Improved dynamics and gravitational collapse of tachyon field coupled with a barotropic fluid

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We consider the spherically symmetric gravitational collapse of a tachyon field with an inverse square potential, which is coupled with a barotropic fluid. By employing the holonomy correction imported from loop quantum gravity, we analyse the dynamics of the collapse within a semiclassical description. Using a dynamical system approach, we find saddle fixed points. These provide a new dynamics in contrast to the classical black hole and naked singularities solutions appearing in the standard general relativistic collapse setting. Our results indicate that the classical singularities are avoided by the herein quantum gravity induced holonomy effects. In addition, thorough numerical studies shows that there exists a threshold scale which distinguishes an outward energy flux from a non-singular black hole forming at the collapse final stages.

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I. INTRODUCTION

The spherically symmetric gravitational collapse, with a variety of matter fields, has been well studied in general relativity (see Ref. 1 and references therein). Those investigations indicate that the gravitational collapse, depending on the initial condition, may produce a black hole with a singularity inside or a naked singularity as its final state 1-3. However, these results are not expected to hold in a quantum theory of gravity. Among the candidates for a theory of quantum gravity, loop quantum gravity (LQG) 4-6 is a non-perturbative background independent theory. From the effective constraints approach used in the LQG program, there are two general types of quantum corrections, namely the ‘inverse triad’ and ‘holonomy’ types.

The study of late time stages of gravitational collapse has thus been considered in LQG, by means of these corrections. It was shown that inverse triad modifications resolve the classical singularities that arise at the final state of gravitational collapse, whose matter source are a standard scalar field 7,8 or a tachyon field 9. One the one hand, in an isotropic and spherically symmetric model, loop gravity effects, within a holonomy correction, modify the standard Friedmann equation by adding a $-\rho^2/\rho_{\text{crit}}$ correction term into it. In a cosmological context, these effects resolve the big bang singularity and replace it by a bounce 10. Within a gravitational collapse of a scalar field 11, non interacting particles (dust) and a perfect fluid describing radiation 12, it was shown that those quantum gravity effects provide a threshold scale for a non-singular black hole formation.

More recently, a self interacting tachyon field coupled with a barotropic fluid has been studied in a classical 13 and a semiclassical 9 context, regarding gravitational collapse. A dynamical system analysis was employed therein, where by making use of the specific kinematical features of the tachyon (which are rather different from a standard scalar field) within the classical setting, asymptotic solutions, corresponding to a naked singularity or a black hole, were investigated. In addition, the semiclassical collapse was discussed, where an inverse triad correction induced from LQG was employed.

In this letter, we follow the analysis of 13, by means of investigating how, due to the holonomy effects, results from the general relativistic collapse is changed. To this aim, we will employ dynamical system techniques to study the asymptotic behaviour of the semiclassical interior space-time of the collapse. This provides a contrasting analysis regarding the other quantum gravity correction, with the inverse triad type.

II. GRAVITATIONAL COLLAPSE: IMPROVED DYNAMICS

We consider a spherically symmetric gravitational collapse whose interior space-time is the marginally bound case, i.e., the $k = 0$ FLRW 13. Let $t$ be the proper time for a falling observer whose geodesic trajectories are labeled by the comoving radial coordinate $r$, and $R(t, r) := r a(t)$ is the area radius of the collapsing cloud. Then, for a continuous collapsing scenario, we take $R = r a < 0$, implying that the area radius of the collapsing shell, for a constant $r$, decreases monotonically.

The corresponding Hamiltonian constraint for the interior geometry is provided as 9

$$\mathcal{H} = -\frac{6}{\kappa \gamma^2} c^2 \sqrt{|p|} + \mathcal{H}_{\text{matt}} \, ,$$

(2.1)

where $c := \gamma a < 0$ and $p := a^2$ are, respectively, the con-
jugate connection and triad satisfying the non-vanishing Poisson bracket \( \{ c, p \} = \kappa^2 \gamma / 3 \), with \( \gamma \approx 0.23 \) being the Barbero-Immirzi dimensionless parameter. Moreover, \( \kappa = 8 \pi G \), and \( H_{\text{matt}} = \rho V \) is the matter Hamiltonian with \( V \) being the volume of the fiducial cell \( [5] \).

A pertinent scenario to investigate semiclassically effects suggested from LQG (as far as a gravitational collapse is concerned) is the so-called holonomy correction. The algebra generated by the holonomy of phase space variables \( c \) is just the algebra of the almost periodic function of \( c \), i.e., \( e^{i\mu c / 2} \) (where \( \mu \) is inferred as kinematical length of the square loop since its order of magnitude is similar to that of length), which together with \( p \), constitutes the fundamental canonical variables in quantum theory \([5]\). This consists semiclassically in replacing \( c \) in Eq. \((2.1)\), with the phase space function, by means of

\[
\frac{1}{2i\mu} \left( e^{i\mu c} - e^{-i\mu c} \right) = \frac{\sin(\mu c)}{\mu} .
\]

It is expected that the classical theory is recovered for small \( \mu \); we therefore obtain the effective semiclassical Hamiltonian \([10, 14]\)

\[
\mathcal{H}_{\text{eff}} = -\frac{6}{\kappa^2 \gamma^2 \mu^2} \sqrt{|p|} \sin^2(\mu c) + H_{\text{matt}} .
\]

