Effective mass signatures in multiphoton pair production

Christian Kohlfurst

Helmholtz-Institut Jena & Friedrich-Schiller-Universität Jena

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CK, H. Gies and R. Alkofer, Phys. Rev. Lett. 112, 050402 (2014)
arxiv.org/abs/1310.7836;

CK, PhD thesis, arXiv:1512.06082;
Why Pair Production?

Cite: www.slac.stanford.edu/exp/e144
QED Vacuum
Electron-Positron Pair

\[ dt \approx 10^{-21} \text{s} \]
\[ dx \approx 10^{-12} \text{m} \]

- Vacuum fluctuations
- QED scale

F. Sauter: Z. Phys. 69(742), 1931
W. Heisenberg et al.: Z. Phys. 98(714), 1936
J. S. Schwinger: Phys. Rev. 82(664), 1951
Multiphoton Pair Production

Electric field $E$ (photons $\gamma$) $\rightarrow$ charge separation
Particles $\rightarrow$ measurable
Critical field strength $E_{cr} \approx 10^{16} \text{ V/cm}$
Critical energy $\mathcal{E}_{e^+e^-} \approx 1 \text{ MeV}$

E. Brezin and C. Itzykson: Phys. Rev. D 2, 1191 (1970)
V. S. Popov: Sov. Phys. JETP 35, 659 (1972)
Electric Field: \( E(t) = \varepsilon E_{cr} \cos(\omega t) \)

**Keldysh-Parameter**

\[
\gamma_K = \frac{\omega \sqrt{2E_{IP}}}{eE}
\]

Absorption (\( \gamma_K \gg 1 \))

\[
P \sim \left( \frac{eE}{2\omega \sqrt{2E_{IP}}} \right)^{2(E_{IP}/\omega)}
\]

Tunneling (\( \gamma_K \ll 1 \))

\[
P \sim \exp\left( -\frac{2}{3} \frac{2(E_{IP})^{3/2}}{eE} \right)
\]

**Pair Production**

\[
\gamma_K = \frac{\omega}{eE}
\]

\[
P \sim \left( \frac{eE}{2\omega} \right)^{4/\omega}
\]

\[
P \sim \exp\left( -\pi \frac{E_{cr}}{eE} \right)
\]

L. V. Keldysh: Sov. Phys. JETP 20(1307), 1965
Motivation

Features
- Non-equilibrium physics
- Non-perturbative effect
- Probing strong-field QED

Experiment
- SLAC
- Extreme Light Infrastructure (ELI)
- FAIR, HiBEF, XFEL
Sketch of an Experimental Setup

- Two colliding laser fields
- Electric field in an antinode of a standing-wave

M. Marklund: Nature Photonics 4, 72-74 2010
Considerations

Goal
Describe $\bar{e}e$ pair production

Requirement
- Electric background field
- Dynamics - field is rapidly oscillating
- Particle statistics
- Observables
Quantum Kinetic Theory

Equation of motion

\[ \overline{D}_t \overline{w} = \overline{M} \overline{w} \]  

(1)

Wigner components \( \overline{w} \)
Source and interaction matrix \( \overline{M} \)
Pseudo-differential operator \( D_t \)

Mean-Field approximation

\[ F_{\mu \nu} \approx \langle \hat{F}_{\mu \nu} \rangle \rightarrow \text{classical background field} \]

Observables

Combination of \( \overline{w} \rightarrow \text{particle density} \ F(q) \)

I. Bialynicki-Birula: Phys. Rev. D 44(1825), 1991
Positive Aspects

- **Electric** fields as input
- Time evolution
- Phase-space approach $\rightarrow$ particle spectrum

Challenges

- Beyond **mean-field** approximation
- Back-reaction and particle collisions
- Numerics
Model for the Field

Electric field: \( E(t) = \varepsilon \exp\left(-\frac{t^2}{2\tau^2}\right) \cos(\omega t) \)

Photon energy \( \omega \); Field strength \( \varepsilon \); Pulse duration \( \tau \)
Effective Mass Model

- $e^- e^+$ interact with electric background field
- Particles behave as if they had a higher mass

inspired by Prof. Miller, University College London
Effective Mass Model

- "Ionization" energy depends on laser field: \( E_{\text{Kin}} = n\omega - m_* \)
- Effective mass: \( m_* = m\sqrt{1 + \varepsilon^2/(2\omega^2)} \)
- Above-Threshold peak position: \( \left( \frac{n\omega}{2} \right)^2 = m_*^2 + q_n^2 \)

