Noise Standard Deviation Estimation for Additive White Gaussian Noise Corrupted Images using SVD Domain

Sridhar P, R.R Sathiya

Abstract: During denoise an image; noise level estimation is one of the most important key factors. The accurate noise level estimation is needed before processing the image. The prior knowledge of noise level estimation is also used for restoring the image without degradation. In this proposed work, the noise level is estimated by observed singular values on noisy images. The proposed work has two new methods for addressing the main challenges of the noise level estimation. 1. The tail magnitude value of the noisy images singular values has high compare with signal image. This aspect is used for estimate the noise level. 2. The visual based Gaussian noise estimation is used for pre-processing the many 2D signals processing application which enhance the range of this work. The experimental result for this noise level estimation provides reliable and also applicable for real time images/frames and some special images such as cartoon. The proposed work is needed a simple processing unit for implementing in hardware and results are more accurate. It can be used to pre-processing all kinds of real time images.

Keywords –Noise level estimation, Gaussian noise, noise standard deviation, de noising

I. INTRODUCTION

During image acquisition and transmission noise is inevitable. It alters the intensity of the pixel as a random manner which causes unpleasant to see and process the image data. The noise generation sources are vision sensor with its circuit, acquisition equipment such as digital scanner and cameras, analog to digital converter (quantizer), transmission medium. Once the image/frame is corrupted by noise it cannot process easily [2]-[4]. However further step forward pre-processing the image such as noise level estimation and suitable filter to remove the noise after that processing the image [1]. Further the noise estimation is the preliminary step before reliable denoising. The noise estimation in an image is a very tedious task. The other works such as motion estimation, high resolution, and extraction of feature are benefits by noise level estimation [15]-[19]. The Gaussian distribution is used mostly for noise estimation. Gaussian noise model such as noise in amplifier, finite account of particle which has a less energy causes shot noise, the photographic noise. The noise estimation is hard for an image. The brightness or texture of the image is varied by noise. The survey of denoising works clearly indicates consider noise as zero mean additive white Gaussian noise [5]-[8].

The noisy image is mathematically represented by equation (1).

\[
I_N(i, j) = I_o(i, j) + N(i, j)
\]

Where \( I_o(i, j) \) represents the true image, \( N(i, j) \) is the Gaussian noise. The Gaussian noise distribution is given by equation (2)

\[
p(z) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}
\]

Where \( \sigma \) indicates standard deviation of the Gaussian noise and \( \mu \) is the mean value. In this case mean value is zero so the Gaussian noise estimation, the measuring parameter is \( \sigma \). The two main issues for measuring the Gaussian noise of the separate image. [9]-[11](1) how to collect the dataset for estimation noise in an image (2) The various image such real time image, texture type and standard image how to create the adaptive approach for estimating the noise. The Gaussian noise estimation approaches can be classified into 3 types. (1) Filtering methods (2) Transform based approach (3) Patch based works. In filter based approaches, the noise can be removed by low pass filter then estimation of the noise is measured by standard deviation between true image and noisy image. The limitation of this filter based method has not adaptive for all types of images. The primary difficulties of this method the different content of the image consider as noise but commonly we are not considering like that. This provides small original image signal influence on the noise estimation. Histogram based noise estimation [11] has more computational complexity and more no of parameters using. In patch based method images are separated into blocks. The noise level is measured in each block which has homogeneous. The homogeneous block consider as a smooth part of the image add with noise. The homogeneity is relatively connected with the real world images. The identification of homogeneity block based method [12] is a difficult task. Commonly noise estimation block based approaches are used for estimation of noise. Even though this kind approach is easy, noisy estimation is varying predominantly depending upon the type of input image and level of noise.
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Filtering approaches provides the good results for large amount of noise. But they need large memory and high computation costs.

In the transform based work, the frequently used approach is a Mean Absolute Deviation (MAD) [13]. The noise is estimated by using the equation (3).

\[ \hat{\sigma}_n = \frac{\text{median}(HH)}{0.6745} \]  

(3)

Where HH indicates the high frequency sub band coefficients of wavelet transform. The interpretation of this equation noise signal is lying in a high frequency subband. But the estimated output is more than actual value of the noise. The high frequency subband is influenced by noise compare to signal. Consider the case which is more image content compare to noise; this case estimation is less accuracy. The little modification was performed which yields the better results. This estimation is performing is based Donoho’s approach.

