Pomeron in the $\mathcal{N} = 4$ SYM at strong couplings

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Abstract. We show the result for the BFKL Pomeron intercept at $\mathcal{N} = 4$ Supersymmetric Yang-Mills model in the form of the inverse coupling expansion $j_0 = 2 - 2\lambda^{-1/2} - \lambda^{-1} + 1/4\lambda^{-3/2} + 2(1 + 3\zeta_3)\lambda^{-2} + O(\lambda^{-5/2})$, which has been calculated recently in [1] with the use of the AdS/CFT correspondence.

1. Introduction

The investigation of the high energy behavior of scattering amplitudes in the $\mathcal{N} = 4$ Supersymmetric Yang-Mills (SYM) model [2]-[?] is important for our understanding of the Regge processes in QCD. Indeed, this conformal model can be considered as a simplified version of QCD, in which the next-to-leading order (NLO) corrections [8, 9] to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [10]-[14] are comparatively simple and numerically small. In the $\mathcal{N} = 4$ SYM the equations for composite states of several reggeized gluons and for anomalous dimensions (AD) of quasi-partonic operators turn out to be integrable at the leading logarithmic approximation [15, 16, 17]. Further, the eigenvalue of the BFKL kernel for this model has the remarkable property of the maximal transcendentality [3]. This property gave a possibility to calculate the AD $\gamma$ of the twist-2 Wilson operators in one [18, 19], two [3, 20], three [21], four [22, 23] and five [24] loops using the QCD results [24] and the asymptotic Bethe ansatz [26] improved with wrapping corrections [23] in an agreement with the BFKL predictions [2, 3].

On the other hand, due to the AdS/CFT-correspondence [28, 29, 30], in $\mathcal{N} = 4$ SYM some physical quantities can be also computed at large couplings. In particular, for AD of the large spin operators Beisert, Eden and Staudacher constructed the integral equation [31] with the use the asymptotic Bethe-ansatz. This equation reproduced the known results at small coupling constants and is in a full agreement (see [32, 33, 34]) with large coupling predictions [35, 36, 37].

With the use of the BFKL equation in a diffusion approximation [2, 5], strong coupling results for AD [35, 36, 37] and the pomeron-graviton duality [38, 39] the Pomeron intercept was calculated at the leading order in the inverse coupling constant (see the Erratum [40] to the paper [21]). Similar results were obtained also in Ref. [41]. The Pomeron-graviton duality in the $\mathcal{N} = 4$ SYM gives a possibility to construct the Pomeron interaction model as a generally covariant effective theory for the reggeized gravitons [42].

Below we present the strong coupling corrections to the Pomeron intercept $j_0 = 2 - \Delta$ in next orders. These corrections were obtained in Ref. [4] with the use of the recent calculations [43]-[47] of string energies.
2. BFKL equation at small coupling constant

The eigenvalue of the BFKL equation in $\mathcal{N} = 4$ SYM model has the following perturbative expansion \[2, 3\] (see also Ref. \[5\])

$$ j - 1 = \omega = \frac{\lambda}{4\pi^2} \left[ \chi(\gamma_{BFKL}) + \delta(\gamma_{BFKL}) \frac{\lambda}{16\pi^2} \right], \quad \lambda = g^2 N_c, $$

where $\lambda$ is the t'Hooft coupling constant. The quantities $\chi$ and $\delta$ are functions of the conformal weights $m$ and $\tilde{m}$ of the principal series of unitary Möbius group representations, but for the conformal spin $n = m - \tilde{m} = 0$ they depend only on the BFKL anomalous dimension

$$ \gamma_{BFKL} = \frac{m + \tilde{m}}{2} = \frac{1}{2} + i\nu $$

and are presented below \[2, 3\]

$$ \chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1 - \gamma), $$

$$ \delta(\gamma) = \Psi''(\gamma) + \Psi''(1 - \gamma) + 6\zeta_3 - 2\zeta_2 \chi(\gamma) - 2\Phi(\gamma) - 2\Phi(1 - \gamma). $$

Here $\Psi(z)$ and $\Psi'(z)$, $\Psi''(z)$ are the Euler $\Psi$-function and its derivatives. The function $\Phi(\gamma)$ is defined as follows

$$ \Phi(\gamma) = 2 \sum_{k=0}^{\infty} \frac{1}{k + \gamma} \beta'(k + 1), \quad \beta'(z) = \frac{1}{4} \left[ \Psi'(\frac{z}{2}) - \Psi'(\frac{z}{2}) \right]. $$

