Hu, Jinsong, Yan, Shihao, Shu, Feng, Wang, Jiangzhou, Li, Jun and Zhang, Yijun (2017) Artificial-Noise-Aided Secure Transmission with Directional Modulation based on Random Frequency Diverse Arrays. IEEE Access. ISSN 2169-3536.

Downloaded from
https://kar.kent.ac.uk/59862/ The University of Kent's Academic Repository KAR

The version of record is available from
https://doi.org/10.1109/ACCESS.2017.2653182

This document version
Publisher pdf

DOI for this version

Licence for this version
UNSPECIFIED

Additional information

Versions of research works

Versions of Record
If this version is the version of record, it is the same as the published version available on the publisher's web site. Cite as the published version.

Author Accepted Manuscripts
If this document is identified as the Author Accepted Manuscript it is the version after peer review but before type setting, copy editing or publisher branding. Cite as Surname, Initial. (Year) 'Title of article'. To be published in Title of Journal, Volume and issue numbers [peer-reviewed accepted version]. Available at: DOI or URL (Accessed: date).

Enquiries
If you have questions about this document contact ResearchSupport@kent.ac.uk. Please include the URL of the record in KAR. If you believe that your, or a third party's rights have been compromised through this document please see our Take Down policy (available from https://www.kent.ac.uk/guides/kar-the-kent-academic-repository#policies).
Artificial-Noise-Aided Secure Transmission With Directional Modulation Based on Random Frequency Diverse Arrays

JINSONG HU1, SHIHAO YAN2, (Member, IEEE), FENG SHU1,3,4, (Member, IEEE), JIANGZHOU WANG5, (Fellow, IEEE), JUN LI1, (Senior Member, IEEE), AND YIJIN ZHANG1 (Member, IEEE)

1School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China
2Research School of Engineering, Australian National University, Canberra, ACT 0200, Australia
3National Key Laboratory of Electromagnetic Environment, China Research Institute of Radiowave Propagation, Qingdao 266107, China
4National Mobile Communications Research Laboratory, Southeast University, Nanjing 210018, China
5School of Engineering and Digital Arts, University of Kent, Canterbury CT2 7NT, U.K.

Corresponding author: F. Shu (shufeng@njust.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61472190, Grant 61501238, Grant 61301107, and Grant 61271230, in part by the Open Research Fund of National Key Laboratory of Electromagnetic Environment, China Research Institute of Radiowave Propagation under Grant 201500013, in part by the Open Research Fund of the National Mobile Communications Research Laboratory, Southeast University, China, under Grant 2013D02, in part by the Jiangsu Provincial Science Foundation Project under Grant BK20150786, in part by the Specially Appointed Professor Program in Jiangsu Province, 2015, in part by the Fundamental Research Funds for the Central Universities under Grant 30916011205, and in part by the Australian Research Council’s Discovery Projects under Grant DP150103905.

Abstract
In this paper, a random frequency diverse array-based directional modulation with artificial noise (RFDA-DM-AN) scheme is proposed to enhance physical layer security of wireless communications. Specifically, we first design the RFDA-DM-AN scheme by randomly allocating frequencies to transmit antennas, thereby achieving 2-D (i.e., angle and range) secure transmissions, and outperforming the state-of-the-art 1-D (i.e., angle) phase array (PA)-based DM scheme. Then we derive the closed-form expression of a lower bound on the ergodic secrecy capacity (ESC) of our RFDA-DM-AN scheme. Based on the theoretical lower bound derived, we further optimize the transmission power allocation between the useful signal and artificial noise (AN) in order to improve the ESC. Simulation results show that: 1) our RFDA-DM-AN scheme achieves a higher secrecy capacity than that of the PA-based DM scheme; 2) the lower bound derived is shown to approach the ESC as the number of transmit antennas N increases and precisely matches the ESC when N is sufficiently large; and 3) the proposed optimum power allocation achieves the highest ESC of all power allocations schemes in the RFDA-DM-AN.

Index Terms
Physical layer security, directional modulation, frequency diverse array, power allocation.

I. INTRODUCTION
As a promising physical layer security technique, directional modulation (DM) has attracted extensive studies due to its unique characteristic. This characteristic is that DM projects modulated signals into a predetermined spatial direction while simultaneously distorting the constellation of these signals in all other directions. This can significantly decrease the probability of these signals being eavesdropped on. As such, the DM technique is an ideal candidate to achieve physical layer security [1]–[9]. In general, there are two main types of methods to implement the DM technique in wireless communications. The first one is to adopt DM on the radio frequency (RF) frontend (e.g., [10]–[13]). For example, [10], [11] obtained the phase and amplitude of DM signal at the predefined direction through varying the effective length and scattering property of a reflector. A similar approach was proposed in [12] and [13], where the phase of each antenna element was shifted accordingly in order to construct the DM signal. However, the flexibility of implementing DM on the RF frontend is limited, which leads to high complexity in the design of constellation diagram for DM. Against this background, the second method was developed in the literature (e.g., [14]), which implemented the DM technique on the baseband instead of on the RF frontend.
Specifically, an approach to applying the DM technique on the baseband based on an orthogonal vector was proposed in [14]. In addition, [15] and [16] provided a robust baseband DM algorithm by considering estimation errors on the direction angles. Compared with the design in the RF frontend, it is more efficient to realize DM technique on the baseband by utilizing beamforming operation and adding the artificial noise, thereby enabling dynamic DM transmissions to send the different patterns of a constellation point at different time slots. Therefore, implementing DM on baseband can make eavesdroppers hard to track and decode useful signals; thus, can further improve physical layer security.

