Properties of the $\pi^0$, $\eta$, $\eta'$, $\sigma$, $f_0(980)$ and $a_0(980)$ mesons and their relevance for the polarizabilities of the nucleon

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Abstract

The signs and values of the two-photon couplings $F_{M\gamma\gamma}$ of mesons ($M$) and their couplings $g_{MNN}$ to the nucleon as entering into the $t$-channel parts of the difference of the electromagnetic polarizabilities ($\alpha - \beta$) and the backward angle spin polarizabilities $\gamma_\pi$ are determined. The excellent agreement achieved with the experimental polarizabilities of the proton makes it possible to make reliable predictions for the neutron. The results obtained are

- $\alpha_n = 13.4 \pm 1.0$, $\beta_n = 1.8 \pm 1.0 \times 10^{-4}$ fm$^3$, and
- $\gamma_\pi^{(n)} = 57.6 \pm 1.8 \times 10^{-4}$ fm$^4$.

New empirical information on the flavor wave functions of the $f_0(980)$ and the $a_0(980)$ meson is obtained.

PACS. 11.55.Fv Dispersion relations – 11.55.Hx Sum rules – 13.60.Fz Elastic and Compton scattering – 14.70.Bh Photons

1 Introduction

Due to recent research the electromagnetic polarizabilities $\alpha$ and $\beta$ and the backward spin polarizabilities $\gamma_\pi$ of the proton and the neutron are known to a level of precision, that details of the $t$-channel contributions may be investigated which previously were not of relevance. The $t$-channel contributions $\gamma_\pi^t$ are given by the pseudoscalar mesons $\pi^0$, $\eta$, and $\eta'$ whereas the $t$-channel contributions $(\alpha - \beta)^t$ are given by the scalar mesons $\sigma$, $f_0(980)$, and $a_0(980)$. The individual contributions depend on the mass $m_M$ of the meson ($M$), on their two-photon couplings $F_{M\gamma\gamma}$ and on their couplings $g_{MNN}$ to the nucleon. In addition to the values of these couplings also their signs have to be known.

The $\sigma$-meson enters into the amplitude for Compton scattering as a $t$-channel exchange. This means that the $\sigma$-meson resonant excited state is located in the unphysical region of the Compton scattering amplitude $A_1(s,t)$ at positive $t$. We consider this as an argument in favor of treating the $\sigma$-meson contribution to $(\alpha - \beta)^t$ in terms of a pole in the complex $t$-plane with properties of the $\sigma$ meson as the particle of the $\sigma$ field, having a definite mass of $m_{\sigma} = 666.0$ MeV [1, 2]. However, for the obvious reasons discussed in [1, 3] it is equivalent to treat the $\sigma$-meson contribution to $(\alpha - \beta)^t$ in terms of a cut in the complex $t$-plane with properties as obtained for the on-shell $\sigma$ meson. This equivalence may be formulated in terms of a sum rule, where $(\alpha - \beta)^t$ connects properties of the $\sigma$-meson as the particle of the $\sigma$ field with properties of the on-shell $\sigma$ meson.

The theoretical investigations of the $t$-channel pole contributions carried out in the past [1, 3, 4] remained incomplete because there were no firm predictions of the signs. Furthermore, there was only vague information on the contributions of the $f_0(980)$ and $a_0(980)$ mesons. The purpose of the present investigation is to provide the necessary complementary information.
2 Outline of the problem

The t-channel contributions entering into the backward spin polarizability \( \gamma_{\pi} \) and the difference of the electric and magnetic polarizabilities \((\alpha - \beta)\) may be written in the form

\[
\gamma_{\pi}^t = \frac{1}{2\pi m_N} \left[ \frac{g_{\pi NN} F_{\pi\gamma\gamma}}{m_\pi^2} + \frac{g_{\eta NN} F_{\eta\gamma\gamma}}{m_\eta^2} \right], \tag{1}
\]

\[
(\alpha - \beta)^t = \frac{g_{\eta NN} F_{\eta\gamma\gamma}}{2\pi m_\eta^2} + \frac{g_{a_0 NN} F_{a_0\gamma\gamma}}{2\pi m_{a_0}^2} + \frac{g_{a_9 NN} F_{a_9\gamma\gamma}}{2\pi m_{a_9}^2} \tau_3. \tag{2}
\]

In (1) and (2) \( m_N \) is the nucleon mass, \( m_\pi \) etc. the meson mass, \( g_{\pi NN} \) etc. the meson nucleon coupling constant, \( F_{\pi\gamma\gamma} \) etc. the meson to two-photon coupling and \( \tau_3 = +1, -1 \) for the proton and neutron, respectively.

In addition to the absolute values of the quantities entering into (1) and (2) the signs of the quantities have to be known. The meson-nucleon coupling constants are known to be positive whereas the signs of the two-photon couplings of the mesons have partly been uncertain. Recently, it has been shown [1] that the options

\[
F_{\pi\eta\gamma} = -|M(\pi^0 \rightarrow \gamma\gamma)| \tag{3}
\]

\[
F_{\eta\gamma\gamma} = +|M(\eta \rightarrow \gamma\gamma)| \tag{4}
\]

\[
F_{\eta'\gamma\gamma} = +|M(\eta' \rightarrow \gamma\gamma)| \tag{5}
\]

\[
F_{\sigma\gamma\gamma} = +|M(\sigma \rightarrow \gamma\gamma)| \tag{6}
\]

lead to good agreement with experiment where the quantities \( M(M \rightarrow \gamma\gamma) \) are the meson \( M \rightarrow \gamma\gamma \) decay matrix elements. In case of the large contributions corresponding to the couplings \( F_{\pi\eta\gamma} \) and \( F_{\sigma\gamma\gamma} \), the signs have been known before from analyses of Compton scattering data, whereas in case of the small contributions corresponding to the couplings \( F_{\eta\gamma\gamma} \) and \( F_{\eta'\gamma\gamma} \) the signs formerly have been adopted as negative [4]. Therefore, it appears desirable to have theoretical arguments which firmly predict the signs in all four cases. These arguments will be given in the following. In addition also the contributions of the mesons \( f_0(980) \) and \( a_0(980) \) will be determined.