The dynamics of the fundamental variables is then obtained by solving the system of Hamilton equations; i.e.,

\[
\dot{p} = \{ p, \mathcal{H}_{\text{eff}} \} = -\frac{\kappa^2}{3} \frac{\partial \mathcal{H}_{\text{eff}}}{\partial c} = \frac{2a}{\gamma \mu} \sin(\mu c) \cos(\mu c). \tag{2.4}
\]

Furthermore, the vanishing Hamiltonian constraint \((2.3)\) implies that

\[
\sin^2(\mu c) = \frac{\kappa^2 \mu^2}{3a} H_{\text{matt}} . \tag{2.5}
\]

Thus, using Eqs. \((2.4)\) and \((2.5)\), we subsequently obtain the modified Friedmann equation, \( H = \dot{a}/a = \dot{p}/2p \):

\[
H^2 = \frac{\kappa}{3} \rho \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right) , \tag{2.6}
\]

where \( \rho_{\text{crit}} = 3/(\kappa^2 \gamma^2 \lambda^2) \approx 0.41 \rho_1 \), and \( \rho \) is the total (classical) energy density of the collapse matter content. Eq. \((2.6)\) implies that the classical energy density \( \rho \) is limited to the interval \( \rho_0 < \rho < \rho_{\text{crit}} \) having an upper bound at \( \rho_{\text{crit}} \), where \( \rho_0 \ll \rho_{\text{crit}} \) is the energy density of the star at the initial configuration, \( t = 0 \). Hence, the effective energy density reads

\[
\rho_{\text{eff}} := \rho \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right) . \tag{2.7}
\]

We see that the effective scenario, provided by holonomy corrections, leads to a \(-\rho^2\) modification of the energy density, which becomes important when the energy density becomes comparable to \( \rho_{\text{crit}} \). In the limit \( \rho \to \rho_{\text{crit}} \), the Hubble rate vanishes; a classical singularity is thus replaced by a bounce.

The time derivative of the Hubble rate can be written in the effective dynamics as below,

\[
\dot{H} = \frac{\kappa}{2} \left( \rho + p \right) \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right) . \tag{2.8}
\]

The corresponding pressure of the system is given by

\[
p_{\text{eff}} = p \left( 1 - \frac{2\rho}{\rho_{\text{crit}}} \right) - \frac{\rho^2}{\rho_{\text{crit}}} . \tag{2.9}
\]

The effective energy conservation for \( \rho_{\text{eff}} \) is given by the relation \( \rho_{\text{eff}} = -3H(p_{\text{eff}} + p_{\text{eff}}) \), so that, we can define the effective equation of state as

\[
w_{\text{eff}} := \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = \frac{p}{\rho} \left( \frac{\rho_{\text{crit}} - 2\rho}{\rho_{\text{crit}} - \rho} \right) - \frac{\rho}{\rho_{\text{crit}} - \rho} . \tag{2.10}
\]

In general relativity, the equation for an apparent horizon in a spherical symmetric space-time is given by \( g^{ij} R_i R_j = 0 \), which \( F/R = 1 \). Here \( F(R) = \kappa \rho R^3 / 3 \) is the mass function of the collapsing matter, and, the space-time is said to be trapped or untrapped if \( F > R \) or \( F < R \), respectively \([11]\). Since in the effective scenario herein, the energy density \( \rho \) in the Friedmann equation \((2.8)\) is modified as \( \rho_{\text{eff}} \), hence the mass function \( F(R) \) is modified in the semiclassical regime as \([11]\)

\[
F_{\text{eff}} = \frac{\kappa}{3} \rho_{\text{eff}} R^3 = F \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right) . \tag{2.11}
\]

A possible perspective into it is to consider that the phase space trajectories are classical whereas the matter content is assumed to be effective due to the semiclassical effects. The \( \rho/\rho_{\text{crit}} \) term in Eq. \((2.11)\) can be written as

\[
\frac{\rho}{\rho_{\text{crit}}} = \frac{a_0^3}{a^3} \frac{F}{F_{\text{crit}}} , \tag{2.12}
\]

where \( a_0 \) and \( F_{\text{crit}} := (\kappa/3) \rho_{\text{crit}} r^3 \) are respectively, the values of scale factor and the mass function at the bounce. It is seen from Eq. \((2.12)\) that, the mass function \( F \) changes in the interval \( F_0 \leq F \leq F_{\text{crit}} \) along with the collapse dynamical evolution, so that, it remains finite during the semiclassical regime: \( F_0 = (\kappa / 3) \rho_{\text{crit}} r^3 a_0^3 \) is the initial data for the mass function at \( t = 0 \). Furthermore, the effective mass function \((2.11)\) vanishes at the bounce.

Let us follow Refs. \([9,13]\) and consider the total energy density, \( \rho \), of the collapse to be

\[
\rho = \rho_\phi + \rho_1 , \tag{2.13}
\]

which constitutesthe classical energy densities of the ta-chyon field and the barotropic fluid. In a strictly classical setting, the energy density \( \rho_\phi \) and pressure \( p_\phi \) of the ta-chyon field are given by

\[
\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \phi^2}} , \quad p_\phi = -V(\phi) \sqrt{1 - \phi^2} , \tag{2.14}
\]
with a dot denoting a derivative with respect to the proper time \( t \), and \( V(\phi) \) is the tachyon potential. Furthermore, the energy density of the barotropic fluid, \( \rho_\gamma \), reads

\[
\rho_\gamma = \rho_{\gamma0} \left( \frac{a}{a_0} \right)^{-3\gamma}, \tag{2.15}
\]

where \( \rho_{\gamma0} \) is a positive constant denoting the fluid density at the initial configuration, \( a_0 \), of the collapse, and \( \gamma \) is an adiabatic index satisfying \( p_\gamma = (\gamma - 1)\rho_\gamma \), with \( \rho_\gamma \) being the pressure of the barotropic fluid.

