Mixed convection heat transfer in multi-Lid-driven trapezoidal annulus filled with hybrid nanofluid

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Abstract: Numerical study of mixed convection heat transfer in multi-Lid driven concentric trapezoidal annulus filled with H₂O-Cu-Al₂O₃ hybrid nanofluid has been investigated. Three cases for multi-Lid driven have been studied: single lid-driven, double lid-driven move in the same direction, double lid-driven move in the opposite direction. The lid-driven walls move with a constant speed with constant cold temperature T_C and the other inclined walls are insulated while the inner trapezoidal cylinder heated at constant temperature T_h. Finite volume method used to solve the continuity, momentum, and energy equations by SIMPLE algorithm. The results validated by comparing with previous study with a good agreement of accuracy. The working fluids was: water with hybrid nanoparticles (volume fraction ϕ = 0 to 10%). The Richardson numbers changed from 0.01 to 10, to cover all convection heat transfer modes, and aspect ratios were 0.5 and 1. The results show that, the opposing flow produced highest maximum stream function. Moreover, in aiding flow (case 2) produced a heat transfer coefficient on the top and bottom walls of outer cylinder higher than that produced by the opposing flow (case 3). Generally, the skin friction increases with increase in the volume fraction of nanoparticles due to increasing the viscosity of fluid causes increase in shear stress and leads to increasing the pressure drop. Additionally, the aiding flow produced fiction factor higher than the opposing flow.

Key words: Mixed convection, Heat transfer, Lid-driven, Trapezoidal cavity, Hybrid Nanofluid.

1. Introduction

Mixed convection heat transfer in lid-driven enclosures with different geometries, boundary conditions, and enhancement techniques has received great attention by authors because of its importance in the industrial applications. Examples of applications can be found in nuclear reactors, cooling of electronic equipment’s, solar energy, and technology of lubrication [1]. When the flow is induced by external means such as fan, or by rotating/moving parts in the system itself, the process is called forced convection. The thermal gradient between the solid wall and the fluid layers causes...
decrease in the density of fluid leading to rise the lighter fluid particles upwards and descend the heavy particles. The fluid movement resulted from buoyancy forces is called natural convection. The forced convection is characterized by Reynolds number, while the natural convection is characterized by Grashof number.

Recently, many authors studied combined convection heat transfer in a lid-driven cavity and used various techniques to enhance heat transfer. Some of them tried to increase the thermal conductivity of the base fluid by adding nanoparticles [1-10], and porous materials [11-16]. Other techniques included using of magneto-hydrodynamic (MHD) and vibration with or without nanofluids [17-24].

Abdelkader et al. (2015), [1] proved that the heat transfer in lid-driven square cavity filled with Ag-water nanofluid increases with increase in Richardson number. Habib et al. (2015), [2] concluded that the heat transfer rates enhance with increase in cavity inclination angle. Zoubair et al. (2016), [3] investigated the mixed convection heat transfer around hot triangular cylinder enclosed by a lid driven square cavity. It was concluded that, the average Nusselt number increases with decrease in Richardson number. Zhimeng et al. (2017), [4] studied the mixed convection in a lid-driven square cavity having sinusoidal bottom wall. Ishrat (2019), [5] used hybrid nanofluid to study the influence of magnetohydrodynamic (MHD) on mixed convection heat transfer in a lid driven triangular cavity. It was concluded that the heat transfer rate increases with increase in wave number. MS Rahman et al. (2019), [6] concluded that the increasing of Nusselt number depends on Richardson number. Md Shajedul et al. (2019), [7] noticed that the average heat transfer rate around circular cylinder enclosed by lid-driven square cavity increases as Reynolds number and solid volume fraction of nanoparticle increase. Neşe et al. (2020), [8] used various viscosities and thermal conductivities to enhance the heat transfer in a square cavity. Mostafa et al. (2020), [9] concluded that the decrease in frequency of the wall oscillation produced high heat transfer rate in an oscillatory lid-driven rectangular. Abdullah et al. (2020), [10] studied the heat transfer characteristics of nanofluid in a trapezoidal cavity saturated with porous media. It was observed that the average Nusselt number increases with increase in Darcy number for all aspect ratios.

