Exploring the self-mode locking and vortex structures of nonplanar elliptical modes in selectively end-pumped Nd:YVO₄ lasers: manifestation of large fractional orbital angular momentum

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Abstract: An end-pumped Nd:YVO₄ laser under selective pumping is used to excite lasing modes with transverse patterns performed to exhibit the characteristics of multiple spots arranged on elliptical features near degenerate cavities. The spatial distribution of elliptical lasing modes is clearly revealed to be localized on the nonplanar ray orbits, so-called nonplanar elliptical modes, which possess large fractional orbital angular momentum. Moreover, temporal dynamics for the output emission of nonplanar elliptical modes are verified to obtain self-mode-locked operation. We further numerically manifest not only the influence of radial-asymmetry distributions on the vortex structures of nonplanar elliptical modes, but also the vector field of transverse lasing modes altered with twisting phase structures in the propagation direction.

OCIS codes: (140.4050) Mode-locked lasers; (050.4865) Optical vortices; (140.3480) Lasers, diode-pumped.

References and links

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1. Introduction

Techniques of self-mode locking with a decrease of cavity length for developing high-repetition-rate optical pulses is an interesting phenomenon without using active elements or saturable absorbers [1]. The efficient self-mode-locked operation has recently been realized in various diode-pumped Yb-doped [2–5] and Nd-doped [6–9] crystal lasers. In particular, the
self-mode-locked high-order Hermite-Gaussian (HG) modes were experimentally confirmed
in a selectively diode-end-pumping scheme [10]. Moreover, it has been found that when the
cavity length is adjusted to be the degenerate cavities, the self-mode locking with the
simultaneously longitudinal and transverse mode locking can be achieved. To fulfill the
degenerate cavities, the cavity configuration requires satisfying the degenerate condition of
\( \frac{\Delta f_T}{\Delta f_L} = \frac{P}{Q} \), where \( P \) and \( Q \) are co-prime integers, \( \Delta f_T \) is the longitudinal mode spacing,
and \( \Delta f_L \) is the transverse mode spacing. The simultaneously self-mode locking can be seen to
be a time-dependent wave packet with dynamical behaviors corresponding to the planar
periodic ray trajectories inside the cavity, known as planar geometric modes [11,12]. Thus far
the optical configuration producing these multipass beams has been widely used in diverse
experiments, such as optical delay lines [13], absorption spectroscopy [14–16], Raman
conversion [17–20], high-power laser systems [21] and low repetition rate ultrafast laser
cavities [22–24].

However, the generalized multipass beams in a three-dimensional (3D) laser resonator
near the degenerate cavities can be localized on the nonplanar ray orbits, described as
nonplanar geometric modes. The transverse pattern of nonplanar geometric modes is
performed to demonstrate the property of multiple spots arranged on the topological
trajectories. The topological trajectories are altered with the relative phase \( \phi \) between the
degenerate eigenmodes from the line feature at \( \phi = 0 \) (i.e., planar geometric modes) through
the elliptical shape in the range of \( 0 < \phi < \pi / 2 \) and to the circular form at \( \phi = \pi / 2 \). Hence,
the nonplanar geometric modes can be generally represented by nonplanar elliptical modes.
More interestingly, these nonplanar elliptical modes exhibit the existence of light fields with
spiral characteristics during propagation. Several properties of spiral optical beams developed
by Abramochkin and Volostnikov [25,26] are of practical interest for laser technologies,
medicine, and microbiology. Specifically, in recent years elliptical modes with possessing
fractional orbital angular momentum (OAM) and lacking for radial symmetry have been
observed to be a rapidly growing interest in the field of quantum communication to achieve
high security of quantum cryptography and quantum teleportation [27–34]. It is believed that
the investigation of nonplanar elliptical modes with the self-mode-locked operation and the
spiral features carrying huge fractional OAM is an important step in the structure light to be
exploited in numerous applications.

In this work we exploit a diode-end-pumped Nd:YVO₄ laser with selective pumping to
produce the nonplanar elliptical modes near the degenerate cavities. The experimental far-
field patterns of nonplanar elliptical modes with the characteristics of multiple spots reveal to
obtain the self-mode locking. To analyze the average OAM and the vortex structures of nonplanar
elliptical modes, we theoretically perform that spatial distributions of nonplanar elliptical modes can be perfectly reconstructed with the numerical calculations. Theoretical
exploration validates the influence of radial-asymmetry distributions on the vortex structures
of nonplanar elliptical modes that the primary singularities can be separated by controlling the
phase factor between the degenerate eigenmodes. It is further observed that the vector field of
transverse lasing modes is displayed swirling features along the elliptical structure at the
beam waist, and evidently altered with the twisting phase angle field in the propagation
direction.

