Electromagnetic dipole moments and radiative decays of particles from exchange of fermionic unparticles

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Abstract

We construct the propagator for a free fermionic unparticle field from basic considerations of scale and Lorentz invariance. The propagator is fixed up to a normalization factor which is required to recover the result of a free massless fermion field in the canonical limit of the scaling dimension. Two new features appear compared to the bosonic case. The propagator contains both $\gamma$ and non-$\gamma$ terms, and there is a relative phase of $\pi/2$ between the two in the time-like regime for arbitrary scaling dimension. This should result in additional interference effects on top of the one known in the bosonic case. The non-$\gamma$ term can mediate chirality flipped transitions that are not suppressed by a light fermion mass but are enhanced by a large bosonic mass in loops, compared to the pure particle case. We employ this last feature to set stringent bounds on the Yukawa couplings between a fermionic unparticle and an ordinary fermion through electromagnetic dipole moments and radiative decays of light fermions.

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Recently, Georgi suggested [1] that a high energy scale theory with a nontrivial infrared fixed point may manifest itself at low energies as an effective degree of freedom that is scale invariant. Since such an identity cannot be a particle of definite mass, he termed it unparticle. The suggestion has stimulated in the past year intense activities, exploring various aspects of unparticles interacting with standard model particles. Most studies, with very few exceptions [2, 3, 4, 5, 6], have dealt with unparticles of integer spin. This is perhaps partly because such unparticles can more easily couple as a standard model singlet to particles and partly because their propagators as a necessary ingredient for any physics analysis are known at the very start [7, 8, 9]. On the other hand, since unparticles must interact with particles to be physically relevant, it should not be surprising that they could carry standard model charges. There have been indeed interesting attempts in this direction very recently [10, 11, 12, 13, 14], though delicacies arise with gauge interactions of unparticles due to the highly nonlocal nature of the latter [11, 15]. In such a circumstance, it is no more difficult to couple fermionic unparticles to particles. Furthermore, as it is to be shown in this brief note, the propagator for a fermionic unparticle can also be constructed from basic considerations [3, 5]. Though simple, the propagator brings in new features not seen in its bosonic counterparts. It is also a purpose of this note to investigate some of their phenomenological implications as exemplified in chirality flipped processes.

The first attempt for a fermionic unparticle was made by Luo and Zhu [2]. They treat its propagator as a Green’s function of effective operators and parameterize it in terms of unknown constants through considerations of scale invariance and Lorentz symmetry. There are two points concerning that treatment. First, the propagator of a massless fermion should be recovered as the unparticle’s scaling dimension goes to the canonical limit [3, 2, 4]. Second, the unparticle field is free when its interactions with particles are ignored at the leading order in the low energy effective theory. Therefore, its propagator should not contain unknown parameters just like its bosonic counterparts except that the scaling dimension is fixed from a high energy theory and that the absolute normalization is immaterial [3, 5]. As we shall show below, the propagator can indeed be constructed for a free fermionic unparticle field through a similar procedure as for a canonical fermion field.

Similar to the canonical case, a free fermionic unparticle field as a function in spacetime can be Fourier decomposed as [3]:

\[
\mathcal{U}(x) = \int \frac{d^4 p}{(2\pi)^4} \Xi_d(p) \sum_s \left( a^s_p \bar{u}^s(p) e^{-i p \cdot x} + b^{s\dagger}_p \bar{v}^s(p) e^{i p \cdot x} \right), \\
\Xi_d(p) = F(d) \theta(p^0) \theta(p^2) (p^2)^{d - \frac{5}{2}},
\]

(1)

(2)

where the scaling property for an unparticle field of dimension \( d \) has been taken into account. \( F(d) \) is a normalization factor appropriate for a fermionic \( \mathcal{U} \). The operators \( a^s_p, b^s_p \) of dimension \( (\frac{1}{2} - d) \) are assumed to satisfy the anti-commutation relations, e.g.,

\[
\Xi_d(p) \Xi_d(q) \{ a^s_p, a^{s\dagger}_q \} = \Xi_d(p) (2\pi)^4 \delta^4(p - q) \delta^{rs},
\]

(3)
with the normalized single-unparticle states, e.g.,
\[ |p, s\rangle = \Xi_d(p)\alpha_p^{rs}|0\rangle, \]  
\[ \langle p, q, s| p, r, s\rangle = \Xi_d(p)(2\pi)^4\delta^4(p-q)\delta^{rs}. \]

\[ U^s(p), V^s(p) \] are spinor wavefunctions of dimension \( \frac{d}{2} \) to be built below.

