Crystallographic modelling of protein loops and their heterogeneity with Rappertk *

Swanand Gore and Tom Blundell
{swanand,tom}@cryst.bioc.cam.ac.uk
Department of Biochemistry, University of Cambridge
Cambridge CB2 1GA England

Abstract

Background All-atom crystallographic refinement of proteins is a laborious manually driven procedure, as a result of which, alternative and multiconformer interpretations are not routinely investigated.

Results We describe efficient loop sampling procedures in Rappertk and demonstrate that single loops in proteins can be automatically and accurately modelled with few positional restraints. Loops constructed with a composite cns/Rappertk protocol consistently have better $R_{\text{free}}$ than those with cns alone. This approach is extended to a more realistic scenario where there are often large positional uncertainties in loops along with small imperfections in the secondary structural framework. Both ensemble and collection methods are used to estimate the structural heterogeneity of loop regions.

Conclusion Apart from benchmarking Rappertk for the all-atom protein refinement task, this work also demonstrates its utility in both aspects of loop modelling - building a single conformer and estimating structural heterogeneity the loops can exhibit.

*This document is very similar to a chapter in SG’s PhD thesis submitted in Sept. 2007 to the University of Cambridge, England.
1 Introduction

X-ray crystallography has been the most popular protein structure determination technique of both pre- and post-genomic eras. The challenges of macromolecular crystallography are manifold - after the difficult steps of expression, purification, crystallization and data collection, there remains the final and important task of data interpretation in order to build a model which explains the observed diffractions. Structural interpretation requires overcoming the phase problem and often starts with partial and incorrect phases. Typically, semi-automatic iterative refinement is carried out, gradually improving the model’s quality as indicated by the $R$ and $R_{\text{free}}$ factors as well as decrease in covalent geometry and excluded volume violations. Although excellent softwares like CCP4 ([CCP4 (1994)]), Phenix ([Adams et al. (2002)]) and CNS ([Brunger et al. (1998)]) make this task possible, the structure refinement procedure remains manually-driven hence laborious and subjective. Due to this, the heterogeneity in structural interpretation of diffraction data is often ignored in favour of a single-conformer isotropic B-factor model.

Protein structure is important for its function. But very stable, rigid proteins cannot exhibit enzymatic activity. This suggests that proteins have to be stable enough to retain their fold yet dynamic enough to be functional. Both experimental and computational studies indicate that single-conformer interpretation of crystallographic data is not adequate to capture the native state dynamics which is largely conserved even in a crystal owing to its high solvent content ([Petsko (1996), Jensen (1997)]). Reporting a multiconformer interpretation of data will make use of the structure less misleading, especially in the analyses that depend on geometry such as shapes of binding sites, orientations of sidechains, detection of non-covalent interactions and so on. While multiple interpretations are necessary, they should be free from any bias such as that introduced when different crystallographers solve the same diffraction data. Multiconformer interpretation will be greatly facilitated by automated methods.

Thus multiple persuasive justifications emerge for automating the protein crystallographic refinement task: (a) capturing the dynamics of protein in the crystalline state (b) removing subjective bias from the refinement process and (c) reducing the need for precious human resource. But this goal is hard to achieve in practice. The under-determined nature of the problem (number of independent observation < number of parameters) prevents a straightforward solution by minimization. Even when sufficient restraints exist, minimization methods like conjugate gradient, steepest descent etc. suffer from the problem of local minima. Hence use of well-known features of proteins is unavoidable. Automatic pattern recognition in electron density is very successful in presence of high resolution data and good phases because it looks for such features ([Perrakis et al. (1997)]). But at medium resolution or given poor
phases, this strategy can get misled.

Our recent efforts with automated crystallographic refinement started with rappeR, which is a conformation sampling program for proteins and uses a genetic algorithm cum branch-and-bound (gabb) algorithm. DePristo et al. (2004) showed that multiple interpretations similar to the deposited structure are possible given the deposited data, and the divergence in interpretation is correlated to resolution. With rappeR, it was demonstrated (DePristo et al. (2005)) that when a protein structure is approximately known, it can be refined to native-like quality, unlike mdsa in CNS which may get stuck in local minima. Fundamental features of rappeR responsible for avoidance of local minima traps were (a) use of fine-grained, propensity-weighted $\phi - \psi$ maps for backbone sampling (b) use of backbone-dependent rotameric libraries (c) use of ideal Engh and Huber covalent geometry (d) mild use of electron density and positional restraints to guide the sampling process. Later Furnham et al. (2006) demonstrated that a low-resolution dataset can be rescued and interpreted semi-automatically to obtain structure of a system with great biological significance.

