Double self-Kerr scheme for optical Schrödinger-cat state preparation

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Abstract

We propose a scheme for preparing optical Schrödinger-cat states in a traveling wave setting. Two states are similarly prepared via the self-Kerr effect, and after mixing them, one mode is measured by homodyne detection. In the other mode, a superposition of coherent states is conditionally prepared. The advantage of the scheme is that assuming a small Kerr effect, one can prepare with a high probability one of a set of Schrödinger-cat states. The measured value of the quadrature provides information about which one of the set of states is actually prepared.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Superpositions of coherent states (CSSs) are often referred to as Schrödinger-cat states, because the constituents of the superposition are two quasi-classical states. It would be desirable to prepare quantum systems in such states for both practical and fundamental purposes. Their nonclassical properties could be exploited in, for example, quantum communication. In traveling wave optics, there have been a number of proposals to prepare Schrödinger-cat states \cite{1}.

Photon subtraction from a squeezed vacuum state is a method that has been successfully applied to produce approximate CSS states \cite{2, 3}. Recently, squeezed CSSs were also generated with a similar technique \cite{4}. In another proposal for conditional preparation, squeezed single photon states were allowed to interfere on a beamsplitter, and the states in one arm were conditionally selected by photon detection \cite{5}.

Propagation through nonlinear media exhibiting the Kerr effect was suggested in one of the first proposals to prepare CSS states \cite{6}. There have been several proposals employing the cross-Kerr effect \cite{7}, as well as the self-Kerr effect \cite{8}. A double cross-Kerr scheme was recently proposed by us \cite{9}, exploiting the interference of two similar states on a beamsplitter for a conditional CSS preparation scheme.

In this paper, we propose a scheme with two identically prepared states by propagating a traveling wave coherent state through a self-Kerr medium. The states are allowed to interfere on a balanced beamsplitter and one of them is then measured by balanced homodyne detection.

2. Double self-Kerr scheme

The proposed scheme is depicted in figure 1. Two identical states are prepared from identical coherent states by letting them propagate through a medium exhibiting the self-Kerr effect. After the interaction, both modes are prepared in the state \cite{10}

\[
|\Psi\rangle = \sum_{k=1}^{N} c_k |\alpha_k\rangle, \quad \alpha_k = \alpha \exp(i2k\pi/N),
\]

where

\[
c_k = \frac{e^{i\xi_k}}{\sqrt{N}} = \frac{1}{N} \sum_{l=0}^{N-1} (-1)^l \exp \left[ -\frac{i\pi l}{N} (2k - l) \right] .
\]
The proposed scheme. Two coherent states with the same amplitude are fed into media exhibiting the self-Kerr effect. The two identically prepared states are then mixed on a balanced beamsplitter. One of the outputs is measured by balanced homodyne detection.

The balanced beamsplitter transforms coherent states in the following way:

\[ |\alpha_1\rangle|\beta_2\rangle \rightarrow \left| \frac{\alpha + \beta}{\sqrt{2}} \right\rangle_3 \left| \frac{\alpha - \beta}{\sqrt{2}} \right\rangle_4. \tag{3} \]

The two-mode state at the outputs of the beamsplitter reads

\[ |\Phi\rangle_{3,4} = \sum_{k=1}^{N} \sum_{l=1}^{N} c_k c_l |\alpha_k + \alpha_l\rangle_3 \left| \frac{\alpha_k - \alpha_l}{\sqrt{2}} \right\rangle_4. \tag{4} \]

In mode 3, we measure by balanced optical homodyne detection the real quadrature. If the measurement results in the value \(|X_m\rangle\), then the state after homodyne detection in mode 4 will have the form

\[ |\phi\rangle = N_\phi \sum_{k=1}^{N} \sum_{l=1}^{N} c_k c_l \left( X_m \left| \frac{\alpha_k + \alpha_l}{\sqrt{2}} \right\rangle \right) \left| \frac{\alpha_k - \alpha_l}{\sqrt{2}} \right\rangle. \tag{5} \]

