Coherent quantum transport through ferromagnetic graphene structures: Effects of Rashba spin–orbit coupling

Kobra Hasanirokh1,2,∗ and Fezzeh Naderi3

1Azarbaijan Shahid Madani University, Tabriz 53714-161, Iran
2Research Institute for Applied Physics and Astronomy, University of Tabriz, Tabriz 51665-163, Iran
3Young Researchers and Elite Club, Marand Branch, Islamic Azad University, Marand, Iran
∗E-mail: zhasanirokh@yahoo.com

Received January 9, 2020; Revised March 1, 2020; Accepted March 4, 2020; Published July 2, 2020

In a system consisting of a monolayer ferromagnetic graphene structure, based on the scattering matrix approach, we study the spin-dependent transmission coefficients, group delay time, magnetoresistance and giant magnetoresistance of spin-polarized electron tunneling through the Rashba barrier in single-layer graphene. The results show that Rashba spin–orbit coupling can cause a natural spin filter mechanism; it thus has a significant role in controlling the transmission probabilities. In addition, the quantum transport properties of our system depend critically on the structural parameters. The incidence angle, energy, barrier number, and exchange energies can strongly control the transport properties of multi-layer graphene. It is predicted that controlling spin-dependent transport in single layer graphene results can develop the well-known spintronics.

1. Introduction

Graphene, a zero-gap semiconductor with a thickness of one atom of carbon densely packed in a honeycomb lattice, is a two-dimensional electronic system that has attracted vast interest both from the scientific and investigation sides [1]. The energy dispersion relation in graphene is approximately linear close to the Dirac points K and K' of the Brillouin zone where the electron and hole bands touch. This peculiar band structure makes graphene different from a two-dimensional electron gas (2DEG) and causes many unique and fascinating electronic and transport properties, such as Klein tunneling [2], universal minimal conductivity [3,4], half-integer quantum Hall effect [5], and special Andreev reflection [6]. The ability to manipulate the charges and the spin of the electrons has developed a new field of application in spintronics. Some interesting findings have been achieved in electron transport through a graphene barrier without considering the spin states of the electron [7–11]. The study of spin-dependent transport properties is one of the most important problems in graphene structures, and many interesting results have been achieved in the spin-polarized transport of electrons through a ferromagnetic single-layer graphene barrier [12,13] and graphene superlattice [14,15].

Ferromagnetism in graphene structure and spin inversion has been extensively studied theoretically and experimentally [16–21]. Ferromagnetic barriers are a useful method for generating spin-polarized currents, but such structures need an external magnetic field. Spin-polarized current generation is a fundamental prerequisite for spintronic devices [22].
First, Datta and Das proposed the concept of the spin-polarized field effect transistor [23] and after that the study of spintronics has focused on tunneling through nonmagnetic semiconductors. Spin transport is one of the active fields in graphene research.

Spin–orbit interaction (SOI) can control the electrical influence of the spin degree of freedom by transferring the orbital angular momentum taken by an electric current to the electron spin. Spin–orbit coupling (SOC), particularly Rashba spin–orbit coupling (RSOC), plays a key role in the well-known Hall effect [5,6]. The Rashba-type effective interfacial spin–orbit Hamiltonian was offered by E. I. Rashba to investigate the influence of asymmetric interfaces in 2DEG [2]. Breaking the structure inversion symmetry, such as asymmetric interfaces, proposes a potential gradient that is normal to the interfaces. Rashba SOI creates an effective magnetic field perpendicular to the linear momentum. In graphene, the Rashba SOI originates from the interaction of carbon atoms and the substrate [24,25] or electric field, or by an external electric field perpendicular to the graphene layer (generated by a gate) [26,27]. It has been reported experimentally that the Rashba SOI strength $\lambda_R$ in graphene can reach values up to 200 meV [25].

The time aspect of quantum tunneling is also another interesting problem, and many investigations have been focused on the tunneling time of a particle passing through a barrier [26–28], such as group delay time $\tau_g$. This is an important quantity, defined as the delay time for the peak of the transmitted wave packet at $x = 0$ to appear at $x = L$ (the barrier width) [29], and can be calculated by the stationary phase method with the energy derivative of the transmission phase shift [29–33]. The independence of the group delay time from the barrier width of the wide barrier is known as the Hartman effect in quantum tunneling [30–32], first reported by Hartman.

Although the tunneling time through semiconductors [33–36] and graphene superlattices [37] have been studied, we focus here on the group delay time through a ferromagnetic graphene layer with Rashba SOI, which to the best of our knowledge has not been reported. These phenomena can benefit graphene-based spin-polarized electron devices. It is expected that our theoretical results will provide an important and interesting reference for basic physics and device applications.

