Entropic corrections to Friedmann equations and bouncing universe due to the zero-point length

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We employ Verlinde’s entropic force scenario to extract the modified Friedmann equations by taking into account the zero-point length correction to the gravitational potential. Starting from the modified gravitational potential due to the zero-point length, we first find the logarithmic corrections to the entropy expression and then we derive the modified Friedmann equations. Interestingly enough, we observe that the corrected Friedmann equations are similar to the Friedmann equations in braneworld scenario. In addition, from the corrected Friedmann equations, we pointed out a possible connection to the GUP principle which might have implications on the Hubble tension. To this end, we discuss the evolution of the scale factor under the effect of zero-point length. Finally, having in mind that the minimal length is of the Planck order, we obtain the critical density and the bouncing behavior of the universe with a critical density and a minimal scale factor of the order of Planck length.

I. INTRODUCTION

Since the discovery of black holes thermodynamics in 1970’s [1], physicists have been speculating that there should be a profound connection between the gravitational field equations and the law of thermodynamics. This is due to the fact that thermodynamic quantities such as entropy and temperature are, respectively, proportional to the horizon area and surface gravity, which are pure geometrical quantities. Jacobson was the first who disclosed that Einstein field equation of gravity is indeed an equation of state for the spacetime [2]. According to Jacobson’s argument, one can derive the hyperbolic second order partial differential equations of general relativity by starting from the Clausius relation $\delta Q = T \delta S$, together with the relation between horizon area and entropy [2]. Jacobson’s discovery was a great step toward understanding the nature of gravity and supports the idea that gravitational field equations are nothing but the first law of thermodynamics for the spacetime. The profound connection between the first law of thermodynamics and the gravitational field equations were also generalized to other gravity theories including $f(R)$ gravity [3], Gauss-Bonnet gravity, the scalar-tensor gravity, and more general Lovelock gravity [4, 5]. In the cosmological background, it has been shown that the differential form of the Friedmann equation on the apparent horizon can be rewritten in the form of the first law of thermodynamics and vice versa [6–14]. Although Jacobson’s derivation is logically clear and theoretically sound, the statistical mechanical origin of the thermodynamic nature of gravity remains obscure.

The next great step towards understanding the nature of gravity put forwarded by Verlinde who claimed that gravity is not a fundamental force and can be regarded as an entropic force [15]. Verlinde proposal based on two principles. The equipartition law of energy for the degrees of freedom of the system and the holographic principle. Using these two principles, he derived the Newton’s law of gravity, the Poisson equations and in the relativistic regime, the Einstein field equations of general relativity. Similar discoveries were also made by Padmanabhan [16] who observed that the equipartition law for horizon degrees of freedom combined with the Smarr formula leads to the Newton’s law of gravity. This may imply that the entropy is to link general relativity with the statistical description of unknown spacetime microscopic structure when the horizon is present.

It is important to note that Verlinde’s proposal changed our understanding on the origin and nature of gravity, but it considers the gravitational field equations as the equations of an emergent phenomenon and leave the spacetime as a background geometry which already exists. In line with studies to understand the nature of gravity, Padmanabhan [17] argued that the spacial expansion of our Universe can be understood as a consequence of the emergence of space. Equating the difference between the number of degrees of freedom in the bulk and on the boundary with the volume change, he extracted the Friedmann equation describing the evolution of the Universe [17]. The idea of emergence spacetime was also extended to Gauss-Bonnet, Lovelock gravities [18–20].

In the present work, we adopt the viewpoint of Verlinde and consider gravity as an entropic force caused by the changes in the information of the system. Using this scenario, we shall investigate the effects of zero-point length corrections to the gravitational potential on the cosmological field equations. The concept of duality and zero point length was derived on the framework of quantum gravity by Padmanabhan [21]. In his reasoning, the spacetime manifold can be taken as a large distance limit of the main quantum spacetime. In this manner, the discrete to continuum transition should have a memory of the fluctuations of the quantum spacetime. It was recently used to obtain black hole solutions [22–25]. In order to incorporate such quantum gravity effects we need to know the modified entropy. Toward this goal, we shall use Verlinde’s entropic force scenario [15] to obtain the corrected entropy.

