**Bose-Bose mixtures in reduced dimensions**

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**Abstract.** The two-body scattering is greatly modified in reduced dimensions. With ultracold atoms, low dimensional configurations are routinely accessible thanks to the use of optical lattices which allow confinements sufficiently strong to freeze the motion along chosen directions. With two different atomic species, we use a species-selective optical potential, in the form of a standing wave, to confine only one species in 2D disks and study the scattering between particles existing in different dimensions, i.e., we realize a 2D-3D mix-dimensional configuration, reminiscent of a brane world.

We review the scattering theory specific to this configuration and derive an effective scattering length \(a_{\text{eff}}\) in terms of the free-space scattering length \(a\) and the confinement parameters. We detect experimentally the enhancement of inelastic collisions arising at particular values of \(a\) and relate these values to the divergences of \(a_{\text{eff}}\). Unlike the confinement-induced resonances predicted and observed for identical particles, our mixed-dimensional resonances occur in a series of several resonances, because the relative and centre-of-mass motion are coupled.

**1. Introduction**

Some of the most spectacular developments witnessed in the domain of quantum degenerate gases have been made possible by the control of interatomic interactions achieved by means of scattering resonances. Altering the two-body scattering length has been instrumental both for the creation of strongly correlated many-body states [1] and for the investigation of few-body physics, most notably for the observation of Efimov resonances [2].

While Fano-Feshbach resonances represent the most widely used method to modify the scattering length between two atoms, more than a decade ago M. Olshanii realized that strong confinement also affects the scattering in a non-trivial manner [3]. The most dramatic prediction was that the effective coupling strength of two identical particles tightly confined along two directions, and therefore moving only in 1D, diverges for one particular value of the transverse-confinement length to the scattering length ratio, \(\ell/a\) [3]. Such a scattering resonance was therein termed a confinement-induced resonance (CIR). The work of Olshanii also provided a much needed interpretation of the CIR [4], as a resonance arising from the degeneracy of the two colliding atoms, both in the ground state of the transverse potential, with the bound state in the closed channel formed by the set of excited levels of the transverse potential. The scattering of identical particles moving on a 2D plane also displays non-trivial features [5, 6, 7]:

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the coupling strength diverges for a single negative value of the free-space scattering length, \( \ell/|a| = (1/\sqrt{2\pi})\ln(1/q^2\ell^2) \), which however depends on the relative momentum \( q \).

Following the above-mentioned works, the modifications of the scattering in reduced dimensions was later confirmed by experimental observations. First, Moritz et al. observed confinement-induced molecules, i.e., a 1D bound state existing even at negative values of the scattering length \( a < 0 \) for which, in the absence of the external confinement, no weakly bound state exists \[8\]. The same group also studied the modifications of \( p \)-wave interactions in low dimensionality \[9\]. Haller et al. observed the presence of a CIR and employed it to enhance the interactions of a 1D degenerate gas of Cs atoms \[10\]. The dramatic increase of atomic interactions afforded by the CIR led to the creation of a strongly correlated Tonks-Girardeau gas of repulsive 1D bosons and of a metastable, strongly-correlated state of attractive bosons, the so-called “super-Tonks gas” \[10\]. Finally, a CIR between two spin states of \(^{6}\text{Li}\) atoms has been observed in a quasi-2D gas confined in a disk-shaped trap \[11\].

In this work, we describe a novel type of resonances arising between two different particles when their confinement is strongly asymmetric. Specifically, we studied a system of K and Rb atoms, such that the K atoms are confined in an array of 2D disks whereas the Rb atoms move freely in 3D. We will refer to these resonances as mix-dimensional resonances (MDR) \[12\]. Such resonances occur whenever the effective scattering length \( a_{\text{eff}} \) between the two colliding particles diverges. Although \( a_{\text{eff}} \) pertains to the two-body collisions, we detect the divergence of \( a_{\text{eff}} \) by monitoring the entity of three-body inelastic collisions that expel the collisional partners from the trap. Throughout, our working assumption is that, like in free-space, inelastic 3-body collisions are greatly enhanced as \( a_{\text{eff}} \to \infty \).

A crucial difference with respect to all the above mentioned systems is that the asymmetric confinement couples the relative motion with the centre-of-mass (COM) motion. Indeed, for two particles subject to equal harmonic confinement, the COM motion is completely decoupled and, as such, can be ignored.

