Cross sections of inclusive $\psi(2S)$ and $X(3872)$ production from $b$-hadron decays in $pp$ collisions and comparison with ATLAS, CMS, and LHCb data

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We study the cross sections for the inclusive production of $\psi(2S)$ and $X(3872)$ hadrons in $pp$ collisions at the LHC at two different center-of-mass energies and compare them with experimental data obtained by the ATLAS, CMS, and LHCb Collaborations.

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I. INTRODUCTION

Some time ago, Paolo Bolzoni and two of us calculated the cross section for the inclusive production of $J/\psi$ and $\psi(2S)$ mesons originating from decays of $B$ mesons produced in $p\bar{p}$ collisions with center-of-mass energy $\sqrt{S} = 1.96$ TeV at the Fermilab Tevatron and in $pp$ collisions with $\sqrt{S} = 7$ TeV at the CERN LHC at next-to-leading order (NLO) in the framework of the general-mass variable-flavor-number scheme (GM-VFNS) in connection with nonrelativistic-QCD (NRQCD) factorization [1]. In Ref. [1], the transverse momentum ($p_T^\prime$) distributions of such nonprompt $J/\psi$ mesons measured by the CDF II [2,3], CMS [4,5], LHCb [6,7], ATLAS [8], and ALICE [9] Collaborations were found to be very well described by our predictions, with respect to both absolute normalization and line shape. Similarly, the $p_T$ distributions of $\psi(2S)$ nonprompt production measured by CDF II [3], CMS [5], and LHCb [7] were rather well described by our calculations.

In 2003, a narrow charmonium-like state was discovered in inclusive $B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$ decays by the Belle Collaboration [10]. The subsequent development was described in detail in publications by CMS [11] and ATLAS [12], where the first measurements of $X(3872)$ at the LHC were reported, and in two review articles [13,14].

In 2017, ATLAS published measurements of the $\psi(2S)$ and $X(3872)$ cross sections in $pp$ collisions at $\sqrt{S} = 8$ TeV, for both prompt and nonprompt production [12]. Both the $\psi(2S)$ and $X(3872)$ hadrons were detected via their decays to $J/\psi \pi^+ \pi^-$. The experimental results for prompt production were found to be in agreement with predictions of nonrelativistic QCD (NRQCD) factorization [15–17]. The $\psi(2S)$ nonprompt production cross section was compared with FONLL predictions [18] and also found to agree very well with the ATLAS data [12].

Also, the cross section of $X(3872)$ nonprompt production measured by ATLAS [12] was compared with theory. Specifically, the nonprompt $\psi(2S)$ production cross section evaluated in the FONLL scheme was rescaled by the ratio of branching fractions,

$$ R_B = \frac{\text{Br}(B \rightarrow X(3872) + X) \cdot \text{Br}(X(3872) \rightarrow J/\psi \pi^+ \pi^-)}{\text{Br}(B \rightarrow \psi(2S) + X) \cdot \text{Br}(\psi(2S) \rightarrow J/\psi \pi^+ \pi^-)}, $$

which was evaluated using the result $\text{Br}(B \rightarrow X(3872) + X) = (1.9 \pm 0.8) \times 10^{-4}$ extracted in Ref. [20] from Tevatron data [21] and the values $\text{Br}(B \rightarrow \psi(2S) + X) = (3.07 \pm 0.21) \times 10^{-3}$ and $\text{Br}(\psi(2S) \rightarrow J/\psi \pi^+ \pi^-) = 0.3446 \pm 0.0030$ quoted by the Particle Data Group [22] to give

$$ R_B = 0.18 \pm 0.08. $$

Notice that Eq. (1) implies a summation over the various $B$-hadron species and that the tacitly assumed universality of $R_B$ is based on the assumption that the $B$-hadron fragmentation functions (FFs) are process independent, as they should by the factorization theorem [19].

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FIG. 1. Double differential cross sections times branching fractions for (a) $\psi(2S)$ (left) and (b) $X(3872)$ (right) nonprompt hadroproduction at $\sqrt{s} = 8$ TeV, averaged over $y$ in the region $|y| \leq 0.75$, as a function of $p_T$. The solid points with error bars represent the ATLAS data [12]. The solid and the upper and lower dashed histograms represent our NLO GM-VFNS predictions for $\xi = 1, 0.5, 2$, respectively.

This led to an overestimation of the ATLAS data by a large factor, increasing with $p_T$ from about 4 to about 8 [12].