This paper is organized as follows. Our method and some standard calculations of the electron spin-dependent transmission coefficients, the group delay time, the magnetoresistance (MR), and the giant magnetoresistance (GMR) in a ferromagnetic graphene structure in the presence of Rashba SOI are described in Sect. 2. We present and discuss our results in Sect. 3, and we end the paper with a brief summary and conclusion study on MR with a non-homogenous Rashba barrier.

2. Model and formula

In the present study we consider a graphene structure in which the barrier region with Rashba SOI is separated by ferromagnetic graphene with no Rashba SOI. We model an FG/RFG/FG junction locating in the $x−y$ plane [37], with FG the ferromagnetic graphene region and RFG the ferromagnetic graphene layer with RSOC. The $x$-axis is supposed to be the growth direction, as shown in Fig. 1. We also assume that the dimensions of the graphene layer are large enough that we can neglect the edge effects. Here, the Rashba SOI originates from the effective electric field at the graphene and substrate interface.

At the low-energy limit, massless fermions near Dirac points obey the Dirac Hamiltonian

$$H_0 = -i\hbar v_F (\partial_x \sigma_x + \partial_y \sigma_y) S_0 - E_F,$$

(1)

where $\hbar$ is the Planck constant, $\sigma$ ($\sigma_x, \sigma_y$) are Pauli matrices in the pseudo-spin space, $S_0$ is the unit matrix in the electron spin space, and $v_F (= 10^6 \text{ m s}^{-1})$ is the Fermi velocity in graphene.
The Hamiltonian of regions (1) and (3) that have different ferromagnetic strength reads as
\[ H = H_0 + H_{\sigma 1(3)}, \]  
(2)
where \( H_{\sigma 1(3)} = h^{(\sigma)} \sigma \) is the exchange Hamiltonian in the ferromagnetic regions with different exchange interaction magnitude \( h^{(\sigma)} \) for regions (1) and (3).

The Hamiltonian in the central region (RFG) is
\[ H = H_0 + H_R, \]  
(3)
where \( H_R \) is the Rashba coupling and in the rotational symmetry state of the interfaces plane has the form
\[ H_R = \lambda_R S_x S_y \]  
where \( \lambda_R \) represents the Rashba SOI constant and \( S_x, S_y \) are the Pauli matrices in the spin subspace.

It is useful to write down the total Hamiltonian \( H = H_0 + H_R + H_{\sigma 1(3)} \) in the basis \( \{ \psi_A^\uparrow, \psi_B^\uparrow, \psi_A^\downarrow, \psi_B^\downarrow \} \) explicitly as
\[
H = \begin{pmatrix}
-h^{(\sigma)} & h \nu_F k e^{(-i\psi)} & 0 & 0 \\
h \nu_F k e^{(i\psi)} & -h^{(\sigma)} & i\lambda_R & 0 \\
0 & -i\lambda_R & h^{(\sigma)} & h \nu_F k e^{(-i\psi')} \\
0 & 0 & h \nu_F k e^{(i\psi')} & h^{(\sigma')}
\end{pmatrix}.
\]  
(4)

We assume a spin-up electron with energy \( E \) and wave vector \( k \) that propagates from the left ferromagnetic graphene and is incident with angle \( \varphi \) on the interface with the RSOC region.

The solution of the Dirac equation in Eq. (2) provides a simple expression for the spinor forms in the \( i \)th ferromagnetic region:
\[
\psi_{F_i}^\pm (x) = (\pm e^{\mp \frac{i\varphi}{2}} (0), \pm e^{\pm \frac{i\varphi}{2}} (0), 0 (e^{\pm \frac{i\varphi'}{2}}), 0 (e^{\pm \frac{i\varphi'}{2}})) \exp(\pm ik_{\downarrow(\downarrow)}x + ik_y y).
\]  
(5)

The ferromagnetic layers lead to splitting of the energies of the electrons into two sub-bands, so spin-dependent current is induced that passes through the heterojunctions.

