ON THE ORBITAL EVOLUTION OF A JOVIAN PLANET EMBEDDED IN A SELF-GRAVITATING DISK

HUI ZHANG,1,2 CHI YUAN,1 D. N. C. LIN,3,4 AND DAVID C. C. YEN1,5

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ABSTRACT

We performed a series of hydrodynamic simulations to investigate the orbital evolution of a Jovian planet embedded in a protostellar disk. In order to take into account of the effect of the disk’s self-gravity, we developed and adopted an ANTARES code which is based on a 2D Godunov scheme to obtain the exact Reimann solution for isothermal or polytropic gas, with nonreflecting boundary conditions. Our simulations indicate that in the study of runaway (type III) migration it is important to carry out a fully self-consistent treatment of the gravitational interaction between the disk and the embedded planet. Through a series of convergence tests, we show that adequate numerical resolution, especially within the planet’s Roche lobe, critically determines the outcome of the simulations. We consider a variety of initial conditions and show that isolated, noneccentric protoplanets do not undergo type III migration. We attribute the difference between our and previous simulations to the contribution of a self-consistent representation of the disk’s self-gravity. Nevertheless, type III migration cannot be completely suppressed, and its onset requires finite-amplitude perturbations such as that induced by planet-planet interaction. We determine the radial extent of type III migration as a function of the disk’s self-gravity.

Subject headings: accretion, accretion disks — hydrodynamics — methods: numerical — planetary systems: formation — planetary systems: protoplanetary disks

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1. INTRODUCTION

More than 200 planets have been discovered around nearby solar-type stars. Their kinematic properties are characterized by diversities in their mass, period, eccentricity, and physical radius. An important dynamical process which may have led to these properties is the migration of protoplanets due to their tidal interaction with their nascent disks (Lin & Papaloizou 1993; Papaloizou & Terquem 2006). The progenitor cores of gas giant planets undergo type I migration due to a torque imbalance between different regions of the disks in which they are embedded (Goldreich & Tremaine 1980; Ward 1984). After they have acquired sufficient mass to open a gap near their orbit, gas giant planets’ orbital evolution is locked to that of the disk gas through a type II migration (Lin & Papaloizou 1986). But under some circumstances the gap may be partially cleared, and the disk gas which leaks through this region can induce the gas giants to undergo runaway (type III) migration (Masset & Papaloizou 2003, hereafter MP03).

All of these processes can relocate protoplanets far from their birthplace. The rapid timescale for type I migration (Ward 1997; Tanaka et al. 2002) poses a challenge to the formation of gas giant planets (Thommes & Murray 2006; Ida & Lin 2008). However, several potential retardation mechanisms have been proposed. They include variation in the surface density and temperature gradient (Masset et al. 2006a), intrinsic turbulence in the disk (Laughlin et al. 2004; Nelson & Papaloizou 2004), and non-linear radiative and hydrodynamic feedbacks (Masset et al. 2006b). If these cores are formed in the turbulence-free dead zone (Gammie 1996), self-induced unstable flow (Koller 2004, p. 789; Li et al. 2005; de Val-Borro et al. 2007) would reduce the efficiency of type I migration by an order of magnitude (I. Dobbs-Dixon et al. 2008, in preparation).

Type II migration has been invoked as a mechanism for the formation of close-in gas giants (Lin et al. 1996). Both 1D (Lin & Papaloizou 1986) and 2D (D’Angelo et al. 2006) simulations have shown that before the disk mass decays to a value comparable to the planet mass, the planet migrates with (unperturbed) disk accretion on a viscous diffusion timescale, and when the disk mass is comparable to the planet mass, only a fraction of the total (viscous plus advective) angular momentum flux transported by the disk gas (which is assumed to be independent of the disk radius) is utilized by the planet in its orbital evolution (Ivanov et al. 1999). The inclusion of type II migration in the planet formation models has yielded a mass-period distribution which is similar to that observed (Ida & Lin 2004a, 2004b, 2005, 2008).

Type III migration is driven by a strong corotation torque very near the planet (Ida et al. 2000; MP03). The timescale for this process is much shorter than that for both type I and type II migration. If it commonly occurs, type III migration would greatly erase any signature in the dynamical structure of planetary systems from the disk initial surface density distribution of their nascent disks. Due to its dramatic effects, type III migration has been extensively studied over the past few years (Ogilvie & Lubow 2006). This process can only be maintained if there is an uninterrupted flow across the planet’s orbit so that saturation of the corotation resonances can be avoided. MP03 show that the radial motion of a rapidly migrating planet can indeed be self-sustained; i.e., its motion leads to a fresh supply to the corotation region which provides a torque to induce further migration. Ida et al. (2000) demonstrated a similar effect for planetary migration...
induced by residual planetesimals. They suggest that the critical condition for the onset of this process is that the mass of the residual planetesimals contained within the feeding zone (with a half-width up to a few times that of the Roche radius) must exceed that of the planet. The results of the hydrodynamic simulations (MP03) also show that a planet would undergoes runaway migration in disk regions with $\dot{m} \geq M_{\text{planet}}$, where $\dot{m}$ is the mass of the disk gas in the planet’s co-orbital region. For a Jupiter-mass planet, this requirement implies a mass ratio between the entire disk and the central star of $\mu \equiv M_\text{d}/M_\ast \gtrsim 0.02$.

However, there is some controversy about type III migration. D’Angelo et al. (2005) claimed that the fast migration obtained by MP03 was a consequence of inadequate resolution within the planet’s Roche lobe. They adopted a nested-grid code to achieve high resolution within the vicinity of the planet. At their highest resolution, they found that the migration rate is much lower than the type I rate but comparable to (a little larger than) the type II rate even in a massive disk.

