The Application of Lagrange Relaxation on Taxi Dispatchments During Evening Rush Hours

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Abstract. The city transportation status is directly related to the taxi supply arrangements during evening rush hours. The operation times, operation nature, operation requirements, vehicle resource types and quantities will have big influence on taxi drivers' dispatchments, it is comparatively difficult to dispatch reasonably. This paper uses Lagrange Relaxation to solve the problem of taxi supply and demand during evening rush hours. NP-Hardness is the difficult point of dispatchments, thus Lagrange method is used to result an optimal solution on taxi dispatchments within reasonable calculating time, while the bottom limits of the approximate optimal solution and the approximate solution are resulted as well. This method can effectively solve the problem of the balance of taxi supply and demand during evening rush hours. Based on this, the dual problem is solved by using incremental subgradient method, while an improved strategy is proposed to rise the convergence rate.

Keywords: Lagrange Relaxation; Evening rush hours; Taxi; Supply and Demand Controlling; Incremental Subgradient Method; Convergence Rate.

1. Introduction
At present, the taxi supply exceeds the demand in our country, the vacancy rate is very high in most periods, and the efficiency of taxi supply and distribution is very low at the evening peak, so it is easy to produce the problem of transportation inconvenience. With the increase of taxi vacancy rate and the aggravation of the contradiction between taxi supply and demand in evening peak, the problem of traffic congestion is becoming more and more serious, the contradiction between supply and demand of urban road traffic is more prominent, and the traffic pressure is increasing day by day. At the same time, the taxi industry also faces the problem of difficult taxi-taking and overcrowding in the evening peak. In order to further strengthen the management of urban taxis, improve the efficiency of taxi use, and reduce the phenomenon of taxi empty driving in rush hour, it is suggested that Lagrange relaxation method be used to balance supply and demand in the evening peak period. In literature [1], the relationship between legal input, legal operation and related resources and effective output of taxi is studied deeply, the evaluation model and index system of taxi operation efficiency are established, and the evaluation results are analyzed. The results show that the combination of Lagrange relaxation method and single learning method is one of the most widely used and successful scheduling methods at home and abroad. As mentioned in literature [2], the computational complexity decreases with the increase of the group
number when the original problems are decomposed into sub-problems. At the same time, the economy of Lagrange multiplier has an important influence on the quotation and scheduling of transportation market. Literature [3] points out that the current market-oriented reform of transportation has entered a stage of regional market-oriented construction. The regional market settlement price actually takes the quotation curve of each unit as the cost curve. The Langer relaxation algorithm has strong optimization ability and solution efficiency, which can provide strong support for calculating regional market settlement price.

2. Analysis and Model

2.1. Current situation of taxi supply and demand scheduling in evening peak

In the past five years, China's social economy has made great progress, but the development of taxis lagged behind the level of social and economic development. In addition, with the acceleration of urban road construction, the road and community construction between the second ring road, the third ring road and the high speed expressway around the city is becoming more and more perfect, and the taxi management area and service market are expanding, but the number of taxis has not changed. Due to the short supply of taxis, it not only cannot meet the increasing demand, but also increase the operating, contracting and transaction prices of urban taxis [4]. Considering the current supply and demand gap of taxi, the improvement of operation efficiency and the future operation mode, the construction of roads and residential areas is becoming more and more perfect, and the taxi management area and service market are expanding, but the taxi ownership is still low.

At present, the average use area of taxis is larger than that of traditional taxis. But if modern taxi develops too fast, it will bring great impact to traditional taxi. As the main body of passenger transport, the road resources generated by empty vehicles will not consume output, which will greatly increase the pressure of road traffic and energy consumption, and make the social cost of taxis far higher than that of urban public transport. On the contrary, if we blindly restrict the development of taxis and cannot meet the needs of consumers, it will not only affect the service level of the whole public transport, but also lead to the increase of road occupation area and traffic flow, thus causing a series of other social problems. For example, taxi charges are rising because of the current low number of taxis, and drivers pay taxi companies more and more contract fees. In order to increase income, drivers have to extend working hours and increase labor intensity. There are low income, high risk and heavy social burden for drivers [5]. There are many unstable factors in the industry, which will naturally lead to a lot of social problems. Therefore, the taxi increase should be controlled within a certain range.