DePristo et al. (2005) observed that automatic refinement becomes less satisfactory as positional restraints become weaker: the structures could not be refined if the initial $C_\alpha$ perturbation was of order of 3Å or more. This is not unexpected because larger positional restraints dilute the information and would make the search harder. But often a practical problem encountered in crystallography is that of missing loops, i.e. knowing loop regions with far less $C_\alpha$ positional certainty than the regions with regular secondary structure. By definition, loops exhibit rich variability in backbone torsion angles. They are thought to be more dynamic than the protein secondary structural framework and also functionally more interesting. Thus it is important to use the available restraints as efficiently as possible to build loops despite weaker electron density and greater positional uncertainty while tolerating small positional errors in the framework.

After determining a single-conformer loop structure, the second important challenge is to estimate the structural variability of the loops. It is easy to see by generating artificial data that existence of structural heterogeneity for a loop results in confusing electron density. In general, partial occupancies result in weaker density than full occupancy. Sidechains of the same residue may occupy different density contours. Overlaps in conformations may lead to significant loss of shape information. These challenges can be expected to make the task only harder for minimization-based programs when refining a multiconformer structure.

Following the reformulation of rappeR as a versatile modular software called Rappertk (Gore et al. (2007)), it was essential to benchmark its performance for all-atom protein crystallographic refinement. Hence the first result in this work is the all-atom knowledge-based crystallographic refinement given the positional restraints for the entire protein, establishing
that a similar result as rapper \cite{DePristo2005} can be achieved. We then demonstrate that single loops in proteins can be reconstructed to a high quality with Rappertk using little positional information. This case is then extended to include all loop regions and a small error in the framework to show that the composite cns/Rappertk refinement approach is suitable in a realistic scenario. Finally, we ask whether single-loop heterogeneity can be modelled with collections of independently generated models or ensembles of conformers.

2 Methods

2.1 Overview of iterative refinement

Each step in refinement procedure consists of: (a) identification of residues which do not fit well into density, (b) finding contiguous bands of such residues, (c) rebuilding the bands with knowledge-based conformational sampling within restraints, (d) optimal sidechain placement of rebuilt sidechains and (e) refining the resulting model with cns.

The fit of a set of atoms to electron density is calculated as the correlation coefficient between the $\sigma_A$-weighted omit map and $F_c$ map for a region around 1Å of the atoms. The maps used are both generated by cns refinement script, hence they are described on the same grid. Following \cite{Kleywegt1996} and \cite{DePristo2005}, the correlation coefficient between the maps is calculated on the grid neighbourhood around atoms of interest:

$$CorrCoef = \frac{\sum \sigma_{omnit} \sigma_c}{\sqrt{\sum \sigma_{omnit}^2 \sum \sigma_c^2}}$$  \hspace{1cm} (1)

If the correlation is below 0.9, the atoms are flagged for rebuilding. Correlation is calculated on all atoms in residues and then on mainchains only, sidechains only and peptide atoms only. Ill-fitting sidechains are marked for sidechain reassignment whereas residues with ill-fitting peptide or mainchain or all-atoms are marked for all-atom reconstruction.

Once the residues are flagged for all-atom rebuilding, contiguous bands are identified and marked either as N-terminal, C-terminal or intermediate. Bands are then sampled in random order using the PopulationSearch algorithm. Each band is attempted 5 times and left as it was if it cannot be sampled within given restraints. Previously sampled bands are considered while sampling later bands. N and C terminal bands are built using forward and reverse techniques and weighted sampling of $\phi - \psi$ propensities. Building of intermediate regions is described in a later section.

Once all bands are sampled, all the resampled sidechains are reassigned using the optimal sidechain placement procedure described elsewhere \cite{Gore2007}.
2.2 Electron Density Ranker

Generally a single conformer model would refine better if its occupation of the $2F_o - F_c$ map is better - so a model within $1\sigma$ contour is more reliable than $0.5\sigma$. Thus, on output of each builder, a binary electron density restraint can be applied with a $\sigma$-level cutoff. But the quality of map is not uniform over all residues and hence such binary restraint is useful only for ensuring that model remains within positive electron density. Hence, in addition to that restraint, an analog ranker is used for electron density that ranks the possibilities and chooses the better ones. In the population search algorithm, the ranker asks more children to be generated at each conformation extension step than the population size (typically 5 times more), ranks them and chooses top-ranking ones to fill the conformation pool. The ratio of number of children generated to the population size is termed as the enrichment ratio. The electron density ranking scheme calculates score of a set of atoms by summing up the $\sigma$ values in a 1Å region around their coordinates. The effective $\sigma$ value is calculated by penalising the negative $\sigma$ and flattening the peaks by upper cutoff, the latter for better recognition of shape of density rather than spikes, say due to waters or ions. In addition to filtering children at each step, the ranker also chooses the best member from the conformational pool generated, which is returned as the sampled model for the band.

