Using time-dependent indirect $CP$ asymmetries to measure $T$ and $CPT$ violation in $B^0-\bar{B}^0$ mixing

Anirban Karan $^a$, Abinash Kumar Nayak $^a$, Rahul Sinha $^a$, and David London $^a$

$a$: The Institute of Mathematical Sciences, HBNI, Taramani, Chennai 600113, India,
b: Physique des Particules, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

(March 3, 2021)

Abstract

Quantum field theory, which is the basis for all of particle physics, requires that all processes respect $CPT$ invariance. It is therefore of paramount importance to test the validity of $CPT$ conservation. In this Letter, we show that the time-dependent, indirect $CP$ asymmetries involving $B$ decays to a $CP$ eigenstate contain enough information to measure $T$ and $CPT$ violation in $B^0-\bar{B}^0$ mixing, in addition to the standard $CP$-violating weak phases. Entangled $B^0\bar{B}^0$ states are not required (so that this analysis can be carried out at LHCb, as well as at the $B$ factories), penguin pollution need not be neglected, and the measurements can be made using $B^0_d$ or $B^0_s$ mesons.
CPT invariance is one of the fundamental principles of quantum field theory: all physical processes are expected to respect this symmetry. Indeed, CPT violation would have a profound impact on physics in general, as it would also lead to a violation of Lorentz symmetry \cite{1,2}. Given its importance to the theoretical framework underlying all of particle physics, much attention has been devoted to experimentally testing the validity of CPT invariance.

One of the consequences of CPT invariance in quantum field theory is that a particle and its antiparticle should have the same mass and lifetime. However, these quantities are mostly dominated by the strong or electromagnetic interactions. Given that CPT violation, if nonzero, is certainly a very small effect, it is very difficult to test it by measuring the differences of masses or lifetimes. A more promising area for testing CPT violation is in $P^0 - \bar{P}^0$ mixing, where $P^0$ is a neutral pseudoscalar meson (e.g., $K^0$, $D^0$, $B^0_d$, $B^0_s$) \cite{3}. Since this mixing is a second-order electroweak process, small CPT-violating effects may be easier to detect.

Now, it is known that, in addition to incorporating CPT violation, the most general $B^0 - \bar{B}^0$ mixing matrix also involves $T$ and $CP$ violation. As a consequence, studying CPT violation is impossible without discriminating it from the effects of $CP$ and $T$ violation. That is, the effects of $CP$, $T$ and CPT violation must be considered together.

A first step was taken in Refs. \cite{4,5}, with followup papers in Refs. \cite{6-11}. The proposed method uses entangled $B^0 - \bar{B}^0$ states produced in the decay of the $\Upsilon(4S)$, with one meson decaying to a $CP$ eigenstate ($J/\psi K_S$ or $J/\psi K_L$) and the other used to tag the flavour. Using this technique, true $T$- and CPT-violating asymmetries can be measured. The BaBar Collaboration implemented this strategy \cite{12,13}, culminating in the measurement of $T$ violation \cite{14}.

At present, all experimental results are consistent with CPT conservation. On the other hand, an important improvement in statistics is expected at LHCb and Belle II, so that it will be possible to measure the $CP$, $T$- and CPT-violating parameters with greater precision. However, the method using entangled states produced in the decay of the $\Upsilon(4S)$ cannot be used at LHCb. An alternate approach is needed.

In this Letter, we re-examine the possibilities for measuring $T$ and CPT violation in $B^0 - \bar{B}^0$ mixing using the decays of $B^0$ or $\bar{B}^0$ to a $CP$ eigenstate. As we will show, the time-dependent indirect $CP$ asymmetry contains sufficient information to measure the conventional $CP$-violating effects and extract the $T$- and CPT-violating parameters \cite{15,16}. Since no true $T$- and CPT-violating asymmetries are measured, this is an indirect determination of the $T$- and CPT-violating parameters. In this sense, this method is complementary to that using entangled states.