This paper is outlined as follows. In Section II, we derive the corrected entropy and the modified Friedmann equations. In Section III, we discuss the evolution of the scale factor under the effect of zero-point length. In Section IV, we explore the bouncing behavior of the universe with the critical density and a minimal scale factor of the order of Planck length.
We comment on our results in Section IV. Through the paper we set $G = c = \hbar = 1$.

II. ENTRISTIC CORRECTIONS TO FRIEDMANN EQUATIONS

According to Verlinde’s argument, when a test particle or excitation moves apart from the holographic screen, the magnitude of the entropic force on this body has the form

$$ F = T \frac{\Delta S}{\Delta x}, \quad (1) $$

in this equation $\Delta x$ gives the displacement of the particle from the holographic screen, on the other hand $T$ and $\Delta S$ are the temperature and the entropy change on the screen, respectively. Verlinde’s derivation of Newton’s law of gravitation at the very least offers a strong analogy with a well understood statistical mechanism. It is important to note that in Verlinde discussion, the black hole entropy $S$ plays a crucial role in the derivation of Newton’s law of gravitation. Indeed, the derivation of Newton’s law of gravity depends on the entropy-area relationship $S = A/4$ of black holes in Einstein’s gravity, where $A = 4\pi R^2$ represents the area of the horizon. However, this definition can be modified from the inclusion of quantum effects, here we shall use the following modification [26, 27]

$$ S = \frac{A}{4} + S(A). \quad (2) $$

It can be seen that if we take the second term zero, the standard Bekenstein-Hawking result is reproduced. Assuming a test mass $m$ with a distance $\Delta x = \eta \lambda_m$ away from the surface $\Sigma$, where $\lambda_m$ is the reduced Compton wavelength of the particle given by $\lambda_m = 1/m$ in natural units, $\eta$ is some constant of proportionality, then the entropy of the surface changes by one fundamental unit $\Delta S$ fixed by the discrete spectrum of the area of the surface via the relation

$$ dS = \frac{\partial S}{\partial A} dA = \left[ \frac{1}{4} + \frac{\partial S}{\partial A} \right] dA. \quad (3) $$

Here we note that the energy of the surface $S$ is identified with the relativistic rest mass $M$ of the source mass:

$$ E = M. \quad (4) $$

On the surface $\Sigma$, we can relate the area of the surface to the number of bytes according to

$$ A = QN, \quad (5) $$

where $Q$ is a fundamental constant and $N$ is the number of bytes. Let us assume that the temperature on the surface is $T$, by means of the equipartition law of energy [28], we get the total energy on the surface via

$$ E = \frac{1}{2} N k_B T. \quad (6) $$

We also need the force, which, according to this picture, it is the entropic force obtained from the thermodynamic equation of state

$$ F = T \frac{\Delta S}{\Delta x}, \quad (7) $$

where $\Delta S$ is one fundamental unit of entropy when $|\Delta x| = \eta \lambda_m$, and the entropy gradient points radially from the outside of the surface to inside. Note that we have $\Delta N = 1$, hence from (5) we have $\Delta A = Q$. Now, we are in a position to derive the entropic-corrected Newton’s law of gravity. Combining Eqs. (3)-(7), we can get $F$ using $\Delta S \cong dS$, furthermore as we noted $\Delta x = \eta \lambda_m = \eta/m$ and $T$ is found from Eq. (6), where $N = A/Q$ and $T = MQ/(2\pi k_B R^2)$, we get

$$ F = - \frac{Mm}{R^2} \left( \frac{Q^2}{2\pi k_B \eta} \right) \left[ \frac{1}{4} + \frac{\partial S}{\partial A} \right]_{A=4\pi R^2}, \quad (8) $$

