The development of smoothed particle hydrodynamics (SPH) method for modeling the interaction movement of fluid layers in between two soil layers in flat surface condition

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Abstract. The liquefaction phenomenon most often happens in saturated, loose, and low density such as sandy soils. When the bearing capacity drops, loose sand tends to become compressed and behaves like a liquid. This behavior generates an interaction between sandy soils and clay soils which has a higher permeability and include its water content. Therefore, it required a method that can model large deformation movements, namely the free particle method, Smoothed Particle Hydrodynamics (SPH). The SPH Method is used to determine the layer of water formed during the cyclic load to approach the liquefaction phenomenon. The model use clay soils as solid and water layers as fluid with model geometry 0.1x0.1x0.05 m. Program parameters represent clay soils with stiffness coefficient, solid volume, and critical shear strength also fluid layers with damping coefficient, fluid volume, and viscosity. The calculation is done with Fortran Program and visualized in 2 Dimension and 3 Dimension with Gnuplot. Both will be evaluated from the movement of solid and fluid also numeric stability. In the final model the parameters $K_s = 20000$ N/m, $K_s = 53.3$ Ns/m, dan $x$ particle = 50 reach numeric stability and all fluid particles can recognize the solid layer.

1. Introduction

An earthquake of 7.4 magnitudes occurred in Central Sulawesi on 28 September 2018 at 18:02 local time. The epicenter of the earthquake in Palu was located on the Palu-Koro fault in Donggala, around 80 km to the north of Palu City with the earthquake focus to a depth of 10 km. This earthquake generated a tsunami around the coast and a motion of lateral ground such as liquefaction in a considerable manner in some regions, namely Petobo, Sibalaya, Jono Oge, and Bolaroa [1].

Palu is dominated by Alluvium soil layer, Beach Sediment (Qap) and Molasa Celebes Serasin (QTms). Alluvium consists of three soil layers, namely sand, silt, and clay. In which, the soil layer is a layer that dominates or the thickest with characteristics features of having greyish color and loose with a sand thickness of 1 – 7.2 meters and the groundwater level can be found in the depth of 0.5 – 16 meters below ground level [1].

Phenomenon happens as a result of soil bearing capacity disappearance dominated by sandy soil that is impermeable and has a very low permeability as the outcome of accepting cyclic loads. Earthquake vibration that happens conduces soil particles move rapidly with the condition of being undrained, that triggers the disappearance of soil bearing capacity and the increasing of pore water pressure until it is equal to zero soil stress, with the result that effective stress equal to zero. This condition causes soil to undergo a decrease in shear strength and then collapse [1].

Accordingly, liquefaction modeling is needed to analyze the movement of water particles between impermeable layers. Therefore, the model’s movements are analyzed with Smoothed Particle
Hydrodynamics (SPH) method that has the advantage to model the movement with large deformation. This research is focused on finding out the effect of the stiffness coefficient (Ks), and damping coefficient (Kd) toward fluids behavior and the interactions happen in impermeable layers.

2. Literature Review

According to Koester (1988) [2], in general, liquefaction may occurs if the condition of a region is, 1. The dominant soil layer is sand or silt, 2. Soil layers are saturated with water and loose (not solid), 3. There was an earthquake with a magnitude above 5.0 SR, 4. Earthquake acceleration is more than 0.1 g.

The SPH Method represents a group of particles that carry material properties and interact with each other and are controlled by smoothing function. Descriptions of equations are obtained from particle descriptions to calculate the fluid’s initial density, velocity, and acceleration. Fluid pressure is derived from the density equation, in which the acceleration of the particle is accumulated from pressure and density itself [3].

