Fluctuations in an Ideal Modified Bosonic Gas Trapped in an Arbitrary 3–dim Power–Law Potential

Elías Castellanos∗ and Claus Lämmerzahl†
ZARM, Universität Bremen, Am Fallturm, 28359 Bremen, Germany

We analyze the number of particle fluctuations within the semiclassical approximation of an ideal bosonic gas with an anomalous single–particle dispersion relation suggested in several quantum–gravity approaches trapped in a generic 3–dimensional power–law potential as a criterium of thermodynamic stability for these systems. We show that the analysis of the fluctuations in the number of particles caused by a deformation in the dispersion relation, leads to an observable consequence associated to the thermodynamic stability for a specific choice of the trap parameters. Additionally, we deduce the shift in the critical temperature in the thermodynamic limit associated with this “modified bosonic gas” and show that this shift expressed in function of the number of particles, can be used as an amplifier for some quantum gravity manifestations for stable systems.

I. INTRODUCTION

The search for small manifestations of quantum gravity or “quantum gravity phenomenology” in our low energy world is a very controversial topic in modern physics. In some schemes, the possibility that the space-time could be quantized, can be characterized, from a phenomenological point of view as a modification in the dispersion relation of microscopic particles [1–3]. A modified dispersion relation emerges as an adequate tool in the search for phenomenological consequences caused by this type of quantum gravity models. Nevertheless, the principal problem in the search of manifestations of quantum gravity is the smallness in the predicted effects [3]. If this kind of deformations are characterized by some Planck scale, then the quantum gravity effects become very small [1, 2], but in principle, different from zero. In the non–relativistic limit, the dispersion relation can be expressed as follows [2]

\[ E \approx m + \frac{p^2}{2m} + \frac{1}{2M_p}\left(\xi_1 m p + \xi_2 p^2 + \xi_3 \frac{p^2}{m}\right), \]  

(1)

Equation (1) is expressed in units with the speed of light \( c = 1 \), and \( M_p \approx 1.2 \times 10^{28} \text{eV} \) is the Planck mass. The three parameters \( \xi_1, \xi_2, \) and \( \xi_3 \), depend of the quantum gravity model in question [1, 2], and should take positive or negative values close to 1. Equation (1), is the starting point in the search for small manifestations of quantum gravity at low energies.

The Bose–Einstein condensation phenomenon, from the theoretical and experimental point of view has produced an enormous amount of publications associated to this topic [4–20]. Among the issues addressed we may find, its possible use as tools in the search of quantum–gravity manifestations [2, 21–25]. For this propose, let us define the next “modified Hamiltonian”

\[ H = \frac{p^2}{2m} + \alpha p + U(\vec{r}). \]  

(2)

Where \( p \) is the momentum, \( m \) is the mass of the particle, and the term \( \alpha p \), with \( \alpha = \xi_1 \frac{m}{2M_p} c \) in ordinary units, is the leading order modification in expression (1), being \( c \) the speed of light.

The potential term

\[ U(\vec{r}) = \sum_{i=1}^{d} A_i \left| \frac{r_i}{a_i} \right|^s, \]  

(3)

*Electronic address: elias@zarm.uni-bremen.de
†Electronic address: laemmerzahl@zarm.uni-bremen.de
in the Hamiltonian (2) is the so–called generic 3–dimensional power–law potential, where $A_i$ and $a_i$ are energy and length scales associated to the trap [13]. On the other hand, $r_i$ are the $d$ radial coordinates in the $n_i$–dimensional subspace of the 3–dimensional space. The sub–dimensions $n_i$ satisfy the following expression in three spatial dimensions

$$\sum_{i=1}^{d} n_i = 3. \quad (4)$$

If in equation (4) $d = 3$, $n_1 = n_2 = n_3 = 1$, then the potential becomes in the so–called Cartesian trap. If $d = 2$, $n_1 = 2$ and $n_2 = 1$, then we obtain the cylindrical trap. If $d = 1$, $n_1 = 3$, then we have the spherical trap. On the other hand, if $s_i \to \infty$, we have a free gas in a box. In this sense the potential included in the Hamiltonian (2) is quite general. Different combinations of these parameters give different classes of potentials, according to (3).