In a massive disk, the effect of the disk’s self-gravity is important, especially in the determination of the torque applied to the gas in the planet’s co-orbital region by that in other regions of the disk. However, in previous numerical simulations the effect of the disk’s self-gravity has been neglected. In this paper we develop a new numerical scheme which takes the effect of the disk’s self-gravity into account. With this scheme, our main objectives are to examine the conditions under which type III migration is launched and sustained. In § 2 we provide a description of our computational method and model parameters.

The simulations of MP03 showed that type III migration is spontaneously excited in relatively massive disks. We consider two limiting initial conditions. In the first set of simulations, we consider the emergence of an isolated planet on a circular orbit. In the core accretion scenario, the growth of gas giants occurs on timescales much longer than the synodic timescale in most nearby disk regions such that the stream lines can adjust adiabatically. In the second set of simulations, we introduce a set of “dynamically quiescent” initial conditions by gradually increasing the planet’s mass over many orbits; the initial angular velocity of planet is not exact Keplerian (the centrifugal force will balance the gravity from both the center star and the whole gas disk) so that the disk gas can adjust to the tidal potential of the planet and attain a dynamical equilibrium. During this transition stage, the semimajor axis of the planet is artificially held fixed. This “quiet-start” prescription is introduced to minimize the impulse felt by the gas at the onset of the simulation. With this initial condition, we carried out several series of simulations to determine the dependence of the disk flow and the planet’s migration on the numerical resolution and the degree of the disk’s self-gravity. In § 3 we present the results of these simulations.

The quiet-start models provide a test of whether type III migration may be spontaneously launched under optimum conditions as indicated by the numerical simulation by MP03. They also provide a powerful analytic argument to suggest that type III migration can be sustained once it is initiated. Ida et al. (2000) found that this process may need an “initial push” due to some large perturbations. In the second series of simulations, we consider this possibility by introducing an initial jolt as a trigger for type III migration. One potential mechanism for this impulsive perturbation is a close encounter between two protoplanets (Zhou et al. 2005). In § 4 we present the simulated results of the initial-push models. Finally, we summarize our results and discuss their implications in § 5.

2. PHYSICAL AND NUMERICAL MODEL

2.1. Computational Units

For numerical convenience we set the gravitational constant $G = 1$, solar mass $M_\odot = 1$, and the radius of planet’s initial orbit $R_0 = 1$, where $R_0 = 5.2$ AU. The unit of time is $1/2\pi$ of the planet’s initial orbit period $P_0$. Most of our simulations are carried out over a period of $1000P_0$. Since the unit of density $\rho_0$ drops out of the equations of motion, we can normalize it to any specified density.

2.2. Physical Model

Following conventional procedures, we simulate the dynamical response of a gas disk around a star which is located at the origin of the coordinates. We construct a 2D numerical hydrodynamic scheme to solve the continuity and momentum equations, neglecting the effect of any explicit viscosity. We place a protoplanet which is initially embedded in the disk with a circular orbit around the central star. In order to avoid some well-known problems (see below) at the inner boundary (close to the central star), we solve the governing equations in the Cartesian coordinates.

The vertically averaged continuity equation for the disk gas is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_x)}{\partial x} + \frac{\partial (\rho u_y)}{\partial y} = 0.$$  \hspace{1cm} (1)

The equations of motion in the Cartesian coordinates are

$$\frac{\partial (\rho u_x)}{\partial t} + \frac{\partial (\rho u_x^2)}{\partial x} + \frac{\partial (\rho u_x u_y)}{\partial y} = -\frac{\partial P}{\partial x} - \sigma \frac{\partial \Phi}{\partial x},$$  \hspace{1cm} (2)

$$\frac{\partial (\rho u_y)}{\partial t} + \frac{\partial (\rho u_x u_y)}{\partial x} + \frac{\partial (\rho u_y^2)}{\partial y} = -\frac{\partial P}{\partial y} - \sigma \frac{\partial \Phi}{\partial y},$$  \hspace{1cm} (3)

where $P$ is the pressure and $\Phi$ is the gravitational potential of the star-planet-disk system, which includes the softened potential of the central star ($\Phi_c$), the softened potential of the planet ($\Phi_p$), the potential of the disk itself ($\Phi_d$), and the indirect potential ($\Phi$) due to the acceleration of the origin by the planet and the disk. The softened potential of the central star is given by $\Phi_c = -\frac{GM_\odot}{(x^2 + y^2 + \epsilon_\text{star})^{3/2}}$, and the softened potential of planet is $\Phi_p = -\frac{GM_p}{(x-x_p)^2 + (y-y_p)^2 + \epsilon_p^2}^{1/2}$, where $\epsilon_\text{star}$ and $\epsilon_p$ are the softening length to the central star and planet, respectively. In all the models presented here we adopt $\epsilon_p$ as half of the planet’s Roche radius. Figure 1 shows the rotation curves of the disk when we adopt different $\epsilon_\text{star}$ values. During our simulations it is 0.1 units where the initial semimajor axis of the planet is unity (in some other simulations we reduce it to 0.05 and find little difference).

