nu} \ d\sigma,$$

and the total physical length of the string is given by

$$L = \int_{\sigma_i}^{\sigma_f} \sqrt{g_{\mu\nu}(\partial_\tau x^\mu)(\partial_\tau x^\nu)} \ d\sigma.$$  \hfill (6)

Note that thanks to the conformal invariance, (4) still allows the residual reparametrizations where $\tau + \sigma \to g(\tau + \sigma)$ and $\tau - \sigma \to h(\tau - \sigma)$. In this paper, we neglect back reaction of the string on the geometry.

We consider 4-dimensional Schwarzschild black hole in the advanced Eddington-Finkelstein coordinates

$$ds^2 = -f(r)dv^2 + 2dvdr + r^2d\Omega_2^2 \ + \ g_{ij}(x)dx^idx^j,$$  \hfill (7)

where $f = (1-2M/r)$. Here, we assume a direct product structure with an internal space parametrized by $x^i$ which has the metric $g_{ij}$ which can be taken to be flat. Our results can easily be generalized to higher dimensional analogs of the Schwarzschild black hole.

We start with the following ansatz

$$v = v(\tau, \sigma), \quad r = r(\tau, \sigma),$$  \hfill (8)

where all other coordinates are set to arbitrary constants. Then (2) implies that

$$\dot{\tau} r' + \dot{\sigma} v' = f \dot{\sigma} v',$$

$$\dot{\tau} r' + \dot{\sigma} v' r' = \frac{1}{2} f v'^2 + \frac{1}{2} f \dot{\tau}^2,$$  \hfill (9)

where the dot and the prime denotes partial differentiation with respect to $\tau$ and $\sigma$, respectively. A simple solution to these constraints is given by

$$v = \tau - \sigma, \quad r = r(\sigma),$$  \hfill (10)

where

$$\frac{dr}{d\sigma} = -f.$$  \hfill (11)

It is easy to verify that, (11) also obeys the field equations (3).

Eq. (12) can be solved to obtain an explicit relation between $r$ and $\sigma$

$$\sigma = -r - 2M \ln |2M - r|,$$  \hfill (13)

where we ignore an additive integration constant. As $r \to 2M$ $\sigma \to +\infty$. Therefore, the strings cannot cross the event horizon and are located either inside or outside. It is possible to view them as one-dimensional semi-infinite throats stretching through the black hole horizon. The string located inside can be extended through the singular region $r = 0$ for finite $\sigma$. For a generic case, they are pictured in figure 1.

It is worth to note that the string inside the horizon is oriented along a time-like line and its motion violates space-time causality. Indeed, from (11) the induced metric on the world-sheet can be found to be $ds^2 = f(\dot{\sigma}^2 + \dot{\tau}^2)$. Inside the horizon $f < 0$ and therefore it seems that the time and the space coordinates are interchanged. Viewing $\sigma$ as the time coordinate alters the interpretation of the motion such that in figure 1 the string inside the horizon should be imagined to be stretched between the two $r = \text{const.}$ lines and is moving upwards. This is pictured in figure 2. The situation is very similar to open-closed string duality where an open string loop can be expressed in terms of a sum over all closed string states by interchanging the roles of $\tau$ and $\sigma$. In this case, one dimensional throat interpretation should be understood along the time direction. This solution can also be obtained by choosing $r = r(\tau)$ in (11).

![FIG. 1: The strings in Schwarzschild black hole located inside and outside the horizon pictured for two different values of $\tau$. The string inside the horizon can be extended through the singular region $r = 0$ for finite $\sigma$.](image-url)

As pointed out earlier, since we are working with open strings, suitable boundary conditions should be imposed on the boundary of the world-sheet. For a string extending through the event horizon $\sigma \in [\sigma_i, +\infty]$ and thus the only boundary is located at $\sigma = \sigma_i$. In the generic case, one should also impose a boundary condition at the maximum value of $\sigma$. Of course we would like to impose a condition that is obeyed by our solution (11). From the
variation of the action one obtains the following surface terms
\[
\delta v (-f \partial_x v + \partial_x r) + \delta r (\partial_x v), \quad (14)
\]
which can be canceled by imposing
\[
\delta r = 0 \quad (\partial_x r - f \partial_x v) = 0 \quad \text{at } \sigma = \sigma_i, \sigma_f. \quad (15)
\]
Other coordinates may obey either Dirichlet \((\delta x = 0)\) or Neumann boundary conditions \((\partial_x x = 0)\).

