Evolution of perturbations in distinct classes of canonical scalar field models of dark energy

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Dark energy must cluster in order to be consistent with the equivalence principle. The background evolution can be effectively modeled by either a scalar field or by a barotropic fluid. The fluid model can be used to emulate perturbations in a scalar field model of dark energy, though this model breaks down at large scales. In this paper we study evolution of dark energy perturbations in canonical scalar field models: the classes of thawing and freezing models. The dark energy equation of state evolves differently in these classes. In freezing models, the equation of state deviates from that of a cosmological constant at early times. For thawing models, the dark energy equation of state remains near that of the cosmological constant at early times and begins to deviate from it only at late times. Since the dark energy equation of state evolves differently in these classes, the dark energy perturbations too evolve differently. In freezing models, since the equation of state deviates from that of a cosmological constant at early times, there is a significant difference in evolution of matter perturbations from those in the cosmological constant model. In comparison, matter perturbations in thawing models differ from the cosmological constant only at late times. This difference provides an additional handle to distinguish between these classes of models and this difference should manifest itself in the ISW effect.

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I. INTRODUCTION

Various observation have confirmed that the expansion rate of the universe is accelerating [1]. These observations include those of Supernova type Ia [2], observations of Cosmic Microwave Background [3, 4] and large scale structure [5]. The accelerated expansion of the universe can be explained by introducing a cosmological constant Λ in the Einstein’s equation [6, 7]. However, the cosmological constant model is plagued by the fine tuning problem [8]. This has motivated the study of dark energy models to explain the current accelerated expansion of the universe (for reviews see [8]). An alternative to the cosmological constant model is to assume that this accelerated expansion is driven by a canonical scalar field with a potential $V(\phi)$, namely the quintessence field [9–14]. There exists another class of string theory inspired scalar field dark energy models known as tachyon models [15, 16] and there are models which allow $w < -1$ are known as phantom models [20]. Phantom type dark energy can also be realized in a scalar tensor theory of gravitation [21]. Other scalar field models include k-essence field [17], branes [22] and fluid models like the Chaplygin gas model and its generalizations [23]. There are also some phenomenological models [27], field theoretical and renormalization group based models (see e.g. [28]), models that unify dark matter and dark energy [29], holographic dark energy models [27], QCD dark energy [30] and many others like those based on horizon thermodynamics (e.g. see [31]).

Different models of dark energy which have the same background evolution are indistinguishable purely from distance measurements. Evolution of perturbations in these models is expected to break this degeneracy. The Integrated Sachs Wolfe (ISW) effect can distinguish a cosmological constant from other models of dark energy, especially ones with a dynamical dark energy [16]. Dark energy perturbations have been extensively studied in the linear approximation [24, 32–34]. Perturbations in dark energy affect the low $l$ quadrupole in the CMB angular power spectrum through the ISW effect. For models with $w > -1$ this effect is enhanced while for phantom like models it is suppressed. In these models dark matter perturbations and dark energy perturbations are anti-correlated for large effective sound speeds. This anti-correlation is a gauge dependent effect [32]. There are several other studies of perturbations in dark energy [35], including some that deal with evolution of spherical perturbations [36, 57]. For canonical scalar field dark energy, the perturbations in matter are enhanced by the presence of dark energy perturbations in comparison with smooth dark energy model [10]. The matter perturbations in fluid models are suppressed compared to corresponding homogeneous dark energy scenario [11]. As long as the speed of propagation of perturbations $c_s^2$ is positive the evolution of matter perturbations is indistinguishable from a smooth dark energy model. This is true for scales smaller then the Hubble radius. Dark energy perturbations in a fluid model with an appropriate $c_s^2$ emulate that of a scalar field model very well below the Hubble scale but start to differ at larger scales. Therefore the fluid model is not

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a good approximation at these scales. This also implies that the growth of perturbations at large scales depends on the details of the model even though the background evolution is the same. A separate analysis is therefore required for every model.

