Reconfigurable Controller Design in Descriptor Systems Obtained from Second Order Dynamic Systems via State Feedback Eigenstructure Assignment

Amir Parviz Valadbeygi1,* and Vahid Pourgharibshahi1

1 Department of Mechanical Engineering Tiran Branch, Islamic Azad University, Tiran, Iran

* Corresponding author E-mail: amir.p.valad@gmail.com

Received 12 Jun 2012; Accepted 26 Jul 2012

DOI: 10.5772/51894

© 2012 Valadbeygi and Pourgharibshahi; licensee InTech. This is an open access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract this paper proposes a new method for designing a reconfigurable controller for descriptor systems which is obtained from the second order dynamics of a robot control situation. Using mathematical tools the equations of robot control situation have been translated to descriptor systems, thus, after this change the controller has been designed for control performances. The proposed reconfigurable controller can recover the nominal closed-loop performances after fault occurrence in the system. The dynamics of descriptor systems contain infinite and finite elements, so a complete response of descriptor systems can be represented by an eigenstructure which involves finite and infinite elements. In this paper eigenstructure assignment is used to design a reconfigurable controller in general such that the reconfigured system can recover the complete response of a nominal system as much as possible. Finally, an example represents the effectiveness of the new method.

Keywords Descriptor systems, Eigenstructure Assignment, Reconfigurable Controller

1. Introduction

Reconfigurable control system is a system that is capable of dealing with large variations in the system being controlled by means of adjusting or modifying the nominal control law. The stability and performance of the original closed-loop system is maintained as much as possible using the reconfigurable control systems. The reconfigurable control law must provide stability and excellent performance under conditions of failure and damage, as well as during normal operation [1]. In recent years, reconfigurable control has attracted much research attention and many new approaches have been proposed. In [2], the linear quadratic regulator method has been considered. In addition [2], pseudo inverse method [3], feedback linearization[4], the Lyapunov method [5], sliding mode control [6] are all considered, in [10] a reconfigurable controller design via output feedback in the case of post order fault is represented. [11] considers a modified approach that guarantees the stability of the
closed-loop system by using an appropriate Lyapunov equation - all examples of approaches for designing reconfigurable control.

The eigenstructure assignment is one of the most powerful tools for control systems. According to the fact that the response of the system can be stated based on eigenvalues and corresponding eigenvectors, in this paper the eigenstructure assignment method is considered.

We know that the second order dynamic system is one of the most important systems in the dynamics of robot manipulators [12]. It is well known that these systems can be translated as descriptor systems by changes in variables. Designing a reconfigurable controller in these systems is important because fault occurrence in these systems may disturb the appropriate performances. Designing a reconfigurable controller in a descriptor system and high order dynamics system has been considered in some research. In [7], designing a reconfigurable controller in a second order dynamic system via p-d feedback eigenstructure assignment is considered. By changing variables in the second order system, this system is translated to a descriptor system. Based on finite eigenvalues and corresponding eigenvectors, the reconfigurable system has been designed. However, we know that the complete response of the descriptor system [8] consists of finite and infinite eigenstructures. So this method does not give any information about infinite eigenstructures.

The major contribution of this paper is to develop using eigenstructure assignment in second order dynamic systems in general case. In the proposed method there isn’t any constraint on eigenvalues and eigenvectors of system, while in the previous work [7] eigenvalues are distinct. The proposed method can be used for descriptor systems which are important in network and other systems. According to the fact that the complete response of descriptor systems involves finite eigenstructure and infinite eigenstructure, the proposed method considers both finite eigenstructure and infinite eigenstructure. In this paper, a new method is suggested for the design of a reconfigurable controller in descriptor systems using finite and infinite eigenstructure assignment. Based on parametric eigenstructure assignment by state feedback in the descriptor system [9], the reconfigurable controller is designed.

This paper is organized as follows: Section 2 describes the parametric approach. In section 3 the main problem is represented. Section 4 demonstrates the effectiveness of the proposed method.

