Analysis of Interdependence of Arctic Critical Infrastructures as Transportation Networks

S A Timashev¹,²,³, A V Bushinskaya¹,²,³

¹Sci.& Engng Center “Reliability and Safety of Large Systems and Machines”, Ural Branch, Russian Academy of Sciences, Yekaterinburg 620016, Russia
²Ural Federal University, Yekaterinburg 620016, Russia
³ N. Laverov Federal Center for Integrated Arctic Research, Arkhangelsk, Russia
⁴Old Dominion University, Norfolk, VA, United States

E-mail: timashevs@gmail.com

Abstract. The Arctic zone of the Russian Federation (AZRF) is characterized by the specificity of the development of its interconnected critical infrastructures' objects (ICI) - power generation and transmission systems, social infrastructure facilities, transportation (land, air, water) and communication systems, by the exceptional importance of their stable operation. The spatial isolation of the ICI objects predetermines, practically for each municipality, formation of its specific configuration, defining the range of needed capacities, corresponding set of ICI objects, etc. The main functional of Arctic ICI objects is to provide and sustain life support. Authors developed a model that permits high resolution description of ICI operation on the local level. The accuracy of the model is enough, especially for describing electrical and water supply grids and nets. Its substantial advantage is the generality of its mathematical description of the ICI of different physical nature and its capacity to account for the randomness of ICI parameters. The model permits simulation of the ICI behavior on the local level when exposed to ordinary, everyday conditions and in emergency situations (i.e., during an industrial disaster, Natural catastrophe or a terrorist attack). Several specific problems were solved using this approach for the AZRF region.

1. Introduction

In order to solve the problem outlined in the title of the paper, it is necessary to create a model of CI operation which would allow, for fixed external forces and actions, to assess the character and volume of damage, and produce numerical assessments of all possible consequences.

From the description of the problem it is obvious that the model should allow finding in a comparatively simple way and without big errors numerical values of all types of damage and how they change in time. The described model permits modeling ICI operation on the local level, which is very important when applying the “from-bottom-up” approach to assessing the resilience of the ICI [6], [7]. Most adequately fit models to solve these kinds of problems are the logic-structural models proposed by I.A. Ryabinin, different transportation network models, Petri networks and Bayesian nets [1], [3]. In this paper the advanced transportation network model [8] is used.

This synthetic practical model describes functioning of interdependent systems of critical infrastructures (ISCI), each of which is represented in the form of a transportation/services network comprised of nodes and directed links. Nodes typically represent physical infrastructure components
(the so-called assets, such as electrical power grids and plants, oil, gas and water transmission pipeline systems, sewage systems, railway and highway systems, hospitals, industrial and office buildings, living quarters and the like), which are directly involved in supplying the population and different industries with different products or commodities, and services. The links of the network are modeling the flow (product transportation) between the nodes and may present electrical power lines, main and distribution pipelines of gas and oil; water supply and sewage systems, as well as railways and highways, etc.

Below, when describing the supply process, the generalized notion of "product" will be used for all types of consumption. Due to this factor it has a higher resolution as compared to ordinary methods, constructed for analyzing ICI on a national scale, as it is capable of accounting for local specifics of ICI, such as presence of product storages and emergency operators.

The model considers the conditions and specifics of supply, delivery, overhaul, and demand of resources, such as electricity, hot and cold water, and removal of the human activity waste or industrial waste [1].

It should be noted that the described approach ensures only the continuity of product flow in the nodes; at the same time, the physical laws, which define the patterns of product flow may not be complied with. But the accuracy of such simplified models is sufficient for most practical cases, especially for the electricity and water supply systems [4]. Considerable advantage of such models is the generality of the mathematical description of CI of different physical nature and their ability to account for the probabilistic character of the CI model parameters.

2. Generalized probabilistic model of ICI in the form of a network flow

The described below CI model is a generalization of existing models of transportation networks (flows). The infrastructure is presented in Fig. 1 by a digraph \( G(V, E) \), i.e., by a set of nodes \( V \), which are connected with directing edges (links) \( E \). In the standard model nodes are the physical components of the infrastructure, which are responsible for the supply and, overhaul of the product, and the edges are responsible for the transportation (flow) of the product between the nodes [8].

