LDPC Coded Multi-User Massive MIMO Systems with Low-Complexity Detection

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ABSTRACT We design a low-density parity-check (LDPC) coded multi-user (MU) massive multiple-input multiple-output (MIMO) system with an iterative joint detection and decoding (JDD) algorithm. As a low-complexity MU detection scheme, we consider a factor graph based belief propagation detection with Gaussian approximation of interference, called a FG-GAI BP detection. We introduce a factor graph representation of LDPC coded MU massive MIMO system and define the message updating rule in the JDD process. We devise a design tool for analyzing extrinsic information transfer (EXIT) characteristics of messages exchanged in JDD, based on which degree distribution of LDPC codes and a JDD strategy are efficiently designed for coded MU massive MIMO systems. A JDD strategy and LDPC codes are designed such that a fast convergence of JDD and a low bit error probability are attained. It is observed that the coded MU massive MIMO system equipped with LDPC codes and the JDD strategy designed by the proposed method shows a lower bit error rate than conventional ones with a given number of iterations.

INDEX TERMS Massive MIMO, multi-user, LDPC codes, joint detection and decoding, low complexity, density evolution.

I. INTRODUCTION

Along with the continuous increase of demand for wireless and mobile services, performance requirements for wireless communication system including data rate, spectral efficiency and energy efficiency are getting strengthened [1], [2]. As one of promising solutions to meet such tight performance requirements, a multi-input and multi-output (MIMO) technology was proposed and tremendous amount of research works have been conducted in various aspects of MIMO systems [3], [4]. Recently, it was found that the use of massive number of antennas at transmitter and receiver can improve the spectral efficiency and save energy significantly in wireless communication systems [5], [6]. As a result, the massive MIMO technology has been under active study in both academia and engineering fields, and it was chosen as one of key technologies for the next generation cellular networks, known as the fifth generation (5G) systems [7]. Among various forms of massive MIMO schemes, a multi-user (MU) massive MIMO system, in which a base station (BS) is equipped with a large number of antennas to serve many user equipments (UE) simultaneously over the same time-frequency resource, has been actively studied to be practically adopted in 5G systems [8]–[10].

Since the signal detection in massive MIMO systems requires high amount of computations, the complexity reduction of signal detection algorithm has been a great concern to implement a massive MIMO technique in practical systems [11]–[20]. Suboptimal linear detection algorithms have been intensively studied for the purpose of complexity reduction, where zero forcing (ZF) detection [11], [12] and minimum mean squared error (MMSE) detection [13]–[16] are well known examples. When implementing these linear detectors, various alternatives of matrix inversion are applied [14]–[16]. Low-complexity detection algorithms based on belief propagation (BP) over factor graph (FG) have also been proposed [19], [20]. Among those, the FG-based BP detection with Gaussian approximation of interference (GAI), called FG-GAI BP detection, is known as one of promising solutions for complexity reduction in the receiver of massive MIMO systems [20]. Despite using low amount of computational complexity, the FG-GAI BP detection shows sufficiently low probability of error through iterative message passing on factor graphs [20].

Error control coding is widely used in a variety of commu-
The transmission reliability of massive MIMO systems because the performance requirement for data rate, spectral efficiency and energy efficiency is obviously keeping tight. Thus, it is a natural approach to apply LDPC codes having a powerful error-correction capability to the massive MIMO systems, by which LDPC coded massive MIMO system is devised. To obtain a maximum performance gain, a joint detection and decoding (JDD) scheme needs to be implemented at the receiver. Since decoding process requires additional computational complexity, a low-complexity detection algorithm had better be applied to JDD. In [30], non-binary LDPC codes are designed for coded massive MIMO systems considering modified MMSE and matched filter (MF) soft-output detectors. In [31]–[33], binary and non-binary LDPC codes for massive MIMO systems using the FG-GAI BP detection are designed. Protograph LDPC codes for massive MIMO systems [34] and LDPC coded space shift keying (SPK) for massive MIMO system with a low-complexity detector [35] are also proposed. In [17], [18], iterative soft-input soft-output (SISO) MMSE detectors are proposed to enhance the signal detection of massive MIMO systems.

In JDD mechanism, one JDD iteration is composed of a detection phase followed by a decoding phase, where each phase may consist of multiple local iterations. JDD had better converge by a smaller number of local iterations, i.e., with a faster convergence speed, to meet the latency requirement. Thus, we need to design LDPC coded MU massive MIMO system to achieve lower BER with faster convergence speed. In designing LDPC codes, the threshold, meaning the value of $E_b/N_0$ over which BER improves abruptly, is used as an indicator for evaluating BER. The threshold is determined by the structure of factor graph representing the overall system. The convergence speed is controlled by the ratio of local iteration numbers in detection phase and decoding phase composing one JDD iteration, which will be called a JDD strategy. There exist various techniques to predict the threshold of LDPC coded systems. However, there exist few works considering JDD convergence speed when designing LDPC codes in the system. Thus, there is a strong need for an efficient design tool by which we can check if messages flowing in JDD evolve in the way of satisfying two design goals: lower threshold and faster JDD convergence.
antenna and a BS is equipped with \(n_R\) multiple antennas. Each UE encodes \(K\) information bits to a \(N\)-bit codeword with the code rate \(R = K/N\) and modulates it as \(M_o\)-ary QAM symbols. Then, \(L = N/\log_2 M_o\) symbols generated at each UE are transmitted to BS over \(L\) channel uses. We suppose that all UEs transmit codeword symbols to BS in a synchronous manner.

Let \(x_k^{(l)}\) denote the \(l\)-th symbol generated at the UE \(k\), where \(k = 1, \ldots, n_U\) and \(l = 1, \ldots, L\). We let \(x^{(l)} = [x_1^{(l)} \cdots x_{n_U}^{(l)}]^T \in \mathbb{C}^{n_U \times 1}\) denote a symbol vector transmitted from \(n_U\) UEs to BS at the \(l\)-th channel use. We also let \(y^{(l)} = [y_1^{(l)} \cdots y_{n_R}^{(l)}]^T \in \mathbb{C}^{n_R \times 1}\), \(w^{(l)} = [w_1^{(l)} \cdots w_{n_R}^{(l)}]^T \in \mathbb{C}^{n_R \times 1}\) and \(H^{(l)} \in \mathbb{C}^{n_R \times n_U}\) denote the received signal vector, the additive noise vector and the channel gain matrix, respectively, all at the \(l\)-th channel use. Let \(h_{ij}^{(l)}\) denote the \((i, j)\)-th entry of \(H^{(l)}\). Suppose that all \(w_k^{(l)}\) are independent and identically distributed (i.i.d.) zero-mean circular symmetric complex white Gaussian noise with variance of \(\sigma^2\), and all \(h_{ij}^{(l)}\) are i.i.d. complex Gaussian with zero mean and unit variance. The input-output relation of the MU massive MIMO system at the \(l\)-th channel use is expressed as

