ERRATA

One-Loop Threshold Effects in String Unification

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As originally published, the article contained a few minor errors. Unfortunately, these
errors have lately caused a not-so-minor confusion among several physicists working on the
phenomenology of string unification. To avoid further confusion, I would like to make the
following corrections:

(1) The renormalization scheme referred throughout the article as \( \overline{\text{MS}} \) is in fact the modified
minimal subtraction scheme for the \textit{dimensional reduction} and not the standard dimensional
regularization. The proper name for the scheme I used is \( \overline{\text{DR}} \).

(2) In the second paragraph on page 154 the formula for \( \xi'' \) should be
\[
\xi'' = 1 + \log\left(\frac{2}{\sqrt{27\pi}}\right) - \gamma \approx -1.6767
\]
instead of \( \xi'' = 1 - \log\left(\frac{2}{\sqrt{27\pi}}\right) - \gamma \approx -0.532 \).

The rest of the mistakes involve misplaced factors of 2:

(3) Formula (26) for the effective scale of string unification should read
\[
M_{\text{GUT}} \equiv \frac{2e^{\left(1 - \gamma\right)/3 - 3/4}}{\sqrt{2\pi\alpha'}} \approx \frac{e^{(1 - \gamma)/3 - 3/4}}{4\pi} g_{\text{string}} M_{\text{Planck}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17} \text{ GeV.} \quad (26)
\]

(In the original publication, the factor 2 shown here in the boldface was missing.) Note
that the correct formula here uses the tree-level relation \( k_a g_a^2 = g_{\text{GUT}}^2 = 32\pi/\alpha' M_{\text{Planck}}^2 \)
which differs by a factor of 2 from a similar formula given in ref. [6]. This difference is
due to different normalization conventions for the gauge generators \( Q_a \) and gauge couplings

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$g_a$ — unlike Ginsparg, I used the phenomenological convention\(^\S\) throughout the paper and adjusted the string-theoretical formulæ to work with this convention.

(4) In formulæ (1), (2), (7) and (24) coefficients printed as $4\pi^2$ should be $16\pi^2$. Similarly, in formulæ (5) and (21) coefficients in front of the respective integrals should be $1/16\pi^2$ instead of $1/4\pi^2$ and in formula (16) the coefficient printed as $2\alpha'^2$ should be $\alpha'^2/2$.

(5) In formulæ (6) and (23), the expressions for the coefficients $b_a$ of the low-energy $\beta$-functions were too small by a factor of 2. Correspondingly, the field-theoretical functions $B_a(t)$ and the string-theoretical functions $B_a(\tau, \bar{\tau})$ also missed that factor. The correct form of the field-theoretical eq. (5) is

$$W_a^{\text{field}} = \frac{1}{16\pi^2} \int_0^\infty \frac{dt}{t} C_\Lambda(t) \cdot \left[ B_a(t) \overset{\text{def}}{=} 2 \text{str} \left( Q_a^2 \left( \frac{1}{12} - \chi^2 \right) e^{-tM^2} \right) \right]. \quad (5)$$

Consequently, the super-traces in eqs. (7) and (8) should also be multiplied by two; in particular, the correct formula for the one-loop threshold corrections in GUTs is

$$\Delta_a = 2 \text{str}_{M \sim M_{\text{GUT}}} \left( Q_a^2 \left( \frac{1}{12} - \chi^2 \right) \log \frac{M_{\text{GUT}}^2}{M^2} \right). \quad (8)$$

Similarly, the correct form of the string-theoretical formula (22) is

$$B_a(\tau, \bar{\tau}) = \frac{2}{|\eta(\tau)|^4} \sum_{\text{even } s} (-)^{s_1 + s_2} \frac{dZ_\Psi(\bar{\tau}, s)}{2\pi i d\bar{\tau}} \cdot \text{Tr}_{s_1} \left( Q_a^2 \cdot (-)^{s_2} F q^H \bar{q} \bar{H} \right)_{\text{int}}. \quad (22)$$

The relation (25) between these $B_a$ functions and the

\(^\S\) Most phenomenologists normalize the non-abelian gauge generators to $\text{tr}(Q_a^2) = \frac{1}{2}$ where the trace is taken over the fundamental representation of an $SU(N)$ group such as $SU(3)_{\text{color}}$ or $SU(2)_{\text{weak}}$; correspondingly, the generators of a GUT group are normalized to $\text{tr}_5(Q_a^2) = \frac{1}{2}$ for the $SU(5)$ or $\text{tr}_{10}(Q_a^2) = 1$ for the $SO(10)$. On the other hand, many string theorists normalize the generators to $\text{tr}(Q_a^2) = 2$ where the trace is taken over a vector representation of an $SO(2N)$ group such as $SO(32)$ or $SO(16) \subset E_8$; according to this convention, phenomenologically-normalized $Q_a$ should be multiplied by $\sqrt{2}$. In both conventions, the gauge connection $A_\mu$ is $igQ_aA_\mu^a$, $A_\mu^a$ being canonically-normalized vector fields; therefore, the string-theoretical convention has to compensate for the $Q_a$ being bigger by a factor $\sqrt{2}$ by having the gauge couplings $g$ being smaller by the same factor.
string threshold corrections $\Delta_a$ remains unchanged, which means that the actual values of $\Delta_a$ should be doubled.

As to the formulæ (6) and (23) for $b_a$ and the infra-red limits of $B_a(t)$ and $B_a(\tau, \bar{\tau})$, in addition to missing an overall factor of 2, they also used inconsistent conventions for the traces. The correct formulæ should read

$$B_a(t) \xrightarrow{t \to \infty} -\frac{11}{3} \text{tr}_{V,M=0}(Q_a^2) + \frac{2}{3} \text{tr}_{F,M=0}(Q_a^2) + \frac{1}{3} \text{tr}_{S,M=0}(Q_a^2) \equiv b_a, \quad (6)$$

$$B_a \xrightarrow{\tau \to \infty} -\frac{11}{3} \text{tr}_{V,M=0}(Q_a^2) + \frac{2}{3} \text{tr}_{F,M=0}(Q_a^2) + \frac{1}{3} \text{tr}_{S,M=0}(Q_a^2) = b_a = \lim_{t \to \infty} B_a(t), \quad (23)$$

where the traces are taken over the massless charged particles and count each CPT-conjugate particle-antiparticle pair only once.

Most of the above errors were either obvious or irrelevant to most of the readers of the paper. Unfortunately, the missing factor of 2 in eqs. (22) and (23) was neither, which lead to its propagation via papers concerned with the threshold corrections in specific string models. I would like to use this opportunity to apologize to the authors of those papers for leading them into the error. I would also like to apologize for propagating this error myself, in the article “Moduli-Dependence of String Loop Corrections to Gauge Coupling Constants” I co-authored with L. Dixon and J. Louis (Nuclear Physics B355 (1991), p. 649). Fortunately, only some of the intermediate results of that article are affected by this missing factor of 2; the final results — the relations between $\Delta_a$ of an orbifold and the $b'_a$ ($b_a$ coefficients of the $N = 2$ partial orbifold) are correct, provided one uses the correct definitions of the $b_a$ and $b'_a$ coefficients.

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