Lens-axicon separation to tailor aberration free focused Bessel-Gaussian beams in the paraxial regime

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Abstract: Lens-axicon doublets have been used to produce Bessel-Gaussian beams, a narrow non-diffracting beam of relatively constant width. One problem of using Bessel-Gaussian beams is that there is a compromise between achieving a long effective focal length with a small central core radius and distributing the beam intensity between the central core and the off-axis rings. Here, we explore the advantage of tuning the lens-axicon separation, which allows us to have an additional degree of freedom to tailor the beam profile. Moreover, the separation between the lens and the axicon reduces the spherical aberrations in the beam profile, which can then be modeled within the paraxial regime. We study the detrimental effects of the spherical aberrations and provide several options to minimize them. We examine both sharp and shallow axicons used in combination with different converging lenses. We perform a series of detailed experiments to image the structure of the beam through the Bessel region. The spatial light distribution of the lens-axicon system is analyzed by using high dynamic range imaging and complemented with consistent theoretical calculations within the paraxial regime.

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1. Introduction

The use of Bessel beams has opened a way of overcoming traditional problems in diffractive optics. In order to sustain those thin beams, they must be surrounded by, in principle, an infinite set of rings, which also provide their self-reconstructing nature [1, 2]. Although ideal Bessel beams, much as plane waves, cannot be implemented, good approximations can be generated which present a similar behavior at finite propagating distances. They are commonly produced by using a Gaussian beam as a transversal envelope to the ideal Bessel beam, so that the transversal beam intensity can remain finite [3]. As such, these approximated Bessel beams are known as quasi-Bessel beams or Bessel-Gaussian beams [4]. A common way to generate Bessel-Gaussian beams is with an axicon, although it can also be engineered using a range of experimental techniques including diffractive optics, computer generated holograms, spatial light modulators (SLMs) [5] and other optical elements [6]. Roughly, the design of the Bessel-Gaussian beam can be controlled by the apex angle of the axicon that accounts for the width of the central spot, while the Gaussian beam width controls the axial length where the beam can be well approximated by a Bessel function. Therefore, by controlling the apex angle of an axicon and the input Gaussian beam diameter, it is possible to tailor the focal beam profile of the Bessel-Gaussian beam.

One practical problem of the Bessel-Gaussian beam is due to its inherent structure; a Bessel beam consists of a central core and many concentric rings, \( N \), where the central core contributes to the total power, \( P_{\text{total}} \), in the same way as each of the rings [7–9]. Then, when engineering Bessel-Gaussian beams, one has to balance three competing parameters: the central core radius,
\[ \Delta r, \text{ the diffraction-free working distance, } z_{\text{max}}, \text{ and the power of the central core } P_{\text{core}}: \]

\[
N \approx \frac{z_{\text{max}}}{2\Delta r}, \quad P_{\text{core}} \approx \frac{P_{\text{total}}}{N}.
\]  

(1)

In this way a sharp angle axicon can produce a tight focal spot with a confined central core but the diffraction-free working distance or Bessel zone becomes short. On the other hand, a shallow angle axicon can be used to produce long effective Bessel zone with the consequence of a large central core width. Increasing the incoming Gaussian beam diameter, will increase the Bessel zone, however the number of concentric rings also increases proportionally resulting in a reduced intensity in the central core. Additionally, in the implementation of Bessel-Gaussian beams the role of aberrations can completely distort the final result. All these elements are essential in any practical implementation of Bessel beams, but are particularly important in those applications where the power of the central core and the light field distribution have to be precisely controlled, such as optical trapping [10, 11], non-linear optics [12] and photo-patterning [5, 13].