![Figure 1. Basic parameters of the net flow model.](image)

The model is developed for modeling behavior of the operating ICI on the local level in normal and emergency situations (such as industrial incidents, Natural catastrophes, or terrorist attacks). In order to reach this goal following modifications were introduced into the model [8]:

- if the CI is damaged, the modeling it network may become unbalanced, i.e., the overall demand of a specific product \( k \) may be larger than its on-hand supply. In order to be able to conduct the modeling some variables \( \lambda_k \) are introduced, which describe the cropped-up unsatisfied demand/deficit (the difference between the volume of actual demand of the product and the volume of its actual availability), as well as the corresponding to the cropped-up deficit rise of prices or fines \( c^k \);
- resilience of the ICI systems may be increased by using local storages and/or production of needed products (like, spare pumps or generators).
Parameters that describe the quality of operation of CI physical components (assets) of the infrastructure network such as productivity, transportation capacity, demand) may be described at first approximation as random variables (RV). It is also possible to give to each network node (asset) scientifically justified/supported values of the failure rate. These probabilities are found using the statistical data on assets failures or through data obtained by corresponding probabilistic calculus/design.

To take these into account variables representing storage, $s_k$, and production, $\pi_k$ (with the corresponding costs, $c_s$ and $c_p$, respectively) have also been added to the model.

To take unmet demand/shortage into account variables $s_k$ and $\pi_k$, representing storage and production, respectively, with the corresponding costs $c_s^k$ and $c_p^k$ are added to the model. Thus, the model distinguishes a commodity received from sources outside of the local infrastructure, from a commodity of local production and storage. However, to produce a commodity $k$ another commodity (or commodities), $n$, is usually consumed.

To distinguish between traditional supply and internal production, the variable $\pi_k$ is considered as the sum of variables:
- $\pi_{k,k}$, representing the traditional supply (i.e., supply from sources outside the boundaries of the local infrastructure);
- $\pi_{n,k}$, representing the internal production (i.e., consumption of commodity $n$ to produce $k$ is taken into account).

In a similar way, the variable $\kappa_k$, representing consumption of a commodity $k$, is the sum of variables:
- $\kappa_{k,k}$, which denotes traditional demand $k$;
- $\kappa_{n,k}$, denoting consumption of $k$ for production of a commodity $n$.

The (links) edges of the considered model are analogous to the edges that are used in traditional transportation network model: each edge is a flow/supply channel of one product.

The volume of products stored in the node may change in time. Particularly, when the infrastructure is damaged due to an incident or catastrophe, and the available on-hand products are used to compensate for the cropped up undersupply. In order to account for this most important circumstance the time factor is explicitly introduced into the model.

The process of solving the problem starts from the moment of time $t_0$ when the initiating the CI failure event (infrastructure incident or catastrophe) occurs. The solutions are found for discrete moments of time with increment $\Delta t$ (which could be variable) to the moment of full restoration of the damaged CI. On each time increment all the variables and constants do not change [5]. When forecasting the possible CI damage (i.e., assessment of its partial or full failure), independent from the character of the type of forces acting on the infrastructure, it is necessary to quantitatively assess the considerable uncertainty of parameters of the model equations. This assessment is performed either by establishing the probabilities of failure of the CI assets, as given by the nodes, or by considering the asset productivity and/or the volume of supply as random variable (RV). As the demand of products could be uncertain, it also is regarded as a RV.

3. **A multifunctional nodes model**

If the same node could simultaneously act both as a consumer and a producer, have storage facilities, emergency generators and, at the same time, belong to several different CIs, it is called a multifunctional node.

Let us describe the relationship between variables and constants, representing four functions of a multifunctional node of an infrastructure, as shown in Fig. 1.

