PRIMORDIAL NON-GAUSSIANITY AND DARK ENERGY CONSTRAINTS FROM CLUSTER SURVEYS

EMILIANO SEFUSATTI, CHRIS VALE, KENJI KADOTA,1 AND JOSHUA FRIE MAN2

Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL; emiliano@fnal.gov

Received 2006 October 4; accepted 2006 November 28

ABSTRACT

Galaxy cluster surveys will be a powerful probe of dark energy. At the same time, cluster abundance is sensitive to any non-Gaussianity of the primordial density field. It is therefore possible that non-Gaussian initial conditions might be misinterpreted as a sign of dark energy or at least degrade the expected constraints on dark energy parameters. To address this issue, we perform a likelihood analysis of an ideal cluster survey similar in size and depth to the upcoming South Pole Telescope survey and Dark Energy Survey. We analyze a model in which the strength of the non-Gaussianity is parameterized by the constant $f_{NL}$; this model has been used extensively to derive cosmic microwave background (CMB) anisotropy constraints on non-Gaussianity, allowing us to make contact with those works. We find that the constraining power of the cluster survey on dark energy observables is not significantly diminished by non-Gaussianity, provided that cluster redshift information is included in the analysis. We also find that even an ideal cluster survey is unlikely to significantly improve current and future CMB constraints on non-Gaussianity. However, when all systematic uncertainties are under control, such surveys could constitute a valuable cross-check on CMB observations.

Subject headings: cosmology: theory — galaxies: clusters: general

Online material: cosmology: theory — galaxies: clusters: general

1. INTRODUCTION

Of the many fascinating discoveries in cosmology over the last decade, perhaps none has aroused more interest than the discovery of the accelerating expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999). Probing the nature of the dark energy thought to be driving this acceleration has become a top priority for the community, and among the promising tools under consideration are surveys of galaxy clusters. Since the number of clusters as a function of redshift and mass depends on both the growth of structure and on the volume of space, the cluster abundance is sensitive to the matter density, the density fluctuation amplitude, and the expansion history of the universe. For this reason, upcoming cluster surveys will be powerful probes of cosmology (see, e.g., Haiman et al. 2001; Holder et al. 2001; Battye & Weller 2003; Molnar et al. 2004; Wang et al. 2004; Rapetti et al. 2005; Marian & Bernstein 2006).

Although constraining dark energy is a leading motivator for much of the interest in cluster surveys, it is worth noting that the cluster abundance is potentially sensitive to various cosmological parameters beyond those associated with dark energy. For example, it has been recognized for some time that slight deviations from Gaussianity in the primordial matter distribution would cause a significant change in the high-mass tail of the halo distribution (Lucchin & Matarrese 1988; Colafrancesco et al. 1989; Chiu et al. 1998; Robinson et al. 1998; Koyama et al. 1999; Robinson & Baker 2000; Robinson et al. 2000; Matarrese et al. 2000). In this paper, we use a maximum likelihood analysis to investigate the extent to which dark energy constraints from cluster surveys are degraded by including the possibility of non-Gaussian initial conditions, in particular when considered within the limits allowed by present and future cosmic microwave background (CMB) observations.

The specific form of non-Gaussian initial conditions we consider here is of the local type, described in position space by a primordial curvature perturbation of the form

$$\Phi(x) = \phi(x) + f_{NL}[\phi^2(x) - \langle \phi^2(x) \rangle]$$

(Verde et al. 2000; Komatsu & Spergel 2001), where $\phi(x)$ is a Gaussian random field and the degree of non-Gaussianity is parameterized in terms of the constant $f_{NL}$. For this model, tight constraints, of order $\Delta f_{NL} \sim 40$, are provided by CMB observations (Komatsu et al. 2003; Creminelli et al. 2006; Chen & Szapudi 2006; Spergel et al. 2007), while constraints that are somewhat weaker but closer in physical scale to that of clusters are expected from higher order galaxy correlations (Scoccimarro et al. 2004). From a theoretical point of view, the non-Gaussian model of equation (1) is motivated in part by studies of the generation of density perturbations in inflationary scenarios; while single-field inflation models typically predict an unobservably small value for $f_{NL}$ (e.g., Acquaviva et al. 2003; Maldacena 2003), multifield inflation models can lead to much higher values (e.g., Lyth et al. 2003; Creminelli 2003; Dvali et al. 2004; Zaldarriaga 2004; Arkani-Hamed et al. 2004; Alishahiha et al. 2004; Kolb et al. 2006; Sasaki et al. 2006). For a review, see Bartolo et al. (2004).

While we believe it is worthwhile to keep an open mind regarding other forms of non-Gaussianity that may not be properly described by the simple expression in equation (1) and which might make the extrapolation of current CMB constraints to cluster scales less straightforward than we assume here (see, e.g., Mathis et al. 2004), we note that the physical scale probed by clusters differs from that of the Planck survey by roughly a factor of 2, so that the two probes are likely to be affected more or less equally by deviations from equation (1).

As we discuss below in greater detail, the parameters in our likelihood analysis include $f_{NL}$, the matter density $\Omega_m$, and the matter fluctuation amplitude $\sigma_8$, while we consider both constant and time-varying dark energy equations of state, described in terms of one ($w$) and two ($w_0$ and $w_a$; Chevallier & Polarski 2001;
Linder (2003) parameters, respectively. For definiteness, we assume a fiducial ideal survey similar in size and depth to the upcoming South Pole Telescope (SPT) survey and Dark Energy Survey (DES) (Ruhl et al. 2004; Abbott et al. 2005). We assume a ΛCDM fiducial cosmology, for two values of σ8, since cluster number counts are extremely sensitive to this parameter.

This paper is organized as follows: In § 2, we introduce our model for the non-Gaussian mass function and describe our analysis of the dependence of the expected errors on cosmological parameters on the non-Gaussian component. In § 3 we present our results, and we conclude in § 4.

2. THE MODEL

In this section, we present the methods applied in the present work. We begin with a brief review of previous works dealing with non-Gaussian initial conditions in galaxy cluster observations, and then we describe in detail our treatment of the non-Gaussian mass function. We conclude with a discussion of the likelihood analysis, the results of which are given in § 3.

