Analysing Vibrations of Dissipative Structures with Connection Disruption

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Abstract. The article contains the mathematical models of vibrations and the algorithm to calculate a constructive nonlinear system in case of a sudden failure process in a connection. The calculation model of the structure is viewed as a discreet dissipative system. From the position of time analysis, the author derives the governing equation of the system reaction under static load before the failure. The author has conducted the analysis of the system reaction and has proved certain dependence for its parameters at the moment of connection fracture. For the critical time point we obtained the kinematic (movements, velocities and accelerations) and force parameters of reaction (restoring, dissipative and inertial forces) in two states of the calculation model: before and after the fracture. The article contains the derivation of analytical expressions defining the leap size of the dynamic reaction parameters of the calculation model at the moment of its damage. The author has presented the auxiliary reaction which allows one to evaluate the side effect caused by the constructive element disruption. The side effect reaction equation presents a vector difference of the reaction after and before the damage. The results are illustrated by the example of a two span steel beam the vibrations of which were caused by the sudden destruction of an intermediate support. The evaluation of the solution accuracy is also presented.

1. Introduction

The work of the construction in conditions of intensive dynamic exposure (impacts, impulses etc) often leads to the destruction of connections and alterations in the design scheme of the structure in the process of vibrations. Connection failure presents a destructive process which changes the ratio between internal and external system forces. Therefore the system needs a quick internal redistribution of internal forces aimed at preserving its integrity.

In Russia one of the first studies in the field of collapsed constructions refers to earthquake-resistant structures where for assessing the dynamic effect the researchers suggested replacing the dynamic reaction at the moment of disruption by the action of a certain impulse load [1]. In brittle material systems the dynamic effect was assessed by means of static analysis [2]. The calculation of progressive collapse of monolith residential buildings for taking into account the immediacy of construction elements removal was performed with the help of static multiplier 2 [3].

The problem of connection destruction has taken an increasing interest in the last decades due to more frequent terrorist attacks. The studies are mainly concerned with the development of methods of reliability evaluation. For this matter flat [4,5] and spatial [6-8] collapse models are used. Another part of the research is aimed at the development of collapse resistance measures [9,10]. In some cases the ideas of designing
and calculating progressive collapse are borrowed from the theory of seismic resistant structures. The literature review shows that the majority of studies are connected with the collapse analysis of steel and reinforced concrete frame structures [4,5,7,11,12]. In [13] the drawbacks of simplified methods based on the experiment are criticized because of their low effectiveness while analyzing random structures. That is why such methods can be applied only for the structures they were calibrated for. We can state that there is a lack of theoretical studies concerning the nature of fundamental ratios in damaged systems. To create a well-grounded mathematical model one should have a clear understanding of the theoretical base of ongoing processes at connection failure. Existing approaches at present do not allow to explain (and model) with the unified scientific voice the ongoing processes in such systems. It is connected with the lack of working out in the field of analytical methods of modeling the system’s response taking into account the effect of connection fracture [8,14].

The regulatory documents in the part concerning the protection of buildings form progressive collapse [15-17] do not contain any universally acclaimed and well-grounded calculation algorithms of suddenly damaged structures, which could provide an accurate quantitative assessment of internal forces and stresses in the structure elements. That is why the development of analytical methods of designing response of the damaged systems is of great importance [14].

The author of the article derives the governing equations for the reaction of the damaged system, which was at static equilibrium before the failure. To achieve this goal he uses the time analysis theory of discreet dissipative systems (DDS), based on the study of characteristic matrix quadratic equation (MQE) [18]. It should be noted that at notation mathematical models nonlinear factors prior to the connection failure are not taken into account, instead we model the subsequent event which occurred as a result of a sudden removal of structural elements [8,7].

2. General comments

Suppose in a certain structural element there happens a sudden failure of connection. Simultaneously it leads to the absence of stress in this element, which results in the static unbalance of the system, as the balance between external and internal forces is broken. The disproportion in the ratio of the given forces demands a quick redistribution of the system by means of its internal resources. To preserve its integrity the system is forced to come to the state of motion accompanied by the appearance of dynamic forces. It is worth mentioning that in new state the system may undergo both vibrational and non-vibrational motion. The character of the motion after the failure may depend on the loading capacity of the structure. On condition that the strength reserve of the damaged system is preserved, its nodes will undergo free decaying vibrations. In the course of time when the vibrations are over, the system will be in a new state of static balance.

3. Reaction of a Statically Loaded System under communication breakdown

The following equation is valid for the state of static balance

$$\nu_{st} = K^{-1}f_Q$$

(1)

where $Q$, $Y_{st}$ are the nodal load and the displacement vectors; $K$ is the stiffness matrix of the model.

