Scalar-vector Lagrangian without nonlinear self-interactions of bosonic fields in the relativistic mean-field theory

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(Dated: July 3, 2008)

Abstract

A new Lagrangian model without nonlinear scalar self-interactions in the relativistic mean-field (RMF) theory is proposed. Introducing terms for scalar-vector interactions (SVI), we have developed a RMF Lagrangian model for finite nuclei and nuclear matter. It is shown that by inclusion of SVI in the basic RMF Lagrangian, the nonlinear $\sigma^3$ and $\sigma^4$ terms can be dispensed with. The SVI Lagrangian thus obtained provides a good description of ground-state properties of nuclei along the stability line as well as far away from it. This Lagrangian model is also able to describe experimental data on the breathing-mode giant monopole resonance energies well.

PACS numbers: 21.30.Fe, 21.10.Dr, 21.60.-n, 24.10.Cn, 24.10.Jv, 21.65.+f
The relativistic mean-field (RMF) theory \cite{1, 2, 3} of nuclear interaction based upon exchange of mesons between nucleons has established itself as a successful approach to describing properties of nuclei along the stability line as well as far away from it \cite{4, 5, 6}. The Dirac-Lorentz structure of nucleons provides a built-in spin-orbit interaction with its advantage over the non-relativistic Skyrme theory in describing properties such as anomalous isotope shifts in Pb nuclei \cite{7}, which depend upon shell structural effects. An isospin dependence of the spin-orbit interaction or rather a lack of it is responsible for the anomalous behaviour of the isotope shifts \cite{8}. The idea of pseudospin symmetry in nuclei has been attributed to the relativistic (Dirac) nature of nucleons \cite{9}.

With the advent of the Walecka model \cite{1} for nuclei and nuclear matter, it was realized that the linear RMF Lagrangian with its large value of nuclear incompressibility was unable to describe properties of finite nuclei properly. A lack of proper ingredients for a suitable description of surface properties was cited as a main reason for this drawback. In order to remedy this problem, Boguta and Bodmer \cite{10} introduced nonlinear scalar self-coupling terms of the form $\sigma^3 + \sigma^4$. Consequently, the RMF Lagrangian has proved to be successful and the nonlinear scalar terms have thus become indispensable for an adequate treatment of finite nuclei and nuclear matter saturation. The nonlinear $\sigma$-field seems to provide an essential density dependence of nuclear force in a finite nucleus. With the inclusion of the nonlinear $\sigma$-field, the theory also becomes renormalizable.

One of the first successful nuclear forces within this model is NL-SH \cite{5}. Within this model an improved set NL3 has been obtained for finite nuclei \cite{6}. However, as is well known, the model with $\sigma^3 + \sigma^4$ gives an equation of state (EOS) of nuclear matter that is very stiff and is consequently untenable for the spectrum of observed neutron star masses.

The quartic vector coupling of the form $\omega^4$ was introduced \cite{11} in the RMF Lagrangian. An appropriate description of finite nuclei was obtained with the vector self-coupling of $\omega$ meson \cite{12, 13}. The inclusion of the $\omega^4$ coupling has also helped to improve the shell effects along the stability line \cite{13}. A desired softening of the EOS of nuclear matter due to vector self-coupling of $\omega$ meson was shown in ref. \cite{14}.

The density dependence of the nuclear interaction in the RMF theory remains an open problem. The Walecka model offers an appropriate avenue to construct a theory that should be suitable for describing various aspects of finite nuclei all along the periodic table (an ambitious goal) as well as properties of nuclear matter concomitantly. Point-coupling models
have been introduced [15, 16] to describe finite nuclei. Attempts have been made to broaden
the basis of the RMF Lagrangian by including terms of higher orders in the scalar and
vector fields with inclusion of interaction terms amongst various mesonic degrees of freedom
[17, 18, 19]. Density-dependent meson couplings [20, 21, 22, 23, 24] have been introduced
with a view to modify density dependence of the nuclear interaction in an explicit form
with a good degree of success. This requires inclusion of additional parameters to model
density-dependence of meson couplings. Notwithstanding the above, the RMF theory serves
as an ideal platform for an effective field theoretical approach for many-body problems of
nuclei with sufficient space for innovation.

