Galactic Orbits of Globular Clusters in the Region of the Galactic Bulge

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Abstract—Galactic orbits have been constructed over long time intervals for ten globular clusters located near the Galactic center. A model with an axially symmetric gravitational potential for the Galaxy was initially applied, after which a non-axially symmetric potential corresponding to the central bar was added. Variations in the trajectories of all these globular clusters in the $XY$ plane due to the influence of the bar were detected. These were greatest for the cluster Terzan 4 in the meridional $(RZ)$ plane. The globular clusters Terzan 1, Terzan 2, Terzan 4, Terzan 9, NGC 6522, and NGC 6558 always remained within the Galactic bulge, no farther than 4 kpc from the Galactic center.

1 INTRODUCTION

Globular clusters are an important source of information for studies of the structure and evolution of the Galaxy [1,2]. Distant globular clusters are of considerable interest for studies of the properties of the halo, the distribution of the matter density in the Galaxy, and estimates of its mass [3–5]. For example, globular clusters located near the Galactic center [6] have been used to investigate the properties of the bulge and bar [7,8] or star-formation processes after the passage of a globular cluster through the disk [9].

Many globular clusters in the Milky Way have measured absolute proper motions. For example, the proper motions of globular clusters were determined in [10–13] based on ground photographic and CCD observations of stars in the Southern hemisphere [14], combined with data from modern HIPPARCOS catalogs [15] to provide second-epoch observations. These data were used in [7,8] to construct Galactic orbits for more than 50 globular clusters. Proper motions for 92 globular clusters were determined in [16] using data from the UCAC2 catalog [17]. Absolute proper motions of more than 140 globular clusters were computed in [18] using data from the PPMXL catalog [19]. The UCAC2 and PPMXL catalogs contain absolute proper motions of stars. They extend the HIPPARCOS system to weaker stars, but contain significant errors, in the brightness equation for UCAC2 and in the form of zonal inhomogeneities of order 2 milliarcsecond/year (2 mas/yr) for PPMXL, as was shown for example in [20] and [21].

When studying the kinematics of Galactic globular clusters, most of which are far from the Sun, it is of most interest to use proper motions obtained using the largest ground telescopes at appreciably different epochs, or with the Hubble Space Telescope (HST), where uncertainties in the measurements and absolute values are determined directly using images of distant galaxies. We have made use of precisely such observations in our current study.
The construction of Galactic orbits for globular clusters requires a good model for the gravitational potential of the Galaxy. In [22], we refined the parameters of three models for the Galactic potential with different forms for the dark-matter halo. We used modern sets of observational data encompassing a wide range of distances to the rotation axis $R$, from 0 to $\sim 200$ kpc. Moreover, it was shown in [7,8] that the central bar of the Galaxy appreciably influences the motions of globular clusters. Therefore, it is especially important to take this influence into account when analyzing the motions of globular clusters located near the bulge and bar. Our goal in the current study was to investigate the three-dimensional kinematics of globular clusters in the Milky Way using measured distances, radial velocities, and proper motions. We constructed their Galactic orbits using a refined model for the gravitational potential of the Galaxy. The orbits were constructed for two cases: using an axially symmetric potential, and adding the non-axially symmetric potential of the bar.

2 METHOD

2.1 Model for the Galactic Potential

The axially symmetric gravitational potential of the Galaxy was represented as the sum of three components — the central, spherical bulge $\Phi_b(r(R, Z))$, the disk $\Phi_d(r(R, Z))$, and the massive, spherical dark-matter halo $\Phi_h(r(R, Z))$:

$$\Phi(R, Z) = \Phi_b(r(R, Z)) + \Phi_d(r(R, Z)) + \Phi_h(r(R, Z)).$$

Here, we used a cylindrical coordinate system $(R, \psi, Z)$ with its origin at the Galactic center. In Cartesian coordinates $(X, Y, Z)$ with their origin at the Galactic center, the distance to a star (the spherical radius) is $r^2 = X^2 + Y^2 + Z^2 = R^2 + Z^2$, where the $X$ axis is directed from the Sun toward the Galactic center, the $Y$ axis is perpendicular to the $X$ axis and points in the direction of the Galactic rotation, and the $Z$ axis is perpendicular to the Galactic $(XY)$ plane and points in the direction of the North Galactic pole. The gravitational potential is expressed in units of $100$ km$^2$ s$^{-2}$, distances in kpc, masses in units of the mass of the Galaxy, $M_{\text{gal}} = 2.325 \times 10^7 M_\odot$, and the gravitational constant is taken to be $G = 1$.

