A determination of the parton distributions for flavor asymmetric light sea in the proton

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Abstract

Using the DGLAP equation with corrections of parton recombination, it is the first time that the distributions of the flavor asymmetric sea in the proton are extracted from the available experimental data in a model-independent way. We find that the distribution shape of the asymmetric sea quark is similar to that of the corresponding valence quark. Based on the separation of the flavor symmetric and asymmetric sea, the possible relations between the strange quark distribution and symmetric light quark distribution in the proton are discussed. The results are used to explain the recent HERMES results for the strange quark distribution. The comparisons of our results with the predictions of the CTEQ-, MRS-, GRV-databases and some models for flavor asymmetric sea are also presented.

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1 Introduction

The observations of the violation of the Gottfried sum rule [1], and a surprisingly large asymmetry between the up (u) and down (d) antiquark distributions of the proton in deep-inelastic scattering (DIS) of lepton-nucleon [2], Drell-Yan pair production in nucleon-nucleon collisions [3] and semi-inclusive [4] measurements revealed new light on the origin of the nucleon sea. In order to gain deeper and more precise understanding of the nucleon structure, the flavor asymmetric sea as a new recognized partonic component should be extracted from the experimental data. While we still have not a quantitative answer for this basic question. For example, several databases [5∼7] of the parton distribution function (PDF) in the proton are updated using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [8]. Unfortunately, the symmetric and asymmetric sea quarks have not yet completely separated from experiments in those new PDF-databases. The reason is that the solutions of QCD evolution equation depend on the initial parton distributions, which are fixed at an arbitrary scale \( Q_0^2 \sim 1 \text{ GeV}^2 \), where the asymmetric sea is always mixed with the symmetric sea since the gluons are already generated.

The flavor asymmetric sea is created by the nonperturbative QCD process since perturbative processes cannot cause a significant difference between \( \bar{d} \) and \( \bar{u} \). A straightforward way to separate the asymmetric sea from the symmetric sea is that the QCD evolution starts from an low enough resolution scale \( \mu^2 \), where the nucleon contains only the nonperturbative valence and asymmetric sea quarks, while the flavor symmetric sea and gluons naturally disappear in the initial distributions. Once asymmetric sea distributions are fixed at \( \mu^2 \), they will be known at \( Q^2 > \mu^2 \), and can be used to explain the experiments or make predictions.
It is well known that the application of the standard DGLAP equation is not applicable in the region $Q^2 << 1 \text{ GeV}^2$ and this equation should be modified at small $x$ and low $Q^2$ due to the correlation among partons. These nonlinear effects are discussed in the perturbative QCD [9∼13], in particular, such nonlinear corrections including complete parton recombinations in the leading order (LO) approximation are derived by Zhu, Ruan and Shen (ZRS) in [11∼13].

The purpose of this work is to extract the parton distributions including flavor asymmetric sea quarks in the proton using the DGLAP equation with the ZRS corrections. The numerical calculations [14] show that the negative nonlinear corrections can improve the perturbative stability of the QCD evolution equation at low $Q^2$. Thus the parton distributions evolution starts from a lower resolution scale $\mu^2$, where the nucleon consists entirely of the nonperturbative components, and all symmetric sea and gluons are radioactively created in the evolution at $Q^2 > \mu^2$. Although the contributions of the nonperturbative QCD effects to the measured structure functions at $\mu^2 < Q^2 < 1 \text{ GeV}^2$ (for example, the hadronic components in the virtual photon) are necessary, however these nonperturbative effects are strongly $Q^2$-power suppressed and they don’t participate the evolution of the parton distributions. We find that our predicted parton distributions at $Q^2 > 1 \text{ GeV}^2$ in this schema are acceptable. In particular, the symmetric sea vanishes in the input distributions since the gluons are absent. Thus, a complete separation of the asymmetric sea quark distributions becomes possible.

The distributions of the asymmetric light sea in the proton extracted by ZRS corrections are consistent with the available experimental data. We find that the distribution shape of the asymmetric light sea is almost similar to that of the corresponding valance
quark. Based on the separated symmetric light sea distributions, we present the possible relation between the strange quark distribution and symmetric light quark distribution in the proton. Our results can be used to check the predictions of various models for the flavor structure of the parton distributions and to improve the PDF-databases.