The DM technique can be achieved by phase array (PA) [12]–[14]. Considering security, previous studies on the DM technique only investigated the system where a legitimate user locates at the desired direction and an eavesdropper locates in another direction (that is different from the desired direction). However, it is common to assume that the location information of the eavesdropper is not available at the transmitter in the context of physical layer security. The eavesdropper may be passive and never transmit signals, thus means it is hard to obtain such location information. Considering a realistic scenario, an eavesdropper may exactly locate in the desired direction as the legitimate user. In this scenario, the aforementioned DM based on PA can no longer guarantee the secure transmission for the legitimate user. This is due to the fact that the DM based on PA can only distort signals at the directions that are different from the desired one.

A linear frequency diverse array (LFDA) in [17]–[19] creates new possibilities for DM to guarantee a secure transmission in the aforementioned scenario where the legitimate user and eavesdropper locate in the same direction (but different ranges). This is due to the fact that LFDA can produce a beam-pattern with controllable direction and range, by linearly shifting the carrier frequencies across different transmit antennas. However, as discussed in [18] and [19], the direction and range achieved by LFDA are coupled. This means that there may exist multiple direction-range pairs at which the eavesdropper can receive identical signals as the legitimate user, which compromises the secure transmission. Recently, [20] and [21] developed a new type of frequency diverse array, namely the random frequency diverse array (RFDA), of which each transmit antenna is randomly (instead of linearly) allocated a narrow band frequency or subchannel frequency [22]–[24]. As shown in [20] and [21], RFDA owns one property that it can decouple the correlation between the direction and range (this correlation exists in LFDA and cannot be decoupled). This property enables RFDA to be a good candidate for DM to achieve a robust secure transmission (i.e., physical layer security).

In [3], the authors discussed two main metrics, i.e., ergodic secrecy capacity (ESC) and secrecy outage probability, which are often adopted to measure the performance of secure transmissions over fading channels. ESC applies for delay tolerant systems which allows for the adoption of an ergodic version of fading channels. On the other hand, secrecy outage probability, which measures systems with probabilistic formulations, is more appropriate for scenario under stringent delay constraints. In our work, the instantaneous value of the secrecy capacity at the eavesdropper is not available due to the strategies of randomly allocating frequencies to the transmit antennas in the proposed scheme. Averaging over all the realizations of the frequencies allocation, we can capture the ergodic features of the secrecy capacity. The concept of ESC bears the similar significance to the one adopted in this paper.

In this work, for the first time, we utilize the DM with artificial noise based on RFDA (referred to as the RFDA-DM-AN scheme) to enhance physical layer security of wireless communications. Due to the fact that RFDA can decouple the correlation between the range and angle, the proposed RFDA-DM-AN scheme can significantly outperform the PA-DM-AN and LFDA-DM-AN schemes in terms of secrecy capacity. In this scheme, in addition to maximizing the signal-to-noise ratio (SNR) of useful signals at the desired direction, the transmitter also sends artificial noise (AN) in all other directions to cause interferences to the eavesdropper. In order to fully examine the secrecy performance of the RFDA-DM-AN scheme, we first derive a lower bound on its ESC. Based on this lower bound, we can determine the optimal transmit power allocation between the useful signal and AN more efficiently relative to using the ESC. As shown in the simulation, this lower bound is in agreement with the ESC when the number of transmit antennas is sufficiently large, which confirms the validity and effectiveness of using this lower bound to perform transmit power allocation. Moreover, the proposed optimum power allocation achieves the highest ESC compared with other power allocations in the RFDA-DM-AN. In addition, we investigate two strategies of randomly allocating frequencies to the transmit antennas in the RFDA (i.e., frequency allocations based on the continuous and discrete uniform distributions). The simulation results demonstrate that the continuous uniform frequency allocation outperforms the discrete one in terms of average ESC.

The remainder of this paper is organized as follows. In Section II, we detail our system model for the RFDA-DM-AN scheme. Then the secrecy performance of the RFDA-DM-AN scheme is analyzed in Section III, based on which the transmit power and frequency allocations are examined. The secrecy performance of the proposed scheme is numerically evaluated in Section IV, and Section V draws conclusions.

Notations: Scalar variables are denoted by italic symbols. Vectors and matrices are denoted by lower-case and upper-case boldface symbols, respectively. Given a complex number, $| \cdot |$ and $(\cdot)^*$ denote the modulus and conjugation, respectively. Given a complex vector or matrix, $(\cdot)^H$, tr$(\cdot)$, and $\| \cdot \|$ denote the transpose, conjugate transpose, trace, and norm, respectively. The $N \times N$ identity matrix is referred to as $\mathbb{I}_N$ and $\mathbb{E}[(\cdot)]$ denotes expectation operation.
where $\theta_n$ is the angle from the receiver to the $n$-th element relative to the reference element. The phase of the transmit signal at the $n$-th element is denoted as $\psi_n(\theta, R)$. In this work, we consider a uniform linear array (ULA) at the transmitter and the element spacing of the ULA is given by $d = \frac{\lambda}{N}$. As such, the phase shift defined in (6) can be approximated by

$$
\psi_n(\theta, R) \approx \frac{2\pi}{c} (-b_n \frac{f_c d \cos \theta}{c} + k_n \Delta f R - b_n k_n \Delta f \frac{d \cos \theta}{c}).
$$

Then, the normalized steering vector of RFDA to a specific location $(\theta, R)$ is given by

$$
h(\theta, R) = \frac{1}{\sqrt{N}} \left[ e^{j\Psi_0(\theta, R)}, e^{j\Psi_1(\theta, R)}, \ldots, e^{j\Psi_{N-1}(\theta, R)} \right]^T.
$$

