Quantum speed-up based on classical-field and moving-velocity

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In this work, we provide a method of solving the model of a dissipative moving qubit under the classical field. We obtain the analytic solution of the density operator of the qubit and investigate the quantum speed limit time (QSLT) and the non-Markovianity based on the classical field and the moving velocity of the qubit. The results show that, the transition from Markovian to non-Markovian dynamics is the intrinsic physical reason of the quantum speed-up process, and both of the driving field and the strong-coupling can enhance the non-Markovianity in the dynamics process and speed up the evolution of the qubit, but the moving velocity of the qubit can decrease the non-Markovianity in the dynamics process and delay the evolution of the qubit.

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I. INTRODUCTION

During the past several decades, quantum effect in open quantum systems has always been a hot issue in the quantum optics field and has been extensively studied [1][2]. Most of these researches are based on quantum models of stationary qubits, but a qubit in practical environment has always a tiny movement and how to treat a moving qubit in a dissipative cavity is always a very complicated problem. The authors in Ref. [3] proposed a method of solving this problem which the coupled multimode equations associated with laser oscillation can be reduced to an equation in the usual single-quasimode theory, based on the normal modes for the combined system of a maser cavity coupled to the outside world, in which the mirrors at $z = l$ and $-L$ are completely reflective while the one at $z = 0$ is semitransparent (Fig.1). C. Leonard et al. investigated non-markovian dynamics and spectrum of a moving atom strongly coupled to the field in a damped cavity by means of the single-quasimode theory. A. Mortezapour and D. Park et al. also studied quantum entanglement, non-Markovianity and coherence of this model by assuming $l \to \infty$ [4][6]. Inspired by these works and considered the experimental conditions ($l$ is about 20 cm path) in Ref. [7], we choose a parameter ($l = 23$ cm) consistent with the experimental device, which is the first motive of this paper. Meanwhile, we add an external classical field along the $y$-direction for region $l$ in order to effectively control the quantum effect of a moving qubit, which is the second motive of this paper.

Though the physical mode of a moving qubit under the classical field in open system is a very complicated, we obtain an analytical solution of this qubit by using some approximate conditions, which is the third motive of this paper.

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![FIG.1. Cavity with partially reflecting mirror imbedded in large ideal cavity ($L \to \infty$).](image)

On the other hand, in recent years, the quantum speed limit time and the non-Markovian dynamic process of an open quantum system are have been widely concerned. Quantum speed limit (QSL) can effectually characterize the maximal speed of evolution of a quantum system from a given initial state to a target state [8]. Quantum speed limit time (QSLT) is defined as the minimal evolution time of a quantum system. It plays an important role in many fields of quantum physics, such as quantum optical control [9], quantum communication and quantum commutation [10][12], quantum metrology [13]. For a unitary process, there are two common bounds of the QSLT. One is expressed as $\tau_{qsl} = \pi \hbar/(2\Delta E)$, where $\Delta E$ represents the energy fluctuation of the initial state, which is propose by Mandelstam and Tamm (the MT bound), and the other is $\tau_{qsl} = \pi \hbar/(2\langle E \rangle)$, where $\tau_{qsl}$ depends on the average energy $\langle E \rangle$, which is derive by Margolus and Levitin (the ML bound). By combining the two bounds, the QSLT of the two orthogonal pure states in the closed system is $\tau_{qsl} = \max\{\pi \hbar/(2\Delta E), \pi \hbar/2\langle E \rangle\}$ [14][19]. One often uses the non-Markovianity to quantify non-Markovian eects of dynamical behaviors of an open system. For example, the measures for the non-Markovianity of quantum processes for an open two-level system have been presented in Refs. [20][22]. The non-Markovianity of the dynamics of an open quantum system [23] and the non-Markovian character of colored noisy channels [24] have also been addressed. Meanwhile, some schemes have been proposed to study the QSL of quantum systems in non-Markovian environments. For
instance, Defner and Lutz acquires the unified bound of an open system by using the Bures angle based on the ML and MT bounds, and their result shows that the non-Markovian effects could speed up the quantum evolution[25]. The MT-QSL bound based on the relative purity, the ML-QSL and NI-QSL dependent on initial states as also as the quantum speed limit in a nonequilibrium environment have also been investigated in succession[26–28]. N. Mirkin et al. investigates the QSL bound in terms of the quantum Fisher information, different operator norms and the notion of quantumness, respectively[29]. In addition, many valuable effort have also been devoted to the relationships between the non-Markovianity and the QSL, such as quantum speedup in a memory environment[30] and quantum speedup in open quantum systems[31], and so on. In particular, Y. J. Zhang and W. Han propose a method of accelerating the speed of evolution of an open system by an external classical drifting field of a qubit in a zero-temperature structured reservoir[32].

