On the role of hard rescattering in exclusive diffractive Higgs production

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Abstract

We discuss the contribution of so-called semi-enhanced hard rescattering corrections to central exclusive diffractive Higgs production, $pp \rightarrow p + H + p$, at the LHC. We present arguments to show that these corrections are small. We confirm these expectations by considering HERA data for leading neutron production.

1 Introduction

Central exclusive diffractive processes offer an excellent opportunity to study the Higgs sector at the LHC in an exceptionally clean environment; for recent reviews see, for example, \cite{1}. The process we have in mind is

$$pp \rightarrow p + H + p$$ (1)

where the $+$ signs denote large rapidity gaps. Demanding such an exclusive process (1) leads to a small cross section \cite{2}. At the LHC, we predict

$$\sigma_{\mathrm{excl}}(H) \sim 10^{-4} \sigma_{\mathrm{incl}}(H).$$ (2)

In spite of this, the exclusive reaction (1) has the following advantages:

(a) The mass of the Higgs boson can be measured with high accuracy (with mass resolution $\sigma(M) \sim 1$ GeV) by measuring the missing mass to the forward outgoing protons, provided that they can be accurately tagged far away from the interaction point. Such a measurement can be done irrespective of the decay mode, and is at the heart of an LHC proposal \cite{3} to complement the central detectors by forward proton taggers in the 420m region from the interaction point.
(b) The leading order $b\bar{b}$ QCD background is suppressed by the P-even $J_z = 0$ selection rule \cite{4}, where the $z$ axis is along the direction of the proton beam. Therefore one can consider the observation of a Standard Model Higgs boson via $H \rightarrow b\bar{b}$, which is the main decay mode for a mass $M \lesssim 140$ GeV. Moreover, a measurement of the mass of the decay products must match the ‘missing mass’ measurement. It should be possible to achieve a signal-to-background ratio of the order of 1. For an integrated LHC luminosity of $\mathcal{L} \sim 60$ fb$^{-1}$ we expect about a dozen or so observable events for a Standard Model Higgs, after accounting for signal efficiencies and various cuts\footnote{See Ref. \cite{4} for early estimates of the signal-to-background ratio.}

(c) The quantum numbers of the central object (in particular, the C- and P-parities) can be analysed by studying the azimuthal angle distribution of the tagged protons \cite{6}. Due to the selection rules, the production of $0^{++}$ states is strongly favoured.

(d) There is a very clean environment for the exclusive process – the soft background is strongly suppressed.

(e) Extending the study to SUSY Higgs bosons, there are regions of SUSY parameter space where the signal is enhanced by a factor of 10 or more, while the background remains unaltered. Indeed, there are even regions where the conventional inclusive Higgs search modes are suppressed, whereas the exclusive diffractive signal is enhanced, and even such that both the $h$ and $H$ $0^{++}$ bosons may be detected \cite{7}.

2 The KMR estimate of $pp \rightarrow p + H + p$ at the LHC

The basic mechanism for the exclusive process, $pp \rightarrow p + H + p$, is shown in Fig. 1(a). The left-hand gluon $Q$ is needed to screen the colour flow caused by the active gluons labelled by $x_1$ and $x_2$. The $t$-integrated cross section is of the form \cite{2,8}

$$\sigma \sim \frac{\tilde{S}^2}{b^2} \left| N \int \frac{dQ_t^2}{Q_t^2} f_g(x_1, x'_1, Q_t^2, \mu^2) f_g(x_2, x'_2, Q_t^2, \mu^2) \right|^2,$$  \hspace{1cm} (3)

where $b/2$ is the $t$-slope of the proton-Pomeron vertex, and the constant $N$ is known in terms of the $H \rightarrow gg$ decay width. The factor, $\tilde{S}^2$, is the probability that the rapidity gaps survive against population by secondary hadrons. It has been omitted ($\tilde{S}^2 = 1$) in Fig. 1(a). We will consider it in a moment. The amplitude-squared factor, $|M_0|^2$, however, may be calculated using perturbative QCD techniques, since the dominant contribution to the integral comes from the region $\Lambda_{QCD}^2 \ll Q_t^2 \ll M_H^2$. The probability amplitudes, $f_g$, to find the appropriate pairs of $t$-channel gluons $(x_1, x'_1)$ and $(x_2, x'_2)$, are given by the skewed unintegrated gluon densities at a hard scale $\mu \sim M_H/2$.

