Hard X-ray or Gamma Ray Source Based on the two-stream instability and Backward Raman Scattering

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A new scheme of hard x-ray or gamma ray light source is considered. The excitation of the Langmuir wave in an ultra dense electron beam via the two-stream instabilities and the interaction of the excited Langmuir wave with the visible light-laser results in hard x-ray or gamma ray via three-wave interaction. The analysis suggests that the hard x-ray with the wave-length as small as 0.03 nm can be achieved. The plausible parameter for the practical use is proposed and the comparison with the conventional methods are provided.

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Since its initial discovery by Wilhelm Rontgen, the x-ray has been utilized in many applications and its numerous light sources have been invented [3–5]. With the great advances in other scientific areas, there have been exponentially growing interests for intense x-ray light applications. An intense hard x-ray or gamma ray source would open up many possible commercial applications, including the atomic spectroscopy [10–12], the dynamical imaging of fast biological processes [13, 14] and the next-generation semi-conductor lithography [15–18]. Yet, it is very hard to achieve intense hard x-ray or gamma ray source [6–9, 19–22].

Noting the recent progress in the dense electron beam source [23, 24] and the intense visible-light lasers [25–27], the author proposes a new scheme of a hard x-ray or gamma ray laser based on the two-stream instability and the backward Raman scattering (BRS). If two electron beams propagate in the same direction with the different drift velocities, the Langmuir waves could be excited inside the electron beams via the two-stream instability, and the BRS of the excited Langmuir waves with a counter-propagating visible-light laser could result in the hard x-ray or gamma ray. The current work has two major merits. First, the seed x-ray is very hard to generate but pre-requisite for the x-ray generation via the BRS, and the technological options to create an intense seed are severely limited so far. If the Langmuir waves (or the seed x-ray) are generated via the two-stream instability, there is no need to create an intense seed. Second, as the Langmuir waves are directly excited by the two-stream instability, not by the ponderomotive interaction between a seed x-ray and an intense laser, the required intensity of a visible light laser is considerably lower. The analysis suggests that the hard x-ray up to the wave length of 0.03 nm is possible and the requirement of the laser intensity is moderate. The regime of the practical interest is identified and the advantage (disadvantage) over the straight use of the BRS [28–31] are discussed.

To begin with, Consider two dense relativistic electron beams with the relativistic factor $\gamma_0 < \gamma_1$, where $\gamma_0 = (1 - \beta_1^2)^{-1/2}$, $\gamma_1 = (1 - \beta_2^2)^{-1/2}$, $\beta_1 = v_1/c$ ($\beta_2 = v_2/c$) and $v_1$ ($v_2$) is the velocity of the electron beam 0 (1). For simplicity, the electron density of two beams are assume to be the same $n_0$. Due to the initiative from the fast igniter concept of the inertial confinement fusion [23, 24, 32–37], the electron density as high as $n_0 \equiv 5 \times 10^{23}$ /cc are being contemplated, to which the electron density is assumed to be comparable. It is most convenient to use the co-moving frame with the electron beams. In this frame, one of the electron beam is stationary and the other beam has the drift velocity given as $v_0 = (v_1 - v_2)/(1 - \beta_1 \beta_2)$. In the co-moving frame, the both beams have the density $n_e \equiv n_0/\gamma_0$. An appropriate Langmuir wave excited by the two-stream instability and the visible-light laser could emit the x-ray (seed pulse) in the beam direction via the BRS. Conventionally, the visible-light laser (the emitted hard x-ray) is called as the pump laser (seed laser). Denoting the wave vector of the visible-light laser in the laboratory frame as $k_0$, it will be derived later that the appropriate Langmuir wave for the BRS has the wave vector $k_3 \equiv 4\gamma_0 k_0$. Then, the higher the $k_3$ is, the higher the frequency of the x-ray can be emitted; it is important to estimate the possible highest wave vector of the Langmuir wave excitable by the two-stream instability. The criteria of the two-stream instability would be that 1), for a fixed wave-vector $k$, there is the local maxima of the dielectric function $\epsilon$ as a function of the wave frequency $\omega$ and 2), the value of the local maxima is less than zero. The longitudinal dielectric function of a plasma is given as

