A new finger replacement technique which is applicable for RAKE receivers in the soft handover (SHO) region has been proposed and studied in [1], [2] under the ideal assumption that the fading is both independent and identically distributed from path to path. To supplement our previous work, we present a general comprehensive framework for the performance assessment of the proposed finger replacement schemes operating over independent and non-identically distributed (i.n.d.) faded paths. To accomplish this object, we derive new closed-form expressions for the target key statistics which are composed of i.n.d. exponential random variables. With these new expressions, the performance analysis of various wireless communication systems over more practical environments can be possible.

Index Terms

Diversity channels, joint PDF, order statistics, i.n.d fading channels.

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I. INTRODUCTION

Multipath fading is an unavoidable physical phenomenon that affects considerably the performance of wideband wireless communication systems. While usually viewed as a deteriorating factor, multipath fading can also be exploited to improve the performance by using RAKE-type receivers. However, in the soft handover (SHO) region, due to the limited number of fingers in the mobile unit, we are faced with a problem of how to judiciously select a subset of paths for the RAKE reception to achieve the required performance.

Finger replacement techniques for RAKE reception in the SHO region have been proposed and analyzed over independent and identical distributed (i.i.d.) fading environments with two base stations (BSs) in [1] which was extended to the case of multiple BSs in [2]. The proposed schemes in [2], as shown in Fig. 1, are basically based on the block comparison among groups of resolvable paths from different BSs and lead to the reduction of complexity while offering commensurate performance in comparison with previously proposed schemes in [3], [4]. However, in practice, the i.i.d. fading scenario on the diversity paths is not always realistic due to, for example, the different adjacent multipath routes with the same path loss and the resulting unbalance among paths. Although this non-identical consideration is important from a practical standpoint, [2] was able to investigate the effect of the non-uniform power delay profile of the finger replacement schemes only with computer simulations due to the high complexity of the analysis. Note that the applied method in [5] to derive the required key statistics for i.i.d fading assumptions can not be directly adopted to the case of independent and non-identically distributed (i.n.d.) fading environments. The major difficulties lie in deriving the target statistics with non-identical parameters.

With this observation in mind, we mathematically attack these main difficulties in this report.
More specifically, we address the key mathematical formalism which are the statistics of partial sums and the two-dimensional joint statistics of partial sums of the i.n.d. ordered random variables (RVs) for the accurate performance analysis of the finger replacement scheme with non-identical parameters. The rest of this report is organized as follows. In Section II, we present the system models as well as the mode of operation of the finger replacement scheme under consideration and provide the results of a general comprehensive framework for the outage performance based on statistical results over i.n.d. fading channels. We then provide in Section III some closed-form expressions of the required key statistics. Finally, Section IV provides some concluding remarks.

II. SYSTEM MODELS AND PERFORMANCE MEASURES

Among the path scanning schemes proposed in [2], we consider the full scanning method. With this method, if the combined output signal-to-noise ratio (SNR) of current assigned fingers is greater than a certain target SNR, a one-way SHO is used and no finger replacement is needed. Otherwise, the receiver attempts a two-way SHO by starting to scan additional paths from the serving BS as well as all the target BSs.

We assume that $L$ BSs are active, and there are a total of $N_{(L)}$ resolvable paths, where $N_{(L)} = \sum_{n=1}^{L} N_n$ and $N_n$ is the number of resolvable paths from the $n$-th BS. In the SHO region, only $N_c$ out of $N_{(n)}$ ($1 \leq n \leq L$) paths are used for RAKE reception. Without loss of generality, let $N_1$ be the number of resolvable paths from the serving BS and $N_2, N_3, \ldots, N_L$ be those from the target BSs. In the SHO region, the receiver is assumed at first to rely only on $N_1$ resolvable paths and, as such, starts with $N_c/N_1$-generalized selection Combining (GSC) [6]. These schemes are based on the comparison of blocks consisting of $N_s (< N_c < N_n)$ paths from each BS.

Let $u_{i,n}$ ($i = 1, 2, \ldots, N_n$) be the $i$-th order statistics out of $N_n$ SNRs of paths from the
n-th BS by arranging $N_n$ ($L \geq 2$) nonnegative i.n.d. RVs, $\{\gamma_{j,n}\}_{j=1}^{N_n}$, at $n$-th BS, where $\gamma_{j,n}$ is the SNR of the $j$-th path from $n$-th BS, in decreasing order of magnitude such that $u_{1,n} \geq u_{2,n} \geq \cdots \geq u_{N_n,n}$. If we let

$$Y = \sum_{i=1}^{N_c-N_s} u_{i,1}$$

and

$$W_n = \begin{cases} 
\sum_{i=N_c-N_s+1}^{N_c} u_{i,n}, & n = 1 \\
\sum_{i=1}^{N_s} u_{i,n}, & n = 2, \ldots, L,
\end{cases}$$

then the received output SNR after GSC is given by $Y + W_1$. At the beginning of every time slot, the receiver compares the GSC output SNR, $Y + W_1$, with a certain target SNR. If $Y + W_1$ is greater than or equal to the target SNR, a one-way SHO is used and no finger replacement is needed. On the other hand, whenever $Y + W_1$ falls below the target SNR, the receiver attempts a two-way SHO by starting to scan additional paths from the target BSs.

To study the performance of the finger replacement scheme for i.n.d. fading assumptions, we look into the outage performance. Based on the mode of operation in Section [2, II-B], an overall outage probability is declared when the final combined SNR, $\gamma_F$, falls below a predetermined threshold, $x$, as

$$F_{\gamma_F}(x) = \Pr[\gamma_F < x]$$

where

$$\gamma_F = \begin{cases} 
Y + W_1, & Y + W_1 \geq \gamma_T \\
Y + \max\{W_1, W_2, \cdots, W_L\}, & Y + W_1 < \gamma_T.
\end{cases}$$

Considering two cases that i) the final combined SNR is greater than or equal to the target SNR, $\gamma_T$, (i.e., $x \geq \gamma_T$) and ii) the final combined SNR falls below the target SNR, (i.e., $0 < x < \gamma_T$),
separately, we can rewrite (3) as

\[
F_{\gamma P}(x) = \begin{cases} 
\Pr [Y + \max \{W_1, W_2, \ldots, W_L\} < x], & 0 < x < \gamma_T \\
\Pr [\gamma_T \leq Y + W_1 < x]\\n\quad + \Pr [Y + W_1 < \gamma_T, \gamma_T \leq Y + \max \{W_1, W_2, \ldots, W_L\} < x], & x \geq \gamma_T.
\end{cases}
\]  

(5)

The detailed derivation is presented in the Appendix I.

