Emulating hyperbolic-media properties with conventional structures

Constantinos A Valagiannopoulos and Sergei A Tretyakov
Department of Radio Science and Engineering, School of Electrical Engineering, Aalto University, PO Box 13000, FI-00076 Aalto, Finland
E-mail: konstantinos.valagiannopoulos@aalto.fi and sergei.tretyakov@aalto.fi

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Abstract
Hyperbolic media (media with hyperbolically-shaped dispersion surfaces) possess interesting properties which have inspired numerous application concepts. In this study, we try to identify conventional materials which resemble the behavior of hyperbolic media. We start from biaxial concepts and conclude that a necessary condition for that purpose is the high permittivity contrast between the two axes. Since components with substantial permittivity magnitude are behaving like PEC boundaries, we consider an electrically dense mesh of tilted PEC strips formulating numerous parallel TEM waveguides as a possible substitute to hyperbolic structures. Indeed, this wave-guiding medium is found to exhibit hyperbolic properties via semi-analytical approximations which have been verified by full-wave simulations. The design flexibility provided by the increased independence between the formulated consecutive waveguides can be exploited in a variety of different objectives when controlling electromagnetic fields.

Keywords: controlling EM fields, high-contrast anisotropic materials, hyperbolic media, spatial dispersion
1. Introduction

Hyperbolic metamaterials are uniaxial anisotropic media whose transverse and longitudinal permittivities (the real parts) have opposite signs. This feature yields a hyperbolically-shaped dispersion curve (in two dimensions) instead of an ellipsoidally-shaped one which corresponds to common anisotropic dielectrics. The hyperbola is topologically different from an ellipse: it extends to infinity, which allows a number of very interesting electromagnetic phenomena. An overview of the electromagnetic properties of hyperbolic materials (also called ‘uniaxial backward-wave media’ [1] and ‘indefinite’ materials [2]) and the new physics that is hidden behind its concept of these materials can be found in [3]. Moreover, a study that examines the same topic, but from the quantum photonics point of view, has been given in [4]. Furthermore, certain analytical investigations in general-purpose configurations comprising indefinite media, such as the rigorous determination of the Green function, have been also performed [5].

The rapidly increasing interest in hyperbolic metamaterials is motivated by a vast variety of applications that hyperbolic media can enable. Besides the well-known phenomena of huge enhancement in radiation resistance of small radiators [6, 7] and negative refraction [1, 2], a technique of converting the evanescent modes of an arbitrary source to propagating radiation through a hyperbolic prism has been developed in [8]. Also, the superlensing and focusing features of indefinite materials have been investigated in [9]. In addition, the phenomenon of radiative heat transfer in micron-thick multilayer stacks of hyperbolic metamaterials has been studied in [10], where the adopted design is suitable for prospective thermophotovoltaic systems. Moreover, nonlinear effects in hyperbolic media have been analyzed [11] and the phenomenon of total absorption by slabs of tilted indefinite media has been scrutinized in [12].

Due to the application potential of hyperbolic metamaterials, several attempts to actually construct media with such properties have been made. In particular, stacks of alternating gold and silicon layers have been employed in [13], where the samples are manufactured using magnetron sputtering. The same idea but using silver instead of gold is implemented in [14]. Furthermore, multilayers comprising graphene sheets separated by subwavelength-thick dielectric spacers have been fabricated and found to exhibit hyperbolic isofrequency wavevector dispersion at far- and mid-infrared frequencies, allowing propagation of waves that would be otherwise evanescent in an isotropic dielectric [15]. Finally, a metamaterial formed by a racemic mixture of left- and right-handed long metallic helices has been proposed as a hyperbolic medium [16], while a complete overview of several constructing techniques for indefinite media is given in [17].

