Accelerated process optimization for laser-based additive manufacturing by leveraging similar prior studies

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\textbf{ABSTRACT}
Manufacturing parts with target properties and quality in Laser-Based Additive Manufacturing (LBAM) is crucial toward enhancing the “trustworthiness” of this emerging technology and pushing it into the mainstream. Most of the existing LBAM studies do not use a systematic approach to optimize process parameters (e.g., laser power, laser velocity, layer thickness, etc.) for desired part properties. We propose a novel process optimization method that directly utilizes experimental data from previous studies as the initial experimental data to guide the sequential optimization experiments of the current study. This serves to reduce the total number of time- and cost-intensive experiments needed. We verify our method and test its performance via comprehensive simulation studies that test various types of prior data. The results show that our method significantly reduces the number of optimization experiments, compared with conventional optimization methods. We also conduct a real-world case study that optimizes the relative density of parts manufactured using a Selective Laser Melting system. A combination of optimal process parameters is achieved within five experiments.

\textbf{1. Introduction}
Additive Manufacturing (AM) processes can be utilized to generate physical objects directly from a digital model, layer by layer—providing an opportunity to generate complex-shaped, functionally graded, or custom-tailored parts that can be utilized in a variety of engineering applications. New AM technologies have been recently identified as potentially disruptive by manufacturing researchers, practitioners, and public media (Campbell \textit{et al.}, 2011; Berman, 2012; Petrick and Simpson, 2013). AM processes are now capable of generating metallic parts such as stainless steel (Spierings and Levy, 2009; Averyanova \textit{et al.}, 2011; Kempen \textit{et al.}, 2011), Ti-6-Al-4V (Hooreweder \textit{et al.}, 2012; Leuders \textit{et al.}, 2013), and nickel-based alloys (e.g., Inconel 625 and 718; Frazier (2014)) and this has allowed AM to transition from solely producing visualization and functional prototypes to producing end parts for direct use. Many real-world end parts produced using Laser-Based Additive Manufacturing (LBAM) have recently been reported. These include fuel nozzles produced using Selective Laser Melting (SLM) for GE Aviation’s LEAP engine (Foust \textit{et al.}, 2012; Wohlers and Caffrey, 2013) and Airbus’s brackets in its A300/A310 models, which promise to achieve 30–55% weight reduction and 90% raw material usage (Camisa \textit{et al.}, 2014). Direct end part production has contributed to 34.7% of the total product and service revenue from AM, a significant increase from only 3.9% in 2003 (Wohlers and Caffrey, 2013). Although LBAM has been used for generating parts for real-world industrial application, many challenges remain in achieving its full potential and unlocking the many opportunities that it offers (Tapia and Elwany, 2014).

LBAM relies on the successful layer-to-layer adhesion of melted powder with solid material. A directed energy source, a laser beam being a common example, is utilized to ensure phase-change of the utilized powders. Powders can either be deposited co-axially with the directed energy source—that is, Directed Energy Deposition—or deposited to form a bed first and then selectively melted; that is, Laser Powder Bed Fusion (PBF-L). There are many controllable process parameters that can affect the layer-to-layer adhesion and solidification heat transfer during LBAM—and such parameters have a direct impact on the properties of the produced part (Bian \textit{et al.}, 2015). For example, in SLM (a well-known PBF-L process), these parameters include laser power, laser velocity, layer (bed) thickness, hatch distance between successive passes of the laser within the same layer, etc. (Van Elsen, 2007). Most existing studies that are related to the optimization of LBAM processes can be categorized into two groups: physics-based methods and data-driven methods. Physics-based methods primarily rely on differential equations that govern the underlying thermomechanical process (Vasinonta, 2000; Bontha \textit{et al.}, 2009). Data-driven methods, on the other hand, circumvent the challenge of characterizing the underlying thermomechanical processes and are based on empirical experimental data and
Recent efforts in data-driven methods for LBAM center on using a traditional Design Of Experiment (DOE) approach, such as two-level fractional factorial design (Kummailil, 2004; Kummailil et al., 2005), full-factorial design (Averyanova et al., 2011; Averyanova et al., 2012; Spierings et al., 2012), and response surface methods (Lynn-Charney and Rosen, 2000; Zhou et al., 2000). Other papers have studied the effect of SLM process parameters on the resulting density of SLM parts; however, the process parameters were selected arbitrarily without the use of formal experimental design (Spierings and Levy, 2009; Averyanova and Bertrand, 2010; Gu et al., 2013). Despite these successful efforts, one important drawback of DOE methods is that most of them do not utilize results from existing studies or the domain of public knowledge pertaining to the LBAM process. As a result, when the experimental conditions (such as, materials, process type/scale, etc.) change, new sets of experiments are required prior to process optimization, thus leading to higher experimental costs.

The purpose of this study is to present a methodology that accelerates process parameter optimization in LBAM by leveraging prior information from similar, but non-identical, studies, of which there are numerous examples in the literature, due to recent advances in AM technologies. Instead of seeking an unknown global optimal value, the objective of the proposed optimization method is to develop an efficient experimental procedure that seeks to identify the optimal process parameters that result in a targeted value of a mechanical/physical property, which is usually specified by the design or application of the part. A DOE method that utilizes data from prior studies may have first been presented by Vastola (2013), who presented a two-step optimization process requiring, first, a large batch of initial experiments and, second, multiple smaller batches of sequential experiments. In this particular method, a large number of experiments may be required; for example, in its initial/first phase, 18 experiments are needed when only two process parameters are taken into account (Vastola, 2013). In contrast, the method presented herein, which is a generalization of the Sequential Minimum Energy Design (SMED) method first proposed by Dasgupta (2007), directly utilizes prior data from previous studies as the initial experimental data and incorporates it into sequential optimization experiments. In other words, the process optimization begins with sequential experiments directly without generating initial experiments, resulting in a smaller number of time- and cost-intensive experiments.

One challenge in leveraging prior data is to account for the differences in experimental conditions from which they originated, such as the original equipment manufacturer of the LBAM system, scale of machines, raw material characteristics, etc. Due to these differences, the experimental results for a given material (e.g., density, mechanical strength, geometric accuracy, etc.) between prior and current studies may be different. Figure 1 illustrates the concept of our proposed method. In this figure, the horizontal axis represents a given process parameter $(x)$, and the vertical axis represents the objective function (relative density as an example). The $f_2(x)$ curve is the true unknown density function in the current study, and we aim to reach its optimum.

