HEAVY QUARK EXPANSIONS FOR INCLUSIVE HEAVY-FLAVOUR DECAYS AND THE LIFETIMES OF CHARM AND BEAUTY HADRONS

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ABSTRACT

Inclusive heavy-flavour decays can be described through $1/m_Q$ expansions derived from QCD with the help of an operator product expansion. I sketch their methodology and apply them first to semileptonic $B$ decays; $|V_{cb}|$ can be extracted from $\Gamma_{SL}(B)$ and $\bar{B} \rightarrow l\nu D^*$ with the result: $|V_{cb}|_{\text{incl}} = 0.0413 \pm 0.0016_{\text{exp}} \pm 0.002_{\text{theor}}$, $|V_{cb}|_{\text{excl}} = 0.0377 \pm 0.0016_{\text{exp}} \pm 0.002_{\text{theor}}$. The lifetimes of charm and beauty hadrons are discussed. The charm lifetimes are predicted/reproduced as well as could be expected. Predictions on $B$ meson lifetimes agree with available data; $\Lambda_b$ baryons are predicted to be shorter lived than $B_d$ mesons by no more than $\sim 10\%$ – in marked contrast to present measurements. I evaluate the situation and comment on recent theoretical criticism. The importance of the concepts of global vs. local quark-hadron duality is pointed out.

1 Motivations

Although I am certainly preaching to the converted here, I want to start out with some very general statements on the virtues of heavy-flavour physics. Analysing it in detail plays a central role in our attempts to uncover Nature's Grand Design. For this dynamical sector provides us with an extremely rich phenomenology possessing fundamental implications with CP violation as the ultimate prize. Few other (if any)
areas have a comparable potential for fundamental discoveries. Some predictions can be made with high \textit{parametric} reliability; for example the CP asymmetry in $B_d \to \psi K_S$ can be predicted to be given by $\sin 2\beta$ with very high reliability. Furthermore new theoretical tools apply enabling us to translate the \textit{parametric} into a \textit{numerical} accuracy. This will be the subject of my talk here.

As explained later I find it quite realistic that the numerical values of the KM angles of most direct relevance for beauty physics can be extracted with the following accuracy:

$$\delta |V(cb)| \leq \pm \text{very few \%}$$  \hspace{1cm} (1)

$$\delta \frac{|V(ub)|}{|V(cb)|} \leq \pm \text{few \%}$$  \hspace{1cm} (2)

$$\delta \frac{|V(td)|}{|V(cb)|} \leq \pm (10 - 15) \%$$  \hspace{1cm} (3)

The objective set in eq.(1) is a near-term goal we are close to achieving; eqs.(2) and (3) amount to mid-term and long-term goals, respectively.

These benchmarks represent tall orders; our hope to attain them is based on the presence of the heavy-flavour quark mass: expanding transition amplitudes in powers of $\mu/m_Q$ – with $\mu$ denoting an ordinary hadronic scale not exceeding 1 GeV – should lead to meaningful results for beauty decays when only the first few terms are retained since $\mu/m_Q \ll 1$; for charm on the other hand, the situation is a priori unclear.

This hope can be formulated through four second-generation theoretical technologies, namely (i) QCD sum rules, (ii) Lattice QCD, (iii) Heavy Quark Effective Theory (=HQET) \footnote{It will be pointed out later that methods (iii) and (iv) are distinct and should not be equated.} and (iv) Heavy Quark Expansions; methods (i) - (iii) deal primarily with exclusive decays whereas (iv) treats inclusive transitions \footnote{It will be pointed out later that methods (iii) and (iv) are distinct and should not be equated.}. The message I want to convey is the following:

- There are several self-consistent methods that to the best of our knowledge are genuinely based on QCD.

- They allow a \textit{quantitative} treatment of important aspects of the decays of heavy-flavour hadrons.

- Even failures teach us important lessons on QCD, however saddening they might be in that case.
The remainder of my talk will be organized as follows: in Sect. 2 I introduce the theoretical tools, which in Sect. 3 are applied to semileptonic beauty decays; in Sect. 4 I discuss the lifetimes of charm and beauty hadrons before presenting my conclusions in Sect. 5.

2 The \(1/m_Q\) Methodology for Inclusive Decays of Heavy-Flavour Hadrons

2.1 The Operator Product Expansion

Analogous to the treatment of \(e^+e^- \rightarrow \text{hadrons}\) one can describe the decay rate into an inclusive final state \(f\) in terms of the imaginary part of a forward scattering operator evaluated to second order in the weak interactions [2, 3, 4]:

\[
\hat{T}(Q \rightarrow f \rightarrow Q) = i \text{Im} \int d^4x \{ \mathcal{L}_W(x) \mathcal{L}_W^\dagger(0) \} \tag{4}
\]

where \(\{ \cdot \}_T\) denotes the time ordered product and \(\mathcal{L}_W\) the relevant effective weak Lagrangian expressed on the parton level. If the energy released in the decay is sufficiently large one can express the non-local operator product in eq. (4) as an infinite sum of local operators \(O_i\) of increasing dimension with coefficients \(\tilde{c}_i\) containing higher and higher powers of \(1/m_Q\). This operator product expansion (OPE) [3] and its consistent realization is the central theoretical tool in the Heavy Quark Expansions. The width for \(H_Q \rightarrow f\) is then obtained by taking the expectation value of \(\hat{T}\) for the heavy-flavour hadron \(H_Q\):

\[
\langle H_Q | \hat{T}(Q \rightarrow f \rightarrow Q) | H_Q \rangle \propto \Gamma(H_Q \rightarrow f) = G_F^2 |KM|^2 \sum_i \tilde{c}_i^{(f)}(\mu) \langle H_Q | O_i | H_Q \rangle (\mu), \tag{5}
\]

where I have used the following notation: \(|KM|\) denotes the appropriate combination of KM parameters; the c-number coefficients \(\tilde{c}_i^{(f)}(\mu)\) are determined by short-distance dynamics whereas long-distance dynamics control the expectation values of the local operators \(O_i\). Such a separation necessitates the introduction of an auxiliary scale with \(\text{long distance} > \mu^{-1} > \text{short distance}\) [5]. While this is a conceptually and often also practically important point I will not refer to it explicitly anymore in this article [5].

\[3\]It should be kept in mind, though, that it is primarily the energy release rather than \(m_Q\) that controls the expansion.

\[4\]Observables of course do not depend on \(\mu\). Yet one has to choose \(\Lambda_{QCD} \ll \mu \ll m_Q\) if one wants to calculate perturbative as well as non-perturbative corrections in a self-consistent way.
The master formula eq.(5) holds for a host of different integrated heavy-flavour decays: semileptonic, nonleptonic and radiative transitions, KM favoured or suppressed etc. For semileptonic and nonleptonic decays, treated to order $1/m_Q^3$, it takes the following form:

$$\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} |KM|^2 \left[ c_3 \langle H_Q|\bar{Q}Q|H_Q \rangle + c_5 \frac{\langle H_Q|\bar{Q}i\sigma \cdot GQ|H_Q \rangle}{m_Q^2} + \right.$$  

$$\left. + \sum_i c_{6,i} \frac{\langle H_Q|((\bar{q}\Gamma_i q)(q\Gamma_i Q)|H_Q \rangle}{m_Q^3} + O(1/m_Q^4) \right]$$  

(6)

Four comments might elucidate this expression:

1. We know which local operators can appear in the operator product expansion and what their dimensions are; this determines how they scale with $m_Q$. We also follow the usual procedure of actually calculating the short-distance coefficients $c_i(f)$ in perturbation theory.

2. The expectation values of the local operators are shaped by long-distance dynamics and in general we cannot derive their size from first principles. We will employ $1/m_Q$ expansions to relate these matrix elements to other observables of a typically static nature like hadron masses. One can also rely on the findings from QCD sum rules and lattice QCD concerning these expectation values; this will be of increasing value in the future.

