When Auction Meets Fixed Price: A Theoretical and Empirical Examination of Buy-it-Now Auctions

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A Theoretical and Empirical Examination of Buy-it-Now Auctions

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Recently fixed pricing and auctions have been brought together in a new pricing format that offers bidders the option of prematurely ending an auction at a fixed price. eBay calls this a “Buy-it-Now” auction, uBid calls this “UBuy it” and Yahoo auctions refer to this as a “Buy Now”. The growing popularity of auctions presents an interesting pricing decision for managers: whether to sell at a fixed price, sell in a regular auction, or to sell through a buy-it-now auction. By posting a fixed-price at an auction, the seller provides customers a convenient option to buy the item directly without bidding. However, in doing so, the seller implicitly imposes a maximum bidding level that could potentially limit revenue. This paper studies this new pricing format and answers the following research questions: why is fixed price used at traditional auctions, will buy-it-now increase the seller’s profit, how is an optimal price determined, and how is the buy-it-now decision influenced by key factors such as the customer’s cost of participating in the auction, the seller’s reserve price, and the number of potential customers. Our results show that buy-it-now auctions can increase both customers’ utility and sellers’ profit under certain conditions. We empirically test the predictions of our theoretical model using data collected from eBay.
1 Introduction

Auctions are quintessentially a variable pricing format, but recently the largest online auction sites have introduced a fixed-pricing element known as a buy-it-now option. This feature allows a bidder to immediately stop an ongoing auction and purchase the item at a fixed, posted price set by the seller. This feature is known as “Buy-it-Now”, “uBuy”, and “Buy-Now” at eBay, uBid, and Yahoo, respectively. For clarity we refer to all these services generically as buy-it-now. These buy-it-now features have become an important component of online auctions. eBay introduced its service in 2000. Subsequently it has been adopted by 45% of eBay’s U.S. auctions by the end of its first year. These buy-it-now auctions accounted for 29% of gross merchandise sales at eBay in 2003. Additionally, empirical work shows (Park and Bradlow 2003) that the buy-it-now feature is an important element in auction design.

Economists advocate the efficiency of auctions over fixed price markets when auction costs are low (Wang 1993). The principle being that auctions match products to customers with the highest valuation in the market. However, the buy-it-now feature effectively limits the maximum auction price which would seem to lower the profitability of auctions for sellers. This leads to several perplexing problems: why would sellers choose to adopt buy-it-now auctions, when should they be used, and how should an optimal buy-it-now price be set? To answer these questions we develop an analytical model of auctions. Surprisingly, our results show that the buy-it-now option can actually increase the expected profits from an auction if customer participation costs are substantial.

Our model considers a potential customer’s optimal bidding strategy, willingness to pay, prob-
ability of winning, and the costs of participating in an auction. These participation costs are borne by the customer and refer to the opportunity cost associated with waiting for the auction to close, transaction costs associated with bidding, and the customer’s cognitive effort expended in the bidding process. These participation costs lower the number of bidders, which in turn reduce the seller’s profit. The buy-it-now option reduces these participation costs and attracts buyers who might otherwise bypass the auction.

To illustrate this problem consider a customer bidding for an ideal gift for her friend’s birthday coming up in one week. The buy-it-now auction presents two options: to bid and wait for the auction to close in three days to find out if she will win, or to purchase at the posted price right away (provided her valuation is higher than the price). If she chooses to bid, then there is a chance that she may not win, in which case she has to start over and bid again or buy at another store. In this example the auction participation cost borne by the customer includes the opportunity cost associated with waiting, checking emails to see if she has been outbid, potentially having to pay an extra charge for quick shipping if she loses and must buy elsewhere, and the possibility of not having a gift on time for her friend’s birthday. The buy-it-now option allows the auctioneer to lower the transactions costs to potential buyers, which increases the potential customer pool and the potential selling price. Hence, buy-it-now options are most valuable when a seller faces a customer base with participation costs.

The use of buy-it-now auctions also helps address the question of when sellers should use auctions versus fixed, posted price formats found in most retail contexts. If sellers set a buy-it-now price that is ridiculously high then these formats are immaterial since the buy-it-now option is never exercised. If the buy-it-now price is set too low then consumers never purchase through
a bidding process, but instead always exercise the buy-it-now option. Additionally, the simplicity of the fixed, posted price found in most retail environments may generate higher profits than the hybrid auction format which buy-it-now pricing affords.

In order to test our theory we conduct an empirical analysis using real auction data from eBay. We conduct our analysis using two methods. First, we examine the adoption of buy-it-now auctions by sellers at an aggregate level and see if their behavior is consistent with our theoretical predictions. Second, we test the assumptions of our consumers' equilibrium strategies using a choice model. We predict which auction a consumer chooses given the set available at the time they made their bid. Auction choices reveal the trade-offs customers make between different auction formats and attributes.

Our paper contributes to both the existing auction and marketing literatures. To our knowledge this work is the first to incorporate participation or transaction costs to explain the use of buy-it-now auctions and additionally to empirically test such a theory. In the past, consumer reactions to marketing elements at auctions have been largely ignored (Chakravarti, et al. 2002). Although consumers’ limited-information acquisition-ability in a traditional retail setting has gained considerable acceptance (e.g. Ratchford 1982; Mehta, Rajiv and Srinivasan, 2003), in marketing literature consumer uncertainty about the surplus associated with the choice of bidding or a buy-it-now purchase has not been studied. This paper not only sheds light on the trade-off faced by consumers but also the profit-maximizing pricing strategy for sellers who use auctions as a channel to sell their products.
2 Literature Review

Formal research on buy-it-now auctions is limited given its recent introduction. However, several papers consider this problem directly. Mathews (2003) argues that the buy-it-now option is beneficial to impatient bidders who discount future utility. Impatience refers to the notion that customers want their product in a shorter span of time than others. However, impatience does not reflect the limited cognitive resources that most consumers appear to apply in making decisions (Ratchford 1982, Mehta, Rajiv and Srinivasan 2003).

Additionally, recent online auction studies have empirically and experimentally documented such transaction costs that are an important component of our participation costs. Bajari and Hortascu (2003) empirically studied common-value auctions at eBay for mint and proof sets of U.S. coins. They found the average cost of bidding in those categories to be $3.20 (with a standard error of $1.48, and a mean book value of $50.10). Additionally, in a field experiment for nearly $10,000 worth of sports cards, List and Lucking-Reiley (2002) found that cognitive costs influence subjects’ strategic behavior. Hence, we believe participation costs should be reflected in a model of bidder behavior, which contrasts with past work on buy-it-now auctions which has focused on risk aversion and impatience as potential explanations.

