Magnetic and superconducting instabilities in the periodic Anderson model: an RPA study

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Abstract. We study the magnetic and superconducting instabilities of the periodic Anderson model with infinite Coulomb repulsion $U$ in the random phase approximation. The Néel temperature and the superconducting critical temperature are obtained as functions of electronic density (chemical pressure) and hybridization $V$ (pressure). It is found that close to the region where the system exhibits magnetic order the critical temperature $T_c$ is much smaller than the Néel temperature, in qualitative agreement with some $T_N/T_c$ ratios found for some heavy-fermion materials. In our study, the magnetic and superconducting physical behaviour of the system has its origin in the fluctuating boson fields implementing the infinite on-site Coulomb repulsion among the $f-$ electrons.

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1. Introduction

The superconducting and magnetic properties of heavy-fermion materials have recently attracted much attention because of their non-conventional character. These materials have very large specific heat coefficients $\gamma$, indicating very large effective quasiparticle masses, hence the designation heavy fermions. Some of these materials, order antiferromagnetically at low temperatures (examples are $UAgCu_4$, $UCu_7$, $U_2Zn_{17}$) while others (such as $UBe_{13}$, $CeCu_2Si_2$, $UPt_3$) order in a superconducting state and others show no ordering (such as $CeAl_3$, $UAuPt_4$, $CeCu_6$, $UAl_2$). Some compounds exhibit phases where antiferromagnetic order coexists with unconventional superconductivity. Examples are: $UPd_2Al_3$ ($T_N = 14.3$ K and $T_c = 2$ K), $UNi_2Al_3$ ($T_N = 4.5$ K and $T_c = 1.2$ K), $CePd_2Si_2$ ($T_N \sim 10$ K and $T_c \sim 0.5$ K) and $CeIn_3$ ($T_N \sim 10$ K and $T_c \sim 0.15$ K). In the prototype heavy-fermion system $CeCu_2Si_2$ the coexistence of $d-$wave superconductivity and magnetic order was clearly identified in a small range $x$ values around $x \simeq 0.99$. 


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Systems that exhibit both superconductivity and antiferromagnetism at low temperature have ratios between the Néel temperature $T_N$ and the superconducting critical temperature $T_c$ of the order of $T_N/T_c \sim 1 - 100$. The coexistence of both types of order can be tuned by external parameters such as external pressure or changes in the stoichiometry. [3, 4]

A description of the normal state properties of the heavy-fermion systems has been attempted assuming a generalization of the impurity Anderson model to the lattice. [5, 6] In the Anderson lattice the energy of a single electron in an $f-$orbital (e. g. $4f^1$) is $\epsilon_0$, and the energy of two electrons in the same $f-$orbital ($4f^2$) is $2\epsilon_0 + U$, where $U$ is the on-site Coulomb repulsion. The energy of the $4f^2$ state is much larger than the energy of the $4f^1$ state. Thus, if the charge fluctuations at the $f-$orbital are small, the $4f^1$ electron may behave as a local moment.

The complexity of heavy-fermion systems arises from the interplay between Kondo screening of local moments, the antiferromagnetic (RKKY) interaction between the moments and the superconducting correlations between the heavy quasi-particles. The local moments form in partially filled $f$ shells of Ce and U ions. The absence of magnetic order in some cases could perhaps be due to complete Kondo screening (below the Kondo temperature $T_K$) or to a spin liquid arrangement of the local moments. In the normal non-magnetic state the Anderson lattice model predicts Fermi-liquid like behaviour and explains the main features at low temperatures, such as the large effective masses and the Kondo resonance near the Fermi level. But the main technical difficulty is the competition between the Kondo compensation of the localized spins and the magnetic interaction between them. This interaction is mediated by the conduction electrons (RKKY-type). Related to this competition is the effectiveness of the compensating cloud around each $f$-site. The size of this cloud has been subject of controversy. While some arguments show that it should be a large scale of the order of $v_F/T_K$, other arguments claim to be $\sim a$ ($a$ is the lattice constant). [8] This is a relevant issue and is related to Nozières exhaustion problem which states that there are not enough conduction electrons to screen the $f$-moments.

