Optimal broad-band frequency conversion via a magnomechanical transducer

F. Engelhardt,1,2,3 V. A. S. V. Bittencourt,1 H. Huebl,4,5,6 O. Klein,7 and S. Viola Kusminskiy3,1

1Max Planck Institute for the Science of Light, Staudtstr. 2, PLZ 91058 Erlangen, Germany
2Department of Physics, University Erlangen-Nuremberg, Staudtstr. 7, PLZ 91058 Erlangen, Germany
3Institute for Theoretical Solid State Physics, RWTH Aachen University, 52074 Aachen, Germany
4Walther-Meißner-Institut, Bayerische Akademie der Wissenschaften, D-85748 Garching, Germany
5Physik-Department, Technische Universität München, D-85748 Garching, Germany
6Munich Center for Quantum Science and Technology (MCQST), D-80799 Munich, Germany
7Université Grenoble Alpes, CEA, CNRS, Grenoble INP, Spintec, 38054 Grenoble, France

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Developing schemes for efficient and broad-band frequency conversion of quantum signals is an ongoing challenge in the field of modern quantum information. Especially the coherent conversion between microwave and optical signals is an important milestone towards long-distance quantum communication. In this work, we propose a two-stage conversion protocol, employing a resonant interaction between magnetic and mechanical excitations as a mediator between microwave and optical photons. Based on estimates for the coupling strengths under optimized conditions for yttrium garnet, we predict close to unity conversion efficiency without the requirement of matching cooperativities. We predict a conversion bandwidth in the regions of largest efficiency on the order of magnitude of the coupling strengths which can be further increased at the expense of reduced conversion efficiency.

I. INTRODUCTION

Modern quantum information technology relies on quantum control across several platforms, ranging from single atoms, ions and spins, to superconducting circuits. A given system is, however, usually only able to excel at a specific task. For example, superconducting qubits and superconducting cavities, which typically operate in the microwave energy range, have been identified as one of the front-runners [1] for quantum information processing, whereas transferring quantum information over long distances [2] is a task best accomplished with optical signals travelling inside photonic fibers [3]. Critically coupling microwave quantum circuits with optical communication networks represents a step forward in the development of a quantum network [1, 5] and requires the design of a hybrid system tailored for optics-to-microwave multi- 

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In this article, we propose an approach to optics-to-microwave conversion based on optomechanics by inserting a magnetic element that maintains high cooperativity both with a mechanical mode and with the microwave mode. The cost of the increased setup complexity comes at the gain of substantial increase of the coupling strength: through magnetoelastic and optomechanical effects it is possible to achieve a much stronger coupling to the microwave mode, compared to purely mechanical systems, and to optical photons, compared to the purely magnetic system. We show that such a optomechanomagnomechanical setup, schematized in Fig. 1, yields conversion efficiencies close to unity, while simultaneously providing large bandwidths. The latter is due to the fact that we expect all relevant coupling strengths to be large compared to any internal linewidths. Our estimates are based on the magnetic insulator yttrium iron garnet (YIG). This material is the staple choice for magnonic experiments, since it provides long lifetime of magnon excitations and a large spin density [48–51].

The strong coupling of the considered modes leads to hybridization and the formation of frequency-shifted normal modes. Whereas the phonon frequencies are mainly determined by the choice of a specific geometry, magnons provide a desirable tunability for optimization. When all modes are tuned to resonance, the efficiency is given only in terms of the cooperativities between the modes. The cooperativity relates the coupling strength between modes to their linewidths, therefore characterizing the dissipative behaviour of a coupled system. Contrary to setups based on one-stage conversion, matching cooperativities are not required in order to obtain efficiencies close to unity. Here, optimization can be performed by altering the optical pump power (which determines the effective optomechanical coupling rate) and the respective port-coupling rates of the optical and microwave mode. Moreover, the total linewidth of the optical modes determines the optomechanical coupling regime and therefore the hybridization of the coupled system.

The article is structured as follows. In Section II we introduce the model Hamiltonian and give a brief description of each relevant coupling term together with an estimate of the coupling strength for material-specific parameters for YIG. These values together with relevant frequencies and linewidths are summarized in Table I. In Section III we set up the two-stage conversion setup based on resonant magnon-phonon coupling and calculate the expected conversion efficiency. We give a description of the expected hybridization of the interaction chain and discuss the resulting bandwidths of the conversion spectrum for different regimes of the driven optomechanical interaction. Finally, in Section IV we summarize our results.

![FIG. 1. Sketch of a two-stage interconversion protocol between optical and microwave photons represented respectively by \( \hat{a} \) and \( \hat{c} \). The two intermediary states are \( \hat{b} \), phonons and \( \hat{m} \), the magnons, both with frequencies in the microwave range. The thick lines denote the dominant interaction scheme.](image)

**II. MODEL**

Our model is based on the interaction scheme given in Fig. 1. Four different bosonic modes are coupled, with frequencies \( \omega_a, \omega_b, \omega_m, \omega_c \) (where \( \omega_a \gg \omega_b, \omega_m, \omega_c \)) corresponding to optical photons, phonons, magnons and microwave photons, respectively. Optical photons couple to phonons and magnons via optomechanical and magneto-optical effects, respectively. Due to a large frequency mismatch, these interactions are parametric. Magnons couple to phonons and microwave photons via magnetoelastic and magnetic dipole interactions, respectively. We consider a setup in which magnons, phonons and microwave photons can be all three brought into degeneracy, leading to linear interaction terms. In this work, we consider no piezoelectric interactions and assume that the magneto-optical coupling is weak compared to all other types of interactions. This leads to an effective interaction chain comprising optomechanics, magnumechanics and inductive coupling to magnetization dynamics.

The undriven Hamiltonian of the system is given by

\[
\hat{H}_1 = \omega_a \hat{a}^\dagger \hat{a} + \omega_m \hat{m}^\dagger \hat{m} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + g_{ab} \hat{a}^\dagger \hat{b}^\dagger + g_{mb} \hat{m}^\dagger \hat{b} + g_{mc} \hat{m}^\dagger \hat{c} + \frac{g_{ab}}{\hbar} (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}) + \frac{g_{mc}}{\hbar} (\hat{m}^\dagger \hat{c} + \hat{c}^\dagger \hat{m}) \]

(1)

