Novel Teaching-learning-based Optimization Algorithm for Design of Digital Filters

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Abstract. An improved teaching-learning-based optimization algorithm named NTLBO is proposed for IIR digital design in this paper. Conventional mathematical methods have failed when reduced order adaptive models were used for the purposes of identification of problems. NTLBO utilizes a multi-learning strategy and opposition learning to overcome this disadvantage of the basic TLBO. The quasi-opposition-based learning has been applied to increase the diversity of solutions and broaden the search space to improve the global search ability. The multi-learning strategy makes local search more effective so as to speed up the convergence. To make a tradeoff between exploration and exploitation properly, the teaching factor is redesigned to increase the likelihood of solutions jumping out of local optima. Experiments are carried out on the classical examples and comparisons are made as well. The results indicate that the NTLBO algorithm achieved preferable performance in both reduced and same order models of IIR digital filters.

Keywords: digital filters; Teaching-Learning-Based Optimization algorithm; random multi-learning strategy; quasi-opposition learning.

1. Introduction

Design of digital filter has become a hot issue for many researchers due to the popularization of digital system and as well as due to the demand for date processing. The digital filter consists of two basic filters, namely Finite Impulse Response (FIR) filters and Infinite Impulse Response (IIR) filters. IIR filter has higher precision, lower system delay, fewer number of calculations per time step etc. It has been used to solve several problems including communication systems, image processing, signal processing, biomedical imaging, system identification and modeling, and seismic survey[1] etc. These methods can be divided into two classes: the conventional design methods and heuristic optimization methods. One design analog filters first and then utilizing the bilinear transformation method to transform them into digital filter, which is very simple, but sometimes the result is not ideal and usually trapped into the local optima. In order to offset the defect, several meta-heuristic optimization algorithms are introduced to design the IIR filter[2-7]. However, these algorithms are easily getting trapped into local optima and have slow convergence and stagnation. Teaching-Learning-Based Optimization algorithm (TLBO)[8-9] with different strategies is introduced to overcome all these problems. TLBO mimics the teaching-learning process and through the interaction among the students in one class to improve the performance of all, which is not only has characteristics of parameter-free, fast convergence and simple computation, but also its capacity to global search is better than other algorithms. However, this algorithm was easily trapped into...
minimum or backwater status. A novel TLBO called NTLBO is proposed to identify the IIR filter. Multi-learning strategy and quasi-opposition-based learning are utilized for adaptive IIR filtering applications.

Section 2 expounds the identification problem clearly. A brief overview of TLBO is presented in Section 3. The novel TLBO (NTLBO) algorithm is given in Section 4. Section 5 provides the experimental settings and analyses the results in detail. Finally, in Section 6, the conclusions and the future direction of research are offered.

2. Overview of Filter Model

The input-output relationship of IIR digital filters is given as follows:

\[ a_0 y(p) + \sum_{k=1}^{v} a_k y(p-k) = \sum_{k=0}^{u} b_k x(p-k) \]  
(1)

where \(x(p)\) is the input of the filters and \(y(p)\) is the output of the filters; \(v\) is the order of the filter. \(a_0=1\), the transfer function is expressed as:

\[ H(z) = \frac{\sum_{k=0}^{u} b_k z^{-k}}{1 + \sum_{k=1}^{v} a_k z^{-k}} \]  
(2)

In fact, the design of IIR filter can be viewed as an identification problem of system coefficients. Figure 1 shows the design of IIR digital filter.

\[ MSE = J(\omega) = E[e^2(p)] \approx \frac{1}{N} \sum_{p=1}^{N} e^2(p) \]  
(3)

\(MSE(dB)=10\log_{10}(J)\) in dB. The error signal can be calculated as \(e(p)=d(p)-y(p)\). \(d(p)\) is the responses of the unknown plant and \(y(p)\) for IIR filter. \(N\) samples are used for the calculation of the objective function. Here, \(w\) is the filter coefficient vector: \(w=[a_0, a_1, \cdots, a_v, b_0, b_1, \cdots, b_u]\).

3. The Basic TLBO Algorithm

TLBO[8] was proposed in 2011 by the Indian scholars Rao et al. It simulates the process of the teaching and learning in daily life and has teaching phase and learning phase. During teaching, class students learn from the teacher and students with the average values to improve their performance; in the learning stage, students communicate with other more knowledgeable students, which improve their performance accordingly.

3.1. Teaching Phase

During teaching, a teacher is selected from the best learners in the class to improve the average score from the initial level to his own level depending on their capabilities, which effectively incorporates
the influence of the population mean. The teacher tries his or her best to share his or her knowledge with the students, which will not only increase the knowledge level, but also enhance the students’ results. However, average result of the students is usually not up to the level of the teacher and only reaches some extent in terms of the capability of the class. The mathematic expression of the teaching process as below.

\[
X_{\text{new}} = X_{\text{old}} + \text{rand} \cdot (X_{\text{teacher}} - \text{Mean})
\]

(4)

Where, \(X_{\text{teacher}}\) represents the best learner in current population. \(\text{rand}\) is a uniform random number between 0 and 1. \(X_{\text{new}}\) is updated value of \(X_{\text{old}}\). The teaching factor \(T_F\) is gotten as \(T_F = \text{round}(1+\text{rand})\), \(\text{Mean}\) is the average result of the class.

