Comparative Study of $B_c \to D_s^* \ell^+ \ell^-$ Decays in Standard Model and Supersymmetric Models

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ABSTRACT: A comparative study of the exclusive rare $B_c \to D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) decays has been made in the minimal supersymmetric models (MSSM) and the SUSY SO(10) GUT models. In this context, various physical observables such as branching ratios ($BR$), forward-backward asymmetries ($A_{FB}$), lepton polarization asymmetries ($P_{L,N,T}$) and helicity fractions ($f_{L,T}$) of $D_s^*$ meson by using the the QCD sum rules form factors have been investigated. It is found that the SUSY effects are characteristically prominent to that of the SM values for these observables. For instance, in SUSY I and SUSY II, the forward-backward asymmetry does not cross zero which is mainly due to the same sign of the $C_{eff}^7$ and $C_{eff}^9$ Wilson coefficients. Similarly in SUSY SO(10) GUT models due to the complex nature of the new Wilson coefficients – corresponding to the new operators arising due to the contribution of neutral Higgs bosons (NHBs) – the above mentioned observables are sizably affected. Therefore the analysis of said observables in charmed semileptonic $B$ meson decays can put some stringent constraints on the parameter space of SUSY variants and can serve as a windowpane to look beyond the SM.

KEYWORDS: B-Physics, Supersymmetric Standard Model, Beyond Standard Model
1 Introduction

The standard model (SM) of particle physics is considered to be one of the most successful theories of the twentieth century but we cannot accept it as the ultimate theory of nature since there are many open questions beyond the scope of the SM to be addressed. These questions include: gauge and fermion mass hierarchy, matter-antimatter asymmetry, number of generations of quarks and leptons, the nature of the dark matter and the unification of fundamental forces. Therefore, we need to search new physics (NP) beyond the SM that may help us to answer these open problems of the SM. One of the plausible extensions of the SM is supersymmetry (SUSY) [1] which is a promising candidate of NP and can shed light on some of the issues mentioned above [2]. For example, SUSY solves the hierarchy problem of the SM by the cancellation of the quadratic divergences in the radiative loop corrections since in SUSY fermions and bosons contribute with opposite sign [3, 4]. It is also considered that the lightest supersymmetric particles (LSP) are the dark matter candidates because they are stable and interact very weakly with the ordinary matter. It is also an important ingredient in the superstring theory which is a suitable candidate for the unification of all the known forces including gravity.

There are two ways to search for the SUSY, one is to discover the SUSY particles (sparticles) at high energy colliders directly [5] and the other is to search for its effects through indirect methods. Since no sparticle has been seen so far, they must have higher masses than their SM partners, implying that the SUSY is a broken symmetry. The other

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option is to investigate the SUSY through the indirect searches where SUSY can show its footprints. In this regard, the processes involving the flavor changing neutral currents (FCNC) are the elegant way to probe the indirect searches of the SUSY. So the rare decays, involving FCNC, induced by $b \rightarrow s(d)$ transitions provide a windowpane to look for the physics beyond the SM. This lies in the fact that FCNC processes are not allowed at the tree level in the SM but can occur at the loop level through Glashow-Iliopoulos-Maiani (GIM) mechanism [6] thereby making them quite sensitive to the indirect searches of the SUSY [7, 8]. The sparticles can contribute to FCNC transitions through the quantum loop due to R-parity conservation, hence, making these processes a handy tool for studying the possible effects of sparticles in the rare decays [9–11]. This gives us a good motivation to study the rare decays both theoretically and experimentally to look beyond the SM especially in the LHC era [12, 13].

Since FCNC processes are only possible at the loop level and have small branching ratios as compared to the tree level transitions but they have implications for the results obtained in the ongoing experiments after the observations of the rare radiative $b \rightarrow s\gamma$ decay at CLEO [14]. Since then there have been many studies on the rare semileptonic, radiative and leptonic decays of $B_{u,d,s}$ mesons induced by the FCNC transitions of $b \rightarrow s(d)$ [15]. These studies will be even more complete if one considers similar decays of the charmed $B$ mesons ($B_c$). The charmed $B_c$ meson is a bound state of two heavy quarks, the bottom $b$ and the charm $c$, and was first observed in 1998 at the Tevatron in Fermilab [16]. Because of the two heavy quarks, the $B_c$ mesons are rich in phenomenology compared to the other $B$ mesons. At Large Hadron Collider (LHC) the expected number of events for the production of $B_c$ mesons are about $10^8−10^{10}$ per year [17–22] which is a reasonable number to work on the phenomenology of the $B_c$ meson. In the literature, some of the possible radiative and semileptonic exclusive decays of $B_c$ mesons such as $B_c \rightarrow (\rho, K^*, D_{s,1}^*, B_{u,d,s}^*)\gamma$, $B_c \rightarrow \ell\nu\gamma$, $B_c \rightarrow B_{u,d,s}^*\ell^+\ell^−$, $B_c \rightarrow D_{s,0}^0\ell^+\ell^−$ and $B_c \rightarrow D_{s,d}^*\ell^+\ell^−$ have been studied using the framework of relativistic constituent quark model, QCD Sum Rules, the Light Cone Sum Rules and the Ward identities [23–33].

Theoretically what makes the $B_c \rightarrow D_{s}^*\ell^+\ell^−$ more important compared to the other $B$ meson decays, such as $B^0 \rightarrow (K^*, K_1, \rho, \pi)\ell^+\ell^−$, is the fact that it can have two types of contributions, one is through the penguin (loop level) and the other is due to weak annihilation (WA). In the ordinary $B$ meson decays the WA contributions are very small and can be ignored as compared to the corresponding penguin contributions. However, for the $B_c$ meson these WA contributions are proportional to the CKM matrix elements $V_{cb}V_{cs}^*$ and hence can not be ignored as compared to penguin contributions [33]. In perspective to the new physics (NP) effects in the presence of weak annihilation (WA) contribution, it is shown [34] that for $B^0 \rightarrow K^*\mu^+\mu^−$ the helicity fractions are mildly affected under the implication of SM4 while in the case of $B_c \rightarrow D_{s}^*\mu^+\mu^−$ the SM4 effects are very optimistic [35]. Similarly for this observables when tauns are the final state the effects are comparable i.e. for $B_c \rightarrow D_{s}^*\mu^+\mu^−$ the shift in the maximum(minimum) values in the longitudinal(transverse) helicity fractions is 0.23 and in the case of $B^0 \rightarrow K^*\tau^+\tau^−$ this shift is 0.20. With this motivation it is also interesting to study the SUSY effects for the lepton polarization asymmetries in this decay channel which is calculated in the present manuscript.
The present study shows that the lepton polarization asymmetries for $B_c \to D_s^{(*)} l^+ l^-$ more influenced due to the supersymmetric models in comparison of $B^0 \to K^{(*)} l^+ l^-$ [36] and one can clearly see from the graphs that in the transverse lepton polarization for muons and tauns the NP effects are more prominent for $B_c \to D_s^{(*)} l^+ l^-$. This feature arises due to the interference between the WA and the Supersymmetric contribution which is encapsulated in the $f_7$ and $f_8$ auxiliary functions. In the study of the exclusive $B$-meson decays the form factors which are the non-perturbative quantities and are the scalar functions of the square of momentum transfer are important ingredients. In literature the form factors for $B_c \to D_s^{(*)} \ell^+ \ell^-$ decay were calculated using different approaches including the light front constituent quark models, the relativistic quark models, the Ward identities and the QCD sum rules [25–33, 37, 38]. In this study we borrow the form factors calculated by the QCD sum rules approach for our numerical calculations from Azizi et al. [38]. However, in the context of hadronic uncertainties which enter through the relevant form factors, it needs to be stressed that various asymmetries such as forward-backward asymmetries, polarization asymmetries of the final state meson and leptons (which are calculated in the manuscript submitted) have almost negligible influence. Therefore, any deviation from the SM in these observables is the clear indication of NP.

It is important to emphasize here that the NP effects manifest themselves in the rare $B$ decays in two different ways, one is due to the new contribution to the Wilson coefficients and other is due to the appearance of new operators in the effective Hamiltonian, which are absent in the SM. The manifestations of the NP due to the SUSY is unique in a sense that it modifies the Wilson coefficients as well as it introduces the new operators in the effective Hamiltonian. In the present study, the NP effects are analyzed by studying the branching ratio $BR$, the forward-backward asymmetry $A_{FB}$, the lepton polarization asymmetries and the helicity fractions of $D_s$ meson for $B_c \to D_s^{(*)} l^+ l^-$ decays both in the SM and in different MSSM scenarios with special emphasis on the effects of neutral Higgs bosons (NHBs) [39–44]. A sizeable deviations to the SM results due to the SUSY effects are observed in the above mentioned observables for many rare decays [45–50].

It is worth mention that previously an anomaly has been observed by Belle [51] in the lepton forward-backward asymmetry $(A_{FB})$ in the exclusive decay $\bar{B} \to \bar{K}^{*} \mu^+ \mu^-$ [52–55]. It was noticed that in the large $q^2$ region $q^2 \geq 14$ GeV$^2$, the $A_{FB}(q^2)$ measurements tend to be larger than that of the SM predictions, although the behavior was same for the both cases. The discrepancy was more severe at the low $q^2$ region where the SM prediction is negative [56], whereas the data favor positive values and Belle claimed that this is the clear indication of NP [59]. Contrary to this the LHCb recently announced the first result on the lepton forward-backward asymmetry $(A_{FB})$ in the exclusive decay $\bar{B} \to \bar{K}^{*} \mu^+ \mu^-$ which is close to the SM predictions. In particular, the position of the zero crossing of the $A_{FB}(q^2)$ lie on the same position as predicted by the SM [9, 57, 58]. This situation is really exciting and tighten the screws on scenarios those have not zero-crossing point of the $A_{FB}(q^2)$. In this context, SUSY which previously become a prime candidate to explain the Belle anomaly because in some of the scenarios of MSSM such as SUSY-I and SUSY-II, the $A_{FB}(q^2)$ do not cross the zero position [45–50] now seems to be ruled out after the LHCb data. However, before to say any final remarks about SUSY-I nd SUSY-II, it needs to
be required some confirmation through complementary information as LHCb collaboration said that they will continue to collect more data and try to see deviation in the observable data if there is any NP exist. With this motivation we have predicted the MSSM effects in the $B_c \to D_s^* \ell^+ \ell^-$, where $\ell = \mu, \tau$, decay not only for the $A_{FB}(q^2)$ but also for the other asymmetries which depends on the polarization of the final state particles such as lepton polarization asymmetries and the helicity fractions.

