A universal Regge pole model for all vector meson exclusive photoproduction by real and virtual photons

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A model based on a dipole Pomeron framework for vector meson exclusive photoproduction by real and virtual photons shows very good agreement with the experimental data. The model does not violate unitarity constraints and describes in a universal manner all the available data for $\rho$, $\omega$, $\varphi$, $J/\psi$ and $\Upsilon$ vector meson photoproduction in the region of energies $1.7 \leq W \leq 250$ GeV and photon virtualities $0 \leq Q^2 \leq 35$ GeV$^2$.

1 Introduction

Exclusive vector meson production by real and virtual photons is deemed to provide important information on the transition region from the “soft” dynamics at low virtualities of the photon $Q^2$ to the “hard” perturbative regime at high $Q^2$. An evidence of such a transition was the rapid growth of $J/\psi$ meson production discovered at HERA which raised the question of the applicability and validity of the Regge pole model in such a region.

Similarly the question of how to reconcile within QCD the transition from the perturbative regime (hard diffraction) and the one of hadron-hadron scattering cross-sections (soft diffraction) remains still quite open.

The effect discovered at HERA caused a vivid response by theorists. Vector meson photoproduction is described in phenomenological and QCD based approaches.

Based on the idea of soft Pomeron exchange, a model of Donnachie and Landshoff \cite{1} is capable of describing $\rho_0$ and $\varphi$ meson photoproduction using vector meson dominance with a scale factor 0.84. At the same time it fails to describe $J/\psi$ meson photoproduction cross section since the modest rise of soft Pomeron exchange with the intercept $\Delta_{FP} \simeq 0.08$ could not account for the steep behaviour of the $J/\psi$ cross section. To cope with such a steep behaviour they proposed a model of two Pomerons \cite{2} in which an additional hard Pomeron

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is present with a high intercept $\Delta_{P_0} \simeq 0.4$ and the introduction of such an exchange helps to describe the data on $J/\psi$ photoproduction and charm component $F_2^c(x, Q^2)$.

The model of Haakman, Kaidalov and Koch [3], taking into account both (‘eikonal’) multiple-Pomeron exchanges and a more complete set of diagrams including the interactions among the exchanged Pomerons, is capable of describing not only the photoproduction of vector mesons but also the $Q^2$ dependent $\rho_0$ meson photoproduction by virtual photons. The model is based on a Pomeron with intercept $\Delta_{P_0} \simeq 0.2$ which is softened by multi Pomeron exchanges in hadron-hadron interactions and the effective intercept grows with $Q^2$ leading to an increase of the growth of vector meson cross sections with increasing $Q^2$. The authors relate the vector meson differential cross section to the proton structure function $F_2^p(x, Q^2)$ and obtain a good description of the $\rho_0$ meson photoproduction by virtual photons using their model for $F_2^p(x, Q^2)$ [4].

Using a method of off shell extension of Regge eikonal model, Petrov and Prokudin [5] described $\rho_0$ and $J/\psi$ vector meson production by real and virtual photons using the universal Pomeron with intercept $\Delta_{P_0} \simeq 0.1$ obtained from the description of $pp$ and $\bar{p}p$ total, elastic and differential cross sections. The model uses a non $Q^2$ dependent Pomeron trajectory, takes into account unitarity corrections and does not violate unitarity for hadron-hadron processes and vector meson production.

A model based on dipole Pomeron by Jenkovszky, Martynov and Paccanoni [6] describes the vector meson photoproduction cross section for $\rho_0$, $\omega$, $\phi$ and $J/\psi$ vector meson using the Pomeron with intercept equal to 1, thus not violating unitarity restrictions.

The Dipole Pomeron was used in Ref. [7] to describe heavy meson photoproduction. The model [7] does not violate unitarity and shows good agreement with the data.

At the same time vector meson photoproduction at large $Q^2$ is seen as a test of perturbative QCD and of exchanges of the so called BFKL Pomeron.

Brodsky, Frankfurt, Gunion, Mueller and Strikman [8] using pQCD calculations show that with increasing $Q^2$, the vector meson photoproduction cross section starts rising more steeply as compared with photoproduction by real photons. They argue that the cross section is dominated (at large $Q^2$) by two gluon exchange through its longitudinal component

$$\sigma \sim \sigma_L \sim \frac{1}{Q^4} |xg(x, Q^2)|^2,$$

(1)

(where $xg(x, Q^2)$ is the gluon distribution) with respect to which the transverse component is suppressed by a factor of $Q^2$

$$\sigma_T \sim \frac{1}{Q^8} |xg(x, Q^2)|^2.$$

(2)

Martin, Ryskin and Teubner [9] show that the diffractive electroproduction of $\rho$ meson at high $Q^2$ can be described by pQCD and argue that the conflict between the QCD of prediction for the ratio

$$\frac{\sigma_L}{\sigma_T} \sim \frac{Q^2}{2m_\rho^2}$$

(3)

and the existing data [10] may be resolved introducing the relation

$$\frac{\sigma_L}{\sigma_T} = \frac{Q^2}{M^2} \left( \frac{\gamma(Q^2)}{\gamma(Q^2) + 1} \right)^2,$$

(4)
where $M$ is the invariant mass of the $q\bar{q}$ pair and $\gamma$ is the effective anomalous dimension of the gluon.

