Agile Missile Autopilot Design for High Angle of Attack Maneuvering with Aerodynamic Uncertainty*

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The paper proposes a new autopilot design for agile missiles flying at a high angle of attack (AoA). A maneuver strategy applicable to 90° AoA flight for agile turning is described prior to the missile modeling. Accounting for the disturbance rejection, the extended state observer (ESO) technique is employed for online estimation of the system uncertainties due to the aerodynamic unpredictability at high AoA regimes. Under the circumstances, linearization with dynamic compensation and non-singular terminal sliding mode control are applied to achieve controllability during 90° AoA flight. Numerical simulation results demonstrate the effectiveness and robustness of the proposed scheme. Additionally, the chattering caused by unmodeled dynamics is obviously mitigated with the action of the ESO.

Key Words: Agile Missile, Autopilot, High Angle of Attack, Disturbance Rejection, Extended State Observer

1. Introduction

Current high-performance fighter aircraft are required to achieve omni-directional attack of the launch platform, which implicates the ability to engage targets in the rear hemisphere for an agile missile. The fast 180° turn of a missile is performed under high angle of attack (AoA) maneuvering. The dynamics of an agile missile at high AoA are inherently nonlinear, fast time varying and extremely uncertain. In practice, the uncertainties of missile dynamics are, to a large extent, caused by the unpredictability of aerodynamics. When AoA increases, the asymmetric flow separation appearing on the leeside of the missile makes aerodynamic data unpredictable. The uncertainties in dynamics of the missile increase the difficulty of high AoA flight control. In addition, alternative control technology such as thrust vector control (TVC) or a reaction-jet control system (RCS) must possess super-agility for agile missiles because of the ineffectiveness of aerodynamic control at high AoA.

Research about agile turn control has progressed in different perspectives. The dynamics and technical challenges of an agile missile at high AoA were discussed by Wise and Broy.1 A 90° AoA command was used to turn the missile into the rear hemisphere, with sideslip angle regulated to zero. The simulation results for the missile using RCS thrusters showed the feasibility of such high AoA flight control. Thukral and Innocenti2 proposed an autopilot based on variable structure control (VSC) to achieve a fast 180° heading reversal in the vertical plane. Furthermore, robustness of the VSC system for the agile missile was investigated by Innocenti and Thukral.3 These studies, however, only considered the longitudinal dynamics, regardless of the roll and yaw channel control, which is a crucial factor for high AoA autopilot due to the serious aerodynamic uncertainties. McFarland and Calise4,5 applied neural network-based adaptive control to agile turning of anti-air missiles, resulting in an improvement in approximate dynamic inversion for the control of uncertain nonlinear systems. A bank-to-turn (BTT) steering technique also used by Wise and Broy1 was applied in the six-degrees-of-freedom (6-DOF) simulation for a head-on merge scenario. Various nonlinear autopilot schemes were also considered using the $H_{\infty}$ method,6 pole placement method,7 backstepping method8 and so on. Recently, a novel strategy for agile turn control of high-performance missiles was first proposed by Ratliff et al.9 and further developed by Kim et al.10 In this strategy, fast 180° change in the missile pitch angle was accomplished by purely aerodynamic control. This aero-surface control mode, only verified by longitudinal motion simulation without the consideration of aerodynamic disturbance, is questionable in practical terms.

As mentioned above, the majority of research has paid close attention to the nonlinearities of plants to meet challenges of nonlinear characteristics at high AoA, while ignoring how to deal with the system uncertainties which actually play a critical role in improving control performance. The effects of uncertainties in plant dynamics are counteracted by the robustness of the control system itself. In a distinct way, to address the problem that aerodynamic coefficients cannot be predicted in the high AoA domain, the control laws designed by Kim and Kim11 did not require the aerodynamic data. In literature, nonetheless, the pitching moment was assumed to vary according to a sinusoidal function bounded by a very small constant, which does not conform to real-world conditions.

In a different way than in the past studies, this paper designs an autopilot following the thought that the aerodynamic data regarded as the uncertainty of missile dynamics can be estimated online using an effective mechanism. The information on the true values of aerodynamic coefficients...
is not used in the control laws, abiding by the viewpoint presented by Kim and Kim.\textsuperscript{11} What makes the proposed control scheme different is the disturbance rejection capability resulting from the real-time acquisition of the unknown system information. As the key part of the active disturbance rejection control (ADRC) method, extended state observer (ESO) is employed to estimate the uncertainties of the control system in real time. As a means of accounting for the unknown information of system dynamics, the ESO proves the possibility of online compensation of the uncertainties for controller design.

