Metacognitive therapy for mathematics disorder

S S Faradiba*, C Sadijah, I N Parta, and S Rahardjo
Graduate School, Universitas Negeri Malang, Jl. Semarang No. 5, Malang, Indonesia

*surya.faradiba.1503119@students.um.ac.id

Abstract. Mathematical disorders are learning disabilities that greatly affect a person's mathematical abilities so that they need to be recognized as early as possible. This aims to minimize the negative impacts caused by mathematics disorder. This study is a descriptive qualitative research. The participants of this study were two university students of the department of mathematics education, Universitas Islam Malang. This paper describes Metacognitive therapy (MCT) for students who have mathematics disorder. MCT therapy is done by asking metacognitive questions which consist of comprehension, connection, strategic, and reflection questions. The purpose of MCT is to find out what students believe about their own thoughts and how their thoughts work about mathematics, then show students how this belief leads to an unsupported response to thoughts that inadvertently prolong or worsen symptoms, and ultimately give way alternative response to the mind to allow for symptom reduction. Need to be further investigated whether the results of metacognitive therapy are only temporary or not.

1. Introduction
Mathematics anxiety (MA) is a feeling of tension that disturb with the manipulation of mathematical problems in everyday and academic life [1]. MA is a main factor in blocking students’ reasoning, memory, understanding of general concepts, and appreciation for mathematics [2]. MA can lead to the rejection of mathematics and the decline of mathematical performance. So, MA can influence to the future learning.

The research MA had been widely experienced by many researchers in every grade of education [3-7]. Moreover, mathematics is an important subject in the school curriculum in every country. From a young age children must learn the basic concepts of mathematics in order to function well in their daily life [8]. Mathematics has been taught so that children can understand the numerical data presented to them, and able to perform simple and complex calculations in day-to-day encounters. It is also a common belief among students that mathematics is a challenging subject and difficult to learn.

In the recent research, MA is distinguishable from other types of anxiety symptoms. It can exist in the absence of more general anxiety traits. Besides that, MA has a direct and deleterious effect on underlying cognitive processes as the individual performs a mathematics task. We also show that this is especially true for individuals with mathematics disorder [9]. So, the researchers conclude that special treatment is needed to overcome MA in students with mathematical disorders.

In mathematics education, many researchers propose innovative ways of teaching, linking concept and real-life applications and motivating the students to take more interest in the subject to overcome MA [10]. However, not all MA can be overcome in such a way. For MA that cannot be solved immediately in these ways, it takes special therapy, one of them with metacognitive therapy.
The word *meta* comes from the Greek word meaning "above", cognition refers to mental processes such as attention, memory and problem-solving. So it can be concluded that metacognition is thinking about how we think [11]. Metacognition is used to refer to the awareness individuals have of their own thinking; their evaluation of that thinking; and their regulation of that thinking [12]. Metacognitive therapy for students with MA can be described as therapy that can improve the way students think of their cognitive biases. This cognitive bias can lead to the formation of false beliefs. Metacognitive therapy aims to sharpen students' awareness of the spectrum of cognitive bias [13]. In this therapy, a teacher first introduces new concepts, theorems, and formulas by modeling a self-addressed questioning technique that employs four types of metacognitive questions, that is: comprehension, connection, strategic, and reflection questions [14]. Comprehension questions help students to understand mathematics problems. Connection questions help them to build a bridge between a given problem and similar or related problems in the past. In this case, the role of student learning experience is very important. Strategic questions help students to determine the right strategy to solve a problem. Finally, a reflection question that leads the student to check the problem-solving process and its solutions

2. Method

This is an explorative research with a qualitative approach. The qualitative approach was chosen because the study was natural. In addition, the data are descriptive in the form of a series of words and pictures taken from interviews, field notes, photos, videos, and documents. The purpose of this research is to describe Metacognitive therapy (MCT) for the student who has mathematics disorder.

Two undergraduate students of mathematics education from Universitas Islam Malang were selected as the participants of the study. Selection of the participants using four criteria of mathematics disorders in DSM 5 at the level mild and severe. While at the moderate level is not done because the researchers did not find a subject that meets these criteria [15]. This study uses indicators of specific learning disorder with impairment in mathematics as an indicator of MA. The general criterion of MA indicated by the presence of at least one of the following symptoms that have persisted for at least six months, despite the provision of interventions that target those difficulties: (1) Difficulties mastering number sense, number facts, or calculation, for examples has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the mathematics fact as peers do; and get lost in the most of arithmetic computation and may switch procedures, and (2) Difficulties with mathematical reasoning, namely has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems.

