THE INFLUENCE OF WORLD-SHEET BOUNDARIES ON CRITICAL CLOSED STRING THEORY

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ABSTRACT

This paper considers interactions between closed strings and open strings satisfying either Neumann or constant (point-like) Dirichlet boundary conditions in a BRST formalism in the critical dimension. With Neumann conditions this reproduces the well-known stringy version of the Higgs mechanism. With Dirichlet conditions the open-string states correspond to either auxiliary or Lagrange multiplier target-space fields and their coupling to the closed-string sector leads to constraints on the closed-string spectrum.

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Conventional closed-string theory is formulated in terms of a sum over world-sheets with no boundaries. Adding boundaries in the usual manner, i.e.
with Neumann boundary conditions on the space-time coordinates, \( X^\mu(\sigma, \tau) \), leads to a theory with both closed and open strings with free end-points. Adding boundaries with constant Dirichlet conditions leads to a theory with no physical open strings but with a radical modification of the closed-string theory (in which, for example, fixed-angle scattering behaves in a point-like manner). With orientable world-sheets (which is all that is considered here) the boundaries carry quantum numbers of the defining representation of a unitary group, denoted \( U(m) \) (or ‘flavour’) in the Neumann case and \( U(n) \) (or ‘colour’) in the Dirichlet case.

Figure 1. (a) A representation of the interaction of a number of on-shell closed-string states on a world-sheet with the topology of a disk. The thick line indicates the world-sheet boundary. (b) The same process in a configuration in which the boundary represents a closed string disappearing into the vacuum. With Neumann conditions the boundary couples to a linear combination of the dilaton and the trace of the graviton in the cylinder while with Dirichlet conditions only the dilaton couples to the boundary. (c) Illustration of an intermediate open string. With Dirichlet conditions this string has both end-points fixed at the same space-time point.

Earlier work (see [1] and references therein) has pointed out certain special features of the Dirichlet theory and its relationship, via space-time duality, to the Neumann theory. A world-sheet with a single boundary (the disk illustrated in fig.1(a)) may be parametrized so that the boundary is the end-state of a cylindrical section of world-sheet (fig.1(b)). The process may then be expressed in terms of the evolution of a closed string coupling to the boundary, which is represented by a state, \( \langle B \rangle \) (the double ket notation indicates a state defined in the product of the spaces of left and right-moving modes), where \( \langle B \rangle | \partial_{\parallel} X^\mu = 0 \) in the Neumann theory (the subscript \( n \) indicates a derivative normal to the boundary), whereas in the Dirichlet theory \( \langle B \rangle | (X^\mu - y^\mu_B) = 0 \), where the position of each boundary must be integrated (and the total momentum passing through the boundary vanishes as
The end-state bra, given in terms of oscillator states by $[2, 3, 1]$

$$\langle \langle B | = \langle \langle \uparrow \uparrow | (B_0 - \tilde{B}_0) \exp \sum_{n=1}^{\infty} \left( \pm \frac{\alpha_{-n} \cdot \tilde{\alpha}_{-n}}{n} - C_{-n} \tilde{B}_{-n} - \tilde{C}_{-n} B_{-n} \right),$$

(1)

satisfies $\langle \langle B | Q_c = 0$ (where $Q_c \equiv Q + \tilde{Q}$ is the closed-string BRST operator). The plus sign corresponds to the Neumann theory and the minus sign to the Dirichlet theory (whereas the ghost coordinate boundary conditions are determined by the world-sheet geometry, independent of the boundary conditions of the space-time coordinates). The harmonic oscillators $\alpha_n^\mu, \tilde{\alpha}_n^\mu$ are the modes of the left-moving and right-moving closed-string embedding coordinates and $B_n, \tilde{B}_n, C_n, \tilde{C}_n$ are the modes of the antighost and ghost coordinates. The ground state $\langle \langle \uparrow \uparrow | (\equiv \langle \langle \downarrow \downarrow | C_0 \tilde{C}_0)$ is the product of the ghost-number $\frac{1}{2}$ ground states in the left-moving and right-moving closed-string space of states which is annihilated by $C_0, \tilde{C}_0$ and has zero momentum (which follows in the Dirichlet case after integration over the boundary position, $y_B^\mu$). The closed-string propagator contains a factor of $(C_0 - \tilde{C}_0)$ so that (1) implies in the Neumann case that a linear combination of the trace of the graviton and the dilaton propagate in the cylinder. This leads to a divergence that signals the breakdown of conformal invariance from a boundary of moduli space that may be compensated by a change in the background fields that introduces a cosmological term in the target-space theory [4]. By contrast, in the Dirichlet theory the only massless state that propagates in the cylinder is the dilaton causing a zero-momentum divergence that will be addressed later. The term ‘dilaton’ is here reserved for the massless general coordinate scalar state.

