Transparallel mind:
Classical computing with quantum power

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Abstract

Inspired by the extraordinary computing power promised by quantum computers, the quantum mind hypothesis postulated that quantum mechanical phenomena are the source of neuronal synchronization, which, in turn, might underlie consciousness. Here, I present an alternative inspired by a classical computing method with quantum power. This method relies on special distributed representations called hyperstrings. Hyperstrings are superpositions of up to an exponential number of strings, which – by a single-processor classical computer – can be evaluated in a transparallel fashion, that is, simultaneously as if only one string were concerned. Building on a neurally plausible model of human visual perceptual organization, in which hyperstrings are formal counterparts of transient neural assemblies, I postulate that synchronization in such assemblies is a manifestation of transparallel information processing. This accounts for the high combinatorial capacity and speed of human visual perceptual organization and strengthens ideas that self-organizing cognitive architecture bridges the gap between neurons and consciousness.

Keywords
cognitive architecture; distributed representations; neuronal synchronization; quantum computing; transparallel computing by hyperstrings; visual perceptual organization.

Highlights

• A neurally plausible alternative to the quantum mind hypothesis is presented.
• This alternative is based on a classical computing method with quantum power.
• This method relies on special distributed representations, called hyperstrings.
• Hyperstrings allow many similar features to be processed as one feature.
• Thereby, they enable a computational explanation of neuronal synchronization.
1 Introduction

Mind usually is taken to refer to cognitive faculties, such as perception and memory, which enable consciousness. In this article at the intersection of cognitive science and artificial intelligence research, I do not discuss full-blown models of cognition as a whole, but I do aim to shed more light on the nature of cognitive processes. To this end, I review a powerful classical computing method – called transparallel processing by hyperstrings (van der Helm 2004) – which has been implemented in a minimal coding algorithm called PISA. The algorithm takes just symbol strings as input but my point is that transparallel processing might well be a form of cognitive processing (van der Helm 2012). This approach to neural computation has been developed in cognitive science – in research on human visual perceptual organization, in particular – and is communicated here to the artificial intelligence community. For both domains, the novel observations in this article are (a) that this classical computing method has the same computing power as that promised by quantum computers (Feynman 1982), and (b) that it provides a neurally plausible alternative to the quantum mind hypothesis (Penrose 1989). Next, to set the stage, five currently relevant ingredients are introduced briefly.

1.1 Human visual perceptual organization

Visual perceptual organization is the neuro-cognitive process that enables us to perceive scenes as structured wholes consisting of objects arranged in space (see Fig. 1). This process may seem to occur effortlessly and we may take it for granted in daily life, but by all accounts, it must be both complex and flexible. To give a gist (following Gray 1999): For a proximal stimulus, it usually singles out one hypothesis about the distal stimulus from among a myriad of hypotheses that also would fit the proximal stimulus (this is the inverse optics problem). To this end, multiple sets of features at multiple, sometimes overlapping, locations in a stimulus must be grouped in parallel. This implies that the process must cope simultaneously with a large number of possible combinations, which,

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1 PISA is available at https://perswww.kuleuven.be/~u0084530/doc/pisa.html. Its worst-case computing time may be weakly-exponential (i.e., near-tractable), but in this article, the focus is on the special role of hyperstrings in it. The name PISA, by the way, was originally an acronym of Parameter load (i.e., a complexity metric), Iteration (i.e., repetition), Symmetry, and Alternation.
Fig. 1 Visual perceptual organization. Both images at the top can be interpreted as 3D cubes and as 2D mosaics, but as indicated by "Yes" and "No", humans preferably interpret the one at the left as a 3D cube and the one at the right as a 2D mosaic (after Hochberg and Brooks 1960)

in addition, seem to interact as if they are engaged in a stimulus-dependent competition between grouping criteria. This indicates that the combinatorial capacity of the perceptual organization process must be high, which, together with its high speed (it completes in the range of 100–300 ms), reveals its truly impressive nature.

I think that this cognitive process must involve something additional to traditionally considered forms of processing (see also Townsend and Nozawa 1995). The basic form of processing thought to be performed by the neural network of the brain is parallel distributed processing (PDP), which typically involves interacting agents who simultaneously do different things. However, the brain also exhibits a more sophisticated processing mode, namely, neuronal synchronization, which involves interacting agents who simultaneously do the same thing – think of flash mobs or choirs going from cacophony to harmony.

1.2 Neuronal synchronization

Neuronal synchronization is the phenomenon that neurons, in transient assemblies, temporarily synchronize their firing activity. Such assemblies are thought to arise when neurons shift their allegiance to different groups by altering connection strengths, which
may also imply a shift in the specificity and function of neurons (Edelman 1987; Gilbert 1992). Both theoretically and empirically, neuronal synchronization has been associated with various cognitive processes and 30–70 Hz gamma-band synchronization, in particular, has been associated with feature binding in visual perceptual organization (Eckhorn et al. 1988; Gray 1999; Gray and Singer 1989).

Ideas about the meaning of gamma-band synchronization are, for instance, that it binds neurons which, together, represent one perceptual entity (Milner 1974; von der Malsburg 1981), or that it is a marker that an assembly has arrived at a steady state (Pollen 1999), or that its strength is an index of the salience of features (Finkel et al. 1998; Salinas and Sejnowski 2001), or that more strongly synchronized assemblies in a visual area in the brain are locked on more easily by higher areas (Fries 2005). These ideas sound plausible, that is, synchronization indeed might reflect a flexible and efficient mechanism subserving the representation of information, the regulation of the flow of information, and the storage and retrieval of information (Sejnowski and Paulsen 2006; Tallon-Baudry 2009).

I do not challenge those ideas, but notice that they are about cognitive factors associated with synchronization rather than about the nature of the underlying cognitive processes. In other words, they merely express that synchronization is a manifestation of cognitive processing – just as the bubbles in boiling water are a manifestation of the boiling process (Bojak and Liley 2007; Shadlen and Movshon 1999). The question then still is, of course, of what form of cognitive processing it might be a manifestation. My stance in this article is that neuronal synchronization might well be a manifestation of transparallel information processing, which, as I explicate later on, means that many similar features are processed simultaneously as if only one feature were concerned.

Apart from the above ideas about the meaning of synchronization, two lines of research into the physical mechanisms of synchronization are worth mentioning. First, research using methods from dynamic systems theory (DST) showed that the occurrence of synchronization and desynchronization in a network depends crucially on system parameters that regulate interactions between nodes (see, e.g., Buzsáki 2006; Buzsáki and Draguhn 2004; Campbell et al. 1999; van Leeuwen 2007; van Leeuwen et al. 1997). This DST research is relevant, because it investigates – at the level of neurons – how synchronized as-
Assemblies might go in and out of existence. Insight therein complements research, like that in this article, into what these assemblies do in terms of cognitive information processing. Second, the quantum mind hypothesis postulated that quantum mechanical phenomena, such as quantum entanglement and superposition, are the subneuron source of neuronal synchronization which, in turn, might underlie consciousness (Penrose 1989; Penrose and Hameroff 2011; see also Atmanspacher 2011). This hypothesis is controversial, mainly because quantum mechanical phenomena do not seem to last long enough to be useful for neuro-cognitive processing, let alone for consciousness (Chalmers 1995; Chalmers 1997; Searle 1997; Seife 2000; Stenger 1992; Tegmark 2000). Be that as it may, the quantum mind hypothesis had been inspired by quantum computing which is a currently relevant form of processing.

