Possible complex annihilation and $B \to K\pi$ direct CP asymmetry

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We point out that a sizable strong phase could be generated from the penguin annihilation in the soft-collinear effective theory for $B$ meson decays. Keeping a small scale suppressed by $O(\Lambda/m_b)$, $\Lambda$ being a hadronic scale and $m_b$ the $b$ quark mass, the resultant strong phase can accommodate the data of the $B^0 \to K^\pm \pi^\mp$ direct CP asymmetry. Our study reconciles the opposite conclusions on the real or complex penguin annihilation amplitude drawn in the soft-collinear effective theory and in the perturbative QCD approach based on $k_T$ factorization theorem.

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The effect of scalar penguin annihilation on charmless nonleptonic $B$ meson decays has attracted intensive attention. This power-suppressed contribution is chirally enhanced, i.e., proportional to $\mu_p/m_b$ in $B \to PP$ decays, where $\mu_p$ is the chiral scale associated with the pseudoscalar meson $P$ and $m_b$ the $b$ quark mass. Since it involves endpoint singularities, it was parameterized as a free parameter $X_A = \ln(m_b/\Lambda)[1 + \rho_A \exp(i\phi_A)]$ in QCD-improved factorization (QCDF) [1], with $\Lambda$ being a hadronic scale, and $\rho_A$ and $\phi_A$ varied arbitrarily within some artificially specified ranges. In order to fit data such as the $B^0 \to K^\pm \pi^\mp$ direct CP asymmetry $A_{CP}(B^0 \to K^\pm \pi^\mp)$, $\phi_A$ must take a sizable value. On the other hand, the contribution from scalar penguin annihilation has been found to be almost imaginary in the perturbative QCD (PQCD) approach based on $k_T$ factorization theorem [2,3], and the resultant strong phase leads to a prediction consistent with the measured $A_{CP}(B^0 \to K^\pm \pi^\mp)$. The annihilation amplitude was not considered in the leading-power formalism of soft-collinear effective theory (SCET) [1,5,6]. Instead, a nonperturbative complex charming penguin was introduced to accommodate the data of $A_{CP}(B^0 \to K^\pm \pi^\mp)$. In the recent SCET formalism with the zero-bin subtraction [7], the annihilation contribution becomes factorizable, and has been concluded to be almost real [8].

The motivation of this paper is to reconcile the opposite theoretical observations on the almost imaginary or real $penguin$ annihilation derived in PQCD and in SCET. We shall first point out that the comparison of the measured $A_{CP}(B^\pm \to K^{\mp}\pi^0)$ and $A_{CP}(B^\pm \to K^{\mp}\rho^0)$ indicates an imaginary penguin annihilation amplitude [9,10]: The $B^\pm \to K^{\mp}\pi^0$ ($B^\pm \to K^{\mp}\rho^0$) decays involve a $B \to P$ ($B \to V$) transition, so the penguin emission amplitude is proportional to the constructive (destructive) combination of the Wilson coefficients $a_4 \pm (-)2(\mu_K/m_b)a_6$, $\mu_K$ being the chiral scale associated with the kaon. The annihilation effect is then less influential in the former than in the latter. If the penguin annihilation is real, both decays will exhibit small direct CP asymmetries, i.e., $A_{CP}(B^\pm \to K^{\mp}\pi^0) \approx A_{CP}(B^\pm \to K^{\mp}\rho^0) \approx 0$. If imaginary, it will cause a larger $A_{CP}(B^\pm \to K^{\mp}\rho^0)$. The current data $A_{CP}(B^\pm \to K^{\mp}\pi^0) = 0.050 \pm 0.025$ and $A_{CP}(B^\pm \to K^{\mp}\rho^0) = 0.31^{+0.11}_{-0.10}$ favor an imaginary penguin annihilation.