The mention challenges in transform domain cases and break the limitation in the existing system, we explore the techniques for noise estimation using singular value decomposition (SVD). The SVD transform domain has well covered in recognition and restoration problems in images. The SVD based noise estimation is strong influence for noise estimation and gives less significance in original image data.

The various kind of dataset is tested against the proposed work which provides reliable result for different level standard deviation of Gaussian noise with zero mean.

II NOISE INFLUENCE IN SVD DOMAIN

A. Singular values of Additive white Gaussian Noise

With the help of Linear algebra the theory of SVD in which the rectangular matrix \( A \) is decomposed into three matrices. It has an orthogonal matrix \( U \), diagonal element matrix \( S \) and one more orthogonal matrix \( V \).

\[ A = U \times S \times V^T \]  

(4)

Where \( U^T U = I_m \) and \( V^T V = I_n \) and \( I_m \) and \( I_n \) indicate the identity square matrices. The dimension of \( A \) is \( m \times n \). The column of \( U \) and \( V \) matrices are orthonormal and \( S \) is the diagonal matrix which square roots are eigen values of the \( AA^T \). The Columns of orthogonal matrix are orthonormal Eigen vector. The singular values of diagonal matrix \( S \) is denoted by descending order. The singular values are \( S(1) > S(2) > ... > S(r) \) and then \( S(1) > S(2) > ... > S(r) \).

The signal and noise is denoted by notation \( S_s \) and \( S_n \) and its SVD transform is denoted by equation (5) and (6).
In Eq (2) are shown different test image’s singular values and its noise levels. The test image size is 512x512.All test image are standard grayscale image. The singular value is increasing when add Gaussian noise in an image and this was indicated in [20].Conversely, if the higher the noise standard deviation, the magnitude of the singular values is larger. The earlier singular values are not influencing if increasing the noise level. But the noise can be identified by later singular values part. This is the important part of the Gaussian noise estimation.

B. Noise Analysis

Additive white Gaussian noise is represented by N of matrix size and its variance $\sigma^2$. The SVD transform of the noise matrix is

$$N = U \times S_n \times V^T$$

Variance ($\sigma^2$) = \frac{1}{\min} \sum S(i)^2

The number of tail end singular values is denoted by M. The function of noise standard deviation is the average of these last M singular values and it’s estimated by

$$L_M(\sigma) = \frac{1}{M} \sum_{i=M+1}^r S(i)$$

Where 1 $\leq M \leq r$ if M=2, the end of two singular value will be taken. If M=r, all M values will be considered. The noise function $L_M(\sigma)$ is linearly dependent on standard deviation. It can be indicated by equation (9) and it was satisfied superposition principle.

$$L_M(c\sigma) = c \times L_M(\sigma)$$

$$L_M(c_1\sigma + c_2\sigma_1) = c_1L_M(\sigma) + c_2L_M(\sigma)$$

Where $c_1$, $c_2$, c are constants.

Table 1 shows the linear relation between the function $L_M(\sigma)$ and $\sigma$. The linear relation is given by Eq (10).

$$L_M(\sigma) = c\sigma$$

Where ‘c’ is the slope or proportionality constant. It can change by the chosen value of M. The chosen value of M<4/3 the function $L_M(\sigma)$ is not a linear relationship with the standard deviation. The resolution of the image is 512x512 added with additive white Gaussian Noise we consider 128 values to 512 singular values for showing the linear relationship. For different resolution of image and proper slope value of ‘c’ it provides the linear relation. The real world image sequences are entirely different from the standard dataset. Since, the equation (10) is not satisfied the function $L_M(\sigma)$. We can modify the equation (10) for real world image is

$$L_M = \sum_{i=M+1}^r S(i) = c\sigma + \beta$$

\beta is related to the content of the image. We combine the equation (8) and (11) it gives the equation (12).

$$L_M = \sum_{i=M+1}^r S(i) = c\sigma + \beta$$

The function $L_M$ can be divided into two components based on the signal and noise in image content. $L_{Ms}$ is the signal component and $L_{Mn}$ is the noise component. It can be represented by equation (13) and (14).

$$L_{Ms} = \sum_{i=M+1}^r S(i)$$

$$L_{Mn} = \sum_{i=M+1}^r S(i)$$

We examine the signal and noise contribution for Standard Lena Image with 512x512. The chosen singular values is from 128 to 512. The noise distribution function is almost parallel to image with noise function. But the signal content is eventually different from this two noise function. The signal function is almost constant with respect to coefficients. It shows in Fig.3.

