Large Quantum Fluctuations in Planar Josephson Junctions

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We investigate the stochastic phase dynamics of planar Josephson junctions (JJs) and superconducting quantum interference devices (SQUIDs) defined in epitaxial InAs/Al heterostructures. The interplay of large quantum fluctuations and phase diffusion results in an intriguing regime where the mean switching current of a JJ is a small fraction of the critical current and macroscopic quantum tunneling is the most relevant switching mechanism up to the critical temperature. Phase locking between two JJs further modifies the phase dynamics, resulting in a different switching current between that of a JJ measured in isolation and that of the same JJ measured in an asymmetric SQUID loop. The relative contribution of quantum tunneling and phase diffusion is further tuned by a magnetic flux in the SQUID loop.

Two-dimensional superconductor/semiconductor hybrid systems are a promising platform for scalable quantum computation and for the study of novel physical phenomena. The possibility to produce transparent interfaces [1–9], combined with flexible lithographic patterning, is paving the way to a new generation of voltage-tunable qubit architectures [10–16], with planar Josephson junctions (JJs) and superconducting quantum interference devices (SQUIDs) as core elements. Furthermore, the interplay between superconducting phase, spin-orbit interaction and Zeeman energy results in a rich playground for fundamental physics [17–19], including the realization of topological states of matter [20–25]. In this context, understanding the phase dynamics of hybrid JJs and SQUIDs, which ultimately determine their switching currents, is crucial.

Here we investigate the stochastic phase dynamics of hybrid JJs in an InAs/Al planar heterostructure [3]. We show that our JJs are characterized by prominent quantum fluctuations, resulting in a mean switching current $I_M$ which is a small fraction of the JJ critical current $I_C$. In JJs with small $I_C$, $I_M$ suppression is strong enough for phase-diffusion effects to dominate at low-temperature. We further show that embedding a JJ in an asymmetric SQUID, an approach intensively pursued for realizing topological states [22–25], alleviates the impact of quantum fluctuations. Consequently, $I_M$ may significantly vary when a JJ is measured in isolation or in a SQUID (by a factor of 2.5 or higher, in the present case). The dominant phase-escape mechanism is further tuned via temperature, gate voltages and fluxes threading the SQUID. Contrary to conventional metallic JJs, no indication of thermal phase activation is observed. Characteristic experimental features are reproduced with a Monte Carlo simulation of the phase dynamics. Our observations contribute to the understanding of high-quality planar JJs, both in isolation and within SQUID loops, and guide towards the realization of novel quantum architectures.

Figure 1(a) shows a micrograph of the device under study and measurement configuration. The InAs is highlighted in pink and the Al in blue. Gates are drawn on the image and highlighted in yellow. (b) Differential resistance $R$ as a function of $B_\perp$ and $I_{DC}$ obtained with $V_{G1} = -180$ mV and $V_{G2} = -140$ mV. The amplitude of the switching current oscillations, $\Delta I$, is marked. (c) Differential resistance of JJ1 in isolation, with $V_{G1} = -180$ mV and $V_{G2} = -450$ mV. Large fluctuations close to $B_\perp = 0$ are marked with an arrow. (d) Differential resistance of JJ2 in isolation, with $V_{G1} = -550$ mV and $V_{G2} = -140$ mV. The peak at $B_\perp = 0$ is less than half $\Delta I/2$ in (b).
V_{G2} allowed tuning of JJ1 and JJ2, respectively. The gate voltage $V_{\text{global}}$ was kept constant at $-600 \text{ mV}$ to prevent parallel conduction in the semiconductor. The design was optimized to reach a critical current in JJ1 ($I_{C,1}$) that was much larger than the critical current in JJ2 ($I_{C,2}$) [22, 26]. This was achieved by changing the lateral extent of the Al electrodes (5 $\mu$m in JJ1 vs. 1.6 $\mu$m in JJ2) and their separation (50 nm in JJ1 vs. 100 nm in JJ2).

Electronic measurements were conducted in a dilution refrigerator with a mixing chamber base temperature $T$ below 20 mK. Results presented here were confirmed on a second SQUID device and on several individual JJs.

We first present switching currents obtained with low-frequency lock-in techniques, similar to previous work [18, 22, 26]. A source-drain current $I_{\text{DC}}$ was swept over timescales of seconds, while the SQUID differential resistance $R$ was recorded. Figure 1(b) shows $R$ as a function of out-of-plane magnetic field $B_\perp$ with $V_{G1} = -180 \text{ mV}$ and $V_{G2} = -140 \text{ mV}$, which is the configuration where $I_{C,1}$ and $I_{C,2}$ were independently maximized. The SQUID switching current $I_1$ had a periodicity of $350 \mu \text{T}$, corresponding to a flux quantum $\hbar/2e$ threading the loop. The amplitude of the SQUID oscillations, $\Delta I$, reveals the switching current of JJ2 as $I_2 = \Delta I/2 = 350 \text{ nA}$, while the mean value gives the switching current of JJ1, $I_1 = 850 \text{ nA}$. Figure 1(c) shows $R$ when JJ2 is closed and with JJ1 in the gate configuration of Fig. 1(b). The well-known Fraunhofer interference pattern emerges, with a periodicity in $B_\perp$ consistent with one flux quantum in the area of JJ1 [27] and with a maximum of $I_1$ matching the mean switching current of Fig. 1(b). Furthermore, large switching current fluctuations were present at $B_\perp = 0$ (black arrow). Figure 1(d) shows similar measurements performed with $I_{\text{DC}}$ flowing in JJ2 only. Surprisingly, the maximum of $I_2$ is 120 nA; a significant difference with the 350 nA deduced from Fig. 1(b).