The $f_1(x)$ and $f_3(x)$ curves represent unknown density functions in two different prior studies. The true objective functions from the current and prior studies do not exactly overlap, due to different experimental conditions. Clearly, we cannot fully rely on the reported data in the prior studies (two diamonds on $f_1(x)$ and one triangle on $f_3(x)$) because they do not exactly sit on the $f_2(x)$ curve.

The method presented herein characterizes such difference of responses (i.e., experiment results) between the prior and current studies (DRPCS for short) using probability distributions. With this characterization, results from prior studies are used to guide the optimization experiments of the current study and data from the current study are, in turn, used to update the DRPCS distribution. In this presented method, a closed-form expression for updating this distribution to further reduce the computational load and streamline the optimization process is developed. The efficiency and performance of the presented method is examined using a series of simulation studies and a real-world case study using an experimental test based on the SLM process.

The remainder of this article is organized as follows: Section 2 describes a knowledge-guided optimization process via the efficient utilization of data from related prior studies. In Section 3, a series of simulation studies and a real-world case study are presented to evaluate the performance of the proposed method. Finally, in Section 4, concluding remarks and directions for future work related to LBAM process optimization are provided.

2. Methodology: Accelerated process optimization by leveraging related but non-identical prior studies

We propose an accelerated process optimization methodology for LBAM by leveraging related, but non-identical, prior studies and designing experiments for the current study based on SMED (Dasgupta, 2007; Joseph et al., 2014). In what follows, we first review the SMED method developed by Dasgupta and then generalize the SMED method to utilize knowledge from prior studies and account for the differences in experimental conditions.

2.1 SMED

SMED is a novel DOE method that balances the objectives of optimization and space-filling. For example, assume that we need to determine the parameter values for $n$ potential design points $s_1, s_2, \ldots, s_n$ that maximize a response function $y(s)$. Here, the response can be any experimental result of interest, such as relative density, strength, etc., and each design point $s_i$ is a vector that represents a combination of process parameters to be tested via experimentation. To account for the optimization objective (maximizing $y(s)$ for instance), more design points of $s_1, s_2, \ldots, s_n$ should be placed in the range of that have a higher potential of resulting in optimal values of $y(s)$. On the other hand, the range of $s$ with a lower optimization potential should also be tested—that is, space-filling—to avoid the case where the optimization algorithm is trapped in local optima.
To do this, SMED assigns a positive electrical charge to each design point (i.e., combination of parameters). The charged design points repel each other and occupy positions within the design space, so as to minimize the total potential energy. The resulting positions of these electrical charges correspond to the minimum energy design points. The mathematical formulation of the SMED is summarized below. Let \( q(s) \) be the charge of the \( i \)th design point \( s_i \). The potential energy between any two design points \( s_i \) and \( s_j \) is equal to \( q(s_i)q(s_j)/d(s_i, s_j) \), where \( d(s_i, s_j) \) represents the Euclidean distance between \( s_i \) and \( s_j \). Therefore, the total potential energy for \( n \) charged design points, \( s_1, s_2, \ldots, s_n \), is given by

\[
E_n = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{q(s_i)q(s_j)}{d(s_i, s_j)}. \tag{1}
\]

The optimal parameter values of \( s_1, s_2, \ldots, s_n \), based on SMED, can be obtained by choosing \( s_1, s_2, \ldots, s_n \) that minimize the total potential energy \( E_n \).

The charge function \( q(s) \) is selected based on the optimization objective. In the previous example, in which the objective is to choose \( s \) to maximize \( y(s) \), \( q(s) \) should be selected to be a decreasing function of \( y(s) \). Hence, any design point \( s \) with a low \( y(s) \) value results in a high value of the charge \( q(s) \) and thus it strongly pushes other design points away. In contrast, a design point with a high \( y(s) \) value corresponds to a low charge, which allows for more design points to exist in the neighborhood of \( s \). Therefore, the number of design points \( s_1, s_2, \ldots, s_n \) will be higher in the range of \( s \) that tends to maximize the objective function \( y(s) \).

SMED allows the addition of \( s_1, s_2, \ldots, s_n \) in a sequential manner under the constraint of minimizing the total potential energy \( E \), represented by Equation (1). To add a new design point \( s_{n+1} \), the response at \( s_{n+1} \) (i.e., \( y_{n+1} \)) is predicted using inverse distance weighting (IDW) as follows:

\[
\hat{Y}_{n+1} = \frac{\sum_{j=1}^{n} |s_{n+1} - s_j|^{-2} y_j}{\sum_{j=1}^{n} |s_{n+1} - s_j|^{-2}}, \tag{2}
\]

where \(|s_{n+1} - s_j|^{-2}\) is the coefficient that captures the effect of \( s_j \) on \( \hat{Y}_{n+1} \) for \( j = 1, \ldots, n \).

The predicted \( \hat{Y}_{n+1} \) value is used to calculate the corresponding charge function \( q(s_{n+1}) \) for all possible choices of \( s_{n+1} \). The new design \( s_{n+1} \) that minimizes total potential energy \( E \) will be selected for the next experiment.

Figure 2 demonstrates an illustrative example of applying SMED to LBAM process optimization based on some available real experimental data. The vertical axis represents laser power and the horizontal axis represents laser velocity. These are two major factors that significantly affect part density. In this contour plot, the black color corresponds to design points resulting in high build with low density, and the light gray color represents design points (i.e., combination of process parameters) resulting in low part density. It can be observed that the SMED method tends to assign larger charges to design points that result in lower part density, so that these design points push other design points.
points away to reduce the number of experiment runs in low density areas, and vice versa. One limitation of SMED is the fact that it was not developed to account for experimental data from prior studies, thus requiring the generation of all design points within the current study. When the experimental conditions (e.g., original equipment manufacturer of the LBAM system, scale of machine, raw material characteristics, etc.) or target properties (e.g., relative density, tensile strength, elastic modulus, fatigue life, roughness, etc.) change, SMED cannot be used to incorporate the results from prior non-identical studies.