B. DIRECTIONAL MODULATION WITH ARTIFICIAL NOISE

Since DM is a transmitter-side technology, this work considers a multiple-input single-output (MISO) wiretap channel as shown in Fig. 2. In this wiretap channel, the transmitter (Alice) is equipped with $N$ antennas, the legitimate user (Bob) is equipped with a single antenna, and the eavesdropper (Eve) is equipped with a single antenna. We assume that the location of Bob, denoted by $(\theta_B, R_B)$, is available at Alice, while the location of Eve, denoted by $(\theta_E, R_E)$, is unavailable at Alice (which potentially exists in anywhere). In addition, free space channel model has been widely adopted in the literature for the DM technique (e.g., [14]–[16]). Without loss of generality, we normalize the channel gain to be one.
Beamforming with AN has been widely used in the context of physical layer security due to its robustness and desirable secrecy performance [25], [26]. Therefore, for the first time, we adopt the AN-aided secure transmission in the DM technique based on RFDA. Considering beamforming with AN, the transmitted signal can be expressed as

$$s = \sqrt{\alpha P_s} x + \sqrt{1-\alpha} P_s w,$$  \hspace{1cm} (9)

where $x$ is a symbol chosen from the complex signal constellation with average power constraint (i.e., $\mathbb{E}[|x|^2] = 1$). $P_s$ is the transmit power of Alice and $\alpha$ is the parameter that determines the power allocation between the useful signal and AN. In addition, $v$ is the beamforming vector for the useful signal. Since Alice does not know Eve’s location, in order to maximize the SNR at Bob, $v$ is given by

$$v = h(\theta_B, R_B),$$  \hspace{1cm} (10)

where $h(\theta_B, R_B)$ is the steering vector of the RFDA at Alice to Bob, which can be obtained by replacing $(\theta, R)$ with $(\theta_B, R_B)$ in (8). Furthermore, the artificial noise vector $w$ in (9) should lie in the null space of $h(\theta_B, R_B)$ (i.e., $h(\theta_B, R_B)w = 0$) in order to avoid interference to Bob. As such, $w$ can be expressed as [15]

$$w = \frac{(I_N - h(\theta_B, R_B)h^H(\theta_B, R_B))z}{\| (I_N - h(\theta_B, R_B)h^H(\theta_B, R_B))z \|},$$  \hspace{1cm} (11)

where $z$ consists of $N$ i.i.d. circularly-symmetric complex Gaussian random variables with zero-mean and unit-variance, i.e., $z \sim CN(0, I_N)$.

Following (9), the received signal at Bob is given by

$$y(\theta_B, R_B) = h^H(\theta_B, R_B)s + n_B$$

$$= \sqrt{\alpha P_s} h^H(\theta_B, R_B)x + n_B$$

$$= \sqrt{\alpha P_s} x + n_B,$$  \hspace{1cm} (12)

where $n_B$ is the additive white Gaussian noise (AWGN), distributed as $n_B \sim \mathcal{CN}(0, \sigma_B^2)$. As shown in (12), Bob can restore the original signal $x$ from Alice easily without knowing the random mapping rule. To be fair, we assume that Eve cannot obtain this random mapping rule. Following (12), the SNR at Bob is given by

$$\gamma_B = \frac{\alpha P_s}{\sigma_B^2} = \alpha \mu_B,$$  \hspace{1cm} (13)

where $\mu_B = P_s/\sigma_B^2$.

Likewise, the received signal at Eve can be expressed as

$$y(\theta_E, R_E) = h^H(\theta_E, R_E)s + n_E$$

$$= \sqrt{\alpha P_s} h^H(\theta_E, R_E)x$$

$$+ \sqrt{(1-\alpha)P_s} h^H(\theta_E, R_E)w + n_E,$$  \hspace{1cm} (14)

where $n_E$ is the AWGN with the distribution $n_E \sim \mathcal{CN}(0, \sigma_E^2)$ and $h(\theta_E, R_E)$ is the steering vector of the RFDA at Alice to Eve, which can be obtained by replacing $(\theta, R)$ with $(\theta_E, R_E)$ in (8).

As per (14), the item $\sqrt{(1-\alpha)P_s} h^H(\theta_E, R_E)w$ in (14) distorts the amplitude and phase of the signals at Eve. In addition, the item $h^H(\theta_E, R_E)w$ is nonzero since $h^H(\theta_E, R_E)$ is not orthogonal with $w$. This further distorts the constellation of $x$ at Eve.

Following (14), the signal-to-interference-plus-noise ratio (SINR) at Eve is given by

$$\gamma_E = \frac{\alpha P_s |h^H(\theta_E, R_E)h(\theta_B, R_B)|^2}{(1-\alpha)P_s |h^H(\theta_E, R_E)w|^2 + \sigma_E^2}$$

$$= \frac{\alpha \mu_B |h^H(\theta_E, R_E)h(\theta_B, R_B)|^2}{(1-\alpha)\mu_B |h^H(\theta_E, R_E)w|^2 + \beta},$$  \hspace{1cm} (15)

where

$$\beta \triangleq \frac{\sigma_E^2}{\sigma_B^2}.$$  \hspace{1cm} (16)

### III. SECRECY PERFORMANCE OF THE RFDA-DM-AN SCHEME

In this section, we analyze the secrecy performance of the RFDA-DM-AN scheme. Specifically, we first determine its ESC and then derive a lower bound on this ESC. Based on this lower bound, we determine the optimal power allocation between the useful signal and AN. Then, two strategies of randomly allocating frequencies to the transmit antennas are studied.

#### A. ERGODIC SECRECY CAPACITY

In the context of physical layer security, the secrecy capacity is defined as $\{0, C_B - C_E\}^+$, where $C_B$ is the capacity at Bob,
which is given by
\[ C_B = \log_2(1 + \gamma_B) \]  
(17)
and \( C_E \) is the capacity at Eve, which is given by
\[ C_E = \log_2(1 + \gamma_E) \]  
(18)
In the considered system model without path loss \( C_B \geq C_E \) can be guaranteed, since \( \gamma_B \) is the maximized by Alice.