The wave vectors and incident angles are defined as
\[
k_{\downarrow(\downarrow)} = \left| \frac{E - E_F + (-)h^{(\sigma)}}{h \nu_F} \right|, \quad \varphi' = \sin^{-1}(k \tan \varphi/k_{\downarrow}).
\]

The general solution to the Hamiltonian in the ferromagnetic regions is given by
\[
\psi (x) = A_1 \Psi_{F_i}^+ + A_2 \Psi_{F_i}^- + A_3 \Psi_{F_i}^- + A_1 \Psi_{F_i}^-\]  
(6)
where \( A_j (j = 1, 2, 3, 4) \) are the coefficient probabilities in the ferromagnetic regions.
The wave function of the Rashba layers is given by

\[ \psi(x) = \sum_{j=1}^{4} B_j \exp(i k_j x \cos \varphi_j) \left( \frac{\hbar v_F k_j}{E + h'} e^{-i \varphi_j} \left( E + h' - \frac{\hbar^2 v_F^2 k_j^2}{E + h'} \right) \frac{1}{i \lambda_R} \right) \]  

(7)

with following wave vectors and incident angles:

\[ q_{1(2)} = \sqrt{E^2 + h'^2 - (+)(E^2 + h'^2)^2 + (E^2 - h'^2)(h'^2 - E^2 + \lambda_R^2)^2} \frac{1}{\hbar v_f}, \]

\[ \varphi_j = \tan^{-1} \left[ k \tan \varphi / q_j \right], \]

where \( h' \) indicates the exchange interaction in Rashba layers, and \( B_j \) (\( j = 1, 2, 3, 4 \)) are the coefficients of the wave function for the Rashba ferromagnetic regions.

From the conservation of the transverse momentum in the interfaces we have

\[ k_F \sin \varphi = k_F^i \sin \varphi_i. \]  

(9)

By employing the conservation of the transverse momentum and the transition matrix (T-matrix) method, we can calculate the transmission amplitudes \( T \) and \( T' \), respectively, for the up and down spins.

The total group delay time \( \tau_g \) for a quasi-particle through the system is obtained from the spin- dependent transmission and reflection amplitudes as

\[ \tau_g = 1/2(\tau_{g\uparrow} + \tau_{g\downarrow}), \]

where

\[ \tau_{gs} = T^{\uparrow \rightarrow s} \tau_{gs\uparrow\uparrow} + R^{\uparrow \rightarrow s} \tau_{gs\uparrow\downarrow}, \]

\[ \tau_{gs\uparrow\uparrow} = \hbar \frac{d}{dE} (k_x^L) L(2N) + \theta_{t\uparrow\downarrow}, \quad \tau_{gs\uparrow\downarrow} = \hbar \frac{d}{dE} \theta_{r\uparrow\downarrow}. \]  

(10)

The index \( S \) shows spin up or down, \( \theta_{t\uparrow\downarrow} \) and \( \theta_{r\uparrow\downarrow} \) represent the arguments of the \( t_{\uparrow\downarrow} \) and \( r_{\uparrow\downarrow} \), respectively, and \( L(2N) \) is the length of the system that a particle, in the absence of the barrier, passes in time \( \hbar \frac{d}{dE} (k_x L(2N)). \)

2.1. Non-homogenous Rashba barrier

We now consider a graphene-based ferromagnetic–non-homogenous Rashba barrier–ferromagnetic heterostructure, and by using the T-matrix method we calculate the MR of the structure for different forms of non-homogenous Rashba barrier. It should be noted that the MR usually refers to the contribution of the external magnetic field or intrinsic magnetization to the resistivity. However, it is also used to refer to the effect of spin-dependent interactions such as SOCs or magnetic impurities by some authors [39]. In the current work, employing the second definition, we consider the magnetoresistance as the contribution of the Rashba interaction to the resistivity.
The T-matrix is extracted by wave function continuity at the junction interfaces [40]. This matrix can connect the wave functions of the two sides, i.e. it connects mth region to the (m + 1)th region. We divide the non-homogenous Rashba region into some homogenous Rashba regions and use the T-matrix method as follows:

$$
\begin{pmatrix}
A_{1}^{(m+1)} \\
A_{2}^{(m+1)} \\
A_{3}^{(m+1)} \\
A_{4}^{(m+1)}
\end{pmatrix} = M_{m+1,m}
\begin{pmatrix}
A_{1}^{(m)} \\
A_{2}^{(m)} \\
A_{3}^{(m)} \\
A_{4}^{(m)}
\end{pmatrix},
$$

(11)

where the transfer matrix is defined as

$$
M_{m+1,m} = (M_{m+1})^{-1}_{4 \times 4} \times (M_{m})_{4 \times 4}.
$$

(12)

