Effect of Tail Capacitor on Phase Noise in LC Cross-connected Oscillators: An Analytical Investigation

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1. INTRODUCTION

Oscillators are one of the important blocks in many applications such as RF electronics and digital systems. Since the oscillators have a non-linear behavior, it is very hard to model and analyze them. One of the most important parameters in oscillators is phase noise. The noise injected into the circuit by active and passive elements can show itself as a phase (frequency) perturbation in the desired signal which is called phase noise (or Jitter). Phase noise is one of the key parameters to determine the spectral purity of a signal generated by an oscillator. Depicted in Figure 1(a), single sideband (SSB) phase noise is defined as the ratio of the spectral power density measured at an offset frequency from the carrier (in 1Hz bandwidth) to the total power of the carrier signal and is stated as dBc/Hz. New communication circuits need low phase noise oscillators to satisfy the strict requirement of the modern communication standards. Among different type of oscillators [1], LC-VCOs attract many attentions due to their superior phase noise performance, reliable startup and ability for integration above standard CMOS technologies.

So far, several models for the prediction of phase noise were presented. Among these models, the most well-known phase noise model is Leeson’s equation [2] in which the noise behavior of an oscillator is assumed linear-time-invariant (LTI). As reported in [2, 3] the verified Leeson’s phase noise equation at offset frequency $\Delta \omega$ from the oscillation frequency $\omega_0$, is expressed as Equation (1).

$$L_1(\Delta \omega) = 10 \log \left( \frac{2\pi F_1 P}{P_e} \cdot \left( 1 + \frac{\omega_0}{2Q \omega_s} \right) \cdot \left( 1 + \frac{\Delta \omega}{\text{corner}} \right) \right)$$

in which $k$, $T$, $P$, and $Q$, are Boltzmann constant, absolute temperature, signal power and quality factor of the inductor respectively. $\Delta \omega / \omega_s$ is also the corner frequency and $F$ denotes an experimental noise factor parameter. According to Figure 1(b) and (1), the plot can be divided into three regions. First region where $\Delta \omega >> \omega / 2Q$ has a
flat profile and is dominated by the thermal noise. In regions two and three ($\Delta \omega << \omega / 2Q$) the white thermal noise and flicker noise make the phase noise with the slope of $1/\Delta \omega^2$ and $1/\Delta \omega^3$, respectively [4].

Alper Demir’s model is one of the complex and accurate phase noise model (but almost without enough circuit intuition) [5]. In addition, the Demir’s model can predict cyclostationary noise and also can present a fast simulation CADs [4].

As explained by Hajimiri et al. [3], in their phase noise model introduces a general theory of the phase noise for different kind of voltage-controlled oscillators (VCO) [4]. This model has a lower complexity with enough circuit intuition and can explain up and down conversion of noise in the close frequencies to the carrier. Another advantage of this model is that it introduces impulse-sensitivity-function (ISF) concept to consider the linear-time-variance (LTV) and cyclostationary behavior of noise in oscillators. The ISF is calculated by injecting an impulse current as the noise source of the device and measuring the phase shift (zero crossing) at the output voltage of the oscillator [6].

One of the oscillators which has the best performance in terms of phase noise amongst all CMOS VCOs is cross-connected LC-tank oscillator depicted in Figure 2 [3]. Figures 2(a) and 2(b) show two possible configurations of a cross-connected oscillator, i.e. without and with tail current source [7]. Using tail transistor is one of the ideas in the design of the cross-coupled LC-tank VCOs which was ignored in past decades. Later, it was considered more in [6, 8, 9] and discovered that it plays a prominent role in phase noise of the LC-tank VCO.

The output spectrum of a practical oscillator and (b) different regions of phase noise vs frequency offset

![Figure 1](image_url)

\[ V_{out} = \frac{4}{\pi} \times (RI_b) \]  

(2)

While eliminating tail transistor in Figure 2(a) results in higher voltage headroom, using tail transistor in Figure 2(b) is preferred due to several reasons: first it creates a high impedance in series to the cross-connected switching transistors, reduces the loading of LC resonator and prohibits tank quality factor degradation [9]. Second, it defines the bias current $I_b$ of the cross-connected pair and the output voltage of the oscillator as Equation (2) that results in more controllable and robust design against supply variations [6, 7, 10]. ($R_I$ is loss of the LC-tank and $I_b$ is the tail bias current.)

