Buckling of composite sandwich cylindrical shell with lattice anisogrid core under hydrostatic pressure

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Abstract. Anisogrid structural elements offer significant advantages over conventional stringer-stiffened analogues due to an exceptional strength and stiffness of unidirectional composite ribs, which are used as the main load-bearing elements. The paper presents a numerical solution for buckling of a sandwich cylindrical shell with composite lattice core loaded with hydrostatic pressure. To model the buckling of the shell, the finite element method is used. A parametric analysis on the effect of the number of helical ribs and the orientation angle on the critical hydrostatic pressure is performed. It is revealed that the local, global or coupled buckling modes can be realized in the buckled shell depending on the geometric parameters of the lattice core structure. The parameters at which the shell provides the maximum mass efficiency are determined.

1. Introduction

Anisogrids are lattice structures consisting of a repeating symmetrical pattern of intersecting circumferential, helical and/or axial ribs [1]. Continuous filament winding is generally used to manufacture these structures. Owing to superior weight efficiency and damage tolerance properties, anisogrid structural elements have found a wide use in aviation, aerospace and marine industries. They offer significant advantages over conventional stringer-stiffened analogues resulting from an exceptional strength and stiffness of unidirectional composite ribs, which are used as the main load-bearing elements [2]. Particularly, the lattice structures demonstrate a high buckling resistance due to ability to redistribute the load in multiple paths [3]. The history of development of composite lattice structures, approaches for their design and manufacture are comprehensively reviewed in [1, 2].

Despite numerous analytical [4-6], numerical [7-9] and experimental [10-12] studies concerned with structural analysis of anisogrid lattice structures, some issues of their buckling behavior for a specific application are still open. Solving buckling problems is crucial as the nominal stress at the buckling failure can be much less than the ultimate compressive strength of the materials used, thus the overall load carrying capacity of lattice structures is significantly reduced.

Depending on the design requirements, there are two main types of composite lattice structures currently used for various applications - with and without the integrated skin layer(s). A payload attach fitting (adapter) and spokes of umbrella-type antenna are examples of the former type [1]. A three-layered cylindrical hull of the underwater vehicle with the bottoms attached at the ends is a representative of double-skin (sandwich) lattice structures. The core comprises of helical and circumferential ribs wound into grooves on a low-density foam.

Upon immersing, the external surface of underwater vehicle is exposed to hydrostatic pressure. At such conditions its overall structural performance is mostly determined by the buckling resistance of the cylindrical part, which is subjected to a combined load: external pressure acting on cylindrical
Several studies were focused on the buckling analysis of composite cylindrical shells subjected to external pressure. The buckling and postbuckling behavior of anisotropic multilayer cylindrical shells under the combined loading were analyzed in [13] using governing equations derived from classical shell theory and von Karman-Donnell strain displacement relations. The results revealed a pronounced effect of the laminate lay-up and Batdorf shell parameter on buckling failure modes. The influence of geometrical imperfection on the shell postbuckling response was also shown. Presented in [14] an analytical solution for buckling of composite cylindrical shells ended with rigid disks was obtained using Fourier decomposition and the Galerkin method assuming the membrane stress state before the buckling. Based on the analytical formulas, the buckling hydrostatic pressure for the shells with different combinations of elastic and geometric properties was calculated. Regardless the length of angle-ply laminated shells, the shells wound with helical angles close to 65° were found to deliver the maximum critical pressure. Buckling of laminated cylindrical shells loaded with hydrostatic pressure was also investigated in [15] using analytical and numerical approaches. The boundary conditions similar to those in [14] were employed. It was shown that the buckling pressure and buckling shape are strongly influenced by interrelationships between the ply angle and shell length. In particular, comparing the dependence of buckling pressure on the change of ply orientation for short and long shells, the higher sensitivity of former was demonstrated. Results of analytical and numerical buckling analysis of sandwich cylindrical shells with the isotropic core and orthotropic skins were presented in [16]. The effect of geometrical imperfections on bucking pressure for shells with simply supported ends under uniform lateral pressure was addressed. The buckling pressure reduction factor, which was computed as the ratio between critical loads for the imperfect shell for and the intact structure, evidenced that thick sandwich shells are more prone to buckling due to imperfections than thin ones.

