Production of a tensor glueball in the reaction $\gamma\gamma \rightarrow G_2\pi^0$ at large momentum transfer

N. Kivel * and M. Vanderhaeghen

Helmholtz Institut Mainz, Johannes Gutenberg-Universität, D-55099 Mainz, Germany
Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany

Abstract

We study the production of a tensor glueball in the reaction $\gamma\gamma \rightarrow G_2\pi^0$. We compute the cross section at higher momentum transfer using the collinear factorisation approach. We find that for a value of the tensor gluon coupling of $f_T^g \sim 100$ MeV, the cross section can be measured in the near future by the Belle II experiment.

1 Introduction

The hadronic states made up only from gluons are known as glueballs [1]. The spectrum of such states have been computed using lattice QCD techniques, see e.g. [2–5]. However, identifying glueballs in experiment is not an easy task because they have the same quantum numbers as quark-antiquark mesons. At present time there are few scalar candidates which can be associated with the scalar glueballs, see e.g. [6–8] and references therein. Much less is even known about tensor glueballs. The lattice calculations predict the mass of the lowest $2^{++}$ glueball state to be around $2.3 - 2.4$ GeV. There are some experimental evidences that such state have may been seen in various processes [9–11].

In this Brief Report we would like to study the production of tensor $2^{++}$ glueballs in two-photon collisions at high momentum transfer. Despite the fact that the cross section of such process is relatively small it can potentially be observed at high luminosity $e^+e^-$ colliders like Belle II. The production of glueballs in the reaction $\gamma\gamma \rightarrow G\pi^0$ has already been studied long time ago in Refs. [12–14]. However the tensor glueball was considered only in Ref. [14] but within the specific approach where the glueball state is described as a weakly bound state of two non-relativistic gluons. In the present work the production amplitude is computed using the QCD factorisation approach [16–18]. The coupling of quarks and gluons to the final mesonic states is described by the distribution amplitudes (DAs) describing the momentum fraction distribution of partons at zero transverse separation in a two-particle Fock state. Our calculation is presented in Sec.2 and results for the cross section are shown in Fig.4. Concluding remarks are given in Sec.3.

2 Calculation

The production amplitude for the $\gamma\gamma \rightarrow G\pi^0$ process can be described in terms of the helicity amplitudes

$$iA_{\pm\pm} = \varepsilon_{1\mu}\varepsilon_{2\nu}(\pm) \int d^4x \, e^{-i(q_1x)} \langle G_2(p), \pi^0(k) | T\{ J^{\mu}_{em}(x), J^{\nu}_{em}(0) \} | 0 \rangle$$

The cross section of the process is given by [15]

$$\frac{d\sigma_{\gamma\gamma}[\pi^0G_2]}{d\cos\theta} = \frac{1}{64\pi} \frac{s + m^2}{s^2} \left( |A_{++}|^2 + |A_{+-}|^2 \right)$$

*On leave of absence from St. Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia
where we assume for the quark flavors \( \bar{q} q \) traceless, and satisfies the condition

\[
|A_{\pm\pm}|^2 = \sum_{\lambda=-2}^{2} A_{\pm\pm}(\lambda) A_{\pm\pm}^*(\lambda),
\]

(3)

We choose the momenta as \( \gamma(q_1)\gamma(q_2) \rightarrow \pi(k)G_2(p) \) and consider the center mass system (cms) \( \bar{k} + \bar{p} = 0 \) with the pion and glueball momenta directed along \( z \)-axis, see Fig.1. We also introduce light-like vectors \( n = (1, 0, 0, -1) \) and \( \bar{n} = (1, 0, 0, 1) \) so that \( (V n) \equiv V_+ = V_0 + V_3 \). The light-cone expansion of the particle momenta in the region with \( s \sim -t \sim -u \gg \Lambda_{QCD} \) reads

\[
p \simeq \sqrt{s} \bar{n}, \quad k \simeq \sqrt{s} \bar{n}^2,
\]

