COINCIDENCE LIMIT AND GENERALIZED INTERACTION TERM STRUCTURE IN MULTIGRAVITY

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The generalized structure of the interaction term of multigravity is analyzed in detail. The coincidence limit of any multigravity theory is defined and the compatibility equation for the interaction potential is derived which is studied in the weak perturbation limit of metric. The most general properties of the invariant volume and the scalar potential of multigravity are investigated. The general formula for multigravity invariant volume using three means (arithmetic, geometric and harmonic) is derived. The Pauli-Fierz mass term for bigravity in the weak field limit is obtained.

KEY WORDS: coincidence limit, compatibility equation, invariant volume, weak field limit, scalar potential

The multigravity extension of General Relativity (in first papers it was called “f-g theory” or “strong gravity” [1–3]) is important both from theoretical constructions (quantum gravity and branes [4–6], discrete dimensions [7,8], renormalization [9], massive gravity [10] etc.) and experimental facts (dark matter and dark energy [11–13], cosmic acceleration [14,15] etc.). In this respect it is worthwhile to consider non-linear formulation of multigravity [16]. The shape of interaction term plays the most crucial role in constructing models.

The goal of this paper is to consider the generalized structure of the interaction term in detail (see also [17]). That is, we introduce the coincidence limit of a multigravity theory and obtain the compatibility equation for the interaction potential and analyze it in the weak perturbation limit. Note that a particular case of our general construction, a “perturbative limit” which corresponds to critical points of interacting potential and depends from their special form of interaction potential, was considered in [16] for bigravity only. Here we propose the multigravity generalizations and do not consider any restrictions on the metric, as in [16] (where only spaces with constant curvature were considered).

Also we study the most general properties of invariant volume in the interaction term and the scalar potential of multigravity. We generalize the invariant volume for multigravity for three means and obtain the Pauli-Fierz mass term [18] for bigravity in the weak field limit [16], as an example.

MULTIGRAVITY AND THE COINCIDENCE LIMIT

We consider several Universes (labelled by \(i = 1, \ldots, N\)) each described by the metric \(g_{\mu\nu}^{(i)}\) (we use the signature \(+---\)), the set of matter fields \(\Phi^{(i)}\) (scalar, spinorial, vector ones) and the action

\[
S_{G(i)} = \int d\Omega^{(i)} \left[ F^{(i)}(\Phi^{(i)}) + L\left(\Phi^{(i)}, \Phi^{(i)}\right) \right],
\]

where \(d\Omega^{(i)} = d^4 x \sqrt{g^{(i)}}\), \(g^{(i)} = -\det\left(g^{(i)}_{\mu\nu}\right) > 0\) (distinguishing \(g^{(i)}\) as a positive number and \(g^{(i)}\) as a tensor) is the invariant volume and \(F^{(i)}(\Phi^{(i)})\) is pure gravity Lagrangian of \(i\)-Universe, \(L\left(\Phi^{(i)}, \Phi^{(i)}\right)\) describes coupling of matter fields and gravity. In the concept of Weakly Coupled Worlds [16] due to the no-go theorem of [19] the only consistent nonlinear theory of \(N\) massless gravitons is the sum of decoupled gravity actions (1)

\[
S_0 = \sum_{i=1}^{N} S_{G(i)}
\]

which has the huge symmetry \(\prod_{i=1}^{N} \text{diff}^{(i)}\) (each \(\text{diff}^{(i)}\) acts on its metric \(g^{(i)}_{\mu\nu}\) and matter fields \(\Phi^{(i)}\)). The full action of multigravity, as Weakly Coupled Worlds mixing by their gravitational fields only, is

\[
S_{mG} = \sum_{i=1}^{N} \int d\Omega^{(i)} \left[ F^{(i)}(\Phi^{(i)}) + L\left(\Phi^{(i)}, \Phi^{(i)}\right) \right] + \int d^4 x W\left(g^{(1)}, g^{(2)}, \ldots, g^{(N)}\right),
\]

