Detection of a Schrödinger’s Cat State in an rf-SQUID

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We present experimental evidence for a coherent superposition of macroscopically distinct flux states in an rf-SQUID. When the external flux $\Phi_x$ applied to the SQUID is near 1/2 of a flux quantum $\Phi_0$, the SQUID has two nearly degenerate configurations: the zero- and one-fluxoid states, corresponding to a few microamperes of current flowing clockwise or counterclockwise, respectively. The system is modeled as a particle in a double-well potential with equal well depths and a sufficiently high barrier, the system has a set of quantized energy levels localized in each well. The relative energies of these levels can be varied with $\Phi_x$. External microwaves are used to pump the system from the well-localized ground state of one well into one of a pair of excited states nearer the top of the barrier. We spectroscopically map out the energy of these levels in the neighborhood of their degeneracy point by varying $\Phi_x$ as well as the barrier height. We find a splitting between the two states at this point, when both states are below the classical energy barrier, indicating that the system attains a coherent superposition of flux basis states that are macroscopically distinct in that their mean fluxes differ by more than 1/4 $\Phi_0$ and their currents differ by several microamperes.

In 1935, Schrödinger attempted to demonstrate the limitations of quantum mechanics through a thought experiment in which a cat is put in a quantum superposition of alive and dead states. [1] The idea remained an academic curiosity until the early 1980s when Leggett and co-workers [2–4] proposed that under suitable conditions a macroscopic object with many microscopic degrees of freedom could behave quantum mechanically provided that it was sufficiently decoupled from its environment. While much progress has been made in demonstrating the macroscopic quantum behaviour of various systems such as superconductors, [5–9] nanoscale magnets, [10–12] laser-cooled trapped ions, [13] photons in a microwave cavity [14] and C$_60$ molecules, [15] heretofore there has been no experimental demonstration of a quantum superposition of truly macroscopically distinct states. Here we present the first experimental evidence that a superconducting quantum interference device (SQUID) can be put into a superposition of two magnetic-flux states, one corresponding to a few microamperes of current flowing clockwise, the other corresponding to the same amount of current flowing counterclockwise.

The basic rf-SQUID is a superconducting loop of inductance $L$ broken by a Josephson tunnel junction with capacitance $C$ and critical current $I_c$. Classically, when an external magnetic flux $\Phi_x$ applied to the SQUID, a superconducting (dissipationless) current will flow around the loop to screen out the external flux. When the external flux reaches a certain critical value (i.e. when the screening current reaches $I_c$), a flux quantum $\Phi_0$ will enter the loop and the SQUID will again be in a stable state. Quantum-mechanically, however, flux can tunnel into or out of the SQUID before this critical value is reached. The dynamics (both classical and quantum) of the SQUID are analogous to that of a particle of mass $C$ moving in a double-well potential given by

$$U = U_0 \left[ \frac{1}{2}(\varphi - \varphi_x)^2 + \beta_L \cos(\varphi) \right],$$

where $\varphi$ and $\varphi_x$ are the flux through the SQUID loop and the external flux, respectively, in units of $\Phi_0/2\pi$ and measured with respect to $\Phi_0/2$ ; $U_0 \equiv \Phi_0^2/4\pi^2 L$ and $\beta_L \equiv 2\pi L I_c/\Phi_0$ characterize the energy barrier between flux states. This potential is shown in Figure 1a, where the left well corresponds to zero $\Phi_0$ in the SQUID and the right well one $\Phi_0$. The system has energy levels localized in each well. When the external flux is exactly $\Phi_0/2$ ($\varphi_x = 0$), the potential is symmetric. Any additional external flux tilts the potential, as shown in the figure. At various values of external flux, levels in opposite wells will align, giving rise to resonant tunnelling between flux states in the SQUID. [5] However, until now, there has been no evidence that the tunnelling process was coherent, that is, that the SQUID could be put into a superposition of the two flux states.

Such a superposition would manifest itself in an anticrossing, as illustrated in Figure 1b, where the energy level diagram of two levels of different flux states (labelled $|0\rangle$ and $|1\rangle$) is shown in the neighbourhood in which they would become degenerate without coherent interaction (dashed lines). Coherent tunnelling lifts the degeneracy (solid lines) so that at the degeneracy point the energy eigenstates are close to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, the symmetric and antisymmetric superpositions.

