Self-Organised Criticality and Emergent Hyperbolic Networks—Blueprint for Complexity in Social Dynamics

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Online social dynamics based on human endeavours exhibit prominent complexity in the emergence of new features embodied in the appearance of collective social values. The vast amount of empirical data collected at various websites provides a unique opportunity to quantitative study of the underlying social dynamics in full analogy with complex systems in the physics laboratory. Here, we briefly describe the extent of these analogies and indicate the methods from other science disciplines that the physics theory can incorporate to provide the adequate description of human entities and principles of their self-organisation. We demonstrate the approach on two examples using the empirical data regarding the knowledge creation processes in online chats and questions-and-answers. Precisely, we describe the self-organised criticality as the acting mechanisms in the social knowledge-sharing dynamics and demonstrate the emergence of the hyperbolic geometry of the co-evolving networks that underlie these stochastic processes.

1. INTRODUCTION

One of the most apparent features of complexity is the emergence of a new functional property of the ensemble of interacting units which does not exist at any of its constituents. In this regard, complex systems have multi-scale dynamics and exhibit collective phenomena that mostly resemble self-organized criticality in physical systems \cite{1-0}. The majority of physical and biological systems with these properties link their dynamics with complex topological characteristics in the coordinate or phase space \cite{6-10}. Therefore, one of the fundamental questions of the science of complexity regard the role of topology in the emergence of new properties.

Mapping the dynamics of a complex system onto a graph (network) is the first step to address this problem. In this context, a suitable mapping results in a mathematical graph with nodes and connections that contain some essential attributes of the original system and its dynamics. On the other hand, it provides an objective analysis using graph theory \cite{11-12}. The naturally evolving structures obey a particular kind of optimisation principle at all stages of growth. Therefore, the architecture of a graph receive interpretations regarding the fundamental dynamics and interactions between the constitutive units of the system. A striking example of this principle is the community structure in the protein interaction \cite{13} and gene expression \cite{14} networks that can be detected by graph theory methods \cite{15}.

Recently, the study of graphs representing various complex systems has been extended beyond the standard graph-theoretic metric. The use of methods of algebraic topology \cite{16, 17} enabled revealing the higher organised structures and related hidden geometries that can appear in the graph. More specifically, the Q-analysis based on the algebraic topology of graphs identifies elementary geometric shapes or simplexes (triangles, tetrahedrons and higher order cliques) and how they connect to each other making the more massive structures, simplicial complexes \cite{18-20}.

A particular composition of these basic geometric shapes can lead to emergent hyperbolicity or a negative curvature \cite{21, 22}, a measure of the nodes proximity in the metric graph space, which often associates with improved collective dynamics in natural and technological networks \cite{23-25}.

Among complex systems that have recently driven much attention of the science community are different types of networks, which embody human social experience \cite{26} from patterns of brain activity of participating individuals during social communications \cite{27, 28} to large-scale online social graphs \cite{29-34}. These studies are primarily enabled by the vast amount of empirical data obtained by brain imaging in the laboratory \cite{35} as well as from online social interaction sites \cite{36, 37} where users leave information about themselves and their actions that are continuously being stored at the server. In contrast to the abovementioned networks representing dynamical systems in the physical world, the principles governing social interactions are more complex and depend on various human attitudes during the communication process as well as the contents (cognitive, emotional) contained in the exchanged messages. Nevertheless, the high-resolution data on social websites contain sufficient information to extract relevant dynamical quantities, which can be analysed by approaches based on the formal analogy to the physical systems in the laboratory. Precisely, the response of a physical system to the external driving force can be measured in the laboratory and often represents a noisy signal (time series of a relevant dynamical variable), see an example in Fig\textsuperscript{1} from which the avalanche dynamics can be studied. Extracting the time-series of the relevant quantities from the empirical data then enables the use of time-series analysis, entropy measures, information divergence and self-organised criticality (see \cite{29, 45} and references there).

