Functional Space $C(\Omega)$, $C_0(\Omega)$

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Summary. In this article, first we give a definition of a functional space which is constructed from all complex-valued continuous functions defined on a compact topological space. We prove that this functional space is a Banach algebra. Next, we give a definition of a function space which is constructed from all complex-valued continuous functions with bounded support. We also prove that this function space is a complex normed space.

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The terminology and notation used here have been introduced in the following articles: [6], [24], [25], [1], [26], [5], [4], [2], [21], [15], [3], [18], [19], [23], [22], [17], [7], [11], [12], [9], [10], [13], [8], [14], [20], and [16].

Let $X$ be a topological structure and let $f$ be a function from the carrier of $X$ into $\mathbb{C}$. We say that $f$ is continuous if and only if:

(Def. 1) For every subset $Y$ of $\mathbb{C}$ such that $Y$ is closed holds $f^{-1}(Y)$ is closed.

Let $X$ be a 1-sorted structure and let $y$ be a complex number. The functor $X \mapsto y$ yielding a function from the carrier of $X$ into $\mathbb{C}$ is defined by:

(Def. 2) $X \mapsto y = (\text{the carrier of } X) \mapsto y$.

One can prove the following proposition

(1) Let $X$ be a non empty topological space, $y$ be a complex number, and $f$ be a function from the carrier of $X$ into $\mathbb{C}$. If $f = X \mapsto y$, then $f$ is continuous.

Let $X$ be a non empty topological space and let $y$ be a complex number. Observe that $X \mapsto y$ is continuous.

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Let $X$ be a non empty topological space. One can verify that there exists a function from the carrier of $X$ into $\mathbb{C}$ which is continuous.

The following propositions are true:

(2) Let $X$ be a non empty topological space and $f$ be a function from the carrier of $X$ into $\mathbb{C}$. Then $f$ is continuous if and only if for every subset $Y$ of $\mathbb{C}$ such that $Y$ is open holds $f^{-1}(Y)$ is open.

(3) Let $X$ be a non empty topological space and $f$ be a function from the carrier of $X$ into $\mathbb{C}$. Then $f$ is continuous if and only if for every point $x$ of $X$ and for every subset $V$ of $\mathbb{C}$ such that $f(x) \in V$ and $V$ is open there exists a subset $W$ of $X$ such that $x \in W$ and $W$ is open and $f \circ W \subseteq V$.

(4) Let $X$ be a non empty topological space and $f, g$ be continuous functions from the carrier of $X$ into $\mathbb{C}$. Then $f + g$ is a continuous function from the carrier of $X$ into $\mathbb{C}$.

(5) Let $X$ be a non empty topological space, $a$ be a complex number, and $f$ be a continuous function from the carrier of $X$ into $\mathbb{C}$. Then $a \cdot f$ is a continuous function from the carrier of $X$ into $\mathbb{C}$.

(6) Let $X$ be a non empty topological space and $f, g$ be continuous functions from the carrier of $X$ into $\mathbb{C}$. Then $f - g$ is a continuous function from the carrier of $X$ into $\mathbb{C}$.

(7) Let $X$ be a non empty topological space and $f, g$ be continuous functions from the carrier of $X$ into $\mathbb{C}$. Then $f \cdot g$ is a continuous function from the carrier of $X$ into $\mathbb{C}$.

(8) Let $X$ be a non empty topological space and $f$ be a continuous function from the carrier of $X$ into $\mathbb{C}$. Then $|f|$ is a function from the carrier of $X$ into $\mathbb{R}$ and $|f|$ is continuous.

Let $X$ be a non empty topological space. The $\mathbb{C}$-continuous functions of $X$ yields a subset of $\mathbb{C}$-Algebra(the carrier of $X$) and is defined by:

(Def. 3) The $\mathbb{C}$-continuous functions of $X = \{f : f$ ranges over continuous functions from the carrier of $X$ into $\mathbb{C}\}$.

Let $X$ be a non empty topological space. Observe that the $\mathbb{C}$-continuous functions of $X$ is non empty.

Let $X$ be a non empty topological space. Observe that the $\mathbb{C}$-continuous functions of $X$ is $\mathbb{C}$-additively linearly closed and multiplicatively closed.

