POSITIVE LAWS ON GENERATORS IN POWERFUL PRO-$p$ GROUPS

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ABSTRACT. If $G$ is a finitely generated powerful pro-$p$ group satisfying a certain law $v \equiv 1$, and if $G$ can be generated by a normal subset $T$ of finite width which satisfies a positive law, we prove that $G$ is nilpotent. Furthermore, the nilpotency class of $G$ can be bounded in terms of the prime $p$, the number of generators of $G$, the law $v \equiv 1$, the width of $T$, and the degree of the positive law. The main interest of this result is the application to verbal subgroups: if $G$ is a $p$-adic analytic pro-$p$ group in which all values of a word $w$ satisfy positive law, and if the verbal subgroup $w(G)$ is powerful, then $w(G)$ is nilpotent.

1. Introduction

If $\alpha$ and $\beta$ are two group words, we say that a group $G$ satisfies the law $\alpha \equiv \beta$ if every substitution of elements of $G$ by the variables gives the same value for $\alpha$ and for $\beta$. If the words $\alpha$ and $\beta$ are positive, i.e. if they do not involve any inverses of the variables, then we say that $\alpha \equiv \beta$ is a positive law. We can similarly speak about a law holding on a subset $T$ of $G$, if we only substitute elements of $T$ by the variables. Groups satisfying a positive law have received special attention in the past decade. The main result is due to Burns and Medvedev, who proved in Ref. [2] that a locally graded group $G$ satisfies a positive law if and only if $G$ is nilpotent-by-(locally finite of finite exponent). This applies in particular to residually finite groups.

A similar kind of problem has been considered by Shumyatsky and the second author in Ref. [5]. If $G$ is a finitely generated group and $T$ is a set of generators satisfying a positive law, they ask whether the whole of $G$ will also satisfy a (possibly different) positive law, provided that $T$ is sufficiently large in some sense. In this direction, they obtain a positive answer if $T$ is a normal subset of $G$ which is closed under taking commutators of its elements ($commutator$-$closed$ for short), under the assumption that $G$ satisfies an arbitrary law and is residually-$p$ for some prime $p$. More precisely, the result is proved for all primes outside a finite set $P(n)$ depending only on the degree $n$ of the law (that is, the maximum of the lengths of $\alpha$ and $\beta$).

The result in the previous paragraph can be applied to verbal subgroups $w(G)$ in a group $G$, where $T$ is considered to be the set $G_w$ of all values of the word $w$ in $G$. Note that $G_w$ is always a normal subset. Among

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other results, Shumyatsky and the second author prove that, if $G$ is a $p$-adic analytic pro-$p$ group and $p \not\in P(n)$ then, for every word $w$ such that $G_w$ is commutator-closed, a positive law on $G_w$ implies a positive law on the whole of $w(G)$. Now two questions naturally arise: (i) can we get rid of the restriction $p \not\in P(n)$?; (ii) can we get rid of the condition that $G_w$ should be commutator-closed? If we can give a positive answer to both these questions, then the result will hold in $p$-adic analytic pro-$p$ groups for all primes and for all words.

If $G$ is a $p$-adic analytic pro-$p$ group, Jaikin-Zapirain has proved (see Theorem 1.3 of Ref. [7]) that the set $G_w$ has finite width for every word $w$, and then, by Proposition 4.1.2 of Ref. [9], the verbal subgroup $w(G)$ is closed in $G$. (See Section 2 for the definition of width.) Thus $w(G)$ is again a $p$-adic analytic pro-$p$ group and, according to Interlude A of Ref. [3], it contains a powerful subgroup of finite index. One of the main results of this paper is the solution of the problem raised in the last paragraph in the case when $w(G)$ itself is powerful.

**Theorem 1.1.** Let $G$ be a $p$-adic analytic pro-$p$ group, and let $w$ be any word. If all values of $w$ in $G$ satisfy a positive law and the verbal subgroup $w(G)$ is powerful, then $w(G)$ is nilpotent.

Observe that the conclusion in the previous theorem that $w(G)$ is nilpotent is actually stronger than $w(G)$ satisfying a positive law.

