Partial Observations, Einstein Locality and Bell Inequalities in Quantized Detector Networks

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Abstract. Quantized detector networks (QDN) deals with quantum information exchange between observers and their apparatus rather than with systems under observation. Partial observations in QDN involve subsets of the elementary signal detectors which constitute an apparatus. We use them to prove that QDN is consistent with Einstein locality and violations of Bell-type inequalities.

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1. Introduction

QDN (quantized detector networks) \[1\] is an approach to the description of quantum experiments which emphasizes the role of observer and apparatus, rather than the properties of any imagined SUO (system under observation). QDN is based on the core principles underpinning Heisenberg’s approach to QM (quantum mechanics) \[2, 3\] and asserts that the only physically relevant quantities in quantum physics are signals from apparatus.

Non-locality has always been a fundamental issue in QM and is the source of various apparent paradoxes, such as wave-particle duality and the super-luminal transmission of certain types of information \[4\]. QDN interprets quantum non-locality as originating from the fact that apparatus is invariably non-local, as are the processes of extracting information from it, rather than reflecting strange, non-classical properties of SUOs.

There is a fundamental constraint on all quantum theories, known as Einstein locality or the principle of local causes \[5\]. This principle asserts that “events occurring in a given spacetime region are independent of external parameters that may be controlled, at the same moment, by agents located in distant spacetime regions” \[5\]. The aim of this paper is to demonstrate that QDN indeed provides a consistent, physically correct account of quantum physics, capable of satisfying Einstein locality on the one hand and the demands of quantum non-locality, such as is seen in violations of Bell-type inequalities, on the other.

The plan of this paper is as follows. In §2 we briefly review the core formalism of QDN. In §3 we discuss labstates and maximal questions, generalizing the latter in §4 to the notion of partial questions, which is central to this paper. In §5 we discuss local operations on apparatus and show how Einstein locality can be encoded into QDN. In §6, we apply these ideas to a QDN discussion of local spatial rotations of quantization axes in Stern-Gerlach experiments. This prepares the ground for a discussion in §7 of EPR spin-pair experiments and Bell-type inequalities. Finally, in §8, we discuss the implication of these ideas.

2. QDN basics

In QDN, time is measured in terms of quantum information exchange between observer and apparatus and is discrete on that account. At any given time \( n \), the observer’s apparatus \( A_n \) consists of a finite number \( r_n \) of ESDs (elementary signal detectors), each of which is represented by a corresponding single signal qubit. In a classical approach, \( A_n \) would be represented by the Cartesian product of all the signal bits, but in order to reflect quantum properties such as superposition and entanglement, \( A_n \) is represented by a quantum register \( R_n \equiv Q^1_n \otimes Q^2_n \otimes \ldots \otimes Q^n_n \), the tensor product of all the signal qubits at that time. Such a register, together with the contextual information as to what each signal qubit means, is called a Heisenberg net. Even if the rank \( r_n \) remains constant in time, an observer’s Heisenberg net changes at each time step: \( R_{n+1} \) is always
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distinct from $R_n$.

In QDN, the observer calculates quantum outcome probabilities not via quantum states of SUOs but via quantum states of their apparatus, which are referred to as labstates. A labstate at time $n$ is denoted by $|\Psi, n\rangle$ and is a normalized element of $R_n$. Labstates can be separable or entangled, but apparatus itself is not entangled normally (although that possibility may exist but this cannot be discussed here).

In QDN, observations are answers to questions asked by observers of their apparatus, and we shall use the two terms, observation and question, to mean the same thing. A maximal question is one involving all the ESDs in a given Heisenberg net, which invariably implies that such an observation is a non-local operation. However, what is possible in principle and in practice is that an observer could decide to look at only a subset of the ESDs available to them at a given time, and then such an observation is called a partial observation.

