Holomorphic families of Fatou-Bieberbach domains and applications to Oka manifolds

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Abstract We construct holomorphically varying families of Fatou-Bieberbach domains with given centres in the complement of any compact polynomially convex subset $K$ of $\mathbb{C}^n$ for $n > 1$. This provides a simple proof of the recent result of Y. Kusakabe to the effect that the complement $\mathbb{C}^n \setminus K$ of any polynomially convex subset $K$ of $\mathbb{C}^n$ is an Oka manifold. The analogous result is obtained with $\mathbb{C}^n$ replaced by any Stein manifold with the density property.

Keywords Fatou-Bieberbach domain, polynomially convex set, Oka manifold

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1. Introduction

A Fatou-Bieberbach domain in $\mathbb{C}^n$ is a proper subdomain $\Omega \subsetneq \mathbb{C}^n$ which is biholomorphic to $\mathbb{C}^n$. No such domains exists for $n = 1$, but they are plentiful for any $n > 1$; see the survey of this topic in [3, Chapter 4]. In particular, the basin of attraction of an attracting fixed point of a holomorphic automorphism of $\mathbb{C}^n$ (or in fact of any complex manifold) is biholomorphic to $\mathbb{C}^n$, cf. [11] and [3, Theorem 4.3.2]. Furthermore, for any compact polynomially convex set $K \subset \mathbb{C}^n$ for some $n > 1$ and point $p \in \mathbb{C}^n \setminus K$ there is a Fatou-Bieberbach domain $\Omega \subset \mathbb{C}^n$ such that $p \in \Omega$ and $K \cap \Omega = \emptyset$; this is a special case of [5, Proposition 9] where the same is shown with $p$ replaced by any compact convex set.

In this note we prove the following more general result in this direction.

Theorem 1.1. Let $K$ be a compact polynomially convex set in $\mathbb{C}^n$ for some $n > 1$, $L$ be a compact polynomially convex set in $\mathbb{C}^N$ for some $N \in \mathbb{N}$, and $f : U \to \mathbb{C}^n$ be a holomorphic map on an open neighbourhood $U \subset \mathbb{C}^N$ of $L$ such that $f(z) \in \mathbb{C}^n \setminus K$ for all $z \in L$. Then there are an open neighbourhood $V \subset U$ of $L$ and a holomorphic map $F : V \times \mathbb{C}^n \to \mathbb{C}^n$ such that for every $z \in V$ we have that $F(z, 0) = f(z)$ and the map $F(z, \cdot) : \mathbb{C}^n \to \mathbb{C}^n \setminus K$ is injective. Hence, $\Omega_z := \{F(z, \zeta) : \zeta \in \mathbb{C}^n\}$ is a Fatou-Bieberbach domain in $\mathbb{C}^n \setminus K$ for each $z \in V$.

A proof of this result based solely on Andersén-Lempert theory is given in Section 2; it also applies if $\mathbb{C}^N$ is replaced by an arbitrary Stein manifold, and also to variable fibres $K_z \subset \mathbb{C}^n$, $z \in L$, with polynomially convex graph (see Remark 2.2). For a convex parameter space $L \subset \mathbb{C}^N$ we prove the analogous result with $\mathbb{C}^n$ replaced by an arbitrary Stein manifold having the density property; see Theorem 3.1.

These two theorems immediately imply the following recent and very interesting result of Yuta Kusakabe.

Theorem 1.2. (Kusakabe, [9, Theorem 1.2 and Corollary 1.3].) For any compact holomorphically convex subset $K$ in a Stein manifold $Y$ with the density property the complement $Y \setminus K$ is an Oka manifold. In particular, the complement $\mathbb{C}^n \setminus K$ of any compact polynomially convex set $K$ in $\mathbb{C}^n$ for $n > 1$ is an Oka manifold.
This is the first result in the literature which gives a large class of Oka domains in $\mathbb{C}^n$ for any $n > 1$, and it provides an affirmative answer to a long-standing problem. As noted in [9] Corollary 1.4, it follows from Theorem 1.2 and [4] Theorem 1.1 that for any compact polynomially convex set $K$ in $\mathbb{C}^n$ ($n > 1$), the complement $\mathbb{C}^n \setminus K$ (like any $n$-dimensional Oka manifold) is the image of a strongly dominating holomorphic map $\mathbb{C}^n \to \mathbb{C}^n \setminus K$.