2. Parametric Eigenstructure Assignment

Consider the second order dynamic system

\[ E \dot{q} - A \dot{q} - C q = B u \]

(1)

Where \( q \in R^n \) and \( u \in R^r \) are state and input vectors respectively. Suppose that \( \text{rank}(E_i) = n_i, \text{rank}(B_i) = r_i \). Consider the following transformation

\[ E = \begin{bmatrix} I & 0 \\ 0 & E_i \end{bmatrix}, A = \begin{bmatrix} 0 & I \\ C_i & E_i \end{bmatrix}, B = \begin{bmatrix} 0 \\ B_i \end{bmatrix}, x = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} \]

(2)

By applying the above transformation the following descriptor system is obtained

\[ E \dot{x} = Ax + Bu \]
\[ y = Cx \]

(3)

Where \( x \in R^n, u \in R^r, n = 2n_i, r = 2r_i \) are respectively state vector and control vector \( E, A, B \) and \( C \) are the real matrices with appropriate dimensions, such that the closed-loop finite eigenvalues \( \lambda_i \) and \( (i=1,...,m') \), infinite eigenvalues and corresponding eigenvectors are designed to satisfy the special control specifications. Note that the closed system \((E, A+BK)\) is regular, i.e., \( \text{det}(sE-A-BK) \) is not identically zero [9] where \( \text{rank}(E) = m \leq n, \text{rank}(B) = r \), also suppose that, (1) is controllable, i.e., \( \text{rank} = [sE-A, B] = n \) for all \( s \in C \).

Assume the following state feedback which is design to satisfy special performances

\[ u = kx \]

(4)

Suppose that due to the fault occurrence in the system, the dynamics of the nominal system (3) is changed. The new model of the faulty system is described as follows

\[ E_f \dot{x}_f = A_f x_f + B_f u_f \]

(5)

\[ y_f(t) = C_f x_f \]

Where \( A_f, E_f, B_f \) are real matrices with appropriate dimension and \( x_f \in R^n, u_f \in R^r, y_f \in R^r \). Also \( \text{rank}(E_f) = m \leq n, \text{rank}(B_f) = r \) and the faulty system is controllable. Our objective is to design a new state feedback
such that the performances of the nominal system are recovered. The closed-loop system is given by

$$ E_f \dot{x}_f = (A_f + B_f K_f) x_f $$

In order to solve this problem, first we describe the parametric eigenstructure assignment by state feedback which is presented in [9].

Let $\theta = \{ \lambda_1^f, \lambda_2^f, ..., \lambda_m^f \}$, where $\theta$ is the set of finite eigenvalues of the closed-loop system $(E_f, A_f + B_f K_f)$. The eigenvalue $\lambda_i^f$ has the algebraic multiplicity $m_i$ and a geometric multiplicity $q_i$. The Jordan canonical form $J_f$ of $(E_f, A_f + B_f K_f)$ has $\sum_i q_i$ Jordan blocks. The order of Jordan block corresponds to $\lambda_i^f$ is $p_j (j = 1, 2, ..., q_i)$. According to the above definition the following equation is satisfied [9]

$$ P_{11} + P_{12} + ... + P_{iq} = m_i $$

Also $m_1 + m_2 + ... m_m = m$ according to definition of eigenvalues and eigenvector in descriptor system, the following equations are satisfied

$$ (A_f + B_f K_f - \lambda_i^f E_f) g_{ij} = E g_{ij}^{k-1} g_{ij}^0 = 0 $$

$$ k = 1, 2, ..., p_j, \; j = 1, 2, ..., q_i $$

$$ i = 1, 2, ..., m' $$

Where $g_{ij}^0$ and $\lambda_i^f$ are eigenvalue and corresponding eigenvector of faulty system. Based on [9], (9) can be written as

$$ A_f G_f + B_f M_f = E_f V_f $$

$$ M_f = K G_f $$

Where

$$ G_f = [G_1, G_2, ..., G_m] $$

$$ G_i = [G_{i1}, G_{i2}, ..., G_{iq}] \quad m_i^k = k_f g_{ij}^k $$

By applying singular value decomposition to the matrix $[A_f - \lambda_i^f E_f \quad B_f]$, the following equation is obtained

$$ P_f(\lambda_i^f)[A_f - \lambda_i^f E_f \quad B] = [\psi_f(\lambda_i^f) \quad 0] $$