Note that the first term in \((12)\) is a condition on fluctuations and the second one is satisfied by the string solution \((11)\). The first condition implies that there are two D-branes located at two different values of \(r\), unless \(r\) is not on the horizon (in this case \(\sigma\) has an infinite extend and thus there is no need to impose a boundary condition). Coordinate \(r\) is time-like inside the horizon and in this case it would be more appropriate to name these as S-branes (i.e. branes having spatial world-volumes).

\[
\begin{align*}
\text{FIG. 2: The cosmic string inside the black hole horizon where } \sigma \text{ plays the role of world-sheet time. The string leaves the horizon at } \sigma = -\infty \text{ and reaches out the singularity at some finite } \sigma \text{ forming a semi-infinite throat along time direction.}
\end{align*}
\]

Viewing \(\sigma\) as the time and \(\tau\) as the space coordinates, appropriate boundary conditions should be imposed at \(\tau = \tau_i, \tau_f\). In this case the surface terms coming from the variation of the action equal to \((13)\) in which \(\partial_x\) is replaced by \(\partial_\sigma\). These unwanted terms can be canceled by imposing
\[
\begin{align*}
\partial_\sigma r &= 0 \quad \text{at } \tau = \tau_i, \tau_f. \quad (16)
\end{align*}
\]
The first term in \((13)\) is the Neumann condition on \(r\) and obeyed by our solution \((11)\) and the second one is a Dirichlet condition on the coordinate defined by \(dy = f^{-1} dr - dv\). Inside the horizon \(y\) is a space-like coordinate and thus the second condition implies that there are two time-like D-branes located at two different values of \(y\).

Let us now discuss some of the classical properties of these cosmic strings. From \((13)\) one can calculate the length as
\[
L = \int_i^f f^{-1/2} dr, \quad (17)
\]
which converges at \(r = 2M\). Thus, even though \(\sigma\) diverges on the horizon, the string has a finite proper length. For strings located outside, \((17)\) diverges linearly as \(r \to \infty\) since string stretches out to spatial infinity. Note that inside the horizon \((13)\) gives an imaginary result consistent with the fact that the string is oriented along a time-like line. Ignoring the complex number, \((13)\) measures the proper time-like length which converges at \(r = 0\).

From \((13)\) one can easily calculate non-zero components of momentum vector
\[
\begin{align*}
P_v &= r_f - r_i, \quad (18) \\
P_\tau &= \sigma_f - \sigma_i. \quad (19)
\end{align*}
\]
These are conserved quantities. \(P_v\) is finite for a string stretched between two finite values of \(r\) and \(P_\tau\) diverges for a string extending through the horizon. This suggests that, even though the string has a finite proper length through the horizon, one may need to introduce a cutoff (a D-brane for the string to end) just before the string stretches out the horizon.

On the other hand, inside the event horizon viewing \(\tau\) as the space and \(\sigma\) as the time coordinate, \((13)\) and \((14)\) should be modified by replacing \(\partial_x\) with \(\partial_\sigma\) and \(\sigma\) integral by \(\tau\) integral. In this case the total length of the string becomes
\[
L = \sqrt{-f} (\tau_f - \tau_i). \quad (20)
\]
The proper length of the string vanishes on the horizon. As the string moves through the singularity in the black hole its length increases. The length finally diverges at \(r = 0\). On the other hand the momentum vector can be calculated as
\[
\begin{align*}
P_v &= f (\tau_f - \tau_i), \quad (21) \\
P_\tau &= (\tau_f - \tau_i). \quad (22)
\end{align*}
\]
Although \(P_\tau\) is conserved (i.e. \(\partial_\tau P_\tau = 0\)) \(P_v\) is not a constant of motion. On the horizon \(P_v = 0\) and it diverges when the string hits the singularity at \(r = 0\).

In both pictures the space time mass of the string can be calculated from the center of mass momentum using \(m^2 = -g^{\mu\nu} P_\mu P_\nu\). For the strings
picted in figure 1, this is a conserved quantity, for but for the one pictured in figure 2 it is not a con-
stant of motion.

