Anomalous wave propagation in quasiisotropic media

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Abstract

Based on boundary conditions and dispersion relations, the anomalous propagation of waves incident from regular isotropic media into quasiisotropic media is investigated. It is found that the anomalous negative refraction, anomalous total reflection and oblique total transmission can occur in the interface associated with quasiisotropic media. The Brewster angles of E- and H-polarized waves in quasiisotropic media are also discussed. It is shown that the propagation properties of waves in quasiisotropic media are significantly different from those in isotropic and anisotropic media.

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I. INTRODUCTION

In classic electrodynamics, it is well known that the electrodynamic properties of anisotropic materials are significantly different from those of isotropic materials. For anisotropic materials, one or both of the permittivity \( \varepsilon \) and the permeability \( \mu \) are second-rank tensors. The recent advent of a new class of material with negative permittivity and permeability has attracted much considerable attention [1, 2, 3, 4, 5, 6, 7]. Lindell et al. [8] have shown that anomalous negative refraction can occur at an interface associated with an anisotropic materials, which does not necessarily require that all tensor elements of \( \varepsilon \) and \( \mu \) have negative values. The studies of such anisotropic materials have recently received much interest and attention. Experimentally, an anisotropic metamaterial, which have negative parameters of \( \varepsilon \) and \( \mu \) tensors in the microwave regime, has been constructed and measured [10, 11]. Theoretically, anomalous negative refractions [8, 9, 10, 11, 12, 13, 14, 15], partial focus lens [16, 17, 18, 19], unusual quantum optical properties [20], oblique total transmission [21], and inverse Brewster angle [21, 22] can be realized by anisotropic metamaterials.

In general, E- and H-polarized waves propagate in different directions in anisotropic media. Now a question arises: whether there is a kind of anisotropic media in which E- and H-polarized waves propagate in the same direction? Recently, Shen et al. [23] proposed a new kind of anisotropic media, which was called quasiisotropic material. The quasiisotropic media have some special properties: First, E- and H-polarized waves have the same wave-vector surface. Second, in any allowed propagation directions, E- and H-polarized waves propagate with the same phase velocity and Poynting vector. Third, the wave vector and the Poynting vector generally do not coincide with direction.

In this work, we present a detailed investigation on the characteristics of electromagnetic wave propagation in quasiisotropic media. We are interested in understanding the condition under which the anisotropic media can be regarded as quasiisotropic one. To obtain a better physical picture of the total reflection and oblique total transmission, we introduce the Brewster angles of E- and H-polarized waves in a quasiisotropic media. We show that the propagation properties of waves in quasiisotropic media are significantly different from those in isotropic or anisotropic media.
II. DISPERSION RELATIONS OF QUASIISOTROPIC MEDIA

In this section, we will present a brief investigation on the refraction behavior at the interface between isotropic media and quasiisotropic media. For anisotropic materials, one or both of the permittivity and permeability are second-rank tensors. To simplify the proceeding analyses, we assume the permittivity and permeability tensors are simultaneously diagonalizable:

\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z \\
\end{pmatrix}, \quad \mu = \begin{pmatrix}
\mu_x & 0 & 0 \\
0 & \mu_y & 0 \\
0 & 0 & \mu_z \\
\end{pmatrix}.
\]

where \(\varepsilon_i\) and \(\mu_i\) are the permittivity and permeability constants in the principal coordinate system \((i = x, y, z)\). We consider the propagation of a planar wave of frequency \(\omega\) as \(E = E_0 e^{i k_i r - i \omega t}\) and \(H = H_0 e^{i k_i r - i \omega t}\), through an isotropic media toward a quasiisotropic LHM. In the principal coordinate system, Maxwell’s equations yield a scalar wave equation. In isotropic media, the accompanying dispersion relation has the familiar form

\[
k^2_i = \frac{\varepsilon_i \mu_i}{\varepsilon_i + \mu_i} \frac{\omega^2}{c^2},
\]

where \(k_i\) is the \(i\) component of the propagating wave vector, \(\omega\) is the frequency and \(c\) is the speed of light in vacuum. In the quasiisotropic medium, the E- and H- polarized incident waves have the same dispersion relation [23]

