On the comparison of plans:
Proposition of an instability measure for dynamic machine scheduling

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ABSTRACT. On the basis of an analysis of previous research, we present a generalized approach for measuring the difference of plans with an exemplary application to machine scheduling. Our work is motivated by the need for such measures, which are used in dynamic scheduling and planning situations. In this context, quantitative approaches are needed for the assessment of the robustness and stability of schedules.

Obviously, any ‘robustness’ or ‘stability’ of plans has to be defined w. r. t. the particular situation and the requirements of the human decision maker. Besides the proposition of an instability measure, we therefore discuss possibilities of obtaining meaningful information from the decision maker for the implementation of the introduced approach.

KEYWORDS. Robust scheduling; dynamic scheduling; rescheduling; stability measure.

1 Scheduling in dynamic environments

Planning and scheduling generally considers a set of activities (operations) for which (i) assignments to resources, such as machines, and (ii) definition of starting times must be found, thus defining a schedule $x$ for a given problem. While the assignment of operations to machines is often given by technical side constraints, the processing order on the machines, and thus the precise starting times of the jobs is determined by the planning procedure/algorithm. W. r. t. the terminology used in machine scheduling, operations, denoted as $O_{jk}$, are often grouped into ‘jobs’ $J_j = \{O_{j1}, \ldots, O_{joj}\}$, and precedence constraints among the operations belonging to a particular job implement the technical requirements of processing.

Besides these general, static considerations, practical problems of scheduling in manufacturing environments are in most cases of dynamic nature. In these situations, a schedule needs to be found that is modified during the actual execution, adapting to the dynamically changing conditions. Changes of relevance to the scheduling are referred to as rescheduling factors (Dutta, 1990). Along with these issues come modifications of the relevant data and the characteristics of the problem. Depending on the changes and on the initial schedule $x_t$ at time $t$, a revision of $x_t$ might be necessary.

Such dynamic adaptations raise stability and robustness issues. Obviously, an impact on the organization has to be expected when making changes to once defined and committed plans. Therefore, quantitative approaches of measuring the difference of plans, and thus the robustness of schedules, are needed for such practical applications.

Rescheduling factors
Changes to the machine environment are the most commonly discussed factors of dynamic scheduling in the literature. Machine failures are reported to play a role in dynamic situations (Abumaizar and Svestka, 1990).
Besides the machines failures due to technical problems, Church and Uzsoy (1992) report the possibility of operator absenteeism which may lead to an unavailability of the machine. Job related rescheduling factors comprise the arrival of new jobs, and especially rush jobs (Jain and Elmaraghy, 1997). Besides, the cancelation of jobs is another source of dynamics in scheduling. In the case of defined due date, changes may also address the due dates (Fang and Xi, 1997).

In addition to the basic data of the problem, preferences of the decision maker and priorities of jobs may change over time, an example being unexpectedly arriving rush orders which implicitly reduce the relative importance of known jobs from the previous planning period (Jain and Elmaraghy, 1997).

Classification of solution approaches

Common to all dynamic scheduling concepts is the distinction between an initial scheduling and a recurring schedule adaptation phase. Two aspects of schedules play an important role when dynamically revising schedules, utility and stability. While the term utility refers to the quality of the schedule, expressed by the objective function(s), the stability measures the changes of an initial schedule compared to a revised schedule. The goal of any rescheduling concept is the maximization of both the achieved utility as well as the stability. However these two are often of conflicting nature, they lead to a situation where a tradeoff between the two aspects has to be made.

The general treatment of dynamic scheduling problems can be broadly classified into three categories: on-line scheduling, predictive-reactive and robust scheduling (Aytug et al., 2005). On-line approaches make assignment and sequencing decision on the job floor in real time (Mehta and Uzsoy, 1999), e.g. by the means of dispatching rules (Haupt, 1989). While this concept is highly flexible, a drawback lies in the missing predictability of the schedule's performance as the precise production schedule is only available after the operations have been assigned to the resources.

