On gauge coupling constant
in linearization of nonlinear supersymmetry

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Abstract

We study in two space-time dimensions ($d = 2$) the relation between $N = 2$ supersymmetric (SUSY) QED theory and $N = 2$ nonlinear (NL) SUSY model by linearizing $N = 2$ NLSUSY generally based upon the fundamental notions of the basic theory. We find a remarkable mechanism which determines theoretically the magnitude of the bare gauge coupling constant from the general structure of auxiliary fields. We show explicitly in $d = 2$ that the NL/linear SUSY relation (i.e. a SUSY compositeness condition for all particles) determines the magnitude of the bare electromagnetic coupling constant (i.e. the fine structure constant) of $N = 2$ SUSY QED.

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1 Introduction

Towards the unified description of space-time, all forces and matter beyond the standard model (SM) it is promising and essential to study (new) physics based upon super-Poincaré (SP) symmetry.

From the group theoretical survey of $SO(N)$ SP algebra for $N > 8$ [1], it was shown that $SO(10)$ SP group accommodates minimally (and correctly) the SM with just three generations of quarks and leptons and the graviton in the single irreducible representation. According to $SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$ and the decomposition of 10 supercharges $10_{SO(10)} = 5_{SU(5)} + 5^{*}_{SU(5)}$ we regarded $\tilde{5}$ as a hypothetical spin-$\frac{1}{2}$ new particles (called superons) possessing the same quantum numbers as $\tilde{5}$ of $SU(5)$ GUT and proposed superon-quintet model (SQM) [2] for matter and forces.

For the field theoretical arguments of SQM including gravity, called superon-graviton model (SGM), a fundamental action has been constructed in nonlinear supersymmetric general relativity (NLSUSY GR) theory [3], which is based upon the general relativity (GR) principle and the nonlinear (NL) representation [4] of supersymmetry (SUSY) [5, 6]. Also, the NLSUSY GR action has a priori promising large symmetries isomorphic to $SO(N)$ ($SO(10)$) SP group [7].

The NLSUSY Volkov-Akulov (VA) model [4] is an almost unique action which represents $N = 1$ global SUSY nonlinearly and describes the dynamics of massless Nambu-Goldstone (NG) fermion of the spontaneous breakdown of space-time SUSY, i.e. the spontaneous SUSY breaking (SSB) mechanism is encoded a priori in the geometry of ultimate flat space-time. (Alternatively, the geometry of ultimate flat (tangential) space-time is defined by the invariant volume action and induces self-contained phase transition to ordinary Minkowski space-time accompanying NG fermion, whose invariant action is given by the NLSUSY VA model.) While, the NLSUSY model is recasted (related) to various linear (L) SUSY theories with SSB (abbreviated as NL/L SUSY relation) by linearizing NLSUSY, which has been shown by many authors in the various cases both for free theories [8]-[13] and for interacting (Yukawa interactions and SUSY QED) theories [14]-[16]. (In Ref.[8] SUSY QED theory in NL/L SUSY relation is discussed by using a constrained gauge superfield in the context of the coupling of the VA action to the gauge multiplet action.)

From the above group and field theoretical aspects in SGM, it is interesting and worthwhile investigating the NLSUSY GR theory as a fundamental theory beyond
the SM, which proposes a new paradigm, SGM scenario \([2, 3, 7, 17]\), for describing the unity of nature.

Indeed, the NLSUSY GR action is constructed in the Einstein-Hilbert form on new (generalized) space-time, \(SGM\) space-time \([3]\), where tangent space-time has the NLSUSY structure, i.e. flat space-time is specified not only by the \(SO(3,1)\) Minkowski coordinates but also by \(SL(2,C)\) Grassmann coordinates for NLSUSY as coset parameters of \(\frac{superGL(4,R)}{GL(4,R)}\). The locally homomorphic non-compact groups \(SO(3,1)\) and \(SL(2,C)\) for the new \(SGM\) space-time degrees of freedom is analogous to compact groups \(SO(3)\) and \(SU(2)\) for gauge degrees of freedom (d.o.f.) of \(\text{t'}\) Hooft-Polyakov monopole. The cosmological term (for the cosmological constant \(\Lambda > 0\)) in the NLSUSY GR action can be interpreted as the potential which induces the SSB of super-\(GL(4,R)\) down to \(GL(4,R)\) due to the NLSUSY structure, i.e. the phase transition of \(SGM\) spacetime to Riemann spacetime with NG-fermion superons matter, called \(Big\ Decay\) \([17]\) which subsequently ignites the Big Bang and the inflation of the present universe. Since the cosmological term gives the NLSUSY model \([4]\) in asymptotic Riemann-flat (i.e. the ordinary vierbein \(e^a_{\mu} \rightarrow \delta^a_{\mu}\)) space-time, the SSB scale of NLSUSY, arbitrary so far, is now related to the Newton gravitational constant \(G\) and the cosmological term of GR.

All (observed) particles, which are assigned uniquely into the single irreducible representation of \(SO(N)\) \((SO(10))\) SP group as an on-shell supermultiplet of \(N\) LSUSY, are considered to be realized as (massless) eigenstates of \(SO(N)\) \((SO(10))\) SP composed of \(N\) \((\text{spin}-\frac{1}{2})\) massless NG-fermion superons through the NL/L SUSY relation after Big Decay. Note that the no-go theorem is overcome (circumvented) in a sense that the nontrivial \(N\)-extended SUSY gravity theory with \(N > 8\) has been constructed in the NLSUSY invariant way, i.e. by the degenerate vacuum (flat space-time).

