FedFA: Federated Learning With Feature Anchors to Align Features and Classifiers for Heterogeneous Data

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Abstract—Federated learning allows multiple clients to collaboratively train a model without exchanging their data, thus preserving data privacy. Unfortunately, it suffers significant performance degradation due to heterogeneous data at clients. Common solutions involve designing an auxiliary loss to regularize weight divergence or feature inconsistency during local training. However, we discover that these approaches fall short of the expected performance because they ignore the existence of a vicious cycle between feature inconsistency and classifier divergence across clients. This vicious cycle causes client models to be updated in inconsistent feature spaces with more diverged classifiers. To break the vicious cycle, we propose a novel framework named Federated learning with Feature Anchors (FedFA). FedFA utilizes feature anchors to align features and calibrate classifiers across clients simultaneously. This enables client models to be updated in a shared feature space with consistent classifiers during local training. Theoretically, we analyze the non-convex convergence rate of FedFA. We also demonstrate that the integration of feature alignment and classifier calibration in FedFA brings a virtuous cycle between feature and classifier updates, which breaks the vicious cycle existing in current approaches. Extensive experiments show that FedFA significantly outperforms existing approaches on various classification datasets under label distribution skew and feature distribution skew.

Index Terms—Federated learning, data heterogeneity, feature anchor, feature alignment, classifier calibration.

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I. INTRODUCTION

With massive data located at mobile clients of large-scale networks such as the Internet of Things (IoT) networks, mobile networks and vehicular networks, federated learning [1] enables clients to jointly train a machine learning model without collecting client data into a centralized server, thus preserving data privacy. However, the private data are typically heterogeneous across clients, resulting in slower convergence [2], [3], [4], [5] and degraded generalization performance [6], [7], [8]. This is because data heterogeneity makes the local objectives inconsistent with the global objective and causes drifts in client updates to slow down convergence. The drifts then deviate the converged model from the expected optima and degrade its generalization performance according to [9], [10].

Existing works have observed that data heterogeneity induces weight divergence (from the view of parameter space) and feature inconsistency (from the view of feature space) when clients train their models. Furthermore, the implementation of federated learning in wireless mobile networks may exacerbate the negative impact of data heterogeneity due to limited wireless resources [11], [12]. Common solutions add a regularizer to control weight divergences such as [2], [13] or feature inconsistency across clients such as [14], [15]. See more discussion in Section II. Nevertheless, recent works like [7] found that these methods did not show clear advantages over the canonical FedAvg [1] on various classification tasks.

To unravel the underlying reasons for the ineffectiveness of existing methods, we first observe that data heterogeneity (including heterogeneous label and feature distributions across clients) induces feature inconsistency and classifier divergence concurrently across clients. We then theoretically and empirically identify the existence of a vicious cycle between feature inconsistency and classifier divergence across clients, as shown in Fig. 1(a). Specifically, inconsistent features diverge the classifier updates, and then the diverged classifiers force feature extractors to map to more inconsistent feature spaces, thus diverging client updates. Therefore, the vicious cycle between feature inconsistency and classifier divergence causes client models to be updated in inconsistent feature spaces with more diverged classifiers.

To overcome the vicious cycle, we propose a novel and effective framework called Federated learning with Feature
Anchors (FedFA) for classification tasks to address the skewed label and feature distributions across clients. FedFA introduces the feature anchors to unify the extraction of features by clients from a shared feature space and to calibrate classifiers into this space during local training. We show theoretically and empirically that FedFA enables smoother classifier updates and polymerized features, which brings a virtuous cycle between classifier similarity and feature consistency, as shown in Fig. 1(b), contrary to the above vicious cycle. Meanwhile, we analyze the non-convex convergence rate of FedFA. Finally, our experiments show that FedFA significantly outperforms the existing methods under label distribution skew, feature distribution skew, and their combined skew. To the best of our knowledge, we are the first to study the combined label and feature distribution across clients. FedFA introduces a model-contrastive regularizer to maximize (or minimize) the agreement of the features extracted by the local and global models as a regularizer. FedDyn [18] modifies the local objective with a dynamic regularizer based on the first-order condition to make clients’ local minima consistent with the global minima. SCAFFOLD [3] uses the variance-reduction technique found in standard convex optimization to create a control variate that adjusts client updates to be more similar to the global update. According to [19], instead of adjusting the weights of the entire model, it is more effective to focus on the classifier layer (which is the final layer of the model) as it is most affected by label distribution skew. The solution proposed is to calibrate the classifiers using virtual features after training. Additionally, in [20], a fine-grained calibrated classifier loss is incorporated to address the issue of over-fitting of underrepresented classes in clients’ datasets that are affected by the long-tail effect. Some methods enable clients to share their data with privacy guarantees, such as sharing a synthesized dataset in [19], [21], [22] and sharing coded data in [23], [24].

2) Tackle Data Heterogeneity by Controlling Feature Inconsistency: Recent studies have discovered feature inconsistency among clients from the perspective of feature space. To control model divergence, certain contrastive learning techniques like feature alignment and logit distillation are employed. For instance, MOON [14] introduces a model-contrastive regularizer to maximize (or minimize) the agreement of the features extracted by the local model and that by the global model (or the local model of the previous round). In place of the model-contrastive term, FedProc [15] and FedProto [25] add a prototype-contrastive term to regularize the features within each class with class prototypes [26]. In [27], [28], clients share their own models with other clients and take logit distillation to align the logit outputs of all client models.

3) Tackle Data Heterogeneity by Improving Aggregation Schemes: Some works have developed alternative aggregation schemes at the server to tackle data heterogeneity in federated learning. For instance, unbalanced data induces a different number of local updates and causes an objective inconsistency problem found in [10], which propose FedNova to eliminate the
inconsistency by normalizing the local updates before averaging. Besides, adaptive momentum updates on the server side was adopted in [29] to mitigate oscillation of global model updates when the server activates the clients with a limited subset of labels. Beyond layer-weighted averaging, some works like FedMA [30] introduce neuron-wise averaging because there may exist neuron mismatching from permutation invariance of neural networks in federated learning. These ideas complement our work and can be integrated into our method because our method only adds an auxiliary loss at the client side.

