Fluid acceleration in the bulk of turbulent dilute polymer solutions

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New Journal of Physics 10 (2008) 123015 (10pp)
Received 19 August 2008
Published 16 December 2008
Online at http://www.njp.org/
doi:10.1088/1367-2630/10/12/123015

Abstract. We studied the effects of long-chain polymers on the small scales of turbulence by experimental measurements of Lagrangian accelerations in the bulk of turbulent flows of dilute polymer solutions. Lagrangian accelerations were measured by following tracer particles with a high-speed optical tracking system. We observed a significant decrease in the acceleration variance in dilute polymer solutions as compared with in pure water. The shape of the normalized acceleration probability density functions, however, remained the same as in Newtonian water flows. We also observed an increase in the turbulent Lagrangian acceleration autocorrelation time with polymer concentration. The decrease of acceleration variance and the increase of acceleration autocorrelation time are consistent with a suppression of viscous dissipation, and cannot be explained by a mere increase of effective viscosity due to the polymers.

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1. Introduction

It has long been known that the addition of a very small amount of long-chain polymers to a fluid can have a dramatic effect on the turbulence near solid boundaries and result in significantly reduced flow resistance [1]. This ‘drag-reduction’ phenomenon has been the subject of extensive studies [2]–[12]. On the other hand, the effect of polymers on the bulk turbulence, away from boundaries, has received less attention.

In the classical Richardson–Kolmogorov energy cascade picture, kinetic energy is injected into turbulence at the large, forcing scales. It is then transferred to smaller and smaller scales until it is finally dissipated to thermal energy. In Newtonian fluids, viscosity is the only dissipation mechanism and it acts on very small spatial and temporal scales—the Kolmogorov scales. Between the energy injection scales and the Kolmogorov scales, energy is transferred down scale without loss, since the effect of viscosity at these scales is negligible. Therefore, in statistically stationary turbulence of Newtonian fluids, the energy injection rate $\epsilon_I$, the energy transfer rate $\epsilon_T$, and the energy dissipation rate $\epsilon_D$ are the same, $\epsilon_I = \epsilon_T = \epsilon_D$, and are usually not distinguished. In the presence of polymers, however, the situation becomes less clear. Since polymers can be stretched by the flow, they may extract energy at certain scales (or a range of scales) and thus interrupt the energy cascade. In addition, the internal friction among different parts of a polymer chain or between the polymer and the fluid provide another way of dissipating kinetic energy. Tabor and De Gennes [5, 13], Lebedev and co-workers [14]–[16], and Casciola and De Angelis [17] proposed theories for homogeneous isotropic turbulence in dilute polymer solutions. In particular the effect of polymers on energy transfer has been discussed.

Experiments [10, 18, 19] and numerical simulations [20, 21] of modeled polymers in turbulent flows have provided evidence that in bulk turbulence the small scale Eulerian quantities are damped by the polymer additives. The relatively low Reynolds numbers of the flows investigated in these previous works make it difficult to compare them quantitatively with available theories.

Recently, there has been growing interest in studying turbulence from a Lagrangian viewpoint, i.e. by following the motion of the fluid particles [22]. It has been shown, in particular, that the dynamics of polymers in a turbulent flow is closely related to the correlation of the velocity gradient following Lagrangian trajectories [23].

Here, we report experimental measurements of fluid acceleration and its Lagrangian time-correlations in intense turbulent flows of dilute polyacrylamide (PAM) (molecular weight $18 \times 10^6$) solutions with various concentrations between 0 and 10 parts per million (ppm) by weight. The Taylor microscale Reynolds number of the turbulent flows before adding polymers was in the range of $140 \leq R_{\lambda,w} \leq 485$. The Reynolds number was obtained by measuring the
velocity fluctuation and the energy transfer rate through the Eulerian structure functions [24]. We do not report a Reynolds number for the polymer flows. It is not clear what the appropriate definition of Reynolds number in the presence of polymers is. The Reynolds number is a measure of the separation of the integral scale and the Kolmogorov scale in Newtonian fluid turbulence and the presence of polymers may affect the energy cascade and hence the scale separation. To avoid confusion, we add subscripts ‘w’ or ‘water’ to those quantities that are measured from the water flow.

