Adomian decomposition method for modelling the dissipative higher-order rogue waves in a superthermal collisional plasma

Noufe. H. Aljahdaly, S. A. El-Tantawy, Abdul-Majid Wazwaz and H. A. Ashi

Abstract

A reductive perturbation technique (the derivative expansion technique (DET)) is employed to derive a linear damped nonlinear Schrödinger equation (LDNLSE) for investigating the feature properties of the dissipative modulated nonlinear dust-acoustic wavepacket including rogue waves (RWs) in an electron-ion dusty plasma having superthermal electrons and ions. The modulation instability (MI) of the dissipative envelope structures is investigated and the (un)stable domain of the modulated structures is mapped precisely. The LDNLSE is solved numerically using Adomian decomposition method (ADM) in order to study the impact of the related plasma parameters on the features of the dissipative RWs. The influence of superthermal parameters and the dust-neutral collisional frequency on the profile of the dissipative dust-acoustic RWs is numerically investigated. Moreover, the accuracy of the numerical solutions is examined by estimating the global residual error in the whole space-time domain.

1. Introduction

The partial differential equations (PDEs) are useful mathematical tools to describe a variety of many applications and phenomena in multiple branches of science [1–18]. The nonlinear Schrödinger equation (NLSE) is considered one of the most important and most famous global PDE due to its applications in various fields of science. This equation and many related equations helped the authors over the past decades in explaining and describing several natural phenomena in optical fibre, oceanography, physics of plasma, quantum mechanics, water waves, etc. [19–21]. The standard form of the following planar NLSE is considered an integrable Hamiltonian system [22, 23]

$$i\hbar\psi_t + \frac{1}{2}P\psi_{xx} + Q|\psi|^2\psi = 0,$$

where $\psi = \psi(x, t)$ represents the wave function and the coefficients ($P, Q$) are functions of the physical parameters related to the system under consideration and are not functions in ($x, t$).

The NLSE describes the propagation of modulated envelope waves such as freak waves (FWs) or sometimes called rogue waves (RWs), breathers (Akhmediev breathers and Kuznetsov-Ma breathers), dark solitons, bright solitons, modulated cnoidal waves in the weakly nonlinear and dispersive medium [24–26]. Researchers found that the most appropriate way to explain the dynamic mechanism of the generation and propagation of the RWs is to study the modulated instability (MI) of the envelope wavepackets. A number of authors used the rational solution of Equation (1) to investigate the salient features of the non-dissipative/undamping RWs in optical fibre, laser fibre, water tank, physics of plasmas, etc. [24–26].

Rogue wave (RW) is a very ambiguous mechanism since it appears in the fluid media from nowhere and disappears without any sign [27]. It is localized wave in both time and space domain and propagates with long amplitude. For the first time, the first-order RWs have been generated experimentally and theoretically by Pathak [28] and Sharma and Bailung [29]. The authors compared the experimental observations with the NLSE RW solution and they found that there is a great compatibility between them. Also, the higher-order/multi-RWs (here, we main the second-order RWs) in a multicomponent plasma have been created and investigated experimentally [31]. Recently and in a multidipole double plasma device, the impact of Landau damping on the evolution of the IA RWs in a multicomponent plasma has been studied experimentally and theoretically by Pathak [32]. The experiment is performed in a multidipole double plasma device. Moreover, in a water tank, the super RWs have
been studied experimentally and theoretically by Chabchoub et al. [27]. It was observed that the amplitude of first-/second-order RW is greater than the surrounding wave amplitudes by three/five times. Moreover, the amplitude of the RW can be infinite by a slight periodic perturbation on the fluid [33].

Sometimes, if some effects associated with realist applications are accounted to problems, Equation (1) cannot be valid to describe the dynamic mechanism of the RWs or to describe such modulated waves in all cases. For example, in all fluids, we can not neglect the friction forces except in very few cases which are called superfluid. In a collisional plasma physics, if the collisions between the charged and neutral particles are taken into consideration, in this case, the following linear damped NLSE (LDNLSE) could be obtained [34, 35]

$$i\partial_t \psi + \frac{1}{2} P \partial_x^2 \psi + Q |\psi|^2 \psi + iR\psi = 0.$$  \tag{2}

Equation (2) is the most realistic model to study the dissipative/damping RWs by taking the friction force into account. The LDNLSE is not an integrable Hamiltonian system. Nevertheless, several attempts have been made to solve Equation (2) in order to understand the dynamic mechanism of the dissipative/damping RWs in several nonlinear and dispersive media. For instance, Onorato and Proment [36] introduced the powerful transformation with Taylor expansion to transfer the LDNLSE to the standard NLSE (1) for investigating the behaviour of the dissipative/damping RWs and breathers in water. After that, this transformation has been used widely by many researchers in order to study the dissipative envelope structures in different plasma models [37–41]. Herein, we only mention some contributions that used the Onorato-Proment transformation for studying the dissipative RWs and breathers. The collisional effect on the MI of the compressional electromagnetic wave and the dissipative RWs in an electron-positron quantum magnetoplasma, as well as the effect of viscosity on the MI of the dissipative positron-acoustic wave and the associated dissipative RWs in an electron-positron quantum plasma, have been studied by analyzing the LDNLSE using the Onorato-Proment transformation [37, 38]. Also, Guo et al. [39] followed the same technique to investigate the effect of the ionic viscosity and the ion-neutral particles collision on the MI of the ion-acoustic modulated waves and the dissipative RWs in a strongly coupled collisional plasma having nonthermal electrons and strongly coupled ions. Guo and Mei [40] studied the MI of the dissipative IAWs, the dissipative RWs, and super RWs in multi-ion plasmas composed of positive and negative ions and positive ion beam in addition to superthermal electrons. El-Tantawy [41] used the Onorato-Proment transformation for studying the MI of the IAWs and the dissipative RWs in strongly coupled ultracold plasmas.