3.2. Learning Phase

Learning large amounts of knowledge from the teacher is not only one way of learning knowledge. In order to gain more knowledge, the students learn something new from their classmates who have more profound knowledge than through interaction among themselves. The following expressions are for minimization problem. The arithmetic expression of the phase is

\[
X_{\text{new}} = \begin{cases} 
X_{\text{old}} + \text{rand} \cdot (X^i - X_{\text{old}}), & f(X^i) < f(X_{\text{old}}) \\
X_{\text{old}} + \text{rand} \cdot (X_{\text{new}} - X_{\text{old}}), & \text{otherwise}
\end{cases}
\]

(5)

Where \(X^i\) and \(X^2\) are two different learners, \(X_{\text{new}}\) is replaced by \(X_{\text{old}}\) if \(X_{\text{new}}\) gives a better fitness values, otherwise, \(X_{\text{old}}\) is retained to be a parent vector in the next generation.

4. The Proposed NTLBO Algorithm

Multi-learning is utilized to overcome the disadvantage of the basic TLBO that the local search capacity is poor and the solutions are intend to be trapped around the local optima. At the same time, quasi-opposition based learning introduced can broaden the search space and increase the diversity of the individuals to raise the probability of jumping out of local optima. Besides, considers the fact in our teaching-learning process, a new teaching factor is designed, which can balance the aggregation and divergence of the algorithm.

4.1. Analysis of Teaching Factor and Improvement

The teaching factor in TLBO is randomly selected and can be either 1 or 2. At the early iteration, the learners improve their results through learn something from their teacher. The individuals move towards the best individual in current population so as to accelerate the convergence and the results obtained have better solutions. When one is better than the mean level of the class, it can get close to the best one in current population. However, assumed that a learner is superior to the mean level of the class, the learner will get close to the best one among current population in the opposite direction with a certain probability which can lead to low optimization efficiency and longer computation time. According to the above shortcomings of the TLBO algorithm, the teaching factor is redesigned using the form of the piecewise function. The modified teaching factor is given by

\[
T_F = \begin{cases} 
\text{round}(1 + \text{rand}(0,1)), & \text{if } \text{fitness}_{\text{mean}} \geq \text{fitness}_i \\
\text{round}(1.05 + \frac{\text{rand}(0,1)}{2}), & \text{otherwise}
\end{cases}
\]

(6)

where \(\text{fitness}_i\) is fitness value of current individual.
4.2. Random Multi-learning

The learning way of the learners in basic TLBO algorithm is simple. It will slow the convergence and may lead to be trapped into the local optima. In view of these shortcomings, random multi-learning strategy is employed. Through the random multi-learning strategy, the learners gain knowledge not only through interaction among them, but also learn new things through the difference between two difference learners. The expression of random multi-learning strategy is given by:

\[
X_{i}^{\text{new}} = \begin{cases} 
X_{i}^{\text{old}1} + \text{rand} \cdot (X_{i}^{\text{old}1} - X_{i}^{\text{old}2}) & f(X_{i}^{\text{old}1}) < f(X_{i}^{\text{old}2}) \\
X_{i}^{\text{old}2} + \text{rand} \cdot (X_{i}^{\text{old}2} - X_{i}^{\text{old}1}) & \text{otherwise}
\end{cases}
\]

where \(X_{i}^{\text{old}1}\) and \(X_{i}^{\text{old}2}\) are random select two different individuals. \(X_{i}^{\text{new}}\) is the updated value of \(X_{i}^{\text{old}1}\).

4.3. Quasi-opposition Learning

In 2007, Rahnamayan[10] raised a quasi-opposition-based learning technique. However, compared with the opposite point, a quasi-opposite point gets closer to the solution[11-12]. To take advantage of different strategies, the local search and quasi-opposition learning are combined to broaden the searching space and enhance the individual diversity to avert premature convergence. Thus, we use the quasi-opposite point instead of opposite point to generate an opposite population when meet a certain probability \(J_r\). The arithmetic expression is given by:

\[
X_{i}^{\text{new}} = \begin{cases} 
\frac{\text{rand}(X_{i}^{\text{old}}, X_{i}^{\text{old}}) + X_{i}^{\text{old}}}{2} & \text{if rand} < J_r \\
X_{i}^{\text{old}} & \text{otherwise}
\end{cases}
\]

5. Simulation Results and Discussions

Two classical IIR plants are carried out with MATLAB (2009b) to examine the performance of NTLBO in comparison with PSO-w, DE and HS. The input signal of system is a uniform white sequence, in order to compare with CSO, a Gaussian white signal with variance \(10^{-3}\) is set as the additive noise. In each case, the experimental results are obtained on account of 50 independent runs. Parameters of NTLBO is set as follows: population size=30, iteration cycle=150; the parameters setting of the WPSO, DE and HS algorithms are given. For the PSO-w, population size=30, iteration cycle=300, \(w_{\text{max}}=0.9, w_{\text{min}}=0.1, c_1=c_2=2.05\). For the DE, the population size is 30; iteration cycles is 300; \(C_r=0.8; F=0.6\). For the CSO, the parameters setting and simulation results are taken from [2].

