Quantum correlations of light from a room temperature mechanical oscillator for force metrology

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The coupling of laser light to a mechanical oscillator via radiation pressure leads to the emergence of quantum mechanical correlations between the amplitude and phase quadrature of the laser beam. These correlations form a generic non-classical resource which can be employed for quantum-enhanced force metrology, and give rise to ponderomotive squeezing in the limit of strong correlations. To date, this resource has only been observed in a handful of cryogenic cavity optomechanical experiments. Here, we demonstrate the ability to efficiently resolve optomechanical quantum correlations imprinted on an optical laser field interacting with a room temperature nanomechanical oscillator. Direct measurement of the optical field in a detuned homodyne detector (“variational measurement”) at frequencies far from the resonance frequency of the oscillator reveal quantum correlations at the few percent level. We demonstrate how the absolute visibility of these correlations can be used for a quantum-enhanced estimation of the quantum back-action force acting on the oscillator, and provides for an enhancement in the relative signal-to-noise ratio for the estimation of an off-resonant external force, even at room temperature.

The radiation pressure interaction of light with mechanical test masses has been the subject of early theoretical research in the gravitational wave community [1, 2], leading for example, to an understanding of the quantum limits of interferometric position measurements. For a mechanical oscillator parametrically coupled to an optical cavity, the trade-off between radiation pressure quantum fluctuations of the meter beam (i.e. measurement back-action) and detected shot noise establishes a minimum uncertainty in the detection of the test mass (i.e. mirror) position, commonly referred to as the standard quantum limit [3, 4]. However the two noise contributions – measurement back-action and imprecision – are in general correlated. From the perspective of the transmitted light, the interaction with the mechanical oscillator causes quantum correlations among its degrees of freedom via the same radiation pressure quantum fluctuations. The fluctuations in the amplitude quadrature drive the mechanical oscillator, and this back-action driven motion is transduced into the phase quadrature. Correlations thus established form a valuable quantum mechanical resource: the optomechanical system may be viewed as an effective Kerr medium emitting squeezed states of the optical field [5, 6], or the correlations can be directly employed for back-action cancellation in the measurement record via “variational measurements” [7–9].

In practice, the difficulty in observing, and ultimately utilizing, these optomechanical quantum correlations is compounded by the presence of thermal noise. For a high-quality-factor (Q) mechanical oscillator, its intrinsic thermal Brownian motion poses the largest source of contamination. In the past decade, the emergence of cavity opto- and electromechanical systems [10], incorporating high-Q mechanical oscillators operated at cryogenic temperatures, have enabled experiments in the quantum coherent regime where the optical field is the dominant bath seen by the mechanical oscillator. In particular, cryogenic experiments have accessed the regime where mechanical motion driven by quantum fluctuations in the optical field is comparable to the thermal noise [11–13], enabling the study of various manifestations of optomechanically generated quantum correlations. In a heterodyne measurement, these correlations can give rise to an asymmetry of the mechanical sidebands generated by the optomechanical interaction [14–19], while in a homodyne measurement they lead to optical squeezing [11, 19–22]. Despite these advances, directly observing such quantum correlations at room temperature has remained elusive.

Here we describe an experiment that observes optomechanically generated quantum correlations at room temperature (ca. 300 K), and a proof-of-principle demonstration of their use in enhancing the ability to estimate forces. Homodyne detection of the transmitted meter field near the amplitude quadrature (“variational measurement” [7]), together with the ability to probe far away from the mechanical resonance frequency, allows us to detect signatures of the correlations established by a mechanical oscillator, circumventing the large small compared to the thermal occupation $n_{th}$, i.e. $n_{QBA} \ll n_{th}$, we show that signature of quantum correlations in a variational measurement of the meter beam scales as $\sqrt{n_{QBA}/n_{th}}$, thus yielding a square-root enhancement in the ability to estimate the back-action force compared to conventional detection on phase quadrature [11–13]. Finally, we show how the signal-to-noise ratio for the estimation of an external force applied on the mechanical oscillator is improved by the presence of the same correlations. These experiments herald cavity optomechanics as a platform for room temperature quantum optics and quantum metrology.

Our system consists of a Si$_3$N$_4$ nanomechanical oscillator coupled dispersively to a whispering gallery mode of a sil-
increase compared to recent cryogenic experiments [12, 19]. Together with the high mechanical $Q \approx 3 \cdot 10^4$ (corresponding to a decay rate of $\Gamma_m \approx 2 \pi \cdot 12$ Hz) and an optical cavity in the bad-cavity limit (cavity decay rate, $\kappa \approx 2 \pi \cdot 4.5$ GHz, mechanical resonance frequency $\Omega_m \approx 2 \pi \cdot 3.4$ MHz) the device attains a near unity single photon cooperativity, $C_0 = 4g_0^2/\kappa \Gamma_m \approx 0.27$, at room temperature. Specific aspects of the device fabrication and general room temperature performance are detailed elsewhere [25].

In the experiment (see fig. 1A), the optomechanical device is placed in a high-vacuum chamber and probed on resonance using a Ti:Sa laser. The transmitted phase quadrature fluctuations, $\delta p_{\text{out}} = -\delta p_{\text{in}} + \sqrt{2C}\Gamma_m (\delta x/\partial_\text{p})$, in the frame rotating with the meter laser (see SI), carries information regarding the total motion $\delta x$ of the mechanical oscillator; here $C = C_{\text{QBA}}$ is the multi-photon cooperativity, $n_c$ is the mean intracavity photon number, and $x_{\text{zp}} = \sqrt{h/2m\Omega_m}$ is the zero-point motion of the oscillator. The total motion $\delta x = \delta x_{\text{th}} + \delta x_{\text{QBA}}$, contains a component due to the thermal Brownian motion of the oscillator, $\delta x_{\text{th}} = \chi[\delta F_{\text{th}}(\Omega)]$, and a quantum back-action driven motion, $\delta x_{\text{QBA}} = \chi[\delta F_{\text{QBA}}(\Omega)] = \sqrt{2C}\Gamma_m (h\chi[x_{\text{zp}}/\partial_\text{p})\delta p_{\text{in}} (\Omega)]$, that is due to the quantum fluctuations in the amplitude of the meter field. Here $\chi[\Omega] = m^{-1}(\Omega^2 - \Omega^2 - \Omega^2_{\text{m}})^{-1}$ is the susceptibility of the mechanical oscillator’s position to an applied force at frequency $\Omega$. Importantly, the optomechanical interaction establishes quantum correlations between the light’s amplitude and phase: the symmetrized cross-correlation spectrum [4], $S_{pq}[\Omega] = \int \left(\frac{1}{2} \{\delta q_{\text{out}}(t), \delta q_{\text{out}}(0)\}\right) e^{i\Omega t} dt$ characterizes the magnitude of these correlations. Explicitly (see SI),

$$S_{pq}[\Omega] = C\Gamma_m \Re \frac{h\chi[\Omega]}{x}_{\text{zp}},$$

where $\eta$ is the detection efficiency. The quantum correlations in the output field (third term in eq. (2)) can be observed despite the large thermal noise (second term in eq. (2)) by exploiting, firstly, the difference in the dependence of the correlation term on the local oscillator phase, and secondly, its dependence on the mechanical susceptibility. Specifically, by operating close to the amplitude quadrature ($\delta = 0$) and at Fourier frequency detuning, $\delta = \Omega - \Omega_m$, far from mechani-

$S_{pq}[\Omega] = \cos^2 \theta \, S_{pq}^{\text{out}}[\Omega] + \sin^2 \theta \, S_{pp}^{\text{out}}[\Omega] + \sin(2\theta) \, S_{pq}^{\text{out}}[\Omega]$

$\propto 1 + \frac{4\eta C\Gamma_m}{x_{\text{zp}}^2} \left( \sin^2 \theta \, |\chi[\Omega]|^2 (S_{FF}^{\text{QBA}}[\Omega] + S_{FF}^{\text{QBA}}[\Omega]) + \sin(2\theta) \frac{h}{2} \Re \chi[\Omega] \right),$