The study of fluctuations in the number of particles in the Bose–Einstein condensation phenomenon is a very important topic [15–20]. First of all, the fluctuations in the number of particles are directly related with the equivalence or non–equivalence of the statistical ensembles which is a non–trivial and deep topic [26]. On the other hand, the fluctuations in the number of particles are directly related with the thermodynamic stability of the system in question [18, 19]. The fluctuations in a Bose–Einstein condensate depend strongly on the single–particle energy spectrum of the trap [20] and, consequently, the modification in the dispersion relation (1) shall affect the thermodynamic stability of the condensate, and could lead to an observable manifestation with an appropriate manipulation of the trap parameters, defined in the generic potential (3).

In this work, we analyze the Bose–Einstein condensation phenomenon within the formalism of the semiclassical approximation, assuming as a fundamental fact a deformation in the dispersion relation. Afterwards, we analyze the effects of this assumption upon the thermodynamics of the considered bosonic gas, explicitly, over the critical temperature. Additionally, we study the thermodynamic stability of these systems through the fluctuations caused by the deformed dispersion relation (1). Finally, we give the main results and adds some comments about the possible detection of some manifestations of quantum–gravity using different kinds of potentials.

II. SEMICLASSICAL APPROXIMATION FOR AN IDEAL MODIFIED BOSONIC GAS

The first part of the present work addresses the issue of the effects of a deformed dispersion relation over the condensation temperature for an ideal bosonic gas, trapped by the generic potential given by expression (3) in the semiclassical approximation [4, 27].

The semiclassical energy associated with the modified Hamiltonian (2) is given by

$$\epsilon(\vec{r}, \vec{p}) = \frac{p^2}{2m} + \alpha p + U(\vec{r}). \quad (5)$$

In the semiclassical approximation, the single–particle phase–space distribution may be written as [4, 27]

$$n(\vec{r}, \vec{p}) = \frac{1}{e^{\beta(\epsilon(\vec{r}, \vec{p}) - \mu)} - 1}. \quad (6)$$

Where $\beta = 1/\kappa T$, being $\kappa$ the Boltzmann constant and $T$ the temperature. Additionally, $\mu$ is the chemical potential.

The number of particles in the 3–dimensional space obey the normalization condition [4, 27],

$$N = \frac{1}{(2\pi \hbar)^3} \int d^3 \vec{r} d^3 \vec{p} n(\vec{r}, \vec{p}) = \int d^3 \vec{r} n(\vec{r}) = \int d^3 \vec{p} n(\vec{p}), \quad (7)$$

where

$$n(\vec{r}) = \int d^3 \vec{p} n(\vec{r}, \vec{p}), \quad (8)$$

$$n(\vec{p}) = \int d^3 \vec{r} n(\vec{r}, \vec{p}), \quad (9)$$
are the spatial and momentum densities, respectively \cite{4,27}. Using the expression \cite{5}, and integrating expression \cite{3} over the momentum space, using expression \cite{8}, allows us to obtain the spatial distribution associated with the modified semi–classical spectrum \cite{4,27}.

\[
n(\vec{r}) = \lambda^{-3} g_{3/2} \left( e^{\beta (\mu_{eff} - U(\vec{r}))} \right) - \alpha \lambda^{-2} \left( \frac{m}{\pi \hbar} \right) g_1 \left( e^{\beta (\mu_{eff} - U(\vec{r}))} \right) + \alpha^2 \lambda^{-1} \left( \frac{m^2}{2 \pi \hbar^2} \right) g_{1/2} \left( e^{\beta (\mu_{eff} - U(\vec{r}))} \right),
\]

where

\[
\lambda = \left( \frac{2 \pi \hbar^2}{m \kappa T} \right)^{1/2}, \tag{11}
\]

is the de Broglie thermal wavelength, \( \mu_{eff} = \mu + m \alpha^2 / 2 \), and \( g_\nu(z) \) is the Bose–Einstein function defined by \cite{26}.

