A Simple Method for Measuring Plastic Properties of Power Hardening Metals via the Indentation Curve with a Large Depth

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The spherical indentation technique provides an easy way to evaluate the integrity of in-service structures because it is non-destructive. In this study, a simple method was proposed to measure mechanical properties such as the yield strength, the ultimate tensile strength, and the strain hardening exponent from the indentation curve at a large indentation depth, which is 0.4 times of the indenter radius. Based on finite element analyses, a simple function was proposed to relate representative stress to indentation data. Besides, representative strains at different indentation depths were identified according to the load-depth curves from simulations. The calculated plastic properties from the developed method were compared well with experimental results.

1. Introduction

Mechanical properties of materials such as ultimate tensile strength (UTS) and yield strength are considered to be important for evaluating the integrity of structures [1, 2]. However, conventional tensile tests are destructive and need preparation of specimens. In order to obtain mechanical properties of local domains and in-service components, the instrument indentation test (IIT) was proposed [3–6]. Compared with sharp indenter, spherical indenter is more practical since it can measure elastoplastic properties by just one indentation [7–10]. Many studies have been carried out with the purpose of extracting mechanical properties from spherical indentation parameters [11–13]. Some researchers directly relate the indentation parameters to the tensile strain-stress points. In the direct method, the representative stress was defined as the averaged true stress under the indenter, while the representative strain was defined by using tangent function or sine function [14], which leads to different accuracies of the calculation results. Jeon et al. [15] found that, compared with the tangent function, the sine function results in a too large work hardening exponent for materials that obey Hollomon hardening law. Besides, both the sine function and the tangent function contain the parameter of contact depth, which is difficult to be measured in practical. Other researchers extract the elastoplastic properties from a closed-form expression of the indention load [16] or work [17] as a function of parameters of the Hollomon hardening law. Cao and Lu [16] derived the indention load as a function of representative stress. However, the expression contains dozens of coefficients, which need to be fitted and lack physical meanings. Ogasawara et al. [18] established a relatively simple equation of the loading work and the representative stress. However, the strain hardening exponent in the equation is in contradiction with the definition of the representative stress.

Indentation curves with a shallow depth cannot guarantee the uniqueness of the solution since different materials may generate identical indentation curves when the indentation depth is small [19]. Chen et al. [20] suggest that a deep indentation with the depth-radius ratio \((h/r)\) of 0.3 may...
distinguish the indentation curves. Lee et al. [17] demonstrate that the load-depth curves of different materials clearly separated from each other when the $h/r$ is 0.4, which may be the largest depth ratio in literatures.

In the current study, a simple method was proposed to obtain the yield strength, the ultimate tensile strength, and the strain hardening exponent from an indentation curve with a large indentation depth. Based on finite element analyses, the developed representative stress function is independent with the strain hardening exponent. Besides, the representative strains are determined at different depth-radius ratios with a maximum value of 0.4. The effectiveness of the method is verified on the Q235 steel and the 2219Al alloy by indentation tests and tensile tests.

2. Materials and Experiment

One of the experimental materials is a Q235 low carbon steel with the typical compositions of 0.17C-0.26Si-0.46Mn (wt %). The other material is an annealed 2219Al alloy with the chemical compositions of 5.80Cu-0.20Mn-0.20Si-0.30Fe-0.10Zn-0.02Mg (wt%).

The samples used in indentation tests and tensile tests were cut from the same blank. The tensile specimens were machined into cylindrical bars with a gage section of 5 mm in diameter and 30 mm in length. The indentation specimens were cut into cuboids with the dimensions of 30 mm $\times$ 30 mm $\times$ 20 mm. The uniaxial tests were performed on an Instron 5582 universal testing machine at room temperature. The indentation tests were performed on a Shimadu AG-X testing machine, whose load and displacement resolution are 0.2 N and 0.2 $\mu$m, respectively. A constant loading rate of 0.01 mm/s was applied for all indentation tests. The indenter is a tungsten carbide spherical with a radius of 0.25 mm. Four indentation and tensile tests were carried out for each material.

