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Studying the energy variation in the powered Swing-By in the Sun-Mercury system

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Abstract. A maneuver where a spacecraft passes close to Mercury and uses the gravity of this body combined with an impulse applied at the periapsis, with different magnitudes and directions, is presented. The main objective of this maneuver is the fuel economy in space missions. Using this maneuver, it is possible to insert the spacecraft into an orbit captured around the Sun or Mercury. Trajectories escaping the Solar System are also obtained and mapped. Maps of the spacecraft energy variation relative to the Sun and the types of orbits resulting from the maneuver are presented, based in numerical integrations. The results show that applying the impulse out of the direction of motion can optimize the maneuver due to the effect of the combination of the impulse and the gravity.

1. Introduction

Mercury is the smallest and most internal planet in the Solar System. It has an equatorial radius of approximately 2440 km, 0.055 times the Earth's mass and an orbital period around 88 days. It is approximately 57,910,000 km away from the Sun and, among all the planets of the solar system, it is the one with the largest eccentricity, in the order of 0.2056 [1].

Missions such as Mariner 10 and Messenger were sent for observations of Mercury [2-5]. Along the way, Swing-By maneuvers were made to help the approach of the probes to the destination and then to insert them into the planet's orbit. Mariner 10 was the first probe to use the gravity assisted maneuver with a celestial body in a mission. Both probes had close encounters with the planet for observations and data collection.

Swing-By maneuvers or gravity assisted maneuvers are techniques that use a close approach of a spacecraft with a celestial body to give or to remove energy of the spacecraft. The powered Swing-By maneuver combines the gravity of the celestial body with the application of an impulse during the close encounter to change the trajectory of the spacecraft, and it is recommended in situations where only the gravity part the maneuver is not sufficient to meet the mission needs.

There are works about this subject available in the literature. Prado (1996) [6] studied the powered Swing-By maneuver and its efficiency for systems in circular orbits. In 1999, Casalino et al. [7] made an analytical extension of Prado's work, considering the impulse applied in different positions of the orbit. The detailed result for the case with the impulse applied in the periapsis of the orbit of the spacecraft, including an analysis of the maximum gain and loss of energy, was presented in [8].
Ferreira et al. (2017a) [9] made a study detailed showing the effects of the eccentricity of the orbit of the primary bodies of the system in the maneuver. A numerical map of the energy gain due to the powered Swing-By maneuver, for the circular case and assuming different magnitudes, directions and positions of the impulse, was present in [10].

The present paper is an application of this maneuver, for the Sol-Mercury system, based in the study made in references [9] and [10]. The Sun-Mercury system is chosen because it has high eccentricity and the fact that it is the focus of many studies and missions. The main reason to perform the present study, and not only using the results shown in references [9-10], is that important aspects, like the occurrence of collisions and escapes, are highly dependent on the particular system. The exact values of the mass and dimensions of the primaries has strong effects in the behavior of the trajectories, and general results like the ones shown in references [9-10] give only a first view of the problem. Therefore, the present publication looks in detail the Sun-Mercury case, skipping general description and studies already published in the cited papers.

2. Statement of the problem

The work was developed by numerically integrating the equations of motion of the spacecraft, based in the Elliptical Restricted Three-Body Problem (ERTBP) [11]. The advantage of this model is the increase of the accuracy in the results, since it does not depend on approximations made by the “patched-conics” or circular models. It also helps to identify trajectories resulting in captures and collisions. We consider $M_1$ the Sun, $M_2$ Mercury and $M_3$ the spacecraft with negligible mass and that moves under the gravitational forces of the two primaries. The equations of motion [11] are given by:

$$\dot{x} = -\frac{(1-\mu)(x-x_1)}{r_1} - \mu(x-x_2)$$ and $$\dot{y} = -\frac{(1-\mu)(y-y_1)}{r_1} - \mu(y-y_2),$$

where $x_1, y_1$ are the Sun positions coordinates, $x_2, y_2$ represents the Mercury position coordinates; $\mu$ is the mass parameter of $M_2$; $r_1$ is the distance between the spacecraft and the Sun and $r_2$ is the distance between the spacecraft and Mercury.