Closely related to our work and appearing soon afterwards, is a recent experiment where the CIR between identical Cs atoms is found to split in two separate resonances as the 2D optical lattice is transformed into a 1D lattice, i.e., the system undergoes a dimensional crossover from 1D tubes to 2D disks \[13\]. The interpretation of the resonance splitting proposed in \[13\] is under debate \[14\].

2. Scattering in mixed dimensions

We apply here the theory of scattering to two particles of unequal masses \( m_1 \) and \( m_2 \), the first being confined along the \( z \)-axis by a strong potential, the second being free. First, we consider the case of harmonic confinement and the interactions are modelled by a delta-like pseudo-potential, similar to the work of Massignan and Castin \[15\]. We then introduce the effective range correction to the pseudo-potential approximation, since this turns out to be important for our experiment. Finally, we replace the harmonic potential with the 1D lattice: while the effective scattering length is no longer available, we can still identify the values of \( a \) that make \( a_{\text{eff}} \) diverge by using a simple argument relating the energy of closed and open channels. Although an approximation, such an argument holds in the case of harmonic confinement: it yields the position of MDRs in perfect agreement with the formal theory. Thus, we use it for the case of 1D lattice confinement as well and compare the predictions with the observed data.

2.1. Harmonic confinement

The relevant Schrödinger equation is given by

\[
\left[ -\frac{\hbar^2}{2m_1} \nabla^2_{r_1} - \frac{\hbar^2}{2m_2} \nabla^2_{r_2} + \frac{1}{2}m_1\omega_1^2z_1^2 + gV \right] \psi(r_1, r_2) = E\psi(r_1, r_2)
\]  

(1)
where \( g = 2\pi \hbar^2 a / \mu \), \( a \) being the s-wave scattering length, \( \mu \) the reduced mass, and \( V \) the pseudo-potential

\[
V\psi(r_1, r_2) = \delta(r_1 - r_2) \frac{\partial}{\partial |r_1 - r_2|} (|r_1 - r_2| \psi(r_1 - r_2)) \equiv \chi(r)|_{r=r_1=r_2}.
\] (2)

Due to the translational symmetry, the COM motion in the \( xy \) plane can be eliminated and the Schrödinger equation reduces to

\[
\left[ -\frac{\hbar^2}{2m_1} \frac{\partial^2}{\partial z_1^2} + \frac{1}{2} m_1 \omega^2 z_1^2 - \frac{\hbar^2}{2m_2} \frac{\partial^2}{\partial z_2^2} - \frac{\hbar^2}{2\mu} \nabla^2 \mathbf{\rho} + gV \right] \psi(z_1, z_2, \mathbf{\rho}) = E \psi(z_1, z_2, \mathbf{\rho}),
\] (3)

where \( \mathbf{\rho} = (x_1 - x_2, y_1 - y_2) \). The general solution of this equation can be written as

\[
\psi(z_1, z_2, \mathbf{\rho}) = \psi_0(z_1, z_2, \mathbf{\rho}) + g \int_{-\infty}^{\infty} dz' G_E(z_1, z_2, \mathbf{\rho}; z', 0) \chi(z', 0) \] (4)

with \( \psi_0 \) the solution of the non-interacting Schrödinger equation \((V = 0)\) and \( G_E \) the two-particle retarded Green’s function:

\[
G_E(z_1, z_2, \mathbf{\rho}; z'_1, z'_2, \mathbf{\rho'}) = \sum_{n=0}^{\infty} \int_{n=0}^{\infty} \frac{d^2 k d k_z}{(2\pi)^3} \frac{(z_1, z_2, \mathbf{\rho}| k, n, k_z)}{(k, n, k_z; z'_1, z'_2, \mathbf{\rho}')} \left( \frac{E - \hbar k_z^2}{2\mu} + \frac{(n + \frac{1}{2}) \hbar \omega}{2m_2} + \frac{(\hbar k_z^2)^2}{2m_2} \right) + i0^+ \]
\[
= -\frac{2\mu}{\hbar^2} \sum_n e^{-\sqrt{2m_2(n+1/2)}|\mathbf{\rho} - \mathbf{\rho}'|/\hbar} \frac{1}{4\pi|\mathbf{\rho} - \mathbf{\rho}'|} \phi_n(z_1) \phi_n^*(z'_1),
\] (5)

where \( \phi_n(z) \) are the wave functions of the 1D harmonic oscillator with mass \( m_1 \) defined as

\[
\phi_n(z) = \left( \frac{2^{n+1}}{(n! \sqrt{\pi})} \right)^{1/2} e^{-\frac{z^2}{2}} H_n(z), \quad \ell = \sqrt{\frac{\hbar}{m_1 \omega}}
\] (6)

\( H_n \) being the Hermite polynomials. We have also defined \( \mathbf{\tilde{\rho}} \equiv \left( -\frac{\mu}{m_2} \mathbf{\rho}, z_2 \right) \), rescaled and anisotropic coordinates of the free atom relative to the confined atom.

