The Growth of Correlations in the Matter Power Spectrum

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ABSTRACT
We find statistically significant correlations in the cosmological matter power spectrum over the full range of observable scales. While the correlations between individual modes are weak, the band-averaged power spectrum shows strong non-trivial correlations. The correlations are significant when the modes in either one or both bands are in the non-linear regime, and approach 100% for pairs of bands in which all the modes are non-linear. The correlations are weaker, but not absent, when computed in redshift space. Since estimates of the power spectrum from galaxy surveys require band-averaging, the correlations must be taken into account when comparing a measured power spectrum with theoretical models.

Key words: cosmology:theory – large scale structures

1 INTRODUCTION
In our current paradigm for cosmological structure formation, initially small, primordial density perturbations grow through gravitational instability to form the large-scale structures that we see in the distribution of galaxies today. The most popular assumption is that the primordial fluctuations were distributed according to a homogeneous Gaussian random process. Under this assumption all of the statistical information is encoded in the power spectrum. The assumption of initially Gaussian modes is a prediction of the simplest models of inflation, and has received some limited observational support through measurements of the CMB (Kogut et al. 1996; Heavens 1998) and large-scale structure (Bouchet et al. 1993; Gaztanaga 1994; Nusser, Dekel & Yahil 1994; Feldman, Kaiser & Peacock 1994; Colley 1997). However, it is known that as the perturbations grow and become non-linear, the modes become coupled. In this paper, we study this process quantitatively using \( N \)-body simulations.

Since it is the lowest order non-vanishing statistical description of the density or galaxy field, the power spectrum has been at the focus of much recent attention. Many galaxy surveys have attempted to determine the nature of the power spectrum in two and three dimensions, and two massive surveys, the Anglo-Australian Two Degree Field (2dF\(^\ast\)) and the Sloan Digital Sky Survey (SDSS\(^\dagger\)) are moving the subject toward high-precision measurements. Several methods for performing detailed comparisons between a measured power spectrum from a galaxy survey and a theoretical power spectrum have been devised (see Tegmark et al. 1998 for a review). Critical to assessing the accuracy of the measurements is an evaluation of the statistical properties of the power spectrum. To date most estimates of the precision to which the power spectrum may be measured have assumed that the Fourier components of the density fluctuations are still distributed as a Gaussian random process. In that case the standard deviation of the power spectrum is determined by the value of the power spectrum. We expect this assumption to be valid at early times and on sufficiently large scales for models with Gaussian initial conditions. The question of how large is “sufficiently large” requires a detailed calculation.

We show here that non-linear clustering gives rise to significant correlations in the band-averaged power spectrum at different scales in the regime of observational interest. The correlations increase with increasing (spatial) frequency, reaching levels near 100% when the perturbations are non-linear. Moreover, the correlations require the modes in only a single band to be non-linear: we find significant correlations between high and low frequencies even if the modes in the low frequency band are still in the linear regime. A consequence of the mode coupling is an increase in the dispersion of the power spectrum, suggesting that estimates of the accuracy to which the power spectrum may be measured should be revisited.

The outline of the paper is as follows: in \( \S \) we discuss the expectations for correlations in the power spectrum between modes. In \( \S \) we describe our numerical experiments,
while in [3] we discuss the statistical properties of the correlations. We finish in [4] with some comments on the impact of our results for future surveys designed to measure the matter power spectrum.

### 2 CORRELATIONS IN THE POWER SPECTRUM

The density contrast is defined by \( \delta(\mathbf{r}) = [\rho(\mathbf{r}) - \bar{\rho}] / \bar{\rho} \), where \( \rho(\mathbf{r}) \) is the matter density at position \( \mathbf{r} \), and \( \bar{\rho} \) is the mean density. We consider a large spatial volume \( V \) and impose periodic boundary conditions, thus \( \mathbf{r} \) is continuous but its Fourier conjugate \( \mathbf{k} \) is quantized. One can take the continuum limit by replacing \( V^{-1} \sum_k \) with \( \int d^3k / (2\pi)^3 \). Our Fourier transform convention has

\[
\delta_k = V^{-1} \int d^3r \, \delta(\mathbf{r}) \exp[\mathbf{i}\mathbf{k} \cdot \mathbf{r}],
\]