These investigations mentioned above are mainly focused on the QSLT and the non-Markovianity of stationary qubits. However, how does the velocity of moving qubits act on the QSLT and the non-Markovianity? Can the external classical field regulate the QSLT and the non-Markovianity? These problems are also important and meaningful in experimental researches of open systems.

In this paper we investigated the quantum speedup and the non-Markovianity based on the classical field and the moving velocity. By considering the dissipative moving qubit under the classical field, we find that the classical field can accelerate the evolution of the qubit and increase the non-Markovianity of the dynamics process, but the velocity of the moving qubit can delay the evolution of the qubit and decrease the non-Markovianity. Our results show that the QSLT of a moving qubit can be effectively controlled by the driving strength and the velocity.

This paper is organized as following. In Section 2, we present a physical model and its analytical solution of a moving qubit under the classical field for an open system. In section 3, we introduce the quantum speed limit and the non-Markovianity. In section 4, we give the results and discussions. Finally, we conclude with a brief summary of our work in section 5.

II. PHYSICAL MODEL AND ANALYTICAL SOLUTION

We considered a moving qubit interacting with the multimode cavity, where the qubit is driven by the classical field. We choose a parameter \( l = 23cm \) consistent with the experimental device and assuming \( L \rightarrow \infty \).

In this thesis, the electric field is polarized along the x-direction and qubit moving direction along with the z-direction. Meanwhile, the external classical field acting on the QSLT and the non-Markovianity of stationary qubits affects the QSLT and the non-Markovianity? These problems are also important and meaningful in experimental researches of open systems.

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\[ \hat{H} = \frac{1}{2} \omega_0 \sigma_z + \sum_k \omega_k a_k^\dagger a_k + \Omega e^{-i \omega_L t} \sigma_+ + \Omega e^{i \omega_L t} \sigma_- + \sum_k f_k(z) g_k a_k \sigma_+ + h.c. \]  

where \( \sigma_\pm \) are the Pauli operators, \( \omega_0 \) is the transition frequency of the qubit, \( a_k^\dagger (a_k) \) and \( \omega_L \) are creation (annihilation) operator and the frequency of the k-th mode. In addition, \( g_k \) denotes the coupling constant between the qubit and the k-th mode and \( \Omega \) is the classical driving strength. The parameter \( f_k(z) \) describes the dependency function of the qubit motion along with the z-direction, and it is given by

\[ f_k(z) = f_k(v t) = \sin[k(z-l)] = \sin[\omega_k(\beta t - \tau_0)], \]

where \( \beta = \frac{z}{l} \) and \( \tau_0 = \frac{l}{c} \), \( v \) and \( c \) are respectively the velocities of the moving qubit and the light, \( l \) is the length of the right side cavity[33]. Note that the dependency function is not zero when \( z = 0 \), while it is zero when \( z = l \) (perfect boundary).

FIG.2. Schematic illustration of a setup where a single qubit is moving inside a cavity and driven by the classical field. The qubit is a two-level atom with transition frequency \( \omega_0 \) traveling with constant velocity \( v \).

In the basic states \{ |φ_0 \rangle = |g \rangle_S |0 \rangle_B, |φ_1 \rangle = |e \rangle_S |0 \rangle_B, |φ_k \rangle = |g \rangle_S |1_k \rangle_B \}, |g \rangle_S = \sigma_- |e \rangle_S \) and \( |e \rangle_S = \sigma_+ |g \rangle_S \) indicate the ground and excited states of the qubit, the state \( |0 \rangle_B \) denotes the vacuum state of the cavity and \( |1_k \rangle_B = b_k^\dagger |0 \rangle_B \) represents the cavity state containing one photon in the k-th mode.