This is nothing but the Newton’s law of gravitation to the first order provided we define $\eta = 1/8\pi k_B$, we get $Q^2 = 1$. Thus we reach

$$ F = - \frac{Mm}{R^2} \left[ 1 + 4 \frac{\partial S}{\partial A} \right]_{A=4\pi R^2}, \quad (9) $$

In T-duality, it was argued that the gravitational potential due to the zero-point length is modified as [22, 24]

$$ \phi(r) = - \frac{M}{\sqrt{r^2 + l_0^2}} \bigg|_{r=R}, \quad (10) $$

where $l_0$ is the zero-point length and its value is expected to be of Planck length (see, [22]). Now by using the relation $F = -m \nabla \phi(r)|_{r=R}$, we obtain the modified Newton’s law of gravitation as

$$ F = - \frac{Mm}{R^2} \left[ 1 + \frac{l_0^2}{R^2} \right]^{-3/2}. \quad (11) $$

Thus, with the correction in the entropy expression, we see that

$$ 1 + \left( \frac{1}{2\pi R} \right) \frac{dS}{dR} = \left[ 1 + \frac{l_0^2}{R^2} \right]^{-3/2} \quad (12) $$

Solving the above equation, for the entropy we obtain

$$ S = \pi R^2 + S = \pi R^2 \left( 1 + \frac{l_0^2}{R^2} \right)^{-1/2} + 3\pi l_0^2 \left( 1 + \frac{l_0^2}{R^2} \right)^{-1/2} - 3\pi l_0^2 \ln(R + \sqrt{R^2 + l_0^2}), \quad (13) $$

It is very interesting to see that we obtained the log corrections to the entropy in accordance with quantum effects. This is the first important results in the present work. Let us now extend our discussion to the cosmological setup. Assuming the background spacetime to be spatially homogeneous and
isotropic which is given by the Friedmann-Robertson-Walker (FRW) metric
\[ds^2 = h_{\mu\nu}dx^\mu dx^\nu + R^2(d\theta^2 + \sin^2 \theta d\phi^2),\]  
where \( R = a(t)r, \) \( x^0 = t, x^1 = r, \) the two dimensional metric
\[h_{\mu\nu} = \text{diag}(-1, a^2/(1 - kr^2))\]  
Here \( k \) denotes the curvature of space with \( k = 0, 1, -1 \) corresponding to flat, closed, and open universes, respectively.

