Numerical approach to reproduce instabilities of partial cavitation in a Venturi $8^\circ$ geometry

Boris Charriere$^1$ and Eric Goncalves$^2$

$^1$ University of Grenoble-Alpes, Laboratory LEGI, Grenoble, France
$^1$ Department of Aeronautical Engineering ENSMA, Laboratory Pprime, Poitiers, France
E-mail: boris.charriere@gmail.com
E-mail: eric.goncalves@ensma.fr

Abstract. Unsteady partial cavitation is mainly formed by an attached cavity which present periodic oscillations. Under certain conditions, the instabilities are characterized by the formation of vapour clouds, convected downstream the cavity and which collapse in higher pressure region. In order to gain a better understanding of the complex physics involved, many experimental and numerical studies have been carried out. These identified two main mechanisms responsible for the break-off cycles. The development of a liquid re-entrant jet is the most common type of instabilities, but more recently, the role of pressure waves created by the cloud collapses has been highlighted. This paper presents a one-fluid compressible Reynolds-Averaged NavierStokes (RANS) solver closed by two different equations of state (EOS) for the mixture. Based on experimental data, we investigate the ability for our simulations to reproduce the instabilities of a self-sustained oscillating cavitation pocket. Two cavitation models are firstly compared. The importance of considering a non-equilibrium state for the vapour phase is also exhibited. To finish, the role played by the added transport equation to compute void ratio is emphasised. In case of partially cavitating flows with detached cavitation clouds, the reproduction of convective mechanisms is clearly improved.

1. Introduction

Partial cavitating flows are characterized by the formation of an attached cavity which the unsteadiness present oscillations and, sometimes, unstable dynamics. Under certain conditions, the instabilities create structures of vapour torn from the pocket, convected by the flow before collapsing in re-compression area. These shedding of vapour clouds may have different origins. In some cases, the clouds are generated by vortex shedding filled with bubbles or produced by periodic disturbances imposed by the mean flow. For pumps and turbines, these disturbances can be caused by rotor-stator interactions. The inducer geometries also reveal coupled pocket oscillations frequencies between the blades [14]. Other observations notice the presence of instabilities caused by the cavity itself. For such unsteadiness, the experimental and numerical studies identify the presence of two main mechanisms: a liquid re-entrant jet and pressure waves generated by the collapse of larger structures.

The development of a liquid re-entrant jet is the most common type of instability and was primary described by Knapp et al. [16]. De Lange et al. [5, 6] proposed an experimental study of the phenomenon on NACA profiles. Initially, sheet cavity extends on the suction side. When it reaches a sufficient length, the re-entrant jet set up at the closure of the cavity and is directed...
toward the leading edge. Then, it divides the cavity in a rear part transformed into bubble cloud and convected by the main stream and a front part reduced to a tiny sheet cavity which grows again and starts a new cycle. In high pressure zone, the clouds collapse and generate pressure waves as it has been measured by Le et al. [17]. This mechanism is reported by many studies on different geometries such as a hydrofoil [20, 19], a cylindrical orifice [24] or a Venturi [22, 9]. All these investigations enhance the theory of two phenomenon combination and corroborate the works conducted by Arndt et al. [2]. From experimental measurements coupled with LES calculations, the study distinguishes each mechanism using the parameter $\sigma/2\alpha$, where $\sigma$ is the number of cavitation and $\alpha$ the angle of attack. For smaller values the re-entrant jet dominates while for larger values, the periodic shedding is coupled to pressure waves. The transition amounted to a border value $\sigma/2\alpha = 4$.