We focus on $B^0_d - \bar{B}^0_d$ mixing but the same approach can be modified and applied to the $B^0_s$ system. Note that we restrict the analysis to $T$ and CPT violation arising from the $B^0 - \bar{B}^0$ mixing matrix alone. If there are new-physics contributions to $B$ decays, we assume they are CPT-conserving.
We begin by reviewing the most general formalism for $B^0-\bar{B}^0$ mixing, in which $CPT$ and $T$ violation are incorporated. The $2 \times 2$ hermitian matrices $M$ and $\Gamma$, respectively the mass and decay matrices, are defined in the $(B^0, \bar{B}^0)$ basis. When $M - (i/2)\Gamma$ is diagonalized, its eigenstates are the physical light ($L$) and heavy ($H$) states $B^0_L$ and $B^0_H$. Now, any $2 \times 2$ matrix can be expanded in terms of the three Pauli matrices $\sigma_i$ and the unit matrix with complex coefficients:

$$M - \frac{i}{2} \Gamma = E_1 \sigma_1 + E_2 \sigma_2 + E_3 \sigma_3 - iD1 \ .$$

Comparing both sides of this equation, we obtain

$$E_1 = \text{Re} M_{12} - \frac{i}{2} \text{Re} \Gamma_{12} ,$$

$$E_2 = -\text{Im} M_{12} + \frac{i}{2} \text{Im} \Gamma_{12} ,$$

$$E_3 = \frac{1}{2} (M_{11} - M_{22}) - \frac{i}{4} (\Gamma_{11} - \Gamma_{22}) ,$$

$$D = \frac{i}{2} (M_{11} + M_{22}) + \frac{1}{4} (\Gamma_{11} + \Gamma_{22}) .$$

We can define complex numbers $E$, $\theta$ and $\phi$ as follows:

$$E = \sqrt{E_1^2 + E_2^2 + E_3^2} ,$$

$$E_1 = E \sin \theta \cos \phi ,$$

$$E_2 = E \sin \theta \sin \phi ,$$

$$E_3 = E \cos \theta .$$

The eigenvalues of $M - (i/2)\Gamma$ are $E - iD$ and $-E - iD$, with eigenstates

$$|B_L\rangle = p_1 |B^0\rangle + q_1 |\bar{B}^0\rangle ,$$

$$|B_H\rangle = p_2 |B^0\rangle - q_2 |\bar{B}^0\rangle ,$$

where $p_1 = \cos \frac{\theta}{2}$, $q_1 = e^{i\phi} \sin \frac{\theta}{2}$, $p_2 = \sin \frac{\theta}{2}$ and $q_2 = e^{i\phi} \cos \frac{\theta}{2}$.

In Ref. [17] (pgs. 349-358), T.D. Lee discusses the $CPT$ and $T$ properties of $M$ and $\Gamma$. First, if $CPT$ invariance holds, then, independently of $T$ symmetry,

$$M_{11} = M_{22} , \quad \Gamma_{11} = \Gamma_{22} \implies E_3 = 0 \implies \theta = \frac{\pi}{2} .$$

In addition, if $T$ invariance holds, then, independently of $CPT$ symmetry,

$$\frac{\Gamma^*_{12}}{\Gamma_{12}} = \frac{M^*_{12}}{M_{12}} \implies \text{Im}(E_1 E_2^*) = 0 \implies \text{Im} \phi = 0 .$$
From Eqs. (2) and (3), we have
\[ e^{i\phi} = \pm \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}. \] (7)

Thus, if \( T \) is a good symmetry, the left-hand quantity is a pure phase, and the modulus of the square root is one. Note that it is usually said that the absence of \( CP \) violation implies \( |e^{i\phi}| = 1 \). However, strictly speaking, this is due to the absence of \( T \) violation. The two reasons are equivalent only if \( CPT \) is conserved.

Second, defining \( \zeta \equiv \langle S|L \rangle = \langle B_L|B_H \rangle \),
\[ \zeta \text{ is } \begin{cases} \text{real} & \text{if } CPT \text{ holds,} \\ \text{imaginary} & \text{if } T \text{ holds.} \end{cases} \] (8)

Using Eq. (4), we have
\[ \langle B_L|B_H \rangle = \cos \frac{\theta}{2} \sin \frac{\theta^*}{2} - e^{i\phi} \sin \frac{\theta}{2} e^{-i\phi^*} \cos \frac{\theta^*}{2} \\
= \frac{1}{2} \left[ (1 - e^{-2i\text{Im}(\phi)}) \sin \{\text{Re}(\theta)\} - i \{1 + e^{-2i\text{Im}(\phi)}\} \sinh \{\text{Im}(\theta)\} \right]. \] (9)

Now, if \( CPT \) is a good symmetry, then \( \theta = \frac{\pi}{2} \) [Eq. (5)], so that \( \sinh \{\text{Im}(\theta)\} = 0 \) and \( \langle B_L|B_H \rangle \) is real. And if \( T \) is a good symmetry, then \( \text{Im}(\phi) = 0 \) [Eq. (6)], so that \( 1 - e^{-2i\text{Im}(\phi)} = 0 \) and \( \langle B_L|B_H \rangle \) is imaginary. With this, we see that Eq. (8) is verified. (Obviously, if both \( CPT \) and \( T \) are good symmetries, \( \langle B_L|B_H \rangle = 0 \), i.e., the states are orthogonal.)