Predictive-reactive approaches introduce a two-step-process by first constructing an initial schedule which is then to be modified given dynamic changes of the problem. Here, an important question is the appropriate schedule adaptation to the changed circumstances, taking into consideration both utility and stability. Potential changes in the manufacturing environment are however not anticipated or integrated into the scheduling procedure. Concepts of robust scheduling try to anticipate possible changes and sources of disturbances when proposing a schedule for the problem at hand. While at a first glimpse this concept seems overall promising, knowledge about possible changes has to be available or at least sensible assumptions have to be made. One possibility is the consideration of a range of scenarios (Kouvelis et al., 2000).

2 Stability of schedules

Stability measures

Besides the evaluation of schedules with respect to known and initially defined optimality criteria (T’kindt and Billaut, 2002), the stability of a schedule plays a role in the context of dynamic scheduling. Various stability measures have been proposed in order to express transition between two schedules in a quantitative way. Common to all is the proposition of a cost function based on the operations and/or jobs of the problem. Associated with changing starting times is a negative impact which is penalized in a quantitative way. The idea behind this can be seen in resulting adaptations that have to be made in the manufacturing environment, resulting in costs.

Most approaches go back to the early proposal of Wu et al. (1993). Here, the difference between two schedules is computed as the sum of the absolute differences of starting times of all operations, given in Expression 1. Stability is therefore independent from whether operations start earlier or later than initially planned. The stability stab is expressed depending on the initial schedule x with starting
times \(s_{jk}\) and the revised schedule \(x'\) with starting times \(s'_{jk}\) of operations \(O_{jk}\).

\[
\text{stab}(x, x') = \sum |s_{jk} - s'_{jk}|
\]

(1)

The stability measure of Wu et al. (1993) comes with the implicit assumption, that a discrimination between operations being shifted forward or backward in time has an equal impact on the stability. It therefore is only applicable in situations in which this assumption holds. In many production situations however, a difference between delayed operations and earlier executed operations can be observed. Delaying operations does not necessarily result in a higher organizational overhead, but it has an obvious effect on the completion of the jobs, and is in this sense already captured by other optimality criteria. An earlier execution of operations however often results in a considerable effort. For example, the required material and tools have to be made available, computerized numerical control (CNC) programs need to be completed upon start of production, etc.

In the work of Lin et al. (1994), only operations with earlier starting times are considered in the proposed stability measure. This implies that delays do not affect the schedules stability while earlier starting times do, see Expression (2).

\[
\text{stab}(x, x') = \sum \max(0, s_{jk} - s'_{jk})
\]

(2)

To some extent, the approach of Lin et al. (1994) can be seen as an answer to the criticism mentioned above with regard to the work of Wu et al. (1993). The stability measure is therefore suitable for manufacturing environments where only earlier executions of operations present a relevant change to the production plan.

The approaches of Wu et al. (1993) and Lin et al. (1994) are combined in the work presented by Rangsaritratsanee et al. (2004) into the definition of an overall cost function. As a result, a more general way of measuring the stability of schedules is derived, making it possible to individually discriminate between the effect of earlier and later scheduled operations.

As opposed to starting times of operations, Cowling and Johansson (2002) propose a combination of start and completion times of the entire job, leading to an overall cost function. In a later work of Cowling et al. (2004), only the completion times \(C_j\) of jobs are relevant for the proposed stability measure. As in this stability measure it is not the operations that are considered but the completion of the job as a whole, this concept differs significantly from other approaches of measuring the stability of schedules. Its use can be seen in manufacturing environments where changes of the completion of the jobs are more important than the operations themselves.

The approach of Watatani and Fujii (1992), later also used by Iima (2005), is based on the sequences of the operations. Here, schedules \(x\) and \(x'\) are considered to be different if the sequences of the operations differ. This is the case for operations \(O_{jk}, O_{lo}\) for which \(s_{jk} < s_{lo} \land s'_{jk} > s'_{lo}\). This measure is useful when changing the sequence of operations is difficult with respect to the organization of the manufacturing environment and therefore results in a necessary organizational effort. An example of such a production situation would be a flow-oriented manufacturing environment where jobs are transported in a fixed sequence, e.g. by means of a belt. Changes to the sequence through jobs overtaking others impose problems here.

