Evaluations of some terminating hypergeometric $2F_1(2)$ series

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Abstract

Explicit expressions for the hypergeometric series $2F_1(-n, a; 2a \pm j; 2)$ and $2F_1(-n, a; -2n \pm j; 2)$ for positive integer $n$ and arbitrary integer $j$ are obtained with the help of generalizations of Kummer’s second and third summation theorems obtained earlier by Rakha and Rathie. Results for $|j| \leq 5$ derived previously using different methods are also obtained as special cases.

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1. Introduction

In a problem arising in a model of a biological problem, Samoletov [9] obtained by means of a mathematical induction argument the following sum containing factorials

$$
\sum_{k=0}^{n} \frac{(-1)^k (2k+1)!!}{(n-k)! k!(k+1)!} = \frac{(-1)^n}{\sqrt{n!(n+1)!}} \left( \sqrt{n+1} \frac{(n-1)!!}{n!!} \right)^{(-1)^n},
$$

where throughout $n$ denotes a positive integer and, as usual,

$$(2n)!! = 2 \cdot 4 \cdot 6 \cdots = 2^n n!, \quad (2n+1)!! = 1 \cdot 3 \cdot 5 \cdots = \frac{(2n+1)!}{2^n n!}.$$

Samoletov also expressed the above sum in the equivalent hypergeometric form

$$
2F_1 \left[ \left. \begin{array}{c} -n, \frac{3}{2} \\ 2 \end{array} \right| 2 \right] = \begin{cases} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{n+1}{2}\right)} & (n \text{ even}) \\ \frac{-\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{n+1}{2}\right)} & (n \text{ odd}) \end{cases}
$$
Subsequently, Srivastava [12] pointed out that this result could be easily derived from a known hypergeometric summation formula [7, Vol. 2, p. 493] for \( _2F_1(-n, a; 2a - 1; 2) \) with \( a = \frac{3}{4} \), which is a contiguous result to the well-known summation

\[
_2F_1 \left[ \frac{-n, a}{2a} : 2 \right] = \frac{2^{n \sqrt{-\pi}} \Gamma(1 - a)}{(2a)_{n} \Gamma(\frac{1}{2} n) \Gamma(1 - a - \frac{1}{2} n)} \quad (n = 0, 1, 2, \ldots).
\]

The aim in this note is to obtain explicit expressions for

\[
_2F_1 \left[ \frac{-n, a}{2a + j} : 2 \right] \quad \text{and} \quad _2F_1 \left[ \frac{-n, a}{-2a + j} : 2 \right]
\]

for arbitrary integer \( j \). We shall employ the following generalizations of Kummer’s second and third summation theorems given in [8] (we correct a misprint in Theorem 6 of this reference). These are respectively

\[
_2F_1 \left[ \frac{\alpha, \beta}{2(\alpha + \beta + j)} : 1 \right] = \frac{\sqrt{\pi} \Gamma(\frac{1}{2} \alpha + \frac{1}{2} \beta + \frac{1}{2} + \frac{1}{2} j)}{\Gamma(\frac{1}{2} \alpha + \frac{1}{2} \beta) \Gamma(\frac{1}{2} \beta + \frac{1}{2})} \frac{\Gamma(\frac{1}{2} \alpha - \frac{1}{2} \beta + \frac{1}{2} + \frac{1}{2} j)}{\Gamma(\frac{1}{2} \alpha - \frac{1}{2} \beta + \frac{1}{2} + \frac{1}{2} j)}
\]

\times \sum_{r=0}^{j} \frac{1}{(\alpha + \beta + j)_{r/2}} \quad \text{for } j \geq 0.
\]

and

\[
_2F_1 \left[ \frac{\alpha, 1 - \alpha + j}{\gamma} : 2 \right] = \frac{2^{\pm j} \Gamma(\frac{1}{2} \gamma + \frac{1}{2}) \Gamma(\frac{1}{2} \gamma - \frac{1}{2} \alpha + \frac{1}{2}) \Gamma(\frac{1}{2} \gamma - \frac{1}{2} \beta + \frac{1}{2})}{\Gamma(\frac{1}{2} \gamma + \frac{1}{2} \alpha) \Gamma(\frac{1}{2} \gamma + \frac{1}{2} \beta)}
\]

