The determination of redundant test patterns based on binary matrix symmetry properties

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Abstract. The object of research are digital circuit control tests. The model of a digital circuit is of a grey-box type. Test patterns (impacts) are combinations of logical zeros and ones, applied to circuit inputs. A binary matrix is the response of a digital circuit to input impacts. The examined faults are of «bridging faults» and «stuck-at faults» types. We distinguish the structural units of a test, as well as transformations, maintaining its structure. The measure of binary matrix symmetry is defined. Based on the measure obtained, the criteria are introduced, which help to calculate the utility of another test impact upon a digital circuit. To investigate the criteria, both pseudo-random and deterministic test sequences have been used. Among others, we have observed such algorithmic sequences as a sliding-ONE test, a logarithmic test, a galloping test pattern and a checkerboard test. The proposed approach gives the possibility to identify and exclude the impacts, not carrying useful information, from a test of combinational circuit control. The developed criteria can also be applied to construct memory tests. The analysis of experimental data allows us to make a conclusion about the advisability of using the proposed mathematical apparatus in the theory and practice of digital device testing, as well as in processes of test development.

1. Introduction
The testing of digital devices is an important task, contributing to successful implementation of advanced engineering projects in electronics and related fields. As a rule, the basic quality criterion of a digital circuit control test is its coverage of the specified class of faults. Another criterion of test quality is its information redundancy (or non-redundancy).

Information redundancy is understood as a state of information, obtained in the process of control over an object, in which the amount of information formally exceeds the one, necessary for judgement about the object’s state [1].

With the same information capacity, real messages possess certain redundancy of elements, compared with optimal messages. In some cases, information redundancy can be harmful, while, in other cases, it is useful. In functional testing of digital devices, certain redundancy of input impacts is of use. When testing a digital device, aimed at detecting the faults of «short-circuit» and «bridging faults» types, input impacts with identical information are usually excluded from the test.

Test compression will be understood as an algorithmic transformation of a test, in which its length is reduced due to decreasing redundancy of test impacts. In this article, the consideration of this task is based on the principles of symmetry, which represents the novelty of the approach.
In the general case, symmetry is understood as a category, indicating the preservation of certain features of an object with respect to selected transformations [2]. The use of symmetry principles assumes the existence of some rule for dividing a test into structural units, the assignment of transformations, acting on the basis of these structural units and preserving a test, as well as the criterion of quality, allowing for ranking test impacts in order of preference. Based on the number of such transformations, the measure of test symmetry will be examined, introducing a number of criteria, according to which, input impacts, not carrying useful information, can be excluded from the test.

2. Materials and method

2.1. The survey of solutions to the problem of assessing the quality of a digital circuit control test based on the information approach

The problem, considered in this article, assumes the presence of a certain reference circuit, in which test impacts are applied to inputs, and the response to test impacts is recorded in control points. The circuit inputs, its outputs and inner points can serve as control points. A binary matrix is formed on the basis of circuit response to test impacts. The analysis of reference circuit responses allows us to further exclude low-informative test impacts, reduce the control test length, and even construct a near-optimal test.

To assess the quality of tests and particular test impacts, this article will employ the information approach. It involves calculating the entropy of probability distribution of specified structural elements in a test sequence.

To rank tests in order of preference, the method can be employed, proposed by V.D. Agraval in the paper [3], in which the output entropy $H(p_1, p_2, ..., p_R)$ is used to improve the quality of the test and reduce its length. It is defined as:

$$H(p_1, p_2, ..., p_R) = - \sum_{i=1}^{R} p_i \cdot \ln p_i,$$

where $N$ is the number of outputs (control points) in a digital circuit; $p_i$ is the probability of response to external impact $(u_1, ..., u_m)$, located in the region of permitted values, $u_i \in \{0, 1\}$, $i = 1, ..., m$; $m$ is the number of inputs in a digital circuit, and $R$ is the number of possible responses of a digital circuit to test impacts, defined by the formula (2):

$$R = 2^N.$$

In the paper [3], the problem of output entropy maximization was solved in order to optimize the probability distribution of input impacts upon a digital circuit.

In the paper [4], V.D. Speransky and N.V. Cherevko considered the problem, similar to the one, presented in [3], however, the test quality was assessed on the basis of the formula (3):

$$H(q_1, ..., q_N) = - \sum_{i=1}^{N} (q_i \cdot \ln q_i + \beta (1 - q_i) \cdot \ln(1 - q_i)),$$

where $N$ is the number of outputs (control points) in a digital circuit; $q_i$ is the probability of a logic $1$, occurring in an $i$ control point of a digital circuit as a response to external impact, $i = 1, ..., N$.