This study aims to numerically investigate the buckling behavior of a sandwich cylindrical shell with anisogrid lattice core loaded with hydrostatic pressure. Finite element analysis is used to assess the value of the buckling pressure depending on the number of helical ribs and their orientation angle. The influence of geometrical parameters of the lattice core on the mass efficiency and resulting buckling modes is also addressed.

2. Finite element model

A three-dimensional numerical model of the sandwich cylinder is built using the MSC Nastran package [17] on the basis of finite element model for a periodic unit cell of the three-layer shell. The cell consists of a segment of the lattice structure, segments of the outer and inner skins and lightweight core segments. The anisogrid lattice consists of helical ribs oriented at ± φ angle with respect to the shell axis and circumferential ribs, which pass through the midpoints of the segments located between the intersection nodes of the helical ribs.

The steps to construct the unit cell model are shown in figure 1. First, the lattice structure segment model is created (Fig. 1a) followed by forming the corresponding segments of the inner (Fig. 1b) and outer skins (Fig. 1c). Then, fragments of the lightweight core are filled in the space between the skins and ribs (Fig. 1d) by means of the Extrude command. Linear three or four nodes plate elements are used to model the ribs of the lattice structure and skins, while six or eight nodes 3D solid elements are employed to represent the foam core. The finite element model of the unit cell is completed by specifying the mechanical properties of the materials used.

Owing to the periodicity of the unit cell, the finite element model of the whole sandwich cylindrical shell is prepared by copying the unit cell model in circumferential and axial directions. To simulate the stiff bottoms attached to the shell ends, finite elements of the Rigid type are used. A general view of the finite element shell model with imposed boundary conditions is shown in figure 2. The boundary conditions allow the movement of rigid disks towards each other when buckling of the shell occurs, while the disks remain orthogonal to the shell axis. The axial displacement of the shell is constrained at the midspan. The cylindrical surface of the shell is loaded with uniform pressure $P$. An axial force $F$
is applied at the center of each of the rigid disks, the value of which is calculated as $F = P\pi D^2/4$ (figure 3). Here $D$ - is the shell diameter.

![Image](image1.png)

Figure 1. Building the finite element model for the unit cell of the sandwich cylinder.

![Image](image2.png)

Figure 2. Finite element model of the sandwich cylinder with imposed boundary conditions.

![Image](image3.png)

Figure 3. Axial forces and uniform pressure applied to the finite element model.

3. Numerical analysis
Using the finite element model described above, the buckling pressure for composite sandwich shells with various parameters of the lattice core can be determined. Consider the shell with the length $L$ of 2 m and the diameter $D = 2$ m. The width of helical and circumferential ribs forming the lattice structure $\delta_1 = \delta_2 = 2$ mm, the height of the ribs (foam core thickness) $h = 20$ mm, the thickness of the skins $h_1 = h_2 = 1$ mm. The skins and ribs are made of a carbon fiber-reinforced polymer matrix composite. The following properties are used in calculations: the longitudinal and transverse elastic moduli $E_1 = 70$ GPa, $E_2 = 70$ GPa, in-plane shear modulus $G_{12} = 20$ GPa, Poisson's ratios $\nu_{12} = 0.32$, $\nu_{21} = 0.32$, density $\rho = 1550$ kg/m$^3$. The modulus of elasticity of the foam core is 30 MPa, density 100 kg/m$^3$. The angle of orientation of the helical ribs takes values of $15^\circ$, $20^\circ$, $25^\circ$ and $30^\circ$, while the number of pairs of helical ribs is 48, 72 and 96. Thus, the helical angle $\phi$ and the number of helical ribs $n_s$ are the
variable parameters in the calculations. The results of calculating the critical pressure $P_{cr}$ for the given sandwich cylindrical shells are given in Table 1.

