Transport Coefficients in Gluodynamics: From Weak Coupling towards the Deconfinement Transition

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Abstract. We study the ratio of bulk to shear viscosity in gluodynamics within a phenomenological quasiparticle model. We show that at large temperatures this ratio exhibits a quadratic dependence on the conformality measure as known from weak coupling perturbative QCD. In the region of the deconfinement transition, however, this dependence becomes linear as known from specific strongly coupled theories. The onset of the strong coupling behavior is located near the maximum of the scaled interaction measure. This qualitative behavior of the viscosity ratio is rather insensitive to details of the equation of state.

Keywords: transport coefficients, viscosity, quasiparticle model, effective kinetic theory

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INTRODUCTION

Transport coefficients, such as bulk (ζ) and shear (η) viscosities, represent fundamental quantities specifying the physical properties of the matter. In the hydrodynamic description of the medium created in high-energy nuclear collisions they are important input parameters. First-principle lattice QCD calculations of the viscosity coefficients have been performed recently in SUc(3) gluodynamics in the deconfinement region [1, 2]. These lattice QCD results were shown to be successfully describable within a phenomenological quasiparticle model (QPM) for the gluon plasma [3, 4]. This model is based on an effective kinetic theory approach to QCD and includes non-perturbative effects via a thermal quasigluon mass. In particular, in the vicinity of the deconfinement transition temperature Tc the shear viscosity to entropy density ratio exhibits a minimum [3] with a value as small as the predicted universal lower bound [5] η/s ∼ 1/4π. This result is in clear contrast to naive extrapolations of standard perturbative QCD approaches into the non-perturbative regime close to Tc. The specific bulk viscosity ζ/s is instead found to rapidly increase in the deconfinement transition region [3]. Moreover, the ratio ζ/η is known to exhibit a distinct dependence on the conformality measure Δv2s = 1/3 − v2s with v2s being the squared speed of sound. At weak coupling, i.e. at high temperatures T, the viscosity ratio depends quadratically on Δv2s [6]. On the other hand, at strong coupling, i.e. close to Tc, this dependence is expected to be linear [7]. The QPM reproduces the above asymptotic dependencies and exhibits a gradual interpolation between both regimes [4]. In this work, we analyze the structure of the viscosity ratio ζ/η in SUc(3) gluodynamics in the vicinity of Tc. In particular, we discuss its sensitivity on details in the equation of state (EoS) and the corresponding v2s.

THERMODYNAMICS AND TRANSPORT COEFFICIENTS

In the QPM, the EoS, which describes changes of the pressure P with energy density ε, as well as related thermodynamic quantities are obtained from the local thermal equilibrium limit of the underlying effective kinetic theory. This model was shown to be very successful in describing the thermodynamic quantities obtained in lattice QCD [8]. Essential feature of the QPM is that the quasiparticle excitations follow a medium-modified dispersion relation, E2 = p2 + Π(T). In a quasigluon plasma, the effective mass Π(T) = T2G2(T)/2 is quantified by the coupling G2(T) = 16π2/(11 ln[λ(T − Ts)/Tc])2, which near Tc accommodates non-perturbative effects via only two
parameters, $\lambda$ and $T_c$. At large $T$, it reproduces the perturbative running coupling in QCD. To determine $P$ and $\epsilon$ in the QPM, one needs to introduce the integration constant $B(T_c)$ as an additional parameter [8]. The squared speed of sound follows then from $v_s^2 = \partial P/\partial \epsilon$.

Figure 1 (left panel) shows comparisons of the QPM predictions for the scaled interaction measure $(\epsilon - 3P)/T^4$ as a function of $T/T_c$ with corresponding lattice QCD results [9, 10, 11]. The model parameters read as $T_c/T_c = 0.73$, $\lambda = 4.3$ and $B(T_c) = 0.19 T_c^4$ (Fit 1) for data from [9, 10], while we use $T_c/T_c = 0.52$, $\lambda = 2.5$ and $B(T_c) = 0.48 T_c^4$ (Fit 2) for data from [11]. The conformality measure obtained from these QPM results for the EoS is depicted in Fig. 1 (right panel) as a function of $T/T_c$. The two different fits result in visible deviations in $\Delta v_s^2$ for $T_c < T < 2.5T_c$.

At the first-order phase transition at $T = T_c$, the squared speed of sound $v_s^2(T)$ is discontinuous, which results in a discontinuity in $\Delta v_s^2$. We note that the limiting value $\lambda = \lim_{T \to T_c^+} \Delta v_s^2$, which is linearly approached with $T/T_c$ as seen in Fig. 1 (right panel), is in general different from $\Delta v_s^2(T_c) = 1/3$.

