Polarized pass band for longitudinal waves in solid phononic crystals

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Abstract
Elastic waves possess a rich variety of polarizations, and the wave propagation in solids is intricate due to the possible coupling and conversion between each wave mode. In this paper, we investigate the canonical solid phononic crystals (PCs) that can filter transverse waves in a purely longitudinal polarization band (PLPB). We numerically study the effects of the geometric parameters and material properties of the crystal components on the width and location of the PLPB. We find that the PCs considered here can be easily modulated to exhibit a wide PLPB due to their simple structures, that there is no connection between the PLPB and band gaps of the XY mode, and that the PLPB is independent of the Dirac cone. The reported findings could provide an alternative way for filtering certain types of elastic waves with the use of PCs.

1. Introduction
Much research has been conducted in recent years on the propagation of elastic waves in periodic materials known as phononic crystals (PCs) or acoustic metamaterials (AMMs). They have man-made structures exhibiting extraordinary properties which surpass conventional materials found in nature, such as outstanding sound insulation performance [1, 2], negative elastic parameters [3–8], self-collimation [9], and superlensing [10–12]. PCs are composite structures that are usually composed of two or more kinds of periodically arranged conventional materials whose elastic properties are distinct from one another, such as metals and plastics. A frequency band structure, analogous to an electronic band diagram, is often used to describe their dynamic characteristics [13]. PCs were first proposed to eliminate the propagation of waves owing to the band gaps [14, 15]. Many investigations have been conducted since then to generate or enlarge the band gaps for practical applications [16–29]. Compared with PCs of which the periodicity is of the same order as the wavelength, AMMs are composed of subwavelength units. The periodicity in space is not required for AMMs and they are often described by effective parameters [30–34]. However, the band theory is still applicable for the AMMs with periodicity.

The wave motions in the pass band are complex. In fact, elastic waves possess a rich variety of polarization characteristics, and the wave propagation in solids is intricate in view of the concurrence of P (longitudinal) and S (transverse) polarizations as well as the possible coupling and conversion between them. Elastic wave mode separation in solids has been theoretically investigated for many years in the field of geophysics [35–38]. In comparison, different wave modes can be decoupled physically with the use of the man-made structures, for which a typical example is the pentamode metamaterial (PMM). The PMMs are man-made structures consisting of solids, but they behave ideally like a fluid, thus allowing only the propagation of compressive waves [39–42]. This fluid-like behavior is found to be caused by a single mode frequency band [41, 42]. Within the single mode regime, longitudinal wave modes are permitted, but any transverse wave modes are forbidden. Resonant AMMs have been proved to be able to filter transverse waves in recent years. Lai and coworkers [3] proposed a two-dimensional (2D) multi-resonator metamaterial in which the band structure has two negative bands. Numerical calculations for the wave transmission suggest that these two bands can only support longitudinal waves, which
resembles the single mode regime in PMMs. Later in another research effort [7], they investigated a three-dimensional single-resonator model of which the unit is composed of epoxy matrix and silicone coated steel cylinders. A finite frequency regime that only allows longitudinal waves was observed in their study.

As yet, no research with respect to the Bragg PCs has been carried out into the fluid-like behavior. In this paper, we start with numerical calculation for the band structures of a canonical PC, followed by an eigenstate analysis and a set of transmission simulations. We find a purely longitudinal polarization band (PLPB) through the numerical simulations. We then explore how the structural geometry and material property will affect the PLPB. The inclusion radius, the lattice constant, the material property, and the inclusion shape are taken into account, and numerical investigation are performed to evaluate their effects.