3. Eq.(6) does not contain a contribution of order $1/m_Q$ since there is no independent gauge-invariant dimension-four operator. The leading non-perturbative corrections are then of order $(\mu/m_Q)^2$. As we will see in more detail this means they amount to no more than a few percent in beauty decays. This is one major reason why the hope to achieve the benchmark accuracies listed above is realistic: one has to control the non-perturbative corrections only on the, say, 30% level to describe an integrated width with a few percent accuracy.

4. The expansion parameter is actually the inverse of the energy release rather than $1/m_Q$ although this is not manifest in the expression given in eq.(5). This distinction will become relevant for the discussion of $b \to c\bar{c}s$.

5Expanding $\langle H_Q|\bar{Q}i\sigma \cdot GQ|H_Q \rangle/m_Q^2$ also yields contributions of order $1/m_Q^3$; those are however practically insensitive to the light quark flavours.

6The situation is more subtle for final-state spectra.
The three terms appearing on the right-hand side of eq. (6) allow an intuitive interpretation: (i) The leading operator $\bar{Q}Q$ contains the spectator contribution that dominates for $m_Q \to \infty$, yet goes beyond it: for example in incorporates the motion of the heavy quark relative to the rest frame of the hadron. (ii) $\langle H_Q|\bar{q}i\sigma \cdot GQ|H_Q\rangle$ describes the spin interaction of the heavy quark $Q$ with the light degrees of freedom inside the hadron. This term had been overlooked in the earlier phenomenological descriptions. (iii) $\langle H_Q|\bar{Q}\Gamma_i q\rangle_{\bar{Q}}^* GQ|H_Q\rangle$ contains the so-called Pauli Interference (PI) \[7\], Weak Annihilation (WA) \[8\] and Weak Scattering (WS) \[9\] contributions which had been introduced by the earlier phenomenological descriptions. However there is little ‘wiggle space’ here: WA is helicity suppressed \[10\]; PI and WS scale (at least formally) like $1/m_Q^3$.

### 2.2 Determination of the Expectation Values

Using the equation of motion one can expand the local operator $\bar{Q}Q$ in powers of $1/m_Q$ and finds \[7\]:

$$
\langle H_Q|\bar{Q}Q|H_Q\rangle_{\text{norm}} = 1 + \frac{\langle H_Q|\bar{Q}\frac{i}{2}\sigma \cdot GQ|H_Q\rangle_{\text{norm}}}{2m_Q^2} - \frac{\langle (\vec{p}Q)^2 \rangle_{H_Q}}{2m_Q^2} + O(1/m_Q^3)
$$

(7)

The first term on the right-hand side, which reflects the flavour charge carried by $H_Q$, represents the spectator contribution that dominates for $m_Q \to \infty$.

The expectation values of the chromomagnetic operator are known. Since the light di-quark system inside $\Lambda_Q$ and $\Xi_Q$ (but not inside $\Omega_Q$) baryons carries no spin, there can be no spin-interaction:

$$
\langle \Lambda_Q|\bar{Q}^{\frac{i}{2}}\sigma \cdot GQ|\Lambda_Q\rangle \approx 0 \approx \langle \Xi_Q|\bar{Q}^{\frac{i}{2}}\sigma \cdot GQ|\Xi_Q\rangle
$$

(8)

The expectation value for pseudoscalar mesons $P_Q$ is given by the observed hyperfine splitting between the masses of the vector $V_Q$ and pseudoscalar mesons:

$$
\langle P_Q|\bar{Q}^{\frac{i}{2}}\sigma \cdot GQ|P_Q\rangle_{\text{norm}} \approx \frac{3}{4} \left( M^2(V_Q) - M^2(P_Q) \right)
$$

(9)

For beauty and charm one then has

$$
G_b \equiv \frac{\langle B|\bar{b}\frac{i}{2}\sigma \cdot Gb|B\rangle_{\text{norm}}}{m_b^2} \approx 0.016
$$

(10)

$$
G_c \equiv \frac{\langle D|\bar{c}\frac{i}{2}\sigma \cdot Gc|D\rangle_{\text{norm}}}{m_c^2} \approx 0.21
$$

(11)

\[7\] I use here a relativistic normalization: $\langle H_Q|O_i|H_Q\rangle_{\text{norm}} = \langle H_Q|O_i|H_Q\rangle / 2M(H_Q)$. 

5
This representing a second order correction one infers that the expansion parameter is small albeit not tiny for beauty decays: \( \sqrt{G_b} \sim 0.13 \); for charm it is not small though at least smaller than unity: \( \sqrt{G_c} \sim 0.46 \).

The expectation values of the other independant dimension-five operator

\[
\langle H_Q | \bar{Q}(i\bar{D})^2 Q | H_Q \rangle_{\text{norm}} \equiv \langle (\bar{p}_Q)^2 \rangle_{H_Q}
\]  

with \( D_\mu \) denoting the covariant derivative – can be interpreted as the average kinetic energy of the heavy quark \( Q \) inside the hadron \( H_Q \). Its numerical value is not known precisely. From an analysis based on QCD sum rules \([11]\) one obtains

\[
\langle (\bar{p}_b)^2 \rangle_B \simeq 0.5 \pm 0.1 \, \text{(GeV)}^2
\]

in agreement with a lower bound \([12, 13]\)

\[
\langle (\bar{p}_b)^2 \rangle_B \geq \langle B | \bar{b} \sigma \gamma_5 B | B \rangle_{\text{norm}} \geq 0.15 \, \text{(GeV)}^2 = (0.37 \pm 0.15) \, \text{(GeV)}^2
\]

The differences in the mesonic and baryonic expectation values can be related to the ‘spin averaged’ meson and baryon masses: \( \langle \bar{p}_Q \rangle^2 \rangle_{\Lambda_Q} - \langle \bar{p}_Q \rangle^2 \rangle_{P_Q} \simeq \frac{2m_m}{m_b - m_c} \cdot \left\{ [\langle M_D \rangle - M_{\Lambda_c} - [\langle M_B \rangle - M_{\Lambda_b}] \right\} \) \([14]\). Present data yield:

\[
\langle \bar{p}_Q \rangle^2 \rangle_{\Lambda_Q} - \langle \bar{p}_Q \rangle^2 \rangle_{P_Q} = -(0.015 \pm 0.030) \, \text{(GeV)}^2
\]

i.e., no significant difference. In deriving eq.(15) it was assumed that the \( c \) quark can be treated as heavy; in that case \( \langle \bar{p}_c \rangle^2 \rangle_{H_c} \simeq \langle \bar{p}_b \rangle^2 \rangle_{H_b} \) holds.

The expectation values of the two classes of four-fermion operators that appear – one coupling two colour singlets, the other two colour octets – are not known accurately. To estimate their size for mesons one usually invokes factorization or vacuum saturation:

\[
\langle P_Q(p) | J^{(i)}_{\mu} \cdot J^{(i)}_{\nu} | P_Q(p) \rangle \equiv \langle P_Q(p) | (\bar{Q}_L \gamma_{\mu \nu} q_L) (\bar{q}_L \gamma_{\nu} \gamma_{\mu} q_L | P_Q(p) \rangle_{\text{norm}} \simeq
\]

\[
\langle P_Q(p) | (\bar{Q}_L \gamma_{\mu} q_L) | 0 \rangle_{\text{norm}} \langle 0 | (\bar{q}_L \gamma_{\nu} Q_L | P_Q(p) \rangle_{\text{norm}} = \frac{1}{8M_{P_Q}} f^2_P p_{\mu} p_{\nu}
\]

(16)

\[
\langle P_Q(p) | J^{(i)}_{\mu} \cdot J^{(i)}_{\nu} | P_Q(p) \rangle \equiv \langle P_Q(p) | (\bar{Q}_L \gamma_{\mu \nu} \lambda_i q_L) (\bar{q}_L \gamma_{\nu} \lambda_i Q_L | P_Q(p) \rangle_{\text{norm}} \simeq
\]

\[
\langle P_Q(p) | (\bar{Q}_L \gamma_{\mu} \lambda_i q_L) | 0 \rangle_{\text{norm}} \langle 0 | (\bar{q}_L \gamma_{\nu} \lambda_i Q_L | P_Q(p) \rangle_{\text{norm}} = 0
\]

(17)

Such an ansatz cannot be an identity; it can hold as an approximation, though – at certain scales. Invoking it at \( \sim m_Q \) does not make sense at all. For as far as QCD is concerned, \( m_Q \) is a completely foreign quantity. A priori it has a chance to hold
at ordinary hadronic scales $\mu \sim 0.5 \div 1$ GeV [13]; various theoretical analyses based on QCD sum rules, QCD lattice simulations, $1/N_C$ expansions etc. have indeed found it to apply in that regime. It would be inadequate conceptually as well as numerically to renormalize merely the decay constant: $f_Q(m_Q) \rightarrow f_Q(\mu)$. Instead the full set of operators has to be evaluated at $\mu$. One proceeds in three steps (for details see [15]):

(A) Ultraviolet renormalization translates the weak Lagrangian defined at $M_W$, $\mathcal{L}_W(M_W)$, into one effective at $m_Q$, $\mathcal{L}_W(m_Q)$.