$M_m$ is given by

$$
M_m = \begin{pmatrix}
e^{ik_1 \cdot x_m} W_1(x_m) & e^{ik_2 \cdot x_m} W_2(x_m) & e^{ik_3 \cdot x_m} W_3(x_m) & e^{ik_4 \cdot x_m} W_4(x_m) \\
e^{ik_1 \cdot x_m} X_1(x_m) & e^{ik_2 \cdot x_m} X_2(x_m) & e^{ik_3 \cdot x_m} X_3(x_m) & e^{ik_4 \cdot x_m} X_4(x_m) \\
e^{ik_1 \cdot x_m} Y_1(x_m) & e^{ik_2 \cdot x_m} Y_2(x_m) & e^{ik_3 \cdot x_m} Y_3(x_m) & e^{ik_4 \cdot x_m} Y_4(x_m)
\end{pmatrix}, \quad m = 1, \ldots, n,
$$

(13)

where

$$
W_j(x) = \frac{\hbar v_f k_j(x)}{E} e^{-i\phi_j},
$$

$$
X_j(x) = \left( E - \frac{\hbar^2 v_f^2 k_j(x)^2}{2} \right) \frac{1}{i\lambda_R(x)}, \quad Y_j(x) = W_j(x) \times X_j(x).
$$

Therefore, the transmission amplitudes for spin up ($t$) and down ($t'$) can easily be obtained by

$$
\begin{pmatrix}
1 \\
0 \\
r \\
r'
\end{pmatrix} = (N_{4 \times 4})^{-1} \left( \prod_i M_{i+1,i} \right)^{-1}_{4 \times 4} K_{4 \times 4} \begin{pmatrix}
t \\
0 \\
0 \\
r'
\end{pmatrix}.
$$

(14)

Here, $N$ and $K$ are the wave functions of the FG and RFG regions, respectively:

$$
N = \begin{pmatrix}
e^{ik \cdot x} e^{-i\phi'/2} & e^{-ik \cdot x} e^{-i(\pi - \phi)/2} & 0 & 0 \\
e^{ik \cdot x} e^{i\phi'/2} & e^{-ik \cdot x} e^{i(\pi - \phi)/2} & 0 & 0 \\
0 & 0 & e^{ik \cdot x} e^{-i\phi'/2} & e^{-ik \cdot x} e^{-i(\pi - \phi)/2} \\
0 & 0 & e^{ik \cdot x} e^{i\phi'/2} & e^{-ik \cdot x} e^{i(\pi - \phi)/2}
\end{pmatrix},
$$

$$
K = \begin{pmatrix}
e^{ik \cdot x} e^{i\phi'/2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & e^{ik \cdot x} e^{i\phi'/2} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.
$$

The conductance for the spin-up incident electrons is given by Ref. [2] as

$$
G = \int [T(\phi) + T'(\phi)] \cos(\phi) d\phi,
$$

(15)
where $T(\phi) = t^*t$ (for spin up) and $T'(\phi) = t^*t'$ (for spin down) are the spin-dependent transmission probabilities.

MR is defined as

$$
MR(\lambda) = \frac{G(\lambda = 0) - G(\lambda)}{G(\lambda)},
$$

and GMR is obtained from the transmission amplitudes with and without spin inversion, defined as

$$
GMR = \frac{T - T'}{T + T'}.
$$

3. **Numerical results and discussion**

We apply the T-matrix method to investigate the spin-dependent transmission probabilities and the group delay time through a single layer of ferromagnetic graphene with the Rashba SOI. We assume incoming electrons with up spin, energy $E$, and incident angle $\varphi$ propagated from the left ferromagnetic graphene to interface with Rashba spin–orbit region. The calculated results for this graphene structure are illustrated below.

As the incident energy is easily changed by controlling the gate voltage, we show the transmission probabilities as a function of the Rashba coupling strength $\lambda_R$ in Fig. 2 for different values of the incident electrons’ energy. As expected, both probabilities $T$ and $T'$ increase by increasing the electron energy $E$. Figure 2 demonstrates that by increasing the Rashba strength $\lambda_R$, the probabilities of transmission increase, then both decrease for high values of the Rashba coupling. Figures 2a and 2b illustrate that for the same system conditions, the spin flip probability is greater than the without spin flip probability, and electrons lose their spins in the transmission time. This property is very important

![Fig. 2. Transmission probabilities with and without spin-flip $T(T^\uparrow\rightarrow\uparrow)$ and $T'(T^\uparrow\rightarrow\downarrow)$ as a function of the Rashba coupling strength $\lambda_R$ for $\hbar = -1.5 \text{meV}$, $\hbar' = 2 \text{meV}$, $\hbar'' = 1 \text{meV}$, $\varphi = \frac{\pi}{4}$, $L = 4 \text{nm}$.](https://academic.oup.com/ptep/article-abstract/2020/7/073I01/5867885)
Fig. 3. Transmission probabilities $T(T^{↑→↑})$ and $T'(T^{↑→↓})$ as functions of the exchange energy of the Rashba region for $h' = -0.3$ meV, $h = -0.2$ meV, $L = 2$ mm, $\lambda_R = 0.6$, $\varphi = \pi/8$, and (a) $E = 4$ meV, (b) $E = 5$ meV.