SPH Method is based on Navier-Stokes equation for incompressible and isothermal fluid which include density, internal forces, and external forces, this equation presented by [4]:

\[ \rho \frac{\partial u}{\partial t} = -\nabla p + \mu \nabla^2 u + f \]  \hspace{1cm} (1)

Equation (1) can be modified into,

\[ \rho \frac{\partial u}{\partial t} = F_{\text{internal}} + F_{\text{external}} = F \]  \hspace{1cm} (2)

Then, for each particle can measure the acceleration (m/s²),

\[ a_i = \frac{du}{dt} = \frac{F_i}{\rho_i} \]  \hspace{1cm} (3)

Where \( a_i \) and \( u_i \) are acceleration and velocity particle, also \( F_i \) is a sum of forces field, \( \rho_i \) is the density on a particle. If the pressure is known for each particle, at particle \( i \), the SPH pressure equation is [5],

\[ f_{\text{pressure}} = -\nabla p(r_i) \]  \hspace{1cm} (4)

If the material density is defined, changes in density during modeling can be analyzed by,

\[ \rho_i = \rho(r_i) \]  \hspace{1cm} (5)

\[ = \sum_j \rho_j \frac{m_j}{\rho_j} W(r_i - r_j, h) \]  \hspace{1cm} (6)

Resistance to flow is called viscosity, \( \mu \), defined as fluid viscosity force. Equation of viscosity presented by

\[ f_{\text{Viscosity}} = \mu \nabla^2 u(r_i) \]  \hspace{1cm} (7)

\[ = \mu \sum_{j \neq i} \frac{m_j}{\rho_j} \nabla^2 W(r_i - r_j, h) \]  \hspace{1cm} (8)

Each particle is affected by the same gravitational force, the gravitation force is,
\[ f_{\text{gravitation}} = \rho_i g \]  

(9)

Where, \( g = 9.81 \text{ m/s}^2 \). For gaseous fluid, particle in it will be evaporated because temperature diffusion. For isothermal fluid, buoyancy forced is,

\[ f_{\text{buoyancy}} = b(\rho_i - \rho_0)g \]  

(10)

For isothermal water is used during simulation, buoyancy coefficient \( b = 0 \).

The surface tension that occurs will flatten the curvature of the surface by minimizing the surface area. Surface tension force is,

\[ f_{\text{Surface Tension}} = -\sigma \nabla^2 c_i n \|n\| \]  

(11)

Bouyancy force equation consist of two parameter, No-Penetration and No-Slip [6]. Bouyancy force equation is,

\[ f_{\text{boundary}} = f^{np} + f^{ns} \]  

(12)

No-Penetraton force prevents the particle move outside the model area or dissipation through impermeable layer. No-penetration force is,

\[ f^{np} = (K_d - (v \cdot n)K_0)n \]  

(13)

No-Slip force used to maintain particle velocity on the surface remain zero. No-slip force is,

\[ f^{ns}(r) = \sum_b L_{bn}^2 \tau_{\text{viscosity}}(|r - r_b|) \]  

(14)

Where \( L_{bn} \) is the distance between particle and \( \tau_{\text{viscosity}} \) is traction, where presented by,

\[ \tau_{\text{viscosity}}(r) = -\mu v \nabla^2 W_v(r, h) \]  

(15)

Where \( \mu \) is the friction content that contained at the boundary and \( W_v \) is the viscosity kernel. The amount of friction parameter between fluid and solid particles is \( \mu = \eta = 0.1 \).

3. Method and Materials

3.1. Algorithm

To run the simulation, solid and water were used as the materials. Where the input parameters were material parameter, position and velocity, density, with the existing input, it will be processed according to the Navier-Stokes equation using the Fortran program [7]. The running program’s result is the total force from internal force (density, viscosity) and external force (gravity, buoyancy, boundary). Velocity will be obtained from the total force then it will be reprocessed into final velocity and final position. The position and velocity that are already processed will be evaluated towards the movement of fluid-particle dissipation against the impermeable layer it is expected that the dissipation value is close to 0%, which means that the fluid particles recognize the impermeable layer.
3.2. Boundary Condition
To model liquefaction phenomenon, 3-dimensional space geometry in the form of block with the size of 0.1 x 0.1 x 0.05 meter was used. The use of geometrical space was done for simulation of horizontal flow, which was chosen to approach the particle movement.