Following the approach of Ref. [5], we obtain Theorem 1.1 from a more general result not involving directly word values. In this case, we work with $G$ a finitely generated powerful pro-$p$ group for an arbitrary prime $p$, and the set of generators $T$ has to be normal and of finite width, but not necessarily commutator-closed. Recall that, as mentioned above, if $G$ is a $p$-adic analytic pro-$p$ group, then $G_w$ has finite width for every word $w$.

**Theorem 1.2.** Let $G$ be a powerful $d$-generator pro-$p$ group which satisfies a certain law $v \equiv 1$. Suppose that $G$ can be generated by a normal subset $T$ of width $m$ that satisfies a positive law of degree $n$. Then $G$ is nilpotent of bounded class.

Here, and in the remainder of the paper, when we say that a certain invariant of a group is bounded, we mean that it is bounded above by a function of the parameters appearing in the statement of the corresponding result. Thus, in Theorem 1.2 the nilpotency class of $G$ is bounded in terms of the prime $p$, the number $d$ of generators of $G$, the law $v \equiv 1$, the width $m$ of $T$, and the degree $n$ of the positive law. If we want to make explicit the set $S$ of parameters in terms of which a certain quantity is bounded, then we will use the expression ‘$S$-bounded’.

We want to remark that, contrary to what happens in Theorem 1.2, we cannot guarantee that the nilpotency class of $w(G)$ is bounded in Theorem
The reason is that we are using the above-mentioned result of Jaikin-Zapirain, which provides the finite width of $G_w$, but not bounded width for that set.

2. The action on abelian normal sections

Our first step is to translate the positive law on the normal generating set $T$ into a condition about the action of the elements of $T$ on the abelian normal sections of $G$. More precisely, we have the following consequence of Lemma 2.1 in Ref. [5]. (Let $f(X)$ be the product of the polynomials $f_1(X)$ and $f_{-1}(X)$ in the statement of that lemma.)

**Lemma 2.1.** Let $T$ be a normal subset of a group $G$, and assume that $T$ satisfies a positive law of degree $n$. Then there exists a monic polynomial $f(X) \in \mathbb{Z}[X]$ of degree $2n$, depending only on the given positive law, which satisfies the following property: if $A$ is an abelian normal section of $G$, then $f(t)$, viewed as an endomorphism of $A$, is trivial for every $t \in T \cup T^{-1}$.

If $T$ is a subset of a group $G$, we say that $T$ has finite width if there exists a positive integer $m$ such that every element of the subgroup $\langle T \rangle$ can be expressed as a product of no more than $m$ elements of $T \cup T^{-1}$. The smallest possible value of $m$ is then called the width of $T$.

In our next theorem, we show how some properties of the generating set $T$ of $G$ are hereditary for the natural generating set of $\gamma_k(G)$ which can be constructed from $T$. For simplicity, if $A = K/L$ is a normal section of a group $G$, we say that two elements $g, h \in G$ commute modulo $A$ if $gL$ and $hL$ commute modulo $A$ (or, equivalently, if $g$ and $h$ commute modulo $K$).

**Theorem 2.2.** Let $G$ be a $d$-generator finite $p$-group, and let $T$ be a normal generating set of $G$. Then $T_{k} = \{[t_1, \ldots, t_k] \mid t_i \in T\}$ is a normal generating set of $\gamma_k(G)$, and furthermore:

(i) If $T$ has finite width $m$, then the width of $T_k$ is at most $md^{(k-1)}$.

(ii) If $T$ satisfies a positive law of degree $n$, then there exists a monic polynomial $h(X) \in \mathbb{Z}[X]$ of $n$-bounded degree such that $h(t_k)$ annihilates $\gamma_{k+1}(G)/\gamma_{k+1}(G)'$ for every $t_k \in T_k \cup T_k^{-1}$.

