Optimal power flow solution in direct current grids using Sine-Cosine algorithm

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Abstract. In the next years, Colombian power system will have the connection of distributed generators and constant power loads; Therefore, it is necessary to propose the analysis methods that allow establishing the minimum requirements that have to be satisfied for the power system to guarantee an optimal power flow in order to preserve safe and reliable operation of it. For this purpose, in this paper presents an optimal power flow in direct current resistive grids with constant power loads and distributed generators. The optimal power flow problem is formulated as a master-slave optimization algorithm. The master stage covers the dispatching of all the distributed generators by using the Sine-Cosine algorithm. In the slave stage an efficient power flow method based on successive approximations is employed to determine the voltage variables and evaluate the objective function of the problem, which corresponds to the power loss minimization. A direct current distribution network composed by 21 nodes is used as test case by comparing its numerical performance with nonlinear optimization packages and two metaheuristic approaches named black-hole optimization and continuous genetic algorithm. All the simulations are conducted via MATLAB software.

1. Introduction

Direct-current (DC) electrical grids are emerging and promissory networks with potential of using renewable energy resources as photovoltaic plants and battery energy storage system without using inversion stages [1]; additionally, those grids present lower power losses due to low resistive effects in the distribution lines and, frequency and reactive power do not exist [2, 3].

To analyze electrical networks under DC paradigm there are two main approaches, the first one is related with the control of all the power electronic converters that interface nonlinear loads and renewable generation [4, 5]; the second one corresponds to the power flow [6] and optimal power flow analysis [7–10], which are the interest area in this research.

In the specialized literature the optimal power flow (OPF) problem in DC grids has been solved with exact and heuristic approaches; in the framework of the exact methods, the main approaches correspond to semidefinite programming methods [11], second order cone programming [7], or sequential quadratic programming [10, 12]; among others. In the combinatorial (heuristic) context, hybrid methods are shown as the solution for the OPF problem by integrating classical power flow methods with metaheuristic algorithms. This is the case of
the hybrid Gauss-Seidel and continuous algorithm reported in [13], or the hybrid black hole and Gauss-Seidel approach reported in [14].

Based on that situation, this paper deals with the problem from an easy-to-implement combinational and metaheuristic perspective. In this regards, it is proposed a new hybrid master-slave optimization algorithm, composed in the master stage by a combinatorial method named Sine-Cosine algorithm (SCA), which is adequate to analyze continuous problems in constrained solution spaces. In the slave stage a classical power flow problem is solved using the successive approximation method, which allows to work with radial and mesh grids with multiple slack nodes. It is important to mention that the master stage defines the total power output of each generator and the slave stage is responsible to solve the power flow equations and compute the objective function of the problem (i.e., power loss minimization in this paper).

The remainder of this document is organized as follows: Section 2 presents the mathematical formulation of the optimal power flow problem in DC networks with a nonlinear non-convex structure. Section 3 defines the proposed hybrid methodology based on the SCA and the successive approximation power flow method; section 4 presents the main characteristics of the test system and shows the numerical validation of the master-slave optimization algorithm and its comparison with nonlinear solvers and some metaheuristic approaches, with the corresponding analysis and discussion. Finally, section 5 presents the concluding remarks derived from this research.

2. Mathematical model

The OPF problem in DC networks corresponds to a nonlinear non-convex optimization problem, which has the power loss minimization as classical objective function, subject to power balance, voltage regulation and elements capability constraints. The complete mathematical model of the OPF problem is presented in the objective function, Equation (1).

$$\min p_{\text{loss}} = \sum_{j=1}^{n} \sum_{k=1}^{n} g_{jk} v_j v_k,$$

where $p_{\text{loss}}$ is the objective function value associated with the power loss in all the resistive effects along the DC grid conductors, $g_{jk}$ is the $jk$th component of the conductance matrix, and $v_j$ and $v_k$ are the voltage values at nodes $j$ and $k$, respectively. Note that $n$ corresponds to the total number of nodes in the grid.

