Observational Tests of FRW World Models

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Abstract. Observational tests for the Cosmological Principle are reviewed. Assuming the FRW metric we then summarize estimates of cosmological parameters from various data sets, in particular the Cosmic Microwave Background and the 2dF galaxy redshift survey. These and other analyses suggest a best-fit \( \Lambda \)-Cold Dark Matter model with \( \Omega_m = 1 - \Omega_\Lambda \approx 0.3 \) and \( H_0 \approx 70 \) km/sec/Mpc. It is remarkable that different measurements converge to this 'concordance model', although it remains to be seen if the two main components of this model, the dark matter and the dark energy, are real entities or just 'epicycles'. We point out some open questions related to this fashionable model.

1. Introduction

The Cosmological Principle (CP) was first adopted when observational cosmology was in its infancy; it was then little more than a conjecture, embodying 'Occam's razor' for the simplest possible model. Observations could not then probe to significant redshifts, the 'dark matter' problem was not well-established and the Cosmic Microwave Background (CMB) and the X-Ray Background (XRB) were still unknown. If the Cosmological Principle turned out to be invalid then the consequences to our understanding of cosmology would be dramatic, for example the conventional way of interpreting the age of the Universe, its geometry and matter content would have to be revised. Therefore it is important to revisit this underlying assumption in the light of new galaxy surveys and measurements of the background radiations.

Like with any other idea about the physical world, we cannot prove a model, but only falsify it. Proving the homogeneity of the Universe is in particular difficult as we observe the Universe from one point in space, and we can only deduce isotropy directly. The practical methodology we adopt is to assume homogeneity and to assess the level of fluctuations relative to the mean, and hence to test for consistency with the underlying hypothesis. If the assumption of homogeneity turns out to be wrong, then there are numerous possibilities for inhomogeneous models, and each of them must be tested against the observations.

Here we examine the degree of smoothness with scale by considering redshift and peculiar velocities surveys, radio-sources, the XRB, the Ly-\( \alpha \) forest, and the CMB. We discuss some inhomogeneous models and show that a fractal model on large scales is highly improbable. Assuming a Friedmann-Robertson-Walker (FRW) metric we evaluate the 'best-fit Universe' by performing a joint analysis of cosmic probes.

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2. The Cosmological Principle(s)

Cosmological Principles were stated over different periods in human history based on philosophical and aesthetic considerations rather than on fundamental physical laws. Rudnicki (1995) summarized some of these principles in modern-day language:

- The Ancient Indian: The Universe is infinite in space and time and is infinitely heterogeneous.
- The Ancient Greek: Our Earth is the natural centre of the Universe.
- The Copernican CP: The Universe as observed from any planet looks much the same.
- The Generalized CP: The Universe is (roughly) homogeneous and isotropic.
- The Perfect CP: The Universe is (roughly) homogeneous in space and time, and is isotropic in space.
- The Anthropic Principle: A human being, as he/she is, can exist only in the Universe as it is.

We note that the Ancient Indian principle can be viewed as a ‘fractal model’. The Perfect CP led to the steady state model, which although more symmetric than the CP, was rejected on observational grounds. The Anthropic Principle is becoming popular again, e.g. in ‘explaining’ a non-zero cosmological constant. Our goal here is to quantify ‘roughly’ in the definition of the generalized CP, and to assess if one may assume safely the FRW metric of space-time.

3. Probes of Smoothness

3.1. The CMB Temperature at High Redshift

The CMB provides the strongest evidence for the CP. Most of the observational tests for the CP really constrain isotropy rather than homogeneity, as they are done from our point in space. However, one can use atoms and molecules at high redshift as ‘hypothetical observers’ that ‘report’ to us the CMB temperature at that redshift. According to the standard Hot Big Bang model, which assumes the FRW metric, the CMB temperature varies with redshift as $T(z) = T(0)(1 + z)$, where $T(0) = 2.726 \pm 0.010$ K. Recent measurements (Srianand et al. 2000) of the relative populations of atomic fine-structure levels, which are excited by the background radiation, find from atoms and molecules in an isolated cloud which happens to be along the line of sight to a quasar, $6 < T(z = 2.34) < 14$ K, in very good agreement with the predicted temperature of 9.1 K. This provides yet another strong evidence for the Hot Big Bang, together with the Hubble recession of galaxies and the Big Bang Nucleosynthesis.

