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On the analysis of Caputo fractional order dynamics of Middle East Lungs Coronavirus (MERS-CoV) model

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Abstract The current paper deals with the transmission of MERS-CoV model between the humans populace and the camels, which are suspected to be the primary source for the infection. The effect of time MERS-CoV disease transmission is explored using a non-linear fractional order in the sense of Caputo operator in this paper. The considered model is analyzed for the qualitative theory, uniqueness of the solution are discussed by using the Banach contraction principle. Stability analysis is investigated by the aid of Ulam-Hyres (UH) and its generalized version. Finally, we show the numerical results with the help of generalized Adams-Bashforth-Moulton Method (GABMM) are used for the proposed model, for supporting our analytical work.

1. Introduction

Epidemic like Coronaviruses are a huge community of viruses that are common in several different species of animals, including camels, cattle, cats, and bats. Coronaviruses have been tested by the medical laboratories in various countries. Among them is the Middle East respiration syndrome coronavirus (MERS-CoV) found in December 2012 in Saudi Arabia [1–3].
Mostly the said virus is tested in animals of middle Eastern countries. After that it was very threatening when the said virus also tested in human beings [4]. This viirus may also have caused infection and transmission through small drops of saliva or coughing. The MERS-CoV infection also spreading through close contact with already infected peoples [5]. For the last decades MERS-CoV causes 150 death cases and an about 550 infected cases. The death percentage is about 30% which is very high as compared to the new Covid-19 which to human. Nearly 25% from animals to humans and 75% from human to human. Scientists trying from the very beginning to collects its data and provide some predictions for present and future to reduce its infection. For this the simplest way is the conversion of the said problem to a mathematical formulation in the for differential or integral equation. Form the analysis of these equation we can easily gains a lot of information. The largest MERS-CoV transmission from human to human was first time formulated by Assire et al. [7]. The animal to human transmission is due to indirect exposition was described by Zumla et al. [8]. They also analyzed the said transmission caused by the taking non-pasteurized milk of the camels in the middle Eastern countries. Some scientists like Poletto et al. [9] claim the reason for the said infection, the overcrowding of pilgrims in the offering the Manasik Hajj and Umrah. He also reported the other reasons like camels contacts in the markets, Eid festivals, racing festivals, closing and opening ceremonies.

In this paper, we consider a mathematical model [10] for MERS-CoV transmission dynamics between human and camel as:

\[
\begin{align*}
\frac{dS(t)}{dt} &= \lambda - \xi_1 IS - \xi_2 p \phi AS - \xi_4 p \phi HS - \xi_4 CS - \nu_0 S, \\
\frac{dI(t)}{dt} &= \xi_1 IS + \xi_2 p \phi AS + \xi_4 p \phi HS + \xi_4 CS - (\delta + \nu_0) I, \\
\frac{dH(t)}{dt} &= \delta I - (\xi_3 + \nu_0) I, \\
\frac{dR(t)}{dt} &= \xi_3 I + \xi_4 A - (\xi_3 + \nu_0) H, \\
\frac{dC(t)}{dt} &= \nu_1 I + \nu_2 A - \nu_0 R, \\
\frac{dC(t)}{dt} &= \phi_1 I + \phi_2 A - \nu_0 C,
\end{align*}
\]

with initial condition

\[
S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, A(0) \geq 0, H(0) \geq 0, R(0) \geq 0, C(0) \geq 0.
\]

The above system that include parameters with whole descriptions are is, \(\lambda\) is the rate of new born. Where \(\xi_1, \xi_2, \xi_3, \xi_4\) are the disease transmission rate, \(\phi\) represent the asymptomatic patient rate, \(\delta\)represents the progression to infectious class, the average rate of symptomatic individuals hospitalize is represented by \(\xi_1\), infectious individual recovered without hospitalization at rate \(\xi_3\). \(p\) is the approximate rate of transmissibility of hospitalize patient, the recovery rate of hospitalize individual is presented by \(\xi_3\), the camel life time rate is \(\nu_1, \nu_2, \nu_3, \nu_4\) and \(\nu_5\) is the rate of symptomatic and asymptomatic infected individual spread virus from C.

