Spooky Work at a Distance: an Interaction-Free Quantum Measurement-Driven Engine

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Recent progress in the science of quantum measurement has focused on the energy associated with the wavefunction collapse process. Energy may be stochastically transferred from the measurement probe to the system being measured, such that a highly efficient quantum measurement powered engine can be realized with cyclic feedback. Here we show that this work extraction can be done in a nonlocal way using interaction free measurements, despite a local interaction Hamiltonian. By putting an Elitzur-Vaidman bomb in one arm of a tuned Mach-Zehnder interferometer, the detection of a photon in the dark port of the interferometer indicates the bombs presence without blowing it up. Treating the bomb quantum mechanically, the bombs ground state exists in superposition of inside and outside the interferometer arm. If the optical dark port fires, the bombs wavefunction must collapse inside the interferometer arm, which raises the bombs energy. The energy can then be extracted in the engine cycle. Crucially, the wavefunction collapse of the bomb inside the interferometer arm indicates the photon could not have taken the path the bomb was localized in, otherwise it would have absorbed the photon and exploded. Therefore, the work done on the bomb by the photon is seemingly nonlocal. We complement this discussion by calculating the anomalous measurable energy gain when postselecting realizations where the dark port fires. Regardless of interpretation, this interaction free quantum measurement engine is able to lift the most sensitive bomb without setting it off.

Interaction-free measurements are often presented as a way of gaining information. Given prior knowledge, we can classically learn something nonlocally about a system by measuring where it is not. This is simply an updating of our knowledge given prior ignorance. A quantum mechanical version was introduced by Elitzur and Vaidman [1], where no prior knowledge was required. They envisioned an ultra sensitive bomb that would explode whenever it encountered a photon. When this bomb was placed into one arm of a tuned interferometer, detecting a photon at the dark port, i.e. the port that would normally never fire, brought simultaneously two pieces of information. First, that the photon arrived in the first place, and second, that the interference pattern was modified, revealing that one of the arms of the interferometer must have been blocked. From a retrodictive analysis, observing the photon at the dark port indicated the presence of the bomb, while not exploding it, therefore having had to take the other path of the interferometer (consequently seemingly never interacted with the bomb).

This type of quantum interaction-free measurement has since been generalized to a wide variety of situations, such as counter-factual quantum computation [2] or communication [3], seemingly paradoxical experiments [4], etc. Some of them were successfully implemented experimentally [5-8]. In all such cases, information about objects or actions is obtained, seemingly without having interacted with them.

The purpose of the present paper is to show that one can make physical changes on the objects playing the role of the bomb, while performing an interaction-free measurement of their presence. That is, not just to gain information, but to change their energy. To be specific, we consider an engine able to do useful work on an Elitzur-Vaidman bomb, despite never having (seemingly) interacted with it. Thus we make the prediction of being able to do useful work on an extremely sensitive bomb that would explode if that same working photon were present.

For our theoretical purposes, we model the bomb as a zero-temperature reservoir with finite spatial extent, and a motional degree of freedom. If a photon is located inside the bomb area, it is absorbed, which counts as an explosion in our treatment. Further, we treat the ‘bomb’ position quantum mechanically, and put it in a spatial superposition inside and outside of the photon’s beam path. Assuming that the bomb rests on a floor in the presence of a gravitational field, its ground state is such a superposition. Consider that we now send a single photon in a spatial mode that has some overlap with the bomb’s wavefunction and detect whether it is transmitted. There is some probability amplitude for the photon to be absorbed, and some probability amplitude for it to go through, see Fig. [1] If it is absorbed, no subsequent photon measurement happens, indicating the bomb was in the photon’s path. Conversely, if it is detected by the photon counter, this indicates it was not absorbed, and consequently the bomb must have been out of the photon’s path. In both instances, whether the bomb detonates or not, its motional degree of freedom is altered, thus departing from its ground state, and therefore gaining energy. Indeed, its wavefunction collapses to be inside or outside the path, resulting in an excited motional state. The energy exchange from such a posi-

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ation quantum measurement is the source of the quantum measurement engine proposed by some of the authors [9]. When the measured object is found localized above the floor, the latter can be raised with no work cost, realizing a useful engine with efficiency approaching unity. This gain of energy must be compensated by the corresponding energy loss in the meter, here the photon.

An interesting question is whether this measurement-induced energy exchange is local or not. One may argue that even though the bomb was deduced not to be in the path of the photon, the update of the photon wavefunction comes from terms involving the photon-bomb interaction. The locality of the energy exchange has been highlighted by Dicke in a very similar situation where the photons scattered by an atom are monitored [10]. In that case, virtual absorption and reemission of the photons are necessary to explain the measurement-induced wavefunction collapse. However, we can turn this deductive logic on its head by considering the experiment shown in Fig. 2. There, a bomb modeled as in Fig. 1 is placed inside a Mach-Zehnder interferometer, which is tuned so that no dark port photon is ever detected in case no “bomb” were inside of the interferometer arm. If the bomb is entirely localized inside the interferometer arm I, the photon will be absorbed if it travels along this arm I, and transmitted if it travels along the other arm II.

As we will show below, this scheme, conditioned on a dark port photon detection event (which happens one eighth of the times), permits us to deduce that the photon seemingly went solely through arm II, while still performing work to raise the atom against the gravitational field, in what appears to be a non-local way. In the first section, we present the model for the photon-bomb interaction and analyze the conditions leading to the bomb’s position measurement. We then detail the energy balance. We show that the useful extracted work is associated with a red-shift of the photon frequency. In the second section, we consider that the photon’s beam path is one of the two arms of an interferometer, and show that this allows us to implement a seemingly interaction-free position measurement on the atom. In the third section, we calculate the post-selected anomalous energy gains, that are insightful about how the energy exchange takes place, and highlight the contextual aspect of the phenomenon. We finally discuss the role of energy uncertainty and coherence of the bomb.

FIG. 1. Position measurement of a bomb using the transmission of a single photon. The bomb’s motional degree of freedom is initially in the ground state \( |0\rangle_m \) of the quantum bouncing ball Hamiltonian, i.e. in the linear potential associated with the gravity field \( \vec{g} \), and above a rigid platform enforcing its altitude to be \( z > 0 \). The photon solely interacts with the part of the bomb’s wavefunction that overlaps with its own spatial wavefunction \( \phi_0 \) (see SI Appendix [11] A). An absorption of the photon is associated with a localization of the bomb’s wavefunction within this overlap area.

