Nano $a_{g_{\mathcal{I}}}$-open set

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ABSTRACT: The main objective of this paper is to define the notion $Nano-a_{g_{\mathcal{I}}}$-open set by using nano topological space and some properties of this set are studied also, $Nano-a_{g_{\mathcal{I}}}-\beta$ set and $nano-a_{g_{\mathcal{I}}}-\delta$-closed set are two concepts that are defined by using $Nano-a_{g_{\mathcal{I}}}$-open set many examples have been cited to indicate that the reverse of the proposition and remarks is not true. Also an applied example was presented explains how to benefit from $nano-a_{g_{\mathcal{I}}}$-closed set.

1- Introduction

An $\alpha$-open was studied in 1965 by O. Njastad, as a subset $\mathcal{C}$ is an $\alpha$-open set where $\mathcal{C} \subseteq int(cl(int(\mathcal{C}))) ^{[1,2]}$. The notion of ideal was studied by Kuratowski $^{[3,4]}$, that $\mathcal{I}$ is an ideal on $\mathcal{L}$, when $\mathcal{I}$ is a collection of all subsets of $\mathcal{L}$ and $\mathcal{I}$ have two properties (if $\mathcal{C}, \mathcal{D} \in \mathcal{I}$, then $\mathcal{C} \cup \mathcal{D} \in \mathcal{I}$) and (if $\mathcal{C} \in \mathcal{I}$ and $\mathcal{D} \subseteq \mathcal{C}$, then $\mathcal{D} \in \mathcal{I}$).

There are many types for the ideal $^{[5-8]}$

i. $\mathcal{I}_\emptyset$: the trivial ideal where $\mathcal{I} = \{\emptyset\}$.

ii. $\mathcal{I}_n$: the ideal of all nowhere dense sets

$\mathcal{I}_n = \{\mathcal{C} \subseteq \mathcal{X} : \text{int}(cl(\text{int}(\mathcal{C}))) = \emptyset\}$.

iii. $\mathcal{I}_f$: the ideal of all finite subsets of $\mathcal{X}$

$\mathcal{I}_f = \{\mathcal{C} \subseteq \mathcal{X} : \mathcal{C}$ is a finite set$\}$.

The collection of all $\alpha$-open sets denoted by $\overline{\alpha}$ and the collection of all $\alpha$-closed denoted by $\overline{\alpha}$.

By using the nano topological space, a new type of near nano open set is presented which is $Nano-a_{g_{\mathcal{I}}}$-closed set with remarks interpretative table for this type of set. Also concept which is $nano-a_{g_{\mathcal{I}}}$-kernel were given by remarks and example with interpretative table, then other connotation are given; $Nano-a_{g_{\mathcal{I}}}-\beta$ set, $Nano-a_{g_{\mathcal{I}}}-\delta$-closed set with some advantage of those connotation with examples. Finally an introductory example of a specific disease is provided showing how to use the $nano-a_{g_{\mathcal{I}}}$-closed set with clarifying in a table.

2- Preliminaries

Definition 2.1 $^{[9]}$ For equivalence relation $\mathfrak{R}$ on a set $\mathcal{X} \neq \emptyset$, let $\mathcal{C} \subseteq \mathcal{X}$:

i. The lower approximation of $\mathcal{C}$ via $\mathfrak{R}$ denoted by $\mathfrak{R}(\mathcal{C})$ where

$\mathfrak{R}(\mathcal{C}) = \bigcup_{\mathfrak{R}(\mathcal{C}) \subseteq \mathcal{C}} \{\mathfrak{R}(\mathfrak{C}) : \mathfrak{R}(\mathfrak{C}) \subseteq \mathcal{C}\}$, and $\mathfrak{R}(\mathfrak{C})$ defined by the equivalence class by $\mathfrak{C}$.

ii. The upper approximation of $\mathcal{C}$ via $\mathfrak{R}$ denoted by $\overline{\mathfrak{R}}(\mathcal{C})$ where

$\overline{\mathfrak{R}}(\mathcal{C}) = \bigcup_{\mathfrak{R}(\mathcal{C}) \subseteq \mathcal{C}} \{\mathfrak{R}(\mathfrak{C}) : \mathfrak{R}(\mathfrak{C}) \cap \mathcal{C} \neq \emptyset\}$.

iii. The boundary of $\mathcal{C}$ via $\mathfrak{R}$ denoted by $\mathfrak{B}(\mathcal{C})$ where