In order to discuss partial observations, we first need to understand how labstates are described. The two most useful representations of a current labstate $|\Psi, n\rangle$ are in terms of the computational basis $B_n \equiv \{|i, n\rangle : i = 0, 1, 2, 3, \ldots, 2^n - 1\}$ and via the signal operators $\{A_{i,n}^+ : i = 1, 2, \ldots, r_n\}$ and their adjoints. The computational basis is useful for mathematical calculations whilst the signal operators are directly tied in with the intuitive physics of the situation.

Given a rank-$r$ quantum register $R^r \equiv Q^1 \otimes Q^2 \otimes \ldots \otimes Q^r$, there are $r$ signal creation operators $A_i^+ : i = 1, 2, \ldots, r$, each of which has a corresponding signal destruction operator $A_i : i = 1, 2, \ldots, r$. These operators are constructed from tensor products of various individual signal qubit operators, as discussed in detail in [1].

In QDN, there is no concept of ground state. Instead, the nearest equivalent to it is the void state, or information vacuum, which represents an apparatus in its quiescent state, i.e., one such that none of its constituent ESDs would be in its signal (i.e., fired) state if examined by the observer. The signal destruction operators annihilate the void state $|0\rangle$, i.e., $A_i|0\rangle = 0, i = 1, 2, \ldots, r$ whilst the signal creation operators create signal states, i.e., $A_i^+|0\rangle = |2^{-1}\rangle, A_i^+A_j^+|0\rangle = |2^{-1} + 2^{i-1}\rangle, i \neq j$, etc., using the computational basis representation.

If $I^r$ denotes the identity operator for $R^r$ then the signal operators satisfy the signal algebra

$$\{A_i, A_i\} = \{A_i^+, A_i^+\} = 0, \quad \{A_i, A_j\} = I^r, \quad i = 1, 2, \ldots, r,$$

$$[A_i, A_j] = [A_i, A_j^+] = 0, \quad i \neq j,$$

(1)

where square brackets denote commutators and curly brackets denote anticommutators. We refer to the above as quadratic relations, as they involve products of two signal operators. The signal algebra (1) is based on the physics of quantum observation, i.e., on what happens in the laboratory, and is unique on that account.

It is convenient to define corresponding elementary projection operators (EPOs). We define $P_i \equiv A_i^+A_i, \quad \overline{P}_i \equiv A_iA_i^+, \quad i = 1, 2, \ldots, r$. These operators satisfy the quadratic
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relations
\[ P_i + P_i^\dagger = 1, \quad P_i|0\rangle = 0, \quad (0|P_i = 0, \quad i = 1, 2, \ldots, r, \quad (2) \]
the cubic relations
\[ P_i A_i = A_i^\dagger P_i = P_i A_i^\dagger = A_i P_i = 0, \quad i = 1, 2, \ldots, r, \quad (3) \]
and the quartic relations
\[ P_i P_i = P_i, \quad P_i P_i = P_i P_i, \quad P_i P_i = P_i P_i = 0, \quad i = 1, 2, \ldots, r, \quad (4) \]

3. Labstates and maximal questions

In this section, dependence on the temporal index \( n \) is suppressed. Given a rank-\( r \) Heisenberg net, a pure labstate \(|\Psi\rangle\) is of the general form
\[ |\Psi\rangle = |\Psi_0\rangle|0\rangle + \sum_{i=1}^{r} \Psi_i A_i^\dagger|0\rangle + \sum_{1 \leq i < j \leq r} \Psi_{ij} A_i^\dagger A_j^\dagger|0\rangle + \cdots + \Psi_{12\ldots r} A_1^\dagger A_2^\dagger \ldots A_r^\dagger|0\rangle. \quad (5) \]
Labstates are generally normalized to unity, so the coefficients satisfy the condition
\[ (\Psi, \Psi) = |\Psi_0|^2 + \sum_{i=1}^{r} |\Psi_i|^2 + \sum_{1 \leq i < j \leq r} |\Psi_{ij}|^2 + \cdots + |\Psi_{12\ldots r}|^2 = 1. \quad (6) \]
For example, an arbitrary labstate for a rank-2 Heisenberg net is of the form
\[ |\Psi\rangle = \{ \Psi_0 + \Psi_1 A_1^\dagger + \Psi_2 A_2^\dagger + \Psi_{12} A_1^\dagger A_2^\dagger \}|0\rangle, \quad (7) \]
with \(|\Psi_0|^2 + |\Psi_1|^2 + |\Psi_2|^2 + |\Psi_{12}|^2 = 1. \]
The interpretation of these coefficients is based on the Born rule in standard quantum mechanics (SQM) \([6]\); if the apparatus is in labstate \(|\Psi\rangle\) prior to the observer looking at both ESDs “simultaneously” (which is possible in QDN by definition), then the probability of each ESDs being found in its void state is \(|\Psi_0|^2\), the probability of ESD1 being in its fired state and ESD2 being in its void state is \(|\Psi_1|^2\), the probability of ESD1 being in its void state and ESD2 being in its fired state is \(|\Psi_2|^2\), and the probability of both ESDs being in their fired states is \(|\Psi_{12}|^2\).

