Blending measurements and numerical models: a novel methodological approach for land subsidence prediction with uncertainty quantification

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Abstract. The use of numerical models for land subsidence prediction above producing hydrocarbon reservoirs has become a common and well-established practice since the early ’90s. Usually, uncertainties in the deep rock behavior, which can affect the forecast capability of the models, have been taken into account by running multiple simulations with different constitutive laws and mechanical properties. Then, the most uncertain parameters were calibrated to reproduce available subsidence measurements. The objective of this work is to propose a novel methodological approach for land subsidence prediction and uncertainty quantification by integrating the available monitoring information in numerical models using ad hoc Data Assimilation techniques. The proposed approach allows to: (i) train the model with the available data and improve its accuracy as new information comes in, (ii) quantify the prediction uncertainty by providing confidence intervals and probability measures instead of deterministic outcomes, and (iii) identify the most appropriate rock constitutive model and geomechanical parameters. The methodology is tested in synthetic models of production from hydrocarbon reservoirs. The numerical experiments show that the proposed approach is a promising way to improve the effectiveness and reliability of land subsidence models.

1 Introduction

The interest and capability of analyzing real world phenomena in a stochastic way have increased in recent years. Focusing on land subsidence, deterministic studies do not allow to take care of uncertainties that unavoidably affect the characterization of the porous medium, e.g., the choice of the constitutive law and values of the governing mechanical parameters. A way to address this issue relies on the integration of Data Assimilation (DA) techniques into the numerical model, in order to train the model through the available measurements and improve the reliability and accuracy of the solution by reducing the uncertainties. In petroleum engineering, DA approaches have been originally used for history matching purposes, with recent applications for subsidence prediction (Baù et al., 2016; Fokker et al., 2016; Zoccarato et al., 2016). In a very recent work by Gazzola et al. (2019), different kinds of DA approaches are combined in a novel robust methodological approach for the stochastic analysis of land subsidence. It allows for the quantification and reduction of uncertainties affecting this process, with the aim to build more realistic and reliable models. Here, that idea has been further developed and improved with the definition of a three-stage workflow that has been applied to a synthetic model of land subsidence above a producing hydrocarbon reservoir, in order to investigate strengths, weaknesses and potentials. The procedure is based on the generation of many ensembles of Monte Carlo realizations, differing, for example, for the constitutive law that describes the porous medium behavior and the values of the geomechanical parameters. Then, the methodology consists of: (i) a first model diagnostic stage based on the mismatch between the
model outcomes and the available measurements, which al-

The paper is organized as follows. First, the methodolog-
ical approach is described in detail, then the synthetic, but
realistic, test case is presented with a discussion of the out-
comes of every step of the proposed workflow. Finally, a few
conclusive remarks close the presentation.

2 Methodological approach

At the very beginning of a land subsidence study, the num-
ber of available pieces of information concerning the deep
rock behavior is usually low. For example, the most appro-
 priate constitutive law that defines the behavior of the porous
medium is generally unknown and the mechanical proper-
ties of the porous medium, e.g., stiffness or anisotropy, can
be estimated only within a variability range by laboratory
tests and preliminary in-situ analyses. Consequently, the first
step for a stochastic subsidence analysis is to propagate all
these input uncertainties to the subsidence prediction, that is
the output of the geomechanical model. This is done by the
generation of many forecast ensembles of Monte Carlo re-
alizations for each constitutive law that could be suitable to
describe the behavior of the porous medium, and consider-
ing different variability ranges for the main mechanical pa-
rameters. When the production program starts, displacement
records are collected and used to train the model through
different DA approaches in order to reduce the uncertainties
and make the analysis more accurate and reliable. In particu-
lar, following the methodological approach proposed in Gazzo-
la et al. (2019), a Bayesian approach, i.e., RF technique,
and a smoother formulation are suggested. Here, a prelimi-
mary model diagnostic is added, in order to evaluate the most
suitable ensemble for the successive and more expensive DA
techniques.