### 2.2 Accelerated process optimization based on SMED

We extend and generalize the SMED method to accelerate LBAM process optimization by utilizing data from prior studies. Hence, the availability of a few prior studies including some experimental data is a major assumption of the presented method. However, directly using data from prior studies may cause error in the optimal process parameters of the current study, due to variations in experimental conditions. Our method characterizes the DRPCS using probability distributions. The prior data, with DRPCS, are used to guide the optimization experiments of the current study before any experiment data of the current study are generated. Once data are obtained from the current study, prior data, current data, and DRPCS are used to generate the next design point for the optimization experiment. In particular, responses for the next batch of experiments are predicted using a generalized formula of IDW. Based on the predicted responses, design points that minimize the total potential energy are selected for the next batch of experiments, and the corresponding experimental data are collected. If the experimental results meet a pre-specified criterion, the optimization algorithm is stopped; otherwise, these data are added to the existing database to update the distribution of DRPCS and generate the next batch of experiment designs using the Bayesian method. The Bayesian method allows for updating the parameters of the posterior distribution of DRPCS using empirical data, which eventually converge to their true values even though the initial values of the parameters of prior distributions may not be accurate. In other words, as long as reasonable values of the prior parameters are used, the posterior parameters updated using experimental data will eventually be close to their true values. An overall framework of our proposed accelerated process optimization method is summarized in Fig. 3.

#### 2.2.1 Characterizing DRPCS

Denote data from prior studies by \((x_i, u_i)\) for \(i = 1, 2, \ldots, n_p\), where \(x_i\) is the \(i\)th design point from prior studies, \(u_i\) the corresponding response, and \(n_p\) the number of data points from prior studies. Pairs of \((x_i, u_i)\)'s have been reported in the literature. In the rest of this article, we use lowercase letters to represent deterministic known variables; and use uppercase letters for random variables. Let random variable \(Y_i\) represent the part property corresponding to design point \(x_i\) under the experimental conditions of the current study. Due to the difference in experimental conditions between the current and prior studies, the true value of \(Y_i\) is unknown and is thus represented by a random variable. At design point \(x_i\), the difference between the reported response from prior studies \(u_i\) and the actual unknown response from the current study \(Y_i\)—that is, DRPCS—is characterized by another random variable \(\lambda_i\). That is, \(\lambda_i = Y_i - u_i\), for \(i = 1, 2, \ldots, n_p\). The value of DRPCS \(\lambda_i\) is unknown. In other words, we define \(\lambda_i\) as \(\lambda_i = Y_i - u_i\) for the \(i\)th design point. \(u_i\) is the deterministic experimental data collected in the prior experiments, and \(Y_i\) is the unknown response of the current experimental conditions. Without further knowledge, we assume that \(Y_i\) follows a normal distribution, which is a common assumption about uncertainty. Hence, \(\lambda_i = Y_i - u_i\) also follows a normal distribution. Therefore, we assume that \(\lambda_i\) follows a prior normal distribution \(N(\beta_i, \omega^2)\), as the normal distribution is the most natural choice in this case. \(\beta_i\) represents the prior mean and \(\omega^2\) is the prior variance of \(\lambda_i\), which may be obtained via expert opinions and domain knowledge. We specify the parameters values of \((\beta_i, \omega^2)\) based on two cases:

- Case 1: If the domain or expert knowledge about the difference between the current and prior studies is available, the values of \(\beta_i\) and \(\omega^2\) can be specified using the domain/expert knowledge.
2.2.2 Predicting the response at new design points

In some real-world applications, it may be more efficient to design experiments in the form of batches, so that the total time spent on experiments may be reduced. Our proposed method allows for predicting the responses of design points in batches and generating batch experiments with any integer size \( b \). In what follows, we develop a formula for predicting experiment responses at new design points of the current study \( \tilde{Y}_j \). We present the formula for two phases: Phase 1 (Initialization), in which the data of the current study have not been generated, and Phase 2 (Iterative Experimentations), in which the data of the current study have been generated in an iterative manner.

Phase 1 (Initialization): In the initialization phase—that is, before experimental data are generated from the current study—we only use the prior results, with DRPCS, to optimize the current study. We directly apply Equation (2), the IDW formula by Dasgupta (2007) to predict \( \hat{Y}_j \)'s, as shown below:

\[
\hat{Y}_j = \frac{\sum_{i=1}^{n_p} |s_j - x_i|^{-2} (u_i + \lambda_i)}{\sum_{i=1}^{n_p} |s_j - x_i|^{-2}}.
\]

Phase 2 (Iterative Experimentations): When data are generated from the current study, the original IDW formula does not apply because DRPCS are included in the data of prior studies, whereas the data of the current study does not include DRPCS terms. We generalize the IDW formula so as to simultaneously account for batch experiments and prior data.

Assume that the available experimental data from the current study are represented by \((s_j, y_j)\), for \( j = 1, 2, \ldots, n_c \), where \( n_c \) represents the number of data points available in the current study. Note that \( n_c \) is not a fixed constant; instead, the value of \( n_c \) increases by batch size \( b \) after each batch of experiments in the current study. In what follows, we predict the response values of the next batch of experiments; that is, \( Y_{j+b} \) for \( j = n_c+1, \ldots, n_c+b \). If the experimental data from the current study are generated in batches, we need to define the index of its experimental batch. For \( Y_j \), \( 
\left(\frac{j-1}{b}\right) \) is the index of the last batch of experiments before \( Y_j \), where \( [ \cdot ] \) is the floor function. For example, for experiments with a batch size of \( b = 4 \), there are two batches of experiments before \( Y_9 \), and \( Y_9 \) is the first experiment in its experimental batch, because \( Y_9 = Y_{2 \times 4+1} \). With this notation, \( n_c = \left\lfloor \frac{j-1}{b} \right\rfloor b \).

The predicted value of \( Y_j \) is dependent on response data in previous batches \( y_1, y_2, \ldots, y_{\left\lfloor \frac{j-1}{b} \right\rfloor b} \) and data from prior studies \( u_1, \ldots, u_{n_c} \). We predict \( Y_j \) by generalizing the IDW formula represented by Equation (2) to account for its dependency on prior data and batch experiments. The generalized IDW formula of our approach for a new design point \( s_j \) is expressed as follows:

\[
\hat{Y}_j = \frac{\sum_{i=1}^{n_p} |s_j - x_i|^{-2} (u_i + \lambda_i) + \sum_{j'=1}^{\left\lfloor \frac{j-1}{b} \right\rfloor} \sum_{i=1}^{n_p} |s_j - s_{i'}|^{-2} y_{j'}}{\sum_{i=1}^{n_p} |s_j - x_i|^{-2} + \sum_{j'=1}^{\left\lfloor \frac{j-1}{b} \right\rfloor} \sum_{i=1}^{n_p} |s_j - s_{i'}|^{-2}}
\]