We emphasize that strong phases, generated by subleading corrections, are the leading effect for direct CP asymmetries of $B$ meson decays. For example, the prediction for the direct CP asymmetry $A_{CP}(B^\pm \to K^{\mp}\pi^0)$ is sensitive to the strong phase of the ratio $C/T$ [12,13], where $C$ ($T$) is the color-suppressed (color-allowed) tree amplitude, though the branching ratio $B(B^\pm \to K^{\mp}\pi^0)$ is not. Assuming this ratio to be real as in the leading-power SCET [3], it is difficult to explain the data. Therefore, the study of strong phases requires a

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careful treatment of subleading corrections. It will be explained that the different penguin annihilation effects observed in PQCD and SCET arise from whether parton transverse momenta $k_T$ and other intrinsic mass scales in particle propagators are expanded or not. If these small scales are neglected or expanded, the internal particles in an annihilation amplitude are on their mass shell only at the endpoints of parton momentum fractions, where hadron distribution amplitudes usually vanish, or the zero-bin subtraction applies. An annihilation amplitude is then real. Including $k_T$, the on-shell condition of internal particles does not occur at the endpoints, so that there is a potential to generate a sizable strong phase. We claim that when $m_b$ approaches infinity, the on-shell region coincides with the endpoints, and the same vanishing results for strong phases will be derived, irrespective of whether the small scales are expanded into a power series. For the physical value of $m_b$, however, a formally power-suppressed correction may have a significant numerical effect on strong phases, and lead to large direct CP asymmetries in $B$ meson decays.

As argued in Ref. [14], a parton, carrying a transverse momentum $k_T$ as small as a hadronic scale $\Lambda$ initially, accumulates its $k_T$ after emitting infinitely many collinear gluons. When the parton participates in a hard scattering eventually, $k_T$ can become as large as the hard scale. Such an accumulation is described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution [15] for a parton distribution function in inclusive processes and by the Sudakov evolution [16] for a hadron wave function in exclusive processes. For two-body nonleptonic $B$ meson decays, $k_T^2$ of internal particles in a hard kernel reaches the hard scale of $O(m_b \Lambda)$. That is, the effect resulting from $k_T^2$ is suppressed by a power of $r = k_T^2/m_b^2 \sim O(\Lambda/m_b)$. In SCET, the power counting rule for $k_T$ is different, which is always treated as being $O(\Lambda)$, and expanded. However, there exists a scale of $O(m_b \Lambda)$ from the hard-collinear modes, which is also suppressed by $\Lambda/m_b$ compared to $m_b^2$. To verify the above claim, we shall keep a small scale in particle propagators, which can be regarded as an averaged parton transverse momentum in PQCD or the hard-collinear scale in SCET, and examine its effect on the penguin annihilation in the SCET formalism with the zero-bin subtraction [7].

Before computing the direct CP asymmetry of the $B^0 \to K^\mp \pi^\pm$ decays, we illustrate why a formally power-suppressed correction of $O(r)$ could produce a sizable strong phase in an annihilation amplitude. Expand a kernel of the form

$$\frac{1}{x - r + i\epsilon} = \frac{1}{x + i\epsilon} + O\left(\frac{r}{x}\right),$$

which appears in a convolution with a meson distribution amplitude. Eq. (1) holds in principle as long as the contribution from the small $x$ region is suppressed by the meson distribution amplitude, namely, as the main contribution comes from the region with $r/x \ll 1$. On the other hand, we have the principle-value prescription without expansion,

$$\frac{1}{x - r + i\epsilon} = P \frac{1}{x - r} - i\pi\delta(x - r).$$

Convolving the kernel with the distribution amplitude $\phi(x) = 6x(1 - x)$, the real parts from Eqs. (1) and (2) differ by only 15%. The imaginary part from Eq. (1) vanishes, but that from Eq. (2) reaches half of the real part for a typical value of $r \sim \Lambda/m_b \sim 0.1$. Obviously, in order that the imaginary part becomes negligible, i.e., about 5% of the real part, $r$ must decrease to 0.01 (or $m_b$ increases up to 50 GeV). The lessons we learn from this simple example are 1) as $x$ has the substantial probability to be close to $r$, which is small but away from the endpoint, the expansion in a power series of $r$ breaks down, and an imaginary piece could develop; 2) the expansion is reliable only for sufficiently small $r$ such that the contribution from $x \sim r$ is highly-suppressed like the endpoint one; 3) $r$ is expected to give a minor (larger) effect on branching ratios (direct CP asymmetries) of $B$ meson decays.