Let $X$ be a non empty topological space. The $\mathbb{C}$-algebra of continuous functions of $X$ yielding a complex algebra is defined by the condition (Def. 4).

(Def. 4) The $\mathbb{C}$-algebra of continuous functions of $X = \{$(the $\mathbb{C}$-continuous functions of $X$, mult$(\mathbb{C}$-continuous functions of $X$, $\mathbb{C}$-Algebra$(\text{the carrier of } X))$), Add$(\mathbb{C}$-continuous functions of $X$, $\mathbb{C}$-Algebra$(\text{the carrier of } X))$), Mult$(\mathbb{C}$-continuous functions of $X$, $\mathbb{C}$-Algebra$(\text{the carrier of } X))$, One$(\mathbb{C}$-continuous functions of $X$, $\mathbb{C}$-Algebra$(\text{the carrier of } X))$\}.
Next we state the proposition

(9) Let \( X \) be a non empty topological space. Then the \( \mathbb{C} \)-algebra of continuous functions of \( X \) is a complex subalgebra of \( \mathbb{C}\text{-Algebra}(\text{the carrier of } X) \).

Let \( X \) be a non empty topological space. Observe that the \( \mathbb{C} \)-algebra of continuous functions of \( X \) is strict and non empty.

Let \( X \) be a non empty topological space. One can check that the \( \mathbb{C} \)-algebra of continuous functions of \( X \) is Abelian, add-associative, right zeroed, right complementable, vector distributive, scalar distributive, scalar associative, scalar unital, commutative, associative, right unital, right distributive, vector distributive, scalar associative, and vector associative.

Next we state several propositions:

(10) Let \( X \) be a non empty topological space, \( F, G, H \) be vectors of the \( \mathbb{C} \)-algebra of continuous functions of \( X \), and \( f, g, h \) be functions from the carrier of \( X \) into \( \mathbb{C} \). Suppose \( f = F \) and \( g = G \) and \( h = H \). Then \( H = F + G \) if and only if for every element \( x \) of the carrier of \( X \) holds \( h(x) = f(x) + g(x) \).

(11) Let \( X \) be a non empty topological space, \( F, G \) be vectors of the \( \mathbb{C} \)-algebra of continuous functions of \( X \), and \( f, g \) be functions from the carrier of \( X \) into \( \mathbb{C} \), and \( a \) be a complex number. Suppose \( f = F \) and \( g = G \). Then \( G = a \cdot F \) if and only if for every element \( x \) of \( X \) holds \( g(x) = a \cdot f(x) \).

(12) Let \( X \) be a non empty topological space, \( F, G, H \) be vectors of the \( \mathbb{C} \)-algebra of continuous functions of \( X \), and \( f, g, h \) be functions from the carrier of \( X \) into \( \mathbb{C} \). Suppose \( f = F \) and \( g = G \) and \( h = H \). Then \( H = F \cdot G \) if and only if for every element \( x \) of the carrier of \( X \) holds \( h(x) = f(x) \cdot g(x) \).

(13) For every non empty topological space \( X \) holds
\[ 0_{\text{the } \mathbb{C}\text{-algebra of continuous functions of } X} = X \mapsto 0_{\mathbb{C}}. \]

(14) For every non empty topological space \( X \) holds
\[ 1_{\text{the } \mathbb{C}\text{-algebra of continuous functions of } X} = X \mapsto 1_{\mathbb{C}}. \]

(15) Let \( A \) be a complex algebra and \( A_1, A_2 \) be complex subalgebras of \( A \). Suppose the carrier of \( A_1 \subseteq \text{the carrier of } A_2 \). Then \( A_1 \) is a complex subalgebra of \( A_2 \).

(16) Let \( X \) be a non empty compact topological space. Then the \( \mathbb{C} \)-algebra of continuous functions of \( X \) is a complex subalgebra of the \( \mathbb{C} \)-algebra of bounded functions of the carrier of \( X \).