After determining these classical properties, let us now study linearized fluctuations around the so-
lution (11). This is a saddle point approximation to the full quantum theory defined by the non-
linear sigma model (1). In this way we can also decide if the solutions are stable. We define

\[ v = \tau - \sigma + \delta v, \]  
\[ r = \tilde{r}(\sigma) + \delta r, \]  

where \( \tilde{r} \) is given by (13). It turns out that to the linearized order (2) involves only \( \delta v \) and \( \delta r \), and gives two constraints

\[ \partial_+ \delta v = 0, \]  
\[ \partial_- \delta r - \frac{1}{2} \tilde{f} \partial_- \delta v - (\partial_\tau \tilde{f}) \delta r = 0, \]

where \( \partial_\pm = (\partial_\tau \pm \partial_\sigma) \). On the other hand, \( v \) and \( r \) components of the field equations (3) imply

\[ \partial_+ \partial_- \delta v = 0, \]  
\[ \partial_+ \left[ \partial_- \delta r - \frac{1}{2} \tilde{f} \partial_- \delta v - (\partial_\tau \tilde{f}) \delta r \right] = 0. \]

Therefore, to this order these field equations are obeyed provided that the constraints (25) and (26) are satisfied.

It turns out that fixing residual reparametrisation invariance one can set \( \delta v = \delta r = 0 \). This can be achieved as follows. From (23) we see that \( \delta v \) is a function of \( \tau - \sigma \). On the other hand (24) can be rewritten as

\[ \partial_- \left( \frac{\delta r}{\tilde{f}} - \frac{\delta v}{2} \right) = 0. \]

One can now perform the following infinitesimal reparametrizations

\[ \tau - \sigma \rightarrow \tau - \sigma - \delta v \]  
\[ \tau + \sigma \rightarrow \tau + \sigma + \epsilon(\tau + \sigma) \]

where \( \epsilon = 2 \tilde{f}^{-1} \partial_\tau \delta v - \partial_\sigma \delta r \). Note that (29) implies that \( \epsilon \) is a function of \( \tau + \sigma \). One can easily verify that these transformations set \( \delta v = \delta r = 0 \). This is very similar to the light cone gauge where two longitudinal modes were eliminated using residual reparametrization invariance before quantization. We expect that it is possible to impose \( \delta v = \delta r = 0 \) systematically order by order in perturbation theory.

Following [12], it would be interesting to see how the longitudinal modes decouple in a path integral quantization. As shown by Polyakov in [21], an arbitrary world-sheet metric can be parametrized as

\[ h_{\alpha \beta} = \tilde{h}_{\alpha \beta} \delta \lambda + \nabla_{(\alpha} \eta_{\beta)} \]  

where \( \tilde{h}_{\alpha \beta} \) is the induced metric and \( \nabla \) is the corresponding covariant derivative. Then the functional integration measure over the metrics can be decomposed as

\[ D h_{\alpha \beta} = D \lambda D \eta_{\alpha} \det[L^1 L]^{1/2}, \]

where \( L^1 L \) is the following operator acting on one-forms

\[ (L^1 L) \eta_{\alpha} = (\tilde{h}_{\alpha \beta} \nabla^2 + [\nabla_{\beta}, \nabla_\alpha]) \eta^\beta. \]

The integral over \( \lambda \) decouples since the background obeys \( \beta \)-function equations to first order in \( \alpha' \) (since the black hole is Ricci flat). We now show that the Fadeev-Popov determinant involving the longitudinal modes cancels against the determinant arising from the integration of longitudinal modes.

This can easily be seen by a normal coordinate expansion of the action (1) around the classical solution (11). To quadratic order the terms involving the longitudinal modes can be written as

\[ S = \int d\tau d\sigma \eta_{\alpha \beta} \eta^{\alpha \beta} D_{\alpha} \eta^a D_{\beta} \eta^b \]

\[ + \eta^{\alpha \beta} R_{\mu \nu ab} \eta^a \eta^b \partial_\alpha \eta^\mu \partial_\beta \eta^\nu, \]

where

\[ D_{\alpha} \eta^a = \partial_\alpha \eta^a + w_{\mu}^{ab} (\partial_\alpha \eta^\mu) \eta^b, \]

\( R_{\mu \nu ab} \) and \( w_{\mu}^{ab} \) are the curvature tensor and spin connection of the space-time metric, \( \eta^\mu \) is the solution (11) and \( \eta^a \) represents \( (\delta v, \delta r) \) in normal coordinate form. It would be more convenient to change the coordinates using \( du = dv - 2 dr/\tilde{f} \) to recast \( \eta^\mu \) in a much simpler form. Note that \( u \) is the retarded null coordinate. In \( (u, v) \) coordinates, the solution (11) becomes \( u = \tau + \sigma \) and \( v = \tau - \sigma \). Thus \( (u, v) \) plane can be identified with the \( (\tau, \sigma) \) plane which implies that \( D_{\alpha} = \nabla_{\alpha} \) and the last term in (33) becomes \(-R_{\mu \nu ab} \eta^a \eta^b \) where \( R_{\mu \nu} \) is the Ricci tensor of \( \tilde{h}_{\alpha \beta} \). Integrating by parts, one exactly obtains the operator \( L^1 L \) in (34) contracted by two \( \eta^a \) from left and right. The functional integral over \( \eta^a \) then gives \( \det[L^1 L]^{-1/2} \) exactly canceling the Fadeev-Popov determinant in (33).