From the past few decades, several researcher and scientists have given more importance to fractional calculus (FC) and shown that FC better explain the natural phenomena then the integer order. FC has taken advantages and popularity of modelling with memory effects [11–14]. Further, FC and mathematical modelling are applicable in many areas of science, biology and engineering for details see, [15–20]. The current work will investigate the system (3) in the form of system (3) for fractional order analysis under Caputo fractional operator. Arbitrary order calculus have provided the knowledge about the whole spectrum for any of the dynamical system, lying between any two integer values [21–28].

The application of various real globe problems have been formulated by arbitrary order differential or integral equation like, mathematical fractional model for small-organism populace, logistic non-linear model for human populace, TB, diny problem, hepatitis B, C and the basic Lotka-Volterra models being the fundamental of all the contagious problems [29–35,63–70]. The model (3) will be investigated for qualitative analysis with the aid of some known theorems of fixed point as already mentioned in several papers see, [36–43].

The stability and feasibility stability study have also been done through several theorems for model (3). Additional, the FODEs have been checked for numerical, semi-analytical and analytical solution using different methods. Some of the well-known methods have been studied by several researchers such as Euler, Taylor, Adams-Bashforth, predictor-corrector and various transforms, for details see, [44–51]. After studying the existing literature on fractional differential equation (FODEs) we have found that some area of FODEs need further investigation and discussion. In the first step we will find stability of the considered random fractional order problem (3) by various techniques. Qualitative analysis of positive solution for fractional order linear and non-linear boundary value problems will be studied. For this we will use fixed point theory and functional analysis like Leray-Schauder theory, fixed point theorems of Schauder’s, Krasnosilki’s, Banach, etc. We will also provide some iterative method for the numerical approximation of the proposed fractional order problem.

Having been motivated from the above research work in fractional mathematical epidemiology for details see [52–54]. We have presented the following MERS-CoV model in the sense of Caputo and the dimensional inconsistency of the model has been removed by carrying the fractional order \(\theta\) and also in the power of all of its parameters

\[
\begin{align*}
^{\mathbf{C}^{\theta}D}_{0+} S(t) &= \lambda t^{\theta} - \xi_1 IS t^{\theta} - \xi_2 p \phi AS t^{\theta} - \xi_4 p \phi HS t^{\theta} - \xi_4 CS t^{\theta} - \nu_0 S t^{\theta}, \\
^{\mathbf{C}^{\theta}D}_{0+} E(t) &= \xi_1 IS t^{\theta} + \xi_2 p \phi AS t^{\theta} + \xi_4 p \phi HS t^{\theta} + \xi_4 CS t^{\theta} - (\delta + \nu_0) E t^{\theta}, \\
^{\mathbf{C}^{\theta}D}_{0+} I(t) &= \delta I t^{\theta} - (\xi_3 + \nu_0) I t^{\theta}, \\
^{\mathbf{C}^{\theta}D}_{0+} H(t) &= \xi_3 I t^{\theta} + \xi_4 A t^{\theta} - (\xi_3 + \nu_0) H t^{\theta}, \\
^{\mathbf{C}^{\theta}D}_{0+} A(t) &= \nu_1 I t^{\theta} + \nu_2 A t^{\theta} - \nu_0 R t^{\theta},
\end{align*}
\]

where \(^{\mathbf{C}^{\theta}D}_{0+}\) stands for noninteger-order Caputo operator with fractional order \(\theta \in (0, 1)\).

Our original contribution in the article are consideration of the paper in the sense of Caputo fractional derivative to check the dynamical behavior inside the integer orders like the
continuous spectrum instead of discrete behavior. We have also analyzed the problem for existence, stability and approximate solution along with comparison to the discrete or classical order. By taking different fractional orders, we have simulated all the compartments at different fractional orders.

