Giant atoms with modulated transition frequency

Lei Du,1,2 Yan Zhang,3,* and Yong Li1,4,†

1 Center for Theoretical Physics and School of Science, Hainan University, Haikou 570228, China
2 Beijing Computational Science Research Center, Beijing 100193, China
3 Center for Quantum Sciences and School of Physics, Northeast Normal University, Changchun 130024, China
4 Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China

Giant atoms are known for the frequency-dependent spontaneous emission and associated interference effects. In this paper, we study the spontaneous emission dynamics of a two-level giant atom with dynamically modulated transition frequency. It is shown that the retarded feedback effect of the giant-atom system is greatly modified by a dynamical phase arising from the frequency modulation and the retardation effect itself. Interestingly, such a modification can in turn suppress the retarded feedback such that the giant atom behaves in some sense like a small one. By introducing an additional phase difference between the two atom-waveguide coupling paths, we also demonstrate the possibility of realizing chiral and tunable temporal profiles of the output fields. The results in this paper have potential applications in quantum information processing and quantum network engineering.

I. INTRODUCTION

Spontaneous emission is a basic and important process that arises from the interaction between an excited quantum system and the surrounding environment [1–3]. This process is typically irreversible and thus plays a negative role in quantum information processing, e.g., leading to the so-called quantum decoherence effect. On the other hand, controlling the spontaneous emission of an open quantum system has great significance for many applications, such as quantum switch engineering [4–9], high-frequency coherent light generation [10, 11], clock frequency estimation [14], and chiral quantum optics [12, 13]. As a result, the control of spontaneous emission has attracted a plethora of research interest, with common strategies including designing the density of states of the reservoir [15–21], changing the system-reservoir couplings [22–24], and using strong pulse sequences [25–27] or low-frequency coherent fields [28, 29], to name a few.

Besides the common methods mentioned above, it is also possible to couple a single quantum emitter to the environment (waveguide) at multiple discrete points. The spontaneous emission can be significantly modified, depending on both the transition frequency of the emitter and the spacing distances between different coupling points [30]. This scheme is closely related to the emerging quantum optical paradigm called “giant atoms”, where the size of the (artificial) atom can be much larger than the wavelength of the field it interacts with [31]. In this case, the propagation phases of the field between different coupling points should be considered, as they play a vital role in determining the spontaneous emission of the emitter. This is in some sense similar to an atom in front of a mirror [23, 32–36], but giant atoms allow for richer interference effects and more advanced scatterings [31]. To date, giant atoms have witnessed a series of intriguing quantum optical phenomena, such as decoherence-free atomic interactions [37–39], unconventional bound states [40–44], chiral quantum optics [42, 45, 46], synthetic dimension manipulation [47], and phase-dependent single-photon scatterings [48–54], and photon storage [55].

In this paper, we consider a two-level giant atom with modulated transition frequency. If the system is operated in the non-Markovian regime, where the propagation time of the field (e.g., photons) between the two atom-waveguide coupling points is nonnegligible compared with the relaxation time of the atom, the frequency modulation imprints a time-dependent modification on the retarded feedback and thereby changes the spontaneous emission dynamics of the system. The modification effect depends on both the concrete form of the frequency modulation and the propagation time between the two coupling points. Such a modification tends to disappear if the system enters the Markovian regime with negligible propagation time. As will be shown below, the combination of the frequency modulation and the giant-atom interference effect not only enables richer dynamics of the atom, but also shows the possibility of engineering chiral single-photon source with tunable output profiles.

II. MODEL AND METHOD

We consider in this paper a two-level giant atom whose transition frequency $\omega(t)$ is dynamically modulated around a constant value $\omega_0$ (i.e., $|\omega(t) - \omega_0| \ll \omega_0$). Experimentally, two-level systems with modulated transition frequencies can be implemented via, e.g., driven superconducting qubits [56, 57] or quantum dots [58]. Assuming that the atom is coupled to the one-dimensional
waveguide at two points \( x = 0 \) and \( x = d \), the Hamiltonian of the system can be written as \((\hbar = 1 \text{ hereafter})\)

\[
H(t) = \omega(t)\sigma_+\sigma_- + \int_{-\infty}^{+\infty} dk \omega_k a_k^\dagger a_k + \int_{-\infty}^{+\infty} dk \left[ g \left( 1 + e^{i\varphi_e e^{ikd}} \right) \sigma_+ a_k + \text{H.c.} \right],
\]

(1)

where \( \sigma_+ (\sigma_-) \) is the raising (lowering) operator of the two-level atom; \( a_k^\dagger (a_k) \) is the creation (annihilation) operator of the waveguide mode with frequency \( \omega_k \) and wave vector \( k \); \( g \) is the coupling coefficient between the atom and the waveguide, which is assumed to be \( k \)-independent and identical at the two coupling points. In Eq. (1), we have also introduced an additional phase difference \( \varphi \) between the two atom-waveguide coupling paths at \( x = 0 \) and \( x = d \), which can be achieved experimentally via some artificial methods \([42, 46, 59–62]\). Such a phase difference mimics a synthetic gauge field so that it imprints a momentum kick on the emitted photons. In this case, the atom-waveguide interaction and thereby the spontaneous emission of the atom becomes chiral \([63]\) (although the photon scatterings are still symmetric if other decay channels are negligible \([49, 54]\)).