Here, \( \hat{a} (\hat{a}^\dagger) \), \( \hat{b} (\hat{b}^\dagger) \), \( \hat{m} (\hat{m}^\dagger) \), \( \hat{c} (\hat{c}^\dagger) \) are the bosonic annihilation (creation) operators for optical photon, phonon, magnon and microwave photons, respectively. Moreover, \( g_{ab}, g_{mb} \) and \( g_{mc} \) are the optomechanical, magnomechanical and inductive coupling strength, respectively. In order to realize the Hamiltonian of Eq. (1), a microstructure serving as an optical cavity and at the same time supporting mechanical and magnetic modes in the GHz regime is needed. In the following we briefly describe the coupling mechanisms subject to our model and give estimates for the optimal expected couplings using material specific constants for YIG. Full derivations of the coupling terms can be found in the Appendix, as well as a...
The coupling between magnons and phonons is mediated by magnetoelastic effects, where the magnetization inside a material couples dynamically to the mechanical stress and thus causing lattice deformations. State-of-the-art experiments [22, 55] have involved magnetic spheres which yield mechanical modes of MHz frequencies, due to their relatively large diameters on the order of 100 μm. This limits the parametric interaction strengths with a uniform magnetic Kittel mode (frequency on the order of GHz) to a few mHz. In contrast, resonant magnomechanical interaction yields much larger coupling strengths. For mode frequencies of 2π × 10 GHz we expect an optimal coupling strength on the order of g_{mb} = 2π × 10 MHz. If the mode volumes of the magnon (V_m) and phonon (V_b) do not match, the coupling scales as (2B_1 + B_2) \rho^{-1/2}\sqrt{V_m/V_b} (if V_m < V_b) or as (2B_1 + B_2) \rho^{-1/2}\sqrt{V_b/V_m} (if V_m > V_b), where B_1 = 3.48 J m^{-3} and B_2 = 6.4 J m^{-3} are the magnetoelastic constants for YIG [54] and \rho = 5110 kg m^{-3} is the material density [55]. Detailed derivations of all coupling terms can be found in the Appendix.

The optical cavity is connected to a coherent drive, leading to the total Hamiltonian

$$\hat{H}_2 = \hat{H}_1 + \hat{H}_{\text{drive}},$$

with an optical drive term \(\hat{H}_{\text{drive}} = i\alpha \left(\hat{a}e^{i\omega_D t} - \hat{a}^\dagger e^{-i\omega_D t}\right)\). We perform an unitary transformation to the rotating frame of the optical driving field via \(U\hat{H}U^\dagger = i\hbar \partial_t \hat{a}^\dagger \hat{a}\) and linearize the Hamiltonian by considering fluctuations on top of the coherent steady-state induced by the drive. For small coupling rates, only the optical field exhibits a coherent part, such that \(\hat{a} = \alpha + \delta\hat{a}\) with \(\alpha = \sqrt{n_a}\), where \(n_a\) is the mean photon number in the optical cavity. The mean photon number \(n_a\) can be estimated by

$$\overline{n_a} = \frac{\kappa_{a,\text{out}} P}{\hbar \omega_a \left[\Delta_a^2 + (\gamma_a + \kappa_{a,\text{out}})^2 / 4\right]},$$

where \(P\) is the drive power, \(\gamma_a\) the internal optical mode linewidth, \(\kappa_{a,\text{out}}\) the dissipation rate to the external port and \(\Delta_a = \omega_D - \omega_a\) the detuning from the drive frequency. The confinement of photons yields a coupling enhancement of the parametric processes which scales with the square root of the mean photon number, specifically, the cavity enhanced coupling is given by \(G_{ab} = \overline{n}_a g_{ab}\). Next we apply a rotating wave approximation, neglecting all terms of the form \(\hat{m}^{(i)}(\hat{b}^{(i)} + \hat{c}^{(i)})\) and \(\hat{m}^{(i)}(\hat{c}^{(i)} + \hat{b}^{(i)})\). As long as the coupling rates remain smaller than about 10% of the mode frequencies, so \(G_{ab}, g_{mb}, g_{mc} < 0.1 (\Delta_a, \omega_m, \omega_b, \omega_c)\), they can be safely neglected [57, 58]. For a red-detuned drive \(\Delta_a \approx -\omega_b\), the terms \(\delta\hat{a}^{(1)}\hat{b}^{(1)}\) can also be neglected under the rotating wave approximation. This leads to the Hamiltonian used in the following (from now on we adopt the notation \(\delta\hat{a} \to \hat{a}\))

$$\hat{H}_2/\hbar = -\Delta_a \hat{a}^\dagger \hat{a} + \omega_m \hat{m}^\dagger \hat{m} + \omega_b \hat{b}^\dagger \hat{b} + \omega_c \hat{c}^\dagger \hat{c} + G_{ab} (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}) + g_{mb} (\hat{m}^\dagger \hat{b} + \hat{b}^\dagger \hat{m}) + g_{mc} (\hat{m}^\dagger \hat{c} + \hat{c}^\dagger \hat{m}).$$

### III. Conversion Setup

Based on the Hamiltonian of Eq. (4) we consider a conversion scheme as depicted in Fig. [2](b). The quantum
TABLE I. Table of used values for mode frequencies, linewidths/dissipations and coupling strengths. $g_{mb}$ and $g_{ab}$ assume a perfect mode overlap at a phonon, photon and magnon mode volume of $1$ µm$^3$. The estimate for $g_{mc}$ assumes 1% mode overlap between the magnon and the microwave field, so the microwave mode volume is $100$ µm$^3$.

| Quantity | Symbol | Value |
|----------|--------|-------|
| Optical photon freq. | $\omega_{a/2\pi}$ | 200 THz |
| Optical detuning (red) | $\Delta_{a/2\pi}$ | -10 GHz |
| Phonon freq. | $\omega_{/2\pi}$ | 10 GHz |
| Magnon freq. | $\omega_{m/2\pi}$ | 10 GHz |
| Internal optical linewidth | $\gamma_{a/2\pi}$ | 0.1 GHz |
| External optical dissipation | $\kappa_{a/\text{out}/2\pi}$ | 1.5 GHz |
| Phonon linewidth | $\gamma_{b/2\pi}$ | 1.0 kHz |
| Magnon linewidth | $\gamma_{m/2\pi}$ | 1.0 MHz |
| Internal mw cavity loss rate | $\gamma_{/2\pi}$ | 3.0 MHz |
| External mw cavity coupling rate | $\kappa_{c/\text{out}/2\pi}$ | 100 MHz |
| Magnon-Microwave photon coupling | $\kappa_{a/c}$ | 10.0 MHz |
| Optomechanical coupling | $\kappa_{o/a/\text{out}}$ | 180 MHz |
| Mode volume (photon, phonon, magnon) | $V_{a/b/m}$ | $1$ µm$^3$ |
| Mode volume (microwave) | $V_c$ | $100$ µm$^3$ |