Budish and Takeyama (2001) investigated a two-bidder model and show that if the customers are risk averse then the seller has an incentive use post a buy-it-now option. Furthermore, Hidvégi et al (2002) study the bidding strategies at ascending English auctions with buy-it-now options, and conclude that risk-averse sellers should prefer setting buy-it-now prices at auctions while risk-neutral sellers should be indifferent between a regular auction and an auction with a buy-it-now
option when the price is appropriately set. Although some auctioneers could be risk averse, many large retailers that have opened eBay stores, like Dell, IBM, Sun Microsystems, and Sony are unlikely to be risk averse. Hence, we feel that risk aversion is not a universal explanation.

Furthermore, past work assumes full and exogenous participation of bidders (Budish and Takeyama 2001, Mathews 2003). This assumption is quite important in their results, and our model endogenizes this participation decision, which leads to a participation threshold. Additionally, our work incorporates a buy threshold. Compared to the conditional strategies proposed by prior research,\(^1\) we believe our model better reflects consumer behavior and more realistically models typical online auction formats such as eBay. For example, the bidding strategy we derive in the next section allows customers to arrive sequentially and make a one-time decision upon arrival.

## 3 A Theoretical Model

In this section, we develop a theoretical model to study the strategies of the seller and customers at a buy-it-now auction. Our model is set up as a two-stage sequential game (Figure 1). In the first stage, the seller decides whether to incorporate a buy-it-now option with knowledge about the number of potential customers, an estimate of the customers’ bidding costs, and the seller’s own reservation price for the product. In the second stage, the customers observe the seller’s decision, and make decisions about participation and purchase.

\(^1\)The previously derived conditional strategies (e.g. Hidvégi, 2002; Reynolds and Wooders, 2002) assumed that all the potential bidders are present at the same time and monitoring the auction throughout the whole time while the auction is open and are able to revise their bids immediately once a certain bid level is reached.
Figure 1: Sequential Moves of the Firm and Consumers.

To clarify our terminology, we use bidding to imply that a customer has chosen not to exercise a buy-it-now option but to participate in the usual auction process. If at the close of the auction the bidder has placed the high bid, that winning bidder becomes a buyer, since there is only one product there is only one buyer. If a customer exercises a buy-it-now option then the customer becomes a buyer and buys the product at the seller’s stated buy-it-now price. The assumptions we make in this model are stated as follows.

Assumption 1: Auction Setup. We assume a single-period auction for a risk-neutral seller who uses a second-price sealed bid auction to sell one item. The second-price sealed bid auction is effectively equivalent to eBay’s proxy bidding format. We assume that the seller owns the item and exogenously sets the reservation price. In the following model, we assume that seller’s reservation price is zero, and we keep the discussion of the non-negative reservation case in the Appendix. Additionally, we assume that once a bid has been placed that exceeds the reservation price the buy-it-now option disappears or effectively the auction
is converted back to a regular auction. This follows eBay’s implementation of the buy-it-now auction.

The seller’s decision is whether to incorporate an buy-it-now option into a traditional second-price sealed bid auction. There are $n$ potential customers in the market who are aware of the auction and may be interested in purchasing the item, where $n \geq 2$. We assume that this is a private-value auction in that each customer knows their own valuation for the auctioned item perfectly, but only knows the distribution from which the other customers’ valuations are drawn. For simplicity, we assume that these valuations are independently and identically drawn from a uniform distribution on the interval $[0, 1]$. In summary, the common knowledge of customers includes the number of potential customers at the auction, the customers’ valuation distribution, the participation cost, and the seller’s reservation price posted at the auction.

Assumption 2: Customers’ auction participation cost at auctions. The search cost $c$ is assumed to fall between the upper and lower bounds of the valuation distribution $c \in [0, 1)$. This is a mild assumption since we assume that after paying the auction participation cost a customer can still derive some positive surplus. It is a trivial case when the auction participation cost is higher than the upper bound of the valuation distribution, because no one will participate in the auction. A customer only incurs their participation cost upon submitting a bid to the auction. This contrasts with the entry cost present in other auction research which requires a customer to pay a fee to know their valuation. We believe this assumption better represents the online auctions observed in practice.

Assumption 3: Customer’s purchase decisions are endogenously made. Because cus-
tomers incur a cost when they engage in bidding, it is natural to assume that they follow a rule when making “bid” or “buy” decisions. We assume that they solve for “cutoff points” or “thresholds” internally by calculating the expected utility for each choices and act optimally given their own valuations. This assumption will be discussed further in the following section.

3.1 Consumers’ Bidding Strategy

We use backward induction to solve this game. The first problem that needs to be solved is the bidders’ optimal bidding strategy in the second stage. For tractability, we limit our attention to pure strategies.

We begin by studying a customer’s decision process when she arrives at a particular regular auction. We index the customer by subscript $i$. Since the auction is open, the customer has the option of submitting a bid. If customer $i$ participates (by submitting a bid) and wins then her utility is the surplus (excess value above second highest bid) less the auction participation cost $c$.

If the customer participates and loses then her utility is decreased by the auction participation cost $c$:

$$U_i = \begin{cases} 
(v_i - z)^+ - c, & s_a < v_i \\
-c, & s_a \geq v_i 
\end{cases}$$

(1)

where $(x)^+$ refers to the positive part of the $x$,

$$z = \max \{v_1, ..., v_{i-1}, v_{i+1}, ..., v_n\},$$

and $n$ refers to the number of potential bidders for this auction.
A participation threshold \( (s_a) \) is defined as the valuation at which the customer is indifferent between bidding and not bidding. In the following analysis, the “participation-threshold” is used to refer to the borderline valuation, cross which a customer will participate in the bidding process. An equivalent way of understanding this threshold is to consider that with a valuation equal to \( s_a \), a customer can win the item only if she is the only bidder (Riley and Samuelson 1981, Samuelson 1985, Menezes and Monterio 2000). A customer’s expected utility of bidding given the number of bidders is

\[
EU_i(v_i) = v_i[F(s_a)]^{n-1} + (n-1) \int_{s_a}^{v_i} (v_i - z) [F(z)]^{n-2} f(z) \, dz - c
\]  

(2)

After integrating by parts, (2) becomes

\[
EU_i(v_i) = s_a[F(s_a)]^{n-1} + \int_{s_a}^{v_i} [F(x)]^{n-2} dx - c,
\]  