3.2. The CMB Fluctuations

Ehlers, Garen and Sachs (1968) showed that by combining the CMB isotropy with the Copernican principle one can deduce homogeneity. More formally the EGS theorem (based on Liouville theorem) states that “If the fundamental observers in a dust spacetime see an isotropic radiation field, then the spacetime is locally FRW”. The COBE measurements of temperature fluctuations $\Delta T/T = 10^{-5}$ on scales of $10^\circ$ give via the Sachs Wolfe effect ($\Delta T/T = \frac{1}{3}\Delta \phi/c^2$) and Poisson equation rms density
fluctuations of $\frac{\delta \rho}{\rho} \sim 10^{-4}$ on 1000 $h^{-1}$ Mpc (e.g. Wu, Lahav & Rees 1999; see Fig 3 here), i.e. the deviations from a smooth Universe are tiny.

3.3. Galaxy Redshift Surveys

The distribution of galaxies in local redshift surveys (e.g. ORS and IRAS) is highly clumpy, with the Supergalactic Plane seen in full glory. However, deeper surveys such as LCRS and 2dFGRS (Figure 1) show that the fluctuations decline as the length-scales increase. Peebles (1993) has shown that the angular correlation functions for the Lick and APM surveys scale with magnitude as expected in a universe which approaches homogeneity on large scales.

Multifibre technology now allows us to measure redshifts of millions of galaxies. Two major surveys are underway. The US Sloan Digital Sky Survey (SDSS) will measure redshifts to about 1 million galaxies over a quarter of the sky. The Anglo-Australian 2 degree Field Galaxy Redshift Survey (2dFGRS) survey has already measured redshifts for 210,000 galaxies selected from the APM catalogue (as of November 2001). The median redshift of both the SDSS and the 2dFGRS galaxy redshift surveys is $\bar{z} \sim 0.1$. While they can provide interesting estimates of the fluctuations on intermediate scales (e.g. Peacock et al. 2001; Percival et al. 2001; see Fig 2 here), the problems of biasing, evolution and $K$-correction, would limit the ability of SDSS and 2dF to ‘prove’ the Cosmological Principle. (cf. the analysis of the ESO slice by Scaramella et al 1998 and Joyce et al. 1999). But the measurement of the power spectrum of 2dFGRS (Percival et al. 2001) shows good agreement with
what is expected in both shape and amplitude from the \( \Lambda + \) Cold Dark Matter (CDM) model (which assumes an underlying FRW metric).

Assuming a Gaussian prior on the Hubble constant \( h = 0.7 \pm 0.07 \) (based on Freedman et al. 2000) the shape of the recovered power spectrum within \( 0.02 < k < 0.15h/\text{Mpc} \) (Figure 2) was used to yield 68\% confidence limits the shape parameter \( \Omega_m h = 0.20 \pm 0.03 \), and the baryon fraction \( \Omega_b/\Omega_m = 0.15 \pm 0.07 \), in accordance with the popular ‘concordance’ model \( \ddagger \), in particular as the derived from the CMB data (Efstathiou et al. 2001; Lahav et al. 2001).

3.4. Peculiar Velocities

Peculiar velocities are powerful as they probe directly the mass distribution (e.g. Dekel et al. 1999). Unfortunately, as distance measurements increase with distance, the scales probed are smaller than the interesting scale of transition to homogeneity. Conflicting results on both the amplitude and coherence of the flow suggest that peculiar velocities cannot yet set strong constraints on the amplitude of fluctuations on scales of hundreds of Mpc’s. Perhaps the most promising method for the future is the kinematic Sunyaev-Zeldovich effect which allows one to measure the peculiar velocities of clusters out to high redshift.