Let \( X \) be a non empty compact topological space. The \( \mathbb{C} \)-continuous functions norm of \( X \) yields a function from the \( \mathbb{C} \)-continuous functions of \( X \) into \( \mathbb{R} \) and is defined by:
(Def. 5) The C-continuous functions norm of $X = \langle \text{C-BoundedFunctionsNorm (the carrier of } X) \rangle$ \{the C-continuous functions of $X$\}.

Let $X$ be a non empty compact topological space. The C-normed algebra of continuous functions of $X$ yields a normed complex algebra structure and is defined by the condition (Def. 6).

(Def. 6) The C-normed algebra of continuous functions of $X = \langle \text{the C-continuous functions of } X, \text{mult}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{Add}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{Mult}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{One}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{Zero}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{the C-continuous functions norm of } X \rangle$.

Let $X$ be a non empty compact topological space. Note that the C-normed algebra of continuous functions of $X$ is non empty and strict.

Let $X$ be a non empty compact topological space. Observe that the C-normed algebra of continuous functions of $X$ is unital.

Next we state the proposition

(17) Let $X$ be a non empty compact topological space. Then the C-normed algebra of continuous functions of $X$ is a complex algebra.

Let $X$ be a non empty compact topological space. One can check that the C-normed algebra of continuous functions of $X$ is right complementable, Abelian, add-associative, right zeroed, vector distributive, scalar distributive, scalar associative, associative, commutative, right distributive, right unital, and vector associative.

One can prove the following proposition

(18) Let $X$ be a non empty compact topological space and $F$ be a point of the C-normed algebra of continuous functions of $X$. Then $(\text{Mult}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)))(1_c, F) = F$.

Let $X$ be a non empty compact topological space. Observe that the C-normed algebra of continuous functions of $X$ is vector distributive, scalar distributive, scalar associative, and scalar unital.

We now state a number of propositions:

(19) Let $X$ be a non empty compact topological space. Then the C-normed algebra of continuous functions of $X$ is a complex linear space.

(20) Let $X$ be a non empty compact topological space. Then $X \mapsto 0 = \langle \text{the C-normed algebra of continuous functions of } X \rangle$.

(21) Let $X$ be a non empty compact topological space and $F$ be a point of the C-normed algebra of continuous functions of $X$. Then $0 \leq \|F\|$.

(22) Let $X$ be a non empty compact topological space, $f, g, h$ be functions from the carrier of $X$ into $\mathbb{C}$, and $F, G, H$ be points of the C-normed algebra of continuous functions of $X$. Then

\[ \text{Mult}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X), \text{Add}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{Mult}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{One}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{Zero}(\text{the C-continuous functions of } X, \text{C-Algebra (the carrier of } X)), \text{the C-continuous functions norm of } X \rangle. \]
algebra of continuous functions of $X$. Suppose $f = F$ and $g = G$ and $h = H$. Then $H = F + G$ if and only if for every element $x$ of $X$ holds $h(x) = f(x) + g(x)$.

(23) Let $a$ be a complex number, $X$ be a non empty compact topological space, $f$, $g$ be functions from the carrier of $X$ into $\mathbb{C}$, and $F$, $G$ be points of the $\mathbb{C}$-normed algebra of continuous functions of $X$. Suppose $f = F$ and $g = G$. Then $G = a \cdot F$ if and only if for every element $x$ of $X$ holds $g(x) = a \cdot f(x)$.

(24) Let $X$ be a non empty compact topological space, $f$, $g$, $h$ be functions from the carrier of $X$ into $\mathbb{C}$, and $F$, $G$, $H$ be points of the $\mathbb{C}$-normed algebra of continuous functions of $X$. Suppose $f = F$ and $g = G$ and $h = H$. Then $H = F \cdot G$ if and only if for every element $x$ of $X$ holds $h(x) = f(x) \cdot g(x)$.

(25) Let $X$ be a non empty compact topological space. Then $\|0\|_{\text{the } \mathbb{C}\text{-normed algebra of continuous functions of } X} = 0$.

(26) Let $X$ be a non empty compact topological space and $F$ be a point of the $\mathbb{C}$-normed algebra of continuous functions of $X$. Suppose $\|F\| = 0$. Then $F = 0$ in the $\mathbb{C}$-normed algebra of continuous functions of $X$.