We now consider the low-energy scattering for which \( E - \hbar \omega/2 = (\hbar^2 k_z^2)/(2m_2) + (\hbar^2 k'^2)/(2\mu) \ll \hbar \omega \). Then \( \psi_0 \) becomes

\[
\psi_0(z_1, \mathbf{\tilde{\rho}}) = Ce^{ik\mathbf{\hat{r}} \cdot \phi_0(z_1), \quad k \equiv (k_\rho, k_z)}
\] (7)

We can also write the asymptotic form of \( \psi \) at a large separation \(|\mathbf{\tilde{\rho}}| \gg \ell \) as

\[
\psi(z_1, \mathbf{\tilde{\rho}}) \to C \left[ e^{ik\mathbf{\hat{r}}} + \frac{e^{ik\mathbf{\hat{r}}}}{\ell} f(k'_z) \right] \phi_0(z_1),
\] (8)

where \( f(k'_z) \) with \( k'_z \equiv k\mathbf{\hat{r}} \) defines the two-body scattering amplitude in the 2D-3D mixed dimensions:

\[
f(k'_z) \equiv -\frac{1}{C} \int dz' e^{-ik'_z z'} \phi_0^*(z') \chi(z').
\] (9)

We note that \( \chi \) has an implicit \( k_z \) dependence and both \( \chi \) and \( f \) depend on \( k \) through the Green’s function (5). The unknown function \( \chi \) can be determined by substituting the solution
(4) into the Bethe-Peierls boundary condition. We isolate the regular part of the Green’s $G_E$ and denote it with $G$:

$$G_E(z_1, z_2, \rho; z', z', 0)|_{z_1, z_2 \rightarrow z; \rho \rightarrow 0} = -\frac{\mu}{2\pi \hbar^2 \sqrt{(z_1 - z_2)^2 + \rho^2}} \delta(z - z') + g(z; z'),$$

and obtain

$$\frac{1}{\alpha} \chi(z) = Ce^{ikz} \phi_0(z) + \frac{2\pi \hbar^2}{\mu} \int dz' g(z; z') \chi(z').$$

This integral equation determines $\chi/C$, which in turn provides $f$ from Eq. (9).

Because the system has reflection symmetry about the $z$-axis, $f$, $\chi$, and $G$ can be decomposed into their even and odd-parity components, i.e., $f_{\pm}(k') = [f(k') \pm f(-k')]/2$, $\chi_{\pm}(z) = [\chi(z) \pm \chi(-z)]/2$ and $G_{\pm}(z; z') = [G(z; z') \pm G(z; -z')]/2$. At the lowest order in $k$, we only need to consider even components. At finite temperature, odd components might, however, play a role as they feature an independent set of resonances. From the explicit calculation using the Green’s function in Eq. (5), we can show that $f_+$ has the following low-energy expansion:

$$\lim_{k \rightarrow 0} f_+ = -\frac{1}{a_{\text{eff}}} - \frac{1}{2} r_{\text{eff}} k^2 + O(k^4) + ik [1 + O(k^2)] + O(k^2, k^2)$$

where $a_{\text{eff}}$ and $r_{\text{eff}}$ are effective scattering length and range parameters in the even parity channel.

At zero-energy $k \rightarrow 0$, $f_+(0) = -a_{\text{eff}}$. Then from (9) and (11), we obtain the following integral equation

$$\frac{1}{\alpha} \chi(z) = \frac{1}{a_{\text{eff}}} \phi_0(z) \int dz' \phi_0^*(z') \chi(z') + \frac{2\pi \hbar^2}{\mu} \int dz' G_{\pm}(z; z')|_{k \rightarrow 0} \chi_{\pm}(z')$$

3. Secular equation

Now we briefly show how to turn the above integral equation (13) into a linear secular equation. We first evaluate the regular part of the Green’s function $G$ and then expand both $G$ and $\chi$ into an orthonormal set of functions.

