Structure optimization of a micro drill bit with nonlinear constraints considering the effects of eccentricity, gyroscopic moments, lateral and torsional vibrations

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Abstract. A micro drill structure was optimized to give minimum lateral displacement at its drill tip, which plays an extremely important role on the quality of drilled holes. A drilling system includes a spindle, chuck and micro drill bit, which are modeled as rotating Timoshenko beam elements considering axial drilling force, torque, gyroscopic moments, eccentricity and bearing reaction force. Based on our previous work, the lateral vibration at the drill tip is evaluated. It is treated as an objective function in the optimization problem. Design variables are diameter and lengths of cylindrical and conical parts of the micro drill, along with nonlinear constraints on its mass and mass center location. Results showed that the lateral vibration was reduced by 15.83% at a cutting speed of 70000 rpm as compared to that for a commercial UNION drill. Among the design variables, we found that the length of the conical part connecting to the drill shank plays the most important factor on the lateral vibration during cutting process.

1. Introduction
Nowadays, with the development of science and technology, there is a huge demand for micro drill bits to machine micro holes. Hence, many researchers have been making efforts to improve their cutting performance. The quality of micro drills is evaluated by many factors, for instance, material, burrs, chip removing, accuracy of drilled holes, length to diameter ratio, tool lifetime, etc. Several researchers investigated these factors in the past. To name a few, new material was created, e.g., the metal matrix composite of nickel ions and fine diamond powder was created to promote wear resistance of micro drills [1,2]. With coating technique, surfaces of micro drills were coated by special material to improve cutting performance. For example, the approach of coating Zr-C:H:N17% extended the tool lifetime by a factor of at least four compared to the uncoated micro drill, and obtained better cutting performance [3-5]. On the other hand, since drill point angle has a great effect on the tool lifetime of a micro drill, many researches have been conducted on this area [6]. The optimization of the geometry of micro drill point has been investigated theoretically and experimentally. In particular, helix angle and web thickness of a micro drill were optimized by Taguchi methods and response surface methodology to promote its cutting performance [7]. With the development mathematical formulation for the drill flute and geometrical features, the optimization was implemented by software Pro/Machanica to minimize the Von Mises stress of the micro drill below the compressive strength of the material property [8]. Generally, all these methods only focused on material, geometrical point angles, and flute of the drill to
enhance its cutting ability. However, studies on the other portion of the drill bit, such as shank, and conical parts have been paid little attention in the past. The structure of a micro drill is to be optimized to obtain the least lateral vibration at its drill tip for better performance in this research. Design variables are the diameter, and lengths of its cylindrical and conical parts. Constraints are its length, mass as well as the mass center location. The optimization problem is solved by function Fmincon in software Matlab. Results showed that the lateral vibration was reduced by 15.83% at the cutting speed of 70000 rpm as compared to that of a commercial UNION drill. From the sensitivity test, it is found that the length of the conical part connecting to the micro drill shank play the most important role on the lateral displacement during cutting process.

2. Optimization of the micro drilling system

The structure of the micro drilling system can be simplified as shown in figure 1(a). The micro drill E is grasped in the chuck C-D that is connected to the spindle A by a small shaft B. The system is supported by two identical linear bearings, and applied by a motor torque Tm for running the system at rotating speed \( \Omega \). It is assumed that the tip of the micro drill is applied by an axial force \( F_a \), and a torque \( T \) at the steady-state during drilling machining. The whole system dimensions and parameters are referred from previous works [9-11]. Because the axial displacement is much smaller than the other displacements, it is neglected in this research [12-14]. The micro drill, chuck and spindle of micro drilling system are modeled as rotating Timoshenko beam elements to evaluate the lateral displacement at the drill tip. This vibration has a great impact to the accuracy of drilling process, so it is chosen as the objective function of optimization problem. Hence, the objective function is set as the common logarithm of lateral amplitude.

\[
F = \log_{10}(\text{lateral amplitude})
\]

where lateral amplitude = maximum value of \( \sqrt{x^2 + y^2} \), in which \( x \) and \( y \) are evaluated in our previous work [11]. The vibration at the micro drill tip is affected by many factors, such as micro drill structure, chuck, machine spindle, and cutting velocity, and so on. The micro drill structure is mainly concerned in this research. The design variables are lengths of cylindrical and conical parts of the micro drill \( (X_1, X_2, X_3, X_4) \), and the diameter of the cylindrical part \( X_5 \) as shown in figure 1(b).