Inspired by Prof. Miller, University College London
Particle Distribution

Parameters: \( \tau = 300\, [1/m], \ \epsilon = 0.2\, E_{cr}, \ \omega = 0.3\, [m] \)

- Above-Threshold peaks
- Peak position predictable via effective mass concept
Particle Yield

Parameters: $\tau = 100[1/m]$, $\varepsilon = 0.1 E_{cr}$

- Resonant at n-photon frequencies: $\omega_n = 2m_* / n$
Effective Mass

Parameters: $\tau = 100\,[1/m],\, n = 7$

Parameters: $\tau = 100\,[1/m],\, n = 5$

Comparison: numerical simulation - $m_*$ model
Channel Closing

Parameters: $\tau = 300[1/m] \ \omega = 0.322[m]$

- Above-Threshold peak position varies with $\varepsilon$
- Resonance: Peak at threshold ($q = 0$)
Takeaways

Summary

- Phase-space formalism → pair production processes in the non-perturbative threshold domain
- Effective mass
- Channel closing

Outlook

- Back-reaction
- Beyond mean-field
- 3D-simulation
Thank you!

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Quasi-probability Distribution

**Wigner operator**

\[
\mathbb{W}(x, p) = \frac{1}{2} \int d^4y \, e^{ip \cdot y} \, U(A_\mu, x, y) \left[ \bar{\psi}(x - \frac{y}{2}), \psi(x + \frac{y}{2}) \right]
\]  

(2)

- \( A_\mu \) in mean field approach, \( \mathbb{W}(x, p) \) is gauge invariant

**Equal-time Approach**

\[
\mathcal{W}(x, p, t) = \int \frac{dp_0}{2\pi} \mathbb{W}(x, p)
\]

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D. Vasak et al.: Annals of Physics 173(462-492), 1987
I. Bialynicki-Birula et al.: Phys. Rev. D 44(6), 1991
Quasi-probability Distribution

Wigner operator

\[ \mathcal{W}(x, p) = \frac{1}{2} \int d^4 y \ e^{ip \cdot y} \ U(A_\mu, x, y) \left[ \bar{\psi}(x - \frac{y}{2}), \psi(x + \frac{y}{2}) \right] \]  \tag{2}

- \( A_\mu \) in mean field approach, \( \mathcal{W}(x, p) \) is gauge invariant

Equal-time Approach

\[ \mathcal{W}(x, p, t) = \int \frac{dp_0}{2\pi} \mathcal{W}(x, p) = \frac{1}{4} (s + i\gamma_5 \mathbf{p} + \gamma^\mu \mathbf{v}_\mu + \gamma^\mu \gamma_5 a_\mu + \sigma^{\mu\nu} t_{\mu\nu}) \]

D. Vasak et al.: Annals of Physics 173(462-492), 1987
I. Bialynicki-Birula et al.: Phys. Rev. D 44(6), 1991
Dirac-Heisenberg-Wigner Formalism

Equation of motion

\[
(D_t \mathbb{1} + D_1 \overline{A}_1 + D_2 \overline{A}_2 + D_3 \overline{A}_3 + \Pi_1 \overline{B}_1 + \Pi_2 \overline{B}_2 + \Pi_3 \overline{B}_3) \mathbf{w} = \overline{M} \mathbf{w} \tag{3}
\]

Wigner vector \( \mathbf{w} \); Matrices \( \mathbb{1}, \overline{A}_i, \overline{B}_i \) and \( \overline{M} \)

Pseudo-differential operators

\[
D_t = \partial_t + e \int d\xi \mathbf{E} (\mathbf{x} + i\xi \mathbf{∇}_p, t) \cdot \mathbf{∇}_p
\]

\[
D = \mathbf{∇}_x + e \int d\xi \mathbf{B} (\mathbf{x} + i\xi \mathbf{∇}_p, t) \times \mathbf{∇}_p
\]

\[
\Pi = \mathbf{p} - ie \int d\xi \xi \mathbf{B} (\mathbf{x} + i\xi \mathbf{∇}_p, t) \times \mathbf{∇}_p.
\]

