A q-spin Potts model of markets: Gain–loss asymmetry in stock indices as an emergent phenomenon

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Abstract
Spin models of markets inspired by physics models of magnetism, as the Ising model, allow for the study of the collective dynamics of interacting agents in a market. The number of possible states has been mostly limited to two (buy or sell) or three options. However, herding effects of competing stocks and the collective dynamics of a whole market may escape our reach in the simplest models. Here I study a q-spin Potts model version of a simple Ising market model to represent the dynamics of a stock market index in a spin model. As a result, a self-organized gain–loss asymmetry in the time series of an index variable composed of stocks in this market is observed.

1. Introduction
This article is dedicated to the memory of Dietrich Stauffer who taught me the beauty of simple computer models. He was very supportive of the formation of a socio- and econophysics community in the German physical society and inspired many with his quest for universality in human nature. In this brief paper I will study a variation of a spin model for markets which, 20 years ago, appeared in the International Journal of Modern Physics C, after Dietrich edited and reviewed the manuscript within hours of submission.

Collective effects of traders in a market are of particular interest for the dynamics of markets and are notoriously hard to grasp with traditional economic equilibrium models of representative agents [1]. Agent based models filled the gap in representing the non-rational and collective elements of the dynamics of markets [2] [3] [4]. Stylized facts of market dynamics, for example, were first reproduced in agent based models [5]. Extreme simplifications of such models, as in the Ising model variant we come back to here [6], then allowed to isolate the particular underlying herding mechanism among agents, at the roots of stylized facts.

A curious property of real stock markets is an observed gain–loss asymmetry in the time series data of stock indices, but not in the time series of single stocks.

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This phenomenon is largely still not fully understood and could be an interesting case for simple econophysics models to test possible mechanisms at the roots of such a collective effect.

We do have an intuitive understanding of why such an observation could be plausible: Downward movements of stock markets are often in synchrony, due to external shocks, or bad news affecting the whole of the market, while, on the other side, upward movements of stocks happen in a more uncorrelated fashion, with hopes and fantasy of investors often tied to the ideas and fate of single enterprises. A phenomenological model with an external “fear factor” that randomly lowers the stocks of a model market from time to time, with stocks otherwise performing independent random walks with an upward bias, demonstrated that this intuition works [9].

As a next step, it would be interesting to see how such a phenomenon could emerge in a dynamical market model itself. A first model that exhibits the gain–loss asymmetry as an emergent effect of its intrinsic dynamics is the block-spring conveyor belt model by Bulcsù and Nédà [10]. It is inspired by the classical self-organized critical block spring model originally discussed for earthquakes [11].

We here go a step further and ask if this effect could emerge in a minimalistic market model inspired by the Ising model of statistical physics [6]. However, in a two state spin model a Dow Jones like index variable cannot be defined as there is only one stock in this minimal version. Let us therefore generalize the spin model market to a q-spin Potts model version and define a stock market index variable to study its time series. Let us now briefly recapitulate the Ising spin model, then define the Potts model extension, followed by some numerical experiments.

2. A spin market model

The attractive nearest neighbor interaction in the ferromagnetic phase of the Ising model can be viewed as a simple model for imitating the actions of others, a tendency assumed to be at work in the stock market. Adding a second interaction that represents the opposing tendency of the trader to escape the crowd, makes a particularly simple model for markets [6]. The magnetic alignment between neighboring spins is countered with an additional coupling $\alpha > 0$ between the absolute value of the magnetization, acting as a destabilizing "external" field, to the local field of each spin as defined in a local field at spin $S_i$ given by

$$h_i = \sum_{j=1}^{N} J_{ij} S_j - \alpha \left| \frac{1}{N} \sum_{j=1}^{N} S_j \right| .$$

The overall model consists of $i = 1, ..., N$ spins with orientations $S_i(t) = \pm 1$ and we consider a 2 dimensional lattice with couplings $J_{ij} = 1$ for the four nearest neighbors of each spin and 0 else. The dynamics of each spin $S_i(t)$ depends on
its local field $h_i(t)$ and is determined by a heat-bath dynamics according to

$$
S_i(t + 1) = +1 \quad \text{with} \quad p = 1/[1 + \exp(-2\beta h_i(t))]
$$

$$
S_i(t + 1) = -1 \quad \text{with} \quad 1 - p.
$$

(2)