Cudell and Royen [11] using the lowest-order QCD calculation show that it reproduces correctly the ratios of cross sections for $\rho$, $\varphi$ and $J/\psi$. At the same time, the data for $\frac{\sigma}{\sigma_T}$ are not reproduced as in most of the models and they propose a new approach [12] where the asymptotic form of the transverse cross section gives $\sigma_T \sim \frac{1}{Q^6}$ leading to a good description of the ratio $\frac{\sigma}{\sigma_T}$.

The energy dependence of the vector meson photoproduction cross section was investigated in a paper by Donnachie, Gravelis and Shaw [13] who use pQCD calculations, the model of Cudell and Royen [12], soft Pomeron contribution modelling non-perturbative two-gluon exchange and several models for the vector meson wave function and achieve good global description of the vector meson data.

Nemchik, Nikolaev, Predazzi and Zakharov [14] use color dipole phenomenology and predict for the dipole cross section

$$\frac{\sigma(\gamma^* \to \omega)}{\sigma(\gamma^* \to \rho_0)} = \frac{1}{9}$$

independent of energy and $Q^2$. The elastic dipole cross section is in accord with the QCD prediction

$$\sigma(\gamma^* \to V) = \sigma_T(\gamma^* \to V) + \epsilon \sigma_L(\gamma^* \to V) \sim \frac{1}{(Q^2 + m_V^2)^4} \left(1 + \epsilon R_{LT} \frac{Q^2}{m_V^2}\right).$$

Nemchik, Nikolaev and Zakharov [15] observe that elastic production of vector mesons is of great potentiality for probing the BFKL Pomeron [16].

Vanderhaeghen, Guichon and Guidal [17] estimate the leading order amplitudes for exclusive meson electroproduction at large $Q^2$ in terms of skewed quark distributions and obtain a good description of $\sigma_L$ for $\rho_0$ meson exclusive production.

The $|t|$ dependence of $J/\psi$ production is discussed in [18] using DGLAP equation with one and two Pomeron exchanges being taken into account.

High $t$ vector meson production, finally, is analyzed in a paper by Laget [19]. Good description of elastic cross section for $\rho$, $\omega$ and $\varphi$ meson is obtained using the soft Pomeron approach, but the $J/\psi$ cross section is not well reproduced.

The problem of reconciling “soft” and “hard” regimes is still present despite the substantial progresses of the theory. It is not clear if the hard Pomeron is ultimately needed to describe the data or not (see for example [20]). We attempt to describe the data in the framework of “soft” physics, so that not we do not violate unitarity. In addition, we describe hadron-hadron processes thus answering in the positive the question: is it possible to use the universal soft Pomeron not only for hadron-hadron processes but also for photoproduction of light and heavy vector mesons?

## 2 General formalism

We utilize the following picture of the interaction: a photon fluctuates into a quark-antiquark pair and as the lifetime of such a fluctuation is quite long (by the uncertainty principle it grows with the beam energy $\nu$ as $2\nu/(Q^2 + M_V^2)$ [21]), the proton interacts via Pomeron or secondary Reggeon exchange with this quark-antiquark pair. After the interaction this
pair forms a vector meson \[8\]. We may conclude that such an interaction must be very close to that among hadrons and following the principle of Regge pole theory that the Pomeron is universal in all hadron-hadron interactions, we use the same Pomeron which is used to describe hadron-hadron interaction. Thus if Pomeron exchange is present in some interaction, then it has the same properties (is the form of singularity, position of such a singularity in the \(J\)-plane, trajectory etc.) as in hadron-hadron interaction. This is true at least for on shell particles. Concerning the \(Q^2 = 0\) photon, we consider it as a hadron (according to the experimental data). Then for \(Q^2 \neq 0\) we assume that no new singularity appears \[22\]. More precisely, even if we assume a new singularity at \(Q^2 \neq 0\), its contribution must be equal to zero for \(Q^2 = 0\). Indeed the analysis of the data \[20\] shows that there is no need for such a new contribution.

Our choice of the Pomeron contribution is the so called dipole Pomeron which gives a very good description of all hadron-hadron total cross sections \[23\] and was used already to describe photoproduction of vector mesons \[4\]. However the dipole Pomeron was applied in this paper only at \(Q^2 = 0\) in a qualitative rather than in a detailed description of the data. In addition, a large wealth of new data have appeared since \[4\].

The basic diagram is depicted in Figure 1; \(s\) and \(t\) are the usual Mandelstam variables, \(Q^2 = -q^2\) is the virtuality of the photon.

\[
\gamma^*(Q^2) \rightarrow p p \rightarrow p + V
\]

Figure 1: Photoproduction of a vector meson.

We introduce the “scaling” variable \(\tilde{Q}^2\)

\[
\tilde{Q}^2 = Q^2 + M_V^2,
\]

where \(M_V^2\) is the mass of the vector mesons. One assumes that a large meson mass plays the same rôle as the photon virtuality setting a “hard” scale for the reaction.