2.1. Historical Overview

Expressions for the cluster mass function in the presence of non-Gaussian initial conditions have been derived as extensions to the Press-Schechter Ansatz (Press & Schechter 1974, hereafter PS), first by Lucchin & Matarrese (1988) and Colafrancesco et al. (1989), with a simpler approach adopted later by Chiu et al. (1998) and Robinson et al. (1998).

The original PS formula describes the comoving number density \( n(M) dM \) of clusters with masses in the interval \((M, M + dM)\) as

\[
n_{\text{PS}}(M) dM = -\frac{2\bar{\rho}}{M} \frac{d\ln M}{dM} \left[ \int_{\delta_c}^{\infty} P_G(y) dy \right] dM, \tag{2}
\]

where we have suppressed, for clarity, the redshift dependence, \( \bar{\rho} \) is the critical density, \( \sigma_M \) is the rms of mass fluctuations in spheres of radius \( R = (3M/4\pi\bar{\rho})^{1/3} \), \( \delta_c = 1.686 \) is the critical linear overdensity in the spherical-collapse model, and \( P_G \) is the Gaussian probability distribution function (PDF), \( P_G(y) = e^{-y^2/(2\sigma^2)} \). Since the function \( P_G(y) \) does not depend explicitly on the mass \( M \) or, therefore, on the scale \( R \), equation (2) reduces to

\[
n_{\text{PS}}(M) dM = -\frac{2\bar{\rho}}{M} \frac{\delta_c}{\sigma_M} \frac{d\ln M}{dM} P_G(\delta_c/\sigma_M) dM. \tag{3}
\]

The PS formalism assumes that the scale dependence of the PDF of the density field is completely described by the scale dependence of the variance \( \sigma^2 \). Lucchin & Matarrese (1988), Colafrancesco et al. (1989), and, later, Matarrese et al. (2000) considered a derivation of the non-Gaussian mass function, based on equation (2), that takes into account the scale dependence of higher order cumulants, thereby allowing for a generic dependence of the PDF on the smoothing scale \( R \). Specifically, Matarrese et al. (2000, hereafter MVJ) derived the mass function corresponding to the model described by equation (1). The non-Gaussianity of the mass function is described, to first approximation, in terms of the skewness \( S_{3,R} \) of the smoothed density field \( \delta_R \),

\[
S_{3,R} = \langle \delta_R^3 \rangle / \langle \delta_R^2 \rangle^2,
\]

and is obtained from the cumulant generator of the distribution as

\[
n_{\text{MVJ}}(M) dM \\
\approx -\frac{2\bar{\rho}}{M^2} \frac{1}{\sigma_M^2} \left( \frac{\delta_c}{2} \frac{d\ln \sigma_M}{d\ln M} + \delta_c \frac{d\ln \sigma_M}{d\ln M} \right) e^{-\delta_c^2/(2\sigma^2)} dM,
\]

where \( \delta_c = 3(1 - S_{3,R} \delta_c/3)^{1/2} \).

It is worth noting here that although equation (1) should be seen as a truncated expansion in powers of \( \phi \), the mass function provided by equation (5) is not linear in the non-Gaussianity parameter \( f_{\text{NL}} \) (since \( S_{3,R} \sim f_{\text{NL}} \)); rather, it describes the non-Gaussian PDF by its proper dependence on the skewness while neglecting all higher order cumulants.

The simpler extension to non-Gaussian initial conditions introduced by Chiu et al. (1998) consists instead of replacing the Gaussian function \( P_G(y) \) in equation (3) by the appropriate, non-Gaussian PDF \( P_{\text{NG}}(y) \), assumed to be scale independent. The resulting mass function, which we will denote here as “extended PS” or EPS, therefore reads

\[
n_{\text{EPS}}(M) dM = -\frac{2\bar{\rho}}{M^2} \frac{\delta_c}{\sigma_M} \frac{d\ln \sigma_M}{d\ln M} P_{\text{NG}}(\delta_c/\sigma_M) dM. \tag{6}
\]

This approach has been tested in \( N \)-body simulations by Robinson & Baker (2000) for several non-Gaussian models; they find that equation (6) agrees with measurements of the cumulative mass function \( n(M) \) in the simulations to within 25%. While this error is slightly larger than the differences between the PS formula (eq. [2]) and simulation results for Gaussian initial conditions, it is much smaller than the model-to-model differences between the cumulative mass functions. As a measure of the non-Gaussianity of the tail of the distribution function \( P_{\text{NG}}(y) \), Robinson et al. (1998) introduced a parameter (there called \( T \)) defined as

\[
G = \frac{\int_{-\infty}^{\infty} P_{\text{NG}}(y) dy}{\int_{-\infty}^{\infty} P_G(y) dy}, \tag{7}
\]

with \( G = 1 \) corresponding to the Gaussian case.

Following this approach, Robinson et al. (1998), Koyama et al. (1999), and Willick (2000) placed constraints on primordial non-Gaussianity from X-ray cluster survey observations (Henry & Arnaud 1991; Ebeling et al. 1996; Henry 1997), and Amara & Refregier (2004) relate primordial non-Gaussianity to the normalization of the dark matter power spectrum. In particular, assuming that the non-Gaussian primordial field can be generically described by a lognormal distribution, Robinson et al. (2000) found, for a ΛCDM cosmology, the constraint \( G < 0 \) at the 2 \( \sigma \) level. An analysis of the constraining power of future Sunyaev-Zel’dovich (S-Z) cluster surveys on cosmological parameters that includes the possibility of primordial non-Gaussianity is provided by Benson et al. (2002). Specifically, that work assumes the lognormal PDF studied by Robinson et al. (2000) and performs a Fisher-matrix analysis that includes the matter and baryon density parameters \( \Omega_m \) and \( \Omega_b \), and the non-Gaussianity factor \( G \). The results for the 1 \( \sigma \) errors on \( G \), assuming priors from CMB, large-scale structure (LSS), and supernova observations, are \( \Delta G \approx 2 \) and \( \Delta G \approx 0.1 \) for the Bolocam and Planck experiments, respectively.