An upsurge in experimental data especially in the domain of extreme regions of the
periodic table provides an incentive to devise new and improved approaches and models
to be able to describe the same. Savushkin et al. [17] have incorporated various meson-
meson interactions in their approach especially those between $\sigma$ and $\omega$ meson in addition
to nonlinear couplings of both these mesonic fields. This problem has been approached [18]
from a more general point of view by taking expansion in and interactions amongst various
mesonic fields. This approach has led to an improvement for finite nuclei and nuclear matter
with a larger number of parameters required.

In this work, we have sought to explore the possibility of dispensing with the nonlinear
scalar self-couplings which have so far remained essential for finite nuclei. We ask ourselves:
whether it is possible to mock the scalar self-couplings and their inherent density dependence
in nuclei by employing meson-meson interactions especially between $\sigma$ and $\omega$ mesons instead?
Keeping the issue of renormalizability in abeyance, we have added couplings between $\sigma$ and
$\omega$ mesons of the form $\sigma\omega^2 + \sigma^2\omega^2$ to the basic (linear) RMF Lagrangian based upon exchange
of $\sigma$, $\omega$ and $\rho$ mesons. Properties of nuclear matter for the scalar-vector interaction (SVI)
of this form were explored in ref. [25]. Recently, the Lagrangian model SIG-OM with the
inclusion of the coupling of the form $\sigma^2\omega^2$ whilst retaining the scalar self-couplings $\sigma^3 + \sigma^4$
has been developed [26]. In the present work, we have narrowed down the space by excluding
the self-couplings at the expense of the scalar-vector meson-meson couplings.

The basic RMF Lagrangian density that describes nucleons as Dirac spinors interacting
with the meson fields is given by

$$L_0 = \bar{\psi} \left( \gamma \partial - g_\sigma \sigma - g_\omega \omega - g_\rho \rho - \frac{1}{2} e(1 - \tau_3) A - M_N \right) \psi$$

$$+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$

$$- \frac{1}{4} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

(1)

where $M_N$ is the bare nucleon mass and $\psi$ is its Dirac spinor. Nucleons interact with $\sigma$, $\omega$, and $\rho$ mesons, with coupling constants being $g_\sigma$, $g_\omega$, and $g_\rho$, respectively. The photonic field is represented by the electromagnetic vector $A^\mu$. The effective Lagrangian for finite nuclei that is used commonly is given by

$$L_{\text{eff}} = L_0 - U_{\text{NL}}.$$  

(2)

The nonlinear $\sigma$-meson self-couplings which have so far been an integral part of the RMF Lagrangian are given by

$$U_{\text{NL}} = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4.$$  

(3)

The parameters $g_2$ and $g_3$ are the nonlinear couplings of $\sigma$-meson in the conventional $\sigma^3 + \sigma^4$ model [10]. Here, we put $g_2 = g_3 = 0$, thus eliminating the self-couplings $U_{\text{NL}}$ of $\sigma$ meson. Instead, we introduce the meson-meson interaction terms of the form

$$U_{\text{mm}} = \frac{1}{2} g_4 \sigma \omega_\mu \omega^\mu + \frac{1}{2} g_5 \sigma^2 \omega_\mu \omega^\mu.$$  

(4)

where $g_4$ and $g_5$ represent the respective coupling constants for meson-meson interactions between $\sigma$ and $\omega$ mesons. The effective Lagrangian in our case is then

$$L_{\text{eff}} = L_0 + U_{\text{mm}}.$$  

(5)