2. Experimental results for the self-mode locking of nonplanar elliptical modes

Here we demonstrate nonplanar elliptical modes generated from nearly degenerate cavities by
exploiting the selective pumping in a concave-plane resonator as shown in Fig. 1. The front
mirror is a 10-mm radius-of-curvature concave mirror with antireflection coating at 808 nm
on the entrance face and with high-reflectance coating at 1064 nm (99.8%) and high
transmittance coating (T>95%) at 808 nm on the second surface. The gain medium is an \( a \)-cut
2.0-at.% Nd³⁺:YVO₄ crystal with a length of 2 mm. One side of the gain medium is coated for
antireflection at 808 nm and 1064 nm (reflection < 0.1%) and the other side is coated to be an output coupler with a transmission of 0.5% at 1064 nm. The pump source is an 808-nm fiber-coupled laser diode with a core diameter of 100 μm, a numerical aperture of 0.16, and a maximum output power of 3 W. A focusing lens with 25 mm focal length and 85% coupling efficiency is used to focus the pump beam into the gain medium. The pump radius is estimated to be approximately 25 μm. We use the lens to reimage the transverse patterns at the different longitudinal positions inside the cavity.

When the optical cavity length $L$ is adjusted to be approximately 11 mm and 7 mm, the measured transverse patterns at the different longitudinal positions for $(P,Q) = (1,3)$ and $(P,Q) = (1,5)$ could be obtained as shown in the upper row of Fig. 2 and Fig. 3, respectively. The $z$-dependent experimental patterns clearly display a characteristic of multiple spots well localized on elliptical shapes that is consistent with the periodic ray orbit depicted in the lower row of Fig. 2 and Fig. 3. Furthermore, the pump profile at the location of the crystal is a Gaussian distribution satisfying sufficient overlap with the noticeable bright spot of the lasing mode distribution shown in Fig. 2 for $z = -L/5$ and Fig. 3 for $z = -L/3$. Since the cavity mode possessing the biggest overlap with the gain region will dominate the laser emission, the nonplanar elliptical mode can be generated by precisely manipulating and persistently shifting the pump position on the gain medium. The experimental results of the nonplanar elliptical mode for $(P,Q) = (1,3)$ can be found under the off-center pumping with accurately transverse displacements of $\Delta x = 0.47$ mm and $\Delta y = 0.45$ mm along the $x$- and $y$- directions; the nonplanar elliptical mode for $(P,Q) = (1,5)$ is produced with $\Delta x = 0.43$ mm and $\Delta y = 0.80$ mm along the $x$- and $y$- directions, respectively. On the whole, nonplanar elliptical modes are usually difficult to be flexibly generated in the present approach that the strictly transverse displacements are required to balance the astigmatism induced by the birefringence of the gain medium and to achieve the isotropic effective cavity lengths in $x$- and $y$- directions.
Fig. 2. Upper row: experimental transverse patterns with \((P, Q) = (1, 3)\) measured at the different longitudinal positions inside the concave-plano resonator. Lower row: periodic ray orbit in geometric optics.