Langevin equations (QLE’s) for the bosonic annihilation operators are obtained via the Heisenberg equation of motion $\dot{A}(t) = -i[A, H]/\hbar$ (for an arbitrary operator $A$), as

$$\dot{\hat{a}}(t) = -i\left(-\Delta_a - i\frac{\gamma_{a,\text{tot}}}{2}\right)\hat{a}(t) - iG_{ab}\hat{b}(t) - \sqrt{\gamma_a}\hat{a}_{\text{in}}(t) - \sqrt{\gamma_a}\hat{a}_{\text{out}}\hat{a}_{\text{in}}(t), \quad (5a)$$

$$\dot{\hat{b}}(t) = -i\left(\omega_b - i\frac{\gamma_b}{2}\right)\hat{b}(t) - iG_{ab}\hat{a}(t) - ig_{mb}\hat{m}(t) - \sqrt{\gamma_b}\hat{b}_{\text{in}}(t), \quad (5b)$$

$$\dot{\hat{m}}(t) = -i\left(\omega_m - i\frac{\gamma_m}{2}\right)\hat{m}(t) - iG_{mb}\hat{b}(t) - ig_{mc}\hat{c}(t) - \sqrt{\gamma_m}\hat{m}_{\text{in}}(t), \quad (5c)$$

$$\dot{\hat{c}}(t) = -i\left(\omega_c - i\frac{\gamma_c,\text{tot}}{2}\right)\hat{c}(t) - ig_{mc}\hat{m}(t) - \sqrt{\gamma_c}\hat{c}_{\text{in}}(t) - \sqrt{\kappa_{c/\text{out}}+\kappa_{c/\text{in}}}(t), \quad (5d)$$

where we have included the internal noise operators $\hat{a}_{\text{in}}$, $\hat{b}_{\text{in}}$, $\hat{m}_{\text{in}}$, $\hat{c}_{\text{in}}$ to describe the open dynamics of the system, and $\hat{a}_{\text{in/\text{out}}}$, $\hat{c}_{\text{in/\text{out}}}$ for the input fields from the external ports. The description of the mechanical mode in terms of the annihilation operator $\hat{b}$ instead of the quadrature $\hat{x}$ is justified since the setup operates in the regime $\omega_b \gg \gamma_b$ [59] (compare Tab. 1). The rates $\gamma_{a/b/m/c}$ define the internal linewidths of the modes. Furthermore, $\gamma_{a/c/\text{tot}} = \gamma_{a/c} + \kappa_{a/c/\text{out}}$ gives the total linewidth of the optical/microwave mode which includes the coupling rates $\kappa_{a/c/\text{out}}$ to the two external ports. All noise operators $\hat{\beta} = \hat{a}_{\text{in/\text{out}}}, \hat{b}_{\text{in/\text{out}}}, \hat{m}_{\text{in/\text{out}}}, \hat{c}_{\text{in/\text{out}}}$ describe thermal noise in each mode, given by

$$\langle \hat{\beta}(t)\hat{\beta}^\dagger(t') \rangle = (n_{th,\beta} + 1)\delta(t-t'), \quad$$

$$\langle \hat{\beta}^\dagger(t)\hat{\beta}(t') \rangle = n_{th,\beta}\delta(t-t'), \quad (6)$$

where $n_{th,\beta} = [\exp(h\omega_b/k_B T) - 1]^{-1}$ is the Bose-Einstein distribution determining the thermal occupancy of the respective baths, with $k_B$ the Boltzmann constant and $T$ the temperature. For the optical photons the thermal occupancy is $n_{th,\alpha} \approx 0$ even at room temperature due to the high frequency. In the next step, we define the $4 \times 4$ matrix $A$ and the $4 \times 6$ matrix $B$ to bring the QLE’s in the form

$$\dot{\mathbf{v}}(t) = A\mathbf{v}(t) + B\mathbf{v}_{\text{in}}(t), \quad (7)$$

with $\mathbf{v}(t) = \begin{pmatrix} \hat{a}, \hat{b}, \hat{m}, \hat{c} \end{pmatrix}^T$ and $\mathbf{v}_{\text{in}}(t) = \begin{pmatrix} \hat{a}_{\text{in/\text{ext}}}, \hat{b}_{\text{in/\text{ext}}}, \hat{m}_{\text{in/\text{ext}}}, \hat{c}_{\text{in/\text{ext}}} \end{pmatrix}^T$, where we distinguish between internal and external dissipation paths for optical photons and microwaves.

In order to obtain the conversion spectrum of our coupled system, we compute the scattering matrix $S(\omega)$ which relates input and output in frequency space via $\mathbf{v}(\omega)_{\text{out}} = S(\omega)\mathbf{v}(\omega)_{\text{in}}$. For that, we Fourier transform the QLE’s (Eqs. (5a) to (5d)) using the convention

$$\int_{-\infty}^{\infty} dt e^{i\omega t}\hat{O}(t) = \hat{O}(\omega), \quad (8a)$$

$$\int_{-\infty}^{\infty} dt e^{i\omega t}\hat{\beta}(t) = -i\omega\hat{\beta}(\omega), \quad (8b)$$

FIG. 2. (a) Schematic and notation of the quadri-partite coherent interconversion protocol showing the interdependence of the four different waveforms. The chain arises from selecting the dominant coupling mechanisms in Fig. 1. Here, $\kappa_{a/c/\text{out}}$ denotes coupling to the optical/microwave port and $\gamma_{a/b/m/c}$ are the intrinsic linewidth of each waveform. We consider an optical pump at frequency $\omega_D$ and therefore the optomechanical coupling is enhanced and denoted by $G_{ab}$.

(b) Pumping scheme for driving of the red sideband of the optical cavity. Here $\omega_a, \omega_b, \omega_m$, and $\omega_c$ correspond to the optical photon, phonon, magnon and microwave photon mode, respectively.
and obtain
\[
\hat{a} = -iG_{ab}\chi_a[\omega]\hat{b} - \sqrt{\kappa_a}\chi_a[\omega]\hat{b}_{\text{in,int}},
\]
\[
\hat{b} = -iG_{ab}\chi_b[\omega]\hat{a} - ig_{mb}\chi_b[\omega]\hat{m},
\]
\[
\hat{m} = -ig_{mb}\chi_m[\omega]\hat{b} - ig_{mc}\chi_m[\omega]\hat{c},
\]
\[
\hat{c} = -ig_{mc}\chi_c[\omega]\hat{m} - \sqrt{\gamma_c}\chi_c[\omega]\hat{c}_{\text{in,int}},
\]
where the susceptibilities have the form e.g.
\[
\chi_c[\omega] = (-i(\omega - \omega_c) + (\gamma_c/\kappa_{c,\text{out}})/2)^{-1}.
\]
We obtain the scattering matrix as a function of frequency \(\omega\) referenced to a red sideband tone via
\[
S(\omega) = B^T[-i\omega I_4 - A]^{-1}B - I_6,
\]
where \(I_N\) is the \(N \times N\) identity matrix. The conversion efficiency from optics to microwave is then obtained as
\[
\xi_{ac}(\omega) = |S_{1,5}(\omega)|^2
\]
and explicitly reads
\[
\xi_{ac}(\omega) = \eta_a\eta_c \frac{4C_{ab}C_{mb}C_{mc}}{(1 + C_{ab})(1 + C_{mc}) + C_{mb}}^2,
\]
where \(\eta_a/\eta_c = \kappa_{a,c,\text{out}}/\kappa_{a,c,\text{tot}}\) are the extraction factors defined as the ratio between port dissipation and total dissipation. A cooperativity value of \(C_{ij} \gg 1\) is a necessary criterion for an efficient energy transfer \([59, 60]\).

