Adaptive covariance relaxation methods for ensemble data assimilation: experiments in the real atmosphere

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Covariance inflation plays an important role in the ensemble Kalman filter because the ensemble-based error variance is usually underestimated due to various factors such as the limited ensemble size and model imperfections. Manual tuning of the inflation parameters by trial and error is computationally expensive; therefore, several studies have proposed approaches to adaptive estimation of the inflation parameters. Among others, this study focuses on the covariance relaxation method which realizes spatially dependent inflation with a spatially homogeneous relaxation parameter. This study performs a series of experiments with the non-hydrostatic icosahedral atmospheric model (NICAM) and the local ensemble transform Kalman filter (LETKF) assimilating the real-world conventional observations and satellite radiances. Two adaptive covariance relaxation methods are implemented: relaxation to prior spread based on Ying and Zhang (adaptive-RTPS), and relaxation to prior perturbation (adaptive-RTPP). Both adaptive-RTPS and adaptive-RTPP generally improve the analysis compared to a baseline control experiment with an adaptive multiplicative inflation method. However, the adaptive-RTPS and adaptive-RTPP methods lead to an over-dispersive (under-dispersive) ensemble in the sparsely (densely) observed regions compared with the adaptive multiplicative inflation method. We find that the adaptive-RTPS and adaptive-RTPP methods are robust to a sudden change in the observing networks and observation error settings.

Key Words: ensemble Kalman filter; data assimilation; covariance relaxation method; observation-space statistics

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1. Introduction

The ensemble Kalman filter (EnKF) finds the best estimate of the numerical model state based on the observations and model forecasts with their error covariance (Evensen, 1994, 2003). The EnKF is an advanced data assimilation method using the flow-dependent forecast error covariance estimated by an ensemble of model forecasts. It is known that the EnKF generally underestimates the error variance in practical geophysical applications due to various factors such as the limited ensemble size and model errors. Therefore, it is a common practice to inflate the underestimated variance, the technique known as covariance inflation (Houtekamer and Mitchell, 1998).

The covariance inflation methods can be categorized into the following three approaches: multiplicative inflation (Anderson and Anderson, 1999), additive inflation (Mitchell and Houtekamer, 2000; Corazza et al., 2003), and relaxation to prior (Zhang et al., 2004; Whitaker and Hamill, 2012). The multiplicative inflation multiplies ensemble perturbations by a factor. The inflation factor may depend on the location and variable, and can be estimated either on-line or off-line. The additive inflation adds random perturbations to the analysis ensemble perturbations. The relaxation-to-prior methods relax the reduction of the ensemble spread at analysis. Zhang et al. (2004) proposed the relaxation to prior perturbation (RTPP) method, which blends the forecast and analysis ensemble perturbations. Whitaker and Hamill (2012) proposed the relaxation to prior spread (RTSP) method, which relaxes the reduction of the analysis ensemble spread by multiplication without blending perturbations. RTPS is similar to multiplicative inflation, but it is applied after the analysis. Many studies have pointed out that the data assimilation performance with the EnKF is considerably sensitive to the choice of the inflation parameters (e.g. Whitaker and Hamill, 2012; Nerger, 2015). However, manual tuning of the inflation parameters by trial and error is computationally expensive, especially for realistic geophysical problems.
Several studies have proposed adaptive covariance inflation methods to avoid the expensive tuning and to estimate the inflation parameters adaptively based on the observation data or observation-space statistics (Desroziers et al., 2005). For multiplicative inflation, Anderson (2007, 2009) proposed to estimate the inflation parameters with observation data based on the Bayesian estimation theory. Alternatively, Li et al. (2009), hereafter LKM09) proposed to estimate the multiplicative inflation parameter based on the observation-space statistics. Miyoshi (2011, hereafter M11) extended LKM09’s approach and hereafter LKM09) proposed to estimate the multiplicative inflation parameters with observation data based on observation-space statistics. We perform NICAM-LETKF experiments with two different observation networks. Section 5 provides discussions with additional experiments. Finally, a summary is presented in section 6.

2. Method

2.1. Ensemble Kalman filter

This study uses the local ensemble transform Kalman filter (LETKF: Hunt et al., 2007), a kind of ensemble square root filter to update the ensemble mean and perturbations with a transform matrix (Tippett et al., 2003). However, we note that the adaptive inflation methods used in this study can be applied to any kind of EnKF such as the ensemble adjustment Kalman filter (Anderson, 2001), ensemble transform Kalman filter (Bishop et al., 2001), serial ensemble square root filter (Whitaker and Hamill, 2002), and perturbed observation method (Burgers et al., 1998; Houtekamer and Mitchell, 1998). Here, we consider an ensemble of m members. The ensemble perturbation matrix X is defined by

\[ X = \begin{pmatrix} x^{(1)} - \bar{x} \\ x^{(2)} - \bar{x} \\ \vdots \\ x^{(m)} - \bar{x} \end{pmatrix}, \]

where \( \bar{x} \) and \( \bar{x} \) are the ith ensemble state vector and the ensemble mean, respectively. In the LETKF, the analysis ensemble mean \( \bar{x} \) and ensemble perturbations \( X \) are given by

\[ \bar{x} = x + \alpha^b \hat{P} (Hx^b)^T R^{-1}(y - H\bar{x}), \]

\[ X = x^b ((m - 1) \hat{P}^b)^{1/2}, \]

where \( y \), \( H \), \( R \) and \( \hat{P} \) denote the observation vector, linear observation operator, observation error covariance matrix, and analysis error covariance matrix in the ensemble space, respectively. The m-dimensional ensemble space is spanned by the ensemble perturbations and is indicated by tilde. The superscripts a and b denote the background (prior), analysis (posterior), and observation, respectively. Here, the analysis error covariance matrix in the ensemble space is defined by

\[ \hat{P}^a = ((m - 1)I + (Hx^b)^T R^{-1}Hx^b)^{-1}. \]

The LETKF computes Eqs. (1) and (2) efficiently by an eigenvalue decomposition of the matrix inside the curly brackets of Eq. (3), or equivalently, \( (\hat{P}^a)^{-1} \). Covariance localization within the LETKF is applied to the observation error covariance, so that the errors of distant observations are enlarged (Hunt et al., 2007; Miyoshi and Yamane, 2007).