\times \sum_{r=0}^{j} \frac{1}{(\gamma - \alpha + j)_{r/2}} \quad \text{for } j \geq 0.
\]

for \( j = 0, 1, 2, \ldots \), where \( \epsilon_{j} = 0 \) (resp. \( j \)), \( \delta_{j} = j \) (resp. \( 0 \)) when the upper (resp. lower) signs are taken and \( (a)_{k} = \Gamma(a + k) / \Gamma(a) \) is the Pochhammer symbol defined for arbitrary index \( k \). When \( j = 0 \), the summations (1.2) and (1.3) reduce to the well-known second and third summation theorems due to Kummer [11, p. 243]

\[
_2F_1 \left[ \frac{\alpha, \beta}{2(\alpha + \beta + 1)} : 1 \right] = \frac{\sqrt{\pi} \Gamma(\frac{1}{2} \alpha + \frac{1}{2} \beta + \frac{1}{2})}{\Gamma(\frac{1}{2} \alpha + \frac{1}{2} \beta) \Gamma(\frac{1}{2} \beta + \frac{1}{2})}
\]

and

\[
_2F_1 \left[ \frac{\alpha, 1 - \alpha}{\gamma} : 2 \right] = \frac{\Gamma(\frac{1}{2} \gamma + \frac{1}{2}) \Gamma(\frac{1}{2} \gamma - \frac{1}{2} \alpha + \frac{1}{2})}{\Gamma(\frac{1}{2} \gamma + \frac{1}{2} \alpha) \Gamma(\frac{1}{2} \gamma - \frac{1}{2} \beta + \frac{1}{2})}
\]

In addition, we shall make use of the transformation [6, (15.8.6)]

\[
_2F_1 \left[ \frac{-n, \beta}{\gamma} : 2 \right] = \frac{(-2)^{n} \Gamma(\beta n)}{\Gamma(n) \Gamma(1 - \beta - n)} _{2}F_{1} \left[ \frac{-n, 1 - \gamma - n}{1 - \beta - n} : 2 \right]. \quad (1.4)
\]

Expressions for the series in (1.1) for arbitrary integer \( j \) have recently been obtained by Chu [2] using a different approach. This involved expressing the series for \( j \neq 0 \) as finite sums of \( _2F_1(2j) \) series in (1.1) with \( j = 0 \). The cases with \( |j| \leq 5 \) have also been given by Kim and Rathie [3] and Kim et al. [4]. An application of the first series in (1.1) for \( j = 0, 1, \ldots, 5 \) has been discussed in [5].

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\[1\] In [11, p. 243], this summation formula is referred to as Bailey’s theorem. However, it has been pointed out in [1] that this theorem was originally found by Kummer.
2. Statement of the results

Our principal results are stated in the following two theorems.

**Theorem 1** Let \( n \) be a positive integer, \( a \) be a complex parameter and define \( j_0 = \lfloor \frac{1}{2} j \rfloor \). Then we have

\[
\binom{-2n, a}{2a \pm j} = \frac{2^{2n}(\frac{1}{2} a)_n}{(2a \pm j)_n} \sum_{r=0}^{j_0} (-1)^r \binom{j}{2r} (-n)_r (a + \delta_j)_{n-r} \tag{2.1}
\]

and

\[
\binom{-2n-1, a}{2a + j} = \frac{2^{2n}(\frac{3}{2} a)_n}{(2a + j)_{2n+1}} \sum_{r=0}^{j_0} (-1)^r \binom{j}{2r + 1} (-n)_r (a + \delta_j)_{n-r} \tag{2.2}
\]

for \( j = 0, 1, 2, \ldots \), where \( \delta_j = j \) (resp. \( 0 \)) when the upper (resp. lower) signs are taken.