The potentials of the bulge $\Phi_b(r(R, Z))$ and disk $\Phi_d(r(R, Z))$ were taken to have the form proposed by Miyamoto and Nagai [23]:

$$\Phi_b(r) = -\frac{M_b}{(r^2 + b_b^2)^{1/2}},$$

$$\Phi_d(R, Z) = -\frac{M_d}{\left[R^2 + (a_d + \sqrt{Z^2 + b_d^2})^2\right]^{1/2}},$$

where $M_b$ and $M_d$ are the masses of the corresponding components and $b_b, a_d,$ and $b_d$ are scale parameters of the components in kpc. According to [24], the halo component can be represented

$$\Phi_h(r) = -\frac{M_h}{r} \ln \left(1 + \frac{r}{a_h}\right).$$

Table 1 presents the parameters of the model for the Galactic potential (2)–(4) derived by Bajkova and Bobylev [22] using a Galactic rotation curve constructed for objects located at
Table 1: Parameters of Model III for the Galactic potential, according to [22], with $M_{gal} = 2.325 \times 10^7 M_\odot$.

| $M_b$ | 443 $M_{gal}$ |
|-------|----------------|
| $M_d$ | 2798 $M_{gal}$ |
| $M_h$ | 12474 $M_{gal}$ |
| $b_b$ | 0.2672 kpc |
| $a_d$ | 4.40 kpc |
| $b_d$ | 0.3084 kpc |
| $a_h$ | 7.7 kpc |
| $M_{bar}$ | 43.1 $M_{gal}$ |
| $q_b$ | 5.0 kpc |
| $a_b/b_b$ | 1/0.42 |
| $a_b/c_b$ | 1/0.33 |

Distances $R$ out to $\sim 200$ kpc. The local parameter values $R_\odot = 8.3$ kpc and $V_\odot = 244$ km s$^{-1}$ were used when constructing this rotation curve. The model (2)–(4) was denoted Model III in [22].

We chose to describe the potential of the central bar using the triaxial ellipsoid model [25]:

$$
\Phi_{bar} = \frac{M_{bar}}{(q_b^2 + X^2 + [Y a_b/b_b]^2 + [Z a_b/c_b]^2)^{1/2}},
$$

where $X = R \cos \vartheta$, $Y = R \sin \vartheta$, $a_b, b_b, c_b$ are the three semi-axes of the bar; $q_b$ is the length of the bar; $\vartheta = \theta - \Omega_{bar} t - \theta_{bar}$, $tg(\theta) = Y/X$, $\Omega_{bar}$ is the angular speed of the bar; $t$ is the integration time; and $\theta_{bar}$ is the inclination of the bar relative to the $X$ and $Y$ axes, measured from the line joining the Sun and the Galactic center (the $X$ axis) to the major axis of the bar in the direction of the Galactic rotation. We adopted the angular speed of the bar $\Omega_{bar} = 55$ km s$^{-1}$ kpc$^{-1}$, in accordance with the estimates of Bobylev and Bajkova [26].

### 2.2 Construction of the Orbits

The equation of motion of a test particle in an axially symmetric gravitational potential can be obtained from the Lagrangian $\mathcal{L}$ of the system (see Appendix A of [27]):

$$
\mathcal{L}(R, Z, R, \psi, Z) = 0.5(\dot{R}^2 + (R \dot{\psi})^2 + \dot{Z}^2) - \Phi(R, Z).
$$

Introducing the canonical momenta $p_R = \partial \mathcal{L} / \partial \dot{R} = \dot{R}$, $p_\psi = \partial \mathcal{L} / \partial \dot{\psi} = R^2 \dot{\psi}$, and $p_Z = \partial \mathcal{L} / \partial \dot{Z} = \dot{Z}$, we obtain the Lagrangian equations in the form of a system of six first-order differential equations:

$$
\dot{R} = p_R,
\dot{\psi} = p_\psi / R^2,
\dot{Z} = p_Z,
p_R = -\partial \Phi(R, Z) / \partial R + p_\psi^2 / R^3,
p_\psi = 0,
p_Z = -\partial \Phi(R, Z) / \partial Z.
$$
We integrated Eqs. (7) using a fourth-order Runge–Kutta algorithm.