2.3 Symmetry-related clash checking

As described in [Gore et al. (2007)], Rappertk uses geometric caching implemented as Clashchecking grid for efficiently deciding whether atomic van der Waals spheres are overlapping. This excluded-volume restraint rules out many unproductive sampling trajectories. When loop positions are largely uncertain, the existence of symmetry-related images of the molecule around it acts as an excluded volume restraint to loop sampling. Rappertk uses the Clipper (Cowtan (2003)) libraries for crystallographic computing for symmetry-related calculation. Clashchecking grid uses Clipper’s Spacegroup class and symmetry operators therein to calculate the images of atoms to be added into the grid. Images within 20Å of the bounding box of given protein coordinates are considered. First the grid looks for any clashes between sampled coordinates and their images. Then it is verified that they do not clash against the rest of the coordinates or their images. In case of no clashes, the new coordinates and their images are added to the grid. Removing coordinates from the grid removes their images too.

2.4 Loop closure

The typical incremental sampling step in Rappertk builds $C_\alpha^i$ and $(i - 1)^{th}$ sidechain in the forward mode or $C_\alpha^i$ and $(i + 1)^{th}$ sidechain in the reverse mode. In this context, loop closure
can be formulated as finding the locations of mainchain atoms \( \{ C^{i-1}, O^{i-1}, N^i, C^{i}_{\alpha}, C^{i}, O^i, N^{i+1} \} \) and sidechains of \((i-1)^{th}, i^{th}\) and \((i+1)^{th}\) residues. Seamless loop closure of this kind is challenging because many conditions need to be met: (a) the covalent angles and lengths should be correct (b) \( \phi, \psi \) states of 3 residues should be in the allowed regions (c) two \( \omega \) angles should be adopt cis or trans conformation, but not be restricted to one or the other (d) 3 sidechains should be rotameric and (e) van der Waals restraints should be obeyed.

A sampling procedure was devised for meeting conditions (a), (b), (c) and (d), while (e) is met using clash-checking restraints. The sampling procedure is similar to the one described in Gore et al. (2007), but modified to meet the cis \( \omega \) state too. First, the two \( \omega \) angles are sampled, leading to the center, plane and radius of the circle on which the middle \( C^{\alpha} \) is sampled. The circle is uniformly sampled. For each sample, the \( \{ r, \alpha, \theta, \} - \{ \phi, \psi, \omega \} \) mapping is used to build the mainchain atoms. Then the three sidechains are sampled from a rotamer library. Sampling is continued until a conformation satisfying all restraints is found.

The problem with this sampling is that the restraint density abruptly increases at loop closure because it is not clear how to back-propagate the geometric requirements (a) and (c). Due to this, the sampling procedure fails often and takes a long time to find a valid sample. Often an incorrect conformation is built in case of imperfect density because sampling of \( \phi, \psi, \omega \) is not propensity-weighted. Hence after significant experimentation with this approach, it was abandoned in favour of a simpler approximate approach.

In the simpler approach, the loop closure is formulated as finding the coordinates of mainchain atoms \( \{ C^i, O^i, N^{i+1} \} \) and sidechains of residues \( i \) and \( i+1 \). A \( \phi \) sampler is used to build the \( C^i \) atom which is required to lie between 0.5 to 2Å from the \( N^{i+1} \) atom. Covalent angles \( N^i - C^i_{\alpha} - C^i \) and \( C^i - N^{i+1} - C^{i+1}_{\alpha} \) are restrained to lie between 90° and 150°. \( \omega \) dihedral angle \( C^i_{\alpha} - C^i - N^{i+1} - C^{i+1}_{\alpha} \) is allowed a maximum deviation of 30° from cis or trans conformation. Two sidechains are sampled for each \( C^i \) sampled. It is observed that it is more efficient to close a loop with this method than the previous.

### 2.5 Both-sided sampling

Bands to be rebuilt can be of three types: the N terminal band, the C terminal or intermediate. For the C and N terminal bands, forward and reverse sampling are used respectively. For the intermediate regions, the most efficient way is to use a both-sided sampling approach as opposed to only forward sampling. As explained previously (see Gore et al. (2007) in the context of \( \beta \)-sheet sampling), in both-sided sampling, residues are sampled in the order \( i, k, i+1, k-1, i+2, k-2, \ldots \). In case of forward or reverse sampling, only a weak loop-closure distance restraint informs the sampling process of the other end of the loop, but with both-sided sampling, information at both N and C termini is actively used. A distance restraint is
used between $C_\alpha$ atoms at the same sequence distance from both termini, so that the chance of loop closure remains high despite both sided sampling. Initial experiments with crystallographic loop building clearly showed that refinement was better with both-sided sampling than forward-only sampling, especially with larger loops and weaker positional restraints. Thus, in this work, we have used forward-only, backward-only and both-sided sampling for C terminal, N-terminal and intermediate bands respectively.

### 2.6 Multiconformer sampling

This type of sampling constructs many conformations of the same band simultaneously. Instead of incrementally sampling one band, multiple models of the band are extended simultaneously. This is achieved by re-implementing the PopulationSearch algorithm in its plural form in which each builder is replaced by a set of builders that have same input and output atoms in different models. Clash checking is not performed across models. Electron density ranker uses the combined output of a set of corresponding builders to calculate the score of a child conformation. Due to this, the possibility of getting attracted into higher density is reduced and the chance of occupying the density generated due to genuine heterogeneity increases. The disadvantage of this kind of sampling is the obvious increase in conformational freedom and execution time.