At low energy $E - \frac{\hbar^2}{2} \equiv \frac{\hbar^2}{2m_2} \leq 0$, it is useful to represent $G_E$ as

$$G_E(z_1, z_2, \rho; z'_1, z'_2, \rho') = -\langle z_1, z_2, \rho | \int_0^{\infty} d\tau h^{-1} e^{(E-H_0)\tau/h} | z'_1, z'_2, \rho' \rangle =$$

$$= -\int_0^{\infty} d\tau h^{-1} e^{\frac{\hbar^2}{2m_2} \tau} \sqrt{\frac{m_1 \omega \omega'}{2\pi \hbar \sinh \omega}} \sqrt{\frac{m_2}{2\pi \hbar \tau}} \frac{\mu}{2\pi \hbar \tau} e^{-\frac{m_2}{2\pi \hbar} \frac{(z_1 + z'_1)^2 \cosh \omega - 2z_1 z'_1}{\sinh \omega}} - \frac{m_2}{2\pi \hbar} \frac{(z_2 - z'_2)^2}{\sinh \omega^2} - \frac{\mu}{2\pi \hbar} (\rho - \rho')^2.$$

Therefore

$$G(z; z') = -\int_0^{\infty} d\tau h^{-1} e^{\frac{\hbar^2}{2m_2} \tau} \sqrt{\frac{m_1 \omega \omega'}{2\pi \hbar \sinh \omega}} \sqrt{\frac{m_2}{2\pi \hbar \tau}} \frac{\mu}{2\pi \hbar \tau} e^{-\frac{m_2}{2\pi \hbar} \frac{(z_1 + z'_1)^2 \cosh \omega - 2z_1 z'_1}{\sinh \omega}} - \frac{m_2}{2\pi \hbar} \frac{(z_2 - z'_2)^2}{\sinh \omega^2} - \frac{\mu}{2\pi \hbar} (\rho - \rho')^2$$

$$+ \int_0^{\infty} d\tau h^{-1} \left( \frac{\mu}{2\pi \hbar \tau} \right)^{3/2} \delta(z - z').$$

(15)
the positive solution of the secular equations are given by

\[ \text{one resonance is expected. In the limit of large mass imbalance} \]

\[ m \rightarrow \infty \]

an infinite series of resonances for positive \( a \)

\[ a > 0 \]

values, in stark contrast to the case of CIR, where only

\[ a < 0 \]

are given in units of the harmonic oscillator length \( \ell \)

\[ \ell = \sqrt{\hbar /m_1 \omega} \]

We notice that in addition to one resonance, i.e., divergence of \( a_{\text{eff}} \) at \( a < 0 \), there is an

\[ a_{\text{eff}} \]

in the limit of large mass imbalance \( m_1 \gg m_2 \), it can be shown that the positive solution of the secular equations are given by

\[ h \omega \sqrt{\frac{m_1}{m_1 + m_2}} (n' + \frac{1}{2}) - \frac{h^2}{2ma^2} = \frac{1}{2} h \omega, \]

\[ n' \]

for \( i, j \) even integers, where \( _2F_1 \) is the hypergeometric function, \( 2X \equiv 1 + m_2/(m_1 \omega) \) + 1/\( \tanh \omega \tau \) and \( 2Y \equiv m_2/(m_1 \omega \tau) + 1/\sinh \omega \tau \). Expanding also \( \chi_+ \) in terms of \( \phi_n \), the integral equation (13) reduces to the following secular equation

\[ \frac{1}{a} b_i = \left[ \frac{1}{a_{\text{eff}}} \delta_{ij} \delta_{\bar{a} \bar{b}} + M_{ij} \right] b_j \]

In figure 1 we show the numerical solution of the above secular equation. Inspection of (17) reveals that, once the scattering lengths are expressed in units of the harmonic oscillator length \( \ell = \sqrt{\hbar /m_1 \omega} \), the solution depends only on the \( m_1/m_2 \) mass ratio.

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\[ a_{\text{eff}} \]
for \( n' = 2, 4, \ldots \). Actually, even for the small mass ratio of our experiment \( m_1/m_2 = 41/87 \), the above equation (19) is very accurate. Based on this equation, we propose the interpretation of MDR given below.

