Softly broken conformal symmetry  
and the stability of the electroweak scale

Piotr H. Chankowski\(^1\), Adrian Lewandowski\(^1\), Krzysztof A. Meissner\(^1\) and Hermann Nicolai\(^2\)
\(^1\) Faculty of Physics, University of Warsaw
Hoża 69, Warsaw, Poland
\(^2\) Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)
Mühlenberg 1, D-14476 Potsdam, Germany

We point out a novel possible mechanism by which the electroweak hierarchy problem can be avoided in the (effective) quantum field theory. Assuming the existence of a UV complete underlying fundamental theory and treating the cutoff scale \(\Lambda\) of the effective field theory as a real physical scale we argue that the hierarchy problem would be solved if the coefficient in front of quadratic divergences vanished for some choice of \(\Lambda\), and if the effective theory mass parameters fixed at \(\Lambda\) by the fundamental theory were hierarchically smaller than \(\Lambda\) itself. While this mechanism most probably cannot work in the Standard Model if the scale \(\Lambda\) is to be close to the Planck scale, we show that it can work in a minimal extension (Conformal Standard Model) proposed recently for a different implementation of soft conformal symmetry breaking.

PACS numbers: 12.60.Fr,14.80.Ec,14.80Va

The problem of stability of the electroweak scale with respect to the Planck scale (the so-called hierarchy problem) has for almost 40 years been one of the main driving forces of theoretical research in high energy physics. Over the years various mechanisms for solving it at the effective quantum field theory level have been proposed and investigated in detail, of which the most notable are technicolor and low energy supersymmetry. With the discovery of a spin-zero particle at the LHC, and after establishing its basic characteristics, it has become clear that a solution which departs little from the simplest mechanism of the electroweak symmetry breaking realized in the Standard Model (SM) may be preferred. In particular, extensions of the SM which predict only elementary scalars and no new higher spin particles other than right-chiral neutrinos seem distinguished at present. It is therefore of interest that there exists an alternative way (which does not require new spin \(s \geq \frac{1}{2}\) degrees of freedom) by which the problem of stability of the electroweak scale could be avoided in the low energy effective theory. It is based on a novel implementation of ‘near conformal symmetry’ in the effective low energy theory.

As is well known, the classical conformal symmetry of the SM is spoiled only by the scalar field mass term necessary to induce phenomenologically viable electroweak symmetry breaking. Moreover, as in any generic quantum field theory, conformal symmetry of the SM is broken by quantum effects. Yet, the idea that ‘softly broken conformal symmetry’ (SBCS) might be relevant for the solution of the hierarchy problem was expressed already long ago \(^{[1]}\). As one possible concrete implementation of this idea a minimal extension of the SM, the Conformal Standard Model (CSM), has been proposed in \(^{[2]}\). Besides the known particles this model only involves the right-chiral neutrinos and one extra (complex) scalar field. Originally it was assumed that its conformal symmetry is broken only by the anomaly, inducing electroweak symmetry breaking via the Coleman-Weinberg mechanism \(^{[3]}\). However, although there do exist perturbatively stable minima of the potential of this model giving rise to a Higgs mass equal to 125 GeV (as we have checked by carefully investigating the 2-loop effective potential of the model), the mixing with the second heavier spin-zero particle in all cases turned out too large to be in agreement with the LHC data. For this reason, and because of another serious drawback of this implementation (related to quadratic divergences, see below) we here propose a different way in which SBCS can be at work to solve the hierarchy problem, and show how this mechanism can be realized in the model \(^{[2,4]}\) with explicit small mass parameters. We also note some similarities with the scheme proposed in \(^{[2]}\) in the framework of the asymptotic safety program.