Where $P_f(\lambda_i^f) \in C^{n \times n}$ and $Q_f(\lambda_i^f) \in C^{(n+m)(n+m)}$ are orthogonal matrices, and $\psi_f(\lambda_i^f)$ is a non singular diagonal matrix, such that the diagonal elements are singular values of $[A_f - \lambda_i^f E_f \quad B_f]$. The parametric form of eigenstructure assignment is

$$ g_{ij}^k = \begin{bmatrix} N_i(\lambda_i^f) & & \\ D_i(\lambda_i^f) & & \\ & ... & \\ & & D_i(\lambda_i^f) \\ & & & N_i(\lambda_i^f) \\ & & & D_i(\lambda_i^f) \\ & & & & & & \end{bmatrix} \begin{bmatrix} h_{ij}^k \\ h_{ij}^{k-1} \\ \vdots \\ h_{ij}^1 \\ h_{ij}^0 \end{bmatrix} $$

$$ k = 1, 2, ..., p_j, \; j = 1, 2, ..., q_i, \; i = 1, 2, ..., m' $$

Where $h_{ij}^k \in C^r$ are parameter vectors and $N_i(\lambda_i^f)$, $D_i(\lambda_i^f)$ are obtained as follows

$$ N_i(\lambda_i^f) = (Q_{i1}(\lambda_i^f))^{-1}(\lambda_i^f) P_f(\lambda_i^f) E_f (Q_{i2}(\lambda_i^f))^{-1} Q_{i2}(\lambda_i^f) $$

$$ D_i(\lambda_i^f) = \begin{bmatrix} Q_{i1}(\lambda_i^f) & & \\ (Q_{i2}(\lambda_i^f))^{-1} P_f(\lambda_i^f) E_f (Q_{i3}(\lambda_i^f))^{-1} P_f(\lambda_i^f) E_f \end{bmatrix} $$
Assume that the infinite eigenvalue of 
\((E_f, A_f + B_f K_f)\) is shown by \(\lambda_{\infty}^f\), so \(s_{\infty}^f = \frac{1}{\lambda_{\infty}^f} = 0\)
is the infinite eigenvalue of \((A_f + B_f K_f, E_f)\). According to definition of eigenstructure assignment, we have 
\[(E^f - s_{\infty}^f (A^f + B^f K^f) j) g_{sj} = 0 \quad j = 1, 2, ..., (n-m)\]  
(15)

Note that the algebraic and geometric multiplicities are \(n-m\). Corresponding eigenvector of \(S_{\infty}\) is \(g_{sj}\).

Consider 
\[m_{sj} = k_f g_{sj} \quad j = 1, 2, ..., n-m\]  
(16)

Based on the results of [9], the parametric infinite eigenvector is

\[
\begin{bmatrix}
g_{sj} \\
m_{sj}
\end{bmatrix} = \begin{bmatrix}
N_{\infty}^f \\
D_{\infty}^f
\end{bmatrix} h_{sj} \quad j = 1, 2, ..., n-m
\]  
(17)

Where \(N_{\infty}^f, D_{\infty}^f\) are obtained from the following equations

\[E_f S_{\infty}^f = 0 \quad \text{rank}(s_{\infty}^f) = n-m\]  
(18)

\[
\begin{bmatrix}
N_{\infty}^f \\
D_{\infty}^f
\end{bmatrix} = \begin{bmatrix}
S_{\infty} \\
0
\end{bmatrix} \quad I_f
\]

Define the matrix \(G_{\infty}^f = [g_{s1}, g_{s2}, ..., g_{s,n-m}]\) and 
\(M_{\infty} = [m_{s1}, m_{s2}, ..., m_{s,n-m}]\)

\[m_{sj} = k g_{sj} \quad j = 1, 2, ..., n-m\]  
(19)

Similar to (10), we have

\[M_{\infty} = K_f G_{\infty}\]  
(20)

