Reformulation of Matching Equation in Potential Energy Shaping

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Abstract—Stabilization of an underactuated mechanical system may be accomplished via energy shaping. Interconnection and damping assignment passivity-based control is an approach based on total energy shaping by assigning desired kinetic and potential energy to the system. This method requires solving a partial differential equation (PDE) related to the potential energy shaping of the system. In this short article, we focus on reformulating this PDE in order to be solved easier. For this purpose, two sufficient conditions are proposed such that under the satisfaction of one of them, it is possible to merely solve the homogeneous part of potential energy PDE. Besides, the class of systems that simply verify these conditions are discussed. The results are applied to two benchmark systems.

Index Terms—Energy shaping, mechanical systems, passivity-based control, potential energy partial differential equation (PDE).

I. INTRODUCTION

Stabilization of an underactuated mechanical system is a challenging problem since it is not fully feedback linearizable due to fewer actuators than the degrees of freedom (DOF) [1]. Nevertheless, interconnection and damping assignment passivity-based control (IDA-PBC) is a general approach to stabilize an underactuated system by shaping the total energy of the system [2]. In this method, the desired structure of the closed-loop system is assigned by the designer, compared with the usual classical passivity, and then all the corresponding energy functions are derived. For this purpose, two sets of partial differential equations (PDEs) related to the shaping of potential and kinetic energy functions, which are called matching equations, should be solved analytically [3]. By this means, the desired inertia matrix should be derived from the kinetic energy PDE, and then by knowing this matrix, the potential energy PDE is solved. Since the analytic solutions of these equations shall be derived, the applicability of this approach is significantly restricted.

Several papers have focused on solving or reformulating the matching equations. In [4], solving the matching equations of systems with one degree of underactuation has been reported. It has been shown that upon certain conditions, the solution of the kinetic and potential energy PDEs could be derived analytically. Furthermore, solving the PDEs of 2-DOF underactuated systems satisfying some conditions has been reported in [5]. Simplification of kinetic energy PDE through coordinate transformation has been studied in [6]. In [7], the total energy shaping without solving the matching equations for a class of underactuated systems has been investigated (see also [8]). A method for simplifying the kinetic energy PDE has been reported in [9] in which a particular structure for desired inertia matrix has been considered to streamline the related matching equation. Recently, in [10], it has been shown that replacing a PDE with some Pfaffian differential equations may ease up this stumbling block of solving matching equations. Furthermore, it has been shown that the solution of potential energy PDE may be divided into homogeneous and nonhomogeneous components, whereas Harandi et al. [11] reported the general structure of the homogeneous solution, see also [12] for more details about the literature of IDA-PBC. In some cases, deriving desired potential energy, especially the nonhomogeneous component, is a prohibitive task.

In this note, we reformulate the potential energy PDE, and derived two sufficient conditions to merely compute the homogeneous solution of the PDE. Both of the conditions are based on the physical parameters of the system, namely, the desired inertia matrix and the controller gains in the homogeneous part of the desired potential function, while one of them has an extra free parameter that makes it easy to satisfy the condition. It is shown that satisfaction of the conditions leads to a particular solution for the nonhomogeneous part of PDE, and therefore, it is possible to represent the closed-loop system in the port-Hamiltonian formulation. Furthermore, the class of systems that naturally satisfy the sufficient conditions are discussed in detail, and the method is verified on the cartpole without feedback linearization and pendubot with any desired inertia matrix. Besides, it is shown that in the cases where the extra free parameter exists but may not be computed, the stabilization error has an ultimate bound, which may be forced to zero by using available robust controller schemes.

II. REVIEW OF IDA-PBC FOR MECHANICAL SYSTEMS

Dynamic equations of a mechanical system are as follows [2]:

\[
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0_{n \times n} & I_n \\
-I_n & 0_{n \times n}
\end{bmatrix}
\begin{bmatrix}
\nabla_q H \\
\nabla_p H
\end{bmatrix}
+ \begin{bmatrix}
0_{n \times m} \\
G(q)
\end{bmatrix} u
\]

where \(q, p \in \mathbb{R}^n\) denote generalized position and momentum, respectively, \(H(q, p) = \frac{1}{2} p^T M^{-1}(q) p + V(q)\) denotes the Hamiltonian of the system, which is the summation of kinetic and potential energy, \(M(q) \in \mathbb{R}^{n \times n}\) denotes the inertia matrix, \(u \in \mathbb{R}^m\) is the input, \(G(q) \in \mathbb{R}^{n \times m}\) denotes full rank input mapping matrix, and the operator \(\nabla\) denotes the gradient of a function, which is represented by a column vector. Presume that the target dynamic of the closed-loop system may be written in the following form:

\[
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0_{n \times n} & M^{-1} M_d \\
-M_d M^{-1} & J_d - G K_c G^T
\end{bmatrix}
\begin{bmatrix}
\nabla_q H_d \\
\nabla_p H_d
\end{bmatrix}
\]

in which \(H_d = \frac{1}{2} p^T M_d^{-1}(q) p + V_d(q)\) is the summation of the desired kinetic and potential energy, \(J_d \in \mathbb{R}^{n \times n}\) is a free skew-symmetric matrix, \(K_c \in \mathbb{R}^{n \times m}\) is positive definite damping gain, and \(V_d\) should be designed such that \(q_d = \arg \min q V_d(q)\) with \(\begin{bmatrix} q_d^T \\ 0_{m \times 1}^T \end{bmatrix}^T\) being the desired equilibrium point where \(0_{m \times 1}\) denotes a column vector of zeros. Note that \(q_d\) must satisfy \(G^T \nabla V = 0\).
0_{n-m} in which \( G^i \in \mathbb{R}^{(n-m) \times m} \) denotes the left annihilator of \( G \) such that \( G^i G = 0_{(n-m) \times n} \).

By setting (1) equal to (2), the control law is derived as

\[
\begin{align*}
u &= (G^i G)^{-1} G^i (\nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 \nabla_p H_d) \\
&\quad - K_v G^T \nabla_p H_d 
\end{align*}
\tag{3}
\]

while the following PDEs shall be satisfied

\[
\begin{align*}
G^i (\nabla_q (p^T M^{-1} (q)p) - M_d M^{-1} (q) \nabla_q (p^T M^{-1} (q)p)) \\
+ 2J_2 M_d^{-1} p &= 0_{n-m} \\
G^i \nabla_q V(q) - M_d M^{-1} \nabla_q V_d(q) &= 0_{n-m}.
\end{align*}
\tag{4}
\]

The first PDE is related to kinetic energy shaping, and the second one corresponds to the potential energy PDE. First, \( M_d \) should be derived from the nonlinear PDE (4), and then \( V_d \) will be obtained from (5). Stability of \( [q_0^T, 0_{n-m}^T] \) is ensured by considering \( H_d \) as a Lyapunov candidate whose derivative is \( \dot{H}_d = -(\nabla_p H_d)^T G K_v G^T \nabla_q H_d \). In the next section, we focus on the reformulation of PDE (5).

### III. MAIN RESULTS

As indicated in [10] and [11], the solution of potential energy PDE can be divided into homogeneous \( (V_{dh}) \) and nonhomogeneous \( (V_{dn}) \) components. Furthermore, \( V_{dh} \) is derived from the following PDE:

\[
G^i M_d M^{-1} \nabla_q V_d(q) = 0_{n-m} \tag{6}
\]

which has the following form:

\[
V_{dh} = \phi (V_{dh1}, V_{dh2}, \ldots) 
\]

where \( V_{dh}, \phi \) are functions satisfying (6). Typically, the free design function \( \phi \) is set to

\[
V_{dh} = \sum_{i=1}^{m'} k_i (V_{dh_i} - V_{dh0})^2 
\tag{7}
\]

in which \( V_{dh_i} = V_{dh} |_{q=q_d}, k_i > 0 \) and \( k_i \) are free positive gains. Furthermore, \( V_{dh0} \) is the particular solution of the PDE (5).

Clearly, the solution of (5) depends on \( M_d \), which is derived from PDE (4). In general, deriving the analytical solution of the nonlinear PDE (4) is a prohibitive task, especially in the cases where the first term of (4) is nonzero. Hence, one may try to solve (4) without considering (5). This typically leads to the complexity of (5), and the difficulty to derive the nonhomogeneous solution. To tackle this problem, in the following theorem a set of sufficient conditions are proposed, through which it is enough to solve the PDE (6) instead of (5), and therefore, derivation of the homogeneous solution would be sufficient. Notice that in the following theorem, the matrix \( G^i \) should be considered as an orthonormal matrix, i.e.,

\[
\| G^i \| = 1 \quad i \in \{1, \ldots, n-C\} \\
G^i G^j = 0 \quad \text{for} \quad i \neq j
\tag{8}
\]

where \( G^i \) denotes the \( i \)-th row of \( G^i \).