(B) The operators $J^{(0)}_\mu \cdot J^{(0)}_\nu$ and $J^{(i)}_\mu \cdot J^{(i)}_\nu$ undergo hybrid renormalization [15] down to $\mu$.

(C) At scale $\mu$ one invokes factorization.

To be more specific I state the final result for the PI contribution:

$$\Delta \Gamma_{PI} \simeq \Gamma_0 \cdot 24\pi^2 \frac{f_{H_Q}^2}{M_{H_Q}^2} \kappa^{-4} \left[ (c_+^2 - c_-^2)\kappa^{9/2} + \frac{c_+^2 + c_-^2}{3} - \frac{1}{9}(\kappa^{9/2} - 1)(c_+^2 - c_-^2) \right]$$

(18)

where $\Gamma_0$ denotes the width for the decay of a free quark $Q$ and $c_\pm$ the usual UV operator renormalization coefficients; hybrid renormalization is described by

$$\kappa \equiv \left[ \frac{\alpha_s(\mu^2)}{\alpha_s(m_Q^2)} \right]^{1/b},\ b = 11 - \frac{2}{3}n_F$$

(19)

From eq.(18) one reads off for the colour factor

$$\left[ (c_+^2 - c_-^2)\kappa^{1/2} + \frac{c_+^2 + c_-^2}{3\kappa^4} - \frac{(\kappa^{9/2} - 1)}{9\kappa^4}(c_+^2 - c_-^2) \right] \rightarrow \left[ \frac{4}{3}c_+^2 - \frac{2}{3}c_-^2 \right] \rightarrow \frac{2}{3}$$

(20)

when first ignoring hybrid renormalization $- \kappa = 1$ – and then UV renormalization as well $- c_+ = 1 = c_-$. 

Some comments are in order for proper evaluation:

- The factorizable contributions to PI largely cancel at scales around $m_Q$ – in particular in the case of beauty and apparently for accidental reasons. The ratio of non-factorizable to factorizable contributions is thus large and numerically unstable there.

- No such cancellation occurs around scales $\mu_{had}$ making factorizable contributions numerically more stable and dominant over non-factorizable ones.

- Contributions that are factorizable (in colour space) at $\mu_{had}$ are mainly non-factorizable at $m_Q$. 

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• The role of non-factorizable terms has been addressed in the literature over the years, most explicitly and in a most detailed way in [17, 18].

I will later comment on the criticism expressed in [19].

The situation becomes much more complex for baryon decays. To order $1/m_Q^3$ there are several different ways in which the valence quarks of the baryon can be contracted with the quark fields in the four-quark operators; furthermore WS is not helicity suppressed and thus can make a sizeable contribution to lifetime differences; also the PI effects can now be constructive as well as destructive. Finally one cannot take recourse to factorisation as a limiting case. Thus there emerge three types of numerically significant mechanisms at this order in baryon decays – in contrast to meson decays where there is a single dominant source for lifetime differences – and their strength cannot be expressed in terms of a single observable like $f_{H_Q}$.

At present we do not know how to determine the relevant matrix elements in a model-independant way. Guidance and inspiration has traditionally been derived from quark model calculations with their inherent uncertainties. This analysis had already been undertaken in the framework of phenomenological models [20, 21, 22].

One thing should be obvious already at this point: with terms of different signs and somewhat uncertain size contributing to differences among baryon lifetimes one has to take even semi-quantitative predictions with a grain of salt!

There is another relation that will become highly relevant in the discussion of semileptonic beauty decays. The mass difference which is free of renormalon ambiguities and well-defined can be expressed as follows:

$$m_b - m_c \simeq \frac{1}{4}(M_B + 3m_{B^*}) - \frac{1}{4}(M_D + 3m_{D^*}) + \langle (\vec{p}_Q)^2 \rangle_{P_Q} \cdot \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)$$

Using the observed mass values for the charm and beauty mesons and varying $\langle (\vec{p}_Q)^2 \rangle_{P_Q} \simeq \langle (\vec{p}_b)^2 \rangle_{B} \simeq \langle (\vec{p}_c)^2 \rangle_{D}$ over a reasonable range one obtains

$$m_b - m_c \simeq (3.46 \pm 0.04) \text{ GeV}$$

3 Semileptonic $B$ Decays

There are three topics I want to address here, namely the semileptonic branching ratio of $B$ mesons and the extractions of $|V_{cb}|$ and $|V_{ub}|$.

3.1 $BR_{SL}(B)$

Without radiative QCD corrections one finds $BR(b \to l\nu c) \sim 15\%$. Including them lowers $BR(b \to l\nu c)$ down to $\sim 12 - 13.5\%$. Yet this is still significantly higher than
Table 1: The measured values of \( n_c \).  

|        | CLEO       | ALEPH     |
|--------|------------|-----------|
| \( n_c \) | \( 1.134 \pm 0.043 \) | \( 1.23 \pm 0.07 \) |

the observed branching ratio for beauty mesons [23]:

\[
BR(B \rightarrow l\nu X) = 10.43 \pm 0.24\% \quad (23)
\]

The weak link in the theoretical treatment is the estimate of \( \Gamma(\bar{B} \rightarrow c\bar{s}q) \) since the energy release in \( b \rightarrow c\bar{s}s \) is not very large; therefore the nonperturbative corrections might not be under good control. It is then quite conceivable that \( BR(\bar{B} \rightarrow c\bar{s}q) \) is considerably larger than is usually inferred from \( BR(b \rightarrow c\bar{s}s) \) computed on the parton level; for this latter quantity a ‘canonical’ value of 0.15 is often adopted [23]. There are actually various theoretical indications that an enhancement of this nonleptonic channel indeed takes place [24]. One can then entertain the idea that \( BR(\bar{B} \rightarrow c\bar{s}q) \sim 0.3 \) holds rather than 0.15; this would bring the predicted semileptonic branching ratio into agreement with the observed one. One has to keep in mind that this transition rate is particularly sensitive to which values one adopts for the quark masses: using ‘low’ values for the quark masses, namely \( m_b = 4.6 \text{ GeV}, m_c = 1.2 \text{ GeV} \) and \( m_s = 0.15 \text{ GeV} \), rather than ‘high’ values - \( m_b = 5.0 \text{ GeV}, m_c = 1.7 \text{ GeV} \) and \( m_s = 0.30 \text{ GeV} \) - would enhance \( BR(b \rightarrow c\bar{s}s) \) by a factor of about two to a value 1.23 [25, 26]. This magnified dependance on the quark mass values provides us with another illustration that nonperturbative corrections can be expected to be large here since they control the proper usage of quark masses. It might still turn out that we will be able to calculate \( \Gamma(\bar{B} \rightarrow c\bar{s}q) \) reliably – once we understand how the quark masses are to be evaluated for this transition.

