Analysis of the strong coupling form factors of $\Sigma_b NB$ and $\Sigma_c ND$ in QCD sum rules

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In this article, we study the strong interaction of the vertices $\Sigma_b NB$ and $\Sigma_c ND$ using the three-point QCD sum rules under two different dirac structures. Considering the contributions of the vacuum condensates up to dimension 5 in the operation product expansion, the form factors of these vertices are calculated. Then, we fit the form factors into analytical functions and extrapolate them into time-like regions, which giving the coupling constant. Our analysis indicates that the coupling constant for these two vertexes are $G_{\Sigma_b NB} = 0.43 \pm 0.01 GeV^{-1}$ and $G_{\Sigma_c ND} = 3.76 \pm 0.05 GeV^{-1}$.

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1 Introduction

By this time, many heavy baryons have been observed by BaBar, Belle and CLEO Collaborations$^{1-4}$ such as $\frac{1}{2}^+$ antitriplet states($\Lambda_c^+, \Xi_c^+, \Omega_c^0$), the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ sextet states ($\Omega_c, \Sigma_c, \Sigma_c'$) and ($\Omega_c^*, \Sigma_c^*, \Sigma_c'^*$)$^2$. Besides, several S-wave bottom baryon states such as $\Lambda_b, \Sigma_b, \Xi_b$ and $\Omega_b$ have also been observed by CDF and LHCb Collaborations$^5, 6$. The SELEX collaboration even have reported the observation of the signal for the doubly charmed baryon state $\Xi_{cc}^+$ $^7, 8$. Since then, people have showed great interest in studying the properties of these heavy baryons which contains at least a heavy quark$^9-13$. The charm and bottom baryon states which contain one (two) heavy quark(s) are particularly interesting for studying dynamics of the light quarks in the presence of the heavy quark(s), and serve as an excellent ground for testing predictions of the quark models and heavy quark symmetry.

The properties of the heavy baryons such as mass spectrum, radiative and strong decays have been studied by many researches, which is very important for us to further understand the heavy flavor physics$^{14-21}$. In this regards, the strong coupling constants associating with heavy baryons play an important role in describing the strong interaction among the heavy baryons and other participated hadrons. In addition, the properties of $B$ and $D$ mesons in nuclear medium are closely related with
their interactions with the nucleons\textsuperscript{22,23}, i.e.

\[ D^0 + p \text{ or } n \rightarrow \Lambda^+_c, \Sigma^+_c \text{ or } \Sigma^0_c \]

\[ B^- + p \text{ or } n \rightarrow \Lambda^0_b \text{ or } \Sigma^-_b \]

From these processes, we can see that it is significant to know the values of the related strong coupling constants \( G_{\Sigma_b N B} \) and \( G_{\Sigma_c N D} \) which is essential to determine the modifications on the masses, decay constants and other parameters of the \( B \) and \( D \) mesons in nuclear medium. Up to now, only a few works on the strong coupling constants of the heavy baryons with the nucleon and heavy mesons have been reported\textsuperscript{19,20,25,26}.

On the other hand, QCD sum rules is one of the most powerful non-perturbative methods, which is also independent of model parameters. In recent years, numerous research articles have been reported about the precise determination of the strong form factors and coupling constants via QCDSR\textsuperscript{27–41,44–46}. In this work, we use the QCDSR formalism to obtain the coupling constants of the strong vertexs \( \Sigma_b N B \) and \( \Sigma_c N D \), where the contributions of vacuum condensates up to 5 in the OPE are considered.

The outline of this paper is as follows. In Sect.2, we study the strong vertexs \( \Sigma_b N B \) and \( \Sigma_c N D \) using the three-point QCDSR under two different dirac structures \( \gamma^5 \gamma^5 \) and \( \gamma^5 \gamma^5 \). Besides of the perturbative contribution, we also consider the contributions of \( \langle \overline{\tau q} q \rangle \), \( \langle \overline{G} \gamma \gamma G \rangle \) and \( \langle \overline{q} q \gamma \gamma G \rangle \) at OPE side. In Sect.3, we present the numerical results and discussions, and Sect.4 is reserved for our conclusions.