In this work, we focus on the small scale quantities by measuring in a small observation volume at high spatial and temporal resolution. Our results are directly related to the dissipation range and the energy dissipation rate $\varepsilon_D$. In the rest of the text, we will simply use the symbol $\varepsilon$ when referring to $\varepsilon_D$. We measured fluid accelerations by optically following tracer particles with a high-speed three-dimensional particle tracking system. We observed strong decreases of turbulent Lagrangian accelerations and an increase in acceleration anisotropy in these flows. Similar effects on Eulerian small scale quantities have been reported previously [18, 19]. Moreover, we observed that both effects increase with polymer concentration. To experimental accuracy, the probability density function (PDF) of acceleration normalized by the standard deviation, however, is not affected by the polymer additives. The measured autocorrelation time of Lagrangian acceleration components increases with polymer concentration. The change in acceleration variance and acceleration autocorrelation time with polymer concentration favors the hypothesis that the effect of polymers affect the small scales of bulk turbulence through a decrease in the energy dissipation rate rather than through a change in the effective viscosity.

2. Experiments

We studied a turbulent water flow generated between two counter-rotating coaxial baffled disks. The blades on the disks force the flow inertially rather than through a boundary layer [25]. The flow was seeded with 50 $\mu$m or 26 $\mu$m approximately neutrally buoyant polystyrene particles. These particles have previously been shown to faithfully follow the turbulent flow in pure water at Reynolds numbers up to $O(10^3)$ [24]. In this study, no difference was seen between measurements made with the two different sized particles in water or in the polymer solutions. In addition, within the range of polymer concentrations studied in this work (0–10 ppm), we observed only small changes in fluctuating velocity with polymer concentration, comparable with the estimated uncertainties of velocity measurement. Later, larger observation volume measurements in the same flow at a comparable Reynolds number ($R_{\lambda,w} = 350$) using multiple high speed cameras showed that the fluctuating velocity decreased by less than 10% at polymer concentration up to 10 ppm [26].

A 35 W pulsed frequency-doubled Nd:YAG laser was used to illuminate the tracer particles. A $4.1 \times 4.1 \times 2.05 \text{mm}^3$ volume at the center of the apparatus, a place with the least possible effect of the container walls, was imaged on to four silicon strip detectors. The measurement volume was 24 cm from the sides of the apparatus and 18 cm from the baffled disks. Each silicon strip detector measured one coordinate of the particle position with spatial resolution 8 $\mu$m pixel$^{-1}$. The silicon strip detectors can record up to 70 000 frames per second. The recording frequencies were chosen such that the time resolution of the measurement is at least 100 frames per Kolmogorov time $\tau_{\eta,w}$, as measured in pure water. As $\tau_{\eta,w}$ is the smallest timescale in the flow, this is sufficient for accurate acceleration measurement [24]. The details of the flow and imaging system have been described in [24, 27].
Table 1. Parameters of the experiments with pure water. \( f_p \) is the rotation speed of the baffled disks. \( R_{\lambda,w} \) is the Taylor microscale Reynolds number. \( \varepsilon \) is the energy dissipation rate per unit mass, measured from the Eulerian structure function. \( \eta_w \) and \( \tau_{\eta,w} \) are the Kolmogorov length and timescales, respectively. \( R_{\lambda,w}, \varepsilon, \eta_w \) and \( \tau_{\eta,w} \) refer to the turbulent water flows before adding polymer solutions, and are measured as in [21]. \( N_f \) is the temporal resolution of the recording system, in frames per \( \tau_{\eta,w} \). \( Wi \equiv \tau_p/\tau_{\eta,w} \) is the Weissenberg number.

| \( f_p \) (Hz) | \( R_{\lambda,w} \) (m² s⁻³) | \( \varepsilon \) (µm) | \( \tau_{\eta,w} \) (ms) | \( N_f \) (frames/\( \tau_{\eta,w} \)) | \( Wi \) |
|----------------|-----------------|-------------|-----------------|----------------|-------|
| 0.15           | 140             | 9.0 × 10⁻⁵  | 322             | 105            | 699   | 0.4  |
| 0.3            | 200             | 7.2 × 10⁻⁴  | 191             | 37             | 373   | 1.2  |
| 0.6            | 285             | 5.8 × 10⁻³  | 114             | 13.1           | 175   | 3.3  |
| 1.75           | 485             | 0.14        | 51.0            | 2.63           | 102   | 16   |

The measured particle positions were connected to form three-dimensional particle trajectories using the algorithms described in [24]. Velocities and accelerations were computed by convolution of the trajectories with an appropriate kernel based on successive derivatives of a Gaussian filter of width \( \sigma_k \), which performs simultaneously a low-pass filtering and the differentiation [28]. The variances of acceleration components, \( \langle a_i^2 \rangle \) were estimated as in [24], i.e. \( \langle a_i^2 \rangle \) is computed as a function of \( \sigma_k \) and then is extrapolated to \( \sigma_k \) going to zero, in order to remove the effect of measurement noise on acceleration variance.