2.2. Covariance inflation methods

The EnKF needs variance inflation to mitigate the under-dispersive ensemble as described in the introduction. The EnKF estimates the background (analysis) error covariance \( P^a \) (\( P^b \)) by an ensemble:

\[ p^{(a)} = \frac{1}{m - 1} X^{(a)}(X^{(a)})^T. \]

This study focuses on the multiplicative inflation and relaxation-to-prior methods. The multiplicative inflation method (Anderson and Anderson, 1999) inflates the background error covariance by a factor \( \gamma \):

\[ p_{\text{inf}}^b = \gamma P^b. \]

The subscripts tmp and inf denote temporary (i.e. before inflation) and inflated, respectively. The relaxation-to-prior methods relax the reduction of the ensemble spread after updating the ensemble perturbations with Eq. (2). Zhang et al. (2004) proposed the RTPP, which blends the background and analysis ensemble perturbations as

\[ X_{\text{inf}}^a = \alpha_{\text{RTPP}} X^a + (1 - \alpha_{\text{RTPP}}) X_{\text{tmp}}^a, \]

where \( \alpha_{\text{RTPP}} \) denotes the relaxation parameter of the RTPP. Alternatively, the RTPS (Whitaker and Hamill, 2012) relaxes the reduction of the ensemble spread as

\[ X_{\text{inf}}^{a(i)} = \left( \frac{\alpha_{\text{RTPS}} \sigma^{b(i)} + (1 - \alpha_{\text{RTPS}}) \sigma^{a(i)}_{\text{tmp}}}{\sigma^{a(i)}_{\text{tmp}}} \right) X_{\text{tmp}}^{a(i)}, \]

where \( \alpha_{\text{RTPS}} \) and \( \sigma \) denote the relaxation parameter of the RTPS and the ensemble spread, respectively. The superscript i denotes the ith variable of the state vector \( x \). For both RTPP and RTPS, \( \alpha = 0 \) corresponds to no inflation, and \( \alpha = 1 \) corresponds to relaxing to the background ensemble spread. The parameter \( \alpha \) is generally defined between 0 and 1, so that the resulting perturbation size is bounded by the analysis (before inflation) and background perturbation sizes. The optimal parameters \( \alpha_{\text{RTPP}} \) and \( \alpha_{\text{RTPS}} \) can be different for the RTPP and RTPS.
The RTPP blends the background and analysis ensemble perturbations; therefore, the RTPP preserves grown perturbations during the forecasts similarly to the breeding method (Toth and Kalnay, 1993). With increasing the RTPP parameter α, the ensemble tends to be over-dispersive due to the fast growth of perturbations. By contrast, the RTPS inflates the analysis perturbations multiplicatively, Whitaker and Hamill (2012) compared the RTPP and RTPS, and found that analysis accuracy with the RTPS was less sensitive to the parameter α than that with the RTPP.

Generally, we need a large inflation factor in a densely observed region. Relative to multiplicative inflation, the relaxation-to-prior methods have an advantage that spatially varying inflation factors can be obtained using a spatially uniform (i.e. globally constant) relaxation parameter. Because the relaxation methods relax the reduction of the ensemble spread due to data assimilation, the relaxation parameter. Because the relaxation methods relax the spatially uniform relaxation parameter provides larger (smaller) reduction of the ensemble spread due to data assimilation, the relaxation parameter. Namely, the OMB-OMB statistics need more realistic observation error variances compared to the AMB-OMB statistics. However, the AMB-OMB statistics need an additional computation to obtain the analysis in the observation space (\(H\mathbf{x}^\text{b}\) or \(y^\text{b}\)) by applying the observation operator to the analysis state (\(H\mathbf{x}^\text{a}\)), or performing data assimilation in the observation space (\(y^\text{o}\)). The OMB-OMB statistics may thus be easier to apply for some EnKF implementations including the LETKF.

For covariance relaxation methods, YZ15 proposed to use the following statistics:

\[
\mathbf{d}^{\text{a-b}}(\mathbf{d}^{\text{a-o}})^T = H\mathbf{P}_{\text{inf}}^{-1} H^T. \quad (13)
\]

Hereafter, we denote Eq. (13) as AMB-OMA statistics. Taking the matrix trace of both sides of Eq. (13), we get:

\[
\text{tr}[\mathbf{d}^{\text{a-b}}(\mathbf{d}^{\text{a-o}})^T] = \text{tr}[H\mathbf{P}_{\text{inf}}^{-1} H^T] = \frac{1}{m-1}\text{tr}[H\mathbf{X}_{\text{imp}}^a(H\mathbf{X}_{\text{imp}}^a)^T]. \quad (14)
\]

YZ15 proposed an adaptive-RTPS method based on Eq. (14) as follows:

\[
\text{tr}[\mathbf{d}^{\text{a-b}}(\mathbf{d}^{\text{a-o}})^T] \approx \frac{1}{m-1}\text{tr}[\beta^2H\mathbf{X}_{\text{imp}}^a(H\mathbf{X}_{\text{imp}}^a)^T], \quad (15)
\]

\[
\beta = \frac{\alpha_{\text{RTPS}}^2 \gamma + (1 - \alpha_{\text{RTPS}})\hat{\sigma}_{\text{y,imp}}^2}{\hat{\sigma}_{\text{y,imp}}^2}. \quad (16)
\]

We can estimate the unique \(\alpha_{\text{RTPS}}\) by solving Eq. (16). Using Eqs (4) and (7), the approximation of Eq. (15) is equivalent to the following approximation:

\[
\beta = \frac{\alpha_{\text{RTPS}}^2 \sigma^2 + (1 - \alpha_{\text{RTPS}})\hat{\sigma}^2_{\text{y,imp}}}{\hat{\sigma}^2_{\text{y,imp}}}. \quad (17)
\]

\[
\hat{\sigma}^2_{\text{y,imp}} = \sqrt{\text{tr}[H\mathbf{P}_{\text{inf}} H^T]/p}. \quad (18)
\]

