RETAINING BLACK HOLES WITH VERY LARGE RECOIL VELOCITIES

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ABSTRACT

Recent numerical simulations of binary black hole mergers show the possibility of producing very large recoil velocities (>3000 km s⁻¹). Kicks of this magnitude should be sufficient to eject the final black hole from virtually any galactic potential. This result has been seen as a potential contradiction to observations of supermassive black holes residing in the centers of most galaxies in the local universe. Using an extremely simplified merger tree model, we show that, even in the limit of very large ejection probability, after a small number of merger generations there should still be an appreciable fraction (>50%) of galaxies with supermassive black holes today. We go on to argue that the inclusion of more realistic physics ingredients in the merger model should systematically increase this retention fraction, helping to resolve a potential conflict between theory and observation. Finally, we develop a more realistic Monte Carlo model to confirm the qualitative arguments and to estimate occupation fractions as a function of the central galactic velocity dispersion.

Subject headings: black hole physics — galaxies: nuclei

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1. INTRODUCTION

In the standard model of hierarchical structure formation, the most overdense regions of the universe collapse at early times to form small, gravitationally bound clumps of dark matter and baryons. These protogalaxies subsequently merge with each other and form larger and larger objects, up to the galaxies and clusters we see today (Press & Schechter 1974; Efostathiou & Rees 1988; Haehnelt & Rees 1993; Cole et al. 2000). Observations of the local universe suggest that most galaxies at redshift z = 0 are also hosts to supermassive black holes (SMBHs) near their centers (Richstone et al. 1998). Observations of distant quasars in the Sloan Digital Sky Survey suggest that these SMBHs are common up to redshifts of at least z ~ 6 (Fan et al. 2001). One natural explanation is that the SMBHs originally formed in the early universe and then simply followed their host galaxies throughout multiple generations of hierarchical mergers: whenever a pair of galaxies merged, their central black holes would sink together through dynamical friction and eventually merge, resulting in a single, more massive galaxy with a single, more massive BH at its center (Kauffmann & Haehnelt 2000; Cattaneo 2001; Menou et al. 2001).

In the absence of special symmetries, the final black hole will receive a linear momentum recoil from the gravitational radiation emitted during the binary merger process (Bekenstein 1973; Fitchett 1983). Recent numerical relativity simulations suggest that this recoil cannot exceed 175 km s⁻¹ for nonspinning black holes (Herrmann et al. 2007a; Baker et al. 2006; González et al. 2007b) but may be as large as ~3600 km s⁻¹ for maximally spinning equal-mass black holes with spins antialigned in the orbital plane (González et al. 2007a; Campanelli et al. 2007). In this latter case, the resulting BH would almost certainly be ejected from the gravitational well of its host galaxy (Merritt et al. 2004; Libeskind et al. 2006).

However, since observations of the local universe suggest that SMBHs are quite common, there appears to be something preventing these ejections. One possibility is the evolution of the spins through spin-orbit resonance toward an orientation favoring a smaller recoil (Schnittman 2004). More recently, Bogdanović et al. (2007) proposed a mechanism in which a circumbinary gas accretion disk would align the spins of the two black holes with the orbital axis of the disk prior to merger, thereby limiting the maximum recoil to 200 km s⁻¹ (Baker et al. 2007). Initial surveys of red- and blueshifted quasar lines support this theory in the case of gas-rich mergers (Bonning et al. 2007).

In this Letter, we take a rather different approach to the problem and question the very premise of the apparent conflict: Does a large recoil velocity necessarily imply a small occupation fraction? Furthermore, does a small occupation fraction at high redshift imply that only a small number of SMBHs will survive to the present day? If, for example, 95% of all BH mergers result in an ejection from the host galaxy, does this really mean that at least 95% of galaxies today should be devoid of SMBHs? Using a binary tree merger model for the hierarchical growth of BHs and their host galaxies, we show that the answer to these questions is a resounding no!

The formal mathematical argument is given in § 2, but a simple qualitative reasoning is that in every generation of the merger tree, the number of galaxies decreases, and so the fraction with BHs can easily increase (Menou et al. 2001). If every BH host galaxy mergers with an empty galaxy, then the fraction of galaxies with SMBHs can double from 0.5 to 1. Including the losses due to ejected BHs, we find a steady state solution with occupation fraction f = 1/(1 + pₑ), where pₑ is the probability that a merging BH is ejected due to gravitational recoil. In § 3, we discuss a number of potential physics ingredients that could be added to the simple model, arguing that each one would systematically increase the fraction of observed SMBHs today. In § 4 we attempt to confirm some of these arguments with a more astrophysically realistic Monte Carlo merger model, and in § 5 we present our conclusions.

2. SIMPLE MERGER TREE MODEL

We begin with the simplest possible model for a galactic merger tree: a binary tree where in each generation every galaxy merges with one “spouse” and produces a single “child” (see Fig. 1 for a schematic). Each parent galaxy may or may
mergers, beginning with \( f_1 = 1 - \rho_{ij} \). In both cases, the system approaches the equilibrium fraction \( f_e \) of equation (2) after only a few generations.