Recall that a complex manifold $Y$ is said to be an Oka manifold if every holomorphic map from a neighbourhood of a compact (geometrically) convex set $L$ in a Euclidean space $\mathbb{C}^N$ into $Y$ is a uniform limit on $L$ of entire maps $\mathbb{C}^N \to Y$ (see [3] Definition 5.4.1); this is also called the convex approximation property and denoted CAP. By [3] Theorem 5.4.4, holomorphic maps $S \to Y$ from any reduced Stein space $S$ to an Oka manifold $Y$ satisfy all natural Oka-type properties. In his recent paper [8], Kusakabe showed that a complex manifold $Y$ is Oka if (and only if) it satisfies the following condition:

(*) For any compact convex set $L \subset \mathbb{C}^N$, open set $U \subset \mathbb{C}^N$ containing $L$, and holomorphic map $f : U \to Y$ there are an open set $V$ with $L \subset V \subset U$ and a holomorphic map $F : V \times \mathbb{C}^N \to Y$ with $F(\cdot, 0) = f|_V$ such that

$$\frac{\partial}{\partial \zeta} |_{\zeta=0} F(z, \zeta) : \mathbb{C}^N \to T_{f(z)}Y$$

is surjective for every $z \in V$.

A map $F$ with these properties is called a dominating holomorphic spray over $f$. This is a restricted version of condition Ell$_1$ introduced by Gromov [6, p. 72] (see also [7]). In [8], Kusakabe used the technique of gluing sprays from [3, Sect. 5.9] to show that this condition implies CAP, so $Y$ is an Oka manifold. Conversely, it has been known before that every Oka manifold satisfies condition Ell$_1$ (see [3, Corollary 8.8.7]).

Theorem 1.1 provides a very special dominating spray with values in $\mathbb{C}^n \setminus K$ over any given holomorphic map $f : L \to \mathbb{C}^n \setminus K$, thereby proving Theorem 1.2 in the case $Y = \mathbb{C}^n$. In exactly the same way, Theorem 3.1 implies the general case of Theorem 1.2.

Kusakabe also proved in [9] Theorem 4.2 that certain closed noncompact sets in Stein manifolds $Y$ with the density property have Oka complements. He constructed a holomorphically varying family $f(z) \in \Omega_z \subset Y \setminus K$ of nonautonomous basins with uniform bounds (i.e., basins of random sequences of automorphisms of $Y$ which are uniformly attracting at $f(z) \in Y \setminus K$); these are elliptic manifolds as shown by Fornæss and Wold [2], hence Oka. When $Y = \mathbb{C}^n$, the domains $\Omega_z$ can be chosen Fatou-Bieberbach domains by using Theorem 1.1 with variable fibres (cf. Remark 2.2). Kusakabe’s proof of [9] Theorem 4.2] can also be modified so as to provide a family of Fatou-Bieberbach domains in the general situation under consideration.