Fig. 4. Spin-dependent transmission probabilities with and without spin inversion $T'(T^{↑→↓})$ as functions of the Rashba region length $L$ for $E = 3$ meV, $h' = 7$ meV, and $\lambda_R = 0.5$ meV.

and useful for magnetic information in fabrication. Therefore, RSOC acts as a spin-dependent barrier and transmission probabilities depend strongly on the Rashba coupling constant.

The variation of the spin-dependent transmission probabilities with the exchange energy of the central layer $h'$, the dependence of the probabilities on the incident electron energy $E$, and the touching of the two probabilities at $h' \approx +0.33$ meV are plotted in Fig. 3. It is important, indicating that the RSOC acts as a spin-dependent barrier and that transmission processes depend strongly on the incident energy and Rashba coupling constant. This figure shows that at high exchange energy ($|h'| \gtrsim \pm 0.33$ meV) the spin flip probability is greater than without spin flip, but at low values of the exchange energy ($|h'| \lesssim \pm 0.33$ meV) spin flip probability decreases, i.e. the electrons keep their spins with high probability in the transmission time.

The probabilities of electron transmission are plotted as functions of the Rashba region length $L$ in Fig. 4. The transmission $T$ oscillates with the length of the Rashba region as reported in Ref. [40]. The Rashba SOC acts like an effective magnetic field. When the carrier is moving in the $x$-direction, this effective magnetic field orients in the $y$-direction and therefore the spin of the electrons moving in the $x$-direction rotates along a Larmor-like circle around the $y$-axis. Figure 4 shows that the probability $T'$ takes higher values compared with the probability $T$ with respect to the Rashba region length $L$, so one can have great spin-transmission $T'(T^{↑→↓})$ control on the system.
The transmission probabilities $T$ and $T'$ are plotted as functions of $L$ and $\lambda_R$ in Fig. 5, which clearly shows the transmission coefficients' dependence on the Rashba coupling. We found that the transmission probabilities show an oscillatory behavior with increasing length and strength of the Rashba SOI.

That is to say, when the length and strength of the RSOC are large, due to the influence of the RSOC the present graphene structure changes into a more complex structure and so the frequency and amplitude of the oscillation are changed. This phenomenon originates from the interference effects between the electron waves, which have two different momenta in the RFG region. For a fixed Rashba strength, a finite RSOC will increase the difference between the two propagating modes. Thus, a small incident energy interval can achieve the response condition, which leads to a sharper oscillation showing.

We now present the group delay time $\tau_g$ as a function of the barrier width, strength, Rashba SOI exchange energy, and incident energy in Figs. 6–8. First, we plot $\tau_g$ as a function of $L$ for different values of the Rashba constant. It is found that for different Rashba strengths the electrons spend quite different times passing through the same barrier. The parameters are given in the caption of Fig. 6. For the FG/RFG/FG structure, increased amplitude of the $\tau_g$ oscillations is observed. That is to say, the maximum value of $\tau_g$ for $\lambda_R = 0.5$ meV and $L \approx 15.5$ nm is 2 ns, almost 20 times larger than for $\lambda_R = 0.2$ meV. Additionally, the amplitude of the $\tau_g$ oscillation depends on the distance $L$.

To further understand the electron tunneling properties through a barrier in monolayer graphene, two plots of $\tau_g$ as a function of the Rashba barrier thickness $L$ and (a) the Rashba constant and (b) the energy of the incident electron $E$ for $\lambda_R = 0.5$ meV, $h = -1$ meV, $h' = 0.04$ meV, $h'' = 0.04$ meV, and $\varphi = \pi/18$ are shown in Fig. 7.
Fig. 7. Group delay time $\tau_g$ as a function of (a) the region length and strength of the Rashba SOI for $E = 7\text{meV}$; (b) the energy $E$ and length of the Rashba region for $\lambda_R = 0.5\text{meV}$, $h = -1\text{meV}$, $h' = 0.04\text{meV}$, $h'' = 0.04\text{meV}$, and $\varphi = \pi/18$.

