The growth index of matter perturbations using the clustering of dark energy

Spyros Basilakos*

Academy of Athens, Research Center for Astronomy & Applied Mathematics, Soranou Efessiou 4, 11-527 Athens, Greece

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ABSTRACT

We have put forward a new unified framework which provides a consistent and rather complete account of the growth index of matter perturbations in the regime where the dark energy is allowed to have clustering. In particular, we find that the growth index is not only affected by the cosmological parameters but rather it depends on the choice of the considered dark energy (homogeneous or clustered). Using the Planck priors and performing a standard $\chi^2$-minimization between theoretical expectations and growth data, we statistically quantify the ability of the growth index to represent the observations. Finally, based on the growth index analysis we find that the growth data favour the clustered dark energy scenario.

Key words: dark energy.

1 INTRODUCTION

The available high-quality cosmological observational data (e.g. Type Ia supernovae, CMB, galaxy clustering, etc.), accumulated during the last two decades, have enabled cosmologists to gain substantial confidence that modern cosmology is capable of quantitatively reproducing the details of many observed cosmic phenomena, including the late time accelerating stage of the Universe. A variety of studies have converged to a cosmic expansion history involving a spatially flat geometry and a cosmic dark sector formed by cold dark matter and some sort of dark energy (DE), endowed with large negative pressure, in order to explain the observed acceleration of the Universe (cf. Blake et al. 2011; Hinshaw et al. 2013; Komatsu et al. 2011; Farooq, Mania & Ratra 2013; Spergel, Flauger & Hlozek 2015; Planck Collaboration XVI 2014, and references therein).

Although there is a common agreement concerning the basic ingredients of the universe, there are different ideas regarding the underline physical mechanism which is responsible for the cosmic acceleration. These patterns are based either on the existence of new fields in nature (DE) or in some modification of Einstein’s general relativity, with the present accelerating stage appearing as a sort of geometric effect. In order to test the latter possibilities, it has been proposed that measuring the so-called growth index, $\gamma$, could provide an efficient tool to discriminate between modified gravity models and DE models which adhere to general relativity (Linder & Cahn 2007; Nesseris & Perivolaropoulos 2008; Polarski & Gannouji 2008; Wei 2008; Fu, Wu & Yu 2009; Gong, Ishak & Wang 2009; Tsujikawa et al. 2009; Basilakos & Pouri 2012; Basilakos, Nesseris & Perivolaropoulos 2013; Nesseris et al. 2013; Pouri, Basilakos & Plionis 2014; Steigerwald, Bel & Marinoni 2014, and references therein).

It is interesting to mention that most of the attempts towards estimating the growth index have a common basis, namely at the sub-horizon scales the DE component is expected to be smooth and thus we consider perturbations only on the matter component of the cosmic fluid. But how would the growth index predictions change in the case where the DE is allowed to cluster? In other words can we estimate the growth index of matter fluctuations within the framework of clustered DE (hereafter CLDE)? Indeed, this is an interesting question because an affirmative answer opens naturally a new path towards understanding the structure formation mechanism in the DE regime.

Recently, there are efforts towards investigating the linear growth of matter perturbations in the context of CLDE (Bean & Dore 2004; Ballesteros & Riotto 2008; Sapone & Kunz 2009; Sapone & Majerotto 2012; Batista & Pace 2013; Dossett & Ishak 2013; Batista 2014; Basse et al. 2014; Pace, Batista & Del Popolo 2014; Steigerwald et al. 2014). From the observational viewpoint, although it is difficult to measure the DE perturbations, it has been found that the homogeneous DE scenario fails to reproduce the observed concentration parameter of the massive galaxy clusters (Basilakos, Sanchez & Perivolaropoulos 2009). Similar results have also been found recently by Mehrabi, Malekjani & Pace (2015) who claim that the CLDE models fit better the growth data than the homogeneous DE models (see also Nesseris & Sapone 2015). Generally, it has been shown that in order to search for observational effects of DE clustering we need to use the growth of matter fluctuations (Sapone, Kunz & Amendola 2010).

The layout of the current paper is the following. In Section 2, we provide for the first time (to our knowledge) the growth index of matter fluctuations as a function of the DE perturbations. In Section 3, we compare the theoretical predictions with the growth...
data in order to check the range of validity of the CLDE models and we discuss the variants from the homogeneous case. Finally, we summarize our results in Section 3.