The dynamical apparent horizon, a marginally trapped surface responding to flat, closed, and open universes, respectively. Here \( k \) denotes the curvature of space with \( k = 0, 1, -1 \) corresponding to flat, closed, and open universes, respectively. The dynamical apparent horizon radius for the FRW universe is determined by the relation
\[R = \frac{\dot{a}r}{a^2} = \frac{1}{\sqrt{H^2 + k/a^2}}.\]  
A simple calculation gives the apparent horizon radius for the FRW universe
\[R = \frac{a}{a} = \frac{1}{\sqrt{H^2 + k/a^2}}.\]  
For the matter source in the FRW universe we shall assume a perfect fluid described by the stress-energy tensor
\[T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}.\]  
On the other hand, the total mass \( M = \rho V \) in the region enclosed by the boundary \( S \) is no longer conserved, one can compute the change in the total mass using the pressure \( dM = -p dV, \) and this leads to the continuity equation
\[\dot{\rho} + 3H(\rho + p) = 0,\]  
with \( H = \dot{a}/a \) being the Hubble parameter. Let us now derive the dynamical equation for Newtonian cosmology. Toward this goal, let us consider a compact spatial region \( V \) with a compact boundary \( S, \) which is a sphere having radius \( R = a(t)r, \) where \( r \) is a dimensionless quantity. Going back and combining the second law of Newton for the test particle \( m \) near the surface, with gravitational force \( (11) \) we obtain
\[F = m\ddot{R} = m\ddot{r} = -\frac{Mm}{R^2} \left[ 1 + \frac{l_0^2}{R^2} \right]^{-3/2},\]  
We also assume \( \rho = M/V \) is the energy density of the matter inside the the volume \( V = \frac{4}{3}\pi a^3 r^3. \) Thus, Eq. \( (20) \) can be rewritten as
\[\ddot{a} = -\frac{4\pi}{3} \rho \left[ 1 + \frac{l_0^2}{R^2} \right]^{-3/2},\]  
this result represent the entropy-corrected dynamical equation for Newtonian cosmology. In order to derive the Friedmann equations of FRW universe in general relativity, we can use the active gravitational mass \( \mathcal{M}, \) rather than the total mass \( M. \) It follows that, due to the entropic corrections terms via the zero-point length, the active gravitational mass \( \mathcal{M} \) will be modified. Using Eq. \( (21) \) and replacing \( M \) with \( \mathcal{M}, \) it follows
\[\mathcal{M} = -\frac{\ddot{a}a^2 r^3}{3} \left[ 1 + \frac{l_0^4}{R^2} \right]^{3/2}\]  
In addition, for the active gravitational mass we can use the definition
\[\mathcal{M} = 2 \int_V dV \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) u^\mu u^\nu.\]  
From here it is not difficult to show the following result
\[\mathcal{M} = (\rho + 3p)\frac{4\pi}{3} a^3 r^3.\]  
By means of Eqs. \( (22) \) and \( (24) \) we find
\[\frac{\ddot{a}}{a} = -\frac{4\pi}{3} (\rho + 3p) \left[ 1 + \frac{l_0^2}{R^2} \right]^{-3/2}.\]  
This is the modified acceleration equation for the dynamical evolution of the FRW universe. To simplify the work, since \( l_0 \) is a very small number, we can consider a series expansion around \( l_0 \) via
\[\left[ 1 + \frac{l_0^2}{R^2} \right]^{-3/2} = 1 - \frac{3}{2} \frac{l_0^2}{R^2} + ...\]  
For the Friedmann equation we therefore obtain
\[\frac{\ddot{a}}{a} = -\frac{4\pi}{3} (\rho + 3p) \left[ 1 + \frac{l_0^4}{2 R^2 a^2} + ... \right].\]  
Next, by multiplying \( 2\dot{a}\ddot{a} \) on both sides of Eq. \( (25) \), and by means of continuity equation \( (19) \), we obtain
\[\ddot{a}^2 + k = \frac{8\pi}{3} \rho \left[ 1 - \frac{l_0^2}{2 R^2 a^2} + ... \right],\]  
where \( k \) is a constant of integration and physically characterizes the curvature of space. Using the expression for density
\[\rho = \rho_0 a^{-3(1+\omega)},\]  
we obtain in leading order terms
\[H^2 + k \frac{a^2}{a^2} = \frac{8\pi}{3} \rho \left[ 1 - \frac{l_0^2}{2 R^2 a^2} + ... \right].\]  
It is convenient, to simplify the above equation by rewriting it as
\[H^2 + k \frac{a^2}{a^2} = \frac{8\pi}{3} \rho \left[ 1 - \Gamma \rho \right],\]  
where \( \Gamma \) is a constant defined as
\[\Gamma = \frac{4\pi l_0^2}{3} \left( 1 + 3\omega \right).\]  
We see that in the limit \( l_0 \rightarrow 0 \) the standard Friedmann equation is obtained. Eq. \( (31) \) is similar to the Friedmann equations in braneworld scenario \( [30] \) (see, in particular \( [12, 31] \)).

This is one of the most interesting results found in the present paper, and the correspondence or the link between these models can be seen from the fact that string theory has a
T-duality symmetry relating circle compactifications of large and small radius. Put in other words, T-duality, identifies string theories on higher-dimensional spacetimes with mutually inverse compactification radii. The idea of extra spatial dimensions is not new and goes back to Kaluza and Klein. As we noted, these extra dimensions can be compactified for example on a small enough radius (in the traditional Kaluza-Klein sense), however, these extra dimensions can be also large, in the sense that the ordinary matter is confined onto a three-dimensional subspace, known as the brane, embedded in a larger space, known as the bulk. The idea of these large extra dimension, was used in braneworld cosmology to explain the weakness of gravity relative to the other fundamental forces of nature.