(3)

where \( F(.) \) denotes the cumulative distribution function of the valuations. The first part of equation (2) represents the case when all other bidders’ values are less than the threshold \( (s_a) \). In other words, this is the case where the customer with a value equal to the threshold wins the auction. The second part captures the more general case, the expected gain from bidding when a customer is facing other opponents whose valuations are above the threshold \( s_a \). The participation threshold by definition solves the zero-profit condition defined by equation (3):

\[
EU_i(v_i = s_a) = 0
\]  

(4)

Under the uniform assumption we made on customers’ valuations, this reduces to

\[
s_a = c^{1/n}.
\]  

(5)

If upon arrival the buy-it-now option is available, the bidder may execute this option and immediately purchase the item for the stated price. The buy-option is available if no other
customers have submitted a bid or have already executed the buy-it-now option. When the buy-it-now option is present, a customer must ex ante evaluate both options before making the purchase decision. She has to evaluate the expected gains from both the “buy” and “bid” options. In this case, a buy-threshold $s_b$ defines the valuation at which the customer is indifferent between the two options, conditional on participation. Therefore, the buy-threshold solves the following equation:

$$EU_i^{bid}(v_i = s_b) = EU_i^{buy}(v_i = s_b)$$

(6)

Again using the uniform assumption and the results of the participation threshold ($s_a = c^{1/n}$) the expected utility of submitting a bid for an eligible bidder with a valuation $v_i > s_a$ simplifies to:

$$EU_i^{bid}(v_i) = \frac{v_i^n - c}{n}$$

(7)

It is straightforward to see that the expected utility from purchasing the product directly at the buy-it-now price for this bidder is

$$EU_i^{buy}(v_i) = v_i - B$$

Therefore the buy-threshold defined in (6) solves

$$s_b - B = \frac{s_b^n - c}{n}.$$  

(8)

Lemma 1  The consumer’s surplus curves for buying and bidding intersect at most once (in the non-negative domain).

Note that the bidding surplus is convexly increasing with a slope less than or equal to 1.

$$\frac{\partial \frac{v_i^n - c}{n}}{\partial v_i} = v_i^{n-1} \leq 1$$

(9)
Figure 2: Expected Utility Surplus for Bidding and Buying.

The buying surplus is a straight line with the slope being exactly equal to one. Figure 2 plots customer’s expected surplus from bidding (the dashed line) and buying (the straight line) over the range of possible valuations. The intersection of the bidding surplus curve and the zero-utility line (the horizontal line) defines the participation threshold, and the intersection of the utility of bidding and buying curves indicates the buy-threshold. The above analysis on the thresholds and Figure 2 are useful in understanding consumer behavior at an auction with a fixed price. The position of these two curves guides the optimal action that a customer should take: to bid or to exercise the buy-it-now option. The customer’s optimal strategy in the buy-it-now auction is a two-threshold strategy, summarized in the following theorem:

**Theorem 1** In a buy-it-now auction, the following symmetric equilibrium exists for the potential bidders, which involves two thresholds, the buy-threshold ($s_b$) and the participation-threshold ($s_a$):

(1a). If $B > s_a$: A customer with a valuation $v_i \in [s_b, 1]$ will exercise the buy-it-now option and buy at the posted price upon arrival. If $v_i \in [s_a, s_b)$ then the customer will place a bid. The
customer will leave the auction without submitting a bid if \( v_i \in [0, s_a) \).

(1b). If \( B < s_a \): A customer with a valuation \( v_i \in [B, s_a] \) will exercise the buy-it-now option and buy at the price upon arrival. The customer will leave the auction without submitting a bid if \( v_i \in [0, B) \).

(2). When the buy-it-now price is not available: the auction becomes a standard second-price sealed bid auction, in which the optimal bidding strategy for a customer is to bid their true valuation conditional upon their participation.

In summary, each customer utilizes a rational expectation to anticipate the strategies of other customers and computes \textit{ex ante} utilities from the available options.

The two thresholds in (5) and (8) offer the following insights:

**Proposition 2** At a buy-it-now auction, the participation-threshold \( s_a \) increases in the auction participation cost \( c \) and the total number of potential bidders \( n \). The buy-threshold \( s_b \), however, decreases in \( n \) and the auction participation cost \( c \). It increases in the posted buy-it-now price \( B \).

\textit{Ceteris paribus}, as the auction participation cost increases, it becomes more costly to bid and the buy-it-now option becomes more attractive. For illustration purposes, in both plots in Figure 3 the buy-it-now price is set at 0.7. Figure 3(a) illustrates that when participation costs increase from 0.1 to 0.4, the participation threshold increases from 0.63 to 0.83. Notice that if \( c = 0.4 \) and the buy-it-now price is 0.7, then the customer will not bid but instead will either exercise the buy-it-now option and buy outright or not buy at all. Similarly, as the number of potential customers increases (in Figure 3(b), \( n \) increases from 5 to 8) competition in the bidding process
intensifies. As a result, the participation threshold required to bid increases, and fewer potential customers can afford to bid ($s_a$ increases from 0.63 to 0.75). In both examples, the buy threshold reduces to the price set by the seller.

3.2 The Seller’s Profit Maximization Problem

Now we consider the game’s first stage: the seller’s decision about whether to include a buy-it-now option and if so what price. The seller’s decision is based upon the customers’ strategy discussed in the previous section. Additionally, we assume that if the seller selects a buy-it-now auction, then this option disappears after the first buyers. This mirrors eBay’s implementation of a buy-it-now format, since we assume that the seller’s reserve is equal to 0.

Another insight from Figure 3 is that various pricing strategies can lead to different action spaces for customers. The seller not only needs to pick a buy-it-now price, but also needs to consider the resulting selling format. For example, if the buy-it-now price is set lower than the
participation threshold, then the “bid” option disappears from a customer’s decision space. This happens since a customer with a valuation greater than the participation threshold would not want to bid because the potential gain from bidding never exceeds directly exercising the buy-it-now option. For customers with a valuation lower than the posted price, they will not buy since this generates negative surplus. Bidding is not an option either since it requires an even higher threshold. Therefore, a price set lower than the participation threshold will result in a “pure” fixed-post format similar to the usual retail environment. If on the other hand the buy-it-now price is set too high then bidding is preferred.