(1)

and

\[
\delta(\mathbf{r}) = \frac{V}{(2\pi)^3} \int d^3k \, \delta_k \exp[-i\mathbf{k} \cdot \mathbf{r}]
\]

(2)

both dimensionless. We define the power spectrum as \( P(k) = V \langle |\delta_k|^2 \rangle - 1/n \), where the angled brackets (\( \langle \cdot \rangle \)) denote an ensemble (and Poisson) average, and the shot noise term \( 1/n \) has been subtracted. Here, \( n \) is the mean number density of objects used to measure the power spectrum. The power spectrum is related to the 2-pt spatial correlation function \( \xi(\mathbf{r}) \) by

\[
P(k) = \int d^3\mathbf{r} \, \xi(\mathbf{r}) \exp[\mathbf{i}\mathbf{k} \cdot \mathbf{r}].
\]

(3)

We denote our (unbiased) estimate of the power spectrum at \( \mathbf{k} \), which we obtain from the simulations, as

\[
\hat{P}(k) = V |\delta_k|^2 - 1/n
\]

(4)

Typically we reduce the scatter in this quantity by averaging over a thin shell in \( k \)-space with \( |\mathbf{k}| \approx k \).

The covariance between two random variables \( x \) and \( y \) is defined as \( \text{cov}[x, y] = \langle xy \rangle - \langle x \rangle \langle y \rangle \). The variance is \( \text{var}(x) = \text{cov}[x, x] \). The correlation between \( x \) and \( y \) is defined as \( \rho(x, y) = \text{cov}[x, y] / [\text{var}(x)\text{var}(y)]^{1/2} \). The covariance of our power spectrum estimator, \( \hat{P}(k) \), depends on the 2-pt function (power spectrum), the 3-point function (bi-spectrum) and the 4-point function (tri-spectrum). We obtain, to leading order in \( (nV)^{-1} \) as \( nV \to \infty \) (Peebles 1980, §36),

\[
\text{cov}[\hat{P}(k), \hat{P}(k')] = \left[ P(k) + \frac{n}{V} \right] \{ \delta_{kk'} + \delta_{k-k'} \} + T(k, -k, -k') / V
\]

(5)

with \( \delta_{kk'} = 1 \) for \( k = k' \), and 0 otherwise, and where we have written the tri-spectrum as \( T \). The tri-spectrum is defined as the Fourier transform of the 4-point function \( \eta \) according to

\[
T(k_1, k_2, k_3, k_4) = \frac{1}{V} \int d^3x_1 d^3x_2 d^3x_3 d^3x_4 \eta(x_1, x_2, x_3, x_4) \times \exp[ik_1 \cdot x_1 + ik_2 \cdot x_2 + ik_3 \cdot x_3 + ik_4 \cdot x_4].
\]

The first term in Eq. (3) is the usual result for Gaussian fluctuations, viz \( \text{var}[x^2] = 2\langle x^2 \rangle^2 \) for a real variable \( x \). Because \( \delta_k \) is complex with uncorrelated real and imaginary parts, \( P = 2[\text{Re}(\delta_k)]^2 = 2[\text{Im}(\delta_k)]^2 \), and the Gaussian contribution to the error on \( P \) becomes equivalent to the total power spectrum: the sum of the signal and the noise. Only the tri-spectrum contributes to the off-diagonal \( (k \neq k') \) elements of the covariance.