The resulting state can be written as a superposition of CSS states and the vacuum:

\[ |\phi\rangle = N_\phi \left[ \left( \sum_{k=1}^{N} \left| \alpha_k - \alpha_l \right\rangle \left| \alpha_l + \alpha_k \right\rangle \right) X_m \right| \frac{\alpha_k + \alpha_l}{\sqrt{2}} \rangle \left| \frac{\alpha_k - \alpha_l}{\sqrt{2}} \right\rangle + \sum_{k,l=1, k<l}^{N} c_k c_l \left( X_m \left| \frac{\alpha_k + \alpha_l}{\sqrt{2}} \right\rangle \right) \left| \frac{\alpha_k - \alpha_l}{\sqrt{2}} \right\rangle. \tag{6} \]

where we have defined the CSS states

\[ |\text{CSS}_{k,l}\rangle = \mathcal{N}_{\text{CSS}} \left( \left| \frac{\alpha_k - \alpha_l}{\sqrt{2}} \right\rangle + \left| \frac{\alpha_l - \alpha_k}{\sqrt{2}} \right\rangle \right), \tag{7} \]

and the normalization factor reads \( \mathcal{N}_{\text{CSS}} = (2 + 2 \exp(-|\alpha_k - \alpha_l|^2))^{-1/2} \). The CSS states may be distinguished only if we choose \( N = 2n + 1 \). Further conditions of distinguishability stem from the requirement that the overlaps of the Gaussian quadrature wave functions in mode 3 are minimal.

The constituent coherent states of the resulting CSS states in mode 4 form \( n \) circles in phase space. This can be easily seen by calculating the absolute values of the coherent state amplitudes. The radii of the circles read

\[ |\alpha_k - \alpha_l| = \alpha \sqrt{1 - \cos \frac{2(k-l)\pi}{2n+1}}. \tag{8} \]

The largest circle has a radius smaller than, but close to, \( \sqrt{2}\alpha \).

In this section, we demonstrate with a concrete example how the proposed scheme works. We choose a moderate number \( N = 5 \), in order to explicitly show the working mechanism of selection. In practice, a weak self-Kerr effect might require one to work with higher values of \( N \), but the principle for preparation will be the same. The state incident on both inputs of the beamsplitter reads

\[ |\Phi_3\rangle = c_1 |\alpha e^{i0\pi/10}\rangle + c_2 |\alpha e^{-7i\pi/10}\rangle + c_3 |\alpha e^{-3i\pi/10}\rangle + c_4 |\alpha e^{i\pi/10}\rangle + c_5 |\alpha i\rangle. \tag{9} \]

The beamsplitter transforms the states according to equation \( \text{(4)} \). We illustrate the complex amplitudes of the participating coherent states for both modes in figure \( 2 \).

The state after homodyne detection will be a superposition of
the vacuum and ten different CSSs

$$|\phi_5\rangle = N_\alpha \left[ d_0|0\rangle + \sum_{k,l=1, k<l}^5 d_{kl}|\text{CSS}_{l,k}\rangle \right].$$

The coefficients $d$ can be determined from equation (6). The CSS states in mode 4, after homodyne measurement of the value $X_m$ in mode 3, are multiplied by the constants

$$d_{k,l} = \frac{c_k c_l}{N_{\text{CSS}}} \left( X_m \left| \frac{\alpha_l + \alpha_k}{\sqrt{2}} \right| \right) dX.$$

The actual ratio of the constants depends on the value of the measured quadrature. A good choice for homodyne measurement is to pick the most isolated coherent state. For $N = 5$ the best choice is to measure around the peak corresponding to the coherent state with amplitude

$$\alpha_{3,4} = \frac{\alpha_3 + \alpha_4}{\sqrt{2}} = \frac{e^{i\pi/10} + e^{-3i\pi/10}}{\sqrt{2}}.$$

In this way the state $|\text{CSS}_{3,4}\rangle$ will be approximately prepared. In mode 4 the constant multiplying the vacuum reads

$$d_0 = \sum_{k=1}^N c_k^2 (X_m \sqrt{2} \alpha_k) dX.$$

Examining its absolute value as a function of $\alpha$, we find that it exhibits oscillations, and at certain points it can be exactly zero. The function is depicted in figure 3.

Let us fix, as an example, $\alpha = 7.23$. With this choice the amplitudes of the other states in the superposition are negligible compared to the target CSS state (the largest ones have the following numerical values: $|d_0/d_{3,4}| = 9.0 \times 10^{-4}$, $|d_{4,5}/d_{3,4}| = 1.2 \times 10^{-4}$). We note that with increasing amplitude $\alpha$ the weight of the other CSS state in the superposition decays monotonically; thus by an appropriate choice of the intensity of the input coherent state, one can achieve high-fidelity preparation.

4. Conclusions

We have proposed a conditional scheme for preparing coherent state superpositions. The scheme is based on a double self-Kerr scheme, where two identically prepared states interfere on a balanced beamsplitter and subsequently one of the outputs is measured by balanced homodyne detection. The detection event in general prepares a superposition of CSS states and the vacuum. We have shown that due to destructive interference the vacuum amplitude can be zero, depending on the amplitude of the initial coherent state and the selected CSS state. The separation of the selected CSS state can vary, conditional on the measurement result.

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