The paper is organized as follows: In Section 2, results are presented from FC. In Section 3, the qualitative theory for the fractional order model is presented. In Section 4, by the aid of fixed point theory, we have found the solution for both stability UH and generalized UH. We present a brief introduction of GABMM for the solution of fractional order Merse-CoV model in Section 5. Numerical simulation are performed of the GABMM with comparing results with Runge-Kutta method. At the end of the paper a brief conclusion is given in Section 6.

2. Preliminaries

Here, we recall some basic results from. [60,61].

**Definition 2.1.** The Reimann-Liouville fractional integral for a function say $X \in L^1(R^+)$, of order $\theta$ is defined as
\[
\Gamma_X^\theta(t) = \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} X(\tau)d\tau.
\]

**Definition 2.2.** the Caputo fractional order derivative for a function say $X$, is defined as
\[
\mathcal{D}_X^\theta(t) = \frac{1}{\Gamma(n - \theta)} \int_0^t (t - \tau)^{n - \theta - 1} X^{(n)}(\tau)d\tau,
\]
the integral part on the RHS exists and $n - 1 < \theta \leq n$ $n = [\theta] + 1$. If $\theta \in (0, 1)$, then we have
\[
\mathcal{D}_X^\theta(t) = \frac{1}{\Gamma(1 - \theta)} \int_0^t \frac{X(\tau)}{(t - \tau)^\theta}d\tau.
\]

**Lemma 2.3.** The following holds, in case of fractional differential equation;
\[
\Gamma_X^\theta\left[\mathcal{D}_X^\theta(q)(t)\right](t) = q(t) + \eta_n + \eta_{n-1}t + \ldots + \eta_1t^{n-1},
\]

3. Existence and uniqueness results

In this section, using the fixed point theorem we will show the existence and uniqueness solution for the system (3). The considered system (3) can be written into the following form as
\[
\begin{align*}
\mathcal{D}_X^\theta(S(t)) &= \mathcal{K}_1(t, S, E, I, A, H, R, C) + \mathcal{D}_X^\theta(S(t)), \\
\mathcal{D}_X^\theta(E(t)) &= \mathcal{K}_2(t, S, E, I, A, H, R, C) + \mathcal{D}_X^\theta(E(t)), \\
\mathcal{D}_X^\theta(I(t)) &= \mathcal{K}_3(t, S, E, I, A, H, R, C) + \mathcal{D}_X^\theta(I(t)), \\
\mathcal{D}_X^\theta(A(t)) &= \mathcal{K}_4(t, S, E, I, A, H, R, C) + \mathcal{D}_X^\theta(A(t)), \\
\mathcal{D}_X^\theta(H(t)) &= \mathcal{K}_5(t, S, E, I, A, H, R, C) + \mathcal{D}_X^\theta(H(t)), \\
\mathcal{D}_X^\theta(R(t)) &= \mathcal{K}_6(t, S, E, I, A, H, R, C) + \mathcal{D}_X^\theta(R(t)) + \mathcal{D}_X^\theta(C(t)).
\end{align*}
\]
Thus, the system (3) may in the following
\[
\begin{align*}
\mathcal{D}_X^\theta(\Omega(t)) &= \mathcal{W}(t, \Omega(t)), \\
\Omega(0) &= \Omega_0 > 0,
\end{align*}
\]
if
\[
\begin{align*}
\begin{cases}
\Omega(t) = (S, E, I, A, H, R, C)^T, \\
\Omega(0) = (S_0, E_0, I_0, A_0, H_0, R_0, C_0)^T, \\
\mathcal{W}(t, \Omega(t)) = (K, S, E, I, A, H, R, C)^T, i = 1, 2, 3, 4, 5, 6, 7,
\end{cases}
\end{align*}
\]
where $(\cdot)^T$ denoting the transpose. Next, the system (5) can write as
\[
\begin{align*}
\Omega(t) &= \Omega_0 + \mathcal{J}^\theta\mathcal{W}(t, \Omega(t)), \\
&= \Omega_0 + \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} \mathcal{W}^\theta(\Omega, \Omega(\tau))d\tau.
\end{align*}
\]
Let us suppose that the Banach space on $[0, b]$ is a set of continuous functions from on $\mathcal{W}$ with given norm $||\Omega|| = sup_{t \in \mathcal{J}} ||\Omega(t)||$ be $F = C([0, b] ; \mathcal{W})$. Next, we will use the following assumption:
\[
\begin{align*}
(C_1) &\text{ There exist a constant } \mathcal{L}_\mathcal{W} > 0 \text{ such that for each } \\
&\text{ } \Omega_1, \Omega_2 \in C([\mathcal{J}, \mathcal{B}]), \\
&|\mathcal{W}(t, \Omega_1(t)) - \mathcal{W}(t, \Omega_2(t))| \leq \mathcal{L}_\mathcal{W}||\Omega_1(t) - \Omega_2(t)||.
\end{align*}
\]
(C2) $\exists$ a constant $K \in (C[0, b], \mathcal{B})$ for all $(t, \Omega) \in \mathcal{J} \times \mathcal{B}$
\[
|\mathcal{W}(t, \Omega)| \leq K(t).
\]
Now, to find the solution is unique, take into account the following theorem [7].