The peculiar velocity of the Sun relative to the Local Standard of Rest (LSR) was taken to be \((u_\odot, v_\odot, w_\odot) = (11.1, 12.2, 7.3) \pm (0.7, 0.5, 0.4) \text{ km s}^{-1}\), in accordance with [28]. Here, the heliocentric velocity is given in a moving Cartesian coordinate system with \(u\) directed toward the Galactic center, \(v\) in the direction of the Galactic rotation, and \(w\) perpendicular to the Galactic plane toward the North Galactic pole.

We denoted the initial values of the positions and space velocities of a test particle in the heliocentric coordinate system \((x_0, y_0, z_0, u_0, v_0, w_0)\). These initial positions and velocities are then given in the fixed Cartesian coordinates of the Galactic system by the formulas

\[
\begin{align*}
X &= R_0 - x_0, \quad Y = y_0, \quad Z = z_0, \\
U &= u_0 + u_\odot, \\
V &= v_0 + v_\odot + V_0, \\
W &= w_0 + w_\odot, \\
\Pi &= -U \cos \psi_o + V \sin \psi_o, \\
\Theta &= U \sin \psi_o + V \cos \psi_o,
\end{align*}
\]

where \(R_0\) and \(V_0\) are the Galactocentric distance and linear velocity of the LSR about the center of the Galaxy and \(\tan \tan \psi_o = Y/X\).

### 3 DATA

Our main source of data was [6], where both ground observations on telescopes of the European Southern Observatory and space observations with the HST (as a first epoch for NGC 6540) were used to derive the absolute proper motions of selected globular clusters of the Galactic bulge. The mean difference between the epochs was 25 yrs.

The proper motions of two globular clusters located near the Galactic center \((R < 4 \text{ kpc})\) — NGC 6652 and NGC 6681—were determined using HST observations. The proper motion of NGC 6652 was obtained in [29] with a mean difference between epochs of about seven years, and the proper motion of NGC 6681 in [30] with a mean difference between epochs of about 5.5 yrs.

### Table 2: Input data on the globular clusters according to [6]

| Cluster    | \(l\), deg | \(b\), deg | \(\mu_\alpha \cos \delta\), mas yr\(^{-1}\) | \(\mu_\delta\), mas yr\(^{-1}\) | \(V_r\), km s\(^{-1}\) | \(d\), kpc |
|------------|------------|------------|-----------------------------|-----------------------------|---------------------|--------|
| Terzan 1   | 357.57     | 1.00       | 0.51 \pm 0.31               | -0.93 \pm 0.29             | 114 \pm 14          | 6.2 \pm 0.6 |
| Terzan 2   | 356.32     | 2.30       | -0.34 \pm 0.30              | 0.15 \pm 0.42              | 109 \pm 15          | 8.7 \pm 0.8 |
| Terzan 3   | 356.02     | 1.31       | 3.50 \pm 0.69               | 0.35 \pm 0.58              | -50.0 \pm 2.9       | 9.1 \pm 0.9 |
| Terzan 4   | 3.61       | -1.99      | 0.00 \pm 0.38               | -3.07 \pm 0.49             | 59 \pm 10           | 7.7 \pm 0.7 |
| NGC 6522   | 1.02       | -3.93      | 3.35 \pm 0.60               | -1.19 \pm 0.34             | -21.1 \pm 3.4       | 7.8 \pm 0.7 |
| NGC 6540   | 3.29       | -3.31      | 0.07 \pm 0.40               | 1.90 \pm 0.57              | -17.7 \pm 1.4       | 3.7 \pm 0.3 |
| NGC 6558   | 0.20       | -6.02      | -0.12 \pm 0.55              | 0.47 \pm 0.60              | -197.2 \pm 1.5      | 7.4 \pm 0.7 |
| NGC 6652   | 1.53       | -11.38     | 4.75 \pm 0.07               | -4.45 \pm 0.10             | -1117.5 \pm 5.8    | 9.6 \pm 0.9 |
| NGC 6681   | 1.53       | -12.51     | 1.58 \pm 0.18               | -4.57 \pm 0.16             | 220.3 \pm 0.9       | 9.0 \pm 1.8 |
| Palomar 6  | 2.10       | 1.78       | 2.95 \pm 0.41               | 1.24 \pm 0.19              | 181.0 \pm 2.8       | 7.3 \pm 0.7 |
Table 3: Initial velocities in the fixed Cartesian coordinates $U, V, W$ and the cylindrical coordinates $\Pi, \Theta$