It is clear that the total excitation number of the system, which is defined by the operator \( N = \int dk \omega_k a_k^\dagger a_k + \sigma_+\sigma_- \), is conserved due to \( [N, H(t)] = 0 \). Therefore, if the system is initialized in a single-excitation state, the state of the system at time \( t > 0 \) can be written as

\[
|\psi(t)\rangle = \int_{-\infty}^{+\infty} dk c_k(t)a_k^\dagger e^{-i\omega_k t}|G\rangle + c_e(t)\sigma_+ e^{-i\int_0^t dt'\omega(t')}|G\rangle,
\]

(2)

where \( c_k(t) [c_e(t)] \) is the probability amplitude of creating a photon with wave vector \( k \) in the waveguide (of the atom in the excited state); \( |G\rangle \) denotes the ground state of the whole system. In this paper, we focus on the spontaneous emission dynamics of the modulated giant atom, thus the initial state is always assumed to be \( |\psi(0)\rangle = \sigma_+|G\rangle \), i.e., the atom is in the excited state and the waveguide is empty at the initial time. By solving the Schrödinger equation, one obtains

\[
\dot{c}_k(t) = -i \int_{-\infty}^{+\infty} dk g \left( 1 + e^{i\varphi_e e^{ikd}} \right) c_k(t) \times e^{-i\omega_k t} e^{i\int_0^t dt'\omega(t')},
\]

(3)

\[
\dot{c}_e(t) = -ig \left( 1 + e^{-i\varphi e^{-ikd}} \right) c_e(t) \times e^{i\omega_k t} e^{-i\int_0^t dt'\omega(t')}.
\]

(4)

Substituting the formal solution of Eq. (4), i.e.,

\[
c_k(t) = -i \int_0^t dt' g \left( 1 + e^{-i\varphi e^{-ikd}} \right) \times c_e(t') e^{i\omega_k t'} e^{-i\int_0^{t'} dt''\omega(t'')},
\]

(5)

into Eq. (3), one arrives at

\[
\dot{c}_e(t) = -2 \int_0^t dt' \int_{-\infty}^{+\infty} dk g^2 \left[ 1 + \cos(\varphi + kd) \right]
\]

\[
\times c_e(t') e^{-i\omega_k(t-t')} e^{i\int_0^{t'} dt''\omega(t'')},
\]

(6)

Note that we have taken \( c_k(0) = 0 \) in Eq. (5) due to the initial state \( |\psi(0)\rangle = \sigma_+|G\rangle \) and exchanged the integration order in Eq. (6) as usual \([3]\). By assuming \( \omega_k \approx \omega_0 + \nu = \omega_0 + (k - k_0)v_g \) \([4, 64]\), where \( v_g \) \((k_0)\) is the group velocity (wave vector) of the field at frequency \( \omega_0 \), and changing the integration variable as \( \int_{-\infty}^{+\infty} dk \rightarrow 2 \int_{-\infty}^{+\infty} dv / v_g \), Eq. (6) becomes

\[
\dot{c}_e(t) = -\frac{2g^2}{v_g} \int_0^t dt' \int_{-\infty}^{+\infty} dv' \left[ 2 + \cos \varphi \left( e^{-ikd} e^{i\omega_0 t'} + e^{ikd} e^{-i\omega_0 t'} \right) \right]
\]

\[
\times c_e(t') e^{-i(\omega_0 + \nu)(t-t')} e^{i\int_0^{t'} dt''\omega(t'')},
\]

(7)

with

\[
\phi(t, \tau) = \phi_0 - \omega_0 \tau + \int_{t-\tau}^t dt'\omega(t'),
\]

(8)

where \( \Gamma = 4\pi g^2 / v_g \) is the radiative decay rate of the atom; \( \phi_0 = k_0d \) describes a static phase accumulation that exists even without modulations \([65, 66]\); \( \tau = d / v_g \) is the time delay (propagation time) of photons traveling between the two coupling points; \( \Theta(x) \) is the Heaviside step function. Clearly, Eq. (7) describes the non-Markovian dynamics of the giant atom with a retarded coherent feedback. In contrast to the common situation where the transition frequency of the atom is constant and there is no additional phase difference between the two coupling points, the present model has two interesting hallmarks: (i) the retarded feedback term contains a dynamical phase \( \phi(t, \tau) \) that is determined by the concrete form of \( \omega(t) \) as well as the value of \( \tau \); (ii) the amplitude of the feedback term is further modified by the additional phase difference \( \varphi \) in terms of a cosine func-
tion. Note that $\phi(t, \tau)$ becomes trivial if $\tau$ is exactly zero since in this case the model reduces to a small-atom system with $d = 0$. Moreover, we have assumed that $\omega(t)$ is always in the vicinity of $\omega_0$, justifying the linearized dispersion relation of the waveguide employed above.

In the case of $\omega(t) \equiv \omega_0$ and $\varphi = 0$, Eq. (7) becomes

$$\dot{c}_e(t) = -\Gamma c_e(t) - \Gamma c_e(t - \tau)e^{i\phi_0}(t - \tau), \quad (9)$$

which recovers the dynamic equation of a common giant atom that has been studied in detail [65, 66]. In this case, the retardation effect simply postpones the onset of the giant-atom interference effect determined by the value of $\phi_0$. For example, partial decay of the atom can be observed if $\phi_0 = (2m+1)\pi$ ($m$ is an arbitrary integer). This can be seen by solving the Laplace transformation of Eq. (9) and using the final value theorem, which yields $c_e(t \to +\infty) = 1/(1 + \Gamma \tau)$ in this case. This phenomenon cannot be observed if $\text{mod}(\varphi, \pi) \neq 0$, since the retarded feedback (whose amplitude is modified by $\cos \varphi$) cannot cancel the instantaneous decay of the atom exactly in this case.