### 3 Results

#### 3.1 Reproducing rapper/cns refinement

The utility of knowledge-based refinement has been demonstrated by [DePristo et al. (2005)] with automatic refinement of perturbed starting structures of 9ILB and 1KX8 to an $R_{free}$ almost same as the deposited structure. When that refinement protocol was closely reproduced in Rappertk, very similar results were obtained. Five proteins were selected in the 2Å-3Å resolution range from the PDB: 9ILB (Yu et al. (1999)), 1KX8 (Lartigue et al. (2002)), 1MB1 (Taylor et al. (1997)), 1BYW (Cabral et al. (1998)) and 1RN7 (Alvarez-Fernandez et al. (2005)). Five perturbed structures were generated for each of them within 2Å $C_\alpha$ and 3Å sidechain centroid restraints respectively. 20 rounds of both cns-only and cns/Rappertk refinement protocols were performed on these starting models to obtain the $R_{free}$ statistics summarized in Table 1. $R_{free}$ figures reported for 9ILB and 1KX8 by [DePristo et al. (2005)] for same restraints were 0.27(0.01) and 0.32(0.01) respectively - the corresponding statistics observed here are comparable.
Table 1: Full protein testset and refinement statistics for 5 starting models generated within 2Å Cα and 3Å sidechain restraints.

| PDB Id | Resolution (Å) | #AA | Baseline\(^a\) \(R_{\text{free}}\) | CNS-only \(R_{\text{free}}\) | \(R_{\text{free}}\) | CNS/Rappertk AA-RMSD\(^b\) | \(\chi_1^c\) | \(\chi_{12}^d\) |
|--------|----------------|-----|-----------------|-----------------|-----|------------------|-----|------|
| 9ILB   | 2.28           | 153 | 0.225           | 0.266 (0.016)   | 0.253 (0.011) | 1.29 (0.08) | 75.2 (2.63) | 53.0 (4.64) |
| 1KX8   | 2.80           | 99  | 0.294           | 0.317 (0.018)   | 0.310 (0.010) | 0.60 (0.02) | 76.2 (1.93) | 50.2 (4.06) |
| 1MB1   | 2.10           | 98  | 0.292           | 0.333 (0.021)   | 0.302 (0.003) | 0.60 (0.02) | 75.8 (2.31) | 56.4 (2.05) |
| 1BYW   | 2.60           | 110 | 0.292           | 0.342 (0.020)   | 0.316 (0.013) | 0.58 (0.06) | 78.6 (3.32) | 57.4 (5.60) |
| 1RN7   | 2.50           | 112 | 0.270           | 0.366 (0.054)   | 0.318 (0.009) | 0.61 (0.04) | 81.0 (2.09) | 53.2 (1.93) |

\(^a\)Baselines \(R_{\text{free}}\) values are computed by running a long CNS-only refinement starting from the deposited structure.

\(^b\)All-atom RMSDs are in Å units. First two numbers are the mean and standard deviations of RMSDs between the PDB and 5 models. The third number is the mean of RMSDs among 5 models based on \(^5C_2 = 10\) pairwise comparisons.

\(^c\)\(\chi_1\) accuracy of a protein structure with respect to another is defined as the percentage of sidechains in the former that have a \(\chi_1\) differing by less than 40° from the corresponding sidechain in the latter. First two numbers are the mean and standard deviation of the \(\chi_1\) accuracies between 5 models and the deposited structure. The third number is the mean of \(\chi_1\) accuracies from \(^5C_2 = 10\) pairwise comparisons among the 5 models.

\(^d\)Same as \(^c\), but with \(\chi_1\) and \(\chi_2\) both instead of only \(\chi_1\).
Table 1 shows the variation in all-atom \textit{RMSD} and $\chi_1, \chi_{12}$ values as function of resolution. The reported \textit{RMSD} is the average of pairwise unsuperposed \textit{RMSD} between the deposited and each of the composite models and thus can be said to indicate the inaccuracy in retrieving the deposited model from an approximate starting model. This inaccuracy does not seem to be sensitive to the resolution, suggesting that at least in the 2.1Å-2.8Å resolution range, an approximate model can be corrected to a similar quality with respect to the deposited one irrespective of the resolution.

When the models are compared among themselves in a pairwise manner, the average \textit{RMSD} and $\chi_1, \chi_{1,2}$ figures can be said to represent heterogeneity. This heterogeneity is slightly lower than the inaccuracy, but the difference is insignificant, i.e. each model is as far from the deposited structure as from any other model. Recently the heterogeneity defined similarly has been suggested to be the minimum uncertainty expected in the coordinates of a single-conformer model of that structure (Terwilliger et al. (2007)).

