Tests for the existence of horizon through gravitational waves from a small binary in the vicinity of a massive object

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In this letter we calculate the gravitational waves (GWs) emitted from a small binary (SB) by solving the Teukolsky equation in the background of a massive exotic compact object (ECO) which is phenomenologically described by a Schwarzschild geometry with a reflective boundary condition at its “would-be” horizon. The “continuous echo” waves propagating to infinity due to reflectivity of ECO at its “would-be” horizon provide an exquisite probe to the nature of the ECO’s horizon.

Introduction. Black holes are predicted by general relativity as the compact stellar remnants of gravitational collapse of massive stars. Detection of event horizon, marking the boundary within which future null infinity cannot be reached, is supposed to be the decisive evidence for black hole. Both the gravitational wave (GW) detections [1–4] and the observations of supermassive black holes by the Event Horizon Telescope [5–9] provide the elegant avenues toward the physics near the event horizon of black hole. However, no experiment yet is able to reveal the geometry structure near event horizon although the black hole paradigm is consistent with hitherto all electromagnetic and GW observations [5, 6, 10, 11]. In particular, Barausse and Cardoso et.al.[12, 13] argued that GW from a binary coalescence, especially the ringdown waveform, could not be regarded as a conclusive evidence for the existence of event horizon after the merger. Furthermore, event horizon plays a central role in the black hole information paradox [14], and the behavior of event horizon is important to the theory of quantum gravity. Thus, probing the existence of event horizon is one of the frontier researches of fundamental physics.

On the other hand, the concept of horizonless object, namely exotic compact object (ECO), was put forward as an alternative to the classical black hole paradigm. ECO is motivated either by candidate theories of quantum gravity, including the ECO models of firewall, wormhole, fuzzball, and black holes modified by generalized uncertainty principle [15–22], or by the physics beyond standard model of particle physics, including ECO models of boson star and gravastar [23–26]. Despite different model of ECO, it may be phenomenologically described by a standard black hole with a reflective boundary condition given at its “would-be” horizon”, i.e., its physical surface close to the event horizon predicted by general relativity, as a result of our beliefs that the difference between ECO and black hole would be only significant in the Planckian or near-Planckian scale near the event horizon [27].

In literature there are several methodologies to test the nature of dark compact objects based on GW astronomy: (i) detecting the small corrections to the GW phase during binary insparal stage, which is driven by different multipolar structures, tidal deformation, tidal heating, resonance excitation, and the correction of gravitational waves from extreme-mass-ratio inspiral systems [28–42]; (ii) late-time echoes in the GW waveforms afterwards the merger stage, induced by the presence of structure close to the gravitational radius of the ECO [43–53]; (iii) stochastic GW background caused by ergoregion instability of ECOS [54–56]. See [57, 58] for some recent review.

In this letter, we propose a new method to test the existence of event horizon through GWs emitted by a small binary (SB) in the background of a massive ECO. The idea is sketched out in Fig. 1. Usually the Center of Galaxy harbors a supermassive black hole (SMBH) [59–61], and SB could form in a close orbit to the SMBH due to abundant astrophysical phenomena (see, e.g., [62–68]) and even merger in the inner-most stable circular orbit (ISCO) of the SMBH [68]. The orbital radius of SB is much smaller than the orbital radius of their center of mass revolving around the SMBH, and thus compose a stable hierarchical triple system, the so called “binary-EMRI” system [66]. The gravitational radiation in such system has been studied in large amount of contexts, the waveforms are characterized by many unique features [69–83]. If the centering object is a massive ECO rather a SMBH, GWs emitted by the SB will be reflected to infinity at the “would-be” horizon of ECO, resulting the “continuous echo” waves, and thus provides a new probe to the “would-be” horizon of the massive ECO.
where \( f(r) = 1 - 2M/r \), the potential \( U(r) \) is given by
\[
U(r) = f^{-1} \left[ (\omega r)^2 - 4i\omega (r - 3M) \right] - (\ell - 1)(\ell + 2). \tag{6}
\]