III. CONTROLLABLE SPONTANEOUS EMISSION

In this section, we consider a simple cosine-type modulation

$$\omega(t) = \omega_0 + \alpha \cos (\Omega t + \theta) \quad (10)$$

around the background frequency $\omega_0$, where $\alpha$, $\Omega$, and $\theta$ are the amplitude, frequency, and initial phase of the modulation, respectively. In this case, the dynamical phase $\phi(t, \tau)$ can be written as

$$\phi(t, \tau) = \phi_0 + \chi [\sin (\Omega t + \theta) - \sin (\Omega t - \Omega \tau + \theta)] \quad (11)$$

with $\chi = \alpha/\Omega$ the modulation depth [67]. Clearly, the spontaneous emission dynamics of the giant atom can be controlled by tuning $\Omega$, $\chi$, and $\theta$. To study the non-Markovian retardation effect, we consider a finite time delay ($\Gamma \tau = 0.2$) that is nonnegligible compared with the relaxation time of the atom. Moreover, we assume $\varphi = 0$ tentatively in order to demonstrate the effect of merely the frequency modulation. We will discuss the influence of $\varphi$ on the spontaneous emission dynamics of the atom at the end of this section and on the output fields of the model in the next section.

We first focus on the influence of the modulation frequency and plot in Fig. 1 the dynamic evolutions of the atomic population probability $P_\epsilon(t) = |c_e(t)|^2$ for different values of $\Omega$. For $\phi_0 = (2m+1)\pi$, as shown in Fig. 1(a), the cosine-type modulation markedly modifies the dynamics of the atom. When $\Omega \tau = 2n\pi$ ($n$ is another arbitrary integer that is in general unequal to $m$), the spontaneous emission of the atom is inhibited after $t = \tau$, just as it would be in the absence of modulations (see the coincident blue solid and yellow dashed lines). As $\Omega \tau$ approaches $(2n-1)\pi$, the decay of the atom tends to be exponential-like but with a slight oscillation. This can be well understood from Eq. (11): when $\Omega \tau = 2n\pi$, $\phi(t, \tau) \equiv \phi_0$ becomes time independent as if there were no modulation; for other cases of $\Omega \tau \neq 2n\pi$, $\phi(t, \tau)$ changes periodically in time due to the cosine-type modulation. Moreover, as shown in Fig. 1(b), one can tune the dynamic evolution of the atom between the superradiant-like form (arising from the constructive interference between the two atom-waveguide coupling paths; see the blue solid and yellow dashed lines) and the typical exponential form (corresponding to a small atom with a double decay rate $2\Gamma$; see the gray solid line) in the case of $\phi_0 = 2m\pi$.

It is also clear from Eq. (11) that the modulation effect is strongly influenced by the modulation depth $\chi$. If $\chi$ is very small, the modulation effect is limited because the dynamical part of $\phi(t, \tau)$ changes within the finite range $[-2\chi, 2\chi]$. This is also why the spontaneous emission of the atom cannot be further boosted (suppressed) in Fig. 1(a) [Fig. 1(b)]. In view of this, we examine in Figs. 2(a) and 2(b) the evolutions of $P_\epsilon(t)$ for $\Omega \tau = (2n+1)\pi$ and different values of $\chi$ [here we limit ourselves to the case of $\chi \in [0, 2]$, the reason of which can be understood from the result in Fig. 2(c)]. As shown in Fig. 2(a), when $\phi_0 = (2m+1)\pi$, the atom exhibits a nearly linear decay for small $\chi$ (see, e.g., the blue solid and red dot-dashed lines), whereas the atomic decay becomes exponential-like as $\chi$ increases (see, e.g., the green dotted and yellow dashed lines). This suggests a way to engineer richer spontaneous emission dynamics for quantum emitters. Similarly, when $\phi_0 = 2m\pi$ as shown in
FIG. 2. (a)-(b) Dynamic evolutions of atomic population probability \( P_e(t) \) with different values of \( \chi \) and \( \phi_0 \). Panels (a) and (b) share the same legend. (c) \( P_e(t = 2/\Gamma) \) as a function of \( \chi \) for \( \phi_0 = 2m\pi \). (d) Dynamic evolutions of atomic population probability \( P_e(t) \) with different values of \( \theta \) and \( \phi_0 = 2m\pi \). We assume \( \theta = 0 \) in panels (a)-(c) and \( \chi = 1 \) in panel (d). The vertical dotted lines in (a), (b), and (d) correspond to \( t = \tau \) as those in Fig. 1. Other parameters are \( \tau\Gamma = 0.2 \), \( \Omega/\Gamma = 5\pi \), and \( \varphi = 0 \). In Fig. 2(b), the atomic decay can be further suppressed upon increasing \( \chi \) properly. The modulation effect, however, does not keep growing if we further increase \( \chi \) (i.e., \( \chi > 2 \)). As shown in Fig. 2(c), the atomic population \( P_e(t = 2/\Gamma) \) exhibits a damped oscillation as \( \chi \) increases and the maximum is found around \( \chi = 2 \). Such a non-monotonic behavior can be understood from the Jacobi-Anger extension of the dynamical phase factor, which will be discussed in detail below. Moreover, we plot the evolutions of \( P_e(t) \) for different values of modulation phase in Fig. 2(d), which shows that the spontaneous emission of the atom is quite insensitive to \( \theta \). Tuning \( \theta \) only leads to a slight phase shift for the oscillating evolution curve.

Before proceeding, we would like to point out that the above modulation effects tend to disappear as the time delay \( \tau \) decreases gradually. This can be seen clearly from Eq. (11): \( \phi(t, \tau) \approx \phi_0 \) if \( \tau \) is much smaller than the other timescales. In view of this, the controllable spontaneous emission here is closely related to the non-Markovian retardation effect arising from the giant-atom structure. Moreover, our scheme is quite different from that in Ref. [68], where a structured reservoir with a narrow band is required in order to suppress the spontaneous emission of a modulated small atom. In our scheme the frequency spectrum of the waveguide modes can be very broad and flat.