**Theorem 3.1.** Using the assumption $(C_1), \mathcal{W} \in C([\mathcal{J}, \mathcal{B}])$ and with maps $\mathcal{J} \times \mathcal{B}$ bounded subset to relatively compact subset to $\mathcal{B}$. If $\Theta \mathcal{L}_\mathcal{W} < 1$, then the system (3) has a unique solution and
\[
\Theta = \frac{b^\theta}{\Gamma(\theta + 1)}.
\]

**Proof.** Let the operator $G_0 : Z \rightarrow Z$ expressed as
\[
(G_0)(t) = \Omega_0 + \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} \mathcal{W}(\Omega, \Omega(\tau))d\tau.
\]
The above Eq. (8) prove that the unique solution for (3) represent the fixed point of the operator $G$. Additionally, $sup_{t \in \mathcal{J}} |\mathcal{W}(t, 0)|| = \mathcal{M}_t$ and $b \geq ||\Omega_0|| + \Theta \mathcal{M}_t$. Thus, its enough to show that $GP_0 \subset P_0$ and the set given by $P_0 = \{ \Omega \in Z : ||\Omega|| \leq \mathcal{B} \}$ is convex and closed.

Next, for every $\Omega \in P_0$, we obtain
\[
\begin{align*}
(G_0)(t) &\leq \Omega_0 + \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} |\mathcal{W}(\Omega, \Omega(\tau))|d\tau, \\
&\leq \Omega_0 + \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} |\mathcal{W}(\Omega, \Omega(\tau)) - \mathcal{W}(\Omega, 0)| + |\mathcal{W}(\Omega, 0)|d\tau, \\
&\leq \Omega_0 + \frac{sup_{t \in \mathcal{J}} |\mathcal{W}(t, 0)||}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} d\tau, \\
&\leq \Omega_0 + \frac{sup_{t \in \mathcal{J}} |\mathcal{W}(t, 0)||}{\Gamma(\theta)} b^\theta, \\
&\leq \Omega_0 + \Theta \mathcal{L}_\mathcal{W} + \mathcal{M}_t, \\
&\leq \mathcal{B}_t.
\end{align*}
\]
Next, for $\Omega_1, \Omega_2, Z$, we have
\[
| \mathcal{L}(\Omega_1)(t) - \mathcal{L}(\Omega_2)(t) | \leq \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} |\mathcal{W}(\Omega_1, \Omega_2)| + |\mathcal{W}(\Omega_1, 0)|d\tau, \\
\leq \frac{sup_{t \in \mathcal{J}} |\mathcal{W}(t, 0)||}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} |\mathcal{W}(\Omega_1, \Omega_2)| + |\mathcal{W}(\Omega_1, 0)||d\tau, \\
\leq \Theta \mathcal{L}_\mathcal{W} |\Omega_1(t) - \Omega_2(t)|,
\]

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showing that \( \| (G_i \Omega) - (G_i \Omega) \| \leq \Theta L \| \Omega_i - \Omega_i \| \). Therefore, by Banach contraction the system (3) has a unique solution on \( \mathcal{J} \).