| Cluster      | $U$, kpc | $V$, kpc | $W$, kpc | $\Pi$, kpc | $\Theta$, kpc |
|--------------|----------|----------|----------|------------|--------------|
| Terzan 1     | $125 \pm 14$ | $236 \pm 12$ | $-18 \pm 4$ | $-153 \pm 14$ | $219 \pm 12$ |
| Terzan 2     | $117 \pm 14$ | $233 \pm 20$ | $47 \pm 4$  | $-128 \pm 19$ | $-227 \pm 16$ |
| Terzan 4     | $-30 \pm 1$  | $355 \pm 38$ | $-112 \pm 16$ | $-247 \pm 24$ | $-257 \pm 30$ |
| Terzan 9     | $74 \pm 11$  | $163 \pm 25$ | $-50 \pm 6$  | $41 \pm 18$  | $174 \pm 21$  |
| NGC 6522     | $-19 \pm 3$  | $278 \pm 22$ | $-121 \pm 17$ | $91 \pm 6$   | $263 \pm 21$  |
| NGC 6540     | $-7 \pm 2$   | $285 \pm 12$ | $23 \pm 2$   | $20 \pm 2$   | $284 \pm 12$  |
| NGC 6558     | $-184 \pm 1$ | $268 \pm 28$ | $39 \pm 7$   | $191 \pm 1$  | $263 \pm 28$  |
| NGC 6652     | $-151 \pm 1$ | $160 \pm 10$ | $-247 \pm 25$ | $-112 \pm 2$ | $-190 \pm 10$ |
| NGC 6681     | $203 \pm 4$  | $116 \pm 31$ | $-179 \pm 28$ | $227 \pm 21$ | $52 \pm 23$   |
| Palomar 6    | $191 \pm 4$  | $353 \pm 16$ | $-52 \pm 11$ | $-94 \pm 5$  | $390 \pm 15$  |

The input parameters for the globular clusters are presented in Table 2, whose columns give (1) the name of the cluster, (2)–(3) the Galactic coordinates $l$ and $b$, (4)–(5) the proper motions $\mu_\alpha \cos \delta$ and $\mu_\delta$ in mas yr$^{-1}$, (6) the heliocentric radial velocity $V_r$, and (7) the heliocentric distance $d$. The initial values of the space velocities $U, V, W$ and $\Pi, \Theta$ are given in Table 3.

### 4 RESULTS AND DISCUSSION

The Galactic orbits of the globular clusters are presented in Figs. 1 and 2. Figure 1 presents the orbits of the first six clusters in our list: (a) Terzan 1, (b) Terzan 2, (c) Terzan 4, (d) Terzan 9, (e) NGC 6522, and (f) NGC 6540. Figure 2 presents the orbits for the remaining four clusters: (a) NGC 6558, (b) NGC 6652, (c) NGC 6681, and (d) Palomar 6. Two projections, onto the $XY$ and $RZ$ planes, are given for each cluster, for the axially symmetric potential (upper) and with the addition of the bar potential (lower).

Table 4 presents the perigalactic $a_{\min}$ and apogalactic $a_{\max}$ distances and the orbital eccentricities $e$ for both potential models corresponding to the orbits in Figs. 1 and 2. The parameters of the globular cluster Terzan 4 varied most strongly due to the influence of the bar potential: $a_{\max}$ increased by 0.12 kpc. For the remaining clusters, the variation in this parameter was 0.05 kpc or less.

