STUDY OF RESONANT STRUCTURE OF $S$-WAVE AMPLITUDES IN $\pi^- p^\uparrow \rightarrow \pi^- \pi^+ n$ MEASURED ON POLARIZED TARGET AT 17.2 GeV/c IN THE MASS RANGE OF 580–1080 MeV.

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Abstract

We have performed a new model independent amplitude analysis of reaction $\pi^- p \rightarrow \pi^- \pi^+ n$ measured at CERN at 17.2 GeV/c on polarized target. The analysis extends the range of dipion mass from 600–880 MeV used in the previous study to 580–1080 MeV. Our purpose is to study the role of $f_0(980)$ interference and to better determine the resonance parameters of $\sigma(750)$ resonance from fits to both $S$-wave amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$ with recoil nucleon transversities “up” and “down”, respectively. This work is the first attempt to determine resonance parameters of a resonance from Breit-Wigner fits to production amplitudes of opposite transversity. We used several fitting procedures to understand the resonant structure of the $S$-wave amplitudes in this mass range. First we performed separate fits to $|S|^2\Sigma$ and $|S|^2\Sigma$ using a Breit-Wigner parametrization for $\sigma(750)$ and $f_0(980)$, and a constant coherent background. We obtained an excellent fit to the data with a low $\chi^2$ per

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data point ($\chi^2/dpt$). However the width of $\sigma$ is narrow 110–76 MeV in $|\bar{S}|^2\Sigma$ while it is broad 217–233 MeV in $|S|^2\Sigma$. To decide whether the apparent dependence of $\Gamma_{\sigma}$ on nucleon spin is a real physical effect or an artifact of separate fits, we performed simultaneous fits to $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$. First we assumed a common single $\sigma$ pole. We found a good fit with $m_{\sigma} = 775 \pm 17$ MeV, $\Gamma_{\sigma} = 147 \pm 33$ MeV but the $\chi^2/dpt$ is larger than $\chi^2/dpt$ from separate fits. We concluded that the results of separate fits may indicate existence of two $\sigma$ poles, one with a narrow and the other with a broad width. Self-consistency requires that both poles contribute to both $S$-wave amplitudes. The simultaneous fit to $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ with two common $\sigma$ poles yields an excellent fit with a lower $\chi^2/dpt$ than the separate fits. All amplitudes are dominated by a $\sigma$ state with $m_{\sigma} = 786 \pm 24$ MeV and $\Gamma_{\sigma} = 130 \pm 47$ MeV. All amplitudes receive a weaker contribution from a narrower $\sigma'$ state with $m_{\sigma'} = 670 \pm 30$ MeV and $\Gamma_{\sigma'} = 59 \pm 58$ MeV. We propose to identify $\sigma'(670)$ and $\sigma(786)$ with colour-electric and colour-magnetic modes of lowest mass scalar gluonium $0^{++}(gg)$. 

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I. INTRODUCTION

In 1978, Lutz and Rybicki showed [1] that almost complete amplitude analysis of reactions $\pi N \rightarrow \pi^+ \pi^- N$ and $KN \rightarrow K^+ \pi^- N$ is possible from measurements in a single experiment on a transversely polarized target. More recently it was shown [2,3] that amplitude analyses of reactions $\pi^- p \rightarrow \pi^0 \pi^0 n$ and $\pi^- p \rightarrow \eta \pi^- p$ are also possible from measurements on transversely polarized target. The work of Lutz and Rybicki opened a whole new approach to hadron spectroscopy by enabling to study the production of resonances on the level of spin amplitudes rather than spin-averaged cross-sections.

In amplitude analyses of these reactions resonances are observed in mass distribution of recoil nucleon transversity amplitudes $|A|^2 \Sigma$ and $|A|^2 \Sigma$ corresponding to recoil nucleon spin “up” or “down” relative to the scattering plane. In traditional hadron spectroscopy, the nucleon spin is irrelevant and resonance should appear in both spin amplitudes with the same mass, with the same width and with similar mass distributions. The measurements of reactions $\pi^- p \rightarrow \pi^- \pi^+ n$, $\pi^+ n \rightarrow \pi^+ \pi^- p$ and $K^+ n \rightarrow K^+ \pi^- P$ at CERN on polarized targets seem to challenge these expectations. These CERN experiments reveal shapes of resonant mass distributions in amplitudes of opposite transversity that are different and dependent significantly on the nucleon spin [4–18].

An important advantage of amplitude analyses is that new resonances can be detected on the level of spin amplitude that are not visible in the spin-averaged measurements on unpolarized targets. Such is the case of the new scalar resonance $\sigma (750)$ found in amplitude analyses of reactions $\pi^- p \rightarrow \pi^- \pi^+ n$ at 17.2 GeV/c at low momentum transfers $-t = 0.005–0.2$ (GeV/c)$^2$ and in $\pi^+ n \rightarrow \pi^+ \pi^- p$ at 5.98 and 11.85 GeV/c at larger momentum transfers $-t = 0.2–0.4$ (GeV/c)$^2$ [16 [18]. Using Monte Carlo method of amplitude analysis, it was found in [17] that in $\pi^- p \rightarrow \pi^- \pi^+ n$ reaction both solutions for spin “up” $S$-wave amplitude $|S|^2 \Sigma$ resonate below 880 MeV while both solutions for spin “down” $S$-wave amplitude $|S|^2 \Sigma$ appeared nonresonating. This result is in perfect agreement with $\chi^2$ minimization method of amplitude analysis used in [15] (see Fig. 1 and 2 of [18]). Detailed quantitative Breit-
Wigner fits to $|S|^2\Sigma$ in [18] showed the importance of coherent background for determination of the resonance parameters of $\sigma(750)$. The best fit in [18] gives $m_\sigma = 753 \pm 19$ MeV and $\Gamma_\sigma = 108 \pm 53$ MeV. The excellent agreement of these values for $m_\sigma$ and $\Gamma_\sigma$ with the Ellis-Lanik theorem [19] strongly suggests that $\sigma(750)$ is the lowest mass scalar gluonium [18].

Törnqvist expressed concern [20,21] that the low mass and the narrow width of $\sigma(750)$ is due to the neglect of interference of $\sigma(750)$ with $f_0(980)$ in our Breit-Wigner fits which initially extended only to 880 MeV in dipion mass using the data set of Ref. [5]. The apparent nonresonating behaviour of the amplitude $|S|^2\Sigma$ below 880 MeV was also puzzling. To investigate these questions we have performed a new amplitude analysis of $\pi^- p \rightarrow \pi^- \pi^- n$ reaction at 17.2 GeV/c and small momentum transfers $-t = 0.005-0.20$ (GeV/c)$^2$ extending the dipion mass range up to 1080 MeV using the data set of Ref. [7].

In this report we present the results of our detailed study of resonant structure of $S$-wave amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$ in the mass region of 580–1080 MeV. This work is the first attempt to determine resonance parameters of a resonance from fits to production amplitudes of opposite nucleon transversity. All previous determinations of resonance parameters used spin-averaged mass distributions or, in the case of Ref. [18], only one spin amplitude ($|S|^2\Sigma$).

The new results for $|S|^2\Sigma$ shows a narrow resonance below 880 MeV, followed by an enhancement above 900 MeV and a dramatic dip at $\sim$1000 MeV corresponding to $f_0(980)$. The amplitude $|S|^2\Sigma$ shows a broader structure with a dip at $\sim$1000 MeV.

First we performed separate fits to $|S|^2\Sigma$ and $|S|^2$ using a coherent sum of $\sigma(750)$ and $f_0(980)$ Breit-Wigner amplitudes and constant background. We find that $\sigma$ contributes to both amplitudes but with different widths. Solutions 1 and 2 of amplitude $|S|^2\Sigma$ have a narrow width of 100 and 76 MeV, respectively. Solutions 1 and 2 of amplitude $|S|^2\Sigma$ have a broader width of 217 and 233 MeV. The interference with $f_0(980)$ is crucial in reproducing the enhancements above 900 MeV and the dip at 1010 MeV.

The narrow width in the spin “up” amplitude $|S|^2\Sigma$ and the broad width in the spin
“down” amplitude $|S|^2\Sigma$ is inconsistent with the traditional view of a resonance as a particle with single mass and width. To see if we can describe the mass spectra of $|S|^2\Sigma$ and $|S|^2\Sigma$ with a single $\sigma$ with the same resonance parameters, we have performed a simultaneous fit to $|S|^2\Sigma$ and $|S|^2\Sigma$ hoping that the differences in the resonant shapes can be accounted for by differences in the background. We obtained an acceptable fit but the $\chi^2$ per data point ($\chi^2/dpt$) was significantly higher than $\chi^2/dpt$ in separate fits. We thus obtained a poorer fit.