The SQUID used in the experiments is made up of two Nb/AIOx/Nb tunnel junctions in parallel, as shown in Fig. 1c; this essentially acts as a tuneable junction in which $I_c$ can be adjusted with an applied flux $\Phi_x$. Thus, with $\Phi_x$ we control the tilt $\varepsilon$ of the potential in Figure 1a, while with $\Phi_x$ we control $\Delta U_0$, the height of the energy barrier at $\varepsilon = 0$. The flux state of our sample is measured by a separate dc-SQUID magnetometer induc-
tively coupled to the sample. The sample is encased by a PdAu shield that screens it from unwanted radiation; a coaxial cable entering the shield allows the application of controlled external microwaves. The set up is carefully filtered and shielded, as described elsewhere.\[5,6\] and cooled to 40 mK in a dilution refrigerator.

In our experiments, we probe the anticrossing of two excited levels in the potential by using microwaves to produce photon-assisted tunnelling. Figure 1a depicts this process for the case where the levels \(|0\rangle\) and \(|1\rangle\) are each localized in opposite wells. The system is initially placed in the lowest state in the left well (labelled \(|i\rangle\)) with the barrier high enough such that the tunnelling rate is small. Microwave radiation is then applied. When the energy difference between the initial state and an excited state matches the radiation frequency, the system has an appreciable probability of making a transition to the right well, which can be detected by the magnetometer. Figure 2 shows schematically the photon-assisted process when the excited levels have an anticrossing. Here the energy of the relevant levels is plotted as a function of tilt \(\varepsilon\). The dashed line represents level \(|i\rangle\) shifted upward by the energy of the microwaves. At values of \(\varepsilon\) for which this line intersects one of the excited levels (indicated by the arrows), the system can absorb a photon and make an interwell transition. When the barrier is reduced, the excited levels move to lower energy (dotted lines in the figure) relative to \(|i\rangle\) and photon absorption occurs at different values of \(\varepsilon\). For a fixed frequency, we can map out the anticrossing by progressively reducing the barrier and thus moving the levels through the dashed photon line.

We use pulsed microwaves to excite the upper levels. Before each pulse, the system is prepared in state \(|i\rangle\) and the values of \(\Delta U_0\) and \(\varepsilon\) are set. Millisecond pulses of 96-GHz microwave radiation at a fixed power are applied and the probability of making a transition is measured. The experiment is repeated for various values of \(\varepsilon\) and \(\Delta U_0\). Data from these measurements is shown in Figure 3, where the probability of making a photon-assisted interwell transition is plotted as function of \(\Phi_2\). Each curve for a given \(\Delta U_0\) is shifted vertically for clarity. Two sets of peaks are clearly seen. As \(\Delta U_0\) is decreased, these peaks move closer together and then move apart again. For \(\Delta U_0 = 9.117\) K (red curve), one peak is clearly higher than the other. Here the right peak roughly corresponds to level \(|0\rangle\), which is localized is the same well as \(|i\rangle\). The matrix element for photon absorption is larger for the \(|i\rangle \rightarrow |0\rangle\) transition than for the \(|i\rangle \rightarrow |1\rangle\) transition, resulting in the asymmetry between the peaks. When \(\Delta U_0\) is decreased to 8.956 K (green curve), the peaks move closer together and the asymmetry disappears. The two peaks now approximately correspond to the coherent superpositions of the \(|0\rangle\) and \(|i\rangle\) states; that is, \(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\) and \(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\). As the barrier is decreased further (8.797 K - violet curve), the peaks move apart again and the asymmetry reappears, now with the left peak being larger. The two levels have thus passed through the anticrossing, changing roles without actually intersecting. The inset of the figure shows the positions of the peaks in the main figure (as well as others peaks) in the \(\Delta U_0 - \Phi_2\) plane. Two anticrossings are clearly seen. The red lines are the results of a calculation and the violet line represents the position of the classical energy barrier relative to state \(|i\rangle\). The position of this line relative to the data indicates that both anticrossings correspond to levels that are below the barrier and thus represent the superposition of macroscopically distinct states with mean fluxes differing by about \(\frac{1}{4} \Phi_0\).