A buy-it-now auction only occurs when customers perceive a chance for the auction to change format, namely, a chance for bidding as well as the opportunity to exercise a buy-it-now option. These cases are shown in Figure 4. The seller gets an expected revenue equal to that of a regular auction when posting a price $B \geq p_2$, and a revenue of the standard fixed-price, if $B \leq p_1$. The revenue associated with the buy-it-now price is achieved when $p_1 < B < p_2$. According to Lemma 1, the expected surplus curves of purchasing outright from the exercise of a buy-it-now option and bidding intersect at most once. Therefore, we can solve for the bounds on the pricing solution to identify when a buy-it-now auction occurs:

$$p_1 = c^{1/n}$$  \hspace{1cm} (10)

$$p_2 = 1 - \frac{1 - c}{n}$$  \hspace{1cm} (11)

The above analysis suggests that posting a fixed price can result in different auction formats, hence different expected revenue functions. The seller needs to consider the impact of the selected price on the auction format and adopts the price and format that yields the highest profit. Now
we consider each of these three formats and determine the optimal pricing decision for the seller. As we noted earlier, any buy-it-now price lower than the participation-threshold, i.e. $B < c^{1/n}$, yields a pure, fixed-price strategy. Furthermore if $B > 1 - \frac{1-c}{n}$, the price posted at the auction has no effect since bidding gives customers higher expected utility than buying for all possible valuations. Consequently, the buy-it-now auction turns into a regular auction. Therefore, the seller’s expected revenue is a step function:

\[
\pi = \begin{cases} 
\pi_c, & B \in [0, c^{1/n}] \\
\pi_b, & B \in (c^{1/n}, 1 - \frac{1-c}{n}] \\
\pi_a, & B \in (1 - \frac{1-c}{n}, 1]
\end{cases}
\]  

(12)

We denote $\pi_c$, $\pi_b$, and $\pi_a$ as the expected revenue from fixed-price, a buy-it-now auction, and a regular auction respectively. We derive the profitability of each type as follows.

The expected revenue a monopoly seller facing $n$ customers in a fixed-price setting is

\[
\pi_c = B (1 - B^n).
\]  

(13)
This function is concave and has a unique optimum. The expected revenue of a regular auction with endogenous participation equals the profit from those who participate in bidding:

$$\pi_a = n (n - 1) \int_{s_a}^{1} v [1 - F (v)] [F (v)]^{n-2} f (v) dv.$$  \hspace{1cm} (14)

If customers’ valuations follow a uniform distribution this simplifies to:

$$\pi_a = \frac{(n - 1) \left[1 - (n + 1) c + nc^{(n+1)/n}\right]}{n + 1}.$$  \hspace{1cm} (15)

Next we examine the revenue from a buy-it-now auction. As mentioned earlier in Assumption 1: Auction Setup, after the reserve price is met the buy-it-now option disappears, hence the arrival process is important in assessing the auction. A customer’s action potentially alters the format of the auction. For instance, by submitting a bid at an auction that the seller initially offered with a buy-it-now auction, the auction is converted into a regular auction. Alternatively if the consumer exercises the buy-it-now option then the auction ends and is no longer available to other customers.

Therefore, to obtain the seller’s expected profit we need to condition upon the first customer’s action.\footnote{In our model, customers arrive randomly at an auction. We do not consider the case where customers strategize concerning the sequence of entry in this paper.} Customers are indexed as \(i = 1, 2, \ldots n\). If the first customer quits the auction without doing anything, i.e. \(v_1 < s_a\), then the second customer faces the same decision: to buy at the buy-it-now price, to bid, or to not participate. Note that \(v_1 < s_a\) is a sufficient condition for the first customer to take not participate. The price \(B\) must be higher than the participation threshold in order for buy-it-now auction to occur. This process continues until the auction receives a customer who either submits a bid or ends the auction by exercising the buy-it-now
option, or the auction ends without receiving any bids from any of the \( n \) potential customers. Knowing this process, the seller can compute expected revenue for each customer conditional upon the previous one. We denote \( V_i \) as the seller’s expected revenue starting from customer \( i \).

His \textit{ex ante} expected revenue from a buy-it-now auction \( \pi_b \) is just \( V_1 \). The recursive nature of the process means that the seller anticipates expected revenue \( \{V_1, V_2, ..., V_n\} \) from the customer \( \{1, 2, ..., n\} \), where

\[
V_1 = s_a V_2 + (1 - s_a) [s_b \pi_a + (1 - s_b) B]
\]

\[
V_2 = s_a V_3 + (1 - s_a) [s_b \pi_a + (1 - s_b) B]
\]

\[\vdots\]

\[
V_n = s_a \cdot 0 + (1 - s_a) [s_b \pi_a + (1 - s_b) B]
\]

Solving recursively gives the seller’s \textit{ex ante} buy-it-now auction revenue:

\[
\pi_b = V_1 = (1 - s_a^n) [s_b \pi_a + (1 - s_b) B]
\]

The seller’s problem is to choose an optimal buy-it-now price \( B^* \) to maximize total expected revenues:

\[
B^* = \arg \max \left[ \max_{B < p_1} \pi_c, \ \max_{p_1 \leq B \leq p_2} \pi_b, \ \max_{B > p_2} \pi_a \right]
\]

In order to decide on the optimal buy-it-now price, the seller first computes the optimal price in each of the three intervals of \( B \): fixed-price interval \([0, p_1]\), buy-it-now auction interval \((p_1, p_2)\), and auction-only interval \([p_2, 1]\). Note that in the auction-only interval the expected revenue \( \pi_a \) is a constant, and no price level has to be determined. Then the seller picks the pricing decision
that yields the greatest expected revenue from the three. Figure 5 shows an example \((n=4, c=0.02)\) in which the buy-it-now price is set at 0.68 and achieves the optimal expected revenue for the seller.

![Figure 5: Seller's Expected Revenue](image)

The above analysis requires that the seller know the size of the customer pool and the participation cost of bidding to make an optimal decision concerning auction format (fixed price, auction or buy-it-now auction) and the corresponding buy-it-now price or fixed-price as appropriate. Table 1 outlines the seller’s pricing options and the conditions under which each would be implemented.

