Hybrids as a Signature of Quark-Gluon Plasma

Afsar Abbas\textsuperscript{1} and Lina Paria\textsuperscript{2}

Institute of Physics, Sachivalaya Marg,
Bhubaneswar-751005, India.

March 26, 2022

Abstract

We show that the dynamics of the Quark-Gluon Plasma is such that during hadronization the creation of hybrids will predominate over the creation of mesons, giving a novel signature of the existence of QGP. At $T = 0$ the ($q\bar{q}g$) hybrids are known to decay strongly into a pair of mesons. We find that at temperatures relevant to the QGP, this channel is forbidden. This would lead to significant modifications of the photonic signals of the QGP.
In addition to the conventional mesons and baryons, on very general arguments based on QCD one expects non-conventional systems like glueballs (made up of two or more gluons) and hybrids (like $q\bar{q}g$). There have been some claims that these glueballs and hybrids may already have been seen in the laboratory [1-4]. Here we concentrate upon the $(q\bar{q}g)$ hybrid [1,2]. These are expected to decay strongly into a pair of mesons [1,2]. More work has been done recently [3,4] to understand the structure of these objects and their possible experimental identification in low energy hadron spectroscopy. The hybrids remain an exciting and open problem.

Is there any other place where hybrid may manifest themselves more explicitly? Perhaps finite temperatures may hold the clue? With this in mind we look at the Quark-Gluon Plasma (QGP) as a possible laboratory for studying the hybrids. The field of QGP is currently a very active area of research, both experimentally as well as theoretically (see ref. 5 & 6 for recent reviews). It is extremely important to be able to identify QGP if and when it is formed. Several signature of QGP have been suggested [5,6] and looked for, but none of the suggested signals have been unambiguous to demonstrate in a clearcut manner whether QGP has been formed in the laboratory or not. Hence we ask the question- could a hybrid be that signal of QGP? We shall show that indeed it may very well be so!

In this paper we consider a system of quarks, antiquarks and gluons placed in a heat bath with which it can exchange the energy and particle number. We study the thermodynamics of the system confined in a MIT bag [7]. All the thermodynamic quantities are obtained by using discrete single particle states of the quarks and gluons which are obtained by solving the equation of motion with linearised boundary conditions [8].

The number of quarks and gluons of the system at finite temperature $T$ is given by
\[ N_q = d_q \sum_i g_i f_i \]  \hspace{1cm} (1)

\[ N_g = d_g \sum_i g_i b_i \]  \hspace{1cm} (2)

where \( f_i = 1/(e^{(\epsilon_i - \mu)/T} + 1) \), the F - D distribution function for quarks and \( b_i = 1/(e^{\epsilon_i/T} - 1) \), the B - E distribution function for gluons, \( d_q = 2.3/2 \) and \( d_g = 8 \). Here in the case of the quarks, the factor of 2 is for the flavours, 3 for the color and the division by 2 is for the particular spin of \( q \) and \( \bar{q} \). For the gluon case, 8 comes from the color configuration and \( g_i \) is the spin degeneracy factor for the \( i^{th} \) single particle state with energy \( \epsilon_i \) and \( \mu \) being the chemical potential for quark. All the quantities can be calculated for the antiquark by replacing \( \mu \) by \(-\mu\). As our aim is to study the thermodynamic properties of the hybrid \((q\bar{q}g)\), we ensure \( N_q = 1 \) and \( N_{\bar{q}} = 1 \). The gluon arises due to the thermal excitations as the temperature \( T \) increases.

The energy and the free energy of the quark and gluon is given by

\[ E_q = d \sum_i g_i \epsilon_i f_i \]  \hspace{1cm} (3)

\[ E_g = 8 \sum_i g_i \epsilon_i b_i \]  \hspace{1cm} (4)

\[ F_q = -d T \sum_i g_i \ln \left( 1 + e^{-(\epsilon_i - \mu)/T} \right) \]  \hspace{1cm} (5)

\[ F_g = 8T \sum_i g_i \ln \left( 1 - e^{-\epsilon_i/T} \right) \]  \hspace{1cm} (6)

where \( d \) is the quark degeneracy factor. Now adding the well known zero point energy term \((BV + C/R)\) [7] we get the total free energy of the system as
\[ F = F_q + F_{\bar{q}} + F_g + 2\mu + BV + C/R \]  \hspace{1cm} (7)

The pressure generated by the participant gas (q, \( \bar{q} \), g) is

\[ P = -\left( \frac{\partial F}{\partial V} \right)_T \]  \hspace{1cm} (8)

This is balanced by the bag pressure constant B leading to the stability condition of the system. From this equilibrium condition one gets the total energy \( E = 4BV \).