The problem of output entropy maximization has also been solved in [5]. The test quality was assessed on the basis of the entropy criterion, obtained by principles of symmetry and defined by the formula (4):

$$H = \frac{\gamma}{N} \cdot \sum_{j=1}^{R} (p_j \cdot \ln p_j) + \frac{\beta}{N} \cdot \sum_{i=1}^{N} (q_i \cdot \ln q_i) + \sum_{i=1}^{N} ((1 - q_i) \cdot \ln(1 - q_i)),$$
where $R$ is the number of possible responses of a digital circuit to test impacts, calculated by the formula (2): $p_j$ is the probability of response to external impact ($u_1 \ldots u_m$), located in the region of permitted values. $u_i \in \{0, 1\}$, $i = 1, \ldots, m$; $N$ is the number of columns (control points) in a binary matrix; $q_i$ is the probability of a logic 1, occurring in an $i$ control point of a digital circuit as a response to external impact, $i = 1, \ldots, N$; $\gamma$ and $\beta$ are fixed parameters.

In papers [6] and [7], the task of output entropy maximization is not solved, however, the algorithm is proposed for detecting redundant test patterns and reducing the test length on the basis of the introduced entropy criterion of quality (5):

$$H(k_1, k_2, \ldots, k_N) = - \sum_{i=1}^{s} k_i \cdot \ln k_i,$$

where $k_i$ is the number of $i$-type columns in a binary matrix, representing a response of a digital circuit to input impacts; $s$ is the number of column types in a binary matrix, representing a response of a digital circuit to input impacts; $K$ is the number of control points (matrix columns).

Herewith, the following condition must be satisfied:

$$\sum_{i=1}^{s} k_i = K.$$

The criterion is obtained on the basis of symmetry principles.

The results, presented in papers [3-7], are relevant for automated development of digital circuit control tests and digital circuit testing.

2.2. Problem statement

The measure of the object’s symmetry will be understood as a number of its automorphisms (i.e. transformations, maintaining the structure) [8].

Suppose there is a binary matrix, composed of $K$ columns and $N$ rows:

$$
\begin{array}{cccc}
Y_{11} & Y_{12} & \cdots & Y_{1K} \\
Y_{21} & Y_{22} & \cdots & Y_{2K} \\
\cdots & \cdots & \cdots & \cdots \\
Y_{N1} & Y_{N2} & \cdots & Y_{NK}
\end{array}
$$

The matrix is the response of a digital circuit to input test impacts. Test patterns (impacts) are combinations of logical zeros and ones, applied to circuit inputs. Let us assume that its structural units are rows, columns and the elements of both. We will review the following as automorphisms of the binary matrix:

- the reciprocal permutations of identical elements in a matrix row;
- the reciprocal permutations of identical elements in a matrix column;
- the reciprocal permutations of identical rows;
- the reciprocal permutations of identical columns.

It is necessary to introduce the measure of binary matrix symmetry and, using it as a basis, develop and experimentally investigate the criteria of quality, making it possible to exclude the impacts, not carrying useful information, from the test of digital circuit control.

2.3. The measure of matrix symmetry

The measure of binary matrix symmetry, based on specified automorphisms, can be found by analogy with methodology, presented in paper [5]. The measure of symmetry, introduced in paper [5], is presented by two factors only, since automorphisms were viewed as reciprocal permutations of identical matrix column elements, and reciprocal permutations of identical rows. For the task under review, the measure of binary matrix symmetry is defined by the expression (8):
\[ S = \left( \prod_{i=1}^{N} (K - m_i)! m_i! \right)^{\alpha} \left( \prod_{i=1}^{K} (N - n_i)! n_i! \right)^{\beta} \left( \prod_{i=1}^{R} (l_i!) \right)^{\gamma} \left( \prod_{i=1}^{s} (k_i!) \right)^{\mu}, \] (8)

where \( \alpha, \beta, \gamma, \mu \) are coefficients, defining the way of binary matrix division, \( \alpha, \beta, \gamma, \mu \in \{0, 1\}; \) \( m_i \) is the number of 1’s in the \( i \)-row of a matrix; \( n_i \) is the number of 1’s in the \( i \)-column of a matrix; \( l_i \) is the number of \( i \)-type rows in a matrix; \( s \) is the number of \( i \)-type columns in a binary matrix; \( k_i \) is the number of \( i \)-type columns in a binary matrix; \( N \) is the number of rows in a binary matrix (i.e. test length); \( K \) is the number of columns in a binary matrix (i.e. the number of control points); \( R \) is the possible number of various rows, \( R = 2^k \).

