MASSIVE NEUTRINO DECAY AND
SHAPE OF THE GALACTIC DARK HALO

Srdjan Samurović
Dipartimento di Astronomia,
Università degli Studi di Trieste,
Via Tiepolo 11,
I-34131 Trieste, ITALY

Milan M. Ćirković
Astronomical Observatory,
Volgina 7,
11160 Belgrade-74, YUGOSLAVIA

and
Department of Physics & Astronomy
S.U.N.Y. at Stony Brook
Stony Brook, NY 11794-3800, USA

Abstract

In this Letter we investigate the basic assumptions of the decaying dark matter (DDM) theory in the light of recent advances in observational and theoretical cosmology and physics, i.e. detection of massive astrophysical compact halo objects (MACHOs) and Super-Kamiokande results. Specifically, the consequences pertaining to the shape of the Milky Way galaxy dark halo are discussed. We find that, by taking into account the values of the main constituent of the mass in DDM theory, massive neutrino, with the mass of 30 eV, and lifetime $\sim 10^{23}$ s, the initially proposed value of extreme halo flattening $q \sim 0.2$ is no longer necessary, and that one can easily accommodate a much larger value of $q \sim 0.6$, that is in accord with all available observational data.

1E-mail address: srdjan@ts.astro.it
2E-mail address: cirkovic@mail.ess.sunysb.edu
1 Introduction

It is well known that every spiral galaxy consists of three main parts: central bulge, disk and dark halo (e.g. [4]). In this Letter we wish to investigate in more detail the idea that the halo of our Galaxy, Milky Way (in the further text also denoted as the Galaxy) is made mainly of two components: baryonic (MACHOs, see the discussion later) and non-baryonic (massive neutrinos with the mass $m_\nu \sim 30$ eV), and impact of such a composition of matter on the shape of the halo.

The problem of the mass composition in spiral galaxies is well known and still unsolved (e.g. Ref. [7]). Rotation curve of the Galaxy (Ref. [20]) and non-existence of the Keplerian fall-off points out that there exists a large amount of matter that still has to be accounted for. It is a general opinion that a bulk of this matter resides in the dark halo of each spiral galaxy (Ref. [2, 42, 1]), although there are opposite standpoints that suggest the modification of the Newtonian dynamics (MOND) (Refs. [21, 22, 23]) that recently produced quite good agreements with the observations of 15 rotation curves (Ref. [3]).

Massive neutrinos remain viable candidates for the "missing light" problem (as defined by Schramm and Steigman [31]). It can be shown that under certain assumptions (Ref. [11]) massive neutrinos with the mass of the order of 30 eV can account for the mass in spiral galaxies (e.g. Ref. [28]). These neutrinos play the central role in the decaying dark matter (DDM) theory usually associated with the name of Dennis W. Sciama [34].

One of the major motivations for undertaking this research is recent work of the present authors (Ref. [30]), who marshalled lots of evidence pointing to a moderately flattened dark halo $q \sim 0.6$ for both our Galaxy and spi-
ral galaxies in general. Most of those arguments apply to an essentially baryonic (MACHO or gaseous) halo, but some are applicable to the general dynamically dominant halo component. Sciama's DDM theory is among the plausible and interesting theories predicting an extreme $q \lesssim 0.2$ flattening for the dark neutrino component.

It should be noted that the presence of MACHOs detected through microlensing searches does not automatically discard the DDM theory, neither its applications in the galactic context. It is still possible (albeit more and more difficult to maintain) that MACHOs are dynamically insignificant or only marginally significant. The estimates of the MACHO contribution to the cosmological mass density are still dependent on several not completely watertight assumptions [12, 30]. We shall discuss some possibilities for a combined DDM + MACHO model in the Section 4.

2 DDM theory: Fundamentals

According to the DDM theory it is assumed that neutrinos do have masses and that the more massive neutrino, $\nu_1$ radiatively decays into a photon and less massive neutrino, $\nu_2$:

$$\nu_1 \rightarrow \gamma + \nu_2$$  \hspace{1cm} (1)

(e.g. Ref. [3]). If one further wishes to discuss the kinematics of this equation, one would obtain the following simple relation [34]:

$$E_\gamma = \frac{1}{2} m_\tau \left( 1 - \frac{m_{\nu_e,\mu}^2}{m_\tau^2} \right),$$  \hspace{1cm} (2)

where Greek letters in indices are related to the various types of neutrinos, i.e. tau, electron and mu neutrinos. It is also assumed that

$$m_{\nu_e,\mu} \lesssim 5 \text{ eV}$$  \hspace{1cm} (3)
therefore giving the following simple relation:
\[ E_\gamma \sim \frac{1}{2} m_{\nu_\tau}. \] (4)

