Magnetohydrodynamics with Forced Convection in Micropolar Fluid Flows Pass a Magnetic Porous Sphere

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Abstract. The research of fluid flows is growing. The physical research requires a high cost. So, it is needed science studies that can solve the problem. One of them is mathematical modeling. This research considers magnetic field effect in unsteady magnetohydrodynamics of micropolar fluid through a magnetic porous sphere with forced convection. Governing equations are derived from continuity, momentum, angular momentum, and energy equations. The form of these equations are two dimensional partial differential equations. Dimensional equations are transformed to non-dimensional equations and then, it will be converted to similarity equations. The similarity equations are solved numerically using Keller-Box scheme. It could be concluded that velocity curve decreases when magnetic, porosity and micropolar parameters increase. Microrotation curve increases with the increasing of magnetic, porosity, and micropolar parameters.

1. Introduction
The diversity of people's needs in the manufacture increases. Industrial manufacturing processes use heat transfer principle over a stretching surfaces that be through by fluid [1]. However, the physical research in industrial's field requires an expensive cost. So, we need a method to represent the problem. One of them is mathematical modeling. In mathematics field research about heat transfer over a stretching surfaces have been done. Magnetic application is one of the solutions that often offered [2]. Magnetohydrodynamics is the study that discussed a motion of electrically conducting fluid flow caused by the magnetic field [3]. Based on the fluid shear stress is divided into two types. There are Newtonian and non-Newtonian fluids [4]. On the daily activities, non-newtonian fluid is often encountered. One of them is micropolar fluid. The micropolar fluid has a microstructure and can microrotate with itself. Characteristics of micropolar potentially will be applied in engineering fields [5]. Therefore, the growth of micropolar fluid is interesting to expand.

The previous research discusses magnetohydrodynamics on a micropolar fluid with a magnetic fluid. Therefore, this research will construct a mathematical model from the effect of unsteady magnetohydrodynamics on a micropolar fluid with forced convection pass a magnetic porous sphere. The discussion of this research focuses on the effect of the parameter on the velocity, microrotation, and temperature profile. Parameter that used are magnetic, micropolar, porosity, and Prandtl number. The mathematical model then will be solved numerically using Keller-Box scheme.
2. Numerical Methods

The problem of this research is micropolar fluid flows pass a magnetic porous sphere. Here is the illustration of unsteady magnetohydrodynamics with forced convection on micropolar fluid flows pass a magnetic porous sphere.

Fluid flows from the bottom to up and then pass a magnetic porous sphere. This research focuses on stagnation point where \( x = 0 \). When fluid is rubbing with a porous sphere, it will make a layer along to the surface of sphere. It is called boundary layer[4].

The governing equations of this system are obtained from continuity, momentum, momentum angular, and energy equations. These equations are derived from some of laws of physics. It is conservation mass, Newton II, and Thermodynamics I. In this research, part that will be inspected is the surfaces of porous sphere. 3D coordinates of a porous sphere will simplify to 2D. Therefore, the velocity on the ordinat \( z \) is ignored. We used ordinat \( x \) and \( y \) to modeling the system.

Continuity Equation:

\[
\frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial y} = 0
\]

Momentum Equation

Momentum equation on the \( x \)-axis :

\[
p \left[ \frac{\partial \vec{u}}{\partial t} + \vec{u} \frac{\partial \vec{u}}{\partial x} + \vec{v} \frac{\partial \vec{u}}{\partial y} \right] = - \frac{\partial \vec{p}}{\partial x} + (\mu + \kappa) \left[ \frac{\partial^2 \vec{u}}{\partial x^2} + \frac{\partial^2 \vec{u}}{\partial y^2} \right] - \rho \beta (\vec{T} - T_\infty) g_x + \kappa \frac{\partial \vec{N}}{\partial y} + \sigma B_0^2 \vec{u} + \frac{\mu}{K^*} \vec{u}
\]

Momentum equation on the \( y \)-axis :

\[
p \left[ \frac{\partial \vec{v}}{\partial t} + \vec{u} \frac{\partial \vec{v}}{\partial x} + \vec{v} \frac{\partial \vec{v}}{\partial y} \right] = - \frac{\partial \vec{p}}{\partial y} + (\mu + \kappa) \left[ \frac{\partial^2 \vec{v}}{\partial x^2} + \frac{\partial^2 \vec{v}}{\partial y^2} \right] - \rho \beta (\vec{T} - T_\infty) g_y + \kappa \frac{\partial \vec{N}}{\partial x} + \sigma B_0^2 \vec{v} + \frac{\mu}{K^*} \vec{v}
\]

Angular Momentum Equation:

\[
p \left[ \frac{\partial \vec{N}}{\partial t} + \vec{u} \frac{\partial \vec{N}}{\partial x} + \vec{v} \frac{\partial \vec{N}}{\partial y} \right] = \gamma \left[ \frac{\partial^2 \vec{N}}{\partial x^2} + \frac{\partial^2 \vec{N}}{\partial y^2} \right] - \kappa \left[ 2 \vec{N} + \frac{\partial \vec{u}}{\partial y} - \frac{\partial \vec{v}}{\partial x} \right]
\]