Such a resolution of the puzzle would have another observable consequence: it would lead to a larger than previously expected charm yield in \( B \) decays. To be more specific: for \( n_c \) – the number of charm states emerging from \( B \) decays – one has \( n_c \approx 1 + BR(\bar{B} \rightarrow c\bar{s}q) \). This quantity can be measured where one assigns charm multiplicity one to \( D, D_s, \Lambda_c \) and \( \Xi_c \) and two to charmonia. There are two new experimental studies which I juxtapose in Table 1 to the two theoretical expectations sketched above: While both experimental numbers [27] are consistent with each other and the ‘canonical’ value 1.15, the CLEO number clearly favours 1.15 over 1.3. Yet in view of the ALEPH number one can say that a higher value of

\footnote{Being ‘canonical’ does not make it necessarily right – even for someone from Notre Dame.}

\footnote{One should note that both sets of mass values satisfy \( m_b - m_c \approx 3.4 \text{ GeV} \).}
1.25 - 1.30 that would lead to predicting the observed semileptonic branching ratio is not ruled out yet.

The plot thickens even further: the channel $\bar{B} \to c\bar{s}q$ can be accessed more directly by observing the decays of $B$ into ‘wrong-sign’ $D$ mesons as done by CLEO:

$$BR(\bar{B} = (b\bar{q}) \to \bar{D} = (c\bar{q}) + X) = 0.081 \pm 0.026 \quad (24)$$

Such an inclusive transition is fed almost completely by $b \to c\bar{s}q$ since the strong fragmentation reaction $q \to qc\bar{c}$ is highly suppressed. Combining eq.(24) with the findings on $\bar{B} \to \bar{D}_s + X, \psi(0) + X$, etc. yields

$$BR(\bar{B} \to c\bar{s}q) = 0.239 \pm 0.038 \quad (25)$$

The measurement given in eq.(24) means that the channel $\bar{B} \to c\bar{s}q$ is not dominated by $\bar{B} \to \bar{D}_s + X$ – contrary to earlier expectations! It should be noted that a recent theoretical analysis [28] invoking factorization finds $\Gamma(\bar{B} \to \bar{D}_s + X) \leq \frac{1}{2}\Gamma(\bar{B} \to c\bar{s}q)$ due to the production of higher-mass $\bar{D}_s^{*+}$ resonances decaying preferentially into $D + K + X$.

My conclusions are the following: The experimental situation is still somewhat in limbo with various intriguing possibilities. The present discrepancy between the data and the theoretical expectation will probably be resolved through a combination of factors. One also has to allow for a larger theoretical uncertainty in predicting the absolute value of this branching ratio than the ratio of semileptonic branching ratios.

### 3.2 Extracting $|V(cb)|$

There is near-universal consensus in the community that the KM parameter $|V(cb)|$ is best extracted from semileptonic $B$ decays (and likewise for $|V(ub)|$). This consensus gets dissipated, however, when one discusses what specifically is the most reliable method for that. I will sketch two complementary analyses which I consider the most reliable ones from a theoretical perspective.

#### 3.2.1 $|V(cb)|$ from $\Gamma_{SL}(B)$

The total semileptonic width of $B$ mesons being proportional to $|V(cb)|^2$ is a prime candidate:

$$\Gamma_{SL}(B) = \frac{G_F^2 m_b^5}{192\pi^3} |V(cb)|^2 \cdot \left[ F(\alpha_s, m_c^2/m_b^2, (\langle p_b\rangle)^2_B/m_b^2) + \mathcal{O}(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3) \right]$$

(26)
The function $F$ in eq.(26) containing perturbative, mass and nonperturbative corrections is known; in general nonperturbative corrections are found to be small and under control. Yet the dependance on the fifth power of the $b$ quark mass with its intrinsic uncertainties would appear – at first sight – to severely limit or even vitiate the quantitative usefulness of $\Gamma_{SL}(B)$. However it turns out that $\Gamma_{SL}(B)$ depends mainly on the difference $m_b - m_c$ rather than on $m_b$ and $m_c$ separately although this is not manifest in eq.(26). It is obvious in the so-called Small Velocity (SV) limit that is realized for $(m_b - m_c)/m_b \ll 1$, i.e. $m_c \approx m_b$; it is shown in [29] that there is an extended SV limit, i.e. the SV limit becomes relevant precociously for the real value of $m_c/m_b \sim 1/3$. As pointed out above, the difference $m_b - m_c$ is well-defined and well-known numerically to within 1 % roughly, see eq.(22).

The new criticism was put forward that the perturbative expansion is particularly ill-behaved, namely that the coefficient for the second order term $(\alpha_s/\pi)^2$ is around 10 for $b \to c$ and even 20 for $b \to u$ [30]. Yet considerable care has to be applied in treating quark masses. Usage of the pole mass is not appropriate when including perturbative as well as nonperturbative corrections [31]; running masses on the other hand can be employed. When extracting $m_b$ from $\Upsilon$ spectroscopy [32] and applying it to $\Gamma_{SL}(B)$ one has to keep track at which scale $m_b$ is evaluated. Using $m_b$ evaluated at the high scale $m_b$ indeed leads to the appearance of large second order contributions in $\Upsilon$ spectroscopy as well as in $B$ decays! However, once one evaluates $m_b$ at a low scale around 1 GeV, the coefficients of the second order corrections become small, namely less than unity [33]. It suggests that the natural scale is not the heavy mass, but considerably smaller. This observation can be explained through a careful analysis of the phase space available in semileptonic $B$ decays [29].

Putting everything together one arrives at

$$|V(cb)|_{incl} = (0.0413 \pm 0.002_{\text{theor}}) \times \sqrt{\frac{1.57 \text{ psec}}{\tau(B)}} \times \sqrt{\frac{BR_{SL}(B)}{0.1043}}$$ (27)

where I have listed the theoretical uncertainty only, estimated to be around 5% based on the following considerations:

1. The main error is in the value for $m_b - m_c$; its $\sim 1\%$ uncertainty stated in eq.(22) translates into a $\sim 5\%$ one for $\Gamma_{SL}(B)$ and thus $\sim 3\%$ for $|V(cb)|$.

2. The remaining separate sensitivity to $m_b$ generates a $\sim 1\%$ error.
3. The first two points can be expressed through a simple scaling law \[34\]

\[
\delta |V(cb)|_{m_b - m_c, m_b} \simeq \left(1 - 0.012 \cdot \frac{\langle (\vec{p}_b)^2 \rangle_B - 0.4 (\text{GeV})^2}{0.1 (\text{GeV})^2} \right) \left(1 - 0.006 \cdot \frac{\delta m_b}{30 \text{ MeV}} \right)
\]  

(28)

keeping in mind that at present the main uncertainty on \( m_b - m_c \) originates in the value of \( \langle (\vec{p}_b)^2 \rangle_B \).

4. Finally, one has to allow for a \( \sim 1\% \) error in \( |V(cb)| \) due to the not completely known higher order perturbative corrections.

The purpose of this ‘anatomy’ is not to claim that the present theoretical error cannot exceed 5\% by a single iota, but to elucidate the bases for the estimate – and to indicate how it can be improved in the future:

1. The value of \( m_b - m_c \) can be determined also from a detailed study of the shape of the lepton spectra in \( B \to l\nu + X \) \[3, 35\]; a precision of \( \delta (m_b - m_c) \sim 0.5\% \) seems to be achievable generating \( \delta |V(cb)| \sim 1\% \). Measuring the mass difference in this way would also avoid one assumption inherent in eq. (22), namely that charm is sufficiently heavy, at least in this instance, to make an expansion in \( 1/m_c \) numerically reliable.

2. Likewise one will be able to extract \( m_b \) from the lepton spectra well enough such that \( \delta |V(cb)| \sim 0.5\% \) from this source alone.

3. A full calculation of the \( \alpha_s^2 \) corrections beyond the BLM prescription appears to be a feasible, though technically non-trivial project; the remaining perturbative error would be reflected in \( \delta |V(cb)| \sim 0.5\% \).