The disk amplitude in fig.1(a) can also be expressed as a sum over intermediate open strings, as illustrated in fig.1(c). In the Neumann theory this leads to the mixing of open-string and closed-string particle states but in the Dirichlet theory the presence of intermediate open strings leads to further divergences. This problem (noted and discussed in [1]) arises because the intermediate open string in fig.1(c) has both end-points fixed at the same space-time point. The level-one vector state, in particular, should be identified with a space-time Lagrange multiplier field. The divergences arise in string perturbation theory due to the fact that such a field has no mass or kinetic terms and hence has a singular propagator. Obviously, perturbation theory is inadequate in this situation and the presence of such fields is a signal that constraints should be imposed on the closed-string states to which they couple. One purpose of this paper is to elucidate these constraints in more detail.

This requires a discussion of the cohomology of the BRST operator in the open-string sector of the Dirichlet theory together with a description of the coupling
of open strings to a Dirichlet boundary. It will prove useful to compare the Dirichlet case with the more familiar case of Neumann boundary conditions.

**BRST cohomology and free string fields.**

Physical states of Neumann strings (open strings with free end-points) represent particles of definite mass (such as the massless photon) with wave-functions that satisfy equations of motion when the BRST constraints, $Q_o|\Psi\rangle = 0$, are imposed (where $Q_o$ is the open-string BRST operator which will be denoted by $Q_N$ and $Q_D$ in the Neumann and Dirichlet cases, respectively). These constraints may be interpreted as the free-field equations for the open-string field, $|\Psi\rangle$ (which is an $m \times m$ hermitian matrix). The physical states have ghost-number $-\frac{1}{2}$ and are defined up to gauge transformations, $\delta|\Psi\rangle = Q_o|\epsilon\rangle$ (where $|\epsilon\rangle$ is an arbitrary ghost-number $-\frac{3}{2}$ state). For example, the massless states at level 1 satisfy

$$Q_N (A_\mu \beta_{\mu}^{\mu} + \omega c_0 b_{-1}) |\downarrow\rangle = 0,$$

where $|\downarrow\rangle$ is the ghost-number $-\frac{1}{2}$ state annihilated by all the positive modes of the open-string conformal gauge antighost and ghost coordinates, $b_n, c_n$ (with positive $n$), as well as by $b_0$ (and the open-string modes are denoted $\beta_n$ in order to distinguish them from the closed-string modes). The gauge transformations that relate these states are associated with the gauge parameter $\lambda b_{-1} |\downarrow\rangle$ (which is the massless component of the general ghost-number $-3/2$ gauge parameter).

Equation (2) requires $\omega = ik_\mu A_\mu$ and $k^2 A_\mu + ik_\mu \omega = 0$ and Maxwell’s equations result upon eliminating the auxiliary field $\omega$. These equations follow from a target-space field theory action principle based on the action,

$$S'_0 = \int d^D x \left( -\frac{1}{2} \omega^2 + i \omega k_\mu A^\mu - \frac{1}{2} A^\mu k^2 A_\mu \right),$$

(3)

(where $k \equiv -i \partial/\partial x$ and the subscript 0 indicates that the action refers to the massless, or low-energy, sector of the complete string theory), which illustrates the characteristic feature of auxiliary fields that their action does not contain kinetic terms. The cohomology of $Q_N$ also contains the $SU(1,1)$-invariant ground state which is a scalar of ghost-number $-3/2$ and may be represented by $A b_{-1} |\downarrow\rangle$. This is an isolated state since the BRST constraint requires $k^\mu \Lambda = 0$. The remaining non-trivial cohomology classes are the duals of the above states, with ghost-numbers $1/2$ and $3/2$.

World-sheets with Dirichlet boundaries are correlation functions of boundaries at different space-time points. They can be constructed by sewing together open-string vertices with propagators that describe the world-sheet evolution of a string.
with end-points fixed in space-time. The propagator for such a string is given by
\[ \Delta_D = (L_0D - 1)^{-1}, \]
where (in units in which \( \alpha' = 1 \))
\[ L_0D = \left( \frac{y_2 - y_1}{2\pi} \right)^2 + N \tag{4} \]
and \( N \) is the level number for the states of a string with end-points fixed at space-time points \( y_1^\mu \) and \( y_2^\mu \). The singularities of \( \Delta_D \) determine the space-time singularity structure of the correlation functions [1]. The BRST cohomology of these open-string states is isomorphic to those of the Neumann theory with the momentum \( k^\mu \) in \( Q_N \) replaced by the separation \( (y_2 - y_1)^\mu / 2\pi \) in \( Q_D \). This relationship between the Neumann and Dirichlet theories is a manifestation of space-time duality in the context of open string theory.

