Bound on the tau neutrino magnetic moment from the TRISTAN experiments

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Abstract

We set limits on the magnetic moment and charge radius of the tau neutrino by examining an extra contribution to the electroweak process $e^+e^- \rightarrow \nu\nu\gamma$ using VENUS, TOPAZ and AMY results. We find that $\kappa(\nu_\tau) < 9.1 \times 10^{-6}$ (i.e. $\mu(\nu_\tau) < 9.1 \times 10^{-6} \mu_B$, $\mu_B = e/2m_e$) and $\langle r^2 \rangle < 3.1 \times 10^{-31}$cm$^2$ with Poisson statistics by combining their results. Whereas, similar to this method, with the Unified Approach we find that $\kappa(\nu_\tau) < 8.0 \times 10^{-6}$ and $\langle r^2 \rangle < 2.7 \times 10^{-31}$cm$^2$.

Key words: tau neutrino, magnetic moment, charge radius, Unified Approach
The electromagnetic properties of neutrinos have been vigorously examined in recent years, since they are related to the neutrino mass and to solar neutrino problems. Neutrino oscillation between $\nu_\mu$ and $\nu_\tau$, which means a finite neutrino mass, has become realistic since evidential results by Super-Kamiokande [1]. Gninenko [2] has shown the bound on the tau neutrino magnetic moment from the S-K atmospheric neutrino data to be $1.3 \times 10^{-7} \mu_B$. Chua and Hwang [3] recently remarked that the third-generation neutrino magnetic moment induced by leptoquarks might be of the order of $10^{-10} \sim 10^{-13} \mu_B$.

If neutrinos have mass, the tau neutrino would be the most massive among $\nu_e$, $\nu_\mu$ and $\nu_\tau$. Therefore, the tau neutrino magnetic moment might show a relatively large value because the neutrino magnetic moment is estimated to be proportional to its mass according to the standard model extended to have the right-handed neutrino singlet ($\nu_R$), $\mu_\nu = 3eG_F m_\nu/(8\pi^2\sqrt{2}) = (3.20 \times 10^{-19})m_\nu\mu_B$ [4]. Assuming $m_\nu < 18.2$ MeV [5], the upper limit on the magnetic moment is less than $5.8 \times 10^{-12} \mu_B$, where $e$, $G_F$, $m_\nu$ and $\mu_B$ are the electron charge magnitude, the Fermi coupling constant, the neutrino mass in eV and the Bohr magneton ($= e/2m_e$), respectively. Thereupon, we estimated the tau neutrino magnetic moment and charge radius limits with the $e^+e^-\rightarrow \nu\sigma\gamma$ results from three TRISTAN experimental groups.

We classify the methods to evaluate the tau neutrino magnetic moment as follows: (A) cosmological estimation, (B) fixed target experiments ($\nu_\tau e^-\rightarrow \nu_\tau e^-$) and (C) $e^+e^-$ colliding beam experiments. Method(A) gives a very strong upper limit of the magnetic moment such as $\mu_\nu < 6.2 \times 10^{-11} \mu_B$ [6], $2 \times 10^{-12} \mu_B$ [7] and $6 \times 10^{-14} \mu_B$ [8]. However, these values are based on many cosmological assumptions. Incidentally, Grifols and Massó [9] have argued that primordial nucleosynthesis also constrains the neutrinos charge radii to satisfy $\langle r^2 \rangle < 7 \times 10^{-33}$ cm$^2$. Their argument, however, also has an implicit dependence on the neutrino mass which may allow them to be evaded. Method(B) gives $\mu_\nu < 5.4 \times 10^{-7} \mu_B$ [10]. It assumes the form factor ratio of $f_{D_s}/f_\pi = 2$ and $D_s$, $\overline{D_s}$ production cross section $= 2.6 \mu$b to calculate $\nu_\tau$ flux, because $\nu_\tau$ beam flux has to be produced and estimated by $D_s$, $\overline{D_s}$ production. Method(C) is the most direct. Groth and Robinett [11] combined the results from ASP, MAC and CELLO experiments well below the $Z^0$ resonance and set the limits at the 90% confidence level on the magnetic moment and the charge radius of the tau neutrino, $\mu_\nu < 4 \times 10^{-6} \mu_B$ and $\langle r^2 \rangle < 2 \times 10^{-31}$ cm$^2$, respectively. The other is from the experiments at the $Z^0$ resonance. They give $\mu_\nu < 4.4 \times 10^{-6} \mu_B$ [12] and $3.3 \times 10^{-6} \mu_B$ [13].