Teukolsky radial equation can be solved by the Green function method with standard procedure. Firstly, we need two linearly independent homogeneous solutions which satisfy proper boundary conditions. For a massive BH, the two homogeneous functions are denoted by \( R_{\omega tm}^H \) and \( R_{\omega tm}^\infty \) satisfying the following pure in-going condition at the event horizon and pure out-going condition at spatial infinity
\[
R_{\omega tm}^H \sim \begin{cases} 
B_{\omega tm}^{\text{hole}} \Delta^2 e^{-i\omega r} & r \to 2M \\
B_{\omega tm}^{\text{out}} e^{\omega r} + r^{-1} B_{\omega tm}^{\text{in}} e^{-i\omega r} & r \to +\infty
\end{cases}
R_{\omega tm}^\infty \sim \begin{cases} 
D_{\omega tm}^{\text{out}} e^{\omega r} + \Delta^2 D_{\omega tm}^{\text{in}} e^{-i\omega r} & r \to 2M \\
D_{\omega tm}^{\infty} r^3 e^{i\omega r} & r \to +\infty
\end{cases} \tag{7}
\]
where \( r_* \) is the tortoise coordinate given by \( r_* = r + 2M \ln \left( \frac{r}{2M} - 1 \right) \). The Wronskian of Eq. (5) can be obtained from the asymptotic behavior at spatial infinity in Eq. (7), namely
\[
\lim_{r \to \infty} \frac{R_{\omega tm}^H R_{\omega tm}^{\infty} - R_{\omega tm}^H R_{\omega tm}^{\infty}}{r^2 f(r)} = 2i\omega B_{\omega tm}^{\text{in}} D_{\omega tm}^{\infty}, \tag{8}
\]
where the prime denotes the derivative with respect to \( r \). Therefore, the Green function \( G^{\text{BH}}(r, r') \) in BH case could be constructed by
\[
G^{\text{BH}}(r, r') = \frac{1}{2i\omega D_{\omega tm}^{\infty} B_{\omega tm}^{\infty}} \times \left[ \theta (r - r') \frac{R_{\omega tm}^H (r') R_{\omega tm}^{\infty} (r)}{r^2 f^2(r')} + \theta (r' - r) \frac{R_{\omega tm}^H (r) R_{\omega tm}^{\infty} (r')}{r'^2 f^2(r')} \right] \tag{9}
\]
where \( \theta (r - r') \) is the Heaviside step function, and the radial function \( R_{\omega tm} \) at spatial infinity reads
\[
R_{\omega tm}^H(r) = \int_{2M}^{\infty} dr' G^{\text{BH}}(r, r') T_{\omega tm}(r') = \int_{2M}^{\infty} dr' \frac{R_{\omega tm}^H(r') T_{\omega tm}(r')}{r^2 f^2(r')}, \tag{10}
\]
In order to get the homogenous solution \( R_{\omega tm}(r) \), we usually solve the so-called Regge-Wheeler equation which has a form of wave equation as follows:
\[
\left( \frac{d^2}{dr^2} + \omega^2 - V(r) \right) X_{\omega tm}(r) = 0, \tag{11}
\]
where
\[
V(r) = \left( 1 - \frac{2M}{r} \right) \left( \frac{\ell(\ell + 1)}{r^2} - \frac{6M}{r^3} \right), \tag{12}
\]
and \( R_{\omega tm}(r) \) is related to \( X_{\omega tm}(r) \) by the Chandrasekhar transformation \([85]:\)
\[
R_{\omega tm}(r) = r^3 \left[ \frac{2}{r} \left( 1 - \frac{3M}{r} \right) + 2i\omega \right] \left[ \frac{d}{dr_*} + i\omega \right] X_{\omega tm}(r) + r^3 V(r) X_{\omega tm}(r). \tag{13}
\]
The asymptotic behaviors of the Regge-Wheeler function $X_{\omega \ell m}(r)$ is then inherited from Eq. (7) as