(27) Let $a$ be a complex number, $X$ be a non empty compact topological space, and $F$ be a point of the $\mathbb{C}$-normed algebra of continuous functions of $X$. Then $\|a \cdot F\| = |a| \cdot \|F\|$.

(28) Let $X$ be a non empty compact topological space and $F$, $G$ be points of the $\mathbb{C}$-normed algebra of continuous functions of $X$. Then $\|F + G\| \leq \|F\| + \|G\|$.

Let $X$ be a non empty compact topological space. Observe that the $\mathbb{C}$-normed algebra of continuous functions of $X$ is discernible, reflexive, and complex normed space-like.

The following propositions are true:

(29) Let $X$ be a non empty compact topological space, $f$, $g$, $h$ be functions from the carrier of $X$ into $\mathbb{C}$, and $F$, $G$, $H$ be points of the $\mathbb{C}$-normed algebra of continuous functions of $X$. Suppose $f = F$ and $g = G$ and $h = H$. Then $H = F - G$ if and only if for every element $x$ of $X$ holds $h(x) = f(x) - g(x)$.

(30) Let $X$ be a complex Banach space, $Y$ be a subset of $X$, and $s_1$ be a sequence of $X$. Suppose $Y$ is closed and $\text{rng } s_1 \subseteq Y$ and $s_1$ is $\mathbb{C}$-Cauchy. Then $s_1$ is convergent and $\lim s_1 \in Y$.

(31) Let $X$ be a non empty compact topological space and $Y$ be a subset of the $\mathbb{C}$-normed algebra of bounded functions of the carrier of $X$. If $Y$ is the $\mathbb{C}$-continuous functions of $X$, then $Y$ is closed.

(32) Let $X$ be a non empty compact topological space and $s_1$ be a sequence
of the $C$-normed algebra of continuous functions of $X$. If $s_1$ is $C$-Cauchy, then $s_1$ is convergent.

Let $X$ be a non empty compact topological space. One can verify that the $C$-normed algebra of continuous functions of $X$ is complete.

Let $X$ be a non empty compact topological space. Observe that the $C$-normed algebra of continuous functions of $X$ is Banach Algebra-like.

Next we state three propositions:

(33) For every non empty topological space $X$ and for all functions $f$, $g$ from the carrier of $X$ into $C$ holds $\text{support}(f + g) \subseteq \text{support } f \cup \text{support } g$.

(34) Let $X$ be a non empty topological space, $a$ be a complex number, and $f$ be a function from the carrier of $X$ into $C$. Then $\text{support}(a \cdot f) \subseteq \text{support } f$.

(35) For every non empty topological space $X$ and for all functions $f$, $g$ from the carrier of $X$ into $C$ holds $\text{support}(f \cdot g) \subseteq \text{support } f \cup \text{support } g$.

Let $X$ be a non empty topological space. The $CC_0$-functions of $X$ yielding a non empty subset of the $C$-vector space of the carrier of $X$ is defined by the condition (Def. 7).

(Def. 7) The $CC_0$-functions of $X = \{f; f$ ranges over functions from the carrier of $X$ into $C$: $f$ is continuous $\land \lor_{Y: \text{non empty subset of } X} (Y$ is compact $\land \land_{A: \text{subset of } X} (A = \text{support } f \Rightarrow \overline{A}$ is a subset of $Y))\}$.

The following propositions are true:

(36) Let $X$ be a non empty topological space. Then the $CC_0$-functions of $X$ is a non empty subset of $C$-Algebra(the carrier of $X$).

(37) Let $X$ be a non empty topological space and $W$ be a non empty subset of $C$-Algebra(the carrier of $X$). Suppose $W = \text{the } CC_0$-functions of $X$. Then $W$ is $C$-additively linearly closed.

(38) For every non empty topological space $X$ holds the $CC_0$-functions of $X$ is add closed.

(39) For every non empty topological space $X$ holds the $CC_0$-functions of $X$ is linearly closed.

Let $X$ be a non empty topological space. Observe that the $CC_0$-functions of $X$ is non empty and linearly closed.

