Nonlinear self-duality in $\mathcal{N} = 2$ supergravity

Sergei M. Kuzenko

School of Physics M013, The University of Western Australia
35 Stirling Highway, Crawley W.A. 6009, Australia
sergei.kuzenko@uwa.edu.au

Abstract

For nonlinear models of an Abelian vector supermultiplet coupled to $\mathcal{N} = 2$ supergravity in four dimensions, we formulate the self-duality equation which expresses invariance under U(1) duality rotations. In the flat space limit, this equation reduces to the $\mathcal{N} = 2$ self-duality equation proposed in hep-th/0001068. We also give an example of a self-dual locally supersymmetric model containing a higher-derivative extension of the Born-Infeld action at the component level.
Given a model for nonlinear electrodynamics described by a Lagrangian $L(F_{ab})$, its invariance under $U(1)$ duality rotations is known to be equivalent to the requirement that the Lagrangian should obey the self-duality equation

$$G^{ab} \tilde{G}_{ab} + F^{ab} \tilde{F}_{ab} = 0 ,$$

where

$$\tilde{G}_{ab}(F) := \frac{1}{2} \varepsilon_{abcd} G^{cd}(F) = 2 \frac{\partial L(F)}{\partial F_{ab}} , \quad G(F) = \tilde{F} + O(F^3) .$$

This equation was originally derived by Gibbons and Rasheed in 1995 [1]. Two years later, it was re-derived by Gaillard and Zumino [2] building on on their 1981 work [3]. Such self-dual theories possess interesting properties [2, 4] reviewed in [5] (see also [6] for a more recent review). In particular, the action functional is automatically invariant under Legendre transformation. The self-duality equation (1) can be reformulated to be suitable for theories with higher derivatives [5].

The concept of self-dual nonlinear electrodynamics was generalized to the cases of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ rigid supersymmetric theories in [7]. This generalization has turned out to be very useful, since the families of actions obtained include all the known models for partial breaking of supersymmetry based on the use of a vector Goldstone multiplet. In particular, the $\mathcal{N} = 1$ supersymmetric Born-Infeld action [8], which is a Goldstone multiplet action for partial supersymmetry breakdown $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ [9, 10] is, at the same time, a solution to the $\mathcal{N} = 1$ self-duality equation [5, 7]. Furthermore, the model for partial breaking of supersymmetry $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ [11], which nowadays is identified with the $\mathcal{N} = 2$ supersymmetric Born-Infeld action, was first constructed in [5] as a unique solution to the $\mathcal{N} = 2$ self-duality equation possessing a nonlinearly realized central charge symmetry.

Models for self-dual nonlinear $\mathcal{N} = 1$ supersymmetric electrodynamics [5, 7] were generalized to supergravity in [12]. For several years, however, an extension to the case of $\mathcal{N} = 2$ supergravity was not feasible to achieve, due to non-existence of a useful superspace formulation for $\mathcal{N} = 2$ supergravity-matter systems (although bits and pieces of curved superspace constructions had been known for quite a while). Such a formulation has recently been developed [13, 14]. Using the results of these and related works [15, 16], in this note we present a general setting for duality invariant theories of an Abelian $\mathcal{N} = 2$ vector multiplet coupled to $\mathcal{N} = 2$ supergravity. Throughout this paper, we will use the superspace formulation for $\mathcal{N} = 2$ conformal supergravity developed in [13] and based on the curved superspace geometry introduced by Grimm [17]; its salient points are summarized in the Appendix. Our results can naturally be extended to more general superspace formulations for $\mathcal{N} = 2$ conformal supergravity,
with larger structure groups of the curved superspace, which were developed by Howe [18] and Butter [19].

The Abelian vector multiplet coupled to $\mathcal{N} = 2$ conformal supergravity can be described by its covariantly chiral field strength $W$,

$$\mathcal{D}_{\alpha i} W = 0 ,$$

subject to the Bianchi identity\[13, 18\]

$$\left( \mathcal{D}^{ij} + 4S^{ij} \right) W = \left( \overline{\mathcal{D}}^{ij} + 4\overline{S}^{ij} \right) \overline{W} ,$$

where $\mathcal{D}^{ij} := \mathcal{D}^{\alpha(i} \mathcal{D}^{\alpha j)}$ and $\overline{\mathcal{D}}^{ij} := \overline{\mathcal{D}}^{\dot{\alpha}(i} \overline{\mathcal{D}}^{\dot{\alpha} j)}$; $S^{ij}$ and its conjugate $\overline{S}^{ij}$ are special dimension-1 components of the torsion, see the Appendix. In the flat superspace limit these relations reduce to those given in [20].