All mentioned studies used the Onorato-Proment transformation in order to investigate the salient features of the modulated dissipative structures (e.g. RWs and breathers) in different plasma models. On the other side, there are a few researchers who have obtained some semi-analytical solutions to Equation (2) without using Onorato-Proment’s transformation [42–45]. The propagation of the dissipative dust-acoustic RWs and breathers in a dusty plasma having two different superthermal ions and with taking the dust kinetic viscosity into consideration has been reported numerically with the help of wolfram mathematica command ‘DSolve’ [42]. Salas et al. [43] used the finite difference method (FDM) as well as the hybrid finite difference method with the moving boundary method (FDM-MBM) to analyze the LDNLSE in order to investigate both the dissipative and non-dissipative RWs and breathers in a complex plasma taking the effect of dust viscosity into account. Moreover, the dissipative dust-acoustic RWs and breathers in an unmagnetized collisional pair-ion plasma have been investigated numerically and analytically by El-Tantawy et al. [44]. For the numerical simulation, the hybrid FDM-MBM was devoted for analysing the LDNLSE while for the analytical solution, the authors derived for the first time a semi-analytical solution with high accuracy for modelling the dissipative RWs and breathers. Also, the authors compared the two approximate solutions and found that there is a great agreement between the accuracy of the two solutions. Furthermore, Douanla et al. [45] derived a semi-analytical solution to the three-dimensional LDNLSE in order to investigate the characteristics behaviour of the dissipative ion-acoustic RWs in an electron-ion magnetoplasma. However, the accuracy of the obtained solution is not good in the whole space-time domain.

Motivated by the mentioned papers, the main goal of the present study is to investigate the dissipative dust-acoustic RWs (first-order RWs, second-order RWs, etc.) in a collisional complex plasma having superthermal electrons and ions. In this plasma model, the effect of dust-neutral collision will be taken into consideration. Accordingly, the basic equation of the model can be reduced to the LDNLSE using the derivative expansion technique (DET) and for modelling the dissipative RWs, we will solve the evolution equation numerically using some high-accurate numerical methods such as the Adomian decomposition method (ADM) [46–50].

Most mathematical techniques have been used to find some solutions to the integrable PDEs in different forms such as exact solution, analytical, equivalent, approximate or numerical solutions [51–53]. However, for the non-integrable equations, the numerical or approximate solutions can be computed. Thus, in this work, we aim to use the ADM to investigate the dynamic mechanism of the dissipative super RWs numerically. In fact the ADM method converges to the exact solution.
for the integrable NLSE if its Taylor expansion is well-known form. While if the Taylor expansion form of the evolution equation is unknown, the result will converge to the solution locally about the neighbourhood of the initial point. Thus, the solution is obtained in a small time domain. However, ADM is simple, fast and reliable method and it is still worth using it if the problem looks for the solution in small domain or at fixed point of t.

2. The physical model and the lineal damped NLSE

Let us consider the propagation of nonlinear structures (say modulated envelope structures) in an unmagnetized collisionless dusty plasma consisting of inertial negatively charged dust grains (with mass \(m_d\), velocity \(u_d\), and density \(n_d\)) as well as inertial superthermal singly charged positive ions (with mass \(m_i\), velocity \(u_i\), density \(n_i\), and temperature \(T_i\)) and electrons (with mass \(m_e\), velocity \(u_e\), density \(n_e\), and temperature \(T_e\)). The charge variations/fluctuations are predicted to have a minor impact on the dust-acoustic modes, thus we can assume the dust charge is constant [54]. Also, in most dusty plasma experiments, the impact of the dust-neutral collisions becomes compulsory when the plasma frequency (\(\omega_p\)) becomes larger than the dust-neutral collisional frequency \(\nu_d\), i.e., \(\omega_p > \nu_d\). Accordingly, this effect should be taken into account in the fluid equations of the dust impurities. The heavy mobile charged dust particles satisfy the continuity and momentum equations by taking the effect of dust-neutral collisions into consideration

\[
\begin{align*}
\partial_t n_d + \partial_x (n_d u_d) &= 0, \\
\partial_t u_d + u_d (n_d u_d + \partial_x u_d) - \partial_x \phi &= 0,
\end{align*}
\]

where \(n_d\) and \(u_d\) are, respectively, the normalized density and velocity of the dust grains, and \(\phi\) is the normalized electrostatic potential.

The normalized number density of both the superthermal electrons and ions are, respectively, given by

\[
\begin{align*}
n_e &= \mu_e \left( 1 - \frac{\sigma}{a_e} \right)^{-b_e}, \\
n_i &= \mu_i \left( 1 + \frac{1}{a_i} \right)^{-b_i},
\end{align*}
\]

with \(a_{e,i} = (\kappa_{e,i} - 3/2)\), \(b_{e,i} = (\kappa_{e,i} - 1/2)\), where \(\mu_e = n_e^{(0)} / z_d n_d^{(0)}\), \(\mu_i = n_i^{(0)} / z_d n_d^{(0)}\), and \(\sigma = T_i / T_e\) refers to the temperature ratio, \(\kappa_e\) (\(\kappa_i\)) indicates the spectral index of the electrons (ions). Physically, the spectral index \(\kappa_e\) (\(\kappa_i\)) must be larger than 3/2. Also, for \(\kappa_{e,i} \to \infty\), the Maxwellian distribution of the electron and ion could be recovered.