It will be useful to define the complex \( \theta \) and \( \phi \) in terms of real parameters as \( \theta = \theta_1 + i\theta_2 \) and \( \phi = \phi_1 + i\phi_2 \). In the absence of both \( T \) and \( CPT \) violation in \( B^0-\bar{B}^0 \) mixing, the parameters take the following values:
\[ \theta_1 = \frac{\pi}{2}, \quad \theta_2 = 0, \quad \phi_1 = -2\beta_{\text{mix}}, \quad \phi_2 = 0, \] (10)

where \( \beta_{\text{mix}} \) is the weak phase describing \( B^0-\bar{B}^0 \) mixing. In the standard model, \( \beta_{\text{mix}} = \beta \) for the \( B_d^0 \) meson. Now, if \( T \) and \( CPT \) violation are present in the mixing, the parameters \( \theta_1, \theta_2 \) and \( \phi_2 \) will deviate from these values. We define \( \epsilon_{1,2,3} \) via
\[ \theta_1 \rightarrow \frac{\pi}{2} + \epsilon_1, \quad \theta_2 \rightarrow \epsilon_2, \quad \phi_2 \rightarrow \epsilon_3. \] (11)

\( \epsilon_1 \) and \( \epsilon_2 \) are \( CPT \)-violating parameters, whereas \( \epsilon_3 \) indicates \( T \) violation.

The values for \( \epsilon_1, \epsilon_2 \) and \( \epsilon_3 \) have been reported by the BaBar and Belle Collaborations \cite{18, 19}. Their notation is related to ours as follows:
\[ \cos \theta \leftrightarrow -z, \quad \sin \theta \leftrightarrow \sqrt{1 - z^2}, \quad e^{i\phi} \leftrightarrow \frac{q}{p}, \] (12)
so that
\[
\epsilon_1 = \text{Re}(z), \quad \epsilon_2 = \text{Im}(z), \quad \epsilon_3 = 1 - \left|\frac{q}{p}\right|.
\] (13)

\(\epsilon_1\) and \(\epsilon_2\) are expected to be very small, as they are \(CPT\)-violating parameters. As for \(\epsilon_3\), note that \(|q/p|\) has been measured at the \(\Upsilon(4S)\) using the same-sign dilepton asymmetry, assuming \(CPT\) conservation [20]:

\[
\left|\frac{q}{p}\right| = 1.0010 \pm 0.0008 \implies \epsilon_3 = -(1.0 \pm 0.8) \times 10^{-3}.
\] (14)

Thus, \(\epsilon_3\) is also very small.

Above, we have called \(\epsilon_1\) and \(\epsilon_2\) the \(CPT\)-violating parameters. But one must be careful about such names. \(\epsilon_1\) and \(\epsilon_2\) do not contribute only to observables measuring \(CPT\) violation. They also lead to \(CP\)- and \(T\)-violating effects. Similarly, the \(T\)-violating parameter \(\epsilon_3\) also contributes to \(CP\)-violating observables. And the reverse is true: recall that, in Ref. [14], the BaBar Collaboration measured a large true \(T\)-violating asymmetry. This does not suggest that \(\epsilon_3\) is large, as there are also large contributions to the asymmetry coming from \(CP\)-violating effects (assuming \(CPT\) invariance). The point is that \(\epsilon_1, \epsilon_2\) and \(\epsilon_3\) are also sources of \(CP\) violation, and it is this fact that allows their measurement in the time-dependent indirect \(CP\) asymmetry, as we will see below.

In the presence of \(T\) and \(CPT\) violation in \(B^0-\bar{B}^0\) mixing, the time evolution of the flavor eigenstates \((|B^0\rangle \equiv |B^0(t=0)\) and \(|\bar{B}^0\rangle \equiv |B^0(t=0))\) is given by

\[
|B^0(t)\rangle = (g_+ + g_- \cos \theta)|B^0\rangle + e^{i\phi} g_- \sin \theta |\bar{B}^0\rangle,
\]

\[
|\bar{B}^0(t)\rangle = e^{-i\phi} g_- \sin \theta |B^0\rangle + (g_+ - g_- \cos \theta)|\bar{B}^0\rangle,
\] (15)

where

\[
g_+ = e^{-i(M - i\frac{\Gamma}{2})t} \cos \left(\left(\Delta M - i\frac{\Delta \Gamma}{2}\right) t\right),
\]

\[
g_- = e^{-i(M - i\frac{\Gamma}{2})t} \sin \left(\left(\Delta M - i\frac{\Delta \Gamma}{2}\right) t\right).
\] (16)

Here \(M \equiv (M_H + M_L)/2, \Delta M \equiv M_H - M_L, \Gamma \equiv (\Gamma_H + \Gamma_L)/2\) and \(\Delta \Gamma \equiv \Gamma_H - \Gamma_L\).