Both the fluctuations in Fig. 1(c) and the switching current enhancement in Fig. 1(b) with respect to Fig. 1(d) are manifestations of the unconventional phase dynamics in our devices. To address this, we evaluate the phase escape mechanisms in JJ1 and JJ2 separately (Fig. 2), and in the SQUID loop formed by their combination (Fig. 3). Finally, we demonstrate gate and flux tunability of the escape dynamics (Fig. 4). To capture the stochastic characteristics of phase escape, we modulate the input current with a sawtooth function using a ramp rate $\nu = 240 \mu \text{As}^{-1}$ and monitor the voltage across the SQUID with an oscilloscope. This technique allows us to record the switching current $I_{\text{SW}}$ for 10,000 switching events in approximately ten minutes, and produce the switching probability distribution (SPD), that is the probability for a switch to occur per unit of input current. Similar techniques were used for detailed studies of conventional [28, 29] and hybrid JJs [30–33], metallic nanowires [34–36] and SQUIDs [37–41].
as \[12\]:

\[
\Gamma(I_{SW}) = \text{SPD}(I_{SW}) \nu \left[1 - \int_0^{I_{SW}} \text{SPD}(I) dI \right]^{-1} \tag{1}
\]

are shown in Fig. 2(b) and (d), respectively.

Figure 2(e) and (f) show the mean value of the SPDs in JJ1 \((I_{M,1})\) and its standard deviation \((\sigma_1)\), respectively, both as a function of \(T\). For \(T < 400\) mK, \(\sigma_1\) is constant and large, and \(\Gamma\) increases exponentially with \(I_{SW}\), indicating that macroscopic quantum tunneling (MQT) dominates the phase dynamics. For higher \(T\), \(\sigma_1\) decreases as \(T\) increases, signaling the crossover to a regime of phase diffusion (PD), where escape and retrapping events have similar probabilities to occur, so that many escape events are required to transition to the resistive state. The temperature \(T^* \sim 0.55\) K marks the crossover between a regime dominated by MQT and one dominated by PD.

The temperature dependence of \(I_{M,1}\) and \(\sigma_1\) is well captured by a Monte Carlo simulation of the phase dynamics (gray line) \[13\], an approach previously adopted for the study of moderately damped JJs \[14, 15\]. Details on our simulation, which requires the JJ capacitance \(C\), temperature-dependent critical current \(I_{C,1}\) and the zero-temperature quality factor \(Q_0\) as input parameters, are discussed in the Supplemental Material \[16\].

The best fit to the low-temperature data was obtained with \(C = 0.95\) fF, \(I_{C,1} = 3.05\) mA and \(Q_0 = 6.9\). Thus, JJ1 is moderately damped and has a small intrinsic capacitance, leading to large quantum fluctuations. Furthermore, \(I_{C,1}\) was 2.5 times higher than \(I_{M,1}\), indicating that moderate input currents already result in a high switching probability. The ratio between \(I_{C,1}\) and \(I_{M,1}\) decreases towards one for \(T > T^*\), as signaled by the kink in \(I_{M,1}\) at \(T = T^*\). This is because the chance of phase retrapping increases with \(T\) more than the tunneling probability. The small value of \(C\) is consistent with the estimated geometrical capacitance between the Al electrodes. Previous studies of planar JJs in different physical systems obtained a fit capacitance much larger than the geometrical capacitance (and consequently a less marked importance of quantum fluctuations) \[17, 18, 19\].

The discrepancy was explained by considering an effective capacitance associated to diffusive electron motion in the normal region. In our devices, the highly-transmissive supercurrent-carrying Andreev bound states (ABSSs) might make such a term negligible. The result \(Q_0 = 6.9\) indicates moderate damping and is consistent with measuring a finite resistance at \(I_{DC} = 0\) for \(T > 1\) K, which is well below the critical temperature of the Al \[16\].