The two systems (3) and (4) are closed through the Poisson's equation

\[
\partial_x^2 \phi - n_d - \rho = 0,
\]

where \(\rho = n_e - n_i\).

At the equilibrium, the neutrality condition is fulfilled, i.e., \(n_d^{(0)} z_d + n_i^{(0)} - n_e^{(0)} = 0\), which equivalents \(\mu_d = z_d n_d^{(0)} / n_i^{(0)} = 1 - f\), where \(f = n_e^{(0)} / n_i^{(0)}\) and \(\mu_d\) express the concentration of the electron and negative dust grains, respectively. The ratios \(\mu_e\) and \(\mu_i\) could be rewritten as function of \(f\) as: \(\mu_e = f / (1 - f)\) and \(\mu_i = 1 / (1 - f)\). For simplicity, \(\rho\) can be written in the following form:

\[
\rho = n_e - n_i = \alpha_0 + \alpha \phi + \beta \phi^2 + \gamma \phi^3 + \cdots,
\]

with

\[
\begin{align*}
\alpha_0 &= (\mu_e - \mu_i), \\
\alpha &= \frac{\beta}{\partial_e + \frac{\partial_i \mu_i}{\partial_i}}, \\
\beta &= \frac{1}{2} \left( \frac{\sigma^2 b_e (b_e + 1) \mu_e - b_i (b_i + 1) \mu_i}{\partial_e^2} \right), \\
\gamma &= \frac{1}{6} \left( \frac{\sigma^3 b_e (b_e + 1) (b_e + 2) \mu_e}{\partial_e^3} + \frac{b_i (b_i + 1) (b_i + 2) \mu_i}{\partial_i^3} \right).
\end{align*}
\]

The DET is employed to reduce the fluid Equations (3)–(5) to the LDNLSE for investigating the MI of the dissipative dust-acoustic envelope waves and the associated dissipative higher-order/multi-FWs that can exist and propagate in this model. According to this technique, the independent variables \((x, t, \nu)\) could be stretched as

\[
\begin{align*}
\xi &= \varepsilon (x - \lambda_g t), \\
\tau &= \varepsilon^2 t, \\
\nu &= \varepsilon^2 \nu,
\end{align*}
\]

where \(\varepsilon\) is a real positive parameter \((\varepsilon < 1)\) and \(\lambda_g\) gives the group velocity of the dust-acoustic waves (DAWs). Also, the dependent quantities \(F(x, t) = [n_e, \phi]^T\) in the new frame are expanded as [55, 56]

\[
F = F^{(0)} + \sum_{m=-\infty}^{\infty} \xi^m \sum_{l=-m}^{m} f^{(m)} \theta^l,
\]

with

\[
\begin{align*}
f^{(0)} &= \left[ 1 \quad 0 \quad 0 \right]^T, \\
f^{(m)}_l &= \left[ n_{il}^{(m)} \quad u_{il}^{(m)} \quad \phi_{il}^{(m)} \right]^T, \\
\theta &= (kx - \omega t), \omega \text{ and } k \text{ indicate the normalized frequency and wavenumber of the carrier wave, respectively. Note that the perturbed quantities } f^{(m)}_l \text{ must recover the reality condition: } f^{(m)}_l = f^{* (m)}_l, \text{ where } f^{*} \text{ refers to the complex conjugate, and } i \equiv \sqrt{-1}. \]
According to the DET, the following operators are introduced
\[
\begin{pmatrix}
\partial_t \\
\partial_x \\
\partial_x^2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\partial_t - \varepsilon \lambda_g \partial_x + \varepsilon^2 \partial_x^2 \\
\partial_x \\
\partial_x^2 + 2\varepsilon \partial_x^3 + \varepsilon^2 \partial_x^4
\end{pmatrix}
\]  
(9)

Moreover, after operating on any vector (say \( \vec{v} \)) of the component \( f_j^{(m)} \), we get
\[
\begin{pmatrix}
\partial_t (k_j^{(m)}) e^{i\omega t} \\
\partial_x (k_j^{(m)}) e^{i\omega t} \\
\partial_x^2 (k_j^{(m)}) e^{i\omega t}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\partial_t (k_j^{(m)}) e^{i\omega t} - \varepsilon \lambda_g \partial_x (k_j^{(m)}) + \varepsilon^2 \partial_x^2 (k_j^{(m)}) e^{i\omega t}, \\
\partial_x (k_j^{(m)}) e^{i\omega t} + i(k_j^{(m)}) e^{i\omega t}, \\
\partial_x^2 (k_j^{(m)}) e^{i\omega t} - (k_j^{(m)})\partial_x (k_j^{(m)}) + \varepsilon^2 \partial_x^2 (k_j^{(m)}) e^{i\omega t}
\end{pmatrix}
\times
\begin{pmatrix}
\partial_x \\
\partial_x^2 \\
\partial_x^3
\end{pmatrix}
\]  
(10)

The substitution of Equation (10) into system (3)–(5), and after a simplified mathematical manipulation, we get some reduced equations with different orders of \( \varepsilon \). The below steps for obtaining the LDNLSE are summarized as:

- The first-mode \((m = 1)\) with the first-harmonic \((l = 1)\) of the reduced equations gives the values of the quantities \((n_1^{(1)}, u_1^{(1)})\) and linear dispersion relation \(\omega\) as
  \[
  n_1^{(1)} = \frac{k^2}{\omega^2} u_1^{(1)} = \frac{k}{\omega} \phi_1^{(1)},  \\
  \omega = \frac{1}{\sqrt{k^2 + \alpha}}.
  \]  
  (11)

- The second-mode \((m = 2)\) with the first-harmonic \((l = 1)\) of the reduced equations gives the values of the quantities \((n_2^{(2)}, u_2^{(2)})\) and the group velocity \(\lambda_g\) as
  \[
  n_2^{(2)} = -\frac{k^2}{\omega^2} \phi_1^{(2)} - \frac{2ik}{\omega^3} \partial_x \phi_1^{(1)},  \\
  u_2^{(2)} = -\frac{k}{\omega} \phi_1^{(2)} - \frac{i(k^2 - \omega^2)}{\omega^2} \partial_x \phi_1^{(1)},  \\
  \lambda_g = \frac{\alpha}{\sqrt{(k^2 + \alpha)^3}} \equiv \partial_x \omega.
  \]  
  (12)

- The second-mode \((m = 2)\) with the second-harmonic \((l = 2)\) of the reduced equations gives the values of the quantities \((n_2^{(2)}, u_2^{(2)}, \phi_2^{(2)})\) as
  \[
  n_2^{(2)} = C_1 \phi_1^{(1)},  \\
  u_2^{(2)} = C_2 \phi_1^{(1)},  \\
  \phi_2^{(2)} = C_3 \phi_1^{(1)},
  \]  
  (13)

with
\[
C_1 = \frac{k^2 (2\beta \omega^2 + 3\alpha k^2 + 12k^4)}{2\alpha \omega^4 + k^4 (8\alpha^2 - 2\omega^2)},  \\
C_2 = \frac{k (2\beta \omega^4 + \alpha k^2 \omega^2 + k^4 (4\alpha^2 + 2))}{2\alpha \omega^5 + k^2 (8\alpha \omega^2 - 2\omega^3)},  \\
C_3 = -\frac{(2\beta \omega^4 + 3k^4)}{2\alpha \omega^5 + k^2 (8\alpha^2 - 2\omega^2)}.
\]

- The second-mode \((m = 2)\) with the zeroth-harmonic \((l = 0)\) of the reduced equations gives the values of the quantities \((n_0^{(2)}, u_0^{(2)}, \phi_0^{(2)})\) as
  \[
  n_0^{(2)} = S_1 \phi_1^{(1)},  \\
  u_0^{(2)} = S_2 \phi_1^{(1)},  \\
  \phi_0^{(2)} = S_3 \phi_1^{(1)},
  \]  
  (14)

with
\[
S_1 = \frac{(2\beta \omega^3 + 2\alpha \lambda k^3 + \alpha k^2 \omega)}{\omega^3 (\alpha \lambda^2 - 1)},  \\
S_2 = \frac{(2\beta \lambda \omega^3 + \alpha \lambda k^2 \omega + 2k^3)}{\omega^3 (\alpha \lambda^2 - 1)},  \\
S_3 = -\frac{(2\beta \lambda^2 \omega^3 + 2\alpha \lambda k + k^2 \omega)}{\omega^3 (\alpha \lambda^2 - 1)}.
\]

- The third-mode \((m = 3)\) with the first-harmonic \((l = 1)\) of the reduced equations and with the help of the values of the quantities given in Equations (10)–(14), the following compatibility condition (the LDNLSE) is obtained
  \[
  i\partial_t \Psi + \frac{1}{2} \beta \partial_x^2 \Psi + Q |\Psi|^2 \Psi + iR \Psi = 0,
  \]  
  (15)

with
\[
\beta = \frac{3k^2 \lambda^2 - 4k\omega \lambda_g - \omega^4 + \omega^2}{4k^2 \omega},
\]
\[
Q = \frac{1}{2} \left( \frac{\omega^3}{k^2} (2\beta C_3 + 3\gamma + 2\beta S_3) - 2k (C_2 + S_2) - (C_1 + S_1) \omega \right),
\]
where \(\Psi \equiv \phi_1^{(1)}\) and \(R = \nu_{dn}/2\).

Equation (15) describes the dissipative modulated envelope structures that can propagate with group velocity \(\lambda_g\), e.g. bright solitons, dark solitons, cnoidal waves, FWs, breathers, etc. It is clear that the coefficients \(P\) and \(Q\) are not functions in the dust-neutral collisional frequency \(\nu_{dn}\) and the effect of \(\nu_{dn}\) appears only in the coefficient of the linear damping term \(R\). Thus, the criteria of the MI of the dissipative envelope structures described by Equation (15) are different from the MI criteria of the non-dissipative/undamping modulated structures described by standard NLSE, i.e. Equation (15)
for $R = 0$. In the non-dissipative case ($R = 0$), the necessary and sufficient conditions of the (un)stable modulated structures read [57, 58]

$$
PQ < 0 \quad \Omega^2 > 0 \rightarrow \text{Stable envelope waves},
$$
$$
PQ > 0 \quad \Omega^2 < 0 \rightarrow \text{Unstable envelope waves},
$$
(16)

with

$$
\begin{align*}
\Omega^2 &= (2PK^2)^2 \left(1 - \frac{K^2}{2R^2}\right), \\
K^2 &= \left(\frac{Q}{P}\right) \Phi_0^2,
\end{align*}
$$
(17)

where $\Omega$ acts the frequency of the modulated structures, $K$ and $K_c$ indicate the wavenumber and critical wavenumber of the modulated structures, respectively, and $\Phi_0$ is the pump carrier wave.

