Numerical study of non-stationary radiation-conductive ice heat transfer taking into account radiation scattering

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Abstract. A numerical study of the formation of the temperature field and the melting rate of a layer of ice scattering radiation vertically located on an opaque substrate during radiation heating has been carried out. To solve the radiation part of the problem, a modified method of average fluxes is used, taking into account the volume absorption and scattering of radiation into the medium, as well as the selective nature of the radiation source. Anisotropic scattering radiation is taken into account by the method of expanding the scattering indicatrix in a series according to Legendre polynomials. The effect of the spectral volumetric properties of ice on the melting and temperature increase of the non-irradiated side is presented. Comparison of calculation results with experimental data demonstrates satisfactory agreement. The proposed method for taking into account anisotropic scattering does not show any significant difference from isotropic scattering.

1. Introduction

Ice and the snow thickness are translucent media in which heat is transported together by radiation and thermal conductivity. Krass and Merzlikin [1] described early studies of thermophysical processes in the snow and ice cover. In [2] and [3], the current state of theoretical and numerical simulation in the snow-ice mass at solar irradiation, when snow and ice are considered as absorbing and scattering media, was presented. The ice melting simulation was based on Stefan's problem for a translucent medium. The authors of [4–6] validated the solution of the single-phase Stefan problem in a translucent medium using the experimental data [7] as an example of melting of pure, non-scattering ice. Seki et al. [7] performed calculation and experimental investigation in a climatic chamber at a constant temperature of 0 °C under the action of radiation of two types of lamps (halogen and with nichrome filament). The ice layer was on a vertical opaque substrate. Thus, the radiation melting of pure (non-scattering) ice and ice scattering radiation was simulated under the conditions of an incident short-wave and long-wave radiation flux. In the mathematical model of the process, the authors neglected the presence of a water film formed on the ice surface and the calculation was carried out in a single-phase formulation of the Stefan problem. A comparison of the melting rate of the ice layer and the heating of the non-irradiated ice surface demonstrated satisfactory agreement between the experimental and numerical results. When calculating radiation heat transfer in [7], the fitting parameters and direct integration according to the Bouguer law were used. For scattering ice, the authors of [7], unfortunately, did not indicate the nature of radiation scattering, the albedo, and the scattering indicatrix.
The aim of this work is to further develop the methodology of [4–6], taking into account the bulk optical properties of a translucent medium. The mathematical model and computational algorithm described in [4–6] are used. The selectivity of the radiation source, as well as the selective volumetric absorption and scattering of radiation, are taken into account. The calculation of anisotropic scattering is carried out by expanding the scattering indicatrix in a series according to Legendre polynomials. The results of the numerical calculation are compared with the experimental data given in [7].

2. Formulation of the problem and methods of solution

Fig. 1 shows a geometrical scheme of the problem in which a layer of scattering ice of thickness \( L_0 \) is located on a vertical opaque substrate and is in a medium with a constant temperature \( T_\infty \). The right surface of the flat ice layer is illuminated by a lamp with an incandescent filament temperature of 3200 K with a constant incident radiation flux \( E^* = 4648.88 \text{ W/m}^2 \). The spectral composition of the radiation source is similar to [6]. The authors of [3, 7] indicate that in the radiation wavelength range from 0.3 to 1.2 \( \mu \text{m} \) (hereinafter referred to as a short-wavelength range), radiation scattering significantly prevails over absorption, and in the wavelength range from 1.2 \( \mu \text{m} \) and higher (long-wavelength range) on the contrary, the absorption of radiation in ice significantly prevails over scattering.