3. Finite Element Modelling

Finite element simulations were performed using ABAQUS software. An axisymmetric 2D model was constructed to simulate the indentation load-depth curves of materials. The Mises yield criterion and isotropic hardening law were used for the materials in simulations. The model contains more than 50,000 4-node bilinear axisymmetric quadrilateral elements (CAX4R in ABAQUS). Fine meshes of size 2 $\mu$m were used on the region beneath the indenter, which was modeled as an elastic material with Young’s modulus 534 GPa and Poisson ratio 0.3.

The maximum indentation depth-radius ratio is 0.4, which is deep enough to obtain unique solution of material properties in reverse analysis [17]. 720 simulations were carried out for different combinations of material properties within the range of $E$ from 70 GPa to 210 GPa, $E/\sigma_s$ from 0.001 to 0.04, and $n$ from 0 to 0.5. Figure 1 shows the geometry, mesh, and boundary conditions of the finite element model. The dimensions of the blank are 12 mm in radius and 8 mm in height, which are large enough to get rid of the size effect of the samples.

4. Verification of the Finite Element Model

The mechanical properties obtained from tensile tests are listed in Table 1. Figure 2 illustrates the experimental true stress-strain curves of Q235 steel and 2219Al, which were used for the simulation. As can be seen in Figure 2, the 2219Al alloy and Q235 steel have different behaviors between the elastic and plastic transitions. 2219Al shows a smooth curve, while the Q235 steel curve contains a passage of Lüders strain.

The friction coefficient $f$ is an important parameter in indentation simulation. Numerous studies have discussed the influence of friction coefficient on indentation curves based on simulation results. However, the reported research studies rarely compared the simulation load-depth curve ($p$-$h$ curve) with that of experiment when the friction coefficient is fixed. In the present work, the coulomb’s friction law is assumed between the contact surfaces. Figure 3 shows the experimental and numerical indentation curves of 2219Al alloy and Q235 steel when the friction coefficients in numerical simulation are 0.2, 0.3, and 0.4, which are reasonable values in experiment. It can be seen that simulated curves with various $f$ (0.2 $\leq f \leq$ 0.4) are compared well with experimental curves. Therefore, the friction coefficient was set as 0.3 for simulations in this paper.

5. Measuring Method

Ogasawara et al. [18] proposed a dimensionless function to calculate the representative stress as follows:

$$\frac{W}{h^3 \sigma_s (1 + sn)} = \left( \frac{1}{m_e \left( \frac{E}{\sigma_s (1 + sn)} \right)} + \frac{1}{m_p} \right)^{-1},$$

(1)

where $W$ is the indentation work during loading; $h$ is the indentation depth; $\sigma_s$ is the representative stress; $n$ is the strain hardening exponent; $s$ is the sphere factor, which represents the work hardening effect; and $m_e$ and $m_p$ are coefficients. The values of $W$, $\sigma_s$, $s$, $m_e$, and $m_p$ are influenced
by $h/r$ for a certain material. $E$ is the plain strain modulus, which is as follows [18]:

$$E = \frac{E}{1 - \nu^2},$$

(2)

where $\nu$ is the Poisson ratio and $E$ is Young’s modulus of the test material. It should be noted that equation (2) did not contain the term that reflects the effect of indenter properties. Thus, the reduced elastic modulus $E^*$ was introduced in this paper to replace $E$. The $E^*$ is given as follows [16]:

$$E^* = \left(1 - \nu^2\right)\left(\frac{1 - \nu^2}{E^i} + \frac{1 - \nu^2}{E_i}\right)^{-1},$$

(3)

where $\nu$ is the Poisson ratio and $E_i$ is Young’s modulus of the indenter material.

