An optimized parameter control method of resonance corrector for improving the output characteristics of single-phase inverter

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Abstract. According to the principle of internal mode, the resonant corrector is applied to the control system of single-phase inverter, which can effectively improve the output characteristics of the system. Based on the mathematical model, the dc input voltage fluctuation and load change are taken as the uncertain factors affecting the performance of the inverter, and the parameter optimization design of the resonant corrector is carried out based on pole assignment. The simulation results show that compared with the traditional quasi-PR controller, the controller designed with the resonance corrector parameter optimization can better adapt to the continuous large range of voltage fluctuation at the DC input end, and the system has good dynamic performance, waveform stability, and the ability to resist load disturbance.

1. Introduction

According to the principle of internal mode [1], PI control can only realize the static-free tracking of DC signal, while to realize the static-free tracking of ac signal of single-phase inverter, resonance link needs to be introduced. Because the resonance corrector has infinite gain at the specified frequency and small gain in other frequency domains, it can track the specified sinusoidal signal without static difference, so it is widely used in ac signal control. In practical application, the PR controller only has infinite gain at a single frequency, which cannot prevent the influence of power grid frequency offset and make the stability of the system worse.

In order to improve the traditional PR control method, literature [2] adopts the quasi-PR control method to design the controller which increases the frequency bandwidth and improves the adaptability of the system to the frequency. However, there is no specific research on the influence of parameter selection on the system. Literature [3] studied and discussed the influence of parameter changes of resonance corrector on the dynamic performance and stable performance of the system. However, the adjustment of parameters lacks the theoretical basis of the system, and the parameter design cannot reach the goal of optimization. Literature [4] adds multiple resonance links to the quasi-PR control to suppress the specific harmonic components in the current through multiple resonance links. The control process does not need complex decoupling operation, and the dynamic performance is good. However, when the DC input fluctuates in a wide range, its adaptability is not strong enough and its resistance to load disturbance is poor.
In this paper, a method for parameter optimization design of resonance corrector based on pole assignment is presented. In the case of load mutation and DC input voltage fluctuation, the simulation results of the quasi-PR controller are compared to show that the parameter optimization method of inverter resonant corrector based on pole assignment is correct and effective.

2. Mathematical model of single-phase full-bridge inverter

Figure 1 shows the schematic diagram of single-phase full-bridge inverter topology and control system.

![Schematic diagram of single-phase full bridge inverter topology control system](image)

L is the inductance, C is the capacitance, and R is the equivalent resistance of the filter. The input voltage on the DC side is $U_{dc}$, the capacitor terminal voltage is $U_c$, the inductor terminal voltage is $U_l$, the current flowing out of the inductor is $I_l$, and the current flowing into the rear load direction is $I_o$.

The following equations can be derived from KCL and KVL:

$$
\begin{bmatrix}
U_c \\
I_l
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{1}{C} \\
\frac{1}{L} & -\frac{1}{L}
\end{bmatrix}
\begin{bmatrix}
U_c \\
I_l
\end{bmatrix} +
\begin{bmatrix}
0 & -\frac{1}{C} \\
\frac{1}{L} & 0
\end{bmatrix}
\begin{bmatrix}
U_i \\
I_o
\end{bmatrix}
$$

(1)

Capacitive voltage and inductive current are adopted as state variables, and the continuous state space model can be obtained as follows:

$$
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} =
\begin{bmatrix}
\dot{x} = A x + Bu \\
y = C x
\end{bmatrix}
$$

(2)

Among them: $A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L} & -\frac{1}{L} \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$; $C = [1 \ 0]$; $x = \begin{bmatrix} U_c \\
I_l
\end{bmatrix}$; $y = U_c$; $u = \begin{bmatrix} U_i \\
I_o
\end{bmatrix}$;

Thus, the transfer function between the terminal voltage of the connecting capacitor and the output voltage of the converter can be calculated as follows:

$$
G_0(s) = \frac{1}{LCs^2 + rCs + 1}
$$

(3)

3. Optimization design of resonator corrector parameters based on pole assignment

3.1 Parameter design of resonance corrector based on generalized Floating time method

Resonance corrector is a kind of controller based on resonance effect. The control loop generated by it has an infinite gain mode at a certain special frequency. At this frequency, any nonlinear or interference effect will be eliminated, so it can better solve the problem of ac control. For a second-order transfer function, the resonance corrector has the following form:

$$
C(s) = \frac{C_0 + C_1 s + C_2 s^2 + C_3 s^3}{(D_0 + D_1 s)(s^2 + \omega_0^2)}
$$

(4)
Among them, $\omega_p$ is the angular frequency of the controlled quantity, the unit is $\text{rad/s}$. $C_0$, $C_1$, $C_2$, $C_3$, $D_0$, $D_1$ is the calibrator parameter. The control loop of the system can be expressed in the form shown in Figure 2:

![System control loop](image)

Therefore, the characteristic polynomial of the closed-loop transfer function is:

$$
\Delta(s) = C_0 s^5 + C_1 s^4 + C_2 s^3 + C_3 s^2 + (D_0 + D_1 s)(s^2 + \omega_p^2)(1 + rCs + LCs^2) \tag{5}
$$

The system is a fifth order system with five closed-loop poles. Many factors such as response speed and frequency band width of the controller need to be considered when using general methods such as Byrd diagram method and root locus method to design the closed-loop controller. Using pole assignment not only simplifies the design process but also reduces the amount of manual computation, and the designed parameters can meet the requirements of the system performance index.