We developed a protocol to prepare the polymer solutions such that the experiments are robust and reproducible. First, a high concentration stock solution was prepared by mixing the dry polymer powder with water. A magnetic stir was used to mix the solution at slow speed for 16 h. Then the stock solution stood for several hours and was depressurized to remove soluble gases. Finally, it was gently added to the turbulence chamber to form a dilute solution at the desired concentration. We typically waited 10–15 min before starting the measurement so that the solution was well mixed. We present here measurements on solutions of (PAM) 18522 from Polysciences. All initial stock solutions were 400 ppm except one experiment with a final concentration of 1ppm, which had a stock solution of only 200 ppm.

According to the manufacturer, the weight average molecular weight, \( M_w \), for the 18522 PAM is \( M_w = 18 \times 10^6 \) amu. The manufacturer also gave a measure of the polydispersity of the sample \( M_w/M_n = 3–4 \), where \( M_n \) is the number average molecular weight. Using the Zimm model [29] we calculated that the radius of gyration \( R_g = 0.5 \) µm, the maximum extension \( R_{\text{max}} = 77 \) µm and the polymer relaxation time \( \tau_p = 43 \) ms for this polymer. The relaxation time \( \tau_p \) calculated from the Zimm model has been shown to match experimental observations [30] and has been widely used to characterize polymeric solutions [31]. The most important dimensionless number that characterizes the polymer dynamics in turbulence is the Weissenberg number \( Wi \equiv \tau_p/\tau_{\eta,w} \), which measures the ratio of the polymer relaxation time to the Kolmogorov time of the flow \( \tau_{\eta,w} \equiv (\nu/\varepsilon_w)^{1/2} \), where \( \nu \) is the fluid viscosity and \( \varepsilon_w \) is the turbulence energy dissipation rate per unit mass. When \( Wi \gtrsim 1 \), polymer molecules can be stretched by the turbulent flow and may affect the turbulence. The corresponding Weissenberg number for each Reynolds number that we have measured is shown in table 1. In the calculation of \( Wi \), we used the Kolmogorov time measured in pure water before adding...
Figure 1. PDF of acceleration components at $R_{\lambda,w} = 285$. ○ are for water. • are for a 5 ppm polymer solution. The width of the filter used to calculate acceleration is $\sigma_k = 0.23\tau_{p,w}$. (a) $P(a_z)$, (b) $P(a_x)$, (c) $P(a_x/\langle a_x^2 \rangle^{1/2})$ and (d) $P(a_z/\langle a_z^2 \rangle^{1/2})$.

polymers. Some experiments have shown that the effect of polymer on turbulence is sensitive to the polydispersity of the polymer sample with the longer polymers in the sample having a greater effect on the turbulence [32]. In view of this, the $\tau_p$ and $\tilde{W}/\tilde{M}$ calculated from the $M_w$ may be treated as a lower bound.

At the largest Reynolds number $R_{\lambda,w} = 485$ (also the largest $\tilde{W}/\tilde{M}$), we noticed a slow increase in acceleration variance over time. This is an indication that the polymers might have degraded due to mechanical forcing or other reasons. For measurements at this Reynolds number, only data collected in the first 3.3 h after adding polymers into the apparatus were used in the statistics. The acceleration variance changed less than 10% over this time. There was no appreciable change in acceleration variances at other lower Reynolds numbers that we measured.