**Table 1.** Buckling pressure (kPa) of sandwich cylindrical shells with anisogrid lattice core.

| $\phi$ | $n_r$ | 48   | 72   | 96   |
|--------|-------|------|------|------|
| 15°    | 325.33| 398.55| 501.81|
| 20°    | 367.63| 460.13| 576.68|
| 25°    | 415.35| 519.66| 636.92|
| 30°    | 483.23| 599.86| **662.60**|

As seen from table 1, the critical pressure increases with an increase in the orientation angle of helical ribs. It is not surprising that increasing their number also improves the buckling resistance of the shells.

Depending on the parameters of the lattice structure, different buckling mode shapes can be realized after the critical pressure is reached: a) global buckling, which is a collapse of the shell as the whole structure without buckling of the lattice elements (ribs); b) local buckling of ribs and c) coupled buckling, which occurs when the shell buckling and local buckling of ribs take place simultaneously. The shapes of sandwich cylindrical shells with lattice anisogrid core buckled under external pressure are presented in figure 4.

The occurrence of a particular buckling mode is highly attributed to how dense is the lattice structure. The buckling is likely to be global if the number of helical ribs and their orientation angle are high, while the local buckling of ribs is the dominant failure mode for sandwich shells with a sparse lattice core structure. The coupled buckling mode takes place at the boundary between the global and local modes. The influence of the lattice pattern on resulting buckling mode was also demonstrated in [8, 18].

![Figure 4](image-url)

**Figure 4.** Buckling mode shapes of the sandwich cylindrical shell with lattice anisogrid core: (a) local mode, (b) global mode, (c) coupled mode.
Determine the mass efficiency of the considered sandwich shells. For this purpose the mass efficiency coefficient is used in the following form:

\[ K_p = \frac{P_{cr}\pi DL}{Mg} \]

where \( g = 9.81 \text{ m/s}^2 \) is the gravitational acceleration, \( M \) is the shell mass.

The mass of shells with different geometric parameters of the lattice core are shown in table 2. The corresponding calculated values of the mass efficiency coefficients are listed in table 3.

### Table 2. Mass of sandwich cylindrical shells (kg).

| \( \phi \) | \( n_s \) | 48   | 72   | 96   |
|----------|----------|------|------|------|
| 15°      |          | 79.50| 87.23| 94.93|
| 20°      |          | 81.02| 89.75| 97.96|
| 25°      |          | 82.63| 91.91| 101.18|
| 30°      |          | 84.91| 94.74| 105.16|

### Table 3. Mass efficiency of sandwich cylindrical shells.

| \( \phi \) | \( n_s \) | 48   | 72   | 96   |
|----------|----------|------|------|------|
| 15°      |          | 5239.35| 5849.75| 6767.94|
| 20°      |          | 5809.51| 6563.97| 7537.14|
| 25°      |          | 6435.72| 7238.98| 8059.55|
| 30°      |          | 7286.44| 8106.57| 8067.18|

As clear from table 3, the shell with the lattice structure parameters \( n_s = 72 \) and \( \phi = 30^\circ \) is optimal from the point of view of mass efficiency as it provides the highest specific buckling resistance among those considered in the study.

### 4. Conclusions

The article presents a numerical solution to the buckling problem of a three-layer cylindrical shell exposed to external pressure, which core consists of helical and circumferential ribs wound into grooves on a low-density foam. The effect of the number of helical ribs and their orientation angle on the critical buckling pressure was investigated. The critical pressure was found to increase with an increase in the orientation angle of helical ribs. It was revealed that depending on the parameters of the lattice structure different buckling mode shapes can be realized, namely: shell global buckling, local buckling of ribs and coupled buckling. The shell with the lattice core structure having 72 helical rib pairs and oriented at 30° to the shell axis provides the highest mass efficiency. The develop approach to modeling a sandwich cylindrical shell with lattice anisogrid core can be used in the design of composite pressure hulls of underwater vehicles.
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