The basic values of the DDM theory are the following (Ref. [34], updates in Ref. [37]):

- According to the latest estimates the mass is: \( m_{\nu_\tau} = 27.4 \pm 0.2 \text{ eV} \).
- These neutrinos decay radiatively with the lifetime \( \tau = 2 \pm 1 \times 10^{23} \text{ s} \). For the latest correction of this value, see the discussion later (based upon Ref. [24]).
- A decay photon energy is \( 13.7 \pm 0.1 \text{ eV} \) (obtained from the eq. [4]), so these photons can ionize hydrogen, but not helium.

3 Flattening in the DDM theory and observational data

The significant flattening of the halo was introduced by Binney, May and Ostriker (Ref. [3]) who found that it is likely that if the kinematics of the objects in the halo are similar to the kinematics of the extreme Population II objects, then the massive halo should have the axis ratio \( c/a < \sim 0.5 \). One can now define the flattening parameter \( \psi = \cos \psi = q = c/a \), i.e. its cosine determines the shape of the halo \( En \). The \( En \) notation is related to \( q \) as \( q = 1 - n/10 \).

As for the DDM theory, Sciama [32] proposed that the dark halo had to be extremely flattened \( q \sim 0.2 - 0.3 \), in order to obtain scale height of electrons responsible for pulsar dispersion that is determined by the scale height of the dark matter (i.e. massive neutrinos). By scale height we assume the column
density on one side of the galactic plane divided by the volume density in the plane. Later on, Sciama [33] argued that the scale height of the ionized gas should be reduced to $900 \pm 100$ pc, thus eliminating the need of the strongly flattened halo. However, Nordgren, Cordes and Terzian [26] found that the electron scale height is $670^{+170}_{-140}$ pc, and the mean electron density for the interstellar medium $n_e = 0.033 \pm 0.002$ cm$^{-3}$. This is also in accordance with earlier results of Cordes et al. [8].

This leads us to important questions concerning the ionization problem in the spiral galaxies (e.g. Refs. [34, 29]). We just mention here that the conventional sources such as O stars or supernovae have much smaller scale heights ($\sim 100$ pc) than the aforementioned ones of the free electron component of the interstellar matter. Following Ref. [34] we start with the equation that represents ionization equilibrium:

$$\frac{n_\nu(0)}{\tau} = \alpha n_e^2,$$

(5)

where $n_\nu(0)$ is the neutrino density near the Sun, $n_e$ is the free electron density, $\tau$ is the predicted value of the decaying neutrino lifetime and $\alpha$ is the hydrogen recombination coefficient excluding transition directly to the ground state (cf. Ref. [36]). If one adopts $\alpha = 2.6 \times 10^{-13}$ cm$^3$s$^{-1}$ and $n_e = 0.033$ cm$^{-3}$ this leads to the following neutrino density:

$$n_\nu(0) = 2.83 \times 10^7 \tau_{23} \text{ cm}^{-3}$$

(6)

where $\tau$ is expressed as $\tau = 10^{23} \tau_{23}$ s. This can give the following value of the mass density $\rho_\nu(0)$, assuming the neutrino mass of $27.4 \pm 0.2$ eV:

$$\rho_\nu(0) = 1.384 \times 10^{-24} \tau_{23} \text{ g cm}^{-3} = 0.02\tau_{23} M_\odot \text{ pc}^{-3}$$

(7)

where we, as usual, expressed mass density in the Solar masses ($M_\odot = 1.989 \times 10^{33}$ g) over cubic parsec (1 pc $= 3.0857 \times 10^{18}$ cm).
Now we can see why the extremely flattened halo was needed in the DDM theory; namely using equation (5) one has:

$$n_e = \left( \frac{n_\nu}{\alpha \tau} \right)^{\frac{1}{2}}$$  \hspace{1cm} (8)

that gives, after inserting appropriate values, the value for the electron density:

$$0.016 \leq n_e \leq 0.028 \text{ cm}^{-3}$$  \hspace{1cm} (9)