In this paper, we investigate the decay processes $B_c \to D_s^* \ell^+ \ell^-$ in the context of the SUSY SO(10) GUT models [60]. Since the effects of the counterparts of usual chromo-magnetic and electro-magnetic dipole moment operators as well as semileptonic operators with opposite chirality are suppressed by $m_s/m_b$ and as such become insensitive in the SM. However, in the SUSY SO(10) GUTs their effects can be significantly large, since complex flavor non-diagonal down-type squark mass matrix elements of 2nd and 3rd generations in the RR sector ($\delta_{23}^{dRR}$) can be as large as 0.5 [60, 61]. Furthermore, $\delta_{23}^{dRR}$ can induce new operators, the counterparts of usual scalar operators ($Q_{1,2}$) in the SUSY models, due to the NHBs penguins with gluino-down type squark propagated in the loop. The values of the relevant new Wilson coefficients for the MSSM and the SUSY SO(10) GUT models are collected from Refs. [45, 46]. The NHBs could contribute largely to the inclusive processes $B \to X_s \ell^+ \ell^-$, because part of the SUSY contributions is proportional to the $\tan^3 \beta$ [62, 63]. Subsequently, the physical observables, such as the branching ratio $BR$, forward-backward asymmetries $A_{FB}$, lepton polarization asymmetries and helicity fractions of the final state meson, in the large $\tan \beta$ region of parameter space in the SUSY models can be quite different from that in the SM. Hence, the measurement of these observables will give us some hints of the SUSY effects in these decays.

The paper is structured as follows. In Sec. 2 we present the theoretical framework for the decay $B_c \to D_s^* \ell^+ \ell^-$ necessary for the study of the different SUSY variants. In Sec. 3 we present the basic formulas for physical observables such as decay rate, forward-backward asymmetries $A_{FB}$, lepton polarization asymmetries and helicity fractions of $D_s^*$ meson. The numerical analysis and discussion on these observables is given in Sec. 4. Section 5 gives the summary of the results of our study.

2 Theoretical Framework

In this section we give the effective Hamiltonian for $B_c \to D_s^* \ell^+ \ell^-$ decays. We can split the contributions of the total decay amplitude into two, one is the penguin (FCNC trasitions) and the second is weak annihilation contribution.

2.1 Penguin Amplitude

At quark level, the semileptonic decay $B_c \to D_s^* \ell^+ \ell^-$ is governed by the transition $b \to s \ell^+ \ell^-$ for which the general effective Hamiltonian in the SUSY SO(10) GUT model, can
be written, after integrating out the heavy degrees of freedom in the full theory, as \[ H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1}^{10} \left\{ C_i(\mu)O_i(\mu) + C'_i(\mu)O'_i(\mu) \right\} \\
+ \sum_{i=1}^{8} \left\{ C_{Q_i}(\mu)Q_i(\mu) + C'_{Q_i}(\mu)Q'_i(\mu) \right\} , \] where \( O_i(\mu) \) (\( i = 1, 2, \cdots, 10 \)) are the four quark operators and \( C_i(\mu) \) are the corresponding Wilson coefficients at the energy scale \( \mu \) [39–44] which is usually taken to be the \( b \)-quark mass (\( m_b \)). The theoretical uncertainties related to the renormalization scale can be reduced when the next to leading logarithm corrections are included. The new operators \( Q_i(\mu) \) (\( i = 1, 2, \cdots, 8 \)) come from the NHBs exchange diagrams, whose manifest forms and corresponding Wilson coefficients can be found in [64–68]. The primed operators are the counterparts of the unprimed operators, which can be obtained by flipping the chiralities in the corresponding unprimed operators. It is worth mentioning that these primed operators will appear only in the SUSY SO(10) GUT model and are absent in the SM and the MSSM [45].

The explicit forms of the operators responsible for the decay \( B_c^- \rightarrow D_s^-\ell^+\ell^- \), in the SM and the SUSY models, are

\[ O_7 = \frac{e^2}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} R b) F^{\mu\nu} \] \[ O'_7 = \frac{e^2}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} L b) F^{\mu\nu} \] \[ O_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu L b) \bar{\ell}\gamma^\mu \ell \] \[ O'_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu R b) \bar{\ell}\gamma^\mu \ell \] \[ O_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu L b) \bar{\ell}\gamma^\mu \gamma^5 \ell \] \[ O'_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu R b) \bar{\ell}\gamma^\mu \gamma^5 \ell \] \[ Q_1 = \frac{e^2}{16\pi^2} (\bar{s} R b) \bar{\ell}\ell \] \[ Q'_1 = \frac{e^2}{16\pi^2} (\bar{s} L b) \bar{\ell}\ell \] \[ Q_2 = \frac{e^2}{16\pi^2} (\bar{s} R b) \bar{\ell}\gamma^5 \ell \] \[ Q'_2 = \frac{e^2}{16\pi^2} (\bar{s} L b) \bar{\ell}\gamma^5 \ell \] with \( L, R = \frac{1}{2} (1 + \gamma^5) \).
Using the effective Hamiltonian given in Eq. (2.1) the free quark amplitude for $b \to s\ell^+\ell^-$ can be written as

$$
\mathcal{M}_{\text{PEENG}}(b \to s\ell^+\ell^-) = -\frac{G_{\text{F}}\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ C_9^{\text{eff}}(\mu) (\bar{s}\gamma_\mu Lb)(\bar{\ell}\gamma^\mu \ell) + C_9^{\text{eff}}(\mu) (\bar{s}\gamma_\mu Rb)(\bar{\ell}\gamma^\mu \ell) + C_{10}(\bar{s}\gamma_\mu Lb)(\bar{\ell}\gamma^\mu \ell) \right]
$$

where $q$ is the momentum transfer. Note that the operator $O_{10}$ given in Eq. (2.2e) can not be induced by the insertion of four quark operators because of the absence of Z-boson in the effective theory. Therefore, the Wilson coefficient $C_{10}$ does not renormalize under QCD corrections and is independent of the energy scale $\mu$. Additionally the above quark level decay amplitude can get contributions from the matrix element of four quark operators, $\sum_{i=1}^{6} |(\ell^+\ell^- s) |O_i|b\rangle$, which are usually absorbed into the effective Wilson coefficient $C_9^{\text{eff}}(\mu)$ and can be written as [69–75]

$$
C_9^{\text{eff}}(\mu) = C_9(\mu) + Y_{SD}(z, s') + Y_{LD}(z, s').
$$

where $z = m_c/m_b$ and $s' = q^2/m_b^2$. $Y_{SD}(z, s')$ describes the short distance contributions from four-quark operators far away from the $c\bar{c}$ resonance regions, and this can be calculated reliably in the perturbative theory. However the long distance contribution $Y_{LD}(z, s')$ cannot be calculated by using the first principles of QCD, so they are usually parametrized in the form of a phenomenological Breit-Wigner formula making use of the vacuum saturation approximation and quark hadron duality. The expressions for the short-distance and the long-distance contributions $Y_{SD}(z, s')$ is given as

$$
Y_{SD}(z, s') = h(z, s') \left[ 3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right]
$$

$$
- \frac{1}{2} \left[ 3C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right]
$$

$$
- \frac{1}{2} \left[ 3C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right]
$$

$$
Y_{LD}(z, s') = \frac{3}{\alpha_{\text{em}}^2} (3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu))
$$

$$
\times \sum_{j=\psi, \psi'} \omega_j(q^2) k_j \frac{\pi \Gamma(j \to l^+\ell^-) M_j}{q^2 - M_j^2 + i M_j \Gamma_{j,\text{tot}}},
$$

with

$$
\omega_j(q^2) = \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2} \left\{ \begin{array}{ll} 
\ln \frac{\sqrt{1 + x} + 1}{\sqrt{1 + x} - 1} - i \pi & \text{for } x \equiv 4z^2/s' < 1 \\
2 \arctan \frac{\sqrt{x}}{\sqrt{1 - x}} & \text{for } x \equiv 4z^2/s' > 1 
\end{array} \right.,
$$

$$
h(z, s') = \frac{8}{27} - \frac{8}{9} \left( \frac{m_b}{\mu} \right) - \frac{4}{9} \ln s' + \frac{4}{9} i \pi.
$$

(2.5)
Here $M_j(\Gamma_j^{tot})$ are the masses (widths) of the intermediate resonant states and $\Gamma(j \to l^+l^-)$ denote the partial decay width for the transition of vector charmonium state to massless lepton pair, which can be expressed in terms of the decay constant of charmonium through the relation [76]

$$\Gamma(j \to l^+l^-) = \pi\alpha_{em}^2 \frac{16 f_j^2}{27 M_j}.$$  

The phenomenological parameter $k_j$ in Eq.(2.5) is to account for inadequacies of the factorization approximation, and it can be determined from

$$BR(B_c \to D_s^* J/\psi \to D_s^* \ell^+\ell^-) = BR(B_c \to D_s^* J/\psi) \cdot BR(J/\psi \to \ell^+\ell^-).$$  

The function $\omega_j(q^2)$ introduced in Eq.(2.5) is to compensate the naive treatment of long distance contributions due to the charm quark loop in the spirit of quark-hadron duality, which can overestimate the genuine effect of the charm quark at small $q^2$ remarkably \(^1\). The quantity $\omega_j(q^2)$ can be normalized to $\omega_j(M_{\psi}^2) = 1$, but its exact form is unknown at present. Since the dominant contribution of the resonances is in the vicinity of the intermediate $\psi_i$ masses, we will simply use $\omega_j(q^2) = 1$ in our numerical calculations.

Moreover, the non factorizable effects from the charm quark loop brings further corrections to the radiative transition $b \to s\gamma$, and these can be absorbed into the effective Wilson coefficients $C_{eff}^\gamma$ which then takes the form [76, 82-86]

$$C_{eff}^\gamma(\mu) = C_\gamma(\mu) + C_{b\to s\gamma}(\mu)$$

with

$$C_{b\to s\gamma}(\mu) = i \alpha_s \left[ \frac{2}{9} \eta^{14/23} G_1(x_t) - 0.1687 - 0.03 C_2(\mu) \right]$$

$$G_1(x_t) = \frac{x_t \left( x_t^2 - 5x_t - 2 \right)}{8 \left( x_t - 1 \right)^3} + \frac{3x_t^2 \ln^2 x_t}{4 \left( x_t - 1 \right)^3}$$

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, $x_t = m_t^2/m_W^2$ and $C_{b\to s\gamma}$ is the absorptive part for the $b \to scc \to s\gamma$ rescattering.

