The influence of joint geometry of toneholes with the main bore of wind musical instruments on their frequency characteristics

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Abstract. The article discusses the influence of geometric parameters (the presence and magnitude of the radius of curvature) at the junction of the toneholes with the main bore of the air column on the frequency characteristics of woodwind musical instruments. A theoretical calculation of the eigenfrequencies of an air column with one tonehole in the case of sharp edges has been carried out. The resonance frequencies were also found using computer simulation in the COMSOL Multiphysics 5.5 program for the case of sharp edges and joints with a radius of curvature. An empirical dependence of the frequency of the main tone of the air column on the radius of curvature of the edges of the tonehole is obtained. All simulation were carried out for two models: excluding and including viscous drag and thermal exchange losses.

1. Introduction
The function of side holes (toneholes) in woodwind musical instruments is to shorten the resonating air column, thereby greatly increasing the number of notes which can be generated by a single pipe. Changing the location and geometric size and proportions of toneholes makes it possible to control the frequency and timbre characteristics of the instrument. Therefore, precise modeling of the toneholes of woodwind instruments is essential for predicting the playing characteristics of an instrument.

When studying the acoustic characteristics of air columns with side holes, only a simple geometry is usually considered, when the main channel is a straight cylinder with a side outgrowth (one or more) in the form of the same straight cylinder of smaller size [1-8]. In practice, woodwind instruments often have a radius of curvature at the junction between the tonehole and the main bore. It is known that the toneholes of good instruments made of wood, such as clarinets, oboes, renaissance flutes, and other similar instruments, are known to be undercut, which means, the material was removed from under the tonehole, effectively reducing the sharpness of the edges. The paper [6] presents the experimental characteristics (series and shunt impedance) of an open side hole for the case of a large vibration amplitude (sound pressure), i.e. when nonlinear effects occur at the boundaries of the junction of toneholes with the main bore. In such cases, the acoustic flow in the tonehole may be subject to a greater convective acceleration (the term $\nabla \cdot \nabla \hat{V}$ in the Navier-Stokes equations), especially in the case of short holes, a term that is dropped in the development of linear acoustic equations. It is said in the paper of the same authors, that nonlinear effects become less noticeable if there is a certain radius
of curvature at the junction of the hole with the main bore. On metal instruments, the corners are also rounded, but instrument makers try to minimize these nonlinear losses empirically by smoothing out at the junction between the tonehole and the main bore.

The presence of smoothness of the edges of the tone hole, even in the miss of nonlinear effects, will affect the frequency characteristics of the instrument. Therefore it is important to know how the position of the eigenfrequencies of the air column depends on the degree of rounding of the edges of the side holes. Therefore, it is important to know the dependence of the values of the eigenfrequencies of the air column on the degree of rounding of the edges of the side holes. In this paper, we investigate the dependence of the frequency of the fundamental tone of the air cylindrical column vs. the radius of curvature at the junction between the tone hole and the main bore at the linear acoustic model. All simulations were carried out for two models: excluding and including viscous drag and thermal exchange losses.

2. Model. Theoretical calculation of eigenfrequencies

In this work, we considered a cylindrical air column of length \( L \) and radius \( a \) with a perpendicular side hole (in the middle of pipe) of height \( t \) and radius \( b \) with sharp edges, or having a radius of curvature \( r \) at the junction of the hole with the main bore (figure 1).

![Figure 1](image)

(a) (b) (c)

Figure 1. Schematic view of a straight pipe of length \( L \) with a perpendicular side hole with sharp edges (a); side hole with rounded edges (b); cross-sectional diagram of the pipe at the side hole location and notations (c).

For the theoretical calculation of the column with a perpendicular side hole with sharp edges, the Transmission-Matrix Method (TMM) was used. In this method each section of an instrument (waveguide) is represented by a matrix \( T \) relating the pressure \( p \) and volume flow \( U \) from the output to the input plane and is expressed as:

\[
\begin{bmatrix}
 p_{\text{in}} \\
 Z_0 U_{\text{in}}
\end{bmatrix} =
\begin{bmatrix}
 T_{11} & T_{12} \\
 T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
 p_{\text{out}} \\
 Z_0 U_{\text{out}}
\end{bmatrix},
\]