In this paper we consider different scalar field models to study evolution of dark energy perturbations. We consider two different types of potentials, classified as ‘thawing’ and ‘freezing’ in Ref. [38]. For potentials with a thawing behavior, the scalar field is frozen at early times and starts to roll down the potential at late time. Hence the equation of state of dark energy starts near \( w = -1 \) at early times to \( w > -1 \) at late times. In contrast, in the case of potentials with freezing behavior, the scalar field rolls down the potential and approaches the mini-

In this case we can directly relate the metric perturbation \( \delta \phi \) to the gravitational potential perturbation. In the absence of anisotropic stress, the perturbed metric can be written in the form

\[
ds^2 = (1 + 2\Phi) dx^2 - a^2(t) [(1 - 2\Phi)\delta_{\alpha\beta} dx^\alpha dx^\beta]
\]

where \( \Phi \) is the gauge invariant potential defined in [38].

The linearized Einstein equations obtained from the above metric are given by

\[
\frac{k^2}{a^2}\Phi + \frac{\ddot{a}}{a} \Phi + \frac{\dot{a}^2}{a^2} \Phi = -4\pi G \left[ \rho_{NR}\delta_{NR} + \rho_{DE}\delta_{DE} \right]
\]

\[
\Phi + \frac{\dot{a}}{a} \Phi = -4\pi G \left[ \rho_{NR}\nu_{NR} + \rho_{DE}\nu_{DE} \right]
\]

\[
4\frac{\dot{a}}{a} \Phi + 2\frac{\ddot{a}}{a} \Phi + \frac{\dot{a}^2}{a^2} \Phi + \ddot{\Phi} = 4\pi G \delta P
\]

where dot denotes derivative with respect to the coordinate time \( t \) and \( v_{NR} \) is the potential for the matter peculiar velocity such that \( \delta u_i = \nabla_i v_{NR} \). In these equations, we have Fourier decomposed the perturbed quantities such as \( \Phi, \delta \phi, \delta_{NR} \) and \( v_{NR} \) and replaced \( \nabla^2 \) by \(-k^2\), where \( k \) is the wave number defined as \( k = 2\pi/\lambda \) with \( \lambda \) being the comoving length scale of perturbation. In these equations all the perturbed quantities correspond to the amplitude of perturbations in the \( k^0 \)th mode.

We assume a flat spatial cosmology and we choose the initial equation of state \( w \) to be very close to \(-1 \) at early times (say redshift of \( z_i = 100 \)). The initial value of the scalar field needs to be fine tuned such that universe begins to accelerate at late times. We fine tune the parameters such that the matter density parameter is within the range allowed by the present day observations. For a system of pressureless matter and a scalar field, the dynamics of perturbations is uniquely determined by \( \Phi(t) \) and \( \delta \phi(t) \). Hence we need two second order equations which connect \( \Phi(t) \) and \( \delta \phi(t) \). As one of these equations, we choose the third equation in system (2). The dynamical equation for the perturbations in the scalar field \( \delta \phi(t) \) is given by:

\[
\delta \phi + \frac{\dot{a}}{a} \delta \phi + \frac{k^2\delta \phi}{a^2} + 2\Phi V'(\phi) - 4\Phi \dot{\phi} + V''(\phi) \delta \phi = 0
\]

These equations complete the system and we can then calculate the fractional density perturbation for dark energy and for nonrelativistic matter as

\[
\delta \phi = \frac{1}{2\dot{\phi}^2 + V(\phi)} \left[ \Phi \delta \phi - \Phi V'(\phi) \delta \phi \right]
\]

\[
\delta_{NR} = -\frac{1}{4\pi G\rho_{NR}a^{-3}} \left\{ 3\frac{\dot{a}^2}{a^2} \Phi + 3\frac{\dot{\phi}}{a} \Phi + \frac{k^2\Phi}{a^2} \right\}
\]

\[
+ \frac{\delta \phi}{\rho_{NR}a^{-3}} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right]
\]

To solve the equations numerically, we introduce the following dimensionless variables

\[
\Phi_N = \frac{\Phi}{\Phi_{in}}, \quad \delta y = \frac{\delta \phi}{\Phi_{in}M_p}
\]

