Prospects of Fast Flavor Neutrino Conversion in Rotating Core-collapse Supernovae

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Received 2021 October 15; revised 2021 November 5; accepted 2021 November 9; published 2022 January 18

Abstract

There is mounting evidence that neutrinos undergo fast flavor conversion (FFC) in core-collapse supernova (CCSN). In this paper, we investigate the roles of stellar rotation on the occurrence of FFC by carrying out axisymmetric CCSN simulations with full Boltzmann neutrino transport. Our result suggests that electron neutrino lepton number (ELN) angular crossings, which are the necessary and sufficient condition to trigger FFC, preferably occur in the equatorial region for rotating CCSNe. By scrutinizing the neutrino–matter interaction and neutrino radiation field, we find some pieces of evidence that the stellar rotation facilitates the occurrence of FFC. The low-electron-fraction region in the post-shock layer expands by centrifugal force, enhancing the disparity of neutrino absorption between electron-type neutrinos ($\nu_e$) and their anti-particles ($\bar{\nu}_e$). This has a significant impact on the angular distribution of neutrinos in momentum space, in which $\nu_e$ tends to be more isotropic than $\bar{\nu}_e$; consequently, ELN crossings emerge. The ELN crossing found in this study is clearly associated with rotation, which motivates further investigation on how the subsequent FFC influences explosion dynamics, nucleosynthesis, and neutrino signals in rotating CCSNe.

Unified Astronomy Thesaurus concepts: Core-collapse supernovae (304); Supernova neutrinos (1666)

1. Introduction

Core-collapse supernovae (CCSNe) are massive stellar explosions that announce the death of stars. During the last decade, remarkable progress has been made in the theory of CCSNe, which is greatly attributed to efforts on both multi-dimensional (multi-D) numerical modeling and improvements of input physics. On the other hand, there is a growing interest in collective neutrino oscillation in CCSN theory. If it happens, the flavor conversion results in mixing the energy spectrum and angular distributions of each species of neutrinos, indicating that this potentially affects CCSN dynamics.

Fast flavor conversion (FFC), one of the modes of collective neutrino oscillation (Sawyer 2005, 2016; Chakraborty et al. 2016), has been of great interest recently. This mode offers the fastest flavor conversion in dense neutrino gases such as CCSN cores. The quantum kinetic equation provides a way to investigate the property of FFC (Volpe 2015; Blaschke & Cirigliano 2016). However, the consistent treatment of neutrino flavor conversion, transport, and collision term is a formidable technical difficulty. Nevertheless, much effort has been expended in the study of nonlinear features (Capozzi et al. 2019; Martin et al. 2019; Johns et al. 2020; Martin et al. 2020a, 2020b, 2021; Bhattacharyya & Dasgupta 2021; Duan et al. 2021; Kato et al. 2021; Richers et al. 2021a, 2021b; Sasaki & Takewaki 2021; Sigl 2021; Tamborra & Shalgar 2021; Wu et al. 2021; Zaizen & Morinaga 2021). On the other hand, linear stability analyses of flavor conversion, which can be carried out based on neutrino data from classical neutrino transport, provide an indication of whether FFC occurs in neutrino radiation fields (Izaguirre et al. 2017). Very recently, Morinaga (2021) proved that a necessary and sufficient condition for the instability is the electron neutrino lepton number (ELN) crossing, which means that the angular distributions of electron-type neutrinos ($\nu_e$) and their anti-particles ($\bar{\nu}_e$) are crossing each other in momentum space. According to the recent studies of searching ELN crossings or linear stability analysis, FFC commonly occurs in the CCSN environment (Abbar et al. 2019, 2020; Nagakura et al. 2019b; Capozzi et al. 2021; Delfan Azari et al. 2020; Glas et al. 2020; Morinaga et al. 2020; Abbar et al. 2021; Nagakura et al. 2021), which is why FFC has gained increased attention from the community.

It should be noted, however, that the linear stability analysis or ELN crossing search thus far has been focused only on CCSN models without stellar rotation. For rotational CCSNe, on the other hand, there are some characteristic properties of rotation in both fluid dynamics and neutrino radiation fields (Summa et al. 2018; Harada et al. 2019; Powell & Müller 2020; Takewaki et al. 2021). By scrutinizing neutrino radiation fields computed from axisymmetric CCSN simulations with full Boltzmann neutrino transport, we investigate how the rotation gives impacts on the occurrence of FFC.

This paper is organized as follows. In Section 2, we provide the essence of our CCSN models and FFC analysis. Section 3 provides information related to the supernova properties. In Section 4, we present our main findings. In Section 5, we conclude and discuss the significance of our findings in the CCSN theory.