Since it is hard to find the exact Riemann solution for a locally isothermal system, a more idealized and simpler model is adopted in our simulation. We assume the disk gas has an isothermal equation of state. The sound speed is $c_s = (H/r)\sqrt{\rho}$, where we set $H/r = 0.04$ and $\epsilon_k = 1$ (Keplerian velocity at $r = 1$). Since we adopt a constant-density disk, at the beginning of simulations the Toomre parameter $Q \propto r^{-3/2}$, while in a locally isothermal system it should be $Q \propto r^{-2}$. The difference in the initial $Q$ value is not significant at the outer part of the disk (from $r = 1$ to 2.5): in a low-mass disk ($\sigma_0 = 0.0006$) $Q_{\text{loc}} |r=1| = Q_{\text{loc}} |r=1/2| = Q_{\text{loc}} |r=2| = 3$, and in a high-mass disk ($\sigma_0 = 0.0012$) $Q_{\text{loc}} |r=1| = Q_{\text{loc}} |r=1/2| = Q_{\text{loc}} |r=2| = 2.6$, $Q_{\text{loc}} |r=2.5| = 1.8$. We do not add any explicit viscosity in the
In many models, the planet’s orbit undergoes extensive decay. A natural system to solve the governing equations is polar coordinates. However, the axial symmetry of the disk flow is broken by the presence of the planet. The inner boundary conditions can only be an approximate function of the disk radius well inside the orbit of the planet. Although the Fargo prescription provides a solution for this technical challenge over some regions of the simulation. There is, however, some numerical viscosity associated with our computational scheme and grid effects. In our low-resolution cases ($512 \times 512$), extensive tests indicate that the magnitude of the artificial viscosity is equivalent to $\alpha < 10^{-4}$. Due to the absence of viscosity, type II migration will be absent too. In fact, in most of our simulations, after the gap is well cleared the planet will barely migrate unless it undergoes runaway migration. To focus on the migration and reduce variables, we do not allow the planet to accrete gas from the disk.

### 2.3. Numerical Method

The ANTARES code we have developed is adopted in the calculations. It is a 2D Godunov code based on the exact Riemann solution for isothermal or polytropic gas, featured with nonreflecting boundary conditions. The details of this code have been described elsewhere (Yuan & Yen 2005).

Full self-gravity of the disk is calculated by fast Fourier transform (FFT). To avoid the imbalance of gravity at the boundaries we expand our domain to a larger disk, the radius of which is $R = 5$ (see Fig. 2). When in a constant-density disk, the self-gravity potential within this enlarged disk only depends on the distance to the center. Thus, we separate the self-gravity potential into two parts: one is the initial potential which comes from the enlarged disk and will be constant versus time, and the other is the perturbation potential due to the changes of the density field within our computational domain. This part of the self-gravity is calculated by FFT, and we adopt a softened potential kernel instead of a point-point potential to take into account the finite thickness of the disk and the grid size. For the orbit of the planet we adopt RK78 to integrate it.

### 2.4. Computational Mesh Configuration and Domain

In many models, the planet’s orbit undergoes extensive decay. A natural system to solve the governing equations is polar coordinates. However, the axial symmetry of the disk flow is broken by the presence of the planet. The inner boundary conditions can only be an approximate function of the disk radius well inside the orbit of the planet. Although the Fargo prescription provides a solution for this technical challenge over some regions of the disk, it is nonetheless difficult to extend the computational domain to very small $R_{\text{inner}}$ with polar coordinates because the computational time $T_{\text{com}} \sim R_{\text{inner}}^{-3/2}$.

A Cartesian coordinate system introduces some advantages over the polar coordinates. It is easy to achieve high resolution without the bottleneck in the azimuthal direction. It is also straightforward to calculate the self-gravitating effect with FFT. During some of our simulations we adopt a $512 \times 512$ grid, while in some high-resolution cases it increases to $1024 \times 1024$. To carry out the calculation, however, a softening length $\epsilon_{\text{star}}$ is assigned to the central star, where $\epsilon_{\text{star}}$ is 1 order smaller than the length unit. The computational domain is from $-2.5$ to $2.5$ in both the $x$- and the $y$-directions. The primary star locates at the origin, where $x = 0$ and $y = 0$. To avoid the symmetric problem of the four corners we extend the disk to $R = 5$ and assume the area outside the computational square $(xy = 5 \times 5)$ will stay constant (see Fig. 2) during evolution, while the gravity is taken into account all over the area within $R \leq 5$.

#### 2.5. Initial Conditions

At the beginning of evolution, the disk surface density is set to be uniform. We adopt four different initial surface densities: $\sigma_0 = 0.6 \times 10^{-3}$, $0.9 \times 10^{-3}$, $1.2 \times 10^{-3}$, and $1.5 \times 10^{-3}$. The mass of the thinnest disk is about $0.012 \, M_\odot$, which is about the minimum mass of a solar nebula, while the thickest one is about $0.03 \, M_\odot$. The angular velocity of the gas $v_\theta = r \Omega_\theta$ is slightly different from the Keplerian velocity, since the flow is in a centrifugal balance with both the softened gravity of the star and the self-gravity of the disk (when a self-gravitating effect is included) such that $v_\theta = \Omega_\theta \sqrt{\left[\frac{3GM_\odot}{r^2 + \epsilon^2} + r f_{\text{gas}}\right]}^{1/2}$. In disks with an isothermal equation of state and a homogenous surface density distribution, the pressure gradient effect does not contribute to the initial azimuthal speed. The initial radial velocity of gas is set to be 0. These initial disk conditions do not take into account the gravitational perturbation by the planet.
The initial azimuthal velocity of the planet is also set to balance the gravity of the central star and that of the disk. The initial location of planet is at \((x = 1, y = 0)\). In the simplest models, the planet’s initial mass is \(10^{-2} M_J\) (1 \(M_J\)) and is fixed during the evolution. This approach introduces a gravitational impulse which can strongly perturb the stream line, especially near the planet’s orbit.

For quiet-start models, we adjust the initial velocity to set up a dynamical equilibrium in which the planet’s orbit is circular and the stream lines are closed. To do so, we adopt an negligible initial mass for the planet \((3 \times 10^{-7} M_J\), or equivalently \(0.1 M_J\)). We specify the planet’s growth rate to be \(\sim 3\%\) during every orbit period until it grows to \(1 M_J\) within the first 250 fixed circular orbit periods. With this prescription the planet gains mass through adiabatic growth and the disk has enough time to make a smooth response. Before the planet is launched, we further adjusted the azimuthal speed of the planet so that it retains a circular orbit despite the presence of the gas.