Similar to JJ1, MQT is the dominant phase escape mechanism in JJ2. However, large dissipation results in a significant retrapping probability and places JJ2 in the PD regime down to base temperature. This is evident from \(I_{M,2}\) and \(\sigma_2\) shown in Figs. 2(g) and (h), respectively, where \(\sigma_2\) does not saturate for \(T \to 0\), and from the deviation of \(\Gamma\) from an exponential in Fig. 2(d). We could not accurately fit the behavior of JJ2 with Monte Carlo simulations, as the low \(I_{C,2}\) likely sets \(Q \sim 1\), while available theories are reliable for \(Q \gtrsim 5\). Thermal phase activation, which would result in \(\sigma_1\) increasing with \(T\), is not observed neither in JJ1 nor JJ2. The absence of thermal activation is highly unusual \[17\], as it requires quantum fluctuations to dominate over thermal energy scales up to the critical temperature \[16\]. It also indicates that the PD regime is of quantum mechanical nature, and might serve as basis for future studies on quantum random walks and dissipation \[15, 19\].

We now present the phase dynamics when the JJs are simultaneously activated. Figures 3(a) and (b) show the mean, \(I_{M,S}\), and standard deviation, \(\sigma_S\), of each SPD obtained in the gate configuration of Fig. 1(b) as a function of \(B_\perp\) and \(T\) \[20\]. In Fig. 2(b), SQUID oscillations are clearly captured by \(I_{M,S}\). In Fig. 2(b), the curves at low \(T\) have a large \(\sigma_S\), independent of \(B_\perp\). As \(T\) increases further, \(\sigma_S\) is modulated by \(B_\perp\) and ultimately becomes small and independent of \(B_\perp\). In Fig. 2(c) we compare \(I_{M,2}\) [squares, as in Fig. 2(g)] to the half-amplitude of the oscillations in \(I_{M,S}\) [circles]. In the absence of fluctuations, the two quantities would coincide. Instead, we find a significant discrepancy, highlighted by green shading, which is large at low \(T\) and vanishes above \(T^*\) of JJ1. By tuning \(T^*\) via \(V_{G1}\), we confirm that the enhancement of \(\Delta I_{M,S}/2\) with respect to \(I_{M,2}\) was always correlated to \(T^*\) in JJ1 \[16\]. The mean value of \(I_{M,S}\) matched \(I_{M,1}\) \[16\] and the mean of \(\sigma_S\), \(\langle \sigma_S \rangle\), was similar to \(\sigma_1\) [Fig. 2(d)].

The results presented in Fig. 3 are intuitively understood by considering phase-locking by the loop inductance. For JJ2 alone, quantum fluctuations are large compared to the Josephson energy, so phase escape is already likely at moderate currents. Coupling JJ2 to JJ1 effectively realizes a new JJ with higher Josephson energy and similar phase dynamics to JJ1, so that the dominant switching mechanism is MQT and, consequently, the suppression of \(I_M\) is reduced. However, protection of \(I_{M,2}\) is maintained while JJ1 stays in the MQT regime \((T < T^*)\), where phase uncertainty is less than in the PD regime. Consistent with this interpretation, phase dynamics in the asymmetric SQUID configuration are well described by a Monte Carlo simulation of a fictitious JJ with a field-dependent critical current \(I_{C,S}(B_\perp)\), and with \(C\) and \(Q_0\) as derived for JJ1. The sole fit parameter was \(I_{C,S}\) for \(T = 20\) mK, which is shown in Fig. 3(e) as a function of \(B_\perp\) [circles]. Also for the SQUID, critical current \(I_{C,S}\) and mean switching current \(I_{M,S}\) [Fig. 3(a)] differ by a factor of approximately 2.5. After obtaining \(I_{C,S}(B_\perp)\) for \(T = 20\) mK, the entire dataset of Figs. 3(a) and (b) was simulated without free parameters. We show the simulated half-amplitude \(\Delta I_{M,S}/2\) and the mean of \(\sigma_S\) as gray lines in Figs. 3(c) and (d) respectively. Despite the simplicity of our model, experimental results are reproduced
FIG. 3. Mean $I_{M,S}$ (a) and standard deviation $\sigma_S$ (b) of the SPDs in the SQUID configuration as a function of $B_\perp$, for temperatures between 20 and 800 mK. (c) Mean switching current of JJ2 as a function of $T$, derived from the SQUID oscillations (circles) and measured with JJ2 in isolation (squares). The solid line is $\Delta I_{M,S}/2$ obtained from a Monte Carlo simulation fitted to the experimental results. (d) Standard deviation of the SPD in JJ1 measured in isolation (squares) together with the mean of $\sigma_S$ from (b) (circles) as a function of temperature. The solid line is the result of the Monte Carlo simulation presented in (c). (e) SQUID critical current obtained by fitting the SPDs for $T = 20$ mK to an MQT escape rate (circles) together with a fit of an ABSs model. (f) Color map of fitted standard deviation $\sigma_S$, with transition temperature $T^*$ marked by a dashed line.

FIG. 4. (a) Switching currents of JJ2 as a function of $V_{G2}$ when measured in isolation (squares) and in the SQUID configuration (circles), together with the critical current derived from Monte Carlo simulations (diamonds). (b-d) Standard deviation of SPDs measured in the SQUID configuration for three values of $V_{G1}$. Blue shading highlights MQT regimes.