The agreement between the CMB dipole and the dipole anisotropy of relatively nearby galaxies argues in favour of large scale homogeneity. The IRAS dipole (Strauss et al 1992, Webster et al 1998, Schmoldt et al 1999) shows an apparent convergence of the dipole, with misalignment angle of only 15\°. Schmoldt et al. (1999) claim that 2/3 of the dipole arises from within a 40\,h^{-1}\,Mpc, but again it is difficult to ‘prove’ convergence from catalogues of finite depth.

3.5. Radio Sources

Radio sources in surveys have typical median redshift \( \bar{z} \sim 1 \), and hence are useful probes of clustering at high redshift. Unfortunately, it is difficult to obtain distance information from these surveys: the radio luminosity function is very broad, and it is difficult to measure optical redshifts of distant radio sources. Earlier studies claimed that the distribution of radio sources supports the ‘Cosmological Principle’. However, the wide range in intrinsic luminosities of radio sources would dilute any clustering when projected on the sky. Recent analyses of new deep radio surveys (e.g. FIRST) suggest that radio sources are actually clustered at least as strongly as local optical galaxies (e.g. Cress et al. 1996; Magliocchetti et al. 1998). Nevertheless, on the very large scales the distribution of radio sources seems nearly isotropic. Comparison of the measured dipole, quadrupole and higher harmonics in a radio sample in the Green Bank and Parkes-MIT-NRAO 4.85 GHz surveys to the theoretically predicted ones (Baleisis et al. 1998) offers a crude upper limits of the fluctuations on scales \( \sim 600h^{-1}\,\text{Mpc} \), consistent with the \( \Lambda - \text{CDM} \) model.

3.6. The XRB

Although discovered in 1962, the origin of the X-ray Background (XRB) is still unknown, but is likely to be due to sources at high redshift (for review see Boldt \ddagger As shown in Percival et al. 2001, the likelihood analysis gives a second (non-standard) solution, with \( \Omega_m h \sim 0.6 \), and the baryon fraction \( \Omega_b/\Omega_m = 0.4 \), which generates baryonic ‘wiggles’. The ‘wiggles’ are probably due to ‘noise’ correlated by the survey window function.
Figure 2. The observed (convolved) 2dFGRS power-spectrum (Percival et al. 2001), and a linear theory Λ-CDM (real space, convolved with the window function) with $\Omega_m h = 0.2, \Omega_b/\Omega_m = 0.15, h = 1, n = 1$ and best-fitting (linear) redshift space normalization $\sigma_8 = 0.94$. Only the linear regime $0.02 < k < 0.15 h/$Mpc was used to derive the above parameters (roughly corresponding to CMB harmonics $200 < l < 1500$ for a flat $\Omega_m = 0.3$ universe).

1987; Fabian & Barcons 1992). The XRB sources are probably located at redshift $z < 5$, making them convenient tracers of the mass distribution on scales intermediate between those in the CMB as probed by COBE, and those probed by optical and IRAS redshift surveys.

The interpretation of the results depends somewhat on the nature of the X-ray sources and their evolution. By comparing the predicted multipoles to those observed by HEAO1 (Lahav et al. 1997; Treyer et al. 1998; Scharf et al. 2000) we estimate the amplitude of fluctuations for an assumed shape of the density fluctuations (e.g. CDM models). The observed fluctuations in the XRB are roughly as expected from interpolating between the local galaxy surveys and the COBE and other CMB experiments. The rms fluctuations $\delta\rho/\rho$ on a scale of $\sim 600 h^{-1}$Mpc are less than 0.2%.