where we put Sciama’s value $n_\nu = 2 \times 10^7 \text{ cm}^{-3}$ (cf. Ref. [34]) and let lifetime to take the values $\tau = 2 \pm 1 \times 10^{23} \text{ s}$. One can see that there are two ways to achieve agreement of theory with observations: one can either increase $n_\nu$ as suggested by Sciama (Ref. [34]), by reducing the assumed scale height of the neutrino distribution of 8 kpc by a factor of $\sim 4$ to $\sim 2$ kpc which means that one must introduce extremely flattened halo, or, as we will show one can assume modified form of the equation (5) (see the discussion later). We first note that we obtained lower value than Sciama (Ref. [34]) for $\rho_\nu(0)$, $0.02\tau_{23}M_\odot \text{ pc}^{-3}$ vs. $0.03\tau_{23}M_\odot \text{ pc}^{-3}$, because we used Nordgren et al. (Ref. [26]) lower value for $n_e$ ($0.033 \text{ cm}^{-3}$), rather than Sciama ($0.04 \text{ cm}^{-3}$). In the Section, we shall see that this has profound consequences for flattening, even if we accept all other Sciama’s premises.

4 Uncertainties inherent in the flattening parameter determination

We would like now to investigate the general applicability of the decay–ionization equilibrium equation, and in order to do it, we would like to isolate all possible sources of uncertainty. Therefore we propose the following improvements concerning equation (5):
1) Additional ionizing sources should be taken into account, regardless on the assumed scale height. These can be O and B stars (the question of galactic disks opacity not being completely clear at present), as discussed, for instance, by Mathis [18] and others [13], metagalactic background, cosmic rays, large-scale shocks, etc.

In this case, the equation for decay-ionization equilibrium (5) should read
\[
\frac{n_\nu(0)}{\tau} + F = \alpha n_e^2,
\]
where \(F\) is the ionizing contribution of all other sources. Thus, we see that
\[
n_\nu(0) = (\alpha n_e^2 - F)\tau,
\]
i.e. required neutrino density is decreased. Accordingly, \(\rho_\nu(0)\) is also decreased, and necessity for flattening is decreased, as we shall see in more detail below.

2) Assumption of "maximal neutrino" halo is unwarranted, even if desirable. The discovery of MACHOs, and substantial mass contribution of these objects to the required dynamical mass of the Galaxy (e.g. Ref. [12]), immediately invalidates this assumption. So far, using the notation of the Sciama [36], we have to use the decomposition
\[
\Sigma_{\text{rot}} = \Sigma_{\text{rot}}(\nu) + \Sigma_{\text{rot}}(\text{other})
\]
Accordingly, the required lengthscale is decreased by a factor
\[
\frac{\Sigma_{\text{rot}}(\nu)}{\Sigma_{\text{rot}}} = f < 1.
\]
This offers an interesting opportunity of building of more complex model incorporating other observed phenomena. If we suppose that the rest of
dynamical mass of the Milky Way halo is made of baryonic MACHOs of the same population as the one detected in the microlensing searches, then one should be able, in principle, to tightly constraint the parameter $f$, i.e. the decaying neutrino fraction of the dynamical mass. This is to be achieved by simultaneous consideration of (i) microlensing optical depths and their ratios in various directions, which would put constraints on the shape of the MACHO component \([30]\); (ii) ionization of Reynolds’ layer as in the present paper; and (iii) constraints on the MACHO abundance stemming from the Big Bang nucleosynthesis constraints.

We consider this to be a good bargain, since for price of introducing one additional parameter ($f$ ratio) in the theory, we get multiple advantages: accounting for observed microlensing events and baryonic dark matter on one hand, and non-baryonic dark matter on the other, as well as retaining some advantages of the classical DDM theory, like explanation of cosmological reionization. Therefore, in the course of future work, we shall try to show how this extension can be specifically realized.

3) Column density uncertainty ($\delta \Sigma_{1,1}$ may vary within a factor or $\sim 2$) is introduced.

4) Galactocentric distance of the Sun, as well as the rotational velocity of the LSR are still subject to some uncertainty (e.g. Ref. [4]).

5) The least significant source of uncertainty is the uncertainty in the mass of decaying neutrino. It is manifested in transition from the neutrino number-density (as a discrete value quantifying ionization equilibrium) to the mass-density distribution.

