Optimal Quantization of TV White Space Regions for a Broadcast Based Geolocation Database

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Abstract—In the current paradigm, TV white space databases communicate the available channels over a reliable Internet connection to the secondary devices. For places where an Internet connection is not available, such as in developing countries, a broadcast based geolocation database can be considered. This geolocation database will broadcast the TV white space (or the primary services protection regions) on rate-constrained digital channel.

In this work, the quantization or digital representation of protection regions is considered for rate-constrained broadcast geolocation database. Protection regions should not be declared as white space regions due to the quantization error. In this work, circular and basis based approximations are presented for quantizing the protection regions. In circular approximation, quantization design algorithms are presented to protect the primary from quantization error while minimizing the white space area declared as protected region. An efficient quantizer design algorithm is presented in this case. For basis based approximations, an efficient method to represent the protection regions by an ‘envelope’ is developed. By design this envelope is a sparse approximation, i.e., it has lesser number of non-zero coefficients in the basis when compared to the original protection region. The approximation methods presented in this work are tested using three experimental data-sets.

I. INTRODUCTION

The wireless spectrum is a limited and valuable resource. The demand for spectrum is increasing due to the increase in the number of wireless devices and this demand has led to research for efficient utilization techniques of the spectrum. The usage of TV white space by unlicensed secondary users is an example of efficient utilization of spectrum. The spectrum licensing agencies, Federal Communications Commission (FCC) in the United States and Office of Communication (Ofcom) in the United Kingdom, have permitted access of TV white space by an unlicensed secondary device [1], [2].

According to the existing regulations of FCC and Ofcom, TV white space can be accessed by a secondary or white space device (unlicensed user) via TV white space (geolocation) database access. A certified TV white space database service is queried before operation by the secondary device. This query includes the location of secondary device, and database registers the secondary (white space) device if it is allocated a ‘white’ TV channel. The TV transmitter protection regions are calculated by the TV white space database service providers to avoid harmful interference to the primary devices of the licensed broadcasting services. The available TV white space changes with time and space, and it is mandatory for the secondary device to know the availability at the location and time of current operation. The access of TV white space database takes place over the Internet [1]. Fig. 1 depicts the scheme for accessing TV white space database over the Internet. By design, TV white space database access is inaccessible for secondary devices in areas where there is unreliable or no Internet connection. The lack of internet connection is especially prevalent in many developing or under-developed countries where internet services are limited. In such areas, an alternate scheme for communication of the protection regions of TV transmitters are needed.

Fig. 1. Three step explanation of database operation is explained. A white space (WS) device queries the database with geolocation information r and device id M. The database assigns an available channel and registers the device M.

In this work, a broadcast based database service for communicating TV white space availability is proposed. A broadcast based database, such as one using a satellite, can transmit the TV transmitters’ protection regions, or simply the protection regions. In this broadcast based database, the white space device will not query and only receive the broadcast message from the database. The proposed broadcast based database will communicate the protection regions to all the white space devices in a region. This broadcast based approach is assumed to make use of a wireless or digital channel, and the transfer of information to a secondary device will be rate constrained (see Fig. 2). Accordingly, quantization of protection regions is of interest for a broadcast-style TV white space database. When the protection regions are quantized, some TV white space...
A broadcast based geolocation database is illustrated. It is assumed that white space devices can receive the broadcast over a rate constrained channel. Thus, the database should quantize the protection regions, while ensuring the protection of primary from quantization error. Area will be ‘lost’ due to quantization error. This quantizer has to be designed to minimize the TV white space area lost due to quantization. In summary, protection region quantizer’s design and performance are the key problems addressed in the current work. To the best of our knowledge, this is the first work of its kind.

Examples of the protection regions are illustrated in Fig. 3, which are obtained from the iconectiv website for Channel 22 in the New York region. Protection regions such as 1 are nearly circular while protection regions labeled 2 and 3 are non-circular in shape.

In both circular and non-circular models, the protection region has to be quantized or represented to a larger or envelope-style region to protect the incumbent. That is, TV white space can be labeled as protected (due to quantization) but protection region must not be labeled as unprotected (due to quantization). For this important regulatory reason, a traditionally well-studied mean-squared error optimal quantization method cannot be employed.