On the other hand, tail transistor can impose extra noise to the VCO and degrade the phase noise. Since the tail node ($x$) is a common mode node, the even harmonics especially second harmonic are usually dominant in that node. The switching cross-coupled pair, which acts as a single-balance mixer, up/down converts low frequency noise into two correlated sidebands around the fundamental frequency. It should be noted that the low-frequency noise in tail current source does not affect the phase noise directly. In fact, noise frequencies around the second harmonic is down converted close to the oscillation frequency [7, 10, 11]. It should be considered that because the level of the third and higher order harmonics is low and can be filtered by the LC-tank resonator, the effect of the second order harmonic is dominant and significant in phase noise [6].

Filtering technique is one of the best options for eliminating the unwanted (second) harmonics caused by
tail transistor and improving the phase noise of the oscillators [7]. Therefore, several techniques for attenuating the second harmonic in LC-tank oscillators have been proposed [7–9, 12–14], but the preferred technique is usually putting a capacitor in parallel to the tail transistor to bypass the second harmonic noise to ground. The tail capacitor acts as follows: (a) it attenuates the high-frequency noise components at tail node \( x \), (b) prevents the up-conversion of the low-frequency noise of tail transistor into phase noise [9], and (c) reduces voltage variation at tail node and decreases the channel length modulation [9].

While using tail in Figure 2(b) produces a high impedance path, big shunt capacitor \( (C_T) \) bypasses it and results in loading the LC tank with lower impedance. In other words, LC tank is loaded through switching transistors by a low impedance and its quality factor is degraded. So, although a shunt capacitor to the tail node results in lower harmonic distortion in the output of the oscillator, it may degrade the quality factor of the LC-tank and accordingly the phase noise caused by switching transistors [6, 9]. In next section, we will conclude that the effect of harmonics filtering by \( C_T \) is more dominant than degradation of the quality factor on the phase noise.

It is worth mentioning as reported in literature [6, 9] the effect of capacitive noise filtering on phase noise is only investigated by simulation but no analysis is presented. Further, Andreani et al. [15] used a closed-form symbolic formula for phase noise of cross-connected oscillators in the case of negligible \( C_T \) is obtained by using phase noise relation in (3) which was introduced by Hajimiri [6] and others [15].

\[
L(\Delta \omega) = 10 \times \log_{10} \left( \sum \frac{\Gamma_{\text{rms}}^2 \times \tilde{i}_{\text{rms}}^2 / \Delta f}{2 \times q_{\text{rms}}^2 \times \Delta \omega^2} \right)
\]

(3)

where \( \tilde{i}_{\text{rms}}^2 / \Delta f \) and \( \Gamma_{\text{rms}} \) represent the power spectral density of the current noise source and the root-mean-square of the ISF respectively. The maximum charge at output capacitor is denoted by \( q_{\text{rms}} \). By neglecting \( C_T \) (i.e. very small tail capacitances), a phase noise closed-formula obtained from literature [6, 15] as follows:

\[
L(\Delta \omega) = 10 \times \log \left( \frac{k_B T}{N^2 C_A A \Delta \omega^2 R_i} \times (y + 1 + \frac{\eta[\Phi]}{N} \gamma g_m R_i) \right)
\]

(4)
in which \( k_B, g_m, A \) and \( C \) are, respectively, the Boltzmann constant, the transconductance of each transistor, the amplitude of output voltage and total capacitance of LC tank. \( \eta[\Phi] \) represents the tail current phase noise coefficient [16], \( N=1 \) for single-ended and \( N=2 \) for differential oscillators. Apparently, \( C_T \) and its effect are not presented in (4).

Recently, Razavi [16] has been presented an intuitive but very instructive discussion for the effect of tail capacitance on oscillator phase noise. It was explained that the tail capacitance bypasses the second harmonics of tail node, produces a doublet around each zero-crossing, results in up-conversion of flicker noise of cross-connected transistors and also shunts the noise of tail transistor (at \( 2\omega_0 \)) to ground.

On the other hand, along with different phase noise analyses, several researches were also devoted to the phase noise reduction of LC oscillators and different techniques have been introduced in literature. Since zero crossing points are strongly vulnerable to the noise, in literature [17] a phase noise reduction technique is presented by pushing high closed-loop gain to the non-zero-crossing points of the outputs. In order to reduce the close-in phase noise caused by the flicker noise of tail transistor, a resistive feedback is used in literature [18] and the flicker noise of the tail transistor has been suppressed. However, such a diode-connected tail transistor reduces the output impedance of tail and may degrade the quality factor of the tank. Current-switching as well as capacitive-degeneration techniques are utilized simultaneously to reduce the flicker and thermal noise of tail and cross-connected transistors [19].