where \(W\left(g^{(1)}, g^{(2)}, \ldots, g^{(N)}\right)\) is the interaction term which is a scalar density made up from metrics taken at the same point, i.e. in ultralocal limit [16]. The symmetry of (3) reduces to only one diffeomorphism, because of the no-go theorem [19].
Therefore, it is interesting to consider the case when also the Universities are described by the same metric. So let us introduce the coincidence limit, when $g_{\mu\nu}^{(1)} = g_{\mu\nu}^{(2)} = \ldots = g_{\mu\nu}^{(N)} \equiv g_{\mu\nu}$. In case of the absence of interaction ($W = 0$) and matter, we have

$$S_0 = \int d\Omega \sum_{i=1}^N F_i(g).$$

(4)

If $F_i^{(1)}(g) = F_i^{(2)}(g) = \ldots = F_i^{(N)}(g) \equiv F_i(g)$, then $S_0 = N \int d\Omega F_i(g_{\mu\nu})$, and therefore noninteracting full theory coincides with the initial one. But in the case of interacting theory and moreover nonvanishing interacting term in the coincidence limit the multigravity can be equivalent to some effective gravity theory described by the effective metric $\bar{g}_{\mu\nu}$ and effective function $\bar{F}(g)$. Thus we arrive to the compatibility equation

$$\sqrt{g} \left( F(g) + U(g) \right) = \sqrt{g} \bar{F}(\bar{g}),$$

(5)

where $\sqrt{g} U(g) = W(g, g, \ldots, g) \neq 0$, and all functions are taken in the same ‘point’. The equation (5) is defined up to covariant divergence of any function, because it will not contribute to the equations of motion. In [1, 2, 16] the only case where...
It is convenient to use covariant derivatives by $g_{\mu\nu}$, then
\[
\tilde{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{1}{2} g^{\mu\sigma} (p_{\sigma\nu\rho} + p_{\sigma\rho\nu} - p_{\nu\rho\sigma}),
\]
(14)
\[
\tilde{R}_{\nu\rho\sigma} = R_{\nu\rho\sigma} + \frac{1}{2} g^{\mu\alpha} (p_{\alpha\nu\rho\sigma} + p_{\alpha\nu\sigma\rho} - p_{\alpha\rho\nu\sigma} - p_{\alpha\rho\sigma\nu} + p_{\nu\rho\sigma\alpha}),
\]
(15)
\[
\tilde{R} \equiv \tilde{g}^{\alpha\beta} R_{\alpha\beta} = R - p^{\alpha\beta} R_{\alpha\beta} - \tilde{\Box} p + \frac{1}{2} \tilde{g}^{\alpha\beta} g^{\mu\nu} (p_{\beta\mu\nu\alpha} - p_{\beta\mu\alpha\nu} + p_{\beta\nu\mu\alpha} + p_{\mu\nu\alpha\beta}),
\]
(16)
where $\tilde{\Box}$ is covariant D’Alambertian defined as $\tilde{\Box} = \nabla_\mu \nabla^\mu$ and $\nabla_\mu$ is covariant derivative by $g_{\mu\nu}$, i.e. $\tilde{\Box} = \tilde{p}^{\alpha\beta} \tilde{\Box} \cdot$

After substitution to (5) we obtain
\[
\int \tilde{R} \sqrt{\tilde{g}} d^4x = \int \sqrt{g} d^4x R
+ \int \sqrt{\tilde{g}} d^4x \left[ -p^{\alpha\beta} R_{\alpha\beta} - \tilde{\Box} p + \frac{1}{2} \tilde{g}^{\alpha\beta} g^{\mu\nu} (p_{\beta\mu\nu\alpha} - p_{\beta\mu\alpha\nu} + p_{\beta\nu\mu\alpha} + p_{\mu\nu\alpha\beta}) - \frac{R}{2\sqrt{g}} p_{\alpha\beta} F^{\alpha\beta} \right].
\]
(17)

### GENERALIZED INVARIANT VOLUME IN MULTIGRAVITY

In consideration of the interaction term of multigravity it is important to choose consistently the invariant volume which in coincidence limit transforms to the standard invariant volume $d^4x \sqrt{g}$. For simplicity, first we consider the bigravity case [16].