This shows that the longitudinal modes decouple and we are left with the transverse fluctuations of the string along internal directions parametrized by \( \delta x \) and the angular directions on
\( \Omega_2 \) parametrized by \( \delta \theta \). In a normal coordinate expansion, (36) gives field equations for these perturbations which read

\[
\partial^\alpha \partial_\alpha \delta \theta - 2M(\bar{r})^{-3} \bar{f} \delta \theta = 0 \quad (37)
\]
\[
\partial^\alpha \partial_\alpha \delta x = 0. \quad (38)
\]

Thus the perturbations along internal directions, \( \delta x \), obey free massless scalar equation on the world-sheet. On the other hand, there is a mass term for angular perturbations, \( \delta \theta \), on \( \Omega_2 \). Not surprisingly (37) is ill defined at \( \bar{r} = 0 \) i.e. when the string hits the singularity.

For the string outside the horizon the “mass squared” term in (37) has the correct sign. Thus small fluctuations around this solution does not run away which shows that the configuration is stable. The mass depends on the radial position of the string which vanish on the horizon and at the spatial infinity at \( r = \infty \). However, contrary to the macroscopic string considered in [13], there is a time translation symmetry on the world-sheet and thus there is a globally well defined notion of a particle. Thus there is no particle creation by the gravitational field and Hawking radiation cannot be realized along this string.

For the string inside the horizon pictured in figure 2 (where the roles of the \( \tau \) and \( \sigma \) are interchanged as world-sheet space and time) the “mass squared” term in (37) has also the correct sign. Thus, this solution is also stable under small perturbations. (Note that in this case \( \bar{f} < 0 \).) However, the mass term depends on world-sheet time \( \sigma \), therefore this is not a static configuration. We expect that there will be particle creation on the world-sheet by gravitational field. One can define the Fock vacuum when the string leaves the horizon at \( \sigma = -\infty \). Since creation and annihilation operators are time dependent, this vacuum state will evolve to some many particle state. However this radiation is confined in the horizon and again black hole radiance cannot be realized along such a string.

For the string inside the horizon pictured in figure 1, the “mass squared” term has the wrong sign which indicates that there are tachyonic excitations. This is a highly unstable configuration and one expects that it will decay to a stable ground state by giving out radiation. However, world-sheet is embedded in space-time so that some of the perturbations move on classically forbidden paths. More precisely, the right moving and left moving massless modes follow \( \tau - \sigma = \text{const.} \) and \( \tau + \sigma = \text{const.} \) lines in the world-sheet, respectively.

By the map (11), these modes follow \( v = \text{const.} \) and \( u = \text{const.} \) lines, where \( u \) is the retarded null coordinate define below (36). The directions for these modes are pictured in figure 3. The left moving modes follow constant \( u \) lines and propagate on causally well defined paths. They hit the singularity at \( r = 0 \) in a finite proper world-sheet time. On the other hand, the right moving modes propagate through the horizon on classically forbidden \( v = \text{const.} \) paths. Since \( \sigma \) diverges on the horizon these modes can never reach the horizon, but they can carry radiation from \( r = 0 \) to an arbitrarily close distance to the horizon on a constant \( v \) line. Assuming small fluctuations of the light cone on the horizon (corresponding to small perturbations of the metric), we expect that these modes can escape out to infinity. Therefore, these strings can carry radiation from inside the horizon to the horizon and that radiation can escape out to infinity in the form of Hawking radiation. The thermal nature of the radiation can be related to the thermodynamics of the massless modes moving on the world-sheet (for instance as discussed in [24]). It is interesting to note that in this scenario we are not dealing with the full quantum gravitational effects. Namely, we still cannot exactly solve the theory defined by (6). Thus the singularity problem at \( r = 0 \) still survives. However, information puzzle can be resolved since the radiation which escape out to infinity can carry information about the collapsed matter at \( r = 0 \). The information transfer is achieved in a semiclassical context by the help of the radiation tunneling through the horizon along classically forbidden paths.