**Proof.** From the result (1.4) we have

\[
\binom{-n, a}{2a \pm j} = \frac{(-2)^n(a)_n}{(2a \pm j)_n} \binom{-n-1 - 2a \mp j - n, 1}{1 - a - n}. \tag{2.3}
\]

The hypergeometric function on the right-hand side can be summed by the generalized second Kummer summation theorem (1.2), where we put \( \alpha = 1 - 2a - j - n \) and \( \beta = -n \). After some straightforward algebra we obtain

\[
\binom{-n, a}{2a + j} = \frac{2^n \sqrt{\pi}}{(2a + j)_n (\frac{1}{2} a + \frac{3}{2})} \sum_{r=0}^{j_0} (-1)^r \binom{j}{r} \frac{\Gamma(-\frac{1}{2} n + \frac{3}{2} r)}{(1-a-j)_{(r-n)/2}}
\]

and

\[
\binom{-n, a}{2a - j} = \frac{2^n \sqrt{\pi}}{(2a - j)_n (\frac{1}{2} a + \frac{1}{2})} \sum_{r=0}^{j_0} \binom{j}{r} \frac{\Gamma(-\frac{1}{2} n + \frac{1}{2} r)}{(1-a)_{(r-n)/2}}.
\]

Changing \( n \) to \( 2n \) and \( 2n + 1 \) and using the properties of the gamma function and

\[(a)_{2n} = 2^{2n}(a)_n (a + \frac{1}{2})_n, \quad (a)_{2n+1} = 2^{2n}a (\frac{1}{2} a + \frac{1}{2})_n (\frac{3}{2} a + 1)_n,
\]

we then find the results stated in the theorem. \( \square \)

We remark that in (2.2) the upper limit of summation can be replaced by \( \lfloor \frac{1}{2} j \rfloor - 1 \) when \( j \) is even. Also, since \( (-n)_r \) vanishes when \( r > n \), it is possible to replace the upper summation limit in both (2.1) and (2.2) by \( n \) whenever \( n > \lfloor \frac{1}{2} j \rfloor \).

**Theorem 2** Let \( n \) be a positive integer and \( a \) be a complex parameter. Then we have

\[
\binom{-n, a}{-2n + j} = \frac{2^{2n-j}(n-j)!}{(2n-j)!} \sum_{r=0}^{j} \binom{j}{r} (\frac{1}{2} a + \frac{1}{2} - \frac{1}{2} r)_n \tag{2.4}
\]

provided \( j \) does not lie in the interval \([n+1, 2n]\) where the hypergeometric function on the left-hand side of (2.3) is, in general, not defined, and

\[
\binom{-n, a}{-2n - j} = \frac{2^{2n+j} j!}{(2n+j)!} \sum_{r=0}^{j} (-1)^r \binom{j}{r} (\frac{1}{2} a + \frac{1}{2} - \frac{1}{2} r)_{n+j} \tag{2.4}
\]

for \( j = 0, 1, 2, \ldots \). When \( j \geq 2n + 1 \) in (2.3), the ratio of factorials \((n-j)!/(2n-j)!\) can be replaced by \((-1)^n(j-2n-1)!/(j-n-1)!\).
Proof. From the result (1.4) we have
\[
\binom{n}{a} = \frac{(a+1)_n}{a^n} = \frac{n!}{(2n+1)_n}
\]
when the parameters are such that the hypergeometric functions make sense. The hypergeometric function on the right-hand side can be summed by the generalized third Kummer theorem (1.3), where we put \( \alpha = -n \) and \( \gamma = 1 - a - n \). Some straightforward algebra using the properties of the gamma function then yields the results (2.3) and (2.4) in the theorem.