By combining (10) and (20) the following equation is obtained

\[
\begin{bmatrix}
M \\
M_{\infty}
\end{bmatrix} = K_{\infty} \begin{bmatrix}
G \\
G_{\infty}
\end{bmatrix}
\]  
(21)

If \(\text{det}[V_{\infty} V_{\infty}] \neq 0\) then \(K_{\infty}\) is obtained as follows

\[k_f = [M_{\infty}][G_{\infty}]^{-1}\]  
(22)

In order to calculate the matrix \(K_f\), the following constraints must be satisfied

1. \(g_{0f}^\ell \in R^r\) for real eigenvalue \(\lambda_{\infty}^f\), whereas \(g_{0f}^\ell = \overline{g_{0f}^k} \in C^r\) for complex conjugate of eigenvalues \(\lambda_{\infty}^f, \lambda_{\infty}^k\).
2. \(\text{det}[G_{\infty}] \neq 0\)
3. \(\text{det}[F_{\infty} h_{\infty1}, F_{\infty} h_{\infty2}, ..., F_{\infty} h_{\infty}] \neq 0\)

Where \(T_{\infty}\) is the left eigenvector matrix of the closed-loop system associated with the finite closed-loop eigenvalue \(s_{\infty}\).

\[T_{\infty} E_f = 0 \quad \text{rank}(T_{\infty}^f) = n-m.\]  
(23)

3. Main problem

Based on [8], the closed-loop response of descriptor systems (1) using state feedback and in terms of eigenstructure assignment can be stated as follows

\[x_f(t) = x_{\text{initial}}^f + x_{\text{external}}^f(t)\]

\[x_{\text{initial}}^f(t) = Ge^{J_f t} T_f^T E_f x(0)\]

\[x_{\text{external}}^f = -Ge((T_{\infty}^f)^T A_{\infty}^f (G_{\infty})^{-1}(T_{\infty}^f)^T B_f u_f^\ell) + \int_0^t Ge^{J_f(t-\tau)} B_f u_f^\ell d\tau\]  
(24)

Where \(T_f\) and \(T_{\infty}\) are left eigenvector matrices of the closed-loop system associated with the finite closed-loop eigenvalues \(\lambda_{\infty}^f\) and left eigenvector matrices of the closed-loop system associated with the finite closed-loop eigenvalue \(s_{\infty}\) respectively, such that

\[T_{\infty}^T E_f G = I\]  
(25)

The complete response of closed-loop system can be rewritten as follows

\[x_f^f(t) = x_{\text{fin}}^f + x_{\text{inf}}^f\]

\[x_{\text{fin}}^f = Ge^{J_f T_f^T E_f x(0)} + \int_0^t Ge^{J_f(t-\tau)} B_f u_f^\ell d\tau\]  
(26)

\[x_{\text{inf}}^f = -Ge((T_{\infty}^f)^T A_{\infty}^f (G_{\infty})^{-1}(T_{\infty}^f)^T B_f u_f^\ell\]
Clearly, this response consists of two parts, the finite part \( X_{fin}^f \) that related to finite eigenstructure and the infinite part \( X_{inf}^f \) related to infinite eigenstructures.

According to the above discussions, similarly the closed-loop response of the nominal system contains two parts \( X_{inf}^r , X_{fin}^r \). Based on (24) and (26), it is clear that the response of the system is related to its eigenstructure, so in order to recover nominal performance, the state feedback controller \( K_f \) must be designed such that the behaviour of the faulty system with this controller is closed to nominal system much as possible. The reconfiguration objective can be translated to the following equations

\[
\lambda_i = \lambda_i^f \\
\min[(g_{ij}^f - v_i^f)^T (g_{ij}^f - v_i^f)] \\
\min[(g_{i,j}^r - v_i^r)^T (g_{i,j}^r - v_i^r)]
\]

(27)