We consider a final state \(f\) to which both \(B^0\) and \(\bar{B}^0\) can decay. Using Eq. (15), the time-dependent decay amplitudes for uncorrelated or tagged neutral mesons are given by

\[
\mathcal{A}(B^0(t) \to f) = (g_+ + g_- \cos \theta) A_f + e^{i\phi} g_- \sin \theta \bar{A}_f,
\]

\[
\mathcal{A}(\bar{B}^0(t) \to f) = e^{-i\phi} g_- \sin \theta A_f + (g_+ - g_- \cos \theta) \bar{A}_f,
\] (17)
where $A_f \equiv \langle f | \mathcal{H}_{\Delta F = 1} | B^0 \rangle$ and $\bar{A}_f \equiv \langle f | \mathcal{H}_{\Delta F = 1} | \bar{B}^0 \rangle$. The differential decay rates $\Gamma_f(\bar{B}^0(t) \to f)$ and $\Gamma_f(B^0(t) \to f)$ are given by

$$
\frac{d\Gamma}{dt}(\bar{B}^0(t) \to f) = \frac{1}{2} e^{-\Gamma t} \sinh(\Delta\Gamma t/2) \left\{ 2\text{Re} \cos\theta |A|_t^2 + e^{i\phi} \sin \theta A_t^* \bar{A}_t \right\} + \cosh(\Delta\Gamma t/2) \left\{ |A_f|^2 + |\cos\theta| |A_f|^2 + |e^{i\phi} \sin \theta|^2 |\bar{A}_f|^2 + 2\text{Re} (e^{i\phi} \cos \theta^* \sin \theta A_f^* \bar{A}_f) \right\} + \cos(\Delta M t) \left\{ |A_f|^2 - |\cos\theta| |A_f|^2 - |e^{i\phi} \sin \theta|^2 |\bar{A}_f|^2 - 2\text{Re} (e^{i\phi} \cos \theta^* \sin \theta A_f^* \bar{A}_f) \right\} - \sin(\Delta M t) \left\{ 2\text{Im} (\cos \theta |A|_t^2 + e^{i\phi} \sin \theta A_t^* \bar{A}_t) \right\} , \tag{18}
$$

$$
\frac{d\Gamma}{dt}(B^0(t) \to f) = \frac{1}{2} e^{-\Gamma t} \sinh(\Delta\Gamma t/2) \left\{ 2\text{Re} (-\cos\theta^* |\bar{A}|_t^2 + e^{i\phi^*} \sin \theta^* A_t^* \bar{A}_t) \right\} + \cosh(\Delta\Gamma t/2) \left\{ |\bar{A}_f|^2 + |\cos\theta|^2 |\bar{A}_f|^2 + |e^{-i\phi} \sin \theta|^2 |A_f|^2 - 2\text{Re} (e^{i\phi^*} \cos \theta^* \sin \theta^* A_f^* \bar{A}_f) \right\} + \cos(\Delta M t) \left\{ |\bar{A}_f|^2 - |\cos\theta|^2 |\bar{A}_f|^2 - |e^{-i\phi} \sin \theta|^2 |A_f|^2 + 2\text{Re} (e^{i\phi^*} \cos \theta \sin \theta^* A_f^* \bar{A}_f) \right\} + \sin(\Delta M t) \left\{ 2\text{Im} (-\cos \theta^* |\bar{A}|_t^2 + e^{i\phi^*} \sin \theta^* A_t^* \bar{A}_t) \right\} . \tag{19}
$$

If we set $\theta = \frac{\pi}{2}$ and $\text{Im} \phi = 0$ in the above expressions, we recover expressions for the differential decay rates that are commonly found elsewhere in the literature.

We now write $\theta = \theta_1 + i\theta_2$ and $\phi = \phi_1 + i\phi_2$, with $\text{[see Eqs. (10) and (11)]}$

$$
\theta_1 \to \frac{\pi}{2} + \epsilon_1 , \quad \theta_2 \to \epsilon_2 , \quad \phi_1 = -2\beta^{\text{mix}} , \quad \phi_2 \to \epsilon_3 , \tag{20}
$$

where the $\epsilon_{1,2,3}$ are very small. In order to probe $T$ and $CPT$ violation in $B^0-\bar{B}^0$ mixing, one must measure the parameters $\epsilon_{1,2,3}$. Below, we illustrate how this can be done in the $B^0_d$ system. For $B^0_s$ mesons, the procedure is similar, though a bit more complicated.