![Figure 1. Geometrical scheme of the problem.](image)

It is assumed that the boundary surfaces of the ice layer diffusely absorb, reflect and transmit radiation, so that \( A_i + R_i + D_i = 1 \), where \( A_i \), \( R_i \), \( D_i \) are the absorption, reflective and transmission hemispherical abilities of the ice surface, \( i = 1, 2 \). The validity of Kirchhoff’s law is also assumed: \( A_i = \varepsilon_i \). The left surface of the ice layer on the substrate is maintained at a constant temperature \( T_{\text{sub}} = 260.15 \text{ K} \), which coincides with the initial temperature of the flat ice layer \( T(x,0) \), and the temperature inside the chamber is \( T_\infty = 273.15 \text{ K} \) [7].

The problem is solved in two stages. At the first stage, radiation-conductive heat transfer is considered; it continues until the right surface of the ice layer \( T(L_0,t) \) and reaches the phase transition temperature \( T_f \). At the second stage, the Stefan problem is solved with a fixed value of the temperature of the right boundary \( T(L(t),t) = T_f \), and on the irradiated surface it is assumed that a film of water flows off under the action of gravity. The temperature of the film \( T_{\text{fil}} \) is believed to be higher than the temperature of the phase transition of ice and, therefore, the boundary condition on the irradiated surface considers its own radiation and convective heat transfer taking into account the water film.

The non-stationary energy equation in a flat ice layer with temperature \( T(x,t) \) taking into account the energy transfer by radiation is written as follows:
\[
  \frac{c_p \rho}{\partial T(x,t)} = \frac{\partial}{\partial x} \left( \frac{\lambda}{\partial x} \frac{\partial T(x,t)}{\partial x} - E_r(x,t) \right), \quad 0 < x < L(t).
\]  

Here \( c_p \) is the heat capacity at constant pressure, \( \rho \) is the density, \( \lambda \) is the thermal conductivity, and \( E_r(x,t) = E_r^+(x,t) - E_r^-(x,t) \) is the flux density of the resulting radiation.

The boundary conditions for equation (1) at the first stage of the process are written as follows:

\[
  \frac{\partial T}{\partial x} \biggr|_{x=0} = \left( 1 - \gamma \right) E(x,t) \text{ at } x = 0, \quad \frac{\lambda}{\partial x} \frac{\partial T}{\partial x} - h(T_m - T) - |E_{res,2}| = 0 \text{ at } x = L_t.
\]  

Here \( |E_{res,2}| = A \left( E_r^+(x,t) + E_r^-(x,t) \right) - \varepsilon_2 \sigma_0 T^4(x,t) \). According to [7], the left surface of the substrate is maintained at a constant temperature \( T_{sub} \). On its right surface, on the border with ice, heat transfer is carried out by heat conduction and radiation. The right boundary of the ice is exposed to radiation from a radiation source; cooling associated with convection is also taken into account. Equations (1) and (2) are supplemented by the initial condition: \( T(x,0) = T_{sub} \).

At the second stage of the process, the temperature of the surface of the right boundary at \( x = L(t) \) is fixed: \( T(x,t) = T_f \). The boundary condition (2) is transformed into the Stefan condition with allowance for a thin film of water formed on the surface. We assume that the water film is isothermal, and the temperature difference over its thickness is negligible:

\[
  \frac{\lambda}{\partial x} \frac{\partial T}{\partial x} + h(T_{film} - T_m) - |E_{res,fil}| = \rho_f \frac{\partial L}{\partial t}
\]  

Here \( |E_{res,fil}| \) has the following form:

\[
  |E_{res,fil}| = A \left( E_r^+(x,t) + E_r^-(x,t) \right) - \varepsilon_2 \sigma_0 \left( T^4(x,t) - T_{film}^4 \right) \text{ at } x = L(t).\]

Here, \( T_f = 273.15 \) K is the melting temperature of ice, \( T_{film} = 277.15 \) K is the temperature of the water film, and \( \gamma \) is the latent heat of the phase transition. In condition (3), heat transfer from the outer surface of the water film is taken into account, and (4) is the natural radiation of the film at the right surface.

The assumption that there is a thin film of water on the ice surface does not contradict the single-phase approximation of the Stefan problem, since radiation is not absorbed in the film itself and it is taken into account only in an additional boundary condition on the interface. The thermal problem is solved only in the ice on a vertical substrate.