As shown in Eq. (7), the inflation factor of RTPS depends on the prior and posterior ensemble spreads of the \(i\)th variable. YZ15 introduced the inflation factor \(\beta\) by solving Eq. (14), but Eq. (14) is not strictly satisfied due to the approximation of Eq. (19). YZ15 pointed out that their formulation is not directly applicable to treat different types of observations with different units. For realistic geophysical problems, we extend the YZ15’s approach as follows:

\[
\text{tr}[\mathbf{d}^{\text{a-b}}(\mathbf{d}^{\text{a-o}})^T] \approx \frac{1}{m-1}\text{tr}[\beta^2H\mathbf{X}_{\text{imp}}^a(H\mathbf{X}_{\text{imp}}^a)^T \circ R^{-1}]. \quad (20)
\]

\[
\alpha_{\text{RTPP}} = \lambda_1 \sigma_{\text{RTPP}}^2 + \lambda_2 \sigma_{\text{RTPP}}^2 + \lambda_3 = 0, \quad (21)
\]

\[
\sigma_{\text{RTPP}} = -\frac{\lambda_2}{2} \pm \sqrt{(\lambda_2)^2 - 4\lambda_1\lambda_3}. \quad (22)
\]

The extension enables us to consider different kinds of observations by the normalization with \(R^{-1}\).

Motivated by YZ15, this study proposes an adaptive-RTPP based on the same observation-space statistics. For the RTPP, we can substitute Eq. (6) into Eq. (14) without approximation, and obtain the following quadratic equation and solution for \(\alpha_{\text{RTPP}}\):

- \(\gamma = \hat{\sigma}_{\text{y,imp}}^2 - \hat{\sigma}_{\text{y,imp}}^2 = \hat{\sigma}_{\text{y,imp}}^2\)
- \(\hat{\sigma}_{\text{y,imp}}^2 = \sqrt{\text{tr}[H\mathbf{P}_{\text{inf}} H^T]/p}
- \lambda_1 = \alpha_{\text{RTPP}}^2 + \lambda_2 \sigma_{\text{RTPP}}^2 + \lambda_3 = 0
- \sigma_{\text{RTPP}} = -\frac{\lambda_2}{2} \pm \sqrt{(\lambda_2)^2 - 4\lambda_1\lambda_3}
The detailed derivation with the three coefficients ($\lambda_1$, $\lambda_2$ and $\lambda_3$) is described in the Appendix. Equation (22) generally gives a pair of positive and negative solutions (cf. Appendix). Since the tuning parameter $\alpha_{\text{RTPP}}$ is defined between 0 and 1, we use the positive solution of Eq. (22) as a diagnosed RTPP parameter. Since no approximation is used for adaptive-RTPP, the diagnosed parameter $\alpha_{\text{RTPP}}$ satisfies Eq. (14) strictly. This can be an advantage of adaptive-RTPP relative to adaptive-RTPS. Another possible advantage of RTPP over RTPS is that perturbations with RTPP are more balanced than those of RTPS. RTPS inflates each element separately, so that the inflated ensemble perturbations may not be necessarily balanced.

3. Experimental design

3.1. NICAM-LETKF data assimilation system

This study uses a real-world atmospheric data assimilation system NICAM-LETKF (Terasaki et al., 2015) to test the adaptive covariance inflation methods. The NICAM-LETKF comprises the Nonhydrostatic Icosahedral Atmospheric Model (NICAM: Tomita and Satoh, 2004; Satoh et al., 2008; Satoh et al., 2014) for the atmospheric model, and the LETKF (Hunt et al., 2007; Miyoshi and Yamane, 2007) for data assimilation. NICAM is constructed based on the icosahedral grid points, so that the horizontal resolution of NICAM is described by a grid division level. This study uses the NICAM with the 6th grid division level (approximately 112 km horizontal resolution), and 38 vertical layers up to about 40 km. We use the cumulus parametrization scheme of Arakawa and Schubert (1974). The minimal advanced treatments of surface interaction and runoff model (Takata et al., 2003) and the slab ocean are used for the surface and ocean boundaries, respectively.

The LETKF can consider temporally distributed observations over a 6 h time window centred at the analysis time (also known as 4D-LETKF: Fertig et al., 2007; Miyoshi et al., 2010). We run 9 h ensemble forecasts and assimilate observations in the last 6 h period. The treatment of temporally distributed observations considerably improves the data assimilation performance to assimilate asynoptic observations such as aircraft data and satellite radiances (Terasaki and Miyoshi, 2017; personal communication). For the 4D-LETKF, temporal localization is applied to the observation error covariance, so that the errors of temporarily distant observations from the analysis time are enlarged. Due to the temporal localization, the transform matrices of the LETKF are different at each time slot of the 4D-LETKF. Although we can compute the transform matrices and analyses at every time slot, this study computes the analysis state only at the analysis time which is the minimum necessary. This study performs the 4D-LETKF with 40 ensemble members.

3.2. Implementation of adaptive covariance inflation methods

For the NICAM-LETKF, M11’s adaptive multiplicative inflation method (adaptive-MULT) was already implemented and tested (Terasaki et al., 2015; Terasaki and Miyoshi, 2017; personal communication). Adaptive-MULT estimates inflation factors locally at every model grid point with the OMB-OMB statistics within the localization radius. Therefore, adaptive-MULT estimates horizontally and vertically varying inflation factors. At each grid point, the same inflation factor is applied for all model variables in adaptive-MULT. We consider adaptive-MULT as the baseline control experiment, which is compared with adaptive-RTPS and adaptive-RTPP. To obtain a realistic relaxation parameter, the diagnosed parameter $\alpha$ is forced to be bounded by [0, 1]. Then, temporal smoothing is applied as follows:

$$\alpha_{\text{RTPP,RTPS}}(t) = \tau \cdot \alpha_{\text{RTPP,RTPS}}(t) + (1 - \tau) \cdot \alpha_{\text{smooth}}(t - 1),$$

where $\tau$ is the time-smoothing parameter chosen to be 0.03 in this study. At analysis time $t$, the RTPP and RTPS use the smoothed parameter obtained at the last analysis time $t - 1$. After the analysis, the smoothed parameter at $t$ is diagnosed based on the AMB-OMA statistics. The initial relaxation parameter is set to be 0.5 for all the experiments. We apply the observation operator for the temporary analysis perturbations $X_{\text{inp}}$, to obtain $H_{\text{inp}}$, which is needed to estimate the relaxation parameter with Eqs (20) and (A1). Because we obtain the analysis states only at the analysis time (i.e. at the centre of seven time slots of the 4D-LETKF; cf. Terasaki and Miyoshi, 2017; personal communication), the AMB-OMB statistics only at the analysis time are used to diagnose the relaxation parameters. In adaptive-MULT, the OMB-OMB statistics use all observations over the 6 h window.