**Theorem 1:** Consider IDA-PBC methodology introduced in Section II with \( V_d = V_{dh} \) as the solution of (6). Then, \( [q_0^T, 0_{n-m}^T] \) is (locally) stable if one of the following conditions is satisfied:

1) \( \left. \left( \frac{\partial^2 (V_{dh} + \eta)}{\partial q^2} \right) \right|_{q=q_d} > 0 \)

in which the function \( \eta(q) \in \mathbb{R} \) is defined as

\[
\eta := \int_{q_0}^{q} (\nabla_q V(\nu))^T G^i (\nu) G^i (\nu) M_d^{-1} (\nu) M(\nu) d\nu
\tag{9}
\]

exists, i.e., the matrix \( \partial^2 \eta / \partial q^2 \) is symmetric.

2) There exists \( \xi(q) \in \mathbb{R}^m \) and \( \xi(q_d) = 0_m \) such that the function \( \mu \in \mathbb{R} \) is defined as

\[
\mu := \int_{q_0}^{q} \left( (\nabla_q V(\nu))^T G^i (\nu) G^i (\nu) M_d^{-1} (\nu) M(\nu) + \xi(\nu)^T G^T (\nu) \right)
\times M_d^{-1}(\nu)M(\nu) d\nu
\tag{10}
\]

exists and

\[
\left. \left( \frac{\partial^2 (V_{dh} + \mu)}{\partial q^2} \right) \right|_{q=q_d} > 0.
\tag{11}
\]

In this case, the term \(-\xi(q)\) is added to the control law.\[\]

Notice that the verification of the abovementioned conditions is quite simple since under the existence of the integrals, which will be discussed later, only positive definiteness of a constant matrix should be checked. Furthermore, since the left annihilator of a matrix is not unique, it is always possible to select \( G^i \) such that (8) is satisfied.\[\]

**Proof:** 1) Substitute the control law (3) with \( V_d = V_{dh} \) as the solution of (6). Then, the closed-loop equations are given by

\[
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix}
= 
\begin{bmatrix}
0_{n \times n} & M_d^{-1} M_d & -M_d M^{-1} J_2 - G K_v G^T \\
M_d^{-1} & -J_2 & 0 \\
0 & 0 & G^i G^i \nabla_q V
\end{bmatrix}
\begin{bmatrix}
\nabla_q H_d \\
\nabla_p H_d \\
V_d
\end{bmatrix}
\tag{12}
\]

with \( H_d = \frac{1}{2} p^T M_d^{-1} p + V_{dh} \). To verify (13), it is clear that in (1) and (13) we have \( \dot{q} = M_d^{-1} p \). Multiply the dynamic of \( \phi \) in (1) and (13) from left side to the full rank matrix \( [G, G^i \nabla V]^T \). By multiplying it into \( G^i \), the control law (3) is derived and its multiplication to \( G^i \) results in the matching equations (4) and (6). Note that the last term in (13) is resulted from dissatisfaction of (5) and is interpreted as the natural gravity torques/forces of the system in the unactuated coordinates. Now, consider the following function:

\[
V = H_d + \eta.
\tag{14}
\]

It is easy to verify that its derivative along the trajectories of the system is

\[
\dot{V} = - (\nabla_p H_d)^T G K_v G^i \nabla_q H_d
\tag{15}
\]

Thus, if it is shown that \( V \) is a suitable Lyapunov function, the proof is completed. For this purpose, we should show that under the existence of \( \eta \), the following terms are satisfied:

\[
\left. \frac{\partial V}{\partial x} \right|_{x=x_d} = 0_n, \quad \left. \frac{\partial^2 V}{\partial x^2} \right|_{x=x_d} > 0 \quad \text{with} \quad x = [q^T, p^T]^T.
\tag{16}
\]

The only \( p \)-dependent term in \( V \) is \( \frac{1}{2} p^T M_d^{-1} p \), which clearly satisfies (16). Therefore, we verify conditions (16) with respect to \( q \) for the other terms of \( V \). By this means, we have

\[
\left. \frac{\partial (V_{dh} + \eta)}{\partial q} \right|_{q=q_d} = (\nabla V_{dh})^T |_{q=q_d} + (\nabla V(\nu))^T |_{q=q_d} G^i (\nu) G^i (\nu) M_d^{-1}(\nu) M(\nu) (\nu) d\nu = 0_n.
\tag{17}
\]