## 2 QCD sum rules for \( \Sigma_b N B \) and \( \Sigma_c N D \)

The three-point correlation functions of these two vertices \( \Sigma_b N B \) and \( \Sigma_c N D \) can be written as:

\[ \Pi(p, p', q) = i^2 \int d^4 x \int d^4 ye^{-i p' x e^{i p' y}} \langle 0 | \tau(J_N(y)J_{B[D]}(0)\overline{J}_{\Sigma_b[\Sigma_c]}(x))|0 \rangle \]  

Where \( \tau \) is the time ordered product and \( J_{\Sigma_b[\Sigma_c]}(x), J_N(y) \) and \( J_{B[D]}(0) \) are the interpolating currents of the hadrons \( \Sigma_b[\Sigma_c], N \) and \( B[D] \) respectively:

\[ J_{\Sigma_b[\Sigma_c]}(x) = \epsilon_{ijk} \left( u^{iT}(x)C \gamma_\mu d^j(x) \right) \gamma_5 \gamma_\mu b^k(x) \]  

\[ J_N(y) = \epsilon_{ijk} \left( u^{iT}(y)C \gamma_\mu u^j(y) \right) \gamma_5 \gamma_\mu d^k(y) \]  

\[ J_{B[D]}(0) = \overline{\tau}(0)\gamma_5 b^i(0) \]  

where \( C \) is the charge conjugation operator, and \( i, j \) and \( k \) are color indices.

According to the QCD sum rules, the three-point correlation function can be calculated in two different ways. In the first way, the calculation is carried out in hadron degrees of freedom, called the phenomenological side. Secondly, it is called OPE side which is calculated in quark degrees of freedom. Then, invoking the quark-hadron duality, we equate the phenomenological and OPE sides from which the QCD sum rules for the strong coupling form factors is attained.
2.1 The phenomenological side

We insert a complete set of intermediate hadronic states with the same quantum numbers as the operators $J_{\Sigma_0}[\Sigma_c](x), J_N(y)$ and $J_{[D]}(0)$ into the correlation function Eq(1) to obtain the phenomenological representations. After isolating the ground-state contributions, the correlation function is written as:

\[
\Pi^{HAD}(p, p', q) = \frac{\langle 0|J_N|N(p')\rangle \langle 0|J_{[D]}[B[D]|(q)\rangle \langle \Sigma_b[\Sigma_c](p)|\bar{T}_{\Sigma_b[\Sigma_c]}|0\rangle}{(p^2 - m_{\Sigma_b[\Sigma_c]}^2)(q^2 - m_{\Sigma_b[\Sigma_c]}^2)(q^2 - m_{[D]}^2)}
\]

\[
\langle N(p')B[D](q)|\Sigma_b[\Sigma_c](p)\rangle + \cdots
\]

(5)

Where \(h.r.\) stands for the contributions of higher resonances and continuum states. And the matrix elements appearing in the above equation can be parameterized as the following formulas:

\[
\langle 0|J_N|N(p')\rangle = \lambda_N u_N(p', s')
\]

(6)

\[
\langle 0|J_{[D]}|B[D](q)\rangle = \frac{m_{[D]}^2 f_{[D]}}{m_u + m_{b[c]}}
\]

(7)

\[
\langle \Sigma_b[\Sigma_c](p)|\bar{T}_{\Sigma_b[\Sigma_c]}|0\rangle = \lambda_{\Sigma_0[\Sigma_c]} \bar{\Sigma}_{\Sigma_0[\Sigma_c]}(p, s)
\]

(8)

\[
\langle N(p')B[D](q)|\Sigma_b[\Sigma_c](p)\rangle = G_{\Sigma_0[NB]|\Sigma_0[ND]} \bar{\Sigma}_{\Sigma_0[NB]}(p', s') i\gamma_5 u_{\Sigma_0[ND]}(p, s)
\]

(9)