The ESC is commonly used for the fading channel with statistical channel state information at the transmitter. In general, the ESC is defined as the instantaneous secrecy capacity averaged over \( \gamma_B \) and/or \( \gamma_E \). From (13), \( \gamma_B \) does not depend on the frequency allocation at the RFDA (i.e., the values of \( k_n \)). However, in (15) \( \gamma_E \) is a function of \( k_n \), since both \( h(\theta_B, R_B) \) and \( h(\theta_E, R_E) \) are functions of \( k_n \). As the distribution of \( k_n \) is available at Alice (the transmitter), we adopt the ESC, which is obtained by averaging the secrecy capacity over \( \gamma_E \), as the main performance metric to evaluate the secrecy performance of different schemes. We would like to mention that the randomness in \( \gamma_E \) is caused by the random frequency allocation instead of the fading in our work. Accordingly, this ESC is given by
\[ C = \mathbb{E}[C_B - C_E] = C_B - \mathbb{E}[C_E]. \]  
(19)
Note that this ESC \( C \) is dependent on a specific location of Eve. However, as assumed in this work, Alice does not know Eve’s location. As such, we define \( \overline{C} \) as the average value of \( C \) over all possible locations of Eve, which is determined by the region where Eve potentially exists. For example, the location of Eve can be assumed at an annular region centered on the location of Bob, which is similar to the annulus threat model mentioned in [27]. The average value of \( C \) can be calculated through
\[ \overline{C} = \int_{R_E \in R} \int_{\theta_E \in \Theta} C(\theta_E, R_E) d\theta_E dR_E, \]  
(20)
where \( f(\theta_E, R_E) \) is the joint probability density function (pdf) of \( \theta_E \) and \( R_E \) in the sets \( \Theta \) and \( R \), respectively. Then, the optimal value of the power allocation parameter \( \alpha \) that maximizes \( \overline{C} \) can be obtained through
\[ \alpha^* = \arg \max_{0 \leq \alpha \leq 1} \overline{C}. \]  
(21)
In order to efficiently determine \( \alpha^* \), we have to derive a closed-form expression for \( \overline{C} \). However, due to the high complexity of \( \gamma_E \), as shown in (15), the closed-form expression for \( \overline{C} \) is mathematically intractable (not to mention the closed-form expression for \( C_E \)). In order to facilitate the power allocation, we derive a lower bound on the ESC \( C \) in the following subsection.

**B. A LOWER BOUND ON THE ERGODIC SECRECY CAPACITY**

A lower bound on the ESC \( C \) is derived in the following theorem in order to facilitate the transmit power allocation between the useful signal and AN at Alice.

**Theorem 1:** The lower bound on the ESC of the RFDA-DM-AN scheme is
\[ C_{LB} = \log_2 \left( \frac{\alpha^2 \mu_B^2 + \alpha \mu_B (\beta F + \mu_B - 1) + \beta F + \mu_B}{\alpha \mu_B (F - \frac{1}{\eta} - 1) + \beta F + \mu_B} \right), \]  
(22)
where
\[ F \triangleq \frac{N^2}{\eta (N^2 - N(1 - \Phi^2(2\pi p)) + S_N^2(q)\Phi^2(2\pi p))}, \]  
(23)
\[ q \triangleq \frac{f(\theta_E - \cos \theta_B)}{N}, \]  
(24)
\[ p \triangleq \frac{\Delta f(R_E - R_B)}{N}, \]  
(25)
\[ S_N(x) \triangleq \frac{\sin(N\pi x)}{\sin(x)}, \]  
(26)
\[ \eta \triangleq \frac{1}{tr} \left\{ \left[ I_N - \mathbf{h}(\theta_B, R_B)\mathbf{h}^H(\theta_B, R_B) \right] \right\}, \]  
(27)
and \( \Phi(\cdot) \) is the moment generating function (MGF) of \( k_n \).

**Proof:** The cross correlation coefficient between \( \mathbf{h}(\theta_E, R_E) \) and \( \mathbf{h}(\theta_B, R_B) \) is
\[ \mathbf{h}^H(\theta_E, R_E)\mathbf{h}(\theta_B, R_B). \]  
(29)
The mean of \( \rho(q, p, k_n) \) over \( k_n \) is derived as
\[ \mathbb{E}_{k_n}[|\rho(q, p, k_n)|^2] = \mathbb{E}_{k_n}[\rho^*(q, p, k_n)\rho(q, p, k_n)] \]
\[ = \frac{1}{N^2} \mathbb{E}_{k_n} \left\{ \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{2\pi i [b_nq - k_n]p} e^{2\pi i [b_nq - k_n']p} \right\} \]
\[ = \frac{1}{N^2} \mathbb{E}_{k_n} \left\{ \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{-2\pi i [b_nq - k_n]p} e^{2\pi i [b_nq - k_n']p} \right\} + \frac{1}{N^2} \]
\[ \cdot \mathbb{E}_{k_n, k_{n'}} \left\{ \sum_{n=0, n \neq n'}^{N-1} \sum_{n'=0, n \neq n'}^{N-1} e^{-2\pi i [b_nq - k_n]p} e^{2\pi i [b_nq - k_n']p} \right\} \]
\[ = \frac{N}{N^2} + \frac{1}{N^2} \left\{ \int_{k_n \in \mathcal{K}} g(k_n) e^{2\pi i k_n p} dk_n \int_{k_{n'} \in \mathcal{K}} g(k_{n'}) \right\}
\[ \cdot e^{-2\pi i k_{n'} p} dk_{n'} \}
\[ \cdot e^{2\pi i b_nq} \sum_{n=0, n \neq n'}^{N-1} e^{-2\pi i b_nq} e^{2\pi i b_{n'}q} \}
\[ = \frac{1}{N} + \frac{1}{N^2} \Phi^2(2\pi p) \left( \frac{\sin^2(N\pi q)}{\sin^2(\pi q)} - N \right) \]
\[ = \frac{1}{N^2} \left[ N(1 - \Phi^2(2\pi p)) + S_N^2(q)\Phi^2(2\pi p) \right], \]  
(30)
where \( g(k_n) \) is the pdf of \( k_n \) in the set \( \mathcal{K} \).
Next, we can derive the lower bound of the ESC $C$ by using the Jensen’s inequality, i.e., $\log_2 \mathbb{E}[x] \geq \mathbb{E}[\log_2 x]$. Then, using (15) we have

