An efficient estimator for source localization using TD and AOA measurements in MIMO radar systems

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Abstract—By utilizing time delay (TD) and angle of arrival (AOA) measurements, three dimensional (3D) target localization based on multi-input and multi-output (MIMO) radar systems is studied in this letter. For such a positioning problem, the typical closed-form solution firstly establishes pseudo-linear equation and then applies the weighted least squares (WLS) to determine the final position. However, the WLS solution often suffers performance degradation when the noise is large. To alleviate this problem, the cost function of the WLS is re-represented and a quadratic constraint is imposed such that the expectation of the cost function can attain the minimum value at the true position. Moreover, simulation experiments show that the proposed method performs better than the state-of-art algorithms.

Index Terms—multi-input and multi-output (MIMO) radar, closed-form solution, target localization, expectation of the cost function, quadratic constraint.

1. Introduction

Research on multiple-input multiple-output (MIMO) systems has been drawing more and more attention from both the communication and radar communities [1-4]. Especially, MIMO radar systems with distributed antennas [5] can provide spatial diversity, leading to improve localization and detection performance. Due to this advantage, target positioning based on multiple-input and multiple-output (MIMO) radar systems with
distributed antennas has attracted considerable attention in recent years.

Localization methods [6-13] in MIMO systems can be attributed to use different measurements, including time delay (TD), bistatic range (BR), angle of arrival (AOA), and their combinations. The doppler shift (DS) and bistatic range rate (BRR) can be also utilized when the source and/or receivers are moving [14-17]. TD-based one is mainly the most extensively used technique due to its high accuracy and simplicity in a MIMO localization systems. A BR measurement converted by Each TD measurement induces an ellipsoidal locus, on which the target is located, with the associated transmitter and receiver as its foci. Intersection of the ellipsoids from multiple bistatic pairs (transmitter-receiver) gives the estimated target position. Pioneers have researched many methods of the target positioning in MIMO radar systems. Similar to the hyperbolic location [18-20], the maximum likelihood (ML) estimator corresponding to elliptic localization leads to a nonlinear and nonconvex optimization problem, whose global solution is difficult to obtain. There are some exist iterative algorithms, such as [6-7] directly solving the ML problem, but they need burdensome computation and an initial point sufficiently close to the actual target position to ensure convergence. To overcome these disadvantages, the closed-form solutions have been developed. The measurements were first divided the into several groups based on the different transmitter or receivers in [8]. Then, it employs two-stage weighted least squares (TSWLS) estimator for each group to independently produce an estimate of target position. Finally, the results from different groups are combined to form a composite solution. In [9] a different WLS algorithm has been proposed which solves the problem in a centralized manner. By introducing nuisance parameters, a pseudo-linear set of BR equations is established in [10]. And the final solution was obtained by using multistage weighted least squares estimation. These closed-form methods can reach the Cramer–Rao lower bound (CRLB) [21] performance under small noise levels. But the closed-form solutions have poor performance when the noise is large. In addition, the localization problem was formulated as a constrained weighted least squares (CWLS) problem, which was a quadratically constrained quadratic programming (QCQP) problem, and the QCQP problem was then converted into the linear constrained quadratic programming (LCQP) problem in [11]. Finally, the LCQP problem was solved iteratively by using
the basic properties of the matrix pseudo-inverse [22]. This technique has the advantage of efficient computation and performs much better than closed-form solutions under the Gaussian noise model.

Compared to the methods using only either BR measurements or TD measurements, hybrid TD-AOA methods perform better for the stationary target positioning. In [12], a unit vector was introduced to eliminate the extra variables and a pseudo-linear equation was established. Then, the problem was solved by WLS. Reference [13] firstly established pseudo-linear equation with nuisance parameters, the second step refined the final solution by estimating the error of the first step to improve the estimated accuracy. Both the two methods can achieve Cramér-Rao lower bound (CRLB) a low noise level. However, the performance degradation occurs to the method [12] at a high noise level due to the presence of measurement noises in both the regressor and regressand. Similarly, the squaring operations of the Second stage in [13] may also lead to increased bias as well as poor positioning accuracy when the noise is large.