![Figure 1](image-url)

**Figure 1.** (a) Structure of micro drilling system, (b) design variables

The length, mass, and mass center location of the micro drill are constrained in the optimization problem. For reducing the vibration during cutting process, the drill mass location needs to be constrained inside the tool holder of the machine spindle. The drill mass is set as constant. The micro drill length \( L_d \) is set as variable within \([L_d^1, L_d^2]\), where, \( L_d^1 \) and \( L_d^2 \) are given parameters. The above nonlinear and linear constraints are described in the following:
The values of design variables are shown in (6)

\[
L_{d} = L_{g} + L_{f} + X_{1} + X_{2} + X_{3} + X_{4} \leq L_{g}^{2} .
\]

(2)

\[
\begin{align*}
&f_{m}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) = m_{0} \\
&L_{\text{mass-center}}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) \leq L_{g}
\end{align*}
\]

(3)

where \( f_{m}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) \) and \( L_{\text{mass-center}}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) \) are the functions of mass and mass center location of the micro drill, respectively. The notation \( m_{0} \) is the mass of the micro drill. The notations of \( L_{d}, L_{g}, \) and \( L_{f} \) represents lengths of the micro drill, the shank part grasped by the machine spindle, and flute part, respectively.

The micro drill mass function \( f_{m}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) \) in Eq. 3 is the sum of the drill element masses. A drill element mass \( m_{i}' \) is evaluated by

\[
m_{i}' = \rho \cdot A \cdot L_{\text{drill element}},
\]

(4)

where \( \rho, A, \) and \( L_{\text{drill element}} \) are the mass density, cross sectional area, and element length, respectively.

The cross sectional area \( A \) is derived from our previous work [11].

The mass center location function \( L_{\text{mass-center}}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) \) in Eq. 3 is calculated as follows:

\[
L_{\text{mass-center}}(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}) = \frac{\sum_{i=1}^{N} m_{i}' \times \left( L_{i} + \frac{L_{\text{drill element}}}{2} \right)}{m_{0}},
\]

(5)

where \( L_{i}' \) is the position evaluated from the original \( B \) as shown in figure 1(b) of drill element \( i \).

The optimization problem is resolved by Matlab function Fmincon to solve constrained linear and nonlinear optimization problems. The interior-point algorithm is chosen to solve the optimization problem [15-18].

A commercial Union micro drill is adopted for benchmarking and its parameters denoted as original variables are given in tables 1 and 2. The lower and upper bounds, as well as starting values of design variables are given in table 2. Applying the parameters in tables 1 and 2 and, objective function Eq. 1, and constraint functions Eqs. 2 and 3, one obtains the results as shown in figure 2(a). The optimum objective function is \( F_{\text{Optimal}} = -5.4932 \). The values of design variables are shown in table 1. The length of the optimal micro drill is

\[
L_{d,\text{new}} = L_{g} + X_{1} + X_{2} + X_{3} + X_{4} + L_{f} = 19.67 + 4.13 + 7 + 2.5 + 2 + 1.7 = 37 \text{[mm]}.
\]

(6)

Its mass and mass center location are 1.7 \( g \) and 11.75 mm, respectively. The percentage reduction \( \Delta X \) in table 1 is calculated by

\[
\Delta X = \left( \frac{X_{i}^{\text{op}} - X_{i}^{\circ}}{X_{i}^{\circ}} \right) \times 100\% ,
\]

(7)

where \( X_{i}^{\text{op}}, \) and \( X_{i}^{\circ} \) are values of original and optimal variables, respectively. "+" and "−" signs denote that the increase and decrease, respectively, of the optimal value in comparison with the original value.

The original objective function is \( F_{\text{Original}} = -5.4183 \), so the reduction percentage of lateral amplitude at the micro drill tip is

\[
\Delta F = \left( \frac{\text{Lateral amplitude(Original)} - \text{Lateral amplitude(Original)}}{\text{Lateral amplitude(Original)}} \right) \times 100\% = -15.83\% .
\]

(8)
Table 1. Values of design variables

| Design variable | Lower bound (mm) | Upper bound (mm) | Starting value (mm) | Original variable value (mm) | Optimal variable value (mm) | Percentage deviation ($\Delta X$) (%) |
|-----------------|-----------------|-----------------|--------------------|-----------------------------|-----------------------------|-----------------------------------|
| $X_1$           | 2               | 10              | 6                  | 5                           | 4.13                        | -17.40                            |
| $X_2$           | 2               | 7               | 4.5                | 4.43                        | 7                           | +58.01                            |
| $X_3$           | 2.5             | 8               | 5.25               | 6.03                        | 2.5                         | -58.54                            |
| $X_4$           | 1               | 2               | 1.5                | 1.27                        | 2                           | +57.48                            |
| $X_5$           | 0.132           | 2.86            | 1.49               | 0.8                         | 0.58                        | -27.50                            |

