Analytical evaluation of equivalent energy loss in causal frequency selection filters in the processing of quadrature amplitude modulation signals

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Abstract. In this paper, we consider the need to assess the influence of the parameters of a linear time-invariant causal system in the processing of quadrature amplitude modulation signals of various positionality and constellation diagram irregularity. We describe the equations that determine the energy balance of a radio line and the total losses arising in route. We have developed mathematical expressions to assess the equivalent energy losses caused by the irregularity of amplitude-frequency and phase-frequency characteristics of the frequency selection filters. We propose a procedure to assess the non-noise losses in the frequency selection filters when designing high-speed radio systems for transmitting the information.

1. Introduction
Modern high-speed data transmission systems widely use high-position quadrature amplitude modulation (QAM) increasing the spectral efficiency of using communication channels. A decrease in the noise immunity of a telecommunication system is a significant disadvantage of increasing the modulation ratio, other conditions being equal. This phenomenon is due to a reduction in the decision-making area for the demodulator decision device output due to various factors that are taken into account when forming a modified energy balance equation.

It should be noted that when constructing radio complexes based on QAM with positionalties higher than 2048, it is necessary to consider energy losses, which were previously neglected due to their insignificant influence [1].

It is noted from the researches [2,3] that a 5% reduction in losses caused by imperfect parameters of causal filters for signals with a modulation ratio of 10 reduces the probability of a symbol reception error from 10⁻¹ to 10⁻⁶. This fact corresponds to a qualitative transition from complete loss of signal to normal reception and demodulation.

The purpose of this work is to assess the influence of the order of a digital linear time-invariant causal system on the equivalent energy loss in signal-to-noise ratio at the input of the decision device when organizing radio systems for transmitting information with QAM of various positionality and constellation diagram irregularity.

2. Problem-solving methods and assumptions
The conducted research is based on the probability theory, mathematical statistics, statistical radio engineering and computational mathematics. Irregularity ratio of the constellation diagram \( \chi \leq 5 \) and
positionality $L$ of no more than 8192 are considered as the accepted assumptions. This decision is justified by their use in modern and future radio systems for transmitting the information.

### 3. Results

As follows from the equation of the energy balance of information transmission radio line, the resulting signal-to-noise ratio at the input of the demodulator decision device (DDD) depends both on the application conditions of information transmission systems and on the parameters (controlled and uncontrolled).

The disadvantages of the energy balance equation formation include the energy losses (EL) of radio signal along the radio waves propagation path and the equivalent energy losses (EEL) in the radio transmitting (RTD) and radio receiving device (RRD).

Considering modern achievements in the theory of space-time processing of radio signals and the theory of noise-immune coding of signals in digital radio systems for transmitting information, the classical energy balance equation has been supplemented. The left part of the equation includes:

- an additional term describing a negative factor which is a decrease in the signal-to-noise ratio due to the mismatch of the amplitude spectrum of the signal with the amplitude-frequency characteristic of the causal filter.

In that case, the energy balance equation of the radio line takes the following form:

$$q^i = P_u^i + G_{an}^i + G_{ar}^i + M_{oc} + V - L_i(R^i) - L_i(k^i) - \Delta - P_{ny}, \quad (1)$$

where $P_u^i$ is the power of the $i$-th transmitting station, dB;

$G_{an}^i$ is antenna power gain of the $i$-th transmitting station, dB;

$G_{ar}^i$ is receiving-antenna gain, dB;

$L_i(R^i)$ is space attenuation of electromagnetic wave, dB;

$L_i(k^i)$ is attenuation caused by multipath nature of radio wave propagation, dB;

$M_{oc}$ is additional energy gain through the use of spatially-polarization signal processing methods at the reception point in case of multipath nature of radio wave propagation;

$V$ is additional energy gain through the use of noise-immune signal decoding methods in digital communication lines;

$k^i$ is the number of beams arriving at the reception point from the $i$-th station;

$R^i$ is the distance between the receiving and the $i$-th transmitting station;

$q^i$ is an additional signal-to-noise ratio required to ensure the detection of the $i$-th signal with quality not lower than required, dB;

$P_{np}$ is limiting sensitivity of RRD, which is determined by the signal quantity at the input to achieve the signal-to-noise ratio $q = 1$ and is equal to the noise power brought to its input;

$\Delta = \Delta_{afu} + \Delta_{rmu} + \Delta_{dmu}$ is EL in RRD;

$\Delta_{afu}$ is EL caused by imperfect parameters of antenna-feeder device (AFD), dB;

$\Delta_{rmu}$ is EL caused by imperfect parameters of the radio receiving device (RRD), dB;

$\Delta_{dmu}$ is EL caused by imperfect parameters of DDD, dB;

