Effect of splitting of the neutron and proton effective mass on nuclear symmetry energy at finite temperature

Li Ou\textsuperscript{a,b,c}, Zhuxia Li\textsuperscript{a,d}, Yingxun Zhang\textsuperscript{e}, Min Liu\textsuperscript{b}

\textsuperscript{a}China Institute of Atomic Energy, Beijing, 102413, P. R. China
\textsuperscript{b}College of Physics and Technology, Guangxi Normal University, Guilin, 541004, P. R. China
\textsuperscript{c}onlyouil@gmail.com
\textsuperscript{d}lizwux@ciae.ac.cn

Abstract

We present the temperature and density dependence of symmetry energy for nuclear matter at finite temperature based on the approach of the thermodynamics with Skyrme energy density functional. We first classify the Skyrme interactions into 7 groups according to the range of neutron and proton effective mass in neutron matter limit (99.99 per cent neutron in the matter). We find that there is obvious correlation between the temperature dependence of the symmetry energy and the splitting of the neutron and proton effective mass. For some Skyrme interactions with $m_{n}^{*} > m_{p}^*$ and strong splitting of the neutron and proton effective mass in asymmetric nuclear matter, a transition of the temperature dependence of symmetry energy from decreasing with temperature at low densities to increasing with temperature at high densities appears. For other Skyrme interactions, we do not observe such phenomenon. Our study show that the symmetry energy in hot asymmetric matter not only depends on symmetry potential part but also on the splitting of the neutron and proton effective mass to a certain extent.

Keywords: nuclear symmetry energy, nuclear symmetry energy at finite temperature, Skyrme density functional, Splitting of neutron and proton effective mass

The symmetry energy for nuclear matter $E_{\text{sym}}$ is very important for understanding not only the structure of nuclei far from $\beta$ stability line, the dynamics and many interesting phenomena in reactions of isospin asymmetric nuclei but also many critical issues in astrophysics\cite{1}. Recent research on the symmetry energy for cold nuclear matter has already obtained a great progress and some constraints on the density dependence of the symmetry energy at subnormal densities have been set \cite{2, 3, 4, 5, 6}. But the density dependence of the symmetry energy, the temperature dependence of the symmetry energy at fixed temperature $T$ and density $\rho$.

In addition to the symmetry energy, the splitting of the neutron and proton effective mass (EFM) is also a hot topic in the study of the properties of asymmetric nuclear matter\cite{13, 14}. The neutron and proton EFM splitting predicted by different effective interactions are rather different\cite{9, 13, 17, 18, 19, 20}. Both the symmetry energy and the neutron and proton EFM splitting closely relate to the isospin dependence of the nuclear interactions. Then one would naturally ask whether the splitting of the neutron and proton EFM influences the temperature and density dependence of the symmetry energy? In this letter we will first study the symmetry energy at finite temperature and then investigate the correlation between the EFM splitting and the density and temperature dependence of symmetry energy. The approach of thermodynamics with Skyrme energy density functional is applied in this work.

The Skyrme energy density functional reads

\[
H = \frac{\hbar^2}{2m_n} (\frac{\partial}{\partial \rho_n} + \frac{\partial}{\partial \rho_p})(\tau_p + \tau_n) \\
+ \frac{1}{12} \rho_0 [2(2 + x_0) \rho^2 - (2x_0 + 1)(\rho_p^2 + \rho_n^2)] \\
+ \frac{1}{12} \rho [2(2 + x_3) \rho^2 - (2x_3 + 1)(\rho_p^2 + \rho_n^2)] \\
+ \frac{1}{4} \rho (2(2 + x_1) + t_2(2 + x_2)) \tau_p \\
+ \frac{1}{4} \rho (2(2 + x_1) - t_1(2x_1 + 1))(\tau_p \rho_p + \tau_n \rho_n),
\]

where $\rho = \rho_n + \rho_p$, and $\tau = \tau_n + \tau_p$. The $\rho_q$ and $\tau_q$ ($q$ denotes

\begin{equation}
E(\rho, T, \delta) = E(\rho, T, \delta = 1) - E(\rho, T, \delta = 0).
\end{equation}
protons or neutrons) are computed by

\[ \rho_q = 2 \int_0^{\infty} n_q(p) \frac{4 \pi p^2}{m_q^*} dp, \]  
\[ \text{(3)} \]

and

\[ \tau_q = 2 \int_0^{\infty} n_q(p) \frac{\pi p^2}{m_q^*} dp. \]  
\[ \text{(4)} \]

