The MSSM at the low $\tan\beta$ fixed point is meta-stable

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Abstract

We analyse the mass spectrum of the Constrained Minimal Supersymmetric Standard Model at the low $\tan\beta$ fixed point. We find that the model only satisfies experimental and dark matter bounds in regions where the vacuum is meta-stable – i.e. where it violates ‘unbounded from below’ (UFB) bounds. Adding a small amount of $R$-parity violation solves these problems but the absolute upper bound on the lowest higgs mass $m_{h^0} < 97$ GeV remains. We present the predicted sparticle mass spectrum as a function of the gluino mass $m_g$. 
Introduction

Fixed point behaviour (or rather ‘quasi-fixed’) is a striking feature in the Minimal Supersymmetric Standard Model (MSSM) \[1, 2, 3\]. Broadly speaking, it is a focusing of the parameters in the infra-red regime which occurs when the top quark Yukawa coupling, $h_t$, is large. Its existence has been examined in detail for $h_t$ itself in Refs.\[2, 3\] where it was found that $h_t(m_t) = 1.1$ independently of $h_t(M_{GUT})$ as long as it is big, say bigger than about 1.2, at the GUT scale. There are three parameters which always have quasi-fixed points (QFPs) regardless of the pattern of supersymmetry breaking, and which are strongly attracted towards them (although others formally have fixed points as well) \[3, 4\]:

\[
\begin{align*}
R & \equiv h_t^2/g_3^2 \\
A_t & \equiv A_{U33} \\
3M^2 & \equiv m_{U_{33}}^2 + m_{Q_{33}}^2 + m_{2}^2.
\end{align*}
\]

When $h_t$ is high at the GUT or Planck scale, these three parameters are completely determined at the weak scale;

\[
\begin{align*}
R_{QFP} &= 0.87 \\
A_{tQFP} &= -1.60M_{1/2} \\
(M^2)_{QFP} &= 1.83M_{1/2}.
\end{align*}
\]

They govern the running of the MSSM at low $\tan\beta$ \[4\] and indeed all of the soft supersymmetry breaking parameters may easily be solved (to one loop) in terms of them. In the appendix, we list the solutions for the running MSSM mass parameters in terms of the GUT scale values and these three parameters. Writing the solutions in this form is particularly useful for finding combinations of parameters which have QFPs with various patterns of supersymmetry breaking including non-universal GUT scale conditions (e.g. in Carena and Wagner of ref. \[4\]). Many of these combinations are flavour off-diagonal and first and second generation which leads to a natural reduction in FCNCs at low $\tan\beta$ \[4\].

At large $h_t$, therefore, quasi-fixed behaviour pervades the entire renormalisation group running of the MSSM, offering the possibility of a considerably reduced parameter space. Moreover it was also observed that the recent precise determination of $m_t = 175 \pm 5 \text{ GeV}$ \[5\] means that the top Yukawa coupling must be large at the
GUT scale ($M_{GUT} \sim 10^{16}$ GeV) in the low $\tan \beta < 30$ regime \cite{3, 8}. If $m_t$ lies within (or above) these quoted $1\sigma$ errors, then quasi-fixed behaviour is indeed going to be a dominant feature. It has also been shown numerically \cite{10}, that bottom-tau Yukawa unification in SUSY GUTs forces the solutions to be near the QFP. (We will explain why with a simple analytic argument below.) We should stress that quasi-fixed behaviour is a one loop effect and in principle it could be destroyed by two loop and higher corrections. However the quasi-fixed behaviour persists to two loop order (including Yukawa corrections other than that of $h_t$).

In this letter, we use these predictions to simplify the analysis of the mass spectrum for the specific case of the ‘Constrained’ MSSM (CMSSM). This is a minimal version of the MSSM with the usual $R$-parity invariant MSSM superpotential,

$$W_{MSSM} = h_U Q H_2 U^c + h_D Q H_1 D^c + h_E L H_1 E^c + \mu H_1 H_2,$$

and a soft SUSY breaking sector which depends on only four high scale parameters; $A$ (the degenerate trilinear coupling), $m_0$ (the degenerate scalar mass), $M_{1/2}$ (the degenerate gaugino mass) and $\tan \beta$ (the ratio of higgs VEVs). The degeneracy is motivated in part by minimal supergravity but we shall, as is usual, impose it at $M_{GUT}$. The fact that we are working close to the QFP means that the ratio of higgs VEVs, $\tan \beta$, is fixed by the QFP value $h_t(m_t) = 1.1$ and the relation