Due to the dynamic nature of the buy-it-now auction, i.e. the auction format is determined by action of the first bidder (which terminates the buy-it-now option) or the first buyer (which ends the auction), it is useful to study the properties of the expected revenue function over the
### Table 1: Seller’s Optimal Pricing Format and Prices

| Selling Format and Range | Seller’s Expected Revenue Function \( \pi \) | Optimal Buy Price \( B^* \) | Optimal Revenue |
|--------------------------|------------------------------------------|-----------------------------|-----------------|
| Fixed Price \([0, c^{1/n}]\) | \( B(1 - B^n) \) | \((1 + n)^{-1/n}\) | \((1 + n)^{-1/n}(1 - (1 + n)^{-1}) \) if \( B^* \in [0, c^{1/n}] \) \( c^{1/n}(1 - c) \) otherwise |
| Buy-It-Now Auction \((c^{1/n}, 1 - \frac{1-c}{n})\) | \( q(s_B A + (1 - s_B)B) \) | \( \frac{2n + An - 2 + 2c}{3n} \) | \[ \begin{cases} 
q(A + \frac{2\sqrt{2/3}(n-1+c-An)^{3/2}}{3n\sqrt{n(n-1)}}), & \text{if } B^* \in (c^{1/n}, 1 - \frac{1-c}{n}] \\
q(A + \frac{\sqrt{2(1+c+n-cn^{1/n})}(-A+c^{1/n})}{\sqrt{n(n-1)}}, & \text{if } B^* \in [0, c^{1/n}] \\
qA, & \text{if } B^* \in (1 - \frac{1-c}{n}, 1] 
\end{cases} \] |
| Pure Auction \((1-\frac{1-c}{n}, 1)\) | \( A \) | - | \( A \) |

where \( s_B = 1 - \frac{\sqrt{2(1 - B)n + c - 1}}{\sqrt{n(n-1)}} \), \( A = \frac{(n-1)(1-(n+1)c+nc^{1/n})}{n+1} \), \( q = 1 - c \).

The expected revenue of a buy-it-now auction is concave and has a unique optimum in the buy-it-now price \( B \).

Proposition 3. The expected revenue of a buy-it-now auction is concave and has a unique optimum in the buy-it-now price \( B \).

Maximizing the expected profit defined in (17), we obtain the optimal buy-it-now price, denoted as \( B^*_b \):

\[
B^*_b = \frac{2 + \pi a}{3} - \frac{2(1 - c)}{3n} \tag{19}
\]

Proposition 4. The optimal buy-it-now price increases in the number of potential customers \( n \).
A comparative static analysis on the optimal buy-it-now price with respect to $n$ shows that as the number of potential customers increases then the optimum buy-it-now price increases. The intuition is that demand increases, it is less likely that the product will go unsold and the seller can potentially gain more profit from an auction than selling it at a fixed price. Hence the seller relies less on the buy-it-now option to achieve the optimal revenue. Nonetheless, when $n$ increases, a customer’s buy-threshold decreases—holding everything else constant—the customer is more likely to buy. To prevent the item being sold at a lower price than it necessary, the seller charges a higher buy-it-now price.

**Proposition 5**  *As the auction participation cost of bidders (c) increases, it is not always better for the seller to increase the buy-it-now price.*

The increase in the auction participation cost has two effects: it raises the participation threshold and decreases the buy-threshold for potential customers. By posting a fixed-price at the auction, the seller can increase the chance of a sale knowing that the expected revenue from selling in the standard auction format is low. This intuition is that the high auction participation cost directly reduces the seller’s expected auction revenue. Lowering the buy-it-now price reduces the chance that a buy-it-now auction is turned into a regular auction by the first bidder which leads to less expected profit.
4 Empirical Analysis

The main purpose of the empirical analysis is to test the findings of our theory concerning seller and the bidder behavior, particularly if the properties of the two thresholds are supported. We take two quite different approaches to analyze the data to reduce our reliance upon a single methodology. We start by examining the data at an aggregate level to see if the seller’s buy-it-now decision is influenced by the variables as predicted by our theoretical model. Secondly, we estimate an individual-level choice model to measure the consumer’s revealed preferences for various auction formats and attributes.

4.1 Data

We collected auction data from eBay during the period April 1 to May 20, 2003 for the following four product categories: Lexar memory stick 128 MB (only new items), Apple iPod MP3 player 10 GB (including both old and new items), KitchenAid 525W Mixer (only refurbished), and KitchenAid KSM103 Professional Mixers (all new). In the first two categories, sellers consist of both individual sellers and eBay’s “power sellers”. KitchenAid is the only seller for the two mixer products. At the time our data was collected, the average retail prices for the four categories was respectively $60, $400, $399 and $329. We queried eBay many times per hour since the buy-it-now price disappears during the course of the auction once the seller’s reserve price is met. Additionally, the reserve status, original duration, and the feedback ratings of both buyers and sellers can change during the auction. If the data was collected only after the auction close, we would not fully recover this information.
During one and a half months, we collected without interruption a total of 328 auctions for Lexar memory sticks, 273 auctions for iPod players, 292 auctions for KitchenAid 525W mixers, and 176 auctions for KitchenAid KSM103 professional mixers. The data set contains detailed auction information such as complete bidding history, the auction starting date and time, end date and time, buy-it-now prices, bidder ID, feedback ratings for both the sellers and buyers, the minimum bid, the number of bids received, and the reserve status (i.e., whether the auction has a secret reserve, and if so whether it has been met). We also have the item condition information (new or used) for the four product categories.

4.2 An Aggregate Level Analysis

We look at the descriptive statistics of some variables that are proxies for the parameters in the theoretical model. The results are consistent with the predictions (see Table 2). Across the four categories (Lexar, iPod, KA-525, KA-KSM103), the sellers’ adoption of buy-it-now auctions varies greatly (listed in an ascending order): 21% for Lexar memory sticks, 34% for iPod auctions, and 78% for the two KitchenAid mixers.

Our theory predicts that when the number of potential customers increases, we should observe the sellers using the buy-it-now option less frequently. We use the average number of unique customers per auction in each of the categories as a proxy for the number of potential customers in that category. The means and the standard errors are shown in the third column. Indeed, as the number of customers increases (from 5.2 for KA-KSM103 to 8.5 for iPod), the use of the buy-option decreases.
Our theory also suggests that as auction participation cost increases and when the seller has a reserve for the product, we should observe the buy-option being used more often. We use customer’s experience (feedback ratings) as an inverse proxy for the auction participation cost (e.g., List and Lucking Reiley, 2002), and the average medians are respectively 36.3, 49.8, 15.8, and 11.6 for the four categories. This indicates that the buy-it-now option adoption goes in the same direction as the auction participation cost. We use the percentage of sellers using the reserve option to approximate the reservation price that the seller has for the item. (We cannot directly observe the seller’s reservation price.) The result shows that compared to the two KitchenAid products the iPod and Lexar memory stick sellers use reservation and buy-it-now options less frequently.

