Upper Bounds on the Lightest Higgs Boson Mass in General Supersymmetric Standard Models *

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Abstract
In a general supersymmetric standard model there is an upper bound $m_h$ on the tree level mass of the $CP = +1$ lightest Higgs boson which depends on the electroweak scale, $\tan \beta$ and the gauge and Yukawa couplings of the theory. When radiative corrections are included, the allowed region in the $(m_h, m_t)$ plane depends on the scale $\Lambda$, below which the theory remains perturbative, and the supersymmetry breaking scale $\Lambda_s$, that we fix to 1 TeV. In the minimal model with $\Lambda = 10^{16}$ GeV: $m_h < 130$ GeV and $m_t < 185$ GeV. In non-minimal models with an arbitrary number of gauge singlets and $\Lambda = 10^{16}$ GeV: $m_h < 145$ GeV and $m_t < 185$ GeV. We also consider supersymmetric standard models with arbitrary Higgs sectors. For models whose couplings saturate the scale $\Lambda = 10^{16}$ GeV we find $m_h < 155$ GeV and $m_t < 190$ GeV. As one pushes the saturation scale $\Lambda$ down to $\Lambda_s$, the bounds on $m_h$ and $m_t$ increase. For instance, in models with $\Lambda = 10$ TeV, the upper bounds for $m_h$ and $m_t$ go to 415 GeV and 385 GeV, respectively.

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The most outstanding challenge for present (Tevatron, LEP) and future (LEP-200, NLC, LHC, SSC) colliders is the discovery of the Higgs boson \[1\], which might confirm the standard model as the final theory of the electroweak interactions. However, though the standard model (SM) is in excellent agreement with all precision measurements at present energies \[1, 2\], extensions thereof are not excluded at higher scales. The most appealing of these extensions, which provides a technical solution to the hierarchy puzzle, is the supersymmetric standard model \[3\].

Supersymmetric models have well constrained Higgs sectors \[4\] which can provide crucial tests of them. In particular, the most constraining feature of the minimal supersymmetric standard model (MSSM) is the existence of an absolute upper bound on the tree-level mass of the $CP = +1$ lightest Higgs boson

\[
m_h \leq m_Z |\cos 2\beta|,
\]

where $\tan \beta \equiv v_2/v_1$, $v_i \equiv \langle H^0_i \rangle$. Therefore a negative result on the Higgs search would seem to exclude phenomenological supersymmetry at all making its search at future accelerators unnecessary. However relation (1) is spoiled by two effects: i) Radiative corrections, and ii) The enlargement of the Higgs sector in non-minimal supersymmetric standard models (NMSSMs). Only the simultaneous consideration of both effects can provide reliable bounds in general supersymmetric models.

Radiative corrections have been computed in the MSSM by different groups using different methods: standard diagrammatic techniques \[5\], the one-loop effective potential \[6\] and the renormalization group (RG) approach including one \[7\] and two \[8\] loop corrections. All approaches provide remarkably coincident results. The latter is reliable provided that

\[
\Lambda_s^2/m_W^2 \gg 1,
\]

where $\Lambda_s$ is the scale of supersymmetry breaking, since it amounts to a resummation of all leading logarithms in the effective potential. On the other hand this method is universal because we are assuming that the standard model holds below $\Lambda_s$ and the supersymmetric theory beyond $\Lambda_s$. Under the condition (2) supersymmetric particles decouple from the low-energy theory and the RG procedure is expected to provide
a good enough description of the radiative contribution to the lightest Higgs boson mass in a general supersymmetric standard model. Our definition of lightest Higgs boson is the $CP = +1$ bosonic state whose mass is not controlled by $\Lambda_s$ in the sense that it has a finite limit when $\Lambda_s \to \infty$. In that limit supersymmetry decouples and the latter state becomes the SM Higgs boson. We will include radiative corrections for $\Lambda_s = 1 \, TeV$ using the RG approach \cite{8}. The radiative squared mass $\Delta m^2_r$ is $\beta$-dependent and has to be added to the tree-level mass.

