New Time Distributions of $D^0$-$\bar{D}^0$ or $B^0$-$\bar{B}^0$ Mixing and $CP$ Violation

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Abstract

The formulae for $D^0$-$\bar{D}^0$ or $B^0$-$\bar{B}^0$ mixing and $CP$ violation at the $\tau$-charm or $B$-meson factories are derived, for the case that only the decay-time distribution of one $D$ or $B$ meson is to be measured. In particular, we point out a new possibility to determine the $D^0$-$\bar{D}^0$ mixing rate in semileptonic $D$ decays at the $\Psi(4.14)$ resonance; and show that both direct and indirect $CP$ asymmetries can be measured at the $\Upsilon(4S)$ resonance without ordering the decay times of two $B_d$ mesons or measuring their difference.

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1 It is well known that mixing between a neutral meson and its \( CP \)-conjugate counterpart can arise if both of them couple to a subset of real and (or) virtual intermediate states. Such mixing effects provide a mechanism whereby interference between the decay amplitudes of two mesons may occur, leading to the phenomenon of \( CP \) violation. To date the \( K^0 - \bar{K}^0 \) and \( B_d^0 - \bar{B}_d^0 \) mixing rates have been measured \([1]\), and the \( CP \)-violating signals in neutral \( K \)-meson decays have unambiguously been established \([2]\). A preliminary but encouraging result for the observation of \( CP \) violation in \( B_d^0 \) vs \( \bar{B}_d^0 \rightarrow J/\psi K_S \) decay modes has recently been reported by the CDF Collaboration \([3]\). In contrast, the present experiments have only yielded the upper bound on \( D^0 - \bar{D}^0 \) mixing and the lower bound on \( B^0_s - \bar{B}^0_s \) mixing \([1]\), which are respectively expected to be rather small and large in the standard model. Today the \( B_d^0 - \bar{B}_d^0 \) and \( B^0_s - \bar{B}^0_s \) systems are playing important roles in the study of flavor mixing and \( CP \) violation beyond the neutral kaon system. The \( D^0 - \bar{D}^0 \) system is, on the other hand, of particular interest to probe possible new physics that might give rise to observable \( D^0 - \bar{D}^0 \) mixing and \( CP \) violation in the charm sector.

The most promising place to produce \( B_d^0 \) and \( \bar{B}_d^0 \) events with high statistics and low backgrounds is the \( \Upsilon(4S) \) resonance, on which the asymmetric \( B \)-meson factories at KEK and SLAC as well as the symmetric \( B \)-meson factory at Cornell are based. Similarly \( B_s^0 \) and \( \bar{B}_s^0 \) events may coherently be produced at the \( \Upsilon(5S) \) resonance. At a \( \tau \)-charm factory \( D^0 \) and \( \bar{D}^0 \) events will in huge amounts be produced at the \( \Psi(4.14) \) resonance. To measure \( CP \) violation on any resonance, where the produced meson pair has the odd charge-conjugation parity \( (C = -1) \), a determination of the time interval between two meson decays is generally needed. This has led to the idea of asymmetric \( e^+e^- \) collisions at the \( \Upsilon(4S) \) resonance, i.e., asymmetric \( B \)-meson factories, in which the large boost allows to order the decay times of two \( B_d \) mesons and to measure their difference.

Recently a new idea, that \( CP \) violation can be measured on the \( \Upsilon(4S) \) resonance without ordering the decay times of two \( B_d \) mesons or determining their difference, has been pointed out by Foland \([4]\). If this idea is really feasible, it implies that the time-dependent measurement of \( B_d^0 - \bar{B}_d^0 \) mixing and \( CP \) violation may be realized at a symmetric \( e^+e^- \) collider running at the \( \Upsilon(4S) \) resonance, such as the one operated by the CLEO Collaboration at Cornell. It also implies that the time-dependent measurement of \( D^0 - \bar{D}^0 \) mixing and \( CP \) violation may straightforwardly be carried out at the \( \Psi(4.14) \) resonance with no need to build an asymmetric \( \tau \)-charm factory. Therefore a further and more extensive exploration of Foland’s idea and its consequences is desirable.