The occupation number distribution for species q, \( n_q(p) \), obeys the Fermi-Dirac distribution function, which reads

\[ n_q(p) = \frac{1}{1 + \exp[\beta (\varepsilon_q - \mu_q)]}. \]  
\[ \text{(5)} \]

\( \varepsilon_q = \frac{p^2}{2m_q^*} + U_q \) is the single particle energy for species q. The EFM \( m_q^* \) for species q is expressed as

\[ \frac{m_q^*}{m_q} = \frac{1}{2} h_0[(2 + x_0)p - (1 + 2x_0)\rho_q] + \frac{1}{2} h_1[(2 + x_1)(2 + \alpha)p^{\alpha+1} - (1 + 2x_1)[2\rho^\alpha \rho_q + \alpha p^{\alpha-1}(\rho_q^\alpha + \rho^\alpha)]] + g_1[(2 + x_1) + x_2(2 + x_2)] + \frac{1}{2} g_2[(2 + x_2) + (2 + x_1)] \tau_q. \]  
\[ \text{(6)} \]

The mean field \( U_q \) for species q reads

\[ U_q(r) = \frac{1}{2} h_0[(2 + x_0)p - (1 + 2x_0)\rho_q] + \frac{1}{2} h_1[(2 + x_1)(2 + \alpha)p^{\alpha+1} - (1 + 2x_1)[2\rho^\alpha \rho_q + \alpha p^{\alpha-1}(\rho_q^\alpha + \rho^\alpha)]] + g_1[(2 + x_1) + x_2(2 + x_2)] + \frac{1}{2} g_2[(2 + x_2) + (2 + x_1)] \tau_q. \]  
\[ \text{(7)} \]

Introducing effective chemical potential \( \mu_q' = \mu_q - U_q \), then

\[ n_q(p) = \frac{1}{1 + \exp[\beta (p^2/2m_q^* - \mu_q')]} \].  
\[ \text{(8)} \]

By solving equation \((3)\) and \((8)\) with EFM depending on density iteratively, for any pair of \( \mu_q \) and \( \mu_q' \), we can obtain the proton and neutron density \( \rho_q \) and \( \rho_q' \) at temperature \( T \). Then, the energy per nucleon in neutron matter(NM) and SM can be calculated by using equation \((3)\) and \((7)\) and finally symmetry energy at finite temperature can be calculated.

Since the nucleon EFM is explicitly involved in the iteration process we first make a general survey of the density dependence of the neutron, proton EFM for various Skyrme interactions. 94 Skyrme interactions are tested and they can be divided into 7 groups according to their corresponding ranges of the neutron and proton EFMes varying with density. Table I presents the groups of Skyrme interactions and the corresponding ranges of \( m^*/m \) in SM and NM. The \( m \) is the bare nucleon mass. In this work, the NM is always referred to a nuclear matter limit with 99.99 percent neutrons in the matter. For convenience, the EFM \( m^* \) discussed in the following text always refer to the ratio \( m^*/m \) (the scaled effective mass), which should not be confused with the EFM \( m_q^* \) appeared in expression \((6)\) and \((8)\). In SM, the neutron and proton EFM are equal and all Skyrme interactions are simply in the groups: (I) \( m^* > 1 \) and it increases with density increasing; (II) \( m^* = 1 \) and it is independent on density; and (III) \( m^* < 1 \) and it decreases with density increasing. Most of Skyrme interactions belong to the group (III). For asymmetric nuclear matter, the situation becomes more complex because of neutron-proton EFM splitting. The splitting of the neutron and proton EFM in asymmetric matter is currently not known empirically[9]. Theoretical results on the splitting of the neutron and proton EFM are highly controversial among different approaches and different effective interactions. One can see from Table I that only few Skyrme interactions belong to groups I,II, and III and most of Skyrme interactions belong to groups IV to VII with splitting of neutron and proton EFM. Large part of them belong to groups IV and \( V (m_n^* > m_p^*) \), others belong to groups VI and VII (\( m_n^* < m_p^* \)).

