Superfluid Fermi liquid in a unitary regime

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A short popular review of theoretical and experimental results about properties of ultracold Fermi gases in vicinity of the Feshbach resonance is presented. The paper is based on the authors talk on the meeting of the Physical Department of the Russian Academy of Sciences dedicated to 100th anniversary of the birth of L. D. Landau (Moscow, January 22, 2008). A version of this article has been published in Physics - Uspekhi 51 603 (2008).

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When choosing the subject of my presentation at this session dedicated to the 100th anniversary of the birth of Landau, I wanted to speak about something that would have surprised Landau. I believe that the recently created physical object - a universal superfluid Fermi liquid - meets this requirement in the best way possible.

As is well known, Landau did not regard the microscopic theory of fluids as a problem worth being occupied with. I quote a well-known passage from Statistical Physics (see [1], page 187):

"Unlike gases and solids, liquids do not allow a calculation in a general form of the thermodynamic quantities or even of their dependence on temperature. The reason lies in the existence of a strong interaction between the molecules of the liquid while at the same time we do not have the smallness of the vibrations which makes the thermal motion in solids especially simple. The strength of the interaction between molecules makes it necessary to know the precise law of interaction in order to calculate the thermodynamic quantities, and this law is different for different liquids."

This statement is perfectly correct for all liquids existing in nature. However, progress in experimental techniques has recently enabled preparing liquids with properties independent of any quantities that characterize the interaction. This situation emerges because the interatomic interaction in these bodies is, in a sense, infinitely strong. The case in point is ultracold gases near the so-called Feshbach resonances.

First of all, one can pose the question: what is the word "liquid" taken to imply? We accept a natural definition: a liquid is a fluid body with a strong interaction between its particles. We emphasize that fluidity implies the absence of strict periodicity, of a crystalline long range order. The liquids of interests to us are made from gases whose atoms obey the Fermi statistics. The gas is dilute in the sense that the average interatomic distance \( n^{-1/3} \), where \( n \) is the atomic number density, is much greater than the characteristic range \( r_0 \) of interatomic forces:

\[
0 \ll n^{-1/3} .
\]

Condition (1) is always satisfied for the objects under consideration. However, the fulfillment of this condition does not yet mean that we are dealing with a gas in the sense that the interaction is weak. Let the temperature be sufficiently low, such that the gas is degenerate, \( T \leq E_F \) [2]. It is then valid to say that all properties of the body depend on one parameter \( f \), the amplitude of the scattering of atoms with the orbital momentum \( l = 0 \) by each other. The interaction is weak, i.e. the body is indeed a gas, if the amplitude is small in comparison with the interatomic distances:

\[
|f| \ll n^{-1/3} .
\]

The quantities \( r_0 \) and \( |f| \) are typically of the same order of magnitude and conditions (1) and (2) are practically equivalent. However, this is not the case when a system of two atoms has an energy level close to zero. According to the general theory, the scattering amplitude then takes the form (see, for example [3])

\[
f(k) = -(a^{-1} + ik)^{-1} ,
\]

where \( k \) is the wave vector and \( a = -f(0) \) is the scattering length, a constant that characterizes the scattering completely. When \( a > 0 \), the system of two atoms has a bound state with the negative energy \( \epsilon = -\hbar^2/ma^2 \). When \( a < 0 \), the system is said to have a virtual level. If \( |a| \) is high enough, \( |a| \geq k^{-1} \sim n^{1/3} \), the interaction weakness condition (2) is certainly violated and we are by definition dealing with a liquid, although a dilute liquid in the sense of condition (1). In this case, the properties of the liquid are characterized by the sole parameter \( a \). When \( |a| \gg k^{-1} \),
FIG. 1: The dimer recombination coefficient $a_{dd}$ as a function of the scattering length $a$ (from [5]). The slope of the dashed line corresponds to the theoretical dependence $a_{dd} \propto a^{-2.5}$.

the scattering amplitude reaches its “unitary limit”

$$f \approx \frac{i}{k}. \quad (4)$$

The length $a$ then also drops out of the theory and we are dealing with a universal liquid, which properties do not depend on interaction at all. Of course, the picture under discussion implies the possibility of changing the scattering amplitude. This opportunity arises in the presence of Feshbach resonances, in the vicinity of which the position of the energy level of the system of two atoms depends on magnetic field [4]. Here the scattering length as a function of magnetic field can be represented as

$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right). \quad (5)$$

Near the resonance $B \approx B_0$, the scattering length is large and the system is a universal "unitary" liquid.

