Anisotropic transport properties of Hadron Resonance Gas in magnetic field

Ashutosh Dash\textsuperscript{1}, Subhasis Samanta\textsuperscript{2}, Jayanta Dey\textsuperscript{3}, Utsab Gangopadhyay\textsuperscript{1}, Sabyasachi Ghosh\textsuperscript{3}, Victor Roy\textsuperscript{1}
\textsuperscript{1}School of Physical Sciences, National Institute of Science and Research, Bhubaneswar, HBNI, Jatni, 752050, India
\textsuperscript{2}Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland
\textsuperscript{3}Indian Institute of Technology Bhilai, GEC Campus, Sejbahar, Raipur 492015, Chhattisgarh, India

An intense transient magnetic field is produced in high-energy heavy-ion collisions mostly due to the spectator protons inside the two colliding nuclei. The magnetic field introduces anisotropy in the medium and hence the isotropic scalar transport coefficients become anisotropic and split into multiple components. Here we calculate the anisotropic transport coefficients shear, bulk viscosity, electrical conductivity, and thermal diffusion coefficients for a multicomponent Hadron-Resonance-Gas (HRG) model for a non-zero magnetic field by using the Boltzmann transport equation in a relaxation time approximation (RTA). The anisotropic transport coefficients along the magnetic field remains unaffected by the magnetic field, while perpendicular dissipation is governed by the interplay of the collisional relaxation time and the magnetic time scale, which is inverse of the cyclotron frequency. We calculate the anisotropic transport coefficients as a function of temperature and magnetic field using the HRG model. The neutral hadrons are unaffected by the Lorentz force and do not contribute to the anisotropic transports, we estimate within the HRG model the relative contribution of isotropic and anisotropic transports as a function of magnetic field and temperature. We also give an estimation of these anisotropic transport coefficients for the hadronic gas at finite baryon chemical potential ($\mu_B$).

PACS numbers:

I. INTRODUCTION

In the initial stage of heavy ion collisions an intense transient magnetic field $eB \sim (1-10)m_{\pi}^2$ (for $\sqrt{s_{NN}} = 200$ GeV collisions) is expected to be produced \cite{1,2}. Theoretically it was also shown that the magnitude of the magnetic field almost linearly rise with center of mass energy collisions \cite{2,6}.

A general consensus is that the initial large magnetic field will decay quickly (within a few fm) and becomes so weak that its effect may be negligible in any bulk observables. However, the initial hot and dense phase of Quark-Gluon-Plasma (QGP) and later time hadronic phase both have finite electric conductivities, this finite conducting medium will definitely modify the decay of magnetic field according to the laws of magneto-hydrodynamics (MHD) \cite{6,10} or through a transport simulation \cite{11}, a matter which is still under investigation Refs. \cite{9,12,13,6}. Usually the transport coefficients such as shear, bulk viscosity, and electrical conductivity are taken as an input to dynamical models such as relativistic MHD. Hence it is important to calculate these transport coefficients possibly the temperature dependence in presence of strong electro-magnetic fields from the underlying microscopic theories. The calculation of transport coefficients in quark and hadronic matter in presence of a magnetic field were carried out in recent Refs. \cite{14,30,31}, where shear viscosity \cite{14,29}, electrical conductivity \cite{18,32}, and bulk viscosity \cite{33,37} were calculated in presence of a magnetic field. The dynamics of heavy quark in presence of magnetic field within the framework of Fokker-Planck equation was studied in \cite{39,40}. In the present work, we carry out a similar investigation, where we consider a multi-component Hadron Resonance Gas and evaluate the shear viscosity and electrical conductivity in the presence of a magnetic field. In principle, one can calculate these transport coefficients in the presence of a magnetic field by solving QCD on a space-time lattice, but due to the current computational limitation and some technical difficulties it is unlikely to obtain the accurate result of these quantities in the low-temperature regime. However, it is well known that HRG model successfully reproduces Lattice data just below the crossover temperature ($T_c$) \cite{41} and it is expected that at much lower temperatures HRG as an effective model can be reliably used to calculate transport coefficients of hadronic matter. Since the magnetic field is non-zero in the hadronic phase it motivates us to calculate the transport coefficients in presence of the magnetic field.

Here we would like to mention that recently in Refs. \cite{20,30,31} transport coefficients (electrical conductivity and shear viscosity) for a HRG were studied in presence of the magnetic field using the relaxation time approximation. The relaxation time was obtained from the constant cross section of hadrons. In the present study we relax this constraint and treat the relaxation time as a free parameter. Also, we calculate here the shear, bulk, electrical conductivity, and the diffusion coefficients for the HRG as a function of magnetic field and temperature for neutral and electrically charged hadrons. Due to the Lorentz force the transport coefficients for electrically charged hadrons becomes anisotropic, whereas, the neutral hadrons only contribute to the isotropic transport processes. We give some estimate of the relative contribution of such anisotropic and the isotropic transport coefficients within the HRG model for zero and non-zero $\mu_B$.

The article is organized as follows: in Sec. \textsuperscript{II} we
briefly discuss the thermodynamics of the HRG model. In Sec. III we introduce the Boltzmann transport equation in relaxation time approximation and the ansatz for the off-equilibrium distribution function required to calculate the transport coefficients. In the same section we discuss the transport coefficients obtained from relaxation time approximation with and without the magnetic field. Next, in Sec. IV we discuss numerical results obtained for HRG. We give a summary of our work in Sec. (V) At the end detailed derivation of various transport coefficients are given in appendix. Throughout the paper we use the natural unit, the four vectors are denoted by the greek indices and three vectors are denoted by the latin indices unless stated otherwise.

II. FORMALISM

A. Thermodynamics

Here, we start with a brief discussion of the hadron resonance gas (HRG) model to define the thermodynamical quantities like entropy density \( s \), enthalpy per particle \( h \), etc. which are used for the calculations of different transport coefficients. All thermodynamic quantities are derived from the grand canonical partition function \( Z \) of the hadronic matter with volume \( V \) at temperature \( T \) and baryon chemical potential \( \mu \):

\[
\ln Z = V \sum_i \int \frac{d^3 \vec{p}}{(2\pi)^3} g_i r_i \ln \left[ 1 + r_i e^{\beta(p_i^0 - \mu)} \right],
\]

where \( g_i \), \( p_i^0 = (p_i^2 + m_i^2)^{1/2} \) are degeneracy factors and energy of the hadrons of species \( i \) with mass \( m_i \); \( r_i = \pm \) stands for fermion or bosons respectively. The total degeneracy factor of a particular species of hadron is obtained as \( g_i = g_i^s \times g_i^l \), where \( g_i^s \), \( g_i^l \) are the spin and iso-spin degeneracy factors respectively.

Once the partition function is defined, the thermodynamic quantities pressure \( (P) \), energy density \( (\epsilon) \), net baryon density \( (\rho) \) are calculated from the following standard definitions:

\[
\begin{align*}
P & = \frac{T}{V} \ln Z, \\
\epsilon & = \frac{T^2}{V} \frac{\partial}{\partial T} \ln Z, \\
\rho & = \frac{T}{V} \frac{\partial}{\partial \mu} \ln Z.
\end{align*}
\]

Using Eqs. (2), we can further define the entropy density \( s \), and the enthalpy per particle \( h \) by using the relations

\[
\begin{align*}
s & = \frac{(\epsilon + P - \mu \rho) / T}, \\
h & = \frac{(\epsilon + P) / \rho}.
\end{align*}
\]

III. BOLTZMANN TRANSPORT EQUATION

The calculation of all the transport coefficients considered here are based on relaxation time approximation of the collision kernel of the Boltzmann equation, hence, it is worthwhile to discuss the method for the sake of completeness. The general form of the Boltzmann equation in the presence of external fields, in the relaxation time approximation is given by \[16, 18, 19, 47],

\[
p^n_{\mu} \partial_{\mu} f + qF^{\mu\nu} p_{\nu} \frac{\partial f}{\partial p^{\mu}} = -\frac{U \cdot p}{\tau_c} \delta f,
\]

where, \( F^{\mu\nu} \) is the electromagnetic field strength tensor. For our case, only magnetic field is present, hence \( F^{\mu\nu} = -B^{\mu\nu} \) with \( B^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} B_{\rho\sigma} \). \( B \) is the magnetic field strength and \( b^\mu \) is the unit four vector defined as \( b^\mu = \frac{B^\mu}{B} \). So, for a small deviation of the distribution function from the equilibrium, Eq. (4) can be written as,

\[
p^n_{\mu} \partial_{\mu} f_0 = \left( -\frac{U \cdot p}{\tau_c} \right) \left[ 1 - \frac{qB_{\rho\sigma} F^{\mu\nu} p_{\nu}}{U \cdot p} \frac{\partial}{\partial p^{\mu}} \right] \delta f.
\]

The equilibrium distribution function is \( f_0 = (e^{\beta(U \cdot p - \mu)} + r)^{-1} \), where \( r = \pm 1 \) depending on the statistics.

