The primordial “$f_{NL}$” non-Gaussianity, and perturbations beyond the present horizon

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(Dated: June 14, 2009)

We show a primordial non-linear (“$f_{NL}$”) term may produce unphysically large CMB anisotropy for a red-tilted primordial power spectrum ($n < 1$), because of coupling to primordial fluctuation on the largest scale. We consider a primordial power spectrum models of a running spectral index, and a transition at very low wavenumbers. We find that only negative running spectral index models are allowed, provided that there is no transition at a low wavenumbers (i.e. $k \ll 1$). For models of a constant spectral index, we find $\log(k_c/k_0) \gtrsim -184$, at 1$\sigma$ level, on the transition scale of sharp cut-off models, using recent CMB and SDSS data.

PACS numbers: 96.10.+i, 98.70.Vc, 98.80.Cq, 98.80.-k, 98.80.Es

I. INTRODUCTION

Recently, there have been great successes in measurement of Cosmic Microwave Background (CMB) anisotropy by ground and satellite observations [1, 2, 3, 4, 6, 7, 8]. The five year data of the Wilkinson Microwave Anisotropy Probe (WMAP) [1, 2, 3] is released and the recent ground-based CMB observations such as the ACBAR [4, 5] and QUAD [6, 7, 8] provide information complementary to the WMAP data. In near future, PLANCK surveyor [9, 10] is going to measure CMB temperature and polarization anisotropy with great accuracy Planck scale $14, 15$. Another important feature of the recent CMB observations is testing non-Gaussianity and statistical anisotropy of the CMB sky $16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28$. They provide a unique opportunity to test the modern theories of inflation through the observational data (see for review [29]).

The fluctuations of the gravitational potential $\Phi(x)$ (equivalent to Bardeen’s gauge invariant variable $\Phi_H$ [30]) is related to primordial perturbation in complicated ways [31, 32]. When considered up to the second order, there exists a nonlinear term $f_{NL}\Phi_L(x)$, where $f_{NL}$ is a coupling constant. The nonlinear term $f_{NL}\Phi_L(x)$ leads to coupling between largest scales and scales relevant to observable Universe. The recent constraint of the WMAP data shows $f_{NL} \sim 60 \pm 30$ (see [32, 33, 32, 34, 35] for the recent analysis).

Much of studies have been focused on the behavior of the primordial power spectrum on small scales. In this paper, we focus on the ‘infrared’ asymptotic behavior of a red-tilted ($n < 1$) primordial power spectrum. Given the red-tilted primordial power spectra $32, 33$, coupling to the fluctuation on largest scales may produce unphysically large CMB anisotropy, which could be in disagreement with CMB observational data. There have been attempts to remove the divergence of the nonlinear term by renormalization [37], and the author notes that the residual $k$-dependent term, which is not removed by renormalization, is negligible on observable scales for galaxy surveys. Unlike galaxy surveys, the residual $k$-dependent term produce very large excess power on CMB anisotropy of low multipoles. Not to produce unphysical excess power for CMB anisotropy, we require a primordial power spectrum to satisfy: 1) a spectral index of negative running or 2) a transition at very large scale (e.g. sharp cutoff in the power spectrum at a very low wavenumber). We find at least one of them should be satisfied to make agreement with the recent CMB observational data. As will be discussed in this paper, the imprints of the largest scales due to $f_{NL}\Phi_L(x)$ term may improve our understanding on the properties of primordial perturbations on the scales larger than the present particle horizon.

The outline of this paper is as follows. In Section II, we discuss the primordial power spectrum associated with a primordial nonlinear (“$f_{NL}$”) term. In Section III we discuss the effect of a ‘$f_{NL}$’ term on CMB power spectra. In Section IV we show the primordial power spectrum should satisfy some requirement not to produce unphysically large CMB power spectra. In Section V we summarize our investigation and discuss prospects.

