Ruling out real-number description of quantum mechanics

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Abstract

Standard quantum mechanics has been formulated with complex-valued Schrödinger equations, wave functions, operators, and Hilbert spaces$^{1-4}$. However, previous work has shown possible to simulate quantum systems using only real numbers by adding extra qubits and exploiting an enlarged Hilbert space$^{5-13}$. A fundamental question arises: are the complex numbers really necessary for the quantum mechanical description of nature? To answer this question, a non-local game has been developed to reveal a contradiction between a multiqubit quantum experiment and a player using only real numbers$^{14}$. Here, based on deterministic and high-fidelity entanglement swapping with superconducting qubits, we experimentally implement the Bell-like game and observe a quantum score of 8.09(1), which beats the real number bound of 7.66 by 43 standard deviations. Our results disprove the real-number description of nature and establish the indispensable role of complex number in the quantum mechanics.
Physicists use mathematics to describe nature. In classical physics, real number appears complete to describe the physical reality in all classical phenomenon, whereas complex number is only sometimes employed as a convenient mathematical tool. In quantum mechanics, the complex number was introduced as the first principle in Schrödinger's equation and Heisenberg’s commutation relation\textsuperscript{1,2}. The complex wavefunction has been shown to represent physical reality of quantum objects\textsuperscript{3}. Experimentally, the real and imaginary parts of the wavefunction has been directly measured\textsuperscript{4}. Today, the quantum mechanics with complex wavefunction seems the most successful theory to describe nature.

However, whether the complex number is necessary to represent the theory of quantum mechanics is still an open question. Starting with von Neumann in 1936, previous works\textsuperscript{5-12} have shown possible to simulate quantum systems using only real numbers by adding extra qubits and exploiting an enlarged Hilbert space. For example, by adding an extra qubit $|\pm i\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$, a single-quantum system with a complex density matrix $\rho$ and Hermitian operator $H$ can be simulated through $tr(\rho H) = tr(\tilde{\rho} \tilde{H})$, where $\tilde{\rho}$ and $\tilde{H}$ are real and of the form:

$$\tilde{\rho} = (\rho \otimes |i\rangle \langle i| + \rho^* \otimes |-i\rangle \langle -i|)/2,$$

$$\tilde{H} = H \otimes |i\rangle \langle i| + H^* \otimes |-i\rangle \langle -i|.$$

Therefore, in this case one cannot distinguish between the real- and complex-number representations. It has also been shown that real-number quantum state is universal for quantum computing\textsuperscript{13}, which again challenged the fundamental role of complex-number quantum mechanics.

Very recently, Renou et al. developed an elegant scheme to provide an observable effect in quantum experiments to distinguish between the two representations\textsuperscript{14}. The scheme
is a Bell-like\textsuperscript{15-19} three-party game based on deterministic entanglement swapping\textsuperscript{20}. With the assumption that the quantum states produced by two independent sources is a tensor product of the states produced by each of the sources, the real-number quantum mechanics cannot obtain maximal violation of a Bell CHSH-like inequality, thus being falsified.

Figure 1a shows an illustration of the non-local game for Alice, Bob, and Charlie. First, two pairs of Einstein-Podolsky-Rosen (EPR) entangled qubits\textsuperscript{21} are distributed between Alice and Bob, and between Bob and Charlie, respectively. Bob then performs a joint Bell-state measurement (BSM) on his two received qubits, which randomly projects them into one of the four Bell states: 

- \( |\phi^+\rangle = (|0\rangle|0\rangle + |1\rangle|1\rangle)/\sqrt{2}, \)
- \( |\psi^+\rangle = (|0\rangle|1\rangle + |1\rangle|0\rangle)/\sqrt{2}, \)
- \( |\phi^-\rangle = (|0\rangle|0\rangle - |1\rangle|1\rangle)/\sqrt{2}, \)
- \( |\psi^-\rangle = (|0\rangle|1\rangle - |1\rangle|0\rangle)/\sqrt{2}. \)

The four results are correspondingly registered as \( b \in \{00, 01, 10, 11\} \). Alice performs a single-qubit local measurement on her received qubit in the eigenstate bases of operator \( A_x \ (x \in \{1, 2, 3\}) \) which yields an outcome \( a \in \{0, 1\} \). Similarly, Charlie measures his qubit using operator \( C_z \ (z \in \{1, 2, 3, 4, 5, 6\}) \) and obtain his outcome \( c \in \{0, 1\} \). The score \( \Gamma \) of the game is defined as the weighted sum of the conditional joint probability distribution \( P(abc \mid xz) \), given by

\[
\Gamma = \sum_{abc,xz} w_{abc,xz} P(abc \mid xz),
\]

where \( w_{abc,xz} \in \{-1, +1\} \) are the weights (see Supplementary Information), \( abc \in \{0, 1\}^{\otimes 4} \) are the measurement results bit strings, and \( xz \in \{11, 12, 21, 22, 13, 14, 33, 34, 25, 26, 35, 36\} \) are 12 combinations of local single-
qubit measurement settings $x$ and $z$.