$$\epsilon(k, \omega) = 1 + \frac{4\pi\epsilon^2}{k^2} \sum \chi_i. \quad (1)$$

where the summation is over the group of particle species and $\chi_i$ is the particle susceptibility. In classical plasmas, the susceptibility is given as

$$\chi_i(k, \omega) = \frac{n_i Z_i^2}{m_i} \int \frac{[k \cdot \nabla f_i]}{\omega - k \cdot \mathbf{v}} \mathrm{d}^3\mathbf{v} \quad (2)$$

where $m_i$ ($Z_i$, $n_i$) is the particle mass (charge, density) and $f_i$ is the distribution with the normalization
\[ \int f \, d^3v = 1. \] For the case of our two group of electrons, it is given as

\[ \epsilon = 1 + \left(4\pi e^2/k^2\right)(\chi_e^C(\omega, k) + \chi_e^C(\omega - k \cdot v_0, k)). \]  

One example, where the Langmuir wave is susceptible to the two-stream instability, is shown in Fig. 1. In Fig. 2, we draw the threshold wave vector \( k_e \) over which all the plasma Langmuir waves become stable to the two-stream instability. In the figure, we plot the maximum \( k_e(v_0) \) as a function of the drift velocity \( v_0 \) for three different electron temperature. Such analysis suggests that the optimal regime is characterized by 2.5 < \( kv_0/\sqrt{\omega_{pe}} \) < 3.5 and \( k\lambda_{de} < 0.5 \). In general, the most prominent condition is

\[ k_3\lambda_{de} \leq 0.5. \]  

For a fixed electron temperature and the electron density, there is a lower bound of the drift velocity \( v_0/c \) under which the plasma is stable to the two-stream instability for any wave vector, which is also illustrated in Fig. 2.

Eq. (4) suggests that the lower electron temperature the plasma has, the higher Langmuir wave vector can be excited via the two-stream instability. However, the electron temperature is limited by the energy spread of the electron beams, which cannot be lowered to a certain degree. If the beam has the energy spread \( \delta E/E \) in the laboratory frame, the electron temperature spread in the co-moving frame is given as \( T_e \approx (\delta E/E)^2m_e c^2 \) from the fact that \( \delta E/E \approx \delta v/c \). If the energy spread is 0.01 < \( \delta E/E < 0.1 \), then the electron temperature would be 25 eV < \( T_e < 2.5 \) keV. As shown in the figure, for the electron temperature of 200 eV, the maximum wave vector is given as \( k_e \approx 0.5/\lambda_{de} \) when \( v_0/c \approx 0.15 \).

If \( c k_3/\omega_3 \approx \omega_0/\sqrt{\omega_{pe}} \gg 1 \), then \( k_3 \approx 4\omega_0 k_{p0} \) and \( \omega_3 \approx \omega_{pe}/\sqrt{70} \). The condition given in Eq. (4) can be rewritten as

\[ \lambda_{de}^L < \frac{1}{k_{p0}} \times \left(\frac{k_{p0}}{k_{p0}}\right), \]  

where \( \lambda_{de}^L = \sqrt{T_e/4\pi n_{e0} e^2} \), \( k_{p0} \) is the wave vector of the x-ray and \( k_{p0} \) is the wave vector of the visible-light laser. For a fixed \( k_{p0} \), the lower the \( k_{p0} \) is, the higher the electron temperature can be. For this reason, the infra-red laser with the wave length of 10 \( \mu \)m to 20 \( \mu \)m is more advantageous than ND:YAG laser with the wave length of 1 \( \mu \)m.