\[
g_\nu(z) = \frac{1}{\Gamma(\nu)} \int_0^\infty x^{\nu-1} e^{\frac{x}{z} e^{-x} - 1}. \tag{12}
\]

Where \( z \) is the so–called fugacity, which is related to the chemical potential through \( z = e^{\beta \mu} \) \cite{26,27}. The Bose–Einstein function \cite{12} diverges for \( z = 1 \) when \( \nu \leq 1 \) \cite{26}.

If we set \( \alpha = 0 \) in equation \cite{10} we recover the usual result for the spatial density in the semiclassical approximation \cite{4,27}.

\[
n(\vec{r}) = \lambda^{-3} g_{3/2} \left( e^{\beta (\mu - U(\vec{r}))} \right). \tag{13}
\]

Integrating the normalization condition \cite{4}, using expression \cite{10}, with the corresponding potential \cite{3}, allows us to obtain an expression for the number of particles in function of the chemical potential \( \mu \), the temperature \( T \), and the deformation parameter \( \alpha \), given by

\[
N = N_0 + C \prod_{l=1}^d A_l^{-\frac{n_l}{s_l}} a_s A \Gamma \left( \frac{n_l}{s_l} + 1 \right) \left[ \frac{m}{2 \pi \hbar^2} \right]^{3/2} g_\gamma(z_{eff})(\kappa T)^\gamma - \alpha \left( \frac{m^2}{2 \pi \hbar^2} \right) g_{\gamma - 1/2}(z_{eff})(\kappa T)^{\gamma - 1/2} + \alpha^2 \left( \frac{m}{2 \pi \hbar^2} \right)^{1/2} g_{\gamma - 1}(z_{eff})(\kappa T)^{\gamma - 1}. \tag{14}
\]

Where

\[
\gamma = \frac{3}{2} + \sum_{l=1}^d \frac{n_l}{s_l}, \tag{15}
\]

is the parameter that defines the shape of the potential. \( N_0 \) are the particles in the ground state, \( \Gamma(y) \) is the Gamma function, and \( C \) is a constant associated to the potential in question. In the case of Cartesian traps, and in consequence, for a three dimensional harmonic oscillator potential \( C = 8 \). Additionally, \( z_{eff} = e^{\beta (\mu + m \alpha^2 / 2)} = ze^{m \alpha^2 / 2 \kappa T} \) is an effective fugacity. Setting \( \alpha = 0 \) we recover the usual expression for the fugacity \( z \) \cite{26}.

Accepting that \( m \alpha^2 / 2 << \kappa T \), we can expand the expression \cite{14} around \( \alpha = 0 \) to first order, using the properties of the Bose–Einstein functions \cite{26}.

\[
x \frac{\partial}{\partial x} g_\nu(x) = g_{\nu - 1}(x). \tag{16}
\]

Using this fact we can re–write \cite{14} as follows
\[ N = N_0 + C \Pi_{l=1}^d A_l^{\frac{\gamma l}{s_l}} a_l^{\gamma l} \Gamma \left( \frac{n_l}{s_l} + 1 \right) \left[ \left( \frac{m}{2\pi \hbar^2} \right)^{3/2} g_\gamma(z)(\kappa T)^\gamma - \alpha \left( \frac{m^2}{2\pi^2 \hbar^3} \right) g_{\gamma - 1/2}(z)(\kappa T)^{\gamma - 1/2} \right]. \] (17)

Let us define
\[ V_{\text{char}} = C \Pi_{l=1}^d A_l^{\frac{\gamma l}{s_l}} a_l^{\gamma l} \Gamma \left( \frac{n_l}{s_l} + 1 \right), \] (18)
as the characteristic volume associated with the system. We can see that from expression (18), when \( s_l \to \infty \), then \( V_{\text{char}} \) becomes the volume associated with a free gas in a box. In this sense \( V_{\text{char}} \) can be interpreted as the available volume occupied by the gas \([13][19]\). On the other hand, it is noteworthy to mention that the most general definition of thermodynamic limit can be expressed as
\[ N \to \infty, \quad V_{\text{char}} \to 0, \] (19)
keeping the product \( NV_{\text{char}} \to \text{const} \), and is valid for all power law potentials in any spatial dimensionality \([19]\).