Fig. 8. Dependence of $\tau_g$ on the exchange energy at $E = 3\text{meV}$; $h = -0.2\text{meV}$, $h' = 0.1\text{meV}$ $\varphi = \pi/8$, $\lambda = 2\text{nm}$, $\lambda_R = 0.6\text{meV}$.

As shown in these figures, the group delay time $\tau_g$ takes constant values independent of $L$ when the barrier thickness becomes large enough, i.e., the Hartman effect [41]. This effect is the saturation of the $\tau_g$ with barrier length. Therefore, it cannot be a propagation delay and should not be associated with a traversal velocity.

In Fig. 8, where the dependence of $\tau_g$ on the barrier exchange energy at different incident angles $\varphi$ is presented, we clearly see that $\tau_g$ has the same value ($\approx 1\text{nm}$) for three angles at $h' = -0.4\text{meV}$, then $\tau_g$ shows different behavior for each angle by increasing $h'$. As can be seen, $\tau_g$ is not only an oscillating function of the strength and length of the Rashba region, it can also oscillate with a change in the exchange energy.

3.1. GMR in a graphene superlattice

In this section, exploiting the advantages of the graphene structure, we study a graphene-based superlattice of alternating Rashba coupling to gain full control of the transport properties and hence full control of the giant magnetoresistance. In order to investigate the possible GMR, which may create an oscillatory behavior, we begin with the spin-dependent transmission probabilities. $T'$ is plotted as a function of the Rashba strength in Fig. 9. We take the exchange energies $h = h'' = -0.1\text{meV}$, $h' = -0.3\text{meV}$, and the width of the barriers and wells as $L = 0.3\text{nm}$ and $d = 0.2\text{nm}$, respectively.

For a structure with more layers, decreased magnitude of the transmission probability is observed. Comparing the major figures with the inset ones shows that the decrease for $N = 7$ is more than for $N = 13$. We also find that transmission coefficient is modulated to zero at a large RSOI strength with increasing number of layers of the structure due to strong reflection. Therefore, the behavior...
Fig. 9. Transmission probability $T'$ as a function of the Rashba strength $\lambda_R$ for $E = 2 \text{meV}$, $\varphi = \frac{\pi}{4}$, $h' = 0.3 \text{meV}$, $h = -0.1 \text{meV}$, $L = 0.3 \text{nm}$, $d = 0.2 \text{nm}$.

of the transmission coefficients depends critically on $N$. We can easily control the Rashba coupling by the external field and thus control the spin-dependent transmission in a single layer of graphene superlattice.

We present the results for GMR in the following. The dependence of the GMR on the barrier numbers $N$, the energy $E$, the length of the Rashba region $L$, and the exchange energy $h'$ as functions of the Rashba coefficient $\lambda_R$ are presented in Fig. 10. It is clear that the GMR properties can be understood very well by the important fact that the transmission coefficients are related to the structure parameters. The difference between these curves obviously indicates the importance of these parameters, so it means that we can tune GMR by the transport properties of ferromagnetic graphene.

3.2. MR of the graphene layer with non-homogenous Rashba coupling

Some results of the numerical study of spin transport properties of the ferromagnetic–non-homogenous Rashba–ferromagnetic graphene heterostructure are presented in this section. The length of the Rashba region $L$ is taken to be $2.0 \text{nm}$, divided into $n$ different regions with nearly homogenous Rashba coupling strength.

In Fig. 11, the probability $T$ is plotted as a function of the incident angle $\varphi$ for $h = h' = 0.001 \text{meV}$, $h'' = -0.04 \text{meV}$, $E = 2 \text{meV}$ and $\lambda_0 = 0.01 \text{meV}$. Since the Rashba coupling depends on the wave vector, the asymmetry $T(\varphi) \neq T(-\varphi)$ increases with increasing $k_y$ (i.e. by increasing the incidence angle $\varphi$) [40]. It is noteworthy that this anisotropy indicates that in the present structure, some incidence directions are considered as spin selective paths. At low incident angles $|\varphi|$, the two solid and dashed curves (corresponding to non-homogenous Rashba couplings) show similar behavior, but in the limit of $|\varphi| \to \pi/2$, a significant distinction appears. This means that the functionality of the Rashba coupling with respect to particle position, $x$, has an important contribution in the transmission function $T$.