The above decomposition would be completely similar to that for a complex scalar unparticle field [3] (for a discussion of the scalar case, see also [16]) in the absence of \( U^s(p), V^s(p) \). The other factors each carry a dimension that has the symmetry, \( (d - \frac{1}{2}) \leftrightarrow d \), with their scalar counterparts. This suggests a natural choice for the normalization factor, \( F(d) = B(d - \frac{1}{2}) \), where \( B(d) \), usually denoted as \( A_d \), is the original normalization chosen by analogy in Ref. [1]. This choice guarantees automatically that the massless canonical propagator is recovered as \( d \rightarrow \frac{3}{2} \).

We should emphasize that except for this welcome property, normalization is arbitrary. While it affects apparent couplings, it does not modify for example the cross section of a physical process. This is obvious for unparticles appearing as an intermediate state where the normalization convention cancels between couplings and propagators, or in the final state where the cancelation occurs for a squared single vertex and phase space. For unparticles appearing in the initial state, the latter still holds true [17]: their flux or density is convention dependent.

Although an unparticle has no dispersion relation, the standard construction for a spinor wavefunction still works by the definition of its Lorentz properties [5]. For a physical unparticle with \( p^2 > 0 \), we go to its rest frame, where \( \sqrt{p^2} \) plays the role of mass for a canonical fermion. A Lorentz boost then gives the wavefunction for a general \( p \) but with the same \( p^2 \) as in the rest frame of course. The \( U(p) \) thus constructed satisfies the equations
\[ (\hat{p} - \sqrt{p^2})U(p) = 0, \quad \sum_{\text{spin}} U(p)\bar{U}(p) = \hat{p} + \sqrt{p^2}, \]
and similarly for \( V(p) \). Note that the Klein-Gordon equation becomes a trivial identity due to the lack of dispersion relation.

We are now ready to compute the propagator from its definition:
\[ \tilde{S}^U_{F\alpha\beta}(x-y) = \langle 0| T\bar{U}_\alpha(x)U_\beta(y)|0\rangle, \]
where, using the relations elaborated so far,
\[ \langle 0| U_\alpha(x)\bar{U}_\beta(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4} \Xi_d(p)e^{-ip(x-y)}(\hat{p} + \sqrt{p^2})_{\alpha\beta} \]
\[ = +i(\not{p})_{\alpha\beta}\int \frac{d^4p}{(2\pi)^4} \Xi_d(p)e^{-ip(x-y)} \]
\[ +\delta_{\alpha\beta}\int \frac{d^4p}{(2\pi)^4} \Xi_d(p)(p^2)^{\frac{1}{2}}e^{-ip(x-y)}. \]

Similarly,
\[ \langle 0| \bar{U}_\beta(y)U_\alpha(x)|0\rangle = -i(\not{p})_{\alpha\beta}\int \frac{d^4p}{(2\pi)^4} \Xi_d(p)e^{ip(x-y)} \]
\[ -\delta_{\alpha\beta}\int \frac{d^4p}{(2\pi)^4} \Xi_d(p)(p^2)^{\frac{1}{2}}e^{ip(x-y)}. \]
The non-$\gamma$ term of the propagator is worked out in detail in Appendix to be,

$$\delta_{\alpha\beta} \frac{F(d)}{2\sin(d\pi)} \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{(-p^2 - i\epsilon)^{2-d}},$$

(10)

which is the same as the propagator for a scalar unparticle except for the factors $F(d)$ vs $B(d)$. The $\gamma$ term is

$$i(\partial^\alpha)_{\alpha\beta} \int \frac{d^4p}{(2\pi)^4} \Xi_d(p) \left[ \theta(x_0 - y_0)e^{-ip(x-y)} + \theta(y_0 - x_0)e^{ip(x-y)} \right],$$

(11)

where the terms resulting from moving the derivatives across the step functions sum to zero. The integral is again of the same form as for a scalar field, which can be obtained from our previous result by shifting $d \rightarrow d - \frac{1}{2}$ in all factors except $F(d)$. Acting the derivative inside yields the term,

$$\frac{F(d)}{2\sin[(d - \frac{1}{2})\pi]} \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \frac{i\dot{p}}{(-p^2 - i\epsilon)^{2-d}}.$$

(12)

To summarize, the propagator is

$$\tilde{S}_F(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} S_F^\gamma(p),$$