Where \(\lambda_N\) and \(\lambda_{\Sigma_0[\Sigma_c]}\) are residues of \(N\) and \(\Sigma_b[\Sigma_c]\) baryons, \(f_{[D]}\) is the leptonic decay constant of \([D]\) meson and \(G_{\Sigma_0[NB]|\Sigma_0[ND]}\) is the strong coupling form factor of the vertices \(\Sigma_b[NB]\) and \(\Sigma_c[ND]\). Considering these parameters, Eq.(5) can be written as:

\[
\Pi^{HAD}(p, p', q) = \frac{i^2 m_{[D]}^2 f_{[D]}}{m_{b[c]} + m_u} \frac{\lambda_N \lambda_{\Sigma_0[\Sigma_c]} g_{\Sigma_0[NB]|\Sigma_0[ND]}}{(p^2 - m_{\Sigma_b[\Sigma_c]}^2)(q^2 - m_{\Sigma_b[\Sigma_c]}^2)(q^2 - m_{[D]}^2)} \times \left\{ (m_N m_{\Sigma_b[\Sigma_c]} - m_{\Sigma_b[\Sigma_c]}^2) \gamma_5 + (m_{\Sigma_0[\Sigma_c]} - m_N) \gamma_5 \gamma_5 + \gamma_5 \gamma_5 - m_{\Sigma_b[\Sigma_c]} \gamma_5 \right\}
\]

+ \cdots

(10)

2.2 The OPE side

Now, we briefly outline the operator product expansion (OPE) for the three-point correlation Eq.(1). Firstly, we contract the quark fields in the correlation with Wich’s theorem.

\[
\Pi(p, p', q)^{OPE} = i^2 \int d^4x \int d^4y e^{-ip.x} e^{ip'.y} \epsilon_{abc} \epsilon_{ijk}
\]

\[
\times \left\{ \gamma_5 \gamma_\nu S^{\nu}_{d}(y - x) \gamma_\mu C S^{\mu}_{a}(y) \gamma_5 S^{h}_{b[c]}(-x) \gamma_5 \gamma_5 - \gamma_5 \gamma_\nu S^{\nu}_{d}(y - x) \gamma_\mu C S^{h}_{b[c]}(-x) \gamma_5 \gamma_5 \right\}
\]

(11)

Secondly, we replace the heavy and light quark propagators with the following full propagators [15,47],

\[
S^{mn}_{b[c]}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik.x} \left\{ \delta_{mn} \frac{g_{m\alpha\beta}}{k - m_{b[c]} - (k + m_{b[c]}) \sigma_{\alpha\beta}} \right\}
\]

\[
\frac{\alpha_s G_F}{\pi} \delta_{mn} m_{b[c]} \frac{k^2 + m_{b[c]}^2}{(k^2 - m_{b[c]}^2)^2} + \cdots
\]

(12)
\[ S_{u[d]}^{mn}(x) = \frac{i}{2\pi^2 x^2}\delta_{mn} - \frac{m_{u[d]}^4}{4\pi^2 x^2} \delta_{mn} - \frac{<q\bar{q}>}{12} \left( 1 - i \frac{m_{u[d]}^4}{4} \right) - \frac{x^2}{192m_0^2<q\bar{q}>} \left( 1 - i \frac{m_{u[d]}^4}{6} \right) \] (13)

Where \( m,n \) are the color indices, and \(<q\bar{q}>\) is the \( \langle u\bar{u} \rangle \) and \( \langle d\bar{d} \rangle \) in Eq(13). After these above substitutions in Eq(11), we carry out Fourier transformation in D=4 dimensions using the following formulas:

\[ \frac{1}{(|y-x|^2)^n} = \int \frac{d^Dk}{(2\pi)^D} e^{-it.(y-x)} (-1)^{n+1} 2^{D-2n} \pi^D \Gamma(D/2-n) \Gamma(n) \left( \frac{1}{t^2} \right)^{D/2-n} \] (14)

\[ \frac{1}{|y|^2} = \int \frac{d^D\ell'}{(2\pi)^D} e^{-i\ell'.y} (-1)^{n+1} 2^{D-2n} \pi^D \Gamma(D/2-n) \Gamma(n) \left( \frac{1}{t^2} \right)^{D/2-n} \] (15)