The necessity to realize, in one material sample, both positive and negative values of the permittivity with specific requirements on the absolute values of the real parts and on the level of losses is a serious challenge which hinders potential applications. In this paper, we consider possible alternative scenarios which can enable at least some of the exciting hyperbolic-media effects but do not require materials with both negative and positive eigenvalues of the permittivity tensor. The main idea is to mimic the hyperbolic media properties using ordinary materials (dielectrics or metals) but with extreme anisotropy. One key feature of hyperbolic media is the existence of propagating waves with very large wavenumbers, due to the hyperbolic shape of the dispersion curve. This feature can be emulated (in part) in ordinary anisotropic media if at least one of the eigenvalues of the permittivity tensor is very large in the absolute value, although the real parts of all eigenvalues are positive and the dispersion curves are elliptical. The same feature (propagating waves with very large wavenumbers along some
directions) is found also in composite materials with strong spatial dispersion, so called wave-guiding media. A typical example is an electrically dense array of thin conducting wires, the wire medium (see a review e.g. in [18]). Electromagnetic properties of these structures have a number of common features with hyperbolic materials (enhancement of radiation from small sources, negative refraction at interfaces [19, 20], etc), although wave-guiding media are formed by ordinary metals and dielectrics, and, moreover, due to their strongly spatially dispersive nature, cannot be described by a local permittivity tensor. Realizations of wave-guiding media do not necessarily require exotic materials with negative and positive permittivities.

The other characteristic feature of hyperbolic media is their potentially extreme propagation asymmetry. This can be clearly seen in the electromagnetic properties of hyperbolic-media slabs with the permittivity tensor axis tilted with respect to the slab surfaces. In this configuration, eigenwaves propagating in the two opposite directions with respect to the normal to the slab interfaces (with a fixed value of the tangential wavenumber) have very different wavenumbers. For some specific values of the permittivities and tangential wavenumbers, the ratio of the normal components of the two propagation constants can be equal to zero or infinity, which results in extreme properties in terms of reflection and transmission coefficients [12]. We expect that this property can be also at least partially emulated using ordinary but highly anisotropic materials or structures. In this scenario, reversing the sign of the normal component of the propagation direction vector (assuming the same tangential wavenumber) corresponds to changing the direction of the eigenwave electric field vector from being predominantly orthogonal to the optical axis to being predominantly parallel to that axis (or the other way around), which should lead to a huge difference in the value of the normal components of the propagation constant.

Based on the above considerations, we expect that at least some of the interesting properties of hyperbolic materials can be emulated using highly anisotropic configurations of ordinary materials. As a 'test-bed' for our investigations we choose a slab with the optical axis tilted with respect to the interfaces (figure 1). In this configuration we can effectively test the primary characteristic of hyperbolic media and its emulations which defines most of the practically interesting phenomena: the strong asymmetry of the normal components of the propagation factor. Furthermore, we can compare reflection and transmission phenomena in different highly anisotropic structures, including negative refraction. It is

![Figure 1. The physical configuration of the investigated structure. An obliquely incident, TM plane wave illuminates a uniaxially anisotropic slab with tilted optical axis.](image-url)
directly inferred that hyperbolic media behavior can be approximated if the two eigenvalues of the permittivity tensor of a uniaxial medium are very different. In particular, large contrast between the waves is observed when one of the two permittivity eigenvalues has a very large magnitude. Thus, we will consider uniaxial dielectrics such that one of the two permittivity eigenvalues has a very large magnitude, either due to its real (lossless dielectrics) or its imaginary (lossy dielectrics or good conductors) part. To include the case of strong spatial dispersion and based on the fact that materials of extremely high dielectric constants react similarly to perfect electric conductors (PEC), we will also consider a two-dimensional, dense mesh of parallel PEC strips with an arbitrary length and tilted by an arbitrary angle with respect to the slab boundaries. This configuration is analytically solvable with the use of an approximate consideration that ignores the effects of the reactive fields near the strip edges and closed-form expressions for the reflection and transmission coefficients are derived. The response of the anisotropic or composite slabs for some characteristic cases of excitation is verified via simulations of the actual device and similarities with the behavior of respective hyperbolic media are remarked and discussed. It is found that a simple periodical array of PEC strips to some extent behaves similarly to hyperbolic materials and can offer a variety of means to control electromagnetic waves with flexibility and robustness. In contrast to earlier studies of tilted wire-medium slabs [19, 20], which are based on corresponding homogenized models, we consider the actual TEM waveguides formed between the PEC strips.