Finally, Sadeh et al. (2006) apply the same extended PS formalism to the \( \chi^2_{\text{m}} \) non-Gaussian model (White 1999; Koyama...
et al. 1999). Here, however, much attention is devoted to highly non-Gaussian models, for example, with \( m = 1 \) and \( m = 2 \), which are already excluded by measurements of the galaxy bispectrum in the PSCz survey (Feldman et al. 2001).

### 2.2. The Non-Gaussian Mass Function

In our analysis we will make use of the EPS approach (eq. [6]), since it can be more easily implemented [once the probability function \( P_{NG}(y) \) is known] and avoids problems with small regions of the parameter space where the MVJ expression for the mass function (eq. [5]) is beyond its limits of validity. For most of the cases considered in § 3, however, we performed the analysis using both approaches, finding almost identical results.

Since the PS and EPS expressions are known to differ by up to 25% from \( N \)-body results, we use the EPS non-Gaussian mass function only to model departures from the Gaussian case; for the latter, we use an analytic mass function fit to the \( N \)-body results. Specifically, we consider the non-Gaussian mass function \( n(M, z, f_{NL}) \) to be given by the product

\[
n(M, z, f_{NL}) = n_G(M, z)F_{NG}(M, z, f_{NL}),
\]

where \( n_G(M, z) \), corresponding to the Gaussian case, is the fit to \( N \)-body simulations provided by Jenkins et al. (2001):

\[
n_G(M, z)\, dM = -0.301 \frac{\rho_m}{M_{\sigma M}} \log |D(z)\sigma_M| \exp \{ -|0.64 - \log (D(z)\sigma_M)|^{1.42} \} \frac{d\sigma_M}{dM},
\]

with \( D(z) \) the linear growth factor computed by solving the differential equation governing structure evolution. The non-Gaussian factor \( F_{NG}(M, z, f_{NL}) \) is derived from the EPS mass function and simply given by

\[
F_{NG}(M, z, f_{NL}) = \frac{n_{EPS}(M, z, f_{NL})}{n_{PS}(M, z, f_{NL})},
\]

where \( n_{PS} \) is the Gaussian PS mass function. Note that for \( f_{NL} = 0 \), we have \( n_{EPS} = n_{PS} \).

It can be easily shown that the predictions of the EPS and MVJ methods are very close by comparing them for relevant values of the parameter \( f_{NL} \). In Figure 1, we plot the ratio of the non-Gaussian mass function to the Gaussian one at different redshifts and as a function of the mass \( M \) for the 2 \( \sigma \) limits

\[-27 < f_{NL} < 121\]

obtained from a bispectrum analysis of the first-year Wilkinson Microwave Anisotropy Probe (WMAP) data by Creminelli et al. (2006), yielding constraints that are slightly tighter than, but consistent with, those obtained from the WMAP 3 year data by Spergel et al. (2007). We plot as well the limits

\[-243 < f_{NL} < 337,\]

obtained from a bispectrum analysis of the first-year Wilkinson Microwave Anisotropy Probe (WMAP) data by Creminelli et al. (2006), yielding constraints that are slightly tighter than, but consistent with, those obtained from the WMAP 3 year data by Spergel et al. (2007). We plot as well the limits

\[-243 < f_{NL} < 337,\]

and redshift, corresponding to extremely rare events (7 \( \sigma \)) limits obtained from the WMAP first-year constraints on \( f_{NL} \), from Creminelli et al. (2006), while the outer solid lines correspond to the limits \(-243 < f_{NL} < 337 \) from the expected SDSS galaxy bispectrum constraints (Scoccimarro et al. 2004) computed with the EPS approach (eq. [6]). The dashed lines, which almost coincide with the solid ones, correspond to the same limits computed by means of the MVJ formula (eq. [5]).

[See the electronic edition of the Journal for a color version of this figure.]

![Fig. 1.—Uncertainty on the mass function \( n(M, z) \) due to non-Gaussianity, expressed as the ratio of the non-Gaussian to the Gaussian mass function at four different redshifts (0, 0.5, 1, and 1.5) for different values of \( f_{NL} \). The inner solid lines correspond to the (2 \( \sigma \)) limits --27 < \( f_{NL} < 121 \) derived from the WMAP (first-year) constraints on \( f_{NL} \), from Creminelli et al. (2006), while the outer solid lines correspond to the limits --243 < \( f_{NL} < 337 \) from the expected SDSS galaxy bispectrum constraints (Scoccimarro et al. 2004) computed with the EPS approach (eq. [6]). The dashed lines, which almost coincide with the solid ones, correspond to the same limits computed by means of the MVJ formula (eq. [5]).]
the first moments of the primordial distribution, if not by the skewness alone. It is worth stressing, however, that both prescriptions for the non-Gaussian mass function, even when limited to modeling deviations from the Gaussian case, need to be properly tested against N-body simulations. New results in this direction will soon be available (S. Matarrese 2006, private communication).

By means of these measured probability functions, in the framework of the EPS approach it is also possible to translate the constraints on the parameter \( f_{NL} \) into constraints on the parameter \( G \) defined in equation (7). As an example, the WMAP 1 \( \sigma \) error \( \Delta f_{NL} = 37 \) (Creminelli et al. 2006) corresponds to \( \Delta G \simeq 0.06 \), while the expected 1 \( \sigma \) error \( \Delta f_{NL} = 145 \) from SDSS galaxy bispectrum measurement corresponds to \( \Delta G \simeq 0.25 \).

2.3. Likelihood Analysis

In this subsection, we describe the likelihood analysis that we use to obtain our results. We consider two simple models depending on four and five parameters. In addition to \( f_{NL} \), we consider the matter density parameter \( \Omega_m \) and fluctuation amplitude parameter \( \sigma_8 \), and we will separately consider the cases of dark energy with either a constant equation-of-state parameter \( w \) or a time-varying equation of state described by two parameters \( (w_0, w_\text{eff}) \). In all cases we assume a spatially flat cosmological model for simplicity.