This note aims at reformulating the phenomenology of meson-antimeson mixing and \( CP \) violation at the \( \Upsilon(4S) \), \( \Upsilon(5S) \) or \( \Psi(4.14) \) resonance, for the case that only the decay-time distribution of one meson is to be measured. We take both \( C = -1 \) and \( C = +1 \) cases of the produced meson pair into account, and make no special assumption in deriving the generic formulae. In particular, we point out a new possibility to determine the \( D^0 - \bar{D}^0 \) mixing rate in
semileptonic $D$ decays at the $\Psi(4S)$ resonance; and show that both direct and indirect $CP$ asymmetries can be measured at the $\Upsilon(4S)$ resonance without ordering the decay times of two $B_d$ mesons or measuring their difference.

Let us make use of $P$ to symbolically denote $D$, $B_d$ or $B_s$ meson. In the assumption of $CPT$ invariance, the mass eigenstates of $P^0$ and $\bar{P}^0$ mesons can be written as

$$|P_L\rangle = p|P^0\rangle + q|\bar{P}^0\rangle,$$
$$|P_H\rangle = p|P^0\rangle - q|\bar{P}^0\rangle,$$

(1)
in which the subscripts “L” and “H” stand for Light and Heavy respectively, and $(p, q)$ are complex mixing parameters. The proper-time evolution of an initially $(t = 0)$ pure $P^0$ or $\bar{P}^0$ meson is given as

$$|P^0(t)\rangle = g_+(t)|P^0\rangle + \frac{q}{p}g_-(t)|\bar{P}^0\rangle,$$
$$|\bar{P}^0(t)\rangle = g_+(t)|P^0\rangle + \frac{p}{q}g_-(t)|\bar{P}^0\rangle,$$

(2)

where

$$g_+(t) = \exp\left[-\left(im + \frac{\Gamma}{2}\right)t\right] \cosh\left[(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}\right],$$
$$g_-(t) = \exp\left[-\left(im + \frac{\Gamma}{2}\right)t\right] \sinh\left[(i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}\right],$$

(3)

with the definitions $m = (m_L + m_H)/2$, $\Delta m = m_H - m_L$, $\Gamma = (\Gamma_L + \Gamma_H)/2$, and $\Delta\Gamma = \Gamma_L - \Gamma_H$. Here $m_{L(H)}$ and $\Gamma_{L(H)}$ are the mass and width of $P_{L(H)}$, respectively. In practice it is more popular to use two dimensionless parameters for the description of $P^0$-$\bar{P}^0$ mixing: $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/(2\Gamma)$.

For a coherent $P^0\bar{P}^0$ pair at rest, its time-dependent wave function can be written as

$$\frac{1}{\sqrt{2}} \left[ |P^0(K, t)\rangle \otimes |\bar{P}^0(-K, t)\rangle + C|P^0(-K, t)\rangle \otimes |\bar{P}^0(K, t)\rangle \right],$$

(4)

where $K$ is the three-momentum vector of the $P$ mesons, and $C = \pm 1$ denotes the charge-conjugation parity of this coherent system. The formulae for the time evolution of $P^0$ and $\bar{P}^0$ mesons have been given in Eq. (2). Here we consider the case that one of the two $P$ mesons (with momentum $K$) decays to a final state $f_1$ at proper time $t_1$ and the other (with $-K$) to $f_2$ at $t_2$. $f_1$ and $f_2$ may be either hadronic or semileptonic states. The amplitude of such a joint decay mode is given by

$$A(f_1, t_1; f_2, t_2)_C = \frac{1}{\sqrt{2}} A_{f_1} A_{f_2} \xi C \left[ g_+(t_1)g_-(t_2) + C g_-(t_1)g_+(t_2) \right] + \frac{1}{\sqrt{2}} A_{f_1} A_{f_2} \xi C \left[ g_+(t_1)g_+(t_2) + C g_-(t_1)g_-(t_2) \right],$$