Recall that \( [q_0^T, 0_{n-m}^T]^T \) is an equilibrium point, and therefore, it is on the manifold \( G^i \nabla V = 0_{n-m} \). Hence, the second condition of (16) will be reformulated as (9), and this completes the proof of this part.
2) Similar to 1), by applying \( u = u_{IDA-PBC} - \xi \) with \( V_d = V_{db} \), the closed-loop equations are as follows:

\[
\begin{bmatrix}
\dot{q} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0_{n \times n} & M^{-1}M_d \\
-M_dM^{-1} & J_d - GK_\nu G^T
\end{bmatrix}
\begin{bmatrix}
\nabla_q H_d \\
\nabla_\theta H_d
\end{bmatrix} \\
- \begin{bmatrix}
0 \\
G^{-T}G\nabla V + G\xi
\end{bmatrix}
\]

(17)

with \( H_d = \frac{1}{2}T^T M_d^{-1} p + V_{db} \). Under the existence of \( \mu \), it is easy to verify that the time derivative of \( V = H_d + \mu \) as is (15). Besides, \( V \) is a suitable Lyapunov function since (16) is satisfied due to (12) and \( \xi(q_d) = 0_m \). Hence, \([q_d^T, \Omega_d^T]^T \) is a stable equilibrium point.

**Remark 1:** The first part of Theorem 1 may be interpreted from another point of view. Upon existence, the function \( \eta \) is indeed the non-homogeneous solution of (5). Hence, by considering \( V_d = V_{db} + \eta \), the closed-loop equations are as (2). The abovementioned explanations also apply to \( \mu \) [i.e., it also satisfies (5)] with the benefit of a slightly easier matching equation due to free term \( \xi \). Note that the general relation of energy shaping for mechanical systems is given in [13, eq. (25)], in which gyroscopic forces are replaced by dissipative forces that result in the simplification of kinetic energy PDE. One may easily verify that the results of Theorem 1 are matched with [13, eq. (25)], and thus, it is possible to apply them to simultaneous IDA-PBC.

**Discussion:** In Theorem 1, two sufficient conditions related to potential energy shaping were proposed. Considering the definition of \( \eta \) and \( \mu \), it is clear that these functions depend on the physical parameters of the system and to the matrix \( M_d \), which is a design parameter that should satisfy (4). Therefore, the existence of these functions is related to desired inertia matrix. Since the derivation of \( M_d \) is out of the scope of this article, generally, it is not possible to determine how we may find \( \eta \) or \( \mu \). However, in the sequel, we discuss the situations under which the existence of \( \eta \) or \( \mu \) is verified easily.

For simplicity, let us focus on the systems with underactuation degree one. To satisfy the existence of \( \eta \), it is required to consider the PDE (4) simultaneously. A suitable class of systems is those whose first term of (4) is zero. In this case, a general structure for \( M_d \) is derived based on the existence of \( \eta \) and then its exact value is derived from (4). In particular, the problem is straightforward if \( G^T \nabla V \) and \( M \) depend merely on one of the \( q \)'s, such as in inertia wheel pendulum, inverted pendulum, and VTOL aircraft, and also some systems with a feedback linearization [4]. Furthermore, the condition of existence of \( \mu \) is satisfied in a larger class of systems, including the previous aforementioned class. Moreover, it is not necessary to derive \( \xi \) and \( M_d \) at the same time. In addition, it seems that the second part of Theorem 1 is suitable to be integrated with the method of solving the PDE (4) proposed in [9]; see example IV-A. Note that generally, (11) results in a PDE, which may be simplified easily in the systems with particular structures, such as some underactuated serial robots; see example IV-B.

**Remark 2:** As in the abovementioned Discussion, the method given in Theorem 1 may be applied to a number of benchmark underactuated systems. For the systems that \( \xi \) (and thus, \( \mu \)) exists but may not be computable, it is possible to modify Theorem 1 to ensure stability or boundedness of error. Consider the control law of the first part of Theorem 1 (i.e., the IDA-PBC control law with \( V_d = V_{db} \)) and \( V = H_d + \mu \) with sufficiently large \( k \)'s as a Lyapunov function. Time derivative of \( V \) may be computed as

\[
V = -(\nabla_q H_d)^T G K_\nu G^T \nabla_q H_d + (\nabla_\theta H_d)^T G \xi.
\]

This equation state that \( \xi \) may be interpreted as an unknown matched disturbance and may be rejected or attenuated using the methods proposed in the literature, such as adding the term \( K_\nu \text{sign}(G^T \nabla_q H_d) \) or increasing the damping gain \( K_\nu \).