There are several ways to realize $W$ as a gauge invariant field strength. In this paper, we will solely use a curved-superspace extension of Mezincescu’s prepotential [21] (see also [22]), $V_{ij} = V_{ji}$, which is an unconstrained real SU(2) triplet, $(V_{ij})^* = \varepsilon^{ik}\varepsilon^{jl}V_{kl}$. The expression for $W$ in terms of $V_{ij}$ was shown in [16] to be

$$W = \bar{\Delta} \left( \mathcal{D}^{ij} + 4S^{ij} \right) V_{ij} .$$

Here $\bar{\Delta}$ is the covariantly chiral projection operator [23]

$$\bar{\Delta} = \frac{1}{96} \left( (\mathcal{D}^{ij} + 16\overline{S}^{ij}) \mathcal{D}_{ij} - (\mathcal{D}^{\dot{\alpha}\dot{\beta}} - 16\overline{Y}^{\dot{\alpha}\dot{\beta}}) \mathcal{D}_{\dot{\alpha}\dot{\beta}} \right) = \frac{1}{96} \left( \bar{\mathcal{D}}_{ij} (\mathcal{D}^{ij} + 16S^{ij}) - \bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}} (\mathcal{D}^{\dot{\alpha}\dot{\beta}} - 16\overline{Y}^{\dot{\alpha}\dot{\beta}}) \right) ,$$

where $\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} := \bar{\mathcal{D}}^{(\dot{\alpha} \dot{\beta})k}$; $\overline{Y}^{\dot{\alpha}\dot{\beta}}$ is a special dimension-1 component of the torsion, see the Appendix. The fundamental property of $\bar{\Delta}$ is that $\bar{\Delta} U$ is covariantly chiral, for any scalar and isoscalar superfield $U$, that is $\bar{\mathcal{D}}^{ij} \bar{\Delta} U = 0$. This operator relates an integral over the full superspace to that over its chiral subspace:

$$\int d^4x\, d^4\theta\, d^4\bar{\theta}\, E\, U = \int d^4x\, d^4\theta\, E\, \bar{\Delta} U , \quad E^{-1} = \text{Ber}(E_A) ,$$

with $E$ the chiral density, see [15] for a derivation.

Let $S[W, \bar{W}]$ be an action functional describing the dynamics of the $\mathcal{N} = 2$ vector multiplet. Suppose that $S[W, \bar{W}]$ can be unambiguously defined as a functional of unconstrained (anti) chiral superfields $W$ and $\bar{W}$. Then, one can introduce a covariantly chiral superfield $M$ as

$$i\, M := 4\frac{\delta}{\delta W} S[W, \bar{W}] , \quad \bar{\mathcal{D}}_{\alpha i} M = 0 ,$$

\footnote{Such a superfield is often called reduced chiral.}
where the variational derivative $\delta S/\delta W$ is defined by

$$\delta S = \int d^4xd^4\theta \mathcal{E} \delta W \frac{\delta S}{\delta W} + \text{c.c.} \quad (9)$$

In terms of $M$ and its conjugate $\bar{M}$, the equation of motion for $V_{ij}$ is

$$\left(D^{\alpha(i}\mathcal{D}_{\alpha}^{i)} + 4S^{ij}\right)M = \left(D_{\alpha}^{(i}\mathcal{D}^{j)\alpha} + 4S^{ij}\right)\bar{M} \quad (10)$$

Here we have used the representation (5).

