Mathematical Modelling of Vortex Dust Separator

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Abstract. The problem of the motion of a swirling flow in an axisymmetric channel is investigated numerically. The flow field calculation results have been obtained as the solutions of the Navier-Stokes equations. Various flow regimes with the formation of axial recirculation zones are presented. The convection-diffusion model is used for the determination of the flow particle concentration and the formation of typical sedimentation zones.

1. Introduction
Swirling flows are widely used in various technical applications and are observed in natural phenomena. They have been the object of a great deal of theoretical and experimental study. The most important properties of swirling flows are described in [1, 2]. Swirling flows in channels were investigated numerically in [3–5], in an unbounded medium in [6, 7] and experimentally in [8-10]. The stability of the swirling axisymmetric flows is considered in [11-14].

Figure 1. Schematic of the problem.

Direct-flow cyclone dust-separating devices are used in various technological processes, for example, in air purification plants. The principle of operation of the dust separator is as follows. Dusty gas through tangential swirler 1 enters to the pre-separation chamber 2 (figure 1). In this part, coarse dust is separated and then it is ejected through tangentially located exits 3. Fine dust moves further with the gas in the radial direction to the axis of the vortex chamber 5. Due to the action of centrifugal forces the dust was separated and removed through a series of annular slots 6 in the lateral surface of the channel.
Experiments in a vortex dust separation device were carried out in [15]. The purpose of this work is to develop a mathematical model and investigate deposition particles process under the action of a swirling flow.

2. Formulation of the problem and numerical procedure

The present analysis is based upon the numerical solution of the full Navier-Stokes equations for laminar axisymmetric viscous flow. In the cylindrical coordinate system \( r, \varphi, z \), the Navier-Stokes equation can be represented in terms of the stream function \( \psi \), the vorticity \( \Omega \) and azimuthal velocity \( V_\varphi \) in form

\[
\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) = -\Omega, \quad \text{(1)}
\]

\[
\frac{\partial \Omega}{\partial t} + \frac{\partial}{\partial z} (V_\varphi \Omega) + \frac{\partial}{\partial r} (V_r \Omega) = \frac{1}{Re} \left[ \frac{\partial^2 \Omega}{\partial z^2} + \frac{\partial^2 \Omega}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{\Omega}{r} \right) \right] + G^2 \frac{1}{r} \frac{\partial (V_\varphi)^2}{\partial z}, \quad \text{(2)}
\]

\[
\frac{\partial V_\varphi}{\partial t} + \frac{\partial}{\partial z} (V_r V_\varphi) + \frac{1}{r} \frac{\partial}{\partial r} \left( r V_r V_\varphi \right) + \frac{V_r V_\varphi}{r} = \frac{1}{Re} \left[ \frac{\partial^2 V_\varphi}{\partial z^2} + \frac{\partial^2 V_\varphi}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r V_\varphi}{r^2} \right) \right], \quad \text{(3)}
\]

\[
V_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad V_\varphi = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad \Omega = \frac{\partial V_r}{\partial z} - \frac{\partial V_\varphi}{\partial r}. \quad \text{(4)}
\]

The system of equations (1)-(4) is written in the conservative form and contains two dimensionless parameters: the Reynolds number \( Re = UR/\nu \) and the swirl ratio \( G = W_0/U \), where \( \nu \) is the kinematic viscosity. Here the radius \( R \) is taken as the characteristic length, the axial and radial velocities are related to a given axial velocity \( U \) at the channel inlet, and the azimuthal velocity is related to its maximal value \( W_0 \) at the channel inlet.

The flow is considered in the cylindrical domain \( D \) \( (0 \leq z \leq z_k, \ 0 \leq r \leq 1) \). The velocity distribution in the initial section \( z = 0 \) was set as follows:

\[
V_z(0) = 1, \quad V_r(0) = 0, \quad V_\varphi(0) = \frac{b_1}{r}(1-\exp(-b_2 r^2)), \quad 0 \leq r \leq 1, \quad z = 0, \quad (5)
\]

where \( b_1 = 0.554, \ b_2 = 8 \). The boundary conditions include the specification of the uniform profile for the axial velocity \( V_z \) as well as the formulation of no-slip conditions on the rigid surfaces of the channel and the formulation of the symmetry conditions on the axis \( r = 0 \). In addition, soft boundary conditions should be given at the outlet section \( z = z_k \). Thus, the set of boundary conditions can be written down as

\[
\psi = \frac{r^2}{2}, \quad V_\varphi = V_\varphi(r), \quad \frac{\partial \psi}{\partial z} = 0, \quad 0 \leq r \leq 1, \quad z = 0, \quad (6)
\]

\[
\psi = \frac{1}{2}, \quad V_\varphi = 0, \quad \frac{\partial \psi}{\partial r} = 0, \quad 0 \leq z \leq z_k, \quad r = 1, \quad (7)
\]

\[
\psi = 0, \quad V_\varphi = 0, \quad \Omega = 0, \quad 0 \leq z \leq z_k, \quad r = 0, \quad (8)
\]
\[ \frac{\partial \psi}{\partial z} = \frac{\partial \Omega}{\partial z} = \frac{\partial V_{\phi}}{\partial z} = 0, \quad 0 \leq r \leq 1, \quad z = z_k. \]  