### 2.2 Weak Annihilation Amplitude

The charmed B-meson ($B_c$) is made up of two different heavy flavors, $b$-quark and $c$-quark, which brings WA contributions into the play. Using the procedure developed in refs. [87, 88] for $B_c \to D_s^* \gamma$, the WA amplitude for the decay $B_c \to D_s^* \ell^+\ell^-$ can be written as

$$M_{B_c \to D_s^* \ell^+\ell^-}^{WA} = \frac{G_F\alpha}{2\sqrt{2}\pi V_{cb}V_{cs}^*} \left[ -i e_{\mu\nu\alpha\beta} \varepsilon_{\nu}^{\mu} q^\alpha p^\beta K_1^{ann}(q^2) + (\varepsilon \cdot q p_\mu + p \cdot q \varepsilon_\mu) K_2^{ann}(q^2) \right] \ell^\mu \ell$$

where $K_1^{ann}(q^2)$ and $K_2^{ann}(q^2)$ are the weak annihilation form factors.

\(^1\)For a more detailed discussion on long-distance and short-distance contributions from the charm loop, one can refer to references [9, 76-82].
Before proceeding further we would like to mention that we use parametrizations for weak annihilation form factors $K_{1,2}^{ann}(q^2)$ and $K_{3}^{ann}(q^2)$, i.e.

$$K_{1,2}^{ann}(q^2) = \frac{K_{1,2}^{ann}(0)}{1 + \alpha \frac{q^2}{M_{c}} + \beta \frac{q^4}{M_{c}^2}}. \quad (2.10)$$

The values of $K_{1}^{ann}(0)$ and $K_{2}^{ann}(0)$ are calculated by using QCD sum rules [37, 38] and the values of the parameter $\alpha$ and $\beta$ are given in Ref. [26–31, 37, 38], which are summarized in Table 1.

| $K_{1}^{ann}(q^2)$ | $K_{0}^{ann}$ | $\alpha$   | $\beta$   |
|---------------------|--------------|------------|------------|
| $K_{1}^{ann}(q^2)$  | 0.23         | -1.25      | -0.097     |
| $K_{2}^{ann}(q^2)$  | 0.25         | -0.10      | -0.097     |

Table 1. $B_c \rightarrow D_s^*$ form factors corresponding to WA in the QCD Sum Rules. $K_{1,2}^{ann}(0)$ denote the value of form factors at $q^2 = 0$ while $\alpha$ and $\beta$ are the parameters in the parametrizations shown in Eq. (2.10) [38].

### 2.3 Parameterizations of the Matrix Elements and Form Factors

The exclusive $B_c \rightarrow D_s^* \ell^+ \ell^-$ decay involves the hadronic matrix elements which can be obtained by sandwiching the quark level operators give in Eq. (2.3) between initial state $B_c$ meson and final state $D_s^*$ meson. These can be parametrized in terms of form factors which are the scalar functions of the square of the four momentum transfer($q^2 = (p - k)^2$).

The non vanishing matrix elements for the process $B_c \rightarrow D_s^*$ can be parametrized in terms of the seven form factors as follows

$$\langle D_s^*(k, \varepsilon) | s \gamma_{\mu} b | B_c(p) \rangle = \frac{2A_V(q^2)}{M_{B_c} + M_{D_s^*}} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu\rho} p^\alpha k^\beta \quad (2.11)$$

$$\langle D_s^*(k, \varepsilon) | s \gamma_{\mu} \gamma_5 b | B_c(p) \rangle = i \left( M_{B_c} - M_{D_s^*} \right) \epsilon^{\mu\nu} A_0(q^2) - i \frac{A_+ (q^2)}{M_{B_c} + M_{D_s^*}} (\varepsilon^\ast \cdot p)(p + k)^\mu$$

$$- i \frac{A_- (q^2)}{M_{B_c} + M_{D_s^*}} (\varepsilon^\ast \cdot p)q^\mu \quad (2.12)$$

$$\langle D_s^*(k, \varepsilon) | s (1 \pm \gamma_5) b | B_c(p) \rangle = \mp i \frac{2M_{D_s^*}}{m_b + m_s} (\varepsilon^\ast \cdot p) \tilde{A}_0(q^2) \quad (2.13)$$

where $p$ is the momentum of the $B_c$, $\varepsilon$ and $k$ are the polarization vector and momentum of the final state $D_s^*$ vector meson. Whereas $\tilde{A}_0(q^2)$ can be parametrized as

$$\tilde{A}_0(q^2) = - \frac{M_{B_c} + M_{D_s^*}}{2M_{D_s^*}} A_0(q^2) + \frac{M_{B_c} - M_{D_s^*}}{2M_{D_s^*}} A_+(q^2) + \frac{q^2}{(M_{B_c} - M_{D_s^*}) M_{D_s^*}} A_-(q^2) \quad (2.14)$$

In addition to the above form factors there are some penguin form factors, which we
can write as

\[
\langle D^+_s(k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q'^* b \rangle | B_c(p) \rangle = 2i T_1(q^2) \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^\alpha k^\beta
\]

\[
\langle D^+_s(k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q'^* b \rangle | B_c(p) \rangle = \left( M_{D^+_s}^2 - M_{B_c}^2 \right) \epsilon^*_\mu - (\varepsilon^* \cdot p)(p + k)_\mu \right] T_2(q^2)
\]

\[
+ (\varepsilon^* \cdot p) \left[ q_\mu - \frac{\lambda_2(q^2)}{M_{B_c}^2 - M_{D^+_s}^2} (p + k)_\mu \right] T_3(q^2).
\]

The form factors $A_V(q^2)$, $A_0(q^2)$, $A_+(q^2)$, $A_-(q^2)$, $T_1(q^2)$, $T_2(q^2)$, $T_3(q^2)$ are the non-pertubative quantities and to calculate them one has to rely on some non-perturbative approaches in our numerical analysis we use the form factors calculated by using QCD Sum Rules [38]. The dependence of these form factors on square of the momentum transfer ($q^2$) can be written as

\[
F(q^2) = \frac{F(0)}{1 + a \frac{q^2}{M_{Bc}^2} + b \frac{q^4}{M_{Bc}^4}}
\]

where the values of the parameters $F(0)$, $a$ and $b$ is given in Table 2.

| $F(q^2)$ | $F(0)$ | $a$ | $b$ |
|---|---|---|---|
| $A_V(q^2)$ | 0.54 ± 0.018 | -1.28 | -0.23 |
| $A_0(q^2)$ | 0.30 ± 0.017 | -0.13 | -0.18 |
| $A_+(q^2)$ | 0.36 ± 0.013 | -0.67 | -0.066 |
| $A_-(q^2)$ | -0.57 ± 0.04 | -1.11 | -0.14 |
| $T_1(q^2)$ | 0.31 ± 0.017 | -1.28 | -0.23 |
| $T_2(q^2)$ | 0.33 ± 0.016 | -0.10 | -0.097 |
| $T_3(q^2)$ | 0.29 ± 0.034 | -0.91 | 0.007 |

Table 2. $B_c \to D^+_s$ form factors corresponding to penguin contributions in the QCD Sum Rules. $F(0)$ denotes the value of form factors at $q^2 = 0$ while $a$ and $b$ are the parameters in the parametrizations shown in Eq. (2.17) [38].

From Eq. (2.3) it is straightforward to write the penguin amplitude

\[
\mathcal{M}^{\text{PENG}} = -\frac{G_F^\alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ T_1^1 (\bar{c} \gamma^\mu \ell) + T_2^0 (\bar{c} \gamma^\mu \gamma^5 \ell) + T (\bar{c} \ell) \right]
\]

where

\[
T_1^1 = f_1(q^2) \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^\alpha k^\beta - if_2(q^2) \varepsilon^*_\mu + if_3(q^2)(\varepsilon^* \cdot p) P_\mu
\]

\[
T_2^0 = f_4(q^2) \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^\alpha k^\beta - if_5(q^2) \varepsilon^*_\mu + if_6(q^2)(\varepsilon^* \cdot p) P_\mu + if_7(q^2)(\varepsilon^* \cdot p) P_\mu
\]

\[
T = 2i f_8(q^2)(\varepsilon^* \cdot p)
\]

with $P_\mu = p_\mu + k_\mu$.

The next task is to calculate the decay rate and the helicity fractions of $D^+_s$ meson in terms of these auxiliary functions which contains both long distance (form factors) and
short distance (Wilson coefficients) effects and these can be written as

\[ f_1(q^2) = 4(C_{7}^{eff} + C_{7}^{eff}) \frac{m_b + m_s}{q^2} T_1(q^2) + (C_9^{eff} + C_9^{eff}) \frac{2A_V(q^2)}{M_{B_s} + M_{D_s}} \]  
\[ f_2(q^2) = 2(C_{7}^{eff} - C_{7}^{eff}) \frac{m_b - m_s}{q^2} T_2(q^2) \left( M_{B_s}^2 - M_{D_s}^2 \right) + (C_9^{eff} - C_9^{eff}) A_0(q^2) (M_{B_s} + M_{D_s}) \]  
\[ f_3(q^2) = 4(C_{7}^{eff} - C_{7}^{eff}) \frac{m_b - m_s}{q^2} T_3(q^2) \left( M_{B_s}^2 - M_{D_s}^2 \right) + (C_9^{eff} - C_9^{eff}) \frac{A_+(q^2)}{M_{B_s} + M_{D_s}} \]  
\[ f_4(q^2) = (C_{10} + C_{10}') \frac{2A_V(q^2)}{M_{B_s} + M_{D_s}} \]  
\[ f_5(q^2) = 2(C_{10} - C_{10}') A_0(q^2) (M_{B_s} + M_{D_s}) \]  
\[ f_6(q^2) = 2(C_{10} - C_{10}') \frac{A_+(q^2)}{M_{B_s} + M_{D_s}} \]  
\[ f_7(q^2) = 4(C_{10} - C_{10}') \frac{A_-(q^2)}{M_{B_s} + M_{D_s}} + (C_{Q2} - C_{Q4}) \frac{M_{D_s}}{m(m_b + m_s)} \] \[ A_0(q^2) \]  
\[ f_8(q^2) = - (C_{Q1} - C_{Q1}') \frac{M_{D_s}}{m(m_b + m_s)} A_0(q^2) \]

Here the NHBs contribution are encoded in the auxiliary functions \( f_7 \) and \( f_8 \).