**Key contributions:**

For the circular model, efficient quantizer design algorithm will be discussed in this work to ensure primary’s protection even after quantization! By efficient, it is meant that for a given quantizer precision, the TV white space area mislabeled (lost) as protection region is minimum. For the non-circular model, a basis (such as Fourier) based approach is detailed to obtain an envelope-approximation to the non-circular protection region. As explained in Fig. 4, a basis based approximation will minimize or reduce the loss in TV white space area beyond the circular (radius based) approximation. The main goal of our developed techniques is to obtain the qualitative graph illustrated in Fig. 5. There is a fundamental limit of how much TV white space area is present. With larger transmission rate, the database can facilitate a more accurate recovery of the TV white space region or the protection region. While using the circular approximation, the recovery of TV white space region will have a gap from the actual TV white space area.

To clarify further, the chart shown in Fig. 6 will be useful to identify our contributions.

**Related work:** Geolocation database are well known in the literature. Circular protection regions for TV white space regions are well known (see [6]). Primary service contours are available as databases for countries such as US and UK [1, 2]. To the best of our knowledge, a broadcast style geolocation database has not been studied in the literature.
Finally, conclusions are presented in Section IV.

II. CIRCULAR PROTECTION REGIONS AND THEIR OPTIMAL QUANTIZATION

This section deals with the quantization of (circular) protection regions. The quantization has to be designed to minimize the white space area lost due to quantization across all transmitters. It is assumed that the centers of these protection regions are already available at the receiver, so that only radius of protection regions have to be quantized and communicated. In case the protection region is not circular, the naïve radius based approximation scheme depicted in Fig. [4] can be used to obtain a radius.

Let \( R := \{r_1, r_2, \ldots, r_n\} \) be the radius of protection regions. For simple exposition in this paper, it is assumed that this set \( R \) is fixed. These radius have to be quantized in the set \( Q = \{q_1, q_2, \ldots, q_m\} \), where \( \log_2 m \) will be the number of bits being spent to communicate each circular region. These \( \log_2 m \) bits will index various quantization levels in the set \( Q \). Without loss of generality, it is assumed that \( q_1, q_2, \ldots, q_m \) and \( r_1, r_2, \ldots, r_n \) are both in an increasing order. The quantized radius set \( Q \) is known to the broadcast-database transmitter as well as all the secondary white space devices. The protection region radius set \( R \), on the other hand, is only known to the transmitter.

To ensure protection for the primary, \( Q(r_i) \geq r_i \), for all protection radius \( r_i \in R \). Unlike in traditional mean-squared or minimax optimal quantization, where \( Q(r) \) is mapped to the closest quantization level \( [\cdot] \), quantization level for protection region’s radius is always the nearest larger level. Thus, the quantizer design for protection region radius is different than traditional mean-squared optimal quantizers.

The actual area of a circular protection region with radius \( r \) is \( \pi r^2 \). After quantization, the circular protection region will have a radius of \( \pi Q(r)^2 \). Since \( Q(r) \geq r \) by design requirements, so a part of TV white space region will be lost or mislabeled as protection region. This motivates the following cost function

\[
C(R, Q) = \pi \sum_{r \in R} \left\{ Q^2(r) - r^2 \right\}
\]

which signifies the white space area lost due to quantization. This is explained with an example having four protection radius and two quantization levels in Fig. [7]. The radius \( r_1, r_2 \) get mapped to \( q_1 \). Even though \( q_1 \) is nearer to \( r_3 \), still \( r_3 \) gets mapped to \( q_2 \). For any given number of quantization levels \( m \), an optimal quantization map \( Q : R \rightarrow Q \) has to be designed to minimize the lost TV white space area, subject to a primary protection condition—the actual (unquantized) protection region should be a subset of the quantized protection region.

The cost function in (1) can be rewritten as

\[
C(R, Q) = \pi \left\{ \sum_{i=1}^{n} Q^2(r_i) - r_i^2 \right\}
\]
Using the Intermediate value theorem, with the assumption that \( p(r) \) is continuous, an extrema of \( C(p(r), Q) \) with respect to \( q_j \) lies between \( q_{j-1} \) and \( q_{j+1} \). To argue that the extrema is a point of minimum, the Hessian in (8) should be positive definite, which requires more assumptions on \( p(r) \). At this point it isn’t clear what assumptions should be made on \( p(r) \), so an empirical approach is taken.

An iterative algorithm will be obtained to find the set \( Q \) which minimizes \( C(r, Q) \) or \( C(R, Q) \). This iterative algorithm is explained next for minimizing \( C(R, Q) \), i.e., assuming that \( R \) is given.

