CELLULARIZATION OF CLASSIFYING SPACES AND FUSION PROPERTIES OF FINITE GROUPS

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Abstract. One way to understand the mod $p$ homotopy theory of classifying spaces of finite groups is to compute their $B\mathbb{Z}/p$-cellularization. In the easiest cases this is a classifying space of a finite group (always a finite $p$-group). If not, we show that it has infinitely many non-trivial homotopy groups. Moreover they are either $p$-torsion free or else infinitely many of them contain $p$-torsion. By means of techniques related to fusion systems we exhibit concrete examples where $p$-torsion appears.

Introduction

Let $A$ be a pointed space. Although the idea of building a space as a homotopy colimit of copies of another fixed space $A$ goes back to Adams ([Ada78]) in the framework of the classification of the acyclics of a certain generalized cohomology theory, it was in the early 90’s that Dror-Farjoun ([Far96]) and Chachólski ([Cha96]) formalized and developed this idea in the wider context of cellularity classes. Thus a space will be called $A$-cellular if it can be built from $A$ by means of pointed homotopy colimits. There exists an $A$-cellularization functor $CW_A$ that provides the best possible $A$-cellular approximation: the natural map $CW_AX \to X$ induces a weak equivalence between pointed mapping spaces $\text{Map}_\ast(A,CW_AX) \simeq \text{Map}_\ast(A,X)$.

Our interest lies essentially in using the cellularization functor to study the $p$-primary part of the homotopy of the classifying space of a finite group $G$. This approach was already suggested by Dror-Farjoun in [Far96, Example 3.C.9], and proved to be very fruitful in the last years; we can remark work of Bousfield ([Bou97]) which describes cellularization of nilpotent spaces with regard to Moore spaces $M(\mathbb{Z}/p,n)$, or the relationship recently discovered ([BS01]) between the $B\mathbb{Z}/p$-cellularization of spaces and the $\mathbb{Z}/p$-cellularization in the category of groups.

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In the present paper we focus our attention in describing the $B\mathbb{Z}/p$-cellularization of $BG$, where $G$ is a finite group. From the above description of the cellularization functor, we know this is a space which can be built from $B\mathbb{Z}/p$ by means of push-outs, wedges, and telescopes, and which encodes all the information about the pointed mapping space $\text{Map}_*(B\mathbb{Z}/p, BG)$, which is isomorphic to $\text{Hom}(\mathbb{Z}/p, G)$. A first attempt to understand $CW_{B\mathbb{Z}/p}BG$ was undertaken in [Flo], where in particular the fundamental group of the cellularization was described as an extension of the group-theoretical $\mathbb{Z}/p$-cellularization of a subgroup of $G$ by a finite $p$-torsion free abelian group (see Section 1 for details). In the present note we concentrate thus on the higher homotopy groups $\pi_n(CW_{B\mathbb{Z}/p}BG)$, for $n \geq 2$.

Our first result establishes that the homotopy of $CW_{B\mathbb{Z}/p}BG$ is subject to a dichotomy finite-infinite. This contrasts with the results in [CCS]: for easier spaces to work with, such as $H$-spaces or classifying spaces of nilpotent groups, the assumption that the mapping space $\text{Map}_*(B\mathbb{Z}/p, X)$ is discrete implies that $CW_{B\mathbb{Z}/p}X$ is aspherical.

**Theorem 2.1.** Let $G$ be a finite group. Then $CW_{B\mathbb{Z}/p}BG$ is either the classifying space of a finite $p$-group, or it has infinitely many non-trivial homotopy groups.

An important role in the proof of this result is played by Levi’s dichotomy theorem [Lev95, Theorem 1.1.4] about the homotopy structure of the Bousfield-Kan $p$-completion of $BG$, and just like this, it is very much in the spirit of the McGibbon-Neisendorfer theorem ([MN84]) and other similar results that have arisen in the last years in the context of localization ([CP93, BD02]). All of these dichotomies share the common feature that they essentially concern objects whose homotopy, if infinite, is a $p$-torsion invariant. In the case of $B\mathbb{Z}/p$-cellularization, however, it cannot be deduced from the construction of the functor that the homotopy groups of a $B\mathbb{Z}/p$-cellular space should be necessarily $p$-groups. Hence it becomes evident that one should start a more precise study of the higher homotopy of $CW_{B\mathbb{Z}/p}BG$. In fact Chachólski’s description of the cellularization as a small variation of the homotopy fiber $\mathcal{P}_{B\mathbb{Z}/p}$ of the $B\mathbb{Z}/p$-nullification map makes us investigate the difference between these two functors.
**Theorem 2.3.** Let $G$ be a finite group. Then either the cellularization $CW_{\mathbb{Z}/p}BG$ has infinitely many homotopy groups containing $p$-torsion or it fits in a fibration

$$CW_{\mathbb{Z}/p}BG \rightarrow BG \rightarrow \prod_{q \neq p}(BG)_{q}^{\wedge},$$

where the (finite) product is taken over all primes $q$ dividing the order of $G$, and the right map is the product of the completions.

Note that in the second case the upper homotopy of the cellularization is that of $\Omega \prod_{q \neq p}(BG)_{q}^{\wedge}$. In particular, if $G$ is $\mathbb{Z}/p$-cellular, the results [Cha96, 20.10] and [Flo, 3.5] imply that $CW_{\mathbb{Z}/p}BG$ is different from $\bar{P}_{\mathbb{Z}/p}$ exactly when there appears (a lot of) $p$-torsion in the higher homotopy groups. Curiously enough this yields many examples of $\mathbb{Z}/p$-cellular spaces whose higher homotopy is $p$-torsion free, including the cellularization of classifying spaces of all symmetric groups. The main question which remains unanswered up to this point is thus to determine if there actually exist groups for which $CW_{\mathbb{Z}/p}BG$ does contain $p$-torsion in its higher homotopy groups! This turns out to depend heavily on the $p$-complete classifying space $BG_{p}^{\wedge}$.