The fiducial values assumed for the likelihood analysis are given in Table 1. Since the expected number of observable clusters is highly dependent on the value of \( \sigma_8 \), for the four-parameter model we perform the analysis assuming as well the lower value \( \sigma_8 = 0.75 \), while in every other case we assume \( \sigma_8 = 0.9 \). The choice of the fiducial value \( f_{NL} = 47 \) for the non-Gaussianity parameter does not substantially affect any of the results of the present work.

Unless otherwise stated, we consider an ideal survey with limiting mass \( M_{\text{lim}} = 1.75 \times 10^{14} \) \( h^{-1} M_{\odot} \) and a sky coverage of 4000 deg\(^2\) \( (\text{fsky} \simeq 10\%) \) out to a maximum cluster redshift of 1.5, corresponding to the expectations for the SPT and DES projects. For our fiducial model with \( \sigma_8 = 0.9 \) and \( f_{NL} = 47 \), this yields a total of 21,000 clusters in 15 redshift bins; if we had instead chosen \( f_{NL} = 0 \) for the fiducial model, we would obtain 20,000 clusters, consistent with earlier estimates (Wang et al. 2004).

We study the dependence on cosmology and on the constant \( f_{NL} \) of the total number and mass distribution of clusters above a certain fixed—i.e., redshift-independent—threshold mass \( M_{\text{lim}} \) and explore the degeneracies introduced by varying non-Gaussian initial conditions. While the redshift dependence of the threshold mass should be included when making precise predictions for a given survey, this dependence is weak for S-Z-selected cluster samples; as a result, our neglect of such dependence here does not significantly affect our conclusions.

The total number of clusters with mass \( M \) above \( M_{\text{lim}} \), per unit redshift, is given by

\[
\frac{dN}{dz} = \Delta \Omega \frac{dV}{dz d\Omega}(z) \int_{M_{\text{lim}}}^{\infty} n(M, z, f_{NL}) dM ,
\]

where

\[
\frac{dV}{dz d\Omega}(z) = \frac{1}{H(z)} \left[ \int_{z'}^{\infty} \frac{dz'}{H(z')} \right]^2 
\]

is the cosmology-dependent volume factor for flat models and \( \Delta \Omega \) is the solid angle subtended by the survey area.

We show in Figure 2 the sensitivity of the total number of clusters above \( M_{\text{lim}} \) per unit redshift and unit area \( (\text{top}) \) and of the comoving number density \( (\text{bottom}) \) to different values of the non-Gaussianity parameter \( f_{NL} \), compared with the sensitivity to different values of the dark energy equation-of-state parameter \( w_0 \) and of the fluctuation amplitude parameter \( \sigma_8 \).

The top left panel of Figure 2 shows that varying \( f_{NL} \) over the range allowed by current CMB observations yields changes in the cluster counts comparable to a 10% variation in the dark energy equation-of-state parameter \( w \). However, the top right panel shows that the redshift dependence of the mass function variations due to non-Gaussianity are different from the variations due to changes in \( w \). This is essentially a result of the fact that \( w \) affects both the mass function and the volume factor. On the other hand, the redshift dependence of variations due to changes in \( f_{NL} \) appears more similar to variations induced by changes in \( \sigma_8 \), so we expect a stronger degeneracy between these two parameters.

In Figure 3, we show the dependence of the mass function \( n(M, z) \) on the same parameters, this time as a function of the mass \( M \) for \( z = 0 \) \( (\text{top}) \) and \( z = 1 \) \( (\text{bottom}) \). In this case the behavior of the cluster density as we vary \( f_{NL} \) and \( \sigma_8 \) is quite different. One can clearly see how non-Gaussianity is particularly significant for
Throughout this paper, we consider the high-mass tail of the distribution. This fact suggests that it might be relevant to consider a likelihood analysis that takes into account the full functional shape of the mass function by dividing the observable clusters into mass bins (see, e.g., Hu 2003; Lima & Hu 2004, 2005; Francis et al. 2005; Kravtsov et al. 2006) and in cluster redshift determination (e.g., Huterer et al. 2004). We excluded as well statistical uncertainties related to sample variance (e.g., Hu & Kravtsov 2003) and theoretical uncertainties in the cluster mass function and its cosmological dependence (e.g., Heitmann et al. 2005; Warren et al. 2006; Crocce et al. 2006; Reed et al. 2007). Finally, we note that other degeneracies with parameters such as those describing spatial curvature (Abbott et al. 2005) and the effect of massive neutrinos on the dark matter power spectrum (Huterer & Linder 2006) might be relevant for future high-precision analyses.

3. RESULTS

In this section, we estimate the impact of marginalizing over the non-Gaussianity parameter $f_{\text{NL}}$ on the determination of the dark energy equation of state, as well as on two other relevant cosmological parameters, the matter density $\Omega_m$ and fluctuation amplitude $\sigma_8$. We separately consider the case of a dark energy equation of state determined by a single parameter ($w$) and the case of a two-parameter description of a time-varying equation of state (at 0 and $w_0$).

We derive the marginalized errors on the parameters with fixed $f_{\text{NL}} = 47$ (no marginalization) and with three different Gaussian priors on $f_{\text{NL}}$, two corresponding to the constraints from CMB bispectrum measurements expected from the Planck experiment (Komatsu & Spergel 2001; Liguori et al. 2006) and measured in the WMAP experiment (Creminelli et al. 2006), with

\[
\begin{align*}
\sigma_{f_{\text{NL}}} &= 47 \pm 5 \quad (1 \sigma, \text{Planck}), \\
\sigma_{f_{\text{NL}}} &= 47 \pm 37 \quad (1 \sigma, \text{WMAP}), \\
\sigma_{f_{\text{NL}}} &= 47 \pm 145 \quad (1 \sigma, \text{SDSS forecast}).
\end{align*}
\]

This last case is motivated by a possible strong scale dependence of primordial non-Gaussianity, not captured by the model defined by equation (1), which could result in a stronger non-Gaussian effect at smaller scales, thereby escaping the CMB constraints. As a rough estimate of the smallest scale probed by the mentioned experiments, we note that for WMAP the maximum multipole $l_{\text{max}} \approx 1000$ corresponds to $\sim 50 h^{-1}$ Mpc, while Planck is expected to probe a scale one-third this size; in the SDSS case, a maximum comoving wavenumber $k_{\text{max}} \approx 0.3 h$ Mpc$^{-1}$ corresponds to $20 h^{-1}$ Mpc. The typical scale probed by clusters is about $5-10 h^{-1}$ Mpc, with the most massive clusters approaching the lowest scale probed by Planck.