This paper begins with a description of the proposed control strategy for 90° AoA maneuvering in Section 2. Furthermore, the dynamics of an agile missile equipped with a RCS system are studied, followed by a detailed presentation of the three-channel independent autopilot design using ADRC/ESO and the non-singular terminal sliding mode (NTSM) method in Section 3. In Section 4, simulation results for an agile missile are presented to validate the proposed autopilot design technique. Finally, conclusions are summarized in Section 5.

2. Missile Dynamics

2.1. Maneuver description

A 90° AoA command\textsuperscript{1,4,5} is used to achieve agile turning with a 180° off-boresight trajectory. Traditionally, an aircraft flying in endo-atmosphere performs maneuvers by virtue of the normal aerodynamic overload generated by a proper AoA. Nevertheless, the normal overload will be enhanced for the effect of the main engine thrust during a 90° AoA flight (Fig. 1). If the AoA equals exactly 90°, the missile will obtain the maximal normal overload provided by the engine thrust, which is absolutely used in the normal direction.

High AoA flight leads to some control challenges for agile missiles including strong nonlinearity, model parameter perturbation and serious aerodynamic coupling. For the purpose of solving the relevant problems in an implementable and reasonable way, a quasi BTT autopilot similar to widely used BTT is adopted here. Before a detailed description, two attitude angles, quasi roll angle $\phi^*$ and quasi yaw angle $\psi^*$, are defined. $\phi^*$ ($\psi^*$) denotes the angle which the missile body moves around the $x$-axis ($z$-axis) in the body coordinate system. $\phi^*$ ($\psi^*$) is positive when the rotation vector is in the same direction as the $x$-axis ($z$-axis). At the beginning of implementing the quasi BTT maneuver, the roll control of the agile missile is performed after separating from the carrier aircraft to adjust the $z$-axis of the body into the expected maneuvering plane. Then the autopilot controls the missile to track the AoA command in the pitch channel. In the subsequent process, only stability control is necessary in the roll and yaw channels, aiming to hold $\phi^*$ and $\psi^*$ near zero. According to the above approach, the autopilot is synthesized with a three-channel independent design to make the missile fly in the preferred orientation as much as possible. In this paper, the interchannel coupling is treated as an unknown disturbance whose relevant discussions are described in detail in Section 3.

The quasi BTT proposed in this paper controls the rolling and yawing attitude of the missile without confining the missile sideslip angle. Actually, under the influence of dynamic model uncertainties, the missile cannot fly in the expected maneuvering plane absolutely, regardless of whether BTT or quasi BTT is chosen for the flight control. However, it is unnecessary to pose the strict requirement that both the missile axis and velocity direction aim at the target after the agile turn, since the top priority is to make the missile turn around. Simultaneously, the missile axis and velocity direction could be adjusted by guidance command in the subsequent guidance phase. Compared with BTT control, quasi BTT control is capable of dominating the rolling and yawing attitude of the missile, which is beneficial to enable the missile seeker to capture the target for terminal guidance.

2.2. Modeling

The system dynamic models are based on a slender body with the RCS system and the rear control dominated by cruciform rudders, as shown schematically in Fig. 2. RCS is used for the missile attitude control on the assumption that the RCS can provide roll, pitch and yaw moment control respectively. The magnitude of the reaction-jet is adjusted by the RCS nozzle throat area valve and changes continuously. In the current scenario, RCS is the only means to control the missile for an agile maneuver in this paper, as a consequence of the low effectiveness of rudder control in the high AoA region.

The motion of a missile can be described using the following equations:
\[ \dot{V} = (a_c \cos \alpha + a_s \sin \alpha) \cos \beta + a_r \sin \beta \\
\dot{\alpha} = q - (p \cos \alpha + r \sin \alpha) \tan \beta \\
\dot{p} = \frac{p \sin \alpha - r \cos \alpha - (a_c \cos \alpha + a_c \sin \alpha) \sin \beta / V}{(V \cos \beta)} \\
\dot{q} = \frac{p \sin \alpha - r \cos \alpha - (a_c \cos \alpha + a_c \sin \alpha) \sin \beta / V}{(V \cos \beta)} + a_r \cos \beta / V \\
\dot{r} = \frac{M / I_{yy} + (1 - I_{xx} / I_{yy}) pr}{L / I_{xx}} \\
\dot{\psi} = N / I_{yy} - (1 - I_{xx} / I_{yy}) pq \tag{1} \]

where \( V \) is velocity, \( \alpha \) is angle of attack, and \( \beta \) is sideslip angle. \( a_r, a_s, \) and \( a_c \) are the body-axis components of acceleration. \( p, q, \) and \( r \) are the body-axis angular rates. \( L, M \) and \( N \) are aerodynamic moments about the body axis. \( I_{xx} \) and \( I_{yy} \) are rolling and pitching moments of inertia. Additionally, it is assumed that \( V, \alpha, p, q, \) and \( r \) are available or effective estimates for control implementation in this paper. Finally, according to the definition proposed above, the differential equations of the quasi roll angle and the quasi yaw angle can be expressed as

\[ \begin{align*}
\dot{\phi}^* &= p \\
\dot{\psi}^* &= r \tag{2}
\end{align*} \]