$$C = C_B - \mathbb{E}[C_E] \geq \log_2 (1 + \alpha \mu_B) - \log_2 \left(1 + \frac{\alpha \mu_B \mathbb{E} \left[|h^H(\theta_E, R_E)h(\theta_B, R_B)|^2\right]}{(1 - \alpha) \mu_B \mathbb{E} \left[|h^H(\theta_E, R_E)w|^2 + \beta\right]} \right) $$

where $\frac{\alpha}{1 - \alpha} \mu_B$ is achieved by

$$\mathbb{E} \left[|h^H(\theta_E, R_E)w|^2\right] = \mathbb{E} \left[|h^H(\theta_E, R_E)w|^2\right] = \mathbb{E}_{\theta_E} \left[|h^H(\theta_E, R_E)P(\theta_B, R_B)\mathbb{E}[zz^H]|\right] = \mathbb{E}_{\theta_E} \left[|h^H(\theta_E, R_E)P(\theta_B, R_B)\mathbb{E}[zz^H]|\right] \geq 1 - \mathbb{E}_{\theta_E} \left[|h^H(\theta_E, R_E)h(\theta_B, R_B)|^2\right] $$

where $P(\theta_B, R_B) \triangleq I_N - h(\theta_B, R_B)h^H(\theta_B, R_B)$. Note that $z \sim CN(0, I_N)$, and $\frac{b}{n}$ is obtained based on $\mathbb{E}[zz^H] = I_N$.

From (31), after some algebraic manipulations we obtain the lower bound as given by (22), which completes the proof of this theorem. □

We note that the results provided in Theorem 1 is valid for arbitrary values of $N$. Due to the distance concentration phenomenon [28, 29], we know that $|\rho(q, p, k_n)|^2$ approaches its mean $\mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right]$ when $N \to \infty$, i.e., $|h^H(\theta_E, R_E)h(\theta_B, R_B)|^2$ in (15) approaches its mean $\mathbb{E}_{\theta_E} \left[|h^H(\theta_E, R_E)h(\theta_B, R_B)|^2\right]$ when $N \to \infty$. As such, we can conclude that the lower bound approaches the ESC when $N \to \infty$. Therefore, we next determine the expression of the ESC when $N \to \infty$ in the following corollary.

Corollary 1: As $N \to \infty$, the asymptotic ESC of the RFDA-DM-AN scheme is

$$C_\infty = \log_2 \left(\frac{-\alpha \mu_B \beta F_\infty + \mu_B - 1}{\alpha \mu_B F_\infty - \frac{1}{\eta} - 1 + \beta F_\infty + \mu_B} \right),$$

where

$$F_\infty \triangleq \frac{N^2}{\eta (N^2 - S^2_N(q) \Phi^2(2\pi p))}.$$  \hspace{1cm} (34)

Proof: The value of $\mathbb{E}_{k_n} \left[|\rho(q, p, k_n)|^2\right]$ can be expressed as

$$\mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] = \mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] + \mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] = 0.$$

As $N \to \infty$, we will have $\mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] \to 0$ due to the distance concentration phenomenon [28, 29]. As such, from (35) we have

$$\mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] = \mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] = \mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] + \mathbb{E}_{\theta_E} \left[|\rho(q, p, k_n)|^2\right] = 0,$$

Then, substituting (36) into (31) and performing some algebraic manipulations, we can obtain the asymptotic ESC as given in (33). This completes the proof of Corollary 1. ■

Similar to (20), we can determine the average value of $C_{LB}$ over all possible locations of Eve as

$$\overline{C}_{LB} = \int_{R_B \in \mathcal{R}} \int_{\theta_B \in \Theta} f(\theta_B, R_B) C_{LB} d\theta_B dR_B.$$  \hspace{1cm} (37)

Then, the optimal value of $\alpha$ that maximizes $\overline{C}_{LB}$ can be obtained through

$$\alpha_{LB}^* = \arg \max_{0 \leq \alpha \leq 1} \overline{C}_{LB}.$$  \hspace{1cm} (38)

C. CONTINUOUS AND DISCRETE UNIFORM FREQUENCY ALLOCATIONS

We note that the lower bound derived in Theorem 1 is valid for any MGF of $k_n$, i.e., for any random frequency allocation method. In this paper, we consider the continuous uniform and discrete uniform frequency allocations, in which $k_n$ follows a continuous uniform distribution and a discrete uniform distribution, respectively. The MGF of a continuous uniform random variable $t$ is given by

$$\Phi(t) = \frac{e^{at} - e^{bt}}{t(a - b)}.$$  \hspace{1cm} (39)

where $t \in [a, b]$. Therefore, when $k_n$ is a continuous uniform random variable within $[-M, M]$, its MGF is given by

$$\Phi(2\pi p) = \frac{e^{-\frac{M}{2}} - e^{-\frac{M}{2}}}{\sin(M \pi p)},$$  \hspace{1cm} (40)
where $M$ is determined by the total available frequency bandwith for the antenna array at Alice.

The MGF of a discrete uniform random variable $t$ is given by

\[
\Phi(t) = \frac{e^{it} - e^{(b+1)t}}{K(1 - e^t)},
\]

where $K$ is the number of all possible values of $t$ subject to $t \in [a, b]$. As such, when $k_n$ is within a discrete uniform set \([-M-\frac{1}{2}, -M+\frac{1}{2}, \ldots, M-\frac{1}{2}\}$, its MGF is given by

\[
\Phi(j2\pi p) = \frac{e^{-\frac{M-\frac{1}{2}}{2}\pi p} - e^{(\frac{M-\frac{1}{2}}{2}+1)2\pi p}}{M(1 - e^{2\pi p})} = \frac{\sin(M\pi p)}{M\sin(\pi p)}.
\]

By substituting (40) and (42) into Theorem 1, we can obtain the lower bound on the secrecy capacity $C$ for the continuous uniform frequency allocation and discrete uniform frequency allocation, respectively. Accordingly, we can obtain the secrecy performance of these two frequency allocations, which will be evaluated in the following section.