3.3. Initial Conditions and Properties
The researcher did a modeling to find out the soil and water motions when liquefaction occurs. That goal is served by using 2500 solid particles and 625 fluid particles in the first simulation and 5000 solid particles and 625 fluid particles in the second simulation. Below is the parameter used for each material.
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Table 1. Initial material parameters simulation

| Parameter         | Fluida - Water | Solid - Clay |
|-------------------|----------------|-------------|
| Density \( \rho \) | 998.29 kg/m³  | 2800 kg/m³  |
| Buoyancy \( b \)  | 0 n/a          |             |
| Viscosity \( \mu \) | 0.001 Pa.s     |             |
| Surface Tension \( \sigma \) | 0.0736 N/m |             |
| Smoothing Length \( h \) | 0.01 m        |             |

Simulation was done in the span of 0.001 s, obtained with a trial in the span of 0.001 s – 0.01 s. That trial was influenced by CFL value in which time changes will affect the stability of the model that can be seen in fluid particles’ direction of motion toward solid particles.

4. Results and Discussions

SPH Program performs two types of modeling, the response of fluid particles’ movement to one impermeable layer and two impermeable layers. The fluid analysis movement from fluid particle accumulation that is dissipated out of the impermeable layer, when the number of fluid particles is close to 0%, it can be said that the simulation is achieved.

4.1. Simulation 1

In simulation one successfully runs 14 variations of the model, which aims to assess the stiffness coefficient and damping coefficient parameters that are stable close to be the properties of clay. The objective is viewed from the dissipation of the fluid particles to the impermeable layer, as seen from the particles’ final position.

4.1.1. Stiffness Coefficient Analysis. The model used for stiffness coefficient uses the \( x = 50 \) and damping coefficient (Kd) = 1.4 Ns/m. The simulation results are as follows.

Table 2. Average dissipated fluid particles through impermeable layer

| Stiffness Coefficient (N/m) | x Particle on Support Radius | % Average Dissipated Fluid Particles |
|-----------------------------|-----------------------------|-------------------------------------|
| 10000                       | 50                          | 82.40%                              |
| 20000                       | 50                          | 1.60%                               |
| 30000                       | 50                          | 0.48%                               |
| 40000                       | 50                          | 0.00%                               |
| 50000                       | 50                          | 0.16%                               |

![Relationship of Effect Stiffness Coefficient on Particle Acceleration](a)
Figure 3. (a) Average acceleration and (b) average velocity experiment result effected by stiffness coefficient onimulation 1

From Figure 3 (a) can conclude that particles’ acceleration stable at $K_s = 40000 - 50000$ N/m and these changes affects the change in velocity at certain intervals. From Figure 3 (b) the changes of the velocity is stable at $K_s = 30000 - 60000$ N/m. Both are seen in the fluid particle dissipation results where the results of fluid dissipation parameters are close to 0%.

4.1.2. Damping Coefficient Analysis. The damping coefficient analysis models are $K_s = 40000$ N/m and $x = 50$ with two variance of damping coefficient.

Table 3. Average dissipated fluid particles on damping coefficient analysis

| Stiffness Coefficient (N/m) | x Partikel on Support Radius | Damping Coefficient (Ns/m) | % Average Dissipated Fluid Particles |
|-----------------------------|-------------------------------|----------------------------|--------------------------------------|
| 40000                       | 50                            | 1.4                        | 0.00%                                |
|                             |                               | 53.3                       | 0.00%                                |
In velocity result, the damping coefficient \( K_d = 53.3 \text{ Ns/m} \) has smaller average. This indicates there is very small moving fluid particles interact with solid later. Therefore, the damping coefficient \( K_d = 53.3 \text{ Ns/m} \) can withstand the movement of fluid particles.

4.2. Simulation 2

The modeling in simulation 2 aims to evaluate the movement of the solid layer and fluid layer also the parameters that influence it. The difference between simulation two and simulation 1 is the addition of solid particles, namely 5000 solid particles and solid volume.