M. A. Waheed et al. (2011), [11] used a fluid-saturated porous medium to enhance the heat transfer process in a cavity. Wael et al. (2012), [12] investigated the influence of sinusoidal heating on heat transfer in a square lid-driven enclosure filled with saturated porous material. Abdalla (2013), [13] proved that the porous block inside a square lid-driven cavity enhances the heat transfer rate by mixed convection comparing with the case of no block. Anirban et al. (2014), [14] examined the influence of moving walls direction and the nonuniform heating on the heat transfer rate of saturated porous medium in a lid-driven cavity. Satyajit et al. (2016), [15] concluded that the increase in heat transfer in a lid-driven L-shaped cavity containing a porous medium depends on Grashof number, Darcy number and Reynolds number. Ahmad et al. (2018), [16] showed that the heat transfer rate inside a right-angled trapezoidal enclosure increases with decrease in as the Lewis number.

T. Javed et al. (2016), [17] investigated the influence of MHD and Rayleigh number on the heat transfer process in a lid-driven trapezoidal enclosure. N. A. Bakar et al. (2016), [18] concluded that the magnetic field has a strong effect on the thermal patterns and fluid field in square cavity. M. Borhan et al (2016), [19] applied the uniform magnetic field in negative horizontal direction of a trapezoidal cavity. It is noticed that the uniformity heating of bottom wall produced higher heat transfer rate than that in case of non-uniformity. M. Rashad et al. (2017), [20] concluded that the improving of heat transfer depended on the thermal patterns in a square lid driven enclosure filled with a nanofluid-saturated porous medium. Sameh et al. (2018), [21] studied the effect of inclination angle of (MHD) on thermal patterns and fluid flow field in lid-driven trapezoidal cavity filled with different nanofluids. It was observed that the heat transfer rate depended strongly on the heat source length and the solid volume fraction. M. Humaun et al. (2019), [22] concluded that heat transfer rate in a lid-driven porous rectangular cavity filled with nanofluid and containing three square heating blocks increases with increase in Darcy number and Richardson. Md. Fayz et al. (2019), [23] analyzed the influence of MHD on mixed convection in a lid-driven enclosure having a single fin placed on its hot
wall. The results reveal that Richardson number and Hartmann number with existence of fin have a strong effect on the heat transfer process. Priyajit Monda and Tapas Ray Mahapatra (2020), [24] concluded perfect shape of trapezoidal cavity under effects of MHD mixed convection heat transfer and diffusion to produce minimum entropy and maximum efficiency.

Numerical analysis of mixed convection heat transfer in trapezoidal lid-driven concentric annulus filled with H$_2$O-Cu-Al$_2$O$_3$ hybrid nanofluid has been studied. The inner cylinder is heated at constant temperature $T_h$, while the horizontal cold walls move as lid-driven with constant speeds in the same and opposing directions. The inclined walls are insulated. Finite volume method was used to solve the continuity, momentum, and energy equations by SIMPLE algorithm. Three Richardson numbers were studied: 0.01 (forced convection), 1 (mixed convection), and 10 (natural convection).

## 2. Mathematical modeling

Consider the physical domain shown in Fig. 1 which represents a trapezoidal enclosure having two moving lid walls at isothermal cold temperature. The top and bottom lid walls are continually moved at constant velocity $U_1$ and $U_2$; respectively, and kept at constant cold temperature $T_c$. The heat source produced by the inner trapezoidal cylinder which is located concentrically inside the annulus. The inclined walls are insulated adiabatically. Two aspect ratios were chosen (0.5 and 1) with two working fluids: water with nanoparticles ($\varphi = 0.1$) and without nanoparticles. The mathematical modeling for the mixed convection heat transfer in a trapezoidal lid-driven cavity was analyzed using the finite volume method.