We introduce the following notation to encode the above ideas. Suppose the observer looked at the \( i^{th} \) ESD, \( E_i \), and obtained the answer to the basic signal question What is the signal state of this detector? If the answer is “void”, i.e., no signal, then we write \( s_i = 0 \). Otherwise, if the answer is “fired”, i.e., there is a signal, then we write \( s_i = 1 \). What was expressed in words in the preceding paragraph can now be expressed in terms of conditional probabilities. For example, \( P\{\{s_1 = 1\} \& \{s_2 = 0\}|\Psi\} = |\Psi_1|^2 \), and so on. A further simplification is to express the various propositions symbolically. We write \( S_i \equiv \{s_i = 1\} \), \( \bar{S}_i \equiv \{s_i = 0\} \) and denote conjunctions such as \( \{s_1 = 1\} \& \{s_j = 0\} \) by \( S_i \bar{S}_j \), etc. Then for example we write \( P\{\{s_1 = 0\} \& \{s_2 = 0\}|\Psi\} \equiv P(S_1 S_2) = |\Psi_0|^2 \),
and so on. We may use the properties of the projection operators given above to relate answers to all these questions to expectation values of products of EPOs. For example, for the rank-2 apparatus discussed above, we have four maximal questions and answers, such as \( P(S_1 S_2 | \Psi) \equiv (\Psi | P_1 P_2 | \Psi) = |\Psi_1|^2 \), etc.

4. Partial questions

The above probabilities represent answers to maximal questions, i.e., questions which are asked of each and every ESD in the Heisenberg net at a given time. For a rank-\( r \) Heisenberg net, any maximal question involves a product of \( r \) distinct EPOs. For each ESD, \( \mathcal{E}_i \), there are two related EPOs, \( P_i \) and \( \overline{P}_i \), which form a conjugate pair. Therefore there are exactly \( 2^r \) distinct maximal questions.

In the real world, however, observers could choose to ask partial questions, which involve looking at only some (or even none) of the ESDs. An extreme example of a partial question is the normalization condition \( (\Psi | \Psi) = 1 \). This is equivalent to asking for the probability of finding anything at all, including no signals, without bothering to look. This probability is obviously unity, conditional on the apparatus existing in the first place and on a normalized labstate having been prepared.

It will be clear from the above that the set of all partial questions involves expectation values of all possible products of the projection operators. For the rank-2 example discussed above, there are four non-trivial partial questions, such as \( P(S_1 | \Psi) \equiv (\Psi | P_1 | \Psi) = |\Psi_1|^2 + |\Psi_{12}|^2 \), etc. Trivial partial questions are those for which the answer is always zero. For example, the answer to the question What is the probability of ESD \( \mathcal{E}_i \) being in its void state and in its signal state? is given by \( P(\overline{S}_i S_i | \Psi) \equiv (\Psi | \overline{P}_i P_i | \Psi) = 0 \), which arises from the property that \( \overline{P}_i P_i = 0 \) for each \( i \).