\[
y^{(l)} = H^{(l)} x^{(l)} + w^{(l)}
\]

and its real-valued representation is written by

\[
y^{(l)} = H^{(l)} x^{(l)} + \tilde{w}^{(l)},
\]

where

\[
x^{(l)} = \begin{bmatrix} \Re\{x_1^{(l)}\} & \Im\{x_1^{(l)}\} & \cdots & \Re\{x_{n_U}^{(l)}\} & \Im\{x_{n_U}^{(l)}\} \end{bmatrix}^T \in \mathbb{R}^{2n_U \times 1},
\]

\[
y^{(l)} = \begin{bmatrix} \Re\{y_1^{(l)}\} & \Im\{y_1^{(l)}\} & \cdots & \Re\{y_{n_R}^{(l)}\} & \Im\{y_{n_R}^{(l)}\} \end{bmatrix}^T \in \mathbb{R}^{2n_R \times 1},
\]

\[
w^{(l)} = \begin{bmatrix} \Re\{w_1^{(l)}\} & \Im\{w_1^{(l)}\} & \cdots & \Re\{w_{n_R}^{(l)}\} & \Im\{w_{n_R}^{(l)}\} \end{bmatrix}^T \in \mathbb{R}^{2n_R \times 1},
\]

\[
H^{(l)} = \begin{bmatrix} H_{11}^{(l)} & H_{12}^{(l)} & \cdots & H_{1n_U}^{(l)} \\
H_{21}^{(l)} & H_{22}^{(l)} & \cdots & H_{2n_U}^{(l)} \\
\vdots & \vdots & \ddots & \vdots \\
H_{n_R1}^{(l)} & H_{n_R2}^{(l)} & \cdots & H_{n_Rn_U}^{(l)} 
\end{bmatrix} \in \mathbb{R}^{2n_R \times 2n_U},
\]

with

\[
H_{ij}^{(l)} = \begin{bmatrix} \Re\{h_{ij}^{(l)}\} & -\Im\{h_{ij}^{(l)}\} \\
\Im\{h_{ij}^{(l)}\} & \Re\{h_{ij}^{(l)}\} \end{bmatrix} \in \mathbb{R}^{2 \times 2}.
\]

Note that \(\Re\{\}\) and \(\Im\{\}\) represent real and imaginary part of a complex variable, respectively, and \(\mathbb{A}\) denotes the set of values for real-valued transmit symbols.

The receiver of coded MU massive MIMO system with the real-valued representation given in (2) can be expressed by a bipartite graph shown in Fig. 2. The receiver consists of \(L\) detectors and \(n_U\) decoders, where each detector corresponds to each channel use and each decoder corresponds to each UE. Each detector consists of \(2n_R\) observation nodes and \(2n_U\) symbol nodes connected through edges. Each decoder is composed of \(N\) variable nodes (or bit nodes) and \(N - K\) check nodes. Let \(o_i^{(l)}\) denote an observation node belonging to the \(l\)-th detector, whose input is \(\bar{y}_i^{(l)}\), \(1 \leq i \leq 2n_R\), \(1 \leq l \leq L\). We also let \(s_{2k-1}^{(l)}\) and \(s_{2k}^{(l)}\), \(1 \leq k \leq n_U\), denote symbol nodes belonging to the \(l\)-th detector, which correspond to \(\Re\{x_k^{(l)}\}\) and \(\Im\{x_k^{(l)}\}\), respectively. Note that \(s_{2k-1}^{(l)}\) and \(s_{2k}^{(l)}\) is connected to the first half and the second half of the \(l\)-th group of \(\log_2 M_o\) bit nodes in the \(k\)-th decoder, respectively.

**III. JOINT DETECTION AND DECODING**

For given received signals over \(L\) channel uses, i.e., \(\bar{y}^{(1)}, \bar{y}^{(2)}, \ldots, \bar{y}^{(L)}\), the receiver performs a JDD process in an iterative manner over the factor graph presented in Fig. 2. Detectors perform \(N_{det}\) iterations and pass resulting messages to decoders, then decoders perform \(N_{dec}\) iterations and pass computed messages to detectors. This cycle is called one JDD iteration or a global iteration. Note that each detection iteration and each decoding iteration will be called a local iteration. We perform \(N_g\) global iterations for a whole JDD process.

We consider a low-complexity FG-GAI BP-based detection algorithm [33] and a sum-product decoding algorithm in a JDD process. We focus on the \(l\)-th detector, in which \(\bar{y}_i^{(l)}, \bar{x}_i^{(l)}\) and \(\bar{w}_i^{(l)}\) denote the \(i\)-th entry of \(\bar{y}^{(l)}, \bar{x}^{(l)}\) and \(\bar{w}^{(l)}\) respectively.
respectively, and $\bar{x}_{ij}^{(l)}$ denotes the $(i, j)$-th entry of $\mathbf{H}^{(l)}$. We rewrite (2) as \[ y_{ik}^{(l)} = h_{ik}^{(l)} \bar{x}_{ik}^{(l)} + z_{ik}^{(l)}, \quad i = 1, \ldots, 2n_R, \] where $z_{ik}^{(l)} \triangleq \sum_{j \neq k}^{2n_U} h_{ij}^{(l)} \bar{x}_{ij}^{(l)} + \bar{w}_i^{(l)}$ is an interference plus noise in case of detecting symbol $\bar{x}_{ik}^{(l)}$, $k = 1, \ldots, 2n_U$, from the received signal $y_{ik}^{(l)}$. If $n_U$ is large enough, $z_{ik}^{(l)}$ can be approximated as a Gaussian random variable $\mathcal{N}(0, \sigma^2)$ with a mean $\mu_{ik}$ and a variance $\sigma_{ik}^2$, where
\[ \mu_{ik} = \sum_{j \neq k}^{2n_U} h_{ij}^{(l)} \sum_{s \in A} s \cdot p(r^{(l)}(x_j^{(l)}) = s) \] and
\[ \sigma_{ik}^2 = \sum_{j \neq k}^{2n_U} \left( h_{ij}^{(l)} \right)^2 \left( \left( \sum_{s \in A} s \cdot p(r^{(l)}(x_j^{(l)}) = s) \right)^2 - \left( \sum_{s \in A} s \cdot p(r^{(l)}(x_j^{(l)}) = s) \right)^2 \right) + \sigma^2 / 2. \]

Note that $p(r^{(l)}(x_j^{(l)}) = s)$ denotes the a priori probability of $x_j^{(l)}$ from the viewpoint of the observation node $n_0$. Let us define $\alpha_{ik}^{(l)}(s) \triangleq p(r^{(l)}(\mathbf{H}^{(l)}, \bar{x}_{ik}^{(l)}) = s)$ and $\beta_{ik}^{(l)}(s) \triangleq \Pr(x_{ik}^{(l)} = s | \mathbf{H}^{(l)}, \bar{y}^{(l)})$ representing the likelihood and the extrinsic probability, respectively, of $x_{ik}^{(l)} = s$ evaluated at the observation node $n_0$ for a given $\mathbf{H}^{(l)}$, where $\bar{y}^{(l)}$ denotes a received signal vector excluding an entry $y_{ik}^{(l)}$.