We qualitatively consider the properties of the system at $T = 0$ in different ranges of the scattering length $a$. When this length is positive and relatively small, $r_0 \ll a \ll n^{-1/3}$, the system of two atoms has a bound state and the atoms combine to form molecules with a binding energy $\epsilon$. The system is a Bose gas of weakly bound diatomic molecules, or dimers. It is significant that the dimer-dimer scattering length $a_{dd}$ is also positive, i.e. these molecules experience mutual repulsion. Calculating $a_{dd}$ is an intricate problem, which was solved by Petrov, Salomon and Shlyapnikov [5]. It turned out that $a_{dd} = 0.6a$. Therefore, in this regime, the system is a weakly nonideal Bose gas of molecules described by the Bogoliubov theory[7], with the obvious change $m \to 2m, a \to 0.6a$.

The question of the lifetime of this system is of paramount importance for the entire area of physics involved. This lifetime is limited by transition from the weakly bound level to deep molecular levels in molecular collisions accompanied by the release of large amount of energy. The molecular number loss in these inelastic processes is described by the equation $\dot{n}_d = -\alpha_{dd}n_d^2$. The dependence of the recombination coefficient $\alpha_{dd}$ on $a$ was also investigated in [5]. It turned out that $\alpha_{dd} \propto a^{-2.25}$. Therefore, the system becomes more stable with an increase in the scattering length, i.e., as the resonance is approached. This paradoxical result stems from the Fermi nature of atoms or, to be more precise, from the fact that fermions with parallel spins cannot reside at the same point. In a Bose gas, which was also studied in experiments, the lifetime decreases sharply as the resonance is approached. This is the reason why only the Fermi liquid can actually be investigated in the unitary regime. The experimentally measured dependence of $a_{dd}$ on $a$ is presented in Fig. 1. It is in satisfactory agreement with the theory.

We now consider the opposite limit case, where the scattering length is negative and small in modulus, $a < 0, r_0 \ll |a| \ll n^{-1/3}$, as is the case on the opposite side of the resonance. The system is then a weakly nonideal Fermi gas with attraction between the atoms. According to the theoretical concepts of Bardeen - Cooper - Schrieffer and Bogoliubov, the occurrence of a Fermi surface gives rise to Cooper pairs in this case. As a result, a gap appears in the fermion
FIG. 2: Schematic of the device employed at Duke University to investigate the properties of a Fermi gas in an optical trap near a Feshbach resonance (from [8]).

energy spectrum and the system becomes superfluid. In the immediate vicinity of the resonance, the system is a universal unitary Fermi liquid. Because the system is superfluid in both limit cases considered, it is reasonable to assume that it is superfluid in all of the interval of a values. (Different arguments are presented below.) Of course, the system is then assumed to be stable in the unitary regime. This assumption is supported by the wealth of experimental data and theoretical calculations.

Prior to discussing these results, I briefly describe the typical experimental arrangement using the example of a device at Duke University [8] (see Fig. 2). The atoms are confined in an optical trap formed by a focused laser beam. The chosen light frequency is somewhat lower than the absorption line frequency, and therefore the atoms are ‘attracted’ to the intensity peak. Because the intensity near the focus decreases rapidly in the radial direction and slowly in the axial direction, the sample was elongated and cigar-shaped. Solenoids induce the magnetic field required to attain the resonance. Since the main objective of the experiments was to investigate superfluidity, two kinds of fermions were needed. In superconductivity theory, electrons with opposite values of spin projection are usually considered. In our case, atoms in different hyperfine structure states were used.