Through the unitary transformation, in the interaction picture, the Hamiltonian reads

\[ \hat{H}_I = \sum_k f_k(z) g_k a_k \sigma_+ e^{i (\omega_L + \omega_0 - \omega_k) t} \]

\[ + \Omega \sigma_+ e^{i \omega_L t} + h.c. \]  

Suppose the initial state of the total system is \( |\Psi(0)\rangle = C_0(0) |\phi_0 \rangle + C_1(0) |\phi_1 \rangle \), then at any time \( t > 0 \) the total system state becomes \( |\Psi(t)\rangle = C_0(t) |\phi_0 \rangle + C_1(t) |\phi_1 \rangle + \)

\[ \cdots \]
$\sum_k C_k(t)|\varphi_k\rangle$, the probability amplitudes $C_i(t)(i = 0, 1, k)$ is time dependent. Where $|C_1(0)\rangle^2 + |C_0(0)\rangle^2 = 1$ and $|C_1(t)\rangle^2 + |C_0(t)\rangle^2 + |\sum_k C_k(t)\rangle^2 = 1$. From the Schrödinger equation, we can obtain the differential equations $C_j(t)(j = 1, 2, 3)$ are

$$\dot{C}_1(t) = -i \sum_k f_k(z)g_k C_k(t)e^{i(\omega_L + \omega_0 - \omega_k)t}$$
$$-i\Omega e^{i\omega_dt}C_0(t),$$
$$\dot{C}_0(t) = -i\Omega e^{-i\omega_dt}C_1(t),$$
$$\dot{C}_k(t) = -if_k(z)g_k C_1(t)e^{-i(\omega_L + \omega_0 - \omega_k)t}.$$  

From Eqs. (4-5), one can obtain

$$\dot{C}_1(t) = -\Omega^2 \int_0^t dt_1 C_1(t_1)e^{i\omega(t-t_1)}$$
$$- \int_0^t dt_1 F(t-t_1)C_1(t_1),$$

the correlation function $F(-t_1)$ can be expressed as the following form

$$F(t-t_1) = \int_0^\infty J(\omega_k)\sin[\omega_k(\beta t - \tau_0)]\times$$
$$\sin[\omega_k(\beta t_1 - \tau_0)]e^{i(\omega_L + \omega_0 - \omega_k)(t-t_1)}d\omega_k,$$

where $J(\omega)$ is the spectral density of the reservoir. If the structure of the reservoir has the Lorentzian form

$$J(\omega_k) = \frac{\gamma\lambda^2}{2\pi[(\omega_0 - \omega_k)^2 + \lambda^2]},$$

where $\lambda$ is the spectral width of the reservoir, $\gamma$ is the dissipative rate. The condition $\lambda > 2\gamma$ means the weak-coupling regime, while the condition $\lambda < 2\gamma$ indicates the strong-coupling regime that the non-Markovian effect is very obviously [38-40]. The correlation function $F(-t_1)$ can be calculated as

$$F(t-t_1) = \sum_{i=1}^4 F_i(t-t_1),$$

where

$$F_1(t-t_1) = \xi \exp\{\eta(\beta - 1)(t-t_1)\}$$
$$F_2(t-t_1) = \xi \exp\{-\eta(\beta + 1)(t-t_1)\}$$
$$F_3(t-t_1) = -\xi \exp\{-\eta(\beta(t+t_1) - 2\tau_0 + (t-t_1))\}$$
$$F_4(t-t_1) = -\xi \exp\{\eta(\beta(t+t_1) - 2\tau_0 + (t-t_1))\},$$

and $\xi = \frac{1}{2}\frac{\lambda}{\beta}$. In experiments, the evolution time $t$ is usually $t < 1 \times 10^2$s and $\beta = v/c \sim 10^{-11}$ [11][12]. According to the above conditions, we can known that $\beta(t + t_1)$ can be ignored. So Eq. (10) can be simplified as

$$F(t-t_1) = \xi \left[1 - e^{-2\tau_0}e^{-\eta(\beta-1)(t-t_1)}\right]$$
$$+ \xi \left[1 - e^{2\tau_0}e^{-(\beta+1)\eta(\tau_1-t_1)}\right].$$