Figures 1 and 2 show that the trajectories of virtually all of the clusters vary appreciably in the $XY$ plane due to the influence of the bar. These variations are smaller in the $RZ$ plane, apart from the cluster Terzan 4, where they are clearly visible in Fig. 1c. The influence of the bar becomes less important with increasing distance of the cluster from the Galactic center. For example, the bar exerts virtually no influence for NGC 6540, which is the most distant cluster from the center ($R = 4.6$ kpc, Fig. 1f). A similar picture can be seen in Fig. 1 of [7], where a different axially symmetric Galactic potential and different bar potential were used: the bar in [7] was longer ($a_b = 3.13$ kpc) and rotated more rapidly ($\Omega_{\text{bar}} = 60$ km s$^{-1}$ kpc$^{-1}$) than ours. Figures 7 and 8 of [13] also show that the influence of the bar is maximum for the orbits of clusters located closer to the Galactic center, but is present out to $R \approx 6$ kpc.

Uncertainties in the input data for the globular clusters could have a larger influence
Рис. 1: Galactic orbits of the first six globular clusters from our list over five billion years in the past. The circle marks the current position of the cluster.
Рис. 2: Same as Fig. 1 for the remaining four globular clusters from our list.
Table 4: Characteristics of the globular-cluster orbits computed for the axially symmetric potential (upper rows) and with the addition of the bar potential (lower rows)

| Cluster     | $a_{\text{min}}$ | $a_{\text{max}}$ | $e$   |
|-------------|------------------|------------------|-------|
| Terzan 1    | 1.41             | 3.80             | 0.46  |
| Terzan 2    | 0.51             | 1.15             | 0.39  |
| Terzan 4    | 0.60             | 4.15             | 0.75  |
| Terzan 9    | 0.42             | 0.89             | 0.36  |
| NGC 6522    | 0.50             | 1.55             | 0.51  |
| NGC 6540    | 4.58             | 6.70             | 0.19  |
| NGC 6558    | 0.70             | 3.82             | 0.69  |
| NGC 6652    | 1.64             | 5.83             | 0.56  |
| NGC 6681    | 0.67             | 4.69             | 0.75  |
| Palomar 6   | 1.00             | 5.29             | 0.68  |
| Terzan 1    | 1.42             | 3.85             | 0.46  |
| Terzan 2    | 0.52             | 1.21             | 0.40  |
| Terzan 4    | 0.59             | 4.27             | 0.76  |
| Terzan 9    | 0.42             | 0.93             | 0.38  |
| NGC 6522    | 0.51             | 1.61             | 0.52  |
| NGC 6540    | 4.59             | 6.72             | 0.19  |
| NGC 6558    | 0.71             | 3.89             | 0.69  |
| NGC 6652    | 1.64             | 5.89             | 0.56  |
| NGC 6681    | 0.65             | 4.74             | 0.76  |
| Palomar 6   | 1.01             | 5.35             | 0.68  |

than the effect of the bar. For example, the characteristics $a_{\text{min}} = 0.96$ kpc, $a_{\text{max}} = 2.78$ kpc, $z_{\text{max}} = 1.43$ kpc, and $e = 0.49$ were found for NGC 6522 for the axially symmetric potential in [7]. A comparison with the corresponding data for NGC 6522 in Table 4 shows that our values for $a_{\text{min}}$ and $a_{\text{max}}$ are nearly half these values, although the two eccentricities are similar. The orbit parameters for all the globular clusters we have considered, obtained for both the axially symmetric potential and with the addition of the bar potential, taking into account the observational uncertainties, are presented in Table 5. We determined the mean parameters and their rms deviations using Monte Carlo simulations, generating 100 independent realizations of random errors in the data for each object, with these errors having a Gaussian distribution with a specified rms deviation and zero mean. A comparison of Tables 4 and 5 confirms our conclusion that uncertainties in the data exert a larger influence than does the bar.

