The Flow Crisis and an Inner Source of Heating in the Vortex Tube

M Kh Pravdina, I K Kabardin, V I Polyakova, M R Gordienko and N I Yavorsky
Kutateladze Institute of Thermophysics, SB RAS, Novosibirsk 630090, Russia
E-mail: pmargarita@mail.ru

Abstract. The analysis of current complex experimental investigation of the vortex tube with square cross section is made focusing on the criticality of the inner flow in the framework of the flow crisis concept. The criticality condition is proved to take place in the vortex tube by operating with LDA obtained velocity experimental data. As the measured axial velocity exceeds the calculated velocity of centrifugal elastic waves on the boundary between the longitudinal circulation zone and the near wall swirling jet, two scenarios are supposed possible for the transition of the supercritical state to a subcritical one, similar to the vortex breakdown in a swirling jet at its discharge into the quiet liquid volume: the mild oscillations or a series of hydraulic jumps become possible, leading to a flow pattern choke change with the transformation to heat of excess kinetic energy.

1. Introduction
Recently there has been a new rise of interest in the Ranque effect due to the development of noncontact or low disturbing flow investigation methods. The previous lack of detailed experimental data on the inner flow pattern in fact stopped the attempts to explain the physical mechanism of the effect. Still the novel experimental data based on noncontact velocity measurements inside the vortex tubes give hope to change the situation.

In recent works [1-5] the detailed experimental investigation was presented for the flow in a vortex tube with fixed construction parameters as its regime modes were widely changed.

![Figure 1. a) The schematic of the vortex tube with square cross section.](image1.png)

![Figure 1. b) The orthogonal coordinates.](image2.png)

The tube chosen for the investigation was identical to the tube with square cross section in which the helical global structures near the hot exit were previously visualized [6]. The schematic of the tube
is shown in Fig.1 a). The square side of the working channel cross section was $D=34$ mm, the channel length being 390 mm. The vortex chamber for the air input had a two-slit guiding apparatus and two orifices in its top covers: one of 34 mm diameter was adjusted to the working channel, and the other of 10 mm diameter served for passing of the cold exit pipe. The inner diameter of the guiding apparatus was 67 mm, and the total area of slits made 112 mm$^2$. The hot exit was arranged as a radial diffuser with 1.5 mm clearance. It was packed inside a buffer volume connected to the atmosphere by a valve that could regulate the cold to total flow ratio $\mu$ from 0.2 to 0.8. The ratio of the inlet to atmosphere pressures, i.e., the expansion ratio $\pi=P_{in}/P_a$ was varied in experiments from 2 to 7. The orthogonal coordinates are shown in Fig.1 b). The used LDA experimental facilities were investigated and manufactured in the Kutateladze Institute of Thermophysics, Siberian branch of Russian Academy of science. The LAD-07 unit measured two projections of the velocity vector with a relative error of no more than 0.5%. The size of the intersection area was 0.1x0.1x0.5 (mm). The coordinate spacer moved the measurement unit with an accuracy of 0.1 mm. No less than 500 particles were registered to average the tool response in each measuring volume. The measurements of the axial and circumferential velocity profiles were provided with especially small steps of 10 mm along the vortex tube and of 1-2 mm in transverse direction. The velocity measurements made in $X=0$ plane are further interpreted in the more accustomed polar coordinates with substitution of the absolute values of coordinate $Y$ and velocity component $V_X$ by the radius $r$ and azimuthal velocity $V_A$.

The fact that all the data obtained belonged to one certain tube enabled the authors of [1-5] to find out the flow features never noticed before. The data presented in [2] for $\mu=0.26$ indicated that the flow structure (namely, velocity profiles, the reverse flow radius $r_0(Z)$, the radius of the near wall swirling jet boundary $r_1(Z)$) is weakly dependent on expansion ratio, not only qualitatively but also quantitatively. The radius $r_0(Z)$ was defined as the modulus of $Y$ coordinate where the azimuthal velocity had its extremum value. The reverse flow radius $r_0$ was defined as $Y$ modulus at which the longitudinal velocity $V_Z$ was zero.