![Figure 1. Physical domain of trapezoidal enclosure subjected to double moving lids driven.](image)

### 2.1 Governing Equations

The schematic diagram of lid-driven trapezoidal annulus with hot inner cylinder is shown in Fig. 1. The trapezoidal annular gap is filled with a water-based nano-fluid. The thermo-physical properties of the nano-fluid are shown in Table 1 [25]:

| Material | $C_p$ (J/kg.K) | $\rho$ (kg/m$^3$) | $k$ (W/m.K) | $\beta$ (1/K) |
|----------|---------------|-----------------|-------------|--------------|
| H$_2$O   | 4179          | 997.1           | 0.613       | 21×10$^{-5}$ |
| Cu       | 385           | 8933            | 401         | 1.67×10$^{-5}$ |
| Al$_2$O$_3$ | 765          | 3970            | 40          | 0.85×10$^{-5}$ |

In this regard, the density $\rho_{hnf}$, buoyancy coefficient $(\rho \beta)_{hnf}$, and specific heat energy $(\rho C_p)_{hnf}$ for the hybrid nanofluid are given below [25]:

$$\rho_{hnf} = (1 - \varphi_{Cu} - \varphi_{Al_2O_3})\rho_f + \varphi_{Cu}\rho_{Cu} + \varphi_{Al_2O_3}\rho_{Al_2O_3}$$ (1)
\((\rho \beta)_{hnf} = (1 - \varphi_{Cu} - \varphi_{Al_2O_3})(\rho \beta)_{f} + \varphi_{Cu}(\rho \beta)_{Cu} + \varphi_{Al_2O_3}(\rho \beta)_{Al_2O_3}\) \hspace{1cm} (2)

\((\rho C_p)_{hnf} = (1 - \varphi_{Cu} - \varphi_{Al_2O_3})(\rho C_p)_{f} + \varphi_{Cu}(\rho C_p)_{Cu} + \varphi_{Al_2O_3}(\rho C_p)_{Al_2O_3}\) \hspace{1cm} (3)

The thermal conductivity of hybrid nanofluid is defined as:

\[k_{hnf} = k_f \left( \frac{\varphi_{Cu} + \varphi_{Al_2O_3}}{\varphi_{Cu} + \varphi_{Al_2O_3} + 2k_f} \right) - 2(\varphi_{Cu} + \varphi_{Al_2O_3})(k_f \frac{\varphi_{Cu} + \varphi_{Al_2O_3}}{\varphi_{Cu} + \varphi_{Al_2O_3}}) + 2(\varphi_{Cu} + \varphi_{Al_2O_3})(k_f \frac{\varphi_{Cu} + \varphi_{Al_2O_3}}{\varphi_{Cu} + \varphi_{Al_2O_3}})\] \hspace{1cm} (4)

The dynamic viscosity ratio of nanofluid is defined from Corcione et al. [26] as:

\[\mu_{hnf} = \mu_f / \left(1 - \varphi_{Cu} + \varphi_{Al_2O_3}\right)^{2.5}\] \hspace{1cm} (5)

The problem of two-dimensional laminar airflow and mixed convection heat transfer in the double lid-driven trapezoidal cavity is characterized by continuity, momentum, and energy equations. The fluid properties are assumed to be constant except the density variation in the buoyant force according to Boussinesq approximation. The governing equations are given below.

\[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\] \hspace{1cm} (6)

\[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial x} + \theta_{hnf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)\] \hspace{1cm} (7)

\[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial y} + \theta_{hnf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \beta_{hnf}(T - T_c)\] \hspace{1cm} (8)

\[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)\] \hspace{1cm} (9)

The dimensionless variables can be written as follows:

\[X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{u_1}, V = \frac{v}{u_1}, T^+ = \frac{T - T_c}{T_h - T_c}, Re = \frac{u_1 H}{v_{hnf}}, Gr = \frac{g \beta_f (T_h - T_c) H^3}{v_f^2}\]

\[P = \frac{p L^2}{\rho_f u_1^2}, Pr = \frac{\theta_f}{\mu_f}, Ri = \frac{Gr}{Re^2}\]