In all the different cases considered, we include as well the results obtained with two independent Gaussian priors on $\Omega_m$ and $\sigma_8$, with errors roughly corresponding to the knowledge provided by WMAP observations for a $\Lambda$CDM model (Spergel et al. 2007) in combination with other probes, such as the LSS power spectrum,

\[
\begin{align*}
\sigma_{\sigma_8} &= 0.9 \pm 0.05, \\
\Omega_m &= 0.27 \pm 0.035,
\end{align*}
\]
and by future constraints from Planck in combination with other probes,
\[ \sigma_8 = 0.9 \pm 0.01, \quad \Omega_m = 0.27 \pm 0.0035 \]  
(Efstathiou et al. 2006). As an extreme example, in the bottommost rows of the tables that follow, we give results corresponding to fixing \( \Omega_m \) and \( \sigma_8 \), therefore studying a likelihood function for the dark energy parameters and \( f_{\text{NL}} \) alone.

We caution that these priors are purely illustrative and were chosen here for the sake of simplicity. A proper treatment of external data sets, which is beyond the scope of this paper, would naturally involve the parameters’ covariance and would directly affect the dark energy parameters as well. On the other hand, even rigorous analyses of CMB or LSS galaxy power spectra would probably be insensitive to the non-Gaussianity parameter \( f_{\text{NL}} \).

### 3.1. One-Parameter Dark Energy Equation of State

The main results of this paper are shown in Table 2, where we present the expected 1 \( \sigma \) errors from the cluster survey for the three parameters \( \Omega_m, \sigma_8 \), and \( w \) with no marginalization on \( f_{\text{NL}} \) (\( \Delta f_{\text{NL}} = 0 \)) and with a marginalization that includes the three Gaussian priors discussed above (\( \Delta f_{\text{NL}} = 5, 37, \) and 145), assuming in this case a fiducial \( \sigma_8 = 0.9 \). The percentages in parentheses express the increase in the error with respect to the case without marginalization on \( f_{\text{NL}} \). Although the derived marginalized likelihoods for single parameters are quite close to Gaussian functions, the estimated errors reported in Table 2, as in the following tables, are given for clarity by the mean between upper and lower errors.

The major conclusion from Table 2 is that inclusion of a possible non-Gaussian component at the level allowed by present and future CMB constraints will not have an appreciable impact on the determination of dark energy parameters from cluster surveys. On the other hand, if we use only a single cluster mass bin (i.e., no information about the shape of the cluster mass function) and the WMAP prior on \( f_{\text{NL}} \), the inclusion of non-Gaussianity degrades the cluster constraint on \( \sigma_8 \) by 50%. Moreover, using only the projected SDSS bispectrum constraint on \( f_{\text{NL}} \), we do see degeneracies between \( f_{\text{NL}} \) and dark energy: the error on \( w \) from clusters increases by \( \sim 70\% \), and the error on \( \sigma_8 \) grows by a factor of more than 3 compared with the purely Gaussian case. In all cases, the determination of \( \Omega_m \) remains largely unaffected.

The expected degeneracy between \( f_{\text{NL}} \) and \( \sigma_8 \) is evident from the two-parameter 95% confidence level (CL) contour plots shown in Figure 4, where we marginalized over the remaining parameters. The same overall behavior is observed when we impose priors on \( \Omega_m \) and \( \sigma_8 \). In Figure 4, the marginalized likelihood plot for \( w \) shows that inclusion of the WMAP prior on \( f_{\text{NL}} \) leads to essentially the same dark energy sensitivity for the cluster survey as one would obtain by fixing \( f_{\text{NL}} \) (i.e., by not including non-Gaussianity).

### Table 2

| Parameter | \( \Delta f_{\text{NL}} = 0 \) | \( \Delta f_{\text{NL}} = 5 \) | \( \Delta f_{\text{NL}} = 37 \) | \( \Delta f_{\text{NL}} = 145 \) |
|----------|-----------------|-----------------|-----------------|-----------------|
| \( \Delta w \) | 0.045 (0%) | 0.045 (0%) | 0.049 (9%) | 0.079 (76%) |
| \( \Delta \Omega_m \) | 0.0085 (0%) | 0.0085 (0%) | 0.0085 (0%) | 0.0087 (2%) |
| \( \Delta \sigma_8 \) | 0.0051 (2%) | 0.0052 (2%) | 0.0083 (63%) | 0.0256 (340%) |
| \( \Delta f_{\text{NL}} \) | ... | 5.0 (0%) | 37 (0%) | 123 (0%) |
| Gaussian Priors: \( \Omega_m = 0.27 \pm 0.0035, \sigma_8 = 0.9 \pm 0.05 \) | | | | |
| \( \Delta w \) | 0.044 (0%) | 0.044 (0%) | 0.048 (9%) | 0.076 (73%) |
| \( \Delta \Omega_m \) | 0.0082 (0%) | 0.0082 (0%) | 0.0082 (0%) | 0.0083 (1%) |
| \( \Delta \sigma_8 \) | 0.0050 (2%) | 0.0050 (2%) | 0.0081 (62%) | 0.0205 (310%) |
| \( \Delta f_{\text{NL}} \) | ... | 5.0 (0%) | 36 (0%) | 113 (0%) |
| Gaussian Priors: \( \Omega_m = 0.27 \pm 0.0035, \sigma_8 = 0.9 \pm 0.01 \) | | | | |
| \( \Delta w \) | 0.023 (4%) | 0.024 (4%) | 0.031 (35%) | 0.042 (83%) |
| \( \Delta \Omega_m \) | 0.0032 (0%) | 0.0032 (0%) | 0.0032 (0%) | 0.0032 (0%) |
| \( \Delta \sigma_8 \) | 0.0021 (10%) | 0.0023 (10%) | 0.0055 (60%) | 0.0091 (330%) |
| \( \Delta f_{\text{NL}} \) | ... | 5.0 (0%) | 31 (0%) | 54 (0%) |
| Fixed \( \Omega_m = 0.27 \) and \( \sigma_8 = 0.9 \) | | | | |
| \( \Delta w \) | 0.0172 (3%) | 0.0177 (3%) | 0.0184 (9%) | 0.0184 (9%) |
| \( \Delta f_{\text{NL}} \) | ... | 5.6 (0%) | 5.6 (0%) | 5.7 (0%) |