![Figure 1 - Geometry of the powered Swing-By maneuver.](image)

Figure 1 shows the geometry of the powered Swing-By maneuver. It starts from the periapsis of the trajectory of the spacecraft around Mercury, defined by the initial conditions: $\psi$, the approach angle; $V_{inf}$, the approach speed given by $V_{inf} = \sqrt{V_{inf}^2 - 2\mu/r_p}$; and $r_p$, the periapsis radius. The equations were numerically integrated reversely in time [12-15], without the impulse, until it reaches a
distance where the effect of $M_2$ on the spacecraft can be neglected and the movement between the spacecraft and the Sun is considered Keplerian. Then, the two-body energy is calculated. Returning the spacecraft to the periapsis of the orbit, the impulse is applied according to the magnitude ($\delta V$) and direction ($\alpha$) defined, and the trajectory is propagated forward in time until it reaches the distance where the spacecraft-Sun movement is considered Keplerian. The new orbit is then obtained. Based on the parameters before and after the maneuver, the variations generated by the maneuver can be obtained by subtracting directly the quantities involved, therefore mapping the variations of the desired quantities as a function of all variables. In this work the energy variation and the type of the orbit of the spacecraft is analyzed, considering Mercury in the periapsis and in the apoapsis of its orbit around the Sun.

3. Results
The eccentricity of Mercury has been considered and its main effect is that the velocity of Mercury ($V_2$) around the Sun is not constant. It varies according to its position, being faster in the periapsis and slower in the apoapsis of its orbit. The energy variation measured with respect to the main body of the system is analyzed. For positive variation values, the second orbit after the impulse increased its energy compared to the first orbit. If the variation is negative, the spacecraft loses energy in the second orbit compared to the same first orbit.

The results are obtained for the initial conditions: $\mu = 1.65 \times 10^{-7}$, $r_p = 1.1$ Mercury’s radius, $V_{inf_{-}} = 1.0$ canonical units, $\psi = 90^\circ$ and $\psi = 270^\circ$. Figure 2 shows the energy variations, considering Mercury positioned in the periapsis of its orbit at the time of the Swing-By. Figure 2(a) considers $\psi = 270^\circ$, and, in this case, the gravity works for the energy gain. Note that the maximum energy variations occur for $\alpha$ between $-45^\circ$ and $0^\circ$. If the goal is to gain energy, it is more consistent to send the spacecraft towards the secondary body, to enhance the effect of gravity. Figure 2(b), made for $\psi = 90^\circ$, has the gravity working for the energy loss. The maximum energy variations have $\alpha$ positive, to move the spacecraft away from the body and to minimize the effect of gravity. In general, the magnitude of the energy variation increases with the magnitude of the impulse. For impulses with 1.5 c.u. and 2.0 c.u., there were cases of captures or collisions of the spacecraft. This type of analyses needs to be made for each system of primaries involved, because the results are highly dependent on the particular physical characteristics of the system under study. The minimum energy variation occurs for retrograde impulses.

![Figure 2](image2.png)

Figure 2 – Energy variation for $\nu = 0^\circ$.

Figure 3 shows the solutions when Mercury is positioned in the apoapsis of its orbit at the time of the Swing-By. The main difference compared to Fig. 2 is the magnitude of the energy variation. Note
that, even $M_2$ being in a position which does not favor energy gains, the combination impulse-gravity assist are more significant, providing greater energy variations. The direction to apply the impulse, in this case, is further from the direction of the spacecraft’s motion.

Note that the results presented in Figures 2 and 3 confirm the study presented in [9], made for a system similar to the Earth-Moon, but they add the particularities of the specific system, like their sizes and masses. Similar behaviors are noted in the energy variations. The difference in the magnitude of the energy variation and in the amount of captures and collisions are due to different size, mass and distance of the systems, as already explained.

The effect of the maneuver in the type of the orbit of the spacecraft is also presented. The descriptions are obtained from the analysis of energy (ellipse or hyperbola) and angular momentum (direct or retrograde) of the spacecraft with respect to the main body of the system.

Table 1: Orbits descriptions

| Before:                | After:             |
|-----------------------|--------------------|
| Direct ellipse        | Direct ellipse     |
| Retrograde ellipse    | Direct hyperbole   |
| Direct hyperbole      | Retrograde hyperbole |

| Direct ellipse | A  | E  | I  | M  |
|----------------|----|----|----|----|
| Retrograde ellipse | B  | F  | J  | N  |
| Direct hyperbole | C  | G  | K  | O  |
| Retrograde hyperbole | D  | H  | L  | P  |