2. Proof of Theorem 1.1

We shall use some standard facts concerning polynomial convex sets; we refer the reader to the monograph by E. L. Stout [12]. Firstly, if $K_1, K_2 \subset \mathbb{C}^n$ is a pair of disjoint compact sets such that $K_1 \cup K_2$ is polynomially convex, then for any polynomially convex set $K'_1 \subset K_1$ the union $K'_1 \cup K_2$ is also polynomially convex. Secondly, every compact polynomially convex set $K \subset \mathbb{C}^n$ is the zero set of a nonnegative plurisubharmonic exhaustion function $\rho : \mathbb{C}^n \to [0, +\infty)$ which is strongly plurisubharmonic on $\mathbb{C}^n \setminus K$. Choosing a sequence $c_1 > c_2 > \cdots > 0$ with $\lim_{n \to \infty} c_i = 0$ and setting $K_i = \{\rho \leq c_i\} \subset K$ yields a decreasing sequence of compact polynomially convex sets with $K_{i+1}$ contained in the interior of $K_i$ for every $i \in \mathbb{N}$.
Let $f$, $K$ and $L$ be as in the theorem. We replace $L$ by a slightly bigger polynomially convex set (still denoted $L$) contained in $U$ and such that $f(z) \in \mathbb{C}^n \setminus K$ for all $z \in L$. Choose a sequence $K_i \supset K$ as above, with $K_i$ chosen close enough to $K$ such that $f(z) \in \mathbb{C}^n \setminus K_1$ for every $z \in L$. The compact set $L \times K_i \subset \mathbb{C}^{N+n}$ is polynomially convex for every $i \in \mathbb{N}$. Applying the change of coordinates $\psi(z, \zeta) = (z, \zeta - f(z))$ replaces $f$ by the zero function, and for every $i \in \mathbb{N}$ the set

$$S_i = \psi(L \times K_i) \subset L \times \mathbb{C}^n \subset \mathbb{C}^{N+n}$$

is polynomially convex and does not intersect $\mathbb{C}^N \times \{0\}^n$. Hence, $(L \times \{0\}^n) \cup S_1$ is polynomially convex. Therefore, there is a small closed ball $B \subset \mathbb{C}^n$ centred at $0 \in \mathbb{C}^n$ such that $(L \times B) \cap S_1 = \emptyset$ and $(L \times B) \cup S_1$ is polynomially convex. Since $S_1 \subset S_i$ is polynomially convex, it follows that $(L \times B) \cup S_i$ is polynomially convex for each $i \in \mathbb{N}$.

The following lemma will be used in the inductive construction.

**Lemma 2.1.** (Assumptions as above.) Let $B' \subset \mathbb{C}^n$ be a closed ball centred at the origin with $B \subset B'$. Then, there are an open neighbourhood $U' \subset U$ of $L$ and a biholomorphic map $\Phi : U' \times \mathbb{C}^n \to U' \times \mathbb{C}^n$ of the form $\Phi(z, \zeta) = (z, \phi(z, \zeta))$ such that

(a) $\Phi$ approximates the identity map as closely as desired on $L \times B$ and $\Phi(z, 0) = (z, 0)$ for all $z \in U'$,

(b) $\Phi(S_1) \cap (L \times B') = \emptyset$, and

(c) the set $\Phi(S_2) \cup (L \times B')$ is polynomially convex.

**Proof.** Choose $r > 1$ such that $rB = B'$. Letting $\theta_r(z, \zeta) = (z, r\zeta)$ for $z \in \mathbb{C}^N$ and $\zeta \in \mathbb{C}^n$, we have that $(L \times B') \cap \theta_r(S_1) = \emptyset$ and

$$\text{(2.1)} \quad (L \times B') \cup \theta_r(S_1) = \theta_r((L \times B) \cup S_1) \text{ is polynomially convex.}$$

Consider the isotopy of biholomorphic maps $\phi_t$ on a neighbourhood of $(L \times B) \cup S_1$ in $\mathbb{C}^{N+n}$ for $t \in [1, r]$ which equals the identity map on a neighbourhood of $L \times B$ and equals $\theta_t$ on a neighbourhood of $S_1$. Note that $\theta_t(S_1)$ is disjoint from $L \times B$ and the union $(L \times B) \cup \theta_t(S_1)$ is polynomially convex for all $t \in [1, r]$ (since it is contained in $\theta_t((L \times B) \cup S_1) = (L \times tB) \cup \theta_t(S_1)$ which is polynomially convex). Hence, by the parametric Andersén-Lempert theorem (see [10] and [3, Theorem 4.12.3]) there is a holomorphic automorphism of $\mathbb{C}^{N+n}$ of the form $\Phi(z, \zeta) = (z, \phi(z, \zeta))$ which approximates the identity map on $L \times B$ and it approximates $\theta_r$ on $S_1$. Hence, conditions (a) and (b) in the lemma hold. Assuming that the approximations are close enough, we have $\Phi(S_2) \subset \theta_r(S_1)$. Note that $\Phi(S_2)$ is polynomially convex. In view of (2.1) it follows that $\Phi(S_2) \cup (L \times B')$ is polynomially convex as well which gives condition (c). \hfill $\square$