The corresponding Klein-Gordon equations can be written as

$$(-\Delta + m_\sigma^* \sigma = -g_\sigma \bar{\psi} \psi$$

$$(-\Delta + m_\omega^* \omega_\nu = g_\omega \bar{\psi} \gamma_\nu \psi$$

$$(-\Delta + m_\rho^* \rho_\nu = g_\rho \bar{\psi} \gamma_\nu \tau_3 \psi$$

$$-\Delta A_\nu = \frac{1}{2} e \bar{\psi} (1 + \tau_3) \gamma_\nu \psi,$$

(6)

where the effective meson masses $m_\sigma^*$ and $m_\omega^*$ can be obtained as

$$m_\sigma^* = m_\sigma^2 - g_4 \omega_0^2/(2\sigma) - g_5 \omega_0^2$$

$$m_\omega^* = m_\omega^2 + g_4 \sigma + g_5 \sigma^2.$$  

(7)
These equations represent an implicit density dependence of $\sigma$ and $\omega$ meson masses and effectively that of the nuclear interaction therein.

The parameters of the new Lagrangian model SVI are obtained by a multi-dimensional search in the parameter space by fitting experimental binding energies and charge radii of a set of a few nuclei (cf. [5] for a detailed procedure). The nuclei included are $^{16}\text{O}$, $^{40}\text{Ca}$, $^{90}\text{Zr}$, $^{116}\text{Sn}$, $^{124}\text{Sn}$ and $^{208}\text{Pb}$. The isotopes $^{116}\text{Sn}$ and $^{124}\text{Sn}$ are included in order to span the broad range of isospin. No conditions have been put on nuclear matter properties and thus the parameters are allowed to vary freely without any bias to the nuclear matter properties. The $\omega$ and $\rho$ meson masses have been fixed at their empirical values.

The parameters of the Lagrangian obtained as a result of a free variation in the multi-dimensional space are shown in Table I. We have obtained two parameter sets SVI-1 and SVI-2 which are deemed as appropriate for ground-state binding energies and charge radii of nuclei. The parameter of the forces NL-SH and NL3 with the nonlinear scalar couplings are also shown for comparison.

The nuclear matter properties of SVI-1 and SVI-2 are shown in the lower section of Table I. The sets SVI-1 and SVI-2 are close to each other in the nuclear matter properties with a slight difference in the incompressibility with $K = 264\text{ MeV}$ for SVI-1 and $K = 272\text{ MeV}$ for SVI-2. There is only a minor difference in the effective mass $m^\ast$. The $m^\ast$ values for SVI interactions are clearly higher than those of the Lagrangian sets NL-SH and NL3 with the scalar self-interactions.

The saturation density for both SVI-1 and SVI-2 is slightly higher than that of NL-SH and NL3. One notable difference between the two SVI sets is the difference in the asymmetry energy $J$ (or $a_4$). One can note that even in an interaction that is different from the scalar self-interactions, it is not possible to bring down the asymmetry energy in the acceptable range of 30-33 MeV.

The binding energy of key spherical nuclei along with a few representative ones as obtained with SVI-1 and SVI-2 is shown in Table II. For a comparison, we also show the results due to NL-SH and NL3. With the exception of the very light nucleus of $^{16}\text{O}$, both SVI-1 and SVI-2 show an excellent agreement with the experimental binding energies over a large range of mass. This is reflected by the smaller value of the $rms$ deviation $\delta$ of SVI-1 and SVI-2 values from the experimental data vis-a-vis NL3 and NL-SH shown at the bottom of Table II. A marked improvement with SVI interactions is in the binding energy of doubly magic
TABLE I: The parameters and nuclear matter (NM) properties of the scalar-vector Lagrangians SVI-1 and SVI-2 without nonlinear scalar self-couplings. The sets NL-SH and NL3 with the scalar self-couplings are also shown for comparison.