3.3. Experiments

We perform two experiments with different observation networks. The first experiment assimilates only conventional observations from the NCEP operational system (also known as NCEP PREPBUFR; hereafter labelled as ‘CONV’). The second experiment assimilates the Advanced Microwave Sounding Unit-A (AMSU-A) brightness temperature observations in addition to the conventional observations (hereafter labelled as ‘AMSU’). The experimental settings (e.g. the boundary and initial conditions) follow Terasaki and Miyoshi (2017; personal communication). We test 13 covariance inflation methods for the two observation networks: (1) adaptive-MULT, (2) adaptive-RTPS, (3) adaptive-RTPP, and (4)–(13) RTPS and RTPP with five fixed relaxation parameters: 0.5, 0.6, 0.7, 0.8 and 0.9. Horizontal and vertical covariance localization is applied based on the Gaussian function with their standard deviations LS being 400 km and 0.4 natural-log-pressure, respectively. The localization function is replaced by zero beyond $2\sqrt{10/3}$ LS (cf. eq. 13 of Miyoshi and Yamane (2007)). We use the diagonal $\mathbf{R}$ for all experiments.

We assimilate the AMSU radiances for channels 6, 7 and 8, whose observation error standard deviations (SD) are set to be 0.5 K, which is larger than those used by the Japan Meteorological Agency (0.30 K; Japan Meteorological Agency, 2013) and Bornmann and Bauer (2010) (0.35 K). One may diagnose the observation error SD by the observation-space statistics (e.g. LKM09), but diagnosing the observation error SD in data assimilation cycles is beyond the scope of this study. Alternatively, this study investigates the sensitivity of the adaptive relaxation methods to the observation error settings for the AMSU radiances (cf. section 5.2). We assimilate the AMSU radiances only over the ocean between 60°S and 60°N.

For data assimilation, inflating the observation error variance while assuming the diagonal $\mathbf{R}$ is a commonly used technique to mitigate the observation error correlations (Liu and Rabier, 2003). One may consider using different observation error variances for data assimilation and for adaptive inflation diagnostics. However, in this study, we use the same observation error variances for both data assimilation and observation space statistics.

We perform data assimilation cycles for 3 months from June to August 2014, and validate the results in August relative to the ERA-Interim reanalysis (Dee et al., 2011).

4. Results

4.1. Experiments with conventional observations

We first examine how the proposed adaptive estimation methods work. Figure 1 shows the time series of the estimated relaxation parameters $\alpha_{\text{RTPP}}$ and $\alpha_{\text{RTPS}}$ over 3 months. Although the diagnosed relaxation parameters show noisy estimates, the smoothed parameters become stable after 15 days. A rapid drift
Figure 1. Time series of estimated relaxation parameters of (a) RTPS and (b) RTPP for the CONV experiment. Red circles and black curves show diagnosed and smoothed relaxation parameters, respectively. The abscissa shows month/day in 2014. [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 2. Time-mean background RMSDs and ensemble spreads of (a) RTPS and (b) RTPP methods for $T$ (K) at 500 hPa relative to the ERA-Interim reanalysis for the global domain, averaged over August 2014. Grey, yellow and blue bars show RMSDs of adaptive-MULT, RTPS (RTPP) with fixed parameters (0.5, 0.6, 0.7, 0.8, and 0.9), and adaptive-RTPS (adaptive-RTPP). Red dots indicate the ensemble spreads. The panels show the time-mean RMSDs for the CONV experiment. [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 3. Time-mean background ensemble spreads of (a) adaptive-MULT, (b) adaptive-RTPS, and (c) adaptive-RTPP for $T$ (K) at 500 hPa, averaged over August 2014. Panels (d)–(f) show differences in ensemble spread: (d) adaptive-RTPS – adaptive-MULT, (e) adaptive-RTPP – adaptive-MULT, and (f) adaptive-RTPP – adaptive-RTPS. [Colour figure can be viewed at wileyonlinelibrary.com].

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of the diagnosed parameters is observed during the initial 15 days when the data assimilation system spins up. The smoothed relaxation parameters averaged over August are 0.775 and 0.735 for adaptive-RTPS and adaptive-RTPP, respectively.

Figure 2 shows the time-mean analysis root-mean-square differences (RMSDs) for temperature \((T)\) at 500 hPa relative to the ERA-Interim reanalysis for the global domain. For the RTPS method, the mean smoothed parameter 0.775 is far from the empirically obtained optimal relaxation parameter 0.9. Consequently, the mean RMSD of adaptive-RTPS is significantly larger than that of the optimal relaxation parameter. On the other hand, the mean smoothed parameter 0.735 for adaptive-RTPP is close to the optimal parameter 0.8. The mean RMSD of adaptive-RTPP is significantly larger than that of the optimal relaxation parameter. On the other hand, the mean smoothed parameter 0.735 for adaptive-RTPP is close to the optimal parameter 0.8. Adaptive-RTPP shows a significantly improved RMSD relative to the control experiment with adaptive-MULT.

Figures 3 and 4 show the horizontal patterns of the time-mean analysis ensemble spreads and RMSDs of adaptive-MULT, adaptive-RTPS and adaptive-RTPP for \(T\) at 500 hPa in August 2014. Adaptive-RTPS and adaptive-RTPP show smaller ensemble spreads over land, and larger ensemble spreads over the ocean than adaptive-MULT (Figures 3(d) and (e)). In other words, adaptive-RTPS and adaptive-RTPP have smaller (larger) spreads in densely (sparsely) observed regions. Adaptive-RTPS and adaptive-RTPP show improved RMSDs over the tropical Pacific Ocean relative to adaptive-MULT (Figures 4(d) and (e)). We see the differences between adaptive-RTPS and adaptive-RTPP for the ensemble spread in the Southern Hemisphere (Figure 4(f)), where adaptive-RTPP outperforms adaptive-RTPS (Figure 4(f)).