### 1.3 Quantum computing

Quantum computing, an idea from physics (Deutsch and Jozsa 1992; Feynman 1982), is often said to be the holy grail of computing: It promises – rightly or wrongly – to be exponentially faster than classical computing. More specifically, the difference between classical computers and quantum computers is as follows. Classical computers, on the one hand, work with binary digits (*bits*) which each represent either a one or a zero, so that a classical computer with *N* bits can be in only one of $2^N$ states at any one time. Quantum computers, on the other hand, work with quantum bits (*qubits*) which each can represent a one, a zero, or any quantum superposition of these two qubit states. A quantum computer with *N* qubits can therefore be in an arbitrary superposition of $O(2^N)$ states simultaneously. A final read-out will give one of these states but, crucially, (a) the outcome of the read-out is affected directly by this superposition, which (b) effectively means that, until the read-out, all these states can be dealt with simultaneously as if only one state were concerned.

The latter suggests an extraordinary computing power. For instance, many computing tasks (e.g., in string searching) require, for an input the size of *N*, an exhaustive search

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2 Formally, for functions $f$ and $g$ defined on the positive integers, $f$ is $O(g)$ if a constant $C$ and a positive integer $n_0$ exist such that $f(n) \leq C \cdot g(n)$ for all $n \geq n_0$. Informally, $f$ then is said to be in the order of magnitude of $g$.
among $O(2^N)$ candidate outputs. A naive computing method, that is, one that processes each of the $O(2^N)$ options separately, may easily require more time than is available in this universe (van Rooij 2008), and compared with that, quantum computing promises an $O(2^N)$ reduction in the amount of work and time needed to complete a task.

However, the idea of quantum computing also needs qualification. First, it is true that the quest for quantum computers progresses (e.g., by the finding of Majorana fermions which might serve as qubits; Mourik et al. 2012), but there still are obstacles, and thus far, no scalable quantum computer has been built. Second, it is true that quantum computing may speed up some computing tasks (see, e.g., Deutsch and Jozsa 1992; Grover 1996; Shor 1994), but the vast majority of computing tasks cannot benefit from it (Ozhigov 1999). This reflects the general tendency that more sophisticated methods have a more restricted application domain. Quantum computing, for instance, requires the applicability of unitary transformations to preserve the coherence of superposed states. Third, quantum computers are often claimed to be generally superior to classical computers, that is, faster than or at least as fast as classical computers for any computing task. However, there is no proof of that (Hagar 2011), and in this article, I in fact challenge this claim. To put this in a broader perspective, I next review generic forms of processing.

1.4 Generic forms of processing

In computing or otherwise, a traditional distinction is that between serial and parallel processing. Serial processing means that subtasks are performed one after the other by one processor, and parallel processing means that subtasks are performed simultaneously by different processors. In addition, however, one may define subserial processing, meaning that subtasks are performed one after the other by different processors (i.e., one processor handles one subtask, and another processor handles the next subtask). For instance, the whole process at a supermarket checkout is a form of multi-threading, but more specifically, (a) the cashiers work in parallel; (b) each cashier processes customer carts serially; and (c) the carts are presented subserially by customers. Compared to serial processing, subserial processing yields no reduction in the work and time needed to complete an entire task, while parallel processing yields reduction in time but not in work. Subserial processing as such is perhaps not that interesting, but taken together with serial and parallel
processing – as in Fig. 2 – it calls for what I dubbed transparallel processing. Transparallel processing means that subtasks are performed simultaneously by one processor, that is, as if only one subtask were concerned. The next pencil selection metaphor may give a gist. To select the longest pencil from among a number of pencils, one or many persons could measure the lengths of the pencils in a (sub)serial or parallel fashion, after which the lengths can be compared to each other. However, a smarter – transparallel – way would be if one person gathers all pencils in one bundle upright on a table, so that the longest pencil can be selected in a glance. This example illustrates that, in contrast to (sub)serial and parallel processing, transparallel processing reduces the work – and thereby the time – needed to complete a task.

In computing, transparallel processing may seem science-fiction and quantum computers indeed reflect a prospected hardware method to do transparallel computing. However, in Sect. 2, I review an existing software method to do transparallel computing on single-processor classical computers. This software method does not fit neatly in existing process taxonomies (Flynn 1972; Townsend and Nozawa 1995). For instance, it escapes the distinction between task parallelism and data parallelism. It actually relies on a special kind of distributed representations, which I next briefly introduce in general.

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Fig. 2 Generic forms of processing, defined by the number of subtasks performed at a time (one or many) and the number of processors involved (one or many)

| One processor | Many processors |
|---------------|-----------------|
| Subserial processing | Parallel processing |
| Serial processing | Transparallel processing |

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3 The pencil selection example is close to the spaghetti metaphor in sorting (Dewdney 1984) but serves here primarily to illustrate that, in some cases, items can be gathered in one bin that can be dealt with as if it comprised only one item (hyperstrings are such bins).
1.5 Distributed representations

In both classical and quantum computing, the term distributed processing is often taken to refer to a process that is divided over a number of processors. Also then, it refers more generally (i.e., independently of the number of processors involved) to a process that operates on a distributed representation of information. For instance, in the search for extraterrestrial intelligence (SETI), a central computer maintains a distributed representation of the sky: It divides the sky into pieces that are analyzed by different computers which report back to the central computer. Furthermore, PDP models in cognitive science often use the brain metaphor of activation spreading in a network of processors which operate in parallel to regulate interactions between pieces of information stored at different places in the network (e.g., Rumelhart and McClelland 1982). The usage of distributed representations in SETI is a form of data parallelism, which, just as task parallelism, reduces the time but not the work needed to complete an entire task. In PDP models, however, it often serves to achieve a reduction in work – and, thereby, also in time.

Formally, a distributed representation is a data structure that can be visualized by a graph (Harary 1994), that is, by a set of interconnected nodes, in which pieces of information are represented by the nodes, or by the links, or by both. One may think of road maps, in which roads are represented by links between nodes that represent places, so that possibly overlapping sequences of successive links represent whole routes. Work-reducing distributed representations come in various flavors, but for \( N \) nodes, they typically represent superpositions of \( O(2^N) \) wholes by means of only \( O(N^2) \) parts. To find a specific whole, a process then might confine itself to examining only the \( O(N^2) \) parts. Well-known examples in computer science are the shortest path method (Dijkstra 1959) and methods using suffix trees (Gusfield 1997) and deterministic finite automatons (Hopcroft and Ullman 1979).

In computer science, such classical computing methods are also called smart methods, because, compared to a naive method that processes each of those \( O(2^N) \) wholes separately, they reduce an exponential \( O(2^N) \) amount of work to a polynomial one – typically one between \( O(N) \) and \( O(N^2) \). Hence, their computing power is between that of naive methods and that of quantum computing, the latter reducing an exponential \( O(2^N) \) amount of work to a constant \( O(1) \) one. The main points in the remainder of this
article now are, first, that hyperstrings are distributed representations that take classical computing to the level of quantum computing, and second, that they enable a neurally plausible alternative to the quantum mind hypothesis.