In the case that all modes are on resonance and the cavity is driven with a red detuning (see Fig. 2(b)), \(\omega = \omega_m = \omega_c = -\Delta_a\), we can express Eq. (11) solely in terms of the cooperativities \(C_{ij} = 4g_{ij}^2/\gamma_i\), \(\tau_{i,tot}\) and \(\tau_{j,tot}\)
\[
\xi_{ac} = \eta_a\eta_c \frac{4C_{ab}C_{mb}C_{mc}}{(1 + C_{ab})(1 + C_{mc}) + C_{mb}}^2,
\]
where \(\eta_a/\eta_c = \kappa_{a,c,\text{out}}/\kappa_{a,c,\text{tot}}\) are the extraction factors defined as the ratio between port dissipation and total dissipation. A cooperativity value of \(C_{ij} \gg 1\) is a necessary criterion for an efficient energy transfer \([59, 60]\).

The single-particle couplings and intrinsic decay rates are fixed by the design of the system and by the material. Therefore, we consider a setup in which the free variables are the photon-phonon cooperativity \(C_{ab}\), which can be tuned via the power of the external optical drive, and the magnon-microwave cooperativity \(C_{mc}\), which can be changed by adjusting the coupling of the microwave cavity to the external port. By setting the partial derivatives of Eq. (12) with respect to \(C_{mc}\) and \(C_{ab}\) to zero while fixing \(C_{mb}\), we obtain a constraint for the cooperativities
\[
C_{ab} \equiv 1 + \frac{C_{mb}}{C_{mc} + 1},
\]
which in turns gives the maximum efficiency at
\[
C_{ab}^{\text{max}} = C_{mc}^{\text{max}} = \sqrt{1 + C_{mb}}.
\]

The optimal value for the conversion efficiency in terms of the fixed cooperativity \(C_{mb}\) is then given by
\[
\max \left[\xi_{ac}^{C_{mb}}\right] = \eta_a\eta_c \frac{(1 + C_{mb})C_{mb}}{(1 + C_{mb} + \sqrt{1 + C_{mb}})^2},
\]
which approaches \(\eta_a\eta_c\) as \(C_{mb}\) grows. In Fig. 3(a), we show the conversion efficiency as a function of the cooperativities \(C_{ab}\) and \(C_{mc}\) at \(C_{mb} = 4 \times 10^5\), corresponding to our estimates shown in Table 1. The indicated global maximum is obtained from Eq. (15), which for our adopted parameters yields a maximum conversion efficiency of \(\sim 0.997 \times \eta_a\eta_c\). The optimal cooperativities obtained with Eq. (14) determine the required microwave port dissipation \(\kappa_{mc}^{\text{max}} \approx 2\pi \times 202\) MHz and an optical pump power \(P_{\text{max}} \approx 5.6 \times 10^{-5}\) W, corresponding to \(\sim 6300\) circulating photons (see Eq. (3)) at a total optical linewidth of \(\gamma_{a,\text{tot}} \approx 2\pi \times 1.6\) GHz. However, as long as Eq. (15) is fulfilled, high efficiencies are still possible even if Eq. (14) is not satisfied.

It is important to note that the discussion exclusively in terms of cooperativities only holds true in the case where all mode frequencies (and the optical detuning) are on resonance. In a non-resonant case, the optimal conversion efficiency has to be found by optimizing Eq. (11) with respect to the relevant parameters. This can be achieved by following the same procedure outlined above, but potentially requires numerical solutions for the obtained set of equations.

When we leave out the phonon mode for the conversion scheme, transduction takes place only through the magnon mode, via magneto-optical effects and magnon-microwave coupling. This setup has been used in recent experiments \([57, 59]\). In such a conversion scheme with only one mediating element, the efficiency in terms of the cooperativities reads \([61]\)
\[
\eta_a\eta_c \frac{4C_{ab}C_{mc}}{(1 + C_{ab} + C_{mc})^2},
\]
where \(C_{am}\) is the magnon-optical photon cooperativity. Compared with the efficiency via optomagnonics, given in Eq. (12), the efficiency for a single mediator requires matching cooperativities for maximum conversion. Since the optomagnonic coupling achieved experimentally is very low \([58, 62]\) compared to magnon-microwave coupling, it is difficult to approach a good matching, even for very high pump powers. This limits the efficiencies obtained so far to values between \(10^{-11}\) \([57]\) and \(10^{-7}\) \([58]\).
and optical photon and phonon (the different microwave waveforms $\omega_i$). All considered modes are strongly coupled, except for the interacting modes is larger than their linewidths, are considered in the strong coupling regime. Based on our estimates and anticipated linewidths (see Tab. [39]), all considered modes are strongly coupled, except phonons and optical photons. Here, the coupling regime depends on the optical pump power. Strongly coupled systems undergo normal mode splitting when they are close to resonance. In particular, for a system with only two coupled modes $i$ and $j$ at $\omega_i = \omega_j = \omega$ the shifted eigenfrequencies are given by $\omega_k = \omega \pm g_{ij}$, where $g_{ij}$ is the coupling strength. If the total linewidth $\gamma_{i,tot}$ of a coupled mode $j$ exceeds the coupling strength $g_{ij}$, the shifted hybrid peaks cannot be resolved. If, however, $4g_{ij} > \gamma_{i,tot}$, the system is considered to be in the strong coupling regime, and the two shifted hybrid peaks can be clearly resolved.

In Fig. 4(a) we show the normal mode spectrum of Eq. (4) of the interaction chain shown in Fig. 2(a). At the resonance point ($\omega_c = \omega_b = \omega_m = -\Delta_a$) we obtain two sets of hybrid modes - one mode partly magnon and microwave and the other partly phonon and optical photon. Since the coupling between magnons and microwaves is the dominant coupling, the phonons can only hybridize with the driven optical cavity mode. Hybridization between magnons and phonons does not occur as long as magnon-microwave interaction dominates. However, if we tune the phonon frequency instead (Fig. 4(b)), we can bring it in resonance with the magnon-microwave hybrid mode. The phonon then couples with the magnon-part and the spectrum is split by twice the magnomechanical coupling rate $g_{mb}$. Simultaneous tuning of magnon and phonon frequency in the case $\omega_b = \omega_m$, as shown in Fig. 4(c), only shows magnon-phonon hybridization when both are tuned away from the microwave frequency.

