Towards the quark–gluon plasma Equation of State with dynamical strange and charm quarks

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Abstract. We present an ongoing project aimed at determining the thermodynamic Equation of State (EoS) of quark–gluon matter from lattice QCD with two generations of dynamical quarks. We employ the Wilson twisted mass implementation for the fermionic fields and the improved Iwasaki gauge action. Relying on $T = 0$ data obtained by the ETM Collaboration the strange and charm quark masses are fixed at their physical values, while the pion mass takes four values in the range from 470 MeV down to 210 MeV. The temperature is varied within a fixed–lattice scale approach. The values for the pseudocritical temperature are obtained from various observables. For the EoS we show preliminary results for the pure gluonic contribution obtained at the pion mass value 370 MeV, where we can compare with previously obtained results with $N_f = 2$ degenerate light flavours.

The Equation of State (EoS) plays a key role in the description of the quark–gluon plasma, serving as important and necessary input for hydrodynamical models describing the space–time evolution of hot QCD matter in relativistic heavy ion collisions. At the moment, lattice QCD is the only known way to non–perturbatively calculate the EoS from first principles and without application of phenomenological models, although this calculation is currently possible only in the limit of an (almost) vanishing baryon chemical potential [1]. Detailed EoS results were presented recently by the Budapest–Wuppertal [2] and HotQCD [3] collaborations, both with $N_f = 2+1$ flavours of staggered fermions. This type of fermions on the lattice is attractive due to the relatively low computational costs, but is known to have certain conceptual problems from the theoretical point of view ("rooting trick“ problem). As an alternative, one can consider a Wilson–type fermionic action, which is theoretically safe but computationally demanding. In this talk we report on the current status of the "twisted mass at finite temperature“ (tmfT) project [4] devoted in particular to the calculation of the EoS with $N_f = 2 + 1 + 1$ flavours of twisted mass Wilson fermions. To our knowledge, this project represents a first study of the Equation of State with almost realistic masses of strange and charm quarks implemented with a Wilson–type fermion discretization. In the works [5] and [6], where the $c$ quark was also taken into account, the staggered approach was used, and in Ref. [7] charm effects were estimated on the basis of a $N_f = 2 + 1$ analysis. The recent progress for the EoS with Wilson fermions includes $N_f = 2$ twisted mass results [8] and $N_f = 2 + 1$ studies with various action improvements [9, 10].

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The Wilson twisted mass fermionic action for light and heavy quark doublets has the following form: 

\[ S_f^{\text{light}}[U, \chi_l, \bar{\chi}_l] = \sum_{x,y} \bar{\chi}_l(x) \left[ \delta_{x,y} - \kappa D_W(x, y)[U] + 2i\kappa a \mu \gamma_5 \delta_{x,y} \tau_3 \right] \chi_l(y), \]

\[ S_f^{\text{heavy}}[U, \chi_h, \bar{\chi}_h] = \sum_{x,y} \bar{\chi}_h(x) \left[ \delta_{x,y} - \kappa D_W(x, y)[U] + 2i\kappa a \mu \gamma \delta_{x,y} \tau_3 \right] \chi_h(y), \]

where \( D_W[U] \) is the usual Wilson operator, \( a \) is the lattice spacing, and \( \chi_{l,h} \) are quark spinors in the twisted basis. The hopping parameter \( \kappa \) is set to its coupling dependent critical value \( \kappa_c(\beta) \) leading to the so-called "maximal twist" of the action \([14,12]\) with the property of automatic improvement for expectation values of any operator \([12]\). The parameter \( \mu_l \) describes the mass of the degenerate light quark doublet, which is still unphysically large in our study: the charged pion mass values \( m_{\pi^\pm} \) considered at present are 210, 260, 370 and 470 MeV. The heavy twisted mass parameters \( \mu_h \) and \( \mu_\delta \) have been tuned in the unitary approach to reproduce approximately the physical \( K \) and \( D \) meson mass values within the accuracy of 10%, thus allowing for a realistic treatment of \( s \) and \( c \) quarks. Three lattice spacing values \( a \) are available for the bare gauge coupling parameter \( \beta = 6/g^2 = 1.90, 1.95 \) and 2.10 corresponding to \( a = 0.094, 0.082 \) and 0.065 fm, respectively \([13]\).

The temperature \( T = 1/(aN_\tau) \) is varied by changing \( N_\tau = 3, 4, \ldots, 24 \), the latter being the lattice extent in the fourth Euclidean direction (fixed–scale approach \([9]\)). The linear spatial extent varies between \( N_\tau = 24 \) and 48, such that the bound of the aspect ratio \( N_\tau/N_\tau \geq 2 \) is guaranteed as the worst case. In comparison with the more often used fixed–\( N_\tau \) approach, such a choice allows to reduce the amount of required \( T = 0 \) computations, necessary to carry out the obligate subtractions in computing the EoS. Fortunately, we can rely on the mass parameters and on the scale setting by the European Twisted Mass (ETM) Collaboration (see \([13]\) and references therein).