The parameter $\beta = 1/T$ is the inverse of a formal temperature $T$ and determines the heat bath dynamics. The newly introduced global coupling constant $\alpha > 0$ can be interpreted as a "fear" factor, rendering each spin’s actions more random, the more asymmetry has accumulated in the model market. Note that this term always is proportional to $-S_i(t)$, thus pulls the spin to its opposite orientation, in particular when magnetization is large. Magnetization can be thought of as a convenient indicator of the overall asymmetry, or a “bubble” away from equilibrium, in the spin orientations.

When running this model numerically below the critical temperature of the Ising model, it does not settle down to an equilibrium state, but rather exhibits broad fluctuations with intermittent phases of “bubbles” and “crashes”, quite similar to the statistical features of financial markets called stylized facts. The features also occur in higher dimensions [12] and in non-lattice topology versions of the model [13, 14, 15]. A detailed analysis of its dynamics has been done e.g. in Refs. [16, 17], and its multifractal behavior is characterized in Ref. [18, 19]. Further applications to financial markets and models include [20, 21, 22, 23, 24]. Discussions in the context of other market models can be found in Refs. [27, 28, 4].

Some model variants include the introduction of more than binary variables, as for example three state models to model the three options of buy, sell, or hold a stock [29, 30, 31]. In order to model a market with a set of many stocks, an ensemble of spin models has been considered [32, 24]. However, curiously, a version proposed to model markets in the original article [6], the q-spin Potts model, did not catch interest so far. Thus let us try this now.

3. A q-state Potts model for markets

Consider a model with $i = 1, \ldots, N$ spins with now $q$ possible states $\sigma_i(t) \in \{0, \ldots, q-1\}$. The different states now symbolize not only two but $q > 2$ different stocks or assets in the model market and each agent will choose one of them to invest in (a model where agents can diversify their investments has been explored by Takaishi [32, 24] who considers an ensemble of binary spin models instead). Let us here consider this minimalistic idealization of a market as a q-spin Potts model, as the simplest extension of a buy-sell spin model to more that two states we can think of. Note that it solely focuses on the imitation dynamics of agents, as also the binary spin models do, and does not itself contain economic quantities as stock prices. Rather, a central observable is the popularity of a state which is taken as an analogy for the over- or undervaluation of the corresponding asset. Prices can be introduced through a market maker mechanism, for example, as has been demonstrated for the two state model in [33].
As in the Ising spin model, two neighbors exhibiting the same preference (state) will be energetically favored (this is the herding interaction: “buy the stock that your friends buy”), and this is already conveniently expressed in the \(q\)-state version of the original Potts model \[34, 35\] as defined by the Hamiltonian

\[
H = -J \sum_{<ij>} \delta_{\sigma_i, \sigma_j},
\]

with \(\delta_{\sigma_i, \sigma_j} = 1\) if \(\sigma_i = \sigma_j\) and \(\delta_{\sigma_i, \sigma_j} = 0\) if \(\sigma_i \neq \sigma_j\).

Let us always set \(J = 1\). For \(q = 2\), this model maps onto the Ising model with \(S_i = 2\sigma_i(t) - 1\) and \(\sigma_i(t) \in \{0, 1\}\). To see this, first rewrite the heatbath update \[2\] as a flip probability

\[
p_{flip}(S_i \to -S_i) = \frac{1}{1 + \exp(2\beta S_i(t)h_i(t))}.
\]

For the Ising spins \(S_i(t)\), this corresponds to the heatbath flip probability

\[
p_{flip} = \frac{1}{1 + \exp(\beta \Delta E)}
\]

as a function of the energy difference

\[
\Delta E = 2S_i(t)h_i(t).
\]