4. Altogether one can state as an ambitious, though feasible expectation for the midterm future:

\[
\delta |V(cb)| \sim 2\%
\]  

(29)

3.2.2 \( |V(cb)| \) from \( \bar{B} \to l\nu D^* \) at Zero-Recoil

The exclusive channel \( \bar{B} \to l\nu D^* \) provides us with an intriguing opportunity to extract \( |V(cb)| \) by adopting the following strategy \[36, 37\]:

1. One measures the rate for \( \bar{B} \to l\nu D^* \) as a function of the momentum transfer, extrapolates to the kinematical point of zero-recoil for the \( D^* \) and extracts
$|F_{B\to D^*}(0)V(cb)|$, where $F_{B\to D^*}(0)$ denotes the form factor at zero-recoil. The present world average yields [27]

$$|F_{B\to D^*}(0)V(cb)| = 0.0339 \pm 0.0014$$

(30)

2. One then has to calculate the size of the formfactor. Asymptotically, i.e. for $m_b, m_c \gg \mu_{had}$, $F_{B\to D^*}(0) = 1$ holds as a consequence of the heavy quark symmetry. For finite values one then has [36, 38]:

$$F_{B\to D^*}(0) = 1 + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{m_c^2}, \frac{1}{m_b m_c}, \frac{1}{m_b^2}\right),$$

(31)

i.e. perturbative as well as non-perturbative corrections will drive the form factor away from unity. Originally it had been claimed that $F_{B\to D^*}(0) = 0.98 \pm 0.02$ holds. The perturbative corrections are indeed small (though care has to be applied to their treatment); however the leading non-perturbative corrections should be given by $(\mu/m_c)^2 \sim O(10\%)$ rather than by $(\mu/m_b)^2 \sim O(2\%)$.

This issue can be addressed not only within HQET [1], but also through a judicious application of Heavy Quark Expansions to inclusive semileptonic rates. As discussed in [12] one can derive SV sum rules: from QCD one calculates the $n$th moments of certain transition rates $\bar{B} \to l\nu + X_c$ as they are produced by specified weak currents in the SV limit and – invoking quark-hadron duality – equates them with the same moments of observable semileptonic rates. In particular one considers the case of axialvector currents which produce $D^*$ and its higher excitations:

$$\Gamma^{(n)}(\bar{B} \to l\nu(\text{quarks & gluons})_{A \times A}) = \Gamma^{(n)}(\bar{B} \to l\nu(D^* + \text{excitations}))$$

(32)

Invoking the positivity of transition rates one can derive bounds on individual exclusive rates although a priori it is not guaranteed that such bounds are useful phenomenologically. In this instance, however, it turns out to be useful; this is not completely unexpected since even in the ‘extended’ SV limit [29] a small number of channels dominates the inclusive transition:

$$\xi_A(\mu) - |F_{B\to D^*}(0)|^2 = \frac{1}{3} \frac{\langle \mu_{T^2} \rangle_B}{m_c^2} + \frac{\langle (\vec{p})^2 \rangle_B - \langle \mu_{T^2} \rangle_B}{4} \left( \frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b} \right) +$$

$$+ \sum_{\epsilon_f < \mu} |F_{B\to f}|^2 + O(1/m_c^3, ...).$$

(33)

\[10\] The necessary theoretical tools go beyond those described in Sect.2.
Table 2: Estimates on the size of $F_{\bar{B} \rightarrow D^*(0)}$.

| $F_{\bar{B} \rightarrow D^*(0)}$ | $\langle (\bar{p}_b)^2 \rangle_B$ | $\langle (\bar{p}_b)^2 \rangle_B \gtrsim 0.5 (\text{GeV})^2$ | Estimate |
|-------------------------------|----------------|----------------|---------|
| $\leq 0.94$                       | $\geq 0.92$            | $0.90 \pm 0.03$           |

The $F_{\bar{B} \rightarrow f}$ represent the form factors for those charm excitations beyond the $D^*$ that are produced by the axialvector current with a mass $M_f = M_{D^*} + \epsilon_f$; $\xi_A(\mu)$ denotes a perturbative renormalization factor depending on $\mu$, the scale separating the long and short distance domains that was introduced in eq.(5); a detailed discussion of this point can be found in ref. [34]. With the possible exception of corrections of order $1/m_Q^3$ all terms on the right-hand side of eq.(33) are positive, see eq.(14); thus a model independent upper bound can be placed on $F_{\bar{B} \rightarrow D^*(0)}$ using $\langle (\bar{p}_b)^2 \rangle_B \geq \langle \mu_G^2 \rangle_B$ established by another SV sum rule. The bound can be further strengthened by using the QCD sum rule result from eq.(13). Finally one can make an educated estimate on the contributions from the higher excitations. The results are given in Table 2; we see that indeed the deviation from unity is closer to 10% than 2%, as suspected!

Using the estimate for the formfactor one then infers from eq.(30):

$$|V(cb)|_{\text{excl}} = 0.0377 \pm 0.0016_{\text{exp}} \pm 0.002_{\text{theor}}$$

(34)

which is consistent with the inclusive value, eq.(27). The agreement would have been much more iffy if $F_{\bar{B} \rightarrow D^*(0)} = 0.98 \pm 0.02$ were to hold.

Some comments on the estimate of the theoretical uncertainty:

- The estimate of $\sum_{\epsilon_f \leq \mu |F_{\bar{B} \rightarrow f}|^2$ is certainly not above reasonable suspicion.
- Corrections of order $1/m_Q^3$ of which the leading term is presumably controlled by $1/m_c^3$ are not known in both magnitude and sign. They could modify the inequality.
- I am not optimistic that we can significantly reduce the theoretical uncertainty here – unlike in the inclusive analysis.

### 3.3 Extracting $|V(ub)|$

CLEO has seen the exclusive charmless channels $B \rightarrow l\nu\pi$, $l\nu\rho$ [27]; to determine $|V(ub)|$ there, one has to rely on models and I will not comment on them. The first evidence for $|V(ub)/V(cb)| \neq 0$ came from observing leptons with energies beyond
the kinematical limit for \( \bar{B} \to l\nu X_c \). The quantitative analysis at present still requires some model elements although their weight will be reduced in the future.

The total width \( \Gamma(\bar{B} \to l\nu X_u) \), on the other hand, can be expressed reliably in terms of \( |V(ub)|, \langle(p_b^2)\rangle_B \) and \( m_b(1\text{GeV}) \) as discussed before; the last quantity is known from Υ spectroscopy. Once \( \langle(p_b^2)\rangle_B \) has been determined from the lepton spectrum in \( \bar{B} \to l\nu X_c \), one can extract \( |V(ub)| \) reliably – if \( \Gamma(\bar{B} \to l\nu X_u) \) can be separated out and measured. Taking the recent ALEPH findings \( \Gamma(\bar{B} \to l\nu X_u)/\Gamma(\bar{B} \to l\nu X_c) = 0.016 \pm 0.004 \pm 0.004 \) at face value one obtains \( |V(ub)/V(cb)| \approx 0.098 \pm 0.013 \pm 0.013 \) where the theoretical uncertainty is considerably smaller than the two experimental ones stated.

## 4 Lifetimes of Heavy-Flavour Hadrons

### 4.1 Generalities

The lifetimes of weakly decaying charm and beauty hadrons can of course be measured accurately without theoretical input. There is also no apparent qualitative disaster in the pattern observed since

\[
\frac{\tau(B^-)}{\tau(B_d)} - 1 \ll \frac{\tau(D^+)}{\tau(D^0)} - 1 , \quad 1 - \frac{\tau(\Lambda_b)}{\tau(B_d)} > 1 - \frac{\tau(\Lambda_c)}{\tau(D^0)},
\]

i.e., the relative difference in the \( B^- - B_d \) lifetimes is considerably smaller than for the \( D^+ - D^0 \) case and the \( \Lambda_Q \) are shorter lived than the \( P_Q \) with the effect much more pronounced in the charm than in the beauty sector.

Yet the heavy quark expansions should be applicable in a quantitative way, at least to beauty lifetimes. Inversely a failure in describing these inclusive quantities would be quite instructive – even if not welcome – regarding our theoretical control over QCD. Yet such a failure which could be caused by a violation of local quark-hadron duality to be defined later does not prejudice our ability to treat semileptonic decays through a heavy-quark expansion.