The tree-level bound (1) does not hold in NMSSMs. The case of the MSSM plus a singlet with coupling $\lambda$ to $H_1 \cdot H_2$ was first studied in \cite{9, 10} where a tree-level bound was found as

$$m^2_h \leq \left( \cos^2 2\beta + \frac{2\lambda^2 \cos^2 \theta_W}{g^2} \sin^2 2\beta \right) m^2_Z,$$

which is $\Lambda_s$-independent. From (3) we see that the bound on $m_h$ is linked to the bound on $\lambda$ if we require the theory to remain perturbative between $\Lambda_s$ and $\Lambda$. For $\Lambda = \Lambda_{GUT}$, the unification scale of gauge coupling constants, the bound (3) was studied in \cite{11, 12} and \cite{13}, where radiative corrections were properly included. The case of one extra singlet has been recently reconsidered in \cite{14}, where the dependence of $m_h$ on $\Lambda$ was studied; \cite{15}, where radiative corrections where considered in the effective potential approach; and \cite{16}, where comparison with the corresponding non-supersymmetric scenario was established. All of these results agree, when they overlap, with our previous calculation \cite{13} within less than 5%. More general models, e.g. the MSSM plus any number of singlets or three $SU(2)$ triplets (whose vacuum expectation values (VEVs) can respect the custodial symmetry at tree-level), were presented by ourselves in \cite{11, 13} and, more recently, also considered in \cite{17}.

In this paper we will present upper bounds on the lightest Higgs boson mass in a general class of models: supersymmetric standard models with an arbitrary Higgs sector. We will assume:
• Two doublets $H_1^{(1)}$, $H_2^{(1)}$, with hypercharges $Y = \pm 1/2$, coupled to quarks and leptons in the superpotential

$$f_m = h_t Q \cdot H_2^{(1)} U^c + h_b Q \cdot H_1^{(1)} D^c + h_L L \cdot H_1^{(1)} E^c,$$

plus an arbitrary number of extra pairs $H_1^{(j)}$, $H_2^{(j)}$, $j = 2, ..., d + 1$, decoupled from quarks and leptons in order to avoid dangerous flavor changing neutral currents [

• Gauge singlets $S^{(\sigma)}$, $\sigma = 1, ..., n_s$.

• $SU(2)$ triplets $\Sigma^{(a)}$, $a = 1, ..., t_o$, with $Y = 0$.

• $SU(2)$ triplets $\Psi^{(i)}_1$, $\Psi^{(i)}_2$, $i = 1, ..., t_1$, with $Y = \pm 1$.

Notice that the above extra Higgses are the only ones that can provide renormalizable couplings to $H_1^{(1)} \cdot H_2^{(1)}$, $H_1^{(1)} H_1^{(1)}$ and $H_2^{(1)} H_2^{(1)}$ in the superpotential. Other (more exotic) Higgs representations will only contribute to the $\beta$-functions of the gauge couplings and give lower values of the upper bound $m_h$. Since we are only interested in absolute upper bounds we can disregard them. Therefore the above class of extra Higgs fields defines the most general Higgs sector in supersymmetric standard models as far as the issue of putting upper bounds on their lightest Higgs boson mass is concerned.

The most general renormalizable superpotential for the above Higgses can be written as $f_h + g$ where:

$$f_h = \lambda_{ijs} H_1^{(i)} \cdot H_2^{(j)} S^{(s)} + \lambda_{ija} H_1^{(i)} \cdot \Sigma^{(a)} H_2^{(j)} + \chi_{1kb} H_1^{(j)} \cdot \Psi_1^{(b)} H_1^{(k)}$$

$$+ \chi_{2kb} H_2^{(j)} \cdot \Psi_2^{(b)} H_2^{(k)},$$

$$g = \lambda_{abk} tr(\Sigma^{(a)} \Psi_1^{(j)} \Psi_2^{(k)}) + \frac{1}{6} \kappa_{abc} tr(\Sigma^{(a)} \Sigma^{(b)} \Sigma^{(c)}) + \frac{1}{6} \kappa_{\sigma \mu} S^{(\sigma)} S^{(\mu)} S^{(\nu)}.$$

Summation over repeated indices is understood and traces are taken over matrix indices in the field decomposition:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^- \end{pmatrix},$$

(7)
\[
\Sigma = \begin{pmatrix}
\xi^o / \sqrt{2} & -\xi^+_2 \\
-\xi^-_2 & -\xi^o / \sqrt{2}
\end{pmatrix}, (8)
\]

\[
\Psi_1 = \begin{pmatrix}
\psi^+_1 / \sqrt{2} & -\psi^+_1 \\
\psi^-_1 & -\psi^+_1 / \sqrt{2}
\end{pmatrix}, (9)
\]

\[
\Psi_2 = \begin{pmatrix}
\psi^-_2 / \sqrt{2} & -\psi^-_2 \\
\psi^-_2 & -\psi^-_2 / \sqrt{2}
\end{pmatrix}. (10)
\]

where we have dropped the multiplicity indices.