After that, a cheap and fast analysis of the forecast ensem-
bles can be performed by the RF technique, as introduced in
Nepveu et al. (2010). It is a statistical approach that com-
putes the probability of every Monte Carlo realization by
combining prior information with the likelihood of the mea-
surements, without solving the inverse problem.

Finally, the ES technique is applied, a non-sequential DA
algorithm originally proposed by Van Leeuwen and Evensen
(1996). It is an ensemble-based technique used to update both
state variables and model parameters and evaluate their un-
certainties by combining prior information, measurements,
and the solution of the forward model. Results are optimal
when the probability distribution of uncertain parameters is
Gaussian (Van Leeuwen and Evensen, 1996). The ES has
been widely used in subsidence applications, some recent
eamples are Baù et al. (2016), Fokker et al. (2016) and
Zoccarato et al. (2018). The analyses that follow are carried
out with the ES implementation according to the approach
by Evensen (2003). The outcomes of the ES application are
evaluated by two performance indices, the average absolute
error (AE) and the average ensemble spread (AES) as pro-
posed by Hendricks Franssen and Kinzelbach (2008) and
used in Zoccarato et al. (2016). The AE index is a measure of
the algorithm capability to approach the truth, as it compares
the model outcome with the true reference value for each ob-
ervation. This index can be computed only in a synthetic
case where the true reference is known. The AES index ac-
counts for the deviation of the model results from the ensem-
ble mean, hence it provides an indication of the spread of the
distribution, i.e., a measure of the confidence in the predicted
value. Generally, results of the assimilation are satisfactory
when AE and AES of the update ensembles decrease with
respect to the corresponding indices of the forecast ensem-
bles. For this reason, in the sequel the relative variation $J$ of
these indices is also computed.

3 Test case

3.1 Test case model

The methodology described in Sect. 2 has been carried
out for a synthetic test case. The model configuration has
been created focusing on an off-shore hydrocarbon reservoir
buried in a sedimentary basin with geometries and proper-
ties typical of the Northern Adriatic, Italy. The model do-
main is a parallelepiped with a square basis of about 50 km
and depth of about 5 km. It has been subdivided in four layers with different properties: overburden, reservoir, aquifer and underburden. For the sake of simplicity, the first and the last are characterized by a linear elastic constitutive law, in fact their behavior does not significantly affect surface movements (Ferronato et al., 2010; Teatini et al., 2010). A nonlinear elasto-plastic constitutive law, namely the modified Cam Clay (De Souza Neto et al., 2008), has been used for the definition of the reservoir and the aquifer behavior. The model domain has been discretized into a 3D finite element mesh composed of 71 734 nodes and 410 030 tetrahedrons. The entire domain has been characterized by a unitary Biot coefficient, a Poisson coefficient equal to 0.30 and a vertical compressibility $c_m$ that follows the law vs. the effective vertical stress $\sigma_z$ developed by Baù et al. (2002) and Ferronato et al. (2013) for the Northern Adriatic basin, Italy:

$$c_{m, I}^{\text{cycle}} = 1.0044 \times 10^{-2} \times \sigma_z^{-1.1347}$$
$$c_{m, II}^{\text{cycle}} = 2.9087 \times 10^{-4} \times \sigma_z^{-0.4315}$$

where $c_m$ and $\sigma_z$ are in [MPa$^{-1}$] and [MPa], respectively. Homogeneous boundary conditions have been prescribed for displacements, on the lateral and the bottom boundaries, and for stress, on the top surface. As external forces, pore pressure variations have been imposed on the reservoir and aquifer nodes with the aim to simulate a general program of hydrocarbon production with two phases of production separated by a temporary stop from year 3 to year 5 and a final period of natural pressure recovery. Additional details on the geometry and properties of the test case are given in Gazzola et al. (2019).