Experiments with fermions are difficult and the number of groups working with them is smaller than the number of groups investigating the Bose-Einstein condensation. The work is undertaken at the JILA (Joint Research Institute of the National Institute of Standards and Technology and the University of Colorado) (Boulder), Massachusetts Institute of Technology (MIT) (Boston), Duke University (Durham), and Rice University (Houston) in the USA, the École Normale Supérieure (Paris) in France, and the University of Innsbruck in Austria. It is a pleasure for me to mention that A Turlapov, one of the leading experimenters at Duke University, has returned to Nizhni Novgorod and is making a facility there.

Two types of Fermi atoms were actually used in the experiments, $^6\text{Li}$ and $^{40}\text{K}$ isotopes. The isotope choice was dictated by the presence of a Feshbach resonance in a convenient range of the magnetic field and the occurrence of spectral lines in a convenient wavelength range.

I give the typical parameters of recent experiments. The number of atoms in the trap is $N \sim 3 \times 10^6 - 10^7$ and the atom density at its center is $n \sim 2 \times 10^{12}$. Accordingly, the Fermi energy is $E_F \sim 200-500 \text{nK}$ and the magnitude of the Fermi wave vector is $k_F \sim 0.3 \mu m^{-1}$. The parameters of the trap are conveniently characterized by the frequencies of atomic oscillations in it. The radial frequency $\nu_\perp$ normally lies in the 60 - 300 Hz range and the longitudinal frequency $\nu_z \sim 20 \text{ Hz}$. The lowest attainable temperature turns out to be under $0.06 E_F$, i.e., of the order of 10 nK. As is evident from the subsequent discussion, it has been possible not only to conduct experiments at these prodigiously low temperatures but also to set up a thermodynamic temperature scale in this domain. I cannot discuss here the techniques of gas cooling, and only mention that during the final stage, the gas is cooled due to the evaporation of the faster atoms from the trap, much like tea is cooled in a cup left on a table.

One of the most important experimental tasks was to prove that the system was superfluid. An immanent property of superfluidity is the existence of quantized vortexes. The velocity circulation around a vortex in a Fermi liquid is
FIG. 3: Schematic of the device used at MIT for investigating the rotation of a superfluid Fermi gas in the experiment [9]. Two laser beams aligned with the axis set the gas in rotation. Separately shown is the scheme for observing the vortexes from the resonant absorption imaging of the expanding fermionic cloud.

Gamma = \frac{\pi \hbar}{\text{m}}\), two times smaller than in a Bose liquid. Accordingly, in the rotation with a sufficiently high angular velocity \(\Omega\), the number of vortexes per unit area must be equal to \(2\Omega m/(\pi \hbar)\). How can the liquid be set in rotation? MIT experimenters positioned a pair of thin laser beams along the trap axis, which were shifted from the axis (Fig. 3) [9]. This 'mixer' rotated about the axis and stirred the liquid. At some instant, the trap was disengaged, the liquid expanded, and observations of the density distribution were made. The result is shown in Fig. 4. The vortex cores are observed as dark reduced-density domains. A simple calculation of the number of vortexes confirms the theoretical value of the circulation given above.

We now consider the liquid precisely at the resonance point, when \(a \rightarrow \pm \infty\). (It is appropriate to note that this is not a phase transition point.) We begin from the properties of a uniform liquid at \(T = 0\). Apart from the density, there are no parameters at our disposal on which the thermodynamic functions may depend. Dimensionality considerations suggest, e.g., that the chemical potential of the liquid must be of the form

\[
\mu(n) = \xi \mu^{id}(n),
\]

where \(\mu^{id}(n) = (3\pi^2n)^{2/3}(\hbar^2/m)\) is the chemical potential of an ideal Fermi gas with the density \(n\) for \(T = 0\) and \(\xi\) is a dimensionless coefficient independent of the kind of liquid. The theoretical task consists in the calculation of \(\xi\) and the experimental task involves its measurement. The first estimates of \(\xi\) were made proceeding from the Bardeen-Cooper-Schrieffer-Bogoliubov (BCSB) theory. This theory is a mean-field theory and, needless to say, is inapplicable near the unitarity point. But its ingenious generalization to the strong-coupling case has allowed obtaining formulas sound in both limit cases (see, e.g., [10, 11]). At exactly the unitarity point, this theory yields \(\xi = 0.59\). The most reliable result is provided by calculations involving the quantum Monte Carlo (QMC) technique: \(\xi = 0.42\) [12].
noteworthy that the absence of a small parameter in the theory is substantially favorable to numerical calculations. The existence of such a parameter often impairs convergence. An attempt has been made to apply the $\varepsilon$-expansion technique, which relies on the fact that $\xi = 0$ in a four-dimensional space \([13]\). The theory is constructed in the space of $D = 4 - \varepsilon$ dimensions under the assumption that $\varepsilon$ is small, and the results are then extrapolated to $\varepsilon = 1$. This technique, which is highly beneficial in the theory of phase transitions, supposedly yields poor accuracy in this case.