Here we construct \( \delta f \) as a linear combination of the thermodynamic forces times appropriate tensorial coefficients so that \( \delta f \) turns out to be a Lorentz scalar,

\[
\delta f = AX + B^{\mu} X_{\mu} + C^{\mu\nu} X_{\mu\nu}.
\]

Where \( X_{\mu\nu} \) represents the thermodynamic forces. Replacing the above form of \( \delta f \) in the Boltzmann transport equation and comparing the coefficients of the thermodynamic forces we get the unknown coefficients \( A, B^\mu \) and \( C^{\mu\nu} \) in the expression for \( \delta f \). Using the \( \delta f \) in the thermodynamic flows we obtain the transport coefficients as discussed in details in the appendix.

A. Transport coefficients without a magnetic field

After the short discussion on the thermodynamical quantities, we discuss here about the transport coefficients of a relativistic systems of particles in absence of any external magnetic fields. The electrical conductivity \( (\sigma) \), shear viscosity \( (\eta) \), bulk viscosity \( (\zeta) \) and the diffusion coefficient \( (\kappa) \) for a HRG are given in terms of the
temperature and the relaxation time of hadrons,

\[ \sigma = \sum_i q_i^2 \frac{1}{3T} \int \frac{d^3 \varphi_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \tau_c f_0(1 - r_i f_0) \]

\[ \eta = \sum_i q_i^2 \frac{1}{15T} \int \frac{d^3 \varphi_i}{(2\pi)^3} \frac{|\vec{p}_i|^4}{(p_i^0)^2} \tau_c f_0(1 - r_i f_0) \]

\[ \zeta = \sum_i q_i \int \frac{d^3 \varphi_i}{(2\pi)^3} \frac{|\vec{p}_i|^3}{(p_i^0)^2} Q_2^2 \tau_c f_0(1 - r_i f_0) \]

\[ \kappa = \sum_i q_i \int \frac{d^3 \varphi_i}{3h_i} \int \frac{d^3 \varphi_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \tau_c (h - p_i^0 f_0(1 - r_i f_0) \right) \]

(7)

where \( q_i \) stands for electric charge of hadrons type \( i \), \( \tau_c \) is the relaxation time of hadrons, which is taken to be same for all hadrons for the sake of simplicity. The \( Q_i \) is a function of speed of sound along with other thermodynamic quantities the details of which is given in Appendix B. The derivation of the transport coefficients given in Eq. (7) can be found in Refs. [42, 43] as well as in the Appendix B. Similar expressions can also be obtained in Kubo relation [44, 45].

In the present article, we aim to calculate the transport coefficients of HRG in presence of a magnetic field, the values of these coefficients without the magnetic fields are obtained by taking the limit of vanishing magnetic field. The expression for the transport coefficients in the presence of magnetic fields are given in the next few subsections and the corresponding detailed derivation for the same is given in Appendix [13].

B. Electrical conductivity in magnetic field

In presence of a magnetic field, the transport coefficients involve another time scale, cyclotron time \( \tau_B = p^0/(eB) \) along with the usual relaxation time \( \tau_c \) which usually depends on the rate of contact collisions between the constituents. The non-zero Lorentz force due to the magnetic fields give rise to an anisotropic transport phenomenon (as the force along the magnetic field is zero and non zero in other directions), it is obvious that if the collision time \( \tau_c \) is much smaller the cyclotron time \( \tau_B \) the effect of magnetic field is negligible i.e., the system is almost isotropic when \( \tau_c/\tau_B \ll 1 \), and it becomes anisotropic when \( \tau_c/\tau_B \sim 1 \) or greater. We also note that along the magnetic field, Lorentz force does not work, so the parallel component of any transport coefficient (denoted by \( || \) ) remains the same as without the magnetic field, given in Eqs. (7). Here we need a little bit more clarification, in linear theory, any thermodynamic fluxes are proportional to the corresponding thermodynamic forces and the proportionality constants are known as transport coefficients. If the system is isotropic the transport coefficients are scalar, but for an anisotropic medium, the transport coefficients are components of a tensor. The decompositions of the transport coefficient tensor in terms of the available basis \( (u^\mu, g^{\mu\nu}, b^\mu, b^{\mu\nu}) \) are not unique and we choose here a particular combination such that the decomposition has a component parallel to the magnetic field which is denoted with a subscript \( \parallel \). Whereas, the remaining components can have two or more components, usually denoted with a subscript \( \perp \) and \( \times \). The \( \times \)-component is basically Hall component, which was absent for \( B = 0 \), while \( \perp \)-component at \( B = 0 \) will still exist and it will be exactly equal to \( \parallel \)-component, which restore the isotropic property of the medium at \( B = 0 \). For electrical conductivity, the expressions of parallel (\( \sigma_{||} \)), perpendicular (\( \sigma_{\perp} \)) and cross (\( \sigma_{\times} \)) components for hadron resonance gas are given below

\[ \sigma_{||} = \sum_i q_i^2 \frac{1}{3T} \int \frac{d^3 \varphi_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \tau_c f_0(1 - r_i f_0) \]

\[ \sigma_{\perp} = \sum_i q_i^2 \frac{1}{3T} \int \frac{d^3 \varphi_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \tau_c \left( f_0(1 - r_i f_0) \right) \]

As mentioned earlier, the detail derivation is given in Appendix A. To compare our results for electrical conductivities Eq. (8) to some of the earlier findings Refs. [18, 19, 21], where the conductivities are denoted with \( \sigma_{0,1,2} \), we found the following relations hold

\[ \sigma_{||} = \sigma_0 + \sigma_2, \]

\[ \sigma_{\perp} = \sigma_0, \]

\[ \sigma_{\times} = \sigma_1. \]

(9)

C. Shear Viscosity in magnetic field

The most general form of the \( \delta f \) in presence of a magnetic field where only shear stress is present is given by,

\[ \delta f = \sum_{n=0}^{4} \sum_{\mu \nu \alpha \beta} c_n C^{(n)}_{\mu \nu \alpha \beta} p^\mu p^\nu V^{\alpha \beta} \]

(10)

where \( V_{\alpha \beta} = \frac{1}{2} \left( \frac{\partial \rho}{\partial \sigma_{\alpha \beta}} + \frac{\partial \mu}{\partial \sigma_{\alpha \beta}} \right) \), the form of projectors \( P_{\mu \nu}^{\alpha \beta} \) will be given in Appendix [13] for \( n = -2, -1, 0, 1, 2 \). Using this expression for \( \delta f \), the shear
viscous coefficients turn out to be,

\[
\eta_\parallel = \sum_i \frac{2g_i}{15} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 p_0} |\vec{p}_i|^4 c_{i0}, \tag{12}
\]

\[
\eta_\perp = \sum_i \frac{2g_i}{15} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 p_0} |\vec{p}_i|^4 c_{i1}, \tag{13}
\]

\[
\eta'_\perp = \sum_i \frac{2g_i}{15} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 p_0} |\vec{p}_i|^4 c_{i3}, \tag{14}
\]

\[
\eta_\times = \sum_i \frac{2g_i}{15} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 p_0} |\vec{p}_i|^4 c_{i2}, \tag{15}
\]

\[
\eta'_\times = \sum_i \frac{2g_i}{15} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 p_0} |\vec{p}_i|^4 c_{i4}. \tag{16}
\]

The detailed calculation along with the value of \(c_0, c_1, c_2, c_3\) and \(c_4\) are given in Appendix B. The coefficients \(\eta_\parallel, \eta_\perp, \eta'_\perp\) are even functions of magnetic field \(B\). The two coefficients \(\eta_\times, \eta'_\times\) may have either sign and they are odd functions of \(B\). The later two coefficients are also called transverse viscosity coefficients [46]. We note the expressions for shear viscosities given in Eq. (12) are identical to those given in Refs. [16, 18, 19, 47].

### D. Bulk Viscosity in magnetic field

Similarly for bulk viscosity we restrict ourselves to only the divergence of the fluid four velocity and neglect the other thermodynamic forces,

\[
\delta f = \sum_{n=1}^{3} c_n C_{\mu \nu}^n \partial_\mu U_\nu. \tag{17}
\]

Using this \(\delta f\) the bulk viscous coefficients turns out to be,

\[
\zeta_\parallel = \zeta_\perp = \sum_i \frac{g_i}{3\hbar} \frac{T_c}{T} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 p_0} Q_i^2 f_0 (1 - r_i f_0), \tag{18}
\]

\[
\zeta_\times = 0. \tag{19}
\]

The bulk viscous coefficients remains unchanged under the influence of the magnetic field as also shown in Ref. [38] using Grad’s 14 moment approximation. The detailed derivation of Eq. (19) is given in the Appendix B.