At this point one may simply suggest to discard this cosmic string solution since it gives rise to tachyonic excitations. However, we now argue that such a configuration naturally exists in the formation of a black hole. Let us recall from our discussion in the introduction that at weak string coupling there exist an open string stretched be-
between two D-branes i.e. between $r = 0$ and $r = d$ ($d > 2M$), and as we increase the coupling, the back reaction of the heavy D-branes on the geometry should be included in the description. This can be achieved by switching on the function $f$ in the metric (4). Note that when $f = 0$ we have the flat space. This gives a geometric flow from flat space to the curved black hole background. In this flow the open string stretched between the two D-branes can now be obtained from (11) where $f$ is changing with the string coupling. There would form a horizon when $f$ has a zero. Before this happens, open string is oriented along a space-like direction and there is no violation of causality and no tachyonic excitations. The string extends smoothly from $r = 0$ to $r = d$. As $f$ tends to zero at $r = 2M$, (12) implies that $\sigma$ starts to diverge and the string inside the horizon tends to be stretched more and more through a time-like direction. When $f$ acquires the zero, the string splits into two semi-infinite parts and the one located inside the horizon now finds itself to be oriented along a time-like direction. As discussed above this is not a stable configuration and this string decays by giving out radiation.

In (12) a different mechanism for black hole evaporation along macroscopic strings were proposed where the Hawking radiation was interpreted as the thermal bath of the string modes. For completeness let us also discuss the cosmic strings considered in (12). In Kruskal coordinates the metric of the Schwarzschild black hole becomes

$$ds^2 = F(r) dU dV + r^2 d\Omega_2^2 + dx^i dx^i,$$  \hspace{1cm} (39)

where

$$F(r) = \frac{16M^2}{r} e^{-r/2M}$$  \hspace{1cm} (40)

and $r$ is determined by the relation

$$UV = (r - 2M) e^{r/2M}.$$  \hspace{1cm} (41)

On this background, one can easily verify that there is a cosmic string solution to (2) and (3) which is given by

$$V = \tau + \sigma, \ U = -\tau + \sigma,$$  \hspace{1cm} (42)

where all other coordinates are set to arbitrary constants. In fact, one can find a more general solution $U = g(\tau + \sigma)$ and $V = g(-\tau + \sigma)$, where $g$ is an arbitrary function. However, by a coordinate transformation on the world-sheet, that preserves the conformal gauge (4), one can set $g$ to identity.

In (12), the authors considered an infinitely long open string. However, it is possible to obtain strings having finite width by imposing suitable boundary conditions at $\sigma = \sigma_i$ and $\sigma = \sigma_f$ to cancel unwanted surface terms in the variation. A convenient choice is

$$\delta(U + V) = 0$$
$$\partial_\tau (V - U) = 0$$  \hspace{1cm} (43)

where the other coordinates may obey Dirichlet or Neumann boundary conditions. The first term in (43) is a condition on the fluctuations and the second term is satisfied by the cosmic string solution (12). On the other hand, the first condition implies that there are two D-branes located at $U + V = 2\sigma_i$ and $U + V = 2\sigma_f$.

Note that $t = V - U$ and $x = V + U$ are time-like and space-like coordinates, respectively. Each point on the string follows the non-geodesic time-like curve parametrized by $t$. It starts from the past singularity at $\tau = -\sqrt{\sigma^2 + 2M}$ and reaches out the future one at $\tau = +\sqrt{\sigma^2 + 2M}$. Depending on $\sigma_i$ and $\sigma_f$, the string extends across the past and the future event horizons in the maximally extended black hole background. For a generic case, the motion of the string is pictured in the Penrose diagram in figure 4. For an infinitely long string $(t, x)$ plane can be identified with the world-sheet spanned by $(\tau, \sigma)$ coordinates.

![FIG. 4: The maximally extended black hole and the cosmic string of (4).](image)

From (4), the only non-zero components of the momentum vector become $P_v = -P_u = P$, where $P$ is given by

$$P = \frac{1}{2} \int_{\sigma_i}^{\sigma_f} F d\sigma.$$  \hspace{1cm} (44)

One can easily verify that $\partial_\tau P \neq 0$. $P$ diverges when the string hits the past or the future singularities since $F$ blows up, but otherwise it is finite. In $(t, x)$ coordinates, the momentum vector becomes, $P_t = P$ and $P_x = 0$. Therefore (44) measures the total energy of the string with respect to
t. Since $\partial_t$ is not a Killing vector of (49), it is not surprising that the energy is not conserved. On the other hand, the total length of the string is given by the integral (15) in which $F$ is replaced by $\sqrt{F}$. The length is not a constant of motion and is finite unless the string hits the singularities.

