Complete one-loop electroweak corrections to $\ZZZ$ production at the ILC

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Abstract

We study the complete $\mathcal{O}(\alpha_{\text{ew}})$ electroweak (EW) corrections to the production of three $Z^0$-bosons in the framework of the standard model (SM) at the ILC. The leading order and the EW next-to-leading order corrected cross sections are presented, and their dependence on the colliding energy $\sqrt{s}$ and Higgs-boson mass $m_H$ is analyzed. We investigate also the LO and one-loop EW corrected distributions of the transverse momentum of final $Z^0$ boson, and the invariant mass of $Z^0Z^0$-pair. Our numerical results show that the EW one-loop correction generally suppresses the tree-level cross section, and the relative correction with $m_H = 120\ GeV (150\ GeV)$ varies between $-15.8\% (-13.9\%)$ and $-7.5\% (-6.2\%)$ when $\sqrt{s}$ goes up from $350\ GeV$ to $1\ TeV$.

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I. Introduction

To discover the signature of new physics beyond the standard model (SM)\cite{1,2} is one of the main goals for the forthcoming collider experiments. The precision measurements of the trilinear gauge-boson couplings are helpful for verification of non-abelian gauge structure, and the investigation of the quartic gauge-boson couplings can either confirm the symmetry breaking mechanism or present the direct test on the new physics beyond the SM\cite{3}. The direct study of quartic gauge-boson couplings requires the investigations of the processes involving at least three external gauge-bosons. In Refs.\cite{4,5,6,7,8}, the precise predictions for the $VVV$ productions at hadron colliders were provided. It shows that the QCD corrections increase the $WWZ$ cross section at the LHC by more than 70\%, and the QCD corrections to $ZZZ$ production at the LHC increase the LO cross section by about 50\% \cite{6}. Thus, any quantitative measurement of the concerned gauge couplings will have to take QCD corrections into account.

Due to heavy backgrounds, the precise measurement at a hadron collider is more difficult than at linear collider. The proposed International Linear Collider (ILC) by the particle physics community will be built with the entire colliding energy in the range of $200\, GeV < \sqrt{s} < 500\, GeV$ and an integrated luminosity of around 500 ($fb$)$^{-1}$ in four years. The machine should be upgradeable to $\sqrt{s} \sim 1\, TeV$ with an integrated luminosity of 1 ($ab$)$^{-1}$ in three years\cite{9}. Among all the ILC physics goals, the verification of gauge theory in the SM and finding the evidence of new physics via experimental measurement of the electroweak gauge boson couplings are crucial tasks too. The measurement will be able to be improved considerably at ILC compared with at Fermilab Tevatron and CERN LHC, and therefore a precise understand of the SM phenomenology at ILC to at least one-loop order is necessary\cite{10}. Without the accurate theoretical predictions and reliable error estimates for important observables at ILC, it is impossible to interpret experimental data properly.

The process of $ZZZ$ production with the subsequential leptonic decays of vector bosons
at ILC is not only an important process as a background for various new physics processes, but also possible to provide further tests for the quadrilinear gauge boson couplings, including the four gauge boson coupling, such as between $ZZZZ$, which does not exist at tree-level in the SM, because this kind of quadrilinear couplings would induce deviations from the SM predicted observables\cite{11}.

In this paper we present the calculations of the cross sections for the process $e^+e^- \rightarrow Z^0Z^0Z^0$ at the leading order (LO) and involving complete electroweak (EW) one-loop ($O(\alpha_{ew})$) corrections. The paper is organized as follows: In the next section we present the calculation descriptions for the tree-level process $e^+e^- \rightarrow Z^0Z^0Z^0$. The calculation of the electroweak corrections at one-loop level is reported in section III. The numerical results and discussions are given in section IV. In the last section we give a short summary.