2.6. Boundary Conditions

We adopt the nonreflecting boundary condition, which means we do wave decomposition at each boundary and set all the waves that propagate inward to the computational domain to be zero. Waves at the boundaries can only propagate outward. While the boundaries are not closed to mass flow, gas may flow through the boundary freely according to the equation of motion. The area outside the boundary is assumed to be uniform and maintain the initial condition without evolution. The details of this boundary condition have been described elsewhere (Godon 1996).

3. SIMULATIONS OF FLOW WITH QUIET STARTS

A series of hydrodynamic simulations had been performed (see Table 1). At first, we consider a series of simplest models (S1–S4). In these models, the numerical resolution is relatively low (512 \(\times\) 512), and the prescription for self-gravitating and quiet-start effects is not included. We specify the planet’s mass to be \(1 M_J\) and test four disks with different surface density.

From the lowest to the highest surface density \(\alpha_0 = 0.6 \times 10^{-3}, 0.9 \times 10^{-3}, 1.2 \times 10^{-3}, \) and \(1.5 \times 10^{-3}\) (in units of solar mass divided by the square of the planet’s initial orbit radius). The results show that the migration rate is proportional to the surface density of the disk. In the limit in which the disk’s surface density is higher than \(1.2 \times 10^{-3}\), a very rapid migration occurs (see Fig. 3).

This result is in agreement with that obtained by MP03. For the critical model, the disk-to-primary mass ratio \((\pi \sigma R^2 / M_p)\) is a little above 0.01, and the planet-to-primary mass ratio is 0.001. According to the criterion specified by MP03, this set of parameters is at the boundary of the “runaway domain,” so the migration curve shows a critical property. When the surface density becomes higher, the model parameters are totally in the “runaway area” and the migration is much faster. MP03 suggested that this rapid migration is due to the planet’s corotation torque being consistently replenished by the disk gas which flow through the planet’s orbit. In our high-mass disk cases \((\sigma \gg 1.2 \times 10^{-3})\), gas accumulated within the co-orbital zone of the planet is comparable to the planet. We present detailed analysis for the high-resolution models to support this conjecture.

![Fig. 3.—Migration in different disk surface densities. From top to bottom, \(\alpha_0 = 0.6 \times 10^{-3}, 0.9 \times 10^{-3}, 1.2 \times 10^{-3}, \) and \(1.5 \times 10^{-3}\). There are 12 Jupiter masses within the disk when the surface density is \(0.6 \times 10^{-3}\) in our units. The solid lines show the cases without self-gravity, while the dash-dotted lines denote the cases with self-gravity. The critical density to trigger runaway migration becomes higher in a self-gravitating disk.](image-url)
The above simulations show that the torque density is most intense near the orbit of the protoplanet, especially within its Roche radius. With the adopted mass ratio, the planet’s Roche radius is about 0.069 in our dimensionless unit. For the relatively low resolution (512 × 512) models, there are only seven grids within the planet’s Roche radius. Torques associated with large mass resonances cannot be well resolved in this limited resolution, and the underresolved torques may lead to unreasonable effect.

In principle, the nearly symmetric flow pattern within the Roche radius is expected to lead to a large cancellation of the net torque applied by the circumplanetary disk on the planet at its center. D’Angelo et al. (2005) have shown that the direct consequence of inadequate numerical resolution is that the gas accumulated in the Roche lobe will generate large artificial fluctuations in the magnitude of the torque applied to the planet which cannot be easily canceled. When the disk is massive this artificial effect becomes more pronounced, since the gas accumulated within the planet’s vicinity is comparable to the planet. They concluded that the migration rate will slow down and become comparable to (a little larger than) the type II migration when high resolution is adopted. The disk in our simulations is more massive: the thinnest disk has 0.012 solar masses inside 12 AU, which is 2 times higher than the low-mass disk in D’Angelo’s simulation. And to achieve the convergence of our simulations, as well as high resolution in the full co-orbital range (instead of the vicinity of planet), we also test some high-resolution cases. In Figure 4 we show the ratio ($\Gamma$) of the tidal torque (on the planet) by the gas within the planet’s Roche lobe to the tidal torque (on the planet) by the gas within the entire disk. The rapid oscillation of this ratio is clearly shown. This inadequate resolution of the flow pattern introduces an inconsistency in which the planet and the gas flow within its Roche lobe are dragged along by each other.

The issue with resolving the flow within the Roche lobe is clearly illustrated by the sensitive dependence of the migration rate on the softening length for the planet’s potential (Nelson & Benz 2003a, 2003b; Cresswell & Nelson 2006). In the low-mass limit where the Roche radius is smaller, the lack of resolution introduces even more severe problems for both type I and type III migration because they are strongly determined by the flow close to the planet. Inadequate resolution is less serious for type II migration, in which case a gap is clearly formed and the gas in the planet’s Roche lobe is depleted. Nevertheless, inadequate resolution may also lead to artificial diffusion of gas into the gap. Figure 4 shows a large fluctuation in the magnitude of $\Gamma$ after the gap has formed (at $\sim 100 P_\odot$) in the low-resolution models. There is gas flow across the planet’s Roche lobe where torque imbalance is amplified by the coarsely resolved mesh. Leakage of fresh gas into the planet’s co-orbital region can also sustain a gradient in the potential vorticity and suppression of corotation saturation (Masset et al. 2006b) which may reduce the efficiency of type II migration from disk gas accretion (Crida & Morbidelli 2007). The corotation torque scales with the gradient of the potential vorticity (Goldreich & Tremaine 1979; Ward 1991, 1992):

$$\Gamma_c \propto \frac{d \log (\sigma/B)}{d \log r}.$$  \hspace{1cm} (4)

For a sufficiently smooth, monotonic transition of surface density from $\sigma_1$ to $\sigma_\sigma$, the vorticity logarithmic gradient is therefore (Masset et al. 2006a)

$$\frac{d \log (\sigma/B)}{d \log r} = \frac{r}{\lambda} \log \left( \frac{\sigma}{\sigma_1} \left( a + b \frac{H^2}{r^2} \right) \right),$$  \hspace{1cm} (5)

where $a$ and $b$ are constants of numerical functions of $r$ of unity that depend on the shape of the surface density profile; $H$ is the scale height of the disk and $\lambda \ll r$ is the length scale of the density transition, both of which are constant too.