2 DE PERTURBATIONS AND THE GROWTH INDEX

Let us first present the basic linear equations that govern the evolution of the matter and DE perturbations. Following the notations of Abramo et al. (2007, 2009) we write the following system

\[
\delta_m + 2H \dot{\delta}_m = \frac{3H^2}{2} \left[ \Omega_m \delta_m + \Omega_d \delta_d (1 + 3w) \right],
\]

\[
\delta_d + \left( 2H - \frac{\dot{w}}{1 + w} \right) \dot{\delta}_d = \frac{3(1 + w)H^2}{2} \left[ \Omega_m \delta_m + \Omega_d \delta_d (1 + 3w) \right],
\]

(1)

(2)

where an overdot means cosmic time differentiation, \(H(a) = H_0 E(a)\) is the Hubble parameter, \(w\) is the equation-of-state (EoS) parameter, \(\Omega_m(a) = 1 - \Omega_d(a) = \Omega_m a^{-3}/E^2(a)\) and \((\delta_m, \delta_d)\) denote the corresponding fluctuations. We would like to stress that in order to obtain the above-mentioned system, Abramo et al. (2007) used the framework of the spherical collapse model with the traditional top-hat approximation.1 Moreover, for simplicity these authors restrict their analysis to \(c_{\text{eff}}^2 = w\) (where \(c_{\text{eff}}^2\) is the fluid’s effective sound speed) assuming that the EoS parameter remains the same either for the background expansion or for the spherical perturbations. In principle, since \(c_{\text{eff}}^2\) can take negative values one may expect that there are instabilities in the growth of DE perturbations. However, in the context of the spherical collapse model with the top-hat profile if we consider that \(c_{\text{eff}}^2\) is a scale-independent quantity, then it is easy to prove that the pressure and the density gradients vanish which implies that DE instabilities do not exist (for more details, see Abramo et al. 2007). We will elaborate on the details of the general linear equations that lead the evolution of the matter and DE perturbations in a forthcoming paper, but as a first step it is important to study the performance of the DE fluctuations in the most simple case.

For a constant EoS parameter \(\dot{w}(a) = 0\), the normalized Hubble function is written as

\[
E(a) = \sqrt{\Omega_m a^{-3} + \Omega_{d0} a^{-3(1 + w)}}.
\]

(3)

Let us first concentrate on equation (1) [for a more general analysis see the appendix]. Inserting into the latter equation the growth rate of clustering \(f = \frac{\delta_m}{\delta_0 \Delta_{\text{m}}(0)}\) (Peebles 1993; Wang & Steinhardt 1998) into equation (4), using a slow varying EoS parameter and performing simultaneously a first-order Taylor expansion around \(\Omega_m(a) = 1\) (for a similar analysis in the case of homogeneous DE, see Linder & Cahn 2007; Nesseris & Perivolaropoulos 2008; Gong et al. 2009; Tsujikawa, De-Felice & Alcaniz 2013), we find the following new approximate solution

\[
\gamma \simeq \gamma_{\text{IDE}} + \frac{3\Delta_{\text{de}}(1 + 3w)}{6w - 5} \Omega_{\text{de}},
\]

(5)

where

\[
\gamma_{\text{IDE}} \simeq \frac{3(w - 1)}{6w - 5} + \frac{3(1 - w)(2 - 3w)}{2(6w - 5)^2(5 - 12w)} \Omega_{\text{de}}.
\]

Obviously, from the above analysis it becomes evident that within the framework of the CLDE scenario the corresponding growth index is written in terms of the nominal growth index, namely \(\gamma_{\text{IDE}}\) (based on homogeneous DE) plus an additional component which is related with the DE perturbations. Of course, assuming a homogeneous dark energy (\(\Delta_{\text{de}} = 0\)) the above first-order solution reduces to the usual growth index functional form (see Gong et al. 2009; Tsujikawa et al. 2009) as it should. Also, as it is expected at high enough redshifts (\(z \gg 1\)) since, \(\Omega_m \simeq 1\) (or \(\Delta_{\text{de}} \simeq 0\)) we find that the asymptotic value of the growth index is not really affected by the clustering properties of the DE: \(\gamma_{\infty} \approx 3(w - 1)/(6w - 5)\).