The terms sub-dominant in \( \bar{n}V \), all shot noise terms, are also straightforward to derive. There is a constant term \( (nV)^{-1} \), contributions from the 2-pt function

\[
\frac{1}{nV} \left[ P(|k + k'|) + P(|k - k'|) + 2P(k) + 2P(k') \right]
\]

(7)

and the 3-pt function

\[
\frac{1}{nV} \left[ B(k, -k, 0) + B(0, k', -k') + B(k + k', -k, k') + B(k + k', -k, -k') \right]
\]

(8)

where the bi-spectrum is defined as the Fourier transform of the 3-pt function \( \zeta \)

\[
B(k_1, k_2, k_3) = \frac{1}{V} \int d^3x_1 d^3x_2 d^3x_3 \zeta(x_1, x_2, x_3) \times \exp[ik_1 \cdot x_1 + ik_2 \cdot x_2 + ik_3 \cdot x_3]
\]

(9)

Finally we consider the result for band-averaged estimates of \( P(k) \). Denote by \( \hat{P}_i \) our estimator \( \hat{P}(k) \) averaged over a set of \( N \), \( k_{i,\alpha} \) with \( k_{i,\alpha} \approx k_i \) for \( \alpha = 1 \) to \( N \). Straightforwardly

\[
\text{cov} \left[ \hat{P}_i, \hat{P}_j \right] = \frac{1}{N^2} \sum_{\alpha=1}^{N_1} \sum_{\beta=1}^{N_2} \text{cov} \left[ \hat{P}(k_{i,\alpha}), \hat{P}(k_{j,\beta}) \right]
\]

(10)

Even when the bands \( i \) and \( j \) are disjoint, we shall see that the band-averaging introduces non-trivial correlations in the power spectrum. These correlations are given by the tri-spectrum averaged over the configurations in the shell. In principle one could apply different weights to the modes in the band, but we have not pursued this line of inquiry. It is traditional to merely average over the directions in a \( k \)-shell and this is what we have chosen to do in the simulations.

### 3 THE GROWTH OF CORRELATIONS

Gravitational perturbation theory (PT) has given some insight into the growth of clustering beyond the linear regime, and in particular the effect of clustering on the evolution of the power spectrum, the bi-spectrum, and the tri-spectrum (Peebles 1980, Juszkiewicz 1984, Vishniac 1983, Fry 1984). Perturbation theory shows that the variance in the power spectrum will grow for modes that are still well in the linear regime. In addition, PT shows that the non-linear growth of density fluctuations will give rise to a tri-spectrum [E1984]. The form the tri-spectrum takes in PT involves multiples of the first order power spectrum and no powers of \( V \). While this is shown explicitly by Fry to the lowest non-vanishing order in the tri-spectrum, it is straightforward to extend the result to all orders. Consequently, the contribution of the tri-spectrum to the power spectrum covariance between any off-diagonal pair of \( k \)-modes is of order \( 1/V \) in

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PT, and so vanishes in the large volume limit according to equation (3). Fan & Bardeen (1995) have argued that the power covariance matrix should remain diagonal even in the non-linear regime.

Nonetheless, as we describe below, correlations in the band-averaged power spectrum may remain even in the limit $V \rightarrow \infty$. This is a consequence of the decrease in the variance of the power spectrum with the increased number of modes in the band. Since the number of modes in a band is proportional to $V$, the $1/V$ suppression of the covariance between modes cancels in the expression for the correlation $\rho$, leading to a finite value even as $V \rightarrow \infty$. We investigate the development of the correlations in the non-linear regime using $N$-body simulations.

We remark that for a finite-volume survey, we would expect the presence of both covariances and correlations between individual modes according to equation (3) and the associated shot-noise terms.

Using a particle-mesh (PM) code, described in detail in Meiksin, White & Peacock (1999), we have performed a series of several thousand realizations of the evolution of clustering in a $\Lambda$CDM universe. The parameters chosen are $\Omega_0 = 0.4$, $\Omega_\Lambda = 0.6$, $h = 0.65$, $\Omega_0h^2 = 0.03$, and $n = 1.030$, where $n$ is the primordial spectral index. The power spectrum is COBE-normalized using the method of Bunn & White (1997). The slight tilt has been chosen to reproduce the abundance of rich clusters of galaxies today (White, Efstathiou & Frenk 1993), specifically the rms mass fluctuation in an $8 h^{-1}$Mpc sphere is $\sigma_8 \simeq 0.92$. The non-linear scale, where the variance becomes unity, is $k_{nl} = 0.19 h$ Mpc$^{-1}$. Tests involving the higher order moments indicate that the code should accurately reproduce the bi- and tri-spectrum on the scales of interest (White 1998).