In general, the dipole Pomeron model generalized for virtual external particles can be written as \[24\]

\[
A(W^2, t; \tilde{Q}^2) = IP(W^2, t; \tilde{Q}^2) + f(W^2, t; \tilde{Q}^2) + \ldots,
\]

where \(W^2 = (p + q)^2 = m_p^2 + 2m_p\nu - Q^2\). \(IP\) is the Pomeron contribution, with \(s = W^2\)

\[
IP(W^2, t; \tilde{Q}^2) = ig_0(t; \tilde{Q}^2)\left(\frac{-i s}{s_0(\tilde{Q}^2)}\right)^{\alpha P(t)-1} + ig_1(t; \tilde{Q}^2)\ln\left(\frac{-i s}{s_1(\tilde{Q}^2)}\right)\left(\frac{-i s}{s_1(\tilde{Q}^2)}\right)^{\alpha P(t)-1}
\]

(8)
and
\[ g_i(t; \tilde{Q}^2) = g_i(\tilde{Q}^2) \exp(b_i(\tilde{Q}^2)t). \] (9)

A similar expression applies to the contribution of the \( f \)-Reggeon
\[ f(W^2, t; \tilde{Q}^2) = ig_f(t; \tilde{Q}^2) \left( \frac{-is}{s_f(\tilde{Q}^2)} \right)^{\alpha_f(t)-1}. \] (10)

It is important to stress that in this model the intercept of the Pomeron trajectory is equal to 1
\[ \alpha_{IP}(0) = 1. \] (11)
Thus the model does not violate the Froissart-Martin bound [25].

The parameters in \( \alpha_{IP}(t) \) and \( \alpha_f(t) \) are universal and independent of the reaction; \( g_i, b_i, s_i \) are functions of the variable \( \tilde{Q}^2 \) and the same for all reactions. Universality is a good approximation, which turns out to work very well in the present case, though in the framework of Regge theory strict universality does not hold and, in principle, most parameters could depend on the type of reaction as well. It may happen that new detailed set of data will need such a dependence; this of course would not automatically imply that the model is not able to describe the data or that the framework is a false one.

In hadronic phenomenology \( g_i \) and \( b_i \) are constants. Here, they may, in general, be \( Q^2 \)-dependent functions. The slope \( B(s, t; Q^2) = 2b_i(Q^2) + 2\alpha'_IPln(s/s_i(Q^2)) \) contains a universal energy-dependent term, while the parameter \( b_i(Q^2) \) is responsible for the quark content. In what follows we find that the experimental data do not demand any \( Q^2 \) dependence in \( b_i(Q^2) \); as a consequence we use \( Q^2 \) independent parameters \( b_i \), where \( i = IP, f \).

Let us stress here that the only variable that differentiates among the various vector meson elastic cross sections in this framework is the mass of the vector mesons in the case of photoproduction (\( Q^2 = 0 \)) or \( \tilde{Q}^2 \), i.e. the sum of the mass and virtuality of the photon in the case of DIS scattering. Thus, we propose a universal description for all vector mesons. We will see that such a description is very good, for example, we predict the correct cross sections for heavy vector meson \( \Upsilon \) in the case of photoproduction, and the correct \( Q^2 \) and \( W^2 \) dependence for \( \omega, \varphi \) and \( J/\psi \) vector mesons in the case of DIS when we fit only the \( \rho \) meson cross sections. The differential cross sections in all cases are described without any additional fitting. Such set of predictions give credit to the present model, given that most approaches have been able to make similar predictions in a correct way.

3 The Model

3.1 Photoproduction of vector mesons by real photons

For \( \rho, \varphi \) and \( J/\psi \) meson photoproduction we write the scattering amplitude as the sum of a Pomeron and \( f \) contribution. According to Okubo-Zweig rule, the \( f \) meson contribution ought to be suppressed in the production of \( \varphi \) and \( J/\psi \) mesons, though, taking into account the present crudeness of the state of the art, we added the \( f \) meson contribution even in the \( J/\psi \) meson case. The result of the fit proved that the \( f \) contribution is indeed negligible for \( J/\psi \) meson production (see the end of this Section for details) whereas it is not irrelevant for \( \varphi \) meson production. We believe, in fact, that due to \( \omega - \phi \) mixing \( f \) contribution is present in the latter; indeed in the \( \varphi \) decay mode, more than 15% is due to non strange particles...
and the $KK$ decay mode is present for $f$ meson decay. Thus, we conclude that the $f$ meson may contribute to $\varphi$ meson production.

For $\omega$ meson photoproduction, we include also $\pi$ meson exchange (see also the discussion in [1]), which is needed to describe the low energy data given that we try to describe the data for all energies $W$, starting from its threshold. Initially we did not expect model to work in the threshold area but it turned out to be capable of describing well the reactions even around threshold.