(4)

\[
q_1 = \sqrt{s} \frac{1}{2} (1 - \eta) \frac{\bar{n}}{2} + \sqrt{s} \frac{1}{2} (1 + \eta) \frac{\bar{n}}{2} + q_\perp, \quad q_2 = \sqrt{s} \frac{1}{2} (1 + \eta) \frac{\bar{n}}{2} + \sqrt{s} \frac{1}{2} (1 - \eta) \frac{\bar{n}}{2} - q_\perp,
\]

(5)

\[
q_\perp^2 = \sqrt{s} \frac{1}{4} (1 - \eta^2), \quad \eta \equiv \cos \theta,
\]

(6)

where \( \theta \) is the scattering angle in the cms, see Fig.1 and where we neglected the small power suppressed terms.

In the region \( s \sim -t \sim -u \gg \Lambda_{QCD} \) the amplitudes can be computed in terms of convolution integrals of the hard coefficient function with the mesonic distribution amplitudes. The typical diagrams are shown in Fig.2. The blobs in Fig.2 denote the light-cone matrix elements which define the DAs of the outgoing mesons. The pion DA is defined as

\[
\sqrt{2} f_\pi \phi_\pi(y) = i \int \frac{d\lambda}{\pi} e^{-i\lambda(k-1)k} \langle \pi^0(k) | \bar{q}(\lambda \bar{n}) \bar{\gamma}_- \gamma_5 q(-\lambda \bar{n}) | 0 \rangle,
\]

(7)

where we assume for the flavor structure \( \bar{q} q = \bar{u}u - \bar{d}d \). The pion DA has normalisation \( \int_0^1 dy \phi_\pi(y) = 1 \) so that the pion decay constant \( f_\pi \) is defined as

\[
\langle \pi^0(k) | \bar{\pi}(0) \bar{\gamma}_- \gamma_5 (0) | 0 \rangle = -i \sqrt{2} f_\pi k_-, \quad f_\pi = 131\text{MeV}.
\]

(8)

The light-cone matrix element of tensor glueball can be defined in the similar way as for the tensor meson \( 2^+ \), see, e.g. Ref. [20]. In general case there are three light-cone matrix elements which define two gluon DAs and one quark DA. In the quark case the distribution amplitudes is defined as

\[
\langle G_2(p, \lambda) | \bar{q}(\lambda \bar{n}) \gamma_+ q(-\lambda \bar{n}) | 0 \rangle = \sqrt{2} f_3 m^2 \frac{e^{(\lambda)}}{p^+} \int_0^1 dx e^{i\lambda(2x-1)p^+} \phi_2(x),
\]

where we assume for the quark flavors \( \bar{q} q = \bar{u}u + \bar{d}d \). The polarisation tensor \( e^{(\lambda)}_{\alpha\beta} \) is symmetric and traceless, and satisfies the condition \( e^{(\lambda)}_{\alpha\beta} p^\beta = \epsilon_1 \). The polarisation sum is given by

\[
\sum_\lambda e^{(\lambda)}_{\mu\nu} e^{(\lambda)*}_{\rho\sigma} = \frac{1}{2} M_{\mu\rho} M_{\nu\sigma} + \frac{1}{2} M_{\mu\sigma} M_{\nu\rho} - 1 \frac{3}{3} M_{\mu\nu} M_{\rho\sigma},
\]

(9)

\(^1\text{Here } p \text{ is the exact momentum with } p^2 = m^2.\)
Figure 2: Typical diagrams which describe the amplitudes $A_{++}$. The blobs denote the distribution amplitudes.