To test sellers’ behavior more carefully we use a logistic regression to examine the seller’s decision of adopting the buy-it-now option at an auction-level. We use all the auctions collected in the four product categories and study the effects of the key variables: the number of customers, the reserve price and bidder’s auction participation cost, on the auction format decision. The reserve price variable is the maximum of the starting bid and the secret reserve of an auction. The secret reserve is recovered from the bid price when the message “the reserve price is met” is shown. The dependent variable is a dummy variable, equal to one if the buy-option is used and zero otherwise. The results (in Table 3) show that all the estimates have the expected sign and are all significant at the 0.001 level. This provides strong evidence that sellers’ behavior is consistent with the findings of our theoretical model.
| Buy-it-Now Adoption | # of Unique Bidders per Auction | Secret Reserve Option | Bidder experience (avg. median) |
|---------------------|--------------------------------|-----------------------|-------------------------------|
| Lexar 128MB         | 21%                            | 8.47                  | 4%                            | 36.25                         |
| memory stick        | (0.41)                         | (3.02)                | (0.20)                        | (58.36)                       |
| IPOD 10GB           | 34%                            | 8.07                  | 25%                           | 49.78                         |
| MP3 player          | (0.47)                         | (5.43)                | (0.43)                        | (280.92)                      |
| KitchenAid          | 78%                            | 7.56                  | 98%                           | 15.81                         |
| 525W Mixer          | (0.41)                         | (2.99)                | (0.13)                        | (30.83)                       |
| KitchenAid          | 78%                            | 5.21                  | 77%                           | 11.62                         |
| KSM103              | (0.41)                         | (4.75)                | (0.42)                        | (29.60)                       |

Table 2: Descriptive Statistics of the Key parameters of the Model

| Proxy variable | Hypothesized Sign (for the proxy variable) | Parameter Estimates     |
|----------------|--------------------------------------------|-------------------------|
| Intercept      |                                            | 2.91 (0.48)             |
| Reserve Price  | Max(starting bid, secret reserve price)    | +                       |
| Number of Potential Customers | Original Auction Duration | -                       |
| Hassle Cost    | (Inversely) Bidder’s Feedback Ratings     | -                       |

Table 3: A Logistic Regression on the Seller’s Decision of Using Buy-it-Now Option (N=1069)

4.3 Predicting Consumer Choice of Auctions

In our data set, we observe the time at which each bid is placed. This time together with the auction starting and ending times was used to infer the list of auctions that were available when the bidder made the decision of which auction to select (choice set $\Omega$). Using a multinomial logit model formulation we estimate the parameters of the auction covariates such as the “time-to-end” of an auction, buy-it-now price, starting bid, and the auction duration set by the seller. These
variables are chosen to serve as the proxy respectively for the participation cost, buy-it-now price, reserve price, and the number of potential customers as specified in the theoretical model. Other variables not included in our theory but that we control for include seller’s rating, quality of the product (used or new), and the secret reserve option. Let \( i = 1, 2, \ldots, I \) denote the bidder and let \( j = 1, 2, \ldots, J \) denote the auction in the choice set \( \Omega \), the probability that bidder \( i \) chooses auction \( j \) is:

\[
P(Y_{ij} = 1) = \frac{\exp(X_j \beta)}{\sum_{j' \in \Omega_i} \exp(X_{j'} \beta)}
\]  

(20)

where \( X_j \) is the auction covariates for the \( j^{th} \) auction when bidder \( i \) is making an auction choice, and \( \beta \) is the vector of the parameters associated with these covariates.

The parameter estimates of this reduced form model are presented in Table 4. The estimation results across four product categories consistently show evidence of endogenous participation. The original duration set by the seller at the beginning of the auction is found significant and negatively related to the choice probability of an auction. This variable is used to capture the number of the potential bidders at an auction (i.e. competition among customers). The longer the duration, the more bidders will arrive, hence the higher the participation threshold, everything else being equal. If the “full and exogenous participation” assumption were true then we would not observe a negative relationship between the auction choice probability and the original duration.

Another result that lends support to our theory about auction participation cost is that the “time-to-end” of an auction is also significantly and negatively related to the probability of an auction being chosen. “Sniping” behavior is an often-observed behavior at online auctions (Wilcox, 2000; interested readers are referred to Roth and Ockenfels, 2002 for a more in-depth discussion.
on late bidding). It occurs when many bidders place bids close to the end of the auction. We believe this interesting behavior may be partly due to the participation costs associated with bidding. Should these costs vanish, we would not observe such a strong preference for auctions almost ready to close.

The minimum bid variable, used to directly test the reserve price effect, is again shown to significantly decrease the choice probability. The preference toward the buy-it-now option is tested using three-level dummy variables: regular auction and a buy-it-now auction with a buy-price lower or higher than the average of the closing prices in the data set. The results show that customers welcome the buy-it-now option for the KitchenAid auctions. This is consistent with the fact that the auction participation cost in this category is higher than the others as we conjecture from Table 2. Customers at iPod and Lexar auctions prefer bidding over exercising the buy-it-now option.

5 Summary and Conclusions

This paper examines the effects of the buy-it-now price for private value, second-price sealed bid auctions. Our research findings have important implications for understanding consumer behavior in the competitive environments found within auctions. We show that an increase in customer’s auction participation costs can reduce the seller’s revenue under a traditional auction format. The buy-it-now feature offers the seller a flexible marketing tool to reduce the inefficiency of the traditional auction. It can increase both the seller’s profit and the customers’ utility. Auction participation costs are clearly an important consideration for sellers in deciding which auction
|                                | IPOD          | LEXAR         | KA-525        | KA-KSM        |
|--------------------------------|---------------|---------------|---------------|---------------|
| Constant                       | 1.9759***     | 2.3966***     | 2.0110***     | 0.5665        |
|                                | (0.1101)      | (0.1116)      | (0.2286)      | (0.3532)      |
| Time to end                    | -0.4344***    | -0.7681***    | -0.6161***    | -0.4874***    |
|                                | (0.0204)      | (0.0195)      | (0.0209)      | (0.0301)      |
| Ln(start bid)                  | -0.1565***    | -0.0733***    | -0.1791***    | -0.2777***    |
|                                | (0.0091)      | (0.0123)      | (0.0535)      | (0.0466)      |
| Reserve met                    | -0.5465*      | 1.1329***     | -1.4734***    | -9.7173       |
|                                | (0.2986)      | (0.3258)      | (0.2865)      | (45.4420)     |
| Reserve Not Met                | 0.5013***     | -0.0904       | -0.1349       | 1.3359**      |
|                                | (0.0691)      | (0.1355)      | (0.0984)      | (0.3169)      |
| ln(seller rating)              | 0.0315**      | 0.0275**      | -            | -             |
|                                | (0.0158)      | (0.0092)      |              |              |
| Quality=1 if new               | 0.0316        | -             | -             | -             |
|                                | (0.0579)      |              |              |              |
| Original duration              | -0.0819***    | -0.0530***    | -0.0417*      | -0.0256       |
|                                | (0.0161)      | (0.0124)      | (0.0243)      | (0.0373)      |
| BIN price _low                 | -0.9836***    | -0.2692***    | 0.1610**      | 0.1381***     |
|                                | (0.0799)      | (0.0800)      | (0.0640)      | (0.0131)      |
| BIN price_high                 | -0.4055***    | -0.0587       | -            | 0.0778        |
|                                | (0.0597)      | (0.0595)      |              | (0.1164)      |
| N                              | 1661          | 2438          | 1887          | 912           |
| LL                             | -2479.0757    | -5413.8228    | -2780.2681    | -1189.4304    |
| LL_0                           | -2849.2794    | -5961.6414    | -3055.6191    | -1313.6448    |
| d.f                            | 9             | 8             | 6             | 7             |
| χ2 test p-value                | 0.0000        | 0.0000        | 0.0000        | 0.0000        |

Table 4: Multinomial Logit Model Estimation

format to employ. In contrast most auction research ignore customer participation costs and assume full participation. Moreover, our empirical findings complement and confirm the findings of our theoretical model.