2. Anisotropic slabs with tilted optical axis

2.1. General

Let us consider the simplest possible two-dimensional configuration depicted in figure 1, where the used Cartesian coordinate system \((x, y, z)\) is also defined. A single slab of thickness \(d\) is filled with a homogeneous anisotropic material and excited by a TM-polarized plane wave of the unitary magnitude, traveling along the direction forming an angle \(\xi\) with the horizontal positive semi-axis \(x\). In figure 1, we sketch also an auxiliary capital-lettered Cartesian coordinate system \((X, Y, z)\) with respect to which, the relative permittivity tensor of the anisotropic medium is written as:

\[
\begin{bmatrix}
\alpha & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \alpha
\end{bmatrix}
\]

where \(\alpha \in \mathbb{C}\) can be an arbitrary complex number. This corresponds to a material that is transparent for electric fields polarized along the \(Y\) direction and behaves as a dielectric with the relative permittivity \(\alpha\) for the orthogonal polarizations. The optical axis of the aforementioned material, namely the axis \(X\) of the capital-lettered coordinate system, is rotated with respect to the common \(z\) axis by the angle \(\theta\), as is also shown in figure 1. The normal to the interfaces components of the propagation constant of TM waves inside the medium are analytically evaluated as follows [21]:
\[ k_\pm = \frac{k_0}{2} \left[ (1 - \alpha) \cos \xi \sin (2\theta) \pm \sqrt{2\alpha \left[ \alpha + (1 - \alpha) \cos (2\theta) - \cos (2\xi) \right]} \right], \quad (2) \]

where \( k_0 = 2\pi f \sqrt{\varepsilon_0 \mu_0} = 2\pi /\lambda_0 \) is the free-space wavenumber for the suppressed harmonic time dependence \( e^{j2\omega t} \). The asymmetry of the two wavenumbers is due to the tilt of the optical axis with respect to the normal to the slab boundaries: if the axis is along \( y \) direction, \( \sin (2\theta) = 0 \), and the two solutions simply differ by sign, corresponding to the up- or down-propagating eigenwaves having the same absolute value of the wavenumber. The ratio of the two wavenumbers can be written as \( \frac{k_+}{k_-} = \frac{1-r}{1+r} \), where

\[ r = r (\alpha, \theta, \xi) = \frac{\sqrt{2\alpha \left[ \alpha + (1 - \alpha) \cos (2\theta) - \cos (2\xi) \right]}}{(1 - \alpha) \cos \xi \sin (2\theta)}. \quad (3) \]

### 2.2. Hyperbolic media

The general configuration described by figure 1, corresponds to a lossless hyperbolic medium if \( \alpha \) is negative: \( \alpha < 0 \). The underlying principle beneath most of the interesting applications of these hyperbolic media is the asymmetry between the two supported modes which admits the construction of asymmetric devices. In particular, the most intriguing scenario concerns one unboundedly increasing propagation constant magnitude, while the other one remains moderate. If we confine ourselves to the investigation of acute angles of tilt and incidence, namely \( 0 < \theta, \xi < 90^\circ \), we have no loss of generality since in the rest of the cases \( (\theta + \xi) > 180^\circ \), the modes simply change positions. Under this assumption, the ratio \( k_+/k_- \) possesses magnitudes less than unity and the optimal scenario (highest difference between the modes) corresponds to \( r = 1 \) (according to (3)). The permittivities \( \alpha \) that achieve this result are given by:

\[ r (\alpha, \theta, \xi) = 1 \Rightarrow \left\{ \alpha = \frac{\tan^2 \theta}{\sec^2 \theta \sec^2 \xi - 1} > 0 \quad \text{or} \quad \alpha = -\cot^2 \theta < 0 \right\}. \quad (4) \]

As we see, in order to obtain infinitely large contrast between the two modes, which is independent from the incidence angle \( \xi \), we should choose \( \alpha = -\cot^2 \theta < 0 \). In other words, this infinitely huge and excitation-independent asymmetry takes place only in hyperbolic materials.