4) Tackle Data Heterogeneity in Wireless Networks: When implemented in a realistic wireless network, the performance of federated learning is further affected by wireless factors and client availability [11]. To enhance the effectiveness of federated learning, wireless resource allocation and client selection can be optimized together using a closed-form expression for the expected convergence rate on FedAvg, as described in [11], or a hierarchical training architecture, as discussed in [12]. A recent study [31] delved into the concept of model quantization as a means to enhance wireless communication. This study suggests a reinforcement learning approach using a model-based method to determine the participating clients and the bitwidths used for model quantization. To further preserve privacy, the truth-discovery technique and homomorphic cryptosystem are introduced by [32] to identify the client reliability and thereby decrease the impact of anomalous clients. Besides, DetFed is a recent work [33] that presents a deterministic federated learning framework for industrial IoT, which integrates 6G-oriented time-sensitive networks to improve the reliability and latency of the training process.

According to [34], existing works may not provide stable better performance gains over FedAvg [1] in classification tasks, which motivates us to analyze the relationship between classifier updates and features in local training. We find that existing methods ignore the inherent relationship (i.e., a vicious cycle) between these two updates and then still suffer from either feature inconsistency or classifier divergence. Different from these methods, our method breaks the vicious cycle by taking feature anchors to align both feature and classifier updates across clients. Moreover, our method addresses both label and feature distribution skew and label distribution skew. In this paper, we consider a federated learning framework with $N$ clients, each with its own dataset $D_i \sim \mathbb{P}_i : \mathbb{R}^d \times \mathbb{R}$ with $n_i$ data samples. The global dataset is the union of all client datasets and denoted by $D = \bigcup_{k=1}^{K} D_k \sim \mathbb{P}$ on $\mathbb{R}^d \times \mathbb{R}$ with $n = \sum_{i=1}^{N} n_i$ data samples. The objective of federated learning is to minimize the expected global loss $\mathcal{L}(w)$ on $D$, which is formulated as:

$$\min_{w \in \mathbb{R}^d} \mathcal{L}(w) := \mathbb{E}_{i}[\mathcal{L}_i(w)] = \sum_{i} \frac{n_i}{n} \mathcal{L}_i(w)$$

$$= \sum_{i} \frac{n_i}{n} \mathbb{E}_{(x,y) \in D_i} [l_i(w; (x,y))],$$

(1)

where $\mathcal{L}_i(w)$ is the expected local objective function on the local dataset $D_i$ of the $i$-th client. When heterogeneous data exist at clients i.e., $\mathbb{P}_i \neq \mathbb{P}$, FedAvg [1], a canonical method, takes client models $\{w_i\}_{i=1}^{N}$ to solve (1) by minimizing the client loss function $\mathcal{L}_i(w)$ locally and obtaining the global model $w = \frac{1}{N} \sum_{i=1}^{N} n_i w_i$, round by round until $w$ converges.

C. Data Heterogeneity in Federated Learning

According to [9], there are two types of data heterogeneity: feature distribution skew and label distribution skew. In this work, we may refer to them as feature skew and label skew for saving pages, respectively. Suppose that the $i$-th client data distribution follows $P_i(x, y) = P_i(x|y)P_i(y) = P_i(y|x)P_i(x)$, where $x$ and $y$ denote the feature and label, respectively. Here, $P_i(x)$ and $P_i(y)$ denote the input feature marginal distribution and label marginal distribution of the $i$-th client distribution, respectively. Our research delves into three distinct forms of data heterogeneity, in contrast to prior studies which only investigated one of these forms. These include:

- **Label distribution skew:** The label marginal distribution $P_i(y)$ varies across clients while $P_i(x|y) = P_i(x|y)$ for all clients $i$ and $j$. For example, the $i$-th client holds different labels from that of the $j$-th client when $i \neq j$.

- **Feature distribution skew:** The input feature marginal distribution $P_i(x)$ varies across clients while $P_i(y|x) = P_j(y|x)$ for all clients $i$ and $j$. For instance, the samples held by $i$-th client have different styles from that of the $j$-th client when considering the same label and $i \neq j$.

- **Label and feature distribution skews:** The $i$-th client and $j$-th client hold $P_i(y) \neq P_j(y)$ when sharing the same $P(x)$, and hold $P_i(x) \neq P_j(x)$ when sharing the same $P(y)$. Specifically, there is an occurrence of label distribution skew and feature distribution skew.

When these distribution skews exist across clients (i.e., $D_i \neq D_j$ when $i \neq j$) as per [6], federated learning by optimizing (1) is incomparable to centralized training with the global dataset.

III. PRELIMINARIES AND PROBLEM FORMULATION

A. Terminology

Suppose that the global dataset $\mathcal{D}$ consists of $C$ classes indexed by $[C]$ for classification tasks. Let $(x, y) \in \mathcal{D}$ and $\mathcal{D} \subseteq \mathcal{X} \times [C]$ where $(x, y)$ denotes a sample $x$ in the input-feature space $\mathcal{X}$, with the corresponding label $y$ in the label space $[C]$. We represent $[C_i]$ as a subset of $[C]$ (i.e., $\bigcup_{i=1}^{K} [C_i] = [C]$) and $D_{i,c} = \{(x, c) \in D_i | c \in [C_i]\}$ as the subset of $D_i$ with the label $c$ at the dataset $D_i$.

Furthermore, we decompose the classification model parameterized by $w = \{\theta, \phi\}$ into a feature extractor (i.e., other layers except for the last layer of the model denoted by $f_{\phi} : \mathcal{X} \rightarrow \mathcal{H}$) and a linear classifier (i.e., the last layer of the model denoted by $f_{\phi} : \mathcal{H} \rightarrow \mathbb{R}^{|C|}$). Specifically, the feature extractor maps a sample $x$ into a feature vector $h = f_{\phi}(x)$ in the feature space $\mathcal{H}$, and then the classifier generates a probability distribution $f_{\phi}(h)$ as the prediction for $x$. 

B. Federated Learning

We consider a federated learning framework with $N$ clients, each with its own dataset $D_i \sim \mathbb{P}_i : \mathbb{R}^d \times \mathbb{R}$ with $n_i$ data samples. The global dataset is the union of all client datasets and denoted by $D = \bigcup_{k=1}^{K} D_k \sim \mathbb{P}$ on $\mathbb{R}^d \times \mathbb{R}$ with $n = \sum_{i=1}^{N} n_i$ data samples. The objective of federated learning is to minimize the expected global loss $\mathcal{L}(w)$ on $D$, which is formulated as:

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When these distribution skews exist across clients (i.e., $D_i \neq D_j$ when $i \neq j$) as per [6], federated learning by optimizing (1) is incomparable to centralized training with the global dataset.
Specifically, these skews cause client updates to diverge and degrade the performance of federated learning, such as decreasing convergence speed [10] and degrading the generalization of trained models [2]. However, there is still a lack of work to observe the impact of these skews on different architectures of the trained model.