Using the Laplace transform, Eq. (7) becomes

$$C_1(0)s - C_1(s) + \xi(1 - \frac{1}{s} + \epsilon_0)C_1(s)$$
$$+ (1 - b)C_1(s) = \Omega^2$$
$$s + \epsilon_1 + \frac{\Omega^2}{1 - i\omega_0}C_1(s),$$

where $\epsilon_0 = \lambda - \chi$ and $\epsilon_1 = \lambda + \chi$. Let $\chi = \beta(\lambda + i\omega_L + i\omega_0)$ and $b = e^{\pi\tau_0(\lambda + i\omega_L + i\omega_0)}$, then the equation Eq. (13) is

$$C_1(s) = \frac{b(s + \epsilon_0)(s + \epsilon_1)}{\Omega^2 s^2 + \epsilon_0^2 + \epsilon_1^2 + \Omega^2}C_1(0) = \frac{A(s)}{B(s)}C_1(0),$$

where $d_1 = \epsilon_0 + \epsilon_1 + \frac{\Omega^2}{1 + \epsilon_0}$, $d_2 = b\epsilon_0\epsilon_1 - a(1 - b)^2 + \frac{\Omega^2}{1 + \epsilon_0}b\epsilon_0+ \epsilon_1$ and $d_3 = a(b - 1)(\epsilon_1 - \epsilon_0) + \frac{\Omega^2}{1 + \epsilon_0}b\epsilon_0$. We can get $C_1(t)$ by using the residue theorem,

$$C_1(t) = C_1(0)\sum_{k=1}^3 \frac{A_k(s)}{B_k(s)}e^{s_k t},$$

here $s_k(k = 1, 2, 3)$ is the root of the equation $B(s) = 0$, the density matrix of the qubit in the basis $|e\rangle, |g\rangle$ is

$$\rho(t) = \left(\begin{array}{cc} |C_1(t)|^2 & C_0(0)C_1(t) \\ C_0(0)^*C_1(t)^* & 1 - |C_1(t)|^2 \end{array}\right),$$

Taking the derivative of Eq. (16), we get

$$\frac{d}{dt}\rho(t) = -i\frac{S(t)}{2}[\sigma_+\sigma_-, \rho(t)] + \frac{\Gamma(t)}{2}\left[2\sigma_-\rho(t)\sigma_+ - \sigma_+\sigma_-. \rho(t) - \rho(t)\sigma_+\sigma_-. \right].$$

This is master equation for the reduced system dynamics. Obviously, $S(t)$ is a time-dependent Lamb shift and $\Gamma(t)$ is a time-dependent the decay rate.

### III. QUANTUM SPEED LIMIT AND NON-MARKOVIANITY

In this section, we will briefly review the definitions of the QSLT and the non-Markovianity for an open quantum system. As a measure of statistical distance between quantum states, the Bures angle is defined as $B(p_0, p_1) = \arccos|F(p_0, p_1)|$, where $F(p_0, p_1) = \text{Tr}[\sqrt{p_0}\sqrt{p_1}]$. In Ref. [25], the Bures angle was simplified as $B(p_0, p_1) = \arccos|\langle\psi_0|\rho_1|\psi_0\rangle|$ in open quantum systems. Here we will introduce the relative purity function to measure the trace distance [38], thus the Bures angle $B(p_0, p_1)$ can be written as

$$B(p_0, p_1) = \arccos\left(\frac{\text{Tr}[p_0\rho_1]}{\sqrt{\text{Tr}[p_0^2]}}\right).$$

Based on the von Neumann trace inequality and the Cauchy-Schwarz inequality, the QSLT is obtained as follows:

$$\tau_{\text{rel}} = \max\left\{\frac{1}{1 + \Gamma}, \frac{1}{1 + \Gamma}, \frac{1}{1 + \Gamma}\right\} \sin^2\left(\text{Tr}[p_0^2]\right),$$