In figure 2(a) we represent the ratio \( |k_+/k_-| \) as a function of the tilt angle \( \theta \) for various permittivities \( \alpha \). It is clear that the tilt of the optical axis which yields huge asymmetry between the two modes is fully controllable via the axial relative permittivity. The points where the quantity \( |k_+/k_-| \) is nullified are determined by the aforementioned equation \( \alpha = -\cot^2 \theta \). In figure 2(b), we represent the ratio \( |k_+/k_-| \) again as a function of the tilt angle \( \theta \) for several incidence angles \( \xi \). Since in this example we have kept fixed \( \alpha = -1 \), it is natural to obtain zero ratio \( |k_+/k_-| \) at the same tilt angle \( \theta = 45^\circ \), regardless of the choice of \( \xi \). As stated above, the opposite situation is recorded for \( \theta, \xi > 90^\circ \), where the two waves change positions with respect to each other and the ratio \( |k_+/k_-| \) tends to infinity in the vicinity of the region of interest.
When the ratio equals unity, the two wavenumbers \( k_+, k_- \) are complex conjugate to each other [22].

### 2.3. High-contrast media

With the goal to emulate the behavior of a hyperbolic material, we consider a high-contrast medium possessing the permittivity tensor (1) with large \( \alpha \), but positive \( \Re\{\alpha\} \). In particular, we are going to investigate the behavior of the structure of figure 1 in cases when the complex value of \( \alpha \) moves along the two paths (blue and red) on the complex \( \alpha \) plane depicted in figure 3. The blue path corresponds to a lossless anisotropic material with positive permittivities whose contrast gets gradually stronger. In figures 4(a) and (b), we consider this case and represent the magnitude ratio \( 10 \log \left| k_-/k_+ \right| \) (in dB) by drawing the contour plots with respect to the input parameters of the problem. By inspecting figure 4(a), we reach the conclusion that the

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**Figure 2.** The magnitude ratio of the supported waveguides \( \left| k_-/k_+ \right| \) as function of the tilt angle \( \theta \), (a) for various relative permittivities \( \alpha (\xi = 45^\circ) \) and (b) for various incidence angles \( \xi (\alpha = -1) \).

**Figure 3.** The two alternative paths on the complex \( \alpha \) plane which correspond to high-contrast materials in an attempt to emulate the behavior of the hyperbolic medium. Blue path: \( \alpha = \Re\{\alpha\} \) with \( \Re\{\alpha\} > 1 \), red path: \( \alpha = 1 + j\Im\{\alpha\} \) with \( \Im\{\alpha\} < 0 \). The black dots corresponds to the free-space (right) and a typical hyperbolic metamaterial (left).
The depicted quantity gets minimized for moderate tilt angles $\theta$ and high permittivity $\Re \{\alpha\}$. Similarly, from figure 4(b), we notice that this regime should be combined with a very oblique incidence angle $\xi$ in order to become possible.

The red path of figure 3 corresponds to a uniaxial lossy material with the fixed real part of the permittivity $\Re \{\alpha\} = 1$ and gradually increasing losses (or conductivity). In figures 4(c) and (d) we consider this case and show the corresponding contour graphs. The conclusions are...
similar to those which follow from figures 4(a) and (b) for the the lossless case. Consequently, for both the considered paths of figure 3, strong asymmetry of the modes is observed when the tilt angle $\theta$ and the incidence angle $\xi$ are small and a high-contrast anisotropy is assumed.

In figure 5 we show the variation of the ratio $\frac{k_+}{k_-}$ as a function of the incidence angle $\xi$ for three different slabs that exhibit large asymmetry between the two supported modes. More specifically, we consider a hyperbolic medium with $\alpha = -1$ and the incidence angle close but not equal to $45^\circ$ ($\theta = 43^\circ$), in order to avoid zero-valued ratios according to figure 2(b). In addition, we study two ordinary anisotropic materials with substantial permittivity contrasts: one lossless ($\alpha = 100$) and one lossy ($\alpha = 1 - j100$) with a tilt of $\theta = 5^\circ$. It is noteworthy that the values of the wavenumber ratio are very small for moderate incidence angles $\xi$ in all cases. However, the growth trend of the represented quantity with $\xi$ is more rapid in the cases of conventional anisotropic materials, contrary to the hyperbolic medium.