**Proposition 3.4.** Let $G$ be a group generated by order $p$ elements which is not a $p$-group. Then the universal cover of $CW_{\mathbb{Z}/p}BG$ is $p$-torsion free if and only if the $p$-completion of $BG$ is $\mathbb{Z}/p$-cellular.

We remark (see Proposition 3.1) that specific representations of $G$ in some $p$-completed compact Lie group detect the presence of $p$-torsion in the higher homotopy groups of the cellularization of $BG$. This observation definitively shifts the problem to the study of fusion systems, a notion which has recently led Broto, Levi, and Oliver to the concept of $p$-local finite groups in a topological context (see [BLO03a] and [BLO03b]). The map $BG_{p}^{\wedge} \rightarrow BK_{p}^{\wedge}$ we are looking for –where $K$ is some compact Lie group– is indeed best understood as a fusion preserving representation of the Sylow $p$-subgroup of $G$ in $K$ (this relationship is well explained by Jackson in [Jac04]), and turns out to be trivial when precomposing with any map $\mathbb{Z}/p \rightarrow BG_{p}^{\wedge}$. To our knowledge, the kind of representations we are looking for had not yet been described in the extensive literature devoted to the classification of representations of finite groups.
We prove that the Suzuki group $Sz(8)$ admits such a representation into $U(7)$ at the prime 2, and so there exist infinitely many homotopy groups of $CW_{\mathbb{Z}/2}BSz(8)$ containing 2-torsion. The same phenomenon actually occurs for all Suzuki groups, and moreover the tools used can be applied to other groups at odd primes (see Example 4.8). Then, we can establish

**Theorem 4.9.** For every integer $n$ of the form $2^{2k+1}$ the $\mathbb{Z}/2$-cellularization of $BSz(n)$ has 2-torsion in an infinite number of homotopy groups. Likewise, if $p$ is an odd prime and $q$ is any integer of the form $mp^k + 1$ with $k \geq 2$, then the $\mathbb{Z}/p$-cellularization of $BPSL(q)$ has $p$-torsion in an infinite number of homotopy groups.

We point out that the representation $BSz(8) \to BU(7)_2$ cannot be induced by a homomorphism of groups $Sz(8) \to U(7)$, not even composed with an Adams operation. Thus our example could be compared with the map $BM_{12} \to BG_2$ constructed by Benson and Wilkerson in [BW95]. It is also related with work of Mislin-Thomas [MT89], and more recently of Broto-Møller [BM].

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1. **Background**

There are two main ingredients used to analyze the cellularization $CW_{\mathbb{Z}/p}BG$. The first one is a general recipe, the second a specific simplification in the setting of classifying spaces of finite groups.

One of the most efficient tools to compute cellularization functors is Chachólski’s construction of $CW_A$ out of the nullification functor $P_A$, see [Cha96]. His main theorem states that $CW_AX$ can be constructed in two steps. First consider the evaluation map $\vee_{[A,X]}, A \to X$ (the wedge is taken over representatives of all pointed homotopy classes of maps) and let $C$ denote its homotopy cofiber. Then $CW_AX$ is
the homotopy fiber of the composite map \(X \to C \to P_{\Sigma A}C\). We will actually need a small variation of Chachólski’s description of the cellularization.

**Lemma 1.1.** Let \(A\) and \(X\) be pointed connected spaces and choose a pointed map \(A \to X\) representing each unpointed homotopy class in \([A, X]\). Denote by \(D\) the homotopy cofiber of the evaluation map \(ev : \bigvee_{[A, X]} A \to X\). Then \(CW_A X\) is weakly equivalent to the homotopy fiber of the composite \(X \to D \to P_{\Sigma A} D\).

**Proof.** According to [Cha96, Theorem 20.3] one has to check that composing any map \(f : A \to X\) with \(ev : X \to D\) is null-homotopic. But such a map \(f\) is freely homotopic to one in \([A, X]\). Thus \(ev \circ f\) is freely homotopic to the constant map, and so it must also be null-homotopic in the pointed category. \(\square\)

In [Flo], the first author focused on the situation when \(A = B\mathbb{Z}/p\) and \(X\) is the classifying space of a finite group \(G\). A first reduction can always be done by considering the subgroup \(\Omega_1(G)_p\) of \(G\) generated by the elements of order \(p\). This notation is the standard one in group theory, see for example [Gor80]. In [Flo] and [RS01] the terminology “socle” and the corresponding notation \(S_{\mathbb{Z}/p} G\) was used instead. We have indeed an equivalence of pointed mapping spaces \(\text{Map}^*_*(B\mathbb{Z}/p, BG) \simeq \text{Map}^*_*(B\mathbb{Z}/p, B\Omega_1(G)_p)\), which means that \(B\Omega_1(G)_p \to BG\) is a \(B\mathbb{Z}/p\)-cellular equivalence. A second simplification consists then in computing the group theoretical cellularization \(CW_{\mathbb{Z}/p} G \simeq CW_{\mathbb{Z}/p} \Omega_1(G)_p\), which is known to be a (finite) central extension of \(\Omega_1(G)_p\) by a group of order coprime with \(p\). Since \(BCW_{\mathbb{Z}/p} G \to BG\) is a \(B\mathbb{Z}/p\)-cellular equivalence, we will from now assume that \(G\) is a finite \(\mathbb{Z}/p\)-cellular group (it can be constructed out of the cyclic group \(\mathbb{Z}/p\) by iterated colimits). Examples of such groups are provided at the prime 2 by dihedral groups, Coxeter groups, and in general by finite \(p\)-groups generated by order \(p\) elements.