The following propositions are true:

(40) Let $V$ be a complex linear space and $V_1$ be a subset of $V$. Suppose $V_1$ is linearly closed and $V_1$ is not empty. Then $(V_1, \text{Zero}(V_1, V), \text{Add}(V_1, V), \text{Mult}(V_1, V))$ is a subspace of $V$.

(41) Let $X$ be a non empty topological space. Then $(\text{the } CC_0$-functions of $X$, $\text{Zero}(\text{the } CC_0$-functions of $X$, the $C$-vector space of the carrier of $X$),$\text{Add}(\text{the } CC_0$-functions of $X$, the $C$-vector space of the carrier of $X$),$\text{Mult}(\text{the } CC_0$-functions of $X$, the $C$-vector space of the carrier of $X))$ is a subspace of the $C$-vector space of the carrier of $X$. 
Let $X$ be a non empty topological space. The $\mathbb{C}$-vector space of $C_0$-functions of $X$ yielding a complex linear space is defined by the condition (Def. 8).

(Def. 8) The $\mathbb{C}$-vector space of $C_0$-functions of $X = \{\text{the } \mathbb{C}C_0\text{-functions of } X, \text{Zero}(\text{the } \mathbb{C}C_0\text{-functions of } X, \text{the } \mathbb{C}\text{-vector space of the carrier of } X), \text{Add}(\text{the } \mathbb{C}C_0\text{-functions of } X, \text{the } \mathbb{C}\text{-vector space of the carrier of } X), \text{Mult}(\text{the } \mathbb{C}C_0\text{-functions of } X, \text{the } \mathbb{C}\text{-vector space of the carrier of } X)\}$. 

Next we state the proposition 

(42) Let $X$ be a non empty topological space and $x$ be a set. If $x \in \text{the } \mathbb{C}C_0\text{-functions of } X$, then $x \in \mathbb{C}$-BoundedFunctions (the carrier of $X$).

Let $X$ be a non empty topological space. The $\mathbb{C}C_0$-functions norm of $X$ yielding a function from the $\mathbb{C}C_0$-functions of $X$ into $\mathbb{R}$ is defined by:

(Def. 9) The $\mathbb{C}C_0$-functions norm of $X = (\mathbb{C}$-BoundedFunctionsNorm (the carrier of $X)) (\text{the } \mathbb{C}C_0\text{-functions of } X$.

Let $X$ be a non empty topological space. The $\mathbb{C}$-normed space of $C_0$-functions of $X$ yielding a complex normed space structure is defined by the condition (Def. 10).

(Def. 10) The $\mathbb{C}$-normed space of $C_0$-functions of $X = \{\text{the } \mathbb{C}C_0\text{-functions of } X, \text{Zero}(\text{the } \mathbb{C}C_0\text{-functions of } X, \text{the } \mathbb{C}\text{-vector space of the carrier of } X), \text{Add}(\text{the } \mathbb{C}C_0\text{-functions of } X, \text{the } \mathbb{C}\text{-vector space of the carrier of } X), \text{Mult}(\text{the } \mathbb{C}C_0\text{-functions of } X, \text{the } \mathbb{C}\text{-vector space of the carrier of } X), \text{the } \mathbb{C}C_0\text{-functions norm of } X\}$. 

Let $X$ be a non empty topological space. One can check that the $\mathbb{C}$-normed space of $C_0$-functions of $X$ is strict and non empty.

One can prove the following propositions:

(43) Let $X$ be a non empty topological space and $x$ be a set. Suppose $x \in \text{the } \mathbb{C}C_0\text{-functions of } X$. Then $x \in \mathbb{C}$-continuous functions of $X$.

(44) For every non empty topological space $X$ holds

$0_{\text{the } \mathbb{C}\text{-vector space of } C_0\text{-functions of } X} = X \mapsto 0$.

(45) For every non empty topological space $X$ holds

$0_{\text{the } \mathbb{C}\text{-normed space of } C_0\text{-functions of } X} = X \mapsto 0$.