The realization of 90° AoA command flight requires high AoA aerodynamic modeling as the basis for autopilot design. For a missile with a high slenderness ratio, when the AoA increases from 0° to 90°, four different lee side flow patterns appear orderly in accordance with the effect of the axis flow component. The asymmetric flow separation on the missile lee side induces considerable lateral force and yawing moment with strong randomness at high AoA, mainly from 30° to 60°. However, the effect of the axis flow component obviously declines when the AoA is close to 90°. Compared to the situation with random lateral loads of 30° to 60° AoA, the uncertainty of 90° AoA aerodynamics is significantly reduced.

This paper divides the missile aerodynamic mathematical model into two parts: the estimated value and disturbed value. The estimated value is generated by the Missile Datcom code to estimate a set of general aerodynamic data in the preliminary design phase, even for a high AoA region. The plots of lift and drag coefficients for a reference velocity of Mach = 0.6 are shown in Fig. 3. It can be concluded that the normal load is the leading aerodynamic influence on the missile for 90° AoA flight. Moreover, other aerodynamic forces are treated as uncertainties including aerodynamic errors between the missile configuration and the cylinder, aerodynamics induced by the change in missile attitude and the coupling between channels. Such uncertainties may induce lateral force, yaw and roll moments at a high AoA. Because the aerodynamic uncertainties are difficult to measure, the disturbed values need to be set manually and are then incorporated into the aerodynamic models. The specific forms used to reflect the uncertainties are described in Section 4.

3. Autopilot Design with ESO

3.1. ESO methodology

ADRC was first proposed by Han at the end of the last century. The theory was explained carefully by Han and analyzed in depth by Gao and Huang et al. As the result of years of investigation, the ADRC technique has been applied in real-world applications, especially in the industrial field. Playing a predominant role in the ADRC method, ESO is able to estimate the total disturbance of plant dynamics online. The ESO technique proposes a new viewpoint on how to deal with system uncertainties. With the action of ESO, ADRC can compensate system uncertainties in real-time to realize linearization for an uncertain nonlinear control system.

Consider an \( n \)-dimensional SISO nonlinear system with the following structure:

\[ \begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
& \vdots \\
\dot{x}_n(t) &= f(t, x_1(t), \ldots, x_n(t)) + w(t) + u(t) \\
y(t) &= x_1(t) \tag{3}
\end{align*} \]

where \( u \) is the input (control), \( y \) is the output (measurement), \( f \) is a possibly unknown system function, and \( w \) is a uncertain external disturbance. \( f + w \) models the total disturbance. The ESO is used to provide the approximations of the state \( x_i \) for \( i = 1, 2, \ldots, n \) and the total disturbance \( f + w \). A kind of nonlinear ESO is written as

\[ \begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + \epsilon^{n-1} h_1 \left( \frac{y(t) - \hat{x}_1(t)}{\epsilon^n} \right) \\
\dot{\hat{x}}_2(t) &= \hat{x}_3(t) + \epsilon^{n-2} h_2 \left( \frac{y(t) - \hat{x}_1(t)}{\epsilon^n} \right) \\
& \vdots \\
\dot{\hat{x}}_{n}(t) &= \hat{x}_{n+1}(t) + h_n \left( \frac{y(t) - \hat{x}_1(t)}{\epsilon^n} \right) + u(t) \\
\dot{\hat{x}}_{n+1}(t) &= \frac{1}{\epsilon} h_{n+1} \left( \frac{y(t) - \hat{x}_1(t)}{\epsilon^n} \right) \tag{4}
\end{align*} \]

Fig. 3. Aerodynamic forces versus angle of attack.
where $\epsilon$ is a constant gain, and $b_i(\cdot)$, $i = 1, 2, \ldots, n$ are pertinent chosen functions. The states of the observer $\hat{x}_i$, $i = 1, 2, \ldots, n$ and $\hat{x}_{n+1}$ can be considered as the approximations of corresponding state $x_i$ for $i = 1, 2, \ldots, n$ and total disturbance $f + w$, respectively.

The following assumptions\textsuperscript{23) are presumed:

Assumption (H1). The possibly unknown functions $f, w$ are continuously differentiated with respect to their variables, and 
\[
|u| + |f| + |w| + \left| \frac{\partial f}{\partial x_i} \right| + \left| \frac{\partial f}{\partial y_i} \right| \leq c_0 + \sum_{j=1}^{n} c_j |x_j|^k
\]

for some positive constants $c_j$, $j = 0, 1, \ldots, n$ and positive integer $k$.