(9)

The finite-difference relaxation method was used to solve boundary value problem (1)-(4) with boundary conditions (5)-(9); this method was successfully applied in [16] for the analysis of swirling flows of various types. We used a uniformly spaced grid 81x256 with the grid spacing \( z_k = 10 \). The time step \( \delta t \) was taken from the range between 0.05 and 0.2. The flow fields corresponding to the boundary-value problem (1)-(9) were calculated for \( Re = 1000-2500 \) and \( G = 3.6 \). This swirl ratio was taken for comparison with experiment [15].

3. Convection-diffusion model

Among all the forces acting on a particle in vortex dust separator the Stokes viscous drag force is the most significant. In cylindrical coordinates the equations of motion of the particles under the action of this force have the dimensionless form:

\[
\begin{align*}
\frac{dV_{zs}}{dt} & = \frac{1}{2St}(V_z - V_{zs}), \\
\frac{dV_{rs}}{dt} & = G^2 \frac{V_{\phi s}^2}{r} + \frac{1}{2St}(V_r - V_{rs}), \\
\frac{dV_{qs}}{dt} & = -\frac{V_{rs}V_{qs}}{r} + \frac{1}{2St}(V_\phi - V_{q\phi}).
\end{align*}
\]

(10)

(11)

(12)

Here the parameter \( St = (\rho_s r_s^2 U)/(9\nu_p R) \) is the Stokes number and the subscript \( s \) indicates the variables associated with particles. For \( St \ll 1 \) from (10) and (12) it follows that the axial and azimuthal particle velocities \( V_{zs} \) and \( V_{q\phi} \) coincide with the corresponding velocities of the basic flow. The radial particle velocity \( V_{rs} \) can be determined from (11). Under the conditions \( St << 1 \) and \( G^2 St = O(1) \) from this equation we obtain

\[
V_{rs} - V_r = 2St G^2 \frac{V_{q\phi}^2}{r}.
\]

(13)

As the flow in question includes the recirculation zone and significantly inhomogeneous velocity and concentration fields, we also investigated two approaches using averaging with respect to \( r \) for calculating \( V_{rs} \). In the first approach the deposition rate was determined as a function of \( V_{rs}(z) \) by averaging (13) with respect to \( r \) in the form

\[ V_s(z) = 2St G^2 \frac{\langle V_{\phi s}^2 / r \rangle + \langle V_r \rangle}{\langle V_r \rangle}. \]

(14)

In the second approach the effective deposition rate \( V_{rs}^* \) was introduced by means of the following relation

\[ \frac{1}{r} \int_0^r V_{rs} c dr = V_{rs}^* \int_0^r c dr. \]

(15)
Here, on the left side we have the flux of \( V_{rs} c_r \) averaged over the interval \( [0, r] \) and on the right an effective flux calculated on the basis of the velocity \( V_{rs}^* = V_{rs}^*(z, t) \) and the total concentration of the particles on the interval \( [0, r] \).

We will consider the process of particle transfer for the flows calculated. Following the method based on the convection-diffusion model for gas mixtures with fine low-inertia particles, we can neglect the inverse effect of the particles on the fluid flow. This passive admixture approximation will be employed in the present paper. The velocity field can be found on the basis of (10)-(12). The particle-mass conservation equation can be transformed into the diffusion equation for a passive scalar

\[
\frac{\partial c}{\partial t} + \frac{\partial}{\partial z}(V_{z} c) + \frac{1}{r} \frac{\partial}{\partial r}(r V_{rs} c) = \frac{1}{Re \text{Sc}} \left[ \frac{\partial^2 c}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial c}{\partial r} \right) \right].
\]

(16)

Here \( c \) is the concentration of particles, \( \text{Sc} = v/D \) is the Schmidt number, and \( D \) is the diffusion coefficient. For example, in [17] the convection-diffusion equation (16) was used to simulate mixture flows with loss of particle mass due to deposition on the walls.

In order to determine \( V_{rs} \) we will consider three models in which in (16) the deposition rate is determined either as \( V_{rs}^* \) from (14), or as \( V_{rs}^*(z, t) \) from (15), or as \( V_{rs}(z, r, t) \) from the solution of (11) with the conditions \( V_{rs}(z, 0, t) = 0 \).