Since a synthetic test case has been taken into consideration, the geomechanical model has been run both to create the ensembles of Monte Carlo realizations and to define a set of hypothetical measurements. Consequently, a set of statistical distributions and a true configuration of the main geomechanical parameters have to be defined. The modified Cam Clay constitutive law requires the definition of the modified compression index $\lambda^*$, i.e. the slope of the normal consolidation line in the plot of volumetric strain vs. axial stress in natural logarithmic scale, and the modified swelling index $k^*$, i.e. the slope of the unloading line (Nguyen et al., 2017). In Spiezia et al. (2017), more details on model properties and implementation have been provided. To simulate a set of GPS recordings (green stars in Fig. 1), the true configuration is selected as follows:

$$\lambda^* = 0.004992$$
$$k^* = 0.000700$$

In this application, three different cases have been taken into consideration: uncertainties related only to $\lambda^*$, only to $k^*$ and to both. The forecast ensembles of vertical displacements $u_z$ (grey lines in Fig. 1) have been generated with the following Gaussian distributions:

$$\ln(\lambda^*) \sim \mathcal{N}(-4.9363; 0.33295)$$
$$\ln(k^*) \sim \mathcal{N}(-6.7271; 0.50975)$$

The covariance matrix of measurement error is required for the application of the DA techniques. It can be computed as the sum of a measurement and an idealisation noise, according to NAM (2017). The first term is an index of the accuracy of the tool for recording the data and the representativeness of the mathematical model for reproducing the observed processes. It has been defined from a normal distribution with zero mean and standard deviation equal to 1.5 mm. The second component takes into consideration the spatial and temporal correlation between measurements. It has been computed following the fractional Brownian motion approach (NAM, 2017) through a fitting process of the variogram of the synthetic observations.

3.2 Model Diagnostic

Preliminarily, a model diagnostic procedure, i.e. the $\chi^2$-test, has been carried out to evaluate every ensemble through the comparison between the model outcomes and the available measurements. Generally, a $\chi^2$ value close to one is desirable, while larger values may point out difficulties in constraining the model through the available measurements. Values of $\chi^2$ smaller than one should be avoided, as they denote a variance associated to the measurements higher than the variance associated to the ensemble. This means that the DA approaches cannot improve the forecast ensemble. Two scenarios have been taken into consideration with the results shown in Table 1. In the first case (third column of Table 1) the state forecast ensembles have been generated with the same constitutive law of the synthetic measurements, i.e. the modified Cam Clay, while in the second case (fourth column) a different constitutive law, i.e. a visco-elasto-plastic one, has been used to create the ensembles. As a matter of fact, in a real study, the constitutive law that describes the behavior of the porous medium is not known a priori. In both scenarios, the $\chi^2$ value is always close to one, but values of the ensembles generated with the wrong constitutive law are always larger. This suggests that the $\chi^2$-test could be effectively used as a tool to select the most suitable constitutive law to describe the porous medium behavior. Only in one case, i.e. when uncertainties are associated to the modified swelling index $k^*$, $\chi^2$ is smaller than one and this could be related to the production program that has been prescribed. As a matter of fact, $k^*$ is a parameter that controls the rock behavior in unloading/reloading conditions and consequently does not significantly influence the surface displacements for a hydrocarbon production program.
Table 1. Analyzed configuration sets with their associated uncertain parameters and $\chi^2$ values for two scenarios: the first one with the right match between the constitutive law that generates the synthetic measurements and the ensembles, and the second one with the wrong match.

| Set | Parameters | Right Match | Wrong Match |
|-----|------------|-------------|-------------|
| 1   | $\lambda^*$ | 1.22        | 1.38        |
| 2   | $k^*$      | 0.95        | 1.49        |
| 3   | $\lambda^*, k^*$ | 1.32       | 1.73        |

Table 2. RF approach: maximum and minimum probabilities of occurrence $P$ for the analyzed ensembles and combination of parameters with maximum $P$.