IV. MOTIVATION: INCONSISTENT FEATURES AND DIVERGED CLASSIFIERS ACROSS CLIENTS

In this section, we explore the effect of data heterogeneity in federated learning in view of the relationship between feature and classifier updates. We empirically and theoretically demonstrate the simultaneous occurrence of feature inconsistency and classifier divergence across clients during training.

A. Experimental Demonstration

We consider the FMNIST [35] task with label skews and the Mixed Digits task [36] with feature skews at ten clients in federated learning. For feature visualization, as shown in the first row of Fig. 2, we visualize feature maps of different methods of federated learning using t-SNE visualization [37]. For classifier-update visualization, we input the same samples into all client models to compute the mean distance of class feature and classifier update similarity at the end of each round during training, as shown in the second row of Fig. 2. Note that features are visualized as per the classes (or digit dataset) owned by a client under label (or feature) distribution skew.

Fig. 2 shows that MOON aiming at feature alignment still suffers from feature inconsistency under both label and feature skews. Specifically, there exists significant feature inconsistency of class 1 (i.e., dark blue), class 5 (i.e., dark red) and class 9 (i.e., dark purple) samples, but our method FedFA introduced in Section V overcomes the inconsistency. Moreover, we find that feature inconsistency also exists in other existing methods, where the visualizations are provided in our supplementary materials. This indicates that the existing methods cannot fully generate a consistent feature space for client models even though they focus on aligning features or controlling classifier divergence across clients.

Furthermore, Fig. 2 reveals the simultaneous occurrence of feature inconsistency and classifier divergence. Specifically, the lower the similarity of classifier updates, the more inconsistent the feature mapping between clients in Fig. 2(b2) and (b3). Meanwhile, as the global model converges, these two issues are only slightly alleviated under label skew, while those under feature skew even get worse. These findings indicate an interactive relationship between feature and classifier updates, making it impossible to solve the heterogeneous data problem of federated learning by only controlling any feature extractors or classifiers.

B. Theoretical Demonstration

We follow [38] to represent the classifier parameters $\phi_i$ of the $i$-th client as $C$ weight vectors $\{\phi_i(c)\}_{c=1}^C$, where $\phi_{i,c}$ refers to the proxy for the $c$-th class samples. For simplicity, we set all the bias vectors of the classifier as zero vectors and use the cross-entropy loss as the supervised loss. The supervised loss of the $i$-th client on its classifier is represented as:

$$L_{\sup_i}(\phi_i) := \mathbb{E}_{(x,y) \in D_i} [l_{\sup_i}(\phi_i(x,y))] = -\frac{1}{n_i} \sum_{j=1}^{n_i} \log \sum_{c=1}^C \exp \left( \phi_{i,c}^T \mathbf{h}_i(y_j) \right) \exp \left( \phi_{i,c}^T \mathbf{h}_i(y_j) \right).$$

(2)
where \( h_{a,j} \) is the feature mapping of a sample \((x_j, y_j)\). We will demonstrate the relationship between classifier and feature updates across clients as follows.

First, the classifier updates diverge across clients. For the \( c \)-th local proxy \( \phi_{c,a} \), the positive features and negative features denote the features from the \( c \)-th class samples and other classes, respectively. Let \( p_{y_{(c)}}^{(j)} = \exp(\phi_{y_{(c)}}^{(j)} h_{a,j}) / \sum_{y} \exp(\phi_{y}^{(j)} h_{a,j}) \) and we follow a mild assumption in [20], [39] that the extracted feature \( h_{a,c} \) of samples and their corresponding prediction output \( p^{(j)}_{c,a} = \frac{1}{n_{c}} \sum_{y_{(c)}=c} p_{y_{(c)}}^{(j)} h_{a,j} \) where \( p_{y_{(c)}}^{(j)} = \frac{1}{n_{c}} \sum_{y_{(c)}=c} p_{y_{(c)}}^{(j)} \) and \( h_{a,c} = \frac{1}{n_{c}} \sum_{y_{(c)}=c} h_{a,j} \). Without losing the generality, we characterize classifier update deviation across clients as follows.

**Lemma 1:** (Classifier update deviation. See proof in Appendix B, available online.) For client \( a \) and client \( b \) with the same sample number \( n_a = n_b = n \), the deviation of classifier update \( \Delta_{\phi_c} = \Delta\phi_{a,c} - \Delta\phi_{b,c} \) is:

\[
\Delta_{\phi_c} = \frac{\eta}{n} \left[ (n_{a,c} - n_{b,c}) \left( 1 - p_{a,c}^{(c)} \right) \left( 1 - p_{b,c}^{(c)} \right) \hat{h}_{a,c} - \left( \sum_{i_{a,c} \neq c} n_{a,c} p_{a,c}^{(c)} h_{a,i_{a,c}} - \sum_{i_{b,c} \neq c} n_{b,c} p_{b,c}^{(c)} h_{b,i_{b,c}} \right) \right]
\]

where \( \eta \) is the learning rate, \( n_{a,c} \) is the \( c \)-th class sample number, and \( i_{a,c} \) is one of the negative classes of the \( c \)-th class of one client (e.g., \( c \in \{C_a \} \backslash \{c\} \) for client \( a \)).

**Lemma 1** formulates the deviation of classifier updates between any two clients. Then, let clients \( a \) and \( b \) hold the same samples if their datasets have the same class under label distribution skew, and let clients \( a \) and \( b \) have all classes under feature distribution skew. With **Lemma 1**, we demonstrate how data heterogeneity diverges classifier updates.

**Theorem 1:** (Classifier update divergence under data heterogeneity. See proof in Appendix C, available online.) For label distribution skew (different label sets \( \{C_a \} \neq \{C_b \} \), when \( c \in \{C_a \} \cap \{C_b \}, \hat{h}_{a,c} = \hat{h}_{b,c}, p_{a,c}^{(c)} = p_{b,c}^{(c)} \) and \( \Delta_{\phi_a} = 0 \), and then \( \|\Delta_{\phi_c}\|^2 = \frac{n^2}{n^2} \|\Delta_{\phi_a}\|^2 > 0 \); when \( c \in \{C_a \} \setminus \{C_b \} \), \( n_{a,c} = n_{b,c} = 0 \) and \( \Delta_{\phi_a} = 0 \), and then \( \|\Delta_{\phi_c}\|^2 = \frac{n^2}{n^2} \|\Delta_{\phi_a}\|^2 > 0 \); when \( c \in \{C_b \} \setminus \{C_a \} \) or \( c \in \{C_a \} \setminus \{C_b \} \), \( n_{a,c} = 0 \) or \( n_{b,c} = 0 \), and then \( \|\Delta_{\phi_c}\|^2 > 0 \). For feature distribution skew, \( \hat{h}_{a,c} \neq \hat{h}_{b,c} \) and then \( \|\Delta_{\phi_c}\|^2 > 0 \).