### 3 Physical Observables for \( B_c \rightarrow D_s^{*} \ell^+ \ell^- \)

In this section we will present the calculations of the physical observables such as the branching ratios \( \mathcal{BR} \), the forward-backward asymmetries \( A_{FB} \), the lepton polarization asymmetries \( P_{L,N,T} \) and the helicity fractions \( f_{L,T} \) of \( D_s^{*} \) meson in \( B_c \rightarrow D_s^{*} \ell^+ \ell^- \) decay by incorporating both the weak annihilation (WA) and the penguin amplitudes.

#### 3.1 The Differential Decay Rate of \( B_c \rightarrow D_s^{*} \ell^+ \ell^- \)

In the rest frame of \( B_c \) meson the differential decay width of \( B_c \rightarrow D_s^{*} \ell^+ \ell^- \) can be written as

\[
\frac{d\Gamma(B_c \rightarrow D_s^{*} \ell^+ \ell^-)}{dq^2} = \frac{1}{(2\pi)^3} \frac{1}{32M_{B_c}^3} \int_{-u(q^2)}^{+u(q^2)} du |\mathcal{M}|^2
\]

where

\[
\mathcal{M} = \mathcal{M}^{WA} + \mathcal{M}^{PENG}
\]

\[
q^2 = (p_{\ell^+} + p_{\ell^-})^2
\]

\[
u = (p - p_{\ell^-})^2 - (p - p_{\ell^+})^2
\]
Now the limits on \( q^2 \) and \( u \) are

\[
4m^2 \leq q^2 \leq (M_{B_c} - M_{D_s^*})^2 \\
-u(q^2) \leq u \leq u(q^2)
\]

with

\[
u(q^2) = \sqrt{\lambda \left(1 - \frac{4m^2}{q^2}\right)}
\]

and

\[
\lambda \equiv \lambda(M_{B_c}^2, M_{D_s^*}^2, q^2) = M_{B_c}^4 + M_{D_s^*}^4 + q^4 - 2M_{B_c}^2 M_{D_s^*}^2 - 2M_{D_s^*}^2 q^2 - 2q^2 M_{B_c}^2
\]

Here \( m \) corresponds to the mass of the lepton which for our case are the \( \mu \) and \( \tau \). The total decay rate for the decay \( B_c \to D_s^* \ell^+ \ell^- \) can be expressed in terms of WA, penguin amplitude and the interference of these two, which takes the form

\[
\frac{d\Gamma}{dq^2} = \frac{d\Gamma_{WA}}{dq^2} + \frac{d\Gamma_{PENG}}{dq^2} + \frac{d\Gamma_{WA-PENG}}{dq^2}
\]

with

\[
\frac{d\Gamma_{WA}}{dq^2} = \frac{G_F^2 |V_{cb}V_{ts}^*|^2}{2^{11} \pi^5 3M_{B_c}^2 M_{D_s^*}^2 q^2} u(q^2) \times A(q^2)
\]

\[
\frac{d\Gamma_{PENG}}{dq^2} = \frac{G_F^2 |V_{cb}V_{ts}^*|^2}{2^{11} \pi^5 3M_{B_c}^2 M_{D_s^*}^2 q^2} u(q^2) \times B(q^2)
\]

\[
\frac{d\Gamma_{WA-PENG}}{dq^2} = \frac{G_F^2 |V_{cb}V_{ts}^*| |V_{cb}V_{ts}^*|^2}{2^{11} \pi^5 3M_{B_c}^2 M_{D_s^*}^2 q^2} u(q^2) \times C(q^2)
\]

The function \( u(q^2) \) is defined Eq. (3.7) and \( A(q^2) \), \( B(q^2) \) and \( C(q^2) \) are

\[
A(q^2) = \frac{1}{2} \left(2m^2 + q^2\right) \kappa^2 \left[8\lambda M_{D_s^*}^2 q^2 (\kappa_{1ann}^2) + (\kappa_{2ann}^2)^2 \right] \begin{pmatrix} 12 M_{D_s^*}^2 q^2 (\lambda + 4M_{B_c}^2 q^2) \\
+ \lambda^2 + \lambda \left(\lambda + 4q^2 M_{D_s^*}^2 + 4q^4\right) \end{pmatrix}
\]

\[
B(q^2) = 8M_{D_s^*}^2 q^2 \lambda \left\{(2m^2 + q^2) \left| f_1(q^2) \right|^2 -(4m^2 - q^2) \left| f_4(q^2) \right|^2 \right\} + 4M_{D_s^*}^2 q^2 \left\{(2m^2 + q^2) \right.
\]

\[
\left. \times \left(3 \left| f_2(q^2) \right|^2 - \lambda \left| f_3(q^2) \right|^2 \right) - (4m^2 - q^2) \left(3 \left| f_5(q^2) \right|^2 - \lambda \left| f_6(q^2) \right|^2 \right) \right\} + \lambda(2m^2 + q^2) \left| f_2(q^2) \right| + \left(M_{B_c}^2 - M_{D_s^*}^2 - q^2 \right) f_3(q^2) + 24m^2 M_{D_s^*}^2 \lambda \left| f_7(q^2) \right|^2
\]

\[
- (4m^2 - q^2) f_5(q^2) + \left(M_{B_c}^2 - M_{D_s^*}^2 - q^2 \right) f_6(q^2) + (q^2 - 4m^2) \lambda \left| f_8(q^2) \right|^2
\]

\[
- 12m^2 q^2 \left[\Re(f_5 f_7^*) - \Re(f_6 f_8^*)\right]
\]

\[
C(q^2) = 2\kappa f_2(q^2) \kappa_{2ann}^2 (2m^2 + q^2) \left\{\lambda + 6M_{D_s^*}^2 \left(M_{B_c}^2 - M_{D_s^*}^2 + q^2\right) \right\}
\]

\[
- \kappa \lambda \left[2 f_1(q^2) \kappa_{1ann}^2 M_{D_s^*}^4 + f_3(q^2) \kappa_{2ann}^2 (2m^2 + q^2) (\lambda + 4 M_{B_c}^2 M_{D_s^*}) \right].
\]
where
\[ \kappa = \frac{8\pi^2 M_{D^*_s} f_{B_c} f_{D^*_s}}{(m_{\ell}^2 - m_{D^*_s}^2)q^2}. \]  

### 3.2 Forward-Backward Asymmetries

The differential forward-backward asymmetry \( A_{FB} \) of final state lepton for the said decay can be written as
\[
\frac{dA_{FB}(s)}{dq^2} = \int_0^1 d^2\Gamma \frac{d^2\Gamma}{dq^2 d\cos \theta} d\cos \theta - \int_{-1}^0 d^2\Gamma \frac{d^2\Gamma}{dq^2 d\cos \theta} d\cos \theta \tag{3.16}
\]

From experimental point of view the normalized forward-backward asymmetry is more useful, defined as
\[
A_{FB} = \int_0^1 d^2\Gamma \frac{d^2\Gamma}{dq^2} d\cos \theta - \int_{-1}^0 d^2\Gamma \frac{d^2\Gamma}{dq^2} d\cos \theta \frac{d^2\Gamma}{d^2\cos \theta} d\cos \theta
\]

The normalized \( A_{FB} \) for \( B_c \to D^*_s \ell^+ \ell^- \) can be obtained from Eq. (3.1), as
\[
A_{FB} = \frac{1}{d\Gamma/dq^2} G_F^2 \alpha^2 [V_{tb} V_{ts}^*] q^2 u(q^2) \left\{ 2\kappa \Re(f_4 K_{2}^{anm}) (M_{B_c}^2 - M_{D^*_s}^2 + q^2) + 2\kappa \Re(f_5 K_{1}^{anm}) 
+ 4 Re[f_2^* f_4 + f_1^* f_5] + 2\lambda \Re(f_3 f_8) + 4 \Re[f_2 f_8^*] \left( -M_{B_c}^2 + M_{D^*_s}^2 + q^2 \right) 
+ 2\kappa \Re[f_8 K_2^{anm}] \left( \lambda + \left( M_{D^*_s}^2 + q^2 \right) M_{B_c}^2 - q^4 \right) \right\} \tag{3.17}
\]

where \( \kappa \) is defined in Eq. (3.15) and \( d\Gamma/dq^2 \) is given in Eq. (3.8).

### 3.3 Lepton Polarization Asymmetries

In the rest frame of the lepton \( \ell^- \), the unit vectors along longitudinal, normal and transversal component of the \( \ell^- \) can be defined as [47, 48, 89]:
\[
s_{L}^{-\mu} = (0, \vec{e}_{L}^{-\mu}) = \left( 0, \frac{\vec{p}_-}{|\vec{p}_-|} \right), \tag{3.18a}
\]
\[
s_{T}^{-\mu} = (0, \vec{e}_{T}^{-\mu}) = \left( 0, \frac{\vec{k} \times \vec{p}_-}{|\vec{k} \times \vec{p}_-|} \right), \tag{3.18b}
\]
\[
s_{N}^{-\mu} = (0, \vec{e}_{N}^{-\mu}) = \left( 0, \vec{e}_T \times \vec{e}_L \right), \tag{3.18c}
\]

where \( \vec{p}_- \) and \( \vec{k} \) are the three-momenta of the lepton \( \ell^- \) and \( D^*_s \) meson respectively in the center mass (c.m.) frame of \( \ell^+ \ell^- \) system. Lorentz transformation is used to boost the longitudinal component of the lepton polarization to the c.m. frame of the lepton pair as
\[
\left(s_{L}^{-\mu}\right)_{CM} = \left( \frac{|\vec{p}_-|}{m}, \frac{E\vec{p}_-}{m|\vec{p}_-|} \right) \tag{3.19}
\]

where \( E \) and \( m \) are the energy and mass of the lepton. The normal and transverse components remain unchanged under the Lorentz boost. The longitudinal \( (P_L) \), normal \( (P_N) \)
and transverse ($P_T$) polarizations of lepton can be defined as:

$$P_i(\pm)(q^2) = \frac{d\Gamma}{dq^2} (\xi^\pm = \varepsilon^\pm) - \frac{d\Gamma}{dq^2} (\xi^\pm = -\varepsilon^\pm)$$