Fig. 3. Upper row: experimental transverse patterns for the case of \((P, Q) = (1, 5)\) measured at the different longitudinal positions inside the concave-plano resonator. Lower row: periodic ray orbit in geometric optics.
Figure 4 shows the schematic of the time-dependent photon wave packet related to the mode-locked operation with dynamical behaviors corresponding to the nonplanar elliptical modes in the laser cavity. It indicates that when the ratio $\Delta f_T/\Delta f_L$ is a simple fraction $P/Q$ with $P$ and $Q$ to be co-prime integers, the wave packet inside the cavity periodically returns to its original position and direction after $(P \cdot Q)$ round trips for the round-trip time $t_r = 2L/c$. It has recently been confirmed that when the cavity length is adjusted to be the degenerate cavity, the self-mode locking can be achieved to generate a localized wave packet traveling along the planar geometric trajectories inside the cavity by identifying the relative shifts of the time sequences between different spots of the far-field pattern [11,12]. Here a similar approach is utilized to build a self-mode-locked laser in Fig. 1. The far-field patterns of nonplanar elliptical modes for $(P,Q) = (1,3)$ and $(P,Q) = (1,5)$ display a characteristic of multiple spots depicted in the insets of Figs. 5(c) and 5(d), respectively. The high-speed InGaAs photodetector (Electro-optics Technology Inc. ET-3500 with rise time 35 ps) is employed to measure the temporal behavior in different spots of the far-field pattern. The output signal of the photodetector is connected to a digital oscilloscope (Agilent DSO 80000) with 10 GHz electrical bandwidth and a sampling interval of 25 ps. The output signal of the photodetector is also analyzed by an RF spectrum analyzer (Agilent) with a bandwidth of 26.5 GHz. Figures 5(a) and 5(b) show the experimental results of the mode-locked pulse trains for one of the spots in the far-field patterns for $(P,Q) = (1,3)$ and $(P,Q) = (1,5)$ measured with a 5 GHz bandwidth real-time oscilloscope, respectively. The corresponding power spectrum is measured by an RF spectrum analyzer with a span of 26.5 GHz in Figs. 5(c) and 5(d), demonstrating the signal to noise to be greater than 35 dB. The fundamental frequency peaks can be seen to be 4.529 GHz in Fig. 5(c) and 4.289 GHz in Fig. 5(d), which correspond accurately to $\Delta f_T$ shown that a wave packet returns exactly to the same position and direction after 3 and 5 round trips in the cavity, respectively. Since nonplanar elliptical modes require simultaneously longitudinal and transverse mode locking, the mode-locked condition is certainly more rigorous than fundamental modes with purely longitudinal mode locking. Even though the performance of mode locking for nonplanar elliptical modes is not perfect as the fundamental modes, it can be seen that the pulse trains display full modulation without CW background, indicating that the mode-locked operation is achieved. It is worthwhile to mention that the above results are all for the isotropic effective cavity lengths in $x$- and $y$-directions inside the laser resonator. To increase the effective path length exploited in numerous applications, the present approach can be further justified to introduce an appropriate astigmatism into the laser cavity to achieve mode-locked states corresponding to nonplanar periodic ray orbits with marvelous structures.
3. Theoretical analysis of vortex structures generated from nonplanar elliptical modes

Considering the paraxial approximation, the eigenmodes for the laser cavity with a concave mirror at \( z = -L \) and a plane mirror at \( z = 0 \) can be divided into two waves traveling in opposite directions: 

\[
\Phi_{m,n,l} = \Phi_{m,n,l}^{(1)} + \Phi_{m,n,l}^{(2)} \sqrt{2},
\]

where 

\[
\Phi_{m,n,l}^{(1)}(x,y,z) = \psi_{m,n}(x,y,z) e^{-i (2k_{m,n}) z} e^{i \phi/2},
\]

the transverse distribution \( \psi_{m,n}(x,y,z) \) is the HG mode:

\[
\psi_{m,n}(x,y,z) = \frac{\sqrt{2}}{w(z)} \left( 2^{m+n} m! n! \pi \right)^{-1/2} e^{-(x^2+y^2)/2} H_n(\tilde{x}) H_n(\tilde{y}),
\]

\( H_n(\cdot) \) is the Hermite polynomials, \( \tilde{x} = \sqrt{2} \xi_x / w(z) \), \( \tilde{y} = \sqrt{2} \xi_y / w(z) \), \( \xi_x = (x+y) / \sqrt{2} \), \( \xi_y = -(x-y) / \sqrt{2} \), \( \tilde{z} = z \left[ 1 + (x^2 + y^2)/(2z_R^2) \right] \), \( w(z) = w_0 \sqrt{1 + (z/z_R^2)^2} \), \( w_0 \) is the beam radius at the waist, \( z_R = \pi w_0^2 / \lambda \) is the Rayleigh range, \( k_{m,n,l} \) is the eigenvalue of the wave number, \( \ell \) is the longitudinal mode index, \( m \) and \( n \) are the transverse mode indices, and \( \theta_G(z) = \tan^{-1}(z/z_R) \) is the Gouy phase. For the concave-plano resonator, the longitudinal and transverse mode spacings are respectively given by

\[
\Delta f_L = c/2L \quad \text{and} \quad \Delta f_T = (\Delta f_L / \pi) \sin^{-1} \sqrt{L' / R},
\]

where \( L = L_c + (n_x - 1) L_g \), \( L' = L_c + [(1/n_x) - 1] L_g \), \( L_c \) is the
is the relative phase between the waves traveling backward and forward in the cavity. Therefore, the relationship between the parameters \( N \) and \( e_{xyz} \) is provided with /5 \( \phi \).