### E. Diffusion coefficient in magnetic field

For the case of diffusion we keep only the term containing the spacial derivative of \(\mu/T\) in the expression for \(\delta f\),

\[
\delta f = K^{\mu \nu} p_\mu \partial_\nu \left( \frac{\mu}{T} \right). \tag{20}
\]

Using this \(\delta f\) the diffusion coefficients turn out to be,

\[
\kappa_\parallel = \sum_i \frac{g_i}{3\hbar} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 (p_0^0)^2} |\vec{p}_i|^2 \tau_c (h - p_0 f_0) (1 - r_i f_0),
\]

\[
\kappa_\perp = \sum_i \frac{g_i}{3\hbar} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 (p_0^0)^2} \tau_c (h - p_0 f_0) \frac{1 + (\frac{r_i}{\tau_c})^2}{\tau_i} f_0 (1 - r_i f_0),
\]

\[
\kappa_\times = \sum_i \frac{g_i}{3\hbar} \int \frac{d^3 \vec{p}_i}{(2\pi)^3 (p_0^0)^2} \tau_c (\frac{r_i}{\tau_c}) (h - p_0 f_0) \frac{1 + (\frac{r_i}{\tau_c})^2}{\tau_i} f_0 (1 - r_i f_0),
\]

where \(h = \frac{\mu + p_0}{T}\) is the enthalpy density to net baryon density ratio or enthalpy per particle. Due to the anisotropy induced by the magnetic field we have three diffusion coefficients. Here again, the details can be found in Appendix B.

### IV. RESULTS

In the formalism section, we have summarized the analytic expressions for the anisotropic components of the
shear viscosity, bulk viscosity, thermal diffusion and the electrical conductivity for a finite magnetic field. In this section, we will explore the temperature and magnetic field dependence of these transport coefficients for HRG model calculations.

Before discussing the results for HRG with physical masses of hadrons let us first consider the simpler massless case for quark gluon plasma (QGP). Here we also compare the result obtained from our numerical implementation of the HRG model to that of a Lattice QCD (LQCD) result for a sanity check. In the massless limit (also known as the Stefan-Boltzmann (SB) limit) the thermodynamical quantities like Pressure ($P$), energy density ($\epsilon$), and entropy density ($s$) varies as $T^3$, more explicitly

\begin{align}
P_{SB} &= g \frac{\zeta(4)}{\pi^2} T^4,
\epsilon_{SB} &= g \frac{3\zeta(4)}{\pi^2} T^4,
s_{SB} &= g \frac{4 \zeta(4)}{\pi^2} T^3,
\end{align}

where $\zeta(4)$ stands for zeta function. Here the subscript SB stands for the Stefan-Boltzmann (SB) limit, in this limit the interaction measure ($\epsilon - 3P$)/$T^4$ becomes zero and we consider the HRG to be a non-interacting gas. It is clear that in the SB limit $P/T^4$, $\epsilon/T^4$, and $s/T^3$ are constants for a given degeneracy. For example, a 3 flavor quark-gluon-plasma with the degeneracy factor $g = 16 + \frac{2}{3}(24 + 12) = 47.5$ yields $P/T^4 = 5.2$, $\epsilon/T^4 = 15.6$ and $s/T^3 = 20.8$. However, for the physical masses of hadrons all these thermodynamics quantities have a smaller value than their corresponding SB values and approaches SB value from below as $m/T \to \infty$. This is shown in the top panel of Fig. 1(a) for the normalized entropy density, where the result obtained from HRG model is shown by the blue dotted line, the corresponding $s_{SB}/T^3$ is shown by the blue horizontal solid line. For comparison we also show the LQCD (shown by green band) result from Ref. [18] in the temperature range 120-180 MeV. It is clear from the Fig. 1(a) that the normalized entropy density obtained from the Lattice QCD calculation and HRG matches very well in the temperature range considered here, also both results approaches SB value as temperature increases.

Now let us discuss the shear viscosity and the electrical conductivity of a massless gas without any magnetic field as given in Eqs. (7). In the massless limit the corresponding expressions are [19]:

\begin{align}
\eta &= g \frac{4 \zeta(4)}{5 \pi^2} \tau_c T^4,
\sigma &= g_q q^2 \frac{\zeta(2)}{3 \pi^2} \tau_c T^2,
\end{align}

where $g_q q^2 = 12 \times \left( \frac{4 \pi^2}{9} + \frac{s_e^2}{9} + \frac{s_q^2}{9} \right) = 8e^2$ for 3 flavor QGP. We note that similar to the thermodynamic quantities, in the SB limit, the normalized shear viscosity and electrical conductivity $\eta/(\tau_c T^4)$ and $\sigma/(\tau_c T^2)$ are constants only depends on the degeneracy factor. These normalized SB values $\eta_{SB}/(\tau_c T^4)$ and $\sigma_{SB}/(\tau_c T^2)$ are shown by the red dash-dotted and black dash horizontal lines in Fig. 1(b) and (c) respectively. For an HRG both $\eta/(\tau_c T^4)$ and $\sigma/(\tau_c T^2)$ has smaller values compared to their corresponding SB values and approaches to SB value from below in the large temperature limit as shown by the red dot and black dash-dotted lines in Fig. 1(b) and (c) respectively.

The striking similarity between the temperature dependence of thermodynamic quantity $s/T^3$ and the transport coefficients, like $\eta/(\tau_c T^4)$, $\sigma/(\tau_c T^2)$ clearly shows that we may gain information about the degrees of freedom of the system under consideration. Alternatively, we might get information about relaxation time $\tau_c$ if the temperature dependence of $\eta$ and $\sigma$ are known from other means.

Next, we explore the role of $B$ and $T$ on shear viscosity as shown in Fig. 2. For reference, we have also shown the values of $\eta/(\tau_c T^4)$ for a massless QGP (black dotted line) and that of HRG with $B = 0$ (shown by the red solid line). The $\eta_{SB}/(\tau_c T^4)$ of charged hadrons for $eB = 10m_\pi^2$ and $\tau_c = 5$ fm is shown by the dash-dotted line in Fig 2(a). Since HRG is composed of both charged and neutral hadrons, it is interesting to study the relative contribution of the charged and uncharged hadrons to the total shear viscosity. Since the neutral hadrons are unaffected by the Lorentz force and they only contribute to the isotropic shear viscosity which is $\propto \tau_c$, this comparison gives us the idea about the relative contribution of the anisotropic transport in the HRG. The anisotropic contribution $\eta_{\perp}/\tau_c T^4$ is given by $A_{\perp} = 1/[1 + (\tau_c/\tau_B)^2]$. It is clear from Fig. 2(a) that the anisotropic shear viscous coefficients from the charged hadrons contribution is quite smaller than that of the isotropic shear viscosity which also contains contributions from the neutral hadrons. However, the above fact is only true for large magnetic fields (in fig. 2(a) $B = 10m_\pi^2$) because $A_{\perp} \to 0$ in that limit. For a smaller magnetic fields the $\eta_{\perp}/\tau_c T^4$ becomes comparable or even larger than the isotropic $\eta/\tau_c T^4$ as shown in fig. 2(b). The $\parallel$ (red solid line) and $\perp$ (blue dash-double-dotted line) components of shear viscosity are plotted against $B$-axis in fig 2(b). The neutral hadrons contribution, which is independent of $B$ is shown by dash line, while the charged hadrons contribution is shown by dashed-dotted line. Blue dash-double-dotted line is basically summation of dash (neutral hadrons) and dash-dotted (charge hadrons) lines. To get some numerical estimate we note that for $B = 0$ the charged hadron contribution in the viscosity is more than 50% than the neutral hadrons. As $B$ increases, the charge hadron contribution decreases due to the decreasing anisotropic factor $A_{\perp}$ and for $eB \gtrsim 10m_\pi^2$ this contribution reduces to $\sim 4\%-8\%$.

Let us now consider the electrical conductivity, where gluons in the QGP phase and the neutral hadrons in the
HRG phase plays no role due to the charge neutrality. The results for the electrical conductivity as function of $T$ and $B$ are plotted in Figs. 2(a) and (b). For comparison, here also we show the massless SB limit for QGP (horizontal black dotted line) and HRG (red solid line) for $B = 0$. We found that the $T$ and $B$ dependence of the electrical conductivity and the shear viscosity are very similar in nature. They mostly differ due to the different contribution from the neutral hadron’s. For example, the neutral hadrons does not contribute to the electrical conductivity but plays a role in the transport phenomenon related to the shear viscosity. At this point we would like to add a few comments: (i) we note that both $\eta/\tau c T^4$, and $\sigma/\tau c T^2$ has the largest value for massless QGP, (ii) in presence of the magnetic field the transport coefficient becomes anisotropic and among the various components the $\parallel$ component is the largest and equals to the corresponding isotropic value of the transport coefficient (i.e., for $B = 0$). (iii) there is a small difference in the temperature dependence of the isotropic and the anisotropic transport coefficients.