By writing the amplitude as $A = A_P + A_{IR}$, where $IR$ stands for the secondary Reggeon contribution, we obtain for the integrated elastic cross section $\sigma_{el}$

$$
\sigma(W^2, M_V^2)_{\gamma p \to Vp} = 4\pi \int_{-\infty}^{0} dt |A_{\gamma p \to Vp}(W^2, t; M_V^2)|^2 .
$$

We propose the following parametrizations for Pomeron and Reggeon couplings:

$$
g_0(t; M_V^2) = \frac{g_0 M_P^2}{(W_0^2 + M_P^2)^2} \exp(b_P^2 t) ;
$$

$$
g_1(t; M_V^2) = \frac{g_1 M_P^2}{(W_0^2 + M_P^2)^2} \exp(b_P^2 t) ;
$$

$$
g_{IR}(t; M_V^2) = \frac{g_{IR} M_P^2}{(W_0^2 + M_P^2)^2} \exp(b_{IR}^2 t) ,
$$

where $g_0, g_1, W_0^2 (GeV^2), b_P^2 (GeV^{-2})$ are adjustable parameters; $M_P^2$ is the proton mass. $IR = f$ for $\rho, \varphi$ and $J/\psi$, and $IR = f, \pi$ for $\omega$. $g_f, g_{\pi}, b_{IR}^2 (GeV^{-2})$ are also adjustable parameters. We use the same slope $b_{IR}^2$ for $f$ and $\pi$ Reggeon exchanges.

The parametrizations of the Pomeron contribution is the following

$$
A_P(W^2, t; M_V^2) = ig_0(t; M_V^2) \left(-i \frac{W^2 - M_P^2}{W_0^2 + M_P^2}\right)^{\alpha_{P}(t)-1} +
$$

$$
i g_1(t; M_V^2) \ln \left(-i \frac{W^2 - M_P^2}{W_0^2 + M_P^2}\right)^{\alpha_{P}(t)-1} ,
$$

where we use a linear Pomeron trajectory

$$
\alpha_{P'}(t) = 1 + \alpha_{P}(0) t
$$

with $\alpha_{P}(0) = 0.25 (GeV^{-2})$.

The contribution of the secondary Reggeons to the amplitude is written as

$$
A_{IR}(W^2, t; M_V^2) = ig_{IR}(t; M_V^2) \left(-i \frac{W^2 - M_P^2}{W_0^2 + M_P^2}\right)^{\alpha_{IR}(t)-1} .
$$

In order to take into account the threshold behaviour \footnote{We believe that without such a factor it is not possible to obtain the correct parameters for the secondary Reggeons (and correspondingly those of the Pomeron), and thus a correct description of the data for $J/\psi$ meson production at high energies $W$ (let alone the data near threshold).} we multiply the amplitude $A$ by a threshold factor

$$
\left(1 - \frac{(M_P + M_V)^2}{W^2}\right)^{\frac{1}{2}}
$$

\footnote{We believe that without such a factor it is not possible to obtain the correct parameters for the secondary Reggeons (and correspondingly those of the Pomeron), and thus a correct description of the data for $J/\psi$ meson production at high energies $W$ (let alone the data near threshold).}
where $M_0^2 (GeV^2)$ is an adjustable parameter and $M_p + M_V$ is the reaction threshold. The threshold behaviour of the amplitude is also discussed in [24] where the exponent (calculated in terms of the number of quark spectators $n_s$) is $2n_s$, where $n_s = 0, 1, \text{or } 2$. The empirical value we find in our model, 1.3 for the $J/\psi$ threshold behaviour (see Table 1), is well compatible with the result of [24].

We take into account the isostructure of vector mesons using the following relation between $\omega$ and $\rho_0$ cross-sections ($\omega$ is an isoscalar while $\rho_0$ is an isovector) which is predicted to hold in [14]

$$\frac{\sigma_\omega}{\sigma_\rho} = \frac{1}{9}. \quad (18)$$

Accordingly, given that the masses of $\omega$ and $\rho_0$ are very close to each other, we multiply the cross-section in Eq. (12) by a factor of 9 for the case of $\rho_0$ meson exclusive production.

In the fit we use all available data starting from the threshold for each meson and we do not discriminate among sets of data having different normalizations. It is, however, quite evident that, especially at low energies, different experiments have different normalizations. This implies that the $\chi^2$/d.o.f. will not be very good while the overall agreement is quite satisfactory.

The whole set of data is composed of 272 experimental points \footnote{The data are available at REACTION DATA Database \url{http://durpdg.dur.ac.uk/hepdata/reac.html} CROSS SECTIONS PPDS database \url{http://wwwppds.ihep.su:8001/c1-5A.html}} and having a grand total of 8 parameters, we find $\chi^2$/d.o.f = 1.32. The main contribution to the $\chi^2$ comes from the low energy region ($W \leq 4 GeV$); had we started fitting from $W_{min} = 4 GeV$, the resulting $\chi^2$/d.o.f = 0.96 would be much better and more appropriate for a high energy model.