The balance equations at a node $i$ for commodity $k$ are

$$f_k^{i-\epsilon} + \pi_k^i - \kappa_k^i - t_k^i - s_k^i = 0,$$

(1)
\[ \tau_k^i - f_k^{i\rightarrow} = 0, \]

where \( f_k^{i\leftarrow}, f_k^{i\rightarrow} \) are the commodities inflow and outflow rates; \( \pi_k^i \) is the production rate; \( \tau_k^i \) is the transhipment rate; \( s_k^i \) is the rate of transferring the commodity to storage. If no cost is associated with the transhipment of the commodity and there is no risk of damage of the transhipment function of the node \( i \), then these two balance equations can be replaced by a single one:

\[ f_k^{i\leftarrow} + \pi_k^i - \kappa_k^i - f_k^{i\rightarrow} - s_k^i = 0 \tag{2} \]

It is worth noting that different commodities can be processed at a single multifunctional node. Hence, the balance equations should be satisfied at the node for each of these commodities.

The in-flow and out-flow rates, \( f_k^{i\leftarrow}, f_k^{i\rightarrow} \) are the sums of the flow rates over all edges transferring commodity \( k \) into and out of the node \( i \), respectively. The production rate of commodity \( k \) at a node \( i \) is expressed as

\[ \pi_k^i = \pi_{k,k}^i + \sum_{n \neq k} \pi_{n,k}^i, \tag{3} \]

where \( \pi_{k,k}^i \) is the supply rate, when the need in another commodity to produce the commodity \( k \) is not taken into account (usually, at nodes that represent sources of supply of the commodity \( k \) outside the boundaries of the local infrastructure network under consideration); \( \pi_{n,k}^i \) is the rate of production of the commodity \( k \) that involves consumption of another commodity \( n \). It is assumed that there is a linear relationship between the amounts of produced and consumed commodities so that

\[ \pi_{n,k} = \alpha_{n,k}^i \pi_{n,k}^i, \tag{4} \]

where \( \alpha_{n,k}^i \) is the coefficient relating the production rate of the commodity \( k \) to the corresponding consumption rate \( \kappa_{n,k}^i \), of the commodity \( n \). Thus, Eq. (3) takes the form

\[ \pi_k^i = \pi_{k,k}^i + \sum_{n \neq k} \alpha_{n,k}^i \pi_{n,k}^i, \tag{5} \]

The consumption rate of commodity \( k \) at the node \( i \) is formulated as:

\[ \kappa_k^i = \kappa_{k,k}^i + \sum_{n \neq k} \kappa_{n,k}^i, \tag{6} \]

where \( \kappa_{k,k}^i \) is the rate of consumption of the commodity \( k \) to satisfy demand of consumers; \( \kappa_{n,k}^i \) is the rate of consumption of the commodity \( k \) to produce another commodity \( n \).

In order to be able to take into account the possibility that not all demands are met when infrastructure is damaged, the variable \( \kappa_{k,k}^i \) is presented as:

\[ \kappa_{k,k}^i = \kappa_{k,k}^{i,max} - \lambda_k^i, \tag{7} \]

where \( \kappa_{k,k}^{i,max} \) is the actual demand rate, i.e., the maximum/required rate of consumption of the commodity \( k \) at the node \( i \) at a given time, \( \lambda_k^i \) is the rate of unmet demand (or shortage).

Eq. (7) can then be written as:

\[ \kappa_k^i = \kappa_{k,k}^{i,max} - \lambda_k^i + \sum_{n \neq k} \kappa_{n,k}^i. \tag{8} \]
The introduction of storage function even in one of the nodes makes the whole model time-dependent. The amount of commodity $k$ stored at the node $i$ at time $t$ is denoted as $\omega_k^i(t)$

$$\omega_k^i(t) = \omega_k^i(t - \Delta t) + s_k^i \Delta t \leq \omega_k^i_{\text{max}},$$

where $s_k^i$ is the rate of change in the amount of stored commodity; $\Delta t$ is the time increment and $\omega_k^i_{\text{max}}$ is the maximum storage capacity for the commodity $k$ at the node $i$.