With (5), we now need to investigate the following three probabilities, a) \(\Pr [\gamma_T \leq Y + W_1 < x]\), b) \(\Pr [Y + W_1 < \gamma_T, \gamma_T \leq Y + \max \{W_1, W_2, \ldots, W_L\} < x]\), and c) \(\Pr [Y + \max \{W_1, W_2, \ldots, W_L\} < x]\). Note that the major difficulty in the analysis is to derive the required key statistics of ordered RVs. In [1] and [2], the required statistics were obtained by applying the conditional probability density function (PDF) based approach proposed in [5] which is only valid for an assumption of i.i.d. fading from path to path. However, in this report, our concern is that the average SNR of each path (or branch) is different, which means more practical channel models. For i.n.d. consideration unlike the i.i.d. case, we need to consider realistic frequency selective channels which have non-uniform delay profile, for example, exponentially decaying power delay profile, and to deal with order statistics of i.n.d. RVs. As results, the proposed method in [5] can not be directly adopted in case of i.n.d. fading environments here.

Recently, a unified framework to determine the joint statistics of partial sums of ordered i.i.d. RVs has been introduced in [7]. With this proposed approach, the required key statistics of any partial sums of ordered RVs can be obtained systematically in terms of the moment generating function (MGF) and the PDF. The extension of the mathematical approach proposed in [7] to i.n.d. fading channels can be found in [8], [9]. With the help of [7]–[9], the required key statistics to investigate the outage probability in (5) over i.n.d. fading channels can be obtained.

Note that based on the mode of operation, \(Y\) and \(W_1\) are correlated while \(W_n\) (for \(n = 2, \ldots, L\)) is independent of \(Y\). Hence, by adopting the proposed approach in [8], [9] instead of
applying [5], required key statistics in [5] can be evaluated as

a) For $\Pr [\gamma_T \leq Y + W_1 < x]$, 

$$\Pr [\gamma_T \leq Y + W_1 < x] = F_{Y+W_1} (x) - F_{Y+W_1} (\gamma_T).$$ (6)

b) For $\Pr [Y + \max \{W_1, W_2, \cdots, W_L\} < x]$, 

$$\Pr [Y + \max \{W_1, W_2, \cdots, W_L\} < x] = \int_0^x \int_0^{x-y} f_{Y,W_1} (y,y_1) \int_0^{x-y} f_{W_2} (w_2) dw_2 \cdots \int_0^{x-y} f_{W_L} (w_L) dw_L dw_1 dy$$ (7)

Similar to the identical case in [2], it is also very important to study the complexity of finger replacement schemes over i.n.d. case by accurately quantifying the performance measures such as the average number of path estimations, the average number of SNR comparisons, and the SHO overhead, which are required during the SHO process of these schemes over i.n.d. case. Note that with these performance measures, a comprehensive investigation of the tradeoff between complexity and performance over i.n.d. fading channels can be feasible. These important design parameters can be evaluated by directly applying the defined formulas presented in [2] with the required key statistics for i.n.d. ordered RVs which will be derived in this work. Hence, based on the mathematical approach proposed in [7]–[9], we here focus on the derivation of the following key statistics such as the cumulative distribution function (CDF) of the $N_c/N_1$-GSC output SNR, $F_{Y+W_1} (\cdot)$, the 2-dimensional joint PDF of two adjacent partial sums, $Y$ and $W_1$, of
order statistics, $f_{Y,W_1}(\cdot,\cdot)$, and the CDF of the sum of the $N_s$ strongest paths from each target BS, $F_{W_n}(\cdot)$, (i.e., $2 \leq n \leq L$).

III. KEY STATISTICS

In this section, we introduce the key statistics which are essential to solve Eqs. (6), (7), and (8) in Sec. II. More specifically, in these three cases, only the best $N_c$ or $N_s$ among $N_n$ ($N_s \leq N_c \leq N_n$) ordered RVs are involved in the partial sums. Thus, based on the unified framework in [7] and the extended work for i.n.d. case in [8], [9], each key statistics for three cases can be derived by applying the special step approach based on the substituted groups instead of original groups for each cases (i.e., starting from 2-dimensional joint statistics, 4-dimensional joint statistics, and 2-dimensional joint statistics, respectively) as

1) $F_{Y+W_1}(x)$: If we let $Z' = Y + W_1$ where $Y + W_1 = \sum_{i=1}^{N_c} u_{i,1}$ for convenience, then we can derive the target CDF of $Z'$ with the 2-dimensional joint PDF of $Z_1 = \sum_{i=1}^{N_c-1} u_{i,1}$ and $Z_2 = u_{N_c,1}$ as

$$F_{Y+W_1}(x) = \int_0^x f_{Z'}(z) \, dz = \int_0^x \int_0^{N_c} f_{Z_1,Z_2}(z-z_2,z_2) \, dz_2 \, dz.$$

(9)

2) $f_{Y,W_1}(x,y)$: In this case, we can derive the target 2-dimensional PDF of $Y = \sum_{i=1}^{N_c-N_s} u_{i,1}$ and $W_1 = \sum_{i=N_c-N_s+1}^{N_c} u_{i,1}$ by transferring the 4-dimensional joint PDF of $Z_1 = \sum_{i=1}^{N_c-N_s-1} u_{i,1}$, $Z_2 = u_{N_c-N_s,1}$, $Z_3 = \sum_{i=N_c-N_s+1}^{N_c-1} u_{i,1}$, and $Z_4 = u_{N_c,1}$ with the help of a function of a marginal PDF as

$$f_{Y,W_1}(x,y) = \int_0^{N_s} \int_0^{N_c-N_s} f_{Z_1,Z_2,Z_3,Z_4}(x-z_2,z_2,y-z_4,z_4) \, dz_2 \, dz_4. \tag{10}$$