Note that $d^4x W(g^{(1)}, g^{(2)})$ is a scalar, while $d^4x W(g^{(1)}, g^{(2)})$ are the scalar densities of opposite weights. By analogy with usual invariant volume $d\Omega = d^4x \sqrt{g}$, we can present $d^4x W(g^{(1)}, g^{(2)})$ as a product $d^4x \cdot f (\sqrt{g_1}, \sqrt{g_2}) \cdot V(g^{(1)}, g^{(2)})$, where $V(g^{(1)}, g^{(2)})$ is a scalar interaction potential.

Now we demand that the ‘interaction’ invariant volume $d\Omega_{int} = d^4x f (\sqrt{g_1}, \sqrt{g_2})$ should be a scalar which in the coincidence limit $g_{\mu\nu}^{(1)} \equiv g_{\mu\nu}^{(2)} \equiv g_{\mu\nu}$ gives the standard invariant volume $d\Omega_{int} = d\Omega$. To satisfy these conditions we require the following general properties of the function $f(u, v)$:

1) Idempotence $f(u, u) = u$; 2) Monotony; 3) Homogeneity $f(\lambda u, \lambda v) = \lambda f(u, v)$; 4) Symmetry $f(u, v) = f(v, u)$.

From homogeneity and symmetry it follows that $f(u, v)$ can be expressed through the function of one variable, the ratio $u/v$, as
\[
f(u, v) = u \cdot f\left(\frac{v}{u}, 1\right) = v \cdot f\left(\frac{u}{v}, 1\right) = \sqrt{u/v} \cdot f\left(\sqrt{\frac{u}{v}}, 1\right).
\]
(18)
Thus, the interaction invariant volume can be presented as
\[
d\Omega_{int} = d^4x f (\sqrt{g_1}, \sqrt{g_2}) = d^4x \sqrt{g_1 g_2} \cdot f \left(\sqrt{\frac{g_1}{g_2}}, \sqrt{\frac{g_2}{g_1}}\right) = d^4x \sqrt{g_1 g_2} \cdot f \left(\frac{g_2}{g_1}\right).
\]
(19)
From symmetry of $f(u, v)$ it follows that $f(u) = f(u^{-1})$.

Let us consider an example. The simplest functions satisfying (18) are usual averages: arithmetic mean, harmonic mean and geometric mean. It is reasonable to consider their linear combination, which gives for the generalized ‘interaction’ invariant volume the following expression
\[
d\Omega_{int} (a, b, c) = d^4x f (\sqrt{g_1}, \sqrt{g_2}) = d^4x \sqrt{a b + c} \left(\frac{1}{\sqrt{g_1}} + \frac{1}{\sqrt{g_2}}\right)
\]
\[
= d^4x \sqrt{g_1 g_2} \cdot \frac{1}{a + b + c} \left[ a \left(\sqrt{\frac{g_1}{g_2}} + \sqrt{\frac{g_2}{g_1}}\right) + b + 2 c \right]
\]
\[
= d^4x \sqrt{g_1 g_2} \cdot \frac{1}{a + b + c} \left[ a \left(\sqrt{\frac{g_1}{g_2}} + \sqrt{\frac{g_2}{g_1}}\right) + b + 2 c \right],
\]
(20)
\[\text{usually one considers the geometric mean only (e.g. see [16]).}\]
where $a$, $b$, $c$ are arbitrary real constants and $y = \sqrt{\frac{2x}{g_2}}$. Similar formulas are valid for $N$-multigravity

$$d\Omega_{int} = d^4x \cdot f (\sqrt{g_1}, \ldots, \sqrt{g_N}) = d^4x \sqrt{g_1 \ldots g_N} \cdot f \left( \frac{g_1^{N-1}}{g_2g_3 \ldots g_N}, \frac{g_2^{N-1}}{g_1g_3 \ldots g_N}, \ldots, \frac{g_N^{N-1}}{g_1g_2 \ldots g_{N-1}} \right).$$ (21)

Evidently, this formula for $N = 2$ (bigravity) gives (19).