Assumption (H2). $w$ and the solution $x_i$ of Eq. (3) satisfy 
\[
|w| + |x_i(t)| \leq B \text{ for some constant } B > 0 \text{ and all } i = 1, 2, \ldots, n, \text{ and } t \geq 0.
\]

Assumption (H3). There exist constants $R, a > 0$ and positive definite, continuous differentiated functions $Z$, 
\[
W: R^{n+1} \rightarrow R \text{ such that } \|y\| Z(y) \leq \delta \text{ for any } \delta > 0,
\]

\[
\sum_{i=1}^{n} \frac{\partial Z}{\partial y_i} (y_i(t) - g_i(y_i)) - \frac{\partial Z}{\partial y_{n+1}} g_{n+1}(y_{n+1}) \leq -W(y),
\]

\[
\left| \frac{\partial Z}{\partial y_{n+1}} \right| \leq a W(y) \text{ for } \|y\| > R.
\]

The weak convergence result stated in the following lemma can be obtained.

**Lemma 1.**\textsuperscript{21) Under Assumptions (H1), (H2) and (H3), the nonlinear ESO Eq. (4) is convergent in the sense that for any $\delta \in (0, 1)$, there exists $\varepsilon_\delta \in (0, 1)$ such that for any $\varepsilon \in (0, \varepsilon_\delta)$, $|x_i(t) - \hat{x}_i(t)| < \delta$, $\forall t \in [T_\varepsilon, \infty)$, where $T_\varepsilon > 0$ depends on $\varepsilon, x_i, \hat{x}_i$ denote the solutions of Eqs. (3) and (4) respectively, and $i = 1, 2, \ldots, n + 1, x_{n+1} = f + w$ is the extended state variable for the system Eq. (3).

Specifically, for a first-order nonlinear system 
\[
\begin{align*}
\dot{\hat{x}}_1(t) &= f_0(t, x_1(t)) + f_1(t, x_1(t)) + w(t) + u(t) \\
y(t) &= x_1(t)
\end{align*}
\]

where $f_0$ and $f_1$ are known and unknown system functions, respectively. By Lemma 1, we can construct an ESO following:

\[
\begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + g_1 \left[ \frac{y(t) - \hat{x}_1(t)}{\epsilon} \right]^a + f_0(t, y(t)) + u(t) \\
\dot{\hat{x}}_2(t) &= \frac{1}{\epsilon} g_2 \left[ \frac{y(t) - \hat{x}_1(t)}{\epsilon} \right]^{2a-1}
\end{align*}
\]

where $[\cdot]^a := \text{sgn}(\cdot)|\cdot|^a$ and $a$ is a constant. $g_1$ and $g_2$ are also constants to be selected. $\hat{x}_1$ and $\hat{x}_2$ are approximations of the state $x_1$ and $f_1 + w$.

In this paper, ESO is introduced in the control system to estimate the system uncertainties online (i.e., aerodynamic relevant terms in system dynamics) to deal with the aerodynamic unpredictability in the high AoA region. It should be emphasized that aerodynamic data is only used in simulation for demonstrating the effectiveness of the online estimation scheme, but unknown to autopilot design.

### 3.2. Pitch autopilot design

First, we consider the following missile dynamics of the pitch channel:

\[
\begin{align*}
\dot{\alpha} &= f_{a0} + f_{a1} + q \\
\dot{q} &= f_{q0} + f_{q1} + b_q u_{RC}
\end{align*}
\]

where 
\[
\begin{align*}
f_{a0} &= -(p \cos \alpha + r \sin \alpha) \tan \beta + (-\sin \alpha T + \cos \alpha u_{RC})/(mV \cos \beta), \\
f_{a1} &= (-\sin \alpha X/m + \cos \alpha Z/m + g \sin \alpha \sin \theta + g \cos \alpha \cos \theta \cos \phi)/(V \cos \beta), \\
f_{q0} &= (1 - I_{xT}/I_{yy}) pr, \\
f_{q1} &= QSDC_m/I_{yy}, \\
b_q &= -l_R/I_{yy}.
\end{align*}
\]

$f_{a0}$ and $f_{q0}$ model the known part, while $f_{a1}$ and $f_{q1}$ model the unknown part of the missile dynamics. $g$ is the gravitational acceleration, $\theta$ and $\phi$ are Euler angles, $u_{RC}$ is the force produced by RCS thrusters in the $z$-body direction, $m$ is the missile mass, $X$ and $Z$ are the aerodynamic forces, $Q$ is the dynamic pressure, $S$ is the reference area, $D$ is the reference length, $C_m$ is the pitching moment coefficient, and $l_R$ is the distance between the point of RCS action and the center of mass.