The organizations of this paper are as follows. In section 2, we present the ZRS corrections to the DGLAP equation and our analysis method. Our results and comparisons with the experimental data and the predictions of various theoretical models are presented in section 3. Finally, the discussions and summary are given in section 4. The simplification of the evolution equation is presented in appendix.
2 Calculation method

We denote the parton distributions in the proton for valence quarks \(q^v(x, Q^2)\) (q=u,d), symmetric sea quarks \(q^s(x, Q^2) = \overline{q}^s(x, Q^2)\) (q=u,d,s...), asymmetric sea quarks \(q^{as}(x, Q^2) = \overline{q}^{as}(x, Q^2)\) (q=u,d) and gluon \(g(x, Q^2)\). We define the flavor non-singlet (NS) distribution \(q^{NS}(x, Q^2) = q^v(x, Q^2) + q^{as}(x, Q^2) + \overline{q}^{as}(x, Q^2)\) and the flavor-singlet distribution \(\Sigma(x, Q^2) = \sum_q[q^{NS}(x, Q^2) + q^s(x, Q^2) + \overline{q}^s(x, Q^2)]\). We take the isospin symmetry \(u_s = d_s\) and parton-antiparton symmetry. The possible asymmetry of the strange sea \(s(x) \neq \overline{s}(x)\) in this work is neglected since it may refer to a different mechanism and we focus only on the flavor asymmetric light sea.

The DGLAP equation with the ZRS corrections at leading order is derived in [11-13]. In this work we use its simplified form (see appendix)

\[
Q^2 \frac{d x q^{NS}(x, Q^2)}{d Q^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{qq} \otimes q^{NS},
\]

(2.1)

for flavor non-singlet quarks;

\[
Q^2 \frac{d x q^s(x, Q^2)}{d Q^2} = \frac{\alpha_s(Q^2)}{2\pi} [P_{qq} \otimes q^s + P_{qg} \otimes g] - \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [y g(y, Q^2)]^2 \\
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{x} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [y g(y, Q^2)]^2, \quad \text{(if } x \leq 1/2),
\]

\[
Q^2 \frac{d x q^s(x, Q^2)}{d Q^2}
\]
\[ \frac{\alpha_s(Q^2)}{2\pi} \left[ P_{qq} \otimes q^s + P_{gg} \otimes g \right] \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{dy}{y} x P_{gg\rightarrow g}(x, y)[yg(y, Q^2)]^2, \text{ if } 1/2 \leq x \leq 1, \]  
(2.2)

for symmetric sea quarks;

\[ Q^2 \frac{d x g(x, Q^2)}{d Q^2} \]
\[ = \frac{\alpha_s(Q^2)}{2\pi} \left[ P_{qq} \otimes \Sigma + P_{gg} \otimes g \right] \]
\[ - \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x}^{1/2} \frac{dy}{y} x P_{gg\rightarrow g}(x, y)[yg(y, Q^2)]^2 \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{x} \frac{dy}{y} x P_{gg\rightarrow g}(x, y)[yg(y, Q^2)]^2, \text{ if } x \leq 1/2, \]

(2.3)

for gluon, where the factor \( 1/(4\pi R^2) \) is from normalizing two-parton distribution, \( R \) is the correlation length of two initial partons, the linear terms are the standard DGLAP evolution and the recombination functions in the nonlinear terms are

\[ P_{gg\rightarrow g}(x, y) = \frac{9}{64} \frac{(2y - x)(72y^4 - 48xy^3 + 140x^2y^2 - 116x^3y + 29x^4)}{xy^5}, \]

\[ P_{gg\rightarrow q}(x, y) = P_{gg\rightarrow \bar{q}}(x, y) = \frac{1}{96} \frac{(2y - x)^2(18y^2 - 21xy + 14x^2)}{y^5}. \]  
(2.4)

Although a next-to leading order (NLO) extraction would be preferred, a LO extraction is an important first step for understanding the flavor structure of the nucleon.
The input distributions are constrained by the following two sum rules

\[ \int_0^1 dx [u^{NS}(x, \mu^2) + d^{NS}(x, \mu^2)] = 1, \quad (2.5) \]

for parton momentum and

\[ \int_0^1 dx u^{NS}(x, Q^2) = 2 + 2 < u^{as} >_1, \quad \int_0^1 dx d^{NS}(x, Q^2) = 1 + 2 < d^{as} >_1, \quad (2.6) \]

for initial parton number, where \(< q >_1\) is the first moment of the distribution \(q\). Note that

\[ u^s(x, \mu^2) = \bar{u}^s(x, \mu^2) = 0, \quad d^s(x, \mu^2) = \bar{d}^s(x, \mu^2) = 0, \]

\[ s(x, \mu^2) = \bar{s}(x, \mu^2) = 0, \quad g(x, \mu^2) = 0. \quad (2.7) \]

Thus, our input distributions decrease to \(u^{NS}, d^{NS}\) and an adjustable parameter \(< d^{as} >_1\)
(or \(< u^{as} >_1\), since the value \(< d^{as} >_1 - < u^{as} >_1\) is fixed).

