Harmonized planning of business development management programmes

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Abstract. The paper presents the numerical methods for solving harmonized optimization problems, which are considered based on an example of compiling of business development management programmes.

1. Introduction

The idea of harmonized planning is that the target function of a business management subsystem (the Center) is optimized based on a set of harmonized business development programmes, i.e. the programmes with the target functions of subprogrammes being not less than a certain value [1-3].

Let a programme consist of m subprogrammes of different directions. The status of each direction is assessed. \( F_i \) - status assessment of direction \( i \) (target function of active production element \( i \)), \( F \) – target function of the Center. The target function of the Center depends on the target functions of managed subsystems of a business (active production elements): \( F = \Phi(F_1:F_2:...:F_m) \).

It can be linear, adaptive or matrix convolution.

The problem of the Center is the development of a programme (a set of projects) with target function \( F \) reaching the maximum at limited resources \( R \) allocated for the programme. Each active production element \( i \) is interested in the development of a subprogramme, which will maximize its target function \( F_i \).

If the Center does not take into account the interests of managed business subsystems when developing the programme, it will bring about the negative consequences such as concealment or misinterpretation of information submitted by the active production elements to the Center, non-execution of programme projects, etc.

To write down the problem of optimum harmonized planning, let us denote essential estimate of direction \( i \) by \( F_i^0 \). The assurance of increase of criterion \( F_i \) by value \( \Delta F_i = \gamma_i F_i^0 \) (i.e. increasing by \( 100 \cdot \gamma_i \) percent) can be the condition of harmonization. In such situation, the problem of harmonized planning takes on the following form (1):

\[
F = \Phi(F_1:F_2:...:F_m) \rightarrow \max
\]

under constraints (2):

\[
F = \Phi(F_1:F_2:...:F_m) \rightarrow \max
\]
\[ F_i \geq (1 + \gamma_i) F_i^0, \quad i = 1, m. \]  

(2)

2. Setting of Problem of Harmonized Planning of Programmes with Single-Purpose Projects

There are \( n \) projects to be included into a programme. The implementation costs, \( c_k \), and the effects given by the project for direction \( i \), \( \alpha_{ki} \) (the effect is understood as the increment of criterion \( F_i \)), are specified for each project \( k \).

Let us denote \( x_k = 1 \), if project \( k \) is included into the programme, and \( x_k = 0 \), if not.

One needs to define \( x = \{ x_k : k = 1, n \} \), which maximize (3):

\[ \phi(y_1, y_2, \ldots, y_m) \]  

(3)

where \( y_i = \sum_k \alpha_{ki} x_k \), \( i = 1, m \) under constraints (4) and (5):

\[ \sum_k c_k x_k \leq R \]  

(4)

\[ \sum_k \alpha_{ki} x_k \geq \gamma_i F_i^0, \quad i = 1, m. \]  

(5)

Let us consider a particular case of the problem when there is a set of projects \( Q_i \) for each direction \( i \), and these sets do not intersect.

A numerical method of problem solving is suggested which includes the solving of classical knapsack problem by dichotomous programming [4].

3. Solution Algorithm for Problem of Harmonized Planning of Programmes with Single-Purpose Projects

Step 1. Let us solve \( m \) knapsack problems: one needs to maximize:

\[ y_i = \sum_{k \in Q_i} \alpha_{ki} x_k \]  

(6)

under constraints

\[ \sum_{k \in Q_i} c_k x_k \leq R_i, \]  

(7)

\[ \sum_{k \in Q_i} \alpha_{ki} x_k \geq \gamma_i F_i^0, \quad i = 1, m. \]  

(8)

where \( 0 \leq R_i \leq R \).

Let us solve knapsack problem (6), (7) at \( R_i = R \), which gives the optimum solution at all \( R_i < R \). Let us denote value \( Y_i(R_i) \) (6) in the optimum problem solution as function \( R_i \). Let us define the minimum \( R_i = d_i \), at which \( Y_i(d_i) \geq b_i \).

One obtains dependence \( Y_i(R_i) \), where \( d_i \leq R_i \leq R \).