Now we pass to determination of the flattening parameter of the neutrino halo, using the method and notation of Sciama [36]. The vertical lengthscale
Figure 1: The flattening parameter $q$ as a function of the electron density of the Reynolds' layer in the DDM theory. Vertical line denotes the recently obtained observational upper limit on the electron density.
can be written as

\[ z_0 = \frac{\Sigma_\nu}{(\alpha n_e^2 - F) \tau m_\nu}. \]  

(14)

Flattening parameter \( q \) can be approximated as

\[ q = \frac{z_0}{r_0}, \]  

(15)

where \( r_0 \) is the lengthscale in the galactic plane. If we accept the usual form of softened isothermal distribution of neutrino dark matter [2, 36]

\[ \rho_\nu(r) = \frac{\rho_\nu(0) a^2}{a^2 + r^2}, \]  

(16)

where \( a \) is the core radius. Horizontal lengthscale is, then, given as

\[ r_0 = \frac{\pi a}{2}. \]  

(17)

Thus, we obtain the general expression for flattening parameter in the "maximal neutrino" halo assumption with additional ionizing sources:

\[ q = \frac{2\Sigma_\nu}{\pi a(\alpha n_e^2 - F) \tau m_\nu}. \]  

(18)

For \( a = 8 \) kpc, we obtain

\[ q = 6.601 \left\{ \tau_{23} \left[ 2.6 \left( \frac{n_e}{10^{-2} \text{ cm}^{-3}} \right)^2 - \frac{F}{10^{-17} \text{ cm}^{-3} \text{ s}^{-1}} \right] \right\}^{-1}. \]  

(19)

Now we have cast the problem in the most tractable form. One may see that the following assumptions

1. purely DDM ionization \((F = 0)\),

2. desirable lifetime of the unstable particle \((\tau_{23} \simeq 1)\), and

3. Sciama’s original value for the electron density of the Reynolds’ layer \((n_e = 0.04 \text{ cm}^{-3})\),
truly lead to extreme flattening of the dark halo \( q = 0.16 \). We would like to investigate the consequences of the relaxing of these assumptions. Keeping the assumption (1) and relaxing (2) and (3) leads to results shown in the Fig. 1. For the electron density, we use the more realistic values of the eq. (9).

We see that the ”classical” value of \( \tau_{23} = 1 \) does not warrant an extreme degree of the neutrino halo flattening, contrary to the claim in the Ref. [36]. Instead, a moderate flattening \( q = 0.4 - 1 \) seems to be quite acceptable.

5 Other ionizing sources

If we relax the assumption (1), it is necessary to ask: which ionizing sources are viable for creating the electron density in (9)? There are several possibilities, main ones being:

- O and B stars of the galactic disk;
- metagalactic UV ionizing background;
- soft X-ray background, either of galactic or extragalactic origin;
- cosmic rays which penetrate the Reynolds’ layer;
- large-scale ionizing shock waves.

Some of them we can eliminate almost immediately, since we are not interested in the detailed ionization structure, only in influence of a specific ionizing source. Cosmic ray ionization, which has played such a prominent role in the first comprehensive ISM model of Field, Goldsmith and Habing in 1969 [11], is now considered to be of secondary importance: an average ionization
per H atom is now considered to be \( \lesssim 3 \times 10^{-17} \text{ s}^{-1} \) (in contradistinction to earlier estimates of \( \sim 10^{-15} \text{ s}^{-1} \)). This makes cosmic rays unimportant for the total ionization and heating budget of ISM, with important exception of the interiors of giant molecular clouds. If, as we have seen, the density of electrons is \( n_e \sim 10^{-2} \text{ cm}^{-3} \) (order of magnitude estimates), the total density can not be much higher, certainly \( < 10^{-1} \text{ cm}^{-3} \). This makes resulting value of \( F \) in the eq. (19) quite unimportant in comparison with the DDM term, and resulting decrease in flattening is negligible.

Similar considerations apply to soft X-ray ionization, which was first proposed by Silk and Werner [40]. The justification is that, like the cosmic rays, X-rays penetrate vast amount of matter before knocking out a photoelectron which is subsequently capable of ionizing and exciting hydrogen. Interestingly enough, for this ionization mechanism, more electrons are released from He than from H, since helium has larger photoionization cross-section at energies \( E_\gamma \sim 100 \text{ eV} \), situation opposite from the one in case of DDM ionization. However, primary ionization rate for this process is estimated as about \( 10^{-16} \text{ s}^{-1} \) in an unshielded region [39]. Thus, resulting value of \( F \) is \( \lesssim 10^{-17} \text{ cm}^{-3} \text{ s}^{-1} \), which does not significantly influence the required flattening of the neutrino halo. The same conclusion applies \textit{a fortiori} when we consider effects of finite opacity of the Reynolds’ layer.