**Theorem 1** reveals that both label and feature skews diverge classifier updates across clients (i.e., \( \Delta_{\phi_c} \neq 0 \)). This explains the classifier divergence as shown in Fig. 2. For example, when both clients have the label \( c \), i.e., \( c \in \{C_a \} \cap \{C_b \} \), the classifier divergence is induced by mean negative features; when both clients do not have the label \( c \), i.e., \( c \in \{C \} \setminus \{C_a \} \cup \{C_b \} \), the classifier divergence is induced by mean positive features.

Second, the diverged classifiers would induce feature inconsistency across clients. We characterize the feature deviation of the same samples across clients as follows.

**Lemma 2:** (Feature update deviation. See proof in Appendix D, available online.) For client \( a \) and client \( b \) with \( n \) samples of one class, the deviation of mean class features \( \Delta_{\phi_c} = \Delta\phi_{a,c} - \Delta\phi_{b,c} \) is:

\[
\Delta_{\phi_c} = \eta \left[ (1 - p_{a,c}^{(c)}) \phi_{a,c} - (1 - p_{b,c}^{(c)}) \phi_{b,c} \right]
\]

where \( \bar{e} \in \{C \} \setminus \{c\} \).

**Lemma 2** formulates the deviation of feature updates between any two clients. According to **Theorem 1**, all classifier proxies under data heterogeneity have \( \phi_{a,c} \neq \phi_{b,c} \), inducing \( \Delta_{\phi_a} \neq \Delta_{\phi_b} \). Therefore, \( \|\Delta_{\phi_c}\|^2 > 0 \) and inconsistent feature updates occur across clients under heterogeneous data.

Finally, we conclude the relationship between classifier divergence and feature inconsistency as follows.

**Theorem 2:** (Relationship between classifier deviation and feature deviation. See proof in Appendix D, available online) Combining **Lemmas 1** and **2**, for client \( a \) and client \( b \) with output \( p_{a,c}^{(c)} = p_{b,c}^{(c)} \) and \( p_{a,c}^{(c)} = p_{b,c}^{(c)} \) on the \( c \)-th class and the same sample numbers \( n_{a,c} = n_{b,c} \), the relationship between their classifier update deviation and feature update deviation is:

\[
\Delta_{\phi_c} = -\eta \Delta_{\phi_a} + \sum_{\bar{e} \in \{C \} \setminus \{C \}} p_{a,c}^{(c)} \phi_{a,\bar{e}}
\]

**Theorem 2** unravels a negative effect of data heterogeneity:

**Observation 1:** (A vicious cycle) Data heterogeneity first induces classifier update divergence, which then leads to inconsistent feature maps; these inconsistent features in turn force different classifiers to diverge worse, as illustrated in the toy example of Fig. 1(a). Moreover, as shown in Fig. 2(a1) and (a3), the vicious cycle does not disappear even when the training of the global model converges.

In summary, feature inconsistency and classifier divergence are coupled to degrade the performance of federated learning.

To break the vicious cycle, it is necessary to address both issues simultaneously.

V. FEDERATED LEARNING WITH FEATURE ANCHORS

We propose FedFA to train client models in a consistent feature space with the classifiers corresponding to this space in order to break the vicious cycle revealed in **Theorem 2**.
A. Propose Method: FedFA

With a total of $C$ classes in the whole dataset, the server initiates $C$ feature anchors $\{a_c\}_{c=1}^C \in \mathcal{H} \times [C]$ indexed by $c \in [C]$ before training. We introduce a feature anchor loss to align the each-class features across clients and formulate it as:

$$l_{fa}(\theta_i; a_c, (x, c) \in D_i) = \|h_{i,c} - a_c\|^2.$$  \hfill (3)

where $h_{i,c}$ denotes the feature mapped by a feature extractor $f_\theta(x)$ for a sample $(x, c)$. The feature anchor loss measures the average distance between features and their corresponding feature anchors. When minimizing (3), the intra-class feature distance for a given client, as well as across all clients, can be reduced since the anchors are the same across clients, as shown in Figs. 1(b) and 5(c), available online. Thereafter, we show the whole training process of FedFA as follows.

1) Minimizing Local Objective With Feature Anchor Loss: In client local objectives, FedFA introduces the feature anchor loss in addition to a supervised loss (e.g., the cross-entropy loss as shown in (2) represented as $\ell_{sup}$). At the start of the $t$-th round, the server sends the current global model $w_t^{(t)}$ and feature anchors $\{a_c^{(t-1)}\}_{c=1}^C$ to a set $S$ of active clients. Each client then $i \in S$ locally updates $w_t^{(t-1)}$ to $w_t^{(t)}$ by optimizing the following local objective:

$$\min_{w_i} L_i(w_i) := \mathbb{E}_{(x,y) \in D_i} [\ell_{sup}(w_i) + \mu l_{fa}(\theta_i)]$$  \hfill (4)

where $\theta_i \in w_i = \{\theta_i, \phi_i\}$ and $\mu$ is a hyper parameter to balance $l_{sup}$ and $l_{fa}$. The feature inconsistency across clients found in Fig. 5, available online, can be alleviated by minimizing the feature anchor loss $l_{fa}$ at each mini-batch update.

2) Calibrating Local Classifiers With Feature Anchors: Beyond aligning features, feature anchors are also used to calibrate the updates of classifier proxies. Specifically, at the end of each mini-batch update, the active client $i \in S$ takes feature anchors $\{a_c^{(t-1)}\}_{c=1}^C$ as one mini-batch input of its classifier $f_\theta$ and their corresponding classes as the label set $[C_i]$ to calibrate classifiers by the following objective:

$$\min_{\phi} l_{cal}(\phi; a_c) = -\frac{1}{C} \sum_{c \in [C]} \log \frac{\exp (\phi_i^\top a_c)}{\sum_{c=1}^C \exp (\phi_i^\top a_c)},$$  \hfill (5)

where $l_{cal}$ is the classifier calibration loss. The loss corrects the classifier divergence and keeps classifiers similar at the beginning of each mini-batch update by reducing the distance between the $c$-th class proxy and feature anchor. The calibration corrects classifier divergence and then mitigates feature inconsistency across clients, as their relationship demonstrated in Theorem 2. In return, consistent features aid in aligning classifiers across clients as illustrated in Fig. 1(b).