Finally we discuss the diffusion coefficient $\kappa$. Since $\kappa$ is divergent at $\mu = 0$ because of its connection with the enthalpy per particle $h$, therefore, we have to estimate it.
at non-zero $\mu$. Similar to the electrical conductivity, in presence of a magnetic field the thermal diffusion coefficient also have three components - $\kappa_\parallel$, $\kappa_\perp$ and $\kappa_\times$. As usual the $\kappa_\parallel$ by construction is independent of the magnetic field but $\kappa_\perp$ and $\kappa_\times$ are function of the magnetic field. In fig. 4 we show the diffusion coefficients as a function of temperature for $B = 10 m_\pi^2$ and $\mu_B = 300$. From fig. 4 we see that $\kappa_\perp$ and $\kappa_\times$ are always smaller than $\kappa_\parallel$ for the temperature range considered here. A non-zero Hall diffusion coefficient $\kappa_\times$ can be attributed to the non-zero $\mu_B$, because for finite $\mu_B$ the particles and the anti-particles flow due to the Hall effect do not cancel out. Similarly, one can get non-zero Hall shear viscosities $\eta_\times$, $\eta_\times'$ and the Hall electrical conductivity for non-vanishing $\mu_B$. All of these Hall like transport coefficients vanishes for a net-baryon free medium because the contribution from the particles and the anti-particles are exactly equal and opposite. Fig. 5 demonstrate this $\mu_B$ dependent Hall viscosity ($\kappa_\times$) and the Hall conductivity ($\sigma_\times$) for $T = 150$ MeV, $eB = 10 m_\pi^2$, and $\tau_\varepsilon = 5$ fm. It is clearly seen that both $\eta_\times$ (black dashed line) and $\sigma_\times$ (blue dashed dotted line) increase monotonically from zero at $\mu_B = 0$. The growing tendency can be understood from the $\mu_B$ dependent of the net baryon density of HRG system, which is roughly proportional to $\text{sinh}(\mu/T)$ for the Maxwell-Boltzmann distribution which at high temperature fairly well describe the Fermi-Dirac or Bose-Einstein distribution function.

The present methodology is semi-classical (as we consider quantum statistical distribution function) in nature and does not include the Landau quantization - a quantum aspects, which is visible in the strong magnetic field. This effect is separately addressed in Ref. [49], but the complete understanding is still missing and we need further theoretical research in this direction. The physics of the anisotropic dissipation of the relativistic fluid in a magnetic field is also applicable for non-relativistic fluid, such as different condensed matter and biological systems.

V. SUMMARY

In high energy heavy-ion collisions, large transient magnetic fields are produced predominantly in the perpendicular direction to the reaction plane. This magnetic field breaks the isotropy of the system and as a result, the transport coefficients become anisotropic. We evaluate the anisotropic transport coefficients of the HRG and massless QGP by using the relaxation time approximation method. We use a unique tensorial decomposition of the anisotropic thermodynamic forces which reduces the computational complexity for evaluating anisotropic transport coefficients. Along with the usual relaxation time, which appears in the collision kernel of the Boltzmann equation and controls the rate of reaching equilibrium for systems that are initially away from the equilibrium, in magnetic fields, we have another timescale equals to the inverse of the cyclotron frequency. The measure of anisotropy (denoted in the text as anisotropic factor $A$) turned out to be a function of the ratio of these two time scales. It is not surprising that we found the anisotropy increases with magnetic field, and due to the specific choice of tensorial decomposition the $\parallel$ components of the anisotropic transport coefficients turned out to be the same with the isotropic case (i.e., for $B = 0$). We estimate the relative contribution of electrically charged and neutral hadrons to the various transport coefficients using HRG model. Since the neutral hadrons are unaf-
fected by the Lorentz force, they do not contribute in the anisotropic transport phenomenon. We have shown that the charged hadron contribution in the viscosity is more than 50% than the neutral hadrons. As B increases, the charge hadron contribution decreases and for $\epsilon B \geq 10n_\pi^2$ this contribution reduces to 4%-8%. In case of diffusion constant we need to consider a medium with finite $\mu_B$, in this study we show the result for $\mu_B = 300$ MeV. We also find that non-dissipative Hall like shear viscosity and conductivity increases monotonically with $\mu_B$ from zero at $\mu_B = 0$. It turned out that there are three diffusion coefficients in non-zero magnetic fields and among them the || component is the largest one. It is interesting to note that in calculating the diffusion coefficients we do not explicitly take into account the electric charge of the hadrons but we observe the anisotropic diffusion coefficients due to the imbalance of particle and anti-particle numbers. We also sketch chemical potential dependence of Hall transport coefficients - how they grow from their numbers. We also sketch chemical potential dependence due to the imbalance of particle and anti-particle hadrons but we observe the anisotropic diffusion coefficients.

**Acknowledgment:** JD and SG acknowledge to MHRD funding facility in IIT Bhilai for supporting this theoretical work. UG and VR are supported by the DST INSPIRE Faculty research grant, India. AD and VR acknowledge support from the department of atomic energy, Govt. of India. SS is supported from Polish National Agency for Academic Exchange through ULM Scholarship with AGREEMENT NO: PPN/ULM/2019/1/00093/U/00001.

**Appendix A: Electrical conductivity in presence of magnetic field**

Electrical conductivity in absence of the magnetic field for a quasi-particle system having degeneracy $g$, electric charge $q$, four momentum $p^\mu = (p^0, \mathbf{p})$ is $[21, 45, 47]$

$$\sigma = gq^2 \frac{\beta}{3} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{p^0} \tau_c f_0 (1 - r f_0), \quad (A1)$$

where $r = \pm$ stand for the fermion/boson, $\tau_c$ is the thermal relaxation time.

For deriving the expression of the electrical conductivity in presence of a magnetic field, let’s start with the Ohm’s law,

$$J^i = \sigma^{ij} E_j, \quad (A2)$$

Here, $J^i = J_0^i + J_3^i$, with $J_0^i, J_3^i$ are the ideal and the dissipative part of the three electric current density respectively. $\sigma^{ij}$ is the electric conductivity tensor, $E_j$‘s are the electric field components in the $j$-th field and $i, j$ runs from 1 to 3.

Now, the dissipative part of the current density according to the microscopic definition can be expressed as,

$$J_3^i = gq \int \frac{d^3p}{(2\pi)^3} \frac{p^i}{p^0} \delta f, \quad (A3)$$

Here, $\delta f$ is deviation of the distribution function $f$ from its equilibrium part $f_0 = \frac{1}{e^{\beta(p^0 - \mu) + \mathbf{p} \cdot \mathbf{B}}}$.

Comparing the Ohm’s law and the microscopic definition of the dissipative current density we get,

$$\sigma^{ij} E_j = J_3^i = gq \int \frac{d^3p}{(2\pi)^3} \frac{p^i}{p^0} \delta f. \quad (A4)$$

To find the $\delta f$ we use relativistic Boltzmann equation (RBE) $[18, 19, 21, 47],$

$$\frac{\partial f}{\partial t} + \frac{p^j}{p^0} \frac{\partial f}{\partial x_j} + \frac{\partial f}{\partial p^j} \frac{\partial x_j}{\partial p^0} = I[\delta f]. \quad (A5)$$

Where, $I[\delta f]$ is the linearized collision integral. Use of the relaxation time approximation (RTA) corresponds to $I[\delta f] = -\frac{\delta f}{\tau_c}$ and we also note that the term $\frac{\partial f}{\partial x_j}$ on the L.H.S of the above equation represents the force due to the electric $\mathbf{E}$ and the magnetic field $\mathbf{B}$. So, Eq. (A5) can be written as (assuming vanishing $\frac{\partial f}{\partial x_j}$ and $\frac{\partial f}{\partial x_i}$)

$$-q(\mathbf{E} + \frac{\mathbf{p}}{p^0} \times \mathbf{B}) \frac{\partial f}{\partial p^i} = -\frac{\delta f}{\tau_c},$$

$$\Rightarrow q\mathbf{E} \frac{\partial f}{\partial p^i} + (\frac{\mathbf{p}}{p^0} \times \mathbf{B}) \frac{\partial f}{\partial p^i} = \frac{\delta f}{\tau_c},$$

$$\Rightarrow q\mathbf{E} \frac{\partial f}{\partial p^i} + (\frac{\mathbf{p}}{p^0} \times \mathbf{B}) \frac{\partial f}{\partial p^i} = \frac{\delta f}{\tau_c}. \quad (A6)$$

Since the second term of L.H.S., $(\frac{\mathbf{p}}{p^0} \times \mathbf{B}) \frac{\partial f}{\partial p^i} = 0,$ so we have considered the $\delta f$ term.

Now, we assume $\delta f = -\phi \frac{\partial f_0}{\partial p^i}$, where $\phi = \frac{\mathbf{p}}{p^0} \cdot \mathbf{F}$ with $\mathbf{F} = (l\hat{e} + m\hat{b} + n(\hat{e} \times \hat{b}))$, where $\hat{e}$ and $\hat{b}$ are unit vector along $\mathbf{E}$ and $\mathbf{B}$.