where $M_{\mu\nu} = g_{\mu\nu} - p_{\mu} p_{\nu} / m^2$ and the normalization condition reads $\epsilon_{\mu\nu}^{(\lambda)} (\lambda')^* = \delta_{\lambda\lambda'}$. The distribution amplitude is antisymmetric function $\phi_2(1-x) = -\phi_2(x)$ and describes transition into the tensor glueball with helicity $\lambda = 0$. The normalization of the quark DA is given by

$$\int_0^1 dx (2x - 1) \phi_2(x) = 1.$$  

(10)

Therefore the constant $f_g$ is defined as a matrix element of the local operator

$$\langle G_2(p, \lambda) \mid q \{ \gamma_\mu (i\not{D}_\nu - i\not{D}_\nu) + (\mu \leftrightarrow \nu) \} \rangle q(0) = \sqrt{2} f_g m^2 \epsilon_{\mu\nu}^{(\lambda)} ,$$  

(11)

where $D_\mu$ is the covariant derivative.

The gluon DAs are defined as

$$\langle G_2(p, \lambda) \mid G_{\alpha\beta}^g (\lambda n) G_{\mu\nu}^g (-\lambda n) \rangle = \int_0^1 dx e^{i(2x-1)p_+} \left( f_g^{T} e_{(\lambda)}^{(\lambda)} p_+ \phi^T_g (x) - f_g^S m^2 \frac{1}{2} g_{\mu\nu}^{\lambda} e_{(\lambda)}^{(\lambda)} \phi^S_g (x) \right) ,$$  

(12)

where $g_{\mu\nu}^{\perp} = g_{\mu\nu} - (n^\mu n^\nu + n^\nu n^\mu)/2$ and the short notation $e_{(\mu\perp \nu \perp)}^{(\lambda)}$ denotes the transverse traceless projection

$$e_{(\mu\perp \nu \perp)}^{(\lambda)} = e_{(\mu\perp \nu \perp)}^{(\lambda)} - \frac{1}{2} g_{\mu\nu} m^2 e_{(\mu\perp \nu \perp)}^{(\lambda)} , \quad g_{\mu\nu}^{\perp} e_{(\mu\perp \nu \perp)}^{(\lambda)} = 0.$$  

(13)

The distribution amplitudes $\phi^T_g (x)$ and $\phi^S_g (x)$ are symmetric with respect to the interchange of $x \leftrightarrow 1-x$ and describe the momentum fraction distribution of the two gluons having the same and the opposite helicity, respectively.

The constants $f_g^{T}$ and $f_g^{S}$ are defined through the matrix element of the local two-gluon operator:

$$\langle G_2(p, \lambda) \mid G_{\alpha\beta}^a (0) G_{\mu\nu}^a (0) \rangle = f_g^{T} \left\{ \left[ (p_\alpha p_\mu - \frac{1}{2} m^2 g_{\alpha\mu}) e_{(\alpha \leftrightarrow \beta)}^{(\lambda)} (\alpha \leftrightarrow \beta) \right] + \frac{1}{2} f_g^S m^2 \left[ g_{\alpha\mu} e_{(\lambda)}^{(\lambda)} - (\alpha \leftrightarrow \beta) \right] - (\mu \leftrightarrow \nu) \right\} .$$  

(14)

Using these definitions one can compute all necessary diagrams some of which are shown in Fig 2.

The glueball with the tensor polarisation $\lambda = \pm 2$ is only produced if the colliding photons have the same helicities.\footnote{Note that use \pm notations both for the helicity amplitudes (in case of amplitudes $A_{\pm\pm}$) and for the light-cone projections. This should not lead to a notational confusion.}

$$A_{++}(s, \eta) = e_{(\perp \perp)}^{(\lambda)} \frac{q_{\perp \perp}^2}{s(1-\eta^2)} \left( \frac{1}{2} i e^{\beta\sigma\mu\nu} \frac{q_{\perp \sigma} n_{\mu} \bar{n}_{\nu}}{s} \right) f_x \int \frac{f_g^{T}}{s} 64\pi^2 \alpha_s (\mu^2) (c_1^2 - c_2^2) \frac{\sqrt{2}}{N_c} I_g^{++}(\eta) .$$  