As for the low energy physics of NLSUSY GR in the asymptotic Riemann-flat space-time, we showed that the phase transition from the massless superon-graviton (SGM) phase to the composite eigenstates-graviton phase corresponds to the transition to the true vacuum (the minimum of the potential) \([18, 19]\). In those works the SSB scale induces (naturally) a fundamental mass scale originated from the cosmological constant and gives through the NL/L SUSY relation a simple explanation of the mysterious (observed) numerical relation between the (four dimensional) dark energy density of the universe and the neutrino mass \([17, 18]\) in the vacuum of the \(N = 2\) SUSY QED theory (in two-dimensional space-time \(d = 2\) for simplicity)
(Note that the $N = 2$ SUSY gives the minimal and realistic model [11] in the SGM scenario.) These are particle physics consequences of NLSUSY GR, i.e. the relation between the large scale structure of space-time and the low energy particle physics.

In the above SGM scenario (the Big Decay process from NLSUSY GR), it is also interesting and crucial problem how the (bare) gauge coupling constant of the SUSY QED theory is determined or whether it is calculable or not, provided the fundamental theory is that of everything. We focus on this problem in this paper by studying the NL/L SUSY relation for the $N = 2$ SUSY QED theory in $d = 2$ with a general structure of auxiliary fields from more general scheme [20, 21]. The relation between the $N = 2$ NLSUSY model and the $N = 2$ SUSY QED ($U(1)$ gauge) theory has been shown in $d = 2$ [14, 16] under the adoption of the simplest SUSY invariant constraints and the subsequent SUSY invariant relations [13, 15]. The SUSY invariant relations, which are obtained systematically from the SUSY invariant constraints [8, 10, 13] in the superfield formulation [22], describe all component fields in the LSUSY multiplets as the composite eigenstates in terms of the NG-fermion superons.

In this paper we study for simplicity and without loss of generality the $N = 2$ SUSY QED, which is physically minimal case, theory in $d = 2$ by linearizing $N = 2$ NLSUSY under general SUSY invariant constraints which induce the subsequent general SUSY invariant relations. In the general NL/L SUSY relation (i.e. the overall SUSY compositness condition) we find a remarkable mechanism which determines the magnitude of the bare gauge coupling constant from vacuum expectation values (vevs) (constant terms in SUSY invariant relations) of auxiliary scalar fields. We show explicitly in $d = 2$ that the NL/L SUSY relation determines the magnitude of the bare electromagnetic coupling constant (i.e. the fine structure constant) of $N = 2$ SUSY QED.

This paper is organized as follows. In Section 2 we present a $N = 2$ general SUSY QED gauge action in $d = 2$ both in the superfield formulation and in the component form. In Section 3 we show in the linearization of $N = 2$ NLSUSY in $d = 2$ the SUSY invariant constraints and the SUSY invariant relations in the most general form. After some reductions of those constraints and relations to more simple (but nontrivial and general) expressions in Section 3 (for simplicity of arguments), the relation between the $N = 2$ NLSUSY action and the $N = 2$ SUSY QED ($U(1)$ gauge) action is discussed in Section 4, where the gauge coupling constant depends
on the vevs (constant terms in SUSY invariant relations) of auxiliary scalar fields. Summary and discussion are given in Section 5.

2 N = 2 SUSY QED action in d = 2

Let us first introduce the superfield formulation of N = 2 SUSY QED theory in d = 2, in which a N = 2 general SUSY QED action is constructed from N = 2 general gauge and N = 2 scalar matter superfields on superspace coordinates (x\(^a\), \(\theta\_a\)) \((i = 1, 2)\). The d = 2, N = 2 general gauge superfield \([23, 24]\) is defined by

\[
\mathcal{V}(x, \theta) = C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \theta^j M^{ji}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \phi(x)
\]

where among component fields \(\varphi^i(x) = \{C(x), \Lambda^i(x), M^{ij}(x), \ldots\}\) in the superfield (2.1), we denote \((C, D)\) for two scalar fields, \((\Lambda^i, \lambda^i)\) for two doublet (Majorana) spinor fields, \(\phi\) for a pseudo scalar field, \(\nu^a\) for a vector field, and \(M^{ij} = M^{(ij)} = \frac{1}{2}(M^{ij} + M^{ji})\) for three scalar fields \((M^{ii} = \delta^{ij} M^{ij})\). The d = 2, N = 2 scalar superfields are expressed as

\[
\Phi^i(x, \theta) = B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij} \bar{\theta}^j \nu(x) - \frac{1}{2} \bar{\theta}^i \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i \bar{\theta}^i \partial B^i(x) \theta^j
\]

where among component fields \(\varphi^i(x) = \{B^i(x), \chi^i(x), \nu^i(x), F^i(x)\}\) in the superfield (2.2), we denote \(B^i\) for doublet scalar fields, \((\chi, \nu)\) for two (Majorana) spinor fields and \(F^i\) for doublet auxiliary scalar fields. The supertransformations of the gauge and scalar superfields with constant (Majorana) spinor parameters \(\zeta^i\) are given as

\[
\delta_\zeta \mathcal{V}(x, \theta) = \bar{\zeta}^i Q^i \mathcal{V}(x, \theta), \quad \delta_\zeta \Phi^i(x, \theta) = \bar{\zeta}^i Q^i \Phi^i(x, \theta),
\]

where \(Q^i = \frac{\partial}{\partial \theta^i} + i \partial \theta^i\) are the generators of LSUSY, which determine LSUSY transformations of the component fields in the power series expansion with respect to \(\theta^i\).