So, Eq. (A6) becomes

$$\frac{1}{p^0} \left[ -q\mathbf{E} \hat{e} + qB\hat{b} \times (l\hat{e} + m\hat{b} + n(\hat{e} \times \hat{b})) \right] = \frac{1}{p^0} \left[ (l\hat{e} + m\hat{b} + n(\hat{e} \times \hat{b}))/\tau_c \right]. \quad (A7)$$

Now, comparing coefficients of $\hat{e}$, $\hat{b}$ and $n(\hat{e} \times \hat{b})$ and solving for $l$, $m$ and $n$ we get

$$l = \left( \frac{-qE\tau_c}{p^0} \right) \frac{1}{1 + (\tau_c/\tau_B)^2},$$

$$m = \left( \frac{-qE\tau_c}{p^0} \right) \frac{(\tau_c/\tau_B)^2}{1 + (\tau_c/\tau_B)^2}(\hat{e} \cdot \hat{b}),$$

$$n = \left( \frac{-qE\tau_c}{p^0} \right) \frac{(\tau_c/\tau_B)}{1 + (\tau_c/\tau_B)^2}, \quad (A8)$$

where $\tau_B = p_0/(eB)$ is inverse of cyclotron frequency.
Hence, $\phi$ can be expressed as,

$$
\phi = \frac{q\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{p_i}{p_0} \left[ \delta_{ij} - \left( \frac{\tau_c}{\tau_B} \right)^2 \epsilon_{ijk} b_k \right] \left( \tau_c/\tau_B \right)^2 b_j f_0(1 - f_0) + (\tau_c/\tau_B)^2 b_i b_j \right) E_j f_0(1 - f_0),
$$

(A9)

and,

$$
\delta f = -\frac{\partial f_0}{\partial \rho^0} = \phi \beta f_0(1 - f_0)
$$

$$
\Rightarrow \delta f = \frac{q\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{p_i}{p_0} \left[ \delta_{ij} - \left( \frac{\tau_c}{\tau_B} \right)^2 \epsilon_{ijk} b_k \right] \left( \tau_c/\tau_B \right)^2 b_j f_0(1 - f_0)
$$

(A10)

Now, using the above expression of $\delta f$ in Eq. (A1), we get

$$
\sigma_{ij} = gq^2 \beta \frac{d}{d^3 \beta} \left( \frac{\tau_c}{\tau_B} \right)^n \left( \tau_c/\tau_B \right)^2 f_0(1 - f_0)
$$

and $n = 0, 1, 2$. One can identify $\parallel, \perp$ and $\times$ components from $\sigma^n$ by using relations [16, 18, 21, 47].

$$
\sigma_{\parallel} = \sigma_0 + \sigma_2 = gq^2 \beta \frac{d}{d^3 \beta} \left( \frac{\tau_c}{\tau_B} \right)^n \left( \tau_c/\tau_B \right)^2 f_0(1 - f_0)
$$

$$
\sigma_{\perp} = \sigma_0 = gq^2 \beta \frac{d}{d^3 \beta} \left( \frac{\tau_c}{\tau_B} \right)^n \left( \tau_c/\tau_B \right)^2 f_0(1 - f_0)
$$

$$
\sigma_{\times} = \sigma_1 = gq^2 \beta \frac{d}{d^3 \beta} \left( \frac{\tau_c}{\tau_B} \right)^n \left( \tau_c/\tau_B \right)^2 f_0(1 - f_0)
$$

(A11)

(A12)

(A13)

So, the left hand side of the above equation can be written as,

$$
p^\mu \partial_\mu f_0 = p^\mu U_\mu D f_0 + p^\mu \nabla_\mu f_0
$$

$$
= \frac{\partial f_0}{\partial T} \left( (U \cdot p) DT + p^\mu \nabla_\mu T\right) + \frac{\partial f_0}{\partial (\mu/T)} \left( (U \cdot p) D(\mu/T) + p^\mu \nabla_\mu (\mu/T)\right) + \frac{\partial f_0}{\partial U^\mu} \left( (U \cdot p) DU^\mu + p^\mu \nabla_\mu U^\nu\right)
$$

(B3)

Where, $U^\mu$ is four velocity of particle, $D \equiv U^\mu \partial_\mu$, $\nabla^\mu \equiv \Delta^\mu\nu \partial_\nu$ with $\Delta^\mu\nu = g^\mu\nu - U^\mu U^\nu$, $g^\mu\nu \equiv \text{diag}(1,-1,-1,-1)$. Now, using the energy-momentum conservation ($\partial_\mu T^\mu_0 = 0$), current conservation ($\partial_\mu N^\mu_0 = 0$) equations and the Gibbs Duhem relation we get,

$$
p^\mu \partial_\mu f_0 = \frac{f_0(1 - r f_0)}{T} \left\{ Q \nabla_\nu U^\sigma - p^\mu p^\nu \left[ \nabla_\mu U_\nu - \frac{1}{3} \Delta^\mu\nu \nabla_\sigma U^\sigma\right] + \left[ 1 - \frac{(U \cdot p)}{h} \right] p^\mu T \nabla_\nu \left( \frac{\mu}{T} \right) \right\}
$$

(B4)

Where $Q = (U \cdot p)^2 \left( \frac{1}{3} \gamma - \gamma' \right) + (U \cdot p) \left( \gamma'' - 1 \right) h - \gamma''' T - \frac{1}{2} m^2$ and $h = m S^3 / S^2_0$. The expressions for $\gamma'$, $\gamma''$, $\gamma'''$ and

### Appendix B: Structure of RBE in RTA

In presence of magnetic field RBE with RTA can be written as [16, 18, 19, 47],

$$
p^\mu \partial_\mu f + q F^\mu\nu p_\nu \frac{\partial f}{\partial p^\mu} = -\frac{U \cdot p}{\tau_c} \delta f,
$$

(B1)

where, $F^{\mu\nu}$ is field strength tensor, carry only magnetic field term $F^{\mu\nu} = -B^{\mu\nu}$ with $B^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} B_\rho U_\sigma$. $B$ is the magnetic field strength and $b^\mu$ is the unit four vector. So, for a small deviation of the distribution function from the equilibrium, eq. (B1) can be written as,

$$
p^\mu \partial_\mu f_0 = \left( -\frac{U \cdot p}{\tau_c} \right) + \frac{1}{\epsilon^{\mu
u-\rho-\sigma} + 1} \right] \right) \delta f.
$$

(B2)

where, chemical potential $\mu$ have space time dependency.
where $S^\alpha_n$ are

$$\gamma' = \frac{(S_0^0/S_1^0)^2 - (S_0^0/S_1^0)^2 + 4z^{-1}S_0^0S_1^0/(S_1^0)^2 + z^{-1}S_0^0/S_1^0}{(S_2^0/S_1^0)^2 - (S_0^0/S_1^0)^2 + 3z^{-1}S_0^0S_1^0/(S_1^0)^2 + 2z^{-1}S_0^0/S_1^0 - z^{-2}}$$

$$\gamma'' = 1 + \frac{(S_0^0/S_1^0)^2 - (S_0^0/S_1^0)^2 + 3z^{-1}S_0^0S_1^0/(S_1^0)^2 + 2z^{-1}S_0^0/S_1^0 - z^{-2}}{(S_2^0/S_1^0)^2 - (S_0^0/S_1^0)^2 + 3z^{-1}S_0^0S_1^0/(S_1^0)^2 + 2z^{-1}S_0^0/S_1^0 - z^{-2}}$$

$$\gamma''' = \frac{S_0^0/S_1^0 + 5z^{-1}S_0^0S_1^0/(S_1^0)^2 + 2z^{-1}S_0^0/S_1^0 - z^{-2}}{(S_2^0/S_1^0)^2 - (S_0^0/S_1^0)^2 + 3z^{-1}S_0^0S_1^0/(S_1^0)^2 + 2z^{-1}S_0^0/S_1^0 - z^{-2}}$$

where $z = m/T$ and $S^\alpha_n(z) = \sum_{k=1}^{\infty} (-r)^{k-1}e^{k\nu/T}K_n(kz)$, $K_n(x)$ denoting the modified Bessel function of order $n$.

### 1. Shear Viscosity

In presence of magnetic field, the general expression of $\delta f$ for shear viscosity is considered as

$$\delta f = \frac{4}{n=0} c_n C_{n(\mu\alpha\beta)} p^\mu p^\nu V^{\alpha\beta}$$

$$= \left[ c_0 P_{\mu\nu}^{(\alpha\beta)} + c_1 (P_{(\mu\nu)}^{(\alpha\beta)} + P_{(\mu\nu)}^{(\beta\alpha)}) + ic_2 (P_{(\mu\nu)}^{(\alpha\beta)} - P_{(\mu\nu)}^{(\beta\alpha)}) + ic_4 (P_{(\mu\nu)}^{(\alpha\beta)} - P_{(\mu\nu)}^{(\beta\alpha)}) \right] V^{\alpha\beta}$$

where $V^{\alpha\beta} = \frac{1}{2} \left( \frac{\partial n}{\partial \nu} + \frac{\partial n}{\partial \mu} \right)$, and $P^{(m)}_{\mu\nu\alpha\beta} = P^{(m)}_{\rho\sigma\alpha\beta} + P^{(m)}_{\nu\mu\alpha\beta}$. The fourth rank projection tensor is defined in terms of the second rank projection tensor as

$$P^{(m)}_{\mu\nu,\mu'\nu'} = \sum_{m_{1} = -1}^{1} \sum_{m_{2} = -1}^{1} P^{(m_{1})}_{\mu\nu,\mu'\nu'}$$

and the second rank projection tensor is defined as,

$$P_{\mu\nu} = b_{\mu}b_{\nu},$$

$$P_{\mu\nu}^{1} = \frac{1}{2} (\Delta_{\mu\nu} - b_{\mu}b_{\nu} + ib_{\mu}b_{\nu}),$$

$$P_{\mu\nu}^{1} = \frac{1}{2} (\Delta_{\mu\nu} - b_{\mu}b_{\nu} - ib_{\mu}b_{\nu}).$$

where $\Delta_{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$. The second rank projection tensor satisfies the following properties,

$$P_{\mu\nu}^{(m)\mu\nu} = \delta_{m\alpha\beta} P_{\mu\nu}^{(m\alpha\beta)},$$

$$P_{\mu\nu}^{(m\alpha\beta)} = P_{\mu\nu}^{(m\beta\alpha)},$$

$$\sum_{m=1}^{4} P_{\mu\nu}^{(m\alpha\beta)} = \delta_{m\alpha\beta},$$

$$P_{\mu\nu}^{(m\alpha\beta)} = 1.$$ (B11)