(15)
where $\alpha$ is the electromagnetic coupling, $N_c$ denotes the number of colors. The convolution integral $I_{g}^{++}(\eta)$ reads

$$I_{g}^{++}(\eta) = \int_{0}^{1} dy \frac{\phi_+(y)}{y y} \int_{0}^{1} dx \frac{\phi_g^T(x)}{x x} \frac{2}{2 xy - x - y + \eta(x - y)},$$  \hspace{1cm} (16)$$

where we assume that $\bar{x} = 1 - x$. The factor $(1 - \eta^2)^{-1}$ which is explicitly shown in Eq. (15) cancels after summation over polarisation $\lambda$ in the computation $|A_{++}|^2$ defined in (3). Therefore the $\eta$-dependence of the cross section (2) is completely defined by the convolution integral $I_{g}^{++}(\eta)$.

The production of a glueball with $\lambda = 0$ (scalar polarisation) is described by the amplitude $A_{+-}$ which reads

$$A_{+-}(s, \eta) = -\epsilon^{(\lambda)} \frac{m^2}{s} \frac{s}{f_{\pi}} 8\pi^2 \alpha \alpha_s(\mu^2)(e_u^2 - e_d^2) \frac{1}{N_c} \left( C_F f_q I_{++}^+ + \sqrt{2} f_g I_{g}^{++} \right),$$  \hspace{1cm} (17)$$

where

$$I_{g}^{--}(-\eta) = \int_{0}^{1} dy \frac{\phi_-(y)}{y y} \int_{0}^{1} dx \frac{\phi_2(x)}{x x} \frac{\eta(1 - \eta^2)(y - x)(1 - x - y)^2}{(1 - x - y)^2(1 - \eta^2) + 4x\bar{x}yy},$$  \hspace{1cm} (18)$$

$$I_{g}^{+-}(-\eta) = \int_{0}^{1} dy \frac{\phi_+(y)}{y y} \int_{0}^{1} dx \frac{\phi_g^S(x)}{x x} \frac{1 - 2x - \eta}{x\bar{y} + y(1 - 2x + \eta)}.$$  \hspace{1cm} (19)$$

In order to write the convolution integrals in this form we used the symmetry properties of the DAs with respect to interchange $x \rightarrow 1 - x$. The hard coefficient functions for various processes with gluons have also been computed in Ref. 19. We have checked up to a general factor that our results shown in Eqs. (15) and (17) are in agreement with the corresponding hard kernels in Ref. 19.

In order to make numerical estimates we need to specify models for the DAs and provide numerical values for the low energy glueball couplings. In the following we suppose that the states $f_2(2300)$ and $f_2(2340)$ which have been recently observed by the Belle [10] and BESIII [11] collaborations are good candidates to be tensor glueball. In our numerical estimates we use the following models of DAs. For pion we take

$$\phi_+(y) \simeq 6y\bar{y} + 6a_2(\mu)y\bar{y}C_2^{3/2}(2y - 1),$$  \hspace{1cm} (20)$$

with the second moment

$$a_2(\mu = 1\text{GeV}) = 0.20.$$  \hspace{1cm} (21)$$

This value is close to many phenomenological estimates and lattice QCD result [21].

For the glueball DAs we take the simplest asymptotic models

$$\phi_2(x) = 30x\bar{x}(2x - 1),$$  \hspace{1cm} (22)$$

$$\phi_g^T(x) = \phi_g^S(x) = 30x^2\bar{x}^2.$$  \hspace{1cm} (23)$$

Let us first consider the properties of the convolution integrals $I_{g}^{++}$ (of Eqs. (16), (18) and (19)). In Fig. 3 we show the values of the convolution integrals as a function of $\cos \theta$. Below we assume that factorisation works reasonably for such values of $\theta$ where $|u|, |t| \geq 2.5 \text{GeV}^2$. This region corresponds to the inner area between the two vertical lines on the plots in Fig. 3. One can easily see that in the vicinity of $\theta = 90^\circ$

$$|I_{g}^{++}| \gg |I_{g}^{+-}| \gg |I_{g}^{+-}|.$$  \hspace{1cm} (24)$$

The integrals $I_{g}^{+-}$ vanish at $\theta = 90^\circ$ therefore we can conclude that at least around this point the dominant contribution will be given by the amplitude $A_{++}$ which describes the production of the glueball in the tensor polarisation.