Figure 1 The model of a micropolar fluid flows pass a magnetic porous sphere. Figure 1(a) represent 3D a porous sphere, while figure 1(b) show a micropolar fluid flows pass a porous sphere.
Energy Equation :
\[ \rho C_p \left[ \frac{\partial \hat{T}}{\partial t} + \hat{u} \frac{\partial \hat{T}}{\partial x} + \hat{v} \frac{\partial \hat{T}}{\partial y} \right] = c \left[ \frac{\partial^2 \hat{T}}{\partial x^2} + \frac{\partial^2 \hat{T}}{\partial y^2} + \frac{\partial^2 \hat{T}}{\partial z^2} \right] \] (5)

Where \( u \) and \( v \) represent velocity on the x-axis and y-axis, \( \rho \) is the density of the fluid, \( p \) is the pressure, \( \mu \) is the dynamic viscosity, \( \kappa \) is vortex, \( \beta \) is the thermal expansion coefficient, \( \sigma \) is the electrical conductivity, \( j \) represent the micro inertia density, \( \gamma \) is spin gradient, \( c \) is the thermal conductivity, and \( C_p \) is specific heat.

The dimensional equations are difficult to solve because it has different dimension. The dimensional equations will be changed to non-dimensional equation with non-dimensional variables and parameters[6] are given by

\[
x = \frac{x}{a}, y = \frac{y}{a}, r(x) = \frac{r(x)}{a}, t = \frac{U_\infty t}{a}, u = \frac{U_\infty u}{a}, v = \frac{U_\infty v}{a}, p = \frac{\rho U_\infty^2}{a}, T = \frac{T - T_\infty}{T_w - T_\infty}, N = \frac{Re a N}{U_\infty} \]

\[ g_x = -\frac{g}{a} \sin \left( \frac{x}{a} \right), g_y = g \cos \left( \frac{x}{a} \right) \]

\[ M = \frac{a a B_0^2}{\rho U_\infty}, \alpha = \frac{Gr}{Re}, Gr = \frac{g \beta (T_w - T_\infty) a^3}{\nu^2}, Pr = \frac{\nu \rho C_p}{a}, \phi = \frac{\mu}{\rho U_\infty K}, K = \frac{\kappa}{\mu} \] (6)

In this research \( M \) represents the magnetic parameter, \( \alpha \) is the forced convection parameter, \( Pr \) shows the prandtl number, \( \phi \) is the porosity parameter, and \( K \) represents the micropolar parameter.

Then substitution of (6) into (1) - (5) to obtain non-dimensional equations. Non-dimensional of the system are defined by

Continuity Equation :
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \] (7)

Momentum Equation
\[
\left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{(1 + K) \partial^2 u}{\partial x^2} + \frac{(1 + K) \partial^2 u}{\partial y^2} + \frac{a T \sin(x)}{\partial y} + K \frac{\partial N}{\partial y} + (M + \phi) u \] (8)

Angular Momentum Equation
\[ \left[ \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right] = \left( 1 + \frac{K}{2} \right) \left[ \frac{1}{Re} \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right] - K \left[ 2 N + \frac{\partial u}{\partial y} - \frac{1}{Re} \frac{\partial v}{\partial x} \right] \] (10)

Energy Equation
\[
\left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \frac{1}{Pr} \frac{\partial^2 T}{\partial x^2} + \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} \] (11)

To connect the velocity on two axes we use the approach of potential theory with variable :
\[ u = \frac{1}{r} \frac{\partial \psi}{\partial y}, v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \] (12)

Based the assumption on equation (12), then substitute equation (12) into (7) - (11) to get flow function
\[ \frac{1}{r} \frac{\partial^2 \psi}{\partial t \partial y} + \frac{1}{r^2} \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{1}{r^2} \frac{\partial^2 \psi}{\partial x^2} \left( \frac{\partial \psi}{\partial y} \right)^2 - \frac{1}{r^2} \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial u_e}{\partial x} + \frac{1}{r} \frac{\partial u_e}{\partial r} = (M + \phi) \left[ \frac{1}{r} \frac{\partial u_e}{\partial r} - \frac{1}{r \partial \psi}{\partial y} \right] + \left( 1 + K \right) \frac{\partial^2 \psi}{r \partial y^2} + a T \sin(x) + K \frac{\partial N}{\partial y} \]

\[ \frac{\partial N}{\partial t} + \frac{1}{r} \frac{\partial N}{\partial y} \frac{\partial u_e}{\partial x} - \frac{1}{r} \frac{\partial N}{\partial x} \frac{\partial u_e}{\partial y} = \left( 1 + \frac{K}{2} \right) + \frac{\partial^2 N}{\partial y^2} - K \left[ 2 N + \frac{1}{r} \frac{\partial^2 \psi}{\partial y^2} \right] \] (14)
\[
\frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial}{\partial y} \left( r \frac{\partial T}{\partial x} \right) - \frac{1}{r} \frac{\partial}{\partial x} \left( r \frac{\partial T}{\partial y} \right) = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} \tag{15}
\]