$\mathfrak{B}(\mathcal{C}) = \overline{\mathfrak{R}}(\mathcal{C}) - \mathfrak{R}(\mathcal{C})$.
Definition 2.2 [11]: For equivalence relation $\mathcal{R}$ on a set $X \neq \emptyset$, let $C \subseteq X$ and $\overline{\mathcal{R}}(C) = \{X, \emptyset, \overline{\mathcal{R}}(C), \overline{\mathcal{R}^b}(C)\}$ is topology on $X$, then $\overline{\mathcal{R}}(C)$ is called nano topology and $(X, \overline{\mathcal{R}}(C))$ is called nano topological space. Every element in this prior topology is called nano-open set (denoted by $\overline{\text{-open set}}$) and its complement is nano-closed set (denoted by $\overline{\text{-closed set}}$). The nano-interior and the nano-closure of $C$ denoted by the following symbols $\text{n-int}(C)$ where $\text{n-int}(C) = \bigcup \{O \subseteq X; O$ is an n-open set, where $O \subseteq C\}$ and $\text{n-cl}(C) = \bigcap \{F \subseteq X; F$ is an n-closed set, where $C \subseteq F\}$, in respectively.

For any ideal $I$, the space $(X, \overline{\mathcal{R}}(C), I)$ is nano ideal topology space.

Example 2.3: Let $X = \{e_1, e_2, e_3\}$ and $\mathcal{R} = \{e_1, e_2, e_3\}, (e_1, e_2), (e_2, e_3), (e_1, e_3), (e_1, e_2), (e_2, e_1)\}$. Then, $\overline{\mathcal{R}} = \{X, \emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_1, e_2\}, \{e_2, e_3\}, \{e_1, e_3\}, \{e_1, e_2, e_3\}\}$. Then the following table:

| $C$   | $\mathcal{R}(C)$ | $\overline{\mathcal{R}}(C)$ | $\mathcal{R}^b(C)$ | $\overline{\mathcal{R}}(C)$ |
|-------|------------------|------------------|-----------------|------------------|
| $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $X$    | $X$              | $X$              | $\emptyset$    | $\{X, \emptyset\}$ |
| $\{e_1\}$ | $\emptyset$     | $\{e_1\}$      | $\{e_1\}$     | $\{X, \emptyset, \{e_1\}\}$ |
| $\{e_2\}$ | $\emptyset$     | $\{e_2\}$      | $\{e_2\}$     | $\{X, \emptyset, \{e_2\}\}$ |
| $\{e_3\}$ | $\{e_3\}$      | $\{e_3\}$      | $\emptyset$    | $\{X, \emptyset, \{e_3\}\}$ |
| $\{e_1, e_2\}$ | $\{e_1, e_2\}$ | $\{e_1, e_2\}$ | $\emptyset$    | $\{X, \emptyset, \{e_1, e_2\}\}$ |
| $\{e_2, e_3\}$ | $\{e_2, e_3\}$ | $\{e_2, e_3\}$ | $\emptyset$    | $\{X, \emptyset, \{e_2, e_3\}\}$ |
| $\{e_1, e_3\}$ | $\{e_1, e_3\}$ | $\{e_1, e_3\}$ | $\emptyset$    | $\{X, \emptyset, \{e_1, e_3\}\}$ |
| $\{e_1, e_2, e_3\}$ | $\{e_1, e_2, e_3\}$ | $\{e_1, e_2, e_3\}$ | $\emptyset$    | $\{X, \emptyset, \{e_1, e_2, e_3\}\}$ |

Table 2.1
**Definition 2.4**[11] For a space $(X, \mathcal{T}_\alpha(\mathcal{C}))$, the set $\mathcal{E} \subseteq X$ is \textit{nano-$\alpha$-open} (denoted by $n\alpha$-open) whenever $\mathcal{E} \subseteq n\text{-int}(n\text{-cl}(\text{n\text{-int}(F))))$, where its complement is \textit{nano-$\alpha$-closed} (denoted by $n\alpha$-closed). The family of all \textit{nano-$\alpha$-closed} symbolize it $n\alpha\mathcal{C}(X)$ and the family of all \textit{nano-$\alpha$-open} symbolize it $n\alpha\mathcal{O}(X)$.

From the table 2.1, the family of all $n\alpha$-closed and $n\alpha$-open can be determined, according to the given $\mathcal{T}_\alpha(\mathcal{C})$ in the previous table as the following table:

| $\mathcal{C}$       | $\mathcal{T}_\alpha(\mathcal{C})$ | $n\alpha\mathcal{O}(X)$ | $n\alpha\mathcal{C}(X)$ |
|----------------------|-------------------------------------|--------------------------|--------------------------|
| $\emptyset$          | $\{X, \emptyset\}$                 | $\{X, \emptyset\}$      | $\{X, \emptyset\}$      |
| $X$                  | $\{X, \emptyset\}$                 | $\{X, \emptyset\}$      | $\{X, \emptyset\}$      |
| $\{\check{e}_1\}$   | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1\}\}$ |
| $\{\check{e}_2\}$   | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1\}\}$ |
| $\{\check{e}_3\}$   | $\{X, \emptyset, \{\check{e}_3\}\}$ | $\{X, \emptyset, \{\check{e}_3, \check{e}_2, \check{e}_3, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ |
| $\{\check{e}_1, \check{e}_2\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1\}\}$ |
| $\{\check{e}_2, \check{e}_3\}$ | $\{X, \emptyset, \{\check{e}_2, \check{e}_3\}, \{\check{e}_1\}\}$ | $\{X, \emptyset, \{\check{e}_2, \check{e}_3\}, \{\check{e}_1\}\}$ | $\{X, \emptyset, \{\check{e}_2, \check{e}_3\}, \{\check{e}_1\}\}$ |
| $\{\check{e}_1, \check{e}_3\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_3\}, \{\check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_3\}, \{\check{e}_2\}\}$ | $\{X, \emptyset, \{\check{e}_1, \check{e}_3\}, \{\check{e}_2\}\}$ |

**Table 2.2**

**Definition 2.5**[10] In a space $(X, \mathcal{T}_\alpha(\mathcal{C}))$, if $\mathcal{E} \subseteq X$, then $n\text{-Ker}(\mathcal{E}) = \cap \{ \mathcal{S} \mid \mathcal{E} \subseteq \mathcal{S}, \mathcal{S} \in \mathcal{T}_\alpha(\mathcal{C}) \}$ which is shortcut for \textit{nano-kernal} of $\mathcal{C}$ at $\mathcal{E}$.

From Table 2.2, if the set $\mathcal{C} = \{\check{e}_1, \check{e}_2\}$ then $\mathcal{T}_\alpha(\mathcal{C}) = \{X, \emptyset, \{\check{e}_1, \check{e}_2\}\}$, then according to the given $\mathcal{E} \subseteq X$, $n\text{-Ker}(\mathcal{E})$ can be determined in the following table:
Table 2.3

| \( \mathcal{F} \) | \( n\text{-Ker}(\mathcal{F}) \) |
|-----------------|------------------|
| \( \emptyset \) | \( \emptyset \) |
| \( \mathcal{X} \) | \( \mathcal{X} \) |
| \{\( \mathcal{E}_1 \)\} | \{\( \mathcal{E}_1, \mathcal{E}_2 \)\} |
| \{\( \mathcal{E}_2 \)\} | \{\( \mathcal{E}_1, \mathcal{E}_2 \)\} |
| \{\( \mathcal{E}_3 \)\} | \( \mathcal{X} \) |
| \{\( \mathcal{E}_1, \mathcal{E}_2 \)\} | \{\( \mathcal{E}_1, \mathcal{E}_2 \)\} |
| \{\( \mathcal{E}_2, \mathcal{E}_3 \)\} | \( \mathcal{X} \) |
| \{\( \mathcal{E}_1, \mathcal{E}_3 \)\} | \( \mathcal{X} \) |

**Definition 2.6**:\([10]\) In a space \((X, \mathcal{I}_\mathcal{R}(\mathcal{C}))\), if \( \mathcal{F} = n\text{-Ker}(\mathcal{F}) \), where \( \mathcal{F} \subseteq X \), then \( \mathcal{F} \) is said \( n\text{-}\beta \) set and briefly \( n\text{-}\beta \) set.

From Table 2.3, the set \( \emptyset, X \) and \( \{\mathcal{E}_1, \mathcal{E}_2\} \) are \( n\text{-}\beta \) sets since every one of those sets is equal to its \textit{nano-kernel}.

**Remark 2.7**:\([10]\) For a space \((X, \mathcal{I}_\mathcal{R}(\mathcal{C}))\), if \( \mathcal{F} \subseteq X \), if and only if \( \mathcal{F} \) is a \( n\text{-open} \) set, then \( \mathcal{F} \) is a \( n\text{-}\beta \) set.

**Definition 2.8**:\([10]\) In a space \((X, \mathcal{I}_\mathcal{R}(\mathcal{C}))\), if \( \mathcal{O} = \mathcal{V} \cap X \), where \( \mathcal{O} \subseteq X \), \( \mathcal{V} \) is \( n\text{-closed} \) set and \( \mathcal{F} \) is \( n\text{-}\beta \) set, then \( \mathcal{O} \) is said \( n\text{-}\theta\text{-}\text{closed} \) set and in shortly \( n\text{-}\theta\text{-\text{closed}} \).

