THE DYNAMICS OF INTERNAL WORKING SURFACES IN MAGNETOHYDRODYNAMIC JETS

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ABSTRACT

The dynamical effects of magnetic fields in models of radiative Herbig-Haro jets have been studied in a number of papers. For example, magnetized radiative jets from variable sources have been studied with axisymmetric and three-dimensional numerical simulations. In this paper, we present an analytical model describing the effect of a toroidal magnetic field on the internal working surfaces that result from the presence of variability in the ejection velocity. We find that for parameters appropriate to Herbig-Haro jets, the forces associated with the magnetic field dominate over the gas pressure force within the working surfaces. Depending on the ram pressure radial cross section of the jet, the magnetic field can produce a strong axial pinch or, alternatively, a broadening of the internal working surfaces. We check the validity of our analytical model with axisymmetric numerical simulations of variable magnetized jets.

Subject headings: ISM: Herbig-Haro objects — ISM: jets and outflows — ISM: kinematics and dynamics — stars: magnetic fields — stars: pre–main-sequence — stars: winds, outflows

1. INTRODUCTION

It is now relatively certain that some Herbig-Haro jets have knot structures that are the result of a time variability in the ejection. For example, observations of some jets with organized structures of knots of different size (e.g., HH 30, 34, and 111; see Esquivel et al. 2007; Masciadri et al. 2002; Raga et al. 2002) can be reproduced surprisingly well with variable ejection velocity jet models. In this paper, we study the effect of the presence of a magnetic field on the evolution of a variable jet.

It is still an open question to what extent magnetic fields are important in determining the dynamics of Herbig-Haro jets. The associated problem of radiative MHD jets has been explored in some detail in the existing literature. Cerqueira & de Gouveia Dal Pino (1999) computed three-dimensional simulations of radiative MHD jets with different magnetic field configurations (at the injection point). Frank et al. (1998) carried out axisymmetric simulations of similar flows.

The problem of an MHD radiative jet ejected with a time-variable velocity has been explored with axisymmetric simulations by Gardiner & Frank (2000), Gardiner et al. (2000), Stone & Hardee (2000), O’Sullivan & Ray (2000), Frank et al. (2000), De Colle & Raga (2006), and Hartigan et al. (2007). Variable MHD jets were also explored with three-dimensional simulations by Cerqueira & de Gouveia Dal Pino (2001a, 2001b). The general conclusions that can be drawn from these simulations is that the internal working surfaces produced by the ejection variability are not affected strongly by a poloidal magnetic field. On the other hand, if the magnetic field is toroidal (or, alternatively, has a strong toroidal component), then the material within the working surfaces of the jet flow has a stronger concentration toward the jet axis.

Gardiner & Frank (2000) showed that in a jet with variable ejection velocity, the “continuous jet beam” sections between the working surfaces have a low toroidal magnetic field, which grows in strength quite dramatically when the material goes through one of the working-surface shocks into one of the knots. Here we present a simple analytical model from which we obtain the conditions under which the toroidal magnetic field will produce an axial compression of the internal working surfaces. The model is described in § 2. In § 3, we present axisymmetric numerical simulations in which we compare the working surfaces with and without a toroidal magnetic field, showing the effect described by the analytical model. Finally, in § 4 we present our conclusions.

2. RADIAL MOTION OF THE MATERIAL WITHIN AN INTERNAL WORKING SURFACE

2.1. General Considerations

Time variability in the ejection velocity leads to the formation of two-shock “internal working surfaces,” which travel down the jet flow. In a frame of reference that moves with one of these working surfaces, the flow takes the configuration shown in Figure 1, with material entering the shocked layer from both the upstream and downstream directions.

Let us consider an internal working surface within a cylindrically symmetric jet with a toroidal magnetic field configuration. The material in the jet beam’s cross section or within the working surface is subject to two radial forces: the magnetic pinch force,

\[ F_m = -\frac{B}{4\pi r} \frac{d}{dr} (rB), \]

where \( B \) is the toroidal magnetic field and \( r \) is the cylindrical radius, and the force due to the pressure gradient,

\[ F_p = -\frac{dP}{dr}, \]

through one of the working-surface shocks into one of the knots.
against the jet cocoon. The material in the working surface exits laterally, shocking this layer, the velocity along the jet axis is discontinuous jet segments enters the shocked layer from both the upstream and downstream directions. The $[B(r), P(r), v(r), \rho(r)]$ radial cross section of the preshock region produces a $[B_w(r), P_w(r), \rho_w(r)]$ cross section within the shocked layer (in this layer, the velocity along the jet axis is $0$; in the reference frame moving with the working surface). The material in the working surface exits laterally, shocking against the jet cocoon.

where $P$ is the gas pressure. The cross section of the jet is in lateral equilibrium when $F = F_m + F_p = 0$; it will be subject to a lateral expansion when $F > 0$ and to a compression when $F < 0$.