4.2. Experiments with satellite radiances

Here we conduct the additional AMSU experiments to validate whether or not the results of the CONV experiment are consistent with a different observation network. Figure 5 shows the time series of the estimated relaxation parameters of the AMSU experiment. We also obtain stable relaxation parameters after 15 days, similarly to the CONV experiment. The mean smoothed relaxation parameters in August are 0.816 and 0.750 for adaptive-RTPS and adaptive-RTPP, respectively. We generally need larger inflation for denser observations. The relaxation parameter; alpha

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parameters of the RTPS and RTPP are reasonably estimated to be larger in the AMSU experiment.

Figure 6 is similar to Figure 3 but for the AMSU experiment. For adaptive-RTPP, the mean smoothed parameter 0.750 is close to the optimal parameter 0.80, and its RMSD is equivalent to that of the optimal RTPP. However, adaptive-RTPS of the AMSU experiment shows a significantly improved RMSD relative to adaptive-MULT (Figure 6(a)), unlike the CONV experiment. The RMSD of adaptive-RTPS is still worse than the optimal RTPS.

Figures 7 and 8 are similar to Figures 4 and 5, but for the AMSU experiment. Similarly to the CONV experiment, adaptive-RTPS and adaptive-RTPP show larger ensemble spreads and smaller RMSDs over the ocean than adaptive-MULT (Figures 7(d, e) and 8(d, e)). Also, adaptive-RTPP shows a larger ensemble spread than adaptive-RTPS in the Southern Hemisphere. However, adaptive-RTPP does not outperform adaptive-RTPS for the RMSDs in the Southern Hemisphere (Figure 8(f)), contrary to the CONV experiment.

5. Discussions

5.1. Spatial patterns of the ensemble spread

Here we focus on the ensemble spreads of the three adaptive inflation methods. As shown in Figures 3 and 7, adaptive-RTPS and adaptive-RTPP show smaller ensemble spreads in the densely observed regions than adaptive-MULT. Also, adaptive-RTPS and adaptive-RTPP show general improvements over the ocean, and insignificant changes or slight degradations over land relative to adaptive-MULT (Figures 4 and 8). These slight degradations in adaptive-RTPS and adaptive-RTPP may be related to an under-dispersive ensemble over land.
Figure 8. Similar to Figure 4, but for the AMSU experiments. [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 9. Time-mean background RMSDs and ensemble spreads of (a) CONV and (b) AMSU experiments for $T$ (K) at 500 hPa relative to the ERA-Interim reanalysis for the GL, NH, TR and SH, averaged over August 2014. Grey, yellow and green bars show the RMSDs of adaptive-MULT, adaptive-RTPS and adaptive-RTPP, respectively. Red dots indicate the ensemble spreads. [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 9 shows the time-mean background RMSDs and ensemble spreads for the global domain (GL), Northern Hemisphere (NH; between 20°N and 90°N), Tropics (TR; between 20°S and 20°N), and Southern Hemisphere (SH; between 90°S and 20°S). The ensemble spreads are generally smaller than the RMSDs except for adaptive-RTPP in the SH. This agrees with Hamill (2001) who suggested that the ensemble spreads be smaller than RMSDs unless verified against the unknown true state since the RMSDs include the additional error from the verifying analysis. When the verifying analysis (ERA-Interim reanalysis in this study) has a similar error relative to the unknown true state, the RMSDs may become smaller. Compared to adaptive-MULT, adaptive-RTPS and adaptive-RTPP show larger ensemble spreads in the SH and smaller ensemble spreads in the NH. As mentioned earlier, adaptive-MULT estimates the inflation factor locally at each model grid point, and adaptive-RTPS and adaptive-RTPP estimate their spatially homogeneous parameters with the observation-space statistics in the global domain. For the multiplicative inflation methods, it is well known that the spatially homogeneous (i.e. globally constant) parameter would lead to an under-dispersive ensemble in densely observed regions (cf. Fig. 7 of Miyoshi et al. (2010)). In contrast, relaxation methods achieve spatially dependent inflation factors with a spatially homogeneous relaxation parameter. However, the relaxation methods still lead to an under-dispersive ensemble in densely observed regions. Figure 10 compares the inflation patterns for adaptive-MULT, adaptive-RTPS, and adaptive-RTPP. Even though adaptive-RTPS and adaptive-RTPP provide spatially varying inflation factors, their land–sea contrasts are not as strong as adaptive-MULT. Adaptive-RTPS and adaptive-RTPP satisfy the AMB-OMA statistics in the global domain, but their observation-space statistics at each location may not be satisfied as strictly.
RTPP would never deflate the analysis error covariance unless the adaptive-RTPP showed no deflation. By definition, the RTPS and RTPP would be overestimated. On the other hand, adaptive-RTPS and (Figure 10(a)), the observation error SD of the AMSU radiances shows broader deflation areas than the CONV experiment value are used. Because the AMSU experiment (Figure 10(d)) shows larger values of the observation error variance than the true values, the OMB-OMB statistics would underestimate the inflation factor over tropical ocean (Figures 10(a) and (d)). As indicated by Eq. (9), we compute \[ \alpha \text{var}_{\text{diag}} \] if \( p < 100 \) at the grid point to avoid the sampling noise. We compute the normalized differences for the background and analysis: \( \frac{\text{var}_{\text{diag}}}{\text{var}_{\text{diag}}} \) and \( \frac{\text{var}_{\text{EnKF}}}{\text{var}_{\text{diag}}} \) (Figures 11 and 12). Figure 13 shows the number of observations used for the observation-space statistics of the AMSU experiment in adaptive-MULT. Here, we use Eqs (24) and (25) instead of the commonly used OMB-OMB statistics because the OMB-OMB statistics would not be suitable when imperfect \( R \) is used.