With respect to the dissipative modulated structures, the MI criteria are related to the time of propagation and the nonlinear dispersion relation reads (more details can be found in Refs. [59]):

$$
\begin{align*}
\Omega (\tau)^2 &= (2PK^2)^2 \left(1 - \frac{K^2}{2R^2}\right), \\
K^2 (\tau) &= \left(\frac{Q}{P}\right) \Phi_0^2 e^{-2\tau R^2},
\end{align*}
$$
(18)

The MI criteria of the dissipative envelope structures are summarized as follows:

$$
\begin{cases}
PQ > 0 & \Omega^2 < 0, \\ K_c (\tau) > K & \tau < \tau_{\text{max}}, \\
\text{Otherwise} & \rightarrow \text{Stable envelope waves}, \\
\end{cases}
$$
(19)

where $\tau_{\text{max}} = (1/2R) \ln(Q \Phi_0^2 / PK^2)$ refers to the maximum value of the period of the MI.

In this case, the local instability growth rate of MI is given by:

$$
\tilde{\Omega} = \text{Im} (\Omega) = |2PK^2 | \sqrt{K^2 (\tau)/K^2} - 1.
$$
(20)

It is shown that $\tilde{\Omega}$ shrinks for time going on until $\tau$ reaches to $\tau_{\text{max}}$ in this case, $\tilde{\Omega} = 0$ and the wave growth stops and for time $\tau$ increasing until $\tau > \tau_{\text{max}}$, then $\tilde{\Omega}^2 > 0$ and the modulated structures become stable even if $PQ > 0$.

The regions of the stable and unstable modulated dissipative structures according to the MI criteria (18) are quantified and defined based on some physical parameters as illustrated in Figures 1–5. It is noticed at first glance that the stable regions are more dominant than the unstable regions. Which means that RWs can be generated and propagated within narrow regions in this system. In Figures 1–5, it should be mentioned that the coloured regions indicate the unstable structures which in these regions, all conditions of the MI ($PQ > 0, \Omega^2 < 0, K_c (\tau) > K, \text{and} \tau < \tau_{\text{max}}$) are fulfilled, while the white regions represent the stable structures of the modulated wave packets. The dependence of the (un)stable regions of the modulated DAWs on the plasma configuration parameters is illustrated in Figures 1–5). In this plasma model, we can see that there is only one critical line for the wavenumber $k_c$ (at $k = k_c$, the product $PQ = 0$) which above it, the product $PQ > 0$ and below it, the product $PQ < 0$. This does not mean that above $k_c$ ($PQ > 0$) or below $k_c$ ($PQ < 0$), the modulated wavepackets become stable or unstable because there are other conditions mandatory with the sign of the product $PQ$ to determine the MI regions of the DAWs, as mentioned above. The effect of the superthermal parameters ($\kappa_e, \kappa_i)$ on the stability regions is depicted in Figure 1. In this figure, we can notice that electron superthermality $\kappa_e$ has a not very significant effect on the MI regions, on the contrary, with respect to the ion superthermality, which is an increase in the ion superthermality $\kappa_i$, a significant change occurs in the MI regions as shown in Figure 1(b). In general, the unstable regions of the dissipative modulated DAWs expand with the increase of the values of the spectral indices ($\kappa_e, \kappa_i$) (see Figures 1 and 2(a)) while the ion temperature ratio $\sigma$ has an opposite effect, i.e. the unstable regions of the dissipative modulated DAWs shrink with the enhancement of $\sigma$ (see Figure 2(b)). Also, the spectral indices ($\kappa_e, \kappa_i$), the ion temperature ratio $\sigma$, and the collisional dust-neutral frequency $\nu_{\text{dn}}$ have significant modifications on the MI regions of the DAWs and the maximum value of the MI period $\tau_{\text{max}}$. In the unstable regions of the modulated DAWs, a random perturbation of the amplitude grows and thus the RWs may be excited and propagated in this plasma model. Thus, for investigating the features of the dissipative RWs in this model, the ADM is introduced to analyze the LDNLSE (15) as shown in the section below.

3. ADM for analysing the dissipative RWs in a collisional plasma

It is known that the LDNLSE (15) is not integrable equation and for examining the features of the dissipative RWs (first and second RWs) in the present model, the ADM is introduced for solving Equation (15) numerically. Before embarking on the application of the ADM, let us rewrite Equation (15) in the form of an initial value problem (i.v.p)

$$
\begin{align*}
i \partial_\tau \Psi + & \frac{1}{2} \nabla^2 \Psi + Q |\Psi|^2 \Psi + iR \Psi = 0, \\
\Psi (\eta, \tau_0) &= \varphi (\eta, \tau_0), \\
\Psi (\eta, \tau) &= F (\tau) \Psi (\eta, \tau) = G (\tau),
\end{align*}
$$
(21)

which $\Psi = \Psi (\eta, \tau)$ and $\varphi (\eta, \tau)$ represent any analytical solution to the integrable NLSE (1) and $\Psi (\eta, \tau_0)$ represents the initial condition at $\tau = \tau_0$ where the domain is given by $(\eta, \tau) \in [\eta_0, \eta_f] \times [\tau_0, \tau_f]$. 
Figure 1. The positive and negative regions of the modulational instability dissipative DAWs according the product $PQ$ and $\Omega^2$ given in Equation (18) for different values of (a) the electron spectral index $\kappa_e$ and (b) the ion spectral index $\kappa_i$. 