The most recent studies on cavity shedding have been proposed on a Venturi $8^\circ$ by Ganesh and Ceccio [9]. Based on LDV and X-ray densitometry measurements, their experimental works identify two mechanisms: a re-circulating flow and a condensation shock waves which is the most dominant for a periodic dynamic. This second mechanism involves propagating void ratio discontinuities that results in leading edge shedding of vapour clouds. From similar study conditions (same inlet velocity and number of cavitation), Stutz [25] and Aeschlimann [1] had also measured a common frequency closed to 45 Hz which characterized the periodic oscillations downstream the attached cavity. Numerically, this periodic behaviour has been reproduced by LeMartelot et al. [18] using a two-phase model where pressure oscillations frequency are between 40 and 50 Hz.

In this article, a special emphasis is made on the numerical reproduction of partial cavitation with a Venturi geometry. An in-house finite-volume code solving a four-equation RANS compressible system was developed [11, 4]. This article studies the ability for simulations to reproduce the instabilities of a self-sustained oscillating cavitation pocket. It extends previous numerical works [7, 8, 12] based on three conservation laws (three-equation system) for mixture quantities in which void ratio was deduced from the density mixture values. In our four-equation system, two cavitation models are compared, and the role of the added void ratio transport equation is investigated by comparison with a three-equation system. Validation and analysis are done with experimental measurements (time-averaged void ratio and velocity profiles, RMS wall and frequency pressure fluctuations). The importance of the propagation of pressure waves in the physical mechanism is also exhibited.

### 2. A four-equation model
#### 2.1. The added transport equation

The homogeneous mixture approach is used with the assumption of thermal and mechanical local equilibrium between pure phases, liquid and vapour. The model consists in three conservation laws for mixture quantities (mass, momentum and total energy) and an additional equation for the void ratio. The expression for the void ratio equation is:

$$\frac{\partial \alpha}{\partial t} + \text{div}(\alpha \vec{V}) = \left( \frac{\rho_l c_l^2 - \rho_v c_v^2}{\rho_l c_l^2 + \rho_v c_v^2} + \alpha \right) \text{div} \vec{V} + \left( \frac{c_l^2 + c_v^2}{\rho_l c_l^2 + \rho_v c_v^2} \right) \dot{m}$$  \hspace{1cm} (1)

where $\dot{m}$ is the mass transfer between phases, $\vec{V}$ is the velocity vector and $\rho_k, c_k$ are the pure phase density and speed of sound, respectively. Pure phases follow the stiffened gas equation of state. By assuming that the mass transfer is proportional to the divergence of the velocity, it is possible to build a family of models in which the mass transfer $\dot{m}$ is expressed as [10]

$$\dot{m} = \frac{\rho_l \rho_v}{\rho_l - \rho_v} \left( 1 - \frac{c^2}{c_{\text{wallis}}^2} \right) \text{div} \vec{V}$$  \hspace{1cm} (2)

where $c_{\text{wallis}}$ is the propagation of acoustic waves without mass transfer.
2.2. The cavitation models
The system is closed by an equation of state for the mixture. Two formulations are considered to determine the pressure in the mixture. A sinusoidal law deduces the pressure from the void ratio:

\[ p_m(\alpha) = P_{vat}(T_{ref}) + \left( \frac{\rho_{sat}^v - \rho_{sat}^l}{2} \right) c_{min}^2 \arcsin(1 - 2\alpha) \]  \hspace{1cm} (3)

while the stiffened gas EOS, written with mixture quantities, proposes the following formulation:

\[ p_m(\rho_m, \epsilon_m, \alpha) = (\gamma_m(\alpha) - 1)\rho_m(\epsilon_m - q_m(\alpha)) - \gamma_m(\alpha)p_{m,\infty}(\alpha) \]  \hspace{1cm} (4)

The parameter \( c_{min} \) refers to the minimal speed of sound in the mixture. The EOS properties are detailed in [13]. Each pressure closure expression has been studied on an expansion tube [10] and a relatively stable cavity behaviour on a Venturi 4° flow [4]. This paper proposes to extend these investigations on the unstable cavity dynamic of the Venturi 8°.

2.3. The turbulence model
The \( k - \ell \) model proposed by Smith [23] is used to solve the turbulent kinetic energy and the specific dissipation with the standard values of the different parameters.