From the modified equations we can see that there is an apparent singularity encoded in $\Gamma$ and an asymptotic behavior is obtained when $\omega \rightarrow -1$ then $\Gamma \rightarrow \infty$, which signals a phase transition of the universe. To summarize, we derived the entropy-corrected Friedmann equation of FRW universe by considering gravity as an entropic force caused by changes in the information associated with the positions of material bodies. From the entropy-corrected Friedmann equation of the FRW universe we expect these corrections to play an important role in the early stage of the universe. Furthermore, we can derive the modified Raychaudhuri equation using

$$\dot{H} = -H^2 + \frac{\ddot{a}}{a},$$

(33)

This equation can also be written as

$$\dot{H} = \frac{k}{a^2} - \frac{8\pi}{3} \left\{ \rho [1 - \Gamma \rho] + \frac{\rho + 3\rho}{2 \left[ 1 + l_0^2 \left( H^2 + \frac{k}{a^2} \right) \right]^{3/2}} \right\}.$$  

(34)

In addition, there is also the deceleration parameter which is defined as

$$q = -1 - \frac{\dot{H}}{H^2},$$

(35)

where depending on the sign of $q$ we can have deceleration or acceleration scenario, respectively. Let us show another interesting results from Eq. (31) and for simplicity we choose a flat universe with $k = 0$ along with the definition $H_0^2 = 8\pi\rho/3$. From Eq. (31) we obtain

$$H = H_0 \left( 1 - \alpha H_0^2 a_0 \right)^{1/2},$$

(36)

where $\alpha$ is some constant of proportionality. Considering a series expansion around $l_0$, we obtain

$$H \simeq H_0 \left( 1 - \Delta_{GUP} H_0^2 \right),$$

(37)

where $H$ has the form of a Generalized Uncertainty Principle (GUP) modified Hubble parameter with $\Delta_{GUP} = a H_0^2/2$. It is expected that the Cosmic Microwave Background (CMB) carries fingerprints of quantum gravity therefore for $H$ we can take the value reported by the Planck collaboration, $H = 67.40 \pm 0.50$ km s$^{-1}$ Mpc$^{-1}$ [33]. On the other hand $H_0$ can be viewed as the unmodified Hubble parameter, and we can take the value reported by the Hubble Space Telescope (HST), $H_0 = 74.03 \pm 1.42$ km s$^{-1}$ Mpc$^{-1}$ [34]. This problem of disagreement between the current value of Hubble parameter is known as the Hubble tension in modern cosmology. The last equation can also be written as

$$\frac{H_0 - H}{H_0^3} = \Delta_{GUP}.$$

(38)

Quite interesting, if we further replace $\Delta_{GUP} = 432\lambda_2 a^4$, the last equation coincides exactly with the GUP modified Hubble parameter given in [37]. The GUP modified Hubble parameter can have interesting implications in cosmology. In particular, according to this picture, one way to see the Hubble tension is to associate the corrected CMB Hubble parameter ($H$) to be the one measured by the Planck collaboration which uses the CMB data. This is related to the fact that $H$ is expected to encode the quantum gravity effects measured by CMB. On the other hand, we associate the unmodified Hubble parameter ($H_0$), to be measured by the HST group which uses the SNeIa data. That being said, one can now use the values for the Hubble parameter given in [33, 34] along with Eq. (38), to constrain the GUP parameter $\Delta_{GUP}$ (see, [37]). Here we note that other authors as well studied the cosmological Hubble tension using the Heisenberg uncertainty, for example, the interested reader can see [38].