In order to manipulate the peak intensity and the Bessel zone length, lens-axicon doublets have been proposed. A more complex combination using a lens-axicon doublet with a second axicon was also proposed in order to obtain infinitely long Bessel beams with central core waist of the order of hundreds of micrometers and above without spherical aberrations [14, 15]. However, due to spherical aberrations appearing in the Bessel region [16] there are few studies on tightly confined Bessel beam cores on the order of a micrometer. Moreover, the predominant study of these beams was on the far field ring that Bessel beams produce [16–19]. 4f telescope systems after an axicon have been used to produce magnified versions of Bessel-Gaussian beams, reducing the Bessel beam core diameter and Bessel zone length by the factors \((\frac{f_1}{f_2})\) and \((\frac{f_1}{f_2})^2\), respectively where \(f_1\) and \(f_2\) are the focal lengths of the telescope lenses [20]. This system is equivalent to a single axicon with angle \(\alpha_2 \approx \alpha_1 \frac{f_2}{f_1}\) and therefore offers the same trade-offs described in Eq. (1) without allowing for variation in the axial beam profile [21]. Using only a single lens separated from an axicon has been investigated for a number of reason; The advantage of tailoring the diameter of the far field ring by changing the distance [19]; The behaviour in the Bessel region experimentally for high power lasers using long focal length lenses and shallow axicons to avoid spherical aberrations [22]; and also enabling non-linear harmonic generation by optimizing the phase matching condition [23, 24]. In this work, we present an analytical scalar model that accounts for the separation between the lens and the axicon for generating aberration free focused Bessel-Gaussian beams. By controlling the separation between the lens and the axicon we show that it is possible to engineer tightly confined focal regions with various tight focal spot sizes and increase the central core intensities while maintaining longer effective Bessel zone. In particular, it allows for more symmetrical axial beam profiles, which are of interest for applications with counter-propagating beams where the intensities of the two beams need to mode matched along a large focal region, such as optical trapping and two-photon non-linear processes. The introduction of the lens-axicon separation allows for the use of longer focal length lenses thereby, essentially achieving similar results than with shorter focal length lenses in the doublet, but reducing the effect of spherical aberrations. We also show that when short focal length lenses are still required to generate a strongly focused Bessel-Gaussian field, we can eliminate the effects of spherical aberration and have an excellent agreement between the paraxial theory and the experiment by using parabolic lenses instead of spherical lenses. In the remaining of the article, we will first present the theoretical background, which will be then followed by the numerical calculations, tuning the distance between axicon and lens. Then we will present the experimental results and their comparison with the numerics to evaluate the effect of aberrations.
2. Theoretical model using the Kirchhoff-Fresnel integral

We will calculate the shape of the Bessel-Gaussian beams after the lens-axicon system, by using the Kirchhoff-Fresnel (K-F) integral formalism. The model considers perfect optical elements and in particular the tip of the axicon. The effect of non-ideal axicon tips are considered in [20, 25, 26]. The K-F integral assumes a cylindrically symmetric system and a field without any azimuthal phase with a profile given at \( z = 0 \) by the function \( A(\rho) \), \( \rho \) being the radial coordinate. Then, it allows us to calculate the field at position \((r, z)\) produced by diffraction. In the integral one can include a phase retardation \( \phi(\rho) \), given by [27]:

\[
u(r, z) = \frac{k}{z} e^{-ikz^2} \int_0^\alpha A(\rho)e^{-ik\phi(\rho)} e^{-ik\rho^2} J_0 \left( \frac{kr\rho}{z} \right) \rho d\rho,
\]

where \( k = \frac{2\pi}{\lambda} \) is the wavenumber with wavelength \( \lambda \), \( a \) is the aperture size and \( J_0 \) is the zeroth order Bessel function.

The analytical solution for the lens-axicon combination with separation \( d \) and Gaussian input involves two K-F integrals. Geometry and parameters of the optical system are represented in Fig. 1. The first K-F integral evaluates, over the radius of the lens \( a_1 \), the electric field diffraction from the lens plane with radial coordinate \( \rho_1 \) to the axicon plane with radial coordinate \( \rho_2 \) with a separation \( d \), such that,

\[
u_1(\rho_2, d) = \frac{k}{d} e^{-ik\rho_2^2} \int_0^{a_1} A(\rho_1)e^{-ik\phi_{\text{lens}}(\rho_1)} e^{-ik\rho_1^2} J_0 \left( \frac{kp_2\rho_1}{d} \right) \rho_1 d\rho_1,
\]

with Gaussian input, \( A(\rho_1) = \sqrt{I_0} e^{-\rho_1^2/W^2} \), and lens phase retardation, \( \phi_{\text{lens}}(\rho_1) = (n_{\text{lens}} - 1) \left( R_1 \left[ 1 - \sqrt{1 - (\rho_1/R_1)^2} \right] \right) \) [28], where \( I_0 \) is the input beam intensity with beam radius \( W \), \( n_{\text{lens}} \) is the refractive index of the lens and \( R_1 \) is the radius of curvature of the plano-convex lens [29].