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FIG. 2. Interaction-free measurement-driven engine. The bomb is prepared in its motional ground state that has some spatial overlap with the photon mode in the arm I of a Mach-Zehnder interferometer. A single photon is sent through the input beamsplitter. In the absence of a bomb, the photon exits the interferometer always in the mode \( |br\rangle_{ph} \) and only detector \( D_1 \) clicks. If the photon goes through arm I and the bomb is present, the photon is absorbed. The presence of the bomb modifies the interference pattern, and detector \( D_2 \) can fire. This latter event is associated with a projection of the bomb inside the arm I, causing an increase of its average potential energy, compensated by a variation of the photon’s frequency. One can then raise the balanced platform without doing any work. At the end of the process, work has been done against the gravity potential and is stored in the particle’s gravitational potential energy.

### POSITION DETECTION OF A BOMB

We consider a bomb described by one motional and many internal degrees of freedom. The bomb has some spatial extension along the photon’s path characterized by the length \( L \), in the \( x \) direction (see Fig. 1). Decomposing the bomb into elementary slices of width \( dx \), we describe the internal degree of freedom in each of them by a collection of harmonic modes of Hamiltonian

\[
H_b = \sum_k \hbar \omega_k b_k^\dagger b_k,
\]

where \( b_k \) is the bosonic annihilation op-
erator in mode $k$. If the bomb is present along the photon’s path, we assume that the photon interacts weakly, locally with each of these elementary slices. Moreover, we assume that the collection $H_b$ has a short correlation time $\tau$, and therefore behaves as a zero-temperature reservoir able to absorb the photon. On the other hand, the motional degree of freedom is characterized by Hamiltonian $H_m = \frac{p^2}{2m} + V_m(z_m)$, where $z_m, p_m$ are the position and momentum of the bomb’s center of mass along the $z$-axis and $m$ its mass. $V_m(z)$ is a potential that is assumed to be infinite for $z < 0$ and linearly increasing for $z > 0$. This potential is realized in particular in the quantum bouncing ball problem, i.e. for a massive particle in the gravity field on top of a rigid floor. We also assume that the bomb is tightly trapped in the $x$ direction such that the corresponding dynamics can be neglected. It is useful to introduce the ground and first excited eigenstates $\{|0\rangle_{m}, |1\rangle_{m}\}$ of $H_m$ associated with energies $0$ and $\hbar \omega_m$. The bomb is initially in the ground state of all degrees of freedom, denoted as $|0\rangle_{m}|0\rangle_{b}$.

In order to detect the position of the bomb, a single photon is sent with an average momentum parallel to the $x$-axis, in a mode with finite spatial overlap with the initial bomb’s wavefunction. A photon-counter is positioned to detect when the photon is transmitted (see Fig. 1). We assume that the interaction between the photon and the atom is proportional to the projector $\Pi_m = |\text{in}\rangle_{m}\langle \text{in}|$, where we have introduced a state of the bomb’s center of mass $|\text{in}\rangle_{m}$, located inside the photon’s beam path and fulfilling together with its orthogonal state $|\text{out}\rangle_{m}$:

$$|0\rangle_{m} = \frac{1}{\sqrt{2}}(|\text{in}\rangle_{m} + |\text{out}\rangle_{m})$$

$$|1\rangle_{m} = \frac{1}{\sqrt{2}}(|\text{in}\rangle_{m} - |\text{out}\rangle_{m}).$$

This condition is approximately verified with a good precision for the two first energy eigenstate of the quantum bouncing ball and a Gaussian photon spatial mode (see SI Appendix B.1 A). Here, both states $|\text{in}\rangle_{m}$ and $|\text{out}\rangle_{m}$ have the same average energy $\hbar \omega_m/2$. The initial photon state is assumed to be a wavepacket of central frequency $\omega_{ph}$ and spectral width $\Delta \omega_{ph}$. This state is denoted as $|\text{in}\rangle_{ph} \equiv \sum_k \phi_0(\omega_k)a^\dagger_{1,k}|0\rangle_{ph}$, while $|0\rangle_{ph}$ refers to the electro-magnetic vacuum and $a^\dagger_{1,k}$ creates a photon in a mode of energy $\omega_k$, and $\phi_0$ is the photon wavepacket in frequency representation.

The coupling Hamiltonian $V$ between the photon and each slice of the bomb induces the absorption of the photon when the atom is localized in state $|\text{in}\rangle$:

$$V = i\Pi_m \sum_{j,k} g_k(a^\dagger_{1,k}b_k - b^\dagger_k a_j),$$

where $g_k$ is the coupling strength, assumed real and independent of photon frequency for the sake of simplicity.

The interaction is switched on at time $t = 0$ when the beginning of the wavepacket reaches the bomb, and switched off at time $t = \tau = L/c$ when the end of the wavepacket leaves the bomb. In SI Appendix B, we derive the dynamics of the photon and bomb center of the mass, given the bomb does not explode, using standard approximations for a quantum open system interacting weakly with a reservoir. We focus on the regime:

$$\omega_{ph}, \Delta \omega_{ph} \gg \omega_m \gg \Gamma.$$  

Here $\Gamma = \sum_k \frac{\delta(\nu_k - \omega_{ph})}{2\hbar}$ is the decay rate of the photon. As it will be clearer below, the inequality $\Delta \omega_{ph} \gg \omega_m$ is required for the interaction-free measurement to work. We find that if the photon leaves the bomb area without exploding it, which happens with 50% probability, the system is in state (see SI Appendix B.1 B):

$$|\tilde{\Psi}_1\rangle = \frac{1}{N_1(\tau)} \sum_j \left( \phi_0(\omega_j)|0\rangle_{m} + \phi_0(\omega_j + \omega_m)|1\rangle_{m} \right)
+ e^{-\Gamma \tau / 2} \left( \phi_0(\omega_j)|0\rangle_{m} - \phi_0(\omega_j + \omega_m)|1\rangle_{m} \right) a^\dagger_{1,j}|0\rangle_{ph}|0\rangle_{b}. \quad (5)$$

