Design of rectangular sandwich panels with metal skins based on post-buckling state in compression and shear

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Abstract. Solutions of geometrically non-linear tasks concerning post-buckling behavior of sandwich panels with thin metal skins and lightweight filler loaded by compression and shear flows are shown. Based on obtained solutions methods of aircraft structures sandwich panels design based on post-buckling state with acceptable global buckling in case of ultimate loading are suggested.

1. Introduction
Sandwich panels (SP) bending stiffness is significant, they are widely used for aircraft structures. SP usually consist of thin load-bearing skins and lightweight filler layer. Buckling is not allowed with loads under limit level in SP design. But buckling is acceptable with loads close to ultimate level. Moreover, tasks concerning SP load-bearing capacity are of interest for experimental determination of aircraft structures actual safety margins when testing them up to failure. It should be noted that SP best possible design tasks are actual for minimal weight aircraft structures development. With that both types of panels can be considered: panels of main load-bearing structures and of secondary elements such as fairings of different purpose.

Let us consider further tasks of SP post-buckling behavior with loads above limit level under compression and tangential flows. Then methods for SP parameters definition with acceptable post-buckling behavior for ultimate loading and limit stress \( \sigma \) are presented. It should be noted that limit stress can be understood to be either yield stress \( \sigma = \sigma_y \) or ultimate stress limit \( \sigma = \sigma_u \) depending on structural peculiarities of the panel when designing aircraft structures. It should be noted that in this investigation only geometrical non-linearity of the task but not its physical non-linearity will be taken into account. In order to take into account influence of loose aggregate basic Reissner correlations [1] are used further on. Method suggested in investigation [2] will be used as basic one for design based on post-buckling state.

2. Main correlations
Let us put down main geometrically non-linear correlations for SP [1] of the following form

\[
\frac{1}{E\delta} \nabla^2 \nabla^2 F = \left( \frac{\partial^2 W}{\partial \alpha \partial \gamma} \right)^2 - \frac{\partial^2 W}{\partial \alpha^2} \frac{\partial^2 W}{\partial \gamma^2},
\]
\[ D \nabla^2 \nabla^2 W = \left( 1 - \frac{D}{hG_c} \right) L(W, F). \]

With functional
\[
L(W, F) = \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 W}{\partial x \partial y}.
\]

\[
\nabla^2 \nabla^2 \theta = \frac{\partial^4 \theta}{\partial x^4} + 2 \frac{\partial^4 \theta}{\partial x^2 \partial y^2} + \frac{\partial^4 \theta}{\partial y^4}.
\]

SP bending stiffness being \( D = \frac{Eh^2}{2\left(1 - \mu^2\right)} \), \( F \) being stress function, \( t \) being SP single skin thickness \((\delta = 2t)\), \( h \) being distance between skins middle surfaces, \( G_c \) being filler shear modulus.

3. Sandwich panels design for compression

Let us examine rectangular panel with geometrical parameters \((a, b)\) and hinge support. Let us further on assume due to structural and technological reasons that known value is filler height and it is necessary to define skin thickness.

Panel flexure is of the following form:
\[
W = f \sin \alpha \sin \beta y,
\]
with \( \alpha = \frac{\pi m}{a}, \beta = \frac{\pi n}{b}; m, n \) being half waves quantity.

After substitution of flexure (2) into equation of strain compatibility (1) Airy stress function can be obtained:
\[
F = f^2 \left[ A_1 \cos 2\alpha x + A_2 \cos 2\beta y \right] + \frac{T_{xy} y^2}{2}
\]
with \( A_1 = \frac{E\delta}{32} \frac{\beta^2}{a^2}, A_2 = \frac{E\delta}{32} \frac{\alpha^2}{\beta^2} \).

Equation for Bubnov-Galerkin method application procedure is written in the following form:
\[
\iint_{00} \left[ D \left( \frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} \right) - \left( 1 - \frac{D}{hG_c} \right) L(W, F) \right] \sin \alpha \sin \beta y = 0.
\]

As a result of substituting flexure (2) and stress function (3) into equation (4) following non-linear equation is derived
\[
D \frac{\pi^4}{16a^3b^3} \left( a^2 n^2 + b^2 m^2 \right)^2 + \frac{\pi^4 f^2}{2ab} m^2 n^2 \left( A_1 + A_2 \right) + \frac{D}{hG_c} \frac{\pi^6 f^2 m^2 n^2}{2a^3b^3} A_{mn} = T_{xy} \frac{\pi^2 m^2 b^2}{4a}
\]
with \( A_{mn} = A_1 a^2 n^2 + 5A_2 b^2 m^2 + 5A_2 a^2 n^2 + A_2 b^2 m^2 \).

Longitudinal membrane stress due to buckling are written based on stress function definition
\[ \sigma_x = \frac{1}{2} \frac{\partial^2 F}{\partial y^2} = -\frac{4\pi^2 A_2 f^2 n^2}{\delta b^2} \cos \frac{2\pi ny}{b} \frac{T_x}{\delta}. \] (6)

Let us examine further on SP design method assuming that normal stress reaches limits \( \sigma_u \) with ultimate compression load. Flexure amplitude is derived from the last equation

\[ f = \sqrt{\left( \sigma_u - \frac{T_x}{\delta} \right) \frac{\delta b^2}{4\pi^2 n^2 A_2}}. \] (7)

It should be noted that half waves quantity \( m \) and \( n \) being part of equations (5)–(7) are defined when solving linear stability task for equation (5) with provision \( f^2 \rightarrow 0 \) and minimizing in above mentioned parameters according to known procedures [3] and defining critical values \( m_{cr} \) and \( n_{cr} \). After substituting these values and flexure (7) into equation (5) following non-linear equation in skin thickness is derived:

\[ \frac{E\delta h^2}{4(1 - \mu^2)} \pi^4 \left( a^2 n^2 + b^2 m^2 \right)^2 \left( \sigma_u - \frac{T_x}{\delta} \right) \frac{\delta b^2}{4\pi^2 n^2 A_2} + \frac{E\delta h^2}{4(1 - \mu^2)} 1 \pi^6 m^2 n^2 \]

\[ \times A_{mn} \right] - \tau_x^{ext} \frac{\pi^2 m^2 b}{4a} = 0. \]

Which is correlated to given ultimate compression force \( T_x^{ult} \).

