Vortex configurations of bosons in an optical lattice

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The single vortex problem in a strongly correlated bosonic system is investigated self-consistently within the mean-field theory of the Bose-Hubbard model. Near the superfluid-Mott transition, the vortex core has a tendency toward the Mott-insulating phase, with the core particle density approaching the nearest commensurate value. If the nearest neighbor repulsion exists, the charge density wave order may develop locally in the core. The evolution of the vortex configuration from the strong to weak coupling regions is studied. This phenomenon can be observed in systems of rotating ultra-cold atoms in optical lattices and Josephson junction arrays.

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Vortices in charged or neutral superfluids (SF) can be created by applying an external magnetic field or by rotating the system. Properties of vortices are essential for understanding superfluidity. Generally, vortices are topological defects in the SF order parameter. Thus, the minimum particle density is given by the square of the amplitude of the SF order parameter. Near the SF-Mott insulator (MI) transitions, the G-P-B method ceases to work well. Theoretically, many investigations based on the Bose-Hubbard model are available. Theoretically, many investigations based on the Bose-Hubbard model are available. Theoretically, many investigations based on the Bose-Hubbard model are available. Theoretically, many investigations based on the Bose-Hubbard model are available. Theoretically, many investigations based on the Bose-Hubbard model are available. Theoretically, many investigations based on the Bose-Hubbard model are available. Theoretically, many investigations based on the Bose-Hubbard model are available."
SF and CDW order parameters are defined as \( \langle f \rangle \) and \( \langle g \rangle \) respectively. The MF ground state is the product wave function \( |\Psi\rangle = \prod_i |\tilde{\Psi}(\vec{r}_i)\rangle \). We cutoff each single-site Hilbert space up to 10 particles which is sufficient for experimental values \( \langle N \rangle \approx 1 \sim 3 \). The SF and CDW order parameters are defined as \( \langle a(\vec{r}_i) \rangle \) and \( \langle -\frac{i}{\sqrt{2}}(\langle n(\vec{r}_i) \rangle - \langle N \rangle) \rangle \) respectively, where \( \langle N \rangle \) is the bulk average particle density. This simple MF theory describes the SF-MI transitions and extrapolates well into the intermediate coupling region. For example, the relation of the SF order \( \langle a \rangle \) vs. \( t/U \) from the MF theory is in good agreement with that of the Monte-Carlo simulations at both commensurate (integer) and incommensurate fillings.

Before discussing the vortex problem, it is helpful to recall the well-known phase diagram when \( \Omega = 0 \). From the equal density contours we can see an approximate particle-hole (p-h) symmetry with respect to \( \langle N \rangle = 2 \) near the phase boundary: \( \mu \) drops or increases with increasing \( t/U \) when \( \langle N \rangle \geq 2 \) or \( < 2 \) respectively. These can be viewed as particle or hole SF, but this difference no longer exists as \( t/U \) becomes large, where \( \mu \) drops with increasing \( t/U \) for both cases, i.e., the system evolves from the strong to weak coupling region. In Fig. 1(b), SF order vs. \( \langle N \rangle \) is shown at different values of \( t/U \). The SF order parameter increases monotonically with \( \langle N \rangle \) at large \( t/U \), while the commensurate filling suppresses the SF order prominently at small \( t/U \), eventually leading to the MI phase. At \( t/U \approx 0.08 \), the suppression disappears even at \( \langle N \rangle = 1 \), which marks the cross-over into the weak coupling region.

We study the single vortex problem in a \( 40a_0 \times 40a_0 \) \((a_0\) being the lattice constant) system around which the circulation of \( A \) is \( 2\pi \) and thus \( \Omega = h/(2mL^2)(L = 40a_0) \) correspondingly. The rotation center is located at the center of the central plaquette. We further simplify the problem by dropping the \( V_{xx} \) and \( V_{ij} \) terms, since they behave smoothly near the center of the trap and they can cancel each other if the \( \Omega \) is close to the trap frequency.

Two typical vortex configurations near the SF-MI transition are shown in Fig. 2(a) \( \sim \) (d) with \( t/U = 0.02 \), where the particle densities in cores are maximal or minimal respectively. The vortex core is located at the center of the plaquette with the reduced SF order on the sites.
FIG. 4: The evolution of vortex particle density distribution at fixed $t/U=0.03$ (a) and 0.02 (b) with varying bulk values $\langle N \rangle$ along the same path in Fig. 3. (a) From top to bottom, $\langle N \rangle=2.3 \sim 1.7$. (b) From top to bottom, $\langle N \rangle=1.9 \sim 1.1$.

nearby. Roughly speaking, the square of the superfluid amplitude, $|\langle a \rangle|^2$, is proportional to $|\langle N \rangle - N_0|$ near the transition, where $N_0=2$ here is the nearest commensurate value. The local particle density in the core should be closer to the commensurate value in order to suppress the SF order. As a result, the particle density reaches a maximum or minimum when the bulk density $\langle N \rangle$ is slightly smaller or larger than the commensurate value. The former case can also be understood as the vortex of the hole superfluid, where the hole density goes to a minimum at the core. This contrasts with the case of fermionic superfluidity, where the Cooper pairs are broken into normal particles in the core with total particle density almost unchanged, and also with the case of the weak coupling bosonic system where the only possibility is the depletion of the core particle density. As we approach the vortex core from outside, the hopping process is frustrated and thus effectively $t/U$ becomes small because of the phase winding of the SF order. As a result, the vortex core is driven closer to the MI state than the bulk area. We also check the vortex configuration with integer value of $\langle N \rangle=2$ at the same value of $t/U$, where the particle density distribution is uniform with suppressed SF order in the core.