II. THE EFFECT OF THE “$f_{NL}$” TERM ON A PRIMORDIAL POWER SPECTRUM

Up to second order, primordial perturbation is given by: $32, 33, 34, 40$;

$$\Phi(x) = \Phi_L(x) + f_{NL} \left[ \Phi_L(x) - \langle \Phi_L(x) \rangle \right],$$

where $\Phi_L(x)$ is a linear part of primordial perturbation, and $f_{NL}$ is the non-linear coupling parameter. The last term on the right hand side ensures $\langle \Phi(x) \rangle = 0$, and is
Finally, by using Eq. 4 and 5 we may easily show
\[ \langle \Phi_L(k) \Phi_{NL}(k') \rangle = 0, \]
\[ \langle \Phi_{NL}(k) \Phi_{NL}(k') \rangle = 8 \pi^2 (f_{NL})^2 \delta(k - k'), \]
\[ \times \int P_{\Phi}(k + p) P_{\Phi}(p) p^2 dp. \]

Finally, by using Eq. 4 and 6 and 7 we find:
\[ \langle \Phi^*(k) \Phi(k') \rangle = (2\pi)^3 \left[ P_{\Phi}(k) + P_{\Phi,NL}(k) \right] \delta(k - k'), \]
\[ P_{\Phi,NL}(k) = \frac{(f_{NL})^2}{\pi^2} \int P_{\Phi}(k + k') P_{\Phi}(k') k'^2 dk'. \]

\[ \text{III. THE EFFECT OF THE 'f_{NL}' TERM ON CMB POWER SPECTRA} \]

The Stokes parameters of CMB anisotropy are conveniently decomposed in terms of spin 0 and spin ±2 spherical harmonics:
\[ T(\hat{n}) = \sum_{lm} a_{T,lm} Y_{lm}(\hat{n}), \]
\[ Q(\hat{n}) \pm i U(\hat{n}) = \sum_{lm} \left[ a_{E,lm} \pm i a_{B,lm} \right] Y_{lm}(\hat{n}), \]
where \( a_{T,lm}, a_{E,lm} \) and \( a_{B,lm} \) are decomposition coefficients. The decomposition coefficients are related to primordial perturbations as:
\[ a_{T,lm} = 4\pi \int d^3k \phi(k) g_{Tl}(k) Y_{lm}^*(\hat{k}), \]
\[ a_{E,lm} = 4\pi \int d^3k \phi(k) g_{El}(k) Y_{lm}^*(\hat{k}), \]
\[ a_{B,lm} = 4\pi \int d^3k \phi(k) g_{Bl}(k) Y_{lm}^*(\hat{k}), \]
where \( g_{Tl}(k), g_{El}(k) \) and \( g_{Bl}(k) \) are the radiation transfer functions and can be numerically computed by a computer software \texttt{CAMB}. In the absence of tensor perturbation, CMB power spectra are given by:
\[ C^T_l = \frac{2}{\pi} \int k^2 dk \left[ P_{\Phi}(k) + P_{\Phi,NL}(k) \right] g_{Tl}(k)^2, \]
\[ C^{EE}_l = \frac{2}{\pi} \int k^2 dk \left[ P_{\Phi}(k) + P_{\Phi,NL}(k) \right] g_{El}(k)^2, \]
\[ C^{EE}_l = \frac{2}{\pi} \int k^2 dk \left[ P_{\Phi}(k) + P_{\Phi,NL}(k) \right] g_{Bl}(k) g_{El}(k), \]
where \( P_{\Phi,NL}(k) \) is the primordial power spectrum associated with the ‘\( f_{NL} \)’ term and given by Eq. 10. Note that CMB anisotropy, excluding the dipole, is sensitive to primordial perturbation of wavenumbers \( k \lesssim 2/\eta_0 \), where \( \eta_0 \) is the present conformal time.

\[ \text{IV. THE SHAPE OF A PRIMORDIAL POWER SPECTRUM} \]

Inflation models predict the power spectrum of primordial perturbation nearly follow a power law \[3, 11, 12, 13, 52, 53] of the form
\[ \frac{\mathcal{P}(k)}{\mathcal{P}(k_0)} = \left[ \frac{k}{k_0} \right]^{n_0 - 1} \left[ 1 + \epsilon + \epsilon \cos \left( \frac{k}{k_0} + \phi \right) \right], \]
where the pivot scale \( k_0 \) is set to the WMAP team’s pivot scale 0.002/Mpc \[36\], and the spectral index \( n \) is given by:
\[ n = n(k_0) + \frac{1}{2} \frac{d n}{d \ln k} \ln \left( \frac{k}{k_0} \right). \]
\( \epsilon_{TP}, \nu, \) and \( \phi \) are the amplitude, the frequency and the phase of trans-Planckian effect. We denote the spectrum in Eq. \( 17 \) and \( 18 \) respectively as ‘the model I’ and ‘the model II’, which differ in the parametrized form of trans-Planckian corrections. Using Eq. \( 11 \), \( 17 \) and \( 18 \) we find