The amplitudes for the decay of pseudoscalar mesons \( P \) and scalar mesons \( S \) into two photons are given by [5,6]

\[
\mathcal{A}(P \rightarrow \gamma(\epsilon_1, k_1)\gamma(\epsilon_2, k_2)) = M(P \rightarrow \gamma\gamma) \epsilon_{\mu\nu\alpha\beta} \epsilon_1^* \epsilon_2^* k_1^{\nu} k_2^{\alpha}, \tag{7}
\]

\[
\mathcal{A}(S \rightarrow \gamma(\epsilon_1, k_1)\gamma(\epsilon_2, k_2)) = M(S \rightarrow \gamma\gamma) \epsilon_{2\mu} \epsilon_{1\nu} (g^{\mu\nu} k_2 \cdot k_1 - k_1^{\mu} k_2^{\nu}), \tag{8}
\]

where \( \epsilon \) is the polarization vector for linear polarization and \( k \) the 4-momentum of the photon. Here, the following notation is adopted: \( g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \ g^{\mu\nu} = 0 \) for \( \mu \neq \nu, \ \epsilon_{\mu\nu\alpha\beta} = 1 \) for even permutations of 0123, \( \epsilon_{\mu\nu\alpha\beta} = -1 \) for odd permutations of 0123, \( \epsilon_{\mu\nu\alpha\beta} = 0 \) if at least two indices are equal to each other. The kinematical factors on the r.h.s of (7) and (8) for the two-photon decays of the pseudoscalar and scalar mesons may be evaluated. For this purpose we may write

\[
k_1 = (\omega, 0, 0, k), \tag{9}
\]

\[
k_2 = (\omega, 0, 0, -k), \tag{10}
\]

\[
\epsilon_1 = (0, 1, 0, 0), \tag{11}
\]

\[
\epsilon_2 = (0, 1, 0, 0) \quad \text{for scalar mesons}, \tag{12}
\]

\[
\epsilon_2 = (0, 0, -1, 0) \quad \text{for pseudoscalar mesons}. \tag{13}
\]

\[\text{1 For the phase convention see [7].}\]
The different expressions in \(12\) and \(13\) are due to the fact that scalar mesons decay into two photons with parallel planes of linear polarization and pseudoscalar mesons into two photons with perpendicular planes of linear polarization. Inserting \(9\) to \(13\) into the kinematical factors of \(7\) and \(8\) we arrive at 

\[
\epsilon_{\mu\nu\alpha\beta} k_1^{*\mu} k_2^{*\nu} k_2^{\beta} = \epsilon_{\mu\nu\alpha\beta} (g_{\mu\nu} k_2 \cdot k_1 - k_1^{\mu} k_2^{\nu}) = -2\omega^2 = -\frac{1}{2} m_M^2.
\]

This means that the kinematical factors in \(7\) and \(8\) are the same except for the fact that they distinguish between the two cases of relative linear polarization of the two photons.

In a quark model the structure of pseudoscalar and scalar may be described as follows

\[
|P\rangle = (a_p|u\bar{u}\rangle + b_p|d\bar{d}\rangle + c_p|s\bar{s}\rangle) (1S_0), \quad J^{PC} = 0^{-+}, \quad (14)
\]

\[
|S\rangle = (a_s|u\bar{u}\rangle + b_s|d\bar{d}\rangle + c_s|s\bar{s}\rangle) (3P_0), \quad J^{PC} = 0^{++}, \quad (15)
\]

with the constraints \(a^2 + b^2 + c^2 = 1, a^2 = b^2\) in both cases. Eqs. \(14\) and \(15\) suggest that the flavor parts of the wave functions may have very similar properties in case of the pseudoscalar and scalar mesons, whereas the angular momentum parts are different. For pseudoscalar mesons we have \(S = 0, L = 0, L = J = 0\) and for scalar mesons \(S = 1, L = 1, J = 0\). A further difference is that the pseudoscalar particles may be considered as quasistable because of the relatively long lifetimes whereas the scalar mesons are comparatively shortlived because of the decay of these particles into meson pairs. This is especially true for the \(\pi\) meson showing up as a very broad resonance in the \(\pi\pi\) channel. As will be shown later, for isoscalar mesons we have \(a = b = 0\), whereas for isovector mesons we have \(-a = b = 0\). In principle there are further constraints on the wave functions stemming from SU(3) symmetry. It will turn out that these constraints lead to very intricate problems with the strange quark content in case of the scalar mesons. We will introduce here a novel way to solve these problems by assuming that these constraints are strongly violated. The justification for this is that in case of scalar mesons the \(q\bar{q}\) core state is strongly coupled to two-meson states, thus leading to a strong distortion of the flavor wave function. This coupling is known to be also responsible for a major portion of the meson mass [8–11]. The quantum numbers given in \(14\) and \(15\) are in line with the fact that pseudoscalar mesons decay into two photons with perpendicular planes of linear polarization whereas scalar mesons decay into two photons with parallel planes of linear polarization [12].

3 Pseudoscalar mesons

Using the arguments contained in [13] (see p. 27, 28, 54 and 55) the flavor wave functions of our present interest may be written in the \(|SU(3), SU(2)\rangle\) form

\[
|\eta_1\rangle = |1, 1\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle, \quad (16)
\]

\[
|\eta_0\rangle = |8, 3\rangle = \frac{1}{\sqrt{2}}| - u\bar{u} + d\bar{d}\rangle, \quad (17)
\]

\[
|\eta_8\rangle = |8, 1\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle, \quad (18)
\]

Using these wave functions and retaining SU(3) symmetry the following relations may be obtained (see p. 70 of [13])

\[
M(\eta_8 \to \gamma\gamma)/M(\pi^0 \to \gamma\gamma) = -\frac{1}{\sqrt{3}}, \quad (19)
\]

\[
M(\eta_1 \to \gamma\gamma)/M(\pi^0 \to \gamma\gamma) = -2\sqrt{\frac{2}{3}}, \quad (20)
\]
The same result for the ratios of matrix elements as given in (19) and (20) may be obtained using arguments contained in [5,14,15]: For pseudoscalar mesons $P$ having the constituent-quark structure

$$|q\bar{q}\rangle = a|u\bar{u}\rangle + b|d\bar{d}\rangle + c|s\bar{s}\rangle, \quad a^2 + b^2 + c^2 = 1$$

the two-photon amplitude may be given in the form

$$M(P \to \gamma\gamma) = \frac{\alpha_e}{\pi f_P} N_c \sqrt{2} \langle e_q^2 \rangle$$

with

$$\langle e_q^2 \rangle = a e_u^2 + b e_d^2 + c (\hat{m}/m_s) e_s^2,$$

$$\alpha_e = 1/137.04, \ f_P \text{ the pseudoscalar decay constant}, \ N_c = 3 \text{ the number of colors}, \ \hat{m} \text{ the constituent mass of the light quarks and } m_s \text{ the constituent mass of the strange quark. Numerically we have } m_s/\hat{m} \simeq 1.44 \ [15,16].$$

Defining $C_P = \langle e_q^2 \rangle_P / |\langle e_q^2 \rangle_{\pi^0}|$ we obtain

$$C_P = \left\{ -1, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}} \right\}$$

for the $\pi^0$, the $\eta_8$ and $\eta_1$ meson, respectively, in the limit $\hat{m}/m_s \to 1$. This is in agreement with the findings in [5] except for the minus sign in front of the first term. This shows that the ratios given in (19) and (20) can be obtained from (22) including the signs, and that $M(\pi^0 \to \gamma\gamma)$ is negative whereas the two other matrix elements are positive. Furthermore, this consideration makes transparent that it is the minus sign in front of the term $uu$ in (17), i.e. the flavor component with the larger electric charge, which leads to the minus signs in (19), (20) and (21).