4.2.1. Review of Simulation Parameter 1

To start simulation 2, it is necessary to re-test simulation parameter 1, because there is an increase in the number of solids in simulation 2. Adding parameters can change the existing stability, because the ratio of the amount of fluid and solid changes.

| Model | Stiffness Coefficient (Ks) [N/m] | Damping Coefficient (Kd) [Ns/m] | x Particle | % Average Dissipated Fluid Particles |
|-------|---------------------------------|---------------------------------|------------|-------------------------------------|
| 1     | 20000                           | 1.4                             | 10         | 8.48                                |
| 2     | 30000                           | 1.4                             |            | 9.52                                |
| 3     | 40000                           | 1.4                             |            | 9.28                                |
| 4     | 20000                           | 53.3                            |            | 0.80                                |
| 5     | 30000                           | 53.3                            |            | 2.56                                |
| 6     | 40000                           | 53.3                            |            | 3.20                                |
| 7     | 20000                           | 1.4                             | 50         | 21.44                               |
| 8     | 30000                           | 53.3                            |            | 11.04                               |

The selection of simulation 2 parameters in terms of the results of fluid-particle dissipation, which is obtained for simulation 2 is carried out with \( K_s = 20000 \text{ N/m} \), \( K_d = 53.3 \text{ Ns/m} \), and \( x \text{ particle} = 10 \).

4.2.2. Solid Movement

Solid movement is a condition in which the solid moves slowly and presses the fluid particles that are between solid clay particles, which is moving 1 mm in a span of 15 - 30 minutes. It is found that the solid movement is only affected by the difference in the time interval.
Table 5. Solid movement experiment results

| Initial Position (m) | Final Solid Position (m) | dt (s) | Iteration t (s) | Δx (m) | Velocity (m/s) |
|----------------------|--------------------------|--------|-----------------|--------|----------------|
| 0.030                | 0.0299411                | 0.001  | 200             | 0.2    | 0.0000589      |
| 0.030                | 0.0299411                | 0.002  | 200             | 0.4    | 0.0000589      |
| 0.030                | 0.0299411                | 0.003  | 200             | 0.6    | 0.0000589      |
| 0.030                | 0.0297646                | 0.004  | 200             | 0.8    | 0.0002354      |
| 0.030                | 0.0296321                | 0.005  | 200             | 1.0    | 0.0003679      |
| 0.030                | 0.0294703                | 0.006  | 200             | 1.2    | 0.0005297      |
| 0.030                | 0.0290582                | 0.008  | 200             | 1.6    | 0.0009418      |
| 0.030                | 0.0285285                | 0.010  | 200             | 2.0    | 0.0014715      |
| 0.030                | 0.0278810                | 0.012  | 200             | 2.4    | 0.0021190      |

After the experiment was carried out, the time interval above 0.003 s made the fluid particles move unstable; that is, in the 14th iteration the fluid particles were completely dissipated. There needs to be a review of the sensitivity of fluid particles to differences in time intervals.

4.2.3. Solid Gravity Force. The force of gravity is one that affects the movement of the solid layer. After experimenting, the parameters that affect the magnitude of solid gravitational force are different time intervals, solid volume, and initial solid density. From there parameters, it has been determined that time interval = 0.001 s and the density of clay soil is 2800 kg/m³.

Table 6. Solid gravity force experiment result

| Initial Density (kg/m³) | Volume Solid [m³] | Solid Gravity Force (fgS) [Joule] |
|-------------------------|-------------------|----------------------------------|
| 2800                    | 0.00008992        | 27698.28                         |
|                         | 0.00009132        | 27637.68                         |
|                         | 0.00009347        | 28288.18                         |
|                         | 0.00009792        | 29634.49                         |
|                         | 0.00009831        | 29752.66                         |
|                         | 0.00010310        | 31202.78                         |

Solid gravity force experimenting results have to approach clay parameters, the solid volume used in simulation 2 is 0.0000934 m³ with a solid gravitational force 28,288.18 Joule.