Let us denote the $N$ arguments of the function $f$ as

$$y_1^{(N)} = \frac{2^{N-1}}{\sqrt{g_1}} g_2^{-1} \ldots g_N^{-1}, \quad y_2^{(N)} = \frac{2^{N-1}}{\sqrt{g_1}} g_2^{-1} g_3^{-1} \ldots g_N^{-1}, \ldots, \quad y_N^{(N)} = \frac{2^{N-1}}{\sqrt{g_1}} g_2^{-1} \ldots g_{N-1}^{-1} g_N^{-1},$$ (22)

which obviously satisfy

$$y_1^{(N)} \cdot y_2^{(N)} \cdot \ldots \cdot y_N^{(N)} = 1.$$ (23)

Therefore the function $f$ has actually $N - 1$ independent arguments, and so

$$d\Omega_{int} = d^4x \cdot f (\sqrt{g_1}, \ldots, \sqrt{g_N}) = d^4x \sqrt{g_1 \ldots g_N} \cdot f \left( y_1^{(N)}, y_2^{(N)}, \ldots, y_{N-1}^{(N)} \right),$$ (24)

which for $N = 2$ gives (19). Using the means as in (20) we obtain its $N$-analog

$$d\Omega_{int} = d^4x \cdot \sqrt{g_1 \ldots g_N} \cdot \frac{1}{a+b+c} \left[ \frac{a}{N} \sum_{i=1}^{N} y_i^{(N)} + b + c \frac{N}{\sum_{i=1}^{N} y_i^{(N)}} \right],$$ (25)

which can be considered as the most general 'interaction' invariant volume for multigravity.

**GENERALIZED INTERACTION POTENTIAL IN MULTIGRAVITY**

Let us construct the most general expression for the interaction term in (3)

$$S_{int} = \int d^4x W(g^{(1)}, g^{(2)}, \ldots, g^{(N)}).$$ (26)

It is convenient to extract the generalized invariant volume (presented in previous section*)

$$S_{int} = \int d\Omega_{int} V(g^{(1)}, g^{(2)}, \ldots, g^{(N)}),$$ (27)

where $V(g^{(1)}, g^{(2)}, \ldots, g^{(N)})$ is the scalar interaction potential of multigravity.

As we noted before, the symmetry of the full action (3) can be reduced to only one diffeomorphism group which is the diagonal subgroup of common diffeomorphisms acting on metrics as Lie derivative $\delta g^{(i)} = L_\chi g^{(i)}$ or in manifest form

$$\delta h^{(i)} = \varepsilon^\rho h^{(i)}_{\rho\nu} + \varepsilon_\mu h^{(i)}_{\rho\nu} + \varepsilon^\rho g^{(i)}_{\rho\nu},$$ (28)

where $\varepsilon^\rho$ is the same for all metrics. This symmetry restricts the shape of the scalar interaction potential: it should depend from invariant which can be constructed from $N$ metrics $g^{(i)}_{\mu\nu}$.