3. Tilted lattice of parallel PEC strips

3.1. Approximate model

From the analysis above, it is inferred that uniaxial anisotropic structures with high permittivity contrasts ($|\alpha| \to \infty$), either of lossless or lossy nature, are suitable to mimic the behavior of hyperbolic media, when it comes to the asymmetry of the two supported modes. On the other hand, is well-known [23] that materials with extremely large permittivities $\alpha$, either due to its real or due to its imaginary part, can imitate the behavior of the PEC. Furthermore, structures with strong spatial dispersion can also have features similar to those of hyperbolic media. Based on this, as an alternative structure to that of figure 1, we consider a tilted (by the angle $\theta$) lattice of parallel PEC strips as depicted in figure 6(a). The configuration is again two-dimensional and
the horizontal distance between two consecutive strips equals $w$. The lattice (of vertical size $d$) is electrically dense, so that only the TEM mode can propagate inside the parallel-plate waveguides formed by each pair of two successive strips ($k_0w\sin\theta < \pi$). The two supported modes in the $n$th representative cell of the mesh are schematically shown in figure 6(b). The excitation is identical to that of figure 1.

A common difficulty in using the concept of perfect electrically conducting (PEC) boundaries in structural modeling, is its limited applicability in optical frequencies, where the metals are highly penetrable and lossy. Indeed, our component seems to be more suitable for radio frequencies where parallel metallic surfaces or rods behave as nearly perfect conductors regardless of the employed metal (copper, aluminum, silver etc). However, certain techniques have been developed with which we can partially overcome this difficulty. In particular, for special architectures of photonic crystals light is not permitted to penetrate within the band gap, namely it can act similarly to a ‘perfect wall’ within some frequency range [24]. In addition, resonance responses of nanoscale metallic apertures could act as an effective PEC cavity in the visible [25]; therefore, elongated closed surfaces could possible mimic the PEC strips operation in our model.

Referring to figure 6(a), the magnetic fields in regions 1 and 2 (upper and lower vacuum regions, above and below the lattice respectively) are written as: $H_1 = z\left(e^{jk_y\sin\xi} + Re^{-jk_y\sin\xi}\right)e^{jk_x\cos\xi}$ and $H_2 = zTe^{jk_y\sin\xi}e^{jk_x\cos\xi}$ respectively. The reflection and transmission coefficients $R$, $T$ can be approximately determined if we ignore the interaction of the electric field component that is parallel to the strips with them and adopt the following expression for the magnetic field in the $n$th cell of the grid (figure 6(b)):

$$H_3(n) = z\left(A_ne^{jk_x(x\cos\theta+y\sin\theta)} + B_ne^{-jk_x(x\cos\theta+y\sin\theta)}\right).$$

The complex coefficients $A_n$, $B_n$ are containing the suitable $x$-variation (translated into $n$-variation), dictated by the phase matching: $e^{jk_x\cos\xi}$. By imposing the necessary boundary conditions at the (representative) isolated points $L_n$ (at $x = x_{\text{lower}}(n) = nw$) and $U_n$ (at $x = x_{\text{upper}}(n) = nw + d/tan\theta$), the following explicit formulas are derived:

![Figure 6.](image-url)
Note that the final result is not dependent on the inter-strip distance \( w \), which is compatible with our assumption that the fields are traveling through the mesh solely using the TEM modes of the formed thin planar waveguides.

The results of calculations with the above formulas are shown in figure 7, where the magnitude of the reflection (6) and the transmission coefficients (7) are represented as functions of the incidence angle \( \xi \) for several tilt angles \( \theta \). Obviously, the two graphs are complementary to each other since the structure is lossless \( |R|^2 + |T|^2 = 1 \). Both the depicted quantities \( |R| \), \( |T| \) are symmetric with respect to \( \xi \) even though the configuration is highly asymmetric; furthermore, the reflection coefficient vanishes (the matched regime) not only for \( \xi = \theta \), where the electric field is normal to the PEC surfaces, but also for \( \xi = 180^\circ - \theta \). It should be additionally stressed that the reflections are locally maximized for the normal incidence regardless of the tilt angle. Finally, \( |R| \) is very small along almost the entire \( \xi \) axis when \( \theta = 80^\circ \).