(19)
where $\Omega^{op, tr}$ is the critical value $\Omega$ for the standard evolution process when the driving strength $\Omega$ is bound. Owing to the relationship $\Omega^{op} \leq \Omega^{h.s} \leq \Omega^{tr}$, the greater QSL velocity is $\Omega^{op}$ and QSL bound is $\tau_{op}$. The QSLT in Eq. (10) is the tightest bound,

$$\tau_{qsl} = \frac{1}{2\gamma} \sin^2(B(\rho_0, \rho_1)|\text{Tr}[\rho_0^2]).$$

(20)

The non-Markovianity ($\mathcal{N}$) is defined as the total backflow of information[37, 44]

$$\mathcal{N} = \max_{\rho_1(0), \rho_2(0)} \int_{\sigma > 0} \sigma \left[ t, \rho_1(0), \rho_2(0) \right] dt,$$

(21)

where $\sigma \left[ t, \rho_1(0), \rho_2(0) \right] = dD[\rho_1(t), \rho_2(t)]/dt$ and $D[\rho_1(t), \rho_2(t)] = \frac{1}{2} \text{Tr}[\rho_1(t) - \rho_2(t)]$. When $\rho_1(0) = |e\rangle\langle e|$ and $\rho_2(0) = |g\rangle\langle g|$, the $\mathcal{N}$ in Eq. (21) can obtain in the maximum value. $\mathcal{N} > 0$ is non-Markovian process and $\mathcal{N} = 0$ is Markovian process. The trace distance $D[\rho_1(t), \rho_2(t)]$ of the evolved states can be written as $D(t) = |G(t)|^2 = G(t)$. Thus the $\mathcal{N}$ in Eq. (21) can be rewritten as

$$\mathcal{N} = \frac{1}{2} \int_{\sigma > 0} \partial G(t) dt + G(\tau) - 1.$$

(22)

Consequently, the QSLT is reduced to

$$\tau_{qsl} = \frac{\tau}{2\mathcal{N} - G(\tau) - 1} \sin^2[B(\rho_0, \rho_1)|\text{Tr}[\rho_0^2]],$$

(23)

the QSL time related to the $\mathcal{N}$ within the driving time and the atomic excited population $G(\tau)$.

IV. RESULTS AND DISCUSSION

![Fig. 3](image-url) Fig. 3. $\tau_{qsl}$ as the function of driving strength $\Omega$. a in the weak-coupling regime ($\lambda > 2\gamma$) and b in the strong-coupling regime ($\lambda < 2\gamma$). The velocity ratio $\beta = 0$. The transition frequency $\omega_0 = 5.1 \times 10^9$ and frequency $\omega_L = 1 \times 10^4$. The dissipative rate $\gamma = 1$. The actual evolution time $\tau = 1$.

Fig. 3 exhibits the variation curves of the $\tau_{qsl}$ with respect to the driving strength $\Omega$ when $\beta = 0$ in the weak-coupling and strong-coupling regimes, respectively. It is worth noting that Fig. 3(a) shows the significant speedup behavior can occur in quantum evolution when the driving strength $\Omega$ reaches a certain critical value $\Omega_c$ in the weak-coupling regime. Namely, the system undergoes a standard evolution process when the driving strength $\Omega$ is less than the critical value $\Omega_c$, while the evolution can be accelerated very obviously when $\Omega > \Omega_c$. For different $\lambda$, the critical value $\Omega_c$ is the same, but the smaller the value

of $\lambda$, the more obvious the acceleration effect of quantum evolution is. Fig. 3(b) shows the evolution curves of $\tau_{qsl}$ vs $\Omega$ in the strong-coupling regime. From Fig. 3(b), we see that, the smaller the value of $\lambda$ is, the smaller the critical value $\Omega_c$, and the speedup phenomenon of the system evolution is more obvious at the same time. In addition, under the strong-coupling regime, the quantum evolution curve appears obviously collapse and recovery. Comparing Fig. 3(b) with Fig. 3(a), we find that the critical value $\Omega_c$ in the strong-coupling regime is significantly less than that in the weak-coupling regime. Thus both of the driving field and the strong-coupling can accelerate the quantum evolution.