The parameters are given in Table 1. The errors of the parameters are those obtained by MINUIT.
| N | Parameter | Value       | Error         |
|---|-----------|-------------|---------------|
| 1 | $g_1$     | 0.18487E-01 | 0.67523E-03   |
| 2 | $g_0$     | -0.45223E-01| 0.24077E-02   |
| 3 | $g_f$     | 0.19866     | 0.43331E-02   |
| 4 | $g_\pi$   | 0.38904     | 0.10219E-01   |
| 5 | $b_P^2$ ($GeV^{-2}$) | 0.51532 | 0.10462       |
| 6 | $b_f^2$ ($GeV^{-2}$) | 0.76365 | 0.62196E-01   |
| 7 | $W_0^2$ ($GeV^2$)  | 0.90696    | 0.10059E-01   |
| 8 | $M_0^2$ ($GeV^2$)  | 5.6044     | 0.45157       |

| Trajectory | $\alpha(0)$ (FIXED) | $\alpha'(0)$ GeV$^{-2}$ (FIXED) |
|------------|----------------------|----------------------------------|
| 1 Pomeron  | 1                    | 0.25                             |
| 2 $f$ Reggeon | 0.8                  | 0.85                             |
| 3 $\pi$ Reggeon | 0.0                  | 0.85                             |

| Meson        | # of points | $\chi^2$ per point |
|--------------|-------------|---------------------|
| $\rho_0(770)$ | 129         | 1.47                |
| $\omega(782)$  | 56          | 1.51                |
| $\phi(1020)$   | 38          | 0.82                |
| $J/\Psi(3096)$ | 49          | 0.84                |

| All mesons  | # of points | $\chi^2$/d.o.f. |
|-------------|-------------|-----------------|
| $\rho_0, \omega, \phi, J/\Psi$ | 272 | 1.32 |

Table 1: Parameters obtained by fitting $\rho_0, \omega, \phi$ and $J/\psi$ photoproduction data

The results are presented in Fig. 2, which shows also the prediction of the model for $\Upsilon(9460)$ photoproduction.

As a temporary comment, we stress that the model describes the data on vector meson exclusive photoproduction without the need of Pomeron contribution with intercept higher than 1.

Moreover, the rapid rise of the $J/\psi$ cross section at low energies is described as a transition phenomenon, a delay of the onset of the real asymptotics.

Most remarkable, the data on $\Upsilon(9460)$ production is very well predicted in this framework, as seen in Fig. 2.
Figure 2: Elastic cross-sections of vector mesons photoproduction. The curve for $\Upsilon(9460)$ production is a prediction of the model.

Let us address once more the issue of $J/\psi$ meson photoproduction. In the framework of the model, the rapid growth of the cross section is a transition effect of the onset of the asymptotic behavior.

In Fig. 3 one can see the cross section and the available corridor due to the uncertainty in the parameters (see Table I). We suppose that with the growth of $W$, the energy dependence of elastic cross section will change to a mild $\ln(s)$ behavior thus growing no faster than hadron-hadron elastic cross-sections. Another interesting question is the suppression of secondary Reggeons. As the quark content of $J/\psi$ (c$\bar{c}$) suggests that $f$ Reggeon exchange (composed of light quarks) must be suppressed, we show in Fig. 4 corresponding cross section without $f$ Reggeon contribution. We may conclude that in the model we have this suppression, as the contribution of the $f$ Reggeon is just some percent in the region of low $W$ and negligible for $W \geq 20$ GeV.
3.2 Photoproduction of vector mesons by virtual photons

In the case of nonzero virtuality of the photon, we have a new variable in play \( Q^2 = -q^2 \). At the same time, we have a nonzero contribution of \( \sigma_L \) for the cross section. According to [8], QCD predicts the following dependence for \( \sigma_T, \sigma_L \) and their ratio as \( Q^2 \) goes to infinity:

\[
\begin{align*}
\sigma_T & \sim \frac{1}{Q^8} (x_F g(x_F, \bar{Q}^2))^2; \\
\sigma_L & \sim \frac{1}{Q^6} (x_F g(x_F, \bar{Q}^2))^2; \\
R & \equiv \sigma_L/\sigma_T \sim Q^2/M_V^2; \\
\sigma & = (\sigma_T + \sigma_L)|_{Q^2 \to \infty} \sim \sigma_L. 
\end{align*}
\] (19)

Here \( x_F g(x_F, \bar{Q}^2) \) is the gluon distribution function, \( \bar{Q}^2 \equiv \frac{Q^2 + M_V^2}{4} \), \( x_F = \frac{Q^2 + M_V^2}{W^2 + M_V^2} \).

Enforcing these predictions, we use the following (most economical) parametrization for \( R \) (which cannot be deduced from the Regge theory)

\[
R(Q^2, M_V^2) = c \frac{Q^2 + M_V^2}{Q_T^2 + Q^2 + M_V^2} \frac{Q^2}{M_V^2} 
\] (20)

where \( c \) and \( Q_T^2 \) (GeV\(^2\)) are adjustable parameters. Thus, the asymptotic behaviour of \( R(Q^2, M_V^2) \) reproduces that of Eq. [19]

\[
R(Q^2, M_V^2)|_{Q^2 \to \infty} \sim \frac{Q^2}{M_V^2}. \] (21)
Accordingly, in the case \( Q^2 \neq 0 \) we use the following parametrizations for Pomeron and Reggeons couplings (compare with Eq. [13]):

\[
\begin{align*}
g_0(t; Q^2, M_V^2) &= \frac{g_0 M_V^2}{(W_0^2 + Q^2 + M_V^2)^2} \sqrt{\frac{Q_0^2}{Q_0^2 + Q^2}} \exp(b_{\text{p}}^2 t); \\
g_1(t; Q^2, M_V^2) &= \frac{g_1 M_V^2}{(W_0^2 + Q^2 + M_V^2)^2} \exp(b_{\text{p}}^2 t); \\
g_{\text{R}}(t; Q^2, M_V^2) &= \frac{g_{\text{R}} M_p^2}{(W_0^2 + Q^2 + M_V^2)^2} \sqrt{\frac{Q_{\text{R}}^2}{Q_{\text{R}}^2 + Q^2}} \exp(b_{\text{R}}^2 t),
\end{align*}
\]

where \( Q_0^2 \) (GeV\(^2\)) and \( Q_{\text{R}}^2 \) (GeV\(^2\)) are adjustable parameters.