Let us consider bigravity as an example [14, 16]. The scalar potential should depend from invariants of the mixed tensor

$$\mathbf{Y}_{\nu} = g^{(1)}_{\mu\nu} g^{(2)\rho\nu},$$

which can be treated as tensorial analog of the scalar variable $y$ from (20). Note that $\mathbf{Y}_{\nu}$ is diffeomorphism invariant, i.e. under transformation (28) we have $\delta \mathbf{Y} = L_\chi \mathbf{Y}$, because of the same $\varepsilon^\rho$ for all metrics. To calculate invariants of the tensor (29) we take powers of traces of the the matrix $\mathbf{Y}$ corresponding to the tensor $\mathbf{Y}_{\nu}$, and the number of invariants in 4 dimensions is 4 by the Cayley theorem, which can be taken as

$$\kappa_1 = \text{Tr} \mathbf{Y}, \quad \kappa_2 = \text{Tr} \mathbf{Y}^2, \quad \kappa_3 = \text{Tr} \mathbf{Y}^3, \quad \kappa_4 = \text{Tr} \mathbf{Y}^4.$$ (30)

Let $\lambda_i$ ($i = 0, 1, 2, 3$) are eigenvalues of the tensor $\mathbf{Y}_{\nu}$, which can be treated as relative eigenvalues of the metric $g^{(1)}$ relatively $g^{(2)}$. In the special bi-orthogonal vierbein $e^{(1)}_{\mu}$ the metrics can be written as follows

$$g^{(1)}_{\mu\nu} = \lambda(0) e^{(0)}_{\mu} e^{(0)}_{\nu} - \lambda(1) e^{(1)}_{\mu} e^{(1)}_{\nu} - \lambda(2) e^{(2)}_{\mu} e^{(2)}_{\nu} - \lambda(3) e^{(3)}_{\mu} e^{(3)}_{\nu},$$

$$g^{(2)}_{\mu\nu} = e^{(0)}_{\mu} e^{(0)}_{\nu} - e^{(1)}_{\mu} e^{(1)}_{\nu} - e^{(2)}_{\mu} e^{(2)}_{\nu} - e^{(3)}_{\mu} e^{(3)}_{\nu},$$ (31)

(32)
and so the matrix $Y$ is diagonal. In case of real and positive eigenvalues $\lambda_{(i)}$ it is convenient to introduce

$$\mu_{(i)} = \ln \lambda_{(i)}$$

and consider their powers

$$\sigma_1 = \mu(0) + \mu_{(1)} + \mu_{(2)} + \mu_{(3)}, \quad \sigma_2 = \mu_{(0)}^2 + \mu_{(1)}^2 + \mu_{(2)}^2 + \mu_{(3)}^2,$$

$$\sigma_3 = \mu_{(0)}^3 + \mu_{(1)}^3 + \mu_{(2)}^3 + \mu_{(3)}^3,$$

Then the scalar potential of bigravity is a function of the introduced invariants $\sigma_n$ as

$$V(g^{(1)}, g^{(2)}) = \hat{V}(\sigma_1, \sigma_2, \sigma_3, \sigma_4).$$

An important class of bigravity models has the symmetry $g^{(1)} \leftrightarrow g^{(2)}$. In this case for eigenvalues we have $\lambda_{(i)} \to \lambda_{(i)}^{-1}$ and $\mu_{(i)} \to -\mu_{(i)}$, therefore

$$\hat{V}(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = \hat{V}(\sigma_1, \sigma_2, -\sigma_3, \sigma_4).$$

In the weak field limit $\sigma_1 \to 0$ it is sufficient to take into account only two first invariants $\sigma_1, \sigma_2$ and consider $\hat{V}(\sigma_1, \sigma_2) = \hat{V}(\sigma_1, \sigma_2, 0, 0)$, which appears naturally in brane models [6] and “Pauli-Fierz-like” bigravity [14]. For the latter we expand

$$g^{(1)}_{\mu\nu} = \eta_{\mu\nu} + \sqrt{2k_1} h_{\mu\nu}^{(1)}, \quad g^{(2)}_{\mu\nu} = \eta_{\mu\nu} + \sqrt{2k_2} h_{\mu\nu}^{(2)},$$

where $\eta_{\mu\nu}$ is the same flat metric. In this limit the mixed tensor (29) is

$$\gamma^\mu_{\nu} = \delta^\mu_{\nu} + \sqrt{2k_1} h_{\nu}^{(1)} - \sqrt{2k_2} h_{\nu}^{(2)}.$$