The Galactic orbits of 63 Galactic globular clusters were computed in [31], using both axially symmetric and non-axially symmetric Galactic potentials. The non-axially symmetric part of the potential included contributions from the bar and spiral density wave. NGC 6522 was included in both [31] and our study. Figure 2 of [31] shows that the orbits (in the ZR plane) constructed for the axially symmetric potential are very close to those for NGC 6522 shown in Fig. 1. It is interesting, however, that the appearance of this plane constructed taking into account the joint influence of the bar and spiral wave differs appreciably from our results. The orbital characteristics found for NGC 6522 in [31] using the axially symmetric potential were $a_{\text{min}} = 0.81$ kpc, $a_{\text{max}} = 2.23$ kpc, and $e = 0.47$ (these values can also be
Table 5: Characteristics of the globular-cluster orbits computed for the axially symmetric potential (upper rows) and with the addition of the bar potential (lower rows), taking into account the measurement uncertainties

| Cluster   | $a_{\text{min}}$  | $a_{\text{max}}$  | $e$   |
|-----------|-----------------|-----------------|-------|
|           | kpc             | kpc             |       |
| Terzan 1  | 1.39 ± 0.10     | 3.80 ± 0.32     | 0.46 ± 0.04 |
| Terzan 2  | 0.51 ± 0.07     | 1.21 ± 0.25     | 0.40 ± 0.09 |
| Terzan 4  | 0.60 ± 0.11     | 4.44 ± 1.27     | 0.75 ± 0.05 |
| Terzan 9  | 0.44 ± 0.10     | 0.91 ± 0.07     | 0.36 ± 0.08 |
| NGC 6522  | 0.51 ± 0.09     | 1.64 ± 0.35     | 0.52 ± 0.07 |
| NGC 6540  | 4.59 ± 0.07     | 6.71 ± 0.56     | 0.19 ± 0.04 |
| NGC 6558  | 0.70 ± 0.15     | 3.92 ± 0.53     | 0.70 ± 0.03 |
| NGC 6652  | 1.64 ± 0.09     | 5.73 ± 0.75     | 0.55 ± 0.05 |
| NGC 6681  | 0.67 ± 0.13     | 4.92 ± 0.82     | 0.75 ± 0.07 |
| Palomar 6 | 1.00 ± 0.08     | 5.29 ± 0.63     | 0.68 ± 0.03 |
|           | 1.42 ± 0.12     | 3.92 ± 0.31     | 0.47 ± 0.03 |
|           | 0.51 ± 0.09     | 1.32 ± 0.24     | 0.44 ± 0.11 |
|           | 0.58 ± 0.11     | 4.65 ± 1.29     | 0.77 ± 0.05 |
|           | 0.46 ± 0.10     | 0.92 ± 0.08     | 0.34 ± 0.07 |
|           | 0.48 ± 0.08     | 1.74 ± 0.41     | 0.56 ± 0.08 |
|           | 4.56 ± 0.09     | 6.87 ± 0.56     | 0.20 ± 0.04 |
|           | 0.72 ± 0.16     | 4.03 ± 0.52     | 0.70 ± 0.04 |
|           | 1.61 ± 0.07     | 5.97 ± 0.75     | 0.57 ± 0.05 |
|           | 0.64 ± 0.14     | 4.89 ± 0.75     | 0.76 ± 0.07 |
| Palomar 6 | 0.98 ± 0.08     | 5.66 ± 0.77     | 0.70 ± 0.03 |

compared with the estimates of [7] mentioned above, and those found using the non-axially symmetric potential were $a_{\text{min}} = 0.37$ kpc, $a_{\text{max}} = 3.87$ kpc, and $e = 0.83$.

As can be seen in the upper part of Table 4, the six clusters Terzan 1, Terzan 2, Terzan 4, Terzan 9, NGC 6522, and NGC 6558 all have $a_{\text{max}} < 4.2$ kpc, so that they are always located in the bulge. It was recently established that the Galactic bulge has an X-like shape [32,33]. Two hypotheses have been proposed to explain this. According to the first, the X-like shape is not a physical property of the bulge, and is a consequence of bimodality in the distribution in the Hertzsprung–Russell diagram of the giant stars used to study the bulge [34–36]. Note, however, that pronounced X-like bulges are observed in other galaxies without any particular difficulties [37,38]. The second hypothesis is that the X-like shape has a dynamical nature, and is due to the characteristic form of stellar orbits in the central region of the Galaxy [39,40]. Another dynamical approach based on modelling disk instability whose development leads to the formation of an X-shaped bulge has also been considered [41]. A final choice of one or the other hypothesis is not currently possible. In this connection, the orbits of globular clusters in the bulge constructed over long time intervals using high-accuracy observational data are of considerable interest.