Next, using Schauder fixed point theory we will find the existence of solutions for system (3).

**Lemma 3.2.** Consider a bounded, convex and closed subset of a Banach space \( Z \) be \( \mathcal{M} \).

Consider the operator be \( G_1, G_2 \), then the following:

- \( G_i \Omega_i + G_2 \Omega_2 \), only if \( \Omega_i, \Omega_2 \in \mathcal{M} \),
- The operator \( G_i \) is a continuous and compact;
- A contraction mapping be \( G_2 \).

Then there exist \( u \in \mathcal{M} \), so that \( u = G_1 u + G_2 u \).

**Theorem 3.3.** Using the assumption (C) and (C) with \( \mathcal{W} : \mathcal{J} \times \mathbb{R}^2 \to \mathbb{R}^2 \). The system (3) has at least one solution on \( \mathcal{J} \) if

\[ L_i \| (\Omega_i(t_0) - \Omega_i(t_0)) \| < 1. \]

**Proof.** Suppose that \( \sup_{t \in \mathcal{J}} |X(t)| = \| X \| \) and \( \rho \geq \| \Omega_0 \| + \Theta \| K \| \), with \( B_r = \{ \Omega \in E : \| \Omega \| \leq \rho \} \). For every \( G_1, G_2 \) on \( \mathcal{J} \), then there is a unique solution \( u \in \mathcal{M} \) with the following properties:

\[
\| (G_i \Omega)(t) - (G_i \Omega)(t) \| \leq \| (\Omega_i(t_0) \Omega_i(t_0)) \| \quad \leq \rho < \infty.
\]

Hence, \( G_i \Omega_i + G_2 \Omega_2 \in B_r \).

Next, the contraction of \( G_2 \) will be proved.

Given any \( t \in \mathcal{J} \) and \( G_1, G_2 \in B_r \), it gives

\[
\| (G_i \Omega_i)(t) - (G_2 \Omega_2)(t) \| \leq \| (G_i \Omega_i)(t_0) - G_2 \Omega_2(t_0) \|.
\]

Having a continuous function \( \mathcal{W} \), thus \( G_i \) is continuous. Moreover, for all \( t \in \mathcal{J} \) and \( \Omega_i \in B_r \),

\[
|G_i \Omega| \leq \Theta \| K \| < +\infty,
\]

hence, \( G_i \) is bounded uniformly. Finally, we will show that \( G_1 \) is compact. Let us suppose that \( \sup_{(t,B) \in \mathcal{J} \times B} |\mathcal{W}(B, \Omega)| = \mathcal{W} \), which gives

\[
\| (G_i \Omega)(t_2) - (G_i \Omega)(t_1) \| = \frac{1}{\Gamma(\theta)} \int_0^1 \left( (t_2 - \beta)^{\delta-1} - (t_1 - \beta)^{\delta-1} \right) \mathcal{W}(B, \Omega) \, dB + \int_0^1 (t_2 - \beta)^{\delta-1} \mathcal{W}(B, \Omega) \, dB
\]

\[
\leq \frac{\Gamma(\theta)}{\Gamma(\delta)} \left[ (t_2 - t_1)^{\delta} + (t_1 - t_2)^{\delta} \right] \to 0 \text{ as } t_2 \to t_1.
\]

In view of well known “Arzela-Ascoli theorem”, the operator \( G_i \) is relatively compact \( B_r \) and thus \( G_i \) completely continuous. Satisfying all the claims of **Lemma 3.2**, we deduce that the proposed system (3) has at least one solution.