3) Fixing Feature Anchors in Local Training but Computing Their Momentum: Feature anchors $\{a_c^{(t-1)}\}_{c=1}^C$ are fixed in local training to keep the feature space consistent across clients under heterogeneous data, instead of being updated by gradient descent. To obtain the latest state of the feature space, clients compute the momentum of class features in local training, and the server aggregates the momentum to update feature anchors at the end of one round. Specifically, although client $i$ does not update $a_c^{(t-1)}$, it accumulates the $c$-th class features of the $\tau$-th batch $B_{t,c}^{(t,\tau)}$ as:

$$m_{c,i}^{(t,\tau-1)} = m_{c,i}^{(t,\tau-1)} + \frac{1}{B} \sum_{(x,c) \in B_{t,c}^{(t,\tau)}} f_{\theta}(x)$$  \hfill (6)

where $B$ represents the total mini-batch number of one epoch and $m_{c,i}^{(t,0)} = 0$. Furthermore, we take epoch momentum $m_{c,i}^{(t)}$ to estimate the class features by

$$\bar{a}_{c,i}^{(t,k)} = \lambda m_{c,i}^{(t,k-1)} + (1 - \lambda)m_{c,i}^{(t,k)}.$$  \hfill (7)

The estimation reduces the computation overhead of FedFA since it does not need to compute the latest class feature with the training dataset after local training.

4) Feature Anchor and Model Aggregation at Server: The server performs weighted averaging on all the $c$-th class feature $\bar{a}_{c,i}^{(t,K)}$ of active clients to generate the next-round feature anchors $\{a_c^{(t)}\}_{c=1}^C$, where $K$ represents the total number of the local epoch. The update of feature anchors is represented as $a_c^{(t)} = \sum_{i \in S} \frac{n_i}{N} \bar{a}_{c,i}^{(t,K)}$. Meanwhile, model aggregation in FedFA is the same as FedAvg, i.e., the global model is $w = \sum_{i=1}^N \frac{n_i}{N} \bar{w}_i$.

Note that the above four procedures are a one-round process of FedFA. FedFA performs the process along rounds until the global model converges, where Algorithm 1 illustrates the pseudo-code of FedFA.

B. Non-Convex Convergence Analysis of FedFA

To show the convergence results, we first make the following commonly used assumptions as per [10], [25].

**Assumption 1:** (Lipschitz Smoothness) Each local objective function is Lipschitz smooth, that is, $\|\nabla L_i(w_1) - \nabla L_i(w_2)\|_2 \leq L_1 \|w_1 - w_2\|_2, \forall i \in \{1, 2, \ldots, N\}$.

**Assumption 2:** (Unbiased Gradient With Bounded Variance) For any stochastic gradient $g_i(w)$, there exists a constant $\sigma$ such that $\mathbb{E}[g_i(w)] = \nabla L_i(w)$ and $\mathbb{E}[\|g_i(w) - \nabla L_i(w)\|^2] \leq \sigma^2, \forall i \in \{1, 2, \ldots, N\}$.

**Assumption 3:** (Bounded Expectation of Norm of Stochastic Gradient) The expectation of any stochastic gradient norm is bounded by $G$ such that $\mathbb{E}[\|g_i(w)\|_2^2] \leq G^2, \forall i \in \{1, 2, \ldots, N\}$.

**Assumption 4:** (Lipschitz Continuity of Feature Extractors) Each local feature-extractor function is $L_2$-Lipschitz continuous, that is, $\|f_i(\theta_1) - f_i(\theta_2)\|_2 \leq L_2 \|\theta_1 - \theta_2\|_2, \forall i \in \{1, 2, \ldots, N\}$.

According to the update of FedFA at one round, we have:

**Lemma 3:** (One-round loss deviation of FedFA. See proof in Appendix A, available online). Let Assumption 1–4 hold, for a one-round update of FedFA, we denote as the total iteration number of one-client updates as $\tau_K$ and have:

$$\mathbb{E}[L(w^{(t+1)})] - L(w^{(t)}) \leq \frac{\tau_K L_1^2}{4} \|\nabla L(w^{(t)})\|^2 + \tau_K L_1 \eta G^2 + \frac{\tau_K L_1^2 \eta^2 G^4}{4} + \mu L_2 \eta^2 \tau_K G^2.$$

**Theorem 3:** (Non-convex convergence rate of FedFA. See proof in Appendix A, available online). With Lemma 3, let
Algorithm 1: FedFA (Proposed Framework).

Input: initial model $\mathbf{w} = \{\theta, \phi\}$, initial feature anchors $\{\mathbf{a}_c\}_{c=1}^C$, learning rate $\eta$, local epoch $K$, client number $N$, class number $C$

for each round $t = 1, \ldots, R$ do
Server samples clients $\mathcal{S} \subseteq \{1, \ldots, N\}$
Server communicates $\mathbf{w}^{(t-1)}$ and $\{\mathbf{a}_c^{(t-1)}\}_{c=1}^C$ to all clients $i \in \mathcal{S}$

on client $i \in \mathcal{S}$ in parallel do
Initialize the local model $\mathbf{w}_i \leftarrow \mathbf{w}^{(t-1)}$, the local feature anchor $\mathbf{a}_{c,i} \leftarrow \mathbf{a}_{c,i}^{(t-1)}$
for local epoch $k = 1, \ldots, K$ do
for each mini-batch do
% feature alignment with feature anchors $\%$
Calculate the local loss $l_i \leftarrow l_{sup,i} + \mu l_{fa,i}$
Compute mini-batch gradient $g_i(\mathbf{w}_i) \leftarrow \nabla_w l_i$
Update local model $\mathbf{w}_i \leftarrow \mathbf{w}_i - \eta g_i(\mathbf{w}_i)$
% classifier calibration with feature anchors $\%$
Calculate the calibration loss $l_i \leftarrow l_{cal,i}$
Compute mini-batch gradient $g_i(\phi_i) \leftarrow \nabla_{\phi_i} l_{cal,i}$
Calibrate classifier proxies $\phi_i \leftarrow \phi_i - \eta g_i(\phi_i)$
% Accumulate class features $\%$
Estimate feature anchors $m_{c,i}^{(t,k)}$
end for
Communicate $\mathbf{w}_i^{(t)}$ and $\{a_{c,i}^{(t)}\}_{c=1}^C$ back to the server
end on client
Server aggregates the global model $\mathbf{w}^{(t)} = \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} \mathbf{w}_i^{(t)}$, and the feature anchors $\mathbf{a}_c^{(t)} = \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} \mathbf{a}_{c,i}^{(t)}$ with weighted averaging
end for