The first author described the fundamental group of \(CW_{\mathbb{Z}/p} BG\):

**Proposition 1.2.** [Flo, Theorem 4.14] Let \(G\) be a finite \(\mathbb{Z}/p\)-cellular group. Then the fundamental group \(\pi = \pi_1 CW_{\mathbb{Z}/p} BG\) is described as an extension \(H \hookrightarrow \pi \twoheadrightarrow G\) of \(G\) by a finite \(p\)-torsion free abelian group \(H\).

He also showed that \(BG\) is \(B\mathbb{Z}/p\)-cellular if and only if \(G\) is a finite \(\mathbb{Z}/p\)-cellular \(p\)-group. This paper is an attempt to understand \(CW_{\mathbb{Z}/p} BG\) when \(G\) is not a \(p\)-group.
2. A cellular dichotomy

We establish in this section a first dichotomy result. The space $CW_{BZ/p}BG$ has infinitely many non-trivial homotopy groups unless it is aspherical. We will say more about the higher homotopy groups in the next section.

**Theorem 2.1.** Let $G$ be a finite group. Then $CW_{BZ/p}BG$ is either the classifying space of a finite $p$-group, or it has infinitely many non-trivial homotopy groups.

**Proof.** If $CW_{BZ/p}BG$ is the classifying space of a group, then it must be a finite $p$-group as shown by the first author in [Flo, Theorem 4.14]. Assume therefore that it is not so, i.e. there exists a prime $q$ different from $p$ dividing the order of $G$. Recall that we may always suppose that $G$ is $Z/p$-cellular. Therefore the evaluation map $\ast Z/p \to G$ taken over all homomorphisms $Z/p \to G$ is surjective. Hence the homotopy cofiber $C$ of the evaluation map $\vee_{(BZ/p,BG)} BZ/p \to BG$, as defined above, is a simply-connected space and so is its nullification $P_{\Sigma BZ/p}C$.

The fibration from [Cha96]

$$CW_{BZ/p}BG \to BG \to P_{\Sigma BZ/p}C$$

shows that the cellularization shares many homotopical properties with the nullification $P_{\Sigma BZ/p}C$. We want to analyze the $q$-completion of this last space. But since $BZ/p$ is $HZ/q$-acyclic, we see that the composite

$$BG \to C \to P_{\Sigma BZ/p}C$$

is an equivalence in homology with coefficients $Z/q$. This implies that $(P_{\Sigma BZ/p}C)^\wedge_q \simeq (BG)^\wedge_q$.

By R. Levi’s dichotomy result [Lev95, Theorem 1.1.4] for $q$-completions of classifying spaces of finite groups one infers that $(BG)^\wedge_q$ must have infinitely many non-trivial homotopy groups (because $G$ is $q$-perfect as its abelianization must be $p$-torsion). □

In view of the proof of the preceding theorem, the main question is to determine whether or not the higher homotopy groups of the cellularization of classifying spaces may contain $p$-torsion. We make next an observation about $\Sigma BZ/p$-local spaces which play such an important role for the $BZ/p$-cellularization because of Chachólski’s theorem 1.1. The proof is very much in the spirit of the Dwyer-Wilkerson dichotomy result [DW90, Theorem 1.3], Levi’s one [Lev95, Theorem 1.1.4]; compare also with Grodal’s work on Postnikov pieces [Gro98].
Proposition 2.2. Let $X$ be a simply-connected torsion $\Sigma B\mathbb{Z}/p$-local space. Then it is either $p$-torsion free or has infinitely many homotopy groups containing $p$-torsion.

Proof. Let $n$ be an integer $\geq 2$ and consider the Postnikov fibration 

$$X(n) \longrightarrow X(n-1) \longrightarrow K(\pi_n X, n).$$

Since the base point component of the iterated loop space $\Omega^{n-1} X$ and $\Omega^{n-1} X(n-1)$ are weakly equivalent there is a fibration

$$\Omega_0^{n-1} X \longrightarrow K(\pi_n X, 1) \longrightarrow \Omega^{n-2} X(n)$$

in which the fiber is $B\mathbb{Z}/p$-local. Thus the total space is $B\mathbb{Z}/p$-local if and only if the base is so, i.e. the homotopy group $\pi_n X$ contains $p$-torsion if and only if the $n$-connected cover $X(n)$ has some $p$-torsion. We use here that a simply-connected $p'$-torsion space $Y$ has trivial $p$-completion, therefore the pointed mapping space $\text{Map}_*(B\mathbb{Z}/p, Y)$ is contractible. $\square$

We turn now to a more detailed study of the case when $G$ is not a $p$-group. We have seen that $CW_{B\mathbb{Z}/p}BG$ has infinitely many non-trivial homotopy groups, but they can arise in two different ways, because the space $P_{\Sigma B\mathbb{Z}/p}C$ may contain $p$-torsion or not. If it is $p$-torsion free, it is weakly equivalent to $P_{B\mathbb{Z}/p}BG \simeq \prod_{q \neq p} BG_q^\wedge$. This means precisely that the cellularization coincides with $P_{B\mathbb{Z}/p}BG$, the homotopy fiber of the nullification map.

Theorem 2.3. Let $G$ be a finite group. Then either the cellularization $CW_{B\mathbb{Z}/p}BG$ has infinitely many homotopy groups containing $p$-torsion or it fits in a fibration

$$CW_{B\mathbb{Z}/p}BG \longrightarrow BG \longrightarrow \prod_{q \neq p} (BG)^\wedge_q,$$

where the (finite) product is taken over all primes $q$ dividing the order of $G$, and the right map is the product of the completions.