All the simulations were run with a $128^3$ force grid and either $128^3$ or $256^3$ particles to isolate the effects of shot noise. Two box sizes were used, $200 h^{-1}$ Mpc and $400 h^{-1}$ Mpc, to test for finite volume effects. For each box size, $N \sim 10^4$ realizations with different Gaussian initial conditions were evolved from $1 + z = 20$ to the present. For the $256^3$ simulations, $N \sim 10^4$ realizations were performed. We verified that starting the simulations at $1 + z = 30$ did not alter our results. For each realization the power spectrum was calculated in 20 bins logarithmically spaced in $k$ from the fundamental mode to the Nyquist frequency of the force grid, as described in Meiksin et al. (1999). The ensemble averages of the previous section were approximated by an average over the $N$ realizations. Convergence of the correlations was typically obtained with several hundred realizations.

The resulting correlations in the real space power spectrum at $z = 0$ for several selected frequency bands from the 200 $h^{-1}$ Mpc simulations are shown in Table 1. For each entry we may assess the likelihood of no correlation using the Spearman rank order correlation coefficient $r_s$ (Siegel & Castellan 1989). The probabilities are $< 0.001$ for pairs of bands with both central frequencies exceeding 0.14 $h$ Mpc$^{-1}$. An estimate of the probability for all the correlations is provided by the Kendall concordance statistic $W$ (Siegel & Castellan 1989), which is linearly related to the average of the $r_s$ values for all the band pairs. For the 12 frequency bands shown in Table 1, we obtain $W = 0.33$, with a vanishingly small probability for obtaining a value so large assuming no correlations. Indeed, it is only for correlations for which both bands are confined to frequencies $k \leq 0.093$ h Mpc$^{-1}$ that the collective probability for non-correlation is found to exceed 0.001.

We also compute the correlations in redshift space by including the peculiar velocities of the particles. The correlations are weaker ($W = 0.26$ for the 12 bands), but are still highly significant. We expect that the correlations are weaker in redshift space both because of the reduced power on small scales in redshift space and because the peculiar velocities introduce extra randomness which destroys the correlations.

In Figure 1 the correlations are shown between each band centred at $k$ and the bands centred at $k = 0.031 h$ Mpc$^{-1}$ (triangles) and $k = 0.17 h$ Mpc$^{-1}$ (squares). The correlations are shown both in real space (filled) and redshift space (open). The correlations become significant when the modes in one of the bands enter the non-linear regime. Redshift space distortions reduce the correlations on small scales.
Table 1. Power spectrum correlation matrix for 200 $h^{-1}$ Mpc simulation. The last row shows $\sigma = [\Delta^2(k)]^{1/2}$.

| $k$ ($h$ Mpc$^{-1}$) | 0.031 | 0.044 | 0.058 | 0.074 | 0.093 | 0.110 | 0.138 | 0.169 | 0.206 | 0.254 | 0.313 | 0.385 |
|-----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.031                 | 1.000  | -0.017 | 0.023  | 0.024  | 0.042  | 0.154  | 0.176  | 0.188  | 0.224  | 0.264  | 0.265  | 0.270  |
| 0.044                 | -0.017 | 1.000  | 0.001  | 0.024  | 0.056  | 0.076  | 0.118  | 0.180  | 0.165  | 0.228  | 0.234  | 0.227  |
| 0.058                 | 0.023  | 0.001  | 1.000  | 0.041  | 0.027  | 0.086  | 0.149  | 0.138  | 0.177  | 0.206  | 0.202  | 0.205  |
| 0.074                 | 0.024  | 0.024  | 0.041  | 1.000  | 0.079  | 0.094  | 0.202  | 0.229  | 0.322  | 0.343  | 0.374  | 0.391  |
| 0.093                 | 0.042  | 0.056  | 0.027  | 0.079  | 1.000  | 0.028  | 0.085  | 0.177  | 0.193  | 0.261  | 0.259  | 0.262  |
| 0.110                 | 0.154  | 0.076  | 0.086  | 0.094  | 0.028  | 1.000  | 0.205  | 0.251  | 0.314  | 0.355  | 0.397  | 0.374  |
| 0.138                 | 0.176  | 0.118  | 0.149  | 0.202  | 0.085  | 0.205  | 1.000  | 0.281  | 0.396  | 0.488  | 0.506  | 0.508  |
| 0.169                 | 0.188  | 0.180  | 0.138  | 0.229  | 0.177  | 0.251  | 0.281  | 1.000  | 0.484  | 0.606  | 0.618  | 0.633  |
| 0.206                 | 0.224  | 0.165  | 0.177  | 0.322  | 0.193  | 0.314  | 0.396  | 0.484  | 1.000  | 0.654  | 0.720  | 0.733  |
| 0.254                 | 0.264  | 0.228  | 0.206  | 0.343  | 0.261  | 0.355  | 0.488  | 0.606  | 0.654  | 1.000  | 0.816  | 0.835  |
| 0.313                 | 0.265  | 0.234  | 0.202  | 0.374  | 0.259  | 0.397  | 0.506  | 0.618  | 0.720  | 0.816  | 1.000  | 0.902  |
| 0.385                 | 0.270  | 0.227  | 0.205  | 0.391  | 0.262  | 0.374  | 0.508  | 0.633  | 0.733  | 0.835  | 0.902  | 1.000  |