The general N = 2 SUSY QED gauge action with SSB is written in terms of the general gauge and the scalar matter superfields (2.1) and (2.2) (for the massless case) as

\[
L_{N=2\text{SUSYQED}}^{\text{gen.}} = L_{\text{kin}} + L_{\text{FI}} + L_{\Phi\text{kin}} + L_e
\]
with

$$L_{\text{V kin}} = \frac{1}{32} \left\{ \int d^2 \theta^i \left( D^i \bar{W}^{jk} D^j \bar{W}^{i \bar{k}} + D^i \bar{W}^{i \bar{k}} D^j \bar{W}^{j \bar{k}} \right) \right\}_{\theta^i = 0},$$

(2.5)

$$L_{\text{V FI}} = \frac{1}{2} \int d^4 \theta^i \frac{\xi}{\kappa} \mathcal{V},$$

(2.6)

$$L_{\Phi \text{kin}} + L_e = -\frac{1}{16} \int d^4 \theta^i e^{-4e\mathcal{V}(\Phi^i)^2},$$

(2.7)

where $L_{\text{V kin}}$, $L_{\text{V FI}}$, $L_{\Phi \text{kin}}$ and $L_e$ are the kinetic terms for the vector supermultiplet, the Fayet-Iliopoulos (FI) $D$ term, the kinetic terms for the scalar matter supermultiplet and the gauge interaction terms, respectively. In Eq.(2.5) $W^{ij}$ and $W^{ij}_5$ are scalar and pseudo-scalar superfields defined by

$$W^{ij} = \bar{D}^i D^j V,$$

$$W^{ij}_5 = \bar{D}^i \gamma^5 D^j V$$

(2.8)

with the differential operators $D^i = \frac{\partial}{\partial \bar{\theta}^i} - i \partial \theta^i$. In Eq.(2.6) $\xi$ is an arbitrary dimensionless parameter and $\kappa$ is a constant with the dimension (mass)$^{-1}$, while in Eq.(2.7) $e$ is a gauge coupling constant whose dimension is (mass)$^1$ in $d = 2$.

The explicit component form of the $N = 2$ SUSY QED action (2.4), i.e. the actions from (2.5) to (2.7), is

$$L_{\text{V kin}} = -\frac{1}{4} (F_{0ab})^2 + \frac{i}{2} \bar{\lambda}_0^i \partial \lambda^i_0 + \frac{1}{2} (\partial_a A_0)^2 + \frac{1}{2} (\partial_a \phi_0)^2 + \frac{1}{2} D_0^2 \equiv L_{\text{V kin}}^0,$$

(2.9)

$$L_{\text{V FI}} = -\frac{\xi}{\kappa} (D_0 - \Box C) \equiv L_{\text{V FI}}^0 + \frac{\xi}{\kappa} \Box C,$$

(2.10)

$$L_{\Phi \text{kin}} = \frac{i}{2} \bar{\lambda}_0 \partial \lambda^i_0 + \frac{1}{2} (\partial_a B^i)^2 + \frac{i}{2} \bar{\nu} \partial \nu + \frac{1}{2} \mathcal{F}^2 - \frac{1}{4} \partial_a (B^i \partial^a B^i)$$

$$\equiv \frac{1}{4} \partial_a (B^i \partial^a B^i),$$

(2.11)

$$L_e = e \left\{ iv_0 \bar{\lambda}^\alpha \nu - e^{ij} v_0^a B^i \partial_a B^j + \bar{\lambda}_0^i \lambda^i_0 B^i + e^{ij} \bar{\lambda}_0^i \nu B^j - \frac{1}{2} D_0 (B^i)^2 \right.$$

$$+ \frac{1}{2} A_0 (\bar{\lambda} \nu + \bar{\nu} \lambda) - \phi_0 \bar{\lambda} \gamma_5 \nu + \cdots \right\}$$

$$+ \frac{1}{2} e^2 \left\{ (v_0^2 - A_0^2 - \phi_0^2) (B^i)^2 + \cdots \right\} + \cdots,$$

$$\equiv L_e^0 + \cdots,$$

(2.12)

where gauge invariant quantities [15, 22] are denoted by

$$\{ A_0, \phi_0, F_{0ab}, \lambda^i_0, D_0 \} \equiv \{ M^{ii}, \phi, F_{ab}, \lambda^i + i \phi \Lambda^i, D + \Box C \}$$

(2.13)
with $F_{0ab} = \partial_a v_{0b} - \partial_b v_{0a}$ and $F_{ab} = \partial_a v_b - \partial_b v_a$, which are invariant ($v_{0a} = v_a$ transforms as an Abelian gauge field) under a SUSY generalized gauge transformation, $\delta_g V = \Lambda^1 + \alpha \Lambda^2$ [22, 24], with an arbitrary real parameter $\alpha$ and generalized gauge parameters $\Lambda^i$ in the form of the $N = 2$ scalar superfields. The component fields $\varphi^I_{V0} = \{A_0, \phi_0, v_{0a}, \lambda^i_0, D_0\}$ in Eq.(2.13) correspond to the degrees of freedom (d.o.f.) for a minimal off-shell vector supermultiplet.