3. Results and discussion

Figure 1 shows the component acceleration PDF for water and the polymer solution of 5 ppm by weight at $R_{\lambda,w} = 285$. The unnormalized PDFs for the dilute polymer solution are strongly
Figure 2. Ratio of the acceleration variance in polymer solution to that of water versus concentration of the polymer solution. □, \( R_{\lambda, w} = 140 \), axial component; ○, \( R_{\lambda, w} = 200 \), axial component; ●, \( R_{\lambda, w} = 200 \), horizontal component; △, \( R_{\lambda, w} = 285 \), axial component; ▲, \( R_{\lambda, w} = 285 \), horizontal component; ✷, \( R_{\lambda, w} = 485 \), axial component; For the data point at \( R_{\lambda, w} = 485 \) and concentration 5 ppm, statistics from the first 3.3 h of the run were used. The variance changed less than 10% over this time. Data points at \( R_{\lambda, w} = 285 \) and concentration 5 ppm were shifted to 5.25 ppm for clarity. Inset: ratio of the horizontal acceleration variance to the axial acceleration variance as a function of concentration of the polymer solution. ●, \( R_{\lambda, w} = 200 \); ▲, \( R_{\lambda, w} = 285 \).

Suppressed, which indicates damping of dissipation scale fluctuations by the presence of polymers. It is interesting to note that the PDFs of the normalized acceleration, \( a_i/\langle a_i^2 \rangle^{1/2} \), for water and for the dilute polymer solution overlap with each other in the range that we have measured, as shown in figures 1(c) and (d), which suggests that the accelerations are suppressed only by a scaling factor.

The rms acceleration, \( \langle a_i^2 \rangle^{1/2} \), in the polymer solutions is dramatically lower than in water. Figure 2 shows the ratio of the acceleration variance in polymer solutions to that of water for the same rotation rate of the propellers. In the range of polymer concentrations investigated, the dependence on concentration is roughly linear. There is no clear dependence on \( R_{\lambda, w} \) or \( Wi \).

It should be pointed out that the decrease in acceleration variance cannot be explained by a decrease in energy injection rate \( \epsilon_1 \). In earlier experiments carried out in similar geometry, it has been shown that, if baffled disks were used to provide the driving, the total energy injection rate remains unchanged when polymers were added into the flow \[10, 25\]. The inertial forcing by the vanes was not altered by the presence of polymers. In our recent experiments performed in the same apparatus that we used in this work, we observed that the fluctuating velocity decreased less than 10% at the highest polymer concentration we used (10 ppm), which implies that the energy injection rate \( \epsilon_1 \) is at most decreased by 27%. If the decrease in \( \epsilon_1 \) is the only mechanism for the decrease in acceleration variance, then the decrease in \( \langle a_i^2 \rangle \) would be no larger than 38%. The observed decrease in \( \langle a_i^2 \rangle \) clearly exceeds that estimate by several times. As we will
show later, the decrease of acceleration variance suggests that the energy dissipation rate \( \epsilon_D \) was suppressed by polymers even though \( \epsilon_t \) might have remained the same.

The anisotropy of the small scales is also increasing upon addition of the polymer. The inset in figure 2 shows the ratio of the horizontal acceleration variance to the axial acceleration variance as a function of polymer concentration. If the polymers were causing the energy cascade to be truncated at a smaller wavenumber, then the separation between large and small scales would be reduced and the anisotropy of the large scales would affect the small scales more. This is similar to seeing the anisotropy increase with decreasing \( R_{\lambda,w} \). The increase of anisotropy might also come from the alignment of polymers in particular directions due to the large scale anisotropic flow. It has been shown that polymer responses in longitudinal and transverse directions (relative to the alignment of polymers) are different [33]. If polymers are aligned in a particular direction, this difference could appear as anisotropy in acceleration.

Since the dynamics of polymer solutions depends on the velocity field following Lagrangian trajectories, we studied the Lagrangian acceleration autocovariance \( c_{ij}(\tau) = \langle a_i(t)a_j(t+\tau) \rangle \) and the Lagrangian autocorrelation \( C_{ij}(\tau) = \frac{\langle a_i(t)a_j(t+\tau) \rangle}{\langle a_i(t)^2 \rangle} \). To account for the effect of the finite volume bias, we calculated a modified autocovariance \( \tilde{c}_{ij}(\tau) \)

\[
\tilde{c}_{ij}(\tau) = \frac{\langle a_i(t)a_j(t+\tau) \rangle_{t&\tau}}{\langle a_i(t)^2 \rangle_{t&\tau}^{1/2} \langle a_j(t+\tau)^2 \rangle_{t&\tau}^{1/2}},
\]

where \( \langle a_i^2 \rangle \) is determined from the procedure described above.

