Radiation reflection from star surface reveals the mystery of interpulse shift and appearance of high frequency components in the Crab pulsar

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Abstract. A new mechanism of radiation emission in the polar gap of a pulsar is discussed. It is based on the curvature radiation which is emitted by positrons moving towards the surface of neutron star along field lines of the inclined magnetic field and reflects from the surface. This mechanism explains the mystery of the interpulse shift and appearance of additional components in the emission of Crab pulsar at high frequencies discovered by Moffett and Hankins twenty years ago. We discuss coherence, energy flux and spectrum of the reflected radiation, appearance and disappearance of the interpulse position shift with the frequency increase. It is also possible that a nonlinear reflection (stimulated scattering) from the star surface is observed in the form of HF components. The frequency drift of these components, discovered by Hankins, Jones and Eilek, is discussed. The nonlinear reflection is associated with “Wood’s anomaly” at the diffracted waves grazing along the star surface. Two components can arise due to slow and fast waves which are present in the magnetospheric plasma. The possible scheme of their appearance due to birefringence at the reflection is also proposed.

1. Introduction
Twenty years later after the discovery of interpulse shift and detection of the high frequency (HF) components in the radiation of the Crab pulsar by D Moffet and T Hankins [1], T Hankins, G Jones and J Eilek returned to observation of these phenomena at even higher frequencies [2]. As one of the results, a frequency drift of HF components has been discovered. The detailed analysis of physical results and problems is discussed in the authors’ review [3]. The problems of interpulse shift and HF components appearance are marked there as unresolved.

The figure (figure 1) from the paper [1] by Moffett and Hankins, 1996 shows that at around 3 GHz there appears a phase shift $\delta$ of interpulse (IP) of about 7 degrees comparing to the initial IP positon at lower frequencies. Moreover, two more distinct pulses – high-frequency components [2] – appear at the same frequencies [1, 2]. In our preprint [4] (and subsequent papers [5, 6]) and in the current work we propose the explanation of these phenomena applying the idea of the radiation reflection from the neutron star surface by returning positrons moving in the polar gap region toward the surface of the star in the inclined magnetic field (figure 2).

Presently we consider in detail the curvature radiation (CR) reflected from the pulsar surface.
Figure 1. Fragment from the average light curves obtained at multi-frequency observations of Crab pulsar in [1]. The shifted (to about $\tau^0$) interpulse and high-frequency components are marked. By the courtesy of the authors. ©AAS. Reproduced with permission.

Figure 2. Schematic picture of motion and radiation by electrons and positrons in the polar gap of the pulsar in the case of a tilted magnetic axis. The directions of radiation by electrons and reflected radiation by positrons are shifted at the angle of mirror reflection

We show that such mechanism of (linear, mirror) reflection also provides the possibility of explaining the intensity of IP coherent radiation, its energy flux and spectrum.\(^1\)

2. Estimation of the reflected radiation flux forming the shifted IP

The positrons moving towards the surface of the pulsar in the polar gap can be returned from the lower layers of the magnetospheric plasma by the same electric field which accelerates the electrons outwards the star. We estimate the flux of the reflected radiation above the entire area of the polar cap. The observations indicate the necessity of coherent character of radiation (see the recent review on the problem, [8]). We make estimation taking into account this fact which is discussed in detail in the Appendix of [6]. For the estimation of CR intensity and derivation of the radiation energy spectrum at first it is enough to accept the very fact of existence of inhomogeneous (clumped) positron flux in the gap. In this case the main contribution to radiation is made by coherently radiating volumes, which we estimate further. The number of such volumes can be rather roughly estimated through ‘division’ of the total radiating volume by the coherence volume. The dependence of such volume on the wavelength, Lorentz-factor and the curvature of the positrons trajectories is taken into account.

The radiation spectral density at frequency $\omega$ produced by a single positron, which moves along an arc of a circle of the (curvature) radius $R$ per unit path can be estimated as $W_\omega \sim \left(\frac{\omega R}{c}\right)^{1/3} e^2/(2\pi R^3)$. It directly follows from the known expression for spectral density of radiative energy loss by a particle moving along a circular trajectory.