3.7. The Lyman-α Forest

The Lyman-α forest reflects the neutral hydrogen distribution and therefore is likely to be a more direct trace of the mass distribution than galaxies are. Unlike galaxy surveys which are limited to the low redshift Universe, the forest spans a large redshift interval, typically $1.8 < z < 4$, corresponding to comoving interval of $\sim 600 h^{-1}$Mpc. Also, observations of the forest are not contaminated by complex selection effects such as those inherent in galaxy surveys. It has been suggested qualitatively by Davis (1997) that the absence of big voids in the distribution of Lyman-α absorbers is inconsistent with the fractal model. Furthermore, all lines-of-sight towards quasars look statistically similar. Nusser & Lahav (2000) predicted the distribution of the flux in Lyman-α observations in a specific truncated fractal-like model. They found that indeed in this model there are too many voids compared with the observations.
and conventional (CDM-like) models for structure formation. This too supports the common view that on large scales the Universe is homogeneous.

3.8. The Isotropy of the Distribution of SN Ia

Another test for isotropy, based on the distribution of 79 Supernovae Ia out to redshift \( z \approx 1 \) is described in Kolatt & Lahav (2001). They divided the sky into two hemispheres that give the most discrepant values of \( \Omega_m \) and \( \Omega_\Lambda \). For a perfect FRW Universe, Monte Carlo realizations that mimic the observed set of SN Ia, yield values higher than the measured discrepancy in about 20% of the case. It would be interesting to repeat this isotropy test with future SN Ia experiments, e.g. SNAP.

4. Is the Universe a Fractal?

The question of whether the Universe is isotropic and homogeneous on large scales can also be phrased in terms of the fractal structure of the Universe. A fractal is a geometric shape that is not homogeneous, yet preserves the property that each part is a reduced-scale version of the whole. If the matter in the Universe were actually distributed like a pure fractal on all scales then the Cosmological Principle would be invalid, and the standard model in trouble. Current data already strongly constrain any non-uniformities in the galaxy distribution (as well as the overall mass distribution) on scales > 300 h\(^{-1}\) Mpc.

If we count, for each galaxy, the number of galaxies within a distance \( R \) from it, and call the average number obtained \( N(< R) \), then the distribution is said to be a fractal of correlation dimension \( D_2 \) if \( N(< R) \propto R^{D_2} \). Of course \( D_2 \) may be 3, in which case the distribution is homogeneous rather than fractal. In the pure fractal model this power law holds for all scales of \( R \).

The fractal proponents (Pietronero et al. 1997) have estimated \( D_2 \approx 2 \) for all scales up to \( \sim 500 h^{-1}\) Mpc, whereas other groups have obtained scale-dependent values (for review see Wu et al. 1999 and references therein).

Estimates of \( D_2 \) from the CMB and the XRB are consistent with \( D_2 = 3 \) to within \( 10^{-4} \) on the very large scales (Peebles 1993; Wu et al. 1999). While we reject the pure fractal model in this review, the performance of CDM-like models of fluctuations on large scales have yet to be tested without assuming homogeneity \textit{a priori}. On scales below, say, 30 h\(^{-1}\) Mpc, the fractal nature of clustering implies that one has to exercise caution when using statistical methods which assume homogeneity (e.g. in deriving cosmological parameters). We emphasize that we only considered one ‘alternative’ here, which is the pure fractal model where \( D_2 \) is a constant on all scales.

5. More Realistic Inhomogeneous Models

As the Universe appears clumpy on small scales it is clear that assuming the Cosmological Principle and the FRW metric is only an approximation, and one has to average carefully the density in Newtonian Cosmology (Buchert & Ehlers 1997). Several models in which the matter in clumpy (e.g. ‘Swiss cheese’ and voids) have been proposed (e.g. Zeldovich 1964; Krasinski 1997; Kantowski 1998; Dyer & Roeder 1973; Holz & Wald 1998; Célerièr 1999; Tomita 2001). For example, if the line-of-sight to a distant object is ‘empty’ it results in a gravitational lensing de-magnification of the object. This modifies the FRW luminosity-distance relation, with a clumping
factor as another free parameter. When applied to a sample of SNe Ia the density parameter of the Universe $\Omega_m$ could be underestimated if FRW is used (Kantowski 1998; Perlmutter et al. 1999). Metcalf & Silk (1999) pointed out that this effect can be used as a test for the nature of the dark matter, i.e. to test if it is smooth or clumpy.