With an appropriate Langmuir wave excited by the two-stream instability, the 1-D BRS three-wave interaction in the co-moving frame between the pump, the seed and a Langmuir wave is described by [8, 9]:

\[ \left(\frac{\partial}{\partial t} + v_p \frac{\partial}{\partial x} + \nu_1\right) A_p = -ic_p A_s A_3, \]  

where

\[ A_i = eE_{i1}/m_e \omega_{i1} c \]  

is the ratio of the electron quiver velocity of the pump pulse \( (i = p) \) and the seed pulse \( (i = s) \) relative to the velocity of the light c, \( E_{i1} \) is the electric field of the E&M pulse, \( A_i = \delta n_i/\nu_i \) is the the Langmuir wave amplitude, \( \nu_i \) is the plasma decay rate, \( c_i = \omega_i^2/(2\omega_{i1} \) for \( i = p, s \), \( c_i = (ck_3)^2/2\omega_3 \), \( \omega_3 \) is the wave frequency of the x-ray (the pump laser) and \( \omega_3 \approx \omega_{pe}/\sqrt{70} \) is the plasmon wave frequency. In the co-moving frame with the electron beam, the wave vector of the light wave satisfies the usual dispersion relationship, \( \omega_i^2 = 2\omega_{pe}^2/\gamma_0 + c^2 k_i^2 \), where \( \omega_1 (k_1) \) is the wave frequency (vector). Denote the wave vector (the corresponding wave frequency) of the pump laser (the seed pulse or soft x-ray) in the co-moving frame as \( k_{p1}, k_{s1}, \omega_{p1} \) and \( \omega_{s1}, \) and the laboratory-frame counterparts as \( k_{p0}, k_{s0}, \omega_{p0} \) and \( \omega_{s0} \). The Lorentz transform prescribes the following relationships:

\[ \omega_{p0} = \gamma_0 \sqrt{2\omega_{pe}^2/\gamma_0 + c^2 k_{p1}^2} - vk_{p1}, \]  

\[ k_{p0} = \gamma_0 \left[k_{p1} - \frac{\omega_{p1} v_0}{c} \right], \]  

\[ \omega_{s0} = \gamma_0 \left[\sqrt{2\omega_{pe}^2/\gamma_0 + c^2 k_{s1}^2} + vk_{s1} \right], \]  

\[ k_{s0} = \gamma_0 \left[k_{s1} + \frac{\omega_{s1} v_0}{c} \right]. \]  

Using Eqs. (7), (8), (9) and (10), the pump laser (seed pulse or soft x-ray) can be transformed from the co-moving frame to the laboratory frame or vice versa. The
energy and momentum conservation of Eq. (10) leads to
\[ \omega_{p1} = \omega_{s1} + \omega_3, \]
\[ k_{p1} = k_{s1} + k_3, \]  
(11)
where \( k_3 \) is the plasmon wave vector. For a given pump frequency \( \omega_{p0} \), \( k_{p1} \) (\( \omega_{p1} \)) is obtained from Eq. (7), \( k_{s1} \) (\( \omega_{s1} \)) is from Eq. (11) and \( k_{s0} \) (\( \omega_{s0} \)) is from Eqs. (9) and (10). In the limit when \( c k_{s1} \gg \omega_3, \omega_{s0} \approx 2 \gamma_0 (\omega_{p1} - \omega_3) \) or
\[ \omega_{s0} \approx 4 \gamma_0^2 \left[ \omega_{p0} - 2 \sqrt{2} \omega_{pe} (\gamma_0)^{-3/2} \right], \]  
(12)
using \( \omega_{p1} \approx 2 \gamma_0 \omega_{p0} \) and \( \omega_3 \approx \omega_{pe} / \sqrt{\gamma_0} \). The equation (12) describes the frequency up-shift of the pump pulse into the hard x-ray by the relativistic Doppler’s effect.

The first one in Eq. (11) is the most relevant for us as the excited Langmuir wave is given by \( A_3 = \delta n_e / n_e \). The mean-free path of the laser to the BRS is estimated to be
\[ l_b \approx c (2 \sqrt{\omega_3} \omega_{p1} / \omega_3^2) (1/A_3). \]  
(13)
The mean-free path from the Thomson scattering (the Compton scattering) is \( l_t = 1 / n \sigma_t \) with \( \sigma_t = (m c^2 / e^2)^2 \). For an example, when \( n_l = 5 \times 10^{23} / \text{cc} \), \( l_t = 0.2 \text{ cm} \) and \( l_b \approx (10^{-6} / A_3) \omega_{s1} / \omega_{s3} \) cm. Even for \( A_3 \approx 0.001 \), the hard x-ray radiation by the BRS is considerably stronger than the Thomson scattering or \( l_t \gg l_b \).