Looking at small perturbations around this classical string background, one can show that it is possible to gauge away the two longitudinal modes using residual reparametrization invariance. Thus, one ends up with the transverse fluctuations of the string obeying

$$\partial^\alpha \partial_\alpha \delta \theta - \vec{r}^{-1} (\partial^\alpha \partial_\alpha \vec{r}) \delta \theta = 0, \quad (45)$$

$$\partial^\alpha \partial_\alpha \delta x = 0, \quad (46)$$

where $\vec{r}$ is the background value of $r$ which can be determined from (13). The “mass squared” term in (13) has the correct sign in the massively extended black hole and thus the classical string is stable under small perturbations. The mass term vanishes on the horizon and at the spatial infinity. Since this is not a static configuration, one expects particle creation effects by gravitational field. Indeed looking at the part of the string that is located outside the horizon it is possible to recover black hole radiance by carefully defining observer dependent “in” and “out” vacua [2]. The relation between this approach and the one proposed in this letter is not clear to us. However, note that (12) cannot represent the string stretched between the D-branes that make up the hole and a test D-brane located outside the horizon since it does not intersect $r = 0$ (where the D-branes inside the horizon are located) for some time during its motion.

### III. OPEN COSMIC STRINGS IN BLACK p-BRANE BACKGROUNDS

In this section, we consider non-dilatonic black p-brane backgrounds of [22]. They can be obtained as magnetically charged solutions of a theory involving only the metric and a $q$-form field $F_q$ which has the following simple action

$$S = \int d^4 x \sqrt{g} (R - F_q^2), \quad (47)$$

where $q = d - p - 2$. The metric can be written as

$$ds^2 = -\Delta_+ \Delta_-^b \, dt^2 + (\Delta_+ \Delta_-)^{-1} \, dr^2$$
$$+ \Delta_-^{b+1} (dy_1 dy_2) + r^2 d\Omega_q^2, \quad (48)$$

where

$$\Delta_\pm = 1 - \left[ \frac{r_\pm}{r} \right]^{q-1}, \quad (49)$$

and $b = (1 - p)/(1 + p)$. The non-extremal solutions have $r_+ > r_-$, where there are regular event horizons at $r = r_+$ and curvature singularities at $r = r_-$. The solutions cannot be extended beyond $r < r_-$ when $b$ is not an integer (i.e. unless $p = 0, 1$). The extreme solutions have $r_+ = r_-$ and $r = r_+$ is a regular event horizon. These are known to be stable when embedded in a suitable supergravity since they preserve some supersymmetry. Thus there is no Hawking radiation on extreme backgrounds. For appropriate values of $d$ and $p$, (18) includes non-dilatonic extended object solutions in string/M theory like the the membrane and the five-brane solutions of $D = 11$ supergravity and the self-dual three-brane of $D = 10$ type IIB supergravity.

It is possible to introduce an advanced null coordinate defined by

$$dv = dt + \Delta_+ \Delta_-^{-(b+1)/2} \, dr$$

so that there is no apparent coordinate singularity on the horizon at $r = r_+$. The metric (13) then becomes

$$ds^2 = -\Delta_+ \Delta_-^b \, dv^2 + 2\Delta_+ \Delta_-^{(b-1)/2} \, dv \, dr$$
$$+ \Delta_-^{b+1} (dy_1 dy_2) + r^2 d\Omega_q^2. \quad (51)$$

In this coordinate system unless $p = 0$ (i.e. $b = 1$) the region $r < r_-$ is not included in space-time. Similarly, in the extreme limit, $r < r_+$ region is only well defined for $b = 1$.