There is an intriguing pattern in how the lifetimes evolve and get differentiated order by order in \( 1/m_Q \): I sketch it here for the widths of charged and neutral pseudoscalar mesons and the lowest baryons:

\[
\Gamma(\Lambda_Q) = \Gamma(P_Q^0) = \Gamma(P_Q^\pm) + \mathcal{O}(1/m_Q)
\]

\[
\Gamma(\bar{\Lambda}_Q) = \Gamma(P_Q^0) = \Gamma(P_Q^\pm) + \mathcal{O}(1/m_Q^2)
\]

\[
\Gamma(\Lambda_Q) > \Gamma(P_Q^0) \simeq \Gamma(P_Q^\pm) + \mathcal{O}(1/m_Q^3)
\]

\[
\Gamma(\bar{\Lambda}_Q) > \Gamma(P_Q^0) > \Gamma(P_Q^\pm) + \mathcal{O}(1/m_Q^4)
\]
### Table 3: QCD Predictions for Charm Lifetime Ratios

| Observable | QCD Expectations (1/$m_c$ expansion) | Ref. | Data from [16] |
|------------|--------------------------------------|------|----------------|
| $\tau(D^+)/\tau(D^0)$ | $\sim 2$ [for $f_D \simeq 200$ MeV] (mainly due to destructive interference) | 10  | $2.547 \pm 0.043$ |
| $\tau(D_s)/\tau(D^0)$ | $1 \pm \text{few } \times0.01$ | 17  | $1.12 \pm 0.04$ |
| $\tau(\Lambda_c)/\tau(D^0)$ | $\sim 0.5^*$ | 39  | $0.51 \pm 0.05$ |
| $\tau(\Xi^+_c)/\tau(\Lambda_c)$ | $\sim 1.3^*$ | 39  | $1.75 \pm 0.36$ |
| $\tau(\Xi^+_c)/\tau(\Xi^0_c)$ | $\sim 2.8^*$ | 39  | $3.57 \pm 0.91$ |
| $\tau(\Xi^+_c)/\tau(\Omega_c)$ | $\sim 4^*$ | 39  | $3.9 \pm 1.7$ |

### 4.2 The Lifetimes of Charm Hadrons – Predictions without Guarantees

With an expansion parameter as large as $\mu/m_c \sim 0.4$ one can hope for a $1/m_c$ expansion to provide us with at best a semi-quantitative description of charm lifetimes.

In Table 3 I juxtapose the data with the theoretical expectations obtained from the heavy quark expansion described above. The numbers for baryon lifetimes are based on quark model evaluations of the four-fermion expectation values; this is indicated by an asterisk. Details can be found in [16].

The agreement between the expectations and the data, within the uncertainties, is respectable or even remarkable considering the large theoretical expansion parameter and the fact that the lifetimes for the apparently shortest-lived hadron – $\Omega_c$ – and for the longest-lived one – $D^+$ – differ by an order of magnitude! Of course the experimental uncertainties in $\tau(\Xi_c)$ and $\tau(\Omega_c)$ are still large; the present agreement could fade away – or even evaporate – with the advent of more accurate data. Yet at present I conclude:

- The observed difference in $\tau(D^0)$ vs. $\tau(D^+)$ is understood as due mainly (though not exclusively) to a destructive interference in $\Gamma_{NL}(D^+)$ arising in order $1/m_c^2$. This is not contradicted by the data showing $BR_{SL}(D^+) \simeq 17\%$. For the corrections of order $1/m_c^2$ reduce the number obtained in the naive spectator model – $BR_{SL}(D) \simeq BR_{SL}(c)$ – from around 16% down to around 9% [3]!

- The observed near-equality of $\tau(D^0)$ and $\tau(D_s)$ provides us with strong, though circumstantial evidence for the reduced weight of WA. It puts a severe bound...
on the size of the non-factorizable parts in the expectation values of the four-fermion operators, as given in [17].

- The lifetimes of the charm baryons reflect the interplay of destructive as well as constructive PI and WS intervening in order $1/m_c^3$ [20, 21, 22].

\begin{align*}
\Gamma(\Lambda_c^+) &= \Gamma_{\text{decay}}(\Lambda_c^+) + \Gamma_{WS}(\Lambda_c^+) - |\Gamma_{PI,-}(\Lambda_c)| \\
\Gamma(\Xi_c^0) &= \Gamma_{\text{decay}}(\Xi_c^0) + \Gamma_{WS}(\Xi_c^0) + |\Gamma_{PI,+}(\Xi_c)| \\
\Gamma(\Xi_c^+) &= \Gamma_{\text{decay}}(\Xi_c^+) + |\Gamma_{PI,+}(\Xi_c^+)| - |\Gamma_{PI,-}(\Xi_c^+)| \\
\Gamma(\Omega_c) &= \Gamma_{\text{decay}}(\Omega_c) + |\Gamma_{PI,+}(\Omega_c)|
\end{align*}

with both quantities on the right-hand-side of the last equation differing from the corresponding ones for $\Lambda_c$ or $\Xi_c$ decays [16]. On rather general grounds one concludes:

\[ \tau(\Xi_c^0) < \tau(\Xi_c^+), \quad \tau(\Xi_c^0) < \tau(\Lambda_c^+) \] (44)

To go beyond this qualitative prediction one has to evaluate the expectation values of the various four-fermion operators. No model-independent manner is known for doing that for baryons; we do not even have a concept like factorization allowing us to lump our ignorance into a single quantity. Instead we have to rely on quark model computations and thus have to be prepared for additional very sizeable theoretical uncertainties.

- The $\Omega_c$ naturally emerges as the shortest-lived charm hadron due to spin-spin interactions between the decaying $c$ quark and the spin-one $ss$ di-quark system.

Finally one should note that the ratios $BR_{SL}(\Xi_c)/BR_{SL}(D^0)$ and $BR_{SL}(\Omega_c)/BR_{SL}(D^0)$ will not reflect their lifetime ratios; for $\Gamma_{SL}(\Xi_c)$ and $\Gamma_{SL}(\Omega_c)$ get significantly enhanced relative to $\Gamma_{SL}(D^0)$ in order $1/m_c^3$ due to constructive PI in $\Gamma_{SL}(\Xi_c, \Omega_c)$ among the $s$ quarks [10]. Thus $\Omega_c$ – despite its short lifetime – could well exhibit a larger semileptonic branching ratio than $D^0$!

### 4.3 The Lifetimes of Beauty Hadrons – Predictions Without Plausible Deniability

Most of the obvious theoretical caveats one can express about charm lifetimes cannot be used as excuses for failures in beauty decays. Due to $m_b \gg m_c > \mu$ the heavy

\[ m_c \] (16)
Table 4: QCD Predictions for Beauty Lifetimes

| Observable | QCD Expectations (1/m_b expansion) | Ref. | Data from [10] |
|------------|-----------------------------------|------|---------------|
| \( \tau(B^-)/\tau(B_d) \) | \( 1 + 0.05(f_B/200 \text{ MeV})^2 [1 + \mathcal{O}(30\%)] > 1 \) (mainly due to destructive interference) | [10] | 1.04 ± 0.04 |
| \( \bar{\tau}(B_s)/\tau(B_d) \) | \( 1 \pm \mathcal{O}(0.01) \) | [41] | 0.97 ± 0.05 |
| \( \tau(\Lambda_b)/\tau(B_d) \) | \( \simeq 0.9^* \) | [41] | 0.77 ± 0.05 |

quark expansion would be expected to yield fairly reliable predictions on lifetime ratios among beauty hadrons. Since we do better than expected for charm lifetimes, one feels doubly confident about making predictions for the lifetimes of beauty hadrons. The actual computations proceed in close analogy to the charm case; details can be found in [10]. The \( B_d - B^- \) lifetime difference is again driven mainly by destructive PI, namely in the \( b \to c \bar{u}d \) channel; similarly, \( \tau(\Lambda_b) \) is reduced relative to \( \tau(B_d) \) by WS winning out over destructive PI in \( b \to c \bar{u}d \):

\[
\Gamma(B_d) \simeq \Gamma_{\text{decay}}(B_d), \quad \Gamma(\Lambda_b) \simeq \Gamma_{\text{decay}}(\Lambda_b) + \Gamma_{WS}(\Lambda_b) - |\Gamma_{PI,-}(\Lambda_b)|
\]

In Table 4 I list the world averages of published data together with the predictions. First a few short comments for orientation:

- These are predictions in the old-fashioned sense, i.e. they were made before data (or data of comparable sensitivity) became available.