II. Leading-order $e^+e^- \rightarrow Z^0Z^0Z^0$ process

In the calculations of the tree-level and one-loop level cross sections for the $e^+e^- \rightarrow Z^0Z^0Z^0$ process, we use the ’t Hooft-Feynman gauge. The analytically calculation of the leading order cross section for $e^+e^- \rightarrow Z^0Z^0Z^0$ process is presented in this section. We describe the lowest order $e^+e^- \rightarrow Z^0Z^0Z^0$ process adopted for evaluating the cross section as

$$e^+(p_1) + e^-(p_2) \rightarrow Z^0(p_3) + Z^0(p_4) + Z^0(p_5), \quad (2.1)$$

where $p_i$ ($i = 1 - 5$) label the four-momenta of incoming positron, electron and outgoing $Z^0$-bosons, respectively. Since the mass of electron/positron is negligible comparing with the colliding energy and the Yukawa coupling strength between Higgs/Goldstone and fermions is proportional to the fermion mass, in our work we ignore the contributions of the Feynman diagrams involving the couplings of $H^0 - e^+ - e^-$ and $G^0 - e^+ - e^-$. We depict the tree-level Feynman diagrams contributing to the cross section of the production process of three $Z^0$-bosons at the ILC in Fig.\textsuperscript{11}. There we have 9 generic tree-level diagrams for the process $e^+e^- \rightarrow Z^0Z^0Z^0$ in the framework of the SM. All the Born-level diagrams
can be grouped in two different topologies. Figs.1(a-c) belong to s-channel, Figs.1(d-i) are grouped in t(u)-channel. The differential cross section for the process $e^+e^- \rightarrow Z^0Z^0Z^0$ at the tree-level is then obtained as

$$d\sigma_{LO} = \frac{1}{3!} \frac{1}{4} \sum_{\text{spin}} |M_{LO}|^2 d\Phi_3,$$  \hspace{1cm} \text{(2.2)}$$

where $M_{LO}$ is the amplitude of all the tree-level diagrams in Fig.1. The factors $\frac{1}{3!}$ and $\frac{1}{4}$ are due to three identical final $Z^0$-bosons and spin-averaging of the initial particles, respectively. The summation in Eq.(2.2) is taken over the spins of the initial and final particles, and $d\Phi_3$ is the three-particle phase space element defined as

$$d\Phi_3 = \delta^{(4)} \left( p_1 + p_2 - \sum_{i=3}^5 p_i \right) \prod_{j=3}^5 \frac{d^3p_j}{(2\pi)^3 2E_j}.$$ \hspace{1cm} \text{(2.3)}$$

### III. Electroweak ($O(\alpha_{ew})$) corrections

The $O(\alpha_{ew})$ order electroweak corrections to the Born-level $e^+e^- \rightarrow Z^0Z^0Z^0$ process consist of two parts, i.e.,

- The virtual contributions to the leading order process $e^+e^- \rightarrow Z^0Z^0Z^0$ from the electroweak one-loop and their corresponding counterterm diagrams;
• The contribution from the real photon emission process \( e^+e^- \rightarrow Z^0Z^0Z^0 \gamma \). The soft photon emission process \( e^+e^- \rightarrow Z^0Z^0Z^0 (\gamma) \) consists IR singularities, which will be cancelled by the IR singularities in the contributions of the one-loop diagrams. There is no collinear IR singularity since we keep the nonzero mass of electron(positron).

In the following subsections, we describe in detail the calculation procedure and discuss the calculation of each contribution part.

