Influence of attenuation on the generation of optical vortices in multihelicoidal optical fibers

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Abstract. In this paper we have studied influence of attenuation on conversion processes of the fundamental mode (FM) in multihelicoidal optical fibers (MHF) in the vicinity of the point of accidental spectrum degeneracy within the framework of the scalar approximation. To this end, we have obtained expressions for modes of the MHF, which consist of the FM and an optical vortex (OV), and shown that conversion of the FM into the OV takes place. The difference in the attenuation coefficients for the partial fields of MHF’s modes leads to deterioration in the conversion process even with an ideal system’s tuning. At sufficiently large values of attenuation coefficients the conversion of the incoming FM into the vortex vanishes. Also we have shown the presence of exceptional point (EP) in the spectra of modes of the MHF and demonstrated enhanced sensitivity of the fiber in the vicinity of the EP to perturbations.

1. Introduction

Modern fiber optics is characterized by attention to the generation of optical vortices (OVs) [1] with fiber optic systems. OVs are distinguished from all optical fields by the presence of a helical wavefront. If its helical surface has l branches then the topological charge (TC) of the OV is equal to l.

The helical nature of OVs allows one to use them in optical sensors, astrophysics and other fields [2-5]. Particularly promising is the usage of beams with TC in information technologies for information transmission based on fiber-optic lines or in free space [6, 7]. The advantage of transmitting information with the help of OVs is that the same frequency range can be used for transmission of a variety of signals, each of which corresponds to vortex with some value of the TC. It should be noted that theoretically the number of such channels is not limited since the TC can be arbitrarily large.

From a practical point of view, optical fiber transmission is complicated by two factors. First, OVs in standard fibers are unstable with respect to disturbances caused by, for example, their bending or twisting. However, the recent studies have shown the ability of some types of fibers with discrete rotational symmetry to maintain a stable propagation of OVs along them [8, 9]. Second, in spite of the abundance of methods for exciting vortex fields, it is desirable to use optical fibers while generating OVs in the case of their subsequent transmission through an optical fiber since the fiber-to-fiber coupling minimizes input losses. Therefore, it is urgent to search for fiber methods for generating OVs.

One of the possible fiber devices that can generate OVs from regular input beams without wavefront dislocations are devices based on multihelical fibers (MHF) (figure 1a). Since such fibers differ in the direction of their twisting they are usually called chiral.

The OV generation in such fibers is influenced, first of all, by the geometric parameters of the fiber, as well as by the distribution of the refractive index. The quantitative and qualitative
characteristics of this influence have been repeatedly discussed in the literature [10-14]. However, in addition to the factors mentioned in these works, energy dissipation is always present in real chiral fibers. In this connection, the question arises: how does the energy dissipation caused by the absorption of the medium affect the processes of OV generation in chiral fibers? We will demonstrate that the presence of absorption in a medium can significantly change the character of OV generation from regular beams.

2. Model
Consider the M HF with a core, which has the l-th order of rotational symmetry (in figure 1 l =5). The core has a refractive index $n_{co}$ and is immersed in a media (cladding) with a refractive index $n_{cl}$ (figure 1b). The deviation of the shape of the core from a circle with a radius $r_0$ is characterized by the dimensionless parameter $\alpha$. This parameter defines the intensity of the coupling between fields with azimuthal numbers differing by $l$.

![Figure 1](image)

**Figure 1.** a) The geometry of M HF, (X,Y,Z) is the laboratory coordinate frame. b) The cross-section of the M HF with axial symmetry of 5-th order. The deviation of cross-section’s form from a circle with the radius $r_0$ is characterized by the parameter $\alpha$. The core (dark green area) has the refractive index $n_{co}$, while the cladding (light green area) has the refractive index $n_{cl}$.

3. Hybrid modes
Let us obtain the expressions for M HF’s modes in the presence of absorption of the medium. It should be noted that absorption can be caused by either medium’s properties itself or a strong coupling of some guided modes to the leaky ones. The latter case can be effectively referred to as absorption.

