Microwave-assisted magnetization reversal\(^{1-10}\) is a fascinating subject in magnetism for practical applications such as high-density recording. An oscillating field generated by microwaves excites a small-amplitude oscillation of the magnetization in the ferromagnet and significantly reduces the switching field to, typically, half of the uniaxial anisotropy field for an optimized microwave frequency. However, zero-field switching has not been reported experimentally; this is an outstanding problem in this field. A proposal of a theoretical possibility for switching induced solely by microwaves, as well as a deep understanding of its physical picture, will be an important guideline for further development in this field.

In previous works on microwave-assisted magnetization reversal\(^{1-10}\), the microwave source is isolated from the ferromagnet. Recently, however, an alternative system has been investigated both experimentally and numerically\(^{1,12}\) in which a spin torque oscillator (STO) is used as the microwave source. An oscillating dipole field emitted from the STO acts as microwaves on the ferromagnet and induces switching. Simultaneously, the dipole field from the ferromagnet changes the oscillation angle, as well as the oscillation frequency, in the STO. Therefore, in this situation, the microwave frequency from the STO depends on the magnetization direction in the ferromagnet. This motivated us to investigate the possibility of switching the magnetization solely by microwaves, the frequency of which depends on the magnetization direction itself. The purpose of this letter is to propose a theoretical framework for the switching process.

Figure 1 schematically shows the energy landscape of a ferromagnet. When the microwave frequency \(\nu\) is close to but slightly different from the oscillation frequency of the magnetization. By efficiently absorbing energy from the microwaves, the magnetization climbs up the energy landscape to synchronize the precession with the microwaves. We introduce a dimensionless parameter \(\epsilon\) that determines the difference between the microwave frequency and the instantaneous oscillation frequency of the magnetization. We analytically derive the condition of \(\epsilon\) required to switch the magnetization and confirm its validity by comparison with numerical simulations.

The magnetization dynamics in a ferromagnet is described by the Landau–Lifshitz–Gilbert (LLG) equation,

\[
\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times \mathbf{H} - \alpha \gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}),
\]

where \(\mathbf{m}\) is the unit vector pointing in the direction of the magnetization. The gyromagnetic ratio is denoted as \(\gamma\). The second term on the right-hand side of Eq. (1) is damping with the damping constant \(\alpha\). The magnetic field \(\mathbf{H}\) is related to the magnetic energy density \(E\) via \(\mathbf{H} = -\partial E/\partial (\mathbf{M} \cdot \mathbf{m})\), where \(\mathbf{M}\) is the saturation magnetization. The explicit form of the magnetic field in the present system is given by

\[
\mathbf{H} = H_{ac} \cos \psi \mathbf{e}_z + H_{ac} \sin \psi \mathbf{e}_y + H_K \mathbf{m},
\]

where we assume that the ferromagnet has uniaxial anisotropy along the \(z\)-axis with the anisotropy field \(H_K\). The microwave amplitude is denoted as \(H_{ac}\). In the absence of microwaves, the ferromagnet has two stable states given by \(\mathbf{m} = \pm \mathbf{e}_z\). In the following, we assume that the magnetization...

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**Magnetization switching by microwaves synchronized in the vicinity of precession frequency**

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We propose a theoretical framework of magnetization switching induced solely by microwaves. The microwave frequency is always close to but different from the oscillation frequency of the magnetization. By efficiently absorbing energy from the microwaves, the magnetization climbs up the energy landscape to synchronize the precession with the microwaves. We introduce a dimensionless parameter \(\epsilon\) that determines the difference between the microwave frequency and the instantaneous oscillation frequency of the magnetization. We analytically derive the condition of \(\epsilon\) required to switch the magnetization and confirm its validity by comparison with numerical simulations.

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initially points in the positive z-direction, for convention. When the microwave frequency $f$ is constant, the phase $\psi$ is related to $f$ via

$$\psi = 2\pi ft. \quad (3)$$

The torque due to the anisotropy term on the right-hand side of Eq. (1), $-\gamma m \times H_K m_e$, describes the precession of the magnetization on a constant energy curve of $E = -MH_K m_e^2/2$. The precession frequency on this constant energy curve of $E$ is given by

$$f(E) = \frac{\gamma}{2\pi} H_K m_e. \quad (4)$$

Note that $f(E)$ decreases with increasing energy. In other words, the oscillation frequency of the magnetization decreases as the magnetization climbs up the energy landscape. The FMR frequency is given by $f_{\text{FMR}} = \frac{\gamma E}{\gamma - E_m} f(E) = \gamma H_K / (2\pi)$. Then, in analogy to Eq. (3), we consider the phase of the microwaves as given by the relation,

$$\psi = \gamma H_K (m_z + \epsilon)t. \quad (5)$$

The microwave frequency will be

$$\nu = \frac{1}{2\pi} \frac{df}{dt} = \frac{\gamma}{2\pi} H_K \left( m_z + \epsilon + \frac{dm_z}{dt} t \right). \quad (6)$$