FIG. 1. Quantum correlations due to a room temperature optomechanical system. (A) Schematic of the experiment. Light from a Ti:Sa laser operating at 780 nm resonantly probes an optomechanical system maintained at room temperature ($T \approx 300$ K) in a low pressure (≈ $10^{-7}$ mbar) vacuum chamber. The transmitted laser light is analysed in a balanced homodyne detector with a local oscillator phase $\delta x_{\text{th}} = \chi[\delta F_{\text{th}}(\Omega)]$, and a quantum back-action driven motion, $\delta x_{\text{QBA}} = \chi[\delta F_{\text{QBA}}(\Omega)] = \sqrt{2C}\Gamma_m (h\chi[x_{\text{zp}}/\partial_\text{p})\delta p_{\text{in}} (\Omega)]$, that is due to the quantum fluctuations in the amplitude of the meter field. Here $\chi[\Omega] = m^{-1}(\Omega^2 - \Omega^2 - \Omega^2_{\text{m}})^{-1}$ is the susceptibility of the mechanical oscillator’s position to an applied force at frequency $\Omega$. Importantly, the optomechanical interaction establishes quantum correlations between the light’s amplitude and phase: the symmetrized cross-correlation spectrum [4], $S_{pq}[\Omega] = \int \left(\frac{1}{2} \{\delta q_{\text{out}}(t), \delta q_{\text{out}}(0)\}\right) e^{i\Omega t} dt$ characterizes the magnitude of these correlations. Explicitly (see SI),

$$S_{pq}[\Omega] = C\Gamma_m \Re \frac{h\chi[\Omega]}{x}_{\text{zp}},$$

i.e. a large correlation between the transmitted phase and amplitude, proportional to the multi-photon cooperativity $C$, is established around the mechanical frequency.

These correlations can be directly observed by measuring the transmitted optical field in a homodyne detector with a local oscillator phase $\delta$, corresponding to a measurement of the rotated quadrature, $\deltaq_{\text{out}} = \deltaq_{\text{out}} \cos \delta + \deltaq_{\text{out}} \sin \delta$. In this case, the homodyne photocurrent spectrum (referred to electronic shot noise) takes the form (see SI),

$$S_{pq}[\Omega] = \cos^2 \theta \, S_{pq}^{\text{out}}[\Omega] + \sin^2 \theta \, S_{pp}^{\text{out}}[\Omega] + \sin(2\theta) \, S_{pq}^{\text{out}}[\Omega]$$

$\propto 1 + \frac{4\eta C\Gamma_m}{x_{\text{zp}}^2} \left( \sin^2 \theta \, |\chi[\Omega]|^2 (S_{FF}^{\text{QBA}}[\Omega] + S_{FF}^{\text{QBA}}[\Omega]) + \sin(2\theta) \frac{h}{2} \Re \chi[\Omega] \right),$

where $\eta$ is the detection efficiency. The quantum correlations in the output field (third term in eq. (2)) can be observed despite the large thermal noise (second term in eq. (2)) by exploiting, firstly, the difference in the dependence of the correlation term on the local oscillator phase, and secondly, its dependence on the mechanical susceptibility. Specifically, by operating close to the amplitude quadrature ($\delta = 0$) and at Fourier frequency detuning, $\delta = \Omega - \Omega_m$, far from mechani-
Asymmetry in homodyne spectrum. (A) Measured variation of the photocurrent signal-to-noise, $\bar{S}_II[\Omega_m]$ (normalized to shot-noise), as the homodyne angle, $\theta$, is varied. A $32 \text{ dB}$ suppression of the resonant signal, proportional to the total motion, is achieved in the amplitude quadrature, limited by residual fluctuations in the homodyne angle ($\theta_{\text{rms}} < 0.01 \text{ rad}$). (B) Examples of spectra taken near the phase (green) and amplitude (blue) quadratures, together with the shot-noise background (gray) estimated by blocking the meter laser path in the homodyne detector. For all measurements, feedback is used to stabilize the mode, as discussed in the text. (C) Zoom-in of the spectrum at two quadratures, $\pm \theta$, approximately symmetric about the amplitude quadrature, shown as blue ($+\theta$) and yellow ($-\theta$) cuts in (A). Quantum correlations manifest as a slight asymmetry between the two spectra, leading to $\bar{S}_{II}[\delta < 0] > \bar{S}_{II}[\delta < 0]$ and visa versa for frequencies $\delta > 0$. Larger asymmetry is observed for frequency offsets further from mechanical resonance, i.e. $|\delta| > \Gamma_m$, as predicted by eq. (3). The data shown here corresponds to a resolution bandwidth of $1 \text{ kHz}$, much smaller than the frequency bands over which the asymmetry is observed. The shaded gray regions show cuts used for all data sets to systematically analyze the asymmetry as the homodyne angle is varied; see text for details.

![Diagram](image)

FIG. 2. Asymmetry in homodyne spectrum. (A) Measured variation of the photocurrent signal-to-noise, $\bar{S}_II[\Omega_m]$ (normalized to shot-noise), as the homodyne angle, $\theta$, is varied. A $32 \text{ dB}$ suppression of the resonant signal, proportional to the total motion, is achieved in the amplitude quadrature, limited by residual fluctuations in the homodyne angle ($\theta_{\text{rms}} < 0.01 \text{ rad}$). (B) Examples of spectra taken near the phase (green) and amplitude (blue) quadratures, together with the shot-noise background (gray) estimated by blocking the meter laser path in the homodyne detector. For all measurements, feedback is used to stabilize the mode, as discussed in the text. (C) Zoom-in of the spectrum at two quadratures, $\pm \theta$, approximately symmetric about the amplitude quadrature, shown as blue ($+\theta$) and yellow ($-\theta$) cuts in (A). Quantum correlations manifest as a slight asymmetry between the two spectra, leading to $\bar{S}_{II}[\delta < 0] > \bar{S}_{II}[\delta < 0]$ and visa versa for frequencies $\delta > 0$. Larger asymmetry is observed for frequency offsets further from mechanical resonance, i.e. $|\delta| > \Gamma_m$, as predicted by eq. (3). The data shown here corresponds to a resolution bandwidth of $1 \text{ kHz}$, much smaller than the frequency bands over which the asymmetry is observed. The shaded gray regions show cuts used for all data sets to systematically analyse the asymmetry as the homodyne angle is varied; see text for details.

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In order to discern any asymmetry in the photocurrent spectra, \( R_\text{Asymmetry ratio} \) (eq. (4)), as a function of homodyne angle. From top to bottom, \( R_\theta \) is plotted as the ratio of intracavity photon number. At each power, \( \theta \) is increased, \( \eta C \) is measured in units of \( \text{GHz} \) with respect to the local oscillator phase tuned through the amplitude quadrature (\( \theta \approx 0 \)), i.e. \( R_\theta - 1 \approx -(R_{-\theta} - 1) \), thus providing a robust experimental signature for the presence of such correlations, when noise in the amplitude and phase quadrature of the meter laser is sufficiently small (see SI and [19]). The ratio \( R_\theta \) (defined in eq. (4)) is measured by recording the spectral power in windows of finite bandwidth symmetric about resonance (indicated as gray vertical lines in fig. 2A; insets to the left and to the right show portions of the photocurrent spectra symmetric about resonance, and at an offset \( |\delta| \approx 2 \times 10^3 \cdot \Gamma_m \). An asymmetry between the spectra at the level of \( \approx \pm 5\% \) is observed, consistent with the theoretically predicted effect due to quantum correlations.

