ProjectManagement: an R package for managing projects

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Outline

1. Project Management
   - Deterministic Projects
   - Resource Management
   - Delay Costs Allocation

2. Stochastic Projects
   - Stochastic Project Management
   - Delay Costs Allocation
Formally, a project is a 3-tuple \((N, \prec, x^0)\) where:

- \(N\) is the finite set of activities.
- \(\prec\) is a binary relation over \(N\) satisfying asymmetry and transitivity. For every \(i, j \in N\), we interpret \(i \prec j\) as "activity \(j\) cannot start until activity \(i\) has finished".
- \(x^0 \in \mathbb{R}^N\) is the vector of estimated durations. For every \(i \in N\), \(x_i^0\) is a non-negative real number indicating the estimated duration of activity \(i\).
### An Example

| $N$ | 1  | 2  | 3  | 4  | 5  |
|-----|----|----|----|----|----|
| Immediate precedence | -  | -  | -  | 2  | 3  |
| Durations            | 2  | 1.5| 1  | 1.5| 2  |

| $N$ | 6  | 7  | 8  | 9  | 10 |
|-----|----|----|----|----|----|
| Immediate precedence | 3  | 1, 4| 2  | 5, 8| 6  |
| Durations            | 2.5| 3  | 4  | 2  | 5  |

**Table:** Example of a deterministic project.
An Example

Example

```r
> prec<-matrix(0,nrow=10,ncol=10)
> prec[1,7]<-1; prec[2,4]<-1; prec[2,8]<-1;
> prec[3,5]<-1; prec[3,6]<-1; prec[4,7]<-1;
> prec[5,9]<-1; prec[6,10]<-1; prec[8,9]<-1;
```

Example

```r
> dag.plot(prec)
```
An Example

Example

```r
> prec<-matrix(0,nrow=10,ncol=10)
> prec[1,7]<-1; prec[2,4]<-1; prec[2,8]<-1;
> prec[3,5]<-1; prec[3,6]<-1; prec[4,7]<-1;
> prec[5,9]<-1; prec[6,10]<-1; prec[8,9]<-1;

Example

> dag.plot(prec)
```
An Example

Figure: AON graph of the project.
An Example

Example

> duration<-c(2,1.5,1,1.5,2,2.5,3,4,2,5)

Example

> schedule.pert(duration,prec)
An Example

Example

> duration<-c(2,1.5,1,1.5,2,2.5,3,4,2,5)

Example

> schedule.pert(duration,prec)
An Example

Example

‘Total duration of the project’

[1] 8.5

|               | Duration | Earliest start | Latest start | Earliest completion |
|---------------|----------|----------------|--------------|--------------------|
| 1             | 2.0      | 0.0            | 3.5          | 2.0                |
| 2             | 1.5      | 0.0            | 1.0          | 1.5                |
| 3             | 1.0      | 0.0            | 0.0          | 1.0                |
| 4             | 1.5      | 1.5            | 4.0          | 3.0                |
| 5             | 2.0      | 1.0            | 4.5          | 3.0                |
| 6             | 2.5      | 1.0            | 1.0          | 3.5                |
| 7             | 3.0      | 3.0            | 5.5          | 6.0                |
| 8             | 4.0      | 1.5            | 2.5          | 5.5                |
| 9             | 2.0      | 5.5            | 6.5          | 7.5                |
| 10            | 5.0      | 3.5            | 3.5          | 8.5                |
## An Example

| Latest completion | Slack  | Free Slack | Independent Slack |
|-------------------|--------|------------|-------------------|
| 1                 | 5.5    | 3.5        | 1.0               | 0.0 |
| 2                 | 2.5    | 1.0        | 0.0               | 0.0 |
| 3                 | 1.0    | 0.0        | 0.0               | 0.0 |
| 4                 | 5.5    | 2.5        | 0.0               | 0.0 |
| 5                 | 6.5    | 3.5        | 2.5               | 0.0 |
| 6                 | 3.5    | 0.0        | 0.0               | 0.0 |
| 7                 | 8.5    | 2.5        | 2.5               | 0.0 |
| 8                 | 6.5    | 1.0        | 0.0               | 0.0 |
| 9                 | 8.5    | 1.0        | 1.0               | 0.0 |
| 10                | 8.5    | 0.0        | 0.0               | 0.0 |
An Example

Figure: AON graph of the project. Nodes in red indicate critical activities.
Resource Management