Where \( \lambda_i \) is eigenvalue of the nominal system and \( v_i^f \) is corresponding eigenvector. By substituting the parametric form of \( g_{ij}^k \) and \( g_{jxc} \) from (12) and (17), the above equations change to

\[
J_{fin} = \min[(N_1^f (\lambda_i^f) h_{ij}^f + N_2^f (\lambda_i^f) h_{ij}^{k-1} + ... + N_k^f (\lambda_i^f) h_{ij}^f - v_{ij}^f - v_{ij}^{k-1} - ... - v_{ij}^1)]^T
\]

(28)

\[
J_{inf} = \min[(N_1^r (\lambda_i^r) h_{jxc} - v_{i,j}^r)^T (N_1^r h_{jxc} - v_{i,j}^r)]
\]

(29)

Where \( V_{ij}^k \) are eigenvectors of closed-loop of nominal system. Note that the parametric form of \( V_{ij}^k \) obtains similar to \( g_{ij}^k \). In order to solve the above problem, taking gradient with respect to \( h_{ij}^k \), \( h_{jxc} \) and then set to zero we have

\[
\frac{\partial J_{fin}}{\partial h_{ij}^k} = 0 , \quad \frac{\partial J_{inf}}{\partial h_{jxc}} = 0
\]

(30)

In order to describe the method, consider \( k = 2 \) so the \( J_{fin} \) is stated as follows

\[
J_{fin} = \min[(N_1^f (\lambda_i^f) h_{ij}^f + N_2^f (\lambda_i^f) h_{ij}^{1} - v_{ij}^f - v_{ij}^{1})]^T
\]

(28)

\[
[(N_1^r (\lambda_i^r) h_{jxc} + N_2^r (\lambda_i^r) h_{jxc} - v_{ij}^r - v_{ij}^{1})]
\]

(29)

According to (30)

\[
\frac{\partial J_{fin}}{\partial h_{ij}^1} = 0
\]

(31)

Taking the above gradient and then by simplifying the results, we have

\[
\left( (N_2^r (\lambda_i^f))^T (N_1^f (\lambda_i^f)) h_{ij}^f + (N_2^r (\lambda_i^f))^T (N_2^r (\lambda_i^f)) h_{ij}^f \right) v_{ij}^f + \left( (N_1^f (\lambda_i^f))^T (N_1^f (\lambda_i^f)) h_{ij}^f \right) v_{ij}^2
\]

(32)

\[
\left( (N_1^r (\lambda_i^r))^T (N_1^r (\lambda_i^r)) h_{jxc} + (N_1^r (\lambda_i^r))^T (N_1^r (\lambda_i^r)) h_{jxc} \right) v_{ij}^f + \left( (N_1^r (\lambda_i^r))^T (N_1^r (\lambda_i^r)) h_{jxc} \right) v_{ij}^2
\]

(33)

The parametric eigenstructure of distinct eigenvalues is computed as

\[
g_{i1}^f = N_1^f (\lambda_i^f) h_{i1}^f \\
g_{i,j}^r = N_1^r h_{i,j}^r
\]

(34)

By substituting (35) in (28) and then take gradient the solution is obtained as

\[
h_{i1}^f = (N_1^f (\lambda_i^f))^T N_1^f (\lambda_i^f) h_{i1}^f)^T (N_1^f (\lambda_i^f))^T v_{i1}^f
\]

(35)

Where \( V_{ij}^1 \) is eigenvector of faulty system in distinct case.
One of the most important specifications that should be recovered by the reconfigured system is steady state response of the nominal systems. The following theorem states how a feedforward matrix should be designed to minimize the difference between the after fault and pre-fault steady state response.

Theorem 1: For the faulty system (5), consider the control law

$$ u_f(t) = K_f x_f + LR_f(t) \quad (37) $$

Where $L \in R^{mxm}$ is feedforward matrix. The steady state response of the nominal system to step input can be recovered if $R$ is selected as

$$ L = (R^T R)^{-1} R^T H \quad (38) $$

Where

$$ H = C(A + BK)^{-1} B $$
$$ R = C_f (A_f + B_f K_f)^{-1} B_f $$

Proof: The closed-loop systems of nominal and faulty systems are given by

$$ \begin{cases} 
E \dot{x}(t) = (A + BK)x(t) + Br(t) \\
y = Cx(t)
\end{cases}, u(t) = Kx(t) + r(t) \quad (40) $$

$$ \begin{cases} 
E_f \dot{x}_f(t) = (A_f + B_f K_f)x_f(t) + B_f Lr(t) \\
y_f = C_f x_f(t)
\end{cases} \quad (41) $$

