Study on the Calculation Method for Pumping Process in as Vacuum System

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Abstract. The calculation methods for vacuum system pumping were usually based on some simple theoretical models, the corresponding results had significant deviations from actual situations. In this study, medium and low vacuum systems (including vacuum chambers, pipes and pumps) were taken as research objects. With a measured vacuum system, and relationship between pump’s suction flowrate and inlet pressure, a new calculation method for vacuum pumping time was proposed, in which laminar or turbulent model was selected according to the pipeline’s flow state. New and traditional laminar method were used to calculate the pumping process of the measured system, which found that in the middle and high pressure stage, the pipeline flow was in turbulent state and the resistance was non-negligible. If the influence of turbulence was ignored, the calculated pressure drop would be faster than actual situation. The calculation result was verified by actual measurement result, indicating that new method is practical for vacuum pumping time calculation.

Keywords. Vacuum pump, vacuum system, pumping process, calculation method.

1. Introduction
The vacuum system is mainly composed of vacuum pumps, vacuum chambers, pipelines, valves, and program controllers. It is widely used in chemical, electronic, medical, and aerospace technologies [1-5]. The characteristic relationship between the suction rate of the vacuum pump \(S_p\) and the suction pressure \(P_o\) is the main factor that determines the performance of the vacuum system [6, 7]. In recent years, scholars have carried out many research on vacuum-system pumping. Zhou [8] used mathematical modeling to check the performance of the compressor in the theoretical method, and verified the effect of the new testing-process through simulation tests; Xu [9] developed a calculation program for the constant-speed pumping, and obtained the required pumping time to the specific vacuum degree in the vacuum circulation-degassing process; Ma [10] used a CFD (Computational Fluid Dynamics) method to simulate the high-altitude environment simulation system with specific Mach number; Sui [11] studied the pumping process of a space-environment-simulator with laminar flow model, and obtained the relationship between the pressure in the simulator and the pumping time.

In summary, there were few calculation methods for system vacuuming, and simple theoretical models were generally used, thus the corresponding results deviated greatly from the actual situation. Therefore, in this paper, a quick and effective calculation method was proposed. This method was based on the characteristic relationship between the suction rate of the vacuum pump \(S_p\) and the suction pressure \(P_o\), and laminar or turbulent model would be selected according to the flow state of
the pipeline, which provided a theoretical basis for the design and engineering application of the system vacuum.

2. Calculation Method

2.1. Theoretical Model of Vacuum System Pumping
As shown in figure 1, the research object was a low-medium vacuum system, which could be simplified to be composed of a vacuum chamber, a main pumping pipeline and a vacuum pump [12]. The theoretical calculation was based on the following assumptions: (1) The fluid media was regarded as ideal gas, and the pumping process was approximately an isothermal process; (2) The gas was a continuous Newtonian fluid.

![Figure 1. Simplified vacuum system model.](image)

The gas passing through any cross section of the pipeline per unit time, satisfies the following continuous equation [13, 14]:

\[ Q = Q_o - Q_i = -V_0 \frac{dP}{dt} \]  

(1)

\( Q_o \) satisfies:

\[ Q_o = S_P P_o = SP = C(P - P_o) \]  

(2)

Equation (2) can be rewritten as:

\[ S = S_p C / (S_p + C) \]  

(3)

The conductance of the pipeline \( C \) is defined by the equation (2), as the gas flow divided by the change value of pressure of pipeline ports, that is, the volumetric flow rate through the pipeline per unit pressure. So the conductance of the pipeline can be used to indicate the passing capacity of air flow within the pipeline, and the value of conductance can be regarded as the resistance of the pipeline to air flow. So the physical meaning of equation (3) is that after the volumetric flow of pump at pipeline outlet \( S_p \) affected by the conductance of the pipeline \( C \), the actual volumetric flow at the pipeline inlet can be calculated.

\[ Q_i = S_i P_a = \frac{S_m P_a}{P_a} \]  

(4)

From equations (1), (2), (3) and (4), we can get the pumping equation of the vacuum system:

\[ V_0 \frac{dP}{dt} = \frac{S_m P_a}{P_a} - \frac{S_p C}{S_p + C} \]  

(5)

2.2. Conductance Equation of Pipeline
The flow state in the pipeline needs to be judged according to the Reynolds number. When less than 2000, it is laminar, otherwise it is turbulent. The expression of Reynolds number is [14]:

\[ Re = \frac{V_0 D}{\nu} \]
The ideal gas in isothermal state satisfies [14]:

\[ P_a = \rho \frac{\bar{p}}{\rho} \]  

(7)

the average gas pressure \( \bar{p} \) in the pipeline was defined as:

\[ \bar{p} = \frac{P + P_a}{2} \]  

(8)

From equations (6), (7), and the average gas flow speed \( u \) in the pipeline was calculated by the volumetric flow of the vacuum pump \( S_p \), so there was:

\[ Re = \frac{D u \bar{p}}{\mu} \]  