**IV. NUMERICAL RESULTS**

In this section, we numerically evaluate the secrecy performance of the RFDA-DM-AN scheme with the PA-DM-AN and LFDA-DM-AN schemes as benchmarks. Without other notes, our system settings used in this section are as follows. The carrier frequency $f_c$ is set to 1 GHz (i.e., $f_c = 1$ GHz), the frequency increment is set to 3 MHz (i.e., $\Delta f = 3$ MHz), the element spacing is half of the wavelength (i.e., $d = c/2f_c$), the location of Bob is set at (45°, 120 m), and $\beta = 1$.

**FIGURE 3.** The ergodic secrecy capacity of the RFDA-DM-AN scheme and secrecy capacities of the PA-DM-AN and LFDA-DM-AN schemes versus $\mu_B$, where $N = 32$, Eve’s location is (45°, 239 m), and $\alpha = 0.5$.

Fig. 3 plots the ESC of the RFDA-DM-AN scheme and secrecy capacities of the PA-DM-AN and LFDA-DM-AN schemes versus $\mu_B$ for a specific location of Eve. Note that this Eve’s location is only for the performance evaluation, which is unknown to Alice. As expected, it can be seen that the secrecy capacity of the PA-DM-AN scheme is zero since Eve is in the same direction as Bob relative to Alice. In addition, the secrecy capacity of the LFDA-DM-AN
scheme is much lower than the ESC of the RFDA-DM-AN scheme, especially when \( \mu_B \) is large. This indicates that our proposed RFDA-DM-AN scheme can significantly outperform both the PA-DM-AN and LFDA-DM-AN schemes. It can be seen that Eve may select the locations (not the same as Bob’s location) that guarantee a zero secrecy capacity in the PA-DM-AN and LFDA-DM-AN schemes, since Eve may know Alice’s location in practice. However, there are no such locations that Eve can select to ensure a zero ESC in the RFDA-DM-AN scheme. This is due to the fact that RFDA can decouple the correlation between the range and angle in DM, which is detailed in the following figure.

Fig. 4 plots the absolute value of the correlation coefficient between \( \mathbf{h}(\theta_E, R_E) \) and \( \mathbf{h}(\theta_B, R_B) \) in the PA-DM-AN, LFDA-DM-AN, and RFDA-DM-AN schemes in order to further explain the observations seen in Fig. 3. It can be seen from Fig. 4 (a), the maximum values (i.e., yellow areas) of this coefficient \( |\mathbf{h}(\theta_E, R_E)\mathbf{h}(\theta_B, R_B)| \) appear in the direction of Bob, which means that if Eve exists along the desired direction, the secrecy capacity is zero (i.e., the received signals at Bob and Eve are identical). This explains why the secrecy capacity of the PA-DM-AN scheme is zero in Fig. 3. As shown in Fig. 4 (b), the maximum values of \( |\mathbf{h}(\theta_E, R_E)\mathbf{h}(\theta_B, R_B)| \) appear periodically around Bob’s location, which demonstrates that the range and angle are coupled in this scheme. The periodical peak values indicate that the LFDA-DM-AN scheme may not achieve positive secrecy capacity even when Eve is not at the same location. In Fig. 4 (c), we observe that the unique maximum value of \( |\mathbf{h}(\theta_E, R_E)\mathbf{h}(\theta_B, R_B)| \) only occurs at the location of Bob, which means that a positive ESC can be achieved as long as Eve is not at the location of Bob. In practice, if Eve locates at the same location as Bob, Bob can inform Alice about this information in order to avoid Eve’s attacks. As such, the aforementioned observations intuitively demonstrate the advantages of the RFDA-DM-AN scheme.

Fig. 5 illustrates the average value of the ESC, i.e., \( \overline{C} \), and the average value of the lower bound on the ESC, i.e., \( \overline{C}_{LB} \), versus \( \alpha \). For this figure, the potential location of Eve is uniformly distributed at \([0^\circ, 44^\circ] \cup [46^\circ, 180^\circ]\) in angle and \([0 \text{ m}, 119 \text{ m}] \cup [121 \text{ m}, 250 \text{ m}]\) in range. From Fig. 5, we first observe that the gap between \( \overline{C} \) and \( \overline{C}_{LB} \) decreases as the number of antennas at Alice (i.e., \( N \)) increases. When \( N \) is sufficiently large (e.g., \( N = 256 \)), we can see \( \overline{C}_{LB} \) precisely matches \( \overline{C} \), which can be explained by our Corollary 1. When \( N \) is not very large (e.g., \( N = 16 \)), we can see that the optimal value of \( \alpha \) determined based on \( \overline{C}_{LB} \) is still close to that determined based on \( \overline{C} \). This demonstrates the validity of using \( \overline{C}_{LB} \) as an approximation of \( \overline{C} \) to determine the transmit power allocation between the useful signal and AN at Alice. The optimal \( \alpha \) takes 0.62 (0.68), 0.76 (0.79), and 1 (1) for \( N = 8, 16, \) and 256, respectively, thus means that the optimal power allocation factor \( \alpha \) increases as the number of antennas \( N \) increases and approaches one when \( N \) is very large. This demonstrates that Alice does not have to transmit AN when \( N \) is sufficiently large, which is due to the fact that Alice can construct an ultra-narrow beam towards Bob when \( N \) is large enough to avoid information leakage to Eve. In Fig. 6, we plot the exact and asymptotic ESCs versus different values of \( N \). In this figure, we can observe that the exact ESC approaches the asymptotic one as \( N \) increases, which confirms our Corollary 1.

