Isospin Dependence of Mechanical and Chemical Instabilities in Neutron-Rich Matter

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Within nuclear thermodynamics and an isospin-dependent transport model we investigate respective roles of the nuclear mean field and the 2-body stochastic scattering on the evolution of density and isospin fluctuations in either mechanically or chemically unstable regions of neutron-rich matter. It is found that the mean field dominates overwhelmingly the fast growth of both fluctuations, while the 2-body scattering influences significantly the later growth of the isospin fluctuation only. Moreover, both fluctuations grow in mechanically unstable systems, while only the density fluctuation grows significantly in chemically unstable ones. Furthermore, the magnitude of both fluctuations decreases with the increasing isospin asymmetry because of the larger reduction of the attractive isoscalar mean field by the stronger repulsive neutron symmetry potential in the more neutron-rich matter. Finally, several experimental measurements are proposed to test these findings.

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The rapid advance in experiments with rare isotopes has opened up several new frontiers in nuclear science [1]. The planned Rare Isotope Accelerator (RIA) will further enhance the exploration of these forefronts dramatically [2]. Besides detailed information on the structure of exotic nuclei in many unexplored regions of the periodic chart, it is now possible to study novel properties of isospin-asymmetric nuclear matter with extreme neutron to proton ratios. Prospects for discovering new physics in warm neutron-rich matter of various densities that can be created transiently in nuclear reactions induced by exotic neutron-rich nuclei have generated much interest in the nuclear science community [3–9]. In particular, the density dependence of the symmetry term in the equation of state (EOS) has recently received much attention since it is among the most important but very poorly known properties of neutron-rich matter [10–15]. This term is very important to the mechanisms of Type II supernova explosions and neutron-star mergers. It also determines the proton fraction and electron chemical potential in neutron stars at β equilibrium. These quantities consequently influence the cooling rate of protoneutron stars and the possibility of kaon condensation in dense stellar matter [16–20].

Based on the EOS obtained from various microscopic many-body theories and phenomenological models [21–26], it has long been predicted that isospin-asymmetric nuclear matter under certain conditions can be mechanically or chemically unstable, i.e.,

\[
\left( \frac{\partial P}{\partial \rho} \right)_{T,\delta} \leq 0 \quad \text{(mechanical)},
\]

or

\[
\left( \frac{\partial \mu_n}{\partial \delta} \right)_{P,T} \leq 0 \quad \text{(chemical)},
\]

where \(P\), \(\mu_n\) and \(\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)\) are the pressure, neutron chemical potential and isospin asymmetry, respectively. In the mechanically or chemically unstable regions, small fluctuations in density and/or isospin-asymmetry are generally expected to grow. However, information about the respective growth rates of these fluctuations, how and why they may depend on the isospin-asymmetry of nuclear matter, seeds of fluctuations and their
isospin-dependence, how and which aspects of nuclear dynamics cause the growth of these fluctuations is rather rare. This information is vitally important for understanding the structure and stability of both neutron stars and radioactive nuclei as well as mechanisms of nuclear multifragmentation in reactions induced by neutron-rich nuclei. In this Rapid Communication, we obtain such information within nuclear thermodynamics and an isospin-dependent transport model. We also propose several experimental measurements to test our findings.

The boundaries of the mechanical and chemical instabilities in the configuration space of density $\rho$, isospin asymmetry $\delta$ and temperature $T$ can be established readily by using a thermal model for isospin asymmetric nuclear matter. We use here a Skyrme-type phenomenological EOS for isospin-asymmetric nuclear matter in the parabolic approximation. The energy per nucleon at $T = 0$ can be written as

$$e(\rho, \delta) = a \frac{\rho}{1 + \sigma} u^\sigma + \frac{3}{5} e_F u^{2/3} + S_0(\rho_0) \cdot u^\gamma \cdot \delta^2.$$  \hfill (3)

