Theoretical nuclear structure and astrophysics at FAIR

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Abstract. Next generation of radioactive ion beam facilities like FAIR will open a bright future for nuclear structure and nuclear astrophysics research. In particular, very exotic nuclei (mainly neutron rich) isotopes will be produced and a lot of new exciting experimental data will help to test and improve the current nuclear models. In addition, these data (masses, reaction cross sections, beta decay half-lives, etc.) combined with the development of better theoretical approaches will be used as the nuclear physics input for astrophysical simulations. In this presentation I will review some of the state-of-the-art nuclear structure methods and their applications.

1. Introduction
The main goal of the nuclear astrophysics is the study of the origin of the elements that are found in Nature and their abundance. Hence, both the different nucleosynthesis processes, as well as the astrophysical scenarios where the proper conditions to obtain such processes are found, are investigated.

Concerning the nucleosynthesis processes, the existence of several of them are well established [1, 2], namely (see Fig. 1): a) Big Bang nucleosynthesis that produced basically hydrogen and helium; b) nuclear burning of light nuclei (hydrogen, helium, carbon, etc.) that produces elements up to the iron group where the nuclear fusion is not energetically favorable any longer; c) s-process, whose main mechanism is the slow neutron capture -slow compared to the beta decay half-lives- and produces the peaks observed in the elementary abundance curve around mass numbers A = 90, 138 and 208. This mechanism occurs through nuclei in the valley of stability; d) r-process or rapid neutron capture [3], which produces the peaks around A = 80, 130 and 196. This process flows through very neutron rich nuclei away from the stability; e) rp-process or rapid proton capture [4] and νp-process, where the antineutrino absorption plays a key role [5]. In both cases, the nuclei involved are neutron deficient.

On the other hand, each process requires compositional and thermodynamical conditions to make possible the nuclear and electroweak reactions which characterize them. Hence, while the nuclear burning and s-process occur mainly in the interior of stars, the other processes are found in X-Ray bursts in binary systems, supernova explosions or neutron star collisions, for instance.

To understand all of these processes and establish the astrophysical sites, the nucleosynthesis simulations require input parameters coming from nuclear physics, such as masses and half-lives, reaction rates, fission products, neutrino reactions, etc. These quantities are taken from
Stellar nucleosynthesis processes

In 1957 Burbidge, Burbidge, Fowler and Hoyle and independently Cameron, suggested several nucleosynthesis processes to explain the origin of the elements.

- **νp-process**: "neutrino-proton process" via unstable proton-rich nuclei through proton capture
- **rp-process**: "rapid proton process" via unstable proton-rich nuclei through proton capture
- **r-process**: "rapid process" via unstable neutron-rich nuclei through proton capture
- **s-process**: "slow process" via chain of stable nuclei through neutron capture
- Fusion up to iron

**Figure 1.** Chart of nuclei and the paths followed by the different nucleosynthesis processes. Black, orange and green colors represent stable, synthesized unstable and non-synthesized in earth nuclei, respectively. Figure adapted by G. Martínez-Pinedo from material available at https://www.gsi.de/en/start/fair/forschung_an_fair/nuclear_structure_physics.htm.

the experimental data where available. To do so, stable and radioactive ion beams facilities play a key role. In particular, one of the main goals of the NUSTAR collaboration at FAIR is the study of the properties of the most relevant nuclei in astrophysical processes, especially in the r-process [6]. Nevertheless, although in near future both FAIR and other similar facilities such as FRIB (MSU-USA), SPIRAL-2 (GANIL-France) or RIBF (RIKEN-Japan) will reach a wide region of nuclei relevant for the r-process, they will not be able to cover the full range of nuclei involved in such a nucleosynthesis mechanism. Hence, the properties of these experimentally out of reach nuclei will have to be studied only theoretically. Therefore, it is important to continue developing nuclear models and theories to provide a reliable nuclear physics input for astrophysical simulations. In this contribution I will introduce briefly the most relevant theoretical approaches to study currently the nuclear structure and their range of applicability.