An ideal refinement method should start with approximate models and yield a set of high heterogeneity models each of which agrees at least as well with the data as the deposited structure, i.e. a combination of results in DePristo et al. (2004) and DePristo et al. (2005). Clearly, the protocol used is similar to DePristo et al. (2005) and perhaps expectedly, does not yield greater heterogeneity than inaccuracy. But when the models are assigned partial occupancies and combined to create a multiconformer model (Fig. 1, Section 3.4), the collection $R_{free}$ values for 1MB1, 1BYW and 1KX8 drop significantly by 1.5 – 2% than the deposited structure. This drop suggests that perhaps structural heterogeneity is captured to some extent.

### 3.2 Rebuilding missing loops

Five structures of various resolutions and no obvious homology were selected from the PDB: 1MB1 (Taylor et al. (1997)), 1BYW (Cabral et al. (1998)), 1KXB (Choi et al. (1996)), 1RN7 (Alvarez-Fernandez et al. (2005)) and 2DBO (Ishii et al. (2005)). All structures have a single continuous peptide chain between 100-200 residues and no ligands. For each structure, a loop at least 10 residues long was chosen for rebuilding (Table 2). Unlike the previous exercise, there are no positional restraints on loop sidechains. $C_\alpha$ atoms are positionally restrained, in the first case with 5Å restraints and later with 10Å restraints. The loops were rebuilt using the both-sided loop sampling and loop closure techniques within the iterative refinement protocol. For 4 of 5 loops considered, the 5Å perturbation of the loop can be corrected to within one point of the baseline $R_{free}$. In the 10Å case, this performance drops marginally to two points from the baseline $R_{free}$ for the same cases.

Fig.1 shows the large difference in the quality of \textit{cns-only} and \textit{cns/Rappertk} refinement.
protocol. Every starting model refines to a structure very similar to the deposited using the composite protocol whereas it gets trapped in local minima during CNS-only refinement. The 1RN7 case (Fig. 2) is unsatisfactory due to a difficult 5-residue segment in the loop (Pro-80, Asn-81, Leu-82, Asp-83, Asn-84). As noted by Alvarez-Fernandez et al. (2005), the density for this segment is confusing, perhaps due to underlying heterogeneity, and consistently misleads the band sampling into a conformation different from the deposited.

3.3 Framework and loops

Perhaps a more frequently encountered scenario than the previous two is the one in which both the secondary structure and loops have positional uncertainty. In such cases, the loop regions invariably are more unreliable than the secondary structure framework. In order to simulate this scenario, the framework was restrained to 1Å $C_\alpha$ and 3Å sidechain centroid restraints, whereas loops were restrained to 3Å $C_\alpha$ restraints and no sidechain restraints. Five models were built for each protein and then iteratively refined using both CNS-only and CNS/Rappertk protocols. The refinement statistics are summarized in Table 3. Note that the refinement composite refinement statistics are not as good as in the previous exercises, but still better than the CNS-only refinement. Fig. 3 shows a typical contrast between the CNS-only and the composite refinement protocols in this scenario.

3.4 Variation of $R_{free}$ with collection size

We define a collection as a set of independently refined structures which when taken together may capture some aspects of structural heterogeneity. This term is introduced to distinguish the collection from an ensemble (see Section 3.6) which is also a set of structures, but refined in an interdependent manner.

For the previous exercises of refinement (whole chain, 5Å loop, 10Å loop and loops with framework), single best $R_{free}$ models from five CNS/Rappertk trajectories are chosen and combined to create collections, e.g., a three model collection is created by choosing three lowest $R_{free}$ models from the five and assigning occupancy of 0.33 to each of them. A collection is subjected to a short CNS refinement and $R_{free}$ at its end is noted as collection $R_{free}$. Fig. 4 shows the variation of such $R_{free}$ values as a function of collection size. The drop in $R_{free}$ is modest and the highest when going from collection size of 1 to 2 or 3. The collection $R_{free}$ generally rises for sizes 4 and 5. This indicates the danger of overfitting due to increase in number of parameters. Thus a straightforward combination of models does not seem to be the correct way of describing heterogeneity. Intelligent schemes for parameter reduction need to be investigated in this regard, such as upper-bounding $B$-factors, enforcing
Table 2: Dataset for loop building and refinement statistics.