The gauge-singlet Neumann open-string states mix with closed-string states [5-7] giving rise to a stringy version of the Higgs mechanism that will be considered in more detail below. In particular, the massless closed-string antisymmetric tensor mixes with the massless level-one open-string vector and gains a mass of order \( g^2 \) (where \( g \) is the open-string coupling constant and \( g^2 \) is proportional to the closed-string coupling constant). The effect of this mixing can also be seen by considering the coupling of a background electromagnetic potential to an end-state [8-10]. In the corresponding discussion of the mixing of closed and open strings in the Dirichlet theory we will see that a special rôle is played by gauge-singlet Dirichlet open-string states with \( y_1 = y_2 = y \) – i.e. states in which the string end-points are fixed at the same space-time point. For every value of \( y^\mu \) the BRST cohomology is the same as that of the \( k^\mu = 0 \) states of the Neumann theory, even though their wave functions depend on \( y^\mu \) and therefore do not have zero momentum. The only BRST non-trivial physical state at ghost-number \( -3/2 \) is the level-one vector, \( A^\mu \). Since it is physical without the need for any constraint it is interpreted in field theory as a Lagrange-multiplier field – it does not appear in the free-field action but enters only via its coupling to the closed-string sector. The wave function of the other level-one ghost-number \( -1/2 \) state remains auxiliary but now vanishes (\( \omega = 0 \)), which is reproduced by the free field action \( S_0' = \int d^Dx \frac{1}{2} \omega^2 \). The constraint on the ghost-number \( -3/2 \) wave function, \( \Lambda \), is no longer required for it to be BRST non-trivial. Its wave function is therefore promoted from a constant to the status of a Lagrange multiplier field.

Before discussing the interaction between open and closed strings in more detail I will briefly review the closed-string sector. In current formulations of closed-string field theory* it is necessary to restrict the closed-string field, \( |\Phi\rangle \), so

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* For the latest word on this subject and an up-to-date review of the literature see [11]. The discussion in this paper will not involve interactions between closed strings.
that

\[ B_0^- |\Phi\rangle \rangle = 0, \quad L_0^- |\Phi\rangle \rangle = 0, \tag{5} \]

where \( B_0^\pm \equiv (B_0 \pm \tilde{B}_0)/\sqrt{2} \) and \( L_0^\pm \equiv (L_0 \pm \tilde{L}_0)/\sqrt{2} \) are the zero-mode Virasoro generators. Physical states satisfying (5) are ‘semi-relative’ cohomology classes of \( Q_c \) (i.e. they also satisfy \( Q_c |\Phi\rangle \rangle = 0 \)) and have total ghost-number \(-1\). The free theory is invariant under the gauge transformations \( \delta |\Phi\rangle \rangle = Q_c |\Xi\rangle \rangle \), where \( |\Xi\rangle \rangle \) is an arbitrary ghost-number \(-2\) state satisfying \( L_0^- |\Xi\rangle \rangle = B_0^- |\Xi\rangle \rangle = 0 \).

The massless states (level one in both the left-moving and right-moving sectors) consist of a second-rank tensor, two vectors and two scalars,

\[
\left( h_{\mu\nu} a_{\mu-1} a_{\nu-1}' + \eta_{\mu} a_{\mu-1} C_0^+ \tilde{B}_{-1} + \bar{\eta}_{\mu} a_{\mu-1} C_0^+ B_{-1} + \phi \tilde{C}_{-1} B_{-1} + \tilde{\phi} C_{-1} \tilde{B}_{-1} \right) |\downarrow\downarrow\rangle \rangle .
\tag{6}
\]

Correspondingly the massless gauge invariances are defined by the ghost-number \(-2\) state \( \left( \xi_{\mu} a_{\mu-1} \tilde{\phi}_{-1} + \bar{\xi}_{\mu} a_{\mu-1} B_{-1} + \rho B_{-1} \tilde{B}_{-1} C_0^+ \right) |\downarrow\downarrow\rangle \rangle \), which contains one scalar and two vector gauge parameters. The dilaton field is the combination \( D = h_{\mu-1} - \phi \tilde{\phi} \), which is invariant under all these transformations.

The massless physical closed-string states satisfy free equations of motion that may be derived from the gauge-invariant free-field action,

\[
S''_0 = \int d^Dx \left( -\frac{1}{2} h_{\mu\nu} k^2 h^{\mu\nu} + \phi k^2 \tilde{\phi} - \frac{1}{2} (k^\nu h_{\mu\nu} - k_\mu \tilde{\phi})^2 \right.
\]

\[
\left. -\frac{1}{2} (k^\nu h_{\nu\mu} - k_\mu \phi)^2 + \frac{1}{2} (i\eta_\mu - k^\nu h_{\nu\mu} + k_\mu \tilde{\phi})^2 + \frac{1}{2} (i\bar{\eta}_\mu - k^\nu h_{\nu\mu} + k_\mu \phi)^2 \right) .
\tag{7}
\]

The parentheses involving the vector auxiliary fields \( \eta \) and \( \tilde{\eta} \) are gauge invariant and are eliminated by shifting these fields and integrating them, whereupon the combination \( \phi - \tilde{\phi} \) decouples. The remaining terms make up the usual linearized action for a second-rank tensor \( h_{\mu\nu} \) interacting with a dilaton \( D \).

The coupling between open and closed strings.