By making a unitary transformation in \( j \) space to the Higgs doublets \( H^{(j)}_1, H^{(j)}_2 \) we can assume, without loss of generality, that only \( H^{(1)}_1 \) and \( H^{(1)}_2 \) take a non-zero VEV \[19\]. This requires the cancellation of various Yukawa couplings in eq. (5). In particular we find the condition

\[
\lambda^{1j\sigma}_1 = \lambda^{1j\alpha}_2 = \lambda^{1j\beta}_1 = \lambda^{1j\beta}_2 = 0 \ (j \neq 1), (11)
\]

that will be assumed from here on.

After imposing condition (11) we can write \( f_h = f + \ldots \), where the ellipsis involves only fields which do not acquire VEV, \( i.e. \langle f_h \rangle = \langle f \rangle \), as

\[
f = \vec{\lambda}_1 \cdot \vec{SH}^o_1 H^o_2 - \frac{\vec{\chi}_1}{\sqrt{2}} \cdot \vec{\xi}^o H^o_1 H^o_2 + \vec{\chi}_1 \cdot \vec{\psi}^o_1 H^o_1 H^o_1 + \vec{\chi}_2 \cdot \vec{\psi}^o_2 H^o_2 H^o_2. (12)
\]

Where we use the notation

\[
(\vec{\lambda}_1)^\sigma = \lambda^{11\sigma}_1,
\]

\[
(\vec{\lambda}_2)^a = \lambda^{11a}_2,
\]

\[
(\vec{\chi}_i)^b = \chi^{11b}_i, (13)
\]

\[
H^o_i = H^{(1)o}_i \ (i = 1, 2).
\]

Using the superpotential (12), and the fact that the smallest eigenvalue of a real, symmetric \( n \times n \) matrix is smaller than the smallest eigenvalue of the upper left \( 2 \times 2 \)
submatrix, one can easily prove that the lightest Higgs boson mass has an upper bound given by [11]

\[ m_h^2/v^2 \leq \frac{1}{2}(g^2 + g'^2) \cos^2 2\beta + \left(\tilde{\chi}_1^2 + \frac{1}{2} \tilde{\chi}_2^2\right) \sin^2 2\beta + \tilde{\chi}_1^2 \cos^4 \beta + \tilde{\chi}_2^2 \sin^4 \beta, \quad (14) \]

where \( v^2 \equiv v_1^2 + v_2^2 \) and \( g, g' \) are the \( SU(2) \times U(1)_Y \) gauge couplings. In particular the bound (3) is recovered when \( \tilde{\chi}_2 = \tilde{\chi}_1 = \tilde{\chi}_2 = 0 \) and the bound (1) in the MSSM when also \( \tilde{\chi}_1 = 0 \).

As it has been repeatedly noticed [11, 17] the bound (14) is independent of the soft-supersymmetry breaking parameters, or any supersymmetric mass terms. It is only controlled by \( v \) and dimensionless parameters \( (g, g', \tan \beta, \text{and Yukawa couplings}) \).

Since the former is fixed by the electroweak scale, the latter will determine the bound (14). In particular the upper bound on the right hand side of (14) comes from the requirement that the supersymmetric theory remains perturbative below \( \Lambda \). To guarantee this condition we need to solve the renormalization group equations (RGEs) of all gauge and Yukawa couplings of the theory.

In the general supersymmetric theory defined by the superpotential (4-6) with the condition (11), the relevant RGEs to the bound (14) can be written, for the Yukawa couplings as:

\[ 4\pi^2 \frac{d}{dt} \tilde{\chi}_1^2 = \left( -\frac{7}{2}g^2 - \frac{1}{2}g'^2 + \frac{3}{2} (h_t^2 + h_b^2) + \frac{3}{2} \tilde{\chi}_2^2 + 3(\tilde{\chi}_1^2 + \tilde{\chi}_2^2) + 2 \tilde{\chi}_1^2 \right) \tilde{\chi}_1^2 
+ \frac{1}{2} \sum_{k,l \neq 1} (\lambda_{\sigma}^{11} \lambda_{\sigma}^{kl})^2 + \frac{1}{4} \sum_{\mu,\nu} (\lambda_{\sigma}^{11} \kappa_{\sigma\mu\nu})^2, \quad (15) \]