As the first example, consider the electron beams with \( n_0 = 5 \times 10^{23} / \text{cc}, \gamma_0 = 200 \) and \( T_e = 25 \text{ eV} \). For the visible-light laser with \( \lambda = 10 \mu \text{m}, k_3 \lambda_{de} = 0.37 \). The Langmuir wave for the BRS will be unstable when \( 0.06 < v_0/c < 0.08 \) as shown in Fig. 2. For the visible-light laser with \( \lambda = 20 \mu \text{m}, k_3 \lambda_{de} = 0.18 \). The Langmuir wave for the BRS will be unstable when \( 0.12 < v_0/c < 0.16 \). The emitted light will have the wave length of \( 0.06 \text{ nm} \) (0.12 nm) for \( \lambda = 10 \mu \text{m} \) (\( \lambda = 20 \mu \text{m} \)). As the second example, consider the same beam but with \( \gamma_0 = 100 \) and \( T_e = 200 \text{ eV} \). For the visible-light laser with \( \lambda = 10 \mu \text{m} \) (\( \lambda = 20 \mu \text{m} \)), \( k_3 \lambda_{de} = 0.37 \) (\( k_3 \lambda_{de} = 0.18 \)). The Langmuir wave for the BRS is unstable for the visible-light laser with when \( 0.1 < v_0/c < 0.15 \) (\( 0.1 < v_0/c < 0.3 \)). The emitted light will be the wave length of 0.25 nm (0.5 nm) for \( \lambda = 10 \mu \text{m} \) (\( \lambda = 20 \mu \text{m} \)). As the third example, consider the same electron beam with \( T_e = 1 \text{ keV} \). For the visible-light laser with \( \lambda = 10 \mu \text{m} \) (\( \lambda = 20 \mu \text{m} \)), \( k_3 \lambda_{de} = 0.83 \) (\( k_3 \lambda_{de} = 0.43 \)) and the plasma Langmuir waves is stable to the two-stream instability for \( \lambda = 20 \mu \text{m} \) when \( 0.3 < v_0/c < 0.35 \). The emitted light will be the wave length of \( 0.5 \text{ nm} \) for \( \lambda = 20 \mu \text{m} \).

In summary, we propose a new scheme of a gamma ray or hard x-ray based on the two-stream instability and the backward Raman scattering. The excitation of Langmuir waves in an ultra dense relativistic electron beam via the two-stream instabilities and the subsequent interaction of the excited Langmuir waves with the infra-red laser via the backward Raman scattering results in the hard x-ray and gamma ray. With the highest electron density possible with the current technologies, the gamma ray or hard x-ray in the range of 1 keV < \( \hbar \omega < 50 \) keV is possible.

In comparison to the previous schemes, the current scheme has some disadvantages and advantages. One disadvantage is the requirement of the low-energy spread of the electron beams; two electron beams with very low-energy spread are needed in order to excite the Langmuir wave with the highest possible wave vector. There are a few advantages. First, the required intensity of the visible-light laser is lowered considerably in the current scheme. As discussed in Eq. (5), the required laser intensity is too high for the infra-red laser under the previous scheme but achievable under the current scheme. Second, the previous methods need an uniform electron beam but the current scheme has much relaxed requirement of the uniformity. Third, the inverse bremsstrahlung is not a concern in the current scheme. In the regime of our interest (\( n_e = 10^{23} - 10^{24} / \text{cc} \)), the inverse bremsstrahlung rate is as high as \( 0.01 \times \omega_{pe} \) and the heating by the inverse bremsstrahlung increases the electron temperature in a few hundred Langmuir periods if the pump laser is very intense as in the direct BRS. After the heating from the inverse bremsstrahlung, the appropriate Langmuir wave could be no longer excited due to the Landau damping. But, in the current scheme, the intensity of the pump laser can be very low, so that the inverse bremsstrahlung is less of concern.

In this paper, it is assumed that the Langmuir wave for the BRS will be excited as long as it unstable to the two-stream instability. However, the future work needs to check through the simulation and the theoretical analysis how intense the excited Langmuir wave is. The full adequacy of the current scheme can be further validated.
by the study along this line, which is beyond the scope of this paper.

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