$$\sin \beta = \frac{m_t(m_t)}{v h_t(m_t)}$$

where $m_t(m_t) \approx 160 \pm 5$ GeV is the DR running top quark mass extracted from experiment and $v = 174.1$ GeV is the higgs vacuum expectation value parameter extracted from $M_Z$. Note that this value of $m_t(m_t)$ is lower than in some of the literature \cite{3, 2} because of the effect of gluino and stop corrections \cite{8}. Since the top quark trilinear coupling, $A_t$, also has a fixed point and is the trilinear coupling which predominates in the mass matrices at low $\tan \beta$, the supersymmetric mass spectrum depends only upon $m_0$ and $M_{1/2}$ at the QFP \cite{2, 3, 8}.

We test the spectrum against experimental bounds and in particular the bound on the lightest higgs. In addition we consider whether there is a charge and/or colour breaking minimum which can compete with the physical vacuum \cite{11, 12, 13, 4}. The most restrictive constraints come from so called ‘unbounded from below’ (UFB) directions \cite{12, 13, 4} in which a minimum can be generated radiatively essentially because, at some point during the running, the mass-squared term for $H_2$ must become negative.
in order to drive radiative electroweak symmetry breaking. The bound which is usually imposed comes from requiring that the physical minimum should not be meta-stable. A more relevant (and sufficient) condition is to require that there be no local minima other than the physical one \[4\]. As shown in Ref.\[4\], the two conditions are in any case numerically very close so we shall use the ‘traditional’ meta-stability bound. The UFB bounds depend only on \(m_0\) and \(M_{1/2}\) at the QFP and are expected to be \[13, 4\]

\[m_0 \gtrsim M_{1/2} \quad \text{(5)}\]

We then compare the remaining parameter space with that allowed by dark matter constraints at the QFP \[14, 15\] and find that the only allowed regions are meta-stable. We stress that the MSSM at the low \(\tan \beta\) QFP is not yet ruled out by Ref.\[16\]. There the bounds on \(\tan \beta\) were \(1.4\) and \(1.7\) for \(\mu < 0\) and \(\mu > 0\) respectively (note that we are using the Ref.\[15\] definition of the sign of \(\mu\) which is opposite to that of Ref.\[15\]). However at the QFP \(\tan \beta \approx 1.4 \rightarrow 1.5\) for \(\mu < 0\), and is largest in the region where \(m_0 \lesssim M_{1/2}\), i.e. precisely where the UFB bounds are relevant. The UFB bounds (which were not included in Ref.\[13\]) are therefore an additional and restrictive constraint at the QFP. (As noted in Ref.\[4\] they drop quite quickly away from the QFP although they are still significant.)

We finish by discussing how this fact should be interpreted and also by pointing out that two of these problems (i.e. meta-stability and the dark matter constraints) can be removed by adding \(R\)-parity violating terms just below experimental bounds \[4\] (albeit at the expense of losing the neutralino as a dark matter candidate).

Before tackling the spectrum, we first expand on the reason why Yukawa unification leads to fixed point behaviour. For example, many SUSY GUTs \[10\] predict the existence of the unification of the bottom and tau Yukawa couplings at the GUT scale, \(\lambda_b(M_{GUT}) = \lambda_\tau(M_{GUT})\); why does this constraint favour the QFP? The RGE for \(R_{b/\tau} \equiv \lambda_b/\lambda_\tau\) to one loop order is

\[
\frac{dR_{b/\tau}}{d \ln r} = \frac{R_{b/\tau}}{6} \left[ R - 16/3 + \frac{4}{3} \frac{\alpha_1}{\alpha_3} \right]
\]

\[
\text{(6)}
\]

where, for convenience, we have expressed the running in terms of

\[
r(Q) \equiv \frac{\alpha_3(M_{GUT})}{\alpha_3(Q)} = 1 - 6 \frac{\alpha_3(M_{GUT})}{\alpha_3(M_{GUT})} \log(\frac{Q}{M_{GUT}}).
\]

\[
\text{(7)}
\]