The coefficient \( \alpha \) assumes the division into rows within a binary matrix, which represents the response of the tested object to test impacts. In this case, the automorphisms are reciprocal permutations of 1’s and 0’s in a matrix row.

The coefficient \( \beta \) assumes the division into columns within a binary matrix. In this case, the automorphisms are reciprocal permutations of 1’s and 0’s in a matrix column.

The coefficient \( \gamma \) assumes the division of a binary matrix into rows. In this case, the automorphisms are reciprocal permutations of identical rows.

The coefficient \( \mu \) assumes the division of a binary matrix into columns. In this case, the automorphisms are reciprocal permutations of identical columns.

\section*{2.4. The criteria of test quality}

If we find the logarithm for the expression (8), the criterion of entropy can be obtained, defined by the formula (9):

\[ H = \frac{\alpha}{N} \left( \sum_{i=1}^{N} (w_i \cdot \ln w_i) + \sum_{i=1}^{N} ((1 - w_i) \cdot \ln(1 - w_i)) \right) + \]
\[ + \frac{\beta}{K} \left( \sum_{i=1}^{K} (q_i \cdot \ln q_i) + \sum_{i=1}^{K} ((1 - q_i) \cdot \ln(1 - q_i)) \right) + \frac{\gamma}{R} \sum_{i=1}^{R} (p_i \cdot \ln p_i) + \]
\[ + \frac{\mu}{N \cdot K} \sum_{i=1}^{s} (k_i \cdot \ln k_i), \] (9)

where \( w_i \) is the frequency with which a logical 1 occurs in the \( i \)-row of a matrix (7); \( q_i \) is the frequency with which a logical 1 occurs in the \( i \)-column of a matrix; \( p_i \) is the frequency with which a row of \( i \)-type occurs in a matrix; \( K \) is the number of columns in a binary matrix (i.e. the number of control points); \( q_i \) is the probability of a logical 1 in the \( i \)-column of a binary matrix; \( k_i \) is the number of columns of the \( i \)-type in a binary matrix; \( N \) is the number of rows in a binary matrix (test length); \( R \) is the total number of possible types of binary matrix rows; \( R = 2^k \); \( s \) is the number of column types, found in a matrix; \( \alpha, \beta, \gamma, \mu \) are coefficients, which determine the way of binary matrix division.

Hereafter, the following conditions must be satisfied:

\[ \sum_{i=1}^{s} p_i = 1, \] (10)
\[ \sum_{i=1}^{s} k_i = K. \] (11)

For ease of presentation, it is advisable to introduce the following notation:
As a result, we have a generalized criterion (9) in the following form:

$$ H = \alpha \cdot H_1 + \beta \cdot H_2 + \gamma \cdot H_3 + \mu \cdot H_4. \tag{16} $$

Special criteria, defined on the basis of the expression (9), are used to assess the quality of tests. To determine the redundant code patterns, the coefficient in the generalized criteria must be necessarily equated to 1. To determine the redundant code patterns, the coefficient in the generalized criteria must be necessarily equated to 1. The component $H_4$ tracks the coverage of faults of the «bridging faults» type in control points \[7\] and has to be present in any special criterion. Therefore, the generalized criterion determines 8 special ones, obtained by picking $\alpha$, $\beta$, $\gamma$, $\mu$ coefficients. Herewith, $\alpha$, $\beta$, $\gamma$, $\mu \in \{0, 1\}$.

The informativeness of each test pattern can be assessed by checking the condition (17):

$$ |H(L) - H(L + 1)| < \varepsilon, \tag{17} $$

where $L$ is the testing step, based on which the number of rows in a matrix (7) is determined, i.e. it is the length of the text fragment under review; $H(L)$, $H(L + 1)$ – is the value of the criterion (17) at testing steps $L$ and $L + 1$, (in expressions of individual criteria, $N$ is substituted with $L$ and $L + 1$, respectively); $\varepsilon$ is the preset, sufficiently small value, selected in such a way, as to provide test coverage of all faults of the «bridging faults» type.

