A hybrid Hausdorff distance track correlation algorithm based on time sliding window

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Abstract. In multi-sensor target tracking, track correlation is the key to the unification of global situation. Hausdorff distance has been applied to power fault elimination, point cloud data, medical measurement, image segmentation, vehicle trajectory recognition and other directions. To solve the problem of track correlation, a hybrid Hausdorff distance track correlation algorithm based on time sliding window is proposed. The hybrid Hausdorff distance based on position, speed and azimuth is defined, and the time sliding window is added on this basis. Simulation results show that the proposed algorithm can maintain a high correct correlation rate under the conditions of target maneuver, time asynchronism, and inconsistent radar sampling frequency, the algorithm is superior to traditional track correlation algorithm.

1. Introduction

Hausdorff distance can be used to measure the similarity of two sets, which is a measurement method of set distance. Hausdorff distance has been applied to power fault troubleshooting, point cloud data, medical measurement, image segmentation, vehicle trajectory recognition and other directions[1-2], and rich results have been achieved. In this paper, Hausdorff distance is applied to the multi-sensor track association problem, to compare the similarity degree between different track sets and determine whether the track is related or not[3]. Because Hausdorff distance is sensitive to wild values, data fluctuation will greatly reduce the accuracy of track correlation. Therefore, this paper improves the traditional Hausdorff distance, and introduces location, speed, azimuth and other information to propose a mixed Hausdorff distance judgment track correlation. In order to solve the time asynchronous problem caused by different starting time and sampling frequency of sensors in different regions, the Hausdorff distance based on time sliding window is proposed to change the traditional one-to-one point trace distance into one-to-three point trace distance, and time sliding window is added to improve the accuracy of track correlation under the time asynchronous situation.

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2. Algorithm principle

2.1 Traditional definition of Hausdorff distance

For two sets \( A = \{a_1, a_2, \ldots, a_m\} \), \( B = \{b_1, b_2, \ldots, b_n\} \), the traditional Hausdorff distance is defined as follows

\[
H(A, B) = \max(h(A, B), h(B, A))
\]

\[
h(A, B) = \max(\min \|b_j - a_i\|), \quad h(B, A) = \max(\min \|a_i - b_j\|), \quad a_i \in A, \quad b_j \in B.
\]

\( h(A, B) \) is called the forward Hausdorff distance between set \( A \) and set \( B \), is the maximum value of the minimum distance between data in set \( A \) and set \( B \). Similarly, \( h(B, A) \) is called the reverse Hausdorff distance, and the larger of \( h(A, B) \) and \( h(B, A) \) is called the Hausdorff distance between two sets.

2.2 Hausdorff distance based on location

Hausdorff distance is strongly influenced by a single point, because the outside noise in the process of radar detection, environment and other factors will cause the outliers, a dramatic increase of Hausdorff distance between collection and subsequent related calculation accuracy, thus to reduce the wild values. This paper uses the weighted thought improve the traditional Hausdorff distance, based on the location of the Hausdorff distance, assuming that there are two groups of track set \( X_A \), \( X_B \), \( X_A = \{(x_1^A, y_1^A), (x_2^A, y_2^A), \ldots, (x_n^A, y_n^A)\} \), \( X_B = \{(x_1^B, y_1^B), (x_2^B, y_2^B), \ldots, (x_n^B, y_n^B)\} \). The Hausdorff distance based on location is defined as follows

\[
h(X_A^{\text{dis}}, X_B^{\text{dis}}) = \sum (\min \|x_j^B - x_i^A\|) + \sum (\min \|y_j^B - y_i^A\|)
\]

\[
h(X_B^{\text{dis}}, X_A^{\text{dis}}) = \sum (\min \|x_j^A - x_i^B\|) + \sum (\min \|y_j^A - y_i^B\|)
\]

Among them, \( x_i^A, y_i^A \in X_A^{\text{dis}}, x_j^B, y_j^B \in X_B^{\text{dis}} \).