It appears counter-intuitive that the reflection coefficient is symmetric with respect to the direction normal to the slab \( (R \) is the same for the incidence angles \( \theta \) and \( 180^\circ - \theta \)), although the internal structure of the slab is highly asymmetric. However, this property can be explained from the fact that the reflection and transmission properties are determined by TEM modes in the planar waveguides formed by each period. Figure 8 illustrates the fact that the voltage \( V_{\text{inc}} = |E_{\text{inc}}| w \cos \theta \) applied to the input of parallel-plate waveguides forming each period of the lattice, is the same for both incidence angles \( \xi = \theta \) and \( \xi = 180^\circ - \theta \), which means that in
both cases the waveguides are excited in the same way, leading to identical reflection and transmission response.

Similar structures have been rigorously analyzed in [26], where slanted mixed gratings made of dielectric materials are considered employed in conical diffraction mounts. Also the solution of an array of tilted metallic wires is provided in [27] where the structure is utilized to achieve near-field transport. Furthermore, an alternative modeling for one-dimensionally periodic, multilayer, inhomogeneous, anisotropic diffraction gratings is presented in [28], while significant contribution in treating similar lattices from the photonics point of view has been made by the works [29, 30]

3.2. Numerical simulations

In order to validate the approximate formulas and made conclusions, at least for the three representative cases of incidence angle: $\xi = \theta, 90^\circ, 180^\circ - \theta$, we utilized the commercial simulation package COMSOL Multiphysics [31] to computationally solve a finite-length cluster of PEC strips. We used the scattering boundary conditions for the surrounding walls which works as a perfectly matched layer with negligible reflections, while the concept of a field-defined port has been employed as excitation. Due to the finite size of the structure, we use a finite-power source instead of plane-wave illumination. In particular, we use the model of the complex point source which has been introduced as a horn antenna model [32]. The produced field exactly satisfies the Helmholtz equation and the mathematical expression of the axial (equaling total) magnetic field is given by:

$$H_{\text{inc}} = z H_0^{(2)} \left[ k_0 \sqrt{(x - (D + jb) \cos \xi)^2 + (y - (D + jb) \sin \xi)^2} \right],$$  \hspace{1cm} (8)

where $H_0^{(2)}$ is the Hankel function of zero order and second type. It corresponds to a beam with the aperture equal to $b$, at the distance $D$ from the origin with the orientation forming an angle $\xi$ with the positive horizontal semi-axis $x$. In figure 9 we depict the variations of the normalized
magnitude of such an incident field with the same incidence angle $\xi = 60^\circ$ and distance $D = 6\lambda_0$ but with different aperture lengths $b$. The white lines show the boundaries of the (absent) slab with the thickness $d$. It is clear that the field behind the source is almost zero, while the beam attenuates and widens as the observation points get more distant from the aperture. One can also notice that the larger the aperture, the stronger the relative field far from it.

In figure 10 we show the axial magnetic field in contour plots as evaluated by simulating the free-space (figures 10(a), (c), (e)) and the PEC strips lattice (figures 10(b), (d), (f)), when the complex point source excitation is adopted. The first pair of figures 10(a), (b) corresponds to the incidence angle $\xi = 45^\circ$ equal to the tilt angle $\theta$. It is apparent that the primary wave passes almost perfectly through the PEC grid in figure 10(b) and the field distribution is almost identical to that of figure 10(a) when the mesh is absent. In the second pair of figures 10(c), (d), the incident wave propagates normally to the structure (figure 10(d)) and substantial reflections occur if one compares the obtained graph with figure 10(c). Strong response of the structure is also indicated by a horizontal shift of the main lobe within and outside the grid. Finally, in figures 10(e) and (f) we consider the incidence angle $\xi = 180^\circ - \theta = 135^\circ$ which, according to our approximate model of (6), (7), should yield negligible reflections. It is clear that the field variation outside the system of parallel strips is very similar to the response of the source radiating alone (figure 10(e)). This verifies our approximate model since the densely populated grid definitely distorts the front of the incident wave but for a certain combination of the thickness $d$ and tilt angle $\theta$ manages to restore the phase of the background field (propagating along the angle $\xi$) below the lattice.