In this paper the effect of different values of tail capacitor on total phase noise is analytically studied and compared with simulation results. The rest of the paper is organized as follows: the phase noise analysis in a cross-connected VCO for three scenarios of \( C_T \) is described in Section 2 and a closed-formula for each ISF is presented. Section 3 compares simulation results with numerical values obtained by derivations. Finally, conclusions are given in Section 4.

2. PHASE NOISE ANALYSIS FOR SMALL, MEDIUM AND LARGE VALUES OF TAIL CAPACITANCES

In the LC-tank oscillator shown in Figure 3(a), the differential cross-connected transistors make a negative transconductance that can eliminate the loss of the LC tank [6, 8, 9]. For an LC-tank oscillator with arbitrary phase of sinusoidal output, the output voltage can be described by Equation (5):

\[
V_{\text{OUT},x}(\phi) = A_{\text{TANK}} \times \sin(\phi)
\]

\[
V_{\text{OUT},y}(\phi) = -A_{\text{TANK}} \times \sin(\phi)
\]

(5)

Denoting DC bias current of tail transistor by \( 2I_b \), the current of \( M_1 \) and \( M_2 \), and the total current of them can be written as Equations (6), (7) and (8), respectively.

\[
I_x(\phi) = \frac{\beta}{2} (A_{\text{TANK}} \times \sin(\phi) + V_s(\phi))^2
\]

(6)

\[
I_y(\phi) = \frac{\beta}{2} (-A_{\text{TANK}} \times \sin(\phi) + V_s(\phi))^2
\]

(7)

\[
2I_b = I_x(\phi) + I_y(\phi)
\]

(8)
where $\beta = \mu C_{\infty} \frac{w}{l}$ and $V_\alpha(\phi)$ is given by Equation (9).

$$V_\alpha(\phi) = \frac{2I_2}{\beta} A_\text{tan} \sin^2(\phi)$$  \hfill (9)

The current of each cross-connected transistor (i.e. $I_1$ and $I_2$) is depicted in Figure 3(b) for one period in which $2\Phi$ is the conduction angle. By substituting Equation (9) into Equation (7) and setting the equation to zero, the half conduction angle $\Phi$ is obtained as Equation (10).

$$\Phi = \arcsin \frac{I_2}{\beta A_\text{tan}} \sin^2(\phi)$$  \hfill (10)

Considering (6), (7) and (9), the transconductance of $M_1$ and $M_2$ can be given by Equations (11) and (12).

$$g_{m1}(\phi) = \beta A \sin^2(\phi) + \sqrt{2} \sin^2(\phi)$$  \hfill (11)

$$g_{m2}(\phi) = \beta A \sin^2(\phi) + \sqrt{2} \sin^2(\phi)$$  \hfill (12)

According to Figure 3(b) the cross-connected transistors operate in four different zones during each period. Due to switching of $M_1$ and $M_2$ the current of each transistor is usually supposed as a square waveform shown in Figure 3(b) (though the LC tank operates as a narrowband filter and generates sinusoidal output voltages). As discussed earlier, in order to reduce the noise of tail transistor, a capacitor $C_T$ is placed between node $x$ and ground. In this section, we will discuss about the effect of tail capacitive filtering by derivation of the ISF for three scenarios of $C_T$, i.e. very small, medium and big $C_T$.

The ISF of tank resistance $R_t$ at nodes $V_{OUT+}$ and $V_{OUT-}$ (denoted by $\Gamma_{R_t,+}$, $\Gamma_{R_t,-}$) is independent of tail capacitance and has been derived Andreani and Wang [20] as Equation (13).

$$\Gamma_{R_t,+}(\phi) = \frac{\cos(\phi)}{N}, \quad \Gamma_{R_t,-}(\phi) = -\frac{\cos(\phi)}{N}$$  \hfill (13)

### 2.1. Derivation of ISF When CT Is Medium

In prior works, the ISF of oscillator has been calculated with the assumption of negligible $C_T$. In the case of non-negligible $C_T$, the charge of $C_T$ cannot be discharged completely during each period. In order to model it, we define a factor $\alpha$ as the ratio of discharged $\Delta Q_x$ to the total charge $\Delta Q_T$ of $C_T$. The amount of charge for $C_T$ during a period (while discharging through transistors $M_{1,2}$) is obtained by Equation (14).

$$\Delta Q_x(t) = \Delta Q \times e^{-\frac{t}{\tau}}$$  \hfill (14)

where $\tau$ is the time constant of tail node $x$ ($\tau = C_T g_{m1,2}$).