Taking radar \( A \) as an example, one-way Hausdorff distance is used to define the distance similarity between track sets as follows

\[
\text{Sim}_{\text{dis}} = 1 - \frac{h(X_A^{\text{dis}}, X_B^{\text{dis}})}{S_{\text{max}}}, \quad S_{\text{max}} = \max \|x_j^B - x_i^A\|
\]

2.3 Hausdorff distance based on speed

It is easy to realize track correlation in the case of small target batch number and simple track, but in the case of dense target, track crossing, bifurcation, merger, etc, it is easy to produce wrong correlation and missing correlation. Therefore, only considering the location factor as the standard for determining the track correlation is not comprehensive enough. The Hausdorff distance is introduced into the speed and azimuth Angle at all times of the track set as the standard for determining the track correlation.
For the track point \((x_i^A, y_i^A)\) at a certain time in the track set \(X_A\), \(t\) is the radar detection time interval, its velocity calculation formula is

\[
v_i^A = \sqrt{(y_i^A - y_{i-1}^A)^2 + (x_i^A - x_{i-1}^A)^2}
\]

(5)

The Hausdorff distance based on speed is defined as follows

\[
h(X_A^v, X_B^v) = \sum \left( \min \left\| v_i^A - v_j^B \right\| \right), h(X_B^v, X_A^v) = \sum \left( \min \left\| v_i^B - v_j^A \right\| \right), v_i^A, v_j^B \in X_A^v, v_j^B \in X_B^v
\]

(6)

The velocity similarity between track sets is defined as follows

\[
Sim_{v} = 1 - \frac{h(X_A^v, X_B^v)}{V_{\text{max}}} \quad V_{\text{max}} = \max \left\| v_j^B - v_i^A \right\|
\]

(7)

### 2.4 Hausdorff distance based on azimuth

When the target motion model is uniform linear motion, the realization of track correlation is relatively simple. However, in the actual situation, most targets do variable speed non-linear motion. Therefore, the Hausdorff distance based on azimuth is proposed as one of the indicators to measure the track correlation, which can improve the accuracy of track correlation under complex motion state.

For the track point \((x_i^A, y_i^A)\) at a certain time in the track set \(X_A\), its azimuth Angle is calculated as follows

\[
\theta_i^A = \arctan \left( \frac{y_i^A - y_{i-1}^A}{x_i^A - x_{i-1}^A} \right)
\]

(8)

The Hausdorff distance based on azimuth is defined as follows

\[
h(X_A^\theta, X_B^\theta) = \sum \left( \min \left\| \theta_i^A - \theta_j^B \right\| \right), h(X_B^\theta, X_A^\theta) = \sum \left( \min \left\| \theta_i^A - \theta_j^B \right\| \right)
\]

(9)

Among them, \(\theta_i^A \in X_A^\theta, \theta_j^B \in X_B^\theta\).

The azimuth similarity between track sets is defined as follows

\[
Sim_{\theta} = 1 - \frac{h(X_A^\theta, X_B^\theta)}{\theta_{\text{max}}} \quad \theta_{\text{max}} = \max \left\| \theta_j^B - \theta_i^A \right\|
\]

(10)

### 2.5 Mixed Hausdorff distance based on time sliding window

To sum up, combined with the Hausdorff distance based on location, velocity and azimuth, the mixed Hausdorff distance is defined as follows

\[
h(X_A, X_B) = W_1h(X_A^{\text{dis}}, X_B^{\text{dis}}) + W_2h(X_A^{v}, X_B^{v}) + W_3h(X_A^\theta, X_B^\theta)
\]

(11)

\[W_1 + W_2 + W_3 = 1, W_1 = 0.5, W_2 = 0.25, W_3 = 0.25\].

The mixing similarity between track sets is defined as follows

\[
Sim = R_1Sim_{\text{dis}} + R_2Sim_{v} + R_3Sim_{\theta}
\]

(12)

\[R_1 + R_2 + R_3 = 1, R_1 = 0.5, R_2 = 0.25, R_3 = 0.25\].

Different starting time and sampling frequency of radar in different places will lead to asynchronous time and misposition of track data, which will reduce the accuracy of track correlation. In order to solve this problem, a hybrid Hausdorff distance based on time sliding
window is proposed. The original one-to-one Hausdorff distance is changed into one-to-three Hausdorff distance to improve the accuracy of track correlation in the case of time asynchronism. As shown in figure 1, assuming that there is track sequence \( T_{A} \), \( T_{B} \) in track set \( X_{A} \), \( X_{B} \), the mixed Hausdorff distance \( h_{i}(X_{A},X_{B}) \) among \( (x_{i}^{A},y_{i}^{A}) \), \( (x_{i-1}^{A},y_{i-1}^{A}) \), \( (x_{i}^{B},y_{i}^{B}) \) and \( (x_{i-1}^{A},y_{i-1}^{A}) \) after adding the time sliding window is calculated according to Equations (11)-(12). Three sampling intervals are taken as the length of the sliding window, and all the track points in the sliding time window \( T_{A} \) are traversed successively.