At energies well below $Z^0$, the dominant contribution to the process $e^+e^-\rightarrow \nu\tau\gamma$ involves the exchange of a virtual photon [11]. The dependence on the magnetic moment comes from its direct coupling to the virtual photon, and the observed photon is the result of the initial-state Bremsstrahlung.
While the results of the TRISTAN experiments (VENUS, TOPAZ and AMY [14] collaborations) have been used to set limits on supersymmetric particles, we will make use of them here to set limits on the tau neutrino magnetic moment and charge radius.

The standard expression [15] for the cross section for the process $e^+e^- \rightarrow \nu\gamma$ due to $Z^0$ and $W$ exchange (Fig.1(a)) is

$$\frac{d\sigma}{dx \, dy} = \frac{G_F^2 \alpha}{6\pi^2} \cdot \left\{ \frac{M_Z^2 \left[ N_\nu (g_V^2 + g_A^2) + 2(g_V + g_A) [1 - s(1 - x)/M_Z^2]\right]}{[s(1 - x) - M_Z^2]^2 + (M_Z \Gamma_Z)^2} \right\} \cdot \frac{s}{x(1 - y^2)} \left[ (1 - x)(1 - x/2)^2 + x^2(1 - x)\frac{y^2}{4}\right], \tag{1}$$

where $x = E_\gamma/E = 2E_\gamma/\sqrt{s}$ is the photon energy in units of the incident beam energy, $y = \cos \theta_\gamma$ is the direction cosine of the photon momentum with respect to the incident beam direction, $\alpha$ is the fine-structure constant, $s$ is the square of the center of mass energy, $N_\nu$ is the number of low-mass neutrino generations, $M_Z$ is the mass of the $Z^0$, $\Gamma_Z$ is the total width of $Z^0$, $g_V = -1/2 + 2\sin^2 \theta_W$ ($\theta_W$ is the weak mixing angle) and $g_A = -1/2$. It is worth noting that the $(g_V^2 + g_A^2)$ term of equation (1) arises from the square of the s-channel $Z^0$ amplitude, the ‘2’ term from the square of the t-channel $W$-exchange amplitude, and the $(g_V + g_A)$ term from $Z^0 - W$ interference.

We now allow for a neutrino electromagnetic interaction given by the vertex $-ie(\gamma_\mu F_1(q^2) + (\kappa/2m_e)\sigma_{\mu\rho}q^\rho)$, where we express $F_1(q^2)$ as $q^2\langle r^2 \rangle/6$ in order to extract a limit on a possible charge radius $\langle r^2 \rangle$. We will include such a contribution only for the tau neutrino because the limits which were already obtained for $\nu_e$ and $\nu_\mu$ are more stringent than the limit we will be obtaining for the tau neutrino. We obtain the additional contributions from the diagram of Fig.1(b) to the cross section,

$$\frac{d\sigma}{dx \, dy} = \frac{\alpha^3}{3} \left\{ \frac{2\langle r^2 \rangle^2 s(1 - x)}{9} + \frac{\kappa^2}{m_e^2} \right\} \frac{g_V \langle r^2 \rangle M_Z^2 s(1 - x)(1 - s(1 - x)/M_Z^2)}{3\sin^2 \theta_W \cos^2 \theta_W [s(1 - x) - M_Z^2]^2 + (M_Z \Gamma_Z)^2} \cdot \frac{[(1 - x/2)^2 + x^2 y^2/4]}{x(1 - y^2)}, \tag{2}$$

\[^1\text{We consider here only Dirac neutrinos since Majorana neutrinos are well known to have quite different electromagnetic properties, in particular, they cannot possess a magnetic moment.}\]
We have integrated (2) over the relevant range given in Table 1 for each experiment [14]. In Table 1, \( x_T = E_{T\gamma}/E \) is the photon transverse energy normalized to the beam energy, and \( \epsilon \) is the overall efficiency for each data sample. For instance, integrating (2) over the VENUS kinematical region, we changed the variable from \( x \) to \( x_T \) with Jacobian, then integrated it over the region \( 0.13 < x_T < 1 \) and \( \cos 130.3^\circ \leq y \leq \cos 50.0^\circ \). We also applied a similar method to the other experiments. Table 2 is a summary of the number of single-photon candidates for each experimental result. It is worth noting that there is no interference between (1) and (2), since the anomalous contribution given in equation (2) flips helicity, but the standard model contribution given in equation (1) does not [16].