Next, we systematically investigate this asymmetry. The asymmetry in the observed spectrum (red in fig. 1B) traces its root to the asymmetric contribution of the quantum correlations (green in fig. 1B, and third term in eq. (3)). This asymmetry can be characterized by the ratio, \( R_\text{Asymmetry ratio} = S_{1/2}[\delta]/S_{1/2}[-\delta] \), which is explicitly given by (see SI),

\[
R_\theta = \frac{1 + 4\eta C \Gamma_{\text{tot}} \Gamma_m \sin \theta/\delta^2 (1 - (\delta/\Gamma_{\text{tot}} \Gamma_m) \cot \theta)}{1 + 4\eta C \Gamma_{\text{tot}} \Gamma_m \sin \theta/\delta^2 (1 + (\delta/\Gamma_{\text{tot}} \Gamma_m) \cot \theta)}
\]

Note that quantum correlations render \( R_\theta \) anti-symmetric with respect to the local oscillator phase tuned through the amplitude quadrature (\( \theta \approx 0 \)), i.e. \( R_\theta - 1 \approx -(R_{-\theta} - 1) \), thus providing a robust experimental signature for the presence of such correlations, when noise in the amplitude and phase quadrature of the meter laser is sufficiently small (see SI and [19]). The ratio \( R_\theta \) is measured by recording the spectral power in windows of finite bandwidth symmetric about resonance (indicated as gray vertical bands in fig. 2C), as a function of the homodyne angle \( \theta \). Figure 3 shows \( R_\theta \) as function of homodyne angle for several probe powers. At low probe powers (i.e. low cooperativity, \( C \approx 8 \times 10^2 \)), shown in the top panel of fig. 3, the anti-symmetric feature around the amplitude quadrature (i.e. \( R_\theta - 1 \)) is diminished by the low measurement impre-
cision. As the probe power is increased, shown in the two subsequent panels of fig. 3, the relative contribution of quantum correlation increases, leading to a progressively larger anti-symmetry near amplitude quadrature. The observed anti-symmetric feature around the amplitude quadrature is a characteristic of amplitude-phase correlations in the meter field. Classical sources of such anti-symmetric correlations, for example from laser phase noise, are negligible in our experiments, as are other sources of systematics (see SI for details).

For the scenario in our experiments, where the back-action occupation is appreciable, yet not larger than the thermal motion, i.e. $n_{th} \gg n_{QBA} \gg 1$, the maximum deviation of the asymmetry ratio $R_{\theta}$, can be shown to take the form,

$$\Delta R \equiv \max_{\theta} R_{\theta} - \min_{\theta} R_{\theta} \approx 4 \sqrt{\frac{n_{QBA}}{n_{th}}}.$$  \hspace{1cm} (5)

On the one hand, this relation implies that despite $n_{QBA}/n_{th} \approx 10^{-3}$, it is eminently possible to estimate the back-action occupation via the square-root enhancement provided by the quantum correlations in the measurement record. On the other hand, this relation implies that a square-root scaling of $\Delta R$ with laser power is an unambiguous signature of the quantum mechanical origin of the correlations that lead to the spectral asymmetry. Figure 4 depicts the scaling of $\Delta R$ with probe power. Importantly, the agreement with a square-root scaling suggests that a dominant portion of the asymmetry witnessed in our experiments arises due to quantum correlations in the probe beam. Note that classical sources of noise would lead to a linear scaling of $\Delta R$ with meter power. For all data reported in fig. 4, $\Delta R$ is extracted by observing the asymmetry in the same spectral window around $|\delta| \approx 2 \cdot 10^{3}$, $\Gamma_{m}$ (shown as gray regions in fig. 2C).

These results signal the emergence of cavity quantum optomechanics at room temperature, and the possibility of room temperature quantum-enhanced metrology using such a platform. Quantum correlations form a generic resource for enhancing the precision with which parameters of a system can be estimated [28]. In our case, the ability to estimate a force $\delta F_{\text{ext}}$ applied on a mechanical oscillator is hindered by fundamental sources of force-equivalent noise. It can be shown (see SI) that the spectral density of the unbiased force estimator $\delta F_{\text{ext}}^{\text{est}}$, based on the observed homodyne photocurrent $\delta I_{h}$, takes the form (see SI),

$$\tilde{S}_{FF}^{\text{est}}{\theta}[\Omega] = \tilde{S}_{FF}[\Omega] + \tilde{S}_{FF}^{\text{tot}}[\Omega] + \tilde{S}_{FF}^{\text{imp}}{\theta}[\Omega] + \hbar \cot \theta \frac{\text{Re} \chi[\Omega]}{|\chi[\Omega]|^2}.$$  \hspace{1cm} (6)

The uncertainty in the estimate of the applied force, $\tilde{S}_{FF}^{\text{est}}$, is due to a contribution from thermal motion and measurement back-action (second term, $\tilde{S}_{FF}^{\text{tot}} = \tilde{S}_{FF}^{\text{th}} + \tilde{S}_{FF}^{\text{QBA}}$), a contribution due to measurement imprecision due to shot-noise in homodyne detection (third term), and a contribution due to quantum correlations (last term). By detecting away from the phase quadrature (i.e. $\theta \neq \pi/2$), non-zero correlations, potentially negative in magnitude, can be used to reduce the uncertainty in force estimation. In fact, in the limit where $\tilde{S}_{FF}^{\text{QBA}} \gg \tilde{S}_{FF}^{\text{imp}}{\theta}$ (practically, $C \gg 1$), and at an optimal measurement quadrature at angle $\theta_{\text{opt}}$, the force estimator spectrum is given by (see SI),

$$\tilde{S}_{FF}^{\text{est}}{\theta_{\text{opt}}}[\Omega] = \tilde{S}_{FF}^{\text{est}}[\Omega] + \tilde{S}_{FF}^{\text{tot}}[\Omega] + \tilde{S}_{FF}^{\text{imp}}{\pi/2}[\Omega]$$

$$+ \left[1 - \eta \left(\frac{\text{Re} \chi[\Omega]}{|\chi[\Omega]|^2}\right)^2\right] \tilde{S}_{FF}^{\text{QBA}}[\Omega];$$  \hspace{1cm} (7)

i.e., complete cancellation of measurement back-action is possible in the estimator at frequencies away from mechanical resonance, limited by the efficiency with which the noise that originally caused the back-action is detected (see SI). Note that in this scheme, back-action is cancelled only in the measurement record – different from back-action evasion [29]. In our experiment, since thermal motion is still the dominant contribution to the uncertainty in the force estimator, back-action cancellation by variational measurement gives only a meagre 0.01% improvement compared to the standard quantum limit for conventional detection. A contemporary cryogenic experiment projects a metrological gain of a few percent [30].

Despite the large thermal occupation, we are able to demonstrate the underlying concept of quantum-enhanced force metrology. To this end, we perform an experiment that shows
an improvement in the relative signal-to-noise ratio for the estimation of a coherent force. We consider an external force, $\delta F_{\text{ext}} = \delta F_+ + \delta F_-$, consisting of two coherent components $\delta F_\pm$ at frequencies symmetric about $\Omega_F$. For the variational measurement strategy, the signal-to-noise ratio, $SN_{\pm}$, is used to apply two coherent radiation pressures on the oscillator, nominally balanced in intensity at $850\text{ nm}$.

For the variational measurement strategy, the signal-to-noise ratio, $SN_{\pm}$, for the detection of either component, benefits from the presence of quantum correlations in the meter beam. In fact, the relative signal-to-noise ratio of the two components (see SI),

$$\frac{SN_+}{SN_-} \approx \frac{1}{R_0} \left| \frac{\chi[\Omega_F + \delta]}{\chi[\Omega_F - \delta]} \right|^2 \left( \frac{\langle \delta F_+^2 \rangle}{\langle \delta F_-^2 \rangle} \right)$$  \hspace{1cm} (8)

directly related to the magnitude of the asymmetry $R_0$.