2 Classical computing with quantum power

The idea that the brain represents information in a reduced, distributed, fashion has been around for a while (Hinton 1990). Fairly recently, ideas have emerged – in both cognitive science and computer science – that certain distributed representations allow for information processing that is mathematically analogous to quantum computing (Aerts et al. 2009), or might even allow for classical computing with quantum power (Rinkus 2012). Returning in these approaches is that good candidates for that are so-called holographic reduced representations (Plate 1991). To my knowledge, however, these approaches have not yet achieved actual classical computing with quantum power – which hyperstrings do achieve. Formally, hyperstrings differ from holographic reduced representations, but conceptually, they seem to have something in common: Hyperstrings are distributed representations of what I called holographic regularities (van der Helm 1988; van der Helm and Leeuwenberg 1991). Let me first briefly give some background thereof.

2.1 Background

The minimal coding problem, for which hyperstrings enable a solution, arose in the context of structural information theory (SIT), which is a general theory of human visual perceptual organization (Leeuwenberg and van der Helm 2013). In line with the Gestalt law of Prägnanz (Wertheimer 1912, 1923; Köhler 1920; Koffka 1935), it adopts the simplicity principle, which aims at economical mental representations: It holds that the simplest organization of a stimulus is the one most likely perceived by humans (Hochberg and McAlister 1953). To make quantifiable predictions, SIT developed a formal coding model for symbol strings, which, in the Appendix, is presented and illustrated in the form it has since about 1990. The minimal coding problem thus is the problem to compute guaranteed simplest codes of strings, that is, codes which – by exploiting regularities – specify strings by a minimum number of descriptive parameters.
SIT, of course, does not assume that the human visual system converts visual stimuli into strings. Instead, it uses manually obtained strings to represent stimulus interpretations in the sense that such a string can be read as a series of instructions to reproduce a stimulus (much like a computer algorithm is a series of instructions to produce output). For instance, for a line pattern, a string may represent the sequence of angles and line segments in the contour. A stimulus can be represented by various strings, and the string with the simplest code is taken to reflect the preferred interpretation, with a hierarchical organization as described by that simplest code. In other words, SIT assumes that the processing principles, which its formal model applies to strings, reflect those which the human visual system applies to visual stimuli.

The regularities considered in SIT’s formal coding model are repetition (juxtaposed repeats), symmetry (mirror symmetry and broken symmetry), and alternation (nonjuxtaposed repeats). These are mathematically unique in that they are the only regularities with a hierarchically transparent holographic nature (for details, see van der Helm and Leeuwenberg 1991, or its clearer version in van der Helm 2014). To give a gist, the string \textit{ababbaba} exhibits a global symmetry, which can be coded step by step, each step adding one identity relationship between substrings – say, from \textit{aba \textit{S}(b) \textit{aba}} to \textit{ab \textit{S}(a)(b) ba}, and so on until \textit{S\{(a)(b)(a)(b)\}}. The fact that such a stepwise expansion preserves symmetry illustrates that symmetry is holographic. Furthermore, the argument \textit{(a)(b)(a)(b)} of the resulting symmetry code can be hierarchically recoded into \textit{2 \star (ab)(ab)} of the original string illustrates that symmetry is hierarchically transparent.

The formal properties of holography and hierarchical transparency not only single out repetition, symmetry, and alternation, but are also perceptually adequate. They explain much of the human perception of single and combined regularities in visual stimuli, whether or not perturbed by noise (for details, see van der Helm 2014; van der Helm and Leeuwenberg 1996, 1999, 2004). For instance, they explain that mirror symmetry and Glass patterns are better detectable than repetition, and that the detectability of mirror symmetry and Glass patterns in the presence of noise follows a psychophysical law that improves on Weber’s law (van der Helm 2010). Currently relevant is their role in the minimal coding problem – next, this is discussed in more detail.
2.2 Hyperstrings

At first glance, the minimal coding problem seems to involve just the detection of regularities in a string, followed by the selection of a simplest code (see Fig. 3). However, these are actually the relatively easy parts of the problem – they can be solved by traditional computing methods (van der Helm 2004, 2012, 2014). Therefore, here, I focus not so much on the entire minimal coding problem, but rather on its hard part, namely, the problem that the argument of every detected symmetry or alternation has to be hierarchically recoded before a simplest code may be selected (repetition does not pose this problem).

As spelled out in the Appendix, this implies that a string of length $N$ gives rise to a superexponential $O(2^{N \log N})$ number of possible codes. This has raised doubts about the tractability of minimal coding, and thereby, about the adequacy of the simplicity principle in perception (e.g., Hatfield and Epstein 1985).

Yet, the hard part of minimal coding can be solved, namely, by first gathering sim-
Fig. 4 Distributed representations of similar regularities. (a) Graph representing the arguments of all symmetries into which the string \textit{ababfabbabfabbaba} can be encoded. (b) Graph representing the arguments of all symmetries into which the slightly different string \textit{ababfabbabfabbabab} can be encoded. Notice that sets \(\pi(1, 5)\) and \(\pi(6, 10)\) of substrings represented by the subgraphs on vertices 1–5 and 6–10 are identical in (a) but disjoint in (b).

Similar regularities in distributed representations (van der Helm 2004; van der Helm and Leeuwenberg 1991). For instance, the string \textit{ababfabbabfabbaba} exhibits, among others, the symmetry \(S[(aba)(b)(f)(aba)(b)]\). Its argument \((aba)(b)(f)(aba)(b)\) is represented in Fig. 4a by the path along vertices 1, 4, 5, 6, 9, and 10. In fact, Fig. 4a represents, in a distributed fashion, the arguments of all symmetries into which the string can be encoded. Because of the holographic nature of symmetry, such a distributed representation for a string of length \(N\) can be constructed in \(O(N^2)\) computing time, and represents \(O(2^N)\) symmetry arguments. In Fig. 4b, the same has been done for the string \textit{ababfabbabfabbabab}, but notice the difference: Though the input strings differ only slightly, the sets \(\pi(1, 5)\) and \(\pi(6, 10)\) of substrings represented by the subgraphs on vertices 1–5 and 6–10 are identical in Fig. 4a but disjoint in Fig. 4b.

The point now is that, if symmetry arguments (or, likewise, alternation arguments) are gathered this way, then the resulting distributed representations consist provably of one (as in Figs. 4a and 4b) or more independent hyperstrings (see van der Helm 2004, 2014, or the Appendix, for the formal definition of hyperstrings and for the proofs). As I clarify next, this implies that a single-processor classical computer can hierarchically recode up to an exponential number of symmetry or alternation arguments in a transparallel fashion, that is, simultaneously as if only one argument were concerned.