It has been proposed that the mechanism for superconductivity lies in the strong Coulomb interaction between the $f$-electrons, not in a phonon mediated attraction. Using Coleman’s [9] slave boson formalism together with a large-$N$ approach, various attempts have been made to search for the existence of an effective interaction which might be responsible for superconductivity in the infinite-$U$ Anderson-lattice model. It was proposed [10] that slave boson fluctuations can provide an effective attraction between the electrons to leading order in $1/N$. Later, a calculation of the electron-electron scattering amplitude to order $1/N^2$ revealed an effective attractive interaction in the $p$ and $d$ channels, which was interpreted as a manifestation of the RKKY interaction, showing that spin fluctuations are an important mechanism. [11] The inclusion of $f^0$, $f^1$ and $f^2$ states, using two sets of slave bosons, was also considered in the context of the Anderson lattice as a possible description of high-$T_c$ superconductors. [12]

The magnetic order in the ground state of Kondo Insulators has been studied by
Dorin and Schlottmann in the framework of the Anderson-lattice model\cite{13}. The same authors have later studied the effect of orbital degeneracy and finite $U$ on a ferromagnetic ground state (their approach did not generate RKKY interactions, thus preventing the study of antiferromagnetic order)\cite{14}.

In this work we consider the slave-boson approach to the infinite-$U$ Anderson-lattice model. We treat the boson fields at the mean-field level, thereby enforcing the constrain of one $f$-electron (at most) per site only on the average. By splitting the boson operator into a condensate part and an above-the-condensate term, which describes fluctuations, we compute the magnetic and pairing susceptibilities at the random-phase approximation (RPA) level. For spin 1/2 particles the condensate density at moderate temperatures does not change much relative to its ground-state value. Therefore we do not expect our results to be of less quality then those characterizing the ground-state properties. We search for the critical temperatures ($T_N$ and $T_c$) at which antiferromagnetic order or superconducting ($s$ or $d$-wave) pairing occurs in a normal non-magnetic system. We find that the value of $T_c$ is much smaller than the magnetic temperature $T_N$. Unlike $T_N$, the superconducting temperature monotonically increases with externally applied pressure.

2. The model and the RPA solution

The PAM Hamiltonian is given by

$$H = H_c^0 + H_f^0 + H_{cf} + H_U,$$

where

$$H_f^0 = \sum_{i,\sigma}(\epsilon_0 - \mu)f_{i,\sigma}^\dagger f_{i,\sigma},$$

$$H_c^0 = \sum_{k,\sigma}(\epsilon_k - \mu)c_{k,\sigma}^\dagger c_{k,\sigma},$$

$$H_{cf} = V\sum_{i,\sigma}\left( c_{i,\sigma}^\dagger f_{i,\sigma} + f_{i,\sigma}^\dagger c_{i,\sigma} \right),$$

$$H_U = U\sum_{i} n_{i,\uparrow} n_{i,\downarrow}.$$  

The $c$ and $f$ operators are fermionic and obey the usual anti-commutation relations. The hybridization potential $V$ is assumed to be momentum independent. The term $H_U$ represents the strong on-site repulsion between the $f$-orbitals. We consider $U = \infty$. We implement the condition $U = \infty$ within the slave-boson formulation due to Coleman\cite{9}, in which the empty $f$-site is represented by a slave boson $b_i$ and the physical operator $f_i$ in equation (4) is replaced with $b_i^\dagger f_i$ together with the constrains of only one $f$-electron per site. The implementation of this constraint amounts to introducing a Lagrange multiplier $\lambda$ which will renormalize the bare $f$-level energy from $\epsilon_0$ to $\epsilon_f = \epsilon_0 + \lambda$. We split the boson operators in two terms

$$b_i^\dagger = \sqrt{N}\sqrt{z}\delta_{0,q} + B_i^\dagger,$$
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where \( z \) represents the boson condensate and \( B^\dagger_{\vec{q}} \) represents the fluctuations above the condensate. This procedure leads in leading order to a mean field Hamiltonian [15, 16]. The corresponding mean field equations can be written in terms of the Fourier transform of the Green’s functions