In this letter, we propose an explicit solution based on TD and AOA measurements for target localization in MIMO radar systems. To the best of our knowledge, the expectation value of the cost function is not considered in the WLS solution [12] and [13]. To better handle the randomness of the regressor and regressand introduced by noise in the WLS solution, the proposed method aims to minimize the cost function by imposing a quadratic constraint such that the expectation of the cost function will attain its minimum value in the case of the true solution. Numerous simulations confirm the effectiveness of the proposed method.

This paper is mainly organized into the following sections. In Section 2, the methods used in this paper are introduced in general. In Section 3, the localization model is described. In Section 4, we summarize the typical closed-form solution and develop the proposed method. Section 5 theoretically proves that our method achieves the CRLB at a moderate noise level. Simulations conducted to verify the effectiveness of the proposed method are reported in Section 6. Section 8 draws the conclusions.

2. Methods

Owing to the poor performance of the typical closed-form solution under large noise levels, we present a novel closed-form solution using TD and AOA measurements for the target localization in MIMO systems
in this paper. To better handle the randomness of the regressor and regressand in the WLS solution, the proposed method is aiming to minimize the cost function by imposing a quadratic constraint such that the expectation of the cost function will attain its minimum value in the case of the true solution.

We exhibit the performance of the proposed method and compare it with the existing TSWLS methods via numerical Monte Carlo experiment. Two indicators are presented to evaluate the estimated performance, one is the root mean square error (RMSE) and another is estimated bias. Besides, we use the root CRB as a benchmark for performance evaluation.

This paper does not contain any studies with human participants or animals performed by any of the authors.

3. Localization Model

Consider a localization scenario in a 3D space where widely separated MIMO radars are employed to determine a single target at position \( u^o = [x^o, y^o, z^o]^T \). The \( m \)th transmitter is arbitrarily located at coordinates \( t_m = [x_{tm}, y_{tm}, z_{tm}]^T \), with \( m = 1, \ldots, M \) and the \( n \)th receiver is also arbitrarily located at coordinates \( s_n = [x_{sn}, y_{sn}, z_{sn}]^T \), with \( n = 1, \ldots, N \). The coordinates of all transmitters and receivers are precise and available. Besides, the signals obtained directly and indirectly by each receiver can produce one AOA pair and \( M \) TDs.

Let the true AOA pair at the \( n \)th receiver be denoted by \( (\theta_n^o, \phi_n^o) \), where \( \theta_n^o \) represents the azimuth angle and \( \phi_n^o \) denotes the elevation angle. These true angle values are related to the source coordinates and the coordinates of receiver \( n \) through following equations

\[
\tan \theta_n^o = \frac{x^o - x_{sn}}{y^o - y_{sn}}, \tan \phi_n^o = \frac{z^o - z_{sn}}{\sqrt{(y^o - y_{sn})^2 + (x^o - x_{sn})^2}}, \quad n = 1, \ldots, N
\]  

The measured versions of AOA pairs at the \( n \) th receiver are modeled as \( \theta_n = \theta_n^o + \Delta \theta_n \) and \( \phi_n = \phi_n^o + \Delta \phi_n \),
where \( \Delta \theta_n \) and \( \Delta \phi_n \) are the error terms. By combing all AOA measurements introduced above, the AOA measurement vectors can be constructed by

\[
\begin{align*}
\theta &= [\theta_1, \ldots, \theta_N]^T = \theta^o + \Delta \theta \\
\phi &= [\phi_1, \ldots, \phi_N]^T = \phi^o + \Delta \phi
\end{align*}
\]

where \( \Delta \theta \) and \( \Delta \phi \) are assumed to be zero-mean Gaussian noise vectors with covariance matrices

\[
E(\Delta \theta^T \Delta \theta) = Q_\theta \quad \text{and} \quad E(\Delta \phi^T \Delta \phi) = Q_\phi.
\]

After some simple operations, each true TD can be converted into a BR \( r^o_{m,n} \) and it is expressed by

\[
r^o_{m,n} = R^o_s + R^o_t
\]

where \( R^o_s = ||u - s_n|| \) and \( R^o_t = ||u - t_m|| \). Collecting all the true BRs into a matrix form, we have

\[
r^o = [r^o_{1T}, \ldots, r^o_{mT}]^T
\]

where \( r^o_{mT} = [r^o_{m1}, \ldots, r^o_{mN}]^T \).

