A Novel Predictor-Corrector Hally Technique for Determining the Parameters for Nonlinear Solar Cell Equation

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Abstract. This paper presents an iterative method Accelerated Predictor-Corrector Hally's Method (AHM) for finding the voltage of a single-diode model for a solar cell from the equivalent circuit. The purposes of the obtained results are to reduce the number of iterations. Two numerical methods are applied and compared; Newton's and Hally's methods. The results showed that the proposed is the most efficient compare with NRM. The purpose of the present work is to acquire the results of photovoltaic parameters using two numerical models and the comparison between them. The acquired results presented the suggested technique (NRM) is a sufficient tool, powerful method to solve this model with the least iteration. All the calculations are achieved using Matlab program.

1. Introduction
The topics of numerical analysis are very important topics in all different fields of science and it is indispensable for all scientific branches, especially applied from the study of numerical analysis and multiple programming languages. Therefore, scientific problems have been solved based on this science in the scientific working life [1-14]. Nowadays, many researchers have developed different types of solar cells based on cost and efficiency. These solar cells were prepared using different and easy methods of preparation, depending on the materials used and the quality of solar cells and the goal is to reduce cost and increase efficiency [15-25]. Other researchers can be utilizing the mathematical techniques in order to solving nonlinear equations of various of solar cells structures depending on the technique of fabrication for inorganic and organic materials for example ceramic materials, silicon solar cells, thin films solar cells [26-36]. Different techniques have been utilized to calculate the parameters of photovoltaic cell and mathematical algorithms for example fuzzy set and logic techniques and visual studio program [37-47]. Recently researchers can be utilizing numerical techniques to solve nonlinear equations in space depending on Kepler's and Barker equations [48-64].

This article aims is to present a new iterative method Accelerated Predictor-Corrector Hally method (AHM) for solving nonlinear equation of a solar cell. The propose of reducing the root of iteration in the numerical solution of the nonlinear equation root. Systematic as the following points: in section two characterizing the model of a solar cell type single-diode; in section three establishing the root finding of Newton Raphson technique; whereas; in section four Accelerated Predictor-Corrector Hally method has been studied; in section five discussion of the results; section six conclusions of the acquired results.

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2. Properties of The equation for a Single Diode Solar Cells [50-61]

Figure 1 presents the equivalent electronic circuit of the photovoltaic cell solar cell.

![Figure 1. Single-diode solar cell (electrical circuit).](image)

By applying Kirchhoff’s current law of Figure 1; the final equation of the solar cell current can be extracted according to this equivalent as follows [52-61]

\[
I = I_{\text{ph}} - I_D \quad (1)
\]

\[
I_D = I_0 \left( e^{\frac{V_{\text{pv}}}{mV_T}} - 1 \right) \quad (2)
\]

\[
I = I_{\text{ph}} - I_0 \left( e^{\frac{V_{\text{pv}}}{mV_T}} - 1 \right) \quad (3)
\]

where:

- \( I_{\text{ph}} \): the photocurrent in (A), \( I_0 \): the diode’s reverse saturation current in (A), \( V_{\text{pv}} \) and \( I \) are the delivered voltage and current in (A) and (V) respectively.

Thermic voltage can be calculated using the equation

\[
V_T = \frac{kT}{q} = 0.0259 \text{ V} = 26 \text{ mV}, \text{ at room temperature, air-mass is 1.5; m: (1 < m < 2), k: Boltzmann constant}= 1.38 \times 10^{-23} \text{ J/K}; T: \text{ the temperature of the junction (p-n) (K); q = 1.6 \times 10^{-19} C: the electron charge}[36-37].
\]

\[
I_{\text{ph}} = I_{\text{source}} \quad (4)
\]

\[
I_D = I_s \times \left( e^{\frac{V_{\text{ph}}}{e^{nV_T}} - 1} \right) \quad (5)
\]

Substitute Eq. 4 in Eq. 5 yield

\[
(I_{\text{source}}) - 10^{-12} \left( e^{\frac{V}{e^{0.026} + 0.026} - 1} \right) = \frac{V}{R} \quad (6)
\]

where: \( I_s \): reverse saturation current= \( 10^{-12} \text{A} \). In the case of parallel, \( V_D = V_{\text{pv}} = V \)

By the first derivative of Eq. 6, the voltage of the cell is calculated numerically.

3. Newton’s Method [50-55]

The following algorithm suggestion for solving Eq. 6 by using NRM

INPUT initial approximate solution \( x_0 = 1 \), \( \varepsilon \) (tolerance), \( N \): the maximum number of iterations.