The M HF modes in the absence of energy dissipation in the medium [10, 13] in the basis of linear polarizations can be written in the form of OVs:

$$\left| \sigma,m \right> = F_m(r) e^{i\nu} \left( \frac{1}{i\sigma} \right),$$

where $\sigma=\pm 1$ defines the sign of circular polarization, $m$ is the TC, $F_m$ is the radial function, which satisfies the standard equation [15], $(r,\phi)$ - cylindrical polar coordinates connected with the center of the cross-section. In the following we use the scalar approximation therefore we set $\sigma=1$. This means that we neglect changing of the polarization state of field in the fiber. Note that the field $\left| 1,0 \right>$ is the FM. The modes’ spectra have the form:

$$\vec{\beta}_{1,2} = \vec{\beta}_0, \vec{\beta}_{3,4} = \vec{\beta}_l \pm ql, \vec{\beta}_{5,6} = -\vec{\beta}_l \pm ql.$$ 

Here $q=2\pi/H$ is the modulus of the reciprocal lattice vector, $H$ is the step of fiber’s twist, $\vec{\beta}_{\text{in}}$ is the scalar propagation constant for the ideal (that is with a round cross-section) straight optical fiber.

As is seen from (2) the M HF’s spectra are not degenerate at arbitrary $q$ but if condition

$$q = q_0 = \left( \vec{\beta}_0 - \vec{\beta}_l \right)/l,$$

is satisfied then spectral branches $\vec{\beta}_l$ and $\vec{\beta}_3$ intersect and the modes of the system become degenerate. In this case the resonant coupling between the modes with differing by $l$ TCs begins to
play a significant role. To take it into account we should use the perturbation theory with degeneration [16]. To this end, we, firstly, write down the spectral equation in the scalar approximation [13]:

$$
\begin{pmatrix}
\beta_0^2 - \beta^2 & A \\
A & \beta_i^2 - (\beta - q_i)^2
\end{pmatrix} = 0.
$$

(4)

Here $A = k^2 n_c^2 \alpha \Delta / \sqrt{N_0 N_1}$, $N_m = \int_0^\infty r F^2(r) dr$. The dimensionless parameter $\Delta$ is the optical contrast between the core and the cladding. To take into account the attenuation in the system, we should make the standard substitution:

$$
\tilde{\beta}_m \rightarrow \tilde{\beta}_m + i \gamma_m,
$$

(5)

where $\gamma_m$ is the attenuation coefficient for the mode of $m$-th order, also we suppose $\gamma_m \ll \tilde{\beta}_m$. While taking into account (5) the equation (4) becomes

$$
\begin{pmatrix}
\beta_0^2 + 2i \tilde{\beta}_0 \gamma_0 - \beta^2 & A \\
A & \tilde{\beta}_i^2 + 2i \tilde{\beta}_i \gamma_1 - (\beta - q_i)^2
\end{pmatrix} = 0.
$$

(6)

In (6) we neglect $\gamma_m^2$ due to its smallness in comparison with the other terms. Since we also neglect the reflected (backward propagating) waves, in this equation for $q$ and $\beta$ we can set:

$$
q = q_0 + \epsilon, \beta = \tilde{\beta}_0 + \delta,
$$

(7)

where $\epsilon, \delta \ll \beta_m$ are detunings. As a result, for the corrections to the scalar propagation constant near the point of intersection $q_0$ of the branches one obtains:

$$
\delta_{1,2} = i \bar{\gamma} + \frac{1}{2} \left( |\epsilon| \pm \sqrt{|i \gamma_0 - \epsilon|^2 + \bar{Q}^2} \right),
$$

(8)

where $\bar{\gamma} = (\gamma_0 + \gamma_1)/2$ and $\gamma = \gamma_0 - \gamma_1$, $\bar{Q} = A / \tilde{\beta}_0$, and we use $\tilde{\beta}_0 \approx \tilde{\beta}_1$. As is seen from this expression, the first term in (8) defines the same attenuation for each of the hybrid modes. Moreover, hybrid modes are attenuated in the same way if the coefficients $\gamma_0$ and $\gamma_1$ are equal. Thus, the general picture of the evolution of the incoming field reduces to the already known results: the conversion of the incoming FM in the transmitted light into an OV [10-14].