$$
X^H_{\omega \ell m} \sim \begin{cases} e^{-i\omega r}, & r \to 2M, \\ A_{\omega \ell m} e^{-i\omega r} + A_{\omega \ell m}^{in} e^{-i\omega r}, & r \to +\infty \end{cases}
$$

$$
X^{\infty}_{\omega \ell m} \sim \begin{cases} e^{i\omega r}, & r \to 2M, \\ C_{\omega \ell m} e^{i\omega r} + C_{\omega \ell m}^{in} e^{i\omega r}, & r \to +\infty \end{cases}
$$

where $A_{\omega \ell m}^{in}$ is related to $B_{\omega \ell m}^{in}$ by

$$
B_{\omega \ell m}^{in} = \frac{[12iM\omega - (l - 1)(l + 1)(l + 2)] A_{\omega \ell m}^{in}}{4\omega^2}.
$$

From now on, we switch to the gravitational radiation of an SB in the massive ECO background. The difference between ECO and BH is phenomenologically described by the reflective boundary condition at the “would-be” horizon of ECO which can be well-defined in the Regge-Wheeler equation due to its standard form of wave equation. The boundary conditions of the homogeneous Regge-Wheeler functions for ECO take the form

$$
X^H_{\omega \ell m} \sim e^{-i\omega r}, \quad X^{\infty}_{\omega \ell m} \sim e^{i\omega r},
$$

for $r \to 2M$,

where $R_{\omega \ell m}^{ECO}$ is the reflectivity which equals to zero for BH. Since the boundary condition for $X^{\infty}_{\omega \ell m}$ is the same as that for BH, we have $X^{\infty}_{\omega \ell m} = X^{\infty}_{\omega \ell m}$. 

Similar to Eq. (9), the Green function of Teukolsky radial equation for ECO can be constructed with the homogeneous functions $R_{\omega \ell m}^{ECO}$ and $R_{\omega \ell m}^{BH}$, and $R_{\omega \ell m}^{ECO}$ is related to $X^H_{\omega \ell m}$ by the Chandrasekhar transformations as well. Since both $R_{\omega \ell m}^{ECO}$ and $R_{\omega \ell m}^{BH}$ satisfy with the homogeneous Teukolsky equation, the homogeneous function $R_{\omega \ell m}^{ECO}$ can be taken as a linear superposition of $R_{\omega \ell m}^{ECO}$ and $R_{\omega \ell m}^{BH}$

$$
R_{\omega \ell m}^{ECO} = R_{\omega \ell m}^{ECO} + \mathcal{K} R_{\omega \ell m}^{BH},
$$

Consequently, the homogeneous Regge-Wheeler function $X^H_{\omega \ell m} = X^H_{\omega \ell m} + \mathcal{K} X^{\infty}_{\omega \ell m}$. By demanding that $X^H_{\omega \ell m}$ satisfies the boundary condition at the ECO “would-be” horizon in Eq. (16), the coefficient $\mathcal{K}$ is given by [86]

$$
\mathcal{K} = \frac{\mathcal{R}_{\omega \ell m}^{ECO} \mathcal{T}_{\omega \ell m}^{BH}}{1 - \mathcal{R}_{\omega \ell m}^{ECO} \mathcal{R}_{\omega \ell m}^{BH}},
$$

where $\mathcal{R}_{\omega \ell m}^{ECO}$ and $\mathcal{T}_{\omega \ell m}^{BH}$ are the reflection and transmission amplitude of the BH potential barrier, namely

$$
\mathcal{T}_{\omega \ell m}^{BH} = \frac{1}{C_{\omega \ell m}^{out}}, \quad \mathcal{R}_{\omega \ell m}^{BH} = \frac{C_{\omega \ell m}^{in}}{C_{\omega \ell m}^{out}}.
$$