It has been shown in Refs. [66, 69] that a common giant atom described by Eq. (9) enables periodic population revivals in the deep non-Markovian regime (i.e., \( \Gamma\tau \gg 1 \)), with adjacent revivals equally spaced by \( \tau \). This non-Markovian effect, however, can be markedly suppressed by the cosine-type frequency modulation. In Fig. 3, we demonstrate the long-time evolutions of \( P_e(t) \) with large enough \( \tau \) and different modulation parameters. As shown in Fig. 3(a), the atom shows evident population revivals in the absence of modulations (i.e., \( \chi = 0 \)), but the revivals tend to fade as the modulation depth \( \chi \) grows. This can be understood from the dynamical phase factor \( F = \exp[i\phi(t, \tau)] \), which dynamically modifies the retarded feedback term in Eq. (7). The evolutions of the real part of \( F \) with different values of \( \chi \) are plotted in Fig. 3(b) (the imaginary part of \( F \) shows similar evolutions, which are not demonstrated here). For small \( \chi \), the dynamical phase factor \( F \) changes slowly within a small range that is away from zero. For large \( \chi \) (with \( \Omega \) remaining invariant), however, the retarded feedback term (with the prefactor \( F \)) oscillates rapidly with a nearly vanishing average contribution, such that the model behaves in some sense like a small atom with negligible population revivals. In view of this, the result in Fig. 3(a) shows the possibility of protecting quantum emitters from unwanted environment backactions.

Moreover, we would like to interpret this result with the Jacobi-Anger expansion

\[
\exp[-i\chi \sin(\Omega t)] = \sum_{q=-\infty}^{+\infty} J_q(\chi)e^{-iq\Omega t},
\]

where \( J_q(\chi) \) is the Bessel function of the first kind. Clearly, the influence of the dynamical phase factor \( F \) becomes negligible if \( \chi \) is large enough because all \( J_q(\chi) \) are small for \( \chi \to +\infty \). This is also why in Fig. 2(c) the modulation effect exhibits a damped oscillating as \( \chi \) increases. It is worth pointing out that the suppression of the population revivals cannot be achieved if \( \Omega\tau = 2n\pi \) since the modulation effect disappears in this case.
We also plot in Fig. 3(c) the evolutions of $P_e(t)$ in the deep non-Markovian regime with two very different values of $\Omega$, i.e., $\Omega/\Gamma = 5.5\pi$ and $80.3\pi$. The two evolution curves show good agreement, illustrating that the population revivals cannot be eradicated by using larger modulation frequency (here we fix the value of $\chi$ by changing $\alpha$ and $\Omega$ simultaneously, otherwise $\chi$ should decrease as $\Omega$ increases, leading to negligible modulation effects for large enough $\Omega$). This can be understood again from the evolutions of Re($\tilde{F}$) shown in Fig. 3(d): the dynamical phase factor oscillates faster for higher modulation frequency, yet its average contribution is almost unchanged. Although we have used $\Omega \tau = (2n + 1)\pi$ in Figs. 3(c) and 3(d), the conclusion here also holds for other values of $\Omega \tau$. This can be seen from the inset in Fig. 3(c), where $P_e(t = 11/\Gamma)$ changes slightly with $\Omega$ in a periodic manner.

Before moving to the next section, we would like to briefly discuss the influence of $\varphi$ on the spontaneous emission dynamics of the atom. It is clear from Eq. (7) that the feedback term is modified by $\varphi$ in terms of a cosine function: the amplitude of the feedback term becomes $\Gamma \cos \varphi$ in this case. In other words, the phase difference between the two atom-waveguide coupling paths alters the effective decay rate of the atom rather than introducing any new physics to the decay dynamics. When $\text{mod}((\varphi, 2\pi) \neq 0$, the results above can also be observed by tuning other parameters such as the atom-waveguide coupling strengths (the amplitude of the feedback term can differ from that of the instantaneous term if the coupling strengths at the two coupling points are different [70]). However, as will be seen in the next section, such a phase difference enables an effective chiral interaction between the atom and the waveguide field and thereby leads to chiral output fields.

IV. CHIRAL AND TUNABLE OUTPUT FIELDS

In experiments, the spontaneous emission of the (giant) atom can be examined by measuring the output fields at the ports of the waveguide. In view of this, it is convenient to transform the field amplitude $c_k(t)$ to the real space via

$$c(x, t) = \frac{1}{\sqrt{2\pi}} \int dk c_k(t) e^{ikx},$$

whose square modulus can be measured by a photon detector placed at position $x$. To derive the real-space field amplitude in Eq. (13), we rewrite Eqs. (3) and (4) as

$$\dot{c}_e = -i\omega(t)c_e - i \int_{-\infty}^{+\infty} dk g \left(1 + e^{i\varphi} e^{ikd}\right) c_k, \quad (14)$$

$$\dot{c}_k = -i\omega_k c_k - ig \left(1 + e^{-i\varphi} e^{-ikd}\right) c_e. \quad (15)$$