The density of radiation fluxes \( E^\pm(x,t) \) included in equations (1) – (4), reduced to the dimensionless form \( \Phi^\pm = E^\pm / (4 \sigma_0 T_f^4) \) and \( \Phi = \sum_j (\Phi_j^- - \Phi_j^+) \), is determined from the solution of the radiation transfer equation in a flat layer of a radiating, absorbing, and scattering medium with a known temperature distribution over the layer, and \( j \) is the number of the spectral band [2, 4].

As in the previous works of the authors, the calculation of radiation transfer is carried out using a simple and accurate modified mean fluxes method [2, 6]. According to this approach, the integral and differential equation of radiation transfer is reduced to a system of two nonlinear differential equations for a plane layer of a translucent absorbing medium. The differential analog of the transport equation for hemispherical fluxes \( \Phi_j^+ \) taking into account anisotropic radiation scattering is presented in the form [2, 6]:
The boundary conditions for the system of equations (5) in dimensionless variables are written as follows [2]:

\[
\begin{align*}
\tau_{j,1} = 0: & \quad \Phi_j^0 = (1 - R_j) \Phi_j^* + \left(1 - \frac{n^*}{n^0} \right) \Phi_j^* + R_j \frac{n^2}{n^0} \Phi_j^*; \\
\tau_{j,1} = \alpha \beta L(t) : & \quad \Phi_j^0 = \frac{\Phi_j^0}{4} + R_j \Phi_j^*; \\
\tau_{j,0} = \alpha \beta L(t) + \infty: & \quad \Phi_{j,0}^* = \Phi_j^*.
\end{align*}
\]

In (6), the selective radiation source \( \Phi_v^* = E_v^0 \left( 4\sigma_j T_j^4 \right) \) is taken into account. Here \( \Phi_v^0 = n^2 B_v \left( 4\sigma_j T_j^4 \right) \) is the dimensionless flux density of equilibrium radiation, \( B_v \) is the Planck function, \( n \) is the refractive index of ice, \( n^* \) is the refractive index of the environment, and \( \tau_j = \alpha \beta L(t) \left( 1 - \omega_j \right) \) is the spectral optical thickness of the layer at the moment of time \( t \), \( \omega_j = \beta_j / (\beta_j + \alpha_j) \) is a single scattering spectral albedo, \( \beta_j \) is the spectral scattering coefficient, \( \bar{\zeta} = 1/2 \int_{-1}^{1} p_v(\mu) \mu d\mu \) is the mean cosine of the scattering angle, \( p_v(\mu, \mu') = \sum_{l=0}^{N} a_l P_l(\mu) \) is the spectral indicatrix of scattering, \( a_l \) is the expansion coefficients of the scattering indicatrix in a series according Legendre polynomials, \( P_l \) is the Legendre polynomial of order \( l \), \( \mu_0 \) is the cosine of the angle between incident and scattered rays, and \( v \) is the radiation frequency index. The values of the coefficients \( m^*, l^* \) are determined from the recurrence relation obtained using the formal solution of the radiation transfer equation, where \( j \) is the number of the spectral band [2, 4]. Layer I refers to ice, and layer II refers to the external space (Fig. 1).

The solution of the boundary value problem is reduced to determining the dimensionless temperatures \( \Theta(\xi, \eta) = T(x, t)/T_j^* \), where \( \xi = x/L(t) \) and \( \eta = (a \cdot t)/L(t) \), and flux densities of the resulting radiation \( \Phi_v(\xi, \eta) \) in the region \( G = \{0 \leq \xi \leq 1; 0 \leq \eta \leq \eta_1\} \), which is a flat layer of clean, absorbing, radiating and non-scattering ice. The position of the phase transition front \( s(\eta) = L(t)/L_0 \) varies from 1 to 0. The boundary value problem (1) – (4) is solved by a finite difference method, the nonlinear system of implicit difference equations is a sweep method and iterations. Solving the radiation problem requires iterations; at each step the boundary value problem (5) – (6) is solved by the matrix factorization method. The rapid convergence of this method of solution allows obtaining fairly accurate results.