We set Alice’s operator as $A_z \in \{Z, X, Y\}$, where $Z, X, Y$ are the standard Pauli matrices, and Charlie’s operator as $C_z \in \{Z \pm X, Z \pm Y, X \pm Y\}/\sqrt{2}$. In the standard complex-number quantum mechanics, the score of the Bell-CHSH-like game$^{16,17}$ reaches its maximum of $6\sqrt{2}$ ($\approx 8.49$). However, if one assumes the players can only use real-number quantum resources (real-number states and real-number local operators), the numerically found optimal score$^{14}$ has an upper bound of 7.66, which sits between the classical limit of 6 and the quantum limit of $6\sqrt{2}$, as shown in Fig. 1b. This contradiction opens a way to a direct experimental test to distinguish between the complex-number and real-number representations of quantum mechanics.

A generic quantum circuit for the experimental test is shown in Fig. 2a which involves three deterministic entangling gates for preparing two maximum entangled EPR pairs (1-2 and 3-4) and implementing the BSM between qubit 2 and 3. Very high quantum state and gate fidelities are required to surpass the real number players$^{14}$. We design and fabricate a superconducting quantum processor (see Fig. 2b for its optical image) to implement the quantum game. To keep Alice’s and Charlie’s qubits (1 and 4) isolated from each other, we use I-shape transmon qubit design$^{22}$ to increase the spacing of the qubits for reducing the non-neighbor coupling and with large frequency detuning, the maximum population exchange is smaller than $10^{-9}$ (see Supplementary Information). The qubits are arranged in a linear array$^{23}$ and each has individual control and readout$^{24,25}$. The qubits work at the transition frequency $f_{01} \sim 5$ GHz and are capacitively coupled to their nearest neighbours with $g/2\pi \sim 5$ MHz.

Single qubit is controlled by microwave pulses on the Rabi driving (XY) line and fast flux-bias current on the frequency tuning (Z) line. Each qubit is dispersively coupled to
a readout resonator (R1-R4) and is readout through a single coplanar waveguide. At the idle frequency, the qubits average lifetime $T_1$ is measured to be 34.8 μs and the average coherence time $T_2^*$ is 8.0 μs. The qubits are initialized by idling 200 μs to decay into the ground states. The average fidelity of single-qubit gates using 23 ns driving pulses is measured to be 99.88%. The average error in the readout of the 16 computational bases after correcting the local bit-flip error is 0.34%.

To implement the quantum circuit in Fig. 2(a), we design the control pulse sequences as shown in Fig. 2(c). The entangling interactions are in the form of iSWAP gates where the qubits fully exchange their populations as $|01\rangle \rightarrow i|10\rangle, |10\rangle \rightarrow i|01\rangle$ in ~50 ns by tuning two nearest-neighbor qubits into resonance. The quantum process fidelity of the iSWAP gate is measured to be 96.7%.