Since the momentum fraction $x'$ transferred through the screening gluon $Q$ is much smaller than that ($x$) transferred through the active gluons ($x' \sim Q_t/\sqrt{s} \ll x \sim M_H/\sqrt{s} \ll 1$), it
Figure 1: Schematic diagrams for central exclusive Higgs production, $pp \rightarrow p + H + p$. The presence of Sudakov form factors ensures the infrared stability of the $Q_t$ integral over the gluon loop in diagram (a). It is also necessary to compute the probability, $\hat{S}^2$, that the rapidity gaps survive soft and semi-hard rescattering; the two possible types of contributions are shown in diagrams (c) and (d) respectively, where the dashed lines represent Pomeron exchanges (as in version (b) of diagram (a)). In addition to diagram (d), there is a ‘mirror-imaged’ enhanced diagram with the additional Pomeron instead being emitted from the upper proton, and an enhanced diagram with additional Pomerons being emitted from both protons and coupling to intermediate partons of the other proton. The expectation is that diagram (c) gives $\hat{S}^2 \approx 0.026$ at the LHC, whereas in the text we argue that the enhanced diagrams do not give a significant contribution.
is possible to express $f_g(x, x', Q_t^2, \mu^2)$ in terms of the conventional integrated density $g(x)$. A simplified form of this relation is \[ f_g(x, x', Q_t^2, \mu^2) = R_g \frac{\partial}{\partial \ln Q_t} \left[ \sqrt{T_g(Q_t, \mu)} x g(x, Q_t^2) \right], \] which holds to 10–20% accuracy. The factor $R_g$ accounts for the single log $Q^2$ skewed effect. It is found to be about 1.4 at the Tevatron energy and about 1.2 at the energy of the LHC.

Note that the $f_g$'s embody a Sudakov suppression factor $T$, which ensures that the gluon does not radiate in the evolution from $Q_t$ up to the hard scale $\mu \sim M_H/2$, and so preserves the rapidity gaps. The Sudakov factor is \[ T_g(Q_t, \mu) = \exp \left( - \int_{Q_t^2}^{\mu^2} \frac{\alpha_s(k_t^2)}{2\pi} dk_t^2 \right) \left[ \int_1^{1-\Delta} z P_{gg}(z) dz + \int_0^1 \sum_q P_{qg}(z) dz \right], \] with $\Delta = k_t/\mu$. The square root arises in (4) because the (survival) probability not to emit any additional gluons is only relevant to the hard (active) gluon. It is the presence of this Sudakov factor which makes the integration in (3) infrared stable, and perturbative QCD applicable.

In fact, the $T$-factors have been calculated to single log accuracy [4]. The collinear single logarithms may be summed up using the DGLAP equation. To account for the ‘soft’ logarithms (corresponding to the emission of low energy gluons) the one-loop virtual correction to the $gg \to H$ vertex was calculated explicitly, and then the scale $\mu = 0.62 M_H$ was chosen in such a way that eq.(5) reproduces the result of this explicit calculation. It is sufficient to calculate just the one-loop correction since it is known that the effect of ‘soft’ gluon emission exponentiates. Thus (5) gives the $T$-factor to single log accuracy.

Now we discuss the rapidity gap survival factor, $\hat{S}^2$. It has been calculated using an eikonal model which embodies all the main features of soft diffraction. A schematic diagram is shown in Fig. 1(c). The additional Pomeron may couple the upper and lower proton lines in all possible configurations. It is found to be $\hat{S}^2 \simeq 0.026$ for $pp \to p + H + p$ at the LHC. The uncertainty in the eikonal evaluation of $\hat{S}^2$ has been estimated to be $\pm 50\%$ [7, 10]. In this connection it is interesting to note that an alternative determination, based on a Monte Carlo calculation, also yields $\hat{S}^2 = 0.026$ at the LHC [11]. A review of the various determinations of $\hat{S}^2$, showing general agreement, can be found in [12]. Moreover the value $\hat{S}^2 = 0.024$ was found in the recent study described in [13], where the amplitude shown in Fig. 1(a) was denoted TPF (Two Pomeron Fusion). Actually the exclusive cross section is proportional to the factor $\hat{S}^2/b^2$, which is almost constant in the relevant interval $b = 4 - 6 \text{ GeV}^{-2}$ [14], where $b/2$ is the $t$-slope of the proton-Pomeron vertex.