Before the preformation of four-\( x \) and four-\( y \) integrals, the replacements \( x_\mu \rightarrow i \frac{\partial}{\partial p_\mu} \) and \( y_\mu \rightarrow -i \frac{\partial}{\partial p_\mu} \) are carried out. After these processes, the integrals turn into Dirac delta functions which are used to simplify the four-integrals over \( k \) and \( t' \). The following step is to perform the Feynman parametrization, after which the following function is used to carried out the remaining four-integral over \( t \).

\[ \int \frac{d^D t}{(2\pi)^D} \frac{1}{|t-M^2|^\alpha} = \frac{i(-1)^{\alpha}}{(4\pi)^{D/2}} \frac{\Gamma(\alpha-D/2)}{\Gamma(\alpha)} \frac{1}{(M^2)^{\alpha-D/2}} \]

Where \( M^2 = m_{b+c}^2 + p^2 x (x + y - 1) + p'^2 y (x + y - 1) - q^2 x y \).

After further simplification, the three-point correlation in OPE side show the following Dirac structures:

\[ \Pi_{OPE}(p, p', q) = \Pi_1(q^2)\gamma_5 + \Pi_2(q^2)\not{p}\gamma_5 + \Pi_3(q^2)\not{q}\gamma_5 + \Pi_4(q^2)\not{q'}\gamma_5 \] (16)

Where each \( \Pi_i \) denotes contributions coming from perturbative and nonperturbative parts. In general, we expect that we can choose either dirac structure \( \Pi_i \) (with \( i = 1, 2, 3, 4 \)) of the correlations \( \Pi(p, p', q) \) to study the hadronic coupling constants. In our calculations, we observe that the structure \( \not{y}\gamma_5 \) and \( \not{q'}\gamma_5 \) are the pertinent dirac structures.

After taking its imaginary parts of \( \Pi_i \), we get the spectral densities \( \rho_i(s, s', Q^2) \) of the corresponding Dirac structure. Using dispersion relations, each \( \Pi_i \) can be written as:

\[ \Pi_{i OPE}(Q^2) = \int ds \int ds' \frac{\rho_{i\text{pert}}(s, s', Q^2) + \rho_{i\text{non-pert}}(s, s', Q^2)}{(s-p^2)(s'-p'^2)} \] (17)

Where \( s = p^2, s' = p'^2 \) and \( Q^2 = -q^2 \). As examples, we give the perturbative and nonperturbative
parts of the spectral densities for the two Dirac structures $\rho_{\gamma_5}$ and $\hat{\rho}_{\gamma_5}$

$$\rho_{\rho_{\gamma_5}}(s, s', Q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{-1}{32\pi^4(x+y-1)} \{ -[2m_{b[c]}(x+y) + m_d(3x + 3y - 1) - m_u(x+y-1)] \\
\times [x(m_{b[c]}^2 + Q^2y) + sx(x + y - 1) + s'x(y + y - 1)] + s(x + y)[m_{b[c]}x(2x + 2y - 1) \\
+3m_d(x - 1)(x+y-1) - m_u(x^2 + x(y-4) - 2y + 3)] + s'(x+y)[m_{b[c]}x(2y - 1) \\
+2(y-1)y] + y[3m_d(x+y-1) - m_u(x+y-2)] \} + 9m_{b[c]}m_am_u + 9m_{b[c]}m_d m_u y \\
-6m_{b[c]}m_dm_u - 3m_{b[c]}m_d^2x - 3m_{b[c]}m_d^2y + 2m_{b[c]}Q^2 x^2 y - m_{b[c]}Q^2 x^2 y^2 \\
-m_{b[c]}Q^2 x^2 y - m_{b[c]}Q^2 x^2 y - 6m_{b[c]}m_d^2 y + 6m_{b[c]}m_d^2 y + 3m_d Q^2 x^2 y + 3m_d Q^2 x^2 y - 3m_d Q^2 x^2 y \\
-3m_d Q^2 x^2 y + 3m_d Q^2 x^2 y - m_d Q^2 x^2 y + 2m_d Q^2 x^2 y - 3m_d Q^2 x^2 y \\
\times \Theta[H_2(s, s', Q^2)]
$$

