Motion Attitude Analysis Method of Bullets Based on Field Shadow Image

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Abstract. Aimed at obvious interference noise of shadow images of bullet shot in outside field, and larger extraction error of bullet attitude features of shadow images and other problems, intersection point location method based on axis line and edge is adopted to extract features of bullet tip, and the method based on ellipse fitting is adopted to extract features of bullet tail in the Paper, and then extracted feature information is used to calculate motion attitude of bullets. The method can be applied to shadow image analysis of bullets under field environment well with accurate feature extraction, good consistency and high analysis precision and other features.

1. Introduction

At present, measurement methods of motion attitude of bullets mainly include paper target method, built-in sensor measurement method, radar measurement method and high-speed imaging measurement method [1, 2]. Bullet image obtained by shadow imaging method based on high-speed imaging technology has high resolution ratio, small distortion and clear contour, which is applicable to motion attitude measurement of bullets with high accuracy [3, 4].

Indoor range with good shooting and light environment is mainly adopted to measure small caliber bullets for existing bullet attitude analysis method based on shadow images [5, 6, 7]. However, as for field test of large caliber bullets with interference factors such as flying dust, stray light and others, existing analysis method cannot be applicable to the environment well. Existing attitude analysis method of shadow images of bullets is mainly divided into two-point method and three-point method. Intersection point of axis line and contour (or target minimum rectangular frame) [8] of bullets in the image is taken as feature point for two-point method; bullet tip point, upper end point and lower end point of bullet tail in shadow image are taken as original feature points, upper end point and lower end point of bullet tail are taken as bullet tail points and finally bullet tip point and bullet tail point are taken as feature points of shadow image [9].

At the time of using two-point method to extract image feature points of bullet with large caliber, parallax error of orthographic camera in shadow graph station is neglected, which causes that the line between corresponding image feature point of two cameras and light source point is skew line of space, and there is large solution error. At the time of using three-point method to extract image feature points of bullet with large caliber, as small round corners exist in the end face of bullet tail, contour curvature
change is slow, features of upper and lower points in bullet tail are not obvious in field shadow image of bullets with large image noise, artificial pickup is usually required, which has large extraction error. Attitude analysis method of bullet motion based on field shadow image proposed in the Paper can be applicable to shadow image analysis of bullets under field environment and realize high accuracy measurement of motion attitude of large caliber bullets under severe field environment.

2. Method for extracting attitude feature of bullets based on field shadow image

2.1. Attitude feature selection of bullets image

Rotating stabilized bullet is composed by bullet head segment, cylindrical segment and bullet tail segment. As bullet tip of rotating stabilized bullet is sharp, the parallax error in the imaging process of orthogonal shadow measurement is smaller, intersection point between axis line and contour of bullet in the image is taken as feature point of bullet tip.

It can be known from imaging system structure that arc segment in end surface of bullet tail accounts for 700 pixl in shadow image approximately, and edge contour of bullets in shadow image is clear, therefore, bullet tail point can be determined by projection rules of end surface of bullet tail on reflecting screen. Pitch angle and yaw angle of bullets usually vary within the scope of 0-5°. According to projection rules, there are three projection conditions of bullet tail on reflecting screen: 1) when pitch angle and yaw of bullets are 0°, projection of end surface of bullet tail on both reflecting screens is line; 2) when one of the pitch angle or yaw of bullets is 0°, one of the projection of end surface of bullet tail on both reflecting screens is ellipse, the other is a line; 3) when pitch angle and yaw of bullets are the scope of 0-5°, projection of end surface of bullet tail on both reflecting screens is ellipse.

For the case where the projection of end surface of bullet tail is line, the central axis can be intersected with the bullet contour as the point of bullet tail; and for the case where the projection of end surface of bullet tail is ellipse, the Center point of the ellipse as the point of bullet tail.

2.2. Feature extraction of bullet tip

The key of feature extraction of bullet tip in shadow image lies in accurate extraction of axle line in image. There are two main methods of obtaining axis line of axisymmetric objects of large target, namely area-based method and edge-based method. According to outfield shadow image features, information of bullet area in shadow image is incomplete and noise of bright spot is serious, which has large influences on rotational inertia of bullet area in the image. Therefore, edge-based method is adopted to extract axis line in bullet image in the Paper. Projection of cylindrical segment and conical tail segment of large caliber bullet is straight line, and edge information of bullet in shadow image is intact, therefore, axis line of bullets can be calculated by fitting of straight lines of cylindrical segment and conical tail segment according to edge line.

According to the edge point fitting, the equations of the four lines (lines are shown in the Fig.1) on the shadow image are:

\[
\begin{align*}
    l_1 : & \quad A_1x + B_1y + C_1 = 0 \\
    l_2 : & \quad A_2x + B_2y + C_2 = 0 \\
    l_3 : & \quad A_3x + B_3y + C_3 = 0 \\
    l_4 : & \quad A_4x + B_4y + C_4 = 0
\end{align*}
\] (1)
Projection line of the cylindrical segment $l_1$ and $l_2$ are parallel, the equation of the shadow axis $l_{12}$ is determined by finding the center lines of the cylindrical portions $l_1$ and $l_2$:

$$l_{12} : A_{12}x + B_{12}y + C_{12} = 0$$

In equation (2): $A_{12} = A_1 + A_2$, $B_{12} = B_1 + B_2$, $C_{12} = C_1 + C_2$.