As illustrated in Fig. 5, a hyperstring is an st-digraph (i.e., a directed acyclic graph...
Fig. 5 A hyperstring. Every path from source (vertex 1) to sink (vertex 9) in the hyperstring at the top represents a normal string via its edge labels. The two hypersubstrings indicated by bold edges represent identical substring sets $\pi(1, 4)$ and $\pi(5, 8)$, both consisting of the substrings $abc$, $xc$, and $ay$. For the string $h_1...h_8$ at the bottom, with substrings defined as corresponding one-to-one to hypersubstrings, this implies that the substrings $h_1h_2h_3$ and $h_5h_6h_7$ are identical. This single identity relationship corresponds in one go to three identity relationships between substrings in the normal strings, namely, between the substrings $abc$ in string $abcfabcg$, between the substrings $xc$ in string $xcfxcg$, and between the substrings $ay$ in string $ayfayg$.

with one source and one sink) with, crucially, one Hamiltonian path from source to sink (i.e., a path that visits every vertex only once). Every source-to-sink path in a hyperstring represents some normal string consisting of some number of elements, but crucially, substring sets represented by hypersubstrings are either identical or disjoint (as illustrated in Fig. 4). This implies that every identity relationship between substrings in one of the normal strings corresponds to an identity relationship between hypersubstrings, and that inversely, every identity relationship between hypersubstrings corresponds in one go to identity relationships between substrings in several of the normal strings.

The two crucial properties above imply that a hyperstring can be searched for regularity as if it were a single normal string. For instance, the hyperstring in Fig. 5 can be treated as if it were a string $h_1...h_8$, with substrings that correspond one-to-one to hypersubstrings (see Fig. 5). The latter means that, in this case, the substrings $h_1h_2h_3$ and $h_5h_6h_7$ are identical, so that the string $h_1...h_8$ can be encoded into, for instance,
the alternation \( ((h_1h_2h_3)) / (h_4)h_8) \). This alternation thus in fact captures the identity relationship between the substring sets \( \pi(1, 4) \) and \( \pi(5, 8) \), and thereby it captures in one go several identity relationships between substrings in different strings in the hyperstring. That is, it represents, in one go, alternations in three different strings, namely:

\[
\langle (abc) \rangle / \langle (f)(g) \rangle \quad \text{in the string} \quad abc fab cg
\]

\[
\langle (xc) \rangle / \langle (f)(g) \rangle \quad \text{in the string} \quad xcf xcg
\]

\[
\langle (ay) \rangle / \langle (f)(g) \rangle \quad \text{in the string} \quad ayf ayg
\]

Returning to hyperstrings of symmetry or alternation arguments, one could say that they represent up to an exponential number of hypotheses about an input string, which, as the foregoing illustrates, can be evaluated further simultaneously. That is, such a hyperstring can be encoded without having to distinguish explicitly between the arguments

![Hyperstring of symmetry arguments](image)

![Regularity search](image)

![Selection of simplest code for the symmetry hyperstring](image)

**Fig. 6** Hyperstring encoding. The hyperstring represents the arguments of all symmetries into which the string *ababfabababababa* can be encoded. The recursive regularity search yields subcodes capturing regularities in hypersubstrings (here, only a few are shown).
represented in it (see Fig. 6). At the front end of such an encoding, one, of course, has to establish the identity relationships between hypersubstrings, but because of the Hamiltonian path, this can be done in the same way as for a normal string. At the back end, one, of course, has to select eventually a simplest code, but by way of the shortest path method (Dijkstra 1959), also this can be done in the same way as for a normal string. Thus, in total, recursive hierarchical recoding yields a tree of hyperstrings, and during the buildup of this tree, parts of it can already be traced back to select simplest codes of (hyper)substrings – to select eventually a simplest code of the entire input string.  

2.3 Discussion

Transparallel processing by hyperstrings has been implemented in the minimal coding algorithm PISA (see Footnote 1). PISA, which runs on single-processor classical computers, relies fully on the mathematical proofs concerning hyperstrings (see the Appendix) and can therefore be said to provide an algorithmic proof that those mathematical proofs are correct. Hence, whereas quantum computers provide a still prospected hardware method to do transparallel computing, hyperstrings provide an already feasible software method to do transparallel computing on classical computers. To be clear, transparallel computing by hyperstrings differs from efficient classical simulations of quantum computing (Gottesman 1998). After all, it implies a truly exponential reduction from $O(2^N)$ subtasks to one subtask, that is, it implies that $O(2^N)$ symmetry or alternation arguments can be hierarchically recoded as if only one argument were concerned. This implies that it provides classical computers with the same extraordinary computing power as that promised by quantum computers. Thus, in fact, it can be said to reflect a novel form of quantum logic (cf. Dunn et al. 2013), which challenges the alleged general superiority of quantum computers over classical computers (see Hagar 2011).

In general, the applicability of any computing method depends on the computing task at hand. This also holds for transparallel computing – be it by quantum computers or via hyperstrings by classical computers. Therefore, I do not dare to speculate on whether,  

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1 For a given input string, the tree of hyperstrings and its hyperstrings are built on the fly, that is, the hyperstrings are transient in that they bind similar features in the current input only. This contrasts with standard PDP modeling, which assumes that one fixed network suffices for many different inputs.
for some tasks, hyperstrings might give super-quantum power to quantum computers. Be that as it may, for now, it is true that quantum computers may speed up some tasks but it is misleading to state in general terms that they will be exponentially faster than classical computers. As shown in this section, for at least one computing task, hyperstrings provide classical computers with quantum power and it remains to be seen if quantum computers also can achieve this power for this task. As I next discuss more speculatively, this application also is relevant to the question – in cognitive neuroscience – of what the computational role of neuronal synchronization might be.

3 Human cognitive architecture

Cognitive architecture, or unified theory of cognition, is a concept from artificial intelligence research. It refers to a blueprint for a system that acts like an intelligent system – taking into account not only its resulting behavior but also physical or more abstract properties implemented in it (Anderson 1983; Newell 1990). Hence, it aims to cover not only competence (what is a system’s output?) but also performance (how does a system arrive at its output?), or in other words, it aims to unify representations and processes (Byrne 2012; Langley et al. 2009; Sun 2004; Thagard 2012). The minimal coding method in the previous section – developed in the context of a competence model of human visual perceptual organization – qualifies as cognitive architecture in the technical sense. In this section, I investigate if it also complies with ideas about neural processing in the visual hierarchy in the brain. From the available neuroscientific evidence, I gather the next picture of processing in the visual hierarchy.

3.1 The visual hierarchy

The neural network in the visual hierarchy is organized with 10–14 distinguishable hierarchical levels (with multiple distinguishable areas within each level), contains many short-range and long-range connections (both within and between areas and levels), and can be said to perform a distributed hierarchical process (Felleman and van Essen 1991). This process comprises three neurally intertwined but functionally distinguishable subprocesses (Lamme and Roelfsema 2000; Lamme et al. 1998). As illustrated in the left-hand
Fig. 7 Processing in the visual hierarchy in the brain. A stimulus-driven perception process, comprising three neurally intertwined subprocesses (left-hand panel), yields hierarchical stimulus organizations (right-hand panel). A task-driven attention process then may scrutinize these hierarchical organizations in a top-down fashion.