(13)

$$S_F^\gamma(p) = \frac{iF(d)}{2\sin(d\pi)} \left[ 1 \frac{1}{(-p^2 - i\epsilon)^{2-d}} - \tan(d\pi) \frac{\dot{p}}{(-p^2 - i\epsilon)^{\frac{d}{2} - d}} \right].$$

(14)

Using the convention,

$$F(d) = B(d - 1/2) = \frac{16\pi^{\frac{d}{2}}}{(2\pi)^{2d-1}} \frac{\Gamma(d)}{\Gamma(d - \frac{1}{2})\Gamma(2d - 1)},$$

(15)

it can be checked that $S_F^\gamma$ recovers the standard result for a massless fermion as $d \rightarrow \frac{3}{2}$. The discontinuity in $S_F^\gamma(p)$ across the cut for $p^2 > 0$ is found to be, using $(-|r| \pm i\epsilon)^\alpha = |r|^\alpha e^{\mp i\pi\alpha}$,

$$F(d)(p^2)^{d-\frac{5}{2}}(\dot{\phi} + \sqrt{p^2}),$$

(16)

exactly as expected.

Two new features are worth mention. It is well known that the bosonic unparticle propagator develops an imaginary part in the time-like regime for non-integral dimension $d$. This has been shown to cause interesting interference between unparticle and particle contributions to a physical process [7]. In addition to this, we find that in the fermionic case the two terms in the propagator has a relative phase of $\pi/2$ in the time-like regime for any $d$. This can result in even more interesting phenomena. Furthermore, the mass term in an ordinary fermionic propagator is now replaced by a momentum dependent one free of the $\gamma$ matrices. It is known in the case of particles that the mass of an intermediate fermion can flip the chirality of its connected
When the latter are light, the contribution of such a term is naturally suppressed either directly by a light intermediate fermion, or indirectly by the small mixing between light and heavy fermions in the opposite case. In the present case where a fermionic unparticle appears in the intermediate state, this non-\(\gamma\) term is generally not suppressed since its momentum can be typically of the order of a heavy particle mass involved in the intermediate state. In the second part of this work, we explore its implications on chirality flipped processes of light fermions, namely the electromagnetic transitions induced by interactions with fermionic unparticles.

Consider the effective interaction,

\[
\mathcal{L}_\text{int}^{\mathcal{U}} = \Lambda_{\mathcal{U}}^{3-d} \mathcal{U} (a_j + b_j \gamma_5) \psi_j \varphi + \Lambda_{\mathcal{U}}^{3-d} \bar{\psi}_j (a_j^* - b_j^* \gamma_5) \mathcal{U} \varphi^+, \tag{17}
\]

where \(\varphi, \psi_j\) are the ordinary scalar and fermion fields with mass \(m\) and \(m_j\) respectively. \(\Lambda_{\mathcal{U}}\) characterizes the energy scale of unparticle physics, and \(a_j, b_j\) are unknown pure numbers. We have deliberately chosen \(\mathcal{U}\) to be electrically neutral to avoid its gauge interactions as the effect on chirality flip can be easily seen via the above interaction together with ordinary QED. Then, in units of \(e > 0\), the electric charges are, \(Q(\varphi) = -Q(\psi_j) \equiv -Q_j\).

The interaction induces the flavor-changing, chirality-flipped transition, \(\psi_1 \rightarrow \psi_2 \gamma\), via the graph shown in Fig. 1, where the arrowed double line indices the \(\mathcal{U}\) field. The amplitude is,

\[
i\mathcal{A}_\mu = \frac{Q_1 e \Lambda_{\mathcal{U}}^{3-2d} F(d)}{2 \sin(d \pi)} \int \frac{d^4k}{(2\pi)^4} \frac{(2k + q)_{\mu}(a_2^* - b_2^* \gamma_5)}{[k^2 - m^2][(k + q)^2 - m^2]} \times \frac{1}{[-(k + p)^2 - i\epsilon]^{2-d}} - \tan(d \pi) \frac{k + \hat{p}}{[-(k + p)^2 - i\epsilon]^{\frac{5}{2} - d}} (a_1 + b_1 \gamma_5). \tag{18}\]