**4. Ulam-Hyers stability**

In this section of the paper, the stability of the system (3) will scrutinize with the aspect of UH and generalized UH [58,59]. For approximate solution stability analysis is very important. Let us assume \( \epsilon \) with the following inequality

\[
\epsilon \mathcal{D}^\gamma \Omega(t) - \mathcal{W}(t, \Omega(t)) \leq \epsilon, t \in \mathcal{J},
\]

and \( \epsilon = \max (\epsilon_i)^T, i = 1, 2, 3, \ldots 7. \)

**Definition 4.1.** System (3) is UH stable if there exist \( \mathcal{U} \) for all \( \epsilon > 0 \) solution of \( \Omega \in \mathcal{Z} \) holds for (14), then there is a unique solution \( \Omega \in \mathcal{Z} \) for Eq. (5) with the following

\[
\Omega(t) - \Omega(t) \leq \mathcal{U} \epsilon, t \in \mathcal{J},
\]

where \( \mathcal{U} \) = \( \max (\Omega_{W_i}) \).

**Definition 4.2.** System (3) is generalized UH stable if there exist a continuous function \( \Phi : \mathbb{R}^+ \to \mathbb{R}^+ \) and \( \Phi(0) = 0 \), so that for all solution \( \Omega \in \mathcal{Z} \) of (14), then there is a unique solution \( \Omega \in \mathcal{Z} \) for (5) with the following

\[
\Omega(t) - \Omega(t) \leq \Phi W \epsilon, t \in \mathcal{J},
\]

where \( \Phi W = \max (\Phi_{W_i}) \).

**Remark 4.3.** Let \( \Omega \in \mathcal{Z} \) be function that satisfies (14) if and only if there exist a function \( \partial Z \) with the following properties:

(I) \( |\partial(\epsilon)| \leq \epsilon, \partial = \max (\epsilon_i), t \in \mathcal{J}; \)

(II) \( \mathcal{C} \mathcal{D}^\gamma \Omega(t) = \mathcal{W}(t, \Omega(t)) + \partial(\epsilon), t \in \mathcal{J}. \)

**Lemma 4.4.** If \( \Omega \in \mathcal{Z} \) holds for Eq. (14), then \( \Omega \) also holds for the following

\[
\Omega(t) - \Omega(t) - \frac{1}{\Gamma(\delta)} \int_0^1 (t - \beta)^{\delta-1} \mathcal{W}(\beta, \Omega(t)) \, d\beta \leq \Theta e.
\]

**Proof.** Using (II), we have

\[
\mathcal{C} \mathcal{D}^\gamma \Omega(t) = \mathcal{W}(t, \Omega(t)) + \partial(\epsilon),
\]

along with **Lemma 4.4**, we obtain
\[\Omega(t) = \Omega_0(t) + \frac{1}{\Gamma(\theta)} \int_0^t (t - B)^{\theta - 1} \mathcal{W}(B, \Omega(B)) dB + \frac{1}{\Gamma(\theta)} \int_0^t (t - B)^{\theta - 1} \phi(B) dB. \quad (16)\]

Next, using (I) gives
\[\mathcal{G}(t) = \mathcal{G}_0(t) - \frac{1}{\Gamma(\theta)} \int_0^t (t - B)^{\theta - 1} \mathcal{W}(B, \Omega(B)) dB \leq \frac{1}{\Gamma(\theta)} \int_0^t (t - B)^{\theta - 1} \phi(B) dB \leq \Theta e. \quad (17)\]

Thus, the proof is completed. \(\square\)

**Theorem 4.5.** For all \(\Omega \in \mathbb{Z}\) and \(\mathcal{W} : \mathcal{J} \times \mathbb{R}^2 \to \mathbb{R}\) with the assumption \((C_1)\) holds and \(1 - \Theta \mathcal{L}_W > 0\). Eq. (5) is equal to Eq. (3) is UH and consequently, generalized UH stable.