2. Numerical Setups

The axisymmetric CCSN simulations presented in this paper are performed by a multi-D neutrino-radiation-hydrodynamics code. We simultaneously solve the Boltzmann equation for neutrino transport, the Euler equation for fluid, and the Poisson equation for gravity. The details of the code and their basic equations are described in Sumiyoshi & Yamada (2012), and Nagakura et al. (2014, 2017, 2019c) and references therein.
In this study, we employ a progenitor model of $15 M_\odot$ evolved without rotation presented in Woosley et al. (2002). We run two CCSN simulations (with and without rotation) and start the simulations from the iron-collapse phase. For the rotating model, we add the rotational velocity of

$$v^\phi(r) = \frac{4 \text{ rad s}^{-1}}{1 + (r/10^3 \text{ km})^2}$$

by hand at the onset of the collapse, where $r$ is the radial coordinate measured from the grid center, not from the symmetry axis. We employ a multi-nuclear equation of state based on the variational method (Togashi et al. 2017). The neutrino reactions are taken from the standard set described inBruenn (1985) with some extensions (Nagakura et al. 2019a). We consider three species of neutrinos: $\nu_e$, $\bar{\nu}_e$, and the other heavy-lepton-type neutrinos ($\nu_l$). Following the assumption, we take the two-flavor approximation in FFC analysis. We refer readers to Capozzi et al. (2020, 2021) for cases distinguishing $\mu$ and $\tau$ neutrinos. Throughout the simulation, we divide the radial range of $0 \leq r \leq 5000 \text{ km}$, neutrino energy of $0 \leq \epsilon \leq 300 \text{ MeV}$, and neutrino momentum angle of $0 \leq \theta < \pi$ and $0 \leq \phi < 2\pi$ into 384, 20, 10, and 6 grid points, respectively. The meridian plain $0 \leq \theta \leq \pi$ is divided into 64 and 128 grid points until and after the core bounce, respectively. At the core bounce, we imposed a random velocity perturbation of 0.1%.

To catch the essence of the roles of rotation on FFC, we focus on a representative time snapshot (165 ms after bounce) and analyze the neutrino data by comparing the nonrotating and rotating models. We also estimate the linear growth rate of FFC by following an empirical formula given in Nagakura et al. (2019b) and Morinaga et al. (2020), which is written as

$$\sigma = \sqrt{-\left(\int_{G(\Omega_\nu) > 0} \frac{d\Omega_\nu}{4\pi} G(\Omega_\nu)\right)\left(\int_{G(\Omega_\nu) < 0} \frac{d\Omega_\nu}{4\pi} G(\Omega_\nu)\right)},$$

where $\Omega_\nu = (\theta_{\nu}, \phi_{\nu})$ is the neutrino flight direction. In the expression, $G(\Omega_\nu)$ denotes ELN angular distribution, which is defined as

$$G(\Omega_\nu) = \sqrt{2} G_F \int_0^\infty \frac{r^2 de}{2\pi^2} (f_{\nu_e} - f_{\bar{\nu}_e}),$$

where $G_F$ denotes the Fermi constant, and $f_{\nu_e/\bar{\nu}_e}$ is the distribution function of $\nu_e/\bar{\nu}_e$.

### 3. Supernova Properties

Before entering details of the analysis of FFC, we describe overall properties of our CCSN models to catch the foundation on which we will give the subsequent discussion. Figure 1 shows the time trajectory of shock radii, neutrino luminosities, and mean energies for both the rotating and nonrotating models. In the panel displaying the neutrino luminosity, we change the perpendicular scales for the upper and lower halves in order to see the neutronization burst and subsequent evolution of each flavor at once.

The shock radii in the rotating model expand further than the nonrotating counterpart. The nonrotating model shows a gradual decrease of the shock radii after reaching its maximum, failing in the shock revival. The rotating model shows continuous shock expansion. However, the shock evolution is quite gradual, and the average value reaches $\sim 300 \text{ km}$ at the latest time. Hence, whether the supernova successfully explodes or not is still uncertain. More time is required for judging the fate of the dynamics. We are currently running the long-term simulation for the rotating model, and the results will be reported in another paper.

The neutrino luminosities for the rotating model are smaller than the nonrotating counterpart. This trend, the non-exploding model has high neutrino luminosities, is commonly observed in the supernova simulations (see, e.g., Summa et al. 2018; Vartanyan et al. 2018). Correspondingly, the neutrino mean
energies (the angular average of the ratio of the neutrino energy density to the number density) for the rotating model is also smaller than the nonrotating model.