**Proof of Theorem 1.1** We apply the push-out method described in [3, Section 4]. Using Lemma 2.1 we inductively construct a decreasing sequence of open neighborhoods $U_k$ of $L$ and holomorphic automorphisms $\Phi_k(z, \zeta) = (z, \phi_k(z, \zeta))$ of $U_k \times \mathbb{C}^n$ such that, setting

$$\Phi^k = \Phi_k \circ \Phi_{k-1} \circ \cdots \circ \Phi_1 : U_k \times \mathbb{C}^n \to U_k \times \mathbb{C}^n,$$

the following conditions hold for every $k \in \mathbb{N}$.

(i) $\Phi_k$ approximates the identity map as closely as desired on $L \times kB$ and $\Phi_k(z, 0) = (z, 0)$ for all $z \in U_k$.

(ii) $\Phi^k(S_k) \cap (L \times (k+1)B) = \emptyset$.

(iii) The set $\Phi^k(S_{k+1}) \cup (L \times (k+1)B)$ is polynomially convex.
Indeed, the lemma furnishes the first map $\Phi_1$ with $B' = 2B$ and the sets $S_2 \subset S_1$; every subsequent step is of the same form by just increasing the indices. Assuming that the approximations are close enough, [3 Proposition 4.4.1 and Corollary 4.4.2] show that the limit $\Phi = \lim_{k \to \infty} \Phi^k$ exists uniformly on compacts on the domain

$$
\Omega = \{(z, \zeta) \in L \times \mathbb{C}^n : \Phi^k(z, \zeta) \text{ is a bounded sequence}\} = \bigcup_{k=1}^{\infty} (\Phi^k)^{-1}(L \times kB),
$$

and for every $z \in L$, $\Phi(z, \cdot)$ maps the fibre $\Omega_z = \{\zeta \in \mathbb{C}^n : (z, \zeta) \in \Omega\}$ biholomorphically onto $\mathbb{C}^n$. By condition (ii) the set $S = \psi(L \times K)$ does not intersect $\Omega$ (it has been pushed to infinity by the sequence $\Phi^k$). Hence, the inverse map $\Phi^{-1}(z, \zeta) = (z, \varphi(z, \zeta))$ provides a holomorphic family of Fatou-Bieberbach maps $\varphi(z, \cdot) : \mathbb{C}^n \to \mathbb{C}^n$ ($z \in L$) such that $\varphi(z, 0) = 0$ and its image does not intersect the set $K - f(z)$. The function $F(z, \zeta) = \varphi(z, \zeta) + f(z)$ for $z \in L$ and $\zeta \in \mathbb{C}^n$ satisfies the conclusion of the theorem. $\square$

**Remark 2.2.** The above proof also applies in the case when the product $L \times K$ is replaced by a compact polynomially convex set $K \subset \mathbb{C}^{N+n}$ projecting onto $L$ whose fibres $K_z$ ($z \in L$) depend on $z$. The conclusion remains the same, that is, given a holomorphic map $f : L \to \mathbb{C}^n$ with $f(z) \in \mathbb{C}^n \setminus K_z$ for all $z \in L$, there is a holomorphically variable family of Fatou-Bieberbach domains $f(z) \in \Omega_z \subset \mathbb{C}^n \setminus K_z$ for all $z \in L$.

### 3. Fatou-Bieberbach domains in Stein manifolds with the density property

In this section we give a version of Theorem [11] with $\mathbb{C}^n$ replaced by an arbitrary Stein manifold with the density property. (See Varolin [14] or [3 Definition 4.10.1] for this notion.) Every such manifold has dimension $> 1$. The following result is similar to Theorem [11], but we impose the extra condition that the set $L$ is geometrically convex.