At first, note that the largest quantization level must be equal to the largest protection radius, i.e.,

\[
q_m = r_n
\]

since the largest values of \( Q(r_i) \) need not exceed \( r_n \) (the maximum protection radius) in (2). Apart from \( q_m \), each \( q_j \) will be chosen from the discrete-set \( R \) only. This can be understood using Fig. 7. If \( q_1 \) is chosen between \( r_2 \) and \( r_3 \), then the cost contribution of \( q_1 \) is \( q_1^2 - r_2^2 + q_2^2 - r_3^2 \), which gets minimized when \( q_1 = r_2 \).

At a high-level, one notes that \( q_1 \) has \((n - m)\) choices in \( R \), subsequently \( q_2 \) has \((n - m - 1)\) choices in \( R \), and so on. So the total number of choices for the entire set \( Q \) is \((n - m)(n - m - 1)\ldots(1) = (n - m)!\), which is quite large. A fast algorithm will be developed next to solve the selection of \( Q \).

From (6), for minimum (or an extrema) point the following equation should be satisfied

\[
2q_j \int_{q_{j-1}}^{q_j} p(r)dr - p(q_j)[q_{j+1}^2 - q_j^2] = 0.
\]

From the above, it is noted that \( q_j \) depends only on \( q_{j-1} \) and \( q_{j+1} \). That is, if the odd elements in the set \( Q \) are fixed, then the even elements can be found separately. Similarly, if the even elements in the set \( Q \) are fixed, then the odd elements can be found separately. This results in a separable optimization algorithm as detailed next. The counterpart of (11) for the discrete case will be highlighted first. The cost function in (2) can be rewritten as

\[
\frac{1}{\pi} C(R, Q)
= \sum_{r \in R} Q^2(r) - \sum_{r \in R} r^2
= \sum_{r \in (q_{j-1}, q_j]} Q^2(r) + \sum_{r \in (q_j, q_{j+1})} Q^2(r) + E_j - \sum_{r \in R} r^2
= \sum_{r \in (q_{j-1}, q_j]} q_j^2 + \sum_{r \in (q_j, q_{j+1})} q_{j+1}^2 + E_j - \sum_{r \in R} r^2.
\]
where the term $E_j$ is positive, depends on $q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_m$, and is independent of $q_j$. In [12], it is also noted that $r \in (q_{j-1}, q_j]$ will get quantized to $q_j$ and $r \in (q_j, q_{j+1}]$ will get quantized to $q_{j+1}$. Let the number of protection region radius $r \in (q_{j-1}, q_{j+1}]$ be $n_j$ and number of radius $r \in (q_{j-1}, q_j]$ be $k_j$. Then [12] can be rewritten as

$$ \frac{1}{\pi} C(R, \mathbb{Q}) = k_j q_j^2 + (n_j - k_j) q_j^2 + E_j - \sum_{r \in R} r^2 $$  \hspace{1cm} (13)

The last term is independent of $\mathbb{Q}$, while the second last term $E_j$ is independent of $q_j$ and can be ignored during optimization. Since $q_{j-1}$ and $q_{j+1}$ are fixed, so is $n_j$. Therefore, the only choice variables are $q_j$, which subsequently determines $k_j$ as well. The minimization of $k_j q_j^2 + (n_j - k_j) q_j^2 + \sum_{r \in R} r^2$ can be performed by an exhaustive search over various values of $q_j$ in between $q_{j-1}$ and $q_{j+1}$. In summary, for given fixed values of $q_{j-1}$ and $q_{j+1}$, the value of $q_j$ that (locally) minimizes $C(R, \mathbb{Q})$ can be found out by an exhaustive search.

This motivates the following Even-Odd algorithm for the minimization of cost function in [2], subject to the condition that quantized protection radius is always larger than the actual protection radius:

1) A random initialization for the quantization levels in the set $\mathbb{Q}$ is assumed. It must be noted that the quantization levels belong in the set $\mathcal{R}$.
2) The largest quantization level $q_m$ is fixed to the largest protection radius $r_n$.
3) After a random initialization, the even quantization levels are fixed and the odd quantization levels are exhaustively searched according to the process outlined in [13]. It is restated that the exhaustive search for each quantization level is separate. This results in optimal values for odd quantization levels with respect to the cost function in [2].
4) The (locally) optimized values of the odd quantization levels, as obtained in the previous step, are fixed. The even quantization levels are now exhaustively searched according to the process outlined in [13]. This results in optimal values for the even quantization levels with respect to the [2].
5) The steps 3 and 4 above are used in an iterative manner, till the quantization levels do not change. The resultant quantization levels minimize the desired cost function stated in [2].

The simulation results are presented next. The data used in our simulation experiments is outlined first.