The $\nu_e$ and $\bar{\nu}_e$ luminosities are similar for both rotating and nonrotating models, while $\nu_e$ has lower mean energy than $\bar{\nu}_e$, indicating that the $\nu_e$ numbers are larger than $\bar{\nu}_e$ numbers. Furthermore, the difference in the $\nu_e$ and $\bar{\nu}_e$ mean energies are smaller for the nonrotating model than the rotating model, and hence the rotating model is more $\nu_e$ rich than the nonrotating model. This condition may be considered unfavorable for the FFC in the rotating model. However, it is compensated by the angular-dependent variation in the rotating model, as will be discussed in the following section.

4. FFC

As shown in the left panel of Figure 2, the ELN crossing appears in the equatorial region for the rotating model, and the resultant growth rate of FFC seems to be fast ($\mathcal{O}(10^{-2})$ cm$^{-1}$). It should be noted that we have never seen this trend before in the previous studies; ELN crossings appear only in the equatorial region (see, e.g., Nagakura et al. 2021). In fact, these crossings are not observed in the nonrotating model (see the right panel of Figure 2). These facts suggest that the stellar rotation has some roles on the trend. Providing its physical interpretation is the subject in this paper.3

The absence of the FFC in the post-shock region of the nonrotating model is in line with the previous studies. Nagakura et al. (2021) conducted a systematic study of occurrences of the FFC in the nonrotating CCSNe models. They investigated many 3D CCSNe simulations with momentscheme neutrino transport. They reported that the FFC occurs at $\sim 100$ ms after the core bounce for all supernova models and that mainly for exploding models. They also discussed the possibility of the FFC by the proto-neutron star convection, but it is hardly seen because of the limited spatial resolution (Glas et al. 2020). Their study is consistent with other ELN crossing searches in CCSN simulations with the full neutrino distribution functions (Abbar et al. 2019, 2020; Delfan Azari et al. 2019, 2020; Nagakura et al. 2019b) and approximate crossing searches (Abbar 2020; Abbar et al. 2021). Considering the fact that the nonrotating model in our model fails to explode (see Section 3), no FFC at 165 ms is compatible with these previous works. In order to see the origin of the ELN crossing, let us first take a look at the angular distribution of ELN for the rotating model. The representative example ($r = 120$ km and $\theta = 1.18$ rad) is displayed in Figure 3. There are more $\bar{\nu}_e$ in the outgoing direction than $\nu_e$, while vice versa for the other directions. Following the language used in Nagakura et al. (2021), this is type II ELN crossing. We note that, although the stellar rotation directs the neutrino flux to the rotational direction (Harada et al. 2019), it is not strong enough to have an influence on the ELN crossing.

Panel (a) of Figure 4 shows the radial profiles of the energy-integrated distribution function of neutrinos in the outgoing direction ($F_{\nu_e}/\nu_e = \int_0^\infty f^{FD}_{\nu_e/\nu_e} e^2 d\epsilon$), and their Fermi–Dirac (FD) distributions determined locally from thermal and chemical equilibrium with matter ($F^{FD}_{\nu_e/\nu_e} = \int_0^\infty f^{FD}_{\nu_e/\nu_e} e^2 d\epsilon$), where $f^{FD}_{\nu_e/\nu_e}$ denotes the FD distribution function for $\nu_e/\bar{\nu}_e$. We select these profiles on the equator in the rotating model. As shown in the panel, although $\nu_e$ is more populated than $\bar{\nu}_e$ at $\lesssim 100$ km, it decreases sharply with radius. Eventually, $\bar{\nu}_e$ dominates over $\nu_e$ at $\sim 150$ km (albeit tiny). The increase or decrease of neutrinos should be dictated by neutrino–matter interactions; hence, we look into the collision term in detail. We note that nucleon scatterings play negligible roles in generating the crossings (we confirmed); thus, we focus only on emission and absorption processes hereafter.

Both $F_{\nu_e}$ and $F_{\bar{\nu}_e}$ coincide with $f^{FD}_{\nu_e/\nu_e}$ and $f^{FD}_{\bar{\nu}_e/\nu_e}$, respectively, at the central region (optically thick region). In the outer regions, on the other hand, $F^{FD}_{\nu_e/\nu_e}$ becomes smaller than $F_{\nu_e}$ regardless of neutrino species. This indicates that neutrino absorption dominates the emission, considering that the collision term (negative means absorption) is expressed as

$$S_{\text{rad},\nu_e/\bar{\nu}_e} = -\frac{1}{\lambda_{\text{abs},\nu_e/\bar{\nu}_e}} \left( f_{\nu_e/\nu_e} - f^{FD}_{\nu_e/\nu_e} \right),$$

where $\lambda_{\text{abs},\nu_e/\bar{\nu}_e}$ is the mean free path for the absorption of $\nu_e/\bar{\nu}_e$. As shown in panel (b) of Figure 4, the absorptivity of $\nu_e$ is higher than $\bar{\nu}_e$, and more importantly, the difference is much