### III.1 Virtual corrections

There are totally 2313 electroweak one-loop and corresponding counterterm Feynman diagrams being taken into account in our calculation, and they can be classified into self-energy, triangle, box, pentagon and counterterm diagrams. We depict some representative samples among 66 pentagon diagrams in Fig.2. In our calculation of the electroweak \( (\mathcal{O}(\alpha_{ew})) \) corrections to \( e^+e^- \rightarrow Z^0Z^0Z^0 \) process, all the one-loop Feynman diagrams and their relevant amplitudes are created by using \textit{FeynArts} 3.3\cite{FeynArts}, and the Feynman amplitudes are subsequently implemented by applying FormCalc5.3 programs\cite{FormCalc} and our in-house routines. The electroweak one-loop amplitude involves five point tensor integrals up to rank 4. The numerical calculation of the integral functions\( (n \leq 4) \) are implemented by using the expressions presented in Refs.\cite{Integrals1, Integrals2}. We use our independent Fortran subroutines following the expressions for the scalar and tensor five-point integrals in Ref.\cite{Integrals3}, and find agreement with LoopTools2.2\cite{LoopTools}. The Grace2.2.1 program\cite{Grace} is used to accomplish five-body phase-space integration for hard photon radiation process \( e^+e^- \rightarrow Z^0Z^0Z^0\gamma \).

The total unrenormalized amplitude corresponding to all the one-loop Feynman diagrams contains both ultraviolet (UV) and infrared (IR) singularities. We regulate all singularities adopting dimensional regularization (DR) scheme \cite{DimReg}, where the dimensions of spinor and space-time manifolds are extended to \( D = 4 - 2\epsilon \). The relevant fields are renormalized by taking the on-mass-shell (OMS) scheme \cite{OMS1, OMS2}. The IR singularity is regularized by introducing a infinitesimal fictitious mass \( m_\gamma \). All the tensor coefficients
Figure 2: Some representative pentagon Feynman diagrams for the process $e^+ e^- \rightarrow Z^0 Z^0 Z^0$. 
of the one-loop integrals can be calculated by using the reduction formulae presented in Refs. [16, 24]. As we expect, the UV divergence contributed by virtual one-loop diagrams can be cancelled by that contributed from the counterterms exactly both analytically and numerically.

**III..2 Real photon emission process** \( e^+e^- \rightarrow Z^0Z^0Z^0\gamma \)

In our calculation for one-loop diagrams, there exists soft IR divergence. In order to get an IR finite cross section for \( e^+e^- \rightarrow Z^0Z^0Z^0 \) up to the order of \( \mathcal{O}(\alpha_{ew}^4) \), we should consider the \( \mathcal{O}(\alpha_{ew}) \) corrections to \( e^+e^- \rightarrow Z^0Z^0Z^0 \) process due to real photon emission. The soft IR divergence in virtual photonic corrections for the process \( e^+e^- \rightarrow Z^0Z^0Z^0 \) can be exactly cancelled by adding the real photonic bremsstrahlung corrections to this process in the soft photon limit. In the real photon emission process

\[
e^+(p_1) + e^-(p_2) \rightarrow Z^0(p_3) + Z^0(p_4) + Z^0(p_5) + \gamma(p_6),
\]

(3.1)
a real photon radiates from the electron/positron, and can be soft or hard. The general phase-space-slicing (PSS) method [25] is adopted to isolate the soft photon emission singularity part in the real photon emission process \( e^+e^- \rightarrow Z^0Z^0Z^0\gamma \), and the cross section of the real photon emission process (3.1) is decomposed into soft and hard terms

\[
\Delta\sigma_{\text{real}} = \Delta\sigma_S + \Delta\sigma_H = \sigma_{\text{LO}}(\delta_S + \delta_H).
\]

(3.2)

