Capabilities of the Maple computer algebra system in the study of set theory and combinatorics

A A Olenev, K A Kirichek, E V Potekhina and O V Pelikh
Stavropol State Pedagogical Institute, 417a, Lenin ave., Stavropol, 355029, Russian Federation

E-mail: olenevalexandr@gmail.com

Abstract. The basic branch of both discrete and continuous mathematics is Set theory, the study of which is a difficult task. Designed and developed in the Maple computer algebra system, the Maplets package - the set theory application – enables to provide assistance both in teaching (by demonstrating the performance of operations on sets, testing knowledge and skills in using operations), and in studying set theory and solving the simplest combinatorial tasks (can be used as a simulator for students). By using interactive worksheets and animated images in Maple, students are given the opportunity for numerous experiments that will contribute to the development of their math education as well as computational skills. The application will form a sustainable need for the use of computer systems by students in the study of various branches of mathematics.

1. Introduction
The Maple software package is a computer algebra system (CAS) that makes it possible to represent and process mathematical information, both in symbolic and algebraic forms, to perform accurate analytical and numerical calculations in most branches of mathematics [1]. This makes Maple a powerful universal program used for solving mathematical problems. Despite the fact that the Maple math libraries are quite extensive in terms of functionality, they do not fully explain the procedure and stages of solving problems. To use the Maple CAS in the process of mastering mathematical disciplines, learners need to learn a new (unfamiliar) language of this program, which takes the time provided for the study of a particular subject [2].

Set theory is one of the most important sections for both continuous and discrete mathematics. Set theory is the basis for studying both discrete mathematics itself [3] and its separate sections, for example, number theory [4], cryptography [5, 6], modular operations [7], etc.

The Maple CAS has a Maplet package, the tools of which allow one to create ergonomic interactive applications - Maplets, containing windows, buttons, dialogs and other visual elements of the graphical user interface (GUI) [8, 9]. Using the Maplet package the authors of the article have developed the set theory application in the Maple CAS which contains the following Maplets: demonstration of set theory operations, solving problems on knowing how to perform set operations and solve the simplest combinatorial tasks. The developed application "Set theory" is an effective didactic tool in the study of set theory and solving the simplest problems of set theory and combinatorics [10]. It allows one to demonstrate the performance of basic set theory operations, create tasks, both algorithmically generated and entered by the users. Learners can refer to reference material, receive feedback, and perform self-
control activities. Working with the developed application, learners have the opportunities to study theoretical material, complete tasks, and in case of incorrect solution, find their own mistakes and correct them. Thus, the developed application "Set Theory" can be a simulator in performing tasks in set theory and combinatorics, be useful when doing homework, ensure cooperation between learners and schoolchildren in completing assignments and creating various projects, balance the development of awareness of the material presented and problem solving skills [11, 12, 13].

2. Materials and methods

The developed application "Set theory" has the following features which allow one:

- To introduce or automatically create new problems using various operations such as addition, union, intersection, difference, symmetric difference, Cartesian product.
- To check the correctness of the solution to a problem in set theory and the simplest problems in combinatorics.
- To solve problems for individual operations with instructions for each operation.
- To refer to reference material and receive a message about the correctness of assignments.

The developed Maplets graphical interface of the Set Theory application contains the following sections and buttons:

1. Section “Set theory. Set operations” (figure 1) enables to make a choice to demonstrate the performance of a selected operation on sets or a set of tasks to control the assimilation of material on set theory or combinatorics.

![Figure 1. A general view of the Set theory application.](image)

2. Pressing the "Demonstrating_Set_theory" button leads to the Set operations section (figure 2), which allows learners to choose to demonstrate how to perform the following set operations: addition, union, intersection, difference, symmetric difference, Cartesian product. The execution of the selected operation is accompanied by a graphical representation (Euler - Venn diagrams) for two sets [14].

![Figure 2. Contents of Maplets “Set theory” application.](image)
Figure 3 shows a Maplets demonstration of the addition operation. It allows one to enter the initial set in the "Entering the set A" window and, by pressing the corresponding “Addition” button, get the result of the operation. What is more, it is possible to view theoretical information (“Help” button), as well as enter another example to view the execution of an operation with other source data.

By analogy with Maplets “Addition”, the work with Maplets “Union”, “Intersection”, “Difference”, “Symmetric difference” and “Cartesian product” is organized.

3. Section “Self-control” contains two buttons: “Tasks. Set theory” and “Tasks. Combinatorics” (figure 4). They make it possible to select tasks for self-testing of the ability to perform operations on sets or to solve problems in combinatorics. The self-test results are displayed as Correct and Incorrect. To form a new task, click on the “Drop all” button and enter any data. It is possible to get help with a detailed solution by clicking on the “Help” button.

Figure 4. Task selection section.