$$\rho_{\rho_{\gamma_5}}(s, s', Q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{1}{32\pi^4(x+y-1)^2} \{(x+y-1)[s(m_{b[c]}xy(2x + 2y - 1) + 3m_d(x^2(y-1) \\
+ x(y^2 - 3y - 1) - (y-1)y) - m_u(x^2(y-3) + x(y^2 - 7y + 3) + (3 - 2y)y)] \\
+ s'y(m_{b[c]}(x(2y - 1) + 2(y - 1)y) + 3m_d(x+y-1)(x+y-1) - m_u(x(y-3) + y^2 - 5y + 3)] \\
+ 9m_{b[c]}m_dm_u y - 6m_{b[c]}m_dm_u - 3m_{b[c]}m_d^2 y + 2m_{b[c]}Q^2 x^2 y - m_{b[c]}Q^2 x^2 y - 6m_{b[c]}m_d^2 y + 3m_d Q^2 x^2 y \\
- 3m_d Q^2 x^2 y - 3m_d Q^2 x^2 y - m_d Q^2 x^2 y + 3m_d Q^2 x^2 y + 2m_d Q^2 x^2 y - 3m_d Q^2 x^2 y \\
- \left[ m_{b[c]}(x(6y - 1) + 6(y - 1)y) + 3m_d(3y - 2)(x+y-1) + m_u(-3x(y-2) - 3y^2 + 10y - 6) \right] \\
\times \left[ x(m_{b[c]} + Q^2 y) + sx(x + y - 1) + s'x(y + y - 1) \right] \Theta[H_2(s, s', Q^2)]
$$