Let us now assume that the jet beam has a generic cross section of the form

$$\rho(r) = \rho_0 \rho(r), \quad B(r) = B_0 B(r), \quad v(r) = v_0 v(r),$$

where $\rho(r)$ is the density, $B(r)$ is the magnitude of the (toroidal) magnetic field, and $v(r) = v_j - v_0$ is the relative velocity at which the jet material (moving at velocity $v_j$) enters the working surface (which moves at a velocity $v_0$; see Fig. 1). The constants $\rho_0, B_0$, and $v_0$ correspond to characteristic values of the respective quantities, and $\rho(r), B(r)$, and $v(r)$ are dimensionless functions of the radius giving the radial dependence of the flow variables from $r = 0$ (the symmetry axis) out to $r = r_j$ (the outer radius of the jet beam). In principle, these three dimensionless functions are of order unity unless very strong changes in the flow variables occur across the jet cross section.

Let us now suppose that the material goes through the “Mach disk” shock of an internal working surface. If we assume that the shock is strong (i.e., that it is highly supersonic and super-Alfvénic), from the Rankine-Hugoniot equations for MHD (e.g., Draine & McKee 1993), the postshock radial cross section is given by

$$P^w_{\text{nr}}(r) = \frac{2}{\gamma + 1} \rho_0 v_0^2 \rho(r) v^2(r),$$

$$P^w_{\text{rad}}(r) = \frac{(8\pi)^{1/2} \rho_0^{3/2} v_0^{3/2} \rho(r) v^2(r)}{B_0 B(r)};$$

$$\rho^w_{\text{nr}}(r) = \frac{\gamma + 1}{\gamma - 1} \rho_0 \rho(r),$$

$$\rho^w_{\text{rad}}(r) = \frac{(8\pi)^{1/2} \rho_0^{3/2} v_0^{3/2} \rho(r) v^2(r)}{B_0 B(r)};$$

Here $P^w_{\text{nr}}(r), \rho^w_{\text{nr}}(r),$ and $B^w_{\text{nr}}(r)$ are the postshock gas pressure, density, and magnetic field cross sections for the case of a non-radiative shock, and $P^w_{\text{rad}}(r), \rho^w_{\text{rad}}(r),$ and $B^w_{\text{rad}}(r)$ are the cross sections for the case of a radiative shock in which the postshock gas instantaneously cools to the isothermal sound speed $c_w$. As stated above, equations (4)–(6) are derived in the case of a strong shock. In order to obtain these relations, it is also necessary to assume that the preshock Alfvénic Mach number has values smaller than $\sim M^w_a = (v/c_w)^2$.

The factors including the specific heat ratio $\gamma$ in equations (4)–(6) take the numerical values $2/(\gamma + 1) = \frac{1}{4}$ and $\frac{\gamma + 1}{\gamma - 1} = 4$ for the case of a monoatomic, nonrelativistic gas (i.e., for $\gamma = 5/3$). From here on, we will use these numerical values in order to simplify the equations.

Combining equations (4)–(6) with equations (1)–(2), we obtain the magnetic and gas pressure forces acting radially on the post–Mach disk material. The resulting magnetic force is

$$F^m_{\text{nr}} = \frac{4B_0^2}{r_{\text{f}}} F^m_{\text{nr}}(r), \quad F^m_{\text{rad}} = \frac{2\rho_0 v_0^2}{r_{\text{f}}} F^m_{\text{rad}}(r),$$

where the dimensionless force $f^m_{\text{nr}}(r)$ is given by

$$f^m_{\text{nr}}(r) = -B(r) \frac{r}{r_{\text{f}}} \frac{d}{dr} [\rho B(r)],$$

$$f^m_{\text{rad}}(r) = -\rho^{1/2}(r) v(r) \frac{r}{r_{\text{f}}} \frac{d}{dr} [\rho^{1/2}(r) v(r)].$$