\[
= \sum_{i=1}^{n_p} w_{j,i} (u_i + \lambda_i) + \sum_{j'=1}^{\left\lfloor \frac{j-1}{b} \right\rfloor} \sum_{i=1}^{n_p} v_{j',j} y_{j'},
\]

where

\[
w_{j,i} = \frac{|s_j - x_i|^{-2}}{\sum_{i=1}^{n_p} |s_j - x_i|^{-2} + \sum_{j'=1}^{\left\lfloor \frac{j-1}{b} \right\rfloor} |s_j - s_{i'}|^{-2}}
\]

represents the weight coefficient of data from prior studies, and

\[
v_{j',j} = \frac{|s_j - s_{i'}|^{-2}}{\sum_{i=1}^{n_p} |s_j - x_i|^{-2} + \sum_{j'=1}^{\left\lfloor \frac{j-1}{b} \right\rfloor} |s_j - s_{i'}|^{-2}}
\]

represents the weight coefficient of data from previous batches of experiments in the current study.

The charge function \( q(s) \) is a positive, decreasing function of the objective function \( y(s) \). For instance, to maximize the relative density of parts built via LBAM, the objective function \( y(s) \) is the relative density of parts. Since the relative density of a part ranges from zero to one, a natural choice of charge function is \( q(s) = (1 - y(s))^2 \), where \( \gamma \) is a positive tuning constant. A new SMED point \( s_j \) minimizes the total potential energy \( E \), which consists of three components: the potential energy of the design points from prior studies, the potential energy of the design points from the current study, and the potential energy between these two groups. The total potential energy function including \( Y_j \) is expressed as follows:

\[
E_j = \sum_{i=1}^{n_{c+1}} \sum_{j=a+1}^{b} \frac{q(x_i)}{d(x_i, x_j)} + \sum_{j=1}^{j-1} \sum_{j'=j+1}^{j} \frac{q(s_j)}{d(s_j, s_{j'})}
\]

\[
+ \sum_{j=1}^{n_c} \sum_{j=1}^{b} \frac{q(s_j)}{d(x_j, s_j)}.
\]

The new design point in the current study can be obtained by solving \( s_j = \text{argmin}_j \ E_j \). Finding SMED for the next batch of experiments, namely, \( Y_{j+b} \) for \( j = n_c+1, \ldots, n_c+b \), is a computationally difficult problem. We can adapt the optimal design algorithms used in the literature, such as simulated annealing.
(Morris and Mitchell, 1995) and stochastic evolutionary algorithm (Jin et al., 2005) for our purpose. Different from these methods, we implemented the exchange algorithm to generate Y, s for \( j = n_c + 1, \ldots, n_c + b \) based on Equation (4). The main idea of the exchange algorithm is that an initial proposed batch of potential design points is updated by exchanging one of its design points with a potential one, given that the new design point decreases the total energy. The exchange algorithm has been successfully implemented to generate the batches of SMED (Casciato, 2013; Casciato et al., 2013; Vastola, 2013). The details of the exchange algorithm can be found in the textbook by Fedorov and Hackl (2012). We do not include the detailed algorithm and the corresponding properties in this article due to page limitations.

### 2.2.3. Updating DRPCS

Theorem 1 presents the analytical form of the posterior distribution of DRPCS, \( \lambda_i \), updated using data from the current study \( \{s_j, Y_j\} \), for \( j = 1, \ldots, n_c \), which are generated in batches of size \( b \). Theorem 1 shows that the posterior distribution of DRPCS still follows a multivariate normal distribution with specific parameters.

**Theorem 1.** Assume that the DRPCS terms, \( \lambda_i \), are independent and identically distributed random variables following the prior distribution \( N(\beta_i, \omega_i^2) \) for \( i = 1, \ldots, n_p \). The posterior distribution of \( \lambda_i \), with \( \lambda = (\lambda_1, \ldots, \lambda_n) \), updated using experimental data from the current study \( \{s_j, Y_j\} : j = 1, \ldots, n_c \) still follows a multivariate normal distribution with mean vector \( \mu_{\lambda} = \Lambda^{-1} \eta \) and covariance matrix \( \Sigma_{\lambda} = \Lambda^{-1} \), where \( \eta_i \) is the \( i \)th element of vector \( \eta \) and \( \Lambda_{i,k} \), the \((i, k)\)th element of matrix \( \Lambda \). \( \Lambda_{i,k} \) and \( \eta_i \) can be expressed as follows:

\[
\eta_i = \sum_{j=1}^{n_c} \frac{w_{j,i} \delta_j}{\sigma_j^2} + \frac{\beta_i}{\omega_i^2}
\]

\[
\Lambda_{i,k} = \begin{cases} 
\sum_{j=1}^{n_c} \frac{w_{j,i}^2}{\sigma_j^2} + \frac{1}{\omega_i^2} & \text{and } i = k \\
\sum_{j=1}^{n_c} \frac{w_{j,i}w_{j,k}}{\sigma_j^2} & \text{and } i \neq k 
\end{cases}
\]

where

\[
w_{j,i} = \frac{|s_j - x_i|^2}{\sum_{j=1}^{n_c} |s_j - x_i|^2 + \sum_{j=1}^{n_c} |s_j - s_f|^2},
\]

\[
\delta_j = y_j - \sum_{i=1}^{n_p} w_{j,i} u_i - \sum_{j=1}^{n_c} v_{j,f} y_j,
\]

\[
v_{j,f} = \frac{|s_j - s_f|^2}{\sum_{j=1}^{n_c} |s_j - x_i|^2 + \sum_{j=1}^{n_c} |s_j - s_f|^2}.
\]

**Proof.** Let \( g(\lambda | y) \) represent the posterior function of \( \lambda \) given data from the current experiment \( y \), where \( y = (y_1, \ldots, y_n)' \). We estimate the expression of \( g(\lambda | y) \) using the Bayesian formula. That is,

\[
g(\lambda | y) \propto \pi (\lambda) L(\lambda | y),
\]

where \( \pi (\lambda) \) represents the prior distribution of \( \lambda \) and \( L(\lambda | y) \) the likelihood function of \( \lambda \) given \( y \). Recall that \( \lambda_i \) follows a normal prior distribution \( N(\beta_i, \omega_i^2) \). Since it is assumed that all prior data are from different independent sources, the corresponding DRPCS terms are also assumed to be independent. Thus, the expression of \( \pi (\lambda) \) can be represented as the product of \( n_p \) probability density functions as follows:

\[
\pi (\lambda) = \prod_{i=1}^{n_p} \left( 2\pi \omega_i^2 \right)^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\omega_i^2} (\lambda_i - \beta_i)^2 \right\}.
\]

By expanding the prior distribution of \( \lambda_i \), we can re-write \( \pi (\lambda) \) according to the terms involving \( \lambda_i \)s as follows (see Appendix A available in the online supplement for a detailed calculation):

\[
\pi (\lambda) \propto \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n_p} \frac{\lambda_i^2}{\omega_i^2} + \sum_{i=1}^{n_p} \frac{\lambda_i \beta_i}{\omega_i^2} \right\}.
\]

In what follows, we determine the closed-form expression of the likelihood function \( L(\lambda | y) \) and the probability distribution determined by \( g(\lambda | y) \).