The growth of traffic demand is limitless. It is impossible to meet unlimited traffic demand by increasing traffic supply alone. In order to achieve this goal, we must use the systematic management thought, break through the traditional idea of simply increasing supply to meet the demand growth, and turn the emphasis to "demand management". On the one hand, we can meet a certain taxi demand through moderate supply, rather than all demand; on the other hand, through the means and measures of traffic demand management, we can fundamentally solve the demand problem and reduce the taxi scheduling demand in night peak time. On this basis, a more reasonable taxi supply and demand scheduling scheme in peak time is proposed.

We should further combine the association rules to conduct in-depth mining of the scheduling values, and convert a large number of irregular raw data into meaningful knowledge data, so as to discover the potential association relationships between projects. The frequent itemset obtained by the Lagrange relaxation method can be used to generate association rules. However, the Lagrange relaxation method requires repeated scanning of the database and produces a large number of candidates sets [6]. The traditional serial algorithm is difficult to meet the performance requirements because of the large amount of traffic data. Based on Lagrange relaxation algorithm, the advantages of Lagrange relaxation method in CUDA framework are analyzed, and the data structure of Lagrange relaxation method is optimized. Based on this, the performance of Lagrange relaxation method is introduced to accelerate Lagrange relaxation algorithm, and the shortcomings of Lagrange relaxation method in various CUDA
frameworks are expounded. And how to use graph join method to generate candidate set and calculate dynamic queue holding degree of candidate set is expounded, and the test data set is optimized [7]. Secondly, by using the actual taxi data, the taxi operation is guided by the correlation between the passenger car behavior, and the taxi scheduling is completed by predicting the taxi travel demand.

2.2. Taxi supply and demand dispatching

The influence on taxi behavior mainly includes travel start time, return time, daily mileage and so on. Lagrange relaxation method is used to estimate the maximum likelihood coefficient. The last driving end time and the driving start time of the taxi can be approximately expressed as the normal distribution function; the daily mileage of the lognormal distribution function can be approximately expressed as the probability density function of the vehicle return time and the driving start time of the day. The specific algorithm is shown as follows:

\[ f(x) = \begin{cases} 
\frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{(x + 24 - \mu)^2}{2\sigma^2} \right) & \text{if } 0 < x \leq \mu_0 - 12 \\
\frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{(x - \mu_0)^2}{2\sigma_0^2} \right) & \text{if } x > \mu_0 - 12
\end{cases} \]

(1)

Wherein, \( \sigma \) indicates the taxi demand parameter; \( \mu \) represents the maximum dispatching number; \( x \) represents the taxi operation load index. \( \mu_0 - 12 < x \leq 24 \). The probability density function of daily mileage is:

\[ f_c(x) = \frac{1}{\sqrt{2\pi\sigma_m x}} \exp \left( -\frac{(\ln x - \mu_m)^2}{2\sigma_m^2} \right) \]

(2)

Based on the above algorithm, a taxi evening peak scheduling mechanism similar to the previous day is further adopted to optimize the scheduling function of all taxis on the same day. After the implementation of the strategy, each taxi must report the next day's dispatch plan to the taxi collector by the next day. The objective function of the taxi collector is set as follows:

\[ \max C = \sum_{i=1}^{N} \sum_{j=1}^{\infty} S_{i,j} \cdot \Delta t \cdot c \]

(3)

In the above algorithm, \( s \) is accidental function and \( c \) is constrained value. For the original vehicle scheduling problem, Lagrange relaxation method does not need to consider the constraint of objective function [3]. However, in order to make the relaxation problem easier to solve and solve the dual problem, the relaxation problem must be solved by iteratively updating the Lagrange multiplier. In the process of scheduling, if the relaxation problem cannot be solved, the solution time of the whole problem is still very long. Therefore, we usually require LRP iteratively update the optimization variables, transform the LRP values into univariate optimization problems, and optimize the variables. In general, the non-smooth optimization correlation method is usually used in the double solution Lagrange relaxation method. However, because the double solution is not differentiable, the subgradient method is often used. The specific algorithm is shown as follows:
In view of the relaxation problem of taxi peak scheduling, we should choose the relaxation constraint. Lagrange multipliers corresponding to different constraints have different physical meanings, and the subproblems generated under different constraints are also different, resulting in different complexity of understanding. The optimal value of the dual problem is affected by different constraints, that is, the quality of LR lower bound is recorded as $J$. for the relaxation constraint $M$ of taxi, the constraint quantity is $m$. There are two options. And it's divided into $n$ decision variables [1]. Each decision variable only corresponds to the completion time of each job. The coupling of variables between groups is caused by constraints. Therefore, when the constraints are relaxed to the objective function, the original decision variables can be decomposed into $n$ independent subproblems and Lagrange multipliers are introduced. Corresponding vectors are recorded as $k$- relaxation constraints.