In Eq. [24] we have introduced a new factor that differentiates virtual from real photoproduction

\[
f(Q^2) = \sqrt{\frac{Q_{0,R}^2}{Q_{0,R}^2 + Q^2}} f(0) = 1.
\]

Since in Eq. [13] we have complete control in the \( Q^2 \) dependence (recall that we use the sum \( \hat{Q}^2 = Q^2 + M_V^2 \) as a variable) up to any dimensionless function \( f(Q^2) \), such that \( f(0) = 1 \).

The parametrization of the Pomeron contribution is now the following (compare with Eq. [14])

\[
\begin{align*}
A_{\text{P}}(W^2, t; M_V^2, Q^2) &= ig_0(t; Q^2, M_V^2)\left(-i \frac{W^2 + Q^2 - M_p^2}{W_0^2 + Q^2 + M_V^2}\right) \alpha_p(t)^{-1} + \\
&+ ig_1(t; Q^2, M_V^2) \ln \left(-i \frac{W^2 + Q^2 - M_p^2}{W_0^2 + Q^2 + M_V^2}\right) \alpha_p(t)^{-1},
\end{align*}
\]

while the parametrization of the Reggeon contribution is (compare with Eq. [14])

\[
\begin{align*}
A_{\text{R}}(W^2, t; M_V^2, Q^2) &= ig_{\text{R}}(t; Q^2, M_V^2)\left(-i \frac{W^2 + Q^2 - M_p^2}{W_0^2 + Q^2 + M_V^2}\right) \alpha_{\text{R}}(t)^{-1}.
\end{align*}
\]

For simplicity, we multiply both components by the same threshold factor already used for real photoproduction (Eq. [17])

\[
\left(1 - \frac{(M_p + M_V)^2}{W^2}\right) \sqrt{\frac{M_V^2}{M_0^2}}.
\]

The (integrated) elastic cross-section can now be written as

\[
\sigma_{\text{el}}^{pp \rightarrow pp}(W^2; M_V^2, Q^2) = \sigma_T(W^2; M_V^2, Q^2) + \sigma_L(W^2; M_V^2, Q^2),
\]

where

\[
\begin{align*}
\sigma_T(W^2; M_V^2, Q^2) &= 4\pi \int_{-\infty}^{0} dt \left| A_{\text{P}}(W^2, t; M_V^2) + A_{\text{R}}(W^2, t; M_V^2)\right|^2 \big|_{Q^2 \to \infty} \sim \frac{1}{Q^8}; \\
\sigma_L(W^2; M_V^2, Q^2) &= R(Q^2, M_V^2) \sigma_T(W^2; M_V^2, Q^2) \big|_{Q^2 \to \infty} \sim \frac{1}{Q^6}; \\
\sigma &= (\sigma_T + \sigma_L) \big|_{Q^2 \to \infty} \sim \sigma_L.
\end{align*}
\]
Notice that we have directly enforced the asymptotical behaviour expected from QCD (Eq. 19) for $\sigma_T$ and $\sigma_L$ through the (empirical) choice of $R$ made earlier (Eq. 20).

We have, altogether 4 additional adjustable parameters as compared with real photoproduction.

In order to obtain the values of the parameters for the case $Q^2 \neq 0$, we choose to fit just the data on $\rho_0$ meson photoproduction in the region $0 \leq Q^2 \leq 35$ GeV$^2$; the parameters for photoproduction by real photons are the same as in Table 1. In order to avoid the low $W$ region where nucleon resonances may spoil the picture of $\rho$ meson exclusive production, we restrict the energy region to the domain $W \geq 4$ GeV. The parameters thus obtained are shown in Table 2.

| N | Parameter       | Value     | Error     |
|---|-----------------|-----------|-----------|
| 1 | $c$             | 1.6569    | 0.27570   |
| 2 | $Q_r$ (GeV$^2$) | 7.2822    | 1.3013    |
| 3 | $Q_0$ (GeV$^2$) | 7.7921    | 4.9477    |
| 4 | $Q_R$ (GeV$^2$) | 1.1209    | 0.69151E-01 |

Fit by a meson

Meson

| Parameter | Value     | Error     |
|-----------|-----------|-----------|
| $\rho_0(770)$ | 200       | 1.33      |
| $J/\Psi(3096)$ | 75        | 0.83 (no fit!) |

Table 2: Parameters obtained by fitting to $\rho_0$ photoproduction by virtual photons data

The results of the fit are depicted in Figs. 5, 6, 7, 8. The description of the data is very good in all the range of energies. Both high energy data from ZEUS and H1 Fig. 5 and low energy data from HERMES Fig. 7 are described. In the region of the HERMES data (Fig. 7) our description is comparable to the one of Haakman, Kaidalov and Koch [3] (see [27] for details).