where $\Delta^2(k) = k^3P(k)/(2\pi^2)$. While the correlations between pairs of low frequency bands, both well in the linear regime ($k < k_{ni}$), are consistent with zero, the power spectrum correlation between low frequency bands and high frequency bands rapidly rises as the frequency of the latter increases. The correlations involving the higher frequency bands are significantly reduced in redshift space.

In Figure 2, we show that no significant difference is found between the 200 $h^{-1}$ Mpc and 400 $h^{-1}$ Mpc box simulations, suggesting that the correlations have converged and are not due to the finite volume of the simulations. Similarly, we show in Figure 3 that the correlations do not decrease as the number of particles increases, demonstrating that the correlations are not dominated by shot noise contributions.

As an additional test, we performed a similar set of $\sim 10^5$ runs using the Zel’dovich approximation with the same box sizes and particle numbers as for the PM code. With the Zel’dovich calculations we could displace the particles from random positions within the simulation volume, from a regular grid or from random positions near grid zones. This allowed us to test that our initial conditions did not contribute spuriously to the correlations. We found, see Figure 4, that the correlations appeared much more strongly in the PM runs than the Zel’dovich runs, suggesting that they are induced by gravitational instability, as shown in Figure 5 and are not an artifact of our numerical technique. We note, however, that evolving the particles in the Zel’dovich approximation, even when the particles were initially placed completely at random within the simulation box, again resulted in statistically significant correlations.

We have also tested the random number generation by using two separate generators for 512 Zel’dovich runs each, one based on a multiplicative congruential generator (Press et al. 1992), and the other based on a lagged Fibonacci generator (Bertschinger 1993), the periods of both of which should be ample for our purposes. The results were statistically indistinguishable.

Figure 2. The correlations in real space power between the bands centred at $k$ and $k = 0.17h$ Mpc$^{-1}$ (open circles) in the 200 $h^{-1}$ Mpc simulation. Also shown are the correlations for the $k = 0.16h$ Mpc$^{-1}$ band (filled triangles) and the $k = 0.19h$ Mpc$^{-1}$ band (filled circles) in the 400 $h^{-1}$ Mpc simulation. The filled points are slightly offset in frequency for clarity. No significant reduction in the correlations is found in the larger box compared with the smaller, suggesting that the correlations are not a finite volume effect.