Consider an infinitesimal super-Weyl transformation of the covariant derivatives [13] given by

$$\delta_{\sigma}D_{\alpha}^{i} = \frac{1}{2}\sigma D_{\alpha}^{i} + (D^{\gamma\iota}\sigma)M_{\gamma\alpha} - (D_{\alpha k}\sigma)J^{ki} \quad (11a)$$

$$\delta_{\sigma}\bar{D}_{\dot{\alpha}i} = \frac{1}{2}\sigma \bar{D}_{\dot{\alpha}i} + (\bar{D}_{\dot{\alpha}i}\sigma)\bar{M}_{\dot{\alpha}\dot{\beta}} + (\bar{D}_{\dot{\alpha}i}\sigma)J_{\dot{\alpha}\dot{\beta}} \quad (11b)$$

where the parameter $\sigma$ is an arbitrary covariantly chiral superfield, $\bar{D}_{\dot{\alpha}i} \sigma = 0$. The Lorentz generators, $M_{\alpha\beta}$ and $\bar{M}_{\dot{\alpha}\dot{\beta}}$, and the SU(2) generators, $J_{ij}$, are defined in the Appendix. Under the transformation (11), $W$ varies as [13]

$$\delta_{\sigma}W = \sigma W \quad (12)$$

This transformation law is induced by the following variation of Mezincescu’s prepotential:

$$\delta_{\sigma}V_{ij} = -(\sigma + \bar{\sigma})V_{ij} \quad (13)$$

We assume that the action $S[W, \bar{W}]$ is super-Weyl invariant $\delta_{\sigma}S[W, \bar{W}] = 0$. Making use of (12) and the super-Weyl transformation of the chiral density $\mathcal{E}$ [13], $\delta_{\sigma}\mathcal{E} = -2\sigma \mathcal{E}$, we obtain the super-Weyl transformation of $M$:

$$\delta_{\sigma}M = \sigma M \quad (14)$$

Since the Bianchi identity (4) and the equation of motion (10) have the same functional form, and also since the super-Weyl transformation laws (12) and (14) are identical, one can consider infinitesimal U(1) duality transformations

$$\delta W = \lambda M, \quad \delta M = -\lambda W \quad (15)$$

$^{2}$The full superspace density, $\mathcal{E}$, is inert under the super-Weyl transformations [13], $\delta_{\sigma}\mathcal{E} = 0$. The left-hand side of (17) is super-Weyl invariant if $\delta_{\sigma}U = 0$. This implies that $\delta_{\sigma}(\Delta U) = 2\sigma \Delta U$. 

3
with \( \lambda \) a constant parameter. In complete analogy with the rigid supersymmetric case \([5, 7]\), the theory with action \( S[W, \bar{W}] \) can be shown to be duality invariant if the following reality condition holds

\[
\text{Im} \int d^4x \, d^4\theta \, \mathcal{E} \left( W^2 + M^2 \right) = 0 .
\] (16)

In the flat superspace limit, this reduces to the \( N = 2 \) self-duality equation \([5, 7]\).

Any solution \( S[W, \bar{W}] \) of the self-duality equation (16) describes a vector multiplet model in curved superspace which is invariant under U(1) duality rotations. Properties of such locally supersymmetric theories are analogous to those in flat superspace \([5, 7]\). The key observation is that the action itself is not duality invariant, but

\[
\delta \left( S - \frac{i}{8} \int d^4x \, d^4\theta \, \mathcal{E} \, W \, M + \frac{i}{8} \int d^4x \, d^4\bar{\theta} \, \bar{\mathcal{E}} \, \bar{W} \, \bar{M} \right) = 0 .
\] (17)

The invariance of the latter functional under a finite U(1) duality rotation by \( \pi/2 \), is equivalent to the self-duality of \( S \) under Legendre transformation,

\[
S[W, \bar{W}] - \frac{i}{4} \int d^4x \, d^4\theta \, \mathcal{E} \, W W_D + \frac{i}{4} \int d^4x \, d^4\bar{\theta} \, \bar{\mathcal{E}} \, \bar{W} W_D = S[W_D, W_D] ,
\] (18)

where \( W_D \) is the dual chiral field strength,

\[
W_D = \bar{\Delta}(D_{ij} + 4S^{ij}) V_D^{ij} ,
\] (19)

with \( V_D^{ij} \) a real unconstrained prepotential.

Suppose that the action of our self-dual theory, \( S[W, \bar{W}; g] \), depends on a duality invariant parameter \( g \). Then, the functional \( \partial S/\partial g \) is duality invariant. The proof of this result is analogous to the non-supersymmetric or rigid supersymmetric cases, see e.g. \([3]\). Let us consider the supercurrent of the theory,

\[
\mathcal{J} = \frac{\delta S}{\delta \mathcal{H}} ,
\] (20)

where \( \mathcal{H} \) is the real scalar prepotential describing the Weyl multiplet of \( N = 2 \) supergravity, see \([24]\) for more details. Since \( \mathcal{H} \) is duality-invariant, we conclude that the supercurrent of any self-dual theory is duality invariant.