| Parameters | SVI-1 | SVI-2 | NL-SH | NL3 |
|------------|-------|-------|-------|-----|
| \( M \) (MeV) | 939.0 | 939.0 | 939.0 | 939.0 |
| \( m_\sigma \) (MeV) | 524.527 | 524.024 | 526.0592 | 508.194 |
| \( m_\omega \) (MeV) | 783.0 | 783.0 | 783.0 | 782.501 |
| \( m_\rho \) (MeV) | 763.0 | 763.0 | 763.0 | 763.0 |
| \( g_\sigma \) | 9.6762 | 9.641 | 10.4436 | 10.217 |
| \( g_\omega \) | 11.6028 | 11.565 | 12.9451 | 12.8675 |
| \( g_\rho \) | 4.464 | 4.492 | 4.3828 | 4.4744 |
| \( g_2 \) (fm\(^{-1}\)) | 0 | 0 | −6.9099 | −10.432 |
| \( g_3 \) | 0 | 0 | −15.8337 | −28.885 |
| \( g_4 \) (fm\(^{-1}\)) | 17.1537 | 16.962 | 0.0 | 0.0 |
| \( g_5 \) | 33.8565 | 32.819 | 0.0 | 0.0 |

| NM properties |
|------------|-------|-------|-------|
| \( \rho_0 \) (fm\(^{-3}\)) | 0.149 | 0.149 | 0.146 | 0.148 |
| \( a_v \) (MeV) | −16.30 | −16.31 | −16.33 | −16.24 |
| \( K \) (MeV) | 263.9 | 271.5 | 354.9 | 271.6 |
| \( m^* \) | 0.616 | 0.621 | 0.597 | 0.595 |
| \( J \) (MeV) | 37.6 | 37.0 | 37.0 | 37.4 |

nuclei \( ^{100}\text{Sn} \), \( ^{132}\text{Sn} \) and \( ^{208}\text{Pb} \) over those of NL-SH and NL3. This may have consequences on the shell effects in nuclei especially in the vicinity of the r-process path and drip lines.

The charge radii of nuclei obtained with SVI-1 and SVI-2 are shown in Table III. These are compared with the values obtained with NL-SH and NL3. The SVI interactions describe the experimental data on nuclei well. An improvement on the charge radii of Pb isotopes over those of NL3 can be seen.

We have calculated the ground-state properties of the isotopic chains of Sn and Pb. Especially, the chain of Sn isotopes offers experimental binding energies over the whole
TABLE II: The binding energy (in MeV) of nuclei calculated with SVI-1 and SVI-2 and compared with NL3 and NL-SH. The empirical values (exp.) are shown in the last column. The \( \text{rms} \) deviation (\( \delta \)) of theoretical values from the experimental data is shown at the bottom of the table.

| Nucleus | SVI-1 | SVI-2 | NL-SH | NL3 | exp. |
|---------|-------|-------|-------|-----|------|
| \(^{16}\text{O}\) | -129.7 | -129.7 | -128.4 | -128.8 | -127.6 |
| \(^{40}\text{Ca}\) | -343.2 | -343.2 | -340.1 | -342.0 | -342.1 |
| \(^{48}\text{Ca}\) | -415.3 | -415.0 | -415.1 | -415.2 | -416.0 |
| \(^{90}\text{Zr}\) | -783.0 | -783.0 | -782.9 | -782.6 | -783.9 |
| \(^{100}\text{Sn}\) | -827.3 | -827.1 | -830.6 | -829.2 | -824.5 |
| \(^{116}\text{Sn}\) | -988.3 | -988.4 | -987.9 | -987.7 | -988.7 |
| \(^{124}\text{Sn}\) | -1049.7 | -1049.6 | -1050.1 | -1050.2 | -1050.0 |
| \(^{132}\text{Sn}\) | -1103.8 | -1103.3 | -1105.9 | -1105.4 | -1102.9 |
| \(^{202}\text{Pb}\) | -1591.2 | -1591.8 | -1595.8 | -1592.6 | -1592.2 |
| \(^{208}\text{Pb}\) | -1637.0 | -1637.3 | -1640.4 | -1639.5 | -1636.7 |
| \(^{214}\text{Pb}\) | -1661.7 | -1662.3 | -1664.3 | -1661.6 | -1663.3 |
| \(\delta\) | 1.33 | 1.20 | 2.70 | 2.00 |      |