The governing equations in dimensionless form are:

\[\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0\] \hspace{1cm} (10)

\[U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \frac{\mu_{hnf}}{\mu_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)\] \hspace{1cm} (11)

\[U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \frac{\mu_{hnf}}{\mu_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \left( \frac{\rho \beta}{\rho_{hnf} \beta_f} \right) Ri T^+\] \hspace{1cm} (12)

\[U \frac{\partial T^+}{\partial X} + V \frac{\partial T^+}{\partial Y} = \frac{\alpha_{hnf}}{Pr Re} \left( \frac{\partial^2 T^+}{\partial X^2} + \frac{\partial^2 T^+}{\partial Y^2} \right)\] \hspace{1cm} (13)

2.2 Boundary conditions

The boundary conditions in dimensionless form are presented in Table 2.

| Table 2. Boundary conditions |
|-----------------------------|
| **U** | **V** | **T^+** |
| Top wall | A=1 | 0 | 0 |
| Bottom wall | A=0, 1, -1 | 0 | 0 |
| Left and right walls | 0 | 0 | \(\partial T^+/\partial X = 0\) |
| Inner cylinder wall | 0 | 0 | 1 |

2.3 Heat transfer and Friction Factor calculations

The average Nusselt number on the moving lid are expressed in Eqs. (14 & 15) respectively:
\[ Nu_{ave} = \int_0^L Nu_x \, dx \quad , \quad Nu_{ave} = \frac{1}{A} \int_0^L A^{-k_f \phi} \frac{\partial T}{\partial x} \big|_{x=0} \, dY \] (14)

\[ f = 8. \frac{\tau_w}{\rho u^2} \] (15)

2.4 Validation case

The results obtained by Ali and Eiyad [27] for average Nusselt number in two square cavities with single and double lid driven was used to validate the present work as shown in Fig. 2. The comparison includes different volume fractions of nanoparticles \( \phi (\text{Al}_2\text{O}_3) \) and Richardson numbers. It shows a good agreement between the two works.

![Figure 2. Validation test with Ali and Eiyad work [27], single lid drive (left) and double lid driven (right).](image)

3. Results and Discussion

3.1 Streamlines and isotherms

The influence of Richardson number (\( \text{Ri}=0.01, 1, \) and 10) and aspect ratio (\( AR=1 \) and 0.5) for three cases (1, 2, and 3) of lid-driven on the characteristics of fluid field and thermal patterns are shown in Figures 3-6; respectively.

For case 1 (single lid driven), it is obvious that at low Richardson number, the shear influences due to moving the top wall is prevalent for both aspect ratios. The movement of the top lid produced primary recirculating eddy. It is evident that, the isotherms produced higher temperature gradients in the region adjacent to the hot cylinder walls. The isothermal contour lines are rotated about the inner hot cylinder. Whereas, these lines fade away from the inner cylinder and this region becomes cold because the dominant forced convection in the heat transfer process. At low Richardson number, the vorticity seems to be strong at the upper region of enclosure, and the maximum stream function values are 0.682 at the right lower part of annulus with high aspect ratio \( \text{AR}=1 \), and 0.59 at the left lower part of annulus with small aspect ratio \( \text{AR}=0.5 \), as shown in Table 3. So, it is expected that the heat transfer coefficient will be higher than that at \( \text{Ri}=1 \) and 10. The natural convection grows with increase in Richardson number and creates circulation. Richardson number equals to one means that the primary and secondary hybrid nanofluid flows have a neutral effect. The isothermal lines extend towards the inner walls of outer cylinder. The distortion in thermal plum occurs because of the higher velocity of hybrid nanofluid near the moving wall. The maximum stream function greatly
increases with increase in Richardson number from 0.01 to 1 for both aspect ratios. Moreover, it is evident from Table 3 that the maximum stream function value decreases as aspect ratio decreases. The behavior of isotherms and streamlines depends on their change according to change of Richardson number, aspect ratio, and volume fraction for all cases. Increasing Richardson number to 10 produces smaller shear driven circulation and indicates that the natural convection is the dominating factor in the heat transfer. The direction of thermal plum distortion is in the same direction of moving wall, and creeps parallel to the top wall of inner hot cylinder for small aspect ratio. The isotherms lines get bigger towards the inner walls of outer cylinder and the values of maximum stream function get smaller ($\Psi_{\text{max}} = 0.0335$ and 0.3) for AR=1 and 0.5; respectively.