First, as regards $\Gamma_d$, the value of $y_d = \Delta\Gamma_d/2\Gamma_d$ has been measured to be small: $y_d = -0.003 \pm 0.015$ with the $B^0_d$ lifetime of $1.520 \pm 0.004$ ps $^2$. This means that we can approximate $\sinh(\Delta\Gamma t/2) \simeq \Delta\Gamma t/2 = y_d \Gamma_d t$ and $\cosh(\Delta\Gamma t/2) \simeq 1$. In principle, for large enough times, this approximation will break down. However, even at time scales of $O(10)$ ps, the approximation holds to $\sim 10^{-4}$, and by this time most of the $B^0_d$s will have decayed.

The observable we will use to extract the $T$- and $CPT$-violating parameters $\epsilon_{1,2,3}$ is the time-dependent indirect $CP$ asymmetry $A^f_{CP}(t)$ involving $B$-meson decays to a $CP$ eigenstate. It is defined as

$$
A^f_{CP}(t) = \frac{d\Gamma/dt(\bar{B}^0(t) \to f_{CP}) - d\Gamma/dt(B^0(t) \to f_{CP})}{d\Gamma/dt(\bar{B}^0(t) \to f_{CP}) + d\Gamma/dt(B^0(t) \to f_{CP})} . \tag{21}
$$

In the limit of $CPT$ conservation, $T$ conservation in the mixing, and $\Delta\Gamma = 0$, one has the familiar expression

$$
A^f_{CP}(t) = S \sin(\Delta M_d t) - C \cos(\Delta M_d t) , \tag{22}
$$

---

$^5$In Ref. [11] it was pointed out that the coefficient of $\cos(\Delta M t)$ includes a $CPT$-violating piece.
where

\[ \varphi \equiv \phi_1 - \arg[A_f] + \arg[\bar{A}_f] , \quad (23) \]

\[ C \equiv |A_f|^2 - |\bar{A}_f|^2 \bigg/ |A_f|^2 + |\bar{A}_f|^2 , \quad (24) \]

\[ S \equiv \sqrt{1 - C^2} \sin \varphi . \quad (25) \]

Here, \( C \) is the direct \( CP \) asymmetry and \( \varphi \) is the measured weak phase, which differs from the mixing phase \( \phi_1 \) if \( \arg[A_f] \neq \arg[\bar{A}_f] \). If there is no penguin pollution, then \( \varphi \) cleanly measures a weak phase and \( C = 0 \). But if there is penguin pollution, then neither of these holds.

In the presence of \( T \) and \( CPT \) violation in the mixing, we use Eqs. (18) and (19) to obtain the time-dependent \( CP \) asymmetry. We first expand the various functions in the two equations, keeping only terms at most linear in the small quantities \( \epsilon_1, \epsilon_2, \epsilon_3 \) and \( \Delta \Gamma_d \):

\[ \frac{d\Gamma}{dt}(B^0(t) \rightarrow f) - \frac{d\Gamma}{dt}(\bar{B}^0(t) \rightarrow f) = e^{-\Gamma_d t}(|A_f|^2 + |\bar{A}_f|^2) \left[ \epsilon_3 + \sqrt{1 - C^2} \epsilon_1 \cos \varphi \right. \]

\[ + \cos(\Delta M_d t) \left\{ -C - \epsilon_3 - \sqrt{1 - C^2} \epsilon_1 \cos \varphi \right\} + \sin(\Delta M_d t) \left\{ -\epsilon_2 + \sqrt{1 - C^2} \sin \varphi \right\} \bigg] , \quad (26) \]

\[ \frac{d\Gamma}{dt}(B^0(t) \rightarrow f) + \frac{d\Gamma}{dt}(\bar{B}^0(t) \rightarrow f) = e^{-\Gamma_d t}(|A_f|^2 + |\bar{A}_f|^2) \]

\[ \left[ 1 + C\epsilon_3 + \frac{1}{2} \sqrt{1 - C^2} \Delta \Gamma_d t \cos \varphi - \sqrt{1 - C^2} \epsilon_2 \sin \varphi \right. \]

\[ + \cos(\Delta M_d t) \left\{ -C\epsilon_3 + \sqrt{1 - C^2} \epsilon_2 \sin \varphi \right\} + \sin(\Delta M_d t) \left\{ C\epsilon_2 + \sqrt{1 - C^2} \epsilon_3 \sin \varphi \right\} \bigg] . \quad (27) \]