To include the linewidths of the bare modes, we look at the resonances of the coupled system of Eqs. (2a) - (2d). The imaginary part of the eigenvalues corresponds to a resonance frequency $\omega_{res}$, while the real part gives the width of the resonance peak $\gamma_{res}$. The resonance is given by the Lorentzian $1/2 \left( (\omega - \omega_{res})^2 + (\gamma_{res})^2 \right)^{-1}$. In Fig. 4 we plot the resonance spectrum for an optical pump power of $10^{-4}$ W and $5 \times 10^{-2}$ W. This enhances the optomechanical coupling rate to about 21 MHz and 474 MHz, respectively. The latter case corresponds to the strong optomechanical coupling regime, where $4G_{ab} > \gamma_{a,tot}$. The total optical linewidth is set to $\gamma_{a,tot} = 2\pi \times 1.6$ GHz (corresponding to a quality factor of $Q = 125000$). As expected, in the first case ($P = 10^{-4}$ W) we observe no hybridization between photon and phonon, hence no shift of the green and yellow resonance peaks. Conversely, the resonance peaks of the magnon-microwave hybrid modes are clearly resolved and separated by twice the coupling rate of $g_{mc} = 2\pi \times 180$ MHz. In the second case ($P = 5 \times 10^{-2}$ W) the transition to the strong optomechanical coupling regime also leads to two frequency shifted peaks. The peaks in the resonance spectrum correspond to the peaks of the conversion efficiency. This is, for example, depicted in Fig. 6 which shows the conversion efficiency as a function of frequency $\omega$ and magnon frequency $\omega_m$. One can observe a resemblance to Fig. 4(a). The largest

A. Hybridization and resonances

Coupled systems, where the coupling strength between the interacting modes is larger than their linewidths, are considered in the strong coupling regime. Based on our coupling estimates and anticipated linewidths (see Tab. [39]), all considered modes are strongly coupled, except for...

![Fig. 3](image)

** FIG. 3.** (a) Conversion efficiency $\xi$ at full degeneracy between the different microwave waveforms ($\omega = \omega_m = \omega_b = \omega_c = -\Delta_a$). The expression is given by Eq. (12) as a function of the cooperativity between magnon and microwave photon ($C_{mb}$) and optical photon and phonon ($C_{ab}$). The dashed line in the lower plot denotes the maximum conversion efficiency. The grey dot marks the global maximum. The magnon-phonon cooperativity is fixed at $C_{mb} = 4 \times 10^7$, corresponding to the estimates of Table [39]. (b) Maximum conversion efficiency given in Eq. (15) as a function of $C_{mb}$, $C_{ab}$ and $C_{mc}$ fulfill Eq. (14).
conversion efficiency is given at the point of full resonance, which is highlighted in the inset. Optimization of the maximum conversion efficiency would be performed following the discussion in terms of the cooperativities presented before.

### B. Conversion bandwidth

In order to characterize the performance of the proposed conversion system, we quantify the conversion bandwidth at frequencies with high conversion efficiency. In general, the conversion bandwidth is limited by the slowest process in the conversion chain. This is what limits e.g. state-of-the-art electromechanical and optomechanical systems whose bandwidths are defined by the small total linewidths of the vibrational modes [20, 24, 63]. Conversely, modern broadband conversion setups suffer from small overall conversion efficiencies due to weak coupling [39].

The conversion bandwidth can be calculated from the scattering matrix element given in Eq. (11). The half width at full maximum (HWFM) \( \Delta \omega_{1/2} \) of a peak in the transmission spectrum at a frequency \( \omega_{\text{max}} \) is defined by

\[
\xi (\Delta \omega_{1/2}) = \frac{\xi (\omega_{\text{max}})}{2}.
\]

The conversion bandwidth is therefore defined as \( \Delta \omega = 2 \Delta \omega_{1/2} \). Conversion spectra can be obtained using Eq. (11). In Fig. 7(a) (Fig. 7(b) zoomed in) we show a plot of the conversion efficiency as a function of frequency and optical pump power (mean photon number). Based on the parameters in Tab. 1 for small pump powers we notice high efficiency (close to unity) at the point of full resonance \( (\omega_\ell = \omega_m = \omega_c = -\Delta_a) \) with a bandwidth \( \Delta \omega \) of up to 0.6 MHz at an optical pump power of \( 2.8 \times 10^{-5} \text{W} \), which corresponds to the point of maximum conversion efficiency (see Fig. 7(d)). As the pump power increases, the resonance peak widens while the efficiency decreases. At pump powers \( \sim 10^{-3} \text{W} \), conversion at the sidebands created from the strong magnon-microwave coupling are enhanced. This also marks the transition to the strong optomechanical coupling regime. In this region, we obtain a very large bandwidth (several hundred MHz) at the expense of a reduced conversion efficiency that does not drop below \( 10^{-4} \). It reaches \( 10^{-2} \) peak efficiency at the magnon-microwave sidebands that are limited in width by the microwave port coupling rate \( \gamma_c \). Since \( \gamma_c \gg \gamma_c \), the width is \( \approx \gamma_c/2 = 50 \times 50 \text{MHz} \). The bandwidth at \( \omega_\ell = \omega_m = \omega_c = -\Delta_a \) can be increased up to about 10 MHz before the efficiency drops off to \( 10^{-1} \) (see Fig. 7(c)).

Optimization of the conversion bandwidth requires improvement of the limiting coupling processes in the chain. In the weak optomechanical coupling regime, where the conversion efficiency is maximum, the bandwidth is limited by the smallest mode coupling rate, which is either \( g_{mb} \) or \( G_{ab} \), depending on the pump power. In the strong optomechanical regime, the bandwidth is increased by enhancing the coupling to the microwave port. Altering the magnon-microwave coupling rate only determines the spectral position of the sidebands, not their width. An enhancement of conversion bandwidth is achieved at the expense of reduced conversion efficiency. To optimize both, one has to take into account the interplay of the cooperativities between the coupled modes (Fig. 3) and find the optimal configuration.
C. Resolved vs. unresolved sideband regimes

A large density of circulating photons in the optical resonator enhances the effective optomechanical coupling, but can be limited by the maximum amount of photons that the cavity supports. However, this limitation is partially lifted under optimization of the single photon-phonon coupling. Furthermore, Eq. (3) implies that, aside from the pump power, the total optical linewidth has a crucial impact on the mean photon number for a given mode. In general, cavities with larger linewidths (low quality factors) require stronger pump powers for achieving a large density of circulating photons.

For the driven optical cavity we have to further distinguish between two regimes of the optical cavity with respect to the optical linewidth. In the resolved sideband regime, the total optical linewidth $\gamma_{a,\text{tot}} = \gamma_a + \gamma_{a,\text{out}}$ is smaller than the mode frequencies of magnons, microwave, and phonons: $\gamma_{a,\text{tot}} < \omega_b$. In this case, the sidebands can be directly addressed by choosing the optical drive frequency (and therefore the optical detuning) to be $\pm \omega_b$. (compare Fig. 2 (b)). If, however, $\gamma_{a,\text{tot}} > \omega_b$ the sidebands cannot be resolved and therefore cannot be individually addressed by a detuned drive. As a consequence, the precise value of the detuning has less impact on the obtained conversion efficiency.