From Table 2.3 where \( \mathcal{C} = \{\mathcal{E}_1, \mathcal{E}_2\} \) then \( \mathcal{I}_\mathcal{R}(\mathcal{C}) = \{X, \emptyset, \{\mathcal{E}_1, \mathcal{E}_2\}, \{\mathcal{E}_1\}\} \) then the family of all \( n\text{-}\theta\text{-closed} \) sets is \( \{X, \emptyset, \{\mathcal{E}_1, \mathcal{E}_2\}, \{\mathcal{E}_1\}\} \).

**Example 2.9**: Let \( X = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\} \) with \( X/\mathcal{R} = \{\{\mathcal{E}_1\}, \{\mathcal{E}_3\}, \{\mathcal{E}_2, \mathcal{E}_4\}\} \) and \( \mathcal{C} = \{\mathcal{E}_1, \mathcal{E}_2\} \). Then \( \mathcal{I}_\mathcal{R}(\mathcal{C}) = \{X, \emptyset, \{\mathcal{E}_1\}, \{\mathcal{E}_2, \mathcal{E}_4\}, \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_4\}\} \) Then \( \{\mathcal{E}_1\} \) is \( n\text{-}\beta \) set and \( \{\mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\} \) is \( n\text{-}\theta\text{-\text{closed}} \).
Proposition 2.10:[10]

i. Every \( n \beta \)-set is \( n \theta \)-closed set.

ii. Every \( n \text{open} \)-set is \( n \theta \)-closed set.

iii. Every \( n \text{closed} \)-set is \( n \theta \)-closed set.

The converse of proposition 2.10, is not true by the example.

Example 2.11: From Table 2.3, \( \mathcal{E} = \{ \mathcal{E}_1 \} \) where \( \mathcal{I} = \{ \mathcal{I}_1, \mathcal{I}_2 \} \) \( \mathfrak{B}(\mathcal{I}) = \{ \mathfrak{B}, \emptyset, \mathcal{I}_1, \mathcal{I}_2 \} \) and \( n \text{-Ker}(\mathcal{E}) = \mathfrak{B} \), then \( \mathcal{E} \) is not \( n \beta \)-set, so it is not \( n \text{-open} \)-set, but \( \mathcal{E} \) is \( n \theta \)-closed set since \( \mathcal{E} = \mathcal{E} \cap \mathfrak{B} \). If we take \( \mathcal{E} = \{ \mathcal{E}_1, \mathcal{E}_2 \} \) with the same set \( \mathcal{C} \) then \( n \text{-Ker}(\mathcal{E}) = \{ \mathcal{E}_1, \mathcal{E}_2 \} \) then \( \mathcal{E} \) is a \( n \beta \)-set and \( \mathcal{E} \) is a \( n \theta \)-closed set, but \( \mathcal{E} \) is not \( n \text{-closed} \)-set.

Remark 2.12:[10] In \((X, \mathfrak{B}(\mathcal{I}))\), if \( \mathcal{O} \subseteq \mathfrak{B} \), \( \mathcal{O} \) is \( n \theta \)-closed set, then \( \mathcal{O} = n \text{-Ker}(\mathcal{O}) \cap \mathcal{Y} \) where, \( \mathcal{Y} \) is \( n \text{-closed} \)-set.

3- On Nano \( \alpha g_1 \)-closed set

Definition 3.1: In \((X, \mathfrak{B}(\mathcal{C}), I)\), \( \mathcal{C} \) is said nano-\( \alpha g_1 \)-closed set (denoted by, \( n \alpha g_1 \)-closed), if \( \mathcal{C} \cdot \mathcal{O} \in I \) then, \( cl(\mathcal{C}) \cdot \mathcal{O} \in I \) where \( \mathcal{O} \subseteq X \) and \( \mathcal{O} \) is a nano-\( \alpha \)-open set.

Now, \( \mathcal{C}^c \) is a nano-\( \alpha g_1 \)-open sets denoted by "\( n \alpha g_1 \)-open". The collection of all nano-\( \alpha g_1 \)-closed sets, denoted by "\( n \alpha g_1 \text{C}(X) \). The collection of all \( \alpha g_1 \)-open sets "\( n \alpha g_1 \text{O}(X) \)."