With the criterium given above, the critical temperature in the thermodynamic limit is well defined.

At the critical temperature in the thermodynamic limit \( \mu = 0 \) and \( N_0 = 0 \), which implies that the Bose–Einstein functions are given by the corresponding Riemann Zeta functions \( \zeta(x) \) \([20]\), then from expression (17) and the definition (18) we obtain that the number of particles at the condensation temperature can be written as
\[ N = V_{\text{char}} \left[ \left( \frac{m}{2\pi \hbar^2} \right)^{3/2} \zeta(\gamma)(\kappa T_c)^\gamma - \alpha \left( \frac{m^2}{2\pi^2 \hbar^3} \right) \zeta(\gamma - 1/2)(\kappa T_c)^{\gamma - 1/2} \right]. \] (20)

where \( T_c \) is the critical temperature for a modified bosonic gas in the thermodynamic limit. It is noteworthy to mention that the phenomenon of condensation for a modified bosonic gas in the thermodynamic limit is possible when \( \gamma > 3/2 \) according to the properties of the Bose–Einstein functions \([12][26]\). When \( s_l \to \infty \), \( \gamma = 3/2 \), we have a free gas in a box, in this case apparently the Bose–Einstein condensation is not possible for a free modified gas in a box, because the divergent behavior of the Bose–Einstein functions \([12]\). To make possible the condensation of a modified free gas in a box we have to take into account the minimal energy associated to this system \([25]\).

From expression (20), setting \( \alpha = 0 \), we may obtain the expression for the critical temperature \( T_0 \) for a gas trapped in a generic 3–dim power–law potential in the thermodynamic limit \([13]\)
\[ T_0 = \left[ \frac{N}{V_{\text{char}} \zeta(\gamma)} \left( \frac{2\pi \hbar^2}{m} \right)^{3/2} \right]^{1/\gamma} \frac{1}{\kappa}. \] (21)

From equations (20) and (21) we obtain the shift in the critical temperature for our deformed bosonic gas
\[ (\kappa T_c)^\gamma = (\kappa T_0)^\gamma + \alpha \left( \frac{2m}{\pi} \right)^{1/2} \frac{\zeta(\gamma - 1/2)}{\zeta(\gamma)} (\kappa T_c)^{\gamma - 1/2} \] (22)

We can notice from expression (22), that \( T_c \) increases for \( \alpha > 0 \), compared with the corresponding critical temperature of the usual Bosonic gas when \( \alpha = 0 \). The opposite case, \( \alpha < 0 \), corresponds to a decrease in the critical temperature, compared with the usual case. The effect of the external potential is to concentrate particles in the center of the trap and a positive \( \alpha \) increases the critical temperature which means that the effect of \( \alpha > 0 \) “reinforce” the main effect of the external potential. When \( \alpha < 0 \) we have the opposite behavior, a negative \( \alpha \) tends to “weaken” the effect of the external potential. We can also express the shift in the critical temperature as a function of the number of particles \( N \),
\[ \frac{\Delta T_c}{T_0} \approx \alpha \Omega N^{-1/2}. \] (23)

Where
\[ \Omega = \left( \frac{2m}{\pi} \right)^{1/2} \frac{\zeta(\gamma - 1/2)}{\gamma \zeta(\gamma)} \left( \frac{(2m\hbar^2)^{3/2}}{V_{\text{char}} m^{3/2} \zeta(\gamma)} \right)^{-1/2\gamma} \]  

It is noteworthy to mention that the correction in the critical temperature depends strongly on the functional form between the number of particles and the parameters of the potential. This fact could be used as an amplifier of some quantum gravity manifestations in systems with finite number of particles [25].