- The minimal cost expediting considers that the duration of some activities can be reduced by increasing the resources allocated to them and thus the implementation costs.
- Levelling of resources: execute the project in its minimum duration time whilst the use of resources is as uniform as possible over time.
- Allocation of resources: the level of resources available in each period of time is limited.
Resource Management

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- Levelling of resources: execute the project in its minimum duration time whilst the use of resources is as uniform as possible over time.
- Allocation of resources: the level of resources available in each period of time is limited.
An Example

Example

```r
> minimum.durations<-c(1,1,0.5,1,1,2,2,3,1,3)
> activities.costs<-c(1,2,1,1,3,2,1,2,3,5)
```

Example

```r
> mce(duration,minimum.durations,prec,
      activities.costs,duration.project=NULL)
```
An Example

**Example**

```r
> minimum.durations<-c(1,1,0.5,1,1,2,2,3,1,3)
> activities.costs<-c(1,2,1,1,3,2,1,2,3,5)
```

**Example**

```r
> mce(duration,minimum.durations,prec,
activities.costs,duration.project=NULL)
```
An Example

Example

necessary negative increase
1: 0.5
Read 1 item
Project duration =
[1] 8.0 7.5 7.0 6.5 6.0 5.5
An Example

Example

necessary negative increase
1: 0.5
Read 1 item
Project duration =
[1] 8.0 7.5 7.0 6.5 6.0 5.5
## An Example

### Example

**Estimated durations =**

|   | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
|---|-----|-----|-----|-----|-----|-----|
| 2.0 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 2.5 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 |
| 4.0 | 4.0 | 3.5 | 3.0 | 3.0 | 3.0 | 3.0 |
| 5.0 | 5.0 | 4.5 | 4.0 | 3.5 | 3.0 | 3.0 |

**Costs per solution =**

|   | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
|---|-----|-----|-----|-----|-----|-----|
| 2.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
An Example

Example

```r
> resources<-c(2,3,4,3,3,4,2,2,5,2)
```

Example

```r
> levelling.resources(duration,prec,resources,int=0.5)
Earliest start times =
[1] 0.0 0.5 0.0 2.0 4.5 1.0 3.5 2.5 6.5 3.5
Resources by period=
[1] 6 9 9 9 7 9 9 6 6 9 9 9 7 7 7 7
```
An Example

Example

> resources<-c(2,3,4,3,4,2,2,5,2)

Example

> levelling.resources(duration,prec,resources,int=0.5)
Earliest start times =
[1] 0.0 0.5 0.0 2.0 4.5 1.0 3.5 2.5 6.5 3.5
Resources by period=
[1] 6 9 9 9 7 9 6 6 9 9 9 7 7 7 7
An Example

Figure: Levelling resources.
An Example

Example

```r
> max.resources <- 8
```

Example

```r
> resource.allocation(duration, prec, resources, max.resources, int = 0.5)
```

Project duration =
```
[1] 10
```

Earliest start times =
```
[1] 1.5 0.0 0.0 3.5 5.5 1.0 5.0 1.5 8.0 3.5
```

Resources by period =
```
[1] 7 7 7 8 8 8 7 7 7 4 7 5 5 5
```
An Example

Example

> max.resources<-8

Example

> resource.allocation(duration,prec,resources, max.resources,int=0.5)
Project duration =
[1] 10
Earliest start times =
[1] 1.5 0.0 0.0 3.5 5.5 1.0 5.0 1.5 8.0 3.5
Resources by period =
[1] 7 7 7 8 8 8 8 7 7 7 4 7 5 5 5
A project with delays

A project with delays \( P \) is a tuple \( (N, \prec, x^0, x, C) \) where:

- \( N \) is the finite set of activities.
- \( \prec \) is a binary relation over \( N \) satisfying asymmetry and transitivity.
- \( x^0 \in \mathbb{R}^N \) is the vector of estimated durations. For every \( i \in N \), \( x_i^0 \) is a non-negative real number indicating the estimated duration of activity \( i \).
- \( x \in \mathbb{R}^N \) is the vector of actual durations. For every \( i \in N \), \( x_i \geq x_i^0 \) indicates the actual duration of activity \( i \).
- \( C : \mathbb{R} \rightarrow \mathbb{R} \) is the delay cost function. We assume that \( C \) is non-decreasing and that \( C(D(N, \prec, x^0)) = 0 \).
The Proportional rule for projects with delays is defined by

\[ \varphi_i(P) = \frac{x_i - x_i^0}{\sum_{j \in N} x_i - x_i^0} C(D(N, \prec, x)) \]

for all \( i \in N \).