### 3 Enhanced diagrams: theoretical uncertainties

Besides the uncertainties in the gap survival factor $\hat{S}^2$ caused by the soft eikonal rescattering of the incoming (outgoing) protons there is the possibility of an additional effect. The gap
may be filled by the secondaries created in the rescattering of the intermediate partons; see, for example, [15]. Formally this effect is described by the semi-enhanced (and/or enhanced) reggeon diagrams. One such diagram\(^2\) is shown schematically in Fig. 1(d). Since the intermediate gluons have a relatively large transverse momenta, there a possibility that the contribution may be evaluated within the framework of perturbative QCD. It is proportional to the QCD coupling \(\alpha_s\) times the density of gluons, generated by the lower proton in the rapidity interval occupied by the intermediate partons of the upper proton, that is \(f_g(x_4, k_{t,4}^2, ...)\). Here \(x_4\) and \(k_{t,4}\) are the momentum fraction of the lower proton and the transverse momentum carried by the \(t\)-channel gluon in the upper cell of the gluon ladder corresponding to the additional Pomeron in Fig. 1(d). Note that \(x_4\) can become very small, \(\sim 10^{-5}\).

The first detailed attempt to calculate such a contribution within the perturbative QCD framework has been performed in ref.[13]. They evaluated an amplitude of the form\(^3\)

\[
M_1 \sim \int \frac{dx_4}{x_4} \int \frac{d^2 q_t}{2\pi^2} \int \frac{d^2 k_{t,4}}{k_{t,4}^4} f_g(x_4, k_{t,4}^2, ...) \ V_{3P} \ M_0,
\]

where the unintegrated gluon density \(f_g(x_4, ...)\) was calculated using the Balitsky-Kovchegov equation [16], and where the leading log expression for the QCD triple-Pomeron vertex, \(V_{3P}\), was used [17]. Their result was that the enhanced diagrams give a rather large (negative) correction to exclusive Higgs boson production at the LHC energy. That is the exclusive Higgs signal may be significantly reduced.

However the computation of the enhanced diagrams has, itself, many unresolved uncertainties. First, the next-to-leading logarithmic (NLL) corrections to the triple-Pomeron vertex are not known at present. To see the possible effect that these could have, we note that in the original Reggeon phenomenological calculations a “threshold” was usually introduced, such that the rapidity interval between two Reggeon vertices must exceed \(\Delta Y = 2 - 3\) [18, 19]. An analogous effective repulsion between the two vertices of gluon emission has also been observed in the calculation of the NLL BFKL corrections [20]. The NLL correction, \(\omega_1\), to the intercept of the Pomeron trajectory is such that

\[
\omega_{\text{NLL}} = \omega_0 + \omega_1 \approx \omega_0(1 - 6.2\alpha_s),
\]

where \(\omega \equiv \alpha_P(0) - 1\), and where \(\omega = \omega_0 = (N_c\alpha_s/\pi)4\ln 2\) is the LO BFKL result. It turns out that the major part of this NLL correction is of pure kinematical origin [21]. On the other hand, in the presence of the “threshold” \(\Delta Y\) we have a behaviour \(\exp(\omega Y) \sim x^{-\omega}\) where the intercept is given by [22]

\[
\omega = \omega_0 \ e^{-\omega \Delta Y} = \omega_0(1 - \omega_0 \Delta Y + ...).
\]

\(^2\)The term enhanced diagram originates from Reggeon Field Theory. It means that, contrary to eikonal rescattering, we have an additional integration over the rapidity of \(V_{3P}\) vertex. This integration enhances the contribution of the given graph (rather than the whole amplitude) by an extra logarithm, arising from the available space in rapidity. Really Fig. 1(d), with one \(V_{3P}\) vertex is called a semi-enhanced diagram, whereas an enhanced diagram contains two \(V_{3P}\) vertices and hence two integrations over their rapidities.

\(^3\)Note that our triple-Pomeron vertex \(V_{3P}\) is defined slightly differently to that in ref. 13.
Thus, if we assume that the whole NLL correction is explained by the $\Delta Y$ threshold, then, on comparing the decrease of the intercept given by (7) and (8), we obtain the value

$$\Delta Y = 6.2/((4 \ln 2) N_c/\pi) \simeq 2.3,$$

which is very close to that coming from the original Regge phenomenology.