In social dynamics, the sequence of events is stored in the empirical data and often contains much other valuable information: id of each user, the action on a particular arte-
FIG. 1: The noisy response of a driven system in the laboratory contains information on how the number of elementary events contributes to the collective response. The example is Barkhausen noise simulated in the model of the random-field ferromagnet slowly driven by the external magnetic field along the hysteresis loop. Temporal clustering of events—avalanches, can be recognised as the bursts of the signal above the red baseline. For a particular avalanche, the duration $T$ is the distance between the two consecutive intersections of the baseline with the signal while the enclosed area under the signal between these two points gives the avalanche size $s$. In a self-organised critical state, the avalanche sizes and durations exhibit power-law distributions $^{[43]}$.

2. EMERGENCE OF SOCIAL NETWORKS VIA SOCIAL KNOWLEDGE-SHARING PROCESSES

The social communications can create new knowledge through so-called meaningful interactions, which social psychology defines as communications intended to meet the needs of others $^{[48][49]}$. Modern technology greatly facilitates these processes by enabling easy access and fast communication between the participants and preserving the data. At the same time, it transforms the system into the online social dynamics, which is still not well understood. Different websites created with the purpose of knowledge-sharing have their own action rules, which can influence the course of the process and its outcome. Here, we illustrate such differences in two types of knowledge-sharing systems: Internet-Relay-Chat (IRC) Ubuntu and Questions & Answers (Q&A) site Mathematics from StackExchange. The corresponding empirical data are mapped onto the network as illustrated in Fig. 2. In particular, depending on the objectives of the study, the considered contents of the messages exchanged among users can be associated either to the links between the nodes or another type of nodes introduced.

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FIG. 2: (left) Top panel: Schematic representation of User-to-User interaction via the exchange of messages, whose contents can be extracted by Machine-learning methods and attributed to the link between the Users. Bottom: A close-up of the network of Users (nodes) connected via emotion-carrying links extracted from data of chats in IRC Ubuntu channel; red positive, black negative, blue–neutral emotion messages [1]. (right) Top panel: Schematic representation of the indirect interaction between Users (blue) mediated by Questions (red) nodes, which contain the tagged contents of the exchanged information. Bottom: A close-up of the bipartite networks of users (blue) and questions (red nodes) emerging from the sequence of events in Q&A site StackExchange Mathematics. In the data, the contents of each Question is tagged according to standard Mathematics Classification Scheme.

Furthermore, analysing the emotional arousal, the degree of the excitement, in the messages exchanged along different links, we have found [41] that the high-arousal links efficiently keep the network together. More specifically, its giant cluster appears via a percolation-type transition when the arousals are exceeding a threshold value. We have shown that the emergent structure in Ubuntu chats data as well as the similar networks obtained through the agent-based simulations of chats [47, 53] fulfil the test of ‘social hypothesis’, thus representing a right social graph. For this reason, it was not surprising that the emotional Bot, designed in the experimental conditions [44], was able to impact the emotional state of the users. Performing large scale simulations of chats with emotional Bots [34, 53], we have shown how the Bots manage to polarise the collective emotional states of agents, splitting the networks into two layers with positive and negative emotion flow. The hyperbolicity of the structure of these layers is analysed in Sec. 3, see Fig. 4.

In the case of social knowledge-creation via Questions-and-Answers, the data from the StackExchange Mathematics site were considered [45, 46, 55]. Here, each user has its unique ID, and its actions are stored in the open log file. In contrast to chat channels, the process is much slower, approximately ten questions per hour, and evolves in a self-organised manner without any central authority. Also, the users often search for previously answered items when their cognitive content is related to a currently active issue. By its first appearance, each question is tagged by up to 5 tags according to the standard Mathematics Classification Scheme, for example, Graph theory, Differential geometry, Number theory, Abstract algebra, Probability, Statistics, and so on. In total, over one thousand different tags appear in all questions in the considered period of four years. Each posted question and answer has a unique ID and content as well as the time of publishing and the user who posted or referred to it. Hence, the interaction between users is effectively going through the questions, or better to say, through the contents of these questions. For these reasons, we present the sequence of events as a bipartite graph, see Fig. 2, where the user-nodes make one partition while the questions-and-answers are the nodes of the other partition. It is important to notice that in such a graph no direct link can occur between the nodes of the same partition, which corresponds to the nature of the interactions on this site.