\[
G_{ff,\sigma}(\vec{k}, \tau - \tau') = -\langle T_\tau f_{\vec{k},\sigma}(\tau) f^\dagger_{\vec{k},\sigma}(\tau') \rangle, \\
G_{cc,\sigma}(\vec{k}, \tau - \tau') = -\langle T_\tau c_{\vec{k},\sigma}(\tau) c^\dagger_{\vec{k},\sigma}(\tau') \rangle, \\
G_{cf,\sigma}(\vec{k}, \tau - \tau') = -\langle T_\tau c_{\vec{k},\sigma}(\tau) f^\dagger_{\vec{k},\sigma}(\tau') \rangle,
\]

as

\[
z = 1 - \frac{T}{N_s} \sum_{\vec{k},\sigma} \sum_{i\omega_n} G_{ff,\sigma}(\vec{k}, i\omega_n),
\]

and

\[
\epsilon_f = \epsilon_0 - \frac{VT}{\sqrt{z}N_s} \sum_{\vec{k},\sigma} \sum_{i\omega_n} G_{cf,\sigma}(\vec{k}, i\omega_n),
\]

where \( N_s \) denotes the number of lattice sites. Equation (10) states that the mean number of electrons at an \( f \)-site is \( n_f = 1 - z \). For a given number of particles per site, \( n \), these equations must be supplemented with the particle conservation condition which yields the chemical potential \( \mu \) for any temperature:

\[
n = 1 - z + \frac{T}{N_s} \sum_{\vec{k},\sigma} \sum_{i\omega_n} G_{cc,\sigma}(\vec{k}, i\omega_n).
\]

The fluctuations beyond the mean field approach are described by the Hamiltonian

\[
H_{\text{fluct}} = \frac{V}{\sqrt{N}} \sum_{\vec{k},\vec{q},\sigma} (c^\dagger_{\vec{k},\sigma} f_{\vec{q},\sigma} B_{\vec{k}-\vec{q}}^\dagger B_{\vec{k},\sigma} + B_{\vec{k}-\vec{q}} c^\dagger_{\vec{k},\sigma} f_{\vec{q},\sigma}^\dagger c_{\vec{k},\sigma}),
\]

and will be considered in the calculation of the magnetic susceptibility and superconducting correlation functions below. The calculation, even at the RPA level, of the correlation functions requires the knowledge of the boson propagator. The full calculation of the latter is a technically difficult problem by itself, and is still unsolved. There are, however, \( 1/N \) calculations of \( D(\vec{k}, \tau - \tau') \). [6, 10, 11] Here we follow the work of Evans [17] and use an asymptotic form for the boson propagator given by

\[
D(\vec{k}, \tau - \tau') = \langle T_\tau B_{\vec{k},\sigma}(\tau) B_{\vec{k},\sigma}^\dagger (\tau') \rangle \sim \frac{1}{\lambda}.
\]

We also adopt the same approximation for the propagator \( \tilde{D}(\vec{k}, \tau - \tau') = \langle T_\tau B_{\vec{k},\sigma}^\dagger(\tau) B_{\vec{k},\sigma}(\tau') \rangle \). In the calculation below we shall use mean field fermionic propagators.

The transverse spin susceptibility for the \( f \) electrons is defined as

\[
\chi_{\perp}(\vec{q}, i\omega_n) = \mu_B^2 \int_0^\beta d\tau e^{i\omega_n \tau} \langle T_\tau S^-(-\vec{q}, \tau) S^+(\vec{q}, 0) \rangle,
\]

\[
\chi_{\perp}(\vec{q}, i\omega_n) = \mu_B^2 \int_0^\beta d\tau e^{i\omega_n \tau} \langle T_\tau S^-(-\vec{q}, \tau) S^+(\vec{q}, 0) \rangle,
\]

\[
\chi_{\perp}(\vec{q}, i\omega_n) = \mu_B^2 \int_0^\beta d\tau e^{i\omega_n \tau} \langle T_\tau S^-(-\vec{q}, \tau) S^+(\vec{q}, 0) \rangle.
\]
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where $\beta = 1/T$ is the inverse temperature, $T_\tau$ is the chronological order operator (in imaginary time), $S^-(\vec{q}) = \sum_{\vec{p}} S^+_{\vec{p}+\vec{q}}$ and $S^+(\vec{q}) = [S^-(\vec{q})]^\dagger$. The calculation at the RPA level yields

$$
\chi^f_{\pm}(\vec{q}, i\omega_n) = \frac{\Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n)[1 - J\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n)]}{[1 - J\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n)]^2 - J^2 \Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n)\Gamma^{cc}_{\vec{q}}(\vec{q}, i\omega_n)},
$$