Keeping only terms that contribute to the on-shell amplitude yields in the limit \(m \gg m_{1,2}\),

\[
\mathcal{A}_\mu = -\frac{1}{(4\pi)^2} \frac{Q_1 e \Lambda_{\mathcal{U}}^{3-2d} F(d)}{2 \sin(d \pi)} m^{2(d-2)} B_\mu, \tag{19}\]

\[
B_\mu = \tan(d \pi) \Gamma(d + 1/2) \Gamma(7/2 - d) \frac{p_\mu (q - 2\hat{p})}{6m} \times [ (a_1 a_2^* + b_1 b_2^*) + (a_2^* b_1 + a_1 b_2^*) \gamma_5 ]
+ \Gamma(d) \Gamma(3 - d) p_\mu [ (a_1 a_2^* - b_1 b_2^*) + (a_2^* b_1 - a_1 b_2^*) \gamma_5 ]. \tag{20}\]
When sandwiched between the external spinors, it becomes the standard dipole form,

\[ \mathcal{B}_\mu = \tan(d\pi) \frac{1}{6m} \Gamma(d + 1/2) \Gamma(7/2 - d) \times \left[ -\hat{m}\left( a_1 a_2^* + b_1 b_2^* \right) i\sigma_{\mu\nu} q^\nu + \Delta m(a_2^* b_1 + a_1 b_2^*) i\sigma_{\mu\nu} q^\nu \gamma_5 \right] + \Gamma(d) \Gamma(3 - d) \frac{1}{2} \left[ (a_1 a_2^* - b_1 b_2^*) i\sigma_{\mu\nu} q^\nu + (a_2^* b_1 - a_1 b_2^*) i\sigma_{\mu\nu} q^\nu \gamma_5 \right], \]  

(21)

where \( \hat{m} = (m_1 + m_2)/2 \), \( \Delta m = (m_1 - m_2)/2 \).

For \( \psi_1 = \psi_2 \), the above amplitude gives directly the electromagnetic dipole moments. The anomalous magnetic moment (coefficient of \( -ei\sigma_{\mu\nu} q^\nu/(2m_1) \) in \( \mathcal{B}_\mu \)) is,

\[ a_{\psi_1} = \frac{1}{2}(g_{\psi_1} - 2) = Q_1 \frac{2m_1}{(4\pi)^2} \frac{\Lambda_{\psi}^{3-2d} F(d)}{2\sin(d\pi)} m^{2(d-2)} \left[ \frac{1}{2} \Gamma(d) \Gamma(3 - d)(|a_1|^2 - |b_1|^2) - \tan(d\pi) \frac{m_1}{6m} \Gamma(d + 1) \Gamma \left( \frac{7}{2} - d \right) \right], \]

(22)

while the coefficient of \( -\sigma_{\mu\nu} q^\nu \gamma_5 \) (corresponding to \( \mathcal{L}^{\text{EDM}} = -\frac{1}{2} d \psi_i \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \)) gives the electric dipole moment,

\[ d_{\psi_1} = -Q_1 \frac{1}{(4\pi)^2} \frac{e\Lambda_{\psi}^{3-2d} F(d)}{2\sin(d\pi)} m^{2(d-2)} \Gamma(d) \Gamma(3 - d) \Im(a_1^* b_1). \]

(23)

It is evident from the above results that the non-\( \gamma \) term in the unparticle propagator enhances the amplitude by a factor of \( m/m_j \) which is large for transitions of light fermions. This occurs because the chirality flip implemented usually by a mass in the pure particle case is now operated by a momentum, which in loops can be of order the mass scale of heavy particles; in short, an internal fermion mass is effectively traded for a large boson mass for chirality flip. For better appreciation of this effect, we record here the dipole moments of \( \psi \),

\[ a_{\psi} = -Q_{\psi} \frac{m_\psi}{(4\pi)^2} m \left[ m_\chi(|a|^2 - |b|^2) + \frac{1}{3} m_\psi(|a|^2 + |b|^2) \right], \]

\[ d_{\psi} = Q_{\psi} \frac{m_\psi}{(4\pi)^2} m \Im(a^* b), \]

(24)

due to pure particle interactions,

\[ \mathcal{L}^\chi_{\text{dir}} = \bar{\chi}(a + b\gamma_5) \psi \varphi + \bar{\psi}(a^* - b^* \gamma_5) \chi \varphi^\dagger, \]

