Nuclear matter properties and equation of state of neutron matter

Satoshi Yoshida\textsuperscript{1} and Hiroyuki Sagawa\textsuperscript{2}

\textsuperscript{1} Science Research Center, Hosei University, 2-17-1 Fujimi, Chiyoda, Tokyo 102-8160, Japan
\textsuperscript{2} Center for Mathematical Sciences, the University of Aizu, Aizu-Wakamatsu, Fukushima 965-8580, Japan

E-mail: s_yoshi@hosei.ac.jp

Abstract. Correlations among symmetric nuclear matter properties, the neutron matter EOS and the pressure of neutron matter are studied in the Skyrme Hartree-Fock (SHF) and the relativistic mean field (RMF) models. At present, many different parameter sets for the Skyrme interaction and RMF model have been proposed. As a result, the property of neutron matter which is one of extreme conditions depends on the parameter set. We found clear linear correlations between the energy per neutron of neutron matter and the symmetry energy and between the pressure of neutron matter and $L$ which is one of isovector nuclear matter properties at the saturation density of symmetric nuclear matter. We show that values of the energy per neutron and the pressure of neutron matter at the saturation density of symmetric nuclear matter can be obtained by accurate experimental data of the symmetry energy and $L$.

1. Introduction

Nuclear mean field models have been successful to describe properties of the ground state, collective excitations and so on. Two different models are widely used. The first one is the Skyrme Hartree-Fock (SHF) model \cite{1} which is one of non relativistic models. The second one is the relativistic mean field (RMF) model with terms of $\sigma$, $\omega$ and $\rho$ meson fields in an effective Lagrangian \cite{2}. The nuclear binding energy and the saturation density precisely measured are used to determine the parameter sets of the Skyrme interactions and RMF models. As a result, many different parameter sets for the Skyrme interaction and RMF model have been proposed. Other nuclear matter properties such as the incompressibility, the symmetry energy are less well measured by experiments and are only used as rough guides in the determination of parameter sets. Different parameter sets give fairly similar results for nuclear ground state properties. However in the extreme condition such as ground state properties of neutron rich unstable nuclei and properties of infinite nuclear matter, results depend on parameter sets. We have discussed about correlations among isovector nuclear matter properties and the neutron skin thickness \cite{3}. In this paper, we study relations between properties of infinite neutron matter and nuclear matter properties of the Hamiltonian density for a large number of different parameter sets of the Skyrme interaction and RMF model.

2. Results and discussion

The Hamiltonian density $H$ consists of the isoscalar part $H_S$ and the isovector part $H_V$ generally and in microscopic calculations $H$ is described by the following equation,
\[ H(\rho) = H_S(\rho) + H_V(\rho) I^2 + \rho D(\rho, I), \]  
(1)

where \( I \) and \( D(\rho, I) \) are the asymmetry parameter \( I = (\rho_n - \rho_p)/\rho \) and the term proportional to \( I^4 \), respectively and \( \rho_n, \rho_p \) and \( \rho = \rho_n + \rho_p \), respectively. The isovector part \( H_V \) can be expressed as a Taylor expansion around the saturation density \( \rho_{nm} \):

\[ \frac{H_V(\rho)}{\rho} = J + \frac{L}{3} \left( \frac{\rho - \rho_{nm}}{\rho_{nm}} \right) + \frac{K_{sym}}{18} \left( \frac{\rho - \rho_{nm}}{\rho_{nm}} \right)^2 + \cdots. \]  
(2)

The symmetry energy \( J \) and \( L \) can be obtained from following equations:

\[ J = \frac{H_V(\rho_{nm})}{\rho_{nm}}, \]  
(3)

\[ L = 3\rho_{nm} \left( \frac{\partial}{\partial \rho} \left( \frac{H_V(\rho)}{\rho} \right) \right)_{\rho=\rho_{nm}}. \]  
(4)

On the other hand, the nuclear saturation density \( \rho_{nm} \), the binding energy per nucleon \( E_0 \), and the incompressibility \( K \) relate to the isoscalar part of the Hamiltonian density \( H_S \). \( \rho_{nm} \) and \( E_0 \) are defined by following equations, respectively:

\[ 0 = \left. \frac{\partial}{\partial \rho} \left( \frac{H_S(\rho)}{\rho} \right) \right|_{\rho=\rho_{nm}}, \]  
(5)

\[ -E_0 = \frac{H_S(\rho_{nm})}{\rho_{nm}}. \]  
(6)