According to the principle of the representative stress-strain method [7], representative stress function should be independent with $n$. Therefore, the introduction of $n$ in equation (1) is in contradiction with the principle of this method. The specific form of the representative stress function depends on the load-depth data from simulations. In the current study, it was found that the $W/(h^3\sigma_r)$ and $E^*/\sigma_r$ curves are in coincidence with each other under different $n$ values when a representative strain $\varepsilon_r$ is given. Figure 4 shows the relationship between $W/(h^3\sigma_r)$ and $E^*/\sigma_r$ on the condition of $E$ 70 GPa, $\varepsilon_r$ 0.055, and $h/r$ 0.4. It can be seen from Figure 4 that the $W/(h^3\sigma_r)$ – $E^*/\sigma_r$ curve is independent of $n$. Thus, equation (1) can be replaced by the following function:

$$\frac{W}{h^3\sigma_r} = \left(\frac{1}{m_\varepsilon (E^r/\sigma_r) + \frac{1}{m_p}}\right)^{-1}.$$  

(4)

Equation (4) is independent of $n$ and more simplified than equation (1). According to the simulation data, the values of $m_\varepsilon$ and $m_p$ under different indentation depths and $E$ were fitted by the following equations:

$$m_\varepsilon = A(E_d) \times \frac{r}{h}^B(E_d), \quad 0.1 \leq h/r \leq 0.4,$$

$$m_p = M(E_d) \times \frac{r}{h} + N(E_d), \quad 0.1 \leq h/r \leq 0.4.$$  

(5)

Based on the indentation data obtained from finite element simulations, the coefficients $A(E_d)$, $B(E_d)$, $M(E_d)$, and $N(E_d)$ can be fitted as

$$A(E_d) = 0.71031 + 0.24281 E_d,$$

$$B(E_d) = 0.46839 - 0.1232 E_d,$$

$$M(E_d) = 0.78397 - 0.434447 E_d + 0.504087 E_d^2 - 0.56863 E_d^3,$$

$$N(E_d) = 12.48498 + 9.87187 E_d - 42.8181 E_d^2 + 90.6233 E_d^3.$$  

(6)

where $E_d$ is a dimensionless coefficient which is defined as $E_d = E/E_1$, in which $E_1$ is $10^6$ MPa.

In this paper, the representative strain $\varepsilon_r$ was defined to be equal to the plastic strain $\varepsilon_{r,p}$. Correspondingly, based on the Hollomon equation, the relationship between the representative stress and strain is as follows [18]:

$$\sigma_r = \sigma_y \left(\frac{E}{\sigma_y}\right)^n \left(\varepsilon_r + \frac{\sigma_r}{E}\right)^n,$$

(7)

where $\sigma_y$ is the theoretical yield strength of Hollomon equation. According to the simulation data that was shown in Figure 5, the $\varepsilon_r$ under different $h/r$ can be fitted as

$$\varepsilon_r = 0.00607 + 0.16048 \frac{h}{r} - 0.09524 \left(\frac{h}{r}\right)^2, \quad 0.1 \leq h/r \leq 0.4.$$  

(8)

The reverse analysis means calculation of mechanical properties from a $p-h$ curve. In the inverse analysis, the $W$ and $h$ were obtained from the $p-h$ curve, while the $E$ and $h$ were taken as known conditions in the current study. Then, the $\sigma_y$ and $\varepsilon_r$ can be calculated using equations (4) and (8), respectively. The $n$ was calculated by using equation (7). According to the relationship between engineering and true stress and strain [21], 0.2% offset yield stress $R_{p0.2}$ is calculated by using the following equation:

$$R_{p0.2} = \frac{\sigma_{0.2}}{e^{(0.02+\sigma_{0.2}/E)}},$$

(9)

in which $\sigma_{0.2}$ was solved by the following equation:
In this study, the Lüders strain $\varepsilon_L$ is calculated using the following equation for materials with Lüders elongations [22]:

$$\ln\left(\frac{K}{\sigma_L}\right) = \varepsilon_L - n \cdot \ln \sigma_L,$$  \hspace{1cm} (11)

where [22]

$$\sigma_{0.2} = \sigma_y \left(\frac{E}{\sigma_y}\right)^n \left(0.002 + \frac{\sigma_{0.2}}{E}\right)^n.$$  \hspace{1cm} (10)