**Data used for experiments:** For circular protection regions, two sets of data were available to us. The first data-set contains the protected service contours’ bounding radius calculated using protection and pollution viewpoints [6]. They make use of the latitude, longitude, EIRP and HAAT to calculate this radius. This set of radius is available for all the channels between 2 to 51 (49 channels) of United States, where transmission by a white space device is permitted by the FCC. There were 8141 protection regions, and hence radius, in total. The second data-set contains the protected service contours’ bounding radius obtained for India [3]. There are no white space regulations in India as of now. The protection radius are available only for 15 channels in the UHF Band-III (470-500MHZ). There were 374 protection regions, and hence radius, in total for India.

These datasets are used to analyse the recovered TV white space area and test our optimal quantization Even-Odd algorithm. As India has extensive rural areas with negligible broadband services, Internet connection is extremely unreliable. It is once again emphasized that a broadcast based TV white space geolocation database will be quite useful for such scenarios.

If $b$ bits are used to index each radius in the set $\mathcal{R}$, then $m = 2^b$. It is recalled that the radius set $\mathcal{R}$ is only known to the database, points in $\mathcal{R}$ will be mapped into $m$ quantization levels, and $\mathbb{Q}$ is agreed upon between the broadcasting geolocation database and the secondary devices. For comparison of our Even-Odd algorithm discussed in Section II, a uniform scalar quantizer is used. For uniform scalar quantizer, the quantization levels are $\mathbb{Q} = \{r_{\min} + \Delta, r_{\min} + 2\Delta, \ldots, r_{\max}\}$ where

\[
\Delta = \frac{r_{\max} - r_{\min}}{m}. \hspace{1cm} (14)
\]

All the values of radius $(r)$ greater than $i - 1^{th}$ level and less than or equal to $i^{th}$ level are translated to the $i^{th}$ level in order to protect the primary.

The results obtained by applying our algorithm on the dataset from United States are illustrated in Fig. 8. Increasing the number of bits decreases the area of lost white space region to quantization, and with 5 bits (per protection radius) or 32 quantization levels most of the white space area can be recovered. Since there are about 8141 protection radius, they can be communicated in a lossless manner with 14 bits per protection radius. The results obtained by applying our algorithm on the data-set from India are illustrated in Fig. 9. With 4 bits (per protection radius) or 16 quantization levels recover most of the white space area. Since there are about 373 protection radius, they can be communicated in a lossless manner with 8-9 bits per protection radius.

The evolution of quantization levels in our Even-Odd algorithm is illustrated in Fig. 10. The initial quantization levels are obtained by using a scalar quantizer for $b = 3$. The quantization levels obtained using the algorithm, help in recovering more white space area as compared to uniform quantization for every bit that is sent as illustrated in Fig. 8 and Fig. 9.

**III. ENVELOPE APPROXIMATION FOR NON-CIRCULAR PROTECTION REGIONS**

For some non-circular protection regions, its circular approximation area can be as large as 30% when compared to the protection region’s area. This area can be scavenged by the secondary if a better approximation method is used. In this section, protection region representation will be considered beyond the circular approximation explained in the Introduction.
As explained in Fig. 6 for non-circular regions quantization is not considered in this work. This section will focus on obtaining basis-based envelope approximation as depicted in Fig. 4. To the best of our knowledge, such approximations are not present in the quantization literature. Consider a non-circular but smooth shape as depicted in Fig. 11. Let its centroid be at the origin. Then, the shape can be parametrized by a waveform $r(t)$ as shown in the figure. With the knowledge of origin (the center), the waveform $r(t)$ is equivalent to the non-circular protection region. The waveform $r(t)$ is periodic (or has finite support) and it is expected to be smooth; so, it can be represented via any basis function suitable for smooth signals on a compact support [9]. That is,

$$r(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t)$$ (15)

where $\phi_k(t), k \in \mathbb{Z}$ form the set of basis functions suitable for representing smooth waveforms with support in $[0, 2\pi]$. The set $\mathbb{Z}$ denotes the set of integers. Fourier series happens to be one choice of basis functions, which will be used in this current work for simplicity of exposition. For Fourier basis, the expansion in (15) reduces to

$$r(t) = \sum_{k=-\infty}^{\infty} a_k \exp(jkt)$$ (16)

The Fourier series coefficients are given by

$$a_k = \frac{1}{2\pi} \int_{0}^{2\pi} r(t) \exp(-jkt) dt$$ (17)