For a physically reasonable matter content for the collapsing cloud, the tachyon field and the barotropic fluid would have to satisfy the weak and dominant energy conditions. It is straightforward to show that the tachyon matter satisfies the weak and dominant energy conditions. For a fluid herein with the barotropic parameter \( \gamma > 0 \), the weak energy condition is satisfied, however, concerning the dominant energy condition, it follows that \( \gamma \) must hold the range \( \gamma \leq 2 \) \[13\].

From the total energy conservation equation for collapse matter source, we could write, generally, that

\[
\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = -\Gamma_{\text{int}}, \tag{2.16}
\]

\[
\dot{\rho}_\gamma + 3\gamma H\rho_\gamma = +\Gamma_{\text{int}}, \tag{2.17}
\]

but we will assume that the tachyon field is just self interacting and not coupled to the barotropic fluid, i.e., \( \Gamma_{\text{int}} = 0 \). The conservation energy density \( \dot{\rho}_\phi \) for the tachyon field gives

\[
\dot{\phi} = -\left(1 - \phi^2\right) \left[3H\dot{\phi} + \frac{V_\phi}{V}\right], \tag{2.18}
\]

where \( \dot{\phi} \) denotes the derivative with respect to \( \phi \). Furthermore, the equation of state \( w_\phi \) for tachyon field is given by

\[
w_\phi := \frac{p_\phi}{\rho_\phi} = -\left(1 - \phi^2\right). \tag{2.19}
\]

In addition, one can define a barotropic index for the tachyon fluid: \( \gamma_\phi := (\rho_\phi + p_\phi)/\rho_\phi = \phi^2 \).

III. HOLOMONY EFFECTS AND PHASE SPACE ANALYSIS

The use of dynamical system techniques to analyse a tachyon field in gravitational collapse has been considered in \[9, 13\]. In what follows, a dynamical system analysis of the tachyon field gravitational collapse within the improved dynamics approach of LQG will be studied.

We assume the time variable (instead of the proper time \( t \) present in the comoving coordinate system \( \{t, r, \theta, \varphi\} \))

\[
N := -\ln \left( \frac{a}{a_0} \right)^3, \tag{3.1}
\]

defined in \[13\]; therein \( 0 < N < \infty \) where the limit \( N \to 0 \) corresponds to the initial condition of the collapsing system \( (a \to a_0) \), and the limit \( N \to \infty \) corresponds to \( a = 0 \), i.e., the classical singularity identified in \[13\]. For an arbitrary function \( f \) we get

\[
\frac{df}{dN} = -\frac{f}{3H}. \tag{3.2}
\]

To analyze the dynamical behaviour of the collapse, we further introduce the following variables:

\[
x := \dot{\phi}, \quad y := \frac{\kappa V}{3H^2}, \quad z := \frac{\rho}{\rho_{\text{crit}}}, \quad s := \frac{\kappa_\gamma}{3H^2}, \quad \lambda := -\frac{V_\phi}{\sqrt{\kappa V}}, \quad \Gamma := \frac{V_\phi}{(V_\phi)^2}. \tag{3.3}
\]

The Friedmann constraint \( \dot{} \) \( x \) \( x \), in terms of the new variables \( (3.3) \), can be rewritten as

\[
1 = \left( \frac{y}{\sqrt{1 - x^2}} + s \right) \left(1 - z\right), \tag{3.4}
\]

in which, the dynamical variables \( x, y \) and \( z \) must satisfy the constraints \( -1 \leq x \leq 1, y \geq 0 \) and \( 0 \leq z \leq 1 \). Furthermore, the time derivative of the Hubble rate, Eq. \( (2.8) \), in terms of dynamical variables \( (3.4) \), can be written

\[
\dot{H} \frac{3H^2}{3H^2} = -\frac{1}{2} \left(1 - 2z\right) \left[\frac{x^2y}{\sqrt{1 - x^2}} + \gamma s\right]. \tag{3.5}
\]

Using the Eq. \( (3.3) \) and the constraint \( (3.4) \), the equations of state \( (2.19) \) and \( (2.10) \), in terms of new variables can be written as

\[
w_\phi = -(1 - x^2), \tag{3.6}
\]

\[
w_{\text{eff}} = -(1 - x^2) \left(1 - \frac{2z}{1 - z}\right) \tag{3.7}
\]

\[
+ s(\gamma - x^2)(1 - 2z) - \frac{z}{1 - z}.
\]

Moreover, the fractional densities of the two fluids are respectively defined as:

\[
\Omega_\phi := \frac{\kappa_\phi}{3H^2} = \frac{y}{\sqrt{1 - x^2}}, \quad \Omega_\gamma := \frac{\kappa_\gamma}{3H^2} = s. \tag{3.8}
\]