Fig. 1. From left to right through \( z \)-axis a collimated Gaussian beam incident on the lens with aperture radius \( a_1 \) and focal length \( f \). The beam is then propagated through the lens, adding the phase factor, by distance \( d \) and integrated over \( \rho_1 \). Then, we have a cylindrically symmetric intensity profile at distance \( d \) (axicon plane with radius aperture \( a_2 \) and apex angle \( \alpha \)) from the lens and at a distance \( \rho_2 \) from the optical axis. Finally, we have a cylindrically symmetric intensity profile from the tip of the axicon along the axial distance and at distance \( r \) from the optical axis. The final ellipse represents out of plane region of the far field ring.

The second K-F integral then evaluates the electric field diffraction from the axicon plane to the Bessel zone with coordinate \((r, z)\) such that,

\[
u_2(r, z) = \frac{k}{z} e^{-ikz^2} \int_0^{a_2} u_1(\rho_2, d) e^{-ik\phi_{\text{axicon}}(\rho_2)} e^{-ik\rho_2^2} J_0 \left( \frac{kr\rho_2}{z} \right) \rho_2 d\rho_2.
\]
with axicon phase retardation given by \( \phi_{\text{axicon}}(\rho^2) = (n_{\text{axicon}} - 1)\alpha \rho^2 \) where \( \alpha \) is the radius, \( n_{\text{axicon}} \) is the refractive index and \( \alpha \) is the apex angle of the axicon.

Unfortunately, this double integral is computationally expensive for \( d \neq 0 \) due to a rapidly varying phase factor which requires the sampling of \( \rho^2 \) of the order of \( \lambda \). In order to calculate the focused Bessel beam profile, we replaced the first K-F integral with Gaussian beam optics in the paraxial regime using the thin lens approximation with a focal length \( f \), assuming that the Rayleigh range is far from the axicon tip, \( (f - d) \gg z_R = \frac{\lambda f^2}{\pi W^2} \). In this way we can replace \( u_1 \) in Eq. (4) with,

\[
u_1(\rho, d) = \sqrt{I_0}\left(\frac{f}{f-d}\right) e^{-\frac{\rho^2}{2w^2(f-d)^2}} e^{-ik\frac{\rho^2}{2(f-d)^2}}.\tag{5}\]

Contrary to the exact analytical solution using two K-F integrals, the Gaussian approximation assumes perfect parabolic lenses and as a result, will not include spherical aberrations. Note that in the instance where \( d=0 \), i.e. lens-axicon doublet, both phase terms can be included in a single K-F integral which can then be numerically solved including spherical aberrations [16]. We have found an excellent agreement between theory and experiments for the largest apex angle axicon used, which is of 20°, and a short focal length lens of about 30mm showing that the theory still does not require a vectorial treatment.

3. Numerical calculations of the lens-axicon system

Now we proceed to integrate Eq. (4) using Eq. (5) in order to analyze the effect of tuning the separation \( d \) between the lens and the axicon on quasi-Bessel beams for the peak intensity, the peak position and the Bessel zone length. The freedom of controlling the separation \( d \) in the lens-axicon combination allows us generate a relatively tight focal spot size with an increased central core intensity and longer working Bessel zone. We show that with a suitable choice of lens, angle axicon and distance between them, we can match the spot size and the Bessel zone generated by a sharp axicon alone while having higher central core intensity. We make a quantitative analysis of the intensity profile for a 30mm-5° lens-axicon combination, where the first value is the focal length of the lens and the second is the angle of the axicon. This notation is maintained throughout the paper. Figure 2 shows six different lens-axicon separation distances. In all calculations, the wavelength is 532nm and the beam radius is \( W=2.5mm \) unless otherwise mentioned.

Firstly, the presence of a lens focuses the Bessel beam structure such that the radial width of the core and the concentric rings along the axial direction are no longer constant. The width of the central core and the concentric side lobes narrow down and tilt towards the vicinity of the focus of the lens as we move along the axial direction. Consequently, the effective Bessel zone length reduces [22]. The reduction can be mitigated by reducing the distance between the lens and the axicon. Secondly, due to the focusing, the effective incident beam diameter reduces through the propagation when reaches the axicon. Therefore, the total number of concentric rings decreases, similarly to what would happen with a collimated incident beam with narrow beam width (Fig. 2).