(5)
where \( A_{fi} = \langle f_i | \mathcal{H} | P^0 \rangle \), \( \lambda_i = (q/p)(\langle f_i | \mathcal{H} | \bar{P}^0 \rangle / \langle f_i | \mathcal{H} | P^0 \rangle) \) (for \( i = 1, 2 \)), and

\[
\zeta_C = \frac{p}{q} \left(1 + C \lambda_{f_1} \lambda_{f_2} \right),
\]

\[
\zeta_C = \frac{p}{q} \left( \lambda_{f_2} + C \lambda_{f_1} \right). \tag{6}
\]

After a lengthy calculation \[5\, 6\], we obtain the time-dependent decay rate as follows:

\[
R(f_1, t_1; f_2, t_2) \propto |A_{f_1}|^2 |A_{f_2}|^2 \exp(-\Gamma t_+) \times \left[ \left( |\xi_C|^2 + |\zeta_C|^2 \right) \cosh(y \Gamma t_C) - 2 \text{Re}(\xi_C^* \zeta_C) \sinh(y \Gamma t_C) \right.
\]

\[
- \left( |\xi_C|^2 - |\zeta_C|^2 \right) \cos(x \Gamma t_C) + 2 \text{Im}(\xi_C^* \zeta_C) \sin(x \Gamma t_C) \right] , \tag{7}
\]

where \( t_C = t_2 + Ct_1 \) has been defined.

Now we integrate the decay rate \( R(f_1, t_1; f_2, t_2) \) over \( t_1 \in [0, \infty) \), i.e., only the time distribution of \( P \)-meson decays into the final state \( f_2 \) is kept \[3\]. The result, with the notation \( t_2 = t \), is given as

\[
R(f_1, f_2; t) \propto |A_{f_1}|^2 |A_{f_2}|^2 \exp(-\Gamma t) \times \left[ \left( |\xi_C|^2 + |\zeta_C|^2 \right) \cosh(y \Gamma t + C \phi_y) - 2 \text{Re}(\xi_C^* \zeta_C) \sinh(y \Gamma t + C \phi_y) \right.
\]

\[
- \left( |\xi_C|^2 - |\zeta_C|^2 \right) \cos(x \Gamma t + C \phi_x) + 2 \text{Im}(\xi_C^* \zeta_C) \sin(x \Gamma t + C \phi_x) \right] , \tag{8}
\]

where the phase shifts \( \phi_x \) and \( \phi_y \) are defined by \( \tan \phi_x = x \) and \( \tanh \phi_y = y \), respectively.

The joint decay rate obtained above is a new result and serves as the master formula of this paper. In the following we shall specifically investigate meson-antimeson mixing and \( CP \) violation in \( D \)- and \( B \)-meson decays into the semileptonic final states, the hadronic \( CP \) eigenstates, and the hadronic non-\( CP \) eigenstates.

3 Let us first consider the joint decays of \( (P^0 \bar{P}^0)_C \) pairs into two semileptonic states \( (l^\pm X^\pm_a) \) and \( (l^\pm X^\pm_b) \), i.e., the dilepton events in the final states. Keeping the validity of the \( \Delta Q = \Delta P \) rule and \( CPT \) invariance, we have \( \langle l^- X^+_a | \mathcal{H} | P^0 \rangle = \langle l^+ X^-_b | \mathcal{H} | \bar{P}^0 \rangle = 0 \) and \( \langle l^+ X^-_a | \mathcal{H} | P^0 \rangle = \langle l^- X^+_b | \mathcal{H} | \bar{P}^0 \rangle \neq 0 \). The latter is denoted later by \( A_{ai} \) for \( i = a \) or \( b \). With the help of Eq. (8), we arrive at the same-sign and opposite-sign dilepton rates as follows:

\[
N_{C^+}^+(t) \propto \left| \frac{p}{q} \right|^2 |A_{ta}|^2 |A_{tb}|^2 \exp(-\Gamma t) \left[ \frac{\cosh(y \Gamma t + C \phi_y)}{\sqrt{1-y^2}} - \frac{\cos(x \Gamma t + C \phi_x)}{\sqrt{1+x^2}} \right] , \tag{9}
\]

\[
N_{C^-}^-(t) \propto \left| \frac{q}{p} \right|^2 |A_{ta}|^2 |A_{tb}|^2 \exp(-\Gamma t) \left[ \frac{\cosh(y \Gamma t + C \phi_y)}{\sqrt{1-y^2}} - \frac{\cos(x \Gamma t + C \phi_x)}{\sqrt{1+x^2}} \right] ; \tag{9}
\]

and

\[
N_{C^-}^+(t) \propto 2 |A_{ta}|^2 |A_{tb}|^2 \exp(-\Gamma t) \left[ \frac{\cosh(y \Gamma t + C \phi_y)}{\sqrt{1-y^2}} + \frac{\cos(x \Gamma t + C \phi_x)}{\sqrt{1+x^2}} \right] . \tag{10}
\]
Obviously the relationship \( N_{i+1}^+(t)N_{i-1}^-(t) = N_{i-1}^+(t)N_{i+1}^- \) holds.

The measure of CP violation in \( P^0 - \bar{P}^0 \) mixing turns out to be

\[
A_{C}^+(t) = \frac{N_{C}^{++}(t) - N_{C}^{--}(t)}{N_{C}^{++}(t) + N_{C}^{--}(t)} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4},
\]

independent of both the decay time \( t \) and the charge-conjugation parity \( C \). Within the standard model the magnitude of \( A_{C}^+(t) \) is estimated to be of \( \mathcal{O}(10^{-3}) \) or smaller, for either the \( D^0 - \bar{D}^0 \) system \([3]\) or the \( B^0 - \bar{B}^0 \) system \([1, 2]\). But it might significantly be enhanced if there were new physics contributions to \( P^0 - \bar{P}^0 \) mixing \([6-9]\).

On the other hand, the rate of \( P^0 - \bar{P}^0 \) mixing can be determined from

\[
S_{C}^{+-}(t) = \frac{N_{C}^{++}(t) + N_{C}^{--}(t)}{N_{C}^{++}(t)} = \frac{1}{2} \left( \frac{|p|^2}{|q|^2} + \frac{|q|^2}{|p|^2} \right) \cosh(y \Gamma t + C \phi_y) - z \cos(x \Gamma t + C \phi_x) \cosh(y \Gamma t + C \phi_y) + z \cos(x \Gamma t + C \phi_x),
\]

where \( z = \sqrt{(1 - y^2)/(1 + x^2)} \). As for \( S_{C}^{+-}(t) \), the approximation \((|p/q|^2 + |q/p|^2)/2 \approx 1\) is rather safe in the standard model.

For the \( B^0_d - \bar{B}^0_d \) system we show the dependence of \( S_{C}^{+-}(t) \) on the decay time \( t \) in Fig. 1, where \( x \approx 0.723 \) and \( y \approx 0 \) \([4]\) (accordingly, \( \phi_x \approx 0.626 \) and \( \phi_y \approx 0 \)) have been taken. We find that \( S_{C}^{+-}(t) \) and \( S_{C}^{++}(t) \) become maximal at the positions \( \Gamma t = (\pi + \phi_x)/x \approx 5.2 \) and \( \Gamma t = (\pi - \phi_x)/x \approx 3.5 \), respectively. The phase interval between these two line shapes, amounting to \( 2\phi_x/x \), also measures the rate of \( B^0_d - \bar{B}^0_d \) mixing \([4]\).