3.1. Effective range corrections

The pseudo-potential approximation is insufficient to accurately describe the binding energy of the closed-channel state in the range experimentally explored. Therefore, following [16] we introduce the effective range correction by replacing the inverse scattering length \( 1/a \) with the energy-dependent quantity

\[
\frac{1}{a(E_c)} = \frac{1}{a} - \frac{\mu r_0}{\hbar^2} \left[ E + \frac{\hbar^2}{2(m_1 + m_2)} \frac{\partial^2}{\partial z^2} - \frac{1}{2} m_1 \omega^2 z^2 \right]
\]

where \( r_0 \) denotes the effective range of the interaction potential: in our case, \( r_0 = 168.37 a_0 \) (\( a_0 \) Bohr radius) is evaluated by fitting the measured binding energy [17] with the formula [16]

\[
E_b = \frac{\hbar^2}{\mu r_0^2} \left( 1 - \frac{r_0}{a} - \sqrt{1 - 2 \frac{r_0}{a}} \right). \tag{21}
\]

As a consequence, in the secular equation (18) we must replace \( M_{i,j} \) with the following matrix:

\[
\tilde{M}_{ij} = M_{i,j} + \frac{\mu r_0}{\hbar^2} \int_{-\infty}^{\infty} dz \phi_i^*(z) \left[ \frac{\hbar \omega}{2} + \frac{\hbar^2}{2(m_1 + m_2)} \frac{\partial^2}{\partial z^2} - \frac{1}{2} m_1 \omega^2 z^2 \right] \phi_j(z). \tag{22}
\]

3.2. 1D lattice confinement

The above analysis refers to harmonic confinement; experimentally the K atoms are confined by a 1D lattice, i.e., in an array of 2D disks. In the direction orthogonal to the disks, the confinement can be assumed to be harmonic if the individual sites are isolated from their neighbours. It turns out that this is not the case, for the lattice strengths considered in the experiment: on one hand, the tunneling time of K atoms is of the same order (or shorter) than the experiment duration; on the other hand, the K atoms start from delocalized states before the 1D lattice turns on. For this reason, we modify the above results to include lattice physics, i.e., we replace the harmonic oscillator levels with energy bands. In doing so, we lack a derivation of \( a_{\text{eff}} \), but we can still invoke the energy degeneracy of open and closed channels to find the values of \( a \) that make \( a_{\text{eff}} \rightarrow \infty \).

In the open channel, the incoming state can be written in terms of mixed coordinates: the coordinates of individual atoms \((z_1, z_2)\) along the direction of the 1D lattice plus the COM and relative coordinates along the orthogonal directions \((X, Y, r_x, r_y)\). The incoming state is a product of plane waves in \((X, Y, r_x, r_y)\), a plane wave of momentum \( p \) in \( z_2 \) and a \( 0-\text{th} \) band Bloch wave \( \psi_{q,n=0}(z_1) \). In the closed channel, instead, it is more convenient to use COM and relative coordinates along all directions: the states that couple to the open channel are products of the plane waves of incoming channel in \((X, Y)\), times a \( n'-\text{th} \) band Bloch wave \( \psi_{q+p,n'}(Z) \), times the wavefunction of the s-wave weakly bound molecule \( \phi_b(|\rho|) \). For simplicity, we drop the energy of all plane waves and write the energy degeneracy of open and closed channel states as:

\[
\epsilon(q, n = 0; V_K) = \epsilon(q + p, n'; V_{K'}) - E_b \tag{23}
\]

\( E_b \) denotes the binding energy as in (21), while \( V_K \) denotes the lattice strength. Notice that the lattice strength for the weakly bound molecules is equal to that of K atoms. However, due to the mass difference, the band structures of K and KRb are completely different.
4. Species-selective confinement

A mix-dimensional configuration might seem unnatural or unrealistic, yet in atomic physics it can be easily realized, as it is extremely simple to confine one atomic species without affecting the other [18]. Such a possibility is granted by optical potentials, whose strength depends on the detuning of the trapping laser from the atomic transitions. In our specific experiment, we use a mixture of two alkali atoms, whose $D_1$ and $D_2$ transitions are actually quite close, lying in a wavelength interval ranging from 766.49 nm to 794.76 nm. Therefore the most obvious choice of tuning the trapping dipole laser next to a specific atomic transition is unsuitable, due to the strong spontaneous emission. Instead, we have chosen a wavelength corresponding to the weighted-average of the $D_1$ and $D_2$ lines of $^{87}$Rb, such that the AC-Stark (light) shifts induced by the $D_1$ and $D_2$ transitions cancel out [19].