Let the momenta of the outgoing quark $u$ and antiquark $\bar{u}$ in opposite directions be $k_2 = (0, yP_2^- , 0_T)$ and $k_3 = (\bar{x}P_3^- , 0, 0_T)$, respectively, for the decay $B^0 \to K^- \pi^+$, where $P_2$ ($P_3$) is the pion (kaon) momentum and $\bar{x} = 1 - x$. We quote the expression for the penguin annihilation amplitude in the SCET formalism with the
where $G_F$ is the Fermi constant, $f_{B,K,\pi}$ the meson decay constants, $\lambda_{c,s}^{(s)}$ the products of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\mu_h \sim m_b$ the hard scale, $C_i$ the Wilson coefficients, and $\mu_\pi$ the chiral scale associated with the pion. The logarithmic terms $\ln \mu_\pm$ in Ref. [8] have been dropped since they are cancelled by the corresponding logarithms in the convolutions. Because of the large theoretical uncertainty shown below, the constant $\kappa$ resulting from the above logarithmic cancellation will be neglected [8]. The three-parton twist-3 contribution to the penguin annihilation, being numerically smaller by one order of magnitude than Eq. (3)[17], is not included.

Motivated by the illustration based on Eqs. (1) and (2), we introduce a small constant $r$ into internal quark propagators involved in the factorizable piece of Eq. (3), corresponding to Fig. 1. Inserting $r$ into internal gluon propagators generates a strong phase down by a factor three. The strong phase from the nonfactorizable annihilation amplitude is smaller by two orders of magnitude. We stress that adding $r$ in the aforementioned way causes a double counting of the contributions from higher-order operators in SCET, and should be regarded as only a test of our claim. Applying the principle-value prescription, we obtain the extra imaginary pieces via the following substitutions,

\[
\langle \bar{x}^{-2} \rangle^M \to \langle \bar{x}^{-2} \rangle^M + i \text{Im} \langle \bar{x}^{-2} \rangle^M,
\]

\[
\text{Im} \langle \bar{x}^{-2} \rangle^M = -\pi \int_0^1 dx \frac{\phi_M(x) + \bar{x} \phi_M'(1)}{\bar{x}} \delta (\bar{x} - r),
\]  

\[
\langle y^{-2} \bar{y}^{-1} \rangle_{pp}^M \to \langle y^{-2} \bar{y}^{-1} \rangle_{pp}^M + i \text{Im} \langle y^{-2} \bar{y}^{-1} \rangle_{pp}^M,
\]

\[
\text{Im} \langle y^{-2} \bar{y}^{-1} \rangle_{pp}^M = -\pi \int_0^1 dy \left[ \frac{\phi_{pp}^M(y)}{y(1-y)} - \frac{y \phi_{pp}^M(0)}{y} \right] \delta (y - r).
\]  

Employing the parameterizations for the leading-twist distribution amplitude $\phi_M(x)$ and for the two-parton
twist-3 distribution amplitudes $\phi_{pp}^M(x)$ \cite{8} \cite{17}

$$
\phi_M(x) = 6x(1-x) \left[ 1 + a_1^M(6x - 3) + 6a_2^M(1 - 5x + 5x^2) - 10a_3^M(1 - 9x + 21x^2 - 14x^3) + 15a_4^M(1 - 14x + 56x^2 - 84x^3 + 42x^4) + \cdots \right],
$$

$$
\phi_{pp}^M(x) = 6x(1-x) \left[ 1 + a_1^{M,pp}(6x - 3) + 6a_2^{M,pp}(1 - 5x + 5x^2) + \cdots \right],
$$

with $M = \pi, K$, it is easy to find that both Eqs. (4) and (5) are proportional to $r$ as expected.

The importance of the penguin annihilation contribution relative to the full penguin one has been estimated in SCET \cite{8}, and found to be about 10% with large uncertainty in the $B^0 \rightarrow K^\pm \pi^\mp$ decays. The full penguin contribution does not come from an explicit evaluation in the same SCET framework, but from a fitting to the $B \rightarrow K\pi$ data. We can certainly follow this approach. However, the factorization formulas for the emission amplitudes have been available in Ref. \cite{7}, so they will be adopted in the numerical analysis below. The feature of generating strong phases does not depend on how we estimate the emission amplitudes. Besides, we shall not include the free parameters associated with the long-distance charming penguin, which is not factorizable in SCET. As demonstrated later, a decay amplitude under the zero-bin subtraction is very sensitive to higher Gegenbauer moments $a_n^M$ and $a_{n,pp}^M$ in Eq. (4,18), which are mostly unknown. Hence, we shall determine these moments by fitting the SCET formulas to data of branching ratios, which are then used to predict direct CP asymmetries. If a strong phase from the source considered here is sizable, the whole CP asymmetry cannot be attributed to the nonperturbative charming penguin alone.