Figure 11 shows the normalized differences for the background. Adaptive-MULT generally overestimates (underestimates) the error variances over land (ocean) for the AMSU experiment. This disagreement can be caused by the suboptimal observation error variances because the OMB-OMB statistics used in adaptive-MULT include \( R \) explicitly (Eq. (9)). For example, we may force larger observation error variances to stabilize the filter, but this may cause an underestimation of the forecast error variances. As mentioned in section 3.3, the observation error standard deviation of the AMSU radiances in this study is larger than those used by Japan Meteorological Agency (2013) and Bormann and Bauer (2010). Although the optimal observation error settings should be different for different systems, adaptive-MULT may underestimate the forecast error variance due to larger values of the observation error variance for the AMSU radiances. Adaptive-RTPS and adaptive-RTPP generally outperform adaptive-MULT in the AMSU experiment over the ocean (Figures 8(d) and (e)), where the AMSU radiances have more impact (Terasaki and Miyoshi, 2017; personal communication).

By contrast, adaptive-RTPS and adaptive-RTPP underestimate (overestimate) the error variances over land (ocean). The land–sea contrast is clearer in adaptive-RTPS and adaptive-RTPP than in adaptive-MULT. We also compare \( \text{var} \) for the analysis (Figure 12). The overestimation of the analysis error variances over the ocean in adaptive-RTPS and adaptive-RTPP is even greater than that of the background error variances (Figure 11). The diagnosis implies that the relaxation methods overestimate

![Figure 10](image-url). Global inflation patterns for \( T \) at 500 hPa at 0000 UTC 1 August 2014 for (a, d) adaptive-MULT, (b, e) adaptive-RTPS and (c, f) adaptive-RTPP. (a–c) and (d–f) show the CONV and AMSU experiments, respectively. [Colour figure can be viewed at wileyonlinelibrary.com].

\[ \alpha = \text{var}_{\text{diag}} \]
Figure 11. Normalized differences of the background error variances between the EnKF estimation and diagnosis from observations in August 2014 for (a, d) adaptive-MULT, (b, e) adaptive-RTPS, and (c, f) adaptive-RTPP. Panels (a–c) and (d–f) show the results of the CONV and AMSU experiments, respectively. Green (brown) colours indicate overestimate (underestimate) of the error variances compared to the diagnosed values. A white area indicates the number of observations used for the diagnosis is less than 100. [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 12. Similar to Figure 11, but for the analysis error variances. [Colour figure can be viewed at wileyonlinelibrary.com].
Compared to adaptive-RTPS and adaptive-RTPP, the inflation decrease of the observation error settings for the AMSU radiances. The inflation factors generally increase with the worst inflation methods, among which adaptive-MULT is always the worst among the three experiments for all three adaptive observation error settings for the AMSU radiances. AMSU-0.2 K.

Figure 14 compares the time-mean background RMSDs with three observation error SDs of the AMSU radiances. The abscissa shows the observation error SD for the AMSU radiances.

Figure 15 compares the inflation fields of the three adaptive observation error settings for the AMSU radiances. The leftmost bars (ERR: 0.5 K) are exactly the same as Figure 9(b) for the global domain. (Colour figure can be viewed at wileyonlinelibrary.com).

5.2. Sensitivity to observation error settings for the AMSU radiances

We conduct additional experiments to investigate the sensitivity to the observation error settings for the AMSU radiances. The observation error SDs of the AMSU radiances are set to be 0.3 K (hereafter AMSU-0.3 K) as used by the Japan Meteorological Agency (2013), and 0.2 K (hereafter AMSU-0.2 K) as diagnosed by Bormann and Bauer (2010). The two experiments are compared with the original AMSU experiment (hereafter AMSU-0.5 K). Figure 14 compares the time-mean background RMSDs with three observation error settings for the AMSU radiances. AMSU-0.2 K is the worst among the three experiments for all three adaptive inflation methods, among which adaptive-MULT is always the worst.

Figure 15 compares the inflation fields of the three adaptive inflation methods. The inflation factors generally increase with the decrease of the observation error settings for the AMSU radiances. Compared to adaptive-RTPS and adaptive-RTPP, the inflation factors of adaptive-MULT are more sensitive to the observation error settings. This may be related to the use of the AMB-OMA statistics which do not include R explicitly. In addition, adaptive-RTPS and adaptive-RTPP estimate the globally uniform parameters using the observations in the global domain, so that the estimated relaxation parameters would be more robust than adaptive-MULT which uses only local observations to estimate inflation parameters at each grid point. Since the true observation errors are unknown, it is difficult to conclude which affects more significantly.

5.3. Adaptive and fixed relaxation parameters

Here we discuss on the reason why adaptive-RTPS could not outperform the tuned RTPS. As noted above, adaptive-RTPS and adaptive-RTPP adjust the analysis error covariance based on the AMB-OMA statistics. A possible reason is that adaptive-RTPS and adaptive-RTPP do not consider the model error explicitly. Figures 2 and 6 imply that the optimal parameters may be beyond the tested parameter range. To estimate the optimal parameter for RTPS, we conducted additional CONV and AMSU experiments with fixed relaxation parameters 0.95, 1.00, 1.05, 1.10, 1.15 and 1.20 (Figure 16), even though the range of relaxation parameters is usually bounded by [0, 1]. The optimal RTPS parameters are estimated around 1.00–1.15 for the CONV and AMSU experiments. α > 1.0 is beyond the normal range, in which the analysis ensemble spread becomes larger than the background ensemble spread. This agrees with Figure 16 showing the time-mean analysis and background ensemble spreads. Namely, the ensemble spread decreases during the forecast. Whitaker and Hamill (2012) showed that the ensemble spread of the RTPS analysis grows more slowly than that of the RTPP analysis because the RTPS gives analyses less physically balanced than the RTPP. The optimal parameter for the RTPS is beyond 1.0 probably because of the slower growth of the ensemble spread than the RTPP. Whitaker and Hamill (2012) hypothesized that the RTPPS accounts for the errors associated with data assimilation, while the model errors can be treated by additive inflation. This study tests the RTPS and RTPP without applying additive inflation. The optimal relaxation parameters would be different if additive inflation is applied simultaneously.