Let us consider the combinations

$$h_{\mu\nu}^0 = q_1 h_{\mu\nu}^{(2)} + q_2 h_{\mu\nu}^{(1)}, \quad h_{\mu\nu}^{mass} = q_1 h_{\mu\nu}^{(2)} - q_2 h_{\mu\nu}^{(1)},$$

where $q_1^2 + q_2^2 = 1$, then it can be shown that $h_{\mu\nu}^0$ is massless and $h_{\mu\nu}^{mass}$ contains the Pauli-Fierz term. Indeed,

$$\sigma_1 = \sqrt{2k_1} h_{\mu}^{(1)} - \sqrt{2k_2} h_{\mu}^{(2)} + k_2 h_{\mu}^{(2)} h_{\mu\nu}^{(2)} - k_1 h_{\mu}^{(1)} h_{\mu\nu}^{(1)},$$

$$\sigma_2 = 2k_1 h_{\mu}^{(1)} h_{\mu\nu}^{(1)} + 2k_2 h_{\mu}^{(2)} h_{\mu\nu}^{(2)} - 4\sqrt{k_1 k_2} h_{\mu}^{(1)} h_{\mu\nu}^{(2)}.$$

Finally we obtain

$$h_{\mu\nu}^{mass} h_{\mu\nu}^{mass,\mu\nu} - (h_{\mu\nu}^{mass})^2 = \frac{1}{2(k_1 + k_2)} (\sigma_2 - \sigma_1^2).$$

Thus, if we choose the interaction in the form

$$S_{int} = -\frac{1}{k_1 + k_2} \int d\Omega_{int} \hat{V}_0 (\sigma_1, \sigma_2),$$

where $d\Omega_{int}$ is defined in (20) and the scalar interaction potential is

$$\hat{V}_0^{PF} (\sigma_1, \sigma_2) = \frac{m_{PF}^2}{8} (\sigma_2 - \sigma_1^2),$$

then the weak field limit of bigravity generates the Pauli-Fierz mass term [18] of the shape

$$S_{int} = -\frac{m_{PF}^2}{4} \int d^4x \left( h_{\mu\nu}^{mass} h_{\mu\nu}^{mass,\mu\nu} - (h_{\mu\nu}^{mass})^2 \right).$$

For the brane motivated bigravity scenario [4, 21] the scalar potential has the form [16]

$$\hat{V}_0^{brane} (\sigma_1, \sigma_2) = m^2 \left( \cosh \frac{\sigma_1}{4} - \cosh \frac{\sqrt{4\sigma_2 - \sigma_1^2}}{4\sqrt{3}} \right).$$

In the weak field limit it reproduces the Pauli-Fierz mass term (45), indeed

$$\hat{V}_0^{brane} (\sigma_1, \sigma_2) \bigg|_{m=\sqrt{3}m_{PF}} = \hat{V}_0^{PF} (\sigma_1, \sigma_2).$$

Note that the “perturbative limit” which corresponds to existence of critical point of potential and from which for bigravity (with potential of form (29) only) it follows that $g^{(1)}_{\mu\nu} = g^{(2)}_{\mu\nu}$, was considered in [16]. Here we present a more general case which is not connected with any concrete form of the interaction potential and does not demand consideration of spaces with constant curvature.
CONCLUSIONS

So in this paper we have analyzed the generalized structure of the interaction term of multigravity. We introduced the coincidence limit and obtained the compatibility equation for the interaction potential which was studied in the weak perturbation limit. We considered the most general properties of invariant volume and the scalar potential. As an example, we derived the Pauli-Fierz mass term for bigravity in the weak field limit.

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