The values of the couplings $f_q^{T,S}$ and $f_q$ are not known. It is natural to assume that the glueball state strongly overlaps with the gluon wave function and the value of the gluon couplings are relatively large and can be of the same order as the quark coupling $f_q \sim 100 \text{MeV}$ for quark-antiquark mesons, i.e.
Figure 3: The convolution integrals as a functions of $\cos \theta$. The shaded area between the vertical lines corresponds to the region where $|u|, |t| \geq 2.5$ GeV$^2$ for $s = 13$ GeV$^2$. The factorisation scale is fixed to be $\mu^2 = 3.2$ GeV$^2$. For the model of the DAs used to make these predictions, see text.

Figure 4: The cross section as a function of $\cos \theta$ at $s = 13$ GeV$^2$ (left) and $s = 16$ GeV$^2$ (right) in the region $|u|, |t| \geq 2.5$ GeV$^2$. The solid, dashed and dotted lines correspond to $f_g^{T}(1\text{GeV}) = 150, 100, 50$ MeV, respectively.

$f_g^S \sim f_g^T \sim 100$ MeV. For the glueball quark coupling $f_q$ we consider the different scenarios with $f_q \ll f_g$ and $f_q \sim f_g$ corresponding to the small and to the large quark-antiquark component, respectively. Such scenarios will be described by the following numerical values

$$f_q(\mu = 1 \text{ GeV}) \simeq 10 - 100 \text{ MeV},$$  \hspace{1cm} (25)

$$f_g(\mu = 1 \text{ GeV}) \simeq 100 \text{ MeV},$$ \hspace{1cm} (26)

$$f_g^T(\mu = 1 \text{ GeV}) \simeq 50 - 150 \text{ MeV}.$$ \hspace{1cm} (27)

The evolution of these coupling is the same as the evolution of the corresponding coupling for the tensor meson $f_2(1270)$ except for flavor mixing and can be found in Ref. [20]. Let us notice that the tensor gluon DA $\phi_5^T$ does not mix under evolution with quark contributions and therefore it describes the genuine gluon component of the glueball wave function.

The numerical estimates show that the value of the cross section is practically saturated by the contribution from the amplitude $A^{++}$ describing the production of a glueball in the tensor polarisation. The contribution of the amplitude $|A^{+-}|$ is always about two orders of magnitude smaller for all numerical values of the couplings $f_q$ and $f_g$ shown in Eqs. (25) and (26). Therefore we can conclude that the contribution with $|A^{+-}|$ does not provide significant numerical impact. Hence the cross section is only sensitive to the value of tensor coupling $f_g^T$. This can also be seen, for instance, from the analysis of the decay $G_2 \to \phi \phi$ which can be used for identification of the glueball state.

In Fig. 4, we show the cross section as a function of $\cos \theta$ at fixed values of energy $s$. In the numerical calculations we take $\alpha_s = 3$ and $\alpha_s(m_z^2) = 0.297$. The cross section are shown for the energy values $s = 13$ GeV$^2$ and $16$ GeV$^2$. The factorisation scale is fixed to be $\mu^2 = 3.2$ GeV$^2$ and $\mu^2 = 4$ GeV$^2$, respectively. The values of $\cos \theta$ correspond to the region $||t|, |u| \geq 2.5$ GeV$^2$. We obtain that for $f_g^T(1 \text{GeV}) = 100$ MeV