(25)

with \( Q(\varphi) = -Q(\psi) \equiv -Q_{\psi}, Q(\chi) = 0 \). It is assumed again that \( m \gg m_{\psi,\chi} \). Note that the \( m_{\psi} \) suppressed term in \( a_{\psi} \) can be obtained from the corresponding term in the unparticle result in the limit \( d \to \frac{3}{2} \), while the \( m_\chi \) suppressed terms in \( a_{\psi} \) and \( d_{\psi} \) are replaced in the unparticle case by the enhanced ones.
We can obtain some stringent constraints on the couplings appearing in (17). Since the non-$\gamma$ term overwhelmingly dominates, we only retain its contribution in (21, 22, 23) for our numerical analysis. We start with the dipole moments:

$$a_{\psi_1} = Q_1 f(d) \left( \frac{m}{\Lambda_{\psi}} \right)^{2d-3} \frac{m_1}{m} (|a_1|^2 - |b_1|^2),$$  

$$\frac{d_{\psi_1}}{e} = -Q_1 f(d) \left( \frac{m}{\Lambda_{\psi}} \right)^{2d-3} \frac{1}{m} \Re(a_1^* b_1),$$

where

$$f(d) = \frac{1}{2^{2d} \pi^{2d-\frac{3}{2}}} \frac{[\Gamma(d)]^2 \Gamma(3-d)}{\sin(d\pi) \Gamma(d-\frac{3}{2}) \Gamma(2d-1)}.$$  

The current potential discrepancies between experiments and standard model expectations for leptons’ magnetic moments are, $\delta a_i = a_i^{\text{expt}} - a_i^{\text{SM}}$,

$$|\delta a_e| < 15 \times 10^{-12} \quad |\delta a_\mu| = 22(10) \times 10^{-10}.$$  

Here, unless otherwise stated, we use the numbers of the Particle Data Group, version 2006. We assume the gap is filled by the unparticle contribution. The most precise experimental results for the electric dipole moments are those of the electron, muon and neutron:

$$d_e = (0.07 \pm 0.07) \times 10^{-26} \text{ e cm},$$

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm},$$

$$d_n < 0.63 \times 10^{-25} \text{ e cm}.$$  

Since the standard model contributions to the above are ignorable, we assume that the unparticle saturates the central values or the upper bound. For the purpose of illustration, we take $\Lambda_{\psi} = 1 \text{ TeV}$, $m = 200 \text{ GeV}$, and $d \in (1.5, 2.0)$. The constraints on the couplings are shown in table 1. We have assumed that the neutron electric dipole is dominated by those of the $u$, $d$ quarks, and ignore factors of $2/3$, $1/3$ in forming the neutron’s moment from those of quarks. It is evident that the constraints are rather stringent for the most precisely measured quantities, i.e., $d_e, d_n$. We remind that in the case of pure particle interactions the Yukawa couplings’ contribution to dipole moments is completely ignorable.

Now we move to the flavor changing electromagnetic transitions. The dominant term for $\psi_1 \rightarrow \psi_2 \gamma$ is,

$$s_{\mu} = \frac{f(d)Q_1 e}{2m} \left( \frac{m}{\Lambda_{\psi}} \right)^{2d-3} \left[ (a_1 a_2^* - b_1 b_2^*) i \sigma_{\mu\nu} q^\nu + (a_2^* b_1 - a_1 b_2^*) i \sigma_{\mu\nu} q^\nu \gamma_5 \right],$$

yielding the rate,

$$\Gamma(\psi_1 \rightarrow \psi_2 \gamma) = 2^{-7} m_1 q \left[ f(d)Q_1 \left( \frac{m}{\Lambda_{\psi}} \right)^{2d-3} \frac{m_1}{m} \right]^2 X_{12},$$
where \( m_1 \gg m_2 \) is assumed in kinematics, and
\[
X_{ij} = |a_i a_j^* - b_i b_j^*|^2 + |a_j^* b_i - a_i b_j^*|^2. \tag{33}
\]

The most stringent experimental bounds are those on lepton flavor changing radiative decays:
\[
\begin{align*}
\text{Br}(\mu \to e\gamma) &< 1.2 \times 10^{-11} \quad [19], \\
\text{Br}(\tau \to \mu\gamma) &< 4.5 \times 10^{-8} \quad [20], \quad \text{Br}(\tau \to e\gamma) < 1.2 \times 10^{-7} \quad [20]. \tag{34}
\end{align*}
\]

Again we assume that these transitions are dominated by the fermionic unparticles. This produces the bounds on various \( X_{ij} \) shown in table 2. Note that \( \text{Br}(\tau \to \nu\tau\ell\bar{\nu}_\ell) \sim 17\% \) (\( \ell = e, \mu \)) has been taken into account. The bound on \( X_{\mu e} \) is enhanced not only by a more precise measurement but also by a less power dependence of the fermion mass in the transition amplitude compared to the particle case. The bound on \( X_{bs} \) from the decay \( b \to s\gamma \) is less stringent.