3.2. Convergence Tests

In order to highlight the problems introduced by inadequate numerical resolution, we carry out several high-resolution (1024 × 1024) simulations. Figure 4 (top) shows that the fluctuation in the magnitude of $\Gamma$ declined greatly after the gap has formed (at $\sim 100 P_\odot$) in the high-resolution models. A combination of the reduction of artificial numerical viscosity and adequately resolved torque greatly reduces the artificial torque imbalance on the planet due to the gas within the Roche lobe (also see D’Angelo et al. 2005). Consequently, the planet’s migration is also significantly reduced. This is in agreement with the conclusion of D’Angelo et al. (2005).

Figure 5 shows the planet’s orbital evolution for simulations with different resolutions. In two separate sets of initial surface densities, other than a modest initial radial decay during the epoch of gap formation, type III (runaway) migration is essentially eliminated. The dichotomy between these models (S3 and H2) is particularly dramatic for the models with sufficiently high disk mass that the type III migration is spontaneously launched in the low-resolution simulations.

Another useful diagnostic is the surface density distribution. In Figures 6 and 7 we show the density evolution of models S3 and H2. Figure 7 clearly shows the presence of gas in the “horseshoe” region and an anaxisymmetric structure at the edge of the gap. The high-resolution simulation in Figure 7 shows a sharper disk edge and more clearly defined wave pattern than...
3.3. Self-Gravity

The results in the previous subsection illustrate the contribution to the torque applied on the planet by the gas within the Roche lobe. Although the amount of material in this region is small compared with the mass of the planet (in the high-mass disk cases, the mass accumulated in the planet’s Roche lobe is about 0.2–0.5 \( M_J \)), the gravity between it and the planet is strong due to their proximity. In fact, a fraction of this gas actually resides in a disk around and is gravitationally attached to the planet. As we have indicated above, the net torque on the planet by the gas in the protoplanetary disk is expected to be mostly canceled, and its mass should share the torque from the rest of the disk with the planet.

In most of the published numerical simulations, the effect of the disk’s self-gravity is not included. This approximation introduces an inconsistent gravitational field felt by the planet and by the gas which shares its orbit. In this section we consider the effect of self-gravity in models SG1–SG4 and QG1–QG4. Due to limited computational resources, these models are simulated with low resolutions (512 \( \times \) 512). Although inadequate resolution continues to plague the proper determination of the tidal torque, we use these models to demonstrate that a self-consistent treatment of the disk’s self-gravity couples the flow within the planet’s Roche lobe to it and reduces the rate of type III migration.

Results in Figure 3 show that the migration slows down slightly (a few percent) in a low-\( \sigma \) model (SG1 and SG2) when a self-gravitating effect is included (similar results are obtained by Nelson & Benz 2003a, 2003b). However, in the high-\( \sigma \) models SG3 and SG4, the difference brought by the self-consistent...
treatment of the disk’s self-gravity is much more pronounced. At $10^3 P_0$, the migration in case SG3 slows down by almost 50% relative to that in model S3 where the effect of the disk’s self-gravity is neglected. And more importantly, runaway migration does not occur with sufficiently high values of $\sigma$ which did lead to runaway migration in a non-self-gravitating disk.

We note that, in all models with identical $\sigma$ distribution, the planet has the same orbital decay rate regardless of whether the effect of self-gravity is included. This similarity is probably due to the slightly artificial impulse initial conditions adopted here. In these series of simulations, gas in the disk is forced to respond to the planet’s gravity for the first time at the onset of the computation. This initial impulse leads to a large potential vorticity gradient, which ensures a strong contribution from the corotation resonances. Under some circumstances, the planet migrates inward by a sufficiently large increment, and a fresh supply of disk gas with new values of potential vorticity is brought into the co-orbital region of the planet such that further migration is promoted. For most disks, however, the replenishment of fresh disk material is inadequate to self-sustain runaway migration.

The difference between these two series of models becomes more pronounced after gap formation. In this limit, replenishment of the co-orbital region is quenched. As gas librates on horseshoe orbits, any initial potential vorticity gradient is erased such that the contribution from the corotation resonance becomes saturated and weakened.

In Figure 8 we plot the $\sigma$ distribution for the self-gravitating model SG3. In this model, the clearing of the gap strongly enhances the effect of self-gravity near the boundary of the gap. Although the $Q$ value of the disk near the planet’s orbit is initially $Q \gg 1$, the clearing and accumulation of gas beyond the gap leads to a local $Q \sim 2$ near the outer edge of the gap. With such a low $Q$ value at a relatively sharp disk edge, unaxisymmetric gravitational instability (Papaloizou & Lin 1989) and shearing (Balmforth & Korycansky 2001; Li et al. 2005; de Val-Borro et al. 2007) may be excited. These instabilities can significantly reduce the migration speed (Koller et al. 2003; I. Dobbs-Dixon et al. 2008, in preparation).

Finally, the treatment of self-gravity closely ties together the planet and the gas within its Roche lobe. With a self-consistent treatment of the gas’s self-gravity, the interaction between the planet and a significant fraction of the gas within its Roche lobe becomes a binding rather than a dispersive force. The self-gravity of the gas beyond the gap region can only start to dominate the torque after the flow has established an equilibrium pattern such that the gas in the co-orbital region migrates together with the planet as integral parts.

4. ISOLATED PROTOPLANET VERSUS PERTURBED SYSTEM

The results in Figure 3 suggest a transition in the protoplanet-disk tidal interaction from being dominated by corotation resonances to their saturation as gas beyond the co-orbital region is being cleared out. We consider two limiting possibilities: a “quiet” and an “impulsive” start.