Table 2. Parameters of the micro drilling system

| Original Micro Drill | Radius $R$ (mm) | Helical angle $\theta_h$ | Semi point angle $\rho_s$ | Length $L_d$ (mm) | Flute length $L_f$ (mm) | Drill mass $m_0$ (g) | Mass center location $L_{mass\_center}$ (mm) | Cross section area of flute $A$ (m$^2$) | Area moment of inertia of cross-section flute $I_u$ (m$^4$) | $I_p$ (m$^4$) | $I_v$ (m$^4$) | Young’s modulus $E$ (N/m$^2$) | Shear modulus $G$ (N/m$^2$) | Mass density $\rho$ (Kg/m$^3$) | Timoshenko’s shear coefficient $k_s$ | Eccentricity $e_u$ (mm) | $e_v$ (mm) |
|---------------------|-----------------|-------------------------|---------------------------|-------------------|-------------------------|----------------------|-----------------------------------------------|----------------------------------------|-------------------------------------------------|----------------|----------------|--------------------------|--------------------------|-----------------------------|-------------------------------|-------------------|----------------|
|                     | 0.06            | 30°                     | 65°                       | 38.1              | 1.7                     | 1.7                  | 11.9                                         | 19.67                                  | $4.929 \times 10^{-9}$                                | $1.242 \times 10^{-18}$ | $8.783 \times 10^{-18}$ | $7.541 \times 10^{-18}$ | $650 \times 10^6$ | $280 \times 10^6$ | 15250                        | 0.835                  | $6 \times 10^{-7}$                        | $-6 \times 10^{-7}$ | $e_u = 10^{-5} \cdot r$, $e_v = -10^{-5} \cdot r$, where $r$ is radius of each cross section, |
| Flute Part           |                 |                         |                           |                   |                         |                      |                                               |                                        |                                                         |                   |                           |                           |                           |                           |                                               |                     |                                                        |
|                     |                 |                         |                           |                   |                         |                      |                                               |                                        |                                                         |                   |                           |                           |                           |                           |                                               |                     |                                                        |
| Other Parts          |                 |                         |                           |                   |                         |                      |                                               |                                        |                                                         |                   |                           |                           |                           |                           |                                               |                     |                                                        |
|                     |                 |                         |                           |                   |                         |                      |                                               |                                        |                                                         |                   |                           |                           |                           |                           |                                               |                     |                                                        |

Bearing stiffness Bearing damping

$K_{xx} = K_{yy} = 7 \times 10^7$, $K_{xy} = K_{yx} = K_{0xx} = 0$ [N/m] $C_{xx} = C_{yy} = 1.7513 \times 10^3$ [Ns/m]

$K_{0xy} = K_{bxy} = 0$, $K_{xy} = 3 \times 10^4$ [Nm/rad] $C_{xy} = C_{yx} = C_{0xx} = C_{0xy} = C_{bxy} = C_{byx} = 0$ [Nms/rad] $C_{q} = 1$ [Nms/rad]

Order of whirling at the drill tip $n = 2$, $\Omega = 7000$ [rpm], $F_u = 10^{-5}$[N], $T = 10^{-7}$[Nm], $L_d^1 = 37$ [mm], $L_d^1 = 39$ [mm],
The sensitivity of design variables to the objective function is carried out to examine their effect on the lateral displacement. As shown in Figure 2(b), horizontal axis represents percentage change of design variables calculated by

$$X_i = X_i^O \pm \delta \cdot X_i^O, \quad i = 1, 5,$$

where $X_i$, $X_i^O$ and $\delta$ are the design variable, optimal value of a design variable and percentage change, respectively, whereas vertical axis represents the objective function values corresponding to each design variable.

Since the lengths of cylindrical and conical parts of the micro drill affect the drill stiffness, i.e., the longer the lengths are, the smaller the stiffness is, increasing these lengths will enlarge the lateral vibration at the drill tip during cutting process. Consequently, the objective functions corresponding to design variable $X_i$, $i = 1, 4$, the lengths of cylindrical and conical parts, increase when the percentage change increases as shown in Figure 2(b). The increase of the cross sectional diameter makes the unbalance force and stiffness to increase. These two factors counteract with each other on the lateral displacement. Therefore, the objective function corresponding to the shank diameter ($X_5$) does not change much. One observes that the design variable $X_2$ has the highest sensitivity. In consequence, compared with other design variables, the length of conical part connecting to the shank has the greatest influence on the lateral vibration at the drill tip. This finding is very valuable in micro drill design.

3. Conclusions
The structure optimization of the micro drill bit was presented to give better cutting quality as compared to the original one. The results showed that the lateral vibration at the micro drill tip was decreased by 15.83% at the cutting speed of 70000 rpm. Compared with other design variables, the length of the conical part connecting to the drill shank is the most important factor that affects the lateral displacement at the drill tip. Furthermore, the usage of the transfer matrix approach to analyze the dynamics of the micro drilling system modeled as rotating Timoshenko beam elements with effects of axial force, torque, gyroscopic moments, and eccentricity provides an effective way in the design of micro drill.

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