![Fig. 4](image-url) Fig. 4. $\tau_{qsl}$ as the function of dissipative rate $\gamma$. The driving strength $\Omega = 0$ in a, and $\Omega = 1 \times 10^4$ in b. The transition frequency $\omega_0 = 5.1 \times 10^9$ and frequency $\omega_L = 1 \times 10^4$. The spectral width parameter $\lambda = 1$. The actual evolution time $\tau = 1$.

In Fig. 4, the evolution curves of $\tau_{qsl}$ vs the dissipative rate are plotted for different velocity ratio when the driving strength $\Omega = 0$ and $\Omega = 1 \times 10^4$, respectively. Fig. 4(a) shows that without classical field driving, the system is always in the standard evolution if the dissipative rate $\gamma$ is less than the critical value $\gamma_c$, while the evolution can be speeded up when $\gamma > \gamma_c$. In addition, the critical value $\gamma_c$ gradually increases as the moving velocity become faster. This also indicates that the moving velocity of the qubit may play an important role in stabilizing quantum evolution. Fig. 4(b) exhibits the dependence of the $\tau_{qsl}$ on the dissipative rate $\gamma$ under the classical field driving. One can find that, in the presence of the classical field driving, there has been a significant acceleration evolution when the dissipation rate $\gamma$ is very small. With the dissipative rate $\gamma$ increasing, the system
will transit from a speedup evolution to a standard evolution process and then again undergo a speedup process. After the dissipation rate $\gamma$ reaches the critical value $\gamma_c$, the evolution of the system is similar to Fig. 4 (a). From Fig. 4, one can find that the moving velocity of the qubit can delay its evolution.

Fig. 5 exhibits the variation curves of non-Markovianity $N'$ with respect to driving strength $\Omega$ when $\beta = 0$ in the weak-coupling and strong-coupling regimes, respectively. Fig 5. (a) shows that, in the weak-coupling regime, the non-Markovianity $N'$ in the dynamics process is always equal to zero when $\Omega < \Omega_c$, and the non-Markovianity $N'$ increases monotonously with the driving strength $\Omega$ when $\Omega > \Omega_c$. When $\lambda$ takes different values, there is the same critical value $\Omega_c$ that $N'$ suddenly increases from 0. In addition, the smaller the value of $\lambda$ is, the more obvious non-Markovian characteristics. The evolution of the $N'$ with the $\Omega$ in the strong-coupling regime is shown in Fig. 5(b). We see that, the smaller the value of $\lambda$ is, the smaller the critical value $\Omega_c$ is, the larger the $N'$ is. And the critical value $\Omega_c$ in the strong-coupling regime is smaller than that in the weak-coupling regime. Thus both of the driving field and the strong-coupling can enhance the non-Markovianity $N'$ in the dynamics process.

![Fig. 6](image1)

**Fig. 6.** $N'$ as the function of dissipative rate $\gamma$. The driving strength $\Omega = 0$ in a, and $\Omega = 1 \times 10^4$ in b. The velocity ratio $\beta = 0$. The transition frequency $\omega_0 = 5.1 \times 10^9$ and frequency $\omega_L = 1 \times 10^3$. The spectral width parameter $\lambda = 1$. The actual evolution time $\tau = 1$.

In Fig. 6, we plot the non-Markovianity $N$ against the dissipative rate $\gamma$ under the driving strength $\Omega = 0$ in Fig. 6(a) and $\Omega = 1 \times 10^4$ in Fig. 6(b). Fig. 6(a) shows that the system undergoes the Markovian evolution process when the dissipative rate $\gamma$ is less than the critical dissipative rate $\gamma_c$, while the system evolution will transit from the Markovian to the non-Markovian processes when $\gamma \geq \gamma_c$. In addition, the $\gamma_c$ gradually increases as $\beta$ adds. From Fig. 6(b), we can find that the non-Markovianity $N$ changes to zero from 0.02 when the dissipation rate $\gamma$ is very small. And then, the non-Markovianity $N$ again increases from zero when the dissipation rate $\gamma \geq \gamma_c$, and the curves of the non-Markovianity is similar to Fig. 6(a). From Fig. 6, one know that the moving velocity of the qubit can decrease the non-Markovianity $N'$ in the dynamics process.