\[ 4\pi^2 \frac{d}{dt} \tilde{\chi}_2^2 = \left( -\frac{7}{2}g^2 - \frac{1}{2}g'^2 + \frac{3}{2} (h_t^2 + h_b^2) + 2 \tilde{\chi}_2^2 + 3(\tilde{\chi}_1^2 + \tilde{\chi}_2^2) + \tilde{\chi}_1^2 \right) \tilde{\chi}_2^2 
+ \frac{1}{2} \sum_{k,l \neq 1} (\lambda_{a}^{11} \lambda_{a}^{kl})^2 + \frac{1}{2} \sum_{k,l} (\lambda_{a}^{11} \lambda_{akl})^2 + \frac{1}{4} \sum_{k,l} (\lambda_{a}^{11} \kappa_{akl})^2; \quad (16) \]

\[ 4\pi^2 \frac{d}{dt} \tilde{\chi}_1^2 = \left( -\frac{7}{2}g^2 - \frac{3}{2}g'^2 + 3h_b^2 + \frac{3}{2} \tilde{\chi}_2^2 + 7 \tilde{\chi}_1^2 + \tilde{\chi}_1^2 \right) \tilde{\chi}_1^2 
+ \frac{1}{2} \sum_{a,r} (\lambda_{11}^{11} \lambda_{air})^2 + \sum_{m,r \neq 1} \left( \lambda_{11}^{11} \lambda_{mri}^{11} \right)^2, \quad (17) \]
\[
4\pi^2 \frac{d\chi^2}{dt} = \left(-\frac{7}{2}g^2 - \frac{3}{2}g'^2 + 3h_t^2 + \frac{3}{2} \chi^2 + 7 \frac{\lambda_2^2}{2} + \frac{\lambda_1^2}{2}\right) \chi^2,
\]
\[
+ \frac{1}{2} \sum_{a,r} \left(\chi^{1i}_{a} \chi^{1i}_{a}\right)^2 + \sum_{m,r \neq 1} \left(\chi^{1i}_{m} \chi^{1i}_{r}\right)^2,
\]
(18)

\[
8\pi^2 \frac{dh_t}{dt} = \left(-\frac{3}{2}g^2 - \frac{13}{18}g'^2 - \frac{8}{3}g_s^2 + 3h_t^2 + \frac{1}{2}h_b^2 + \frac{3}{4} \lambda_2^2 + 3 \chi^2 + \frac{1}{2} \chi^2\right) h_t
\]
\[
+ \left(\frac{3}{4} \sum_{k \neq 1} \lambda_2^{k_2} \lambda_2^{k_2} + \frac{3}{4} \sum_{k \neq 1} \chi^{k_2}_{2} \chi^{k_2}_{2} + \frac{1}{2} \sum_{k \neq 1} \chi^{1k_2}_{1} \chi^{1k_2}_{1}\right) h_t,
\]
(19)

\[
8\pi^2 \frac{dh_b}{dt} = \left(-\frac{3}{2}g^2 - \frac{7}{18}g'^2 - \frac{8}{3}g_s^2 + 3h_t^2 + \frac{1}{2}h_b^2 + \frac{3}{4} \lambda_2^2 + 3 \chi^2 + \frac{1}{2} \chi^2\right) h_b
\]
\[
+ \left(\frac{3}{4} \sum_{k \neq 1} \lambda_2^{k_2} \lambda_2^{k_2} + \frac{3}{4} \sum_{k \neq 1} \chi^{k_2}_{2} \chi^{k_2}_{2} + \frac{1}{2} \sum_{k \neq 1} \chi^{1k_2}_{1} \chi^{1k_2}_{1}\right) h_b,
\]
(20)

and for the gauge couplings as:

\[
16\pi^2 \frac{dg}{dt} = (1 + 2t_0 + 4t_1 + d) g^3,
\]
(21)

\[
16\pi^2 \frac{dg'}{dt} = (11 + 6t_1 + d) g'^3,
\]
(22)

\[
16\pi^2 \frac{dg_s}{dt} = -3g_s^3,
\]
(23)

where \(g_s\) is the SU(3) gauge coupling. Notice that condition (11) is stable under the RGEs.