Eigenvector equation for vector $\{\psi\}$ in the form $x|1,0\> + y|1,1\>$ can be written as

$$
\begin{pmatrix}
(\gamma_0 - \delta) & \bar{Q}/2 \\
\bar{Q}/2 & (\gamma_1 + \epsilon - \delta)
\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix} = 0.
$$

(9)

As follows from (9) the hybrid modes have the form:

$$
|\psi_+\> = (\bar{Q}|1,0\> + (i \gamma + \epsilon - R)|1,1\>) e^{i(\tilde{\beta}_0 + \delta_1)},
$$

$$
|\psi_-\> = (\bar{Q}|1,0\> + (i \gamma + \epsilon + R)|1,1\>) e^{i(\tilde{\beta}_0 + \delta_1)}.
$$

(10)

Here $R = \sqrt{|i \gamma + \epsilon|^2 + \bar{Q}^2}$. It should be noted, that due to the non-Hermitian character of the matrix in (9) the eigenvectors $|\psi_{\pm}\>$ are not orthogonal: $\langle \psi_+ | \psi_- \rangle \neq 0$. Also, the expressions (10) are not normalized. One more peculiarity still exists. Indeed, if $\epsilon = 0$ then at $\gamma = \bar{Q}$ the modes (10) become identical. In this case one can say that an exceptional point (EP) in their spectra appears. The study of
behavior of the system in this point requires a special treatment, which we recently made in [17]. Now we are in a position to study the evolution of the FM in the MHF in the presence of attenuation.

4. Fundamental mode’s conversion
Let the FM be incident on the input end of the MHF. Then at the left boundary it can be represented as the sum of $|\psi_{\pm}\rangle$:

$$|1,0\rangle = c_1 |\psi_+\rangle + c_2 |\psi_-\rangle.$$  \hspace{1cm} (11)

Here $c_i$ are unknown coefficients, which one can easily establish from (11) with the account of (10). Thus, in the MHF the incoming FM $|1,0\rangle$ becomes

$$|\Phi\rangle = c_1 |\psi_+\rangle e^{i\beta_0 z} + c_2 |\psi_-\rangle e^{i\beta_2 z} =$$

$$= \left[\cos \frac{R\phi}{2} - i \frac{\Delta e^{i\epsilon z}}{R} \sin \frac{R\phi}{2}\right] |1,0\rangle + \frac{i \tilde{Q} e^{-i\epsilon z}}{R} \sin \frac{R\phi}{2} |1,l\rangle \exp \left[i\beta_0 - \frac{i\epsilon}{2} z\right].$$  \hspace{1cm} (12)

Here it should be noted that zero approximation modes (1) of the MHF are written in the local frame rotated through an angle $qz$ with respect to laboratory frame, and one must make the substitution $l \phi \rightarrow l(\phi - qz)$ in (1). More details can be found in [13] as well as in other abovementioned papers. Expression (12) allows us to make two conclusions: (1) in the presence of attenuation the transmitted power decreases, which follows from the multiplier $\exp[-\beta z]$; (2) the quality of conversion decreases, even at $\epsilon = 0$ and $\cos R\phi = 0$ in the outcoming field there are both partial fields. These statements are illustrated by figure 2. As is seen from it, even at a sufficiently small ratio $\gamma / \tilde{Q}$ and zero detuning $\epsilon$ the FM $|1,0\rangle$ is present in the outcoming field (figure 2a). As the attenuation $\gamma$ reaches $\tilde{Q}$, the FM’s intensity begins to significantly exceed the OV’s intensity (figure 2b). Qualitatively, this picture remains the same even at $\gamma / \tilde{Q} > 1$.

![Figure 2](image)

**Figure 2.** Intensity of the outcoming field vs $q$; the incoming field is the FM $|1,0\rangle$; the solid red curves correspond to the FM $|1,0\rangle$, and the dashed blue ones – to the OV $|1,l\rangle$. The green dot on the horizontal axis corresponds to $q = q_0$. a) $\gamma = 0.3\tilde{Q}$; b) $\gamma = 0.99\tilde{Q}$; The parameters: $n_{\text{co}} = 1.5$, $\Delta = 0.001$, $n_0 = 6\lambda_0$, $\lambda_0 = 632.8$ nm, $\tilde{Q} = 15.12$ m$^{-1}$, $\epsilon = 0$, $q_0 = 7930$ m$^{-1}$, $z \approx 21.8$ cm, $l = 1$.