TABLE III: The \( \text{rms} \) charge radius (in fm) obtained with SVI-1 and SVI-2. The values for NL-SH and NL3 are also shown. The \( \text{rms} \) deviation (\( \delta \)) of theoretical values from the experimental data is shown at the bottom of the table.

| Nucleus | SVI-1 | SVI-2 | NL-SH | NL3 | exp. |
|---------|-------|-------|-------|-----|------|
| \(^{16}\text{O}\) | 2.698 | 2.700 | 2.699 | 2.728 | 2.730 |
| \(^{40}\text{Ca}\) | 3.438 | 3.442 | 3.452 | 3.470 | 3.450 |
| \(^{48}\text{Ca}\) | 3.451 | 3.457 | 3.452 | 3.470 | 3.450 |
| \(^{90}\text{Zr}\) | 4.277 | 4.285 | 4.289 | 4.287 | 4.258 |
| \(^{116}\text{Sn}\) | 4.593 | 4.601 | 4.599 | 4.599 | 4.626 |
| \(^{124}\text{Sn}\) | 4.644 | 4.652 | 4.651 | 4.661 | 4.673 |
| \(^{208}\text{Pb}\) | 5.501 | 5.508 | 5.509 | 5.523 | 5.503 |
| \(^{214}\text{Pb}\) | 5.562 | 5.568 | 5.562 | 5.581 | 5.558 |
| \(\delta\) | 0.021 | 0.019 | 0.021 | 0.020 |      |
range from the doubly magic nucleus $^{100}$Sn to the doubly magic $^{132}$Sn, thus encompassing the space between the two magic numbers $N = 50$ and $N = 82$. The difference $\Delta E$ of binding energy of nuclei obtained with RMF+BCS calculations from the experimental value is shown for Sn and Pb isotopes in Fig. 1. For the BCS pairing, neutron pairing gaps have been obtained from the experimental masses of neighbouring nuclei.

The problem of the arches and a predominance of shell energy at the magic numbers is well-known. It pervades both the microscopic theories as well as macroscopic-microscopic mass formulae. Viewing the results for Sn isotopes in Fig. 1(a), one can notice unambiguous arches at the two magic numbers especially with the force NL3. With exception of the region near $^{100}$Sn, both SVI-1 and SVI-2 describe the data well for nuclei including those near $^{132}$Sn ($N = 82$). For nuclei in the vicinity of $^{100}$Sn ($N = 50$) SVI-1 and SVI-2 show a significant improvement over the results of NL3. The $rms$ deviation from the experimental data is 1.04 MeV and 1.07 MeV for SVI-1 and SVI-2, respectively. In comparison, it is 1.83 MeV for NL3. This indicates a significant improvement in the binding energies for Sn isotopes with SVI-1 and SVI-2, especially near the magic numbers $N = 50$ and $N = 82$. The arch-like behaviour with SVI-1 and SVI-2 is reduced considerably.

This pattern is also visible for the isotopic chain of Pb in Fig. 1(b). Both SVI-1 and SVI-2 exhibit a significant improvement in the binding energies over NL3. The $rms$ deviation of the theoretical values with SVI-1 and SVI-2 is 0.88 MeV and 0.69 MeV, respectively. This is much smaller than the corresponding value of 1.82 MeV with NL3 for the Pb isotopes. Thus, SVI interactions provide a better description of the binding energies of the Pb isotopes. In comparison, NL3 values overestimate the data near the magic number and gives a well-formed arch about the magic number. With NL3 divergences of the binding energies near the magic number are displayed strongly as has been observed also for the Sn isotopes above. It is a matter of further investigation as to what ingredients in the RMF theory would lead to divergences or a lack thereof at shell closures.