Here, we introduce a parameter $\epsilon$. The difference between the microwave frequency and the oscillation frequency of the magnetization is $\nu - f(E) = \gamma H_K (\epsilon + (dm_z/dt)t)/(2\pi)$. In particular, when the magnetization precesses on a constant energy curve of $E$, as in the case of resonance, $\nu - f(E) = \gamma H_K (\epsilon)/(2\pi)$. Thus, the parameter $\epsilon$ determines the frequency difference. We must note that the term $(dm_z/dt)t$ is also necessary, as mentioned below.

Before proceeding to further discussion, we comment briefly on the above model. As mentioned above, the present model is motivated by a ferromagnet coupled to an STO.\textsuperscript{11,12} To strictly investigate the possibility of switching, the coupled LLG equations between the ferromagnet and the STO should be solved.\textsuperscript{13} It is, however, difficult to solve such LLG equations analytically because of their complexity. The present system might be regarded as a simplified model of this system, instead of solving the coupled equations exactly. We assume that any complexity of the coupled equations is attributable to the parameter $\epsilon$. For example, $\epsilon$ might be related to the delay of the response (frequency change) of the STO due to its finite relaxation time and can be changed, for example, by using various materials or device geometries. Note that the present model is not restricted to a coupled system between a ferromagnet and an STO. The use of an arbitrary wave generator as a microwave source is another candidate for the present proposal. The introduction of the parameter $\epsilon$ provides a wide variety of dynamics in a model and will enable us to characterize experimental results by a limited numbers of parameters. Although $\epsilon$ is assumed to be constant here, it will be interesting to study a model with time-dependent $\epsilon$. In fact, as mentioned below, the total frequency difference $[\nu \epsilon + (dm_z/dt)t]$ should be time-dependent for switching.

Next, we consider the analytical condition of $\epsilon$ required to switch the magnetization. The energy of the system should increase with time for switching. To study the energy change of the ferromagnet, it is useful to use a rotating frame $x'y'z'$, where the $z'$-axis is parallel to the $z$-axis, and the $x'$-axis always points in the direction of the microwaves.\textsuperscript{18} The LLG equation in the rotating frame is given by

$$\frac{dm'}{dt} = -\gamma m' \times B - \alpha (m' \times (m' \times B)) + \alpha \frac{df}{dt} m' \times (e_z \times m'), \quad (7)$$

where $m' = (m_x, m_y, m_z)$ is the unit vector pointing in the magnetization direction in the rotating frame. The magnetic field in the rotating frame is

$$B = H_e e_z + \left( -\frac{1}{\gamma} \frac{df}{dt} + H_K m_z \right) e_z. \quad (8)$$

The second term on the right-hand side of Eq. (7) is the damping in the rotating frame. A mathematical analogy between the third term and the spin torque was pointed out recently.\textsuperscript{14} We define the energy density in the rotating frame as $\delta = -M \int dm' \cdot B$. Then, from Eq. (7), the energy change, $d\delta/dt = (dm'/dt) \cdot (d\delta/dm') + (d\delta/dt)$, is described as

$$\frac{1}{\gamma M} \frac{d\delta}{dt} = -\alpha \left[ \frac{1}{\gamma} \frac{df}{dt} + H_K m_z \right] (H_K m_e - \alpha H^K e_z^2 \times)$$
$$+ \alpha \left[ H_{\text{ac}} m_e + \left( \frac{1}{\gamma} \frac{df}{dt} + H_K m_z \right) m_z \right]$$
$$\times (H_K e_z + H_K m_z^2)$$
$$+ \frac{1}{\gamma} \frac{d\delta}{dt} m_z. \quad (9)$$

Note here that the microwave amplitude $H_e$ is usually much smaller than the uniaxial anisotropy field $H_K$. In addition, $m \approx e_z$ near the initial state. Then, the dominant part of the energy change is given by

$$\frac{1}{\gamma H_K} \frac{d\delta}{dt} \sim \alpha (1 - m_z^2) m_e \left( \epsilon + \frac{dm_z}{dt} t \right)$$
$$+ \frac{1}{\gamma H_K} \frac{dm_z}{dt} m_z. \quad (10)$$

where we used Eq. (6) in the derivation. We note that $dm_z/dt < 0$ because we are interested in switching from $m = +e_z$ to $m = -e_z$. Because the energy should increase for switching, $\epsilon$ should at least satisfy the following condition near the initial state:

$$\epsilon > 0. \quad (11)$$

We note that Eq. (11) is roughly derived without solving the LLG equation exactly and by neglecting the higher-order terms of $H_{\text{ac}}/H_K$. Equation (11), nevertheless, implies the possibility of switching the magnetization solely by microwaves. Regarding the above derivation, Eq. (11) should be regarded as a necessary but not sufficient condition for switching. Equation (11) also implies that the maximum frequency of the STO should be higher than the FMR frequency if the coupled system between a ferromagnet and an STO is used to test the present model. This is because $\nu$ with a positive $\epsilon$ at $t = 0$ is larger than $f_{\text{FMR}}$ from Eq. (6). The sign of $\epsilon$ should be changed for switching in the opposite direction because $dm_z/dt > 0$ and $m_z < 0$ in this case. We note that $dm_z/dt < 0$ also implies another necessary condition for switching, $H_{\text{ac}} > aH_K/2$.\textsuperscript{15}
Then, a term such as climb up the energy landscape by synchronizing its direction. The role of the term the maximum point of the energy landscape and switches occurs continuously, the magnetization chronize the magnetization precession with the microwaves.

Unfortunately, it is difficult to find this limit from Eq. (9) analytically, if it ever exists. Instead, we confirmed the switching for $e \leq 100$ numerically.

We also study the effect of thermal fluctuation by adding a random torque given by $-\gamma m \times h$ to the right-hand side of Eq. (1). The component of the random field $h$ satisfies the fluctuation–dissipation theorem,

$$\langle h(t)h(r') \rangle = \frac{\gamma kT}{\gamma MV} \delta(t - t'),$$

where the temperature is chosen as room temperature $T = 300 K$. The volume of the ferromagnet is $V = \pi r^2 \times 5 \text{ nm}^3$, where $r$ and $d = 5 \text{ nm}$ are the radius and thickness, respectively. The other material parameters and simulation conditions are identical to those in the above calculations. The magnetization dynamics is averaged over $N = 10^5$ samples. Figure 4(a) shows the time evolution of the averaged $m_z$ for $e = 0.1, 0.2, 0.3, 0.4$, and $0.5$. The radius is chosen as $r = 35 \text{ nm}$. This value makes the cross-sectional area almost identical to that in the experiment, in which the cross-sectional area was an ellipse. As shown, magnetization switching occurs even in the presence of thermal fluctuation. We also investigate the switching probability at $t = 100 \text{ ns}$, which is defined as the number of samples showing $m_z(t = 100 \text{ ns}) < -0.9$ divided by the total number of samples $N = 10^5$. Figure 4(b) shows the relation between the switching probability, the parameter $e$, and the radius $r$. When $e$ is $0.1$, the switching probability is small for small $r$. This is because the switching time is relatively long for $e = 0.1$, and the thermal fluctuation becomes large for a small ferromagnet. On the other hand, for $e \geq 0.2$, the switching probability at $t = 100 \text{ ns}$ is $100\%$ even in the presence of thermal fluctuation and for a small volume. Therefore, we concluded that switching occurs even in the presence of thermal fluctuation when $e$ is in an appropriate range.

We emphasize that the present model provides a comprehensive method of analytically studying the possibility of
switching. For example, let us consider autoresonance model in which the microwave frequency of this model is \( \nu = f_0 - a t \) with constants \( f_0 \) and \( a \). Because \( \nu - f(E) \) should be negative for switching, as mentioned above, the constant \( a \) should be positive. The other switching condition, i.e., \( d\varepsilon/dt \) should be positive near the initial state, then requires that \( f_0 > \gamma H_K/(2\pi) \). These conclusions are consistent with Ref. 18. The other approach for switching is to restrict \( \nu \) to \( f(E) \) and neglect the damping. In this case, the magnetization is always in the resonance state. Then, from Eq. (9), \( d\varepsilon/dt \) is zero up to the zeroth order of \( H_{ac}/H_K \). This means that the energy change is unnecessary to move from a certain state to the other; thus, the magnetization can move freely. Then, periodic switching between \( \mathbf{m} = +\mathbf{e}_z \) and \( \mathbf{m} = -\mathbf{e}_z \) is achieved.

In conclusion, we proposed a theoretical framework for magnetization switching induced solely by microwaves. The microwave frequency depends on the magnetization direction and is close to but slightly different from the instantaneous oscillation frequency of the magnetization. We introduced a dimensionless parameter \( \varepsilon \) that determines the difference between the microwave frequency and the oscillation frequency. We analytically derived the necessary condition of \( \varepsilon \) to switch the magnetization from the evolution equation of the energy. When \( \varepsilon \) is in a certain range, the magnetization climbs up the energy landscape to synchronize the magnetization precession with the microwaves and finally switches its direction. We also presented a numerical simulation that confirmed the validity of the analytical theory and provided evidence of switching.

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