Figure 2. The positive and negative regions of the modulational instability dissipative DAWs according the product $PQ$ and $\Omega^2$ given in Equation (18) for different values of (a) the superthermal parameter $\kappa$ (here $\kappa_e = \kappa_i = \kappa$) and (b) the ion temperature ratio $\sigma$.

In the unstable regions ($PQ > 0$&$\Omega^2 < 0$&$\Omega(\tau)$ > $K$&$\tau < \tau_{\text{max}}$), the FWs can exist and propagate. Thus, the non-dissipative FW solutions of the integrable NLSE (Equation (15) for $R = 0$) is introduced [60]

$$\phi_j = \sqrt{\frac{P}{Q}} \left[ (-1)^j + \frac{G_j + iP\tau H_j}{F_j} \right] \exp(iP\tau), \quad (22)$$

where $j$ denotes the solution order and $G_j \equiv G_j(\eta, P\tau)$, $H_j \equiv H_j(\eta, P\tau)$, and $F_j \equiv F_j(\eta, P\tau) \neq 0$ are polynomials in the variables $\eta$ and $\tau$. The maximum amplitude amplification at the origin $(\eta, \tau) = (0, 0)$ equals $(2j + 1)$ times from the height of the carrier wave $\sqrt{P/Q}$.

The first-order RW solution ($j = 1$), reads

$$\begin{cases} H_1 = 2G_1 = 8, \\ F_1 = 1 + 4\eta^2 + 4(P\tau)^2, \end{cases} \quad (23)$$

The polynomials $(G_j, H_j, F_j)$ of the second-order RW solution ($j = 2$) are defined by

$$\begin{cases} G_2 = \frac{3}{8} - 3\eta^2 - 2\eta^4 - 9(P\tau)^2 - 12\eta^2(P\tau)^2 - 10(P\tau)^4, \\
H_2 = \frac{15}{4} + 6\eta^2 - 4\eta^4 - 2\eta^2(P\tau)^2 - 8\eta^2(P\tau)^4 - 4(P\tau)^4, \\
F_2 = \frac{1}{8} \left[ 3 + 9\eta^2 + 4\eta^4 + \frac{16}{3}\eta^6 + 33(P\tau)^2 - 24\eta^2(P\tau)^2 + 16\eta^4(P\tau)^2 + 16\eta^2(P\tau)^4 + 36(P\tau)^4 + \frac{16}{3}(P\tau)^6 \right]. \end{cases} \quad (24)$$

In general, all orders of RW solutions are localized in the space-time as shown in Figure 6 (comparison between the non-dissipative first- and second-order RW solutions).

Now, let us anatomy Equation (15) using the ADM for investigating the dynamic mechanism of the non-stationary dissipative RWs at fixed $\tau$. The algorithm of this method is summarized in the following briefly steps:
Figure 3. The MI maximum period $\tau_{\text{max}}$ and the critical wavenumber $K_c(\tau)$ are mapped out for different values of the superthermal parameter $\kappa$ (here $\kappa_e = \kappa_i = \kappa$).

Figure 4. The MI maximum period $\tau_{\text{max}}$ and the critical wavenumber $K_c(\tau)$ are mapped out for different values of the ion temperature ratio $\sigma$. 
Figure 5. The MI maximum period $\tau_{\text{max}}$ and the critical wavenumber $K_c(\tau)$ are mapped out for different values of the collisional frequency $\nu_{dn}$.

Figure 6. The profile of the non-dissipative first-order RW $|\Psi_1|$ and the non-dissipative second-order RW $|\Psi_2|$ is plotted in the $(\eta, \tau)$ plane. Here, $(k, \sigma, k_p, \kappa, f) = (3.5, 1, 3, 3, 0.2)$.

First step, multiplying Equation (15) by $i = \sqrt{-1}$ and applying the integration over $\tau$ in the interval $[\tau_0, \tau]$, we get

$$\Psi(\eta, \tau) = \Psi(\eta, \tau_0) + \int_{\tau_0}^{\tau} \left( \frac{i}{2} \partial_\eta^2 \Psi + i N(\Psi) - \overline{\Psi} \right) d\tau,$$

where $N(\Psi) = \Psi^2 \overline{\Psi}$ and $\overline{\Psi}$ is the conjugate of $\Psi$ and $\Psi(\eta, \tau_0) = \Psi^{(0)} \equiv \psi^{(0)} \equiv \psi(\eta, \tau_0)$ is an initial solution/condition (say, dark solitons, bright solitons, modulated cnoidal waves, RWS, etc.).

Second step, we assume the solution $\Psi(\eta, \tau)$ can be approximated by infinite Taylor expansion

$$\left\{ \begin{array}{l}
\Psi(\eta, \tau) = \sum_{i=0}^{\infty} \Psi^{(i)} \alpha^i, \\
\overline{\Psi}(\eta, \tau) = \sum_{i=0}^{\infty} \overline{\Psi}^{(i)} \alpha^i.
\end{array} \right.$$

The nonlinear term $N(\Psi)$ is approximated by Adomian polynomial

$$N(\Psi) = \sum_{i=0}^{\infty} D_i(\eta, \tau).$$
Figure 7. Profile of the dissipative first-order RW $|\Psi_1|$ for the first two-approximations $\Psi(\tau)_2$ is plotted against different values of (a) the electron spectral index $\kappa_e$, (b) the ion spectral index $\kappa_i$, (c) electron concentration $f$, (d) the ion temperature ratio $\sigma$, and (e) the collisional frequency $R$.