Figure 3 shows \( \tilde{c}_{ij}(\tau) \) and \( \tilde{C}_{ij}(\tau) \) for water and polymer. The height of the autocovariance is depressed because, as we have discussed above, \( \langle a_i^2 \rangle \) is dramatically decreased in the polymer solutions. It has been shown that the integral of the acceleration correlation vanishes [34] and the zero-crossing time of the correlation curve is usually used as a measure of the correlation time [35, 36]. The curve at the zero-crossing has a shallow slope. Small measurement errors in the acceleration correlation can cause large deviations in the zero-crossing time. A more robust measure is the exponential decay time \( \tau_{1/e} \), defined such that \( \tilde{C}(\tau_{1/e}) = 1/e \). The inset of figure 3 shows that the decay time \( \tau_{1/e} \) increases in the presence of polymers.

For turbulence in Newtonian fluids (e.g. pure water), it has been shown that the decay time of the acceleration autocorrelation is proportional to the dissipative timescale \( (\nu/\epsilon)^{1/2} \) [36]. The acceleration variance, also a small scale quantity, is related to the energy dissipation rate as \( \langle a_i^2 \rangle \sim \epsilon^{3/2} \nu^{-1/2} \) [37, 38]. Therefore, if we assume epsilon stays the same and only the viscosity changes then we would have the following scaling relation between \( \langle a_i^2 \rangle \) and \( \tau_{1/e} \):

\[
\langle a_i^2 \rangle \sim \epsilon^{3/2} \nu^{-1/2} \sim \epsilon \tau_{1/e}^{-1} \sim \tau_{1/e}^{-1}.
\]
Figure 3. Autocovariance of the axial acceleration component for $R_{\lambda,w} = 285$ at different polymer concentrations. • water; □ 3.5 ppm. △ 5 ppm; ▲ 10 ppm; the dashed line indicates the polymer relaxation time $\tau_p$. Inset: Corresponding autocorrelations.

Figure 4. Change of the axial acceleration variance $\langle a^2 \rangle$ versus change of the decay time $\tau_{1/e}$ of acceleration autocorrelation for $R_{\lambda,w} = 200, 285, \text{ and } 485$; at polymer concentrations. The dashed line is the best fit of the power law $\langle a^2 \rangle \sim \tau_{1/e}^{-1}$ and the solid line is a fit of the power law $\langle a^2 \rangle \sim \tau_{1/e}^{-3}$.

dissipation rate $\varepsilon$ was suppressed by the presence of polymers [10, 18, 19]. In that case, we would keep the viscosity $\nu$ constant. Hence the decrease in the acceleration variance would scale with the increase in the correlation time as

$$\langle a^2 \rangle \sim \varepsilon^{3/2} \nu^{-1/2} \sim \nu \tau_{1/e}^{-3} \sim \tau_{1/e}^{-3}.$$  \hspace{1cm} (4)
Figure 4 shows the change of \( \langle a^2 \rangle \) with \( \tau_{1/e} \) for the experiments at three different Reynolds numbers. The data clearly favor equation (4). The presence of polymers suppresses the energy dissipation rate rather than increasing the effective viscosity, as has been proposed for the mechanism of drag reduction by polymer additives in wall bounded flows (see e.g. [7, 12]). Note that in figure 4, we used the measured time scale \( \langle a^2 \rangle_{\text{water}}^{1/3} v^{-1/3} \) to normalize the decay time \( \tau_{1/e} \), where \( \langle a^2 \rangle \) is measured from the axial component. Since all quantities involved in this timescale, namely acceleration and viscosity, are dissipative range quantities, this timescale is a dissipative scale quantity and is related to \( \tau_{\eta,w} \) by a numerical factor of order 1. The advantage of using this timescale is that \( \langle a^2 \rangle_{\text{water}} \) is a directly measured quantity from our experiment. There is no ambiguity associated with inferring \( \tau_{\eta,w} \) from other measurements.

4. Conclusion

In summary, we studied the effect of polymers on the small scales of turbulence by measuring the fluid acceleration in turbulent flows of dilute polymer solutions. We observed strong decreases of acceleration variance in polymer solutions while the shape of the acceleration PDFs remained the same as in water. The effect on acceleration variance was insensitive to the Reynolds number or the Weissenberg number. The Lagrangian correlation time of acceleration components increased with polymer concentration. The relation between the observed decrease of acceleration variance and the increase of acceleration autocorrelation time suggests that the net effect of polymers is not a mere increase of viscosity, but a suppression of the energy dissipation rate at least.

Acknowledgments

This research was supported by the National Science Foundation under grants PHY-9988755 and PHY-0216406 and by the Max Planck Society.

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