If, indeed, the NLL correction to the triple-Pomeron vertex has the form of a $\Delta Y = 2.3$ threshold, then it follows that the semi-enhanced correction will only contribute when the rapidity interval $\delta y = y_p - y_H = \ln(1/x_H)$ between the incoming proton and the vertex of Higgs boson emission becomes larger than $2\Delta Y$; since the interval between the rapidity of the triple-Pomeron vertex ($y_V$) and the proton, and the interval between the triple-Pomeron and Higgs vertices, both must exceed $\Delta Y$. That is, we must have

$$y_p - y_V > \Delta Y, \quad \text{and} \quad y_V - y_H > \Delta Y.$$

If we impose these requirements, then the semi-enhanced correction (considered in [13]) will not contribute significantly\(^5\) to the central ($y_H = 0$) exclusive production of a Higgs boson of mass $M_H > 140$ GeV at the LHC energy $\sqrt{s} = 14$ TeV, since the available rapidity interval $\delta y = \ln(\sqrt{s}/M_H) < 4.6$ is less than $2\Delta Y$. Even for $M_H = 120$ GeV the available phase space is minute.

Secondly, at the moment there are no experimental data which determine the partons in the region with $x < \sim 10^{-4}$. There is a tendency that at low $Q^2 < 2 - 3$ GeV$^2$ and $x < 10^{-3}$ for the gluon density to start to decrease with $x$ decreasing [23, 24]. Moreover, in some global analyses the gluon distribution is even negative for $Q^2 = 2$ GeV$^2$ and $x < 3 \times 10^{-4}$ [24]. A more detailed discussion of our present knowledge (and uncertainties) of the low-$x$ parton distributions can be found, for example, in [25].

Finally, we recall that infrared stability of the calculation of (6) is only provided by the so-called ‘saturation momentum’ $Q_s(x_4)$, below which the unintegrated gluon density $f_g$ becomes proportional to $k_t^2$. That is

$$f_g(x_4, k_{t, 4}, ...) \propto k_{t, 4}^2 \quad \text{for} \quad k_{t, 4} < Q_s(x_4).$$

Indeed, the dimension of the Pomeron loop $\int d^2 q_t$ integration is compensated by the infrared-type integral $\int d^2 k_{t, 4}/k_{t, 4}^4$. Here the infrared divergency is not protected by Sudakov factors, and the infrared cutoff is provided either by the inverse proton size or by the saturation momentum $Q_s$.\(^6\) The hope is that at very low values of $x_4$ the momentum $Q_s(x_4)$ is large enough for

\(^4\) $x_H$ is the proton momentum fraction carried by the Higgs boson.

\(^5\) We thank A.B. Kaidalov for emphasizing the crucial role of this threshold effect, see also [15].

\(^6\) When the essential values of the Pomeron loop momentum $q_t$ (and $k_{t, 4}$) are much smaller than the value of the gluon transverse momenta $Q_T$ in the loop which contains the Higgs ($gg \to H$) vertex, we can justify the validity of the same leading order (LO) P-even, $J_z = 0$ selection rule as in the original amplitude, Fig. 1(a), without the semi-enhanced correction.
perturbative QCD to be applicable. This is not excluded; however so far there is no experimental evidence (in the HERA data) to show the explicit growth of $Q_s(x)$ with decreasing $x$.

Thus the size of the correction crucially depends on the gluon density in the saturation (or, even, the infrared) region. The problem is that there is no established theoretical procedure to calculate the parton densities in this region, where many other more complicated multi-Pomeron graphs, not accounted for in the BK-equation, become important. In particular, the interactions between the gluons from two parallel ‘Pomeron-ladders’ are already not negligible at much lower HERA energies [26]. Clearly the series alternates in sign. The second enhanced correction with two Pomeron loops gives a positive contribution, and so on. This is why the authors of Ref. [13] wrote that “we can not consider our results as representing a reliable numerical final answer”. Moreover, note that in [13], just the first semi-enhanced Reggeon graph was considered. It was demonstrated by Abramovsky [27, 28] that the inclusion of more complicated Reggeon diagrams may strongly diminish the effective value of the triple-Pomeron vertex. In particular, it is found that including graphs with one and two extra Pomerons reduces the effective value of the triple-Pomeron vertex $V_{3P}$ by a factor of 4 [27].