The results for the system MR are depicted in Figs. 12 and 13. One can infer how the Rashba coupling can control the MR of the system. The dependence of the MR as a function of a Rashba coupling with an exponential profile of the form $\lambda = \lambda_0 \exp(-2x)$ (main part of Fig. 12) and a constant Rashba coupling (inset of Fig. 12) is shown. This figure shows that the magnitude of the
Fig. 10. GMR as a function of the Rashba strength $\lambda_R$ for $E = 2 \text{ meV}$, $N = 7$, $\varphi = \frac{\pi}{3}$, $h' = 0.1 \text{ meV}$, (a) $h = -0.5 \text{ meV}$, $d = L = 0.2 \text{ nm}$, (b) $h = -0.5 \text{ meV}$, $L = 0.2 \text{ nm}$, $d = 0.4 \text{ nm}$, (c) $h = -0.3 \text{ meV}$, $d = 0.2 \text{ nm}$, and (d) $E = 2 \text{ meV}$, $h = -0.5 \text{ meV}$, $\varphi = \frac{\pi}{3}$, $N = 19$, $d = L = 0.2 \text{ nm}$.

Fig. 11. Transmission probability as a function of incident angle for different profiles of the Rashba coupling.

MR crucially depends on the strength of the RSOC. The magnitude of the MR has been suppressed in comparison to the case in which the Rashba coupling was assumed to be constant.

Meanwhile, remarkably enough, the MR changes with increasing Rashba strength (Fig. 12, inset), since the SOCs behave as a spin-dependent barrier which can modify the band shape, its anisotropy, and the population of a given state.

Figure 13 plots MR as a function of the Rashba coupling strength for different values of (a) the length of the Rashba region and (b) the exchange energy $h''$ for a Rashba coupling profile of $\lambda = \lambda_0 \cos(x)$. It is found that the effect of the non-homogeneous Rashba interaction and the
Fig. 12. MR as a function of Rashba strength $\lambda_0$ for an exponential Rashba coupling profile, $\lambda_R = \lambda_0 \exp(-2x)$ ($E = 2$ meV, $h' = -0.08$ meV, $h = h'' = 0.0001$ meV).

Fig. 13. MR as a function of Rashba strength $\lambda_0$ for a non-homogenous Rashba coupling profile $\lambda_R = \lambda_0 \cos(x)$ for different values of (a) Rashba region length with $h'' = -0.2$ and (b) exchange Hamiltonian $h''$ ($E = 4$ meV, $h = h' = -0.001$ meV).

modulation, its strength with respect to position $x$, manifests itself by decreasing the MR of the system.

As shown in this figure, when $\lambda_0$ changes from $\lambda_0 = 0.1$ meV towards higher values, the MR grows until it reaches a maximum value located at $\lambda_0 = 0.17$ meV. When $\lambda_0$ goes further, the MR decreases with increasing Rashba coupling. The length of the Rashba-varying region measures the effective time in which the Rashba field has been experienced by the moving electron. This effective time determines the amount of spin precession around the local field of the Rashba interaction. In other words, this time determines the spin orientation of electrons emerging from the left side when
they reach the ferromagnetic region (2). This could be considered an important deterministic factor, especially in a coherent transport regime. Accordingly, since the ferromagnetic region has a spin-selective role, either a change in the Rashba region length or a change of the exchange energy sign could result in a modification of the relative direction of the local magnetization and the incoming spins in the FG region (2).

4. Conclusion
The spectrum of electrons moving in a two-dimensional ferromagnetic graphene layer and interacting with a Rashba coupling has been calculated. The interaction is in the middle region of the graphene structure. Different structures of nonhomogeneous Rashba coupling were considered. By using the T-matrix method, the present study dealt with the group delay time, MR, and GMR of spin-polarized electron tunneling through the Rashba barrier in a single layer of graphene. In conclusion, we have shown that, for oblique incidence angles, the transmission probability, group delay time, and GMR are strongly dependent not only on the Rashba SOI strength and barrier width, but also on the exchange energies and incident electron energy. Furthermore, we have shown how the MR in a graphene-based structure can be tuned by the length and strength of the Rashba coupling and the exchange energy for different profiles of non-homogenous Rashba coupling. RSOC creates an effective magnetic field and can act as a spin-dependent barrier. According to our probes, the group delay time, MR, and GMR depend on the Rashba SOI strength, which is tunable. It can thus contribute to the thorough control of response velocity for graphene-based spintronic devices.