An autonomous system of equations, in terms of the dynamical variables of Eq. \( (3.3) \), together with Eqs. \( (3.4) \) and \( (3.5) \), is then retrieved:

\[
\frac{dx}{dN} = (1 - x^2) \left(x - \frac{\lambda}{\sqrt{3}}\sqrt{y}\right), \tag{3.9}
\]

\[
\frac{dy}{dN} = \frac{\lambda x}{\sqrt{3}y}^2 - y(1 - 2z) \left[\frac{x^2}{1 - z} + s(\gamma - x^2)\right], \tag{3.10}
\]

\[
\frac{dz}{dN} = z \left[x^2 + s(1 - z)(\gamma - x^2)\right], \tag{3.11}
\]

\[
\frac{ds}{dN} = s \left[\gamma - (1 - 2z) \left(\frac{x^2}{1 - z} + s(\gamma - x^2)\right)\right], \tag{3.12}
\]

\[
\frac{d\lambda}{dN} = \frac{1}{\sqrt{3}} \sqrt{\kappa}\lambda x \sqrt{y} \left(\Gamma - \frac{3}{2}\right). \tag{3.13}
\]
Notice that, in the limit $\rho \ll \rho_{\text{crit}}$ (i.e., in the absence of $z$), the Eqs. (3.9)-(3.12) reduce to the corresponding classical autonomous system of equations in Ref. [13].

We will assume the tachyon potential to be of an inverse square form $[9, 13]$:

$$V(\phi) = V_0 \phi^{-2}. \quad (3.14)$$

For the choice $[3.14]$ we get $\lambda = \pm 2/\sqrt{V_0}$ and $\Gamma = 3/2$, i.e., as constants. The dynamical system will be four differential equations with variables $(x, y, z, s)$. Let $f_1 := dx/dN, f_2 := dy/dN, f_3 := dz/dN$ and $f_4 := ds/dN$. Then, the critical points $q_c = (x_c, y_c, z_c, s_c)$ are obtained by setting the condition $(f_1, f_2, f_3, f_4)|_{q_c} = 0$. Next we will study the stability of our dynamical system at each critical point by using a standard linearization and stability analysis.

To determine the stability of critical points, we need to perform linear perturbations around each point by using the form $q(t) = q_c + \delta q(t)$; this results in the equations of motion $\delta q' = M \delta q$, where $M$ is the Jacobi matrix of each critical point whose components are $M_{ij} = (\partial f_i/\partial q_j)|_{q_c}$. A critical point is called stable (unstable) whenever the eigenvalues $\zeta_i$ of $M$ are such that $\text{Re}(\zeta_i) < 0$ ($\text{Re}(\zeta_i) > 0$). If neither of these cases are achieved, the critical point is called a saddle point $[15]$. We have summarized the fixed points for the autonomous system (3.9)-(3.12) and their stability properties in Table I.

| Point | $x$ | $y$ | $z$ | $s$ | $\Omega_c$ | Existence | Stability |
|-------|-----|-----|-----|-----|-----------|-----------|----------|
| $P_1$ | 1   | 0   | 0   | 0   | 1         | All $\lambda$ and $\gamma$ | Saddle point |
| $P_2$ | -1  | 0   | 0   | 0   | 1         | All $\lambda$ and $\gamma$ | Saddle point |
| $P_3$ | $\frac{\lambda}{\sqrt{3}}\sqrt{y_0}$ | $y_0$ | 0 | 0 | 1 | All $\lambda$ and $\gamma$ | Saddle point |
| $P_4$ | 0   | 0   | 0   | 1   | 0         | All $\lambda$ and $\gamma$ | Saddle point |
| $P_5$ | $\sqrt{7}$ | $\frac{3\gamma}{2}$ | 0 | $s_0$ | $1 - s_0$ | All $\lambda$ and $\gamma < \gamma_1 < 1$ | Unstable point |
| $P_6$ | $-\sqrt{7}$ | $\frac{3\gamma}{2}$ | 0 | $s_0$ | $1 - s_0$ | All $\lambda$ and $\gamma < \gamma_1 < 1$ | Unstable point |
| $P_7$ | 1   | 0   | 0   | 1   | 0         | All $\lambda$ and $\gamma$ | Saddle point |
| $P_8$ | -1  | 0   | 0   | 1   | 0         | All $\lambda$ and $\gamma$ | Saddle point |

Table I: Summary of critical points and their properties

Point $P_1$— The eigenvalues of this fixed point are $\zeta_1 = -2, \zeta_2 = -1, \zeta_3 = +1$ and $\zeta_4 = \gamma - 1$. All characteristic values of this point are real, but at least one is positive and two are negative, thus, the trajectories approach this point on a surface and diverge along a curve; this is a saddle point.

Point $P_2$— For this fixed point, the characteristic values are $\zeta_1 = -2, \zeta_2 = -1, \zeta_3 = +1$ and $\zeta_4 = \gamma - 1$, which are the same eigenvalues as the fixed point $P_1$, and thus, similar to the $P_1$, this is a saddle point.

Point $P_3$— This fixed point has eigenvalues $\zeta_1 = 0, \zeta_2 = y_0^2 + \lambda^2 y_0/6 > 0, \zeta_3 = \gamma_1$ and $\zeta_4 = (\gamma - \gamma_1)$, where, $y_0 := -\lambda^2/6 + \sqrt{(\lambda^2/6)^2 + 1}$ and $\gamma_1 := \lambda^2 y_0/3$. For $\gamma > \gamma_1$ this point is not stable; for $\gamma < \gamma_1$ this is a saddle point.

Point $P_4$— The eigenvalues read $\zeta_1 = +1, \zeta_2 = -\gamma, \zeta_3 = +\gamma$ and $\zeta_4 = -\gamma$. For $\gamma \neq 0$, this point possesses eigenvalues with opposite signs; therefore, this point is saddle. For the case $\gamma = 0$, this point has one real and positive eigenvalue, and others are zero, so $P_4$ is not a stable point.