From the formal point of view, if we work perturbatively and include only the first Reggeon diagrams, we can estimate the importance of the semi-enhanced correction by relating the ratio of the contributions to exclusive Higgs production, $\sigma_H(\text{Fig. 1(d)})/\sigma_H(\text{Fig. 1(b)})$, to the ratio $\sigma_{\text{SD}}/\sigma_{\text{tot}}$. Here $\sigma_{\text{tot}}$ and $\sigma_{\text{SD}}$ are the total and single diffractive dissociation cross sections respectively, as computed from Fig. 2. We see that we have the ratios of equivalent Regge diagrams. However, in the first ratio we need to include an AGK factor [29] of 4; one factor of 2 since the Higgs boson may be emitted from either the left or right Pomeron in Fig. 1(d), and another factor of 2 as the cross section is given by the square of the amplitude. Thus in terms of the simplest Regge diagrams we obtain

$$\frac{\sigma_H^{(d)}}{\sigma_H^{(b)}} = 4 \frac{\sigma_{\text{SD}}}{\sigma_{\text{tot}}} \left( \frac{\ln \sqrt{s/M_H^2}}{\ln(s/s_0)} \right) \approx 0.1$$

at the LHC, where the ratio in brackets is to allow for the different rapidity intervals available for the triple-Pomeron vertex, $V_{3P}$. The numerical evaluation of 0.1 is obtained using $\sigma_{\text{tot}} \simeq 100 \text{ mb}$ [19, 10, 30], $\sigma_{\text{SD}} \sim 10 \text{ mb}$, $\ln(s/M_H^2) \simeq 9$ and $\ln(s/s_0) \simeq 18$ for the LHC energy. This

$\sigma_{\text{SD}}$ is observed [31] to be already practically independent of energy by $\sqrt{s} \simeq 500 \text{ GeV}$. 
estimate of the size of the semi-enhanced contribution is much less than that given in [13]. The arguments employed in this paragraph, and in [13], are based on perturbative estimates using the simplest Reggeon graphs, whose validity is questionable close to the saturation regime. The true parameter of the perturbative series is not the QCD coupling $\alpha_S$, but the probability of additional interactions, which however tends to 1 as the saturation region is approached.

Let us discuss this in more detail. Note that, starting from perturbative theory, we arrive in the strong coupling regime. The main contribution comes from the rescattering of partons with low $k_t < Q_s(x_4)$, that is from the region where the probability of rescattering is of the order of 1. So we must consider the possibility of double counting. Indeed the calculation of the “soft” survival factor, $\hat{S}^2$, in [8] used the phenomenological $pp$-amplitude obtained from fitting to “soft” data. This amplitude, shown by the left vertical line in Fig. 1(c), already includes the enhanced Reggeon diagrams like that shown in Fig. 2; that is it accounts for the rescattering of the whole proton wave function including all the intermediate and “wee” partons. Thus we do not need to consider the contribution of Fig. 1(d), but instead the difference between the enhanced contributions to exclusive Higgs production and the enhanced corrections hidden in the phenomenological soft $pp$-amplitude. Here we may appeal to the Good-Walker approach [32]. Qualitatively, we expect that the component of the proton wave function, which contains the Higgs boson, will have smaller size and a smaller number of wee partons than in a normal proton. The probability of a soft rescattering for this component is, most probably, smaller, that is the gap survival factor is, most probably, smaller, that is the “experimental” elastic $pp$-amplitude. So, contrary to Ref. [13], we may find that the corrections from the enhanced diagrams could even enlarge the predicted exclusive Higgs cross section. However the probability of a soft rescattering is mainly driven by the spatial distribution of the valence quarks, so we do not expect the effect to be large (see e.g. [15, 33]).

We conclude that there are theoretical and phenomenological reasons why the semi-enhanced corrections are expected to be small at LHC energies, and will not appreciably affect the estimates, outlined in Section 2, obtained for the cross section of the exclusive process $pp \rightarrow p + H + p$. Indeed, first, the correction comes from the ‘saturation’ (or even the infrared) region, where the global parton analyses which include the low $x$ HERA structure function data, show that, at low $Q^2$, the gluon density decreases as $x$ decreases below $10^{-3} - 10^{-4}$. Moreover, the concept “gluon density ($f_g$)” is not well defined in the saturation domain. When we enter the strong coupling regime of saturation we have to rely more on phenomenological arguments. One of these is the ‘$\Delta Y$’ threshold effect, which arises from the NLL correction to the triple-Pomeron vertex; it is expected to strongly suppress the correction when $x_H > 0.01$. However there is a more direct way of checking the smallness of the semi-enhanced hard rescattering correction. To this we now turn.
4 Enhanced diagrams: experimental information