Substituting the formal solution of $c_k(t)$, i.e.,

$$c_k(t) = -i \int_0^t dt' g \left(1 + e^{-i\varphi} e^{-ikd}\right) c_e(t') e^{-i\omega_k(t-t')}, \quad (16)$$

into Eq. (13), one has

$$c(x, t) = \frac{-ig}{\sqrt{2\pi}} \int_0^t dt' \int dk \left(1 + e^{-i\varphi} e^{-ikd}\right) e^{ikx} c_e(t') e^{-i\omega_k(t-t')}$$

$$= \frac{-i\sqrt{2\pi}g}{v_g} \int_0^t dt' \left[ e^{ikx} \delta(t - t' - x/v_g) + e^{-ik\omega_k}(t - t' + x/v_g) + e^{-i\varphi} e^{ik(x-d)\delta(t - t' - x/v_g + d/v_g)}\right.$$  
$$
+ e^{-i\varphi} e^{-ik(x-d)\delta(t - t' + x/v_g - d/v_g)} \right] c_e(t') e^{-i\omega_k(t-t')}$$

$$= \frac{-i\sqrt{2\pi}g}{v_g} \left[ e^{i\varphi}(k_0 - \omega_0/v_g) c_e(x - t/v_g) \Theta(t + x/v_g) \Theta(-x) \Theta(t + x/v_g) + e^{-i\varphi}(k_0 - \omega_0/v_g) c_e(x + t/v_g) \Theta(x - d) \Theta(t - x/v_g + d/v_g)\right.$$  
$$
+ e^{-i\varphi} e^{-i\varphi}(k_0 - \omega_0/v_g) c_e(x + t/v_g - d/v_g) \Theta(d - x) \Theta(t + x/v_g - d/v_g)$$

$$= \frac{-i\sqrt{2\pi}g}{v_g} \left[ e^{i\varphi}(k_0 - \omega_0/v_g) c_e(x - t/v_g) \Theta(t + x/v_g) \Theta(-x) \Theta(t + x/v_g) + e^{-i\varphi}(k_0 - \omega_0/v_g) c_e(x + t/v_g) \Theta(x - d) \Theta(t - x/v_g + d/v_g)\right.$$  
$$
+ e^{-i\varphi} e^{-i\varphi}(k_0 - \omega_0/v_g) c_e(x + t/v_g - d/v_g) \Theta(d - x) \Theta(t + x/v_g - d/v_g)$$

$$= \frac{-i\sqrt{2\pi}g}{v_g} \left[ e^{i\varphi}(k_0 - \omega_0/v_g) c_e(x - t/v_g) \Theta(t + x/v_g) \Theta(-x) \Theta(t + x/v_g) + e^{-i\varphi}(k_0 - \omega_0/v_g) c_e(x + t/v_g) \Theta(x - d) \Theta(t - x/v_g + d/v_g)\right.$$  
$$
+ e^{-i\varphi} e^{-i\varphi}(k_0 - \omega_0/v_g) c_e(x + t/v_g - d/v_g) \Theta(d - x) \Theta(t + x/v_g - d/v_g)$$

If the photon detector is located at the right side of the giant atom, i.e., $x = d + l$ ($l > 0$), we have

$$|c_R(t)| = |c(x = d + l, t)| = \frac{\sqrt{2\pi}g}{v_g} \left| c_e(t) \Theta(t) \right.$$  
$$+ e^{i\varphi} c_e(-t - \tau) \Theta(-t - \tau) \right|,$$  

while if the detector is located at the left side of the atom, i.e., $x = -l$, we have

$$|c_L(t)| = |c(x = -l, t)| = \frac{\sqrt{2\pi}g}{v_g} \left[ c_e(t) \Theta(t) \right.$$  
$$+ e^{i\varphi} c_e(-t - \tau) \Theta(-t - \tau) \right|.$$  

In Eqs. (18) and (19), we have assumed $\bar{t} = t - l/v_g$ and $\varphi_0 = k_0d - \omega_0d/v_g = \varphi_0 - \omega_0\tau$. Note that $v_g =$
depicts the dynamic evolutions of the left and right output intensities (in units of $\Gamma/2v_g$) versus $t - \tau$ with different values of modulation parameters (cosine-type modulation). We assume $\chi = 1$ in panels (a)-(c) and $\varphi = \pi/2$ in panels (d)-(f). Other parameters are $\phi_0 = (2m + 1)\pi$, $\phi_0 = (2n + 1/2)\pi$, $\tau\Gamma = 0.2$, $\Omega/\Gamma = 5\pi$, and $\theta = 0$. We first examine the evolutions of the left and right output intensities $|c_L|^2$ and $|c_R|^2$ versus a renormalized time scale $t - \tau$ with different parameters in Eq. (10) (for $t < \tau$, the emitted photon cannot be detected, while for $0 < t < \tau$, the output fields are always symmetric and exhibit no modulation effect since the retarded feedback has not come into effect). As shown in Figs. 4(a)-4(c), the output fields are symmetric (achiral) if $\varphi = 0$, otherwise the output fields become chiral with the chiral effect being more evident if mod($\varphi, 2\pi$) = mod($\phi_0', 2\pi$). The output fields exhibit oscillating temporal profiles due to the cosine-type modulation [34]. The case without modulation is demonstrated in Fig. 4(d), where the right output is considerably suppressed and the left one shows an exponentially damped profile. Moreover, as shown in Figs. 4(d)-4(f), the modulation depth $\chi$ plays an important role for harnessing the profiles of the output fields, without affecting the chirality. We point out that changing the modulation phase $\theta$ leads to a slight shift of the profiles along the time axis (not shown here), which provides an additional tunability of the output fields.

Besides the cosine-type modulation discussed above, one can also consider aperiodic modulations to engineer richer chiral output profiles. For example, we now consider a linear frequency modulation in the form of $\omega(t) = \omega_0 + \beta t$, with $\beta$ being the modulation rate and having the dimension of Hz$^2$. Although $\omega(t)$ is not a bounded function in this case, we limit ourselves to the case of small $\beta$ and short evolution time to ensure that $\beta t \ll \omega_0$. Now $\phi(t, \tau)$ can be written as

$$\phi(t, \tau) = \phi_0 + \beta\tau \left( t - \frac{\tau}{2} \right).$$

Clearly, the dynamical phase depends linearly on $t$, with the prefactor determined by both $\beta$ and $\tau$. Therefore we consider in this case a relatively larger $\tau$ to achieve stronger modulation effects.