At lowest order in $\alpha_s(m_b)$ with the Wilson coefficients $T^{(+)} = 1$ and $C_{ij}^{(+)} = 1$ in SCET$_1$ \cite{4}, the $B \rightarrow \pi$ transition form factor is decomposed into

$$
f_+(E) = \zeta^{B\pi}(E) + \zeta^{B\pi}_1(E).
$$

(7)

The second term is factorizable, written as

$$
\zeta^{B\pi}_1(E) = \frac{f_B f_\pi m_B}{4E^2} \frac{4\pi \alpha_s(m_\mu)}{9} \left( \frac{2E}{m_B} + \frac{2E}{m_B} - 1 \right) \int_0^1 dy \frac{\phi_\pi(y)}{y} \int_0^\infty dk^+ \frac{\phi_p^+(k^+)}{k^+},
$$

(8)

where $\mu_\mu \sim \sqrt{m_b \Lambda}$ is the intermediate scale, and $k^+$ the momentum of the spectator quark in the $B$ meson. For charmless two-body nonleptonic $B$ meson decays, we take the pion energy $E = m_B/2$, $m_B$ being the $B$ meson mass. The first term also becomes factorizable after implementing the zero-bin subtraction for the endpoint singularity \cite{7},

$$
\zeta^{B\pi}(E) = \frac{f_B f_\pi m_B}{4E^2} \frac{4\pi \alpha_s(m_\mu)}{9} \int_0^1 dy \int_0^\infty dk^+ \left\{ \frac{(1+y)\phi_\pi(y)}{(y^2)_\phi} \frac{\phi_p^+(k^+)}{(k^+)_\phi} + \mu_\pi \frac{\phi_p^+(y)}{(y^2)_\phi} \frac{\phi_p^+(y)}{(k^+^2)_\phi} \right\},
$$

(9)

where only the terms from the two-parton pion distribution amplitudes are kept. The relation among $\phi_p^x$, $\phi_p^y$ and $\phi_p^{pp}$ can be found in Ref. \cite{8}. The formulas for the $B \rightarrow K$ form factor in SCET are similar. We multiply Eq. (4) by the appropriate CKM matrix elements and Wilson coefficients, including a part of next-to-leading-order corrections \cite{19}, to obtain the emission contributions from both the tree and penguin operators. The Wilson coefficient $a_6$ was neglected in the previous SCET analysis, since the associated penguin contribution is power-suppressed. However, it is enhanced by the chiral scale, and numerically crucial. Furthermore, the power-suppressed annihilation has been formulated into SCET, so there is no reason for ignoring $a_6$ \cite{19}.

The zero-bin subtraction for the logarithmic endpoint singularity associated with the pion distribution amplitude $\phi_\pi$ in the first term of Eq. (9) is referred to Ref. \cite{7}, where the term proportional to $y$ in $(1+y)$ does not require subtraction. We also need the zero-bin subtraction for the linear endpoint singularity present in the second term of Eq. (9) \cite{20}:

$$
\int_0^1 dy \frac{\phi_p^x(y)}{(y^2)_\phi} = \int_0^1 dy \frac{\phi_p^x(y) - \phi_p^x(0) - y \phi_p^x(0)}{y^2} = \int_1^\infty dy (y - 1)(y^2)(y^2) \phi_p^x(0) + \phi_p^x(0) + \ln \left( \frac{n \cdot P}{\mu} \right) \phi_p^y(0),
$$

(10)
where $\vec{p} \cdot P_2 = 2E$. The subtraction associated with the derivative of the two-parton twist-3 pion distribution amplitude, $\phi_\sigma^\prime$, is similar.