Some additional remarks concerning the validity of the minus sign in front of the term $uu$ in (17) should be made. The signs in (16) to (18) are a consequence of the use [13] of the matrix

$$e^{i/2} = \cos \theta/2 + i \tau_2 \sin \theta/2$$

for carrying out the rotation of a spin-$1/2$ system through a finite angle $\theta$ about the 2-axis in the isospin space, whereas

$$e^{-i/2} = \cos \theta/2 - i \tau_2 \sin \theta/2$$

as used in other textbooks [17] would lead to $uu$ carrying the plus sign and $d\bar{d}$ carrying the minus sign in (17). In this latter case the three matrix elements $M(\pi^0 \to \gamma\gamma)$, $M(\eta_8^0 \to \gamma\gamma)$ and $M(\eta_1^0 \to \gamma\gamma)$ would have the same sign. On the other hand we have to realize that the matrices (25) and (26) are one component of a 3-vector of matrices for the three axes, so that (26) can be obtained from (25) through the replacement $\tau \to -\tau$ and, therefore, also $\tau_3 \to -\tau_3$. As far as the signs in (1) and (2) are concerned we, apparently, have two equivalent options:

Option 1: We use the sign convention for the pole terms as introduced in (1) and (2) and treat the matrix element $M(\pi^0 \to \gamma\gamma)$ and also $M(a_0 \to \gamma\gamma)$ as negative quantities and the other matrix elements as positive quantities in accordance with [13].

Option 2: We make the replacement $\tau_3 \to -\tau_3$ in (1) and (2) and treat all the matrix elements as positive quantities in accordance with [17].

We will apply Option 1 throughout this paper.

The physical $\eta$ and $\eta'$ states are defined to be [18]

$$|\eta\rangle = \cos \theta_P |\eta_8\rangle - \sin \theta_P |\eta_1\rangle,$$

$$|\eta'\rangle = \sin \theta_P |\eta_8\rangle + \cos \theta_P |\eta_1\rangle,$$
or equivalently

\begin{align}
|\eta\rangle &= \cos \phi_P \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle - \sin \phi_P |s\bar{s}\rangle, \quad (29) \\
|\eta'\rangle &= \sin \phi_P \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle + \cos \phi_P |s\bar{s}\rangle, \quad (30)
\end{align}

with

\begin{align}
\cos \phi_P &= \sqrt{\frac{1}{3}} \cos \theta_P - \sqrt{\frac{2}{3}} \sin \theta_p, \quad (31) \\
\sin \phi_P &= \sqrt{\frac{2}{3}} \cos \theta_P + \sqrt{\frac{1}{3}} \sin \theta_p. \quad (32)
\end{align}

Furthermore, we have

\[
\Gamma_{P\gamma\gamma} = \frac{m_p^2}{64 \pi} |M(P \rightarrow \gamma\gamma)|^2. \quad (33)
\]

The results obtained for pseudoscalar mesons are contained in Tables 1 and 2. With these Tables we wish to find out to what level of precision the physical \(\eta\) and \(\eta'\) mesons can be represented using only one mixing angle \(\phi_P\) or equivalently \(\theta_p\). As we can see in Table 2 the use of only one mixing angle does not lead to a good agreement between theory and experiment. The existence of two different mixing angles means that the impurities in \(SU(3)\) symmetry cannot be taken into account only by a rotation involving only the basic \(SU(3)\) flavor states, even after the main known reason for symmetry breaking, i.e. the \(m/s\) mass ratio in (23) has been taken into account explicitly.

The investigation of the quark structure of pseudoscalar mesons and the properties of \(\eta - \eta'\) mixing have a long history (see e.g. [19]). The previous analyses cited in [19] led to results ranging from \(\theta_p = -10^\circ\) to \(-20^\circ\), i.e. to a range of results between the smaller and the larger value of our present analysis. When trying to solve the problem of the uncertainty of the mixing angle \(\theta_p\) on theoretical grounds the final conclusion was [19] that the \(\eta - \eta'\) mixing cannot be adequately described by a single mixing angle \(\theta_p\), in agreement with our present result. Furthermore, a new mixing scheme is described in [19] based on \(\chi\)PT. In this scheme the difference between the two angles is determined by the difference of the pion and the kaon decay constant. This implies that the connection between bare octet and singlet states and physical \(\eta\) and \(\eta'\) states is not a simple rotation.
Table 1: The quantity $\langle e^2 \rangle_P$ is calculated according to (23) from the flavor wavefunctions. The quantity $F_{P\gamma\gamma} \equiv M(P \rightarrow \gamma\gamma)$ is the corresponding matrix element for two-photon decay.

|   | meson | $\langle e^2 \rangle_P$ | $M(P \rightarrow \gamma\gamma)$ |
|---|-------|----------------|------------------|
| 2 | $\pi^0$ | $-\frac{1}{\sqrt{2}}\frac{1}{3}$ | $-\frac{\alpha}{\pi f_\pi}$ |
| 3 | $\eta$ | $\frac{5}{9\sqrt{2}}(\cos \phi_P - \frac{\sqrt{2}}{5} \frac{m}{m_s} \sin \phi_P)$ | $\frac{\alpha}{\pi f_\pi}3\sqrt{2}\langle e^2 \rangle_\eta$ |
| 4 | $\eta'$ | $\frac{5}{9\sqrt{2}}(\sin \phi_P + \frac{\sqrt{2}}{5} \frac{m}{m_s} \cos \phi_P)$ | $\frac{\alpha}{\pi f_\pi}3\sqrt{2}\langle e^2 \rangle_{\eta'}$ |

Table 2: The meson mass $m_P$ and the two-photon decay widths $\Gamma_{P\gamma\gamma}$ are taken from experiments. The quantity $F_{P\gamma\gamma}^{\text{exp.}}$ is identical with $M(P \rightarrow \gamma\gamma)$ as given in (23). In line 2 the quantity $F_{P\gamma\gamma}^{\text{theor.}}$ is calculated from the corresponding expressions for $M(P \rightarrow \gamma\gamma)$ in Table 1. In lines 3 and 4 the adjustable parameters are given which lead to agreement between the theoretical expression and the experimental value for $F_{P\gamma\gamma}$.

|   | meson | $m_P$ [MeV] | $\Gamma_{P\gamma\gamma}$ [keV] | $F_{P\gamma\gamma}^{\text{exp.}}$ [$10^{-2}\times\text{GeV}^{-1}$] | $F_{P\gamma\gamma}^{\text{theor.}}$ [$10^{-2}\times\text{GeV}^{-1}$] |
|---|-------|------------|----------------|----------------|------------------|
| 2 | $\pi^0$ | 134.98 | $(7.74 \pm 0.55) \times 10^{-3}$ | $-2.52 \pm 0.09$ | $-2.513 \pm 0.007$ |
| 3 | $\eta$ | 547.51 | $0.510 \pm 0.026$ | $2.50 \pm 0.06$ | $\phi_P = 43.1^\circ \pm 1.0^\circ$ | $\theta_P = 11.6^\circ \pm 1.0^\circ$ |
| 4 | $\eta'$ | 957.78 | $4.28 \pm 0.19$ | $3.13 \pm 0.07$ | $\phi_P = 36.1^\circ \pm 1.4^\circ$ | $\theta_P = 18.6^\circ \pm 1.4^\circ$ |
A possible further explanation is given by the admixture of “extra” gluonic states to the “ordinary” $\bar qq$ states [20]. Such an admixture is possible, depending on the dynamics that we do not understand well [21].