Our findings indicate that the seller’s choice of using a buy-it-now auction option depends upon being able to predict a customer’s auction participation costs, the number of customers, and the seller’s own reserve price to configure a profitable auction. The results of this model can
also help auction companies, like eBay, improve their own pricing policy for sellers who wish to use the buy-it-now feature. Currently, sellers are charged five cents for using this option (except for the auto category). We recommend that auctioneers like eBay should price this option differently across various product categories to increase transactions and in turn increase total revenue.

Our research has many limitations. First, in our model we assume that the seller’s reservation price is exogenously given. Hence, the seller’s strategic behavior in choosing a reserve price in conjunction with a buy-it-now price is assumed away. It would be interesting to examine how sellers can jointly set reserve and buy-it-now prices to maximize profits. Unfortunately, we have not been able to find an analytically tractable framework for this analysis. Second, we realize that customers’ participation costs are unlikely to be homogeneous as we have assumed. Allowing the auction participation cost to vary across customers will lead to richer strategic implications both for the customers and the seller. Again we have been unable to find analytically tractable forms without this assumption. Third, in our empirical analysis we ignored the potential for endogenous seller’s decisions in setting buy-it-now prices. Potentially an empirical analysis that follows a structural approach for constructing consumer and seller decisions could overcome this issue. Fourth, studying the role of competition between fixed-posted retail prices and buy-it-now auctions would be useful, especially as more companies adopt both retail and auction channels. We believe that the future study of consumer behavior at auctions is a fruitful area for both researchers and practitioners.
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6 Appendix

**Proof of Theorem 1.** To prove that the equilibrium bidding strategy prescribed in Theorem 1 exists, we need to show that each customer does not have an incentive to unilaterally deviate. Knowing the number of potential competing bidders at the auction, and the auction participation cost, each customer arrives at the auction computes *ex ante* utility of bidding, purchasing at the buy-it-now price, and not participating.

First consider $B > s_a$. Suppose a customer with a valuation higher than the buy-threshold chooses to submit a bid. The auction becomes a regular auction, as a result, this customer is forced to bid in order to obtain the product. The resulting utility is $v_i - c$. From Lemma 1 we know it is lower than $v_i - B$. Not participating is not an option either, as it requires the customer to forgo the positive utility, $v_i - B$, derived from exercising the buy-it-now option and receive zero utility instead. Therefore, the consumer will be worse off from the choice of any option other than purchasing directly. If $v_i < s_b$ and the consumer buys the product at the pre-set buy-price, the consumer gains a negative profit (equal to $v_i - c$) from this transaction. Therefore it does not constitute an equilibrium. Not participating is weakly dominated too, because the consumer could potentially gain a positive surplus $v_i - c$ from bidding. Any action for a customer whose valuation is lower than $s_a$ makes the consumer worse off, causing a negative utility.

Second if $B < s_a$, then there is a case such that $B < v_i < s_a$. In this case a customer’s dominant strategy is to buy directly to gain $v_i - B$. Submitting a bid or leaving the auction without doing anything will result in a negative profit ($-c$, $-(v_i - B)$, respectively). Therefore they are both strictly dominated. Any action from a customer whose valuation is lower than $B$
will result in a negative profit \(-c\) if the customer bids and \(-(B - v_i)\) if the customer exercises the buy-it-now option). In addition, if a customer’s dominant strategy is to buy at the buy-it-now price, waiting before exercising the buy-it-now option is dominated because subsequent customers may arrive and potentially change the auction format to one less favorable for the customer.

Suppose there is no buy-it-now price when a customer arrives. For a customer to bid when \(v_i < s_a\) results in a negative surplus equal to \(-c\). If \(v_i > s_a\) the customer submits a bid equal to their valuation. The proof of this statement is the same as that of Vickrey (1961). ■

**Proof of Proposition 2.** For the participation threshold \(s_a\), it is straightforward to show that:

\[
\frac{\partial s_a}{\partial c} = \frac{c^{1/n-1}}{n} > 0
\]

\[
\frac{\partial s_a}{\partial n} = -\frac{c^{1/n} \ln c}{n} > 0
\]

Define \(f (s_b) = s_b - B - (s_b^n - c) / n = 0\). Using the implicit function theorem, we can obtain the qualitative results regarding its relationship with other parameters:

\[
\frac{\partial s_b}{\partial B} = -\frac{\partial f}{\partial B} / \frac{\partial f}{\partial s_b} = \frac{1}{1 - s_b^{n-1}} > 0
\]

\[
\frac{\partial s_b}{\partial n} = -\frac{\partial f}{\partial n} / \frac{\partial f}{\partial s_b} = -\frac{(s_b^n - c) / n^2 - s_b^n \ln s_b}{1 - s_b^{n-1}} < 0
\]

\[
\frac{\partial s_b}{\partial c} = -\frac{\partial f}{\partial c} / \frac{\partial f}{\partial s_b} = -\frac{1}{n (1 - s_b^{n-1})} < 0
\]

■

**Proof of Proposition 3.** To show that the expected profit function for using the buy-it-now
feature is strictly concave, we need to show that \( \frac{\partial^2 \pi_b}{\partial B^2} < 0 \).

\[
\frac{\partial^2 \pi_b}{\partial B^2} = \frac{q}{1 - s_b^{n-1}} \left( -2 + \frac{(\pi_a - B) (n - 1) s_b^{n-2}}{(1 - s_b^{n-1})^2} \right)
\]

First note that the second term in the brackets \( -2 + \frac{(\pi_a - B)(n-1)s_b^{n-2}}{(1-s_b^{n-1})^2} \), is decreasing in \( n \). We denote it as \( m \). Therefore, it reaches its maximum when \( n = 2 \) (\( n \) should be no less than two is required by the second-price sealed bid auction) when \( n = 2 \) and \( m = \frac{(\pi_a - B)}{(1-s_b)^2} \), where \( s_b = 1 - \sqrt{1 - 2B + c} \). If we can show that \(-2 + m < 0\) then we can prove concavity of \( \pi_b \). This is true because

\[
m < \frac{\pi_a|_{n=2,c=0} - B}{1 - 2B + c} = \frac{1/3 - B}{1 - 2B + c} \leq \frac{1}{3}
\]

Hence \( \pi_b \) is concave in \( B \).