String field theory provides a succinct framework for describing the effect of the interactions between open and closed strings. The free-field action can be written in the form \([12,13]\),

\[
S^{(0)} = \frac{1}{2} \int (\Phi C_{-0}^- Q_c \Phi + \Psi Q_o \Psi) \equiv \frac{1}{2} \int d^Dx \left( \langle \langle \Phi | C_{-0}^- Q_c |\Phi\rangle \rangle + \text{Tr} \langle \Psi | Q_o |\Psi\rangle \rangle \right) ,
\tag{8}
\]

where \( \langle \Psi | = |\Psi\rangle \rangle^\dagger \) and \( \langle \langle \Phi | = -|\Phi\rangle \rangle^\dagger \) \([11]\). The open-string field is a matrix, \( |\Psi\rangle \rangle^A_B \) \((A, B = 1, \ldots, M)\), in the \( M \times M \)-dimensional representation of the Lie algebra
of $U(M)$ and $\text{Tr}$ denotes the trace in this space. The closed-string term and the Neumann theory open-string term (i.e. with $Q_o = Q_N$ and $M = m$) reduce to sums of infinite numbers of massive free-field actions when expanded in component fields. The Dirichlet theory open-string kinetic term (i.e. with $Q_o = Q_D$ and $M = n$) gives an infinite set of mass terms, one for each auxiliary component field.

Figure 2. (a) The coupling of a closed string to an open string in the ‘light-cone-like frame’ in which the vertex is simply the overlap of the closed-string and open-string coordinates. (b) The same coupling in the ‘vertex-operator frame’, in which the process is described by a local vertex for the emission of an arbitrary open-string state from a point on the boundary end-state of the closed string.

The vertex for the $O(g)$ coupling of an arbitrary closed-string state to an open-string state has been described in several ways. It was deduced in the Neumann theory in terms of light-cone gauge field theory (where the Fadeev–Popov ghost contribution is absent) in [14,15] and in a ‘light-cone-like’ ($lcl$) frame in [7] (where the ghosts were included). This is the frame illustrated in fig.2(a), in which the incoming closed string and the outgoing open string have the same width and the interaction is described simply by the change of boundary conditions at the interaction time. The vertex had originally been deduced by factorization of open-string loop amplitudes [16,17] which gave an expression appropriate to the ‘vertex-operator’ ($vo$) frame (but without consideration of ghosts) illustrated in figure 2(b). The generalization to the Dirichlet theory in both these frames (as well as the inclusion of ghost factors in the vertex-operator frame) is given in detail in [18]. [The discussion could equally be given in a frame relevant to Witten’s formulation of open-string field theory [13], coupled to closed strings in the manner suggested in [10].]

The vertex will be denoted $\langle\langle\langle V\rangle\rangle\rangle$ (where the triple bra indicates the product of the states of the closed string and open string), with superscripts $^{lcl}$ or $^{vo}$ distinguishing the frames and subscripts $N$ and $D$ distinguishing the boundary conditions where appropriate. It can be expressed in the form $\langle\langle\langle V\rangle\rangle\rangle = \langle\langle\langle \uparrow \uparrow; \uparrow \mid \delta^A_B \exp \Delta, \}$
where $\Delta$ is a bilinear form in the annihilation modes of the closed-string and open-string spaces and the state $\langle\langle\langle\uparrow\uparrow;\uparrow|\rangle\rangle\rangle\rangle$ is the ground state of ghost-number $3/2$ (the arrows successively label the states of the ghost zero modes in the spaces of the left-moving and right-moving closed-string modes and the open-string modes). The structure of the vertex guarantees the continuity of the space-time and ghost coordinates and their conjugate momenta at the interaction time (apart from an anomaly in the conservation of the antighost coordinates that is required in order that the total ghost number changes by $3/2$ due to the boundary curvature at the interaction point). This in turn implies the BRST condition

$$\langle\langle\langle V|Q_c + Q_o\rangle\rangle\rangle = 0 \quad (9)$$

(where $Q_o = Q_{\text{N}}$ or $Q_{\text{D}}$, depending on the boundary conditions). The explicit expressions for the vertices in $lcl$ or $vo$ frames the Neumann and Dirichlet theories are given in [7] and [18].

The vertex not only determines the interaction between open and closed strings but the free boundary state (1) can also be obtained by coupling the open-string $SU(1,1)$-invariant vacuum state to the vertex, so that in the $vo$ frame,

$$\langle\langle B\rangle\rangle = \langle\langle\langle V_{\text{vo}}| b_{-1} | \downarrow\rangle\rangle\rangle. \quad (10)$$

Although the constraint (9) is an essential element in the BRST invariance of scattering amplitudes on world-sheets with boundaries it is not the whole story. Generally there are anomalies in the free-field BRST symmetry associated with the boundary interaction which are compensated by a new transformation of the string fields.