4 THE STATISTICAL PROPERTIES OF THE CORRELATIONS

To trace the origin of the correlations, we create two new bands with only 32 modes each for the 200 $h^{-1}$ Mpc box simulation. One band is centred at $k = 0.47h$ Mpc$^{-1}$ and the other at $k = 0.71h$ Mpc$^{-1}$. We have first verified that the density fluctuations $\delta_{i}$ themselves are uncorrelated, as they must be by homogeneity. We find $W = 0.008$, with a probability of non-correlation of $p = 0.44$. For the two band-averaged $P(k)$ the correlation is $\rho = 0.23$, with a probability
$p = 6.3 \times 10^{-9}$ that the two are uncorrelated. For the full set of $(64 \times 63)/2$ $P(k)$ correlations, we find $W = 0.025$, with a probability $p = 2.3 \times 10^{-22}$, demonstrating the clear presence of highly significant correlations among the individual $P(k)$. However, we find that none of the values in the $32 \times 32$ inter-band correlation matrix for the individual $P(k)$ estimates is as great as the correlation value $\rho = 0.23$ found between the band-averaged power spectra above. Indeed, the average correlation value in the matrix is 0.0099, and the standard deviation 0.037. We conclude that the strong correlations among the band-averaged power spectra are built up in the band-averaging procedure itself from the much weaker individual mode correlations.

We may understand the origin of the correlations by considering a thin shell of width $\Delta k$ centred at $k_0$. Neglecting the shot-noise terms, the variance in the band-averaged power $P_i(k)$ is $\text{var}[P_i(k)] = 2P_i(k)^2/N_i + T/V$, where $T$ denotes an average over the configurations in the $k-$shell, and $N_i = V(2\pi)^{-3}4\pi k^2\Delta k$ is the number of modes in the shell. When the contribution of the tri-spectrum to the variance is negligible, we find that the correlation in the band-averaged power is given by $\rho[P_i(k), P_j(k')] \sim \langle P_i(k)P_j(k')\rangle/kk'\Delta k$, and $V$ has explicitly cancelled. Here $T$ denotes the tri-spectrum averaged over all the configurations between both $k-$shells. For logarithmically spaced frequencies this becomes $\rho \sim \langle T/P_i(k)P_j(k')\rangle/(kk')^{3/2}\Delta \log k$.

To make further progress, let us assume the hierarchical clustering ansatz. The form of the reduced 4-point function $\eta$ is then $\eta[x_1, x_2, x_3, x_4] = R_0\{\xi(x_{12})\xi(x_{13})\xi(x_{14}) + \text{cyc. (12terms)}\} + R_0\{\xi(x_{12})\xi(x_{13})\xi(x_{14}) + \text{cyc. (4terms)}\}$, where $x_{ij} = x_i - x_j$ and $R_0$ and $R_0$ are constants of order one \cite{fryetal84}. Using equation (11), we obtain for the contribution to the off-diagonal power spectrum covariance

$$V^{-1}T(k, -k, k', -k') =$$

$$\frac{R_0}{V} \left[ P(|k - k'|) + P(|k + k'|) \right] \left[ P(k) + P(k') \right] +$$
\[ \frac{R_b}{V} P(k) P(k') \left[ P(k) + P(k') \right] . \]

We find then that the power correlations for individual modes of frequencies \( k \) and \( k' \) will be on the order of \( [P(k) + P(k')]/V \sim 10^{-3} - 10^{-2} \) for a 200 h^{-1} Mpc box. After band-averaging, and for \( P(k') \gg P(k) \), the correlations become \( \rho \sim P_b(k) k' \Delta k \) for linearly spaced \( k \)-shells, and \( \rho \sim \Delta^2(k')/(k/k')^{3/2} \Delta \log k \) for logarithmic spacing. Thus for logarithmically spaced shells, at a fixed \( k' \) the correlations will increase with \( k \) like \( k^{3/2} \). When \( \Delta^2(k) \sim (\Delta \log k)^{-1} \), the tri-spectrum term will dominate the variance of \( P \), and the correlations will flatten to \( \rho \sim [\Delta^2(k)^{-1}]^{1/2} \Delta^2(k')/(k/k')^{3/2}(\Delta \log k)^{3/2} \). This is just the behaviour displayed in Figure 6. Motivated by the hierarchical ansatz form, we find the correlations for \( k \gg k' \) and \( P(k') \gg P(k) \) are well approximated by