Our next step was to admit that a narrow $\sigma$ contributes to $|S|^2\Sigma$ and a broad $\sigma$ contributes to $|S|^2\Sigma$. But a self-consistency requires that the narrow $\sigma$ contributes also to $|S|^2\Sigma$, and the broad $\sigma$ contributes also to $|S|^2\Sigma$. We expected these additional contributions to be small in a simultaneous two-pole fit to $|S|^2\Sigma$ and $|S|^2\Sigma$. The results of the fit were surprising. Our best fit had $\chi^2/dpt$ better than the separate fits to $|S|^2\Sigma$ and $|S|^2\Sigma$. And the fit required in all amplitudes the presence of two $\sigma$ resonances: higher mass and broader $\sigma$ with average mass $m_\sigma = 786 \pm 24$ MeV and $\Gamma_\sigma = 130 \pm 47$, and a lower mass and narrower $\sigma'$ with average mass of $m_{\sigma'} = 670 \pm 30$ MeV and $\Gamma_{\sigma'} = 59 \pm 58$ MeV. In all $S$-wave amplitudes the higher mass $\sigma$ dominates the $\sigma'$ contribution. Since this solution gives a very low $\chi^2/dpt$, it must be taken seriously. We propose to identify the $\sigma'(670)$ and $\sigma(786)$ as color electric and color magnetic modes of lowest mass scalar gluonium $0^{++}(gg)$, respectively, following the gluonium classification scheme of Bjorken [22–24] elaborated in [25].

The paper is organized as follows. In Section II we present and discuss the results of our new amplitude analysis. In Section III we introduce the parametrization of unnormalized $S$-wave amplitudes with a single $\sigma$ pole and present the results of separate fits to amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$. Simultaneous fits to these $S$-wave amplitudes using a single $\sigma$ pole are presented in Section IV. In Section V we introduce the parametrization with two $\sigma$ poles and discuss the results of simultaneous fits to $|S|^2\Sigma$ and $|S|^2\Sigma$ using this parametrization. In Section VI we discuss the possible constituent nature of the two $\sigma$ states $\sigma'(670)$ and $\sigma(786)$ resulting from the best fit. The paper closes with a summary in Section VII.
II. AMPLITUDE ANALYSIS

The high statistics CERN-Munich measurement of $\pi^- p \rightarrow \pi^- \pi^+ n$ at 17.2 GeV/c on polarized target was reported in four data sets with kinematics given in the following Table:

| Set | $-t$ (GeV/c)$^2$ | $m$ (MeV) | $\Delta m$ (MeV) | Ref. |
|-----|-----------------|----------|-----------------|------|
| 1   | 0.005–0.20      | 600–900  | 20              | [4,5]|
| 2   | 0.01–0.20       | 580–1780 | 40              | [6]  |
| 3   | 0.005–0.20      | 580–1600 | 20              | [7,10]|
| 4   | 0.20–1.00       | 620–1500 | 40              | [8,9]|

In the Table $\Delta m$ is the size of the mass bins. There is only one $t$-bin covering the whole interval of indicated $-t$. Only [4,5] report the results for amplitudes for the Set 1. In [3] and [10] ratios $|S|/|\bar{S}|$ and $S$-wave intensity $I_S$ are given, so it is possible to reconstruct approximately amplitudes $|S|^2 \Sigma$ and $|\bar{S}|^2 \Sigma$ for the Sets 2 and 3. These are shown in Fig. 5 of Ref. [18] and Fig. 2 below, respectively. All amplitude analysis [4–10] use $\chi^2$ minimization method to find solution for amplitudes and their errors.

For invariant masses below 1000 MeV, the dipion system in reactions $\pi N \rightarrow \pi^+ \pi^- N$ is produced predominantly in spin states $J = 0$ ($S$-wave) and $J = 1$ ($P$-wave). The experiments on polarized targets then yield 15 spin-density-matrix (SDM) elements, or equivalently 15 moments, describing the dipion angular distribution. The measured normalized observables are expressed in terms of two $S$-wave and six $P$-wave normalized nucleon transversity amplitudes. In our normalization

$$|S|^2 + |\bar{S}|^2 + |L|^2 + |\bar{L}|^2 + |U|^2 + |\bar{U}|^2 + |N|^2 + |\bar{N}|^2 = 1 \quad (2.1)$$

where $A = S, L, U, N$ and $\overline{A} = \bar{S}, \bar{L}, \bar{U}, \bar{N}$ are the normalized nucleon transversity amplitudes with recoil nucleon transversity “down” and “up” relative to the scattering plane. The $S$-wave amplitudes are $S$ and $\bar{S}$. The $P$-wave amplitudes $L, \bar{L}$ have dimeson helicity $\lambda = 0$ while the pairs $U, \bar{U}$ and $N, \bar{N}$ are combinations of amplitudes with helicities $\lambda = \pm 1$.
and have opposite $t$-channel-exchange naturality \cite{1,14}. The unnatural exchange amplitudes receive contributions from “$\pi$” and “$A_1$” exchanges. The natural exchange amplitudes $N$, $\overline{N}$ are both dominated by “$A_2$” exchange.

Amplitude analysis expresses analytically the eight normalized moduli

$$|S|, |\overline{S}|, |L|, |\overline{L}|, |U|, |\overline{U}|, |N|, |\overline{N}| \quad (2.2)$$

and six independent cosines of relative phases

$$\cos(\gamma_{SL}), \cos(\gamma_{SU}), \cos(\gamma_{LU}) \quad (2.3)$$

$$\cos(\overline{\gamma}_{SL}), \cos(\overline{\gamma}_{SU}), \cos(\overline{\gamma}_{LU})$$

in terms of the measured observables \cite{1,14}. In (2.3) $\cos \gamma$ and $\cos \overline{\gamma}$ are cosines of relative phases between pairs of amplitudes with opposite transversity (e.g. between $S$ and $L$, and between $\overline{S}$ and $\overline{L}$).

There are two similar solutions in each $(m, t)$ bin \cite{1,14}. However in many $(m, t)$ bins the solutions are unphysical: either a cosine has magnitude larger than 1 or the two solutions for moduli are complex conjugate with a small imaginary part. Unphysical solutions also complicate error analysis. Two methods are used to find physical solutions for amplitudes and their errors. They are $\chi^2$ minimization method and Monte Carlo method. The two methods are described in \cite{4,17,18}.

In Ref. [17] we used Monte Carlo method for finding the physical solutions for amplitudes and their errors in $\pi^- p \rightarrow \pi^- \pi^+ n$ using the data Set 1 for dipion masses in the range 600–900 MeV. In Ref. [18] we show that the results obtained using Monte Carlo method agree with the results using $\chi^2$ minimization method but are smoother and give consistently lower $\chi^2/dpt$ in Breit-Wigner fits.

In this work we report results of Monte Carlo amplitude analysis of $\pi^- p \rightarrow \pi^- \pi^+ n$ reaction using the data Set 3 in the mass range 580–1080 MeV in order to study the $\sigma(750) – f_0(980)$ interference. Above 1000 MeV our analysis is only approximate as we neglect the
D-wave. However, below 1080 MeV the D-wave contribution is still small and its neglect does not affect the structure of S-wave amplitudes. This is confirmed by direct comparison with S-wave amplitudes obtained in \([11]\) using the \(\chi^2\) method with D-wave included above 980 MeV (see Fig. 2d below). The Monte Carlo method is based on 40,000 selections of spin-density matrix elements within their errors in each \((m, t)\) bin. No physical solution was found in 3 bins — 650 MeV, 850 MeV and 930 MeV.

The amplitude analysis is carried out for normalized amplitudes \(|A|^2\) and \(|A|^2\), \(A = S, L, U, N\). However, from the spectroscopic point of view the relevant information is contained in the unnormalized amplitudes \(|A|^2\Sigma\) and \(|A|^2\Sigma\) where \(\Sigma \equiv d^2\sigma/dm^2\) is the reaction cross-section. It is the unnormalized amplitudes \(|A|^2\Sigma\) and \(|A|^2\Sigma\) which represent the direct spin-dependent contributions to the mass distribution \(d^2\sigma/dm^2\). The unnormalized moduli \(|A|^2\Sigma\) and \(|A|^2\Sigma\) were calculated using the \(\Sigma = d^2\sigma/dm^2\) from Fig. 12 of Ref. [26].

The results of our new Monte Carlo amplitude analysis of \(\pi^-p \rightarrow \pi^-\pi^+n\) in the mass range 580–1080 MeV are shown in Fig. 1. The first thing we observe is that the \(\rho^0\) peak in \(d\sigma/dm^2\) is not uniformly reproduced in all \(P\)-wave amplitudes. In fact, the amplitudes \(|N|^2\Sigma\) and \(|U|^2\Sigma\) show considerable suppression of \(\rho^0\) production. The shapes of mass distributions with opposite nucleon transversities show considerable differences in both \(S\)- and \(P\)-wave amplitudes. The amplitudes \(|S|^2\Sigma, |L|^2\Sigma, |U|^2\Sigma\) with recoil nucleon transversity “down” are smaller and broader than the amplitudes \(|S|^2\Sigma, |L|^2\Sigma, |U|^2\Sigma\) with recoil nucleon transversity “up”. The opposite is true for the natural exchange amplitudes \(|N|^2\Sigma\) and \(|N|^2\Sigma\).

The \(S\)-wave amplitude \(|S|^2\Sigma\) shows a resonant behaviour below 900 MeV in both solutions. The relative phase \(\gamma_{SL}\) between \(S\) and \(L\) amplitudes is near zero in Solution 1 and a small constant in Solution 2. Since \(|L|^2\Sigma\) clearly resonates, the amplitude \(|S|^2\Sigma\) must resonate as well. The amplitudes \(|S|^2\Sigma\) and \(|L|^2\Sigma\) are both broader and their relative phase \(\gamma_{SL}\) is again near zero in Solution 2. This strongly suggests that the amplitude \(|S|^2\Sigma\) also resonates.