In fig. 5, we show the result of an experiment that evidences the role of quantum correlations in enhancing the signal-to-noise ratio for external force estimation. The auxiliary laser (at $850\text{ nm}$) is used to apply two coherent radiation pressure forces on the oscillator, nominally balanced in intensity ($\langle \delta F_+^2 \rangle / \langle \delta F_-^2 \rangle \approx 1$) and symmetric with respect to mechanical resonance (at $\Omega_F \approx \Omega_m$), as shown in fig. 5A. Performing a variational measurement as before, the homodyne angle $\theta$ is varied, while the ratio of the signal-to-noises of the two forces is measured. Figure 5B shows this ratio as a function of the homodyne angle. For the case where the forces are precisely balanced, eq. (8) predicts that quantum correlations lead to an improvement in the signal-to-noise ratio between the two forces, quantitatively given by $R_0$. As shown in fig. 5B, we observe a modest improvement in the signal-to-noise ratio (by about 3%) on one side of the mechanical resonance compared to the other. In the absence of such correlations, $R_0 \approx 1$, limited by any residual asymmetry due to the mechanical susceptibility; the resulting classical prediction is shown as a red dashed curve in fig. 5B. The closer agreement of the data with the full quantum mechanical prediction suggests the signal-to-noise improvement observed in our experiment is due to quantum correlations in the meter beam.

In future room-temperature experiments, where the back-action force is the dominant contaminant in the estimation of weak external forces, variational measurements, as demonstrated here, could beat the standard quantum limit for force estimation via conventional detection.

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Supplementary Information

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Appendix A: Theoretical model for optomechanically induced quantum correlations

We consider here an optomechanical system consisting of an optical cavity, whose intracavity field is described by the amplitude $a(t)$, disapproversely coupled to a mechanical oscillator, whose position is described by $x(t)$. Following standard linearization procedure [10], the fluctuations in either variable, denoted $\delta a$ and $\delta x$ respectively, satisfy the equations of motion,

$$
\delta \dot{a} = \left( i \Delta - \frac{\kappa}{2} \right) \delta a + i G \bar{a} \delta x + \sqrt{\eta_c} \delta a_{in} + \sqrt{1 - \eta_c} \kappa \delta a_0 \\
\delta \ddot{x} + \Gamma_m \delta \dot{x} + \Omega_m^2 \delta x = \delta F_{th} + \hbar G a (\delta a + \delta a^\dagger).
$$

(A1)

Here $G$ is the cavity frequency pull parameter, the dispersive optomechanical coupling strength. The noise variables $\delta a_{in,0}$ describe the fluctuations in the cavity input at the coupling port and the port modelling internal losses. The cavity coupling efficiency $\eta_c = \kappa_{ex}/\kappa$, describes the relative strength of the external coupling port. The steady state intracavity photon number, $n_c = \bar{a}^2$ is given by,

$$
n_c = \frac{4 \eta_c P_{in}/\hbar \omega_L}{1 + 4 \Delta^2/\kappa^2},
$$

where $P_{in}$ is the injected probe power at optical frequency $\omega_L$.

In the experimentally relevant situation of resonant probing ($\Delta \approx 0$) and bad cavity limit ($\Omega_m \gg \kappa$), the equation of motion for the cavity field in eq. (A1) assumes the form,

$$
\delta a[\Omega] \approx \frac{2ig}{\kappa} \delta z[\Omega] + \frac{2}{\sqrt{\eta_c}} \left( \sqrt{\eta_c} \delta a_{in}[\Omega] + \sqrt{1 - \eta_c} \delta a_0[\Omega] \right),
$$

where we have introduced the normalized position, $\delta z := \delta x/x_{zp}$, and the optomechanical coupling rate, $g := G \bar{x}_{zp}$; $x_{zp} = \sqrt{\hbar/2m\Omega_m}$ is the zero-point variance in the position of the mechanical oscillator of effective mass $m$. Using the input-output relation [31], $\delta a_{out} = \delta a_{in} - \sqrt{\eta_c} \kappa \delta a$, the transmitted fluctuations,

$$
\delta a_{out}[\Omega] = (1 - 2\eta_c) \delta a_{in}[\Omega] - 2 \sqrt{\eta_c (1 - \eta_c)} \delta a_0[\Omega] - i \sqrt{\eta_c CT_m} \delta z[\Omega],
$$

(A2)

carries information regarding the total mechanical motion $\delta z$ consisting of the thermal motion and the quantum back-action driven motion, i.e.,

$$
\delta z[\Omega] = \delta z_{th}[\Omega] + \delta z_{QBA}[\Omega].
$$

In eq. (A2), we have also introduced the multi-photon cooperativity of the optomechanical system:

$$
C := \frac{4g^2}{\kappa \Gamma_m}.
$$
The back-action motion is given by,

$$\delta z_{BA}[\Omega] = \sqrt{2CT_m} \frac{\hbar \chi[\Omega]}{x_{zp}^2} \left( \sqrt{\frac{\hbar}{\kappa}} \delta q_{in}[\Omega] + \sqrt{1 - \eta_c} \delta q_0[\Omega] \right),$$  \hspace{1cm} (A3)

where $\delta q_{in,0}$ are the amplitude quadrature fluctuations from the two cavity input ports, and

$$\chi[\Omega] = m^{-1} (-\Omega^2 + \Omega_m^2 - i\Omega_m)^{-1},$$  \hspace{1cm} (A4)

is the susceptibility of the mechanical oscillator position to applied force. Note that henceforth (as in the main manuscript) photocurrent spectra are implicitly normalized to shot noise. Using the electronic shot noise, inserting eq. (A6) in eq. (A7), and using the above definition, we arrive at the homodyne photocurrent spectrum (normalized to shot noise)

\[ \langle S_{\Omega} \rangle = \frac{1}{2} \left\{ \delta q_{out}^\theta[\Omega], \delta q_{out}^{\theta,-}[\Omega] \right\}, \]

For a general quadrature at angle $\theta$, defined by,

$$\delta q^\theta_{out}[\Omega] := \delta q_{out}[\Omega] \cos \theta + \delta p_{out}[\Omega] \sin \theta,$$

it follows that,

\[ \langle \delta q^\theta_{out}[\Omega] \delta q^{\theta,-}_{out}[\Omega] \rangle = \cos^2 \theta \langle \delta q_{out}[\Omega] \delta q_{out}[-\Omega] \rangle + \sin^2 \theta \langle \delta p_{out}[\Omega] \delta p_{out}[-\Omega] \rangle + \sin(2\theta) \Re \langle \delta q_{out}[\Omega] \delta p_{out}[-\Omega] \rangle. \]

The homodyne photocurrent spectrum is related to this correlator via,

\[ S^{\theta,\text{hom}}_{\Omega}[\Omega] \cdot 2\pi \delta[0] \propto S^{\theta}_{qq} \cdot 2\pi \delta[0] = \frac{1}{2} \left\{ \delta q^\theta_{out}[\Omega], \delta q^{\theta,-}_{out}[\Omega] \right\}. \]

Inserting eq. (A6) in eq. (A7), and using the above definition, we arrive at the homodyne photocurrent spectrum (normalized to electronic shot noise),

\[ S^{\theta,\text{hom}}_{\Omega}[\Omega] = 1 + \frac{4\eta CT_m}{x_{zp}^2} \left( S_{xx}[\Omega] \sin^2 \theta + \frac{\hbar}{2} \sin(2\theta) \Re \chi[\Omega] \right). \]

Inserting eq. (A6) in eq. (A7), and using the above definition, we arrive at the homodyne photocurrent spectrum (normalized to electronic shot noise),

\[ S^{\theta,\text{hom}}_{\Omega}[\Omega] = 1 + \frac{4\eta CT_m}{x_{zp}^2} \left( S_{xx}[\Omega] \sin^2 \theta + \frac{\hbar}{2} \sin(2\theta) \Re \chi[\Omega] \right). \]

Note that henceforth (as in the main manuscript) photocurrent spectra are implicitly normalized to shot noise. Using the fluctuation-dissipation theorem [32] to relate the thermal and back-action force noise to mean phonon occupations $n_{th}$ and $n_{BA}$ respectively, the spectral density of the total motion,

$$S_{xx}[\Omega] = \frac{4x_{zp}^2 (\Omega_m \Gamma_m)^2}{\Gamma_m} \left( \frac{n_{th} + n_{QBA} + \frac{1}{2}}{\Omega_m^2 - (\Omega_m - \Omega)^2} \right),$$  \hspace{1cm} (A10)

where, $n_{th} \approx k_B T/h\Omega_m$ is the average thermal occupation, and, $n_{QBA} = C = C_0n_c$ is the average occupation due to (quantum) back-action arising from vacuum fluctuations in the input amplitude quadrature.