In Eqs. from (2.9) to (2.12) $L^0_{\text{kin}}, L^0_{\text{FI}}, L^0_{\Phi \text{kin}}$ and $L^0_e$ are defined as the actions which are expressed in terms of only the component fields $\varphi^I_{V0}$ and $\varphi^I_{\Phi}$, while the ellipses in Eq.(2.12) mean the terms depending explicitly on the redundant auxiliary fields $\{C, \Lambda, M^{ij}(i \neq j)\}$ in the general gauge superfield and higher order terms of $e^n$ ($n \geq 3$). The $N = 2$ SUSY QED action (2.4) in the Wess-Zumino (WZ) gauge [22, 24] gives the minimal action for the minimal off-shell vector supermultiplet with the arbitrary $e$.

3 Linearization of $N = 2$ NLSUSY in $d = 2$

In order to discuss the general NL/L SUSY relation for the $N = 2$ SUSY QED theory in $d = 2$, let us show in the linearization of $N = 2$ NLSUSY in $d = 2$ the SUSY invariant constraints and the subsequent SUSY invariant relations in the most general form. They are obtained by introducing NG fermions $\psi^i$ under NLSUSY transformations [4],

$$\delta_\zeta \psi^i = \frac{1}{\kappa} \zeta^i - i \kappa \bar{\psi}^j \gamma^a \psi^i \partial_a \psi^i .$$  \hspace{1cm} (3.1)

The $N = 2$ NLSUSY transformations (3.1) make the following action invariant; namely, the $N = 2$ NLSUSY action is

$$L_{N=2\text{NLSUSY}} = - \frac{1}{2\kappa^2} |w| ,$$  \hspace{1cm} (3.2)

where $|w|$ is the determinant [4] describing the dynamics of (massless) $\psi^i$, i.e. in $d = 2$,

$$|w| = \det(w^a_{\ b}) = \det(\delta^a_{\ b} + t^a_{\ b}) = 1 + t^a_{\ a} + \frac{1}{2!}(t^a_{\ a} t^b_{\ b} - t^a_{\ b} t^b_{\ a})$$  \hspace{1cm} (3.3)

with $t^a_{\ b} = - i \kappa^2 \bar{\psi}^j \gamma^a \partial_b \psi^i$ ($\kappa^{-2} \sim \frac{\Lambda}{G}$ in NLSUSY GR [3]).

The SUSY invariant relations which describe all the component fields in the $N = 2$ SUSY QED theory in terms of the NG fermions $\psi^i$ are systematically obtained
by considering the superfields on specific superspace coordinates \([8, 10]\) shifted with a parameter \(\zeta^i = -\kappa \psi^i\), which are denoted by \((x^a, \theta^\alpha)^i\),

\[
x^a = x^a + i\kappa \bar{\theta}^i \gamma^a \psi^i,
\theta^i = \theta^i - \kappa \psi^i.
\] (3.4)

Indeed, we define the \(N = 2\) general gauge and the \(N = 2\) scalar matter superfields on \((x^a, \theta^\alpha)^i\) as

\[
\tilde{V}(x, \theta, \psi(x)) \equiv \tilde{\nu}(x, \theta, \psi(x)), \quad \Phi^i(x, \theta, \psi(x)) \equiv \tilde{\phi}^i(x, \theta, \psi(x)),
\] (3.5)

and their expansions around \((x^a, \theta^\alpha)^i\) which terminate at \(\mathcal{O}(\theta^4)\) are

\[
\tilde{V}(x, \theta, \psi(x)) = \tilde{C}(x) + \bar{\theta}^i \tilde{\lambda}^i(x) + \frac{1}{2} \bar{\theta}^i \bar{\theta}^j \tilde{M}^{ij}(x) - \frac{1}{2} \bar{\theta}^i \bar{\theta}^j \tilde{M}^{ji}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \tilde{\phi}(x)
- \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j \tilde{\nu}(x) - \frac{1}{2} \bar{\theta}^i \bar{\theta}^j \tilde{\lambda}^i(x) - \frac{1}{8} \bar{\theta}^i \bar{\theta}^j \theta^i \bar{\theta}^j \tilde{D}(x),
\] (3.6)

\[
\tilde{\phi}^i(x, \theta, \psi(x)) = \tilde{B}^i(x) + \bar{\theta}^i \tilde{\chi}(x) - \epsilon^{ij} \bar{\theta}^i \tilde{\nu}(x) - \frac{1}{2} \bar{\theta}^i \theta^j \tilde{F}^i(x) + \bar{\theta}^i \theta^j \tilde{F}^j(x) + \cdots.
\] (3.7)

In Eqs.(3.6) and (3.7) the component fields \(\tilde{\varphi}^I_V(x) = \{\tilde{C}(x), \tilde{\lambda}^i(x), \cdots\}\) and \(\tilde{\varphi}^I_\Phi(x) = \{\tilde{B}^i(x), \tilde{\chi}(x), \cdots\}\) are expressed in terms of the component fields \(\varphi^I_V(x)\) and \(\varphi^I_\Phi(x)\) in Eqs.(2.1) and (2.2) and the NG fermions \(\psi^i\) [13, 16].