The resulting gas pressure force is

$$F^p_{\text{nr}} = \frac{3\rho_0 v_0^2}{4r_{\text{f}}} F^p_{\text{nr}}(r), \quad F^p_{\text{rad}} = \frac{(8\pi)^{1/2} \rho_0^{3/2} v_0^2 c_w^2}{r_{\text{f}} B_0} F^p_{\text{rad}}(r),$$

where the dimensionless force $f^p_{\text{nr}}(r)$ is given by

$$f^p_{\text{nr}}(r) = -r_{\text{f}} \frac{d}{dr} [\rho(r) v^2(r)],$$

$$f^p_{\text{rad}}(r) = -r_{\text{f}} \frac{d}{dr} \left[ \frac{\rho^{1/2}(r) v(r)}{B(r)} \right].$$

2.2. Scaling Properties of the Magnetic and Gas Pressure Forces

Let us now consider the ratio $M/P$ between the moduli of the magnetic and gas pressure forces. From equations (7) and (9), we obtain

$$\frac{(M/P)_{\text{nr}}}{M^2_A} = \frac{64}{3M^2_A} \left| f^m_{\text{nr}}(r) \right|,$$

$$\frac{(M/P)_{\text{rad}}}{M^2_A} = \sqrt{2M^2_A} \left| f^m_{\text{rad}}(r) \right|,$$

where $M_A \equiv v_0/c_{A}$ is the Alfvénic Mach number [obtained with the characteristic velocity $v_0$ and the Alfvén velocity $v_A = B_0/(4\pi \rho_0)^{1/2}$], $M_{\text{nr}} = v_0/c_{\text{A}}$ is the sonic Mach number (calculated with the characteristic velocity $v_0$ and the postshock sound speed $c_w$ of the
radiative shock), and the functions \( f_m(r) \) and \( f_p(r) \) are given by equations (8) and (10), respectively. One can argue that if the dimensionless cross section of the jet (described by eq. [3]) is smooth, then \( f_m^\text{nr}(r) \), \( f_m^\text{rad}(r) \), and \( f_p(r) \) will have values of order unity.

In our derivation of the pressure force within the internal working surface, we have only considered the gradient of the postshock gas pressure. Of course, the fact that the working-surface material is free to leave through the sides of the jet beam will lead to an extra gas pressure gradient (directed outward), particularly in the case of a nonradiative flow. The dimensionless pressure cross section due to this effect is still likely to lead to a dimensionless force \( f_p(r) \sim 1 \).

Setting \( f_m^\text{nr}(r) \), \( f_m^\text{rad}(r) \), \( f_p^\text{nr}(r) \), and \( f_p^\text{rad}(r) \) to \( \sim 1 \), from equation (11) we obtain

\[
(M/P)_{\text{nr}} \sim 64/(3M_A^2), \quad (M/P)_{\text{rad}} \sim M_w^2/M_A. \tag{12}
\]

From these two estimates of the ratio between the magnetic and gas pressure forces, we conclude as follows:

1. For the nonradiative case, if the Alfvénic Mach number of the flow entering the Mach disk is large (e.g., \( M_A > 10 \)) we have \((M/P)_{\text{nr}} < 1\), and therefore the lateral expansion or contraction of the gas within the working surface will be governed by the gas pressure force.

2. For the radiative case, if we consider jets with given values for \( v_{\text{A}} \) and \( c_w \), it is clear that as the velocity \( v_0 \) increases, the ratio \((M/P)_{\text{rad}}\) increases (in proportion to \( v_0 \)). In particular, if we have flows with \( v_{\text{A}} \sim c_w \), the ratio of magnetic to gas pressure forces has values \((M/P)_{\text{rad}} \sim M_w\). Thus, for a Mach disk in the strong-shock regime the postshock magnetic pressure force will, under most conditions, dominate over the gas pressure force.

Therefore, for the nonradiative and the radiative cases, whether the jet material within the working surface expands or contracts in the radial direction is determined by the signs of \( f_m^\text{nr}(r) \) and \( f_m^\text{rad}(r) \), respectively (see eqs. [8] and [10]), provided that the Mach number of the jet has a value \( M_w > 10 \) or larger.