For determining the pseudocritical temperature(s) of the crossover region we have computed the renormalized Polyakov loop

\[ \langle \text{Re} \, L \rangle_R = \langle \text{Re} \, L \rangle \exp[V(r_0)/2T], \]

where \( V(r_0) \) is static quark potential at the Sommer scale \( r_0 \simeq 0.5 \) fm and the renormalized subtracted chiral condensate

\[ \Delta_{l,s} = \frac{\langle \bar{\psi}_l \psi_l \rangle_l - \mu_l \langle \bar{\psi}_l \psi_l \rangle_s}{\langle \bar{\psi}_l \psi_l \rangle_{T=0} - \mu_l \langle \bar{\psi}_l \psi_l \rangle_{T=0}}, \]

where \( \langle \bar{\psi}_l \psi_l \rangle_l \) and \( \langle \bar{\psi}_l \psi_l \rangle_s \) are the light and strange quark condensates, respectively. The latter has been obtained in the Osterwalder–Seiler setup \([14,12]\), which avoids mixing in the heavy quark sector. The mass \( \mu_s \) has been determined as to reproduce the physical \( \bar{s} \gamma_\mu s \)–mass. Moreover, we have determined the unrenormalized disconnected chiral susceptibility

\[ \sigma^2_{\bar{\psi}_l \psi_l} = \frac{V}{T} (\langle (\bar{\psi}_l \psi_l)^2 \rangle_l - \langle \bar{\psi}_l \psi_l \rangle_l^2), \quad V = a^4 N_\sigma^3 N_\tau. \]

The inflexion points of \( \langle \text{Re} \, L \rangle_R \) and \( \Delta_{l,s} \) as well as the maxima of \( \sigma^2_{\bar{\psi}_l \psi_l} \) versus \( T \) determine the crossover temperatures \( T_L, T_\Delta \) and \( T_X \), respectively, as shown in Table \([1]\). In Figure \([\dagger]\) as an example, we show the behaviour of \( \Delta_{l,s} \) for the pion mass values \( m_{\pi^\pm} = 370 \) and 210 MeV as well as of \( \sigma^2_{\bar{\psi}_l \psi_l} \) for \( m_{\pi^\pm} = 370 \) MeV (the latter in comparison with the case \( N_f = 2 \) \([8]\)).
Table 1. Crossover temperatures for different pion mass values \( m_{\pi}\). \( N_f = 2 \) results are taken from Ref. [8] (all numbers in units of MeV).

| Number of flavours | \( m_{\pi} \) | \( T_\chi \) | \( T_\Delta \) | \( T_L \) |
|-------------------|-----------|--------|--------|-------|
| \( N_f = 2 + 1 + 1 \) | 210       | 152(5) | 164(3) |       |
|                   | 260       | 170(5) |         |       |
|                   | 370       | 184(4) | 192(2) | 201(3) |
|                   | 470       | 199(6) |         |       |
| \( N_f = 2 \)     | 360       | 193(13)|         | 219(3)(14) |
|                   | 430       | 208(14)|         | 225(3)(14) |

Figure 1. \( \Delta_{l,s} \) (left) and \( \sigma^2_{\psi\psi} \) (right) versus \( T \).

The trace anomaly or interaction measure, from which the pressure \( p \) as well as the energy density \( \epsilon \) can be finally extracted as a function of \( T \), is given by the logarithmic derivative of the lattice partition function \( Z \)

\[
I(T) = \epsilon - 3p, \quad I(T) = \frac{1}{T^4} \left( \frac{d \ln Z}{d \ln a} \right)_\text{sub} \tag{5}
\]

where the subtracted expectation values are defined as \( \langle \ldots \rangle_\text{sub} \equiv \langle \ldots \rangle_T - \langle \ldots \rangle_{T=0} \). This leads to the expression

\[
\frac{I(T)}{T^4} = \frac{1}{T^3V} \sum_i \frac{db_i}{da} \left( \frac{\partial S}{\partial b_i} \right)_\text{sub} = N_f \left\{ \left( -a \frac{d\beta}{da} \right) \left( c_0 \sum_P \text{Re Tr} U_P \right)_\text{sub} + \frac{c_1}{3} \sum_R \text{Re Tr} U_R \text{sub} 
\right. \\
+ \frac{\partial \kappa_c}{\partial \beta} \langle \chi D \bar{W} | \chi \rangle_\text{sub} - 2am \frac{\partial \kappa_c}{\partial \beta} \langle \chi i \gamma_5 \tau_3 \chi \rangle_\text{sub} \\
+ 2\kappa_c \left( a \frac{d(a\mu)}{da} \right) \langle \chi i \gamma_5 \tau_3 \chi \rangle_\text{sub} + \text{heavy terms} \right\},
\]

\[
I(T) = I_{\text{gauge}} + I_{\text{light}} + I_{\text{heavy}} \tag{6}
\]

with the renormalization functions \( \frac{db_i}{da} \) to be fixed from ETM Collaboration \( T = 0 \) data. Preliminary results for \( I/T^4 \) are presented in Figure 2, where the pure gauge field (tree–level corrected) contribution as the most dominant one is shown for the case \( m_{\pi} = 370 \text{ MeV} \) and the three lattice spacings. We compare with the full \( N_f = 2 \) result at approximately the same pion mass [8] as well as with the fitted results for \( N_f = 2 + 1 \) dynamical staggered fermion
flavour degrees of freedom close to the physical point [3]. The comparison demonstrates that our \(N_f = 2 + 1 + 1\) results turned out to be in the right ball park.

The computation of the light and heavy quark contributions — being most interesting at higher temperatures — is under way. Their magnitude turns out to be rather sensitive with respect to the mass renormalization functions to be determined from the ETM Collaboration analysis of the hadronic spectrum at \(T = 0\).

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