Damage detection of simply supported reinforced concrete beam by S transform

Ning Liu\textsuperscript{1,2}, Jiaxin Xi\textsuperscript{2}, Xuebing Zhang\textsuperscript{3} and Zhenzhou Liu\textsuperscript{2}

\textsuperscript{1} College of Geo-Exploration Science and Technology, Jilin University, Changchun 130026, China
\textsuperscript{2} School of Civil Engineering, Jilin Jianzhu University, Changchun 130118, China
\textsuperscript{3} School of Geomatics & Prospecting Engineering, Jilin Jianzhu University, Changchun 130118, China

Abstract. Signal processing is the key component of vibration-based structural damage detection. The S transform is variable window of short time Fourier transform (STFT) or an extension of wavelet transform (WT). The goal of using S transform is to extract subtle changes in the vibration signals in order to detect and quantify the damage in the structure. This paper presents the concentrated load is applied to the simply supported reinforced concrete beam and adopting the stepwise loading method, the vibration signals of each loading and unloading state is obtained by using the hammer impact. Then the vibration data of the reinforced concrete beam pre-damage and post-damage is analysed by S transform. Experimental result shows the potential ability of S transform in identifying peak energy changes and multiple reflections with different loading force state.

1. Introduction
The realization of the building structure function depends largely on its performance throughout the service period, however, the civil engineering structures in the decades-long service process, inevitably will be affected by human or natural environmental factors and lead to structural damage accumulation and resistance attenuation, thus threatening the safety of the entire structure. Therefore, it is necessary to establish a methodological system for early detection of structural damage in order to find out the existence of damage, the location of crack, the type of fracture, the degree of damage and the effect of damage on structural safety.

Although it was early in the 1970s that researchers tried to use vibration information to detect the damage of the offshore platform structure, it was recent 30 years the large-scale damage identification technology based on vibration information in the field of civil and structural engineering is developed [1-2]. The traditional structural reliability evaluation method is mainly through the static load test and fatigue test, the reliability of the structure is evaluated in terms of strength, stiffness, fatigue resistance and stability. In fact, the working environment of large-scale structure is more complex, during the working process, it not only bears the static load, but also subject to a large number of dynamic load, so the traditional static test as the main basis for the evaluation is difficult to fully reflect the structural characteristics [3-7].

In recent years, many time-frequency analysis methods for detection of structural damage are proposed. Including short-time Fourier transform (STFT), Wavelet transform (WT), Hilbert-Huang transform (HHT), and these time-frequency analysis methods that provide more detailed information about non-stationary signals which traditional Fourier analysis miss. S transform is a time-frequency analysis method developed in the field of geophysics, S transform combines the advantages of short-time Fourier
transform and Wavelet transform, but also has a higher resolution [8-11]. Therefore, how to apply the S transform to the actual engineering by structural damage detection method, and obtain a better result at the same time is a challenging and practical work.

2. S transform theory

2.1 Theoretical basis
The Fourier transform of the entire time series contain information about the spectral components, but it cannot detect the time distribution of different frequency, so for a large class of practical applications, the Fourier transform is unsuitable. So the time-frequency analysis is proposed and applied in data processing for many research field. The STFT is most often used, but the STFT cannot track the signal dynamics properly for non-stationary signal due to the limitations of fixed window width. The WT is an effective time-frequency analysis tool, but the traditional WT does not have the correspondence between wavelet series and frequency, and the WT is sensitive to noise. The S transform was proposed by Stockwell and his co-workers in 1996. S transform combines the advantages of WT and STFT and avoids the shortcomings, the size of the Gaussian window scale in the S transform depends on the reciprocal of the frequency and has the advantages of multi-resolution of the wavelet transform. And there is a phase factor in the S transform, which preserves the absolute phase characteristic of each frequency, this is a characteristic that the wavelet transform does not have, while, the same as the Fourier transform, S transform has lossless and reversible characteristics.

One-dimensional continuous S transform is defined as

\[
S(\tau, f) = \int_{-\infty}^{\infty} f(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2}} e^{-2\pi i f \tau} dt
\]  

(1)

Where \( S \) denotes the S transform of \( f(t) \), \( f \) denotes the frequency, \( t \) denotes the time, \( \tau \) controls the position of the Gaussian window on the time axis, equivalent to the shift factor in the WT.

One-dimensional continuous inverse S transform is defined as

\[
f(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\tau, f) e^{2\pi i f \tau} d\tau df
\]  

(2)

The S transform can be derived in two ways, one is obtained by STFT, and the other is from CWT.

2.2 Numerical example
Take the chirp signal as an example, in figure 1, the S transform shows better frequency resolution than the STFT on the low frequency signal, however in the high frequency part, the resolution of the S transform is worse than the STFT.
Figure 1. (a) Chirp signal, (b) STFT of the time series, (c) $S$ transform of the time series.

But observing the time resolution, the $S$ transform can continuously detect the occurred time and its energy variance. The STFT has a trade-off between detecting short-lived high-frequency signals and low-frequency signals. And while the window width of the STFT decreases, it shows the poorly distinguishability of the frequency component.