However, one may see that the CLDE could affect the growth index at intermediate redshifts. As an example, considering the Planck prior (Planck Collaboration XVI 2014) \((w = -1.13)\) we find \(\gamma_{\infty} \simeq 0.542 + 6.7 \times 10^{-3} \Omega_{\text{de}} + 0.609 \Delta_{\text{de}} \Omega_{\text{de}}\), where \(\Delta_{\text{de}} \sim O(0.1)\) (see below).

Since, the solution (5) is valid at relative large redshifts one may want to treat the growth index also in the late universe and indeed various candidates have been proposed in the literature. In this paragraph, we extend the original Polarski & Gannouji (2008) work for a general family of \(\gamma(z)\) parametrization which is also valid in the CLDE regime. In particular, we phenomenologically parametrize \(\gamma(z)\) by the following general relation

\[
\gamma(z) = \gamma_0 + \gamma_1 \gamma(z).
\]

(7)

Simply, the latter equation can be viewed as a first-order Taylor expansion around some cosmological quantity such as \(\gamma(z)\) and \(z\). If we change variables in equation (4) from \(a\) to redshift \([\frac{\text{d}a}{\text{d}z}] = -(1 + z)^{-2} \frac{\text{d}a}{\text{d}z}\) and using \(f(z) = \Omega_m(z)^{\frac{1}{2}}\), we obtain

\[
- (1 + z) \gamma' \ln(\Omega_m) + \Omega_m + 3w \Omega_{\text{de}}(\gamma - \frac{1}{2}) + \frac{3}{2} \Omega_{\text{m}}^{\frac{1}{2}} \gamma \gamma_0 X.
\]

(8)

where \(X = 1 + \frac{\Delta_{\text{de}}(z)(1 + 3w)}{\Omega_m(z)}\).

(9)

Interestingly, for those \(\gamma(z)\) functions which obey \(\gamma(0) = 0\) or \(\gamma(0) = \gamma_0\) one can write the parameter \(\gamma_1\) in terms of \(\gamma_0\). Indeed, at the present epoch \([z = 0, \gamma'(0) = \gamma_1 \gamma_0(0)]\) equation (8) takes the form

\[
\gamma_1 = \frac{\Omega_{\text{m}}^{\gamma_0} + 3w(\gamma_0 \gamma_1 - \frac{1}{2}) \Omega_{\text{de}} + \frac{1}{2} \frac{3}{2} \Omega_{\text{m}}^{\gamma_0 - \gamma_1} X_0}{\gamma_0 \ln(\Omega_{\text{de}})}.
\]

(10)

Clearly, in the case of homogeneous DE (\(\Delta_{\text{de}} = 0, X = 1\)) the above formula reduces to that of Polarski & Gannouji (2008) for \(\gamma(z) = z\). Also, in the case of \(\gamma(z) = 1 - \gamma(z) = \frac{1}{\gamma_0}\) we fully recover literature results (Ishak & dosset 2009; Bellisso, Garcia-Bellido & Sapone 2011; Di Porto, Amendola & Branchini 2012). Since, the formula \(\gamma(z) = z\) goes to infinity at large redshifts, for the rest of the paper we concentrate on \(\gamma(z) = z/(1 + z)\) (Ballesteros & Riotto 2008) and thus \(\gamma'(0) = 1\). Obviously, at large redshifts \(z \gg 1\), we get \(\gamma_\infty \simeq \gamma_0 + \gamma_1\). Therefore, plugging \(\gamma_0 = \gamma_\infty - \gamma_1\) into equation (10)
and utilizing simultaneously $\gamma_{\infty} \approx 3(w - 1)/(6w - 5)$, we can derive the constants $\gamma_{i,0}$ in terms of $(\Omega_{m0}, w, \Delta_{d0})$.

