Configuration mixing for spin-isospin modes

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Abstract. Development of theories of configuration mixing is reviewed, concentrating on their application to spin-isospin modes, especially to the Gamow-Teller transitions. This talk is divided into three historical stages, the first order configuration mixing as the first stage, the second order configuration mixing as the second stage, and the delta-isobar-hole mixing as the third stage.

1. Introduction
Theories of configuration mixing started with the celebrated Arima-Horie paper [1] in 1954. Its background and applications to the magnetic moments were beautifully reviewed by the previous speaker Talmi [2]. The two previous speakers Richter [3] and Wakasa [4] vividly presented the astonishingly developed recent experimental studies with electrons and hadrons, and explained how the configuration mixing theories played active role for their analyses. Therefore I would like to talk about some sentimental oldies around theories of configuration mixing, concentrating on the subjects of the spin-isospin modes characterized by the operator $\tau \sigma$, especially on quenching problems of the Gamow-Teller (GT) transitions.

Development of the theories may be decomposed into three stages. The first could be “the first order configuration mixing”, starting from 1950s. The second could be “the second order configuration mixing”, starting from 1960s, and the third could be “the mixing of the delta-isobar ($\Delta$)-hole configurations, which became active from 1970s. In connection with the spin-isospin modes, all these mixing mechanisms got spotlighted from time to time, and sometimes provoked severe controversies. I would like to review such development and to show a route to the latest results presented by Wakasa [4].

2. First order configuration mixing—Stage I 1950 ~
The first stage of the configuration mixing theories started off with Arima-Horie’s paper of 1954, on the first order configuration mixing. Historical details were given by Igal Talmi. As has been described in his talk, the idea of this theory was born for explaining the deviation of the measured magnetic moments from the Schmidt lines. The $\tau \sigma$ operator is one of the important ingredients of the magnetic moment operator. As to the $\tau \sigma$ modes, this deviation separately appeared as the large delay of the GT-type allowed $\beta$-decays. The single particle values of log $ft$ of the GT-type decays are about 3.3, but most of the observed values are larger or much larger than 4. An old compilation of the decay rates was given in the paper of Ikeda-Fujii-Fujita [5]. I would like to call this delay as the $\tau \sigma$ quenching of the first kind.
The first order configuration mixing for $\tau\sigma$ is formulated by the first order perturbation theory, which mixes particle-hole (ph) states of the spin-orbit partners in a single particle state as
\[
|j\rangle = |j\rangle + \sum_{j'} a_{jj'} |j', (j'_{\pm})^{-1} (1^+); j\rangle ,
\]
where $j_{\pm} = l \pm \frac{1}{2}$. A key concept of the theory, which gives eminent effects, is coherence among the mixed states. I recall that Akito Arima often emphasized this point in his lectures. (See Noya-Arima-Horie [6].)

Calculation of the coefficients $a_{jj'}$ can be elaborated from the first order perturbation to the Tamm-Dancoff approximation and further to the random phase approximation (RPA). Thus we can extend the first order configuration mixing of ph states to mixing of the $1^+$ collective state, which is made of the spin-orbit partners. Thus the first order configuration mixing has gotten to be called "core polarization", including these extensions. The collective state is now known as the GT giant resonance (GTGR). (Possibly M1 giant resonance, too.)

Discovery of the isobaric analogue state (IAS) at the beginning of 1960s gave clear explanation of the strong quenching of the Fermi-type $\beta^-$ decay by the core polarization mechanism due to the coupling of a single particle state with the collective state, IAS in this case. Suggested by this discovery, Ikeda-Fujii-Fujita [5] expected concentration of the GT transition strengths in a certain confined region, though its width could be larger than IAS, that is, the prediction of the GTGR. Assuming this collective state, they explained the quenching of the GT $\beta^-$ decay, because the high lying collective state absorbs most of the transition strengths and thus little remain in the transitions between the low lying states. This is a key explanation of the $\tau\sigma$ quenching of the first kind. Of course there are other important effects such as deformation, pairing, etc., but I do not touch them here.

More than 10 years later in 1975, the GTGR was discovered by a $(p, n)$ reaction [7]. At the beginning of 1980s, systematic studies of the strength distributions of the GTGR were performed across the periodic table by intermediate energy $(p, n)$ reactions at Indiana University Cyclotron Facilities [8, 9]. Those experiments raised another kind of the GT quenching problem, which I will discuss in the next section.

Here I would like to comment on a problem of perturbative approach, that is, the huge quenching implies that the leading and correction terms could be represented as
\[
1 - (1 - \epsilon) = \epsilon .
\]
Therefore we need a method to get the small $\epsilon$ in good accuracy. Fujita and Ikeda proposed a commutator method [10].

3. Second order configuration mixing—Stage II 1960 ~

3.1. Quenching of the Gamow-Teller type $\beta^-$ decay

For the LS-closed shell $\pm 1$ nucleon nuclei, however, the core polarization mechanism does not contribute because the M1 and GT transitions vanish for the LS-closed shell core. However, experimentally observed matrix elements $(\tau\sigma)$ deduced from the GT $\beta^-$ decays were found to deviate appreciably from their single particle values, as is shown in Table 1. All data indicated quenching of $(\tau\sigma)$.