Taking into account Eqs. (5) and (8), Eq. (2) can be written as

$$f_k^{\text{flow}} + \pi_1^{i,k} + \sum_{a \in k} \alpha_{n,k}^l \pi_{n,k}^l - \pi_{n,k}^{l_{\text{max}}} + \lambda_1^{i,k} - \sum_{a \in k} \kappa_{n,k}^l - r_k^l - s_k^i = 0$$

Eq. (2) remains unchanged, while Eq. (3) can be updated in a similar way as Eq. (1).

In this second equation of Eq. (1) remains unchanged, while Eq. (2) can be updated in a similar way as Eq. (1).

4. Formulation of the optimization problem

In the general case the problems related to the optimization of CI operation, can be solved by simple sorting of options (when the dimension of the problem is not large), and also by using different analytical methods (linear and nonlinear programming, greedy algorithms, etc.). The choice of the solving method is not, as yet, formalized and depends on specifics of the problem in consideration. The distinction of the developed in the paper method from the existing models is that it permits optimizing CI operation according to some risk criteria, and particularly, minimizing the overall cost of its operation in each specific case, taking into account possible human losses. In the latter case additional members that quantitatively describe the non-materiel losses (loss of limb, life), have to be introduced into the optimized cost functional.

The solution of the model is formulated as a linear optimization problem that minimizes the cost of maintaining the network infrastructure at the stage of its operation (or design) as follows:

$$\min_{x \in \mathbb{R}^n} c^T x$$

subject to

$$b_l \leq Ax \leq b_u \quad \text{(general constraints)}$$

$$x_l \leq x \leq x_u \quad \text{(bounds on variables)}$$

where $x$ is the vector of variables, $c$ is the objective coefficient vector (i.e., costs); $A$ is the coefficient matrix; $b_l, b_u, x_l, x_u$ are the vectors of lower and upper bounds on the constraints and the variables, respectively.

Costs $C$ for operating the network infrastructure per unit time within the time interval $(t - \Delta t, t)$ can be expressed as:

$$C = \sum_{k=1}^{K} \left[ \sum_{e \in E} c_k^{f,e} f_k^e + \sum_{i \in V} c_k^{\pi,i} \pi_k^i + \sum_{i \in V} c_k^{\lambda,i} \lambda_k^i + \sum_{i \in V} c_k^{\delta,i} \frac{1}{2} \left( \omega_k^i(t) + \omega_k^i(t - \Delta t) \right) \right],$$

where $c_k^{f,e}, c_k^{\pi,i}, c_k^{\lambda,i}, c_k^{\delta,i}$ are the costs associated with the flow, production, shortage and storage of commodity $k$ per unit of time, respectively; $K$ is the total number of commodities.

The cost of storage (the last sum in Eq. (12)) is based on the average amount of stored commodity within the time interval $(t - \Delta t, t)$. The total cost can be calculated by substituting Eqs. (5) and (9) into Eq. (12), which gives
where the last sum is a constant and should be excluded from the objective function; in order to obtain the total cost, including the full cost of storage, this term can be added after optimization.

The optimization problem can then be formulated as:

\[
\begin{align*}
\min C &= \min \sum_{k=1}^{K} \left[ \sum_{e \in E} c_{k}^{e} f_{k}^{e} + \sum_{i \in V} c_{k}^{i} \left( \pi_{k,i}^{j} + \sum_{n \in k} \alpha_{n,i,k}^{j} \right) + \sum_{i \in V} c_{k}^{i} \lambda_{k}^{j} + \right. \\
&\left. + \frac{1}{2} \sum_{i \in V} c_{k}^{i} s_{k}^{i} \Delta t + \sum_{i \in V} c_{k}^{i} \omega_{k}^{i} \left( t - \Delta t \right) \right].
\end{align*}
\]

subject to the following general constraints:

\[
\begin{align*}
\pi_{k,k}^{j} + \sum_{n \in k} \alpha_{n,k,k}^{j} s_{k}^{i} &= \pi_{k,k}^{j} + \sum_{n \in k} \alpha_{n,k,k}^{j} s_{k}^{i} - \pi_{k,k}^{j} - \Delta t, \forall i \in V, \forall k \in K \\
\tau_{k}^{j} - \tau_{k}^{j-1} &= 0, \forall i \in V, \forall k \in K \\
\pi_{k,k}^{j} + \sum_{n \in k} \alpha_{n,k,k}^{j} s_{k}^{i} &\leq \pi_{k,k}^{j} \forall i \in V, \forall k \in K
\end{align*}
\]