2. Model-based simulations and analyses

2.1. Model description

We consider wave propagation in a 2D PC in the XY plane in which the cross section is shown in figure 1(a). The dimension along the z-direction is assumed to be infinite. The binary PC is composed of epoxy (material A) and Al_2O_3 (material B) with the former being the matrix and the latter being the inclusions. The materials are considered to follow linear and isotropic elasticity with the properties being given as follows:

\[ \rho_A = 1142 \text{ kg m}^{-3}, \quad E_A = 4.35 \text{ GPa}, \quad \nu_A = 0.378, \quad \rho_B = 3970 \text{ kg m}^{-3}, \quad E_B = 402.7 \text{ GPa}, \quad \nu_B = 0.23, \]

where \( \rho, E \) and \( \nu \) represent mass density, Young’s modulus, and Poisson ratio, respectively [23]. The inclusions are cylindrical and are periodically embedded in the matrix to form a square lattice. The lattice constant \( a \) is 10 mm and the cylindrical inclusions’ radius \( r = 0.35a \).

2.2. Band structure

The band structure is the relationship between the eigenfrequencies and the Bloch wave vectors, and is also known as the dispersion relation [14]. There are two categories of vibration mode for a 2D lattice [43] when considering wave propagation in the XY plane, i.e., the XY mode (mixed-polarization mode) and the Z mode (shear mode), which can be represented, respectively, by:

\[ \rho \ddot{u}_{xy} = (\lambda + \mu) \nabla \nabla \cdot \mathbf{u}_{xy} + \mu \nabla^2 \mathbf{u}_{xy}, \quad (1a) \]

\[ \rho \ddot{u}_z = \mu \nabla^2 \mathbf{u}_z, \quad (1b) \]

where the vector \( \mathbf{u} \) represents the displacement field, and \( \lambda \) and \( \mu \) are Lamé’s constants.
Figure 1(c) shows the band structure within the TX section for the XY mode. There exists a band gap ranging from 93.07 to 155.82 kHz as shown by the gray shaded area in this figure. No dispersion curves exist within the band gap, i.e., any form of in-plane waves along TX within this frequency range can not propagate in the lattice. The existence of a band gap is a typical feature that distinguishes PCs from ordinary materials [14, 27]. Instead of the band gap, what we are interested here is the band below it, i.e., the frequency range covering 0–93.07 kHz. This is an apparently allowed band for the XY mode. Contrary to the band gap, an allowed band is the frequency range where dispersion curves exist and the waves within this range are allowed to travel in the lattice.

For the Z mode, the first band gap is located between 44.17 and 108.50 kHz as is shown in figure 1(d), and it partially overlaps the XY mode’s first allowed band (0–93.07 kHz). Accordingly, the band structures of the XY mode and the Z mode are not synchronous. As described by equation (1), the XY mode is an in-plane mode and the Z mode is an out-of-plane mode, and they are independent of each other [20, 43, 44].

2.3. Eigenstate analysis
We have mentioned in section 2.2 that the XY mode is a mixed-polarization mode, which means that there exist more than one type of waves for the in-plane XY mode. It is obvious that in figure 1(c) the second branch intersects the third branch, such that the wave modes of these two branches are distinct from each other [45]. Visualizing the eigenstates is important for understanding the wave modes. On the other hand it has been an effective tool for analyzing the mechanism of the formation of band gaps in PCs [2, 3, 25, 26, 29, 46] and AMMs [1, 4–8, 47]. Figure 2 illustrates the mode shapes of the five points marked as A–E that are located on the lowest three branches in figure 1(c). It can be seen that the mode shapes of modes A–C are strikingly different from those of modes D and E. Specifically, the displacements of modes A–C are perpendicular to the wave vector, and these are typical SH wave modes. A closer look into the eigenstates reveals that for the first branch the oscillation is due to the transverse translation, and that for the third branch the inclusion and matrix do not move in unison such that there exist rotations. The modes D and E are P wave modes since the displacements are parallel to the wave vector. As a result, the first and the third branch are SH branches and the corresponding frequency bands allow SH wave motions. For the PC studied in this paper, P waves and SH waves are decoupled on the XY mode’s lowest three branches. The second branch is a P wave branch and only the P wave is allowed in the corresponding frequency band. The eigenstates of the Z mode are not presented here, and according to the definition of the Z mode all of the branches of the band structure support SV waves.