What is the most difficult is to estimate the importance of the most plausible source of ionization: O and B stars in the Milky Way disk [10]. This uncertainty is reflected in the conclusions of Mathis [18], and is connected with the (in)famous problem of the opacity of disks of spiral galaxies [3, 17, 43], which has not been solved to this day. It seems clear that the \textit{shape} of the ionizing spectrum created by galactic early-type stars is capable of explaining the ionization state of the Reynolds’ layer, but more detailed modelling of the
Plausible modifications of DDM theory

Very recently, an attempt has been made to show that the diffuse ionization in the Galaxy can be explained via the decaying of sterile neutrinos with a mass of 27.4 eV and lifetime of $\sim 10^{22}$ s [24]. Sterile neutrinos appear in the theory as a consequence of the recent Super-Kamiokande result according to which the atmospheric neutrino anomaly is mainly due to nearly maximal oscillations between $\nu_\mu$ and $\nu^s_\mu$, where $\nu^s_\mu$ is a sterile neutrino [38]. This approach is conceived in order to overcome the perhaps biggest objection to the DDM theory, i.e. the bulk of the dark matter is assumed to be in neutrinos. Thus, according to Ref. [24], one can state that matter contribution to the critical density of the Universe is $\Omega_m \simeq 0.4$, while the baryonic contribution is $\Omega_B \simeq 0.08$, that leads to halo density of sterile neutrinos of $\simeq 2 \times 10^6$ cm$^{-3}$ that is 10 times less than the value previously assumed (cf. eq. [9]). According to the eq. [5] this would require that the radiative lifetime is significantly lowered for these sterile neutrinos, i.e. $\tau \sim 10^{22}$ s. However, as we have shown, it is not necessary to consider such radical change of this fundamental parameter of the DDM theory – it is possible to obtain lower concentrations of neutrinos by taking into account the term $F$, like in eq. [10].
7 Conclusions

It is clear, from the eq. (18), that larger values for $\tau_{23}$, require huge amount of flattening, which is quite unacceptable. For $\tau_{23} \sim 10$, corresponding flattening parameter is $q < 0.1$. This much flattening will have several consequences contradictory to the observational data: it is extremely improbable that such number of microlensing events would have been observed (Ref. [19]), it would violate the Oort limit (or its absence, see [17]), causing either the excessive local halo density [14], or too small total galactic mass to be consistent with the satellite galaxies measurements [16, 44, 45] (but see [25]). Both addition of other ionizing sources and addition of other dynamically important halo components would only act to increase the discrepancy with the minimal tolerable amount of flattening. Thus, we consider this an indication of non-viability of the large $\tau_{23}$ hypothesis.

We conclude that in view of all the uncertainties considered, as well as improved values for several of the observable quantities, flattening of the dark halo of the Milky Way can be regarded as neither well-defined nor essential prediction of the DDM theory.

The authors are happy to express their gratitude to Slobodan Ninković and Vesna Milošević-Zdjelar for useful discussions. SS acknowledges the financial support of the University of Trieste.

---

3Strictly speaking, this will apply to the addition of another dynamically important halo component only if this additional component is not an ionizing source itself, which is certainly true for most of the envisaged components (CDM, MACHOs, cold gas, etc.). If we stretch our imagination and consider, for example, an additional species of decaying particles and call it X, then by choosing parameters such that $\Sigma_X / (\tau_X m_X) > \Sigma_\nu / (\tau_\nu m_\nu)$ we could increase the resulting flattening parameter. However, any appeal of the original DDM (as well as conforming to the Occam’s razor) would be lost in this case, which would pile unwarranted hypotheses and require fine-tuning of its model parameters.
References

[1] K. M. Ashman, PASP, 104, 1109 (1992).

[2] J. Binney and S. Tremaine, *Galactic Dynamics*, (Princeton University Press, Princeton, 1987)

[3] J. Binney, A. May and J. P. Ostriker, MNRAS, 226, 149 (1987).

[4] J. Binney and M. Merrifield, *Galactic Astronomy*, (Princeton University Press, Princeton, 1998).

[5] J. Bland-Hawthorn and P. R. Maloney, PASA, 14, 59 (1997).