Example 3.2: From table 2.1 let \( I = \{ \emptyset, \{ \mathcal{E}_1 \}, \{ \mathcal{E}_2 \}, \{ \mathcal{E}_1, \mathcal{E}_2 \} \} \) is the ideal, the family of all \( n \alpha g_1 \)-closed sets and it is a complement \( n \alpha g_1 \)-open sets can be determine, according to the given \( \mathfrak{B}(\mathcal{C}) \) and \( n \alpha \mathcal{O}(X) \) in the table 2.2 as the following table;
| $\mathcal{C}$ | $\tilde{I}_{\mathcal{C}}(\mathcal{C})$ | $n-\alpha\mathcal{O}(\mathcal{X})$ | $n-\alpha g_1\mathcal{C}(\mathcal{X})$ | $n-\alpha g_1\mathcal{O}(\mathcal{X})$ |
|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\emptyset$    | $\{X, \emptyset\}$         | $\{X, \emptyset\}$         | $\{X, \emptyset, \{\mathcal{e}_3\}, \{\mathcal{e}_2, \mathcal{e}_1\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1\}, \{\mathcal{e}_2\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ |
| $X$            | $\{X, \emptyset\}$         | $\{X, \emptyset\}$         | $\{X, \emptyset, \{\mathcal{e}_3\}, \{\mathcal{e}_2, \mathcal{e}_1\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1\}, \{\mathcal{e}_2\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ |
| $\{\mathcal{e}_1\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_3\}, \{\mathcal{e}_2, \mathcal{e}_1\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1\}, \{\mathcal{e}_2\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ |
| $\{\mathcal{e}_2\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_3\}, \{\mathcal{e}_2, \mathcal{e}_1\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1\}, \{\mathcal{e}_2\}, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ |
| $\{\mathcal{e}_3\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | | $P(X)$ |
| $\{\mathcal{e}_1, \mathcal{e}_2\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | | $P(X)$ |
| $\{\mathcal{e}_2, \mathcal{e}_3\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | | $P(X)$ |
| $\{\mathcal{e}_1, \mathcal{e}_3\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | $\{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$ | | $P(X)$ |

Table 3.1

Remark 3.3:

i. Every $n$-closed set in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}))$ is $n-\alpha g_1$-closed in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}), 1)$.

ii. Every $n$-open set in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}))$ is $n-\alpha g_1$-open in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}), 1)$.

Reverse of Remark 3.3 is not true. By example 3.2, if the set $\mathcal{C} = \{\mathcal{e}_1, \mathcal{e}_2\}$ then $\tilde{I}_{\mathcal{C}}(\mathcal{C}) = \{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}\}$; then $n-\alpha g_1\mathcal{C}(\mathcal{X}) = \{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}, \{\mathcal{e}_2, \mathcal{e}_1\}\}$ and $n-\alpha g_1\mathcal{O}(\mathcal{X}) = \{X, \emptyset, \{\mathcal{e}_1, \mathcal{e}_2\}, \{\mathcal{e}_2, \mathcal{e}_1\}\}$, the set $\{\mathcal{e}_1\}$ is $n-\alpha g_1$-open in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}), 1)$ but not $n$-open set in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}))$ and the set $\{\mathcal{e}_1, \mathcal{e}_2\}$ is $n-\alpha g_1$-closed in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}), 1)$ but not $n$-closed set in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}))$.

Theorem 3.4: Let $\mathcal{C}$ and $\mathcal{D}$ are two $n-\alpha g_1$-closed sets then $\mathcal{C} \cup \mathcal{D}$ is a $n-\alpha g_1$-closed.

Proof: Let $\mathcal{C}$ and $\mathcal{D}$ are two $n-\alpha g_1$-closed sets in $(X, \tilde{I}_{\mathcal{C}}(\mathcal{C}), 1)$ and $\mathcal{C} \neq \tilde{I}_{\mathcal{C}}(\mathcal{C})$, where $(\mathcal{C} \cup \mathcal{D}) \cap \mathcal{D} \neq \emptyset$ and $\mathcal{D} \cap \mathcal{C} \neq \emptyset$, so $n-cl(\mathcal{D}) \neq \emptyset$ and $\mathcal{C} \cap \mathcal{D} \neq \emptyset$ therefore, $(n-cl(\mathcal{C}) \cap \mathcal{D}) \cup (n-cl(\mathcal{D}) \cap \mathcal{C}) \neq \emptyset$. Hence $\mathcal{C} \cup \mathcal{D}$ is $n-\alpha g_1$-closed sets.
Corollary 3.5: Let $C$ and $D$ are two $n$-$\alpha g_1$-open sets then $C \cap D$ is $n$-$\alpha g_1$-open.

Proof: Let $C$ and $D$ are two $n$-$\alpha g_1$-open set in $X$ then $C^c, D^c$ are two $n$-$\alpha g_1$-closed sets therefore, $C^c \cup D^c$ is $n$-$\alpha g_1$-closed set by theorem 3.4. Hence $(C \cap D)^c$ is $n$-$\alpha g_1$-closed set so $C \cap D$ is $n$-$\alpha g_1$-open set.