The charge variation ($\Delta Q_x$) of node $x$ can be calculated by Equation (15).

$$\Delta Q_x = \Delta Q - \Delta Q_x = \Delta Q (1 - e^{-\frac{t}{\tau}})$$  \hfill (15)

So, the factor $\alpha$ is obtained as follows:

$$\alpha = \frac{\Delta Q_x}{\Delta Q} = (1 - e^{-\frac{t}{\tau}})$$  \hfill (16)

### 2.1.1. Calculation of $\Gamma_{ids}$

#### 2.1.1.1. Zone 2 ($0 < \phi < \pi - \Phi$)

In this zone $M_1$ is on and $M_2$ is off. As shown in Figure 4(a), applying $i_{ids,n}$ as an impulse current of area $\Delta Q$ charges $C_1$ and $C_T$ by $\Delta Q$ and $-\Delta Q$, respectively. If we assume that $C_T$ is not so small, the time constant of node $x$ is comparable with period of oscillation and as a result $C_T$ is not fully discharged as shown in Figure 4(a). If only $\Delta Q_x = -\alpha \Delta Q$ is transferred from $C_T$ to $C_1$ in each period, the final charge of $C_T$ and voltage variation at output node are as Equations (17) and (18), respectively.

$$\Delta Q_t = \Delta Q - \alpha \Delta Q = (1 - \alpha) \Delta Q$$  \hfill (17)

$$\Delta V_t = (1 - \alpha) \Delta V$$  \hfill (18)

Also, the charge and voltage variation of $C_T$ (i.e. tail node) can be express as Equations (19) and (20), respectively.

$$\Delta Q_{t'} = (\alpha - 1) \Delta Q$$  \hfill (19)

$$\Delta V_{t'} = (\alpha - 1) \Delta V_T$$  \hfill (20)

where,
\[ \Delta V_T = \frac{C}{C_T} \Delta V_r \quad (21) \]

As seen in Equations (20) and (21), contrary to the case with negligible \( C_T \), in this case the voltage of \( C_T \) is non-zero and can affect tank voltage through \( M_1 \) (as a common-gate configuration with gain \( AC_0 \)). Since the voltage at \( C_T \) is amplified by \( AC_0 = \frac{g_{m1} R_t}{C_T} \), the total voltage change at output node is obtained as Equation (22).

\[ \Delta V'_r = \Delta V_r + A_{g21} \times \Delta V'_r \quad (22) \]

By substituting Equations (18), (20) and (21) into Equation (22), we have:

\[ \Delta V'_r = (1 - \alpha) \Delta V + A_{g21}(\alpha - 1) \times \frac{C}{C_T} \Delta V \]

\[ = \Delta V'_r = (1 - g_{m2} R_t \frac{C}{C_T}) \times (1 - \alpha) \Delta V \quad (23) \]

As known from [16], \( \Delta V \) results in \( \Gamma_{\alpha} = \frac{\cos(\alpha)}{N} \) and thus according Equation (23) in this zone \( \Gamma_{\alpha1} \) can be obtained as Equation (24).

\[ \Gamma_{\alpha1} = (1 - g_{m2} R_t \frac{C}{C_T}) (1 - \alpha) \Gamma_{\alpha} \quad (24) \]

### 2.1.1.2. Zones 1 and 3 (\(-\Phi < \phi < \Phi, \pi - \Phi < \phi < \pi + \Phi\))

Since \( M_1 \) and \( M_2 \) are simultaneously on in these zones, the charge of \(-\alpha \Delta \phi \) this time is passed from \( C_T \) through both of the transistors to capacitors \( C_1 \) and \( C_2 \), as depicted in Figure 4(b). The final charge and voltage variation of \( C_1 \) are obtained by Equations (25) and (26), respectively.

\[ \Delta Q_1 = \Delta Q - \frac{g_{m1}}{g_{m1} + g_{m2}} \times \alpha \Delta Q = \frac{g_{m1} + (1 - \alpha) g_{m2}}{g_{m1} + g_{m2}} \Delta Q \quad (25) \]

\[ \Delta V_1 = \frac{g_{m1} + (1 - \alpha) g_{m2}}{g_{m1} + g_{m2}} \Delta V \quad (26) \]