**Theorem 3.1.** Let $X$ be a Stein manifold with the density property, $K$ be a compact holomorphically convex set in $X$, $L$ be a compact convex set in $\mathbb{C}^N$ for some $N \in \mathbb{N}$, and $f : U \to X$ be a holomorphic map on an open neighbourhood $U \subset \mathbb{C}^N$ of $L$ such that $f(z) \in X \setminus K$ for all $z \in L$. Then there are a neighbourhood $V \subset U$ of $L$ and a holomorphic map $F : V \times \mathbb{C}^n \to X$ with $n = \dim X$ such that for every $z \in V$ we have that $F(z, 0) = f(z)$ and the map $F(z, \cdot) : \mathbb{C}^n \to X \setminus K$ is injective.

Hence, $\Omega_z := \{F(z, \zeta) : \zeta \in \mathbb{C}^n\} \subset X \setminus K$ is a Fatou-Bieberbach domain of the first kind (i.e., biholomorphic to $\mathbb{C}^n$) for each $z \in V$.

The proof of Theorem 3.1 depends on the following interpolation result for graphs. We denote by $\text{dist}_X$ a distance function on $X$ compatible with the manifold topology.

**Lemma 3.2.** Let $X, K, L, U$ and $f$ be as above, and let $z_0 \in L$ be arbitrary. Then for any $\varepsilon > 0$ there exist a neighbourhood $V \subset U$ of $L$ and a fibred holomorphic automorphism $\phi(z, x) = (z, \varphi(z, x))$ of $V \times X$ such that $\phi(z, f(z)) = (z, f(z_0))$ for all $z \in V$, and $\text{dist}_X(\varphi(z, x), x) < \varepsilon$ for all $z \in L$ and $x \in K$.

**Proof.** We may assume that $z_0 = 0 \in \mathbb{C}^N$. Let $V_1, \ldots, V_m$ be complete holomorphic vector fields on $X$ such that $V_1(x), \ldots, V_m(x)$ span the tangent space $T_xX$ for all $x \in X$ (such exist by [3 Proposition 5.6.23] since $X$ is Stein and has the density property). Let $\psi_{s_1, \ldots, s_m}$ denote their respective flows, $s \in \mathbb{C}$. Consider the map $\Psi : \mathbb{C}^m \times X \to X$ defined for $s = (s_1, \ldots, s_m) \in \mathbb{C}^m$ and $x \in X$ by

$$
\Psi(s_1, \ldots, s_m, x) = \psi_{s_m, s_{m-1}} \circ \cdots \circ \psi_{s_1, s_0}(x).
$$
Note that $Ψ_s := Ψ(s, ·) ∈ \text{Aut}(X)$ for every $s ∈ \mathbb{C}^m$. Then the partial differential $\frac{∂s}{|z|^2} \partial s_{z=0}(Ψ(s, f(0)))$ has maximal rank $n = \dim X$, so there exists an $n$-dimensional linear subspace $Λ ⊂ \mathbb{C}^m$ on which this differential has rank $n$. We may assume that $Λ = \mathbb{C}^n \times \{0\}^{m-n}$. Write $s = (s', s'')$ with $s' ∈ \mathbb{C}^m$ and $s'' ∈ \mathbb{C}^{m-n}$. It follows that there exists $δ > 0$ such that the map $s' ↦ Ψ_s := Ψ((s', 0^m), f(0)) \in X$ is an embedding of the open $δ$-ball $B_δ ⊂ \mathbb{C}^n$ centred at $0 ∈ \mathbb{C}^n$ onto an open neighbourhood of $f(0) ∈ X$.