In the case of full mode resonance and red optical detuning ($\omega_b = \omega_m = \omega_c = -\Delta_a$), we plot the conversion efficiency given in Eq. (12) as a function of the mean optical photon number and the microwave port dissipation in Fig. 8 for two different total optical linewidths $\gamma_{a,\text{tot}}$. For $\gamma_{a,\text{tot}} = 2\pi \times 1.6\ \text{THz}$ the system is in the sideband unresolved regime of the optical cavity, since the linewidth is larger than the mode frequency of the phonon ($\omega_b = 2\pi \times 10\ \text{GHz}$). In this regime it is not important that the optical detuning is chosen precisely. When we compare the two cases, we conclude that both allow to reach the same conversion efficiencies. However, in the sideband unresolved regime we need much stronger optical pump powers to obtain similar results as for the unresolved regime, due to the stronger dissipation.
D. Weak inductive coupling regime

In the previous sections, we have assumed a mode overlap between magnon and microwave of 1%, which, based on our estimates presented in the appendix, yields a coupling value of $g_{mc} = 180$ MHz. Nevertheless, such overlap is challenging to achieve experimentally, since the system's design would require a balance between a sufficiently small sample volume, which yields a high optomechanical coupling, while maintaining the volume big enough as to achieve a high magnon-microwave cooperativity. In this section we address such design limitation by quantifying the performance of our setup under a magnon-microwave mode overlap of 0.01%, corresponding to $g_{mc} = 18$ MHz.

For this value of the coupling, the cooperativity between magnon and microwave is $C_{mc} = 13$. Based on our discussion of optimization in terms of cooperativities (following Eq. (12)), we still expect high conversion efficiency for sufficiently strong optical pump power. The reduced coupling strength puts the system in the weak inductive coupling regime since $g_{mc} < \gamma_{c,\text{tot}}$. For small optical pump powers, magnons hybridize with the phonons instead of with microwaves. As a consequence, the conversion efficiency as a function of both frequency and optical pump power, depicted in Fig. 9 (a), exhibits two peaks shifted by $g_{mb} = 10$ MHz from the resonance point. For increasing pump power the system approaches the strong optomechanical coupling regime. Around $P = 2.5 \times 10^{-3}$ W the system reaches the point...
of largest conversion efficiency (compare Fig. 9 (c), (d)) where the phonon linewidth in the resonance spectrum, shown in Fig. 9 (b), is strongly broadened, yielding a conversion bandwidth of up to 28 MHz. Note that for this pump power the system is not yet in the strong optomechanical regime \( G_{ab} \approx 100 \text{ MHz} < \gamma_a, \omega_c / 4 \). For a stronger pump power, the optomechanical coupling surpasses the magnomechanical coupling, and phonons hybridize strongly with optical photons. After transition to the strong optomechanical regime, the hybridization between magnons and phonons is weak, and as a consequence, both the bandwidth and the overall efficiency decrease.

Compared to the results discussed in the previous sections, we obtain a much larger conversion bandwidth at a similar conversion efficiency. Therefore, smaller magnon-microwave overlaps can still yield good conversion efficiencies at broad bandwidths. However, the high required optical pump power (and therefore large mean photon number on the order of \(10^6\)) strongly limits the configuration to application in the classical regime.

IV. CONCLUSIONS

In summary, we derived the efficiency of a two-stage optics-to-microwave conversion system that incorporates hybridized mechanical and magnetic excitations as mediating elements. The conversion efficiency exhibits a spectrum that can be understood in terms of the hybrid modes formed by the strong interaction of the coupled modes. In a setup in which magnons, microwaves and phonons are resonant while the laser drive is red detuned, the efficiency can be expressed solely in terms of the cooperativities, and we have derived a condition for the maximum efficiency as a function of the cooperativities which can be used to determine optimal values for port coupling rates and optical pump power. Furthermore, we discussed the conversion bandwidth for different optical pump powers and the effect of total linewidth of the optical cavity mode. Our results do not take into account any specific geometry and can be used as a starting point for designing a magnomechanical-based microwave to optics converter.

We have shown that high conversion between optical and microwave modes is possible by the resonant coupling between phonons and magnons. In regions where the conversion efficiency is large we obtain bandwidths on the same order of magnitude as the coupling rates. This is an advantage over state-of-the-art one-stage conversion setups based on e.g. optomechanics/electromechanics, which are generally limited by their small total linewidth of the vibrational mode. Moreover, the inclusion of magnons adds a desirable tunability to the system and their coupling to phonons overcomes the challenges of weak optomagnonic interactions.

The work presented in this article applies to a fully classical framework and application in quantum conversion requires the evaluation of the added noise, which in turn contributes to the total state transduction fidelity. In general we expect that vacuum and thermal noise is added during each step in the conversion chain \(64\). Since the optical mode frequency is in the THz regime, thermal occupation for optics is expected to be negligible or vanish, even for room temperature, thus contributing only with vacuum noise. The added thermal noise for the microwave is suppressed when the port is strongly overcoupled and the device is operated at sufficiently low temperature. To assure low added quantum noise from the intermediate elements (magnons, phonons) we expect constraints that are potentially stricter than those required for high conversion efficiency \(65\). Finally it is important to note that even though the conversion efficiency is the same in both directions, the noise adds differently to the optic and microwave ports.

A prospective geometry where the proposed system could be implemented would have to fulfill strict requirements in terms of frequency matching, and ensure good overlap and symmetry matching of the participating modes in order to achieve high coupling rates. Patterned magnetic dielectrics at the microscale in the form...
FIG. 9. (a) Conversion efficiency as a function of frequency and optical pump power (corresponding mean photon number on right y-axis). The frequency axis is given in units of $\omega_c$, the microwave frequency and we have set $\omega_b = \omega_m = \omega_c = -\Delta_a$. (b) Frequency spectrum of the resonances of the coupled system following Eqs. (9a) to (9d). In this case the blue peaks correspond to the magnon-phonon hybrid modes. Since the system is still in the weak optomechanical coupling regime for the used optical pump power of $P = 2.5 \times 10^{-3}$ W ($C_{ab} = 28118$, $C_{mc} = 13$), the phonon (yellow) and photon (green) are not shifted. (c) Conversion efficiency as a function of frequency for the point of highest efficiency at a pump power of about $P = 2.5 \times 10^{-3}$ W. The remarkably large bandwidth at this point is marked with 28 MHz. (d) Conversion efficiency as a function of optical pump power.

of an optomechanical crystal can be promising in this regard [34]. Magnon-phonon coupling in this kind of structures remains to be investigated. Our choice of 1 $\mu m^3$ for the effective magnon mode volume at 1% mode overlap with the microwave field also requires improvement of microwave resonator design capable to achieve such a small microwave mode volume, for example through a planar geometry. Alternatively, one could build an overall larger geometry, at the expense of a reduced single-particle optomechanical coupling. The drawbacks of both approaches can be partially resolved by increasing the optomechanical cooperativity (see Fig. 3), e.g. via a stronger optical drive. We have discussed the case of a worse mode overlap between magnons and microwaves of only 0.01%, which yields comparable results for conversion efficiency with a larger conversion bandwidth. A well optimized device would represent a viable technique for overcoming the challenge of efficient quantum frequency conversion, and pave the way towards long distance quantum communication.