For successful search of non-informative test impacts, it is necessary to determine $\varepsilon$. Using the criterion (16), it is possible to show the reasonability of selecting $\varepsilon$, based on the expression (18): 

$$ \varepsilon = |A + B + C + D|, \tag{18} $$

where $A, B, C, D$ are determined as:

$$ A = -\left(\beta \cdot \frac{(K - 1)}{K} + \frac{\gamma}{K}\right) \cdot \ln \frac{1}{K - 1}; $$

$$ B = \left(\frac{\gamma}{K} + 2 \cdot \beta \cdot \frac{K - 1}{K}\right) \cdot \ln \frac{1}{K - 2}; $$

$$ C = -\beta \cdot \frac{K - 3}{K} \cdot \ln \frac{1}{K - 3}; $$

$$ D = \frac{2 \cdot \ln 2}{K \cdot (K - 2)}. $$

and $K$ is the number of control points of a digital circuit (i.e. the number of columns in matrix (7)).
In order to determine the informativeness of each test pattern and further compress the test, it is necessary to iteratively calculate the expression (17) for each L (beginning from the second impact, etc.), i.e. for each fragment of the test, composed of consecutive responses of a reference test object to input impacts. If the condition, defined by the expression (17), is not satisfied at any of the steps, it means that the corresponding test impact upon the object and the digital circuit response do not carry useful information.

3. The research results and their discussion
The table 1 presents 8 test sequences, which have been used in the experiments.

| Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 | Sequence 7 | Sequence 8 |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 010101     | 000111     | 111111     | 101010     | 111000     | 010111     | 100000     | 111000     |
| 101010     | 001101     | 000000     | 101010     | 100101     | 101101     | 010000     | 100101     |
| 010101     | 011100     | 111111     | 101010     | 010011     | 011100     | 001000     | 010011     |
| 101010     | 000000     | 101010     | 000111     | 111100     | 000100     | 101010     |
|            | 101010     | 011010     | 110011     | 000010     | 101100     | 101011     |
|            | 010101     | 010101     | 000111*    | 111100*    | 000100*    | 101011*    |
| 010101     | 011100     | 111111     | 101010*    | 010101*    | 110011*    | 000010*    |
| 101010     | 000000     | 101010     | 000111*    | 111100*    | 000100*    |

To investigate the generalized criterion (16) and the condition (17), various test sequences have been selected. Among them, there are randomly constructed test sequences. We also used standard test sequences [9], like «sliding ONE» (sequence 7), «logarithmic test» (sequence 8), «checkerboard test» (sequence 1), «galloping test pattern » (sequence 3). All eight special criteria have been examined. Each criterion is defined by dividing a matrix (7) into structural elements and by coefficients $\alpha$, $\beta$, $\gamma$.

The sequences under consideration are responses of reference objects to input impacts. Test sequences, carrying useful information for combinational circuits, are highlighted in bold. The sequence 1, sequence 3 and sequence 4 are relevant for memory testing, and repetitive codes are not redundant for this type of circuits.

In the following tables, test patterns, defined by criteria as redundant, are marked with the symbol «*». If we consider the following two cases $\alpha = 1$, $\beta = 0$, $\gamma = 0$, $\mu = 1$ and $\alpha = 0$, $\beta = 0$, $\gamma = 1$, $\mu = 1$, then the relevant research results are presented in table 2.

| Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 | Sequence 7 | Sequence 8 |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 010101     | 000111     | 111111     | 101010     | 111000     | 010111     | 100000     | 111000     |
| 101010     | 001101     | 000000     | 101010     | 100101     | 101101     | 010000     | 100101     |
| 010101     | 011100     | 111111     | 101010     | 010011     | 011100     | 001000     | 010011     |
| 101010     | 000000     | 101010     | 000111     | 111100     | 000100     | 101010     |
|            | 101010     | 011010     | 110011     | 000010     | 101100     | 101011     |
|            | 010101     | 010101     | 000111*    | 111100*    | 000100*    | 101011*    |
|            | 010101     | 011100     | 111111     | 101010*    | 010101*    | 110011*    |
| 101010     | 000000     | 101010     | 000111*    | 111100*    | 000100*    |
|            | 101010*    | 011010*    | 110011*    | 000010*    |
|            | 101100*    | 101011*    | 000001*    |
|            | 010101*    | 010101*    |           |            |            |            |            |
The analysis of experimental data allows us to conclude that memory tests (i.e. sequences 1, 3, 4) are not defined as redundant. The examined criteria make it possible to form sequences with a regular structure. After compression (i.e. the choice of test patterns, carrying useful information), the length of the sequence 4 will be reduced by one. The criteria also help to construct tests, covering all faults of the «bridging faults» type. The conclusion is made, based on the analysis of sequences 5, 6, 8 in table 2. The criteria don’t cover all faults of the «stuck-at faults» type in sequence 6.