6. Cosmological Parameters from a Joint Analysis

6.1. A Cosmic Harmony?

Different cosmic probes determine different sets of cosmological parameters. Below we give the approximated dependence on the parameters $\Omega_m, \Omega_b, \Omega_\Lambda, h$ and the (linear theory) normalization $\sigma_8$ of the mass fluctuations in $8 \, h^{-1}$ Mpc spheres.

- CMB: $\Omega_\Lambda + \Omega_m, \Omega_m h^2, \Omega_b h^2, \sigma_8$
- SNIa: $3\Omega_\Lambda - 4\Omega_m$
- Redshift surveys: $\Omega_m^{0.6}/b, b\sigma_8, \Omega_m h$
- Peculiar velocities: $\sigma_8 \Omega_m^{0.6}, \Omega_m h$
- Cluster abundance: $\sigma_8 \Omega_m^{0.6}$
- Weak lensing: $\sigma_8 \Omega_m^{0.6}$
- Baryon fraction: $\Omega_b, \Omega_m, h$
- Cepheids, SZ, time-delay: $h$

The dependence on the factor of roughly $\Omega_m^{0.6}$ in different probes (peculiar velocities, cluster abundance and cosmic shear) is a coincidence. The important point is that by using 'orthogonal' constraints one can significantly improve the estimation of cosmological parameters. By performing joint likelihood analyses, one can overcome intrinsic degeneracies inherent in any single analysis and so estimate fundamental parameters much more accurately. The comparison of constraints can also provide a test for the validity of the assumed cosmological model or, alternatively, a revised evaluation of the systematic errors in one or all of the data sets. Recent papers that combine information from several data sets simultaneously include Webster et al. (1998); Lineweaver (1998); Gawiser & Silk (1998), Bridle et al. (1999, 2001), Eisenstein, Hu & Tegmark (1999); Efstathiou et al. (1999, 2001); Bahcall et al. (1999) and Wang et al. (2000).

6.2. Statistical Issues

While joint Likelihood analyses employing both CMB and LSS data are allowing more accurate estimates of cosmological parameters, they involve various subtle statistical issues:

- There is the uncertainty that a sample does not represent a typical patch of the FRW Universe to yield reliable global cosmological parameters.
- The choice of the model parameter space is somewhat arbitrary.
- One commonly solves for the probability for the data given a model (e.g. using a Likelihood function), while in the Bayesian framework this should be modified by the prior for the model and its parameters.
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• If one is interested in a small set of parameters, should one marginalise over all the remaining parameters, rather than fix them at certain (somewhat ad-hoc) values?
• The ‘topology’ of the Likelihood contours may not be simple. It is helpful when the Likelihood contours of different probes ‘cross’ each other to yield a global maximum (e.g. in the case of CMB and SNe), but in other cases they may yield distinct separate ‘mountains’, and the joint maximum Likelihood may lie in a ‘valley’.
• Different probes might be spatially correlated, i.e. not necessarily independent.
• What weight should one give to each data set?

In a long-term collaboration in Cambridge (Bridle et al. 1999, 2001; Efstathiou et al. 1999; Lahav et al. 2000) we have compared and combined in a self-consistent way various cosmic probes: CMB, galaxy redshift surveys, galaxy cluster number counts, type Ia Supernovae and galaxy peculiar velocities. These analyses illustrate the power of combining different data sets for constraining the fundamental parameters of the Universe. Our analysis suggests, in agreement with studies by other groups (e.g. Bahcall et al. 1999, Wang et al. 2000), that we live in a flat accelerating Universe, with about 30% of the critical density in the form of matter (baryonic and non-baryonic) and 70% in the form of ‘dark energy’ (which may be Einstein’s cosmological constant, or a more complicated dynamical vacuum energy). We have also addressed recently (Lahav et al. 2000; Lahav 2001) the issue of combining different data sets, which may suffer different systematic and random errors. We generalised the standard procedure of combining likelihood functions by allowing freedom in the relative weights of various probes. This is done by including in the joint likelihood function a set of ‘Hyper-Parameters’, which are dealt with using Bayesian considerations. The resulting algorithm, which assumes uniform priors on the logarithm of the Hyper-Parameters, is simple to implement.