(6)

For a uniform circular cross-section pipe, the pressure difference between the two ports can be expressed by Fanning equation [14], which is:

\[ P - P_o = \lambda \frac{L \bar{p} u^2}{D} \]  

(10)

The resistance coefficient \( \lambda \) relates to flow state:

\[ \lambda = \begin{cases} 
64 / Re, & \text{Laminar} \\
0.3164 / Re^{0.25}, & \text{Turbulence} 
\end{cases} \]  

(11)

Thus, from equations (7), (9), (10) and (11), we could get the expression of gas flow in the pipeline \( Q_o \):

\[ Q_o = \bar{p} u A = \begin{cases} 
\frac{a \bar{p} (P - P_o)}{4}, & Re < 2000 \\
\frac{b \bar{p}^{16} (P - P_o)^7}{4}, & Re \geq 2000 
\end{cases} \]  

(12)

Where \( a = \frac{\pi \cdot D^4}{128 \cdot \mu \cdot L} \) and \( b = 2.2526 \cdot \frac{19}{3} \cdot \frac{3}{7} \cdot \frac{1}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{1}{7} \cdot \rho_a^{3/7} \).

From equations (2) and (12), we could get the expression of the conductance of pipeline \( C \):

\[ C = \begin{cases} 
a \bar{p}, & Re < 2000 \\
\frac{b \bar{p}^{16} (P - P_o)^3}{4}, & Re \geq 2000 
\end{cases} \]  

(13)

3. Application of Calculation Methods

In order to solve equation (5), it could be considered that when the time step \( \Delta t \) was small enough, the volumetric flow of the vacuum pump \( S_p \) was approximately unchanged, and there was a linear relationship between \( \Delta t \) and \( \Delta P \), so equation (5) could be rewritten as:

\[ \Delta t_i = \frac{\Delta P (S_{pi} + C_i)}{S_{pi} C_i P_i} \]  

(14)

where \( \Delta t_i = t_i - t_{i-1}, \Delta P = P_i - P_{i-1} \) and \( \bar{p}_i = (P_i + P_{i-1}) / 2 \).
The specific calculation method was: dividing the initial pressure $P_0$ to the end pressure $P_e$ into $n$ segments. With the initial value $P_0 = P_{o1}$ and $\Delta P = P_1 - P_0$, then calculating $\bar{P}_1$. According to $P_{o1}$ and the $S_pP_o$ characteristic curve of the vacuum pump, to calculate the Reynolds number $Re_1$ and the conductance of pipeline $C_1$. Then the corresponding time step $\Delta t_1 = t_1 - t_0$ was calculated by equation (13), $t_0 = 0$. In next iterative calculation, making $P_{o2} = (P_1 + P_{o1})/2$, calculating time step like above, and finally got the total pumping time $\sum_{i=1}^{n} \Delta t_i$.

The tested and measured system of Reference [11] was selected as the calculation case: volume of the vacuum chamber $V_0$ was 740 m$^3$, the total length of the main pipeline $L$ was 9 m, the diameter $D$ was 0.32 m. The gas media in the vacuum chamber was 20 ℃ air, $\mu$ was $1.820 \times 10^5$ Pa·s, $\rho_a$ was 1.205 kg/m$^3$, $S_m$ was 0 kg/s, calculation initial pressure $P_0 = P_{o1}$ was 101325 Pa, calculation end pressure $P_e$ was 10 Pa. At one port of the main pipeline, four same vacuum pumps were connected in parallel, and the equivalent pumping flow was the superposition of four pumps [15]. The actual $S_pP_o$ characteristic curve of single vacuum pump was shown in figure 2.

![Figure 2. The actual characteristic $S_pP_o$ curve of single vacuum pump.](image)

For ease of calculation, the actual $S_pP_o$ characteristic curve in figure 2 was fitted to obtain the following equations:

$$S_p = \left\{ \begin{array}{ll}
-170348.07 \cdot e^{(P_o/1.36)} - 1961.72 \cdot e^{(P_o/212.56)} + 2127.71, & P_o < 384 \\
-3793.80 \cdot e^{(P_o/289.70)} + 2810.98, & 384 \leq P_o < 6218, \\
3434.75 \cdot e^{(P_o/48136.69)} - 218.53, & P_o \geq 6218
\end{array} \right. \quad (15)$$

The entire calculation process was shown in figure 3, programming with Matlab to do numerical calculation, the pressure step $\Delta P_i$ was 1.0 Pa.
4. Analysis and Verification of Calculation Results

Figure 4 shows the comparison between the theoretical calculation and Reference [11]'s measured results. The two curves are the results obtained by the conventional laminar method and the turbulent method considered in this paper. As shown in figure 4, turbulence has a significant effect on the calculation results of the pumping time. In the medium and high pressure stage, the $P-t$ curve calculated by laminar method drops too fast, and the $P-t$ curve calculated by turbulent method drops relatively slowly; in the low pressure stage, the trend of the $P-t$ turbulent curve is basically close to the laminar curve.