$$Z = \begin{bmatrix} 1 & C_y - 1 \\ 0 & \text{others} \end{bmatrix}$$ (4)

$$Z_{LRP}(\pi) = \min \text{ } \sum_{i} J_i + \sum_{k} \pi_{k} \left(1 - M_{kh}\right)$$

$$= \min \left\{ \sum_{i} J_i - \sum_{k} \pi_{k} M_{kh} + \sum_{j} \sum_{k} \pi_{k} \right\}$$

$$= \min \left\{ \sum_{i} J_i - \sum_{k} \pi_{k} M_{kh} + \sum_{j=0}^{n-1} \pi_{km} \right\}$$

$$= \min \sum_{i} \left\{ J_i + \sum_{j=0}^{n-1} \pi_{km} \right\} - \sum_{k} \pi_{k} M_{k}$$ (5)

Usually, if the GPS location data returned by the taxi seriously does not conform to the vehicle operation law, the GPS data points cannot truly reflect their running state and must be eliminated. The distance between GPS data points is calculated in this process, and the distance product between GPS average speed and taxi data points is compared and analyzed to determine the distance between GPS data points. The distance between A point and B point can be calculated in WGS-84 coordinate system if the latitude coordinate of the known point A is $(\text{lat A}, \text{lon A})$ and point B is $(\text{lat B}, \text{lon B})$ is:

$$Q = \sin(MLatA) \sin(MLatB) \cos(MLonA - MLonB) + \cos(MLatA) \cos(MLatB)$$ (6)

If R represents taxi location identification information, then:

Distance $= R \times \arccos(C) / 180$ (7)

By using ArcGIS hot spot analysis tool, the clustering position of the three characteristic points of taxi driving and getting on and off is identified in space. The hot spot area formed by the three characteristic points of empty driving, loading and unloading is determined as three characteristic points of empty driving, loading & unloading. A new algorithm is proposed. The idea is to check each feature point in the specified neighborhood and determine that a high value feature point may be a hot spot, but not necessarily a statistical hot spot. The hotspots of statistics require that each factor must have a high value and be surrounded by other elements with the same high value. $Z$ and $P$ values are calculated to determine the aggregation of low- and medium-value elements in space. The scoring formula is as follows:
\[ G_j^* = \frac{Q \sum_{j=1}^{n} \omega_{x,j} \max \{C - \sum_{j=1}^{n} \omega_{i,j} \}}{n \sum_{j=1}^{n} \omega_{y,j} - T \left( \sum_{j=1}^{n} \omega_{i,j} \right)^2} \]  

(8)

Of these variables, \( \omega \) is the attribute value of the taxi running element, \( T \) the spatial weight of the element \( i \) to the \( j \), \( n \) is the total number of elements, and satisfies:

\[ \sum_{j=1}^{n} x_j = \bar{X} \]  

(9)

It can be seen from the analysis of the sample data of taxi dispatching trajectory that the error of the calculation results is mainly focused on the following aspects: According to GPS latitude and longitude deviation, equipment position is unreasonable to collect time error value. If the traffic is not good, "0" represents the empty car and "1" represents the passenger. These data are unreasonable if there is a non-0 or non-1 passenger or vehicle status value. There are many points of data loss that are relatively continuous in orbit, and not accidental discontinuous point data should be used as the segmentation point of two orbits. The specific algorithm is:

\[ H = G_j^* \sqrt{\frac{\sum_{j=1}^{n} x_j^2}{Zn} - (\bar{X})^2} \]  

(10)

Based on the above algorithm, taxi supply and demand scheduling in the evening peak period can better ensure the smooth traffic of the city, improve the traffic congestion problem, and fully meet the research requirements.

3. Conclusion

In order to solve the urban traffic problem fundamentally, this paper compares different supply and demand adjustment schemes for taxi supply and demand, analyzes the necessity of demand management, and puts forward corresponding countermeasures. According to the current situation of taxi supply and demand, this paper puts forward the Lagrange relaxation method as taxi supply and demand scheduling method in evening peak. On the one hand, the taxi supply should be increased moderately; on the other hand, it is necessary to study the traffic demand management strategy and find effective measures to provide the strategy of settling urban traffic problems fundamentally and provide reference for solving the contradiction between taxi supply and demand.

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