Further experiments (also at $\mu=0.26$) have shown [5] that at increasing of pressure drop after the threshold value of expansion ratio $\pi=5$ was achieved, the quantitative changes of structure parameters were no more indicated. In Fig. 2 the wall swirling jet radius $r_1(Z)$ and the reverse flow radius $r_0(Z)$ are shown at $\pi=6$, which is greater than the prementioned stabilization threshold. The azimuthal and the axial velocities at $r=r_1-V_{Z1}(Z)$ and $V_{Z1}(Z)$, are shown in Fig.3.

![Figure 2](image_url)

**Figure 2.** The reverse flow radius $r_0$ and the radius of the wall swirling jet boundary $r_1$ along the tube at $\pi=6$, $\mu=0.26$. 
Figure 3. Azimuthal $V_A$ and axial $V_Z$ velocities at $r=r_1$ and $\pi=6$, $\mu=0.26$.

Note, that at the position of $Z=70…75$ mm the values $r_1$ and $V_Z(Z)$ become noticeably oscillating.

The explanation of the stabilization feature may be given using the results of the analysis made in [3-4], where the subsonic velocity blocking was proved for the entrance slits of guiding apparatus (see Fig.4). The phenomenon is that at any $\mu$ value at the rise of expansion ratio $\pi$, the threshold for the vortex tube is achieved ($\pi=5$ for the investigated one): the velocity coefficient $\lambda(\mu) = v/a_*$ stops changing as the gas density continues growing ($v$ is the velocity in the inlet slits, and $a_*$ is the critical sound velocity of the incoming flow). The value $\lambda(\mu)$ also represents the volumetric flow rate through the tube, normalized by the total slits area and the critical sound velocity.

Again, this new fact has to be explained. It is not yet obvious that just the sound velocity had to be used for normalizing the tube volume flow rate to make it a similitude parameter. Nevertheless, the temperature change in the cold exit of the tube, normalized by the temperature loss at adiabatic expansion, $\eta=\Delta T_C/\Delta T_S$, is also unchangeable at the expansion coefficient greater than $\pi=5$ (see Fig.5).

Figure 4. The velocity coefficient or the normalized volumetric flow rate $\lambda(\mu)$ through the tube for $\pi=5–7$.

Figure 5. The cooling coefficient $\eta$ (temperature change in the cold exit normalized by the temperature loss at adiabatic expansion) for $\pi=5–7$.

2. The criticality concept and the flow crisis in the Ranque tube

The existence of the axial velocity limit for the swirling flow in a long tube is known in hydrodynamics as the flow crisis [7-11]. It has been established that when the open swirling flow of incompressible liquid propagates along an infinite tube the velocity of centrifugal elastic waves propagating along its free surface is

$$C = \sqrt{\frac{V_A(R^2-r_1^2)}{2r_1^2}}$$

(1)

where $V_A$ is the azimuthal velocity at free surface radius $r_1$, which is also called the whirl radius, and $R$ is the tube radius. The axial velocity $V_Z$ of the viscous fluid at $r=r_1$, cannot continually overcome the value of $C$, which means that at $V_Z=C$ a criticality or so-called crisis of flow occurs. The viscous flow would slow down while $V_Z$ is greater than $C$, and it would be accelerated if $V_Z$ is less than $C$ [7].
On reaching the critical value $C$ at some tube position the viscous flow becomes pulsing, that is the instability development [7, 10].

Another scenario is claimed in [9, 13] for the turbulent flow in a centrifugal liquid nozzle. If the axial velocity at the nozzle entrance is higher than $C$, the hydraulic jump is possible inside the nozzle tube with chock rearrangement of the flow structure, so that in the new flow pattern the axial velocity becomes less than $C$ with the conservation of momentum and a loss of kinetic energy. This scenario was also described in [10] for the vortex breakdown in the leading-edge vortex and observed in [11] for the submerged vortex breakdown in a swirling jet that was discharged into a quiet media.