Also, the final charge and voltage variation of \( C_2 \) are given as follows:

\[ \Delta Q_2 = -\alpha g_{m2} \frac{g_{m1}}{g_{m1} + g_{m2}} \Delta Q \quad (27) \]

\[ \Delta V_2 = -\alpha g_{m2} \frac{g_{m1}}{g_{m1} + g_{m2}} \Delta V \quad (28) \]

Again, considering the effect of non-zero voltage of \( C_T \) on \( C_1 \) and \( C_2 \) and fully correlation of \( \Delta V'_r \) with \( \Delta V'_r \) [15], total voltage change at \( C_1 \) (i.e.\( \Delta V'_r \)) is expressed by Equation (29).

\[ \Delta V'_r = \Delta V'_r + A_{g21}(1 - \alpha) \times \Delta V'_r \]

\[ = \Delta V'_r - A_{g21}(1 - \alpha) \Delta V'_r \quad (29) \]

By replacing Equation (21), (26) and (28) in Equation (29) we have Equation (30).

\[ \Delta V'_r = \frac{g_{m1} + (1 - \alpha) g_{m2}}{g_{m1} + g_{m2}} \Delta V + (g_{m1} R_t \frac{C}{C_T})(1 - \alpha) \Delta V - (g_{m1} R_t \frac{C}{C_T})(\alpha - 1) \Delta V \]

\[ \rightarrow \alpha \times \frac{g_{m2}}{g_{m1} + g_{m2}} \Delta V - (g_{m1} R_t \frac{C}{C_T})(\alpha - 1) \Delta V \quad (30) \]

Simplifying Equation (30), Equation (31) is obtained.

\[ \Delta V'_r = \frac{(g_{m1} + (1 - \alpha) g_{m2}) + (1 + \alpha) \times \frac{C}{C_T}}{g_{m1} + g_{m2}} \Delta V \]

\[ \rightarrow \frac{(1 + \alpha) g_{m2} + (1 - \alpha) \times \frac{C}{C_T}}{g_{m1} + g_{m2}} \Delta V \quad (31) \]

Finally, one can obtain \( \Gamma_{\alpha1} \) as Equation (32).

\[ \Gamma_{\alpha1} = (1 - g_{m2} R_t \frac{C}{C_T}) \Delta V \quad (32) \]

### 2.1.1.3. Zone 4 (\( \pi + \Phi < \phi < 2\pi - \Phi \))

Apparently, in this zone \( M_1 \) is off and does not contribute to the phase noise of output voltage. So, we have: \( \Gamma_{\alpha1}(\phi) = 0 \). Accordingly, the impulse sensitivity function of \( M_1 \) during a period is summarized as follows:

\[ \Gamma_{\alpha1}(\phi) = \begin{cases} "0" & \text{if } \Phi < \phi < \Phi \\ -\Phi < \phi < \Phi \\ \Phi < \phi < \pi - \Phi \\ \pi - \Phi < \phi < 2\pi - \Phi \\ "0" & \text{if } \phi = 0 \end{cases} \quad (33) \]

It is worth mentioning that the channel current noise of \( M_1 \) and \( M_2 \) is a cyclostationary noise and can be written as Equation (34).

\[ \text{Figure 4. ISF derivation when } C_T \text{ is not small, (a) only } M_1 \text{ is on and (b) both transistors are on.} \]
\[
\bar{l}_c^2(t) = \bar{l}_c^2 \times \sigma(t).
\]  
\[(34)\]

where \(\bar{l}_c^2\) is a stationary term and \(\sigma(t)\) is the noise modulation factor (NMF) which can be derived easily from noise characteristic relation [3]. According to Andreani et al. [15] and using \(g_{m1}(\phi)\) in Equation (11), the channel thermal noise of \(M_1\) can be written as (35). Comparing Equation (35) with Equation (34), the NMF is easily obtained as Equation (36).

\[
\bar{l}_c^2(t) = 4kTg_{m1}(\phi) = 4kT \beta A_{m1} \times (\sin^2(\phi) + \sin^2(\phi))
\]  
\[(35)\]

\[
\sigma(t) = \sqrt{\sin^2(\phi) + \sin^2(\phi)}
\]  
\[(36)\]

Based on Hajimiri’s theorem, in the case of cyclostationary noise, effective root mean square (RMS) of ISF (i.e. \(\Gamma_{ids,eff}\)) should be calculated from Equation (33). Since the integral equation of \(\Gamma_{ids,eff}\) has no algebraic closed-form expression, the closed-form symbolic formulas for RMS value of Equation (33) is not presented here. Instead, numerical methods the numerical values of \(\Gamma_{ids,eff}\) for required \(\Phi\) are presented in the next section.