(16)

where $J = V^2/(N\lambda)$. The result (16) holds for all values of $n_f$ and is a generalization of that obtained by Evans [17][18] for the case $n_f = 1$. The functions $\Gamma(\vec{q},i\omega_n)$ above are given by:

$$
\Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n) = -\frac{1}{\beta} \sum_{\vec{p},i\omega_m} G_{ff}(\vec{p}, i\omega_m)G_{ff}(\vec{p} + \vec{q}, i\omega_m + i\omega_n),
\Gamma^{cc}_{\vec{q}}(\vec{q}, i\omega_n) = -\frac{1}{\beta} \sum_{\vec{p},i\omega_m} G_{cc}(\vec{p}, i\omega_m)G_{cc}(\vec{p} + \vec{q}, i\omega_m + i\omega_n),
\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n) = -\frac{1}{\beta} \sum_{\vec{p},i\omega_m} G_{cf}(\vec{p}, i\omega_m)G_{cf}(\vec{p} + \vec{q}, i\omega_m + i\omega_n)
$$

There are three possible superconducting pairing susceptibilities that one can define. These refer to Cooper pairs of either $c$–electrons or $f$–electrons, and a hybrid Cooper pair with a $c$– and an $f$–electron. We consider the correlation function:

$$
\Delta_{dd}(\vec{q}, i\omega_n) = \int_0^\beta e^{i\omega_n\tau} \sum_{\vec{k}_1,\vec{k}_2} \eta(\vec{k}_1)\eta(\vec{k}_2)\langle T_\tau d_{\vec{k}_1,\downarrow}(\tau) d_{-\vec{k}_1+\vec{q},\uparrow}(\tau) d_{\vec{k}_2,\uparrow}^\dagger d_{-\vec{k}_2+\vec{q},\uparrow}^\dagger \rangle,
$$

(17)

where $d = c, f$ and $\eta(\vec{k})$ is the Cooper pair structure factor, assumed to be either extended $s$–wave or $d$–wave. The hybrid pairing correlation function is defined as

$$
\Delta_{cf}(\vec{q}, i\omega_n) = \int_0^\beta e^{i\omega_n\tau} \sum_{\vec{k}_1,\vec{k}_2} \langle T_\tau f^{\uparrow}_{\vec{k}_1}(\tau)c_{-\vec{k}_1+\vec{q},\uparrow}(\tau)c_{\vec{k}_2,\downarrow}^\dagger f_{-\vec{k}_2+\vec{q},\uparrow}^\dagger \rangle.
$$

(18)

This definition has been used previously in a mean field study of the Kondo lattice [19]. At the RPA level the Cooper pair correlation function [17] is given by

$$
\Delta_{dd}(\vec{q}, i\omega_n) = \Gamma^{dd}_{\vec{q}}(\vec{q}, i\omega_n) + \frac{J\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n)\Gamma^{cc}_{\vec{q}}(\vec{q}, i\omega_n)}{1 - J[\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n) + \Gamma^{cc}_{\vec{q}}(\vec{q}, i\omega_n)]},
$$

(19)

and the function (18) is given by

$$
\Delta_{cf}(\vec{q}, i\omega_n) = \Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n) + \frac{J\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n)\Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n)}{[1 - J\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n)]^2 + J^2[\Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n)]^2}.
$$

(20)