The OUTPUT for the approximate solution \( x_{n+1} \)

Step 1: Set \( x = 0 \)
Step 2: while \( i \leq x_0 \)
Step 3: Calculate

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{for} \quad n = 0, 1, 2, \ldots
\]

Step 4: if \( |x_i - x_{i-1}| < \varepsilon \) then OUTPUT \( x_{n+1} \) and stop.
Step 5: put \( i = i + 1 \) and \( n = n + 1 \); and go to Step 2.
Step 6: The OUTPUT
4. Accelerated Predictor-Corrector Hally Method (AHM)

To compare the different numerical methods of iterations, method 1 has been used against the proposed method 2. Also, Eq. 6 has been solved to demonstrate the performance of the new method, and determine the consistency and the stability of results. The results are examined using two iterative methods

Method 1: Newton Raphson Method (NRM)

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, 3, \ldots \]

Method 2: Accelerated Predictor-Corrector Hally Method (AHM)

\[ y_n = x_n - \frac{2f(y_n)x_n - f(y_n)}{2f(y_n)^2 - f(y_n)x_n}, \quad n = 0, 1, 2, 3, \ldots \]
\[ z_n = \frac{x_n + y_n}{2}, \quad n = 0, 1, 2, 3, \ldots \]

We take \( \varepsilon = 10^{-9} \) (tolerance).

For estimating the zero, the following equation is utilized

| \( f(z_n) \) | < \( \varepsilon \), \( \sigma = |z_{n+1} - z_n| < \varepsilon \)

5. Results and Discussion

Suppose Eq. 6 is modelled has obtained the following approximate solutions and the NRM and AHM are achieved based on first initial value \( x_0 \). In Table 1 the NRM and AHM of the solution results; voltage \( V_{pv} \); current \( I_{pv} \); power \( P_{pv} \) of the solar cell. Table 1 shows the obtained results, \( R = 1 \) (load resistance).

| Iterations | \( V_{pv} \cdot \text{NRM} \) | \( I_{pv} \cdot \text{NRM} \) | \( P_{pv} \cdot \text{NRM} \) | \( V_{pv} \cdot \text{AHM} \) | \( I_{pv} \cdot \text{AHM} \) | \( P_{pv} \cdot \text{AHM} \) |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1          | 0.971416861   | 0.971416861   | 0.893473304   | 0.790453895   | 0.850864439   |
| 2          | 0.946732606   | 0.896302627   | 0.918974787   | 0.798294545   |
| 3          | 0.92243135    | 0.850864434   | 0.850864439   |
| 4          | 0.92243135    | 0.850864434   | 0.850864439   |
| 5          | 0.92243135    | 0.850864434   | 0.850864439   |
| 6          | 0.92243135    | 0.850864434   | 0.850864439   |
| 7          | 0.92243135    | 0.850864434   | 0.850864439   |
| 8          | 0.92243135    | 0.850864434   | 0.850864439   |
| 9          | 0.92243135    | 0.850864434   | 0.850864439   |

Figure 2 The obtained values corresponding to Table 1.
Figure 2. The obtained solutions of the study result.

Table 2 shows AHM and NRM results; voltage $V_{pv}$; current $I_{pv}$; power $P_{pv}$ of the solar cell $R = 2$ (load resistance).

**Table 2. The obtained values using NRM and AHM.**

| Iterations | $V_{pv}^{NRM}$ | $I_{pv}^{NRM}$ | $P_{pv}^{NRM}$ | $V_{pv}^{AHM}$ | $I_{pv}^{AHM}$ | $P_{pv}^{AHM}$ |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1          | 0.97103047     | 0.48551524     | 0.47145009     | 0.877625808    | 0.438812904    | 0.385113529    |
| 2          | 0.94542197     | 0.47271098     | 0.44691135     | 0.911522541    | 0.455761271    | 0.415436672    |
| 3          | 0.92683448     | 0.46341724     | 0.42951107     | 0.916798941    | 0.45839947     | 0.420260149    |
| 4          | 0.91843875     | 0.45921937     | 0.42176486     | 0.917034659    | 0.458517329    | 0.420476283    |
| 5          | 0.91706688     | 0.45853344     | 0.42050584     | 0.917035382    | 0.458517691    | 0.420476946    |
| 6          | 0.9170354      | 0.4585177      | 0.42047696     | 0.917035382    | 0.458517691    | 0.420476946    |
| 7          | 0.91703538     | 0.45851769     | 0.42047695     |                |                |                |
| 8          | 0.91703538     | 0.45851769     | 0.42047695     |                |                |                |
| 9          | 0.91703538     | 0.45851769     | 0.42047695     |                |                |                |

Figure 3. The obtained values corresponding to Table 2.
In Table 3 shows AHM and NRM results; voltage $V_{pv}$; current $I_{pv}$; power $P_{pv}$ of the solar cell $R = 3$ (load resistance).

