QUANTUM MECHANICAL DESCRIPTION OF BELL’S EXPERIMENT ASSUMES LOCALITY

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Abstract.

Here it is shown that the simplest description of Bell’s experiment according to the canon of von Neumann’s theory of measurement explicitly assumes the (Quantum Mechanics-language equivalent of the classical) condition of Locality. This result is complementary to a recently published one demonstrating that non-Locality is necessary to describe said experiment within the framework of classical hidden variables theories, but that it is unnecessary to describe it within the framework of Quantum Mechanics. Summing up these and other related results, it is concluded that, within the framework of Quantum Mechanics, there is absolutely no reason to believe in the existence of non-Local effects. In addition to its foundational significance, this conclusion has practical impact in the fields of quantum-certified and device-independent randomness generation and on the security of Quantum Key Distribution schemes using entangled states.

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Important and plentiful research activity in progress is based on the idea that sequences of outcomes of successive measurements performed on a set of identically prepared spatially extended entangled states (or Bell’s experiment, see Figure 1) are random. It is even claimed this to be the only source of “true” randomness. This idea is based on the result that non-Local effects (which seem to exist in such experiment) and the impossibility of faster-than-light signaling imply that said sequences must be unpredictable [1] (to be precise, non-computable [2]). Recently, it has been demonstrated that non-Local effects (let’s call them “quantum non-Localitiy”) arise when the Bell’s experiment is described within the realm of classical hidden variable theories, but that they vanish when said experiment is described within the realm of conventional Quantum Mechanics (QM) [3]. It has been also demonstrated that the violation of Bell’s inequalities has nothing to do with non-Local effects, but that it is the mere consequence of non-commutativity of local observables. A similar result had been conjectured long ago in terms of wave-particle dualism [4]. An even more recent paper shows that it is possible to remove quantum non-Locality by an appropriate interpretation of wavefunction’s properties [5]. Yet, as Prof. A.Khrennikov (the Author of [3]) has warned: “It is clear that to get rid of nonlocality from quantum theory is not a simple task. The present note is just a step towards the common acceptance of the local interpretation of QM’. This paper can be regarded as another step in the same direction.

In this paper, I present the complementary (to [3-5]) result that the description of Bell’s experiment according to the canon of von Neumann’s theory of measurement explicitly requires assuming Locality valid. Locality here means that Hamiltonians generating evolutions in distant places commute. In fact, if these Hamiltonians did not commute, the results of observations could be different for different reference frames (say, if the observer is moving and coming from station A’s side or from station B’s side, see below). The description is simple and straightforward.

Von Neumann’s theory of measurement is based on the definition of an appropriate interaction between the quantum system and a classical system (the “pointer”), which is eventually observed. I follow the conventional approach, as detailed f.ex. in [6], but applying it to a pair of entangled quantum particles instead that to a single one. Consider then a typical Bell’s experiment (Fig.1). Assume source S emits biphotons in the fully symmetrical polarization-entangled Bell state: \( |\phi^+\rangle = (1/\sqrt{2})(|x_A,x_B\rangle + |y_A,y_B\rangle) \). Biphotons are detected at remote stations A and B by projecting them into the bases \( \{|+A\rangle,|-A\rangle\} \) and \( \{|+B\rangle,|-B\rangle\} \). Eigenvalue +1 (-1) corresponds to a photon detected after the transmitted (reflected) output. These bases are related with the linear polarization ones as:

\[
|x_A\rangle = |+A\rangle \cos(\alpha) - |-A\rangle \sin(\alpha), \quad |y_A\rangle = |+A\rangle \sin(\alpha) + |-A\rangle \cos(\alpha)
\]

\[
|x_B\rangle = |+B\rangle \cos(\beta) - |-B\rangle \sin(\beta), \quad |y_B\rangle = |+B\rangle \sin(\beta) + |-B\rangle \cos(\beta)
\]
where \( \alpha \) (\( \beta \)) is the orientation angle of the polarization analyzer in station A (B) with respect to the \( \{x,y\} \) axes. The operators for the observables in each station are then (note the bold typing for operators):

\[
A = (+1) |+A\rangle\langle +A| + (-1) |-A\rangle\langle -A|, \quad B = (+1) |+B\rangle\langle +B| + (-1) |-B\rangle\langle -B|
\]