4.1. Quiet Start

According to the core-accretion scenario, the most favorable location for the first generation of gas giants to form is near the
snow line (Ida & Lin 2004a). Since the growth timescales for their progenitor cores are sensitively dependent on their disk environment and their gas accretion is a runaway process, the first gas giants are likely to form in isolation over many dynamical timescales (Pollack et al. 1996). In principle, the disk can adjust adiabatically to the perturbation due to the emergence of the gas giants. This expectation provides the rationale for a set of simulations with a “quiet start” (for a description of the quiet-start prescription, see § 2).

We adopt a quiet start in eight models (see Table 1): four of them include the effect of self-gravity, while the others do not. With a quiet start, a clear gap is formed near the planet’s orbit such that the corotation resonance is saturated from the onset. Since the disk is able to establish a dynamical equilibrium through adiabatic adjustments, the impulsive perturbation at the epoch of the planet’s release is minimized. In all cases, the migration rate is greatly suppressed and type III migration is halted. This result is consistent with the MP03 conjecture that unsaturated corotation resonances are responsible for inducing the type III migration (see Fig. 9).

The introduction of the quiet-start algorithm reduces the differential motion between the protoplanet and the disk gas at this proximity and enhances the gravitational interaction between them. Consequently, the effect of self-gravity becomes more pronounced earlier than it does in models without the quiet start (see Fig. 10). With a quiet start and self-gravity of the disk, we minimize inconsistencies of the numerical simulations for planets that grow adiabatically in isolation. In all models with this combination, runaway migration is suppressed.

4.2. Impulsive Initial Perturbation

Although, runaway migration is unlikely to be initiated spontaneously, it can nevertheless be self-sustained by mobile planets. Following the framework in MP03’s analysis, let us suppose the planet has already acquired some initial velocity and is moving inward relative to the disk gas. The “first move” can be the result of close encounters between two gas giants or a strong dynamical perturbation by some external stellar perturbation.

![Fig. 8.—Density evolution of SG3. The resolution is 512 × 512, and a self-gravitating effect is included. From top left to bottom right, the evolution time is 50$P_0$, 100$P_0$, 200$P_0$, and 500$P_0$. The color scale is the same as in Fig. 6. Note that the waves at the outer part of the disk become disorderly; this is due to the effect of self-gravity.](image1)

![Fig. 9.—Migration with a quiet start. From top to bottom, the curves represent models Q1, Q3, S1, and S3. The first 250$P_0$ is the quiet-start stage when the planet’s orbit is fixed and the planet grows gently.](image2)
In order to consider such a possibility, we simulated five additional models with the disk and planetary parameters of model Q3. At the end stage of Q3, a Jupiter-mass planet (A) is centered in a severely depleted gap and its migration is essentially halted; i.e., the planet-disk system has established an equilibrium structure. At this instant in time, we assume there is another planet (B) which enters into a close encounter with planet A with an impact parameter $0.5R_{\text{Roche}}$ (where $R_{\text{Roche}}$ is the Roche radius of planet A). The duration of the close encounter is brief ($\approx P_0/20$). In the five test models (IQ1–IQ5), we adopt five different masses of planet B (see Table 2). The results are shown in Figure 11.

The simulated results of models IQ1–IQ3 indicate that planet A is not significantly perturbed by close encounters with a much less massive planet B. In each of these models, planet A moves inward slightly and then regains dynamical equilibrium and its migration is halted. In model IQ4, planet B is sufficiently massive to induce planet A to undergo a 10% decay in its semimajor axis (see Fig. 11). But planet A manages to open a sufficiently clear gap that the corotation resonance become saturated. Thereafter, runaway migration is also halted in model IQ4, but after migration over a substantial radial extent.

Shortly after the scattering event, planet A's eccentricity acquires a finite amplitude (Fig. 12). In models IQ1–IQ3, planet A acquires eccentricities smaller than the ratio of the Roche radius to its semimajor axis. Thus, planet A avoids direct contact with a substantial amount of disk gas. In model IQ4, planet A's modest eccentricity shortly after the perturbation results in its periodic excursion into the disk region beyond the gap. In principle, the disk gas is periodically fed to the planet and the condition for runaway migration is satisfied. (Although the corotation and Lindblad resonances provide modest flux of angular momentum transport per synodic period, these contributions accumulate over time.) However, the tidal interaction between the disk gas and the planet through the corotation resonance also leads to intense eccentricity damping (Goldreich & Tremaine 1980; Goldreich et al. 2004) on a timescale comparable to or shorter than the runaway migration timescale (with the possible exception of model IQ5, in which the planet's runaway migration is launched).

Once the planet's eccentricity is suppressed, gas flow through the co-orbital region, especially through the planet’s Roche lobe, can only be self-sustained with a sufficiently large radial velocity. Our models show that finite-amplitude perturbation excites a positive feedback: (1) the planet's migration inward leads to disk gas flowing past it, then (2) corotation resonance takes away the planet's angular momentum and induces it to further migrate inwards. The comparison between models IQ3–IQ5 shows that the recoil speed of planet A increases with the mass of planet B. The critical amplitude of the perturbation needed to launch this

| Table 2: Simulations of Impact |
|-------------------------------|
| **Case** | **Encounter Planet Mass ($M_{\text{Earth}}$)** | **Runaway Migration** |
|---------|-----------------|-----------------|
| IQ1     | 5               | No              |
| IQ2     | 15              | No              |
| IQ3     | 50              | No              |
| IQ4     | 100             | Critical        |
| IQ5     | 300             | Yes             |

Fig. 10.—Migration in a self-gravitating disk. From top to bottom, the three dashed lines are for QS1, QS3, and QS4, and the three solid lines are for Q1, Q3, and Q4.