\[
\frac{q^2_x}{\varepsilon_z \mu_y} + \frac{q^2_y}{\varepsilon_z \mu_x} + \frac{q^2_z}{\varepsilon_y \mu_x} = \frac{\omega^2}{c^2},
\]

where \(q_i\) represents the \(i\) component of transmitted wave-vector. For quasiisotropic media the permittivity and permeability constants satisfy the quasiisotropy condition

\[
\frac{\varepsilon_x}{\mu_x} = \frac{\varepsilon_y}{\mu_y} = \frac{\varepsilon_z}{\mu_z} = C \quad (C > 0),
\]

where \(C\) is a constant. Based on the dispersion relation one can find E- and H- polarized waves have the same wave-vector surface. If \(C > 0\) the dispersion surface has the following two types: ellipsoid and double-sheeted hyperboloid dispersion relations. While \(C < 0\) the dispersion surface is single-sheeted hyperboloid. It should be noted, as we will see the following, that the media with single-sheeted hyperboloid dispersion relation cannot be regarded as quasiisotropic one.
We choose the $z$ axis to be normal to the interface, the $x$ and $y$ axes locate at the plane of the interface. The $z$-component of the wave vector can be found by the solution of Eq. (3), which yields
\[ q_z = \sigma \sqrt{\varepsilon_y \mu_x \omega^2 - \frac{\varepsilon_y (\mu_x q_x^2 + q_y^2)}{\varepsilon_z}} \]
where $\sigma = +1$ or $\sigma = -1$. The choice of $\sigma$ ensures that light power propagates away from the surface to the $+z$ direction. In principle, the occurrence of refraction requires that the $z$ component of the refracted wave vectors of is real, hence the incident angle must satisfy the following inequality:
\[ \varepsilon_y \mu_x \omega^2 - \frac{\varepsilon_y (\mu_x q_x^2 + q_y^2)}{\varepsilon_z} < \varepsilon_y \mu_x \omega^2. \]
Without loss of generality, we assume the wave vector locate at the $x-z$ plane ($k_y = q_y = 0$). The incident angle is given by
\[ \theta_I = \tan^{-1} \left[ \frac{k_x}{k_z} \right]. \]
The values of refractive wave vector can be determined by using the boundary conditions and dispersion relations. The refractive angle of the transmitted wave vectors of E- and H-polarized waves can be written as
\[ \beta^E_P = \tan^{-1} \left[ \frac{q_x^E}{q_z^E} \right], \quad \beta^H_P = \tan^{-1} \left[ \frac{q_x^H}{q_z^H} \right]. \]
It should be noted that the actual direction of light is defined by the time-averaged Poynting vector $S = \frac{1}{2} \text{Re}(E^* \times H)$. For E- and H-polarized refracted waves, $S_T$ is given by
\[ S_T^E = \text{Re} \left[ \frac{T_E^2 E_0^2 q_x^E}{2\omega \mu_z} e_x + \frac{T_E^2 E_0^2 q_y^E}{2\omega \mu_x} e_z \right], \]
\[ S_T^H = \text{Re} \left[ \frac{T_H^2 H_0^2 q_x^H}{2\omega \varepsilon_z} e_x + \frac{T_H^2 H_0^2 q_y^H}{2\omega \varepsilon_x} e_z \right], \]
where $T_E$ and $T_H$ are the transmission coefficient for E- and H-polarized waves, respectively. The refraction angles of Poynting vectors of E- and H-polarized incident waves can be obtained as
\[ \beta^E_S = \tan^{-1} \left[ \frac{S_{T_x}^E}{S_{T_z}^E} \right], \quad \beta^H_S = \tan^{-1} \left[ \frac{S_{T_x}^H}{S_{T_z}^H} \right]. \]
As shown in Eqs. (9) and (10), one can see that the direction of $z$ component of the Poynting vector is determined by $q_z^E/\mu_x$ for E-polarized wave and $q_z^H/\varepsilon_x$ for H-polarized wave, respectively. Energy conservation requires that the $z$ component of the energy current density of the refracted waves must propagate away from the interface, for instance, $q_z^E/\mu_x > 0$ and
\( q_z^H / \varepsilon_x > 0 \). Since the quasiisotropic condition \( \varepsilon_x / \mu_x = \varepsilon_z / \mu_z > 0 \) and boundary condition \( q_x^E = q_x^H = k_x, q_z^E \) and \( q_z^H \) should have the same sign. From Eqs. (8)-(10) we have

\[
\beta_P^E = \beta_P^H, \quad \beta_S^E = \beta_S^H. \tag{12}
\]

It means that both the wave vector and the Poynting vector of E- or H-polarized field propagate in the same direction. It is one of the significant differences between the quasiisotropic media and anisotropic one.

