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Spin-twisted Optical Lattices: Tunable Flat Bands and Larkin-Ovchinnikov Superfluids

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Moiré superlattices in twisted bilayer graphene and transition-metal dichalcogenides have emerged as a powerful tool for engineering novel band structures and quantum phases of two-dimensional quantum materials. Here we investigate Moiré physics emerging from twisting two independent hexagonal optical lattices of atomic (pseudo-)spin states (instead of bilayers), which exhibits remarkably different physics from twisted bilayer graphene. We employ a momentum-space tight-binding calculation that includes all range real-space tunnelings, and show that all twist angles $\theta \lesssim 6^\circ$ can become magic that support gapped flat bands. Due to greatly enhanced density of states near the flat bands, the system can be driven to superfluid by weak attractive interaction. Strikingly, the superfluid phase corresponds to a Larkin-Ovchinnikov state with finite momentum pairing, resulting from the interplay between flat bands and inter-spin interactions in the unique single-layer spin-twisted lattice. Our work may pave the way for exploring novel quantum phases and twistorics in cold atomic systems.

Introduction.—Twisting two weakly-coupled adjacent crystal layers has been employed as a powerful tool for tailoring electronic properties of two-dimensional quantum materials [1–7], such as the formation of Moiré superlattices and flat bands. This has been evidenced by the recent groundbreaking discovery of superconductivity and correlated insulator phases in twisted bilayer graphene (TBG) [8, 9], which provide a rich platform for exploring strongly-correlated many-body phases [10–15], with the underlying physical mechanisms still under investigation [16–26]. In TBG, the interactions, the inter- and intra-layer couplings are generally fixed with very limited tunability [27–30], and magic flat bands occur only in a narrow range of very small twist angles around $\sim 1.1^\circ$. Going beyond layer degree of freedom in TBG, two questions naturally arise. Can lattices of other pseudo degrees be twisted to realize novel Moiré lattices and flat bands with great tunability? If so, can new physics emerge in such twisted systems?

Ultracold atoms in optical lattices provide a promising platform for exploring many-body physics in clean environments with versatile tunability [31–47]. While it is challenging to realize twisted bilayer lattices, the atomic internal states offer a pseudospin degree, where optical lattice for each spin state can be controlled independently (in particular for alkaline-earth atoms) [48–51], allowing the realization of spin-twisted-lattices and related Moiré physics. Such spin-twisted-lattices have several remarkable difference from TBG. For instance, two spins reside on one layer spatially (instead of bilayer in TBG) with their coupling provided by additional lasers, resulting in different inter-spin (compared with inter-layer in TBG) hopping and other physical parameters. The interaction is dominated by the inter-spin s-wave scattering between fermion atoms in relatively twisted spin lattices, in contrast to the uniform intra-layer interaction without spin twist in TBG. These differences can significantly affect the resulting band structures and many-body quantum states. It is unclear whether extremely flat and gapped bands (i.e., magic-angle behaviors) can exist in spin-twisted single-layer lattice. If yes, how large can the magic angle be tuned to? Can new phases emerge from twisted inter-spin interactions?

In this Letter, we address these important questions by investigating the Moiré physics for cold atoms in two spin-dependent hexagonal lattices twisted by a relative angle, with two spin states coupled by additional uniform lasers. Our main results are:

i) We employ a momentum-space tight-binding method to include all range real-space tunnelings with high accuracy, which is crucial for obtaining the correct flat band structures and low-energy physics.

ii) Because of the tunability of inter-spin coupling strength and lattice depth, all twist angles with $\theta \lesssim 6^\circ$ can become magic and support extremely flat and gapped bands. In general, a smaller magic angle requires weaker inter-spin coupling or a shallower lattice. When $\theta$ is too large, no flat bands exist in the whole parameter space due to strong inter-valley coupling.

iii) The system can be driven to the superfluid phase by very weak attractive interactions at magic angles where the flat bands greatly enhance the density of states (DOS). Strikingly, the superfluid phase corresponds to a Larkin-Ovchinnikov (LO) state [52] with nonzero pairing momentum and staggered real-space pairing order at the hexagonal lattice scale, which does not exist in TBG and results from the interplay between flat bands and the unique inter-spin interactions of atoms in relatively twisted spin lattices.