**Developing the likelihood function \( L(\lambda | y) \):**

The likelihood function \( L(\lambda | y) \) has the same expression as the probability density function (pdf.) of \( Y \) given parameter \( \lambda \) and data from prior studies; that is, \( f(Y | \lambda, u) \). As the design points of the current study are generated in a sequential manner, the resulting responses \( Y_1, Y_2, \ldots, Y_{n_c} \) are dependent. In this regard, the functional form of \( f(Y | \lambda, u) \) cannot be expressed directly. Based on the conditional probability principles, the functional form of \( f(Y | \lambda, u) \) can be calculated by multiplying the conditional distribution of \( Y_1, Y_2, \ldots, Y_{n_c} \). Hence, we can represent \( f(Y | \lambda, u) \) as follows:

\[
f(Y | \lambda, u) = \prod_{j=1}^{n_c} f_j \left( Y_j|Y_1, Y_2, \ldots, Y_{[\frac{j-1}{b}]}, u, \lambda \right).
\]

The conditional distribution of \( Y_j \), for \( j = 1, \ldots, n_c \), follows the normal distribution:

\[
Y_j \left| \left( Y_1, Y_2, \ldots, Y_{[\frac{j-1}{b}]}, u, \lambda \right) \right. \sim N \left( \mu_j, \sigma_j^2 \right),
\]

where

\[
\mu_j = \sum_{i=1}^{n_p} w_{j,i} (u_i + \lambda_i) + \sum_{j=1}^{n_c} v_{j,f} y_j.
\]

After combining terms that involve \( \lambda_i \)s, we express the likelihood function \( L(\lambda | y) \) as follows (see Appendix B available in the online supplement for a detailed calculation):
Developing the posterior probability density function \( g(\lambda|y) \):

Now by multiplying the expanded form of \( L(\lambda|y) \) and \( \pi(\lambda) \), the posterior distribution of \( \lambda_i \) using the Bayesian formula \( g(\lambda|y) \propto \pi(\lambda)L(\lambda|y) \) is expressed as follows (see Appendix C available in the online supplement for a detailed calculation):

\[
\begin{align*}
g(\lambda|y) & \propto \exp \left\{ -\frac{1}{2} \left( \sum_{i=1}^{n_{\lambda}} \lambda_i^2 \left( \sum_{j=1}^{n_{\lambda}} \frac{w_{ij}^2}{\sigma_j^2} + \frac{1}{\omega_i^2} \right) + 2 \sum_{i \neq k} \lambda_i \lambda_k \sum_{j=1}^{n_{\lambda}} \frac{w_{ij}w_{ij,k}}{\sigma_j^2} \right) \right. \\
& \left. + \sum_{j=1}^{n_{\lambda}} \lambda_i \sum_{j=1}^{n_{\lambda}} \frac{w_{ij}\delta_j}{\sigma_j^2} - \frac{1}{2} \sum_{j=1}^{n_{\lambda}} \left( \frac{\delta_j}{\sigma_j} \right)^2 \right\},
\end{align*}
\]

where

\[
\delta_j = y_j - \sum_{i=1}^{n_{\lambda}} w_{ij}u_i, \quad u_i = -\sum_{f=1}^{n_{\lambda}} w_{ij,f}y'_f.
\]

The resulting posterior distribution \( g(\lambda|y) \) involves the first- and second-order terms of \( \lambda_i \)s and thus represents the p.d.f. of a multivariate normal distribution. However, the posterior mean vector and covariance matrix of \( \lambda \) are not in explicit forms.

We utilize the canonical representation of a multivariate normal distribution to determine the posterior mean vector and covariance matrix of \( \lambda \). For a multivariate normal random variable \( z \) with mean vector \( \mu \) and covariance matrix \( \Sigma \), the following two representations of multivariate pdfs are equivalent:

\[
\begin{align*}
h_1(z|\mu, \Sigma) & = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (z - \mu)^T \Sigma^{-1} (z - \mu) \right\}, \\
h_2(z|\eta, \Lambda) & = \exp \left\{ a + \eta^T z - \frac{1}{2} z^T \Lambda z \right\}.
\end{align*}
\]

These two representations of the p.d.f.s are equivalent via the reparameterizations:

\[
\Lambda = \Sigma^{-1}, \quad \eta = \Sigma^{-1} \mu \quad \text{and} \quad a = -\frac{1}{2} \left( n \log(2\pi) - \log |\Lambda| + \eta^T \Lambda \eta \right).
\]

Applying the canonical representation to \( g(\lambda|y) \), we obtain the posterior mean vector and covariance matrix of \( \lambda \) as follows: \( \Sigma_{\lambda} = \Lambda^{-1} \) and \( \mu = \Lambda^{-1} \eta \), where (see Appendix D available in the online supplement for a detailed calculation)

\[
\Lambda_{i,k} = \begin{cases} 
\sum_{j=1}^{n_{\lambda}} \frac{w_{ij}^2}{\sigma_j^2} + \frac{1}{\omega_i^2}, & \text{and } i = k \\
\sum_{j=1}^{n_{\lambda}} \frac{w_{ij}w_{ij,k}}{\sigma_j^2}, & \text{and } i \neq k
\end{cases}
\]

It is worth noting that the posterior distributions of \( \lambda_i \)s are closed-form expressions, which require minimal computational resources and can be embedded in LBAM systems for process optimization. Also, \( \Lambda \) is a real symmetric matrix and can always be inverted. This guarantees the existence of the posterior covariance matrix \( \Sigma_{\lambda} \).