| PDB Id | Resolution (Å) | #AA | Loop range and size | Baseline $R_{free}$ | $R_{free}^{(CNS-only)}$ | Baseline $R_{free}$ | $R_{free}^{(CNS/Rapperkok)}$ |
|--------|----------------|-----|---------------------|---------------------|------------------------|---------------------|-----------------------------|
| 1MBI   | 2.1            | 98  | 66-75 (12)          | 0.292               | 0.334 (0.024)          | 0.292               | 0.310 (0.013)               |
| 1BYW   | 2.6            | 110 | 113-124 (12)       | 0.292               | 0.329 (0.006)          | 0.292               | 0.314 (0.012)               |
| 1KXB   | 2.9            | 158 | 200-218 (15)       | 0.283               | 0.326 (0.007)          | 0.283               | 0.310 (0.010)               |
| 1RN7   | 2.5            | 112 | 76-95 (22)         | 0.270               | 0.347 (0.018)          | 0.270               | 0.316 (0.009)               |
| 2DBO   | 2.76           | 148 | 88-98 (18)         | 0.289               | 0.332 (0.022)          | 0.289               | 0.333 (0.021)               |
| 3HPT   | 3.0            | 106 | 66-75 (12)         | 0.363               | 0.393 (0.021)          | 0.363               | 0.374 (0.016)               |
| 4B00   | 2.93           | 158 | 200-218 (15)       | 0.283               | 0.326 (0.007)          | 0.283               | 0.310 (0.010)               |
| 5B7C   | 2.8            | 112 | 76-95 (22)         | 0.270               | 0.347 (0.018)          | 0.270               | 0.316 (0.009)               |
| 6IDU   | 2.5            | 112 | 76-95 (22)         | 0.270               | 0.347 (0.018)          | 0.270               | 0.316 (0.009)               |
| 7U2H   | 2.7            | 112 | 76-95 (22)         | 0.270               | 0.347 (0.018)          | 0.270               | 0.316 (0.009)               |
| 80000  | 3.0            | 106 | 66-75 (12)         | 0.363               | 0.393 (0.021)          | 0.363               | 0.374 (0.016)               |
Figure 1: Loop building exercise for the 1MB1 loop with 10Å Cα restraints. Top panel shows the loop in the deposited structure (green) and starting models generated for it. Middle panel shows the best $R_{free}$ models (slate) obtained during the CNS-only refinement. Bottom panel shows the CNS/Rappertk models (magenta) and the loop in the deposited structure (green) in all-atom representation.
Figure 2: Loop building exercise for the 1RN7 loop with 5Å $C_\alpha$ restraints. Panels arranged in a similar way to Fig.1.
Figure 3: Framework/loop exercise for 1BYW. The deposited structure is shown as thick green ribbon and starting structures as brown ribbon in the top panel. Middle panel shows the best $R_{free}$ structures from CNS-only trajectories (slate) and bottom panel shows those from the CNS/Rappertk trajectories (magenta).
Table 3: Dataset and refinement statistics for the loop-framework refinement

| PDB | Resolution (Å) | #AA | #Loops | baseline $R_{free}$ | $R_{free}$ Mean (Std.Dev.) | $R_{free}$ Mean (Std.Dev.) |
|-----|----------------|-----|--------|---------------------|-----------------------------|-----------------------------|
| 1MB1| 2.1            | 98  | 27 (28)| 0.292              | 0.352 (0.012)               | 0.325 (0.010)               |
| 1BYW| 2.6            | 110 | 50 (45)| 0.292              | 0.400 (0.046)               | 0.321 (0.016)               |
| 1KXB| 2.9            | 158 | 54 (36)| 0.283              | 0.351 (0.022)               | 0.336 (0.006)               |
| 1RN7| 2.5            | 112 | 33 (29)| 0.270              | 0.342 (0.032)               | 0.328 (0.008)               |
| 2DBO| 2.76           | 148 | 57 (39)| 0.289              | 0.370 (0.020)               | 0.335 (0.017)               |

the same $B$-factor on corresponding atoms across all models and positionally constraining them together when large variability is not expected.

3.5 Mistakes in the composite protocol

Although the $cns$/$Rapper$tk refinement produces a well-refined structure very close to the baseline $R_{free}$, it never betters the latter. This is due to imperfections in various components of the protocol. Identification of residues to rebuild relies on the $\chi^2$ correlation coefficient which can sometimes be an unsatisfactory substitute for human judgement. This can lead to unnecessary resampling of satisfactory bands and sometimes incorrect conformers are not detected. The copying of non-protein atoms from one round to next may sometimes result in their permanent misplacement. The sampling problem may result in unsatisfactory bands because the right conformation must be generated in order to be picked by the electron density ranker. On the other hand, the ranker may not score a correct conformation as the best one. If the density for a band is weak, band sampling may sometimes lead into the density of waters or ligands. In spite of these difficulties, the refinement statistics presented in previous sections are satisfactory.

But when restraint radii are increased beyond those used here, the serious problem of spatial overlap between restraint spheres of two different bands starts affecting the band sampling. The correct density for a band then may be occupied by a band sampled before it. In such case, the correct density is always occupied by the wrong band. For the wrong band, subsequent $cns$ refinement may take it so far from its correct location that the restraint radii may be too small to let the band be built correctly again. Further work is required to get rid of the problem of band overlaps, either by restraint adjustments or change in the sampling strategy.
Figure 4: Variation of $R_{free}$ with collection size in whole chain, loop and framework-loop exercises. For collection size of 0, baseline $R_{free}$ is shown. For collection size 1, the mean and standard deviation of best $R_{free}$ models in CNS/Rappertk trajectories are shown. The rest are calculated by combining the best $R_{free}$ individual models with partial occupancies.
3.6 Modelling loop heterogeneity with collections and ensembles

Conformational diversity is most pronounced in loop regions due to the relatively smaller number of non-covalent interactions to maintain order. Absence of good density for a loop when rest of the structure has good density is a sure indication of the loop’s flexibility. Modelling heterogeneity is challenging because density is generally more confusing for such regions owing to conformer overlaps and subsequent dilution of shape information.