By applying Laplace transformation to (40) and (41), we have

$$ \begin{cases} 
sEX(s) = (A + BK)X(s) \\
X(s) = (Es - (A + BK))^{-1} BR(s)
\end{cases} \quad (42) $$

$$ \begin{cases} 
sE_f X_f(s) = (A_f + B_f K_f)X_f(s) + B_f LR(s) \\
X_f(s) = (E_f s - (A_f + B_f K_f))^{-1} B_f LR(s)
\end{cases} \quad (43) $$

In the above equation suppose that $(E, A + BK)$ and $(E_f, A_f + B_f K_f)$ are regular. The steady state output of the nominal closed-loop system to a unit step input is given by

$$ y_s = \lim_{s \to 0} s[C(sE - (A + BK))^{-1} B \frac{1}{s}] = -C(A + BK)^{-1} B \quad (44) $$

In addition, for the faulty system

$$ y_s' = \lim_{s \to 0} s[C_f(sE_f - (A_f + B_f K_f))^{-1} B_f L \frac{1}{s}] = -C_f(A_f + B_f K_f)^{-1} B_f L \quad (45) $$

So for recovering of the steady state response of the nominal system we have to design $L$ such that

$$ \min \|v_s - y_s\| \rightarrow \min \|H - RL\| = (H - RL)^T (H - RL) \quad (46) $$

The solution is described in (38).

4. Example

Consider the following fifth order descriptor [9]

$$ E = \begin{bmatrix} 0 & 0 & 0 & 1.72 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
1.55 & 0 & 0 & 0 & 0 \end{bmatrix} , \quad A = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\
0 & 1.56 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1.01 & 0 & 0 \end{bmatrix} $$

According to [9], considering the assignment of the following closed-loop eigenstructure

$$ \theta = \{-0.5, -1, -2\} , \quad m = 3, m_i = q_i = 1, i = 1, 2, 3 $$

So the corresponding parameters are as follows

$$ T_\infty = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{T}, \quad S_\infty = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \end{bmatrix} $$

$$ N_\infty = \begin{bmatrix} S_\infty & 0_{3 \times 3} \end{bmatrix} , \quad D_\infty = \begin{bmatrix} 0_{3 \times 2} & I_3 \end{bmatrix} $$

$$ v_{s1} = N_\infty f_{s1} , \quad v_{s2} = N_\infty f_{s2} $$

$$ w_{s1} = D_\infty f_{s1} , \quad w_{s2} = D_\infty f_{s2} $$

$$ N_1(-0.5) = \begin{bmatrix} 0.2347 & -0.1417 & -0.8465 \\
-0.0282 & 0.0371 & -0.3022 \\
-0.6584 & 0.0070 & -0.1263 \\
0.0361 & -0.0475 & 0.3866 \\
0.0969 & 0.9683 & -0.0780 \end{bmatrix} $$
Consider due to the fault the dynamics of the system are changed as follows

\[
E_f = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
A_f = \begin{bmatrix}
0 & 1.1 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 1 & 1 \\
1 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
B_f = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 \\
\end{bmatrix}
\]

According to the proposed method in this work the reconfigured controller is obtained as follows. The following controller is computed based on (…14)