The interaction can be written in the form

$$S^{(1)} = g \int d^D x \langle\langle\langle V|\Phi\rangle\rangle|\Psi\rangle \quad (11)$$

and the combined action $S = S^{(0)} + S^{(1)}$ is the part of the full action that is bilinear in string fields. It is invariant at $O(g)$ under the gauge transformations,

$$\delta|\Psi\rangle = Q_o|\epsilon\rangle - g\langle\langle\Xi|V\rangle\rangle, \quad \delta|\Phi\rangle = Q_c|\Xi\rangle - g\langle\epsilon|B^-|V\rangle\rangle. \quad (12)$$

The invariance of the action at this order is verified with the help of (9) together with the closed-string field constraints, (5). The modification of the free-field symmetries induced by the boundaries is evident from the $O(g)$ terms in (12). These terms mix the transformations of the states at all levels in a complicated manner.
The action $S^{(0)} + S^{(1)}$ generates tree diagrams that mix open and closed strings. The complete action should also include the cubic open-string interaction (of order $g$) as well as the cubic and higher-order interactions between closed strings – which lead to non-linear modifications of the gauge symmetries. There is no consistent way of separating trees from open-string loop diagrams that contribute at the same order in $g$ and such loops are crucial in verifying that the linearized transformations (12) are symmetries of the complete action. In more detail, applying the transformations (12) to the action $S$ results in two $O(g^2)$ terms. One of these is proportional to $\int g^2 \langle\langle\langle V|\Xi\rangle\langle\langle\Phi|V'\rangle\rangle\rangle$, where the $'$ indicates a second set of open and closed-string spaces and the two vertices are contracted on the open-string space to form a world-sheet with the topology of fig.1(c). This term vanishes due to the fact that both $\langle\langle\Phi|$ and $|\Xi\rangle$ are proportional to $B_0^-$ and that $B^-(\sigma)$ is continuous at each vertex. The second $O(g^2)$ term in the variation of $S$ is proportional to $\int g^2 \langle\langle\langle V|\epsilon\rangle B_0^- \langle\Psi|V'\rangle\rangle$, in which two vertices are contracted on the closed-string states so that the world-sheet has two boundaries and the topology of a cylinder with $|\epsilon\rangle$ and $\langle\Psi|$ attached to separate boundaries. This term is non-zero but is cancelled by a quantum open-string loop correction that arises from the double contraction of two cubic open-string vertices. The cancellation of terms in the variation of the complete action that are quadratic or of higher order in the fields will not be considered here.

In principle, the $O(g^2)$ mass shifts could be calculated by integrating out the open-string fields, resulting in the effective quadratic action for $|\Phi\rangle$ (in the Siegel gauge $b_0|\Psi\rangle = 0$),

$$S_{\Phi} = \frac{1}{2} \int d^D x \langle\langle\Phi| \left( C^{-}Q_c - g^2 M \langle\langle V| r\rangle\langle\langle V'| r\rangle\rangle \right) |\Phi\rangle$$

(13)

(where $L^o_0$ is the zero-mode Virasoro generator in the open-string sector and the sum over the complete set of states, labelled $r$, contracts the vertices on the open-string space). The modified closed-string masses should correspond to the zero eigenvalues of the inverse $|\Phi\rangle$ propagator, $\left( L^+_0 - 2 - g^2 M \langle\langle V| r\rangle\langle\langle V'| r\rangle\rangle \right) b_0 \frac{b_0}{L^o_0 - 1} \langle\langle\langle V'| r\rangle B_0^- B^+_0 \rangle$ (in the gauge $B^+_0|\Phi\rangle = 0$). However, in the Dirichlet theory (where $L^o_0 = L_{0D}$) $(L^o_0 - 1)^{-1}$ is singular due to the $N = 1$ vector Lagrange-multiplier field, which implies constraints on the closed-string field that require separate consideration. The resulting low-energy spectrum will now be considered separately in the Neumann and Dirichlet cases since they are so different.

i) The Neumann case

In this case only the massless states in both the open and closed-string sectors
survive the low-energy limit and result in a self-contained gauge-invariant field theory. In the light-cone-like frame (considered in [7]) the vertex has no momentum dependence so there is no direct coupling of $A^\mu$ to $h^{\mu\nu}$ and the interaction (11) couples the gauge-singlet vector field to the auxiliary fields ($\eta^{\mu}, \bar{\eta}^{\mu}, \phi, \bar{\phi}$),

$$S_0^{(1)} = -g\sqrt{m} \int d^D x \left( \hat{A}^\mu (\eta - \bar{\eta})_\mu + \hat{\omega}(\phi - \bar{\phi}) \right). \quad (14)$$

where $\hat{A}^\mu \equiv \text{Tr} A^\mu / \sqrt{m}$ and $\hat{\omega} \equiv \text{Tr} \omega / \sqrt{m}$. The higher-mass string states contribute to effective mass terms for certain massless fields, resulting in $S_0^{(2)} = g^2m \int d^D x \left( -h^{[\mu\nu]} h_{[\mu\nu]} + \frac{1}{2}(\phi - \bar{\phi})^2 - \hat{A}^2 \right)$. The complete action $S' + S_0'' + S_0^{(1)} + S_0^{(2)}$ is invariant under the ‘massless’ gauge transformations,