Decreasing $I_{C,1}$ via $V_{G1}$ made the SQUID more symmetric and shifted JJ1 towards a regime of PD. Figures 4(b-d) show $\sigma_S$ for decreasing values of $V_{G1}$. For $V_{G1} = -300$ mV and $-350$ mV (Figs. 4(c) and (d), respectively) escape dynamics varied between MQT (blue shading) and PD already at base temperature, with $\sigma_S$ persisting. Modeling the curves in Figs. 4(b-d) would require a quantum treatment of the phase escape from a 2D potential, which to a large extent.

The $I_{C,S}(B_\perp)$ curve of Fig. 3(e) is consistent with the presence of highly-transmissive ABSs, resulting in a forward-skewed current-phase relation. The mean transmission $\tau$ is extracted by fitting $I_{C,S}(B_\perp)$ with $I_C \propto \sin(\varphi) / [1 - \tau \sin^2(\varphi/2)]^{1/2}$ (line), obtaining $\tau = 0.77$. The phase $\varphi$ was considered proportional to $B_\perp$. Figure 3(f) shows a colormap of the simulated standard deviation, $\sigma_S(B_\perp,T)$, with $T^*$ indicated by a dashed line and marking the crossover between MQT and PD. The phase dynamics are completely described by MQT and PD for low and high $T$, respectively. For intermediate $T$, the phase escape mechanism periodically varies between MQT and PD as a function of $B_\perp$.

Results presented so far were obtained in a gate configuration where $I_{C,1}$ and $I_{C,2}$ were maximized. In the following, we discuss how phase escape dynamics vary as $I_{C,1}$ and $I_{C,2}$ are tuned via gate voltages. Figure 4(a) summarizes results for JJ2 as $V_{G2}$ was varied. When JJ2 was measured in isolation, switching currents $I_{M,2}$ were small and PD was the dominant escape mechanism throughout the accessible range of $V_{G2}$. We highlight this condition with gray shading. When the SQUID was formed, the switching current of JJ2 deduced from the SQUID oscillations ($\Delta I_{C,S}/2$) was significantly higher than when JJ2 was measured in isolation. We highlight this situation with gray shading. Interestingly, JJ2 was resistive for $V_{G2} < 300$ mV if measured in isolation, but SQUID oscillations were still observed. Finally, the $I_{C,2}$ obtained by fitting the SPDs in the SQUID with the Monte Carlo simulation [as in Fig. 3(e)] is highlighted in yellow.
goes beyond the scope of this work.

In conclusion, planar JJJs with highly transmissive ABSs reside in a unique dynamical regime with moderate dissipation ($Q_0 < 10$) and with quantum fluctuations exceeding thermal excitations up to the critical temperature. As a result, $I_M$ can differ largely from $I_C$, depending on the details of the JJJs and of their electrostatic environment. Phase dynamics can be modified by embedding JJJs in asymmetric SQUID geometries, resulting in significant changes of $I_M$. Furthermore, the dominant phase-escape mechanism in a SQUID can be tuned between MQT and PD via a magnetic flux. This intricate physics becomes relevant for investigating topological phenomena and ABSs, where the use of hybrid JJJs is widespread. Due to their intriguing phase dynamics, hybrid JJJs could also serve as platform for investigating the physics of strong fluctuations and quantum random walk.

We are grateful to C. Müller and W. Riess for helpful discussions. We thank the Cleanroom Operations Team of the Binnig and Rohrer Nanotechnology Center (BRNC) for their help and support. F. N. acknowledges support from the European Research Council (grant number 804273) and the Swiss National Science Foundation (grant number 200021_201082). W. B. acknowledges support from the European Union’s Horizon 2020 FET Open programme (grant number 964398).

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Sample Fabrication

Samples were fabricated from a heterostructure grown on InP (001) substrates by molecular beam epitaxy techniques. The heterostructure consists of a step-graded metamorphic InAlAs buffer and a 7 nm thick InAs quantum well, confined by InₙAl₁₋ₙAs barriers 10 nm below the surface. A 10 nm thick Al layer is deposited on top of the heterostructure, in the same chamber as the III-V growth while maintaining vacuum. The peak mobility is 18,000 cm²V⁻¹s⁻¹ for an electron density of \( n = 8 \times 10^{11} \text{ cm}^{-2} \). This gives an electron mean free path of \( l_e \gtrsim 350 \text{ nm} \), hence we expect all JJs measured here to be ballistic along the length \( L \) of the junction.

The sample is defined by first isolating the large mesa structures on which the device is patterned. This is done by selectively removing the top Al layer (with Al etchant Transene D) before etching \( \sim 250 \text{ nm} \) into the III-V heterostructure using a chemical wet etch \((220 : 55 : 3 : 3 \text{ solution of } \text{H₂O : C₆H₅₂O₃ : H₃PO₄ : H₂O₂)\). The Al device is then patterned on top of the mesa, by selective etching of the Al with Transene D at 50 °C for 4 s. To control the exposed III-V region, we deposit a 15 nm layer of HfO₂ by atomic layer deposition, before evaporating metallic gate electrodes. These are deposited in two steps: the first consists of 5 nm Ti and 20 nm of Au on top of the device region; the second, 10 nm of Ti and 350 nm of Al, contacts the gates on top of the mesa to the bonding pads.