5. Local operations

In this section, we discuss a physical operation \( U_p \) on a rank-\( r \) apparatus \( \mathcal{A}_r \) which affects a number \( p \) of the ESDs in \( \mathcal{A}_r \) and leaves the remaining \( q \equiv r - p \) unaffected. The affected ESDs and their corresponding signal qubits will be called local whilst the unaffected ESDs and their corresponding signal qubits will be called remote. By unaffected, we mean that no possible partial measurements on the remote ESDs alone would detect any changes, given that \( U_p \) had been implemented.

The approach we take is to split the original register \( \mathcal{R}_r \) into two sub-registers \( \mathcal{R}^p \) and \( \mathcal{R}^q \), such that \( \mathcal{R}_r = \mathcal{R}^p \otimes \mathcal{R}^q \). \( \mathcal{R}^p \) is the tensor product \( Q^1 \otimes Q^2 \otimes \ldots \otimes Q^p \) of the local signal qubits whilst \( \mathcal{R}^q \) is the tensor product \( Q^{p+1} \otimes Q^{p+2} \otimes \ldots \otimes Q^r \) of the remote signal qubits. Each of these subregisters comes with its own natural preferred basis \( B_p \equiv \{|i\}_p : i = 0, 1, \ldots, 2^p - 1 \} \) and \( B_q \equiv \{|a\}_q : a = 0, 1, 2, \ldots, 2^q - 1 \} \) respectively. We

\[ \text{‡} \text{ Note that the language here is imprecise. Experiments to detect changes in the remote ESDs would actually involve ensembles of runs, comparing partial measurements on apparatus evolving without the action of } U_p \text{ with partial measurements on apparatus evolving with it.} \]
write $|i\rangle_p \otimes |a\rangle_q \equiv |i, a\rangle$, an element of $B_r$, the natural basis for $R^r$. Then orthonormality of the elements of $B_r$ gives the rule $(i, a|j, b) = \delta_{ij}\delta_{ab}$.

The operation $U_p$ will be assumed here to leave the rank of the local qubits unchanged, but in principle it is possible to consider changes in rank. Such scenarios occur in particle decay experiments, for example, in which case the rank increases monotonically with time [7]. It is also possible to consider reduction in rank, such as occurs when apparatus is destroyed, or when some ESDs are observed in order to transmit classical information, such as occurs in teleportation experiments. In such cases, the operators involved cannot be semi-unitary and non-linear quantum mechanics is involved.

In our case, the action of $U_p$ will be represented by some semi-unitary operator $U_p$ acting on $R^r$, taking it into a copy $R^r'$. Primes will denote objects such as ESDs, signal qubits, EPOs and labstates after the action of $U_p$. To avoid possible confusion, we write $|i, a\rangle' \equiv |i, a\rangle$.

With these points in mind, then the most general local operation satisfying these conditions has the following action on the natural basis elements of $R^r$:

$$U_p|i, a\rangle = \sum_{j=0}^{2^p-1} U_p^{jj'}|j, a\rangle', \quad 0 \leq i < 2^p, \quad 0 \leq a < 2^q,$$

where the coefficients $\{U_p^{jj'}\}$ are complex-valued functions of the externally controlled parameters mentioned in the statement of the principle of local causes in the introduction. These coefficients satisfy the semi-unitarity relations

$$\sum_{j=0}^{2^p-1} [U_p^{jj'}]^* U_p^{jk} = \delta_{ik}. \quad (9)$$

From completeness of the basis set $\{|i, a\rangle\}$ we deduce

$$U_p = \sum_{i=0}^{2^p-1} \sum_{j=0}^{2^q-1} \sum_{a=0}^{2^r-1} |i, a\rangle U_p^{ij}(j, a). \quad (10)$$