4- On Nano $\alpha g_1$-kernal of set

Definition 4.1: Let $(X, \mathcal{T}_\alpha(C), I)$ be a nano ideal topological space and $E \subseteq X$, $n$-$\alpha g_1$-kernal of $E = \cap \{O: E \subseteq O, O \in n$-$\alpha g_1 O(X)\}$ which is shortcut for $n$-$\alpha g_1$-Ker($E$). It is clear that if $E \in n$-$\alpha g_1 O(X)$ then $E = n$-$\alpha g_1$-Ker($E$).

Example 4.2: From example 3.2, if the set $C = \{e_1, e_2\}$ then $\mathcal{T}_\alpha(C) = \{X, \emptyset, \{e_1, e_2\}\}$ then $\alpha g_1 O(X) = \{X, \emptyset, \{e_1\}, \{e_2\}, \{e_1, e_2\}\}$ according to the given $E \subseteq X$, we can determine $n$-$\alpha g_1$-Ker($E$) in the following table:

| $E$   | $n$-Ker($E$) | $n$-$\alpha g_1$-Ker($E$) |
|-------|-------------|--------------------------|
| $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $X$     | $X$         | $X$                      |
| $\{e_1\}$ | $\{e_1, e_2\}$ | $\{e_1\}$ |
| $\{e_2\}$ | $\{e_1, e_2\}$ | $\{e_2\}$ |
| $\{e_3\}$ | $X$         | $X$                      |
| $\{e_1, e_2\}$ | $\{e_1, e_2\}$ | $\{e_1, e_2\}$ |
| $\{e_2, e_3\}$ | $X$         | $X$                      |
| $\{e_1, e_3\}$ | $X$         | $X$                      |

Table 4.1
Proposition 4.3: In $\mathcal{X}, \mathcal{I}$, if $E \subseteq X$, then $n^{-}a_{g_{1}}\text{Ker}(E) \subseteq n^{-}\text{Ker}(E)$.

Proof: Let $e \in n^{-}\text{Ker}(E)$, $e \in X$ then $e \in \mathcal{I}$, implies, $\exists \alpha \in \mathcal{I}(\mathcal{C}), E \subseteq \mathcal{I}(\mathcal{C})$.

Remark 4.4: For any $E \subseteq X$, then $E$ is a $n^{-}a_{g_{1}}$-open set.

Definition 4.5: For any $E \subseteq X$, the set $E$ is said $n^{-}a_{g_{1}}$-closed set.

Example 4.8: From example 4.2, where $\mathcal{C} = \{e_{1}, e_{2}\}$ then $\mathcal{I}(\mathcal{C}) = \mathcal{X}, \emptyset, \{e_{1}\}, \{e_{2}\}$, then every $E \subseteq X$ is $n^{-}a_{g_{1}}$-closed set since $E \subseteq \mathcal{I}$, such that $\mathcal{C}$ is $n^{-}a_{g_{1}}$-closed set and $H$ is $n^{-}a_{g_{1}}$-closed set.

Theorem 4.9: For any space $\mathcal{X}, \mathcal{I}$ then:

i. Every $n^{-}a_{g_{1}}$-closed set is $n^{-}a_{g_{1}}$-closed set.

ii. Every $n^{-}a_{g_{1}}$-open set is $n^{-}a_{g_{1}}$-closed set.

iii. Every $n^{-}a_{g_{1}}$-closed set is $n^{-}a_{g_{1}}$-closed set.

Proof: (i)\text{Let} $E$ is $n^{-}a_{g_{1}}$-closed then $E = n^{-}a_{g_{1}}\text{Ker}(E)$ but $E \subseteq X$, $X$ is $n^{-}a_{g_{1}}$-closed, then $E$ is $n^{-}a_{g_{1}}$-closed set.

(ii)\text{Let} $E$ is $n^{-}a_{g_{1}}$-open set, then $E = n^{-}a_{g_{1}}\text{Ker}(E)$, then $E$ is $n^{-}a_{g_{1}}$-closed set, so $E$ is $n^{-}a_{g_{1}}$-closed set, by (part i) of theorem 4.9.

(iii)\text{Let} $E$ is $n^{-}a_{g_{1}}$-closed set. Since $X$ is $n^{-}a_{g_{1}}$-closed set and $E \subseteq X$, then $E$ is $n^{-}a_{g_{1}}$-closed set.

The converse of Theorem 4.9, is not true.