$\Delta = \mathcal{L}(\mathbf{w}^{(0)}) - \mathcal{L}(\mathbf{w}^{(t)})$ where $\mathbf{w}^{(t)}$ denotes the local optimum. Given any $\epsilon$, we have $\frac{1}{T} \sum_{t=0}^{T-1} \|\nabla \mathcal{L}(\mathbf{w}^{(t)})\|^2 < \epsilon$, when the total communication rounds $T$ of FedFA meet

$T > \frac{4\Delta}{\mu \epsilon} \sum_{i=0}^T \|\nabla \mathcal{L}(\mathbf{w}^{(t)})\|^2$, $\eta < \min \left\{ \frac{4\Delta}{4\epsilon L_1 + \sigma + \kappa G L_1^2 + 4\mu L_2^2 G^2}, \frac{1}{2\pi L_1} \right\}$ and $\mu < \frac{\epsilon L_1(\sigma^2 + L_1 G^2)}{L_2 G^2}$.

C. Loss Smoothness Analysis of FedFA

With the feature polymerization under feature anchor loss (3) [40], we assume $\bar{\mathbf{a}}_c = \mathbf{a}_c$ in the following analysis.

Theorem 4: (The effect of FedFA on the Lipschitzness of the loss on classifier weight. See proof in Appendix E, available online.) Let $\|\nabla_{\phi_c} \mathcal{L}\|^2$ and $\|\nabla_{\phi_c} \mathcal{L}\|^2$ be the gradient norms of FedFA and FedAvg, respectively. For $\mathbf{a}_c \cdot \mathbf{a}_c = 0$, the deviation of gradient norms of the global classifier between FedFA and FedAvg is computed as:

$\|\nabla_{\phi_c} \mathcal{L}\|^2 - \|\nabla_{\phi_c} \mathcal{L}\|^2 = \|\Delta \hat{\phi}_c\|^2 - \|\Delta \phi_c\|^2$

where $A_c = \sum_{i}^N (n_{i,c}(1 - \hat{p}_{i,c})), \hat{A}_c = \sum_{i}^N \left( n_{i,c} \left(1 - \bar{p}_{i,c}\right)\right)$, $B_c = \sum_{i}^N n_{i,c} \bar{p}_{i,c}$, $\hat{B}_c = \sum_{i}^N n_{i,c} \hat{p}_{i,c}$ and $A_c < \hat{A}_c, \hat{B}_c < B_c$. Note that $\mathbf{a}_c \cdot \mathbf{a}_e = 0$ provides an orthogonal initialization for feature anchors.

Theorem 4 suggests that the incorporation of feature alignment and classifier calibration in FedFA leads to an improvement in the Lipschitzness of the loss on classifier weight space, as compared to FedAvg. This results in a smoother loss function and accelerates the convergence of FedFA.

Observation 2: (A virtuous cycle in FedFA) Combining Theorems 2 and 4, feature alignment and classifier calibration together smooth the loss of classifier updates to boost classifier harmony across clients, which in turn promotes feature mapping consistency across clients, as illustrated in the toy example of Fig. 1(b).

Different from Observation 1, FedFA breaks the vicious cycle to obtain a virtuous cycle between feature and classifier updates under both label distribution skew and feature distribution skew, as shown in Fig. 2(b2) and (b3).

D. Computational Overhead of FedFA

Assuming $N$ clients participate in federated learning, we aim to train a fully connected neural network for the sake of simplicity, which can be extended to other network architectures [41]. The computational overhead with $L$ layers is represented as $G_1 = O(\sum_{l=0}^{2L} I_{l+1}^2)$, where $I_l$ denotes the layer index and $I_l$ represents the neuron number of hidden layers. The computational overhead of feature anchor loss (3) is $G_2 = O((I_{L-1} - 1) I_L)$, while the computation of model averaging is $G_3 = O(\sum_l I_l)$.

We then analyze the computational overhead of FedFA based on the client and server sides and formulate it as follows. For the client side, the total computation of all client models is $N \times G_3$, the computation of feature anchor loss (3) is $N \times G_2$, and the computation of classifier calibration is $N \times G_3$. For the server side, the computation of model averaging on $N$ client models is $G_4$. The computation of feature-anchor accumulation on the client side and feature-anchor averaging on the server side is based on the neuron number of the last layer $I_L$, which is small and can be ignored. In summary, the total computational overhead of FedFA is $N \times (G_1 + G_2 + G_3) + G_4$, compared with that of FedAvg $N \times G_1 + G_4$. With low feature dimensions (i.e., small $I_{L-1}$), the computational overhead of FedFA is similar to FedAvg.

E. Advantages of FedFA

When facing heterogeneous data, FedFA breaks the vicious cycle between feature and classifier updates and brings the virtuous cycle between the two with the help of feature anchors. The anchors help FedFA create a shared feature space across...
TABLE I

| Method (\(l = 0.01\)) | Label Distribution Skew |
|------------------------|-------------------------|
|                         | FMNIST                   |
|                         | \(#C = 2\) \(\alpha = 0.1\) \(\alpha = 0.5\) |
|                         | CIFAR-10                 |
|                         | \(#C = 2\) \(\alpha = 0.1\) \(\alpha = 0.5\) |
|                         | CIFAR-100                |
|                         | \(#C = 20\) \(\alpha = 0.1\) \(\alpha = 0.5\) |
| FedAvg w/o skew         | 85.90(0.14)              |
| FedFA w/o skew          | 89.67(0.16)              |
| FedAvg                  | 74.60(1.42)              |
| FedFA                   | 74.63(1.30)              |
| MOON                    | 74.71(1.76)              |
| FedFA (Out)             | 84.08(1.22)              |

We run three trials and report the mean and standard deviation. For FedAvg and FedFA, we also report their Top-1 accuracy without label skew.