1. **Effect of excess laser noise and detuning**

In addition to vacuum fluctuations in the input amplitude quadrature, classical fluctuations in the amplitude quadrature can lead to phase-amplitude correlations in the cavity transmission. Additionally, detuning deviations causing a finite $\Delta/\kappa$ can transduce classical phase fluctuations in the input to excess phase-amplitude correlations in the output.
In order to analyse the two possible classical contributions on the same footing, we consider the quadratures of the cavity transmission, $\delta q_{\text{out}}$, $\delta p_{\text{out}}$ for the case of a finite detuning $|\Delta| \ll \kappa$. In this regime, eq. (A6) contains corrections of order $\Delta/\kappa$, viz.,

$$
\delta q_{\text{out}}[\Omega] = (1 - 2\eta_c)\delta q_{\text{in}}[\Omega] - 2\sqrt{\eta_c(1 - \eta_c)}\delta q_0[\Omega] + \frac{2\Delta}{\kappa} \left( \sqrt{2\eta_c C T_m^* \delta z[\Omega]} + 2\eta_c \delta p_{\text{in}}[\Omega] + 2\sqrt{\eta_c(1 - \eta_c)}\delta p_0[\Omega] \right)
$$

$$
\delta p_{\text{out}}[\Omega] = (1 - 2\eta_c)\delta p_{\text{in}}[\Omega] - 2\sqrt{\eta_c(1 - \eta_c)}\delta p_0[\Omega] - \sqrt{2\eta_c C T_m^* \delta z[\Omega]}
$$

$$
= -\frac{2\Delta}{\kappa} \left( 2\eta_c \delta q_{\text{in}}[\Omega] + 2\sqrt{\eta_c(1 - \eta_c)}\delta q_0[\Omega] \right),
$$

where the total motion $\delta z = \delta z_{\text{th}} + \delta z_{\text{BA}}$, with,

$$
\delta z_{\text{BA}}[\Omega] = \sqrt{2CT_m^*} \frac{\hbar}{x^2 p} \left[ \left( \sqrt{\eta_c \delta q_{\text{in}}[\Omega]} + \sqrt{1 - \eta_c}\delta q_0[\Omega] \right) + 4i\frac{\Omega\Delta}{\kappa^2} \left( \sqrt{\eta_c \delta p_{\text{in}}[\Omega]} + \sqrt{1 - \eta_c}\delta p_0[\Omega] \right) \right],
$$

the motion induced by the quantum and the classical fluctuations in the input laser field. Excess noise in the input amplitude and phase quadratures is modelled by white noise with intensity $C_{qq}$ and $C_{pp}$, respectively, so that,

$$
\tilde{S}_{qq}^{\text{in}}[\Omega] = \frac{1}{2} + C_{qq}, \quad \tilde{S}_{pp}^{\text{in}}[\Omega] = \frac{1}{2} + C_{pp}.
$$

Using eqs. (A11) and (A12) in the definition of the homodyne spectrum (eq. (A8)) to leading order in $\Delta/\kappa$, the shot-noise normalized balanced homodyne spectrum is:

$$
\tilde{S}_{11}^{\text{hom}}[\Omega] \approx 1 + \frac{4\eta C T_m^*}{x^2 p} \left[ \left( \tilde{S}_{xx}^{\text{e}+QBA}[\Omega] + \tilde{S}_{xx}^{CBA,q}[\Omega] + \tilde{S}_{xx}^{CBA,p}[\Omega] \right) \sin(\theta')^2 + \frac{\hbar}{2} \sin(2\theta') \Re \chi[\Omega] + h \sin(2\theta') \sqrt{\eta_c(1 - 2\eta_c)} C_{qq} \Re \chi[\Omega] + 2h \sin(\theta')^2 \sqrt{\eta_c(1 - 2\eta_c)} \frac{4\Omega m_\Delta}{\kappa^2} \Re \chi[\Omega] \right],
$$

where $\theta' \approx \theta - 4\Delta/\kappa$ is the quadrature angle rotated by the cavity. The effect of excess noise is two-fold. Firstly, classical amplitude (phase) noise $C_{qq}$ ($C_{pp}$) causes additional classical back-action motion $\tilde{S}_{xx}^{CBA,q}$ ($\tilde{S}_{xx}^{CBA,p}$), leading to excess back-action occupations,

$$
n_{CBA,q} = C_0 n_c C_{qq}, \quad n_{CBA,p} = C_0 n_c \left( \frac{4\Omega m_\Delta}{\kappa^2} \right)^2 C_{pp}.
$$

Secondly, classical amplitude noise, and phase noise transduced via finite detuning, establish excess correlations, as can be seen from the last two terms in the eq. (A13). It is important to note that the contribution of excess phase noise $C_{pp}$ to the measured homodyne signal is effectively suppressed for the current experimental parameters as $\Delta \cdot \Omega m_\kappa^2 = O(10^{-4})$. Finally, when laser noise is insignificant, the role of a residual detuning from the cavity, i.e. $\Delta \neq 0$, is to rotate the detected quadrature by an angle $\arctan(4\Delta/\kappa)$, without leading to any artificial asymmetry.
Appendix B: Experimental details

1. Experimental platform

The device measured in this work consists of an SiO$_2$ whispering gallery mode microdisk with a high-stress Si$_3$N$_4$ nanobeam centered in the near-field of the microdisk. The sample has been fabricated by a monolithic wafer-scale process that utilizes a sacrificial layer to define an ~50 nm gap between the microdisk and nanobeam, as detailed in [25]. Similar devices have also been used for recent cryogenic experiments [12, 19]. However, in contrast to those devices, in the devices used here both the mechanical and optical resonator shapes are defined by electron-beam lithography. The bare microdisks exhibit very high finesse of ~10$^5$ – nearly an order of magnitude higher than microdisks produced by photo-lithography. However, in this work we do not access this high finesse regime when the nanobeam is placed in the near-field of the disk. We attribute this to the 80 nm thickness of the Si$_3$N$_4$, which is conjectured to lead to excessive scattering and/or waveguiding. The microdisk is 40 $\mu$m in diameter, ~350 nm thick, and has a gently sloping sidewall of ~10° which results from the use of thin photoresist during the wet-etching process.

In previous work [12, 19] the mechanical resonator was formed by a beam with a homogeneous transverse profile. However, the present device has been designed with a central defect that allows for increased overlap with the optical mode while minimizing the effective mass ($m_{\text{eff}} \approx 1.94$ pg). The optical mode of the microdisk samples approximately 9 $\mu$m of the beam at its center (see [25]), however we utilize a defect that is tapered within the sampling region as this resulted in lower optical loss and overall higher $C_0$ than longer defects. This effect may be attributed to the reduced scattering loss on account of a softer dielectric boundary seen by the optical mode. Figure 6B shows the defect geometry and the effect of defect length on the effective mass of the fundamental out-of-plane mode. The beam is 70 $\mu$m long and consists of a narrow (200 nm) beam with a wider (400 nm) rectangular defect at the center which tapers linearly into the thin beam at an angle of ~12°. The defect length of the device used in this paper is 5 $\mu$m, which exhibits an effective mass only 11% larger than that of a standard 200 nm wide beam.

As shown in Figure 6A, two short beams of Si$_3$N$_4$ with dimensions 20 $\times$ 0.2 $\times$ 0.08 $\mu$m are also placed across the channel on either side of the microdisk to support the tapered optical fiber and increase the overall mechanical stability of the experiment.

2. Measurement setup

The essential layout of the experiment is shown in fig. 7. The sample is placed in a high vacuum chamber, at a pressure of ~10$^{-7}$ mbar, and room temperature. Light is coupled in and out of the microdisk cavity using a tapered optical fiber, the position of which is adjusted using piezo actuators to achieve critical coupling into the cavity (i.e. $\eta_c \approx 0.5$).