Now let us come to study the density and temperature dependence of the symmetry energy in hot nuclear matter. We show in Fig. 1 the symmetry energies in nuclear matter at temperatures \( T = 0, 5, 10, 20 \) MeV calculated with different Skyrme interactions, respectively, as typical examples. The 9 subfigures in Fig. 1 is ordered according to the magnitude of \( R_m^0 \) for the Skyrme interaction applied. Here we introduce the ratio \( R_m = R_m^0 \), where the subscripts 0 and 1 indicate the isospin asymmetry \( \delta = 0 \) (for SM) and 1 (for NM), respectively, to characterize the strongness of the splitting of the neutron and proton EFM for the Skyrme interaction applied. The quantity \( R_m^0 \) presented in each subfigure is the value of \( R_m \) at normal density. Obviously, the \( R_m \) is proportional to the neutron and proton EFM splitting in asymmetric matter. The values of \( R_m^0 \) for Skyrme interactions in groups IV and \( V \) are always larger than 1 and those in groups VI and VII are smaller than 1. As is known that the symmetry energies for cold nuclear matter calculated from different Skyrme interactions are largely divergent especially at high densities. We can also classify the Skyrme interactions into two groups A and B according to the trend of the density dependence of the symmetry energy in cold matter. For the group A, namely the Skyrme interactions like SLy7, SkT5, SkMP, and SkM, etc., nuclear symmetry energy increases monotonously with density and for the group B, namely the Skyrme interactions like Skz3, SIII, SKXm, SkP, and v070, the nuclear symmetry energy first increases with density until certain density then it bends down as density further.
increases and finally becomes negative. The group A corresponds to the group I and the group B corresponds to group II and III in ref. [21], respectively. It is seen from Fig.1 that there is no obvious correlation between the trend of the density dependence of the symmetry energy in cold matter and the magnitude of the $R^0_m$ of the corresponding Skyrme interactions. As the temperature dependence of the symmetry energy is concerned, one sees that the nuclear symmetry energy decreases with temperature increase in all densities for Skyrme interactions with small $R^0_m$, such as SLy7, Skz3, Skt5, SkM, etc., in consistency with the results given in [9]. But for SkM, SKXm, SkP, and v070 with large $R^0_m$ the symmetry energy decreases with temperature increasing at low density and increases with temperature when the density is higher than a certain density. We call this phenomenon the transition of temperature dependence of the symmetry energy (TrTDSE). From Table 1 one can find that Skyrme interactions such as SkM, SKXm, SkP, and v070 for which the TrTDSE phenomenon appears, all belong to groups IV and V satisfying $m^*_p > m^*_n$. The density for the onset of the TrTDSE depends on the magnitude of the $R^0_m$ of the corresponding Skyrme interactions. The larger $R^0_m$ is, the lower density is for the onset of the TrTDSE. For the case of SIII, we find that the symmetry energy at finite temperature is very close to that of cold matter up to $\rho \leq 4\rho_0$ where our calculation stops. For Skyrme interactions belonging to the groups I-III and VI, VII (not satisfying $m^*_n > m^*_p$), the TrTDSE phenomenon can not happen.

In order to explore the condition of TrTDSE in hot matter more quantitatively we study the difference between the symmetry energy in a matter at temperature $T$ and that in a cold matter, which reads as

$$\Delta E_{\text{sym}}(\rho, T) = E_{\text{sym}}(\rho, T) - E_{\text{sym}}(\rho, 0) = \left( \frac{\rho^2}{m^*_p} + b_1 + b_2(\tau^*_T - \tau^*_0) \right) (1 - \frac{\rho}{\rho_0}),$$  \hspace{1cm} (9)