Heterogeneity can be modelled with collections or ensembles. Members of a collection are single-conformer isotropic B-factor models determined independently of one another. Members of an ensemble are determined in a highly interdependent manner and have partial occupancies.

Derivation of a collection is a simple way to estimate the unavoidable uncertainty in structure determination, but it cannot be said to represent any structural correlations. A major advantage of collections is their simplicity. A procedure that produces a single model can be executed multiple times with different random seeds or starting models to generate a collection. Thus the time taken increases only linearly as the collection size.

An implicit assumption in the ensemble representation is that the members are in dynamic equilibrium, making ensemble a much stronger statement than the collection. Determining ensembles is very challenging because it is unclear how to determine the number and occupancies of the ensemble members prior to or during the refinement process. Another major challenge is the linear increase in the number of parameters which results in an exponential increase in search space and execution time.

In order that its output be credible, any procedure that aims to model the structural heterogeneity must be first validated using artificial data where the real heterogeneity is accurately known. To that end, we have chosen a significantly simplified kind of heterogeneity by generating artificial diffraction data in which the underlying heterogeneity is restricted to a single loop and consists of two equally-occupied loop conformations. For a loop each from 1MB1, 1BYW and 2DBO, two conformers were generated by perturbing the loop to within 3Å $C_{\alpha}$ restraints and no sidechain restraints. All non-protein atoms (ions, waters etc.) have been removed so as to reduce the density dilution. Artificial diffraction data were created with the same cell, spacegroup and resolution as the deposited structure. For self-consistency in the cns forcefield, data was generated iteratively. The average of the two conformations was considered as the starting conformation for further heterogeneity modelling.

A collection of 4 members was generated for each loop with cns/Rapporttk protocol used previously. An ensemble consisting of 2 members was generated for each loop using multi-conformer sampling described previously. As with the single-conformer protocol, multiconformers were sampled iteratively and alternatingly with cns. Enrichment was increased to 10
and population size to 200 to be able to build reasonable models. Positive electron density restraint was enforced on mainchain.

The performance of collections and ensembles can be visually inspected in Fig.5, Fig.6 and Fig.7 but it is essential to quantify quality of heterogeneity modelling objectively. The two important quantities to measure are: the extent to which both conformers are captured and the extent to which at least one conformer is captured. The former (multiconformer quality index, MQI) should quantify how much of the underlying diversity is represented and the latter (single-conformer quality index, SQI) should quantify how well at least one of the heterogenous states is modelled.

If conformers $H_i$ constitute the true underlying heterogeneity and $M_i$ are the ensemble or collection members which model it, then MQI and SQI can be calculated as:

$$MQI = \sum_i \min_j (Rmsd(H_i, M_j))$$

$$SQI = \min_{i,j} (Rmsd(H_i, M_j))$$

where $Rmsd$ is calculated over the atoms of interest, e.g. a sidechain or all $C_\alpha$ atoms in the loop. Note that these expressions do not consider the occupancies. Table 4 quantifies the performance of ensembles and collections using $R_{free}$, MQI over loop $C_\alpha$ atoms, and MQI, SQI over each sidechain. The ensemble $R_{free}$ values are smaller than those for collections as expected due to greater number of parameters. $C_\alpha$ MQI suggests that mainchain heterogeneity is modelled better in the ensemble method. Sidechain SQIs do not show any systematic difference between the two methods, which suggests that both methods capture the rotameric heterogeneity to a similar extent. But sidechain MQIs tend to be slightly better for ensemble than collection. This is not surprising because in principle, the only limitations on the ensemble method are sampling and ranking of conformational extensions. Generally the higher density option is chosen in single-conformer modelling but a lower density can also be chosen in multiconformer modelling due to a greater number of atoms to place in the density. This is evident from residues Lys-72 and Tyr-73 in 1MB1, Glu-118, Asp-119 in 1BYW and Lys-86, Lys-87 in 2DBO where collection models are biased towards one of the loop conformations due to weak density.