6The data are available at
REACTION DATA Database [http://durpdg.dur.ac.uk/hepdata/reac.htm](http://durpdg.dur.ac.uk/hepdata/reac.htm)
CROSS SECTIONS PPDS database [http://wwwppds.ihep.su:8001/c1-5A.html](http://wwwppds.ihep.su:8001/c1-5A.html)
Figure 5: Elastic cross section of exclusive $\rho_0$ meson photoproduction by virtual photons as a function of $W$ for different values of $Q^2$. 
Figure 6: Elastic cross section of exclusive $\rho_0$ meson photoproduction by virtual photons as a function of $Q^2$ for $W = 75$, 51, and 14 GeV. The data and curves for $W = 51$, and 14 GeV are scaled by factors $10^{-2}$ and $10^{-4}$. 
We can now check the predictions of the model. As stated earlier, we aimed at proposing a unified model for all vector meson production, thus the only variable that changes is the mass of the vector meson. In the following figures we depict our predictions for $\omega$, $\varphi$ and $J/\psi$ mesons and we compare them with the available data. The description of the data is very good for all the three mesons. The $\chi^2 = 0.83$ for $J/\psi$ meson exclusive production follows without any fitting. Both $W$ and $Q^2$ dependences are reproduced very well.

Figure 7: Elastic cross section of exclusive $\rho_0$ meson photoproduction by virtual photons as a function of $W$ for various $Q^2$ in the region of low and intermediate $W$.

Figure 8: Elastic cross section of exclusive $\rho_0$ meson photoproduction by virtual photons as a function of $Q^2$ for $W = 5.4$, and 4.6 $GeV$. The data and curves for $W = 4.6$ $GeV$ are scaled by a factor $10^{-2}$. 
Figure 9: Elastic cross section of exclusive $\omega$ meson photoproduction by virtual photons as a function of $W$ for various $Q^2$.

Figure 10: Elastic cross section of exclusive $\omega$ meson photoproduction by virtual photons as a function of $Q^2$ for $W = 70 \text{ GeV}$.

Figure 11: Elastic cross section of exclusive $\varphi$ meson photoproduction by virtual photons as a function of $W$ for various $Q^2$.

Figure 12: Elastic cross section of exclusive $\varphi$ meson photoproduction by virtual photons as a function of $Q^2$ for $W = 88, 75, \text{ and } 4.9 \text{ GeV}$ (with scaling factors of $10^{-2}$ and $10^{-4}$.)
We can also plot the various ratios $\sigma_L/\sigma_T$ (shown in Fig. 13 and 16, 17) which, we recall, are constrained by our “ad hoc” choice that the large $Q^2$ QCD predictions Eq. 19 be satisfied. The result shows, indeed, a rapid increase of $\sigma_L/\sigma_T$ with increasing $Q^2$ but this, at least for exclusive $\rho_0$ meson virtual photoproduction is not in agreement with the data (see Fig. 15). This leaves open the question already discussed in [12] of modifying appropriately the QCD predictions Eq. 19 to account for the data. This could be easily done within our model by reconsidering the choice of introducing the factor $f(Q^2)$ in Eq. 22.

Indeed, we can write

$$
\sigma = (1 + R)\sigma_T = (1 + R) \frac{f(Q^2)}{f(Q^2)} \sigma_T = (1 + \tilde{R})\tilde{\sigma}_T , \tag{29}
$$

where $\tilde{R} = (1 + R)/f(Q^2) - 1$, $\tilde{\sigma}_T = f(Q^2)\sigma_T$. By setting $f(Q^2) = \left(\frac{W^2 + Q^2 + M^2}{W^2 + M^2}\right)^a$ (see Eq. 22). If we choose $a = 0, 0.5$, or 1, we can obtain various asymptotical behaviors for $\sigma_T$

$$
a = 0, \quad \tilde{\sigma}_T|_{Q^2 \to \infty} \propto \frac{1}{Q^8} ; \\
a = 0.5, \quad \tilde{\sigma}_T|_{Q^2 \to \infty} \propto \frac{1}{Q^7} ; \\
a = 1, \quad \tilde{\sigma}_T|_{Q^2 \to \infty} \propto \frac{1}{Q^6} .
$$

In the figures 13, 16, 17 we show possible adjustments of $Q^2$ behaviour of $\tilde{\sigma}_L/\tilde{\sigma}_T$. Solid line corresponds to $\tilde{\sigma}_T \propto \frac{1}{Q^8}$, dashed line corresponds to $\tilde{\sigma}_T \propto \frac{1}{Q^7}$ and dotted line corresponds to $\tilde{\sigma}_T \propto \frac{1}{Q^6}$.
From this pure phenomenological analysis we would say that the data prefer $a = 0.5$, though we believe that more complete and accurate set of data is needed in order to resolve the question on the $Q^2$ dependence of the ratio.