Similarly to the decomposition of the AOA measurements shown above, the BR measurements vector, defined as \( r = [r^T_1, \ldots, r^T_M]^T \), is contaminated by the additive noise as

\[
r = [r^T_1, \ldots, r^T_M]^T = r^o + \Delta r
\]

where \( \Delta r = [\Delta r^T_1, \ldots, \Delta r^T_M]^T \) is the zero-mean Gaussian measurement errors vector with a covariance matrix

\[
E(\Delta r^T \Delta r) = Q_r. \quad \text{And} \quad \Delta r^o_{mT} = [\Delta r^o_{m1}, \ldots, \Delta r^o_{mN}]^T.
\]

4. Proposed Method

\subsubsection{Typical closed-form solution}

The closed-form solution [12] introduces a unit vector of the actual target position with respect to the \( n \)th receiver for eliminating the extra variables. This unit vector is denoted as

\[
d^o_n = [\cos(\phi^o_n) \cos(\theta^o_n), \cos(\phi^o_n) \sin(\theta^o_n), \sin(\phi^o_n)]^T, \quad n = 1, \ldots, N
\]

In addition, it also gives two vectors orthogonal to \( d^o_n \) through the following equations...
\[ \alpha_n^T d_n^o = 0, \ \beta_n^T d_n^o = 0 \quad (7) \]

where \( \beta_n^o = [\sin(\phi_n^o) \cos(\theta_n^o), -\sin(\phi_n^o) \sin(\theta_n^o), \cos(\phi_n^o)]^T \) and \( \alpha_n^o = [-\sin(\theta_n^o), \cos(\theta_n^o), 0]^T \), with \( n = 1, \ldots, N \).

Thus, a set of linear equations are established below by utilizing (6)-(7) and the details of the derivation can be found in [12].

\[ \mathbf{h} = \mathbf{G} \mathbf{u}^o \quad (8) \]

where \( \mathbf{h} = [\mathbf{h}_r^T, \mathbf{h}_{\text{ang}}^T]^T \) and \( \mathbf{G} = [\mathbf{G}_r^T, \mathbf{G}_{\text{ang}}^T]^T \). \( \mathbf{G}_{\text{ang}} \) and \( \mathbf{h}_{\text{ang}} \) are expressed as

\[
\mathbf{h}_{\text{ang}} = [\alpha_1^o s_1, \ldots, \alpha_N^o s_N, \beta_1^o s_1, \ldots, \beta_N^o s_N]^T \\
\mathbf{G}_{\text{ang}} = [\alpha_1^o, \ldots, \alpha_N^o, \beta_1^o, \ldots, \beta_N^o]^T
\]  

(9)

The \( k \)th entry of \( \mathbf{h}_r \) and the \( k \)th row of \( \mathbf{G}_r \), respectively, are

\[
\begin{align*}
\mathbf{h}_r(k,1) &= r_m^o + \| s_n \|^2 - \| t_m \|^2 + 2r_m^o d_j^o s_n \\
\mathbf{G}_r(k,:) &= 2(s_n - t_m + r_m^o d_j^o)^T
\end{align*}
\]

(10)

where \( k = (m-1)N+n, \ n = 1, \ldots, N \) and \( m = 1, \ldots, M \).

Substituting TD and AOA measurements into (10) yields

\[ \hat{\mathbf{h}} - \hat{\mathbf{G}} \mathbf{u}^o = \mathbf{B} \Delta \mathbf{\eta} \quad (11) \]

where the regressand \( \hat{\mathbf{h}} \) and regressor \( \hat{\mathbf{G}} \) are the same as previously defined in (8) except that true values are replaced by their measurement versions. \( \Delta \mathbf{\eta} = [\Delta r^T, \Delta \phi^T, \Delta \phi^T]^T \) represents the total noise vector with a covariance matrix \( \mathbf{Q}_\eta = E(\Delta \mathbf{\eta}^T \Delta \mathbf{\eta}) \), and \( \mathbf{B} \) is given by