\[
P_{\phi, NL}(k) \approx \frac{f_{NL}^2}{\pi^2} A^2_0 \left( \frac{k}{k_0} \right)^{-4} \left[ 1 + \nu \cos \left( \frac{k}{k_0} + \phi \right) \right] \times \int_0^\infty \left[ 1 + \nu_T \cos \left( \frac{k}{k_0} + \phi \right) \right] \left[ \frac{k}{k_0} \right]^{-1} dk
\]

where \( \theta(k) = v \frac{k}{k_0} + \phi \) for the model I and \( \theta = v \ln \frac{k}{k_0} + \phi \) for the model II.

Most of inflationary models predict that a primordial spectrum is slightly red-tilted (i.e. \( n(k_0) < 1 \)) \( 12, 13 \), which is in good agreement with observations \( 32 \). Given a slightly red-tilted spectral index \( n < 1 \), \( P_{\phi, NL}(k) \) shown in Eq. \( 19 \) may become quite large. It increases with decreasing \( k \) and has strong \( k \)-dependence.

This \( k \)-dependent excess power is not simply removed by renormalization, and may produce very large CMB power spectra on low multipoles (refer to Eq. \( 14 \), \( 15 \) and \( 16 \)). CMB power spectra are well measured by recent satellite and ground observations \( 1, 2, 6, 8, 53 \). Not to produce unphysical large excess power, a primordial power spectrum should satisfy some condition, which will be discussed in the following subsections.

### A. running spectral index

We consider a running spectral index (i.e. \( d n/d \ln k \neq 0 \)). Since significant contribution to the integral comes from \( k' < k_0 \), we find \( P_{\phi, NL}(k) \) for \( k > 2/\eta_0 \) and the model I:

\[
P_{\phi, NL}(k) \approx \frac{f_{NL}^2}{\pi^2} A^2_0 \left( \frac{k}{k_0} \right)^{-4} \left[ 1 + \nu_T \cos \left( \frac{k}{k_0} + \phi \right) \right] \times \int_0^\infty \left[ 1 + \nu_T \cos \left( \frac{k}{k_0} + \phi \right) \right] \left[ \frac{k}{k_0} \right]^{-1} dk
\]

Note that we have set \( k_{\text{max}} \) to \( \infty \), because the integrand converges to zero for \( k' < k_0 \). If \( d n/d \ln k < 0 \), Eq. \( 20 \) is given by

\[
P_{\phi, NL}(k) \approx \frac{f_{NL}^2}{\pi^2} A^2_0 \left( \frac{k}{k_0} \right)^{-4} \left[ 1 + \nu_T \cos \left( \frac{k}{k_0} + \phi \right) \right] \times \sqrt{2 \pi} \exp \left( -\frac{1}{2\alpha} \right) \left[ 1 + \nu_T \cos \phi \right] \exp \left( -\frac{n^2}{2\alpha} \right),
\]

where \( \alpha = d n/d \ln k \). On the other hand, if \( d n/d \ln k > 0 \) and the lower integration bound \( k_{\text{min}} \to 0 \), Eq. \( 20 \) approach an infinity: \( P_{\phi, NL}(k) \to \infty \). Hence, we see that if \( d n/d \ln k < 0 \) is required to keep CMB power spectra finite.

For \( k > 2/\eta_0 \) and the model II, we find:

\[
P_{\phi, NL}(k) \approx \frac{f_{NL}^2}{\pi^2} A^2_0 \left( \frac{k}{k_0} \right)^{-4} \left[ 1 + \nu_T \cos \left( \frac{k}{k_0} + \phi \right) \right] \times \int_0^\infty \left[ 1 + \nu_T \cos \left( \frac{k}{k_0} + \phi \right) \right] \left[ \frac{k}{k_0} \right]^{-1} dk
\]

where \( \nu = \nu_T \cos (k/\nu + \phi) \) for the model I and \( \nu_T \cos (k/\nu + \phi) \) for the model II.