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APPENDIX: COUPLING DERIVATIONS AND ESTIMATES

1. Magnomechanical coupling

Magnetic properties inside a material are defined by the collective interaction of its atomic magnetic moments. Mechanical deformation alters this collective interaction and gives rise to coupling between mechanical and magnetic degrees of freedom. This magnomechanical interaction is captured by the magnetoelastic anisotropy energy, as it is the only energy that relates the magnetization direction with the crystal axes. Formally, the magnetoelastic anisotropy energy is defined as the difference between the energies required to magnetize a material along an easy axis and a hard axis. The magnetoelastic energy is given by

\[ E_{\text{mel}} = \frac{B_1}{M_S^2} \int d^3r \left[ M_x^2 \sigma_{xx} + M_y^2 \sigma_{yy} + M_z^2 \sigma_{zz} \right] + 2 \frac{B_2}{M_S^2} \int d^3r \left[ M_x M_y \sigma_{xy} + M_y M_z \sigma_{yz} + M_z M_x \sigma_{zx} \right]. \] (18)

We quantize this energy in terms of both spin waves (magnons) and mechanical modes (phonons). Let the total magnetization be defined as a static magnetic ground-state plus a dynamic deviation on top, so \( m(r,t) = m_S(r) + \delta m(r,t). \) Note that here we introduced the normalized magnetic field components \( m = M/M_S. \) The temporally dynamic deviation is quantized via bosonic annihilation and creation operators

\[ \delta m_\alpha(r,t) = \sum_\alpha \delta m_\alpha(r) e^{-i\omega_\alpha t} \hat{m}_\alpha + \delta m_\alpha^*(r) e^{i\omega_\alpha t} \hat{m}_\alpha^+, \] (19)

where we are summing over multiple modes \( \alpha. \) The mechanical modes are quantized in the same way

\[ \hat{u}(r,t) = \sum_\eta \hat{u}_\eta(r) \hat{b}_\eta e^{-i\omega_\eta t} + \hat{u}_\eta^*(r) \hat{b}_\eta^+ e^{i\omega_\eta t}, \] (20)

where we again sum over multiple modes \( \eta. \) Note that in the definition of the magnetoelastic energy (Eq. [19]) we have elements of the symmetric strain tensor

\[ \sigma_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \] (21)

where \( i, j \in (x, y, z), \) so we have the spatial derivatives of the mechanical mode profiles entering the coupling. The quantized magnomechanical coupling Hamiltonian reads

\[ \hat{H}_{\text{ME}} = \sum_{\alpha \beta \eta} G_{\alpha \eta} \hat{m}_\alpha (\hat{b}_\eta + \hat{b}_\eta^+) + T_{\alpha \beta \eta} \hat{m}_\alpha \hat{m}_\beta (\hat{b}_\eta + \hat{b}_\eta^+) \]
\[ + P_{\alpha \beta \eta} \hat{m}_\beta \hat{m}_\alpha (\hat{b}_\eta + \hat{b}_\eta^+) + h.c., \] (22)
with the coupling strengths defined as the following overlap integrals

\[ G_{\alpha\eta} = 2B_1 \int d^3r \sum_i (m_i^\alpha \delta m_i^\eta \partial_t u_i^\eta + ) \]

\[ B_2 \int d^3r \sum_{ij} (m_i^\alpha \delta m_j^\eta + m_j^\alpha \delta m_i^\eta) \times (\partial_j u_i^\eta + \partial_i u_j^\eta) \]

\[ T_{\alpha\beta\eta} = B_1 \int d^3r \sum_i (\delta m_i^\alpha \delta m_i^\beta (\partial_j u_i^\eta + \partial_i u_j^\eta) + ) \]

\[ B_2 \int d^3r \sum_{ij} (\delta m_i^\alpha \delta m_j^\beta (\partial_j u_i^\eta + \partial_i u_j^\eta) + ) \]

The couplings are normalized by the total quasi-particle numbers of both magnons and phonons. The phonon number is calculated using the mechanical energy contained in given mode \( \eta \)

\[ N_{\text{phonon} \eta} = \frac{2m^2\omega_b V_{\text{eff}} \max[u(r)]^2}{\hbar \omega_\eta} \]

where we have used the definition of the kinetic energy. The magnetic part is normalized by the average number of spins in a given magnetic mode \( \alpha \)

\[ N_{\text{mag} \alpha} = \frac{M_S V_{\text{mag} \alpha}}{g_B} \]

For linear magnomechanical interaction \( g_{mb} \hat{b} \hat{b}^\dagger \) + \text{h.c.} the coupling strength is given by

\[ g_{mb}^\alpha\beta = 2B_1 \int d^3r \sum_i (m_i^\alpha \delta m_i^\beta \sigma_{ij}^{\alpha\beta} + ) \]

\[ + B_2 \int d^3r \sum_{ij} (m_i^\alpha \delta m_j^\beta + m_j^\alpha \delta m_i^\beta) \sigma_{ij}^{\alpha\beta} \]

where \( m_0 \) denotes the magnetic ground state, \( \delta m \) the magnetic mode and \( \sigma_{ij} \) the symmetric strain tensor. \( B_1, B_2 \) are the material specific magnetoelastic constants and we have modes \( (\alpha, \beta) \) and spatial indices \( (i, j) = (x, y, z) \). Eq. (28) represents an overlap integral over the crystal volume. Normalizing by the total magnon and phonon numbers inside given respective modes, we can estimate the single mode coupling for perfect mode overlap, so \( V_{\text{max}} = V_{\text{eff}} = V \). In this case, the integral is also equal to \( V \) and the coupling is independent of the mode volumes, so

\[ g_{mb} \approx (2B_1 + B_2) \frac{h\delta m \delta m}{2\omega_b \max[u]^2 M_S} \]

Here, \( g \) is the gyromagnetic ratio, \( \mu_B \) the Bohr magneton, \( \rho \) the mass density and \( M_S \) the saturation magnetization. The volumes \( V_m \) and \( V_b \) correspond to spatial mode volumes of the magnetic and mechanical mode respectively. Furthermore, \( \max[u] \) is defined to be the maximum value of the displacement field corresponding to the zero point fluctuation \( x_{\text{ZPF}, \eta} \) of a given mechanical mode \( \alpha \). Using material specific constants for Yttrium Iron Garnet (YIG) and assuming perfect overlap of mechanical and magnetic mode we obtain coupling rates of 2.5 MHz and 12 MHz for the terms proportional to \( B_1 \) and \( B_2 \), respectively. The mechanical mode frequency was chosen to \( \omega_b = 2\pi \times 10 \text{ GHz} \).