It is significant that the parameter $\xi < 1$. This means that the interaction at the unitarity point lowers the fluid pressure, i.e., is an effective attraction. It is therefore reasonable that it leads to fermion pairing and to superfluidity. A quantitative characteristic of the pairing is the gap $\Delta$ in the Fermi branch of the spectrum. Once again, the dimensionality considerations suggest that

$$\Delta(n) = \theta \mu^{id}(n) \, .$$

QMC calculations yield $\theta = 0.5$ \([12]\).

We now turn our attention to the experimental verification of the theory. The most direct method of determining $\xi$ consists in the precise measurement of fluid density in the trap. In the semiclassical approximation, this distribution is given, in view of expression \([6]\), by the equation $\xi \mu^{id}[n(x)] + V(x) = \text{const}$. Fitting to the observed distribution allows determining $\xi$. At Rice University, the value $\xi = 0.46$ was thus found for $^6\text{Li}$ \([14]\). Another method was applied by experimenters at JILA, who worked with $^{40}\text{K}$ \([15]\). They measured the density distribution and calculated the potential energy $U_{pot} = \int n(x) V n(x) dx$ of the liquid, which is proportional to $\sqrt{\xi}$. By this means, they obtained the value $\xi = 0.46$. The proximity of the values for $^6\text{Li}$ and $^{40}\text{K}$ to the theoretical one confirms the universal nature of $\xi$. Reliable measurements of the gap $\Delta$, in my opinion, have not been made to the present day.

Important information about the properties of the liquid may be obtained by investigating its oscillations in the trap. These oscillations are described by the Landau superfluid hydrodynamics \([16]\). (I emphasize that Landau believed from the outset that his equations applied both to Bose and Fermi superfluid liquids.) An especially simple result for the oscillation frequencies in a harmonic trap is obtained for a liquid with the polytropic equation of state $\mu(n) \propto n^\gamma$. We consider an important type of oscillation: axially symmetric radial oscillations whose frequency is $\omega = \sqrt{2(\gamma + 1)\omega_\perp}$ \([17]\). According to this formula, in the molecular limit ($a > 0, na^3 \ll 1$), when $\mu \propto a q d n$, i.e., $\gamma = 1$, the frequency $\omega = 2\omega_\perp$. In the unitary limit and BCSB limit, $\gamma = 2/3$ as in an ideal Fermi gas and $\omega = \sqrt{10/3}\omega_\perp = 1.83\omega_\perp$. For intermediate values of $a$, the frequency cannot be calculated analytically, but it appears reasonable that the frequency for $a > 0$ is monotonically decreasing with increasing $a$. These were precisely the indications of the first experiments. Theories that have this property and rely on the mean-field approximation have also been proposed. However, the situation is not that simple. For $na^3 \ll 1$, the theory permits rigorous calculations of not only the first term in $\mu$ but also a correction, which was first determined in \([18]\). This gives a correction to the frequency equal to \([19]\)

$$\delta \omega/\omega = +0.72 \sqrt{n(V_{0} = 0)a^3_{dd}} \, .$$

The positive correction sign signifies that the frequency must initially increase with increasing $a$ and only then decrease to attain the limit value $1.83\omega_\perp$. This reasoning was disputed on the grounds that molecular dimers are nevertheless not entirely bosons. However, the correction \([8]\) bears a clear physical meaning. It stems from the contribution to the energy made by zero-point phonon oscillations, whose occurrence in the superfluid liquid is beyond question. This is why it is anomalously large, of the order of the square root of the gas parameter $\hbar \omega \sim \mu$.