The \(\tau\)-Yukawa coupling, \(h_\tau\), will be neglected as compared to \(h_b\), since \(h_\tau/h_b = m_\tau/m_b\). The bottom Yukawa coupling can be important for \(\tan \beta \gg 1\) and will be kept along with the top Yukawa coupling. They are given by

\[
h_t = \frac{g}{\sqrt{2} m_w} \frac{m_t}{(1 + \cot^2 \beta)^{1/2}},
\]
(24)

\[
h_b = \frac{g}{\sqrt{2} m_w} \frac{m_b}{(1 + \tan^2 \beta)^{1/2}},
\]

\footnote{For simplicity we did not present the explicit RGEs for the couplings \(\lambda^{ij}_{1} \), \(\lambda^{ij}_{2} \), \(\lambda^{ij}_{1} \), \(\lambda^{ij}_{2} \), \(\lambda_{aik} \), \(\kappa_{abc} \) and \(\kappa_{\sigma \mu \nu} \).}
and fixed by the boundary conditions: $m_t(2m_t) = m_t$ and $m_b(2m_b) = 5 \text{ GeV}$. For the gauge couplings we will take the boundary conditions:

$$
\alpha_{EM}(M_Z) = \frac{1}{127.9}, \quad \sin^2 \theta_W(M_Z) = 0.23, \quad \alpha_s(M_Z) = 0.12.
$$

(25)

The Yukawa couplings involved in (14) will be let to acquire any perturbative value maximizing the bound (14). In general the bound (14) is maximized whenever some of the involved couplings saturate the scale $\Lambda$. A particular coupling $\lambda$ (gauge or Yukawa) is said to saturate a scale $\Lambda$ if

$$
\lambda^2(Q^2)/4\pi \leq 1,
$$

(26)

for $Q^2 \leq \Lambda^2$, and the equality in (26) holds for $Q^2 = \Lambda^2$.

Once we have the set of RGEs (15-23) we can systematically analyze the case of different supersymmetric standard models characterized by different Higgs sectors. The different cases are parametrized by the number of Higgs representations $(n_s, d, t_0, t_1)$. For simplicity we will assume $t_0 = t_1 \equiv t$ such that the custodial symmetry (and so the tree level value $\rho \equiv 1$) can be respected by means of a very simple relation between the VEVs of $\Sigma$, $\Psi_1$ and $\Psi_2$. In this way the gauge couplings will be parametrized by the single parameter

$$
N = 6t + d,
$$

(27)

and the bound will depend on $n_s$ and $N$. In all cases the bound will be a function of $\Lambda$, and so it is important to discuss the criteria to fix it.

- In theories with gauge coupling unification at the scale $\Lambda_{GUT}$, the natural choice is fixing $\Lambda = \Lambda_{GUT}$. The case satisfying this condition is the MSSM plus an arbitrary number of gauge singlets, i.e. $N = 0$, $n_s$ arbitrary. Of course putting $n_s = 0$ we recover the MSSM.

- In theories without gauge coupling unification, $N \neq 0$, we will assign to any scale $\Lambda$ the value of $N$ such that the gauge coupling $g$ saturates it \footnote{We can see from (21-23) and conditions (25-27) that every scale $\Lambda$ is always saturated by $g$: this means that $g$ is the first gauge coupling to go non-perturbative.}. Once we
have fixed $N$, the bound (14) is maximized for $n_s \neq 0$, and so we will assume there are gauge singlets. The actual value of the bound does not depend on the particular value of $n_s$, provided that $n_s > 0$.

The key observation to maximize the bound (14) is to notice that $\lambda_i^2$ and $\chi_i^2$ ($i = 1, 2$) are maximized by the values

$$
\begin{align*}
\lambda_i^{ij} &= \lambda_2^{ija} = \chi_1^{ijb} = \chi_2^{ijb} = 0 \quad (i, j \neq 1), \\
\lambda_{ajk} &= \kappa_{abc} = \kappa_{\sigma\mu\nu} = 0,
\end{align*}
$$

(28)
since they contribute to the renormalization of $\lambda_i^2$ and $\chi_i^2$ ($i = 1, 2$) with positive definite terms. Eq. (28) is stable under the RGEs, so we can impose it in order to obtain upper bounds on $m_h$. It might happen in specific models that we would need to departure from condition (28) to encompass the low energy experimental bounds. For instance, this could be the case when some of the couplings involved in (28) are necessary to explicitly break a global symmetry; in that case, to avoid a massless axion, we should put them to non-zero values. However, since we are dealing only with absolute upper bounds we will use condition (28) in the rest of this paper.