Proof. As in the proof of Theorem 2.1 we may assume that $G$ is a $\mathbb{Z}/p$-cellular group. The homotopy cofiber of the evaluation map $\vee_{[B\mathbb{Z}/p, BG]} B\mathbb{Z}/p \to BG$ is thus 1-connected and so is $P_{\Sigma B\mathbb{Z}/p}C$. Moreover the group $G$ is finite, so its integral homology groups are finite. The space $P_{\Sigma B\mathbb{Z}/p}C$ is constructed out of $BG$ by taking iterated homotopy cofibers of maps out of (finite) wedges of suspensions of $B\mathbb{Z}/p$, whose homology groups are all torsion, and thus $P_{\Sigma B\mathbb{Z}/p}C$ satisfies the conditions of the above proposition.
Assume thus that \( p\Sigma BZ/pC \) is \( p \)-torsion free. As this space is simply-connected, we see that it is actually \( BZ/p \)-local, and so \( P_{BZ/p}BG \simeq P_{\Sigma BZ/p}C \). The computation in \([\text{Flo}, \text{Section 3}]\) yields now that

\[ P_{BZ/p}BG \simeq \prod_{q \neq p} BG_q \]

because \( G \) is \( Z/p \)-cellular (and thus equal to its \( Z/p \)-radical).

\[ \square \]

\textbf{Example 2.4.} Consider the \( C_2 \)-cellular group \( \Sigma_3 \), symmetric group on three letters. The choice of any transposition yields a map \( f : BZ/2 \to B\Sigma_3 \) which induces an isomorphism in mod 2 homology. Therefore the homotopy cofiber \( C_f \) of \( f \) is 2-torsion free and we are in case (i). This shows that \( CW_{BZ/2}B\Sigma_3 \) is a space whose fundamental group is \( \Sigma_3 \) and its universal cover is \( \Omega(B\Sigma_3)_3^\wedge \simeq S^3 \{3\} \), the homotopy fiber of the degree map on the sphere \( S^3 \), compare with \([\text{BK72}, \text{VII, 4.1}]\) and \([\text{RS01}, \text{Theorem 7.5}]\).

3. \textbf{The \( p \)-torsion free case}

The example of the symmetric group \( \Sigma_3 \) might lead us to think that there will be very few cases when the universal cover of \( CW_{BZ/p}BG \) is \( p \)-torsion free. In this section we will see that there are surprisingly many groups for which this occurs and one could even wonder if \( p \)-torsion actually can appear in the \( BZ/p \)-cellularization of \( BG \).

Let us again consider the cofibration \( \vee BZ/p \to BG \to C \) for a \( Z/p \)-cellular group \( G \). The cofiber \( C \) is a simply connected torsion space, hence equivalent to the finite product \( \prod C_q^\wedge \) where \( q \) runs over the primes dividing the order of \( G \). We need to understand the \( \Sigma BZ/p \)-nullification of \( C \). Since nullification commutes with finite products and obviously \( C_q^\wedge \) is \( \Sigma BZ/p \)-local for \( q \neq p \), we only look at \( C_p^\wedge \) and infer that \( CW_{BZ/p}BG \) has infinitely many homotopy groups with \( p \)-torsion if and only if \( P_{\Sigma BZ/p}C_p^\wedge \) does so, which happens if and only if \( P_{\Sigma BZ/p}C_p^\wedge \) is not contractible by Theorem 2.3.

\textbf{Proposition 3.1.} Let \( G \) be a finite \( Z/p \)-cellular group. For \( CW_{BZ/p}BG \) to have infinitely many homotopy groups with \( p \)-torsion, there must exist a \( p \)-complete space \( Z \) which is \( \Sigma BZ/p \)-local and a map \( f : BG \to Z \) which is not null-homotopic such that the restriction \( BZ/p \to BG \to Z \) to any cyclic subgroup of order \( p \) is null-homotopic.

\textbf{Proof.} We need to understand when \( P_{\Sigma BZ/p}C_p^\wedge \) is not contractible, or in other words when the map \( C_p^\wedge \to * \) is not a \( \Sigma BZ/p \)-equivalence. This means by definition
that there exists some $\Sigma B\mathbb{Z}/p$-local space $Z$ for which the pointed mapping space $\text{Map}_*(C^\wedge, Z)$ is not contractible. Because $C^\wedge_p$ is a $p$-torsion space (homotopy cofiber of such spaces) which is simply connected, we can assume, using Sullivan’s arithmetic square, that $Z$ is $p$-complete.

The cofibration sequence $\vee B\mathbb{Z}/p \to BG \to C$ yields a fibration

$$\text{Map}_*(C, Z) \to \text{Map}_*(BG, Z) \to \prod \text{Map}_*(B\mathbb{Z}/p, Z)$$

in which the loop space of the base is trivial since $Z$ is $\Sigma B\mathbb{Z}/p$-local. The existence of the map $f$ tells us that there is a component of the total space (different from the component of the constant map) which lies over the component of the base point in the base. Therefore $\text{Map}_*(C, Z)$ has at least two component as well. As $Z$ is $p$-complete $\text{Map}_*(C, Z)$ is weakly equivalent to $\text{Map}_*(C^\wedge, Z)$.

On the other hand, if no such map $f$ exists, $\text{Map}_*(C, Z)$ is weakly equivalent to the connected component of the constant $\text{Map}_*(BG, Z)$, which is contractible by Dwyer’s result [Dwy96, Theorem 1.2].

Our aim is now to explain why the map $f$ in the above proposition cannot exist in many cases.

**Proposition 3.2.** Let $G$ be a finite $\mathbb{Z}/p$-cellular group whose Sylow $p$-subgroup is generated by elements of order $p$. The universal cover of $CW_{\Sigma B\mathbb{Z}/p}BG$ is then $p$-torsion free.