where the 'soft' and 'hard' describe the energy nature of the radiated photon. The energy \( E_6 \) of the radiated photon in the center of mass system (c.m.s.) frame is considered soft if \( E_6 \leq \Delta E \), and hard if \( E_6 > \Delta E \), respectively. Then both \( \sigma_S \) and \( \sigma_H \) should depend on the arbitrary soft cutoff \( \delta_s \equiv \Delta E/E_b \), where \( E_b \) is the electron beam energy in the c.m.s. frame and equals to \( \sqrt{s}/2 \), but the total cross section of the real photon emission process \( \sigma_{\text{real}} \) is cutoff \( \Delta E/E_b \) independent. Since the soft cutoff \( \Delta E/E_b \) is taken to be a small value in our calculations, the terms of order \( \Delta E/E_b \) can be neglected and the soft contribution can be evaluated by using the soft photon approximation analytically.
$$d\Delta \sigma_S = -d\sigma_{LO} \frac{\alpha_{ew}}{2\pi^2} \int_{|p_6| \leq \Delta E} \frac{d^3p_6}{2E_6} \left( \frac{p_1}{p_1 \cdot p_6} - \frac{p_2}{p_2 \cdot p_6} \right)^2.$$ (3.3)

Our calculation demonstrates that the IR singularity in the soft contribution from Eq. (3.3) is cancelled exactly with that from the virtual photonic corrections. Therefore, $$\Delta \sigma_v + \Delta \sigma_{real}$$, the sum of the $$\mathcal{O}(\alpha_{ew}^4)$$ cross section corrections from virtual, soft and hard photon emission contribution parts, is independent of the cutoff value $$\delta_s$$. The hard contribution, which is UV and IR finite, is computed by using the Monte Carlo technique. Finally, the electroweak corrected cross section for the $$e^+e^- \rightarrow Z^0 Z^0 Z^0$$ process up to the order of $$\mathcal{O}(\alpha_{ew}^4)$$ can be obtained by

$$\sigma_{tot} = \sigma_{LO} + \Delta \sigma_v + \Delta \sigma_{real} = \sigma_{LO} (1 + \delta_{tot}).$$ (3.4)

### III..3 QED and total $$\mathcal{O}(\alpha_{ew})$$ order corrections

In analyzing the originations of the electroweak corrections, we split the full $$\mathcal{O}(\alpha_{ew})$$ corrections to the process $$e^+e^- \rightarrow Z^0 Z^0 Z^0$$ into two parts, the QED correction part and the weak correction part. Correspondingly we define the total relative correction as $$\delta_{tot} = \delta_{QED} + \delta_{weak}$$. The QED correction part is contributed by the diagrams with virtual photon in loop, and real photon emission process $$e^+e^- \rightarrow Z^0 Z^0 Z^0 \gamma$$. For the counter-terms involved in the QED contribution, the electron/positron wave function renormalization constants include only photonic contribution. The remainders of the total virtual electroweak corrections belong to weak correction part. With above definitions we can express the full one-loop electroweak corrected total cross section as a summation of several parts.

$$\sigma_{tot} = \sigma_{LO} + \Delta \sigma_{v,QED} + \Delta \sigma_s + \Delta \sigma_h + \Delta \sigma_{v,weak}$$

$$= \sigma_{LO} (1 + \delta_{QED} + \delta_{weak}) = \sigma_{LO} (1 + \delta_{tot}),$$ (3.5)

where $$\Delta \sigma_v$$ and $$\Delta \sigma_s$$ are the cross section corrections contributed by the virtual electroweak one-loop diagrams and the soft photon emission process respectively, $$\Delta \sigma_{v,QED}$$, $$\Delta \sigma_s$$, $$\Delta \sigma_h$$
and $\Delta\sigma_{v,\text{weak}}$ are the corrections from the virtual QED contribution, the soft photon emission process, the hard photon emission process and the virtual weak contribution, separately. $\delta_{QED}$, $\delta_{\text{weak}}$ and $\delta_{\text{tot}}$ are the relative corrections contributed by the QED correction part, the weak correction part and the total electroweak correction, respectively.

As we mentioned above, there exist both ultraviolet (UV) divergence and infrared (IR) soft singularity in the contributions of the electroweak one-loop diagrams for $e^+e^- \rightarrow Z^0 Z^0 Z^0$ process, but no collinear IR singularity since we keep the nonzero mass of electron/positron in our calculation of one-loop order corrections. After doing the renormalization procedure, we verified that the UV singularity is vanished, and the IR soft divergence appeared in the virtual correction is cancelled by the the soft photon emission process $e^+e^- \rightarrow Z^0 Z^0 Z^0 \gamma$.