The total loss is determined by the following mathematical expression:

$$\Delta = \Delta_{AFU} + \Delta_{PSU} + \Delta_{DMU} + \Delta_{DTU} = \sum_i \Delta_{ak}^i (K_{sh}^i(f)) + \sum_i \Delta_{mshu}^i (K_{sh}^i(f)) + \sum_i \Delta_{sm}^i (K_{sm}^i(f)) + \Delta_{feeder}(K_{sh}(f)) + \Delta_{antenna}(K_{sh}(f)) + \sum_j \Delta_{comm}^j (K_{sh}^j(f)) + \sum_p \Delta_{upch}^p (K) + \sum_e \Delta_{act}^e (K) + \sum_p \Delta_{pf}^p (\delta_{ach}) + \sum_p \Delta_{pf}^p (\delta_{fchh}) + \sum_k \Delta_{get}^k (G_{v}^k(M)) + \Delta_{svn} (\delta_f) + \Delta_{svn}^2 (\delta_0) + \Delta_{usw} (\delta_i) + \Delta_{omsi} (\delta_{msi}), \quad (2)$$

where $\Delta_{ak}$ is EL of the $i$-th antenna multicoupler caused by thermal noise (noise ratio $K_{sh}^i$);

$\Delta_{mshu}$ is EL in low noise amplifiers (LNA) caused by Johnson noise;

$\Delta_{sm}$ is EL caused by Johnson noise of mixers;
Δfeeder is EL in high-frequency feeders caused by thermal noise;  
Δantenna is EL in antenna caused by Johnson noise;  
Δ\text{komm}\text{j} is EL in the j-th multicoupler caused by thermal noise;  
Δ\text{upch}\text{p} is EL caused by the thermal noise of the p-th intermediate-frequency amplifier;  
Δ\text{att}\text{i} is EL in the d-th attenuator which provides a given signal level at the detector input;  
Δ\text{pf}\text{r} is EEL caused by the irregularity of amplitude-frequency characteristic (AFC) of the r-th band-pass filter;  
Δ\text{pf}\text{t} is EEL caused by the irregularity of phase-frequency characteristic (PFC) of the r-th band-pass filter;  
Δ\text{get}\text{k}(G_{v\text{k}}) is EEL caused by imperfection of amplitude-frequency spectrum of oscillation of the k-th heterodyne;  
Δ\text{svn}(\delta_f) is EEL caused by an error in carrier frequency reinsertion system;  
Δ\text{svtch}(\delta_0) is EEL caused by an error in reinsertion DDD clock frequency;  
Δ\text{uvs}(\delta_i) is EEL caused by an error in maintaining the level of the detector input signal;  
Δ\text{omsi}(\delta_{ms}) is EEL caused by residual intersymbol interference.  

EEL caused by the imperfect parameters of linear time-invariant causal systems has not been thoroughly studied in relation to other reasons of non-noise losses. Therefore, it is necessary to develop an analytical model that reflects the relationship between the average signal-to-noise ratio and the probability of error per digit when the QAM signal passes through the studied structural-functional element of the receiving-selecting device.

The studies of the analytical dependence between the EEL in RRD on the degree of distortion (deformation) of AFC were carried out based on the results of work [4].

The output signal of the optimum filter has a spectrum [4]:

\[
\overline{S'_2(\omega)} = \overline{S'_1(\omega)K(\omega)} = \overline{S'_1(\omega)C}S'_1(\omega)e^{-j\omega t_0} = C\overline{D(\omega)}S'_1(\omega)e^{j\omega t_0},
\]

where \(S'_1(\omega) = \) a spectrum of the received undistorted signal;  
\(K(\omega) = \) optimum filter transfer characteristic;  
\(C = \) a constant;  
\(S'_2(\omega) = \) a complex-conjugate signal;  
\(D(\omega) = \) causal filter transfer characteristic;  

So, for example, the loss for the harmonic type of distortion is determined by the expression [1,4]:

\[
G_{AChH} = 1 + \frac{a_1^2}{2a_0^2}
\]

where \(a_1 = \) an amplitude of AFC irregularity in the transition band of a causal filter;  
\(a_0 = \) the ratio of the peak value of the distorted output signal to the peak value of the undistorted output signal.

Determination of the signal-to-noise loss for causal filters with given approximating characteristics (Kaiser, Barlett-Khan, Tukey, etc.) is carried out in a trivial way; therefore, it is not advisable to consider this assessment method in this work.

The expressions do not take into account the constellation diagram, so the following contains developed expressions that take this fact into account.