In the above $u \equiv \rho/\rho_0$ is the reduced density, $e_F$ is the Fermi energy in symmetric nuclear matter at normal density, and $a = -123.6$ MeV, $b = 70.4$ MeV and $\sigma = 2$ corresponding to a stiff nuclear EOS of isospin-symmetric nuclear matter. The last term is the symmetry energy whose density dependence is currently rather uncertain. We adopt here a parameterization used by Heiselberg and Hjorth-Jensen in their recent studies of neutron stars with $S_0(\rho_0) = 30$ MeV and $\gamma$ as a free parameter. We note that a value of about $\gamma = 0.6$ was obtained by fitting to the result of variational many-body calculations. The nucleon chemical potential in asymmetric nuclear matter $\mu_q$ (q=n for neutrons, q=p for protons) at temperature $T$ is thus

$$\mu_q = a u + bu^\sigma + v_q^{\text{asy}} + T \left[ \ln\left( \frac{\lambda_T^2}{2} \rho_2 \right) + \sum_{n=1}^{\infty} \frac{n + 1}{n} b_n \left( \frac{\lambda_T^2}{2} \rho_2 \right)^n \right],$$  \hfill (4)

where $\lambda_T = \left[ \frac{2\pi \hbar^2}{m_q T} \right]^{1/2}$ is the thermal wavelength of a nucleon. The coefficients $b_n$ are obtained from mathematical inversion of the Fermi distribution function. The single particle symmetry potential $v_q^{\text{asy}}$ is
\[ v_{\text{asy}}^0 = \pm 2 \left[ S_0 \cdot u^\gamma - 12.7u^{2/3} \right] \delta + \left[ S_0(\gamma - 1)u^\gamma + 4.2u^{2/3} \right] \delta^2, \] (5)

where “+” and “-” are for neutrons and protons, respectively. The corresponding pressure is

\[
P = \frac{1}{2} a \rho_0 u^2 + b \sigma \rho_0 \sigma + 1 u^{\sigma+1} + (S_0 \rho_0 u^{\gamma+1} - 8.5 \rho_0 u^{\sigma}) \delta^2 \\
+ T \rho \left\{ 1 + \frac{1}{2} \sum_{n=1}^{\infty} b_n (\frac{\lambda^2}{4})^n \left[ (1 + \delta)^{n+1} + (1 - \delta)^{n+1} \right] \right\}. \] (6)

We then use the Gibbs-Duhem relation

\[
\frac{\partial P}{\partial \rho} = \frac{\rho}{2} \left[ (1 + \delta) \frac{\partial \mu_n}{\partial \rho} + (1 - \delta) \frac{\partial \mu_p}{\partial \rho} \right] 
\] (7)

to find boundaries of mechanically unstable regions (ITS: isothermal spinodal) in the \( \rho - \delta \) plane for given temperatures. Since the chemical instability condition has to be evaluated at constant pressures, the following Maxwellian relation

\[
\left( \frac{\partial \mu_n}{\partial \delta} \right)_{T,P} = \left( \frac{\partial \mu_n}{\partial \delta} \right)_{T,\rho} - \left( \frac{\partial \mu_n}{\partial \rho} \right)_{T,\delta} \cdot \left( \frac{\partial P}{\partial \rho} \right)_{T,\delta}^{-1} \cdot \left( \frac{\partial P}{\partial \delta} \right)_{T,\rho} 
\] (8)

is used to find boundaries of the chemically unstable regions (DS: diffusive spinodal).

Shown in Fig. 1 are the boundaries of the mechanical (thick lines) and chemical (thin lines) instabilities in the \( \rho - \delta \) plane with \( \gamma = 0.5 \) at \( T = 5 \) (lower window), 10 (middle window) and 15 MeV (upper window), respectively. It is seen that the diffusive spinodal extends further out into the plane and envelopes the region of mechanical instability; the two regions of instability do no overlap. As the temperature increases, both instabilities become less prominent over a more narrow range of densities at smaller isospin asymmetries. These features are independent of the parameters of the EOS and are in good agreement with those based on more microscopic many-body theories \[21,23\]. In particular, we found that the variation of \( \gamma \) parameter has very little effect on the instability boundaries. Therefore, in the following a constant of \( \gamma = 0.5 \) is used. The results shown in Fig. 1 not only provide the motivations but also serve as a guidance for our following numerical simulations.