Fig.3 Signal and Noise Contribution of $L_M$

The equation (19) is related with the Fig.3. The signal with noise function or noise function ‘c’ values are slope of the function and $\beta$ value depends on the content of the image. The complex image has higher value of $\beta$.

Fig.4 Linear relationship between Singular values and Noise function for different noise levels of 512X512 image

We should not take the M value is larger. Since the starting singular values the signal part is influenced in the noisy image. Noise function $L_M$ is calculated based on the M parameter value. The M value should not choose too small. It does not provide the sufficient data noise estimation. It causes reliability and accuracy issues of the noise estimation.
The various experimental tests were performed from different images at different resolutions. We applied SVD to estimate the noise standard deviation for additive white Gaussian noise corrupted images. The singular values of various test images for different Gaussian noise levels are shown in the figure. (a) Lena (512x512).
image and its singular values. column (b) Barbara (512x512) image and its singular values. column (c) Pepper image and fix the M parameter range from \( r/4 \) to \( r/5 \). In Fig.5 provides the graph between standard deviation and the noise function for different resolution test images. The content does not influence the noise function. In Fig.5 (b) the test images size has 256x256. If the noise standard deviation is increased the \( L_M \) is also increased. So the parameter M value is chosen (M=3r/4). Fig.5 (a) shows the crowd, Lena, pepper and Blank test images graphs. The test result shows that the blank image signal. Furthermore, the content of the test images is varying. So the line of each test image parallel but 

Table-I: Using the slope values of ‘c’ and M=3r/4, the linear relationship between \( L_M \) and \( \sigma \) can be seen for different resolution

| \( \sigma \) | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | c |
|---|---|---|---|---|---|---|---|---|---|---|
| \( L_M \) (512x512) | 137.9833 | 207.5290 | 277.2149 | 344.2644 | 413.6893 | 484.9244 | 552.6644 | 621.6843 | 690.9194 | 12.86 |
| \( L_M \) (256x256) | 97.4532 | 150.3322 | 189.2242 | 242.3334 | 256.4858 | 347.2265 | 389.2456 | 445.1226 | 495.2364 | 9.97 |
| \( L_M \) (128x128) | 216.3034 | 278.8576 | 347.6419 | 412.4439 | 480.9524 | 548.6564 | 616.1221 | 685.8564 | 754.2832 | 6.98 |

Fig.5 the relationship between noise function \( L_M \) and standard deviation \( \sigma \) for different resolution (a) 512x512 resolution test images 512x512 resolution test images (c)
III. PROPOSED NOISE STANDARD DEVIATION ESTIMATION ALGORITHM USING SINGULAR VALUE DECOMPOSITION DOMAIN

The analysis results give the relationship between parameter value M and noise function and linearity of the noise standard deviation. This analysis creates the platform for noise estimation. The different test images used at different resolution levels and fixed the M parameter to maintain the linearity for noise estimation. The different image size the chosen value of M is the only factor for affecting the slope value c. For instance from Table I, if the image size has 512X512 the chosen M value is 3r/4 i.e. M=384 and slope c is 12.86. If the size 256x256 the chosen M parameter is 192 and slope is 9.97. The analysis provides the conclusion for choosing the M parameter value range of [r/4, 3r/4] for reliable calculation. The other size of image the following steps provide the way for calculating the slope c values.

1. Calculating the noise function of additive white Gaussian noise corrupted image at different deviation level we tabulated the values such as Table I
2. Calculating the slope c value by least square method. The slope c value is not dependent on image content.
3. Due to the development of the embedded system, nowadays different resolution cameras are available in the market for various purposes. The calculation of the slope value for different resolution test image is estimated by method such as Table I. Noise standard deviation level is calculated by using equation (11) we find the image content related parameter β. But this is very difficult to find the accurate β value. The proposed algorithm uses 4 steps for estimating the Gaussian noise level.

1. The parameter M value have to choose Properly.
2. Apply the Singular value Decomposition of noise corrupted image
3. Calculate the value of L_M
4. Estimating the noise standard deviation