3) $F_{W_n}(x)$, (i.e., $2 \leq n \leq L$): Similar to case 1), the target one-dimensional CDF of $W_n = \sum_{i=1}^{N_n} u_{i,n}$ with the 2-dimensional joint PDF of $Z'_1 = \sum_{i=1}^{N_c-1} u_{i,n}$ and $Z'_2 = u_{N_s,n}$ can be derived
with the help of a function of a marginal PDF as

$$F_{W_n}(x) = \int_{0}^{x} \int_{0}^{\gamma_{l,n}} f_{Z_{1}', Z_{2}'}(z - z', z')\,dz'dz. \quad (11)$$

The above novel generic results in (9)-(11) are quite general and can be applied for any RVs. In this report, we limit our analysis to the i.n.d. RVs case with a common exponential PDF, $p_{i,n}(x) = \frac{1}{\gamma_{i,n}} \exp\left(-\frac{x}{\gamma_{i,n}}\right)$ and CDF, $P_{i,n}(x) = 1 - \exp\left(-\frac{x}{\gamma_{i,n}}\right)$ for $\gamma \geq 0$, respectively, where $\gamma_{i,n}$ is the average of the $l$-th RV at $n$-th BS. Then, we can obtain the target statistics in a ready-to-use form for i.n.d. exponential RVs cases given as the following subsection. The detailed derivations are presented in the Appendix II and III.

A. CDF of the $N_c/N_1$-GSC Output SNR over i.n.d. Rayleigh Fading, $F_{Y+W_1}(x)$

$$F_{Y+W_1}(x) = \sum_{i_{N_c,1}=1}^{N_c} \sum_{i_{N_c+1,1}, \ldots, i_{N_1,1}}^{1,2, \ldots, N_1} \prod_{k=1}^{N_1-N_c} (-1)^{i_{N_c+1,1}} \prod_{j=1}^{N_1-k+1} \sum_{j_{k'}=j_{k}+1}^{N_1} \left[ \frac{1}{\gamma_{i_{N_c+1,1}}} \sum_{l=1}^{N_c} \frac{1}{\gamma_{l,1}} \left( 1 - \exp\left(-\frac{x}{\gamma_{l,1}}\right) \right) \right]$$

$$- \frac{N_c}{\sum_{l=1}^{N_c} \frac{1}{\gamma_{l,1}}} \left[ \frac{1}{\gamma_{i_{N_c+1,1}}} \sum_{l=1}^{N_c} \frac{1}{\gamma_{l,1}} \left( 1 - \exp\left(-\frac{x}{\gamma_{l,1}}\right) \right) \right]$$

$$\times \left[ \gamma_{l,n} \left( 1 - \exp\left(-\frac{x}{\gamma_{l,n}}\right) \right) - \frac{N_c}{\sum_{l=1}^{N_c} \frac{1}{\gamma_{l,1}}} \sum_{m=1}^{k'} \frac{1}{\gamma_{j_{m},1}} \left( 1 - \exp\left(-\frac{x}{\gamma_{j_{m},1}}\right) \right) \right]$$

$$\times \left[ \gamma_{l,n} \left( 1 - \exp\left(-\frac{x}{\gamma_{l,n}}\right) \right) - \frac{N_c}{\sum_{l=1}^{N_c} \frac{1}{\gamma_{l,1}}} \sum_{m=1}^{k'} \frac{1}{\gamma_{j_{m},1}} \left( 1 - \exp\left(-\frac{x}{\gamma_{j_{m},1}}\right) \right) \right],$$

where $j_0 = 0$,

$$C_{l,n_1,n_2} = \frac{1}{\prod_{l=n_2}^{l_2} \left(-\frac{1}{\gamma_{l,1}}\right) F_{l,n_1,n_2}^{\gamma_{l,1}} \frac{1}{\gamma_{l,1}}}, \quad (12)$$

$$F_{l,n_1,n_2}(x) = \left[ \sum_{l=1}^{n_2-n_1} (n_2 - n_1 - l + 1) \sum_{j_1=j_0+n_1}^{n_2-l+1} \sum_{j_2=j_1+1}^{n_2} \prod_{m=1}^{l} \frac{1}{\gamma_{j_{m},1}} \right] + (n_2 - n_1 + l) x^{n_2-n_1}. \quad (13)$$
B. Joint PDF of Two Adjacent Partial Sums $Y$ and $W_1$ over i.n.d. Rayleigh Fading, $f_{Y,W_1}(x,y)$

\[
\begin{align*}
\frac{1}{2} & \sum_{N_c=1}^{N_c-1} \sum_{N_s=1}^{N_s-1} \sum_{i=1}^{N_1} \sum_{j=1}^{N_c-N_s} \sum_{l=1}^{N_s-1} \sum_{k=1}^{N_c-N_s+1} \sum_{m=1}^{N_c-N_s+1} C_{k,N_c-N_s+1,N_c-1} \\
& \left[ \sum_{h=1}^{N_c-N_s+1} C_{h,1,N_c-N_s-1} \exp \left( -\frac{x}{\gamma_{h,1}} \right) \exp \left( -\frac{y}{\gamma_{h,1}} \right) \right] \\
& \times I \left( z_{2}, \beta', \frac{y}{N_s} - \frac{l}{N_s} \cdot \frac{z_4}{N_s} \right) + I \left( z_{2}, \beta', \frac{y-(N_s-\hat{N}) \cdot z_4}{l} ; z_4, \alpha'' \right) \left( z_{2}, \beta', \frac{y-(N_s-\hat{N}) \cdot z_4}{l} ; z_4, \alpha'' \right)
\end{align*}
\]
C. CDF of the Sums of the $N_s$ Strongest Paths from Each Target BS over i.n.d. Rayleigh Fading, $F_{W_n}(x)$