3. SIMULATIONS OF THE INTERNAL WORKING SURFACE OF A HERBIG-HARO JET

Let us now consider the case of a jet model with a “top hat” density and velocity initial cross section and an initial toroidal magnetic field cross section of the form

\[
B(r) = B_0(r/r_j).
\]

This kind of magnetic field cross section has been used in many previous simulations of radiative MHD jets (e.g., Gardiner & Frank 2000). With this cross section for the jet beam, we have

\[
\begin{align*}
f_m^\text{nr} = -2r/r_j, & \quad f_m^\text{rad} = -r_j/r, \tag{14} \\
f_p^\text{nr} = 0, & \quad f_p^\text{rad} = (r_j/r)^2. \tag{15}
\end{align*}
\]

In other words, the magnetic pressure force is directed toward the axis, and the gas pressure force (acting in the radial direction on the working-surface jet material) is zero for the nonradiative case and points outward for the radiative case.

We now compute models of a jet with this initial cross section and initial scales for the magnetic field \( B_0 = 0 \) (i.e., a purely hydrodynamic jet) and \( B_0 = 5 \mu G \). The jet is injected with a constant density \( n_j = 100 \text{ cm}^{-3} \), temperature \( T_j = 900 \text{ K} \), and radius \( r_j = 2 \times 10^{15} \text{ cm} \), and it moves into a homogeneous, unmagnetized environment of density \( n_{\text{env}} = 10 \text{ cm}^{-3} \) and temperature \( T_{\text{env}} = 9000 \text{ K} \). The injection velocity varies sinusoidally with time, with a period \( r = 20 \text{ yr} \), a half-amplitude of \( 150 \text{ km s}^{-1} \), and an average velocity of \( 300 \text{ km s}^{-1} \).

For the two chosen values of the magnetic field (0 and 5 \( \mu G \); see above and eq. [13]), we ran both nonradiative simulations and simulations in which we include the coronal ionization equilibrium cooling function of Dalgarno & McCray (1972). These simulations were run with the uniform-grid axisymmetric MHD code described in detail in De Colle & Raga (2006). The code uses a second-order upwind scheme, which integrates the MHD equations using a Godunov method with a Riemann solver. The Riemann problem is solved using primitive variables, and the magnetic field divergence is maintained close to zero using the constrained-transport method (Toth 2000). The computational domain, of \((5, 1) \times 10^{16} \text{ cm} \) (axial, radial) extent, is resolved with \( 2000 \times 400 \) grid points. A reflection condition is applied on the jet axis and on the \( z = 0 \) plane in the \( r > r_j \) region. An outflow condition is applied in the remaining grid boundaries.

The time-dependent ejection velocity of the jet leads to the formation of successive internal working surfaces that travel down the jet flow. It is possible to estimate the ratio \( M/P \) between the magnetic and pressure forces within the internal working surfaces by noting that the shock velocity (associated with the two working-surface shocks) has a value \( v \approx 150 \text{ km s}^{-1} \). In other words, the value of the shock velocity is approximately the same as the half-amplitude of the ejection velocity’s variability (see, e.g., Raga et al. 1990).

With this value of \( v \) and the initial jet density and temperature, we can compute \( M_A = v/v_A \approx 31 \) (where \( v_A = 4.8 \text{ km s}^{-1} \) for our \( B_0 = 5 \mu G \) value and our initial jet density) and \( M_w = 150 \) (for an assumed postcooling sound speed of \( 10 \text{ km s}^{-1} \)), and then we use equation (11) to obtain \((M/P)_{\text{nr}} \sim 0.02 \) and \((M/P)_{\text{rad}} \sim 10^3 \).

Therefore, the magnetic force should have little effect in the nonradiative simulations and result in similar structures for the internal working surfaces in the cases of magnetized and nonmagnetized jets.

Figure 2 shows that our numerical simulations do exhibit this effect. In this figure, we show the density stratification obtained for nonradiative jets with \( B_0 = 0 \) (left) and \( B_0 = 5 \mu G \) (right) after a \( t = 90 \text{ yr} \) time integration. It is clear that although the details of the flow are affected by the presence of a toroidal magnetic field, the general features of the two working surfaces produced within the computational domain are quite similar in the magnetized and nonmagnetized cases.