Both solutions in both \(S\)-wave amplitudes dip at 1010 MeV. This dip is accompanied
with a dramatic change of phase $\tau_{SL}$ and $\tau_{SU}$ above 1000 MeV. This behaviour is interpreted as evidence for a narrow resonance $f_0(980)$. Its contribution must be taken into account in fits to amplitudes $|\mathcal{S}|^2\Sigma$ and $|S|^2\Sigma$.

In Figure 2 we summarize the results for the $S$-wave amplitudes. Fig. 2a and 2b are based on Monte Carlo method of amplitude analysis of data Set 1 and 3, respectively. The amplitude $|\mathcal{S}|^2\Sigma$ clearly resonates below 880 MeV in both solutions but shows an enhancement between 880 MeV and 980 MeV which is particularly large on Solution 2. The enhancement is followed by a rapid drop at 990 MeV with a dip at 1010 MeV. The apparent broad structure of $|S|^2\Sigma$ amplitude is due to poor separation of the resonant structure below 880 MeV and the enhancement between 880 and 980 MeV which are of comparable size. The rapid drop of the enhancement at 990 MeV and dip at 1010 MeV are clearly seen even in $|S|^2\Sigma$ amplitude. As we shall see later the enhancement is due to interference of $f_0(980)$ with coherent background.

Fig. 2c and 2d are based on $\chi^2$ minimization methods of amplitude analysis of data Sets 1 and 3, respectively. Fig. 2c is based on Fig. 10 of Ref. [5] and Fig. 2d is based on Fig. 2 of Ref. [10]. The results are consistent with the Monte Carlo amplitude analysis.

The spin-averaged $S$-wave intensity is defined as $I_S = (|S|^2 + |\mathcal{S}|^2)\Sigma$. Since there are two independent solutions for the amplitudes $|S(i)|^2$ and $|\mathcal{S}(j)|^2$, $i, j = 1, 2$, we obtain 4 solutions for $I_S$ which we label as follows

$$I_S(i, j) = (|S(i)|^2 + |\mathcal{S}(j)|^2)\Sigma$$  \hspace{1cm} (2.4)

The results are shown in Fig. 3. We see that only solutions $I_S(1, 1)$ and $I_S(2, 1)$ are clearly resonating while the solutions $I_S(1, 2)$ and $I_S(2, 2)$ do not have a clear resonant structure due to enhancement in the range 880–980 MeV. It is also for this reason that it is better to study directly the resonant structure of the amplitudes $|\mathcal{S}|^2\Sigma$ and $|S|^2\Sigma$ rather than their spin averaged $S$-wave intensity.

**III. SEPARATE FITS TO AMPLITUDES $|\mathcal{S}|^2\Sigma$ AND $|S|^2\Sigma$**
A. Parametrization of amplitudes

To understand the resonant structure of the $S$-wave amplitudes $|\mathcal{S}|^2\Sigma$ and $|S|^2\Sigma$ we first performed separate fits to these amplitudes using a parametrization which included single $\sigma(750)$ Breit-Wigner pole, $f_0(980)$ Breit-Wigner pole and a coherent background. Following Ref. [18], we write the unnormalized $S$-wave amplitudes in the form

\[ |\mathcal{S}|^2\Sigma = q|F|^2 \]  
\[ |S|^2\Sigma = q|F|^2 \]  

where $q$ is the mass dependent phase space factor determined in [18] to be simply the c.m.s. momentum of $\pi^-$ in the $\pi^-\pi^+$ rest frame. The amplitudes $\mathcal{F}$ and $F$ are unnormalized amplitudes which are parametrized in terms of resonance contributions and coherent background. For amplitude $F$ we write

\[ F = R_\sigma(s,t,m)BW_\sigma(m) + R_f(s,t,m)BW_f(m) + Q(s,t,m) \]  

where $BW_R$ is the Breit-Wigner amplitude

\[ BW_R = \frac{m_R\Gamma}{m_R^2 - m^2 - imR\Gamma} \]  

where $m_R$ is the resonant mass, $R = \sigma, f$. In the following $f$ will refer always to $f_0(980)$ resonance. The mass dependent width $\Gamma(m)$ depends on spin $J$ and has a general form

\[ \Gamma = \Gamma_r\left(\frac{q}{q_R}\right)^{2J+1}\frac{D_J(qR^r)}{D_J(qr)} \]  

In (3.4) $q_R = q(m = m_R)$ and $D_J$ are the centrifugal barrier functions of Blatt and Weishopf

\[ D_0(qr) = 1.0 \]  
\[ D_1(qr) = 1.0 + (qr)^2 \]
where $r$ is the interaction radius. We recall that

$$Re\ BW_R = \left(\frac{m_R^2 - m^2}{m_R \Gamma}\right) |BW_R|^2 \equiv w_R |BW_R|^2$$

$$Im\ BW_R = |BW_R|^2$$

(3.6)

In (3.2) the term $Q(s, t, m)$ is the coherent nonresonant background. The amplitude $\mathcal{F}$ has a form similar to (3.2) with obvious replacements $R_\sigma \rightarrow \overline{R}_\sigma$, $R_f \rightarrow \overline{R}_f$ and $Q \rightarrow \overline{Q}$.

The energy variable $s$ is fixed and will be omitted in the following. Since the experimental mass distributions are averaged over a broad $t$-bin $-\Delta t = 0.005$–$0.20$ (GeV/c)$^2$, we will work with amplitudes averaged over the same $t$ interval. The $t$-averaged amplitudes have the same form as (3.2), so we will simply assume that $R_\sigma$, $R_f$ and $Q$ are all $t$-independent. We will also assume that $R_\sigma$, $R_f$ and $Q$ depend only weakly on the dipion mass $m$ and can be taken as constants in our fits. It is then convenient to factor out the phase of $R_\sigma$ and define

$$R_\sigma = |R_\sigma| e^{i\phi} = \sqrt{N_S} e^{i\phi}$$

(3.7)

$$C = R_f/R_\sigma = C_1 + iC_2$$

$$B = Q/R_\sigma = B_1 + iB_2$$

Then the unnormalized amplitude (3.2) has the form

$$F = \sqrt{N_S} \{BW_\sigma + CBW_f + B\} e^{i\phi}$$

(3.8)

and the parametrization of $|S|^2 \Sigma$ reads

$$|S|^2 \Sigma = q N_S \left[ [1 + 2w_\sigma B_1 + 2B_2] |BW_\sigma|^2 + 
+ B_1^2 + B_2^2 + [C_1^2 + C_2^2] |BW_f|^2 + 
+ 2[ (w_\sigma |BW_\sigma|^2 + B_1) (w_f C_1 - C_2) + 
+ (|BW_\sigma|^2 + B_2) (C_1 + w_f C_2) ] |BW_f|^2 \right]$$

(3.9)

The parametrization of $|\overline{S}|^2 \Sigma$ has the same form as (3.9).
B. Results of separate fits to $|S|^2\Sigma$ and $|S|^2\Sigma$

The measured mass distributions $|S|^2\Sigma$ and $|S|^2\Sigma$ were augmented at 650 and 850 MeV mass bins by data points from Monte Carlo amplitude analysis using the data Set 1 \[17\]. Independent fits were performed to both solutions of both $S$-wave amplitudes with mass and width of $\sigma$ resonance being free parameters. The fitting was done using the CERN optimization program FUMILI \[28\]. Initially the $f_0(980)$ was fixed with mass $m_f = 980$ MeV and width $\Gamma_f = 48$ MeV. An improved fit was obtained when $m_f$ and $\Gamma_f$ were allowed to be free parameters to be fitted. The best fit was obtained with fitted $f_0(980)$ using as initial values of parameters $B_i, C_i, i = 1, 2$ the output values from the fit with fixed $f_0(980)$. The results are shown in Fig. 4a. The fitted values of $m_\sigma, \Gamma_\sigma, m_f$ and $\Gamma_f$ and the associated $\chi^2/dpt$ ($\chi^2$ per data point) for each solution are given in Table I.

We see from Fig. 4a that the parametrization (3.9) reproduces well the structure of $S$-wave amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$ in both solutions. The $\sigma$ resonance peaks are clearly visible in both solutions to $|S|^2\Sigma$ while the presence of $\sigma$ resonance in $|S|^2\Sigma$ is required to explain the broad structure observed in this amplitude. The interference of $f_0(980)$ with coherent background and the $\sigma$ resonance is responsible for the enhancements in the mass region 880–980 MeV, the rapid decrease at 990 MeV and the dip at 1010 MeV in all solutions.

We see from the Table I that the masses $m_\sigma$ are similar in all solutions. We might expect the same situation for the width $\Gamma_\sigma$ but this is not the case. The width of $\sigma$ from fits to $|S|^2\Sigma$ is a narrow 110 or 76 MeV. The width of $\sigma$ from fits to $|S|^2\Sigma$ is much broader at 217 or 233 MeV. We obtain a puzzling result that the width of $\sigma$ depends on nucleon transversity.

It is an accepted view that hadron resonance is a particle that, upon its production in a hadronic reaction, freely propagates and decays independently of other particles involved in the reaction. Thus its width cannot depend on the spin of recoil nucleon. However, so far this assumption has not been tested in experiments as all meson widths were determined from fits to spin-averaged mass distributions. This work is the first attempt to determine resonance parameters from different spin dependent mass distributions. It is therefore necessary to
determine whether the apparent dependence of $\Gamma_\sigma$ on nucleon transversity is an artifact created by separate fits to $|S|^2\Sigma$ and $|S|^2\Sigma$, or whether it is a genuine property of resonances. To this end we turn to simultaneous fits of $|S|^2\Sigma$ and $|S|^2\Sigma$ examining these two possibilities. Finally we note that we have performed separate fits to $|S|^2\Sigma$ and $|S|^2\Sigma$ obtained by $\chi^2$ minimization method (Fig. 2d). The fits are very similar to results for Monte Carlo method (Fig. 4a) but the $\chi^2/dpt$ is larger for all solutions.