The characteristics of two subjects (S1 and S2) in this study are made based on mathematics disorder criteria in DSM 5 [15]. At the mild level, it was explained that the subject experienced some difficulties learning skills in one or two academic domains. In this study, S2 has difficulties in mathematics only, whereas in other capabilities (reading and writing) are not. So that it can be concluded that S2 is classified as mild level. Meanwhile, at a severe level, the subject is affecting several academic domains. In this case, S1 has difficulty in mathematics (number sense, memorization of arithmetic facts, accurate or fluent calculation, accurate mathematics reasoning) and reading (word reading accuracy, reading rate or fluency, reading comprehension). So that it can be concluded that S1 is included at the severe level.

Data were collected by task-based interviews where the task contains a mathematical problem. At each stage, subjects (S1 and S2) are interviewed to reveal their metacognitive activities during this stage. The interviews were conducted twice using equivalent problem at two different times. Data from interviews supported by observations, answer sheets, and recordings (audio and video) were analyzed qualitatively, including data review, categorization, reduction, display, and interpretation.

S1 and S2 were given a task containing algebra combine with geometric problems, number one for the first interview and number 2 for the second. The results showed that the S1 responses in both interviews were consistent. Therefore, the following will only be exposed to the results of the first interview.

- A rectangle, the width is 6 cm shorter than its length. If the perimeter is 44 cm, find the length and width!
When the sides of a square are each increased by 2 cm its area increases by 44 cm$^2$. Find the length of the side $S$ before the increase!

3. Results and Discussion
This study uses indicators of specific learning disorder with impairment in mathematics (code 315.1 F81.2) present in DSM-5 [15]. Difficulties learning and using academic skills, as indicated by the presence of at least one of the following symptoms that have persisted for at least six months, despite the provision of interventions that target those difficulties: (1) Difficulties mastering number sense, number facts, or calculation, for examples has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the mathematics fact as peers do; and get lost in the most of arithmetic computation and may switch procedures, and (2) Difficulties with mathematical reasoning, namely has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems. In this study, a teacher first introduces new concepts, theorems, and formulas by modelling a self-addressed questioning technique that employs four types of metacognitive questions: comprehension, connection, strategic, and reflection [14].

MCT is an evidence based psychotherapy approach, where the main emphasis lies on the thought processes rather than the content of the thoughts [16]. Metacognitions consist of beliefs, experiences and strategies [17]. Metacognitive beliefs (knowledge) are related to a person’s own thinking style, and his beliefs about them. These beliefs are further divided by two subcategories, i.e. explicit (declarative) beliefs, and implicit (procedural) beliefs. These beliefs are categorized according to whether the belief associated with them can verbally be expressed or not. In this study, the belief used is a belief that is orally expressed by subjects. Metacognitive experiences refer to the appraisals and feelings a person has regarding his own state of mind. Metacognitive experience was observed during the interview process by using comprehension and connection questions. Metacognitive strategies reflect the emotional and cognitive self-regulatory strategies which are involved with the control responses of thoughts and behaviours. In this study, metacognitive strategies are observed during interviews using strategic and reflection questions to clarify.

3.1. Analysis of Subject 1 (S1)
S1 does not seem to have difficulty describing the rectangle, showing the length and width of the rectangle. Figure 1 shows that S1 has difficulty to show the section referred to as the perimeter of the rectangle by answering the perimeter obtained from the length multiplied by the width.

![Figure 1](image.png)

Figure 1. The answer of S1.

Comprehension questions above guide students to articulate central ideas found in the problem.

1. I : What do you think about a perimeter?
S1 : Perimeter is the word used to describe the distance around the outside of a figure (the rectangle)

1. I : Ok, good, so what do you know to find the perimeter?
S1 : To find the perimeter, multiply the lengths of the sides of the rectangle.