Here $N_1(\tau) = \sqrt{2(1 + e^{-\Gamma \tau})}$ is a normalization factor. The tilde indicates that the state is written in the interaction picture w.r.t. Hamiltonians $\hat{H}_m + \hat{H}_{ph}$. Now, if $\Gamma \tau \gg 1$, i.e. for a bomb that perfectly absorbs the photon when they are in contact, the second line terms vanishing. In addition, condition $\omega_m \ll \Delta \omega_{ph}$ implies that the shifted wavepacket $\phi_0(\omega + \omega_m)$ is almost indistinguishable from the initial photon wavepacket $\phi_0(\omega)$. Consequently, state $|\tilde{\Psi}_1\rangle$ can be well approximated by $|\text{in}\rangle_{ph}|\text{out}\rangle_{m}|0\rangle_{b}$, i.e. an absence of absorption of the photon is associated with the bomb collapsed to the outside state. Note that the motional state $|\text{out}\rangle_{m}$ is not an eigenstates of $H_m$ and therefore evolves coherently when written back in Schrödinger picture. This evolution is a coherent rotation exchanging the roles of states $|\text{in}\rangle_{m}$ and $|\text{out}\rangle_{m}$ at frequency $\omega_m$, which conserves the energies of the bomb and the photon (the “bouncing” of the quantum bouncing ball problem). Such unitary evolution can be corrected by letting the bomb evolve freely after the end of the interaction with the photon, during a time chosen to compensate the accumulated phase.

Energy exchanges

The coupled evolution of the bomb and the photon leads to energy exchanges between the two systems. Interestingly, even when the photon is not absorbed (the bomb is found outside the beam path), the bomb’s energy still increases from its ground state due to the localization of its motional wavefunction. This measurement-induced energy increase can then be extracted to build a measurement-driven engine, for instance by taking advantage of the fact that state $|\text{in}\rangle_{m}$ has
no support on the region immediately near the floor to raise it without paying work. Precisely, the energy exchange is reflected in \[ |\Psi_0\rangle = \frac{1}{N_0^2(\tau)} \left[ |I\rangle_{ph} |0\rangle_m |0\rangle_b + \sum_j \left\{ e^{-\Gamma \tau/2} \langle \phi_0(\omega_j) |0\rangle_m - \phi_0(\omega_j + \omega_m) |1\rangle_m \right\} a_{j,0}^\dagger |0\rangle_{ph} |0\rangle_b \right] \]
\[
+ \frac{1}{2} \left( \langle \phi_0(\omega_j) |0\rangle_m + \phi_0(\omega_j + \omega_m) |1\rangle_m \right) a_{j,0}^\dagger |0\rangle_{ph} |0\rangle_b
\]
\[
+ \frac{e^{-\Gamma \tau/2}}{\sqrt{2}} |I\rangle_{ph} |\text{out}_m\rangle |0\rangle_b + \frac{e^{-\Gamma \tau/2}}{\sqrt{2}} |I\rangle_{ph} |\text{in}_m\rangle |0\rangle_b,
\]
where \( N_0^2(\tau) = \sqrt{3 + e^{-\Gamma \tau}/2} \). As in the previous section, we have used the fact that the photon state is almost unaffected as the frequency shift \( \omega_m \) is much smaller than its initial variance \( \Delta\omega_{ph} \) in order to give an approximate factorized form. This operating condition is crucial to preserve interference between the transmitted photon state in arm \( I \) and the photon state in arm \( II \) at the output of the final beam-splitter.

The two exit ports of the interferometer are monitored by photo-counters. In the absence of the bomb, the photon always goes out of the interferometer in the same mode, called bright (label br), and the other port’s detector never fires: this port is called the dark port (label dk). In the presence of a bomb able to absorb the photon, the interference pattern is modified, leading to a non-zero probability for the dark port to fire. Such an event thus allows us to tell for sure that the bomb was present in arm \( I \). The exit beam-splitter links the inside and outside modes according to:
\[
|I\rangle_{ph} \rightarrow \frac{1}{\sqrt{2}} (|br\rangle_{ph} + |dk\rangle_{ph})
\]
\[
|I\rangle_{ph} \rightarrow \frac{1}{\sqrt{2}} (|br\rangle_{ph} - |dk\rangle_{ph}).
\]

After the photon exits the interferometer, the state of the system is then (in the interaction picture):
\[
|\Psi_2\rangle \simeq \frac{1}{2N_1^2(t)} \left[ |br\rangle_{ph} \left( |1\rangle_{ph} + 2|\text{out}_m\rangle |0\rangle_b \right)
\right.
\]
\[
- \left. (1 - e^{-\Gamma \tau/2}) |dk\rangle_{ph} |\text{in}_m\rangle |0\rangle_b \right].
\]

This allows us to conclude that the dark port fires with probability \( p_{dk} = (1 - e^{-\Gamma \tau/2})^2/8 \) which goes to 1/8 in the limit \( \Gamma \tau \gg 1 \). Moreover, when this happens the bomb is projected inside the interferometer. Specifically, the conditioned state is \( |\phi_{dk}\rangle = |\text{in}_m\rangle |0\rangle_b \); the bomb did not explode, which corresponds to the usual interpretation of an interaction-free measurement, but the motional degree of freedom got collapsed inside the arm. The absence of the bomb is recovered taking \( \Gamma = 0 \), such that this event never occurs. Note that when the bright port fires, which occurs with probability \( p_{br} = (5 + 2e^{-\Gamma \tau/2} + e^{-\Gamma \tau})/8, \)
one cannot conclude with certainty that the bomb is located outside. In this case the bomb state is updated to $|\phi_m\rangle \propto \{(1 + e^{-i\Gamma\tau/2})|in\rangle_m + 2|out\rangle_m\}|0\rangle_b$. Finally, the absence of detection is associated with the explosion of the bomb and the failure of the interaction-free measurement, which occurs with probability $p_{\text{expl}} = (1-e^{-i\Gamma\tau})/4$.

**Retrodictive analysis and energy transfer**

The possibility of the dark port detector firing is associated with the change of the interference pattern with respect to the no-bomb case. Consequently, the measurement outcome provides information about the path taken by the photon. Indeed, in the case where the dark port has fired, one can infer that (i) the bomb was present in the arm $I$ as the interference pattern was modified, and (ii) the photon was not absorbed by the bomb, otherwise it could not have been detected. If one in addition assumes a perfect absorption, i.e. $\Gamma \tau \gg 1$, one concludes that the only term in $|\Psi_f\rangle$ compatible with these conditions is the one proportional to $|II\rangle_{\text{ph}}$, indicating logically that the photon has taken path $II$.