4. Figure 5 shows Maplets to control the knowledge of performing the Merge operation. To check the knowledge of this operation, it is necessary to enter the presumably correct result and get the result of the performed action. Control over the use of other operations is performed in the same way.

Figure 5. Maplets for solving tasks using set theory operations illustrated by the merge operation.
Packages of computer mathematics (algebra), which perform analytical calculations, are practically all suitable for implementing the idea of analytical testing [1, 10]. The analytical testing is based on a comparison of the answer received by the tested person and the reference answer received by means of the CAS, in this case, the Maple CAS. Comparison of answers is made by finding the difference between their formula or numerical expressions. In this case, the test-takers answer can be presented in one of the many equivalent expressions. The testing program establishes the equivalence of the response expressions of the tested and the reference response obtained by means of Maple CAS. If equality is established, the answer received by the test taker is considered correct. Below is a fragment of one of the variants of application of this method, implemented in Maplets Set theory application:

```maple
> response := (m,n) -> if (m=n) then RETURN(CORRECT) else RETURN(INCORRECT) fi;
> randomize();
> F:= rand(8..15):G:=rand(3..5):
> A := F():
> B := G():
> corransw := A*B:
```

**Figure 6.** Section for selecting combinatorics tasks.

Figure 6 shows Maplets allowing selecting tasks for self-control by combinatorics.

Figure 7 shows Maplets for controlling combinatorial problem solving capabilities. To test knowledge on the proposed task, it is necessary to enter the presumably correct result in the upper field and receive a message (estimate) of the result of the performed action. Control for solving other problems is performed in a similar way.

**Figure 7.** Maplets for solving one of the tasks of combinatorics.

3. Conclusions
The “Set Theory” application can significantly facilitate the teacher's work in managing the independent work of a large number of learners, as well as provide the learners with the opportunity for practical
training in performing operations on sets and solving the simplest combinatorial problems. The developed application can be used to demonstrate and implement the main provisions of set theory when explaining new material, along with as a simulator for performing individual tasks in the process of developing skills and acquiring the skill of these positions, as well as to test the ability to solve problems of set theory and combinatorics.

Maplets supports both randomly generated tasks and user-entered tasks. This allows learners to solve specific problems presented in various sources. In addition, the generated tasks enable to work on tasks of various levels of complexity and form a stable skill in solving them. For example, Maplets “Addition” gives a general idea of how to perform and apply a given operation.

Instant feedback and endless repetition of Maplets allows learners to use the Set Theory app as a trainer without a teacher.

References
[1] Lopez R J 2012 Maple via Calculus: A Tutorial Approach (Springer Science & Business Media)
[2] 2012 Maplesoft Maple Student Version 15.0 for Calculus 9 (Brooks/Cole Publishing Co.)
[3] Rosen K H 2012 Discrete Mathematics and Its Applications (New York: Published by McGraw-Hill, a business unit of The McGraw-Hill Companies, Inc., 1221 Avenue of the Americas) p 1071
[4] Erickson M and Vazzana A 2007 Introduction to Number Theory (New York: Chapman and Hall/CRC) p 536
[5] Pardo Gómez J L 2013 Introduction to Cryptography with Maple (New York: Springer) p 705
[6] Klima R E and Sigmon N P 2012 Cryptology: Classical and Modern with Maplets (New York: Chapman & Hall/CRC) p 536
[7] Selivanova M V, Tyncherov K T, Ikhsanova F A, Kalmykov I A and Olenev A A 2019 Proof of the method of paired zeroing of numbers in a residue system J. of Phys.: Conf. Ser. 1333 022016
[8] Meade D B and Yasskin P B 2008 Maplets for Calculus: Improving Student Skills and Understanding in Calculus Elec. Proc.of the 20th Annual Inter. Conf. on Technology in Collegiate Mathematics (San Antonio, Texas)
[9] Monagan M B, Geddes K O et al. 2009 Maple Introductory, Programming Guide, Maplesoft, a division of Waterloo (Maple Inc.)
[10] Kirichek K A, Maliataki V V and Olenev A A 2018 The use of information technologies to introduce students in the basic school with the topic "Set" Computer science at school 8(141) 39-46
[11] Abramovich S, Burns J, Campbell S and Grinshpan A Z 2016 STEM education: action learning in primary, secondary, and post-secondary mathematics IMVI Open Mathematical Education Notes 6 65-106
[12] Friedlander E M, Holm T S, Ewing J et al. 2012 Mathematicians’ central role in educating the STEM workforce Notices of the American Mathematical Society 10
[13] Tompson P W, Artigue M, Törner G and de Shalit E 2014 Collaboration between mathematics and mathematics education Mathematics and Mathematics Education: Searching for the Common Ground 313-33
[14] Tyncherov K T, Olenev A A and Selivanova M V A program for visualizing the basic operations of set theory Certificate of state registration of the computer program RU 2020617848.15.07.2020. Application 2020616926 dated 02.07.2020