2. **The road from QCD to the stars**

The starting point of the microscopic models in nuclear physics is the definition of the interaction between the constituent nucleons (protons and neutrons). These interactions can be defined from bare interactions between the nucleons (sometimes called **realistic** interactions) or can take into account nuclear medium effects (**phenomenological** interactions). In any case, all of them have parameters or coupling constants that must be fitted to reproduce some sets of experimental data.

Hence, belonging to the first category, the bare interactions are fitted to reproduce with high precision the scattering phase shifts in nucleon-nucleon (NN) collisions and deuteron properties. If three body (3N) are included, this channel is normally fitted to reproduce properties of triton and $^4$He. Therefore, the experimental information contained in this kind of interactions stops at four body systems at most since no additional many body forces beyond 3N have been taken into account so far. Among these interactions, those extracted from a chiral effective field theory ($\chi$EFT) are widely used and studied recently for their close connection to QCD [7, 8].
Here, the interactions are expanded systematically in powers of the pion mass with respect to the momentum scale of the effective theory, obtaining a hierarchy in the NN, 3N, etc. forces. Hence, at the lowest two orders of the expansion, only contact and one- and two- pion exchange NN forces are obtained, while 3N forces appear at next-to-next-to-leading order (N^2LO).

Concerning the phenomenological interactions, these are adjusted in different ways depending on the nuclear many body method used to solve the problem. For example, in the interactions used in large scale shell model (LSSM) calculations [9], the single particle energies are taken from the excitation energies of nuclei next to magic numbers -with one nucleon more or less. In the interactions used in energy density functional methods (EDF) [10], the parameters of the functional are fitted to reproduce masses and radii of nuclei in different regions of the nuclear chart.

In the following subsections I will give a brief description of the nuclear many body methods currently used applying either bare or in medium interactions as starting points, according to the previous classification.

2.1. Nuclear structure with bare nucleon interactions
The main obstacle for using bare interactions to calculate nuclei with mass numbers A > 12 is the fact that the nucleons with low momentum can be scattered to high momenta and vice versa due to the strong short-range repulsion and tensor terms present in those forces. This means that the number of states that have to be taken into account in the A-body Hilbert space explodes. Nevertheless, there are some methods like the Similarity Renormalization Group (SRG) -see Ref. [11] and references therein- or the Unitary Correlator Operator Method (UCOM) [12] where the original hamiltonian is unitary transformed to decouple high and low momentum states. The main drawback of these renormalization methods is that they induce many body forces even when the original hamiltonian only contains NN interactions. Therefore, truncations in those induced many body forces have to be done, although the contribution of these missing pieces can be estimated through the dependence of the observables on the renormalization parameters.

These soft interactions have been widely used recently with different many body methods and some of these calculations have been called as ab-initio nuclear structure. Hence, using the no-core shell model method (NCSM), where the hamiltonian is diagonalized in a large valence space without a core, nuclei up to \(^{16}\)O including genuine and induced NN+3N forces have been calculated (see Ref. [13] and references therein). With other methods, which do not perform direct diagonalization, such as Couple-Cluster (CC) [14], in-medium SRG [15] or Self-consistent Green’s function methods [16] heavier magic and semi magic nuclei up to \(A \sim 70\) have been calculated. Recently, these interactions have been also used in combination with many body perturbation theory and standard large scale shell model techniques to compute neutron deficient semi magic nuclei \(A < 50\) [17] or neutron rich calcium isotopes [18].

On the other hand, the neutron matter equation of state and its astrophysical implications in neutron stars have been studied using these soft interactions transformed from the bare one [19].

2.2. Nuclear structure with phenomenological interactions
Despite the range of applicability of the methods shown in the previous section has grown in the last years, most of the nuclei with astrophysical interest are out of reach for those methods. In addition, the precision obtained in the description of binding energies is unacceptable for nucleosynthesis simulations because the absence of forces beyond 3N interactions produces largely over-bound nuclei. The inclusion of these many body forces within the chiral effective field theory framework -and their corresponding coupling constants fitted to few body experimental data- is a work in progress but the results are expected in the long-range plan.