We now give an example of a self-dual theory. It is described by the action

\[
S = \frac{1}{4} \int d^4x \, d^4\theta \, \mathcal{E} \, X + \frac{1}{4} \int d^4x \, d^4\bar{\theta} \, \bar{\mathcal{E}} \, \bar{X} ,
\] (21)

where the chiral superfield \( X \) is a functional of \( W \) and \( \bar{W} \) defined via the constraint

\[
X = \frac{X}{Z^2} \frac{\bar{X}}{\bar{Z}^2} + \frac{1}{2} W^2 .
\] (22)
Here $Z$ denotes the chiral field strength of a vector multiplet which is chosen to be one of the two compensators of $\mathcal{N} = 2$ Poincaré supergravity\(^3\). The superfield $Z$ is reduced chiral, i.e. it obeys the same equations which $W$ is subject to, (3) and (4). The superfield $X$ can be expressed in terms of $W$, $\bar{W}$ and their derivatives by iteratively solving the equation (22) with $1/Z$ considered as a small parameter. In the limit $Z \to \infty$, the above action reduces to that describing Maxwell’s action coupled to $\mathcal{N} = 2$ conformal supergravity

$$S_{\text{Maxwell}} = \frac{1}{8} \int d^4x \, d^4\theta \, \mathcal{E} W^2 + \frac{1}{8} \int d^4x \, d^4\bar{\theta} \, \bar{\mathcal{E}} \bar{W}^2.$$  

(23)

The fact that the system defined by eqs. (21) and (22) is a solution of the self-duality equation (16), can be established by analogy with the rigid-supersymmetric proof given in [7]. In the rigid supersymmetric limit, the system (21) and (22) reduces to the one proposed by Ketov [26]. The latter is a higher derivative extension of the $\mathcal{N} = 2$ supersymmetric Born-Infeld action proposed in [5, 11] as the model for partial breaking of supersymmetry $\mathcal{N} = 4 \to \mathcal{N} = 2$ (the model proposed in [5, 11] is not yet known in a closed form).

Perturbative nonlinear solutions of the self-duality equation (16) may be constructed similarly to the rigid supersymmetric case [5, 27]. We believe our results will be useful in the context of perturbative construction of nonlinear deformations of all classically duality invariant theories, including $\mathcal{N} = 8$ supergravity (see [27, 28] and references therein).

Acknowledgements: The author is grateful to Daniel Butter for reading the manuscript. This work is supported in part by the Australian Research Council.

A  $\mathcal{N} = 2$ conformal supergravity

This appendix contains a summary of the superspace formulation for $\mathcal{N} = 2$ conformal supergravity developed in [13].

Conformal supergravity can be realized in a four-dimensional curved $\mathcal{N} = 2$ superspace parametrized by local coordinates $z^M = (x^m, \theta_\mu, \bar{\theta}_{\dot{\mu}} = (\theta_{\mu})^*)$, where

\(^3\)Within the superconformal tensor calculus, $\mathcal{N} = 2$ Poincaré supergravity is obtained by coupling conformal supergravity to two compensators, of which one is a vector multiplet, and the other can be, e.g., a hypermultiplet or a tensor multiplet, see [26] and references therein.
\( m = 0, 1, \ldots, 3, \mu = 1, 2, i = 1, 2 \). The structure group is chosen to be \( \text{SL}(2, \mathbb{C}) \times \text{SU}(2) \), and the covariant derivatives \( \mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_i^\alpha, \bar{\mathcal{D}}_i^{\dot{\alpha}}) \) read

\[
\mathcal{D}_A = E_A + \Phi_A^{kl} J_{kl} + \frac{1}{2} \Omega_A^{bc} M_{bc} \\
= E_A + \Phi_A^{kl} J_{kl} + \Omega_A^{\beta \gamma} M_{\beta \gamma} + \bar{\Omega}_A^{\dot{\beta} \dot{\gamma}} \bar{M}_{\dot{\beta} \dot{\gamma}}. \tag{A.1}
\]