The method of determining the resonant corrector’s coefficient is called the generalized floating time method. The pole position of the resonant corrector’s transfer function is set by selecting the real part of the transfer function. The characteristic polynomial of the closed-loop transfer function is equivalent to the assumption of a same order number expected standard closed-loop characteristic polynomial $\Delta_p(s)$. This allows the desired dynamic response to be applied in the closed loop mode.

$$
\Delta_p(s) = (s + P + j\omega_p)(s + P - j\omega_p)(s + P + j\omega_n)(s + P - j\omega_n) \tag{6}
$$

Among them, $P$ represents the real part of the pole, $\omega_n$ is the imaginary part of the complex root of LC filter penetration rate.

The relative terms of $\Delta(s)$ and $\Delta_p(s)$ should be obtained:

$$
D_0 = \frac{(5P-r/L)}{LC} \tag{7}
$$

$$
D_1 = \frac{1}{LC} \tag{8}
$$

$$
C_0 = P^5 + P^3(\omega_p^2 + \omega_n^2) + P\omega_p^2\omega_n^2 - \omega_p^2D_0 \tag{9}
$$

$$
C_1 = 5P^4 + 3P^2(\omega_p^2 + \omega_n^2) + P^2\omega_p^2\omega_n^2 - C\omega_p^2D_0 - \omega_n^2D_1 \tag{10}
$$

$$
C_2 = 10P^3 + 3P(\omega_p^2 + \omega_n^2) - D_0 - LC\omega_p^2D_0 - r\omega_p^2D_1 \tag{11}
$$

$$
C_3 = 10P^2 + (\omega_p^2 + \omega_n^2) - D_1 - LC\omega_p^2D_1 - r\omega_p^2D_0 \tag{12}
$$

### 3.2 Parameter design of resonance corrector based on generalized Floating time method

Pole assignment range is limited. In order to avoid system instability caused by pole assignment to an inappropriate position, it is usually necessary to select a pre-set stability margin, and then comprehensively consider the inherent parameters of the system and the limiting factors of pre-set parameters, and finally determine the area scope of pole assignment.
Figure 3 shows the area scope for pole assignment. The choice of $P$ must conform to the two limiting regions shown in the figure 4: the damping limit and the limit representing the stability margin. Let the dominant pole of the closed loop of the system be $\lambda = P + \omega$. $P$ is the value of the real part of the pole, which determines the stable margin, $\omega$ is the imaginary part of the pole, from which the damping ratio of the system can be calculated as:

$$\xi = -\frac{p}{\sqrt{p^2 + \omega^2}} = \cos \theta$$  

(13)

where $\theta$ is the inverse tangent of the ratio between the imaginary part and the real part of the pole. The value of its damping ratio was generally not less than 0.1~0.3 in system design [5]. Therefore, the minimum value of $\xi$ in the following analysis is $\xi_{\min} = 0.3$.

Thus, the scope of pole assignment can be determined:

$$\theta = \arccos \xi_{\min} = \pm 72.54^\circ$$  

(14)

The included angle between the damping limit boundary and the real axis is $\theta$. In order to meet the damping limit requirements of the system, the poles should be arranged between the two damping limit boundaries. According to Equation (14), it can be deduced that:

$$P_{\text{max}} = -\frac{\xi_{\min}}{\sqrt{1-\xi_{\min}^2}}|\omega| \approx -0.31|\omega|$$  

(15)

Under the requirement of system stability margin, the real part of the dominant pole is negative and away from the imaginary axis, so the poles should not be placed on the right side of the limit boundary of system stability margin, that is, in the limit area shown in the shadow in the figure 4, otherwise the system will be unstable.

From this, it can be determined that the ideal system pole assignment region is to the left of the ABCD broken line as shown in Figure 4. In this region, $P_{\text{max}}$ guarantees the stability margin of the system, and poles larger than $P_{\text{max}}$ will make the system unstable. $\theta$ ensures the minimum damping ratio of the system, and the poles in the sector range of $\angle AOD$ can meet the damping ratio requirements of the system.

In engineering practice, the dynamic performance of higher-order systems is more complex to determine, and the second-order systems are usually used to estimate the dynamic performance of higher-order systems[6]. Therefore, when the poles are assigned, the higher-order systems are adjusted to have a pair of closed-loop dominant poles, that is, the two freely assigned poles are assigned as the closed-loop dominant poles of the system. In order to reduce the influence of non-dominant poles on the stability of the system, the non-dominant poles are required to be far away from the dominant poles, so as to achieve the optimal control effect of the system.
4. Simulation Analysis

In this paper, the quasi-PR controller [7] and the resonant corrector based on pole assignment are respectively used as single-phase inverter controllers to conduct Simulink simulation comparison. The basic parameters of the simulation model are as follows: the DC input voltage of the inverter is 600V and the amplitude varies periodically between 300V~900V; the amplitude of the reference output voltage is 200V; the frequency of the reference output voltage is 50Hz; the inductance of the filter is 2mH.; and the filter capacitance is 20μF.