We consider the following experimental data to parameterize the input parton distributions:

(1) \(F_2(x, Q^2)\) data [15-19] are used to determine the non-singlet input quark distributions \(u^{NS}(x, \mu^2)\) and \(d^{NS}(x, \mu^2)\)

(2) For separating \(u^{as}(x)\) and \(d^{as}(x)\) from \(u^v(x)\) and \(d^v(x)\) in \(q^{NS}(x)\), one can use the average structure function of the neutrino and antineutrino scattering on an iso-scalar nucleon

\[ xF_3(x, Q^2) = \frac{1}{2} [xF_3^{\nu N}(x, Q^2) + xF_3^{\bar{\nu} N}(x, Q^2)] = xu^v(x, Q^2) + xd^v(x, Q^2), \quad (2.8) \]
which has been measured by CCFR analysis [20]. Therefore,

\[ u^{as}(x, \mu^2) + d^{as}(x, \mu^2) = \frac{1}{2} [u^{NS}(x, \mu^2) + d^{NS}(x, \mu^2) - u^v(x, \mu^2) - d^v(x, \mu^2)]. \]  \hfill (2.9)

On the other hand, the difference

\[ d^{as}(x, \mu^2) - u^{as}(x, \mu^2) = \overline{d}(x, \mu^2) - \overline{u}(x, \mu^2), \]  \hfill (2.10)

is obtained from the E866 [3] and HERMES [4] measurements. Combining Eqs. (2.9) and (2.10), in principle, we can directly get the input asymmetric light sea distributions if neglecting the errors in the experiments. Unfortunately, the distributions of the asymmetric sea quarks are much smaller than that of the valence quarks. The experimental errors in \( F_2 \) and \( F_3 \) may hinder us to extract the distributions \( q^{as} \). Therefore, the above mentioned results should be adjusted after globally comparing with other experimental data, for example, \( \overline{d}/\overline{u}, \overline{d_v}/u_v \) and \( F_2^n/f_2^p \), in particular, \( \overline{d} - \overline{u} \) and \( \overline{d}/\overline{u} \), which are sensitive to the shape of \( q^{as} \).


3 Results

Our input distributions are

\[ xu^{NS}(x, \mu^2) = 0.321x^{0.418}(1-x)^{1.368}(1 + 22.69x) \]

\[ xd^{NS}(x, \mu^2) = 1.329(1-x)^2 xu^{NS}(x, \mu^2), \] (3.1)

they are plotted in figure 1. The relating parameters in Eqs. (2.1)-(2.3) are \( R = 4.09 \) GeV\(^{-1} \), \( \mu^2 = 0.064 \) GeV\(^2 \), \( \Lambda_{QCD} = 0.204 \) GeV, and \( < d^{as} >_1 = 0.248 \) with \( < d^{as} >_1 - < u^{as} >_1 = 0.148 \) (Note that the experimental data are \( < \overline{d} >_1 - < \overline{u} >_1 = < d^{as} >_1 - < u^{as} >_1 = 0.118 \pm 0.012 \sim 0.148 \pm 0.039 \) [21]).

The description of the DIS structure function with our input parton distributions is shown as follows. The \( x \)- and \( Q^2 \)-dependence of \( F_2^p(x, Q^2) = x \sum_j e_j^2 q_j^{NS}(x, Q^2) + x \sum_i e_i^2 [q_i^x(x, Q^2) + \overline{q}_i^x(x, Q^2)] \) are plotted in figures 2 and 3, where the data is taken from [15~19].

![Figure 1: The input densities \( xq^{NS} \) (for \( q = u, d \)) in eq. (1) at \( \mu^2 = 0.064 \) GeV\(^2 \).](image)

Generally, the two unknown functions \( P_u(x) \) and \( P_d(x) \) can be written as
Figure 2: Comparison of our LO nonlinear QCD fitting results for $x$-dependence of $F_2^p(x, Q^2)$ with the data [15∼19] at $Q^2 > 1$ GeV$^2$.

Figure 3: Comparison of our LO nonlinear QCD fitting results for $Q^2$-dependence of $F_2^p(x, Q^2)$ with the data [15∼19] at $Q^2 > 1$ GeV$^2$. 
\[ u^{as}(x, \mu^2) = u^v(x, \mu^2) P_u(x), \quad d^{as}(x, \mu^2) = d^v(x, \mu^2) P_d(x). \]  \hspace{1cm} (3.2)

In the following global fittings we find that it is acceptable if we set both \( P_u(x) \) and \( P_d(x) \) as constant. Thus, using Eq.(2.6) we have

\[ P_u(x) \simeq <u^{as}> / 2, \quad P_d(x) \simeq <d^{as}>. \]  \hspace{1cm} (3.3)

It implies that \( q^{as}(x, \mu^2) \) and \( q^v(x, \mu^2) \) have similar \((x\text{-dependence})\) shapes, i.e.,

\[ u^{as}(x, \mu^2) = 0.05u^v(x, \mu^2), \quad d^{as}(x, \mu^2) = 0.248d^v(x, \mu^2). \]  \hspace{1cm} (3.4)

Figure 4 shows the input valence quarks and asymmetric sea at \( \mu^2 = 0.064 \text{ GeV}^2 \). The distributions \( x\overline{u}^v(x, Q^2) = x\overline{d}^v(x, Q^2), \quad x\overline{u}^{as}(x, Q^2) \) and \( x\overline{d}^{as}(x, Q^2) \) at different \( Q^2 \)-scale are first presented in figures 5 and 6.