\[
F_{W_n}(x) = \sum_{i=N_s+1}^{N_s-N_{s-k}+1} \sum_{i=N_s+1}^{N_s-N_{s-k}+1} \sum_{i \neq N_s+1, \ldots, N_s} \prod_{i=N_s+1}^{N_s} C_{q,1,N_s-1} \prod_{i=N_s+1}^{N_s-k} \left( \gamma_{q,n} - 1 - \exp \left( -\frac{x}{\gamma_{q,n}} \right) \right) - \sum_{i=N_s}^{N_s-N_{s-k}+1} \left( \sum_{i=N_s}^{N_s-N_{s-k}+1} \prod_{i=N_s+1}^{N_s} C_{q,1,N_s-1} \prod_{i=N_s+1}^{N_s-k} \left( \gamma_{q,n} - 1 - \exp \left( -\frac{x}{\gamma_{q,n}} \right) \right) \right) \]

Note that in this report, we provide all three required key statistics in (12), (15), and (16), in the closed-form expressions to accurately investigating the performance measures mentioned in Sec. III especially, over i.n.d. Rayleigh fading conditions while [2] provides non-closed-form expressions even over i.i.d. fading assumptions since the final results involve finite integrations.

With these joint statistics derived in closed-form expressions, the outage probability as well as other performance measures mentioned in Sec. III can be easily calculated with standard mathematical softwares such as Mathematica.
IV. Conclusions

In this work, we studied the assessment tool of the finger replacement scheme proposed in [2] over i.n.d. fading conditions by providing the general comprehensive mathematical framework with non-identical parameters. Specifically, we provided the closed-form expressions for the required key statistics of i.n.d. ordered exponential RVs by applying a unified framework to determine the target statistics of partial sums of ordered RVs proposed in [8], [9] and the general comprehensive framework for the outage performance based on these statistical results. The proposed approach is quite general to apply to the performance analysis of various wireless communication systems over practical fading channels.

In Fig. 2, we assess the effect of non-identically distributed paths on the outage performance of the replacement schemes. More specifically, instead of the uniform power delay profile (PDP) considered so far, we now consider an exponentially decaying PDP. More specifically, we assume that the channel has an exponential multipath intensity profile (MIP), for which \( \bar{\gamma}_i = \bar{\gamma} \cdot \exp(-\delta (i - 1)), \) (1 \( \leq i \leq N_n, 1 \leq n \leq L \)) where \( \bar{\gamma}_i \) is the average SNR of the \( i \)-th path out of the total available resolvable paths from each BS, \( \bar{\gamma} \) is the strongest average SNR (or the average SNR of the first path), and \( \delta \) is the power decay factor. Note that \( \delta = 0 \) means identically distributed paths. These results show that the effect of path unbalance induces non-negligible performance degradation compared with the results for i.i.d. fading scenario. [2] showed that the proposed scheme can be still applied to the i.n.d. fading scenario. However, this effect must be taken into account for the accurate prediction of the performance over i.n.d. fading environments and with our analytical results, we believe that it is available to accurately predict the performance.
APPENDICES

In here, for analytical convenience, we assume that $N_n = N$, $u_{i,n} = u_i$, and $\bar{\gamma}_{i,n} = \bar{\gamma}_i$ for all $n = 1, 2, \cdots, L$.

APPENDIX I

DERIVATION OF (5)

Based on the mode of operation, we need to consider two cases i) the final combined SNR is greater than or equal to the target SNR, $\gamma_T$ and ii) the final combined SNR falls below $\gamma_T$, separately. For case ii), after scanning the paths from the serving BS as well as all the target BSs, the combined SNR of the resolvable paths from the serving BS and all the target BS falls below $\gamma_T$. Therefore, we can directly re-write (3) for $0 < x < \gamma_T$ as

$$F_{\gamma_T}(x) = \Pr[Y + \max\{W_1, W_2, \cdots, W_L\} < x].$$

(17)

However, for case i), we also need to consider two cases separately a) $Y + W_1 \geq \gamma_T$ and b) $Y + W_1 < \gamma_T$. More specifically, for case a), no finger replacement is needed while for case b), the receiver attempts a two-way SHO by starting to scan additional paths from the target BSs and the final combined SNR should be greater than or equal to $\gamma_T$. By considering the case ii)-a) and ii)-b), we can write (3) for $x \geq \gamma_T$ as

$$F_{\gamma_T}(x) = \Pr[Y + W_1 \geq \gamma_T, Y + W_1 < x]$$

$$+ \Pr[Y + W_1 < \gamma_T, \gamma_T \leq Y + \max\{W_1, W_2, \cdots, W_L\} < x].$$

(18)

As results, after some manipulations, we can re-write (18) in the simplified form given in (5) for $x \geq \gamma_T$. 
APPENDIX II

CDF OF THE $N_c/N$-GSC OUTPUT SNR

Based on the proposed unified framework in [7], noting that $Z' = Y + W_1$, we can obtain the target CDF of $Z' = \sum_{i=1}^{N_c} u_i$ with the 2-dimensional joint PDF of $Z_1 = \sum_{i=1}^{N_c-1} u_i$ and $Z_2 = u_{N_c}$. Specifically, by letting $X = Z_1 + Z_2$, we can obtain the target CDF of $Z' = X$ by integrating over $z_2$ for a given condition $Z_2 \leq \frac{X}{N_c}$ yielding (9). Fortunately, by adopting [9, Eq. (51)] to (9), we can obtain the closed-form expression of (9) over i.n.d. Rayleigh fading conditions by performing the double integrations over $z_2$ and $z$ in order.