\[ \rho \approx \left( \frac{\bar{P}_a + \bar{P}_b}{2} \right) \left( \frac{k}{k'} \right)^{3/2} \frac{\Delta^2(k') \Delta \log k}{1 + \bar{P}_a \Delta^2(k) \Delta \log k} / \Delta^2 / \Delta \log k. \]  

where \( \bar{P}_a = (34/21)^2 \) and \( \bar{P}_b = 682/189 \) are angle-averaged coefficients derived in perturbation theory [Fry 1984]. We find that a direct computation using equation (11) with \( \bar{P}_a \) and \( \bar{P}_b \) agrees less well with the simulations, over-estimating the correlations by as much as a factor of 2. It is unclear why the direct computation fails, but it improves toward the higher \( k \) shells, which may indicate that using the angle-averaged coefficients is an inadequate approximation for the bins with fewer modes. Since performing the full set of integrals in perturbation theory for all the modes in the \( k \)-shells is extremely cumbersome, we will not pursue the hierarchical ansatz any further, but note that it may be worth greater consideration.

A consequence of the correlations is a reduction in the number of degrees of freedom describing the variance of the power spectrum. While the distribution of band-averaged power is still \( \chi^2 \), the modes are no longer independent and the “effective number” of degrees of freedom will be less than the number of modes in the band. We may define the “effective number” of degrees of freedom \( n_{\text{dof}} \) by

\[ n_{\text{dof}} = \frac{2\langle \bar{P} \rangle}{\text{var}(\bar{P})}. \]  

Since the density fluctuation \( \delta_b \) for each mode consists of independent real and imaginary parts, when the modes are Gaussian-distributed each mode will contribute two degrees of freedom to the average. The band-averaged power spectrum will then be distributed like \( \chi^2 \) with \( n_{\text{dof}} \) degrees of freedom. Correlations between modes will reduce \( n_{\text{dof}} \) below the number expected from mode counting. This is a strong effect at high frequencies, as shown in Figure 6. Motivated by the hierarchical ansatz, we find that the reduction factor is approximately described by \( 1 + \bar{P}_a \Delta^2(k) \Delta \log k \). While \( n_{\text{dof}} \) agrees with the number of degrees of freedom expected for Gaussian-distributed modes in the linear to quasi-linear regimes (\( k < 0.1 \) h Mpc^{-1} ), on scales for which the fluctuations are non-linear \( n_{\text{dof}} \) falls substantially short. The errors on the power spectrum in the non-linear regime will correspondingly greatly exceed estimates based on the assumption of Gaussian statistics.

In spite of the presence of strong correlations, the distribution of band-averaged power spectra is still reasonably well-described by a \( \chi^2 \) distribution. We show the maximum-likelihood estimates for the numbers of degrees of freedom in Fig. 8 found by assuming the power spectra in a given band are distributed like \( \chi^2 \). Formally, the KS test rejects the \( \chi^2 \) distribution with 90-99.7% confidence at frequencies in the range \( 0.25 < k < 2 \) h Mpc^{-1} . (We haven’t examined higher frequencies.) The distribution of the redshift-space estimated power spectra, however, is found to be very close to \( \chi^2 \) at all frequencies.

5 CONCLUSIONS

Estimates of the matter power spectrum from galaxy surveys have generally assumed the power in separate modes to be uncorrelated. The gravitational growth of the modes, however, will give rise to a tri-spectrum which will induce correlations in the band-averaged power spectrum. In a set of numerical experiments, we find that significant correlations in power develop when at least one of the modes is non-linear, and approaching 100% when the modes in both bands are non-linear. Although the power spectrum correlations are weaker in redshift space, they are still statistically significant.

In addition to redshift space distortions, the measured galaxy power spectrum will also depend on the bias between the galaxy count fluctuations and the underlying matter fluctuations. We have not computed the correlations in power allowing for bias. Although this is straightforward within any given bias scheme, which scheme, if any, the true galaxy distribution follows is unclear at this time. More direct measurements of the dark matter fluctuations may be obtained through galaxy velocity flows [Dekel 1994; Strauss & Willick 1995] or lensing [Blandford et al. 1991].
The Growth of Correlations in the Matter Power Spectrum

Our results are not directly applicable to these measures, as they involve projections (integrals) of the matter power spectrum. However, we expect that measurement errors will again exceed those estimated assuming Gaussian-distributed density fluctuations as a result of correlations in the matter power spectrum.