In the longitudinal and transverse mode spacings depending on the optical cavity length \( L \), nonplanar elliptical modes can be constructed in degenerate cavities satisfying the condition of \( \Delta f L / \Delta f L = P/Q \) by adjusting the optical cavity length \( L \), where \( P \) and \( Q \) are co-prime integers. In terms of the traveling components \( \Phi^{(s)}_{m,n,t} \) and the degenerate conditions of \( \Delta f L / \Delta f L = P/Q \), the wave distributions of lasing modes in a standing-wave cavity can be expressed as a sum of two traveling waves [35]:

\[
\Psi_{M,N}^{(s)} = \left[ \Psi_{M,N}^{(s)} + \Psi_{M,N}^{(s)} \right] / \sqrt{2},
\]

where

\[
\Psi_{M,N}^{(s)}(x,y,z;\phi) = \sum_{s=0}^{M} \sum_{s=0}^{N+Q} a_{M,s} b_{M,s} e^{i\phi} \Phi_{M,N}^{(s)}(x,y,z)
\]

and

\[
a_{M,s} = 2^{-M/2} \left[ \frac{M!}{s!(M-s)!} \right]^{1/2}
\]

\[
b_{M,s} = 2^{-(N+Q)/2} \left[ \frac{(N+sQ)!}{K!(N+sQ-K)!} \right]^{1/2}
\]

are the weighting coefficients, and the parameter \( \phi \) is the relative phase between the degenerate traveling components. Note that the index \( \ell_o \) indicates the maximum longitudinal order, the integer \( M + 1 \) represents the total number of longitudinal eigenmodes in the lasing modes, and the integer \( N = m_o + n_o \) denotes the sum of the transverse orders in \( x \)- and \( y \)-directions, where \( m_o = \left[ \Delta x / w(z_c) \right]^2 \) and \( n_o = \left[ \Delta y / w(z_c) \right]^2 \), and \( z_c \) is the location of the gain medium [36]. In other words, the values of the parameters \( m_o \) and \( n_o \) signify the magnitudes of the off-center displacements in the \( x \)- and \( y \)-directions, respectively. For the frequency peak corresponding to \( \Delta f f_T \) in Fig. 5, it means a unidirectional wave packet traveling along the nonplanar geometric trajectories as seen in Fig. 4 that implies the traveling waves \( \Psi_{M,N}^{(s)} \) and \( \Psi_{M,N}^{(s)} \) with the same phase factor \( \phi \). Therefore, the relationship between \( m_o \), \( n_o \), and \( \phi \) is given by the expression for OAM: \( h (m_o + n_o) \sin \phi \). With the transverse orders \( m_o \) and \( n_o \) in \( x \)- and \( y \)-directions to be the integers closest to the values of \( \left[ \Delta x / w(z_c) \right]^2 \) and \( \left[ \Delta y / w(z_c) \right]^2 \), the parameters \( m_o \) and \( n_o \) can be shown as 124 and 113 for \( (P,Q) = (1,3) \) in Fig. 2 with \( \Delta x = 0.47 \) mm, \( \Delta y = 0.45 \) mm and \( z_c = -2 \) mm. With the same parameter \( z_c \), the transverse orders can be verified to be \( m_o = 103 \) and \( n_o = 359 \) for \( (P,Q) = (1,5) \) in Fig. 3 with \( \Delta x = 0.43 \) mm and \( \Delta y = 0.80 \) mm. The parameter \( \phi \) is provided with \( \pi / 5 \) due to the ellipticity of the lasing modes in Fig. 2 and Fig. 3. Therefore, the average OAM can be calculated as 139.3 \( h \) and 255.1 \( h \) for \( (P,Q) = (1,3) \) in Fig. 2 and \( (P,Q) = (1,5) \) in Fig. 3, respectively, which indicate the large fractional OAM in such beams. However, the integer \( N \) related to the experimental observations to be 237 for \( (P,Q) = (1,3) \) and 462 for \( (P,Q) = (1,5) \) is so high that it is difficult to be analyzed by numerical calculations. Figure 6 just reveals the numerical transverse patterns to illustrate the contours of the experimental patterns. The upper row in Fig. 6 shows the numerical results for the spatial distribution \( |\Psi_{M,N}(x,y,z;\phi)|^2 \) at different seven longitudinal positions inside the concave-flat resonator for the case of \( (P,Q) = (1,3) \) with \( N =\)
30, $M = 20$ and $\phi = \pi / 5$. It can be seen that the variation of the transverse patterns on the longitudinal positions agrees very well with the experimental results shown in Fig. 2. The lower row in Fig. 6 depicts numerical transverse patterns for $(P,Q) = (1,5)$ with the same parameters used in the first row of Fig. 6. It can be found that the numerical calculation is also in good agreement with the experimental data in Fig. 3.