III. EVOLUTION OF THE SCALE FACTOR

Let us define the energy density due to the curvature via

$$\rho_{curv} = -\frac{3k}{8\pi a^2},$$

(39)

then, we can rewrite the Friedmann equation as

$$H^2 = \frac{8\pi}{3} \left[ \rho(1 - \Gamma \rho) + \rho_{curv} \right].$$

(40)

We are going to consider first the case of flat universe ($\rho_{curv} = 0$). To solve the above equation we can take $\rho = \rho_0 a^{-n}$. For a matter dominated universe we have $n = 3$, which leads to

$$a(t) \sim \left( t^2 + \frac{\Gamma}{6\pi} \right)^{1/3}.$$ 

(41)

In the limit $\Gamma \rightarrow 0$ we get the standard law $a(t) \sim t^{3/2}$. For a radiation dominated we have $n = 4$, which leads to

$$a(t) \sim \left( t^2 + \frac{3\Gamma}{32\pi} \right)^{1/4}.$$ 

(42)

In the limit $\Gamma \rightarrow 0$, again we get the standard law $a(t) \sim t^{3/2}$. Finally, we can find the scale factor for a vacuum dominated universe if we can set $n = 0$, which leads to

$$a(t) \sim e^{C\sqrt{(1 - \Gamma) \rho_0} t},$$

(43)

where $C$ is a constant. In the limit $\Gamma \rightarrow 0$, we can identify $C = H$, yielding $a(t) \sim e^{Ht}$. There is an interesting special case if we consider a universe with curvature dominated
energy and say we take $\rho = 3k/8\pi a^2$, which cancels out the term $\rho_{\text{curv.}}$, then we get

$$a(t) \sim (-k^2 T^2)^{1/4}, \quad (44)$$

which make sense only for $k = \pm 1$, along with a specific domain for the parameter $\omega$. Note also that such a scale factor, is absent in the classical picture with the limit $l_0 \to 0$.

**IV. BOUNCING BEHAVIOR OF THE MODIFIED FRIEDMANN EQUATIONS**

In this section, we are going to use some of these equations, to study the bouncing behavior of the modified Friedmann equations. According to the bouncing paradigms, there might be a nonsingular connection between the contraction phase and the expansion phase. During this phase change the universe goes through its minimal value with non-vanishing value referred as a critical point. During this process, quantum effects can play a dominant role and prevent the universe from collapsing into a singularity and then drive our universe to accelerate expansion. Mathematically, at the critical point we can write the minimal nonzero value ($a_0 > 0$) which has $H|_{a=a_0} = 0$ along with the condition $\dot{a}_0 > 0$ (see, [32, 39]). Setting $H|_{a=a_0} = 0$, from Eq. (31) we find two branches of solution for the critical density

$$\rho_{\text{crit.}} = \frac{1}{2!} \left(1 \pm \sqrt{1 - \frac{3\Gamma k}{2\pi a_0^2}}\right). \quad (45)$$

Here we further impose the condition $1 - 3\Gamma k/2\pi a_0^2 \geq 0$, from where we find the minimal quantity for the scale factor

$$a_0 = \pm \sqrt{\frac{3\Gamma k}{2\pi}}. \quad (46)$$

Furthermore, we can take as a physical solution only the positive branch and having in mind Eq. (32) we obtain

$$a_0 = \sqrt{2} l_0 \sqrt{\frac{k(1 + 3\omega)}{1 + \omega}}. \quad (47)$$

Since $l_0$ is of the order of Planck length, we see that the universe has bouncing behavior at Planck length. As a first example, if we set $k = 0$, we see that the first condition $a_0 > 0$ is not satisfied, hence the bouncing behavior for the flat universe is absent. For a closed universe with $k = +1$, we get

$$a_0 = \sqrt{2} l_0 \sqrt{\frac{1 + 3\omega}{1 + \omega}}, \quad (48)$$

provided $(1 + 3\omega)/(1 + \omega) > 0$. On the other hand for $k = -1$, in order to get $a_0 > 0$, we obtain