Here \( \Phi_N \) is the normalized gravitational potential with \( \Phi_{in} \) being the value of the metric potential at the initial time \( t = t_{in} \). In terms of these two new variables, the equations are

\[
\Phi''_N + \frac{4s'}{s} \Phi'_N + \left( 2\frac{s''}{s} + \frac{s'^2}{s^2} \right) \Phi_N = 0
\]

\[
\delta y'' + 3\frac{s'}{s} \delta y' - 4\Phi_N y'' + \frac{k^2\delta y}{s^2}
\]

\[
+ 2\Phi_N V'(y) + V''(y) y' = 0
\]
FIG. 1: The figure shows evolution of nonrelativistic matter density contrast as a function of the scale factor for the exponential potential (TH1). The thick solid line is the evolution of density contrast in the cosmological constant model. The solid line is for a homogeneous dark energy scenario whereas the dashed line show the evolution if dark energy perturbations are taken into account.

Here \( \bar{k} = kc/H_0 \), is the wave number scaled with respect to the Hubble radius, \( H_0 \) being the Hubble parameter at the present epoch. The prime denotes derivative with respect to the variable \( tH_0 \).

We assume that the perturbation in the scalar field in the matter dominated epoch is negligibly small compared to other perturbed quantities such as \( \Phi \) and \( \delta_{NR} \). Hence we can treat the scalar field to be initially homogeneous. This corresponds to setting the initial condition \( \delta y_{in} = 0 \) and \( \delta y_{in}' = 0 \). The only initial condition that needs to be determined is the value of \( \Phi_{Nin} \) at \( t = t_{in} \). In the matter dominated epoch, the gravitational potential \( \dot{\Phi}(t) = 0 \) for all values of the wave number \( k \), therefore we can set the initial condition \( \Phi_{Nin}'(k) = 0 \).

If dark energy is a cosmological constant, then it does not cluster and the gravitational field \( \Phi \) decays when the cosmological constant dominates. For \( w \neq -1 \) too, the potential \( \Phi \) remains constant in the matter dominated era and starts to decay when dark energy contribution becomes significant. This rate of decay is dependent on the value of \( w \). For a canonical scalar field, dark energy perturbations are correlated with the matter perturbations and they enhance matter perturbations. If dark energy is a barotropic fluid, the matter perturbations are suppressed as compared to the corresponding homogeneous model. This difference is due to the fact the homogeneous limit is achieved differently for the two models. For a scalar field, even if one ignores spatial gradients there still remains a contribution to \( \delta P \). This contribution cancels with a corresponding contribution from the pressure term due to the background. This term cancels for homogeneous scalar field dark energy and leads to a suppression in matter perturbations. For fluid models \( \delta P \) vanishes and the residual pressure term due to the background evolution makes the evolution different from that in a scalar field. Therefore, assuming dark energy to be homogeneous leads to a large difference in matter density contrast.

For sub-Hubble scales, a fluid model effectively emulates a scalar field model. At scales smaller then the Hubble radius, dark energy can be assumed to be homogeneous. It is sufficient to assume a fluid approximation and the number counts of clusters at different redshifts distinguish deviation from the cosmological constant. At larger scales, where dark energy perturbations may play a significant role, the fluid analogy breaks down and the evolution of matter density contrast depends on individual scalar field models. This also implies that the growth of perturbations at large scales depends on the details of the model even though the background evolution is the same. A separate analysis is therefore required for every model. Since dark energy perturbations are significant at large scales, i.e., scales larger then Hubble radius, we will restrict the subsequent discussion to large scales.
In particular, the reference scale we choose here is the length scale $\lambda = 10^5 \text{ Mpc}$.

The scalar field models have been classified as freezing or thawing in [39]. The classification is based on the difference in the way equation of state of dark energy evolves. The freezing type models are fast roll models with a steep potential. The field remains subdominant and at late times becomes dominant and drives the acceleration of the expansion of the universe. The thawing models have a potential which is nearly flat. Therefore, the equation of state starts with the value $w = -1$ at early times and the energy density of the field is negligible. As the universe expands, the energy density of the field becomes comparable to the background energy density. The equation of state of dark energy deviates away from its frozen value, hence the scalar field thaws.