The Shapley rule for projects with delays \( Sh \) is defined by

\[ Sh(P) = \Phi(c^P) \]

- \( c^P \) is the TU-game with set of players \( N \) given by
  \[ c^P(S) = C(D(N, \prec, (x_S, x^0_{N \setminus S}))) \], for all \( S \subset N \), and
- \( \Phi(c^P) \) denotes the proposal of the Shapley value for \( c^P \).
The Proportional rule for projects with delays is defined by

\[ \varphi_i(P) = \frac{x_i - x^0_i}{\sum_{j \in N} x_j - x^0_j} C(D(N, \prec, x)) \]

for all \( i \in N \).

The Shapley rule for projects with delays \( Sh \) is defined by

\[ Sh(P) = \Phi(c^P) \]

- \( c^P \) is the TU-game with set of players \( N \) given by
  \[ c^P(S) = C(D(N, \prec, (x_S, x^0_{N \setminus S}))) \]
  for all \( S \subset N \), and
- \( \Phi(c^P) \) denotes the proposal of the Shapley value for \( c^P \).
Example

```r
> observed.duration<-c(8,3,2,5,2,6,4,6,4,5.5)
> cost.function<-function(x)return(max(x-8.5,0))
```

Example

```r
> delay.pert(duration,prec,observed.duration,delta=NULL,
  cost.function)
There has been a delay of = 5
```
An Example

Example

```r
> observed.duration<-c(8,3,2,5,2,6,4,6,4,5.5)
> cost.function<-function(x)return(max(x-8.5,0))
```

Example

```r
> delay.pert(duration,prec,observed.duration,delta=NULL, cost.function)
There has been a delay of = 5
```
An Example

|              | 1   | 2   | 3   | 4   | 5   |
|--------------|-----|-----|-----|-----|-----|
| The proportional payment | 1.43 | 0.36 | 0.24 | 0.83 | 0.00 |
| The truncated proportional payment | 1.25 | 0.38 | 0.25 | 0.88 | 0.00 |
| Shapley rule | 0.71 | 0.40 | 0.55 | 0.26 | 1.68 |

|              | 6   | 7   | 8   | 9   | 10  |
|--------------|-----|-----|-----|-----|-----|
| The proportional payment | 0.83 | 0.24 | 0.48 | 0.48 | 0.12 |
| The truncated proportional payment | 0.88 | 0.25 | 0.5   | 0.5   | 0.13   |
| Shapley rule | 1.68 | 0.19 | 0.45 | 0.45 | 0.32 |
A Stochastic Project

Formally, a stochastic project is a 3-tuple \((N, \prec, X^0)\) where:

- \(N\) is the finite set of activities.
- \(\prec\) is a binary relation over \(N\) satisfying asymmetry and transitivity. For every \(i, j \in N\), we interpret \(i \prec j\) as "activity \(j\) cannot start until activity \(i\) has finished".
- \(X^0 \in \mathbb{R}^N\) is the vector of random durations. For every \(i \in N\), \(X^i_0\) is a non-negative random variable describing the duration of activity \(i\).
An Example

Example

\[ X^0 = \left( t(1, 2, 3), \exp(2/3), t(1/2, 5/4, 5/4), t(1/4, 7/4, 5/2), t(1, 2, 3), \\
    t(1, 3/2, 5), t(1, 1, 7), t(3, 4, 5), t(1/2, 5/2, 3), t(1, 6, 8) \right), \]

where \( t(a, b, c) \) denotes the triangular distribution with parameters \( a, b, \) and \( c, \) and \( \exp(\alpha) \) denotes the exponential distribution with parameter \( \alpha. \)
An Example

Example

```r
> distribution<-c("TRIANGLE","EXPONENTIAL",rep("TRIANGLE",8))
> values<-
matrix(c(1,3,2,2/3,0,0,1/2,5/4,5/4,1/4,5/2,7/4,1,3,2,1,5,3/2,1,7,1,3,5,4,1/2,3,5/2,1,8,6),nrow=10,ncol=3,byrow=T)

>stochastic.pert(prec,distribution,values,
percentile=0.95,plot.activities.times=c(7))

Average time of the project = 9.070575
Percentile duration of the project = 11.66658
Criticality index by activity 1.3 34.2 64.5 8.2 0 64.5 9.5 26 26 64.5
```
An Example