Owing to estimating the dynamic uncertainties online, the ESO translates system Eq. (7) with unmodeled dynamics into a certain series-connection integral system based on the theory of feedback linearization. For the first equation in Eq. (7), as a virtual control, $q$ is available for controlling $\alpha$. That is, AoA command $\alpha$ determines the controlled variable $q_s$. For the second equation, $u_{RC}$ is controlled to achieve the desired command $q$. This approach divides the second-order system into two first-order controllers including the AoA and the pitch rate control loop.

In terms of the first equation (AoA control loop) in Eq. (7), ESO Eq. (6) is adopted to estimate the uncertainty $f_{a1}$. Here, $x_1, f_0, f_1$ and $u$ in system Eq. (5) are specified as $\alpha$, $f_{a0}$, $f_{a1}$ and $q$, respectively. Then $\hat{x}_2$ in ESO Eq. (6) becomes the approximation of $f_{a1}$. A nonlinear state error feedback is applicable to form the control input, shown as follows:

\[
\begin{align*}
\epsilon_u &= \alpha_c - \alpha \\
u_a &= K_u f_{a1}(\epsilon_u, \alpha_c, \delta_u)
\end{align*}
\]

where $f_{a1}(\epsilon, \alpha, \delta)$ is a continuous power function with a linear segment near the origin:

\[
f_{a1}(\epsilon, \alpha, \delta) = \begin{cases} 
\frac{\epsilon}{\delta^2 - \epsilon^2}, & |\epsilon| \leq \delta \\
|\epsilon|^c \text{sgn}(\epsilon), & |\epsilon| > \delta
\end{cases}
\]

$K_u$, $\alpha_c$, $\delta_u$ are parameters to be designed. The power function is reorganized into a piecewise function, leading to the more
excellent characteristics of quick convergence and small steady-state error compared to the linear combination of feedback.\textsuperscript{17} The virtual control input \( q_c \) is selected as
\[
q_c = u_a - f_{d0} - \hat{f}_{a1}
\]
where \( \hat{f}_{a1} \) is the online approximation of \( f_{a1} \). In the case of perfect estimation with \( \hat{f}_{a1} = f_{a1} \), linearization of the AoA control loop can be realized as
\[
\hat{\alpha} = u_a
\]
(11)

Considering the second equation (pitch rate control loop) in Eq. (7), the same approach is adopted to realize linearization. Here, \( x_1, f_{01}, f_{11} \) and \( u \) in system Eq. (5) are specified as \( q, f_{d0}, f_{q1} \) and \( b, u_{RC} \), respectively. \( \hat{x}_2 \) in ESO Eq. (6) is the approximation of \( f_{q1} \). Then the nonlinear state error feedback law is obtained as:
\[
\begin{align*}
    e_q &= q_c - q \\
    u_q &= K_q f a_l(e_q, e_q, \beta_q)
\end{align*}
\]
(12)
The control input is
\[
u_{RC} = \frac{1}{b_y}(u_q - f_{d0} - \hat{f}_{q1})
\]
(13)
where \( \hat{f}_{q1} \) is the online approximation of \( f_{q1} \). The pitch rate control loop is also linearized as
\[
\hat{q} = u_q
\]
(14)
The nonlinear system Eq. (7) has been linearized for the pitch channel control on account of getting the approximations of the high AoA aerodynamic data in real time. Eventually, the control input is determined by the state error feedback control technique as a common approach for linear system control.

3.3. Roll and yaw autopilot design

The missile dynamics of roll and yaw channels involving quasi roll angle \( \phi^* \) and quasi yaw angle \( \psi^* \) are shown as
\[
\begin{align*}
    \dot{\phi} &= \omega \\
    \dot{\omega} &= f_0 + f_1 + u
\end{align*}
\]
where
\[
\begin{align*}
    \theta &= \begin{bmatrix} \phi^* \end{bmatrix}, \quad \omega &= \begin{bmatrix} p \\ r \end{bmatrix}, \quad f_0 &= \begin{bmatrix} 0 & -(1 - I_{xx}/I_{yy})pq \end{bmatrix}, \\
    f_1 &= \begin{bmatrix} f_{p1} \\ f_{r1} \end{bmatrix}, \quad u = \begin{bmatrix} u_{Mx}/I_{xx} \\ I_{Ry}/I_{yy} \end{bmatrix}
\end{align*}
\]
and \( f_{p0} \) and \( f_{r0} \) model the system certainty and uncertainty, respectively. \( C_{f1} \) is the rolling moment coefficient, and \( C_{r} \) is the yawing moment coefficient. \( u_{Mx} \) and \( u_{Ry} \), produced by RCS thrusters, are the rolling control moment and the control force in the \( y \)-body direction.

