Review of Recent Developments in the CGC

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Abstract

I review recent developments in the CGC approach to high-energy collisions. The focus is on topics related to the Quark Matter conference, specifically on predictions for the p+Pb run at the LHC; as an added bonus some of these predictions are confronted with first data taken during the p+Pb machine performance test at the LHC performed in September 2012. I also highlight recent work related to particle production fluctuations, suppression of nuclear modification factors in p+A from rcBK evolution, (partial) NLO corrections in forward production, and photon-hadron as well as di-hadron correlations.

1. Introduction

In the Bjorken-Feynman parton model the constituents of a boosted hadron are viewed as dilute non-interacting quanta collinear to the beam. This picture has had tremendous success describing the proton structure at very short distances, high-\(E_T\) jets in hadronic collisions, and more. However, for a probe of fixed size \(r\) (well below the confinement scale of \(\sim 1\) fm) in the high-energy limit this picture is expected to break down. This is due to ample radiation in QCD of soft gluons as phase space opens up [1]. As a consequence, although the QCD coupling at the scale \(r\) may be small there are many \(O(\alpha_s^{-1})\) partners to interact with. Thus, at high energies hadrons rather correspond to a dense system of gluons and non-perturbatively strong fields, \(A^\mu \sim 1/g\) [2]. Thanks to the high occupation number these can be described as classical color fields sourced by the “valence” charges at higher rapidities which have been integrated out [3]. The Color Glass Condensate (CGC) is an effective theory describing the dynamics of non-linear color fields at high energies. Exploring its properties and systematically improving its accuracy is one of the main goals of the high-energy nuclear physics programme.

At present, the most suitable process for studies of high gluon density QCD is p+A collisions where a relatively dilute projectile probes a dense target. Particle production in the forward region of d+Au collisions at RHIC energy has discovered an interesting suppression relative to a simple superposition of binary p+p collisions [4]. However, due to limited phase space those measurements are restricted to relatively low transverse momenta and can not probe accurately the tails of the intrinsic gluon transverse momentum distributions. Also, quantitative calculations suffer from large uncertainties from a variety of sources: the large-\(x\) parton distributions of the projectile (\(x \sim 0.1 - 0.8\), the large-\(z\) behavior of parton fragmentation functions (\(z > 0.6\)), NLO corrections to the “hybrid formalism” at high \(p_T\) (see below), and so on. Presently, many eyes in this community are therefore focused on the upcoming p+Pb run at the LHC at \(\sqrt{s} = 5\) TeV which will open up phase space for small-\(x\) physics tremendously. This write-up shall therefore
also focus mainly (but not exclusively) on recent predictions for p+Pb collisions at the LHC based on the CGC approach.

The frozen valence charge density per unit transverse area, \( \rho \), in a hadron or nucleus is a random variable with a distribution determined by an effective action \( S[\rho] \). In the limit of a large number of valence sources this distribution is Gaussian \([3]\),

\[
S_{\text{MV}}[\rho] = \int d^2 r \, dy \, \frac{\rho^2}{2\mu^2} .
\]

The variance \( \mu^2 \) is proportional to the thickness of the target (= \( A^{1/3} \) for a nucleus, on average over impact parameters) since the sources act coherently. The classical field \( A_{cl}^\alpha \) is obtained from \( \rho \) by solving the Yang-Mills equations, analogous to the Weizsäcker-Williams approach in electrodynamics.

At the purely classical level the saturation momentum is energy independent. Energy dependence arises from quantum corrections to the classical field. It is not presently known how to incorporate all quantum corrections but a specific class of quantum fluctuations, those which are approximately boost invariant and are proportional to \( \alpha_s \log 1/x \) can be re-summed. For the two-point correlation function of light-like Wilson line operators, the so-called dipole scattering amplitude

\[
N(r) = \frac{1}{N_c} \langle \text{tr} \, V(0) V'(r) \rangle ,
\]

this is accomplished by the Balitsky-Kovchegov (BK) equation:

\[
\frac{dN(r,x)}{d\ln(x_0/x)} = \int d^2 r_1 \, K(r,r_1,r_2) \left[ N(r_1,x) + N(r_2,x) - N(r,x) - N(r_1,x) N(r_2,x) \right] .
\]