It is worth noting that within the frequency band that is located between 45.03 and 74.66 kHz (filled with yellow color in figure 1(c)) there exists a P wave branch with no SH wave branch or SV wave branch. It is a band gap for both SH waves and SV waves but an allowed band for P waves, in other words, it is a PLPB.

2.4. Transmission analysis
To further examine the wave behaviors in the lattice, we perform the following analysis with longitudinal and transverse excitations, respectively. The dimension of the sample is finite in the wave propagation direction. As is shown in figure 3(a), the 2D sample consists of ten unit cells in the longitudinal direction, and periodicity is considered in the transverse direction to reduce the problem size. Monochromatic excitations with unit magnitude are input at one end, and the responses are detected at the other end. The analyses are conducted with the frequency sweep technique. The transmissions of the in-plane wave motion are examined with longitudinal and transverse excitations, respectively. For each type of input, we evaluate both the longitudinal and transverse transmissions by averaging the quantities obtained at the other end. The results are given in decibel which is $20 \log_{10}(X_{\text{res}}/X_{\text{exc}})$, where $X_{\text{exc}}$ and $X_{\text{res}}$ represent the excitation and the response, respectively.

Figure 3(b) shows the transmissions for both longitudinal (left) and transverse (right) excitations, in which the middle describes the band structure of the XY mode with the lowest three branches. A good agreement between the band structure and the transmission spectra can be observed. Specifically, the frequency range
covered by the $L_0$ branch is an allowed band for $P$ waves so that the longitudinal response (pink solid line) is remarkable for the longitudinal excitation as shown in the left panel. Outside this frequency range, strong attenuation occurs and the response is very weak. Within the frequency ranges that are occupied by the $SH_0$ and $SH_1$ branches, strong transverse response (cyan dashed line) is evoked for the case of transverse excitation in the right panel; outside these ranges, the waves are greatly mitigated. The remarkable response that can be detected within the PLPB (yellow shaded area in figure 3(b)) is the longitudinal response for longitudinal excitation. The transverse response under the longitudinal excitation is attenuated within this frequency regime. The PLPB, which separates the $SH_0$ and $SH_1$ branches, corresponds to attenuation zones for both transverse and longitudinal responses for the transverse excitation.

It is found that within the allowed band for $P$ waves, i.e., the frequency range covered by the $L_0$ branch, the transverse response (cyan dashed line) for the case of longitudinal excitation is very weak as illustrated in the left panel. Within the allowed bands for $SH$ waves, i.e., the frequency ranges covered by the $SH_0$ and $SH_1$ branches, the longitudinal response is much weaker than the transverse response under the transverse excitation in the right panel. In fact, within the whole frequency range considered here, the response with the same polarization as the excitation is much stronger than the response with the different polarizations. As can be seen in figure 3(b), the longitudinal response is remarkable for longitudinal excitation, and the transverse response is remarkable for transverse excitation. The difference reveals that the wave mode conversion is weak in the whole frequency range that we are considering, i.e., 0–100 kHz in this study.

For the $Z$ mode, a good agreement between the transmission spectra and the band structure can be found in figure 3(c). The frequency bands corresponding to the $SV_0$ and $SV_1$ branches are allowed bands, and other bands in figure 3(c) are band gaps. Strong attenuation occurs so that the responses within the forbidden bands are very

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**Figure 3.** (a) Schematic of the numerical calculation for the transmission along $\Gamma X$. (b) Comparison between the $XY$ mode’s band structure (middle) and the transmissions for the in-plane longitudinal (left) and transverse (right) excitation, with the pink solid lines and cyan dashed lines representing the responses in the $x$- and $y$-directions, respectively. (c) Comparison between the $Z$ mode’s band structure (left) and the transmission for the out-of-plane excitation, with the gray area being the first band gap of the $Z$ mode.
weak. The frequency band filled with yellow color is the PLPB, and the Z mode wave motions are forbidden in this range since it is a subset of the first band gap of the Z mode.