![Figure 4: The input valence quark distributions and the input asymmetric sea distributions in the proton at \( \mu^2 = 0.064 \text{ GeV}^2 \).](image-url)
Figure 5: The distributions of symmetric light sea at different scale $Q^2$ in the proton, which are first presented.

Figure 6: Similar to figure 5, but for the flavor asymmetric light sea.
To check the above parametrization is correct or not, we compute the ratios $F_n^2/F_p^2$ and $d^v/u^v$ and then compare to the data [22,23] in Figures 7 and 8. These two plots show that our results are in good agreement with the data very well.

The Drell-Yan cross section ratio from E866 was analyzed to obtain $\bar{d} - \bar{u}$. We present a comparison of our results with the E866 [3] and HERMES [4] data in figure 9, where (a) compares with the global fits in CTEQ5M [24], GRV98 [7] and MRSTW [25]; (b) compares with some theoretical models [26,27]. The possible sign-change of $\bar{d}(x) - \bar{u}(x)$ at $x > 0.3$ is important for understanding the flavor structure of the nucleon sea [28]. In figure 9c (also see figure 6) we show that a very small negative value of this difference is possible at $x > 0.3$ and the $x$-position of $\bar{d}(x) - \bar{u}(x) = 0$ is $Q^2$-dependence.

![Figure 7](image-url)

Figure 7: Comparison of our results for $F_n^2/F_p^2$ with the measured data [22].
Figure 8: Comparison of our results \((Q^2 = 2.5 \text{GeV}^2)\) for the ratio \(d^v/u^v\) with the measured data \((Q^2 = 2.4 \sim 42.9 \text{GeV}^2)\) [23].

Figure 9: Comparisons of the flavor difference \(\bar{d} - \bar{u}\) in the proton as a function of \(x\) at \(Q^2 = 54 \text{GeV}^2\), the data are taken from the Fermilab E866 [3] and HERMES data [4]. (a) Compared to the global fits in CTEQ5M [24], GRV98 [7] and MRSTW [25]; (b) compared to theoretial models [26,27]; (c) Shows the possible sign-change of \(\bar{d}(x) - \bar{u}(x)\).
Similarly, we calculate the ratio $\overline{d}/\overline{u}$ in the proton and compare them with the E866 [3] and NA51 [29] data in figure 10, where (a) compares with the global fits in CTEQ4M [5], CETQ5M [24], GRV98 [7] and MRSTW [25]; (b) compares with some models [30].

The CCFR neutrino data [20] about the isoscalar structure functions $xF_3$ provides the valence quark distributions. Using the input distributions Eqs. (3.1) and (3.4), we plot $xF_3$ distribution in figure 11. Considering the approximation of Eq. (3.3), the results are consistent with the data.

Figure 10: Comparisons of our predicted ratio $\overline{d}/\overline{u}$ in the proton as a function of $x$ at $Q^2 = 54 GeV^2$, the data are taken from the Fermilab E866 [3] and NA51 data. (a) Comparisons with the global fits in CTEQ4M [5], CETQ5M [24], GRV98 [7] and MRSTW [25]; (b) Comparisons with theoretical models [30]; (c) The ratio $\overline{d}/\overline{u}$ at large $x$. 

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Figure 11: Fitting to the CCFR data [20] of $x_{F_3}$. 

$10 < Q^2 < 13$ (GeV$^2$)
Figures 5 and 6 show the big differences in the $x$- and $Q^2$-dependence of $\bar{u}(d)$ and $\bar{u}^{as}(d^{as})$. This result impels us to pay attention to the distribution of the strange quark. The distribution function of the strange quark describes important features of the sea structure in the nucleon. However, the information on the relation between the strange sea and non-strange sea is rather confused [31]. We think that the decomposition of the asymmetric and symmetric sea in this work provides a possible way to study the origin of the strange sea in the proton. In the above mentioned global fits, $s = \bar{s}$ is generated radiatively as same as $\bar{u}$, $\bar{d}$, without any extra parameter. Assuming $SU(3)F$ flavor is symmetry, we have $s + \bar{s} = \bar{u} + \bar{d}$. The result is shown by the solid curve in figure 12. Recently, the strange quark distribution in the proton of HERMES experiment are re-evaluated [32]. The distribution $x(s(x, Q^2) + \bar{s}(x, Q^2)) = x(\bar{u}(x, Q^2) + \bar{d}(x, Q^2))$ evolved to $Q^2 = 2.5$ GeV$^2$ is presented in figure 12. For comparison, we draw the prediction of the CTEQ6L PDFs [33], where $s(x, Q^2_0) + \bar{s}(x, Q^2_0) = 0.4(\bar{u}(x, Q^2_0) + \bar{d}(x, Q^2_0))$ at $Q^2_0 = 1.69$ GeV$^2$ was assumed. We also plot the strange quark distribution in the GRV analysis [7], which also is radiatively generated but starting from $s(x, Q^2_0) = 0$ at $Q^2_0 = 0.26$ GeV$^2 >> 0.064$ GeV$^2$. Although our predicted shape of $x(s + \bar{s})$ is similar to the HERMES data comparing with other two predictions in figure 12 the broken $SU(3)_F$ symmetry due to a large mass of the strange quark is obvious.