The denominator [Eq. (27)] has the form \( A(1 + x) \), with \( x \) small, so we can approximate \( 1/A(1 + x) \simeq (1 - x)/A \). Combining all the pieces, and again keeping only terms at most linear in \( \epsilon_{1,2,3} \) and \( y_d \), we obtain\(^6\)

\[ A_{CP/CPT}^f(t) \simeq c_0 + c_1 \cos(\Delta M_d t) + c_2 \cos(2\Delta M_d t) + s_1 \sin(\Delta M_d t) + s_2 \sin(2\Delta M_d t) \]

\[ + c_1' \Gamma_d t \cos(\Delta M_d t) + s_1' \Gamma_d t \sin(\Delta M_d t) , \quad (28) \]

\(^6\)A time-dependent \( CP \) asymmetry having a complicated form with higher harmonics in \( \Delta M_d t \), similar to that in Eq. (28), was noted in Ref. \(^9\).
where the coefficients are given by

\[
\begin{align*}
c_0 &= \epsilon_1 \cos \varphi + \epsilon_3 - \frac{1}{2} \epsilon_3 \sin^2 \varphi , \\
c_1 &= -C - \epsilon_3 - \epsilon_1 \cos \varphi - \epsilon_2 \sin \varphi , \\
c_2 &= \frac{1}{2} \epsilon_3 \sin^2 \varphi + \epsilon_2 \sin \varphi , \\
s_1 &= \sqrt{1 - C^2} \sin \varphi - \epsilon_2 \cos^2 \varphi - \epsilon_3 \sin \varphi , \\
s_2 &= -\frac{1}{2} \epsilon_2 \sin^2 \varphi + \epsilon_3 C \sin \varphi , \\
c'_1 &= C y_d \cos \varphi , \\
s'_1 &= -\frac{1}{2} y_d \sin 2\varphi .
\end{align*}
\]

(29)

The seven pieces have different time dependences so that, by fitting \( A^f_{\text{CP}/\text{CPT}}(t) \) to the seven time-dependent functions, all coefficients can be extracted.

The five observables \( c_0, c_1, c_2, s_1 \) and \( s_2 \) can be used to solve for the five unknown parameters \( C, \varphi \) and \( \epsilon_{1,2,3} \). In practice, a fit will probably be used, but there is an analytical solution. The parameter \( C \) is simply given by

\[
C = -(c_0 + c_1 + c_2) .
\]

(30)

The solution for \( \sin \varphi \) is obtained by solving the following quartic equation:

\[
\sin^4 \varphi - 2 \left[ \frac{s_1 + 2s_2}{2 - C^2} \right] \sin^3 \varphi + 4C \left[ C + \frac{c_2}{2 - C^2} \right] \sin^2 \varphi \\
- 4 \left[ \frac{2C^2(s_1 + s_2) - s_2}{2 - C^2} \right] \sin \varphi - \left[ \frac{8Cc_2}{2 - C^2} \right] = 0 .
\]

(31)

Of course, there are four solutions, but, since the \( \epsilon_i \) are small, the correct solution is the one that is roughly \( s_1/\sqrt{1 - C^2} \). Finally, \( \epsilon_1, \epsilon_2, \epsilon_3 \) are given by

\[
\begin{align*}
\epsilon_1 &= c_0 \sec \varphi - \frac{(2 - \sin^2 \varphi)(c_2 \sin \varphi + 2C s_2)}{(4C^2 + \sin^2 \varphi) \sin \varphi \cos \varphi} , \\
\epsilon_2 &= \frac{2(2C c_2 - s_2 \sin \varphi)}{(4C^2 + \sin^2 \varphi) \sin \varphi} , \\
\epsilon_3 &= \frac{2(c_2 \sin \varphi + 2C s_2)}{(4C^2 + \sin^2 \varphi) \sin \varphi} .
\end{align*}
\]

(32)

This shows that it is possible to measure the parameters describing \( T \) and \( \text{CPT} \) violation in \( B^0_d-\bar{B}^0_d \) mixing using the time-dependent indirect \( \text{CP} \) asymmetry, and this can be carried out at LHCb.
The parameters $c'_1$ and $s'_1$ depend only on $\varphi$ and $y_d$. Thus, given knowledge of $\varphi$, the value of $y_d$ can be found from measurements of these parameters. Note that, even if the width difference $\Delta \Gamma_d$ between the two $B$-meson eigenstates vanishes, the $T$-violating parameter $\varepsilon_3$ can still be extracted. This is contrary to the claim of Refs. [6] and [9].