The sums on the right-hand sides of (2.3) and (2.4) can be written in an alternative form involving just two Pochhammer symbols containing the index \( n \) by making use of the result
\[
(\alpha - r)_n = \frac{(\alpha)_n(1 - \alpha)_r}{(1 - \alpha - n)_r}
\]
for positive integers \( r \) and \( n \). Then we find, with \( j_0 = \lfloor \frac{1}{2}j \rfloor \),
\[
\binom{-n,a}{-2n+j} = \frac{2^{2n-j}(n-j)!}{(2n-j)!} \left\{ \left( \frac{1}{2}a + \frac{1}{2}j \right)_n \sum_{r=0}^{j_0} \binom{j}{2r} A_r(n,0) + \left( \frac{1}{2}a \right)_n \sum_{r=0}^{j_0} \binom{j}{2r+1} B_r(n,0) \right\}
\]
(2.5)
and
\[
\binom{-n,a}{-2n-j} = \frac{2^{2n+j}n!}{(2n+j)!} \left\{ \left( \frac{1}{2}a + \frac{1}{2}j \right)_{n+j} \sum_{r=0}^{j_0} \binom{j}{2r} A_r(n,j) - \left( \frac{1}{2}a \right)_n \sum_{r=0}^{j_0} \binom{j}{2r+1} B_r(n,j) \right\}
\]
(2.6)
where
\[
A_r(n,j) := \frac{(\frac{1}{2} - \frac{1}{2}a)_r}{(\frac{1}{2} - \frac{1}{2}a - n - j)_r}, \quad B_r(n,j) := \frac{(1 - \frac{1}{2}a)_r}{(1 - \frac{1}{2}a - n - j)_r}.
\]
Again, when \( j \) is even, the upper summation limit in the second sums in (2.5) and (2.6) can be replaced by \( j_0 - 1 \), if so desired.

3. Special cases

If we set \( 0 \leq j \leq 5 \) in (2.1) we obtain the following summations:
\[
\binom{-2n,a}{2a} = \frac{1}{2} \binom{-2n,a}{2a+1} = \binom{-2n,a}{2a+1} = \binom{2n+1}{a+2} = \binom{2n+1}{a+1} = \binom{2n+1}{a+1},
\]
(3.1)
\[
\binom{-2n,a}{2a+2} = \frac{1}{2} \binom{-2n,a}{2a+3} = \frac{1}{2} \binom{-2n,a}{2a+3} = \binom{2n+3}{a+4} = \binom{2n+3}{a+2} = \binom{2n+3}{a+2},
\]
(3.2)
\[
\binom{-2n,a}{2a+3} = \frac{1}{2} \binom{-2n,a}{2a+4} = \frac{1}{2} \binom{-2n,a}{2a+4} = \binom{2n+4}{a+5} = \binom{2n+4}{a+2} = \binom{2n+4}{a+2},
\]
(3.3)
\[
\binom{-2n,a}{2a+4} = \frac{1}{2} \binom{-2n,a}{2a+5} = \frac{1}{2} \binom{-2n,a}{2a+5} = \binom{2n+5}{a+6} = \binom{2n+5}{a+3} = \binom{2n+5}{a+3},
\]
(3.4)
\[ _2F_1 \left[ \frac{-2n, a}{2a + 5} : 2 \right] = \frac{(\frac{3}{2})_n}{(a + \frac{3}{2})_n} \left( 1 + \frac{12n}{a + 3} + \frac{16n(n - 1)}{(a + 3)(a + 4)} \right) \] (3.5)

and

\[ _2F_1 \left[ \frac{-2n, a}{2a - 1} : 2 \right] = \frac{(\frac{1}{2})_n}{(a - \frac{1}{2})_n}, \] (3.6)

\[ _2F_1 \left[ \frac{-2n, a}{2a - 2} : 2 \right] = \frac{(\frac{3}{2})_n}{(a - \frac{3}{2})_n} \left( 1 + \frac{2n}{a - 1} \right), \] (3.7)

\[ _2F_1 \left[ \frac{-2n, a}{2a - 3} : 2 \right] = \frac{(\frac{3}{2})_n}{(a - \frac{5}{2})_n} \left( 1 + \frac{4n}{a - 1} \right), \] (3.8)