Fig. 7 shows the secrecy performance of the RFDA-DM-AN scheme with continuous and discrete uniform frequency allocations. As seen from Fig. 7, the continuous uniform frequency allocation outperforms the discrete one in terms of average ESC. The average ESC increases as \( \mu_B \) increases, which indicates that Alice can enhance physical layer security through increasing her transmit power. Finally, we observe that the optimal value of \( \alpha \) that maximizes the average ESC increases as \( \mu_B \) decreases. This indicates that Alice allocates a larger fraction of her transmit power to the useful signal as her transmit power decreases,
and she allocates all her transmit power to the useful signal (i.e., $\alpha = 1$) when her transmit power is sufficiently low as shown in Fig. 7.

V. CONCLUSION

In this paper, a novel DM scheme based on random frequency diverse arrays with artificial noise is proposed to enhance physical layer security of wireless communications. By randomly allocating frequencies to transmit antennas, the proposed RFDA-DM-AN achieves secure two-dimensional (i.e., angle and range) transmission. A lower bound on the ESC of the proposed method is derived. Using this lower bound, the transmit power is efficiently allocated between the useful signal and AN. Also, we derived an asymptotic ESC when $N$ approaches infinity, which is precisely consistent with our derived lower bound when $N$ is sufficiently large. Simulation results show that: 1) the proposed RFDA-DM-AN scheme can significantly outperform the PA-DM-AN and LFDA-DM-AN schemes in terms of secrecy capacity, 2) the proposed optimum power allocation achieves the highest ESC among all power allocations schemes in the RFDA-DM-AN, 3) the optimal power allocation factor $\alpha$ increases as the number of antennas $N$ increases given a fixed transmit power, 4) it is feasible to use $C_{DL}$ as an approximation of $C$ to allocate the transmit power between the useful signal and AN at Alice, 5) the continuous uniform frequency allocation can achieve a higher average ESC compared to the discrete one. The RFDA-DM-AN scheme proposed in this paper could be applied to several future practical application scenarios including satellite communications, unmanned aerial vehicle communications, millimeter wave communications and so on.

REFERENCES

[1] N. Yang, L. Wang, G. Geraci, M. Elkashlan, J. Yuan, and M. Di Renzo, “Safeguarding 5G wireless communication networks using physical layer security,” IEEE Commun. Mag., vol. 53, no. 4, pp. 20–27, Apr. 2015.

[2] W. Trappe, “The challenges facing physical layer security,” IEEE Commun. Mag., vol. 53, no. 6, pp. 16–20, Jun. 2015.

[3] B. He, X. Zhou, and T. D. Abhayapala, “Wireless physical layer security with imperfect channel state information: A survey,” ZTE Commun., vol. 11, no. 3, pp. 11–19, Sep. 2013.

[4] B. He, X. Zhou, and A. L. Swindlehurst, “On secrecy metrics for physical layer security over quasi-static fading channels,” IEEE Trans. Wireless Commun., vol. 15, no. 10, pp. 6913–6924, Oct. 2016.

[5] S. Yan, N. Yang, R. Malaney, and J. Yuan, “Transmit antenna selection with alamouti coding and power allocation in MIMO wiretap channels,” IEEE Trans. Wireless Commun., vol. 13, no. 3, pp. 1656–1667, Mar. 2014.

[6] Y. L. Zou, J. Zhu, X. Wang, and V. Leung, “Improving physical-layer security in wireless communications through diversity techniques,” IEEE Netw., vol. 29, no. 1, pp. 42–48, Jan. 2015.

[7] S. Yan and R. Malaney, “Location-based beamforming for enhancing secrecy in rician wiretap channels,” IEEE Trans. Wireless Commun., vol. 15, no. 4, pp. 2780–2791, Apr. 2016.

[8] X. Chen, D. W. K. Ng, and H. Chen, “Secrecy wireless information and power transfer: Challenges and opportunities,” IEEE Wireless Commun. Netw., vol. 23, no. 2, pp. 54–61, Apr. 2016.

[9] N. Zhao, F. R. Yu, M. Li, and V. C. M. Leung, “Anti-eavesdropping schemes for interference alignment (IA)-based wireless networks,” IEEE Trans. Wireless Commun., vol. 15, no. 8, pp. 5719–5732, Aug. 2016.

[10] A. Babakhani, D. Rutledge, and A. Hajimiri, “Transmitter architectures based on nearfield direct antenna modulation,” IEEE J. Solid-State Circuits, vol. 43, no. 12, pp. 2674–2692, Dec. 2008.

[11] A. Babakhani, D. B. Rutledge, and A. Hajimiri, “Near-field direct antenna modulation,” IEEE Microw. Mag., vol. 10, no. 1, pp. 36–46, Feb. 2009.

[12] M. P. Daly and J. T. Bernhard, “Directional modulation technique for phased arrays,” IEEE Trans. Antennas Propag., vol. 57, no. 9, pp. 2633–2640, Sep. 2009.

[13] M. P. Daly, E. L. Daly, and J. T. Bernhard, “Demonstration of directional modulation using a phased array,” IEEE Trans. Antennas Propag., vol. 58, no. 5, pp. 1545–1550, May 2010.

[14] Y. Ding and V. F. Fusco, “A vector approach for the analysis and synthesis of directional modulation transmitters,” IEEE Trans. Antennas Propag., vol. 62, no. 1, pp. 361–370, Jan. 2014.

[15] J. Hu, F. Shu, and J. Li, “Robust synthesis method for secure directional modulation with imperfect direction angle,” IEEE Commun. Lett., vol. 20, no. 6, pp. 1084–1087, Jun. 2016.

[16] F. Shu, X. Wu, J. Li, R. Chen, and B. Vuetic, “Robust synthesis scheme for secure multi-beam directional modulation in broadcasting systems,” IEEE Wireless Commun., vol. 14, no. 5, pp. 6614–6623, Oct. 2016.

[17] P. Antonik, “An investigation of a frequency diverse array,” Ph.D. dissertation, Dept. Electron. Electr. Eng., Univ. College London, London, U.K., 2009.