Point $P_5$— This point is located at $(\sqrt{7}, 3\gamma/\lambda^2, s_0)$, where $s_0 := \left(1 - \frac{3\gamma}{\lambda^2\sqrt{1 - \gamma}}\right)$. The eigenvalues for this fixed point are $\zeta_1 = 0, \zeta_3 = \gamma$ and $\zeta_2, 4 = \frac{2 - \gamma \pm \sqrt{(1 - \gamma)(4 - 16s_0\gamma) + \gamma^2}}{4}. \quad (3.15)$

For $\gamma < \gamma_1$, all eigenvalues are non-negative, and for $\gamma = \gamma_1$, we have $\zeta_2 > 0$ and $\zeta_4 = 0$. Therefore, this point is not a stable fixed point. Notice that, since $0 < s_0 < 1$, the barotropic parameter $\gamma$ must hold the range $\gamma < \gamma_1 < 1$ for this fixed point.

Point $P_6$— The eigenvalues of this point are the same as the point $P_5$, so that this is not a stable point. Furthermore, the existence condition for this point implies that the barotropic index satisfies $\gamma \leq \gamma_1 < 1$.

Point $P_7$— The eigenvalues for this fixed point are $\zeta_1 = -2, \zeta_2 = -\gamma, \zeta_3 = \gamma$ and $\zeta_4 = 1 - \gamma$. At least one characteristic value is negative and one is positive,
so $P_7$ is a saddle point.

Point $P_8$ For this point, the eigenvalues are similar to those of point $P_7$, i.e., $\zeta_1 = -2$, $\zeta_2 = -\gamma$, $\zeta_3 = \gamma$ and $\zeta_4 = 1 - \gamma$. Therefore, this is a saddle point.

In the standard general relativistic collapse of a tachyon field with barotropic fluid [13], the fixed points $(x_c, y_c, s_c) = (1, 0, 0)$ and $(x_c, y_c, s_c) = (-1, 0, 0)$ are stable fixed points (attractors) and correspond to a tachyon dominated solution; therein, the collapse matter content behaves, asymptotically, as a homogeneous dust-like matter which leads to a black hole formation at late times [13]. Nevertheless, in the semiclassical regime herein, in the presence of the loop (quantum) holonomy correction term $z \neq 0$, these fixed points become a saddle, so that the stable points (i.e., the singular black hole solution) of the classical collapse disappears here.

The points $(x_c, y_c, s_c) = (1, 0, 1)$ and $(x_c, y_c, s_c) = (-1, 0, 1)$, in the classical regime (in the absence of the $z$ term), correspond to the stable fixed points (attractors), namely the fluid dominated solutions, and lead to the black hole formation as the collapse end state [13]. Nevertheless, holonomy effects, in the presence of the $z$ term induce respectively, the corresponding saddle points $(x_c, y_c, s_c, z_c) = (1, 0, 1, 0)$ and $(x_c, y_c, s_c, z_c) = (-1, 0, 1, 0)$ for the collapsing system, instead. This means that the classical singular black holes are absent in the semiclassical regime herein.

In figure 1 we show a selection of numerical solutions of the dynamical system Eqs. (3.9)-(3.12), in terms of the variables $(x, y, z, s)$. This figure represents trajectories which start from the lower $x - y$ plane and evolve in the phase space. These trajectories will initially converge to a point where $\dot{\rho} \rightarrow -1$, along the $x - y$ plane; however, in the vicinity of this point, they diverge along the $y - z$ plane and move away from it. This point can be identified to be the saddle fixed fixed points $P_2$ or $P_8$.

However, it is pertinent to point the following. Figure 1 involves parametric functions $x(N)$, $y(N)$ and $z(N)$. The numerical solution shows that the variable $N$ is only defined on a finite interval $[0, N_{boune}]$; this can be seen from Eq. (3.1) in which the scale factor is bounded from below, i.e., $u_{min} < a < a_0$. In fact, and contrasting with the classical solution [13], where $x(N \rightarrow \infty) \rightarrow \pm 1$ and $y(N \rightarrow \infty) \rightarrow 0$ are asymptotic limits, in the herein semiclassical scenario, the variable $N$ is bounded at the bounce. This boundary is shown in figure 1 where the curves end at a region where $z \rightarrow 1$ (identified as point B in the plot), which consequently, cannot be classified as a fixed point of the dynamical system. The numerical study supports the analytical discussion that the solutions in [13] for points $P_1$, $P_2$, $P_7$ and $P_8$, is now avoided on the semiclassical trajectories. In addition, for all the trajectories on the phase space shown in figure 1 point B corresponds to a bouncing scenario which we will analyse it on the next section.

IV. SEMICLASSICAL COLLAPSE END STATE

In this section we present additional results related to the numerical studies of Eqs. (2.8)-(2.18).
γ < 1 it is the tachyon field that is dominant. In the right plot we represent the evolution of effective pressure, for the three solutions, until the bounce is reached.