$$s^3 \frac{d\sigma}{d\cos \theta} \simeq 11 - 17 \text{ GeV}^3 \text{nb}.$$  \hspace{1cm} (28)
In Fig. 5, we show the glueball cross section for $f_T^g(1 \text{ GeV}) = 100 \text{ MeV}$ and $s = 13 \text{ GeV}^2$ in comparison with the cross section data for $\gamma \gamma \to \pi^0 \pi^0$ for $s = 13.3 \text{ GeV}^2$. The data are taken from Ref. [22]. For convenience the glueball cross section is scaled by a factor 4. From this picture one can conclude that the measurement of $\gamma \gamma \to G_2^2 \pi^0$ cross section requires a larger luminosity which will be achieved in the Belle II experiment.

3 Conclusions

We calculated the amplitudes and cross sections for the production of a tensor glueball in the reaction $\gamma \gamma \to G_2^2 \pi^0$. We obtained that for the value of the low energy coupling $f_T^g \simeq 100 \text{ MeV}$ the cross section is dominated by the contribution describing the production of a glueball in the tensor polarization. A corresponding measurements allow one to constrain the value of the tensor coupling $f_T^g$. We expect that the corresponding cross section can be observed in the upcoming higher statistic Belle II experiment.

References

[1] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365.
[2] G. S. Bali et al. [UKQCD Collaboration], Phys. Lett. B 309 (1993) 378 [hep-lat/9304012].
[3] C. J. Morningstar and M. J. Peardon, Phys. Rev. D 60 (1999) 034509 [hep-lat/9901004].
[4] Y. Chen et al., Phys. Rev. D 73 (2006) 014516 [hep-lat/0510074].
[5] C. M. Richards et al. [UKQCD Collaboration], Phys. Rev. D 82 (2010) 034501 arXiv:1005.2473 [hep-lat].
[6] E. Klempt and A. Zaitsev, Phys. Rept. 454 (2007) 1 [arXiv:0708.4016 [hep-ph]].
[7] V. Crede and C. A. Meyer, Prog. Part. Nucl. Phys. 63 (2009) 74 [arXiv:0812.0600 [hep-ex]].
[8] W. Ochs, J. Phys. G 40 (2013) 043001 arXiv:1301.5183 [hep-ph].
[9] A. Etkin et al., Phys. Lett. 165B (1985) 217.
[10] S. Uehara et al. [Belle Collaboration], PTEP 2013 (2013) no.12, 123C01 doi:10.1093/ptep/ptt097 arXiv:1307.7457 [hep-ex].
[11] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 93 (2016) no.11, 112011 arXiv:1602.01523 [hep-ex].
[12] G. W. Atkinson, J. Sucher and K. Tsokos, Phys. Lett. **137B** (1984) 407.
[13] A. B. Wakely and C. E. Carlson, Phys. Rev. D **45** (1992) 1796.
[14] M. A. Ichola and J. Parisi, Z. Phys. C **66** (1995) 653 [hep-ph/9501225].
[15] V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo, Phys. Rept. **15** (1975) 181.
[16] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22** (1980) 2157.
[17] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24** (1981) 1808.
[18] A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B** (1980) 245.
[19] V. N. Baier and A. G. Grozin, Z. Phys. C **29** (1985) 161.
[20] V. M. Braun, N. Kivel, M. Strohmaier and A. A. Vladimirov, JHEP **1606** (2016) 039 [arXiv:1603.09154 [hep-ph]].
[21] V. M. Braun, S. Collins, M. Gekeler, P. Prez-Rubio, A. Schfer, R. W. Schiel and A. Sternbeck, Phys. Rev. D **92** (2015) no.1, 014504 [arXiv:1503.03656 [hep-lat]].
[22] S. Uehara *et al.* [Belle Collaboration], Phys. Rev. D **79** (2009) 052009 [arXiv:0903.3697 [hep-ex]].