Fig. 11.—Migration after impact, showing the migration curves after close encounters (IQ1–IQ5). Runaway migration occurs when planet B is comparable to A.

Fig. 12.—Eccentricity evolution after impact, showing the eccentricity evolution after close encounters (IQ1–IQ5). The critical mass of planet B which will break the steady state of planet A is of Saturn's mass, and runaway migration occurs when planet B is comparable to A. [See the electronic edition of the Journal for a color version of this figure.]
self-sustained migration is that the supply into the co-orbital region, during the horseshoe orbits’ libration timescale, must be comparable to or larger than the mass of the planet (MP03).

In model IQ5, the planet undergoes runaway migration over an extensive radial distance. In the co-moving frame of the planet, the disk gas travels from the inner to the outer regions of the disk and removes angular momentum from it through the corotation resonance (or, equivalently, the gas is being scattered to large distances by the planet). The disk-planet interaction is greatly amplified during the passage of the gas through the planet’s Roche lobe where the imbalance of the torque is a direct consequence of a self-sustained potential vorticity gradient. In comparison to model Q3, we find that the disk’s self-gravity, as a global and indirect effect, can only overwhelm the local flow through the planet’s Roche lobe when its vicinity is severely depleted and the impact of the initial impulse has decayed or is suppressed.

We interpret the results in models IQ3–IQ5 in terms of torque due to the corotation resonance from the gas inside the planet’s radius. In the discussion on models S3 and H2, we have already indicated that inadequate resolution can introduce a spurious torque imbalance which may drive type III migration. But we also showed in models SG3 and Q3 that both self-gravity and a quiet start can suppress type III migration even when the flow is simulated with inadequate resolution. In these cases, the torque imbalance may be further reduced in simulations with a more refined numerical resolution. In comparison with the results of models IQ1–IQ3, the launch of type III migration in models IQ4 and IQ5 is due to a physical effect rather than a numerical flaw. In these models, the planet's orbital evolution is dominated by the local torque (including those that have not been well resolved) when there is sufficient mass near it. Figure 13 shows the surface density distribution within planet A's vicinity after the scattering event. Panels a, b, c, and d correspond to cases IQ1, IQ3, IQ4, and IQ5 respectively.

Panel a.—For a small perturbation, the density profile within the vicinity of the planet does not change much and planet A retains its equilibrium (IQ1 and IQ2).

Panel b.—Gas starts to flood into planet A's vicinity (gray lines) shortly after a relatively large perturbation and evokes a potential vorticity gradient. However, the gas depletes soon after 25P0 (light green lines), so there is no runaway migration (IQ3).

Panel c.—In IQ4 gas takes about 400P0 to deplete (red lines), and the corotation resonance which associates with the vorticity gradient was suppressed then. Planet A had undergone fast migration through an extensive radial region; however, the migration is suppressed finally since it cannot self-sustain the vorticity gradient.

Panel d.—A large radial perturbation allows planet A to self-sustain the vorticity gradient in its vicinity, so it keeps losing angular momentum through corotation resonance and undergoes a runaway migration.

5. SUMMARY AND DISCUSSIONS

In this paper we are motivated to consider the origin and evolution of type III (runaway) migration. This process is thought to be important for Saturn-mass planets which may have formed...
in disks more massive than the minimum-mass nebula. We are particularly interested in two issues: (1) whether this process can occur spontaneously for isolated gas giants which formed through gas accretion onto solid cores on timescales much longer than the dynamical timescale in the disk; and (2) whether type III migration can be self-sustained by a planet which is strongly perturbed by a close encounter with another planet.

We performed a series of numerical simulations to investigate the orbit evolution of an embedded planet. In models S1–S4, which have the simplest settings, a Jupiter-mass planet is inserted into an isothermal disk with a constant surface density. In all cases, migration occurs with a speed which is an increasing function of the disk mass (when the gap is cleared the planet will barely migrate, since we do not adopt any artificial viscosity). For disks with more than twice the mass of the minimum-mass nebula model, the planet undergoes type III migration. In this high-mass disk, mass that accumulates within the planet’s co-orbital zone is comparable to that of the planet. Since we hold the planet at the beginning of evolution in quiet-start cases, when we release the planet the co-orbital zone is quite clean. So type III migration never happens in quiet-start cases (Q1–Q4 and QG1–QG4). However, when we include an initial push to the planet after it reaches a steady state (IQ1–IQ5), in some cases the mass within the planet’s co-orbital zone increases and becomes comparable to that of the planet (IQ4–IQ5), and type III migration begins.

Although these results are intriguing, we identify three technical issues which may have led to an artificial outcome for the numerical simulations. In an attempt to carry out a convergence analysis, we note that our low-resolution models (comparable in resolution to most existing simulations) contain only $14 \times 14$ mesh grids across the planet’s Roche lobe. Insufficient numerical resolution introduces an artificial torque imbalance, especially for the gas flow within the planet’s Roche lobe. When this region is intruded by a disk flow, unphysical net torque is generated spuriously. This torque induces the planet to rapidly migrate. When the flow through the identical disks is simulated with twice the resolution in each direction, the torque imbalance of the flows through the Roche lobe and the rate of planetary migration are greatly reduced. These studies suggest that under-resolved simulations may lead to spurious runaway migrations. This agrees with the results and conclusion of D’Angelo et al. (2005). They found that the torque imbalance within the Roche lobe and the large migration rate decrease substantially with higher numerical resolution in their non-self-gravitating cases. In the work presented here, we cannot yet demonstrate that we have reached numerical convergence. It will require more powerful computational algorithms and tools to determine the conditions for adequate resolution.