The studies in this section demonstrate that the results of the transmission analyses are in good agreement with what have been predicted both by the band structure calculations and by the eigenstates analyses. On the lowest three branches of the XY mode, SH waves and P waves are decoupled, while all of the branches of the Z mode support SV waves. The asynchronism in the disperse relations of the SH, SV, and P waves makes the occurrence of the PLPB possible. The mode conversion between the P wave and the S wave is very weak in the whole frequency range that we are considering in this study. In this respect, within the PLPB, only longitudinal excitation can produce a remarkable response that is also longitudinal.

3. Modulating the PLPB

The band structure of the PC is influenced by many factors [19–23, 46], and the PLPB can be modulated via geometrical parameters and/or material properties. In this section, we investigate how these factors will affect the PLPB.

3.1. Inclusion radius

Varying the radius of the inclusions will transform the band structures. In addition, the XY and Z modes are not synchronous, such that the band structure corresponding to each mode should be compared to identify the upper and lower boundaries of the PLPB. In figure 4(a), we plot the band structures of the XY mode (dark cyan solid circle) and the Z mode (pink solid line) for all the radii with the PLPBs shaded in yellow color. The lower boundary as well as the upper boundary is determined by the XY mode for all the radii considered here. The band width, which is represented by the solid squares, increases as the radius of the inclusions increases from $a = 0.05$ to $a = 0.45$. The width is only $1.87$ kHz if $r = 0.05a$, and it becomes $33.55$ kHz if $r = 0.45a$. Meanwhile, the band center fluctuates between 58 and 85 kHz. It is worth noting that within the range considered here, the dependency of the PLPB’s width on the inclusions’ radius is much weaker if the ratio of the inclusions radius to the lattice constant is $0.35$ with the inset showing the shape of the band structures; in (c), $\times$ (◯) represents the existence (nonexistence) of the PLPB.

Figure 4. Effects of (a) inclusion radius, (b) lattice constant, and (c) materials property on the PLPB. In (a), the lowest three branches of the XY mode (dark cyan solid circle) and the first two branches of the Z mode (pink solid line) within the $\Gamma X$ section are put together, in which each solid square represents the width (right axis) of the corresponding PLPB, with the dashed line connecting the squares and the lattice constant $a$ being 10 mm. In (b), the ratio of inclusion radius to the lattice constant is $0.35$ with the inset showing the shape of the band structures; in (c), $\times$ (◯) represents the existence (nonexistence) of the PLPB.
the lattice constant (i.e., \( r/a \)) is greater than 0.25. In a word, we demonstrate in this section that for a given a lattice constant, the band width increases as the inclusion radius is enlarged, but the band location will not change remarkably. The supplementary material is available online at stacks.iop.org/JPCO/1/055026/mmedia offers more information outside the frequency range considered in figure 4(a).

### 3.2. Lattice constant

Figure 4(b) demonstrates the effect of the lattice constant on the PLPB. The lattice constant varies evenly from 5 to 45 mm with the interval being 5 mm, and the ratio of the inclusion radius to the lattice constant is fixed at 0.35 in this study. The unit cells are labeled in ascending order with \#1–9 according to their lattice constant. It can be seen that varying the lattice constant will not transform the band structures, i.e., the shapes of these band structures are similar and the dimensions will change with the lattice constant. The inset shows the shape of the band structures as well as the location of the PLPB. In general, as the lattice constant increases (from \#1 to \#9), the center of the band moves downwards, and the width becomes narrower. This is consistent with the previous findings about the band gap of PCs [1]. We also notice that the ratio of the band width to the band center is constant (the value is 0.495 in this study) as long as the ratio of inclusion radius to the lattice constant remains the same.

