R. Navarro Pérez · J. E. Amaro · E. Ruiz Arriola

Partial Wave Analysis of Chiral NN Interactions

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Abstract We analyze chiral interactions to N2LO on the light of proton-proton and neutron-proton scattering data published from 1950 till 2013 and discuss conditions under which the chiral coefficients can be extracted.

Keywords NN interaction · Chiral symmetry · Two Pion Exchange

1 Introduction

While the NN interaction is traditionally acknowledged as a key building block in Nuclear Physics, the possibility of describing it using chiral symmetry and effective field theory methods has been a fascinating pastime for Nuclear theoreticians for more than 20 years as it offers a link to the underlying quark and gluon dynamics of QCD (see e.g. [1; 2] for reviews). A crucial feature is the correct determination of the chiral constants $c_1$, $c_3$ and $c_4$ which appear both in $\pi N$ as well as in $NN$ scattering as a TPE contributions [3; 4]. Our purpose is to extract them with errors from the analysis of the about 8000 scattering data collected from 1950 till 2013. This is based in our previous works [5; 6; 7; 8; 9; 10; 11; 12; 13].

2 Anatomy of the NN interaction and the number of fitting parameters

In Fig. 1 we show the abundance plots for a total number of 7709 pp and np data (the total number of 8124 fitting data includes 415 normalization data provided by experimentalists), in the LAB energy-angle plane. Most high quality fits [14; 15; 16; 17; 18] which have historically been capable of fitting their contemporary NN scattering data with $\chi^2/d.o.f \lesssim 1$ require about 40 parameters for the unknown part of the interaction.

To understand the rationale of this, the anatomy of the NN interaction below pion production threshold is sketched in Fig. 2. The maximal CM momentum corresponding to the inelastic process $NN \rightarrow NN\pi$, which is roughly $p_{CM} = \sqrt{m_{NN}M_{NN}}$. This corresponds to a de Broglie wavelength, which we identify with the shortest resolution scale $\Delta r \sim h/p_{\text{max}} \sim 0.6 fm$. For comparison we also depict a free spherical wave, $\sin(pr)$ with

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**Fig. 1** Abundance plots for pp (top panel) and np (bottom panel) scattering data. Full data base (left panel). Standard 3σ criterion (middle panel). Self-consistent 3σ criterion (right panel). We show accepted data (blue), rejected data (red) and recovered data (green).

**Fig. 2** Left panel: Anatomy of the NN interaction showing the different regions as a function of the distance (in fm) for a resolution $\Delta r = 0.6$fm (see main text). Right panel: The NN provider Android app, available at Google Play Store.

$p = 2k_F$ relevant for nuclear matter. The idea is to coarse grain the interaction down to that scale. On the other hand, nucleons are composite and extended particles made of three quarks, $p = uud$ and $n = udd$, thus we must distinguish between the overlapping and non-overlapping regions as measured by the interaction. For instance, the classical electrostatic interaction the pp potential at a distance $r$ would be

$$V_{pp,EM}(r) = \int d^3r_1d^3r_2 \frac{\rho_p(r_1)\rho_p(r_2)}{|r_1 - r_2 - r|} = \int \frac{d^3q}{(2\pi)^3} \frac{4\pi e^2}{q^2} |G_{E,p}(q)|^2 e^{iq\cdot r} \sim \frac{e^2}{r} \quad r \geq 2\text{fm} \quad (1)$$
where $G_{E,p}(q)$ is the proton electric form factor (we take a dipole). Thus, regarding EM interaction the proton behaves as a point-like particle for $r \geq 2$fm since $V_{pp,E}(2\text{fm}) = 0.714\text{MeV}$ to be compared to the point-like value 0.719MeV. For np one has $V_{np,E}(2\text{fm}) = 0.001\text{MeV}$ compared to a vanishing point-like electric interaction. A similar situation happens for the strong part of the interaction. Using cluster chiral quark model calculation [19] in the Born-Oppenheimer approximation with finite nucleon and NΔ transition form factors, one sees point-like spin-flavour van der Waals interactions with OPE and TPE above 2fm. Likewise, one can also check that above 3fm the main contribution is just OPE. This is consistent with the regularization used for OPE in high quality potentials [14,15,16]. If we switch off this known piece, we are left with an unknown potential with a finite range $r_c = 3$fm. For such a truncated potential, the maximal angular momentum needed for convergence of the partial wave expansion is $l_{\text{max}} = p_{\text{max}}r_c = r_c/\Delta r = N$. On the other hand, the minimal distance where the centrifugal barrier dominates corresponds to $l(l+1)/r_{\text{min}}^2 \leq p^2$, which is $r_{\text{min}} = 0.7, 1.2, 1.7, 2, 2.2, 2.7$fm for $l = 1, 2, 3, 4, 5$ respectively. Thus, for a given $l \leq l_{\text{max}}$ we can count the number of points between $r_{\text{min}}$ and $r_c$ sampled at a resolution $\Delta r = 1/\sqrt{Nm_{\pi}}$, see Fig. 2, which means $l_{\text{max}} = N = 3, 4, 5$ for $r_c = 1.8, 2.4, 3$. We count partial waves according to their threshold behaviour in coupled channels [21], namely $2\nu + 1 L_j = \mathcal{O}(p^{2L}), E_j = \mathcal{O}(p^{2\nu})$ with $E_j$ being the tensor mixing waves. The number of parameters for an unknown interaction below $r_c > 2$fm and momentum $p \leq p_{\text{max}} = 2$fm$^{-1}$ becomes