Figure 5 depicts the dynamic evolutions of the left and right output intensities versus $t - \tau$ with different values of $\beta$. It is clear that the chiral temporal profiles of the output fields can be engineered upon tuning $\beta$. In particular, the profile of the right output field can be adjusted from an exponential-like shape to a Gaussian-like shape as $\beta$ changes. Moreover, the overall intensity of the left (right) output field reduces (grows) gradually with the increase of $\beta$. As a result, aperiodic frequency modulations provide richer schemes for tuning the chirality of the output dynamics.

**V. CONCLUSIONS**

In summary, we have studied the spontaneous emission dynamics of a two-level giant atom with modulated transition frequency. We have revealed that the non-Markovian retardation effect, which stems from the non-negligible time delay of photons traveling between different coupling points, endows the giant-atom interference effect with a dynamical modification. This thus allows for controlling the spontaneous emission of the atom, depending on both the concrete form of the frequency modulation and the value of the time delay. As an example, we have considered a cosine-type frequency modulation and studied in detail its influence on the dynamic evolutions of the atomic population. Based on the controllable spontaneous emission, we have also demonstrated how to engineer chiral output fields with tunable temporal profiles. This can be achieved by introducing an additional phase difference between the two atom-waveguide coupling coefficients and using various modulation schemes. The results in this paper can be immediately extended to situations of a single multilevel giant atom [30, 48, 49].
and multiple correlated two-level giant atoms \cite{37, 38, 74}, where richer quantum interferences and collective effects can be expected. For the latter situation, it is also possible to achieve efficient dipole-dipole interactions with appropriate frequency modulations even if the atoms are detuned from each other \cite{75}.

[1] G. S. Agarwal, Quantum Statistical Theories of Spontaneous Emission and their Relation to Other Approaches (Springer, Berlin, 1974).
[2] H. J. Carmichael, An Open Systems Approach to Quantum Optics (Springer-Verlag, Berlin, 1993).
[3] C. W. Gardiner and P. Zoller, Quantum Noise, 2nd ed. (Springer, Berlin, 2000).
[4] J.-T. Shen and S. Fan, Coherent Single Photon Transport in a One-Dimensional Waveguide Coupled with Superconducting Quantum Bits, Phys. Rev. Lett. 95, 233001 (2005).
[5] L. Zhou, Z. R. Gong, Y.-x. Liu, C. P. Sun, and F. Nori, Controllable Scattering of a Single Photon inside a One-Dimensional Resonator Waveguide, Phys. Rev. Lett. 101, 100501 (2008).
[6] L. Zhou, L.-P. Yang, Y. Li, and C. P. Sun, Quantum Routing of Single Photons with a Cyclic Three-Level System, Phys. Rev. Lett. 111, 103604 (2013).
[7] H. Zheng, D. J. Gauthier, and H. U. Baranger, Waveguide-QED-Based Photonic Quantum Computation, Phys. Rev. Lett. 111, 090502 (2013).
[8] C. Gonzalez-Ballestero, E. Moreno, F. J. Garcia-Vidal, and A. Gonzalez-Tudela, Nonreciprocal few-photon routing schemes based on chiral waveguide-emitter couplings, Phys. Rev. A 94, 063817 (2016).
[9] M. Mirhosseini, E. Kim, X. Zhang, A. Sipahigil, P. B. Dieterle, A. J. Keller, A. Asenjo-Garcia, D. E. Chang, and O. Painter, Cavity quantum electrodynamics with atom-like mirrors, Nature (London) 569, 692 (2019).
[10] B. W. Adams, C. Buth, S. Cavaletto, J. Evers, Z. Harman, C. H. Keitel, A. Palfly, A. Picon, R. Röhlsberger, Y. Rostovtsev, and K. Tamasaku, X-ray quantum optics, J. Mod. Opt. 60, 2 (2013).
[11] S. Cavaletto, Z. Harman, C. Ott, C. Buth, T. Pfeifer, and C. H. Keitel, Broadband high-resolution X-ray frequency combs, Nat. Phys. 8, 520 (2014).
[12] I. Söllner, S. Mahmoodian, S. L. Hansen, L. Midolo, A. Javadi, G. Kirsanske, T. Pregnolato, H. El-Ella, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, Deterministic photon-emitter coupling in chiral photonic circuits, Nat. Nanotechnol. 10, 775 (2015).
[13] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, Nature (London) 541, 473 (2017).
[14] X.-Z. Qin, J.-H. Huang, H.-H. Zhong, and C. Lee, Clock frequency estimation under spontaneous emission, Front. Phys. 13, 130302 (2018).
[15] D. Kleppner, Inhibited Spontaneous Emission, Phys. Rev. Lett. 47, 233 (1981).
[16] W. Jhe, A. Anderson, E. A. Hinds, D. Meschede, L. Moi, and S. Haroche, Suppression of spontaneous decay at optical frequencies: Test of vacuum-field anisotropy in confined space, Phys. Rev. Lett. 58, 1497 (1987).
[17] D. J. Heinzen, J. J. Childs, J. E. Thomas, and M. S. Feld, Enhanced and inhibited visible spontaneous emission by atoms in a confocal resonator, Phys. Rev. Lett. 58, 1320 (1987).
[18] E. Yablonovitch, Inhibited Spontaneous Emission in Solid-State Physics and Electronics, Phys. Rev. Lett. 58, 2059 (1987).
[19] P. Lambropoulos, G. M. Nikolopoulos, T. R. Nielsen, and S. Bay, Fundamental quantum optics in structured reservoirs, Rep. Prog. Phys. 63, 455 (2000).
[20] P. Lodahl, A. F. van Driel, I. S. Nikolaev, A. Irman, K. Overvaag, D. Vannakselbergh, and W. L. Vos, Controlling the dynamics of spontaneous emission from quantum dots by photonic crystals, Nature (London) 430, 654 (2004).
[21] S. Noda, M. Fujita, and T. Asano, Spontaneous-emission control by photonic crystals and nanocavities, Nat. Photonics 1, 449 (2007).
[22] A. G. Kofman and G. Kurizki, Universal Dynamical Control of Quantum Mechanical Decay: Modulation of the Coupling to the Continuum, Phys. Rev. Lett. 87, 270405 (2001).
[23] U. Dorner and P. Zoller, Laser-driven atoms in half-cavities, Phys. Rev. A 66, 023816 (2002).
[24] M. Kiffner, M. Macovei, J. Evers, and C. H. Keitel, Vacuum-Induced Processes in Multilevel Atoms, Prog. Opt. 55, 85 (2010).
[25] L. Viola and S. Lloyd, Dynamical suppression of decoherence in two-state quantum systems, Phys. Rev. A 58, 2733 (1998).
[26] A. G. Kofman and G. Kurizki, Acceleration of quantum decay processes by frequent observations, Nature (London) 405, 546 (2000).
[27] D. Dhar, L. K. Grover, and S. M. Roy, Preserving Quantum States using Inverting Pulses: A Super-Zeno Effect, Phys. Rev. Lett. 96, 100405 (2006).
[28] J. Evers and C. H. Keitel, Spontaneous-Emission Suppression on Arbitrary Atomic Transitions, Phys. Rev. Lett. 89, 163601 (2002).
[29] U. Akram, J. Evers, and C. H Keitel, Multiphoton quantum interference on a dipole-forbidden transition, J. Phys. B 38, L69 (2005).
[30] A. F. Kockum, P. Delsing, and G. Johansson, Designing frequency-dependent relaxation rates and Lamb shifts for a giant atomic transition, Phys. Rev. A 90, 013837 (2014).
[31] A. F. Kockum, Quantum optics with giant atoms—the first five years, in Mathematics for Industry (Springer, Singapore, 2021), pp: 125-146.
[32] H. Dong, Z. R. Gong, H. Ian, L. Zhou, and C. P. Sun, Intrinsic cavity QED and emergent quasinormal modes for a single photon, Phys. Rev. A 79, 063847 (2009).