(3.20)

where $i = L, N, T$ and $\xi^\pm$ is the spin direction along the leptons $\ell^\pm$. The differential decay rate for polarized lepton $\ell^\pm$ in $B_\ell \to D_{\ell}^* \ell^+ \ell^-$ decay along any spin direction $\xi^\pm$ is related to the unpolarized decay rate (3.8) with the following relation

$$\frac{d\Gamma(\xi^\pm)}{dq^2} = \frac{1}{2} \left[ \frac{d\Gamma}{dq^2} \left[ 1 + (P_L^\pm \varepsilon_L^\mp + P_N^\pm \varepsilon_N^\mp + P_T^\pm \varepsilon_T^\mp) \cdot \xi^\pm \right] \right].$$

(3.21)

The expressions of the numerator of longitudinal, normal and transverse lepton polarizations can be written as

$$P_L(q^2) \propto \frac{4\lambda}{3M_{D_L}^2} \sqrt{\frac{q^2 - 4m^2}{q^2}} \times \left\{ \Re(f_2f_5^* \left[ (1 + \frac{12q^2M_{D_L}^2}{\lambda}) + \lambda\Re(f_3f_6^*) + 8q^2M_{D_L}^2\Re(f_1f_4^*) \right] 
+ \left( -M_{D_L}^2 + M_{D_L}^2 + q^2 \right) \Re(f_3f_6^*) \Re(f_2f_5^*) \right\} 
+ \frac{3}{2} \left[ \Re(f_5f_6^*) + \Re(f_6f_5^*) \left( -M_{D_L}^2 + M_{D_L}^2 \right) - q^2\Re(f_7f_8^*) \right] 
+ 4M_{D_L}^2\kappa\Re(K_1^{ann}f_4^*) + \left( \lambda + q^2(M_{D_L}^2 + M_{D_L}^2 - q^2) \right) \kappa\Re(K_2^{ann}f_6^*) 
+ \left[ M_{D_L}^2 - M_{B_L}^2 - \frac{6q^2M_{D_L}^2}{\lambda} \left( q^2 - M_{D_L}^2 + M_{B_L}^2 \right) \right] \kappa\Re(K_2^{ann}f_2^*) \right\}$$

(3.22)

$$P_T(q^2) \propto \frac{m\pi}{M_{D_L}^2} \sqrt{\frac{\lambda}{2m}} \times \left\{ \lambda^2\Re(f_3f_7^*) + \lambda(M_{D_L}^2 - M_{D_L}^2)\Re(f_3f_6^*) - \lambda\Re(f_3f_5^*) \right\} 
+ \left( -M_{D_L}^2 + M_{D_L}^2 + q^2 \right) \Re(f_2f_5^*) \Re(f_3f_6^*) \Re(f_3f_5^*) 
+ \frac{8q^2M_{D_L}^2\Re(f_1f_2^*) + \frac{\lambda}{2m} \left[ (q^2 - 4m^2)\Re(f_3f_6^*) \right] \Re(f_2f_5^*) 
+ 4q^2M_{D_L}^2\Re(f_1f_2^*) \Re(f_3f_6^*) \Re(f_3f_5^*) 
+ \left[ \lambda + q^2 \left( M_{B_L}^2 + M_{D_L}^2 - q^2 \right) \right] \kappa \Re(K_2^{ann}f_7^*) 
+ 4q^2M_{D_L}^2 \left( M_{B_L}^2 - M_{D_L}^2 + q^2 \right) \kappa^{ann}K_{2}^{ann} \right\}$$

(3.23)

$$P_N(q^2) \propto \frac{m\pi}{M_{D_L}^2} \sqrt{\frac{\lambda}{2m}} \left\{ M_{D_L}^2 \left[ 4\Im(f_2f_4^*) + 4\Im(f_1f_5^*) + 2\kappa\Im(f_3K_1^{ann}) \right] 
+ 2M_{D_L}^2\Re(f_4K_2^{ann}) \left( M_{B_L}^2 - M_{D_L}^2 + q^2 \right) - \lambda\Re(f_3f_6^*) - \frac{\left( \lambda + q^2(M_{B_L}^2 + M_{D_L}^2 - q^2) \right) \Re(f_3f_6^*)}{2m} \right\}$$

(3.24)
where $\kappa$ is defined in Eq. (3.15) along with auxiliary functions $f_1, f_2, \cdots, f_8$ in Eqs. (2.22-2.29). Here we have dropped out the constant factors which are however understood.

### 3.4 Helicity Fractions of $D_s^*$ in $B_c \rightarrow D_s^* \ell^+ \ell^-$

We now discuss helicity fractions of $D_s^*$ in $B_c \rightarrow D_s^* \ell^+ \ell^-$ which are interesting variable and are as such independent of the uncertainties arising due to form factors and other input parameters. The final state meson helicity fractions were already discussed in literature for $B \rightarrow K^* (K_1^0) \ell^+ \ell^-$ decays [90-93]. Even for the $K^*$ vector meson, the longitudinal helicity fraction $f_L$ has been measured by Babar collaboration for the decay $B \rightarrow K^* \ell^+ \ell^-(l = e,\mu)$ in two bins of momentum transfer and the results are [94]

$$f_L = 0.71^{+0.63}_{-0.30} \pm 0.07, \quad 0.1 \leq q^2 \leq 8.41\text{GeV}^2$$

(3.25)

$$f_L = 0.51^{+0.22}_{-0.25} \pm 0.08, \quad q^2 \geq 10.24\text{GeV}^2$$

while the average value of $f_L$ in full $q^2$ range is

$$f_L = 0.63^{+0.18}_{-0.19} \pm 0.05, \quad q^2 \geq 0.1\text{GeV}^2$$

(3.26)

The explicit expression of the decay rate for $B_c^- \rightarrow D_s^{*-} \ell^+ \ell^-$ decay can be written in terms of longitudinal $\Gamma_L$ and transverse components $\Gamma_T$ as

$$\frac{d\Gamma_L(q^2)}{dq^2} = \frac{d\Gamma_{\text{WA}}^L(q^2)}{dq^2} + \frac{d\Gamma_{\text{PENG}}^L(q^2)}{dq^2} + \frac{d\Gamma_{\text{WA-PENG}}^L(q^2)}{dq^2}$$

(3.27)

$$\frac{d\Gamma_+(q^2)}{dq^2} = \frac{d\Gamma_{\text{WA}}^+(q^2)}{dq^2} + \frac{d\Gamma_{\text{PENG}}^+(q^2)}{dq^2} + \frac{d\Gamma_{\text{WA-PENG}}^+(q^2)}{dq^2}$$

(3.28)

$$\frac{d\Gamma_T(q^2)}{dq^2} = \frac{d\Gamma_+(q^2)}{dq^2} + \frac{d\Gamma_-(q^2)}{dq^2}$$

(3.29)

where

$$\frac{d\Gamma_{\text{WA}}^L(q^2)}{dq^2} = \frac{G_F^2}{32\pi^5} \frac{|V_{cb}|^2 |V_{ts}|^2}{M_{B_c}^2} \frac{2\alpha^2 u(q^2)}{3} \times A_{\text{WA}}$$

(3.30)

$$\frac{d\Gamma_{\text{PENG}}^L(q^2)}{dq^2} = \frac{G_F^2}{32\pi^5} \frac{|V_{cb}|^2 |V_{ts}|^2}{M_{B_c}^2} \frac{2\alpha^2 u(q^2)}{3} \times A_{\text{PENG}}$$

(3.31)

$$\frac{d\Gamma_{\text{WA-PENG}}^L(q^2)}{dq^2} = \frac{G_F^2}{32\pi^5} \frac{|V_{cb}|^2 |V_{ts}|^2}{M_{B_c}^2} \frac{2\alpha^2 u(q^2)}{3} \times A_{\text{WA-PENG}}$$

(3.32)

$$\frac{d\Gamma_{\text{WA}}^+(q^2)}{dq^2} = \frac{G_F^2}{32\pi^5} \frac{|V_{cb}|^2 |V_{ts}|^2}{M_{B_c}^2} \frac{2\alpha^2 u(q^2)}{3} \times A_{\text{WA}}$$

(3.33)

$$\frac{d\Gamma_{\text{PENG}}^+(q^2)}{dq^2} = \frac{G_F^2}{32\pi^5} \frac{|V_{cb}|^2 |V_{ts}|^2}{M_{B_c}^2} \frac{2\alpha^2 u(q^2)}{3} \times A_{\text{PENG}}$$

(3.34)

$$\frac{d\Gamma_{\text{WA-PENG}}^+(q^2)}{dq^2} = \frac{G_F^2}{32\pi^5} \frac{|V_{cb}|^2 |V_{ts}|^2}{M_{B_c}^2} \frac{2\alpha^2 u(q^2)}{3} \times A_{\text{WA-PENG}}$$

(3.35)
The different functions appearing in Eqs. (3.30-3.35) can be expressed in terms of auxiliary functions (cf. Eqs. (2.22-2.29)) as

\[
A_{L}^{W(A)} = \frac{\kappa^2}{4q^2M_{D_1}^2} \left[ (\kappa_1^{ann}(q^2))^2 \left\{ q^2\lambda(\lambda + 4q^2M_{D_1}^2) - 4M^2\lambda(2\lambda + 8q^2M_{D_1}^2) \right. \right. \\
- q^2 \left( M_{B_e}^2 - M_{D_1}^2 - q^2 \right)^2 \left( \lambda - 2u^2(q^2) \right) \right] + (\kappa_2^{ann}(q^2))^2 \left\{ -\lambda^2(q^2 - 4m^2) \right. \\
+ 12\lambda q^2((M_{B_e}^2 - M_{D_1}^2)^2 - M_{D_1}^2) + q^2(8q^2M_{D_1}^2 - \lambda)(M_{B_e}^2 - M_{D_1}^2 + q^2)^2 \\
- 2u^2(q^2)q^2((M_{B_e}^2 - M_{D_1}^2)^2 + q^4) + 4m^2((M_{B_e}^2 - M_{D_1}^2)^2 - q^4) \right) \right] \right) \\
(3.36)
\]

\[
A_{L}^{P(E)} = \frac{1}{q^2M_{D_1}^2} \left[ 24f_5(q^2)M_{D_1}^2 \lambda + (2m^2 + q^2) \left( M_{B_e}^2 - M_{D_1}^2 - q^2 \right) f_2(q^2) + \lambda f_3(q^2) \right] \\
+ (q^2 - 4m^2) \left( M_{B_e}^2 - M_{D_1}^2 - q^2 \right) f_5(q^2) + \lambda f_6(q^2) \right] \right] + \frac{1}{2}(q^2 - 4m^2)\lambda |f_8|^2 \\
(3.37)
\]