On the other hand, as explained above, the atom is projected the state $|in\rangle_m|0\rangle_b$ when this outcome is obtained, and this corresponds to an increase of its internal energy by $h\omega_m/2$, provided by the photon. Just as before, this energy could be extracted in a setup similar to the one described in Ref. [9]. A striking difference though is that the obtained outcome indicates that the photon did not enter arm $I$. One therefore faces the following dilemma to interpret the phenomenon: Either (a) the photon was allowed to provide energy to the atom at a distance, without passing through arm $I$, despite a strictly local interaction Hamiltonian $V$, or (b) it is impossible to interpret the photon as taking only path $II$, even though the bomb was present to absorb it in arm $I$. Note that as mentioned earlier, the energy extraction requires one to take into account the free evolution induced by Hamiltonian $H_{\text{opt}}$, which exchanges the roles of states $|in\rangle_m$ and $|out\rangle_m$ at frequency $\omega_m$. Note also that when the bright port fires, the updated state of the bomb $|\phi_{\text{br}}\rangle$ is still an excited state of energy $h\omega_m/10$ (for $\Gamma \tau \gg 1$), and this energy is also in that case provided by the photon (see SI Appendix [11] B).

In the next section, we analyze the situation using the weak values of observables involved in the problem, i.e. their expectation values in ensemble of realizations post-selected on the cases where the dark port fires.

**INSIGHTS FROM WEAK VALUE ANALYSIS**

The weak value of a given operator $A$ on the photon-bomb Hilbert space at time $t$, is defined as:

$$\langle A \rangle_w (t) = \frac{\langle \Psi_f | U_{\text{BS}} U(t - \tau) A U(t) | \Psi_i \rangle}{\langle \Psi_f | U_{\text{BS}} U(\tau) | \Psi_i \rangle},$$

where $|\Psi_f\rangle$ (resp. $|\Psi_i\rangle$) corresponds to a state in which the system is prepared (resp. postselected). Here $U(t)$ is the (non-unitary) propagator encoding the evolution of the photon-bomb state up to time $t$, postselecting on the absence of bomb explosion. $U_{\text{BS}}$ is the unitary transformation describing the action of the beam-splitter, acting on $|I\rangle_{\text{ph}}$ and $|II\rangle_{\text{ph}}$ as described by [8] - [9], and leaving $|0\rangle_{\text{ph}}$ unchanged.

The weak value can be interpreted as the average outcome of a weak measurement of observable $A$ performed at time $t$ [15]. Such weak measurements have the advantage of having negligible back-action on the system state, at the cost of yielding very noisy outputs and therefore requiring many repetitions for the average to be resolved. Here we choose $0 \leq t \leq \tau$, $|\Psi_i\rangle = |\Psi_0\rangle$ and post-select on the dark port firing, i.e. in the interaction picture $|\Psi_f\rangle = |dk\rangle_{\text{ph}}|in\rangle_{\text{ph}}|0\rangle_b$ (this implies that the bomb did not explode and is projected into state $|in\rangle_m$). The weak value is plotted in Fig. 3b) for $A$ being different projectors. The weak values evoked below are computed in SI Appendix [11] C for the current model.

![FIG. 3](image)

**FIG. 3.** Weak values $\langle A \rangle_w (t)$ as a function of time $0 \leq t \leq \tau$ of different observables $A$ involved in the problem. a) Projectors on the photon in arm $I$ and the bomb in the ground motional state (orange), photon in arm $I$ and bomb in the excited motional state (red), photon in arm $I$ (dashed blue), photon in arm $II$ and bomb in the ground motional state (black). b) Energy of the photon in arm $I$ (red), in arm $II$ (orange), total photon energy (dashed brown), energy of the motional degree of freedom of the bomb when the photon is in arm $I$ (cyan), in arm $II$ (green) and total motional energy (dotted blue). Hamiltonian weak values are plotted in units of $\hbar\tau$. Parameters: $\Gamma = 20$, $\omega_m/\Gamma = 2$, $\omega_{\text{ph}}/\Gamma = 10$.

We first note that the weak value of the projector onto arm $II$, $\Pi_{II} = |II\rangle\langle II|$ is one at any time (black solid line in Fig. 3b) while the weak value of the projector onto arm $I$, $\Pi_I = |I\rangle\langle I|$, is always zero (blue dashed line in Fig. 3b), supporting the interpretation that the photon...
solely took path II. We moreover compute the weak values of joint photon-bomb observables involving a projector of the photon onto one of the arms and a projector of the bomb onto state $|\text{in}_m\rangle$ or $|\text{out}_m\rangle$. One finds that for $\Gamma \tau \gg 1$, solely the weak value of the projector $\Pi_{I,\text{in}}$, i.e. of the photon being in arm II while the bomb is projected inside, remains 1.

Surprisingly, another interpretation seems to emerge if one considers weak values of the projectors onto another basis for the bomb, namely the motional energy eigenbasis $\{|0\rangle_m, |1\rangle_m\}$ instead of the $\{|\text{in}_m\rangle, |\text{out}_m\rangle\}$. We find that the weak values of projectors onto states $|I\rangle_{\text{ph}}|0\rangle_m$ and $|I\rangle_{\text{ph}}|1\rangle_m$ both start at zero and acquire finite values during the photon-bomb joint evolution. Strikingly, the weak value $\langle \Pi_{I,0}\rangle_w(t)$ of the photon being in arm I and the bomb in its motional ground state is negative, while the weak value for the photon being in arm I and the bomb being in the excited ground state is always positive and fulfills $\langle \Pi_{I,1}\rangle(t) = -\langle \Pi_{I,0}\rangle(t)$. Consequently, even when postselecting on realizations leading to the dark port firing, there is some activity in arm I if one looks at the bomb motional eigenbasis. This behavior closely resembles the so-called Quantum Cheshire Cat paradox [10], in which a single photon seemingly takes one arm of an interferometer, while still having some of its properties, like its polarization, be present in the other arm. In that case too, the phenomenon can be diagnosed through weak values summing up to zero when tracing over the subspace corresponding to this delocalized property – the “grin” of the Cheshire cat. In our generalized version of the experiment, the property turns out to be energy transferred to the bomb motional state.