After inserting [9, Eq. (51)] in (9), the inner integral term in (9) can be re-written as

$$\int_0^{\frac{X}{N_c}} f_Z(z - z_2, z_2) dz_2 = \sum_{i=1}^{N_c} \frac{1}{\gamma_i} \sum_{i \neq i_{N_c+1}}^2 \left[ \sum_{i_{N_c+1}}^{N_c} \prod_{q=1}^{N_c} C_{q,1,N_c-1} \right]$$

$$\times \left[ - \int_0^{\frac{X}{N_c}} \exp \left( - \frac{z}{\gamma_i} - \left( \sum_{i=1}^{N_c} \frac{1}{\gamma_i} - \frac{N_c}{\gamma_i} \right) z_2 \right) dz_2 \right]$$

$$- \prod_{k'=1}^{N-N_c} (-1) \sum_{j_1=j_0+N_c+1}^{N-k'+1} \sum_{j_{k'}=j_{k'-1}+1}^{N} \int_0^{\frac{X}{N_c}} \exp \left( - \frac{z}{\gamma_i} - \left( \sum_{i=1}^{N_c} \frac{1}{\gamma_i} + \sum_{m=1}^{k'} \frac{1}{\gamma_m} - \frac{N_c}{\gamma_i} \right) z_2 \right) dz_2 \right].$$

In (19), the first and second integral terms can be evaluated as the following closed-form expressions with the help of the basic exponential integration [10]

$$\frac{1}{\gamma_i} \left[ \exp \left( - \frac{z}{\gamma_i} \right) - \exp \left( - \left( \sum_{i=1}^{N_c} \frac{1}{\gamma_i} \right) \frac{z}{N_c} \right) \right],$$

(20)

$$\frac{1}{\gamma_i} \left[ \exp \left( - \frac{z}{\gamma_i} \right) - \exp \left( - \left( \sum_{i=1}^{N_c} \frac{1}{\gamma_i} + \sum_{m=1}^{k'} \frac{1}{\gamma_m} \right) \frac{z}{N_c} \right) \right].$$

(21)

Subsequently, after substituting (20) and (21) in (19), we can obtain a closed-form expression
of the inner integral term in (9) as
\[
f_{Z'}(z) = \sum_{i_{N_c}+1}^{N} \frac{1}{\gamma_{i_{N_c}}} \prod_{j_{i_{N_c}}+1}^{i_{N_c}+1} \sum_{l \neq i_{N_c}+1}^{N} \left\{ \exp \left( -z \frac{1}{\gamma_l} \right) - \exp \left( -z \frac{1}{\gamma_{i_{N_c}}} \right) \right\}
\]
\[
\times \left[ \prod_{l=1}^{i_{N_c}+1} \exp \left( -z \frac{1}{\gamma_l} \right) - \exp \left( -z \frac{1}{\gamma_{i_{N_c}}} \right) \right]^{N_{i_{N_c}+1}}
\]
\[
- \prod_{k'=1}^{N-N_c} (-1)^{k'} \sum_{j_{i_{N_c}+1}}^{N-k'+1} \prod_{j_{i_{N_c}+1}}^{j_{i_{N_c}}-1} \sum_{l \neq j_{i_{N_c}+1}}^{N} \exp \left( -z \frac{1}{\gamma_{j_{i_{N_c}}}} \right) - \exp \left( -z \frac{1}{\gamma_{i_{N_c}}} \right) \right\}
\]
\[
\sum_{k'=1}^{N-N_c} (-1)^{k'} \sum_{j_{i_{N_c}+1}}^{N-k'+1} \prod_{j_{i_{N_c}+1}}^{j_{i_{N_c}}-1} \sum_{l \neq j_{i_{N_c}+1}}^{N} \left\{ \exp \left( -z \frac{1}{\gamma_{j_{i_{N_c}}}} \right) - \exp \left( -z \frac{1}{\gamma_{i_{N_c}}} \right) \right\}
\]
\[
= \prod_{j_{i_{N_c}+1}}^{j_{i_{N_c}}-1} \sum_{l \neq j_{i_{N_c}+1}}^{N} \left\{ \exp \left( -z \frac{1}{\gamma_{j_{i_{N_c}}}} \right) - \exp \left( -z \frac{1}{\gamma_{i_{N_c}}} \right) \right\}
\]

Finally, by simply applying a basic exponential integration [10] over \( z_2 \) after substituting (22) in (9) and then replacing \( z \) in (22) by \( z_2 \), we can obtain the target CDF in closed-form as shown in (12).

Note that, in the case of the closed-form expression of the CDF of the \( N_s/N \)-GSC output SNR given in (16), we can directly apply the same approach for (11) just by replacing \( N_s \) with \( N_c \).

**APPENDIX III**

**JOINT PDF OF \( Y \) AND \( W_1 \)**

In this case, the target 2-dimensional joint PDF of \( Y \) and \( W_1 \) can be obtained starting from the 4-dimensional joint PDF of \( Z_1 = \sum_{i=1}^{N_c-N_c-1} u_i, Z_2 = u_{N_c-N_c}, Z_3 = \sum_{i=N_c-N_c+1}^{N_c-1} u_i, \) and \( Z_4 = u_{N_c} \) based on the proposed unified frame work in [7], [9]. As results, the target 2-dimensional joint PDF of interest, \( f_{Y,W_1}(x,y) \), can be finally obtained from transformed higher dimensional joint PDFs as shown in (10). Here, RVs, \( Z_1, Z_2, Z_3, \) and \( Z_4 \), have the following relationships
\[
\begin{align*}
Y & = u_{1}, \cdots, u_{N_c-N_c-1}, u_{N_c-N_c} \\
Z_1 & = u_{N_c-N_c-1} \\
Z_2 & = u_{N_c-N_c} \\
W_1 & = u_{N_c-N_c+1} \cdots, u_{N_c-1} \cdots, u_{N_c+1}, \cdots, u_N \\
Z_3 & = u_{N_c-1} \\
Z_4 & = u_{N_c+1} \\
\end{align*}
\]
From (23), we can directly obtain the following valid conditions between these RVs i) \( Z_1 \geq (N_c - N_s - 1) Z_2 \), ii) \( Z_3 \geq (N_s - 1) Z_4 \) and iii) \( N_s \cdot Z_2 \geq Z_3 + Z_4 \). For case i) and ii), by adding \( Z_2 \) to both sides of case i), we can obtain the following result as \( Z_1 + Z_2 \geq (N_c - N_s) Z_2 \) while by adding \( Z_4 \) in case ii), we can obtain \( Z_3 + Z_4 \geq N_s \cdot Z_4 \). Therefore, with the 4-dimensional joint PDF of \( Z_1, Z_2, Z_3, \) and \( Z_4 \), letting \( X = Z_1 + Z_2 \) and \( Y = Z_3 + Z_4 \), then we can obtain the target 2-dimensional joint PDF of \( Z' = [X, Y] \) by integrating over \( z_2 \) and \( z_4 \) yielding (10).