$$\rho_{\rho_{\gamma_5}}^{\text{non-pert}}(s, s', Q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{3((\bar{m} m) - \langle \bar{d} d \rangle)}{12\pi^2} \left( 3x + 3y - 1 \right) \Theta[H_2(s, s', Q^2)] \\
- \frac{1}{48x^2(m_{b[c]}^2 + Q^2)} \left\{ \begin{array}{l}
6m_{b[c]}^3 m_d - 12m_{b[c]}^3 m_u - 6m_{b[c]}^2 m_d m_u + 6m_{b[c]}^2 m_u^2 \\
+ s' \left[ 2m_{b[c]}^2 - m_{b[c]} m_u + 2Q^2 \right] - 2m_{b[c]}^2 Q^2 + 6m_{b[c]} m_d Q^2 + s \left[ m_{b[c]} (m_u - 2m_{b[c]}) - 2Q^2 \right] \\
- 11m_{b[c]} m_d Q^2 - 3m_d m_u Q^2 + 3m_{b[c]}^2 Q^2 - 2Q^4 \end{array} \right\} \Theta[H_1(s, s', Q^2)] \\
- \int_0^1 dx \int_0^{1-x} dy \frac{(\bar{q} q)(G^2)}{16\pi^2(x+y-1)} \delta[H_2(s, s', Q^2)] \\
+ \frac{8 \times 4\pi^2 \times 9(m_{b[c]}^2 + Q^2)}{18m_{b[c]}^6 - 18m_{b[c]}^5 m_d - 36m_{b[c]}^5 m_u - 18m_{b[c]}^4 m_d m_u + 18m_{b[c]}^4 m_u^2 \\
+ 39m_{b[c]}^4 Q^2 - 36m_{b[c]}^3 m_d Q^2 - 32m_{b[c]}^3 m_u Q^2 + 30m_{b[c]}^2 Q^4 - 3s \left[ m_{b[c]}^3 (3m_{b[c]} - 2m_u) \right] \\
+ 4m_{b[c]}^2 Q^4 + Q^4 + 3s \left[ 3m_{b[c]}^3 - 2m_{b[c]} m_u + 4m_{b[c]}^2 Q^2 + Q^4 \right] - 18m_{b[c]} m_d Q^4 \\
- 2m_{b[c]} m_u Q^4 + 9Q^6 \right\} \Theta[H_1(s, s', Q^2)]
\begin{align*}
\rho^{\text{non-pert}}_{\gamma\gamma}(s, s', Q^2) &= \int_0^1 dx \int_{0}^{1-x} \frac{3}{16 \times 12 \pi^2} \left\{ (48y - 32)d\bar{d} + (32 - 48y)(u\bar{u}) \right\} \Theta[H_2(s, s', Q^2)] \\
&- \frac{1}{48\pi^2 m_{B[c]}^2 + Q^2} \left\{ -3m_{B[c]} \left[ 2m_{B[c]}^2 (m_d - 2m_u) + m_{B[c]} m_u (m_u - 5m_d) + 2m_d m_u \right] \\
&+ Q^2 \left[ 2m_{B[c]}^2 - 6m_{B[c]} m_d + 11m_{B[c]} m_u + 12m_d m_u \right] + s \left[ m_{B[c]} (2m_{B[c]} - m_u) + 2Q^2 \right] - 2Q^4 \right\} \\
&\times \Theta[H_1(s, s', Q^2)] + \int_0^1 dx \int_{0}^{1-x} \frac{dy^2}{16\pi^2} x^3 \left( 18s'x - x + 18s'y - y + 2 \right) m_{B[c]} \delta[H_2(s, s', Q^2)] \\
&- \frac{1}{8 \times 4\pi^2 \times 9(m_{B[c]}^2 + Q^2)^4} \left\{ 27m_{B[c]}^5 - 54m_{B[c]}^3 m_d - 54m_{B[c]}^3 m_u - 72m_{B[c]}^2 m_d m_u + 18m_{B[c]}^2 m_u^2 \\
&- 60m_{B[c]}^4 Q^2 + 36m_{B[c]}^3 m_d m_u + s \left[ m_{B[c]}^3 (-15m_{B[c]} + 36m_d + 10m_u) \right] \\
& - 4m_{B[c]} Q^2 (6m_{B[c]} - 9m_d - m_u) - 9Q^4 \right\} + 2s' \left[ m_{B[c]}^3 (3m_{B[c]} - 72m_d - 2m_u) \right] \\
& - 2m_{B[c]} Q^2 (3m_{B[c]} + 9m_d + m_u) + 3Q^4 \right\} - 72m_{B[c]}^2 m_d m_u Q^2 \\
& - 72m_{B[c]}^2 m_d m_u Q^2 + 45m_{B[c]}^2 Q^4 - 16m_{B[c]}^2 m_d Q^4 - 16m_{B[c]} m_u Q^4 \\
&+ 12Q^6 \right\} \Theta[H_1(s, s', Q^2)]
\end{align*}

Where \( \Theta \) denotes the unit-step function, and \( H_1[s, s', Q^2] \), \( H_2[s, s', Q^2] \) are defined as:

\begin{align*}
H_1[s, s', Q^2] &= s' \\
H_2[s, s', Q^2] &= x(m_{B[c]}^2 + Q^2 y) + sx(x + y - 1) + s'y(x + y - 1)
\end{align*}

2.3 The strong coupling constant

We perform a double Borel transformation to the physiological as well as the OPE sides. Then, we equate these two sides, invoking the quark-hadron duality from which the sum rule is obtained.

As an example, the form factors for the structure \( \gamma'\gamma_5 \) is:

\begin{align*}
G^{\gamma'\gamma_5}_{\Sigma a, B[\Sigma_c, N D]}(Q^2) &= e^{\frac{m_{\Sigma_a[Nc]}^2}{M_1^2}} e^{\frac{m_{\bar{B}[D]}^2}{M_2^2}} \frac{(m_{B[c]} + m_u)(Q^2 + m_{B[D]}^2)}{m_{B[D]}^2 + m_{B[D]}} \int_{\Sigma a[\Sigma_c]} \lambda_{\Sigma a[\Sigma_c]} \lambda_N (m_{\Sigma a[\Sigma_c]} - m_N) \\
&\times \left\{ \int_{(m_{B[c]} + m_u + m_d)^2}^{s_0} ds' e^{-\frac{s'}{M_1^2}} e^{-\frac{m_{\bar{B}[D]}^2}{M_2^2}} \left[ \rho_{\gamma'\gamma_5}(s, s', Q^2) + \rho_{\gamma'\gamma_5}^{\text{non-pert}}(s, s', Q^2) \right] \right\}
\end{align*}