In our opinion, the shape of the orbits of the six clusters noted above provides support for the second dynamical hypothesis for the origin of the X-shaped form of the bulge. Figure 1 clearly shows that the orbits of the globular clusters Terzan 2, Terzan 4, Terzan 9, and NGC 6522 in the $ZR$ plane resemble each other. Moreover, they resemble a trapezoid with
sharply protruding edges set on its side, so that they trace out an X-like shape upon mirror reflection relative to the vertical axis.

In spite of the unusual form of the orbit of NGC 6558 (ZR plane, Fig. 2), an X-like shape is formed upon mirror reflection relative to the vertical and horizontal axes. The orbit of this cluster can be considered to be banana-like. Such orbits make an appreciable contribution to the formation of the X-like shape of the bulge, as was shown, for example, in the numerical simulations of [39], as is clearly visible in their Figs. 2 and 3.

The spatial morphology of an X-shaped bar was studied in [41] using three models with different input parameters. Numerical solutions were used to trace the evolution of a cloud of particles under the action of a developing disk instability. The distribution of the density was constructed for all three models. It is interesting that the “boxes” in the ZR meridional planes for all the clusters in Figs. 1 and 2 apart from the distant NGC 6540 are in fairly good agreement with the simulation results of Li and Shen [41] (see Fig. 5 in [41]).

5 CONCLUSION

We selected high-accuracy measurements of the proper motions, radial velocities, and distances of ten globular clusters in the Milky Way for the study considered here. These clusters are located in the inner part of the Galaxy, at distances of no more than 5 kpc from the center.

We have constructed their Galactic orbits over a long time interval, first applying a model with an axially symmetric gravitational potential for the Galaxy, then with the addition of the potential of the central bar. Virtually all of the ten globular clusters display appreciable variations in their trajectories in the XY plane due to the influence of the bar. Here, the influence of the bar is mainly manifest through chaosization of the filling of the annular region bounded by the values \( a_{\text{min}} \) and \( a_{\text{max}} \). The influence of the bar in the RZ plane is most clearly visible in the motion of the cluster Terzan 4. Here, we can see a substantial variation in the appearance of the corresponding “box” (the “boxes” coincide after a longer integration interval).

We have identified a number of clusters that have always been located within the central bulge over the past five billion years (no farther than 4 kpc from the Galactic center): Terzan 1, Terzan 2, Terzan 4, Terzan 9, NGC 6522, and NGC 6558. The shapes of their orbits are consistent with the hypothesis that the X-like shape of the bulge has a dynamical nature. The banana-like orbit of NGC 6558 is of the most interest from this point of view.