By inspection, it is clear that

$$|j, a\rangle(i, a|P_b = P_b'|j, a\rangle(i, a), \quad p < b \leq r.$$  

Using this and the representation (10), we readily find that the operator $U_p$ and the EPOs $\{P_a, P_a': p < a \leq r\}$ associated with the remote ESDs satisfy the relations

$$U_p P_a = P_a' U_p, \quad U_p P_a = P_a' U_p, \quad p < a \leq r. \quad (12)$$

To demonstrate that these are consistent with Einstein locality, consider an actual experiment involving such a transformation $U_p$. If $|\Psi\rangle$ is an initial labstate, i.e., before the action of $U_p$, then the final labstate is $|\Psi'\rangle = U_p|\Psi\rangle$. Suppose the observer performs arbitrary partial observations on the remote ESDs after the action of $U_p$. Then for any
choice $P_aP_b \ldots P_z$ of remote EPOs, we find
\begin{align}
(\Psi'|P'_aP'_b \ldots P'_z|\Psi') &= (\Psi|U^+_pP'_aP'_b \ldots P'_zU_p|\Psi)
= (\Psi|U^+_pU_pP'_aP'_b \ldots P'_z|\Psi)
= (\Psi|P_aP_b \ldots P_z|\Psi), \quad p < a, b, \ldots z \leq r,
\end{align}
using the semi-unitarity condition $U^+_pU_p = I_r$, the identity operator for $R^r$. These probabilities are obviously independent of the details of $U_p$, which proves that there is no way that measurements on the remote ESDs alone could detect any effects of the local operation $U_p$ acting on the local ESDs. This is precisely what the principle of local causes requires.

A technical question remains about the evolution of the remote ESDs themselves. The above analysis assumed that the remote ESDs evolved unchanged during the action of $U_p$. This is equivalent to a null test on the remote ESDs, which would be unrealistic in practice. Therefore, the discussion should be extended to two or more independent local operations, one of which is the $U_p$ discussed above and the other is some operation $V_q$ on the remote qubits. As before, basis elements for $R^r$ are written in the form $|i, a\rangle$, where $0 \leq i < 2^p$ and $0 \leq a < 2^q$, where $r = p + q$.

In the following we shall use the summation convention. Then the operator $U_{p,q}$ for the combined simultaneous transformation is given by
\begin{equation}
U_{p,q} = |i, a\rangle U_p^{ij} V_q^{ab} (j, b),
\end{equation}
where the indices $i, j$ are summed from 0 to $2^p - 1$ and $a, b$ are summed from 0 to $2^q - 1$ and the coefficients satisfy the semi-unitarity conditions
\begin{equation}
[U_p^{ij}]^* U_p^{ik} = \delta_{jk}, \quad [V_q^{ab}]^* V_q^{ac} = \delta_{bc}.
\end{equation}
Unlike the previous situation, all partial observations on either set of localized signal qubits are now affected by the transformation. However, how they are affected is still in accordance with Einstein locality, which is proven as follows. Using the summation convention, suppose $|\Psi\rangle = \Psi_{ia}|i, a\rangle$ is an initial normalized labstate and consider a set of partial observations on the first local set of ESDs represented by $(\Psi'|P'_{i_1}P'_{i_2} \ldots P'_{i_k}|\Psi')$, where $1 \leq i_1, i_2, \ldots, i_k \leq p$. Then
\begin{align}
(\Psi'|P'_{i_1}P'_{i_2} \ldots P'_{i_k}|\Psi') &= [U_p^{mn}]^* U_p^{ij} [V_q^{cd}]^* V_q^{ab} \Psi_{ia} \Psi_{jb}(\overline{m, c}|P'_{i_1}P'_{i_2} \ldots P'_{i_k} |i, a). \tag{16}
\end{align}
By inspection, it can be seen that $(\overline{m, c}|P'_{i_1}P'_{i_2} \ldots P'_{i_k} |i, a) = (\overline{m, 0}|P'_{i_1}P'_{i_2} \ldots P'_{i_k} |i, 0)\delta_{ac}$, from which we deduce
\begin{align}
(\Psi'|P'_{i_1}P'_{i_2} \ldots P'_{i_k}|\Psi') &= [U_p^{mn}]^* U_p^{ij} \Psi_{ia} \Psi_{jb}(\overline{m, 0}|P'_{i_1}P'_{i_2} \ldots P'_{i_k} |i, 0) \tag{17}
\end{align}
using the semi-unitarity conditions $[V_q^{ad}]^* V_q^{ab} = \delta_{bd}$. The right hand side of (17) is independent of any of the $V_q^{ab}$ coefficients parametrizing the $V_q$ transformation, which proves that Einstein locality holds for $V_q$. The same argument applies for $U_p$.