A. Tracking solutions: Tachyon versus barotropic fluid

Figure 2 show the behaviour of the scale factor. Therein we observe that in the limit ρ → ρcr, when the Hubble rate vanishes, the classical singularity is replaced by a bounce (cf. figure 2). In figure 3 we represent the energy densities, ρφ(t) (left plot) and ρφ(t) (right plot), for different values of the barotropic parameter γ at the bounce. We see that three scenarios have to be considered. When the energy densities of the tachyon and of the fluid scale exactly at the same power of the scale factor, namely

\[ ρ_φ ≈ ρ_φ ≈ ρ_0 a^{-3γ}, \]  

(4.1)

then the semiclassical solutions display a tracking behaviour [16, 17]. Numerical analysis shows that this happens when the barotropic parameter is approximately γ ∼ 1, that is, the collapse matter content acts like dust. From Eq. (4.1), we have \( a_{\text{crit}} = [ρ_{\text{crit}} / (2 ρ_φ)]^{-1/3} \) at the bounce for the tracking solution. In the case where γ > 1 the solution at the bounce is fluid dominated, whereas for γ < 1, the tachyon field is the dominant component of the energy density content of the system.

In addition, from figure 3 we also observe that, starting from very low values of the energy density (classical regime), a system that is fluid dominated reaches the bounce faster than a system that is tachyon dominated. This seems to point to the fact that a fluid dominant solution will drive the energy density until its critical value more efficiently that when the tachyon field is dominant. In order to explain this result, let us consider what happens to the total pressure \( p = (γ - 1) p_γ \). For the tracking solution (γ ∼ 1), we have p ∼ 0 and the matter content behaves as dust. Finally, for the fluid dominated solutions, the total pressure is approximately p ∼ (γ - 1) ργ, which is positive because in this case γ > 1. Consequently, in this last scenario, the positive pressure drives the fluid dominant content of the energy density rapidly towards its critical value ρ → ρcrit at the semiclassical bounce.

In addition, when we consider Eq. (2.9) for the effective pressure, in particular its value at the bounce (where ρ → ρcrit),

\[ \begin{align*}
\rho_\text{eff}^\phi &\approx −(γ - 1) ρ_γ − ρ_\text{crit} & γ < 1 \\
\rho_\text{eff}^{tr} &\approx −ρ_\text{crit} & γ ≈ 1 \\
\rho_\text{eff}^γ &\approx −(γ - 1) ρ_γ − ρ_\text{crit} & γ > 1
\end{align*} \]

(4.3)

we can establish that \( \rho_\text{eff}^\phi < \rho_\text{eff}^{tr} < \rho_\text{eff}^γ < 0 \) (see right plot of figure 3). In this plot we have that for the fluid dominated solution, the effective pressure start at a positive value (assisting the collapsing system energy density rapidly towards its critical value ρ → ρcrit). However, near the bounce, the effective pressure rapidly switches to negative values. In contrast, for the tachyon dominated solution, the effective pressure starts from negative values from the beginning; this is related to the fact that the initial energy densities of both the tachyon and barotropic fluid are approximate. Moreover, the change near the bounce is less pronounced in this last case. Therefore the evolution of the collapse is slower and the bounce is delayed when compared to the fluid dominated scenario. It is straightforward to verify that the tracking solution provides an intermediate context between the fluid and tachyon dominated solutions.

B. Horizon formation

From the equation \( R^2(t, r_b) = 1 \) (where \( r_b \) is the radius of the boundary shell) we can determine the speed of the collapse, \( |v|_{\text{AH}} \), at which horizons form, i.e., \( |v|_{\text{AH}} = \frac{1}{r_b} \).
When the speed of collapse, $|\dot{a}|$, reaches the value $1/r_b$, then an apparent horizon forms. Thus, if the maximum speed $|\dot{a}|_{\text{max}}$ is lower than the critical speed $|\dot{a}|_{\text{AH}}$, no horizon can form. More precisely, in order to discuss the dynamics of the trapped region in the perspective of the effective dynamics scenario, we consider $|\dot{a}|$ from Eq. (2.6) to be equal to $|\dot{a}|_{\text{AH}} = 1/r_b$. Solving this new equation for $\rho$ and $a$ we get scale factors and energy densities at which the horizon forms. Figure 4 represents the speed of the collapse, $|\dot{a}|$, as a function of the scale factor, reaching the maximum value $|\dot{a}|_{\text{max}}$.

The tachyon field equation (2.18) implies that $\dot{\phi} \equiv \phi (a)$. Therefore, from Eqs. (2.13)–(2.15) we can also establish that the total energy density can be expressed as a function $\rho \equiv \rho (a)$. Then, we can rewrite $|\dot{a}| = \frac{1}{r_b}$ by setting $X := \rho/\rho_{\text{crit}}$ and $a^2 := f(X)$ as

$$f(X) (1 - X) - A = 0 ,$$

where $A := 3/(8\pi G \rho_{\text{crit}} r_b^2)$ is a constant. The study of roots of the Eq. (4.4) enables us to get the values of energy density at which an apparent horizon forms. Considering more closely Eq. (4.4), we need to estimate the behaviour of the function $f(X)$. In figures 2 and 3 we have that $f(X)$ is minimum when $X$ is maximum. It is also expected that, since $f(X)$ is a monotonically decreasing function near the bounce, Eq. (4.4) is essentially described as a second order polynomial. Therefore, depending on the initial conditions, in particular on the choice of the $r_b$, three cases can be evaluated, which correspond to no apparent horizon formation ($A/f(X) > 1/4$), one and two horizons formation ($A/f(X) \leq 1/4$).

Let us introduce a radius $r_*$, defined by

$$r_* := \frac{1}{|\dot{a}|_{\text{max}}} .$$

We see that $r_*$ determines a threshold radius for the horizon formation; if $r_b < r_*$, then no horizon can form at any stage of the collapse. The case $r_b = r_*$ corresponds to the formation of a dynamical horizon at the boundary of the two spacetime regions. Finally, for the case $r_b > r_*$ two horizons will form, one inside and the other outside of the collapsing matter.