Step 2. Let us solve maximization problem (9):

\[ Y(R) = \sum_i Y_i(R_i) \]  

(9)
under constraints (10) and (11)

\[ Y_i \left( R_i \right) \geq b_i, \quad i = 1, \ldots, m, \quad (10) \]

\[ \sum_{i=1}^{m} R_i \leq R. \quad (11) \]

4. Setting of Problem of Harmonized Planning of Programmes with Multi-Purpose Projects

In the general case, there are projects, the implementation of which contributes to several directions [5-8]. Let us refer to such projects as multi-purpose. The numerical method suggested is based on the branch-and-bound algorithm with estimates obtained on the basis of network programming for a inverse problem – minimization of costs at constrained total effect and effects of directions [9].

There are \( n \) projects to be included into a programme. The implementation costs, \( c_k \), and the effects given by the project for direction \( i, \alpha_{ki} \), are specified for each project \( k \).

Let us denote \( x_k = 1 \), if project \( k \) is included into the programme, and \( x_k = 0 \), if not.

Let us define \( x = \{ x_k, k = 1, n \} \), which minimize (12):

\[ C(x) = \sum_k c_k x_k \quad (12) \]

under constraints (13) and (14)

\[ \sum_i y_i \geq B, \quad (13) \]

\[ i = 1, \ldots, m. \quad (14) \]

In accordance with the network programming theory, the costs of multi-purpose projects are randomly divided into two parts.

Figure 1 shows an example of network constraints of the problem.

Let us obtain two estimation problems with single-purpose projects for each direction: one needs to minimize project implementation costs (12) with effect increment constraints (13), (14) fulfilled. The problem is solved by network programming method [10].

By solving the estimation problem of the upper level, let us obtain the upper-bound estimate. If the solution obtained is feasible for the original problem, then it is the optimum solution as well. Otherwise, the optimum solution is given by the branch-and-bound method.
Let us illustrate the suggested method of problem solving for harmonized planning of programmes with single-purpose projects by the following example.

Example. There are two programme directions, for which set of projects $Q_i$ is proposed, and the sets do not intersect. The data on projects are given in Table 1.

### Table 1. Project Data

| $i$ | 1 | 2 | 3 | 4 |
|-----|---|---|---|---|
| $\alpha_k$ | 4 | 6 | 8 | 7 |
| $c_k$ | 2 | 4 | 6 | 7 |

Let us take $b_1 = 5$, $b_2 = 8$, $R = 15$.

Stage 1. Let us solve the knapsack problem for each direction by the dichotomous programming method. The dichotomous representation tree of the problem is given in Figure 2.

![Figure 2. Tree of Dichotomous Problem Representation](image)

Step 1. Let us solve the problem for projects 1 and 2 (Table 2).

### Table 2. Data on Costs and Effect of Projects from the First Direction

| variant | 1 | 2 |
|---------|---|---|
| $R_I$   | 4 | 6 |
| $Y_I$   | 6 | 10 |

Step 2. Let us solve the problem for projects 3 and 4 (Table 3).

### Table 3. Data on Costs and Effect of Projects from the Second Direction

| variant | 1 | 2 |
|---------|---|---|
| $R_{II}$ | 6 | 13 |
| $Y_{II}$ | 8 | 15 |

Stage 2. Let us solve maximization problem $Y_I (R_I) + Y_{II} (R_{II})$ under constraint $R_I + R_{II} \leq 15$.

The solution is given in Table 4.

### Table 4. Data on Combined Projects from the First and Second Directions

| variant | 13;15 | 17;21 | 19;25 |
|---------|-------|-------|-------|
| $II$, $I$ | 6;8 | 10;14 | 12;18 |
|          | 4;6   | 6;10  |       |
In Table 6, let us define a cell with the maximum second number and the first not exceeding $15 - (12;18)$. By the backward method, let us find solution $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0$.

5. Conclusion
The paper suggests, by an example of compiling of business development management programmes, the numerical methods of problem solving for harmonized planning based on dichotomous programming methods, branch-and-bound algorithm with estimate obtained by network programming. It is assumed that the problems considered will be developed in the management of interdependent projects and high-risk projects.

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