**Note:** The percentages in parentheses express the increase in the error with respect to the case without marginalization on \( f_{\text{NL}} \) (\( \Delta f_{\text{NL}} = 0 \)). The mass bin is defined by \( M > M_\text{lim} = 1.75 \times 10^{14} h^{-1} M_\odot \).
about 6000; as a consequence, the cosmological constraints from the cluster survey are weaker than for the high-$C_{27}^8$ model. However, the relative impact of the marginalization over primordial non-Gaussianity is reduced. This result could have been expected, since the effect of imposing the same priors on $f_{NL}$ is relatively smaller when the cosmological errors on the other parameters for the fixed-$f_{NL}$ case increase.

To further illustrate the dependence of the results on survey parameters, in Table 4 we show the constraints obtained when the threshold cluster mass is reduced to $M_{\text{lim}} = 1.75 \times 10^{14} h^{-1} M_\odot$. This lower threshold may be achieved, for example, by supplementing S-Z cluster detection with optical cluster selection using the red galaxy sequence (e.g., Gladders et al. 2007; Koester et al. 2007). In this case, the 4000 deg$^2$ survey to $z = 1.5$ includes about 75,000 clusters, and the forecast cosmological parameter errors (without non-Gaussianity) are smaller by almost a factor of 2 than for the case with larger $M_{\text{lim}}$ considered above. For this more sensitive cluster survey, the impact on cosmological parameters of marginalizing over $f_{NL}$ is correspondingly larger: while the impact on dark energy remains small, including non-Gaussianity with the WMAP prior expands the error on $\sigma_8$ by more than 100%.

As already discussed ($\S$ 2.3), the degeneracy between $\sigma_8$ and $f_{NL}$ could be partially reduced by introducing a number of cluster mass bins and using the information contained in the shape of the mass function. In Table 5, we present the results for an analysis with a fiducial $\sigma_8 = 0.9$ and $M_{\text{lim}} = 1.75 \times 10^{14} h^{-1} M_\odot$ as in Table 2, but dividing the clusters into 10 mass bins and using the likelihood function defined in equation (14). As the last column in the table indicates, the main effect of including mass bins is that the cluster constraint on the non-Gaussianity parameter $f_{NL}$ becomes stronger than that from the SDSS galaxy bispectrum. Even without combining with external data sets, one can reach a 1 $\sigma$ error of $\Delta f_{NL} \approx 50$, not too far from current limits from CMB observations. Further study would be needed in order to determine whether this conclusion remains when realistic uncertainties in the cluster mass-observable relation are included in the analysis.

### 3.2. Two-Parameter Dark Energy Equation of State

Finally we consider the case of a time-varying dark energy equation of state,

$$w(a) = w_0 + (1 - a)w_a$$

(Chevallier & Polarski 2001; Linder 2003), adding the parameter $w_a$ to the likelihood analysis carried out so far. In this case, the strong degeneracy between $w_0$ and $w_a$ enlarges considerably the region of parameter space that has to be covered for the likelihood function evaluation, including unphysical regions where...
the combination \( w_0 + w_a \), representing the equation of state at large redshift, takes large positive values. To avoid such cases, we impose, by hand, a Gaussian prior on the value of \( \Omega_m(z) \), requiring in particular \( 1 - \Omega_m(z) < 0.01 \) at \( z = 30 \), the initial redshift considered for the numerical solution to the differential equation governing the growth factor \( D(z) \). This ensures that the universe is matter dominated at early times, as required by structure growth.

In Table 6, we present the derived 1σ errors on the parameters \( \Omega_m \), \( \sigma_8 \), \( w_0 \), and \( w_a \) with and without marginalization on \( f_{NL} \) and assuming a single mass bin defined by \( M > M_{lim} = 1.75 \times 10^{14} \, h^{-1} \, M_\odot \). Since in this case the uncertainties on the

### Table 3

**Expected 1σ Cosmological Errors for the Four-Parameter Analysis:**

**Fiducial \( \sigma_8 = 0.75 \) and \( M_{lim} = 1.75 \times 10^{14} \, h^{-1} \, M_\odot \)**

| Parameter | \( \Delta f_{NL} = 0 \) | \( \Delta f_{NL} = 5 \) | \( \Delta f_{NL} = 37 \) | \( \Delta f_{NL} = 145 \) |
|-----------|----------------|----------------|----------------|----------------|
| No Priors |                 |                 |                 |                 |
| \( \Delta w \)         | 0.079           | 0.079 (0%)      | 0.083 (5%)      | 0.124 (57%)    |
| \( \Delta \Omega_m \)   | 0.0140          | 0.0140 (0%)     | 0.0140 (0%)     | 0.0144 (3%)    |
| \( \Delta \sigma_8 \)   | 0.00076         | 0.0076 (0%)     | 0.0100 (32%)    | 0.0238 (210%)  |
| \( \Delta f_{NL} \)     | ...             | 5.0             | 37             | 128            |
| Gaussian Priors: \( \Omega_m = 0.27 \pm 0.035 \), \( \sigma_8 = 0.75 \pm 0.05 \) | | | | |
| \( \Delta w \)         | 0.073           | 0.073 (0%)      | 0.079 (8%)      | 0.119 (63%)    |
| \( \Delta \Omega_m \)   | 0.0129          | 0.0129 (0%)     | 0.0129 (0%)     | 0.0129 (0%)    |
| \( \Delta \sigma_8 \)   | 0.0070          | 0.0070 (0%)     | 0.0094 (34%)    | 0.0212 (200%)  |
| \( \Delta f_{NL} \)     | ...             | 5.0             | 37             | 118            |
| Fixed \( \Omega_m = 0.27 \) and \( \sigma_8 = 0.75 \) | | | | |
| \( \Delta w \)         | 0.030           | 0.031 (3%)      | 0.034 (10%)     | 0.034 (10%)    |
| \( \Delta f_{NL} \)     | ...             | 4.2             | 7.5            | 7.6            |