Example 4.10: From example 3.2, if $E = \{e_{1}\}$ where $\mathcal{C} = \{e_{1}, e_{2}\}$ and $\mathcal{I}(\mathcal{C}) = \mathcal{X}, \emptyset, \{e_{1}\}, \{e_{2}\}$, then $n^{-}a_{g_{1}}\mathcal{C}(X) = \mathcal{X}, \emptyset, \{e_{1}\}, \{e_{2}\}$, then $n^{-}a_{g_{1}}\text{Ker}(E) = \{e_{1}\}$. Thus $E$ is neither $n^{-}a_{g_{1}}$-closed set nor $n^{-}a_{g_{1}}$-open set.
Proposition 4.11: In \((X, \mathcal{T}_{\Omega}(\zeta), I))\), if \(X\) is a finite set and \(\mathcal{O} \subseteq X\); \(\mathcal{O}\) is \(n\)-\(\alpha\)-\(g_1\)-\(\beta\)-closed set, then \(\mathcal{O} = n\)-\(\alpha\)-\(g_1\)-\(\beta\)-\(\text{Ker}(\mathcal{O})\) \(\cap H\), \(H\) is \(n\)-\(\alpha\)-\(g_1\)-closed set.

Proof: since \(\mathcal{O}\) is \(n\)-\(\alpha\)-\(g_1\)-\(\beta\)-closed set, then \(\mathcal{O} = H \cap \mathcal{E}\) such that \(\mathcal{E}\) is \(n\)-\(\alpha\)-\(g_1\)-\(\beta\) set and \(H\) is \(n\)-\(\alpha\)-\(g_1\)-closed set. Implies, \(\mathcal{O} \subseteq n\)-\(\alpha\)-\(g_1\)-\(\text{Ker}(\mathcal{E})\) = \(\mathcal{E}\) and \(\mathcal{O} \subseteq n\)-\(\alpha\)-\(g_1\)-\(\text{Ker}(\mathcal{O})\) which is the smallest \(n\)-\(\alpha\)-\(g_1\)-open set containing \(\mathcal{O}\). Then, \(n\)-\(\alpha\)-\(g_1\)-\(\text{Ker}(\mathcal{O})\) \(\subseteq n\)-\(\alpha\)-\(g_1\)-\(\text{Ker}(\mathcal{E})\) = \(\mathcal{E}\) and \(\mathcal{O} = H \cap \mathcal{E}\). Therefore, \(\mathcal{O} = H \cap n\)-\(\alpha\)-\(g_1\)-\(\text{Ker}(\mathcal{O})\).

5- Some application via \(n\)-\(\alpha\)-\(g_1\)-closed sets.

The example that we will deal with in our topic is a viral hepatitis look to the shape 5.1.

\[\text{Shape 5.1}\]

Example 5.1: Hepatitis A is a viral disease that affects the liver and can cause symptoms that range from mild to severe. The infection is transmitted by eating contaminated food and water, or by direct contact with an infected person.

Almost all people with hepatitis A recover completely with lifelong immunity. However, a very small percentage of people with hepatitis A infection may die from a deadly hepatitis infection. The World Health Organization estimates that in 2016, hepatitis A caused about 7,134 deaths (representing 0.5% of all deaths from viral hepatitis).
The risk of contracting hepatitis A is associated with a lack of safe drinking water and poor sanitation and hygiene (such as infected hands). In countries where the risk of food or water transmission is low, have abnormal sex and who inject drugs. Epidemics can persist and lead to heavy economic losses.

A safe and effective vaccine is available to prevent hepatitis A. Safe water supply, food safety, improved sanitation, hand washing and hepatitis vaccine are among the most effective ways to control the disease. People at high risk, such as those traveling to countries with high levels of infection, men who have same-sex relationships, and drug users can be vaccinated intravenously.

The incubation period for hepatitis A ranges from 14 to 28 days. Symptoms of the infection vary from mild to severe, including fever, malaise, loss of appetite, diarrhea, nausea, abdominal pain, dark urine, diarrhea, and yellowing of the skin and the whites of the eyes. Not all of these symptoms appear on every person with this disease.

Sings and symptoms of the disease are more common in adults than in children. Critical illnesses and death rates are higher among older age groups. Infected children under the age of six do not usually show visible symptoms, and the proportion of infected children is limited to 10%. Infection usually causes more severe symptoms severe attack, and he will soon recover from it.

The following table gives input about 4 patients people \{\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4\}, we will indicate to the symbol 1 if the symptoms are clear to the person and refer the symbol 0 if the symptoms do not appear:

| Injured | Yelllowing of the skin (Y) | Abdominal pain (A) | Slenderness (S) | The whites of the eyes (E) | Dark urine (D) | Hepatitis |
|---------|--------------------------|-------------------|----------------|--------------------------|----------------|-----------|
| \dot{e}_1 | 1                        | 1                 | 1              | 1                        | 1              | 1         |
| \dot{e}_2 | 1                        | 1                 | 0              | 1                        | 0              | 1         |
| \dot{e}_3 | 1                        | 1                 | 0              | 0                        | 0              | 0         |
| \dot{e}_4 | 0                        | 1                 | 0              | 0                        | 0              | 0         |