Stability measures are implemented in rescheduling strategies either in a predictive-reactive way, where the problem is treated as a multi-objective optimization problem maximizing both utility and stability, or in concepts of robust scheduling, where they are used to compute a quantitative measure for a schedule robustness with respect to changes of the manufacturing situation.
Critical analysis

Stability of schedules is always measured by comparison of an initial production schedule \( x \) with a revised schedule \( x' \). Various characteristics of the schedules \( x, x' \) may differ from each other. In particular, the following aspects have to be considered to be the basis for a further analysis.

1. Different starting times of operations, along with different completion times of the jobs.
2. Different operation sequences.
3. Different machine assignments of operations.

Consequently, approaches measuring the stability of schedules are based on either one characteristic or combine several characteristics to an overall stability measure.

Stability measures express the difference in a quantitative way, given an overall idea of how ‘different’ schedules are. Existing approaches define stability with respect to starting time changes of operations (Cowling et al., 2004; Cowling and Johansson, 2002; Lin et al., 1994; Rangsaritratsanee et al., 2004; Wu et al., 1993) or changes of operation sequences (Iima, 2005). A distinction between earlier and later starting operations is sometimes made (Lin et al., 1994). This expressed the fact that on one hand an earlier execution is critical in the sense of schedule stability, while on the other hand a later execution does not lead to problems for the organization.

The difference between schedules is of particular importance as changes of the production planning have an organizational impact, e.g. on required material, suppliers, etc. A stability measure of schedules therefore should express the impact of the changes on the organization.

Reviewing existing approaches of schedule stability, one can observe that they do not take into consideration the influence of the point in time when the changes occur. While the absolute deviation of the starting times from the initially scheduled \( s_{jk} \) plays a role, it is not further analyzed whether the changes appear close to the actual rescheduling moment or at the end of the planning horizon. In many real world applications of scheduling, immediate changes will however have a different impact compared to changes happening in e.g. several weeks. The reason behind this can be seen in the time needed to implement the changes made to the production schedule. Communication with suppliers and customers requires time.

3 Proposition of a novel approach

A generalized instability measure

To overcome the limitations described in the previous section, we propose a novel approach measuring the instability of schedules. A measure for instability is defined in such a way that larger values refer to a larger instability. This is in contrast to approaches found in the literature, which wrongly denote the measures as measures of stability, although the computed values increase with increasing instability.

The concept is based on starting times of operations, and measures instability of schedules analyzing two aspects:
1. The size of the deviation of the starting times.
2. The point of time, when the deviation occurs.

The first aspect, the size of the deviation of the starting times, can easily be measured by comparing the starting times \( s_{jk} \) and \( s'_{jk} \) of the operations. Expression (3) computes their absolute difference, comparing the initial schedule \( x \) and the revised one \( x' \).

\[ \Delta_{jk} = |s'_{jk} - s_{jk}| \] (3)
The second aspect, the point of time when the deviation occurs, can be determined as follows. The exact time when a schedule has to be modified, is denoted with $t_0$. The relative impact of starting time changes may be expressed depending on how close either the initially planned start $s_{jk}$ or the revised start $s'_{jk}$ of the operation $O_{jk}$ is scheduled. Expression (4) measures the closeness of the operation $O_{jk}$ to the start of the planning horizon.

$$dist_{jk} = \min(s_{jk} - t_0, s'_{jk} - t_0) = \min(s_{jk}, s'_{jk}) - t_0$$ (4)

The impact of the closeness $dist_{jk}$ may then be expressed as a (monotonic) decreasing function $imp(dist_{jk})$. Operations $O_{jk}$ being close to the start of the current planning horizon, and thus having a small $dist_{jk}$, receive a high impact value $imp(dist_{jk})$, while the impact decreases with increasing distance to the moment of rescheduling.