The bulk and shear viscosity coefficients follow directly from the effective kinetic theory [3, 12]. Their ratio was found in the following form [4]

$$\frac{\zeta}{\eta} = 15 \left( \Delta v_s^2 \right)^2 \left[ 1 - \phi_0 + \frac{1}{4} \phi_2 \right] + 5 \Delta v_s^2 \left[ \phi_0 - \frac{1}{2} \phi_2 \right] + \frac{5}{12} \phi_2, \tag{1}$$

where $\phi_0 = T^2 dG^2/dT^2 \mathcal{F}_0 / \mathcal{F}_2$ and $\phi_2 = \left( T^2 dG^2/dT^2 \right)^2 T^4 \mathcal{F}_2 / \mathcal{F}_2$ depend non-trivially on $T$. The momentum integrals $\mathcal{F}_k$ in $\phi_{0,2}$ read as $\mathcal{F}_k = \int \frac{d^3p}{(2\pi)^3} n(T)|1 + d^{-1}n(T)|^{-\tau} \mu^{2-k}$, where $n(T) = d \left( \exp(E/T) - 1 \right)^{-1}$ is the Bose distribution function for gluons with $d = 16$ and $\tau$ denotes the relaxation time. For $T \geq T_c$, these integrals follow the hierarchy $\mathcal{F}_2 / \mathcal{T}_2 \gg \mathcal{F}_0 \gg T^2 \mathcal{F}_2 > 0$, whereas $dG^2 / dT^2 < 0$. Assuming a momentum independent $\tau$, the ratio $\zeta/\eta$ in Eq. (1) is solely determined by parameters adjusted to equilibrium thermodynamics.

In Fig. 2, the ratio $\zeta/\eta$ from Eq. (1) is quantified for Fit 1 and 2. For both QPM fits one observes that the viscosity ratio for $\Delta v_s^2 \lesssim 0.07$, i.e. for $T \gtrsim 1.5T_c$, is entirely determined by 15 $(\Delta v_s^2)^2$, i.e. by the quadratic, $\phi_{0,2}$-independent term in Eq. (1). This is a direct consequence of the behavior of the conformality measure at large $T$, which in the QPM reads as $\Delta v_s^2 \sim -75/(18d\pi^2)T^2(dG^2/dT^2) + \mathcal{O}(G^2 T^2 (dG^2/dT^2))$ with $|T^2(dG^2/dT^2)| \ll G^2 \ll 1$. Thus, at leading order, all terms in Eq. (1) depend quadratically on $\Delta v_s^2$, where the first, $\phi_{0,2}$-independent term dominates numerically. With increasing $\Delta v_s^2$, the $\zeta/\eta$ ratio is reduced compared to 15 $(\Delta v_s^2)^2$ by the non-perturbative terms in Eq. (1) which are proportional to $(dG^2/dT^2)$. Consequently, one finds $\zeta/\eta < 1$ for $T \to T_c^+$. For $\Delta v_s^2 \gtrsim 0.17$, i.e. for $T \lesssim 1.15T_c$, a linear dependence on $\Delta v_s^2$ develops as seen in Fig. 2. In fact, near $T_c$ the factors $|\phi_{0,2}|$ become large as a consequence of a large $|dG^2/dT^2|$. This results in a cancelation of the quadratic dependence $15 (\Delta v_s^2)^2$ in Eq. (1) by all other $\phi_{0,2}$-dependent terms. Effectively, Eq. (1) sums up near $T_c$ to a linear dependence of the form $\zeta/\eta = \alpha \Delta v_s^2 + \beta$. The transition to this linear behavior, as it is known for specific strongly coupled theories [7], takes place at $T$ in the vicinity of $T_c$. 

![Figure 1](image1.png)  
![Figure 2](image2.png)
FIGURE 2. Bulk to shear viscosity ratio $\zeta/\eta$ from Eq. (1) as a function of the conformality measure $\Delta v^2_s$ from Fig. 1 (right panel) for Fit 1 (long-dashed curve) and Fit 2 (solid curve). The short-dashed curve exhibits the quadratic dependence $\zeta/\eta = 15 (\Delta v^2_s)^2$ and the dash-dotted curve shows a linear fit $\zeta/\eta = \alpha \Delta v^2_s + \beta$ of the QPM result from Fit 1 with $\alpha = 3.78$ and $\beta = -0.305$. For the QPM result from Fit 2, a similar linear fit yields $\alpha = 3.63$ and $\beta = -0.305$.

of the maximum in the scaled interaction measure, cf. Fig. 1. As is evident from Fig. 2, this qualitative behavior is a common feature of the viscosity ratio within the quasiparticle model for the gluon plasma irrespective of details in the EoS near $T_c$. In the interval $0.07 \lesssim \Delta v^2_s \lesssim 0.17$, a gradual change between quadratic and linear dependence on $\Delta v^2_s$ is observed. We note that at $T = T_c$ the $\zeta/\eta$ ratio develops a discontinuity in line with $\Delta v^2_s$.

SUMMARY

Within a phenomenological quasiparticle model for the gluon plasma, the bulk to shear viscosity ratio is found to exhibit, at large $T$, the quadratic dependence on the conformality measure as known from perturbative QCD. In the deconfinement transition region, this dependence becomes linear as found in specific strongly coupled theories. Thus, the quasiparticle model provides a systematic link between both regimes. The onset of the strong coupling behavior is located near the maximum in the scaled interaction measure. This qualitative behavior of the viscosity ratio is insensitive to details in the equation of state.

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