The true stress at Lüders strain is given as

$$\sigma_L = \sigma_y \left(\frac{E}{\sigma_y}\right)^n \varepsilon_L^n,$$  \hspace{1cm} (13)

The engineering stress at Lüders strain is given as

$$R_L = \frac{\sigma_L}{\sigma_y \varepsilon_L + \sigma_L / E}.$$  \hspace{1cm} (14)

Figure 3: Comparison between experimental and numerical indentation curves of 2219Al alloy and Q235 steel: (a) the integral curve; (b) partial enlarged view.

Figure 4: Relationship between $W/(h^3\sigma_y)$ and $E^*/\sigma_y$.

Figure 5: Simulation data of $\varepsilon_r$.  

\[ E = 70\,\text{GPa} \]
\[ h/r = 0.4 \]
\[ n = 0.5 \]
\[ n = 0.4 \]
\[ n = 0.3 \]
\[ h/r = 0.4 \]
\[ n = 0.2 \]
\[ n = 0.1 \]
\[ n = 0.0 \]
The ultimate tensile strength UTS was calculated using the following equation [23]:

\[
UTS = \sigma_y \times \left( \frac{nE}{2.718\sigma_y} \right)^n.
\]

(15)

6. Results and Discussion

In order to verify the measuring method, indentation curves of Q235 steel, 2219Al, and 3 imaginary materials were employed to extract mechanical properties. The experimental stress-strain curves of Q235 steel and 2219Al do not match equation (7) precisely. The 3 imaginary materials are ideal power law hardening materials. The input mechanical properties of the 3 imaginary materials are shown in Table 2. The numerical simulations of indentation were carried out to obtain P-h curves of the 3 imaginary materials. The indentation works W under different depths are collected in Table 3.

Figure 6 is the comparison between calculated stress-strain points and input curves or experimental curves. It can be seen from Figure 6 that the calculated data points are compared well with the input curve for the 3 imaginary materials. However, the stress-strain points of Q235 and 2219Al show deviations from the experimental tensile curves. It should be noted that the combination of E, σ_y, and n is not unique for real materials that do not strictly obey the power law equation. Figure 7 shows comparison between experimental tensile and fitted curves of 2219Al. It can be seen that the same stress-strain curve can be fitted as several combinations such as E = 70 GPa, σ_y = 52.8 MPa, and n = 0.296 and E = 23 GPa, σ_y = 83.9 GPa, and n = 0.296 when E or σ_y was fixed. In this study, the σ_y was an unknown parameter, while the E was taken as a known parameter. Consequently, the σ_y can only be fitted as 52.8 MPa, which is much smaller than the yield strength (83.9 MPa) of 2219Al. This phenomenon was also reported in literatures. Byun et al. [23] introduced an offset constant to calculate the yield strength from the indentation test. However, the proposed constant was determined from tensile tests versus indentation tests on the same material. It is well known that the 0.2% offset yield strength R_{p0.2} was taken as the yield strength of the material without Lüders strain in conventional tensile tests. Thus, instead of the σ_y in the Hollomon equation, the R_{p0.2} extracted from indentation stress-strain curve should be taken as the yield strength and compared with that of the tensile test. As shown in Figure 7, the R_{p0.2} are 83.9 MPa and 80.6 MPa on the experimental curve and fit curve, respectively. Nevertheless, the R_{p0.2} may deviate from the yield strength of materials with Lüders strains, which make the yield strain unknown. In general, the yield strength is directly obtained from the stress-strain curve in the tensile test for the material with a yield point. In this work, the measuring method is based on the Hollomon equation, which cannot describe the passage of Lüders strain. Thus, the stress at the Lüders strain ε_L is proposed to be the yield strength of this kind of materials.