The appearance of \( \alpha \) in expression (22) or (23) may be interpreted as a parameter that modified the thermodynamic stability of the system. One way of quantify the thermodynamic stability of the system is through the fluctuations in the number of particles, which are directly related to the so-called isothermal compressibility [18, 19, 26].

Let us suppose now that our condensate is trapped in an anisotropic three-dimensional harmonic-oscillator potential. For this trap the the shape parameter is given by \( \gamma = 3 \) with \( A_i = \hbar \omega_i / 2 \), \( a_i = \sqrt{\hbar / m \omega_i} \) (see expression (18)), and using the definition \( \bar{\omega} = (\omega_1 \omega_2 \omega_3)^{1/3} \), expression (23) becomes

\[ \frac{\Delta T_c}{T_0} \simeq \alpha \frac{\zeta(5/2)}{3\zeta(3)^{5/6}} \left( \frac{8m}{\hbar \omega \pi} \right)^{1/2} N^{-1/6}. \]  

### III. FLUCTUATIONS IN A MODIFIED BOSONIC GAS

The number of particle fluctuations are characterized by the dispersion \[ (\Delta N)^2 \equiv \langle N^2 \rangle - \langle N \rangle^2 = \kappa T \left( \frac{\partial N}{\partial \mu} \right)_{T,V} \]  

where \( N \) is the total number of particles of the gas. The over–line in expression (26) means an average over the ensemble, which corresponds to the thermodynamic variables [26]. When \( (\Delta N)^2 \sim N \) the fluctuations are called normal, while if \( (\Delta N)^2 \sim N^s \) the fluctuations are anomalous, with \( s > 1 \) [18, 20]. The analysis of fluctuations in statistical systems is important because the fluctuations define for instance, the stability of the system and how this system leads to the state of thermodynamic equilibrium [18, 19].

The expression (26) is directly related to the isothermal compressibility \( \kappa_T \) through the next relation [18, 20]

\[ \kappa_T = \frac{(\Delta N)^2}{\rho N \kappa T}. \]

Where \( \rho \) is a mean particle density. Using expression (17), we may calculated the fluctuations associated with our modified bosonic gas in the thermodynamic limit. A necessary condition for the stability of a system is the semi–positiveness and finiteness of the isothermal compressibility. If the isothermal compressibility is negative or divergent, could lead to a immediate collapse or explosion of the system. In this aim, the analysis of fluctuations caused by the quantum structure of space–time in the number of particles, must lead to a observable consequences for a specific choice of the trap parameters. It is means, that the fluctuations caused by the deformation in the dispersion relation allows us, in principle, discriminate in which kind of systems the manifestations of quantum gravity are more feasible to be detected by analyzing the stability of the system.

Assuming that \( N = N_e \), being \( N_e \) the number of particles in the excited states, allows us to calculate from (17), together with (26), the fluctuations associated with our modofied bosonic gas

\[ \langle \Delta N_e \rangle^2 = \langle \kappa T \rangle V_{\text{char}} \left[ \Gamma(3/2) \frac{2\pi(2m)^{3/2}}{(2\pi \hbar)^3} g_{\gamma-1}(z) - \alpha \frac{8\pi m^2}{(2\pi \hbar)^3} \langle \kappa T \rangle^{-1/2} g_{\gamma-3/2}(z) \right]. \]

This last expression is a direct consequence of \( N = N_0 + N_e \) [15, 19]. From the relation between the critical temperature and the characteristic volume [21], allows us to write (28) above the critical temperature \( (T > T_0) \) as follows.
The appearance of $\alpha$ decreases in the opposite case, $\alpha < 0$. Nevertheless, the corrections caused by the deformation leaves the fluctuations normal, independent of the sign of $\alpha$. The isothermal compressibility is always finite and positive for $\gamma > 1$ and, hence, the system is always stable for temperatures above $T_0$.