VI. EXPERIMENTS

A. Experimental Setup

1) Datasets and Data Heterogeneity Setups: This work aims at image classification tasks under label and feature distribution skews, and it uses federated benchmark datasets as [1], [7], [42], including EMNIST [43], FMNIST, CIFAR-10, CIFAR-100 [44], and Mixed Digits dataset [36]. Specifically, for label distribution skew, we consider two settings: (i) Same size of local dataset: following [1], we split data samples based on class to clients (e.g., \(#C = 2\) denotes that each client holds two class samples); (ii) Different sizes of local dataset: following [42], we set \(\alpha\) of Dirichlet distribution \(Dir(\alpha)\) as 0.1 and 0.5 to generate distribution \(p_{i,c}\) by which the \(c\)-th class samples are split to client \(i\). For feature distribution skew, we consider two settings: (i) Real-world feature skew: we sample a subset with 10 classes of a real-world dataset EMNIST with natural feature skew; (ii) Artificial feature skew: we use a mixed-digit dataset from [36] consisting of MNIST [45], SVHN [46], USPS [47], SynthDigits and MNIST-M [48]. In Table II, we test the top-1 accuracy based on the global model, except for Mixed Digits where we report the average top-1 accuracy on five-benchmark digit datasets.

2) Baselines: We consider two popular research branches in enhancing the performance of federated learning as our baselines in addition to the canonical method FedAvg. One branch is to implement weight regularizers in clients’ local objectives by controlling the distance between client models and the global model. Our two baselines for this branch are FedProx [2] and FedFA ([19], [20]).
FedDyn [18]. Different from this branch, our method FedFA controls the distance by aligning features and calibrating classifiers across clients. A similar branch to FedFA is to align features extracted by client models across clients, where MOON [14] and FedProc [15] are considered as our baselines. However, these two methods do not calibrate classifiers across clients since they overlook the vicious circle between feature and classifier updates. Note that our method exclusively modifies the client-side local updates. Therefore, server-side-based methods, such as improving aggregation schemes [10] and adding server-update momentum [29], are complementary to FedFA and not considered as our baselines.

Moreover, we carefully select the coefficient of local regularization from \{1, 0.1, 0.01\} (i.e., \(\mu/2 = 0.05\) for FedProx and FedDyn, \(\mu = 1\) for MOON except \(\mu = 5\) on CIFAR-10), set the temperature hyperparameter \(\tau = 0.5\) for MOON and FedProc, and report their best results in our experiments.

3) Models: To ensure a fair comparison, our models adhere to the reported baselines. Following [18], we use a CNN model with two convolution layers for EMNIST, FMNIST, and CIFAR-10. We utilize ResNet-18 [49] with a linear projector from [14] for CIFAR-100 and a CNN model with three convolutional layers from [36] for Mixed Digits.

4) FedFA Setup: We set the coefficient of exponential moving average \(\lambda = 0.5\) in momentum accumulation for feature anchors in local training and local loss coefficient \(\mu = 0.1\) in (4). For anchor initialization, we initiate the pairwise orthogonal feature anchors \(a_c\) by sampling column vector from an identity matrix whose dimension is the same as the size of the features. Other settings of FedFA are the same as baselines in all experiments.

5) Federated Simulation Setups: In Tables I and II, 100 clients attend federated training. 10 clients participate in each round, the local batch size is 64, the local epochs number is 5, and the targeted communication round is 200. We use the SGD optimizer with a 0.01 learning rate and 0.001 weight decay for all experiments. Furthermore, in Table III and Fig. 3, we follow the setups of [14] to investigate the impact of different federated setups with 200 rounds and a local SGD with a 0.01 learning rate and 0.9 momentum. All experiments are performed based on PyTorch and one node of the High-Performance Computing platform with 4 NVIDIA A30 Tensor Core GPUs with 24 GB.

B. Experiment Results

1) Performance Under Label Distribution Skew: Table I shows that FedFA provides significant gains in different label-skew settings regardless of the dataset. Compared with \(\alpha = 0.5\), both \#C = 2 and \(\alpha = 0.1\) indicate more severe label distribution skew, but clients under \#C = 2 have the same sample number while the ones with \(\alpha = 0.1\) do not. First, we find that the performance of all methods degrades as the degree of data heterogeneity increases. Nevertheless, the decline of FedFA is much smaller than that of other methods. For example, when \(\alpha\) changes from 0.5 to 0.1, the top-1 accuracy of all the baselines goes down by about 13% on FMNIST and CIFAR-10, which is twice as large as FedFA. Second, under the same label skew, FedFA achieves larger gains over other methods when label distribution skew becomes more severe, up to 18.06% (i.e., MOON: 34.89% and FedFA 52.95% under \(\alpha = 0.1\) in CIFAR-10). Third, to explore more difficult tasks, we test on CIFAR-100 with ResNet18, and our method still achieves the best performance (i.e., about 3% accuracy advance). It is important to note that FedDyn exhibits unstable performance compared to other methods when considering the same hyperparameter setups. For instance, it shows significantly low accuracy in the CIFAR-100 task. This is due to the fact that FedDyn is much more sensitive to hyperparameters.

2) Performance Under Feature Distribution Skew: According to Table II, our method obtains higher accuracy than all baselines on EMNIST and Mixed Digits. Specifically, the accuracy of FedFA in EMNIST reaches 99.28%, which is 0.77% higher than the best baseline (i.e., MOON 98.51%). Moreover, we split each digit dataset of Mixed Digits into 20 subsets, one for each client with the same sample number (i.e., a skewed feature distribution exists between the clients with a subset of SVHN and the ones with a subset of MNIST). Compared with the best baseline (Feddyn: 83.59%) on Mixed Digits, our method achieves performance gains by 7.14%.

3) Performance Under Both Label and Feature Skews: We combine label skew and feature skew to explore the impact of data heterogeneity further. Namely, we not only split each dataset in Mixed Digits into 20 subsets, one for each client but also set different label distributions for various clients (i.e., clients are subject to at least one of label distribution skew and feature distribution skew). The results in Table II show that all the methods are more susceptible under this setting than that of feature distribution skew. For example, the most significant performance drop reaches 31.93% (i.e., FedDyn from 83.59% to 51.66% under \#C = 2). Nevertheless, FedFA significantly mitigates this performance degradation with a mild decrease from 90.73% to 83.46%. Meanwhile, FedFA maintains at least 10% performance advantage over all baselines under this case, with the largest gap reaching 31.8% (i.e., FedFA from 83.59% to FedDyn 51.66% under \#C = 2).