**IV. Examples**

### A. Cartpole

Consider the cart-pole system [4]

\[
\begin{bmatrix}
1 & b \cos(q_1) \\
b \cos(q_1) & m_3
\end{bmatrix}, \quad V = c \cos(q_1), \quad G = [0, 1]^T
\]

\[
c = g/l, \quad b = 1/l, \quad m_3 = (m + M)/ml^2, \quad q = [\theta, \dot{\theta}]^T.
\]

In literature, the designed IDA-PBC methods are based on a partial feedback linearization to transform the system into the so-called Spong’s normal form. Here, our goal is to stabilize the system without partial feedback linearization. For this purpose, we apply the method introduced in [9]. By this means, \( M_d^{-1} \) is in the following form (see [14] for more details):

\[
M_d^{-1} = \begin{bmatrix}
\frac{\alpha_2(q_1) + \beta^2}{a_1} & b_1 \\
b_1 & a_1
\end{bmatrix}
\]

in which \( a_1, b_1, \) and \( \lambda \) are constant and \( a_2(q_1) \) is determined in [14]. Thus, the term (11) is in the following form:

\[
\mu = \int [\beta_1, \beta_2] + [\xi(\beta_1, \beta_2)] dq
\]

\[
\beta_1 = -c \sin(q_1) \frac{[\alpha_2(q_1) + \beta^2]}{a_1 + bb_1 \cos(q_1)}
\]

\[
\beta_2 = -c \sin(q_1) \frac{b \cos(q_1) \alpha_2(q_1) + \beta^2}{a_1 + bb_1}
\]

\[
\beta_3 = b_1 + a_1 b \cos(q_1), \quad \beta_4 = bb_1 \cos(q_1) + a_1 m_3.
\]

Hence, it is sufficient to define \( \xi(q_1) \) such that \( \beta_1(q_1) + \xi(q_1) \beta_3(q_1) \) is a constant.

### B. Pendubot

The system is a 2R serial robot whose first link is actuated. Dynamic parameters of the system are given as [15]

\[
\begin{bmatrix}
c_1 + c_2 + 2c_3 \cos(q_1) & c_2 + c_3 \cos(q_1) \\
c_2 + c_3 \cos(q_2) & c_2
\end{bmatrix}
\]

\[
V = c_4 g \cos(q_1) + c_5 g \cos(q_1 + q_2)
\]

(18)

and \( G = [1, 0]^T \). Assume that \( M_d(q_2) \) is given (see, for example [9], [10], and [15]). The structure of (11) is in the following form:

\[
\mu = \int \sin(q_1 + q_2) [a_1(q_2), a_2(q_2)] + \xi [b_1(q_2), b_2(q_2)] dq
\]

in which

\[
\begin{bmatrix}
b_1 & b_2 \\
a_1 & a_2
\end{bmatrix} = M_d^{-1} M
\]

\mu exists if and only if,

\[
\frac{\partial}{\partial q_2} \left[ \sin(q_1 + q_2) a_1 + \xi b_1 \right] = \frac{\partial}{\partial q_1} \left[ \sin(q_1 + q_2) a_2 + \xi b_2 \right].
\]

(19)

It is suitable to define

\[
\xi := a(q_2) \sin(q_1 + q_2) + \beta(q_2) \cos(q_1 + q_2)
\]

(20)
in which \( \alpha(q_2) \) and \( \beta(q_2) \) should be designed. Substitute (20) in (19) and simplify

\[
\begin{align*}
&b_1 \frac{d\alpha}{dq_2} + \frac{da_1}{dq_2} + \alpha \frac{db_1}{dq_2} - b_1\beta + b_2\beta = 0 \\
&b_2 \frac{d\beta}{dq_2} + \frac{db_1}{dq_2} + a_1 + b_1\alpha - a_2 - b_2\alpha = 0.
\end{align*}
\]

These may be merged into a second-order ordinary differential equation (ODE). This idea may be applied simply to Acrobat [16] since the key point is the structure of potential energy, which is a summation of some sinusoidal functions with the property that the absolute value of the second derivative of such function is equal to the function itself.

Before concluding this article, it is worth mentioning that Remark 2 gives an insight to the rejection of a particular unmatched disturbance, which is currently an open problem [17, 18]. Furthermore, we saw in the simulation that the first part of Theorem 1 is applicable even though \( \eta \) does not exist. Detail analysis of these points are out of the scope of this note, and are currently under investigation.

V. CONCLUSION

In this article, we concentrated on the reformulation of potential energy PDE of IDA-PBC approach. It was shown that upon satisfaction of one of the proposed conditions, it is enough to derive the homogeneous solution of this PDE. Furthermore, the physical interpretation of the conditions and the class of systems that satisfy them together with two examples was proposed. Generalization of the method to simplify the kinetic energy PDE is under current investigation.

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