Model.—To obtain independent optical lattices that can be twisted, we consider two long-lived $^3S_0$ and $^3P_0$ orbital states (denoted as pseudospin states $\uparrow$ and $\downarrow$) of alkaline-earth(-like) atoms [48–51] as shown in Fig. 1a. Atoms in state $\uparrow$ ($\downarrow$) are trapped solely by $\lambda_{\uparrow\downarrow}$-wavelength lasers, which are tuned-out for atoms in state $\downarrow$ ($\uparrow$) (e.g., $\lambda_{\uparrow\downarrow} = 627$nm, 689nm for Sr atoms).
A hexagonal lattice \( V(r) = -V_0 \sum_{j=1}^3 \epsilon_j \exp[i k_{L,j} \cdot (r - r_0)] \) is generated by intersecting three lasers at 120° in the \( xy \) plane with each beam linearly in-plane polarized [37]. Here \( V_0 \) is the trap depth, \( r_0 \) is the hexagonal plaquette center, \( k_{L,j} \) and \( \epsilon_j \) are the laser wave vector and polarization. Hereafter, we set momentum and energy units as \( k_R = 2\pi/\lambda_1 \) and \( E_R = \hbar^2 k_R^2/2m \). The two spin-dependent potentials \( V_{\uparrow,\downarrow}(r) \) are obtained through twisting \( V(r) \) by \( \pm \theta/2 \) (see Fig. 1b). The shorter-wavelength \( \lambda_1 \) lasers have an out-of-plane angle to ensure the same lattice constant for both potentials. The \( z \)-direction is tightly confined by an additional state-independent potential using the so-called magic-wavelength lasers [35], which reduces the dynamics to two dimensions (2D). The two pseudospin states are coupled by a clock laser [35] propagating along \( z \), with \( \Omega \) the Rabi frequency.

We first consider commensurate twists with \( \cos(\theta) = \frac{m^2+n^2+4mn}{2(n^2+m^2+mn)} \) parameterized by two integers \((m, n)\) [1]. In Figs. 1c and 1d, the real-space pattern and Moiré Brillouin zone (BZ) are shown together with the bare BZs of two spins. For typical lattice depth, long-range tunneblings beyond nearest neighbors (especially for the inter-spin couplings where the site separations take various values and are nearly continuously distributed for small twists) should be taken into account to obtain the correct magic flat bands [53]. Small deviations in the tunneling coefficients may result in significant change in the flat band structures due to the narrow bandwidth. Here we adopt the momentum-space Bloch basis \( \{ \phi_{s\ell k}(r) \} \) (with \( k \), the Bloch momentum, \( l \) the band index and \( s = \uparrow, \downarrow \)) of \( V_s(r) \) which spans the same tight-binding Hilbert space as the Wannier basis. When the two spins are decoupled, the lowest two bands of each spin state form two Dirac points for \( k_s \) at valley \( K_s \) and \( K'_s \) in the bare BZs [53].

By projecting onto the basis \( \{ \phi_{s\ell k}(r) \} \), the inter-spin coupling Hamiltonian reads [53]

\[
H_{\uparrow\downarrow}(q) = \sum_{l,r,l',r'} j_{g,l',l}^{\uparrow\downarrow}(q) \alpha_{l'g,r}^\uparrow \alpha_{lq+g,r}^\downarrow + h.c.,
\]

where \( \alpha_{slk}^\uparrow \) are the creation operators of the Bloch states, \( q \) is the superlattice Bloch momentum in the Moiré BZ and \( g \) are the reciprocal lattice vectors of the Moiré superlattice whose summation runs over the bare BZ of state \( s \). The inter-spin coupling coefficients are \( j_{g,l'l}\) which already incorporate all range real-space tunneblings. Another advantage of this momentum-space approach is that if only the low-energy physics is of interest, then we only need to keep \( l \) and \( g \), that correspond to the low-energy Bloch states [1–4], leading to a rather rapid convergence of the basis set.