(1)

where \( Z_0 = \rho c / S \) is the characteristic impedance of the waveguide at the location of the plane, \( \rho \) is the fluid density, \( c \) is the speed of sound in free field and \( S \) is the cross-sectional area of the pipe. The properties of the complete instrument are then calculated from each transmission matrix \( T_n \) and the normalized radiation impedance \( \bar{Z}_{\text{rad}} = p_{\text{out}} / Z_0 U_{\text{out}} \):

\[
\begin{bmatrix}
 p_{\text{in}} \\
 Z_0 U_{\text{in}}
\end{bmatrix} = \left( \prod_{i=1}^{n} T_i \right) \begin{bmatrix}
 \bar{Z}_{\text{rad}} \\
 1
\end{bmatrix}.
\]

(2)

The normalized input impedance is then calculated simply as \( Z_{\text{in}} = p_{\text{in}} / Z_0 U_{\text{in}} \).
The air columns of woodwind instruments are waveguides comprising cylindrical or conical sections with open or closed toneholes. The theoretical expression of the transmission matrix of the cylinder of length $L$ excluding the energy loss is:

$$T_{cyl} = \begin{bmatrix} \cos kL & j \sin kL \\ j \sin kL & \cos kL \end{bmatrix},$$

where $k = 2\pi f/c$ – wavenumber, $j = \sqrt{-1}$.

The tonehole may be represented as a symmetric T-shaped section (figure 2) depending on two parameters, the shunt impedance $Z_s = Z_s/Z_0$ and the series impedance $Z_a = Z_a/Z_0$. An expression of the tonehole transmission matrix is [1]:

$$T_{hole} = \begin{bmatrix} 1 & \frac{Z_s}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_s & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{Z_s}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_s} & \frac{Z_a}{2Z_s} \left(1 + \frac{Z_a}{4Z_s}\right) \\ 1 + \frac{Z_a}{2Z_s} \end{bmatrix}.$$

\textbf{Figure 2.} Equivalent block diagram of the tonehole.

The impedances $Z_s$ and $Z_a$ must be evaluated for the open $(o)$ and closed $(c)$ states of the tonehole as a function of geometry and frequency. The shunt impedance can be expressed as [3, 5]

$$Z_{s(o)}^{(o)} = \frac{1}{\delta^2} \left(jkt_s^{(o)} + \xi_s\right),$$

$$Z_{s(c)}^{(c)} = -j \frac{1}{\delta^2 k_t^{(c)}},$$

where $\xi_s$ is the open tonehole shunt resistance, $t_s$ is the tonehole’s equivalent length and $\delta = b/a$ is the ratio of the radius of the tonehole to the radius of the air column. The shunt resistance $\xi_s$ doesn’t influence the calculated playing frequencies of a woodwind instrument; thus, most research efforts concentrate on the determination of the correction of the shunt equivalent length $t_s$. In the case of an open tonehole, the authors [6] represent equivalent length $t_s$ as

$$kt_s^{(o)} = kt + \tan(k(t + t_m + t_r)),$$

where $t$ is the height of the tonehole as defined in figure 1(c), $t_m$ is the matching volume equivalent length, $t_r$ is the radiation length correction and $t_i$ is the inner length correction. There are many expressions provided in the literature for the inner length correction $t_i$ (for toneholes of large height $t > b$). In this work, for the theoretical calculation, we used the expression of the authors [5]

$$t_i = \left(0.82 - 1.4\delta^2 + 0.75\delta^{2.7}\right)b.$$

The same authors obtained an accurate approximation for the matching volume equivalent length $t_m$
$$t_m = \frac{b\delta}{8} \left(1 + 0.207\delta^2\right). \quad (8)$$

The radiation length correction $t_r$ depends on the external geometry. In the low-frequency approximation, it may be

$$t_r = 0.8216b \quad \text{(flanged pipe),} \quad (9a)$$

$$t_r = 0.6133b \quad \text{(unflanged pipe)} \quad (9b)$$

or another intermediary value for more complicated situations. In this work, we used the equation (9b).

In the case of a closed tonehole, the tonehole’s equivalent length can be represented as

$$k t_i^{(c)} = \tan[k(t + t_m)]. \quad (10)$$

An inner length correction may be considered as well for the closed tonehole, but its influence is small and becomes significant only in the high frequencies [4]. An expression including the inner length correction is [5]

$$Z_s^{(c)} = -\frac{j}{\delta^2}[t_i - \cot(k(t + t_m))]. \quad (11)$$

where $t_i$ is the same as for the open tonehole as defined in equation (7).