We have analyzed different cases characterized by different values of $\Lambda$. In Fig. 1 we plot $m_h$ as a function of $m_t$ for the model $N = 0, n_s > 0$ (i.e. the MSSM enlarged with Higgs singlets) and different values of $\tan \beta$. In Fig. 2 we plot the case saturating the scale $\Lambda = 10^{16}$ GeV, which corresponds to $N = 5$. The model with $t = 1, d = 0$ ($N = 6$), which saturates the scale $\Lambda = 10^{14}$ GeV, is shown in Fig. 3. We see that the value of $m_h$ increases as we let the saturation scale $\Lambda$ to go down to $\Lambda_s$. In particular we show in Fig. 4 the plot corresponding to $\Lambda = 10^{10}$ GeV ($N = 10$). Finally we show in Fig. 5 the allowed region in the $(m_h, m_t)$ plane for the MSSM (long-dashed); the model $N = 0, n_s > 0$ with $\Lambda = \Lambda_{GUT}$ (short-dashed), and different models saturating different scales, from $\Lambda = 10^{16}$ GeV to $\Lambda = 10$ TeV (solid).

In conclusion, we have computed upper bounds on the mass of the lightest Higgs boson in a general class of supersymmetric standard models, characterized by arbitrary Higgs sectors. For a given scale $\Lambda$ the corresponding upper bound is saturated
by the model whose gauge couplings become non-perturbative at that scale. We have taken the scale of supersymmetry breaking $\Lambda_s = 1\ TeV$ and assumed a supersymmetric theory between $\Lambda$ and $\Lambda_s$ and the standard model below $\Lambda_s$. In detailed studies of particular models, threshold effects of supersymmetric particles should be taken into account, as well as physical conditions on the Yukawa couplings that we are not considering (e.g. unbroken color and/or electric charge). These effects could slightly change our absolute upper bounds. Radiative corrections are taken into account using the (universal) renormalization group method. Diagrammatic techniques should give similar results provided that $\Lambda_s^2 \gg m_W^2$. (This has been explicitly checked in the simplest non-minimal model with a singlet.) For lower values of $\Lambda_s$ the detailed calculation should be done for every particular model. We did not analyze the dependence of the upper bounds on $\Lambda_s$ but we expect it not to be too dramatic provided $\Lambda_s$ is kept inside its phenomenological range. For all the above reasons our results should be taken as rough estimates to guide the eye in supersymmetric standard models. They are strongly dependent on the scale $\Lambda$. In fact for $\Lambda$ from $10^{16}\ GeV$ to $10^4\ GeV$ the upper bounds on $m_h$ range from 130 to 415 GeV, and the bounds on $m_t$ from 185 to 385 GeV. For $\Lambda = \Lambda_s$ the bound coincides with that in the non-supersymmetric standard model $[20]$.

We have not considered in this paper other possible generalizations of the MSSM that could, in turn, modify the upper bounds on the lightest Higgs boson mass. One of them is the introduction of extra colored multiplets, as e.g. extra (fractions of) generations. They would provide large radiative corrections, through the Yukawa couplings of the additional up quarks, to the Higgs boson mass for the case of a not-so-heavy top quark $[21]$. Another possible generalization is the presence of an extra gauge group $G$, with couplings $g_a$ and generators $T^a$ $[22]$, which would amount to a correction to the tree-level bound (14) as

$$\Delta m_h^2 = 2g_a^2 v^2 \sum_a \left( T^a_1 \cos^2 \beta - T^a_2 \sin^2 \beta \right)^2$$

(29)

where $T^a_i \equiv \langle H_i^{(1)} T^a H_i^{(1)} \rangle / v^2_i$. In that case the Yukawa couplings in (4) and (5) should respect the gauge symmetry $G$ and all the bounds could change dramatically.
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**Figure Captions**

**Fig. 1** Upper bounds on the lightest scalar Higgs boson in NMSSM with singlets and different values of $\tan \beta$ (solid).

**Fig. 2** The same as in Fig.1 but for a model saturating $\Lambda = 10^{16}$ GeV.

**Fig. 3** The same as in Fig.1 but for a model saturating $\Lambda = 10^{14}$ GeV.

**Fig. 4** The same as in Fig.1 but for a model saturating $\Lambda = 10^{10}$ GeV.

**Fig. 5** Allowed regions in the $(m_h, m_t)$ plane for different supersymmetric standard models. The solid curves correspond to models saturating the scales $\Lambda = 10^{16}, 10^{14}, 10^{10}, 10^{8}, 10^{6}$ and $10^{4}$ GeV.