For our further analysis the results obtained for the $\eta$ and $\eta'$ mesons is very important because it teaches us that constraints from $SU(3)$ symmetry are broken even in the case of pseudoscalar mesons. This means that these constraints may be disregarded to a large extent for scalar mesons where dimeson states are supposed to strongly mix into the $q\bar q$ flavor wave functions.

4 Scalar mesons

In the following we discuss current approaches to low-mass scalar mesons and a convenient way how scalar mesons may be treated in the analyses of their decays to two photons and their coupling to nucleons. As noted before, for the present purpose we do not have to take into account the rather complicated structure of the on-shell $\sigma$ meson but may treat this particle in terms of the dynamical version of the L$\sigma$M.

4.1 Scalar mesons in the ideal mixing approach

In case of ideal mixing the flavor wave-functions of the neutral scalar mesons are

$$|\sigma\rangle = \frac{1}{\sqrt{2}} |u\bar u + d\bar d\rangle$$

(34)

$$|f_0(980)\rangle = |s\bar s\rangle$$

(35)

$$|a_0(980)\rangle = \frac{1}{\sqrt{2}} |-u\bar u + d\bar d\rangle$$

(36)

The structures of these flavor wave functions apparently are in agreement with the sign convention of Option 1, discussed in section 3 and adopted throughout in this paper. Table 3 summarizes the analysis of decay matrix elements of the scalar mesons given in [6, 22]. We find agreement between experiment and prediction for the $\sigma$ and the $f_0$ meson within the errors but a large correction factor of $V_{q\pi} = 0.31 \pm 0.05$ in case of the $a_0$ meson. This correction factor has been related to a special property of the $a_0$ meson. Though the $a_0$ meson has the same flavor wave function as the $\pi^0$ meson the quark-loop calculation may be different for the scalar meson in comparison to the pseudoscalar meson. The following relation is derived for the $a_0(980)$ meson

$$M_{\text{quark-loop}} = 2\xi[2 + (1 - 4\xi)I(\xi)]\frac{\alpha_e}{\pi f\pi} \equiv V_q \frac{\alpha_e}{\pi f\pi},$$

(37)

$$I(\xi) = \frac{\pi^2}{2} - 2\ln^2 \left[ \frac{1}{\sqrt{4\xi}} + \sqrt{\frac{1}{4\xi} - 1} \right]; \quad \xi \leq \frac{1}{4},$$

(38)

where $\xi = (m_q/m_{a_0})^2$. This relation leads to a constituent quark mass $m_q$ as an adjustable parameter. The parameter $V_q = 1$ corresponds to $m_q = 360$ MeV whereas $V_q = 0.31 \pm 0.05$ corresponds to $m_q = 231 \pm 10$ MeV. This means that the small decay matrix element of the $a_0$ meson may be related to a comparatively small constituent quark mass to be inserted into the loop calculation.
Table 3: The quantity $m_S$ is the scalar meson mass taken from experiments, except for $m_\sigma$ where the prediction of the dynamical version of the LσM is given. The two-photon decay widths $\Gamma_{S\gamma\gamma}$ are taken from experiments in all cases. In lines 2 and 3 the quantity $F_{S\gamma\gamma}^{\text{theor.}}$ is calculated from the corresponding expressions for $M(S \rightarrow \gamma\gamma)$. In lines 4 the value for adjustable parameter $V_q$ is given which leads to agreement between the theoretical expression and the experimental value.

| meson   | $m_S$ [MeV] | $\Gamma_{S\gamma\gamma}$ [keV] | $F_{S\gamma\gamma}^{\text{exp.}}$ [10$^{-2}$ GeV$^{-1}$] | $M(S \rightarrow \gamma\gamma)$ [10$^{-2}$ GeV$^{-1}$] | $F_{S\gamma\gamma}^{\text{theor.}}$ |
|---------|-------------|-------------------------------|---------------------------------|-----------------------------|---------------------------|
| $\sigma$ | 666.0       | 3.8 ± 1.5                     | 5.0 ± 1.0                       | $\frac{\alpha_{\pi}}{\pi f_{\pi}}\frac{3}{2}$                  | 4.19                      |
| $f_0(980)$ | 980         | 0.29$^{+0.07}_{-0.09}$        | 0.79 ± 0.11                     | $\frac{\alpha_{\pi}}{\pi f_{\pi}}\sqrt{2}\frac{4m_s}{m_\pi}$     | 0.82                      |
| $a_0(980)$ | 984.7       | 0.30 ± 0.10                   | -0.79 ± 0.13                    | $-\frac{\alpha_{\pi}}{\pi f_{\pi}}V_q(\xi)$                      | $V_q = 0.31 \pm 0.05$    |

4.2 Scalar mesons with strange quarks in the $a_0$ and non-strange quarks in the $f_0$ flavor wave functions in the diquark approach

The weak point of the ansatz of the foregoing subsection is that in contrast to the pseudoscalar mesons $\eta$ and $\eta'$ which both contain strange and nonstrange quarks the scalar mesons $f_0$ and $a_0$ are believed to contain either only strange quarks or only nonstrange quarks. Therefore, attempts have been made to provide a strange-quark component for the $a_0$ meson and a nonstrange component for the $f_0$ meson. The most prominent example is the introduction of a cryptoexotic diquark structure of these two mesons [23]

$$|a_0(980)\rangle = \frac{1}{\sqrt{2}}|sd\bar{s}d - su\bar{s}u\rangle,$$  \hspace{1cm} (39)

$$|f_0(980)\rangle = \frac{1}{\sqrt{2}}|sd\bar{s}d + su\bar{s}u\rangle,$$  \hspace{1cm} (40)

which has a long and heavily debated history. An argument in favor of this diquark structure is that the two mesons $a_0$ and $f_0$ are symmetric with respect to their strange-quark content. But this does not necessarily mean that the diquark structure is the only one which may provide the $a_0$ with strange quarks and the $f_0$ meson with non-strange quarks. An other possibility is provided by the coupling of these two mesons to $K\bar{K}$ meson pairs which is discussed in the next subsection.