The presence of an anomalous weak value, i.e. a value out of the observable’s eigenvalues range has been proved to be a hallmark of the presence of contextuality [17], and is equivalent to the violation of a generalized Leggett-Garg inequality [18][19]. Here, the weak value $\langle \Pi_{I,0}\rangle_w(t)$ is outside the range $[0,1]$ expected for a projector. One can get further insight with the interpretation of the process associated with the $\{|0\rangle_m, |1\rangle_m\}$ basis by studying the anomalous energy gain, or Hamiltonian weak values, plotted in Fig. 3. We find that as expected, the energy of the photon decreases by an amount $\hbar \omega_m/2$, which is in turn provided to the bomb. But, strikingly, the weak value $\langle H_{\text{ph}}\Pi_{II}\rangle_w(t)$ of the energy of the photon if it is in arm II stays constant and equal to the energy of the incoming photon $\hbar \omega_m$. It is actually the energy of the photon if it is present in arm I which decreases from its initial value 0 to the negative value $-\hbar \omega_m/2$ (see red solid curve in Fig. 3b). These values suggest an alternative interpretation of how the energy exchange between the bomb and the photon takes place: during the interaction time $\tau$ an effective “energy hole” or a “negative-energy photon” is created in arm I (remember that all the weak values associated with the projector on arm I are zero at $t = 0$, i.e. when the photon enters the interferometer). Such an effective particle then propagates until the second beam-splitter where it coalesces with the photon amplitude coming from arm II. Besides, the weak values of the bomb motional energy is solely non-zero when associated with the projector onto arm I, which seems to support a local energy exchange in arm I, carried by this effective negative energy particle.

DISCUSSION

An important requirement for the interaction-free measurement presented here to occur is that the atom remains in a coherent superposition of motional energy eigenstates after interaction with the photon and until the dark port detector fires. To emphasize this point, let us assume that a projective measurement in the $\{|0\rangle_m, |1\rangle_m\}$ basis is performed after the interaction, i.e. on state $|\Psi^+_1\rangle$. If state $|1\rangle_m$ is found, the system collapses onto state $|I\rangle_{\text{ph}}|1\rangle_m|0\rangle_B$. As this state has no amplitude on arm II, it leads to no interference at the second beam-splitter: in other words the energy measurement on the atom brought which-path information. Incidentally, the energy exchange between the atom and the photon is in this case localized inside arm I where both systems are present. Thus, one can conclude that the seeming non-locality of the energy exchange is closely related to the uncertainty about the atom’s precise energy in the final state $|\text{in}_m\rangle$. As mentioned earlier, this also requires a large energy uncertainty of the photon state $\hbar \Delta \omega_{\text{ph}} \gg \hbar \omega_m$. On the other hand, if one lets the photon exit the second beam-splitter and be detected at the dark port, and then performs a projective measurement in the $\{|\text{in}_m\rangle, |\text{out}_m\rangle\}$ basis (up to the free rotation induced by $H_m$), one obtains the result $|\text{in}_m\rangle$ with certainty, and one can in principle extract deterministically all the energy stored in this state by doing a unitary rotation back to the ground state $|0\rangle$. In this case, the interference between the two paths do take place: the which-path information is not directly available and can only be inferred indirectly from the fact that the photon was not absorbed. There, the measurement outcome brings no information about the atom’s motional energy (state $|\text{in}_m\rangle$ has maximum uncertainty about the energy of this degree of freedom). The fact that after the dark port detector has fired, the retrodicted path the photons took through the interferometer depends on what basis the atom is later measured in is another manifestation of the contextual aspect of this experiment.

Implementing this setup in order to test the results of this article seems challenging. Indeed, the typical position uncertainty of single atoms in the gravity field is very small, and the bomb-photon interaction requires either a strong coupling or a large spatial extent of the bomb. However, state-of-the-art setups on photonic crystal waveguides [20] enable the engineering of strong coupling between single-photon and atoms. In such a setup, the role of the “inside” and “outside” states could correspond to two different modes of the atom coupled differentially to the photonic modes. Another possibility would
be to use the properties of opto-mechanical devices, in which macroscopic oscillator can be prepared in a non-clasical state \[21\]. In this case, the role of the bomb would be played by a movable mirror mounted on a torsion pendulum that would replace the mirror of arm \(I\) on Fig. 2. The ground state of the pendulum has an intrinsic uncertainty about the torsion angle, such that the reflected photon could be efficiently directed to the output beamsplitter, or conversely diverted out of the interferometer arm. The latter event would play the role of the bomb explosion. Interestingly, for long enough interferometer arms, a small angle uncertainty could result in a large position uncertainty of the outgoing photon, amplifying the phenomenon.

**SUMMARY AND CONCLUSION**

We have analyzed an interaction-free measurement setup in a situation where the measured bomb can gain energy, even though it seemingly did not interact with the meter, a single photon. This energy gain can be understood as a non-local energy transfer triggered by a strictly local interaction Hamiltonian. Alternatively, one can consider the transfer to occur locally, but it corresponds to discarding the retrodictive logical inference and its conclusion that the meter never interacted with the bomb as the photon was not absorbed. In addition, this scenario involves the creation of an effective negative energy photon in the other arm of the interferometer. Regardless of interpretation, this interaction free quantum measurement engine is able to lift the most sensitive bomb without setting it off. These results bring new insights about quantum energy transfers, opening new path to design genuinely quantum engines.

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SUPPLEMENTAL INFORMATION

A. Inside and outside states

The quantum bouncing ball Hamiltonian $H_m$ corresponds to a potential $V_m(z) = mgz$ for $z > 0$, and infinite for $z \leq 0$. The wavefunction of the energy eigenstates $|j\rangle_m$, $j \in \mathbb{N}$ of $H_m$ can be expressed given in term of the Airy function $A(u)$ which is the solution of the ODE $\frac{d^2}{du^2}y(u) - uy(u) = 0$ which does not diverge for $u \to \infty$:

$$\psi_m(j)(z) = \langle z|j\rangle_m = \frac{A(z/z_0 + \zeta_{j+1})}{A'(\zeta_{j+1})},$$

where $z_0 = (\hbar^2/2m^2g)^{1/3}$ is a characteristic length of the problem and $\zeta_j$ is the $j$th zero of the Airy function (which is negative). We consider that the atom is in the ground state $|0\rangle_m$ and one sends a photon towards it. The photon spatial wavefunction has a Gaussian shape in the $z$ direction $\phi_0(z)$. The photon and the atom interaction strength is proportional to the spatial overlap of their wavefunctions, such that $\phi_0(z)$ can be seen as the atom wavefunction that optimally interacts with the photon. It turns out that a Gaussian wavefunction $\chi(z)$ of average $\bar{z} = 2.80z_0$ and standard deviation $\bar{\sigma} = 1.27z_0$ coincides almost exactly with that of the state $|\rangle_m = (|0\rangle_m - |1\rangle_m)/\sqrt{2}$, as quantified by an overlap probability $\langle \int dz \chi(z)|\rangle_m^2 \simeq 0.99$. This justifies Eqs.(1)-(2) of the main text defining states $|\rangle$ and $|\rangle_{out}$. While a similar analysis can be done for an arbitrary superposition of the states $|0\rangle_m$ and $|1\rangle_m$ being coupled to the photon, this particular choice simplifies the calculation and clarifies the analysis.