The $\Gamma(\vec{q},i\omega_n)$ functions appearing in the previous expressions are given by

$$
\Gamma^{cc}_{\vec{q}}(\vec{q}, i\omega_n) = \frac{1}{\beta} \sum_{\vec{p},i\omega_m} \eta^2(\vec{p})G_{cc}(\vec{p}, i\omega_m)G_{cc}(\vec{p} + \vec{q}, -i\omega_m + i\omega_n),
\Gamma^{cc}_{\vec{q}}(\vec{q}, i\omega_n) = \frac{1}{\beta} \sum_{\vec{p},i\omega_m} \eta(\vec{p})G_{cc}(\vec{p}, i\omega_m)G_{cc}(\vec{p} + \vec{q}, -i\omega_m + i\omega_n),
\Gamma^{ff}_{\vec{q}}(\vec{q}, i\omega_n) = \frac{1}{\beta} \sum_{\vec{p},i\omega_m} G_{ff}(\vec{p}, i\omega_m)G_{cc}(\vec{p} + \vec{q}, -i\omega_m + i\omega_n),
\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n) = \frac{1}{\beta} \sum_{\vec{p},i\omega_m} G_{cf}(\vec{p}, i\omega_m)G_{cf}(\vec{p} + \vec{q}, -i\omega_m + i\omega_n),
\Gamma^{cf}_{\vec{q}}(\vec{q}, i\omega_n) = \frac{1}{\beta} \sum_{\vec{p},i\omega_m} G_{cf}(\vec{p}, i\omega_m)G_{cf}(\vec{p} + \vec{q}, -i\omega_m + i\omega_n)
$$
3. Superconducting and magnetic instabilities

The magnetic and superconducting instabilities of the system are signaled by the poles of the corresponding susceptibilities. Therefore, we search for the temperature $T$ at which the denominators in the RPA expressions for the susceptibilities vanish:

$$K_m(Q,0) = [1 - J\Gamma_{cf}(0,0)]^2 - J^2\Gamma_{ff}(0,0)\Gamma_{cc}(0),$$
$$K_{dd}(0,0) = 1 - J[\Gamma_{cf}(0,0) + \Gamma_{cc}(0,0)],$$
$$K_{cf}(0,0) = [1 - J\Gamma_{cf}(0,0)]^2 + J^2[\Gamma_{cc}(0,0)],$$

where $Q = (\pi, \pi, \pi)$ and $K_m(Q,0)$, $K_{dd}(0,0)$, and $K_{cf}(0,0)$ are the Stoner factors of the correlation functions (16), (19), and (20), respectively. Since heavy-fermion materials are antiferromagnetic materials we seek for poles of $K_m(Q,0)$ at the antiferromagnetic wave vector $Q$. From the definitions of the $\Gamma(q, i\omega_n)$ functions we see that the Cooper pair structure factor $\eta(p)$ does not appear in the Stoner factors $K_{dd}(0,0)$ and $K_{cf}(0,0)$. Moreover, we shall see below that the solutions to $K_{dd}(0,0) = 0$ and $K_{cf}(0,0) = 0$ both lead to the same critical temperature. Therefore, the system’s tendency for a certain Cooper pair symmetry only shows up in the intensity of $\Delta_{dd}(0,0)$ or $\Delta_{cf}(0,0)$, which is controlled by the numerator of these functions. We also see that both antiferromagnetism and superconductivity are controlled by the same interaction parameter $J$, which, in turn, depends on hybridization only.

In Figure 1 we show a plot of the Néel and superconducting temperatures as functions of the total electronic density $n$. It is seen that antiferromagnetism can only occur in a very small region of electronic density. Furthermore, the increase of $T_N$ when $n \to 2$ corresponds to an increase of the density of $n_f$ electrons towards the Kondo limit ($n_f = 1$). It is also clear that $T_N$ is not a monotonically increasing function of $J$. Upon reducing $V$, a larger range of electronic densities can be reached where antiferromagnetic order can be found. From the inset of Figure 1 we see that the superconducting temperature $T_c$ is very small (about $T_c \sim T_N/50$) close to the density region where the system exhibits antiferromagnetic order.