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3 ELN crossings commonly appear in the pre-shock region for both rotating and nonrotating models. This comes from the disparity of inward scattering by heavy nuclei between $\nu_e$ and $\bar{\nu}_e$ (see Morinaga et al. 2020, for more detail).
larger than the case with the nonrotating model. Here, we define the absorptivity as the inverse of $\lambda_{\text{abs, } \nu_e}/\rho_e$ and display them for 11 MeV neutrinos. The different absorptivity between $\nu_e$ and $\bar{\nu}_e$ reflects the disparity of the number density of interacting nucleons (see also Nagakura et al. 2021). As displayed in panel (c) of Figure 4, the neutron number density is much higher than proton; consequently, $\nu_e$ experiences strong absorption. These facts suggest that the disparity between $\nu_e$- and $\bar{\nu}_e$-absorption is responsible for the ELN crossing. We also note that the $F^{\text{FD}}_{\nu_e}$ is larger than $F^{\text{FD}}_{\bar{\nu}_e}$ at 60 km $\lesssim r \lesssim 120$ km (see panel (a) of Figure 4), indicating that the chemical potential of $\nu_e$ becomes negative in this region. In such a region, $\nu_e$ absorption is much stronger than $\bar{\nu}_e$, which facilitates the occurrence of ELN crossing.

We now turn our attention to the effect of rotation. The change of matter distribution by the centrifugal force is the key. In general, the density gradient in the equatorial region becomes shallower than the nonrotating cases (due to the centrifugal force), indicating that the low-$Y_e$ (electron fraction) environment in the post-shock region is extended. This provides a preferable condition for the excess of neutrons to protons, which is why $\nu_e$ goes through strong absorption.

It is interesting to point out that $\alpha$ (the ratio of the number density of $\bar{\nu}_e$ to $\nu_e$) in the region with the ELN crossing is $\sim 0.7$ (see panel (d) of Figure 4), which is smaller than that has been observed in nonrotating CCSN models (see, e.g., Abbar et al. 2019; Delfan Azari et al. 2020; Nagakura et al. 2019b, 2021). Instead, the differences in the degree of forward-peaking angular distributions between $\nu_e$ and $\bar{\nu}_e$ seem large enough to make the ELN crossings. To quantify the degree of forward peaking, we compute the flux factor ($\kappa$, defined as the number flux divided by the number density) of $\nu_e$ and $\bar{\nu}_e$ for both the rotating and nonrotating models\(^4\), and the results are displayed in the panel (e) of Figure 4. As shown in the panel, the difference in the flux factors between $\nu_e$ and $\bar{\nu}_e$ for the rotating model is much larger than the nonrotating model. This suggests that $\bar{\nu}_e$ has a much sharper forward-peaked angular distribution than $\nu_e$ in the rotating model; hence, $\nu_e$ in the outgoing direction dominates over $\bar{\nu}_e$ (in spite of low $\alpha$).

The large difference in the angular distributions between $\nu_e$ and $\bar{\nu}_e$ can be understood by considering the positions of neutrinospheres. In panels (b)–(e) of Figure 4, we highlight the positions of neutrinospheres as vertical lines (for comparison, those in the nonrotating model are also displayed). As shown in these panels, the neutrinosphere radii for both $\nu_e$ and $\bar{\nu}_e$ tend to be larger in the rotating model than in the nonrotating model. This is due to the shallow density structure in the equatorial region of the rotating model. It should also be mentioned that the low-$Y_e$ environment in the equatorial region increases the difference in the neutrinosphere radii between $\nu_e$ and $\bar{\nu}_e$ since the large $\nu_e$ absorption rate increases the neutrinosphere radius of $\nu_e$. As a result, the degree of forward-peak in $\nu_e$ angular distribution is substantially reduced.