We let $\gamma_{ik}^{(l)}(s) \triangleq \Pr(x_{ik}^{(l)} = s)$ represent the a priori probability of $x_{ik}^{(l)} = s$. Note that $\alpha_{ik}^{(l)}(s)$ and $\beta_{ik}^{(l)}(s)$ are obtained by [31], [33]
\[ \alpha_{ik}^{(l)}(s) \approx \frac{1}{\sqrt{2\pi\sigma_{ik}^2}} \exp \left( -\frac{(y_{ik}^{(l)} - h_{ik}^{(l)} s - \mu_{ik}^{(l)})^2}{2\sigma_{ik}^2} \right) \] and
\[ \beta_{ik}^{(l)}(s) = \kappa \prod_{j \neq i}^{2n_R} \alpha_{jk}^{(l)}(s) \cdot \gamma_{ik}^{(l)}(s), \] where $\kappa$ is a constant. By the iterative detection principle, the extrinsic probability $\beta_{ij}^{(l)}(s)$ plays the role of the a priori probability $\Pr(x_{ij}^{(l)} | \bar{x}_{ij}^{(l)}) = s$ in (4) and (5) during iterations. So, for a given $\gamma_{ik}^{(l)}(s)$, the message $\beta_{ik}^{(l)}(s)$ is used for updating $\mu_{ik}$ and $\sigma_{ik}^2$ by (4) and (5), and consequently updating $\alpha_{ik}^{(l)}(s)$ by (6). Of course, $\beta_{ik}^{(l)}(s)$ is updated by $\alpha_{ik}^{(l)}(s)$ as in (7). Consequently, $\alpha_{ik}^{(l)}(s)$ and $\beta_{ik}^{(l)}(s)$ are updated in a recursive manner via detection iterations. The FG-GA1 BP detection algorithm is summarized in Algorithm 1.

After $N_{det}$ detection iterations, each symbol node $s_k^{(l)}$ computes the log-likelihood ratios (LLR) of code bits composing the symbol $\bar{x}_k^{(l)}$ and delivers these to corresponding bit nodes in decoders. Let $a[t]$ denote the $t$-th bit in the bit-stream composing a symbol $a$. The LLR of $\bar{x}_k^{(l)}[t]$ is defined by $L(\bar{x}_k^{(l)}[t]) \triangleq \log \frac{p(r^{(l)}(\bar{x}_k^{(l)}[t]=0)}{p(r^{(l)}(\bar{x}_k^{(l)}[t]=1)}$ and obtained as [33]
\[ L(\bar{x}_k^{(l)}[t]) = \log \frac{\sum_{s \in S_0} p(r^{(l)}(\mathbf{H}^{(l)}, \bar{y}^{(l)}) = s)}{\sum_{s \in S_1} p(r^{(l)}(\mathbf{H}^{(l)}, \bar{y}^{(l)}) = s)} \] \[ = \log \frac{\sum_{s \in S_0} \prod_{i=1}^{2n_R} \alpha_{ik}^{(l)}(s)}{\sum_{s \in S_1} \prod_{i=1}^{2n_R} \alpha_{ik}^{(l)}(s)}, \] where $S_0 = \{s|s[t] = 0\}$ and $S_1 = \{s|s[t] = 1\}$, and the last equality comes from $p(r^{(l)}(\bar{x}_k^{(l)}[t]) = s | \mathbf{H}^{(l)}, \bar{y}^{(l)}) \propto \prod_{i=1}^{2n_R} p(r^{(l)}(\mathbf{H}^{(l)}, \bar{x}_{ik}^{(l)} = s)$. If we let $r = (2L + 1) + \text{mod}(k + 1, 2) \cdot \log_2 \sqrt{\kappa} + t$, the message $L(\bar{x}_k^{(l)}[t])$ is delivered to the $r$-th bit node in the $k$-th decoder, denoted by $v_k^r$.

By using $L(\bar{x}_k^{(l)}[t])$ obtained in (8) as the channel LLR of the corresponding bit node $v = v_k^r$, i.e., $L_v = L(\bar{x}_k^{(l)}[t])$ for $v = v_k^r$, the sum-product decoding is performed in an iterative manner. Let $L_{vc}$ and $L_{cv}$ denote the message sent from the bit node $v$ to the check node $c$ and the message sent from the check node $c$ to the bit node $v$, respectively. Then, these messages are updated [24] as
\[ L_{vc} = L_v + \sum_{c \in C_v \setminus c} L_{cv}, \] and
\[ L_{cv} = \prod_{v' \in C_v \setminus c} \text{sign}(L_v^{v'}) \cdot \phi \left( \sum_{v' \in C_v \setminus c} \phi(\{ |L_v^{v'}| \}) \right), \] where $\phi(x) = \log \frac{\exp(x) + 1}{\exp(x) - 1}$. Note that $C_v \setminus c$ denotes the set of check nodes except $c$ connected to the bit node $v$ and $V_c \setminus v$.