\[ \mathbf{B} = \text{blkdiag}(\mathbf{B}_r, \mathbf{B}_{\text{ang}}) \quad (12) \]

where

\[
\begin{align*}
\mathbf{B}_r &= 2\text{diag}([\mathbf{R}_1^o, \ldots, \mathbf{R}_M^o]^T) \otimes \mathbf{I}_N \\
\mathbf{B}_{\text{ang}} &= \text{diag}([\mathbf{R}_1^o \cos(\phi_1), \ldots, \mathbf{R}_N^o \cos(\phi_N), \mathbf{R}_1^o, \ldots, \mathbf{R}_N^o]^T)
\end{align*}
\]

(13)

and the symbol \( \otimes \) stands for Kronecker product.

The WLS solution of \( \mathbf{u} \) that minimizes the cost function \( J = (\hat{\mathbf{h}} - \hat{\mathbf{G}} \mathbf{u}^o) \mathbf{W}(\hat{\mathbf{h}} - \hat{\mathbf{G}} \mathbf{u}^o) \), where \( \mathbf{W} = (\mathbf{B}^T \mathbf{Q}_\eta \mathbf{B})^{-1} \)
is the positive define weighting matrix, is

\[ u = (\hat{G}^T W G)^{-1} \hat{G}^T W \hat{h} \]  

(14)

Notably, there requires the true target position for calculating the weight matrix \( W \). Method [12] first approximates \( W = Q_{\eta}^{-1} \) and uses (14) to obtain an initial position. This initial value is then used to update \( W \) so that a better estimated position can be found.

b) Proposed method

The closed-form algorithm summarized above can achieve CRLB under a low noise level. However, this solution may suffer from performance degradation at high noise levels due to the presence of the noise disturbances in both \( \hat{h} \) and \( \hat{G} \). Therefore, inspired by the BiasRed introduced in [23], a new approach for improving the estimation accuracy is proposed in this subsection.

The proposed method starts from introducing the augmented matrix \( A = [-\hat{G}, \hat{h}] \) and vector \( v = [u^T, 1]^T \) so that the cost function mentioned above can be rewritten as

\[ J = v^T A^T W A v \]  

(15)

\( A \) can be decomposed as \( A = A^o + \Delta A \), where \( A^o = [-G, h] \) is the true augmented matrix and the noise matrix \( \Delta A \), by subtracting the true value \( A^o \) from \( A \) and considering only the linear noise terms, can be reformulated as

\[ \Delta A = T F \]  

(16)

where \( T = \text{blkdiag}(T_1, T_2) \), \( T_1 \) and \( T_2 \) can be obtained by

\[
\begin{align*}
T_1 &= \begin{bmatrix}
diag(\Delta r_1) & I_M \otimes [diag(\Delta \theta), diag(\Delta \phi)] \\
\vdots & \\
diag(\Delta r_M) & 
\end{bmatrix} \\
T_2 &= \begin{bmatrix}
diag(\Delta \theta) & O_{N \times N} & O_{N \times N} \\
O_{N \times N} & diag(\Delta \theta) & diag(\Delta \phi) 
\end{bmatrix}
\end{align*}
\]  

(17)

and the matrix \( F \) can be decomposed into three parts as follows:
\[ F = [F_1^T, F_2^T, F_3^T]^T \] (18)

such that

\[
F_i = \begin{bmatrix}
-2d_{i1}^o & 2d_{i1}^o s_i \\
\vdots & \vdots \\
-2d_{iN}^o & 2d_{iN}^o s_i \\
-2 \text{diag}([r_i^o, r_i^o]^T)f_i & 2 \text{diag}([r_i^o, r_i^o]^T)f_i \\
\vdots & \vdots \\
-2 \text{diag}([r_M^o, r_M^o]^T)f_i & 2 \text{diag}([r_M^o, r_M^o]^T)f_i
\end{bmatrix}
\] (19)

where

\[
f_i = [f_{i1}^T, f_{i2}^T]^T, \quad f_i = [f_{i13}^T, f_{i23}^T]^T
\] (20)