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REFERENCES

1. E. Bica, C. Bonatto, B. Barbuy, and S. Ortolani, Astron. Astrophys. 450, 105 (2006).
2. R. Leaman, D.A. van den Berg, and J.T. Mendel, Mon. Not. R. Astron. Soc. 436, 122 (2013).
3. G. Battaglia, A. Helmi, H. Morrison, P. Harding, E.W. Olszewski, M. Mateo, K.C. Freeman, J. Norris, and S. A. Shectman, Mon. Not. R. Astron. Soc. 364, 433 (2005).
4. Y. Sofue, Publ. Astron. Soc. Jpn. 61, 153 (2009).
5. P. Bhattacharjee, S. Chaudhury, and S. Kundu, Astrophys. J. 785, 63 (2014).
6. L.J. Rossi, S. Ortolani, B. Barbuy, E. Bica, and A. Bonfanti, Mon. Not. R. Astron. Soc. 450, 3270 (2015).
7. C. Allen, E. Moreno, and B. Pichardo, Astrophys. J. 652, 1150 (2006).
8. C. Allen, E. Moreno, and B. Pichardo, Astrophys. J. 674, 237 (2008).
9. D.V. Putte and M. Cropper, Mon. Not. R. Astron. Soc. 392, 113 (2009).
10. D.I. Dinescu, T.M. Girard, and W.F. van Altena, Astron. J. 117, 1792 (1999).
11. D.I. Casetti-Dinescu, T.M. Girard, D. Herrera, W.F. van Altena, C.E. López, and D.J. Castillo, Astron. J. 134, 195 (2007).
12. D.I. Casetti-Dinescu, T.M. Girard, V.I. Korchagin, W.F. van Altena, and C.E. López, Astron. J. 140, 1282 (2010).
13. D.I. Casetti-Dinescu, T.M. Girard, L. Jilková, W.F. van Altena, F. Podestá, and C.E. López, Astron. J. 146, 33 (2013).
14. T.M. Girard, D.I. Dinescu, W.F. van Altena, I. Platais, D.C. Monet, and C.E. López, Astron. J. 127, 3000 (2004).
15. The HIPPARCOS and Tycho Catalogues, ESA SP–1200 (1997).
16. A.K. Damibis, Astron. Astrophys. Trans. 25, 185 (2006).
17. N. Zacharias, S.E. Urban, M.I. Zacharias, G. L. Wycoff, D.M. Hall, M.E. Germain, E.R. Holdenried, and L. Winter, Astron. J. 127, 3043 (2004).
18. N.V. Kharchenko, A.E. Piskunov, E. Schilbach, S. Röser, and R.-D. Scholz, Astron. Astrophys. 558, A53 (2013).
19. S. Röser, M. Demleitner, and E. Schilbach, Astron. J. 139, 2440 (2010).
20. P.N. Fedorov, V.S. Akhmetov, and V.V. Bobylev, Mon. Not. R. Astron. Soc. 416, 403 (2011).
21. J.J. Vickers, S. Röser, and E.K. Grebel, Astron. J. 151, 99 (2016).
22. A.T. Bajkova and V.V. Bobylev, Astron. Lett. 42, 567 (2016).
23. M. Miyamoto and R. Nagai, Publ. Astron. Soc. Jpn. 27, 533 (1975).
24. J.F. Navarro, C.S. Frenk, and S.D.M. White, Astrophys. J. 490, 493 (1997).
25. J. Palouš, B. Jungwirt, J. Kopecký, Astron. Astrophys. 274, 189 (1993).
26. V.V. Bobylev and A.T. Bajkova, Astron. Lett. 42, 228 (2016).
27. A. Irrgang, B. Wilcox, E. Tucker, and L. Schiefelbein, Astron. Astrophys. 549, 137 (2013).
28. R. Schönrich, J. Binney, and W. Dehnen, Mon. Not. R. Astron. Soc. 403, 1829 (2010).
29. S.T. Sohn, R.P. van der Marel, J.L. Carlin, S.R. Majewski, N. Kallivayalil, D.R. Law, J. Anderson, and M.H. Siegel, Astrophys. J. 803, 56 (2015).
30. D. Massari, A. Bellini, F.R. Ferraro, R.P. van der Marel, J. Anderson, E. Dalessandro, and B. Lanzoni, Astrophys. J. 779, 81 (2013).
31. E. Moreno, B. Pichardo, and H. Velázquez, Astrophys. J. 793, 110 (2014).
32. O.A. Gonzalez and D. Gadotti, Astrophys. Space Sci. Lib. 418, 199 (2016).
33. M. Ness and D. Lang, Astron. J. 152, 14 (2016).
34. A. McWilliam and M. Zoccali, Astrophys. J. 724, 1491 (2010).
35. D.M. Nataf, A. Udalski, J. Skowron, M.K. Szymański, M. Kubiak, G. Pietrzyński, I. Soszyński, K. Ulaczyk, et al., Mon. Not. R. Astron. Soc. 447, 1535 (2015).
36. Y.-W. Lee, S.-J. Joo, and C. Chung, Mon. Not. R. Astron. Soc. 453, 3906 (2015).
37. E. Laurikainen, H. Salo, E. Athanassoula, A. Bosma, and M. Herrera-Endoqui, Mon. Not. R. Astron. Soc. 444, 80 (2014).
38. E. Laurikainen and H. Salo, Astrophys. Space Sci. Lib. 418, 77 (2016).
39. M. Portail, C. Wegg, and O. Gerhard, Mon. Not. R. Astron. Soc. 450, 66 (2015).
40. E. Athanassoula, Astrophys. Space Sci. Lib. 418, 391 (2016).
41. Z.-Y. Li and J. Shen, Astrophys. J. 815, 20 (2015).