4.3 The scalar mesons $f_0(980)$ and $a_0(980)$ within realistic meson-exchange models of the $\pi\pi$ and $\pi\eta$ interactions

The structure of the scalar mesons $f_0(980)$ and $a_0(980)$ has been investigated by the Jülich group [24] within realistic meson-exchange models of the $\pi\pi$ and $\pi\eta$ interactions. The formalism developed for the $\pi\pi$ system is consistently extended to the $\pi\eta$ interaction leading to a description of the $a_0(980)$ as a dynamically generated threshold effect, which is therefore neither a conventional $q\bar{q}$ state nor a $K\bar{K}$ bound state.
4.4 The $f_0(980)$ and $a_0(980)$ as $q\bar{q}$ quarkonia coupled to dimeson states

The peak energies of the $f_0(980)$ and $a_0(980)$ resonances are only few MeV below the $K^-K^+$ and $K^0\bar{K}^0$ thresholds ($2m_{K^\pm} = 987.6$ MeV, $2m_{K^0} = 995.4$ MeV). This makes it likely that the $f_0(980)$ and $a_0(980)$ mesons are $q\bar{q}$ quarkonia states strongly coupled to $K^-K^+$ and $K^0\bar{K}^0$ dimeson states. A detailed discussion of the $KK$ fraction in the $f_0(980)$ and the $a_0(980)$ is given in [10]. To give an idea of the order of magnitude discussed there we quote that the $KK$ fraction in the $f_0(980)$ is $\sim 70\%$ and in the $a_0(980)$ $\sim 35\%$. In [9] K-matrix analyses are carried out leading to further arguments which favor the opinion that $f_0(980)$ and $a_0(980)$ are dominantly $q\bar{q}$ states, with a small ($10-20\%$) admixture of a $K\bar{K}$ loosely bound component. A treatment of the coupling of the $q\bar{q}$ quarkonia to dimeson states in terms of explicit models is discussed in [11].

4.5 Strong isospin breaking $a_0(980) - f_0(980)$ mixing

Achasov et al. [25] describe a mechanism through which there should be a strong isospin breaking $a_0(980) - f_0(980)$ mixing. This phenomenon is shown to be determined by the strong couplings of the $f_0(980)$ and $a_0(980)$ mesons with the $K^+K^-$ and $K^0\bar{K}^0$ channels. The origin of isospin breaking is the mass difference between the pairs $K^+K^-$ and $K^0\bar{K}^0$ [25, 26]. Therefore, the $a_0(980) - f_0(980)$ mixing amplitude could be shown [25,26] to be especially large in the 8 MeV wide interval between the $K^+K^-$ and $K^0\bar{K}^0$ thresholds, but remaining sizable outside this interval.

The concept of strong isospin breaking $a_0(980) - f_0(980)$ mixing has also been used to point out that these scalar mesons both have strange and non-strange $q\bar{q}$ components [27]. This leads to the ansatz

$$|f_0(980)\rangle = \cos \phi \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle - \sin \phi |s\bar{s}\rangle, \quad (41)$$

$$|a_0(980)\rangle = \sin \phi' \frac{1}{\sqrt{2}} | -u\bar{u} + d\bar{d}\rangle + \cos \phi' |s\bar{s}\rangle, \quad (42)$$

which is of special interest for our data analysis described in the following.

4.6 Ansatz for the flavor wave functions appropriate for the analysis of data

In view of the many different models available for the $f_0(980)$ and $a_0(980)$ mesons we try to avoid the choice of a very specific model for the data analysis. Instead, we propose to make use of the assumption that the wavefunction may be expanded in terms of $q\bar{q}$ components being strongly coupled to $K\bar{K}$ pairs. This coupling is supposed to be responsible for the masses of the mesons [8–11] and may as well have an impact on the relative sizes of the strange-quark content and the nonstrange-quark content in these two mesons [25–27]. Furthermore, we choose the option to determine the relative sizes of the $q\bar{q}$ components empirically. Details will be discussed in the next section. We will show that within this ansatz the two-photon couplings of the mesons give rather direct information on the flavor content of the states.
5 Empirical flavor wave-functions of pseudoscalar and scalar mesons

We start from Eqs. (14) and (15) and denote pseudoscalar mesons \((P)\) and scalar mesons \((S)\) by the common symbol \((M)\). Under this condition and with \(\hat{m}/m_s = 1/1.44\) we arrive at

\[
M(M \to \gamma\gamma) = \frac{\alpha_e}{\pi f_{\pi}} \frac{1}{3} \sqrt{2} \left[ 4a + b + \frac{\hat{m}}{m_s} c \right] \tag{43}
\]

where \(\frac{\alpha_e}{\pi f_{\pi}} = 0.02513\) GeV\(^{-1}\) and \(M \to \gamma\gamma\) denotes a two-photon decay of a pseudoscalar or scalar meson. The relation between the decay matrix element \(M(M \to \gamma\gamma)\) and the two-photon decay width is given by

\[
\Gamma_{M\gamma\gamma} = \frac{m_M^3}{64\pi} |M(M \to \gamma\gamma)|^2. \tag{44}
\]

The quantity \(\Gamma_{M\gamma\gamma}\) in column 7 of Table 4 is the experimental two-photon decay width, the quantity \(M(M \to \gamma\gamma)\) in column 6 the decay matrix element where the number is taken from the experimental two-photon decay width and the sign from the prediction of the flavor structure of the meson. The amplitudes \(a\), \(b\) and \(c\) in Table 4 have been obtained by adjusting to the experimental two-photon decay widths given in (44) using (43).

| \(m_M\) [MeV] | \(a\) | \(b\) | \(c\) | \(M(M \to \gamma\gamma)\) \([10^{-2} \text{ GeV}^{-1}]\) | \(\Gamma_{M\gamma\gamma}\) \([\text{keV}]\) |
|-------------|-----|-----|-----|-----------------|-----------------|
| \(\eta\)    | 547.75 | 0.518 | 0.518 | -0.681         | +2.50 ± 0.06   | 0.510 ± 0.026 |
| \(\eta'\)   | 957.78 | 0.414 | 0.414 | 0.810          | +3.13 ± 0.05   | 4.29 ± 0.15   |
| \(f_0\)     | 980.0 | 0.262 | 0.262 | -0.929         | +0.79 ± 0.11   | 0.29\(^{+0.07}_{-0.09}\) |
| \(a_0\)     | 984.7 | -0.415 | 0.415 | 0.810          | -0.79 ± 0.13   | 0.30 ± 0.10   |

6 The couplings of the mesons to the nucleon

6.1 The \(SU(2) \times SU(2)\) linear sigma model

In the linear sigma model \(L\sigma M\), fermions have Yukawa couplings with a scalar, isoscalar field \(\sigma\) and a pseudoscalar, isovector field \(\pi\) [29] given by

\[
\mathcal{L}_{\text{int}} = g \bar{\psi}(\sigma' + i\gamma_5 \tau \cdot \pi)\psi \tag{45}
\]

where \(\sigma'\) is the \(\sigma\) field incorporating the effects of chiral symmetry breaking (see Eqs. (5.50) and (5.51) in [29]). In the dynamical version of the \(L\sigma M\) the meson-quark couplings constants for the \(\pi\) and \(\sigma\) meson are given by [1, 2]

\[
g = g_{\pi qq} = g_{\sigma qq} = 2\pi/\sqrt{N_c} = 3.63. \tag{46}
\]

Then with the Goldberger-Treiman relation for the chiral limit (cl)

\[
g f_{\pi}^{\text{cl}} = M \tag{47}
\]
we obtain the constituent-quark mass \( M = 325.8 \text{ MeV} \) in the chiral limit and via \( m^2 = (2M)^2 + m^2_\pi \) with \( f^2_\pi = 89.8 \text{ MeV} \) and \( m_\pi = 138.0 \text{ MeV} \) the \( \sigma \)-meson mass

\[
m_\sigma = 666.0 \text{ MeV}. \tag{48}
\]