We have worked out the propagator of a free fermionic unparticle from basic considerations of scale invariance and Lorentz symmetry. It has a correct particle limit as its scaling dimension goes to the canonical limit. The propagator has a \( \gamma \) dependent term as naively expected, and a momentum-dependent non-\( \gamma \) term that would correspond to the mass term in the particle case. There is a \textit{relative} phase between the two terms in the time-like regime, which can result in interesting interference phenomenon on top of the one known already for a bosonic unparticle. We pointed out that the non-\( \gamma \) term can cause chirality flip that is not suppressed by a fermion mass, in contrast to the case of particles. Instead, when appearing in loops, the term is traded for a large mass of virtual bosonic particles, and can thus be dangerous for precisely measured chirality-flipped quantities, like electromagnetic dipole moments and radiative decays of light fermions. We have employed this to set stringent bounds on the mixing Yukawa couplings between fermionic unparticles and light particles. For the diagonal combinations of couplings, the

| d  | 1.6 | 1.7 | 1.8 | 1.9 |
|----|-----|-----|-----|-----|
| \( 10^4(|a|^2 - |b|^2)_{e} \) | 7.3 | 6.5 | 6.5 | 5.4 |
| \( 10^4(|a|^2 - |b|^2)_{\mu} \) | 5.2 | 4.6 | 4.6 | 3.8 |
| \(-10^5\Im (a^* b)_{e} \) | 8.9 | 7.8 | 7.9 | 6.5 |
| \(-3\Im (a^* b)_{\mu} \) | 4.7 | 4.1 | 4.2 | 3.4 |
| \(-10^7\Im (a^* b)_{n} \) | 8.0 | 7.0 | 7.1 | 5.9 |

Table 1: Couplings needed to saturate the deviations in \( a_j \) or the measured results for \( d_j \).

| d  | 1.6 | 1.7 | 1.8 | 1.9 |
|----|-----|-----|-----|-----|
| \( 10^{14}X_{\mu e} \) | 3.3 | 2.6 | 2.7 | 1.8 |
| \( 10^5X_{\tau e} \) | 5.3 | 4.1 | 4.2 | 2.9 |
| \( 10^5X_{\tau\mu} \) | 2.0 | 1.5 | 1.6 | 1.1 |

Table 2: Bounds on the combination of couplings \( X_{ij} \). Same input parameters as in table 1.
anomalous magnetic moments of the electron and muon give similar constraints on their magnitude, while the electron’s electric dipole moment yields the best constraint on the imaginary part. For the non-diagonal combinations, the radiative decay of the muon sets a limit that is several orders of magnitude stronger than the tau decays.

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Note added. As our phenomenological analysis was going on, we received a preprint [21] in which the propagator of a fermionic unparticle was worked out with the help of a spectral function [7]. The derivation is different but the result is the same as obtained here, up to a different convention for the normalization that also has the correct canonical limit. The main concern of that work is with gauge interactions of unparticles and their contribution to the $\beta$ function, following the work in Refs. [10, 11].

Appendix Derivation of equation (10)

We derive the equation for completeness. Using

$$\theta(\eta) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{ds}{s + i\epsilon} e^{is\eta}, \tag{35}$$

and making appropriate changes of parameters, the first term becomes

$$\delta_{\alpha\beta} F(d) \frac{i}{2\pi} \int \frac{d^4k}{(2\pi)^4} \int_{|k|}^{\infty} \frac{dt}{(k_0 + i\epsilon) - t} (t^2 - k^2)^{d-2} e^{-ik \cdot \vec{x}}. \tag{36}$$

The second term only differs in the sign of the exponent. They sum to

$$-\delta_{\alpha\beta} F(d) \frac{i}{2\pi} \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot \vec{x}} \int_0^{\infty} d\rho \frac{\rho^{d-2}}{\rho - k^2 - i\epsilon}. \tag{37}$$

Using Cauchy theorem, the $\rho$ integral can be evaluated along the ray starting from the origin and passing the point $\rho = -\rho_0 = -(k^2 + i\epsilon)$. The term becomes then

$$-\delta_{\alpha\beta} F(d) \frac{1}{2\pi} \Gamma(2 - d) \Gamma(d - 1) \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot \vec{x}} \frac{i}{(-k^2 - i\epsilon)^2 - d}, \tag{38}$$

which is (10) upon using the relation

$$-\frac{1}{2\pi} \Gamma(2 - d) \Gamma(d - 1) = \frac{1}{2\sin(d\pi)}. \tag{39}$$

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