Figure 8. A comparison between the dissipative first- and second-order dissipative RW solutions for the first fifth-approximations $\Psi(\tau)_5$. 
The combination of Equations (25) and (26), gives us
\[
\Psi(\eta, \tau) = \Psi^{(0)} + \int_{\tau_0}^{\tau} \left( \frac{1}{2} \mathcal{P}_{\eta}^2 \sum_{i=0}^{\infty} \Psi^{(i)}(\eta) + \int_{0}^{\infty} \sum_{i=0}^{\infty} D_i(\eta, \bar{\tau}) \right) \, d\bar{\tau},
\]
where \( D_i \) is the appropriate Adomian’s polynomials which is defined by
\[
D_i(\eta, \tau) = \left( \frac{1}{2} \mathcal{D}^i \partial_{\eta} \left( \sum_{a=0}^{\infty} \Psi^{(a)}(\eta) \right) \right)_{\tau = \tau_0}.
\]

Therefore, the series of \( D_i \) for \( N(\Psi) \) as follows:
\[
\begin{align*}
D_0 &= \Psi^{(0)}(\tau) = \Psi^{(0)}(0), \\
D_1 &= 2\Psi^{(0)}(\eta)\Psi^{(1)}(\tau) + \Psi^{(0)}(\eta)^2 \Psi^{(1)}(\tau), \\
D_2 &= \Psi^{(1)}(\tau)^2 + 2\Psi^{(0)}(\eta)\Psi^{(2)}(\tau) + \Psi^{(0)}(\eta)^2 \Psi^{(2)}(\tau), \\
& \quad \vdots \\
D_k &= \sum_{i=0}^{k} \Psi^{(i)}(\eta)^{k-i} \Psi^{(i)}(\tau)^{i}. 
\end{align*}
\]

\textbf{Third step}, applying the following iteration for \( i = 1 \) to \( k \)
\[
\Psi^{(i)}(\eta, \tau) = \int_{\tau_0}^{\tau} \left( \frac{1}{2} \mathcal{P}_{\eta}^2 \Psi^{(i-1)} + \int_{0}^{\infty} \sum_{i=0}^{\infty} D_i(\eta, \bar{\tau}) \right) \, d\bar{\tau}.
\]

For the RW solutions (22), some first approximations can be obtained as follows
\[
\begin{align*}
\Psi^{(0)}(\eta, \tau_0) &= \sqrt{\frac{P}{Q}} \left[ (1)^{\frac{1}{2}} + \frac{G_j + i PrH_l}{F_j} \right] \exp(iPr) \bigg|_{\tau = \tau_0}, \\
\Psi^{(1)}(\eta, \tau) &= \int_{\tau_0}^{\tau} \left( \frac{1}{2} \mathcal{P}_{\eta}^2 \Psi^{(0)} + \int_{0}^{\infty} \sum_{i=0}^{\infty} \Psi^{(i)}(\eta) \right) \, d\tau, \\
&= \left( \frac{1}{2} \mathcal{P}_{\eta}^2 \Psi^{(0)}(\eta) + \int_{0}^{\infty} \sum_{i=0}^{\infty} \Psi^{(i)}(\eta) \right) \Psi^{(1)}(\tau), \\
\Psi^{(2)}(\eta, \tau) &= \int_{\tau_0}^{\tau} \left( \mathcal{P}_{\eta}^2 \Psi^{(1)} + \int_{0}^{\infty} \sum_{i=0}^{\infty} \Psi^{(i)}(\eta) \right) \, d\tau, \\
&= -\frac{1}{8} \sum_{i=0}^{\infty} \left( -4 \Psi^{(i)}(\eta) \left( R^{2} - 3PQ\mathcal{D}_{\eta}^2 \Psi^{(0)} \right) \right) + 8PQ \Psi^{(0)} \mathcal{D}_{\eta}^2 \Psi^{(0)} + 4Q \mathcal{D}_{\eta}^4 \Psi^{(0)} + 4Q \mathcal{D}_{\eta}^4 \Psi^{(0)} + 16iQR \Psi^{(0)} \right) \\
& \quad + 4Q^2 \Psi^{(0)} \right)^2 + 16iQR \Psi^{(0)} \right) \\
& \quad + 4Q^2 \Psi^{(0)} \right)^2 + 16iQR \Psi^{(0)} \right), \\
& \quad \vdots
\end{align*}
\]

\[
\Psi^{(k)}(\eta, \tau) = \int_{\tau_0}^{\tau} \left( \mathcal{P}_{\eta}^2 \Psi^{(k-1)}(\eta) \right) \, d\tau,
\]

where \( \bar{\tau} = \tau - \tau_0 \). From Equations (27) and (31), the final solution of Equation (15) could be obtained as
\[
\Psi(\eta, \tau) = \Psi^{(0)} + \sum_{i=1}^{\infty} \Psi^{(i)}(\eta, \tau).
\]