IV. SIMULTANEOUS FIT TO $|S|^2\Sigma$ AND $|S|^2\Sigma$ WITH A SINGLE $\sigma$ POLE

Hadron resonance in a production process is represented as a pole in helicity or transversity amplitudes with mass and width independent of helicities or transversities of external particles. In this Section we examine the possibility that the same $\sigma$ resonance contributes to both transversity amplitudes $T$ and $F$ (see (3.8)), and that the difference in the shape of $|S|^2\Sigma$ and $|S|^2\Sigma$ is due to difference in coherent background. Thus we will try to show that the apparent dependence of $\Gamma_\sigma$ on nucleon spin is an artifact of separate fits to $|S|^2\Sigma$ and $|S|^2\Sigma$ distributions.

We use the parametrization (3.9) for $|S|^2\Sigma$ and $|S|^2\Sigma$ with common $m_\sigma$ and $\Gamma_\sigma$ and use the programme FUMILI [28] to perform a simultaneous fit to data on $|S|^2\Sigma$ and $|S|^2\Sigma$. The two solutions for $|S|^2\Sigma$ are labeled $\overline{1}$ and $\overline{2}$, while the two solutions for $|S|^2\Sigma$ are labeled 1 and 2. Thus there are 4 simultaneous fits ($\overline{1}$, 1), ($\overline{1}$, 2), ($\overline{2}$, 1) and ($\overline{2}$, 2). Initially the $f_0(980)$ was fixed with a mass $m_f = 980$ MeV and width $\Gamma_f = 48$ MeV. An improved fit was obtained when $m_f$ and $\Gamma_f$ were allowed to be free parameters. The best fit was obtained with fitted $f_0(980)$ using as initial values of parameters $B_i, C_i, \overline{B}_i, \overline{C}_i, i = 1, 2$ the output values from the fit with fixed $f_0(980)$. The results are shown in Fig. 4b. The fitted values of $m_\sigma, \Gamma_\sigma, m_f$ and $\Gamma_f$ and the associated $\chi^2/dpt$ for each combination of solution are given in Table II.

We see in Fig. 4b that the simultaneous fits are similar to separate fits in Fig. 4a. Also, the cross-fits ($\overline{1}2$) and ($\overline{2}1$) are similar to fits ($\overline{1}1$) and ($\overline{2}2$), respectively. In Table II we notice that all fits have similar values of $m_\sigma, \Gamma_\sigma, m_f$ and $\Gamma_f$. The average values of mass and
width of $\sigma$ resonance are

$$m_\sigma = 775 \pm 17 \text{ MeV}, \quad \Gamma_\sigma = 147 \pm 33 \text{ MeV}$$

(4.1)

This compares with $m_\sigma = 753 \pm 19 \text{ MeV}$ and $\Gamma_\sigma = 108 \pm 53 \text{ MeV}$ obtained in \cite{18} from fits only to $|S|^2\Sigma$ without the interference with $f_0(980)$. The simultaneous fit of $|S|^2\Sigma$ and $|S|^2\Sigma$ and the interference with $f_0(980)$ thus require a slightly higher mass and a broader width of $\sigma$ state.

We now compare the $\chi^2/dpt$ for the separate and simultaneous fits:

| Fit          | (11) | (12) | (21) | (22) | Average |
|--------------|------|------|------|------|---------|
| Separate     | 0.250| 0.177| 0.267| 0.194| 0.222   |
| Simultaneous | 0.386| 0.292| 0.370| 0.272| 0.330   |

In this table the $\chi^2/dpt$ is the average value of $\chi^2/dpt$ for solutions $i$ and $j$, $i, j = 1, 2$ calculated by the program FUMILI. We see that $\chi^2/dpt$ for simultaneous fits is systematically larger than for separate fits. The average $\chi^2/dpt$ over all combinations is 0.222 for separate fits and 0.330 for simultaneous fits, or 50% larger. To reject the hypothesis that $\Gamma_\sigma$ depends on nucleon spin we would need a simultaneous fit with average $\chi^2/dpt$ over all combinations smaller than 0.222. Since this is not the case, we will explore in the next section the possibility that $\Gamma_\sigma$ depends on nucleon spin in another simultaneous fit to $|S|^2\Sigma$ and $|S|^2\Sigma$.

At this point we comment on our use of $\chi^2$. Let $N$ be the number of observations, $M$ the number of free parameters and $\nu = N - M$ the number of degree of freedom. Then $\chi^2$ per degree of freedom, $\chi^2/dof = \chi^2/\nu$, is used to decide the goodness of the fit \cite{29}. The fit is said to be good if $\chi^2/dof \lesssim 1$. In comparing various fits it is more appropriate to use $\chi^2$ per data point, $\chi^2/dpt = \chi^2/N$, calculated by the program FUMILI. $\chi^2/dpt$ reflects the absolute value of $\chi^2$ obtained and fits with lower $\chi^2/dpt$ are said to be better than fits with higher $\chi^2/dpt$. Notice that $\chi^2/dof = (\chi^2/dpt)(N/\nu)$. We have verified that all our fits are good with $\chi^2/dof < 1$. 

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V. SIMULTANEOUS FIT TO $|S|^2\Sigma$ AND $|S|^2\Sigma$ WITH TWO $\sigma$ POLES

A. Two-pole parametrization

Let us accept that the amplitude $|S|^2\Sigma$ has a $\sigma$ pole with a narrower width and that the amplitude $|S|^2\Sigma$ has a $\sigma$ pole with a broader width. We can express the helicity amplitude in terms transversity amplitudes. Using the notation [18]

$$|S|^2\Sigma = q|F|^2 = \frac{1}{2}|F_0 - iF_1|^2$$ (5.1)

$$|S|^2\Sigma = q|F|^2 = \frac{1}{2}|F_0 + iF_1|^2$$

the helicity nonflip and flip amplitudes can be written

$$F_0 = \frac{1}{\sqrt{2}}(F + \bar{F})$$ (5.2)

$$F_1 = -\frac{i}{\sqrt{2}}(F - \bar{F})$$

The helicity amplitudes acquire two $\sigma$ poles from the transversity amplitudes $F$ and $\bar{F}$. Let us tentatively label $m_\sigma$ and $\Gamma_\sigma$ the mass and width of the $\sigma$ pole in amplitude $|S|^2\Sigma$, and $m_\sigma$ and $\Gamma_\sigma$ the mass and width of the $\sigma$ pole in the amplitude $|S|^2\Sigma$. Inverting the relationship (5.2) we get

$$F = \frac{1}{\sqrt{2}}(F_0 + iF_1)$$ (5.3)

$$\bar{F} = \frac{1}{\sqrt{2}}(F_0 - iF_1)$$

The self consistency now requires that the transversity amplitudes $F$ and $\bar{F}$ also possess the two sigma poles $\sigma$ and $\sigma$. We could imagine that while $\sigma$ dominates $|S|^2\Sigma$, the $\sigma$ presents a small contribution. Similarly $\sigma$ could dominate $|S|^2\Sigma$ with $\sigma$ being a small contribution. If this were the case then the apparent dependence of $\sigma$ width on nucleon spin could be explained in terms of two sigma poles. Such explanation would still be in accord with the
standard picture of hadron resonances provided that the resulting $\chi^2/dpt$ would be smaller than the $\chi^2/dpt$ for separate fits.

Consequently we will write the amplitudes $\mathcal{F}$ and $F$ in the following form:

$$
\mathcal{F} = \mathcal{R}_\sigma BW_\sigma + \mathcal{R}_\sigma BW_\sigma + \mathcal{R}_f BW_f + Q
$$

(5.4)

$$
F = R_\sigma BW_\sigma + R_\sigma BW_\overline{\sigma} + R_f BW_f + Q
$$

where $BW_\sigma$, $BW_\sigma$, $BW_f$ are Breit-Wigner amplitudes for $\overline{\sigma}$, $\sigma$ and $f$ resonances and $Q$, $Q$ are background terms. Since we expect the terms $BW_\overline{\sigma}$ and $BW_\sigma$ to be dominant in $|\mathcal{S}|^2\Sigma$ and $|S|^2\Sigma$, respectively, we can factor out the residue terms $\mathcal{R}_\sigma$ and $R_\sigma$

$$
\mathcal{F} = |\mathcal{R}_\sigma|\{BW_\sigma + DBW_\sigma + CBW_f + B\}e^{i\phi}
$$

(5.5)

$$
F = |R_\sigma|\{BW_\sigma + DBW_\overline{\sigma} + CBW_f + B\}e^{i\phi}
$$

In (5.5) $D, D, C, C, B, B$ are complex numbers which we shall write in the general form

$$
D = D_1 + iD_2, \ldots, B = B_1 + iB_2
$$

(5.6)

We will again assume $t$-averaged amplitudes (5.4). This makes the parameters (5.6) $t$-independent and we will assume also their mass independence. The parametrization for $|\mathcal{S}|^2\Sigma$ can be written

$$
|\mathcal{S}|^2\Sigma = \mathcal{Y}_{old} + \mathcal{Y}_{new}
$$

(5.7)

where the $\mathcal{Y}_{old}$ does not contain the terms involving the parameters $\overline{D}_1$ and $\overline{D}_2$ and has the same form as (3.9). With a notation

$$
\mathcal{H}_\sigma = |BW_\sigma|^2, \ H_\sigma = |BW_\sigma|^2, \ H_f = |BW_f|^2
$$