Let the stiffness matrix become $K_0$, when a connection fracture occurs (at $t_0$), then the differential equation of DDS motion will be ($t \geq t_0$)

$$M\ddot{v}(t) + C\dot{v}(t) + K_0v(t) = f_Q$$

(2)

where $M$, $C$ are the mass matrices and the damping matrices; $v(t)$ is the displacement vector.

Integrating (2) and taking into account the initial conditions $v_0 = v_{st,0}, \dot{v}_0 = 0$, we get the reaction equation of DDS in a matrix form of Duhamel’s integral [18,19]:

$$v(t) = 2\operatorname{Re}\{x(t)\} + v_{st,0}, \quad \dot{v}(t) = 2\operatorname{Re}\{S_0x(t)\}, \quad \ddot{v}(t) = 2\operatorname{Re}\{S_0^2x(t)\},$$

$$x(t) = \Phi_0(t - t_0)(U_0S_0)^2\Delta K\nu_{st}.$$
where $\Delta K = K - K_0$ is the difference of the stiffness matrices before and after the connection failure; $v_{st,0} = K_0^{-1}f_Q$ is the vector of static displacements in the damaged DDS; $S_0$ is the matrix square root (the solution of the characteristic MQE); $U = 2iM\Im(S_0)$ (i is the imaginary unit); $\Phi_0(t-t_0) = e^{S(t-t_0)}$.

The structure of vector $v(t)$ in (3) shows that the calculation model moved to the state of free vibrations performed according to the new position of static equilibrium (vector $v_{st,0}$).

4. The side effect
The side effect of connection failure can be obtained with the help of vector displacement difference $\Delta v(t) = v(t) - v_{st}$, where $v(t)$ corresponds to the system’s damaged state of (3).

$$\Delta v(t) = 2\Re\{\Phi_0(t-t_0)(U_0S_0)^{-1}\Delta K v_{st} + \Delta v_{st}\}$$

$\Delta v_{st}$ is the difference of static displacements vectors of the system after and before the disruption

$$\Delta v_{st} = v_{st,0} - v_{st}$$

The auxiliary reaction $\Delta v(t)$ expresses the side effect which occurred as a result of connection failure and contains dynamic and static components. The dynamic component describes free decaying vibrations of the nodes according to the new position of static equilibrium of the system.

For damped vibrations ($t \to \infty$) we have $\Phi_0(t-t_0), x(t) \to 0$, thus: $v(t_0) = v_{st,0}, \ddot{v}(t_0) = \ddot{v}(t_0) = 0$.

Then the vectors of elastic, dissipative and inertial forces [18], taking into account $f_Q = K_0v_{st,0}$, take values

$$f_S(t_0) = K_0 v_{st,0}, \quad f_c(t_0) = C_0 \ddot{v}(t_0) = 0, \quad f_f(t_0) = -M \ddot{v}(t_0) = 0.$$  

Consequently, after the vibrations are over the system acquires a new balance of forces where internal efforts (components $f_S(t)$), counterbalance the immobile static load $f_Q$.

5. The DDS reaction at the moment of connection failure
We will define the reaction of the calculation model at the failure moment (at $t_0$). For this matter the critical point $t_0$ will be $t_0^*, t_0^- -$ the time before and after the connection failure, correspondingly.

The parameters for the reaction of the undamaged and the damaged DDS one look like this [19]:

$$\begin{align*}
v(t_0) &= v_{st}, \quad \dot{v}(t_0) = 0, \quad \ddot{v}(t_0) = 0; \\
f_S(t_0) &= K v_{st}, \quad f_c(t_0) = 0, \quad f_f(t_0) = 0.\end{align*}$$  \hspace{1cm} (4)\

$$\begin{align*}
v(t_0^+) &= v_{st}, \quad \dot{v}(t_0^+) = 0, \quad \ddot{v}(t_0^+) = M^{-1}\Delta K v_{st}, \\
f_S(t_0^+) &= K_0 v_{st,0}, \quad f_c(t_0^+) = 0, \quad f_f(t_0^+) = -\Delta K v_{st}.\end{align*}$$  \hspace{1cm} (5)\

6. Analysis of results
We will present the residuals for the reaction parameters at $t_0$ by means of the following formulas:

$$\begin{align*}
\Delta v(t_0) &= v(t_0^-) - v(t_0^+), \quad \Delta \dot{v}(t_0) = \dot{v}(t_0^-) - \dot{v}(t_0^+), \quad \ldots, \quad \Delta f_f(t_0) &= f_f(t_0^-) - f_f(t_0^+). $$
\end{align*}$$