We consider the models for the $B$ meson distribution amplitudes $\phi_B^\pm$ proposed by Kodaira et al. (KKQT) [21] and by Grozin and Neubert (GN) [22]. The associated zero-bin subtraction is defined by

$$\int_0^\infty dk^+ \frac{\phi_B^+(k^+)}{k^+} = \int_0^\infty dk^+ \frac{\phi_B^+(k^+)}{k^+} - \int_0^\Lambda dk^+ \frac{\phi_B^+(0)}{k^+} + \ln \left( \frac{n \cdot v A}{\mu_+} \right) \phi_B^+(0),$$

(11)

$$\int_0^\infty dk^+ \frac{\phi_B^-(k^+)}{k^+} = \int_0^\infty dk^+ \frac{\phi_B^-(k^+)}{k^+} - \int_0^\Lambda dk^+ \frac{\phi_B^-(0)}{k^+} + \ln \left( \frac{n \cdot v A}{\mu_+} \right) \phi_B^-(0),$$

(12)

with the parameter relation $\omega_0 = 2\Lambda/3$, $\Lambda$ being the $B$ meson and $b$ quark mass difference. In the above expressions $n$ is a light-like vector along the Wilson line in the definition for the $B$ meson distribution amplitudes, and $v$ is the $B$ meson velocity. The terms containing $\ln \mu_\pm$ in Eqs. (10)-(12) are also dropped.

For the numerical analysis, we assume the Gegenbauer moments of the pion and kaon distribution amplitudes, $a_i^\pi = 0.0, a_i^K = -0.05$ consistent with the results in Ref. [23, 24], $a_2^K = a_2^\pi = 0.2$ [24, 25, 26], $a_3^K = 0, a_4^K = a_4^\pi$, $a_7^{\pi pp} = a_7^{K pp} = 0.0$, and $a_2^{K pp} = a_2^{pp}$, among which $a_4^M$ and $a_2^{pp}$ are most uncertain. To simplify the formulas, we do not consider the Gegenbauer moment $a_5^K$ for the two-loop kaon distribution amplitude. That is, we keep one most uncertain parameter from each of $\phi_B^M$ and $\phi_B^{pp}$, whose variation is sufficient for our purpose. The hard and intermediate scales are fixed at $\mu_h = m_b$ and $\mu_i = \sqrt{\Lambda m_b}$, respectively, with $\Lambda = 0.55$ GeV and $m_b = m_b^{1S} = 4.7$ GeV. Other relevant heavy-quark masses are taken to be $m_c = m_c^{1S} = 1.4$ GeV and $m_c = m_c^{1S}(m_b) = 4.2$ GeV. We obtain the chiral scales $\mu_+(\mu_h) = 2.4$ GeV, $\mu_K(\mu_h) = 3.0$ GeV, $\mu_+(\mu_i) = 1.8$ GeV, and $\mu_K(\mu_i) = 2.3$ GeV from the two-loop running for the strong coupling constant with $\alpha_s(M_Z) = 0.11876$ GeV $= 0.118$ and for the light-quark masses with $m_{u,d}(2\text{GeV}) = 5$ MeV and $m_s(2\text{GeV}) = 95$ MeV. We take the Wilson coefficients for four-fermion operators evaluated at $\mu_h = m_b$ and at next-to-leading-logarithmic level: $C_1 = 1.078, C_2 = -0.177, C_3 = 0.014, C_4 = -0.034, C_5 = 0.009, C_6 = -0.040, C_7 = 0.7 \times 10^{-4}, C_8 = 4.5 \times 10^{-4}, C_9 = -9.9 \times 10^{-3};$ $C_{10} = 1.8 \times 10^{-3}$. Those for dipole operators at leading-logarithmic level are $C_{7\gamma} = -0.314$ and $C_{8\gamma} = -0.149$ [27]. We also take the Fermi constant $F_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$, the decay constants $f_B = 0.02$ GeV, $f_K = 0.16$ GeV, and $f_\pi = 0.131$ GeV, the meson masses $m_B = 5.28$ GeV, $m_K = 0.497$ GeV, and $m_\pi = 0.14$ GeV, the $B$ meson lifetime $\tau_B = 1.530 \times 10^{-12}$ sec, and the CKM matrix elements $V_{us} = 0.2257, V_{ub} = (4.2 \times 10^{-3}) \exp(-i\phi_3), V_{cs} = 0.957$, and $V_{cb} = 0.0416$ with the weak phase $\phi_3 = 74^\circ$ [28].