- As far as the meson lifetimes are concerned, data and predictions are completely and non-trivially consistent.

- The average \( B_s \) lifetime, i.e. \( \bar{\tau}(B_s) = [\tau(B_{s,\text{long}}) + \tau(B_{s,\text{short}})]/2 \), as measured in \( B_s \to l\nu D_s^{(*)} \), is predicted to be practically identical to \( \tau(B_d) \).

- The largest lifetime difference among beauty mesons is expected to occur due to \( B_s - \bar{B}_s \) oscillations. One predicts [12]:

\[
\frac{\Delta \Gamma(B_s)}{\Gamma(B_s)} \equiv \frac{\Gamma(B_{s,\text{short}}) - \Gamma(B_{s,\text{long}})}{\Gamma(B_s)} \simeq 0.18 \cdot \frac{(f_{B_s})^2}{(200 \text{ MeV})^2}
\]

- The prediction on \( \tau(\Lambda_b)/\tau(B_d) \) seems to be in conflict with the data.

Next I give a more detailed evaluation of these comparisons.

(A) \( \tau(B^-) \) vs. \( \tau(B_d) \):
The prediction given above that the $B^-$ lifetime exceeds that of $B_d$ by a few percent involves assuming factorization to hold at a low scale $\mu_{\text{had}} \ll m_Q$. That has been criticized in ref.[19] where it was argued that neither $\tau(B^-)/\tau(B_d) < 1$ nor $\tau(B^-)/\tau(B_d) \geq 1.2$ would be surprising due to a failure of the factorization approximation at any scale.

It is conceivable that factorization might provide a poor approximation for the expectation values of these four-quark operators -- at a significant theoretical price:

- The successful treatment of $\tau(D^+) \text{ vs. } \tau(D^0) \text{ vs. } \tau(D_s)$ was based on the factorization approximation at a low scale. Of course, these successes might be a mere coincidence.

- For $\tau(B^-)$ to exceed $\tau(B_d)$ by 20% or more the nonfactorizable contributions have to be of a magnitude that -- if true -- would expose serious limitations in the analytical evaluations of weak matrix elements.

- Destructive interference as the main motor of a $B^-$-$B_d$ lifetime difference can occur only in $B^- \rightarrow c\bar{u}d\bar{u}$ transitions. Since those make up no more than about half of all $B$ decays, a 20% lifetime difference would require a $\sim 40\%$ destructive interference in $B^- \rightarrow c\bar{u}d\bar{u}$ -- again an amazingly huge effect.

- The possible size of nonfactorizable contributions has been studied in a detailed way in ref.[18, 17] (although the reader of ref.[19] would not realize that). It was shown there that the relevant expectation values of the four-quark operators can be determined from comparing the lepton spectra in $D^0 \rightarrow l\nu X$, $D^+ \rightarrow l\nu X$ and $D_s \rightarrow l\nu X$ or in $B_d \rightarrow l\nu X$ and $B^- \rightarrow l\nu X$ decays.

- If factorization passed these tests while $\tau(B^-)/\tau(B_d) < 1$ or $\tau(B^-)/\tau(B_d) \geq 1.2$ were observed, we had to infer that local quark-hadron duality did not hold to a sufficient degree in nonleptonic $B$ decays!

- The concept of quark-hadron duality which is essential to applying QCD is not always clearly defined. For our purposes it can be best illustrated through an analysis of the 'classical' reaction $e^+e^- \rightarrow \text{had}$. Unlike in the case of deep inelastic lepton-nucleon scattering where the relevant large parameters are momenta from the space-like region, the relevant momenta here and in heavy-flavour decays are time-like. The transition amplitude under study thus contains singularities in the physical region, namely poles for resonances and cuts signaling particle production. This has been discussed explicitly for $e^+e^-$
annihilation near $E_{c.m.} \sim 4$ GeV \cite{43}. On the real $E_{c.m.}$ axis there are two poles describing the $\psi$ and $\psi'$ resonances and there is a cut reflecting the production of open-charm hadrons. The cross section can be computed in QCD through an operator product expansion along the imaginary $E_{c.m.}$ axis. A dispersion relation is then used to continue the result into the physical regime; this means, however, that only ‘smeared’ transition rates can be predicted, i.e. transition rates averaged over some finite energy range $\Delta E$:

$$\langle \sigma(e^+e^- \rightarrow \text{had}; E_{c.m.}) \rangle \equiv \frac{1}{\Delta E} \int_{E_{c.m.}-\Delta E}^{E_{c.m.}+\Delta E} d\tilde{E} \sigma(e^+e^- \rightarrow \text{had}; \tilde{E}) \quad (46)$$

Equating the quantity thus calculated with the corresponding observed one constitutes the assumption of (global) duality. It was advocated in \cite{43} to use $\Delta E \simeq \mu \sim 0.5 - 1$ GeV. If the cross section happens to be a smooth function of $\tilde{E}$ – as it happens far away from any production thresholds – then one can effectively take the limit $\Delta E \to 0$ to predict $\sigma(e^+e^- \rightarrow \text{had})$ for a fixed energy $E_{c.m.}$. This scenario is referred to as local duality and clearly represents a stronger assumption than global duality. When one describes semileptonic decays, then one deals with smeared quantities since integration over the neutrino momenta is understood; assuming global duality then suffices. In nonleptonic decays on the other hand such smearing is not guaranteed, there could be unforeseen singularities in the $qq\bar{q}\bar{q}$ matrix elements and in general one has to invoke local duality for equating the results of the heavy quark expansion with observable rates.

\textbf{(B) $\tau(B_s)$ vs. $\tau(B_d)$:}

There is general agreement that the heavy quark expansion predicts that the average $B_s$ lifetime as measured in semileptonic decay modes and the $B_d$ lifetime practically coincide.

\textbf{(C) $\tau(\Lambda_b)$ vs. $\tau(B_d)$:}

The experimental situation has not been settled yet. Let me cite here the CDF results from the full run 1 data sample of 110 $pb^{-1}$:

$$\begin{align*}
\tau(B_d) & = 1.52 \pm 0.06 \ psec \\
\tau(\Lambda_b) & = 1.32 \pm 0.15 \pm 0.07 \ psec \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} & = 0.87 \pm 0.10 \pm 0.05
\end{align*} \quad (47)$$

While this ratio is quite consistent with the stated world average, it would also satisfy the theoretical prediction.
The difference between $\langle \tau(\Lambda_b)/\tau(B_d) \rangle_{\text{exp.}} \simeq 0.77$ and $\tau(\Lambda_b)/\tau(B_d)\big|_{\text{theor.}} \simeq 0.9$ represents a large discrepancy. For once one has established – as we have – that $\tau(\Lambda_b)$ and $\tau(B_d)$ have to coincide for $m_b \to \infty$, then the predictions really concern the deviation from unity; finding a $\sim 23\%$ deviation when one around $10\%$ was predicted amounts to an error of about $200\%$!

(iii) A failure of that proportion cannot be rectified unless one adopts a new paradigm in evaluating baryonic expectation values. Two recent papers [44, 19] have reanalyzed the relevant quark model calculations and found:

$$\tau(\Lambda_b)/\tau(B_d) \equiv 1 - \text{DEV}, \; \text{DEV} \sim 0.03 \div 0.12$$

i.e., indeed there are large theoretical uncertainties in DEV since the baryon lifetimes reflect the interplay of several contributions of different signs in addition to the quark decay expression, namely from $WS$ and destructive as well as constructive $PI$:

$$\Gamma(\Lambda_b) = \Gamma_{\text{decay}}(\Lambda_b) + \Gamma_{WS}(\Lambda_b) - |\Gamma_{PI,-}(\Lambda_b, b \to c\bar{u}d)|$$

$$\Gamma(\Xi^0_b) = \Gamma_{\text{decay}}(\Xi_b) + \Gamma_{WS}(\Xi_b) - |\Gamma_{PI,-}(\Xi_b, b \to c\bar{c}s)|$$

$$\Gamma(\Xi^-_b) = \Gamma_{\text{decay}}(\Xi_b) - |\Gamma_{PI,-}(\Xi_b, b \to c\bar{c}s)| - |\Gamma_{PI,-}(\Xi_b, b \to c\bar{u}d)|$$

Yet one cannot boost the size of DEV much beyond the $10\%$ level. To achieve the latter one had to go beyond a description of baryons in terms of three valence quarks only. A similar conclusion has been reached by the authors of ref.[45] who analyzed the relevant baryonic matrix elements through QCDJsum rules.