Thus, the Green function for ECO, $G^{ECO}(r, r')$, reads

$$
G^{ECO}(r, r') = G^{BH}(r, r') + G^{DEF}(r, r'),
$$

where

$$
G^{DEF}(r, r') = \frac{\mathcal{K}}{2i\omega B_{\omega \ell m}^{in} D_{\omega \ell m}^{\infty}} \mathcal{R}_{\omega \ell m}^{ECO}(r) \mathcal{R}_{\omega \ell m}^{BH}(r') r'' f^2(r'').
$$

Since the Green function in Eq. (20) is separated into the BH part and the part related to the ECO reflectivity, the radial function of the GW emitted by the SB is also separated into

$$
R_{\omega \ell m}^{ECO}(r) = R_{\omega \ell m}^{BH}(r) + R_{\omega \ell m}^{DEF}(r),
$$

where

$$
R_{\omega \ell m}^{DEF}(r) = \int_{2M}^{\infty} dr' G^{DEF}(r, r') T_{\omega \ell m}(r')
$$

$$
= \frac{\mathcal{R}_{\omega \ell m}^{ECO} \mathcal{R}_{\omega \ell m}^{BH}}{1 - \mathcal{R}_{\omega \ell m}^{ECO} \mathcal{R}_{\omega \ell m}^{BH}} R_{\omega \ell m}^{eff}(r),
$$

with $R_{\omega \ell m}^{eff}(r)$ given by

$$
R_{\omega \ell m}^{eff}(r) = \frac{R_{\omega \ell m}^{ECO}(r) C_{\omega \ell m}^{in}}{2i\omega B_{\omega \ell m}^{in} D_{\omega \ell m}^{\infty}} \int_{2M}^{\infty} dr' \frac{R_{\omega \ell m}^{ECO}(r') T_{\omega \ell m}(r')}{{r''}^2 f^2(r')}.
$$

The function $R_{\omega \ell m}^{DEF}(r)$ describes the waveform being reflected by the “would-be” horizon of ECO. Here, $R_{\omega \ell m}^{DEF}(r)$ can be re-written by the geometric series of $\mathcal{R}_{\omega \ell m}^{ECO} \mathcal{R}_{\omega \ell m}^{BH}$ (which is smaller than unity). In this sense, $R_{\omega \ell m}^{DEF}(r)$ can be physically explained as the linear superposition of repeated $R_{\omega \ell m}^{eff}(r)$ with the coefficient $(\mathcal{R}_{\omega \ell m}^{ECO} \mathcal{R}_{\omega \ell m}^{BH})^n$, which leads to the phenomenon called “echo”, similar to that after the ringdown signal of a merger event discussed in [86]. Therefore, the GWs detected by a distant observer include both the waveforms emitted by the SB in the BH background and the “echo” waves reflected by the “would-be” horizon of ECO. This is just what is illustrated in Fig. 1.

**Setting sources.** We model the SB as two point particles of mass $m_1$ and $m_2$, whose energy momentum tensor is given by

$$
T^{\mu\nu}(x) = m_1 \int_{-\infty}^{+\infty} \delta^{(4)}(x - z(\tau_1)) d\tau_1 \frac{dz^\mu(\tau_1)}{d\tau_1} d\tau_1 + (1 \to 2),
$$

where $z^\mu_i = (t(\tau_i), r(\tau_i), \theta(\tau_i), \varphi(\tau_i))$ (with $i = 1, 2$) are the world lines of the two point particles in the SB, and the normalization condition is given by $\int \delta^{(4)}(x) \sqrt{-g} dx = 1$. For simplicity, we consider these two point particles have the same mass $m_1 = m_2 = \mu$. We take the orbital plane for the center of mass (CM) of SB as equatorial plane. For the two point particles of SB in the $\theta, \varphi$ plane, their positions are