\[ _2F_1 \left[ \frac{-2n, a}{2a - 4} : 2 \right] = \frac{(\frac{1}{2})_n}{(a - \frac{5}{2})_n} \left( 1 + \frac{8n}{a - 2} + \frac{8n(n - 1)}{(a - 1)(a - 2)} \right), \] (3.9)

\[ _2F_1 \left[ \frac{-2n, a}{2a - 5} : 2 \right] = \frac{(\frac{3}{2})_n}{(a - \frac{7}{2})_n} \left( 1 + \frac{12n}{a - 2} + \frac{16n(n - 1)}{(a - 1)(a - 2)} \right). \] (3.10)

Similarly, if we set \( 0 \leq j \leq 5 \) in (2.2) we obtain the following summations:

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a} : 2 \right] = 0, \] (3.11)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a + 1} : 2 \right] = \frac{(\frac{3}{2})_n}{(2a + 1)(a + \frac{1}{2})_n}, \] (3.12)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a + 2} : 2 \right] = \frac{2(\frac{3}{2})_n}{(2a + 2)(a + \frac{3}{2})_n}, \] (3.13)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a + 3} : 2 \right] = \frac{(\frac{3}{2})_n}{(2a + 3)(a + \frac{5}{2})_n} \left( 3 + \frac{4n}{a + 2} \right), \] (3.14)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a + 4} : 2 \right] = \frac{(\frac{3}{2})_n}{(2a + 4)(a + \frac{7}{2})_n} \left( 4 + \frac{8n}{a + 3} \right), \] (3.15)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a + 5} : 2 \right] = \frac{(\frac{3}{2})_n}{(2a + 5)(a + \frac{9}{2})_n} \left( 5 + \frac{20n}{a + 3} + \frac{16n(n - 1)}{(a + 3)(a + 4)} \right) \] (3.16)

and

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a - 1} : 2 \right] = -\frac{(\frac{3}{2})_n}{(2a - 1)(a + \frac{1}{2})_n}, \] (3.17)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a - 2} : 2 \right] = -\frac{2(\frac{3}{2})_n}{(2a - 2)(a - \frac{1}{2})_n}, \] (3.18)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a - 3} : 2 \right] = -\frac{(\frac{3}{2})_n}{(2a - 3)(a - \frac{3}{2})_n} \left( 3 + \frac{4n}{a - 1} \right), \] (3.19)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a - 4} : 2 \right] = -\frac{(\frac{3}{2})_n}{(2a - 4)(a - \frac{5}{2})_n} \left( 4 + \frac{8n}{a - 1} \right), \] (3.20)

\[ _2F_1 \left[ \frac{-2n - 1, a}{2a - 5} : 2 \right] = -\frac{(\frac{3}{2})_n}{(2a - 5)(a - \frac{7}{2})_n} \left( 5 + \frac{20n}{a - 2} + \frac{16n(n - 1)}{(a - 1)(a - 2)} \right). \] (3.21)
Finally, from (2.5) and (2.6) we obtain:

\[ 2F_1 \left[ \begin{array}{c} -n, a \\ -2n \end{array} ; 2 \right] = \frac{2^{2n} n! (\frac{1}{2} a + \frac{1}{2})}{(2n)!} (\frac{1}{2} a + \frac{1}{2})_n, \]  

\[ 2F_1 \left[ \begin{array}{c} -n, a \\ -2n + 1 \end{array} ; 2 \right] = \frac{2^{2n-1} (n-1)!}{(2n-1)!} \left\{ (\frac{1}{2} a + \frac{1}{2})_n + (\frac{1}{2} a)_n \right\}, \]  

\[ 2F_1 \left[ \begin{array}{c} -n, a \\ -2n - 1 \end{array} ; 2 \right] = \frac{2^{2n+1} n!}{(2n+1)!} \left( \frac{1}{2} a + \frac{1}{2} \right)_{n+1}, \]  