the \( i \) th row of \( f_2 \) is \( f_2(i,:) = [-\cos \theta_i^o, -\sin \theta_i^o, 0] \), with \( i = 1, \ldots, N \). Besides, the \( i \) th row of \( f_1 \) and \( f_3 \), respectively, are computed by

\[
f_{i1}(i,:) = [-\cos \phi_i^o \sin \theta_i^o, \cos \phi_i^o \cos \theta_i^o, 0]
\]

\[
f_{i2}(i,:) = [-\sin \phi_i^o \cos \theta_i^o, -\sin \phi_i^o \sin \theta_i^o, \cos \phi_i^o]
\]

\[
f_{i3}(i,:) = [-\cos \phi_i^o \cos \theta_i^o, -\cos \phi_i^o \sin \theta_i^o, -\sin \phi_i^o]
\] (21)

and

\[
f_{2s} = [f_{2s}(1,:)s_1, \ldots, f_{2s}(N,:)s_N]^T
\]

\[
f_{3s} = [f_{3s}(1,:)s_1, \ldots, f_{3s}(N,:)s_N]^T
\] (22)

Obviously, substituting \( A = A^o + \Delta A \) into (15), \( J \) can be rewritten by

\[
J = v^T A^o A^o^T W A^o v + 2 v^T \Delta A^T W A^o v + v^T \Delta A^T W \Delta A v
\] (23)

Since the second term on the right side of (23) is a mean of zero, therefore, by taking the expectation of \( J \), we have

\[
E(J) = v^T A^o A^o^T W A^o v + v^T E(\Delta A^T W \Delta A) v
\] (24)
Therefore, $E(J)$ will take its minimum value at the true value $v^\theta$ when the last term on the right side of (24) is constrained to a constant $\kappa$ [23]. Thus, the solution for $v$ can be expressed as

$$
\min_v v^T A^T W A v \quad \text{s.t. } v^T \Omega v = \kappa
$$

(25)

where the constant $\kappa$ can be any positive value since it doesn’t affect the final solution and $\Omega = E(\Delta A^T W \Delta A)$. Substituting (16) into $\Omega$ yields $\Omega = F^T P F$, where $P = E(T^T W T)$. If we divided the weighting matrix $W$ into $(M + 2)^2 \times N \times N$ blocks with the $(i, j)$ block denoted as $W_{ij}$, $i, j = 1, 2, \ldots, M + 2$. Then, the matrix $P$ can be expressed by

$$
P = \text{blkdiag}(P_1, P_2)
$$

(26)

where

$$
P_1 = \sum_{i=1}^{M} (Q_r, (N(i-1)+1: Ni, N(i-1)+1: Ni) \parallel W_{ij})
$$

(27)

similarly, $P_2$ is partitioned into $(M + 2)$ block matrices denoted as $P_2 = [\bar{P}_{21}, \ldots, \bar{P}_{2(M+1)}]^{T}$ and the $i$ th block matrix is expressed as

$$
P_{2i} = \begin{bmatrix}
(Q_{\alpha\alpha} \parallel \text{diag}(W_{i1}, W_{i1}))^{T} \\
\vdots \\
(Q_{\alpha\alpha} \parallel \text{diag}(W_{iM}, W_{iM}))^{T} \\
(Q_{\theta} \parallel W_{(i+1),O_{N \times N}})^{T} \\
(Q_{\theta} \parallel \text{diag}(W_{i(M+2)}, W_{i(M+2)}))^{T}
\end{bmatrix}^{T}
$$

(28)

where $i = 1, \ldots, M, M + 2$ and $P_{2(M+1)}$ is given by

$$
P_{2(M+1)} = \begin{bmatrix}
(Q_{\theta} \parallel W_{(M+1),O_{N \times N}})^{T} \\
\vdots \\
(Q_{\theta} \parallel W_{(M+1)(M+1),O_{N \times N}})^{T} \\
(Q_{\theta} \parallel W_{(M+1)(M+2),O_{N \times N}})^{T}
\end{bmatrix}^{T}
$$

(29)