In (10), we now need to derive the 4-dimensional joint PDF of \( Z_1, Z_2, Z_3, \) and \( Z_4 \). Fortunately, with the help of the derived result over i.n.d. exponential RVs in [9, Eq. (54)], we only need to evaluate the double integrations over \( z_2 \) and \( z_4 \). With [9, Eq. (54)], to evaluate the additional 2-fold integrations, the multiple product expression, \( \prod_{j=N_s+1}^{N} \left( 1 - \exp \left( -\frac{z_4}{\gamma_{ij}} \right) \right) \), need to be converted to the summation expression. In this case, with the help of the property of exponential multiplication, this multiple product expression can be re-written as the following summation expression by adopting the derived result presented in (42)

\[
\prod_{j=N_s+1}^{N} \left( 1 - \exp \left( -\frac{z_4}{\gamma_{ij}} \right) \right) = 1 + \sum_{g=1}^{N-N_s} (-1)^g \sum_{j'_{g+1}}^{N-g+1} \sum_{j'_{g+1}}^{N-N_s} \exp \left( -\sum_{m=1}^{g} \frac{z_4}{\gamma_{ij'm}} \right). \tag{24}
\]

Then, inserting the re-written expression of [9, Eq. (54)] as the summation expression into (10) and then after some manipulations, (10) can be re-written as

\(^1\)The product of two exponential numbers of the same base can be simply represented as the sum of the exponents with the same base.
With (25), we now need to evaluate the following four integral terms:
A) For the first integral term:

\[
\int_0^\infty \int_\frac{x}{N_s}^\infty \frac{x}{N_s} \exp \left( - \left( \sum_{l=1}^{N_c-N_s} \frac{1}{\gamma_{ii}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c-N_s}{\gamma_{ik}} \right) y \right) \exp \left( - \left( \sum_{l=N_c-N_s+1}^{N_c} \frac{1}{\gamma_{ii}} - \frac{(N_s-\bar{N})}{\gamma_{ik}} \right) \frac{l}{\gamma_{ik}} \right) \frac{dz_2}{z_2} dz_2 dz_4
\]

\( (26) \)

B) For the second integral term:

\[
\int_0^\infty \int_\frac{x}{N_s}^\infty \frac{x}{N_s} \exp \left( - \left( \sum_{l=1}^{N_c-N_s} \frac{1}{\gamma_{ii}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c-N_s}{\gamma_{ik}} \right) y \right) \exp \left( - \left( \sum_{l=N_c-N_s+1}^{N_c} \frac{1}{\gamma_{ii}} - \frac{(N_s-\bar{N})}{\gamma_{ik}} \right) \frac{l}{\gamma_{ik}} \right) \frac{dz_2}{z_2} dz_2 dz_4
\]

\( (27) \)

C) For the third integral term:

\[
\int_0^\infty \int_\frac{x}{N_s}^\infty \frac{x}{N_s} \exp \left( - \left( \sum_{l=1}^{N_c-N_s} \frac{1}{\gamma_{ii}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c-N_s}{\gamma_{ik}} \right) y \right) \exp \left( - \left( \sum_{l=N_c-N_s+1}^{N_c} \frac{1}{\gamma_{ii}} - \frac{(N_s-\bar{N})}{\gamma_{ik}} \right) \frac{l}{\gamma_{ik}} \right) \frac{dz_2}{z_2} dz_2 dz_4,
\]

\( (28) \)

D) For the fourth integral term:

\[
\int_0^\infty \int_\frac{x}{N_s}^\infty \frac{x}{N_s} \exp \left( - \left( \sum_{l=1}^{N_c-N_s} \frac{1}{\gamma_{ii}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c-N_s}{\gamma_{ik}} \right) y \right) \exp \left( - \left( \sum_{l=N_c-N_s+1}^{N_c} \frac{1}{\gamma_{ii}} - \frac{(N_s-\bar{N})}{\gamma_{ik}} \right) \frac{l}{\gamma_{ik}} \right) \frac{dz_2}{z_2} dz_2 dz_4.
\]

\( (29) \)

For the first and the third integral terms, closed-form expression can be obtained by simply applying basic exponential integration [10], using the following useful common function

\[
I(x, e, a; b; y, f, c, d) = \int_c^d \int_a^b \exp (e \cdot x) \exp (f \cdot y) dx dy
\]

\( = \frac{1}{ef} \left\{ \exp (e \cdot b) - \exp (e \cdot a) \right\} \left\{ \exp (f \cdot d) - \exp (f \cdot c) \right\}. \)

\( (30) \)

With \( (30) \), by letting \( \alpha = - \left( \sum_{l=N_c-N_s+1}^{N_s} \frac{1}{\gamma_{il}} \right) - \frac{(N_s-\bar{N})}{\gamma_{ik}} \), \( \alpha' = - \left( \sum_{l=N_c-N_s+1}^{N_s} \frac{1}{\gamma_{il}} \right) + \sum_{m=1}^{g} \frac{1}{\gamma_{im}} - \frac{(N_s-\bar{N})}{\gamma_{ik}} \), and \( \beta = - \left( \sum_{l=1}^{N_c-N_s} \frac{1}{\gamma_{il}} \right) - \frac{(N_s-N_c)}{\gamma_{ik}} \), the closed-form expression of the first integral term can be
obtained by simply applying the basic exponential integration [10] as
\[
\int_0^\infty \int_0^\infty \exp(\beta z_2) \exp(\alpha z_4) U(x - (N_c - N_s) \cdot z_2) U(y - (N_s) \cdot z_4) dz_2 dz_4 \\
= \int_0^\infty \int_0^\infty \frac{\exp(\beta z_2) \exp(\alpha z_4)}{N_s} dz_2 dz_4 \\
= I\left(z_2, \beta, \frac{y}{N_s}, \frac{x}{N_c - N_s}, z_4, \alpha, 0, \frac{y}{N_s}\right).
\]