To this end, we would like to stress that the parameterization $f(a) \approx \Omega_{m}(a)^{p(w)}$ plays a significant role in structure formation studies since it greatly simplifies the numerical calculations of equation (1). Indeed, providing a direct integration of the above parametrization we easily find the linear growth factor

$$\delta_{w}(a, \gamma) = a(z) \exp \left[ \int_{m}^{a(z)} \frac{du}{a} \left( \Omega_{m}(u) - 1 \right) \right],$$

(11)

where $a_i$ is the scale factor of the universe at which the matter component dominates the cosmic fluid (here, we use $a_i \approx 10^{-1}$ or $z_i \approx 10$). Then the linear growth factor normalized to unity at the present epoch is $D(a) = (\Omega_{m}(a))^{1/2}$.

Lastly, let us conclude with a brief discussion regarding the functional form of $\Delta_{d0}$. Generally, in order to investigate the evolution of the DE perturbations we need to solve the system of equations (1) and (2). However, one may easily check that the latter system contains a particular solution, namely $\delta_{d0} = (1 + w)\delta_{m}$ (see also Bean & Dore 2004; Abramo et al. 2007; Ballesteros & Riotto 2008; Abramo et al. 2009). Also, analytical solutions under of specific conditions can be found in Sapone & Kunz (2009) and Sapone & Majerotto (2012). Note that the accelerated expansion of the universe poses the restriction $w < -1/3\Omega_{d0}$ which implies $\Delta_{d0} < (3\Omega_{d0} - 1)/3\Omega_{m}$. Based on the above arguments, we find the following two interesting cases: (a) for the quintessence CLDE ($-1 < w < -1/3\Omega_{d0}$) model with $\delta_{d0} > 0$ we can have overdense DE regions ($\delta_{d0} > 0$) and (b) we may have underdense DE regions ($\delta_{d0} < 0$) ‘DE voids’ in the phantom DE regime as long as $\delta_{d0} > 0$. In a forthcoming paper, we attempt to investigate the general solution of the system (1) and (2) and thus to provide a complete classification of the DE structures. To this end, if we consider that the DE fluctuations exist in nature ($\delta_{d0} \neq 0$) then they could potentially play a role in the DE era defined as $z \leq z_{s}$, where $z_{s}$ is the redshift of matter–DE equality which is given by $z_{s} = \left(\frac{\Omega_{d0}}{\Omega_{m}}\right)^{1/3w} - 1$.

3 OBSERVATIONAL CONSTRAINTS

In the following, we briefly present some details of the statistical method and on the observational sample that we adopt in order to constrain the free parameter $\Delta_{d0}$. We use the recent growth rate data for which their combination parameter of the growth rate of structure, $f(z)$, and the redshift-dependent rms fluctuations of the linear density field, $\sigma_{f}(z)$, is available as a function of redshift, $f(z)\sigma_{f}(z)$. The total sample contains $N = 16$ entries (as collected by Basilakos et al. 2013 – see their table 1 and references therein).

Note that the $f\sigma_{f}$ estimator is almost a model-independent way of expressing the observed growth history of the universe (Song & Percival 2009). We use the standard $\chi^{2}$-minimization procedure, which in our case it is defined as follows:

$$\chi^{2}(p) = \sum_{i=1}^{N} \left[ \frac{f\sigma_{f}(z_{i}, p) - f\sigma_{f}(z_{i}, p)}{\sigma_{f}} \right]^{2},$$

(12)

where $\sigma_{f}$ is the observed $\sigma$ uncertainty and the theoretical growth rate is given by $f\sigma_{f}(z_{i}, p) = \sigma_{f}(z_{i})\Omega_{m}(z_{i})^{1/2}$. The statistical vector $p$ provides the free parameters that enter in deriving the theoretical expectations. For the case of constant $w$, it is defined as $p = (\Omega_{m0}, w, \Delta_{d0}, \sigma_{f})$. Since we are interested to check the DE perturbations, at the present time we restrict the likelihood analysis to the choice $\Omega_{m0} = 0.30$ and $\sigma_{f,\Lambda} = 0.18(0.30/\Omega_{m0})^{0.26}$ provided by the Planck analysis of Spergel et al. (2015). Concerning the EoS

Figure 1. Left-hand panel: the variance $\Delta\chi^{2} = \chi^{2} - \chi_{min}^{2}$ around the best-fitting $\Delta_{d0}$ value for the quintessence ($w = -0.9$; dashed line) and phantom ($w = -1.1$; dotted line) models. Right-hand panel: comparison of the observed and theoretical evolution of the growth rate $f(z)\sigma_{f}(z)$. Note that for the curves we utilize the Planck priors provided by Spergel et al. (2015), $(\Omega_{m0}, \sigma_{f,\Lambda}) = (0.30, 0.818)$. The solid curve corresponds to the concordance $\Lambda$CDM model.