A feasible approach for solving (25) is to exploit Lagrange multiplier $\lambda$. After taking its derivative with respect of $v$ and setting it to zero, we have the following equation
\[ A^T W A v = \lambda \Omega v \]  

(30)

Multiplying \( v^T \) on both sides of (30) and using the constrained equation \( v^T \Omega v = \kappa \), we can obtain

\[ v^T A^T W A v = \lambda \kappa \]  

(31)

Consequently, the smallest value of the cost function \( J \) is the minimum generalized eigenvalue \( \lambda_{\text{min}} \) of the pair \((A^T W A, \Omega)\) when \( \kappa = 1 \), and the related parameter vector \( v \) is the generalized eigenvector corresponding to \( \lambda_{\text{min}} \). Thus, the source location is given by

\[ u = v(1:3) / v(4) \]  

(32)

Another point to note is that the true values are needed to construct \( \Omega \). We first approximate the true values in \( \Phi \) by using their noisy versions and the weighting matrix \( W = Q^{-1} \) to obtain an initial estimate. Then, this initial value can be employed to get a better estimated position of \( u^o \). Simulations show that the performance loss due to this approximation is negligible for such problems.

5. Performance Analysis

a) Cramer–Rao lower bound

For the localization model described in Section 3, the unknown parameter is \( u^o \). The CRLB of \( u^o \), for the Gaussian noise model, is expressed as

\[ \text{CRLB}(u^o) = \left( \frac{\partial \eta^o}{\partial u^o} \right)^T Q^{-1} \left( \frac{\partial \eta^o}{\partial u^o} \right) \]  

(33)

where \( \left( \frac{\partial \eta^o}{\partial u^o} \right) = \left[ \left( \frac{\partial r^o}{\partial u^o} \right)^T, \left( \frac{\partial \theta^o}{\partial u^o} \right)^T, \left( \frac{\partial \phi^o}{\partial u^o} \right)^T \right]^T \) and the rows of \( \frac{\partial r^o}{\partial u^o}, \frac{\partial \theta^o}{\partial u^o} \) and \( \frac{\partial \phi^o}{\partial u^o} \) are given by

\[
\begin{align*}
\frac{\partial r^o}{\partial u^o}(k,:) & = \rho^T_{u^o - s_a} + \rho^T_{u^o - s_b} \\
\frac{\partial \theta^o}{\partial u^o}(n,:) & = \frac{a^o_T}{R^o_s} \cos(\phi^o) \\
\frac{\partial \phi^o}{\partial u^o}(n,:) & = \frac{b^o_T}{R^o_s}
\end{align*}
\]  

(34)

where \( \rho_{a,b} = (a - b) / \| a - b \| \) denotes a unite vector from \( b \) to \( a \). \( \alpha^o_n, \beta^o_n, \text{ and } k \) is defined in section 4.
b) Performance analysis of the proposed method

Let us use $v^*$ to denote the solution from (32), thus the source position can be expressed as $u^* = v^*(1:3) / v^*(4)$. Moreover, the equation error at $v^*$ is given by

$$Av^* = \left[ -\hat{G}, \hat{h} \right] u^* \cdot v^*(4)$$

$$= (-Gu^* - \Delta Gu^* + \hat{h}) \cdot v^*(4)$$

After some algebraic operations, we have

$$Av^* = \tilde{A}v^*$$

where $\tilde{A}$ is defined as

$$\tilde{A} = [-G, \hat{h} - \Delta Gu^*]$$

By using the equation (36), we can formulate a new constraint problem that is equivalent to

$$\min_{v^*} v^T \tilde{A}^T W \tilde{A} v^* \quad \text{s.t.} \quad v^T \tilde{\Omega} v^* = 1$$

where $\tilde{\Omega} = E(\Delta A^T W \Delta \tilde{A})$ and $\Delta \tilde{A} = [O_{MN \times 3}, \Delta h - \Delta Gu^*]$. Through some simple calculations we can obtain

$$\tilde{\Omega} = \begin{bmatrix} O_{3 \times 3} & 0_3^T \\ 0_3 & c \end{bmatrix}$$

where $c$ is an irrelevant constant. Similarly, the solution to (38) also satisfies

$$\tilde{A}^T W \tilde{A} v^* = \tilde{\lambda} \tilde{\Omega} v^*$$

where

$$\tilde{A}^T W \tilde{A} = \begin{bmatrix} G^T W G & -G^T W (h - \Delta Gu^*) \\ -(h - \Delta Gu^*)^T W \theta & (h - \Delta Gu^*)^T W (h - \Delta Gu^*) \end{bmatrix}$$