We now discuss the fluid properties at the unitarity point at finite temperatures. In this case, the temperature is assumed to be not too high, and therefore the wavelength of atoms in their thermal motion is long in comparison with the atomic size: $r_0 \ll \hbar/\sqrt{m T}$. We are actually dealing with temperatures of the order of $E_F$.

The question of the temperature of transition to the superfluid state is all-important here. The most reliable data were obtained in \([22]\) using the Monte Carlo technique: $T_c = 0.16\mu^{id}$. This result is in good agreement with experiment. It is noteworthy that the transition temperature is relatively low, and hence at temperatures somewhat higher than $T_c$, we face an interesting research object - a degenerate normal Fermi liquid in the unitary regime.
At finite temperatures, the equation of state cannot be written proceeding from only the dimensionality considerations. But these considerations lead to important similarity relations. For instance, the chemical potential is of the form $\mu(n, T) = \mu^{id}(n)f_\mu[T/\mu^{id}(n)]$; the entropy per atom can be written as $s(n, T) = f_S[T/\mu^{id}(n)]$. The last relation implies that under an adiabatic density variation, the temperature varies as $T \propto n^{2/3}$, as in an ideal monatomic gas.

For a fluid in the trap, these formulas lead to an important integral relation. Following the standard derivation of the virial theorem, it can be shown that

$$2U_{pot} = E,$$

where $U_{pot}$ is the potential energy and $E$ is the total energy, i.e., the sum of potential, internal, and hydrodynamic kinetic energies. As indicated above, $U_{pot}$ can be calculated directly from the measured density distribution. The total energy may be changed in a controllable way. For this, the trap potential was switched off for some ‘heating time’ $t_{heat}$. During this period, the liquid was free to expand. The sum of the kinetic and internal energies was conserved in the process. Then, the trap was turned on again and the system came to equilibrium, and its potential energy, which was measured anew, turned out to be higher. This ingenious method enabled the authors of [21] to verify relation (7) with high precision and thus confirm the similarity laws formulated above.

The total entropy $S = \int n(x)s(x)dx$ of the system as a function of its energy $E$ was measured in a similar experiment in [24]. In the experiment, the energy was varied and measured as described above; to measure the entropy, the magnetic field was adiabatically increased, taking the system away from resonance, where the interaction was insignificant. Measurements of the cloud dimension enabled calculating the entropy from the formulas for an ideal Fermi gas, which, due to the adiabaticity of the process, was equal to the entropy of the liquid before the increase in the magnetic field. It is noteworthy that the derivative $T = dE/dS$ directly yields the absolute temperature of the system. I believe that the capability of measuring the absolute temperature in the nanokelvin domain is a wonderful achievement by itself. Another way of measuring the absolute temperature is described below.

The aforesaid leaves no room for doubt that the theoretical notions about the properties of a ‘unitary’ superfluid liquid are amply borne out by experiments. I believe, however, that the significance of the issue calls for high-precision verification. Such a possibility does exist. For this, the fluid should be placed in a trap that is harmonic and isotropic with a high degree of accuracy. Then, we can state with certainty that the spherically symmetric cloud pulsations are precisely equal to $2\omega h$ in frequency, where $\omega h$ is the eigenfrequency of the trap, and do not attenuate [25]. This theorem is valid both below and above the superfluid transition point and applies to oscillations of arbitrary amplitude. It is a corollary of the hidden symmetry of the system at the unitarity point. (A similar situation occurs for oscillations of a dilute Bose gas in a cylindrical trap [26, 27].) The absence of damping signifies that the second viscosity $\zeta$ of the fluid is equal to zero above the transition point. Of the three second viscosity coefficients introduced by Khalatnikov [28], $\zeta_1$ and $\zeta_2$ turn out to be zero in the superfluid phase [29].