The size of a volume which encloses amount of particles radiating coherently we take as $V_{coh} = \tau_{||}^2 \sim \gamma^2 \lambda^3/(4\pi^3)$, where the distances $r_{||} \sim \lambda/\pi$ and $r_\perp \sim \gamma \lambda/(2\pi)$ define the linear dimensions of such coherently radiating volume both in the direction of the particles motion and in the perpendicular one. It is the square of the average number $N_{coh} \sim \kappa \times n_{GJ} V_{coh}$ of positrons in such volume that radiation flux produced by these particles is proportional to. Here $n_{GJ} \sim 10^{11} \text{ cm}^{-3}$ is the Goldreich-Julian density and $\kappa < 1$ is some coefficient defining the

\(^1\) The idea of (nonlinear) reflection from the pulsar surface is applied in [5] for consideration of high frequency components (HFCs) since they appear at the same frequencies as the shifted IP [1, 2]. (For the independent treatment of the HFCs, not connected with the IP shift, see the article of S. Petrova [7]).
difference of $n_{GJ}$ from the real particle density in the returning positron flux. The amount of the volumes $V_{coh}$ which lie within the surface element $2\pi r dr$ of the polar cap at certain altitude is then $dN_\perp \sim 2\pi r dr/r_\perp^2 = (2\pi)^3 r dr/\gamma^2 \lambda^2$. The upper estimation for the number of such volumes falling on the polar cap surface segment of the order of $r_\perp^2$ per unit of time is then $dN_\parallel/dt \sim c/\lambda$. The effective solid angle which encloses the considered radiation emitted by positrons with the Lorentz-factor $\gamma$ can be estimated as $\Omega_{eff} \sim \pi/\gamma^2$ that gives for the effective area on the distance $d$ from the star: $S_{eff} \sim \pi d^2/\gamma^2$.

Summarizing the previous considerations we can present the expression for the flux of the reflected radiation by positrons as $J(\omega) \sim \int dzdN_\perp \times dN_\parallel/dt \times W_{\omega N_{coh}^2}/S_{eff}$. One can express the positron coordinate through its Lorenz-factor as $dz = \sqrt{mc^2/2eE(r)}d\gamma/\sqrt{\gamma}$ at the linear growth of accelerating electric field strength in the actual region of the trajectories contributing to the coherent radiation flux. Here $\bar{h}$ is an averaged length of the positron trajectory interval of the gap size order, $z = R_\alpha$ (see figure 4) is the positron coordinate along the magnetic field line and $E(r) = E_0(1 - r^2/R_{PC}^2)$ is the accelerating electric field strength, vanishing on the polar cap boundary $R_{PC}$.

The dependence of the curvature radius of the magnetic field lines on $r$ in the vicinity of the star surface in the case of a dipole magnetic field is described by the expression: $R(r) = 4R_*^2/3r$, where $R_* \sim 10^6$ cm is the pulsar radius. Substitution of the explicit expressions for the quantities presented here gives the following:

$$J(\omega) \sim \frac{\kappa^2 n_{GJ}^2 e^2 \lambda^{3-1/3}}{25/3 \pi^{14/3} d^2 R_*^3} \sqrt{\frac{mc^2 \bar{h}}{2eE_0}} \times \frac{R_{PC}}{\sqrt{1 - r^2/R_{PC}^2}} \int_0^{\gamma_{max}} \int_{dN_{\parallel}}^{\gamma_{max}(r,\omega)} d\gamma/\gamma^{7/2}. \tag{1}$$

$\gamma_{max}$ is an effective value of the positron Lorenz-factor at which its radiation reflected from the surface of the star ceases to hit the telescope (figure 3). Here $\gamma_{min}$ is of the order of $10^2$. We assume that the initial angular width of the positron radiation diagram which is $\sim 1/\gamma_{min}$ exceeds the value of the minimal angle between the magnetic axis and the direction to the telescope. With the increase of the positron Lorenz-factor at lower altitudes the characteristic angle of its radiation diagram becomes less than $\theta_{min}$ (at $\gamma \sim \gamma_{max}$) and radiation ceases to be caught by the telescope.