As an interesting additional feature, the state wavefunction of state $|\rangle_m$ turns out to be negligible in the vicinity of the platform up to $z \sim z_0/2$. As a consequence, once the bomb has been projected onto this state, it is possible to raise the floor level of an amount $\Delta z \leq z_0/2$ without paying work to raise the bomb, therefore storing useful gravitational potential energy $\cal{W}$.

B. Photon-bomb interaction

The bomb is a zero temperature reservoir, modeled by a collection of harmonic modes $b_k$ at frequency $\nu_k$. The photon modes are denoted $a_j$ with frequencies $\omega_j$. The coupling Hamiltonian reads:

$$V = i\Pi_n \sum_{j,k} g_k (a_j^d b_k - b_k^d a_j).$$

We consider the weak coupling limit $|g_k|\tau_c \ll 1$ where $\tau_c$ it the correlation time of the bomb and assume a coupling independent of the photon’s frequency for simplicity. We model the evolution the following way: the photon and the bomb interact during $\Delta t$ fulfilling $\tau_c \ll \Delta t \ll \tau$, then the number of excitations in the bomb is checked. In addition, we work in the limit:

$$\omega_{ph}^{-1}, \Delta \omega_{ph}^{-1}, \omega_{b}^{-1}, \omega_{m}^{-1} \ll \Delta t \ll \Gamma^{-1}$$

where

$$\Gamma = \sum_k g_k^2 \delta(\omega_{ph} - \omega_k),$$

is the effective rate of the evolution induced by the interaction, as shown below.
We work in the interaction picture (denoted with tilde) where the coupling Hamiltonian fulfils:

\[ \tilde{V}(t) = i\tilde{\Pi}_{\text{in}}(t) \sum_{j,k} g_k (a_j^\dagger b_k e^{i(\omega_j - \nu_k) t} - b_k^\dagger a_j e^{i(\nu_k - \omega_j) t}). \]

(16)

Starting from the state \( |\tilde{\Psi}(t)\rangle = |\tilde{\phi}(t)\rangle_{\text{ph}} |\tilde{x}(t)\rangle_m |0\rangle_b \) and the bomb in the vacuum \(|0\rangle_b\), the evolved state, after the bond is found containing \( n \) excitations, reads at second order in \( \Delta t \):

\[ |\tilde{\Psi}(t + \Delta t)\rangle = \delta_{n,0} |\tilde{\Psi}(t)\rangle - i \int_t^{t + \Delta t} dt' b_n \langle n | \tilde{V}(t') |0\rangle_b |\tilde{\Psi}(t)\rangle - \frac{1}{2} \int_t^{t + \Delta t} dt' \int_t^{t' \Delta t} dt'' b_n \langle n | \tilde{V}(t') \tilde{V}(t'') |0\rangle_b |\tilde{\Psi}(t)\rangle. \]

(17)

We focus to the case \( n = 0 \) (bomb not exploded), such that the term in first order in \( \tilde{V} \) vanishes. Denoting \( \tilde{M}_0 \) the Kraus operator updating the wavefunction in this case, we get:

\[ \tilde{M}_0 = 1 - \frac{1}{2} \int_t^{t + \Delta t} dt' \int_t^{t' \Delta t} dt'' b_n \langle n | \tilde{V}(t') \tilde{V}(t'') |0\rangle_b = 1 + \frac{1}{2} \int_t^{t + \Delta t} dt' \int_t^{t' \Delta t} \int_t^{t''} \sum_{i,j,k} g_k^2 a_j^\dagger a_i \tilde{\Pi}_{\text{in}}(t') \tilde{\Pi}_{\text{in}}(t'' - t'). \]

(18)

We now use that \( \tilde{\Pi}_{\text{in}}(t) = (1/2)(1 - \sigma_m e^{-i\omega_m t} - \sigma_m^\dagger e^{i\omega_m t}) \) and use that the bomb correlation time is assumed to be much smaller than \( \Delta t \). This allows to replace the upper bound of the integral over \( \tau \) by \(+\infty\). This integral can then be computed and yields Dirac distributions, forming the bomb spectral density taken at various frequencies, i.e. \( S(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k) \), where \( \omega \) takes typical values in the range \([\omega_{\text{ph}} - \Delta\omega_{\text{ph}}, \omega_{\text{ph}} + \Delta\omega_{\text{ph}}]\), i.e. the frequencies typically contained in the initial photon state. We assume that the bomb spectral density is flat on this frequency range, such that we can replace \( S(\omega) \) with \( \Gamma \) defined in (15). Finally, apply the Secular approximation, i.e. we neglect in the integral over \( t' \) all the term rotating at non-zero frequency. We finally get (back in Schrödinger picture):

\[ M_0 - \mathbb{1} = -i(H_{\text{m}} + H_{\text{ph}}) dt - \frac{\Gamma}{4} \sum_{i,j} a_j^\dagger a_i \left[ \delta(\omega_i - \omega_j) - \sigma_m \delta(\omega_i - \omega_j - \omega_m) - \sigma_m^\dagger \delta(\omega_i - \omega_j + \omega_m) \right]. \]

(19)

This Kraus operator solely couples states \(|\omega_j\rangle_{\text{ph}} |0\rangle_m |0\rangle_b\) to \(|\omega_j - \omega_m\rangle_{\text{ph}} |1\rangle_m |0\rangle_b\). As a consequence, if one assumes the ansatz

\[ |\tilde{\Psi}(\tau)\rangle = \sum_j \left[ c_0(\tau) \phi_0(\omega_j) |0\rangle_m + c_1(\tau) \phi_0(\omega_j + \omega_m) |1\rangle_m \right] a_j^\dagger |0\rangle_{\text{ph}} |0\rangle_b, \]