Two lasers are employed in the experiment – a TiSa laser (MSquared Solstis) with wavelength centered around 780 nm which is the meter beam, and an auxiliary 850 nm external cavity diode laser (NewFocus Velocity) which is the feedback beam. Both beams are combined before the cavity and separated after it using dichroic beamsplitters. The feedback beam is detected on an

FIG. 6. (A) False colored scanning electron micrograph of the device design used in this work. Si$_3$N$_4$ is indicated in red and SiO$_2$ in blue. (B) Finite element calculation of effective mass for defect beam design, as a function of the defect length. The data point in orange indicates the defect length (5 $\mu$m) of the experimental device; see text for details.
avalanche photodetector (APD), while the meter beam is fed into a length- and power-balanced homodyne detector. A small portion of the meter beam – stray reflection from the dichroic beam-splitter – is directed onto an APD.

Both lasers are actively locked to their independent cavity resonances using the APD signal. For the meter beam, a lock on cavity resonance (\(|\Delta| < 0.1 \cdot \kappa\)) is implemented using the Pound-Drever-Hall technique. For the feedback beam, a part of the APD signal is used directly to implement a lock red-detuned from cavity resonance.

The other part of the feedback beam APD signal is used to perform moderate feedback cooling of the mechanical oscillator. Specifically, the photosignal is amplified, low-pass filtered and phase-shifted, before using it to amplitude modulate the same laser. As in conventional cold damping [33], the phase-shift in the feedback loop is adjusted to synthesise an out-of-phase radiation pressure force that damps the mechanical oscillator. At the nominal feedback laser power of 5 \(\mu\)W, a damping rate of 1 kHz is realized; the associated increase in the mechanical decoherence rate due to injected imprecision noise was measured to be below 5%.

The path length difference of the homodyne interferometer is actively stabilized using a two-branch piezo translation system. Demodulation of the homodyne signal at PDH frequency also produced interference fringes suitable for locking the homodyne angle near the amplitude quadrature (i.e. \(\theta = 0\)). The residual homodyne angle fluctuations could be estimated \(\theta_{\text{RMS}} \lesssim 1^\circ \approx 0.017\text{ rad}\), inferred from the suppression of thermomechanical signal-to-noise ratio on amplitude quadrature of \(\approx 10^{-4}\) compared to the phase quadrature. An offset DC voltage is applied to the homodyne error signal for deterministic choice of detection quadrature.

Since the feedback cooling exclusively relies on the auxiliary diode laser, the homodyne measurements on the 780 nm meter beam are completely out-of-loop and does not contain electronically-induced correlations.

3. Data analysis

In each experimental run, corresponding to the data shown in one panel of Figure 3 of the main text, the meter laser is locked to cavity resonance at fixed input power, and a series of homodyne photocurrent spectra are taken at various settings of the homodyne angle \(\theta\). From independently measured mechanical and optical parameters of the sample, together with the known input power, the homodyne detection efficiency is inferred in each run by the thermomechanical signal-to-shot-noise ratio (shot noise level was measured by blocking the signal interferometer arm). To account for a small quadrature rotation by the cavity the nominal \(\theta = 0\) quadrature was inferred from the minimum in the transduction of thermomechanical noise.

In order to experimentally access the asymmetry ratio \(R_{\theta}\) discussed in the main text, \(R_{\theta}\) is estimated from an integral over a finite bandwidth \(\Delta \Omega\), i.e.,

\[
R_{\theta} = \int_{\Omega_m+\delta+\Delta \Omega/2}^{\Omega_m+\delta+\Delta \Omega/2} \frac{S_{J}\theta[\Omega]}{\Omega} d\Omega \int_{\Omega_m-\delta-\Delta \Omega/2}^{\Omega_m-\delta-\Delta \Omega/2} \frac{S_{J}\theta[\Omega]}{\Omega} d\Omega .
\]
FIG. 8. (A,C) Illustration of the variation of the experimental asymmetry ratio $R(\theta)$ for different offsets $\delta$ at fixed integration bandwidth $\Delta \Omega/2\pi = 20$ kHz (A) and for different integration bandwidths $\Delta \Omega$ at fixed offset $\delta/2\pi = 21$ kHz (C). Solid red and dashed blue curves show theoretical predictions corresponding to the viscous (eq. (B4)) and structural (eq. (B5)) damping models of the oscillator dissipation. Deviation of the mechanical susceptibility from the former results in $R(\theta)$ being not perfectly asymmetric with degree of distortion increasing with $\delta$. (B,D) Plots show the integration bands used for calculation of the $R(\theta)$ on the left (shaded gray regions). Dark green is a mechanical spectrum at an intermediate homodyne quadrature and light green is the local oscillator trace showing the shot noise level. The data was taken at $P_{\text{in}} = 25 \mu W$.

Theoretically, there is some freedom in the choice of the detuning offset $\delta$ and integration bandwidth $\Delta \Omega$, since the relative contribution of the quantum interference term to the detected signal is maximum within a broad range of detunings $\Gamma_{\text{eff}} \ll \delta \ll 2\Gamma_{m}\sqrt{\eta C n_{\text{th}}}$, here $\Gamma_{\text{eff}} \approx 2\pi \cdot 1 \text{ kHz}$ is the effective damping rate due to feedback. For typical experimental conditions in this work $1 \text{ kHz} \ll \delta/2\pi \ll 500 \text{ kHz}$. Figure 8 shows the ratio $R_\theta$ extracted for various choices of the detuning offset and integration bandwidth. Figure 3 of the main manuscript depicts data extracted for the choice $\delta = 2\pi \cdot 21 \text{ kHz}$ and $\Delta \Omega = 2\pi \cdot 20 \text{ kHz}$.

In the demonstration of external force estimation in the main manuscript, the signal-to-noise ratio for the applied force $\delta F_{\text{ext}}$ is defined by,

$$SN_\theta^\pm \equiv \overline{S}_{II}[\Omega_F \pm \delta]/\overline{S}_{II}[\Omega_F \pm \delta]|_{\delta F_{\text{ext}}=0};$$

i.e., the signal is the photocurrent noise at the frequencies where the force is applied ($\Omega_F \pm \delta$), while the noise is the photocurrent noise at the same frequencies without the force. Practically, we estimate both contributions from finite bandwidth integrals.
over the relevant part of the photocurrent spectrum: for the signal, the photocurrent signal is integrated over a finite bandwidth \( \Delta \Omega_F \) around the applied force, while to estimate the noise, we choose to take averages of the photocurrent spectrum over finite bandwidth \( \Delta \Omega_N \), on either side of the applied force, without turning off the force. Specifically,

\[
\frac{SN^\theta_{\pm}}{\bar{S}^\theta_{II}[\Omega]} = \int_{\Omega \pm \delta + \Delta \Omega_F/2}^{\Omega \pm \delta - \Delta \Omega_F/2} S^\theta_{II}[\Omega] d\Omega + \int_{\Omega \pm \delta - \Delta \Omega_N/2}^{\Omega \pm \delta + \Delta \Omega_N/2} S^\theta_{II}[\Omega] d\Omega.
\]

\[ \text{(B3)} \]

The integration bands used for the Figure 5 in the main text are shown in fig. 9.

4. Role of mechanical susceptibility

In practice however, for detunings from the mechanical resonance \( \delta/2\pi > 30 \text{ kHz} \) analysis is sensitive to possible deviations of the mechanical oscillator susceptibility from a simple velocity damped model (eq. (A4)),

\[
\chi_{\text{velo}}[\Omega] = \frac{1}{m} \left( \frac{\Omega_m^2 - \Omega^2}{\Omega_m^2 - \Omega^2} - i\Omega \Gamma_m \right).
\]

\[ \text{(B4)} \]

For example, a model of the mechanical oscillator taking into account inelastic structural damping, described by the susceptibility \[34–36\],

\[
\chi_{\text{struc}}[\Omega] = \frac{1}{m} \left( \frac{\Omega_m^2 - \Omega^2}{\Omega_m^2 - \Omega^2} - i(\Omega \Gamma_m + \Omega_m^2 \phi[\Omega]) \right)
\]

\[ \text{(B5)} \]

introduces a distortion in the otherwise anti-symmetric function \( R_\theta \), depending on the anelastic loss angle \( \phi[\Omega] \). For a large class of materials, \( \phi[\Omega] \approx Q^{-1} \), and the distortion increases with increasing detuning from mechanical resonance, \( \delta \). For simplicity we chose a band relatively close to the mechanical resonance, the variation of the data analysis result for various choices of the integration band is shown in fig. 8.