It is clear that this result generalizes immediately to apparatus of any rank and to arbitrary splits involving arbitrary localized transformations, provided none of these overlap as far as the ESDs involved are concerned. It should be clear also that this
formalism provides a basis for a discussion of lightcone and causal set structure in a QDN approach to relativity (the objective of future papers).

Two important conclusions can be drawn from this analysis: i) it is consistent to apply QM to parts of the universe, whilst ignoring the rest, even though all of it is subject to the laws of quantum mechanics [8] and ii) it is the possibility of isolating apparatus which gives rise to the SUO concept. From the QDN perspective, it is not that particles such as electrons and photons “exist”, but that apparatus behaves in such as way as to support that notion, most of the time.

6. Local spatial rotations

In this section we use the above results to prepare the ground for a discussion of EPR spin pairs and Bell inequalities. Consider an experiment involving an isolated Stern-Gerlach (S-G) apparatus \( \Sigma(\mathbf{a}) \), where \( \mathbf{a} \) is the associated quantization axis, together with miscellaneous other equipment. \( Q^1 \) and \( Q^2 \) are the two signal qubits associated with the two outcomes of \( \Sigma(\mathbf{a}) \) (known conventionally as spin-up and spin-down respectively), and together constitute our local qubits. Signal qubits \( Q^3, Q^4, \ldots, Q^r, r > 2 \), represent the rest of the apparatus, which is considered remote.

Any S-G apparatus such as \( \Sigma(\mathbf{a}) \) is associated with a definite axis of quantization \( \mathbf{a} \) in physical three-space and this axis can be altered by physically rotating the apparatus whilst doing nothing to the rest of the laboratory. Suppose the initial axis is given by the unit vector \( \mathbf{k} \), which may be imagined to point in the conventional \( z \)-direction. Unprimed quantities will be associated with this orientation of the axis. Now consider a local operation \( U(\mathbf{a}) \) on \( \Sigma(\mathbf{k}) \), rotating its axis \( \mathbf{k} \) into some new direction \( \mathbf{a} \), such that \( \Sigma(\mathbf{k}) \rightarrow \Sigma(\mathbf{a}) \). Primed quantities will be associated with the new orientation \( \mathbf{a} \) of the axis. There are four kind of basis labstate we need to consider, given by \( |0, a\rangle \), \( A^+_1|0, a\rangle \equiv |1, a\rangle, A^+_2|0, a\rangle \equiv |2, a\rangle \) and \( A^+_1 A^+_2|0, a\rangle \equiv |3, a\rangle \), where \( 0 \leq a < 2^q \) and \( q = r - 2 \). These are discussed in turn:

i) When isolated apparatus is in its void state, we would not normally expect it to generate signals spontaneously whilst the apparatus is being moved around in physical space. Hence we require rotations of S-G axes of magnetization to satisfy the condition

\[
U(a)|0, a\rangle = |0, a\rangle, \quad 0 \leq a < 2^q. \tag{18}
\]

This supposes that space is homogeneous and isotropic. We expect this condition to be broken in the presence of what would normally be regarded as a gravitational field. This is analogous to the phenomenon of Rindler radiation, or the spontaneous creation of particles in accelerated frames of reference, as discussed in conventional approaches to quantum physics in the presence of curved spacetime.