We obtained the upper limits on the number of signal events for the observed events and the expected background using two methods: One is Poisson statistics [17],

\[
1 - \alpha = 1 - \frac{\sum_{n=0}^{n_0} \frac{(n_B + N)^n}{n!} e^{-n_B}}{\sum_{n=0}^{n_0} \frac{n_B^n}{n!} e^{-n_B}},
\]

where \( n_0 \) is the number of the single-photon candidates which each experiment has obtained, \( n_B \) is the mean for the sum of all backgrounds and \( N \) is the desired upper limit on the unknown mean for the signal with confidence coefficient \( \alpha \). The other is the Unified Approach [18]. We applied \( n_B = n_0 \) to both methods because each \( n_0 \) in Table 2 could be explained by the sum of the number of physically expected events and that of non-physical backgrounds. Then, we required

\[
N_i > \sigma_i \times \epsilon_i \times \int L_i dt
\]

for each experiment, where \( N_i \) is the upper limit at 90% C.L. on the number of signal events, \( \sigma_i \) is the cross section obtained from integration of (2) over the relevant ranges, \( \epsilon_i \) is the overall efficiency, and \( \int L_i dt \) is the integrated luminosity for each experiment, i.e., \( i \) means VENUS, TOPAZ, and so on. Table 2 contains the upper limits of the tau neutrino magnetic moment for each experiment.

We combined the bounds on the magnetic moment at the 90% C.L. at TRISTAN using Poisson statistics,

\[
\kappa < 9.1 \times 10^{-6},
\]
and also derived the bounds on the charge radius of the tau neutrino for the experiments (Table 2) and combined them at the 90% C.L.,

$$\langle r^2 \rangle < 3.1 \times 10^{-31} \text{cm}^2. \quad (6)$$

In addition, using the Unified Approach, we obtained the combined bound on the magnetic moment at the 90% C.L.,

$$\kappa < 8.0 \times 10^{-6} \quad (7)$$

and charge radius

$$\langle r^2 \rangle < 2.7 \times 10^{-31} \text{cm}^2. \quad (8)$$

The obtained results (5)-(8) give upper limits comparable to those obtained from other $e^+e^-$ colliding beam experiments.

We have reported on the bound on the tau neutrino magnetic moment and charge radius from the TRISTAN experiments and have obtained the bound from single photon production cross section at TRISTAN, $9.1 \times 10^{-6} \mu_B$ and $3.1 \times 10^{-31} \text{cm}^2$ at 90% C.L. using Poisson statistics, and $8.0 \times 10^{-6} \mu_B$ and $2.7 \times 10^{-31} \text{cm}^2$ at 90% C.L. using Unified Approach. They are still far above what is predicted by the standard electroweak theory extended to include massive neutrinos although comparable to the results from other $e^+e^-$ colliding experiments.