The boundary conditions for equation (16) can be written down as

\[
z = 0: \quad c = 1, \quad \eta \leq r \leq r_2; \quad c = 0, \quad r < r_1, r > r_2,
\]

\[
z = z_k: \quad \frac{\partial c}{\partial z} = 0, \quad 0 \leq r \leq 1,
\]

\[
r = 0, \quad r = 1: \quad \frac{\partial c}{\partial r} = 0, \quad 0 \leq z \leq z_k,
\]

(17)

(18)

(19)

where \( r_1 = 0.1, \ r_2 = 0.9 \) according to [15]. Here we do not take into account the effects of repulsion of particles during the process of deposition on the lateral surface; instead we assume that the flow of particles \( j = D \partial c / \partial r + V_{rs} c \) is determined only by the convective component without the influence of diffusion \( (\partial c / \partial r = 0) \).

Boundary value problem (16)-(19) are solved by the finite-difference relaxation method used commonly for transport equations (2) and (3). The basic calculations were carried out using approach (14).

4. Results and discussions

Direct-flow vortex dust-separator in which a dusty gas enters a cylindrical channel through a tangential swirler was investigated [15]. Due to the action of centrifugal forces the dust was separated and removed through a series of annular slots in the lateral surface of the channel. The channel length \( L \) was 1 m, the Reynolds number calculated on the basis of the mean-flow-rate velocity and a channel radius of 0.05 m was \( \sim 10^5 \), and the swirl parameter \( G \) varied from 0 to 4. In figure 2 we have reproduced the results of
measuring the axial velocity profiles \( V_z(r) \) for the flow rates \( Q = 0.3, 0.25, \) and \( 0.2 \) kg/s (symbols 5–7). The results show that the profiles are self-similar with respect to the flow rate.

**Figure 2.** Comparison of the computational model with experiment: mass \( \frac{m_b}{m_0} \) for \( r = 1 \) (models 1–3 for \( V_z \) ) (curves 1–3); data of [15] (6); axial velocity \( V_z(r) \) (curve 4); flow rate [15] \( Q = 0.3, 0.25 \) and \( 0.3 \) kg/s (5–7).

On the basis of equations (1)-(4) and using the method proposed, we calculated channel flows. The numerical results for \( \text{Re} = 1000 \) and swirl \( G = 3.6 \), taken from the experiment [15], show that the velocity profiles do not changed in the cross sections. The typical axial velocity profile \( V_z(r) \) is presented in figure 3 for \( z = 2.5 \) (broken curve 4). This distribution is in agreement with experimental data (although the flow in experiments was turbulent).

Particles of quartz dust with diameter \( d_s = 5.5 \) \( \mu \)m were used in experiments. The dimensionless particle radius \( r_s \) scaled to \( R \) for the calculations is defined as follows. If the particle Reynolds number is \( \text{Re}_s < 1 \), where \( \text{Re}_s = \frac{V_r s}{\nu} \), the drag coefficient is written in the form Stokes law \( \frac{c_s}{\text{Re}_s} = \frac{12}{\text{Re}_s} \) and we have

\[
\frac{r_s}{r_s^*} = \frac{\sqrt{\text{Re}_s}}{\text{Re}_s},
\]

in which the quantities marked by asterisks relate to the experiment. In the experiments [15] \( \text{Re}_s \) is varied in the interval \( 1 < \text{Re}_s < 500 \). For this transient laminar-turbulent regime \( c_s = 12/\text{Re}_s^{0.6} \) and we obtain

\[
\frac{r_s}{r_s^*} = \left( \frac{\text{Re}_s}{\text{Re}_s^*} \right)^{3/8}.
\]

Substituting into the expression (21) the above-mentioned values we get \( r_s = 0.2 - 0.3 \cdot 10^{-4} \).

The particle rates \( Q_k \) and \( Q_b \) through the outlet section and lateral pipe surface for \( r = 1 \) are important flow characteristics. These values and the particle mass \( m_f \) in the considered volume are obtained from:

\[
Q_k(t) = 2\pi \int_0^1 r(V_z)\big|_{z=z_k} dr, \quad Q_b(t) = 2\pi \int_0^{z_k} r(V_r)\big|_{r=1} dz, \quad m_f(t) = 2\pi \int_0^{z_k} \int_0^1 c r dr dz.
\]
Integrating the flow rates $Q_k$ and $Q_b$ with respect to time, we can determine the powder mass $m_k$ escaping

$$m_k = \int_0^T Q_k(t) \, dt, \quad m_b = \int_0^T Q_b(t) \, dt$$

through the exit cross-section and the mass $m_b$ remaining on the lateral surface.

The numerical solution results for the boundary value problem (16)-(19), (21)-(23) are presented in figure 2 for $Re = 1000$, $G = 3.6$ and $r_s = 0.2 \cdot 10^{-4}$. Curves (1)-(3) correspond to the particle mass distribution $m_b/m_0$ from $z/L$ deposited on the channel wall and calculated using the three different models for the determination of the velocity $V_z$. Figure 2 shows that the curves 1-3 are practically the same for $z > 0.3$.

5. Conclusions

The mathematical model used makes it possible qualitatively to describe the basic properties of the flow. The comparison of the obtained results with the experiment indicates a good agreement and confirms the correctness of the suggested model to study particle transport and deposition in a vortex dust separator.

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