and bounds:

\[
\begin{align*}
0 \leq f_{e}^{j} &\leq f_{e}^{\max} \forall e \in E; \\
0 \leq \pi_{k}^{j} &\leq \pi_{k}^{j} \forall i \in V, \forall k \in K; \\
0 \leq \kappa_{n,k}^{j} &\leq \kappa_{n,k}^{j} \forall i \in V, \forall k \in K, \forall n \in K, k \neq n; \\
0 \leq \lambda_{k}^{j} &\leq \lambda_{k}^{j} \forall i \in V, \forall k \in K; \\
0 \leq \tau_{k}^{j} &\leq \tau_{k}^{j} \max, \forall i \in V, \forall k \in K; \\
\frac{\alpha_{k}^{j}}{\Delta t} &\leq s_{k}^{i} \leq \frac{\alpha_{k}^{j} - \alpha_{k}^{j} \max}{\Delta t}, \forall i \in V, \forall k \in K,
\end{align*}
\]

where \( f_{e}^{\max} \) is the maximum flow capacity of the link \( e \); \( \pi_{k}^{j} \max \) is the maximum production capacity of commodity \( k \) at the node \( i \); \( \tau_{k}^{j} \max \) is the maximum transshipment capacity for commodity \( k \) at the node \( i \).

Minimizing the total cost separately at each time step does not yield an optimal solution for the network performance over the whole time period under consideration. For example, when an infrastructure network is undamaged it is obvious that the optimal amount of a stored commodity is zero – if everything works properly there is no need for stored commodities because their storage incurs additional costs. However, when the network is damaged by a hazard, the availability of commodities from storage may reduce the total cost (since there are additional high costs penalizing for unmet demand). This problem can be resolved either by artificially setting negative costs for storage (that will prevent the use of stored commodities under normal conditions) or minimizing the total cost over all considered time increments, i.e., the objective function should become the sum of
the costs associated with each time increment. The second approach seems preferable. However, in order to use this approach it is necessary to know the time required for restoring the infrastructure to its original undamaged condition (as new).

This time can serve as a good criterion for resilience and strategic readiness of the infrastructure, but it can be difficult to predict due to the considerable uncertainty of many parameters. Here, to obtain practical results, it may be necessary to use special probability models - fuzzy logic, interval probabilistic estimates, etc.

5. Accounting for the interdependency of critical infrastructures

This accounting is very important as from it depends the level of likelihood of modeling results, because quite often failures of elements belonging to the same CI or to two and more CI could be interdependent. Such dependency between elements of logic-structural CI models can be of two types:

- failure of one of the elements of a CI changes the operational regime of the whole CI (like, when failure of one controlling element changes the operational regime of other elements);
- when the whole CI in its entirety is exposed to some single random factor (wind, wave pressure, snow load, temperature, vibration etc.).

Specific calculations show that if a simple, without reserve elements and non-renewable system is considered, the dependence of the first type does not affect the reliability of the CI as a whole. If the CI has reserve elements and is renewable, the second-type interdependence must be taken into account. In general, failure to account for the strong dependence of failures in a multi-element CI can lead to large errors. Neglecting of dependent failures in series connection elements can lead to a significant underestimation of CI reliability. When the elements are connected in parallel, ignoring their interdependence leads, on the contrary, to an overestimation of the CI reliability.

If the operating mode of the CI is characterized by some continuous random variable \( V \) (say the wind speed) with a known distribution density function \( f(v) \), then the reliability of the CI is determined by the formula [2]:

\[
P(t) = \int_{f_{P}(v)} P_{f_{P}(v)} f(v) dv,
\]

where \( P_{f_{P}(v)} \) is the conditional reliability of CI, provided that \( V = v \). The integral extends over the entire range of possible values of \( V \).