Although we assume independence among the prior distribution of \( \lambda \), our model can capture the dependence among prior data via specifying the values of the prior mean of \( \lambda_i \), namely, \( \beta_i \). The prior data from the same or similar experimental conditions tend to have similar values for \( \beta_i \). For example, if we have three prior data from the same experimental condition and we know that this experimental condition tends to generate parts with higher density than the current experimental condition, we may specify appropriate positive values of prior means; for example, \( \beta_1 = 0.002, \beta_2 = 0.003, \beta_3 = 0.0025 \). If we have two additional prior data from another experimental condition, which results in lower density, we can set negative values of the corresponding prior means; for example, \( \beta_4 = -0.002, \beta_5 = -0.003 \). Moreover, if the \( \lambda_i \)s are dependent, this dependency will be captured in their posterior distribution. We showed, in Theorem 1, that posterior distribution of \( \lambda_i \)s follows a multivariate normal distribution, which allows for dependence between the \( \lambda_i \) values.

### 3. Numerical studies

We examined the efficiency and robustness of the proposed process optimization method via a series of simulation studies, which simulated the relative density of parts based on an empirical relative density model developed by Spierings et al. (2012), and a real-world case study, which aimed at maximizing the part density of SLM 17-4 PH stainless steel (SS). We measured the efficiency of the optimization method using the number of experiment runs needed to reach a targeted part relative density. In the simulation studies, we tested how different characteristics of DRPCS affected the efficiency of our proposed method. The results were compared to two existing DOE methods: traditional full factorial DOE and SMED.

#### 3.1 Simulation studies

We performed a series of simulation studies so that we could generate prior data with various types of DRPCS and examine how they affect the efficiency of our proposed optimization method. We focused on the effect of four process parameters (laser power, laser velocity, hatch distance, and powder layer thickness) on the relative density of parts built via SLM, simulated using the empirical relative density function developed by Spierings et al. (2012) as shown in Equation (5) and Fig. 4:

\[
y = 0.92 + 12.2a^{-1} - 514a^{-2} + 4318a^{-3}, \tag{5}
\]

in which \( a \) is the energy density with

\[
a = \frac{\text{Laser Power}}{\text{Laser Velocity} \times \text{Hatch Distance} \times \text{Layer Thickness}}.
\]
This empirical model was obtained using a full-factorial DOE for SLM built SS 17-4PH/AISI-630 material. The type of SLM machine used in the experiments was a Concept Laser M2. A chess board scanning structure was applied to fabricate tensile samples. Two levels were assumed for five process parameters as provided in Table 1. In addition, the hatch distance was fixed at 0.0975 mm.

Equation (5) was used to generate the $y_j$ values, the part density given the process parameters of the current study. To simulate $u_1, \ldots, u_{np}$, we artificially added DRPCS to the $y_j$ values computed using Equation (5) that represent the differences in density caused by different experimental conditions in prior and current studies. Five different groups of prior data representing different types of DRPCS were generated as shown in Table 2, and the corresponding performance of our approach was examined and is reported in this section. For example, the first row of Table 1 represents one prior data with a reported relative density of 50%; that is $u_1 = 0.5$. We set the corresponding DRPCS $= -0.33$, meaning that the true value of $\lambda_1 = -0.33$. The true value of $\lambda_i$ was only used for the purpose of data simulation. We did not use this information in the process of optimization. In the first four scenarios, we investigated how different characteristics (e.g., sign and magnitude) of the DRPCS affect the number of experiments conducted. Two prior data points were used for the purpose of demonstration. In the fifth scenario, we increased the number of prior data points from two to five and examine the effect of $n_p$.

We set the values of $\beta_i$ to zero, as there was no prior knowledge available. We also let $\omega_i^2 = 3.5 \times 10^{-2}$, as $\lambda_i$ ranged from zero to one. We estimated $\sigma^2_j$ using generated repeated measurements of $Y_j$ and obtained that $\sigma^2_j = 4 \times 10^{-3}$. For all scenarios, we conducted simulation experiments with a batch size of $b = 2$. We considered $\gamma = 3$ for all simulation and case studies as the tuned parameter in the charge function $q(s) = \left(1 - y(s)\right)^\gamma$, as it is recommended by Dasgupta (2007). (Note that a set of simulated experiments are shown in Appendix F to illustrate the superiority the optimization performance of $q(s) = \left(1 - y(s)\right)^\gamma$ for $\gamma = 3$ in our method.)

### 3.1.1 Comparing our approach with full factorial DOE

We measure the number of experiments needed to reach a targeted relative density of 99.65%, which represents a reasonable target. This target value can be replaced by any other criterion. We first compare our approach with the full factorial design, which has been used by other researchers to optimize the SLM process (Kummailil et al., 2005; Averyanova et al., 2012). Note that the experiments using the full-factorial design approach were performed at the same time, as opposed to a sequential manner. We varied the number of levels for each factor so that the optimal relative density could be achieved. The possible combinations of process parameters and levels that result in the targeted density are summarized in Table 3: 10 levels for laser power, three levels for laser velocity, three levels for hatch distance, and two levels for layer thickness. Without considering replicated measurements, which leads to a higher number of experiments, the minimum number of experiments needed to achieve the target density using the full-factorial design approach was $10 \times 3 \times 3 \times 2 = 180$ experiments. In contrast, the maximum number of experiments needed to achieve the targeted density using our approach was 21.

### 3.1.2 Comparing our approach with SMED

We compare our approach with the SMED approach proposed by Dasgupta (2007). Figure 5 compares the number of experiments under various scenarios needed to achieve the targeted relative density by applying both SEMD and our method. Figure 6 illustrates the percentage improvement in terms of the number of experiments by applying our method rather than...
Table 2. Different types of DRPCS for SLM SS 17-4 PH.

| Scenario | Laser power (W) | Laser velocity (mm/s) | Hatch distance (mm) | Layer thickness (mm) | Relative density from prior studies $u_1$ | DRPCS
|----------|----------------|-----------------------|---------------------|---------------------|----------------------------------------|--------
| 1        | 50             | 1850                  | 0.05                | 0.04                | 0.5                                    | -0.33  |
| 2        | 120            | 1756                  | 0.1                 | 0.032               | 0.6                                    | -0.25  |
| 2       | 120            | 1756                  | 0.1                 | 0.032               | 0.7                                    | -0.13  |
| 3        | 70             | 1500                  | 0.08                | 0.036               | 0.7                                    | -0.13  |
| 3        | 100            | 1200                  | 0.1                 | 0.032               | 0.9                                    | 0.11   |
| 4        | 50             | 1850                  | 0.05                | 0.04                | 0.5                                    | -0.33  |
| 5        | 120            | 1756                  | 0.1                 | 0.032               | 0.9                                    | 0.22   |
| 5        | 120            | 1756                  | 0.1                 | 0.032               | 0.9                                    | 0.22   |
| 5        | 120            | 1756                  | 0.1                 | 0.032               | 0.9                                    | 0.22   |

Table 3. Process optimization using full-factorial DOE.

| Process parameter | Min | Max | Levels |
|------------------|-----|-----|--------|
| Laser power (W)  | 40  | 50  | 10     |
| Laser velocity (mm/s) | 400 | 2000 | 3 |
| Hatch distance (mm) | 0.05 | 0.15 | 3 |
| Layer thickness (mm) | 0.03 | 0.01 | 2 |

SMED. Detailed experimental results are summarized in Appendix E (available in the online supplement).