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References

[1] Super-Kamiokande Collab., Y. Fukuda et al., Phys. Rev. Lett. 81 (1998) 1562.
[2] S.N. Gninenko, Phys. Lett. B 452 (1999) 414.
[3] C.-K. Chua and W.-Y. P. Hwang, Phys. Rev. D 60 (1999) 073002.
[4] B.W. Lee and R.E. Shrock, Phys. Rev. D 16 (1977) 1444; K. Fujikawa and R.E. Shrock, Phys. Rev. Lett. 45 (1980) 963.
[5] ALEPH Collab., R. Barate et al., Eur. Phys. J. C 4 (1998) 433.
[6] P. Elmfors, K. Enqvist, G. Raffelt and G. Sigl, Nucl. Phys. B 503 (1997) 3.
[7] G. Raffelt, Phys. Rev. Lett. 64 (1990) 2856.
[8] S. Nussinov and Y. Rephaeli, Phys. Rev. D 36 (1987) 2278.
[9] J.A. Grifols and E. Massó, Mod. Phys. Lett. A 2 (1987) 205.
[10] A.M. Cooper-Sarkar et al., Phys. Lett. B 280 (1992) 153.
[11] H. Grotch and R.W. Robinett, Z. Phys. C 39 (1988) 553.
[12] DELPHI Collab., P. Abreu et al., Z. Phys. C 74 (1997) 577.
[13] L3 Collab., M. Acciarri et al., Phys. Lett. B 412 (1997) 201.
[14] VENUS Collab., N. Hosoda et al., Phys. Lett. B 331 (1994) 211; TOPAZ Collab., T. Abe et al., Phys. Lett. B 361 (1995) 199; AMY Collab., Y. Sugimoto et al., Phys. Lett. B 369 (1996) 86.
[15] K.J.F. Gaemers, R. Gastmans and F.M. Renard, Phys. Rev. D 19 (1979) 1605.
[16] N.G. Deshpande and K.V.L. Sarma, Phys. Rev. D 43 (1991) 943.
[17] Review of Particle Properties, Phys. Rev. D 54 (1996) 1.
[18] G.J. Feldman and R.D. Cousins, Phys. Rev. D 57 (1998) 3873.
Fig. 1. Diagrams leading to the process $e^+e^- \rightarrow \nu_i \bar{\nu}_i \gamma$ due to (a) standard model processes and (b) contributions from anomalous neutrino electromagnetic coupling.
Table 1
Summary of the criteria for TRISTAN experiments. $\epsilon$ is the overall efficiency for each data sample.

| $\sqrt{s}$[GeV] | $x_T, x$ | $y = \cos \theta_\gamma$ | $E_\gamma$[GeV] | $\epsilon$ |
|----------------|----------|----------------------|-----------------|---------|
| VENUS          | 58       | $x_T > 0.13$          | $50.0^\circ \leq \theta_\gamma \leq 130.3^\circ$ | 0.57    |
| TOPAZ          | 58       | $x_T \geq 0.12$      | $|\cos \theta_\gamma| \leq 0.8$ | $\geq 4.0$ | 0.27 |
| AMY 1          | 57.8     | $x > 0.175$          | $|\cos \theta_\gamma| < 0.7$ |         | 0.44 |
|                |          |                      |                 |         |      |
| AMY 2          | 57.8     | $x > 0.175$          | $|\cos \theta_\gamma| < 0.7$ |         | 0.64 |
| AMY 3          | 57.8     | $x > 0.125$          | $|\cos \theta_\gamma| < 0.7$ |         | 0.58 |
| AMY 4          | 57.8     | $x > 0.125$          | $|\cos \theta_\gamma| < 0.7$ |         | 0.57 |
Table 2
Summary of the upper limits of the tau neutrino magnetic moment and the charge radius for each experimental result with Poisson statistics and Unified Approach, and their combined results. $n_0$ is the number of the single-photon candidates which each experiment has obtained. Numbers in parentheses indicate the expected number of events originating from $W$ and $Z^0$ exchange.

| $\int L \, dt$ [pb$^{-1}$] | $n_0$ | Poisson statistics | Unified Approach |
|---------------------------|------|--------------------|------------------|
|                           |      | $\kappa \times 10^{-6}$ | $\langle r^2 \rangle \times 10^{-31}$cm$^2$ | $\kappa \times 10^{-6}$ | $\langle r^2 \rangle \times 10^{-31}$cm$^2$ |
| VENUS                     | 164.1| (3.9$^{+4.2}_{-2.8}$) | $< 13.$ | $< 4.6$ | $< 13.$ | $< 4.6$ |
| TOPAZ                     | 213  | (3.1)              | $< 13.$ | $< 4.6$ | $< 13.$ | $< 4.5$ |
| AMY 1                     | 55   | 0                  | $< 16.$ | $< 5.9$ | $< 17.$ | $< 6.0$ |
| AMY 2                     | 91   | 2                  | $(7.2)$ | $< 14.$ | $< 4.9$ | $< 14.$ | $< 4.9$ |
| AMY 3                     | 56   | 2                  | $< 16.$ | $< 5.6$ | $< 16.$ | $< 5.6$ |
| AMY 4                     | 99   | 2                  | $< 12.$ | $< 4.2$ | $< 12.$ | $< 4.2$ |
| Combined results          |      | $< 9.1$            | $< 3.1$ | $< 8.0$ | $< 2.7$ |