### 4 Concluding Discussion

The problem of automated crystallographic refinement is interesting, challenging and has significant immediate practical relevance. An automated solution for building single con-
Figure 5: Heterogeneity modelling with collection and ensemble for a loop in PDB 1MB1. The top panel shows the artificially generated loop heterogeneity with corresponding electron density contoured at $1\sigma$. The middle and bottom panels respectively show a 4-member collection and 2-member ensemble model of that heterogeneity.
Figure 6: Heterogeneity modelling with collection and ensemble for a loop in PDB 1BYW. Panels arranged as in Fig.5.
Figure 7: Heterogeneity modelling with collection and ensemble for a loop in PDB 2DBO. Panels arranged as in Fig. 5. In the top panel, the red contours correspond to 0.5σ.
Table 4: Comparison of collection and ensemble heterogeneity modelling styles with artificially generated 2-conformer heterogeneity for 3 loops. MQI and SQI are in Å units.

| PDB    | Baseline \( R_{\text{free}} \) | Collection \( R_{\text{free}} \) Mean (Std.Dev.) | Ensemble MQI-\( C_{\alpha} \) Collection | Ensemble MQI-\( C_{\alpha} \) Ensemble |
|--------|---------------------------------|-----------------------------------------------|------------------------------------------|------------------------------------------|
|        | 0.055                           | 0.127 (0.003)                                | 2.11                                     | 0.094                                    |
| 1MB1   |                                 |                                               |                                          |                                          |
| 1BYW   | 0.159                           | 0.242 (0.004)                                | 0.79                                     | 0.192                                    |
| 2DBO   | 0.292                           | 0.312 (0.005)                                |                                          | 0.271                                    |
|        |                                 |                                               |                                          |                                          |
former models from approximate spatial restraints, combined with an automated method to explore heterogeneity, will allow the crystallographic community to revisit and annotate the entire PDB with structural variability information. Such information will have an impact on all analyses that rely on accurate coordinate information, like in-silico ligand design and binding, non-covalent interactions and sequence-structure conservation, etc. It will also significantly change the understanding of crystalline state and protein flexibility, benefitting the refinement process. The main components for successful heterogeneity annotation are reliable construction of single-conformer models and reliable estimation of heterogeneity that is predominantly seen in loops. This work has attempted to develop methods to that end.

Application of the rapper approach to crystallographic refinement (DePristo et al. (2005), Furnham et al. (2006)) has the primary benefit of crossing the energy barriers in a non-random manner, based on knowledge-based sampling instead of kinetic sampling. As described in Gore et al. (2007), rapper has been reformulated as Rapportk recently, creating possibilities of applying knowledge-based sampling in many different ways. In this work, we showed that Rapportk can be used to automatically refine the whole protein structure starting from reliable positional restraints on mainchain and sidechain. This benchmarked its performance vis-a-vis rapper as reported by DePristo et al. (2005) for a similar task. Then we showed that by efficient use of available restraints, single loops can be modelled in protein structures to native-like quality with few positional restraints. The efficient use of available restraints was a result of symmetry-related clash checking, restraint propagation using loop anchors and sampling from both anchors simultaneously. The same strategy could be extended to a more realistic problem of a large uncertainty in loop regions and an imperfect secondary structure framework. The CNS-only refinements, run as controls, showed the value added by Rapportk to the refinement task.

In addition to determination of single-conformer models under differing restraint qualities, we started addressing the challenge of heterogeneity assessment in loops. This is indeed a very difficult problem with fundamental unknowns like number of conformers, correlations within heterogeneity and relative occupancies. Conformational heterogeneity can be divided into two types: the simpler sidechain-only heterogeneity where mainchain is nearly the same and the all-atom heterogeneity where mainchain also takes distinct conformations. The latter can be further divided based on the extent of spatial overlap between the conformers. Sidechain-only heterogeneity is relatively easy than the all-atom heterogeneity because the density is likely to contain good cues about diversity. But for overlapping conformers, a visual inspection of density is less likely to be helpful. There are two distinct ways to model heterogeneity, which we have termed collections and ensembles, depending upon interdependence of member conformations. For single-loop 2-conformer overlapping heterogeneity, we generated both the
collections and ensembles and assessed how well they modelled the heterogeneity. The main observation was that the collection was generally biased towards the higher electron density. The ensemble method, due to more freedom and parameters available to it, manages to avoid this trap and fit two distinct conformers, leading to better modelling of heterogeneity than the collection.

Various pitfalls of the composite refinement protocol were recognized and they need to be addressed in future. Addressing the problem of overlapping bands will significantly increase the reliability of the method given very approximate positional restraints, and make the method more useful in low resolution, large uncertainty cases. Perhaps many models can be generated for such bands and the best combination of those models can be used. At lower resolution, use of coarse-grained sampling (fragment sampling) may also be useful, followed by fine-grained $\phi - \psi - \chi$ sampling.

From the heterogeneity perspective, a fundamental question to address would be the estimation of the nature of underlying conformations before attempting to model it because ensemble sampling must have prior knowledge of the number and occupancies of its members. Generation of collections seems the only way for such estimation, for which collection modelling method will have to be modified suitably to sample within electron density yet avoid the bias towards higher density. The main challenge of ensemble sampling is the explosion in conformational freedom and the work ahead will have to focus on efficiently scaling this sampling method for larger ensemble size.

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