The roll and yaw autopilot is designed to stabilize \( \phi^* \) and \( \psi^* \) to zero under the disturbance influence of uncertainty \( f_{i1} \). The stabilization control is solved by non-singular terminal sliding mode (NTSM) control and the ESO technique in this paper. According to the proposed agile turn strategy, the lateral-directional control determines the maneuvering plane of the missile, resulting in the roll and yaw channel requiring greater control precision than the pitch channel. NTSM offers some superior properties such as fast, finite time convergence. This controller is particularly useful for high-precision control as it speeds up the rate of convergence near an equilibrium point.\textsuperscript{22} Furthermore, the combination of ESO and the SMC method, whose robustness is prominent, has better performance for avoiding the influence of the system uncertainties. So the proposed approach compares favorably to both the traditional SMC and ADRC methods for the stabilization problem described here.

The sliding mode surface function is selected as
\[
s = [s_1, s_2]^T = \theta + B^{-1} \text{sgn}(\omega)|\omega|^a
\]
(16)
where
\[
B^{-1} = \text{diag} \left[ B_{11}^{-1}, B_{21}^{-1} \right], \quad \text{sgn}(\omega) = \text{diag} \left[ \text{sgn}(p), \text{sgn}(r) \right], \quad |\omega|^a = \left[ |p|^a, |r|^a \right]^T
\]
\( B_{i}, n_i (i=1, 2) \) are constant, and \( B_{i} > 0 \) and \( 1 < n_i < 2 \). If NTSM is applied in the autopilot independently, the NTSM control law based on the exponential approach law is written as
\[
u = -[B_n^{-1} \text{sgn}(\omega)|\omega|^{2-a} + f_0 + ks + \delta \text{sgn}(s)]
\]
(17)
where \( k = \text{diag}[k_1, k_2] \) and \( \delta = \text{diag}[\delta_1, \delta_2], k_i, \delta_i (i = 1, 2) \) are constant, and \( k_1 > 0 \) and \( \delta_1 > 0 \). \( \delta \) models the upper bound of absolute values of system uncertainties, i.e.,
\[
|f_{pi}| \leq \delta_1, \quad |f_{ri}| \leq \delta_2.
\]
The ESO used for online estimating uncertainty \( f_{1} \) is shown as
\[
\begin{align*}
    \dot{X}_1 &= X_2 + G_1 \left[ e^{-1}(\omega - X_1) \right]^T + f_0 + u \\\n    \dot{X}_2 &= e^{-1} G_2 \left[ e^{-1}(\omega - X_1) \right]^{2a-1}
\end{align*}
\]
(18)
where
\[
\begin{align*}
    X_1 &= [y_{p1}, x_{r1}]^T, \quad X_2 &= [y_{p2}, x_{r2}]^T, \\
    G_1 &= \text{diag} \left[ g_{p1}, g_{r1} \right], \quad G_2 = \text{diag} \left[ g_{p2}, g_{r2} \right], \\
    e^{-1} &= \text{diag} \left[ e_{p1}, e_{r1} \right], \\
    [Y]^T &= \text{diag} \left[ \text{sgn}(y_1), \text{sgn}(y_2) \right] \cdot \left[ |y_1|^a, |y_2|^a \right]^T, \\
    Y &= [y_1, y_2]^T.
\end{align*}
\]
\( X_1 \) and \( X_2 \) are the approximations of \( \omega \) and \( f_{i1}. \quad g_{p1}, g_{p2}, \quad g_{r1}, g_{r2}, a_1, a_2, e_1, e_2 \) are constants to be selected. Augmented by the approximation of uncertainty \( f_{1} \), the control law (17) is restructured as
\[
u_{ESO} = -[B_n^{-1} \text{sgn}(\omega)|\omega|^{2-a} + f_0 + X_2 + ks + \delta_{ESO} \text{sgn}(s)]
\]
(19)
where \( \delta_{ESO} = \text{diag} \left[ \delta_{ESO1}, \delta_{ESO2} \right] \) and \( \delta_{ESO1} > 0 \) (i = 1, 2).
Comparing control law Eq. (17) with Eq. (19), we draw a conclusion that with the aid of estimating and compensating the system uncertainties by ESO, switch gains in the control law are clearly decreased. In other words, we can select smaller gains as \( \delta_{ESO1} < \delta_1 \) and \( \delta_{ESO2} < \delta_2 \). In this case, the control input chattering phenomenon caused by the unmodeled system dynamics will be significantly mitigated.
Theorem 1. With the non-singular terminal sliding surface given by Eq. (16), ESO obtained by Eq. (18), the trajectory of the system Eq. (15) can be driven onto the neighborhood around the sliding surface in finite time with the control law Eq. (19). Finally, the system states \( \theta \) and \( \omega \) converge into a residual set of the origin.