**Proof.** Suppose that \(\Omega, \mathcal{G} \in \mathbb{Z}\) be a unique solution of (5), therefore for all \(t > 0\), \(t \in \mathcal{J}\) along with Lemma 4.4, we have
\[
\|\Omega - \mathcal{G}\| = \max_{0 < t < T} \left|\int_0^t (t - B)^{\theta - 1} \mathcal{W}(B, \Omega(B)) dB \right| \leq \max_{0 < t < T} \left|\int_0^t (t - B)^{\theta - 1} \mathcal{W}(B, \Omega(B)) dB \right| \leq \Theta e + \Theta \mathcal{L}_W \|\Omega - \mathcal{G}\|. \quad (18)
\]

From (18), we can write as
\[
\mathcal{G} = \frac{\Theta}{1 - \Theta \mathcal{L}_W}. \quad (19)
\]

Hence, equating \(\mathcal{G}(t) = \mathcal{W}(t)\) so that \(\mathcal{G}(0) = 0\), one may conclude that the system (3) is stable for both UH and generalized UH. \(\square\)

5. Numerical simulation and discussion

In this part, we will show the numerical solution for the Merse-Cov model (3). For this we will use the GABMM [55]. Summarize the method by considering the following nonlinear equation

\[
\mathcal{C}\mathbf{v}(t) = g(t, v(t)), \quad t \in [0, T],
\]

\(v^+(0) = v_0^+, \quad o = 0, 1, 2, \ldots, n, u = [0]\)

The above equation is equal to the following Volterra integral equation
\[
v(t) = \sum_{i=0}^{n} \int_0^t \left( (t - B)^{\theta - 1} g(B, v(B)) dB \right). \quad (20)
\]

To integrate (20), Diethelm et al. used the Adams-Bashforth Moulton method [56,57]. Put \(h = T/N, t_k = nh, n = 0, 1, 2, 3, \ldots, N \in \mathbb{Z}\), we may write the system (3) into the following form as [55]
Fig. 1 Plot of $S(t)$ having initial value for (3) at various fractional order $\theta$.

Fig. 2 Plot of $E(t)$ having initial value for (3) at various fractional order $\theta$.

Fig. 3 Plot of $I(t)$ having initial value for (3) at various fractional order $\theta$.

Fig. 4 Plot of $A(t)$ having initial value for (3) at various fractional order $\theta$.

Fig. 5 Plot of $H(t)$ having initial value for (3) at various fractional order $\theta$.

Fig. 6 Plot of $R(t)$ having initial value for (3) at various fractional order $\theta$. 
In the last we conclude that the successfully analysis for the fractional model of Middle Eastern respiration syndrome corona virus (MERS-CoV) has carried out at different fractional order in sense of Caputo fractional operator. All the nine agents of the proposed system are simulated against the available and estimated data to know about their dynamics at different fractional order. The simulation converges to the integer order as we increases the order \( \theta \). The solution of proposed system is also checked for at least one and unique solution using the technique of fixed point theory. The system is perturbed by very small quantity for very well achievement of UH stability. The numerical solution is obtained by the Modified Adams-Bashforth Moulton techniques. In final section the numerical simulation is provided at two different set of initial condition which shows convergency to the same equilibrium points of all the nine compartments of the considered model. Good comparable results are achieved at different fractional orders. The study is carried out because of need felt for the continuous dynamics of the said model. For this we have applied the fractional derivative in Caputo sense to the problem. As for as the novelty of the paper is concerned we apply the said operator for the first time to the problem and investigating its existence, stability and approximate solution at different fractional orders. We also checked the dynamical behavior of each compartment on different non-integer orders and lying between 0 and 1 along with the comparison with integer value 1 for different sets of data as non-negative values satisfying the conditions of reproductive number.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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