Another prominent feature of this process is that the cognitive contents of each question are systematically observed. Namely, only the users with the expertise in the area mentioned by tags in that question can offer a meaningful answer. In the data, the user’s skill is not explicitly known but can be derived statistically from the history of its actions, as we have done in [45]. In designing the agent-based model of these processes, see [45, 55], we create the agents with given expertise (expressed by a combination of tags) and assume that strict matching of the agent’s knowledge with the contents of the answered question should apply at least in one tag. The comparisons of the prominent features of the stochastic processes in the simulated and empirical
data confirmed \[55\] that, indeed, the expertise matching the question contents plays the crucial role in the creation of knowledge by the expansion of innovation; see precise definition and details in \[45\]. Thus, the users bring new knowledge that accumulates via answers on a particular question. The sequence of events is then mapped onto a growing bipartite network with two types of nodes representing users and artefacts. An example is shown in Fig. 5 corresponding to the subset of the empirical data where each question-node contains the tag Linear algebra among other tags.

The tag-matching between the user’s expertise and the contents of the answered question is illustrated in Fig. 6. For the objectives of this work, it is important to notice that the tag-matching constraint during the social process of questions-and-answers leads to a non-random appearance of different tags. Namely, being a part of the expertise of each user, various combinations of tags occur in a logical connection with each other. Consequently, the network of these knowledge contents emerges reflecting the ways that they were used in the process. In the context of knowledge creation processes, these networks contain what is called explicit knowledge, that is a form of the collective knowledge used by the participants, and remains for others to learn from it \[48\]. In \[46\] we have extracted such networks of tags over consecutive year-long periods and analysed their hierarchical architecture by algebraic topology methods. An example is shown in Fig. 6 corresponding to the contents used during the first year of the users’ activity. Moreover, we have shown that the attachment of new tags to the network occurs through so-called innovation channel, which contains new tags added within a specified short period and the tags to which they attach through the underlying users’ actions. The analysis in \[46\] revealed that the attachment of new contents observes the logical structure of mathematics knowledge. This structure confirms that the network growth principle obeys the tag-matching constraints at each stage. Here, in Sec. 3 we analyse the hyperbolic geometry of these tag networks as well as of their innovation channels appearing over time.

3. HYPERBOLIC GEOMETRY OF NETWORKS EMERGING IN KNOWLEDGE-SHARING SOCIAL ENDEAVOURS

Spontaneous evolution of networks in physical world obeys certain optimisation principles at different stages of the network growth. The developed complex architecture often exhibit hidden geometries with emergent hyperbolicity, the proximity of nodes in the graph-metric space, which facilitates flow between them. In social systems, on the other hand, the co-evolving networks embody human interactions, which are governed by different principles and depend on communicated cognitive or emotional contents. Above, we have described how two types of social networks emerge from knowledge-sharing social endeavours: the graph that contains explicit knowledge derived from the empirical data of knowledge-creation processes by Questions and Answers, and a social graph built through IRC chats devoted to Ubuntu problems solving. Here, by testing the Gromov 4-point criterion of hyperbolicity \[50\], we show that the structure of these empirical networks is \(\delta\)-hyperbolic.

A generalization of the Gromov notion of hyperbolicity \[50\] has been applied to networks \[23, 25\]; it uses the graph metric space (shortest paths) to measure the distance between nodes. Here, we apply 4-point Gromov delta-hyperbolicity \[50\] test to the adjacency matrices of the graphs that we described above. Specifically, for an arbitrary set of four nodes \((A, B, C, D)\) one defines three combinations of the sums of the pairwise distances \(d(A, B), d(C, D), d(A, C), d(B, D), d(A, D), d(B, C)\). Then these combinations are ordered from the largest to smallest \(L \geq M \geq S\), where, for example, the largest sum is \(L = d(A, B) + d(B, C), \) middle \(M = d(A, C) + d(B, D), \) and the smallest \(S = d(A, B) + d(C, D). \) For a \(\delta\)-hyperbolic graph, there is a small fixed value \(\delta\) such that any four nodes of the graph satisfy the condition

\[
2\delta(A, B, C, D) = L - M < \delta. \tag{1}
\]