Figure 11 present the results of another numerical simulation by comparing the electromagnetic behavior of a PEC grid with a hyperbolic slab ($\alpha = -4$) under complex point source excitation. In the case of $\xi = 60^\circ$ (figures 11(a), (b)), the spatial distribution of the magnetic field possesses certain similarities; therefore, it appears that the grid of PEC strips can emulate the response of a hyperbolic medium. The two patterns resemble each other better in
Figure 10. The two-dimensional distributions of the axial magnetic field $H_z = z \cdot H$ in the case of absent ((a), (c), (e)) and present ((b), (d), (f)) PEC lattice for: (a), (b) $\xi = 45^\circ$, (c), (d) $\xi = 90^\circ$, (e), (f) $\xi = 135^\circ$ for the complex point source illumination. Simulation parameters: $\theta = 45^\circ$, $k_d = 2.67$, $D = 4\lambda_0$, $b = 2\lambda_0$. 

ksi(1)=0.785398 axial component of the magnetic field Hz (A/m) ksi(2)=1.570796 axial component of the magnetic field Hz (A/m) ksi(3)=2.356194 axial component of the magnetic field Hz (A/m)
the case of $\xi = 60^\circ$, (c), (d) $\xi = 115^\circ$. Simulation parameters: $\theta = 45^\circ$, $k_0 d = 2.67$, $D = 4\lambda_0$, $b = 2\lambda_0$.

Figure 11. Two-dimensional distributions of the magnetic field magnitude $|H_z| = |H|$ in the case of a PEC lattice ((a), (c)) and a hyperbolic slab ($\alpha = -4$) ((b), (d)) for: (a), (b) $\xi = 60^\circ$, (c), (d) $\xi = 115^\circ$. Simulation parameters: $\theta = 45^\circ$, $k_0 d = 2.67$, $D = 4\lambda_0$, $b = 2\lambda_0$.

the case of $\xi = 115^\circ$ (figures 11(c), (d)), where the reflections are similar and the power concentration into the mesh is almost identical. The negative refraction phenomenon is clearly demonstrated in both cases. The similarity between the responses of the waveguiding medium and hyperbolic materials could be also demonstrated by considering super-lens or hyper-prism structures, which would be the topic of a separate future work.

4. Conclusions

Hyperbolic media have enabled a vast variety of applications which in turn have motivated many attempts to fabricate materials with related characteristics. Layered structures with noble metals and penetrable dielectrics, graphene sheets or metallic helices are only few of the components that have been employed to build absorbers, superlenses or thermophotovoltaic enhancers based on the hyperbolic material features. We perform a study on how one can
emulate these hyperbolic properties with exclusive use of conventional media. In particular, we considered positively-defined anisotropic materials and concluded that sufficient emulation of the hyperbolic properties is achieved when high-contrast media are utilized. Since permittivities with large either real or imaginary part in limiting cases tend to imitate the PEC concept, we regarded another alternative structure comprised of numerous parallel PEC strips formulating a densely populated mesh. We proposed an approximate semi-analytical solution to this configuration, which has been verified via numerical simulations, and indeed can exhibit certain hyperbolic-media characteristics.

We advocate that considered structure (grid of parallel metallic strips) can be of multiple utility when it comes to the purpose of controlling electromagnetic waves. Due to their flexibility and robustness, similar meshes comprised of TEM-mode waveguides with various length or shape can process each spatial sample of the incoming signal differently and thus formulate an output that can vary at will. Tailoring the transmitting pattern, changing the output phase or determining the spatial distribution of the produced power are only few of the controlling objectives that similar operation principles, proofs of concept and design models could be helpful.

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