In our previous paper [4], we studied correlations among the isovector and isoscalar nuclear matter properties by using SHF results for 13 different Skyrme interactions and RMF results for seven parameter sets. In this work, we discuss those correlations by using 111 Skyrme interactions and 15 RMF parameter sets listed in Table 1. The isoscalar and isovector nuclear matter properties affect the equation of state of nuclear matter. We study relations between

| Skyrme interactions | RMF parameter sets |
|---------------------|---------------------|
| SI, SI’, SII, SIII, SIII’, SIV, SV, SVI, SVII, Skya, Skyb, SkM, SkM*, SkM1, SLy4, SLy5, SLy6, SLy7, SLy10, SLy230a, SLy20b, SG1, SGI, SkII, SkI2, SkI3, SkI4, SkI5, SkX, SkX ce, SkXm, SkP, SkT, SkT1, SkT2, SkT3, SkT4, SkT5, SkT6, SkT7, SkT8, SkT9, RATP, E, Es, T, Rs, Gs, Z, Zs, Zs*, SkO, SkO’, LNS, MSkA, MSk1, MSk2, MSk3, MSk4, MSk5, MSk5*, MSk6, MSk7, MSk8, MSk9, v1.10, v1.05, v1.00, v0.90, v0.80, v0.75, v0.70, SK255, SK272, SK271, SkSC1, SkSC2, SkSC3, SkSC4, SkSC4o, SkSC5, SkSC6, SkSC10, SkSC11, SkSC14, SkSC15, SkRA, SkMP, BSki, BSki2, BSki3, BSki4, BSki5, BSki6, BSki7, BSki8, BSki9, BSki10, BSki11, BSki12, BSki13, BSki14, BSki15, BSki16, BSki17, BSki18, BSki19, BSki20, BSki21, KDE0v1, NL1, NL2, NL3, NLSH, NLC, NL–S, NL–Z, NL–Z2, TW99, TM1, TM2, DDME1, DDME2, PK1, PKDD |

Table 1. Skyrme interactions and RMF parameter sets
properties of infinite neutron matter and nuclear matter properties of the Hamiltonian density at the saturation density $\rho_{nm}$ of symmetric nuclear matter which is determined by Eq.(5). Figure 1(a) shows the clear linear correlation between the energy per neutron $H_n(\rho_{nm})/\rho_{nm}$ of neutron matter at the saturation density $\rho_{nm}$ and $(-E_0 + J)$ calculated by 111 Skyrme interactions and 15 RMF parameter sets listed in Table.1. In Fig.1(a), $H_n$ is the Hamiltonian density of neutron matter and $J$ and $-E_0$ are defined by Eqs.(3) and (6), respectively. Because of $I = 1$ in the case of neutron matter, Eq.(1) becomes the following equation,

$$H_n(\rho_n) = H_S(\rho_n) + H_V(\rho_n) + \rho_n D(\rho_n, I = 1).$$  \hfill (7)

We fit the linear function shown by the dotted line in Fig.1(a) by the method of least-squares and obtain the following relation,

$$\frac{H_n(\rho_{nm})}{\rho_{nm}} = 0.9965 (-E_0 + J) + 1.022 \quad (R = 0.9955),$$  \hfill (8)

where $R$ is the correlation coefficient. Because the value of $-E_0$ defined by Eq.(6) is about $-16$ MeV in all the parameter sets of the Skyrme interactions and RMF models listed in Table.1, the correlation between $H_n(\rho_{nm})/\rho_{nm}$ and $J$ becomes linear. Actually Eq.(8) is analytically obtained in the case that the neutron density $\rho_n$ is equal to the saturation density $\rho_{nm}$ of symmetric nuclear matter. By using Eq.(7), the energy per neutron $H_n(\rho_{nm})/\rho_{nm}$ of neutron matter at the saturation density $\rho_{nm}$ is given by the following equation,

$$\frac{H_n(\rho_{nm})}{\rho_{nm}} = (-E_0 + J) + D(\rho_{nm}, I = 1).$$  \hfill (9)