The charge radii of Pb isotopes and the anomalous kink in charge radii represent a characteristic feature related to shell structure of nuclei. It was shown that the RMF theory with NL-SH reproduced the anomalous kink successfully [7]. This feature has since been demonstrated by all the Lagrangian models in the RMF theory. In order to discern the behaviour of SVI in this respect, we show in Fig. 2 the charge radii of Pb isotopes calculated with SVI-1 and SVI-2. The force SVI-1 reproduces the experimental data [27] over the range
FIG. 1: Binding energies for (a) Sn and (b) Pb isotopes with SVI-1 and SVI-2 and compared with the experimental data. The results obtained with NL3 are also shown.}

of Pb isotopes very well. In comparison, SVI-2 values overestimate the data only slightly. The set NL3, on the other hand, overestimates the charge radii of all the Pb isotopes significantly.

In order to test the applicability of the new model for nuclei away from the line of $\beta$-stability, we have performed axially deformed RMF calculations for a number of nuclei. The nuclei encompass representative cases from $^{36}\text{Si}$ to $^{196}\text{Pt}$, which includes nuclei from medium masses through the rare-earth region to higher masses. The results of calculations with SVI-1 and SVI-2 are shown in Table IV. The experimental data on the binding energies are taken from the recent high-precision mass measurements on Si [28], Sr [29] and Mo [30]. The binding energy of other nuclei has been taken from the 2003 compilation of atomic masses [31].

Both SVI-1 and SVI-2 provide an excellent description of the experimental binding energies of nuclei over a large range of atomic mass. The difference between the predictions
FIG. 2: Charge radii of Pb isotopes obtained with SVI-1, SVI-2 and NL3 and compared with the experimental data.

of the two sets are small. For a few cases SVI-1 provides a better description whereas for a few others the SVI-2 does better. The \textit{rms} deviation of the binding energies for both the sets amounts to \( \sim 0.62 \text{ MeV} \).

The original Walecka model (linear) \([1]\) with the nucleon-meson couplings of \( \sigma \) and \( \omega \) mesons has been instructive for achieving saturation of nuclear matter. With the inclusion of nonlinear scalar self-couplings, the saturation is achieved at nuclear matter properties viz., the compressibility within the acceptable range (cf. nuclear matter properties in Table I with e.g. NL-SH and NL3). Here the incompressibility of nuclear matter \( K \) denotes the cardinal point on the saturation curve. The incompressibility \( K \) plays an important role in determining the breathing-mode giant monopole resonance (GMR) energies. It is thus important that an acceptable Lagrangian model should be able to describe the GMR data.

For a comparative analysis of the Lagrangian models involved, we have carried out con-
TABLE IV: The binding energy (in MeV) and quadrupole deformation $\beta_2$ (in parentheses) of nuclei away from the stability line calculated with SVI-1 and SVI-2. The experimental values (exp.) where available are shown for comparison.