we set it either to $w = -1.1$ (phantom Planck Collaboration XVI 2014) or to $w = -0.9$ (quintessence). Also, in order to use the $\sigma_{f}$ prior properly along the DE models, we rescale the values of $\sigma_{f}$ by $\sigma_{f} = \left(\frac{\Omega_{m0}}{0.30}\right)^{1/2}$. In that case, the statistical vector includes only one free parameter, $p = \Delta_{d0}$.

Below we briefly discuss the main statistical results as follows.

(a) Homogeneous DE ($\Delta_{d0} = 0$): for the concordance $\Lambda$ cosmology, we find that the theoretical $(\gamma_{0,1}, \gamma_{1,1}) \simeq (0.556, -0.011)$ values reproduce the growth data with $\chi_{min}^{2}/dof \simeq 18.1/15$. The number of degrees of freedom is $dof = N - k - 1$, where $k$ is the number of the fitted parameters (in this case $k = 0$). Evidently, the above value of the reduced $\chi_{min}^{2}$ suggests that the $\Lambda$CDM model cannot simultaneously accommodate the Planck priors and the growth data (see also Macaulay, Wehus & Eriksen 2013; Mehrabi et al. 2015).

For the quintessence and phantom DE models, we find that $(\gamma_{0,1}, \gamma_{1,1}) = (0.56, -0.012)$ with $\chi_{min}^{2} \simeq 18.4/15$ and $(\gamma_{0,1}, \gamma_{1,1}) = (0.554, -0.011)$ with $\chi_{min}^{2} \simeq 20.2/15$, respectively.

(b) Clustered DE ($\Delta_{d0} \neq 0$, $k = 1$): in the left-hand panel of Fig. 1, we present the variation of $\Delta\chi^{2} = \chi^{2}(\Delta_{d0}) - \chi_{min}^{2}(\Delta_{d0})$ around the best-fitting $\Delta_{d0}$ value. For the quintessence CLDE model (dashed line), we find that the likelihood function of the growth data peaks at $\Delta_{d0} = 0.14 \pm 0.04$ with $\chi_{min}^{2}/dof \simeq 8.2/14$ and thus we obtain $(\gamma_{0,1}, \gamma_{1,1}) \simeq (0.677, -0.129)$ which are in tension with those of homogeneous quintessence DE (see above). Alternatively, if we impose the particular solution $\Delta_{d0} = 1 + w$ and minimize equation (12) with respect to $w$, we find $w = -0.85 \pm 0.05$ with $\chi_{min}^{2}/dof \simeq 7.7/14$. The fitted value of $\Delta_{d0} = 0.14$ is in agreement, within $1\sigma$ errors, with that of $\Delta_{d0} = 1 + w$, which implies that for the quintessence CLDE model the corresponding particular solution of the system (1) and (2) $\delta_{d0} = (1 + w)\delta_{m}$ is consistent with the growth data. In the case of phantom CLDE (with $w = -1.1$) the best-fitting value is $\Delta_{d0} = 0.11 \pm 0.03$ (dotted line: left-hand panel of Fig. 1) with $\chi_{min}^{2}/dof \simeq 10.5/14$ and thus we obtain $(\gamma_{0,1}, \gamma_{1,1}) \simeq (0.667, -0.124)$.

It is interesting to mention that for the quintessence model the above $\Delta_{d0}$ measurements are in agreement with those predicted by previous studies. Indeed, Bean & Dore (2004) found that in the case of $w = -0.8^{2}$ the ratio $\Delta_{d0} = \delta_{d0}/\delta_{m}$ at the present time can reach up to $\sim 0.09$ (see their fig. 1).