On these backgrounds the string propagation is still described by the action (10) since there is no dilaton and these are magnetically charged solutions. One can then easily verify that there is a solution to (10) and (13) which is given by

$$v = \tau - \sigma, \quad r = r(\sigma), \quad (52)$$

where

$$\frac{dr}{d\sigma} = -\Delta_+ \Delta_-^{(b+1)/2}. \quad (53)$$

It is possible to obtain an explicit relation between $r$ and $\sigma$ by integrating this differential equation. As in the Schwarzschild black hole, $\sigma$ diverges at $r = r_+$ and thus the strings are located either inside or outside the horizon. The string inside the horizon can be extended through $r = r_-$ for finite $\sigma$ when $b \neq 1$. When $b = 1$ (i.e. $p = 0$), $\sigma$ also diverges at $r = r_+$. There are now three different pieces which are located in between $(0, r_-)$, $(r_-, r_+)$ and outside the horizon. The one stretched between $(r_-, r_+)$ has $\sigma \in (-\infty, +\infty)$. 

To be able to determine the orientations of the strings in space-time, the induced metric on the world-sheet can be found to be

\[ ds^2_{\text{ind}} = \Delta_+\Delta_- (d\tau^2 + d\sigma^2). \]  

The strings located outside the horizons are always oriented along space-like directions. In non-extreme backgrounds, when the string is located in between \( (r_-, r_+) \), the world-sheet space and time coordinates are interchanged. This string is oriented along a time-like direction, and one expects that this is an unstable configuration. When \( b = 1 \), the string located in the region \( (0, r_-) \) is oriented along a space-like direction. In the extreme limit, for \( b = 1 \), the string located inside the horizon also oriented along a space-like direction.

In the variation of the action, to cancel the unwanted surface terms one can impose

\[ \delta r = 0, \]  
\[ \partial_\sigma r - \Delta_+^{(b+1)} \Delta_- \partial_\sigma v = 0, \]  
where all other coordinates may obey either Dirichlet or Neumann conditions. The first condition implies that the cosmic string stretches between two D-branes located at different values of \( r \), and the second condition is obeyed by the cosmic string solution (52).

We now show that using residual reparametrization invariance one can set \( \delta v = \delta r = 0 \). To see this, expanding (52) to the linearized order in perturbations, we obtain

\[ \partial_\sigma \delta r \]  
\[ \Delta_+^{-{(b-1)}} \partial_- \delta r - \frac{b+1}{2} \Delta_+^{(b-1)} \Delta_- \Delta_+^\prime \partial_\sigma \delta r \]  
\[ - \Delta_+^{(b)} \Delta_- \partial_\sigma \delta r - \frac{1}{2} \Delta_+^{(b+1)} \Delta_- \partial_\sigma \delta v = 0, \]  
where \( \prime \) denotes differentiation with respect to \( r \) and all functions are evaluated on the background cosmic string solution (52). Eq. (55) implies that \( \delta v \) is a function of \( \tau - \sigma \). One can then show that the following residual reparametrizations set \( \delta v = \delta r = 0; \)

\[ \tau - \sigma \rightarrow \tau - \sigma - \delta v \]  
\[ \tau + \sigma \rightarrow \tau + \sigma + \epsilon (\tau + \sigma), \]  
where \( \epsilon = 2 \Delta_+^{-{(b+1)/2}} \Delta_-^{-1} \partial_\tau - \partial_\sigma \). By (58), we have \( \partial_\sigma \epsilon = 0 \).

Therefore, as in the black hole case, longitudinal modes can be gauged away. On the other hand by a normal coordinate expansion, the transverse fluctuations can be determined to obey

\[ \partial_\tau \partial_\tau \delta \theta - m_\theta^2 \delta \theta = 0, \]  
\[ \partial_\tau \partial_\tau \delta y - m_y^2 \delta y = 0, \]  
where \( \delta \theta \) and \( \delta y \) represent perturbations along the transverse \( q \)-sphere and the brane directions in (51) and the mass squared terms are given by

\[ m_\theta^2 = \frac{\Delta_+ \Delta_-}{r} [\Delta_+ \Delta_-^\prime], \]  
\[ m_y^2 = \frac{(b+1)}{2} \Delta_+ [\Delta_+ \Delta_-^\prime], \]  
where \( \prime \) denotes differentiation with respect \( r \). For the strings located outside the horizon, i.e. when \( r > r_+ \), \( m_\theta^2 > 0 \) and \( m_y^2 \) can be zero, positive or negative depending on \( r \). When \( r_- < r < r_+ \) both \( m_\theta^2 \) and \( m_y^2 \) can be positive or negative. And finally when \( b = 1 \) in the region \( r < r_- \), \( m_\theta^2 > 0 \) and \( m_y^2 \) can be positive, negative or zero.