For the \( D^0 - \bar{D}^0 \) system one has the following conservative bound on the mixing rate: \( x < 0.1 \) and \( y < 0.1 \) (satisfying \( x^2 + y^2 < 0.01 \)), which were obtained from the wrong-sign semileptonic decays of neutral \( D \) mesons at the 90\% confidence level \([1, 11]\). The relative magnitude of \( x \) and \( y \) remains unclear, as the theoretical estimates involve too large uncertainty due to the long-distance effects \([12]\). In Fig. 2 we illustrate the time-dependent behavior of \( S_{C}^{+-}(t) \) with three types of inputs: (a) \( x \approx y \approx 0.06 \); (b) \( x \approx 0.08 \) and \( y \approx 0 \); and (c) \( x \approx 0 \) and \( y \approx 0.08 \). We see that the line shape of \( S_{C}^{+-}(t) \) for the \( x \ll y \) case is clearly distinguishable, when \( \Gamma t \geq 5 \), from that for the \( x \gg y \) case. A delicate analysis even allows to discern the relative magnitude of \( x \) and \( y \). This provides us a new possibility, different from those proposed previously in the literature \([13]\), to measure the rate of \( D^0 - \bar{D}^0 \) mixing \([1]\).
Figure 1: Ratios of the same-sign to opposite-sign dilepton events changing with the decay time $t$ at the $\Upsilon(4S)$ resonance, where $x \approx 0.723$ and $y \approx 0$ have been taken.

Figure 2: Illustrative plot for ratios of the same-sign to opposite-sign dilepton events changing with the decay time at the $\Psi(4.14)$ resonance.
For the $B_s^0$-$\bar{B}_s^0$ system we have $x > 14$ from current experimental data at the 95% confidence level [1], and $y \sim 0.03$ from the latest theoretical calculation [17]. Hence the behavior of $S_{C}^{+}(t)$ depends mainly upon the value of $x$. Taking $x \approx 20$ and $y \approx 0$ typically, one finds that the oscillation term of $S_{C}^{+}(t)$ is suppressed by a factor $z \approx 1/x$. As a consequence $S_{C}^{+}(t) \approx 1$ holds for variable values of $x$, i.e., the magnitude of $S_{C}^{+}(t)$ deviates little from unity. This property makes it somehow difficult to determine the precise value of $x$ by measuring the time distribution of $S_{C}^{+}(t)$ at the $\Upsilon(5S)$ resonance [17].

4 Now let us consider $CP$ violation in neutral $B$- or $D$-meson decays into hadronic $CP$ eigenstates at the $\Upsilon(4S)$ or $\Psi(4S)$ resonance. In this case the semileptonic decay of one $P$ meson serves to tag the flavor of the other $P$ meson decaying into a nonleptonic $CP$ eigenstate. There are generally three different types of $CP$ asymmetries, arising from $P^0-\bar{P}^0$ mixing itself, from the interference between two decay amplitudes ($direct CP$ violation), and from the interplay of decay and $P^0-\bar{P}^0$ mixing ($indirect CP$ violation). For the $B^0_d-\bar{B}^0_d$ system the typical magnitudes of these three kinds of $CP$-violating effects are respectively expected to be of $\mathcal{O}(10^{-3})$, $\mathcal{O}(10^{-2})$ to $\mathcal{O}(10^{-1})$, and $\mathcal{O}(1)$ in the standard model. It is more difficult to classify the magnitudes of direct and indirect $CP$ asymmetries in different decay channels of neutral $D$ or $B_s$ mesons, but $CP$ violation in either $B^0_s-\bar{B}^0_s$ or $D^0-\bar{D}^0$ mixing is anticipated to be below $\mathcal{O}(10^{-3})$ within the standard model. Therefore the neglect of tiny mixing-induced $CP$ violation, equivalent to taking $|q/p| \approx 1$ (as well as $y \approx 0$), is a good approximation when we calculate the direct and indirect $CP$ asymmetries in most $B_d$, $B_s$ and $D$ decays. We obtain the time-dependent decay rates as