| Set | max $P$ (%) | min $P$ (%) | $\lambda^*$ | $k^*$ |
|-----|-------------|-------------|--------------|-------|
| 1   | 7.11        | $4.44 \times 10^{-15}$ | 0.005591     |       |
| 2   | 2.84        | 0.14        | 0.001189     |       |
| 3   | 8.27        | $3.19 \times 10^{-33}$ | 0.004990     | 0.000958 |

3.3 Red Flag

After the model diagnostic, an easy and fast technique for a preliminary analysis of the ensembles is RF. It allows to characterize every realization of the ensemble by its own probability of occurrence. In Table 2 the parameters corresponding to the realization with the largest probability are provided. They are not always the closest set to the true parameters and the reason is twofold. First, the comparison is based on the vertical displacements $u_z$, that are the effect through the forward model of the parameter selection and not directly the parameters themselves. Second, an error is associated to the observations in order to consider, for example, the accuracy of the measurement tool or possible temporal correlations between successive measurements. Table 2 also provides the maximum and minimum probabilities of occurrence of the analyzed ensembles. They seem to confirm the outcome of the diagnostic stage, in fact for set #1 and #3 the difference between the largest and smallest probability is significant, while for set #2 the maximum probability is smaller and with a similar order of the minimum one.

3.4 Ensemble Smoother

ES has been applied for the three sets of uncertain parameters previously described, with the outcome shown in Fig. 1. They confirm the results obtained by the model diagnostic procedure and the RF technique. In fact, for parameter set #1 results are satisfactory, i.e. the update ensembles of state and parameter are closer to the true values than the forecast ones. Also the positive values of the relative variations $J$ of the update AE and AES with respect to the forecast ones, reported in Table 3, confirm these results. On the contrary, for set #2, results of ES application seem to be not encouraging. As anticipated by the value of $\chi^2$, the parameter $k^*$ does not significantly affect surface movements, i.e. the forecast state ensemble is already quite close to the measurements. This is also shown by the forecast AES values in Table 3. In particular, notice that similar variations of the parameters $\lambda^*$ ($AES(\lambda^*) = 2.429 \times 10^{-1}$ for set #1) and $k^*$ ($AES(k^*) = 3.706 \times 10^{-1}$ for set #2) produce quite different spreads, measured by AES($u_z$), of the state ensembles. Consequently, the ES is not effective in constraining the model. Finally, in set #3 the available measurements help to reduce uncertainties both for parameter $\lambda^*$ and for surface measurements similarly to set #1, but not for $k^*$. 
Table 3. ES approach: AE and AES for the forecast ensemble and the relative variation $J$ of AE and AES from the update ensembles. The larger $J$, the more effective is the assimilation approach on the parameter or state ensembles indicated in column 2. Negative values are highlighted in bold.

| Set | Ensemble | $\lambda^*$ | $J$ (%) | $u_z$ | $J$ (%) |
|-----|----------|-------------|---------|--------|---------|
| 1   | $\lambda^*$  | $4.159 \times 10^{-1}$ | 51      | $1.432 \times 10^{-1}$ | 62      |
|     | $u_z$     | $2.429 \times 10^{-1}$ | 16      | $9.260 \times 10^{-2}$ | 42      |
| 2   | $k^*$   | $6.488 \times 10^{-1}$ | $-29$   | $6.331 \times 10^{-3}$ | $-37$   |
|     | $u_z$     | $3.706 \times 10^{-1}$ | $-125$  | $3.348 \times 10^{-3}$ | $-150$  |
| 3   | $\lambda^*$  | $3.996 \times 10^{-1}$ | 49      | $1.454 \times 10^{-1}$ | 64      |
|     | $k^*$   | $2.563 \times 10^{-1}$ | $30$    | $1.000 \times 10^{-1}$ | $53$    |

Table 4. Testing ES approach in a predictive sense for set #1. Variations $J$ of AE and AES from the update ensembles assimilating 5, 7 and 10 years of measurements. The AE and AES indices for the forecast ensembles are the same of Table 3. Negative values are highlighted in bold.

| $J$ (%) | 5 years | 7 years | 10 years |
|---------|---------|---------|----------|
| AE($\lambda^*$)  | 40      | 49      | 51       |
| AES($\lambda^*$) | $-2$    | 13      | 16       |
| AE($u_z$)    | 54      | 61      | 62       |
| AES($u_z$)   | 30      | 40      | 42       |

Figure 2. The same as in Fig. 1 for set #1 for the assimilation of 5 years (a) and 7 years (b) of displacements measurements (green stars).