\[
A_{L}^{W(A)-P(E)} = \frac{\kappa}{q^2M_{D_1}^2} \left[ Re(f_1(q^2)\kappa_1^{ann}(q^2)) \left\{ (\lambda + 4M_{D_1}^2)q^2 \right. \right. \\
- 4M_{D_1}^2q^2\lambda \right) + Re(f_2(q^2)\kappa_2^{ann}(q^2)) \left\{ q^2u^2(q^2)(M_{B_e}^2 - M_{D_1}^2 - q^2) \right. \\
+ 6q^2\lambda(M_{D_1}^2 - M_{B_e}^2) + q^2(\lambda - 8q^2M_{D_1}^2)(M_{B_e}^2 - M_{D_1}^2 + q^2) + 4m^2q^2(4q^2M_{D_1}^2 + \lambda) \\
+ Re(f_3(q^2)\kappa_3^{ann}(q^2)) \left\{ \lambda^2(4m^2 - q^2) + q^4u(q^2)\sqrt{\lambda} - 6\lambda(M_{B_e}^2 + M_{D_1}^2) \right) \\
+ q^2(M_{B_e}^2 - M_{D_1}^2)(6\lambda - u^2(q^2)) \right) \right] \\
(3.38)
\]

\[
A_{\pm}^{W(A)} = \kappa^2 \left[ (2m^2 + q^2) \left( \lambda (\kappa_1^{ann}(q^2))^2 + (\kappa_2^{ann}(q^2))^2 \right) \lambda + 4M_{D_1}^2q^2 \right] \left] \right] \\
(3.39)
\]

\[
A_{\pm}^{P(E)} = (q^2 - 4m_1^2) |f_5(q^2) + \sqrt{\lambda}f_3(q^2)|^2 + (q^2 + 2m_1^2) |f_2(q^2) + \sqrt{\lambda}f_1(q^2)|^2 \\
(3.40)
\]

\[
A_{\pm}^{W(A)-P(E)} = -\kappa \left\{ 2\sqrt{\lambda}(q^2 - 4m_1^2)Re(f_2(q^2)\kappa_1^{ann}(q^2)) + 4\lambda(q^2 + 2m_1^2)Re(f_1(q^2)\kappa_1^{ann}(q^2)) \right. \\
\pm 2(q^2 + 2m_1^2)(M_{B_e}^2 - M_{D_1}^2 + q^2)[2Re[|f_1(q^2)\kappa_1^{ann}(q^2)|\sqrt{\lambda}] + 2Re(f_2(q^2)\kappa_1^{ann}(q^2))] \right) \right] \\
(3.41)
\]

Finally the longitudinal and transverse helicity amplitude becomes

\[
f_L(q^2) = \frac{d\Gamma_{L}(q^2)/dq^2}{d\Gamma(q^2)/dq^2} \\
f_{\pm}(q^2) = \frac{d\Gamma_{\pm}(q^2)/dq^2}{d\Gamma(q^2)/dq^2} \\
f_T(q^2) = f_+(q^2) + f_-(q^2) \\
(3.42)
\]
so that the sum of the longitudinal and transverse helicity amplitudes is equal to one, i.e. \( f_L(q^2) + f_T(q^2) = 1 \) for each value of \( q^2 \).

As we have discussed in the Introduction that the WA contribution plays a crucial role in the \( B_c \to D_s^* \ell^+ \ell^- \). This feature can also be seen via the expressions of different observables for example branching ratio which is given in eq. (29), there are three terms \( A(q^2), B(q^2) \) and \( C(q^2) \) correspond to WA, Penguin and the cross term of Penguin and WA, although the WA contribution \( A(q^2) \) trying to suppress the effects of NP but the term \( C(q^2) \) some how compensate and enhanced the NP effects. Same is the case for the other calculated observables. Therefore, the interference term between the NP and WA play a crucial role and make the NP effects distinct from the SM.

4 Numerical Results and Discussion

We present here our numerical results of the branching ratio (BR), the forward-backward asymmetry \( A_{FB} \), lepton polarizations asymmetries \( P_{L,N,T} \) and the helicity fractions \( (f_L, f_T) \) of \( D_s^* \) for the \( B_c \to D_s^* \ell^+ \ell^- \) decays with \( \ell = \mu, \tau \). The numerical values of the input parameters which are used in the subsequent analysis are summarized in Table 3:

\[ m_{B_c} = 6.277 \text{ GeV}, \quad m_{D_s^*} = 2.1123 \text{ GeV}, \quad m_b = 4.28 \text{ GeV}, \quad m_{\mu} = 0.105 \text{ GeV} \]
\[ m_{\tau} = 1.77 \text{ GeV}, \quad |V_{cb}V_{cs}^*| = 4.15 \times 10^{-2}, \quad |V_{tb}V_{ts}^*| = 4.1 \times 10^{-2}, \quad \tau_B = 0.453 \times 10^{-12} \text{ sec} \]
\[ G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha^{-1} = 137, \quad f_B = 0.35 \text{ GeV}, \quad f_D = 0.30 \text{ GeV} \]

Table 3. Values of input parameters used in our numerical analysis [95].

| Wilson coefficients | \( C^7 \) | \( C^9 \) | \( C_{10} \) |
|---------------------|-----------|-----------|-------------|
| SM                  | -0.313    | 4.334     | -4.669      |
| SUSY I              | +0.376    | 4.767     | -3.735      |
| SUSY II             | +0.376    | 4.767     | -3.735      |
| SUSY III            | -0.376    | 4.767     | -3.735      |
| SUSY SO(10) (\( A_0 = -1000 \)) | -0.219 | 4.275 | -4.732 |

Table 4. Wilson Coefficients in SM and different SUSY models but without NHBs contributions [45–48].

| Wilson coefficients | \( C^7 \) | \( C^9 \) | \( C_{10} \) |
|---------------------|-----------|-----------|-------------|
| SM or SUSY I,II,III | 0         | 0         | 0           |
| SUSY SO(10) (\( A_0 = -1000 \)) | 0.039 + 0.038i | 0.011 + 0.072i | -0.075 - 0.67i |

Table 5. Primed Wilson Coefficients in SM and different SUSY models but without NHBs contributions. Where the primed Wilson coefficients corresponds to the operators which are opposite in helicities from those of the SM operators.

The values of Wilson coefficients used in our numerical analysis are taken from Refs. [45–48] and summarized in Tables 4, 5 and 6. In the following analysis, we will focus on
the parameter space of large \( \tan \beta \), where the NHBs effects are important owing to the fact that the Wilson coefficients corresponding to NHBs are proportional to \( (m_b m_t/m_{h^0}^2) \tan^3 \beta \), with \( h = h^0, A^0 \). Here, the \( \tan \beta \) contributes from the chargino-up-type squark loop and the \( \tan^2 \beta \) appears from the exchange of the NHBs. In the Ref. [62, 63] it is pointed out that at large value of \( \tan \beta \) the \( C^{(i)}_{Q_i} \) compete with \( C^{(j)}_{Q_j} \) and can outpace \( C^{(k)}_{Q_k} \) in some region. Depending on the magnitude and sign of the SUSY parameters one can think of many options in the parameter space, but experimental results i.e., the decay rate of \( b \to s \gamma \) and \( b \to s \ell^+ \ell^- \) restrict us to consider the following scenarios for MSSM:

- **SUSY I**: refers to the regions where SUSY destructively contributes and changes the sign of \( C^{\text{eff}}_7 \), which will have drastic effects on the observables, but without contribution of NHBs.

- **SUSY II**: corresponds to the region where \( \tan \beta \) is large and the masses of the particles are relative small.

- **SUSY III**: points to the regions where \( \tan \beta \) is large and the masses of superpartners are also relatively large, i.e. \( \geq 450 \text{ GeV}^2 \).

In SUSY I and SUSY II scenarios one can accommodate the non-zero crossing of the forward-backward asymmetry in \( B \to K^* \mu^+ \mu^- \) decay [52–55]. Since the primed Wilson coefficients are for the primed operators in Eq. (2.2) which appear in the SUSY SO(10) GUT model. In Table 6 it is mentioned that the values of \( C^{(i)}_{Q_i} \) is different for the different choice of final state lepton, which is due to the fact that contributions from the NHBs are proportional to the lepton mass.

| Wilson coeff. | \( C_{Q_1} \) | \( C'_{Q_1} \) | \( C_{Q_2} \) | \( C'_{Q_2} \) |
|---------------|----------------|----------------|----------------|----------------|
| SM            | 0              | 0              | 0              | 0              |
| SUSY I        | 0              | 0              | 0              | 0              |
| SUSY II       | 6.5(16.5)      | 0              | -6.5(-16.5)    | 0              |
| SUSY III      | 1.2(4.5)       | 0              | -1.2(-4.5)     | 0              |
| SUSY SO(10)   | 0.106 + 0i     | -0.247 + 0.242i | -0.107 + 0i   | -0.25 + 0.246i |

\((A_0 = -1000)\) (1.775 + 0.002i) \((-1.418 + 4.074i)\) \((-1.797 - 0.002i)\) \((-4.202 + 4.128i)\)

**Table 6.** Wilson coefficient corresponding to NHBs contributions in different SUSY scenarios. Where the primed Wilson coefficients are for the primed operators from NHBs contribution in SUSY SO(10) GUT model. The values in the parentheses are for the \( \tau \) lepton case.
\( b \rightarrow s\ell^+\ell^- \) due to flip of the sign of \( C_7 \) from positive to negative, within the constraint on \( b \rightarrow s\gamma \). Also it is noticed that when the masses of sparticles are relatively large, say about 450 GeV, there exits considerable regions in the SUSY parameter space where NHBs could contribute dominantly as in the case of SUSY III and the SUSY SO(10) models. However, in these scenarios the sign of \( C_7 \) remains unaltered with respect to the SM sign because of the cancellation of the contributions of charged Higgs and charginos with each other.