$$R(l^\pm, f; t)_C \propto |A_f|^2|A_i|^2\exp(-\Gamma t) \left[ 1 + |\lambda_f|^2 \right] \pm \frac{1 - |\lambda_f|^2}{\sqrt{1 + x^2}} \cos(x\Gamma t + C\phi_x)$$

$$\mp \frac{2\text{Im}\lambda_f}{\sqrt{1 + x^2}} \sin(x\Gamma t + C\phi_x) \right], \quad (13)$$

where $f$ is the $CP$ eigenstate, and $\lambda_f = (q/p)\langle f|\mathcal{H}|\bar{P}^0\rangle/\langle f|\mathcal{H}|P^0\rangle$ as defined before. The $CP$ asymmetry is then given by

$$A_f^C(t) = \frac{R(l^-, f; t) - R(l^+, f; t)}{R(l^-, f; t) + R(l^+, f; t)}$$

$$= \frac{1}{\sqrt{1 + x^2}} \left[ \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(x\Gamma t + C\phi_x) - \frac{2\text{Im}\lambda_f}{1 + |\lambda_f|^2} \sin(x\Gamma t + C\phi_x) \right]. \quad (14)$$

Clearly $A_f^C(t)$ consists of both the direct $CP$ asymmetry ($|\lambda_f| \neq 1$) and the indirect one ($\text{Im}\lambda_f \neq 0$). Measuring the time distribution of $A_f^C(t)$ can distinguish between these two sources of $CP$ violation.

For illustration let us take the gold-plated channels $B_d^0$ vs $\bar{B}_d^0 \rightarrow J/\psi K_S$, which are dominated by the tree-level quark transitions [17], for example. It is well known that $|\lambda_{\psi K_S}| \approx 1$ and $\text{Im}\lambda_{\psi K_S} = \sin(2\beta)$ hold, where $\beta = \arg[-(V^*_{cb}V_{cd})/(V^*_{tb}V_{td})]$ is an inner angle of the quark
mixing unitarity triangle. We are left with

$$A_{\psi K_S}^C(t) = -\frac{\sin 2\beta}{\sqrt{1 + x^2}} \sin(x\Gamma t + C\phi_x),$$ (15)

to a high degree of accuracy. The behavior of this $CP$ asymmetry changing with the decay time $t$ is illustrated in Fig. 3. Certainly the weak phase $\beta$ can well be determined from such a time-dependent measurement at the $\Upsilon(4S)$ resonance.5

5 Finally we consider the case that both $P^0$ and $\bar{P}^0$ mesons decay into a common non-$CP$ eigenstates. For neutral $D$-meson decays, most of such decay modes occur through the quark transitions $c \rightarrow s(u\bar{d})$ and $c \rightarrow d(u\bar{s})$ or their flavor-conjugate processes. For $B_d$ and $B_s$ decays, most of such decay channels take place through the quark transitions $b \rightarrow q(u\bar{c})$ and $b \rightarrow q(c\bar{u})$ or their flavor-conjugate processes (for $q = d$ or $s$). The typical examples of such decay channels include $D^0 \rightarrow \bar{D}^0 \rightarrow K^\pm \pi^\mp$, $B_d^0 \rightarrow D^\pm \pi^\mp$, and $B_s^0 \rightarrow \bar{D}_s^0 \rightarrow D_s^\pm K^\mp$ decays.6

For simplicity we concentrate only on the decay modes in which no direct $CP$ violation exists, i.e., the decay amplitudes of $P^0 \rightarrow f$ and $\bar{P}^0 \rightarrow \bar{f}$ are governed by a single weak phase. We also take $y \approx 0$, as indirect $CP$ violation is primarily associated with the mixing parameter $x$. For coherent $P^0\bar{P}^0$ decays at the resonance, we make use of the semileptonic decay of one $P$.

4The result for the $C = -1$ case has been presented in Ref. [4], where the definition of $CP$ asymmetries is different from ours in Eq. (14).