According to the supertransformations (2.3) and (3.1), the superfields (3.5) transform homogeneously \([8, 10]\) as

\[
\delta_\zeta \tilde{V}(x, \theta, \psi(x)) = \xi^a \partial_a \tilde{V}(x, \theta, \psi(x)), \quad \delta_\zeta \tilde{\phi}^i(x, \theta, \psi(x)) = \xi^a \partial_a \tilde{\phi}^i(x, \theta, \psi(x))
\] (3.8)

with \(\xi^a = i\kappa \bar{\psi}^i \gamma^a \zeta^i\), which mean the components \(\tilde{\varphi}^I_V(x)\) and \(\tilde{\varphi}^I_\Phi(x)\) do not transform each other, respectively. Therefore, the following conditions, i.e. the SUSY invariant constraints eliminating the other d.o.f. than \(\varphi^I_V(x), \varphi^I_\Phi(x)\) and \(\psi^i\), can be imposed,

\[
\tilde{\varphi}^I_V(x) = \text{constant},
\] (3.9)

\[
\tilde{\varphi}^I_\Phi(x) = \text{constant},
\] (3.10)

which are invariant (conserved quantities) under the supertransformations (2.3) and (3.1).
The constraints (3.9) and (3.10) are written in the most general form as follows;

\[
\tilde{C} = \xi, \quad \tilde{N} = \xi, \quad \tilde{M} = \xi, \quad \tilde{\phi} = \xi, \quad \tilde{v} = \xi,
\]

\[
\tilde{D} = \xi, \quad (3.11)
\]

\[
\tilde{B} = \xi, \quad \tilde{\chi} = \xi, \quad \tilde{\nu} = \xi,
\]

\[
\tilde{F} = \xi, \quad (3.12)
\]

where the mass dimensions of constants (or constant spinors) in \( d = 2 \) are defined by \((-1, 1/2, 0, 0, -1/2)\) for \( (\xi, \xi, \xi, \xi, \xi) \), \((0, -1/2, -1/2)\) for \( (\xi, \xi, \xi, \xi) \) and 0 for \( \xi \) for convenience. The general SUSY invariant constraints (3.11) and (3.12) can be solved with respect to the component fields \( \varphi_\lambda \) and \( \varphi_\phi \) in terms of \( \psi^i \); namely, the SUSY invariant relations \( \varphi_\psi = \varphi_\psi(\psi) \) are calculated systematically and straightforwardly as

\[
C = \xi + k \psi^i \xi^i + i/2 k^2 \xi^i \psi^j \psi^j \xi + 1/4 \xi^i \psi^j \gamma_5 \psi^j - i/4 \xi^i \psi^j \gamma_5 \psi^j,  
\]

\[
\Lambda^i = \xi^i + k \psi^j \psi^i \psi^j \xi + \xi^i - k \psi^j \psi^i \psi^j \xi + 1/2 \xi^i \psi^j \gamma_5 \psi^j - i/2 \xi^i \psi^j \gamma_5 \psi^j,  
\]

\[
M = \xi^i + k \psi^j \psi^i \psi^j \xi + \xi^i - k \psi^j \psi^i \psi^j \xi + 1/2 \xi^i \psi^j \gamma_5 \psi^j - i/2 \xi^i \psi^j \gamma_5 \psi^j,  
\]

\[
\phi = \xi^i - i \psi^j \psi^i \psi^j \xi - i/2 \xi^i \psi^j \gamma_5 \psi^j - \psi^j \psi^i \psi^j \xi^i - i/2 \xi^i \psi^j \gamma_5 \psi^j,  
\]

\[
v^a = \xi^a - i \psi^j \psi^i \psi^j \xi^a - i/2 \xi^i \psi^j \gamma_5 \psi^j - \psi^j \psi^i \psi^j \xi^a,  
\]

\[
\lambda^i = \xi^i + k \psi^j \psi^i \psi^j \xi + 1/2 k^2 \psi^j \psi^j \psi^i \psi^i \xi^i \psi^i,  
\]

\[
D = \xi + k \psi^j \psi^i \psi^j \psi^i \xi^i \psi^i,  
\]

where \( \gamma_5 \) is the Dirac gamma matrix in \( d = 2 \).
while the SUSY invariant relations \( \varphi^l_\phi = \varphi^l_\phi(\psi) \) are

\[
B^i = \xi^i_B + \kappa (\bar{\psi}^j \xi_\chi - e^{ij} \bar{\psi}^i \nu_\psi) - \frac{1}{2} \kappa^2 \{ \bar{\psi}^j \psi^i F^i(\psi) - 2 \bar{\psi}^i \psi^j F^j(\psi) + 2 i \bar{\psi}^i \partial B^j(\psi) \psi^j \}
- i \kappa^3 \bar{\psi}^j \psi^i \{ \bar{\psi}^i \partial \chi(\psi) - e^{ik} \bar{\psi}^k \partial \nu(\psi) - \gamma^a \bar{\psi}^i \bar{\psi}^j \partial_a \nu(\psi) \} + \frac{3}{8} \kappa^4 \bar{\psi}^j \psi^i \psi^k \chi B^i(\psi),
\]