Case 2 indicates the heat transfer inside trapezoidal cavity with two horizontal walls move in the same direction. At Ri=0.01 and AR=1, the forced convection is dominant in the heat transfer process. The upper and lower walls movement in the same direction causes the high shearing action which leads to rotating the hybrid nanofluid in the counterclockwise direction at the lower region and in the clockwise at the upper region. The movement walls help to moving the hybrid nanofluid in the same direction, then the fluid collides with the right wall to change its movement towards the left, causing a rotational movement. Two vortices will be generated near to the right side of upper and lower walls. The maximum vortex ($\Psi_{\text{max}} = 0.669$) is located near the lower wall because the narrow space in that part of cavity compared with the upper region causes increasing the hybrid nanofluid. Decreasing the aspect ratio to 0.5 means decreasing the annular space. Main vortex having streamlines rotate around the inner cylinder and minor vortex rotates near the upper moving wall will be generated. This causes rising the strong mini vortex from the bottom to the right of the inner hot cylinder ($\Psi_{\text{max}} = 0.65$). Increasing Richardson number to one means the inertia force and the buoyancy force are equivalent. As shown from the streamlines, the strong vortex bisects into two weak parts. The maximum stream function is located in the lower part ($\Psi_{\text{max}} = 0.14$) for AR=1 and in the right part ($\Psi_{\text{max}} = 0.132$) for AR=0.5. For Ri=10 (an increase in Gr), the natural convection (buoyancy force) is the dominated. Streamlines near the upper-right and lower-right corners tend to rotate in the clockwise and counterclockwise directions; respectively. The values of maximum stream functions will further decrease for AR=1 ($\Psi_{\text{max}} = 0.062$) and for AR=0.5 ($\Psi_{\text{max}} = 0.046$).

As shown from the isothermal contours, increasing Richardson number from 0.01 to 1 (AR=1) cause enlarging the thermal plum at the top and bottom inner hot cylinder because of increasing the fluid temperature. Increasing the thermophoresis near the left and bottom walls of inner cylinder causes a high temperature gradient. Distortion in thermal patterns occurs as Richardson number increases to 10 because of the strong natural convection currents in the left cold region of cavity. Decreasing the aspect ratio from 1 to 0.5 enlarging the hot regions at the expense of cold regions at Ri=1 and 10. Whereas, most of the regions far from the inner cylinder will be kept cold at Ri=0.01 because of dominant forced convection.

Case 3 indicates the heat transfer inside trapezoidal cavity with two horizontal walls move in the opposite direction. It is noticed from Figures 3 and 4 that there is no distortion in the streamlines which rotate in the clockwise direction for all Richardson numbers and aspect ratios. The movement of top wall towards the right wall causes clashing the hybrid nanofluid into this wall and change its direction towards lower part of cavity because of gravitational force and moving the bottom wall towards the left. As a result, the hybrid nanofluid crashes into the left wall and change its direction towards the upper wall. The motion of hybrid nano fluid towards the right on the top region of cavity and towards the left on the bottom region causes a rotational motion in the clockwise direction. For Ri=0.01, Figure 6 shows the thermal plum of warm liquid is located at the left corner of top wall of inner cylinder for AR=1 and at the right corner of top wall and left corner of the bottom wall of inner cylinder because the dominant forced convection in the regions far from inner hot cylinder. As Richardson number increases, the warm liquid ascends along the left wall of inner cylinder and arrives at the upper wall of outer cylinder, whilst the cold liquid descends along the right wall of inner
cylinder. The isothermal lines will not change greatly as Richardson number increases from 1 to 10. With decreasing aspect ratio to 0.5, and for Ri=1 and 10, the isothermal lines will be nearly in elliptic paths around the inner cylinder because of high temperature gradient at this region.