The fact that \((M/P)_{\text{rad}} \sim 700 \) (see above) implies that the magnetic pinch force should dominate the dynamics of the matter within the internal working surfaces. Our two radiative numerical simulations, shown in Figure 3, do exhibit this effect. In the magnetized simulation, the internal working surfaces become more strongly compressed toward the jet axis as they evolve (traveling away from the source), an effect not seen in the nonmagnetized radiative jet simulation.

Figure 4 zooms in on the knot situated at \( z \approx 3.5 \times 10^{16} \text{ cm} \) (the knot in the top half of Figs. 2 and 3) for our four computed models. This figure shows that in the nonradiative case, the two working-surface shocks have a separation that is similar to the diameter of the jet and that the density structures are very similar for the \( B_0 = 0 \) and \( B_0 = 5 \mu G \) models.

As expected, much higher densities are obtained in the radiative jet simulations. In this case, the working surface obtained
from the $B_0 = 5 \mu G$ model shows larger densities, a much higher concentration toward the jet axis, and larger separations between the working-surface shocks than in the $B_0 = 0$ model.

More complex profiles for the magnetic field and the pressure have been explored in the past by several authors (e.g., Gardiner & Frank 2000; Gardiner et al. 2000; Stone & Hardee 2000; O'Sullivan & Ray 2000; Frank et al. 2000; De Colle & Raga 2006), with results similar to those obtained with the simple magnetic field and pressure configuration presented here. As shown in § 2.2, the expansion or contraction of the material in the working surface is nearly independent of the initial profile of the pressure. On the other hand, the preshock magnetic field profile contributes to $f_{\mu}^{\text{rad}}$ (but not to $f_m^{\text{rad}}$) and to the value of

\[ M/P \]

For small values of the magnetic field (e.g., close to the jet axis), $M/P \leq 1$ and the gas pressure force dominates.

Also, we note that Begelman (1998) studied the development of pinch instabilities in nonradiative jets as as a result of the presence of a toroidal magnetic field. He found that the condition necessary for the development of the pinch instability is

\[
\frac{d \ln B}{d \ln r} > \frac{\gamma \beta - 2}{\gamma \beta + 2},
\]

where $\beta = 8\pi P/B^2$. In the case of a radiative working surface with a postshock region with $\beta \ll 1$ [corresponding to $(M/P)_{\text{rad}} \gg 1$], this condition reduces to $f_m < 0$.

4. CONCLUSIONS

It is a known result that internal working surfaces in radiative MHD jets with a toroidal magnetic field configuration form...
dense, axial structures, which do not appear in nonmagnetized jets. We have presented a simple analytical model with which we show that the strong jump conditions (applied to one of the working-surface shocks) imply that the magnetic force dominates over the gas pressure force within a radiative working surface and that the gas pressure force is dominant for a nonradiative working surface (provided that one has a shock Mach number of at least $M_w \sim 10$ and an Alfvénic Mach number not exceeding $M^2_w$).

Interestingly, the radial dependence of the toroidal magnetic field within a radiative working surface depends only on the cross section of the preshock ram pressure, $p_{\text{ram}}(r) = \rho(r)v^2(r)$, impinging on the shocks. From equation (8), one can see that if we have a $p_{\text{ram}}(r)$ that decreases toward the edge of the jet faster than $1/r^2$, then the magnetic force within the working surface will be directed outward and will tend to increase the width of the working surface.

We have run four simulations with a top-hat cross section for $p_{\text{ram}}$, which results in an axially directed magnetic pinch within the working surfaces, in complete consistency with our analytical model. We find that in the nonradiative case, the presence of a toroidal magnetic field has very little effect on the structure of the internal working surfaces. We also find that for the radiative case, the presence of a toroidal magnetic field produces a strong axial compression of the material within the internal working surfaces (see Fig. 4).

The analytical model presented in this paper can thus be used to decide what ram pressure and toroidal magnetic field cross section to use in a magnetized, radiative, variable jet simulation in order to produce internal working surfaces that show narrower or broader structures than what is obtained in nonmagnetized jet simulations. This might be a valuable tool when attempting to model the knots in specific Herbig-Haro jets and might provide a possible method for constraining the strength and the configuration of magnetic fields within such objects.

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