The third consequence is an increase of the peak intensity when \( d \) increases, keeping \( f - d \gg z_R \). These two effects together; tight spot size and less number of rings, results in a significant increase in the Bessel core intensity (Fig. 3(a)). An axicon behaves as a converging optical element. Thus placing an axicon between a positive focusing lens and its focal point further contributes to the focusing and the reduction of the focal length. As the lens-axicon separation distance is increased, the focal point of the lens moves closer to the tip of the axicon and sharply focuses the Bessel-Gaussian beam as can be also shown in Fig. 2. This results in a significant decrease of the effective Bessel zone but without modifying the size of the central core width at
Fig. 2. Numerical calculation of the beam profile after crossing a 30mm-5° lens-axicon combination with six different distances (a) 4mm, (b) 8mm, (c) 12mm, (d) 16mm, (e) 20mm and (f) 24mm. All intensity distributions are normalized to the maximum of (f) with the exception of its inset, which reproduces (a) normalized to its maximum intensity. Notice that in the selected region the number of rings is the same in all cases.

the peak intensity as shown in Fig. 3(b). Nevertheless, Eq. (1) still holds through the Bessel region. Although \(\Delta r\) reduces linearly through the Bessel zone (Fig. 3(c)) the power at each ring and the central core contribute in the same amount to the total power. Conversely, as \(\Delta r\) does not change and the Bessel zone reduces when \(d\) increases, the number of rings has to reduce and the power at the core increases accordingly.

In Fig. 3 we present a plot of the fields calculated from the 30mm-5° system and a comparison with a single 20° axicon. The insets of Fig. 3 presents two combinations of different \(d\) for a 30mm-5° and an axicon of 20°. The beam width incident on the 20° axicon is reduced to be equivalent to the beam width incident on the 5° axicon after crossing the lens and propagating the distance \(d\). Although the 20° axicon has a sharper apex angle and the incident beam diameter has been reduced, the peak intensity and power are higher for the focused beam in the 30mm-5° combination with \(d=24\)mm.

Interestingly, the central core width at maximum intensity is solely dependent on the axicon angle [2, 30] and the focal length of the lens chosen. For a single 5° axicon, the core width is about 5\(\mu\)m in diameter. Hence, the placement of a lens before an axicon reduces the central core width linearly as we move along the Bessel zone towards the end of the Bessel zone (Fig. 3(c)). Although the Bessel zone reduces gradually, the minimum of the core size at the peak intensity is fixed for a given lens and axicon combination, independently of the separation. As an example, for a 30mm-5° lens-axicon combination, the central core width reduces down to 2\(\mu\)m from the peak intensity to the first zero independently of the distance between both optical elements as seen in Fig. 3(b). When the lens-axicon distance is 4mm, a Bessel zone of 22mm is achieved and the central core displaces keeping the width constant. If we compare the peak intensity with a much sharper axicon we still obtain a stronger peak intensity. Although a single 20° axicon generates a smaller core diameter, 1.3\(\mu\)m, the central core intensity is about 40% higher for the 30mm-5° lens-axicon combination at \(d=4\)mm (Fig. 3(a) inset). Moreover, not only does the Bessel zone have similar dimension while the peak intensity increases but it also moves away from the axicon. This could be an advantage in an optical setup by allowing space for additional optical elements, such as polarizers or waveplates, for a final modification in the engineered optical beam.
Curiously, the shape of the intensity distribution through the Bessel length in the axial direction is nearly fixed once the focal length lens and the axicon apex angle are selected. As mentioned, \( d \) displaces both the peak intensity and Bessel zone but without modifying the shape of the intensity distribution, in other words, it re-scales the intensity profile. This can be of benefit in schemes like in counterpropagating beams for optical trapping as shown in [11]. There, both beams have to be axially similar and symmetric to cancel out the scattering forces while the particle is confined along the axial direction. In this scheme once a symmetric beam distribution is designed by the focal length and axicon angle, it is just a matter of changing \( d \) in order of increasing the gradient of the intensity, therefore the gradient forces.