We further discuss the evolution of the vortex core configuration from the strong to the weak coupling regions. The bulk particle density is fixed at $\langle N \rangle=1.95$ with increasing $t/U$ as shown in Fig. 3. The SF order increases and the healing length $\xi$ decreases away from the SF-MI boundary along this direction of evolution. On the other hand, the weak coupling expression $\xi/a_0 = \sqrt{t/(U\langle N \rangle)}$ states that $\xi$ increases with increasing $t/U$. Thus we can infer that $\xi$’s minimum value appears in the intermediate coupling region. The vortex core with extra particle density survives until $t/U \approx 0.05 \sim 0.06$, after which it crosses over gradually to that with depleted particle density at large $t/U$. This transition agrees with the behavior of the SF order vs. $\langle N \rangle$ when the system is not in rotation. In that case, at the same filling level, we check that the suppression due to the commensurate filling also vanishes around a similar value of $t/U$. When $t/U$ is larger than 0.06, the vortex configuration is already similar with the weak coupling case. We also check the above evolution with fixed $\langle N \rangle=2.05$. The feature of SF order is similar with that in Fig. 3 (a), and the minimum particle density is always located in the core. When $t/U$ is less than an intermediate value around 0.05, the core particle density is close to the commensurate value 2.0. When $t/U$ grows larger, it drops further. This also agrees with the evolution picture from strong to weak coupling physics.

Another evolution from the particle-like vortex core to the hole-like core with fixed $t/U$ in the strong coupling region and varying $\langle N \rangle$ is shown in Fig. 4 (a). We choose the region close to the MI phase with $\langle N \rangle=2$, where the approximate p-h symmetry is valid, with $t/U$ fixed at 0.03 and $\langle N \rangle$ varying from 2.3 to 1.7. As a result, the difference between the core particle density and the bulk value changes from negative to positive as the bulk density passes $\langle N \rangle=2$. This point is the closest one to the tip of the MI phase along the evolution, where the minimum of the SF order and the maximum of $\xi$ also
lie. In Fig. 4(b), such evolution is shown along the path $t/U = 0.02$ connecting two neighboring MI phases with $\langle N \rangle = 2$ and 1. The core configuration is close to the MI phase with $\langle N \rangle = 2$ on the top, and becomes close to the MI phase with $\langle N \rangle = 1$ at the bottom. Around $\langle N \rangle = 1.55$, the density distribution becomes almost uniform. This point is the maximum of the SF order and the minimum of $\xi$ because it is the farthest point from the MI, which is just opposite to Fig. 4(a). In both Fig. 4(a) and (b), the tendencies to MI phase in the vortex core are strong when the average $\langle N \rangle$ is close to an integer, and becomes weaker as $\langle N \rangle$ away from commensurate fillings.

Next we turn on the nearest neighbor repulsion $W/U = 0.1$. In the absence of rotation, CDW phases appear between two neighboring commensurate MI phases at half-integer fillings when $t/U$ is small. At large values of $t/U$, SF phases stabilize as usual. Between the CDW and the SF phases, the mean field theory gives a small area of the coexistence of CDW and SF order, i.e., the supersolid phase. These are shown in Fig. 9(a) with the equal density lines around $\langle N \rangle = 1.5$. It is well known that the hard-core boson model can be mapped into spin 1/2 XXZ model in a magnetic field, and thus the CDW and SF orders can be unified in a 3-vector pseudospin picture. With the releasing of the hard core constraint, the above mapping is still approximately valid in the sense that the particle density in the core can lie. Another possible realization is the Josephson junction array. The non-uniform charge distributions in the vortex configuration result in electric fields. Thus it is possible to determine the filling in the vortex core with respect to the outside by measuring electric field distributions.

In summary, we have studied vortex structures of the strong coupling boson systems. Near the SF-MI transition, the vortex core is more strongly coupled compared to the bulk area and is thus closer to the MI phase with suppressed SF order. The particle density in the core can be either the maximum or the minimum of the whole system, always approaching the nearest commensurate density of the Mott insulator. Near the SF-CDW transition, a superfluid meron-like vortex is found with a CDW core. All of these strong coupling vortex configurations evolve to the conventional weak coupling one as $t/U$ increases.

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