The final estimation of the total reflected positron radiation flux for $\gamma_{min} \ll \gamma_{max}$ is the following:

$$J(\omega) \sim \frac{\kappa^2 n_{GJ}^2 e^2 \lambda^{3-1/3}}{40 \pi^{14/3} d^2 R_*^3} \sqrt{\frac{mc^2 \bar{h}}{2eE_0}} \gamma_{9/2, max}^{R_{PC}^8/3}. \tag{2}$$

The electric field strength $E_0$ on the magnetic axis can be estimated as $E_0 \sim a\Omega R_*/cB \sim 10^7$ CGSE, where $\Omega = 2\pi/T$ is the angular frequency of the pulsar rotation, $B \sim 10^{11}$ Gs is the magnetic field strength and $a \sim 10^{-1} \div 10^{-2}$ is a small parameter which takes the effect of geometrical and gravitational factors into account (see [9]). Then, the estimation of the flux reduces to $J(\omega) \sim \kappa^2 \lambda^{3-1/3} \gamma_{max}^{9/2} \times 10^{-40} W/(Hz \times m^2)$ (at $R_{PC} = 10^4$ cm, $\bar{h} = 10^4$ cm, $d = 6 \times 10^{21}$ cm). As this expression shows, the parameter $\gamma_{max}$ can be approximately estimated by the comparison of the calculated flux value with the observational radio emission of the Crab pulsar. Quite a good agreement is achieved if $\gamma_{max} \sim 10^3$ and $\kappa \sim 10^{-1}$ are chosen.

The expression also shows that in the interval of wavelengths considered here the radiation energy spectrum is defined by the term $\lambda^{3-1/3} \approx \lambda^{2.7}$, which is also in a quite good qualitative agreement with observational results (see refs in [6]). Let us note that for $\lambda/L \ll 1$ the estimation $dN_\parallel/dt \sim c/L$ might be more preferable. Here $L$ is a spatial size of longitudinal inhomogeneities in the positron flow (for details see Appendix
in [6]). In this case radiation spectrum is defined by the term $\lambda^{3.7}$ (instead of $\lambda^{2.7}$), which seems to be in a better accordance with the observations (see refs in [6]).

3. Estimation of the highest frequency at which the interpulse shift takes place

In the framework of our model the positron Lorenz-factor value $\gamma_{\text{min}}$ at the beginning of the effective path defines the minimal frequency $\omega$ of its radiation which can reflect from the surface and hit the telescope. The corresponding relation between $\gamma_{\text{min}}$ and $\omega$ can be presented as $\omega \sim c\gamma_{\text{min}}^3/R$. The value of $\gamma_{\text{min}}$ can be expressed through radiated frequency $\nu = \omega/(2\pi)$ and the distance $r$ from the magnetic field axis as $\gamma_{\text{min}}(\nu, r) \sim \left(8\pi\nu R_*^2/3cr\right)^{1/3}$. Here we see that $\gamma_{\text{min}}$ grows with the increase of $\nu$ and decrease of $r$. Therefore, at some value of $r = r_0(\nu)$ the magnitude of $\gamma_{\text{min}}$ reaches the $\gamma_{\text{max}}$ and the internal integral over $\gamma$ in (1) tends to zero. In the region of polar cap $r < r_0$ in this case the considered radiation mechanism does not work. The values of $r$ at which the moving positrons do not contribute to the observed radiation can be estimated as $r < r_0(\nu) \sim 8\pi\nu R_*^2/3c\gamma_{\text{max}}^3$.