One may notice that the diagnosed relaxation parameters seem to oscillate (Figures 1 and 5). Mainly due to the radiosonde
observations of the conventional observations, there is a large difference in the number of assimilated observations at 0000, 0600, 1200 and 1800 UTC. For example, the numbers of assimilated observations are about 35 000 for 0000 and 1200 UTC, and 10 000 for 0600 and 1800 UTC in the CONV experiment. We compute the mean diagnosed RTPS and RTPP parameters in August separately for 0000 and 1200 UTC, and for 0600 and 1800 UTC (Table 1). For adaptive-RTPS, the diagnosed relaxation parameter at 0000 and 1200 UTC is significantly larger than that of the other times. Namely, the optimal RTPS parameter would be different for 0000 and 1200 UTC, and for 0600 and 1800 UTC. In the case of adaptive-RTPS in the CONV experiment (Table 1), the background ensemble spreads at 0600 and 1800 UTC would be under-dispersive because the smaller relaxation parameter 0.770 (all) is used instead of 0.859 estimated for 0000 and 1200 UTC. Generally, the under-dispersive background ensemble is more problematic than the over-dispersive ensemble. A possible solution is to estimate the relaxation parameter for 0000 and 1200 UTC, and 0600 and 1800 UTC separately as done by Miyoshi and Kunii (2012).

Interestingly, the RTPP parameter shows an opposite behaviour to the RTPS parameter. The diagnosed parameter at 0000 and

Figure 15. Similar to Figure 10, but for the AMSU experiments for (a, d, g) adaptive-MULT, (b, e, h) adaptive-RTPS and (c, f, i) adaptive-RTPP. The observation error SD for the AMSU radiances are 0.5, 0.3 and 0.2 K for (a–c), (d–f) and (g–i), respectively. [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 16. Time-mean RMSDs and ensemble spreads for (a) CONV and (b) AMSU for T (K) at 500 hPa relative to the ERA-Interim reanalysis for the global domain, averaged over the month August 2014. Yellow and blue bars show the background RMSDs for RTPS with fixed parameters (0.90, 0.95, 1.00, 1.05, 1.10, 1.15 and 1.20), and adaptive-RTPS. Red and blue dots indicate the background and analysis ensemble spreads, respectively. [Colour figure can be viewed at wileyonlinelibrary.com].
1200 UTC is smaller than that at the other times. In the case of adaptive-RTPP in the CONV experiment (Table 1), the background ensembles at 0000 and 1200 UTC would be under-dispersive because the smaller relaxation parameter 0.733 (all) is used instead of 0.798 estimated for 0600 and 1800 UTC. Adaptive-RTPP outperforms adaptive-RTPS in the CONV experiment. Note that it does not necessarily mean that the RTPP is better than the RTPS if the relaxation parameters are tuned to be optimal. An advantage of adaptive-RTPP is that the diagnosed relaxation parameter strictly satisfies the observation-space statistics unlike adaptive-RTPS. In addition, another advantage is that the analyses produced by the RTPP would be more physically consistent than those by the RTPS. However, adaptive-RTPP could be degraded with a higher relaxation parameter due to the fast growth of perturbations. As mentioned in section 2.2, the RTPP preserves grown perturbations during the forecasts like the breeding method (Toth and Kalnay, 1993). For the moment, we have not obtained sufficient results to conclude which adaptive relaxation method would be better than the other. It would be interesting to test adaptive-RTPS and adaptive-RTPP with simultaneous additive inflation. Further experiments, such as increasing the number of observations and the adaptive estimation of the RTPS and RTPP parameters with additive inflation are important future directions.

5.4. Adaptation to an abrupt change of the observing network

Here we investigate how the adaptive inflation methods react to a sudden change of the observing network. The estimation of the spatially dependent inflation fields (adaptive-MULT) requires temporally homogeneous observing networks (Miyoshi and Kunii, 2012). This does not apply to certain observing platforms such as aircraft and ships, as well as satellites. We perform an additional experiment assimilating conventional and AMSU observations in June and July, and only conventional observations in August. Namely, we suddenly lose AMSU in August, simulating an abrupt change of the observing network. We call it the ABRUPT experiment. Figure 17 shows the time series of the RMSDs of the CONV, AMSU and ABRUPT experiments. Adaptive-MULT shows a degradation relative to the CONV experiment before 21 August, although the same conventional observations were assimilated in August. It takes about 20 days for adaptive-MULT to adapt to the conventional observations after suddenly losing the AMSU observations. On the other hand, adaptive-RTPS and adaptive-RTPP show no significant degradation relative to the CONV experiment. Hence, adaptive-RTPS and adaptive-RTPP are more robust to the sudden change of the observing network mainly because the relaxation methods are robust to the change in observing network. The inflation fields of adaptive-RTPS and adaptive-RTPP turn out to be less sensitive to the observing network and observation error settings than those by adaptive-MULT (Figures 10 and 15).

6. Summary

Previous studies have been seeking better inflation methods for the EnKF. The adaptive approaches would be desirable in terms of the computational cost for manually tuning the inflation parameters. This study proposed two adaptive relaxation methods, adaptive-RTPS and adaptive-RTPP, for the realistic data assimilation problems. The main findings are summarized as follows:

- The relaxation parameters are estimated reasonably well for the RTPS and RTPP based on the AMB-OMA statistics. The estimated RTPP parameter is close to the optimal parameter. The estimated RTPS parameter tends to be smaller than the optimal parameter. It would be better to estimate the relaxation parameter separately for 0000 and 1200 UTC and for 0600 and 1800 UTC to consider the general difference of the number of assimilated observations.
- Adaptive-RTPS and adaptive-RTPP have a spatially homogeneous relaxation parameter and lead to an over-dispersive (under-dispersive) ensemble in the sparsely (densely) observed regions. The observation-space statistics were not satisfied at each location with the adaptive-RTPS or adaptive-RTPP.
- Adaptive-RTPS and adaptive-RTPP are robust to observation error settings and sudden change of the observing network.