According to the design idea of quasi-PR control parameters in this paper, the optimization parameters of the controller are determined as $K_p = 4$, $K_r = 100$, $\omega_c = 2$. According to the optimal design idea of resonant corrector parameters based on pole assignment in this paper, it can be calculated $\theta = \pm 72.54^\circ$, $P_{\text{max}} = -1550$, $P_{\text{min}} = -6200$. Considering the damping ratio $\xi$ and the influence on the system, $P_{\text{min}} = -3000$ is selected as the more appropriate pole position. And $\omega_p = 314\text{rad/s}$, $\omega_n = 5000\text{rad/s}$. By substituting the value of the pole $P$ into equations (7) - (12), the resonant corrector parameters $C_0$, $C_1$, $C_2$, $C_3$, $D_0$, $D_1$ can be obtained.

In view of the changes of DC input voltage and load, the simulation under the following two situations is considered:

4.1 DC voltage input constant, under the case of resistive load, loading and unloading load

The simulation time is 0.3s, pure impedance load resistance is 5Ω, at the time of 0.2s 100Ω dump load. In this case, the quasi-PR controller after parameter optimization and the resonance corrector based on pole assignment were respectively used as the single-phase inverter controller for simulation, and the results were shown in Figure 5-7.

![Figure 4. Comparison of output waveforms during load loading and unloading](image)

Figure 4 shows the waveform comparison of output voltage and current when loading and unloading load abruptly under the two control modes. (a) is output waveform of quasi-PR controller, (b) is output waveform of resonant corrector based on pole assignment. In general, both control methods can effectively track the reference voltage and make rapid changes and adjustments when loading and unloading load. However, it can be seen from the figure 5 that the waveform dynamic performance and stability are poor when quasi-PR control is adopted, and the harmonic distortion rate of output voltage is 2.36%. The harmonic distortion rate of the output voltage is 0.58% when the resonant corrector based on pole assignment is used.

The following will be combined with the simulation waveform for specific analysis of the details.
Figure 5. Comparison of output waveforms during load loading and unloading

Figure 5 shows the comparison of the dynamic performance of output voltage and current under the two control modes when loading and unloading load abruptly. (a) is output waveform of quasi-PR controller, (b) is output waveform of resonant corrector based on pole assignment. It can be seen clearly from the picture, when using quasi PR control, in the early stage of the inverter to work in the output voltage waveform after four cycles of dynamic adjustment process is gradually stabilized, and the use of resonant corrector control based on pole assignment, the output voltage stability tracking quickly, fast dynamic response.

Figure 6. Stability comparison of output voltage waveform under load loading and unloading

Figure 6 shows the stability comparison of transmission voltage waveform under load loading and unloading under two control modes. (a) is output waveform of quasi-PR controller, (b) is output waveform of resonant corrector based on pole assignment. When the quasi-PR controller is used, the steady-state output voltage fluctuation still reaches about 14V. When using the resonant corrector control based on pole assignment, the steady-state output voltage fluctuation is only about 2.8V. Therefore, adopting the resonant corrector based on pole assignment can make the system have better stability and avoid the adverse effect of large voltage fluctuation on load work.

4.2 The input voltage at the DC end changes and the resistance value of the resistive load remains unchanged

In pure impedance load resistance for 5Ω cases, the Chirp signal is used to generate a DC voltage input whose amplitude varies periodically between 300V ~ 900V, and the frequency of change gradually increases, the simulation results as shown in figure 8.
Figure 7 shows the comparison of the output waveforms under the two control modes when the DC input changes. (a) is output waveform of quasi-PR controller, (b) is output waveform of resonant corrector based on pole assignment. It can be clearly seen that the output current and voltage amplitude are not stable when quasi-PR control is adopted, and the output voltage amplitude always has errors compared with the reference voltage amplitude, and there is a certain phase error, which makes it impossible to realize the effective tracking of the reference signal. The harmonic distortion rate of the output voltage is 2.52%. When using the resonant corrector control based on pole assignment, the output voltage and current can be stable, and the effective tracking of the reference signal can be realized well. The harmonic distortion rate of the output voltage is 0.47%, and it has good dynamic performance and robustness.

To sum up, the system has excellent output characteristics when using the resonant corrector based on pole assignment to control the single-phase inverter.

5. Conclusion
Based on single phase inverter as the research object, this paper designs the resonant corrector control strategy based on pole assignment, compared with traditional control strategy of quasi PR. In this paper, the realization process of parameter optimization design of resonant corrector using pole assignment method is derived in detail, and the MATLAB/Simulink simulation model is built to verify the effectiveness and feasibility of the proposed control strategy. The simulation results show that the control strategy can significantly improve the output voltage dynamic response speed and steady state performance when the load is suddenly changed and the DC input is continuously fluctuated in a large range, and the system has good robustness and resistance to load disturbance.

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