It is worth mentioning here that the decay \( B_q \rightarrow \ell^+\ell^- \) is a clean channel to probe for the NHBs effects in SUSY models at large \( \tan \beta \). Since its branching ratio in the SUSY models can be written as

\[
\text{BR}(B_s \rightarrow \mu^+\mu^-) = \frac{\sum^2 m^2_{B_s} \tau_{B_s} f^2_{B_s} |V_{ts}^* V_{tb}|^2 \sqrt{1 - \frac{4m^2}{m_{B_s}^2}}}{64\pi^3 m_{B_s}^3} \left(1 - \frac{4m^2}{m_{B_s}^2}\right) C^2_{Q_1} + \left[C_{Q_2} + \frac{2m}{m_{B_s}} C_{10}\right]^2, \tag{4.1}
\]

where \( C_{Q_1} \) and \( C_{Q_2} \) are referring to the NHBs contributions which are absent in the SM but in MSSM they are proportional to \((m_b m/m_H^2)\tan^3 \beta\), with \( h = h^0, A^0 \). The value of \( C_{10} \) is large in the SM but it is suppressed by the factor \( 2m/m_{B_s} \), therefore, the corresponding SM value of branching ratio is

\[
\text{BR}_{SM}(B_s \rightarrow \mu^+\mu^-) = (3.19 \pm 0.19) \times 10^{-9}. \tag{4.2}
\]

The branching ratio corresponding to different values of the NHBs parameters can be enhanced by a factor ranging from \(10^1 - 10^3\), but the recent upper bound by CDF collaborations, \(3.9 \times 10^{-8}\) at 95% confidence level [97], is about 12 times larger than that of SM prediction. It can be noticed from Eq. (4.1) that the branching ratio for \( B_s \rightarrow \mu^+\mu^- \) is directly proportional to the NHBs contributions. Therefore, this stringently constraints the parameter space of the SUSY models and especially the large value of the Wilson Coefficients \( C_{Q_1} \) and \( C_{Q_2} \) are severely constrained. Therefore, the precise measurement of this decay at the future experiments will help us to get useful constraints on the SUSY parameters and if considerably large deviation from SM prediction is measured, then along with the signal of supersymmetry, this would have important implications on the Higgs searches at LHC.

The purpose of the present study of the SUSY effects in \( B_s \rightarrow D^*_s \ell^+\ell^- \), with \( \ell = \mu, \tau \), is to incorporate the constraints provided by \( B_s \rightarrow \mu^+\mu^- \) as well as to see the effects of NHBs at the larger extent in these FCNC processes. Since this decay mode is unique in a sense that it contains WA contributions along with the penguin and so it is interesting to see how SUSY affects different physical observables in this process. For our numerical analysis we have set the values of \( C_{Q_1} \) and \( C_{Q_2} \) between 0 to 6.5(16.5) for muons and (tauons) to check the dependency of different observables on these NHBs contributions. We hope that in the future experiments such as the Tevaran and the LHC, with the more data on these decays, will help us to test more precisely the constraints obtained from the decay channel \( B_s \rightarrow \mu^+\mu^- \).
First, we discuss the branching ratios ($\mathcal{BR}$) for the decays $B_c \rightarrow D_\ast^s \ell^+\ell^-$, with $\ell = \mu, \tau$, which we have plotted as a function of $q^2$ (GeV$^2$) in Fig. 1, without and with long-distance contributions in the Wilson coefficients, both in the SM and in the SUSY scenarios. This figure depicts that the values of $\mathcal{BR}$, both for the case of muons and tauons as final state leptons, get sizeably influenced due to the SUSY effects which come through the new parameters, i.e., the modified and new Wilson coefficients corresponding to the operators described in Eq. (2.2). One can see clearly from these graphs that the increment in the values of $\mathcal{BR}$ in the SUSY I model is due to the relative change of the sign of $C_7^{eff}$ with respect to that of the SM; while the large deviation in SUSY II model from the SM values is mainly due to the contributions of NHBs and due to the relative change of the sign of $C_7^{eff}$. Since the values of Wilson Coefficients corresponding to NHBs is small for the cases of SUSY III and SUSY SO(10) GUT models, so one should expect small deviation in the $\mathcal{BR}$s from the SM values and this signature is clear in Fig. 1. Moreover, the NP effects due to the SUSY, manifest in the $\mathcal{BR}$s throughout the $q^2$ region irrespective of the mass of the final state leptons. In addition, one can also extract the constructive behavior of SUSY I and SUSY II to the $\mathcal{BR}$ from Table 7. Furthermore, we have also plotted the $\mathcal{BR}$s with the long-distance contribution corresponding to the SM and SUSY models in Fig. 1(b,d).

![Figure 1](image-url)  
*Figure 1.* The dependence of branching ratio of $B_c \rightarrow D_\ast^s \ell^+\ell^-$ ($\ell = \mu, \tau$) on $q^2$ without long-distance contributions (a,c) and with long-distance contributions (b,d) for different scenarios of MSSM and SUSY SO(10) GUT model. In all the graphs, the solid, dashed, dashed-dot, dashed-double dot and dashed-triple dot curves correspond to the SM, SUSY I, SUSY II, SUSY III and SUSY SO(10) GUT model, respectively.
It is important to mention that as an exclusive decay, there are different sources of uncertainties involved in the analysis of the above mentioned decay. The major source of uncertainties in the numerical analysis of $B_c \to D_s^* \ell^+\ell^-$ ($\ell = \mu, \tau$) decays originate from the $B_c \to D_s^*$ transition form factors calculated in the QCD sum rule approach [38] as summarized in Table 2. But it is also important to stress that these hadronic uncertainties have almost no influence on the various asymmetries including the forward-backward asymmetries, lepton polarization asymmetries and helicity fractions of $D_s^*$ in the decays $B_c \to D_s^\ell^+\ell^-$ because of the cancellation among different polarization states and this makes them a good tool to probe beyond the SM.

To illustrate the generic effects due to the SUSY models on the forward-backward asymmetry $A_{FB}$, we plot $\frac{d(A_{FB})}{dq^2}$ as a function of $q^2$ in Fig. 2. For the zero position of $A_{FB}$ it is also argued that the uncertainty in the zero position of the $A_{FB}$ due to the hadronic uncertainties is negligible [9] to leading order in $\alpha_s$. Therefore, the zero position of the $A_{FB}$ can serve as a stringent test for the NP effects including the SUSY. Figures 2(a) and 2(b) describe the $A_{FB}$ for $B_c \to D_s^* \mu^+\mu^-$ with and without long-distance contributions in the Wilson coefficients respectively, where the different SUSY models show clear deviation from the SM curve. Figure 2(a) clearly shows that SUSY I and SUSY II do not cross the zero of the $A_{FB}$ because of the fact that the zero crossing of $A_{FB}$ of SM is solely due to the opposite signs of $C^\text{eff}_7$ and $C^\text{eff}_9$, but in SUSY I and SUSY II the signs of these coefficients are same and hence the zero point of $A_{FB}$ disappears in both cases. Furthermore, in the SUSY III model, due to the opposite sign of $C^\text{eff}_7$ and $C^\text{eff}_9$, the $A_{FB}$ passes from the zero but the zero position shifts mildly to the right (about 0.2 GeV$^2$) from that of the SM value, i.e. 3.3 GeV$^2$, due to the contribution from the NHBs. Moreover, the SUSY SO(10) GUT model also shows the similar behavior except that the zero position of the $A_{FB}$ shifts considerably to the left (about 1 GeV$^2$) from that of the SM value. Hence, the precise measurement of the zero position of $A_{FB}$ for the decay $B_c \to D_s^* \mu^+\mu^-$ will be a very good observable to yield any indirect imprints of NP due to SUSY models and can serve as a good tool to distinguish among the different variants of SUSY models.

Besides the zero position of $A_{FB}$, the magnitude of $A_{FB}$ is also an important tool to establish NP. Particularly, this can be more exciting when the tauons are the final state leptons, where the zero of the $A_{FB}$ is absent. In the Figs. 2(c) and 2(d) the $A_{FB}$ of $B_c \to D_s^\tau^+\tau^-$ is plotted with and without long-distance contributions, respectively, where the different SUSY models present a distinct variations from that of the SM magnitude.
Figure 2. The dependence of forward-backward asymmetry for the decays $B_c \rightarrow D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) on $q^2$ without long-distance contributions (a,c) and with long-distance contributions (b,d) for different scenarios of MSSM and SUSY SO(10) GUT model.

A closer look on the pattern of Fig. 2(c) indicates that the SUSY I and SUSY II models decrease the magnitude of $A_{FB}$ from its SM value. Whereas, SUSY III has very mild deviation from that of the SM behavior but SUSY SO(10) GUT model has an increment in the magnitude of $A_{FB}$ compared to that of the SM value for the case where tauons are in the final state. It is valuable to comment here that just like the zero position of the $A_{FB}$, the magnitude of $A_{FB}$ depends on the values of the Wilson coefficients $C_{eff}^7, C_{eff}^9, C_{10}$ and $C_Q^{(\ell)}$ and the effects due to the hadronic uncertainties on the magnitude of $A_{FB}$ are almost insensitive. Hence, the deviation in the magnitude of $A_{FB}$ due to the SUSY parameters is prominent and can be measured at the experiments which indeed help us to understand the constraints on the parameter space of the SUSY.

We now consider another interesting observable to get the complementary information about NP in $B_c \rightarrow D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) decays, i.e. the lepton polarization asymmetries which are shown in Figs 3, 4 and 5. Since different polarization asymmetries depend on the Wilson Coefficients so one can expect large dependency of these asymmetries on different SUSY variants and hence making these observables fertile to extract the NP. Therefore, we expect that even in the SUSY I model where the value of the Wilson coefficient corresponding to NHBs is zero, the values of these polarization asymmetries would be mainly modified from those of the SM value, because of the change in the sign of the terms pro-
Figure 3. The dependence the probabilities of the longitudinal lepton polarization asymmetries, $P_L$, for the decays $B_c \rightarrow D_s^*\ell^+\ell^-$ ($\ell = \mu, \tau$) on $q^2$ without long-distance contributions (a,c) and with long-distance contributions (b,d), respectively, for different scenarios of MSSM and SUSY SO(10) GUT model.

proportional to $C_7^{\text{eff}}C_{10}$. Now we focus on the longitudinal polarization asymmetry for the decays $B_c \rightarrow D_s^*\ell^+\ell^-$ ($\ell = \mu, \tau$) without and with long-distance contributions plotted in Figs. 3(a,c) and 3(b,d), respectively. Due to this sign alteration of Wilson coefficients in different SUSY models, Fig. 3 evince the variations in the magnitude of the longitudinal polarization asymmetries from those of the SM. This value is expected to increase in SUSY I and SUSY II model compared to that of the SM value due to the opposite sign $C_7^{\text{eff}}C_{10}$ and because of the NHBs contribution in the later case. By looking at the Eq. (3.22) we can see that the contribution of NHBs, coming in the auxiliary functions $f_7$ and $f_8$, is compensated by the mass of the final state lepton. Therefore the large deviation is expected for heavy mass of the final state lepton and large value of NHBs contribution i.e. SUSY I and SUSY II. Whereas, the SUSY III and SUSY SO(10) GUT models show mild effect in the case of muons as the final state lepton while their effects are quite distinguishable in the case of tauons as the final state leptons.