\footnote{In this study, the effective sound speed lies in the interval $0 \sim 1$.}
In the right-hand panel of Fig. 1, we plot the observed $f(z)\sigma_8(z)$ with the estimated growth rate function (see $\Lambda$CDM – solid line, quintessence – dashed line and phantom – dotted line). In the right-hand panel of Fig. 2, we present the evolution of the DE perturbations for the quintessence and phantom models, respectively. Based on the aforementioned $\Delta_\text{det}$ observational constraints and equation (11), we can estimate the DE fluctuations at the present time.\footnote{Inserting the values of $(\Omega_m,\gamma_0,\gamma_1)$ into equation (11), we find that the linear growth factor is $\delta_{\text{lin}} \simeq 0.774$ (for $w = -0.9$) and $\delta_{\text{lin}} \simeq 0.735$ (for $w = -1.1$), respectively.} In particular, we find $\delta_{\text{lin}} = 0.108 \pm 0.031$ and $\delta_{\text{lin}} = 0.081 \pm 0.022$ for the quintessence and phantom models, respectively. In the right-hand panel of Fig. 2, we can also see our determination of $\gamma(z)$ as a function of redshift. The comparison indicates that the growth index of the CLDE models strongly deviates with respect to that of the concordance $\Lambda$ cosmology. As an example close to the present epoch the departure can be of the order of $(\Delta_\gamma) \simeq 25\%$. Note that Ballesteros & Riotto (2008) found an $\sim 10\%$ per cent while Sefusatti & Vernizzi (2011) computed an $\sim 10\%$ per cent difference (see footnote 5 in their paper). On the other hand using equations 1 and 2, Mehrabi et al. (2015) found a relative large amount of deviations, namely $\sim 20–30\%$ per cent. These differences imply that the deviations of the growth index depend on the initial assumptions and limitations imposed in the general system of equations that govern the matter and DE fluctuations. This is something which needs further investigation.

To complete our study, we repeat the analysis by using those priors derived by the Planck team (Planck Collaboration XVI 2014), namely $\Omega_m = 0.315$ and $\sigma_8 = 0.87(0.27)/\Omega_m$\footnote{In particular, we find the linear growth factor is $\delta_{\text{lin}} \simeq 0.774$ (for $w = -0.9$) and $\delta_{\text{lin}} \simeq 0.735$ (for $w = -1.1$), respectively.}. In brief, we find: $\Delta_\text{det} = 0.19 \pm 0.04$ with $\chi^2_{\text{min}}/\text{dof} \simeq 8/14$ and $\Delta_\text{det} = 0.16 \pm 0.03$ with $\chi^2_{\text{min}}/\text{dof} \simeq 11.6/14$ for the quintessence and phantom CLDE models, respectively. Recently, the cosmological results of Planck 2015 appeared in the literature (Planck Collaboration XIII 2015) and thus we utilize the pair $(\sigma_{8,\Lambda}, \Omega_m)_{\text{Planck15}} = (0.308, 0.815)$, which is close to that provided by Spergel et al. (2015). In this case, the likelihood function peaks at $\Delta_\text{det} = 0.15 \pm 0.04$ with $\chi^2_{\text{min}}/\text{dof} \simeq 8.1/14$ and $\Delta_\text{det} = 0.12 \pm 0.03$ with $\chi^2_{\text{min}}/\text{dof} \simeq 10.3/14$ for the quintessence and phantom CLDE models, respectively. We verify that if we increase the value $\Omega_m$ and that of the rms fluctuation of the linear density field on $8h^{-1}\text{Mpc}$ scales, then the corresponding likelihood function peaks at higher $\Delta_\text{det}$ values.

Finally, in order to compare the above DE models we use the corrected Akaike information criterion for small sample size (AIC; Akaike 1974; Suguriya 1978) which is defined for the case of Gaussian errors, as

$$AIC = \chi^2_{\text{min}} + 2k + \frac{2k(k - 1)}{N - k - 1}.$$  

A smaller value of AIC indicates a better model-data fit. We observe that the values of $\text{AIC}_{\text{CLDE}}(\simeq 10.1–12.4)$ are smaller than the corresponding homogeneous DE values, namely $\text{AIC}_{\text{DE}} \simeq 18.1–20.2$ which indicate that the CLDE scenario appears to fit the growth data better than the usual homogeneous DE (see also Mehrabi et al. 2015). At this point, it is interesting to mention that Nesseris & Sapone (2015) proposed a model-independent test in order to check possible departures from the $\Lambda$CDM cosmological model at perturbation level. Assuming that the growth data are free from systematics they found that in order to reproduce the growth data, we need to deal either with a CLDE scenario or with a modified gravity.