So far, we have dealt with experiments in which the numbers of atoms in two spin states were equal. Recently, active work commenced to study polarized systems in which the number of atoms in one spin state (we conventionally speak of ‘spin-up’ atoms) is greater than in the other state. This question had already been discussed for superconductors. In [30] and [31], the existence of spatially inhomogeneous phases (LOFF phases) was predicted, in which the superconducting gap is a periodic function of coordinates. In superconductors, the population difference of the spin states may exist in ferromagnetic bodies or may be induced by an external magnetic field. In both cases, the magnetic field affects the orbital motion and destroys superconductivity.
In our neutral dilute systems, the spin relaxation time is quite long and the numbers of atoms in different states are practically arbitrary parameters, determined by the initial conditions. Theoretical calculations [32] and the experiment [33] show that the liquid in a trap at $T = 0$ near the unitarity point breaks up into three phases. At the center is the superfluid phase with equal numbers of ‘spin-up’ and ‘spin-down’ atoms. It is surrounded by the partially polarized normal phase with unequal densities of the atoms of different polarization. At the periphery is the completely polarized phase, which consists of only the atoms of excess polarization. In this case, the existence of LOFF-type phases in some parameter value ranges is not ruled out.

The measurement data are presented in Fig. 6. The system with $N_1 = 5.9 \times 10^6$, $T/E_F = 0.03$ and the spin state population ratio $N_i/N_1 = 0.39$ was investigated. Figure 6a shows the phase-contrast image of the two-dimensional polarization distribution (the column density) $\delta n_u(x, z)$ along the x axis. Figure 6b shows the ‘weighted’ distribution $\delta n_u(x, z) = \int dy [n_t(r) - n_\uparrow(r)]$ and Fig. 6c shows the ‘weighted’ picture. Figure 6c shows the curves $\delta n_u(0, z) = 0.76 n_\uparrow(0) - 1.43 n_\uparrow(0)$, and Figs 6d and 6e are the plots of integrated linear densities $\delta n_a(z) = \int dx n_a(x, z)$ and $\delta n_a(x) = \int dz n_a(x, z)$.

I emphasize that the measurements were made in situ, i.e. in the trap itself, without prior expansion of the fluid. Processing the measured two-dimensional distribution by the Abel transform enabled reconstructing the three-dimensional polarization distribution and confirmed the three-phase fluid structure. Measurements at different temperatures were also made. Worthy of note in this connection is the special role played by the completely polarized phase. Because slow fermions with parallel spins do not interact with each other, this phase is an ideal Fermi gas. By measuring the density distribution of this phase and fitting it to formulas for the ideal gas, it is possible to determine the thermodynamic temperature of the system. The polarized phase plays the role of an ideal-gas thermometer contacting with other phases. It is significant in this case that the fermions of the polarized phase interact with the fermions of other phases, which ensures thermodynamic equilibrium. These temperature measurements permitted verifying the transition temperature calculated in Ref. [22]. The results under discussion are at some variance with the findings in [14], where a smaller number of atoms was considered. Conceivably, the surface tension at the phase boundaries plays a role under these conditions.

I mention several interesting possibilities for future investigations. One of them involves employing two types of fermions of different masses for which the Feshbach resonance exists [34]. Theory predicts unconventional properties for a superfluid liquid formed as a result of Cooper pairing of the fermions of different masses.

Another possibility is related to the vortex-free rotation of a Fermi liquid [25]. The vortex lattice shown in Fig. 4 is formed due to a strong fluid perturbation by the rotating mixers. If a trap asymmetric about the axis is simply set in rotation, there are grounds to believe that vortices would be formed only for a high rotation rate, when the fluid shape becomes unstable. At lower rotation rates, the fluid would break up into two phases. The center of the weakly deformed trap would be occupied by the superfluid liquid at rest, while the normal phase of the liquid would
rotate in the usual way at the periphery. The existence of the normal phase at absolute zero kept by rotation from transiting into the superfluid state raises difficult theoretical issues.

A very rich area of research opens up when the fluid is placed in a periodic lattice produced by counterpropagating laser beams (see the author’s review Ref. [36]). This research in the unitary domain is still in its infancy.

We see that the investigations of a near-resonance Fermi gas in a trap have opened up entirely new theoretical and experimental opportunities in condensed matter physics, reflecting the modern trend. Work to an increasing extent is shifting to the investigation of specially fabricated objects that do not exist in nature and have surprising new properties. In view of this, I believe, no exhaustion of our realm of physics is to be expected in the foreseeable future.

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