Figures 4(a,c) (without long-distance contribution) and 4(b,d) (with long-distance contribution) show the dependence of normal lepton polarization asymmetries on the square of momentum transfer for the said decays. One can notice that the values of said asymmetries are quite sensitive to the large contribution of NHBs in SUSY II and SUSY III.
models which is also clear from Eq. (3.23) whereas, it is mildly effected for the case of SUSY SO(10) GUT model due to small contributions of NHBs and due to the complex part of the Wilson coefficients. It is important to note that when there is a large contribution from the NHBs in $P_N$ it will changes its sign as indicated in Fig. 4 for the case of SUSY II. As the hadronic uncertainties are insignificant in these asymmetries so the contribution from different SUSY variants are quite distinguishable for the case when the final state leptons are muons and even more prominent when these leptons are tauons. This can be established from Eq. (3.23) when $P_N$ is proportional to the final state leptonic mass. Furthermore, as the normal polarization is proportional to the $\lambda$ which approaches to zero at high $q^2$ region and hence the normal polarization asymmetries are suppressed by $\lambda$ in this region which is depicted in Fig. 4.

We now discuss the dependence of transverse polarization asymmetries on square of momentum transfer for the decays $B_c \rightarrow D_s^* \ell^+ \ell^-$, without and with long-distance effects, plotted in Figs. 5(a,b) and 5(c,d) for $\ell = \mu$ and $\tau$, respectively. One can see from Eq. (3.24), that it is proportional to the imaginary parts of the Wilson coefficients which are absent in the SM and in SUSY I, SUSY II and SUSY III model. But in the SUSY SO(10) GUT model one would expect the non-zero transverse polarization in for the $B_c \rightarrow D_s^* \mu^+ \mu^-$
Figure 5. The dependence the probabilities of the transverse lepton polarization asymmetries, $P_L$, for the decays $B_c \rightarrow D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) on $q^2$ without long-distance contributions (a,c) and with long-distance contributions (b,d), respectively, for different scenarios of MSSM and SUSY SO(10) GUT model.

$(\tau^+ \tau^-)$ decays due to complex flavor non-diagonal down-type squark mass matrix of 2nd and 3rd generations of order one at GUT scale, which can induce the complex couplings and consequently lead to complex Wilson coefficients. But these effects are very small in the said decay channels. These signatures of the transverse polarization asymmetries are depicted in Figs. 5(a,c) and Figs. 5(b,d) without and with long-distributions, respectively. Here we can see that its value is for small to measure experimentally both for the case of muons and tauons as the final state leptons.

Apart from the above mentioned observables there is another physical observable sensitive the NP in $B_c \rightarrow D_s^* \ell^+ \ell^-$ transitions, i.e. the helicity fractions of $D_s^*$ vector meson produced in the final state. The measurement of longitudinally $K^*$ helicity fractions ($f_L$) in the decay modes $B \rightarrow K^* \ell^+ \ell^-$ by the BABAR Collaborations [94] put enormous interest in this observable. Additionally, it is also shown that the helicity fractions of final state meson, just like $BR$, $A_{FB}$ and $P_{L,N,T}$, are also very good observables to dig out the NP [33, 90–93]. In this respect, it is natural to study the helicity fractions for the complementary FCNC processes such as $B_c \rightarrow D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) in and beyond the SM. For this purpose, we have plotted the longitudinal ($f_L$) and transverse ($f_T$) helicity fractions of $D_s^*$ for the SM and different SUSY models in Figs. 6 and 7 for the final state leptons as muons.
and tauons, respectively. In these graphs the values of the longitudinal ($f_L$) and transverse ($f_T$) helicity fractions of $D_s^*$ are plotted against $q^2$ and one can clearly see that at each value of $q^2$ the sum of $f_L$ and $f_T$ is equal to one.

Figure 6 depicts the case of muons as final state leptons, the effects of the different SUSY scenarios on the longitudinal (transverse) helicity fractions of $D_s^*$ are well distinguishable throughout the $q^2$ region. Here one can notice that for the case of SUSY I, when the contributions of NHBs is neglected, the deviation from that of the SM values is prominent. Similarly for the SUSY II scenario, when the NHBs contributions are large and tan $\beta$ is also large, the NP effects are spotlighted. It is also clear from Fig. 6 that although the influence of the SUSY III and SUSY SO(10) models are mild for the case of muons as final state leptons but one can observe that for the case of tauons in the final state these effects are quite enhanced from that of the SM values (see Fig. 7). Moreover, Fig. 6 also manifests the variations in the values of $f_L$ ($f_T$) for the different SUSY variants with respect to that of the the SM values, which can be a good tool to put stringent constraints on the parameter space of different SUSY models.

Now we turn our attention to the case, where tauns are the final state leptons, the helicity fractions of $D_s^*$ are shown in Figs. 7(a,c) and 7(b,d) without and with the long-
distance contributions, respectively. One can easily extract that similarly to the case of

\[ f_{L,T} \] in \( B_c \rightarrow D_s^* \tau^+ \tau^- \) decays on \( q^2 \) without long-distance contributions (a,c) and with long-distance contributions (b,d), respectively, for different scenarios of MSSM and SUSY SO(10) GUT model.

![Figure 7](image-url)

**Figure 7.** The dependence the probabilities of the longitudinal and transverse helicity fractions, \( f_{L,T} \), of \( D_s^* \) in \( B_c \rightarrow D_s^* \tau^+ \tau^- \) decays on \( q^2 \) without long-distance contributions (a,c) and with long-distance contributions (b,d), respectively, for different scenarios of MSSM and SUSY SO(10) GUT model.

muons, there is also prominent deviations in the values of the helicity fractions for all the SUSY models from that of the SM values. However, the effects for SUSY III and SUSY SO(10) models are more prominent as compare to the previous case where the muons are the final state leptons. These figures have also enlightened the variation in the extremum values of helicity fractions from the SM due to the change in the SUSY parameters. The deviation in extremum values are very well marked up at 12.5 GeV² for all the SUSY models, for example, the extremum value of longitudinal (transverse) helicity fraction is changed from its SM value 0.47(0.53) to 0.32(0.68), 0.385(0.615), 0.40(0.60) and 0.53(0.47) for SUSY I, SUSY II, SUSY III and SUSY SO(10), respectively, which is suitable amount of deviation to measure. Hence, the measurement of the extremum values of \( f_L \) and \( f_T \) in the case of \( B_c \rightarrow D_s^* \tau^+ \tau^- \) can be used as a good tool in studying the NP beyond the SM and to distinguish among the different SUSY models.
5 Conclusion

In our study on the rare $B_c \rightarrow D_s^* \ell^+\ell^-$ decays, with $\ell = \mu, \tau$, we have calculated branching ratio ($\mathcal{BR}$), the forward-backward asymmetry $A_{FB}$ of the leptons, the polarization asymmetries $P_{L,N,T}$ of final state leptons and the helicity fractions $f_{L,T}$ of the final state vector meson $D_s^*$ and analyzed the implications of different SUSY models on these observable for the said decays. The main outcomes of our analysis can be summarized as follows:

- We have observed that the $\mathcal{BR}$s deviate sizeably from the SM value in different SUSY models. The study has shown that the $\mathcal{BR}$ is increased considerably for SUSY I and SUSY II mainly because of the change of the sign of Wilson coefficient $C_{7}^{eff}$. But for SUSY III and SUSY SO(10) the values of $\mathcal{BR}$ are mildly effected from that of the SM values because the small contributions of NHBs and due to the fact that the sign of Wilson coefficient $C_{7}^{eff}$ remains the same as the SM. Hence the accurate measurement of the $\mathcal{BR}$s for these decays would help us to say something about the physics beyond the SM including SUSY models.

- Along with the $\mathcal{BR}$, our analysis show that $A_{FB}$, especially the zero position of the $A_{FB}$, is a fertile observable to extract the NP including SUSY. We have found that for SUSY I and SUSY II, $A_{FB}$ does not cross the zero position unlike the SM for $B_c \rightarrow D_s^* \mu^+\mu^-$, because of the same signs of Wilson coefficient $C_{7}^{eff}$ and $C_{9}^{eff}$ for these two scenarios of SUSY. This signature is similar to that of the observed signals for $B \rightarrow K^* \mu^+\mu^-$. Moreover, the shift in the zero positions of $A_{FB}$ for SUSY III and SUSY SO(10) models provide a promising signature of the NP which can be tested experimentally. Hence, the measurement of the magnitude as well as the zero crossing position of $A_{FB}$ of the considered decays can provide a stringent test for MSSM models and SUSY SO(10) GUT models.

- The longitudinal, normal and transverse polarization asymmetries of leptons are calculated in different SUSY scenarios for the rare semileptonic charmed $B_c$ meson which can be tested in experiments with great precision. It is found that the SUSY effects could be measured at future experiments and will shed light on the NP signal beyond the SM. The transverse polarization asymmetry is the most interesting observable to look for the SUSY SO(10) effects where its value is non-zero in almost all $q^2$ region unlike for the cases of the SM and MSSM models. It is measurable at future experiments such as the LHC and the BTeV machines where a large number of $b\bar{b}$ pairs are expected to be produced.

- We have calculated the helicity fractions $f_{L,T}$ of the final state vector meson $D_s^*$ to extract the comparative study of different SUSY models and the SM. The study has shown that the deviation from the SM values of the helicity fractions are quite large with tauons in final state for the MSSM models as well as SUSY SO(10) model. It is also shown that there is a noticeable change due to SUSY parameter in the position of the extremum values of the longitudinal and transverse helicity fractions of $D_s^*$ meson for the case of tauons as a final state leptons. Hence, the helicity fraction of
$D_s^*$ meson in the decays $B_c \to D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) can be a stringent test in finding
the status of the SUSY models at the LHC.

To sum up, the more data to be available from Tevatron and LHCb will provide a
powerful testing ground for the SM and to put some constraints on the SUSY parameter
space in particular the value of $\tan \beta$ and the masses of lightest chargino and lightest
stop squark. Our comparative analysis of the SM and the different SUSY models on the
observables for $B_c \to D_s^* \ell^+ \ell^-$ ($\ell = \mu, \tau$) decays can be useful for probing and its extension
to various SUSY models the existence of the supersymmetry.

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