\[
N_f^T (-0.5) = \begin{bmatrix}
-0.1497 & -0.1497 & -0.9175 \\
-0.0139 & 0.0916 & -0.1167 \\
-0.6486 & 0.6420 & -0.0196 \\
0.0306 & -0.2016 & 0.2568 \\
0.6851 & 0.6329 & -0.1265 \\
\end{bmatrix},
N_f^T (-1) = \begin{bmatrix}
-0.3210 & -0.1240 & -0.9386 \\
0.0988 & 0.2695 & -0.0696 \\
-0.5905 & 0.6805 & 0.1121 \\
-0.1087 & -0.2965 & 0.0763 \\
0.5852 & 0.4504 & -0.2595 \\
\end{bmatrix},
N_f^T (-2) = \begin{bmatrix}
-0.4026 & 0.1872 & -0.6749 \\
0.5156 & 0.4361 & 0.1662 \\
-0.4294 & 0.6581 & 0.4391 \\
-0.2836 & -0.2399 & -0.0914 \\
0.2170 & 0.2801 & -0.4378 \\
\end{bmatrix}
\]

\[
K_f = \begin{bmatrix}
-0.1083 & 0 & -0.7715 & 0 & 0.5683 \\
-0.0243 & 0 & 0.0125 & 0 & 0.4852 \\
0.2061 & 1.0000 & 0.6340 & 0 & -0.7391 \\
\end{bmatrix}
\]

Because the eigenvalues are distinct, \( K_f \) is computed based on 34. So we can simulate the response of this system to the initial conditions \( x(0) = [1 \ 1 \ 0.9 \ 1 \ 0.9]^T \) as follows
In these pictures dashed lines relate to the reconfigured system, which is simulated by using the proposed controller $K_f$, and the solid lines relate to the nominal system. From the above figures it can be seen that the proposed reconfigurable control system can recover the nominal system response.

5. Conclusion

An eigenstructure assignment–based method is suggested to design a reconfigurable controller for descriptor systems via state feedback. The controller can be reconfigured to compensate for the effect of change in the system dynamics. This work deals with the control reconfiguration in the general case where the complete response of systems includes the infinite and finite eigenstructure. Finally, an example demonstrates the effectiveness of the proposed method via simulation.

6. References

[1] A. Esna Ashari and A. Khaki Sedigh (2005) “Output Feedback Reconfigurable Controller Design Using Eignestructure Assignment: Post Order Change.” Int. Conference on Control and Automation, pp. 474-479.
[2] D.P. Looze, J.L. Weiss, J.S .Eterna and N.M. Barret (1985) “An Automatic redesign approach for restructurable control systems,” IEEE Control System Magazine, 5(2), pp. 16-22.
[3] Z. Gao and P.J. Antsaklis (1991) “Stability of pseudo-inverse method for reconfigurable control systems,” Int. Journal of Control, 53(2), pp. 717-729.
[4] Y.Ochi and K.Kanai (1991) “Design of restructurable flight control systems using feedback linearization.” Journal of Guidance, Control and Dynamics, 14(5), pp.903-911.
[5] P.C. Parks (1996) “Lyapunov Redesign of model reference adaptive control systems.” IEEE Trans. Automatic Control, 11(3), pp.362-367.
[6] A. Esna. Ashari, M. J. Yazdanpanah and A. Khaki Sedigh (2005) “Reconfigurable sliding mode control design using genetic algorithm and eigenstructure assignment”. IEEE Int. Conf. on control Automation, pp.38-43.

[7] G. S. Wang, B. Liang, G. R. Duan (2005) “Reconfiguring second-order dynamics systems via P-D feedback Eigenstructure Assignment: A Parametric method”, Int. Journal of Control, Automation, and Systems, 3(1), pp. 109-116, 2005.

[8] G. R. Duan (2009), “Analysis and design of descriptor linear systems” Springer, p.515.

[9] B. Zhang (2008), “Parametric eigenstructure assignment by state feedback in descriptor systems” IET Control Theory Appl. 2(4), pp. 303-309.

[10] A. Esna Ashari, A. Khakisedigh and M.J. Yazdanpanah (2005) “Output feedback reconfigurable design using eigenstructure assignment: post-order fault order change “ in Proc. IEEE Int. Conf. on Control Automation, pp.474-479.

[11] K. Konstantopoulos and P. J. Antaklis (1996) “Eigenstructure assignment in reconfigurable control systems.” Technical Report Interdisciplinary Studies of Intelligent Systems

[12] J.J. Slotine and W. Li (1991) “Applied Nonlinear Control”, Englewood Cliffs, Prentice-Hall.