with $b_1 = \frac{n_1}{4t_1}(2 + x_1) + t_2(2 + x_2)$ and $b_2 = \frac{n_1}{4}[t_2(1 + 2x_2) - t_1(1 + 2x_1)]$. It is easy to extend equation (9) to $E_{\text{sym}}(\rho, \Delta T)$ with $\Delta T = T_2 - T_1$. The $R_0$ in (9) is the ratio between the kinetic energy density increasing from $T = 0$ to $T = T$ in NM and that in SM, $R_0 = \frac{\rho^*_T}{\rho^*_0}$, where the subscripts 0 and 1 indicate the isospin asymmetry $\delta = 0$ and 1, respectively. The first factor in the expression (9) depends on the Skyrme interaction parameters which is always positive up to $\rho = 2\rho_0$ for the 94 Skyrme interactions involved in this work (in fact for some Skyrme interactions this density can be much higher than $2\rho_0$). The second factor is always positive. Consequently the sign of $\Delta E_{\text{sym}}(\rho, T)$ is determined by the third factor. If $R_0/R_T > 1$ the symmetry energy decreases with temperature, otherwise it increases with temperature. The splitting of the neutron and proton EFM influences the $\Delta E_{\text{sym}}(\rho, T)$ explicitly through $R_0$ and indirectly through $R_T$. Next let us investigate how the $R_T$ is eventually also influenced by the EFM splitting. Before coming to this point, we show in Fig. 2 the systematic calculated results with Skz-1, Skz2, and Skz3, respectively. We notice that the predictions from Skz-series Skyrme interactions are the same for the isoscalar part but different for the isovector part of the equation of state and moreover the Skz-1, Skz2, and Skz3 belong to groups IV, V, VI, respectively. Therefore they are very suitable for exploring the correlation between the temperature dependence of the symmetry energy and the nucleon EFM splitting. In Fig. 2 the top panel shows the symmetry dependence of energy densities at $T=0$, 5, 10, 20 MeV, respectively; the second panel shows the energies per nucleon in NM and SM at $T=0$ and 20 MeV, where the energies per nucleon in SM are the same for three interactions because they have the same isoscalar part; The third panel shows the nucleon(neutron) EFMes in SM and NM, and the bottom panel shows the density dependence of $R_m$, $R_0$, and $R_m/R_T$ at $T=20$ MeV, respectively. The results calculated with other Skz-series interactions are similar to the one of these three Skz interactions. We first investigate the calculation results with Skz-1, the strong neutron and proton EFM splitting case with $R^0_m=1.84$, one sees from Fig. 2(c1) that the neutron EFM in NM is larger than 1 and increases with density

| group | SM | NM (99.99 percent neutrons) |
|-------|----|-----------------------------|
| I $m^*_p > m^*_n > 1$ | BSK1~5, MSK2, MSK4~6, SVII, SKXce, v series | BSK1, MSK2, MSK4~6, v105 |
| II $m^*_p = m^*_n = 1$ | MSK1, MSK3, SKp, SKSC1~4, ST1~6 | MSK1, MSK3, SKSC1~4, ST1~6 |
| III $m^*_p > m^*_n < 1$ | BSK4~17, Es, FitB, GS, RAI1P, Rs, SGI, SGIL1~VI, SIII*, SKRA, Sk1l~6, SkM1, SkM, SkM*, SkMP, SkO, ST7~9, SKX, SKXm, Skz series, SLy series, Zs | SkT8, SkT9 |
| IV $m^*_p < 1 < m^*_n$ | BSK2~5, BSK10~13, Es, FitB, SI, SKP, SKX, SKXce, SKXm, Skz-1, Skz0, SVI, SVII, v070, v075, v080, v090, v100 |
| V $m^*_p > m^*_n < 1$ | BSK4~17, GS, RAI1P, Rs, SGI, SGI, SIII, SIVI, SIII*, SKRA, SkM, SkM1, SkMP, SkM*, SkO, ST7, Skz1, Skz2, Zs |
| VI $m^*_n > m^*_p$ | BSK6~9, Sk1l~6, Skz3, Skz4, SLy-series |
| VII $m^*_p > m^*_n > 1$ | v110 |
quickly, the proton EFM is smaller than 1 and decreases with density even faster, and the nucleon EFM in SM is just between them and decreases with density, which leads the $R_{np}$ to increase with density quickly as shown in Fig.2(d1). From Fig.2(b1), one sees that the energy per nucleon in NM increases with density and that in SM decreases with density for both $T=20$ and $T=0$ MeV cases but the increasing slopes in NM and decreasing slopes in SM with density are different for $T=20$ and $T=0$ MeV. As a result, the symmetry energy at $T=20$ MeV becomes higher than that at $T=0$ MeV when $\rho > 0.75\rho_0$ (Fig.2(a1)). It is more evident from Fig.2(d1). For $T=20$ MeV case, the $R_{nm}/R_t$ becomes smaller than one when the density exceeds about $0.75\rho_0$ and therefore the symmetry energy to be larger than that in $T=0$ MeV case from Eq.(9). In the middle and right columns, we show the results calculated with Skz2 and Skz3. For Skz2 ($m_{np}^* > m_n^*$ with $R_{nm}^*=1.13$) and for Skz3 ($m_{np}^* < m_n^*$ with $R_{nm}^*=0.91$), the splitting of the neutron and proton EFM is small and both neutron and proton EFM are close to 1. Accordingly, the $R_{nm}/R_t$ becomes larger than 1 at $T=20$ MeV for both Skz2 and Skz3 cases and thus the symmetry energies for those two cases are all decreasing with temperature in whole densities. The extent of decreasing depends on the magnitude of $R_{nm}/R_t$. The calculation results for $\nu$-series Skyrme interactions have similar behavior as those of Skz-series.