| Iterations | $V_{pv}$ - NRM | $I_{pv}$ - NRM | $P_{pv}$ - NRM | $V_{pv}$ - AHM | $I_{pv}$ - AHM | $P_{pv}$ - AHM |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1          | 0.333333333    | 0.333333333    | 0.691850003    | 0.230616668    | 0.159552142    |                 |
| 2          | 0.323547931    | 0.31404979     | 0.854423029    | 0.284807676    | 0.243346237    |                 |
| 3          | 0.314694744    | 0.297098346    | 0.90112767     | 0.30037589     | 0.270677026    |                 |
| 4          | 0.307864748    | 0.284342109    | 0.909824015    | 0.303274672    | 0.27592658     |                 |
| 5          | 0.304292613    | 0.277781984    | 0.91039934     | 0.303466447    | 0.276275653    |                 |
| 6          | 0.30429613     | 0.276337516    | 0.910403374    | 0.303467791    | 0.276278101    |                 |
| 7          | 0.303467844    | 0.276278197    | 0.910403374    | 0.303467791    | 0.276278101    |                 |
| 8          | 0.303467791    | 0.276278101    |                 |                 |                 |                 |
| 9          | 0.303467791    | 0.276278101    |                 |                 |                 |                 |

Figure 3. The obtained solutions of the study result.

Figure 4 The obtained values corresponding to Table 3.
In Table 4 shows AHM and NRM results; voltage $V_{pv}$; current $I_{pv}$; power $P_{pv}$ of the solar cell $R = 4$ (load resistance).

**Table 4.** The obtained values using NRM and AHM.

| Iterations | $V_{pv}$-NRM | $I_{pv}$-NRM | $P_{pv}$-NRM | $V_{pv}$-AHM | $I_{pv}$-AHM | $P_{pv}$-AHM |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1          | 1            | 0.25         | 0.25         | 0.598809171  | 0.149702293  | 0.089643106  |
| 2          | 0.970256822  | 0.242564205  | 0.235349575  | 0.816819533  | 0.204204883  | 0.166798538  |
| 3          | 0.94271872   | 0.23567968   | 0.222179646  | 0.884826124  | 0.221206531  | 0.195729317  |
| 4          | 0.920123009  | 0.230030752  | 0.211656588  | 0.900161102  | 0.225040276  | 0.202572502  |
| 5          | 0.906346494  | 0.226586624  | 0.205365992  | 0.901713938  | 0.225428485  | 0.203272007  |
| 6          | 0.902077706  | 0.225519427  | 0.203436047  | 0.901740591  | 0.225435148  | 0.203284023  |
| 7          | 0.901742503  | 0.225435626  | 0.203284885  | 0.901740602  | 0.22543515  | 0.203284028  |
| 8          | 0.901740602  | 0.225435151  | 0.203284028  |              |              |              |
| 9          | 0.901740602  | 0.22543515  | 0.203284028  |              |              |              |

Figure 5 The obtained values corresponding to Table 4.
Figure 5. The obtained solutions of the study result.

In Table 5 shows AHM and NRM results; voltage $V_{pv}$; current $I_{pv}$; power $P_{pv}$ of the solar cell $R = 5$ (load resistance).

Table 5. The obtained values using NRM and AHM.

| Iterations | $V_{pv}$-NRM | $I_{pv}$-NRM | $P_{pv}$-NRM | $V_{pv}$-AHM | $I_{pv}$-AHM | $P_{pv}$-AHM |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1          | 0.970256822  | 0.242564205  | 0.235349575  | 0.816819533  | 0.204204883  | 0.166798538  |
| 2          | 0.94271872   | 0.23567968   | 0.222179646  | 0.884826124  | 0.221206531  | 0.195729317  |
| 3          | 0.920123009  | 0.230030752  | 0.211656588  | 0.900161102  | 0.225040276  | 0.202572502  |
| 4          | 0.906346894  | 0.226586624  | 0.205365992  | 0.901713938  | 0.225428485  | 0.203272007  |
| 5          | 0.902077006  | 0.225519427  | 0.203436047  | 0.901740591  | 0.225435148  | 0.203284023  |
| 6          | 0.901747253  | 0.225435626  | 0.203284028  | 0.901740602  | 0.22543515  | 0.203284028  |
| 7          | 0.901740602  | 0.22543515   | 0.203284028  | 0.901740602  | 0.22543515   | 0.203284028  |

Figure 6 The obtained values corresponding to Table 5.
Figure 6. The obtained solutions of the study result.

Obtained results plot number of iteration V, I and P-plane and the initial-output values proves that the proposed method AHM has eight iterations indicated a fast behavior. The comparison between the two methods NRM and AHM is to acquire the results based on the least number of iterations, which confirms that the mathematical method with the least number of iterations is better and faster. The results confirmed with NRM, ten iterations were used, while using the AHM, eight iterations were utilized.

6. Conclusion
In this paper, we observed that the efficiency of the new Accelerated Predictor-Corrector Hally method considerably improve that of Newton method and the given two step method Remark that only 8 iterations are needed to reach the exact solution with small tolerance, while Newton's method requires ten iterations. Data acquired from the proposed method AHM were found to be sufficient and values for single diode solar cell were determined with fast convergence, more capable to determine these parameters and establishing towards the final values.

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