| Images | σ=10  | σ=15  | σ=20  | σ=25  | σ=30  | σ=35  | σ=40  | σ=45  | σ=50  |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Lena   | 9.8315| 14.9106| 19.9450| 24.9956| 30.0751| 35.1242| 40.0321| 45.1530| 50.2167|
| Peppers| 9.6652| 14.6668| 19.6797| 24.7306| 29.7179| 34.7337| 39.7293| 44.7708| 49.7558|
| Crowd  | 7.6176| 12.9417| 18.1816| 23.3610| 28.4766| 33.6535| 38.7309| 43.8501| 48.9776|
| Barbara| 9.1786| 14.4172| 19.4980| 24.6122| 29.6503| 34.6896| 39.7497| 44.7619| 49.7390|
| Cameraman| 9.3022| 14.4623| 19.5854| 24.7041| 29.7218| 34.8377| 39.9021| 44.8898| 49.9531|
| Cartoon| 8.5879| 13.7858| 18.9598| 24.1058| 29.1829| 34.3145| 39.3237| 44.3515| 49.465 |
| Dora   | 11.4242| 16.0352| 20.8403| 25.7705| 30.7491| 35.7404| 40.7900| 45.8461| 50.8522|
| Blank  | 9.9395| 14.9266| 19.9019| 24.8837| 29.8564| 34.9422| 39.8510| 44.8797| 49.8594|

| Images | σ=10  | σ=15  | σ=20  | σ=25  | σ=30  | σ=35  | σ=40  | σ=45  | σ=50  |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Lena   | 0.1198| 0.1547| 0.2018| 0.2215| 0.2358| 0.3020| 0.330 | 0.3876| 0.4432|
| Peppers| 0.1454| 0.1613| 0.1811| 0.2234| 0.2257| 0.2941| 0.3687| 0.4296| 0.4132|
| Crowd  | 0.1254| 0.1535| 0.1875| 0.1961| 0.2488| 0.2900| 0.3436| 0.4111| 0.4477|
| Barbara| 0.1146| 0.1481| 0.1609| 0.2358| 0.2817| 0.3112| 0.3598| 0.4078| 0.4725|
| Cameraman| 0.1215| 0.1421| 0.1833| 0.2108| 0.2488| 0.3167| 0.3169| 0.3918| 0.4532|
IV EXPERIMENTAL RESULTS AND ANALYSIS
The proposed work is tested different types of images such as standard image and real world image Dora. The estimated value of L_{34} is the mean of tail ended singular values (75% of singular values i.e. 3r/4). The proposed method using Lena, Barbara, Peppers, Crowd, Blank Dora images at different resolution level. The Fig.1 isolates the signal and noise content of specific range of singular values. We analyse the simulation results from Fig.2 to Fig 5 how the parameter is noise standard deviation estimation error \( \hat{\sigma} = \sigma - \hat{\sigma} \). In Table IV shows the comparison analysis using two wavelet based works. Algorithm 1 [13] D.L. Donoho proposed wavelet coefficient based noise estimation. Algorithm 2 [14] is modified version of wavelet domain noise estimation. The two algorithms provide the good noise estimation for high standard deviation noise level. But minimum noise standard deviation these two works more deviation was there.

V. CONCLUSION
Singular value decomposition is mathematical tool it has been important for signal processing application for a long time. We used SVD tool for additive white Gaussian noise estimation. In this tool signal and noise are separately distinguished. The experimental results show our proposed methods are outperforms the estimation of noise with the other existing methods. It provides better estimation results. Computer vision, pattern recognition, image processing application noise estimation is important for us to know in advance. These works such as video analytics and denoising are needed for noise estimation. The proposed method good ground work for image processing application. A lot of algorithms don’t provide us favourable results due to noise in an videos or frames. It is initiative for denoising.

| Noise Level | Proposed Method | Algorithm 1 [13] | Algorithm 2 [14] |
|-------------|-----------------|------------------|------------------|
| 10          | 0.1             | 0.9              | 1.38             |
| 15          | -0.1            | 0.35             | 1.5              |
| 20          | 0.01            | 0.58             | 0.9              |
| 25          | -0.12           | 0.5              | 0.8              |
| 30          | 0.2             | 0.22             | 0.7              |
| 35          | 0.15            | 0.16             | 0.6              |
| 40          | 0.1             | -0.1             | 0.58             |
| 43          | 0.01            | -0.3             | 0.4              |
| 50          | 0.18            | -0.35            | 0.2              |

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AUTHORS PROFILE

P. Sridhar is currently the Assistant Professor (Sr.Gr) of the Department of Electronics and Communication Engineering, Sri Ramakrishna Engineering College, Coimbatore. He received Bachelor of Engineering degree in Electronics and Communication Engineering from Periyar University, Salem, India and Master degree in Applied Electronics from Anna University, Coimbatore, India. His area of interest includes Video Analytics and Artificial Intelligence.

Sathiya R. R. currently serves as Assistant Professor in the department of Computer science and Engineering, Amrita School of Engineering, Coimbatore campus. Her research interest Data Mining.