In order to discuss the origin of the large correction when the colliding energy is close to the threshold of the production of three $Z^0$-bosons, we discuss the photonic (QED) corrections and the genuine total electroweak corrections separately. The QED corrections comprise two parts: the QED virtual corrections $\Delta\sigma_{v,QED}$ which contributed by the loop diagrams with virtual photon exchange in loop and the corresponding QED parts of the counterterms, and the real photon emission corrections $\Delta\sigma_{\text{real}}$. Therefore, the QED relative correction $\delta_{QED}$ can be expressed as

$$\delta_{QED} = \delta_{v,QED} + \delta_{\text{real}},$$

(3.6)

where $\delta_{v,QED} = \Delta\sigma_{v,QED}/\sigma_{LO}$, and the genuine weak relative correction $\delta_w$ can be got from

$$\delta_w = \delta_{\text{tot}} - \delta_{QED}.$$  

(3.7)

IV. Numerical results and discussion

For the numerical calculation, we take the input parameters as follows:

$$m_e = 0.51099892 \text{ MeV}, \quad m_\mu = 105.658369 \text{ MeV}, \quad m_\tau = 1776.99 \text{ MeV},$$
\[ m_u = 66 \text{ MeV}, \quad m_c = 1.25 \text{ GeV}, \quad m_t = 174.2 \text{ GeV}, \]
\[ m_d = 66 \text{ MeV}, \quad m_s = 95 \text{ MeV}, \quad m_b = 4.7 \text{ GeV}, \]
\[ m_W = 80.403 \text{ GeV}, \quad m_Z = 91.1876 \text{ GeV}. \]  

Equation (4.1)

There we use the experimental value of W-boson mass as input parameter, but not the 
m_W evaluated from \(G_\mu\) as in \(\alpha_{\text{ew}}\) scheme\[28\]. We take the electric charge defined in the 
Thomson limit \(\alpha_{\text{ew}}(0) = 1/137.036\) and the effective values of the light quark masses \(m_u\) and \(m_d\) which can reproduce the hadronic contribution to the shift in the fine structure 
constant \(\alpha_{\text{ew}}(m_Z^2)\) \[29\]. As we know that the LEP II experiments provide the lower 
limit on the SM Higgs mass as 114.4 GeV at the 95% confidence level from the results 
of direct searches for \(e^+e^- \rightarrow Z^0H^0\) production\[30, 31\], and the electroweak precision 
measurements indicate indirectly the upper bound as \(m_H \lesssim 182 \text{ GeV}\) at the 95% C.L., 
when the lower limit on \(m_H\) is used in determination of this upper limit\[31\]. Therefore, 
in our numerical evaluation it is reasonable to take the mass of Higgs-boson being in the 
range of 115 GeV < \(m_H\) < 170 GeV. Then we shall not encounter the resonance problem 
of Higgs-boson during our calculation.

We checked the correctness of the numerical results of the LO cross section for process 
\(e^+e^- \rightarrow Z^0Z^0Z^0\), by using Grace2.2.1 \[17\] and FeynArts3.3/FormCalc5.3 \[12, 13\] packages 
separately. In adopting Grace2.2.1 and FeynArts3.3/FormCalc5.3 programs, we used both 
\'t Hooft-Feynman and unitary gauges separately in the calculations of the LO cross section 
to check the gauge invariance, and we got coincident numerical results. The numerical 
results of the LO cross section for the process \(e^+e^- \rightarrow Z^0Z^0Z^0\), by using \'t Hooft-Feynman 
gauge and taking \(\sqrt{s} = 500 \text{ GeV}\) and the other input parameters shown in Eqs. (4.1), are 
listed in Table 1. There it is shown that there is a good agreement between the numerical 
results by adopting different packages.