For a more quantitative estimate of the “convergence time” it takes to reach equilibrium, we see from equation (1) that

\[
\begin{align*}
  f_1 &= 1 - \rho_{ij}, \\
  f_2 &= 1 - \rho_{ij} + \rho_{ij}^2 - \rho_{ij}^3, \\
  f_3 &= 1 - \rho_{ij} + \rho_{ij}^2 - \rho_{ij}^3 + \rho_{ij}^4 - \rho_{ij}^5 + \rho_{ij}^6 - \rho_{ij}^7, \\
  &\vdots \\
  f_i &= \sum_{j=0}^{2^i-1} (-1)^j \rho_{ij}^i.
\end{align*}
\]

This series not only converges to equation (2), as expected, but does so at quite a rapid pace. Even for extremely large ejection rates of \( \rho_{ij} = 0.95 \) (i.e., \( f_i = 0.05 \)), \( f_i \) is within 5% of its asymptotic value of \( f_e = 0.513 \) after just five generations. Because of this rapid convergence, we find that the final results are essentially independent of the initial occupation fraction \( f_0 \).

3. ADDITIONAL PHYSICS COMPONENTS

The model as described above is based on two major simplifying assumptions: a perfect binary merger tree and a constant ejection probability. Of course, in reality both of these assumptions are in all likelihood invalid. Here we list a number of possible modifications based on more realistic astrophysics, and for each one we argue that their inclusion will only increase the fraction of SMBHs observed today. A few of these modifications are straightforward enough to implement with a simple Monte Carlo code (§ 4), which lends support to our qualitative arguments.

The binary merger tree assumes that for a given generation, every galaxy has exactly the same mass and merges with exactly one other galaxy, halving the total number of galaxies in each subsequent generation. In practice, in many hierarchical merger simulations, one finds that a single trunk of the merger tree dominates, with a large number of small branches joining in at different redshifts (Volonteri et al. 2003; Malbon et al. 2006). This means that the mass ratio for a typical (proto)galaxy merger can be significantly different than unity. Assuming that the SMBH masses scale with their host masses, this suggests that the BH mass ratio will not be unity (Sesana et al. 2007). Furthermore, as the galaxies evolve along the hierarchical merger tree, their masses will increase, and thus so will their escape velocities. Of course, the BHs are also growing in mass through mergers and accretion during this time, but the kick velocity is a function only of the mass ratio and the dimensionless spin parameters. Thus, if the masses of the BHs and their host bulges all double, the recoil will be the same, but the escape velocity will be larger; thus, \( \rho_{ij} \) will be smaller. In this scenario, even if most of the BHs are ejected at large redshifts, after a few generations \( f_i \) will grow rapidly due to a
large number of “single-parent” mergers, ultimately converging to a larger population fraction based on the smaller value of \( p_0 \) at late times (Libeskind et al. 2006).

In addition to these modifications to the merger tree physics and escape velocities, there are also a number of processes that more directly affect the actual recoil velocity. Since the largest kicks are found in systems with the BH spins in the orbital plane, any systematic effect that tends to avoid this orientation will thus reduce the expected kick velocity. One particularly strong influence on the spin orientation is the torque produced by a single circumbinary accretion disk, which can align both black hole spins with the orbital angular momentum with high efficiency (Bogdanović et al. 2007). This orientation, the maximum recoil should not be more than \( \sim 200 \) km s\(^{-1} \) (Baker et al. 2007), well below the escape velocities of most present-day galaxies. In the event that there is no surrounding accretion disk, the two black hole spins may become aligned via spin-orbit resonances (Schnittman 2004), but this process likely requires somewhat special initial conditions.

Finally, there is also the possibility of creating a new SMBH ex nihilo during the galactic mergers, which typically are accompanied by massive gas inflows to the center of the resulting galaxy (Mihos & Hernquist 1994). This rapid increase in gas density will trigger a burst of massive star formation, which may then proceed to form supermassive stars through runaway mergers (Gurkan et al. 2006), in turn collapsing to form the seeds of SMBHs, which will also be surrounded by copious amounts of fuel to accrete more mass (we note, however, that the high velocity dispersions typical of galactic centers may not be as conducive to this formation mechanism as the high-density, low-velocity-dispersion environment at the centers of globular clusters). In this way, orphan black holes can appear in the merger tree, further increasing the overall occupation fraction.