The research results for the following cases: $\alpha = 0$, $\beta = 1$, $\gamma = 0$, $\mu = 1$ and $\alpha = 1$, $\beta = 1$, $\gamma = 0$, $\mu = 1$ are presented in table 3.

| Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 | Sequence 7 | Sequence 8 |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 010101     | 000111     | 111111     | 101010     | 111000     | 010111     | 100000     | 111000     |
| 101010     | 001101     | 000000     | 101010     | 010101     | 101101     | 010000     | 100101     |
| 010101     | 011100     | 111111     | 101010     | 010011     | 011100     | 001000     | 010011     |
| 101010     | 000000     | 101010*    | 001111*    | 111100*    | 000100     |           |            |
|            |            | 101010*    | 010101*    | 110011     | 000010     |           |            |
|            |            |            | 101100*    | 101011*    | 000001*    |            |            |
|            |            |            | 101010*    | 010101*    |            |            |            |

The results obtained in this experiment are very close to those, presented in table 2. The exception is sequence 4. After compression, this test sequence will be shorter by two test patterns. The experimental data allow us to conclude that the criteria do not define memory tests (i.e. sequences 1, 3) as redundant. The examined criteria make it possible to form sequences with a regular structure. The criteria also help to construct tests, covering all faults of the «bridging faults» type (i.e. sequences 5, 6, 8). The criteria cover all faults of the «stuck-at faults» type in sequence 6.

For the case $\alpha = 1$, $\beta = 0$, $\gamma = 1$, $\mu = 1$ the results of data compression are presented in table 4.

| Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 | Sequence 7 | Sequence 8 |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 010101     | 000111     | 111111     | 101010     | 111000     | 010111     | 100000     | 111000     |
| 101010     | 001101     | 000000     | 101010     | 010101     | 101101     | 010000     | 100101     |
| 010101     | 011100     | 111111     | 101010     | 010011     | 011100     | 001000     | 010011     |
| 101010     | 000000     | 101010*    | 001111*    | 111100*    | 000100     |           |            |
|            |            | 101010*    | 011010*    | 110011*    | 000010     |           |            |
|            |            |            | 101100*    | 101011*    | 000001*    |            |            |
|            |            |            | 101010*    | 010101*    |            |            |            |

The obtained results are very close to those of the previous case. The exception is sequence 6. Unlike the previous case, the criterion don’t cover all faults of the «stuck-at faults» types in sequence 6. The experimental data allow us to conclude that the criteria do not define memory tests (i.e. sequences 1, 3) as redundant. The examined criterion makes it possible to form sequences with a regular structure. After compression, the length of the sequence 4 will be reduced to two. The criteria also help to construct tests, covering all faults of the «bridging faults» type (i.e. sequences 5, 6, 8).
With coefficients $\alpha = 0$, $\beta = 1$, $\gamma = 1$, $\mu = 1$ we obtain the results of test sequence compression, presented in table 5.

| Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 | Sequence 7 | Sequence 8 |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 010101     | 000111     | 111111     | 101010     | 111000     | 010111     | 100000     | 111000     |
| 101010     | 001101     | 000000     | 101010     | 100101     | 101101     | 010000     | 100101     |
| 010101*    | 011100     | 111111     | 101010     | 010011     | 011100     | 001000     | 010011     |
| 101010*    | 000000     | 101010*    | 000111*    | 111100     | 000100     |           |            |
|            | 101010*    | 011010*    | 110011*    | 000010     |           |            |            |
|            | 101100*    | 101011*    | 000001*    |            |            |            |            |
|            | 101010*    | 010101*    |            |            |            |            |            |

Among the tests, having a regular structure, the sequence 3, according to the criterion, is a non-redundant one. After compression, the length of sequence 1 and sequence 4 will be reduced by two. In the case of sequence 5, the test patterns, carrying useful information for combinational circuits, are defined correctly. The criterion don’t cover all faults of the « stuck-at faults » types in sequence 6. The examined criteria make it possible to form sequences with a regular structure.