4. Experimental implementation of the Bessel-Gaussian beam

4.1. Experimental setup

The source we used is a fiber coupled 532nm diode laser, from which we could obtain a very good spatial single mode. We set both the lens and the axicon on a 1D translation stage in order to scan the axial profile of the beam. Particular efforts had to be taken to align laterally...
the beam with both the axicon and the lens, to avoid further aberrations and asymmetries. The beam produced from the tip of the axicon is a focused Bessel-Gaussian beam. In order to image the desired region, we placed a second lens (15mm focal length) and a charge-coupled device (CCD) camera on the optical axis. The single lens imaging provided a 52 times magnification on the CCD pixel array. The object plane was within the proximity of the Bessel zone and the 1D translation stage was moved to scan through the entire Bessel zone.

To experimentally study the relations between the central core width, the effective Bessel zone and the central core beam intensity, we used a combination of two positive spherical plano convex lenses and two apex angle axicons. In order of removing the spherical aberration an aspheric lens is also used in combination with the axicons. The focal length were 30mm and 50mm for the spherical lenses, 32mm for the aspheric lens and the apex angle of the UV fused silica axicons were 5° and 20°. We focused our efforts in the regime that was described previously, when the focal length is larger than the lens-axicon distance , \( f - d > 0 \), and \( d \neq 0 \). We also kept the lens-axicon distance such that the Bessel zone appeared inside the geometric focus of the imaging lens. Increasing the lens axicon separation decreases the length of the Bessel zone. Further away the far field ring is formed at the focus of the lens with no light in the center, which is found beyond the Bessel zone (Fig. 1) out of the scope of this work [16–19]. The measurements were taken combining the 5° axicon with a lens-axicon distance \( d \) of 8mm for the 30mm focal length lens and \( d \) of 35mm for the 50mm focal length lens in order to confirm the theory. In addition the combination 50mm-20° is analyzed.

To have an accurate focal profile analysis, we used the high dynamic range (HDR) imaging technique to extend the bit-depth of the CCD camera. This allowed us to clearly distinguish between maxima and minima of the concentric Bessel rings. Thus, three images with different exposure times were captured at each axial position through the scan of the Bessel zone (patterned region in Fig. 1). Those three images with different exposure times were merged together and a final HDR image is generated for the analysis.

4.2. Results and analysis

We start by showing a configuration where the spherical aberrations are not observed. This can be achieved by combining a long focal length lens and shallow axicon. Figure 4 shows the radial intensity distribution of the axial beam profile for a 50mm-5° lens-axicon with a separation of 35mm that fulfills this condition. Equivalently, Fig. 5 shows the cross section at five different locations in the Bessel region shown in Fig. 4. The experimental result is in excellent agreement with the intensity distribution predicted from Eq. (4). On the right of the experimental images of Fig. 5, it can be observed a subtle formation of 4 lobes. These lobes have their origin in the ellipticity of the axicon tip [20,31]. The central core width is 2.9 \( \mu \)m while the simulation predicts 2.6\( \mu \)m. It can be observed that the central core radius reduces linearly (Fig. 4(c)) through the propagation as shown in Fig. 3(c). Although the agreement is good the deviation is due to non-perfect optical elements. On top of a slight ellipticity of the tip of the axicon, the increase in the core width has its origin on a round end [20,25,26]. This effect also contributes to a non-completely negligible aberrations. The peak intensity is shifted right towards the central focal region compared to a single axicon as expected due to the focusing and at the same time the transversal intensity distribution in stronger compare to the doublet (\( d=0 \)).

If more intensity is required we can gradually increase the lens axicon separation towards the focal length as shown in Figs. 2 and 3. Otherwise, we need to use a shorter focal length lens, a sharper angle axicon or to reduce the incident beam size. However, these changes increase the appearance of spheric aberrations in the Bessel zone.

Next, we will study the Bessel zone region where the effect of the spherical aberration is significant. In Fig. 6 we present the results for the 30mm-5° and the 30mm-20° system, where the presence of spherical aberrations are evident and not predicted in the model. We observe in
Fig. 4. Numerical calculations (a) and experiment (b) of the normalized intensity beam profile generated with a 50mm-5° lens-axicon. Comparison of the simulation (black line) with the experimental data (red line) for (c) the intensity distribution of the central core normalized at their maximum intensity and (d) the central core radius through the axial direction.