We calculate the free energy of the system of a quark, an antiquark and gluons from equation(7) which gives a minimum at a particular value of R arising from the equilibrium condition of the system. At low temperatures the gluonic contribution is small but at higher temperatures which are relevant to QGP, a single gluon arises due to the thermal excitations giving rise to a hybrid (qqg).

Note that our picture of the hybrid at finite temperatures is akin to what Close and Page [3] are considering i.e. it’s a gluonic excitation of meson. So what may have been meson at \( T = 0 \) converts into a hybrid at higher temperatures due to the gluonic excitation arising therein. The flux tube model used in ref.[3] had earlier been developed and used in the case of the glueball [9] and the hybrid [10].

Next we suppress the gluonic part to get a pure mesonic system (qq) whose energy, free energy etc. changes with temperature giving the minimum of \( F \) at a particular value of \( R \). The value of \( C \) taken for pure mesonic system is \( \sim 0.04 \) whereas for (qqg) system, it is \( \sim 0.4 \) [8].

We take \( B^{1/4} = 250 \) MeV, and then the temperature of the system of q, \( \bar{q} \), and g is increased. We calculate the free energy of the system which has a minimum at a certain value of the bag radius \( R \) and notice the variation of it with temperature. At a particular \( T \) at which a single gluon attaches to the quark, antiquark system leading
to a hybrid, we note the corresponding equilibrium radius of the hybrid at which $F$ is minimum. This gives the corresponding energy of the hybrid. Next we calculate the same thermodynamic quantities at the same $T$ for the pure mesonic system. The free energy for the meson and the hybrid cases as a function of radius $R$ are displayed in Fig.1. The equilibrium radius and the energy are given in Table 1.

Looking at Fig.1. we notice that if a mesonic bubble is created at $T = 183$ MeV, then depending upon whether the radius was smaller or larger than 0.658 fm, the bubble will grow or shrink to attain the equilibrium radius to minimise the free energy of the system. But now the temperature is enough to excite a thermal gluon. As we can view our hybrid $(q\bar{q}g)$ to be the gluonic excitation of the meson [3], the gluon will sink in to make the meson into a hybrid of radius 0.807 fm thereby reducing the free energy of the system. If a hybrid of radius greater (or smaller) than 0.807 fm was created, then it would shrink (or expand) to attain the equilibrium radius. Thus one notes that the creation of hybrids would be favoured over the creation of mesons at $T = 183$ MeV for $B^{1/4} = 250$ MeV. Also note that $M_H < 2m_M$.

Similarly for (i) $B^{1/4} = 200$ MeV, we get $m_H = 3.637$ GeV with the radius $R = 1.014$ fm at the temperature $T = 146.5$ MeV, whereas for the pure mesonic case, $m_M = 1.944$ GeV with the radius $R = 0.823$ fm. (ii) for $B^{1/4} = 300$ MeV, we get $m_H = 5.313$ GeV with the radius $R = 0.67$ fm at the temperature $T = 219.5$ MeV, whereas for the pure mesonic case, $m_M = 2.907$ GeV with the radius $R = 0.548$ fm.

It is well known that in the finite temperature case of QGP, one may use the bag constant $B$ as parameter [6,p269]. Hence there may be uncertainty in the quantitative prediction of the model as evident above. But there need be no ambiguity regarding the qualitative prediction. Qualitatively we notice that for any reasonable value of $B$ at finite temperatures, the creation of the hybrids will predominate over the creation of
the mesons and also that $m_H < 2m_M$ holds.

How come at $T = 0$ the hybrid is above two meson production threshold [1,2] and hence decays strongly into these channels, but at finite temperature $m_H < 2m_M$? The answer lies in the temperature dependence of the hybrid and the meson masses in our model calculation. The situation is akin to the $\sigma$ and $\pi$ cases as studied by Hatsuda & Kunihiro [11]. They studied the QGP to hadron phase transition as an extension of the NJL model. They showed that the mass of the $\sigma$ meson ($m_\sigma$) decreases and the mass of the pions ($m_\pi$) increases with temperature. There is a temperature $T_\sigma \sim 190$ MeV at which $m_\sigma < 2m_\pi$. Thence the decay width of $\sigma \to 2\pi$ goes to zero, indicating no decay mode of $\sigma$ into $2\pi$ at that temperature. Hence at this temperature $\sigma$ decays electromagnetically only. Similarly in our calculation (table.1.) at some temperature $T \sim 183$ MeV, we get $m_H < 2m_M$ which implies that the decay channel to two mesons is forbidden. So the hybrids can decay electromagnetically producing photons.