The behaviour of the three possible scenarios (tracking solution, tachyon and fluid dominated solutions) are also represented in figure 3. Therein, we note that only one horizon forms for some particular tachyon dominated solutions. Therefore, for these solutions the bounce will be covered by an horizon. In order to further clarify this aspect, we note that when more than one horizon forms, the speed of the collapse $\dot{\rho}$ must have a local maximum. In that case, the acceleration must be $\ddot{\rho} = 0$, and from equations (2.6), (2.8), we can determine that this local maximum can be found by imposing

$$\ddot{\rho} \equiv \ddot{H} + H^2 = -\frac{\rho_{\text{eff}}}{2} = 0 .$$

This last condition, being equivalent to $\rho_{\text{eff}} = -3 \rho_{\text{eff}}$, must be closely monitored for the three different solutions discussed herein this section. For the fluid dominated solution, and since the effective pressure starts from positive values and evolve to negatives one near the bounce, it is straightforward to verify that the function $-3 \rho_{\text{eff}}$ must intersect $\rho_{\text{eff}}$ at some point before reaching the bounce. For the tracking solution we can use the same argument but with an initial effective pressure starting near zero and reaching $3 \rho_{\text{crit}}$ at the bounce. Finally, the case of the tachyon dominated solution depends on the value of the barotropic parameter $\gamma$. When the initial values for the effective pressure and energy densities are $\rho_{\text{eff}} \approx - (\gamma - 1) \rho_{\gamma 0}$ and $\rho_{\text{eff}} \approx \rho_{\gamma 0} > \rho_{\gamma 0}$, respectively; then, if $\gamma > 2/3$, the argument given for the tracking and fluid dominated solution is also valid for this case. However, if $\gamma < 2/3$, there will be always one horizon forming. Besides taking $\gamma > 2/3$, if we consider an imbalanced initial energy density, with the tachyon being slightly dominant, i.e., $\rho_{\gamma 0} \geq \rho_{\gamma 0}$, a local maximum for $\ddot{\rho}$ will also be present.
Finally, the discussion of the final outcomes related to the herein semiclassical solution follows the one made in [11]. Therein, it is described that the fate of the collapsing star whose shell radius is less than the threshold radius \( r_\star \) points to the existence of an energy flux radiated away from the interior spacetime and reaching the distant observer. Herein, for a collapsing system whose initial boundary radius \( r_b \) is less than \( r_\star \), we analyze the resulting mass loss due to the semiclassical modified interior geometry. In particular, this analysis is only carried for the tracking solution or fluid dominated scenario, since the tachyon dominated solution develops an horizon, for \( \gamma < 2/3 \) and \( \rho_{\gamma 0} \geq \rho_{\gamma 0} \), before reaching the bounce. Let us designate the initial mass function at scales \( \rho \ll \rho_{\text{crit}} \), i.e., in the classical regime, as \( F_0 = (8\pi G/3)\rho_0 R_0^3 \), with \( \rho_0 = \rho_{\gamma 0} + \rho_{\gamma 0} \). For \( \rho \lesssim \rho_{\text{crit}} \) (in the semiclassical regime) we have, instead, an effective mass function \( F_{\text{eff}} \) given by Eq. (2.11). Then, the (quantum geometrical) mass loss, \( \Delta F/F_0 \) (where \( \Delta F = F_0 - F_{\text{eff}} \)), for any shell, is provided by the following expression:

\[
\frac{\Delta F}{F(a_0)} = 1 - \frac{F_{\text{eff}}}{F_0} = 1 - \sqrt{\frac{\rho}{\rho_0}} \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right).
\]

As \( \rho \) increases the mass loss decreases positively until it vanishes at a point. Then, \( \Delta F/F \) continues decreasing (negatively) until it reaches to a minimum at \( \rho = \rho_{\text{crit}}/3 \). Henceforth, in the energy interval \( \rho_{\text{crit}}/3 < \rho < \rho_{\text{crit}} \), the mass loss increases until the bouncing point at \( \rho = \rho_{\text{crit}} \), where \( \Delta F/F \to 1 \); this means that the quantum gravity corrections, applied to the interior region, give rise to an outward flux of energy near the bounce in the semiclassical regime. The previously described behaviour for the mass loss will be qualitatively identical with respect to the solution considered. Therefore the tachyon (when \( \gamma > 2/3 \) or the initial energy densities are \( \rho_{\gamma 0} > \rho_{\gamma 0} \)), fluid dominated or tracking solutions will exhibit the same profile for the mass loss. The only difference between these three cases, shown in the right plot of figure [5] is the value of the radius where the mass loss reaches the maximum \( \Delta F/F \to 1 \). In the last section we discussed the fact that the bounce occurring in the tachyon dominated solution is delayed compared with the other solutions. Consequently, the bounce (where \( \Delta F/F \to 1 \)) take place for a smaller value of the radius \( R \).