Lock-in Measurements

Electrical measurements were performed in a dilution refrigerator with a mixing chamber temperature below 20 mK. Initial characterization was performed by standard lock-in techniques. An AC current of \( I_{AC} = 2 \text{ nA} \) was applied to the source contact of the SQUID device, with a frequency of 233 Hz. The four-terminal differential voltage \( V_{AC} \) across the SQUID was measured at this frequency, via a differential voltage amplifier with 1000 times gain. The differential resistance \( R = V_{AC}/I_{AC} \) was measured as a function of the applied DC current \( I_{DC} \).

In addition to the \( B_1 \)-dependent measurements presented in Fig. 1 of the Main Text, we show temperature- and gate-dependent measurements of JJ1 and JJ2. In Figs. S.1(a) and (b), for JJ1 and JJ2 respectively, we show the differential resistance \( R \) as a function of \( I_{DC} \), swept from negative to positive currents. The color denotes the temperature, which ranges from 20 mK to 1.6 K, at which point both JJs are fully resistive. We offset the vertical axis by 200 Ω between each temperature trace, to highlight the zero-resistance state at low bias currents.

As we increase \( I_{DC} \) from \(-2 \mu\text{A}\), each junction undergoes a transition from the resistive to superconducting state at the retrapping current \( I_R \). At positive bias, the superconducting-to-resistive transition occurs at the switching current \( I_{SW} \). The difference between the two, most notable in JJ1, indicates the underdamped JJ behavior. At high temperatures the superconducting state softens, leading to a finite resistance at bias values below \( I_{SW} \). This is expected from phase diffusive JJs at high temperatures, but makes determination of \( I_{SW} \) less reliable; hence we do not present SPDs at temperatures \( T > 1 \text{ K} \), where this effect is significant.

Figures S.1(c) and (d) show the gate dependence of the differential resistance \( R \) across JJ1 and JJ2, respectively. The normal state resistances for JJ1 and JJ2 are \( R_{N,1} = 100 \text{ Ω} \) and \( R_{N,2} = 300 \text{ Ω} \), respectively. At a small negative gate voltage, the switching current reaches its maximum. The peak occurs at \( V_{G1} = -180 \text{ mV} \) for JJ1 and \( V_{G2} = -140 \text{ mV} \) for JJ2. These define the operating points for each junction in Figs. 1-3 of the
Phase Escape in Single JJs

Phase dynamics in Josephson junctions are often described using the analogy of a massive phase particle in a tilted washboard potential, where the particle mass corresponds to the JJ capacitance and its damping to the inverse of its resistance. The zero-resistance state corresponds to the particle being trapped in an energy minimum while the resistive state corresponds to the particle moving along the potential. A transition to the resistive state generally takes place, via quantum or thermal fluctuations, at switching currents $I_{SW}$ lower than the JJ critical current $I_C$. To measure this transition, we apply a sawtooth signal from a waveform generator through a bias resistor to rapidly ramp $I_{DC}$, and detect a switch to the resistive state by measuring the point at which the voltage across the JJ increases above a threshold. The value of $I_{DC}$ at which this switch occurs is recorded as the switching current, $I_{SW}$. We measure $I_{SW}$ more than 10,000 times and collect the results in a switching probability distribution (SPD). The SPDs are recorded as a function of temperature, from 20 mK to 1 K.

The SPDs are dependent on the rate at which the DC current is increased, $\nu$. To compare with theoretical expressions for phase dynamic mechanisms, we must convert to a measurement-independent quantity: the escape rate, $\Gamma$. This is done via KFD transform using the following equation \[42\]:

$$\Gamma(I_{SW}) = \text{SPD}(I_{SW})\nu \left(1 - \int_0^{I_{SW}} \text{SPD}(I)dI\right)^{-1} \quad (S.1)$$

where $\nu$ is the ramp rate of the DC current, which was $\nu = 240 \mu\text{A s}^{-1}$ for all measurements.

In the following, we will consider equations derived for JJs with a sinusoidal current-phase relation. While our JJs are known to contain highly-transmissive modes, leading to deviations from this sinusoidal behavior, we use the existing theory in the absence of an alternative model and expect only small numerical deviation from the results presented.

The rate of escape of a Josephson junction to the resistive state under the action of a DC bias follows the general dependence $51$:

$$\Gamma(I_{SW}, T) = \Omega(I_{SW}, T)e^{-\Delta U/I_{SW,T}}/k_B T, \quad (S.2)$$

where $\Omega$ is the attempt frequency and $\Delta U$ is the potential barrier height. Under a reduced DC bias of $\gamma = I_{DC}/I_C$, the approximate barrier height is $\Delta U = 2E_{J0}\left(\sqrt{1 - \gamma^2} - \gamma \cos^{-1}\gamma\right)$, with a Josephson energy of $E_{J0} = \hbar I_C/2e$.

