Point-Form Quantum Field Theory and Meson Form Factors

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Recently we have reconsidered the quantization of relativistic field theories on a Lorentz-invariant surface of the form $x_\mu x^\mu = \tau^2$ [1]. With this choice of the quantization surface all components of the 4-momentum operator become interaction dependent, whereas the generators of Lorentz transformations stay free of interactions - a feature characteristic for Dirac’s “point form” of relativistic dynamics. Thus we speak of “point-form quantum field theory” (PFQFT). Old papers on PFQFT (see, e.g., [2, 3]) dealt mainly with the evolution of quantum fields in the parameter $\tau$ and made use of a Fock-space basis which is related to the generators of the Lorentz group. Such a choice for the basis and the “time parameter”, however, gave rise to conceptual difficulties. To avoid these problems we have kept the usual momentum basis and considered evolution of the system as generated by the 4-momentum operator [1]. In this way we were able to show for free fields that quantization on the space-time hyperboloid $x_\mu x^\mu = \tau^2$ leads to the same Fock-space representation of the Poincaré generators as equal-time quantization. Moreover, we have suggested a generalized interaction picture which leads to a manifestly Lorentz covariant expression for the scattering operator as path-ordered exponential of the interaction part of the 4-momentum operator (along arbitrary timelike paths). We furthermore showed that the perturbative expansion of the scattering operator, defined in such a way, is (order by order) equivalent to usual time-ordered perturbation theory.

The nice feature that the operator formalism becomes manifestly Lorentz covariant if fields are quantized on the space-time hyperboloid $x_\mu x^\mu = \tau^2$ was not our only motivation to study PFQFT. PFQFT serves also as a natural starting point for the construction of effective interactions, currents, etc., which can be applied to point-form quantum mechanics. The main difficulty of finding a quantum mechanical realization of the Poincaré algebra, which describes a finite number of interacting particles, is caused by the fact that interaction terms in the Poincaré generators have to satisfy non-linear constraints, in general [1]. A procedure that resolves this problem has been proposed by Bakamjian and

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¹These constraints are automatically satisfied if a local interacting field theory is quantized.
Thomas [4]. Its point-form version amounts to the assumption that the free 4-velocity operator $\hat{V}_{\text{free}}^\mu$ can be factored out of the (interacting) 4-momentum operator

$$\hat{P}^\mu = \hat{P}_{\text{free}}^\mu + \hat{P}_{\text{int}}^\mu = (\hat{M}_{\text{free}} + \hat{M}_{\text{int}})\hat{V}_{\text{free}}^\mu,$$

so that one is left with an interacting mass operator $\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$. Since the mass operator is a Casimir operator of the Poincaré group the interaction term $\hat{M}_{\text{int}}$ is then only restricted by linear constraints, which are easy to satisfy.

As an example of how field theoretical concepts may enter the framework of relativistic quantum mechanics let us consider the electromagnetic form factor of a confined quark-antiquark pair (e.g. the pion). The idea is to work within the Bakamjian-Thomas framework and treat the electromagnetic scattering of an electron by a meson with internal structure as a quantum mechanical 2-channel problem for the mass operator, in which the dynamics of the exchanged photon is explicitly taken into account. The structure of the meson is encoded in a phenomenological vertex form factor which is not known a priori. Similarly we can consider the electromagnetic scattering of an electron by a quark-antiquark pair, which interacts via an instantaneous confining potential, as a 2-channel problem. If we reduce both 2-channel problems to 1-channel problems for the eM and eq\ ¯ q channels, respectively, we end up with 1-photon exchange optical potentials for the two systems (cf. Fig. 1). By comparing appropriate matrix elements of the eq\ ¯ q optical potential (i.e. between states in which q and \ ¯ q are bound with the quantum numbers of the meson) with those of the eM optical potential the electromagnetic form factor can be identified. The only problem with this kind of procedure is that a simple factorization as in Eq. (1) does not hold for the full electromagnetic vertex. We therefore have to resort to the approximation that the total 4-velocity of the system is conserved at electromagnetic vertices. In Ref. [5] it has been demonstrated in some detail that this is a way to implement field theoretical vertex interactions into a Bakamjian-Thomas type framework.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Contributions to the one-photon-exchange optical potential for electron-meson scattering (left) and electron scattering off a q\ ¯ q-cluster (right).

A calculation of the electromagnetic pion form factor along the lines just sketched has been carried out in Ref. [6]. For simplicity quarks have been treated as spinless. What one observes is that the extracted form factor depends not only on the virtuality $-Q^2$ of the photon, but also on the momentum of the meson $|k_M|$ in the center-of-mass of the electron-meson system (Fig. 2 left). This is equivalent to a dependence on Mandelstam $s$ and does not spoil Poincaré invariance. It is merely a consequence of the assumption that the 4-velocity of the system is conserved at electromagnetic vertices. The $|k_M|$-dependence vanishes rather quickly and if one takes the limit $|k_M| \to \infty$ the optical potential assumes
its expected structure $V_{\text{opt}} \propto j_{\mu}j_{\mu}^{\text{M}}/Q^2$ with $j_{\text{M}}^{\mu} = (k_{\text{M}} + k_{\text{q}}')^{\mu}F(Q^2)$. In this limit the expression for the form factor becomes

$$F(Q^2 = q^2) = \int_{\mathbb{R}^3} d^3\tilde{k}_q' \sqrt{\frac{m_{q'q}}{m_{q'q}}} \psi^*(\tilde{k}_q')\psi(\tilde{k}_q),$$

(2)

where $k_{\text{M}} = k_{\text{M}}' - q$, $k_{\text{q}} = k_{\text{q}}' - q$ and $m_{q'q}^2 = (E_{q'} + E_{q})^2 - k_{\text{M}}^2$. Quantities without a tilde refer to the electron-meson center-of-mass and quantities with a tilde to the meson rest system. With a simple harmonic-oscillator wave function $\psi(\tilde{k}_q)$ and a reasonable choice of the two free parameters (the constituent-quark mass $m_q$ and the oscillator parameter $a$) a satisfactory fit of the pion form-factor data up to momentum transfers of a few GeV$^2$ can be achieved (Fig. 2 right). There is, of course, room left for other dynamical ingredients.

The big advantage of our approach is that the electromagnetic current for a hadron with internal structure comes out with the right properties since we work within a manifestly Lorentz covariant framework. These first exploratory calculations for a very simple bound system show the virtues of point-form dynamics. The next step will be the inclusion of the quark spin. Generalizations to vector mesons, (spin–1/2 and spin–3/2) baryons and hadron transition form factors seem to be quite obvious and will be the focus of our further investigations.

References

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