Where \( M_1 \) and \( M_2 \) are the Borel parameters, \( s_0 \) and \( u_0 \) are two continuum threshold parameters which are introduced to eliminate the h.r. terms. These parameters fulfill the following relations: \( m_{i}^2 < s_0 < m_{o}^2 \) and \( m_{i}^2 < u_0 < m_{o}^2 \), where \( m_i \) and \( m_o \) are the masses of the incoming and out-coming hadrons respectively and \( m' \) is the mass of the first excited state of these hadrons.

3 The results and discussions

Present section is devoted to the numerical analysis of the sum rules for the coupling constants. The decay constants parameters used in this work are taken as \( f_{B} = (248 \pm 23_{\text{exp}} \pm 25_{v_{ub}}) \text{MeV} \), \( f_{D} = (205.8 \pm 8.5 \pm 2.5) \text{MeV} \), \( \lambda_{N} = (0.0011 \pm 0.0005) \text{GeV}^2 \), \( \lambda_{\Sigma b} = (0.062 \pm 0.018) \text{GeV}^2 \) and \( \lambda_{\Sigma a} = (0.045 \pm 0.015) \text{GeV}^2 \). We take the masses of the hadronic from reference.
where $m_B = (5279.26 \pm 0.17) \text{MeV}$, $m_D = (1864.84 \pm 0.07) \text{MeV}$, $m_N = (938.272046 \pm 0.000021) \text{MeV}$, $m_{\Sigma_0} = (5811.3 \pm 1.9) \text{MeV}$, $m_{\Sigma_c} = (2452.9 \pm 0.4) \text{MeV}$ and of quark $m_c = (1.275 \pm 0.025) \text{GeV}$, $m_d = (4.8^{+0.5}_{-0.3}) \text{MeV}$, $m_u = (2.3^{+0.7}_{-0.5}) \text{MeV}$. The vacuum condensates are taken to be the standard values $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(0.8 \pm 0.1) \times (0.24 \pm 0.01) \text{GeV}^2$ \cite{54}, $\langle \bar{q}_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$ \cite{54}, $m_0^2 = (0.8 \pm 0.1) \text{GeV}^2$, $(g^2 G G) = (0.022 \pm 0.004) \text{GeV}^4$ \cite{55}. From Eq. (20), we also know that the value of the form factor $G_{\Sigma_B \Sigma_B}^{NB}[\Sigma_c ND]$ is the function of the input parameters, including the Borel parameters $M_1^2$ and $M_2^2$, continuum threshold $s_0$ and $u_0$, the momentum $Q^2$.

The working regions for the $M_1^2$ and $M_2^2$ are determined by requiring not only that the contributions of the higher states and continuum be effectively suppressed, but also that the contributions of the higher-dimensional operators are small. In other words, we should find a good plateau which will ensure OPE convergence and the stability of our results \cite{48}. The plateau is often called "Borel window". Considering these factors, the Borel windows are chosen as $7(3) \text{GeV}^2 \leq M_1^2 < 14(7) \text{GeV}^2$ and $3(2) \text{GeV}^2 \leq M_2^2 < 7(6) \text{GeV}^2$ for the strong vertex $\Sigma_B NB(\Sigma_c ND)$ (see Figures 1-4). From these figures, we can see that the values are rather stable with variations of the Borel parameters, it is reliable to extract the form factors. In addition, the continuum parameters, $s_0 = (m_i + \Delta_i)^2$ and $u_0 = (m_o + \Delta_o)^2$ are employed to include the pole and to suppress the h.r. contributions. The values for $\Delta_i$ and $\Delta_o$ can not be far from the experimental value of the distance between the pole and the first excited state \cite{48}. In general, these two continuum thresholds $s_0$ and $u_0$ are determined by the relations $s_0 \sim (m_i + 0.5 \text{GeV})^2$ and $u_0 \sim (m_o + 0.5 \text{GeV})^2$. According to these considerations, we take $s_0 = 37.4(7.6) \text{GeV}^2$ and $u_0 = 1.99(1.99) \text{GeV}^2$ for the strong vertex $\Sigma_B NB(\Sigma_c ND)$.