2. Photoelastic coupling

The interaction between phonons and optical photons arises mostly from the photoelastic effect \[66\]. However, the total coupling strength is also affected by the deformation of the boundary. This contribution is usually on the order of 10% of that of the photoelastic effect and is difficult to estimate since it has to be evaluated for a specific geometry. For our estimate we will neglect this contribution. A mechanical mode deforms the optical cavity therefore altering its resonance frequency \( \omega_b \). This requires that a potential coupling device serves both as mechanical resonator and optical cavity. We introduce a
a small perturbation in the permittivity tensor of the material via \( \bar{\varepsilon} \) and \( \bar{\delta} \). To first order, we obtain the shift of the resonance frequency for a given optical mode profile \( \hat{E}_\text{optical} \) and write the photoelastic overlap integral as [21]

\[
E_{\text{photoelastic}} = -\varepsilon_0 n^4 \int d^3 r \ e \cdot \bar{\varepsilon} \hat{E}_\text{optical} = -\varepsilon_0 n^4 \int d^3 r \left[ 2 Re \left\{ E_i^* E_j \right\} p_{44} u_{ij} + |E_i|^2 \left( p_{11} \sigma_{ii} + p_{12} \left( \sigma_{jj} + \sigma_{kk} \right) \right) \right].
\]

The optical modes are quantized via bosonic operators \( \hat{E}_\mu, \nu, \eta \)

\[
\hat{E} \left( r, t \right) = \sum_{\mu} \left[ \hat{E}_\mu \left( r \right) e^{-\imath \omega_\mu t} \hat{a}_\mu + \hat{E}_\mu^* \left( r \right) e^{\imath \omega_\mu t} \hat{a}_\mu^\dagger \right].
\]

Together with the quantization of the mechanical mode (Eq. [27]), we obtain the quantized photoelastic interaction Hamiltonian as

\[
\hat{H}_\text{PE} = \sum_{\mu, \nu, \eta} g_{\mu \nu \eta} \hat{a}_\mu^\dagger \hat{a}_\nu \left( \hat{b}_\eta^\dagger + \hat{b}_\eta \right),
\]

with coupling strength defined as the overlap integral

\[
g_{\mu \nu \eta} = -\varepsilon_0 n^4 \int d^3 r \left[ \sum_{ij} \left( E_{\mu}^i \right)^* E_{\nu}^j \sigma_{ij}^{\eta} p_{44} + \sum_{ijk} \left( E_{\mu}^i \right)^2 \left( \sigma_{ij}^{\eta} p_{11} + \left( \sigma_{ij}^k + \sigma_{kk}^\eta \right) p_{12} \right) \right].
\]

where we have modes \((\mu, \nu, \eta)\) and spatial indices \((i, j, k) = (x, y, z)\). This integral is again normalized to coupling per photon (Eq. [26]) and photon. The number of photon contained in an optical mode \( \alpha \) is calculated via

\[
N_{\text{photon}, \mu} = \frac{2V_{\text{eff, photon}, \mu} \cdot \max \left[ |E|^2 \right]}{\hbar \omega_{\text{photon}, \mu}},
\]

where we have used the definition of the total electric energy. The normalized coupling for photoelastic interactions neglecting the boundary contributions reads

\[
g^{\mu \nu \eta}_{\text{eff}} = -\frac{n^4 \hbar \omega_\eta}{\epsilon_r 2 V_a V_b} \sqrt{h \omega_{\text{photon}, \mu}} \left( \frac{2 \omega_{\text{photon}, \mu} V_b \max |u|^2}{\hbar \omega_{\text{photon}, \mu}} \right) \times \int d^3 r \left[ \sum_{ij} \left( e_{\mu}^i \right)^* e_{\nu}^j \sigma_{ij}^{\eta} p_{44} + \sum_{ijk} \left| e_{\mu}^i \right|^2 \left( \sigma_{ij}^{\eta} p_{11} + \left( \sigma_{ij}^k + \sigma_{kk}^\eta \right) p_{12} \right) \right],
\]

where \( n \) is the refractive index, \( \epsilon_r \) the relative permittivity, \( e \) the electric field components and \( p \) the elements of the material specific photoelastic tensor. For perfect mode overlap, the single mode coupling approximates to 0.2 MHz for material specific constant of YIG. We consider a phonon mode frequency of \( \omega_\eta = 2\pi \times 10 \text{ GHz} \) and an optical mode frequency of \( \omega_\alpha = 2\pi \times 200 \text{ THz} \).

### 3. Magnon-microwave coupling

The linear interaction between a microwave cavity field and the magnetic excitations is mediated by magnetic dipole interaction, captured by the energy term

\[
E_{\text{mag}} = \frac{1}{2} \mu_0 \int d^3 r \hat{M} \cdot \mathbf{H}.
\]

The magnetic energy can be quantized in terms of cavity modes. Following Eq. [31], but for a magnetic field profile, we have

\[
\hat{H}_\text{mag} = \sum_{\eta} \hat{H}_\eta \left( r \right) e^{-\imath \omega_\eta t} \hat{c}_\eta + \hat{H}_\eta^* \left( r \right) e^{\imath \omega_\eta t} \hat{c}_\eta^\dagger.
\]

and normalized to a coupling per microwave photon via

\[
N_{\text{mag-photon}, \mu} = \frac{2V_{\text{eff, mag-photon}, \mu} \cdot \max |H|^2}{\hbar \omega_{\text{mag-photon}, \mu}}.
\]

and magnon using Eq. [27]. The normalized overlap integral reads

\[
g_{\text{mc}} = \frac{1}{2} \sqrt{\mu_0 \hbar c g_B M_S} \int d^3 r \delta \mathbf{m}_\alpha \cdot \mathbf{h}_\eta,
\]

where a magnetic mode \( \delta \mathbf{m} \) couples to the magnetic part \( \mathbf{h} \) of an electromagnetic microwave field. For perfect mode overlap, the single mode coupling strength is evaluated to

\[
g_{\text{cm}} \approx \frac{g \mu_B \beta h}{2h} \sqrt{\mu_0 \hbar c \rho_S},
\]

where \( \rho_S \) is the material specific spin density. The coupling can be evaluated up to 1.85 GHz for a microwave mode frequency of \( \omega_c = 2\pi \times 10 \text{ GHz} \). Perfect overlap is challenging to achieve experimentally, therefore we have considered an overlap factor of 1%, such that \( V_m = 0.01 V_c \), yielding a coupling rate of 180 MHz. We further discuss the performance of the system under a worse mode overlap of 0.01% (\( V_m = 0.0001 V_c \)), for which the coupling is 18 MHz.

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