In order to extract the scaling of the maximum asymmetry \( \Delta R \), shown in Figure 4 of the main manuscript, the experimental dependences \( R_\theta \) were transformed by correcting for the contribution due to non-Lorenzian mechanical susceptibility

\[
R_{\theta \text{corr}} = R_\theta \times \sqrt{\int_{\Omega_m + \delta + \Delta \Omega/2}^{\Omega_m + \delta - \Delta \Omega/2} |\chi[\Omega]|^2 d\Omega / \int_{\Omega_m + \delta - \Delta \Omega/2}^{\Omega_m + \delta + \Delta \Omega/2} |\chi[\Omega]|^2 d\Omega}. 
\]

\[ \text{(B6)} \]

5. Laser noise

A MSquared Solstis Ti:Sa laser was used for the measurements presented in the manuscript. The amplitude noise of the laser was characterized via direct photo-detection. In a frequency band 3 MHz wide around the mechanical frequency, \( \Omega_m = 2\pi \cdot 3.4 \text{ MHz} \) at the highest employed power 50 \( \mu \text{W} \) the classical amplitude noise level was < 1% of the shot noise (see

FIG. 9. Integration bands used in the definition of the signal-to-noise ratio \( SN^\theta \) in the main manuscript. The signal bands are shaded gray (\( \Delta \Omega_F = 3 \text{ kHz} \)), the bands for noise estimation are shaded orange (\( \Delta \Omega_N = 5 \text{ kHz} \)).
While the amplitude quadrature of the employed Ti:Sa laser is quantum-noise-limited at Fourier frequencies around the mechanical oscillator resonance, analysis of the displacement spectra reveals that there is an additional background present in the measurement, that reaches 25% of the shot noise level around the mechanical oscillator Fourier frequencies for the largest powers used in the experiment (50 µW). This structured background, extrinsic to the laser, is revealed around the mechanical frequency for the used Ti:Sa laser. The solid line shows fit with $1/P$ dependence, characteristic of shot noise limited behavior.

Figure 10B). This means that, $C_{pp} < 5 \times 10^{-3}$, implying a negligible contribution to excess classical correlations and a negligible fraction of classical back-action motion, $n_{CBA,Q} < 0.005 \cdot n_{QBA}$, compared to quantum back-action.

Laser phase noise was upper-bounded using a self-heterodyne measurement [37] with a 400 m fiber delay line. The self-heterodyne signal can be described by the formula (after shifting the beat-note to zero frequency)

$$S_{II}[\Omega] \propto \frac{\pi}{2} \delta[\Omega] + \sin^2\left(\frac{\Omega \tau_0}{2}\right) S_{\phi\phi}[\Omega],$$

(B7)

where $\tau_0$ is the delay and $S_{\phi\phi}[\Omega]$ is the laser phase noise spectral density. The measured signal for the laser is shown in fig. 10A, where the vertical scale is calibrated using the known mean photon flux in the beat note carrier. For the Ti:Sa laser (the blue curve in Figure 10A) the absence of the characteristic $\sin^2(\Omega \tau_0)$ interference pattern suggests that laser phase noise is below the sensitivity of the measurement. Although the laser is expected to be quantum-noise-limited at frequencies well above the relaxation oscillation frequency ($\approx 400 \text{kHz}$), our measurements can only provide a conservative upper-bound for the frequency noise to be at the level of $2 \text{Hz}^2/\text{Hz}$ (in comparison, frequency noise of a commercial external cavity diode laser, also shown in Figure 10A, is $20 \text{dB}$ larger). This upper bound on the excess phase noise, together with large optical linewidth ($\kappa$) strongly suppresses the influence of $C_{pp}$ and leads to an estimated back-action motion that is below a factor 0.0025 compared to the quantum mechanical contribution. Intrinsic cavity frequency noise, for example from thermoelastic [38] or thermorefractive [39] processes, can also lead to a finite value of $C_{pp}$. In the current experiments, broadband measurements of cavity transmission on phase quadrature, shown in fig. 11, suggests a conservative upper bound of $C_{pp} < 10$ at frequencies around $\Omega_m$. Using a length-balanced homodyne interferometer for detection, classical phase noise in the measurement imprecision could also be bounded by 0.1%.

6. Excess detection noise due to taper vibrations

While the amplitude quadrature of the employed Ti:Sa laser is quantum-noise-limited at Fourier frequencies around the mechanical oscillator resonance, analysis of the displacement spectra reveals that there is an additional background present in the measurement, that reaches 25% of the shot noise level around the mechanical oscillator Fourier frequencies for the largest powers used in the experiment (50 µW). This structured background, extrinsic to the laser, is revealed around the mechanical frequency where sensitivity to broadband thermomechanical noise is significantly reduced, as shown in fig. 11. By analysing the spectral dependence of the noise, we find evidence in support of the hypothesis that it is due to thermal motion of the stressed taper fiber softly clamped on two supports. The inset of fig. 11 plots the free spectral range of the noise peaks as a function of frequency, indicated with orange data points, which is seen to follow a power law $\propto \Omega^{0.31}$. Such a power law scaling is consistent with phase velocity dispersion of the lateral vibrations of an elastic cylinder [40, 41]. As a second check of the hypothesis that
FIG. 11. Broadband homodyne spectrum of cavity transmission at phase (green) and amplitude (blue) quadratures, for a power of $P = 24 \, \mu W$. Local oscillator shot noise is shown in yellow. Inset shows measured free-spectral range of noise peaks as function of frequency (orange), with power law fit, $\propto \Omega^{0.31}$. Finite element model calculation of the free-spectral range is shown in blue. Image shows a fiber harmonic near 3.5 MHz.

the excess noise originates from fiber vibrations, the eigenmodes of a realistic tapered fiber geometry are computed using finite element modeling. The model incorporates the known geometry of the taper, which is ca. 25 mm long and 80 $\mu m$ in diameter at the clamping points. The taper profile is modeled as exponential in cross-section, as expected for a taper pulled with a uniform heat source [42]. The model assumes the center of the taper is 1 $\mu m$ in diameter. The prediction of this mode, shown as blue data point in Figure 11 inset, closely matches the measured data (orange).

Also, in contrast to guided-acoustic wave Brillouin scattering [43], the vibrational noise peaks are only present when the taper is coupled to the microcavity. We suspect that reactive and dispersive coupling of the tapered fiber to the cavity leads to transduction of its motion onto both transmitted amplitude and phase quadratures.

Appendix C: Quantum-enhanced force sensitivity

Consider estimation of an arbitrary force, $\delta F$, acting on the mechanical oscillator. The homodyne photocurrent spectrum carries information about the force eq. (A8), viz.

$$S^\theta,\text{hom}_{II}[\Omega] = 1 + \frac{4\eta C T_m}{x_{2p}^2} \left| \chi[\Omega] \right|^2 \left( S_{FF}[\Omega] + S_{QBA, FF}[\Omega] \right) \sin^2 \theta + \frac{\hbar}{2} \sin(2\theta) \Re \chi[\Omega].$$

The spectrum of the applied force $S_{FF}[\Omega]$ can be estimated from the photocurrent spectrum via,

$$S^\theta_{\text{est}, FF}[\Omega] := \frac{S^\theta,\text{hom}_{II}[\Omega]}{(4\eta C T_m/x_{2p}^2) \left| \chi[\Omega] \right|^2 \sin^2 \theta}$$

$$= S_{FF}[\Omega] + S_{QBA, FF}[\Omega] + \frac{x_{2p}^2}{4\eta C T_m \left| \chi[\Omega] \right|^2 \sin^2 \theta} + \hbar \cot \theta \frac{\Re \chi}{\left| \chi \right|^2}.$$  \hspace{1cm} (C2)

Here, the first term represent the spectral density to be estimated. The second term, positive at all frequencies, is the contamination in the measurement record due to quantum back-action. The third, also positive term, is the imprecision due to shot-noise in the detection. The last term is due to quantum correlations between the back-action and imprecision in homodyne measurement record that can be negative at some frequencies, providing for reduced uncertainty in the ability to estimate the force.