Similarly, for the third integral term, we can also obtain closed-form expressions simply by replacing \( \alpha \) with \( \alpha' \) on (31) as
\[
\int_0^\infty \int_0^\infty \exp(\beta z_2) \exp(\alpha' z_4) U(x - (N_c - N_s) \cdot z_2) U(y - (N_s) \cdot z_4) dz_2 dz_4 \\
= \int_0^\infty \int_0^\infty \frac{\exp(\beta z_2) \exp(\alpha' z_4)}{N_s} dz_2 dz_4 \\
= I\left(z_2, \beta, \frac{y}{N_s}, \frac{x}{N_c - N_s}, z_4, \alpha', 0, \frac{y}{N_s}\right).
\]

However, for the second and the fourth integral terms, we need to consider two cases separately based on the valid integration region of \( z_2 \). More specifically, \( z_2 \) should satisfy two following conditions as i) \( z_2 \leq \frac{x}{N_c - N_s} \) and ii) \( z_2 \leq \frac{y - (N_s - l)z_4}{l} \) and it leads \( z_2 \leq \min\left[\frac{x}{N_c - N_s}, \frac{y - (N_s - l)z_4}{l}\right] \).

Based on it, if \( \frac{x}{N_c - N_s} \leq \frac{y - (N_s - l)z_4}{l} \), then the valid integral regions for \( z_2 \) is unchanged, \( \frac{y}{N_s} < z_2 < \frac{x}{N_c - N_s} \), but the valid integral regions for \( z_4 \) becomes changed to \( 0 < z_4 < \min\left[\frac{y}{N_s}, \frac{y - (N_s - l)z_4}{N_s - l}\right] \) from \( 0 < z_4 < \frac{y}{N_s} \). Otherwise, the valid integral regions for \( z_2 \) is changed as \( \frac{y}{N_s} < z_2 < \frac{y - (N_s - l)z_4}{l} \) and for \( z_4 \), we need to consider both cases of \( 0 < z_4 < \min\left[\frac{y}{N_s}, \frac{y - (N_s - l)z_4}{N_s - l}\right] \) and \( 0 < z_4 < \frac{y}{N_s} \) by considering the unit step function, \( \left\{ 1 - U\left(\frac{x}{N_c - N_s} - \frac{y - (N_s - l)z_4}{l}\right) \right\} \). As results, the second and the fourth integral terms can be re-written, respectively, as
For the second integral term:
\[
\int_0^\infty \left[ \frac{y^{N_s(N_s-1)}}{	heta} \right] \exp \left( - \left( \sum_{l=1}^{N_c} \frac{1}{\gamma_{li}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c - l}{\bar{\gamma}_ih} - \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} \right) z_2 \right) d\theta d\gamma
\]
\[
\int_0^{N_c-N_s} \exp \left( - \left( \sum_{l=1}^{N_c} \frac{1}{\gamma_{li}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c - N_s}{\bar{\gamma}_ih} - \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} \right) z_2 \right) d\theta d\gamma
\]
\[
+ \left\{ \int_0^\infty \left[ \frac{y^{N_s(N_s-1)-N_c}}{\theta} \right] \exp \left( - \left( \sum_{l=1}^{N_c} \frac{1}{\gamma_{li}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c - l}{\bar{\gamma}_ih} - \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} \right) z_2 \right) d\theta d\gamma \right\}
\]

(33)

For the fourth integral term:
\[
\int_0^\infty \left[ \frac{y^{N_s(N_s-1)-N_c}}{\theta} \right] \exp \left( - \left( \sum_{l=1}^{N_c} \frac{1}{\gamma_{li}} + \sum_{m=1}^{g} \frac{1}{\gamma_{ij,m}} - \frac{N_c - l}{\bar{\gamma}_ih} - \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} \right) z_2 \right) d\theta d\gamma
\]
\[
\int_0^{N_c-N_s} \exp \left( - \left( \sum_{l=1}^{N_c} \frac{1}{\gamma_{li}} + \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} - \frac{N_c - N_s}{\bar{\gamma}_ih} - \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} \right) z_2 \right) d\theta d\gamma
\]
\[
+ \left\{ \int_0^\infty \left[ \frac{y^{N_s(N_s-1)-N_c}}{\theta} \right] \exp \left( - \left( \sum_{l=1}^{N_c} \frac{1}{\gamma_{li}} + \sum_{m=1}^{g} \frac{1}{\gamma_{ij,m}} - \frac{N_c - l}{\bar{\gamma}_ih} - \sum_{q=1}^{l} \frac{1}{\gamma_{iq}} \right) z_2 \right) d\theta d\gamma \right\}
\]

(34)

In (33), the closed-form expression of the first double integral term can be obtained with the help of (30). However, for the second and the third double integral terms, the inner integral limit depends on the outer variable. Therefore, for these cases, we can obtain the closed-form
expression with the help of another useful common function as

\[
I'(x, e, a, b - b'y; y, f, c, d) = \int_c^d \exp (f \cdot y) \int_a^{b - b'y} \exp (e \cdot x) \, dx \, dy
\]

\[
= \frac{1}{2} \left[ \exp (e \cdot a) \frac{1}{\exp (f)} \left\{ \exp \left( (f - e \cdot b') \cdot d \right) - \exp \left( (f - e \cdot b') \cdot c \right) \right\} 
- \exp (e \cdot a) \frac{1}{f} \left\{ \exp (f \cdot d) - \exp (f \cdot c) \right\} \right].
\]  