We observe that the value of $R_B$ in Eq. (2) is a factor of 5 larger than the one measured by ATLAS in a two-lifetime fit [12],

$$R_B^{(2)} = (3.57 \pm 0.348) \times 10^{-2}. \quad (3)$$

If this lower value had been used instead of the larger value based on Ref. [20], the differential cross section $d\sigma/dp_T$ of $X(3872)$ nonprompt production calculated in the FONLL framework would have been in approximate agreement with the ATLAS measurement [12]. We are not aware of an explanation for the discrepancy between the two values of $R_B$.

The $\psi(2S)$ and $X(3872)$ processes considered here differ in the mechanisms of both production from $B$-hadron decay and decay to $J/\psi\pi^+\pi^-$. The $\psi(2S)$ meson is a pure $c\bar{c}$ state, whereas the $X(3872)$ hadron is believed to be a quantum-mechanical mixture of the pure $c\bar{c}$ state $\chi_{1c}(2P)$ and a $D^0\bar{D}^{*0}$ molecule, which is predominantly produced via its $\chi_{1c}(2P)$ component [15–17]. These differences are reflected in the fact that $R_B$ is so different from unity. The value of $R_B$ could be predicted from purely theoretical considerations—e.g., on the basis of effective field theories derived from QCD in combination with hadron models. However, this would reach beyond the scope of this work. Instead, we adopt here a more heuristic approach to $R_B$, based on the interpretation of experimental data.

In the present work, we shall present the results of an alternative calculation of $d\sigma/dp_T$ for $\psi(2S)$ nonprompt production in the kinematic range relevant for the CMS [11] and ATLAS [12] measurements, based on the GM-VFNS. The calculation of $d\sigma/dp_T$ for a fixed rapidity ($y$) range was described in detail in Ref. [1], which is based on the earlier analysis [23] based on the zero-mass variable-flavor-number scheme (ZM-VFNS), where bottom is included among the massless quark flavors. An alternative GM-VFNS approach, in the SACOT-$m_T$ scheme, was described in Ref. [24], but cross sections of $J/\psi$ or $\psi(2S)$ production have not yet been calculated in that scheme.

II. RESULTS

We start by considering $\psi(2S)$ nonprompt hadroproduction. Its theoretical treatment in the GM-VFNS was described in Ref. [1]. Without any additional assumptions, we thus obtain the results for the differential cross section $d^2\sigma/(dp_Tdy)$ at $\sqrt{s} = 8$ TeV, averaged over the rapidity range $|y| < 0.75$, which are presented for $10 < p_T < 70$ GeV in Fig. (1a) and compared with the ATLAS data [12]. We adopt the choices of renormalization and (unified) factorization scales, $\mu_R$ and $\mu_F$, from our recent analysis of $B^\pm$-meson hadroproduction at the LHC [25] by setting $\mu_R = \xi \mu_T$ and $\mu_F = 0.49 \mu_T$, with $\mu_T = \sqrt{p_T^2 + 4m_b^2}$ and $m_b = 4.5$ GeV, and varying $\xi$ between 0.5 and 2 about its default value 1 to estimate the theoretical uncertainty. We use set CT14nlo [26] of parton distribution functions (PDFs) and the $B$-meson fragmentation functions (FFs) from Ref. [27]. As in the ATLAS analysis [12], we include the $B^+, B_s^0$, and $B_s^0$ hadrons and their antiparticles [28], taking into account the respective $b \rightarrow B$-hadron branching fractions as given in Ref. [22]. Here, we select the ATLAS dataset corresponding to the long-lifetime contribution.
determined by the two-lifetime fit, where the short-lifetime contribution is attributed to the $B_c \rightarrow \psi(2S) + X$ decay. The data corresponding to the one-lifetime fit are quite similar, except that the cross sections in the first two $p_T$ bins are slightly larger. We observe from Fig. 1(a) that the agreement between experiment and theory is quite good, except for the uppermost $p_T$ bin.

We now turn to $X(3872)$ nonprompt hadroproduction. We convert our results for $\psi(2S)$ nonprompt hadroproduction shown in Fig. 1(a) to the $X(3872)$ case by including the $R_B^{\psi}$ ratio listed in Eq. (3). In Fig. 1(b), we compare the outcome with the respective ATLAS data, again for the two-lifetime fit [12]. The ATLAS data for the single-lifetime fit are somewhat larger in the first two $p_T$ bins. We emphasize that the value of $R_B^{\psi}$ in Eq. (3) comes from the same experimental analysis as the cross sections $d\sigma/dp_T$ of inclusive $\psi(2S)$ and $X(3872)$ production, albeit from a different observable, namely from the ratio of long-lived $X(3872)$ to long-lived $\psi(2S)$ production rates with additional information from the $p_T$ dependencies [see Fig. 4(b) in Ref. [12]]. Of course, one would like to have this information also from an independent experiment—for example, from $\psi(2S)$ and $X(3872)$ production in $e^+e^-$ annihilation. Unfortunately, such information is not available yet.