The peak positions in Figure 3 can also be used to calculate the level energies. For one of the anticrossings, Figure 4 shows the energy of the levels relative to the mean energy of \(|0\rangle\) and \(|1\rangle\) as a function of \(\varepsilon\). The similarity of the data to Figure 1b is now manifest. The figure also shows the results of a calculation of the energy levels. At the middle of the anticrossing, the two levels are separated by \(\approx 0.1\) K in energy and the upper level is \(\approx 0.15\) K below the top of the classical energy barrier. There are three parameters used for the calculations presented in Figures 3 and 4: \(L, L/C, \beta_L\), all of which can be independently determined from measurements of classical phenomena or incoherent resonant tunnelling in the absence of radiation. From independent measurements, we find \(L = 240\pm15\) pH, \(L/C = 2300\pm10\) \(\Omega^2\) and \(\beta_L = 2.33\pm0.01\). The values used in the calculation that yielded the best agreement with the data are L = 238 pH, L/C = 2300 \(\Omega^2\) and \(\beta_L = 2.35\), all in good agreement with the independently determined values.

In closing, we would like to stress two related points regarding these results. The first is that at the anticrossing both levels are below the top of the classical energy barrier. This fact is essential for the system to be in a superposition of macroscopically distinct states since the levels can only be associated with one well (one flux state) if they are below the top of the barrier. The second point concerns the meaning of “macroscopic”. The SQUID exhibits macroscopic quantum behaviour in two senses: 1) The quantum dynamics of the SQUID is determined by the flux through the loop, a collective coordinate representing the motion of \(10^9\) Cooper pairs acting in tandem. Since the experimental temperature is 1000 times smaller than the superconducting energy gap, almost all microscopic degrees of freedom are frozen out and only the collective flux coordinate retains any dynamical relevance. 2) The two classical states that we find to be superposed are macroscopically distinct. We calculate that for the anticrossings measured, the states \(|0\rangle\) and \(|1\rangle\) differ in flux by more than \(\frac{1}{4} \Phi_0\) and differ in current by 2-3 microamperes. Given the SQUID’s geometry, this corresponds to a local magnetic moment of \(10^{10}\) \(\mu_B\), a truly macroscopic moment.

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FIG. 1. a SQUID potential. The left well corresponds to the zero-flux state of the SQUID and the right well to the one-flux-quantum state. Energy levels are localized in each well. Both the tilt ε and energy barrier ΔU₀ can be varied in situ in the experiments. The process of photon-induced interwell transitions is illustrated by the arrows, where the system is excited out of the initial state |i⟩ and into one of two excited states |0⟩ or |1⟩. b Schematic anticrossing. When the two states |0⟩ and |1⟩ would classically become degenerate, the degeneracy is lifted and the states of the system are the symmetric and antisymmetric superpositions of the classical states: 1/√2(|0⟩ + |1⟩) and 1/√2(|0⟩ − |1⟩). c Experimental set-up. Our SQUID contains a “tuneable junction”, a small dc-SQUID. A flux Φₓ applied to this small loop tunes the barrier height ΔU₀. Another flux Φ_y tunes the tilt ε of the potential. A separate SQUID acts as a magnetometer, measuring the flux state of the sample.

FIG. 2. Illustration of experimental procedure. The applied microwaves boost the system out of the initial state |i⟩, bringing it virtually to the dashed line. At certain values of ε for which this line intersects one of the excited states (indicated by the arrows), a photon is absorbed and the system has a large probability of making an interwell transition. When the energy barrier ΔU₀ is reduced, the levels move down relative to |i⟩ (dotted lines) and the values of ε at which photon absorption occurs changes.

FIG. 3. Experimental data. The main figure shows the probability of making an interwell transition when a millisecond pulse of 96-GHz microwave radiation is applied as a function of Φ_y. For clarity, each curve is shifted vertically by 0.3 for each excited state of the system (indicated by the arrows), a photon is absorbed and the system has a large probability of making an interwell transition. When the energy barrier ΔU₀ is reduced, the levels move down relative to |i⟩ (dotted lines) and the values of ε at which photon absorption occurs changes.

FIG. 4. Energy of measured peaks relative to the calculated mean of the two levels as a function of ε. At the midpoint of the figure, the measured splitting between the two states in this anticrossing is ~0.1 K and the upper level is ~0.15 K below the top of the classical energy barrier. Calculated energy levels are also shown.