The complete interpretation of the mathematical model shown in Equation (1) to Equation (7), is described as follows: in Equation (1), it is formulated the objective function of the OPF problem related to the total power loss minimization; Equation (2) and Equation (3) present the power balance at all nodes of the network at all nodes of the network, i.e., slack and constant power load nodes; Equation (4) and Equation (5) show the non-negative variables of the power generation and the constant voltage performance at all the slack nodes, respectively. Finally, in Equation (6) and Equation (7) the minimum and maximum capabilities of the DGs are defined as well as the voltage regulation bounds in all the demand nodes of the network, respectively.

$$p^s_j = v_j \sum_{k=1}^{n} g_{jk} v_j v_k, \quad \forall j \in S,$$

$$p^{d}_j - p^s_j = v_j \sum_{k=1}^{n} g_{jk} v_j v_k, \quad \forall j \in N - S,$$

$$p^s_j \geq 0, \quad \forall j \in S,$$
v_j = v_{js}, \forall j \in S, \quad (5)

0 \leq p_{dg}^j \leq p_{dg,max}^j, \forall j \in \mathcal{N} - S, \quad (6)

v_{jmin}^j \leq v_j \leq v_{jmax}^j, \forall j \in \mathcal{N} - S, \quad (7)

where $p_{s}^j$ represents all the power generation in the slack nodes (contained in the set $S$), $p_{dg}^j$ and $p_{d}^j$ are the power generation in the DGs and the constant power consumption at all demand nodes (contained in the set $\mathcal{N} - S$), $v_{js}$ are the output voltage in all the slack nodes, which are constant values; $p_{dg,max}^j$ is the maximum power generation of a DG located at node $j$; and $v_{jmin}^j$ and $v_{jmax}^j$ are the minimum voltage regulation bounds allowed for the DC grid operation.

Due to the non-convexity of the OPF problem, there are two main ways to solve this problem efficiently: i) using a convex reformulation via sequential quadratic approximations [10, 12], or ii) using metaheurisitic approaches as reported in [13, 14]. Here, we adopted the second option to solve the OPF problem as presented in next section.

3. Proposed methodology

To solve the OPF problem in this paper, we propose a master-slave methodology formed by the SCA in conjunction with the successive approximation power flow method.

3.1. Master stage

In this stage the SCA generates the power outputs in the DGs that minimizes the total power loss in the network [15]. Algorithm 1, presents the pseudocode of the proposed SCA to solve the OPF problem in DC grids with DGs.

For complete details of vectors and matrices used during the evolution of the Sine-Cosine algorithm refers to [15].

**Algorithm 1.** Proposed optimization methodology based on the SCA for OPF analysis.

**Data:** Read data of the network and adjust parameters of the SCA
Generate the initial population $Q^m$;
Evaluate all the individuals $q_i^m$ and find $q_{best,m}$;

**for** $m = 1 : m_{max}$ **do**

**for** $i = 1 : a$ **do**

| Generate the potential individual $x_i^m$;
| Evaluate the fitness function of $x_i^m$, i.e., $z_f(x_i^m)$;
| Determine if the potential individual $x_i^m$ will replace $q_i^m$ and construct the descending population $Q^{m+1}$;

| Evaluate the number of non-consecutive improvements of $z_f$;
| **if** $k \geq k_{max}$ **then**
| Select the best solution contained in $Q^{m+1}$;
| Return the solution of the OPF problem;
| **break**

**end**

**end**

**Result:** Return the solution of the OPF problem
3.2. Slave stage
The slave stage is entrusted of solving the power flow equations by and iterative procedure. Here, we use Equation (8) reported in [6] for power flow analysis named successive approximation approach.

\[
v_{t+1}^d = G_{dd}^{-1} \left[ \text{diag}(v_d^t) \left[ p^{dg} - p^d \right] - G_{ds} v_s \right],
\]

where \(p^{dg}\) and \(p^d\) are vectors that contain all the power generation in the DGs and the constant power consumption at all the demand nodes; \(v_d\) is the vector that contains all the unknown voltage profiles, \(v_s\) is the vector that contains all the slack voltages (i.e., known voltage profiles); and \(G_{ds}\) and \(G_{dd}\) are sub-matrices of the conductance matrix, which define the relation between demands and slacks, and between demands, respectively. Note that \(\text{diag}(v_d)\) generates a diagonal matrix with the components of the vector \(v_d\); \(t\) is the iterative counter that starts from 0 to \(t_{\text{max}}\), and we can say that the successive approximation algorithm converges if \(\max(|v_{t+1}^d - v_t^d|) \leq \epsilon\), being \(\epsilon\) the convergence error, typically set as \(1 \times 10^{-10}\) in DC power flow analysis [6].