The larger is the number of reserve elements in the CI block, the higher is the overestimation of its reliability due to the neglect of the interdependence of the failures of its elements.

If the CI consists of elements connected in series and in parallel (only the most important components of the CI are duplicated), the neglect of the dependence of the failures can lead to both overestimation and underestimation of the reliability of the CI.

For more complex CIs that do not reduce to purely logical-structural schemes, it is necessary to conduct a series of numerical comparative calculations that allow estimating the amount of incident damage that depends on accounting for / not taking into account the interdependence of the elements of one, two or more CIs for each scenario of the development of an emergency situation.

6. Probabilistic analysis of a local interdependent infrastructure system. Real case study

Analyze, using the described above model, operation of a system which consists of two interdependent infrastructures (electricity and water supply) which are located in one of the Russian Far North cities (see. Fig. 2) during an extreme winter snow storm with hurricane wind speeds, which occur once in a 100 years (design event), with simultaneous rise of ambient temperature to levels that produce thick icing on the electrical power lines' wires.
Modeling of the extreme winter snow storm per se is not considered in this paper, because it is a standalone problem which belongs to the analysis of the consequences of natural forces on ICI structures class of problems. It is solved by considering the influence of the joint action of wind, and icing of electrical power grid (EPG) wires on the strength, stability and vibrations of the EPG foundation structures, supports and wiring of the EPG as a large mechanical system (Timashev S.A., 2016). As the result of solving this stochastic mechanics problem the so called fragility curves are produced, which connect the physical properties of wind pressure (average speed and variance), ambient air temperature, intensity and duration of snow fall, etc., with the probability of any kind of damage of EPG supports (warping, capsizing, buckling of individual strut elements, etc.) and wires (sagging, contacting of two adjacent wires, wire rupture).

The probability of failure (PoF) \( P_f \) of each EPG element can be assessed as

\[
P_f(t) = P[Y(t) < 0;\ 0 \leq t \leq t]
\]

where \( Y(t) \) is the limit state function (LSF) of the EPG element. The LSF actually is the difference between the function, which describes the physical load vector on the element and the ultimate permissible value of it.

The local EPG (the first CI) is comprised of four electrical substations--one substation, which is an element of the Federal EPG (node 2), with maximal capacity of 20 MW hours/day and three distribution substations: nodes 4 and 7, with maximal distribution power of 5 MW hours/day, and node 5 with maximal distribution power of 10 MW hours/day. The second CI is comprised of: the water filter station (node 1) with maximal distribution power of 1000 m\(^3\) drinking water per day; water tower (node 3), with maximal capacity 550 m\(^3\); and the pumping station (node 6) with maximal pumping power of 200 m\(^3\)/day. For pumping the water they use electricity. It is assumed that to pump 7.5 m\(^3\) of water 1 KW hour of electricity is needed. Water consumers are: hospital (node 8), old age home and the surrounding it living quarters (node 9), and living quarters (node 10). Their daily consumption of electricity and water are given in Table 1. Nodes and connections between them are schematically shown in Fig. 2.

| Consumer                        | Daily demand | Daily demand |
|---------------------------------|--------------|--------------|
|                                | Electricity, MWhour/day | Water, m\(^3\)/day |
| Hospital (node 8)              | 1.3          | 35           |
| Old age house & surrounding it | 3.1          | 75           |
| Living quarters (node 9)       | 6.0          | 165          |
| Living quarters (node 10)      |              |              |
Two variants were analyzed: 1) CIs without the emergency generators; 2) CI with emergency generators in nodes 3, 6 and 8. Each generator of 12 KW power consumes one liter of diesel oil to produce 3 KW of electricity and is equipped with an oil tank with capacity of 100 liters.

In the considered case the expenditures for transportation, storage, production and the products deficit are not actual expenditures and are assigned in order to correctly distribute the products between the consumers as was intended. In order to prevent usage of the stored products (water and fuel) until the moment, when there would be no other source to satisfy the demand, the expenditures associated with their storage are set negative.