The solution is given by

\[
\frac{R}{R_0} = \left( \frac{R_{b/\tau}(m_t)}{R_{b/\tau}(M_{GUT})} \right)^{12} \frac{\alpha_2(M_{GUT})}{\alpha_2(m_t)}^{\frac{\alpha_1(M_{GUT})}{\alpha_1(m_t)}} \right)^{\frac{\alpha_3(M_{GUT})}{\alpha_3(m_t)}},
\]

\[
\text{(8)}
\]
where \( R_0 \equiv R(M_{\text{GUT}}) \). As is customary, we define a distance \( \rho \) to the QFP,

\[
\rho \equiv 1 - \frac{R}{R^{\text{QFP}}} = \frac{R}{R_0 \Pi r},
\]

where the last relation can be found, for example, in Ref.[4], and where

\[
\Pi = r^{16/9} \left( \frac{\alpha_2(m_t)}{\alpha_2(M_{\text{GUT}})} \right)^{-3} \left( \frac{\alpha_1(m_t)}{\alpha_1(M_{\text{GUT}})} \right)^{-13/99}.
\]

\( \rho \ll 1 \) near the QFP\(^1\).

Yukawa unification (i.e. \( R_{b/\tau}(M_{\text{GUT}}) = 1 \)) then yields a value for \( \rho \) via

\[
\rho = \left( R_{b/\tau}(m_t) \right)^{12} r^{\frac{22}{33}} \left( \frac{\alpha_1(M_{\text{GUT}})}{\alpha_1(m_t)} \right)^{\frac{40}{3}}.
\]

\( R_{b/\tau}(m_t) \) is a number which may be determined from experiment; evaluation to three-loop order in QCD and one loop order in QED yields \( R_{b/\tau} = 1.48 - 1.67 \) for \( \alpha_s(M_Z) \) in the range 0.115-0.121 and \( m_b(m_b) \) in the range 4.1-4.4 GeV. We calculate \( \alpha_1(m_t)^{-1} = 58.62, \alpha_2(m_t)^{-1} = 30.022 \) from \( \sin^2 \theta_{\text{w}}^{\text{MS}} = 0.2315, \alpha(M_Z)^{-1} = 127.9 \) and renormalising from \( M_Z \) to \( m_t \) to one loop accuracy in the Standard Model [17]. Substituting these figures into Eq.[11] we find \( \rho = 7.7 \times 10^{-3} - 2.6 \times 10^{-2} \). If threshold effects imply [10] that \( R_{b/\tau}(M_{\text{GUT}}) = 0.9 \) or 1.1, then \( \rho \) is 3 times smaller or larger respectively. In other words bottom-tau Yukawa unification at low \( \tan \beta \) can only be consistent with experiment if the solutions are very near to their QFPs. A similar situation holds at high \( \tan \beta \).

The sparticle spectrum and constraints

We now turn to the two loop numerical evaluation of the spectrum. In minimal supergravity, the sparticle spectrum depends (generically) upon the six parameters \( \tan \beta, A, m_0, M_{1/2}, \mu, B \). The empirically derived value of \( m_t \) and the QFP prediction sets the first parameter by Eq.[4], and the QFP prediction of \( A_t \) eliminates the spectrum’s dependence upon the second. The parameters were run very close to the quasi-fixed point (taking \( R_0 = 10 \) which corresponds to \( \rho = 1.8 \times 10^{-2} \)) and the full one loop potential minimised to determine the higgs couplings, \( \mu \) and \( B \), by imposing correct electroweak symmetry breaking. (Note that at the QFP the parameter \( \mu \) has a fixed

\(^1\)Constraining \( h_t(M_{\text{GUT}}) < 5 \) yields the ‘perturbativity’ condition \( R/R_0 > \frac{1}{50} \) or \( \rho > 4 \times 10^{-3} \).
Figure 1: The sparticle mass spectrum in the quasi-fixed CMSSM (normalised by the gluino mass) vs. $M_{1/2}$/GeV. We have chosen the line $m_0 = 0.5M_{1/2}$ of Fig. 3 and $\mu > 0$. Note that $m_g \approx 2.7M_{1/2}$. 
Figure 2: As in Fig.1 for $\mu < 0$. 
point prediction of zero which would be incompatible with electroweak symmetry breaking.) The sign of $\mu$ is retained as an additional discrete parameter (see Refs.\cite{18, 19} for details). The derivation of $\tan{\beta}$ ($\sim 1.5$) from $m_t$ was made using the prescription given in Ref.\cite{5}. Analytic expressions for the light higgs masses may be found in Refs.\cite{6, 7}; we used those of Ref.\cite{7} and were able to reproduce the figures of Ref.\cite{8} to within $\pm 2$ GeV (although our lightest higgs mass derived using a full numerical running mostly fell about 1-2 GeV below that in Ref.\cite{8}).