According to the triangle equality, the upper bound of the difference between the two largest sums, \((L - M)/2,\) is given by the minimal distance \(d_{min}\) in the smallest sum, that is

\[
d_{min} = \min\{d(A, B), d(C, D)\} \tag{2}
\]

in the given example. Therefore, plotting \(\delta(A, B, C, D)\) against the corresponding \(d_{min}\) for all 4-tuples of the graph indicates the possible \(\delta\) value if the curve that is indicated by averaging over all points \(\langle \delta \rangle\) saturates for larger distances. A sharp saturation value suggests fixed hyperbolicity. In contrast, the graph exhibits weak or no hyperbolicity if the corresponding curve grows sub-linearly or linearly with the distance \(\langle A, B, C, D\rangle\). For a given network, which is defined by its adjacency matrix, we first compute the matrix of all distances by computing the shortest-paths tree from each node. The distance matrix is then used to determine the pair distances and abovementioned their combinations for each randomly selected 4-tuple of nodes. A large number of 4-tuples is sampled; the sample-average \(\langle \delta \rangle\) corresponding to a given \(d_{min}\) is plotted against \(d_{min}\). Additionally, we also track the largest value \(\delta_{max}\) that occurs in the whole network and plot it against \(d_{min}\).

In the following, we examine \(\delta\)-hyperbolicity of two types of networks derived from the knowledge-sharing dynamics in online chats channel and questions-and-answers site. More specifically, in the online chats simulations in the presence of emotional Bots have been performed \[53\], resulting in emotion polarisation of the communicated messages following the emotion valence which is preferred by the Bot. We analyse the whole network, see Fig. ??, and two
layers of it which contain nodes and their connections along which negative/positive emotion messages flow. The situations corresponding to the Bots preferring positive (posBot) and negative (negBot) are considered, see more details in [53]. The results are shown in Fig. 4 indicating that this social network, as well as its emotion-carrying layers, are $\delta$-hyperbolic with $\delta_{\text{max}} \leq 2$.

Although the knowledge of the participating individuals plays a vital role in building the chat networks, the direct user-to-user contacts contribute to developing its social-graph characteristics. In Q&A data, however, the knowledge contents appear at different levels resulting in two types of networking. First, the users can group around a particular set of tags, representing their primary interest. An example of content-based bipartite community is shown in Fig. 5.
The use of particular knowledge contents (tags) can be systematically analysed in the considered data and agent-based modelling [45], assuming that the individual knowledge of each user can be tagged accordingly. For example, Fig. 6 reveals the knowledge contents that were utilised during the communications by the users U1 and U6, featuring in Fig. 3. For instance, the four tags (Geometry, Differential geometry, Algebraic topology, Ring theory) appear to be the user’s U1 expertise, while three tags (Geometry, Differential geometry, Lie algebra) represent the knowledge of the user U5. Then the interaction via questions (answers) as in Fig. 6 leads to mutual linking of these tags.

Next, we divide the whole set of events into the sequence of the specified period (one year). For each period, we obtain the network of tags, which reflects the way that these knowledge contents were used by the participants in all questions and answers occurring within that period. As it was shown in [46], filtering of these networks is needed to remove spurious connections (precisely defined by null-model) and obtain the network of tags with a given confidence level. The resulting networks, see an example in Fig. 6, contain explicit knowledge built during this social process. Then the new tags added to it within a short time at the beginning of the next period are considered. Together with the previous tags to which they directly link, the newly added contents constitute an innovation channel of the knowledge network, for more details, see [46].

In Fig. 7 we show the results of hyperbolicity analysis of innovation channels of tag networks over three consecutive year-long periods. The results show that the all analysed tag-networks appear to have emergent hyperbolic structure. The origin of the hyperbolicity can be sought in the appearance of simplicial complexes of elementary geometric forms (cliques of different orders); they associate together to form higher organised complexes, which are determined in [46]. As the networks of tags grow over the years by the addition of innovation channels, their structure becomes increasingly more complex as expressed concerning topology structure vectors [46]. Notably, the network after the fourth year (Y4) has a shorter diameter and gradually smaller $\langle \delta \rangle$ while $\delta_{\text{max}}$ remains limited below two. By cutting the Y4-network along the topology level $q = 3$, i.e., removing the cliques of the orders lower than the indicated $q$, the higher-order simplexes that appear in the network remain loosely connected along faces of the order $q$ and larger. Consequently, the hyperbolic structure changes such that $\delta_{\text{max}} = 2$ appears at longer distances, and overall $\langle \delta \rangle$ is larger compared to the whole network. In-depth understanding the links between the mesoscopic structure of the network and its hyperbolic geometry is currently receiving much attention in theoretical investigations [57], and can have many practical consequences.