The dark energy equation of state evolves differently in these classes of models. Therefore, the evolution of perturbations in freezing type and thawing type models is expected to be different. Using present day distance measurement observations, it is not possible to distinguish between different dark energy models from the $\Lambda$CDM model and also between various scalar field models. These observations include Present day Supernova observations and data from the Baryon Acoustic Oscillations measurements [27]. This degeneracy is expected to be removed if dark energy perturbations are included.

The difference in these models is expected to make a significant contribution to integrated effects such as the ISW effect [16].

To study scalar field perturbations, we consider the following potentials:

1. For 'thawing' behavior
   - (a) TH1: Exponential potential:
     \[ V(\phi) = M^4 e^{-\sqrt{\alpha} \phi / M} \]
   - (b) TH2: Polynomial (concave) potential:
     \[ V(\phi) = M^4 + n \phi^n \]

2. For 'freezing' behavior
   - (a) FR1: Inverse power potential:
     \[ V(\phi) = M^4 + n \phi^{-n} \]
   - (b) FR2: \[ V(\phi) = M^4 + n \phi^{-n} \exp(\alpha \phi^2 / M^2) \]

We have labeled the scalar field potentials as TH (thawing) and FR (freezing), and henceforth will be using these labels to refer to the models. Since Einstein equations imply homogeneous dark energy to be inconsistent, we will compare the evolution of perturbations in scalar field dark energy with the cosmological constant. We have however also considered the homogeneous limit in all these models.

The growth of perturbations in scalar field model with an exponential potential (TH1) has been studied in [40].
FIG. 3: The figure shows evolution of nonrelativistic matter density contrast as a function of the scale factor for the inverse $\phi$ potentials (FR1) for the parameter values mentioned.

For this potential, if $\alpha = 1$, the present day equation of state is $w = -0.86$ and the acceleration of the universe starts at redshift $z = 0.81$. For smaller $\alpha$ the models $w$ is smaller, for instance if $\alpha = 0.1$, the equation of state at present is $w = -0.95$. In Fig. 1 we plot the evolution of matter perturbations at scale $\lambda = 10^5$ Mpc as a function of the scale factor $a$. For smaller $\alpha$ the present equation of state is closer to that of a cosmological constant, there is no significant difference if we assume dark energy to be homogeneous or if we assume dark energy to be clustered. If $\alpha = 1$ there is a significant enhancement of matter perturbations [40]. Compared to the cosmological constant model, matter perturbations in this model are suppressed, with approximately 6% deviation.

In Fig. 2 we show the evolution of matter density contrast for the concave potential (TH2) with $n = 1$ and $n = 2$. The present day of equation of state is $w \approx -0.94$ if we choose $n = 1$ and reaches $w \approx -0.3$ for $n = 2$. If the potential is quadratic, the gravitational potential initially decays, and in the future starts to oscillates. This is due to the fact that the scalar field reaches the minimum of the potential and the equation of state for dark energy then oscillates between $w = -1$ and $w = 1$. These oscillations are evident at earlier times at very large scales, say at $\lambda = 10^5$ Mpc. At large scales, there is a significant enhancement in matter perturbation as compared to the case when dark energy is assumed to be homogeneous. In contrast, the gravitational potential $\Phi$ continues to decay if $n = 1$. For $n = 1$ the matter perturbations are suppressed as compared to the potential with $n = 2$ but they continue to grow and become larger when oscillations set in the case of the quadratic potential. Matter perturbations in linear potential follow those in cosmological constant model very closely. In quadratic scalar field potential case, these perturbations are enhanced compared to the $\Lambda$CDM model at early times and are suppressed at late times. At $a \approx 0.4$, the percentage enhancement as compared to the cosmological constant model is $\approx 5\%$.