Since the SMED method does not account for the error in the prior data, we applied SMED to prior data without characterizing and updating DRPCS. We also report the computational time needed for each method and each scenario in Table 4. All

Figure 5. Comparison between the performances of SMED and our method.

Figure 6. The improvement obtained by applying our method rather than SMED.
computations were coded in MATLAB 2013 on a desktop with Intel Core i7 3.60 GHz processor and 16.0 GB RAM. We make the following observations.

1. The simulation studies show consistent improvement in the number of experiment runs of our proposed method, compared with SMED, this is due to our method being able to characterize the DRPCS distribution and update DRPCS using data from the current study. The updated distribution of DRPCS facilitates the generation of new optimization experiments.

2. When the magnitude of DRPCS increases, the improvement of our method is greater; for example, 16 to 27% (see scenarios 1 and 2 in Fig. 6). This is because the strength of our method is the characterization of DRPCS. On the other hand, when the magnitude of DRPCS is relatively insignificant, our method and SMED have a relatively similar performance.

3. The largest improvement in experiment runs (43%) is observed when there are positive and negative DRPCS (see scenario 3 in Fig. 6). A DRPCS with different signs may occur when the prior data come from different sources, which might contradict each other. The simulation results of scenario 3 indicate that our method can be implemented to handle contradicting prior studies, as even if contradicting prior results are included in our database, our method is capable of updating the DRPCS using data from the current study and eventually generating an accurate DOE for the purpose of optimization.

4. For all five scenarios, our method requires less computational time compared with SMED, as our method can obtain the optimal parameters with fewer experiments.

Table 4. Computational time needed to obtain optimal process parameters.

| Scenario                        | Computational time (seconds) |
|---------------------------------|-----------------------------|
| 1. Negative DRPCS               | 44                          |
| 2. Negative DRPCS with small magnitude | 51                          |
| 3. Negative and positive DRPCS  | 38                          |
| 4. Positive DRPCS               | 11                          |
| 5. More prior data points       | 13                          |

Table 5. Number of experimental runs needed to achieve the target density with various choices of charge functions and various scenarios of DRPCS.

| Scenario of DRPCS | Choices of charge function | Method |
|-------------------|-----------------------------|--------|
|                   | \(\gamma=1\)               | \(\gamma=3\) | \(\gamma=5\) |
| Negative DRPCS    | SMED                        | 12      | 22       | 38       |
|                   | Our method                  | 11      | 13       | 26       |
| Negative and positive DRPCS | SMED                        | 11      | 13       | 26       |
|                   | Our method                  | 11      | 13       | 17       |
| Positive DRPCS    | SMED                        | 11      | 14       | 14       |
|                   | Our method                  | 11      | 14       | 14       |

3.2 Case study

We verify the performance of our method in a real-world case study that aims at maximizing the relative density of parts...
(test coupons in this case) built in a ProX 100 SLM System using PH 17-4 SS powder (see Fig. 7). The fabrication of steel alloys has been frequently studied in the literature on metal-based AM, including austenitic 316 L SS (Abe et al., 2001; Spierings and Levy, 2009; Tolosa et al., 2010; Yasa and Kruth, 2011), H13 tool steel (Cormier et al., 2004; Pinkerton and Li, 2005; Cottam et al., 2014), and maraging steel (Casavola et al., 2008; Kempen et al., 2011). In this case study, we focus on precipitation hardening martensitic steel (17-4 PH SS), which is widely used in industrial applications that require a combination of high strength and a moderate level of corrosion resistance (Antony, 1963). This alloy is more common than any other type of precipitation hardening SS and has been frequently used in parts such as oil field valve parts, aircraft fittings, nuclear reactor components, paper mill equipment, missile fittings, and jet engine parts (Hsiao et al., 2002). Some researchers have studied the additive manufacture of 17-4 PH SS using SLM, focusing on its manufacturability and analyzing the resulting microstructural/mechanical properties of the manufactured samples (Kumar and Kruth, 2008; Jerrard et al., 2009; Facchini et al., 2010; Murr et al., 2012).

We found three prior studies that reported on the density of 17-4 SS parts fabricated via SLM (Kumar and Kruth, 2008; Spierings et al., 2012; Gu et al., 2013). Despite using the same type of SLM process, the capacities of these machines are different. For instance, the EOS M270 system used in the study of Gu et al. (2013) can output a maximum laser power of 200 W. The maximum relative part density is achieved when laser power is set at the level of 195 W, which exceeds the maximum laser power capacity of our SLM system and that of Kumar and Kruth (2008). Thus, the optimized process parameters obtained from the study of Gu et al. (2013) cannot be applied directly to manufacturing fully dense parts in our study. Other than the maximum laser power, there exist differences in other experimental conditions, such as laser profile and powder morphology. One of the key differences is the powder feed mechanism. The SLM system used in our study uses a roller that first feeds the powder into the feeding piston and then spreads the layer using counter-rotation, whereas the other systems used in the prior studies use a blade for powder feeding. A detailed summary of the experimental conditions of prior studies and our study are presented in Table 6.