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**Algorithm 1: FG-GA1 BP Detection**

1. for $m = 1$ to $N_{det}$ do
2. for $i = 1$ to $2n_R$ do
3. for $j = 1$ to $2n_U$ do
4. $\xi_{ij}^{(l)} \leftarrow h_{ij}^{(l)} \cdot \sum_{s \in A} s \cdot \beta_{ij}^{(l)}(s)$
5. $\xi_{ij}^{(l)} \leftarrow (\bar{y}_{ij}^{(l)})^2 \cdot \sum_{s \in A} s^2 \cdot \beta_{ij}^{(l)}(s) - \xi_{ij}^{(l)}$
6. $\xi_{ij}^{(l)} \leftarrow \sum_{j=1}^{2n_U} \xi_{ij}^{(l)}$
7. $\xi_{ij}^{(l)} \leftarrow \sum_{j=1}^{2n_U} \xi_{ij}^{(l)} + \frac{\sigma^2}{\sigma_{ik}^2}$
8. for $k = 1$ to $2n_U$ do
9. $\mu_{ik} \leftarrow \xi_{ik}^{(l)} - \xi_{ik}^{(l)}$
10. $\sigma_{ik}^2 \leftarrow \xi_{ik}^{(l)} - \xi_{ik}^{(l)}$
11. $\alpha_{ik}^{(l)}(s) \leftarrow \frac{1}{\sqrt{2\sigma_{ik}^2}} \exp \left( -\frac{(y_{ik}^{(l)} - h_{ik}^{(l)} s - \mu_{ik}^{(l)})^2}{2\sigma_{ik}^2} \right)$, $\forall s \in A$.
12. for $k = 1$ to $2n_U$ do
13. $\beta_{ik}^{(l)}(s) \leftarrow \kappa \prod_{i=1}^{2n_R} \alpha_{ik}^{(l)}(s) \cdot \gamma_{ik}^{(l)}(s), \forall s \in A$
14. for $i = 1$ to $2n_R$ do
15. $\beta_{ik}^{(l)}(s) \leftarrow \beta_{ik}^{(l)}(s) / \alpha_{ik}^{(l)}(s), \forall s \in A$.
FIGURE 3: Factor graph of LDPC coded MU massive MIMO system for the bit-level EXIT analysis of JDD.

denotes the set of bit nodes except $v$ connected to the check node $c$.

After $N_{\text{dec}}$ decoding iterations, the LLR of $\bar{x}^{(l)}_k[t]$ is computed as $L'(\bar{x}^{(l)}_k[t]) = \sum_{c \in \mathcal{C}_{\bar{x}^{(l)}_k}} L_{cv}$ and delivered to the symbol node $s^{(l)}_k$ in the detector. Then, the a priori probability $\gamma^{(l)}_k(s)$ is computed as

$$\gamma^{(l)}_k(s) = \prod_{t=1} \frac{\exp((1-s[t]) \cdot L'(\bar{x}^{(l)}_k[t]))}{1 + \exp(L'(\bar{x}^{(l)}_k[t]))}$$ (11)

and used in (7) at the next global iteration.

After $N_g$ global iterations, we make decision on each code bit such that $\bar{x}^{(l)}_k[t]$ is estimated as 1 if $L(\bar{x}^{(l)}_k[t]) + L'(\bar{x}^{(l)}_k[t]) < 0$ and as 0 otherwise. The overall procedure of JDD is presented in Algorithm 2.

Algorithm 2: Joint Detection and Decoding (JDD)

1. Initialize: $\beta^{(l)}_k(s) = \frac{1}{\sqrt{M_v}}$, $\forall l,k,s$, $P_{r}(\bar{x}^{(l)}_k) = s \leq \frac{1}{\sqrt{M_v}}$, $\forall l,k,s$, and $L_{cv} = 0$, $\forall c,v$.
2. for $l' = 1$ to $N_g$ do
3.   for $l = 1$ to $L$ do
4.     [Run Algorithm 1 in the $l$-th detector.
5.     Compute $L_{cv}$, $\forall v$, by (8).
6.     for $l'' = 1$ to $N_{\text{dec}}$ do
7.       [Update $L_{cv}$ and $L_{cv}$, $\forall v$, $c$, by (9) and (10), respectively.
8.       Compute $\gamma^{(l'')}_k(s)$, $\forall l,k,s$, by (11).
9.     Determine the value of code bit corresponding to $v$, $\forall v$.

IV. EXIT ANALYSIS OF JOINT DETECTION AND DECODING

We propose an analysis tool for studying the behavior of JDD in the coded MU massive MIMO system in terms of extrinsic information transfer (EXIT) characteristics of component units. We investigate the mutual information between code bits and corresponding soft-valued messages traveling in JDD. For this purpose, we construct a bit-level factor graph by decomposing observation nodes and symbol nodes in Fig. 2 as sub-nodes, where each observation node and symbol node is decomposed into $\log_2 \sqrt{M_v}$ sub-nodes. Each symbol sub-node is connected to the corresponding bit node in an one-to-one manner, so that symbol sub-nodes can be simply merged by bit nodes. As a result, we obtain a bit-level factor graph as shown in Fig. 3, where only a part of graph associated with observation nodes $\alpha_0^{(1)}$, $\cdots$, $\alpha_{2n_R}$ and symbol nodes $s_1^{(1)}$, $s_2^{(1)}$ is depicted.

Let $L_{uo}$ and $L_{ov}$ denote bit LLRs sent from a bit node to an observation sub-node from an observation sub-node to a bit node, respectively. For an observation node $o = \alpha^{(l)}_0$ and a bit node $v$ corresponding to $\bar{x}^{(l)}_k[t]$, the bit LLRs are obtained by $L_{uo} = \log \frac{\sum_{x^{(l)}_k} \rho_{x^{(l)}_k}}{\sum_{x^{(l)}_k} \rho_{\overline{x}^{(l)}_k}}$ and $L_{ov} = \log \frac{\sum_{x^{(l)}_k} \sigma_{x^{(l)}_k}}{\sum_{x^{(l)}_k} \sigma_{\overline{x}^{(l)}_k}}$. By letting $U$ denote a code bit, we define $I_{VO} = I(U; L_{uo})$ and $I_{OV} = I(U; L_{ov})$, where $I(U; X)$ is the mutual information between $U$ and $X$. Note that $I_{VO}$ and $I_{OV}$ are input and output, respectively, of observation sub-nodes in terms of bit-level EXIT characteristics. We also define $I_{CV} = I(U; L_{cv})$ and $I_{VC} = I(U; L_{cv})$, where $I_{CV}$ and $I_{VC}$ are input and output, respectively, of bit nodes as well as output and input, respectively, of check nodes in terms of EXIT characteristics.

Allowing slight abuse of notation, we represent $I_{CV}$ associated with degree-$d_v$ check nodes as $I_{CV}(d_v)$. We also represent $I_{VC}$ associated with degree-$d_v$ bit nodes as $I_{VC}(d_v)$. In the similar manner, we use $I_{VO}(d_v)$ and $I_{OV}(d_v)$ to denote $I_{VO}$ and $I_{OV}$, respectively, associated with degree-$d_v$ bit nodes. Then, we define averages of these as $I_{CV} = \sum_{d_v} \rho_d I_{CV}(d_v)$, $I_{VC} = \sum_{d_v} \lambda_d I_{VC}(d_v)$ and $I_{OV} = \sum_{d_v} \mu_{d_v} I_{OV}(d_v)$, where $\lambda_d$ and $\rho_d$ denote the fractions of edges that are connected to degree-$d_v$ bit nodes and degree-$d_v$ check nodes, respectively, and $d_v, \mu_{d_v}$ are maxima of $d_v$ and $d_v, \mu_{d_v}$, respectively.