Let us first define our framework. We assume that there exists a complete and UV finite fundamental theory (which is likely not a quantum field theory) describing all interactions including (quantum) gravity which, after integrating out all degrees of freedom above some large scale \(\Lambda\) (presumably close to the Planck scale \(M_{Pl}\)), fixes the ‘bare’ action of the effective field theory. In particular, we assume that the fundamental theory determines the way the cutoff \(\Lambda\) should be implemented in the effective theory loop calculations. To understand our proposal how the stability of electroweak scale at the level of the effective quantum field theory can be secured by the putative fundamental theory it is crucial to keep in mind that, unlike the usual renormalization program in which \(\Lambda\) is eventually taken to infinity, here \(\Lambda\) is finite; for this reason all ‘bare’ parameters of the effective theory fixed at this scale are also finite. In general the cutoff \(\Lambda\) is \textit{a priori} arbitrary: given an UV finite fundamental theory it should always be possible to integrate
out all (gravitational and matter) degrees of freedom above the scale \( \Lambda \) to obtain a finite ‘bare’ effective theory valid for all energy scales below \( \Lambda \). Even if the fundamental theory does correctly predict (as we assume) the very small ratio \( M_{\text{EW}}^2/M_{\text{Pl}}^2 \) and related low energy observables, that is, even if it completely solves the conceptual aspect of the hierarchy problem, the effective theory generically is still susceptible to ‘technical’ aspect of the problem if it involves scalar fields: if the effective theory is solved (perturbatively or not) directly in terms of the bare parameters defined at the scale \( \Lambda \), such small ratios arise as the result of very precise cancellations of \( \Lambda^2 \) contributions against (bare mass)\(^2\) parameters of the same order.

From this perspective the implementation of SBCS as proposed in [2] (as well as in any other model that relies on radiative symmetry breaking \( \text{a la} \) Coleman-Weinberg) suffers from the same problem: the absence of \( \Lambda^2 \) divergences in the dimensional regularization scheme used there is, in fact, artificial: in terms of bare parameters, there is a huge cancellation between the \( \Lambda^2 \) contributions induced by real fluctuations of the quantum fields and the (bare mass)\(^2\) terms of the effective action fixed at \( \Lambda \) by the fundamental theory which is supposed to produce vanishing or very small mass values at the level of the effective action.

Within this general framework one can envisage two different ways in which the hierarchy problem at the level of the effective field theory can be avoided. The first possibility is that the bare parameters \( m_B^2(\Lambda) \) of the effective theory are hierarchically smaller than \( \Lambda \) and loop corrections to masses of light particles proportional to \( \Lambda^2 \) cancel exactly by some symmetry. This mechanism is realized in supersymmetric theories [6]. In this case the precise value of the cutoff \( \Lambda \) does not matter: the cancellation of the quadratic divergences holds automatically for exactly by some symmetry. This mechanism is realized in supersymmetric theories [6]. In this case the precise value of the cutoff \( \Lambda \) does not matter: the cancellation of the quadratic divergences holds automatically for any choice of \( \Lambda \). For practical purposes one can then formally send \( \Lambda \) to infinity and adopt any convenient regularization in order to set up the standard renormalized perturbative expansion.

The second and novel possibility which we want to point out here is that the putative fundamental theory singles out a particular scale \( \Lambda \), the physical cutoff, at which \( m_B^2(\Lambda) \ll \Lambda^2 \) and at which the complete \( \propto \Lambda^2 \) corrections to the physical spin-zero boson(s) (and thus to the ratio \( M_{\text{EW}}^2/M_{\text{Pl}}^2 \)) vanish. Naturally one expects \( \Lambda \) to be close to the Planck mass \( M_{\text{Pl}} \). We will argue below that this can also be regarded as a solution of the ‘technical’ hierarchy problem. Both mechanisms of avoiding the hierarchy problem in the effective quantum field theory can thus be attributed to SBCS, by small mass terms and by the quantum anomaly. We stress that neither of these mechanisms solves the ‘conceptual aspect’ of the hierarchy problem which probably cannot be solved without knowing the underlying fundamental theory. However, we point out that since both the effective theory parameters at the scale \( \Lambda \) as well as the prescription how the cutoff \( \Lambda \) should be implemented are determined by the same fundamental theory (and thus are necessarily correlated with each other) it is not inconceivable, that the latter theory singles out a scale \( \Lambda \) at which our assumptions are satisfied.