Due to the symmetry of $\omega$ to the substitution $\gamma_{BFKL} \rightarrow 1 - \gamma_{BFKL}$ expression (1) is an even function of $\nu$

$$ \omega = \omega_0 + \sum_{m=1}^{\infty} (-1)^m D_m \nu^{2m}, $$

where

$$ \omega_0 = 4 \ln 2 \frac{\lambda}{4\pi^2} \left[ 1 - \frac{\lambda}{16\pi^2} \right] + O(\lambda^3), $$

$$ D_m = 2 \left( 2^{2m+1} - 1 \right) \zeta_{2m+1} \frac{\lambda}{4\pi^2} + \frac{\delta(2m)(1/2)}{(2m)!} \frac{\lambda^2}{64\pi^4} + O(\lambda^3). $$

According to Ref. \[3\] we have

$$ \tau_1 = 2\zeta_2 + \frac{1}{2\ln 2} \left( 11\zeta_3 - 32L_3 \frac{\pi}{2} - 14\pi \zeta_2 \right) \approx 7.5812, \quad L_3(x) = -\int_0^x \ln^2 \left[ 2\sin \left( \frac{y}{2} \right) \right] dy. $$

Due to the Möbius invariance and hermicity of the BFKL hamiltonian in $\mathcal{N} = 4$ SYM expansion \(6\) is valid also at large coupling constants. In the framework of the AdS/CFT correspondence the BFKL Pomeron is equivalent to the reggeized graviton \[39\]. In particular, in the strong coupling regime $\lambda \rightarrow \infty$

$$ j_0 = 2 - \Delta, $$

where the leading contribution $\Delta = 2/\sqrt{\lambda}$ was calculated in Refs. \[40, 41\]. Below we find NLO terms in the strong coupling expansion of the Pomeron intercept.
3. AdS/CFT correspondence

Due to the energy-momentum conservation, the universal AD of the stress tensor $T_{\mu\nu}$ should be zero, i.e.,

$$\gamma(j = 2) = 0.$$  \hspace{1cm} (11)

It is important, that the AD $\gamma$ contributing to the DGLAP equation \[48\]-\[52\] does not coincide with $\gamma_{BFKL}$ appearing in the BFKL equation. They are related as follows \[8, 53, 54\]

$$\gamma = \gamma_{BFKL} + \frac{\omega}{2} = \frac{j}{2} + i\nu,$$  \hspace{1cm} (12)

where the additional contribution $\omega/2$ is responsible in particular for the cancelation of the singular terms $\sim 1/\gamma^3$ obtained from the NLO corrections \[11\] to the eigenvalue of the BFKL kernel \[8\]. Using above relations one obtains

$$\nu(j = 2) = i.$$  \hspace{1cm} (13)

As a result, from eq. \[6\] for the Pomeron trajectory we derive the following representation for the correction $\Delta$ \[11\] to the graviton spin 2

$$\Delta = \sum_{m=1}^{\infty} D_m.$$  \hspace{1cm} (14)

According to \[10\] and \[14\], we have the following small-$\nu$ expansion for the eigenvalue of the BFKL kernel

$$j - 2 = \sum_{m=1}^{\infty} D_m \left((-\nu^2)^m - 1\right),$$  \hspace{1cm} (15)

where $\nu^2$ is related to $\gamma$ according to eq. \[12\]

$$\nu^2 = -\left(\frac{j}{2} - \gamma\right)^2.$$  \hspace{1cm} (16)

On the other hand, due to the ADS/CFT correspondence the string energies $E$ in dimensionless units are related to the AD $\gamma$ of the twist-two operators as follows \[29, 30\]

$$E^2 = (j + \Gamma)^2 - 4, \quad \Gamma = -2\gamma,$$  \hspace{1cm} (17)

and therefore we can obtain from \[16\] the relation between the parameter $\nu$ for the principal series of unitary representations of the Möbius group and the string energy $E$

$$\nu^2 = -\left(\frac{E^2}{4} + 1\right).$$  \hspace{1cm} (18)