\[
\chi = \xi_\chi + \kappa \{ \bar{\psi}^i F^i(\psi) - i \partial B^i(\psi) \psi^i \}
- \frac{i}{2} \kappa^2 [ \partial \chi(\psi) \bar{\psi}^i \psi^i - e^{ij} \{ \bar{\psi}^i \bar{\psi}^j \partial \nu(\psi) - \gamma^a \bar{\psi}^i \bar{\psi}^j \partial_a \nu(\psi) \} ]
+ \frac{1}{2} \kappa^3 \psi^i \bar{\psi}^j \partial B^i(\psi) + i \frac{1}{2} \kappa^3 \psi^i F^i(\psi) \bar{\psi}^j \psi^j + \frac{1}{8} \kappa^4 \psi^i \bar{\psi}^j \bar{\psi}^k \psi^k \partial B^i(\psi),
\]

\[
\nu = \xi_\nu - i \kappa^{ij} \{ \bar{\psi}^i F^j(\psi) - i \partial B^i(\psi) \psi^j \}
- i \kappa^2 [ \partial \nu(\psi) \bar{\psi}^i \psi^i + e^{ij} \{ \bar{\psi}^i \bar{\psi}^j \partial \chi(\psi) - \gamma^a \bar{\psi}^i \bar{\psi}^j \partial_a \chi(\psi) \} ]
+ \frac{1}{2} \kappa^3 e^{ij} \psi^i \bar{\psi}^j \psi^k \bar{\psi}^k \partial B^i(\psi) + i \frac{1}{2} \kappa^3 \psi^i F^i(\psi) \bar{\psi}^j \psi^j \psi^k + \frac{1}{8} \kappa^4 \psi^i \bar{\psi}^j \bar{\psi}^k \psi^k \partial F^i(\psi).
\]

For simplicity of arguments in NL/L SUSY relation, we reduce the above SUSY invariant constraints and the SUSY invariant relations, i.e. the massless eigenstates in terms of \( \psi^i \), to more simple (but nontrivial and general) expressions. Since in Eqs.(3.13) and (3.14) the constants (the vevs) which do not couple to \( \psi^i \) are only \( \xi_e \) and \( \xi_B \), we put

\[
\xi^i_A = \xi^i_M = \xi^i_\phi = \xi^i_\nu = \xi^i_\lambda = 0, \quad \xi_\chi = \xi_\nu = 0.
\]

except for \( \xi \) and \( \xi^i \) which are the fundamental constants in the simplest and nontrivial NL/L SUSY relation for the \( N = 2 \) SUSY QED theory with the SSB [14, 16]. Further we put

\[
\xi^i_B = 0,
\]

because we would like to attribute straightforwardly the \( N = 2 \) SUSY QED action (2.4) to the \( N = 2 \) NLSUSY action (3.2) up to a normalization factor when the
SUSY invariant relations are substituted into Eq.(2.4). Then, the SUSY invariant constraints (3.11) and (3.12) become

\[
\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{\nu}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad (3.17)
\]

and the SUSY invariant relations (3.13) and (3.14) reduce to

\[
C = \xi_c - \frac{1}{8} \xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j |w|,
\]

\[
\Lambda^i = -\frac{1}{2} \xi \kappa^2 \bar{\psi}^i \bar{\psi}^j \psi^j |w|,
\]

\[
M^{ij} = \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^j |w|,
\]

\[
\phi = -\frac{1}{2} \xi \kappa \epsilon^{ijkl} \bar{\psi}^i \gamma_5 \psi^j |w|,
\]

\[
\nu^a = -\frac{i}{2} \xi \kappa \epsilon^{ijkl} \bar{\psi}^i \gamma^a \psi^j |w|,
\]

\[
\chi = \xi \psi^i |w|,
\]

\[
D = \frac{\xi}{\kappa} |w|, \quad (3.19)
\]

and

\[
\chi = \xi^i \left[ \psi^i |w| + \frac{1}{2} \kappa^2 \partial_a (\gamma^a \psi^i \bar{\psi}^j \psi^j |w|) \right],
\]

\[
B^i = -\kappa \left( \frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|,
\]

\[
\nu = \xi^i \epsilon^{ij} \left[ \psi^j |w| + \frac{i}{2} \kappa^2 \partial_a (\gamma^a \bar{\psi}^j \bar{\psi}^k \psi^k |w|) \right],
\]

\[
F^i = \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \Box (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\}
\]

\[
- \imath \kappa \xi^i \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|), \quad (3.20)
\]

which are written in the form containing some vanishing terms due to \((\psi^i)^5 \equiv 0\).

4 NL/L SUSY relation for \(N = 2\) SUSY QED in \(d = 2\)

In this section we discuss the relation between the \(N = 2\) SUSY QED action (2.4) and the \(N = 2\) NLSUSY action (3.2). Substituting the reduced (but general) SUSY
invariant relations (3.19) and (3.20) into Eqs.(2.5), (2.6) and (2.7) gives the relations among the actions as follows;

\[ L_{\text{V kin}}(\psi) = -\xi^2 L_{N=2\text{NLSUSY}}, \]
\[ L_{\text{V FI}}(\psi) = 2\xi^2 L_{N=2\text{NLSUSY}}, \]
\[ (L_{\Phi kin} + L_e)(\psi) = -\left(\xi^i\right)^2 e^{-4e\xi_c} L_{N=2\text{NLSUSY}}. \]

(4.1)