To see how this second possibility manifests itself in a bottom-up perspective, it is important to realize that for this the finiteness of the bare parameters must be preserved by keeping the cutoff \( \Lambda \) finite (in a way dictated by the fundamental theory), and for this reason one is not allowed to use continuation in space-time dimension to regularize loop integrals in the effective theory calculations. Renormalized running parameters can nevertheless be introduced by the usual splitting of the mass parameters \( m_B^2(\Lambda) = m_R^2(\Lambda, \mu) + \delta m^2(\Lambda, \mu) \) and couplings \( \lambda_B(\Lambda) = \lambda_R(\Lambda, \mu) + \delta \lambda(\Lambda, \mu) \), and by fixing the counterterms involving \( \delta m^2(\Lambda, \mu) \) and \( \delta \lambda(\Lambda, \mu) \) in the \( \Lambda \)-MS subtraction scheme in which by definition they absorb only contributions proportional to \( \Lambda^2 \) and \( \ln(\Lambda^2/\mu^2) \) (the counterterms \( \delta m^2 \) and \( \propto \ln(\Lambda^2/\mu^2) \) (the counterterms \( \delta \lambda) \). Computing physical observables within the effective theory one then finds the following relation between bare and renormalized parameters

\[
\lambda_B(\Lambda) = \lambda_R + \sum_{L=1}^{\infty} \sum_{\ell=1}^{L} a_{L\ell} \lambda_R^{L+1} \left( \frac{\ln \Lambda^2}{\mu^2} \right)^{\ell} . \tag{1}
\]

It follows that \( \lambda_B = \lambda_R \) for \( \mu = \Lambda \), and

\[
m_B^2(\Lambda) = m_R^2(\mu, \lambda_R, \Lambda) \Lambda^2 + m_R^2 \sum_{L=1}^{\infty} \sum_{\ell=1}^{L} c_{L\ell} \lambda_R^{L} \left( \frac{\ln \Lambda^2}{\mu^2} \right)^{\ell} . \tag{2}
\]

The crucial fact, which is at the heart of our proposal [7] is that the coefficient in front of \( \Lambda^2 \)

\[
\hat{f}^{\text{quad}}(\mu, \lambda_R, \Lambda) = \sum_{L=1}^{\infty} \sum_{\ell=0}^{L-1} b_{L\ell} \lambda_R^{L} \left( \frac{\ln \Lambda^2}{\mu^2} \right)^{\ell} , \tag{3}
\]

can be written as a function of the bare coupling(s) only: from the analysis of the \( \phi^4 \) theory [8] (which we assume to hold generally) it follows that the logarithmic dependence on the scale \( \mu \) of the \( \Lambda^2 \) divergence in \( \hat{f} \) is spurious
two relevant functions \( f_{\mu, \lambda, \Lambda} = f_{\lambda_B(\Lambda)} \).

In other words, when corrections to the scalar boson mass are computed in the perturbative expansion in terms of the renormalized parameters, only non-logarithmic pieces proportional to \( \Lambda^2 \) in consecutive orders of the loop expansion correct the form of the function \( f_{\mu, \lambda, \Lambda} \); logarithms multiplying \( \Lambda^2 \) contribute only to converting the renormalized couplings \( \lambda_R \) into the bare ones. Thus, an effective quantum field theory derived from a complete UV finite fundamental theory is free from the (“technical”) hierarchy problem if the condition

\[
\hat{f}_{\mu, \lambda, \Lambda} = f_{\lambda_B(\Lambda)} = 0
\]

is satisfied! This condition, which from the bottom-up perspective looks accidental should, according to our assumptions, be viewed as a manifestation of the intrinsic working of the underlying fundamental theory.