\section*{4 CONCLUSIONS}

To summarize, we derive a new formulation of the growth index of matter perturbations in the regime (sub-horizon) where the DE is allowed to cluster. In this framework, we find that the growth index is indeed affected by the DE perturbations. In order to check the range of validity of such a scenario, we perform a likelihood analysis using the recent growth data. We show that the CLDE models fit the observational data much better than those of homogeneous DE models. With the next generation of surveys, based mainly on Euclid (see also Sapone et al. 2013) we will be able to check whether the DE perturbations do really exist in nature.

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**APPENDIX A:**

In this appendix, we examine the linear equation that describes the evolution of the total (matter and DE) perturbations. Specifically, using \( w = \text{const.} \), the combination of equations (1) and (2) provides

\[
\ddot{\delta} + 2H\dot{\delta} = \frac{3H^2}{2}(2+w)\left[\Omega_m\delta_m + \Omega_d\delta_d(1+3w)\right],
\]

where \( \delta \equiv \delta_m + \delta_d \) is the sum of matter and DE fluctuations. Obviously, for \( (w, \delta_d) = (-1, 0) \) the above equation boils down
to that of the concordance \( \Lambda \) cosmology. Now, considering that the overall growth rate of clustering is given by \( f = \frac{d\ln\delta}{d\ln a} \simeq \Omega_m^{\gamma}(z) \) and following simultaneously the procedure described in Section 2, we arrive at

\[
f = \frac{3(2+w)}{2(1+\Delta_{de})}\left[\Omega_m + (1+3w)\Delta_{de}\Delta_{de}\right]
\]

(A2)

or

\[-(1+z)\gamma'\ln(\Omega_m) + \Omega_m' + 3w\Omega_m + (\gamma - \frac{1}{2}) + \frac{1}{2} + \frac{3}{2}\Omega_m^{-\gamma}X,\]

(A3)

where prime denotes derivative with respect to redshift and

\[\dot{X}(z) = \frac{(2+w)X(z)}{1 + \Delta_{de}(z)}\]

We remind the reader that the function \( X(z) \) is given by equation (9) and \( \gamma' = \gamma_0 + \gamma_1, y(z) \) with \( y(z) = z/(1+z) \). Therefore, replacing \( X_0 = X(z = 0) \) with \( X_0 = X(z = 0) \) in equation (10) we have \( \gamma_1 \) in terms of \( \gamma_0 \). Finally, repeating our statistical analysis we find the following results.

In the case of Spergel et al. (2015) Planck priors (\( \Omega_m, \sigma_{8, \lambda} = (0.30, 0.818) \):

(i) for the quintessence model: \( \Delta_{de0} = 0.13 \pm 0.04 \) with \( \chi^2_{\text{min}}/\text{dof} \simeq 8.2/14, \text{AIC} \simeq 10.2 \) and \( (\gamma_0, \gamma_1) \simeq (0.671, -0.123) \).

(ii) for the phantom model: \( \Delta_{de0} = 0.10 \pm 0.04 \) with \( \chi^2_{\text{min}}/\text{dof} \simeq 10.5/14, \text{AIC} \simeq 12.5 \) and \( (\gamma_0, \gamma_1) \simeq (0.674, -0.131) \).

In the case of the Planck 2015 priors (Planck Collaboration XIII (2015) (\( \Omega_m, \sigma_{8, \lambda} = (0.308, 0.815) \):

(i) for the quintessence model: \( \Delta_{de0} = 0.15 \pm 0.04 \) with \( \chi^2_{\text{min}}/\text{dof} \simeq 8.1/14, \text{AIC} \simeq 10.1 \) and \( (\gamma_0, \gamma_1) \simeq (0.689, -0.141) \).

(ii) for the phantom model: \( \Delta_{de0} = 0.11 \pm 0.04 \) with \( \chi^2_{\text{min}}/\text{dof} \simeq 10.3/14, \text{AIC} \simeq 12.3 \) and \( (\gamma_0, \gamma_1) \simeq (0.684, -0.141) \).

Obviously the above results are in agreement within 1\( \sigma \) with those presented in Section 3.