$$\delta h^{\mu\nu} = ik^{\mu}\xi^\nu + ik^{\nu}\xi^\mu, \quad \delta \phi = ik_{\mu}\xi^\mu + \rho + g\text{Tr}\lambda, \quad \delta \bar{\phi} = ik_{\mu}\bar{\xi}^\mu - \rho - g\text{Tr}\lambda$$
$$\delta \eta^\mu = k^2\xi^\mu - ik^{\mu}\rho - g^2m(\xi^\mu - \bar{\xi}^\mu), \quad \delta \bar{\eta}^\mu = k^2\bar{\xi}^\mu + ik^{\mu}\rho + g^2m(\xi^\mu - \bar{\xi}^\mu)$$
$$\delta A^\mu = ik^{\mu}\lambda + g(\xi^\mu - \bar{\xi}^\mu)\delta A_B, \quad \delta \lambda = -k^2\lambda - 2g\rho \delta A_B - 2g^2\lambda,$$

where the $O(g)$ terms are just the massless components of (12) and the $O(g^2)$ terms are induced by the massive fields in the functional integral. Upon integrating out the auxiliary fields the part of the resulting effective low-energy linearized action involving $h^{[\mu\nu]}$ and $\hat{A}^\mu$ is

$$\int d^D x \left( \frac{3}{4}k_{[\mu}h_{\nu\rho]}k^{[\mu}h^{\nu\rho]} - (g\sqrt{m}h_{[\mu\nu]} - i k_{[\mu} \hat{A}_{\nu]})(g\sqrt{m}h^{[\mu\nu]} - i k^{[\mu} \hat{A}^{\nu]}) \right), \quad (16)$$

which is the gauge-invariant Higgs-like action describing a massive antisymmetric tensor field. The action for the other fields, $h^{(\mu\nu)}$ and $D$ remains unaltered.

In the vertex-operator frame (as originally described in [5]) the boundary simply gives rise to a direct coupling between the vector field, $\hat{A}^\mu$, and the antisymmetric tensor field, $h^{[\mu\nu]}$, of the form $g\sqrt{m} \hat{A}_\mu \partial_\nu h^{[\mu\nu]}$ (where $g$ is the open-string coupling constant). A mass term, $g^2m h_{[\mu\nu]} h^{[\mu\nu]}$ is again generated, leading once more to the action (16) for $h^{[\mu\nu]}$ and $\hat{A}^\mu$ when the auxiliary fields are integrated.

(ii) The Dirichlet case

The systematics of the Dirichlet theory are quite different since it is not possible to argue that the states of the open strings with identified end-points (the auxiliary and Lagrange multiplier fields) decouple at low energies, even though the massive closed-string states do. However, the divergent perturbation theory contributions due to the coupling of the Lagrange multiplier field $A^\mu$ to closed strings, $S_A^{(1)}$, may be dealt with by isolating this term in the interaction.
The vertex-operator frame for the Dirichlet theory has a particularly simple physical interpretation, in which the process in fig.2(b) is described as the amplitude for a closed string to collapse to a single space-time point at the interaction time, whereupon it transforms into the open string. In this frame the interaction of the level-one vector Lagrange multiplier field with the boundary state has the expected simple form of a vertex operator,

\[ S^{(1)vo}_A = g \sqrt{n} \int d^Dx \langle \langle V^{vo}|\Phi \rangle \rangle \beta_{-1}^\mu | \downarrow \rangle A_\mu(x) \]

\[ = g \sqrt{n} \int d^Dx \langle \langle B| \int d\sigma_B (C(\sigma_B) \partial X^\mu(\sigma_B) - \tilde{C}(\sigma_B) \bar{\partial} X^\mu(\sigma_B))|\Phi \rangle \rangle \dot{A}_\mu(x) \]

\[ = g \sqrt{n} \int d^Dx \langle \langle B| \sum_{n=0}^{\infty} C_n^- (\alpha_n^\mu + \tilde{\alpha}_n^\mu)|\Phi \rangle \rangle \dot{A}_\mu(x), \] (17)

(where \( \alpha_0 = \tilde{\alpha}_0 = k/2 \)) so that the coupling to massless closed-string states is simply \( g \sqrt{n} \dot{A}^\mu \partial_\mu D \). [In the light-cone-like frame \( \dot{A}^\mu \) couples to a gauge-invariant combination of massless fields, \( g \sqrt{n} \int d^Dx (\dot{A}_\mu (\eta^\mu + \bar{\eta}^\mu + ik_\nu h^\nu{}^\mu + ik_\nu h^\mu{}^\nu - ik^\mu (\phi + \bar{\phi})) + i \dot{A}_\mu k^\mu D) \). Together with an \( O(g^2) \) mass term for \( \dot{A}^\mu \), the first term can be absorbed into a redefinition of the \( \eta \) and \( \eta' \) terms in \( S'' \) which are then integrated, leaving the previous effective interaction between \( \dot{A}^\mu \) and the dilaton.]