Due to the presence of non-linear effects this equation leads to saturation of the scattering amplitude at large dipole sizes \( r \) or small intrinsic transverse momenta \( k_T \). Evolution thus generates a dynamical scale \( Q_s(x) \) which grows with energy (or \( 1/x \)). In practice, eq. (3) is nowadays solved with running coupling accuracy \([3]\), and one of the main goals in this field is to test whether the evolution speed predicted by rcBK is consistent with observations or if improved accuracy is required. From the Fourier transform of \( N(r,x) \) (transformed to the adjoint representation) one obtains the (dipole) unintegrated gluon distribution (UGD) \( \Phi(k_\perp, x) \sim k_\perp^2 N(k_\perp, x)/\alpha_s(k_\perp^2) \).

Solving (3) requires an initial condition \( N(r,x_0) \), where the reference rapidity \( \log 1/x_0 \) is typically estimated to be \( Y_0 = \log 1/x_0 = \log 100 = 4.6 \). \( N(r,x_0) \) is the dipole scattering amplitude corresponding to the initial classical field at the reference rapidity \( Y_0 \). In the MV model (1) it is given by

\[
N(r,x_0) = 1 - \exp \left[ -\frac{1}{4} \left( r^2 Q_s^2(x_0) \right)^{\gamma} \log \frac{1}{r_\Lambda} \right] \quad (\text{MV: } \gamma = 1; \text{ AAMQS: } \gamma \approx 1.1) ,
\]

with \( Q_s^2(x_0) \sim g_0^4 \mu_0^2 \) and \( \gamma = 1 \). Ref. [6] performed detailed fits to (the most recent) HERA DIS data and found that a larger value for the initial "anomalous dimension" is preferred. Below, we shall see that semi-hard \( p_T \) distributions in p+p collisions are consistent with the AAMQS initial condition coupled with rcBK evolution but clearly exclude the MV model initial condition for protons. As a first quantitative achievement we thus note that the proton rcBK-UGD at small \( x < x_0 \) (averaged over impact parameter) is now known to some degree of accuracy. However, the physical origin of the AAMQS correction to \( \gamma \), which corresponds to a suppression of the
2. Multiplicities and multiplicity distributions

The $p_T$-integrated multiplicity of charged particles is the most basic observable in particle collisions. For a variety of reasons though, the “details” are too many to discuss here, it is also hard to compute precisely, unfortunately. That said, one may hope that the main dependence of $dN/d\eta$ on energy and system size is through the saturation scale $Q_s$ and so it is certainly warranted to check how this compares to data.

This issue was first addressed by the KLN model which indeed is in reasonable agreement, certainly well within its level of accuracy and credibility, with the centrality dependence of the multiplicity at midrapidity in Au+Au and d+Au collisions at RHIC, Pb+Pb collisions at LHC, and even p+p collisions from 0.9 – 7 TeV; for the most recent compilation and references to the original KLN papers we refer to ref. [8]. That paper also presented a prediction for p+Pb collisions at 4.4 TeV; an update for 5 TeV is shown in fig. 1. Other predictions for the multiplicity in p+Pb collisions at the LHC which instead use rcBK or IPsat UGDs, differ in their hadronization prescription or in the treatment of the nuclear geometry and of Glauber fluctuations, lead to very similar predictions at $\eta = 0$ (deviations from the central value are $\pm 15\%$) [9, 10, 11]. This confirms that the energy dependence of particle production is determined mainly by the growth of the saturation scale. It would be interesting also to compare theory and data for different cuts on $N_{\text{part}}$. The prediction is in very good agreement with preliminary ALICE data [12] at midrapidity, with a slightly too steep $\eta$-dependence.

One can also study the entire multiplicity distribution rather than just its mean. Assuming that particle production is due to strong classical fields, multiplicity fluctuations arise due to color charge density fluctuations of the effective sources at midrapidity. For a quadratic action such as in the MV model this leads (approximately) to a negative binomial distribution [13]; corrections are illustrated in ref. [14]. At midrapidity the bulk of the multiplicity distributions from p+p collisions at energies 0.2 – 7 TeV can indeed be described quite well by NBDs [5, 15] which
Figure 2: KNO scaling plot of charged particle multiplicity distributions at |η| < 0.5 in NSD collisions at various energies \[16\] and NBD fits; \( z \equiv N_{ch}/\langle N_{ch} \rangle \) and \( \Psi(z) \equiv \langle N_{ch} \rangle P(N_{ch}) \). Note that the quoted mean multiplicity includes neutral particles; also, that here the fluctuation parameter \( k \) is integrated over the transverse plane of the collision. p+Pb prediction from ref. \[15\].