### B. sharp cut-off in the primordial power spectrum

We consider a model of a constant spectral index (i.e. \( d n/d \ln k = 0 \)), and discuss some requirement to avoid unphysically large \( P_{NL} \). We may consider a transition in the shape of the primordial power spectrum at very low wavenumber, below which Eq. \( 17 \) or \( 18 \) are no longer valid. For instance, the WMAP team have considered a model of a sharp cutoff, and found that the cut-off at \( k_c \sim 3 \times 10^{-4}/\text{Mpc} \) makes a slightly better fit \( 30 \). For
For $k > k_c$, we may write Eq. 19 as follows:

$$P_{\Phi, NL}(k) = \frac{(f_{NL})^2 A_0}{\pi^2} \int^{k_{max}}_{k_c} \frac{dk'}{k_0} \left( \frac{k + k'}{k_0} \right)^{n-4} \times \left[ 1 + \epsilon_{TP} \cos \theta(k + k') \right] P_{\Phi}(k') \frac{k'}{k_0}^2$$

$$\times \left[ 1 + \epsilon_{TP} \cos \theta(k + k') \right] \frac{k'}{k_0}^{-2} \left[ 1 + \epsilon_{TP} \cos \theta(k') \right]$$

where $\theta(k) = v \frac{k}{k_0} + \phi$ for the model I and $\theta = v \ln \frac{k}{k_0} + \phi$ for the model II.

Just as the WMAP team did, we consider a sharp cut-off at $k_c$, and set $P_{\Phi}(k') = 0$ for $k' < k_c$. However, $P_{\Phi}(k')$ of $k' < k_c$ may take on some non-zero value, though they are assumed to differ significantly from Eq. 17 and 18. Therefore, our estimate on a transition scale should be interpreted as a lower bound, since a true $P_{\Phi, NL}(k)$ is more likely to be higher than that of our sharp cut-off model, and a higher $k_c$ is needed to make a true $P_{\Phi, NL}(k)$ equal to that of our sharp cut-off model. In our sharp cut-off model, Eq. 21 is given by

$$P_{\Phi, NL}(k) = \frac{(f_{NL})^2 A_0}{\pi^2} \int^{k_{max}}_{k_c} \frac{dk'}{k_0} \left( \frac{k + k'}{k_0} \right)^{n-4} \times \left[ 1 + \epsilon_{TP} \cos \theta(k + k') \right] P_{\Phi}(k') \frac{k'}{k_0}^2$$

$$\times \left[ 1 + \epsilon_{TP} \cos \theta(k + k') \right] \frac{k'}{k_0}^{-2} \left[ 1 + \epsilon_{TP} \cos \theta(k') \right].$$

We have found that $P_{\Phi, NL}(k)$ of $k > 2/k_0$ is barely affected by the value of $k_{max}$, as long as $\log(k_{max}/k_0) > 5$. Hence, we have fixed $k_{max}$ to $\log(k_{max}/k_0) = 10$, and numerically computed Eq. 22 by Romberg integration method [54].

![FIG. 1: CMB temperature power spectra of log(k_c/k_0) = -120, -100, -60, -5 (from the highest to the lowest), dots denote the WMAP and the ACBAR data.](image1)

By making a small modification to CAMB, we have computed theoretical CMB and matter power spectrum, in which $P_{\Phi, NL}(k)$ is taken into account. We show the theoretical CMB power spectra, TE correlation in Fig. 1[2] and the ACBAR data [5]. We may see that anisotropy on largest scales ($l \lesssim 10$) is affected by $P_{\Phi, NL}$ most. For a E mode power spectrum and TE correlations, we show only low multipoles, since there is no visible effect on higher multipoles. We show a theoretical matter power spectrum and SDSS data in Fig. 4[4]. It also shows that matter inhomogeneities on largest scales ($k \lesssim 10^{-3} h/\text{Mpc}$) are

![FIG. 2: CMB TE correlation of log(k_c/k_0) = -120, -100, -60, -5 (from the highest curve to the lowest), dots denote the WMAP data.](image2)

![FIG. 3: E mode power spectrum of log(k_c/k_0) = -120, -100, -60, -5 (from the highest curve to the lowest), dots denote the WMAP data.](image3)

![FIG. 4: matter power spectrum of log(k_c/k_0) = -120, -100, -60, -5 (from the highest to the lowest), dots denote SDSS data.](image4)
affected by \( P_{NL} \) most. As also noted by [37], these excess power is, however, negligible on observable scales of the SDSS survey.