Comparing these flows with the vortex tube one, it is fair to assume that in the latter there is instability, similar either to a viscous flow crisis in a long tube or to a hydraulic jump in a centrifugal nozzle. Obviously, there is no free surface in the vortex tube flow. Thus, it should be compared to the submerged flow in a gas nozzle rather than to an open liquid flow with a free surface. Note, that if the cold exit is closed ($\mu=0$) the vortex tube flow is similar to the well-known submerged flow in a centrifugal nozzle with a central longitudinal circulation zone. Fortunately, it is proved for the gas vortex chambers or centrifugal gas nozzles [8-9, 12-13] that the near wall swirling jet boundary radius is the same as that for the liquid nozzle, depending purely on the geometry parameters of the tube and of the input. The rest part of the nozzle tube at $0<r<r_1$ is occupied by a secondary gas flow forming a longitudinal circulation zone which is weekly admixed with the main swirling jet flow near the wall. In [8] a series of experiments is presented for a water flow in the long vortex chamber with partly or totally submerged circulation zone. In all the cases, the visualization of the circulation zone radius by air bubbles show that its size is the same as the wall swirling jet radius in the open liquid flow with the boundary radius also depending purely on geometry parameters. Experimentally the circulation zone boundary radius may be estimated as the radius where the circumferential velocity is maximal.

The flow pattern is schematically shown in Fig. 6, where the area I is the swirling jet propagating near the wall from the entrance to the hot exit, II is the part of reverse flow that reaches the cold exit and III is the elongated torus similar to a circulation zone surrounding the part of reverse flow that achieves the cold exit. A simplified pattern is predicted for a laminar flow in [14] and a similar pattern was recently presented in [15] for the water flow in a Ranque vortex tube.

![Figure 6](image-url)

**Figure 6.** The scheme for the vortex tube inner flow for $0<r<34\text{mm}$ at $\mu=0.26$.

I – the near wall swirling jet, II – the part of central reverse flow, that reaches the cold exit, III (dashed) – the secondary flow (circulation zone).

Fig.7 shows the ratio of the axial velocity $V_{Z1}$ at the circulation zone boundary radius $r_1$ to the critical velocity $C$ calculated by the formula (1), where $R$ is the radius of the circle inscribed into the square cross section of the vortex tube. At $Z=70-75\text{ mm}$ a pulsar mode is established that can be seen up to $Z=210\text{ mm}$. The data witnesses for the flow crisis and corresponding instability in the vortex tube. This picture should be compared to the temperature evolution along the axes shown in Fig.8, where the results of [1] near the wall are chosen at $\pi=6$. It is hard to escape a conclusion that the instability development is just the reason for the accelerated temperature rise in the area $80\text{mm}<Z<210\text{mm}$ along the tube.
Figure 7. The axial velocity at the circulation zone radius $V_{Z1}$ normalized by the critical velocity $C$ at $\pi=6$ and $\mu=0.26$.

Figure 8. The temperature evolution along the $Z$ axes near the wall at $\pi=6$ and $\mu=0.3$.

3. Discussion and conclusion
The question still comes up if the elastic waves or just hydraulic jumps are possible in the fluid with no free surface. It should be noticed that the solitary waves with recirculation zones have been evaluated inside the rotating flows [16]. Another example was mentioned above for the submerged swirling jet discharged into a quite liquid [11] where the two scenarios predicted in [10] were observed depending on the swirl level: mild transitions among two possible flow states or hydraulic jumps with essential kinetic energy loss.

The detailed analysis of the vortex tube flow structure is still limited by a rather small value of the cold flow ratio $\mu=0.25-0.3$. In this case the cooling makes the main contribution to the thermal separation and the flow crisis with its pulsing mode probably is a reasonable additional source of heat at the wall region. A more powerful source of heat near the wall may be needed in the case of greater values of $\mu$, when the heating is governing the separation. The measurements are planned for the future and a series of hydraulic jumps rather than mild oscillations in the tube is expected.

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