The interaction Hamiltonian with the classical apparatus in station A is assumed \( H_A = \lambda A \times P_A \), where \( \lambda \) measures the strength of the interaction, and \( P_A \) is the linear momentum operator of the pointer in that station (\( H_B = \lambda B \times P_B \), the interaction strength is assumed the same in both stations for simplicity). The evolution of the system “entangled state plus apparatuses” is:

\[
|\psi_{\text{final}}\rangle = U(t) |\psi_{\text{initial}}\rangle = \exp[-2\pi it/\hbar] (H_A + H_B) \{ (1/\sqrt{2})(|x_A, x_B\rangle + |y_A, y_B\rangle) \} |\psi_A^0\rangle |\psi_B^0\rangle
\]

where \( |\psi_A^0\rangle, |\psi_B^0\rangle \) are the pointers’ initial states (see Fig.1, up). Using the seemingly trivial equality:

\[
\exp[-2\pi it/\hbar] (H_A + H_B) = \exp[-2\pi it/\hbar] H_A \times \exp[-2\pi it/\hbar] H_B
\]

replacing \( H_A, H_B \), using eqs.(1,2) and the property that for operators \( A, B \) with eigenvalues \( a, b \):

\[
\exp(iA.B) \approx A \times \exp(i\alpha B)
\]

one gets:

\[
|\psi_{\text{final}}\rangle = (1/\sqrt{2}) \{ \cos(\alpha) \cos(\beta) |+A\rangle\langle +B| \exp(-i\epsilon P_A) |\psi_A^0\rangle \exp(-i\epsilon P_B) |\psi_B^0\rangle + \\
\sin(\alpha) \sin(\beta) |+A\rangle|+B\rangle \exp(-i\epsilon P_A) |\psi_A^0\rangle \exp(-i\epsilon P_B) |\psi_B^0\rangle + \\
\cos(\alpha) \sin(\beta) |+A\rangle|-B\rangle \exp(-i\epsilon P_A) |\psi_A^0\rangle \exp(i\epsilon P_B) |\psi_B^0\rangle + \\
\sin(\alpha) \cos(\beta) |+A\rangle|-B\rangle \exp(-i\epsilon P_A) |\psi_A^0\rangle \exp(i\epsilon P_B) |\psi_B^0\rangle + \\
[-\sin(\alpha)] \cos(\beta) |A\rangle|+B\rangle \exp(i\epsilon P_A) |\psi_A^0\rangle \exp(-i\epsilon P_B) |\psi_B^0\rangle + \\
\cos(\alpha) \cos(\beta) |A\rangle|+B\rangle \exp(i\epsilon P_A) |\psi_A^0\rangle \exp(-i\epsilon P_B) |\psi_B^0\rangle + \\
[-\sin(\alpha)] [-\sin(\beta)] |A\rangle|-B\rangle \exp(i\epsilon P_A) |\psi_A^0\rangle \exp(i\epsilon P_B) |\psi_B^0\rangle + \\
\cos(\alpha) \cos(\beta) |A\rangle|-B\rangle \exp(i\epsilon P_A) |\psi_A^0\rangle \exp(i\epsilon P_B) |\psi_B^0\rangle
\]

where \( \epsilon = 2\pi t\lambda /\hbar \). Now assume that interaction time is short enough, and interaction strength large enough, so that \( t\lambda \approx \text{constant} \) (“sudden interaction” approximation), therefore \( \epsilon \approx \text{constant} \).
The linear momentum operator produces a pointer’s displacement to the state (f.ex.):

$$\exp(-ieP_A)|\psi_A^0\rangle = |\psi_A^+\rangle$$  \hspace{1cm} (7)

what means “state of the classical apparatus in A with its pointer in the position +”. The pointer no longer moves. It indicates that one single detection occurred in the output “transmitted” of the polarizer in station A. The other operators in eq.(6) produce analogous displacements to the pointers’ final states: $|\psi_B^+\rangle$, $|\psi_A^+\rangle$, $|\psi_B^-\rangle$. Therefore:

$$|\psi_{\text{final}}\rangle = (1/\sqrt{2}) \{ |+A\rangle|+B\rangle|\psi_A^+\rangle|\psi_B^+\rangle - |-A\rangle|+B\rangle|\psi_A^-\rangle|\psi_B^-\rangle - |+A\rangle|-B\rangle|\psi_A^+\rangle|\psi_B^-\rangle + |-A\rangle|-B\rangle|\psi_A^-\rangle|\psi_B^+\rangle \}$$  \hspace{1cm} (8)