(20)

where \( \phi_0(\omega) \) is the initial photon wavefunction, one finds that the amplitudes \( c_0 \) and \( c_1 \) fulfill the evolution equations:

\[ \dot{c}_0 = -\frac{\Gamma}{4}(c_0 - c_1), \]
\[ \dot{c}_1 = \frac{\Gamma}{4}(c_0 - c_1). \]

(21)

In particular, starting from \( c_0 = 1, c_0 = 0 \), one gets \( c_{0,1}(\tau) = (1 \pm e^{-\Gamma \tau/2})/2 \). Finally, the state \(|I\rangle_{\text{ph}} |0\rangle_m |0\rangle_b\) is mapped onto state \( \sum_j (\phi_0(\omega_j) |0\rangle_m + \phi_0(\omega_j + \omega_m) e^{-i\omega_m \tau} |1\rangle_m) |0\rangle_b / \sqrt{2} \) at long times \( \tau \gg \Gamma^{-1} \). This state can be approximated (after being renormalized) by \(|I\rangle_{\text{ph}} |\text{out}\rangle_m |0\rangle_b\) provided the two wavefunctions \( \phi_0(\omega) \) and \( \phi_0(\omega + \omega_m) \) have overlap of almost unity, i.e. provided that the initial photon width \( \Delta\omega_{\text{ph}} \) is much larger than \( \omega_m \).

Note that the bomb-photon interaction time \( \tau \) is set by the longest of the bomb length and photon duration. The photon duration is constrained by its frequency width \( \tau_{\text{ph}} = \Delta\omega_{\text{ph}}^{-1} \ll \omega_m^{-1} \ll \Gamma^{-1} \). Consequently, in order to have \( \Gamma \tau > 1 \), we need a large bomb of width \( L > c/\Gamma \).

Note also that when written in Schrödinger picture, the system’s for \( \Gamma \tau \gg 1 \) is actually a limiting cycle \(|I(\tau)| \langle \text{out}(\tau) \rangle_m\). The time-dependence of the photon state wavefunction solely encode the free propagation of the wavepacket. On the other hand, the bomb motional state evolves \(|\text{out}(\tau)\rangle_m = (|0\rangle_m + e^{-i\omega_m \tau} |1\rangle_m) / \sqrt{2} \) i.e. a coherent rotation exchanging states \(|\text{out}\rangle_m\) and \(|\text{in}\rangle_m\) at frequency \( \omega_m \). In order to obtain state at the end of the protocol \(|\text{out}\rangle_m\), one has to keep track of the phase \( \omega_m \) accumulated during \( \tau \) to correct it by letting the bomb evolve freely.
during a time $\tau_2$ such that $\tau + \tau_2$ is a multiple of $2\pi/\omega_m$.

The probability $p_{ne}(\tau)$ of the bomb not having exploded until time $\tau$ is encoded in the norm of $|\bar{\Psi}(\tau)\rangle$. We find:

$$p_{ne}(\tau) = \frac{1}{2} \left( 1 + e^{-\Gamma \tau} \right).$$  \hspace{1cm} (22)

**Interferometer setup** – When the previous setup is embedded as one of the two arms of an interferometer, the model is modified as follows. The coupling Hamiltonian is modified as follows. The coupling Hamiltonian $V$ is assumed to vanish on the photon subspace corresponding to arm $II$, i.e. the space spanned by $\{a^\dagger_j|0\rangle_{ph}\}_j$. As the consequence, the evolution of the initial state $|\Psi_0\rangle$ in the interaction picture can be deduced by keeping the term involving state $|II\rangle_{ph}$ unchanged and applying to the term involving $|I\rangle_{ph}$ the same evolution as above. The probably of non-explosion in this case can be deduced from (22) which can be understood as the conditional probability for the bomb not exploding given the photon is initially in arm $I$. Using that the probability of explosion is zero if the photon is in arm $II$, we obtain:

$$p_{ne}(\tau) = \frac{1}{2} + \frac{1}{4} \left( 1 + e^{-\Gamma \tau} \right).$$  \hspace{1cm} (23)

The probability of explosion is therefore $p_{expl}(\tau) = 1 - p_{ne}(\tau) = \frac{1}{4} \left( 1 - e^{-\Gamma \tau} \right)$. In order to compute the probability of the dark and bright port detections, we use the conditional probabilities

$p(br|ne) = |\langle br|\bar{\Psi}_2(\tau)\rangle|^2$
\[= \frac{5 + 2e^{-\Gamma \tau/2} + e^{-\Gamma \tau}}{2(3 + e^{-\Gamma \tau})}; \hspace{1cm} (24)\]

$p(dk|ne) = |\langle dk|\bar{\Psi}_2(\tau)\rangle|^2$
\[= \frac{(1 - e^{-\Gamma \tau/2})^2}{2(3 + e^{-\Gamma \tau})}; \hspace{1cm} (25)\]

that the photon is found in the bright and dark port respectively, given the bomb did not explode. The probabilities of these two events is then obtained by multiplying by $p_{ne}(\tau)$, allowing to find the expressions given in the main text.

Transmitted photon energy – At the end of the interaction, the photon and bomb state are entangled as described by Eq. (6) of the main text. One can compute the energy in the photon going into each port using the beam-splitter relation:

$$a_{I,j}^\dagger = \frac{1}{\sqrt{2}} (a_{br,j}^\dagger + a_{dk,j}^\dagger)$$
$$a_{II,j}^\dagger = \frac{1}{\sqrt{2}} (a_{br,j}^\dagger - a_{dk,j}^\dagger), \hspace{1cm} (26)$$

where $a_{br,j}^\dagger$ (resp. $a_{dk,j}^\dagger$) creates a photon of frequency $\omega_j$ at the bright (resp. dark) port. This allows to express the exact form of the output state, introducing the notation $\phi_1(\omega_j) = \phi_0(\omega_j + \omega_m)$:

$$|\Psi_2'\rangle \propto \sum_j \left[ \left( \phi_0(\omega_j) \frac{3 + e^{-\frac{\Gamma \tau}{2}}}{2} |0\rangle_m + \phi_1(\omega_j) \frac{1 - e^{-\frac{\Gamma \tau}{2}}}{2} |1\rangle_m \right) a_{br,j}^\dagger + \frac{1 - e^{-\frac{\Gamma \tau}{2}}}{2} \left( - \phi_0(\omega_j)|0\rangle_m + \phi_1(\omega_j)|1\rangle_m \right) a_{dk,j}^\dagger \right] |0\rangle_{ph} |0\rangle_b.$$  \hspace{1cm} (27)