In the one-loop calculation, we must set the values of the IR regulator \(m_\gamma\), the fictitious 
photon mass, and soft cutoff \(\delta_s = \Delta E/E_b\) besides the parameters mentioned in Eqs. (4.1). 
As we know, the total cross section should have no relation with these two parameters if
The IR divergency does really vanish. Our numerical results show that the cross section correction at $\mathcal{O}(\alpha_{ew}^4)$ order $\Delta\sigma_{tot} = \Delta\sigma_{real} + \Delta\sigma_v$ is invariable within the calculation errors when $m_H = 120$ GeV, $\sqrt{s} = 500$ GeV, $\delta_s = 10^{-3}$ and the fictitious photon mass $m_\gamma$ varies from $10^{-15}$ GeV to $10^{-1}$ GeV.

Table 1: The numerical results of the LO cross sections for the process $e^+e^- \rightarrow Z^0Z^0Z^0$ by using Grace2.2.1 and FeynArts3.3/FormCalc5.3 packages separately, and taking the input parameters as shown in Eqs.(4.1) and $\sqrt{s} = 500$ GeV.

| $m_H$(GeV) | $\sigma_{LO}(fb)$ (Grace) | $\sigma_{LO}(fb)$ (FeynArts) |
|------------|--------------------------|---------------------------|
| 115        | 1.0056(4)                | 1.0055(2)                 |
| 120        | 1.0139(4)                | 1.0138(2)                 |
| 150        | 1.0975(4)                | 1.0975(2)                 |
| 170        | 1.2565(4)                | 1.2564(2)                 |

Fig.3(a) presents a verification of the correctness of our calculation for process $e^+e^- \rightarrow Z^0Z^0Z^0$ including $\mathcal{O}(\alpha_{ew})$ order corrections. The amplified curve for $\Delta\sigma_{tot}$ including calculation errors is depicted in Fig.3(b). Both figures are to show the independence of the total $\mathcal{O}(\alpha_{ew})$ electroweak correction on the soft cutoff $\delta_s$, when we take $m_H = 120$ GeV and $\sqrt{s} = 500$ GeV. From Fig.3(b) we can say that the total EW relative correction $\Delta\sigma_{tot}$ has no relation to the value of $\delta_s$ within the calculation error range. In the further calculations, we set $m_\gamma = 10^{-2}$ GeV and $\delta_s = 10^{-3}$.

In Table 2, we list some representative numerical results of the LO and one-loop EW corrected cross sections($\sigma_{LO}$, $\sigma_{tot}$), the QED and total EW corrections to the cross sections for $e^+e^- \rightarrow Z^0Z^0Z^0$ process($\Delta\sigma_{QED}$, $\Delta\sigma_{tot}$), and their corresponding relative corrections($\delta_{QED}$, $\delta_{tot}$) when $\sqrt{s} = 500$ GeV and the values of Higgs-boson mass are taken to be 115 GeV, 150 GeV and 170 GeV separately. From these data we can see that the one-loop EW corrections suppress the LO cross section of the process $e^+e^- \rightarrow Z^0Z^0Z^0$, and the relative corrections are about minus few percent. We can conclude also the LO and EW corrected cross sections increase with the increment of $m_H$, while the absolute total EW correction decreases when the value of Higgs-boson mass goes up.
Figure 3: (a) The dependence of the $\mathcal{O}(\alpha_{\text{ew}})$ correction to cross section of $e^+e^- \rightarrow Z^0 Z^0 Z^0$ on the soft cutoff $\delta_s$ with $m_H = 120$ $\text{GeV}$ and $\sqrt{s} = 500$ $\text{GeV}$. (b) the amplified plot of the curve for the total correction $\Delta \sigma_{\text{tot}}$ in Fig.3(a), where it includes calculation errors.