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[33] M. Bradford and J.-T. Shen, Spontaneous emission in cavity QED with a terminated waveguide, Phys. Rev. A 87, 063830 (2013).

[34] T. Tufarelli, F. Ciccarello, and M. S. Kim, Dynamics of spontaneous emission in a single-photon waveguide, Phys. Rev. A 87, 013820 (2013).

[35] T. Tufarelli, M. S. Kim, and F. Ciccarello, Non-Markovianity of a quantum emitter in front of a mirror, Phys. Rev. A 90, 012113 (2014).

[36] G. Calajó, Y.-L. L. Fang, H. U. Baranger, and F. Ciccarello, Exciting a Bound State in the Continuum through Multiphoton Scattering Plus Delayed Quantum Feedback, Phys. Rev. Lett. 122, 073601 (2019).

[37] A. F. Kockum, G. Johansson, and F. Nori, Decoherence-Free Interaction between Giant Atoms in Waveguide Quantum Electrodynamics, Phys. Rev. Lett. 120, 140404 (2018).

[38] B. Kannan, M. Ruckriegel, D. Campbell, A. F. Kockum, J. Braumüller, D. Kim, M. Kjaergaard, P. Krantz, A. Melville, B. M. Niedzielski, A. Vepsäläinen, R. Winik, J. Yoder, F. Nori, T. P. Orlando, S. Gustavsson, and W. D. Oliver, Waveguide quantum electrodynamics with superconducting artificial giant atoms, Nature (London) 583, 775-779 (2020).

[39] A. Carollo, D. Cilluffo, and F. Ciccarello, Mechanism of decoherence-free coupling between giant atoms, Phys. Rev. Res. 2, 043184 (2020).

[40] L. Guo, A. F. Kockum, F. Marquardt, and G. Johansson, Oscillating bound states for a giant atom, Phys. Rev. Res. 2, 043014 (2020).

[41] S. Guo, Y. Wang, T. Purdy, and J. Taylor. Beyond spontaneous emission: Giant atom bounded in the continuum, Phys. Rev. A 102, 033706 (2020).

[42] X. Wang, T. Liu, A. F. Kockum, H.-R. Li, and F. Nori, Tunable Chiral Bound States with Giant Atoms, Phys. Rev. Lett. 126, 043602 (2020).

[43] W. Zhao and Z. Wang, Single-photon scattering and bound states in an atom-waveguide system with two or multiple coupling points, Phys. Rev. A 101, 053855 (2020).

[44] C. Vega, M. Bello, D. Porras, and A. González-Tudela, Qubit-photon bound states in topological waveguides with long-range hoppings, Phys. Rev. A 104, 053522 (2021).

[45] A. Soro, and A. F. Kockum, Chiral quantum optics with giant atoms, Phys. Rev. A 105, 023712 (2022).

[46] X. Wang and H.-r. Li, Chiral quantum network with giant atoms, Quantum Sci. Technol. 7, 035007 (2022).

[47] L. Du, Y. Zhang, J.-H. Wu, A. F. Kockum, and Y. Li, Giant Atoms in Synthetic Frequency Dimensions, Phys. Rev. Lett. 128, 223602 (2022).

[48] L. Du and Y. Li, Single-photon frequency conversion via a giant A-type atom, Phys. Rev. A 104, 023712 (2021).