![Fig. 7](image2)

**Fig. 7.** $\tau_{qsl}$ and $N$ as the functions of the driving strength $\Omega$. Here the spectral width parameter $\lambda = 3\gamma$ in Fig. 7(a) and spectral width parameter $\lambda = 0.05\gamma$ in Fig. 7(b). The dissipative rate $\gamma = 1$. The velocity ratio $\beta = 0$. The transition frequency $\omega_0 = 5.1 \times 10^9$ and frequency $\omega_L = 1 \times 10^3$. The actual evolution time $\tau = 1$.

In order to show more clearly the dependency relationship of the $\tau_{qsl}$ and the non-Markovianity, we draw Fig. 7 and Fig. 8.

Fig. 7 gives the curves of the $\tau_{qsl}$ and the non-Markovianity $N$ with respect to the driving strength $\Omega$ when $\beta = 0$ in the weak-coupling and strong-coupling regimes, respectively. We can observe from Fig. 7(a) that, in the weak-coupling regime, $N$ remains zero and $\tau_{qsl}$ stays at $\tau$ when $\Omega < \Omega_c$, but $N$ will increase and $\tau_{qsl}$ experiences a sudden transition from no speed-up to speed-up evolution when $\Omega \geq \Omega_c$. Fig. 7(b) shows that, in the strong-coupling regime, there are also $N = 0$ and $\tau_{qsl} = \tau$ when $\Omega < \Omega_c$ and there are also $N > 0$ and $\tau_{qsl} < \tau$ when $\Omega \geq \Omega_c$. Besides, the quantum evolution curve appears obvious collapse and recovery and $N$ gradually increases.

![Fig. 8](image3)

**Fig. 8.** $\tau_{qsl}$ and $N$ as the functions of dissipative rate $\gamma$. Here the driving strength $\Omega = 0$ in Fig. 8(a) and $\Omega = 1 \times 10^4$ in Fig. 8(b). The spectral width parameter $\lambda = 1$. The velocity ratio $\beta = 0$. The transition frequency $\omega_0 = 5.1 \times 10^9$ and frequency $\omega_L = 1 \times 10^3$. The actual evolution time $\tau = 1$.

Fig. 8 exhibits the curves of the $\tau_{qsl}$ and non-Markovianity $N$ against the dissipative rate $\gamma$ when the driving strength $\Omega = 0$ and $\Omega = 1 \times 10^4$, respectively. In Fig. 8(a), we can observe that, if $\Omega = 0$, $N = 0$ and $\tau_{qsl} = \tau$ when $\gamma < \gamma_c$, while $N > 0$ and $\tau_{qsl} < \tau$ when $\gamma > \gamma_c$. In Fig. 8(b), we can find that, if $\Omega = 1 \times 10^4$, there are $N > 0$ and $\tau_{qsl} < \tau$ when $\gamma$ is very small. Then $N$ decreases to zero from 0.02 and $\tau_{qsl}$ enlarges synchronously to $\tau$ as $\gamma$ increases. When $\gamma \geq \gamma_c$, $N$ will again increase and $\tau_{qsl}$ will again experience a sudden transition from no speed-up to speed-up evolution.

From Fig. 7 and Fig. 8, the transition from Markovian to non-Markovian dynamics is the main physical reason...
of the quantum speed-up process, and both of the driving field and the strong-coupling can enhance the non-Markovianity in the dynamics process and speed up the evolution of the qubit.

V. CONCLUSIONS

In summary, we consider a model of a moving qubit interacting with the multimode cavity, where the qubit is driven by the classical field. We obtain the analytic solution of the density operator of the qubit. We investigate the QSLT of the qubit evolution and the non-Markovianity in the quantum process based on the classical field and the moving velocity of the qubit. The results show that the classical field also obviously accelerates the quantum evolution in both of the weak-coupling and the strong-coupling regimes. Namely, the transition from Markovian to non-Markovian dynamics is induced by the classical field and the qubit-reservoir coupling, and this transition is the main physical reason of the quantum speed-up process. Furthermore the moving velocity of the qubit can decrease the non-Markovianity in the dynamics process and delay the evolution of the qubit. Therefore, the controllable operation of quantum evolution can be realized by adjusting the classical field strength, the qubit-reservoir coupling and the moving velocity of the qubit.

Acknowledgments

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