Given the form of the analytic solutions to the renormalisation group equations, the mass spectrum is expected to become proportional to $M_{1/2}$ along the line $m_0/M_{1/2} = a$, where $a$ is constant. We present the spectrum along the line $m_0 = 0.5M_{1/2}$ in Fig.\,1 for positive $\mu$ and in Fig.\,2 for negative $\mu$. It should be noted that the spectrum is generically virtually proportional to $M_{1/2}$ along a given line of constant $m_0/M_{1/2}$; basically $M_{1/2}$ (or equivalently $m_0$) simply sets the superpartner scale. The spectrum is found to be almost entirely independent of $A$ as expected since the only trilinear coupling entering the spectrum, $A_t$, has a fixed point given by Eq.\,2.

The squark/slepton spectrum has a non-trivial dependence at low $M_{1/2}$ because $M_Z$ appears in the mass matrices and is comparable to $M_{1/2}$ in this region. The heavy neutralinos and charginos are dominated by $\mu$ at low values of $M_{1/2}$ until $M_{1/2}$ becomes large enough at which point their masses are proportional. The lightest neutralino and chargino masses are almost proportional to $M_{1/2}$. In particular we find that the mass of the lightest supersymmetric partner lies in the range,

$$0.15 \lesssim m_{LSP}/m_{g} \lesssim 0.18$$

and agrees well with the empirical analytic approximation

$$m_{\chi^0_1} \approx 0.448M_{1/2} + 12 \sin 2{\beta} - 10 \quad : \mu > 0$$

$$m_{\chi^0_1} \approx 0.452M_{1/2} + 5 \sin 2{\beta} - 13 \quad : \mu < 0$$

reported in Ref.\cite{15}.

We now apply some additional constraints to the ($m_0$, $M_{1/2}$) parameter space. Figs. \,3 and \,4 shows experimental bounds (see e.g. Ref.\cite{21}) and bounds from deep minima appearing in ‘unbounded from below’ (UFB) directions in the potential \cite{11, 12, 13, 4}. Regions of parameter space above the line $M_{1/2} \gtrsim m_0$ have a minimum which can compete with the physical one and which is generally much larger. (The spectra we presented above were in regions of parameter space which violate this bound for reasons which will become apparent in the discussion.)
Figure 3: Constraints upon the quasi-fixed CMSSM: bounds in the $m_0$ and $M_{1/2}$ plane for $\mu > 0$. $m_0$ and $M_{1/2}$ are measured in GeV. The labels correspond to the following requirements: $A$-neutralino is the lightest supersymmetric particle (LSP); $B$-chargino mass bounds satisfied; $C$-CCB and UFB bounds satisfied; $D$ non over closure dark matter bound. The lines marked 70, 75, 80 GeV give contours of lightest higgs mass $m_{h^0}$. 
Figure 4: As in Fig. 3 but for $\mu < 0$. 
The constraint is $m_0 \gtrsim 0.92 M_{1/2}$ at low $m_0$ and falls to $m_0 \gtrsim 0.75 M_{1/2}$ for larger values, mainly because of the larger values of $\mu$. This is in accord with the numerical work of Ref.\cite{13}. The analytic (one loop) estimates of Ref.\cite{4} give $m_0 \gtrsim 1.12 M_{1/2}$ and $m_0 \gtrsim 0.95 M_{1/2}$ respectively and therefore represent an overestimate of roughly 15-25\% in the bound. The UFB bound was not included in the analysis of Ref.\cite{14} and close to the QFP this is the severest bound.

The constraint that neutralino dark matter does not over close the universe should also be applied. The LSP should be able to annihilate quickly enough, for which we require that the masses of sparticles appearing in $s$ and $t$-channel processes be sufficiently small \cite{14}. This places a limit on the supersymmetry breaking scale; a full calculation is outside the scope of this paper and here we shall simply adopt the overall limit found at the QFP in Ref.\cite{13}: $m_0 < 200$ GeV. This is actually quite conservative; as may be seen for example in Refs.\cite{15, 16}; the tendency is for the dark matter bound to confine $M_{1/2}$ as well.