(5.8)

we get

$$
\mathcal{Y}_{old} = qN_S[(1 + 2\pi_\sigma B_1 + 2\overline{B}_2)\mathcal{H}_\sigma + B_1^2 + \overline{B}_2^2]
$$
\[(C_1^2 + C_2^2)H_f + 2\{(w_\sigma H_\sigma + B_1)(w_f C_1 - C_2) + (H_\sigma + B_2)(C_1 + w_f C_2)\}H_f\] (5.9)

\[Y_{\text{new}} = qN_S[2\{w_\sigma H_\sigma + B_1\} + (C_1 w_f - C_2)H_f\} \{D_1 w_\sigma - D_2\}H_\sigma + + 2\{(H_\sigma + B_2) + (C_1 + w_f C_2)H_f\} \{D_1 + D_2 w_\sigma\}H_\sigma + + \{D_1^2 + D_2^2\}H_\sigma\] (5.10)

In (5.9) and (5.10) the coefficients \(w_\sigma, w_\sigma\) and \(w_f\) are defined in (3.6) for \(R = \sigma, \sigma\) and \(f\). The normalization term \(N_S = |R_\sigma|^2\).

The parametrization for \(|S|^{2\Sigma}\) is similar to (5.9) and (5.10) with some changes: the free parameters are without bars, and the role \(H_\sigma, w_\sigma\) is interchanged with \(H_\sigma\) and \(w_\sigma\).

\section*{B. Results}

We have fitted the parametrizations (5.9) and (5.10) for \(|\overline{S}|^{2\Sigma}\) and \(|S|^{2\Sigma}\) simultaneously to the data on \(|\overline{S}|^{2\Sigma}\) and \(|S|^{2\Sigma}\). Since there are two solutions for each distributions \(|\overline{S}|^{2\Sigma}\) and \(|S|^{2\Sigma}\), we obtain 4 independent fits which we label (11), (12), (21) and (22).

We started with the expectation that the poles \(\overline{\sigma}\) in \(|\overline{S}|^{2\Sigma}\) and \(\sigma\) in \(|S|^{2\Sigma}\) will not change much, and that \(\sigma\) and \(\overline{\sigma}\) will be only small contributions to \(|\overline{S}|^{2\Sigma}\) and \(|S|^{2\Sigma}\), respectively. The actual results were different and surprising.

We run the optimization program with well over 20 initial conditions. In each case the fits to combinations (21) and (22) converged to the same solutions for resonance parameters \(m_{\overline{\sigma}}, \Gamma_{\overline{\sigma}}, m_{\sigma}\) and \(\Gamma_{\sigma}\). Since these solutions are very similar, we call them Solution A. The fits to combination (11) resulted in a different solution for the resonance parameters which we call Solution B. The fits to combination (12) produced two solutions very close to solutions A and B. In each combination (1j) the solution for resonance parameters produced always the same \(\chi^2/dpt\).
Even with additional runs we failed to find Solution A in the combination ($\bar{T}1$) in free unconstrained fits. However when we run a constrained fit with $m_{\sigma}, \Gamma_{\sigma}, m_{\sigma}$ and $\Gamma_{\sigma}$ from the Solution A from fits to ($T2$), ($\bar{T}2$) and ($\bar{T}2$) combinations, we obtained excellent fits with $\chi^2/dpt$ nearly equal to $\chi^2/dpt$ for Solution B from combination ($\bar{T}1$). The resonance parameters from the fit to ($T2$) produced the closest $\chi^2/dpt$. We have therefore accepted this constrained fit as the Solution A for the ($\bar{T}1$) combination.

Similarly we were unable to find Solution B in unconstrained fits to combinations ($\bar{T}2$) and ($22$). When we run a constrained fit with $m_{\sigma}, \Gamma_{\sigma}, m_{\sigma}$ and $\Gamma_{\sigma}$ from the Solution B from ($\bar{T}1$) combination, we obtained acceptable fits with somewhat larger $\chi^2/dpt$ in comparison with $\chi^2/dpt$ from Solution A. On the basis of $\chi^2/dpt$ alone the Solution A is much preferable.

The fits from the Solution A and Solution B are shown in Fig. 4c and Fig. 4d, respectively. The figures show two fits to each solution of amplitudes $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$. One fit comes from the diagonal combinations ($\bar{T}1$) or ($22$) (solid lines) while the other is from the off-diagonal combinations ($T2$) or ($\bar{T}2$) (dashed lines). In Fig. 4c and Fig. 4d we see that the two $\sigma$ poles do not result in two peaks in the $|\bar{S}|^2\Sigma$ spectrum. However, there is a hump at 670 MeV in $|S|^2\Sigma$ in Solution A (Fig. 4c) and a small peak at 710 MeV in $|S|^2\Sigma$ in Solution B (Fig. 4d).

Our initial expectation was that the $\bar{\sigma}$ and $\sigma$ resonances will dominate the $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ amplitudes, respectively. Two features of $\bar{\sigma}$ and $\sigma$ poles have emerged in all fits. First, $\bar{\sigma}$ has always a higher mass and a broader width than $\sigma$. Second and surprising feature is that $\bar{\sigma}$ is always the dominant contribution in all fits to both amplitudes $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ while $\sigma$ presents a weaker contribution to both amplitudes. Thus our initial expectation that the amplitudes $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ are dominated by a narrow $\bar{\sigma}$ and a broad $\sigma$ states is not validated. Instead, both mass spectra of $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ can be described in terms of dominant $\bar{\sigma}$ and subdominant $\sigma$. This parametrization with two new $\sigma$ poles leads to $\chi^2/dpt$ that is lower than $\chi^2/dpt$ for separate fits (see below). Thus the hypothesis that the width of $\sigma$ depends on nucleon spin must be abandoned as an artifact of separate fits.

Since $\bar{\sigma}$ and $\sigma$ poles are no longer associated with the nucleon transversities of the amplitudes $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$, we will relabel these states in a more conventional way. We
will make the following change of notation

$$\bar{\sigma} \rightarrow \sigma, \; m_{\bar{\sigma}} \rightarrow m_\sigma, \; \Gamma_{\bar{\sigma}} \rightarrow \Gamma_\sigma$$

$$\sigma \rightarrow \sigma', \; m_\sigma \rightarrow m_{\sigma'}, \; \Gamma_\sigma \rightarrow \Gamma_{\sigma'}$$

(5.11)

The results for \(m_\sigma, \Gamma_\sigma\) (old \(m_{\bar{\sigma}}, \Gamma_{\bar{\sigma}}\)) and \(m_{\sigma'}, \Gamma_{\sigma'}\) (old \(m_\sigma, \Gamma_\sigma\)) are shown in the Table III. The average values of resonance parameters over the combinations give for Solution A

\[ m_\sigma = 786 \pm 24 \text{ MeV}, \; \Gamma_\sigma = 130 \pm 47 \text{ MeV} \]  

(5.12)

\[ m_{\sigma'} = 670 \pm 30 \text{ MeV}, \; \Gamma_{\sigma'} = 59 \pm 58 \text{ MeV} \]

Solution B is similar but the differences between the masses of \(\sigma\) and \(\sigma'\) are smaller. The average values of resonance parameters over the combinations are

\[ m_\sigma = 768 \pm 24 \text{ MeV}, \; \Gamma_\sigma = 128 \pm 53 \text{ MeV} \]  

(5.13)

\[ m_{\sigma'} = 711 \pm 44 \text{ MeV}, \; \Gamma_{\sigma'} = 70 \pm 78 \text{ MeV} \]

The Table III also shows the \(\chi^2/dpt\) for each combination \((ij)\) which is the average of \(\chi^2/dpt\) for \(|\tilde{S}|^2\Sigma\) and \(|S|^2\Sigma\) solutions. The lowest \(\chi^2/dpt\) is always obtained for the \((22)\) combination. The largest \(\chi^2/dpt\) is always obtained for the \((11)\) combination. It is worthwhile to summarize the \(\chi^2/dpt\) of various fits

| Fit       | \((11)\) | \((12)\) | \((21)\) | \((22)\) | Average |
|-----------|---------|---------|---------|---------|---------|
| Separate  | 0.250   | 0.177   | 0.267   | 0.194   | 0.222   |
| Simult. (1σ) | 0.386   | 0.292   | 0.370   | 0.272   | 0.330   |
| Simult. (2σ) |         |         |         |         |         |
| – Solution A | 0.274   | 0.168   | 0.172   | 0.110   | 0.181   |
| – Solution B | 0.268   | 0.168   | 0.246   | 0.180   | 0.216   |
Clearly the best overall fit is the simultaneous fit with two \( \sigma \) poles \( \sigma \) and \( \sigma' \), Solution A. Its \( \chi^2/dpt \) is lower than \( \chi^2/dpt \) for separate fits, and much lower than the \( \chi^2/dpt \) for simultaneous fits with a single \( \sigma \) pole. From this we can conclude that the hypothesis that the width of \( \sigma \) depends on nucleon spin has been invalidated. Equally invalidated is the hypothesis of a single \( \sigma \) pole contributing to both \( |S|^2\Sigma \) and \( |S|^2\Sigma \) amplitudes. Since the Solution B has a larger \( \chi^2/dpt \) close to \( \chi^2/dpt \) for separate fits, we propose to reject it as unphysical. As the result of this \( \chi^2 \) criterion we are left with two \( \sigma \) states \( \sigma \) and \( \sigma' \) from the Solution A. A possible physical interpretation of these states will be discussed in Section VI.