With coefficients $\alpha = 0$, $\beta = 1$, $\gamma = 1$, $\mu = 1$ we obtain the results of test sequence compression, presented in table 6.

| Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 | Sequence 7 | Sequence 8 |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 010101     | 000111     | 111111     | 101010     | 111000     | 010111     | 100000     | 111000     |
| 101010     | 001101     | 000000     | 101010     | 100101     | 101101     | 010000     | 100101     |
| 010101*    | 011100     | 111111     | 101010     | 010011     | 011100     | 001000     | 010011     |
| 101010*    | 000000     | 101010*    | 000111*    | 111100*    | 000100     |           |            |
|            | 101010*    | 011010*    | 110011*    | 000010     |           |            |            |
|            | 101100*    | 101011*    | 000001*    |            |            |            |            |
|            | 101010*    | 010101*    |            |            |            |            |            |
|            | 010101*    |            |            |            |            |            |            |

The results, presented in table 6, are close to those of the previous experiment. Among the tests, having a regular structure, the sequence 3, according to the criterion, is a non-redundant one. After compression, the length of sequences 1 and 4 will be reduced by two. In the case of sequence 5, the test patterns, carrying useful information for combinational circuits, are defined correctly. The criteria cover all faults of the « stuck-at faults » types in sequence 6.

For ease of perception and analysis of research results, the table 7 is presented farther, containing the experimental data for the case when $\alpha = 0$, $\beta = 0$, $\gamma = 0$, $\mu = 1$. The relevant criterion was presented in papers [6] and [7], however, it was investigated on the basis of other test sequences.

This criterion is rather strict, since repetitive patterns, as well as those, obtained by inversion (i.e. sequences 1, 3) are considered as redundant test patterns. The criterion helps to obtain a near-optimal test, covering the faults of the «bridging faults» type. The conclusion was drawn, based on the analysis of sequences 5 and 8.
Traditionally, when compressing code sequences, the repetitive test patterns are considered redundant and do not carry useful information. During the in-circuit control of digital combinational circuits, redundant input impacts (i.e. those, not carrying useful information) are usually not included in the test.

During functional control, test redundancy is welcomed. Memory tests are distinguished by a regular structure and look redundant (in the classical sense of the word).

The criteria, proposed in this article, help to construct memory tests with repetitive patterns. Such criteria include:

- \( H = H_1 + H_4 \),
- \( H = H_3 + H_4 \),
- \( H = H_2 + H_4 \),
- \( H = H_1 + H_2 + H_4 \),
- \( H = H_1 + H_3 + H_4 \),
- \( H = H_2 + H_3 + H_4 \),
- \( H = H_1 + H_2 + H_3 + H_4 \).

Below, the criteria are presented, based on which the tests were constructed for combinational digital circuits, covering all faults of «bridging faults» type and all faults of the «stuck-at faults» types in sequence 6, and not containing redundant patterns:

- \( H = H_2 + H_4 \),
- \( H = H_1 + H_2 + H_4 \),
- \( H = H_1 + H_2 + H_3 + H_4 \).

The analysis of the experimental data showed that, in the process of compression, it was possible to significantly reduce the test length.

The criterion \( H = H_4 \) makes it possible to construct a near-optimal test, checking combinational circuits for faults of the «bridging faults» type. According to this criterion, all repetitive test patterns, as well as those obtained by inversion, are considered redundant.

The research confirmed the reasonability of the practical use of special criteria, obtained by expressions (16) and (17). However, it is worth noting, that not all criteria are sensitive to full test coverage of «stuck-at faults» faults.
4. Conclusion
The article deals with the concept of symmetry and creates the structure of a binary matrix. Based on the introduced measure of binary matrix symmetry, the generalized entropy criterion has been synthesized. A number of special criteria were obtained on the basis of the generalized criterion. The measure was introduced, allowing to assess the information redundancy of code sets.

The developed mathematical apparatus was investigated with respect to tests of digital circuit control. The applicable model of a digital circuit is of a grey-box type. We have considered both control tests, constructed with a pseudorandom number generator, and deterministic ones, covering the faults of «stuck-at faults» and «bridging faults» types. The test model is a binary matrix.

The mathematical apparatus, proposed in the article, makes it possible to construct both memory tests with repeating sets, and the tests for combined digital circuits, covering all faults of «stuck-at faults » and «bridging faults» types, and not containing redundant test sets. As a rule, the traditional methods of information compression do not permit any repetitions of code combinations. The analysis of experimental data has shown that, in the process of compression, it became possible to significantly reduce the test length and even obtain a near-optimal test. It is worth noting, that not all criteria are sensitive to full test coverage of «stuck-at faults» faults.

The analysis of experimental data allows us to make a conclusion about the reasonability of using the proposed mathematical apparatus in the theory and practice of testing objects, which can be represented as a binary matrix. The proposed material can also be applied in processes of test development.

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