As follows from the researches [3,5], the analytical model for determining the probability of error per digit (Posh) when demodulating signals with QAM (\(\chi = 1\)) for frequency selection filters is also valid for matched filters, taking into account the case if the channel memory does not exceed 10 symbols: for QAM-16, QAM-64, QAM-256, QAM-1024, QAM-4096:
\[ P_{osh}(g) = 1 - \frac{4}{L} \times \left( 4 \times 0.25 \times \left( 1 + \Phi \left( \frac{f(g)}{gA} \right) \right)^2 \right) + 4 \times \left( \frac{L}{4} - 2 \right) \times 0.5 \times \left( 1 + \Phi \left( \frac{f(g)}{gA} \right) \right) \times \Phi \left( \frac{f(g)}{gA} \right) + \left( \frac{L}{4} - 2 \right) \times \left( \Phi \left( \frac{f(g)}{gA} \right) \right)^2 \] ; (5)

for QAM-32, QAM-128, QAM-512, QAM-2048:

\[ P_{osh}(g) = 1 - \frac{4}{L} \times \left( 5 \times 0.25 \times \left( 1 + \Phi \left( \frac{f(g)}{gA} \right) \right)^2 \right) + \left( 12 \times \sqrt{2(M-5)} - 10 \right) \times 0.5 \times \left( 1 + \Phi \left( \frac{f(g)}{gA} \right) \right) \times \Phi \left( \frac{f(g)}{gA} \right) + \left( \frac{L}{4} - 5 - \left( 12 \times \sqrt{2(M-5)} - 10 \right) \right) \times \left( \Phi \left( \frac{f(g)}{gA} \right) \right)^2 ; (6)\]

where \( q_1 = U_1^2/\sigma^2 = f(g) \) is minimum signal-to-noise ratio; 
\( U_1^2 \) is the minimum peak value of signal vector; 
\( \sigma^2 \) is noise dispersion; 
\( g \) is the average signal-to-noise ratio; 
\( \Phi(x) \) is Kramp function; 
\( L \) is the number of signal locations in signal space; 
\( M \) is modulation ratio; 
\( gA \) is the decrease value of the average signal-to-noise ratio due to the influence of AFC irregularity (7), (8):

for QAM-16, QAM-64, QAM-256, QAM-1024, QAM-4096:

\[ gA = 1 + \left( 2 \left( \frac{L}{4} - 1 \right) + 1 \right) - \frac{2 \left( \frac{L}{4} - 1 \right) + 1}{G_{achh}} \] ; (7)

for QAM-32, QAM-128, QAM-512, QAM-2048:

\[ gA = 1 + \frac{\sqrt{52 \cdot 2^{M-5} - 20 \cdot \sqrt{2^{M-5}} + 2}}{G_{achh}} - \frac{\frac{52 \cdot 2^{M-5} - 20 \cdot \sqrt{2^{M-5}} + 2}{G_{achh}}}{G_{achh}} \] . (8)

It should be noted that the above analytical models are valid given that the signal points are statistically independent, the appearance of symbols in the constellation is equally probable, and the regularity ratio of the constellation diagram is \( \chi = 1 \).

When a channel memory is over 10 characters, the counting time increases exponentially, so the error probability is advisable to calculate in accordance with the following expression [1,4-9]:

\[ P_{osh} = \frac{1}{2} - \frac{1}{\pi} \int_0^X \frac{\sin(uh_0)}{u} \exp(-0.5u^2\sigma^2) \prod_{k=0}^{X} \cos(uh_k)du, \] (9)

where \( u \) is linear system input voltage; 
\( h_0 \) is the response of system linear part in the \( k \)-th sample to the impact of a single pulse with a unit amplitude; 
\( \sigma \) is root-mean-square deviation.

In order to take into account the irregularity ratio of constellation diagram \( \chi \) of signals from QAM to determine the error per digit probability, it is necessary to put a correction value to the average signal-to-noise ratio (table 1) in the expressions (5), (6).
### Table 1. Correction values to the average signal-to-noise ratio for determination of error per digit probability taking this into account the irregularity ratio of constellation diagram.

