New Method for Taking into Account Radiative Events in the Inclusive Experiment MOLLER

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Abstract. A new method for taking into account of radiative events in inclusive polarized experiments that, in principle, makes no use of an unphysical separation into soft- and hard-photon regions is demonstrated. A procedure for taking into account bremsstrahlung in the MOLLER forthcoming ultraprecise experiment at the Thomas Jefferson Laboratory is developed on the basis of this method.

1. Introduction
Inclusive experiments play a role of paramount importance in high-energy physics. In a number of cases, this is because it is impossible in principle to detect all particles produced in the reaction being considered; in other cases (including deep-inelastic lepton scattering on nucleons, the Drell–Yan reaction, and identical-particle scattering), this is not needed for deducing information about the reaction mechanism. In almost any of the present-day experiments, reliable information can be obtained from experimental data only upon taking into account electroweak radiative corrections, which are higher order effects in the Standard Model. An infrared divergence is a well-known problem encountered in taking into account contributions of extra virtual particles to observables. A path toward solving this problem goes through incorporating the contribution of real bremsstrahlung photons into respective calculations. If bremsstrahlung photons are not detected in the reaction being considered, the problem of taking them into account should be addressed in dealing with inclusive reactions.

A new method (N-method) for precisely taking into account the emission of one bremsstrahlung photon as a component of one-loop radiative corrections is formulated in the present study by considering the example of observables in the inclusive Møller scattering reaction

\[ e^-(k_1) + e^-(p_1) \rightarrow e^-(k_2) + e^-(p_2) + (\gamma(k)) \] (1)

involving polarized electrons [1] (here in brackets the 4-momenta of particles are presented).

The nonradiative and radiative process of polarized Møller scattering was fully described in [2] and [3], correspondingly, and the notation used there is reproduced in the present study where, we place emphasis on the method used to calculate the reaction phase space, which originally has the form

\[ d\Phi = \delta^4(k_1 + p_1 - k_2 - p_2 - k) \frac{d^3p_2}{2p_{20}} \frac{d^3k_2}{2k_{20}} \frac{d^3k}{2k_0}. \] (2)
2. Phase space

In [2], the phase space (2) was reduced to the form presented in [4], which involves a dependence on four (radiative) invariant quantities; that is,

\[ d\Phi = \frac{\pi}{16s\sqrt{-\Delta_4}}dtvdvdv_1, \]  

where \( \Delta_4 = \Delta_4(k_1, p_1, k_2, p_2) \) is Gram determinant. Although the expression for the phase space looks quite simple and admits a clear interpretation in the respective computer code, the phase-space in (3) presents a difficult challenge in integration. This served as a motivation of searches for new ways to improve the accuracy.

To improve the convergence we transform the phase space (2) in the reference frame comoving with the center-of-mass of primary particles, where \( k_1 + p_1 = 0, \ k_{10} = p_{10} = \sqrt{s}/2 \). First of all, we remove the three-dimensional integral with respect to \( d^3p_2 \), whereupon we have

\[ d\Phi = \delta(\sqrt{s} - k_{20} - p_{20} - k_0) \frac{1}{2p_{20}} \frac{d^3k_2}{2k_{20} 2k_0}. \]  

We then go over to the system of spherical coordinates,

\[ d^3k_2 = |k_2|^2d|k_2|d\cos \theta_{k_2} d\varphi_{k_2} \]

and apply the relation \( |k_2|d|k_2| = k_{20}dk_{20} \).

It is now necessary to remove the integral with respect to \( k_{20} \) with the aid of the remaining delta function by using the property \( \delta[f(x)] = \delta(x - x_0)/|f'(x_0)| \), where \( x_0 \) is root of \( f(x_0) = 0 \). In the case being considered, the function \( f(x) \) is the lefthand side of the energy-conservation law specified in the implicit form

\[ f(x) = k_{20} + p_{20} + k_0 - \sqrt{s}, \]  

where \( \sqrt{s} \) is the total reaction energy and \( x \equiv k_{20} \). One can readily evaluate the derivative

\[ f'(x) = 1 + \frac{x(1 - |k|A(x^2 - m^2)^{-1/2})}{\sqrt{x^2 - 2|k|A\sqrt{x^2 - m^2} + |k|^2}}. \]  

In Fig. 1, we depict the 3-momentum vectors of the final-state particles in the c.m. frame by using the auxiliary vector \( p_5 = -k \). The possible energies of the final-state electron can be found by resolving the energy-conservation law \( f(x_0^\pm) = 0 \). The result has the form

\[ x_0^\pm = \frac{BC \pm \sqrt{C^2 + m^2(1 - B^2)}}{1 - B^2}, \]  