To understand what factors will influence the $R_{np}$, let us start from the expression (5) or (8) for $n_q(p)$. For cold matter, the $n_q(p)$ is a step-function, the momenta of all particles are within the Fermi momentum sphere with $k_{nf} = k_{pf} = (3\pi^2 p/2)^{1/3}$ for SM and $k_{nf} = (3\pi^2 p)^{1/3}$ for NM. For hot matter we know from expressions (5) or (8) that the deviation of $n_q(p)$ from a step-function depends on the density (related to the chemical potential term in (5) and (8)) and nucleon EFM in the matter (related to the kinetic energy of nucleon in (8)) when temperature is fixed. More explicitly, the deviation of $n_q(p)$ for hot matter from a step-function is larger for a system with lower density and larger nucleon EFM. In Fig. 3 the left panel shows the $n_q(p)$ at $\rho = 0.3\rho_0$ and the right panel shows the $n_q(p)$ at $\rho = 1.5\rho_0$ at $T=0$, 20 MeV calculated with Skz-1 and Skz2, respectively. The solid lines in the figure denote $n_q(p)$ for neutrons(protons) in SM and dashed lines denote $n_q(p)$ for neutrons in NM at $T=0$. The excessive kinetic energy needed for conversion of all protons in SM into neutrons in NM comes from the excessive neutrons in $n_q(p)$ in NM as compared with that in SM, which is one of the origin of the symmetry energy for cold matter. For the matter at finite temperature, the calculated results of $n_q(p)$ are shown by the lines with symbols in the figure. The calculation results are just as expected. From Fig.3(a) and Fig.3(c) for low density $\rho = 0.3\rho_0$ case we find that the $n_q(p)$ at $T=20$ MeV for both $\delta = 0$ and $\delta = 1$ deviates from step-function largely but there is no obvious difference between the results calculated with Skz-1 and Skz2 because of low matter density. For high density $\rho = 1.5\rho_0$ case, the results of $n_q(p)$ (right panel) in SM ($\delta = 0$) at $T=20$ MeV calculated with Skz-1 and Skz2 are the same because of the same iso-scalar part of two interactions and both deviate from step-function not largely. While, the results of $n_q(p)$ in NM ($\delta = 1$) calculated with Skz-1 and Skz2 become much different. For the case with Skz2 (Fig.3(d)), the deviation of the nucleon $n_q(p)$ in NM from a step-function is almost the same or even weaker than that in SM and the ratio $R_t$ becomes larger than 1 at $T=20$ MeV to be larger than 1 because $R_{nm}/R_t$ at $T=20$ MeV in NM deviates from that at $T=0$ MeV largely (Fig.3(b)) and its very long tail means that a large portion of neutrons inside the Fermi momentum sphere are excited to the outside of the Fermi momentum sphere and quite large part of them have very large momenta. And consequently the kinetic energy increases largely. Thus, the $R_t$ becomes much large at high density and even larger than the $R_{nm}$ which is larger than one. Finally, $R_{nm}/R_t$ at high densities becomes smaller than 1 and the symmetry energy at high density at $T=20$ MeV is enhanced as
compared with cold matter. From above discussion, we find that the splitting of the neutron and proton EFM influences the $R_c$ through the difference between the $m_n(p)$ in SM and that in NM. This effect on $R_c$ can be even amplified as compared with that on $R_m$ for some Skyrme interactions. Thus, ultimately, the splitting of the neutron and proton EFM influences the trend of the density dependence of symmetry energy in hot matter. For some Skyrme interactions like Skz-1 which has strong splitting of the neutron and proton EFM and $m_n^* > m_p^*$, the onset of the TrTDSE at a certain density, usually at supernormal density, leads the symmetry energy to be stiffer in hot matter than that in cold matter. For others, the splitting of the neutron and proton EFM leads the symmetry energy to be more soft in hot matter than that in cold matter. Generally, if the Skyrme interaction belongs to group IV and V with its $R_m^0$ being much larger than 1, the TrTDSE will occur at certain density. The density for the onset of the TrTDSE depends on the magnitude of the splitting of the neutron and proton EFM in asymmetric matter. Concerning experimentally probing the TrTDSE or the temperature and density dependence of the symmetry energy, it requires to simultaneously measure the observables such as the transverse momentum spectrum of protons(pions or other light charged particles) to determine the temperature and those to be sensitive to symmetry energy at high densities such as the ratio between the yields of different charged pions, $\pi^-/\pi^+$, to extract the symmetry potential at high densities in heavy ion collisions with isospin asymmetric nuclei at energies of hundreds to around 1 GeV per nucleon. At present, highly contradictory results about the symmetry energy at high density extracted from experiments may have some relation with the unclear temperature dependence of the symmetry energy.