3. Numerics
The numerical simulations are carried out using an explicit CFD code based on a finite-volume discretization. For the mean flow, the convective flux density vector on a cell face is computed with the Jameson-Schmidt-Turkel scheme [15]. The viscous terms are discretized by a second-order space-centered scheme. For the turbulence transport equations, the upwind Roe scheme [21] is used to obtain a more robust method. Time integration is achieved using the dual time stepping approach and a low-cost implicit method consisting in solving, at each time step, a system of equations arising from the linearization of a fully implicit scheme. The derivative with respect to the physical time is discretized by a second-order formula. The numerical treatment of boundary conditions is based on the use of the preconditioned characteristic relationships. More details are given in [11].

4. Experimental and numerical parameters
4.1. Experimental conditions
The Venturi was tested in the cavitation tunnel of the CREMHyG (Centre d’Essais de Machines Hydrauliques de Grenoble). It is characterized by a divergence angle of 8°, illustrated in Figure 1. The edge forming the throat of the Venturi is used to fix the separation point of the cavitation cavity. This geometry is equipped with eight probing holes (stations 1-8) to allow various measurements such as the local void ratio, instantaneous local speed and wall pressure. The velocity is evaluated as the most probable value and the void ratio is obtained from the signal of the double optical probe using a post-processing algorithm. The relative uncertainty on the void ratio measurement was estimated at roughly 15% [3].

The selected operating point is characterized by the following physical parameters [3]:
- \( U_{inlet} = 7.04 \) m/s, the inlet velocity
- \( \sigma_{inlet} = \frac{P_{inlet} - P_{vat}}{0.5 \rho U_{inlet}^2} \simeq 1.99 \), the cavitation parameter in the inlet section
- \( T_{ref} \simeq 293K \), the reference temperature
- \( L_{ref} = 224 \) mm, the reference length
- \( Re_{ref} = \frac{U_{inlet} L_{ref}}{\nu} = 1.57 \times 10^6 \), the Reynolds number

With these parameters, a cavity length \( L \) closed to 45 mm is obtained. Based on pressure
oscillations measurements, typical self-oscillation behaviour with quasi-periodic vapour clouds shedding is measured close to 50 Hz (see Figure 1).

Figure 1. Scheme of the Venturi 8° and experimental pressure oscillations frequency

4.2. Mesh and numerical parameters
The grid is a H-type topology. It contains 174 nodes in the flow direction and 56 nodes in the orthogonal direction. A special contraction of the mesh is applied in the main flow direction just after the throat to better simulate the two-phase flow area. The $y^+$ values of the mesh, at the center of the first cell, vary between 12 and 27 for a non cavitating computation.

Unsteady computations are performed with the dual time stepping method and are started from the non cavitating numerical solution.

5. Computational results
Computations were performed with two cavitation model. For the sinusoidal law, different values of the tunable parameter $c_{min}$ are tested and summarized in the Table (1). The goal is to obtain a sheet with time-averaged profiles and oscillation frequencies close to experimental values.

| Cavitation model     | $\sigma_c$ | CFL number | $\Delta t \times 10^{-3}$ s | $c_{min}$ (m/s) |
|----------------------|------------|------------|----------------------------|-----------------|
| sinusoidal law       | 2.1        | 0.1        | 0.02                       | 0.47            |
| sinusoidal law       | 2.1        | 0.1        | 0.02                       | 0.92            |
| sinusoidal law       | 2.1        | 0.1        | 0.02                       | 1.36            |
| stiffened gas EOS    | 2.1        | 0.1        | 0.02                       | $\phi$          |

5.1. Model of cavitation
To evaluate the ability for the 4-equation model to correctly reproduce the cloud cavitation shedding, the sinusoidal law (3) and the stiffened gas (4) EOS are successively used to compute the pressure. The influence of the empirical parameter $c_{min}$ is investigated at 0.47 m/s, 0.92 m/s and 1.36 m/s values.