Panel in Fig. 7, these subprocesses are responsible for (a) feedforward extraction of, or tuning to, features to which the visual system is sensitive, (b) horizontal binding of similar features, and (c) recurrent selection of different features. As illustrated in the right-hand panel in Fig. 7, these subprocesses together yield integrated percepts given by hierarchical organizations (i.e., organizations in terms of wholes and parts) of distal stimuli that fit proximal stimuli. Attentional processes then may scrutinize these organizations in a top-down fashion, that is, starting with global structures and, if required by task and allowed by time, descending to local features (Ahissar and Hochstein 2004; Collard and Povel 1982; Hochstein and Ahissar 2002; Wolfe 2007).

Hence, whereas perception logically processes parts before wholes, the top-down attentional scrutiny of hierarchical organizations implies that wholes are experienced before parts. Thus, this combined action of perception and attention explains the dominance of wholes over parts, as postulated in early twentieth century Gestalt psychology (Wertheimer 1912, 1923; Köhler 1920; Koffka 1935) and as confirmed later in a range of behavioral studies (for a review, see Wagemans et al. 2012). This dominance also has been specified further by notions such as global precedence (Navon 1977), configural superiority
(Pomerantz et al. 1977), primacy of holistic properties (Kimchi 2003), and superstructure dominance (Leeuwenberg and van der Helm 1991, Leeuwenberg et al. 1994).

Furthermore, notice that the combination of feedforward extraction and recurrent selection in perception is like a fountain under increasing water pressure: As the feedforward extraction progresses along ascending connections, each passed level in the visual hierarchy forms the starting point of integrative recurrent processing along descending connections (see also VanRullen and Thorpe 2002). This yields a gradual buildup from partial percepts at lower levels in the visual hierarchy to complete percepts near its top end. This gradual buildup takes time, so, it leaves room for attention to intrude, that is, to modulate things before a percept has completed (Lamme and Roelfsema 2000; Lamme et al. 1998). However, I think that, by then, the perceptual organization process already has done much of its integrative work (Gray 1999; Pylyshyn 1999), because, as I sustain next, its speed is high due to, in particular, transparallel feature processing.

3.2 The transparallel mind hypothesis

The perceptual subprocess of feedforward extraction is reminiscent of the neuroscientific idea that, going up in the visual hierarchy, neural cells mediate detection of increasingly complex features (Hubel and Wiesel 1968). Furthermore, the subprocess of recurrent selection is reminiscent of the connectionist idea that a standard PDP process of activation spreading in the brain’s neural network yields percepts represented by stable patterns of activation (Churchland 1986). While I think that these ideas capture relevant aspects, I also think that they are not yet sufficient to account for the high combinatorial capacity and speed of the human perceptual organization process. I think that, to this end, the subprocess of horizontal binding is crucial. This subprocess may be relatively underexposed in neuroscience, but may well be the neuronal counterpart of the regularity extraction operations, which, in representational theories like SIT, are proposed to obtain structured mental representations.

In this respect, notice that the minimal coding method in Sect. 2 relies on three intertwined subprocesses, which correspond to the three neurally intertwined subprocesses in the visual hierarchy (see Fig. 8). In the minimal coding method for strings, the formal counterpart of feedforward extraction is a search for identity relationships between
Hyperstrings

Binding of similar features

Extraction of visual features

All-pairs shortest path method

Hyperstrings

All-substrings identification

### Fig. 8

(a) The three intertwined perceptual subprocesses in the visual hierarchy in the brain.
(b) The three corresponding subprocesses implemented in the minimal coding method

...substrings, by way of an $O(N^2)$ all-substrings identification method akin to using suffix trees (van der Helm 2014). Furthermore, it implements recurrent selection by way of the $O(N^3)$ all-pairs shortest path method (Cormen et al. 1994), which is a distributed processing method that simulates PDP (van der Helm 2004, 2012, 2014). Currently most relevant, it implements horizontal binding by gathering similar regularities in hyperstrings, which allows them to be recoded hierarchically in a transparallel fashion.

This correspondence between three neurally intertwined subprocesses in the visual hierarchy in brain and three algorithmically intertwined subprocesses in the minimal coding method is, to me, more than just a nice parallelism. Epistemologically, it substantiates that knowledge about neural mechanisms and knowledge about cognitive processes can be combined fruitfully to gain deeper insights into the workings of the brain (Marr 1982/2010). Furthermore, ontologically relevant, one of its resulting deeper insights is that transparallel processing might well be an actual form of processing in the brain. That is, horizontal binding of similar features is, in the visual hierarchy in the brain, thought to be mediated by transient neural assemblies, which signal their presence by synchronization of the neurons involved (Gilbert 1992). Synchronization is more than standard PDP and the hyperstring implementation of horizontal binding in the minimal coding method – which enables transparallel processing – therefore leads me to the following hypothesis about the meaning of neuronal synchronization.
The transparallel mind hypothesis

Neuronal synchronization is a manifestation of cognitive processing of similar features in a transparallel fashion.

This hypothesis is, of course, speculative – but notice that it is based on (a) a perceptually adequate and neurally plausible model of the combined action of human perception and attention, and (b) a feasible form of classical computing with quantum power. Next, this hypothesis is discussed briefly in a broader, historical, context.

3.3 Discussion

In 1950, theoretical physicist Richard Feynman and cognitive psychologist Julian Hochberg met in the context of a colloquium they gave. Feynman would become famous for his work on quantum electrodynamics, and Hochberg was developing the idea that, among all possible organizations of a visual stimulus, the simplest one is most likely to be the one perceived by humans. They discussed parallels between their ideas and concluded that quantum-like cognitive processing might underlie such a simplicity principle in human visual perceptual organization (Hochberg 2012). This fits in with the long-standing idea that cognition is a dynamic self-organizing process (Attneave 1982; Kelso 1995; Koffka 1935; Lehar 2003). For instance, Hebb (1949) put forward the idea of phase sequence, that is, the idea that thinking is the sequential activation of sets of neural assemblies. Furthermore, Rosenblatt (1958) and Fukushima (1975) proposed small artificial networks – called perceptrons and cognitrons, respectively – as formal counterparts of cognitive processing units. More recently, the idea arose that actual cognitive processing units – or, as I call them, gnosons (i.e., fundamental particles of cognition) – are given by transient neural assemblies defined by synchronization of the neurons involved (Buzsáki 2006; Fingelkurts and Fingelkurts 2001, 2004; Finkel et al. 1998).

In this article, I expanded on such thinking, by taking hyperstrings as formal counterparts of gnosons, and by proposing that neuronal synchronization is a manifestation of transparallel information processing. Because transparallel processing by hyperstrings provides classical computers with quantum power, it strengthens ideas that quantum-like cognitive processing does not have to rely on actual quantum mechanical phenomena at the subneuron level (de Barros and Suppes 2009; Suppes et al. 2012; Townsend and
Nozawa 1995; Vassilieva et al. 2011). As I argued, it might rely on interactions at the level of neurons. It remains to be seen if the transparallel mind hypothesis is tenable too for synchronization outside the "visual" gamma band, but for one thing, it accounts for the high combinatorial capacity and speed of human visual perceptual organization.