Then, by letting \( \alpha'' = - \left( \sum_{l=N_e-N_s}^{N_e} \left( \frac{1}{\gamma_i} \right) - \frac{(N_e-l)}{\gamma_i} - \frac{l}{\gamma_i q} \right) \) and \( \beta' = - \left( \sum_{l=1}^{N_e-N_s} \left( \frac{1}{\gamma_i} \right) + \frac{l}{\gamma_i q} - \frac{N_e-N_s}{\gamma_i} - \frac{l}{\gamma_i q} \right) \), we can finally obtain the closed-form expression of (33) as

\[
\int \left( z_2, \beta', \frac{y}{N_s}, \frac{x}{N_s-N_e}; z_4, \alpha'', 0, \min \left[ \frac{y}{N_s}, \frac{y}{N_s} - \frac{1}{(N_s-1)} \right] \right) 
+ I' \left( z_2, \beta', \frac{y}{N_s}, \frac{y-(N_s-l)}{l}; z_4, \alpha''', 0, \min \left[ \frac{y}{N_s}, \frac{y}{N_s} - \frac{1}{(N_s-1)} \right] \right)
- I' \left( z_2, \beta', \frac{y}{N_s}, \frac{y-(N_s-l)}{l}; z_4, \alpha''', 0, \min \left[ \frac{y}{N_s}, \frac{y}{N_s} - \frac{1}{(N_s-1)} \right] \right). 
\]  

For (34), by applying a similar approach and with the help of (35), we can also obtain the closed-form expressions given in (34). More specifically, we can also obtain the closed-form expressions simply by replacing \( \alpha'' \) with \( \alpha''' \) on (36) as

\[
\int \left( z_2, \beta', \frac{y}{N_s}, \frac{x}{N_s-N_e}; z_4, \alpha''', 0, \min \left[ \frac{y}{N_s}, \frac{y}{N_s} - \frac{1}{(N_s-1)} \right] \right) 
+ I' \left( z_2, \beta', \frac{y}{N_s}, \frac{y-(N_s-l)}{l}; z_4, \alpha''', 0, \min \left[ \frac{y}{N_s}, \frac{y}{N_s} - \frac{1}{(N_s-1)} \right] \right),
\]  

where \( \alpha''' = - \left( \sum_{l=N_e-N_s+1}^{N_e} \frac{1}{\gamma_i} \right) + \sum_{m=1}^{N_s} \frac{1}{\gamma_i q_m} - \frac{(N_e-l)}{\gamma_i} - \frac{l}{\gamma_i q} \).

Then, by substituting all these closed-form results and after a few manipulations, we can obtain the final closed-form expression of (10) as shown in (15).

**APPENDIX IV**

**DERIVATION OF USEFUL FUNCTION**

In this appendix, we derive (24) by deriving the special case from \( n_1 = 1 \) and \( n_2 = 2 \) and then we extend this result to the general case for arbitrary \( n_1 \) and \( n_2 \). At first, let \( n_1 = 1 \) and
$n_2 = 2$, then we can write (24) as the following summation expression

$$
\prod_{j=1}^{2} (1 - \exp(-a_{i_j})) = (1 - \exp(-a_{i_1}))(1 - \exp(-a_{i_2}))
$$

$$
= 1 - \exp(-a_{i_1}) - \exp(-a_{i_2}) + \exp(-a_{i_1} - a_{i_2}) .
$$

(38)

Similarly, for $n_1 = 1$ and $n_2 = 3$, we can write (24) as

$$
\prod_{j=1}^{3} (1 - \exp(-a_{i_j})) = (1 - \exp(-a_{i_1}))(1 - \exp(-a_{i_2}))(1 - \exp(-a_{i_3}))
$$

$$
= 1 - \exp(-a_{i_1}) - \exp(-a_{i_2}) - \exp(-a_{i_3}) + \exp(-a_{i_1} - a_{i_2}) + \exp(-a_{i_1} - a_{i_3}) + \exp(-a_{i_2} - a_{i_3}) .
$$

(39)

After simplification with a few manipulations, we can re-write the multiple product expressions in (38) and (39) as the following simplified summation expressions, respectively

$$
\prod_{j=1}^{2} (1 - \exp(-a_{i_j}))
$$

$$
= 1 + (-1)^2 \sum_{j'_{i_1}=1}^{2} \exp(-a_{i_{j'_{i_1}}}) + (-1)^2 \sum_{j'_{i_1}=1}^{1} \sum_{j'_{i_2}=j'_{i_1}+1}^{2} \exp(-a_{i_{j'_{i_1}}} - a_{i_{j'_{i_2}}}) ,
$$

(40)

and

$$
\prod_{j=1}^{3} (1 - \exp(-a_{i_j}))
$$

$$
= 1 + (-1)^3 \sum_{j'_{i_1}=1}^{1} \sum_{j'_{i_2}=j'_{i_1}+1}^{2} \sum_{j'_{i_3}=j'_{i_2}+1}^{3} \exp(-a_{i_{j'_{i_1}}} - a_{i_{j'_{i_2}}} - a_{i_{j'_{i_3}}}) .
$$

(41)

As results, after simplifying and generalizing the above equations, we can obtain the generalized expression of (38) and (39) with arbitrary $n_1$ and $n_2$ as

$$
\prod_{j=n_1}^{n_2} (1 - \exp(-a_{i_j})) = 1 + \sum_{l=1}^{n_2-n_1+1} (-1)^l \sum_{j'_{i_1}=j'_{0}+n_1}^{n_2-l+1} \cdots \sum_{j'_{i_{l-1}}=j'_{l-1}+1}^{n_2} \exp\left(-\sum_{m=1}^{l} a_{i_{j'_{m}}} \right) ,
$$

(42)

where $j'_{0} = 0$. 
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Fig. 1. Finger replacement schemes for RAKE receivers in the soft handover region.
Fig. 2. Outage probability of finger replacement schemes for RAKE receivers in the soft handover region over i.n.d. Rayleigh fading channels when $L = 4$, $N_1 = \cdots = N_4 = 5$, and $N_c = 3$. 