The series impedance of the open/closed tonehole is a small negative inertance:

$$Z_{a}^{(o/c)} = j k t_i^{(o/c)} \frac{1}{\delta^2}. \quad (12)$$

which slightly reduces the effective length of the instrument and raises the resonance frequencies. In equation (12) the series length correction $t_i$ has many expressions obtained by different authors. In this work for open tonehole we used the expression [5]

$$t_i^{(o)} = -0.28b\delta^2, \quad (13)$$

for closed tonehole [9]

$$t_i^{(c)} = -\frac{b\delta^2}{1.78 \coth(1.84t/b) + 0.940 + 0.540\delta + 0.285\delta^2}. \quad (14)$$

In total, for a cylindrical pipe with one side hole, based on the TMM, it’s possible to write

$$\begin{bmatrix} p_{in} \\ Z_{0}U_{in} \end{bmatrix} = T_{cyl} T_{hole}^{cyl} \begin{bmatrix} Z_{rad} \\ 1 \end{bmatrix}, \quad (15)$$

where the radiation impedance $Z_{rad}$ is frequency dependent and was calculated by authors for an unflanged pipe [10]. The evaluation of the exact solution $Z_{rad}$ demands performing a number of numerical integrals. An approximate formula for this impedance was given in work [11]:

$$Z_{rad} = 0.6113 jk a - j(k a)^3 \left[0.36 - 0.034\log(k a) + 0.0187(k a)^2\right] + \frac{(k a)^2}{4} +$$

$$+ (k a)^4 \left[0.0127 + 0.082\log(k a) - 0.023(k a)^2\right]. \quad (16)$$

The eigenfrequencies can be calculated, if $Z_{in} = p_{in}/Z_{0}U_{in} \to \infty$.

In the models that take into account the effect of visco-thermal losses, wavenumber takes on a complex value [6]

$$k = 2\pi f/c + (1 - j)\alpha, \quad (17)$$

where $\alpha$ is the attenuation coefficient per unit length of pipe, which is a function of frequency $f$ and radius $a$:

$$\alpha = 2.96 \cdot 10^{-5} \frac{\sqrt{f}}{a}. \quad (18)$$
Figure 3 shows a theoretical calculation of the real part of the normalized input acoustic impedance \( \text{Re}(Z_{in}) \) of a cylindrical air column with an open and closed tonehole (with sharp edges) vs. the oscillation frequency \( f \) near the first three eigenfrequencies in models excluding and including visco-thermal losses.

The calculation of the eigenfrequencies gives values for the fundamental tone in the case of an open tonehole: \( f_0 = 105,529 \) Hz (excluding visco-thermal losses) and \( f_0 = 105,249 \) Hz (including visco-thermal losses); in the case of a closed tonehole: \( f_0 = 82,601 \) Hz (excluding visco-thermal losses) and \( f_0 = 82,358 \) Hz (including visco-thermal losses).

![Diagram](image.png)

**Figure 3.** Real part of the normalized input acoustic impedance of a cylindrical air column with an open (a, b) and closed (c, d) side hole vs. oscillation frequency near the first three eigenfrequencies (excluding visco-thermal losses) (a, c); near the frequency of the main tone of the column excluding (dashed line) and including (solid line) visco-thermal losses (b, d).

3. **Computer simulation**

The evaluation of the input impedance of woodwind instruments using the Finite Element Method (FEM) involves constructing a 3D model of the air column surrounded by a radiation sphere and the solution of the Helmholtz equation for a number of selected frequencies. The body of the instrument itself is considered to be rigid. The mesh occupies the volume inside and outside the instrument.

The surrounding spherical radiation domain uses a non-reflecting boundary conditions that can be implemented in COMSOL Multiphysics 5.5 using the built-in Perfectly Matched Layers (PML) technology – a system of ideally matched layers (at least 8) with fictitious efficient absorption of radiation of arbitrary shape and direction.
An air column (pipe) with a length $L = 1$ m and a radius $a = 6$ cm, surrounded by a sphere with a radius $R = 1.25$ m, was taken as a modeled object. The pipe had one perpendicular cylindrical side hole in the middle with a radius $b = 2$ cm and a height $t = 4$ cm (figure 1(a)). In case of rounded edges, the hole had a radius of curvature $r$ (figure 1(b)) and smoothly connected with the main bore in its cross section (figure 1(c)). The height $t$ of the hole remained unchanged for any radius of curvature $r$. Only one half of the model was solved, taking advantage of the symmetries. On the symmetry plane, a null normal acceleration boundary condition is imposed. The number of degrees of freedom per minimum possible wavelength $\lambda_{\text{min}} \approx 34$ cm ($f_{\text{max}} = 10^3$ Hz) inside a cylindrical region with a tonehole equaled 12 in each direction. The spherical radiation domain was automatically divided into tetrahedrons with linear dimensions from 3 mm to 7 cm. Figure 4 shows the tetrahedral FEM mesh of the air column and the surrounding spherical radiation domain with PML at the boundary of the simulation area.