Fig. 5. Numerical calculation (top row) and experiment (bottom row) cross sections normalized to the peak intensity (fourth column) of the beam profile shown in Fig. 4 at 3.5mm, 4.7mm, 5.9mm, 7.1mm and 8.2mm from the tip of the axicon.
the experimental measurements a modulated central core intensity composed of sharp peaks near
the focus of the lens and a much shorter Bessel zone. The effects of spherical aberrations are
even more pronounced when using a sharper axicon.

![Image](image_url)

**Fig. 6.** Numerical calculation (top row) and experiment (bottom row) of the intensity beam
profile generated with a 30mm-5° lens-axicon (left column) and a 30mm-20° lens-axicon
(right column). All plots normalized at their maximum intensity.

Figure 7 gives a closer look at the sharp peak intensity in the central core region for the
aberrated system. We observe the details of the effect of the spherical aberrations on the highly
focused Bessel beam. An inspection of the peak intensity region shows a beating of the light
intensity both in the radial (Fig. 7(a) and 7(b)) and axial (Fig. 7(c) and 7(d)) directions, typical
of spherical aberrations in Bessel beams [16]. In a lens with strong spherical aberrations the
incoming rays will not meet in a unique focal point after passing through the lens. Only those
rays intersecting the lens at the same height will meet at the same focal point, hence the focal
point is spread. Consequently, focusing a Bessel beam with such a lens can give rise to the
beating phenomenon shown due to the interference through the axis of propagation.

The difference between including or not including the effect of spherical aberrations for
converging lens-axicon doublets was addressed theoretically in [16]. However, their model of
lens-axicon doublets does not account for the lens-axicon separation that we present in our theory.
When the spherical aberration is considered, their model shows a repeating pattern of on-axis
intensity variation and a short propagation distance, similar to what we observe experimentally in
Fig. 7. Moreover, both aberration-free models predicts a smoothly varying central core intensity
profile with longer propagation distances. This corresponds experimentally when the focal length
is relatively long.

This beating effect of the on-axis intensity is more pronounced for the 20° axicon as seen
in Figs. 6(d) and 7(c). Here, the incoming Gaussian beam is sharply focused with a lens with short focal length and then focused again with a sharp angle axicon. The beating pattern becomes weaker and disappears with the use of the 50mm focal length lens as seen with the 50mm-5° lens-axicon combination (Fig. 4). Thus, the source of disagreement between theory and experiment is the spherical aberration. The disappearance of the on-axis beating intensity for a lens with long focal length combined with a shallow angle axicon confirms the hypothesis of the spherical aberrations as the source of the beating effect.

In practice, the lens-axicon separation allows to minimize the effects of spherical aberrations by using long focal length lenses while achieving high intensities in the Bessel region. However, if we need to achieve small width central cores, the use of a shorter focal length may be unavoidable. In such a case, we aim to minimize the spherical aberrations using aspheric lenses. Figures 8(a) and 8(b) show the numerical calculation and experimental results for the axial beam profile generated by a 32mm-20° aspheric lens-axicon combination with a lens-axicon separation distance of 22.15mm. Similarly, Figs. 9(a) and 9(f) show the 32mm-5° aspheric lens-axicon combination with a lens-axicon separation distance of 17.65mm. Figures 8(c) and 9(c) show the comparison of the numerical calculation with the experimental data for the intensity distribution of the central core (focal-line) region for a 32mm-20° and a 32mm-5° aspheric lens-axicon combinations, respectively. The removal of spherical aberrations is even more evident when comparing with
the results shown on Fig. 6. In both measurements the axicon is the same and the differences are
the substitution of the type of lens and just 2mm in the focal length of the lens from 30mm to
32mm and less than half lens-axicon separation. It is worth noting here that by choosing the
appropriate parameters for the lens-axicon combination, the intensity of the beam along the axial
direction varies with respect to a single axicon where the maximum intensity is closer to the tip
of the axicon. In Fig. 8(c) the peak intensity is closer to the centre of the Bessel beam showing a
high symmetry in the axial beam profile whereas in Fig. 9(c) the peak intensity has been shifted
further from the axicon tip producing a profile structurally similar to a Bessel-Gaussian beam
produced in the opposite direction. Finally, as shown in Fig. 3(c) the central core radius reduced
linearly (Figs. 8(d) and 9(d)). As mentioned for the 50mm-5° the small deviation between theory
and experiment is due to a non-perfect end of the tip of the axicon [20], and in particular for Fig.
8(d) can be fixed by increasing the CCD pixel resolution or the 52 times magnification for the
imaging.