4.2.4. Collision Handling. The problem found in this research is fluid behavior, which can’t recognize the solid layer, making fluid particles come out of the solid layer. This is not following the phenomenon liquefaction, where the fluid comes out of the clay and floats into the air. Therefore, an analysis of the boundary setting needs to be done. After experimenting, it was found that boundary conditions contained sensitive parameters, namely xn and zn.

Table 7. Changes in boundary condition on collision handling

| Boundary Condition | Parameter | Value | Unit |
|-------------------|-----------|-------|------|
| Axis X            | x1        | 0.0   | m    |
|                   | p1 = xn   | 0.05  | m    |
| Axis Y            | y1        | 0.0   | m    |
|                   | y2        | 0.1   | m    |
| Axis Z            | z1        | 0.05  | m    |

4.2.5. Simulation Model 2. In the final simulation model 2, the objective is to combine the existing changes and make a model according to the movement of clay layer. The following are the model parameters used.
A change in the solid particles’ position is made, so the cavity between the fluid and solid particles is not too large.

**Table 8. Final simulation parameter 2**

| Parameter              | Variable | Value  | Unit   |
|------------------------|----------|--------|--------|
| Stiffness Coefficient  | Ks       | 20000  | N/m    |
| Damping Coefficient    | Kd       | 53.3   | Ns/m   |
| x particle             |          | 10     |        |
| Support Radius         | h        | 0.00445| m      |
| Fluid Particle         |          | 625    |        |
| Solid Particle         |          | 5000   |        |
| Fluid Volume           | Volume   | 0.0000231481 | m³ |
| Solid Volume           | VolumeS  | 0.00009347 | m³ |
| Time Interval          | dt       | 0.001  | s      |

**Table 9. Changes in position simulation 2**

| Parameter   | Initial Position (m) | Final Position (m) |
|-------------|-----------------------|--------------------|
| Clay Layer  |                       |                    |
| - Layer 1   | 0.015                 | 0.015              |
| - Layer 2   | 0.030                 | 0.020              |
| Fluid Particle | 0.025             | 0.018              |

![Figure 5](image)

**Figure 5.** (a) Side view (b) oblique view, and (c) above view of simulation final model 2
5. Conclusion
Modeling simulation and analysis of influence interaction of the fluid layer on the impermeable layer can be carried out using the Smoothed Particle Hydrodynamics (SPH) method through differences in Stiffness Coefficient and Damping Coefficient parameters as well as a review of both changes in velocity and position fluid particles, where the research is considered ideal if the interaction between fluid particles and clay soil layer form a water layer with 0% fluid particle dissipation.

Table 10. Final Parameters for Simulation 1 and Simulation 2

| Parameter                  | Simulation 1 | Simulation 2 |
|----------------------------|--------------|--------------|
| Fluid Particle             | 625          | 625          |
| Solid Particle             | 2.500        | 5.000        |
| Stiffness Coefficient [Ks] (N/m) | 40,000       | 20,000       |
| Damping Coefficient [Kd] (Ns/m) | 53.3         | 53.3         |
| x particle                 | 50           | 10           |
| Support Radius [h] (m)     | 0.00762      | 0.00445      |
| Time Interval (s)          | 0.001        | 0.001        |
| Fluid-Particle Dissipated (%) | 0.0          | 0.0          |

The final parameters exist because of the change in the initial position of solid layers and boundary conditions, which makes fluid particles that are dissipated do not exist. The initial volume of clay that effects gravitational force is also changing, so the impermeable layer can move and press fluid particles.

After changes were made, the results of fluid-particle dissipation were satisfactory. No fluid particle are coming out of the top, and the bottom layers of clay soil and eater layers are formed between the clay layers. Influence by solid volume and the addition of a compressive force algorithm and gravitational force, the final position of clay shows changes as expected.

6. Suggestion
An analysis of layer movements with an addition of a particular parameter in Fortran calculation is needed for further research and an analysis of fluid-particle behavior to time changes. Moreover, dissipated fluid particles from impermeable layers need collusion handling analysis, which organizes Fortran’s spatial geometry. Furthermore, a real correlation between the stiffness coefficient and liquefaction soil condition is needed.

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