3. Result analysis

Below is an analysis of the results of numerical simulation of a vertical layer of the radiation-scattering ice with the following physical parameters: \( L_0 = 0.045 \text{ m} \) is the initial ice thickness, \( T_{\text{sub}} = 253.15 \text{ K} \) is the temperature of the left boundary of the substrate and the initial temperature of the substrate and ice, the temperature of the atmosphere inside the chamber is maintained at a constant value of \( T_c = 273.15 \text{ K} \).
K, equal to the melting ice temperature $T_f$, and a constant density of the incident radiation flux is $E'_s = 4648.88$ W/m$^2$. The following thermophysical properties of ice scattering radiation are accepted: thermal conductivity $\lambda = 1.87$ W/(m·K); thermal diffusivity $a = 1.31 \times 10^{-3}$ m$^2$/s; and latent heat of the phase transition $\gamma = 335$ kJ/kg. Optical parameters are: the refractive index of ice $n = 1.31$, that of air $n' = 1$; reflection coefficients $R_1 = 0.97$ and $R_2 = 0.063$; and the boundary emissivity $\varepsilon_1 = 1 - R_1$. The spectral characteristics are presented in table 1. The expansion coefficients in the Legendre polynomial are taken equal to $a_0 = 1$, $a_1 = 1.2$, and $a_2 = 0.5$ for forward scattering and $a_1 = -1.2$, and $a_2 = 0.5$ for backward scattering.

Three parameters varied in the calculations: the heat transfer coefficient $h$, the emissivity $\varepsilon_2$ of the irradiated ice surface and the albedo in the long-wavelength part of spectrum $\omega_2$. At the first stage, $h = 17.17$ W/(m$^2$·K) and $\varepsilon_2 = 0.97$. At the second stage, $h = 80$ W/(m$^2$·K) and approximately corresponds to the conditions of heat transfer in [7], $\varepsilon_2 = 0.5$. The above parameter values were obtained in the numerical experiments and corresponded to conditions on very rough surfaces [7]. Values of $\omega_2$ are given in the table.

Figure 2 shows the calculated field of ice temperature at the stages of heating and subsequent melting at an albedo of $\omega_1 = 0.999$ and $\omega_2 = 0.1$ is the most characteristic values for ice scattering radiation (at $\zeta = 1$). Hereinafter, the curves between 1 and 2 refer to the first stage, and those between 2 and 3 refer to the second stage. At the heating stage, the temperature curves are not monotonic; at the boundaries, the influence of the optical properties of the surface, as well as (on the right) the influence of the source radiation, is noticeable. At the second stage, the temperature curves become monotonic and at the end of the calculations, with a decrease in the thickness of the medium, turn out to be linear.

### Table 1. Spectral dependences of ice parameters and radiation source.

| $j$ | $\nu_j$, $10^{14}$ Hz | $\lambda_j$, µm | $\alpha_j$, m$^{-1}$ | $\omega_j$ | $E'_s$, W/m$^2$ |
|-----|----------------------|-----------------|---------------------|-------------|-----------------|
| 1   | 9.09 – 2.02          | 0.33 – 1.2      | 0.001               | 0.999       | 2073            |
| 2   | 2.02 – 1.18          | 1.2 – $\infty$  | 1                   | 0.1/0.4/0.8 | 1883            |

Figure 2. The temperature field at $\zeta = 1$ (1 – beginning of heating, 2 – melting beginning, 3 – end of melting).