4) Performance Without Label and Feature Distribution Skew: We compare our method with FedAvg under more homogeneous data and take the same learning rate of this case as that of data heterogeneity for comparison, where the results are reported in Tables I and II. The results demonstrate that FedFA still brings a significant advance in the presence of data homogeneity. For example, FedFA is 8.64% more accurate than FedAvg on CIFAR-100. Incredibly, FedFA under mild data heterogeneity (e.g., \(\alpha = 0.5\) in FMNIST or Mixed Digits) even obtains higher
accuracy than FedAvg without any label or feature skew (e.g., FedFA: 88.40% versus FedAvg: 85.90% in FMNIST). This reveals that the effect of data heterogeneity on federated learning deserves to be further explored.

5) Performance on Different Client Sample Rate and Local Epoch: Following the setup of Table III, we further explore the impact of federated setups. As shown in Fig. 3(a), a larger client sample rate achieves better test accuracy for all methods. Especially, the accuracy gains (about 10%) when increasing the sample rate from 0.1 to 0.3 is much larger than that from 0.3 to 0.5. As shown in Fig. 3(b), larger local epochs have a negative impact on performance, but FedProx and MOON have worse performance degradation than FedAvg and FedFA. Besides, the performance advantage of FedFA under various batch sizes and client numbers is shown in Figs. 9 and 10, available online. Overall, our method FedFA consistently achieves better than all baselines under different setups.

6) Performance of Lipschitzness of Loss: In [50], [51], it has been found that the local optimizer with momentum is more robust to different smoothness of loss to improve generalization. We explore the effect of FedFA on the Lipschitzness of loss by comparing local SGD optimizers with or without momentum on CIFAR-10. Note that FedDyd and FedProc are not compatible with local SGD with momentum, and thus Table III and Fig. 3 do not include their results. Comparing Table I with Table III, all methods with momentum work better than without momentum, but FedFA without momentum has superior performance than baselines with momentum under #C = 2 and α = 0.1 (e.g., FedFA without momentum: 52.95% versus FedProx with momentum: 49.87% under α = 0.1). As expected by Theorem 4, a smoother loss of FedFA achieves better generalization.

C. Ablation Studies

1) Ablation on Anchor Updates and Classifier Calibration of FedFA: As shown in Table IV, we conduct ablation studies on FedFA without anchor updating in (4) and FedFA without classifier calibration in (5) to give an intuition of FedFA performance. On the one side, feature anchors can be fixed during federated training (i.e., the client would not aggregate any information into feature anchors, which would not bring potential privacy leakage). FedFA performs better than the best baseline under label and feature skew. Meanwhile, the anchor updating brings consistent performance benefits (i.e., at least around 2% boost) since the updated anchors keep more representative in the shared feature space across clients. On the other hand, classifier calibration plays the most crucial role in FedFA because data heterogeneity induces a low classifier update similarity as observed in Fig. 2. For instance, classifier calibration boosts performance by 22.37% in the case combined by label skew and feature skew. Overall, both feature alignment and classifier calibration play an essential role in FedFA to overcome data heterogeneity.

2) The Momentum Update of Feature Anchors: We experiment on the FMNIST with different momentum coefficients (λ) of feature anchor updates with the label-skew case of #C = 2. This experiment setting is the same as Table I. Specifically, when λ = 1, feature anchors will not be updated; when λ = 0, feature anchors will be set as the mean feature of the last epoch. Fig. 4(a) shows that the performance of FedFA with different λ is similar. Meanwhile, although FedFA with λ = 0 introduces more oscillations during training, it performs similarly to other cases. This means that FedFA is not sensitive to the momentum coefficient λ.

3) The Initialization of Feature Anchors: To explore the impact of the initialization of feature anchors in FedFA, we design three experiments, including random initialization, one-round FedAvg initialization (i.e., performing FedAvg but accumulating anchors at first round), and “ideal” initialization (i.e., feature anchors are initiated by the trained feature anchors obtained from a finished training of FedFA with the same setting). Fig. 4 reveals that the initialization of the feature anchors does not affect the convergence speed because the anchors are updated in each communication round so that the impact of initialization of feature anchors is quickly and drastically mitigated. For example, as shown in Fig. 4, FedFA with orthogonal initialization provides better accuracy in the first round but does not obtain the best accuracy finally.

4) Timing to Calibrate Classifiers: Compared with [19] that calibrates classifier with virtual representation (CCVR) after training, we perform it during different phases of training and the setting of Table V is the same as Table I. The result of each mini-batch calibration done by FedFA is the best in all cases. This reveals that the classifier divergence induced by data heterogeneity should be corrected as early as possible. Meanwhile, maintaining the virtuous cycle between feature and classifier updates during local training helps the final model converge at a point that generalizes better, since compared with FedFA without classifier calibration, FedFA with classifier calibration after training only improves little.

VII. CONCLUSION AND FUTURE WORKS

This work proposes FedFA, a framework that aims to alleviate performance degradation caused by label and feature distribution skews in federated learning. FedFA creates a shared feature space across clients, assisted by feature anchors, and keeps the classifier consistent in this space. With the help of the shared feature space, FedFA brings a virtuous cycle between feature and classifier updates and significantly outperforms baselines on various data-heterogeneity tasks. The virtuous cycle in FedFA

Fig. 4. Test accuracy (y-axis) along the training communication round (x-axis). (a) shows the performance of FedFA with different λ on momentum updates of feature anchors. (b) shows the performance of FedFA on feature anchors with different initialization.
provides a fundamental solution to the issue of data heterogeneity in federated learning. This contrasts previous attempts, which often resulted in a vicious cycle between feature inconsistency and classifier divergence. Such attempts only focus on addressing either feature inconsistency or classifier divergence and fail to consider the relationship between the two, leading to a decline in performance. Overall, this work provides insights into how feature and classifier updates are related under data heterogeneity and proposes that FedFA exploit this relationship to improve federated learning effectively.

In future work, it is interesting to explore further the causes of feature inconsistency in tasks beyond classification. For example, it is valuable to verify whether a vicious cycle between encoder and decoder updates exists in encoder-decoder-based tasks with heterogeneous data and to extend the observations of this work into more general tasks. In addition, a promising direction is to investigate potential improvements to FedFA, such as aligning the features of shallow layers instead of the last layer of the feature extractor, to improve the performance of federated learning training deep models, etc.

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