[6] F. Boehm and P. Vogel, *Physics of Massive Neutrinos*, (Cambridge University Press, Cambridge, 1992).

[7] F. Combes, P. Boissé, A. Mazure, A. Blachard, *Galaxies and Cosmology*, (Springer Verlag, New York, 1995).

[8] J. M. Cordes, J. M. Weisberg, D. A. Frail, S. R. Spangler & M. Ryan, Nature, 354, 121 (1991).

[9] W. J. G. de Blok and S. S. McGaugh, preprint astro-ph/9805120 (1998).

[10] J. B. Dove and J. M. Shull, ApJ, 430, 222 (1994).

[11] G. B. Field, D. W. Goldsmith and H. J. Habing, ApJ, 155, L149 (1969).

[12] B. D. Fields, K. Freese and D. S. Graff, New Astronomy, 3, 347 (1998).

[13] C. Fransson and R. A. Chevalier, ApJ, 296, 35 (1985).

[14] E. I. Gates, G. Gyuk and M. S. Turner, ApJ, 449, L123 (1995).
[15] J. Kovalevsky, ARAA, 36, 99 (1998).

[16] A. S. Kulessa and D. Lynden-Bell, MNRAS, 255, 105 (1992).

[17] C. Leitherer, H. C. Ferguson, T. M. Heckman and J. D. Lowenthal, ApJ, 454, L19 (1995).

[18] J. S. Mathis, ApJ, 301, 423 (1986).

[19] Y. Mellier, F. Bernardeau and L. van Waerbeke, to appear in the "IVieme colloque de cosmologie". June 4-6 1997, Observatoire de Paris (preprint astro-ph/9802005) (1998).

[20] M. R. Merrifield, AJ, 103, 1552 (1992).

[21] M. Milgrom, ApJ, 270, 365 (1983a).

[22] M. Milgrom, ApJ, 270, 371 (1983b).

[23] M. Milgrom, ApJ, 270, 384 (1983c).

[24] R. N. Mohaparta and D. W. Sciama, preprint hep-ph/9811446 (1998).

[25] S. Ninković, Bull. Astron. Inst. Czechosl., 41, 236 (1990).

[26] T. E. Nordgren, J. M. Cordes and Y. Terzian, AJ, 104, 1465 (1992).

[27] P. J. E. Peebles, Principles of Physical Cosmology, (Princeton University Press, Princeton, 1993).

[28] S. Samurović and V. Čelebonović, Publ. Astron. Obs. Belgrade, 54, 81 (1996).
[29] S. Samurović, M. M. Ćirković, V. Milošević-Zdjelar and J. Petrović, in 19th Summer School and International Symposium of the Physics of Ionized Gases (SPIG), August 31 – September 4, 1998, Zlatibor, Contributed Papers & Abstracts of Invited Lectures, Topical Invited Lectures and Progress reports, eds. N. Konjević, M. Ćuk and I. Videnović, 635 (1998).

[30] S. Samurović, M. M. Ćirković and V. Milošević-Zdjelar, MNRAS, submitted (1999).

[31] D. N. Schramm and G. Steigman, ApJ, 243, 1 (1981).

[32] D. W. Sciama, MNRAS, 244, 1p (1990a).

[33] D. W. Sciama, ApJ, 364, 549 (1990b).

[34] D. W. Sciama, Modern Cosmology and the Dark Matter Problem, (Cambridge University Press, Cambridge, 1993).

[35] D. W. Sciama, in The Physics of the Interstellar Medium and Interagalactic Medium, ed. by A. Ferrara et al. (ASP Conference Series, San Francisco, 1995), p. 114

[36] D. W. Sciama, preprint astro-ph/9704081 (1997).

[37] D. W. Sciama, A & A, 335, 12 (1998).

[38] D. W. Sciama, preprint astro-ph/9811172 (1998).

[39] J. Silk, PASP, 85, 207 (1973).

[40] J. Silk and M. Werner, ApJ, 158, 185 (1969).
[41] S. Tremaine and J. E. Gunn, Phys. Rev. Lett., 42, 407 (1979).

[42] V. Trimble, ARAA, 25, 425 (1987).

[43] C. Xu and V. A. Buat, A & A, 293, L65 (1995).

[44] D. Zaritsky, R. Smith, C. Frenk and S. D. M. White, ApJ, 405, 464 (1993)

[45] D. Zaritsky, R. Smith, C. Frenk and S. D. M. White, ApJ, 478, 39 (1997)