### 4.4 A Radical Phenomenological Proposal

In a recent paper [40] it was argued that the widths of heavy-flavour hadrons scale with the fifth power of their mass – $M_{HQ}$ – rather than the heavy quark mass $m_Q$. This is inconsistent with the heavy quark expansion based on the operator product expansion: it introduces corrections of order $1/m_Q$ in a prominent way:

$$\Gamma(H_Q) \propto G_F^2 M_{HQ}^5 = G_F^2 (m_Q + \bar{\Lambda} + \ldots)^5 = G_F^2 m_Q^5 \left(1 + 5 \cdot \frac{\bar{\Lambda}}{m_Q} + \ldots\right)$$

However in the spirit of the time-honoured advice of ‘Peccate Fortiter’ it was suggested that local duality does not hold in nonleptonic decays of beauty and charm hadrons.
The recipe leads to \[ \frac{\tau(\Lambda_b)}{\tau(B_d)} \approx \left( \frac{M_{B_d}}{M_{\Lambda_b}} \right)^5 \simeq 0.77 \pm 0.05 \] (55)

Likewise one finds for the average \( B_s \) lifetime:

\[ \frac{\bar{\tau}(B_s)}{\tau(B_d)} \approx 0.93 \pm 0.03 \] (56)

which is a significantly smaller ratio than predicted by the \( 1/m_Q \) expansion, yet quite consistent with present measurements.

When applying this prescription to charm decays one finds that the observed \( \Lambda_c, \Xi_c^0 \) and \( \Omega_c \) lifetimes follow the scaling law of eq.(54) relative to the \( D^0 \) lifetime. However a pattern

\[ \tau(D^+) \approx \tau(D^0) > \tau(D_s) > \tau(\Xi_c^+) \]

since \( M(D^+) \approx M(D^0) < M(D_s) < M(\Xi_c^+) \) is in obvious conflict with the data. This has to be remedied by the a posteriori introduction of destructive \( PI \) in \( D^+ \) and \( \Xi_C^+ \) decays and of destructive \( WA \) in \( D_s \) decays tuned as to reproduce the data. One should note, though, that a large overall destructive \( PI \) contribution is not natural for \( \Gamma(\Xi_c^+) \) since there arises also a constructive \( PI \) term, see eqs.(43); a \( WA \) contribution to \( \Gamma(D_s) \) that is both sizeable and destructive would be surprising as well.

Measurements of the semileptonic branching ratios for \( \Xi_c^0, + \) and \( \Omega_c \) baryons would provide important constraints for this phenomenological model as well as for the OPE based heavy quark expansion.

### 4.5 \( B_c \) Decays

\( B_c \) decays provide a particularly intriguing lab to probe QCD. They are shaped by three classes of reactions, namely the decay of the \( b \) quark, the \( c \) quark and \( WA \) between the two heavy constituents.

It had been suggested that the quark masses entering the quark decay widths should be reduced by the binding energy of the \( \bar{b}c \) bound state; this would imply that the \( B_c \) lifetime is relatively long, namely above 1 psec with beauty decays dominating over charm decays. However the \( 1/m_Q \) expansion \[47, 48\] predicts a lifetime well below 1 psec with unfortunately charm decays dominating, mainly because there are

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\[12\] This suggestion was actually first made in \[10\] when the experimental evidence for it was quite marginal before the authors realized it was inconsistent with the operator product expansion.
no corrections of order $1/m_Q$. These findings are also in agreement with an earlier phenomenological analysis [49]. It is curious to note that the recipe of ref. [46] would also yield a short $B_c$ lifetime: $\tau(B_c) \sim (M_B/M_{B_c})^3 \tau(B_d) \sim 0.6$ psec for an expected mass value of $M_{B_c} = 6.26$ GeV. It is not clear what this ansatz predicts for the relative weight of $b$ and $c$ decays.

A study of $B_c$ decays would thus provide us with crucial tests – alas, the prospects for that to happen soon are quite gloomy [50]!

5 Summary and Outlook

Very considerable progress has been achieved in the theoretical description of heavy-flavour decays both of the inclusive variety, as mainly discussed here, and the exclusive one [1]. We can treat semileptonic transitions of beauty hadrons with a reliability and precision that would have seemed unrealistic a few years ago. This can be illustrated by the values extracted for $|V(cb)|$ from inclusive and exclusive semileptonic $B$ decays:

\[
|V(cb)|_{\text{incl}} = 0.0413 \pm 0.0016_{\text{experim}} \pm 0.002_{\text{theor}} \quad (57)
\]
\[
|V(cb)|_{\text{excl}} = 0.0377 \pm 0.0016_{\text{experim}} \pm 0.002_{\text{theor}} \quad (58)
\]

In each case the experimental and theoretical uncertainties are comparable and the two values are consistent with each other. This is quite remarkable considering that they result from analyses that are quite different systematically in their experimental as well as theoretical elements. This agreement came about in a non-trivial way since it is based on the formfactor $F_{B \to D^*}(0)$ to be substantially smaller than unity. At least as far as the theoretical treatment of $\Gamma_{SL}(B)$ is concerned I am confident that the theoretical uncertainty can be reduced significantly in the foreseeable future.

With respect to nonleptonic decays we can now tackle questions that could not be addressed before or only in an ambiguous way: what is the impact of $WA$; how do $\tau(D_s)$ and $\tau(D^0)$ or $\tau(B_s)$ and $\tau(B_d)$ compare to each other; how does the ratio $\tau(P^+ Q)/\tau(P^0 Q)$ scale with $m_Q$ etc. A failure to describe weak lifetimes will of course never rule out QCD; yet it will still teach us important lessons on QCD and our theoretical control over it. Such failures can arise at different layers leading to different kinds of lesson:

- The apparent agreement between predictions for and measurements of charmed baryon lifetimes might evaporate when the merciful imprecision of the present data is overcome. One could then just shrug the shoulders saying that charm
baryon lifetimes receiving contributions from various sources with different signs provide a **numerically very unstable scenario**.

- A failure in reproducing $\tau(D_s)$ vs. $\tau(D^0)$ could be blamed on $m_c$ being too small to provide us with a reliable expansion parameter.

- The $\Lambda_b$ lifetime remaining ‘short’ signals at the very least the need for a new paradigm in evaluating baryonic matrix elements.

- A failure in $\tau(B^-)$ vs. $\tau(B_d)$ vs. $\bar{\tau}(B_s)$ would cast serious doubts on factorization as useful approximation in this case; it would establish the short comings of local duality in nonleptonic beauty decays if factorization had passed the independant tests referred to before.

Of course there exists still a strong need to expand the data base:

1. In the charm sector one wants to measure $\tau(D_s)$ with a 2% accuracy and $\tau(\Xi^+_c, \Xi^0_c, \Omega_c)$ with about 10%.

2. In the beauty sector one has to probe for percent differences in $\tau(B^-)$ vs. $\tau(B_d)$ vs. $\bar{\tau}(B_s)$.

3. A dedicated effort has to be made to search for $\tau(B_{s, short})$ vs. $\tau(B_{s, long})$.

4. One has to measure $\tau(\Lambda_b)$ with even more precision and study $\tau(\Xi_b^-)$ and $\tau(\Xi_b^0)$ as well.

5. $B_c$ decays will provide an intriguing lab for the discriminating connoisseur!

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