Experimental simulation of tensile fractures from a circular cavity

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Abstract. The authors experimentally study initiation of tensile fractures from a circular cavity in a geomedium subjected to nonuniform compression. It is found that diameter of the cavity has a significant influence on the local fracture of the geomaterial. A new criterion is proposed for the quasi-brittle fracture of a material in the tensile stress concentration zone. The new criterion holds an integrated parameter which characterizes the size of the pre-fracture zone, the structure of a material and its plastic properties, the cavity geometry and the conditions of loading. The calculation and experiment result agree well.

1. Introduction
Rocks feature lower tensile strength as against their strength in compression. In the meanwhile, even under compression, zones of tension stresses and tensile fractures can arise in surround rock mass around underground opening, which is endangers mine safety.

Furthermore, in structurally inhomogeneous materials such as rocks, strength properties depend on the loaded volume (size effect). This dependence is especially pronounced under conditions of stress concentration when the actual loaded volume is governed by the stress concentration zone while the size of the latter is smaller than the size of the body subjected to deformation. In this case, the critical load is calculated using nonlocal and gradient fracture criteria [1–11]. The nonlocal criteria are based on the concept of a pre-fracture zone with local stress re-distribution while the main bulk of the material deforms elastically to failure. An alternative approach to the size effect assumes that the size effect in the brittle materials is connected not with the stress re-distribution but with the actual increase in the local ultimate stress. The gradient criterion of fracture is based on the concept of local strength [12].

This paper presents the experimental and theoretical studies into influence exerted by a circular cavity diameter on fracture of a structurally inhomogeneous geomaterial in the zone of stress concentration under nonuniform compression with regard to the size effect. The authors also analyze applicability of the gradient criterion to the description of quasi-brittle fractures.

2. Experimental procedure
We nonuniform compression tests were carried out on gypsum specimens with circular cavities. The specimens were made of a water solution of alabaster at a S : W ratio of 1.5 : 1. The specimens were made as square slabs 200×200 mm with thickness of 36 mm. The manufactured specimens were dried in the open air for 30–40 days. The density of dry specimens was 1.10 g/cm³. Before testing, circular cavities with different diameters from 1 to 20 mm were drilled in the slabs. The load $p$ was applied to
the specimens through rigid inserts 120 mm in size, placed between a specimen and the testing machine presses (Figure 1).

![Diagram of specimen loading](image)

**Figure 1.** Scheme of specimen loading.

We tested 5 specimens with cavities of each of the chosen diameters. The tests demonstrated the sudden nature of tensile fractures from the cavity boundary in the zones of tensile stress concentration. In the specimens with cavities having diameters from 5 to 20 mm, fractures rapidly grew to a length of 500 mm along the line of the compression application; the growth of these fractures stabilized in further loading. In the specimens with cavities having diameters of 12 mm, tensile fractures immediately propagated along the cavity vertical cross-section. Fracturing was accompanied by local relaxation of the specimen form stresses, which was echoed by a kink in the deformation curve. The strongest relaxation takes place in the specimens with a smaller diameter cavity (1 mm). Figure 2 presents the representative deformation curves for the specimens with cavities of different diameters in the form of a testXpert screenshot with test results. The crucial load at the moment of fracture initiation was determined by the tooth point.

![Deformation curves](image)

**Figure 2.** Deformation curves in specimens with cavities with different diameters, mm:
1—1; 2—5; 3—10; 4—15.

The strength of the material in compression was determined on the specimens of the same size 200×200 mm without cavities. Loading was applied via inserts 200 mm in size. The ultimate compression strength was 11.53 MPa. The ultimate tension strength was determined on corset samples with a radius of the effective part radius of 110–120 mm and with the minimal cross with of 29 mm. The ultimate tension strength was 2.61 MPa.

3. **Theoretical approach**

According to the gradient criterion [12], the local strength of a material is assumed to depend on the stress concentration zone size $L_e$. If $L_e$ is sufficiently large as compared with the size of structural component of the material, the local strength slightly differs from the limit stress $\sigma_0$ determined from
uniform stress tests. Vice versa, if $L_e$ is comparable with the size of the structural elements, the latter have higher influence on the local strength. This influence is stronger with smaller $L_e$ relative to the characteristic structural size $L_0$ of the material. In this manner, the local strength depends not on the stress concentration zone size $L_e$ but on the ratio $L_0/L_e$ which is the size effect characteristic in this problem. In this regard, the fracture criterion is given by $f = f(L_0/L_e)$, where $f$ is the equivalent stress characterizing the internal stress state of the body under deformation. In the problem on tensile fractures under compression, the equivalent stress is determined from the theory of the highest tensile stresses, and the local strength is given by the function:

$$f(\sigma_0, L_0/L_e) = \sigma_0 \left(1 + \left(\frac{L_0}{L_e}\right)^n\right);$$

where $\sigma_0$ is the ultimate tensile strength of the material; $L_e = \frac{\sigma_0}{[\text{grad}\sigma_0]}$. The critical stress of initiation of tensile fracture from the cavity is found from the expression:

$$\sigma_c = \frac{\sigma_0}{k} \left(1 + \left(\frac{L_0}{L_e}\right)^n\right).$$

The coefficient $k$ in (2) is a ratio of the maximal tensile stress at the cavity boundary to the applied stress $\sigma$. For brittle materials, the exponent $n = 1$. At zero $L_0$ formula (2) offers the calculation of the critical stress by the conventional fracture criterion.