The quality of agreement between data and theoretical predictions is similar for $\psi(2S)$ and $X(3872)$ production. It would be interesting to perform a comparative analysis of $\psi(2S)$ and $X(3872)$ production in $e^+e^-$ annihilation. After all, the $R_B$ ratio should not depend on the production mechanism.

Our results for $X(3872)$ nonprompt hadroproduction shown in Fig. 1(b) were obtained using the value of $R_B^{\psi}$ given in Eq. (3). To obtain a cross-check, we determine $R_B$ by fitting our theory prediction to the ATLAS data [12] for the five $p_T$ bins, assuming Gaussian errors. We thus obtain

$$R_B^{\text{ATLAS}} = (3.41 \pm 0.37_{-0.56}^{+0.63}) \times 10^{-2},$$

(4)

where the first error is propagated from the experimental data and the second errors are due to the uncertainty of the theory prediction from $\xi$ variations as explained above. This value agrees within errors with the value given in Eq. (3) obtained from the two-lifetime fit in Ref. [12]. Its central value is slightly smaller and its error somewhat larger than in Eq. (3). We conclude that the determinations of $R_B$ from the two-lifetime fit and from the cross-section analyses by ATLAS are consistent with each other.

To obtain yet another check of our assumption that the $R_B$ value in Eq. (3) [12] is realistic, we use it for a comparison with the inclusive cross section $d\sigma/dp_T$, integrated over $|y| < 1.2$, of nonprompt $X(3872)$ production measured by the CMS Collaboration at $\sqrt{s} = 7$ TeV [11]. In Ref. [11], this cross section was not reported directly, but it can be calculated in four $p_T$ bins from the prompt $X(3872)$ cross section times branching ratio (Table 7 in Ref. [11]) and the $X(3872)$ nonprompt fraction (Table 6 in Ref. [11]). Since both measurements have rather large uncertainties, the same is true also for the nonprompt cross section, but this should be sufficient for the sake of an approximate cross-check. In Ref. [11], CMS used as a benchmark their earlier measurement of the inclusive cross section of nonprompt $\psi(2S)$ production, also at $\sqrt{s} = 7$ TeV and for $|y| < 1.2$, but with a different $p_T$ binning [5]. In Ref. [1], we compared these data with our prediction, which came as a continuous function in $p_T$. We now repeat this comparison, adopting the very $p_T$ binning from Ref. [5] and present the outcome in Fig. 2. The good agreement which is evident from Fig. 2 reassures us that our prediction of nonprompt $\psi(2S)$ production serves as a useful starting point for generating a prediction of nonprompt $X(3872)$ production.

We thus proceed by adjusting the $p_T$ binning according to Ref. [11] and including the $R_B$ value from Eq. (3). In Fig. 3(a), the outcome is compared with the inclusive cross section of nonprompt $X(3872)$ production extracted from Ref. [11], as explained above. We find satisfactory agreement within the rather large experimental errors, albeit not as good as for the ATLAS data [12] in Fig. 1(b). However, our predictions do not deviate more than 2 standard deviations from the CMS data [11]. It is obvious that there is no room for a discrepancy by a factor between 4 and 8 as claimed in Ref. [12]. The $\pm 10\%$ uncertainty in the $R_B$ value in Eq. (3), which is not included in Fig. 3(a), does not affect this conclusion.
In turn, we may now use the CMS data [11] for an independent determination of $R_{B}$, just as we did above with the ATLAS data [12]. We thus find

$$R_{B}^{\text{CMS}} = (1.89 \pm 0.32^{+0.38}_{-0.33}) \times 10^{-2}. \quad (5)$$

As expected from Fig. 3(a), this result is smaller than the one in Eq. (3) from the two-lifetime fit in the ATLAS publication [12] and the one in Eq. (4) from matching our $\psi(2S)$ predictions to the ATLAS $X(3872)$ data [12]. For a consistency check, we compare the CMS $X(3872)$ data [11] with our predictions evaluated with the $R_{B}$ value fitted to these data in Fig. 3(b), to find good agreement as expected.

We also simultaneously fit $R_{B}$ to the nonprompt $X(3872)$ data from ATLAS [12] and CMS [11]. This is possibly problematic because the experimental (first) errors of $R_{B}$ in Eqs. (4) and (5) from the individual fits do not overlap. In other words, the agreement of the underlying ATLAS [12] and CMS [11] data, gauged with respect to our universal NLO GM-VFNS predictions, is marginal. In fact, the combined fit to the $N = 9$ experimental data points yields a minimum $\chi^2$ of $\chi^2_{\text{min}} = 14.70$. Following a recommendation by the Particle Data Group [22], we thus rescale the original Gaussian error of 0.24 with the enhancement factor $\sqrt{\chi^2_{\text{min}}/(N-1)} = 1.36$. We so obtain

$$R_{B}^{\text{ATLAS+CMS}} = (2.54 \pm 0.33^{+0.49}_{-0.43}) \times 10^{-2}, \quad (6)$$

where the theoretical (second) error is determined as in Eqs. (4) and (5).