Substituting the relationship $v^* = v^*(4) * [u^T, 1]^T$ into (40), the first three entries in (41) can be expressed as

$$G^T W Gu^* - G^T W (h - \Delta Gu^*) = 0$$

By substituting $u^* = u^o + \Delta u^*$ into (42) and ignoring the second order terms, we arrive at
\[ \Delta \mathbf{u}^* = (G^T W G) G^T W B \Delta \eta \]  

(43)

This result shows that the proposed method is unbiased when the noise level. By multiplying \( \Delta \mathbf{u}^* \) by its transpose, we can obtain the theoretical covariance matrix

\[ \text{cov}(\Delta \mathbf{u}^*) = (G^T W G)^{-1} \]  

(44)

Next, we will prove that the proposed method can achieve the CRLB at a low noise level. (44) can be rewritten as

\[ \text{cov}(\mathbf{u}) = ((B^{-1} G)^T Q_\eta^{-1} (B^{-1} G))^{-1} \]  

(45)

After some Simple matrix manipulation, we can easily obtain \( B^{-1} G = \partial \eta^o / \partial \mathbf{u}^o \). Note that the covariance matrix in (45) and the CRLB(\( \mathbf{u}^o \)) matrix in (33) have the same form; therefore, the proposed method can achieve CRLB accuracy at a low noise level.

6. Simulation results

In this part, we exhibit the performance of the proposed method and compare it with the methods [12]-[13]. The simulation results are averaged by \( L = 10000 \) Monte Carlo experiments. Two indicators are used to evaluate the estimated performance, one is the root mean square error (RMSE), define as

\[ \text{RMSE}(\mathbf{u}) = \sqrt{\sum_{l=1}^{L} || \mathbf{u}_l - \mathbf{u}^o ||^2 / L} \]  

, and the other is estimated bias, defined by

\[ \text{bias}(\mathbf{u}) = \sqrt{\sum_{l=1}^{L} \mathbf{u}_l / L - \mathbf{u}^o ||^2} \]  

, where the \( \mathbf{u}_l \) represent the results of the \( l \)th Monte Carlo experiment. Meanwhile, the CRLB is also considered as a benchmark in the RMSE comparison. The positions of transmitters and receivers are given in Table I. 100 targets are randomly generated on a circle centered at the origin of the coordinate system with a radius of 1000 to obtain the average RMSE and bias norm. Consistent with [12]-[13], the covariance matrices, respectively, are set to be \( Q_r = \sigma_r^2 I_M N \), \( Q_\theta = \sigma_\theta^2 I_N \) and \( Q_\phi = \sigma_\phi^2 I_N \), where \( \sigma_r^2 = 1600 \sigma^2 \) and \( \sigma_\theta^2 = \sigma_\phi^2 = 0.1 \sigma^2 \). \( \sigma^2 \) is the noise-related multiplier that varies from -40dB to 20dB. For simplicity, the constant \( \kappa \) is set to unity.

Fig 1 compares the RMSE of different algorithms and we find that all the methods perform well at low
noise levels. However, the method [12] suffers performance loss when $\sigma^2 > -30$ dB, which is mainly because the measurement noises dominate the performance. And the method [13] starts to deviate from the CRLB when $\sigma^2 > -10$ dB. In contrast, our proposed achieves near-CRLB performance for over small to relatively high noise levels. Compared with methods [12] and [13], the proposed solution performs better on RMSE of position when they both depart from the CRLB accuracy. In addition, when $\sigma^2 \geq 0$ dB, the RMSE performance from the proposed solution improves averaging 5.2 dB compared with that from method [13].