In Figure 6, we make a comparison between the profile of the first-order RW \( \Psi_1 \) and the second-order RW \( \Psi_2 \) in order to discuss the difference between them. It is observed that all orders of the RW solutions e.g. \( \Psi_1 \) and \( \Psi_2 \), have the same qualitative behavior against the physical parameters related to the model under consideration. However, the profile of the second-order RW \( \Psi_2 \) is more spiky (narrower width and taller amplitude) than the first-order RW \( \Psi_1 \) as seen in Figure 6. In general, the maximum amplitude of the super RWs reads \( |\Psi_2(\eta, \tau)| = \Psi_2|_{\text{max}} = (2\tau + 1)^{\frac{1}{2} - \frac{1}{4} \sqrt{Q}} \). Thus, the maximum amplitude of the first- and second-order RW solutions are, respectively, given by \( \Psi_2|_{\text{max}} = 3\sqrt{Q} \) and \( \Psi_2|_{\text{max}} = 5\sqrt{Q} \). Moreover, the second-order RW solution \( \Psi_2 \) has double structures as compared to the first one \( \Psi_1 \), i.e. \( |\Psi_2(\eta, \tau)| \) has three local maxima while \( |\Psi_1(\eta, \tau)| \) has one local maxima as shown in Figure 6.

The impact of the spectral indices \( (k_x, k_i) \), the ion temperature ratio \( \sigma \), and the collisional dust-neutral frequency \( v_{dn} \) on the profile of the non-stationary dissipative FWSs is illustrated in Figure 7. It is remarked from Figure 7(a) that the RW amplitude grows with increasing the electron superthermality, i.e. the higher values of the electron spectral index \( k_e \) lead to a reduction in the RW amplitude. The ion superthermal parameter \( k_i \) has a slight effect on the profile of the dissipative RWs as is evident in Figure 7(b). The electron concentration \( f \) and the ion temperature ratio \( \sigma \) have a significant effect on the dissipative RWs as shown in Figures 7(c,d), respectively, which the pulse amplitude enhances with the increase of both \( f \) and \( \sigma \). Figure 7(e) demonstrates that the amplitude of the dissipative RWs diminishes with the enhancement of \( v_{dn} \). The amplitude of the dissipative modulated structures always decreases with increasing the time of propagation and the collisional dust-neutral frequency \( v_{dn} \) because the system dissipates its energy due to the collisions between the particles, which leads to a reduction of the nonlinearity of the system and thus a decay of the pulse amplitude.

For studying the accuracy of numerical solutions, the profile of the dissipative RWs according to the first fifth-approximations \( \Psi(\eta, \tau)|_5 = \Psi^{(0)} + \sum_{i=1}^{5} \Psi^{(i)}(\tau) \) is plotted in Figure 8 for different values of the collisional frequency \( R \). Moreover, the maximum residual error \( |\Psi(\eta, \tau)|_\infty \) in the whole space-time domain \( \in \times [\tau_0, \tau_f] \) is estimated for several approximations and with different values of the collisional frequency \( R \) as shown in Table 1.

| No. Approx. | \( L_\infty \) for \( R = 0 \) | \( L_\infty \) for \( R = 0.2 \) | \( L_\infty \) for \( R = 0.5 \) |
|-------------|---------------------|---------------------|---------------------|
| \( \Psi(\tau)|_{1} \) | 0.00288869 | 0.0244995 | 0.0666218 |
| \( \Psi(\tau)|_{5} \) | 8.51709 x 10^{-9} | 0.00000493797 | 0.000713639 |
| \( \Psi(\tau)|_{8} \) | 3.96814 x 10^{-15} | 0.000035583 | 0.00125405 |
One can see from the results in Table 1 that the ADM gives high-accurate solutions in the small time range. Also, it is noted that the accuracy increases with the increase of the number of approximations. However, for studying the phenomena in a long time range, we should introduce some modifications to the standard ADM because this method and most of its family do not give accurate solutions in the long time range.

4. Summary

The dissipative multi-rogue waves (multi-RWs) including the first-order RWs and second-order RWs have been investigated in an unmagnetized collisional complex plasma consisting of inertial cold negative dust grains and inertialess superthermal electrons and positive ions. Using the derivative expansion technique, the linear damped NLSE (LDNLSE) has been derived for investigating the features of the multi-RWs. The criteria of the existence modulational instability (MI) of the dissipative modulated structures have been investigated and the regions of the dissipative (un)stable modulated structures have been precisely defined. The Adomian decomposition method (ADM) has been devoted for solving the LDNLSE numerically in order to study the characteristics behaviour of the dissipative multi-RWs in the model under consideration. The effect of the damping parameters on the profile of the dissipative multi-RWs has been examined. It was observed that the amplitude of the dissipative RWs decays with the increase of the collisional frequency parameter and for the time going on. The influence of superthermal particles (electrons and ions) and the dust-neutral collisional frequency on the propagation of the dissipative RWs has been studied numerically. This study does not relate to a specific type of plasma, but can be applied to any physical system that can be described by the LDNLSE.

**Future work**, if we consider Shamel distribution for the inertialess particles in addition to the collisions between the charged and neutral particles, in this case, the fluid equations of the plasma model will reduce to the damped modified nonlinear Schrödinger equation. There is not a single attempt by any author to get a rogue wave solution to this problem, so we will try to solve this problem in the future.

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**Disclosure statement**

No potential conflict of interest was reported by the author(s).

**Data availability statement**

The datasets generated for this study are available on request to El-Tantawy.

**ORCID**

Nouf H. Aljahdaly [http://orcid.org/0000-0001-6227-5817]
S. A. El-Tantawy [http://orcid.org/0000-0002-6724-7361]
Abdul-Majid Wazwaz [http://orcid.org/0000-0002-8325-7500]

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