**Proof.** By Theorem 2.3 the universal cover of $CW_{\Sigma B\mathbb{Z}/p}BG$ is either $p$-torsion free or it has infinitely many homotopy groups with $p$-torsion. As $CW_{\Sigma B\mathbb{Z}/p}BG$ is the homotopy fiber of a map $BG \to P_{\Sigma B\mathbb{Z}/p}C$, the same is true for $P_{\Sigma B\mathbb{Z}/p}C^\wedge$. We prove now that this space is always contractible because the assumptions on the map $f$ in Proposition 3.1 are never satisfied.

Let $f : BG \to Z$ be a map into a $\Sigma B\mathbb{Z}/p$-local space which is trivial when restricted to any cyclic subgroup $\mathbb{Z}/p$ in $G$. The composite $BG \to Z \to K(\pi_1Z, 1)$ is then null-homotopic because it corresponds to a homomorphism $G \to \pi_1Z$ restricting trivially to all generators of $G$. Therefore the map $f$ lifts to the universal cover of $Z$ and so we might assume that $Z$ is 1-connected. We can also $p$-complete it if necessary, so that $Z$ is actually $H\mathbb{Z}/p$-local. By Dwyer’s theorem [Dwy96, Theorem 1.4], the map $f$ is null-homotopic if and only if the restriction $\tilde{f}$ to some Sylow $p$-subgroup $S$ of $G$ is so. Because we assume $S$ is generated by elements of order $p$, we can consider
as in [Flo, Proposition 4.14] the split extension $T \rightarrow S \rightarrow \mathbb{Z}/p$ given by a maximal normal subgroup generated by all generators but one. By induction on the order the composite

$$BT \rightarrow BS \xrightarrow{f} Z$$

is null-homotopic. Hence using Zabrodsky’s Lemma (see [Dwy96, Proposition 3.5] and [CCS] for a more detailed account in our setting) we see that the map $\tilde{f}$ factors through a map $g : \mathbb{Z}/p \rightarrow Z$ and it is null-homotopic if and only if $g$ is so. As we suppose that the map $f$ is trivial when restricted to any cyclic subgroup $\mathbb{Z}/p$ we can conclude since the above extension is split. □

**Example 3.3.** Consider the $C_2$-cellular group $\Sigma_{2^n}$, symmetric group on $2^n$ letters with $n \geq 2$. The Sylow 2-subgroup is an iterated wreath product of copies of $\mathbb{Z}/2$ which is always generated by elements of order 2. Therefore the above proposition applies and we obtain that $P_{\Sigma_{2^n}/2}C \simeq \prod_{q \neq 2}(B\Sigma_{2^n})_q^\wedge$. Thus $CW_{\mathbb{Z}/2}B\Sigma_{2^n}$ fits into a fibration

$$\prod_{q \neq 2} \Omega(B\Sigma_{2^n})_q^\wedge \rightarrow CW_{\mathbb{Z}/2}B\Sigma_{2^n} \rightarrow B\Sigma_{2^n}.$$

In the next proposition we will see that the existence of $p$-torsion in the upper homotopy of $CW_{\mathbb{Z}/p}BG$ is strongly related with the $\mathbb{Z}/p$-cellularity of $BG_p^\wedge$. Recall that the fundamental group of $BG_p^\wedge$ is always isomorphic to the group theoretical $p$-completion $G_p^\wedge$, i.e. the quotient of $G$ by $O^p(G)$, the maximal $p$-perfect subgroup of $G$.

**Proposition 3.4.** Let $G$ be a finite group generated by order $p$ elements which is not a $p$-group. Then the universal cover of $CW_{\mathbb{Z}/p}BG$ is $p$-torsion free if and only if the $p$-completion of $BG$ is $\mathbb{Z}/p$-cellular.

**Proof.** We know from [BLO03a, Proposition 2.1] that the completion map $BG \rightarrow BG_p^\wedge$ induces a chain of bijections identifying the set of unpointed homotopy classes

$$[\mathbb{Z}/p, BG] \simeq [\mathbb{Z}/p, BG_p^\wedge] \simeq \text{Rep}(\mathbb{Z}/p, G).$$

Choose a set of representatives $f : \mathbb{Z}/p \rightarrow BG$ for all conjugacy classes of elements of order $p$ in $G$ and write $f_p^\wedge$ for the corresponding map into the $p$-completion of $BG$. We have seen in Lemma [□] that $CW_{\mathbb{Z}/p}BG$ can be constructed as the homotopy fiber of the composite $BG \rightarrow D \rightarrow P_{\Sigma_{\mathbb{Z}/p}, D}$, where $D$ is the homotopy cofiber of the
evaluation map \( ev : \sqrt{B\mathbb{Z}/p} \to BG \). Likewise \( CW_{B\mathbb{Z}/p}(BG)^\wedge \) is constructed using the completed version, call the corresponding cofiber \( D' \). In short we have a diagram of cofibrations

\[
\begin{array}{ccc}
\sqrt{B\mathbb{Z}/p} & \xrightarrow{f} & BG & \to D \\
\downarrow & & \downarrow & \downarrow \\
\sqrt{B\mathbb{Z}/p} & \xrightarrow{f'^\wedge} & BG^\wedge & \to D'
\end{array}
\]

Now since \( G \) (and thus its quotient \( G_p^\wedge \) as well) is generated by elements of order \( p \), the cofibers \( D \) and \( D' \) are simply connected. As \( \sqrt{B\mathbb{Z}/p} \) and \( BG \) are rationally trivial, we use Sullivan’s arithmetic square to establish that \( D = \prod_{q \text{ prime}} D_q^\wedge \) and since moreover \( BG^\wedge \) is \( p \)-complete, \( D' \) is so.