The evolution of isospin-asymmetric nuclear matter in the mechanically or chemically unstable regions is studied by using the isospin-dependent transport model \[1,2\]. The model
uses consistently the isospin-dependent EOS and the corresponding potentials as in the thermal model outlined above. Moreover, scatterings among neutrons and protons are fully isospin-dependent in terms of their total and differential cross sections as well as the Pauli blockings. Nucleons are initialized in momentum space using a Boltzmann distribution function in a cubic box of length $L_{\text{box}}$ with periodic boundary conditions. We have checked that the use of Fermi-Dirac distribution does not alter our results at all as we are mainly interested in the evolution of reduced fluctuations with respect to the initial state which is unstable against numerical fluctuations in the mechanical or chemical instability region. Thus, the initialization with the Boltzmann distribution is sufficient and much more efficient numerically. The box is further divided into cells of 1 $fm^3$ volume in which the average density $\rho_{\text{cell}}$ and isospin asymmetry $\delta_{\text{cell}}$ are evaluated. We used $10^4$ test particles per nucleon in evaluating the $\rho_{\text{cell}}$ and $\delta_{\text{cell}}$. Shown in Fig. 2 is an illustration of the evolution of a system initialized in the mechanically unstable region with $T_i = 5$ MeV, $\delta_i = 0.6$ and $\rho_i = 0.05$ $fm^{-3}$. The most interesting feature shown here is the gradually increasing isospin fractionation. This is indicated by the spreading of the initial system into regions with $\rho \leq \rho_i$ and $\delta \geq \delta_i$ and where $\rho \geq \rho_i$ but $\delta \leq \delta_i$. The variations of $\rho$ and $\delta$ with respect to their initial values can be characterized quantitatively by using, respectively, $\sigma_d(t) = (\bar{\rho}^2 - \rho_i^2)^{1/2}$ and $\sigma_\delta(t) = (\bar{\delta}^2 - \delta_i^2)^{1/2}$, where the average is over all cells. Furthermore, the degree of isospin fractionation can be quantified by using the ratio $(N/Z)_{\text{gas}}/(N/Z)_{\text{liquid}}$, where $(N/Z)_{\text{gas}}$ and $(N/Z)_{\text{liquid}}$ is the isospin asymmetry of the low ($\rho/\rho_0 \leq 1/8$) and high ($\rho/\rho_0 > 1/8$) density region, respectively.

Shown in Fig. 3 are the reduced variation with respect to the initial state in isospin asymmetry $\sigma_\delta(t) - \sigma_\delta(0)$, density $\sigma_d(t)/\rho_i$ and the degree of isospin fractionation $(N/Z)_{\text{gas}}/(N/Z)_{\text{liquid}}$ as a function of time for a system initialized at $T_i = 5$ MeV, $\rho_i = 0.05$ $fm^{-3}$ and $\delta_i = 0.2$, 0.6 and 0.9, respectively. As a reference and check of our approach, results (dash-dot lines) are also shown for a system initialized with $T_i = 15$ MeV and $\delta_i = 0.9$ where it is both mechanically and chemically stable. For this system, it is seen that both the isospin and density fluctuations stay almost constant and there is no isospin fractionation at
all. While for the system initialized in the mechanically (chemically) unstable region with $\delta_i = 0.2$ and 0.6 ($\delta_i = 0.9$), it is seen that both the isospin and density fluctuations grow faster with the decreasing isospin asymmetry $\delta_i$, and the opposite trend is observed for the strength of isospin fractionation. Moreover, both the density and isospin fluctuations grow in the mechanically unstable systems, while only the density fluctuation grows significantly in the chemically unstable ones. It is also interesting to note that the isospin fractionation happens later in the more neutron-rich matter. In the latter early fluctuations are smaller and thus take longer time to grow before they can trigger the isospin fractionation.

We now proceed to explore the seeds of fluctuations and their isospin dependence, and to investigate how and which aspects of nuclear dynamics are important in governing the growth of fluctuations. We shall also study the question why the more neutron-rich matter is more stable against both the density and isospin fluctuations. The evolution dynamics of asymmetric nuclear matter is governed by the isospin-dependent nuclear mean field and stochastic nuclear scatterings. In our approach, first-order effects of 2-body stochastic scatterings are included through the collision integral of the BUU equation. Therefore, there are two main seeds of fluctuations in our approach, i.e., the early numerical fluctuations from the random sampling of the initial state and the later 2-body stochastic nuclear scatterings. Both of them may lead to the growth of fluctuations by propagating through the nuclear mean field in the early and later stages of the evolution, respectively. Shown in the upper window of Fig. 4 are the average number of collisions per nucleon as a function of time for $\delta = 0.2$ and 0.9, respectively. Before about 40 fm/c, there is essentially no collision because of the strong Pauli blocking when the phase space nonuniformity due to the initial fluctuation is still small. Later, nuclear scatterings become important, moreover, they are more frequent with the decreasing isospin asymmetry. The less frequent 2-body scatterings observed with the higher $\delta$ is because of the isospin-dependence of both the nucleon-nucleon cross sections and the Pauli blocking rates. It is well known that the cross section for neutron-neutron scatterings is only about 1/3 of that for neutron-proton collisions at beam energies below about 1 GeV \[32\]. One also expects from Fermi statistics that neutron-neutron scatterings
are more strongly Pauli blocked in the more neutron-rich matter. To investigate the respective roles of the nuclear mean field and the 2-body scatterings, model studies by turning off the nucleon-nucleon collisions have been performed. As shown in the middle window of Fig. 4, the collisional seeds of fluctuations lead to the significant growth of the isospin fluctuation in the later stage of the evolution. However, they have very little effect on the growth of density fluctuations as shown in the lower window. A comparison of the results obtained with and without the collision integral indicates that the fluctuations are overwhelmingly dominated by the nuclear mean field. The observed isospin-dependence of the fluctuations can be understood from the interplay between the attractive isoscalar mean field and the repulsive symmetry potential for neutrons. The symmetry potential, as shown in Eq. 5, is repulsive for neutrons and attractive for protons and their magnitudes increase with the increasing isospin asymmetry. Thus, the resultant attractive mean field is weaker in the more neutron-rich matter. Because of the small number of scatterings, particularly in the early stage of the evolution, the growth of fluctuations is thus mainly determined by the strength and sign of the resultant nuclear mean field according to the linear response theory for asymmetry nuclear matter [25,33,34]. Based on the latter, the magnitude of both fluctuations can grow larger with the increasing strength of the attractive resultant mean field, and thus also with the decreasing isospin asymmetry $\delta$.