A transition to the resistive state can be promoted, for currents lower than $I_C$, by macroscopic quantum tunnelling (MQT) or thermal activation (TA) across the barrier $\Delta U$. In the case of MQT, equation $S.1$ is adjusted to the analytical formula for escape $51$:

$$\Gamma_Q = \frac{\omega_P}{2\pi} \sqrt{\frac{864\pi^2}{16\pi^2} \exp\left(-7.2 \left(1 + \frac{0.87}{Q}\right) \frac{\Delta U}{h\omega_P}\right), \quad (S.3)$$

where $\omega_P = \omega_{P0} (1 - \gamma^2)^{1/4}$ is the plasma frequency at $\gamma$ and $Q$ is the $C$ factor of the junction. The junction capacitance $C$ enters in the bare plasma frequency, $\omega_{P0} = \sqrt{2eC/\hbar}$.

When dissipation is large it is possible for the junction to transition from the resistive to the superconducting state, referred to as retrapping. In this regime, many escape events are required to turn the junction resistive. This is referred to as phase diffusion (PD). For $Q \gg 1$, we can use an analytical formula for this retrapping rate.
\[
\Gamma_R = \frac{I_{SW} - I_R}{I_C} \omega \mu_0 \left( \frac{E_{10}}{2 \pi k_B T} \right) \exp \left( - \left( \frac{I_{SW} - I_R}{I_C} \right)^2 \frac{E_{10} Q^2}{2 k_B T} \right)
\]

where \(I_R\) is the retrapping current. We measure immediately damped junctions with \(Q \gtrsim 1\), so this relation can only be considered as an approximation.

The interplay between \(\Gamma_Q\) and \(\Gamma_R\) determines the phase escape regime: \(\Gamma_Q \gg \Gamma_R\) in the MQT regime, whereas the reverse is true for phase diffusion. The rates depend on the junction properties: the critical current \(I_C\), the capacitance \(C\) and the quality factor, which has the zero-temperature value \(Q_0\). We show in Figs. S2(a-c) the relevant rates in the system for the fit parameters for JJ1: \(I_C = 3.05 \mu A, C = 0.95 \text{ fF}\) and \(Q_0 = 6.9\). Each panel corresponds to a different temperature: \(T = 20\) mK, \(T = 500\) mK and \(T = 800\) mK, respectively. These highlight the change in the dominant regime from MQT to PD on increasing \(T\).

The ramp rate \(\Gamma_1(I) = \nu / I\) defines the lowest frequencies at which an escape event can be measured, for a given bias current \(I\). The point at which this intersects the escape rate gives the lowest bias current \(I_{IE}\) at which an escape event is measurable. Since quantum tunneling is the dominant escape mechanism, we define \(I_{IE}\) as the current at which \(\Gamma_1 = \Gamma_Q\), marked in Figs. S2(a-c) with the green dot. The retrapping rate \(\Gamma_R\) is large for low bias currents but quickly decreases with an increase in \(I\). Retrapping is significant when \(\Gamma_R\) reaches a similar magnitude as \(\Gamma_Q\). Escape by thermal activation is given by \(\Gamma_T\), and is much smaller than \(\Gamma_Q\) for all temperatures.

At the lowest temperature, escape by quantum tunneling dominates. This is clear since \(\Gamma_Q \gg \Gamma_R\) for \(I > I_{IE}\). No retrapping of the phase occurs: a single escape event is sufficient to transition to the resistive state. At \(T = 500\) mK, \(\Gamma_Q \lesssim \Gamma_R\) close to \(I_{IE}\). For these low bias currents, the retrapping probability is high so the probability of escape in the junction is reduced relative to quantum tunneling alone. However, \(\Gamma_Q \gg \Gamma_R\) at larger bias so escape occurs unhindered by phase diffusion. As the temperature increases, the effect of retrapping becomes more significant. At the high temperature of \(800\) mK, phase diffusion is dominant since \(\Gamma_Q \ll \Gamma_R\) across the range of escape currents.

The rates in Figs. S2(a-c) are used to model the junction behavior in a Monte Carlo simulation, as in Ref. [13]. As the DC bias current is increased, the simulated junction stochastically switches between the superconducting (0) and resistive (1) states according to the relative escape and retrapping rates. The junction is said to be resistive when the state, averaged over a window of current, exceeds 0.1. This process is performed 20,000 times and the generated \(I_{SW}\) are combined into a switching probability distribution (SPD).