In the Potts model, for each single spin update we choose a site \(\sigma_i\) and randomly pick a state \(\sigma_i(new) = \mu\) with \(\mu \in 0, \ldots, q - 1\). The new local energy minus the old local energy of this site with respect to its 4 neighbors then is

\[
\Delta E = -\sum_{j \in nn(i)} \left( \delta_{\sigma_i(new), \sigma_j} - \sum_{j \in nn(i)} \delta_{\sigma_i(t), \sigma_j} \right).
\]

The flip probability in the \(q=2\) Potts model formulation then is

\[
p_{flip}(\sigma_i(new)) = \frac{1}{1 + \exp \left(-2\beta \left[ 2\sum_{j \in nn(i)} \delta_{\sigma_i(new), \sigma_j} - 4 \right] \right)}.
\]

With \(S_i = 2\sigma_i(t) - 1\) this can be rewritten as

\[
p_{flip}(S_i(new)) = \frac{1}{1 + \exp \left(-2\beta S_i(new) \sum_{j \in nn(i)} S_j \right)}.
\]

This is equivalent to \[4\] thus, for \(q = 2\), the Potts and Ising models yield the identical local field and heatbath updates when both formulated with spins.

Unfortunately, this equivalence does not hold for the \(q > 2\)-state Potts model, so let us briefly recapitulate what this means for the local interaction “buy the stocks that your friends buy” in our market model.

The Ising heatbath flip probability can be read in two ways:
1. The flip probability is determined by the energy difference $\Delta E$ of the flip.

2. The flip probability is determined by the new local energy $\sum_{j \in \text{nn}(i)} \delta \sigma_i(t+1) \sigma_j$ after the flip.

This is equivalent for $q=2$, as we have $\Delta E = 2 \sum_{j \in \text{nn}(i)} \delta \sigma_i(t+1) \sigma_j$. Note that for $q > 2$, $\Delta E \neq 2 \sum_{j \in \text{nn}(i)} \delta \sigma_i(t+1) \sigma_j$, but instead $\Delta E = \sum_{j \in \text{nn}(i)} [\delta \sigma_i(t+1) \sigma_j - \delta \sigma_i(t) \sigma_j]$. From a physical point of view, the straightforward way to generalizing the model to $q > 2$ is to use the Glauber update probability for any general move that changes the energy of the system by some $\Delta E$

$$p_{flip} = \frac{1}{1 + \exp(\beta \Delta E)}.$$ (10)

However, is this interpretation as a local energy budget an accurate account of what happens in the market context, and of what a trader does when flipping one stock to another? One could argue that it reflects the accounting of selling one stock and buying another. Yet, when modeling market dynamics of traders without explicit fundamental knowledge of the stocks, but speculating on future expectations instead, standard accounting is most surely the wrong perspective, as also the more traditional financial market models indicate in failing to reproduce stylized facts in their dynamics. So what could be the underlying microscopic interaction that leads to the formation of expectation bubbles?

New opportunities are best represented by the future characteristics of what we consider opting for, therefore let us choose the second interpretation of the above options when extending our model market to more than two stocks options: This rule favors the new over the old, in fact even when the energy change associated to the flip is zero.

In practical terms, for $q > 2$, we will apply the $q=2$ heatbath update rule of our above Potts version of the Ising model, solely being a function of twice the energy contribution from the new spin orientation. Note that this is not the energetically accurate Glauber flip probability, as the old energy is not subtracted. However, throwing energy conservation overboard, we gain the freedom to incorporate the pull of the future with this step.

To construct a market model, it remains to complement this herding interaction by a counteracting term that represents the fear of the trader in the face of a market bubble. In the Ising model version, a large magnetization is the indication of such a disequilibrium in the market.

The standard order parameter of the $q$-spin Potts model is based on the number of spins $N_{\text{max}}(t)$ exhibiting the state that occurs most frequently at a given time $t$ of the simulation

$$M(t) = \frac{q N_{\text{max}}(t)}{N} - 1.$$ (11)

From the perspective of a stock market, focusing attention on the largest stock may be not completely unrealistic. In our model, let us therefore choose this
Potts model order parameter for the second term in the local field, coupling the agent’s fear of a bubble to the stock with the largest value (or perhaps the largest overvaluation, when thinking of bubbles) in the market.