For potential FR1, the scalar field begins to roll down the potential at early times and begins to freeze at late times. In this freezing type evolution, the equation of motion of dark energy starts away from $w = -1$ and approaches this value as the scalar field slows down. The epoch at which this freezing occurs is subject to fine tuning of parameters. If the scalar field is frozen before the present time, the duration of the matter dominated phase is then too small, i.e., scalar field approaches the kinetic energy dominated phase before reaching $z = 1000$. To achieve a sufficiently prolonged matter dominated phase, the scalar field should freeze in far future. The dark energy equation of state reaches a maximum of approximately $-0.7$, and we tune the initial value of $\phi$ such that we achieve $\Omega_{NR} \approx 0.3$ at present. In Fig. 3 we show the evolution of matter density contrast with scale factor. There is an enhancement in matter perturbations with an increasing value of $n$. We have plotted the density contrast for $n = 1$ and for $n = 2$. In this model, since
the value of equation of state deviates away from \( w = -1 \) at early times, there is a significant enhancement in dark energy perturbations and hence in matter perturbations. With \( n = 1 \), perturbations in nonrelativistic matter are comparable to those in cosmological constant model, and with \( n = 2 \) they are enhanced by approximately 7% at \( a \approx 0.3 \) with respect to the cosmological constant model.

For the potential FR2, the freezing behavior is achieved earlier without trading away a viable duration of the matter dominated era. For a given \( n \), the present value of the equation of state moves further away from \( w = -1 \) as \( \alpha \) increases. Also for a constant \( \alpha \), the value of the equation of state increases with an increase in \( n \). As in all other models, at very early time we assume \( w = w_{\Lambda} \). For instance, if \( \alpha = 0.6 \), for \( n = 1 \) reaches \( w \approx -0.9975 \) at redshift \( \approx 0.4 \) and then starts to approach the freezing behavior. If \( \alpha = 0.8 \) and \( n = 4 \), the maximum value the equation of state reaches is \( w = -0.82 \). In Fig. 4 we show the evolution of matter density contrast at \( \lambda = 10^5 \) for two different parameter sets and also in comparison to that in ΛCDM model.

In Fig. 5 and Fig. 6 we show evolution of matter perturbations in different models considered here at different length scales. At small scales, say at \( \lambda = 50 \) Mpc, the evolution is indistinguishable from that in a homogeneous model of dark energy. The evolution of perturbations in different in different scalar field models but the limit of a smooth dark energy works well at small length scales. At this scale, except for the inverse power law potential model, evolution in the scalar field models closely follows that in the cosmological constant model. The fluid approximation too works well at these scales. Explicit dependence of dark energy perturbations is significant at large scales.

At scales larger than the Hubble radius, the evolution of perturbed quantities are significantly different from those in homogeneous dark energy model. In Fig. 5 we plot the gravitational potential and matter density contrast at length scales \( \lambda = 10^4 \) and \( \lambda = 10^5 \) Mpc. The freezing models of scalar field, at early times have significant enhancement in matter perturbations at early times. There is a significant enhancement of matter perturbations as compared to the other models. This enhancement is more for models with higher \( n \). Among the scalar field models considered here, models FR1 have the highest rate of growth of perturbations. The dashed line denotes FR2, with \( n = 2 \) and \( \alpha = 0.8 \). As compared to the cosmological constant, FR2 has a higher rate of growth of nonrelativistic matter density contrast. The dotted line corresponds to the potentials TH2 with \( n = 2 \). However, the quadratic potential shows a larger growth rate at early times and then shows a downward trend when the scalar field approaches the minimum of the potential. At early times, TH2 has more enhancement in matter density contrast than the FR2 potential. This is because the dark energy equation of state devi-
FIG. 5: This figure shows evolution of the gravitational potential $\Phi$ and matter density contrast $\delta_{NR}$ as a function of scale factor for the four different models at length scale $\lambda = 50$ Mpc. The solid line shows the evolution for the scalar field potential $V(\phi) = M_4^{4+n} \phi^{-n}$ (FR1) with $n = 2$. The dashed line is for the potential $V(\phi) = M_4^{4+n} \phi^{-n} \exp(\alpha \phi^2/M_4^2)$ (FR2) with parameters $\alpha = 0.8$ and $n = 4$. The dotted line corresponds to the thawing exponential potential $V(\phi) = M_4^{4-n} \phi^n$ (TH1) where $\alpha = 1$ and the dot dashed line corresponds to the polynomial potential $V(\phi) = M_4^{4-n} \phi^n$ (TH2) for $n = 2$. At early times, the freezing potentials have matter perturbations enhanced as compared to the thawing potentials. The evolution at this scale is same for a homogeneous dark energy and for a clustering dark energy. The thick solid line corresponds to the $\Lambda$CDM model.