ii) \( |1, a\rangle \) and \( |2, a\rangle \) are labstates representing the spin-up and spin-down outcomes of the S-G sub-experiment respectively, relative to the current quantization axis.
Given that $U(a)$ is an active rotation of the quantization axis from $k$ to $a$, then experience with the SQM description of the S-G experiment leads us to write

$$U(a)|1,a⟩ = α(a)|1,a⟩ + β(b)|2,a⟩,$$
$$U(a)|2,a⟩ = γ(a)|1,a⟩ + δ(a)|2,a⟩,$$  \hspace{1cm} \text{where } 0 ≤ a < 2^n, \hspace{1cm} (19)$$

where the complex-valued coefficients $α(a)$, $β(a)$, $γ(a)$ and $δ(a)$ satisfy the semi-unitarity conditions

$$|α(a)|^2 + |β(a)|^2 = |γ(a)|^2 + |δ(a)|^2 = 1, \hspace{1cm} α(a)^*γ(a) = −β(a)^*δ(a). \hspace{1cm} (20)$$

iii) Labstates of the form $|3,a⟩$ would not normally be encountered in conventional S-G experiments, due to charge conservation. Equivalently, a single photon entering a conventional beam-splitter would not split into a photon pair. Hence we are entitled to assume $U(a)|3,a⟩ = |3,a⟩$, $0 ≤ a < 2^n$, because any phase can always be absorbed by a suitable redefinition of the outcome basis elements.

Together, these conditions give the representation

$$U(a) = \sum_{a=0}^{2^n-1} [\overline{0,a}⟩(0,a) + \{α(a)|1,a⟩ + β(a)|2,a⟩ \} |1,a⟩ + \{γ(a)|1,a⟩ + δ(a)|2,a⟩ \} |2,a⟩ + |3,a⟩(3,a)] \hspace{1cm} (21)$$

7. EPR spin-pair experiments

We now extend the discussion to spin-pair experiments, which have been used to explore issues in QM such as non-locality and violations of Bell-type inequalities. Consider an apparatus consisting of many ESDs, four of which are associated with a spin-zero bound state of two spin-half constituents, such as an electron and a positron. Suppose a quantization axis $k$ is chosen, and let $Q^1$ and $Q^2$ represent the two spin polarization outcomes associated with constituent #1, whilst $Q^3$ and $Q^4$ represent those for constituent #2. It is traditional to describe such experiments in terms of a local observer *Alice* using S-G apparatus $Σ_A(k)$ to observe constituent #1 whilst a remote observer *Bob* uses S-G apparatus $Σ_B(k)$ to observe constituent #2. Other ESDs in the apparatus are isolated from those used by Alice and Bob and are represented by signal qubits $Q^5$, $Q^6$, $\ldots$, $Q^r$, where $r$ is the current rank of the total apparatus.

Experience with spin-zero bound states in SQM leads us to take the initial labstate $|Ψ⟩$ to have the form

$$|Ψ⟩ = \frac{1}{\sqrt{2}} \{A_1^+A_2^+ - A_2^+A_3^+ \} |Φ⟩. \hspace{1cm} (22)$$

Here $|Φ⟩$ is a normalized state in the total register $R^r$ such that $P_1|Φ⟩ = P_2|Φ⟩ = P_3|Φ⟩ = P_4|Φ⟩ = 0$. Equivalently, $P_1|Φ⟩ = P_2|Φ⟩ = P_3|Φ⟩ = P_4|Φ⟩ = |Φ⟩$.

Before Alice and Bob perform any observations on their constituents, each rotates the magnetization axis of their respective S-G apparatus independently of the other. If Alice performs the rotation $Σ_A(k) → Σ_A(a)$ and Bob performs the rotation $Σ_B(k) →$
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$\Sigma_B(b)$, where $a$ and $b$ are unit three-vectors, then the operator $U(a, b)$ representing the combined transformation has the following action:

$$
U(a, b)A^+_a A^+_b |\Phi\rangle = \{\alpha(a)A^+_1 + \beta(a)A^+_2 + \gamma(b)A^+_3 + \delta(b)A^+_4\}|\Phi\rangle,
$$

$$
U(a, b)A^+_2 A^+_3 |\Phi\rangle = \{\gamma(a)A^+_1 + \delta(a)A^+_2\} \{\alpha(b)A^+_3 + \beta(b)A^+_4\}|\Phi\rangle. \quad (23)
$$