Here \( M_{cd} \) and \( J_{kl} \) are the generators of the Lorentz and \( \text{SU}(2) \) groups respectively, and \( \Omega_A^{bc} \) and \( \Phi_A^{kl} \) the corresponding connections. The action of the generators on the covariant derivatives are defined as:

\[
[M_{\alpha \beta}, \mathcal{D}^i_\gamma] = \varepsilon_\gamma (\alpha \mathcal{D}^i_\beta), \quad [\bar{M}_{\dot{\alpha} \dot{\beta}}, \bar{\mathcal{D}}^i_{\dot{\gamma}}] = \varepsilon_{\dot{\gamma}} (\dot{\alpha} \bar{\mathcal{D}}^i_{\dot{\beta}}) \tag{A.2a}
\]

\[
[J_{kl}, \mathcal{D}^i_\alpha] = -\delta^i_{(k} \mathcal{D}^i_{l)}, \quad [J_{kl}, \bar{\mathcal{D}}^i_{\dot{\alpha}}] = -\varepsilon_{i(k} \bar{\mathcal{D}}^i_{l)}. \tag{A.2b}
\]

The algebra of covariant derivatives is [13]

\[
\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = 4S^{ij} M_{\alpha \beta} + 2\varepsilon^{ij} \varepsilon_{\alpha \beta} Y^{\gamma \delta} M_{\gamma \delta} + 2\varepsilon^{ij} \varepsilon_{\alpha \beta} W^{\gamma \delta \delta} M_{\gamma \delta} \\
+ 2\varepsilon_{\alpha \beta} \varepsilon^{ij} S_{kl} J_{kl} + 4Y_{\alpha \beta} J^{ij}, \tag{A.3a}
\]

\[
\{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}^j_{\dot{\beta}}\} = -2i\delta^{i}_{(j} \sigma^{\alpha \dot{\beta}} \mathcal{D}^{j}_{\alpha \beta} + 4\delta^{i}_{\dot{j}} G_{\dot{\alpha} \dot{\beta}} M_{\alpha \delta} + 4\delta^{i}_{\dot{j}} G_{\dot{\alpha} \dot{\beta}} M_{\dot{\gamma} \dot{\delta}} + 8G_{\dot{\alpha} \dot{\beta}} J^{j}. \tag{A.3b}
\]

Here the real four-vector \( G_{\alpha \beta} \), the complex symmetric tensors \( S^{ij} = S^{ji}, W_{\alpha \beta} = W_{\beta \alpha}, Y_{\alpha \beta} = Y_{\beta \alpha} \) and their complex conjugates \( S_{ij} := S^{ij}, \bar{W}_{\dot{\alpha} \dot{\beta}} := \bar{W}_{\dot{\beta} \dot{\alpha}}, \bar{Y}_{\dot{\alpha} \dot{\beta}} := \bar{Y}_{\dot{\beta} \dot{\alpha}} \) obey additional differential constraints implied by the Bianchi identities [17] [13].

References

[1] G. W. Gibbons and D. A. Rasheed, “Electric-magnetic duality rotations in nonlinear electrodynamics,” Nucl. Phys. B454, 185 (1995) [arXiv:hep-th/9506035].

[2] M. K. Gaillard and B. Zumino, “Self-duality in nonlinear electromagnetism,” in Supersymmetry and Quantum Field Theory, J. Wess and V. P. Akulov (Eds.), Springer Verlag, 1998, p. 121 [arXiv:hep-th/9705226].

[3] M.K. Gaillard and B. Zumino, “Duality rotations for interacting fields,” Nucl. Phys. B193, 221 (1981).

[4] M.K. Gaillard and B. Zumino, Nonlinear electromagnetic self-duality and Legendre transformations, in Duality and Supersymmetric Theories, D.I. Olive and P.C. West eds., Cambridge University Press, 1999, p. 33 [hep-th/9712103].

[5] S. M. Kuzenko and S. Theisen, “Nonlinear self-duality and supersymmetry,” Fortsch. Phys. 49, 273 (2001) [arXiv:hep-th/0007231].

[6] P. Aschieri, S. Ferrara and B. Zumino, “Duality rotations in nonlinear electrodynamics and in extended supergravity,” Riv. Nuovo Cim. 31, 625 (2009) [arXiv:0807.4039 [hep-th]].
[7] S. M. Kuzenko and S. Theisen, “Supersymmetric duality rotations,” JHEP 0003, 034 (2000) [arXiv:hep-th/0001068].