| Nucleus | SVI-1       | SVI-2       | exp.      |
|---------|-------------|-------------|-----------|
| $^{36}\text{Si}$ | $-292.8 (-0.03)$ | $-292.7 (0.03)$ | $-292.0$  |
| $^{38}\text{Si}$ | $-300.6 (0.28)$ | $-300.2 (0.28)$ | $-299.9$  |
| $^{40}\text{Si}$ | $-306.9 (0.35)$ | $-306.4 (0.35)$ | $-306.5$  |
| $^{40}\text{S}$  | $-333.2 (0.25)$ | $-332.8 (0.25)$ | $-333.2$  |
| $^{76}\text{Ni}$ | $-632.9 (0.0)$  | $-632.3 (0.0)$  | $-633.1$  |
| $^{86}\text{Sr}$ | $-748.5 (0.0)$  | $-748.5 (0.0)$  | $-748.9$  |
| $^{88}\text{Sr}$ | $-768.1 (0.0)$  | $-768.0 (0.0)$  | $-768.5$  |
| $^{110}\text{Mo}$ | $-918.6 (-0.23)$ | $-918.5 (-0.23)$ | $-919.5$  |
| $^{120}\text{Xe}$ | $-1008.1 (0.28)$ | $-1008.4 (0.27)$ | $-1007.7 (0.22)$ |
| $^{140}\text{Xe}$ | $-1161.3 (0.10)$ | $-1161.2 (0.10)$ | $-1160.7 (0.11)$ |
| $^{154}\text{Sm}$ | $-1267.4 (0.31)$ | $-1267.5 (0.30)$ | $-1266.9 (0.34)$ |
| $^{164}\text{Dy}$ | $-1337.9 (0.35)$ | $-1338.1 (0.35)$ | $-1338.0 (0.35)$ |
| $^{168}\text{Er}$ | $-1365.3 (0.34)$ | $-1365.5 (0.34)$ | $-1365.8 (0.34)$ |
| $^{174}\text{Yb}$ | $-1406.7 (0.31)$ | $-1407.0 (0.31)$ | $-1406.6 (0.33)$ |
| $^{190}\text{W}$  | $-1510.5 (0.19)$ | $-1510.7 (0.20)$ | $-1509.9$  |
| $^{196}\text{Pt}$ | $-1552.2 (0.12)$ | $-1552.5 (0.12)$ | $-1553.6 (0.13)$ |

strained Generator Coordinate Method (GCM) calculations [32] for the isoscalar GMR mode for a set of nuclei. The nuclei included are $^{90}\text{Zr}$, $^{120}\text{Sn}$ and $^{208}\text{Pb}$. The results of GCM calculations are shown in Table V.

The set SVI-1 with $K = 264$ MeV underestimates the experimental values for $^{90}\text{Zr}$ and $^{208}\text{Pb}$ by more than $\sim 0.5$ MeV, whereas for $^{120}\text{Sn}$, the disagreement with the datum is nominal. In comparison, SVI-2 with $K = 272$ MeV gives GMR energies for $^{90}\text{Zr}$ and $^{120}\text{Sn}$, which are very close to the experimental data. For $^{208}\text{Pb}$, its values is slightly smaller than the experimental one. Comparatively, NL3 with $K = 272$ MeV gives values which are systematically smaller than those of SVI-2. This difference in predictions of a nonlinear
TABLE V: The breathing mode GMR energies obtained with constrained GCM calculations using SVI-1 and SVI-2. The experimental data \[33, 34\] are also shown.

| Nucleus | SVI-1 | SVI-2 | NL3 | exp.   |
|---------|-------|-------|-----|--------|
| $^{90}$Zr | 17.2  | 17.5  | 16.9| 17.81 ± 0.30 |
| $^{120}$Sn | 15.2  | 15.4  | 15.0| 15.52 ± 0.15 |
| $^{208}$Pb | 13.3  | 13.5  | 13.0| 13.96 ± 0.28 |

scalar Lagrangian from those of a scalar-vector Lagrangian points to some subtle differences in finite effects, possibly surface, of the two models. A comparison between the two sets of SVI shows that SVI-2 is commensurate with the ground-state properties of finite nuclei as well as with the GMR energies.

In conclusion, we have constructed a Lagrangian model without nonlinear scalar self-couplings in the RMF theory. Incorporating scalar-vector interaction terms in the linear RMF Lagrangian, the Lagrangian model SVI has been developed. We have obtained parameter sets SVI-1 and SVI-2 in the framework of the new model. It is shown that both SVI-1 and SVI-2 provide a good description of the ground-state properties of nuclei along the stability line as well as for nuclei far away from it. With an incompressibility of nuclear matter $K \sim 272$ MeV, SVI-2 is able to reproduce the breathing-mode GMR energies on key nuclei well. Thus, the model SVI without nonlinear interactions becomes viable for finite nuclei and nuclear matter.

I thank Prof. Lev Savushkin for useful discussions. This work is supported by the Research Administration Project No. SP07/03 of Kuwait University.

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