We also impose that the neutralino is the LSP (i.e. lighter than the stau \cite{20}) and the chargino bound from LEP 2\cite{21}. (There are additional bounds coming from slepton searches at low $m_0$ which were not included here.) The most restrictive experimental bounds are those from the LEP2 lower bound on the standard model higgs mass. The CP-odd higgs $A^0$ is always much heavier than the lightest CP-even higgs $h^0$, which results in the Standard Model bounds being applicable to the quasi-fixed MSSM to good accuracy \cite{8}. In Figs. 3 and 4 we show the light higgs contours for $m_{h^0} = 70, 75, 80, 85, 90$, and 90 GeV. The latest lower bound from LEP 2 is 87 GeV \cite{21}, but this is expected to rise. Even this bound rules out both $\mu > 0$ and $\mu < 0$ when combined with the above constraints, unless we allow the physical vacuum to be meta-stable \cite{4} and/or ignore the dark matter bound, perhaps because of thermal inflation \cite{22}.

Discussion

We have presented the spectrum for the constrained MSSM at the low tan $\beta$ fixed point and have found that the model can only satisfy higgs and dark matter bounds in regions of parameter space where the physical vacuum is meta-stable. We should interpret this fact carefully since it does not necessarily exclude the model. To see why, let us first clarify what the UFB bounds mean by summarising the conclusions of Refs.\cite{4, 23}. 
The dangerous charge and colour breaking minima which lead to the UFB bounds form radiatively along $F$ and $D$-flat directions. However the vacuum decay rate is suppressed by a large temperature dependent barrier and the quantum tunneling rate is insignificant except at very small values of $m_0$. Thus a meta-stable vacuum would have survived until the present day. In addition the decay rate out of a meta-stable charge/colour breaking minimum back to the physical vacuum is also very small. Since vacuum decay is ruled out in either direction, the question of meta-stability is probably only of psychological relevance although, rather mystifyingly, it remains the commonly accepted criterion. In Ref. [4], it was suggested that a sufficient condition, that the only minimum be the physical one, is the bound we should use rather than the ‘traditional’ UFB bound. However it was also shown that this condition is numerically very close to the ‘traditional’ UFB bound so that all that is required is a change of emphasis; the correct interpretation is that regions of parameter space which violate a UFB bound have dangerous minima (global or local) which can compete with the physical one whereas those which satisfy the bound don’t. If the UFB bound is violated, one is obliged to explain how the universe ends up in the physical vacuum and not in the charge/colour breaking one which is generally more ‘likely’ (in that it is at least $10^3$ times wider than the physical vacuum). Some cosmological suggestions have been made in Ref. [23] and references therein although none have been worked through in great detail. (They may also entail making assumptions about cosmology, such as a high re-heat temperature, which may be at odds with nucleosynthesis for example [4].)

We favour an alternative remedy for this model which is simply to add a small amount of $R$-parity violation. This would be enough to make the LSP unstable, while still evading current experimental bounds upon the magnitude of $R$-violation [24]. In this case the dark-matter bounds and the sneutrino-as-LSP bound vanish, although obviously we have to look elsewhere for a dark matter candidate. In addition the UFB bounds disappear for the reasons discussed in Ref. [4]. Specifically, there are five dangerous UFB directions which correspond to the sets of invariants \[12, 13\],

\[
L_i H_2 \quad L_i L_3 E_3 \quad ; i = 1, 2 \\
L_i H_2 \quad L_i Q_3 D_3 \quad ; i = 1, 2, 3, 
\]

which are absent from the $R$-parity invariant superpotential. To lift the flat direction we can add the following lepton number violating contribution to the superpotential,

\[
W_B = \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k. \tag{15}
\]
These operators are enough to lift the flat directions provided that they satisfy \[\lambda > \frac{0.26 h_T M_{1/2}}{\mu} \approx 0.007 \frac{M_{1/2}}{\mu} \]
\[\lambda' > \frac{0.26 h_b M_{1/2}}{\mu} \approx 0.009 \frac{M_{1/2}}{\mu}. \]
(Here we find that \(\mu/M_{1/2} \approx 1\).) A suitable selection of non-zero \(R\)-parity violating couplings is \(\lambda_{123}, \lambda'_{113}, \lambda'_{223}, \lambda'_{333} \neq 0\) although other combinations are possible. In Ref. [4], it was shown that, provided they satisfy Eq. 16, these four couplings lift all five would-be UFB directions so that there are no local minima except the physical one, whilst simultaneously evading the experimental limits [24] on \(\lambda_{ijk}, \lambda'_{ijk}\). They are small enough however that they will not significantly affect the spectrum in Figs. 3, 4.