The dependence of the Néel and superconducting temperatures on pressure has been measured in some heavy-fermion systems [4, 20, 21]. In those studies the Néel temperature is found to decrease as the applied pressure increases and superconducting order is found to develop in a limited range of applied pressures, when the Néel temperature is reduced below $\sim 1K$. Let us now see how the critical temperatures in our model vary with the model parameters which, in principle, should depend on externally applied pressure. Increasing pressure should, presumably, make both the hybridization $V$ and the conduction band hopping $t$ increase [22, 23]. In Figure 2 we present $T_N$ and $T_c$ versus $V$, taking the ratio $V/t$ constant. We see that above a certain value of $V$ the magnetic order disappears but the superconducting order remains. We also find that close to the region where the magnetic order vanishes, $T_c$ is much smaller than the maximum value attained by $T_N$. This is in qualitative agreement with experimental data on some Cerium compounds (e.g. CeIn$_3$), where the ratio $T_N/T_c \sim 100$. Other
examples are: CeCu$_2$(Si$_{1-x}$Ge$_x$)$_2$, where $T_c$ as function of pressure displays a positive curvature; and CeRhIn$_5$, where the $T_c$ curve is almost parallel to pressure axis. For CeCu$_2$Ge$_2$ and CeCu$_2$Si$_2$ the $T_c$ curve initially stays almost parallel to the pressure axis, but it shoots up above a certain pressure. Although $T_c$ keeps increasing as $V$ increases, it never reaches values comparable with the maximum value of $T_N$, even for unreasonable values of $V$ as we can see in the right panel of Figure 2. We believe that a better treatment of the boson propagator will lead to a decrease of $T_c$ in agreement with the experiments. We remark that the above calculation of $T_c$ is only valid in the in a situation where the system is non-magnetic because we have not calculated $K_{dd}(0,0)$ or $K_{cf}(0,0)$ in the magnetically ordered phase. Furthermore, when $T_c$ is small, the approximation employed for the boson propagator should be improved by including its low energy part.

For comparison we also plot the temperature $T_K$ defined as the difference between the renormalized $f-$level energy $\epsilon_f$ and the chemical potential $\mu$. This can be very different from the lattice Kondo temperature in the non-magnetic system. Nevertheless, the combined behavior of $T_N$ and $T_K$ represents the well-known Doniach diagram showing the interplay between RKKY and Kondo screening effect. For small values of $V$, $T_K$ is exponentially small and the system shows antiferromagnetic order. On the other hand, as $V$ increases the Kondo temperature grows, leading to Kondo compensation of the $f-$moments and to a decreasing Néel temperature. For even larger values of $V$ complete disappearance of the magnetic order takes place and the system shows paramagnetic behavior (assuming there are enough conduction electrons
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Figure 2. Left: Néel Temperature $T_N$ and superconducting critical temperature $T_c$ as function of $V$, for a constant ratio of $V/t = 1.2$, electronic density $n = 1.8$, and $\epsilon_0 = -0.25D$ . Right: $T_c$ over a very large (nonphysical) values of $V$. Note that $T_c \ll T_N$ always. $\langle cc \rangle$ and $\langle cf \rangle$ means that $T_c$ has been computed using equations 22 and 23 respectively. Both give the same results.

to compensate all the $f$–local moments). We have also computed the superconducting critical temperature from both equations (22) and (23) and obtained the same $T_c$, as can be seen in the right panel of Figure 2. Along the $T_N$ curve, $n_f$ decreases from 1 to 0.8, as $V$ increases, and $n_f \approx 0.85$ when $T_N$ is maximum.

Both Figure 1 and Figure 2 show similar behaviour near the point where $T_N \to 0$. In both cases $T_c$ starts to increase with a positive curvature. Although in many heavy-fermion systems $T_c$ presents a negative curvature, there are examples where a positive curvature have been observed, such as CePd$_2$Si$_2$ under chemical pressure ($T_N = 10$ K, $T_c = 0.2$ K, therefore $T_N/T_c = 50$) [25].

Since our treatment does not take competition between magnetism and superconductivity into account, we cannot tell whether finite values of $T_c$ and $T_N$ imply that both types of order will be present at low temperature. Nevertheless, we found in previous work [23], at the simplest mean field level, that magnetism and superconductivity may coexist in the system. It follows from the above remarks that the calculation of $T_c$ when $T_N$ is finite requires both the introduction of a better
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Approximation for the boson propagator and extra electronic propagators describing the antiferromagnetic order in the system, as was done in the description of spin waves in the magnetically ordered Mott insulator.

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