Under loading as in Figure 1, in the center (beyond the influence zone of the cavity), the stress state is uniform and biaxial: horizontal tension $a\sigma$ and vertical compression $\sigma$ (Figure 3).

The values of $\sigma$ and $a$ were calculated by the finite element method in the center of the specimens without cavities and subjected to loading via inserts of the preset size: $\sigma = 0.764\ p$, $a = 0.187$.

According to the known solution of a Cauchy problem, the distribution of the normal stress $\sigma_y$ along the compressive loading direction is:

$$\sigma_y = \frac{\sigma}{2} \left(\frac{3a^4}{x^4} - \frac{a^2}{x^2}\right) + \frac{a\sigma}{2} \left(2 + \frac{a^2}{x^2} + \frac{3a^4}{x^4}\right),$$

where $a$ is the cavity radius. The coordinate origin is selected at the center of the cavity, the compressive stress is assumed to have a positive value. Then, considering (3), in the problem under discussion, we have:

$$k = 1 + 3a, \quad L_e = a \frac{1 + 3a}{5 + 7a}.$$
The gradient criterion in its original formulation assumes no damage zone in a material and, for this reason, its application is limited to brittle fracture. For the description of quasi-brittle fracture, in accord with [13], we present the structural size \( L_0 \) as:

\[
L_0 = d_0 + \beta L_e, \tag{4}
\]

where \( d_0 \) is the structural dimension; \( \beta \) is a dimensionless parameter characterizing plasticity of a material. For brittle materials, \( \beta = 0 \), for plastic materials, \( \beta >> 1 \). At \( \beta \sim 1 \) materials possess moderate plasticity. The first summand in expression (4) describes the structure of the material, the second summand reflects the input of inelastic strains.

In the original formulation of the criterion (at \( \beta = 0 \)), the parameter \( L_0 \) meant the structural dimension \( d_0 \) and had no associations with the pre-fracture zone. Considering expression (4), \( L_0 \) means the pre-fracture zone dimension \( d \) in the nonlocal criteria [13], and the increase in the local strength (1) is connected with both actual size effect, typical of brittle materials, and an ‘apparent’ increase owing to the elastic stress redistribution in the pre-fracture zone.

In compliance with formula (2) and with regard to the estimates for \( \sigma \) and \( a \), the critical pressure in a specimen with a circular cavity is given by:

\[
p_c = 0.838 \chi C_0 \left(1 + 8.081 \frac{d_0}{l} \beta \right), \tag{5}
\]

where \( \chi = \sigma_0 / C_0 \), \( C_0 \) is the ultimate compression strength of material; \( l \) is the cavity diameter. The first summand in expression (5) is reflective of the conventional calculation of the critical (destructive) load without regard to the size effect, the second summand and the third summands define the input of the actual increase and ‘apparent’ increase of the local strength in the size effect, respectively. At \( l \rightarrow \infty \), \( T_s = T_0 (1 + \beta) \), where \( T_0 = 0.838 \chi C_0 \) is the asymptotic value of the critical pressure for brittle materials.

4. Results and discussion

Figure 4 demonstrates the experimental data (points) on the load value at the moment of initiation of tensile fractures from the cavity depending on the cavity diameter, and also the critical pressure calculations from formula (5) at \( \beta = 0 \), (curve 1) and \( \beta = 0.3 \) (curve 2). The size \( d_0 \) made 0.5 mm. the asymptotic values of the critical pressure in the first and second cases are \( T_0 \) (dashed line) and \( T_s = 1.37T_0 \) (solid line).

![Figure 4. Critical pressure versus cavity diameter.](image)

Figure 4 demonstrates considerable size effect, i.e. influence of the cavity diameter on the local strength of the material. The critical pressure grows up to the ultimate compression strength as the size effect weakens, or asymptotically approaches the stress \( T_s \) as the size effect increases.
The initial gradient criterion of fracture (at $\beta = 0$) used to describe the experimental data produces satisfactory estimates of the critical pressure only at small diameters (1–2 mm) of the cavity. The calculations at large cavity diameters yield underestimated values of critical pressure. According to the experimental results, at larger diameters of the cavity, the critical pressure asymptotically tends to a value 30% higher than the value calculated for an elastic body. Fracture of the test specimens is accompanied by instantaneous sudden initiation and fast growth of tensile fractures from the cavity boundary and along the compression axis. This allows characterizing fracture of the material in the test range of the cavity diameters as quasi-brittle fracture. Such behavior is smoothly described by the modified gradient criterion of fracture, with the parameter $L_0$ determined from formula (4).

5. Conclusions
Fracture of a model geomaterial with a circular cavity is studied theoretically and experimentally under the action of nonuniform compression. The applicability of the gradient criterion of fracture for the destructive load estimation is discussed. The test specimens demonstrate the quasi-brittle fracture behavior. In this case, the conventional gradient criterion gives unsatisfactory estimates of the destructive load. For this reason, the authors have proposed the modified gradient criterion including an integrated parameter which characterizes the size of the pre-fracture zone, structure of the material and its plastic properties, geometry of the specimens and the loading conditions. The calculations using the modified criterion agree with the experimental testing data.

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