Comparing the Mayer-Vietoris sequences in mod \( p \) homology of the two cofibrations, it is easy to see that \( D'^\wedge \simeq D' \). Therefore we can compute

\[
P_{\Sigma B\mathbb{Z}/p}D \simeq \prod_q P_{\Sigma B\mathbb{Z}/p}D_q^\wedge \simeq \prod_{q \neq p} D_q^\wedge \times P_{\Sigma B\mathbb{Z}/p}D'.
\]

Hence the universal cover of the \( B\mathbb{Z}/p \)-cellularization of \( BG \) is \( p \)-torsion free if and only if \( D' \) is \( \Sigma B\mathbb{Z}/p \)-acyclic. This is equivalent for \( BG^\wedge \) to be \( B\mathbb{Z}/p \)-cellular. \( \Box \)

Let us finish this section by showing with some examples the applicability of the last result.

**Example 3.5.** Consider the 2-completion of the classifying space of the symmetric group \( \Sigma_{2^n} \). According to 3.3, the Sylow 2-subgroup of \( \Sigma_{2^n} \) is generated by order 2 elements, and moreover \( \pi_k(CW_{B\mathbb{Z}/2B\Sigma_{2^n}}) \) is 2-torsion free if \( k \geq 2 \). Thus by Proposition 3.2 (\( B\Sigma_{2^n} \)\wedge)^\wedge_2 is \( B\mathbb{Z}/2 \)-cellular.

Unlike to what happens with the cellularization of \( BG \), our study does not give in general information about \( CW_{B\mathbb{Z}/p}(BG^\wedge_p) \) when \( G \) is not generated by order \( p \) elements. In this case, however, it is sometimes possible to reduce the problem to the case of groups for which the hypothesis of the theorem hold. To show this, we compute one last example.

**Example 3.6.** Let us compute the \( B\mathbb{Z}/2 \)-cellularization of \((BA_4)^\wedge_2 \). It is known that the natural inclusion \( A_4 \leq A_5 \) induces an equivalence in mod 2 homology, and then \((BA_4)^\wedge_2 \simeq (BA_5)^\wedge_2 \). Now, as \( A_5 \) is simple with 2-torsion, it is generated by order 2
elements, and moreover the Sylow 2-subgroup of $A_5$ is the Klein group $\mathbb{Z}/2 \times \mathbb{Z}/2$. Hence, $(BA_5)_2^\wedge$ is $B\mathbb{Z}/2$-cellular by Proposition 3.4 and then $(BA_4)_2^\wedge$ is so as well.

4. CELLULARIZATION AND FUSION

To find an example where $CW_{B\mathbb{Z}/p}BG$ has $p$-torsion, one should look by Proposition 3.2 for groups generated by elements of order $p$ with a Sylow $p$-subgroup which is not generated by elements of order $p$. We will see in this section that this is by far not a sufficient condition, as illustrated below by the example of the the finite simple group $PSL_3(3)$. We start with a simple observation which leads then naturally to a closer analysis of the fusion of the groups we look at.

**Proposition 4.1.** Let $G$ be a group generated by elements of order $p$ such that its Sylow $p$-subgroup $S$ is not so. If $S$ is generated by the elements of $\Omega_1(S)$ together with all their conjugates by elements of $G$ which belong to $S$ then $CW_{B\mathbb{Z}/p}BG$ is $p$-torsion free.

*Proof.* We have seen in Proposition 3.1 that the existence of $p$-torsion is detected by a non-trivial map $f : BG^\wedge_p \to Z$ into some $\Sigma B\mathbb{Z}/p$-local space $Z$. Such a map is null-homotopic if and only if the restriction to $BS$ is so. Pick a generator in $S$ and consider the cyclic subgroup $T$ it generates. By assumption this subgroup is conjugate in $G$ to some subgroup of $\Omega_1(S)$. Because conjugation in $G$ induces a weak equivalence on $BG$ we see that $f$ is null-homotopic when restricted to $BT$ (it is so when restricted to $B\Omega_1(S)$). An induction on the order of the Sylow subgroup as in the proof of Proposition 3.2 allows then to conclude that $f$ itself cannot be essential. □

**Example 4.2.** The symmetric group $\Sigma_3$ acts by permutation on $(\mathbb{Z}/4)^3$. The diagonal is invariant, and so is the “orthogonal” subgroup $\mathbb{Z}/4 \times \mathbb{Z}/4$. We define $G$ to be the semi-direct product of $\mathbb{Z}/4 \times \mathbb{Z}/4$ by $\Sigma_3$. It is easy to check that $G$ is generated by elements of order 2, but the Sylow 2-subgroup $S = (\mathbb{Z}/4 \times \mathbb{Z}/4) \times \mathbb{Z}/2$ is not. The subgroup $\Omega_1(S)$ has index 2, and a representative of the generator of the quotient can be taken inside $S$ to have order four (it is inside of $G$ a product of three elements of order 2). This element has a conjugate which lies inside $\Omega_1(S)$, so we may conclude by the above proposition that $CW_{B\mathbb{Z}/2}BG$ is 2-torsion free, i.e. its universal cover is $\Omega(BG_3^\wedge)$ by Theorem 2.3.
Example 4.3. The group $G = \text{PSL}_3(3)$ presents the same features as the above example. It is generated by elements of order 2 (it is simple), but its Sylow 2-subgroup $S$ is semi-dihedral of order 16. The subgroup $\Omega_1(S)$ is dihedral of order 8. Here as well the universal cover of $CW_{\mathbb{Z}/2}B\text{PSL}_3(3)$ is $\Omega(B\text{PSL}_3(3)^\wedge)$.