Figure 3. The resulting flux density field at $\zeta = 1$ (1 – beginning of heating, 2 – melting beginning, 3 – end of melting).
Figure 3 shows the density field of the flux of resulting radiation (RR) in ice at similar scattering albedo values. A positive RR value indicates the predominance of incident radiation over its own, which is characteristic of this material. At the heating stage and partially at the melting stage, the curves are characterized by a constant gradient. Over time, with a subsequent decrease in the ice thickness, the left side is practically fixed at one point, while the right side continues to grow, thereby reducing the gradient.

Figures 4 and 5 show the rates of melting and temperature rise of the left ice boundary with time and their comparison with experimental data [7] at $\zeta = 1$. The simulation was carried out at three albedo values $\omega_2$: 0.1, 0.4, and 0.8, as well as for non-radiation scattering ice ($\omega_1 = \omega_2 = 0$). The albedo values of 0.4 and 0.8 are accepted for a numerical experiment; in reality, such parameters are not observed in ice in this spectral range. For the pure ice, the thermophysical parameters are taken as for the scattering ice.

In fig. 4, the calculated curves lie within the experimental data and are independent of the long-wavelength albedo $\omega_2$. This is due to the fact that the short-wavelength part of the wavelengths, $j=1$, accounts for the majority of the incident radiation $E_i^*$ and scattering in this region significantly prevails over absorption (see table 1). The calculated curves of temperature increase and the corresponding
Experimental data (Fig. 5) are approximately consistent with each other, but have a different character. The calculated lines have a pronounced increase at time $t < 20$ min and then stop, reaching a quasistationary state. Experimental points show monotonous growth throughout the entire melting process. A curve closer to the experimental data is obtained at 0.4; for other albedo values, the difference in the curves is noticeable. Figure 6 shows the melting rate (Fig. 6a) and temperature increase of the non-irradiated side of the ice (Fig. 6b), taking into account anisotropic radiation scattering ($\omega_2 = 0.1$). As can be seen from the calculations, the inclusion of scattering by the method of expanding the indicatrix in a series by Legendre polynomials does not completely differ from the case of isotropic scattering. Similar results were noted in [8, 9].

**Conclusions**
A physical and mathematical model of ice melting taking into account radiation scattering has been presented. In the calculations of the radiation part, the modified mean flux method is used, taking into account the volumetric selective absorption and scattering, as well as the selective nature of the radiation source. The melting rate is shown to be more dependent on the albedo of the short-wavelength part of the spectral range, while the increase in temperature of the left part is more dependent on the long-wavelength spectral range. A comparison of the results with experimental data shows satisfactory agreement with the calculations, however, the model requires refinement taking into account the real characteristics of anisotropic radiation scattering in the medium. Taking into account anisotropic scattering by the method of expanding the scattering indicatrix in a series according to Legendre polynomials does not differ from isotropic scattering, and in the future it is worth using the transport approximation method used in [3].

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**References**
[1] Krass M S and Merzlikin V G 1990 *Radiation Thermophysics of Snow and Ice* (Leningrad: Gidrometeoizdat) 262
[2] Timofeev A M 2018 *Thermophysics and Aeromechanics* 5 765–72
[3] Dombrovsky L A, Kokhanovsky A A and Randrianalisoa J H 2019 *J. Quant. Spectr. Radiat. Transfer* 227 72–85
[4] Sleptsov S D, Rubtsov N A and Savvinova N A 2018 *Thermophysics and Aeromechanics* 3 421–8
[5] Rubtsov N A, Savvinova N A and Sleptsov S D 2015 *J. Engng Thermophysics* 2 123–38
[6] Sleptsov S D and Savvinova N A 2019 *Thermophysics and Aeromechanics* 5 761–8
[7] Seki N, Sugawara M and Fukusaki S 1979 *Wärme- und Stoffübertragung* 2 137–44
[8] Rubtsov N A, Timofeev A M and Savvinova N A 2003 *Combined Heat Transfer in Semitransparent Media* (Novosibirsk: Nauka) 197
[9] Rubtsov N A 1984 *Radiative Heat Transfer in Continuous Media* (Novosibirsk: Nauka) 278