In the other case, where \( r_b \geq r_\star \), in which one or two horizon form, the exterior geometry can be obtained by matching the interior to a generalized Vaidya exterior geometry at the boundary \( r_b \) of the cloud. Following the method provided in Ref. [13], we can write the exterior metric in advanced null coordinates \( (v, R) \) as

\[
dS^2_{\text{ext}} = -f(R, v)dv^2 - 2dvR + R^2 d\Omega^2,
\]

where the exterior function is given by \( f(v, R) = 1 - 2GM(v, R)/R \). By applying the matching conditions at the boundary \( r_b \) we have that

\[
m(v, R) = M - \frac{3}{4\pi \rho_{\text{crit}}} \frac{M^2}{R^3} \]

where we have defined \( M := (4\pi/3)\rho_0 R_0^3 \) is the mass within the volume \( R^3 \). For the tracking solution we have

\[
M = \frac{4\pi}{3} \rho_0 R_0^3 R^{-3(\gamma - 1)}.
\]

In the limit case \( \gamma \sim 1 \), Eq. (4.10) reduces to \( M = M_0 = (4\pi/3)\rho_0 R_0^3 \). Figure [5] shows the numerical behaviour of the boundary function \( f(R) \) in the classical (dashed curve) and semiclassical regime (solid curves) for the cases of the initial masses \( r_b < r_\star \) and \( r_b \geq r_\star \). The later shows the behaviour of an exotic nonsingular black hole geometry which is different than its classical counterpart.

V. CONCLUSION AND DISCUSSION

In this paper we employed an effective scenario imported from LQG, namely the “holonomy” correction to the dynamics of the gravitational collapse whose matter content involves a self interacting tachyon field and a barotropic fluid. In contrast with the inverse triad modification [9], the corresponding effective Hamiltonian constraint lead to a quadratic density modification \( H^2 \propto \rho(1 - \rho/\rho_{\text{crit}}) \). It is expected that the quadratic density modification can dominate over the inverse volume correction [21]. This modification provides an upper limit \( \rho_{\text{crit}} \) for energy density \( \rho \) of the collapse matter, indicating that the gravitational collapse includes a non-singular bounce at the critical density \( \rho = \rho_{\text{crit}} \) (see also Ref. [11]).

Our aim was to enlarge the discussion on tachyon field gravitational collapse, extending the scope analyzed in Ref. [13], by investigating how the quantum gravity correction term \( -\rho^2/\rho_{\text{crit}} \) can alter the fate of the collapse. Using a dynamical system analysis, we subsequently found a class of solutions. Our analysis showed that, the corresponding stable fixed point (attractor) solutions in the classical general relativistic collapse are only saddle points in our semiclassical collapse; hence, the classical black hole and naked singularities provided in Ref. [13] are no longer present within the loop semiclassical regime. We found conditions to define a tachyon or fluid dominated regimes at the bounce, depending on the value of the barotropic parameter of the fluid. The transition from one regime to the other shows the emergence of a tracking solution where the collapsing matter behaves as dust. It was also observed that, considering scenarios starting with the same initial conditions, a fluid dominated solutions is driving the energy density until its
critical value at the bounce more rapidly than a tachyon field solution.

We further investigated, by means of numerical studies, the evolution of trapped surfaces during the collapse in order to determine its final state. We found a threshold radius for the collapsing matter cloud in order to form a black hole at late time stages. The physical modifications related to the semiclassical regime provided three cases for the trapped surfaces formation, depending on the initial conditions of the collapsing star. In particular, our solutions showed that, if the initial boundary radius of the collapsing cloud is less than a threshold radius, namely \( r_* \), no horizon forms during the collapse, whereas for the radius equal and larger than the \( r_* \), one and two horizons form, respectively. It is worthy to mention that for the tachyon dominated solutions the same scenario happens if the barotropic parameter is \( \gamma > 2/3 \) or the initial energy densities are \( \rho_{\phi 0} \geq \rho_{\phi} \). When \( \gamma < 2/3 \) and \( \rho_{\phi 0} < \rho_{\phi} \), one horizon always forms and the bounce will be covered.

The study of this effective scenario for a tachyon field and barotropic fluid matter content presents some similarity with the case where a homogeneous massless scalar field is considered instead [11]. In this context and for the case in which no horizon forms, we have showed that, as the collapse evolves, the energy density increases towards a maximum value \( \rho_{\phi} \) at the bounce. Moreover, in the herein semiclassical collapse, the effective energy density decreases leads to a positive mass loss near the bounce. This results in a positive luminosity near the bounce and gives rise to an outward energy flux from the interior region which may reach to the distant observer. In addition, in the cases in which one or two horizons form, the resulting exterior geometry corresponds to an exotic nonsingular black hole which is different from the Schwarzschild spacetime [17, 21].

The qualitative picture depicted from our toy model is strongly dependent on the choice of an homogeneous interior spacetime. Nevertheless, in a realistic collapsing scenario one should employ a more general inhomogeneous setting (see Ref. [22, 23], where a detailed introduction to recent techniques to handle inhomogeneous systems provides the ingredients on how to extend the limited homogeneous case). Furthermore, the effective theory that comprises modifications in the homogeneous dynamics, for the interior spacetime, may also modify the spacetime inhomogeneous structure [24]. In addition, when we apply homogeneous techniques, the quantum effects are restricted to the interior spacetime; whereas the outside spacetime is assumed to be a generalised Vaidya metric defined in classical general relativity. Some imprint of the interior quantum effects are transported to the outside, by imposing suitable matching conditions at the boundary surface, where it enters the Vaidya solution effectively through a nonstandard energy-momentum tensor. This procedure is also restricted by the fact that the full inhomogeneous quantization, also covering the exterior region, is expected to provide significant modifications to the spacetime structure. It is believed that these effects may not entirely be captured for a general Vaidya mass in a spacetime line element [25]. However, some indications on how the matter content, herein considered, might affect the bounce scenario may still be valid in a more general inhomogeneous setting.

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