Black holes may not constrain varying constants

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Abstract

New and rather controversial observations hint that the fine structure constant $\alpha$ may have been smaller in the early universe, suggesting that some of the fundamental “constants” of physics may be dynamical. In a recent paper [1], Davies, Davis, and Lineweaver have argued that black hole thermodynamics favors theories in which the speed of light $c$ decreases with time, and disfavors those in which the fundamental electric charge $e$ increases. We show that when one considers the full thermal environment of a black hole, no such conclusion can be drawn: thermodynamics is consistent with an increase in $\alpha$ whether it comes from a decrease in $c$, an increase in $e$, or a combination of the two.

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Recent observations of spectral lines of distant quasars have suggested that the fine structure constant \( \alpha = e^2/\hbar c \) may have been slightly smaller in the very early universe \(^2\). Although these claims are still tentative and rather controversial, they have helped rekindle interest in Dirac’s old idea \(^3\) that the fundamental “constants” of physics may vary in time. In a recent Brief Communication to Nature, Davies, Davis, and Lineweaver have argued that black hole thermodynamics favors theories in which the speed of light \( c \) decreases in time, and disfavors those in which the fundamental electric charge \( e \) increases \(^1\). We show here that when one considers the full thermal environment of a black hole, no such conclusion can be drawn.

The fine structure constant depends on the fundamental charge \( e \), Planck’s constant \( \hbar \), and the speed of light \( c \), and it is natural to ask which of these varies. \(^4\) Davies et al. offer an ingenious argument. A black hole with mass \( M \) and charge \( Q = ne \) has an entropy \(^5\)

\[
S/k = \frac{\pi G}{\hbar c} \left[ M + \sqrt{M^2 - n^2e^2/G} \right]^2
\]

Evidently an increase in \( e \) will decrease this entropy, apparently violating the generalized second law of thermodynamics, while a decrease in \( \hbar \) or \( c \) will increase the entropy.

As Davies et al. point out, though, such entropic considerations should take into account not just the black hole, but its surroundings as well. An isolated black hole is never in thermal equilibrium: it will always decay by Hawking radiation and, if it is charged, by Schwinger pair production \(^6\) as well. These processes decrease \( S \), but they do not violate the second law, since the decrease is compensated by an increase in the entropy of the environment.

To study the full thermodynamics of “varying constants,” one may try to examine the detailed dynamics of heat flow and entropy between a black hole and its environment. A simpler alternative is to consider a proper thermodynamic ensemble, that is, a black hole in a heat bath. To obtain an equilibrium, one must consider a black hole in a “box” of radius \( r_B \), with fixed temperature \( T \) and charge \( Q \) (canonical ensemble) or fixed temperature \( T \) and electrostatic potential \( \phi \) (grand canonical ensemble) at the boundary. For charged black holes, such ensembles were first studied by Braden et al. \(^7\). In the canonical ensemble, the total entropy takes the form \( S = \pi r_B^2 x^2 \), where \( x \) is determined by the seventh-order equation

\[
x^5(x - q^2)(x - 1) + b^2(x^2 - q^2)^2 = 0
\]

with \( q = \sqrt{GQ/r_B c^2} \) and \( b = \hbar c/4\pi r_B kT \).

While no analytical solution of (2) is possible for generic charge and temperature, a numerical solution is straightforward. Figure 1 shows a graph of \( S_B = S/r_B^2 \) against \( q^2 \) and \( b \), restricted to the highest-entropy configuration. It is apparent—and may be confirmed with more detailed numerical study—that the total entropy increases with increasing \( q^2 \), and thus with increasing \( \alpha \). For the grand canonical ensemble, exact analytic results can be found, and the conclusion is the same. Black hole thermodynamics thus militates against

\[^*\]Variation of dimensionful constants is inherently ambiguous \(^4\); here, we take “varying \( e \)” to mean “suitable variation of all dimensionless quantities that depend on \( e \).”
How can one reconcile this result with the argument of Davies et al.? Note first that the Hawking temperature,

$$ kT_H = \frac{\hbar c^3}{8\pi G} \frac{\sqrt{M^2 - n^2e^2/G}}{2M^2 - n^2e^2/G + 2M\sqrt{M^2 - n^2e^2/G}} $$

(3)

does not increase with increasing $e$. As $e$ increases, a black hole will thus cool below the ambient temperature of the heat bath, and will absorb heat, increasing its mass. By the first law of thermodynamics, the net change in entropy is

$$ dS = \frac{1}{T} (dE - \phi dQ) $$

(4)

and it may be checked explicitly that the increase in quasilocal energy $E$ more than compensates for the direct effect of changing $Q$.

Of course, thermodynamic arguments of this sort only describe relationships among equilibria, and not the details of the transitions between equilibria. In some ways, this is an advantage: our results are rather insensitive to the details of a theory describing “varying constants,” requiring only that conventional black hole thermodynamics hold at equilibria at which $\alpha$ is constant. On the other hand, this generality allows some loopholes: one could, for instance, imagine a scenario in which an abrupt change in $\alpha$ led to an initial decrease in entropy, which then grew only slowly as heat was redistributed.

To analyze such possibilities, though, there is no substitute for a detailed dynamical model. In particular, any theory with a variable fine structure constant necessarily contains
a new scalar field, $\alpha$ itself. The entropy of that field might be negligible at equilibrium, but it surely cannot be ignored during dynamical processes in which $\alpha$ is changing. Black hole mass quantization may constrain models with “varying constants” [8], but in the absence of a detailed dynamical description, it seems that black hole thermodynamics cannot.

Acknowledgments

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