Usually, the suppression ratio $r_s = (s(x, Q^2) + \bar{s}(x, Q^2))/2\bar{d}(x, Q^2) \neq 1$ is used to describe this symmetry breaking. However, the ratio $\bar{d}(x)/\bar{u}(x)$ strongly depends on $x$ due to $\bar{d}^{as}(x) \neq \bar{u}(x)^{as}$. We should exclude the contributions of $\bar{u}^{as}(x)$ and $\bar{d}(x)^{as}$ in the definition $r_s$. For this sake, we re-define the suppression ratio as
Figure 12: Our predicted strange quark distribution $x(s(x, Q^2) + \bar{s}(x, Q^2))$ at $Q^2 = 2.5$ GeV$^2$ and comparisons with the HERMES data [32], the predictions of CTEQ6L analysis [5] (dashed point curve) and GRV analysis [7] (dashed curve).

We plot our predicted strange quark distribution with different values of $r_s$ at $Q^2 = 2.5$ GeV$^2$ in figure 13a. We find that $r_s = 0.8$ is consistent with the HERMES results at $x < 0.07$ and the ATLAS measurement [34] in W and Z boson production in pp collisions, which obtains $r_s = 1.0 \pm 0.25$ at $x = 0.023$. This $r_s$ value is also consistent with the lattice calculation $r_s = 0.857 \pm 0.040$ at $Q^2 = 2.5$ GeV$^2$ [35]. However, our $r_s$ decreases to $\sim 0.2$ at $x > 0.1$. It indicates there should be an unknown creating mechanism of the strange quarks in the nucleon. For example, we list some possible explanations:

(i) the $x$-dependence of the suppression ratio $r_s(x)$, which is presented in figure 13b;
(ii) the $x$-rescaling, i.e., $x(s(x, Q^2) + \bar{s}(x, Q^2)) = \eta x(\bar{u}(\eta x, Q^2) + \bar{d}(\eta x, Q^2))$ at $Q^2 = 2.5$ GeV$^2$ with $\eta = 0.8$ (figure 14); (iii) the $Q^2$-rescaling, i.e., $x(s(x, Q^2) + \bar{s}(x, Q^2)) = x(\bar{u}(x, \zeta Q^2) + \bar{d}(x, \zeta Q^2))$ with $\zeta = 0.2$ at $Q^2 = 2.5$ GeV$^2$(Figure 15).
Figure 13: (a) The strange quark distribution $x(s(x, Q^2) + \bar{s}(x, Q^2))$ at $Q^2 = 2.5$ GeV$^2$ with different values of $r_s = 1, 0.8, 0.6, 0.4$ and 0.2 (from top to bottom); (b) a possible relation of the suppression ratio $r_s(x, Q^2)$ with $x$ at $Q^2 = 2.5$ GeV$^2$. 
Figure 14: The strange quark distribution $x(s(x, Q^2) + \bar{s}(x, Q^2))$ at $Q^2 = 2.5$ GeV$^2$ assuming $x$-rescaling.

Figure 15: The strange quark distribution $x(s(x, Q^2) + \bar{s}(x, Q^2))$ at $Q^2 = 2.5$ GeV$^2$ assuming $Q^2$-rescaling.
The asymmetric light sea (it is called as the connected sea $u_{cs}(x)$ and $d_{cs}(x)$ in [35]) is calculated by using the QCD lattice method combining with the (old) HERMES results [36] and CTEQ-database:

$$\bar{u}_{cs}(x,Q^2) + \bar{d}_{cs}(x,Q^2) = [\bar{u}(x,Q^2) + \bar{d}(x,Q^2)]_{CTEQ} - \frac{1}{r_s \text{lattice}} [s(x,Q^2) + \bar{s}(x,Q^2)]_{\text{(old)HERMES}}$$

$$= \bar{u}_{cs}(x,Q^2) + \bar{d}_{cs}(x,Q^2) + \bar{u}_{ds}(x,Q^2) + \bar{d}_{ds}(x,Q^2) - \frac{1}{r_s \text{lattice}} [s(x,Q^2) + \bar{s}(x,Q^2)]_{\text{(old)HERMES}},$$

(3.6)

where the disconnected anti-sea $\bar{u}_{ds}(x)$ and $\bar{d}_{ds}(x)$ are identical with our $u_s(x)$ and $d_s(x)$.