$$N_{\text{par}}(r_c) \sim 60, 38, 21 \quad r_c = 3, 2.4, 1.8$$fm \tag{2}$$

This counting argument does not consider that some parameters may be either accidentally small or turn out to be compatible with zero. A polynomial counting to order $\nu$ in momentum, gives a hermitian real potential $V_{S,J}^{\nu}(p', p) = p'\nu \nu \sum_k \sum_l c_{S,J}^{\nu}(N,k)(p')^\nu p^{2\nu} p^2$ with $N(\mathcal{O}(p^\nu)) = 2, 7, 19, 41$ total number of $c_{S,J}^{\nu}(N,k)$ parameters for $\nu = 0, 2, 4, 6$ respectively. The expansion has a convergence radius of $|p'|, |p| < m_{\pi}/2$, which is extended to $nm_{\pi}/2$ after additive inclusion of $n\pi$ exchange. Thus for $p \lesssim 3m_{\pi}/2$ one needs $2\pi$ exchange and just 9 coefficients [22]. This corresponds to take $\Delta r \sim 1$fm and or $E_{\text{LAB}} \sim 90$MeV.

### 3 Delta-shell potential fits

There remains the question on how to encode the unknown part of the interaction which should be sampled, or coarse grained, at least with $\Delta r$ resolution [5]. Following a remarkable and forgotten paper by Aviles [23] we have used delta-shell potentials for the inner unknown part to undertake a simultaneous partial wave analysis (PWA) to proton-proton and neutron-proton scattering data from 1950 to 2013 below pion production threshold up to LAB energies of 350 MeV [11] following the pattern of Fig. 2 and taking a charge dependent one pion exchange (OPE) potential above $r_c$ together with electromagnetic effects, vacuum polarization, magnetic moment effects [12]. The delta-shell potential reduces the numerical effort tremendously and enables a fast determination of the covariance matrix whence errors can be determined and propagated for phase-shifts of nucleon matrix elements. With a total of 46 fitting parameters we obtain $\chi^2/\text{d.o.f.} = 1.06$. The consistent database selected in [12] uses the improved 3$\sigma$ criterion proposed by Gross and Stadler [18] which allows to rescue data which would otherwise have been discarded, see Fig. 1. Data and other amusements can be found at the NN provider Android app, available at Google Play Store, see right panel of Fig. 2.

We have also explored the role of chiral two pion exchange ($\chi$TPE) interactions at intermediate and long distances [24]. Comparison of OPE and TPE results are given in tables [12] and [3]. In table we show the $\chi^2$ values corresponding to a direct fit to all the data. These large values prevent error propagation. In table we show the $\chi^2$ values corresponding to a dynamical data base fit to all the data subjected to the 3$\sigma$ criterion, so that the selection of the data depends on the potential. As we see, there is some improvement but data differ. Finally, in table we use the fixed and consistent data from the OPE $r_c = 3$fm analysis. An acceptable