### 2.1.2. Derivation of Tail ISF (\(\Gamma_{tail}\))

As depicted in Figure 5, the impulse current noise of \(\bar{T}^2_{n,m}\) charges \(C_T\) with charge area of \(\Delta Q\). It should be noted that impulse current only passes through \(C_T\) and does not charge \(C_{1,2}\).

#### 2.1.2.1. Zones 1 and 3

In these zones both of the transistors are on and \(\alpha \times \Delta Q\) from \(C_T\) is discharged through \(M_1\) and \(M_2\) to the output capacitors \((C_1, C_2)\) that results in charge variation of \(\Delta Q_L\) and \(\Delta Q_R\) respectively.

As similar to previous section one can calculate the charge and voltage variation at outputs \((C_{1,2})\) by Equation (37) to Equation (40).

\[
\Delta Q_L = \frac{g_{sd1}}{g_{sd1} + g_{sd2}} \times \alpha \Delta Q
\]  
\[(37)\]

\[
\Delta Q_R = \frac{g_{sd2}}{g_{sd1} + g_{sd2}} \times \alpha \Delta Q
\]  
\[(38)\]

\[
\Delta V_L = \frac{g_{sd1}}{g_{sd1} + g_{sd2}} \times \alpha \Delta V
\]  
\[(39)\]

\[
\Delta V_R = \frac{g_{sd2}}{g_{sd1} + g_{sd2}} \times \alpha \Delta V
\]  
\[(40)\]

Once again, considering the correlation between outputs and effect of residual charge of \(C_T\), the total voltage variation due to \(\bar{T}^2_{n,m}\) at capacitor \(C_1\) can be written as:

\[
\Delta V_L' = \Delta V_L + A_{C1,1}(1-\alpha)\Delta V_L - \Delta V_L
\]  
\[(41)\]

\[
\rightarrow A_{C1,2}(1-\alpha)\Delta V_L
\]  
\[(42)\]

Using Equations (39), (40) and (21), Equation(41) is rewritten as follows:

\[
\Delta V_L' = \Delta V_L + A_{C1,1}(1-\alpha)\Delta V_L
\]  
\[(43)\]

\[
= [\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}] \Delta V_L
\]  
\[(44)\]

So, the ISF of tail in this zone is given by Equation (45).

\[
\Gamma_{tail}(\phi) = [\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}] \times \Gamma_{T}\phi(\phi)
\]  
\[(45)\]

#### 2.1.2.2. Zone 2

In this zone \(M_1\) is on and \(M_2\) is off and similar to previous section \(\alpha \times \Delta Q\) from \(C_T\) is charged to \(C_1\) and total voltage change of output voltage can be express as bellow.

\[
\Delta V_L' = \Delta V_L + A_{C1,1}(1-\alpha)\Delta V_L
\]  
\[(46)\]

2.1.2.3. Zone 4

In this zone \(M_2\) is on and \(M_1\) is off, charge \(\alpha \times \Delta V\) is transferred to \(C_2\) and results in \(\alpha \times \Delta V\). Considering the residual charge on \(C_T\), total voltage variation at \(C_2\) is given by \(\Delta V = [\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}] \Delta V\). Again, with regard to the fully correlation of differential outputs, \(\Delta V = -\Delta V\) and accordingly ISF is obtained. So, the ISF of tail in a period is given by Equation (46):

\[
\Gamma_{tail}(\phi) = \begin{cases} 
\frac{[(\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}] \times \Gamma_{T}\phi(\phi)}{\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}} & \text{if } \phi < \alpha < \Phi \\
\frac{[\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}] \times \Gamma_{T}\phi(\phi)}{\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}} & \text{if } \phi < \pi < \Phi \\
\frac{[(\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}] \times \Gamma_{T}\phi(\phi)}{\alpha + g_{sd1}R_1 (1- \alpha) \frac{C_1}{C_T}} & \text{if } \phi < \pi < 2\pi < \Phi \\
\end{cases}
\]  
\[(47)\]
2.2. ISF Calculation When \( C_T \) Is Big

In the case of big \( C_T \), we can replace \( \alpha = 0 \) in Equation (33) and Equation (46). Thus, the impulse sensitivity functions of cross-connected and tail transistors are simply derived as Equations (47) and (48), respectively.