Although spin-twisted optical lattices share some similarities with TBG, several important differences need be noted: 1) The two twisted optical potentials are spin-dependent and do not affect each other, while in TBG electrons in one layer can feel the potential of the other layer; 2) The inter-spin couplings in the single layer (realized by additional lasers) are different from the inter-layer tunnelings in TBG [1, 53]; 3) The optical lattice potential takes a simple cosine form, therefore the bare bands and inter-spin couplings can be obtained accurately from the Bloch states, while TBG Hamiltonians are usually based on real-space tight-binding approximation expressed in Slater-Koster parameters [1, 54–57]; 4) Long-range tunnelings are more significant due to shallow lattices considered here, which not only improve the atomic lifetime, but also increase the bare Dirac velocity. 5) Interactions are dominated by the \( s \)-wave scattering between fermion atoms in relatively twisted lattices, while electronic interactions in TBG, including both Coulomb repulsive and phonon-mediated attractive interactions, mainly involve electrons in the same layer with no relative twist [16–21]; 6) Finally, the cold-atom parameters (e.g., inter-spin tunnelings, lattice depth, lattice constant, interactions, etc.) are highly tunable, comparing with one tunable parameter, twist angle, in TBG.

**Flat bands.**—We solve the Moiré bands numerically and find that all small twist angles \( \theta \lesssim 6^\circ \) can become magic that support flat bands with proper choice of inter-spin coupling strength or lattice depth. In Figs. 2a and 2b, we plot the band structures for different inter-spin coupling strengths \( \Omega \) with \( V_0 = 6 \) and \( \theta = 5.086^\circ \) \((m = 6, n = 7)\). Similar to the TBG, the system has four low-energy bands, two of which form a Dirac cone at

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**FIG. 1:** (a) Energy level diagram of alkaline-earth(-like) atoms, showing how state-dependent optical lattices can be realized. (b) Laser configuration to generate spin-twisted hexagonal lattices. (c) Moiré pattern and (d) Brillouin zone of spin twisted hexagonal lattices with \( \theta = 9.43^\circ \) \((m = 3, n = 4)\). AA spots form a triangle lattice with \( AB \) or BA spots at the triangles’ centers. \( \mathbf{L}_i \) are the primitive lattice vectors. The large hexagons in (d) correspond to the bare BZs for states \( \uparrow \) (green) and \( \downarrow \) (red), respectively.

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Rabi frequency. shorter-wavelength through twisting the two spin-dependent potentials \( z \)-pled by a clock laser [35] propagating along two dimensions (2D). The two pseudospin states are cou-
FIG. 2: (a) and (b) Moiré bands along high-symmetry lines (the red dashed lines in Fig. 1d) and DOS for $\Omega = 0.1$ and $\Omega = 0.116$, respectively. We set the bare Dirac cone energy as zero. The dashed black lines are bare Dirac bands folded back to Moiré BZ. (c) Flat band width $W$ and gaps $\delta_{K,F}$ with other higher bands at $K$ and $\Gamma$ points. In (a)-(c), $\theta = 5.086^\circ$ and $V_0 = 6$. (d) Critical coupling $\Omega_f$ as a function of $\theta$ with $V_0 = 6$ (circles) and $V_0 = 4$ (plus signs). Color bars show the flatness at $\Omega = \Omega_f$ with flat band width shown by the thick blue markers and lines. The thin solid (dashed) line corresponds to $c = 1.932$ at $V_0 = 6$ ($c = 1.827$ at $V_0 = 4$).

the Moiré $K$ ($K'$) point where the remaining two bands are split by a tiny gap due to the inter-valley ($K_s-K'_s$) coupling. The Dirac cones shift to a higher energy compared to the bare ones, which is due to the couplings with states away from the valleys that have weak nonlinearity in the dispersion. The inter-spin coupling reduces the Dirac velocity significantly and enhances the DOS near the Dirac cones, as shown in Fig. 2a. The peaks in the DOS correspond to the Van Hove singularities near the Moiré $M$ points [21, 58]. The bandwidth $W$ of the low-energy bands and Dirac velocity are reduced further as $\Omega$ increases and may even vanish (i.e., the twist angle becomes magic) at certain inter-spin coupling strength. We are interested in the flat bands associated with magic angles and will focus on the physics around the critical coupling $\Omega_f$ where the narrowest bandwidth occurs (as shown in Fig. 2b). For $\Omega \lesssim \Omega_f$, the four low-energy bands are always separated by an energy gap from other bands in the spectrum, the gap is minimized near the Moiré $\Gamma$ point and would close eventually as we increase $\Omega$ above $\Omega_f$. Shown in Fig. 2c are the bandwidth $W$ and gaps $\delta_{\Gamma,K}$ (with other higher bands) versus $\Omega$.