Therefore the second order perturbation theory was applied to such problems in the middle of 1960s by Ichimura-Yazaki [11] and Mavromatis-Zamick-Brown [12, 13], which gave sizable contribution on the quenching. This approach is called the second order configuration mixing.
An important relation of the theory is that the second order correction on an operator $O$ can be written as

$$
\delta^{(2)} \langle POP \rangle = \langle PV \frac{Q}{E_0 - H_0} \frac{Q}{E_0 - H_0} VP \rangle - \langle PV \left( \frac{Q}{E_0 - H_0} \right)^2 VP \rangle \langle POP \rangle
$$

$$
= \langle PV \frac{Q}{E_0 - H_0} \left[ O, \frac{Q}{E_0 - H_0} V \right] P \rangle
$$

if $POQ = QOP = 0$, where $P$ is the projection operator on the model space and $P + Q = 1$. For instance, if $O = \tau \sigma$ it cannot connect the $0\hbar\omega$ and $n\hbar\omega$, $(n \neq 0)$ excited states in the harmonic oscillator shell model. Therefore the second order contributions exist only when

$$
\left[ O, \frac{Q}{E_0 - H_0} V \right] \neq 0.
$$

This strongly implies that the main contribution to the $\langle \tau \sigma \rangle$ correction comes from the residual interactions $V$, which do not commute with $\tau \sigma$ such as the tensor force.

Shimizu-Ichimura-Arima (SIA) [14] demonstrated that the second order contribution to $\langle \tau \sigma \rangle$ appreciably comes from highly excited intermediate states, because of the short range nature of the tensor force. Hence they emphasized importance of the tensor correlation. Their results are shown in Table 1 with a later calculation with different interactions by Towner-Khanna (TK) [15]. Here I comment that SIA also showed that the second order configuration effects on the isoscalar spin operator $\langle \sigma \rangle$ are the similar size.

### Table 1

| A(state)  | $\delta\langle \tau \sigma \rangle/\langle \tau \sigma \rangle$ (%) | Experiment | SIA [14] | TK [15] |
|-----------|-------------------------------------------------|------------|----------|---------|
| 15(0p_{1/2}) | -13.2                                          | -15.7      | -18.9    |
| 17(0d_{5/2}) | -13.8                                          | -11.9      | -13.9    |
| 39(0d_{3/2}) | -33.7                                          | -21.0      | -25.0    |
| 41(0f_{7/2}) | -26.3                                          | -14.8      | -17.1    |

3.2. Quenching of the total Gamow-Teller transition strength and its sum rule

As to the GT transition the model independent GT sum rule is well known, which was first explicitly written by Gaarde et al. [16] to my knowledge. It is expressed as

$$
S^N(GT) = S^N(GT^-) - S^N(GT^+) = 3(N - Z),
$$

where the total GT-type $\beta^\pm$ transition strengths are given by

$$
S^N(GT^\pm) = \int R^N_{\pm,GT}(\omega)d\omega,
$$

with

$$
R^N_{\pm,GT}(\omega) = \sum_n |\langle \Psi_n | \sum_k t_{k,\pm} \sigma_k | \Psi_0 \rangle|^2 \delta(\omega - (E_n - E_0)).
$$

Here I attached the superscript N, which means that only the nucleon space is taken into account. If we include the $\Delta$ degree of freedom, the sum rule should be modified. The GT sum rule is often
called Ikeda sum rule because Ikeda, Fujii and Fujita [5] first discussed the strength distribution and introduced the factor $3(N - Z)$ in a certain approximation.

Systematic studied of the strength distributions by the $(p, n)$ reactions mentioned above demonstrated that the experimentally observed strengths were much smaller than the sum rule value. The degree of quenching is characterized by the quenching factor

$$Q = \frac{S^+_{\text{GT}}(\omega_{\text{top}}) - S^0_{\text{GT}}(\omega_{\text{top}})}{3(N - Z)}, \quad \text{with} \quad S^0_{\text{GT}}(\omega_{\text{top}}) = \int^{\omega_{\text{top}}}_{-\omega_{\text{top}}} R^\text{exp}_{\pm,\text{GT}}(\omega) d\omega. \quad (8)$$

Taking the upper limit of the integral $\omega_{\text{top}}$ to be just above the GTGR and using the theoretical estimation of $S^+_{\text{GT}}$, it was found that $Q \approx 50\%$ for medium and heavy nuclei [17]. I call this the τσ quenching of the second kind.

As to the origin of this quenching the Δ-hole (Δh) mixing was spotlighted in the beginning of 1980s, which will be discussed in the next section, because the coupling between the ph and Δh states shifts the GT transition strengths to the Δh region around 300 MeV excitation energies.