To characterize our experimental process, we make tomographic measurements of the quantum entangled states created after each step. First, after the first two EPR pulses (see Fig. 2c, at the point of ~120 ns), we measure and reconstruct the density matrix of the product state of the two EPR pairs as shown in Fig. 2d. The experimental data agrees well with the ideal case with a fidelity of 0.975(1). The independence of the two EPR pairs is estimated as $\text{tr}((tr_{12}\rho) \otimes (tr_{12}\rho) \cdot \rho) = 0.982(1)$. Next, after the third entangling pulse (at the point of ~230 ns in Fig. 2c), the four qubits become a fully entangled state. The measured density matrix in Fig. 2e show a fidelity of 0.946(1) compared to the perfect state. Finally, projective measurements on the qubit 2 and 3 are performed, which swap the entanglement onto the independent and non-interacting qubit 1 and 4. Conditioned on random outcomes of the BSM, the qubits 1 and 4 are projected into one of the four Bell states correspondingly, whose quantum state fidelities are measured and plotted in Fig. 2e. The obtained average fidelity is 0.952(1), well above the threshold of $\sim 0.9$. 
Having verified the high quantum operation and state fidelities, we proceed to test the Bell-CHSH quantum game and measure the conditional joint probability distributions \( P(abc \mid xz) \). Alice and Charlie’s qubits are first unitarily transformed using appropriate single-qubit gates (see Supplementary Information) and then measured in the computational bases. There are 12 different combinations of measurement settings and 16 possible measured bit strings. The main experimental results are plotted as a 12×16 probability distribution matrix in Fig. 3(a) together with the ideal values. Summing over the obtained probability \( P(abc \mid xz) \) with their corresponding weight (see Supplementary Information), our quantum experiment gives a score of 8.09(1). As shown in Fig. 4b, the result violates the upper bound set by real-number quantum mechanics by 43 standard deviations, which provides strong evidence to disprove the real-number description of quantum mechanics.

Our experiment presents the first step to rule out the real-number description of nature. Similar to the history of Bell tests where progressively more sophisticated experiments\(^{28-35}\) were performed to refute local hidden variable model, we note that fully ruling out the real-number models is not a single-shot achievement either, but would require future improved work to close the potential loopholes. The current implementation has the feature of deterministic controlled logic gate and single-shot measurement with unity detection efficiency, which eliminates the so-called detection loophole. However, as Alice’s and Charlies’ qubits are located the same superconducting chip, a space-like separation is not guaranteed. A key next step is to fulfill the Einstein locality condition\(^{30}\) and the freedom of measurement choice, which appears feasible in the near future by linking independent superconducting qubits through high-fidelity quantum links\(^{36-38}\).
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Data and materials availability: All relevant data are available from the corresponding author on reasonable request.

Requests for materials should be addressed to C.-Y.L. (cylu@ustc.edu.cn) or J.-W.P. (pan@ustc.edu.cn).
**Fig. 1. A non-local game to disprove real-number model of quantum mechanics.**

(a) A three-party Bell-CHSH-like game based on entanglement swapping. Two EPR pairs are distributed between Alice and Bob, and between Bob and Charlie. The measurement settings are labelled as \( x, y, \) and \( z \), and the outcomes are \( a, b, \) and \( c \), respectively. Bob performs a complete Bell-state measurement. Alice and Charlie perform single-qubit local measurements. (b) The score of the non-local game divides the physics into three regimes: classical, real-number quantum mechanics, and complex quantum mechanics, which are experimental testable. The score \( \Gamma \) is calculated from the weighted sum of the conditional joint probability distribution.
Fig. 2. Experimental implementation. (a) The quantum circuit used to perform the quantum game. The EPR pairs and BSM are realized by a two-qubit iSWAP operation with additional single-qubit gates. (b) Optical image of the superconducting quantum processor. The qubits (Q1-Q4) are placed in a linear array and have direct nearest-neighbour couplings. Each qubit has a XY driving and a Z tuning line and has an individual readout resonator (R1-R4) for measurement. (c) The experimental control pulses. Single qubit is rotated by Rabi resonance and two qubits are entangled by tuning the qubits into resonant spin exchange. Two EPR pulse sequences are used to prepare the two pairs of EPR entangled states. And the BSM pulse sequence is used to implement the joint Bell-state measurement. (d) The density matrix of the product state of the two EPR pairs after the EPR pulses. The state fidelity is 97.5%. (e) The density matrix of the four-qubit entangled state after the BSM pulses. The state fidelity is 94.6%. (f) The entangled states generated between Alice and Charlie conditional on the BSM outcomes. All are above 90% threshold fidelity, and the average is 95.2%.
**Fig. 3. Experimental results.** (a) The conditional joint probability distribution matrices \( P(abc \mid xz) \) from the non-local game (left) and the theoretical prediction from an ideal experiment (right). The row index represents the measurement setting \( \{xz\} \) and the column index is the measured strings \( \{abc\} \). (b) The obtained score of the quantum experiment is 8.09(1), which is 43 standard deviations above the upper bound of the score (7.66) from real-number quantum mechanics.

The boundary of the swapped entangled state fidelity (F) and qubit detection efficiency (\( \eta \)) in order to rule out the real-number quantum mechanics without resorting to assumptions of fair sampling. The marked point presents the current experiment (F= 0.952(1), \( \eta \)=1).