Example

```r
> distribution<-c("TRIANGLE","EXPONENTIAL",rep("TRIANGLE",8))
> values<-matrix(c(1,3,2,2/3,0,0,1/2,5/4,5/4,1/4,5/2,7/4,1,3,2,1,5,3/2,1,7,1,3,5,4,
1/2,3,5/2,1,8,6),nrow=10,ncol=3,byrow=T)

>stochastic.pert(prec,distribution,values,
percentile=0.95,plot.activities.times=c(7))

Average time of the project = 9.070575
Percentile duration of the project = 11.66658
Criticality index by activity 1.3 34.2 64.5 8.2 0 64.5 9.5 26 26 64.5
```
An Example

Figure: Density estimation of project duration time and earliest start and latest completion times for activities 7.
A stochastic project with delays $SP$ is a tuple $(N, \prec, X^0, x, C)$ where:

- $N$ is the finite set of activities.
- $\prec$ is a binary relation over $N$ satisfying asymmetry and transitivity.
- $X^0 \in \mathbb{R}^N$ is the vector random durations. For every $i \in N$, $x_i^0$ is a non-negative random variable describing the duration of activity $i$.
- $x \in \mathbb{R}^N$ is the vector of actual durations. For every $i \in N$, $x_i \geq x_i^0$ indicates the actual duration of activity $i$.
- $C : \mathbb{R} \to \mathbb{R}$ is the delay cost function. We assume that $C$ is non-decreasing and that $C(D(N, \prec, 0)) = 0$. 


The Proportional rule for projects with delays is defined by

\[
\varphi_i(SP) = \frac{x_i - E(X_i^0)}{\sum_{j \in N} x_i - E(X_j^0)} C(D(N, \prec, x))
\]

for all \( i \in N \).

The Shapley rule for projects with delays \( Sh \) is defined by

\[
Sh(SP) = \Phi(c^{SP})
\]

where \( c^{SP} \) is the TU-game with set of players \( N \) given by

\[
c^{SP}(S) = E(C(D(N, \prec, (x_S, X_N^0 \setminus S))))
\]

for all \( S \subset N \), and

\( \Phi(c^{SP}) \) denotes the proposal of the Shapley value for \( c^{SP} \).
The Proportional rule for projects with delays is defined by

\[ \varphi_i(SP) = \frac{x_i - \mathbb{E}(X^0_i)}{\sum_{j \in N} x_i - \mathbb{E}(X^0_j)} C(D(N, \prec, x)) \]

for all \( i \in N \).

The Shapley rule for projects with delays \( Sh \) is defined by

\[ Sh(SP) = \Phi(c^{SP}) \]

- \( c^{SP} \) is the TU-game with set of players \( N \) given by
  \[ c^{SP}(S) = E(C(D(N, \prec, (x_S, X^0_{N\setminus S})))) \]
  for all \( S \subset N \), and
- \( \Phi(c^{SP}) \) denotes the proposal of the Shapley value for \( c^{SP} \).
An Example

Example

> delay.stochastic.pert(prec, distribution, values, observed.duration, percentile=NULL, delta=NULL, cost.function, compilations=1000)

Total delay of the stochastic project = 5
## Example

|               | 1   | 2   | 3   | 4   | 5   |
|---------------|-----|-----|-----|-----|-----|
| Proportional rule | 1.43 | 0.36 | 0.24 | 0.84 | 0.00 |
| Truncated proportional rule | 1.25 | 0.38 | 0.25 | 0.88 | 0.00 |
| Shapley rule   | 0.59 | 0.37 | 0.61 | 0.36 | 0.17 |
| Shapley rule 2 | 0.49 | 0.42 | 0.64 | 0.29 | 0.07 |

|               | 6   | 7   | 8   | 9   | 10  |
|---------------|-----|-----|-----|-----|-----|
| Proportional rule | 0.83 | 0.24 | 0.48 | 0.48 | 0.12 |
| Truncated proportional rule | 0.88 | 0.25 | 0.5  | 0.5  | 0.13 |
| Shapley rule   | 1.36 | 0.24 | 0.51 | 0.50 | 0.31 |
| Shapley rule 2 | 1.52 | 0.18 | 0.48 | 0.46 | 0.47 |
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