$$a_0 = \sqrt{2} l_0 \sqrt{\frac{1 + 3\omega}{1 + \omega}}, \quad (49)$$

provided $(1 + 3\omega)/(1 + \omega) < 0$. Now if we further take at the minimal value $a = a_0$, from Eq. (27) we obtain

$$\frac{\ddot{a}_0}{a_0} = -\left(\frac{4\pi}{3}\right) (\rho_{\text{crit.}} + 3p_{\text{crit.}}) \left[1 - \frac{3 + \omega}{4(1 + 3\omega)} + \ldots\right]. \quad (50)$$

In order to get the condition $\dot{a}_0 > 0$, we can see that there are two possibilities: First possibility is to set $\rho_{\text{crit.}} + 3p_{\text{crit.}} < 0$ and $1 - 3(1 + \omega)/4(1 + 3\omega) > 0$. Second possibility, however, is to take $\rho_{\text{crit.}} + 3p_{\text{crit.}} > 0$ and $1 - 3(1 + \omega)/4(1 + 3\omega) < 0$. In what follows we shall use the second possibility. Let us take now for example the closed universe with $k = +1$, then from the condition $a_0 > 0$, it follows the interval $\omega \in (-\infty, -1) \cup (-1/3, \infty)$, while from the condition $\dot{a}_0 > 0$, we get the interval $\omega \in (-1/3, -1/9)$. This means that only in the interval $\omega \in (-1/3, -1/9)$, a closed universe has bouncing behavior. Finally, let us consider the scenario having a universe with $k = -1$, then from the condition $a_0 > 0$ we get the interval $\omega \in (-1, -1/3)$, while from the condition $\dot{a}_0 > 0$ it follows $\omega \in (-1/3, -1/9)$. We conclude that, the bouncing behavior not possible in this case. We found that there is a great level of similarity between the results found in the present work and the bouncing behavior due to the modified dispersion relation [32, 39]. In particular, it was found for the bouncing condition $\dot{a}_0/a_0 = 1/(\eta_0^2 l_p^2)$, where $\eta_0$ is some dimensionless parameter which is bigger than zero and $l_p$ is the Planck length. This suggest that a closed universe may perform bouncing in the high energy limit when the role of zero-point length is expected to be important. We can show that this conclusion follows directly from our Eq. (50), where we have the proportionality $\dot{a}_0/a_0 \sim \rho_{\text{crit.}} + 3p_{\text{crit.}}$. Furthermore, in this regime, we have

$$\frac{\ddot{a}_0}{a_0} \sim \frac{1}{\Gamma} \sim \frac{1}{l_0^2}, \quad (51)$$

where the zero-point length can be identified with the Planck length $l_0 \sim l_p$ (see for more details [22]). Solving the last equation using $a_0(t) \sim e^{M}$, we obtain for the scale factor

$$a_0(t) \sim A_1 \exp \left(\frac{C}{l_0} t\right) + A_2 \exp \left(-\frac{C}{l_0} t\right), \quad (52)$$

where $C$, $A_1$ and $A_2$ are some constants. Having in mind that there is a minimal value for $a_0$ (see Eq. (47)), at the initial time $t = 0$, we have to set $A_2 = 0$, then we end up with

$$a_0(t) \sim A_1 \exp \left(\frac{C}{l_0} t\right), \quad (53)$$

where $t$ is the time of universe bounce and $A_1$ is proportional to $l_0$. By taking $C \sim l_0^{-1}$ and $l_0 \sim l_p \sim 10^{-35}$ m, one can obtain the huge expansion of the early universe via exponential function. It is interesting that Eq. (52) may suggest that when the universe reaches the minimal scale length, such a state is not stable and, we end up with the bouncing universe. The full mechanism is yet to be found and it is outside the scope of this work. The second term in this equation can describe the inverse scenario, namely, a state of matter that undergoes a collapsing processes. Due to the quantum gravitational bounce,
or zero point length effect, there is no singularity in such a universe described by the critical density. Let us now show that indeed the curvature scalars are regular. Using the Einstein field equations, one can find the Ricci scalar

\[ \mathcal{R} = 8\pi \rho(r). \]  