4. Test system and computational validation
A 21-nodes test feeder described in the specialized literature is employed [2, 12]; complete information of this test system can be found in [14, 16]. In addition, this study analyzes the possibility to install one to three generators considering penetration percentages from 20% to 60% of the total power consumption. Note that, via heuristic search methods, nodes 8, 12 and 21 were selected for DG location in this test feeder. In addition, all the simulations were carried-out on a desktop computer with an INTEL(R) Core(TM) i5-3550 processor at 3.50 GHz, with 8 GB RAM, running the 64-bit Windows 7 Professional operating system. For all the simulations, the population size was considered of 10 individuals with 1000 iterations.

In Table 1 the numerical results are reported when the OPF problem is solved with the proposed master-slave optimization algorithm composed by the SCA and the successive approximation method, considering that the admissible power injection for each distributed generator is 1.5 p.u as maximum. Note that the results of the simulation are compared to the black hole optimizer (BHO) [14], the continuous genetic algorithm (CGA) [13] and the general algebraic modeling system (GAMS) package [17].

| Method | \(p_{8}^{dg}\) (p.u) | \(p_{12}^{dg}\) (p.u) | \(p_{21}^{dg}\) (p.u) | \(p_{\text{loss}}\) (p.u) |
|--------|----------------------|----------------------|----------------------|----------------------|
| BHO    | 1.2910               | 1.3114               | 1.2573               | 0.0641               |
| CGA    | 1.0780               | 1.4865               | 1.1971               | 0.0630               |
| GAMS   | 1.1193               | 1.5000               | 1.1893               | 0.0629               |
| SCA    | 1.1234               | 1.5000               | 1.1886               | 0.0629               |

From results in Table 1 it is possible to affirm that: (i) the proposed SCA reaches the same optimization solution reported by the nonlinear large-scale optimization package GAMS with four decimals of precision, which does not occur with the BHO and the CGA; (ii) the proposed SCA improves the solution reported by the CGA about 0.159% in relation to the power loss, which can be classified in practical terms as a technical draw; and (iii) the BHO approach has the worst result of the comparison methods with an error about 1.908% in comparison with GAMS and the proposed approach.
It is important to mention that the proposed SCA approach and the GAMS package have a similar power injection per node when compared with the CGA; however, the BHO solution behavior is different, due to the algorithm gets stuck in a local solution.

Finally, note that for this test system the power loss minimization respect to the base case (without generation output in all the distributed generators) is 77.210% when GAMS and the SCA approaches are used for OPF analysis, which shows the importance on the effect for correct power generation dispatch in the operative behavior of DC electrical networks.

5. Conclusion and future works

This paper proposed a hybrid master-slave optimization algorithm composed by a Sine-Cosine algorithm in the master stage and the successive approximation power flow method in the slave stage to solve OPF problems in DC grids. This approach showed superior numerical performance when compared with other combinatorial approaches such as black-hole optimizer and continuous genetic algorithm approach in terms of the objective function measurement. Additionally, the SCA algorithm reached the same optimal power flow solution achieved by a large-scale nonlinear optimizer package known as GAMS, which implies that its numerical behavior is adequate and comparable with specialized solvers. The main advantage of the proposed approach was the ease implementation on any programming language or free development software, since it is a pure-algorithmic approach than can be embedded in free applications, that can be used for students, engineers or utility companies without license requirements.

As future work, it will be possible to embed the proposed SCA for OPF analysis inside of a discrete metaheuristic technique to define the optimal location of DGs in electrical distribution systems for planning purposes.

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