Due to these differences in experimental conditions and the sensitivity of part density to process parameters, the optimal process parameters in these prior studies cannot be directly applied in our study. We applied the proposed knowledge-guided process optimization method and used data from prior studies (see Table 7) as prior data in our study. The experiments were designed by batch size two; that is, $b = 2$. For each design point of our study, we printed four samples to account for variability in the data. The experimental data of our study are shown in Table 8. We stopped the optimization process at the fifth experiment at a laser power of 49 W, laser velocity of 350 mm/s, hatch spacing of 0.05 mm, and layer thickness of 0.03 mm, which resulted in an average part density of 99.20%. We believe that this level of relative density is sufficiently high for many engineering applications. Moreover, this combination of process parameters is exactly the recommended process parameters by the manufacturer, which was not used in our optimization procedure. In

### Table 6. Experimental conditions in prior and current studies for SLM of 17-4 PH SS.

| System | Prior study 1 (Gu et al., 2013) | Prior study 2 (Spierings et al., 2012) | Prior study 3 (Kumar and Kruth, 2008) | Our study |
|--------|--------------------------------|--------------------------------------|-------------------------------------|-----------|
| System | M2 Laser CUSING (Concept Laser) | In-house developed SLM machine | ProX 100™ |
| Maximum laser power | 200 W | 150 W | 50 W |
| Laser spot diameter | 100–500 μm | 50–500 μm | 70 μm |
| Raw powder | GP by EOS (Proprietary) | GP by EOS (Proprietary) | PS4542A by 3D Systems (Proprietary) |
| Powder particle size distribution | 37.13 μm mean diameter | D10 = 16.4 μm | 10 < D50 < 13.5 μm |
| | | D50 = 26.8 μm | D80 < 22 μm |
| | | D90 = 42.7 μm | |

### Table 7. Experiment data from prior studies (Kumar and Kruth, 2008; Spierings et al., 2012; Gu et al., 2013).

| Laser power (W) | Scan velocity (mm/s) | Hatch spacing (mm) | Layer thickness (mm) | Relative density (%) |
|-----------------|----------------------|-------------------|----------------------|----------------------|
| Prior study 1 (Gu et al., 2013) | | | | |
| 195 | 1200 | 0.1 | 0.04 | 98.66 |
| 195 | 1000 | 0.1 | 0.04 | 99.75 |
| 195 | 1000 | 0.1 | 0.04 | 99.75 |
| 195 | 800 | 0.1 | 0.04 | 99.58 |
| 195 | 700 | 0.1 | 0.04 | 100.00 |
| 195 | 600 | 0.1 | 0.04 | 99.83 |
| 195 | 800 | 0.1 | 0.04 | 100.00 |
| 170 | 679 | 0.1 | 0.04 | 100.00 |
| 145 | 594 | 0.1 | 0.04 | 100.00 |
| 120 | 492 | 0.1 | 0.04 | 99.63 |
| 95 | 389 | 0.1 | 0.04 | 95.85 |
| 70 | 287 | 0.1 | 0.04 | 94.60 |
| Prior study 2 (Spierings et al., 2012) | | | | |
| 105 | 800 | 0.0975 | 0.03 | 98.44 |
| 190 | 800 | 0.0975 | 0.03 | 100.00 |
| 105 | 1300 | 0.0975 | 0.03 | 89.28 |
| 190 | 1300 | 0.0975 | 0.03 | 99.29 |
| 190 | 800 | 0.0975 | 0.05 | 88.53 |
| 105 | 800 | 0.0975 | 0.05 | 99.12 |
| 105 | 1300 | 0.0975 | 0.05 | 73.33 |
| 190 | 1300 | 0.0975 | 0.05 | 91.53 |
| Prior study 3 (Kumar and Kruth, 2008) | | | | |
| 95 | 350 | 0.14 | 0.03 | 98.00 |

### Table 8. Density optimization data of the case study.

| Laser power (W) | Laser velocity (mm/s) | Hatch spacing (mm) | Layer thickness (mm) | Average relative density (%) |
|-----------------|----------------------|-------------------|----------------------|-----------------------------|
| 1 | 50 | 300 | 0.07 | 0.03 | 98.41 |
| 2 | 40 | 200 | 0.07 | 0.03 | 96.53 |
| 3 | 50 | 200 | 0.14 | 0.03 | 92.27 |
| 4 | 50 | 400 | 0.07 | 0.03 | 97.14 |
| 5 | 49 | 350 | 0.05 | 0.03 | 99.20 |
other words, our proposed optimization method can achieve the manufactured recommended setup within five experiments.

4. Summary and conclusions

This article presented a novel approach for systematically optimizing controllable process parameters in LBAM processes via a DOE approach. The proposed method was developed on the premise that studies related to various metal-based LBAM processes share similar transformations from raw material to final part (that is, powder melting) and thus the microstructural and mechanical properties of LBAM parts may be correlated, despite differences in experimental conditions, material properties, system capacity, etc. We utilized data from prior, related studies to accelerate and characterize the DRPCS using statistical distributions. We developed a closed-form expression to update the distribution of DRPCS using data from the current study. As a result, prior data can be directly used as initial experiments, and this reduces the total number of experiments needed for optimization. To evaluate the performance of our approach, we conducted a series of validation experiments that used an empirical relative density model with four process parameters (laser power, laser velocity, hatch spacing, and layer thickness). We focused on the number of experiments needed to achieve a target relative density level. To test the performance of our method, we studied the effects of two key factors related to prior data: the signs and magnitudes of DRPCS.

The results demonstrate significant promise for accelerating the optimization of LBAM processes. Comparisons with two benchmark models; full-factorial design, which does not account for the optimization objective, and SMED, which does not utilize prior data, illustrate the importance of leveraging prior data. We further verified our method using a real-world case study that aimed at optimizing the relative part density of SS 17-4 PH parts using a SLM system. Optimized process parameters from prior studies could not be applied directly due to difference scales of LBAM systems. We utilized these studies as prior data and applied our optimization method. The optimal process parameters were achieved within five experimental runs.

The proposed method is not limited to solely optimizing part density in parts built via SLM systems. In fact, the proposed methodology provides a formal framework for better understanding the similarities among AM machines and the correlation of microstructural/mechanical properties, not through complex physics-based models that are often computationally cumbersome but through a more practical approach that relies on building a genome (database) of related experiment studies and modeling correlations among the experimental data. The proposed method is established based on two major assumptions: first, the availability of prior experiment data from similar systems and, second, the normally distributed DRPCS together with the prior data to guide the process optimization. Assuming a normal distribution for DRPCS terms may be seen as a limitation of the proposed method; however, it could be an appropriate and natural choice with which to start the study. Future work is needed to characterize the distribution of DRPCS. Also, there is a need to optimize multiple mechanical properties, which may be conflicting. Multi-objective optimization remains an open area, and more research efforts are needed to develop an efficient and effective optimization scheme.

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