Above, the method was described for the $B^0_d$ system. In the case of $B^0_s$ mesons, $\Delta \Gamma_s$ is not that small, so the functions $\sinh (\Delta \Gamma_s t/2)$ and $\cosh (\Delta \Gamma_s t/2)$ must be kept throughout. This modifies the forms of Eqs. (26), (27) and (28), but the idea does not change. $A_{CP/\text{CPT}}(t)$ still depends on seven different time-dependent functions, a fit can be performed to extract their coefficients, and $C$, $\varphi$, $\varepsilon_{1,2,3}$ and $\Delta \Gamma_S$ can be found using the measurements of these coefficients.

Finally, we have another handle for probing CPT violation in $B^0_d$-$\bar{B}^0_d$ mixing. At present, we know that $\varepsilon_3 = -1.0 \pm 0.8 \times 10^{-3}$ [Eq. (14)]. Now, suppose that there is no CPT violation (i.e., $\varepsilon_1 = \varepsilon_2 = 0$). In this case, for the time-dependent CP asymmetry [Eq. (28)], we can eliminate $C$ and $\sin \varphi$ using Eqs. (30)-(32). The coefficients $c_0$, $c_2$ and $s_2$ can then be expressed in terms of the measured quantities $c_1$, $s_1$ and $\varepsilon_3$ as follows:

$$c_0 = \varepsilon_3 \left[ 1 - \frac{2s_1^2}{(2 - c_1^2 + \varepsilon_3^2)^2} \right],$$
$$c_2 = \frac{2s_1^2 \varepsilon_3}{(2 - c_1^2 + \varepsilon_3^2)^2},$$
$$s_2 = -\frac{2s_1^2 (c_1 + \varepsilon_3) \varepsilon_3}{(2 - c_1^2 + \varepsilon_3^2)}.$$

The values of $c_1$ and $s_1$ have been measured for several $B^0_d$ decays to CP eigenstates [20], and the value of $\varepsilon_3$ is independent of the decay mode. Using these values, we can estimate $c_0$, $c_2$ and $s_2$ from Eq. (33), which assumes that CPT is conserved. As an example, for the final state $J/\psi K_S$, we find

$$c_0 = (-15.18 \pm 15.50) \times 10^{-4},$$
$$c_2 = (-4.31 \pm 4.41) \times 10^{-4},$$
$$s_2 = (0.29 \pm 0.43) \times 10^{-4}.$$

Should the measurements of $c_0$, $c_2$ and $s_2$ deviate significantly from the above values, this would indicate the presence of CPT violation in $B^0_d$-$\bar{B}^0_d$ mixing.

To sum up, we have shown that the time-dependent, indirect CP asymmetries involving $B^0, \bar{B}^0 \rightarrow f_{CP}$ contain enough information to extract not only the CP-violating weak phases, but also the parameters describing $T$ and CPT violation in $B^0$-$\bar{B}^0$ mixing. These measurements can be made at the $\Upsilon(4S)$ (e.g., BaBar, Belle) or at high energies (e.g., LHCb). There is no need to neglect penguin pollution in the decay, and the method can be applied to $B^0_d$- or $B^0_s$-meson decays.
Acknowledgments: This work was financially supported in part by NSERC of Canada (DL).

References

[1] O. W. Greenberg, “CPT violation implies violation of Lorentz invariance,” Phys. Rev. Lett. 89, 231602 (2002) doi:10.1103/PhysRevLett.89.231602 [hep-ph/0201258].

[2] V. A. Kostelecky, “Gravity, Lorentz violation, and the standard model,” Phys. Rev. D 69, 105009 (2004) doi:10.1103/PhysRevD.69.105009 [hep-th/0312310].

[3] L. Lavoura and J. P. Silva, “Disentangling violations of CPT from other new physics effects,” Phys. Rev. D 60, 056003 (1999) doi:10.1103/PhysRevD.60.056003 [hep-ph/9902348].

[4] M. C. Bañuls and J. Bernabéu, “CP, T and CPT versus temporal asymmetries for entangled states of the $B_d$ system,” Phys. Lett. B 464, 117 (1999) doi:10.1016/S0370-2693(99)01043-6 [hep-ph/9908353].

[5] M. C. Bañuls and J. Bernabéu, “Studying indirect violation of CP, T and CPT in a B factory,” Nucl. Phys. B 590, 19 (2000) doi:10.1016/S0550-3213(00)00548-4 [hep-ph/0005323].

[6] E. Alvarez and J. Bernabéu, “Correlated neutral $B$ meson decays into CP eigenstates,” Phys. Lett. B 579, 79 (2004) doi:10.1016/j.physletb.2003.10.114 [hep-ph/0307093].