Note that precisely on resonance ($\Omega = \Omega_m$), and/or, for phase quadrature homodyne measurement ($\theta = \pi/2$), correlations do not contribute to the estimator; so any reduction in uncertainty can only be expected away from resonance for quadrature-detuned homodyne measurement.
For a fixed probe strength, i.e. fixed cooperativity $C$, there exists a frequency dependent homodyne phase at which the correlation and the imprecision $S_{FF}^{\text{imp.} \theta}$ achieve an optimal trade-off. This optimal angle $\theta_{\text{opt}}$ is determined by,

$$\cot \theta_{\text{opt}}[\Omega] = -\frac{h}{x_{2p}} 2 \eta \mathcal{C}_m \Re \chi[\Omega] = 4 \eta C \frac{\Omega_m \Gamma_m (\Omega^2 - \Omega_m^2)}{(\Omega^2 - \Omega_m^2)^2 + (\Omega \Gamma_m)^2}.$$  \hfill (C3)

At this optimal angle, the spectrum of the force estimator takes the form,

$$\tilde{S}_{FF}^{\text{est}, \theta_{\text{opt}}}[\Omega] = \tilde{S}_{FF}[\Omega] + \tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \frac{x_{2p}^2}{4 \eta \mathcal{C}_m |\chi[\Omega]|^2} = \eta \mathcal{C}_m \frac{h^2}{x_{2p}} \left( \frac{\Re \chi[\Omega]}{|\chi[\Omega]|} \right)^2.$$  \hfill (C4)

Noting that the third term is simply $\tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega]$, and that $\tilde{S}_{FF}^{\text{QBA}}[\Omega] = C \Gamma_m \frac{h^2}{x_{2p}}$, this equation can be re-expressed in the suggestive form,

$$\tilde{S}_{FF}^{\text{est}, \theta_{\text{opt}}}[\Omega] = \tilde{S}_{FF}[\Omega] + \tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \tilde{S}_{FF}^{\text{QBA}}[\Omega] \left[ 1 - \eta \left( \frac{\Re \chi[\Omega]}{|\chi[\Omega]|} \right)^2 \right].$$  \hfill (C5)

Thus, at the optimal detection angle, quantum correlations conspire to cancel quantum back-action (in the measurement record) and reduce the error in the force estimation compared to the conventional choice $\theta = \pi/2$, for which correlations are absent and $\tilde{S}_{FF}^{\text{est}, \pi/2}[\Omega] = \tilde{S}_{FF} + \tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \tilde{S}_{FF}^{\text{QBA}}[\Omega]$.

(C6)

1. Correlation enhanced thermal force sensing

In the case of an oscillator in thermal equilibrium quantum correlations can yield improved sensitivity in the detection of the thermal force. In such a case the signal is the thermal force noise, i.e. $\tilde{S}_{FF} = \tilde{S}_{FF}^{\text{th.} \theta}$. Assuming that the recorded periodogram of the photocurrent has converged to the theoretical power spectrum, the homodyne angle dependent uncertainty in the spectral estimation of the thermal force may be defined by,

$$\epsilon_{\theta}[\Omega] := \tilde{S}_{FF}^{\text{est}, \theta}[\Omega] - \tilde{S}_{FF}^{\text{th.} \theta}[\Omega].$$  \hfill (C7)

The enhancement in sensitivity attained for measurement at the optimal quadrature $\theta_{\text{opt}}$, compared to the conventional measurement on phase quadrature, is quantified by,

$$\xi_{\text{th}}[\Omega] = \frac{\epsilon_{\pi/2}[\Omega]}{\epsilon_{\theta_{\text{opt}}}[\Omega]} = \frac{\tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \tilde{S}_{FF}^{\text{QBA}}[\Omega]}{\tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \tilde{S}_{FF}^{\text{QBA}}[\Omega] \left[ 1 - \eta \left( \frac{\Re \chi[\Omega]}{|\chi[\Omega]|} \right)^2 \right]^{-1},}$$  \hfill (C8)

where the last approximation is valid when $\tilde{S}_{FF}^{\text{QBA}}[\Omega] \gg \tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega]$, i.e. in the limit of large cooperativity $C \gg 1$ and for frequency offsets around the mechanical resonance $|\Omega - \Omega_m|/\Gamma_m \ll \sqrt{C}$. In this regime $\xi_{\text{th}}[\Omega] > 1$ and quantum-enhanced force sensitivity can be realized, with the enhancement factor being limited by the finite detection efficiency $\eta$ and the imaginary part of the mechanical susceptibility. The back-estimated factors $\xi_{\text{th}}[\Omega]$ for the parameters of our experiment are shown in fig. 12 and demonstrate thermal force sensitivity enhancement up to 25%.

The ability to better estimate the thermal force over a broad range of frequencies may open up opportunities for probing the structure of the weak thermal environment that the oscillator is coupled to.

2. Correlation enhanced external force sensing

If an optomechanical system is used for external incoherent force detection, the thermal force itself becomes a part of the noise background. We now consider the sensitivity enhancement in such a case, i.e. $\tilde{S}_{FF} = \tilde{S}_{FF}^{\text{est}} + \tilde{S}_{FF}^{\text{th.} \theta}$, and the error is,

$$\epsilon_{\theta}[\Omega] := \tilde{S}_{FF}^{\text{est}}[\Omega] - \tilde{S}_{FF}^{\text{th.} \theta}[\Omega].$$  \hfill (C9)

The corresponding expression for the sensitivity enhancement,

$$\xi_{\text{ext}}[\Omega] = \frac{\epsilon_{\pi/2}[\Omega]}{\epsilon_{\theta_{\text{opt}}}[\Omega]} = \frac{\tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \tilde{S}_{FF}^{\text{th.} \theta}[\Omega] + \tilde{S}_{FF}^{\text{QBA}}[\Omega]}{\tilde{S}_{FF}^{\text{imp.} \pi/2}[\Omega] + \tilde{S}_{FF}^{\text{th.} \theta}[\Omega] + \tilde{S}_{FF}^{\text{QBA}}[\Omega] \left[ 1 - \eta \left( \frac{\Re \chi[\Omega]}{|\chi[\Omega]|} \right)^2 \right]}.$$  \hfill (C10)
FIG. 12. Quantum-enhanced sensitivity to thermal force for the parameters realized in the current experiment, assuming input power = 25 µW. Plot shows the enhancement factor $\xi_\theta(\Omega)$, defined in eq. (C8), as a function of Fourier frequency and homodyne angle $\theta$. The dashed black line corresponds to $\xi_{\pi/2}(\Omega)$, where force is estimated by phase quadrature detection, where backaction-imprecision correlations are absent. As the homodyne angle is detuned from phase quadrature, enhancement of up to 25% can be observed, limited by the detection efficiency of similar magnitude. The yellow curve shows the theoretically ideal detection scheme, where the homodyne angle is frequency dependent (eq. (C3)), so that broadband enhancement is realized.

indicates an additional constraint to be met due to the presence of the thermal force – the quantum backaction force needs to be comparable to the thermal force. For the room temperature experiments to date the limit $n_{QBA}/n_{th} \ll 1$ (with $n_{th} \gg 1$) have been relevant, so, again for the case $S_{FF}^{QBA} \gg S_{FF}^{imp,\pi/2}$,

$$\xi_{ext}[\Omega] \approx 1 + \eta \frac{n_{QBA}}{n_{th}} \left( \frac{\text{Re} \chi[\Omega]}{\chi'[\Omega]} \right)^2,$$

and quantum-enhanced sensitivity to external force can be realized far off resonance, if QBA is significant compared to thermal noise.