For the \( t \)-channel part of \((\alpha - \beta)\) due to the \( \sigma \) meson we obtain

\[
(\alpha - \beta)^t_{p,n} = \frac{g_{\sigma NN} F_{\sigma \gamma \gamma}}{2\pi m^2_\sigma} = \frac{5\alpha_e g_{\pi NN}}{6\pi^2 m^2_\sigma f_\pi} = 15.2 \tag{49}
\]

in units of \(10^{-4} \text{ fm}^3\), where \( \alpha_e = 1/137.04\), \( g_{\sigma NN} \equiv g_{\pi NN} = 13.168 \pm 0.057\), \( f_\pi = (92.42 \pm 0.26) \text{ MeV} \). The result given in \[48\] has been obtained through an application of the L\(\sigma\)M and its dynamical version. The justification for using this result in the interpretation of the electromagnetic polarizabilities has been given in \[1\] and \[3\] where it has been shown that this result leads to an excellent agreement with experimental data. In order to obtain \( \alpha^t_{p,n} \) and \( \beta^t_{p,n} \) separately use may be made of \((\alpha + \beta)^t_{p,n} = 0\).

### 6.2 Generalization to \(SU(3) \times SU(3)\)

The properties of meson-baryon coupling constants have been derived for the pseudoscalar meson octet \( \{\pi, K, \eta\} \) and the baryon octet \( \{N, \Sigma, \Lambda, \Xi\} \) using group theory \[30\] and was later adapted to the scalar nonet \( \{\sigma, a_0, f_0, \kappa\} \) \[31\]. A compilation may be found in \[32\]. As a summary of the investigations in \[31\] we may write down the following relations for the coupling constants \( g_{MNN} \) of a meson \( M \) to the nucleon \( N \) in terms of the flavor wave functions:

\[
g_{MNN}\left(\frac{1}{\sqrt{2}}(-u\bar{u} + d\bar{d})\right) = g_{\pi NN}, \tag{50}
\]

\[
g_{MNN}\left(\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\right) = g_{\pi NN}(4\alpha_M - 1), \tag{51}
\]

\[
g_{MNN}(s\bar{s}) = 0. \tag{52}
\]

For the mesons \( a_0(980) \) (see \[53\]) and \( \eta, \eta' \), \( f_0(980) \) (see \[54\]) the meson-nucleon coupling constants, therefore, may be given in the form

\[
g_{a_0 NN} = \sqrt{2}|a|g_{\pi NN}, \tag{53}
\]

\[
g_{MNN} = \sqrt{2}|a|g_{\pi NN}(4\alpha_M - 1), \tag{54}
\]

where the quantity \(|a|\) is absolute value of the amplitude \(a\) given in Table 4. The quantity \(\alpha_M\) is given by

\[
\alpha_M = \frac{F}{F + D}. \tag{55}
\]

The SU(3) axial vector coupling constants \( F \) and \( D \) are determined by neutron and hyperon beta decay. For pseudoscalar mesons the following numbers are given (see \[33\] p. 189)

\[
F \simeq 0.51, \quad D \simeq 0.76, \tag{56}
\]

and for the axial vector coupling constant of the nucleon

\[
g_A = F + D \simeq 1.27. \tag{57}
\]

With these numbers we obtain

\[
\alpha_M = \frac{F}{F + D} \simeq \frac{2}{5}. \tag{58}
\]
Table 5: Meson nucleon coupling constants

| \[a\] | \[g_{MNN}/g_{\pi NN}\] | \[g_{MNN}\] |
|-----|-----------------|----------|
| \[\eta\] | 0.518 (3/5)\sqrt{2}| 5.79 ± 0.15 |
| \[\eta'\] | 0.414 (3/5)\sqrt{2}| 4.63 ± 0.08 |
| \[f_0\] | 0.262 (6/5)\sqrt{2}| 5.8 ± 0.8 |
| \[a_0\] | 0.415 \sqrt{2}| 7.7 ± 1.2 |

For the pseudoscalar states \[\eta_8\] and \[\eta_1\] we then obtain

\[g_{\eta_8 NN} = \frac{\sqrt{3}}{5} g_{\pi NN}, \quad g_{\eta_1 NN} = \frac{\sqrt{6}}{5} g_{\pi NN}\]  

(59)

in agreement with the result given in [4]. Therefore, it appears to be well justified to use \(\alpha_M = 2/5\) for pseudoscalar mesons whereas for scalar mesons \(\alpha_M = 0.55\) has been proposed in [31] and applied here for \(f_0\). The results obtained for the meson-nucleon coupling constants are given in Table 5. The errors given to the quantities \(g_{MNN}\) correspond to the experimental errors of \(\Gamma_{M\gamma\gamma}\) in Table 4.

7 Polarizabilities of the neutron derived from the corresponding values of the proton

In the following we use the well known experimental polarizabilities of the proton and the theoretical results obtained for the proton and the neutron to make more precise predictions for the neutron than obtained directly from the experiments.

7.1 Electromagnetic polarizabilities \(\alpha\) and \(\beta\)

The electromagnetic polarizabilities consist of a resonant part due to the main resonances of the nucleon, of a nonresonant part which is mainly due to electric dipole excitation corresponding to the experimental \(E_{0+}\) CGLN amplitude of meson photoproduction and of the \(t\)-channel part which is mainly due to the \(\sigma\) meson. The resonant parts have formerly been obtained in two different ways [3], from the directly measured parameters of the resonant excited states obtained from the analysis of the total photoabsorption of the proton [34] and from resonance couplings obtained from analyses of meson photoproduction data of the proton and the neutron [35–37]. We believe that for the proton the directly measured parameters of the resonances are more precise than those from the analyses of meson photoproduction data. Therefore, we use the directly measured resonant data of the proton both for the proton and the neutron. This procedure is unquestionable except for the \(F_{15}(1680)\) resonance where the resonance couplings obtained from meson photoproduction data are much smaller for the neutron as compared to the proton. However, a recent measurement of the helicity-dependent photoabsorption cross section of the neutron from 815 to 1825 MeV clearly shows that there is no such difference in resonant photoabsorption strength for the proton and the neutron [38]. Therefore, it appears experimentally justified to use the same prediction for \(\alpha\) and \(\beta\) for the proton and the neutron also in case of the \(F_{15}(1680)\) resonance.

