Finite temperature effects in brane cosmology

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Finite temperature effects in brane world cosmology are studied by considering the interaction between scalar field and bulk gravity. One-loop correction to zero-temperature potential is computed by taking into account, interaction of scalar field and bulk gravity. Phase transitions and high temperature symmetry restoration are examined. Critical temperature of phase transitions depends on the interaction constant of the scalar field and bulk gravity, and these constant is an order parameter. Present study can account for second order phase transition in early universe, in brane world cosmological scenario.

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I. INTRODUCTION

The idea of extra dimensions was proposed in the early twentieth century by Nordstorm and few years later by Kaluza and Klein. It has reemerged over the years in theories combining the principle of quantum mechanics and relativity. In particular theories based on super symmetry, especially super string theories, are naturally expressed in more than four dimensions. Recent developments in string theory and its extension M-theory have suggested another approach to compactify extra spatial dimensions. According to this, the standard model particles are confined on a hyper-surface, called brane, embedded in a higher dimensional space, called bulk. Only gravity and other exotic matter such as dilaton can propagate in the bulk. Our universe may be such a brane like object. This idea was originally motivated phenomenologically and later revival in string theory by Horava and Witten. Heterotic string theory demand eleven dimensional super-gravity to describe low energy. Witten suggested that six of eleven dimensions can be consistently compactified on a Calabi-Yau manifold. Thus, in that limit space-time looks five dimensional with four dimensional boundary brane. This provides the underlying picture for many brane world models proposed so far.

To solve the hierarchy problem, in the brane world scenario, it is assumed that the large extra dimensions may be compact but not small. Recently, the idea that our universe is embedded in higher dimensional space-time has received much attention in cosmology. It was proposed that our observable universe is a three dimensional surface (brane) embedded in a higher dimensional space (bulk). Fundamental structure of brane is well understood by two models Dvali, Gabbadadze and Porrati model (DGP), and Randall-Sundrum model (R-S I and II). In RS models, one non-compact extra dimension is warped by a negative cosmological constant, hence the geometry is warped. In DGP model the extra dimension is flat and the effect of bulk gravity on brane is considered due to interaction between brane matter and graviton.

The introduction of branes into cosmology opened another novel approach to understand the universe and its evolution. This new perception of brane world opened new directions in cosmology, but at the same time imposed some new problems. The cosmological evolution of the universe can take place on brane, but for the whole theory to make sense, the brane should embedded in a consistent way to higher dimensional space time, the bulk. The only physical field in the bulk is the gravitational field, and there are no matter fields.

Brane world cosmology have been examined in problems like, cosmological phase transitions, inflationary scenario, baryogenesis, stochastic background of gravitational waves, singularity, flatness and entropy problems, induced gravity effects on branes, quantum effects of bulk scalar field at non-zero temperature, in early universe.

In brane world scenario localized matter fields on the brane can couple to bulk gravity, which generate a localized term for the coupling. Therefore, if the brane world scenario is correct for cosmology, the effect of bulk on the evolution of matter field, is also be taken into account. In the present work, we examine phase transitions and symmetry restoration occurred in early universe due to massive scalar field which couple with bulk gravity, in the brane world cosmological scenario.

The idea of spontaneous symmetry breaking and its restoration is useful to understand the evolution of the early universe. In this approach, it is believed that at high temperature, symmetries that are spontaneously broken today were restored during the evolution of the universe there were phase transition, perhaps many, associated with the spontaneous breakdown of symmetries. In general, a symmetry breaking phase transition can be first or second order. If the phase transition is first order, the universe may be dominated by the vacuum energy and undergo a period of inflation. In this case, the transition proceeds by the nucleation of bubbles of the true vacuum. If the phase transition is higher order the...
thermal fluctuations may drive the transition.

The paper is organized as follows. Section I gives a brief introduction for the present work. Section II deals with the effects of bulk and finite temperature on the scalar field, is considered and hence examined the phase transition and symmetry restoration in early universe. Conclusions and discussions are presented in section III.