On the other hand Arima strongly argued that the quenching was due to the second order configuration mixing effects [18]. This assertion was well demonstrated by Bertsch and Hamamoto [19] in 1982. They calculated the GT strength spectrum up to about 50 MeV excitation energy by taking into account the 2-particle-2-hole (2p2h) state excitations, whose amplitudes are depicted in Fig. 1. The transition strength is given by the absolute square of the amplitudes of Fig. 1. This can be calculated from the imaginary parts of the diagrams shown in Fig. 2. The figure shows that the mechanism corresponds to the second order configuration mixing discussed above. Here again the tensor force plays an important role to involve the highly excited 2p2h states.

![Figure 1. 2p-2h amplitudes of the GT transition of even-even nuclei.](image)

![Figure 2. Typical diagrams made from those in Fig. 1, whose imaginary parts taken at intermediate states denoted by the dot-dashed lines give the GT transition strength.](image)

The final answer for the longstanding severe controversy between the Δh and the second order configuration mixing as the origin of the quenching was obtained from the multipole decomposition analysis (MDA) of $(p, n)$ reactions carried out by Wakasa et al. [20] in 1997, which was reported by Wakasa [4] in detail with the latest new experiments. They found the strengths of about 90% of the sum rule value below the 50 MeV excitation energy, namely

$$Q \approx 0.88. \quad (9)$$

This means that most of the τσ quenching of the second kind comes from the second order configuration mixing mechanism within the nucleon space. The strength distribution obtained by the MDA are compared with the theoretical calculations in Fig. 3.
4. Mixing of \( \Delta \)-hole states—Stage III 1970 ~

Since the \( \Delta \) has the spin and isospin \( \frac{3}{2} \), transitions from the nucleon to the \( \Delta \) need the isovector and spin-vector transitions. Therefore the \( \Delta \) effects exclusively appear in the spin-isospin modes. In early days the effects were treated as a part of exchange currents [23, 24].

At the beginning of 1970s, quenching of the axial vector weak coupling constant \( g_A' \) in the nuclear medium was discussed on the basis of the partially conserved axial current. It is closely related with the pion propagator in nuclei, which is strongly affected by \( \Delta h \) excitations. In this connection the \( \Delta \) effects on the spin-isospin modes are beautifully formulated [25, 26, 27], but the calculated quenching was not so large. A comprehensive calculation including the configuration mixings, \( \Delta h \) mixing and the exchange currents with the common interaction was carried out by Towner-Khanna [15] for the GT \( \beta \)-decay matrix elements of the LS-closed shell \( \pm 1 \) nucleon nuclei. It showed the effects of the \( \Delta \) were less than 10%.

At the beginning of 1980s, when the quenching of the second kind, namely \( Q \approx 50\% \), was discovered, many people presented the theoretical results, which attribute the quenching to the \( \Delta h \) mixing [28, 29]. Key difference from the older estimations in 1970s was different choice of the transition strength from the nucleon to the \( \Delta \). The calculations in 1980s usually used the Landau-Migdal (LM) interaction with the universality condition \( g_{NN}' = g_{N\Delta}' = g_{\Delta\Delta}' \approx 0.6 - 0.7 \), whose definitions were given in the Wakasa’s talk [4]. This interaction gave much stronger coupling and thus stronger quenching than the older calculations. The universality may be derived from a simple quark-quark interaction [29], if the exchange terms are neglected, but this cannot be justified [30].

As mentioned in the previous section, the recent experimental results \( Q \approx 90\% \) has been explained by the nucleon dynamics. The remaining 10% is now attributed to the \( \Delta h \) mixing, which I would like to call the \( \tau \sigma \) quenching of the third kind. From this 10%, the LM parameters were estimated. Wakasa [4] presented in detail that the latest analysis showed

\[
g_{NN}' \approx 0.6 - 0.7, \quad g_{N\Delta}' \approx 0.2 - 0.4. \tag{10}
\]
5. Summary

I sketched historical development of the theories of the configuration mixings in connection with the quenching phenomena of the spin-isospin modes.

I classified the quenchings into three kinds. The first is the quenching of $\langle \tau \sigma \rangle$ at low lying states, which I named the $\tau \sigma$ quenching of the first kind. This is due to the presence of the Gamow-Teller or M1 giant resonances, around which the transition strengths concentrated, and this has been explained by the core polarization (the first order configuration mixing) theory, which started in 1950s.

The second is the quenching of the total GT transition strength up to just above the GTGR region, which is only about 50% of the GT sum rule value, namely the quenching factor $Q = 50\%$. I named it the $\tau \sigma$ quenching of the second kind. Most of the missing strength are found in highly excited states well above the GTGR by the recent $(p, n)$ and $(n, p)$ reaction experiments, which show $Q \approx 90\%$. This was well explained by the second order configuration mixing theory, which started in 1960s.

The third is the remaining 10% quenching of the total GT strength, which I named the $\tau \sigma$ quenching of the third kind. This has been explained by the $\Delta h$ mixing with appropriate choice of the LM parameters. The role of the $\Delta$ has been actively discussed since 1970s. I leave the latest investigations of the $\tau \sigma$ modes to the previous talks by Wakasa [4] and Richter [3].

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