Finally, it is interesting to compare our results to recent data from the LHCb Collaboration on the dependence on the charged-particle multiplicity $N$ of nonprompt $\psi(2S)$ and $X(3872)$ production, with subsequent decays to $J/\psi \pi^+ \pi^-$, in $p p$ collisions at $\sqrt{s} = 8$ TeV [29]. The study of the $X(3872)$ to $\psi(2S)$ ratio of nonprompt production cross sections as a function of $N$ may help to shed light on the nature of the exotic $X(3872)$ state [30]. This ratio, which we identify with $R_{B}$, as we do in this paper, is presented in Fig. 4 of Ref. [29] for five intervals $[N_{\text{min}}, N_{\text{max}}]$ with weighted centers $N$. The results [31] are compiled in Table I. We observe from Table I a slight increase of $R_{B}$ with $N$, which is, however, not statistically significant. As discussed in Ref. [29], a linear fit to these data points, without considering the correlated systematic uncertainty, gives a positive slope that is consistent with zero within 1.6 standard deviations. For this reason, it

\begin{table}[h]
\centering
\begin{tabular}{cccc}
$N_{\text{min}}$ & $N_{\text{max}}$ & $N$ & $R_{B} \times 10^{2}$ \\
\hline
0 & 40 & 30.8 & 2.65 \(\pm\) 1.32 \(\pm\) 0.20 \\
40 & 60 & 50.3 & 3.17 \(\pm\) 0.60 \(\pm\) 0.24 \\
60 & 80 & 69.3 & 3.47 \(\pm\) 0.71 \(\pm\) 0.26 \\
80 & 100 & 88.8 & 4.43 \(\pm\) 1.20 \(\pm\) 0.32 \\
100 & 200 & 118.3 & 5.33 \(\pm\) 1.59 \(\pm\) 0.39 \\
\end{tabular}
\caption{$R_{B}$ for five different bins of charged-particle multiplicity $N$ as measured by the LHCb Collaboration in $p p$ collisions at $\sqrt{s} = 8$ TeV. The first and second errors are the uncorrelated and correlated uncertainties, respectively. The data are presented in Fig. 4 of Ref. [29].}
\end{table}
appears to be reasonable to fit a horizontal line to these data points. Specifically, we perform three fits using the uncorrelated uncertainties: one to the central data points, and one each to the central data points shifted up and down by their correlated uncertainties. We then take the upward and downward shifts of the central value to be the error due to the correlated uncertainties, which, in want of the correlation matrix, represents a conservative estimate. This yields

\[ R_B^{\text{LHCb}} = (3.48 \pm 0.39 \pm 0.26) \times 10^{-2}, \quad (7) \]

where the first and second errors stem from the uncorrelated and correlated uncertainties, respectively. The fit to the unshifted data points yields a \( \chi^2 \) per degree of freedom as low as \( \chi^2 / 4 = 0.67 \), providing a posteriori a convincing justification for the zero-slope hypothesis. The value in Eq. (7) is in excellent agreement with the two-lifetime fit justification for the zero-slope hypothesis. Among other things, this yielded

\[ R_B^{\text{CMS}} = 1.89 \pm 0.32^{+0.38}_{-0.33} \]

\[ R_B^{\text{ATLAS}} = 2.54 \pm 0.33^{+0.49}_{-0.43} \]

which is consistent with our fit result in Eq. (7).

For the reader’s convenience, we summarize in Table II the various values for \( R_B \) discussed in this paper.

### III. SUMMARY

We updated our prediction of the inclusive cross section of nonprompt \( \psi(2S) \) hadroproduction at NLO in the GM-VFNS [1] and validated it by comparison with ATLAS [12] and CMS [5] data. From this, we obtained an analogous prediction for nonprompt \( X(3872) \) hadrons by including the appropriate ratio \( R_B \) of branching fractions. In turn, this enabled us to determine \( R_B \) by fitting to ATLAS [12] and CMS [11] data of nonprompt \( X(3872) \) production. This also provided us with a useful test bed to assess determinations of \( R_B \) by other authors [12,20,29,30]. Our findings support the results for \( R_B \) obtained by ATLAS [12] and LHCb [29,30], which undershoot a previous result [20] by a factor of about 1/5.

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