For temperatures $T < T_0$ the dispersion (29) becomes

$$\left(\Delta N_e\right)^2 = \left[\frac{g_2(z)}{\zeta(3)} - \alpha \frac{g_3(z)}{\zeta(3)} \left(\frac{8m}{\pi \kappa T}\right)^{1/2}\right] \left(\frac{T}{T_0}\right)^{3} N. \tag{31}$$

Expression (31) shows that the fluctuations are normal, independent of the sign of $\alpha$. Then, the system is stable for temperatures $T > T_0$. On the other hand, for temperatures $T < T_0$, we obtain from expression (30)

$$\left(\Delta N_e\right)^2 = \left[\frac{\zeta(2)}{\zeta(3)} - \alpha \frac{\zeta(3/2)}{\zeta(3)} \left(\frac{8m}{\pi \kappa T}\right)^{1/2}\right] \left(\frac{T}{T_0}\right)^{3} N. \tag{32}$$

The fluctuations in the number of particles for temperatures $T < T_0$ are normal, which means that the isothermal compressibility is finite and positive. Hence the system is stable beyond the critical temperature. We notice from expressions (31) and (32) that the modifications caused by $\alpha$ increases the fluctuations when $\alpha < 0$, and decreases them in the opposite case i.e., $\alpha > 0$. Nevertheless, the fluctuations are normal in both cases. The system is stable for temperatures above and beyond the critical temperature in the case of a harmonic oscillator potential.

Setting $\alpha = 0$ in expression (32) for temperatures $T < T_0$ we recover the result given in [15].

IV. CONCLUSIONS

Using the formalism of the semiclassical approximation, we have analyzed the Bose–Einstein condensation for modified bosonic gas trapped in a 3–D power law potential. We notice that the critical temperature must be corrected as a consequence of the deformation in the dispersion relation. The shift in the critical temperature depends strongly on the sign of $\alpha$. For a positive $\alpha$, the critical temperature increases respect to the usual value when $\alpha = 0$, and decreases in the opposite case, $\alpha < 0$. The main effect of the trapped potential is to concentrate particles in the center of the trap. The insertion of the deformation parameter $\alpha$ modifies this behavior, depending on its sign. This fact allows us to interpret $\alpha$ as a parameter related to the thermodynamic stability of the system, since its appearance modify the fluctuations in the number of particles, and the isothermal compressibility, which is in an intimate relation with such stability. We notice that the fluctuations for $T > T_0$, remains normal for values $\gamma > 1$ of the shape

$$\left(\Delta N_e\right)^2 = \left(\frac{T}{T_0}\right)^{3} \frac{\zeta(2)}{\zeta(3)} N. \tag{33}$$
parameter. On the other hand, a detailed analysis of the different values for the shape parameters, below the critical temperature \( T < T_c \), shows that the system is stable for \( \gamma > 5/2 \). A very important consequence from expressions [17] and [22] is that for values between \( 3/2 < \gamma \leq 5/2 \) we have condensation, but the system is unstable. In this range of values of the shape parameter \( \gamma \) we have, for example, potentials of the type \( V(\vec{r}) \sim \rho^3 + z^3 \) in the case of a Cartesian potential, \( V(\vec{r}) \sim \rho^3 + z^3 \) in the case of cylindrical traps, and \( V(\vec{r}) \sim r^3 \) in the case of spherical traps. We have noticed also, that for potentials of the type \( V(\vec{r}) \sim \rho^3 + z^3 \) or any other combination such that \( 3/2 < \gamma \leq 5/2 \) we have condensation, but the system is unstable. For systems with \( \gamma \leq 3/2 \) (in which is included the ideal modified gas in a box) the condensation apparently is not possible if ones assumes that the minimal energy for the system is zero, or equivalently that the value of the chemical potential \( \mu \) is zero at the transition temperature. Nevertheless, an inspection of the deformed relation dispersion [18] show that the minimal energy is not zero. Then, in principle, the condensation is possible even in the case of a modified gas trapped in a box [25]. In the case of stable systems \( \gamma > 5/2 \), like the harmonic oscillator in 3-D, for example, one possible manner for the analysis of quantum gravity manifestations in a Bose–Einstein condensate is through the study of the thermodynamic variables like the critical temperature in function of the number of particles, as can be seen from expression [22]. Systems with a large but finite number of particles, in the ideal and weakly interacting case, can be used, in principle, as an amplifier of some quantum–gravity manifestations. A detailed analysis of these topics is analyzed in extenso in [27].