II. PHASE TRANSITION AND SYMMETRY RESTORATION

The phase transitions studies due to effective potential scalar field, in the context of conventional four dimensional space time paradigm, usually consider self interaction of the field. But the scalar field interacts with other fields there can be additional one-loop correction to the potential. Since bulk can couple to matter field on the brane it would be useful to study its effects in early universe. In present study, we consider a term arise from the interaction of scalar field and bulk gravity which is on the brane.

The Lagrangian which describe the coupling of massive scalar field with the bulk gravity can be written as

\[ L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi, y), \]  

(1)

where \( A = 0, 1, 2, 3, 4 \) \((t = 0, x^t = 1, 2, 3, y = 4)\) and \( y \) is the extra-dimensional coordinate. The above Lagrangian is assumed invariant under \( Z_2 \) symmetry.

Since the scalar field coupled with bulk gravity the potential, \( V(\phi, y) \), takes the following form

\[ V(\phi, y) = -\frac{1}{2} m^2 \phi^2 - \frac{1}{4} \lambda \phi^4 + \frac{1}{2} D \delta(y) \phi^2, \]  

(2)

where \( \lambda \) is the coupling constant due to self interaction of field \( \phi \). The third term in Eq.(2) is to account for the interaction between the scalar field and bulk gravity, which is assumed on the brane. Where \( D \) is the interaction constant and \( \delta(y) \) due to the fact that the coupling confined on the brane. Since effects of the potential are seen on the brane, Eqn (2) to be considered at \( y = 0 \) for the further study.

Consider the potential(2), therefore we find,

\[ V(\phi_{\pm}) = -\frac{1}{4\lambda} [m^2 - D]^2, \]  

(3)

and

\[ V''(\phi_{\pm}) = 2m^2 - 2D. \]  

(4)

Where prime denotes differentiation with respect to \( \phi \) and \( \phi_{\pm} = \pm \left( \frac{\lambda D}{m^2} \right)^{1/2} \) are the two equivalent minima. The discrete symmetry of the lagrangian is broken by choosing either of the vacuum state \( \phi_{\pm} \) and the mass of physical boson is determined by curvature of the potential about the true ground state \( M^2 = V''(\phi_{\pm}) = 2m^2 - 2D = 2\lambda \phi_{\pm}^2 \).

Our, next aim is to study the finite temperature effects and symmetry restoration due to the potential Eq.(2). In order to study phase transitions in the early universe, the corresponding effective potential of the scalar field to be computed at finite temperature. Thus finite temperature effective potential on field theory can be used to study the phase transition in the early universe, which in turn shows symmetry breaking present in a model under consideration.

The potential given by Eq.(2) is the temperature independent zero-loop effective potential. The finite temperature effective potential \( V_T \) is the free energy density associated with field. The one-loop approximation to the to the finite temperature potential \( V_T \) can be computed as the method of Dolan and Jackiw \( [21] \) and is given by

\[ V_1^T = \frac{T}{2} \sum_n \int \frac{d^3 k}{(2\pi)^3} \ln(k^2 - M^2) \]  

(5)

\[ = \frac{T}{2} \sum_n \int \frac{d^3 k}{(2\pi)^3} \ln(-4\pi^2 n^2 T^2 - \epsilon_M^2), \]  

(6)

where

\[ \epsilon_M^2 = k^2 + M^2, \]  

(7)

\[ M^2(\phi) = -m^2 + D + 3\lambda \phi^2. \]

Following Dolan and Jackiw \( [21] \), the sum on \( n \) diverges. Thus we find

\[ V_1^T = V_1^0 + \bar{V}_1^T. \]  

(8)

Where

\[ V_1^0 = -\frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{D}{2} \phi^2 + \frac{M^4}{64\pi^2} \ln(M^2/\mu^2), \]  