In Fig. 2d, we plot $\Omega_f$ and the corresponding bandwidth $W$ and flatness $F \equiv \delta_{\Gamma}/W$ as functions of the twist angle $\theta$. For small twists, the low-energy bands are mainly determined by the states with $g_s$ around the Dirac valleys, and have a narrow width and high flatness at $\Omega = \Omega_f$. In addition, the inter-valley coupling is weak, thus two conduction or valence bands (one from each valley) are nearly degenerate along the high-symmetric $\Gamma-K$ ($K'$) lines [21]. We find $\Omega_f$ almost linearly increases with $\theta$. Specifically, the magic flat bands occur near $c = \text{const.}$, where $c \equiv \Omega = \frac{\Omega}{\Omega_f}$ is a dimensionless parameter with $k_D = 2k_R \sin(\theta/2)$ the $K-K'$ distance in Moiré BZ and $v_D$ the bare Dirac velocity. This is consistent with the continuum model in the TBG where $c$ is the single parameter [3, 4]. When the twist angles are large $\theta > 6^\circ$, the width and splitting of the four low-energy bands become comparable or larger than the gap with other bands, and no magic flat bands exist for any $\Omega$ since the inter-valley couplings and the effects of states away from the bare Dirac valleys become significant. For incommensurate twist angles, we can generalize the continuum model and only keep $g_s$ around one valley, which should be valid for small $\theta$ [53]. We thus conclude that all small angles $\theta \leq 6^\circ$ can support magic flat bands.

For different lattice depths $V_0$, the magic behaviors discussed above are similar (see Fig. 2d). Meanwhile, a smaller $V_0$ leads to a larger $v_D$ and thereby a stronger $\Omega_f$ (for fixed $\theta$). Long-range tunnelings are also more significant in a shallower lattice, which would effectively enhance the inter-spin couplings, leading to a slightly smaller $c$ where the flat bands occur. The flatness may also be improved by decreasing $V_0$ properly, since a larger $v_D$ leads to a larger gap $\delta_{\Gamma}$ [3, 4] and long-range tunnelings in real space can reduce inter-valley couplings that have large momentum separations. However, in the very shallow region where the dispersion-linearity around the bare Dirac cone becomes poor, the flatness starts to decrease with $V_0$.

**Superfluid orders.**—The narrowly dispersing flat bands suppress the kinetic energy and atom-atom interactions can lead to strongly correlated many-body ground states. Different from TBG [16–21], here the interaction of fermion atoms is dominated by s-wave scattering between atoms in relatively twisted lattices, with strength tunable through Feshbach resonance [46, 47],

$$H_{\text{int}} = U_0 \int d^2 r \bar{\Psi}_\uparrow(r) \bar{\Psi}_\downarrow(r) \Psi_\downarrow(r) \Psi_\uparrow(r).$$

We are interested in the superfluid order driven by attractive interactions. We adopt the mean-field approach [16–
due to the reduced DOS. At the $\Omega < \Omega_f$ side, the flat band DOS peak splits into two peaks (corresponding to the Van Hove singularities near the Moiré $M$ points), therefore the superfluid phase also splits into two regions where $\mu$ matches the DOS peaks. At the $\Omega > \Omega_f$ side, the DOS peak is simply broadened. As the $|U_0|$ decreases, the superfluid phase shrinks to the area around $\Omega \simeq \Omega_f$ and $\mu \simeq 0.005$.

Strikingly, we find that the superfluid phase corresponds to a LO state \[52\], which is very different from that in TBG. The Cooper pairs have nonzero center-of-mass momentum with $\Delta_g$ mainly distributed around the first reciprocal lattice vector shell of the untwisted hexagonal lattice and nearly vanishing around zero momentum, leading to the staggered real-space pairing orders at the hexagonal lattice scale (Figs. 4a and 4b). The attractive $s$-wave interaction pairs atoms from opposite valleys, and the superfluid order is peaked in the AA regions, where the local DOS for the flat bands is strongly concentrated \[53\] and the wavefunction overlap between two spin states is significant. Therefore, the intra-sublattice pairing is dominant. Because atoms at the same sublattices and opposite valleys share opposite angular momenta under the threefold rotation, the pairing order has the same phase factor for the same sublattice.