### 3.3. Material

The material choice for each component is much wider, but we limit our studies to the four materials as given in table 1. The mass density and the wave speed of these material have significant differences. For the sake of clarity, we number every material in the table from 1 to 4. The PCs that are composed of these materials are labeled with two numbers with the first and second numbers standing for the matrix and the inclusion, respectively. For instance, the PC (1, 2) means that the matrix and the inclusion are Epoxy and \( \text{Al}_2\text{O}_3 \), respectively. In this section, we alter the material of each component, and there are 12 PCs in total to be studied here. All the PCs have the geometric parameters as follows: \( a = 10 \text{ mm} \) and \( r = 0.35a \).

As is shown in figure 4(c), nine of the twelve PCs studied here have a PLPB. In order to compare these PLPBs, we calculate the ratio of the width to the center for each of them so that the data are within a narrow range and the effect of the material are taken into account. We arrange the data points in figure 4(c) in ascending order according to the ratios. We note that the PC (3, 1) has a wide PLPB and ranks the top among the nine PCs in figure 4(c) and that the PC (1, 3) does not have a PLPB. However, the former has no band gap for the XY mode, and the latter has very wide band gaps for both the XY mode and Z mode (see the supplementary material). We also note that the PCs (4, 3), (2, 4), (2, 3), and (4, 2) have no obvious PLPB, which is because the materials 2, 3, and 4 are approximately equal in impedance such that the first and third disperse curves can only open up a very narrow gap at the point X for the XY mode. However, the PCs (3, 2) and (3, 4) have no PLPB despite that there exists a narrow gap between the first and third branches for the XY mode. In the supplementary material we will show that the narrow gap is entirely covered by a branch of the Z mode thus eliminating the possibility of the PLPB. To sum up, altering the materials can transform the PLPB, and a remarkable impedance mismatch between the inclusions and the matrix could lead to a wide PLPB. However, it is worth to note that the exchange between the matrix and inclusion may eliminate the PLPB despite that there may exist a wide band gap for the XY mode, which means that no connection exists between the bands of the XY mode and the PLPB.

### 3.4. Inclusion shape

The existence of the PLPB is not limited to cylindrical inclusions. In this section we first consider a squared cross section of the inclusions. The side length of the squared inclusion is set to be 0.62a such that the filling fraction is identical to the cylindrical inclusions for which the radius is 0.35a. We also consider a rectangular cross section with the aspect ratio being 1.5, an arbitrarily set value, and the filling fraction is identical to the referenced cylindrical inclusions. The eigenstates corresponding to the first three branches of the XY mode are presented in

### Table 1. Elastic properties of materials in this section.

| Label | Material name | \( \rho \) (kg \( \text{m}^{-3} \)) | \( c_1 \) (m \( \text{s}^{-1} \))\(^a\) | \( c_2 \) (m \( \text{s}^{-1} \))\(^b\) |
|-------|---------------|-------------------------------|---------------------------------|---------------------------------|
| 1     | Epoxy         | 1142                          | 2655                            | 1176                            |
| 2     | \( \text{Al}_2\text{O}_3 \) | 3970                          | 10844                           | 6421                            |
| 3     | Au [20]       | 19500                         | 3360                            | 1239                            |
| 4     | Steel [20]    | 7890                          | 5780                            | 3220                            |

\(^a\) Longitudinal wave speed.

\(^b\) Transverse wave speed.

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**Figure 5.** (a) Band structures of the PC with square inclusions. The side length of the inclusions is 0.62a for this configuration. (b) The representative eigenstates of the first three branches of the $XY$ mode. (c) Band structures of the PC with rectangular inclusions. The aspect ratio of the inclusions is 1.5 for this configuration and the filling fraction is the same as the one in (a). (d) The representative eigenstates of the first three branches of the $XY$ mode within the $YT$ section (left) and $\Gamma X$ section (right). In parts (a) and (c), the insets are the cross sections of the unit cells, in which the solid lines represent the $XY$ mode and the dashed lines the $Z$ mode. In (b) and (d), the black cones denote the displacements and the white arrows indicate the wave vectors.