Thus the full $q$-spin market model is given by

\[
    h_i = 2 \sum_{j \in nn(i)} \delta_{\sigma_i(new)\sigma_j(t)} - 4 - \alpha M(t). \tag{12}
\]

\[
    M(t) = \frac{q^{N_{\text{max}}(t)} - 1}{q - 1} \tag{13}
\]

\[
    p_{\text{flip}}(\sigma_i(new)) = \frac{1}{1 + \exp(-2\beta h_i)}. \tag{14}
\]

After random initialization of the lattice, spins are serially updated in random order, time is given in number of update steps per spin (sweeps). Temperature has to be chosen below the critical temperature. Note that while the critical point of the Potts model as defined in (4) is given by $T_c = 1/\log(1 + 1/\sqrt{q}) \quad (J = 1)$, its $q = 2$ adaptation to match the Ising model done in (8), however, has the extra factor of 2 of the critical temperature of the Ising model $T_c = 2/\log(1+1/\sqrt{2})$. As we use (8) for our model even with $q > 2$, it is an interesting question how this generalizes to $q > 2$. Numerically one observes a phase transition different from the standard Potts model transition and at a larger temperature than the critical temperature in the Potts model. For the market model, we need to operate well below the critical temperature. Also when switching to the non-physical update rule as compared to the original Potts model, we are on the safe side as that variant is still below critical even at the critical point of the original Potts model, as can be easily seen by switching to the physical Glauber update rule (7). For our simulations we choose $T = 0.2 T_c$ of the Potts model. Figure 1 shows a simulation of this model under random serial single spin update.

4. Dynamics of the model

The dynamics has similarities with the two state Ising market model, namely an intermittent behavior of calm and chaotic phases.

Note that the dynamics of the model at a temperature well below the critical temperature is quite different from the original Potts model which, below $T_c$, settles with one of the $q$ states as the dominant state. Instead our dynamical “rest” state is one where each of the $q$ states occupies approximately $N/q$ states of the system, typically occurring in patches, each dominated by a single state. Fluctuations away from this state are penalized with a higher flip probability (or effective temperature sensu [17]) through the second term in the local field of each agent. Deviations from the symmetric default state exhibit similar activity avalanches as we see in the Ising spin market model.

An overall indicator is the order parameter $M(t)$, see the changes of it per time step in figure 2. One observes intermittent phases with higher and lower
volatility, quite similar to, yet not as pronounced as in the Ising market model (which is also the \( q = 2 \) case of the current model). While the latter reproduces some of the prominent statistical characteristics of financial markets ("stylized facts") quite well, the \( q = 16 \) example of the new model does not. As this article appears in a volume to the memory of Dietrich Stauffer, let us remember that Dietrich always insisted that also negative results should be published.

Therefore, let us consider this an interesting observation that deserves further discussion. It may find a technical explanation from the perspective of our analysis of the spin market model. There we found, for \( q = 2 \), dominant subcritical metastable striped states in the systems dynamics with a vastly fluctuating borderline length that separates the two states (occurring as adjacent stripes) on the lattice, with very large interface lengths in the fragmented phases. For \( q > 2 \), we do not have the situation of the striped states, instead, the calm phases are already exhibiting a much larger interface borderline length around the small patches of different \( q \)-values. In the binary case, a small temperature \( T \) drives the system down to the striped state while, at the same time, a large \( \alpha \) enforces strong intermittency. For large \( q \), the equivalent of striped states can be observed for a small \( \alpha \), where a majority of the \( q \) states disappear, but then \( \alpha \) cannot at the same time be made large to obtain strong intermittency.

With these remarks we leave the question of stylized facts in the new model class and possible modifications for future research and now focus on collective effects in the dynamics of a toy model stock index, which can be defined in the Potts stock model.