Figure [16] shows the comparison of our results $x(\bar{u}_{cs}(x,Q^2) + \bar{d}_{cs}(x,Q^2))$ with the lattice predicted $x(\bar{u}_{cs}(x,Q^2) + \bar{d}_{cs}(x,Q^2))$ at $Q^2 = 2.5$ GeV$^2$. One can find that

(a) $x(\bar{u}_{cs}(x) + \bar{d}_{cs}(x)) < x(\bar{u}_{ds}(x) + \bar{d}_{ds}(x)), \quad x < 0.07,$

and

(b) $x(\bar{u}_{cs}(x) + \bar{d}_{cs}(x)) > x(\bar{u}_{ds}(x) + \bar{d}_{ds}(x)), \quad 0.07 < x < 0.2.$

(3.7)

This difference can be understood as follows. The lattice calculation in [35] sensitively depends on the strange quark distribution. Since $s(x) + \bar{s}(x)$ in the old HERMES data [36] are higher than that in the new HERMES data [32] at $x < 0.07$. The lattice calculation in [34] use the old HERMES data and it leads to the conclusion (a); The lattice calculation assumes that $\bar{u}_{ds}(x) + \bar{d}_{ds}(x) = [s(x) + \bar{s}(x)]/r_s$, where $r_s$ is fixed as a constant and has not considered $r_s \rightarrow 0.2$ at $x > 0.07$ as shown in figure [12]. This leads to the result (b). Therefore, our asymmetric sea distributions are reasonable.
Figure 16: Comparison of our predicted distribution \( x(\bar{u}^a(x, Q^2) + d^a(x, Q^2)) \) with the lattice predicted in [35] \( x(\bar{u}^a(x, Q^2) + \bar{d}^a(x, Q^2)) \) at \( Q^2 = 2.5 \text{ GeV}^2 \).
4 Discussions and Summary

Where is the lowest $Q^2$ bound for the application of perturbative QCD evolution equation? This is an unclear subject. One may think that a large value of $\alpha_s$ at low $Q^2$ will lead to perturbative expansion divergence. However, the authors in [37-39] have emphasized that the DGLAP evolution is still in the perturbative region even at low starting point $\mu^2 \sim 0.064 \text{ GeV}^2$ since the expansion factor $\alpha_s(\mu^2)/2\pi$ is less than 1 in the DGLAP equation and the evolution kernels are non-singular at low $Q^2$. In this work we have shown that these conclusions are still valid for the DGLAP equation with the ZRS corrections and the resummations $\sum_n [\alpha_s/(2\pi) \ln(Q^2/\Lambda_{QCD})]^n$ and $\sum_n [\alpha_s^2/(4\pi R^2 Q^2)]^n$ converge quickly, i.e., the perturbative evolution of eq. (1) is stable at low $Q^2$.

Now let us discuss the possible explanations about eq. (3.4). According to the connected sea model [35], nucleon sea has two sources based on the path-integral formalism of the hadronic tensor: the (asymmetric) connected sea (they distribute $q^{cs}(x)$ and $\overline{q}^{cs}(x)$ for $q=u,d$) and (symmetric) disconnected sea (they distribute $q^{ds}(x)$ and $\overline{q}^{ds}(x)$ for $q = u, d, s...$). In this picture, $u^{cs}$ (or $d^{cs}$) coincides with the valence distribution $u^v$ (or $d^v$) either in the nonperturbative topological structure (figure [17]) or in the perturbative QCD evolution dynamics. Therefore, $q^{cs}(x)$ and $q_v(x)$ have similar shape as Eq. (3.4).

Several explanations for the flavor asymmetric sea are based on the virtual states of the proton containing isovector meson [40,41]. We think that a cloud proton in a quark level at $\mu^2$, which can be described by figure [18], where the quark loop is nonperturbatively created from QCD vacuum rather than the radiative production. We assume that the configuration $u_v u_v d_v$ keeps the proton state at $\mu^2$ and denote the distributions of the cloud quarks $u^{\text{cloud}}(x, \mu^2)$, $d^{\text{cloud}}(x, \mu^2)$ and cloud antiquarks $\overline{u}^{\text{cloud}}(x, \mu^2)$, $\overline{d}^{\text{cloud}}(x, \mu^2)$. One can
Figure 17: One of topologically distinct diagram for the valence and connected sea quarks $q^v + q^{cS}$ in a lattice calculation [35].