Proof. Lemma 1 has shown that the observation errors of ESO converge into the residual set of zero. The observation errors are defined as \( e_1 = f_{p1} - s_{p2} \) and \( e_2 = f_{r1} - s_{r2} \).

The system Eq. (15) represents the dynamic equations for two channels including rolling and yawing motion. We consider the roll channel first for the sake of clarity. Now consider the Lyapunov function candidate

\[
V_s = \frac{1}{2} s_{1s}^2
\]

(20)

Its derivative along the system dynamics Eq. (15) can be rewritten as follows

\[
\dot{V}_{s1} = s_1 \dot{s}_1
= s_1 \left( \dot{s}_1 + n_1 B_1^{-1} p \right)
= s_1 \left[ p + n_1 B_1^{-1} |p|^n - 1 \right]
= s_1 \left[ p + n_1 B_1^{-1} |e_1 - n_1 B_1 \text{sgn}(p)| |p|^n - 1 - \delta_{\text{ESO}} \text{sgn}(s_1) \right]
= n_1 B_1^{-1} |p|^n - 1 - k_1 s_1 \delta_{\text{ESO}} \text{sgn}(s_1) + e_1 s_1
\leq n_1 B_1^{-1} |p|^n - 1 - k_1 \delta_{\text{ESO}} \text{sgn}(s_1) + e_1 s_1
\]

The derivative of the sliding surface.

For the case \( p \neq 0 \), it can be concluded that: 1) If the design parameter \( \delta_{\text{ESO}} \) meets the condition \( \delta_{\text{ESO}} \geq |e_1| \), we get \( \dot{V}_{s1} \leq 0 \), which means the sliding surface \( s_1 = 0 \) will be reached. 2) For the condition \( \delta_{\text{ESO}} < |e_1| \), we have \( \dot{V}_{s1} < 0 \) if \( |s_1| > (|e_1| - \delta_{\text{ESO}})/k_1 \). Decreasing \( s_1 \) eventually drives the system trajectory into a neighborhood around the sliding surface. Therefore, the trajectory of the system is ultimately bounded in the region

\[
|s_1| \leq \frac{|e_1| - \delta_{\text{ESO}}}{k_1}
\]

(21)

For the case \( p = 0 \), we get \( \dot{V}_{s1} = 0 \). Then substituting the control law Eq. (19) into the system dynamics Eq. (15) yields

\[
\frac{\dot{p}}{k_1} = \frac{-k_1 s_1 - \delta_{\text{ESO}} \text{sgn}(s_1) + e_1}{k_1}
\]

(22)

Using Eq. (22), it can be seen that: 1) For \( s_1 > 0 \) and \( s_1 < 0 \), \( \dot{p} < 0 \) and \( \dot{p} > 0 \) are obtained, respectively, if \( \delta_{\text{ESO}} \geq |e_1| \). 2) For the condition \( \delta_{\text{ESO}} < |e_1| \), we analyze \( \dot{p} \) under the assumption \( |s_1| > (|e_1| - \delta_{\text{ESO}})/k_1 \) in the following. If \( s_1 > 0 \), we have

\[
s_1 > \frac{|e_1| - \delta_{\text{ESO}}}{k_1}
\]

(23)

Substituting Eq. (22) into the inequality Eq. (23) yields

\[
\frac{1}{k_1} (\dot{p} + \delta_{\text{ESO}} - e_1) > \frac{|e_1| - \delta_{\text{ESO}}}{k_1}
\]

If \( s_1 < 0 \), we have

\[
-s_1 > \frac{|e_1| - \delta_{\text{ESO}}}{k_1}
\]

(24)

Substituting Eq. (22) into the inequation Eq. (24) yields

\[
\frac{1}{k_1} (\dot{p} - \delta_{\text{ESO}} - e_1) > \frac{|e_1| - \delta_{\text{ESO}}}{k_1}
\]

Hence it is clear that for \( s_1 > 0 \) and \( s_1 < 0 \), we have \( \dot{p} < 0 \) and \( \dot{p} > 0 \), respectively, if \( |s_1| > (|e_1| - \delta_{\text{ESO}})/k_1 \). Under this condition, the system trajectory will cross the line \( p = 0 \) and reach the sliding mode \( s_1 = 0 \).