By defining $J(\sigma_X)$ as [27]

$$J(\sigma_X) = 1 - \int_{-\infty}^{\infty} \frac{e^{-[(\xi-\sigma_X^2)/2\sigma_X^2]}}{\sqrt{2\pi}\sigma_X} \cdot \log_2[1 + e^{-\xi}] d\xi,$$ (12)

we obtain $I(U; X) = J(\sigma_X)$, where $\sigma_X^2$ is the variance of a normally distributed random variable $X$. At bit nodes, the message going to a target node is generated by summing up all incoming messages except one from a target node, so $L_{uo}$ is obtained by summing up $2n_R - 1$ copies of $L_{ov}$ and $d_v$ copies of $L_{cv}$. Then, $I_{VO}(d_v)$ is obtained as

$$I_{VO}(d_v) = J\left(\sqrt{(2n_R - 1) \cdot [J^{-1}(I_{OV}(d_v))^2 + d_v \cdot J^{-1}(I_{CV})]^2}\right),$$ (13)
Algorithm 3: Density Evolution in terms of EXIT Characteristics

1. Initialize: \( I_{CV} = 0 \) and \( I_{OV}(d_v) = 0, \forall d_v \\
2. for \( l' = 1 \) to \( N_g \) do \\
3. for \( d_v = 2 \) to \( d_{v,\text{max}} \) do \\
4. for \( m = 1 \) to \( N_{\text{det}} \) do \\
5. \( I_{OV}(d_v) \leftarrow f_o \left( J \left( \left(2n_R - 1\right) \cdot \left[J^{-1} \left(I_{OV}(d_v)\right)\right]^2 + d_v \cdot \left[J^{-1} \left(I_{CV}\right)\right]^2 \right) \cdot \frac{E_b}{N_0} \right) \) \\
6. \( I_{OV} \leftarrow \sum_{d_v} \lambda_d I_{OV}(d_v) \) \\
7. for \( l'' = 1 \) to \( N_{\text{dec}} \) do \\
8. \( I_{VC} \leftarrow \sum_{d_v=1}^{d_{v,\text{max}}} \lambda_d \cdot J \left( \left(2n_R \cdot \left[J^{-1} \left(I_{OV}(d_v)\right)\right]^2 + (d_v - 1) \cdot \left[J^{-1} \left(I_{CV}\right)\right]^2 \right) \right) \) \\
9. \( I_{CV} \leftarrow \sum_{d_v=1}^{d_{v,\text{max}}} \rho_{d_v} \cdot \left(1 - J \left( \sqrt{d_v} - 1 \cdot J^{-1}(1 - I_{VC}) \right) \right) \)

where \( I_{OV}(d_v) \) is defined as a function of \( I_{VO}(d_v) \) as

\[
I_{OV}(d_v) = f_o \left( I_{VO}(d_v), \frac{E_b}{N_0} \right).
\]

The EXIT function of observation sub-node, \( f_o(\cdot) \), is obtained in a polynomial form by using Monte Carlo simulation [27] and a curve fitting technique. In the same manner, the LLR message \( L_{cv} \) is obtained by summing up \( 2n_R \) copies of \( L_{ov} \) and \( d_v - 1 \) copies of \( L_{cv} \). Thus,

\[
I_{VC}(d_v) = J \left( \sqrt{2n_R \cdot \left[J^{-1} \left(I_{OV}(d_v)\right)\right]^2 + (d_v - 1) \cdot \left[J^{-1} \left(I_{CV}\right)\right]^2} \right),
\]

where the EXIT function of check node is defined as [27]

\[
I_{CV}(d_v) \approx 1 - J \left( \sqrt{d_v - 1} \cdot J^{-1}(1 - I_{VC}) \right).
\]

The density evolution of soft messages exchanged in the JDD process in terms of EXIT characteristics is summarized in Algorithm 3.

By running Algorithm 3, we trace the value of the average mutual information \( I_{VC} \) of output message at the bit node updated through iterations. If the value of \( I_{VC} \) reaches 1 through a sufficient number of iterations at a certain \( E_b/N_0 \), this implies that the JDD converges and the decoding succeeds at this \( E_b/N_0 \). The minimum value of \( E_b/N_0 \) at which \( I_{VC} \) converges to 1 is defined as the threshold.

Let us consider 3-D EXIT chart composed of two EXIT surfaces, each of which represents EXIT characteristics of bit node and check node, respectively. The space between two EXIT surfaces may or may not form a tunnel with an entry and an exit. If a tunnel is formed, JDD trajectory penetrates through the tunnel by 3-D zigzag movements and reaches the point of \( I_{VC} = 1 \). This situation corresponds to the convergence of JDD and a low BER is obtained. On the other hand, if a tunnel is not formed, JDD trajectory gets stuck at a point of \( I_{VC} < 1 \) resulting in high BER.

By a decoding iteration, the trajectory experiences a pair of orthogonal movements on the \( I_{VC}-I_{CV} \) plane. Fig. 4 shows an example of 3-D EXIT chart and some JDD trajectories obtained with different JDD strategies, which means the value of \( N_{\text{det}} : N_{\text{dec}} \) composing one global iteration. It is observed that using different JDD strategies result in distinct JDD trajectories. A JDD trajectory reaching the point of \( I_{VC} = 1 \) with a small number of zigzag movements implies a fast convergence of JDD. Since a tunnel between EXIT surfaces is uneven or bumpy, we need to design an efficient strategy for trajectory movement.

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and check nodes from the edge perspective are represented in check matrix construction. Degree distributions of bit nodes which are the degree distribution optimization and the edge In general, LDPC codes are designed through two steps, by using the proposed EXIT analysis tool so that the lowest strategies result in distinct convergence speeds.