As shown from the figures 3-6 and Table 3, by increasing the volume fraction of the hybrid nanofluid, the maximum stream functions and the fluid temperature increase. The thermophoresis increases near the hot inner cylinder because the high temperature gradient. As a result, high performance of heat transfer is expected.

Figure 3. Streamlines in Trapezoidal lid driven (AR=1) for water (solid black lines) and H₂O-Cu-Al₂O₃ hybrid nanofluid φ=0.1 (dashed red lines).
Figure 5. Isotherms in Trapezoidal lid driven (AR=1) for water (solid black lines) and $\text{H}_2\text{O}\cdot\text{Cu}\cdot\text{Al}_2\text{O}_3$ hybrid nanofluid $\phi=0.1$ (dashed red lines).

Case 1
Case 2
Case 3
Table 3. Maximum stream function different cases

| Case No. | AR | $\varphi = 0$ | $\varphi = 0.1$ |
|----------|----|---------------|------------------|
|          |    | $R_i=0.01$   | $R_i=1$          | $R_i=10$         | $R_i=0.01$   | $R_i=1$          | $R_i=10$         |
| 1        | 1  | 0.682        | 0.098           | 0.0335          | 0.85         | 0.12            | 0.041            |
| 2        | 1  | 0.669        | 0.14            | 0.062           | 0.82         | 0.17            | 0.062            |
| 3        | 1  | 0.83         | 0.13            | 0.04            | 1.07         | 0.16            | 0.05             |
| 1        | 0.5| 0.59         | 0.088           | 0.3             | 0.73         | 0.1             | 0.37             |
| 2        | 0.5| 0.65         | 0.132           | 0.046           | 0.81         | 0.163           | 0.056            |
| 3        | 0.5| 0.84         | 0.15            | 0.047           | 1.03         | 0.18            | 0.057            |

3.2 Average Nusselt number

The variation of average Nusselt number with volume fraction of hybrid nanofluid for inner cylinder is shown in Figure 7, for three studied cases, two aspect ratios, and three Richardson numbers. Generally, the average Nusselt number increases with increase in volume fraction. The results show that the aiding flow case (case 2) produces higher heat transfer coefficients than case 3 (opposed moving walls) and case 1 (one moving wall); respectively, because the lid-driven becomes more active to enhance the heat transfer as the parallel walls move in the same direction. The average Nusselt numbers increase with decrease in Richardson number because of the dominant forced convection in the heat transfer process for both aspect ratios and all cases.

The variation of average Nusselt number with volume fraction of hybrid nanofluid for top and bottom walls of outer cylinder is shown in Figure 8, for the parallel walls move in the same and opposite directions. Two aspect ratios and three Richardson numbers are considered. It is clear that the bottom moving wall produces higher heat transfer coefficients than the top moving wall. Moreover, the aiding flow case (case 2) produces higher heat transfer coefficients than that produced by the opposing flow (case 3). In the opposing flow case (case 3), the values of average Nusselt number for $R_i=1$ (mixed convection) and 10 (dominant natural convection) are much smaller than that for $R_i=0.01$ and close to each other. This convergence in the average Nusselt numbers decreases in the aiding flow case (case 2) and the difference between the two values produced by both Richardson numbers ($R_i=1$ and 10) increases. Moreover, the average Nusselt numbers for the top and bottom walls are close to each other for the opposing flow case (case 3). Generally, the heat transfer coefficient increases with increase in aspect ratio and decrease in Richardson number because of the dominant forced convection in the heat transfer process.
3.3 Friction Factor