A qualitative illustration of cavity pocket motions is shown on Figure 2 by representing the void ratio on a spatio-temporal graph $(x,t)$. The Venturi throat is located at the abscissa $x = 0$.
The cavitating flow behaviour mainly depends on the considered equation of state and, consequently, on the speed of sound formulation. By assuming that pressure oscillations, in the closure region, are mainly created by cloud collapses, it is possible to measure the frequency of cloud cavitation shedding. According to the Figure 3, it increases with the minimal speed of sound value. From a physical point of view, it means that the frequency of cavity destabilisation is amplified by the travelling celerity of pressure waves.

The previous comparisons highlight the modification of sheet cavity behaviour by the propagation of pressure discontinuities. However, as presented in the introductive part, the re-entrant liquid jet [1] is also known as a main responsible of break-off cycles and cloud shedding. In our simulations, this physical mechanism is clearly observed by negative time-averaged velocity profiles near the wall. On Figure 4, it is interesting to notice homogeneous values for all computations, which argue that the pressure and the speed of sound determine the periodic shedding frequency without changing the averaged velocity field. However, void ratio distribution depends on the $c_{\text{min}}$ parameter and averaged vapour fraction diminish when this coefficient is set at highest values. This is linked to the different cavity behaviours: when the frequency pulsations intensify, a greater number of vapour clouds are convected but their void ratio reduces.

![Figure 2](image-url)

**Figure 2.** Maximum void ratio evolution (at each x-plane) over time: the break-off cycle frequency increases with the minimal speed of sound value

5.2. Saturation versus metastable value for $\rho_v$

The added transport equation allows to relax the local thermodynamic equilibrium and to introduce a metastable state for the vapour phase. Comparisons have been done for the sinusoidal EOS (3) between the two cases: equilibrium case with the saturation value $\rho_v = \rho_{v\text{sat}}$
Figure 3. Pressure oscillation frequencies measured in the collapse region: station 2, \(x = 20.9\) mm.

Figure 4. Comparison of averaged void ratio and velocity profiles at station 2, \(x = 20.9\) mm.

and metastable case \(\rho_v = \gamma_v (p_m - p_{\infty})/c_v^2\) according to the stiffened gas equation. The both configurations are summarized in the Table (2).

| Cavitation model      | \(\sigma_c\) | CFL number | \(\Delta t \times 2 \times 10^{-3}\) s | \(c_{min}\) (m/s) | \(\rho_v\) |
|-----------------------|--------------|------------|--------------------------------------|-------------------|----------|
| sinusoidal law        | 2.1          | 0.1        | 0.02                                 | 1.36              | \(\gamma_v (p_m - p_{\infty})/c_v^2\) |
| sinusoidal law        | 2.1          | 0.1        | 0.02                                 | 1.36              | \(\rho_{v sat}\) |

The Figure 5 compares the spatio-temporal distribution of each case. On the one hand, computation with the metastable value \(\rho_v\) involves a more irregular and random behaviour of the cavity. On the other hand, for the saturation case, the attached cavity length does not vary over time and the convected structures keep similar shapes.

The previous qualitative observations can be quantified in the closure region. The instantaneous analysis on Figure 6 compares the pressure evolutions for both cases and demonstrates that a non-equilibrium state for \(\rho_v\) increases the pressure magnitude. A similar result is observed for speed of sound and the Mach number variations in the shedding region. While the flow remains subsonic for the saturations values, it clearly becomes supersonic when the metastable value \(\rho_v\) is computed. These results question the importance of non-equilibrium effect to reproduce the shock waves propagation caused by the vapour structure collapses in our simulations.
Figure 5. Maximum void ratio evolution (at each x-plane) over time: stabilisation of the cavitating flow for the saturation case

Figure 6. Instantaneous evolutions of void ratio, static pressure and the Mach number in the collapse region: station 2, x = 20.9 mm.