Substituting the above expression on the right hand side of the Boltzmann transport equation we get,

$$\left( - \frac{U \cdot p}{\tau_C} \right) \left[ 1 - qB^2C_2 \delta_{\mu\nu} \frac{\partial}{\partial p^\mu} \frac{\partial}{\partial p^\nu} \right] \delta f = \left( - \frac{U \cdot p}{\tau_C} \right) \left[ 1 - qB^2C_2 \delta_{\mu\nu} \frac{\partial}{\partial p^\mu} \frac{\partial}{\partial p^\nu} \right] \sum_{n=0}^{4} c_n C_{n(\alpha\beta\rho\sigma)} p^\alpha p^\beta V^{\rho\sigma}$$

$$= \left( - \frac{U \cdot p}{\tau_C} \right) \left[ p^\alpha p^\beta V^{\rho\sigma} \sum_{n=0}^{4} c_n C_{n(\alpha\beta\rho\sigma)} \right]$$

$$- \frac{qB^2C_2}{U \cdot p} \delta_{\mu\nu} \frac{\partial}{\partial p^\nu} (\Delta^\alpha_{\mu} p^\beta + \Delta^\beta_{\mu} p^\alpha) V^{\rho\sigma} \sum_{n=0}^{4} c_n C_{n(\alpha\beta\rho\sigma)} = T_1 + T_2$$ (B13)

where,

$$T_1 = \left( - \frac{U \cdot p}{\tau_C} \right) \left[ p^\alpha p^\beta V^{\rho\sigma} \sum_{n=0}^{4} c_n C_{n(\alpha\beta\rho\sigma)} \right]$$ and, $T_2 = qB^2 \delta_{\mu\nu} (\Delta^\alpha_{\mu} p^\beta + \Delta^\beta_{\mu} p^\alpha) V^{\rho\sigma} \sum_{n=0}^{4} c_n C_{n(\alpha\beta\rho\sigma)}$. (B14)

Now,

$$T_1 = \left( - \frac{U \cdot p}{\tau_C} \right) p^\alpha p^\beta V^{\rho\sigma} \left[ c_0 p_{(\alpha\beta)\rho\sigma} + c_1 (p_{(\alpha\beta)\rho\sigma} + p_{(\alpha\beta)\rho\sigma}^{(-1)}) + ic_2 (p_{(\alpha\beta)\rho\sigma} - p_{(\alpha\beta)\rho\sigma}^{(-1)}) + c_3 (p_{(\alpha\beta)\rho\sigma}^{(2)} + p_{(\alpha\beta)\rho\sigma}^{(-2)}) \right]$$

$$+ ic_4 (p_{(\alpha\beta)\rho\sigma}^{(2)} - p_{(\alpha\beta)\rho\sigma}^{(-2)}) .$$ (B15)
and,

\[ T_2 = q Bh \mu^\nu p_\nu (\Delta^\alpha_{\mu} P^\beta + \Delta^\beta_{\mu} P^\alpha) V^\rho_{\sigma} \sum_{n=0}^{4} c_n C_{(n)\alpha\beta\rho\sigma} = 2q Bh \mu^\nu p_\nu \Delta^\alpha_{\mu} P^\beta V^\rho_{\sigma} \sum_{n=0}^{4} c_n C_{(n)\alpha\beta\rho\sigma} \]  

(B16)

Since, \( C_{(n)\alpha\beta\rho\sigma} = C_{(n)\beta\alpha\rho\sigma} \).

So,

\[ T_2 = 2q BH \mu^\nu p_\nu \Delta^\alpha_{\mu} p^\beta V^\rho_{\sigma} \left[ c_0 P_{(\alpha\beta)\rho\sigma}^0 + c_1 (P_{(\alpha\beta)\rho\sigma}^1 + P_{(\alpha\beta)\rho\sigma}^{-1}) + ic_2 (P_{(\alpha\beta)\rho\sigma}^1 - P_{(\alpha\beta)\rho\sigma}^{-1}) + ic_3 (P_{(\alpha\beta)\rho\sigma}^2 + P_{(\alpha\beta)\rho\sigma}^{-2}) \right] \]

(B17)

The left hand side of the RBE equation, neglecting the terms that include the spatial gradients of temperature and chemical potential in terms of the projection operator \( P_{(\mu\nu)\alpha\beta} \) turns out to be,

\[ T_1 + T_2 = -\frac{f_0(1 - rf_0)}{T} p^\mu p^\nu V^\rho_{\sigma} \left[ P_{(\mu\nu)\alpha\beta}^0 + P_{(\mu\nu)\alpha\beta}^1 + P_{(\mu\nu)\alpha\beta}^{-1} + P_{(\mu\nu)\alpha\beta}^{-2} \right] \]  

(B19)

Now equating the right hand side with the left hand side of relativistic Boltzmann equation \([\text{B2}]\) with the help of eq. \([\text{B18}], [\text{B17}] \) and \([\text{B3}] \) we get;

\[ c_0 = \frac{1}{2} \frac{f_0(1 - rf_0)}{T(U \cdot p)} \tau_c \]

\[ c_1 = \frac{1}{2} \frac{(U \cdot p) f_0(1 - rf_0)}{T(U \cdot p)^2 + (qB\tau_c)^2} \]

\[ c_2 = \frac{1}{2} \frac{(U \cdot p) f_0(1 - rf_0)}{T(U \cdot p)^2 + (qB\tau_c)^2} \tau_c \]

\[ c_3 = \frac{1}{2} \frac{(U \cdot p) f_0(1 - rf_0)}{T(U \cdot p)^2 + (qB\tau_c)^2} \tau_c \]

\[ c_4 = \frac{1}{2} \frac{(U \cdot p) f_0(1 - rf_0)}{T(U \cdot p)^2 + (qB\tau_c)^2} \]

(B20)

Using the above expressions the shear viscosities turns out to be,

\[ \eta_\parallel = \frac{2}{15} \int \frac{d^3p}{(2\pi)^3 p_0} |p_\parallel|^4 c_0, \]  

(B21)

\[ \eta_\perp = \frac{1}{15} \int \frac{d^3p}{(2\pi)^3 p_0} |p_\parallel|^4 \tau_c f_0(1 - rf_0) \]  

\[ \eta_\perp' = \frac{2}{15} \int \frac{d^3p}{(2\pi)^3 p_0} |p_\parallel|^4 c_1, \]  

(B22)

\[ \eta_\perp = \frac{2}{15} \int \frac{d^3p}{(2\pi)^3 p_0} |p_\parallel|^4 c_2, \]  

(B23)

\[ \eta_\perp' = \frac{2}{15} \int \frac{d^3p}{(2\pi)^3 p_0} |p_\parallel|^4 c_3, \]  

(B24)

\[ \eta_\perp' = \frac{2}{15} \int \frac{d^3p}{(2\pi)^3 p_0} |p_\parallel|^4 c_4. \]  

(B25)

2. Bulke Viscosity
As mentioned earlier in the text, in presence of magnetic field there is three components of the bulk viscosity and the form of $\delta f$ corresponds to them is

$$
\delta f = \sum_{n=1}^{3} c_n C_n(\mu\nu) \partial^\mu U^\nu = \left(c_1 P_{\mu\nu}^0 + c_2 (P_{\mu\nu}^1 + P_{\mu\nu}^{-1}) + c_3 (P_{\mu\nu}^1 - P_{\mu\nu}^{-1})\right) \partial^\mu U^\nu.
$$

(B27)

So, the right hand side of RBE becomes

$$
- \frac{U \cdot p}{\tau_c} \left[ 1 - \frac{qB\tau_c}{(U \cdot p)} b^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} \right] \delta f = - \frac{U \cdot p}{\tau_c} \left[ 1 - \frac{qB\tau_c}{(U \cdot p)} b^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} \right] \left\{ c_1 (b^\mu b^\nu) + c_2 (\Delta^\mu b^\nu - b^\mu b^\nu) + ic_3 b^{\mu\nu} \right\} \partial_\mu U_\nu
$$

$$
= - \frac{U \cdot p}{\tau_c} \left\{ c_1 (b^\mu b^\nu) + c_2 (\Delta^\mu b^\nu - b^\mu b^\nu) + ic_3 b^{\mu\nu} \right\} \partial_\mu U_\nu
$$

$$
= - \frac{U \cdot p}{\tau_c} \left\{ c_2 (\partial^\mu U_\mu) + (c_1 - c_2) b^\mu b^\nu \partial_\mu U_\nu + ic_3 b^{\mu\nu} \partial_\mu U_\nu \right\}.
$$