From Figs. 8 and 9, we obtain a good agreement between theory and experiment, even for
the 20° axicon, proving that the scalar approach of the analytical model is adequate. Significant
effects from the spherical aberrations compared to earlier experiments are removed using the
aspheric lens. We attribute the remaining spherical aberrations in the images due to the spherical
Fig. 9. Numerical calculation (a) and experiment (b) of a normalized intensity beam profile generated with a 32mm-5° aspheric lens-axicon combination. Comparison of the simulation (black line) with the experimental data (red line) for (c) the intensity distribution of the central core normalized at their maximum intensity and (d) the central core radius through the axial direction.

imaging lens and most probably to imperfections of the sharpness of the axicon tip [20, 25, 26].

Next, we quantify the focal spot sizes among the different lens-axicon systems. The results are presented in Table 1. The focal spot sizes (FWHM) were calculated by fitting a Bessel function with the experimental radial beam profile. In the aberration free combinations we obtain a very good agreement between experiment and simulation in all tested cases. Moreover, the engineered focused Bessel beams have a significant reduction of the spot sizes compared with the axicon alone. In detail, we see that the experimental observation of the 50mm-5° lens-axicon has a good agreement with a focal spot size 1.7 times smaller than the 5° axicon alone spot size. Compared with the 20° axicon the spot size is about twice wider but the peak intensity is stronger as shown in the insets of Fig. 3. When the focusing of the Bessel-Gaussian beams increases strong spherical aberrations are present. We accounted for the spherical aberration replacing the spherical lenses with aspheric lenses and an excellent match with the aberration-free theory is obtained as the presence of spherical aberrations are negligible.
Table 1.
Focal spot size (FWHM) of an axicon or a lens-axicon combination for a 532nm incoming Gaussian beam with a beam radius \(W=2.5\)mm.

| Axicon/Lens-Axicon System | Experimental | Simulation |
|---------------------------|--------------|------------|
| 5°                        | (5.0±0.1)\(\mu\)m | 5.0\(\mu\)m |
| 20°                       | (1.3±0.1)\(\mu\)m | 1.3\(\mu\)m |
| Aberration Free            |              |            |
| (aspheric lens) 50mm-5°    | (2.9±0.1)\(\mu\)m | 2.6\(\mu\)m |
| (aspheric lens) 32mm-20°   | (1.8±0.1)\(\mu\)m | 2.0\(\mu\)m |
| Non-Aberration Free        |              |            |
| 30mm-5°                   | (1.4±0.1)\(\mu\)m | Not Available |
| 30mm-20°                  | (0.9±0.1)\(\mu\)m | Not Available |

5. Conclusions
This work on the practical implementation of quasi-Bessel beams offers a new degree of freedom between the single axicon and the lens-axicon doublet by tuning the distance between the lens and the axicon. We have shown using both theory and experimental observations how to significantly tune the effective propagation distance, the spot size and the on-axis central core intensity of Bessel-Gaussian beams and the position. The agreement between theory and experiment justify the use of a scalar analytical solution. Although we have not presented the study of the ring formed at the focal plane, the model confirms the results shown in [19] where the lens-axicon separation can be used to modify the ring size. The results presented can be implemented in any instrument capable to modulate phase as for instance SLMs [5].

We have observed that using spherical lenses, which inherently have spherical aberration, the propagation distance was significantly reduced with a strong intensity peak. Otherwise, if the focal length is long enough the spherical aberrations are negligible. In this work, we have chosen aspheric lenses to account for the spherical aberrations. Using a suitable choice of apex angle axicon, aberration free optical lens and a suitable separation distance, we can tune the Bessel-Gaussian beam to meet any application or specific need of the Bessel-Gaussian beams including those where modifying the intensity along the axial beam profile is needed. This study shows the path to engineer focused Bessel beams in all those applications where narrow Bessel cores were needed and the spherical aberrations were previously a problem.

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