The previous examples share the common property that the index of the subgroup $\Omega_1(S)$ inside the Sylow 2-subgroup is 2. In this case a non-trivial map $BG_2 \rightarrow \mathbb{Z}$ as desired can never exist. This comes from the fact that the only automorphism of the quotient $\mathbb{Z}/2 \cong \Omega_1(S)/S$ is the identity, and thus such a map should actually factor through $B\mathbb{Z}/2$, which is obviously impossible. Hence, we should look for groups where the subgroup $\Omega_1(S)$ has large index in $S$, and is preserved by fusion. The Suzuki group $Sz(2^n)$ with $n$ an odd integer $\geq 3$ is such a group, as we learned from Bob Oliver. The section [Gor80, 16.4] is extremely useful to understand the basic subgroup and fusion properties of the Suzuki groups. In particular the normalizer of the Sylow 2-subgroup of $Sz(2^n)$ is a semi-direct product $S \rtimes \mathbb{Z}/(2^n - 1)$ which is maximal in $Sz(2^n)$.

Lemma 4.4. Let $S$ denote the Sylow 2-subgroup of $Sz(2^n)$. The inclusion of the maximal subgroup $S \rtimes \mathbb{Z}/(2^n - 1) \hookrightarrow Sz(2^n)$ induces a weak equivalence of 2-complete spaces $B(S \rtimes \mathbb{Z}/(2^n - 1))^{\wedge}_2 \simeq BSz(2^n)^{\wedge}_2$.

Proof. The 2-Sylow subgroup of $Sz(2^n)$ can be written as an extension $(\mathbb{Z}/2)^n \hookrightarrow S \twoheadrightarrow (\mathbb{Z}/2)^n$, where the kernel is the center of the group and contains all its order 2 elements. Observe in particular that $S$ is not generated by order 2 elements, an unavoidable condition according to 3.2 and its index is $2^n$.

In order to understand the 2-completion of $BSz(2^n)$ we have learned from [BLO03B] that we need to determine which subgroup of $S$ are 2-centric and 2-radical (see also [Jac01, 2.1] for a definition). Our group $Sz(2^n)$ has the property that all of its 2-Sylow subgroups are disjoint (they have trivial intersection), hence the unique 2-centric 2-radical subgroup (up to conjugation) is $S$ itself. The outer automorphism group of $S$ in the fusion system of the Suzuki group, i.e. the quotient of the normalizer $N_{Sz(2^n)}(S)$ by $S$, is cyclic of order $2^n - 1$, generated by an element $\phi$ which acts fixed-point free and permutes transitively the non-trivial elements of the center of $S$ (see [Gor80, 16.4]).
Now it is clear from the construction that the inclusion $S \times \mathbb{Z}/(2^n - 1) \hookrightarrow S_2(2^n)$ induces an isomorphism of fusion and linking systems in the sense of [BLO03b], and in particular it induces a homotopy equivalence $f : B(S \times \mathbb{Z}/(2^n - 1))^\wedge_2 \simeq BS_2(2^n)^\wedge_2$. □

**Lemma 4.5.** Consider the semi-direct product $(\mathbb{Z}/2)^n \rtimes \mathbb{Z}/(2^n - 1)$ where a generator $\phi$ of $\mathbb{Z}/(2^n - 1)$ acts on the elementary abelian group of rank $n$ by permuting transitively the $2^n - 1$ non-trivial elements. There exists then a faithful representation $\sigma : (\mathbb{Z}/2)^n \rtimes \mathbb{Z}/(2^n - 1) \twoheadrightarrow U(2^n - 1)$.

**Proof.** The representation can be induced from the trivial one on the subgroup $(\mathbb{Z}/2)^n$. When $n = 3$ we can be very explicit. Send the first generator of the elementary abelian group to the diagonal matrix with entries $(-1, 1, -1, -1, -1, 1, 1)$, and the other elements to the cyclic permutations of it. The standard cyclic permutation matrix of order 7 in $U(7)$ is the image of $\phi$ and so $\sigma$ is well-defined. □

**Proposition 4.6.** There exists a non-trivial map $BS_2(2^n) \twoheadrightarrow BU(2^n - 1)^\wedge_2$ such that the composition $B\mathbb{Z}/2 \twoheadrightarrow BS_2(2^n) \twoheadrightarrow BU(2^n - 1)^\wedge_2$ is null-homotopic for every cyclic subgroup $\mathbb{Z}/2$ in $S_2(2^n)$.

**Proof.** We construct actually a map from the 2-completion of $BS_2(2^n)$ and the desired map is then obtained by pre-composing with the completion map $BS_2(2^n) \twoheadrightarrow BS_2(2^n)^\wedge_2$. By Lemma 4.5 we only need to construct a map out of $B((S \times \mathbb{Z}/(2^n - 1))^\wedge_2$ where $S$ denotes the 2-Sylow subgroup of $S_2(2^n)$.

Because $BU(2^n - 1)$ is simply connected the sets of homotopy classes of pointed and unpointed maps into $BU(2^n - 1)$ agree. The fusion system of the Suzuki group (i.e. that of the semi-direct product) is reduced to the Sylow subgroup, so that the set of $[BS_2(2^n), BU(2^n - 1)]$ is isomorphic to the set of fusion preserving representations of $S$ inside $U(2^n - 1)$. This means that for finding a non-trivial map $f : BS_2(2^n) \twoheadrightarrow BU(2^n - 1)^\wedge_2$ it is enough to find a representation $\rho : S \times \mathbb{Z}/(2^n - 1) \rightarrow U(2^n - 1)$ which is non-trivial when restricted to $S$ (compare with Dwyer’s result [Dwy96, Theorem 1.4] which we have already used before).