In figure 4, the trend of turbulent $P-t$ curve is consistent with the measured $P-t$ curve, and the pressure dropping speed is faster till $P = 500\text{ Pa}$, and after the inflection point $P = 500\text{ Pa}$, the pressure dropping speed gradually slows down. The end time of the two curves above reaching $P = 10\text{ Pa}$ are $t = 3.03(10^9)\text{ s}$ and $t = 3.19(11^4)\text{ s}$ respectively, the relative error is $-4.97\%$. Judging from the calculation results, the method considering turbulence is obviously superior to the traditional method only considering laminar flow. When the vacuum degree gradually increases, the phenomenon of gas leakage and deflation in the vacuum system also increases accordingly. The measured pressure drop is slower when it is close to $P = 10\text{ Pa}$, which is the main reason for the deviation from the calculation results. In addition, from the turbulent curve in figure 4, it can be seen that under the same pumping flow speed of the vacuum pump, even if the high pressure stage is affected by the turbulent flow, it has a faster pressure drop speed than the low pressure stage. The pumping time in low pressure stage is more sensitive to variation of pumping speed, so attention should be paid to the pumping speed of the vacuum pump in the low pressure stage to improve vacuum pumping efficiency.
Figure 5 is the Re-t curve calculated according to equation (9) during the entire pumping process. As shown in figure 5, Reynolds number of pipeline is high in the medium and high pressure stage, and the flow is in a turbulent state. Turbulence increases the resistance of the pipeline, which slows down the drop speed of P-t curve. When $t \geq 1.79$ (6450s), that is the pressure vacuum chamber $P$ is less than 384 Pa, the gas density in the pipeline decreases accordingly, and the gas flow in the pipeline changes from turbulent to laminar ($Re < 2000$). In other words, the airflow in the pipeline is in turbulent state for at least half of the entire pumping process, so the influence of turbulent flow should not be ignored in the calculation.

![Figure 4. Comparison between theoretical calculation and measurement.](image)

![Figure 5. Curve about Reynolds number in the pipeline Re and time t.](image)

5. Conclusions
This paper proposed a calculation method for vacuum system degassing, and the following conclusions were obtained by analysis and verification of calculation results:

(1) The influence of turbulence on the calculation results of pumping time is significant and notable, mainly in the middle and high pressure stage of pumping process. At this time, Reynolds number is large, the flow is in turbulent state and the resistance of the pipeline increases correspondingly, the P-t curve drop too fast and there is a big deviation from the actual situation if calculating by laminar method.

(2) During the pumping process, it takes longer time in the low pressure stage. The pumping time is more sensitive to the variation of the pumping speed in the low pressure stage. Therefore, attention should be paid to the pumping speed of the vacuum pump in the low pressure stage to improve vacuum pumping efficiency.

(3) The trend of P-t curve, which is calculated by the turbulent or laminar flow model selected by the airflow state, is close to the measured curve. The error is within the allowable range of actual project, indicating that this is a practical and feasible system method for vacuum pumping time calculation.

Appendices
- $A$: Area of circular pipe of cross-section, m$^2$.
- $C$: Conductance of main pipe, m$^3$/s.
- $C_i$: Conductance of pipeline of certain period, m$^3$/s.
- $D$: Diameter of pipeline, m.
- $L$: Total length of pipeline, m.
- $P$: Pressure in vacuum chamber, Pa.
- $P_0$: Initial pressure of calculation.
\( P_a \)  Local atmospheric pressure, Pa.
\( P_o \)  Suction pressure of vacuum pump, Pa.
\( P_{oi} \)  Suction pressure of vacuum pump inlet of certain period, Pa.
\( P_e \)  End pressure of calculation.
\( Q \)  Gas flow into vacuum system, \( \text{Pa} \cdot \text{m}^3/\text{s} \).
\( Q_l \)  Leakage flow of vacuum chamber, \( \text{Pa} \cdot \text{m}^3/\text{s} \).
\( Q_o \)  Pumping flow of vacuum pump, \( \text{Pa} \cdot \text{m}^3/\text{s} \).
\( S_l \)  Leakage volumetric flow of vacuum chamber, \( \text{m}^3/\text{s} \).
\( S_m \)  Leakage mass flow of vacuum chamber, kg/s.
\( S_p \)  Volumetric flow of vacuum pump inlet, \( \text{m}^3/\text{s} \).
\( S_{pi} \)  Volumetric flow of vacuum pump inlet of certain period, \( \text{m}^3/\text{s} \).
\( V_0 \)  Volume of vacuum chamber, \( \text{m}^3 \).
\( \bar{\rho} \)  Average gas density in pipeline, kg/m\(^3\).
\( \rho_a \)  Corresponding density of local atmospheric pressure, kg/m\(^3\).
\( \Delta P \)  Pressure step of certain period, Pa.
\( \Delta t \)  Time step of certain period, s.

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