**Proof.** Of course, $T_k$ is a normal subset of $G$, and the proof that $T_k$ generates $\gamma_k(G)$ is routine.

(i) We argue by induction on $k$. The result is obvious for $k = 1$, so we assume next that $k \geq 2$. By the Burnside Basis Theorem, we can choose $t_1, \ldots, t_d \in T$ such that $G = \langle t_1, \ldots, t_d \rangle$. If $y$ is an arbitrary element of $\gamma_k(G)$, we can write

(1) $y = [g_1, t_1] \cdots [g_d, t_d]$, for some $g_i \in \gamma_{k-1}(G)$,

by using Proposition 1.2.7 of Ref. [9]. Now, if $g$ is an arbitrary element of $\gamma_{k-1}(G)$, then by the induction hypothesis, we have $g = u_1 \cdots u_s$ for some
$u_i \in T_{k-1} \cup T_{k-1}^{-1}$, where $s \leq md^{k-2}$. Then, for every $t \in T$, we have

$$[g, t] = [u_1, t]^{u_2 \cdots u_s} \cdots [u_{s-1}, t]^{u_s} u[t, t].$$

If $u_i \in T_{k-1}$, then $[u_i, t] \in T_k$; on the other hand, if $u_i \in T_{k-1}^{-1}$ then

$$[u_i, t] = ([u_i^{-1}, t]^{u_i})^{-1}$$

is an element of $T_k^{-1}$. Thus $[g, t]$ is a product of at most $s$ elements of $T_k \cup T_{k-1}^{-1}$, and it follows from (1) that $y$ is a product of no more than $ds$ elements of $T_k \cup T_{k-1}^{-1}$. This completes the proof of (i).

(ii) Set $A = \gamma_{k+1}(G)/\gamma_{k+1}(G)'$. By Lemma 2.3, there exists a monic polynomial $f(X) \in \mathbb{Z}[X]$ of degree $2n$ such that $f(t)$ annihilates $A$ for every $t \in T \cup T^{-1}$.

Let $I$ be the ideal of $\mathbb{Z}[X_1, X_2]$ generated by $f(X_1)$ and $f(X_2)$. Since $f$ is monic, the quotient ring $R = \mathbb{Z}[X_1, X_2]/I$ is a finitely generated $\mathbb{Z}$-module, generated by the images of the monomials $X_1^i X_2^j$ with $0 \leq i, j \leq 2n - 1$. By Theorem 5.3 in Chapter VIII of Ref. [6], $R$ is integral over $\mathbb{Z}$. In particular, there exists a monic polynomial $h(X) \in \mathbb{Z}[X]$ such that $h(X_1 X_2) \in I$. Also, by examining the proof of that result in Ref. [6], it is clear that the degree of $h(X)$ is at most $(2n)^2$.

Now, let $[u, t]$ be an arbitrary element of $T_k$, where $u \in T_{k-1}$ and $t \in T$. Since $(t^u)^{-1}$ and $t$ commute modulo $A$, these elements define commuting endomorphisms of $A$, and hence we can define a ring homomorphism

$$\varphi : \mathbb{Z}[X_1, X_2] \to \text{End}(A)$$

such that

$$X_1 \mapsto (t^u)^{-1},$$

$$X_2 \mapsto t.$$

Since $f((t^u)^{-1})$ and $f(t)$ are both the null endomorphism of $A$, it follows that $f(X_1)$ and $f(X_2)$ are contained in the kernel of $\varphi$, and so the same holds for the ideal $I$. Hence $h(X_1 X_2) \in \ker \varphi$, which means that $h([u, t])$ is the null endomorphism of $A$.

We can similarly prove that $h([t, u]) = 0$ in $\text{End}(A)$, by defining $\psi : \mathbb{Z}[X_1, X_2] \to \text{End}(A)$ via the assignments $X_1 \mapsto t^{-1}$ and $X_2 \mapsto t^u$. Thus $h(t_k)$ annihilates $A$ for every $t_k \in T_k \cup T_k^{-1}$. \hspace{1cm} \Box

Finally, for a certain $k$, we are able to get an Engel action of all $k$-th powers of the elements of $G$ on some abelian normal sections of $G$.