in fact exhibit KNO scaling to a good approximation \[14, 15\], see fig. 2. Approximate KNO scaling has been predicted to persist even for minimum-bias p+Pb collisions at the LHC \[15\]. This represents an important check of our current understanding of multi-particle production. Most importantly such intrinsic particle production fluctuations affect the initial state for hydrodynamics in A+A collisions \[13, 17\] and thus could manifest in final-state flow and angular correlations. In the CGC approach, they are expected to occur on sub-nucleon distance scales on the order of \( \sim 1/Q_s \) \[17\], as shown in fig. 3. In this context it is interesting to note that CGC initial state models which do not incorporate intrinsic particle production fluctuations \[18\] appear to be inconsistent with the distributions of angular flow harmonics presented at this conference by the ATLAS collaboration \[19\].

Figure 3: Fluctuating energy density distribution at midrapidity and time \( \tau = 0.2 \text{ fm/c} \) in an A+A collision \[15\].

3. Nuclear modification factor \( R_{pA}(p_T) \)

Transverse momentum distributions of produced particles provide more detailed information than \( p_T \)-integrated multiplicities. In particular, they can probe the tail of the distribution of gluon
intrinsic $k_T$. To compare to small-$x$ QCD evolution one should restrict to $p_T/\langle z \rangle \sqrt{s} \leq x_0 \sim 0.01$ where $\langle z \rangle$ is the typical momentum fraction in fragmentation.

Figure 4: Left: Single-inclusive $p_T$-distribution in $p+p$ collisions at 7 TeV [11]; the curves show results obtained with rcBK evolved UGDs starting from two different initial conditions at $x_0 = 0.01$. Theory $K$-factor fixed at $p_T = 1$ GeV. Right: nuclear modification factor $R_{pPb}(p_T)$ at $\eta = 0$ with rapidity shift [11].

As a first step, it is important to check the spectrum in $p+p$ collisions to establish consistency with the UGD fitted to HERA DIS data. Furthermore, a correct $p+p$ limit is required for trustworthy minimum-bias $p+A$ spectra and for $R_{pA}$ nuclear modification ratios. The result [11] is shown in fig. 4 for two different initial conditions. Clearly, the MV model initial condition with rcBK evolution is excluded by the LHC data which exhibits strong suppression of the high-$k_T$ tails as compared to this UGD. On the other hand the AAMQS initial condition which has been carefully fitted to HERA data also agrees reasonably well with the CMS spectrum [20] and thus conforms to process independence of the (dipole) UGD.

Fig. 4 also shows a prediction for the nuclear modification factor for $p+Pb$ collisions at 5 TeV. The prediction and its many uncertainties are discussed in detail in ref. [11]. Generically, for all UGDs there is a suppression of particle production at $p_T = 1$ GeV which grows stronger for more “central” collisions (higher $N_{\text{part}}$). $R_{pb}$ then approaches and may even exceed unity at higher $p_T$ as the evolution window shrinks; recall that in hadronic collisions, at fixed rapidity $x \propto p_T$. Also, fluctuations in the thickness of the target amplify higher-twist “anti-shadowing” effects. The shape of the $R_{pb}(p_T)$ curves provides a test for the evolution speed predicted by rcBK; again, within uncertainties the prediction matches the preliminary data from ALICE.

A very important recent development addresses corrections to the so-called “hybrid formalism” for particle production in asymmetric kinematic configurations (such as forward production in $p+Au$ collisions): large-$x$ parton distributions are described within DGLAP while the small-$x$ target field is obtained from rcBK, for example. Until recently only the LO expression corresponding to elastic scattering of collinear projectile partons on the dense target field was known [21]. Ref. [22] computed inelastic corrections corresponding to large-angle emission of a gluon in the projectile wave function which then exchanges little transverse momentum with the target. This correction is formally suppressed by one power of $\alpha_s$, but enhanced by log $p_T/Q_s$. Numerical evaluations [11, 23] show that at high $p_T$ inelastic corrections drive $R_{pA}$ up. However, for realistic values of $\alpha_s$ this contribution can be very large, especially for $p+p$ collisions where $Q_s$ is relatively small [11]. The full NLO expression has been derived recently [24] and awaits numerical evaluation; this is crucial in order to understand the accuracy with which we are able to compute particle production in the forward region.
A prediction for $R_{pA}$ at forward rapidities is shown in fig. 5. As expected, generically the suppression increases with rapidity and could grow much stronger than predicted by some current leading-twist shadowing approaches [25, 26]. Notice the large correction (mainly to the p+p baseline) at high $p_T$ due to the inelastic term, even for very small $\alpha_s = 0.1$; as already mentioned above more reliable predictions at high $p_T$ require full NLO accuracy.

