Algorithmic method for modeling a parametric surface

G M Kravchenko*, E V Trufanova
Don State Technical University, 1, Gagarin Square Rostov-on-Don, 344000, Russia

E-mail: Galina.907@mail.ru

Abstract. The article discusses the peculiarity and relevance of the buildings and structures design using the latest computing technologies and digital modeling methods, which give a possibility to change the shape of a parametric architectural object. The formation and evolution of the surface of the crossed cap obtained from the Mobius strip is investigated. The key features of parametric modeling and the possibility of using this direction in the unique buildings and structures’ design are considered. Recommendations for choosing a rational form of an object based on the frame dynamic characteristics’ analysis and comparison are given. Visualization of the completed building of the sports and entertainment complex emphasizes the aesthetic component of parametric architecture.

1. Introduction

The use of computer technology has become an integral part in the parametric architecture objects’ design. The latest digital modeling methods allow the variation of the surface equation parameters to create the new architectural forms of the object. The opportunity to embody complex geometric shapes, releasing from the images’ linearity and symmetry [1,2] is relevant. A feature of parametric architecture is the consolidation of a complex spatial model and dynamics of natural forms [3].

In the parametric design environment, the following topical methods for obtaining object geometry can be distinguished: combinatorial modeling; algorithmic modeling method; adaptive systems [4]. Algorithmic modeling method is used to solve the design problems; based on changing the spaces parameters to form the new parametric shapes.

The aim of the study is to develop a unique building of unusual geometry by an algorithmic method using the analytical surface of a crosscap [5].

The crosscap is a non-orientable one-side surface obtained on the basis of the Moebius strip. The parametric form of specifying the surface of the crosscap has the form:

\[\begin{align*}
x &= x(u,v) = (1+\cos 2u)\cos 2v; \\
y &= y(u,v) = (1+\cos 2u)\sin 2v; \\
z &= z(u,v) = \sin 2u\sin v,
\end{align*}\]

where the parameters \(u\), \(v\) vary within \(-\pi/2 \leq u \leq \pi/2; 0 \leq v \leq 2\pi\).

Figure 1 shows the parametric shapes of the investigated surface: a disk with a Mobius film, a dissected disk with a Mobius film, a self-intersecting Mobius disk.
Figure 1. Parametric shapes: a) a disk with a Moebius film; b) a dissected disk with a Moebius film; c) self-intersecting disc

During the formation of a crossed cap, a Möbius surface with a circular boundary has two singular points, that is, self-intersection lines. Point with parameters \((u,v)=(0;0)\) and \(u=\pi/2\) are called Pinch points.

2. Materials and Methods

To study the primary forming evolution, the crosscap surface was constructed in the SAPFIR software package. The advantage of the SAPFIR software package is that it is possible to export the resulting file to the LIRA software package, which implements the finite element method.

In the process of studying the shape formation of the crosscap, the influence of the parameters \(u\) and \(v\) on the change in the Pinch point displacement, the shell bend radius was revealed.

The parameter \(u\) affects the displacement of the shell self-intersection singular point, respectively along the axes \(x, y, z\). The parameter \(v\) affects the radius of shell bend during the surface formation along the \(x, y, z\) axes, respectively. The results of the algorithmic method for modeling a parametric surface are presented in Table 1.

| Variation of the \(u\) parameter | Variation of the \(v\) parameter | Algorithmic modeling method |
|----------------------------------|----------------------------------|----------------------------|
| \(x = (1+\cos 1u)\cos 2v\)      | \(x = (1+\cos 2u)\cos 1v\)     | \(x = (2+\cos 2u)\cos 2v\) |
| \(y = (1+\cos 1u)\sin 2v\)      | \(y = (1+\cos 2u)\sin 3v\)     | \(y = (2+\cos 2u)\sin 2v\) |
The result of studying the primary forming evolution is three types of algorithmic models obtained by varying the coefficients simultaneously in three surface equations. The types of algorithmic models are summarized in Table 2.

### Table 2. Types of algorithmic models

| Option 1          | Option 2          | Option 3          |
|-------------------|-------------------|-------------------|
| \( x = (1 + \cos 1.5u) \cos 2v \) | \( x = (1 + \cos 1.1u) \cos 2v \) | \( x = (1 + \cos 5u) \cos 2v \) |
| \( y = (1 + \cos 2u) \sin 2v \) | \( y = (1 + \cos 1.7u) \sin 2v \) | \( y = (1,2 + \cos 5u) \sin 2v \) |
| \( z = \sin 2u \sin v \) | \( z = \sin 1.4u \sin v \) | \( z = \sin 2u \sin v \) |

At the next survey stage, the selected variants of the crosscap surface were proposed for the development of the building design solutions for a sports and entertainment complex. A parametric architecture object modeling is carried out by the finite element method [6]. At the same time, the functional purpose of the object and the internal space zoning are taken into account.