So what does our calculation have to say about the hadronisation of QGP? In the standard picture of QGP consisting of equal number of quarks and antiquarks, as the system hadronises one would expect it to go to a system of mesons [5,6]. However our results here suggest that the dynamics of the system is such that the system would prefer to create hybrids rather than mesons. In an ideal situation, only hybrids would be created. In any condition, the bulk of the matter should go to the hybrids. As such we suggest that the hadronisation of the QGP through the hybrids should be taken as a signal of the existence of QGP.

As $m_H < 2m_M$, the hybrid can not decay into a pair of mesons (as they do at $T = 0$ [1,2]). Hence they decay electromagnetically as $H \to \text{meson} + \text{photon}$. The basic process is $q(\bar{q}) + g \to q(\bar{q}) + \gamma$. On dimensional grounds, we calculate the decay width to be $\Gamma_{\text{decay}} \sim \alpha\alpha_s/m^2V$. Where $\alpha = 1/137$ and $\alpha_s = 0.2$. $V$ is the volume.
in which the decay process takes place (which we take as the volume of the bound state) and \( m \) is the mass of the hybrid. With these values \( \Gamma_{\text{decay}} \sim 0.4 \times 10^{18} \text{sec}^{-1} \) (If we take at the \( B^{1/4} = 200 \text{ MeV} \) case, we get \( \Gamma_{\text{decay}} \sim 0.3 \times 10^{18} \text{sec}^{-1} \)). Hence on the whole we expect the hybrid life time to be \( \sim 10^{-18} \text{ sec} \). This time is much larger than the hadronisation time of the QGP. This will delay creation of mesons substantially. In the mixed phase of QGP, the hybrids are likely to be predominant.

As we showed above, the hybrid state decay electromagnetically as \( H \rightarrow \text{meson} + \text{photon} \) (e.g. \( 0^{--} \rightarrow b_1 + \gamma \) & \( 0^{++} \rightarrow h_1 + \gamma \)). This would effect the photon signal especially in the mixed mode which has been a topic of much current work and has been reviewed recently [6]. This shall require redoing the calculations for the photon signals [6]. In fact the time delay in the photon emission (due to finite life time of the hybrid) may perhaps be accessible through a proper study within the field of photon interferometry [6].

In summary, we show that in QGP at finite temperatures the dynamics of the system prefers to generate hybrids during hadronisation as opposed to mesons. We suggest that this then constitutes a novel signal of QGP. One also finds that the strong decay to mesons is forbidden and hence these hybrids will decay electromagnetically, significantly modifying the photonic signals of QGP.

We would like to thank Dr. M. G. Mustafa for giving us the computer code used herein and helping us in computation in the initial stages.
References

[1] M. Chanowitz and S. Sharpe, Nucl. Phys. B. 222 (1983) 221

[2] T. Barnes, F. E. Close and F. de Viron, Nucl. Phys. B. 224 (1983) 241

[3] F. E. Close and P. R. Page, Nucl. Phys. B. 443 (1995) 233

[4] F. E. Close and P. R. Page, Phys. Rev. D. 52 (1995) 1706

[5] C. P. Singh, Phys. Rep. 236 (1993) 147

[6] Jan-e Alam, S. Raha and B. Sinha, Phys. Rep. 273 (1996) 243

[7] A. Chodes, R. L. Jaffe, K. Johnson, C. B. Thorn
    and V. F. Weisskopf, Phys. Rev. D. 9 (1974) 3471

[8] J. Dey, M. Dey and P. Ghosh, Phys. Lett. B 221 (1989) 161 ;
    M. G. Mustafa and A. Ansari, Z. Phys. C. 57 (1993) 51

[9] R. Kokoski and N. Isgur, Phys. Rev. D. 35 (1987) 907

[10] N. Isgur, R. Kokoski and J. Paton, Phys. Rev. Lett. 54 (1985) 869

[11] T. Hatsuda and T. Kunihiro, Phys. Rep. 247 (1994) 221
CAPTIONS

Table 1
The equilibrium radius\((R)\) and the energy \((E)\) of the hybrid and pure mesonic system is given for bag constant \(B^{1/4} = 250\text{ MeV}\).

Figure 1.
The variation of the free energy of the hybrid and the pure mesonic systems with radius \(R\) are displayed at a particular temperature \(T = 183\text{ MeV}\) and \(B^{1/4} = 250\text{ MeV}\).
Table 1

| System | T(MeV) | R(fm) | E(GeV) |
|--------|--------|-------|--------|
| $(q\bar{q}g)$ | 183 | 0.807 | 4.476 |
| $(q\bar{q})$  | 183 | 0.658 | 2.426 |