**Theorem 2.3.** Let $G$ be a finite $p$-group generated by a normal subset $T$ which has width $m$. Suppose that $A$ is an abelian normal section of $G$ such that the elements of $T$ commute pairwise modulo $A$, and that for some monic polynomial $f(X) \in \mathbb{Z}[X]$, $f(t)$ annihilates $A$ for all $t \in T \cup T^{-1}$. Then:

(i) There exists an $(m, f)$-bounded integer $r$ such that $[A, r, g] \leq A^p$ for every $g \in G$.

(ii) There exist $(m, f)$-bounded integers $n$ and $k$ such that $[A, n, g^k] = 1$ for every $g \in G$. 
Proof. The first part of the proof is similar to the proof of (ii) in the last theorem. Let us write $n$ for the degree of $f(X)$. Consider the quotient ring $R = \mathbb{Z}[X_1, \ldots, X_m]/I$, where $I$ is the ideal generated by the polynomials $f(X_1), \ldots, f(X_m)$. Then $R$ is integral over $\mathbb{Z}$, and there exists a monic polynomial $h(X) \in \mathbb{Z}[X]$ of degree at most $n^m$ such that $h(X_1 \ldots X_m) \in I$. Now let $g$ be an arbitrary element of $G$. Since $T$ generates $G$ and has width $m$, we can write $g = t_1 \ldots t_m$ for some $t_i \in T \cup T^{-1}$. The map $X_1 \mapsto t_1, \ldots, X_m \mapsto t_m$ extends to a ring homomorphism $\varphi : \mathbb{Z}[X_1, \ldots, X_m] \rightarrow \text{End}(A)$, since the elements of $T$ commute pairwise modulo $A$. Since $f(t_1) = \cdots = f(t_m) = 0$, it follows that $I \subseteq \ker \varphi$. Consequently,

$$h(g) = h(t_1 \ldots t_m) = \varphi(h(X_1 \ldots X_m)) = 0.$$ 

Thus we have found a monic polynomial $h(X) \in \mathbb{Z}[X]$ such that $h(g)$ annihilates $A$ for all $g \in G$. Note that the polynomial $h(X)$ only depends on $f(X)$ and $m$, but not on the particular element $g$ or on the section $A$.

(i) Since $G$ is a finite $p$-group, we have $[A_c G] = 1$ for some $c$. Let $(X - 1)^r$ be the greatest common divisor of $(X - 1)^c$ and $h(X)$, when these polynomials are considered in $\mathbb{F}_p[X]$. Since $r \leq \deg h$, it follows that $r$ is $\{m, f\}$-bounded. By Bézout’s identity, we can write

$$(X - 1)^r = p(X)(X - 1)^c + q(X)h(X),$$

for some $p(X), q(X) \in \mathbb{F}_p[X]$. If we consider an element $g \in G$, and substitute $g$ for $X$ in the previous expression, then, as endomorphisms of the $\mathbb{F}_p$-vector space $A/A^p$, we get $(g - 1)^r = 0$. This means that $[A, r g] \leq A^p$, as desired.

(ii) Let $J$ be the ideal of $\mathbb{Z}[X]$ generated by all polynomials $h(X^i)$ with $i \geq 1$. Then, if $j(X) \in J$, it follows that $j(g) = 0$ for every $g \in G$. By Lemma 3.3 of Ref. [10], there exist positive integers $q, k$ and $\ell$ such that

$$qX^\ell(X^k - 1)^\ell \in J,$$

where $q, k, \ell$ depend only on $h(X)$, so only on $f(X)$ and $m$. Then

$$A^{qg^\ell}(g^k - 1)^\ell = 1,$$

for every $g \in G$.

If $p^s$ is the largest power of $p$ which divides $q$, then $A^q = A^{p^s}$, since $A$ is a finite $p$-group. Also, we have $A^q = A$. Hence

$$A^{p^s}(g^k - 1)^\ell = 1$$

or, what is the same,

$$[A^{p^s}, g^k, \ldots, g^k] = 1$$

for every $g \in G$.