4. Sandwich panel design for shear

Let us examine similar SP design task based on post-buckling state with acting tangential force \( S_{xy} \). For rectangular SP flexure is traditionally represented as

\[ W = f \sin \frac{\pi y}{b} \sin \frac{\pi (x - \alpha y)}{s}, \] (9)

with \( \alpha \) being waves angle, \( s \) being distance between waves for buckling.

After substituting flexure (9) into equation of strain compatibility (1) stress function can be derived as

\[ F = f^2 \left[ A_1 \cos \frac{2\pi y}{b} + A_2 \cos \frac{2(x - \alpha y)}{s} + \frac{T_x y^2}{2} + S_{xy} y \right] \] (10)

with \( A_1 = \frac{E\delta b^2}{32 s^2}, \) \( A_2 = \frac{E\delta}{32 b^2} \left( \frac{s^2}{\alpha^2 + 1} \right)^2 \).

Here is also an equation using Bubnov-Galerkin method procedure for considered case

\[ \int_0^b \left[ D \left( \frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} \right) - \left( 1 - \frac{D}{hG_c} \nabla^2 \right) L(W, F) \right] \sin \frac{\pi y}{b} \sin \frac{2\pi (x - \alpha y)}{s} = 0. \] (11)

When substituting flexure (9) and stress function (10) into equation (11) following non-linear equation for tangential force \( S_{xy} \) case is obtained:
\[ D \frac{\pi^4}{4s^3b^3} B_{S1} + f^2 \left[ \frac{\pi^4 f^2 (A_1 + A_2)}{2bs} + \frac{D \pi^6}{hGc2s^5b^3} B_{S2} \right] = S_{xy} \frac{\pi^2 b}{4s}, \]  

with \( B_{S1} = \alpha^4 b^4 + 2\alpha^2 b^4 + 6\alpha^2 b^2 s^2 + b^4 + 2b^2 s^2 + s^4 \), 
\( B_{S2} = A_4 s^4 + 2A_2 \alpha^4 b^4 - 2A_4 \alpha^4 b^4 + A_2 b^2 s^2 + 5A_2 s^2 b^2 + 2A_2 \alpha^2 b^2 s^2 \).

Membrane stress due to buckling is written in general terms as

\[
\sigma_x = \frac{1}{\delta} \frac{\partial^2 F}{\partial y^2} = -\frac{4\pi^2 f^2}{\delta b^2 s^2} \left[ A_1 s^2 \cos \frac{2\pi ny}{b} + A_2 \alpha^2 b^2 \cos \frac{2\pi (x-\alpha y)}{s} \right] \frac{T_x}{\delta},
\]

\[
\tau_{xy} = \frac{1}{\delta} \frac{\partial^2 F}{\partial \alpha \partial y} = \frac{4\pi^2 \alpha f^2}{s^2} \cos \frac{2\pi (x-\alpha y)}{s} + \frac{S_{xy}}{\delta}.
\]

It should be noted that critical parameters of wave generation are defined when considering linear stability task using provision \( f^2 \to 0 \) for equation (12).

Let us further on assume that at the moment of SP buckling skin stress reaches limits \( \tau_u \). Then following equation for flexure amplitude is derived from the last equation:

\[
f = \sqrt{\frac{\tau_u - \frac{S_{xy}}{\delta}}{4\pi^2 A_2 \alpha \cos \frac{2\pi (x-\alpha y)}{s}}}.
\]

Substituting it into equation (12) following non-linear equation in skin thickness with given load and filler height can be derived:

\[
\frac{Eth^2}{2\left(1-\mu^2\right)} \frac{\pi^4}{4s^3b^3} B_{S1} + \left( \frac{\tau_u - \frac{S_{xy}}{\delta}}{4\pi^2 A_2 \alpha} \right) \frac{s^2}{2bs} \left[ \frac{\pi^4 f^2 (A_1 + A_2)}{2bs} + \frac{Eth^2}{2\left(1-\mu^2\right)} \frac{\pi^6}{hGc2s^5b^3} B_{S2} \right] = S_{xy} \frac{\pi^2 b}{4s}.
\]

At conclusion a remark concerning above mentioned methods possible modification should be stated. In considered cases of design based on post-buckling state skin thicknesses \( \delta_{\text{post-buck}} \) were defined for ultimate level. Provided that SP buckling is unacceptable with limit loading linear stability tasks should be considered and thicknesses \( \delta_{\text{fa}} \) should be calculated and further on maximum thickness value should be selected based on more strict design requirement.

5. Conclusion

Shown methods of sandwich panels design based on post-buckling state with compression and shear can be used for aircraft structures design with limitations in stability and load-bearing capacity. Given height and filler characteristics are taken into account in basic correlations.

References

[1] Volmir A 1956 Flexible Plates And Shells (Moscow: Gostekhizdat)
[2] Mitrofanov O 2003 Engineer Composite Load-Bearing Panels Design Methods (Moscow: Sputnik+)
[3] Endogur A, Vainberg M and Ierusalimskiy K 1989 Honeycomb Structures. Parameters Selection and Design (Moscow: Mashinostroenie)