![FIG. 1: $G_{\Sigma_B \Sigma_B}^{NB}$ as a function of $M_1^2$ at average values of the continuum thresholds.](image1)

![FIG. 2: $G_{\Sigma_B \Sigma_B}^{NB}$ as a function of $M_2^2$ at average values of the continuum thresholds.](image2)

However, in order to obtain the coupling constants, it is necessary to extrapolate these results into physical regions ($Q^2 < 0$), which is realized by fit the form factors into suitable analytical functions. It is indicated that we should get the same values for the coupling constants for the different dirac structure $\gamma^5$ or $\bar{q} \gamma_5$ when we take $Q^2 = -m_{B(D)}^2$. This above procedure can help us minimizing the
uncertainties in the calculation of the coupling constant, which will be quite clear in the following section. From our analysis, we observe that the dependence of the form factors on $Q^2$ can be well described by the following fit function (see Figures 5-6):

$$G_{\Sigma_c,ND}(Q^2) = C_1 e^{\frac{Q^2}{C_2}} + C_3 e^{\frac{Q^2}{C_4}}$$

(21)

Where the values of $C_1, C_2, C_3$ and $C_4$ for different dirac structures are presented in Table 1 for these two strong vertexes $\Sigma_b,NB$ and $\Sigma_c,ND$. The fit function is used to determine the value of the strong coupling constant at $Q^2 = -m_{B[D]}^2$ for different structures, and the results are also presented in Table 1. The errors existing in these results arise from the uncertainties of the input parameters together with the uncertainties coming from the determination of the working regions of the auxiliary parameters.

It is indicated from Figure 5, Figure 6 and Table 1 that different dirac structure can give compatible
TABLE I: Parameters appearing in the fit function of the coupling form factor for Σ_bNB and Σ_cND.

| Structure | C_1 (GeV^{-1}) | C_2 (GeV^2) | C_3 (GeV^{-1}) | C_4 (GeV^2) | G  |
|-----------|----------------|-------------|----------------|-------------|----|
| Σ_bNB     | ϒ/γ_5         | -0.40 ± 0.05| -4.89 ± 0.02   | 0.90 ± 0.05 | -26.13 ± 4.00| 0.55 ± 0.01 |
|           | ϒ'γ_5         | -0.78 ± 0.10| -4.99 ± 0.02   | 1.74 ± 0.10 | -24.22 ± 4.00| 0.31 ± 0.01 |
| Σ_cND     | ϒ/γ_5         | 4.21 ± 0.20 | -21.51 ± 3.00  | 0            | 3.58 ± 0.02  |
|           | ϒ'γ_5         | 663.20 ± 63.50| 1728.30 ± 25.60| -660.60 ± 68.70| 0   | 3.94 ± 0.04  |

results for each strong coupling constants when we take \( Q^2 = -m_{B[D]}^2 \) in the fit function(Eq.21). For example, the results of the coupling constant for vertex Σ_bNB are 0.55 and 0.31 for ϒ/γ_5 and ϒ'γ_5 structure respectively. Thus, we can take the average of the coupling constants from two different dirac structure for each vertex, which are 0.43±0.01 and 3.76±0.05 for Σ_bNB and Σ_cND respectively.

4 Conclusion

In this article, we have calculated the form factors of the vertexes Σ_bNB and Σ_cND in the space-like regions by three-point sum rules. Then we fit the form factors into analytical functions, extrapolated them into the time-like regions, and obtained the strong coupling constants \( G_{Σ_bNB} \) and \( G_{Σ_cND} \). These calculated results can be used to analyze the related experimental results at LHC as well as the heavy ion collision experiments like ΨANADA at FAIR.

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