Adopting the above parameters, the two pieces $\zeta^{B\pi}_F$ and $\zeta^{B\pi}_J$ of the $B \rightarrow \pi$ form factor are written as

$$\zeta^{B\pi}_F = \begin{cases} 0.01 + 0.75 a_2^\pi + 2.57 a_4^\pi + 0.43 a_{2pp}^\pi, & \text{for KKQT} \\ 0.09 + 0.65 a_2^\pi + 2.23 a_4^\pi - 2.73 a_{2pp}^\pi, & \text{for GN} \end{cases}$$

(13)

$$\zeta^{B\pi}_J = \begin{cases} 0.016(1.0 + a_2^\pi + a_4^\pi), & \text{for KKQT} \\ 0.024(1.0 + a_2^\pi + a_4^\pi), & \text{for GN.} \end{cases}$$

(14)

Note that the coefficients in Eq. (13) grow quadratically with the order $n$ of the Gegenbauer moments $a_n$. This sensitivity is attributed to the increasing slope of the higher Gegenbauer polynomials at the endpoints of the momentum fraction $x$. The sign flip of the $a_{2pp}^\pi$ terms indicates that $\zeta^{B\pi}$ also depends strongly on the models of the $B$ meson distribution amplitudes in SCET. We mention that the PQCD approach does not suffer such sensitivity, because the endpoint singularity is smeared by including parton transverse momenta $k_T$, whose order of magnitude is governed by the Sudakov factor.
The strong dependence on the higher Gegenbauer moments also appears in the penguin annihilation amplitude,

\[
10^4 \tilde F_{K^\pi} \equiv -10^4 \frac{\sqrt{2}}{G_F m_B^2} \frac{A_{Lann}(K^-\pi^+)}{(1\text{GeV})},
\]

\[
= 2.76(0.07 + a_4^\pi)/(1.20 + a_4^\pi) + a_{2pp}^\pi(27.0 + 413.1 a_4^\pi) - i\pi r a_{2pp}^\pi(53.2 + 1747 a_4^\pi),
\]

with a significant growth of the coefficients of \(a_4^\pi\). The imaginary contribution is proportional to the second moment \(a_{2pp}^\pi\). In fact, it could depend on the zeroth moment, i.e., the normalization of \(\phi_{pp}\), if the denominator \(1 - y\) is not replaced by 1 in the subtraction term in Eq. (5). The denominators \(1 - y\) and 1 correspond to different zero-bin subtraction schemes.

Note that the size of the imaginary part depends on the amount of the subtracted contribution, i.e., on zero-bin subtraction schemes, since it is generated at \(x\) data of the pion transition form factor suggests a dependence on subtraction schemes also exists in all other definitions like Eqs. (10)-(12), which will not be adjusted undetermined. We shall regard these two parameters as being free, and fix them by the strategy stated before: Adjust \(a_4^\pi\) and \(a_{2pp}^\pi\), such that the \(B \to \pi\) form factor has the value around \(f_+ = 0.24 \pm 0.05\) \([13]\), and the \(B^0 \to K^\mp\pi^\pm\) decays have the branching ratio close to the data \(B(B^0 \to K^\mp\pi^\pm) = (19.4 \pm 0.6) \times 10^{-6}\) \([11]\).

Because the last two terms in \(\zeta B\pi\) for the KKQT model are of the same sign, and the coefficient of \(a_4^\pi\) is large, the constraint from the form factor value leads to a smaller \(a_4^\pi\) (considering \(a_4^\pi \approx 0.2\)). The range of \(a_{2pp}^\pi\) is basically undetermined. We shall regard these two parameters as being free, and fix them by the strategy stated before: Adjust \(a_4^\pi\) and \(a_{2pp}^\pi\), such that the \(B \to \pi\) form factor has the value around \(f_+ = 0.24 \pm 0.05\) \([13]\), and the \(B^0 \to K^\mp\pi^\pm\) decays have the branching ratio close to the data \(B(B^0 \to K^\mp\pi^\pm) = (19.4 \pm 0.6) \times 10^{-6}\) \([11]\).

The dependence on subtraction schemes also exists in all other definitions like Eqs. (10)-(12), which will not be discussed in this work.