Tables 4 and 5 summarize the extracted material properties by using the developed method and Ogasawara’s method, respectively. It was shown that the errors of mechanical properties calculated by using the developed method are smaller than these calculated by using Ogasawara’s method. It also can be seen in Table 4 that the errors of R_{p0.2} and R_L of Q235 are ~17.8% and 21.4%. The corresponding strains ε_{0.2} and ε_L can be regarded as the lower and upper limits of the real Lüders strain, which is usually larger than ε_{0.2}. In the present work, the calculated ε_L = 0.0147 is larger than the real ε_L = 0.0098, which leads to an over-estimated R_L of Q235 steel. Thus, an average value (247.3 MPa) of R_{p0.2} and R_L will be a conservative estimate of the yield strength in indentation tests for materials with Lüders strain. It should be noted that the average yield strength is still an conservative value when the calculated ε_L is smaller than the real ε_L [24].

In order to investigate the sensitivity to the possible experimental errors for indentation testing, a 3% error band was artificially added to the indentation curves of Q235 steel and 2219 aluminium alloy (as shown in Figure 8). Table 6 shows the plastic properties obtained from the indentation curves with 3% error. It can be seen that, compared with properties in Table 4, the maximum error is ~4.9%.

7. Conclusions

In this study, a new energy-based approach was proposed to extract the stress-strain curve from the spherical indentation curve. The relationships between indentation stress-strain curve and mechanical properties were derived and discussed. The conclusions can be summarized as follows:
A new representative stress function was proposed for the spherical indentation test. Compared with the original function, the new function is more reasonable since it is independent with the strain hardening exponent. Besides, it can be applied to a maximum depth of 0.4 times of the indenter radius.

Table 4: Plastic properties obtained from the developed method.

| Material   | σ_y (MPa) | R_{p0.2}/R_L (MPa) | ε_{0.2}/ε_L (10^{-3}) | n     | UTS (MPa) | Error R_{p0.2}/R_L/n/UTS (%) |
|------------|-----------|---------------------|------------------------|-------|-----------|-------------------------------|
| 2219Al     | 73.6      | 97.6/—               | 3.4/—                  | 0.241 | 214.4     | 16.4/—18.9/—1.6               |
| Q235       | 138.8     | 199.6/295.0          | 2.9/14.7               | 0.243 | 457.4     | -17.8/21.4/1.6/4.4            |
| Material 1 | 81.0      | 105.5/—              | 3.5/—                  | 0.239 | 227.9     | -1.0/3.9/—                    |
| Material 2 | 227.2     | 290.4/—              | 3.5/—                  | 0.224 | 589.8     | -3.6/4.6/—                    |
| Material 3 | 229.0     | 272.4/—              | 3.3/—                  | 0.157 | 427.1     | -3.0/12.1/—                   |

Figure 6: Comparison between input or tensile curves and stress-strain points calculated by the reverse analysis: (a) 3 imaginary materials; (b) 2219Al and Q235 steel.

Figure 7: Comparison between experimental tensile and fitted curves of 2219Al.
The corresponding representative strain function was also given.

(2) A new method for extracting the yield strength from indentation stress-strain curve was proposed. Instead of the parameter \( \sigma_y \) in the Hollomon equation, the \( R_{p0.2} \) extracted from the indentation stress-strain curve was identified as the yield strength for materials without Lüders strains. It is suggested to take the average value of \( R_{p0.2} \) and the stress at Lüders strains \( R_L \) as the yield strength for materials with Lüders strains.

(3) The mechanical properties calculated by the developed method are more precise than those calculated by Ogasawara’s method. Based on the developed method, the maximum errors of yield strength, strain hardening exponent, and ultimate tensile strength are 21.4%, 18.4%, and 4.38%, respectively.

Data Availability

The data used to reproduce the findings of this study cannot be shared at this time as the work is still in progress.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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