![Figure 3. Schematic picture of angular regions (cones) of concentration of the reflected radiation by positrons at two extreme values of the considered Lorenz-factors. At $\gamma = \gamma_{\text{max}}$ radiation ceases to hit the telescope. Among other reasons is that the magnetic axis does not coincide with the ones of the cones due to its assumed inclination with respect to the surface normal](image1.png)

![Figure 4. Region of space (orange hollow cone) contributing to the positron coherent radiation flux at $\nu < \nu_{\text{max}}$](image2.png)

Thus, the coherence leads to the formation of a hollow cone, restricted (in the vicinity of the star surface) from the inner and outer sides by the field lines situated on distances $r_0$ and $R_{\text{PC}}$ respectively from the magnetic axis (figure 4). The frequency $\nu = \nu_{\text{max}}$, at which $r_0$ reaches the radius $R_{\text{PC}}$ of the polar cap, is the one at which the considered radiation mechanism providing the interpulse shift disappears. It can be estimated as $\nu_{\text{max}} \sim 3cR_{\text{PC}}/(\gamma_{\text{max}}^38\pi R_*^2)$. Note that more accurate estimation of this frequency requires application of self-consistent models of the positron flow motion in the polar gap.

4. Stimulated Scattering and HF components

Appearance of the HF components in the same frequency range, as that in which the interpulse shift takes place, permits us also to associate these components with reflection from the neutron star surface. (See more detailed consideration of the problem with coherence and spectrum discussion in the Appendix of [6]). In the non-linear reflection model [5] (which is the model of induced Raman scattering by surface waves), the frequency shift of the HF
Figure 5. A scheme of the appearance of combinational Raman spectra of the first order at the IP (S pole) at stimulated scattering of radiation by returning positrons

Figure 6. Appearance of the first-order Raman spectra at the MP (N pole). Here the direction of the phase shift is inverse compared to the S pole (schematically).

components appears in the natural way, regardless of the type of surface waves that induce the Raman scattering. (An independent treatment of the HFCs, not connected with the IP shift, see [7]).

5. Wood’s anomaly in Mandel’stam-Raman electromagnetic fields
In our consideration it is only important that the reflection is associated with “Wood’s anomaly” involving the diffracted waves grazing along the star surface (see [11, 12, 13] and refs in [5]). The grazing diffraction component has the largest amplitude with a large dielectric constant of the reflecting medium and makes the greatest contribution to the ray pressure swinging up the surface.

The corresponding estimates for the combinational reflected fields have the form:

$$E_{\pm} \approx (k\zeta)E_0, k_z \neq 0$$  ;  $$E_{\pm} \approx \sqrt{\epsilon}(k\zeta)E_0, k_z = 0$$  \(\epsilon \gg 1\) .

Here \(\zeta\) is the amplitude of a surface Wood’s wave and \(E_0\) is the amplitude of the electromagnetic wave, generated by positrons, incident on the star surface.

In a conducting medium with complex dielectric permittivity, a “Wood’s” resonance of a grazing diffracted component with the surface electromagnetic wave is possible. Since this corresponds to a small (of the surface impedance order) value of \(k_z/k\), the kinematics presented here is practically the same. Dynamic conditions can differ significantly [14]. For the parameters of the neutron star in Crab with strong magnetic field, the surface may apparently consist of iron and have a sharp boundary (by courtesy of A.Potekhin, see refs in [15]).

In accordance with the scheme shown in figure 7, the change in frequency \(\delta\omega\) leads to a change in the wave numbers, the tangential components of the wave vectors and angles in Raman scattering by the Wood’s wave \(q + \delta q\) (\(\theta\) is the azimuth angle of the inclined magnetic moment):

$$\delta k = \delta\omega/V_g, \ \delta k_t = \delta k \sin \theta, \ \delta k_t^- = \delta k(1 - 2\sin \theta).$$

Here \(V_g\) is the group velocity of EM wave, angle \(\theta^-\) in the equatorial plane (differing by a small angle from the “magnetic” one, where both the magnetic poles and the star center lie), indicates the position of the 1-st HF-component HFC1 corresponding to the direction of the diffraction
Figure 7. A scheme of drift of HFC1 position with the increase of frequency due to changing of the wave number of EM waves in the stimulated scattering model.