Next we want to enquire: why the media with single-sheeted hyperboloid dispersion relation can not be regarded as quasiisotropic? Evidently, if \( \varepsilon_x / \mu_x = \varepsilon_z / \mu_z = C < 0 \), the signs of \( q_z^E \) and \( q_z^H \) should be opposite. From Eqs. (8)-(10) we can obtain

\[
\beta_P^E = -\beta_P^H, \quad \beta_S^E = -\beta_S^H. \tag{13}
\]

It means that E- and H-polarized waves exhibit opposite propagating properties in this kind of anisotropic media. The inherent physics is stem from the fact \( \mathbf{S}^E \cdot \mathbf{q}^E \) and \( \mathbf{S}^H \cdot \mathbf{q}^H \) have the opposite sign for E- and H-polarized waves. Considering the properties of the quasiisotropic medium, we conclude that the media with single-sheeted hyperboloid dispersion relation can not be regarded as quasiisotropic medium.

### III. REFLECTION AND TRANSMISSION COEFFICIENTS

In this section, we will discuss the reflection and transmission in the quasiisotropic media. For compactness, we assume the electric filed polarized along the \( y \) axis. For E-polarized incident waves, the incident and reflected fields can be written as

\[
\mathbf{E}_I = e_y E_0 \exp[i(k_x x + k_z z)], \tag{14}
\]

\[
\mathbf{E}_R = R_E E_0 e_y \exp[i(k_x x - k_z z)], \tag{15}
\]

where \( R_E \) is the reflection coefficient. Matching the boundary conditions for each wave-vector component at the plane \( z = 0 \) gives the propagation field in the form

\[
\mathbf{E}_T = T_E E_0 \exp[i(q_x^E x + q_z^E z)]. \tag{16}
\]

Based on the continuity of the tangential component of the electric field yields the equation

\[
1 + R_E = T_E. \tag{17}
\]
A second equation can be found when we require continuity of the tangential components of the \( \mathbf{H} \) fields, which can be obtained from Maxwell’s curl equation

\[
\mathbf{H} = i \frac{c}{\omega} \mu^{-1} \cdot \nabla \times \mathbf{E}.
\]  

Equating the \( x \) component of the \( \mathbf{H} \) vectors corresponding to the incident, reflected, and transmitted fields, we have

\[
\mu_x k_z (1 - R_E) = \mu_I T_E q_z^E,
\]  

(19)

For E-polarized incident waves, combining Eq. (19) with Eq. (17), one can obtain the following expression for the reflection and transmission coefficients

\[
R_E = \frac{\mu_x k_z - \mu_I q_z^E}{\mu_x k_z + \mu_I q_z^E}, \quad T_E = \frac{2\mu_x k_z}{\mu_x k_z + \mu_I q_z^E}.
\]  

(20)

For H-polarized incident waves, the reflection and transmission coefficients can be obtained similarly as

\[
R_H = \frac{\varepsilon_x k_z - \varepsilon_I q_z^H}{\varepsilon_x k_z + \varepsilon_I q_z^H}, \quad T_H = \frac{2\varepsilon_x k_z}{\varepsilon_x k_z + \varepsilon_I q_z^H}.
\]  

(21)

A comparison of Eq. (20) and Eq. (21) shows that if \( \varepsilon_I / \mu_I = C \), one can obtain an interesting property: E- and H-polarized waves will exhibit the same reflection and transmission, namely \( R_E = R_H \) and \( T_E = T_H \).