---

### Table 4

**Expected 1σ Cosmological Errors for the Four-Parameter Analysis:**

**Fiducial \( \sigma_8 = 0.9 \) and \( M_{lim} = 1 \times 10^{14} \, h^{-1} \, M_\odot \)**

| Parameter | \( \Delta f_{NL} = 0 \) | \( \Delta f_{NL} = 5 \) | \( \Delta f_{NL} = 37 \) | \( \Delta f_{NL} = 145 \) |
|-----------|----------------|----------------|----------------|----------------|
| No Priors on \( \Omega_m \) and \( \sigma_8 \) | | | | |
| \( \Delta w \)         | 0.026           | 0.026 (0%)      | 0.030 (15%)    | 0.052 (100%)   |
| \( \Delta \Omega_m \)   | 0.0050          | 0.0050 (0%)     | 0.0050 (0%)    | 0.0052 (4%)    |
| \( \Delta \sigma_8 \)   | 0.0031          | 0.0032 (3%)     | 0.0066 (110%)  | 0.0186 (500%)  |
| \( \Delta f_{NL} \)     | ...             | 5.0             | 36             | 113            |
| Gaussian Priors: \( \Omega_m = 0.27 \pm 0.035 \), \( \sigma_8 = 0.9 \pm 0.05 \) | | | | |
| \( \Delta w \)         | 0.026           | 0.026 (0%)      | 0.030 (15%)    | 0.050 (92%)    |
| \( \Delta \Omega_m \)   | 0.0050          | 0.0050 (0%)     | 0.0050 (0%)    | 0.0051 (2%)    |
| \( \Delta \sigma_8 \)   | 0.0030          | 0.0031 (3%)     | 0.0066 (120%)  | 0.0174 (480%)  |
| \( \Delta f_{NL} \)     | ...             | 5.0             | 36             | 106            |
| Gaussian Priors: \( \Omega_m = 0.27 \pm 0.0035 \), \( \sigma_8 = 0.9 \pm 0.01 \) | | | | |
| \( \Delta w \)         | 0.017           | 0.017 (0%)      | 0.023 (35%)    | 0.032 (88%)    |
| \( \Delta \Omega_m \)   | 0.0028          | 0.0028 (0%)     | 0.0028 (0%)    | 0.0028 (0%)    |
| \( \Delta \sigma_8 \)   | 0.0018          | 0.0020 (11%)    | 0.0051 (180%)  | 0.0087 (380%)  |
| \( \Delta f_{NL} \)     | ...             | 5.0             | 32             | 56             |
| Fixed \( \Omega_m = 0.27 \) and \( \sigma_8 = 0.9 \) | | | | |
| \( \Delta w \)         | 0.0100          | 0.0102 (2%)     | 0.0102 (2%)    | 0.0102 (2%)    |
| \( \Delta f_{NL} \)     | ...             | 3.1             | 4.0            | 4.0            |
### Table 5
**Expected 1σ Cosmological Errors for the Four-Parameter Analysis:**
**Fiducial $\sigma_8 = 0.9$ and 10 Cluster Mass Bins ($M_{\text{lim}} = 1.75 \times 10^{14} h^{-1} M_\odot$)**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| No Priors on $\Omega_m$ and $\sigma_8$ | | | | |
| $\Delta \omega$ | 0.044 | 0.044 (0%) | 0.046 (5%) | 0.048 (9%) |
| $\Delta \Omega_m$ | 0.0082 | 0.0082 (0%) | 0.0083 (1%) | 0.0085 (4%) |
| $\Delta \sigma_8$ | 0.0049 | 0.0050 (2%) | 0.0077 (57%) | 0.0103 (110%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 29 | 45 |

**Gaussian Priors: $\Omega_m = 0.27 \pm 0.035$, $\sigma_8 = 0.9 \pm 0.05$**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $\Delta \omega$ | 0.043 | 0.043 (0%) | 0.045 (5%) | 0.048 (12%) |
| $\Delta \Omega_m$ | 0.0079 | 0.0079 (0%) | 0.0080 (1%) | 0.0082 (4%) |
| $\Delta \sigma_8$ | 0.0048 | 0.0049 (2%) | 0.0075 (56%) | 0.0100 (110%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 29 | 45 |

**Fixed $\Omega_m = 0.27$ and $\sigma_8 = 0.9$**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $\Delta \omega$ | 0.195 | 0.195 (0%) | 0.196 (1%) | 0.208 (7%) |
| $\Delta \Omega_m$ | 0.73 | 0.73 (0%) | 0.74 (1%) | 0.87 (19%) |
| $\Delta \sigma_8$ | 0.0156 | 0.0156 (0%) | 0.0158 (1%) | 0.0185 (19%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 37 | 137 |

### Table 6
**Expected 1σ Cosmological Errors for the Five-Parameter Analysis:**
**Fiducial $\sigma_8 = 0.9$ and One Mass Bin ($M_{\text{lim}} = 1.75 \times 10^{14} h^{-1} M_\odot$)**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| No Priors on $\Omega_m$ and $\sigma_8$ | | | | |
| $\Delta \omega$ | 0.195 | 0.195 (0%) | 0.196 (1%) | 0.208 (7%) |
| $\Delta \Omega_m$ | 0.73 | 0.73 (0%) | 0.74 (1%) | 0.87 (19%) |
| $\Delta \sigma_8$ | 0.0086 | 0.0087 (1%) | 0.0114 (32%) | 0.0296 (240%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 37 | 137 |