C. Spin dependence of free parameters.

It is of interest to examine the dependence of free parameters on nucleon spin. In this discussion we will restrict attention mostly to the Solution A.

A remarkable feature of all fits is that in all amplitudes the contribution of the higher mass and broader \( \sigma \) dominates the contribution of \( \sigma' \). In Table IV we present the real and imaginary parts \( D_1 \) and \( D_2 \) of the coupling of \( \sigma' \) Breit-Wigner amplitude. We see that for recoil transversity “up” amplitudes \( \overline{T} \) and \( \overline{2} \), the real part \( \overline{D}_1 \) is negative and imaginary part \( \overline{D}_2 \) is positive. The opposite is true for \( D_1 \) and \( D_2 \) for the recoil nucleon transversity “down” amplitudes 1 and 2. The average absolute magnitude \( |D|_{av} \) also depends on recoil nucleon transversity:

| \( |D|_{av} \) | \( \overline{T} \) | \( \overline{2} \) | 1 | 2 |
|-----------|------|------|---|---|
| Solution A | 0.08 | 0.40 | 0.62 | 0.60 |
| Solution B | 0.18 | 0.35 | 0.61 | 0.61 |

We see that the coupling of \( \sigma' \) is weakest in the amplitude \( |S|^2\Sigma \), Solution 1. It is strongest in spin down amplitude \( |S|^2\Sigma \) in both solutions.

In Table V we show the real and imaginary parts \( C_1 \) and \( C_2 \) of the coupling of \( f_0(980) \) together with the fitted values of \( m_f \) and \( \Gamma_f \). We see that \( \overline{C}_1 \) and \( \overline{C}_2 \) are both positive in both solutions for spin up amplitude \( |S|^2\Sigma \). In contrast, \( C_1 \) is negative in both solutions.
for spin down amplitude $|S|^2 \Sigma$. The fitted values of $m_f$ and $\Gamma_f$ are in agreement with expectations for $f_0(980)$.

Finally, in Table VII we present the real and imaginary parts $B_1$ and $B_2$ of coherent background and the overall normalization constant. $\overline{B}_1$ and $\overline{B}_2$ are both positive for both solutions of spin up amplitude $|\overline{S}|^2 \Sigma$. For spin down amplitude $|S|^2 \Sigma$ the real part $B_1$ is negative while the imaginary part $B_2$ remains positive. The values of normalization constant $N_S$ show that $\sigma$ couples most strongly to Solution 1 of the spin up amplitude $|\overline{S}|^2 \Sigma$.

VI. POSSIBLE CONSTITUENT INTERPRETATION OF THE SCALAR STATES

$\sigma'(670)$ AND $\sigma(786)$.

In the usual quark model meson resonances are $q\overline{q}$ states. The mass $M$ of the $q\overline{q}$ states increases with its angular momentum $L$ as $M = M_0(2n + L)$ where $n$ is the degree of radial excitation. The lowest mass scalar mesons are $^3P_0$ states with masses expected to be around 1000 MeV or higher. The masses of $\sigma'(670)$ and $\sigma(786)$ are too low to be $q\overline{q}$ states.

The fact that $\sigma'(670)$ and $\sigma(786)$ are not observed in reactions $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ suggests a gluonium interpretation of these states. Since gluons do not couple directly to photons, we expect the $\sigma'$ and $\sigma$ states to be absent in $\gamma\gamma \rightarrow \pi\pi$ reactions. This conclusion is supported by PLUTO and DELCO data [30,31] and more recent results on $\gamma\gamma \rightarrow \pi^0\pi^0$ [32].

Ellis and Lanik discussed the couplings of scalar gluonium $\sigma$ on the basis of the low-energy theorems of broken chiral symmetry and scale invariance, implemented using a phenomenological Lagrangian [19]. They obtained for $\sigma \rightarrow \pi^+\pi^-$ decay the following partial width

$$\Gamma(\sigma \rightarrow \pi^+\pi^-) = \frac{(m_\sigma)^2}{48\pi G_0} \quad (6.1)$$

where $G_0 = \langle 0 | (\alpha_s/\pi) F_{\mu\nu} F^{\mu\nu} | 0 \rangle$ is the gluon condensate term [33] parametrizing the nonperturbative effects of QCD. The numerical values were estimated by the ITEP group [33] to be $G_0 \approx 0.012 \, (\text{GeV})^4$ or up to $G_0 \approx 0.030 \, (\text{GeV})^4$ in later calculations [34,35]. Several recent estimates of $G_0$ all agree on the values around $G_0 \approx 0.020 \, (\text{GeV})^4$ [36,37].
It is very interesting to note that when we take $G_0 = 0.015 \text{ (GeV)}^4$, the Ellis-Lanik theorem (6.1) predicts $\Gamma(\sigma \to \pi^+\pi^-) = 133 \text{ MeV}$ for the $\sigma$ mass $m_\sigma = 786 \text{ MeV}$ and $\Gamma(\sigma' \to \pi^+\pi^-) = 60 \text{ MeV}$ for the $\sigma'$ mass $m_{\sigma'} = 670 \text{ MeV}$. This result is in perfect agreement with Eq. (5.12) where $\Gamma_\sigma = 130 \pm 47 \text{ MeV}$ and $\Gamma_{\sigma'} = 59 \pm 58 \text{ MeV}$. However, $\Gamma_\sigma$ and $\Gamma_{\sigma'}$ are the full widths of $\sigma(786)$ and $\sigma'(670)$, so these results on $\Gamma(\sigma \to \pi^+\pi^-)$ and $\Gamma(\sigma' \to \pi^+\pi^-)$ represent upper limits. When we use $G_0 = 0.020 \text{ (GeV)}^4$, which is the average of the latest values for $G_0 \left[36-38\right]$, we get for the partial width $\Gamma(\sigma \to \pi^+\pi^-) = 0.75\Gamma_\sigma$ and $\Gamma(\sigma' \to \pi^+\pi^-) = 0.75\Gamma_{\sigma'}$ with a very reasonable branching fraction of 75% for the $\pi^+\pi^-$ channel. From this agreement with Ellis-Lanik theorem we can conclude that $\sigma'(670)$ and $\sigma(786)$ are both best understood as the lowest mass scalar gluonium states $0^{++}(gg)$.

For further interpretation of the $\sigma'(670)$ and $\sigma(786)$ states we turn to Bjorken’s classification of gluonium states with no spatial excitation [22–24]. In our discussion we will follow Ref. [25].

Consider gluon field strength tensor

$$F_{\mu\nu}^\alpha = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g_s f_{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu \quad (6.2)$$

where $A^\alpha_\mu$ are QCD gluon gauge fields. The tensor $F_{\mu\nu}^\alpha$ transforms like an octet under SU(3)$_c$ colour symmetry group. In analogy to electrodynamics we define colour electric fields $E_i^\alpha$ and colour magnetic fields $B_i^\alpha$:

$$E_i^\alpha = F_{0i}^\alpha, \quad B_i^\alpha = \frac{1}{2} \epsilon_{ijk} F_{jk}^\alpha \quad (6.3)$$

The colour electric fields correspond to $J^{PC} = 1^{-+}$ states while the colour magnetic fields correspond to $J^{PC} = 1^{++}$ states [23]. Since gluons are Bose particles, their total wave function must be symmetric with respect to the interchange of all labels. We restrict our consideration to $s$-waves with no spatial excitations which have a symmetric space wave function. Thus the product of the spin and colour wave functions must be also symmetric. Since bound gluons are off-shell, we must also admit the longitudinal spin component. Then there are two distinct modes of lowest mass scalar gluonium $0^{++}(gg)$ [25]: colour electric mode gluonium
\[ |0^{++}EE >= \sum_{\alpha,\beta=1}^{8} \sum_{i,j=1}^{3} \delta_{\alpha\beta}\delta_{ij}|E_{i}^{\alpha}E_{j}^{\beta} > \]  
(6.4)

and colour magnetic mode gluonium

\[ |0^{++}MM >= \sum_{\alpha,\beta=1}^{8} \sum_{i,j=1}^{3} \delta_{\alpha\beta}\delta_{ij}|B_{i}^{\alpha}B_{j}^{\beta} > \]  
(6.5)

In (6.4) and (6.5) the indices \( \alpha, \beta = 1, \ldots, 8 \) stand for colour spin, while the indices \( i, j = 1, \ldots, 3 \) stand for the polarization of the field which is related to the third component of the spin of the corresponding off-shell gluon [25].

It is expected that the magnetic mode (6.5) is heavier than the electric mode (6.4) [22,23,25]. Magnetic mode scalar gluonium is also heavier than electric mode scalar gluonium in MIT bag model [39]. On the basis of these considerations we propose to identify the lower mass \( \sigma'(670) \) state with the colour electric mode \( |0^{++}EE > \), and the higher mass \( \sigma(786) \) state with the colour magnetic mode \( |0^{++}MM > \) of the lowest mass scalar gluonium \( 0^{++}(gg) \).