(9)

is the one-loop temperature independent effective potential and \( \mu \) is an arbitrary mass scale and can be related to renormalization constants. In Eq(8) \( \bar{V}_1^T \) can be written as in high temperature limits \( [21] \) as follows

\[ \bar{V}_1^T = \frac{\pi^2}{90} T^4 + \frac{M^2}{24} T^2 - \frac{M^4}{12\pi} T - \frac{M^4}{64\pi^2} \ln \frac{M^2}{T^2}. \]  

(10)

The symmetry restoration can be achieved if the temperature is raised above a certain temperature known as the critical temperature. The critical temperature in the present case is obtained as

\[ T_c = \sqrt{\frac{4 m^2 - D}{\lambda}}. \]  

(11)

The critical temperature vary depending upon the value of \( D \) and the phase transition can occur according to it. Critical temperatures for different values of \( D \) are presented in Table.1.

The behavior of the potential \( V_T \) is shown in Fig1.a,b,c,d,e for fixed value of \( m, \lambda, D \) and different temperatures. Behavior of the effective potential for different \( D \) is shown in Fig2 for a given temperature.
TABLE I:
Critical temperature $T_c$ for corresponding $D$, for fixed
$m = 0.9371$ and $\lambda = 0.008$

| $D$ | $T_c$ | $\frac{D}{T}$ | $T_c$ |
|-----|-------|----------------|-------|
| 0.7 | 9.43  | 0.005          | 20.89 |
| 0.5 | 13.75 | 0              | 20.95 |
| 0.3 | 17.00 | 0.1            | 22.11 |
| 0.1 | 19.72 | -0.3           | 24.27 |
| 0.05| 20.34 | -0.5           | 26.25 |

III. CONCLUSIONS

In this paper we studied finite temperature effects in
brane world cosmology by considering the interaction be-
tween scalar field and bulk gravity. One-loop correction
FIG. 2: Shows the behavior of the effective potential for $D = 0.7, 0.5, 0.37, 0.1, 0.01$, $m = 0.9371, \lambda = 0.008$ and $T = 10$.

to the zero-temperature potential is considered by taking into account of the interaction of scalar field and bulk gravity. Hence the phase transition and high temperature symmetry restoration in brane world scenario is examined. The behavior of finite temperature effective potential is analyzed. Fig1.a,b,c,d,e show behavior of the potential $V_T$ for fixed value of $m, \lambda, D$ and different temperatures. At $T = 0$ the symmetry of finite temperature effective potential is broken (FIG1a). The behaviour of the potential at the critical temperature, $T_C = 15.9$, is shown in FIG1.c. When $T << T_c$ and $T > T_c$, the corresponding behaviour of the potential are shown in FIG1.b and FIG1.e respectively. When $T >> T_c$, the symmetry of the potential became restored (FIG1.f). From the plots it can be see that at the critical temperature the transition occurs smoothly, which is the nature of second order phase transition. Hence from these plots, it is evident that the nature of phase transition in the present study is second order. From the Table.1 it can be concluded that the critical temperature, $T_c$, dependent on the interaction constant, $D$, due to the scalar field and bulk gravity. Plots for the finite temperature effective potential with the scalar field for various values of $D$ are shown in FIG2 for fixed values of $m, \lambda$ and $T$, which has the signature of second order phase transition. These plots show that the symmetry restoration is also dependent on $D$, hence it can be an order parameter for the phase transition. The present study can account for second order phase transition in the early universe, which is due to the interaction of scalar field and bulk, in the brane world scenario. This changes in phase transition due $D$ may have important consequences on inflation. The second order phase transition is very useful to understand the dynamics of very early universe. When $D=0$, the present results reduce to the self interacting scalar field, which is minimally coupled with gravity, in the conventional four dimensional space time paradigm. We hope that the idea used in the present work may be useful to understand inflationary scenario and related issues in early universe, in brane world scenario.

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