As we do not know the scale \( \Lambda \) nor the precise way the cutoff should be implemented, we adopt here a simple smooth cutoff by replacing \( k^\mu \to k^\mu \exp \left( -\frac{k^2}{2\Lambda^2} \right) \), for each momentum in the (renormalizable part of the) action. With this prescription the bottom-up procedure to check whether a given theory with \( n \) physical spin-zero bosons is free from the hierarchy problem consists in fixing its renormalized couplings from fits to the low energy data at \( M_{\text{EW}} \), and then evolving them with the RG equations as functions of the scale \( \mu \) to check whether there exists some scale at which the relevant \( n \) functions \( f_{\mu, \lambda, \Lambda} \) for \( k = 1, \ldots, n \) (determined to the appropriate loop order) vanish simultaneously. One may then identify this scale with \( \Lambda \) and equate \( \lambda_B \) with \( \lambda_R \) at this scale. For consistency, the couplings of the model should then satisfy the following additional conditions over the whole range \( M_{\text{EW}} < \mu < \Lambda \):

- there should be neither Landau poles nor instabilities (manifesting themselves as the unboundedness from below of the effective potential depending on the running scalar self-couplings);
- all couplings \( \lambda_R(\mu) \) should remain small (for the perturbative approach to be applicable and stability of the effective potential electroweak minimum).

In the SM there is only one possible quadratic divergence associated with the Higgs boson. Its vanishing was first conjectured in [10], but the SM couplings were taken at the electroweak scale, leading to a wrong prediction for the top quark mass. The RG evolution of the coefficient in front of this divergence was recently investigated in [11, 12]. This analysis indicates that the SBCS requirements are not met in the SM: the zero of coefficient function \( f_{\mu, \lambda, \Lambda} \) lies around \( 10^{23} \) GeV (it is hard to accept that the scale at which the effective theory should be constructed is so much above the Planck scale) and furthermore the scalar self-coupling \( \lambda_R(\mu) \) becomes negative near \( \mu = 10^{10} \) GeV, signaling an instability of the electroweak minimum. Although these statements depend on the loop order considered, and also (to a considerable extent) on the precise value of the top mass, we conclude that in the SM the hierarchy problem is most likely not solved by the SBCS mechanism.

We now show that all the necessary conditions can be satisfied by the CSM of [2, 4]. With explicit mass terms the potential of this model reads

\[
V = m_H^2 H^\dagger H + m_\phi^2 |\phi|^2 + \lambda_1 (H^\dagger H)^2 + 2 \lambda_3 (H^\dagger H) |\phi|^2 + \lambda_2 |\phi|^4,
\]

where \( H = (H_1, H_2) \) is the \( SU(2)_{\text{EW}} \) doublet and \( \phi \) is the extra gauge singlet. At the minimum \( \sqrt{2}(H_i) = v_H \delta_{i2}, \sqrt{2}(\phi) = v_\phi \), and the physical spin-zero particles are the CP-even \( h^0 \) and \( \phi^0 \), which are mixtures

\[
\begin{pmatrix} h^0 \\ \phi^0 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \sqrt{2} \text{Re}(H_2 - \langle H_2 \rangle) \\ \sqrt{2} \text{Re}(\phi - \langle \phi \rangle) \end{pmatrix},
\]

with masses \( M_h \) and \( M_{\phi^0} \), and the CP-odd axion \( a^0 = \sqrt{2} \text{Im} \phi \). We assume that \( M_h < M_{\phi^0} \). The existing experimental results suggest that \( |\tan \beta| \lesssim 0.3, \) if \( h^0 \) is to mimic the SM Higgs boson (see e.g. [14]).