**Gaussian Priors: $\Omega_m = 0.27 \pm 0.0035$, $\sigma_8 = 0.9 \pm 0.05$**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $\Delta \omega$ | 0.178 | 0.178 (0%) | 0.178 (0%) | 0.183 (3%) |
| $\Delta \Omega_m$ | 0.68 | 0.68 (0%) | 0.68 (0%) | 0.73 (7%) |
| $\Delta \sigma_8$ | 0.0140 | 0.0140 (0%) | 0.0141 (1%) | 0.0151 (8%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 37 | 118 |

**Fixed $\Omega_m = 0.27$ and $\sigma_8 = 0.9$**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $\Delta \omega$ | 0.082 | 0.082 (0%) | 0.85 (4%) | 0.89 (9%) |
| $\Delta \Omega_m$ | 0.43 | 0.43 (0%) | 0.43 (0%) | 0.43 (0%) |
| $\Delta \sigma_8$ | 0.0034 | 0.0034 (0%) | 0.0034 (0%) | 0.0034 (0%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 31 | 54 |

**Gaussian Priors: $\Omega_m = 0.27 \pm 0.0035$, $\sigma_8 = 0.9 \pm 0.01$**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $\Delta \omega$ | 0.082 | 0.082 (0%) | 0.85 (4%) | 0.89 (9%) |
| $\Delta \Omega_m$ | 0.43 | 0.43 (0%) | 0.43 (0%) | 0.43 (0%) |
| $\Delta \sigma_8$ | 0.0034 | 0.0034 (0%) | 0.0034 (0%) | 0.0034 (0%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 31 | 54 |

**Fixed $\Omega_m = 0.27$ and $\sigma_8 = 0.9$**

| Parameter | $\Delta f_{\text{NL}} = 0$ | $\Delta f_{\text{NL}} = 5$ | $\Delta f_{\text{NL}} = 37$ | $\Delta f_{\text{NL}} = 145$ |
|-----------|-----------------|-----------------|-----------------|-----------------|
| $\Delta \omega$ | 0.082 | 0.082 (0%) | 0.85 (4%) | 0.89 (9%) |
| $\Delta \Omega_m$ | 0.43 | 0.43 (0%) | 0.43 (0%) | 0.43 (0%) |
| $\Delta \sigma_8$ | 0.0034 | 0.0034 (0%) | 0.0034 (0%) | 0.0034 (0%) |
| $\Delta f_{\text{NL}}$ | ... | 5.0 | 31 | 54 |
parameters, which are sensitive to the strong \( w_{0} - w_{a} \) degeneracy, are much larger than in the previous case, the effect of the marginalization on \( f_{\text{NL}} \) with a CMB prior is even smaller than in the case of time-independent \( w \). As before, however, marginalization over \( f_{\text{NL}} \) with only the galaxy bispectrum prior substantially increases the error on \( \sigma_{8} \).

As a final example, in Table 7 we consider parameter constraints in the time-varying dark energy cosmology using the 10 cluster mass bins. As noted above in the case of the four-parameter analysis, the \( \sigma_{8} - f_{\text{NL}} \) degeneracy is significantly reduced, and the expected constraints on non-Gaussianity are still of order \( f_{\text{NL}} \sim 50 \).

### 4. CONCLUSIONS

The success of the \( \Lambda \)CDM standard cosmological model in recent years has been nothing short of spectacular. Upcoming surveys either will continue to confirm this model and constrain its parameters with unprecedented accuracy, or they will uncover discrepancies that will point the way toward improvements in our understanding of fundamental physics. Two questions addressing cosmology beyond the standard model that have been the subject of substantial attention in recent years are the nature of the dark energy that is driving the accelerated expansion of the universe and, second, whether fluctuations in the primordial matter distribution are Gaussian and therefore consistent with the predictions of the simplest inflationary models. In this paper, we obtained a rough estimate of the success that one of the most promising cosmological probes, galaxy cluster counts, is likely to have in answering these fundamental questions.

We have assumed an ideal cluster survey with survey parameters expected for the upcoming SPT and DES projects. Our fiducial cosmological model includes both dark energy and primordial non-Gaussianity using popular parameterized models, \( w \) and \( w_{a} \) for the former and \( f_{\text{NL}} \) for the latter. Cluster number counts as a function of mass, redshift, and cosmology were estimated using a standard fit to simulations (Jenkins et al. 2001), which we adjusted to allow for mildly non-Gaussian initial conditions, and all clusters above a threshold mass were considered to be “found” by our fiducial survey. We then performed a simple likelihood analysis on the cluster counts using priors from current \( \Lambda \)CDM and expected \( \Lambda \)CDM and Planck and SDSS constraints on non-Gaussianity, as well as approximate priors on the two other relevant cosmological parameters from other present and future data sets.

Our principal conclusion is that dark energy constraints are in all cases not substantially degraded by primordial non-Gaussianity when the model parameterized by the constant \( f_{\text{NL}} \) and current limits from CMB observations are assumed. This is true despite the fact that variations in \( f_{\text{NL}} \) close to current uncertainties induce differences in the mass function comparable in magnitude to variations of 10% in the dark energy parameter \( w \). A stronger degeneracy is observed instead between \( \sigma_{8} \) and \( f_{\text{NL}} \), which we adjusted to allow for mildly non-Gaussian initial conditions, and all clusters above a threshold mass were considered to be “found” by our fiducial survey.
should be quite robust, since any significant increase in the error budget will reduce the constraining power on dark energy parameters and deemphasize the confusion caused by any non-Gaussian initial conditions. On the other hand, the effectiveness of clusters as a cross-check of primordial non-Gaussianity estimates from the CMB could be dramatically worsened and should therefore be the subject of future work.

We thank Eiichiro Komatsu and Eric Linder for helpful comments on a earlier draft of the paper. C. V. would like to thank Martin White for useful discussions. This work was supported by the US Department of Energy at the University of Chicago and at Fermilab, by the Kavli Institute for Cosmological Physics at the University of Chicago, and by NASA grant NAG 5-10842 at Fermilab.