[8] S. Cecotti and S. Ferrara, “Supersymmetric Born-Infeld Lagrangians,” Phys. Lett. B 187, 335 (1987).

[9] J. Bagger and A. Galperin, “A new Goldstone multiplet for partially broken supersymmetry,” Phys. Rev. D 55, 1091 (1997) [arXiv:hep-th/9608177].

[10] M. Roček and A. A. Tseytlin, “Partial breaking of global D = 4 supersymmetry, constrained superfields, and 3-brane actions,” Phys. Rev. D 59, 106001 (1999) [arXiv:hep-th/9811232].

[11] S. Bellucci, E. Ivanov and S. Krivonos, “Towards the complete N = 2 superfield Born-Infeld action with partially broken N = 4 supersymmetry,” Phys. Rev. D 64, 025014 (2001) [arXiv:hep-th/0101195].

[12] S. M. Kuzenko and S. A. McCarthy, “Nonlinear self-duality and supergravity,” JHEP 0302, 038 (2003) [hep-th/0212039].

[13] S. M. Kuzenko, U. Lindström, M. Roček and G. Tartaglino-Mazzucchelli, “4D N=2 supergravity and projective superspace,” JHEP 0809, 051 (2008) [arXiv:0805.4683].

[14] S. M. Kuzenko, U. Lindström, M. Roček and G. Tartaglino-Mazzucchelli, “On conformal supergravity and projective superspace,” JHEP 0908, 023 (2009) [arXiv:0905.0063 [hep-th]].

[15] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Different representations for the action principle in 4D N = 2 supergravity,” JHEP 0904, 007 (2009) [arXiv:0812.3464 [hep-th]].

[16] D. Butter and S. M. Kuzenko, “New higher-derivative couplings in 4D N = 2 supergravity,” JHEP 1103, 047 (2011) [arXiv:1012.5153 [hep-th]].

[17] R. Grimm, “Solution of the Bianchi identities in SU(2) extended superspace with constraints,” in Unification of the Fundamental Particle Interactions, S. Ferrara, J. Ellis and P. van Nieuwenhuizen (Eds.), Plenum Press, New York, 1980, pp. 509-523.

[18] P. S. Howe, “Supergravity in superspace,” Nucl. Phys. B 199, 309 (1982).

[19] D. Butter, “N=2 conformal superspace in four dimensions,” JHEP 1110, 030 (2011) [arXiv:1103.5914 [hep-th]].

[20] R. Grimm, M. Sohnius and J. Wess, “Extended supersymmetry and gauge theories,” Nucl. Phys. B 133, 275 (1978).

[21] L. Mezincescu, “On the superfield formulation of O(2) supersymmetry,” Dubna preprint JINR-P2-12572 (June, 1979).

[22] P. S. Howe, K. S. Stelle and P. K. Townsend, “Supercurrents,” Nucl. Phys. B 192, 332 (1981).

[23] M. Müller, Consistent Classical Supergravity Theories, (Lecture Notes in Physics, Vol. 336), Springer, Berlin, 1989.

[24] S. M. Kuzenko and S. Theisen, “Correlation functions of conserved currents in N = 2 superconformal theory,” Class. Quant. Grav. 17, 665 (2000) [hep-th/9907107].

[25] B. de Wit, R. Philippe and A. Van Proeyen, “The improved tensor multiplet in N = 2 supergravity,” Nucl. Phys. B 219, 143 (1983).
[26] S. V. Ketov, “A manifestly N=2 supersymmetric Born-Infeld action,” Mod. Phys. Lett. A 14, 501 (1999) [hep-th/9809121]; “Born-Infeld-Goldstone superfield actions for gauge fixed D5- and D3-branes in 6d,” Nucl. Phys. B 553, 250 (1999) [hep-th/9812051].

[27] J. J. M. Carrasco, R. Kallosh and R. Roiban, “Covariant procedures for perturbative non-linear deformation of duality-invariant theories,” Phys. Rev. D 85, 025007 (2012) [arXiv:1108.4390 [hep-th]].

[28] J. Broedel, J. J. M. Carrasco, S. Ferrara, R. Kallosh and R. Roiban, “N=2 supersymmetry and U(1)-duality,” arXiv:1202.0014 [hep-th].