1. I : Well, let’s see if you multiply the lengths of the sides of the rectangle, you will find the area. While we need to find a perimeter. It’s just the distance around the outside of a rectangle. So, any idea?
S1: I think, I will add together the lengths of all of the sides of the figure. The above conversation shows that Interviewer (I) dealing with metacognitive beliefs of Subject 1 (S1) and reality testing of them where appropriate. Then we use connection question to connect the concept of perimeter and the rectangle 

I: Now, please refresh your polygon memories. Remember that "regular polygons" are polygons whose sides are all the same length and whose angles are all the same size. Not all polygons are "regular". How about rectangular? Is it the regular polygon? 
S1: I think it is not regular. 
I: How many sides a rectangle has? 
S1: Four, that are two sides as “long” and two sides as “width’ 
I: Can you show the side of the rectangle called "long" and "width’? 
S1: Yes. 
I: What do you know about the side lengths of a rectangle. 
S1: It always longer than the width. 
I: What the area of the rectangle is? 
S1: Area is the number of square units needed to cover a two-dimensional (flat) region. It is measured in square units. But if we don’t use the square unit, I am so confused with it.

Then we use the strategic question to guide students formulate problems that he encountered in mathematical equations. In this case, we need to a postponement of worry by providing helpful clues to increase students' understanding [17].

I: A parallelogram has two sets of parallel sides. A rhombus is a type of parallelogram in which all sides are congruent. The height of a triangle, trapezoid, and parallelogram makes a right angle with (is perpendicular to) its base(s). The height does not have to be vertical because the shape can be turned. So, what will you do to find the area of a parallelogram? 
S1: I think to find the area of a parallelogram, multiply the base (the length) by its height. It’s true? 
I: Ok, great! 

At the end we use a reflection question to convince the way that the student has obtained is correct. These beliefs are only related to the meaning and significance of cognition, and no further relevance is sought regarding the content of cognition [16]. Among these beliefs, the negative ones are thought to be the most persistent, and acutely these beliefs increase the perception of threat, hopelessness or inefficacy [11]. Questions given at this stage, among others: “Are you sure of your answers?” or “Does your answer apply generally to all types of the rectangle?”. 

At the end of the therapy process, S1 is not able to give the right answer, but S1 can improve their ability to solve mathematics problems. So, we can conclude that students with severe levels can be overcome by metacognitive questions.

3.2. Analysis of Subject 2 (S2) 
Figure 2 shows that S2 solved the problem by symbolizing the width of the rectangle as 6-p. Therefore, therapy begins by providing comprehension questions

I: the length of the rectangle (p) and rectangle width (l), if you symbolize width as 6-p, what do you think is longer p or 6-p? remember p is always a positive number because the length (p) can not be negative. 
S2: I find it difficult because the length of the sides has not been known 
I: you can symbolize a certain alphabet to make it easy, for example” p” as the length of rectangular. 
S2: ok, if the length of p then the width of 6 cm shorter should be 6-p. Is that true? 

Then, connection questions help them to construct bridges between a given problem and problems solved in the past.

I: Check your answers, do not forget the perimeter obtained by summing up the four sides of the rectangle
\[ S2 : p + p + (6 - p) + (6 - p) = 44 \text{ so } 12 = 44 ? \text{ no, this is definitely wrong.} \]

![Figure 2. The answer of S2.](image)

Strategic questions refer to the strategies appropriate for solving a problem. Reflection questions encourage the students to look back during the solving process ("Why am I stuck") and after a solution has been reached ("Does my solution make sense"). Finally, reflection questions can be asked to ensure students are aware of what they are writing, for example: "are you sure of your answers".

![Figure 3. The final answer of S2.](image)

At the end of the therapy process, S2 is able to give the right answer. Figure 3 shows that the answer of S2 is correct. So, we can conclude that students with mild anxiety levels can be overcome by metacognitive questions.

4. Conclusion
MCT provides a new appearance for the treatment of psychological disorders by underlining the significance of how a person thinks, rather than simply focusing on the content of his cognition. Students who experience MA at a mild and severe level can be overcome with metacognitive therapy in the form of giving four types of questions: comprehension questions, connection questions, strategic questions, and reflection questions. It is necessary to further research whether metacognitive therapy can be applied to students with MA levels at moderate levels. In this study, MCT is not applied to the moderate level because we did not find a subject that fit the criteria based on DSM-5.

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