4 Conclusion

Complementary to DST research on how synchronized neural assemblies might go in and out of existence due to interactions between neurons, the transparallel mind hypothesis expresses the representational idea that neuronal synchronization mediates transparallel information processing. This idea was inspired by a classical computing method with quantum power, namely transparallel processing by hyperstrings, which allows up to an exponential number of similar features to be processed simultaneously as if only one feature were concerned. This neurocomputational account thus strengthens ideas that mind is mediated by transient neural assemblies constituting flexible cognitive architecture between the relatively rigid level of neurons and the still elusive level of consciousness.

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Appendix

Structural information theory (SIT) adopts the simplicity principle, which holds that the simplest organization of a visual stimulus is the one most likely perceived by humans. To enable quantifiable predictions, SIT developed a formal coding model to determine simplest codes of symbol strings. This model provides coding rules for the extraction of regularities, and a metric of the complexity, or structural information load $I$, of codes.

The coding rules serve the extraction of the transparent holographic regularities repetition (or iteration I), symmetry (S), and alternation (A). They can be applied to any substring of an input string, and a code of the input string consists of a string of symbols and coded substrings, such that decoding the code returns the input string. Formally, SIT’s coding language and complexity metric are defined as follows.

**Definition 1** A code $X$ of a string $X$ is a string $t_1t_2...t_m$ of code terms $t_i$ such that $X = D(t_1)...D(t_m)$, where the decoding function $D : t \rightarrow D(t)$ takes one of the following forms:

- **I-form:** $n * (y) \rightarrow yyy...y \quad (n \text{ times } y; n \geq 2)$
- **S-form:** $S[(\overline{x}_1)(\overline{x}_2)...(\overline{x}_n), (p)] \rightarrow x_1x_2...x_n p x_n...x_2x_1 \quad (n \geq 1)$
- **A-form (I):** $\langle(\overline{y})\rangle / (\overline{x}_1)(\overline{x}_2)...(\overline{x}_n) \rightarrow yx_1yx_2...yx_n \quad (n \geq 2)$
- **A-form (II):** $\langle(\overline{x}_1)(\overline{x}_2)...(\overline{x}_n)\rangle / (\overline{y}) \rightarrow x_1y x_2y ... x_ny \quad (n \geq 2)$

Otherwise: $D(t) = t$

for strings $y, p, \text{ and } x_i (i = 1, 2, ..., n)$. The code parts $(\overline{y}), (\overline{p}), \text{ and } (\overline{x}_i)$ are chunks. The chunk $(\overline{y})$ in an I-form or an A-form is a repeat, and the chunk $(\overline{p})$ in an S-form is a pivot which, as a limit case, may be empty. The chunk string $(\overline{x}_1)(\overline{x}_2)...(\overline{x}_n)$ in an S-form is an S-argument consisting of S-chunks $(\overline{x}_i)$, and in an A-form, it is an A-argument consisting of A-chunks $(\overline{x}_i)$.

**Definition 2** Let $X$ be a code of string $X = s_1s_2...s_N$. The complexity $I$ of $X$ in structural information parameters (SIP) is given by the sum of (a) the number of remaining symbols $s_i \quad (1 \leq i \leq N)$ and (b) the number of chunks $(\overline{y})$ in which $y$ is neither a symbol nor an S-chunk.

The last part of Definition 2 may seem somewhat ad hoc, but has a solid theoretical basis in terms of degrees of freedom in the hierarchical organization described by a code.
Furthermore, Definition 1 implies that a string may be encodable into many different codes. For instance, a code may involve not only recursive encodings of strings inside chunks – that is, from \( y \) into \( y' \) – but also hierarchically recursive encodings of S- or A-arguments \((x_1)(x_2)...(x_n)\) into \((x_1')(x_2')...(x_n')\). The following sample of codes for one and the same string may give a gist of the abundance of coding possibilities:

String: \( X = abacdacdababacdacdab \)  
\( I = 20 \) sip

Code 1: \( \overline{X} = a \ b \ 2 \ast (acd) \ S[[a](b), \ (a)] \ 2 \ast (cda) \ b \)  
\( I = 14 \) sip

Code 2: \( \overline{X} = \langle(aba)/\langle(cdac)(badacdacdab)\rangle \)  
\( I = 20 \) sip

Code 3: \( \overline{X} = \langle(S[[a], \ (b)])\rangle/\langle(S[[cd], \ (a)])\rangle/S[(b)(a)(cd), \ (a)]\rangle \)  
\( I = 15 \) sip

Code 4: \( \overline{X} = S[[ab](acd)(ac)(bd)] \)  
\( I = 14 \) sip

Code 5: \( \overline{X} = S[S[[ab]](acd)]\] \)  
\( I = 7 \) sip

Code 6: \( \overline{X} = 2 \ast ((a))/\langle(S[[b]], \ ((cd)])\rangle \)  
\( I = 8 \) sip

Code 1 is a code with six code terms, namely, one S-form, two I-forms, and three symbols.  
Code 2 is an A-form with chunks containing strings that may be encoded as given in Code 3. Code 4 is an S-form with an empty pivot and illustrates that, in general, S-forms describe broken symmetry; mirror symmetry then is the limit case in which every S-chunk contains only one symbol. Code 5 gives a hierarchical recoding of the S-argument in Code 4. Code 6 is an I-form in which the repeat has been encoded into an A-form with an A-argument that has been recoded hierarchically into an S-form.

The computation of a simplest code for a string requires an exhaustive search for ISA-forms into which its substrings can be encoded, followed by the selection of a simplest code for the entire string. This also requires the hierarchical recoding of S- and A-arguments: A substring of length \( k \) can be encoded into \( O(2^k) \) S-forms and \( O(k2^k) \) A-forms, and to pinpoint a simplest one, simplest codes of the arguments of these S- and A-forms have to be determined as well – and so on, with \( O(\log N) \) recursion steps, because \( k/2 \) is the maximal length of the argument of an S- or A-form into which a substring of length \( k \) can be encoded. Hence, recoding S- and A-arguments separately would require a superexponential \( O(2^{N\log N}) \) total amount of work.

This combinatorial explosion can be nipped in the bud by gathering the arguments of S- and A-forms in distributed representations. The next definitions and proofs show that S-arguments and A-arguments group naturally into distributed representations consisting of
one or more independent hyperstrings – which enable the hierarchical recoding of up to an exponential number of S- or A-arguments in a transparent fashion, that is, simultaneously as if only one argument were concerned. Here, only A-forms \langle (y)/(x_1)(x_2)...(x_n) \rangle with repeat y consisting of one element are considered, but Definition 4 and Theorem 1 below hold mutatis mutandis for other A-forms as well.

- Graph-theoretical definition of hyperstrings:
  
  **Definition 3** A hyperstring is a simple semi-Hamiltonian directed acyclic graph \((V, E)\) with a labeling of the edges in \(E\) such that, for all vertices \(i, j, p, q \in V:\)
  
  either \(\pi(i, j) = \pi(p, q)\) or \(\pi(i, j) \cap \pi(p, q) = \emptyset\)
  
  where substring set \(\pi(v_1, v_2)\) is the set of label strings represented by the paths between vertices \(v_1\) and \(v_2\); the subgraph on the vertices and edges in these paths is a hypersubstring.