Next we focus on the bias performance in Fig 2. It is clear that the bias of the method [12] is significantly higher than that of the other methods in the whole range of noise levels. This is mainly due to the aforementioned fact that the presence of measurement noises in both the regressor and regressand in method [12]. And the squaring operations of the stage-2 in [13] lead to increased estimation bias when the noise is greater than $-10$ dB. Furthermore, the proposed method offers an average 7.5 dB reduction in the estimation bias as compared to the method [13] when $\sigma^2 \geq 10$ dB.

Moreover, the cumulative distribution function (CDF) of position errors computed by different algorithms is also shown in Fig 3. The multiplier $\sigma^3$ is set to be -20dB. Compared with other methods, the proposed method obviously has smaller position estimation errors.

Finally, the complexity analysis of different methods is also roughly evaluated via average CPU run-times in Table 2. We can see that the proposed method is a little smaller than the other methods (methods [12, 13]) but its performance is better than the other method especially at high noise levels.

### TABLE I

| Positions (m) of transmitters and receivers |
|-----------------|-----------------|-----------------|-----------------|
| $t_m$ | $x_{t_m}$ | $y_{t_m}$ | $z_{t_m}$ | $s_n$ | $x_{s_n}$ | $y_{s_n}$ | $z_{s_n}$ |
| No. | No. | No. | No. | No. | No. | No. | No. |
| 1 | -200 | 300 | 200 | 1 | -450 | -450 | 200 |
| 2 | -200 | 300 | 100 | 2 | 450 | 450 | 100 |
| 3 | 200 | 300 | 80 | 3 | 0 | 600 | 200 |
| 4 | 200 | -300 | 120 | 4 | 600 | 0 | 100 |
| - | - | - | - | 5 | -600 | 0 | 150 |
| - | - | - | - | 6 | 0 | -600 | 100 |
Fig. 1. RMSE comparison of the different methods

Fig. 2. Bias comparison of the different methods

| Method         | CPU run times |
|----------------|---------------|
| method [12]    | 0.51          |
| method [13]    | 0.42          |
| Proposed method | 0.81          |
7. Discussion

To summarize, this paper develops a novel estimator for target localization using TD and AOA measurements in the MIMO radar systems. Simulation results show that the proposed method achieves CRLB at mild noise levels. And the proposed method performs better compared with the existing methods when they both depart from CRLB. Moreover, the proposed method also provides a significantly improvement on bias performance compared with other methods. However, the proposed method is slightly more time consuming than that of the existing methods because the proposed method considers a quadratic constraint, while the existing methods did not.

This solution is only for a stationary source. Hence, we plan to consider moving target localization in MIMO radar systems in our future work.

8. Conclusion

Aiming to the target localization in MIMO radar systems, we present an efficient estimator using TD and AOA measurements in this paper. The cost function is re-formulated and subjected to a quadratic constraint so that the expectation of the cost function will reach its ideal state at the true position. Simulations demonstrate that the proposed method outperforms than existing methods under the Gaussian noise model.
Abbreviations
TD: time delay; AOA: angle of arrival; 3D: three dimensional; MIMO: multi-input and multi-output; WLS: weighted least squares; BR: bistatic range; DS: doppler shift; BRR: bistatic range rate; ML: maximum likelihood; TSWLS: two-stage weighted least squares; CRLB: Cramer–Rao lower bound; CWLS: constrained weighted least squares; QCQP: quadratically constrained quadratic programming; LCQP: linear constrained quadratic programming; RMSE: root mean square error; CDF: cumulative distribution function

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Availability of data and materials
The data in this paper is based on computer simulation as the provided parameters in Section 5.

Authors’ contributions
MML proposed the quadratic constraint which improved the performance of the algorithm and wrote the paper. WG designed the experiments and provided the paper organization. YZ analyzed the performance metric and finished the experiment. XC checked the whole paper and improved the writing. All authors had a significant contribution to the development of early ideas and design of the final methods. All authors read and approved the final manuscript.

Ethics approval and consent to participate
This paper does not contain any studies with human participants or animals performed by any of the authors.

Consent for publication
Informed consent was obtained from all authors included in the study.
Competing interests

The authors declare that they have no competing interests.

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