Therefore, as in the Schwarzschild black hole, the strings located inside the horizons are always unstable. However, the ones located outside are stable for certain values \( r \), i.e. when the D-branes are placed in certain regions. The main instability is related to perturbations along brane directions. Therefore, unlike the Schwarzschild black hole, these strings cannot be interpreted as the long open strings stretched between the D-branes that make up the black \( p \)-brane and a test D-brane located outside the horizon unless \( p = 0 \). It seems one should try different ansatzes (perhaps a non-trivial \( \sigma \) dependence of \( y \) coordinates). We will consider this possibility in a future work. However, one can still insist on realizing Hawking radiation as a result of decaying of an open string stretched between \( r = r_- \) and \( r = r_+ \). As in the black hole case, since this string is oriented along a time-like line by (54), one can show that the right moving massless modes on the world-sheet follow causally forbidden paths, cross the horizon and escape out to infinity. As a consistency check on this interpretation, we note that in the extreme solution for \( b = 1 \) i.e. when \( p = 0 \) (in other extreme cases \( r < r_+ \) region is not included in space-time) the string located inside the horizon is oriented along a space-like direction. Therefore, when the extreme limit is reached then the radiation can no longer escape out to infinity and is confined in the horizon.

IV. CONCLUSIONS

In this paper, we found classical open string solutions in Schwarzschild black hole and non-dilatonic black \( p \)-brane backgrounds. We also studied small fluctuations around these macroscopic configurations. Classical and semiclassical closed string propagation have been studied in the literature before. We extended these considerations to open strings. Working with open strings,
we had to impose boundary conditions and it turns out that suitable conditions exist such that open strings can be thought to stretch between D-branes or between a D-brane and the horizon. While the string solutions located outside the horizons turn out to be stable against small perturbations in the Schwarzschild black hole and black 0-branes, there is a generic stability problem in other black p-brane backgrounds. On the other hand, strings located inside the horizons are always unstable. We argued that such strings decay by giving out radiation and that radiation can escape out to infinity in the form of Hawking radiation.

As discussed in section II, our formulation of black hole radiance might offer a resolution of information paradox. The strings located inside the horizons are embedded in space-time so that the world-sheet times run along spatial directions. In that respect, they are similar to instantons of field theories. Moreover, like instantons, they allow classically forbidden transitions; the strings connect the singular region to the horizon and carry out energy on classically forbidden paths. Note that all these happen inside the horizon and are not visible to an asymptotic observer.

One can try to combine ideas presented in this paper with the others. For instance, in [19], Mathur argued that the bound states of branes have a non-zero size which grows with the number of branes. In this case, non-local effects induced by instantonic strings may operate in a much smaller scale. It would also be interesting to incorporate the idea of the stretched horizon [14] in this picture.

It is possible to extend these considerations in many different directions. For instance, one can try to construct cosmic strings in other black p-brane solutions corresponding to D-branes where the dilaton have non-trivial vacuum expectation values. One can also investigate what would happen when the string couples to other background fields like NS-NS two form potential. In addition, one can consider superstrings to see if they can carry out fermionic information. In all cases, it would be interesting to redrive Hawking’s classical result on black hole radiance.

Finally, let us note that it is possible to construct similar cosmic string solutions in de Sitter space. Indeed in the advanced Eddington-Finkelstein coordinates the de Sitter metric can be written as

\[ ds^2 = f(r) dv^2 - 2dv dr + r^2 d\Omega_2^2 \]  

where \( f = (r^2 - 1) \), which is very similar to the black hole metric [3]. In this coordinate system, the string solution is given by [3], where \( f \) is replaced by \( r^2 - 1 \) (see figure 5). These strings have similar properties with the ones that are propagating in black holes. For instance, they also stretch through the cosmological horizon and it is possible to see that inside the horizon they are located along time-like directions and give rise to tachyonic excitations. However, let us point out that currently it is not known how to embed de Sitter space in string theory. One technical issue related to this problem is that it seems difficult to impose the conformal gauge [9] since it is not possible to satisfy the world-sheet \( \beta \)-function equations. Therefore, our understanding of open cosmic strings propagating in de Sitter space is not complete.

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Assuming that $v = \tau - \sigma$, (9) and (10) give $\dot{r} - r' = f$. A possible way of solving this equation is to assume that $r$ depends on $\sigma$, $\tau$ or $\tau - \sigma$ alone. We choose $r = r(\sigma)$. Choosing $r = r(\tau)$ is equivalent to interchanging the roles of $\tau$ and $\sigma$. On the other hand choosing $r = r(\tau - \sigma)$ gives a collapsed string.