Spatial rod finite element models have been developed in the SAPFIR and LIRA software complexes according to the selected primary forming options. The finite element model is approximated by the spatial bar finite elements with six degrees of freedom at the node. The boundary conditions of the computational models are set at the level of the foundation slab device in the form of constraints on six degrees of freedom.

The steel frame of the building is set by the rods of I-beam 25B1, the box section 80x80x6 mm and pipes TB60x7. Finite element diagrams of the building frame for three variants of algorithmic models are shown in Figure 2.

To select a rational shape of the investigated surface, it is necessary to obtain the results of the stress-strain state and determine the dynamic characteristics of the models under consideration [7].

### 3. Results

Three variants of the projected sports and entertainment complex building were obtained by the algorithmic method of modeling the crosscap surface. Without denying the aesthetic component of the architectural image, it is proposed to include a dynamic criterion corresponding to the characteristics of the building frame as the basis of the analysis. The spectrum of frequencies and modes of natural vibrations were obtained by the finite element method (Table 3).
### Table 3. Modal analysis of finite element schemes for the building frame

| forming No. | Eigenvalues of frequencies, Hz |
|-------------|-------------------------------|
|             | Model 1 | Model 2 | Model 3 |
| 1           | 2.84    | 2.74    | 2.89    |
| 2           | 3.00    | 2.84    | 3.06    |
| 3           | 3.23    | 2.97    | 3.13    |
| 4           | 3.26    | 3.02    | 3.20    |
| 5           | 3.40    | 3.06    | 3.27    |
| 6           | 3.47    | 3.17    | 3.42    |
| 7           | 3.53    | 3.28    | 3.45    |
| 8           | 3.61    | 3.29    | 3.48    |
| 9           | 3.78    | 3.44    | 3.59    |
| 10          | 3.86    | 3.50    | 3.63    |

Modal analysis makes it possible to clarify the design solutions and geometric characteristics of the frame.

The maximum vertical displacements of the finite element model nodes are 35.3; 38.0; 26.4 mm according to the model variant. One of the criteria for choosing a rational analytical surface is the structural stability coefficient. The stability loss of such structures occurs suddenly, with a flap, with the formation of deep dints or the shell snapping. This phenomenon is dangerous both for the frame as a whole and for its individual elements. The stability coefficient of the proposed analytical models in the Lira-CAD software package has been investigated; the stability coefficient varies from 8.7 to 12.4 [8.9]. The dynamic characteristics of options 2 and 3 do not meet the limit values of the design codes. The rational shape of the crosscap surface is a variant of the algorithmic model 1.

### 4. Discussion

A model for designing a unique building obtained by an algorithmic method is recommended. To clarify the dynamic movements of the frame, it is necessary to take into account the effect of the pulsating...
component of the wind load [10]. The preliminary results demonstrate the need to consider the first 10 eigenforms in the obtained spectrum.

Figure 3 shows the dynamic displacements of the finite element mesh nodes taking into account the wind pulsation.

![Figure 3. Dynamic displacements: a) along X-axis; b) Y-axis](image)

It should be noted that the movements increased by an average of 30%. The design characteristics of a complex geometry object meet the current standards.

So, as part of the study:
1. The analysis of the given parametric surface formation in the form of a crosscap has been carried out. The algorithmic method for modeling the surface with varying the given parameters made it possible to choose a variant of the object primary forming for further research of the frame stress-strain state.
2. Using the finite element method in the LIRA software package, the characteristics of the object have been determined taking into account the pulsation component of the wind load.
3. The authors propose, when choosing an algorithmic model of a parametric architecture object, to regulate the stress-strain state, dynamic characteristics of the frame and the stability coefficient of a structure by changing the structural scheme and element stiffnesses.

5. Summary

Thanks to the development of innovative technologies and methods of information modeling, the relevance of creating parametric objects in the urban environment significantly increases [11, 12]. The finite element method allows performing the calculation of unique high-rise and large-span buildings and structures, the primary forming of which was developed with an algorithmic modeling method.

To select a rational shape of the crosscap that meets the requirements for bearing capacity, stiffness and stability, it is necessary to vary the coefficients simultaneously in all three equations and select several variants of the analytical surface for object design.

The stress-strain state study of a unique object of parametric architecture, obtained by the algorithmic method of modeling the crosscap surface, allows choosing the rational constructive solutions for the spatial frame of the building.

Improving the algorithmic method based on the analysis of the stress-strain state and dynamic characteristics will expand the boundaries of parametric architecture in the design of unique buildings and structures. The development of parametric architecture requires the improvement of software systems that implement not only analytical models of complex geometry objects, but also the numerical methods for calculating unique buildings and structures.

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