| $m_H(\text{GeV})$ | $\sigma_{\text{LO}}(fb)$ | $\sigma_{\text{tot}}(fb)$ | $\Delta \sigma_{\text{QED}}(fb)$ | $\Delta \sigma_{\text{tot}}(fb)$ | $\delta_{\text{QED}}(\%)$ | $\delta_{\text{tot}}(\%)$ |
|-------------------|--------------------------|--------------------------|-------------------------------|-------------------------------|--------------------------|--------------------------|
| 115               | 1.0055(2)                | 0.9159(7)                | -0.0451(7)                    | -0.0896(7)                    | -4.49(7)                 | -8.91(7)                 |
| 150               | 1.0975(2)                | 1.0194(8)                | -0.0444(8)                    | -0.0780(8)                    | -4.04(7)                 | -7.11(7)                 |
| 170               | 1.2564(2)                | 1.1989(9)                | -0.0393(8)                    | -0.0575(9)                    | -3.12(7)                 | -4.58(7)                 |

Table 2: The numerical results of $\sigma_{\text{LO}}$, $\sigma_{\text{tot}}$, $\Delta \sigma_{\text{QED}}$, $\Delta \sigma_{\text{tot}}$ (in femto bar), and their corresponding relative EW and QED corrections($\delta_{\text{tot}}$, $\delta_{\text{QED}}$) for the process $e^+e^- \rightarrow Z^0 Z^0 Z^0$, when $\sqrt{s} = 500$ $\text{GeV}$ and $m_H = 115$ $\text{GeV}$, 150 $\text{GeV}$, 170 $\text{GeV}$ respectively.
Figure 4: (a) The LO($\sigma_{LO}$), $\mathcal{O}(\alpha_{ew})$ EW, QED corrected cross sections($\sigma_{tot}$, $\sigma_{QED}$) for the process $e^+e^- \rightarrow Z^0Z^0Z^0$ as the functions of colliding energy $\sqrt{s}$ with $m_H = 120$ GeV, 150 GeV separately. (b) The corresponding relative EW, QED relative corrections($\delta_{tot}$, $\delta_{QED}$) versus $\sqrt{s}$.

The numerical results of the LO, $\mathcal{O}(\alpha_{ew})$ EW, QED corrected cross sections($\sigma_{LO}$, $\sigma_{tot}$, $\sigma_{QED}$) and the total relative EW, QED corrections($\delta_{tot}$, $\delta_{QED}$) for the process $e^+e^- \rightarrow Z^0Z^0Z^0$ with $m_H = 120$ GeV, 150 GeV as the functions of colliding energy $\sqrt{s}$ are plotted in Figs.4(a) and (b) respectively. As indicated in Fig.4(a), The curves for the cross sections of $\sigma_{LO}$, $\sigma_{tot}$ and $\sigma_{QED}$ increase quickly in the $\sqrt{s}$ region of [350 GeV, 550 GeV] and decrease when $\sqrt{s} > 600$ GeV. The two figures show the $\mathcal{O}(\alpha_{ew})$ corrections always suppress the corresponding LO cross sections of process $e^+e^- \rightarrow Z^0Z^0Z^0$ in both cases of $m_H = 120$ GeV and $m_H = 150$ GeV separately, but the QED correction parts can enhance the LO cross sections when $\sqrt{s} > 700$ GeV. We can read out from Fig.4(b) that the corresponding total EW relative corrections for $m_H = 120$ GeV and 150 GeV vary in the ranges of $[-15.8\%, -7.5\%]$ and $[-13.9\%, -6.2\%]$ respectively, when $\sqrt{s}$ runs from 350 GeV to 1 TeV. We can see also from these two plots that in the colliding energy $\sqrt{s}$ region near the threshold of the production of three $Z^0$-bosons, the main contribution to the total EW relative correction($\delta_{tot}$) comes from the QED correction part. That is due to the Coulomb singularity effect coming from the instantaneous photon exchange in loops which has a small spatial momentum.
Figure 5: The distributions of the transverse momenta of $Z^0$-bosons ($P_T^{Z}$) at the LO and up to EW one-loop order with $m_H = 120$ GeV and $\sqrt{s} = 500$ GeV.