We next explore the reflectivity and transmissivity which can be defined by

\[
r = \frac{S_{Rz}}{S_{Iz}}, \quad t = \frac{S_{Tz}}{S_{Iz}},
\]  

(22)

where \( S_{Iz} \) and \( S_{Tz} \) are the \( z \) components of the incident and transmission Poynting vectors, respectively \[24, 25\]. For E- and H- polarized incident waves, the reflectivity and transmissivity can be written as

\[
r_E = R_E^2, \quad t_E = \frac{\varepsilon_I q_z^E}{\varepsilon_x k_z} T_E^2,
\]  

(23)

\[
r_H = R_H^2, \quad t_H = \frac{\mu_I q_z^H}{\mu_x k_z} T_H^2.
\]  

(24)

By using Eqs. (23) and (24), one has \( r + t = 1 \). This demonstrates power conservation for reflection and transmission is satisfied at the boundary surface.

Total reflection means the reflection coefficient is equal to unity. Setting \( R_E = 1 \) and \( R_H = 1 \) in Eq. (20) and Eq. (21), one can obtain the critical angles as

\[
\theta_C^E = \theta_C^H = \sin^{-1} \left( \frac{\varepsilon_x \mu_y}{\varepsilon_I \mu_I} \right),
\]  

(25)
where $\varepsilon_z \mu_y > 0$. If $\varepsilon_z \mu_y < 0$ the total reflection phenomenon never occur for any incident waves.

Total transmission takes place at an incidence angle satisfying $\theta_I + \beta_H^H = \pi/2$ for H-polarized wave incident from free space into an isotropic dielectric RHM ($\mu_I = 1$). Such an angle, determined by $\theta_B^H = \tan^{-1} [\sqrt{\varepsilon_I}]$, is called the Brewster angle [26, 27, 28]. Similarly a simple extension shows that the Brewster angles of E- and H-polarized waves can be obtained in quasiisotropic media. Mathematically the Brewster angles can be obtained from $r_E = 0$ and $r_H = 0$. For E-polarized incident wave if the anisotropic parameters satisfy the relation

$$0 < \frac{\mu_z (\varepsilon_y \mu_I - \varepsilon_I \mu_x)}{\varepsilon_I (\mu_I^2 - \mu_x \mu_z)} < 1,$$

the Brewster angle can be expressed as

$$\theta_B^E = \sin^{-1} \left[ \sqrt{\frac{\mu_z (\varepsilon_y \mu_I - \varepsilon_I \mu_x)}{\varepsilon_I (\mu_I^2 - \mu_x \mu_z)}} \right].$$

For H-polarized wave if the anisotropic parameters satisfy by the relation

$$0 < \frac{\varepsilon_z (\varepsilon_I \mu_y - \varepsilon_x \mu_I)}{\mu_I (\varepsilon_I^2 - \varepsilon_x \varepsilon_z)} < 1,$$

the Brewster angle can be written in the form

$$\theta_B^H = \sin^{-1} \left[ \sqrt{\frac{\varepsilon_z (\varepsilon_I \mu_y - \varepsilon_x \mu_I)}{\mu_I (\varepsilon_I^2 - \varepsilon_x \varepsilon_z)}} \right].$$

It should be mentioned that E- and H-polarized waves may exhibit a Brewster angle simultaneously, which depends on the choice of the anisotropic parameters in respect to the incident medium. The Brewster angle can not exist for a certain material parameters which make the corresponding expressions inside the square root negative.

IV. NEGATIVE REFRACTION AND BACKWARD WAVE PROPAGATION

In this section, we will discuss the negative refraction and backward wave propagation in quasiisotropic media. The refracted wave in quasiisotropic media can be determined by the two principles: First, the boundary conditions require that the tangential component of the wave vector, is conserved across the interface, $q_x = k_x$. Second, causality requires that the energy current of the refracted waves should transmit away from the interface, i.e., the normal component of the Poynting vector, $S_{Tz} > 0$. It is worth mentioning that the
same principles are also used to predict the refracted wave in more complex systems such as multilayers and photonic crystals.