5To extract the weak phase $\beta$ and $\beta'$ a study of $B_d$ and $B_s$ decays into the non-$CP$ eigenstates $D_s^\pm D^\mp$ and $D_s^\pm D_s^\mp$, in which the penguin effects are negligibly small, is also of particular interest [18].
meson to tag the flavor of the other $P$ meson decaying into $f$ or $\bar{f}$. The time-dependent rates of such joint decay modes, with the help of Eq. (8), are given as follows:

$$R(l^-, f; t)_C \propto |A_l|^2 |A_f|^2 \exp(-\Gamma t) \left[ (1 + |\lambda_f|^2) + \frac{1 - |\lambda_f|^2}{\sqrt{1 + x^2}} \cos(x\Gamma t + C\phi_x) \right.$$  
$$- \frac{2\text{Im}\lambda_f}{\sqrt{1 + x^2}} \sin(x\Gamma t + C\phi_x) \right],$$

$$R(l^+, \bar{f}; t)_C \propto |A_l|^2 |A_f|^2 \exp(-\Gamma t) \left[ (1 + |\bar{\lambda}_f|^2) + \frac{1 - |\bar{\lambda}_f|^2}{\sqrt{1 + x^2}} \cos(x\Gamma t + C\phi_x) \right.$$  
$$- \frac{2\text{Im}\bar{\lambda}_f}{\sqrt{1 + x^2}} \sin(x\Gamma t + C\phi_x) \right],$$

where $\bar{\lambda}_f = (p/q)\langle \bar{f}|H|P^0\rangle/\langle \bar{f}|\bar{H}|\bar{P}^0\rangle$, and the relationship $|\bar{\lambda}_f| = |\lambda_f|$ holds. The time-dependent $CP$ asymmetry turns out to be

$$A_{ff}^C(t) = \frac{R(l^-, f; t) - R(l^+, \bar{f}; t)}{R(l^-, f; t) + R(l^+, \bar{f}; t)} = \frac{\text{Im}(\bar{\lambda}_f - \lambda_f)\sin(x\Gamma t + C\phi_x)}{\sqrt{1 + x^2} (1 + |\lambda_f|^2) + F(\lambda_f, \bar{\lambda}_f, x\Gamma t + C\phi_x)},$$

in which $F$ is a function defined by $F(z_1, z_2, z_3) = (1 - |z_1|^2) \cos z_3 - \text{Im}(z_1 + z_2) \sin z_3$. Note that only the difference between $\text{Im}\bar{\lambda}_f$ and $\text{Im}\lambda_f$, which would vanish if the relevant weak phase were zero, measures the $CP$ violation.

Taking the decay modes $B_d^0 \rightarrow D^\pm \pi^\mp$ for example, one finds that measuring the $CP$ violating quantity $\text{Im}(\bar{\lambda}_{D^\pm \pi^\mp} - \lambda_{D^\mp \pi^\pm})$ allows the determination of the weak phase $(2\beta + \gamma)$, where $\gamma = \text{arg}[-(V_{ub}^* V_{ud})/(V_{cb}^* V_{cd})]$ is another angle of the quark mixing unitarity triangle $[19]$. This illustrates that some attention is worth being paid to $CP$ violation in neutral $B$- and $D$-meson decays into hadronic non-$CP$ eigenstates.

6 In summary, we have derived the generic formulae for $P^0 - \bar{P}^0$ mixing and $CP$ violation at the resonance where $P^0 \bar{P}^0$ pairs can coherently be produced, for the case that only the decay-time distribution of one $P$ meson is to be measured. Examples for the $D^0 - \bar{D}^0$, $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ systems are discussed. In particular, we point out a new possibility to measure $D^0 - \bar{D}^0$ mixing in semileptonic $D$-meson decays at the $\Psi(4.14)$ resonance, and show that both direct and indirect $CP$ asymmetries can be determined at the $\Upsilon(4S)$ resonance with no need to order the decay times of two $B_d$ mesons or to measure their difference.

We expect that the formulae and examples presented here will be useful for the physics being or to be studied at the $B$-meson and $\tau$-charm factories.
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