The simulation uses the critical current \(I_C\), capacitance \(C\) and quality factor \(Q_0\) as input parameters for the rates \(\Gamma_Q\) and \(\Gamma_R\). We fit by hand the SPD at low temperature, obtaining \(I_C = 3.05 \mu A, C = 0.95 \text{ fF}\) and \(Q_0 = 6.9\). We use the Bardeen formula for the temperature dependence of \(I_C(T) = I_C(1 - T^2 / T_0^2)^{3/2}\), with \(T_0 = 1.18\) K from experimental results. Since \(Q \propto I_C^{1/2}\), we use \(Q(T) = Q_0(1 - T^2 / T_0^2)^{3/4}\). We use the low temperature fit result and the assumed temperature dependence to simulate the full dataset. Figures S2(d) and (e) show the SPDs and escape rates for JJ1, respectively. The experimental data (circles) is fitted well by the simulated curve (lines). Deviation at high temperatures between the fit and the data is explained by the simple model used for the temperature dependence.

Despite this, we capture the characteristic trend in the data.

In general, the quality factor of the junction is described by \(Q = R C \omega_P\). For \(I_C = 3.05 \mu A\) and \(C = 0.95 \text{ fF}\) we get a resistance of \(R = 2.33\) kΩ, much larger than the normal state resistance at low frequency of \(R_{N,1} = 100\) Ω. We therefore conclude that damping at high frequency is relevant in the case of these junctions. For \(Q \gg 1\), we can relate the quality factor to the ratio of critical and retrapping currents: \(Q = 4 I_C / \pi I_R\). For JJ1, \(I_R = 600\) nA giving \(Q = 6.4\), close to the fit value.

Escape by quantum tunneling is dominant up to the critical temperature. By comparing Eqs. S2 and S3 we obtain an effective temperature of quantum tunneling escape,

\[
k_B T_Q = \frac{h \omega_P}{7.2(1 + 0.87/Q)}.
\]

At low temperature, \(T_Q \approx 3\) K for JJ1. This exemplifies the large scale of quantum fluctuations relative to thermal excitations. The temperature at which \(T_Q < T\) for the parameters of JJ1 is approximately 1 K: thermal activation is not significant up to the critical temperature.

**Full SQUID Data and Monte Carlo Simulation**

Figure 3 in the Main Text shows the switching probability in the SQUID configuration, as a function of \(B_\perp\) and \(T\). From \(I_{M,S}\) and \(\sigma_S\) as a function of \(B_\perp\) we extract \(\Delta I_{M,S}/2\) and \(\langle \sigma_S \rangle\). In Fig. S3 we plot the remaining extracted parameters \((I_{M,S})\) and \(\Delta \sigma_S\) (circles) with the corresponding results for the Monte Carlo simulation (lines).

Figure S3(a) shows the field-averaged value of the oscillations in \(I_{M,S}\) (circles) compared with \(I_{M,1}\), the mean switching current for JJ1 (squares). The two align across all temperatures, confirming that JJ1 dominates the average SQUID behavior.

The Monte Carlo simulation (gray line) follows the data at low temperatures, but some deviations emerge above \(T = 600\) mK when the junction is almost completely phase diffusive. The escape rates are particularly
sensitive to the damping \(Q\) in this regime, so deviations between simulations and experiment at high temperature might be accounted for with a more complex temperature dependence. Figure 3(c) of the Main Text shows a similar deviation in \(\Delta_C\) across different temperatures, \(T\), close to the transition temperature \(T^* \approx 0.55 \text{ K}\), \(\Delta_C\) is more than 35 nA. Both at low and high temperature, \(\Delta_C\) is almost constant across the magnetic field. Oscillations in the simulated \(\Delta_C\) (gray lines) were observed at low-temperature, contrary to the experimental data, hence the larger simulated \(\Delta_C\).

The results are compared with the Monte Carlo simulation (lines). As described in the Main Text, the fit parameter \(I_{C,S}\) is obtained from the low-temperature data. Since the SQUID is in the MQT regime, we fit each escape rate with Eq. (S.3) using a fixed \(C = 0.95 \text{ fF}\) and \(Q_0 = 6.9\) from JJ1.

\[
I_{C,S} = I_{C,1} + I_0 \frac{\sin(\varphi)}{\sqrt{1 - \tau \sin^2(\varphi/2)}}, \tag{S.6}
\]

with \(I_{C,1} = 3.05 \mu\text{A}\), \(I_0 = 480 \text{ nA}\) and \(\tau = 0.77\). This gives \(\Delta I_{C,S}/2 = 650\) nA. The mean value of \(I_{C,S}\) at \(T = 20\) mK is given by the solid line in Fig. S.3(a). We use the low-temperature result to simulate the full dataset by varying \(I_{C,1}\) and \(I_0\) with a Bardeen dependence, as for the isolated junctions.

### Dependence on Gate Voltage 2

In an asymmetric SQUID, the amplitude of oscillations as a function of \(B_\perp\) is an indication of the current flowing through the small junction. This is an approximation to

\[
I_{C,S} = \sqrt{(I_{C,1} - I_{C,2})^2 + 4I_{C,1}I_{C,2}\cos^2\left(\frac{\varphi - \varphi_0}{2}\right)}, \tag{S.7}
\]

when \(I_{C,1} \gg I_{C,2}\). We use this equation to extract the critical current of JJ2 in the asymmetric SQUID, as a function of gate voltage \(V_{G2}\).