First we want to study the anomalous negative refraction in quasiisotropic media. Unlike in isotropic media, the Poynting vector in quasiisotropic media is neither parallel nor antiparallel to the wave vector, but rather makes either an acute or an obtuse angle with respect to the wave vector. In general, to distinguish the positive and negative refraction in quasiisotropic media, we must calculate the direction of the Poynting vector with respect to the wave vector. Positive refraction means $q_x \cdot S_T > 0$, and anomalous negative refraction means $q_x \cdot S_T < 0$. From Eqs. (9) and (10) we get

$$q_x \cdot S_T^E = \frac{T_E^2 E_0^2 q_x^2}{2\omega \mu_z}, \quad q_x \cdot S_T^H = \frac{T_H^2 H_0^2 q_x^2}{2\omega \varepsilon_z}. \quad (30)$$

This anomalous negative refraction phenomenon is one of the most interesting peculiar properties of quasiisotropic media. We can see that the refracted waves will be determined by $\mu_z$ for E-polarized incident waves and $\varepsilon_z$ for H-polarized incident waves. The condition of negative refraction for both E- and H-polarized waves can be given as

$$\varepsilon_z < 0, \quad \mu_z < 0. \quad (31)$$

and other elements of both $\varepsilon$ and $\mu$ do not need to be negative.

Next we want to investigate the backward wave propagation in the quasiisotropic medium. The wave with $q \cdot S_T < 0$ has been called the backward wave or left-handed wave. From Eqs. (9) and (10) we have

$$q^E \cdot S_T^E = Re \left[ \frac{T_E^2 E_0^2 q_x^2}{2\omega \mu_z} + \frac{T_E^2 E_0^2 (q_z^E)^2}{2\omega \mu_x} \right], \quad (32)$$

$$q^H \cdot S_T^H = Re \left[ \frac{T_H^2 H_0^2 q_x^2}{2\omega \varepsilon_z} + \frac{T_H^2 H_0^2 (q_z^H)^2}{2\omega \varepsilon_x} \right], \quad (33)$$

Note that the Poynting vector and the wave vector generally are not parallel or antiparallel in the quasiisotropic media. The electric field $E$, the magnetic field $H$ and the wave vector $q$ can not form a strictly left-handed triplet. Combining Eqs. (3), (32) and (33), we can find that

$$q^E \cdot S_T^E = \frac{T_E^2 E_0^2 \omega}{2c^2} \varepsilon_y, \quad q^H \cdot S_T^H = \frac{T_H^2 H_0^2 \omega}{2c^2} \mu_y. \quad (34)$$

Thus the conditions of backward wave propagation for both E- and H-polarized waves can be written as

$$\varepsilon_y < 0, \quad \mu_y < 0. \quad (35)$$
and other elements of both $\varepsilon$ and $\mu$ need not to be negative. For the quasiisotropic medium ($C > 0$), in which $\varepsilon_y < 0$ means automatically $\mu_y < 0$, so both E- and H-polarized waves can exhibit backward wave propagation simultaneously.

Comparing Eq. (31) with Eq. (35) shows that the sign of $q_x \cdot S_T$ may not coincide with the sign of $q \cdot S_T$. Thus the negative refraction is not necessarily tight to the backward wave propagation in quasiisotropic media. Same has been found to be true in uniaxially anisotropic media [9], and photonic crystal [33, 34].

V. NUMERICAL RESULTS IN QUASIISOTROPIC MEDIA

In this section, we will give the numerical results of waves propagation in quasiisotropic media. The dispersion surface for quasiisotropic media have the following two types: ellipsoid and double-sheeted hyperboloid. In following two subsections we will discuss the anomalous negative refraction, anomalous total reflection and oblique total transmission in the two types of media.

For the purpose of illustration, frequency contour will be used to determine the refracted waves as shown in Fig. 1 and Fig. 3. From the boundary condition $q_x = k_x$, we can obtain two possibilities for the refracted wave vector. Energy conservation requires that the $z$ component of Poynting vector must propagates away from the interface, for instance, $q^E_z/\mu_x > 0$ for E-polarized wave and $q^H_z/\varepsilon_x > 0$ for H-polarized wave. Then the sign of $q_z$ can be determined easily. The corresponding Poynting vector should be drawn perpendicularly to the dispersion contour. This is because the Poynting vector is collinear with the ray velocity [26]. Depending on the sign of $q \cdot S$, the Poynting vector is drawn inwards or outwards. For a certain value of $k_x$ there are always two values of $q_z$ that satisfy both the dispersion relation, while only the Poynting vector with $S_{Tz} > 0$ is causal.

A. Ellipsoid dispersion relation

If all the $\varepsilon_i$ and $\mu_i$ have the same sign, the parameter tensor elements wave-vector surface must be an ellipsoid shown in Fig. 11. The ellipsoid dispersion relation has two subtypes which can be formed from combinations of the material parameter tensor elements.