For the range of \(a_4^\pi\), the crude bound \(a_4^\pi \geq -0.07\) has been determined in Ref. [24]. The analysis based on the data of the pion transition form factor suggests \(a_4^\pi \approx -0.05\) in Ref. [24] and the constraint \(a_4^\pi + a_4^\pi = -0.03 \pm 0.14\) in Ref. [30], both of which prefer a negative value of \(a_4^\pi\) (considering \(a_4^\pi \approx 0.2\)). The range of \(a_{2pp}^\pi\) is basically undetermined. We shall regard these two parameters as being free, and fix them by the strategy stated before: Adjust \(a_4^\pi\) and \(a_{2pp}^\pi\), such that the \(B \to \pi\) form factor has the value around \(f_+ = 0.24 \pm 0.05\) \([13]\), and the \(B^0 \to K^\mp\pi^\pm\) decays have the branching ratio close to the data \(B(B^0 \to K^\mp\pi^\pm) = (19.4 \pm 0.6) \times 10^{-6}\) \([11]\).

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Employing the KKQT model for the \(B\) meson distribution amplitudes, we obtain \(a_4^\pi \approx 0.01\) and \(a_{2pp}^\pi \approx 0.23\), corresponding to which the \(B \to \pi\) form factor, the \(B^0 \to K^\mp\pi^\pm\) branching ratio, and the predicted direct CP asymmetry are given by

\[
\zeta B\pi = 0.29,
\]

\[
\zeta B\pi = 0.02,
\]

\[
B(B^0 \to K^\mp\pi^\pm) = \begin{cases} 
20.5 \times 10^{-6} & \text{for } r = 0.0 \\
20.0 \times 10^{-6} & \text{for } r = 0.1 \\
19.8 \times 10^{-6} & \text{for } r = 0.2 ,
\end{cases}
\]

\[
A_{CP}(B^0 \to K^\mp\pi^\pm) = \begin{cases} 
0.08 & \text{for } r = 0.0 \\
0.05 & \text{for } r = 0.1 \\
0.02 & \text{for } r = 0.2 .
\end{cases}
\]

We do not attempt a fine tuning here, but accept the values of \(a_4^\pi\) and \(a_{2pp}^\pi\) as solutions, when they produce the \(B \to \pi\) form factor and the \(B^0 \to K^\mp\pi^\pm\) branching ratio close to the designated ranges. The results shift with the slight variation of \(a_4^\pi\) and \(a_{2pp}^\pi\), but the behavior for different \(r\) in Eq. (16) has the same pattern. In principle, \(\zeta B\pi\) and \(\zeta B\pi\) have the same scaling law in \(\alpha_s\) and in \(1/m_b\) \([31, 32]\). The numerical hierarchy \(\zeta B\pi \gg \zeta B\pi\) in Eq. (16), consistent with the PQCD results \([31]\), may be altered in different zero-bin subtraction schemes. It is obvious that the power correction associated with \(r\) has a negligible effect on the branching ratio. However, the power correction generates a strong phase: \(A_{CP}(B^0 \to K^\mp\pi^\pm)\) decreases by 40% from \(r = 0\) to \(r = 0.1\). Since the imaginary part is proportional to \(r\), it is difficult to accommodate the data \(A_{CP}(B^0 \to K^\mp\pi^\pm) = -0.097 \pm 0.012\) \([11]\) with a reasonable value of the power-suppressed \(r\) using the KKQT model.
For the GN model, we find two sets of solutions corresponding to $a_4^π ≈ 0.18$ and $a_{2pp}^π ≈ 0.15$,

\[
\begin{align*}
\zeta_B^π &= 0.21, \\
\zeta_J^π &= 0.03, \\
B(B^0 \to K^\mp π^\pm) &= \begin{cases} 
20.1 \times 10^{-6} & \text{for } r = 0.0 \\
20.4 \times 10^{-6} & \text{for } r = 0.1 \\
25.1 \times 10^{-6} & \text{for } r = 0.2, 
\end{cases} \\
A_{CP}(B^0 \to K^\mp π^\pm) &= \begin{cases} 
0.06 & \text{for } r = 0.0 \\
-0.06 & \text{for } r = 0.1 \\
-0.14 & \text{for } r = 0.2, 
\end{cases}
\end{align*}
\]