The \(t\)-channel contributions to \(\alpha\) and \(\beta\) given in lines 2–4 of Table 6 are by far dominated by the contribution from the \(\sigma\) meson. The contributions from the \(f_0\) meson and the \(a_0\) meson cancel
in the case of the proton but interfere constructively with each other and with the contribution of the \( \sigma \)-meson in the case of the neutron. This leads to the conclusion that parts of the difference of the electric and magnetic polarizabilities observed for the proton and the neutron are due to the \( f_0 \) and the \( a_0 \) meson. The other part of this difference stems from the nonresonant contribution in line 11 of Table 6. It is well known that the \( E_{0^+} \) amplitudes for the proton and neutron in the Born approximation and at pion photoproduction threshold differ by a factor \( (1 + \frac{m_\pi}{m_N}) \simeq 1.15 \). This would lead to a ratio \( \alpha_n(E_{0^+})/\alpha_p(E_{0^+}) \simeq 1.30 \) in case the fraction \( (1 + \frac{m_\pi}{m_N}) \simeq 1.15 \) would equally be valid for the empirical \( E_{0^+} \) amplitudes and would extend unmodified to higher energies. For the empirical values \( \alpha_n(E_{0^+})/\alpha_p(E_{0^+}) \simeq 1.28 \) was obtained [3] which is in close agreement with the value expected for the Born approximation. For the total nonresonant parts of the electric and magnetic polarizabilities given in line 11 of Table 6 the value \( \alpha_n(\text{nonres.})/\alpha_p(\text{nonres.}) \simeq 1.25 \) was adopted which preserves the normalizations \( (\alpha + \beta)_p = 13.9 \pm 0.3 \) and \( (\alpha + \beta)_n = 15.2 \pm 0.5 \). Two errors are given for the neutron data in line 12 of Table 6. The first of these errors was chosen to be the same as the one for the proton, since the proton data serve as a measure for the validity of the procedure. The second errors are upper limits of possible additional systematic or model dependent errors.

We consider the results given in line 12 of Table 6 as a new set of data for the electromagnetic polarizabilities of the neutron which supplements to the existing data of line 13. The final result, therefore, is the weighted average of the neutron data in lines 12 and 13:

\[
\alpha_n = 13.4 \pm 1.0, \quad \beta_n = 1.8 \pm 1.0.
\]

These values are also given in line 14 of Table 6 and in the abstract.
7.2 The backward spin polarizability \( \gamma_\pi \)

For the \( t \)-channel parts of the backward spin polarizabilities of the nucleons rather firm information is available after the relative signs of the three contributions have been determined. The numbers obtained for the \( t \)-channel spin-polarizabilities in lines 2 – 4 of Table 7 are obtained from the two-photon couplings given in column 5 of Table 2 and the meson–nucleon couplings given in column 4 of Table 5. The \( s \)-channel parts in line 6 have formerly been precisely determined by L’vov [4] and are used here without modification. Excellent agreement between prediction and experiment is obtained for the proton. This gives us confidence that a similar precision for the agreement between theory and experiment is also given for the neutron. Nevertheless

Table 7: The backward spin polarizabilities \( \gamma_\pi \) for the proton and the neutron. Lines 2–4: \( t \)-channel contributions from the \( \pi^0 \), \( \eta \) and \( \eta' \) meson. Lines 5,6: Total predicted \( t \)- and \( s \)-channel contributions. Line 7: Sum of lines 5 and 6. Line 9: Weighted average of lines 7 and 8.

|   | \( \gamma_\pi^{(p)} \) | \( \gamma_\pi^{(n)} \) |
|---|----------------|----------------|
| 2 | \( t \)-chan. \( \pi^0 \) | -46.7 | +46.7 |
| 3 | \( t \)-chan. \( \eta \) | +1.2 | +1.2 |
| 4 | \( t \)-chan. \( \eta' \) | +0.4 | +0.4 |
| 5 | \( t \)-channel | -45.1 | +48.3 |
| 6 | \( s \)-channel | +\((7.1 \pm 1.8)\) | +\((9.1 \pm 1.8)\) |
| 7 | \( \gamma_\pi \) theor. | -(38.0 \pm 1.8) | +(57.4 \pm 1.8 \pm 0.9) |
| 8 | \( \gamma_\pi \) exp. | -(38.7 \pm 1.8) | +(58.6 \pm 4.0) |
| 9 | weighted av. | | +(57.6 \pm 1.8) |

we take into account a possible systematic or model error in case of the neutron, given by the second error in the neutron data of line 7. This additional error was chosen as an upper limit of possible estimates. We consider this result of line 7 as a new value for the neutron which supplements to the existing one of line 8. The weighted average of the two results given in lines 7 and 8, therefore, is the new final result for the backward spin polarizability of the neutron. This weighted average is

\[
\gamma_\pi^{(n)} = +(57.6 \pm 1.8)
\]

which also is given in line 9 of Table 7 and in the abstract.

8 Summary and Discussion

In the foregoing we have determined the signs and values of the two-photon couplings \( F_{M\gamma\gamma} \) of the pseudoscalar mesons \( \pi^0 \), \( \eta \) and \( \eta' \) and the scalar mesons \( \sigma \), \( f_0(980) \) and \( a_0(980) \) and their couplings to the nucleon \( g_{MNN} \) as entering into the backward angle spin polarizabilities \( \gamma_\pi \) and the \( t \)-channel parts of the electromagnetic polarizabilities \( (\alpha - \beta) \). The quantities \( F_{M\gamma\gamma} \) and \( g_{MNN} \) have been found to be positive numbers except for the two-photon couplings \( F_{\pi\gamma\gamma} \) of the \( \pi^0 \) and \( a_0 \) mesons which are negative within the sign convention of the flavor wave functions adopted here. Because of the high level of precision obtained for the proton rather reliable predictions can be made for the neutron, thus improving on the precision of the polarizabilities of the neutron, well within the limits given by the experimental polarizabilities. The conclusion
we have to draw from these findings is that there are no non-understood contributions to the polarizabilities. For the structure of the nucleon the following conclusions are obtained. The $s$-channel parts can be calculated from the resonant and nonresonant cross sections of the nucleon, where the nonresonant part may be related to the meson cloud. In the case of the meson-cloud contributions 70% are due to nonresonant electric-dipole excitation. The other 30% consist of magnetic-dipole excitation and combined processes consisting simultaneously of nonresonant and resonant excitation processes, resulting in two-pion emission in photoabsorption experiments. The $t$-channel parts can be understood in terms of pseudoscalar and scalar mesons coupled to the constituent quarks. The quantity $(\alpha - \beta)^t$ is given by the scalar mesons which couple to two photons with parallel planes of linear polarization. The quantity $\gamma_{\pi}^t$ is given by pseudoscalar mesons which couple to two photons with perpendicular planes of linear polarization.
References

[1] M. Schumacher, Eur. Phys. J. A 30, 413 (2006); ERRATUM: M. Schumacher, Eur. Phys. J. A 32, 121 (2007) [hep-ph/0609040].

[2] R. Delbourgo, M. Scadron, Mod. Phys. Lett. A 10, 251 (1995) [hep-ph/9910242]; Int. J. Mod. Phys. A 13, 657 (1998) [hep-ph/9807504]; M. Nagy, M.D. Scadron, G.E. Hite, Acta Physica Slovaca 54, 427 (2004); M.D. Scadron, M. Nagy, [hep-ph/0507168].

[3] M. Schumacher, Eur. Phys. J. A 31, 327 (2007) [hep-ph/0704.0200].