image that $\overline{q}^{\text{cloud}}(x, \mu^2) = q^v(x, \mu^2)$ in a same quark loop. At the same time, we have $q^{\text{cloud}}(x, \mu^2) = \overline{q}^{\text{cloud}}(x, \mu^2)$ in a meson configuration due to symmetry. In consequence, we have $u^{\text{cloud}}(x, \mu^2) = \overline{u}^{\text{cloud}}(x, \mu^2) = u^v(x, \mu^2)$ and $d^{\text{cloud}}(x, \mu^2) = \overline{d}^{\text{cloud}}(x, \mu^2) = d^v(x, \mu^2)$ in figure 18 (a)-(c). Assuming the probabilities of these three configurations are $1 - \alpha - \beta$, $\alpha$ and $\beta$, respectively. Thus, the initial quark distributions in the cloud proton are

$$u^{NS}(x, \mu^2) = u^v(x, \mu^2) + u^{\text{cloud}}(x, \mu^2) + \overline{u}^{\text{cloud}}(x, \mu^2)$$

$$= (1 + 2\alpha)u^v(x, \mu^2)$$

and

$$d^{NS}(x, \mu^2) = d^v(x, \mu^2) + d^{\text{cloud}}(x, \mu^2) + \overline{d}^{\text{cloud}}(x, \mu^2)$$

$$= (1 + 2\beta)d^v(x, \mu^2),$$

(4.1)

they are consistent with Eqs. (3.2) and (3.3).

In summary, using the DGLAP equation with corrections of parton recombination, the distributions of the flavor asymmetric sea in the proton are first extracted from the
available experimental data in a model-independent way. We find that the distribution shape of the asymmetric sea quark is similar to that of the corresponding valence quark. We find that the strange sea distribution in the proton is different from the non-strange distributions since the previous one has no nonperturbative asymmetric components. The results are used to explain the recent HERMES data for the strange quark distribution. Based on the separated flavor symmetric sea, we discuss the possible relations between the strange quark distribution and symmetric light quark distribution. The comparisons of our results with the predictions of the CTEQ-, MRS-, GRV- databases and some phenomenological models are presented. We also compare our extracted asymmetric sea with the prediction of a lattice calculation and the difference between two results are discussed.

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Appendix. The simplification of the DGLAP equation with the ZRS corrections.

The DGLAP equation with the ZRS corrections at leading order [11-13] according to figure 19 reads

\[
Q^2 \frac{dx q^{NS}(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{qq} \otimes q^{NS}
\]

\[
- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gq \rightarrow q}(x, y) y g(y, Q^2) y q^{NS}(y, Q^2)
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{gq \rightarrow q}(x, y) y g(y, Q^2) y q^{NS}(y, Q^2)
\]

\[

\int_x^{1/2} \frac{dy}{y} x P_{gq \rightarrow q}(x, y) y g(y, Q^2) y q^{NS}(y, Q^2)
\]

\[

\int_{x/2}^x \frac{dy}{y} x P_{gq \rightarrow q}(x, y) y g(y, Q^2) y q^{NS}(y, Q^2), (i f \ x \leq 1/2),
\]

\[
Q^2 \frac{dx q^{s}(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{qq} \otimes q^s
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gq \rightarrow q}(x, y) y g(y, Q^2) y q^{NS}(y, Q^2)
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{gq \rightarrow q}(x, y) y g(y, Q^2) y q^{NS}(y, Q^2), (i f \ 1/2 \leq x \leq 1),
\]

for non-singlet quarks,
\[ -\frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x}^{1/2} \frac{dy}{y} xP_{gg\rightarrow q}(x,y)[y\Sigma(y,Q^2)]^2 \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x/2}^{x} \frac{dy}{y} xP_{gg\rightarrow q}(x,y)[yyg(y,Q^2)]^2 \]
\[ -\frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x}^{1/2} \frac{dy}{y} xP_{q\overline{q}\rightarrow q}(x,y)y\Sigma(y,Q^2)yyg^*(y,Q^2) \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x/2}^{x} \frac{dy}{y} xP_{q\overline{q}\rightarrow q}(x,y)y\Sigma(y,Q^2)yyg^*(y,Q^2) \]
\[ -\frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x}^{1/2} \frac{dy}{y} xP_{qg\rightarrow q}(x,y)y\Sigma(y,Q^2)yyg^*(y,Q^2) \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x/2}^{x} \frac{dy}{y} xP_{qg\rightarrow q}(x,y)y\Sigma(y,Q^2)yyg^*(y,Q^2), \text{ if } x \leq 1/2, \]