Since there are two scalars in this model, two equations [4] must be simultaneously satisfied [15]. At one loop, the two relevant functions \( f_{\mu, \lambda, \Lambda} \) are straightforward to determine in terms of bare couplings, \( \nu \), viz.

\[
16\pi^2 f_1^{\text{quad}}(\lambda, g, y) = 6\lambda_1 + 2\lambda_3 + \frac{9}{4} g_w^2 + \frac{3}{4} g_y^2 - 6y_t^2,
\]

\[
16\pi^2 f_2^{\text{quad}}(\lambda, g, y) = 4\lambda_2 + 4\lambda_3 - \sum_{i=1}^3 y_{\nu_i}^2.
\]
The beta functions of the remaining couplings read:

\[ \beta_{\lambda_i} = 24\lambda_i^2 + 4\lambda_i^3 - 3\lambda_1 \left( 3g_w^2 + y_t^2 - 4y_t^2 \right) + \frac{9}{8}g_w^4 + \frac{3}{2}g_w^2g_y^2 + \frac{3}{8}g_y^4 - 6y_t^4 \]

\[ \beta_{\lambda_2} = 20\lambda_2^2 + 8\lambda_2^3 + 2\lambda_2 \sum_{i=1}^{3} y_{N_i}^2 - \sum_{i=1}^{3} y_{N_i}^4, \]

\[ \beta_{\lambda_3} = \frac{1}{2}\lambda_3 \left\{ 24\lambda_1 + 16\lambda_2 + 16\lambda_3 - (9g_w^2 + 3g_y^2) + 2 \sum_{i=1}^{3} y_{N_i}^2 + 12y_t^2 \right\} \]

The beta functions of the remaining couplings read:

\[ \beta_{g_w} = -\frac{19}{6}g_w^3, \quad \beta_{g_y} = \frac{41}{6}g_y^3, \quad \beta_{g_t} = -7g_t^3, \]

\[ \beta_{y_t} = y_t \left\{ \frac{9}{2}y_t^2 - 8g_w^2 - \frac{9}{4}g_y^2 - \frac{17}{12}g_t^2 \right\}, \]

\[ \beta_{y_{N_i}} = \frac{1}{2}y_{N_i} \left\{ 2y_{N_i}^2 + \sum_{i=1}^{3} y_{N_i}^2 \right\}. \] (8)

At the electroweak scale the scalar field mass parameters, whose \( \beta \)-functions we give here for completeness

\[ \hat{\beta}_{m_H^2} = \left\{ 12\lambda_1 + 6y_t^2 - \left( \frac{9}{2}g_w^2 + \frac{3}{2}g_y^2 \right) \right\} m_H^2 + 4\lambda_3 m_\phi^2, \]

\[ \hat{\beta}_{m_\phi^2} = 8\lambda_3 m_H^2 + \left\{ 8\lambda_2 + \sum_{i=1}^{3} y_{N_i}^2 \right\} m_\phi^2, \] (9)

are adjusted to give the required values \( v_H = 246 \text{ GeV} \) and \( M_h = 125 \text{ GeV} \). The mixing angle \( \beta \) defined in (8) as well as the \( B - L \) breaking scale \( v_\phi \) are then expressed as functions of heavy particles masses \( (M_\varphi \text{ and } m_{N_j} \approx y_{N_j}v_\phi/\sqrt{2}) \).

We have performed a numerical scan over the values (in the range \( 0 \div 2 \)) of the couplings \( \lambda_1 \) and \( y_N \) at the scale \( \Lambda \), rejecting all points for which one of the couplings \( \lambda_1, \lambda_2 \) becomes negative (or \( \lambda_3 < -\sqrt{\lambda_1\lambda_2} \)) between the scales \( M_{EW} \leq \mu \leq \Lambda \). A typical plot of the running couplings \( \lambda_i(\mu) \) and \( y_N(\mu) \) is shown in Fig.1. Due to the constraints imposed, only solutions with negative values of the mixing angle \( \beta \) in the range \( 0 < |\tan \beta| \lesssim 0.3 \) are found. In Fig.2 we show the predicted correlation of the masses \( m_N \) of the right-chiral neutrinos (here for simplicity assumed to be degenerate) with the mass of the additional scalar \( \varphi^0 \) and negative values of \( \tan \beta \) in the allowed range. The extra scalar \( \varphi^0 \) can decay into the usual SM particles (with small widths [16]), but also into two or three \( h^0h^0 \) (the channels \( a^0a^0 \) and \( h^0a^0a^0 \) are also open), or into the lightest right-chiral neutrinos if this is kinematically allowed (for instance, with non-degenerate neutrino masses, not all of which obey \( M_\varphi < 2m_N \), unlike in Fig.2). This produces calculable deviations from the ‘shadow Higgs’ behavior described in [10]. These very distinctive features of the CSM would clearly allow to discriminate it from other models also predicting new heavy scalar particles.