(54)

Considering now the condition \( \rho(r) = \rho_{\text{crit.}} \), at the minimal value for \( a_0 \), it can be shown that

\[ \mathcal{R} = \frac{8\pi}{2l_0^2} = \frac{3}{1 + 3\omega}. \]  

(55)

This shows that there is no singularity in the expression for the Ricci scalar, provided \( l_0 \) is not zero along with the condition \( \omega \in (-\infty, -1) \cup (-1/3, \infty) \). Note here that as was shown in \([41, 42]\), one can define the minimal length of GUP via \( l_0 = \alpha_0 l_p \) [where \( l_p \) is the Planck length] so \( l_0 \) is proportional to \( l_p \) and the limit to get standard results is to use \( \alpha_0 \). In particular, by taking the dimensionless GUP parameter \( \alpha_0 \to 0 \), we obtain the standard result. However, there is an apparent singularity due to the parameter \( \omega \to -1 \), which signals that the universe undergoes a phase transition in this stage. One can calculate one more scalar invariant, known as the Kretschmann scalar and can be computed via \([40]\)

\[ \mathcal{K} = (8\pi)^2 \left[ \frac{5}{3} \rho_{\text{crit.}}^2 + 2\rho_{\text{crit.}} \rho_{\text{crit.}} + 3\rho_{\text{crit.}}^2 \right]. \]  

(56)

From this equation, one can then show the following result

\[ \mathcal{K} = \frac{3}{l_0^2} \left( 1 + \omega \right)^2 \left( 5 + 6\omega + 9\omega^2 \right) \]  

(57)

Again, we can see from the last equation that there is no singularity due to \( l_0 \), but there are apparent singularities due to the parameter \( \omega \). As we already pointed out, these singularities are a result of the phase transition of the universe.

V. CONCLUSIONS

Adopting the concept of the zero-point length correction to the gravitational potential, and employing the entropic force scenario proposed by Verlinde, we computed the corrections to the Newton’s law of gravity as well as Friedmann equations. By assuming the effect of zero-point length on the gravitational potential, we find several important results.

Firstly, by using the modified Newton law of gravity and the Verlinde entropic force, we found logarithmic correction terms to the entropy and then we found that Friedman equations are indeed affected by the zero-point length.

Secondly, from the modified Friedmann equations we found a correspondence with the Friedmann equations in brane-world scenario. In addition to that, we observed that from the corrected Friedmann equations, we can obtain a form of GUP modified Hubble parameter similar to one reported in \([37]\) and we found an upper bound for the GUP parameter. This might have interesting implications, in particular the GUP modified Hubble parameter might shed some light on the Hubble tension problem in cosmology \([37]\). Specifically, the corrected CMB Hubble parameter \( (H) \) encodes quantum gravity effects and is expected to be measured by the Planck collaboration which uses the CMB data, while the unmodified Hubble parameter is identified to \( (H_0) \), and is expected to be measured by the HST group which uses the SNeIa data \([35, 36]\).

Thirdly, since the effect of \( l_0 \) is expected to play important role in the early universe, we found a bouncing behavior at the minimal value \( a = a_0 \) which is of the order of \( l_0 \). We have elaborated the bouncing condition and it is shown that it is possible only for the closed universe with \( k = +1 \). The bouncing scenario is linked to the high energy limit when the role of zero-point length is expected to be important. i.e., when \( \tilde{a}_0/a_0 \sim 1/l_0^2 \). Similar results were reported in \([32, 39]\) where the bouncing behavior was obtained from the modified dispersion relation. Finally, for the expansion of universe in this regime we found the exponential law

\[ a(t) \sim \exp \left( \mathcal{C}t/l_0 \right), \]  

and the curvature scalars are finite.

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