Lines $l_3$ and $l_4$ are the two busbars of the bullet conical tail over the cone rotation axis projected on the reflective screen, then the rotation axis projection $l_{34}$ equation is:

$$l_{34} : A_{34}x + B_{34}y + C_{34} = 0$$

According to the shadow axis $l_{34}$ of the bullet tail to the line $l_3$ and $l_4$ are equal distance, $A_{34}$, $B_{34}$ and $C_{34}$ can be determined:

$$\frac{|A_3x + B_3y + C_3|}{\sqrt{A_3^2 + B_3^2}} = \frac{|A_4x + B_4y + C_4|}{\sqrt{A_4^2 + B_4^2}}$$

Because of the bullet is an axisymmetric target, the shadow image center axis $l$ can be used as the cylindrical segment shadow axis and the bullet tail segment shadow centerline:

$$l : (A_{12} + A_{34})x + (B_{12} + B_{34})y + (C_{12} + C_{34}) = 0$$

2.3. Feature extraction of bullet tail
After boundary pixel of end face of bullets is obtained, elliptic equation is fitted according to pixel of arc segment. The center of elliptic curve is bullet tail point. As shadow image of large caliber bullets has long arc of end face of bullet rear, accurate edge positioning and other features, least square method can be used for ellipse fitting, which has simple algorithm and high accuracy.

The implicit equation of an elliptic curve can be expressed as:

$$f(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

In order to simplify representation and operation, express equation (6) as a point-multiplied form of two vectors:

$$f(\vec{d}, \vec{z}) = \vec{d} \cdot \vec{z} = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$
In equation (2), ellipse parameter vector: 
\[ \bar{d} = [A, B, C, D, E, F] \], \( \bar{z} = [x^2, xy, y^2, x, y, 1]^T \). The edges of the processed image are jagged and stepped, edge pixel position coordinates are integer values, it can lead to errors in the fitted input data, in order to ensure that the fit curve meets expectations, add a qualification condition during the fitting process: 
\[ 4AC - B^2 = 1 \text{ (equivalent to: } d^T L_0 d = 1) \), the square matrix \( L \) is:

\[
L = \begin{bmatrix}
0 & 0 & 2 & 0 & 0 & 0 \\
0 & -1 & 2 & 0 & 0 & 0 \\
2 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

Substituting the coordinate point \((x_i, y_i)\) of the end face of the bullet end into the elliptic equation, Because the ellipse to be fitted in the shadow image is not a perfect elliptic curve, \( f(\bar{d}, \bar{z}) \neq 0 \). In order to represent the fitting parameter error, Combine \( f(\bar{d}, \bar{z}) \) with the curve represented by the fitting parameter to establish the objective function:

\[
F(\bar{d}) = \sum_{i=1}^{n} f(\bar{d}, \bar{z}_i)^2 \]

Combined with the qualification conditions, the objective function solution is simplified to:

\[
\begin{cases}
F^T Fd = \lambda Ld \\
d^T L_0 d = 1
\end{cases}
\]

Where \( F \) is the matrix of all unknown numerical values of the pixel coordinates to be fitted, \( F = [\bar{z}_1, \bar{z}_2, \cdots, \bar{z}_n]^T \). When the eigenvalue is a positive real number, the fitting error is minimal, corresponding elliptic curve parameter vector is \( \bar{d} = [A, B, C, D, E, F] \), the geometric center of the ellipse is:

\[
\begin{align*}
x_c &= \frac{BE - 2CD}{4AC - B^2} \\
y_c &= \frac{BD - 2AE}{4AC - B^2}
\end{align*}
\]

3. Analysis of Experiment

3.1. Simulation experiment results and analysis

In order to verify extraction accuracy of bullet tail point based on ellipse fitting, a comparison is made between the method based on ellipse fitting and the method determining bullet tail point according to upper end point and lower end point of bullet tail for verification. Three-dimensional software is used to establish simulation model of bullets. Pitch angle of the model is set as 5°, attitude is changed for yaw angle with 15° as interval. 5 ideal images with yaw angle of bullet of 15°, 30° ..., 75° are obtained by parallel projection. At the same time, in order to simulate imaging process of measurement system, Gaussian noise and impulse noise are added in ideal images, therefore simulation image is obtained, it shows as Fig. 2.
Two methods are adopted to extract central point in the front end face and rear end face of cylinder respectively in simulation image. Included angle between central point line of the front and rear end face and image frame is taken as internal pitch angle of model surface to calculate the value. A comparison is made with true value of pitch angle in model surface to obtain included angle error of axis line in the model of image. Thus extraction error of center point of end face can be calculated and obtained further. Extraction condition of center point of model end face determined on the basis of ellipse fitting under different yaw angle is shown in Fig. 3.

