The fragmentation of deuterons into pions with nonzero angle emitted in the kinematical region forbidden for free nucleon-nucleon collisions is analyzed. The inclusive relativistic invariant spectrum of pions and the tensor analyzing power $A_{YY}$ are investigated within the framework of an impulse approximation using different kinds of the deuteron wave function. The influence of P-wave contribution to the deuteron wave function is studied, too. Our results are compared with the experimental data and other calculations performed within both the non-relativistic and relativistic approaches. It was found that theoretical calculations based on IA do not provide a consistent understanding of the new data in a whole cumulative region that can be caused by a influence of non-nucleon degrees of freedom.

1 Introduction

Recently the intriguing preliminary data obtained at “SPHERE” group (LHE JINR) on the deuteron tensor analyzing power $A_{YY}$, measured in the reaction on fragmentation of tensor polarized deuterons into cumulative pions with nonzero pion production angle, have been reported [1, 2]. They are rather precise and correspond to a maximum cumulative variable $x_C$ of 1.75 (internal momentum up to 500 MeV/c). Since in the $pD$ collisions the deuteron gets from proton a momentum comparable with the deuteron mass, these data are probing the truly relativistic dynamics inside the deuteron [3].

The interest of deuterons fragmentation into pions arises from the possibility (I) to experimental study of the polarization observables of deuteron fragmentation into hadrons containing of different quarks, additionally to well-famous proton stripping reaction [3, 4, 5], (II) to theoretical study of nuclear structure at short distances on a based of available deuteron models, (III) to develop in the future the reasonable theory of the relativistic nuclear systems. The measurement of the unpolarized characteristics together with polarization observables allows us to analyze this processes more correctly and to conclude that together with the scheme of the deuteron wave function (DWF) relativization the relativistic description of such processes themselves is of important as well.

In this talk we present a relativistic invariant analysis of the deuteron tensor analyzing power $A_{YY}$ performed in the framework of the relativistic impulse approximation at nonzero pion production angle (see, for review, the paper [8], which contains also our calculation of $T_{20} = -\sqrt{2}A_{YY}$ at zero pion production angle).
2 Relativistic impulse approximation

Let us consider the inclusive reaction of fragmentation of tensor polarized deuteron to pion, $D(p, \pi)X$, where the typical initial energy of a few GeV, and the final pion is detected with nonzero angle $\theta_\pi$. Within spectator mechanism approach this reaction presented by the impulse approximation diagram shown in figure 1, where the upper and lower vertices should factorize and consequently they may be computed separately.

![Impulse approximation diagram](image)

If the initial deuteron is only aligned due to $p^{YY}_D$ component, then the inclusive spectrum of this reaction can be written in the form:

$$\rho^\pi_{pD}(p^{YY}_D) = \rho^\pi_{pD}\left[1 + A_{YY} \cdot p^{YY}_D\right],$$

where $\rho^\pi_{pD}$ is the inclusive spectrum for the case of unpolarized deuterons and $A_{YY}$ is the tensor analyzing power. They can be written in a fully covariant manner within the Bethe-Salpeter formalism. This way is possible to come to general conclusions about the amplitude of the process not to seen in the non-relativistic approach. Such analysis of the deuteron models is very important assuming the searching of nuclear quarks phenomena. One can write the observables in the factorization form:

$$\rho^\pi_{pD} = \frac{1}{(2\pi)^3} \int \frac{\sqrt{\lambda(p, n)}}{\sqrt{\lambda(p, D)}} \left[\rho^\pi_{pN} \cdot \Phi^{(u)}(|q|)\right] \frac{d^3q}{E_q};$$

$$\rho^\pi_{pD} \cdot A_{YY} = \frac{1}{(2\pi)^3} \int \frac{\sqrt{\lambda(p, n)}}{\sqrt{\lambda(p, D)}} \left[\rho^\pi_{pN} \cdot \Phi^{(t)}(|q|)\right] \left(1 - 3 \sin^2 \theta_q \sin^2 \varphi_q \right) \frac{d^3q}{E_q}$$

where $\lambda(p_1, p_2) \equiv (p_1p_2)^2 - m_1^2m_2^2 = \lambda(s_{12}, m_1^2, m_2^2)/4$ is the flux factor; $p, n$ are the four-momenta of the proton-target and intra-deuteron nucleon, respectively; $\rho^\pi_{pN}$ is the relativistic invariant inclusive spectrum of pions produced by interacting the intra-deuteron nucleon with the proton-target.
In the general case, this spectrum can be written as a three-variable function
$$\rho_{\pi p_N} = \rho(x_f, \pi, s_{NN})$$. Feynman’s variable,
$$x_f$$, is defined as
$$x_f = 2 \pi| q||/\sqrt{s_{NN}}$$, where
$$\pi$$ is the pion momentum in the center of mass of two interacting nucleons
and
$$s_{NN} = (p+n)^2$$. As shown in (1), A_{YY} is more sensitive to the DWF form than
this invariant spectrum.