4. FEATURES OF SELF-ORGANISED CRITICALITY IN SOCIAL DYNAMICS

In this regard, by considering the timestamp of the events, we construct different time series of the number of events occurring at a given small time interval as well as the number of events in a small time interval carrying a specified content (cognitive or emotional). Then, performing the analysis of these time series, we can extract the features of the underlying stochastic processes. Here, the assumption is that the time series contains “the breath” of the dynamical social system, in full analogy to laboratory measurements, for instance, measured Barkhausen noise in the field-driven random ferromagnetic materials, given in Fig. 1. Then the analysis of these time series is performed to test the key features of the self-organised criticality [1]. In particular, we identify the avalanches of events and their potential scale-invariance, the temporal correlations, and fractal features of the collective fluctuations [29, 45, 55], see below.

An example of the time series of the number of events in the Q&A data is shown in Fig. 8 top panel. We also show the time series of the number of new users, who bring further questions and thus drive the system’s dynamics. It should be noted that the occurrence of the power-laws as the signature of the mechanisms of SOC in these systems [55], is also compatible with the power-law in the ranking distribution (Zipf’s law) and with it related Heap’s law. Here in the lower panels in Fig. 8, we show the corresponding plots for the frequency of tags and for the appearance of unique combinations of tags, which describes the innovation expansion in this knowledge creation process. A more detailed study can be found in [43].

In the case of Ubuntu chats, which results in the emotion-based social linking, the self-organising dynamics can be detected in the time series of the appearance of new links, as shown Fig. 9. Specifically, the long-range temporal correlations of the appearance of new links manifest themselves
FIG. 8: From Q&A data: Time series of all events (background) and new user arrivals (front curve) with the time interval 10 minutes. Bottom: Zipf’s and Heap’s law plot for all tags (purple) and all distinct combinations of tags—innovation (black).

FIG. 9: From Ubuntu chats data: Time series of the number of new links appearing in a small time window $t$, bottom panel, and its power spectrum, middle, and the fluctuations defining the Hurst exponent, top panel.

in the power-law decay of the power spectrum according to

$$S(\nu) \sim 1/\nu^\phi$$

(3)

where $\phi \sim 1XX$. Moreover, the number of new links fluctuates over time according to the fractal (Hurst) exponent $H_2 \sim 1$. By definition, this exponent is related to the standard deviations of the fluctuations around a local trend $y_\mu(t)$ of the integrated signal $Y(i) = \sum_{k=1}^{i} (N_c(k) - \langle N_c \rangle)$ on the $\mu$-th segment of the length $n$, i.e.,

$$F_2(n) = \left[ \frac{1}{N} \sum_{\mu=1}^{N} F^2(\mu, n) \right]^{1/2} \sim n^{H_2}$$

(4)

where $F^2(\mu, n) = (1/n) \sum_{i=1}^{n} [Y((\mu - 1)n + 1) - y_\mu(i)]^2$. Thus, by varying the length of the segment $n$, the fluctuation function is determined and plotted against $n$. The slope of this plot then defines the Hurst exponent, see more plots in Fig. 10.

FIG. 10: Top-left: Avalanche sequences in Q&A; Bottom-left: Scaling of the standard deviation function for time-series (TS) and avalanche series (AS) in Chats and Q&A data; the corresponding Hurst exponents are shown in the legend. Top-right: Avalanche distributions $P(S)$ and $(T)$ for chats with positive and negative emotion contents; fits according to stretch-exponential distribution $P(X) = aX^{-\tau_S} \exp(-X/b)^{\sigma_X}$ with $\tau_S \sim 1$ and $\tau_T \sim 1.25$ and stretching $\sigma_S \sim 0.9, \sigma_T \sim 0.85$ both for positive and negative contents. Bottom-right: For Q&A data, the distributions of the avalanche sizes and durations, fitted by $q$-exponential with the $q$ values in the legend.