Taking into consideration the formulas (4), (5), we will have:

$$\begin{align*}
\Delta v(t_0) &= 0, \quad \Delta \dot{v}(t_0) = 0, \quad \Delta \ddot{v}(t_0) = M^{-1}\Delta K v_{st}, \\
\Delta f_S(t_0) &= -\Delta K v_{st}, \quad \Delta f_c(t_0) = 0, \quad \Delta f_f(t_0) = -\Delta K v_{st}.\end{align*}$$  \hspace{1cm} (6)\

From (16) it is clear that at the moment of a connection failure part of the reaction parameters ($Y(t)$, $\dot{Y}(t), F(t)$) are continuous time functions. The rest parameters have leaps depending on the residual of the stiffness matrix and a static displacement vector of the undamaged structure.
7. An example of steel beam vibrations caused by the destruction of the middle support

The design scheme of the beam (1-beam size 50) is shown in figure 1 before (a) and after (b) the destruction of the middle support. The stiffness of the beam $EJ = 79454 \text{ kN} \cdot \text{m}^2$, the stiffness of the middle support $k = kEJ / l^3$, where $k = 10^4$. The calculation was performed by breaking the span $l = 15 \text{ m}$ into 10 sections ($a = 1.5 \text{ m}$) and $q = 2,765 \text{ kN} / \text{m}$. The number of the degrees of freedom equals $n = 9$. The elements of the static nodal load $f_Q$ refer to $Q = qa = 4,148 \text{ kN}$.

Figure 2 shows a triangle impulse with the amplitude $R_{05} = 20,699 \text{ kN}$, which presents the reaction in the given support before its destruction.

In figure 3 one can see oscillograms of the kinematic parameters of the beam reaction. The numbers in the graph are the numbers of sections. At $t_0^*$ the values of displacements correspond to the components of the vector $v_{st}$. At $t \rightarrow \infty$ the curves asymptotically strive to the displacements $v_{st,0}$ of the damaged DDS (dash-dotted horizontal straight lines in figure 3, a). The velocities and accelerations (figure 3 b, c) of all the nodes at $t_0^*$ equal zero, excluding: $v_{3}(t_0^*) = 4895.6 \text{ cm} / \text{s}^2$.

Figure 3. Kinematic parameters of the beam reaction at a sudden destruction of the middle support: a, b, c – oscillograms of displacements, velocities and accelerations of the nodes correspondingly; d – the central node displacements (dash-dot – the same influenced by the action of the triangle impulse).
Figure 3d compares the displacements of the central node (section 5) for the beam at a sudden destruction of the support according to the method described in [1], when in node 5 there is an impulse load which changes according to the linear law (figure 2). These curves asymptotically strive to different static displacements: \( v_{5,t_0} = 0.459 \text{ cm} \) (for the damaged beam – a solid curve), \( v_{5,t_0} = 0.879 \cdot 10^{-3} \text{ cm} \) (according to [1] – dotted curve).

The oscillograms of elastic \( R(t) \), dissipative \( F(t) \) and inertial \( I(t) \) forces are given in figure 4 a, b, c, correspondingly. Elastic forces before the support destruction in all sections equal the static value of the load \( R(t_0) = Q \). At \( t_0 \) the value of elastic forces remained the same, except section 5, where \( R(t_0) = -16.551 \text{ kN} \). At \( t \to \infty \) all the curves strive to the value \( Q \).

**Figure 4.** Force parameters of the beam reaction at a sudden destruction of the middle support: a, b, c – oscillograms of elastic, dissipative and inertial forces correspondingly; d – residual oscillograms of the forces from the left and right parts of the equation of motion of the DDS.

The quality of the obtained solution can be assessed with the help of a vector function \( f(t) \), presenting an algebraic sum of all the forces in the left part of the equation of motion (2): \( f(t) = f_e(t) + f_c(t) - f_f(t) \). The character of the solution convergence is shown on the residual oscillograms of the forces from the left and right parts of the equation: \( \Delta f(t) = f(t) - Q \) (figure 4 d).

The given graphs prove high solution accuracy, the error does not exceed the value \( \epsilon \leq 6 \cdot 10^{-12} \text{kN} \).

**8. Conclusion**

From the position of the time analysis theory we introduced a method of calculating discreet dissipative structures with the connection failure under the influence of static load.

1. For the design model of the structure at the moment of connection failure we obtained the analytical expressions of the leaps that characterized the parameters of the dynamic reaction.
2. The leap formulas do not depend on the type of the design model (flat or spatial), as while deriving there were no limitations concerning the equation of motion.
3. The equation for the reaction of the auxiliary effect at the connection failure includes the dynamic and static components. The dynamic component describes free damped nodal vibrations caused by sudden change of model stiffness, while the static component is defined by the vector residual of node displacements after and before the connection failure.

The obtained results of a new analytical approach allow a more profound study of the behavior of structure systems in the conditions of failure process and for modeling effective reliability assessment.
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