(B28)

Equating the coefficients of $\partial^\mu U_\mu$, $b^\mu b^\nu \partial_\mu U_\nu$ and $b^{\mu\nu} \partial_\mu U_\nu$ from eq. (B28) and (B31) we get,

$$
c_1 = \frac{\tau_e Q}{(U \cdot p)} \frac{f_s(1 - r f_0)}{T}
$$

(B29)

$$
c_2 = \frac{\tau_e Q}{(U \cdot p)} \frac{f_s(1 - r f_0)}{T}
$$

(B30)

$$
c_3 = 0
$$

(B31)

Thus the bulk viscosity can be derived from the relation,

$$
\Pi^{\mu\nu} = \Pi \Delta^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 p^0} \delta f
$$

(B32)

II is known as bulk pressure. Therefore,

$$
\Pi = \frac{1}{3} \int \frac{d^3p}{(2\pi)^3 p^0} \Delta^{\mu\nu} p^\mu p^\nu \left\{ c_1 (b^\alpha b^\beta) \right. + c_2 (\Delta^\alpha b^\beta - b^\alpha b^\beta) + c_3 (\alpha b^\beta) \partial_\alpha U_\beta
$$

(B33)

So, there components of bulk viscosity in presence of magnetic field are

$$
\zeta_\parallel = \zeta_\perp = \frac{\tau_e}{T} \int \frac{d^3p}{(2\pi)^3 p^0} Q_0^2 f_0 (1 - r f_0)
$$

(B34)

$$
\zeta_\times = 0
$$

(B35)

Where $Q$ is already addressed in earlier subsection. Since without magnetization, there will be no magnetic field dependent component of bulk viscosity, so its numerical results have not been explored.

3. Diffusion coefficient

In presence of magnetic field for thermal diffusion component of $\delta f$ can be written as

$$
\delta f = K^{\mu\nu} p_\mu \partial_\nu \alpha_0;
$$

(B36)

where, $\alpha_0 = \frac{f}{T}$.

The second order tensor $K^{\mu\nu}$ can be break down into the new projectors:

$$
P_{\mu\nu} = \left( \begin{array}{c} P_{\mu\nu}^0 = b_\mu b_\nu, \\ P_{\mu\nu}^\perp = (P_{\mu\nu}^1 + P_{\mu\nu}^{-1}) = (\Delta_{\mu\nu} - b_\mu b_\nu), \\ P_{\mu\nu}^\times = (P_{\mu\nu}^1 - P_{\mu\nu}^{-1}) = ib_{\mu\nu} \end{array} \right)
$$

(B37)

Using these projectors the $\delta f$ becomes,

$$
\delta f = \left( K_{\parallel} b_\mu b_\nu + K_{\perp} (\Delta_{\mu\nu} - b_\mu b_\nu) + K_{\times} (ib_{\mu\nu}) \right) p^\mu \partial_\nu \alpha_0
$$

(B38)

Now, with this $\delta f$ the right hand side of the Boltzmann transport equation becomes,

$$
- \frac{U \cdot p}{\tau_c} \left[ 1 - \frac{qB\tau_c}{(U \cdot p)} b^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} \right] \delta f = - \frac{U \cdot p}{\tau_c} \left[ 1 - \frac{qB\tau_c}{(U \cdot p)} b^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} \right] \left[ K_{\parallel} b_\alpha b_\beta + K_{\perp} (\Delta_{\alpha\beta} - b_\alpha b_\beta) + K_{\times} (ib_{\alpha\beta}) \right] p^\alpha \partial^\beta \alpha_0
$$

$$
= - \frac{U \cdot p}{\tau_c} \left[ \begin{array}{c} p^\alpha - \frac{qB\tau_c}{(U \cdot p)} b^{\mu\nu} p_\nu \partial^\mu \\ \end{array} \right] \left[ K_{\parallel} b_\alpha b_\beta + K_{\perp} (\Delta_{\alpha\beta} - b_\alpha b_\beta) + K_{\times} (ib_{\alpha\beta}) \right] \partial^\beta \alpha_0
$$

$$
= - \frac{U \cdot p}{\tau_c} \left[ K_{\parallel} b_\alpha b_\beta + K_{\perp} (\Delta_{\alpha\beta} - b_\alpha b_\beta) + K_{\times} (ib_{\alpha\beta}) \right] p^\alpha \partial^\beta \alpha_0
$$

$$
+ qBp_\nu \left[ b^{\mu\nu} K_{\parallel} b_\alpha b_\beta + b^{\alpha\nu} K_{\perp} (\Delta_{\alpha\beta} - b_\alpha b_\beta) + b^{\alpha\nu} K_{\times} (ib_{\alpha\beta}) \right] p^\rho \partial^\beta \alpha_0
$$

(B39)
Using relation Eq. (B39) we have,

\[ \tau^\alpha \rho^\alpha = -i \rho_{\alpha}^\alpha \rho^\alpha = -i \left[ \rho^{1\alpha} - \rho^{-1\alpha} \right] \partial_{\alpha} \tau^\alpha = 0; \]  

(B40)

\[ \tau^\alpha \rho^\alpha = -i \rho_{\alpha}^\alpha \rho^\alpha = -i \left[ \rho^{1\alpha} - \rho^{-1\alpha} \right] \partial_{\alpha} \tau^\alpha = -i \left[ \rho^{\parallel} \rho^\parallel - \rho^{\perp} \rho^\perp \right]; \]  

(B41)

\[ \tau^\alpha \rho^\alpha = -i \rho_{\alpha}^\alpha \rho^\alpha = -i \left[ \rho^{1\alpha} - \rho^{-1\alpha} \right] \partial_{\alpha} \tau^\alpha = -i \left[ \rho^{\parallel} + \rho^{\perp} \right]. \]  

(B42)

Using the above expressions in eq. (B39) R.H.S. of RBE becomes,

\[
\frac{U \cdot p}{\tau_C} \left[ 1 - qB_{\tau_C} p_{\mu} \partial_{p_{\mu}} \right] \delta f = \frac{U \cdot p}{\tau_C} \left[ K^\parallel \partial_{p_{\mu}} \rho_{\mu}^\parallel + K^\perp \partial_{p_{\perp}} \rho_{\perp}^\parallel + K^\perp \partial_{p_{\perp}} \rho_{\perp}^\perp \right] \partial^\beta \alpha_0 \\
+ qB \left[ K_0 \partial_{p_{\mu}} + K^\perp \partial_{p_{\perp}} \rho_{\perp} \right] \tau_C \partial_{\alpha_0} \\
= \partial^\beta \alpha_0 \left[ \left( \frac{U \cdot p}{\tau_C} \right) \rho_{\mu}^\parallel - \left( \frac{U \cdot p}{\tau_C} \right) K^\perp + iqB \right] \partial^\beta \alpha_0 \\
- \left[ \frac{U \cdot p}{\tau_C} K^\perp + iqB \right] \partial^\beta \alpha_0 \\
= \partial^\beta \alpha_0 \left[ \left( \frac{U \cdot p}{\tau_C} \right) b_{\beta} - \left( \frac{U \cdot p}{\tau_C} \right) K^\perp + iqB \right] \Delta_{\beta \alpha} \\
- \left[ \frac{U \cdot p}{\tau_C} K^\perp + iqB \right] \partial^\beta \alpha_0 \\
= \partial^\beta \alpha_0 \left[ \left( \frac{U \cdot p}{\tau_C} \right) + \left( \frac{U \cdot p}{\tau_C} \right) K^\perp + iqB \right] \Delta_{\beta \alpha} \\
- \left[ \frac{U \cdot p}{\tau_C} K^\perp + iqB \right] \partial^\beta \alpha_0. \tag{B43}
\]

So, from eq. (B43) and (B39) the RBE becomes,

\[
\frac{f_0}{1 - f_0} \left[ 1 - \frac{U \cdot p}{h} \right] \partial_{\alpha_0} = \partial^\beta \alpha_0 \left[ \left( \frac{U \cdot p}{\tau_C} \right) + \left( \frac{U \cdot p}{\tau_C} \right) K^\perp + iqB \right] \Delta_{\beta \alpha} \\
- \left[ \frac{U \cdot p}{\tau_C} K^\perp + iqB \right] \partial^\beta \alpha_0. \tag{B44}
\]

Equating the coefficients for different tensorial terms we get,

\[
\frac{U \cdot p}{\tau_C} K^\perp + iqB \right] = f_0 \left[ 1 - \frac{U \cdot p}{h} \right],
\]

(B45)

\[
\frac{U \cdot p}{\tau_C} K^\perp + iqB \right] = 0,
\]

(B46)

\[
\frac{U \cdot p}{\tau_C} K^\perp + iqB \right] = 0.
\]

(B47)

Equating the above three equations we get,

\[
K^\parallel = -\frac{\tau_C f_0 \left( 1 - f_0 \right)}{U \cdot p} \left[ 1 - \frac{U \cdot p}{h} \right],
\]

(B48)