5.1. Example 1

Take the unknown third order plant as the second example, which is considered from [2, 6] and the transfer function is

\[
H_q(z) = \frac{0.05 - 0.4z^{-1}}{1 - 1.131z^{-1} + 0.25z^{-2}}
\]

5.1.1. Case 1. A second order model is employed and its transfer function is given as

\[
H_{q2}(z) = \frac{a_0 + a_1z^{-1}}{1 + b_1z^{-1} + b_2z^{-2}}
\]
Table 1. Results of 2 order filter model

| MSE(dB) | WPSO     | DE       | NTLBO    |
|--------|----------|----------|----------|
| Best   | -24.6883 | -89.8605 | -120.5037|
| Worst  | -15.7917 | -74.7761 | -100.4938|
| Mean   | -18.926  | 2.2363   | 1.0207   |
| Std    | 2.3743   | 1.618    | 3.9586   |

5.1.2. Case 2. A first order (reduced order) model is employed and its transfer function is given as

\[
H_m(z) = \frac{a}{1+bz^{-1}}
\]  

(11)

Table 2. Results of 1 order filter model

| MSE(dB) | WPSO     | DE       | NTLBO    |
|--------|----------|----------|----------|
| Best   | -16.7911 | -17.8605 | -19.5037 |
| Worst  | -15.1355 | -15.7761 | -17.4938 |
| Mean   | -16.0649 | -16.2363 | 18.0207  |
| Std    | 0.7315   | 1.618    | 0.9586   |

Figure 2 and figure 3 depict the convergence characteristic of this test system. Table 1 and table 2 summarize the best and worst variances obtained from NTLBO and other methods. The standard deviation values are also included. For this example, NTLBO has faster convergence and finds greater solutions.

5.2. Example 2
Take the unknown fourth order plant as the second example, which is taken from [2, 6] and the transfer function is.

\[
1 + 0.9z^{-1} + 0.81z^{-2} - 0.729z^{-3} \\
1 + 0.04z^{-1} + 0.2775z^{-2} - 0.2101z^{-3} + 0.14z^{-4}
\]

(12)

5.2.1 Case 1. A fourth order model is employed and its transfer function is given as

\[
H_m(z) = \frac{a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3}}{1 - b_1z^{-1} - b_2z^{-2} - b_3z^{-3} - b_4z^{-4}}
\]

(13)
5.2.2 Case 2. A third order (reduced order) model is employed and its transfer function is given as

\[ H_w(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3}} \]  

(14)

| Table 3. Results of 4 order filter model |
|----------------------------------------|
| MSE(dB) | WPSO | DE | NTLBO |
| Best   | -22.184 | -27.7508 | -89.3209 |
| Worst  | -18.1651 | -21.4617 | -79.5307 |
| Mean   | -19.6603 | -23.8232 | -82.0988 |
| Std    | 1.8853 | 1.9354 | 4.415 |

| Table 4. Results of 3 order filter model |
|----------------------------------------|
| MSE(dB) | WPSO | DE | NTLBO |
| Best   | -10.7256 | -16.4028 | -23.7584 |
| Worst  | -8.1116 | -15.7179 | -21.6125 |
| Mean   | -8.7883 | -16.2088 | -22.3448 |
| Std    | 0.8591 | 0.4679 | 0.5733 |

From table 3 and figure 4, we can know that the proposed NTLBO algorithm has better convergence than others and found best solution among of all. Besides, the standard deviation of the results by NTLBO is very small. Seen from table 4 and figure 5, with the reduced order models, the proposed method could find a better solution around the optimal solution in contrast to other approaches. In summary, the NTLBO algorithm can achieve better performance for two IIR digital filter models. From the results, the proposed approach has higher convergence precision and better robustness.

6. Conclusions
A new algorithm named NTLBO is raised to solve the IIR digital filter design problem. The opposition learning techniques is introduced in NTLBO to enhance the diversity and broaden the searching space. In an attempt to improve the ability of the local search, random multi-learning strategy is proposed which can avert local optima trappings. The teaching factor was redesigned to make a compromise between exploring and exploiting properly. The simulation results on two IIR filters models verified the superiority of the novel NTLBO algorithm. The comparative results illustrate that NTLBO has better convergence and search precision than some other algorithms. In
order to better balance aggregation and divergence, further research could focus on adaptively adjust the teaching factor.

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