We design LDPC codes for the MU massive MIMO system. The first one is the degree distribution optimization process as a criterion (c) is satisfied if the following condition

\[ \lambda(x) = \sum_{d_c=2}^{d_{c,\text{max}}} \lambda_{d_c} x^{d_c-1} \quad \text{and} \quad \rho(x) = \sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c} x^{d_c-1}, \]

respectively. Then, the code rate \( R \) is given by [23]

\[ R(\lambda, \rho) = 1 - \frac{\sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c}/d_c}{\sum_{d_c=2}^{d_{c,\text{max}}} \lambda_{d_c}/d_c}, \]

where \( \lambda = \{\lambda_2, \cdots, \lambda_{d_{c,\text{max}}}\} \) and \( \rho = \{\rho_2, \cdots, \rho_{d_{c,\text{max}}}\} \).

We determine degree distributions \( \lambda \) and \( \rho \) to maximize the code rate \( R(\lambda, \rho) \) guaranteeing the convergence of JDD at a given \( E_b/N_0 \) by using the EXIT analysis introduced in Sec. IV. For a given target code rate, we perform the EXIT analysis with various values of \( E_b/N_0 \) and find the smallest \( E_b/N_0 \) resulting in the maximum \( R(\lambda, \rho) \) exceeding the target code rate. Such \( E_b/N_0 \) is called the threshold and will be denoted by \( (E_b/N_0)^* \). The corresponding degree distributions are considered optimal and will be denoted by \( (\lambda^*, \rho^*) \).

Next, we place edges between bit nodes and check nodes in a factor graph based on \( (\lambda^*, \rho^*) \) to satisfy the following criteria [23]:

(a) Avoid short cycles involving only degree-2 bit nodes.
(b) Length-4 cycles need to be avoided.
(c) All degree-2 bit nodes need to represent only non-systematic bits.

The criterion (c) is satisfied if the following condition

\[ \lambda_2 \leq 2 \sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c}/d_c \quad (19) \]

is met [31]. Furthermore, the condition (19) can be incorporated in the degree distribution optimization process as a constraint [31]. The criteria (a) and (b) can be satisfied by placing edges between nodes based on the progressive edge growth (PEG) algorithm [36]. Then, the degree distribution for a given \( E_b/N_0 \) is determined as

\[
\begin{align*}
\max_{\lambda, \rho} & \quad R(\lambda, \rho) \\
\text{s.t.} & \quad \bar{I}_{VC} = 1 \text{ after running Algorithm 3,} \\
& \quad \lambda_2 \leq 2 \sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c}/d_c, \\
& \quad \sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c} = \sum_{d_c=2}^{d_{c,\text{max}}} \lambda_{d_c} = 1 \quad \text{with} \quad \rho_{d_c}, \lambda_{d_c} \geq 0, \\
& \quad \rho_{d_c}, \lambda_{d_c} \geq 0, \\
& \quad \sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c} = \sum_{d_c=2}^{d_{c,\text{max}}} \lambda_{d_c} = 1 \\
& \quad \rho_{d_c}, \lambda_{d_c} \geq 0, \\
& \quad \sum_{d_c=2}^{d_{c,\text{max}}} \rho_{d_c} = \sum_{d_c=2}^{d_{c,\text{max}}} \lambda_{d_c} = 1 \quad \text{with} \quad \rho_{d_c}, \lambda_{d_c} \geq 0, \\
& \quad \rho_{d_c}, \lambda_{d_c} \geq 0,
\end{align*}
\]

where the first constraint guarantees the convergence of JDD and the second constraint is used to satisfy the criterion (c) introduced earlier. The lowest \( E_b/N_0 \), at which the resultant maximum \( R(\lambda, \rho) \) exceeds the target rate, is considered the threshold \( (E_b/N_0)^* \) and the corresponding degree distributions are defined as \( (\lambda^*, \rho^*) \).
In addition, we take into consideration the convergence speed of JDD when designing LDPC codes. The fastest convergence of JDD algorithm can be achieved by selecting the optimal JDD strategy. The convergence speed is predicted by observing the evolution of $I_{VC}$ as shown in Fig. 5. The degree distribution achieving $(E_b/N_0)_*$ as well as the fastest JDD convergence will be considered the overall optimal and will be denoted as $(\lambda^*, \rho^*)$. Consequently, LDPC codes for MU massive MIMO system are constructed by the following procedure:

(i) perform the optimization process (20) for various $E_b/N_0$ and candidate JDD strategies.
(ii) determine $(E_b/N_0)_*$ and the optimal JDD strategy, and find the corresponding degree distribution $(\lambda^*, \rho^*)$.
(iii) construct the parity-check matrix of LDPC codes from $(\lambda^*, \rho^*)$ by using the PEG algorithm.

By following the above procedure, we can efficiently construct LDPC codes for MU massive MIMO system showing the lower threshold $(E_b/N_0)_*$, or equivalently the better error correcting capability, and the faster convergence of JDD.

### VI. NUMERICAL RESULTS

We consider LDPC coded MU massive MIMO systems with $n_U = 6$ or 10 and $n_R = 16$ or 64. We call the MU massive MIMO system with $n_U$ UE and $n_R$ BS antennas as $n_U \times n_R$ channel. We suppose the channel gain of each pair of UE and BS antenna is i.i.d. complex Gaussian with zero mean and unit variance, while additive noises at BS antennas are assumed to be i.i.d. zero-mean circular symmetric complex white Gaussian. Each UE encodes information bits to rate-1/2 or 3/4 LDPC codes and maps code bits to 4-QAM symbols by Gray-mapping.

We solve the optimization problem (20) for each $E_b/N_0$ and JDD strategy by using the differential evolution algorithm [37]. In degree distribution optimization, we use the concentrated check node degree distribution [24], i.e., $\rho(x) = \rho_d x^{d_c} + (1 - \rho_d) x^{d_c+1}$. For each massive MIMO channel and code rate under consideration, we find degree distributions $(\lambda^*, \rho^*)$ achieving the threshold $(E_b/N_0)_*$ by distinct JDD strategies. Among $(\lambda^*, \rho^*)$ found for various JDD strategies, we determine the overall optimal distribution $(\lambda^*, \rho^*)$ resulting in the fastest JDD convergence.

In Table 1 - Table 4, we list degree distributions $(\lambda^*, \rho^*)$ of LDPC codes for MU massive MIMO systems achieving the threshold with some candidate JDD strategies over $n_U \times n_R$ channels and code rates under consideration. By a practical reason, $\lambda_{d_c}$ and $\rho_{d_c}$ having negligible values are enforced to be null. In each table, the channel capacity and the threshold value obtained by EXIT analysis are also listed. It is found that for a given $n_U \times n_R$ channel, the same threshold is obtained irrespective of the choice of JDD strategy. This implies that equivalent error correcting capabilities are obtained by different JDD strategies if an infinite number of iterations are allowed, while different convergence speeds are obtained with distinct JDD strategies.