| Positionality of constellation diagram (L) | Irregularity ratio of constellation diagram (χ) | Correction value (k) | Positionality of constellation diagram (L) | Irregularity ratio of constellation diagram (χ) | Correction value (k) |
|-----------------------------------------|-----------------------------------------------|---------------------|-----------------------------------------|-----------------------------------------------|---------------------|
|                                         |                                               | 1                   |                                         |                                               | 1                   |
|                                         |                                               | 2                   |                                         |                                               | 2                   |
|                                         |                                               | 3                   |                                         |                                               | 3                   |
|                                         |                                               | 4                   |                                         |                                               | 4                   |
|                                         |                                               | 5                   |                                         |                                               | 5                   |
|                                         |                                               | 1                   |                                         |                                               | 1                   |
|                                         |                                               | 2                   |                                         |                                               | 2                   |
| 16                                      |                                               |                      |                                         |                                               |                      |
|                                         |                                               | 3                   |                                         |                                               | 3                   |
|                                         |                                               | 4                   |                                         |                                               | 4                   |
|                                         |                                               | 5                   |                                         |                                               | 5                   |
|                                         |                                               | 1                   |                                         |                                               | 1                   |
|                                         |                                               | 2                   |                                         |                                               | 2                   |
| 64                                      |                                               |                      |                                         |                                               |                      |
|                                         |                                               | 3                   |                                         |                                               | 3                   |
|                                         |                                               | 4                   |                                         |                                               | 4                   |
|                                         |                                               | 5                   |                                         |                                               | 5                   |
|                                         |                                               | 1                   |                                         |                                               | 1                   |
|                                         |                                               | 2                   |                                         |                                               | 2                   |
| 256                                     |                                               |                      |                                         |                                               |                      |
|                                         |                                               | 3                   |                                         |                                               | 3                   |
|                                         |                                               | 4                   |                                         |                                               | 4                   |
|                                         |                                               | 5                   |                                         |                                               | 5                   |
|                                         |                                               | 1                   |                                         |                                               | 1                   |
|                                         |                                               | 2                   |                                         |                                               | 2                   |
| 1024                                    |                                               |                      |                                         |                                               |                      |
|                                         |                                               | 3                   |                                         |                                               | 3                   |
|                                         |                                               | 4                   |                                         |                                               | 4                   |
|                                         |                                               | 5                   |                                         |                                               | 5                   |
|                                         |                                               | 1                   |                                         |                                               | 1                   |
|                                         |                                               | 2                   |                                         |                                               | 2                   |
| 4096                                    |                                               |                      |                                         |                                               |                      |
|                                         |                                               | 3                   |                                         |                                               | 3                   |
|                                         |                                               | 4                   |                                         |                                               | 4                   |
|                                         |                                               | 5                   |                                         |                                               | 5                   |

The proposed mathematical models make it possible to calculate the error probabilities of the average signal-to-noise ratio for certain causal filters. The results of calculations for QAM-64, -1024 (χ=1) are presented in figures 1-2.
Figure 1. Dependences of the error per digit probability on the average signal-to-noise ratio when signals from QAM-64 ($\chi = 1$) pass through digital causal filters approximated by the Kaiser characteristic ($\beta = 9$) of the following orders: 80, 100, 120.

Figure 2. Dependences of the error per digit probability on the average signal-to-noise ratio when signals from QAM-64 ($\chi = 1$) pass through digital causal filters approximated by the Kaiser characteristic ($\beta = 9$) of the following orders: 80, 100, 120.
As it follows from the researches [1, 2, 10, 11-13], EEL is calculated by the following formula:

\[
EEL = q(X, L) - q(0, L(M)), dB, \text{when } Posh = \text{const},
\]

where \(X\) is AFC irregularity [dB].

From the above, we have formed a procedure to assess the influence of approximating function type and the causal filter order on the equivalent energy losses:

1. initial data: modulation ratio \((M)\) (constellation diagram positionality \((L)\)), AFC irregularity of linear time-invariant causal system \((X)\), probability of the system providing a given error per digit \((Posh)\), irregularity ratio of the constellation diagram \((\chi)\):
   - step 1: calculation of correction value for given irregularity of the constellation diagram and positionality (ratio);
   - step 2: calculation of the probability of error per digit from the average value of the signal-to-noise ratio when a given signal passes through the ideal causal filter of the demodulating device, taking into account the correction values;
   - step 3: calculation of the probability of error per digit from the average value of the signal-to-noise ratio when a given signal passes through the real causal filter with given AFC irregularity, taking into account the correction values;
   - step 4: determination of EEL, taking into account the probability of error per digit \((Posh)\), which is required by regulatory requirements with regard to ensuring proper quality;

result: the value of equivalent energy loss in causal filter [14].

4. Discussion of the results

The analysis of the plotted dependencies illustrates that, if there is a probability of error per digit of \(Posh = 10^{-6}\), EEL, for causal filters approximated by the Kaiser function \((\beta = 9)\), when passing signals from QAM-64, were 3.95; 4.32; 5.26 dB, at orders of 80, 100, 120.

Similarly, for signals from QAM-1024 with a probability of error per digit of \(Posh = 10^{-6}\), EEF for causal filters approximated by the Kaiser function \((\beta = 7)\), were 2.86; 3.18; 4 dB, at orders of 80, 100, 120, respectively.

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