The functions
$$\Phi(u(|q|))$$
and
$$\Phi(t(|q|))$$
depends on the relative momentum
$$q = (n-p')/2$$ and contains full information about the structure of deuteron with one on-shell
nucleon. They can be written in terms of positive-energy,
$$V_s = \frac{1}{2} P^+ - 1$$,
$$V_t = \frac{3}{2} P^- + 1$$, and negative-energy,
$$V_s = \frac{1}{2} P^+ - 1$$,
$$V_t = \frac{3}{2} P^- + 1$$, wave functions:
$$\Phi(u(|q|)) = \frac{2}{\sqrt{3}} m \left[ U - \frac{W}{\sqrt{2}} \right] + \frac{V_s}{\sqrt{2}} \right]$$.

It is intuitively clear that the two nucleons in the deuteron are mainly in states
with angular momentum
$$L = 0, 2$$ (see also numerical analysis of the solutions of the
BS equation in terms of amplitudes within the \( \rho \)-spin basis \( \Phi \)), so the probability
of states with
$$L = 1$$ \( \Phi_{s,t} \) in Eqs.(4,5) is much smaller in comparison with the
probability for the \( U, W \) configurations. Moreover, it can be shown that the \( U \)
and \( W \) waves directly correspond to the non-relativistic S and D ones. Therefore,
Eqs.(4,5) with only \( U, W \) waves can be identified as the main contributions to the
corresponding observables and they may be compared with their non-relativistic
analogies. The other terms posses contributions from the P-waves and they are
proportional to \( q/m \) (the diagonal terms in \( \Phi_{s,t} \) are negligible). Due to their pure
relativistic origin one can refer to them as relativistic corrections.

The natural way to compare the non-relativistic and relativistic calculations is the
Kamada-Glöckle method \( \Psi \) that is used to make relativistic quantum models
using a realistic nonrelativistic nucleon-nucleon interaction as input, which requires
the operator for the kinetic energy is formed out of square roots. This defines the
scale transformation between the relativistic momenta \( q \) in Eqs.(4,5) and nonrelativistic
momenta \( q_{KG} \):
$$2 \sqrt{q^2 + m^2} = 2m + \frac{q_{KG}^2}{2m} \rightarrow q_{KG}^2 = 2m(Eq - m)$$,
and renormalization of the DWF, that simply ensures that the change of variables
is unitary:
$$\Psi_D(q) = \frac{\Psi_{N.R.}[q_{KG}(q)]}{h[q_{KG}(q)]}$$,
where we defined the function \( h(q) \) by the equation:
$$q^2dq = h^2(q_{KG}) q_{KG}^2 dq_{KG} \rightarrow h^2(q_{KG}) = \frac{q}{q_{KG}^2} \left( 1 + \frac{q_{KG}^2}{2m^2} \right)$$.
Then, the transformation involves the simple rescaling and the renormalization, that ensures unitarity of an S-matrix. The minima for deuteron components are shifted towards larger momenta. The effects are large above about \(5 \text{ fm}^{-1} \left( \frac{q^2}{4m^2} \sim 0.25 \right)\).

Let us consider the “minimal relativization scheme” describes rather well the differential cross section for such process as deuteron break-up \(A(D, p)X\). The minimal relativization procedure \([12, 4]\) consist of (i) a replacement of the argument of the non-relativistic wave functions by a light-cone variable \(k = (k_\perp, k_\parallel)\)

\[
k^2 = \frac{m^2 + k_\perp^2}{4x(1-x)} - m^2; \quad k_\parallel = \sqrt{\frac{m^2 + k_\perp^2}{x(1-x)} \left( \frac{1}{2} - x \right)}.
\]  

where \(x = (E_q + |q| \cos \theta_q)/M = (\varepsilon' - p_\parallel')/M\); \(|k_\perp| = p_\perp'\) in the deuteron rest frame, and (ii) multiplying the wave functions by the factor \(\sim 1/(1-x)\). As a results the argument is shifted towards smaller values and the wave function itself decreases less rapidly. This effect of increasing the wave function is compensated by the kinematical factor \(1/(1-x)\).