The coefficient of $(-E_0 + J)$ in Eq.(8) which is 0.9965 is very close to one which is that in Eq.(9). And because the second term in Eq.(8) corresponds to the second term in Eq.(9), the value of the term $D$ at $I = 1$ and $\rho_{nm}$ can be obtained by the following relation,

$$D(\rho_{nm}, I = 1) = 1.022.$$  \hfill (10)
Next, the pressure of neutron matter is as important as its energy per neutron in the discussion of neutron matter EOS. Figure 1(b) shows the clear linear correlation between the pressure $P_{n}(\rho_{nm})$ of neutron matter at the saturation density $\rho_{nm}$ and $L$ defined by Eq.(4) calculated by 111 Skyrme interactions and 15 RMF parameter sets listed in Table.1. We fit the linear function shown by the dotted line in Fig.1(b) by the method of least-squares and obtain the following relation,

$$P_{n}(\rho_{nm}) = 0.05103 L + 0.1700 \quad (R = 0.9975), \quad (11)$$

where $R$ is the correlation coefficient. Actually Eq.(11) is also analytically obtained by using Eq.(7). In the case of neutron matter, the first derivative $\frac{\partial}{\partial \rho_{n}} \left( \frac{H_{n}(\rho_{n})}{\rho_{n}} \right)$ of the energy per neutron of neutron matter at the saturation density $\rho_{nm}$ is given by the following equation,

$$\frac{\partial}{\partial \rho_{n}} \left( \frac{H_{n}(\rho_{n})}{\rho_{n}} \right) \bigg|_{\rho_{n}=\rho_{nm}} = \frac{1}{3\rho_{nm}} L + \frac{\partial}{\partial \rho_{n}} D(\rho_{n}, I) \bigg|_{\rho_{n}=\rho_{nm}}. \quad (12)$$

Because the pressure $P_{n}$ of neutron matter is defined by the following equation as a function of neutron density $\rho_{n}$,

$$P_{n}(\rho_{n}) = \rho_{n}^2 \frac{\partial}{\partial \rho_{n}} \left( \frac{H_{n}(\rho_{n})}{\rho_{n}} \right), \quad (13)$$

we can find that the pressure $P_{n}(\rho_{nm})$ of neutron matter at the saturation density $\rho_{nm}$ is proportional to $L$ as shown in the following equation,

$$P_{n}(\rho_{nm}) = \frac{1}{3} \rho_{nm} L + \rho_{nm}^2 \frac{\partial}{\partial \rho_{n}} D(\rho_{n}, I = 1) \bigg|_{\rho_{n}=\rho_{nm}}. \quad (14)$$

The coefficient of $L$ in Eq.(11) which is 0.05103 is very close to the value of $\rho_{nm}/3$ which is that of $L$ in Eq.(14) because the value of empirical saturation density $\rho_{nm}$ of symmetric nuclear matter is around 0.16 fm$^{-3}$. Because the second term in Eq.(11) corresponds to the second term in Eq.(14), the value of the first derivative of the term $D$ at $I = 1$ and $\rho_{nm}$ can be obtained by the following relation,

$$\rho_{nm}^2 \frac{\partial}{\partial \rho_{n}} D(\rho_{n}, I = 1) \bigg|_{\rho_{n}=\rho_{nm}} = 0.1700. \quad (15)$$

The equation of state and the pressure of neutron matter depend on parameter sets of the Skyrme interaction and RMF model we use. However if we can obtain accurate experimental data of $J$ and $L$, we will be able to find values of the energy per neutron and the pressure of neutron matter at the saturation density of symmetric nuclear matter by using Eqs.(8) and (11).

3. Summary

We found clear linear correlations between the energy per neutron of neutron matter and the symmetry energy and between the pressure of neutron matter and $L$ at the saturation density of symmetric nuclear matter with 111 Skyrme interactions and 15 RMF parameter sets and were able to obtain linear functions fitted by the method of least-squares. And we pointed out that values of the energy per neutron and the pressure of neutron matter at the saturation density of symmetric nuclear matter can be obtained by accurate experimental data of $J$ and $L$ with those linear functions.

References

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