Now, it follows from part (i) that

$$[A^{p^s}, g] \leq A^{p^{i+1}}, \text{ for every } i \geq 0, \text{ and for every } g \in G.$$ 

This, together with (2), shows that

$$[A, n g^k] = 1,$$

for all $g \in G$.
where $n = sr + \ell$.

3. Proof of the Main Theorems

We will begin by proving Theorem 1.2. In order to show that the powerful pro-$p$ group $G$ is nilpotent, we will rely on the following two lemmas. The first one is a classical result of Philip Hall (see, for example, Theorem 3.26 of Ref. [8]), and the other one says that for a finitely generated powerful pro-$p$ group ‘nilpotent-by-finite’ is the same as ‘nilpotent’.

**Lemma 3.1.** Let $G$ be a group, and let $N$ be a normal subgroup of $G$. If $N$ is nilpotent of class $k$ and $G/N'$ is nilpotent of class $c$, then $G$ is nilpotent of $\{k, c\}$-bounded class.

**Lemma 3.2.** Let $G$ be a finitely generated powerful pro-$p$ group. If $G$ has a normal subgroup $N$ of finite index which is nilpotent of class $c$, then $G$ itself is nilpotent of $\{c, e\}$-bounded class, where $e$ is the exponent of $G/N$.

**Proof.** We prove the result for $p > 2$. For $p = 2$, the same proof applies with some little changes. It follows from the hypotheses that $G^e$ is nilpotent of class at most $c$. By Proposition 3.2 and Corollary 3.5 in Ref. [1], we get

$$[G^{e+1}, \ldots, G] = [G, e+1, \ldots, G] = [G, e+1, G^e] = 1.$$ (3)

On the other hand, since $G$ is powerful, we have $\gamma_{i+1}(G) \leq G^{p^i}$ for all $i \geq 1$. As a consequence, for some $\{c, e\}$-bounded integer $k$ we have $\gamma_{k+1}(G) \leq G^{e+1}$. This, together with (3), shows that $G$ is nilpotent of class at most $k + c$, and we are done.

Note that we could have written the previous lemma under the apparently weaker assumption that the exponent of $G/N$ is finite, rather than $N$ being of finite index in $G$. However, if $G$ is a finitely generated powerful pro-$p$ group, these two conditions are equivalent: if $\exp G/N = p^k$, then $G^{p^k}$ is contained in $N$, and then by Theorem 3.6 of Ref. [3], we have $|G : N| \leq |G : G^{p^k}| \leq p^{kd}$, where $d$ is the minimum number of generators of $G$ as a topological group. (In fact, the assumption that $G$ should be powerful is not necessary for this equivalence, since $|G : G^{p^k}|$ is finite for every finitely generated pro-$p$ group. But this is a much deeper result, which needs Zelmanov’s positive solution of the Restricted Burnside Problem.)

We also need the following result of Black (see Corollary 2 in Ref. [1]).

**Theorem 3.3.** Let $G$ be a finite group of rank $r$ satisfying a law $v \equiv 1$. Then, there exists an $\{r, v\}$-bounded number $k$ such that $\gamma_k((G^{kl})') = 1$. In particular, if $G$ is soluble, then the derived length of $G$ is $\{r, v\}$-bounded.

Note that the positive solution to the Restricted Burnside Problem is needed for the conclusion in the soluble case: thus we know that the quotient $G/G^{kl}$ has bounded order, and so also bounded derived length.
We can now proceed to the proof of Theorem 1.2.

Proof of Theorem 1.2. Suppose that the result is known for $G$ a finite $p$-group, so that all finite $p$-groups satisfying the conditions of the theorem have nilpotency class at most $c$, for some bounded number $c$. Now if $G$ is a pro-$p$ group as in the statement of the theorem, and $N$ is an arbitrary open normal subgroup of $G$, it follows that $\gamma_{c+1}(G) \leq N$. Thus necessarily $\gamma_{c+1}(G) = 1$ and the result is valid also for pro-$p$ groups.

Hence we may assume that $G$ is a $d$-generator finite powerful $p$-group. By Theorem 11.18 of Ref. 8, it follows that $G$ has rank $d$, i.e. that every subgroup of $G$ can be generated by $d$ elements. Since $G$ satisfies the law $v \equiv 1$, by Theorem 3.3 we have $G^{(s)} = 1$ for some bounded number $s$. We argue by induction on $s$.