[18] P. F. Sammartino, C. J. Baker, and H. D. Griffiths, “Frequency diverse MIMO techniques for radars,” IEEE Trans. Aerosp. Electron. Syst., vol. 49, no. 1, pp. 201–222, Jan. 2013.

[19] W. Q. Wang, “Frequency diverse array antenna: New opportunities,” IEEE Antennas Propag. Mag., vol. 57, no. 2, pp. 145–152, Apr. 2015.

[20] Y. Liu, “Range azimuth indication using a random frequency diverse array,” in Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP), Mar. 2016, pp. 3111–3115.

[21] Y. Liu, H. Rui, L. Wang, and A. Nehorai, “The random frequency diverse array: A new antenna structure for uncoupled direction-range indication in active sensing,” IEEE J. Sel. Topics Signal Process., to be published, doi: 10.1109/JSTSP.2016.2627183.

[22] H. Zhu and J. Wang, “Chunk-based resource allocation in OFDMA systems—Part I: Chunk allocation,” IEEE Trans. Commun., vol. 57, no. 9, pp. 2734–2744, Sep. 2009.

[23] H. Zhu and J. Wang, “Chunk-based resource allocation in OFDMA systems—Part II: Joint chunk, power and bit allocation,” IEEE Trans. Commun., vol. 60, no. 2, pp. 499–509, Feb. 2012.

[24] H. Zhu, “Radio resource allocation for OFDMA systems in high speed environments,” IEEE Jour. Sel. Area Commun., vol. 30, no. 4, pp. 748–759, May 2012.

[25] S. Goel and R. Negi, “Guaranteeing secrecy using artificial noise,” IEEE Trans. Wireless Commun., vol. 7, no. 6, pp. 2180–2189, Jun. 2008.

[26] N. Yang, S. Yan, J. Yuan, R. Malaney, R. Subramanian, and I. Land, “Artificial noise: Transmission optimization in multi-input single-output wiretap channels,” IEEE Trans. Commun., vol. 63, no. 5, pp. 1771–1783, May 2015.
J. Hu et al.: AN-Aided Secure Transmission With DM Based on RFDA

[27] S. Yan, N. Yang, G. Geraci, R. Malaney, and J. Yuan, “Optimization of code rates in SISOME wiretap channels,” IEEE Trans. Wireless Commun., vol. 14, no. 11, pp. 6377–6388, Nov. 2015.

[28] D. Francois, V. Wertz, and M. Verleysen, “The concentration of fractional distances,” IEEE Trans. Knowl. Data Eng., vol. 19, no. 7, pp. 873–886, Jul. 2007.

JINSONG HU received the B.S. degree from the Nanjing University of Science and Technology, Nanjing, China, in 2013. He is currently pursuing the Ph.D. degree with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing, China. His research interests include wireless communications, array signal processing, and physical layer security.

SHIHAO YAN (S’11–M’15) received the B.S. degree in communication engineering and the M.S. degree in communication and information systems from Shandong University, Jinan, China, in 2009 and 2012, respectively, and the Ph.D. degree in electrical engineering from the University of New South Wales, Sydney, NSW, Australia, in 2015. He was a Visiting Ph.D. Student with the University of South Australia, Adelaide, SA, Australia, in 2014. He is currently a Post-Doctoral Research fellow with the Research School of Engineering, Australian National University, Canberra, ACT, Australia. His research interests are in wireless communications and statistical signal processing, including physical layer security, location verification, and covert communication.

FENG SHU (M’07) received the B.S. degree from the Fuyang Teaching College, Fuyang, China, in 1994, M.S. degree from XiDian University, Xi’an, China, in 1997, and the Ph.D. degree from the Southeast University, Nanjing, in 2002. From 2009 to 2010, he was a held Visiting Post-Doctor position with the University of Texas at Dallas. In 2005, he joined the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing, China, where he is currently a Professor and a Supervisor of the Ph.D. and Graduate students. He is also with Fujian Agriculture and Forestry University and awarded with Mingjian Scholar Chair Professor in Fujian Province. His research interests include wireless networks, wireless location, and array signal processing. He has authored about 200 papers, of which over 80 are in archival journals including over 20 papers on IEEE Journals and over 40 SCI-indexed papers. He holds four Chinese patents.

JIANGZHOU WANG (M’91–SM’94–F’17) is currently a Professor of Telecommunications and the Head of the School of Engineering and Digital Arts, University of Kent, U.K. He has authored over 200 papers in the international journals and conferences in the areas of wireless mobile communications and has written/editied three books. His research interests include wireless multiple access techniques, massive MIMO and small-cell technologies, device to device communications in cellular networks, distributed antenna systems, and cooperative communications. He is a Fellow of the IET. He was an IEEE Distinguished Lecturer from 2013 to 2014. He serves/served as an Editor or Guest Editor of a number of international journals, such as the IEEE Transactions on Communications and the IEEE Journal on Selected Areas in Communications. He was the Technical Program Chair of the IEEE WCNC2013 in Shanghai and the Executive Chair of the IEEE ICC2015 in London. He was a recipient of the Best Paper Award from the IEEE Globecom2012.

JUN LI (M’09–SM’16) received the Ph.D. degree in electronic engineering from Shanghai Jiao Tong University, Shanghai, China, in 2009. From 2009 to 2009, he was with the Department of Research and Innovation, Alcatel Lucent Shanghai Bell, as a Research Scientist. Since 2015, he is with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing, China. His research interests include network information theory, channel coding theory, wireless network coding, and cooperative communications.

YIJIN ZHANG (M’14) received the B.Eng. degree from the Nanjing University of Posts and Telecommunications, China, in 2004, the M.S. degree from Southeast University, China, in 2007, and the Ph.D. degree from the Chinese University of Hong Kong in 2010, all in information engineering. He is currently an Associate Professor with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology. His research interests include sequence design and resource allocation in communication networks.

***