![FIGURE 6: Evolution of $I_{VC}$ with respect to the total number of local iterations for some JDD strategies, where $R = 1/2$, $n_U = 10$, $n_R = 16.$](image)

![FIGURE 7: Evolution of $I_{VC}$ with respect to the total number of local iterations for some JDD strategies, where $R = 3/4$, $n_U = 6$, $n_R = 16.$](image)

Fig. 5 - Fig. 8 show the evolutions of $I_{VC}$ with respect to the total number of local iterations obtained by various JDD strategies for given massive MIMO channel and code rate. Note that evolution curves are obtained by EXIT analysis for MU massive MIMO system equipped with LDPC codes whose degree distributions are listed in Table 1 - Table 4. It is obvious that distinct JDD strategies may result in different convergence speeds. We use boldface fonts to list values of $(\lambda^*, \rho^*)$ resulting in the threshold with the fastest JDD convergence speed for each massive MIMO channel and code rate in Table 1 - Table 4.

We construct parity-check matrices of LDPC codes with specific blocklengths $N$ for MU massive MIMO system with some candidate JDD strategies by applying the PEG algorithm [36] to the obtained degree distributions $(\lambda^*, \rho^*)$. 

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TABLE 1: Optimal degree distributions ($\lambda^* \rho^*$) of rate-1/2 LDPC codes for MU massive MIMO system with some candidate JDD strategies, where $(n_U \times n_R) = (6 \times 16)$ and $(6 \times 64)$ are considered, and $d_{v, \text{max}} = 24$ is used.

| $n_U \times n_R$ | 6 \times 16 | 6 \times 64 |
|------------------|-------------|------------|
| $N_{\text{det}} \times N_{\text{dec}}$ | 1:1 | 1:7 | 2:1 | 3:1 | 1:1 | 1:6 | 1:7 | 2:1 | 3:1 |
| $\lambda_2$ | 0.25766 | 0.25757 | 0.25685 | 0.25666 | 0.25655 | 0.25312 | 0.23019 | 0.23319 | 0.231 | 0.23196 | 0.22944 |
| $\lambda_3$ | 0.2013 | 0.20204 | 0.1999 | 0.20808 | 0.19995 | 0.2086 | 0.18313 | 0.16131 | 0.19338 | 0.16081 | 0.18372 | 0.17621 |
| $\lambda_4$ | 0.02069 | 0.00878 | 0.01077 | 0.00219 | 0.00784 | 0.00435 | 0.00032 | 0.00114 | 0.00254 | 0.004823 | 0.00524 |
| $\lambda_5$ | 0.00124 | 0.00575 | 0.01792 | 0.15288 | 0.22033 | 0.10628 | 0.13497 | 0.16214 | 0.18653 | 0.10778 | 0.16518 |
| $\lambda_6$ | 0.1795 | 0.0525 | 0.04517 | 0.0122 | 0.00247 | 0.13497 | 0.16214 | 0.18653 | 0.10778 | 0.16518 |
| $\lambda_7$ | 0.03416 | 0.09028 | 0.02464 | 0.10835 | 0.02645 | 0.01946 |
| $\lambda_9$ | 0.00771 | 0.03759 | 0.00978 | 0.04268 | 0.02593 | 0.00466 | 0.00667 | 0.00451 | 0.01232 |
| $\lambda_{10}$ | 0.01267 | 0.00183 | 0.00168 | 0.00709 | 0.00535 | 0.0051 |
| $\lambda_{11}$ | 0.00102 | 0.00119 | 0.00499 | 0.01388 | 0.00119 | 0.00604 | 0.00136 |
| $\lambda_{12}$ | 0.98402 | 0.00033 | 0.01192 | 0.97704 | 0.04823 | 0.05778 | 0.00479 |
| $\lambda_{13}$ | 0.000161 | 0.00416 | 0.00745 | 0.00024 | 0.00161 | 0.00071 |
| $\lambda_{14}$ | 0.95179 | 0.00181 | 0.00348 | 0.00363 | 0.00934 | 0.00288 |
| $\lambda_{15}$ | 0.01057 | 0.000232 | 0.000232 |
| $\lambda_{16}$ | 0.00128 | 0.000915 | 0.00363 | 0.00424 | 0.00090 |
| $\lambda_{17}$ | 0.00919 | 0.01374 | 0.01053 | 0.00411 |
| $\lambda_{18}$ | 0.0107 | 0.00224 |
| $\lambda_{19}$ | 0.00555 | 0.01999 | 0.0026 | 0.00782 |
| $\lambda_{20}$ | 0.0014 | 0.00317 | 0.00196 |
| $\lambda_{21}$ | 0.004048 | 0.04048 |
| $\lambda_{22}$ | 0.22699 | 0.23212 | 0.23027 | 0.2393 | 0.24028 | 0.29107 | 0.25326 | 0.25766 | 0.30095 | 0.2847 | 0.29396 |
| $\rho_{7}$ | 0.01947 | 0.03564 | 0.01598 | 0.00479 | 0.02296 | 0.00003 |
| $\rho_{8}$ | 0.98035 | 0.96486 | 0.96402 | 0.95521 | 0.97104 | 0.99997 | 0.436 | 0.95666 | 0.54993 | 0.42131 | 0.48965 | 0.38846 |
| $\rho_{9}$ | 0.541 | 0.60434 | 0.45007 | 0.57869 | 0.51037 | 0.61154 |

Capacity $-3.63$ [dB] $-9.99$ [dB] $-9.77$ [dB]

$\left(\rho_{9}/\rho_{7}\right)^*$

In Fig. 9, we plot the BER of MU massive MIMO system employing optimized LDPC codes with sufficiently large blocklength, i.e., $N = 64000$. It is observed that the threshold prediction based on EXIT analysis agrees quite well
TABLE 3: Optimal degree distributions ($\lambda^*$, $\rho^*$) of rate-3/4 LDPC codes for MU massive MIMO system with some candidate JDD strategies, where $(n_U \times n_R) = (6 \times 16)$ and $(6 \times 64)$ are considered, and $d_{v,\text{max}} = 20$ is used.