If $s \leq 2$, i.e. if $G$ is metabelian, then the elements of $T$ commute pairwise modulo $G'$. Choose generators $g_1, \ldots, g_d$ of $G$. By Lemma 2.3 and Theorem 2.2 since $T$ satisfies a positive law, we know that there exist bounded numbers $n$ and $k$ such that $[G', g_i^k] = 1$ for all $i = 1, \ldots, d$. As a consequence, the subgroups $(g_i^k, G')$ have bounded nilpotency class. Thus $G^k G' = \langle g_1^k, \ldots, g_d^k, G' \rangle$ is the product of $d$ normal subgroups of bounded class, and so has bounded class itself. Since $|G : G^k G'| \leq k^d$, it follows from Lemma 3.2 that $G$ has bounded nilpotency class. This concludes the proof in the metabelian case.

Assume now that $s \geq 3$. We claim that the nilpotency class of $G/\gamma_{k+1}(G)'$ is bounded for all $k \geq 1$ (here, we must also take $k$ into account for the bound). The result is true for $k = 1$, according to the last paragraph. Now we argue by induction on $k$. By Theorem 2.2 $T_k$ is a normal set of generators of $\gamma_k(G)$ of bounded width. Also, the elements of $T_k$ commute pairwise modulo $\gamma_{k+1}(G)$. On the other hand, by (ii) of Theorem 2.2 there exists a monic polynomial $h(X) \in \mathbb{Z}[X]$ such that $h(t_k)$ annihilates the abelian normal section $A = \gamma_{k+1}(G)/\gamma_{k+1}(G)'$ for every $t_k \in T_k$. Thus we may argue as in the metabelian case above with the group $Q = \gamma_k(G)/\gamma_{k+1}(G)'$ and deduce that $Q$ has bounded nilpotency class. Since $G/\gamma_k(G)'$ has also bounded class by the induction hypothesis, the claim follows from Lemma 3.1.

Now that the claim is proved, the result easily follows. Indeed, since $G/G^{(s-1)}$ has bounded class by induction, there exists a bounded integer $\ell$ such that $\gamma_{\ell+1}(G) \leq G^{(s-1)}$. Hence $\gamma_{\ell+1}(G)' = 1$, and $G$ has bounded class by the previous claim.

Now Theorem 1.1 follows readily from Theorem 1.2.

Proof of Theorem 1.1. As already mentioned, the set $G_w$ of all values of $w$ in $G$ is a normal subset of $G$, and in particular of $w(G)$. Also, by Theorem 1.3 of Ref. 7, $G_w$ has finite width, say $m$.

Let $\alpha \equiv \beta$ be the positive law satisfied by the set $G_w$, and suppose that the number of variables used in the law $\alpha \equiv \beta$ and in the word $w$ is $k$ and
ℓ, respectively. Now, consider kl arbitrary elements \( g_1, \ldots, g_{kl} \) of \( G \). Since the \( k \) elements \( w(g_1, \ldots, g_\ell), \ldots, w(g_{(k-1)\ell+1}, \ldots, g_{k\ell}) \) satisfy the law \( \alpha \equiv \beta \), it follows that

\[
\alpha(w(g_1, \ldots, g_\ell), \ldots, w(g_{(k-1)\ell+1}, \ldots, g_{k\ell})) = \\
\beta(w(g_1, \ldots, g_\ell), \ldots, w(g_{(k-1)\ell+1}, \ldots, g_{k\ell})).
\]

This means that the group \( G \) satisfies a law \( v \equiv 1 \), where \( v \) is a word which depends only on \( w \) and on the positive law \( \alpha \equiv \beta \). In particular, the law \( v \equiv 1 \) is also satisfied by \( w(G) \).

Now, we can apply directly Theorem 1.2 to the group \( w(G) \) and the generating set \( G_w \), in order to conclude that \( w(G) \) is nilpotent. \( \square \)

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