Table 1. Time-mean diagnosed RTPS and RTPP parameters in August for the CONV and AMSU experiments at 0000 and 1200 UTC, 0600 and 1800 UTC, and all of the four times.

| Experiment | CONV | AMSU |
|------------|------|------|
|            | 0000, 1200 | 0600, 1800 | All | 0000, 1200 | 0600, 1800 | All |
| Adaptive-RTPS | 0.859 | 0.681 | 0.770 | 0.878 | 0.744 | 0.811 |
| Adaptive-RTPP | 0.668 | 0.798 | 0.733 | 0.689 | 0.807 | 0.748 |

Figure 17. Time series of the background RMSDs for T (K) at 500 hPa relative to the ERA-Interim reanalysis for the global domain for (a) adaptive-MULT, (b) adaptive-RTPS, and (c) adaptive-RTPP. Black, blue and red lines show AMSU, CONV and ABRUPT experiments, respectively. Dashed lines are 6 h RMSDs, and solid lines are the 3-day running means. The abscissa shows month/day in 2014. [Colour figure can be viewed at wileyonlinelibrary.com].
networks. Adaptive-MULT would need more realistic observation error variances compared to adaptive-RTPS and adaptive-RTPP.

- Adaptive-RTPS and adaptive-RTPP generally outperform an adaptive multiplicative inflation method. The improvements would be caused by the use of the observation-space statistics without the observation error covariance and by the use of global observations for the statistics. This study adapts the adaptation methods to the real atmospheric EnKF, therefore, the true observation error covariance is unknown. An OSSE is needed to conduct a clear comparison of the adaptation methods.

Finally, we address a limitation of this study: the treatment of model errors within inflation methods is not explicitly considered in this study. As hypothesized by Whitaker and Hamill (2012), additive inflation would be needed to account for the model error in RTPS and RTPP. It is still an open question how adaptive-RTPS and adaptive-RTPP work with additive inflation. Nevertheless, this study demonstrates adaptive-RTPS and adaptive-RTPP work reasonably well in the real atmospheric EnKF for the first time. Our future study will investigate more about the adaptation methods with additive inflation through OSSES.

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Appendix

Solution of RTPP parameter from observation-space statistics

This appendix describes a detailed derivation to obtain the RTPP parameter aRTPP from the AMB-OMA statistics. By substituting Eq. (6) into Eq. (14) and normalizing with $\mathbb{R}^{-1}$, we obtain a following quadratic equation for $a_{\text{RTPP}}$:

$$
\text{tr}(\mathbf{d}^{-b}(\mathbf{d}^{-a})^T \mathbf{R}^{-1}) = (1 - a_{\text{RTPP}})^2 \text{tr}(\mathbf{H} \mathbf{X}_{\text{imp}} \mathbf{H}^T \mathbf{X}_{\text{imp}})^T \mathbf{R}^{-1} / (m - 1) - 2(1 - a_{\text{RTPP}}) \text{tr}(\mathbf{H} \mathbf{X}_{\text{imp}}, (\mathbf{H} \mathbf{X}_{\text{imp}})^T \mathbf{R}^{-1}) / (m - 1) + (a_{\text{RTPP}})^2 \text{tr}((\mathbf{H} \mathbf{X}_{\text{imp}})^T (\mathbf{H} \mathbf{X}_{\text{imp}})^T \mathbf{R}^{-1}) / (m - 1).
$$

(1A)

By rearranging Eq. (1A), we obtain Eq. (22) with the following coefficients:

$$
\lambda_1 = 1 - 2\Phi_2 + \Phi_3,
$$

(A2)

$$
\lambda_2 = 2\Phi_2 - 2\Phi_3,
$$

(A3)

$$
\lambda_3 = \Phi_3 - \Phi_4,
$$

(A4)

where

$$
\Phi_1 = \text{tr}(\mathbf{H} \mathbf{X}^T (\mathbf{H} \mathbf{X})^T \mathbf{R}^{-1}) / (m - 1),
$$

(A5)

$$
\Phi_2 = \text{tr}(\mathbf{H} \mathbf{X}_{\text{imp}} \mathbf{H} \mathbf{X}_{\text{imp}})^T \mathbf{R}^{-1} / (m - 1),
$$

(A6)

$$
\Phi_3 = \text{tr}((\mathbf{H} \mathbf{X}_{\text{imp}})^T (\mathbf{H} \mathbf{X}_{\text{imp}})^T \mathbf{R}^{-1}) / (m - 1),
$$

(A7)

$$
\Phi_4 = \text{tr}(\mathbf{d}^{-b}(\mathbf{d}^{-a})^T \mathbf{R}^{-1}) / (m - 1).
$$

(A8)

The third coefficient $\lambda_3$ is expected to be non-negative because the normalized temporary analysis error covariance $\text{tr}(\mathbf{H} \mathbf{P}_{\text{tmp}} \mathbf{H}^T \mathbf{R}^{-1})$ is generally underestimated relative to the diagnosis $\text{tr}(\mathbf{d}^{-b}(\mathbf{d}^{-a})^T \mathbf{R}^{-1})$. The second coefficient $\lambda_2$ is always positive since the background perturbation $\mathbf{X}$ is larger than the temporary analysis perturbation $\mathbf{X}_{\text{tmp}}$. The first coefficient $\lambda_1$ is expected to be positive as follows:

$$
\lambda_1 = \frac{1}{m - 1} \sum_{i=1}^{p} \left( \frac{(\mathbf{H} \mathbf{X}^T)^T - 2(\mathbf{H} \mathbf{X}^T)H \mathbf{X}_{\text{imp}}^T + (\mathbf{H} \mathbf{X}_{\text{imp}})^T}{(m - 1) \mathbf{R}^{-1} / (m - 1)} \right)^2.
$$

(1A9)

Summarizing the above, the coefficients $\lambda_1, \lambda_2, \lambda_3$ are expected to be positive, positive and negative, respectively. Therefore, the solutions of the quadratic equation (Eq. 21) are expected to have both positive and negative values. Because the formulations in this appendix assume that the observation operator $\mathbf{H}$ is linear, the expectation could be unsatisfied if the observation operator is nonlinear. Also, the third coefficient $\lambda_3$ can be positive if we pick up subsamples from all observation-space differences. With the positive $\lambda_3$, Eq. (22) can result in imaginary solutions. Note that our data assimilation experiments never give the imaginary solution because all observation-space differences are always used to estimate RTPP parameter $a_{\text{RTPP}}$.

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