ES can be employed also in a predictive sense by assimilating new measurements as they are recorded. Considering set #1, effectiveness of ES improves with the increase of the number of assimilated measurements, but also few observations allow to reduce uncertainties related to parameter and to the state ensembles. Results are shown in Fig. 2 and Table 4.

4 Conclusions

In this work, a novel methodological approach for a stochastic study of land subsidence due to hydrocarbon exploitation has been developed and tested in a synthetic test case inspired by the Northern Adriatic basin, Italy. The aim is to define an efficient and robust workflow that combines DA techniques and geomechanical models in order to quantify and reduce uncertainties and consequently make the model more accurate and reliable.

First, every ensemble has been evaluated considering the comparison with the available measurements through a model diagnostic procedure, i.e. the $\chi^2$-test. An ensemble is considered more appropriate as the associated $\chi^2$ value is closer to one. From the outcomes of numerical experiments, the $\chi^2$-test appears a worthwhile tool to select the most appropriate ensemble for the successive DA techniques, e.g. to define the constitutive law that best approximates the porous medium behavior. After that, RF approach has been used to characterize every Monte Carlo realization by its own probability of occurrence. Despite the parameters of the most probable realization are not always the closest to the true values, RF allows for a preliminary reduction of uncertainties in a fast and cheap way. In fact, if no further analysis is carried out, the least probable realizations can be neglected in the land subsidence study, thus reducing the uncertainty interval. Finally, model state and parameters have been updated through the ES approach. If the forecast ensemble has been properly generated, the ES application to constrain the model through the available measurements proves effective. ES can be used also in a predictive sense, increasing the accuracy of its outcomes while new data are assimilated.

On summary, in the context of a synthetic test case, the proposed workflow seems to be an innovative and suitable approach to study land subsidence above producing hydrocarbon reservoirs, taking into consideration the uncertainties that unavoidably affect numerical models of real world phenomena. Moreover, it allows to reduce these uncertainties in order to obtain more accurate and reliable predictions. Further analyses and improvements on these approaches are currently underway with its application on real-world reservoir problems.
Data availability. This work deals with numerical modeling of a synthetic test case. All data required for the implementation and reproduction are reported in the paper and in the relevant bibliography.

Author contributions. All the authors have equally contributed to the development of the conceptual model. GL, FM and ZC took care of the code implementation and numerical testing. GL, FM and TP are the main contributors for paper writing and proofreading.

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References

Baù, D., Ferronato, M., Gambolati, G., and Teatini, P.: Basin-scale compressibility of the Northern Adriatic by the radioactive marker technique, Geotechnique, 52, 605–616, 2002.

Baù, D., Alzraiee, A., Zoccarato, C., Gambolati, G., Ferronato, M., Bottazzi, F., Mantica, S., and Teatini, P.: Testing a data assimilation approach to reduce geomechanical uncertainties in modelling land subsidence, Environmental Geotechnics, 3, 386–396, https://doi.org/10.1680/envgeo.15.00005, 2016.

Chen, Y. and Oliver, D.: Levenberg-Marquardt forms of the iterative ensemble smoother for efficient history matching and uncertainty quantification, Comput. Geosci., 17, 689–703, https://doi.org/10.1007/s10596-013-9351-5, 2013.

De Souza Neto, E., Peric, D., and Owen, D.: Computational methods for plasticity, theory and application, Wiley, UK, https://doi.org/10.1002/9780470694626, 814 pp., 2008.