[49] L. Du, Y.-T. Chen, and Y. Li, Nonreciprocal frequency conversion with chiral A-type atoms, Phys. Rev. Res. 3, 043226 (2021).

[50] Q. Y. Cai and W. Z. Jia, Coherent single-photon scattering spectra for a giant-atom waveguide-QED system beyond the dipole approximation, Phys. Rev. A 104, 033710 (2021).

[51] S. L. Feng and W. Z. Jia, Manipulating single-photon transport in a waveguide-QED structure containing two giant atoms, Phys. Rev. A 104, 063712 (2021).

[52] W. Zhao, Y. Zhang, and Z. Wang, Phase-modulated Autler-Townes splitting in a giant-atom system within waveguide QED, Front. Phys. 17, 42506 (2022).

[53] X.-L. Yin, Y.-H. Liu, J.-F. Huang, J.-Q. Liao, Single-photon scattering in a giant-molecule waveguide-QED system, arXiv:2203.07812.

[54] Y.-T. Chen, L. Du, L. Guo, Z. Wang, Y. Zhang, Y. Li, J.-H. Wu, Nonreciprocal and chiral single-photon scattering for giant atoms, arXiv:2203.00823.

[55] H. Xiao, L. Wang, Z. Li, X. Chen, and L. Yuan, Excite atom-photon bound state inside the coupled-resonator waveguide coupled with a giant atom, arXiv:2111.06764.

[56] W. D. Oliver, Y. Yu, J. C. Lee, K. K. Berggren, L. S. Levitov, and T. P. Orlando, Mach-Zehnder Interferometry in a Strongly Driven Superconducting Qubit, Science 310, 1053 (2005).

[57] C. M. Wilson, T. Duty, F. Persson, M. Sandberg, G. Johansson, and P. Delsing, Coherence Times of Dressed States of a Superconducting Qubit under Extreme Driving, Phys. Rev. Lett. 98, 257003 (2007).

[58] M. Metcalfe, S. M. Carr, A. Muller, G. S. Solomon, and J. Lawall, Resolved Sideband Emission of InAs/GaAs Quantum Dots Strained by Surface Acoustic Waves, Phys. Rev. Lett. 105, 037410 (2010).

[59] J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, Colloquium: Artificial gauge potentials for neutral atoms, Rev. Mod. Phys. 83, 1523 (2011).

[60] M. Schmidt, S. Kessler, V. Peano, O. Painter, and F. Marquardt, Optomechanical creation of magnetic fields for photons on a lattice, Optica 2, 635 (2015).

[61] P. Roushan, C. Neill, A. Megrant, Y. Chen, R. Babbush, R. Barends, B. Campbell, Z. Chen, B. Chiaro, A. Dunsworth, A. Fowler, E. Jeffrey, J. Kelly, E. Lucero, J. Mutus, P. J. J. O’Malley, M. Neely, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. White, E. Kapit, H. Neven, and J. Martinis, Chiral ground-state currents of interacting photons in a synthetic magnetic field, Nat. Phys. 13, 146 (2017).

[62] L. Jin, P. Wang, and Z. Song, One-way light transport controlled by synthetic magnetic fluxes and P T-symmetric resonators, New J. Phys. 19, 015010 (2017).

[63] T. Ramos, B. Vermerich, P. Hauke, H. Pichler, and P. Zoller, Non-Markovian dynamics in chiral quantum networks with spins and photons, Phys. Rev. A 93, 062104 (2016).

[64] J.-T. Shen and S. Fun, Theory of single-photon transport in a single-mode waveguide. I. Coupling to a cavity containing a two-level atom, Phys. Rev. A 79, 023837 (2009).

[65] S. Longhi, Photonic simulation of giant atom decay, Opt. Lett. 45, 3017 (2020).

[66] L. Guo, A. Grimsmo, A. F. Kockum, M. Pletyukhov, and G. Johansson, Giant acoustic atom: A single quantum system with a deterministic time delay, Phys. Rev. A 95, 053821 (2017).

[67] M. Macovei and C. H. Keitel, Quantum dynamics of a two-level emitter with a modulated transition frequency, Phys. Rev. A 90, 043838 (2014).

[68] M. Janowicz, Non-Markovian decay of an atom coupled to a reservoir: Modification by frequency modulation, Phys. Rev. Lett. 61, 025802 (2000).

[69] G. Andersson, B. Suri, L. Guo, T. Aref, and P. Delsing, Non-exponential decay of a giant artificial atom,
[70] L. Du, Y.-T. Chen, Y. Zhang, and Y. Li, Giant atoms with time-dependent couplings, Phys. Rev. Res. 4, 023198 (2022).

[71] K. Koshino, H. Terai, K. Inomata, T. Yamamoto, W. Qiu, Z. Wang, and Y. Nakamura, Observation of the Three-State Dressed States in Circuit Quantum Electrodynamics, Phys. Rev. Lett. 110, 263601 (2013).

[72] Y. Liu and A. A. Houck, Quantum electrodynamics near a photonic bandgap, Nat. Phys. 13, 48 (2017).

[73] M. Mirhosseini, E. Kim, V. S. Ferreira, M. Kalaee, A. Sipahigil, A. J. Keller, and O. Painter, Superconducting metamaterials for waveguide quantum electrodynamics, Nat. Commun. 9, 3706 (2018).

[74] L. Du, M.-R. Cai, J.-H. Wu, Z. Wang, and Y. Li, Single-photon nonreciprocal excitation transfer with non-Markovian retarded effects, Phys. Rev. A 103, 053701 (2021).

[75] A. A. Clerk, Introduction to quantum non-reciprocal interactions: from non-Hermitian Hamiltonians to quantum master equations and quantum feedforward schemes, arXiv:2201.00894.