Hence the final state $|\Psi\rangle$ on which Alice and Bob perform their measurements is

$$
|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ [\alpha(a)\gamma(b) - \gamma(a)\alpha(b)]A^+_1 A^+_3 + [\alpha(a)\delta(b) - \gamma(a)\beta(b)]A^+_1 A^+_4 + \right.
$$

$$
\left. [\beta(a)\gamma(b) - \delta(a)\alpha(b)]A^+_2 A^+_3 + [\beta(a)\delta(b) - \delta(a)\beta(b)]A^+_2 A^+_4 \right\} |\Phi\rangle \quad (24)
$$

We are going to focus on one particular partial observation, $P(+a, +b|\Psi)$, which asks for the probability that Alice observes a signal in ESD$_1$ and Bob observes a signal in ESD$_3$. In SQM this corresponds to each observer catching their respective constituent particle in its up state. From (24) we immediately read off the required amplitude, giving the probability

$$
P(+a, +b|\Psi) = \langle \Psi|\mathbb{P}_1\mathbb{P}_3|\Psi\rangle = \frac{1}{2}|\alpha(a)\gamma(b) - \gamma(a)\alpha(b)|^2. \quad (25)
$$

Wigner gave an intuitive calculation of a Bell-type inequality for such observations [2], arriving at the classical result

$$
P(+a, +b|\Psi) + P(+b, +c|\Psi) \geq P(+a, +c|\Psi), \quad (26)
$$

for any choice of three-vectors $a$, $b$ and $c$. In our terms, this means that the coefficients have to satisfy the constraint

$$
|\alpha(a)\gamma(b) - \gamma(a)\alpha(b)|^2 + |\alpha(b)\gamma(c) - \gamma(b)\alpha(c)|^2 \geq |\alpha(a)\gamma(c) - \gamma(a)\alpha(c)|^2, \quad (27)
$$

in addition to the semi-unitarity conditions already in force. It is easy to find coefficients which violate this inequality. For example, following Wigner, we take $\alpha(a) = \cos(\frac{1}{2}\theta_a)$, $\beta(a) = \sin(\frac{1}{2}\theta_a)$, $\gamma(a) = -\sin(\frac{1}{2}\theta_a)$ and $\delta(a) = \cos(\frac{1}{2}\theta_a)$, where $\theta_a$ is real, and similarly for the other two rotations. Then each set of rotation coefficients satisfies the semi-unitarity conditions and Wigner’s inequality reduces to

$$
\sin^2(\theta_a - \theta_b) + \sin^2(\theta_b - \theta_c) \geq \sin^2(\theta_a - \theta_c). \quad (28)
$$

It is easy to find three angles for which this condition is violated, which demonstrates that QM is inconsistent with the sort of classical realism which led to the Bell inequality [20].

This result is consistent with Einstein locality because the partial observation used involves both Alice and Bob together, i.e., treats both as simultaneously local. On the other hand, partial observations involving $Q^1$ and $Q^2$ alone (i.e., by Alice alone), or involving $Q^3$ and $Q^4$ alone (i.e., by Bob alone), would be completely unaffected by whatever the other observer had done to the axis of their particular S-G apparatus.
8. Concluding remarks

The above results fully support the position taken by Heisenberg and Bohr: it is the experimental context alone which affects quantum outcome probabilities, both in the preparation of labstates and in how they are observed. Everything else is metaphysical speculation.

By showing that it is really the relationship between observers and apparatus rather than SUOs that matters in quantum physics, these results suggest that the status of quantum mechanics should be changed in a rather serious way. Instead of physical reality being regarded as some “quantized” version of a classical reality, quantum mechanics should be seen as no more and no less than the correct and universal set of rules for information exchange between observers and apparatus.

There are implications of this conclusion for various theoretical disciplines such as quantum gravity and quantum cosmology. In those fields, conventional approaches to quantization start by regarding space and/or the universe as some sort of quantized SUO. The Bohr-Heisenberg vision of reality, supported by QDN, suggests that those fields are ultimately doomed to failure as they are currently formulated, because quantum mechanics cannot be discussed properly without sensible notions of observers and apparatus, and such things could not have existed in proposed early universe scenarios.

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