Acknowledgments

This research was supported by DAAD grant A/09/77687

[1] G. Amelino-Camelia, Quantum-Gravity Phenomenology, gr-qc/0806.0339v1, 2008.
[2] G. Amelino-Camelia, C. Laemmerzahl, F. Mercati, and G. M. Tino, Constraining the Energy–Momentum Dispersion Relation with Planck–Scale Sensitivity Using Cold Atoms, Phys. Rev. Lett, 103, 171302 (2009).
[3] V. A. Kostelecký, R. Lehnert, Phys. Rev. D 63, 065008, (2001).
[4] F. Dalfovo, S. Giordani, L. Pitaevskii, S. Stringari, Theory of Bose–Einstein Condensation in trapped gases, Reviews of Modern Physics, Vol. 71, No. 3, April (1999) pp. 463-512.
[5] V. Bagnato, D. E. Pritchard, D. Kleppner, Bose–Einstein Condensation in an External Potential, Phys. Rev. A 35 (1987).
[6] S. Grossmann and M. Holthaus, On Bose–Einstein condensation in harmonic traps, Phys. Lett. A 208 (1995).
[7] S. Giorgini, L. Pitaevskii, and S. Stringari, Condensate fraction and critical temperature of a trapped interacting Bose gas, Phys. Rev. A 54 (1996).
[8] H. Haugerud, T. Haugset, F. Ravna, Bose-Einstein condensation under external conditions, Phys. Lett. A 225 (1997).
[9] H. Shi and W. M. Zheng, Phys. Rev. A 56 1046, (1996).
[10] Z. Yan, Thermodynamic Properties of an Ideal System Trapped in a Generic Cylindrical Power–Law Potential. Phys. A 298, 455 (2001).
[11] L. Salasnich, Critical Temperature of an Interacting Bose Gas in a Generic Power Law–Potential Int. J. Mod. Phys. B 16, 2185 (2002).
[12] O. Zobay, Mean–field analysis of Bose–Einstein condensation in general power-law potentials J. Phys. B 37, 2593 (2004).
[13] A. Jaouadi, M. Telmini, and E. Charron, Bose–Einstein Condensation with a Finite number of Particles in a Power Law Trap, arXiv:1011.6477v1 [cond-mat.quant-gas] (2010).
[14] W. Ketterle and N. J. van Druten, Bose-Einstein condensation of a finite number of particles trapped in one or three dimensions, Phys. Rev. A 54 (1996).
[15] D. Colladay and P. McDonald, Bose–Einstein Condensates as a probe for Lorentz violation, Phys. Rev. D70 (2004), 125007.
[16] D. Colladay and P. McDonald, Bose–Einstein condensates as a probe for Lorentz violation, Phys. Rev. D73 (2006), 105006.
[17] E. Castellanos, A. Camacho, Critical Points in a Relativistic Bosonic Gas Induced by the Quantum Structure of Spacetime, Gen. Rel. Grav. 41, 2677-2685, (2009).
[24] E. Castellanos, A. Camacho, *Stability of Bose–Einstein Condensates in a Lorentz Violating Scenario*, Modern Physics Letters A, Vol. 25, No. 6, 459–469, (2010).
[25] E. Castellanos, C. Laemmerzahl, *Modified Bosonic Gas Trapped in a Generic 3–dim Power Law Potential* (to be published).
[26] R. K. Phatria, *Statistical Mechanics*, Butterworth Heineman, Oxford (1996).
[27] C. J. Pethick and H. Smith, *Bose-Einstein Condensation in Diluted Gases*, Cambridge University Press, Cambridge (2006).