**VII. SUMMARY.**

We have performed a new model independent amplitude analysis of reaction \( \pi^-p \rightarrow \pi^-\pi^+n \) at 17.2 GeV/c extending the range of dipion mass from 600–880 MeV of the earlier analysis [17,18] to 580–1080 MeV in the present analysis. In this study we focused on the resonant structure of the \( S^- \) wave amplitudes \( \overline{S}|^2\Sigma \) and \( |S|^2\Sigma \) in this new mass range. Our purpose was to understand the role of \( f_0(980) \) interference and to determine better the resonance parameters of \( \sigma(750) \) resonance from fits to both transversity amplitudes \( \overline{S}|^2\Sigma \) and \( |S|^2\Sigma \). This work is the first attempt to determine resonance parameters of a resonance from fits to production amplitudes of opposite nucleon transversity. In the absence of previous experience, we used several fitting procedures to understand the resonant structure of the \( S^- \) wave amplitudes.

The amplitude \( \overline{S}|^2\Sigma \) (recoil nucleon transversity “up”) shows a clear resonant peak at 750 MeV below 880 MeV, an enhancement above 900 MeV followed by a rapid descent to a dip at 1010 MeV. The enhancement is more pronounced in the Solution 2. The enhancement
and the dip are explained by the interference of $f_0(980)$ with coherent background and the contribution from $\sigma(750)$ resonance. The interference of $f_0(980)$ and $\sigma(750)$ does not lead to a high mass and a broad width of $\sigma$ as suggested in Ref. [20,21].

The amplitude $|S|^2\Sigma$ (recoil nucleon transversity “down”) shows a broad structure and a dip at 1010 MeV. The broad structure is a composite of a contribution from $\sigma(750)$ resonance and enhancement due to interference of $f_0(980)$ with background and $\sigma$. The broad structure arises as the two contributions have comparable magnitudes. The resonance behaviour of $|S|^2\Sigma$ was not recognized in the earlier analyses [17,18] because the mass range was limited only to 600–880 MeV. As the result the amplitude $|S|^2\Sigma$ was described in [17,18] as “nonresonating”.

The above description of the structure of $S$-wave amplitudes was first reached by using separate fits to the amplitudes $|\overline{S}|^2\Sigma$ and $|S|^2\Sigma$. However the separate fits suffer from the difficulty that the width of $\sigma$ is narrow 110 or 76 MeV in Solutions 1 and 2 of amplitude $|\overline{S}|^2\Sigma$ but it is broad at 217 and 233 MeV in Solutions 1 and 2 of amplitude $|S|^2\Sigma$.

Next we tried to ascertain whether this apparent dependence of $\sigma$ width $\Gamma_\sigma$ on nucleon spin is a real physical effect or an artifact of separate fits to $|\overline{S}|^2\Sigma$ and $|S|^2\Sigma$. To this end we have first performed a simultaneous fit to $|\overline{S}|^2\Sigma$ and $|S|^2\Sigma$ with a common $\sigma$ pole. The differences in the shape of mass distributions $|\overline{S}|^2\Sigma$ and $|S|^2\Sigma$ were expected to arise purely from the coherent background. The result of the best fit gives

$$m_\sigma = 775 \pm 17 \text{ MeV} , \quad \Gamma_\sigma = 147 \pm 33 \text{ MeV}$$ (7.1)

We note that the previous determinization [18] of $m_\sigma$ and $\Gamma_\sigma$ for $|\overline{S}|^2\Sigma$ alone and without $f_0(980)$ interference gave somewhat smaller mass and a narrower width.

$$m_\sigma = 753 \pm 19 \text{ MeV} , \quad \Gamma_\sigma = 108 \pm 53 \text{ MeV}$$ (7.2)

The values of $\chi^2/dpt$ in the best simultaneous fit show a good fit but nevertheless they are higher than the $\chi^2/dpt$ from the best separate fits. Thus the possibility of the dependence of $\Gamma_\sigma$ on nucleon spin has not been eliminated by these fits. Consequently we turned to
examine the hypothesis that the width $\Gamma_\sigma$ does depend on nucleon spin in a new set of simultaneous fits to $|S|^2\Sigma$ and $|S|^2\Sigma$. If we denote $\sigma$ the narrow $\sigma$ contributing to $|S|^2\Sigma$ and $\sigma$ the broader $\sigma$ contributing to $|S|^2\Sigma$, the self-consistency requires that some $\sigma$ contributes to $|S|^2\Sigma$ and some $\sigma$ contributes to $|S|^2\Sigma$. So instead of one $\sigma$ pole with a width dependent on nucleon spin, we were now looking for two $\sigma$ poles of similar masses with different widths and different relative contributions to the amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$ to explain the results of separate fits.

The results of the simultaneous fits with two $\sigma$ poles turned out differently. We did find two $\sigma$ poles with different widths, but they also have considerably different masses and did not distinguish themselves by contributing preferentially to one or the other $S$-wave amplitude. We called $\sigma$ the $\sigma$ pole with higher mass and broader width, and $\sigma'$ the $\sigma$ pole with lower mass and narrower width. We found that $\sigma$ is the dominating contribution in both solutions in both amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$. The contribution of $\sigma'$ is weaker in all amplitudes, but it does depend on the nucleon spin. It is weak in the spin “up” amplitudes $|S|^2\Sigma$, and much stronger in the spin “down” amplitudes $|S|^2\Sigma$.

We found two solutions for the two poles $\sigma$ and $\sigma'$. The Solution A is much preferred on the basis of very low $\chi^2/dpt$. The averaged resonance parameters are

$$m_\sigma = 786 \pm 24 \text{ MeV} , \quad \Gamma_\sigma = 130 \pm 47 \text{ MeV} \quad (7.3)$$

$$m_{\sigma'} = 670 \pm 30 \text{ MeV} , \quad \Gamma_{\sigma'} = 59 \pm 58 \text{ MeV}$$

The Solution A for $\sigma$ and $\sigma'$ has $\chi^2/dpt$ lower than $\chi^2/dpt$ for separate fits or simultaneous fits with single $\sigma$ pole. On the basis of this $\chi^2$ criterion we recommend that the Solution A for $\sigma$ and $\sigma'$ poles be accepted as the actual physical explanation of the resonant structure of amplitudes $|S|^2\Sigma$ and $|S|^2\Sigma$.

In Section VI we showed that both $\sigma$ and $\sigma'$ poles meet several criteria for gluonium candidates: their masses are too low to be a $q\bar{q}$ scalar states, they are not observed in $\gamma\gamma \rightarrow \pi\pi$ reactions, and they both satisfy Ellis-Lanik theorem (6.1) with the same value of
gluon condensate $G_0$. On the basis of Bjorken’s classification of gluonium states $[22, 23]$ we proposed to identify $\sigma'(670)$ and $\sigma(786)$ with the colour electric and colour magnetic modes of lowest mass scalar gluonium $0^{++}(gg)$.

The CERN measurements of pion production on polarized targets have opened a whole new approach to experimental hadron spectroscopy by making possible the study of resonance production on the level of spin amplitudes. Our results emphasize the need for a dedicated and systematic study of various production processes on the level of spin amplitudes measured in experiments with polarized targets.

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FIGURES

FIG. 1. Mass dependence of physical solutions for moduli squared of $S$-wave and $P$-wave nucleon transversity amplitudes and cosines of their relative phases in reaction $\pi^- p \to \pi^- \pi^+ n$ at 17.2 GeV/c and momentum transfers $-t = 0.995 - 0.2 (\text{GeV/c})^2$. The results are in the $t$-channel dipion helicity frame.

FIG. 2. Mass dependence of unnormalized amplitudes $|\mathcal{S}|^2\Sigma$ and $|S|^2\Sigma$ measured in $\pi^- p \uparrow \to \pi^- \pi^+ n$ at 17.2 GeV/c and $-t = 0.005 - 0.20 (\text{GeV/c})^2$. Figures (a) and (b) show the results using the Monte Carlo method for amplitude analysis of Data Set 1 (Ref. [17]) and Data Set 3 (this paper). Figures (c) and (d) show the results using the $\chi^2$ minimization method for amplitude analysis of Data Set 1 (Ref. [5,18]) and Data Set 3 (Ref. [10]), respectively.

FIG. 3. Four solutions for the $S$-wave partial-wave intensity $I_S$ in the reaction $\pi^- p \to \pi^- \pi^+ n$ at 17.2 GeV/c and $-t = 0.005 - 0.20 (\text{GeV/c})^2$.

FIG. 4. The fits to amplitudes $|\mathcal{S}|^2\Sigma$ and $|S|^2\Sigma$. (a) The separate fits with separate single $\sigma$ pole using the parametrization (3.9). (b) The simultaneous fit with a common single $\sigma$ pole using the parametrization (3.9). Figures (c) and (d) show Solution A and B, respectively, of the simultaneous fit with two common $\sigma$ poles using the parametrization (5.9) and (5.10). The Solution A in (c) gives the best $\chi^2/dpt$. 