We have also checked that the results shown in Figs.1 and 2 are not very sensitive to the precise choice of the scale \( \Lambda \): for example for the same values of the masses \( M_\varphi \) and \( m_N \) varying the scale \( \Lambda \) within one order of magnitude changes the value of \( \tan \beta \) by a few percent at most.

To summarize: we have proposed a novel mechanism by which the effective quantum field theory can avoid the hierarchy problem. We have shown that the CSM of \([2, 10]\) can be consistent with this mechanism because there does exist a range of values of its parameters for which all SBCS requirements can be satisfied with the scale \( \Lambda \) of the order of the Planck scale. Remarkably, with \( \Lambda \) this high, the CSM may provide a complete scenario within which
all problems of particle physics proper can be addressed: strong CP-problem is potentially solved \[13\], neutrinos are naturally massive, non-thermally produced axions can constitute dark matter, and baryogenesis can probably proceed through leptogenesis (whereas the ultimate explanation of the cosmological constant problem, dark energy, and of the mechanism driving inflation must be relegated to a more fundamental theory of quantum gravity). Of course, the real test of the model and of the proposed SBCS scheme would require the detection of the new scalar particle $\varphi^0$, the heavy neutrinos and the axion. In the further perspective, with all the parameters of the model fixed from the low energy data it should become possible to check whether the coefficients in front of quadratic divergences indeed vanish, and to fix the scale $\Lambda$ at which this occurs.

A detailed account of our results will be given elsewhere.

Acknowledgments: AL and KAM thank the AEI for hospitality and support during this work.

[1] W.A. Bardeen, *On Naturalness in the Standard Model*, preprint FERMILAB-CONF-95-391-T; *Mechanisms of Electroweak Symmetry Breaking and the Role of a Heavy Top Quark*, preprint FERMILAB-CONF-95-377-T.
[2] K.A. Meissner and H. Nicolai, Phys. Lett. B 648 (2007) 312; Phys. Lett. B 660 (2008) 260.
[3] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.
[4] K.A. Meissner and H. Nicolai, Eur.Phys.J. C57 (2008) 493-498.
[5] M. Shaposhnikov and C. Wetterich, Phys. Lett. B683 (2010) 196
[6] Another possibility, realized in technicolor-type theories, is that $\Lambda^2$ corrections are absent altogether, and there are no scalar fields in the effective theory close to $\Lambda$; the Higgs boson would then be a composite particle.
[7] The authors of \[8\] did not consider this possibility because of implicitly taking the cutoff to infinity.
[8] M.B. Einhorn and D.R.T. Jones, Phys. Rev. D46 (1992) 5206.
[9] K. Fujikawa, Phys. Rev. D83 (2011) 105012.
[10] M. Veltman, Acta Phys. Pol. B12 (1981) 437.
[11] Y. Hamada, H. Kawai and K. Oda, Phys. Rev. D87 (2013) 053009.
[12] D.R.T. Jones, Phys. Rev. D88 (2013) 098301.
[13] A. Latosinski, K.A. Meissner and H. Nicolai, Nucl.Phys. B868 (2013) 596.
[14] H. Belusca-Maito and A. Falkowski, arXiv:1311.1113 [hep-ph], and references therein.
[15] The mass of $a^0$ which is a (pseudo)Nambu-Goldstone boson of the spontaneously broken lepton number symmetry is automatically protected against corrections $\propto \Lambda^2$.
[16] K.A. Meissner and H. Nicolai, Phys. Lett. B718 (2013) 943.

FIG. 1: Running couplings
FIG. 2: Predicted correlations of masses $M_{\phi}$ with $m_N$.