(46) Let $a$ be a complex number, $X$ be a non empty topological space, and $F, G$ be points of the $\mathbb{C}$-normed space of $C_0$-functions of $X$. Then $\|F\| = 0$ iff $F = 0$ the $\mathbb{C}$-normed space of $C_0$-functions of $X$ and $\|a \cdot F\| = |a| \cdot \|F\|$ and $\|F + G\| \leq \|F\| + \|G\|$.

Let $X$ be a non empty topological space. Note that the $\mathbb{C}$-normed space of $C_0$-functions of $X$ is reflexive, discernible, complex normed space-like, vector distributive, scalar distributive, scalar associative, scalar unital, Abelian, add-associative, right zeroed, and right complementable.

The following proposition is true
(47) Let $X$ be a non empty topological space. Then the $C$-normed space of $C_0$-functions of $X$ is a complex normed space.

References

[1] Grzegorz Bancerek. The ordinal numbers. Formalized Mathematics, 1(1):91–96, 1990.
[2] Czesław Byliński. Binary operations. Formalized Mathematics, 1(1):175–180, 1990.
[3] Czesław Byliński. The complex numbers. Formalized Mathematics, 1(3):507–513, 1990.
[4] Czesław Byliński. Functions from a set to a set. Formalized Mathematics, 1(1):153–164, 1990.
[5] Czesław Byliński. Partial functions. Formalized Mathematics, 1(2):357–367, 1990.
[6] Czesław Byliński. Some basic properties of sets. Formalized Mathematics, 1(1):47–53, 1990.
[7] Agata Darmochwał. Compact spaces. Formalized Mathematics, 1(2):383–386, 1990.
[8] Noboru Endou. Banach algebra of bounded complex linear operators. Formalized Mathematics, 12(3):237–242, 2004.
[9] Noboru Endou. Banach space of absolute summable complex sequences. Formalized Mathematics, 12(2):191–194, 2004.
[10] Noboru Endou. Complex Banach space of bounded linear operators. Formalized Mathematics, 12(2):201–209, 2004.
[11] Noboru Endou. Complex linear space and complex normed space. Formalized Mathematics, 12(2):93–102, 2004.
[12] Noboru Endou. Complex linear space of complex sequences. Formalized Mathematics, 12(2):109–117, 2004.
[13] Noboru Endou. Complex valued functions space. Formalized Mathematics, 12(3):231–235, 2004.
[14] Noboru Endou. Continuous functions on real and complex normed linear spaces. Formalized Mathematics, 12(3):403–419, 2004.
[15] Krzysztof Hryniewiecki. Basic properties of real numbers. Formalized Mathematics, 1(1):35–40, 1990.
[16] Katuhiko Kanazashi, Hiroyuki Okazaki, and Yasunari Shidama. Banach algebra of bounded complex-valued functionals. Formalized Mathematics, 19(2):121–126, 2011, doi:10.2478/v10037-011-0019-0.
[17] Eugeniusz Kusak, Wojciech Leonczuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. Formalized Mathematics, 1(2):335–342, 1990.
[18] Chanapat Pacharapokin, Hiroshi Yamazaki, Yasunari Shidama, and Yatsuka Nakamura. Complex function differentiability. Formalized Mathematics, 17(2):67–72, 2009, doi:10.2478/v10037-009-0007-9.
[19] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. Formalized Mathematics, 1(1):223–230, 1990.
[20] Yasunari Shidama, Hirofumi Suzuki, and Noboru Endou. Banach algebra of bounded functionals. Formalized Mathematics, 16(2):115–122, 2008, doi:10.2478/v10037-008-0017-z.
[21] Andrzej Trybulec. Binary operations applied to functions. Formalized Mathematics, 1(2):329–334, 1990.
[22] Wojciech A. Trybulec. Groups. Formalized Mathematics, 1(5):821–827, 1990.
[23] Wojciech A. Trybulec. Vectors in real linear space. Formalized Mathematics, 1(2):291–296, 1990.
[24] Zinaida Trybulec. Properties of subsets. Formalized Mathematics, 1(1):67–71, 1990.
[25] Edmund Woronowicz. Relations and their basic properties. Formalized Mathematics, 1(1):73–83, 1990.
[26] Edmund Woronowicz. Relations defined on sets. Formalized Mathematics, 1(1):181–186, 1990.

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