For a laser detuning much larger than the atomic hyperfine structure splittings, the AC-Stark shift induced by a linearly polarized laser beam of intensity $I$ is given by [20]

$$U = \frac{\pi e^2}{2 \omega_0^3} \Gamma I \left( \frac{2}{\Delta_2} + \frac{1}{\Delta_1} \right),$$

with $\omega_0 = (\omega_2 + \omega_1)/2$, $\omega_{1,2}$ are the $D$-lines transition wavelengths with the respective detunings $\Delta_{1,2} = \omega_L - \omega_{1,2}$, while $\Gamma$ is the transition linewidth. Therefore for $\Delta_2 = -2\Delta_1$ the cancellation occurs. Experimentally, we optimize the cancellation of Rb light shift by minimizing the Raman-Nath diffraction induced by a pulsed 1D optical lattice, $\lambda = 790.02$ nm. By this means we find an upper limit to the ratio of K to Rb potentials: $V_{Rb}/V_K < 10^{-2}$.

5. Experiment and results

We start the experiment by loading a crossed dipole trap with a cold mixture of $^{87}$Rb and $^{41}$K atoms in their lowest hyperfine states, $(F = 1, m_F = 1)$. Evaporation is continued by decreasing the laser beams' intensity to a temperature of the order of 300 nK. At this stage, we set the external, uniform, magnetic field to a constant value of 1.4 mT where the free-space scattering length is positive. We ramp the species-selective standing wave in 50 ms to values ranging from $s = 10$ to 25, in units of K recoil energy, and then ramp linearly the magnetic field to its final value. Thanks to two Feshbach resonances located at 3.8 mT and 7.9 mT, the magnetic field controls the interspecies scattering length [21].

The mixture is held for 65 ms to 100 ms and then we measure the number of atoms after time-of-flight absorption imaging. We scan the final magnetic field to adjust the free-space scattering length, or equivalently the binding energy. During the hold time, the number of trapped atoms decays, by inelastic losses; therefore minima in the final atom number correspond to maxima of the inelastic collision rate, that we identify as divergences of $a_{eff}$.

In the left panel of figure 2, we show a typical experimental scan where several peaks are visible. The magnetic field is known within 10 T as it is routinely calibrated by Rb hyperfine spectroscopy. The free-space scattering length is obtained from the magnetic field by an empirical formula previously derived [12]

$$a(B) = 208a_0 \left( 1 + \frac{3.09}{B + 3.852} - \frac{4.992}{B - 3.837} - \frac{0.164}{B - 7.867} \right).$$

where $B$ is the magnetic field expressed in mT.

From the magnetic field scans, we extract the position of peaks, i.e., atom number minima, at different lattice strengths. We then compare these values with the predictions of the above theoretical analysis, see right panel in figure 2. It is evident that, if we consider the harmonic confinement of isolated lattice sites, the agreement is rather poor, whereas it is satisfactory when we use the lattice energy bands.
Figure 2. Left: total number of atoms, $N_{\text{Rb}} + N_{\text{K}}$, remaining in the trap after fixed hold time as we scan the external magnetic field, in mix-dimensional configuration with a 1D lattice at $s = 25$. The minima are interpreted as MDR. In 3D we observe a broad asymmetric minimum (not shown) around the resonant field of 3.84 mT (dashed line). Right: Magnetic field values of MDR for different 1D lattice strengths, compared to the theory predictions based on the harmonic oscillator analysis (dashed lines) and the energy band structure (color shaded areas).

6. Interpretation
To understand the origin of MDR, it is useful to first focus on the scattering of 2D identical particles: the confinement induces a single CIR, although the resonance condition depends on the relative momentum $q$ [6]. The COM motion can be ignored since, for harmonic confinement perpendicular to the 2D plane, it decouples from the relative motion.

In the mix-dimensional case, the perpendicular confining potential affects only one particle: $2U = m_1 \omega^2 z_1^2 = m_1 \omega^2 Z^2 + \mu_\omega^2 (m_2/M) z^2 - 2\mu_\omega^2 Zz$. If we neglect the last term, the relative and COM motion are decoupled and we expect a single CIR at $a = \ell' \sqrt{2\pi / \log(\pi q^2 \ell^2)}$ [6], where $\ell' = \sqrt{\hbar/(\sqrt{m_1} \mu_\omega)}$ and $q^2 \simeq 2\mu B T / \hbar^2$. For the harmonic frequencies corresponding to $s = 10(25)$ and a relative momentum $q$ corresponding to $T = 0.25 \mu K$, the CIR would occur at 3.89(4.02) mT, in reasonable agreement with the peak observed at negative scattering lengths for $B > B_0 = 3.84$ mT.