\[
\Gamma_{uu}(\phi) = \begin{cases} 
(1-\frac{g_m}{g_{m,0}})R_T \frac{C_T}{C_i} \Gamma_m(\phi) & -\phi < \phi < \phi \\
(1-\frac{g_m}{g_{m,0}})R_T \frac{C_T}{C_i} & \phi < \phi < \pi - \phi \\
0 & -\pi + \phi < \phi < -\pi \\
-\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} & \phi < \phi < \pi + \phi \\
0 & \pi + \phi < \phi < 2\pi - \phi \\
\end{cases} 
\] (47)

\[
\Gamma_{uu}(\phi) = \begin{cases} 
(\frac{g_m}{g_{m,0}})R_T \frac{C_T}{C_i} \Gamma_m(\phi) & -\phi < \phi < \phi \\
\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} & \phi < \phi < \pi - \phi \\
0 & -\pi + \phi < \phi < -\pi \\
-\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} & \phi < \phi < \pi + \phi \\
0 & \pi + \phi < \phi < 2\pi - \phi \\
\end{cases} 
\] (48)

2.3. ISF Calculation When \( C_T \) Is Small

Although the impulse sensitivity function of cross-connected and tail transistors has been derived for very small values of \( C_T \) in prior works [6, 15], just for the purpose of double checking the derivations obtained in this paper, one can substitute \( \alpha = 1 \) in Equations (33) and (46) and obtain Equations (49) and (50); those are exactly identical to equations obtained by Andreani et al. [15].

\[
\Gamma_{uu}(\phi) = \begin{cases} 
\frac{2g_m}{g_{m,0}}R_T \frac{C_T}{C_i} \Gamma_m(\phi) & -\phi < \phi < \phi \\
\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} \Gamma_m(\phi) & \phi < \phi < \pi - \phi \\
0 & -\pi + \phi < \phi < -\pi \\
-\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} \Gamma_m(\phi) & \phi < \phi < \pi + \phi \\
0 & \pi + \phi < \phi < 2\pi - \phi \\
\end{cases} 
\] (49)

\[
\Gamma_{uu}(\phi) = \begin{cases} 
\frac{2g_m}{g_{m,0}}R_T \frac{C_T}{C_i} \Gamma_m(\phi) & -\phi < \phi < \phi \\
\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} \Gamma_m(\phi) & \phi < \phi < \pi - \phi \\
0 & -\pi + \phi < \phi < -\pi \\
-\frac{g_m}{g_{m,0}}R_T \frac{C_T}{C_i} \Gamma_m(\phi) & \phi < \phi < \pi + \phi \\
0 & \pi + \phi < \phi < 2\pi - \phi \\
\end{cases} 
\] (50)

Comparing ISF obtained for non-small and small \( C_T \) from literature [6, 15] reveals that the tail capacitance can reduce phase noise contribution of tail transistor while it adds some terms to \( \Gamma_{idd} \) and may increase phase noise contribution of cross-connected transistors. However, more investigation is presented by numerical values of phase noise for different tail capacitance in Section 3.

3. NUMERICAL AND SIMULATION RESULTS

For more investigation, an LC-tank cross-connected oscillator has been designed and simulated with the circuit parameters shown in Table 1 in a standard 0.18\( \mu \)m CMOS technology. Simulation results show that this oscillator has an oscillation frequency of 5.2 GHz and the power consumption is 8.87 mW. The output voltages are also depicted in Figure 6. In this circuit the switching angle is obtained \( \Phi = 50^\circ \).

In continue, the simulated and analytical phase noise of the VCO for different values of tail capacitances are presented and the effect of different values of \( C_T \) on phase noise is discussed.

3.1. Case 1: Non-negligible (Medium) \( C_T \)

As mentioned before, the noise of cross-connected transistors is cyclostationary and RMS value of each ISF should be calculated by \( \Gamma_{rms} = \frac{1}{T} \int \Gamma(\phi) d\phi \). Since, for a given \( \Phi \), the integrals have no closed-form expression, their numerical values for different \( \Phi \) are illustrated. Figs 7(a) and (b) show \( \Gamma_{rms} \) and \( \Gamma_{Tail-RMS} \) for different values of \( \Phi \) in the case of \( C_T = 100 \ pF \).

To calculate total phase noise of the circuit by (3), the values of effective ISFs at desired \( \Phi \) are obtained from Figures 7(a) and (b). The calculated phase noise versus offset frequency for three different tail capacitances and \( \alpha \) is shown in Figure 8.