The CMSSM near the QFP is an attractive model in which the sparticle spectrum depends upon only two parameters (modulo a choice of the sign of \(\mu\)). Models such as SUSY GUTs (that have the MSSM as the effective field theory below \(M_{\text{GUT}}\)) which predict bottom-tau Yukawa unification [10] favour the QFP, and we have shown this with a simple analytic argument. For \(m_t = 175\) GeV, the model must be near the QFP [3].

However the CMSSM at the low \(\tan \beta\) fixed point is ruled out by either recent higgs mass bounds or dark matter constraints or the presence of a global UFB minimum. Possible solutions include just living with meta-stable vacuum or adding a small amount of \(R\)-parity violation [4]. \(R\)-parity violation can be made small enough to evade experimental bounds in extensions of the MSSM [25] without invoking very small fundamental dimensionless couplings. In addition the mass spectrum we presented would not change appreciably and hence the \(R\)-parity violating fixed point scenario is still ultimately falsifiable due to the absolute upper bound upon the higgs mass \([8]\) \(m_{h^0} < 97 \pm 2\) GeV. The measurement of the mass of one identified SUSY particle ought to be enough to determine the entire sparticle spectrum (to within a discrete choice of the sign of \(\mu\)) near the QFP.
Appendix

Here, we present the analytic solutions to the one loop RGEs (see for example [28]) for the soft terms of the MSSM, with arbitrary boundary conditions, in terms of the three parameters with QFPs, $A_t$, $R$ and $M^2$. They may easily be found without having to solve explicitly for $A_t$, $R$ and $M^2$. (See e.g. Ref.[4] and references therein for these solutions). Defining

$$\delta_i^{(n)} = (\alpha_i^n - \alpha_i^0)/\alpha_i^0$$

$$G = \frac{R}{R_0} \left( \frac{\alpha_3}{\alpha_3^0} \right)^{-7/9} \left( \frac{\alpha_2}{\alpha_2^0} \right) \left( \frac{\alpha_1}{\alpha_1^0} \right)^{13/99},$$

where the 0-subscript indicates values at the GUT scale, the solutions are

$$A_{U_{ij}} - \frac{1}{2} A_t = M_{1/2} \left( \frac{-8}{9} \delta_3^{(1)} + \frac{3}{2} \delta_2^{(1)} + \frac{13}{99} \delta_1^{(1)} \right) + (A_{U_{ij}} - \frac{1}{2} A_t)|_0$$

$$A_{U_{i3}} - A_t = (A_{U_{i3}} - A_t)|_0 G^{1/6}$$

$$A_{U_{3j}} - A_t = (A_{U_{3j}} - A_t)|_0 G^{1/12}$$

$$A_{D_{\alpha 3}} - \frac{1}{6} A_t = M_{1/2} \left( \frac{-40}{27} \delta_3^{(1)} + \frac{5}{2} \delta_2^{(1)} + \frac{29}{99} \delta_1^{(1)} \right) + (A_{D_{\alpha 3}} - \frac{1}{6} A_t)|_0$$

$$A_{D_{\alpha j}} = M_{1/2} \left( -\frac{16}{9} \delta_3^{(1)} + 3 \delta_2^{(1)} + \frac{7}{99} \delta_1^{(1)} \right) + (A_{D_{\alpha j}})|_0$$

$$A_{E_{\alpha \beta}} = M_{1/2} \left( 3 \delta_2^{(1)} + \frac{3}{11} \delta_1^{(1)} \right) + (A_{E_{\alpha \beta}})|_0$$

$$B - \frac{1}{2} A_t = M_{1/2} \left( \frac{16}{9} \delta_3^{(1)} + \frac{3}{2} \delta_2^{(1)} + \frac{5}{66} \delta_1^{(1)} \right) + (B - \frac{1}{2} A_t)|_0$$

$$m_{U_{33}}^2 - M^2 = M_{1/2} \left( \frac{8}{27} \delta_3^{(2)} + \delta_2^{(2)} - \frac{1}{27} \delta_1^{(2)} \right) + (m_{U_{33}}^2 - M^2)|_0$$

$$m_{Q_{33}}^2 - \frac{1}{2} M^2 = M_{1/2} \left( \frac{16}{27} \delta_3^{(2)} - \delta_2^{(2)} + \frac{5}{297} \delta_1^{(2)} \right) + (m_{Q_{33}}^2 - \frac{1}{2} M^2)|_0$$