Figure 15: Ratio of $\sigma_L/\sigma_T$ for exclusive $\rho_0$ meson photoproduction.

Figure 16: Ratio of $\sigma_L/\sigma_T$ for exclusive $\varphi$ meson photoproduction.

Figure 17: Ratio of $\sigma_L/\sigma_T$ for exclusive $J/\psi$ meson photoproduction.

3.3 Differential cross section of vector meson exclusive production

We have constructed the amplitude $A(W^2, t; Q^2, M^2_V)$ and made sure that the model is valid for integrated observable such as elastic cross section. The natural question of applicability
of such an amplitude to describe also angular distributions arises.

The differential cross section is the following:

\[ \frac{d\sigma}{dt} = 4\pi |A(W, t; Q^2, M_V^2)|^2 \]  \hspace{1cm} (30)

Using the amplitude from the previous section this quantity can be calculated and the comparison with the data is presented in Fig. 18, 19, 23, 20, 21 and 22.

The description is very good without any additional fitting and we find that \( \alpha'_F(0) = 0.25 \ (GeV^{-2}) \) is universal for both hadron-hadron scattering and exclusive vector meson photoproduction.

In view of our result concerning the universality of our approach we are led to conclude that extracting the Pomeron trajectory from the experimental data as proposed in [28] using the data depicted in Fig. 21 cannot be regarded as a valid support to the need of either hard Pomeron contribution or NLO BFKL Pomeron.

Figure 18: Differential cross section of exclusive \( \rho_0 \) meson photoproduction for \( W = 94 \ GeV \).

Figure 19: Differential cross section of exclusive \( \rho_0 \) meson photoproduction for \( W = 71.7, 73, \) and 55 \( GeV \). The data and curves for \( W = 55 \ GeV \) are scaled by a factor \( 10^{-2} \).
**J/ψ(3096) meson photoproduction** $\gamma p \rightarrow J/ψ p$

![Graphs showing differential cross section](image)

Figure 20: Differential cross section of exclusive $J/ψ$ meson photoproduction as a function of $W$ at different $<t>$.

**ω(782) meson photoproduction** $\gamma p \rightarrow ω p$

![Graph showing differential cross section](image)

Figure 21: Differential cross section of exclusive $ω$ meson photoproduction for $W = 16.379$ and $80$ GeV. The data and curves for $W = 80$ GeV are scaled by a factor $10^{-2}$.

**φ(1020) meson photoproduction** $\gamma p \rightarrow φ p$

![Graph showing differential cross section](image)

Figure 22: Differential cross section of exclusive $φ$ meson photoproduction for $W = 13.371$ and $94$ GeV. The data and curves for $W = 94$ GeV are scaled by a factor $10^{-2}$.
4 Conclusion

We have shown that all available data on photoproduction of the various vector mesons can be described in the framework of the Regge approach. Not only the cross sections at $Q^2 = 0$ but also the data at $Q^2 \neq 0$ are described with a good quality. Moreover our model describes without additional fitting (as far as all parameter are determined from the fit to $t = 0$ data) also the differential cross-sections of vector meson production. We would like to emphasize the following important points:

1. Pomeron and secondary Reggeons appear as universal objects in Regge theory. So the corresponding $j$-singularities of $\gamma p$ amplitudes and their trajectories (intercepts $\alpha(0)$ and slopes $\alpha'$ in a linear approximation) at $Q^2 = 0$ coincide with those in pure hadronic amplitudes. They do not depend on the properties of external particles and, consequently, on $Q^2$. The unitarity restrictions on Pomeron contribution obtained strictly for the $hh$ case has to be valid for $\gamma h$ one as far as it is universal.

2. The growth with energy of hadronic total cross sections and the restriction on the Pomeron intercept ($\alpha_{\gamma P}(0) \leq 1$) implied by the Froissart-Martin bound mean that the Pomeron is a more complicated singularity than a simple pole with $\alpha_{\gamma P}(0) = 1$. In accordance with the above mentioned universality this statement can be extended to the DIS case.

We have considered the simplest case when the Pomeron is a double $j$-pole leading to $\sigma(s) \propto \ln s$. Thus double pole (or dipole) Pomeron model describes well hadronic reactions (total and differential cross-sections) and the proton structure function.

3. There are no new $j$-singularities in the DIS amplitudes $Q^2 \neq 0$. In particular, no hard Pomeron contribution appears necessary in the present regime of $Q^2$ and $t$. 

Figure 23: Differential cross section of exclusive $J/\psi$ meson photoproduction for $W = 94.\,\text{GeV}$.
4. The model we suggest is quite economic. The only parameter that makes the transition from one photoproduction process to another is the mass of vector meson $m_V$.

5. Our model describes the data also at low energies due to the threshold factor. This is particularly important for $J/\psi$ production where the bulk of available data are not so far from threshold.

6. Many simplifying assumptions have been implicitly made, for example, some parameters were taken the same for different terms of the amplitudes. When more precise data will become available, some of these simplifying assumptions need not be valid anymore. So the model has a potential to be improved if necessary.

Note added in proof. After this paper was completed, we learned about a paper [29] by Fiore, Jenkovszky, and Paccanoni where some of the arguments discussed here are covered.

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