In the SOC states, the size $s$ and duration $T$ of avalanches (cf. caption to Fig. 1 for the precise definition) are expected to have a power-law decay. We determine these distributions directly from the time series of the number of events in Q&A data and from the chats data in the presence of a Bot which favours positive/negative emotions. The results that are shown in Fig. 10 indicate that the avalanches have a substantial scale invariance in both knowledge-sharing processes. However, we find that different mathematical expressions fit the probability distributions of the avalanches. Explicitly, in the chat data, the power-law decay in a short interval is followed by an exponential cut-off. The exponents and the cut-offs are slightly different for the layer with positive/negative emotion flow. In the case of data Q&A data, however, the avalanches obey a $q$-exponential distribution

$$P(X) = B_X \left[ 1 - (1 - q) \frac{X}{X_0} \right]^{1/1-q}$$

(5)

where the exponent $q \neq 1$ is recognised as the nonextensivity parameter [58, 59]. Moreover, the sequence of avalanches, see the top-left panel in Fig. 10 was shown to have a multifractal spectrum [55].
5. DISCUSSION AND CONCLUSIONS

Knowledge sharing on online social sites represents a particular type of social dynamics resulting in the emergent networks of a characteristic structure. Considering two types of online social interactions which involve knowledge sharing, we have shown that different sorts of networks emerging from these interactions, in particular, the explicit-knowledge networks built via Q&A and the social graph built by Ubuntu chats, are $\delta$-hyperbolic. Furthermore, they exhibit a rather low $\langle \delta \rangle \approx 0.33$ and $\delta_{\text{max}}$ that does not exceed 2. Our analysis [33, 45–47, 60] shows that the occurrence of the higher organised structure with simplicial complexes in these networks can be regarded as the origin of their hyperbolic geometry. The networks underlying knowledge creation via Q&A occur in the stochastic process with the prominent signatures of self-organised criticality [55]. In the case of Ubuntu chats, the temporal correlations, fractality and avalanches of events are also apparent; however, the avalanche distributions indicate possible parameter dependence and a dynamical phase transition, rather than a SOC attractor. These global states emerge from the different use of knowledge contents at the elementary scale of human interactions. This question requires additional study.

The growth of the collective knowledge appears over time by building innovation channels on the network of used knowledge contents. Their structure can be decomposed into cliques of all orders up to an existing $q_{\text{max}}$. Having in mind that these elementary geometric descriptors of the network are $\delta$-hyperbolic [21, 56] with $\delta=0$, it is conceivable that their complexes, which make the graph, also exhibit hyperbolic structure with a low value of $\delta$, as shown in Fig. 4. Note that architecture with simplicial complexes is also found in the social network MySpace [60]. These results open a new direction of research of the origin of hyperbolicity in the online social graphs. The network of Ubuntu chats, on the other hand, grows around an active core of knowledgeable users and Bot by attaching new users, who then stay to serve new arrivals further. In this way, based on the experience of users, a hierarchical architecture of the network is built over time. The characteristic $k$-core structure of this whole network, as well as its layers carrying emotional messages [47], is compatible with the observed hyperbolicity in Fig. 4.

Although the considered networks are of a different nature, in both cases, the knowledge (expertise) of the participating individuals plays an essential role in building the system at each stage. More precisely, in each interaction, specific knowledge content is required to meet the current needs. This knowledge-matching constraints provide a delicate balance at the elementary scale and, consequently, implies the logical attachment of the primary forms into an extensive geometry, eventually resulting in a hyperbolic structure. Note that, such a delicate balance which is apparent in knowledge-sharing processes is not present in various other social interactions; nevertheless, they can result in hyperbolic networks, which are based on another optimisation principle.

Acknowledgments

This work is based on the results of several previous publications, for which I thank the collaborations with Milovan Šuvakov, Miroslav Andjelković, Marija Mitrović Dankulov, Vladimir Gligorjević, Milan Rajković, and Roderick Melnik. Work supported by the Slovenian Research Agency (research code funding number P1-0044).


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cators of collective behavior and functional clusters in gene networks of yeast, *Europhys. J. B—Condens. Matter and Complex Systems* 50(1-2):255-258 (2006)

[15] M. Bastian, S. Heynam, M. Jacomy, Gephi: An Open Source Software for Exploring and Manipulating Networks, *Proceedings of the Third International ICWSM Conference* (2009), pp 361-362 (2009); http://gephi.org

[16] J. Jonsson, Simplicial Complexes of Graphs, *Lecture Notes in Mathematics*, Springer-Verlag, Berlin (2008).