[4] A.I. L’vov, A.M. Nathan, Phys. Rev. C 59, 1064 (1999).

[5] J.F. Donoghue, B.R. Holstein, Y.-C.R. Lin, Phys. Rev. Lett. 55, 2766 (1985).

[6] A.S. Deakin, V. Elias, D.G.C. McKeon, M.D. Scadron, Mod. Phys. Lett. A 9, 2381 (1994).

[7] C.N. Yang, Phys. Rev. 77, 242 (1950).

[8] E. van Beveren, T.A. Rijken, K. Metzger, C. Dullemond, G. Rupp, J.E. Ribeiro, Z. Phys. C 30, 615 (1986); N.N. Achasov, G.N. Shestakov, Phys. Rev. D 49, 5779 (1994); N.A. Törnqvist, Z. Phys. C 68, 647 (1995) [hep-ph/9504372]; N.A. Törnqvist, M. Roos, Phys. Rev. Lett. 76, 1575 (1996) [hep-ph/9511120]; E. van Beveren, G. Rupp, Eur. Phys. J. C 10, 469 (1999) [hep-ph/9806246]; N.A. Törnqvist, Eur. Phys. J C 11, 359 (1999); E. van Beveren, G. Rupp, Eur. Phys. J. C 22, 493 (2001) [hep-ex/0106077]; A. Deandrea et al., Phys. Lett. B 502, 79 (2001) [hep-ph/0012120]; Yu. S. Surovtsev et al., Eur.Phys. J.A. 15, 409 (2002); M. Boglione, M.R. Pennington, Phys. Rev. D 65, 114010 (2002) [hep-ph/0203149].

[9] V.V. Anisovich, Int. J. Mod. Phys. A 21, 3615 (2006) [hep-ph/0510409].

[10] D.V. Bugg, Eur. Phys. J. C 47, 57 (2006) [hep-ph/0603089].

[11] E. van Beveren et al., Phys. Lett. B 641, 265 (2006) [hep-ph/0606022].

[12] M. Schumacher, Prog. Part. Nucl. Phys. 55, 567 (2005) [hep-ph/0501167].

[13] F.E. Close, “An Introduction to Quarks and Partons” Academic Press 1979, Fifth Printing 1982.

[14] S. Cooper, Annu. Rev. Nucl. Part. Sci. 38, 705 (1988).

[15] M. Scadron et al., Phys. Rev. D 69, 014010 (2004) [hep-ph/0309109].

[16] M. D. Scadron, R. Delbourgo, R. Rupp, J. Phys. G 32, 735 (2006) [hep-ph/0603196].

[17] F. Halzen, A.D. Martin, Quarks & Leptons, John Wiley & Sons (1984).

[18] A. Bramon, R. Escribano, M.D. Scadron, Eur. Phys. J. C 7, 271 (1999).

[19] T. Feldmann, Int. Journ. Mod. Phys. A 15, 159 (2000) [hep-ph/9907491].

[20] F. Ambrosino et al., Phys. Lett. B 648, 267 (2007) [hep-ex/0612029]; G. Li, Q. Zhao, C.-H. Chang, (2007) [hep-ph/0701020]; C.E. Thomas, (2007) [hep-ph/0705.1500].

[21] M.S. Chanowitz, Proceedings: Workshop on Photon-Photon Collisions, Shores, Jerusalem Hills, Israel, April 24-28 (1988).
[22] E. van Beveren et al., Mod. Phys. Lett. A 17, 1673 (2002) [hep-ph/0204139]; M.D. Scadron, G. Rupp, F. Kleefeld, E. van Beveren, Phys. Rev. D 69, 014010 (2004) [hep-ph/0309109].

[23] R.L. Jaffe, K. Johnson, Phys. Lett. B 60, 201 (1976); R.J. Jaffe, Phys. Rev. D 15, 267; 281 (1977); 17, 1444 (1978); N.N. Achasov, V.V. Gubin, Phys. Rev. D 56, 4084 (1997); 63, 094007 (2001); M. Alford, R.L. Jaffe, Nucl. Phys. B 578, 367 (2000); D. Black, A.H. Fariborz, J. Schechter, AIP Conf. Proc. 549, 241 (2002) [hep-ph/9911387]; F.E. Close, N.A. Törnqvist, J. Phys. G: Nucl. Part. Phys. 28, R249 (2002); F.E. Close, Int. J. Mod. Phys. A 17, 3239 (2002) [hep-ph/0110081]; R. Jaffe, F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003); S.B. Gerasimov, Proceedings: 12th International Conference on Selected Problems of Modern Physics (Blokhinsev03), Dubna, Russia, 8-11, June 200 [hep-ph/0311080]; F. Wilczek, in From Fields to Strings, edited by M. Shifman et al., Vol 1, pp 77-93 [hep-ph/0409168]; L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. Lett. 93, 212002 (2004) [hep-ph/0407017]; Z.-G. Wang, W.-M. Yang, Eur. Phys. J. C 42, 89 (2004) [hep-ph/0550110]; F. Giacosa, Phys. Rev. D 74, 014028 (2006) [hep-ph/0605191].

[24] G. Janssen, B.C. Pearce, K. Holinde, J. Speth, Phys. Rev. D 52, 2690 (1995); J. Speth, F. P. Sasson, S. Krewald, Nucl. Phys. A 721, 679 (2003).

[25] N. N. Achasov, S. A. Devyanin, G. N. Shestakov, Phys. Lett. B 88, 367 (1979).

[26] C. Hanhart, B. Kubis, J. R. Pelaez, arXiv:0707.0262 [hep-ph]

[27] Z.-G. Wang, W.-M. Yang, S.-L. Wan, Eur. Phys. J. C 37, 223 (2004) [hep-ph/0410046]

[28] W.-M. Yao et al. (Particle Data Group) J. Phys. C 33, 1 (2006) and 2007 partial update for edition 2008.

[29] V. De Alfaro, S. Fubini, G. Furlan, C. Rossetti, “Currents in hadron physics”, North Holland (1973).

[30] J. J. De Swart, Rev. Mod. Phys. 35, 916 (1963).

[31] G. Erkol, Doctoral Thesis, Groningen (2006); G. Erkol, R.G.E. Timmermans, M. Oka, Th.A. Rijken, Phys, Rev, C 73, 044009 (2006).

[32] O. Dumbrajs et al., Nucl. Phys. B 216, 277 (1982).

[33] A.W. Thomas, W. Weise, “The Structure of the Nucleon”, Wiley-VCH Berlin (2000).

[34] T.A. Armstrong et al., Phys. Rev. D 5, 1640 (1972); Nucl. Phys. B 41, 445 (1972).

[35] R.A. Arndt et al., Phys. Rev. C 66, 055213 (2002).

[36] O. Hanstein, D. Drechsel, L. Tiator, Nucl. Phys. A 632, 561 (1998).

[37] D. Drechsel, O. Hanstein, S.S. Kamalov, L. Tiator, Nucl. Phys. A 645, 145 (1999).

[38] H. Dutz and the GDH collaboration, Phys. Rev. Lett 94, 162001 (2005).