A possible way of computing the impact $imp(dist_{jk})$ is given in Expression (5).

$$imp(dist_{jk}) = I^{dist_{jk}}$$ (5)

**On obtaining statements from the decision maker**

The parameter $I$ has to be chosen such that it reflects the length of the planning horizon. The actual value of the parameter $I$ could be obtained from the decision maker, by considering how the relative impact of an operation at the end of the scheduling horizon relates to the impact at the very beginning, expressed by parameter $pc$. For example, the decision maker is enabled to express that changes at the end of the planning horizon have an impact of 30% of changes at the beginning of the horizon, therefore $pc = 0.3$.

With the percentage at the end of the planning horizon $pc$ and the length of the planning horizon $T$, $I$ may be computed as given in Expression (6). A value of $I < 1$ has been chosen to show a decreasing impact over time.

$$I = \sqrt[5]{pc}$$ (6)

In practice, it might be difficult for the decision maker to state the exact percentage of the impact at the end of the planning horizon. An alternative way for obtaining $I$ could be to refer to another period with which the planner is more familiar. From a practical planner’s perspective, such a period could be a (working) week, or a similar time span being used as a time pattern in planning. The decision maker is then asked about the decrease of the impact within this period, denoted by $dec$. With a decrease of e.g. 20%, $dec = 0.2$ and the length of a week of five working days, $I$ is computed as given in Expression (7).

$$I = \sqrt[5]{1 - dec}$$ (7)

In comparison to Expression (5), the alternative way of computing $I$ in Expression (7) only differs with respect to the statement of the decision maker. While in (6) the decision maker has to state the impact at the end of the planning horizon directly, it is indirectly computed in (7) using the decrease $dec$ over an arbitrary period.

It should be noticed that the impact asymptotically approaches zero without ever reaching it. Therefore, the impact of starting time changes decreases over time but always stays positive.
Overall formula
A computation of the total impact of the starting time changes and therefore the instability $\text{instab}(x, x')$ combines the relative importance of the impact with the size of the change. Expression (8) gives the precise formula.

$$\text{instab}(x, x') = \sum_j \sum_k \text{imp}(\text{dist}_{jk}) \Delta_{jk} = \sum_j \sum_k \left( I_{\min(s_{jk}, s'_{jk})} - t_0 \right) |s'_{jk} - s_{jk}|$$

In brief this means that combinations of values $s_{jk}, s'_{jk}$ close to $t_0$ and big changes of the starting times $|s_{jk} - s'_{jk}|$ lead to a high impact. On the other hand, small changes of $s_{jk}$ to $s'_{jk}$ and changes occurring towards the end of the planning horizon do not contribute as much to the impact measure. It becomes clear that the required effort for computing Expression (8) increases with the number of operations. Also, the exponential component adds to the running time of of the formula. For a practical application, e.g. in a metaheuristic approach employing local search, where numerous evaluations of Expression (8) are required, a closer look at the running time behavior of our approach will become necessary.

4 Discussion and conclusions

An approach to measure the instability of schedules in a dynamic manufacturing environment has been presented. The concept integrates two aspects into a single measure, namely the difference of starting times and the significance of the changes depending on when they occur. As a result, the concept generalizes approaches known from literature by providing the possibility of assigning a relative importance of the changes.

The proposed instability measure is more general, yet it includes the special case of Wu et al. [1993] for a chosen value of $I = 1$. In any other case however, the determination of an appropriate $I$ requires considerable more information compared to existing approaches. Small values of $I$ lead to a fast decrease of the impact, while large values tend to discriminate less between the changes.

Given the possibility to collect information about the relative importance of the changes as described, the approach may reflect the actual practical situation of dynamic scheduling in manufacturing environments more closely. Future research will therefore focus on the evaluation of the applicability of the approach in dynamic machine scheduling situations. Such an attempt could be experimentally-driven, investigating the obtained results when making use of the proposed measure, e.g. for an exemplary dynamic job-shop- or flow-shop scheduling environment. Different rescheduling factors should be examined, and the impact of well-chosen rescheduling reasons (disturbances) on the obtained schedule should be studied. This also implies an experimental comparison of the proposed measure to approaches known from the literature.

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