The second technical problem which plagues many existing numerical simulations is an inconsistent treatment of the disk’s self-gravity. In the computation of the force acting on the planet, both the axisymmetric and the nonaxisymmetric components of the gravity from the disk gas are applied to the planet, along with the host star’s gravity. But in the evaluation of the equation of motion of the disk gas, the disk’s own contribution to the gravity is not included. In low-mass disks, the discrepancy introduced by this approximation is negligible. But in disks more massive than the minimum-mass nebula, this inconsistency can lead to a differential motion between the planet and the disk gas near its orbit. The absence of the disk’s self-gravity also modifies the effect of angular momentum transport across both the corotation and Lindblad resonances (Goldreich & Tremaine 1982). A comparison between models S3 and SG3 shows that the extent of radial migration after a time span of $10^5 P_0$ would be reduced by a factor of 2 if the effect of the self-gravity of the disk is included. Nevertheless, runaway migration is not totally suppressed by the disk’s self-gravity. Type III migration is spontaneously launched in model SG4, which is also gravitationally unstable—at the outer part of the disk $Q \sim 1$. But in general, a self-consistent treatment of the disk’s self-gravity can significantly slow down the type III migration rate.

The last technical issue we have considered is the artificial initial conditions. In the standard models S1–S4, the initial motion of the disk gas is set up for Keplerian velocities so that it is in a centrifugal balance with the host star’s gravity. At the onset of the simulation, the introduction of the planet’s gravity induces a strong perturbation to the flow pattern. Consequently, the disk gas can easily enter into the Roche lobe of the planet and intensely exchange angular momentum with the planet. Since the initial potential vorticity gradient is preserved, corotation resonance is artificially intensified at the onset of the simulation (see models SG1–SG4). In the formation of the first-generation, isolated gas giants, the planet’s mass grows over many dynamical timescales. The dynamics and structure of the disk flow adjacent to the planet adjust adiabatically through gap formation and modification of stream lines. In an attempt to simulate this gradual process, we carried out two series of models (Q1–Q4) with a “quiet-start” initial condition. In these models, the mass of the planet is increased gradually over $250 P_0$. In the first series, models Q1–Q4, we neglect the effect of the disk’s self-gravity, whereas in the second series, models QG1–QG4, the effect of the disk’s self-gravity is fully implemented. In both sets of simulations, a quiet start greatly suppress the corotation resonance and hence the planet’s type III migration. In all cases, (including the models for a very massive disk Q4 and QG4), type III migration ceased.

While the quiet-start initial condition is justified for first-generation, isolated planets, it is not appropriate for strongly perturbed planets. A large fraction of all known gas giants reside in multiple planet systems. Indeed, the formation of first-generation planets promotes the build-up of the cores and the formation of gas giant planets beyond the outer edge of the gap around them (Bryden et al. 2000). Despite the gaseous background, if these planets are formed with an initial separation less than about 3 times the sum of their Roche radius, dynamical instabilities can induce them to undergo close encounters well before the gas is depleted (Zhou et al. 2007). In order to investigate these perturbation in the presence of the disk gas, we simulated models IQ1–IQ5. Our results show that the runaway migration of a Jupiter-mass planet can be triggered by its close encounters with another massive planet. Immediately after the encounter, the eccentricity of the planet is excited such that it can undergo radial excursion beyond the edge of the gap (during several orbits after the encounter, and the duration depends on the intensity of the perturbation). Although this motion enables the disk gas to venture into the planet’s Roche lobe, their interaction through the corotation resonances damps the eccentricity faster than it directly induces type III migration. However, the initial impulse may be self-sustained by the modest radial motion of the planet. In the co-moving frame of the planet, the disk gas moves across its orbit, sustains a potential vorticity gradient, and induces the planet to undergo type III migration over large radial distances. D’Angelo et al. (2006) found that a massive planet will undergo outward migration when its eccentricity becomes sufficiently large, $e \gtrsim 0.2$, while in our simulations the planet will undergo inward migration right after the close encounter. This is probably due to our encounter set-up conditions. Our point is
whether type III migration can arise and be self-sustained by a planet which is strongly perturbed. The perturbation may come from any conditions, such as a close encounter with another planet. So we just add a negative torque to the planet, where the torque is impulsive and as if another planet had traveled through the former one’s Roche lobe and then was scattered away. It is equivalent to giving the planet a initial inward push. As we see in IQ1–IQ5, the eccentricities are damped to smaller than 0.05 very quickly and the migration never inverse. This is not contrary to the conclusions of D’Angelo et al. (2006).

In an attempt to account for the wide eccentricity distribution among the extrasolar planets, several authors have consider the possibility of dynamical instability in multiple-planet systems (Papaloizou & Terquem 2001; Juric & Tremaine 2007; Chatterjee et al. 2007; Zhou et al. 2007). The timescale for the onset of dynamical instability is a rapidly increasing function of the planet’s separation. In many simulations, compact systems of planets are imposed initially such that they become dynamically unstable on a timescale much shorter than both the growth timescale for gas giant planets and the gas depletion timescale (a few Myr) in their nascent disks. For example, Juric & Tremaine (2007) present several models to show that dynamical relaxation in a gas-free environment can induce the median separation between gas giant planets to increase from <5 to >12 within $10^5$–$10^6$ yr. The results presented here suggest that close encounters triggered by dynamical instabilities, if occurring frequently prior to the depletion of the disk gas, would launch proto–gas giant planets on type III migration toward either their host stars or the outer edge of their nascent disks. Either outcome may not be compatible with the observed mass-period distribution of extrasolar planets. An alternative scenario is that the formation of first-generation gas giants strongly modified their neighborhood by opening up planetesimal gaps at several Hill radii from themselves (Zhou & Lin 2007), as well as wide gas gaps (Bryden et al. 2000). With moderately large separations, the growth timescale for dynamical instabilities may be lengthened by 1 or more orders of magnitude. Provided the dynamical instability leads to close encounters between gas giants after the depletion of the disk gas, it is possible for most of them to remain in the proximity of their birthplace. Quantitative verification of this conjecture will be presented elsewhere.

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