In the similarity equation, we define

\[
\psi = t^2 u_e(x)r(x)f(x, \eta, t), \quad T = s(x, \eta, t), \quad \eta = y t^{-\frac{1}{3}}, \quad N = t^\frac{1}{2} u_e(x) h(x, \eta, t)
\]

\[
\frac{\partial f}{\partial \eta} = f', \quad \frac{\partial s}{\partial \eta} = s', \quad \frac{\partial h}{\partial \eta} = h' \tag{16}
\]

Then substitute (16) and (17) into (13) – (15), we obtain

Momentum Equation:

\[
(1 + K) f'''' + \frac{n}{2} f' + Kh' + (M + \phi) t (f' - 1) + \frac{2}{3} ats + \frac{3}{2} t (1 - (f')^2 + f'') = t \frac{\partial f'}{\partial t} \tag{18}
\]

Angular Momentum Equation:

\[
\left( 1 + \frac{K}{2} \right) h'' + \frac{n}{2} h' + \frac{1}{2} h + \frac{3}{2} t (f h' - h f') = t \frac{\partial h}{\partial t} + K t \frac{\partial}{\partial \eta} \tag{19}
\]

Energy Equation:

\[
\frac{1}{2} \eta \text{Pr} s'' + s'' + \frac{3}{2} \text{Pr} t f s' = \text{Pr} t \frac{\partial s}{\partial t} \tag{20}
\]

With boundary conditions:

\[
t < 0 : f = \frac{\partial f}{\partial \eta} = h = s = 0 \text{ for } x, \eta
\]

\[
t \geq 0 : f = \frac{\partial f}{\partial \eta}, h = -n \frac{\partial^2 f}{\partial \eta^2}, s = 1 \text{ when } \eta = 0
\]

\[
\frac{\partial f}{\partial \eta} = 1, h = 0, s = 0 \text{ when } \eta \rightarrow \infty
\]

3. Results and Discussion

In this research we will discuss about the effect of magnetic, porosity, and micropolar parameters to velocity and microrotation profile. In 2017 Pratomo [7] made a research about unsteady magnetohydrodynamics micropolar fluid in boundary layer flow past a sphere influenced by magnetic fluid. The discussion focused on the effect of parameters on the velocity and microrotation profile. Mathematic model solved numerically using Keller Box. On his research Pratomo show that when magnetic parameters increases velocity profile increase, but microrotation profile decres. These numerical results are compared to Pratomo’s results.

**Figure 2** The numerical result of Pratomo and Widodo. Figure 2(a) represent velocity profile, while figure 2(b) show microrotation profile.
From the simulations we get that numerical result on this paper is confirm with Pratomo et al. So we are sure that numerical solution on this paper can be use on another parameters and variables.

Figure 3 The effect of magnetic parameter variations where $K = 1$, $\phi = 1$, $Pr = 1$, $\alpha = 0$ and $n = 0$. Figure 3(a) show the profile velocity of fluid Figure 3(b) describe the profile microrotation.

Figure 3 explains that the velocity profile decreases as the magnetic parameters increase, but microrotation increase when the magnetic parameters increase. This is because the magnetic parameter is directly proportional to the magnetic field contained in the porous sphere. While microrotation increase because variations of magnetic parameters cause the fluid density decrease.

Figure 4 The effect of micropolar parameter variations where $M = 1$, $\phi = 1$, $Pr = 1$, $\alpha = 0$ and $n = 0$. Figure 4(a) show the profile velocity of fluid. Figure 4(b) describe the profile microrotation.

Figure 4 show that when the micropolar parameters increase, the velocity profile decreases, but microrotation increase. Based on the mathematical formulation, the micropolar parameter is directly proportional to a motion of microrotation. While microrotation increase because variations of micropolar parameters causes the dynamic viscosity decrease.
Figure 5 The effect of porosity parameter variations where \( M = 1, K = 1, \alpha = 0 \) and \( n = 0 \). Figure 5(a) show the the profile velocity of fluid. Figure 5(b) describe the profile microrotation.

Figure 5 explains that variation of porosity parameter caused the velocity profile decreases as the porosity parameters increase, but microrotation increase when the porosity parameters increase.

4. Conclusion
In conclusion magnetic, micropolar, and porosity parameters take affect to the velocity and microrotation profiles. The profile velocity decreases when the magnetic, micropolar, and porosity parameters increase. The profile microrotation and temperature increase when the magnetic, micropolar, and porosity parameters increase.

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