![Simulation shadow images of bullet](image)

**Figure 2.** Simulation shadow images of bullet.

Pitch angle calculation result in the face obtained by two extraction methods and extraction accuracy of end face center are shown in Table 1.

![Processing result of simulation shadow images of bullet](image)

**Figure 3.** Processing result of simulation shadow images of bullet.

| Model of bullets attitude | Methods (°) | Error (°) | Error of extract central point(πexl) |
|--------------------------|-------------|-----------|-------------------------------------|
| Pitch angle | Yaw angle | ellipse fitting | Center point | ellipse fitting | Center point | ellipse fitting | Center point |
| 5 | 15 | 5.025 | 4.959 | 0.025 | -0.041 | 0.38 | 0.61 |
| 5 | 30 | 4.969 | 4.951 | -0.031 | -0.048 | 0.41 | 0.64 |
| 5 | 45 | 4.959 | 4.946 | 0.041 | -0.054 | 0.44 | 0.58 |
| 5 | 60 | 4.952 | 5.088 | -0.048 | 0.088 | 0.36 | 0.66 |
| 5 | 75 | 5.101 | 5.153 | 0.101 | 0.153 | 0.39 | 0.59 |

It can be seen from Table 1 that extraction error of central point of end face is distributed between 0.36 and 0.44 πexl for the method of determining central point of cylindrical face based on ellipse fitting; while extraction error of central point of end face is distributed between 0.58 and 0.66 πexl for the method of determining central point of target end face by upper and lower end point. It shows that, the method of determining central point of cylindrical face based on ellipse fitting is better, extraction accuracy of central point of bullets can reach 0.44 πexl.

3.2. Measurement experiment of motion attitude of bullets on field

3.2.1 Feature parameter extraction of bullets shadow image. In the field experiment, the original image of the projectile motion image captured by a shadow camera station and the processing extraction result are shown in Fig. 4. According to the cylindrical segment of bullet and the lines of bullet conical
tail can determine the central axis, the point of intersection of the central axis and the bullet contour is used as the feature parameter of bullet tip, based on the least square method to fit curve equation of the arc at the end face of bullets can be used to determine the feature parameter of bullet tail. The result of feature parameter extraction of bullets shadow image are shown in Table 2.

![Shadow image of bullet at the bottom](image1.png) ![Process result of Shadow image of bullet](image2.png)

(a)Shadow image of bullet at the bottom (b) Process result of Shadow image of bullet

![Shadow image of bullet at the top](image3.png) ![Process result of Shadow image of bullet](image4.png)

(c)Shadow image of bullet at the top (d) Process result of Shadow image of bullet

**Figure 4.** Feature point extraction of bullets based on field shadow image

| Unit: pixels | Feature parameter of bullet tip | Feature parameter of bullet tail |
|-------------|---------------------------------|---------------------------------|
| Shadow image of bullet at the bottom | (1933,1066) | (1994.86,5998.70) |
| Shadow image of bullet at the top | (2002,1189) | (1957.70,6300.00) |

**Table 2.** The result of feature parameter extraction of bullets shadow image

3.2.2 Stability analysis to solve the attitude of bullets. In order to verify stability of solving the motion attitude of bullets, repeatedly solve the motion attitude of a certain bullet at the camera stations No. 1 and No. 2, the calculation results are shown in Table 3.

| Camera stations No. 1 | Camera stations No. 2 |
|------------------------|------------------------|
| Pitch angle(°)         | Yaw angle(°)           |
| The 1st solution       | 3.019                  | -0.882                 |
| The 2nd solution       | 3.031                  | -0.871                 |
| The 3rd solution       | 3.010                  | -0.888                 |
| Standard deviation     | 0.011                  | 0.009                  |

From Table 3 we can know that standard deviation of pitch angle of bullet were measured at the camera stations No.1 is 0.011°, standard deviation of yaw angle is 0.009°; standard deviation of pitch angle of bullet were measured at the camera stations No.2 is 0.009°, standard deviation of yaw angle is 0.007°. This shows that standard deviation of motion attitude of bullets less than 0.011°(0.66°).

**Table 3.** The motion attitude of a certain bullet at the camera stations No. 1 and No. 2

4. Conclusion

This article is based on the shape of the bullet and the characteristics of the shadow image in the field, intersection point location method based on axis line and edge is adopted to extract features of bullet tip, and the method based on ellipse fitting is adopted to extract features of bullet tail. Then through a comparison is made between the method based on the method of determining central point of cylindrical face based on ellipse fitting and the method determining bullet tail point according to upper end point and lower end point of bullet tail for verification, the results show that the method of ellipse fitting is better, extraction accuracy of central point of bullets can reach 0.44 pixels. Finally, at the measurement experiment of motion attitude of bullets on field, standard deviation of motion attitude of bullets less than 0.011°, calculation stability of motion attitude is satisfactory.
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