- Definition of distributed representations called A-graphs, which represent all A-forms covering suffixes of strings, that is, all A-forms into which those suffixes can be encoded (see Fig. 9 for an example):
  
  **Definition 4** For a string \(T = s_1s_2...s_N\), the A-graph \(A(T)\) is a simple directed acyclic graph \((V, E)\) with \(V = \{1, 2, ..., N + 1\}\) and, for all \(1 \leq i < j \leq N\), edges \((i, j)\) and \((j, N + 1)\) labeled with, respectively, the chunks \((s_i...s_{j-1})\) and \((s_j...s_N)\) if and only if \(s_i = s_j\).

- Definition of diafixes, which are substrings centered around the midpoint of a string (this notion complements the known notions of prefixes and suffixes, and facilitates the explication of the subsequent definition of S-graphs):
  
  **Definition 5** A diafix of a string \(T = s_1s_2...s_N\) is a substring \(s_{i+1}...s_{N-i}\) \((0 \leq i < N/2)\).

- Definition of distributed representations called S-graphs, which represent all S-forms covering diafixes of strings (see Fig. 10 for an example):
  
  **Definition 6** For a string \(T = s_1s_2...s_N\), the S-graph \(S(T)\) is a simple directed acyclic graph \((V, E)\) with \(V = \{1, 2, ..., \lceil N/2 \rceil + 2\}\) and, for all \(1 \leq i < j \leq \lceil N/2 \rceil + 2\), edges \((i, j)\) and \((j, \lceil N/2 \rceil + 2)\) labeled with, respectively, the chunk \((s_i...s_{j-1})\) and the possibly empty chunk \((s_j...s_{N-j+1})\) if and only if \(s_i...s_{j-1} = s_{N-j+2}...s_{N-i+1}\).
Fig. 9 The A-graph for string $T = \text{akagakag}$, with three independent hyperstrings (connected only at vertex 11) for the three sets of A-forms with repeats $a$, $k$, and $g$, respectively, which cover suffixes of $T$. An A-graph may contain so-called pseudo A-pair edges – like, here, edge $(10, 11)$ – which do not correspond to actual repeat plus A-chunk pairs; they cannot end up in codes but are needed to maintain the integrity of hyperstrings during recoding.

Fig. 10 The S-graph for string $T = \text{ababfdedgpfdedgaba}$, with two independent hyperstrings given by the solid edges, which represent S-chunks in S-forms covering diafixes of $T$. The dashed edges represent the pivots, which come into play after hyperstring recoding.
Theorem 1 The A-graph $\mathcal{A}(T)$ for a string $T = s_1s_2...s_N$ consists of at most $N + 1$ disconnected vertices and at most $\lfloor N/2 \rfloor$ independent subgraphs (i.e., subgraphs that share only the sink vertex $N + 1$), each of which is a hyperstring.

Proof (1) By Definition 4, vertex $i$ ($i \leq N$) in $\mathcal{A}(T)$ does not have incoming or outgoing edges if and only if $s_i$ is a unique element in $T$. Since $T$ contains at most $N$ unique elements, $\mathcal{A}(T)$ contains at most $N + 1$ disconnected vertices, as required.

(2) Let $s_{i_1}, s_{i_2}, ..., s_{i_n}$ ($i_p < i_{p+1}$) be a complete set of identical elements in $T$. Then, by Definition 4, the vertices $i_1, i_2, ..., i_n$ in $\mathcal{A}(T)$ are connected with each other and with vertex $N+1$ but not with any other vertex. Hence, the subgraph on the vertices $i_1, i_2, ..., i_n, N+1$ forms an independent subgraph. For every complete set of identical elements in $T$, $n$ may be as small as 2, so that $\mathcal{A}(T)$ contains at most $\lfloor N/2 \rfloor$ independent subgraphs, as required.

(3) The independent subgraphs must be semi-Hamiltonian to be hyperstrings. Now, let $s_{i_1}, s_{i_2}, ..., s_{i_n}$ ($i_p < i_{p+1}$) again be a complete set of identical elements in $T$. Then, by Definition 4, $\mathcal{A}(T)$ contains edges $(i_p, i_{p+1})$, $p = 1, 2, ..., n-1$, and it contains edge $(i_n, N+1)$. Together, these edges form a Hamiltonian path through the independent subgraph on the vertices $i_1, i_2, ..., i_n, N+1$, as required.

(4) The only thing left to prove is that the substring sets are pairwise either identical or disjoint. Now, for $i < j$ and $k \geq 1$, let substring sets $\pi(i, i+k)$ and $\pi(j, j+k)$ in $\mathcal{A}(T)$ be not disjoint, that is, let them share at least one chunk string. Then, the substrings $s_i...s_{i+k-1}$ and $s_j...s_{j+k-1}$ of $T$ are necessarily identical and, also necessarily, $s_i = s_{i+k}$ and either $s_j = s_{j+k}$ or $j + k = N + 1$. Hence, by Definition 4, these identical substrings of $T$ yield, in $\mathcal{A}(T)$, edges $(i, i+k)$ and $(j, j+k)$ labeled with the identical chunks $(s_i...s_{i+k-1})$ and $(s_j...s_{j+k-1})$, respectively. Furthermore, obviously, these identical substrings of $T$ can be chunked into exactly the same strings of two or more identically beginning chunks. By Definition 4, all these chunks are represented in $\mathcal{A}(T)$, so that each of these chunkings is represented not only by a path $(i, ..., i+k)$ but also by a path $(j, ..., j+k)$. This implies that the substring sets $\pi(i, i+k)$ and $\pi(j, j+k)$ are identical. The foregoing holds not only for the entire A-graph but, because of their independence, also for every independent subgraph. Hence, in sum, every independent subgraph is a hyperstring, as required. ■
Lemma 1 Let the strings $c_1 = s_1 s_2 ... s_k$ and $c_2 = s_1 s_2 ... s_p$ ($k < p$) be such that $c_2$ can be written in the following two ways:

$$
c_2 = c_1 X \quad \text{with} \quad X = s_{k+1} ... s_p
$$

$$
c_2 = Y c_1 \quad \text{with} \quad Y = s_1 ... s_{p-k}
$$

Then, $X = Y$ if $q = p/(p - k)$ is an integer; otherwise $Y = VW$ and $X = WV$, where $V = s_1 ... s_r$ and $W = s_{r+1} ... s_{p-k}$, with $r = p - \lfloor q \rfloor (p - k)$.

Proof (1) If $1 < q < 2$, then $c_2 = c_1 W c_1$, so that $Y = c_1 W$ and $X = W c_1$. Then, too, $r = k$, so that $c_1 = V$. Hence, $Y = VW$ and $X = WV$, as required.

(2) If $q = 2$, then $c_2 = c_1 c_1$. Hence, $X = Y = c_1$, as required.

(3) If $q > 2$, then the two copies of $c_1$ in $c_2$ overlap each other as follows:

$$
c_2 = c_1 X = s_1 \ldots s_{p-k} \ s_{p-k+1} \ldots s_k \ s_{k+1} \ldots s_p
$$

$$
c_2 = Y c_1 = \ Y \ s_1 \ldots s_{2k-p} \ s_{2k-p+1} \ldots s_k
$$

Hence, $s_i = s_{p-k+i}$ for $i = 1, 2, \ldots, k$. That is, $c_2$ is a prefix of an infinite repetition of $Y$.