[17] D. Kozlov, Combinatorial Algebraic Topology, *Springer Series Algorithms and Computation in Mathematics* 21, Springer-Verlag, Berlin, Heidelberg (2008)

[18] R.H. Atkin, An algebra of patterns on a complex II, *Int. Journal Man-Machine Studies* 8, 483-498 (1976)

[19] J. Johnson, Some structures and notations of Q-analysis, *Environment and Planning B Planning and Design* 8, 73-86 (1981)

[20] J.R. Beaumont, A.C. Gatrell, An Introduction to Q-analysis, *Geo Abstracts* Norwich-Printed by Edmund Nome Press, Norwich (1982)

[21] A.G. Askoy, S. Jin, The apple doesn’t fall far from the (metric) tree: The equivalence of definitions, *Arxiv*:1306:6092 (2013)

[22] W.S. Kennedy, I. Sanjee, O. Narayan, On the hyperbolicity of large-scale networks and its estimation, In 2016 IEEE International Conference on Big Data (Big Data), *IEEE Xplore*, pp. 3344-3351 (2016)

[23] R. Albert, B. DasGupta, N. Mobasher, Topological implications of negative curvature for biological and social networks, *Phys. Rev. E* 89, 032811 (2014)

[24] O. Narayan, I. Sanjee, Large-scale curvature of networks, *Phys. Rev. E* 84, 066108 (2011)

[25] M. Šuvakov, M. Andjelković, B. Tadić, Hidden geometries of networks arising from cooperative self-assembly, *Scientific Reports* 8, 1987 (2018)

[26] F. Falk, B. Basset, Brain and social networks: fundamental building blocks of human experience, *Thends Cogn. Sci.* 21:674–90 (2017)

[27] B. Tadić, M. Andjelković, B. Moškoska, Z. Levnajić, Algebraic topology of multi-brain connectivity networks reveals dissimilarity in functional brain patterns during spoken communications, *PLOS One* 11:e0166787 (2016)

[28] B. Tadić, M. Andjelković, M. Šuvakov, Origin of hyperbolicity in brain-to-brain coordination networks, *Frontiers in Physics* 6:7 (2018)

[29] B. Tadić, V. Gligorijević, M. Mitrović, M. Šuvakov, Co-evolutionary mechanisms of emotional bursts in online social dynamics and networks, *Entropy* 15(12):5084–5120 (2013)

[30] M. Mitrović, B. Tadić, Bloggers Behavior and Emergent Communities in Blog Space, *European Physical Journal B* 73(2):293–301 (2010)

[31] M. Mitrović, G. Paltoglou, B. Tadić, Networks and emotion-driven user communities at popular blogs, *European Physical Journal B* 77:591–609 (2010)

[32] M. Mitrović, G. Paltoglou, B. Tadić, Quantitative analysis of bloggers’ collective behavior powered by emotions, *Journal of Statistical Mechanics: Theory and Experiment*, 2011(02):P02005 (2011)

[33] M. Šuvakov, M. Mitrović, V. Gligorijević, B. Tadić, How the online social networks are used: dialogues-based structure of MySpace, *Journal of the Royal Society Interface* 10(79):20120819 (2012)

[34] M. Šuvakov, B. Tadić, Collective emotion dynamics in chats with agents, moderators and bots, *Condensed Matter Physics*, 17(3):33801:1–12 (2014)

[35] N. Liu, C. Mok, E.E. Witt, A.H. Pradhan, J.E. Chen, A.L. Reiss, NIRS-Based Hyperscanning Reveals Inter-brain Neural Synchronization during Cooperative Jenga Game with Face-to-Face Communication. *Frontiers in Human Neuroscience* 10(82):1–11 (2016)

[36] R. Conte et al., Manifesto of computational social science, *Eur. Phys. J. Special topics* 214:325-346 (2012).