$$
\varphi_1 = \Omega_{CM} t + \epsilon_{\varphi} \sin(\omega_0 t), \quad \varphi_2 = \Omega_{CM} t - \epsilon_{\varphi} \sin(\omega_0 t),
$$

$$
\theta_1 = \pi/2 + \epsilon_{\theta} \cos(\omega_0 t), \quad \theta_2 = \pi/2 - \epsilon_{\theta} \cos(\omega_0 t),
$$

where $\epsilon_{\varphi}$ and $\epsilon_{\theta}$ characterize the orbit of the SB. And $\Omega_{CM}$ and $\omega_0$ in Eq. (26) are the angular velocities of CM and SB in the coordinate frame. The coordinate frequency $\omega_0$
is related to the SB proper oscillation frequency \( \omega_0 \) by
\[
\omega_0 = U_{r, CM}^{t} \omega_0,
\]
where \( U_{r, CM}^{t} = (1 - 3M/R)^{-1/2} \) is time-component of the CM four velocity. In the rest frame of the SB, the proper quantity \( \delta R_\varphi = 1/(\omega_0^2)^{2/3} \) is rescaled by \( \delta R_\varphi = \delta R_\varphi / (1 - 2M/\mu) \). We could simply set \( \epsilon_0 \gg \epsilon_\varphi \), or \( \epsilon_\varphi = 0 \) to resemble an elliptic orbit of the SB with a large eccentricity induced by the Kozai-Lidov mechanism [87, 88], and since \( \epsilon_\theta \ll 1 \), we could expand the source term by \( \epsilon_\theta \) and keep to the lowest order. A similar set up could be seen in [83, 89].

**The waveforms in different ECO models.** First of all, in order to sketch out the main feature of the GW waveform reflected by the “would-be” horizon of ECO, we consider a toy model in which the reflectivity is given by
\[
R_{r, t}(\omega) = \epsilon \ e^{-2ib_\omega \omega},
\]
where \( r_* = b_* \) parameterizes the position of the physical surface, and the quantity \( \epsilon \in (0, 1) \) parameterizes the amplitude of reflectivity at the ECO surface. This toy model has a constant reflectivity for different frequencies.

In Fig. 2, we show the GW waveform of the SB emitted in the ECO background for an ECO under this model. The waveform is seen by a distant observer who is setting in the \( \theta = \pi/2 \) plane. From the upper panel of Fig. 2, we see that the GW amplitude of the SB is magnified due to the lensing of the massive body when the SB is moving close to the back of the massive BH/ECO. Compared to the GW emitted by the SB in the BH background, the GW of the SB emitted in the ECO background is enhanced due to the reflection of GWs at the ECO “would-be” horizon. The relative difference of the waveforms in the ECO and BH background become more obvious when the SB is not moving close to the back of the massive body, where no lensing magnification of the amplitude happens while the magnification of the GW amplitude due to the ECO reflectivity dominates. From the Middle and Lower panels of Fig. 2, we see that there are continuous GWs been reflected by the ECO surface, here, we call it “continuous echo” waves, which has a comparable magnitude with the GWs of the SB emitting in a BH background. And since the two echo waves in the middle and lower panels of Fig. 2 are only different by \( b_* \) which characterizes the echo time delay (by \( 2b_* \)) , the only difference of the two echo waveforms is a shifting of a phase of \( \Delta t = 2b_* = 60M \).

For a frequency-dependent reflectivity, such as Lorentzian reflectivity, it has a frequency dependence as follows
\[
R_{r, t}(\omega) = \epsilon \left( \frac{i \Gamma}{\omega + i \Gamma} \right) e^{-2ib_\omega \omega},
\]
where the quantity \( \Gamma \) characterizes a relaxation rate of the ECO surface and imposes a low-pass filtering of waves upon reflection in the frequency domain. For a distant observer, GWs with frequencies \( |\omega| \lesssim \Gamma \) have the highest reflectivity. The GW waveform of the SB emitted in the ECO background with a Lorentzian reflectivity at the ECO “would-be” horizon is shown in Fig. 3. From Fig. 3, we can see that the amplitude of the reflected waves in the Lorentzian reflectivity case is smaller than that in the constant case for the same \( \epsilon \). The reason is that all the GWs of different frequencies are equally reflected by the constant reflectivity, while in the Lorentzian case only the GWs with frequency \( |\omega| < \Gamma \) are efficiently reflected.