3. **Experimental data processing**

Experimental simply supported reinforced concrete beam length is 1000mm, net span is 900mm, section size is 100mm $\times$ 150mm. The strength grade of concrete is C20, tension zone longitudinal reinforcement is $2\phi 10$, compression zone longitudinal reinforcement is $2\phi 6$, stirrup ratio is $\phi 6@100$.

Figure 2. Schematic of reinforced concrete beam

Firstly, the hammer impact method is used to test the initial state of the beam, the three excitation positions are located above the left side of the support, 1/2 span above the longitudinal reinforcement and 50mm on the left side of the mid-span. And then applied the concentrated load on the mid-span, each level increase 5kN force of loading. Load to each level, keep the load for 2 minutes, then use the hammer to stimulate the three points after unloading.

In addition, two acceleration sensors are being deployed on the simply supported reinforced concrete beam, sensor 1 placed on the right side 50mm from the mid-span, sensor 2 placed on the right side above the support.
Figure 3. Received vibration data in different loading force state. (a) Initial state, (b) 5kN force, (c) 15kN force, (d) 25kN force, (e) 30kN force, (f) 35kN force.

Figure 3 shows several typical sets of the received vibration data in sensor 2 from the initial state to the 35kN loading force. From figure 3b to figure 3f, we can see that generally the peak amplitude of the time-domain vibration data is gradually decreased. In order to better observe these sets of data, the time-domain data are being transferred to the time-frequency domain by S transform correspondingly.

Figure 4. S transform of the received vibration data. (a) Initial state, (b) 5kN force, (c) 15kN force, (d) 25kN force, (e) 30kN force, (f) 35kN force.
The $S$ transform spectrum contains time, frequency and energy information of the signal. The transform results are shown in figure 4a to figure 4f. It can be proved that the main frequency of each vibration data distribute across 65Hz to 150Hz. From the time-frequency spectrum, we can see that with the increasing loading force, the peak energy of the vibration response data gradually decreased. This is because of that with the growing of the loading force, the cracks started appearing in the beam, the elastic waves travel from the cracks, and the high-frequency component and energy are being attenuated. Meanwhile, it should be noticed that the number of reflection and refraction waves raised due to the number of the cracks increased, this point in figure 4f is reflected most visibly.

4. Conclusions
We have introduced a time-frequency analysis method, $S$ transform, to analyse the received vibration data under different loading force state. The $S$ transform exhibit the ability of identifying both low frequency and high frequency component in time-frequency domain, and shows high time-frequency resolution. To test the applicability of the $S$ transform in structural damage response vibration data, we use stepwise loading and hammer impact method to collect the vibration data of the simply supported reinforced concrete beam. Result of applying the $S$ transform to the experimental data shows that with the loading force increased gradually, the peak energy in the time-frequency spectrum slightly decreased. In the meantime, with the number of cracks increased, the multiple reflections and refractions are becoming more obvious in the time-frequency spectrum, which displayed the potential ability of $S$ transform in structural damage detection. However, the proposed method needs to be further investigated for locating the position of the cracks.

Acknowledgements
This work is partly supported by 2016 Jilin Jianzhu University students’ innovation and entrepreneurship training program (Grant No. 201610191166). All results are reproducible by Madagascar open-source software [12].

References
[1] Ryther A 1993 Vibration based inspection of civil engineering structures. Ph.D. thesis, Aalborg University, Denmark.
[2] Cawley P and Adams R D 1979 The locations of defects in structures from measurements of natural frequencies. Journal of Strain Analysis 14(2) 49-57.
[3] Pandey A K and Biswas M 1994 Damage detection in structures using changes in flexibility. Journal of Sound and Vibration 169 3-17.
[4] Alvandia A and Cremona C 2006 Assessment of vibration-based damage identification techniques. Journal of Sound and Vibration 292 179-202.
[5] Bernal D 2002 Load vectors for damage localization. Journal of Engineering Mechanics 128(1) 7-14.
[6] Duan Z D, Yan G R and Ou J P 2007 Damage detection in ambient vibration using proportional flexibility matrix with incomplete measured DPFs. Structural Control and Health Monitoring 14 186-196.
[7] Sanchez A and Adeli H 2014 Signal processing techniques for vibration-based health monitoring of smart structures. Archives of Computational Methods in Engineering 23(1) 1-15.
[8] Karasaridis A, Maalej M and Pantazopoulos S 1997 Time-frequency analysis of sensor data for detection of structural damage in instrumented structures. International Conference on Digital Signal Processing Proceedings. IEEE 2 817-820.
[9] Kim H and Melhem H 2004 Damage detection of structures by wavelet analysis. Engineering Structures 26(3) 347-362.
[10] Pai P F, Huang L, Hu J and Langewisch D. R 2008 Time-frequency method for nonlinear system identification and damage detection. Structural Health Monitoring 7(2) 103-127.
[11] Stockwell R G, Mansinha L and Lowe R P 1996 Localization of the complex spectrum: the $S$ transform. IEEE Trans. Signal Process 44(4) 998-1001.
[12] Fomel S, Sava P, Vlad I, Yang L and Bashkardin V 2013 Madagascar: Open-source software project for multidimensional data analysis and reproducible computational experiments. *Journal of Open Research software* 1(1) e8.