We replace $L$ by a slightly larger convex set $L' ⊂ U$ with $L ⊂ (L')^0$ without changing the notation. We also choose a compact holomorphically convex set $K' ⊂ X$ containing $K$ in its interior and such that $f(z) ∈ X \setminus K'$ for all $z ∈ L$. Set $f_t(z) = f(t \cdot z)$ for $z ∈ L$ and $t ∈ [0, 1]$. Consider the isotopy $φ_t(z, x)$ defined to be the identity near $L × K'$ and $φ_t(z, f(z)) = (z, f_{1-t}(z)), 0 ≤ t ≤ 1$ on the graph $Z := \{(z, f(z)) : z ∈ L\} ⊂ \mathbb{C}^N \times X$. The image of $φ_t$ is the disjoint union of $L × K'$ and the holomorphic graph of $f_{1-t}$ over $L$, so it is holomorphically convex in $\mathbb{C}^N \times X$. By using [3] Proposition 3.3.2 (a fibred version of the tubular neighbourhood theorem for Stein manifolds) along with the Oka-Grauert principle we can extend $φ_t$ to a fibred isotopy of injective holomorphic maps on an open neighbourhood of $Z$ in $\mathbb{C}^N \times X$. Since $X$ has the density property, given $η > 0$ there is a fibred holomorphic automorphism $\tilde{φ}(z, x) = (z, \tilde{φ}(z, x))$ of $L × X$ such that $\text{dist}_X(\tilde{φ}(z, f_1(z)), f(0)) < η$ for $z ∈ L$ and $\text{dist}_X(\tilde{φ}(z, x), x) < ϵ/2$ for $z ∈ L$ and $x ∈ K'$ (see [10] and [3] Theorems 4.10.5 and 4.12.3). Note that $f_1 = f$. If $η > 0$ is chosen small enough, there exists for each $z ∈ L$ a unique point $λ(z) ∈ B_δ ⊂ \mathbb{C}^n$ such that $Ψ_{λ(z)}(\tilde{φ}(z, f(z))) = f(0)$. The fibred holomorphic automorphism

$$φ(z, x) = \left(z, Ψ_{λ(z)}^{-1}(\tilde{φ}(z, x))\right), \quad z ∈ L, x ∈ X$$

then satisfies the lemma provided $η > 0$ is chosen small enough.

We will also need the following basic result which we include lacking a reference. (The existence of a Fatou-Bieberbach domain of the first kind containing a point $p ∈ X$ was proved by Varolin [13], but this is not sufficient for our purpose.)

**Lemma 3.3.** Let $X$ be a Stein manifold with the density property, let $K ⊂ X$ be a holomorphically convex compact set, and let $p ∈ X \setminus K$. Then there exists a Fatou-Bieberbach domain $Ω ⊂ X \setminus K$ of the first kind such that $p ∈ Ω$.

**Proof.** Let $K'$ be a holomorphically convex compact set in $X$ containing $K$ in its interior and such that $p \notin K'$. Choose local coordinates $φ : U_p → \mathbb{C}^n$ near $p$ such that $φ(p) = 0$. Let $δ > 0$ be small enough such that $φ^{-1}(B_δ(0)) \cap K' = ∅$ and $φ^{-1}(B_δ(0)) \cup K'$ is holomorphically convex. Let $F : \overline{B_δ(0)} → \overline{B_δ(0)}$ be the map $F(z_1, ..., z_n) = \left(\frac{z_1}{2}, ..., \frac{z_n}{2}\right)$. Utilizing the density property of $X$ we now approximate the map $φ^{-1} \circ F \circ φ$ on $φ^{-1}(\overline{B_δ(0)})$ and the identity map on $K'$ to obtain a sequence $G_j ∈ \text{Aut}(X)$ such that for any $k ∈ \mathbb{N}$ we have that $G_k \circ ... \circ G_1(K) ⊂ K'$, and setting $F_j = φ \circ G_j \circ φ^{-1}$ we have that

$$s \cdot ||z|| ≤ ||F_j(z)|| ≤ r \cdot ||z||, \quad j ∈ \mathbb{N}$$

on $B_δ(0)$, with $r^2 < s < 1$. Now, following [15] proof of Theorem 4] we have that the abstract basin of attraction, or the tail space $\tilde{Ω}$ (see [11] associated to $\{F_j\}$, is biholomorphic to $\mathbb{C}^n$, and the basin of attraction $Ω$ of $\{G_j\}$ is biholomorphic to $\tilde{Ω}$. □

**Proof of Theorem 4.1.** This is an immediate consequence of Lemmas 3.2 and 3.3 □
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