and to $a_4^π ≈ -0.22$ and $a_{2pp}^π ≈ -0.20$,

\[
\begin{align*}
\zeta_B^π &= 0.28, \\
\zeta_J^π &= 0.02, \\
B(B^0 \to K^\mp π^\pm) &= \begin{cases} 
18.6 \times 10^{-6} & \text{for } r = 0.0 \\
19.4 \times 10^{-6} & \text{for } r = 0.1 \\
26.5 \times 10^{-6} & \text{for } r = 0.2, 
\end{cases} \\
A_{CP}(B^0 \to K^\mp π^\pm) &= \begin{cases} 
0.08 & \text{for } r = 0.0 \\
-0.10 & \text{for } r = 0.1 \\
-0.20 & \text{for } r = 0.2. 
\end{cases}
\end{align*}
\]

The existence of the two sets of solutions with opposite signs is understandable. Because the term proportional to the variation of $r = 0$ of the formally power-suppressed terms at the physical former and the almost imaginary annihilation amplitude in the latter are attributed to the different treatments of the penguin annihilation drawn in SCET and in PQCD: The almost real annihilation amplitude in the former is close to that from QCDF in the default scenario [33] (PQCD [2, 13]). Therefore, the strong dependence of the penguin annihilation source in the penguin annihilation could be numerically crucial for the estimation of direct CP asymmetries. We then understand the opposite conclusions on the penguin annihilation) is close to that from QCDF in the default scenario 

As indicated by Eqs. (17) and (18), the branching ratio is stable, while the strong phase is very sensitive to the variation of $r$, so that we easily accommodate the data of $A_{CP}(B^0 \to K^\mp π^\pm)$ with a typical value of $r = 0.1 \sim 0.15$. The predicted $A_{CP}(B^0 \to K^\mp π^\pm)$ for $r = 0$, i.e., real penguin annihilation ($r = 0.1$, i.e., complex penguin annihilation) is close to that from QCDF in the default scenario [33] (PQCD [2, 13]). Therefore, the strong phases resulting from the power-suppressed source in the penguin annihilation could be numerically crucial for the estimation of direct CP asymmetries. We then understand the opposite conclusions on the effect of the penguin annihilation drawn in SCET and in PQCD: The almost real annihilation amplitude in the former and the almost imaginary annihilation amplitude in the latter are attributed to the different treatments of the formally power-suppressed terms at the physical $b$ quark mass. Note that the solutions of $a_4^π$ and $a_{2pp}^π$ in Eqs. (17), (18) will be changed, if higher Gegenbauer moments in Eq. (6) are taken into account, which cause even larger variation of the decay amplitudes. However, the strong dependence of $A_{CP}(B^0 \to K^\mp π^\pm)$ on $r$ will persist.

SCET provides a systematical expansion in powers of $\Lambda/m_b$, which is somewhat twisted here by keeping subleading terms in particle propagators in order to demonstrate a possible mechanism for generating strong phases. This twist of SCET actually violates its power counting rules and other aspects. Hence, our analysis does not imply the breakdown of SCET in its application to $B$ meson decays, but helps clarifying why there are discrepancies in the study of direct CP asymmetries from SCET and PQCD. It hints that more caution is necessary for fixed-power evaluations of direct CP asymmetries at the physical mass $m_b$. The expansion would be reliable for decay rates and direct CP asymmetries, if the $b$ quark mass was 10 times heavier. In that case, the contribution from the on-shell region of internal particles can be really suppressed by hadron distribution amplitudes, or excluded by the zero-bin subtraction. For $m_b ≈ 5$ GeV, a novel method might be demanded.

We have shown that introducing a small scale into denominators of internal quark propagators can accommodate both the measured branching ratio and direct CP asymmetry of the $B^0 \to K^\mp π^\pm$ decays. Keeping a small quantity in denominators without expansion is equivalent to resummation of the associated corrections to all powers. It is similar to resummation of part of higher-order corrections in $\alpha_s$ for many QCD processes. It has been explained that at least the parton transverse momenta can be maintained in denominators consistently in $k_T$ factorization theorem [34, 35]. This treatment is justified by different power counting rules, which hold
in the region of small parton momenta. This alternative power expansion, postulated in $k_T$ factorization theorem, has led to strong phases in more agreement with the indication of data in $B$ meson decays.

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