(7)
where the coefficients
\[ B = \frac{\sqrt{s} - k_0}{A|k|}, \quad C = \frac{|k|^2 - (\sqrt{s} - k_0)^2}{2A|k|} \] (8)
are expressed in terms of the factor
\[ A = \cos(p_5, k_2) = \sin \theta \sin \theta_5 \cos \varphi_5 + \cos \theta \cos \theta_5. \] (9)
This factor specifies that sign on the right-side of Eq. (7) which corresponds to the energy:

at a positive value of \( A \) (in this situation the 3-momentum vector of the emitted photon forms
an obtuse angle with respect to the vector \( k_2 \)) we choose \( x_0^- \), and vice versa. With allowance
for the foregoing, we ultimately obtain the phase space in the form
\[ d\Phi = \frac{|k_2|}{4p_{20}|f'(x_0)|} d\cos \theta_{k_2} d\varphi_{k_2} \frac{d^3k}{2k_0}, \] (10)
where \( d\cos \theta_{k_2} \equiv dc \). Integration with respect to the azimuthal angle \( \varphi_{k_2} \) yields \( 2\pi \) because of
symmetry under rotations of the system about the beam axis. Further, we proceed to perform
integration with respect to \( k \). We have
\[ d^3k = |k|^2d|k|d\cos \theta_{k}d\varphi_{k}, \quad \theta_k = \pi - \theta_5, \quad \varphi_k = \pi + \varphi_5. \]

Upon employing \( p_5 \) the 3-momentum vectors of final-state particles in the c.m. frame assume a
rather simple form; that is,
\[ k_2 = (|k_2| \sin \theta, 0, |k_2| \cos \theta), \]
\[ p_5 = (|k| \sin \theta_5 \cos \varphi_5, |k| \sin \theta_5 \sin \varphi_5, |k| \cos \theta_5), \]
\[ p_2 = p_5 - k_2. \] (11)

It is clear that \( p_{20} = \sqrt{m^2 + |p_2|^2} \) and is readily calculable on the basis of (11).

We will now express all radiative invariants in terms of the proton energy and azimuthal and
polar angles. We have [2]:
\[ z_1 = 2k_0k_{10} + 2|k||k_1| \cos \theta_5, \] \[ v_1 = 2k_0p_{10} - 2|k||p_1| \cos \theta_5, \]
\[ z = 2k_0k_2 + 2|k||k_2|A, \]
\[ v = 2k_0(\sqrt{s} - k_{20}) - 2|k||k_2|A. \] (12) (13) (14) (15)

It is noteworthy that, in fact, \( k_0 \) and \( |k| \) are different in all of the equations used. In other
words, the photon mass \( \lambda \) is retained in these expressions. In the following, this mass will be
used as an infinitesimal parameter to regularize infrared divergences.

3. Integration over the total phase space

Everything is now ready for performing integration over the total bremsstrahlung-photon phase
space. By employing the expressions obtained for the phase space, we remove the integral with
respect to the detected-electron azimuthal angle \( \varphi_{k_2} \), whereupon the R-contribution to the cross
section assumes the form
\[ \sigma^R = \frac{d\sigma^R}{dc} = -\frac{\alpha^2}{4\pi s} \int_0^{k_{\text{max}}} \frac{|k_2|^2}{k_0} \frac{d|k|}{k} \int_{-1}^1 d\cos \theta_k \int_{0}^{2\pi} \frac{d\varphi_k}{p_{20}|f'(x_0)|} \sum_{i,j=\gamma,Z} MM^+. \] (16)
Even in this form, the relevant numerical integration (we will employ the VEGAS Monte Carlo integrator [5]) involves no problems if one pays due attention to the condensation of points in the region of small $|k|$. However, it is possible to represent the result in a more elegant form. First, we proceed to consider the integral with respect to the photon energy by employing the equality $|k|d|k| = k_0 dk_0$. We then have

$$\int_0^{|k|_{\text{max}}} \frac{|k|^2}{k_0} d|k| = \int_0^\Omega |k|dk_0...,$$

where $\Omega$ is the maximum (chosen) bremsstrahlung photon energy. By applying the change of variable (a similar change of variable was used, for example, in [6] to isolate the dependence of the cross section on the minimal photon energy)

$$k_0 = \lambda^{1-x} \Omega^x, \quad \frac{dk_0}{k_0} = \ln \frac{\Omega}{\lambda} dx,$$

we obtain the cross section in the form

$$\sigma^R = -\frac{\alpha^3}{4\pi^8} \ln \frac{\Omega}{\lambda} \int_0^1 dx |k| k_0 \int_{-1}^1 d\cos \theta_k \int_0^{2\pi} d\varphi_k \frac{|k_2|_{P20}|j'(x_0)|}{|f'(x_0)|} \sum_{i,j=\gamma,Z} MM^+.$$

4. Conclusion

In order to obtain reliable data in the MOLLER experiment with a declared relative error below 0.01, not only is it necessary to take precisely into account one-loop and at least leading two-loop radiative effects, but also an exact interpretation of events involving bremsstrahlung-photon emission with allowance for the detector potential is mandatory. The new N-method supplemented with a respective FORTRAN code developed on the basis of this method solves successfully the problem in question. It is the opinion of the author that the N-method may become a standard tool for calculations of the above type, since, by construction, it does not involve unphysical parameters, with the exception of a fictitious photon mass, which cancels the infrared divergence in a natural way. From the methodological point of view, the new method in question is advantageous in the sense that it makes no use of terms such as a “soft photon”, a “hard photon”, and a “parameter that separates the soft and hard regions”, so that a cumbersome and essentially immaterial test for the absence of a dependence on this parameter is not required. One should also bear in mind that the calculation for the region of soft photons within a traditional approach is approximate, inevitably leading to a poorly controllable computational error, but that a calculation on the basis of the N-method involves no such error.

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