\[ Q^2 \frac{dqg^*(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi}[P_{qq} \otimes q^* + P_{gg} \otimes g] \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x/2}^{x} \frac{dy}{y} xP_{gg\rightarrow q}(x,y)[y\Sigma(y,Q^2)]^2 \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x/2}^{x} \frac{dy}{y} xP_{q\overline{q}\rightarrow q}(x,y)y\Sigma(y,Q^2)yyg^*(y,Q^2) \]
\[ + \frac{\alpha_s^2(Q^2)}{4\pi R^2Q^2} \int_{x/2}^{x} \frac{dy}{y} xP_{qg\rightarrow q}(x,y)y\Sigma(y,Q^2)yyg^*(y,Q^2) \]

\[ Q^2 \frac{dxq^*(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi}[P_{qq} \otimes \Sigma + P_{gg} \otimes g] \]

for symmetric sea quarks and
\[-\frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gg \to g}(x, y) [y g(y, Q^2)]^2 \]
\[+ \frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{gg \to g}(x, y) [y g(y, Q^2)]^2 \]
\[-\frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{qg \to g}(x, y) \sum_q [y q^s(y, Q^2)]^2 \]
\[+ \frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{qg \to g}(x, y) \sum_q [y q^s(y, Q^2)]^2 \]
\[-\frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{1/2}^{x/2} \frac{dy}{y} x P_{qg \to g}(x, y) y \Sigma(y, Q^2) y g(y, Q^2) \]
\[+ \frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{qg \to g}(x, y) y \Sigma(y, Q^2) y g(y, Q^2), (if \ x \leq 1/2), \]

\[Q^2 \frac{dx g(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} [P_{gg} \otimes \Sigma + P_{gg} \otimes g] \]
\[+ \frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{gg \to g}(x, y) [y g(y, Q^2)]^2 \]
\[+ \frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{qg \to g}(x, y) \sum_q [y q^s(y, Q^2)]^2 \]
\[+ \frac{\alpha_s(Q^2)}{4\pi R^2 Q^2} \int_{1/2}^{x/2} \frac{dy}{y} x P_{qg \to g}(x, y) y \Sigma(y, Q^2) y g(y, Q^2), (if \ 1/2 \leq x \leq 1), \quad (A.1) \]

for gluon. The linear terms are the standard DGLAP evolution and the recombination functions $P_{ab \to c}$ are

\[P_{gg \to g}(x, y) = \frac{9}{64} \frac{(2y - x)(72y^4 - 48xy^3 + 140x^2y^2 - 116x^3y + 29x^4)}{xy^5}, \]

\[P_{gg \to q}(x, y) = P_{gg \to \bar{q}}(x, y) = \frac{1}{96} \frac{(2y - x)^2(18y^2 - 21xy + 14x^2)}{y^5}, \]
\[ P_{qq\to q}(x, y) = P_{q\bar{q}\to \bar{q}}(x, y) = \frac{2}{9} \frac{(2y - x)^2}{y^3}, \]

\[ P_{q\bar{q}\to q}(x, y) = P_{q\bar{q}\to \bar{q}}(x, y) = \frac{1}{108} \frac{(2y - x)^2(6y^2 + xy + 3x^2)}{y^5}, \]

\[ P_{gq\to q}(x, y) = P_{g\bar{q}\to \bar{q}}(x, y) = \frac{1}{288} \frac{(2y - x)(140y^2 - 52yx + 65x^2)}{y^4}, \]

\[ P_{gq\to g}(x, y) = P_{g\bar{q}\to \bar{g}}(x, y) = \frac{1}{288} \frac{(2y - x)(304y^2 - 202yx + 79x^2)}{xy^4}, \]

\[ P_{\bar{q}q\to g}(x, y) = \frac{4}{27} \frac{(2y - x)(18y^2 - 9yx + 4x^2)}{xy^3}, \quad (A.2) \]

Our numerical calculations show that the following approximation is acceptable

\[ P_{qq\to q}(x, y) = P_{q\bar{q}\to \bar{q}}(x, y) = 0, \quad P_{q\bar{q}\to q}(x, y) = P_{q\bar{q}\to \bar{q}}(x, y) = 0, \]
\[ P_{gq\to g}(x, y) = P_{g\bar{q}\to \bar{g}}(x, y) = 0, \quad P_{gq\to q}(x, y) = P_{g\bar{q}\to \bar{q}}(x, y) = 0. \quad (A.3) \]

In fact, we calculate the parton distributions using the complete form (A.2)(solid curves) and the simplified form (A.3) (dashed curves) with the same parameters in figure \[20\]. The maximum difference between two equations is smaller than \(~5\%).
Figure 19: Schematic diagram for the DGLAP equation with the ZRS corrections.
Figure 20: Some of parton distributions using the complete ZRS corrections eq. (A.1) (solid curves) and simplified ZRS corrections eq. (A.3) (dashed curves).
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