Figure 5 for both the squared and the rectangular inclusions. As discussed in section 2, the first (bottom) and third (top) branches of the $XY$ mode are $SH$ wave modes while the second (middle) branch is $P$ wave mode.

Figure 5(a) shows the band structures of the PC with squared inclusions. There exists a PLPB covering 45.08–76.88 kHz, which is slightly larger than the one as shown in section 2 which is 45.03–74.66 kHz. This is similar to the cases for the first complete band gap investigated in [19, 21, 22]. According to their findings, selecting a squared inclusion will produce the largest complete band gap for squared lattices. The PC is equivalent along the $YT$ and $\Gamma X$ directions for the squared inclusions, but the equivalence is not guaranteed for the rectangular inclusions. As we can see in figure 5(c), the band structures along one direction is quite distinct from those along the other direction. The PLPB within the $\Gamma Y$ section is much wider than the one within the $\Gamma X$ section. Specifically, the former has a width of 55.11 kHz, and the latter 19.94 kHz. With the same filling fraction given, the widths are 21.80 kHz and 19.63 kHz for the squared and cylindrical inclusions, respectively. Therefore, both the squared and the rectangular inclusions exhibit a wider PLPB than the cylindrical inclusions for a given filling fraction. In addition, the PLPB within the $YT$ section for the rectangular inclusions is much wider than that for the squared inclusions.

**4. Concluding remarks**

We have investigated the single mode regime for purely longitudinal waves in canonical PCs. Within the specific frequency domains, the longitudinal wave are permitted while the translational waves are eliminated. Our studies are based on the analyses of band structures, eigenstates, and transmissions. We have demonstrated that the PLPBs, if they exist, are between the first and second translational branches of the mixed-polarization mode band structures. The single mode regime can be easily modulated via changing structural geometry and/or material type. We have evaluated the effect of each factor on the PLPB through model-based simulations. On the other hand, the PLPB can also be modulated collectively via these factors. In fact, the factors that can be controlled to modulate the PLPB are far beyond the scope of this paper. As for the inclusion shapes, we have limited our studies to the rectangular and cylindrical inclusions, while other inclusion shapes such as elliptic cylinder, triangular prism, and hexagonal prism can also be exploited and their orientations can add more freedoms to modulating the band. As for the crystal systems, we have focused our attention on the squared lattice, while the oblique and hexagonal systems may generate multiple PLPBs along different directions. Piezoelectric materials may help realize the active control of the PLPB. The materials in this paper are considered to follow linear and isotropic elasticity, but the band gaps may be swamped by the materials’ damping. However, the previous results as reported in other references have indicated that it is reasonable to ignore the damping effect on the epoxy matrix response [1, 7, 48, 49]. Our analyses are based on the binary PCs, while the ternary or multicomponent PCs can be examined, which is beyond the scope of this work. The reported results are limited to the $\Gamma X$ or $\Gamma Y$ direction, but the filtering effect remains along the $\Gamma M$ direction (see the supplementary material).
We note that in [7] the authors claimed that the PCs having a Dirac cone dispersion can exhibit the single frequency regimes. However, according to our findings in this paper, no Dirac cone is needed. The Dirac cone relies on the accidental degeneracy of the eigenstates resulting from the precise adjustment of the geometric parameters [46]. However, the geometries in this paper are not strictly restricted. In the supplementary materials, we provide a separate section to illustrate the PLPB’s independence of the Dirac cone. The quest for manipulating elastic waves with the use of PCs is unending, and the results of the work are promising for stimulating investigations into the canonical PCs that have been studied for more than 20 years. The findings reported here may provide an alternative way to filter certain types of waves for engineering applications and open a new research direction in advancing the PC technology.

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