The second (resp. third) line terms in (27) can be identified (after renormalization) as the joint photon-bomb state $|\Psi_{br}'\rangle$ (resp. $|\Psi_{dk}'\rangle$) when the photon is found in the bright (resp. dark) port. This allows to compute the corresponding photon energy:

$$E_{br}(ph) = \sum_j \hbar \omega_j \langle \Psi_{br}' | a_{br,j}^\dagger a_{br,j} | \Psi_{br}' \rangle$$
\[= \hbar \omega_{ph} - \frac{\hbar \omega_m}{10}; \hspace{1cm} (28a)\]

$$E_{dk}(ph) = \sum_j \hbar \omega_j \langle \Psi_{dk}' | a_{dk,j}^\dagger a_{dk,j} | \Psi_{dk}' \rangle$$
\[= \hbar \omega_{ph} - \frac{\hbar \omega_m}{2}. \hspace{1cm} (28b)\]
One can check that the variation of the energy of the photon from its initial energy \( \hbar \omega_{\text{ph}} \) compensates in each case the energy gained by the internal degree of freedom of the bomb:

\[
E_{\text{br}}^{(m)} = \hbar \omega_{\text{m}} |m(1|\Psi_{\text{br}}')|^2 = \frac{\hbar \omega_{\text{m}}}{10}
\]

\[
E_{\text{dk}}^{(m)} = \hbar \omega_{\text{m}} |m(1|\Psi_{\text{dk}}')|^2 = \frac{\hbar \omega_{\text{m}}}{2}.
\]

Eq. (10) of the main text is retrieved by making the approximation \( \phi_0(\omega_j) \approx \phi_1(\omega_j) \), i.e. neglecting the photon frequency shift with respect to its much larger initial frequency uncertainty. The same approximation allows to find that \(|\Psi_{\text{br}}'\rangle \approx |\Psi_{\text{br}}\rangle\) and \(|\Psi_{\text{dk}}'\rangle \approx |\Psi_{\text{dk}}\rangle\).

### C. Weak values

The evolution of the photon-atom state during \( t \), postselecting on the absence of explosion up to time \( t \), can be encoded in the propagator:

\[
U(t) = \Pi_{II} + \Pi_1 + \Pi_0 e^{-\Gamma t/2},
\]

where \( \Pi_{II} = \sum_j a_{II,j}^\dagger |0\rangle_{\text{ph}} |0\rangle_{\text{br}} a_j \) is the projector on arm \( II \), and \( \Pi_1 \) (resp. \( \Pi_0 \)) is the projector on state \(|\Phi_1\rangle \) (resp. \(|\Phi_0\rangle\)) defined by:

\[
|\Phi_1\rangle = \sum_j \phi_0(\omega_j) |0\rangle_m - \phi_0(\omega_j + \omega_m) |1\rangle_m a_{II,j}^\dagger |0\rangle_{\text{ph}} |0\rangle_{\text{br}}
\]

\[
|\Phi_0\rangle = \sum_j \phi_0(\omega_j) |0\rangle_m + \phi_0(\omega_j + \omega_m) |1\rangle_m a_{II,j}^\dagger |0\rangle_{\text{ph}} |0\rangle_{\text{br}}.
\]

We first compute the weak values associated with the rank 1 projectors \( \Pi_{I,\text{in}} = |I\rangle_{\text{ph}} \langle I|_{\text{ph}} |m\rangle_{\text{in}} \), \( \Pi_{I,\text{out}} = |I\rangle_{\text{ph}} \langle I|_{\text{ph}} |m\rangle_{\text{out}} \), \( \Pi_{I,0} = |I\rangle_{\text{ph}} \langle I|_{\text{ph}} |0\rangle_m \), and \( \Pi_{I,1} = |I\rangle_{\text{ph}} \langle I|_{\text{ph}} |1\rangle_m \) (and similar for arm \( \Pi \)). We obtain:

\[
\langle \Pi_{I,\text{in}} \rangle_w(t) = -\frac{1}{e^{\Gamma t} - 1}
\]

\[
\langle \Pi_{I,\text{out}} \rangle_w(t) = 0
\]

\[
\langle \Pi_{II,\text{in}} \rangle_w(t) = 1 + \frac{1}{e^{\Gamma t} - 1}
\]

\[
\langle \Pi_{II,\text{out}} \rangle_w(t) = 0
\]

\[
\langle \Pi_{I,0} \rangle_w(t) = -\frac{e^{-\Gamma(t-t)/2} + e^{-\Gamma t/2}}{2(1 - e^{-\Gamma t/2})}
\]

\[
\langle \Pi_{I,1} \rangle_w(t) = \frac{e^{-\Gamma(t-t)/2} - e^{-\Gamma t/2}}{2(1 - e^{-\Gamma t/2})}
\]

\[
\langle \Pi_{II,0} \rangle_w(t) = \frac{1}{1 - e^{-\Gamma t}}
\]

\[
\langle \Pi_{II,1} \rangle_w(t) = 0.
\]

One can then deduce the weak values of the rank 2 projectors \( \langle \Pi_I \rangle_w(t) = \langle \Pi_{I,0} \rangle_w(t) + \langle \Pi_{I,1} \rangle_w(t) \) + \( \langle \Pi_{II,0} \rangle_w(t) = -1/(1 - e^{\Gamma t}) \approx 0 \) for \( \Gamma t \gg 1 \), \( \langle \Pi_{II,1} \rangle_w(t) = 1/(1 - e^{-\Gamma t}) \approx 1 \).

We can then compute the energetic weak-values. Using \( H_m = \hbar \omega_m |m\rangle_m \langle 1| \), one has:

\[
\langle H_{m} \rangle_w(t) = \langle H_{m} \Pi_{II} \rangle (t) = \hbar \omega_m \frac{e^{-\Gamma t/2} - e^{-\Gamma t/2}}{2(1 - e^{-\Gamma t/2})}
\]

\[
\langle H_{m} \Pi_{II} \rangle (t) = 0.
\]
Similarly, one has:

\[ \langle H_{ph} \Pi_I \rangle_w (t) = \hbar \omega_{ph} \langle \Pi_{I,0} \rangle_w (t) + \hbar (\omega_{ph} - \omega_m) \langle \Pi_{I,1} \rangle_w (t) \]

\[ \langle H_{ph} \Pi_{II} \rangle_w (t) = \hbar \omega_{ph} \langle \Pi_{II,0} \rangle_w (t). \]  

(34)