$$m_2^2 - \frac{3}{2} M^2 = M_{1/2} \left( -\frac{8}{9} \delta_3^{(2)} + \frac{2}{99} \delta_1^{(2)} \right) + (m_2^2 - \frac{3}{2} M^2)|_0$$

$$m_{U_{ij}}^2 = (m_{U_{ij}}^2)|_0 G^{1/6}$$

$$m_{U_{i3}}^2 = (m_{U_{i3}}^2)|_0 G^{1/6}$$

$$m_{Q_{ij}}^2 = (m_{Q_{ij}}^2)|_0 G^{1/12}$$

$$m_{Q_{i3}}^2 = (m_{Q_{i3}}^2)|_0 G^{1/12}$$

$$m_{L_{\alpha \alpha}}^2 = M_{1/2} \left( \frac{3}{2} \delta_2^{(2)} - \frac{1}{22} \delta_1^{(2)} \right) + (m_{L_{\alpha \alpha}}^2)|_0$$

$$m_1^2 = M_{1/2} \left( \frac{3}{2} \delta_2^{(2)} - \frac{1}{22} \delta_1^{(2)} \right) + (m_1^2)|_0$$

$$m_{U_{ii}}^2 = M_{1/2} \left( \frac{8}{9} \delta_3^{(2)} - \frac{8}{99} \delta_1^{(2)} \right) + (m_{U_{ii}}^2)|_0$$
\[ m_{Q_{ii}}^2 = M_{1/2}^2 \left( \frac{8}{9} \delta_3^{(2)} - \frac{3}{2} \delta_2^{(2)} - \frac{1}{198} \delta_1^{(2)} \right) + (m_{Q_{ii}}^2)|_0 \]

\[ m_{D_{\alpha\alpha}}^2 = M_{1/2}^2 \left( \frac{8}{9} \delta_3^{(2)} - \frac{2}{99} \delta_1^{(2)} \right) + (m_{D_{\alpha\alpha}}^2)|_0 \]

\[ \mu = \mu|_0 G^{1/4} \left( \frac{\alpha_3}{\alpha_3|_0} \right)^{-1} \left( \frac{\alpha_2}{\alpha_2|_0} \right)^{-3/2} \left( \frac{\alpha_1}{\alpha_1|_0} \right)^{-1/22} \] (18)

where \( ij = 1, 2 \) and \( \alpha = 1, 2, 3 \), and where we assume universal gaugino mass \( (M_{1/2}) \) at the high scale. (The solutions for the off-diagonal terms are only valid in a generic basis, e.g. not the mass basis.) The remaining terms do not run in this approximation.

A more general set of solutions (valid in any basis) for the flavour changing terms was presented in Ref. [27].

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\section*{References}

[1] B. Pendleton and G.G. Ross, \textit{Phys. Lett.} B98 (1981) 21; C.T. Hill, \textit{Phys. Rev.} D24 (1981) 691, C.T. Hill, C.N. Leung and S. Rao, \textit{Nucl. Phys.} B262 (1985) 517.

[2] M. Lanzagorta and G.G. Ross, \textit{Phys. Lett.} B349 (1995) 31; M. Lanzagorta and G.G. Ross, \textit{Phys. Lett.} B364 (1995) 163; B.C. Allanach and S.F. King, \textit{Nucl. Phys.} B473 (1996) 3; M. Carena and C.E.M. Wagner, \textit{Nucl. Phys.} B452 (1995) 45; W.A. Bardeen, M. Carena, S. Pokorski and C.E.M. Wagner, \textit{Phys. Lett.} B320 (1994) 110; V. Barger, M.S. Berger, P. Ohmann and R.J.N. Phillips, \textit{Phys. Lett.} B314 (1993) 351.

[3] S.A. Abel and B.C. Allanach, \textit{Phys. Lett.} B415 (1997) 371.

[4] S.A. Abel and C.A. Savoy, CERN-TH 98/64, hep-ph/9803218.

[5] D.M. Pierce, hep-ph/9407202; J.A. Bagger, K. Matchev and D.M. Pierce, \textit{Phys. Lett.} B348 (1995) 443; A. Donini, \textit{Nucl. Phys.} B467 (1996) 3; J.A. Bagger, K. Matchev, D.M. Pierce and R. Zhang, \textit{Nucl. Phys.} B491 (1997) 3.
[6] M. Carena, J.R. Espinosa, M. Quiros and C.E.M. Wagner, Phys. Lett. B355 (1995) 209; M. Carena, M. Quiros and C.E.M. Wagner, Nucl. Phys. B461 (1996) 539;

[7] H. E. Haber, R. Hempfling and A. H. Hoang, Z. Phys. C 75, 539 (1997)

[8] J.A. Casas, J.R. Espinosa and H.E. Haber, IEM-FT-167-89, CERN-TH-98-12, SCIPP-980-1, hep-ph/9801365.