The $-\mu \omega^2 Zz$ term can be seen as a perturbation coupling the relative and COM motion which, to first order, does not alter the energy levels [15]. Thanks to this term, many COM states of the closed channel are available for the colliding atoms, hence the series of resonances. This interpretation is illustrated in figure 3. The coupling term preserves parity: thus, an initial state with vanishing Rb momentum, $e^{iBzRb \phi_0(z_K)}|p \rightarrow 0\rangle$, being even, is coupled only to even $n' = 0, 2, 4 \ldots$ COM states (because the molecular wavefunction $\phi_b(|r|)$ is itself even under parity transformations).

However, as explained in Section 3.2, the harmonic oscillator description is not sufficiently accurate when compared to the data: introducing the energy bands, the qualitative interpretation remains unchanged and the quantitative agreement with data is striking. Introducing the Bloch states also allows coupling to odd $n'$ states even for $p_{\text{Rb}} = 0$, since Bloch states are not parity eigenstates. Finally, we remark that the coupling of relative and COM motion and, consequently, multiple resonances arises also for identical particles trapped by anharmonic potentials [22].
7. Conclusions

We have shown that in a mix-dimensional configuration the two-body scattering is profoundly modified. The coupling of relative and COM motion multiplies the number of possible resonances, differently from the (1D or 2D) CIR of identical particles. We observed a series of resonances as we tuned the free-space scattering length and we explored different confinement strengths. We have also shown that a simple argument, based on the degeneracy between open and closed channel states, allows us to locate the mix-dimensional resonances in good agreement with the formal theory for harmonic confinement. This argument has therefore been extended to consider the energy bands of the 1D optical lattice that generates the mix-dimensional configuration: this step was crucial in order to achieve a good quantitative agreement with the data.

Besides their specific interest, mix-dimensional configurations are predicted to facilitate the observation of the Efimov effect in 3-body collisions [23]. For bosons the energy of Efimov states are closer in mixed dimensions. In the case of our heteronuclear mixture, the energy ratio of successive Efimov states is given by $E_{n+1}/E_n = \lambda^2$ with $\lambda = 131 (3.48 \times 10^5)$ for the KRbRb (KKRb) trimers in 3D [24]. This ratio decreases to $\lambda = 27.7 (59.3)$ in a 2D/3D dimensional configuration, if the heavier Rb atoms are confined. For the fermionic atoms, Efimov effect can occur at lower mass ratios: in 3D, a binary mixture of fermions features Efimov states only for $m_1/m_2 > 13.6$ in 3D, but the critical mass ratio is 6.35 in 2D/3D and 2.07 in 1D/3D, provided the heavier atom is confined [23].

Appendix A. Analogy with Kaluza-Klein modes

A mix-dimensional configuration immediately prompts us to draw an analogy with the brane world, where extra dimensions are postulated to account for the weakness of gravitational interactions. This analogy can be taken a step further as we consider Kaluza-Klein modes.

Suppose we have a massless particle in four spatial dimensions, with the extra fourth direction compactified with the radius $R$, meaning that the extra direction has length $R$ and is periodic. Because of the periodic boundary conditions, the momentum in the fourth direction is quantized as $p_4 = n\hbar/R$ where $n$ is an integer. The dispersion relation of such a particle is $E = c(p_x^2 + p_y^2 + p_z^2 + p_4^2)^{1/2}$, which makes the discrete $p_4$ equivalent to a mass: the massless 4D particle appears in the 3D world as an infinite series of particles with masses that are integer multiples of $\hbar/(Rc)$. These are Kaluza-Klein modes.

In our case, because of the confinement, the COM motion of the weakly bound molecule along the z-direction is quantized. We can view such quantized energies as a series of new molecular states whose energy, i.e., mass, is quantized as $E_n = n/(m\ell^2)$ where $\ell$ is the confinement length.
In the experiment, we adjust the binding energy by controlling the magnetic field and move successive molecular states to threshold, causing the observed resonances. In this sense, our molecular states are analogous to the Kaluza-Klein modes.

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