The variation of average skin friction coefficient with volume fraction on the top wall, bottom wall, and hot source (inner cylinder) is shown in Fig. 9, for three studied cases, two aspect ratios (1 and 0.5), and three Richardson numbers (0.01, 1, and 10). Generally, the skin friction increases with increase in the volume fraction of nanoparticles due to increasing the viscosity of fluid causes increase in shear stress and leads to increasing the pressure drop. Moreover, the friction factors at Ri=0.01 are much higher than that at Ri=1 and 10 because the dominant forced convection causes high pressure drop on
the moving wall, stationary wall, and inner cylinder wall. It is noticed that the friction factor at the moving wall is larger than the fiction at the stationary wall because of high velocity at the moving wall. Additionally, the aiding flow produced higher friction factor than the opposing flow. This can attribute to moving the upper and lower walls in the same direction (aiding flow) causes the high shearing action (high friction factor) which leads to rotating the hybrid nanofluid in the counterclockwise direction at the lower region and in the clockwise at the upper region. While, the motion of top and bottom walls towards the right and left directions (opposing flow); respectively, causes a rotational motion of hybrid nanofluid in the clockwise direction around the inner cylinder. The vorticity generated in the aiding flow is stronger than that in the opposing flow. As a result, the friction factor will be increase.

AR=1

AR=0.5
4. Conclusions

The following conclusions are remarked:

1. The thermal patterns and the fluid field characteristics inside multi lid-driven concentric trapezoidal annulus depend greatly on the Richardson number.
2. The opposing flow produced highest maximum stream function in the concentric trapezoidal annulus.
3. The maximum stream function value increases with increase in the aspect ratio only for case 1 and case 2, and vice versa for case 3.
4. The maximum stream function value increases with increase in the volume fraction of hybrid nanofluid.
5. The aiding flow case (case 2) produces higher heat transfer coefficients on the inner hot cylinder than case 3 (opposed moving walls) and case 1 (single moving wall); respectively.
6. The bottom moving wall produces higher heat transfer coefficients than the top moving wall.
7. The aiding flow case (case 2) produces higher heat transfer coefficients on the top and bottom walls of outer cylinder than that produced by the opposing flow (case 3).
8. The heat transfer coefficient increases with increase in aspect ratio and with decrease in Richardson number.
9. The skin fiction factor increases with increase in volume fraction and decrease in Richardson number.
10. The aiding flow produced higher skin fiction factor than the opposing flow.

Nomenclature

- \( A \): Constant
- \( U, V \): dimensionless velocity components in x- and y-directions, respectively
- \( u_1, u_2 \): constant velocity of moving lid
- \( x, y \): Cartesian coordinates
- \( X, Y \): dimensionless Cartesian coordinates
- \( \alpha \): fluid thermal diffusivity
- \( \beta \): coefficient of volume expansion for fluid
- \( \mu \): dynamic viscosity of fluid
- \( \theta \): kinematic viscosity of fluid
- \( \phi \): Nanoparticle volume fraction

- \( Cp \): Specific heat capacity
- \( cf, f \): Friction factor and skin friction
- \( g \): Gravitational acceleration
- \( Gr \): Grashof number
- \( k \): Thermal conductivity
- \( L \): Side length of enclosure
- \( p & P \): Pressure and dimensionless pressure
- \( Re_B \): Brownian motion Reynolds number
\begin{center}
\begin{tabular}{ll}
$N_u_{\text{ave}}$ & Average nusselt number \\
$N_u_{x}$ & Local Nusselt number \\
$Pr$ & Prandtl number \\
$Re$ & Reynolds number \\
$Ri$ & Richardson number \\
$T$ & Fluid temperature \\
$T^+$ & Dimensionless temperature \\
$T_h$ & Hot temperature \\
$T_c$ & Cold temperature at base wall \\
u, v & velocity components in x- and y-directions, respectively \\
\end{tabular}
\end{center}

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