Discrepancies also appear with statistical analysis. Indeed, on Figure 7, the vapour cloud shedding frequency and averaged pressure oscillations are reduced for constant values of vapour density. These analysis confirm the initial comments which noticed a more stabilised flow when saturation value is used.

5.3. Influence of the added void ratio transport equation
This part focuses on the added void ratio transport equation influence and compares results obtained with 3- and 4-equation system. The simulations based on a 3-equation system are taken from Decaix [7]. For this case, the void ratio values are deduced from the mixture density according to the following relation: $\alpha = \frac{\rho_m - \rho_{sat}^{\text{in}}}{\rho_{sat}^{\text{in}} - \rho_{sat}^{\text{out}}}$. The pressure computation follows the
sinusoidal law and the minimal speed of sound is set to: $c_{\text{min}} = 0.92 \text{ m/s}$. This value is inferior to $c_{\text{min}} = 1.36 \text{ m/s}$ which gave the best results with the 4-equation system and according to experimental datas. The both cases are summarized in Table 3.

### Table 3. Metastable and saturation parameters

| Cavitation model | $\sigma_e$ | CFL number | $\Delta t \times 2.10^{-3}$ s | $c_{\text{min}}$ (m/s) | $\alpha$ |
|------------------|-----------|------------|--------------------------------|------------------------|---------|
| sinusoidal law   | 2.1       | 0.1        | 0.02                           | 0.92                   |         |
| sinusoidal law   | 2.1       | 0.1        | 0.02                           | 1.36                   | transport equation (1) |

Concerning the average void ratio profiles, the flow topology varies according to the presence or not of the transport equation. The simulations based on 3-equation system over-estimate the vapour density at the station 1 abscissa, i.e. in the attached cavity region. Moreover, the effects of the liquid re-entrant jet, which diminishes the vapour fraction near the wall, is less reproduced. Compared with the topology of 4-equation system, the void ratio of 3-equation simulations decreases quickly downstream to finally reaches zero values at the station 3. These results inform on the influence of void ratio computation. In the case of a 3-equation system, it is estimated directly from the mixture density values and collapse is brutal. Then, for a 3-equation configuration, it is necessary to over-estimate the amount created in the first part of the cavity to obtain void ratio profiles in better agreement with the experiments. However, the transport equation improves the reproduction of travelling structures and homogenizes the vapour distribution.

### 6. Conclusion

An aperiodic and unstable sheet cavity has been studied in a 2D Venturi configuration by numerical one-fluid RANS simulations. Calculations have been carried out with an in-house compressible code and numerical results have been compared with experimental data concerning...
Figure 8. Instantaneous representations of sheet dynamic and maximum void ratio evolution (at each $x$-plane) over time: increase of the break-off cycle frequency with the minimal speed of sound value

Figure 9. Comparison of void ratio profiles at the first three stations: $x = 13.7$ mm, $x = 20.9$ mm and $x = 38.4$ mm

the void ratio, streamwise velocity, wall pressure fluctuations and frequencies. Firstly, two cavitation models, using a barotropic law or a stiffened gas equation, have been compared. It was shown that the the break-off cycle frequency depends on the speed of sound computation. It means that the travelling celerity of pressure waves influences the attached cavity destabilisation. Then, the importance of considering a non-equilibrium state for vapour density is investigated. The metastable case involves more irregular and random behaviour of the cavity. For this configuration, the shock waves following a cloud collapse are better reproduced and Mach number values often exceed the sonic limit. On the other hand, the saturation case diminishes the pressure oscillation frequency and the averaged magnitude. A special emphasis is made on the void ratio transport equation influence. Our present results are compared with previous simulation based on a 3-equation conservation law system where
void ratio is deduced from mixture density values. The results highlight the better reproduction of convective phenomena using the void ratio transport equation.

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