What are the important physical implications of our findings? We expect several effects that can be tested experimentally. It is well known that more neutron-rich systems are less bound and have smaller saturation densities. However, fluctuations and their growth are also important for determining the final state of a neutron-rich system, such as in the projectile fragmentation in producing exotic beams [35]. Our results above indicate that fluctuations actually have a compensating role to the lower binding energy in stabilizing neutron-rich system where fluctuations are smaller and do not grow as fast as in symmetric ones. Moreover, our results on the isospin fractionation accompanying the evolution of fluctuations indicate that the configuration of a more dense, isospin-symmetric region surrounded by a more isospin-asymmetric gas as in halo nuclei is a natural result of the isospin-dependent nuclear
dynamics. Furthermore, we expect that the multifragmentation and isospin fractionation in nuclear reactions induced by neutron-rich nuclei to happen on longer time scales compared to symmetric reactions of the same masses. These expectations can be tested by measuring products of multifragmentation, in particular, the neutron-neutron, proton-proton as well as fragment-fragment correlation functions, in comparative studies of isospin symmetric and asymmetric nuclear reactions [35–38].

In summary, we have investigated the isospin dependence of mechanical and chemical instabilities in neutron-rich matter within nuclear thermodynamics and the isospin-dependent transport model. We found that the mean field dominates overwhelmingly the fast growth of both density and isospin fluctuations, while the 2-body scatterings influence significantly the later, slower growth of the isospin fluctuation only. Moreover, both of the fluctuations grow in the mechanically unstable systems, while only the density fluctuation grows significantly in the chemically unstable ones. Furthermore, the magnitude of both fluctuations decreases with the increasing isospin asymmetry because of the larger reduction of the attractive isoscalar nuclear mean field by the stronger repulsive neutron symmetry potential in the more neutron-rich matter. Finally, several experimental measurements are proposed to test these findings.

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FIG. 1. Boundaries of the mechanical (thick lines) and chemical (thin lines) in the density-isospin asymmetry plane at a temperature of 5, 10 and 15 MeV, respectively.
FIG. 2. An illustration of the evolution of asymmetric nuclear matter in the density-isospin asymmetry plane. Each point in the scatter plots represent one cell of 1 $fm^3$ volume in a cubic box of side length $L_{box} = 10$ fm.
FIG. 3. Evolution of the reduced isospin fluctuation (upper window), reduced density fluctuation (middle window) and strength of isospin fractionation (lower window) in a cubic box of side length $L_{\text{box}} = 30$ fm and density $0.05$ fm$^{-3}$. The dash-dot lines are calculated with $T_i = 15$ MeV and $\delta_i = 0.9$; while the solid, dot and dashed lines are calculated with $T_i = 5$ MeV and $\delta_i = 0.2$, 0.6 and 0.9, respectively.
FIG. 4. The average number of successful nucleon-nucleon collisions per nucleon (upper window), the isospin (middle window) and density (lower window) fluctuations as a function of time. The initial conditions are the same as in Fig. 3. The dot and dash-dot lines are results of calculations without the 2-body scatterings. For comparisons on the same scale, a factor of 7 is multiplied to the isospin fluctuations with $\delta = 0.9$ in the middle window.