[7] E. Alvarez, J. Bernabéu, N. E. Mavromatos, M. Nebot and J. Papavasiliou, “CPT violation in entangled $B^0$-$\bar{B}^0$ states and the demise of flavor tagging,” Phys. Lett. B 607, 197 (2005) doi:10.1016/j.physletb.2004.12.032 [hep-ph/0410409].

[8] E. Alvarez and A. Szynkman, “Direct test of time reversal invariance violation in $B$ mesons,” Mod. Phys. Lett. A 23, 2085 (2008) doi:10.1142/S021773230802728X [hep-ph/0611370].

[9] J. Bernabéu, F. Martinez-Vidal and P. Villanueva-Perez, “Time Reversal Violation from the entangled $B^0$-$\bar{B}^0$ system,” JHEP 1208, 064 (2012) doi:10.1007/JHEP08(2012)064 [arXiv:1203.0171 [hep-ph]].

[10] E. Applebaum, A. Efrati, Y. Grossman, Y. Nir and Y. Soreq, “Subtleties in the $BABAR$ measurement of time-reversal violation,” Phys. Rev. D 89, no. 7, 076011 (2014) doi:10.1103/PhysRevD.89.076011 [arXiv:1312.4164 [hep-ph]].
[11] J. Bernabéu, F. J. Botella and M. Nebot, “Genuine $T$, $CP$, $CPT$ asymmetry parameters for the entangled $B_d$ system,” JHEP 1606, 100 (2016) doi:10.1007/JHEP06(2016)100 [arXiv:1605.03925 [hep-ph]].

[12] B. Aubert et al. [BaBar Collaboration], “Limits on the decay-rate difference of neutral $B$ mesons and on $CP$, $T$, and $CPT$ violation in $B^0\bar{B}^0$ oscillations,” Phys. Rev. Lett. 92, 181801 (2004) doi:10.1103/PhysRevLett.92.181801 [hep-ex/0311037].

[13] B. Aubert et al. [BaBar Collaboration], “Limits on the decay rate difference of neutral $B$ mesons and on $CP$, $T$, and $CPT$ violation in $B^0\bar{B}^0$ oscillations,” Phys. Rev. D 70, 012007 (2004) doi:10.1103/PhysRevD.70.012007 [hep-ex/0403002].

[14] J. P. Lees et al. [BaBar Collaboration], “Observation of Time Reversal Violation in the $B^0$ Meson System,” Phys. Rev. Lett. 109, 211801 (2012) doi:10.1103/PhysRevLett.109.211801 [arXiv:1207.5832 [hep-ex]].

[15] A method similar to ours, but focusing only on $CPT$ violation, was proposed in J. van Tilburg and M. van Veghel, “Status and prospects for $CPT$ and Lorentz invariance violation searches in neutral meson mixing,” Phys. Lett. B 742, 236 (2015) doi:10.1016/j.physletb.2015.01.036 [arXiv:1407.1269 [hep-ex]].

[16] A measurement of $CPT$ violation using the technique of Ref. [15] can be found in R. Aaij et al. [LHCb Collaboration], “Search for violations of Lorentz invariance and $CPT$ symmetry in $B^0(s)$ mixing,” Phys. Rev. Lett. 116, no. 24, 241601 (2016) doi:10.1103/PhysRevLett.116.241601 [arXiv:1603.04804 [hep-ex]].

[17] T. D. Lee, “Particle Physics and Introduction to Field Theory,” Contemp. Concepts Phys. 1, 1 (1981).

[18] T. Higuchi et al., “Search for Time-Dependent $CPT$ Violation in Hadronic and Semileptonic $B$ Decays,” Phys. Rev. D 85, 071105 (2012) doi:10.1103/PhysRevD.85.071105 [arXiv:1203.0930 [hep-ex]].

[19] J. P. Lees et al. [BaBar Collaboration], “Study of $CP$ Asymmetry in $B^0-\bar{B}^0$ Mixing with Inclusive Dilepton Events,” Phys. Rev. Lett. 114, no. 8, 081801 (2015) doi:10.1103/PhysRevLett.114.081801 [arXiv:1411.1842 [hep-ex]].

[20] Y. Amhis et al. [Heavy Flavor Averaging Group (HFAG) Collaboration], “Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton properties as of summer 2016,” [arXiv:1612.07233 [hep-ex], and online updates at http://www.slac.stanford.edu/xorg/hfag]

[21] C. Patrignani et al. [Particle Data Group], “Review of Particle Physics,” Chin. Phys. C 40, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001