7. The Best-Fit Concordance Model

Although the Λ-CDM model with comparable amounts of dark matter and dark energy is rather esoteric, it is remarkable that different measurements converge to ‘concordance model’ with parameters:

• \( \Omega_k \approx 0 \),
• \( \Omega_m = 1 - \Omega_\Lambda \approx 0.3 \),
• \( \Omega_b h^2 \approx 0.02 \),
• \( h \approx 0.7 \)
• the age of the Universe \( t_0 \approx 14 \) Gyr,
• the spectral index \( n \approx 1 \),
• \( \sigma_{8m} \approx 0.7 \).

See for example Figures 2 and 3, that show that roughly the same parameters fit well the 2dFGRS power spectrum and the CMB fluctuations. Perhaps the least accurate estimates on that list are for \( \Omega_m \) and \( \sigma_{8m} \).
Figure 3. A compilation of the latest CMB data points $T^2_\ell(\ell+1)C_\ell/(2\pi)$ (open circles with error bars, from WTZ01) against spherical harmonic $\ell$. The line shows the predicted angular power-spectrum for a $\Lambda$-CDM model with $n=1$, $\Omega_m = 1 - \Omega_\Lambda = 0.3$, $\Omega_b h^2 = 0.02$ (BBN value), $h = 0.7$, COBE normalization $\sigma_8 = 0.90$ (dashed line) and the lower best-fit to WTZ01 data $\sigma_8 = 0.77$ (solid line). Recent results from cluster abundance (e.g. Seljak 2001) are actually in agreement with this low normalisation. The stars indicate this best fit model convolved with the experimental window functions. A similar model is also the best-fit to the shape of the 2dF galaxy power-spectrum (Figure 2).

8. Discussion

Analysis of the CMB, the XRB, radio sources and the Lyman-$\alpha$ which probe scales of $\sim 100 - 1000 h^{-1}$ Mpc strongly support the Cosmological Principle of homogeneity and isotropy. They rule out a pure fractal model. However, there is a need for more realistic inhomogeneous models for the small scales. This is in particular important for understanding the validity of cosmological parameters obtained within the standard FRW cosmology.

While phenomenologically the $\Lambda$-CDM model has been successful in fitting a wide range of cosmological data, there are some open questions:

- Both components of the model, $\Lambda$ and CDM, have not been directly measured. Are they ‘real’ entities or just ‘epicycles’?
- Why is $\Omega_m \sim \Omega_\Lambda$ at the present-epoch? Do we need to introduce a new physics or invoke the Anthropic Principle to explain it?
- There are still open problems in $\Lambda$-CDM on the small scales e.g. galaxy profiles and satellites (e.g. Selwood & Kosowsky 2000).
- The age of the Universe is uncomfortably close to some estimates for the age of the Globular Clusters, when their epoch of formation is also taken into account (Gnedin, Lahav & Rees 2001).
- Could other (yet unknown) models fit the data equally well?
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• Where does the field go from here? Would the activity focus on refinement of the cosmological parameters within Λ-CDM, or on introducing entirely new paradigms?

These issues will no doubt be revisited soon with larger and more accurate data sets. We will soon be able to map the fluctuations with scale and epoch, and to analyze jointly redshift surveys (2dF, SDSS) and CMB (MAP, Planck) data. These high quality data sets will allow us to study a wider range of models and parameters.

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