![Fig. 1. Schematic diagram of time sliding window.](image)

To sum up, the hybrid Hausdorff distance based on time sliding window is defined as follows

\[
h_{i}(X_{A},X_{B}) = \frac{W}{3} \sum \min \left( \text{dist}[x_{i}^{A},(x_{i-1}^{A},y_{i-1}^{A})]+\text{dist}[y_{i}^{A},(y_{i-1}^{A},x_{i-1}^{A})]\right) + \frac{W}{3} \sum \min \left( \text{dist}[y_{i}^{B},(v_{i-1}^{A},v_{i}^{A})]+\text{dist}[\theta_{i}^{A},(\theta_{i-1}^{A},\theta_{i-1}^{A})]\right)
\]

\[
\text{dist}[a,b,c,d]\text{ represents the sum of the Hausdorff distance between } a, b, c \text{ and } d. \]

The comprehensive similarity between track sets is defined as follows

\[
\text{Sim}_{i} = R_{2}(1 - h_{\text{mix}}(X_{A}^{\text{dis}},X_{B}^{\text{dis}})) + R_{3}(1 - h_{\text{mix}}(X_{A}^{\text{sp}},X_{B}^{\text{sp}})) + R_{4}(1 - h_{\text{mix}}(X_{A}^{\text{an}},X_{B}^{\text{an}}))
\]

\[
h_{\text{mix}}(X_{A}^{\text{dis}},X_{B}^{\text{dis}}) = \sum \min \left( \text{dist}[x_{i}^{A},(x_{i-1}^{A},y_{i-1}^{A})]+\text{dist}[y_{i}^{B},(y_{i-1}^{A},x_{i-1}^{A})]\right)
\]

\[
h_{\text{mix}}(X_{A}^{\text{sp}},X_{B}^{\text{sp}}) = \sum \min \left( \text{dist}[v_{j}^{B},(v_{i-1}^{A},v_{i}^{A})]\right)
\]

\[
h_{\text{mix}}(X_{A}^{\text{an}},X_{B}^{\text{an}}) = \sum \min \left( \text{dist}[\theta_{j}^{B},(\theta_{i-1}^{A},\theta_{i-1}^{A})]\right)
\]

### 3. Simulation analysis

#### 3.1 Simulation environment

Assume that two 2D radars configured in different places carry out track correlation for targets in the common area, and the common detection area is a rectangle with a side length of 10km. The coordinates of radar A is (0,0) and radar B is (15km,0). The random error of radar A obeys the Gaussian distribution with mean value of 0 and variance of 2500m². The random error of radar B follows a Gaussian distribution with a mean of 0 and a variance of 3600m². Radar A and B detected data once every 1s, Radar A is started 0.2 seconds later than radar B, randomly generated track data and conducted 500 Monte Carlo simulation experiments.
3.2 Simulation results

Under the condition of simulation environment, the correct correlation rate curve of various algorithms under different target batch Numbers is shown in Figure 2. In simulation environment, the radar boot time is inconsistent, resulting in dislocation of track data. Target group as the increase of the number of all kinds of algorithm of correct association rate is falling, but in this paper, algorithm under the condition of time asynchronous track is correct association rate remained high, when the target group of 100 batches of correct association rate to stay above 80%, the simulation results show that in the case of asynchronous, outliers more time, this algorithm is better than the traditional nearest neighbor method and grey correlation method.

On the basis of simulation environment, change the ratio of sampling rate of radar A and B, and the correct correlation rate curve of various algorithms under different sampling ratio was shown in Figure 3. The nearest neighbor method determines whether the track is related according to the distance of the track points at each time, so it is sensitive to sampling, and the accuracy is greatly reduced when the sampling ratio difference is large. The algorithm presented in this paper and the grey correlation method can still maintain high-precision correlation in the case of different sampling ratios, and the performance of the algorithm presented in this paper is better than that of the grey correlation method. The correct track correlation rate can be maintained above 95% stably.

![Fig. 2. Correct correlation rate of various algorithms under simulation environment 4.](image1)

![Fig. 3. Correct correlation rates of various algorithms under different sampling ratios.](image2)

4 conclusion

In this paper, Hausdorff distance is applied to the track correlation problem, and the traditional Hausdorff distance is improved. A hybrid Hausdorff distance based on position, speed and azimuth is defined. On this basis, time sliding window is added. A hybrid Hausdorff distance track correlation algorithm based on time sliding window is proposed. Simulation results show that the proposed algorithm can maintain high accuracy and robustness under the conditions of target maneuver, time asynchronism, and inconsistent radar sampling frequency, and is superior to the traditional nearest neighbor method and grey correlation method.
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