The visco-thermal losses were taken into account not only in the side hole area, but also along the whole length of the pipe by solving the equations of the “Narrow Region Acoustics” module at wide duct approximation. All experiments were performed by COMSOL Multiphysics 5.5 software using the built-in “Pressure Acoustics” module.

4. Results and discussion

Figure 5 shows as an example the distribution of sound pressure at the fundamental frequency in a cylindrical air column with a side hole with rounded edges in the closed and open states. It can be seen that the sound pressure level drops to almost zero at the open ends.

Table 1 and figure 6 shows the fundamental frequencies $f_0$ of a cylindrical pipe at different radius of curvature of the edges of the side hole ($0 \leq r/b \leq 1.1$) in the models excluding and including visco-thermal losses. It can be seen that with an increasing the degree of rounding of the edges in the case of an open hole, the effective length of the air column gradually decreases, which entails an increase in the resonance frequencies. In case of a closed side hole, the simulation results show that an increase in
the radius of curvature of the edges doesn't significantly affect the change in the frequency of the fundamental tone and doesn't obey a precisely expressed law. The spread of the frequency values in this case is $\Delta f^{(c)} = 0.22$ Hz, which is almost within the computational error, in contrast to the case of an open hole, where the same value is $\Delta f^{(o)} = 1.53$ Hz, and also there is a clear tendency to increase the resonant frequency.

**Table 1.** Comparison of the fundamental frequencies $f_0$ from the reduced radius of curvature $r/b$ of the edges of the side hole in models excluding and including visco-thermal losses.

| Reduced radius of curvature $r/b$ | Fundamental frequency $f_0$ (Hz) |
|-----------------------------------|---------------------------------|
|                                   | Open side hole                  | Closed side hole                |
|                                   | Excluding visco-thermal losses  | Including visco-thermal losses  |
|                                  | Excluding visco-thermal losses  | Including visco-thermal losses  |
| 0 (sharp edges)                  | 105.95 (104,513 – theoretical calculation) | 105.67 (104,235 – theoretical calculation) | 82.783 (82,601 – theoretical calculation) | 82,539 (82,358 – theoretical calculation) |
| 0.1                               | 106.67                          | 106.38                          | 84,871                          | 84,623                          |
| 0.2                               | 106.68                          | 106.40                          | 84,799                          | 84,551                          |
| 0.3                               | 106.76                          | 106.49                          | 84,884                          | 84,636                          |
| 0.4                               | 106.83                          | 106.55                          | 84,853                          | 84,606                          |
| 0.5                               | 106.89                          | 106.63                          | 84,860                          | 84,612                          |
| 0.6                               | 106.99                          | 106.71                          | 84,834                          | 84,587                          |
| 0.7                               | 107.01                          | 106.72                          | 84,662                          | 84,415                          |
| 0.8                               | 107.17                          | 106.88                          | 84,808                          | 84,560                          |
| 0.9                               | 107.28                          | 107.00                          | 84,826                          | 84,578                          |
| 1.0                               | 107.38                          | 107.10                          | 84,816                          | 84,568                          |
| 1.1                               | 107.48                          | 107.20                          | 84,810                          | 84,562                          |

Figure 6 shows linear approximations of computer simulation data and their functional dependencies in case of one open side hole, which are of an estimate tendency, but in a rough approximation can be used in modeling and designing sound holes with rounded edges.

The acoustic properties of woodwind instruments are mainly a consequence of their geometry. Diameter and height of toneholes, their position along the instrument’s body and its geometry, as well
as the height of the hanging pad above toneholes when in the open state, is the most important factor in determining the eigenfrequencies, timbre and the playability of the instrument.

Curvature of the main bore and small details, such as radius of curvature at the junction of the tonehole with the bore, undercutting, the thickness of the wall of the chimney and the type of pad and resonator, also have an influence at the resonant frequencies position. As can be seen from table 1 and figure 6, the displacement from the initial frequency $f_0 = 105.95$ Hz with the appearance of rounded edges can be found in the range of up to 1.5 Hz, which in this case is about 25 cents or $\frac{1}{4}$ of semitone and will be a significant value. If there are several toneholes, this will also additionally influence the displacement of the resonant frequencies. Therefore, the study of the influence of these factors through the development of various theoretical methods and approximations, as well as experimental and computer simulation is an important scientific and engineering task in the development, design and creation of high-quality woodwind musical instruments.

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