Case I. For $\varepsilon_i > 0$ and $\mu_i > 0$, the refraction diagram is shown in Fig. 13. Here $k_z \cdot q_z > 0$ and $q_x \cdot S_T > 0$, so the refraction angles of wave-vector and Poynting vector are always
FIG. 1: The circle and the ellipse represent the dispersion relations of isotropic media and quasi-isotropic media, respectively. The wave vectors and the Poynting vectors of E- and H- polarized waves propagate in the same direction. The ellipsoid wave-vector surface has two subtypes: (a) corresponding to case I: $\beta_E^P = \beta_H^P > 0$, $\beta_E^S = \beta_H^S > 0$. (b) corresponding to case II: $\beta_E^P = \beta_H^P < 0$, $\beta_E^S = \beta_H^S < 0$.

positive. Due to $\mathbf{q} \cdot \mathbf{S}_T > 0$, the backward wave can not propagate in this subtype of media. 

Case II. For $\varepsilon_i < 0$ and $\mu_i < 0$, the refraction diagram is shown in Fig. 1b. Here $k_z \cdot q_z < 0$ and $\mathbf{q}_x \cdot \mathbf{S}_T < 0$, so the refraction angle of wave vector and Poynting vector are always negative. The backward wave can propagate in this subtype of media since $\mathbf{q} \cdot \mathbf{S}_T < 0$.

In principle, the occurrence of refraction requires that the $z$ component of the wave vector of the refracted waves must be real. If $\varepsilon_z \mu_y < \varepsilon_I \mu_I$ the real wave vector only exists for the branch

$$-\theta_C < \theta_I < \theta_C.$$  

(36)

When $\theta_I > \theta_C$ and $r_E = r_H = 1$, the total reflection of E- and H-polarized waves occur in the branch $-\pi/2 < \theta_I < -\theta_C$ and $\theta_C < \theta_I < \pi/2$. For E- and H-polarized waves, $\varepsilon_z \mu_y$ is negative, thus the inequality $\varepsilon_z \mu_y < \varepsilon_I \mu_I$ satisfied for any incident angle. The anomalous total reflection phenomenon thus can not occur.

In the next step, we will discuss the interesting phenomenon of oblique total transmission $T(\theta_I \neq 0) = 1$. The reflectivity of E- and H-polarized waves are plotted in Fig. 2. We choose some simple parameters for the purpose of illustration, i.e., $C = 1$. In Fig. 2a H-polarized incident wave exhibits a Brewster angle. When $\theta_I = \theta_H^T$, one can find $r_H = 0$ and H-polarized incident wave can exhibit oblique total transmission. In Fig. 2b E-polarized wave
FIG. 2: The reflectivity of E- and H-polarized waves as functions of the incident angle $\theta_I$. The quasiisotropic medium with parameters $\varepsilon_x = \varepsilon_y = 1, \varepsilon_z = 1.5$ and $C = 1$. (a) H-polarized incident wave exhibits a Brewster angle. The isotropic medium with $\varepsilon_I = 1$ and $\mu_I = 2$. (b) E-polarized incident wave exhibits a Brewster angle. The isotropic medium with $\varepsilon_I = 2$ and $\mu_I = 1$.

exhibits a Brewster angle. When $\theta_I = \theta_B^E$, one can find $r_E = 0$ and E-polarized incident waves can exhibit oblique total transmission. The appearance of the Brewster’s angle for E-polarized waves is due to the quasiisotropic medium having primarily a magnetic response that reverses the roles of E- and H-polarization.

**B. Double-sheeted hyperboloid dispersion relation**

If only two of $\varepsilon_i$ have the same sign, for instance $\varepsilon_x \cdot \varepsilon_y > 0$, the wave-vector surface is a double-sheeted hyperbola. Also this media has two subtypes:

Case I. For the case of $\varepsilon_x > 0, \varepsilon_y > 0$ and $\varepsilon_z < 0$, the refraction diagram is shown in Fig. 3a. Here $k_z \cdot q_z > 0$ and $q_x \cdot S_T < 0$. It yields that the refraction of Poynting vector refraction is always negative even if the wave-vector refraction is always positive. Due to $q \cdot S_T > 0$, the backward wave can not propagate in this subtype of media.