According to Figure 8, in the case of non-negligible tail capacitance, increasing \( C_T \) results in phase noise reduction. Also, it indicates that for a given tail capacitance, lower time constant at node \( x \) (i.e. higher \( \alpha \)) is led to better phase noise.

| Parameter | Value         |
|-----------|---------------|
| \( M_{ac} \) | 16 \( \mu \)m/0.18 \( \mu \)m |
| \( M_{1,2} \) | 26.8 \( \mu \)m/0.18 \( \mu \)m |
| \( L_{1,2} = L_2 \) | 1 nH |
| \( R_1 \) | 6 \( \Omega \) |
| \( C_T = C_2 \) | 0.837 \( pf \) |
| \( V_{DD} \) | 1.8 V |
| \( V_{s} \) | 1 V |
3.2 Case 2: $C_T$ Is Big

In the case of big tail capacitances, the time constant of the tail node is bigger than the period of oscillation and $\alpha = 0$. Using Equations (39) and (40), numerical values of $\Gamma^2_{\text{eff-RMS}}$ and $\Gamma^2_{\text{Tail-RMS}}$ for different values of $\Phi$ and $C_T = 1\text{pF}$ are shown in Figures 9(a) and 9(b), respectively. As expected, the effective ISF of cross-connected transistors (i.e. their noise contribution) in Figure 9 is increased (deteriorated) by increasing $\Phi$. In contrast with Figure 7(b), effective tail ISF for big tail capacitances is decreased by increasing $\Phi$.

Using Equation (3), total phase noise of the VCO for $C_T = 1\text{pF}$ and $\Phi = 50^\circ$ was calculated using MATLAB software and shown in Figure 10. The simulated phase noise of the oscillator (by ADS) is also depicted in Figure 10 that shows a good agreement with the numerical results.

According to Figure 11, a bigger tail capacitance is led to a lower phase noise, though for very large $C_T$ increase of $C_T$ has a negligible effect on the phase noise. Therefore, for each VCO, there is an optimum value of $C_T$ in which the tail transistor has lowest contribution to the total phase noise. For example, as shown in Figure 11 the optimum value for $C_T$ is $10\text{pF}$ and by increasing $C_T$ the phase noise cannot be decreased anymore.
4. CONCLUSION

This paper presented an analytical study of the impact of capacitive filtering on the phase noise in an LC-tank oscillator. Considering the three scenarios for value of $C_T$ the ISF of each transistor has been calculated and the effect of tail capacitance has been discussed. According to the analysis, tail transistor dramatically deteriorates total phase noise and the use of a parallel capacitance can effectively reduce the noise contribution of the tail. It is worth mentioning; the analysis also shows that the time constant of the tail node ($\chi$) has a very important role in the amount of noise rejection. While bigger tail capacitance caused by cross-connected transistors may be increased. Although the ISF of transistors is usually degraded by increasing conduction angle, our analysis reveals, in the case of large $C_T$, tail ISF is improved by increasing conduction angle.

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چکیده
این مقاله تأثیر خازن دنباله را روی نویز فاز اسیلاتورهای کنترل شده با ولتاژ (LC-VCO) است. تجزیه و تحلیل نویز فاز از طریق روابط میان ولتاژ (VCO) و خازن دنباله (\(C_T\)) انجام شده است. در ابتدا روابط عددی نویز فاز در اندازه‌گیری سیگنال و خازن دنباله در حالت کوچک، متوسط و بزرگ برای مقادیر مختلف خازن دنباله در نظر گرفته شده است. در یک مطالعه موردی، یک اسیلاتور کنترل شده با ولتاژ در فناوری CMOS 180 نانومتر طراحی و نتایج عددی برای مقادیر مختلف خازن دنباله ارائه شده است. در این مقاله نشان داده می‌شود که برای \(T_C = 200 \text{fF}\) (مقدار متوسط خازن دنباله) و \(T_C = 10 \text{pF}\) (مقدار بزرگ خازن دنباله)، مقدار نویز فاز در آفست فرکانسی 1 مگاهرتز (دبیتی برابر با 18 dBc/Hz) و 96 dBc/Hz- است. بر طبق نتایج شبیه‌سازی و تحلیل‌های عددی، مقدار نویز فاز در حساسیت ضربه (ISF) با افزایش خازن دنباله بهبود می‌یابد. مقدار نویز فاز نشان می‌دهد که افزایش جویا ورود خازن دنباله به خاصیت نویز فاز تأثیر مثبتی ندارد. انتخاب خازن بزرگتری می‌تواند به طور مؤثری نویز فاز را کاهش دهد. در واقع، تأثیر می‌تواند با کاهش ضریب کیفیت تانک خروجی کاهش دهد.