Let us now consider the case where the fiber’s length changes. As is seen from figure 3a, in the case of a small ratio $\gamma / \tilde{Q}$, the incoming FM gets converted at some values of $z$ into the OV $|1,l\rangle$. Nevertheless, the conversion disappears if attenuation becomes too large (figure 3b).
Figure 3. Intensity of the outcoming field vs $z$; the incoming field is the FM $\left|1,0\right>$; the solid red curves correspond to the FM $\left|1,0\right>$, and the dashed blue ones – to the OV $\left|1,l\right>$. a) $\gamma = 0.1\bar{Q}$; b) $\gamma = 0.99\bar{Q}$. Parameters are the same as for figure 2.

Changing the value of $\gamma$ also allows one to control the conversion of the FM. As is seen from figure 4a, at zero detuning ($\varepsilon = 0$), the incoming FM at small values of $\gamma$ gets converted into the OV $\left|1,l\right>$, but as $\gamma$ increases the fraction of the FM increases too. The situation gets worse when the detuning $\varepsilon$ appears (figure 4b). Even at low values of $\gamma$ the FM is present in the outcoming field.

Figure 4. a) Intensity of the outcoming field vs $\gamma$; b) Intensity of the outcoming field vs wavelength $\lambda$. Parameters are the same as for figure 2 with the exception of a) $z \approx 2.72$ cm; b) $z \approx 4$ cm and $\bar{Q} = 75.6$ m$^{-1}$. the incoming field is the FM $\left|1,0\right>$; the solid red curves correspond to the FM $\left|1,0\right>$, and the dashed blue ones – to the OV $\left|1,l\right>$.

5. Enhanced sensitivity

One of renowned properties of systems with non-Hermitian Hamiltonians (see (9)) is their increased sensitivity to perturbation of their parameters in the EPs as compared to the one in regular points. Indeed, if, for example, in the regular point the perturbation response scales as $\kappa$, where $\kappa$ is the dimensionless parameter of perturbation, and in the EP it scales as $\sqrt{\kappa}$, then at $\kappa << 1$ one obtains a much stronger relative response in the EP in question. This simple mathematical fact forms the basis for various sensor applications in systems with EPs [18]. In our example, the EP is second-order.

To begin with, let us note that in (12) the EP-sensitive construction $R$ never enters phase factors and rather appears in amplitude coefficients. This dictates studying the amplitudes of partial fields $|0\rangle$ and $|1\rangle$ in those expressions. As a measure of the system’s sensitivity to variations $\Delta\gamma$ of attenuation parameter $\gamma$ we take the following ratio $\xi(\Delta\gamma;\gamma)$

$$\xi(\Delta\gamma;\gamma) = \frac{\|I(\gamma) - I(\gamma + \Delta\gamma)\|}{I(\gamma)},$$

(13)
where $I(\gamma)$ is the output power stored in $|t\rangle$ OV. Figure 5 shows the plot of
\[ \tau = \frac{\xi(\Delta\gamma; \tilde{\mathcal{Q}})}{\xi(\Delta\gamma; \tilde{\mathcal{Q}}/2)}, \]
that is the ratio of relative sensitivities near the points $\gamma = \tilde{\mathcal{Q}}$ (the EP) and $\gamma = \tilde{\mathcal{Q}}/2$. As is seen, the output power features enhanced sensitivity to variations of $\gamma$ near the EP.

**Figure 5.** The ratio of relative sensitivities (13) vs $\log(\Delta\gamma/\tilde{\mathcal{Q}})$; fibre length a) $z = 0.45m$, b) $z = 1m$. $\tilde{\mathcal{Q}} = 148.8 \text{ m}^{-1}$.

**6. Conclusion**

In this work we have studied the influence of attenuation on the processes of optical vortex generation from the fundamental mode in the presence of attenuation in the multihelical optical fiber. We have shown that the presence of attenuation worsens this process and even at an ideal tuning of the system there are always two fields in the outcome field. Also we have demonstrated that changing of attenuation parameter allows one to control conversion of the fundamental mode into the optical vortex. In addition we have shown that in the spectra of hybrid modes of the MHF there is an exceptional point in the vicinity of the fibers demonstrates enhanced sensitivity to external perturbations.

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