ates from that of a cosmological constant for the polynomial potential (TH2) more than for the inverse power exponential potential (FR1). At late times, for TH2 the scalar field reaches the minimum of the potential and the equation of state oscillates between $-1$ and $+1$ and the perturbations in dark energy and consequently matter perturbations begin to oscillate in future. The late time suppression shown in the figure is due to the beginning of the oscillatory phase. In the case of an exponential potential (TH1), matter density contrast is suppressed compared to all other models.

III. CONCLUSIONS

In this paper, we analyze the growth of perturbations in scalar field dark energy scenarios. The assumption that the distribution of dark energy (with $w \neq -1$) is homogeneous at all length scales is inconsistent with the equivalence principle. On length scales comparable to or greater than the Hubble radius, the perturbations in dark energy can become comparable to perturbation in matter if $w_{de} \neq -1$. For scales smaller than the Hubble radius, perturbations in dark energy can be neglected in comparison with the perturbation in matter at least in the linear regime. Hence any deviations from the cosmological constant model can be explored by assuming dark energy as a homogeneous component and the scalar field models can be approximated well by parameterized fluid models. However, on much larger scales, if the equation of state parameter deviates from -1, then perturbations in dark energy do influence matter power spectrum.

For canonical scalar fields, a clustering dark energy enhances matter perturbations as compared to the corresponding homogeneous dark energy scenario. This enhancement more pronounced at scales larger than the Hubble radius. In particular, we study two broad classes of canonical scalar field models, namely the thawing and freezing models. The equation of state of dark energy evolves differently in these two classes of models. This affects the growth of perturbations in these different types. For fast rolling, freezing type models, the dark energy equation of state deviates away from that of a cosmological constant at early times and freezes to $w = -1$ at late times. In slow roll thawing models, the scalar field remains at a constant $w = -1$ and starts to deviate away from this value at late times. The present day observations, which are based on distance measurements, cannot
FIG. 6: The upper panel in this figure shows evolution of the gravitational potential $\Phi$ and matter density contrast $\delta_{NR}$ as a function of scale factor for the four different models at length scale $\lambda = 10^4$ Mpc. The line styles used for different models are same as in Fig. 5. At early times, the freezing potentials have matter perturbations enhanced as compared to the thawing potentials. The lower part of the figure shows the evolution of gravitational potential and matter density contrast at length scale $\lambda = 10^5$. 
distinguish between these models.

We studied evolution of matter perturbations in the presence of a clustering dark energy. The models we have considered in this paper are within the range allowed by distance measurement observations. For corresponding homogeneous dark energy models and clustering dark energy models, there are significant changes in the matter density contrast evolution. All the canonical scalar field models studied here show an enhancement in matter perturbations if dark energy is perturbed. Although we have not considered phantom like models in this paper, it is worth mentioning that in these models, dark energy perturbations suppress matter perturbations.

In general, freezing type models have a higher rate of growth of density contrast at early times. This is due to the fact that in these models, the equation of state of dark energy perturbations suppress matter perturbations.

For thawing type models, in these models, the equation of state of dark energy perturbations suppress matter perturbations. For most of the evolution, the matter perturbations remain close to those in cosmological constant model and being to deviate as the field begins to thaw. There are significant deviations in the way density contrast grows, not only between various models but also from the concordant cosmological constant model. Apart from different way matter perturbations grow in the freezing and thawing classes of models, models within the same class also have variation in the behavior of the density contrast. Observable changes in the angular power spectrum at large scales are limited by the cosmic variance and therefore CMB data alone will be insufficient to distinguish between these models. Since the scales at which dark energy perturbations are relevant are large, the primary contribution to CMB anisotropies is through the ISW effect. Therefore it is important to study the ISW cross correlation with large scale structure indicators.

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