F. Hebenstreit: Dissertation, 2011
Dirac-Heisenberg-Wigner Formalism

\[
\begin{align*}
D_t s & \quad -2\Pi \cdot t_1 = 0 \\
D_t p & \quad +2\Pi \cdot t_2 = -2a_0 \\
D_t v_0 + D \cdot v & \quad = 0 \\
D_t a_0 + D \cdot a & \quad = 2p \\
D_t v + D \cdot v_0 + 2\Pi \times a & \quad = -2t_1 \\
D_t a + D \cdot a_0 + 2\Pi \times v & \quad = 0 \\
D_t t_1 + D \times t_2 + 2\Pi \cdot s & \quad = 2v \\
D_t t_2 - D \times t_1 - 2\Pi \cdot p & \quad = 0
\end{align*}
\]
Observables

Particle Density

\[ N(t \to \infty) = \int n(p, t \to \infty) \, d^3p, \]  
\[ n(p, t) = \int d^3x \frac{s(x, p, t) + p \cdot \nabla(x, p, t)}{\omega(p)} \]  
with one-particle energy \( \omega(p) = \sqrt{1 + p^2} \)

Charge Density

\[ Q(t) = \int d^3x \int d^3p \, \psi_0(x, p, t) \]
Ordinary differential equation

\[
\begin{pmatrix}
\dot{F} \\
\dot{G} \\
\dot{H}
\end{pmatrix} = \begin{pmatrix}
0 & W & 0 \\
-W & 0 & -2\omega \\
0 & 2\omega & 0
\end{pmatrix} \begin{pmatrix}
F \\
G \\
H
\end{pmatrix} + \begin{pmatrix}
0 \\
W \\
0
\end{pmatrix}
\]

(7)

\[W(q, t) = \frac{eE(t) \varepsilon_{\perp}(q, t)}{\omega^2(q, t)}, \quad \varepsilon_{\perp}^2(q, t) = m^2 + q_{\perp}^2\]

\[p = q - eA(t), \quad \omega^2(q, t) = \varepsilon_{\perp}^2(q, t) + (q_z - eA(t))^2\]

- Homogeneous background field, \(E = E(t)\ e_z\)
- Particle density \(F(t)\)

S. A. Smolyansky et al. hep-ph/9712377 GSI-97-72, 1997
S. Schmidt et al.: Int.J.Mod.Phys. E7 709-722, 1998
J. C. R. Bloch et al.: Phys. Rev. D 60(116011), 1999
Problem Statement

Quantum electrodynamic

Only QED-Lagrangian and Dirac equation is known

\[ \mathcal{L} = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \]  
(8)

\[ i \gamma^\mu \partial_\mu \psi - m \psi = e \gamma^\mu A^\mu \psi \]  
(9)

Covariant derivative \( D_\mu = \partial_\mu + ie A_\mu \)

Field strength tensor \( \mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \)

Open question

How to obtain \text{statistical quantity} \( F(q, t) \) from particle description?
Vector Potential

Spatial-independent vector potential in one direction

\[ A_\mu = (0, A(t)e_3) \]  \hspace{1cm} (10)

- Very strong fields \(\rightarrow\) regarded as classical
- **Mean field** approximation (background field)

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S. Schmidt et al.: Int. J. Mod. Phys. E 7(709), 1998
F. Hebenstreit: Dissertation, 2011
Quantization

Dirac Field

Fully quantized

\[ \psi \sim \chi^+ a(q) + \chi^- b^\dagger(q) \]  

Equation of motion

\[ \left( \partial_t^2 + m^2 + (q_3 - eA(t))^2 + ieE(t) \right) \chi^\pm = 0 \]

Creation/Annihilation Operators

- Operators \( a(q), b^\dagger(q) \) hold information about particle statistics
- Fermions \( \rightarrow \) anti-commutation relations
Hamiltonian

- Non-vanishing off-diagonal elements
- Diagonalization by Bogoliubov transformation
- Switching to quasi-particle picture

\[
A(q, t) = \alpha(q, t) a(q) - \beta^*(q, t) b^\dagger(-q) \tag{13a}
\]

\[
B^\dagger(-q, t) = \beta(q, t) a(q) + \alpha^*(q, t) b^\dagger(-q) \tag{13b}
\]

Bogoliubov coefficients have to fulfill

\[
|\alpha(q, t)|^2 + |\beta(q, t)|^2 = 1
\]
One-particle distribution function

\[ F(q, t) = \langle A^\dagger(q, t) A(q, t) \rangle \]  \hspace{1cm} (14)

Fulfills equation of motion

\[ \partial_t F(q, t) = S(q, t) \] \hspace{1cm} (15)

- Gives **distribution** in momentum space
- Time-dependent quantity
- Interpretation as **electron/positron distribution** for \( t \to \pm \infty \) only