### Table 1 Complete NN database from PWA without rejection. $N_{\text{data}} = 8124.$

| $r_c$ [fm] | 1.8 | 2.4 | 3.0 |
|-----------|-----|-----|-----|
| $N_{\text{m}}$ | $\chi^2/\nu$ | $N_{\text{m}}$ | $\chi^2/\nu$ | $N_{\text{m}}$ | $\chi^2/\nu$ |
| OPE       | 31  | 1.80 | 39  | 1.56 | 46  | 1.54 |
| TPE(NLO)  | 31  | 1.72 | 38  | 1.56 | 46  | 1.52 |
| TPE(N2LO) | 30+3| 1.60 | 38+3| 1.56 | 46+3| 1.52 |
One may wonder why should one determine NN interactions by fitting to higher energies than actually resolved in light nuclei. For instance, one can fit the $\alpha$-particle binding energies of $(B_r, B_\alpha) = (5.2, 20.0)$MeV. Variational mean field shell model calculations yield binding energies for $^4$He, $^{16}$O and $^{40}$Ca at 20% level when phases are fitted below LAB energy, $E_{LAB} \leq 125$MeV. This is so because the interaction becomes soft and short distance correlations become marginal. When the full amplitude is fitted in that energy range errors grow dramatically making for instance $\chi^2$ unacceptable.

$\chi^2 = 1.1$ with 30 parameters, see Eq. (2), allows to propagate errors. In GeV$^{-1}$ units we obtain

$$c_1 = -0.41 \pm 1.08 \quad c_2 = -4.66 \pm 0.60 \quad c_3 = 4.31 \pm 0.17$$

and a correlation $r(c_1,c_3) = -1$. This result depends crucially on making the fit up to $E_{LAB} \leq 350$MeV.

4 Discussion

One may wonder why should one determine NN interactions by fitting to higher energies than actually resolved in light nuclei. For instance, one can fit the $^1S_0$ and $^3S_1$-waves scattering length and effective ranges with just one attractive delta-shell [5], yielding a triton and $\alpha$-particle binding energies of $(B_r, B_\alpha) = (5.2, 20.0)$MeV. Variational mean field shell model calculations yield binding energies for $^4$He, $^{16}$O, and $^{40}$Ca at 20% level when phases are fitted below LAB energy, $E_{LAB} \leq 125$MeV. This is so because the interaction becomes soft and short distance correlations become marginal. When the full amplitude is fitted in that energy range errors grow dramatically making for instance $\chi^2$ statistically invisible vs OPE [13]. The binding of light nuclei does not depend explicitly on the high NN scattering data, but the accuracy of the interaction does. Predictive power can still be achieved by solving the many body problem to this accuracy [7,8].

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Table 2 3\sigma-selected NN database from potential analysis.

| $r_c$ [fm] | $N_{accept}$ | $N_{par}$ | $\chi^2/\nu$ | $N_{accept}$ | $N_{par}$ | $\chi^2/\nu$ | $N_{accept}$ | $N_{par}$ | $\chi^2/\nu$ |
|---|---|---|---|---|---|---|---|---|---|
| OPE | 5766 | 31 | 1.10 | 6363 | 39 | 1.09 | 6438 | 46 | 1.06 |
| TPE(NLO) | 5841 | 31 | 1.10 | 6432 | 38 | 1.10 | 6423 | 46 | 1.06 |
| TPE(N2LO) | 6220 | 30+3 | 1.07 | 6439 | 38+3 | 1.10 | 6422 | 46+3 | 1.06 |

Table 3 Consistent NN database from the improved 3\sigma-criterion, $N_{Data} = \chi^2_{accept} = 6713$.

| $r_c$ [fm] | $N_{par}$ | $\chi^2/\nu$ | $N_{par}$ | $\chi^2/\nu$ | $N_{par}$ | $\chi^2/\nu$ |
|---|---|---|---|---|---|---|
| OPE | 31 | 1.37 | 39 | 1.09 | 46 | 1.06 |
| TPE(NLO) | 31 | 1.26 | 38 | 1.08 | 46 | 1.06 |
| TPE(N2LO) | 30+3 | 1.10 | 38+3 | 1.08 | 46+3 | 1.06 |

$\chi^2 = 1.1$ with 30 parameters, see Eq. (2), allows to propagate errors. In GeV$^{-1}$ units we obtain

$$c_1 = -0.41 \pm 1.08 \quad c_2 = -4.66 \pm 0.60 \quad c_3 = 4.31 \pm 0.17$$

and a correlation $r(c_1,c_3) = -1$. This result depends crucially on making the fit up to $E_{LAB} \leq 350$MeV.