**Detectability of the continuous echo.** The GWs emitted by SB studied in this letter could be either LISA or LIGO-Virgo sources when they are moving during inspiral or late inspiral-merger-ringdown phase respectively, and the “continuous echo” waves could be detectable as well. For example, we consider \( \omega_0 = 1/M \), and the GW frequency of SB is typically in LISA band. The detectability of the “continuous echo” waves can be
revealed by comparing the GWs of the SB emitted in an ECO background with those in a black hole background. We denote the former waveform as \( h_1 \), and the later waveform as \( h_2 \). LISA employs a method called the “matched filtering” to discern the difference between two waveforms \([90, 91]\). The inner product of these two waveforms is defined by

\[
\langle h_1 | h_2 \rangle = 2 \int_{0}^{\infty} \frac{\hat{h}_1(f) \hat{h}_2(f) + \hat{h}_1(f) \hat{h}_2^*(f)}{S_h(f)} df, \tag{29}
\]

where \( S_h(f) \) is the spectral noise density \([92]\) and \( \hat{h}_i(f) \) denotes the Fourier transformation

\[
\hat{h}(f) = \int_{-\infty}^{\infty} e^{2\pi i f t} h(t) dt. \tag{30}
\]

These two waveforms are distinguishable if the condition

\[
\langle \delta h | \delta h \rangle = \langle h_1 - h_2 | h_1 - h_2 \rangle > 1 \tag{31}
\]

is satisfied. Or equivalently, the difference between these two waveforms can be described by the “fitting factor” (FF) \([91, 93]\), namely

\[
\text{FF} = \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}, \tag{32}
\]

and then one can compare it with a threshold FFS defined by

\[
\text{FFS} = 1 - \frac{1}{\langle h_1 | h_1 \rangle + \langle h_2 | h_2 \rangle}. \tag{33}
\]

Here FFS is closely related to the signal-to-noise ratio (SNR = \( \sqrt{|h|h} \)). If \( h_1 \approx h_2 \), we have FFS \( \approx 1 - 1/(2 \text{SNR}^2) \), and the criterion given in Eq. (31) is equivalent to FF < FFS. A signal with SNR \( \geq 10 \), or equivalently FFS \( \geq 0.995 \), is needed for claiming a detection by LISA. Therefore a conservative estimation for a distinguishable continuous echo requires FF < 0.995. The fitting factor (FF) vs. the reflective amplitude \( \epsilon \) in the model with a constant reflectivity at the ECO surface is illustrated in Fig. 4 which indicates that the continuous echo is detectable as long as \( \epsilon > 0.17 \).

**Summary.** The SBs could form in the vicinity of a supermassive compact object in galactic nucleus due to abundant astrophysics phenomena (see, e.g., [62–68]), and even merger in the ISCO orbit of the massive object \([68]\). For the SB which orbital radius is much smaller than their CM distance to the centering massive body, which satisfy \( \delta R/R < (m_1 + m_2)^{1/3}/M^{1/3} \), it compose a hierarchical triple system which is stable within a relatively short time (see, e.g. \([94]\) for a long term stability analysis on the relativistic hierarchical triple systems). In this letter, we calculate the GWs from an SB in the vicinity of a massive ECO by solving Teukolsky equation. The reflective boundary condition at the ECO surface causes “continuous echo” waves which last for a relatively long time until the SB merger. These “continuous echo” waves provide an exquisite test for the nature of the ECO “would-be” horizon.

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