Moreover, the pairing is between Moiré states at $\pm \vec{q}$, which are mainly determined by the bare Bloch states $\phi_{\pm \vec{k}}$ at $\pm \vec{k}$ nearest to the valleys (thereby contributing most to the flatbands). In Fig. 4c, the pairing between $\uparrow$ states (green dots at $+\vec{k}$) around valley $K_\uparrow$ and $\downarrow$ states (red dots at $-\vec{k}$) around valley $K_\downarrow$ is illustrated schematically. Due to the relative twist, $\pm \vec{k}$ are at the same side of $K_\uparrow$ and $K_\downarrow$, respectively (see the black arrows in Fig. 4c). Therefore, we have $\phi_{\vec{k}} \propto \begin{bmatrix} 1 & e^{i\gamma_{\uparrow\uparrow}} \end{bmatrix}^T$ and $\phi_{\vec{k}} \propto \begin{bmatrix} 1 & e^{i\gamma_{\uparrow\downarrow}} \end{bmatrix}^T$ on the A and B sublattice basis, with $\gamma_{\uparrow\uparrow} \simeq -\gamma_{\uparrow\downarrow} + \pi$. The relative phases $\gamma_{\uparrow\downarrow}$ are related to the chirality of the valleys (i.e., the Berry phase on loops surrounding the valley), which are responsible for the staggered pairing order $\Delta(r) \propto \langle \phi_{\vec{k}} | \phi_{\vec{-k}} \rangle \propto \begin{bmatrix} 1 & -1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 \end{bmatrix}$ \[53\]. Such LO order is unique for spin-twisted system with pairing between atoms from relatively twisted lattices. In TBG, the pairing between spin-up and spin-down electrons in the same layer (with no relative twist) leads to ordinary BCS order \[17, 18\].

The correlation $C^{ij}_q = \langle \beta^\dagger_{j\vec{q}} \beta_{i\vec{q}} \rangle$ shows $f$-wave structure ($\beta_{i\vec{q}}$ is the annihilation operator for the $j$-th flatband), their combined effects lead to the nearly uniform superfluid gap \[53\] and the pairing is $s$-wave. The valence bands from different valleys become degenerate along the high symmetric $\Gamma-K$ lines with avoided crossing (a tiny gap) due to inter-valley couplings, therefore $C^{ij}_q$ changes from characterizing $K_\uparrow-K_\downarrow$ to $K_\uparrow-K_\downarrow$ correlations across the $\Gamma-K$ lines where its sign flips (see Fig. 4d).

**Discussion and conclusion.**—The ‘magic-angle’ physics in the spin-twisted optical lattice is very robust, supporting magic flat bands and novel LO superfluid
order in a wide range of parameter space ($\theta$, $V_0$, $\Omega$, $U_0$, etc). For $\theta \simeq 5^\circ$ and $V_0 = 6$, the gap between flat bands and other bands is $\sim 10^{-2}E_R$ (about tens of Hz for Sr atoms) and can be improved further using shallower lattices (larger $v_D$) or larger twists. The flat bands and enhanced DOS can be observed within atomic gas lifetime (a few seconds for the shallow lattice considered here) using spectroscopic measurements (e.g., radio-frequency spectroscopy) [65–68]. The critical superfluid temperature $T_{c,BKT}$ is in the nanokelvin region ($\sim 10^{-3}E_R$) which might be possible with the recently developing cold-atom cooling techniques [33, 69–71]. Thanks to the large twist angle $\theta \lesssim 6^\circ$, the Moiré unit-cell may contain less than 100 hexagons; therefore, the magic phenomena can be observed using a small system with tens of hexagons along each direction. The magic-angle physics is similar for different stackings or twist axes [53].

In summary, we study the Moiré flat band physics and the associated superfluid order in spin-twisted optical lattices for ultracold atoms, which showcase magic-angle behaviors for continuum of twists up to 6° and novel LO superfluid phase remarkably different from that in TBG. In future, it would be interesting to study spin-twisted lattices of other types (square, triangle, etc) or with different lattice depth and gapped bands (similar as transition metal dichalcogenide based Moiré systems [72, 73]). Moreover, one could study possible interesting many-body states under repulsive interaction and may even consider the nuclear spin states of alkaline-earth atoms with both nuclear-spin-exchange and inter-spin interactions. In all, our work provides a highly tunable playground for exploring quantum many-body physics and twistronics with novel twisted pseudo degrees of freedom.

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