Evensen, G.: The Ensemble Kalman Filter: Theoretical formulation and practical implementation, Ocean Dynam., 53, 343–367, https://doi.org/10.1007/s10236-003-0036-9, 2003.

Ferronato, M., Gambolati, G., Janna, C., and Teatini, P.: Geomechanical issues of anthropogenic CO2 sequestration in exploited gas fields, Energ. Convers. Manage., 51, 1918–1928, 2010.

Ferronato, M., Castelletto, N., Gambolati, G., Janna, C., and Teatini, P.: II cycle compressibility from satellite measurements, Geotechnique, 63, 479–486, 2013.

Fokker, P., Wassing, B., van Leijen, F., Hanssen, R., and Nieuwland, D.: Application of an ensemble smoother with multiple data assimilation to the Bergermeer Gas Field, using PS-Insar, Geomechanics for Energy and the Environment, 5, 16–28, 2016.

Gazzola, L., Ferronato, M., Frigo, M., Janna, C., Teatini, P., Zoccarato, C., Antonelli, M., Corradi, A., Dacome, M., and Mantica, S.: Uncertainty quantification and reduction through Data Assimilation approaches for the geomechanical modeling of hydrocarbon reservoirs, in: Proceedings of the 53rd US Rock Mechanics/Geomechanics Symposium ARMA, 23–26, 2019.

Hendricks Franssen, H. J. and Kinzelbach, W.: Real-time groundwater flow modeling with the Ensemble Kalman Filter: Joint estimation of states and parameters and the filter inbreeding problem, Water Resour. Res., 44, W09408, https://doi.org/10.1029/2007WR006505, 2008.

NAM: Ensemble Based Subsidence application to the Ameland gas field – long term subsidence study part two (LTS II), Overview Report, EP201701217189, 2017.

Nguyen, S. K., Volontè, G., Musso, G., Brignoli, M., Gemelli, F., and Mantica, S.: Implementation of an elasto-viscoplastic constitutive law in Abaqus/Standard for an improved characterization of rock materials, Proceedings of the 51st U.S. Rock Mechanics/Geomechanics Symposium, ARMA–2017–1043, 2017.

Oliver, D. S. and Alfonzo, M.: Calibration of imperfect models to biased observations, Computat. Geosci., 22, 145–161, https://doi.org/10.1007/s10596-017-9678-4, 2018.

Spiezia, N., Ferronato, M., Janna, C., and Teatini, P.: A two-invariant pseudoequivalent model for reservoir compaction, International Journal of Numerical and Analytical Methods in Geomechanics, 41, 1870–1893, https://doi.org/10.1002/nag.2704, 2017.

Tarantola, A.: Inverse Problem Theory and Methods for Model Parameter Estimation, Siam, Paris, ix + 339, 2005.

Teatini, P., Ferronato, M., Gambolati, G., Baù, D., and Putti, M.: Anthropogenic Venice uplift by seawater pumping into a heterogeneous aquifer system, Water Resour. Res., 46, W11547, https://doi.org/10.1029/2010WR009161, 2010.

Van Leeuwen, P. and Evensen, G.: Data assimilation and inverse methods in terms Of a probabilistic formulation, Mon. Weather Rev., 124, 2898–2913, 1996.

Zoccarato, C., Baù, D., Ferronato, M., Gambolati, G., Alzraiee, A., and Teatini, P.: Data assimilation of surface displacements to improve geomechanical parameters of gas storage reservoirs, J. Geophys. Res.-Sol. Ea., 121, 1441–1461, https://doi.org/10.1002/2015JB012090, 2016.

Zoccarato, C., Ferronato, M., and Teatini, P.: Formation compaction vs land subsidence to constrain rock compressibility of hydrocarbon reservoirs, Geomechanics for Energy and the Environment, 13, 14–24, https://doi.org/10.1016/j.gete.2017.12.002, 2018.