Note that the states of the quantum system and the classical pointers are entangled now. The observer has no access to the quantum system, only to the pointers. What the observer sees is then described by a reduced density matrix, obtained after tracing out the states \{ $|+A\rangle$, $|+B\rangle$, $|-A\rangle$, $|-B\rangle$ \} of the quantum system:

$$\rho_{\text{reduced}} = \sum \langle \text{quantum system}|\psi_{\text{final}}\rangle\langle\psi_{\text{final}}|\text{quantum system}\rangle = \frac{1}{2} \{ \cos^2(\alpha-\beta)|\psi_A^+\rangle|\psi_B^+\rangle\langle\psi_A^+|\langle\psi_B^+| + \sin^2(\alpha-\beta)|\psi_A^-\rangle|\psi_B^-\rangle\langle\psi_A^-|\langle\psi_B^-| + \cos^2(\alpha-\beta)|\psi_A^+\rangle|\psi_B^-\rangle\langle\psi_A^+|\langle\psi_B^-| + \cos^2(\alpha-\beta)|\psi_A^-\rangle|\psi_B^+\rangle\langle\psi_A^-|\langle\psi_B^+| \}$$  \hspace{1cm} (9)

which is a diagonal density matrix with the correct probabilities for the four possible observable outcomes in the experiment in Figure 1.

The assumption of Locality is explicit in the (apparently naïve) eq.(4), valid if $[H_A, H_B] = 0$. The commutation of the Hamiltonians acting on each station is the equivalent, in QM language, to the condition of Locality in classical language. Therefore, to describe the measurement process in Bell’s experiment according to the canon of von Neumann’s theory, \textit{Locality is assumed valid}. Of course, it is still possible that some ad-hoc election of $H_A, H_B$ may lead to the correct result eq.(9) \textit{even if} $[H_A, H_B] \neq 0$, but the simplest, natural way to derive eq.(9) uses $[H_A, H_B] = 0$. In this sense, quantum non-Locality is not refuted, but it is demonstrated unnecessary and unnatural. By Occam’s razor, it must be discarded.

Summing up the arguments in [3-5] and here, it is concluded that \textit{there is absolutely no reason to believe on the existence of quantum non-Locality}. It is worth recalling here that Bohr’s immediate reaction to EPR’s paper was that Realism (instead of Locality) was not valid in QM. Bohr’s point of
view is hence strengthened here. Nevertheless, as stated before, it is to be expected that many papers and discussions will be necessary before the (magic, appealing, apparently fruitful and deeply rooted) belief on the existence of quantum non-Locality is abandoned.

An important consequence of this conclusion is that the ideas of quantum-certified and device-independent randomness, which are derived from the hypothesis of quantum non-Locality, are weakened. This has practical impact on important developments in the field of quantum Random Number Generators and the security of Quantum Key Distribution schemes using entangled states.

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Figure 1: Sketch of a Bell’s experiment. Up: biphotons in the fully symmetrical Bell state $|\phi^+\rangle$ are emitted by source S and propagate to stations A and B, where pointers of classical apparatuses are in their initial states or positions. After passing through polarization analyzers set at angles $\{\alpha, \beta\}$, the biphotons interact with the apparatuses, as described by von Neumann’s theory of measurement. Down: after the interaction is completed (under the “sudden interaction” approximation, see text) the pointer in station A has evolved into the classically observable state “-”, the one in B into the “+”. The outcome “-+” is observed. Probabilities of each outcome $\{++$, $+-$, $-+$, $--\}$ are correctly given by eq.(9), which is derived by explicitly assuming (the QM-language equivalent of) Locality.