[37] P.M.C. De Oliveira, J.S. Samartins, D. Stauffer, S. Moss De Oliveira, editors, *Biological, Sociology, Geology by Computational Physicists*, Elsevier (2006)

[38] B. Tadić, Dynamic criticality in driven disordered systems: role of depinning and driving rate in Barkhausen noise, *Physica A: Statistical Mechanics and its Applications*, 270:125 – 134 (1999)

[39] G. Paltoglou, S. Gobron, M. Skowron, M. Thelwall, D. Thalhammer, Sentiment analysis of informal textual communications in cyberspace, *Springer LNCS State-of-the-Art Survey*, pp. 13-23, Springer, Heidelberg (2011)

[40] A. Garas, D. Garcia, M. Skowron, F. Schweitzer, Emotional persistence in online chatting communities, *Scientific Reports* 2:402 (2012)

[41] V. Gligorijević, M. Skowron, B. Tadić, Structure and stability of online chat networks built on emotion-carrying links, *Physica A: Statistical Mechanics and its Applications* 392:538-543 (2013)

[42] B. Tadić, Modeling behavior of Web users as agents with reason and sentiment in “Advances in Computational Modeling research: Theory, Developments and Applications”, Ed. A.B. Kora, Nova publishing, New York (2013)

[43] M. Mitrović, B. Tadić, Dynamics of bloggers communities: Bipartite networks from empirical data and agent-based modeling, *Physica A: Statistical Mechanics and its Applications*, 391(21):5264 – 5278 (2012)

[44] B. Tadić, V. Gligorijević, M. Skowron, M. Šuvakov, The dynamics of emotional chats with Bots: Experiment and agent-based simulations, *Science Letters* 2:402 (2012)

[45] M. Mitrović Dankulov, R. Melnik, B. Tadić, The dynamics of meaningful social interactions and the emergence of collective knowledge, *Scientific Reports* 5:12197 (2015)

[46] M. Andjelković, B. Tadić, M. Mitrović Dankulov, M. Rajković, R. Melnik, Topology of innovation spaces in the knowledge networks emerging through questions-and-answers, *PLOS ONE* 11(5):e0154655 (2016)

[47] V. Gligorijević, M. Šuvakov, B. Tadić, Building social networks of online chats with Users, Agents and Bots, in Complex Networks and their Applications, Ed. Hichine Cherifi, Cambridge Scholar Pub. (2013)

[48] J. Kimmerle, U. Kress, and Ch. Held, The interplay between individual and collective knowledge: technologies for organisational learning, *Knowledge Management Research & Practice* 8:33-44 (2010)

[49] J.I.M. Carpendale and U. Müller, editors, *Computational Physicists*, Springer series Understanding Complex Systems.
5 DISCUSSION AND CONCLUSIONS

Springer, Berlin (2017)

[52] V. Gligorijević, M. Skowron, B. Tadić, Directed networks of online chats: Content-based linking and social structure, Signal Image Technology and Internet Based Systems, *IEEEExplore* pp. 25-29 (2013)

[53] B. Tadić, M. Šuvakov, Can human-like Bots control collective mood: Agent-based simulations of online chats, *Journal of Statistical Mechanics Theory and Experiment* 10:P10014 (2013)

[54] http://StackExchange/Mathematics

[55] B. Tadić, M. Mitrović Dankulov, R. Melnik, Mechanisms of self-organized criticality in social processes of knowledge creation, *Phys. Rev. E* 96:032307 (2017)

[56] E. Jonckheere, P. Lohsoonthorn, F. Ariaei, Scaled Gromov Four-Point Condition for Network Graph Curvature Computation, *Internet Mathematics* 7:137-177 (2011)

[57] N. Cohen, D. Coudert, G. Ducoffe, A. Lancin, Applying clique-decomposition for computing Gromov hyperbolicity, *Theor. Computer Science* 690:144–139 (2017)

[58] C. Tsallis, The nonadditive entropy Sq and its applications in physics and elsewhere: Some remarks, *Entropy* 13(10):1765–1804 (2011)

[59] G.P. Pavlos, L.P. Karakatsanis, M.N. Xenakis, E.G. Pavlos, A.C. Iliopoulos, D.V. Sarafopoulos, Universality of non-extensive tsallis statistics and time series analysis: Theory and applications, *Physica A: Statistical Mechanics and its Applications* 395:58 – 95 (2014)

[60] M. Andjelković, B. Tadić, S. Maletić, M. Rajković, Hierarchical sequencing of online social graphs, *Physica A: Statistical Mechanics and its Applications* 436:582–595 (2015)