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TABLE I. Mass and width of \( \sigma(750) \) and \( f_0(980) \) from separate fits to solutions \( \overline{1} \) and \( \overline{2} \) of amplitude \( |S|^2 \Sigma \), and to solutions 1 and 2 of amplitude \( |S|^2 \Sigma \).

| Solution | \( m_\sigma \) (MeV) | \( \Gamma_\sigma \) (MeV) | \( m_f \) (MeV) | \( \Gamma_f \) (MeV) | \( \chi^2/\text{dpt} \) |
|----------|-----------------|-----------------|----------------|----------------|----------------|
| \( \overline{1} \) | 773 \( \pm \) 12 | 110 \( \pm \) 25 | 956 \( \pm \) 17 | 81 \( \pm \) 40 | 0.206 |
| \( \overline{2} \) | 775 \( \pm \) 16 | 76 \( \pm \) 37 | 973 \( \pm \) 17 | 57 \( \pm \) 25 | 0.240 |
| 1 | 788 \( \pm \) 33 | 217 \( \pm \) 53 | 982 \( \pm \) 10 | 42 \( \pm \) 39 | 0.294 |
| 2 | 807 \( \pm \) 45 | 233 \( \pm \) 109 | 988 \( \pm \) 10 | 16 \( \pm \) 25 | 0.148 |

TABLE II. Mass and width of \( \sigma(750) \) and \( f_0(980) \) from simultaneous fits to \( |S|^2 \Sigma \) and \( |S|^2 \Sigma \) with a common single \( \sigma \) pole. The combination of solutions of \( |S|^2 \Sigma \) and \( |S|^2 \Sigma \) is given in the parentheses on the left with the notation as in Table I. The \( \chi^2/\text{dpt} \) shown is the average of \( \chi^2/\text{dpt} \) for \( |S|^2 \Sigma \) and \( |S|^2 \Sigma \) solutions.

| Fit | \( m_\sigma \) (MeV) | \( \Gamma_\sigma \) (MeV) | \( m_f \) (MeV) | \( \Gamma_f \) (MeV) | \( \chi^2/\text{dpt} \) |
|-----|-----------------|-----------------|----------------|----------------|----------------|
| \( \overline{11} \) | 771 \( \pm \) 13 | 161 \( \pm \) 22 | 980 \( \pm \) 10 | 49 \( \pm \) 21 | 0.386 |
| \( \overline{12} \) | 771 \( \pm \) 14 | 145 \( \pm \) 25 | 970 \( \pm \) 19 | 60 \( \pm \) 28 | 0.292 |
| \( \overline{21} \) | 780 \( \pm \) 21 | 156 \( \pm \) 43 | 980 \( \pm \) 9 | 47 \( \pm \) 25 | 0.370 |
| \( \overline{22} \) | 780 \( \pm \) 18 | 125 \( \pm \) 40 | 975 \( \pm \) 45 | 20 \( \pm \) 20 | 0.272 |
TABLE III. Mass and width of two $\sigma$ poles from simultaneous fits to $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ using parametrization (5.9), (5.10) for $|\bar{S}|^2\Sigma$ and corresponding parametrization for $|S|^2\Sigma$. Values without errors were obtained in a constrained fit. The $\chi^2/dpt$ shown is the average of $\chi^2/dpt$ for $|\bar{S}|^2\Sigma$ and $|S|^2\Sigma$ solutions.

| Amplitudes | $m_\sigma$ | $\Gamma_\sigma$ | $m_\sigma'$ | $\Gamma_\sigma'$ | $\chi^2/dpt$ |
|------------|------------|-----------------|-------------|-----------------|-------------|
|            | (MeV)      | (MeV)           | (MeV)       | (MeV)           |             |
| Solution A |            |                 |             |                 |             |
| (T1)       | 775        | 118             | 676         | 73              | 0.274       |
| (T2)       | 775 ± 19   | 118 ± 34        | 676 ± 44    | 73 ± 71         | 0.168       |
| (T1)       | 779 ± 27   | 167 ± 63        | 666 ± 12    | 36 ± 38         | 0.172       |
| (T2)       | 795 ± 29   | 120 ± 57        | 665 ± 18    | 55 ± 51         | 0.110       |
| Solution B |            |                 |             |                 |             |
| (T1)       | 761 ± 27   | 140 ± 69        | 717 ± 43    | 72 ± 79         | 0.268       |
| (T2)       | 770 ± 23   | 123 ± 48        | 709 ± 44    | 69 ± 77         | 0.168       |
| (T1)       | 770        | 123             | 709         | 69              | 0.246       |
| (T2)       | 770        | 123             | 709         | 69              | 0.180       |

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TABLE IV. Spin dependence of the $\sigma'$ coupling constants $D_1$ and $D_2$ defined in (5.5). $\overline{D}_1$ and $\overline{D}_2$ are understood for amplitudes $\overline{1}$ and $\overline{2}$.

| Amplitude | $m_\sigma, \Gamma_\sigma$ (MeV) | $m_{\sigma'}, \Gamma_{\sigma'}$ (MeV) | $D_1$ | $D_2$ |
|-----------|-------------------------------|-------------------------------|------|------|
| $\overline{1}(T1)$ | (775, 118) | (676, 73) | $-0.06 \pm 0.10$ | $0.05 \pm 0.10$ |
| $\overline{1}(T2)$ | (775, 118) | (676, 73) | $-0.06 \pm 0.20$ | $0.06 \pm 0.15$ |
| $\overline{2}(T1)$ | (799, 167) | (666, 36) | $-0.36 \pm 0.36$ | $0.20 \pm 0.25$ |
| $\overline{2}(T2)$ | (795, 120) | (665, 55) | $-0.31 \pm 0.36$ | $0.25 \pm 0.29$ |
| $\overline{1}(T1)$ | (775, 118) | (676, 73) | $0.80 \pm 0.60$ | $-0.38 \pm 0.53$ |
| $\overline{1}(T2)$ | (799, 167) | (666, 36) | $-0.06 \pm 0.33$ | $-0.34 \pm 0.36$ |
| $\overline{2}(T2)$ | (775, 118) | (676, 73) | $0.65 \pm 0.88$ | $-0.27 \pm 1.37$ |
| $\overline{2}(T2)$ | (795, 120) | (665, 55) | $0.10 \pm 0.52$ | $-0.47 \pm 0.56$ |

TABLE V. Spin dependence of the $f_0(980)$ coupling constants $C_1$ and $C_2$ defined in (5.5) and the fitted values for $m_f$ and $\Gamma_f$. $\overline{C}_1$ and $\overline{C}_2$ are understood for amplitudes $\overline{1}$ and $\overline{2}$. The ordering of masses is as in Table IV.

| Amplitude | $C_1$ | $C_2$ | $m_f$ (MeV) | $\Gamma_f$ (MeV) |
|-----------|------|------|-------------|-------------|
| $\overline{1}(T1)$ | $0.15 \pm 0.28$ | $0.65 \pm 0.20$ | $968 \pm 16$ | $95 \pm 29$ |
| $\overline{T}(T2)$ | $0.30 \pm 0.55$ | $0.60 \pm 0.26$ | $958 \pm 18$ | $91 \pm 40$ |
| $\overline{2}(T1)$ | $0.92 \pm 1.04$ | $1.34 \pm 1.44$ | $981 \pm 7$ | $35 \pm 24$ |
| $\overline{2}(T2)$ | $1.38 \pm 1.05$ | $0.89 \pm 1.41$ | $973 \pm 16$ | $58 \pm 26$ |
| $\overline{1}(T1)$ | $-1.37 \pm 0.71$ | $-0.21 \pm 0.86$ | $968 \pm 16$ | $95 \pm 29$ |
| $\overline{1}(T2)$ | $-0.83 \pm 0.45$ | $0.35 \pm 0.56$ | $981 \pm 7$ | $34 \pm 24$ |
| $\overline{2}(T2)$ | $-1.09 \pm 1.29$ | $0.33 \pm 1.30$ | $958 \pm 19$ | $91 \pm 41$ |
| $\overline{2}(T2)$ | $-0.66 \pm 0.85$ | $0.56 \pm 0.71$ | $974 \pm 15$ | $58 \pm 26$ |
TABLE VI. Spin dependence of coherent background $B_1$ and $B_2$ and the normalization constant $N_S$ defined in (5.5), (5.9) and (5.10). $\overline{B}_1$, $\overline{B}_2$ and $\overline{N}_S$ are understood for amplitudes $\overline{\Gamma}$ and $\overline{\Omega}$. The ordering of masses as in Table IV.

| Amplitude | $B_1$       | $B_2$       | $N_S$       |
|-----------|-------------|-------------|-------------|
| $\overline{\Gamma}(\overline{\Gamma}_1)$ | $0.82 \pm 0.19$ | $0.27 \pm 0.13$ | $2.81 \pm 0.74$ |
| $\overline{\Gamma}(\overline{\Gamma}_2)$ | $0.81 \pm 0.29$ | $0.22 \pm 0.35$ | $2.91 \pm 0.91$ |
| $\overline{\Omega}(\overline{\Omega}_1)$ | $1.59 \pm 0.64$ | $0.36 \pm 0.75$ | $1.42 \pm 0.82$ |
| $\overline{\Omega}(\overline{\Omega}_2)$ | $1.82 \pm 0.73$ | $0.15 \pm 1.26$ | $1.26 \pm 0.82$ |
| $1(\overline{\Gamma}_1)$ | $-0.92 \pm 0.60$ | $0.44 \pm 0.42$ | $0.76 \pm 0.61$ |
| $1(\overline{\Omega}_1)$ | $-0.12 \pm 0.25$ | $0.35 \pm 0.44$ | $1.92 \pm 1.56$ |
| $2(\overline{\Gamma}_2)$ | $-1.10 \pm 1.42$ | $0.61 \pm 0.98$ | $0.80 \pm 1.26$ |
| $2(\overline{\Omega}_2)$ | $-0.47 \pm 0.74$ | $0.89 \pm 0.65$ | $1.15 \pm 1.06$ |
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