Table 5.1

In this table, let \(X = \{\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4\}\) be the set of injured with hepatitis, let \(I = \{\emptyset, \dot{e}_1\}\), \(\mathcal{C} = \{\dot{e}_1, \dot{e}_3\}\) and \(\mathcal{R}\) be the equivalence on \(X\), where \(\mathcal{R} = \{(\dot{e}_1, \dot{e}_3); \dot{e}_1, \dot{e}_3 \in X\}\) such that \(\dot{e}_1, \dot{e}_3\) have the same symptoms. Then, \(X/\mathcal{R}=\{}\)
If the whites of the eyes (E) column was cancelled, so $\mathcal{E}/\mathcal{R}(E) = \{\{\hat{e}_1\}, \{\hat{e}_2\}, \{\hat{e}_3\}, \{\hat{e}_4\}\}$. Then $\alpha(O) = \{\{\hat{e}_1\}, \{\hat{e}_2\}, \{\hat{e}_3\}, \{\hat{e}_4\}\}$ which is equal from $\alpha(O)$ with respect to $\mathcal{T}_E(\mathcal{C})$.

From all that, $\text{core}(\mathcal{R}) = \{E, Y\}$. That is mean the yellowing of the skin and the whites of the eyes are the needful and enough to inspire injured develop hepatitis.

The previous information as the next table can be shown:
The collection of equivalent classes | Nano topology | $n$-$\alpha O(X)$ | $n$-$\alpha g^cC(X)$ |
---|---|---|---|
$X/\mathcal{R}=\{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\mathcal{I}_{\mathcal{R}}(C) = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ |

$X/\mathcal{R}(E) = \{\{\mathcal{E}_1, \mathcal{E}_4\}, \{\mathcal{E}_2, \mathcal{E}_3\}\}$ | $\mathcal{I}_{\mathcal{R}(E)}(C) = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ |

$X/\mathcal{R}(A) = \{\{\mathcal{E}_1\}, \{\mathcal{E}_2\}, \{\mathcal{E}_3\}, \{\mathcal{E}_4\}\}$ | $\mathcal{I}_{\mathcal{R}(A)}(C) = \{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ |

$X/\mathcal{R}(S) = \{\{\mathcal{E}_1\}, \{\mathcal{E}_2\}, \{\mathcal{E}_3\}, \{\mathcal{E}_4\}\}$ | $\mathcal{I}_{\mathcal{R}(S)}(C) = \{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ |

$X/\mathcal{R}(D) = \{\{\mathcal{E}_1\}, \{\mathcal{E}_2\}, \{\mathcal{E}_3\}, \{\mathcal{E}_4\}\}$ | $\mathcal{I}_{\mathcal{R}(D)}(C) = \{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ |

$X/\mathcal{R}(Y) = \{\{\mathcal{E}_1\}, \{\mathcal{E}_2\}, \mathcal{E}_3, \mathcal{E}_4\}$ | $\mathcal{I}_{\mathcal{R}(Y)}(C) = \{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ | $\{X, \emptyset, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4\}$ |

Table 5.2

References:
[1] O Njastad, 1965 On some classes of nearly open set, Pacific J. Math. 15, pp 961 – 970.
[2] Nadia M Ali, 2004 On New Types of Weakly Open Sets “$\alpha$-Open and Semi-$\alpha$-Open Sets”, M.Sc. Thesis, January, Ibn Al-Haithatham Journal for Pure and Applied Science.
[3] Kuratowski K. 1933 Topology. NewYork: Academic Press. Vol I.
[4] A A Nasef and R B Esmaeael, 2015 Some $\alpha$- operators via ideals, International Electronic Journal of Pure and Applied Mathematics Vol. 9, No. 3, pp 149-159.
[5] A A Nasef, A E Radwan, F A Iprahem and R B Esmaeael, in June-2016 Soft $\alpha$-compactness via soft ideals Vol. 12, No. 4.
[6] M E Abd El-Monsef, A A Nasef, A E Radwan and R B Esmaeael , 2014 On $\alpha$- open sets with respect to an ideal, Journal of Advances Studies in Topology, 5(3), pp 1-9.
[7] R B Esmaeael, 2012 on $\alpha$-$c$-compactness, Ibn Al-Haithatham Journal for Pure and Applied Science, 22, pp 212-218.
[8] R Engling, 1989 "Outline of general topology" Amsterdam.
[9] Z Pawlak, 1982 Rough sets, International journal of computer and Information Sciences. 11, pp 341-356.
[10] M Parimula and S Jafari, 2018 On some new notions in nano ideal topological space, International Balkan Journal of Mathematics (IBJM), 1(3), pp 85-92.
[11] A Steen and J A Seebach, 1970 Counterexamples in Topology, Holt, Rinehart and Winster, New York.