[9] CDF and D0 collab.s, hep-ex/9706011

[10] N. Polonsky, Phys. Rev. D54 (1996) 4537; B. Schrempp Phys. Lett. B344 (1995) 193; B.C. Allanach and S.F. King, Phys. Lett. B328 (1994) 360; V. Barger, M.S. Berger and P. Ohmann, MAD-PH-828, C94-02-11, hep-ph/9404360.

[11] J.-M. Frère, D.R.T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11; M. Claudson, L. Hall and I. Hinchcliffe, Nucl. Phys. B228 (1983) 501; H.-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B120 (1983) 346; J-P. Derendinger and C. A. Savoy, Nucl. Phys. B237 (1984) 307; P. Langacker and N. Polonsky, Phys. Rev. D50 (1994) 2199; H. Baer, M. Brhlik, D. Castano, Phys. Rev. D54 (1996) 6944; A. Kusenko, P. Langacker and G. Segre, Phys. Rev. D54 (1996) 5824; J.A. Casas and S. Dimopoulos, Phys. Lett. B387 (1996) 107; J.A. Casas, A. Lleyda and C. Munoz, Nucl. Phys. B471 (1996) 3; J.A. Casas, hep-ph/9707475.

[12] H. Komatsu, Phys. Lett. B215 (1988) 323.

[13] J. A. Casas, A. Lleyda and C. Munoz, Phys. Lett. B389 (1996) 305

[14] See e.g. M. Drees and M.M. Nojiri, Phys. Rev. D47 (1993) 4226

[15] V. Barger and C. Kao, Phys. Rev. D57 (1998) 3131.

[16] J. Ellis, T. Falk, K.A. Olive and M. Schmitt, Phys. Lett. B413 (355) 1997.

[17] B.C. Allanach et al, Journal of Phys. G 24 (1998) 421.

[18] V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D49 (1994) 4908; G.L. Kane, C. Kolda, L. Roszkowski et al, Phys. Rev. D49 (1994) 6173.

[19] S.A. Abel, W.N. Cottingham, I.B. Whittingham, Phys. Lett. B370 (1996) 106.
[20] S. Sarkar, Rept. Prog. Phys. **59** (1996) 1493; S. Dimopoulos, D. Eichler, R. Esmailzadeh and G. Starkman, Phys. Rev. **D41** (2388) 1990; A. De Rujula, S. Glashow and U. Sarid, Nucl. Phys. **B333** (173) 1990; R.S. Chivukula, A.G. Cohen, S. Dimopoulos and T.P. Walker, Phys. Rev. Lett. **65** (957) 1990.

[21] DELPHI collab., $m_h^0 > 85$ GeV at 95% C.L., L3 collab. $m_h^0 > 87.6$ GeV at 95% C.L., see
http://delphiwww.cern.ch/delfigs/figures/search/moriond.html;
http://l3www.cern.ch/analysis/JoachimMnich/abstracts_1998.html;
OPAL collab. $m_{\chi^\pm} > 89.5$ GeV at 95% C.L., hep-ex/9803026.

[22] D.H. Lyth and E.D. Stewart, Phys. Rev. **D53** (1784) 1996; ibid. Phys. Rev. Lett. **75** (1995) 201.

[23] T. Falk, K.A. Olive, L. Roszkowski, A. Singh, M. Srednicki, Phys. Lett. **B396** (1997) 50; A. Riotto and E. Roulet, Phys. Lett. **B377** (1996) 60; A. Strumia, Nucl. Phys. **B482** (1996) 24.

[24] H. Dreiner, hep-ph/9707435.

[25] J. Ellis, S. Lola and G. Ross, hep-ph/9803308; B.C. Allanach, S.F. King, G.K. Leontaris and S. Lola, Phys. Rev. **D56** (1997) 2632.

[26] S.P. Martin and M.T. Vaughn, Phys. Rev. **D50** (1994) 2282.

[27] P. Brax and C.A. Savoy, Nucl. Phys. **B447** (1995) 227.