This expression for $\nu^2$ can be inserted in the r.h.s. of Eq. \[15\] leading to the following expression for the Regge trajectory of the graviton in the anti-de-Sitter space

$$j - 2 = \sum_{m=1}^{\infty} D_m \left[\left(\frac{E^2}{4} + 1\right)^m - 1\right].$$  \hspace{1cm} (19)

\[1\] Note that our expression \[17\] for the string energy $E$ differs from a definition, in which $E$ is equal to the scaling dimension $\Delta_{sc}$. But eq. \[17\] is correct, because it can be presented as $E^2 = (\Delta_{sc} - 2)^2 - 4$ and coincides with Eqs. (45) and (3.44) from Refs. \[29\] and \[30\], respectively.
4. Graviton Regge trajectory and Pomeron intercept

We assume, that eq. (19) is valid also at large $j$ and large $\lambda$ in the region $1 \ll j \ll \sqrt{\lambda}$, where the strong coupling calculations of energies were performed \[43, 47\]. These energies can be presented in the form

$$\frac{E^2}{4} = \sqrt{\lambda} \frac{S}{2} \left[ h_0(\lambda) + h_1(\lambda) \frac{S}{\sqrt{\lambda}} + h_2(\lambda) \frac{S^2}{\lambda} \right] + O\left(S^{7/2}\right),$$

(20)

where

$$h_i(\lambda) = a_{i0} + \frac{a_{i1}}{\sqrt{\lambda}} + \frac{a_{i2}}{\lambda} + \frac{a_{i3}}{\sqrt{\lambda^3}} + \frac{a_{i4}}{\lambda^2}.$$

(21)

The contribution $\sim \sqrt{S}$ can be extracted directly from the Basso result \[44, 45\] taking $J_{an} = 2$ according to \[46\]:

$$h_0(\lambda) = \frac{I_3(\sqrt{\lambda})}{I_2(\sqrt{\lambda})} + \frac{2}{\sqrt{\lambda}} = \frac{I_1(\sqrt{\lambda})}{I_2(\sqrt{\lambda})} - \frac{2}{\sqrt{\lambda}},$$

(22)

where $I_k(\sqrt{\lambda})$ is the modified Bessel functions. It leads to the following values of coefficients $a_{0i}$

$$a_{00} = 1, \quad a_{01} = -\frac{1}{2}, \quad a_{02} = a_{03} = \frac{15}{8}, \quad a_{04} = \frac{135}{128}.$$ 

(23)

The coefficients $a_{10}$ and $a_{20}$ come from considerations of the classical part of the folded spinning string corresponding to the twist-two operators (see, for example, \[47\])

$$a_{10} = \frac{3}{4}, \quad a_{20} = -\frac{3}{16}. $$

(24)

The one-loop coefficient $a_{11}$ is found recently in the paper \[46\], considering different asymptotical regimes with taking into account the Basso result \[44\] ($\zeta_3$ is the Euler $\zeta$-function)

$$a_{11} = \frac{3}{16}(1 - \zeta_3).$$

(25)

Comparing the l.h.s. and r.h.s. of (19) at large $j$ values gives us the coefficients $D_m$ and $\Delta$ (see Appendix A in \[1\]).

5. Conclusion

We have shown the intercept of the BFKL pomeron at weak coupling regime and demonstrated an approach to obtain its values at strong couplings (for details, see Ref. \[1\]).

At $\lambda \rightarrow \infty$, the correction $\Delta$ for the Pomeron intercept $j_0 = 2 - \Delta$ has the form

$$\Delta = \frac{2}{\lambda^{1/2}} \left[ 1 + \frac{1}{2\lambda^{1/2}} - \frac{1}{8\lambda} - \left( 1 + 3\zeta_3 \right) \frac{1}{\lambda^{3/2}} + \left( 2a_{12} - \frac{145}{128} - \frac{9}{2} \zeta_3 \right) \frac{1}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}}\right) \right].$$

(26)

The fourth corrections in (26) contain unknown coefficient $a_{12}$, which will be obtained after the evaluation of spinning folded string on the two-loop level. Some estimations were given in Section 6 of \[1\].

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\[2\] Here we put $S = j - 2$, which in particular is related to the use of the angular momentum $J_{an} = 2$ in calculations of Refs \[43, 47\].

\[3\] Using a similar approach, the coefficients $\sim \lambda^{-1}$ and $\sim \lambda^{-3/2}$ were calculated also in the paper \[55\]. After correction of some errors, the results in \[55\] coincide with ours.
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