| $n_U \times n_R$ | 6 × 16 | 6 × 64 |
|------------------|--------|--------|
| $N_{\text{dec}}$ | 1 : 1 | 1 : 2 | 1 : 6 | 1 : 8 | 1 : 10 | 1 : 16 | 1 : 18 | 1 : 31 |
| $\lambda_2$     | 0.2513 | 0.25086 | 0.24799 | 0.25131 | 0.25126 | 0.25067 | 0.22195 | 0.22112 | 0.22072 | 0.22134 | 0.22115 | 0.22142 |
| $\lambda_3$     | 0.26405 | 0.25841 | 0.27338 | 0.25148 | 0.25333 | 0.26273 | 0.22886 | 0.24055 | 0.2413 | 0.24071 | 0.2369 | 0.23387 |
| $\lambda_4$     | 0.05149 | 0.02339 | 0.00004 | 0.04076 | 0.08351 | 0.01418 | 0.00016 | 0.00015 | 0.00015 | 0.00015 | 0.00015 | 0.00015 |
| $\lambda_5$     | 0.08901 | 0.01389 | 0.08153 |
| $\lambda_6$     | 0.12351 | 0.22142 | 0.20515 | 0.20359 | 0.11484 | 0.10959 | 0.03423 | 0.21368 |
| $\lambda_7$     | 0.03275 | 0.20663 | 0.12047 | 0.01264 | 0.26284 | 0.14833 | 0.00426 | 0.12453 | 0.09127 | 0.02975 |
| $\lambda_8$     | 0.20208 | 0.03968 | 0.06396 | 0.00723 | 0.08091 | 0.03487 | 0.08481 | 0.01279 |
| $\lambda_9$     | 0.06785 | 0.07289 | 0.01299 | 0.07982 | 0.0973 | 0.02632 | 0.09923 | 0.13317 |
| $\lambda_{10}$  | 0.05706 | 0.05111 | 0.03335 | 0.07036 | 0.03058 | 0.00005 | 0.00005 | 0.03233 |
| $\lambda_{11}$  | 0.01056 | 0.01389 | 0.00178 | 0.00054 | 0.0111 | 0.00128 | 0.02254 | 0.04086 |
| $\lambda_{12}$  | 0.02695 | 0.02488 | 0.00508 | 0.07574 | 0.04426 | 0.02562 | 0.04878 | 0.05029 | 0.00005 |
| $\lambda_{13}$  | 0.00079 | 0.00015 | 0.00101 | 0.00059 | 0.00054 | 0.00525 | 0.00031 | 0.00518 | 0.00092 | 0.00046 |
| $\lambda_{14}$  | 0.09921 | 0.99926 | 0.99985 | 0.99899 | 0.99941 | 0.99946 | 0.03255 | 0.11635 | 0.9997 | 0.99954 |

with the result of BER simulation. This observation verifies the practical effectiveness of the proposed EXIT analysis in predicting the performance of coded MU massive MIMO systems.

In Fig. 10 - Fig. 13, we plot BER performances of coded MU massive MIMO system with short to medium block-length whose LDPC codes are constructed by using ($\lambda^*$, $\rho^*$) listed in Table 1 - Table 4. In each figure, we plot BER curves obtained by three different total numbers of local iterations to show different JDD convergence speeds attained by distinct JDD strategies. It is observed that the convergence speed of JDD strategies can be predicted well by investigating the evolution of $I_{VC}$ obtained by the proposed EXIT analysis as depicted in Fig. 5 - Fig. 8. Using the optimal JDD strategy and overall optimal degree distribution ($\lambda^*$, $\rho^*$) results in the fastest convergence of JDD for each MIMO channel.
In this paper, we designed LDPC coded MU massive MIMO system equipped with an iterative JDD algorithm using the low-complexity FG-GAI BP detection. We defined a factor graph representation of the LDPC coded MU massive MIMO system and defined updating rules for messages flowing in and code rate. The lower BER can be obtained for a given total number of local iterations if the optimal JDD strategy and the corresponding degree distributions are used in the construction of LDPC codes.

In Fig. 14 and Fig. 15, we compare BER performances of the LDPC coded MU massive MIMO systems designed by the proposed scheme and conventional schemes. As conventional schemes, we consider the MU massive MIMO system employing 802.16e LDPC codes [38] and JDD strategy \( N_{\text{det}} : N_{\text{dec}} = 1 : 1 \) as well as the MU massive MIMO system using MMSE-PIC detection [18] with JDD strategy \( N_{\text{det}} : N_{\text{dec}} = 1 : 1 \) and correspondingly designed optimal LDPC codes. It is observed that the MU massive MIMO system designed as proposed outperforms those designed in conventional manners. The coding gain of the proposed system at BER of \( 10^{-5} \) after convergence with \( N = 2304 \), \( R = 1/2 \) over \( 6 \times 16 \) MIMO channel is about 0.2 dB over 802.16e LDPC based system and about 1.6 dB over MMSE-PIC based system. The coding gain of the proposed system at BER of \( 10^{-5} \) after convergence with \( N = 4608 \), \( R = 3/4 \) over \( 6 \times 64 \) MIMO channel is about 0.3 dB over 802.16e LDPC based system and about 0.8 dB over MMSE-PIC based system.

**VII. CONCLUSION**

In this paper, we designed LDPC coded MU massive MIMO system and code rate. The lower BER can be obtained for a given total number of local iterations if the optimal JDD strategy and the corresponding degree distributions are used in the construction of LDPC codes.

In Fig. 14 and Fig. 15, we compare BER performances of the LDPC coded MU massive MIMO systems designed by the proposed scheme and conventional schemes. As conventional schemes, we consider the MU massive MIMO system employing 802.16e LDPC codes [38] and JDD strategy \( N_{\text{det}} : N_{\text{dec}} = 1 : 1 \) as well as the MU massive MIMO system using MMSE-PIC detection [18] with JDD strategy \( N_{\text{det}} : N_{\text{dec}} = 1 : 1 \) and correspondingly designed optimal LDPC codes. It is observed that the MU massive MIMO system designed as proposed outperforms those designed in conventional manners. The coding gain of the proposed system at BER of \( 10^{-5} \) after convergence with \( N = 2304 \), \( R = 1/2 \) over \( 6 \times 16 \) MIMO channel is about 0.2 dB over 802.16e LDPC based system and about 1.6 dB over MMSE-PIC based system. The coding gain of the proposed system at BER of \( 10^{-5} \) after convergence with \( N = 4608 \), \( R = 3/4 \) over \( 6 \times 64 \) MIMO channel is about 0.3 dB over 802.16e LDPC based system and about 0.8 dB over MMSE-PIC based system.
The BER performance over conventional schemes.

Based on the EXIT analysis, we designed the behavior of iterative JDD algorithm of coded massive MIMO receiver. Based on the EXIT analysis, we designed jointly irregular LDPC codes through the optimization of degree distributions and the JDD strategy to achieve the lowest BER and the fastest JDD convergence. The coded MU massive MIMO system equipped with the proposed LDPC codes and the proposed JDD strategy results in the improved BER performance over conventional schemes.

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