Using a modified \textsc{CAMB} and \textsc{CosmoMC} [44, 53], we have estimated \( k_c \) respectively for the model I and II. For data constraints, we have used the SDSS data [56, 57, 58], the recent CMB observations (WMAP + ACBAR + QUaD [1, 2, 4, 5, 6, 7, 8]), Supernovae data [59, 60, 61] and Big-Bang Nucleosynthesis [62]. We show the marginalized likelihood (solid lines) and mean likelihood (dotted lines) distribution of \( \log(k_c/k_0) \) in Fig. 5. In Fig. 6 and 7, we find there is little degeneracy between \( \log(k_c/k_0) \) and other parameters except for \( A_{sz} \). As expected, the best-fit values of the basic \( \Lambda \)CDM parameters are similar to those of the WMAP concordance model.

![FIG. 5: Marginalized likelihood (solid lines) and mean likelihood of \( \log(k_c/k_0) \) for the model I (top) and II (bottom).](image1)

![FIG. 6: Marginalized likelihood in the plane of \( \log(k_c/k_0) \) versus others parameters for the model I. Two contour lines correspond to 1\( \sigma \) and 2\( \sigma \) levels.](image2)

### C. scale-dependent \( f_{NL} \)

The nonlinear coupling parameter ‘\( f_{NL} \)’ is a local parameter, and hence possesses some scale-dependence [63]. In a single-field inflation, for instance, \( f_{NL} \) in Eq. 24 is given by [63]:

\[
 f_{NL} = \frac{5}{6} - \frac{(k \cdot p)^2}{k^4} - \frac{2k \cdot p - p^2}{k^2}. \tag{23}
\]
V. DISCUSSION

We have shown a primordial non-linear term (‘$f_{\text{NL}}$’ term) may produce unphysically large CMB anisotropy, because of coupling to primordial fluctuation on largest scales. Since such large excess power are not observed in CMB data, we have explored the following minimally extended power law models for a primordial power spectrum to explain the absence of the large excess power.

- A spectral index of a negative running: provided a power law model is valid up to the largest scale (i.e. no transition at a very low wavenumber), running of the spectral index should be negative (i.e. $dn/d\ln k < 0$). We may rule out inflationary models of $dn/d\ln k \geq 0$ (e.g. a mass term potential and some models of softly broken SUSY models),

- A transition at a very low wavenumber (e.g. cutoff); provided a spectral index is constant, there should exist some transition at a very low wavenumber, below which the power law is not valid. We have fitted a transition scale of a sharp cut-off model with the recent CMB and SDSS data, and obtained $\log(k_c/k_0) = -1.98^{+0.35}_{-1.08}$ and $\log(k_c/k_0) = -1.98^{+0.35}_{-1.08}$ respectively for two models described by Eq. 17 and 18.

Though it is not clear which condition is true for the primordial power spectrum, it is certain that at least one of two conditions should be met to avoid unphysically large CMB anisotropy.

We shall be able to impose stronger constraints on inflationary models with the data from the upcoming PLANCK surveyor \cite{Planck}. The improved constraints on a running spectral index of scalar perturbation $dn/d\ln k$, tensor-to-scalar ratio $r$, and the spectral index of tensor perturbation $n_t$ will improves our understanding on inflation, and improves our understanding on how unphysically large $P_{\text{NL}}$ is avoided.

VI. ACKNOWLEDGMENTS

We are grateful to V. A. Rubakov for helpful discussion. We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAGNDB), ACBAR, QUaD and SDSS data. This work made use of the Cosmolc package. This work was supported by FNU grant 272-06-0417, 272-07-0528 and 21-04-0355.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure7}
\caption{Marginalized likelihood in the plane of $\log(k_c/k_0)$ versus others parameters for the model II. Two contour lines correspond to 1σ and 2σ levels.}
\end{figure}

In the models of a constant spectral index $n \sim 0.962$ and no transition, all terms of $f_{\text{NL}}$ should have $k$ dependence $k^{\alpha > 0.04}$ not to have unphysically large $P_{\text{NL}}$. However, $f_{\text{NL}}$ predicted by most of inflationary models does not have such $k$ dependence. Therefore, we find a scale-dependent $f_{\text{NL}}$ alone does not provide a way to avoid unphysically large $P_{\text{NL}}$.

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