\[ 2F_1 \left[ \begin{array}{c} -n, a \\ -2n + 2 \end{array} ; 2 \right] = \frac{2^{2n-2} (n-2)!}{(2n-2)!} \left\{ \frac{1}{1-a-2n} \left( \frac{1}{2} a + \frac{1}{2} \right)_n + (\frac{1}{2} a)_n \right\}, \]  

\[ 2F_1 \left[ \begin{array}{c} -n, a \\ -2n - 2 \end{array} ; 2 \right] = \frac{2^{2n+3} n!}{(2n+2)!} \left\{ \left( \frac{1}{1-a-2n-j} \right) \left( \frac{1}{2} a + \frac{1}{2} \right)_{n+2} - (\frac{1}{2} a)_n \right\} \]  

and so on.

The above evaluations agree with those given in [2, 3], although presented in a different format; the results (3.1) and (3.6), together with (3.11), (3.12) and (3.17), are also recorded in [? 1] in another form.

4. An application of Theorem 1

Kummer’s second theorem applied to the confluent hypergeometric function \( 1F_1 \) is [10, p. 12]

\[ e^{-x^2/2} F_1 \left[ \begin{array}{c} a \\ 2a \end{array} ; x \right] = a F_1 \left[ \begin{array}{c} - \frac{x^2}{2} \\ a + \frac{1}{2} \end{array} ; \frac{x^2}{2} \right] = (\frac{1}{2} x)^{\frac{a}{2} - \frac{1}{2}} \Gamma(a + \frac{1}{2}) I_{a - \frac{1}{2}} (\frac{1}{2} x), \]

where \( I_v(z) \) denotes modified Bessel function of the first kind. We now show how the result in Theorem 1 can be used to derive a generalization of the above theorem for the functions

\[ e^{-x^2/2} F_1 \left[ \begin{array}{c} a \\ 2a \pm j \end{array} ; x \right] \]

for arbitrary integer \( j \).

We have upon series expansion

\[ e^{-x^2/2} F_1 \left[ \begin{array}{c} a \\ 2a \pm j \end{array} ; x \right] = \sum_{n=0}^{\infty} \frac{(-\frac{1}{2} x)^n}{n!} \sum_{m=0}^{\infty} \frac{(a)_m}{(2a \pm j)_m} \frac{x^m}{m!} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^n (a)_m x^{m+n}}{2^n (2a \pm j)_m m!}. \]

Making the change of summation index \( n \rightarrow n - m \) and using the fact that \( (n - m)! = (-1)^m m! / (-n)_m \), we find

\[ e^{-x^2/2} F_1 \left[ \begin{array}{c} a \\ 2a \pm j \end{array} ; x \right] = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{(-1)^n (a)_m (-n)_m x^n}{2^{n-m} (2a \pm j)_m m!}. \]

Separation of the above sum into even and odd \( n \), use of the evaluation of the \( 2F_1(2) \) series given in (2.1) and (2.2) followed by inversion of the order of summation then leads to the result

\[ e^{-x^2/2} F_1 \left[ \begin{array}{c} a \\ 2a \pm j \end{array} ; x \right] = \sum_{r=0}^{j} (-1)^r \left( \frac{j}{2} \right) \sum_{n=0}^{\infty} \frac{(-n)_r (a_+ \delta_j)_n x^{2n}}{2^{2n} (2a \pm j)_{2n} n!}. \]
\[
\sum_{r=0}^{j_0} (-1)^r \binom{j}{2r+1} \sum_{n=0}^{\infty} \frac{(-n)_r (a + \delta_j)_n - r x^{2n+1}}{2^{2n+1} (2a \pm j)_{2n+1} n!}, \tag{4.2}
\]

where \(j_0\) and \(\delta_j\) are defined in Theorem 1. When \(j = 0\) it is easily seen that (4.2) reduces to (4.1).

The result (4.2) is given in a different form in terms of modified Bessel functions in [7, Vol. 3, p. 579].

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