4. $\gamma$-hadron and hadron-hadron azimuthal correlations

There has been a lot of activity recently also to predict azimuthal correlations which could be a very powerful tool for detecting high density effects. In essence, $2 \rightarrow 2$ hard scattering should lead to approximately back to back correlations but if the projectile parton scatters off a strong target field that correlation should weaken or even disappear entirely. For the strongest effect, transverse momenta should be on the order of $Q_s$, not far above.

$$d + Au, 2.4 < y_1, y_2 < 4, 1 \text{ GeV} < p_T^{\text{ass}} < p_T^{\text{trig}}, p_T^{\text{trig}} > 2 \text{ GeV}$$

The most straightforward process for theory is $\gamma$-hadron correlations. As only one colored particle interacts with the target, this process, within the hybrid formalism, involves only the
DGLAP PDF of the projectile and the dipole unintegrated gluon distribution mentioned above (which can be obtained from rcBK). Theoretical uncertainties should therefore be reasonably small, allowing for quantitative tests of the theory. A first prediction for $\gamma$-hadron correlations in the forward region of $d+Au$ collisions at RHIC is shown in fig. 6 [27].

Di-hadron correlations in the forward region are more intricate as now two colored particles interact with the target. Aside from the BK dipole the cross section for this process also involves a quadrupole correlation function [28],

$$Q = \frac{1}{N_c} \langle \text{tr} V(0) V^\dagger(r) V(u) V^\dagger(v) \rangle ;$$  \hspace{1cm} (5)

at high $p_T$, the cross section can be expressed in terms of the dipole and the (distinct) Weizsäcker-Williams (WW) UGDs [29]. The evolution of the quadrupole with energy follows from the JIMWLK functional renormalization group equation. A numerical solution has been presented in ref. [30] and turned out to agree rather well, at least for a few simple configurations, with a Gaussian approximation (worked out in [29]) which expresses it as a (complicated) function of the dipole. A Gaussian approximation is extremely useful in practice since numerical solutions of the rcBK equation are rather straightforward; and because the dipole (incl. its initial condition) are rather well constrained by data, see above. The interest in correlations has also initiated new theoretical insight in form of a Gaussian approximation to JIMWLK evolution [31].

To compare to data, ref. [32] also evaluates the pedestal due to double parton scattering. The authors observe broadening of the away side peak, fig. 6, due to the presence of non-linear effects. An earlier analysis using model parametrizations for the dipole and WW gluon distributions was presented in ref. [34].

5. Summary

Tremendous progress has been made in recent years to develop the CGC effective theory into a quantitative framework which can confront data from RHIC and LHC. This includes (but is not limited to) improved computations of multiplicities for a variety of systems and energies; investigations of multiplicity distributions, KNO scaling in $p+p$, $p+A$ and initial-state fluctuations in $A+A$ collisions; semi-hard transverse momentum distributions and nuclear modification factors with rcBK UGDs and including Glauber fluctuations; photon production, $\gamma$-hadron and di-hadron correlations. Currently, the CGC is the only formalism to address such a broad range of observables systematically and with some success.

In the near future we should see further improvements of the accuracy and reliability of some of the predictions, for example a first computation of particle production in the forward region of $p+p$ and $p+A$ collisions at full NLO level. Moreover, there is great potential in understanding initial-state fluctuations in the “little bang”, such as the scale on which they occur and how they reflect in the final state. Further, with respect to gluon correlations in the boosted wave functions which affect multi-hadron correlations. We are also presently lacking a description of the transition from semi-hard $p_T$ at small $x$ to the DGLAP regime. These (and other) questions remain to be solved to achieve a more complete understanding of QCD, and the structure of matter, at short distances and high energies.
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