The presence of power spectrum correlations restricts the range of applicability of many techniques for fitting models to the measured power spectra (e.g. Tegmark et al. 1998) to very large scales. In particular, it may no longer be assumed that the k-space density fluctuations $\delta_k$ are distributed as a Gaussian random process: the tri-spectrum must explicitly be taken into account if any of the modes fit are in the non-linear regime. This constraint may be particularly severe if the window function of the survey is broad, mixing modes across a wide range of scales, and/or the survey is not very deep. Correlations will affect the model-fitting based on all completed galaxy surveys, and may only be circumvented with wide-angle deep surveys like the Sloan Digital Sky Survey and the 2dF survey now underway. For a catalog the depth of the SDSS Northern Polar Cap redshift survey, the minimum frequency spacing for uncorrelated bins is $\Delta k \approx 0.015 h^{-1} \text{Mpc}$. On a scale of $k = 0.19 h^{-1}$, the scale of non-linearity for the model investigated here, the expected correlation with the power at $k' = 0.1 h^{-1}$ will be on the order of $\rho \approx 0.15$. Fitting a given model to measurements of the power spectrum in the non-linear regime will require computing the correlations for the model. Provided there is a sufficient number of modes per band, the Central Limit Theorem guarantees the band-averaged power spectrum estimates will be distributed as a multivariate Gaussian random process, simplifying the fitting process. Designing an efficient fitting procedure in the presence of correlations is a topic worthy of future investigation.

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REFERENCES

Bernardeau F., van Waerbeke L., Mellier Y., 1997, Astron. Astrophys., 322, 1
Bertschinger E., 1995, preprint, astro-ph/9506079
Bland ford R.D., Saust A.B., Brainerd T.G., Villumsen J.V., 1991, MNRAS, 251, 600
Bouchet F., Strauss M., Davis M., Fisher K., Yahil A., Huchra J., 1993, ApJ, 417, 36
Bunn E. F. & White M., 1997, ApJ, 480, 6
Colley W.N., 1997, ApJ, 489, 471
Dekel A., 1994, ARA&A, 32, 371
Fan Z., Bardeen J. M., 1995, Phys. Rev. D, 51, 6714
Feldman H., Kaiser N., Peacock J., 1994, ApJ, 426, 23
Fry J., 1984, ApJ, 279, 499
Gaztañaga E., 1994, MNRAS, 268, 913
Heavens A.P., 1998, preprint, astro-ph/9804222
Jain B., Seljak U., White S.D.M., 1998, preprint astro-ph/9804238
Juszkiewicz R., 1981, MNRAS, 197, 931
Kaiser N., 1992, ApJ, 388, 272
Kaiser N., 1998, ApJ, 498, 26
Kogut A., et al., 1996, ApJ, 464, L29
Meiksin A., White M., Peacock J., 1999, MNRAS, in press astro-ph/9812214
Miralda-Escude J., 1991, ApJ, 380, 1
Nusser A., Dekel A., Yahil A., 1994, ApJ, 449, 439
Peebles P.J.E., 1980, The Large-Scale Structure of the Universe. Princeton Univ. Press, Princeton
Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., 1992, Numerical Recipes, 2nd edn. Cambridge Univ. Press, Cambridge
Scoccimarro R., Zaldarriaga M., Hui L., 1999, submitted to ApJ astro-ph/9901099
Seljak U., 1998, ApJ, 506, 64
Siegel S., Castellan N.S. Jr, 1985, Nonparametric Statistics. McGraw Hill, New York
Strauss M.A., Willick J.A., 1995, Phys. Rep. 261, 271
Tegmark M., Hamilton A., Strauss M., Vogeley M., Szalay A., 1998, ApJ, 499, 555
Vishniac E. T., 1983, MNRAS, 203, 345
White M., submitted to MNRAS, astro-ph/9811227
White S.D.M., Efstathiou G.P., Frenk C.S., 1993, MNRAS, 262, 1023

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