We present the distributions of the transverse momenta of final $Z^0$-bosons at leading order and up to one-loop order, $d\sigma_{LO}/dp_T^{Z}$, $d\sigma_{NLO}/dp_T^{Z}$, when $m_H = 120$ GeV and $\sqrt{s} = 500$ GeV in Fig. 5. There we pick the $p_T^{Z}$ of each of the three $Z^0$-bosons as an entry in this histograms, then the final result of the differential cross section is obtained by multiplying factor $1/3$. In this figure we can see that the EW one-loop correction suppresses obviously the LO differential cross section $d\sigma_{LO}/dp_T^{Z}$ when $p_T^{Z} > 50$ GeV, but the EW correction is small when $p_T^{Z} < 25$ GeV. It also shows that the EW corrections do not observably change the LO distribution line-shape of $p_T^{Z}$, and both the differential cross sections of $d\sigma_{LO}/dp_T^{Z}$ and $d\sigma_{NLO}/dp_T^{Z}$ have their maximal values at about $p_T^{Z} \sim 50$ GeV respectively.

We plot the distributions of the invariant mass of $Z^0Z^0$-pair, denoted as $M_{ZZ}$, at the LO and up to EW one-loop order with $m_H = 120$ GeV and $\sqrt{s} = 500$ GeV in Fig. 6. Here we can see that the EW correction slightly enhances the LO differential cross section when $M_{ZZ} < 250$ GeV, but suppresses $d\sigma_{LO}/dM_{ZZ}$ obviously when $M_{ZZ} > 250$ GeV. The suppression of $d\sigma_{LO}/dM_{ZZ}$ is due to the contribution from the hard photon emission process $e^+e^- \rightarrow Z^0Z^0Z^0\gamma$ at the $O(\alpha_{ew}^4)$ order, in which the momentum balance between
Figure 6: The distributions of the invariant mass of $Z^0Z^0$-pair($M_{ZZ}$) at the LO and up to EW one-loop order when $m_H = 120$ GeV and $\sqrt{s} = 500$ GeV.

the sum of the momenta of three $Z^0$-bosons and that of the radiated hard photon will reduce the value of invariant mass $M_{ZZ}$ and change the line-shape in the range with large $M_{ZZ}$.

V. Summary

In this paper we describe the impact of the complete one-loop EW corrections to the scattering process $e^+e^- \rightarrow Z^0Z^0Z^0$ in the SM. This channel can be used to measure the quartic vector boson coupling at ILC. We investigate the dependence of the LO, $O(\alpha_{\text{ew}})$ EW and QED corrected cross sections on colliding energy $\sqrt{s}$ and Higgs-boson mass, and present the LO and EW one-loop corrected distributions of the transverse momenta of final $Z^0$-bosons and the LO and EW corrected differential cross sections of invariant mass of $Z^0Z^0$-pair. To see the origin of some of the large corrections clearly, we calculate the QED and genuine weak corrections separately. We conclude that both the Born cross section and the EW corrected cross section for $e^+e^- \rightarrow Z^0Z^0Z^0$ process are sensitive to
the Higgs boson mass in the range of $115 \, GeV < m_H < 170 \, GeV$. We find the $O(\alpha_{ew})$ corrections generally suppress the LO cross section, the LO distribution of the momenta of $Z^0$-bosons and the LO differential cross sections of invariant mass of $Z^0Z^0$-pair for process $e^+e^- \rightarrow Z^0Z^0Z^0$. Our numerical results show that when $m_H = 120 \, GeV (150 \, GeV)$ and the colliding energy goes up from $350 \, GeV$ to $1 \, TeV$, the relative EW correction varies from $-15.8\% (-13.9\%)$ to $-7.5\% (-6.2\%)$.

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