(3a) If $q$ is an integer, then $c_2$ is a $q$-fold repetition of $Y$, that is, $c_2 = YY \ldots Y$. This implies (because also $c_2 = Y c_1$) that $c_1$ is a $(q - 1)$-fold repetition of $Y$, so, $c_2$ can also be written as $c_2 = c_1 Y$. This implies $X = Y$, as required.

(3b) If $q$ is not an integer, then $c_2$ is a $\lfloor q \rfloor$-fold repetition of $Y$ plus a residual prefix $V$ of $Y$, that is, $c_2 = YY \ldots YV$. Now, $Y = VW$, so that $c_2$ can also be written as $c_2 = VWVW \ldots VWV$. This implies (because also $c_2 = Y c_1 = VW c_1$) that $c_1 = VW \ldots VWV$, that is, $c_1$ is a $(\lfloor q \rfloor - 1)$-fold repetition of $Y = VW$ plus a residual part $V$. This, in turn, implies that $c_2$ can also be written as $c_2 = c_1 WV$, so that $X = WV$, as required. ■
Lemma 2 Let $S(T)$ be the S-graph for a string $T = s_1s_2...s_N$. Then:

1. If $S(T)$ contains edges $(i, i + k)$ and $(i, i + p)$, with $k < p < \lfloor N/2 \rfloor + 2 - i$, then it also contains a path $(i + k, ..., i + p)$.
2. If $S(T)$ contains edges $(i - k, i)$ and $(i - p, i)$, with $k < p$ and $i < \lfloor N/2 \rfloor + 2$, then it also contains a path $(i - p, ..., i - k)$.

Proof (1) Edge $(i, i + k)$ represents S-chunk $(c_1) = (s_{i+1}...s_{i+k})$, and edge $(i, i + p)$ represents S-chunk $(c_2) = (s_{i+1}...s_{i+p})$. This implies that diafix $D = s_{i+1}...s_{N-i+1}$ of $T$ can be written in two ways:

$$D = c_2 \ldots c_2$$
$$D = c_1 \ldots c_1$$

This implies that $c_2$ (which is longer than $c_1$) can be written in two ways:

$$c_2 = c_1X \quad \text{with} \quad X = s_{i+k}...s_{i+p-1}$$
$$c_2 = Yc_1 \quad \text{with} \quad Y = s_{i-1}...s_{i-p-k-1}$$

Hence, by Lemma 1, either $X = Y$ or $Y = VW$ and $X = WV$ for some $V$ and $W$. If $X = Y$, then $D = c_1Y...c_1$ so that, by Definition 6, $Y$ is an S-chunk represented by an edge that yields a path $(i + k, ..., i + p)$ as required. If $Y = VW$ and $X = WV$, then $D = c_1WV...VWc_1$ so that, by Definition 6, $W$ and $V$ are S-chunks represented by subsequent edges that yield a path $(i + k, ..., i + p)$ as required.

(2) This time, edge $(i - k, i)$ represents S-chunk $(c_1) = (s_{i-k}...s_{i-1})$, and edge $(i - p, i)$ represents S-chunk $(c_2) = (s_{i-p}...s_{i-1})$. This implies that diafix $D = s_{i-p}...s_{N-i+p+1}$ of $T$ can be written in two ways:

$$D = c_2 \ldots c_2$$
$$D = Yc_1 \ldots c_1X$$

with $X=s_{i-p+k}...s_{i-1}$ and $Y=s_{i-p}...s_{i-k-1}$. Hence, as before, $c_2 = c_1X$ and $c_2 = Yc_1$, so that, by Lemma 1, either $X = Y$ or $Y = VW$ and $X = WV$ for some $V$ and $W$. This implies either $D = Yc_1...c_1Y$ or $D = VWc_1...c_1WV$. Hence, this time, Definition 6 implies that both cases yield a path $(i - p, ..., i - k)$, as required. ■
Lemma 3 In the S-graph $S(T)$ for a string $T = s_1 s_2 ... s_N$, the substring sets $\pi(v_1, v_2)$ $(1 \leq v_1 < v_2 < \lfloor N/2 \rfloor + 2)$ are pairwise identical or disjoint.

Proof Let, for $i < j$ and $k \geq 1$, substring sets $\pi(i, i + k)$ and $\pi(j, j + k)$ in $S(T)$ be nondisjoint, that is, let them share at least one S-chunk string. Then, the substrings $s_i ... s_{i+k-1}$ and $s_j ... s_{j+k-1}$ in the left-hand half of $T$ are necessarily identical to each other. Furthermore, by Definition 6, the substring in each chunk of these S-chunk strings is identical to its symmetrically positioned counterpart in the right-hand half of $T$, so that also the substrings $s_{N-i-k+2} ... s_{N-i+1}$ and $s_{N-j-k+2} ... s_{N-j+1}$ in the right-hand half of $T$ are identical to each other. Hence, the diafixes $D_1 = s_i ... s_{N-i+1}$ and $D_2 = s_j ... s_{N-j+1}$ can be written as

$$D_1 = s_i ... s_{i+k-1} p_1 s_{N-i-k+2} ... s_{N-i+1}$$
$$D_2 = s_i ... s_{i+k-1} p_2 s_{N-i-k+2} ... s_{N-i+1}$$

with $p_1 = s_{i+k} ... s_{N-i+k-1}$ and $p_2 = s_{j+k} ... s_{N-j+k+1}$. Now, by means of any S-chunk string $C$ in $\pi(i, i + k)$, diafix $D_1$ can be encoded into the covering S-form $S[C, (p_1)]$. If pivot $(p_1)$ is replaced by $(p_2)$, then one gets the covering S-form $S[C, (p_2)]$ for diafix $D_2$. This implies that any S-chunk string in $\pi(i, i + k)$ is in $\pi(j, j + k)$, and vice versa. Hence, nondisjoint substring sets $\pi(i, i + k)$ and $\pi(j, j + k)$ are identical, as required. ■

Theorem 2 The S-graph $S(T)$ for a string $T = s_1 s_2 ... s_N$ consists of at most $\lfloor N/2 \rfloor + 2$ disconnected vertices and at most $\lfloor N/4 \rfloor$ independent subgraphs that, without the sink vertex $\lfloor N/2 \rfloor + 2$ and its incoming pivot edges, form one disconnected hyperstring each.

Proof From Definition 6, it is obvious that there may be disconnected vertices and that their number is at most $\lfloor N/2 \rfloor + 2$, so let us turn to the more interesting part. If $S(T)$ contains one or more paths $(i, ..., j)$ $(i < j < \lfloor N/2 \rfloor + 2)$ then, by Lemma 2, one of these paths visits every vertex $v$ with $i < v < j$ and $v$ connected to $i$ or $j$. This implies that, without the pivot edges and apart from disconnected vertices, $S(T)$ consists of disconnected semi-Hamiltonian subgraphs. Obviously, the number of such subgraphs is at most $\lfloor N/4 \rfloor$, and if these subgraphs are expanded to include the pivot edges, they form one independent subgraph each. More important, by Lemma 3, these disconnected semi-Hamiltonian subgraphs form one hyperstring each, as required. ■