4. ASTROPHYSICAL MERGER TREE

In an attempt to verify some of these qualitative claims, we now develop a slightly more physical Monte Carlo merger model, motivated by basic astrophysical arguments and observations of local SMBHs. Instead of using a constant ejection probability and a perfect binary merger tree of equal-mass BHs and galaxies, we now consider an initially logarithmically flat BH mass distribution function \( \frac{dn}{dM} \sim M^{-1} \) and assume that the galaxy mass is proportional to the BH mass at all times, with the initial BH mass ranging from \( 10 \) to \( 10^{11} M_\odot \) (we find this mass distribution remains roughly flat after a number of merger generations). If an occupied galaxy merges with an empty galaxy, we adjust the final BH mass to “agree” with the total mass of both parent galaxies (physically represented by some accretion episode). For an isothermal sphere model, the escape velocity is given by \( v_{esc} = 2\sigma \). The velocity dispersion \( \sigma \) is in turn determined from the BH mass via the \( M-\sigma \) relation of Ferrarese & Merritt (2000) and Merritt & Ferrarese (2001): \( M_* \approx 1.3 \times 10^7 (\sigma_v/200 \text{ km s}^{-1})^4 M_\odot \).

In each generation, a fraction \( f_m \) of all the galaxies (selected randomly) merge. In the limit of \( f_\text{m} \to 1 \) (as in the binary merger tree), the low-mass tail of the distribution is prematurely depleted, and if \( f_\text{m} \to 0 \), no evolution takes place at all. However, we find that for the intermediate range \( 0.25 \leq f_\text{m} \leq 0.75 \), the net results are largely independent of \( f_\text{m} \). For each merger, we determine the final occupation as in the binary model: two empty parents create an empty child, a single BH mother or father will create a BH child, and in the case of two BH parents, the child black hole will be ejected if the kick velocity is greater than the escape velocity of the child galaxy.

To determine the kick velocity in this more astrophysically realistic model, we employ the analytic fits presented in Schnittman & Buonanno (2007), assuming that all BHs are rapidly spinning with \( a/M = 0.9 \) and that the spin vectors are oriented with a uniform random distribution. In this case, for a given mass ratio \( q = m_1/m_2 \leq 1 \), 90% of the mergers should produce kick velocities less than \( v_{90}\):

\[
v_{90}(q) \approx \frac{17,900 q^2}{(1 + q)^2} \sqrt{(1 - q)^2 + 1.4(1 + q)^2} \text{ km s}^{-1},
\]

with the actual recoil \( v_{kick} \) selected randomly from the cumulative distribution function (Schnittman & Buonanno 2007)

\[
P_{\text{eff}}(v_{kick}; q) = 10^{-7.2 + 0.5 \log(v_{kick}/v_{90}(q))}.
\]
tion as a function of $\sigma_*$ employing the model described above for two cases: an initial occupation of $f_0 = 0.99$ (solid curves) and $f_0 = 0.4$ (dashed curves). In both plots, the fractions $f_j$ refer to the average number of mergers for each galaxy, not the number of generations $[N_{\text{mergers}} \approx N_{\text{gen}} f_{m}(2 - f_{m})]$. After a few mergers, both cases converge to the steady state case of $f_0$ (thick black curve), just as in the simple binary merger model. In the lower panel, we repeat the same calculation, now multiplying $\nu_0$ by a factor of 3, to see the effect of extremely large kicks. While the occupation fractions clearly decrease somewhat, they remain significantly above 50%, just as predicted by the binary merger model. In the case of $f_0 = 0.1$, we find very similar results, only with somewhat longer convergence times (on the order of 7–8 merger generations). If, on the other hand, we only count major mergers with $q > 0.1$ (astrophysically motivated by the very long tidal capture timescale for small galactic mass ratios; Madau et al. 2004), we find that the occupation fractions $f_j$ converge more rapidly to the asymptotic value. However, we also note that there may not be sufficient time to complete even a few mergers, e.g., in the case of $z \approx 6$ quasars (Volonteri & Rees 2006).

We do not claim that this improved model should be seen as reliable for making quantitative astrophysical predictions, but rather we present it as an application of the simple binary tree model and a confirmation of many of the qualitative arguments from § 3. At the same time, it is noteworthy that the occupation fractions for the standard model (upper panel of Fig. 2) seem to agree with Figure 4 of Volonteri (2007), who employed a much more detailed hierarchical merger model, including accretion. We anticipate that future observations should be able to measure this distribution at larger redshifts and smaller $\sigma_*$, ultimately explaining how the BH occupation fraction evolves in time.

5. Conclusions

Based on a very simple binary merger tree model, we find that the fraction of galaxies hosting a SMBH should be $\approx 50\%$, even in the limiting case of very large recoil velocities (or, alternatively, very small escape velocities). This would be particularly important at high redshifts, when typical galaxies have small escape velocities and when the seeds of today’s SMBHs are presumably formed. Including qualitative arguments about the hierarchical growth of the host galaxies, we see that the ejection probability $p_{\text{ej}}$ will tend to decrease with cosmological redshift, further enhancing the fraction of central BHs observed today.

In § 4, we presented a somewhat more physical Monte Carlo model and were able to confirm the predictions of the binary merger tree and also reproduce some of the basic results of the more detailed calculations of Volonteri (2007). We thus conclude that the very large kicks predicted by numerical relativity should not necessarily be seen as contradictory to the seemingly ubiquitous population of galaxies with SMBHs observed today.

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