In the considered case following decreasing priority hierarchy was established for receiving the necessary resources: hospital, old age home, living quarters. The financial, materiel and social damages, related to the shortage/absence of water and electricity supply for these consumers are established according to the above priorities.

For the three substations in the case of high voltage wires rupture due to a combination of wind pressure and icing following probabilities of failure were obtained: node 4 – 0.4, node 5 – 0.6, and node 7 – 0.2. As the substations can be only in two states (operable or failure/switched off), hence, the whole network has only 23 states and the probabilities can be easily calculated. Infrastructure will operate being partially damaged until the EPG repair is complete, which can take up to several days. Results of the probabilistic analysis of the water and electricity supply process to consumers, as an illustration, are shown in figures 3 and 4, depending on the time of restoration of the EPG integrity (during one –six days, with one day increment). Particularly, the probabilities of delivery to consumer’s necessary volumes (i.e., demand) of electricity and water without using the emergency generators are given.

It can be seen, that the interruption in delivering the electricity can start at the very first day. After that the percentage of the delivery and the corresponding probabilities do not change for the next six days. Interruptions in water delivery during the first day after the incident can take place only in the living quarters (node 10), due to the possible failure of electricity delivery for the pumping station (node 6). Keeping water in the water tower guarantees it will be delivered in quantities that are needed (full demand) to the hospital (node 8) and the old age home, including the surrounding it living quarters (node 9). Interruption of water delivery for these consumers may start at the third day and the situation gets worse on the sixth day.

![Figure 3. Probability of delivering electricity to consumers (without accounting for the emergency generators).](image-url)
Figure 4. Probability of delivering water to consumers during the first six days after the incident occurrence (without accounting for the emergency generators)
7. Conclusion
The results of conducted research are urgent for decision makers, as they allow assessing the consequences of made decisions when managing interconnected CI. On a model but real-life example of two interdependent infrastructure systems of electricity and water supply during a winter snow storm it was shown how the uncertainties related to the damage/downtime of infrastructure elements can be accounted for, using variables which describe the originated unsatisfied demand (deficit) and the corresponding to it costs or fines.

The model can be used for studying productivity of interdependent CI (and via it, the resilience) of much more complicated interdependent network-type infrastructures, in much smaller time increments (every hour, minute or even second, if need be) and consider such parameters as demand and supply, production and/or products flow as non stochastic functions of time of random variables, functions or fields.

Acknowledgments
The study was carried out with the financial support of the Ural Branch of the Russian Academy of Sciences in the framework of the scientific project No. 18-9-17-37 “Modeling the environmental and economic scenarios of the spatial development of the Arctic regions of Russia”.

References
[1] Ahuja R K and Magnanti T L and Orlin J B 1993 Network Flows: Theory, Algorithms and Applications (New Jersey: Prentice Hall)
[2] Bushinskaya A V and Timashev S A 2016 Generalized Probabilistic Network Flow Model of Interdependent Critical Infrastructures J. of Problems of Safety and Emergency Situations 3 pp 9-22
[3] Kachur S A 2011 Modeling of catastrophic situations of nuclear energy facilities using stochastic Petri networks Collection of scientific research works 4(40) (Sevastopol National University of nuclear energy and industry) pp 22–29
[4] Manca A and Sechi G M and Zuddas P 2010 Water supply network optimisation using equal flow algorithms J. Water Resource Management 24 (13) pp 3665–3678
[5] Svendsen N and Wolthusen S 2007 Connectivity models of interdependency in mixed-type critical infrastructure networks Information Security Technical Report 12(1) pp 44–55
[6] Timashev S A 2013 Unified Quantitative Criteria for Management of Regional Risk Proc. of the11-th International Conference on Structural Safety & Reliability
[7] Timashev S A 2016 Infrastructures, Part 1. Reliability and Longevity (Yekaterinburg: Ural Branch, Russian Academy of Sciences) 530 p
[8] Val D V and Holden R and Nodwell S 2013 Probabilistic assessment of failures of interdependent infrastructures due to weather related hazards Proc. of the 11-th International Conference on Structural Safety & Reliability