In summary, we have investigated the temperature and density dependence of the symmetry energy in hot nuclear matter by means of the approach of the thermodynamics with Skyrme energy density functional. We first classify the Skyrme interactions into 7 groups according to the range of the calculated neutron and proton EFM varying with density in the NM limit. Then we study the temperature dependence of symmetry energy for different Skyrme interactions. We find that for some Skyrme interactions the symmetry energy decreases with temperature at whole density region and it means that the symmetry energy in hot matter is softer than that in cold matter. But for Skyrme interactions belonging to groups IV and V (for these two groups $m_n^* > m_p^*$), the TrTDSE phenomenon may occur at certain density, which makes the symmetry energy at high density to be stiffer. In order to explore the correlation between the neutron and proton EFM splitting and the temperature dependence of the symmetry energy in hot nuclear matter we introduce the ratio $R_m$ between the nucleon EFM in NM and that in SM, which is directly related with the neutron and proton EFM splitting in asymmetric matter and the ratio $R_c$ between the kinetic energy density increasing from $T=0$ to $T$ in NM and that in SM. The effect of the effective mass splitting on $R_m$ is indirect but for some Skyrme interactions it can be amplified as compared with that on $R_m$. Only for those Skyrme interactions with $R_m/R_c < 1$ in group IV and V the TrTDSE phenomenon can occur at high densities and it ultimately makes the trend of the density dependence of symmetry energy to be more stiff in hot matter. The onset density of TrTDSE depends on the magnitude of $R_m^0$. The larger $R_m^0$ is, the lower onset density is. Finally, we would stress that the symmetry energy in hot nuclear matter not only depends on symmetry potential part but also on the splitting of the neutron and proton EFM in kinetic part to a certain extent. Therefore, the study of the temperature dependence of the symmetry energy in addition to the density dependence is a matter of significance for extracting the symmetry potential and also the splitting of the neutron and proton EFM in asymmetric nuclear matter.

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