Figure 8. Frequency drift of the HF components. We also see the absence of the MP in this region and very wide HF pulses. By courtesy of the authors of [2]. ©AAS. Reproduced with permission.

maximum on a periodic structure, created by Wood’s wave at frequency $\omega$ of EM wave emitted by returning positrons. Angle $\theta^- + \delta \phi$ indicates the position of the same HF-component at
frequency $\omega + \delta\omega$. Other definitions are clear from the figure 7:

$$\sin(\theta^- + \delta\varphi) = (k^-_t - \delta k^-_t)/(k + \delta k).$$

For drift $\delta\varphi$ of the HFCI, an expression and an estimate in agreement with [2] follows:

$$\delta\varphi = \frac{\delta\omega}{kV_g} \frac{1 - 2\sin\theta^+ - \sin\theta^-}{\cos\theta^-} \approx \frac{\delta\omega}{\omega} \quad (4)$$

6. Drift of HF components

![Figure 9](https://example.com/fig9.png)

**Figure 9.** Two components can arise due to slow ($k_1$) and fast ($k_2$) waves, which are present in the magnetospheric plasma. Only the reflected Stokes waves are shown on the scheme. In reality, the waves, most likely, should arise not directly at reflection, but during the propagation process of the reflected Stokes wave in the anisotropic plasma.

In [5] it has been supposed that different HF components are produced by the opposite magnetic poles. An unidirectional drift, which is intrinsic to both components [2], stipulates one to associate them with the single magnetic pole of the star and, perhaps, with the birefringence in the magnetosphere. The observations to be explained, the second component should match over-horizon spread.

7. Summary

It is shown that the discussed new radiation mechanism can be applied for explanation of such unusual fact of the Crab pulsar radio emission as the shift of its interpulse at several GHz frequency. For this the assumption of the inclined magnetic field is used and returning positron flow is considered. The total flux of the coherent positron radiation is estimated and it (as well as the radiation spectrum) agrees with the results of observation quite well. In the framework of the proposed model the highest frequency at which the interpulse shift takes place is also roughly estimated.

The radiation of high-frequency components of the pulsar in the Crab Nebula can be considered also as a manifestation of an instability in the nonlinear reflection from the neutron star surface [5]. The discussed instability is a stimulated scattering by surface waves, predicted more than forty years ago and still observed nowhere and by no one. Drift of HFCs has a natural explanation in the framework of the stimulated scattering model.
The frequency drift of the components is very important in choosing the correct theoretical model. Particularly, the coincidence of its direction for both components is an argument in favor of the birefringence of the scattered wave in the anisotropic magnetized pulsar plasma. Returned motion of positrons, arising due to penetration of accelerating electric field of the gap in the pair plasma, has been considered in literature in connection with surface heating by the reverse current (see refs in [10]). The difference of a magnetic field from dipole one, which in particular leads to its slope, is also discussed with regard to its toroidal component as well (see refs ibidem). However, the low-frequency radiation of backflow positrons and reflected radiation from the surface of the pulsar have not been considered anywhere before except our papers.

Reflection leading to HF components formation may also be associated with the static periodic structures (“frozen” surface waves) in strong constant electric and magnetic fields [16, 17]. This possibility, as well as the drift of the components in these models, possibly associated with a change in the angle of incidence of radiation at the surface during the positron motion along the force line, and requires a separate study. The explanation of the large width of the HF components also requires an investigation of the dynamic conditions of the surface waves excitation. Possibly, a nonmonochromatic beam of positron radiation incident on the surface should be taken into account. However, the existence of a wide spectrum of the excited surface waves under Wood’s conditions is also a consequence of the purely kinematic conditions when the considered waves do not lie in the plane of incidence. The interaction of these waves with each other, generating an analogue of wave turbulence, can also have a significant effect.

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