We measure SQUID oscillations at \(T = 20\) mK and fixed \(V_{G1} = -180\) mV, with different \(V_{G2}\) [see Fig. S.4(a)]. The SQUID is always in the MQT regime, independent of \(V_{G2}\), and the asymmetry is large. For each \(V_{G2}\), we extract \(\Delta I_{M,S}/2\) as shown, and plot the results in Fig. 4(a) of the Main Text. We extract the critical current \(I_{C,S}\) by fitting each SPD with Eq. (S.3) in the MQT regime, with fixed capacitance \(C\) and quality factor \(Q_0\) defined by JJ1. The result is shown in Fig. S.4(b), from which we extract \(I_{C,2} = \Delta I_{C,S}/2\).

### Dependence on Gate Voltage 1

The regime of the SQUID is dominated by JJ1, the large \(I_C\) component, so we can change the SQUID behavior by varying \(V_{G1}\). For \(V_{G1} = -300\) mV, the SQUID undergoes direct transitions between MQT and PD depending on \(B_\perp\) [Fig. 4(b) in the Main Text]. The full dataset is shown in Fig. S.5 for 20 mK to 900 mK: the mean switching current \(I_{M,S}\) in (a) and the standard deviation in (b).

The oscillation amplitude \(\Delta I_{M,S}/2\) is shown in Fig. S.5(c). The enhancement in switching current at low...
temperatures is again observed, where quantum tunneling is dominant. We also observe the characteristic kink in $\Delta I_{\text{MS}}/2$, in this case at $T \approx 0.5$ K concomitant with the lower transition temperature to the phase diffusive regime.

At $T = 20$ mK, we observe a large variation in the standard deviation $\sigma_S$ depending on the field $B_\perp$. At the maximum of $I_{\text{MS}}$ (diamond), $\sigma_S$ is large at low temperature. This is consistent with quantum tunneling as the dominant mechanism of phase escape. Instead, $\sigma_S = 20$ nA at the minimum (triangle), indicating that phase diffusive effects are strong. The traces in $\sigma_S$ at these field values are shown in Fig. S.5(d) by their respective markers. The large difference in $\sigma_S$ is evident at low temperatures, as indicated by the blue shading, where the external magnetic field determines the extent of phase diffusion in the SQUID. On increasing $T$ towards the transition temperature, the difference in $\sigma_S$ reduces until the SQUID is fully phase-diffusive at all values of $B_\perp$.

Figures S.6 and S.7 show the datasets for $V_{G1} = -350$ mV and $V_{G2} = -140$ mV, as a function of $B_\perp$. Field traces are taken at temperatures ranging from 20 mK up to 900 mK. The maximum (diamond) and minimum (triangle) of the $I_{\text{MS}}$ oscillations are marked. (b) Standard deviation $\sigma_S$ in this SQUID configuration, for temperatures 20 mK to 900 mK. (c) Oscillation amplitude of $I_{\text{MS}}$, $\Delta I_{\text{MS}}/2$ as a function of temperature. A kink in $\Delta I_{\text{MS}}/2$ occurs at the transition temperature of $T^* \approx 0.45$ K. (d) Standard deviation $\sigma_S$ at values of $B_\perp$ corresponding to the $I_{\text{MS}}$ maximum (diamonds) and minimum (triangles) respectively. The large difference at $T < T^*$ is indicated.

On further decrease in $V_{G1}$ to 375 mV, the SQUID
FIG. S.7. (a) Mean switching current $I_{M,S}$ for the SQUID configuration $V_{G1} = -375$ mV and $V_{G2} = -140$ mV, as a function of $B_\perp$. Field traces are taken at temperatures ranging from 20 mK up to 900 mK. The maximum (diamond) and minimum (triangle) of the $I_{M,S}$ oscillations are marked. (b) Standard deviation $\sigma_S$ in this SQUID configuration, for temperatures 20 mK to 900 mK. (c) Oscillation amplitude of $I_{M,S}$, $\Delta I_{M,S}/2$ as a function of temperature. A kink in $\Delta I_{M,S}/2$ occurs at the transition temperature of $T^* \approx 0.3$ K. Figure S.7(d) shows $\sigma_S$ at the maxima (diamonds) and minima (triangles) of $I_{M,S}$, and while some divergence emerges for $T < T^*$, phase diffusion is dominant for all values of $B_\perp$.

is almost symmetric. In this case, a magnetic-field dependence is still observable in the standard deviation [see Fig. S.7(b)] but the SQUID is phase diffusive at $T = 20$ mK for all values of $B_\perp$. The corresponding oscillation amplitude, while no longer representative of the switching current of JJ2, again shows the kink in $\Delta I_{M,S}/2$ at the low transition temperature of $T \approx 0.3$ K. Figure S.7(d) shows $\sigma_S$ at the maxima (diamonds) and minima (triangles) of $I_{M,S}$, and while some divergence emerges for $T < T^*$, phase diffusion is dominant for all values of $B_\perp$. 