These results can be obtained systematically by changing the integration variables in the actions (2.5), (2.6) and (2.7) from \((x, \theta^i)\) to \((x', \theta'^i)\) under the SUSY invariant constraints (3.17) and (3.18) (see, for example, [13]). Therefore, from Eq.(4.1) we obtain a general NL/L SUSY relation for the \(N = 2\) SUSY QED theory in \(d = 2\) as

\[ f(\xi, \xi^i, \xi_c, e) L_{N=2\text{NLSUSY}} = L_{N=2\text{SUSYQED}}^{\text{gen.}} \]

(4.2)

with a normalization factor \(f(\xi, \xi^i, \xi_c, e)\) defined by

\[ f(\xi, \xi^i, \xi_c, e) = \xi^2 - \left(\xi^i\right)^2 e^{-4e\xi_c}. \]

(4.3)

By regarding the NLSUSY GR theory in the SGM scenario as the fundamental theory of space-time and matter, \(L_{N=2\text{SUSYQED}}^{\text{gen.}}\) is attributed to \(L_{N=2\text{NLSUSY}}\) which is the cosmological term of SGM (NLSUSY GR) action for flat space-time, i.e. we put

\[ f(\xi, \xi^i, \xi_c, e) = 1. \]

(4.4)

Remarkably, the condition (4.4) gives the gauge coupling constant \(e\) in terms of \(\xi\), \(\xi^i\) and \(\xi_c\) as

\[ e = \frac{1}{4\xi_c} \ln X, \quad X = \frac{\left(\xi^i\right)^2}{\xi^2 - 1}. \]

(4.5)

Finally we just mention the relation between the general \(N = 2\) SUSY QED action (2.4) and the minimal one for the minimal off-shell vector supermultiplet in the NL/L SUSY relation (4.2) with the normalization condition (4.4). The minimal actions \(L_{\text{V kin}}^0, L_{\text{V FI}}^0, L_{\Phi kin}^0\) and \(L_e^0\) defined in Eqs. from (2.9) to (2.12) is related to the general actions of Eq.(4.1) for the \(N = 2\) SUSY QED theory in NL/L SUSY relation as

\[ L_{\text{V kin}}(\psi) = L_{\text{V kin}}^0(\psi) = -\xi^2 L_{N=2\text{NLSUSY}}, \]
\[ L_{\text{VF1}}(\psi) = L^0_{\text{VF1}}(\psi) + \text{[tot. der. terms]} = 2\xi^2 L_{\text{N=2NLSUSY}}, \]
\[(L_{\Phi\text{kin}} + L_e)(\psi) = (e^{-4e\xi_e} L^0_{\Phi\text{kin}}|_F - F') + L^0_e(\psi) + \text{[tot. der. terms]} = -(\xi^i)^2 e^{-4e\xi_e} L_{\text{N=2NLSUSY}}, \]

where \( L^0_e(\psi) = \frac{1}{4} e\kappa \xi (\xi^i)^2 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \) and the SUSY invariant relations of the auxiliary fields \( F^i \) in Eq.(3.20) have been changed (relaxed) by four NG-fermion self-interaction terms as

\[ F'^i(\psi) = F^i(\psi) - \frac{1}{4} e^{-4e\xi_e} e\kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k. \]

Obviously, the minimal \( N = 2 \) SUSY QED action for the minimal off-shell vector supermultiplet is included in the relations (4.6) at the leading order of the factor \( e^{-4e\xi_e} \).

It can be seen easily that the numerical factor \( e^{-4e\xi_e} \) in the relation (4.6) is absorbed into the action by rescaling the whole scalar supermultiplet \( \Phi' = \{ B^i, \chi^i, \nu^i, F^i \} \) in the scalar superfields \( \Phi^i \) by \( e^{-2e\xi_e} \) and by translating the auxiliary field \( C \) by \( \xi_c \) in the gauge action (2.7) as

\[ \phi'^i(\psi) \to \hat{\phi}'^i(\psi) = e^{-2e\xi_e} \phi'^i(\psi), \]
\[ C(\psi) \to \hat{C}(\psi) = -\frac{1}{8} e\kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|. \]

Indeed, in NL/L SUSY relation the actions (2.5), (2.6) and (2.7) in terms of the component fields \( \{ \phi'^i, \hat{C} \} \) in addition to \( \phi'^0 = \hat{\phi}^0_0 \) in Eq.(2.13) become

\[ L_{\text{Vkin}}(\psi) = \hat{L}_{\text{Vkin}}(\psi) = \hat{L}^0_{\text{Vkin}}(\psi) = -\xi^2 L_{\text{N=2NLSUSY}}, \]
\[ L_{\text{VF1}}(\psi) = \hat{L}_{\text{VF1}}(\psi) = \hat{L}^0_{\text{VF1}}(\psi) + \text{[tot. der. terms]} = 2\xi^2 L_{\text{N=2NLSUSY}}, \]
\[(L_{\Phi\text{kin}} + L_e)(\psi) = (\hat{L}_{\Phi\text{kin}} + \hat{L}_e)(\psi)
= (\hat{L}^0_{\Phi\text{kin}}|_F - \hat{F}) + \hat{L}^0_e(\psi) + \text{[tot. der. terms]}
= -(\xi^i)^2 e^{-4e\xi_e} L_{\text{N=2NLSUSY}}, \]

where \( \hat{L}^0_e(\psi) = \frac{1}{4} e\kappa \xi (\xi^i)^2 e^{-4e\xi_e} \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \) and

\[ \hat{F}^i(\psi) = e^{-2e\xi_e} \left\{ F^i(\psi) - \frac{1}{4} e\kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \right\}. \]
Therefore, we obtain the ordinary $N = 2$ SUSY QED $U(1)$ gauge action for the minimal off-shell vector supermultiplet with the $U(1)$ gauge coupling constant (4.5), i.e.