There is a good way to experimentally probe the importance of the semi-enhanced rescattering correction. It is the observation of leading neutron production in inelastic events at HERA, in which the neutron is measured with Feynman $x$ in the region $x_L \simeq 0.7 - 0.9$. This process, $\gamma p \rightarrow Xn$, is mediated by pion exchange. The gap corresponding to pion exchange may be filled by the secondaries produced in the rescattering of intermediate partons, in exact analogy with the case of exclusive Higgs production. Due to the relatively large values of the momentum fraction $(1 - x_L)$ transferred across the gap, here the rapidity interval available for the triple-Pomeron vertex is already large enough at HERA energies. Since the whole correction, after the integration over the rapidity of the triple-Pomeron vertex, is proportional to the available rapidity interval, which grows with the initial photon energy, one has to expect that the probability to observe a leading neutron (that is to observe a gap) must fall down with energy. However this is not observed experimentally. The leading neutron data can be found in [34]-[38], and a detailed analysis and discussion of the data is given in [39]. These HERA data show a flat dependence on the incoming photon energy; see, for example, Figs. 7 of [37], and Tables 14, 18 and Figs. 11, 12 of [35] which show, for fixed $Q^2$, the same probability\textsuperscript{8} to observe a leading neutron for values of $x_{Bj}$ which decrease by more than an order of magnitude corresponding to an increase of the photon laboratory energy by more than a factor of 10. The flat behaviour provides a strong phenomenological argument in favour of a small semi-enhanced correction.

Soon there will be another way to check experimentally the role of the semi-enhanced rescattering corrections. That is from the measurements of exclusive $\gamma\gamma$ production, $p\bar{p} \rightarrow p + \gamma\gamma + \bar{p}$, at the Tevatron, and subsequently at the LHC where large $\gamma\gamma$ masses should be accessible. Three candidate events have already been observed in Run II at the Tevatron [40]. These hint at a cross section that is even larger than that predicted by a calculation [41] based on a similar mechanism to that described in Section 2, that is without the semi-enhanced correction. Recall that the estimate of the correction in [13] significantly reduces the size of the exclusive cross section. These Tevatron data are preliminary, and we await definitive measurements over a range of masses of the $\gamma\gamma$ system. In particular, if measurements of $\gamma\gamma$ production with $M = 10-20$ GeV were available, it should be possible to confirm the prediction for the exclusive production of a SM Higgs with $M_H = 120-140$ GeV to the order of $30-50\%$. Moreover, if we account for the NLO corrections to $gg \rightarrow \gamma\gamma$ then the uncertainty could be reduced to $10-20\%$.

5 Conclusions

The prediction for the cross section of central exclusive diffractive production of new heavy objects at the LHC is very important. In particular, the Higgs production process $pp \rightarrow p+H+p$\textsuperscript{8}That is the same probability, $S^2$, to observe the rapidity gap associated with pion-exchange.
offers many advantages for experimentally probing the Higgs sector, and, indeed, in some regions of SUSY parameter space can even be the Higgs discovery mode. The expected Signal-to-Background ratio is promising, but the event rate (at least for a Standard Model Higgs) is low. It is therefore crucial to check the existing predictions. One recent check was carried out in Ref. [13]. In this paper the basic ingredients of the calculation outlined in Section 2 were confirmed. However the authors of [13] went a step further. Their aim was to quantify the possible importance of the so-called enhanced diagrams. Indeed, they calculated these contributions perturbatively and came to the conclusion that they could be significant, and could reduce the predicted event rate, although they drew attention to the limited validity of perturbative procedure. We therefore addressed this issue in this Note. In Section 3 we presented arguments which indicate that the enhanced corrections will be small at LHC energies, and will not appreciably affect either the value, or the uncertainty, of the previous predictions. One reason is that there is just not sufficient room in rapidity for the triple-Pomeron vertex. The LHC is a bit below threshold for this contribution to be important. Then, in Section 4, we described how measurements of leading neutrons at HERA clearly confirm the smallness of these enhanced corrections.

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