![Numerical transverse patterns of spatial distribution](image)

In the past, the vortex structures of low-order circular geometric modes can be manifested in both experiment and theory, and the interference patterns are completely consistent with the phase structures [37]. Based on the systematically theoretical analysis for exploring the phase field of circular geometric modes, we can further investigate the intricately optical vortex beams with radial asymmetry. To verify the vortex structures of nonplanar elliptical modes, numerical results of the phase angle field are calculated by

$$\Theta(x,y) = \arctan \left( \frac{\text{Im}[\Psi_{M,N}(x,y,z;\phi)]}{\text{Re}[\Psi_{M,N}(x,y,z;\phi)]} \right)$$

depicted in Figs. 7(e)-7(h), which are corresponding to the numerical patterns for various $\phi$ shown in Figs. 7(a)-7(d), respectively, with the parameters of $(P,Q) = (1,3)$, $z = L$, $N = 6$ and $M = 4$. Considering the phase angle field of circular geometric modes for the case of $N > Q$ [37] as seen in Fig. 7(e), it can be found that in addition to the central singularity at the origin with the topological charges of $N$, there are $MQ$ singularities surrounding on the $N$ groups of branches outside. Figure 7(e) depicts topological charge of 6 at the centrally primary singularity, 12 secondary singularities surrounding on the 6 groups of branches outside. Moreover, it can be observed that the central singularity in Fig. 7(e) gradually splits into $N$ primary singularities with topological charge one represented by black dots with controlling the phase factor $\phi$ shown in Fig. 7(e)-7(h). Although the number of secondary singularities surrounding outside is the same as the circular geometric mode, theoretical analysis reveals the distribution of the secondary singularities is somewhat different from circular geometric modes with the varying ellipticity.

Through the analytical function of $\Psi_{M,N}(x,y,z;\phi)$, the angle pattern and the transverse momentum density at different longitudinal positions inside the cavity are illustrated in the second row and the third row of Fig. 8, respectively, corresponding to the numerical patterns.

![Numerical transverse patterns of spatial distribution](image)
showed in the first row of Fig. 8 with \((P,Q) = (1,3)\), \(N = 15\), \(M = 9\) and \(\phi = \pi / 3\). It is clear that the direction of momentum density displays swirling features along an ellipse at \(z = 0\), and the vector field is obviously varied with the twisting phase angle field in the propagation direction. The nonplanar elliptical modes in the present method are hard to be adjusted that give rise to the inevitably high-order modes with the complex phase structures and the large angle of beam divergence shown in the first row of Fig. 2 and Fig. 3. Even though the interference pattern of nonplanar elliptical modes is difficult to be validated in the experiment, the excellent agreement of circular geometric modes between the results in Ref [37] and Fig. 7(e) confirms the above analysis in vortex structures of nonplanar elliptical modes is reliable.

Fig. 7. Numerical calculation of the transverse pattern (upper row) and the phase angle field (lower row) for various \(\phi\) with the parameters of \((P,Q) = (1,3)\),\(z = L, N = 6\) and \(M = 4\).

Fig. 8. Numerical calculation of the transverse pattern (first row), the phase angle field (second row) and the transverse momentum density (third row) at different longitudinal positions inside the cavity for the case of \((P,Q) = (1,3)\) with \(N = 15, M = 9\) and \(\phi = \pi / 3\).
4. Conclusions

In summary, the diode-pumped solid-state laser under the selective pumping has been employed to excite nonplanar elliptical modes with huge fractional OAM near the degenerate cavities. The temporal dynamics for the output emission of nonplanar elliptical modes reveals to obtain the self-mode-locked states traveling along the nonplanar ray orbits inside the cavity. It has been theoretically indicated for manifesting the influence of the radial asymmetry on the vortex structures of nonplanar elliptical modes that the primary singularities can be separated by controlling the relative phase between the degenerate eigenmodes. We also numerically perform that the vector field of transverse lasing modes is displayed swirling features along an ellipse at the beam waist, and remarkably altered with the twisting phase angle field in the propagation direction.

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