So, the thermal diffusion coefficients \( \kappa \)’s become

\[
\kappa^\parallel = -\frac{1}{3} \int \frac{d^3 \bar{p}}{(2\pi)^3 \bar{p}^0} \bar{p}^2 K^\parallel,
\]

(B49)
[1] A. Bzdak and V. Skokov, “Event-by-event fluctuations of magnetic and electric fields in heavy ion collisions,” Phys. Lett. B 710, 171 (2012).
[2] K. Tuchin, Adv. High Energy Phys. 2013, 490495 (2013) doi:10.1155/2013/490495 [arXiv:1301.0099 [hep-ph]].
[3] W. T. Deng and X. G. Huang, Phys. Rev. C 85, 044907 (2012) doi:10.1103/PhysRevC.85.044907 [arXiv:1201.5108 [nucl-th]].
[4] H. Li, X. I. Sheng and Q. Wang, “Electromagnetic fields with electric and chiral magnetic conductivities in heavy ion collisions,” Phys. Rev. C 94, no. 4, 044903 (2016).
[5] V. Roy and S. Pu, Phys. Rev. C 92, 064902 (2015) doi:10.1103/PhysRevC.92.064902 [arXiv:1508.03761 [nucl-th]].
[6] V. Roy, S. Pu, L. Rezzolla and D. Rischke, Phys. Lett. B 750, 45 (2015) doi:10.1016/j.physletb.2015.08.046 [arXiv:1506.06620 [nucl-th]].
[7] S. Pu, V. Roy, L. Rezzolla and D. H. Rischke, Phys. Rev. D 93, no. 7, 074022 (2016) doi:10.1103/PhysRevD.93.074022 [arXiv:1602.04953 [nucl-th]].
[8] M. Hongo, Y. Hirono and T. Hirano, arXiv:1309.2823 [nucl-th].
[9] G. Inghirami, L. Del Zanna, A. Beraudo, M. H. Moghadam, F. Becattini and M. Bleicher, Eur. Phys. J. C 76, no. 12, 659 (2016) doi:10.1140/epjc/s10052-016-4516-8 [arXiv:1609.03042 [hep-ph]].
[10] G. Inghirami, M. Mace, Y. Hirono, L. Del Zanna, D. E. Kharzeev and M. Bleicher, arXiv:1908.07605 [hep-ph].
[11] S. K. Das, S. Phumari, S. Chatterjee, J. Alam, F. Scardina and V. Greco, Phys. Lett. B 768, 260 (2017) doi:10.1016/j.physletb.2017.02.046 [arXiv:1608.02231 [nucl-th]].
[12] D. Satow, Phys. Rev. D 90, no. 3, 034018 (2014) doi:10.1103/PhysRevD.90.034018 [arXiv:1406.7032 [hep-ph]].
[13] V. Skokov, A. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009).
[14] K. Tuchin, J. Phys. G: Nucl. Part. Phys. 39 (2012) 025010.
[15] S. Li, H-U Yee, Shear Viscosity of Quark-Gluon Plasma in Weak Magnetic Field in Perturbative QCD: Leading Log Phys. Rev. D 97, 056024 (2018).
[16] P. Mohanty, A. Dash, V. Roy, Eur. Phys. J. A 55 (2019) 35.
[17] S. Ghosh, B. Chatterjee, P. Mohanty, A. Mukharjee, H. Mishra, Phys. Rev. D 100 (2019) 034024; arXiv:1804.00812 [hep-ph].
[18] J. Dey, S. Satapathy, A. Mishra, S. Paul, S. Ghosh, From Non-interacting to Interacting Picture of Quark Gluon Plasma in presence of magnetic field and its fluid property, arXiv:1908.04335 [hep-ph].
[19] J. Dey, S. Satapathy, P. Murmu, S. Ghosh, Shear viscosity and electrical conductivity of relativistic fluid in presence of magnetic field: a massless case, arXiv:1907.11164 [hep-ph].
[20] A. Das, H. Mishra, R.K. Mohapatra, Transport coefficients of hot and dense hadron gas in a magnetic field: A relaxation time approach Phys. Rev. D 100,114004 (2019).
[21] A. Harutyunyan and A. Sedrakian, Phys. Rev. C 94, no. 2, 025805 (2016) doi:10.1103/PhysRevC.94.025805 [arXiv:1605.07612 [astro-ph.HE]].
[22] B. O. Kerbikov and M. A. Andreichikov, Phys. Rev. D 91, no. 7, 074010 (2015) doi:10.1103/PhysRevD.91.074010 [arXiv:1410.3413 [hep-ph]].
[23] S. I. Nam, Phys. Rev. D 86, 033014 (2012) doi:10.1103/PhysRevD.86.033014 [arXiv:1207.3172 [hep-ph]].
[24] X. G. Huang, A. Sedrakian and D. H. Rischke, Annals Phys. 326, 3075 (2011) doi:10.1016/j.aop.2010.12.001 [arXiv:1108.0602 [astro-ph.HE]].
[25] K. Hattori, S. Li, D. Satow and H. U. Yee, Phys. Rev. D 95, no. 7, 076008 (2017) doi:10.1103/PhysRevD.95.076008 [arXiv:1610.06830 [hep-ph]].
[26] M. Kurian, S. Mitra, S. Ghosh, V. Chandra, Transport coefficients of hot magnetized QCD matter beyond the lowest Landau level approximation Eur. Phys. J. C 79 (2019) 134.
[27] M. Kurian, V. Chandra, Effective description of hot QCD medium in strong magnetic field and longitudinal conductivity Phys. Rev. D 96 (2017) 114026.
[28] B. Feng, Electric conductivity and Hall conductivity of the QGP in a magnetic field Phys. Rev. D 96, 036009 (2017).
[29] K. Fukushima, Y. Hida, Electric conductivity of hot and dense quark matter in a magnetic field with Landau level resummation via kinetic equations Phys. Rev. Lett. 120, 162301 (2018).
[30] A. Das, H. Mishra, R. K. Mohapatra, Electrical conductivity and Hall conductivity of a hot and dense hadron gas in a magnetic field: A relaxation time approach Phys.Rev. D 99 (2019) 094031.
[31] A. Das, H. Mishra, R. K. Mohapatra Electrical conductivity and Hall conductivity of hot and dense quark gluon plasma in a magnetic field: a quasi particle approach arXiv:1907.05298 [hep-ph].
[32] S. Ghosh, A. Bandyopadhyay, R.L.S. Farias, J. Dey, G. Krein, Anisotropic electrical conductivity of magnetized hot quark matter, arXiv:1911.10005 [hep-ph].
[33] K. Hattori, X. G. Huang, D. H. Rischke and D. Satow, Bulk Viscosity of Quark-Gluon Plasma in Strong Magnetic Fields arXiv:1708.00515 [hep-ph].
[34] X-G Huang, M. Huang, D. H. Rischke, A. Sedrakian, Anisotropic hydrodynamics, bulk viscosities, and r-modes of strange quark stars with strong magnetic fields Phys. Rev. D 81, 045015 (2010).
[35] N.O. Agasian, Phys. Atom. Nucl. 76 (2013) 1382.
[36] N.O. Agasian, JETP Lett. 95 (2012) 171.
[37] M. Kurian, V. Chandra, Bulk viscosity of a hot QCD medium in a strong magnetic field within the relaxation-time approximation Phys. Rev. D 97 (2018) 116008.
[38] G. S. Denicol, X. G. Huang, E. Molnár, G. M. Monteiro, H. Niemi, J. Noronha, D. H. Rischke and Q. Wang, Phys. Rev. D 98, no. 7, 076009 (2018).
[39] M. Kurian, S. K. Das, V. Chandra Heavy quark dynamics in a hot magnetized QCD medium arXiv:1907.09550 [nucl-th].
[40] B. Singh, L. Thakur, H. Mishra Heavy quark complex po-
tential in a strongly magnetized hot QGP medium Phys. Rev. D 97 (2018) 096011.
[41] P. Braun-Munzinger, K. Redlich, J. Stachel, Quark Gluon Plasma 3, eds. R.C. Hwa and X.N. Wang, (World Scientific Publishing, 2004), nucl-th/0304013
[42] S. Gavin, Transport Coefficients In Ultrarelativistic Heavy Ion Collisions, Nucl. Phys. A 435, 826 (1985).
[43] P. Chakraborty and J. I. Kapusta, Phys. Rev. C 83, 014906 (2011).
[44] S. Ghosh, Int. J. Mod. Phys. A 29 (2014) 1450054.
[45] S. Ghosh, Electrical conductivity of hadronic matter from different possible mesonic and baryonic loops Phys.Rev. D 95 (2017) 036018
[46] S. Hess. 2015. Tensors for physics, Undergraduate Lecture Notes in Physics.
[47] E.M. Lifshitz and L.P. Pitaevskii, 1987 Physical kinetics, Pergamon Press, U.K.
[48] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B 730, 99 (2014).
[49] S. Samanta, J. Dey, S. Satapathy, S. Ghosh, Quantum expression of electrical conductivity from massless quark matter to hadron resonance gas in presence of magnetic field, arXiv:2002.04431 [nucl-th].