### 3 Results and discussion

The results of calculation are summarized in figures below, where the tensor analyzing power \(A_{YY}\) as function of the cumulative \(x_C\) (“cumulative number” \([13]\)) is shown. For our reaction, this variable is defined as follows:

\[
x_C = \frac{2(p\pi) - \mu^2/2}{(Dp) - Mm - (D\pi)} \leq 2.
\]  

The value of \(x_C\) corresponds to a minimum mass (in nucleon mass units) of part of the projectile nucleus (deuteron) involved in the reaction. When the pion with \(x_C\) larger than 1 is produced, it is assigned to the cumulative pion. This kinematical region corresponds to the values of the light-cone variable \(x \geq 1\) \([1, 3]\) and internal deuteron momentum \(\geq 0\), as presented in the following table for the planed case \([2]\) of pion production angle \(\theta_\pi = 178\) mrad.

| \(x_C\)     | 0.0 | 1.0 | 1.2 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(|q|_{min}\) (GeV) | 0.0 | 0.14 | 0.29 | 0.39 | 0.48 | 0.61 | 0.75 | 0.96 |
| \(|q_{KG}|_{min}\) (GeV) | 0.0 | 0.14 | 0.29 | 0.38 | 0.47 | 0.58 | 0.70 | 0.87 |
| \(|k|_{min}\) (GeV) | 0.0 | 0.13 | 0.25 | 0.33 | 0.40 | 0.48 | 0.57 | 0.69 |

Deuteron fragmentation into proton \(D + A \rightarrow p(0^+) + X\) is one of the more intensively studied reaction with hadronic probe. As shown in \([14]\), both the differential cross section and \(T_{20}\) for fragmentation \(D + p \rightarrow p(0^+) + X\) can be described within the IA up to \(k \leq 0.2\) GeV/c only. The inclusion of correction to IA related to secondary interactions allows one to describe the experimental data on the deuteron fragmentation \(Dp \rightarrow pX\) at \(k > 0.25\) GeV/c \([5]\).
The calculated results of $A_{YY}$ for the reaction of polarized deuteron fragmentation into cumulative pions are shown in figures 2 and 3. From this figures one can see that it is quite incorrect to use the nonrelativistic DWF for the analysis of deuteron fragmentation into pions. Relativistic effects are sizable, especially in the kinematic region corresponding to short intra-deuteron distances or large $x_C$.

![Fig. 2.](image_url)  

**Fig. 2.** The tensor analyzing power $A_{YY}$ for the planned pion production angle $\theta_\pi = 178$ mrad, calculated using the various types of the DWF. The thin solid line corresponds to the calculus with the Reid DWF [15]. The dot-dashed line represents the calculation with the Gross DWF [16] with the P-wave probability $P_V = \int_0^{\infty} q^2 dq \cdot |V_t^2 + V_s^2| \approx 0.44\%$. The thick solid line and long-dashed line calculated with the Reid DWF using the Kamada-Glöckle (KG) method [11] and the minimal relativization scheme (MRS) [4, 12].

The figure 3 shows a small dependence of the tensor analyzing power $A_{YY}$ to the pion production angle on the contrary to the experimental data. On the other hand, the experimental data on $A_{YY}$ are not described by any DWF used in this calculus over all region $x_C \geq 1$.

### 4 Questions

For a present time, the most sophisticated theoretical approaches are unable to describe properly the experimental data in the cumulative region of this notoriously difficult reaction. Then the following questions still open:

1. Why the measured $T_{20}$ and $A_{YY}$ for deuteron fragmentation into pions are not reproduce by IA even for internal momentum $< 250$ GeV/c where the
Fig. 3. The angle dependence of the tensor analyzing power $A_{YY}$ of the deuterons, calculated using the minimal relativization scheme (MRS) \[4, 12\] for the projectile deuteron momentum $P_D = 9$ GeV/c. The solid, long-dashed, dashed and dot-dashed lines correspond to the calculus at $\theta_\pi = 0, 135, 178$ and 300 mrad, respectively. The experimental data are taken from \[1, 2, 3\].

same observables for deuteron breakup reaction are in a good agreement with IA?

• Additional to IA mechanisms \[17\]?

2. Moreover the data on $T_{20}$ at zero angle have small value as it have to be for case of isotropic source from which pions are emitted. The calculations presented in this work performed in the framework of nucleon model of deuteron. For the small inter-nucleon distances (equal or less of hadron size) the use of nucleon as a quasi-particle seems to be groundless and effects of manifestation of non-nucleon degrees of freedom in deuteron could be expected. In spite of this the data on $A_{YY}$ can be cause by

• non-nucleonic degrees of freedom \[5, 18, 19\].

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