On the other hand we want the map $f$ to be null-homotopic when restricted to any cyclic subgroup $\mathbb{Z}/2$. In other words the composite $\Omega_1(S) \rightarrow S \times \mathbb{Z}/(2^n - 1) \rightarrow U(2^n - 1)$ should be trivial. The subgroup $\Omega_1(S)$ generated by all elements of order 2 is its center, an elementary abelian subgroup of rank $n$. It is normal in $S \times \mathbb{Z}/(2^n - 1)$, and the quotient is isomorphic to $(\mathbb{Z}/2)^n \rtimes \mathbb{Z}/(2^n - 1)$, where the action of the generator $\phi$ of
order $2^n - 1$ permutes transitively the non-trivial elements (one has a bijection between these non-trivial classes in the quotient and the squares of their representatives in the center of $S$). Hence we can define $\rho$ as the composite

$$S \rtimes \mathbb{Z}/(2^n - 1) \longrightarrow (\mathbb{Z}/2)^n \rtimes \mathbb{Z}/(2^n - 1) \longrightarrow U(2^n - 1)$$

where $\sigma$ is the representation constructed in the preceding lemma. It is convenient to remark here that the existence of such a representation does not contradict the theorems in this article since $S \rtimes \mathbb{Z}/(2^n - 1)$ is not generated by order 2 elements.

Observe that the morphism $B(S \rtimes \mathbb{Z}/(2^n - 1))^\wedge_2 \longrightarrow BU(2^n - 1)^\wedge_2$ induced by the representation we have just constructed is clearly trivial when pre-composing with any map $B\mathbb{Z}/2 \longrightarrow B(S \rtimes \mathbb{Z}/(2^n - 1))^\wedge_2$, because the (unpointed) homotopy classes of these last maps can be identified with the conjugacy classes of $\mathbb{Z}/2$ inside $S \rtimes \mathbb{Z}/(2^n - 1)$. □

**Remark 4.7.** Note that the representation $\rho$ constructed in the previous proposition cannot be induced by a homomorphism $Sz(2^n) \longrightarrow U(2^n - 1)$, because the group $Sz(2^n)$ is simple, hence generated by order 2 elements. If it were so, the homomorphism would be zero over the generators, and thus trivial.

The methods above can also be used to obtain examples at odd primes, as we sketch in the sequel. The following example was pointed out to us by Antonio Viruel.

**Example 4.8.** Let $p$ be an odd prime, $n$ an integer $\geq 2$, and $q = mp^n + 1$. Consider the linear group $PSL_2(q)$. According to ([Gor80 15.1.1]), the $p$-Sylow subgroup of $PSL_2(q)$ is cyclic of order $p^n$, and moreover its normalizer $N_{PSL_2(q)}(S)$ is isomorphic to the semidirect product $\mathbb{Z}/p^n \rtimes \mathbb{Z}/2$, where the action is given by the change of sign. As the Sylow subgroup is abelian, it can be deduced from ([DRV 3.4]) that the inclusion $N_{PSL_2(q)}(S) \hookrightarrow PSL_2(q)$ induces a homotopy equivalence between the 2-completions of the classifying spaces. Reasoning as above, it is enough to give, for some $k$, a non-trivial map $\mathbb{Z}/p^n \rtimes \mathbb{Z}/2 \longrightarrow U(k)$ which is trivial when restricted to order $p$ elements. We first construct an inclusion $j : \mathbb{Z}/p \rtimes \mathbb{Z}/2 \hookrightarrow U(2)$ as follows. If $x = e^{2 \pi i/p}$ the image of the generator of order $p$ is the diagonal matrix with entries $(x, \bar{x})$, and the image of the generator of order 2 is the standard permutation matrix. One can then take for example the composition

$$f : \mathbb{Z}/p^n \rtimes \mathbb{Z}/2 \longrightarrow \mathbb{Z}/p \rtimes \mathbb{Z}/2 \longrightarrow U(2),$$
where the first map is the natural projection. Now the induced map at the level of $p$-completed classifying spaces
\[(B\text{PSL}_2(q))^\wedge_p \simeq (B(Z/p^n \rtimes Z/2))^\wedge_p \rightarrow BU(2)^\wedge_p\]
is essential, but homotopically trivial when precomposing with any map from $B\mathbb{Z}/p$.

We are finally able to show that the $B\mathbb{Z}/p$-cellularization of $BG$’s does not always coincide with the fiber of the $B\mathbb{Z}/p$-nullification. Our example is of course given by the groups described above. This implies that our cellular dichotomy Theorem 2.1 is in fact a trichotomy result: the higher homotopy groups of $CW_{B\mathbb{Z}/p}BG$ are either all trivial, or $p$-torsion free and infinitely many are non-trivial, or else infinitely many do contain $p$-torsion.

**Theorem 4.9.** For every integer $n$ of the form $2^{2k+1}$ the $B\mathbb{Z}/2$-cellularization of $BSz(n)$ has 2-torsion in an infinite number of homotopy groups. Likewise, if $p$ is an odd prime and $q$ is any integer of the form $mp^k + 1$ with $k \geq 2$, then the $B\mathbb{Z}/p$-cellularization of $B\text{PSL}(q)$ has $p$-torsion in an infinite number of homotopy groups.

**Proof.** The loop space $\Omega BU(2^n - 1)^\wedge_2$ is $B\mathbb{Z}/2$-local by [Mil84, 9.9]. It is now a direct consequence of Proposition 4.1 that the map $f : BSz(2^n) \rightarrow BU(2^n - 1)^\wedge_2$ constructed in the previous proposition implies the existence of infinitely many homotopy groups $\pi_n BSz(2^n)$ containing 2-torsion. The same argument can be used at odd primes for the linear groups described in example 4.8. □

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