$$L_{\text{N=2NLSUSY}} = L_{\text{gen.}}^{\text{N=2SUSYQED}} = \hat{L}_{\text{N=2SUSYQED}}^0 + [\text{tot. der. terms}],$$

where the minimal $N = 2$ SUSY QED $U(1)$ gauge action $\hat{L}_{\text{N=2SUSYQED}}^0$ is defined by

$$\hat{L}_{\text{N=2SUSYQED}}^0 = \hat{L}_{\text{Vkin}}^0 + \hat{L}_{\text{VFI}}^0 + \hat{L}_{\text{kin}|\hat{F} \rightarrow \hat{F}'}^0 + \hat{L}_e^0.$$

Interestingly $e$ defined by the action (2.7) depends upon the vevs of the auxiliary fields, i.e. the vacuum structures. Note that the bare $e$ is a free independent parameter, provided $\xi_c = 0$ as in the case of adopting the WZ gauge throughout the arguments.

5 Summary and discussion

In this paper we have studied the NL/L SUSY relation for the $N = 2$ SUSY QED theory in $d = 2$ starting from the most general SUSY invariant constraints (3.11) and (3.12) and the subsequent SUSY invariant relations (3.13) and (3.14). After reducing those constraints and relations to more simple (but nontrivial and general) expressions of Eqs. from (3.17) to (3.20), we have obtained the general NL/L SUSY relation (4.2) for the general $N = 2$ SUSY QED gauge action (2.4), which produces the normalization factor (4.3) depending on the gauge coupling constant $e$. The fundamental notions of the NLSUSY GR theory in SGM scenario gives the normalization condition (4.4) and the relation between $e$ and the vevs (the constant terms in SUSY invariant relations) of the auxiliary scalar fields, $\xi$, $\xi^i$ and $\xi_c$ as in Eq.(4.5).

The minimal $N = 2$ LSUSY QED $U(1)$ gauge action (4.12) has been also obtained from the general $N = 2$ SUSY QED action (2.4) related to the $N = 2$ NLSUSY action (3.2) as shown in the NL/L SUSY relation (4.11). This has been achieved substantially by adopting the WZ gauge for LSUSY which can gauge away all auxiliary fields except $D_0$ in the vector supermultiplet and by rescaling the whole scalar supermultiplet in the gauge action (2.7) by the constant numerical factor $e^{-2e\xi_c}$. However, the bare $U(1)$ gauge coupling constant $e$ in the minimal action (4.12) is still defined as Eq.(4.5) in terms of $\xi$, $\xi^i$ and $\xi_c$. 

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The NLSUSY GR action $L_{\text{NLSUSYGR}}(w)$ (in terms of a unified vierbein $w^a_{\mu}$) [3] describes geometrically the basic principle, i.e. the ultimate shape of nature is empty unstable space-time with the constant energy density (cosmological term) $\Lambda > 0$ and decays to (creates) spontaneously (quantum mechanically) ordinary Riemann space-time and spin-$\frac{1}{2}$ massless NG fermion superons matter with the potential (cosmological term) $V = \Lambda > 0$ depicted by the SGM action $L_{\text{SGM}}(e, \psi)$ (in terms of the ordinary vierbein $e^a_{\mu}$ and the NG fermions $\psi^i$). We showed explicitly in asymptotic Riemann-flat ($e^a_{\mu} \to \delta^a_{\mu}$) space-time that the vacuum (the true minimum) of $L_{\text{SGM}}(e, \psi)$ is $V = 0$, which is achieved when all local fields of the LSUSY multiplet of the familiar LSUSY gauge theory $L_{\text{LSUSY}}(A, \lambda, v_a, \ldots)$ are composed of NG-fermion superons dictated by the space-time $N$-extended NLSUSY symmetry, i.e. LSUSY is realized on the true vacuum as the composite eigenstates of SP symmetry.

We can interpret the SM ($SU(5)$ GUT) in terms of superon picture of all particles and we obtain new remarkable insights into the proton decay (stable), neutrino oscillations, the origins of various mixing, CP-violation phase, etc. [25] by replacing the single line of the propagator of a particle in the Feynman diagrams of SM ($SU(5)$ GUT) by the multiple lines of superons.

Our study may indicate that the general structure of the auxiliary fields for the general (gauge) superfield plays a crucial role in SUSY theory by determining not only the true vacuum through the SSB due to $D$ term but also the magnitude of the (bare) gauge coupling constant through the NL/L SUSY relation (i.e. the SUSY compositeness condition for all particles and the auxiliary fields as well), which is favorable to the SGM scenario for unity of nature. The strength of the bare gauge coupling constant may ought to be predicted, provided the fundamental theory is that of everything. The similar arguments in $d = 4$ for more general SUSY invariant constraints and for the large $N$ SUSY, especially $N = 4, 5$ are interesting and crucial.
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