Therefore, to attain more accurate predictions of nuclear masses, spectra or reaction rates in a general way, many body methods based on phenomenological interactions are employed.
Among them, large scale shell model (LSSM) [9] and energy density functionals (EDF) [10] are the most common work horses.

The first one is based on the diagonalization of an effective interaction defined within a restricted valence space. These interactions are adapted specifically to the different valence spaces and fitted to reproduce the evolution of the single particle energies with the number of nucleons included in the space. This method is the best one in reproducing the nuclear spectra. It has been also used in nuclear astrophysics applications such as the calculation of beta decay rates including allowed and first forbidden transitions [20, 21]. Although in the last years the increase of computational power and the improvement in the algorithms have allowed the calculation of nuclei quite far away from shell closures, the main limitation of this method is the combinatorial grow of the number of configurations within the valence space.

On the other hand, the EDF methods do not have this restriction and can be applied all over the nuclear chart, for example, in neutron rich nuclei belonging to the $r$-process path. They are based on the variational principle assuming very simple many body trial states (Hartree-Fock-Bogoliubov -HFB-) and interactions valid in the whole nuclear table, being Skyrme [10], Gogny [22] or Relativistic [23] the most commonly used. Bulk properties such as ground state radii and masses are very well described within this framework [24, 25]. The latter can be used to deduce neutron capture rates to be implemented in nucleosynthesis simulations. In the last years there have been several advances to extend these methods beyond the HFB approach, allowing the study of nuclear spectra (see Ref. [26] and references therein) and nuclear response to electromagnetic probes [27], for example. The most important improvements of the EDF methods which should be addressed to obtain the same degree of precision as the LSSM -in the regions where it can be applied- are the redefinition of the effective interactions including beyond mean field effects and the inclusion of single particle degrees of freedom.

2.3. Nuclear physics impact in $r$-process nucleosynthesis
Finally, I will mention a particular example where the relevance of the nuclear physics input in astrophysical simulations, the nuclear masses in this case [28], is shown. More than the half of elements heavier than iron are produced in the $r$-process through very neutron rich nuclei. Nuclear masses are needed both for computing neutron capture rates and beta decay $Q$-values that enter in the nucleosynthesis simulations. In Fig. 2(a) the two-neutron separation energy $S_{2n}$ for the Cadmium isotopic chain using two mass models, FRDM [29] and Duflo-Zuker [30], is represented. Both mass models reproduce the experimental data where available. However, they differ significantly in the region around neutron number $N = 90$, where FRDM has a minimum

![Figure 2](image_url)

**Figure 2.** (a) Two neutron separation energies for the cadmium isotopic chain calculated for FRDM and Duflo-Zuker mass models. (b) Final elementary abundances obtained by using FRDM or Duflo-Zuker models as part of the nuclear physics input in $r$-process nucleosynthesis simulations ($r$-process solar abundances are plotted to guide the eye). This figure is courtesy of Joel Mendoza-Temis.
and Duflo-Zuker continuously decreases. In Fig.2(b) the elementary abundances obtained after an \( r \)-process nucleosynthesis simulation with the same astrophysical conditions, using these two mass models as inputs, are represented. Noticeable differences around the peaks \( A = 130 \) and 196 are observed. In the FRDM case, an overproduction around \( A = 138 \) and an underproduction around \( A = 190 \) are obtained. This structure is ultimately linked to the minimum observed in the \( S_{2n} \) for the FRDM model because in this point the neutron capture is inhibited and modify the dynamics of the \( r \)-process. This ‘waiting point’ observed in the FRDM produces after beta decays the overabundance at \( A = 138 \) and, correspondingly, a ‘hole’ in the second peak abundances because the matter cannot continue capturing neutrons producing heavier nuclei. More interestingly, the origin of this minimum is a change in the intrinsic deformation in those specific isotopes.

This figure summarizes the necessity of having, on the one hand, experimental results in neutron rich regions to constrain the uncertainties in the astrophysical simulations. This is one of the main goals of NUSTAR at FAIR. On the other hand, we observe that the future facilities will not cover the whole range of nuclei involved in the \( r \)-process and further research will be necessary to improve the nuclear theory and make the models and predictions more reliable.

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