Case II. For the case of $\varepsilon_x < 0, \varepsilon_y < 0$ and $\varepsilon_z > 0$, the refraction diagram is shown in Fig. 3b. Here $k_z \cdot q_z < 0$ and $q_x \cdot S_T > 0$, the refraction of Poynting vector is always positive, even if the refraction of wave vector refraction is always negative. The backward wave can
propagate in this subtype of media since \( \mathbf{q} \cdot \mathbf{S}_T < 0 \).

It is interesting to observe that the anomalous backward waves can propagate in this kind of media. Fig. 3a and Fig. 3b have presented simple examples of possibilities of negative refraction without backward waves, and backward waves without negative refraction, respectively.

For E- and H-polarized waves, \( \varepsilon_z \mu_y \) is negative, thus the inequality \( \varepsilon_z \mu_y < \varepsilon_I \mu_I \) satisfied for any incident angle. From Eq. (36) the real wave vector exists for the branch

\[
-\pi/2 < \theta_I < \pi/2.
\]  

In this case, the anomalous total reflection phenomenon can not occur for any incident angles.

The reflectivity of E- and H-polarized waves are plotted in Fig. 4. As expected, H-polarized wave can exhibit a Brewster angle as shown in Fig. 4a. When \( \theta_I = \theta_B^H \), one can find that \( r_H = 0 \) and H-polarized incident waves can exhibit oblique total transmission. Let us now consider the second case in which E-polarized wave exhibits a Brewster angle, as depicted in Fig. 4b. When \( \theta_I = \theta_B^E \), one can find \( r_E = 0 \) and E-polarized incident waves can exhibit a oblique total transmission. The inherent physics underlying the oblique total transmission are collective contributions of the electric and magnetic responses. The

FIG. 3: The circle and the double-sheeted hyperbola represent the dispersion relations of isotropic media and quasiisotropic media, respectively. The wave vector and Poynting vectors of E- and H- polarized waves propagate in the same direction. The double-sheeted hyperboloid wave-vector surface has two subtypes: (a) corresponding to case I: \( \beta^E_I = \beta^H_I > 0, \beta^E_S = \beta^H_S < 0 \). (b) corresponding to case II: \( \beta^E_I = \beta^H_I < 0, \beta^E_S = \beta^H_S > 0 \).
FIG. 4: The reflectivity of E- and H-polarized waves as functions of the incident angle $\theta_I$. The quasiisotropic medium with parameters $\varepsilon_x = \varepsilon_y = 1$, $\varepsilon_z = -1.5$ and $C = 1$. (a) H-polarized incident wave exhibits a Brewster angle. The isotropic medium with $\varepsilon_I = 1$ and $\mu_I = 2$. (b) E-polarized incident wave exhibits a Brewster angle. The isotropic medium with $\varepsilon_I = 2$ and $\mu_I = 1$.

Brewster condition discussed here requires that the reflected fields of the oscillating electric and magnetic dipoles cancel each other for certain incident angle.

Finally we want to enquire whether E- and H-polarized waves have the same Brewster angle. Obviously, if $\varepsilon_I/\mu_I = C$, Eqs. (26) and (28) are satisfied simultaneously, then E- and H-polarized waves present the same Brewster angle, i.e., $\theta_B^E = \theta_B^H$. For the two polarized waves the material thus present oblique total transmission at the same incident angle.

VI. CONCLUSION

In conclusion, we have investigated the properties of waves propagation in quasiisotropic media. We have performed the detailed analyses of the anomalous negative refraction, anomalous total reflection and oblique total transmission at the interface associated with quasiisotropic media. To obtain a better physical picture of the total reflection and oblique total transmission, we have introduced the Brewster angles for E- and H-polarized waves in quasiisotropic media. We have shown that the properties of waves propagation in quasiisotropic media are significantly different from those in isotropic or anisotropic media. It should
be pointed out that the anisotropic media has been fabricated successfully \cite{10,11}. We are justified to expect that the quasiisotropic medium can be constructed quickly and the anomalous propagation characteristics will be studied experimentally. Finally, it is worth pointing out that the polarization insensitive lens can be designed based on anomalous propagation properties discussed above.

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