Each notation has a color to facilitate the visualization of the figures (Table 1). The notation “Y” (red) represents orbits where the spacecraft collide with Mercury and “Z” (black) are trajectories where the spacecraft remained orbiting Mercury until the final integration time. The orbits of types “A”, “B”, “E” and “F” are closed orbits that remain closed after the impulse. They may or may not have their directions of movement modified. The orbits of type “C”, “D”, “G” and “H” are orbits where the spacecraft was captured by the main body after the impulse. They start with an open orbit (hyperbola) that becomes an ellipse. Since types “I”, “J”, “M” and “N” represent orbits where the spacecraft escaped from the main body. Initially, it was in an elliptical orbit and, after the maneuver, it went into a hyperbolic orbit. Cases “I” and “N” maintained the direction of movement. Finally, the orbits of type “K”, “L”, “O” and “P” are open orbits that remained open, in a hyperbola, after the
maneuver. The results show maps of the orbits having impulses ranging from 0 to 2 c.u. on the horizontal axis and, in the vertical axis, the angle defining the direction of the impulse, ranging from -180° to 180°.

Figure 4 – Type of the orbits for $\nu = 0°$.

Figure 4(a) shows cases where the spacecraft escaped from the main body after the maneuver (“I”), being $\alpha$ approximately between -90° and 90°. In these cases the impulses are prograde. This behavior is consistent considering that $\psi = 270°$ is the region of energy gains due to gravity part of the maneuver. The gaining of more energy results in larger velocities, therefore the greater the chance to generate an escape for the spacecraft. The few cases where the spacecraft collided with the body (“Y”) are for $\delta V > 1.0$ c.u. and with an impulse with a component opposite to the movement of the spacecraft and the other component in the direction of the secondary body. There are also orbits of type “A”, which are direct ellipses, and type “E”, which are ellipses that change the direction of movement after the maneuver; and finally trajectories of type “Z”, which makes the spacecraft to be captured by Mercury.

For the case $\psi = 90°$, the gravity works to reduce energy, increasing the chances of capture. Figure 4(b) shows trajectories where the spacecraft was captured by main body after the maneuver (“C” and “G”). Note that the orbits resulting in capture have retrograde impulses ($\alpha > 90°$ and $\alpha < -90°$), which decelerates the spacecraft and works in favor of the gravity to decrease energy. The orbits of type “G” are captures with direction of the movement inverted after the maneuver. Some cases of collisions with $M_2$ (“Y”) also occurred, as well as orbits of types “K” and “O”. In orbits of type “K”, the spacecraft remained in the same type of trajectory after the maneuver. They occur for impulse direction greater than -90° to just over 90°, and for all magnitudes of the impulse.
In Figure 5, the main change was the position of Mercury, which is now in the apoapsis of its orbit around the Sun. Figure 5(a) has the same orbit type as Figure 4(a), but the impulse configuration for these orbits has some differences. The region with direct ellipse ("A") is smaller, while the region with direct ellipse that has become retrograde ("E") has increased. There were some cases of spacecraft collisions with Mercury ("Y") and all others were escapes from the main body ("I"). Figure 5(b) shows trajectories where the spacecraft was captured by the main body after the maneuver ("C" and "G"). Some cases of collisions with Mercury and orbits of types "K" and "O" also occurred. In orbits of the type "K", the spacecraft remained in the same type of trajectory after the maneuver and in the type "O" only the direction of movement was inverted.

Associating the energy variations with the types of orbits, we see that when $\psi = 270^\circ$, for both positions of Mercury in orbit, the trajectories with maximum energy variation, the spacecraft escaped from the Sun after the complete maneuver. When $\psi = 90^\circ$, in the trajectories with maximum energy variation, the spacecraft came from an open orbit with respect to the Sun and remained in an open orbit with the same direction of the first one after the maneuver.

4. Conclusions
A study of the behavior of the spacecraft energy in a trajectory that performs a powered Swing-By maneuver was presented. Different initial conditions were simulated, considering Mercury in the periapsis and apoapsis of its orbit around the Sun and with the orientation of the Swing-By in the region of maximum gains and losses of energy due to the gravitational part of the maneuver.

The results show the energy variation and the characteristic of the impulse for each solution. For some cases there are captures or collisions of the spacecraft by Mercury. The types of trajectories
performed by the spacecraft before and after the maneuver, with respect to the main body, were also presented. For the cases simulated here, there are trajectories where the spacecraft escaped from the main body and others where it was captured. Collisions with Mercury were also detected and they represent a danger for the mission. This type of risk can only be mapped by studying the specific system of primaries, since general results can only give an idea of the gains and losses of energy.

In general, the solutions show that applying the impulse out of the direction of motion can optimize the maneuver due to the effect of the combination of the impulse and the gravity.

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