References
[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, Nature 438, 197 (2005).
[2] E. I. Rashba, Sov. Phys. Solid 2, 1109 (1960).
[3] Y. Zhang, Y.-W. Tan, H. L. Stormer, and P. Kim, Nature 438, 201 (2005).
[4] K. S. Novoselov, E. McCann, S. V. Morozov, V. I. Fal’ko, M. I. Katsnelson, U. Zeitler, D. Jiang, F. Schedin, and A. K. Geim, Nat. Phys. 2, 177 (2006).
[5] S. Murakami, N. Nagaosa, and S.-C. Zhang, Science 301, 1348 (2003).
[6] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
[7] M. R. Masir, P. Vasilopoulos, A. Matulis, and F. M. Peeters, Phys. Rev. B 77, 235443 (2008).
[8] M. R. Masir, P. Vasilopoulos, and F. M. Peeters, J. Phys.: Condens. Matt. 23, 315301 (2011).
[9] M. R. Masir, P. Vasilopoulos, and F. M. Peeters, Phys. Rev. B 79, 035409 (2009).
[10] M. R. Masir, P. Vasilopoulos, and F. M. Peeters, New J. Phys. 11, 095009 (2009).
[11] M. R. Masir, P. Vasilopoulos, and F. M. Peeters, Phys. Rev. B 79, 035409 (2009).
[12] H. Haugen, D. Huertas-Hernando, and A. Brataas, Phys. Rev. B 77, 115406 (2008).
[13] L. Dell’Anna and A. De Martino, Phys. Rev. B 80, 155416 (2009).
[14] M. Barbier, P. Vasilopoulos, and F. M. Peeters, Phil. Trans. R. Soc. A 368, 5499 (2010).
[15] M. Barbier, F. M. Peeters, P. Vasilopoulos, and J. Milton Pereira, Jr., Phys. Rev. B 77, 115446 (2008).
[16] H. Zhang, C. Lazo, S. Blügel, S. Heinze, and Y. Mokrousov, Phys. Rev. Lett. 108, 056802 (2012).
[17] Q. Shifei, C. Hua, X. Xiaoqiong, and Z. Zhenyu, Carbon 61, 609 (2013).
[18] M. Li, Z. B. Zhao, and L. B. Fan, Phys. Scr. 90, 015806 (2015).
[19] L. Lafetá, A. R. Cadore, T. G. Mendes-de-Sa, K. Watanabe, T. Taniguchi, L. C. Campos, A. Jorio, and L. M. Malard, Nano. Lett. 17, 3447 (2017).
[20] C. Bai, Y. Yang, and X. Zhang, Appl. Phys. Lett. 92, 102513 (2008).
[21] C. Bai, Y. Yang, and X. Zhang, Phys. Rev. B 80, 235423 (2009).
[22] D. Sun, K. J. van Schooten, M. Kavand, H. Malissa, Ch. Zhang, M. Groesbeck, Ch. Boehme, and Z. Vary Vardeny, Nature Mat. 15, 863 (2016).
[23] S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).
[24] S. Ryu, L. Liu, S. Berciaud, Y.-J. Yu, H. Liu, P. Kim, G. W. Flynn, and L. E. Brus, Nano Lett. 10, 4944 (2010).
[25] Yu. S. Dedkov, M. Fonin, U. Rüdiger, and C. Laubschat, Phys. Rev. Lett. 100, 107602 (2008).
[26] Th. Martin and R. Landauer, Phys. Rev. A 45, 2611 (1992).
[27] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. 61, 917 (1989).
[28] M. Büttiker, Phys. Rev. B 27, 6178 (1983).
[29] H. G. Winful, Phys. Rev. Lett. 91, 260401 (2003).
[30] J. C. Martinez and E. Polatdemir, Phys. Lett. A 351, 31 (2006).
[31] J. R. Fletcher, J. Phys. C: Solid State Phys. 18, L55 (1985).
[32] T. E. Hartman, J. Appl. Phys. 33, 3427 (1962).
[33] Y. Aharonov, N. Erez, and B. Reznik, Phys. Rev. A 65, 052124 (2002).
[34] P. Pereyra, Phys. Rev. Lett. 84, 1772 (2000).
[35] H.-C. Wu, Y. Guo, X.-Y. Chen, and B.-L. Gu, J. Appl. Phys. 93, 5316 (2003).
[36] F. Sattari, Appl. Phys. A 117, 1963 (2014).
[37] E. Faizabadi and F. Sattari, J. Appl. Phys. 111, 093724 (2012).
[38] K. S. Yi, D. Kim, and K.-S. Park, Phys. Rev. B 76, 115410 (2007).
[39] K. Výborný, A. A. Kovalev, J. Sinova, and T. Jungwirth, Phys. Rev. B 79, 045427 (2009).
[40] K. Hasanirokh, H. Mohammadpour, and A. Phirouznia, Physica E 56, 227 (2014).
[41] K. Hasanirokh, H. Mohammadpour, M. Esmaelpour, and A. Phirouznia, Physica E 74, 30 (2015).