Next, we determine the optimal buy-it-now price. In order to get an analytically tractable result for the optimal buy-it-now price we use a second-order Taylor series expansion to approximate \( s^n \) around the upper bound of the value distribution:

\[
s^n_b = 1 + n (s_b - 1) + \frac{(n - 1) n (s_b - 1)^2}{2}
\]

Based upon a comparison with simulated numerical results this approximation is very accurate. The corresponding explicit solution for \( s_b = 1 - \sqrt{\frac{2(1-B)n+2c-2}{\sqrt{n(n-1)}}} \). \( s_b \in [0,1] \) is guaranteed by the buy-it-now condition \( B < 1 - \frac{1-c}{n} \) and \( n \geq 2 \). Therefore, solving the first order condition \( \frac{\partial \pi_a}{\partial n} = 0 \), we obtain \( B^* = \frac{2+\pi_a}{3n} - \frac{2(1-c)}{3n} < 1 \).

**Proof of Proposition 4.** \( \frac{\partial B^*}{\partial n} = \frac{2-2c}{3n^2} + \frac{1}{3} \frac{\partial \pi_a}{\partial n} \). The first term on the right hand side of the equation is positive because \( c < 1 \). It is left to show that the second term is also positive in order to prove the proposition. \( \frac{\partial \pi_a}{\partial n} = \frac{n(2-c(1+n)^2)+c^2}{n(1+n)^2} + \frac{1}{n(1+n)^2} \right) \), which decreases
in $c$. Namely, this expression reaches its minimum when $c = 1$. We can further show that
\[
\lim_{c \to 1} \frac{\partial \pi_a}{\partial n} = 0.
\]
Therefore, $\frac{\partial \pi_a}{\partial n} > 0$, $\frac{\partial B^*}{\partial n} > 0$. ■

**Proof of Proposition 5.** See, for example, in the $\pi_b$ region,
\[
\frac{\partial B^*}{\partial c} = \frac{(c^{1/n} - 1)n(n-1) + 2}{3n}.
\]
Its sign depends on the relationship between $n$ and $c$. Furthermore, it can be shown that for most of the cases (especially when $n$ is large) the auction participation cost increases and the optimal buy-it-now price decreases. ■

**Discussion on Reserve Price.** In the above analysis, we have assumed a zero-reserve price for the auctioned good. Although many of the sellers at eBay do not employ a reserve price in order to encourage participation, the use of a reserve price is a common practice for those sellers who want to ensure that their winning bid meets a certain level. For instance a seller who wishes to cover their acquisition cost of the auctioned item. Now we examine how the reserve price influences the seller’s decision regarding the buy-it-now price.\(^3\)

3 We do not consider setting the optimal reserve price in this paper, but rather treat the reserve price as exogenous. We refer interested readers to Riley and Samuelson (1981) for more discussion on optimal auction design by setting the reserve price optimally.

If the seller uses a reserve price at the auction, then it maintains the right not to sell the item if the highest bid is lower than the reserve price. At eBay, a seller can set a reserve price in two ways: use a starting bid or a secret reserve. The starting bid is equivalent to the “open reserve” studied extensively in the traditional auction theory. It is “open” because it is observable to all the potential bidders. Alternatively, the seller can use a secret reserve price. The amount of the secret reserve is not revealed to the public. However, the bidders know that there is a secret reserve need to be met in order to be a winner. At eBay, the status of the reserve being met or
not is public. This reserve option has just begun to receive research attention (e.g. Katkar and Lucking-Reily, 2001; Bajari, and Hortascu, 2003).

First consider the open reserve case. Following the existing auction literature, if the auction participation cost of bidding is zero, a bidder’s weakly dominant strategy in an auction with a positive starting bid (an open reserve) is to bid her true valuation if it is higher than the reserve price. If however, the auction participation cost is strictly positive, then in order to participate in the bidding process, not only must the bidder’s valuation be greater than the reserve price, but she also needs to make sure that after paying for the auction participation cost that the expected gain from bidding remains positive. Therefore, when the open reserve is strictly positive, the participation threshold becomes \( s'_{a} = r + c^{1/n} \), and it increases in \( r \). The seller’s expected profit is similar to (12) except that the lower bound of the integration will be raised from \( s_{a} \) to \( s'_{a} \).

Therefore, we can see that the participation threshold increases in the open reserve (the starting bid). The effect of the open reserve is similar to the auction participation cost (in Proposition 5) in that it increases the customers’ participation threshold. Compared to the regular auction, the seller prefers to use a buy-it-now option when he has a high reserve price, everything else being equal. The reason is that as the reserve goes up, fewer people can participate in this auction due to the reduced gain from bidding. As a result, the seller’s ex ante expected auction profit decreases. Offering a buy opportunity to the customer increases the chance of a sale. We further show that the reserve price increases the seller needs to adjust the price downwards:

\[ B^* = \frac{2 + \pi_{a}^{r}}{3} - \frac{2 (1 - c)}{3n}. \]
where \( \pi_a = \frac{(n-1)[1+(c^{1/n}+r)^n][n(c^{1/n}+r-1)-1]}{n+1} \). To prove that \( \frac{\partial B^*}{\partial r} < 0 \), we only need to show that \( \frac{\partial \pi_a}{\partial r} < 0 \). Since \( r + c^{1/n} \leq 1 \), it follows that \( \frac{\partial \pi_a}{\partial r} = n(n-1)(r+c^{1/n}-1)(r+c^{1/n})^{n-1} < 0 \).

Besides setting a positive starting bid to ensure non-negative profit, the seller can set a secret reserve price, which is assumed to come from the same distribution as that of the valuations of the customers (also see Bajari and Hortacsu, 2003). The value of the secret reserve is unobservable, therefore, \textit{a priori}, customers cannot calculate the participation threshold the same way as in the open reserve case. However, they know that there is one more opponent they are bidding against: the seller. Consequently the only difference brought by the secret reserve case is that there is one more bidder in the game, the main results derived previously still follow. ■