Here, we discuss the latitudinal dependence of ELN crossing. In the left panel of Figure 5, we portray the neutrinospheres on the top of the matter-density color map. This illustrates that the neutrinospheres are oblate deformed, and more importantly, the neutrinospheres of $\nu_e$ and $\bar{\nu}_e$ tend to be close around the polar regions, which is attributed to the sharp decline of the matter density and absorptivity. Consequently, the difference of angular distributions between $\nu_e$ and $\bar{\nu}_e$ becomes small, which suppresses the occurrence of ELN crossings around polar regions. We also note that there is inhomogeneity of ELN crossing in the equatorial region. We first point out that the origin is associated with convective fluid

\(^4\) We note that $\kappa$ is a useful quantity to characterize the full angular distributions of neutrinos in CCSNe; see Nagakura & Johns (2021) for more detail.
motions. Since the neutrino absorption dominates over emission at $\geq 100$ km, the negative entropy gradient in the radial direction is sustained, which drives convection (i.e., neutrino-driven convection). Due to the convective motion, high-$Y_e$ clumps sink into the smaller radius, and low-$Y_e$ buoyant plums propagate outward (see the right panel of Figure 5). In the high-$Y_e$ clumps, the disparity of number density between neutron and proton becomes small; consequently, the difference in $\nu_e$ and $\bar{\nu}_e$ absorption rates is reduced, which suppresses the occurrence of ELN crossings.

5. Summary and Conclusion

ELN crossings emerge around the post-shock equatorial region in our rotating model, whereas they are not observed in the nonrotating model. We confirm that the crossings are induced by rotation; the essential physical mechanism is as follows. Centrifugal force deforms the matter distribution oblately in the post-shock region. As a result, the low-$Y_e$ region is extended to a larger radius than the nonrotating model. Since the neutrons are more abundant than the protons in the low-$Y_e$ region, the outgoing $\nu_e$ experiences more absorption by matter than $\bar{\nu}_e$, which facilitates the generation of the crossings. We also find that $\alpha$ in the ELN crossing region is $\sim 0.7$, which is smaller than reported in other nonrotating CCSN models (Abbar et al. 2019; Nagakura et al. 2019b, 2021; Delfan Azari et al. 2020). This suggests that the ELN crossing reflects the large difference of angular distributions between $\nu_e$ and $\bar{\nu}_e$. The rotation is responsible for the difference; the two conditions, shallow density gradient and low-$Y_e$ environment, push the neutrinosphere of $\nu_e$ outward; consequently, $\bar{\nu}_e$ has a more forward-peaked angular profile than $\nu_e$, leading to ELN crossings. In short, $\nu_e$ is absorbed by low-$Y_e$ matter extended by centrifugal force to be more isotropic than $\bar{\nu}_e$. Finally, we inspect the latitudinal dependence of the ELN crossing and find that neutrino-driven convection primarily accounts for the inhomogeneity of the appearance of ELN crossings around the equatorial region.

Since neutrinos are associated with many ingredients of CCSN, the observables such as neutrino signals, morphology of explosions and nucleosynthesis may be substantially impacted by the rotation-induced FFC. Our finding of this paper opens a new window for the connection between FFC and CCSN theory. For more general arguments, we need a systematic study of FFC in rotating CCSNe by changing rotations, progenitors, and their time dependence; the detailed investigations are postponed to future work.

We thank T. Morinaga, M. Zaizen, S. Yamada, W. Iwakami, and T. Takiwaki for fruitful discussions. This research used high-performance computing resources of K-computer by R-CCS, FX10 and Oakforest-PACS by the University of Tokyo, Grand Chariot by Hokkaido University, and FX10 by Nagoya University, through the HPCI System Research Project (Project ID: hp 140211, 150225, 160071, 160211, 170230, 170301, 170304, 180111, 180239, 190100, 200102), SR16000 and XC40 at YITP of Kyoto University, SR16000 and Blue Gene/Q at KEK under the support of its Large Scale Simulation Program (14/15-17, 15/16-08, 16/17-11), Research Center for Nuclear Physics (RCNP) at Osaka University, and the XC30 and the general common-use computer system at the Center for Computational Astrophysics, CfCA, the National Astronomical Observatory of Japan. This work was supported in part by a Grant-in-Aid for Scientific Research on Innovative areas “Gravitational wave physics and astronomy: Genesis” (17H06357, 17H06365) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan, and in part a Grant-in-Aid for Scientific Research (C; 15K05093, 19K03837, B; 20H01905) and Young Scientists (Start-up, JP19K23435) from the Japan Society for the Promotion of Science (JSPS). This work was also partly supported by research projects at K-computer of the RIKEN R-CCS, HPCI Strategic Program of Japanese MEXT, Priority Issue on Post-K-computer (Elucidation of the Fundamental Laws and Evolution of the Universe), Joint Institute for Computational Fundamental Sciences (JICFus), and by MEXT as “Program for Promoting Researches on the Supercomputer Fugaku” (Toward a unified view of the universe: from large scale structures to planets).

Software: Gnuplot (Williams & Kelley et al. 2020).

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