Adding (17) to the free action and integrating over \( \dot{A}^\mu \) leads to the constraint (using \( \langle \langle B|B_0^- = \langle \langle B|C_0^+ = 0 \),

\[ k^\mu \langle \langle B|B_0^+ C_0^- C_0^+ |\Phi \rangle \rangle = 0. \] (18)

The components of a general closed-string field that enter (18) have the expansion in terms of Fock space component states,

\[ C_0^- C_0^+ |\Phi \rangle = \sum_{\sum n_r^M = \sum \bar{n}_r^M} \phi^{\{n_r^M\}}(x) \{n_r^M\} \{\bar{n}_r^M\} \otimes | \uparrow \uparrow \rangle, \] (19)

where \( \{n_r^M\}, \{\bar{n}_r^M\} \) are occupation numbers for the left and right moving Fock spaces and the non-zero modes, \( \alpha_r^\mu, B_r \) and \( C_r \) have been combined into a vector of \( OSP(1, D-1 | 2) \) labelled by the index \( M \) (and likewise for \( \bar{\alpha}_r^\mu, \bar{B}_r \) and \( \bar{C}_r \)). The condition (18) is a constraint on the graded sum (counting states with an odd number of right-moving fermionic excitations with a minus sign) of those component
fields for which there are equal numbers of left-moving and right-moving Fock space
excitations, i.e. fields of the form $\phi_{\{n^M\}^r}(x)$. The combination of component fields at
level $N$ that is affected by the constraint is $\phi_N \equiv \sum_{\{n^M\}^r} \phi_{\{n^M\}^r}(x)/\sqrt{d_N}$,
where the normalization factor $d_N = \int_0^{2\pi} d\theta \prod_{n=1}^{\infty} (1 - e^{i n \theta})^{-24} e^{-i N \theta}/2\pi$ (which is
equal to the usual degeneracy of open-string states) has been chosen to give the kinetic term
\begin{equation}
\int d^D x \sum_{N,P=0}^{\infty} \frac{1}{2} \phi_N D_{NP} \phi_P,
\end{equation}
with $D_{NP} = \delta_{NP}(k^2 + 4N - 4)$. The constraint takes the form $k^\mu \sum_N \phi_N(x) \sqrt{d_N} = 0$. Naively, this may be argued to eliminate the non-constant part of the dilaton field ($D \equiv \phi_1$) since in the low-momentum limit closed-string states of non-zero mass decouple ($\phi_N \sim \sqrt{\alpha'}$ for $N \neq 1$). The constraint then becomes simply $k^\mu D = 0$ and the dilaton state disappears.

This may be made more explicit by substituting (18) into the free closed-string action by simply making the replacement $k^\mu D \rightarrow -k^\mu \sum_N \phi_N \sqrt{d_N}$ in (20), where the prime indicates that the $N = 1$ state is missing from the sum (if the constraint were used to to eliminate any other component field an equivalent, but more complicated, non-local action would result). This gives a modified action that can then be written as
\begin{equation}
\int d^D x \sum_{N,P=0}^{\infty} \frac{1}{2} \phi_N M_{NP} \phi_P.
\end{equation}
The matrix $M$ is defined by $M_{NP} = D_{NP} + k^2 v_N v_P$, where $v_N = \sqrt{d_N}$. The eigenvalues of $M$, which determine the diagonal form of a modified free-field action, are determined by the zeroes of
\begin{equation}
\operatorname{det}'(M - \lambda I) = \operatorname{det}'(D - \lambda I) \left(1 + k^2 (D - \lambda I)^{-1} v_N v_P \right)
= \operatorname{det}'(D - \lambda I) \left(1 + k^2 \sum_{N=1}^{\infty} \frac{d_N}{k^2 + 4N - 4 - \lambda} \right).
\end{equation}
The zeroes of $\operatorname{det}'(D - \lambda I)$ at $\lambda = k^2 + 4N - 4$ are cancelled by the poles of the sum in parentheses and the eigenvalues of $M$ are therefore determined by the zeroes of the last factor. The masses of particle states are given by the zero eigenvalues, i.e. by the values of $k^2$ at which both $\operatorname{det}'(M - \lambda I) = 0$ and $\lambda = 0$. Since the sum in parentheses in (22) is divergent due to the contribution of the states with large
we shall proceed by regularizing it by replacing the last factor in (22) (setting \( \lambda = 0 \)) by the small-\( \epsilon \) limit of the integral representation,

\[
\sum_{N=0}^{\infty} d_N \frac{k^2 e^{-(k^2+4N-4)\epsilon/2}}{k^2 + 4N - 4} = k^2 \int_{\epsilon}^{\infty} \frac{dl}{\epsilon} e^{(4-k^2)l/2} \prod_{n=1}^{\infty} (1 - e^{-2nl})^{-24} = \frac{\pi}{2} (2\pi)^{13} k^2 \int_0^{1/\epsilon} \frac{dl'}{\sqrt{l'}} e^{l'-\pi^2 k^2/l'} \prod_{n=1}^{\infty} (1 - e^{-nl'})^{-24}
\]

(23)

(where \( l' = 2\pi^2 / l \)) which can be viewed as a functional integral on a cylinder of minimum length \( \epsilon \). The divergence of the series arises from the upper limit of the last expression as \( \epsilon \to 0 \), which illustrates that it can be blamed on the existence of a tachyon state in the open-string annulus that is dual to the cylinder.