Since $r(t)$ is real-valued the coefficients $a_k$ and $a_{-k}$ will be related by conjugate symmetry. That is $a_k = \bar{a}_{-k}$. So, only $a_k, k \geq 0$ needs to be communicated.
Our main innovation in this section is an envelope approximation for the waveform $r(t)$. In other words, $r(t)$ has to be approximated to $r_{ap}(t)$ such that $r_{ap}(t)$ is always larger than $r(t)$. To this end, the following approach is used. Let $r_K(t)$ be the $K + 1$ coefficient based Fourier series approximation, that is,

$$r_K(t) = \sum_{k=-K}^{K} a_k \exp(jkt).$$

(18)

Define $e_K$ as

$$e_K = \max_{0 \leq t \leq 2\pi} r(t) - r_K(t).$$

(19)

It is noted that $e_K$ is the maximum error between $r(t)$ and $r_K(t)$. So if $r_{ap}(t) = r_K(t) + e_K$ (20) then $r_{ap}(t) \geq r(t)$, and $r_{ap}(t)$ has $2K + 1$ Fourier coefficients given by $a_{-K}, \ldots, a_{-1}, e_K + a_0, a_1, \ldots, a_K$. Of these, due to conjugate symmetry, only $K + 1$ coefficients need to be sent. The coefficients that need to be sent are: $[a_0 + e_K, a_1, a_2, \ldots, a_K]$ to communicate an envelope approximation $r_{ap}(t)$ for $r(t)$. Experimental evaluation with this scheme is explained next.

There is no convenient data-set available for the shapes $r(t)$ for the 8141 protected regions in the United States. To test the Fourier series based envelope approximation method, we created a small data-set for Channel 2 using the TV white spaces US Interactive Map of Spectrum Bridge [10]. Across United States, there are 57 protected service contours (excluding Alaska). These contours were hand-picked into images and the coverage region was segmented (using image processing techniques) to obtain various shapes $r(t)$ for these 57 transmitters in Channel 2. Then, the Fourier basis based envelope approximation technique was applied. An example of Fourier basis based envelope is shown in Fig. [12]. Observe that $r_{ap}(t)$ for various values of $K$ always includes the actual protection region as a subset.

In the absence of real-valued data (images obtained have pixels!), a Riemann approximation [7] is used to calculate the Fourier series coefficients as well as the protection region’s area of the extracted service contours. As there is a limitation on the resolution of the images that can be obtained from the Interactive Map, we did observe some issues due to pixelization (see Fig. [13]). With a higher-resolution data-set, we will obtain better results. But such data-set is not available. Due to pixelization, in Fig. [13] the dashed line signifying white space recovered with basis approximation does not converges to the solid line depicting total white space area. Based on the data-set we could scavenge, the each pixel width is 0.412km.

The results obtained by sending Fourier coefficients can be seen in Fig. [13]. This graph is for the white space region in Channel 2 of the TV spectrum in United States (except Alaska). The circular approximation scheme sends one coefficient $K = 1$, but there is a gap between total white space area available and the white space area that can be recovered by circular approximation. As the number of coefficients sent is increased, $K = 2$ and higher, the Fourier basis scheme fills in the gap between circular approximation and actual white space area. It can be observed that the actual protection region is $(7.401 - 7.152) \times 10^6 = 2.49 \times 10^5$ sq. km. Circular approximation (with high bit-rate quantization) labels $(7.401 - 7.030) \times 10^6 = 3.71 \times 10^5$ sq. km. as protection region. So, the basis based approximation method reduces the size of protection region by about $(3.71 - 2.49)/2.49 = 49.0\%$.

IV. CONCLUSIONS

A geolocation database that broadcasts the TV white space or the primary services protection regions on rate-constrained digital channel was considered. The key issue addressed was quantization or digital representation of the protection regions. It was observed that the protection regions can be circular or non-circular. For circular protection regions, a fast algorithm for optimal quantizer design was developed. The algorithm minimizes the white space area identified as protection region, while ensuring that protection region is not labeled as white space region due to quantization. For non-circular protection regions, a basis based approximation was developed. A procedure for obtaining envelope-type approximation for protection region was developed, which depends on very few number of basic coefficients. The approximation methods were tested using three experimental data-sets. These data-sets included circular protection regions across all TV channels in the United States, circular protection regions in the UHF band TV channels in India, and non-circular protection regions in Channel 2 in the United States.

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The approximation in (20) is illustrated. Consider the shape given by solid line in the right graph. The $r(t)$ corresponding to it is given by the solid line in the left graph. Using $K = 2$ and $K = 11$, two different $r_{ap}(t)$ are plotted. Observe that for both values of $K$, the approximation is larger than $r(t)$ for each value of $t$. This property is expected due to design of the approximation in (20). This property also ensures that the approximate protection region includes (or, is a superset of) the actual protection region.

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