5. The dynamics of a market index

Let us finally define a market index for our model market. With \( q \) different competing states in the system which fluctuate around a default value, we consider these as different stocks in a market fluctuating around their fundamental value. An index of a financial market is commonly a selection of stocks. Unsuccessful ones drop out of the index and are replaced by new ones with potential to grow. To mimic this in the simplest way, we define our model index to comprise those of the \( q \) stocks, that have a higher than average share. We include stock \( \mu \) once its share is larger than \( N_\mu(t) > N/q \), thus define our index by

\[
I(t) = \sum_{\mu=0}^{q-1} (N_\mu(t) - N/q) \Theta(N_\mu(t) - N/q) \tag{15}
\]

with the Heaviside function \( \Theta(x) = 1 \) for positive argument \( x \), and else \( \Theta(x) = 0 \). While real stock indices usually consist of a fixed number of stocks, we here choose the threshold definition solely for computational simplicity. Note that it does not affect the dynamics of the model, as it is just a suitably defined observable.

With this index we can now analyze the gain–loss asymmetry of the time series. Figure 3 shows the distribution of waiting times for a five percent gain.
or loss in the index. Both waiting time distributions have a maximum at a certain number of trading days, however, the most likely waiting time for a five percent loss occurs earlier (at about 20 trading days) than the most probable waiting time for a five percent gain (about 30 trading days). This is the main observation of this study.

Another feature of these distributions is a power law decay towards larger times, as one expects for first passage waiting time distributions of random walks. Note that for large waiting times $\tau_\rho$, the asymptotic probability distribution $p(\tau_\rho) \propto \tau_\rho^{-\theta}$ follows an exponent of approximately $\theta = 4/3$ which is a lower value than the value $\theta = 3/2$ for a random walk. The lower value of $\theta$ means that the probability of longer waiting time intervals is increased, pointing to a superdiffusive movement of the index variable $I(t)$.

6. Discussion

The modified $q$-state Potts model we use here as a toy model for markets exhibits a gain–loss asymmetry as is well known from real stock indices. The interpretation is that the model generates a synchronized downward movement across the range of stocks in the index, through the instability inherent to the model. This is a self-organized version of the Donangelo et al. model [9] which used an external “fear factor” instead.

This first small study of a $q$-state Potts model for a market leaves many questions open. The simplistic view of returns in terms of the changes in relative frequencies of $q$-state occurrences among agents has to be further interpreted in a market context for a full understanding. A price definition, for example via a market maker [33], would be a first step.

Another open question is if and how stylized facts of financial markets could be reproduced in a model with more than $q > 2$ states. Earlier analyses of the Ising spin market model could be a starting point, as outlined above. Further development of the model may also involve the definition of the local field $h_i$ in equation (12) and how it depends on the order parameter $M(t)$. I found it crucial for the dynamics that the nature of the new state $\sigma_i(new)$ does not depend on the majority Potts state. However, this might contradict the intuition of how traders act in real markets and other definitions of $h_i$ could turn out to be interesting.

We here studied the model on a two dimensional lattice, mainly because the binary Ising-model version of the market model has been thoroughly analyzed in the two dimensional version and forms an interesting background model, where interface length plays a central role in the model. Alternatively, the regular lattice could be replaced with a (random) graph, opening up for questioning the role of topology and even for the possibility of evolving topologies.

Last, not least, different socio-economic phenomena or systems, other than markets, could be addressed with this approach as, for example, Potts model versions of voter models for dynamics of opinion formation.
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Figure 1: The $q$-state Potts model inspired market model. Simulation on a $128 \times 128$ lattice with $q = 16$ possible states, temperature $T = 0.33$, and “fear”-parameter $\alpha = 100$. 
Figure 2: The change per time step of the order parameter $M(t)$ in the $q$-state market model. Simulation on a 128x128 lattice with $q = 16$ possible states, temperature $T = 0.33$, and fear parameter $\alpha = 100$. 
Figure 3: The probability distributions $p(\tau_\rho)$ of waiting times $\tau_\rho$ for positive and negative relative changes of $\rho = \pm 0.05$ as derived from the index time series $I(t)$. Note that the loss occurs on average earlier than the gain of same size. Simulation parameters are the same as in Figure 1.