At \( k^2 = 0 \) the regulated sum is given by the \( N = 1 \) term which is non-zero so that there is no massless scalar state, as anticipated. In fact, there is exactly one zero between each pair of neighboring poles (all \( d_N \) are positive). However, the divergence of the unregulated sum confuses the issue. As the regulator is removed (\( \epsilon \to 0 \)) the far away poles contribute a positive infinite constant which causes each of the zeroes to move down towards the adjacent pole with the lower value of \( \lambda \), approaching the pole as \( e^{-1/\epsilon} \). The net result is that the zeroes of the determinant approach the same positions as the zeroes of \( \text{det} D \) so that the diagonalised kinetic terms approach those of the usual free-field theory, including a scalar with \( (\text{mass})^2 \sim e^{-1/\epsilon} \).

This discussion of the constraints has singled out the Lagrange multiplier component of the open-string field, thereby accounting for the divergences of the separate perturbation theory diagrams. The effect of the coupling of the infinite number of other components of the open-string field – auxiliary fields – to the boundary must also be taken into account in order to understand the closed-string spectrum in any detail.

Comments.

This paper has discussed the occurrence of constraints on closed-string fields induced by their coupling to fixed end-point open-string fields via Dirichlet world-sheet boundaries. These constraints are not taken into account in string perturbation theory which is consequently plagued by divergences that arise from the region of moduli space in which the intermediate open string in fig.1(c) becomes infinitely long. The solution of the constraints mixes terms arising at all orders in perturbation theory which makes it difficult to estimate the details of the closed-string spectrum. However, the elimination of the dilaton is a feature that can be argued
for in the low-energy limit provided a world-sheet regulator is introduced. Without such a regulator the situation is obscured by a divergence that originates from the presence of the ‘tachyon’ state of the fixed end-point open string. If the dilaton field is indeed constrained to be constant then there is no longer a divergence due to massless states in the cylinder coupling to the boundary state (recalling that the dilaton is the only massless state to which the boundary state couples in the Dirichlet theory).

Apart from leading to a modification of the massless spectrum, Dirichlet boundaries are also intimately connected with a modification of the short-distance properties of the theory, as noted in earlier work. For example, in the presence of a single Dirichlet world-sheet boundary the fixed-angle high-energy cross section decreases as a power of the energy, in contrast to the exponential decrease characteristic of conventional closed-string theories and theories with Neumann boundaries. Furthermore, this power behaviour arises from the region of moduli space in which the vertex operators in fig.1(a) approach the boundary — a region that is larger than (but includes) the region that gives rise to the open-string divergences. For this reason the elimination of these divergences by solving the constraints on the closed-string field should not destroy the power behaviour of fixed-angle scattering. Higher order effects due to the insertion of multiple Dirichlet boundaries gives rise to modifications of the leading power behaviour reminiscent of the logarithmic corrections to naive scaling behaviour in renormalizable field theory ([1] and references therein). Further evidence of modified short-distance properties is provided by the high-temperature behaviour of the theory. At finite temperature the fixed end-point open string can wind around the compact temperature direction and states of winding \( q \) develop an effective mass proportional to \( \beta \) (the inverse temperature) since \( L_{\alpha D} = \beta^2 q^2/4\pi^2 + N \). In the Dirichlet theory such open strings arise as intermediate states in the closed-string propagator (as in fig.1(c)). Once again the presence of an \( N = 1 \) vector state (now with ‘mass’ \( \beta q/2\pi \)) leads to an important temperature-dependent renormalization of the closed-string spectrum as \( \beta \rightarrow 0 \) [19]. This makes contact with the expression deduced in the large-\( n \) limit of \( U(n) \) Yang–Mills theory by Polchinski[20]

The possibility that the presence of point-like structure may be related directly to the massless spectrum of the theory is most intriguing. Alas, as yet there is no evidence that the massless graviton (that seems to be an intrinsic feature of usual string theory) is absent in the Dirichlet theory as it would be in any realistic description of large-\( n \) Yang–Mills theory.

The simplest bosonic theory discussed here is ill-defined due to the presence of tachyonic states that indicate instabilities requiring a more sophisticated treatment which would take into account a condensate of Dirichlet boundaries as well as handles. It might be that a supersymmetric theory such as the one outlined in [21]
could form the basis of a more consistent, tachyon-free theory. Another possible
arena for these ideas is in the context of sub-critical string theory – indeed, the
usual arguments concerning the critical dimension do not apply in an obvious
manner since the constraints mix different orders in the string perturbation theory.
Generalizations of the boundary conditions are also possible. For example, a theory
with both Neumann and Dirichlet conditions describes open strings with end-points
which may be fixed or free. Another possibly important variant is a theory which
involves a sum over all possible ways of dividing each boundary into segments
with Neumann or constant Dirichlet conditions where the target-space position
and the length of each Dirichlet segment is integrated [1] (which may make use
of ideas in [22-24]). This introduces point-like structure at the string end-points.
More general backgrounds than the Minkowski space considered in this paper may
be introduced by considerations of boundary conditions in conformal field theory
based on [25].

Acknowledgments

I am grateful to Ed Corrigan, Chris Hull and other visitors to the Isaac Newton
Institute for useful discussions.

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