In conclusion, the system trajectory is ultimately bounded in the region Eq. (21). For the yaw channel, the proof procedure omitted here is logically the same as that of the roll channel shown above. It is also concluded that the trajectory of the yaw channel control system is bounded ultimately as

\[
|s_2| \leq \frac{|e_2| - \delta_{\text{ESO}}}{k_2}
\]

(25)

According to TSM theory, the states in the sliding mode will reach the origin in finite time. For the system Eq. (15), by the above analysis, the states \( \theta \) and \( \omega \) finally converge into a residual set of the origin. The proof is complete.

Remark 1. Since the ESO cannot completely track the signal in any practical system, asymptotic stability is lost and it can only guarantee the bounded motion about the sliding surface. In Eqs. (21) and (25), it can be seen that the boundary layer about the sliding surface is determined by the estimation error of ESO. Thus the parameter selecting of ESO is very important, because it not only determines the observation performance, but also impacts the behavior of the sliding surface.

Remark 2. The parameter selecting of ESO is associated with the observer error dynamics. For the ESO employed in the roll channel, after subtracting equation set (18) from Eq. (15), we get an expression for the observer error dynamics:

\[
\begin{align*}
\dot{e}_0 &= e_1 + \frac{e_0}{e_1} \dot{e}_1 \\
\dot{e}_1 &= f_{p1} - \frac{e_0}{e_1} \dot{e}_1 \\
\end{align*}
\]

(26)

where \( f_{p1} \) is the derivative of \( f_{p1} \) and \( e_0 = p - s_{p1} \). When the observer is stable, we have \( \dot{e}_0 = 0, e_1 = 0 \). Then the observation errors can be written as

\[
\begin{align*}
|e_0| &= e_1 \left[ \frac{e_1}{e_0} \right]^{2n_1} \\
|e_1| &= g_{p1} \left[ \frac{e_1}{e_0} \right]^{2n_1+1} \\
\end{align*}
\]

(27)
It can be seen that the errors of estimation are determined by \( g_{p1}, g_{p2}, \epsilon_1 \) and \( a_1 \). Via tuning these parameters appropriately, the estimation errors of the observer can be forced small enough such that uncertainty \( f_{p1} \) can be effectively observed by ESO. Especially, an appropriate \( \epsilon_1 \) can be selected small enough such that \( |e_0| \) and \( |e_1| \) are small enough despite \( f_{p1} \) being unknown. The fundamental selection of the parameters can be chosen as \( g_{p1} > 0, g_{p2} > 0, 0 < \epsilon_1 < 1 \) and \( 0.5 < a_1 < 1 \).

4. Simulation Results

In this section, the autopilot synthesis is tested with a 6-DOF simulation of an agile missile. A typical engagement scenario simulated here is the fast 180° turn for attacking a target in the rear hemisphere. The simulation program is performed to test 90° AoA flight performance. For this reason, whole target interception maneuvering is unnecessary to discuss and show here. With RCS control for each channel, the pitch channel follows a 90° AoA command, while the roll and yaw channels perform stabilization control. Here we select the local vertical plane as the missile expected maneuvering plane, so the missile need not perform a roll motion before turning around. For a simulation with large attitude angle variation, it is unsuitable for the rotation motion to be modeled by the Euler angle coordinate system due to the singular problem. Hence, the quaternion algorithm is employed to establish the simulation model, which prevents the degeneration of the dynamic equations in any case.

As previously mentioned, the missile aerodynamic data is obtained by two means. One is Missile Datcom calculation for the estimated values, and the other is manual setting of the disturbed values (i.e., uncertainty \( f_1 \) in system Eq. (15), reflecting the aerodynamic uncertainty of roll and yaw channels). Time histories of \( C_l \) and \( C_n \) set as three different variational forms including constant function, step function and sinusoidal function are shown in Fig. 4. The aerodynamic coefficients change more drastically during 0 to 1 s in order to show the aerodynamic disturbance with strong variation and randomness at rapid changing AoA. Note again that the rolling and yawing moment coefficient values are scarcely possible to get in advance with the current state-of-the-art of aerodynamics. So it is assumed that the aerodynamic moment coefficients vary according to the artificial functions, just as the pitching moment presented by Kim and Kim.\(^{11}\) Correspondingly, the proposed control law Eq. (19) does not use any true value of aerodynamic information.

In the simulation, the initial parameters are selected as the flight altitude 5 km, the initial velocity 0.8 Ma, the main engine thrust 15,000 N and the maximum steady thrust of RCS 2,000 N. The simulation time is set as a fixed value, 3 s. The main geometric and physical characteristics of the configuration are a length of 3 m, diameter of 0.15 m and mass of 105 kg.

The missile responses in pitch angle \( \theta \), angle of attack \( \alpha \), quasi roll angle \( \phi^* \) and quasi yaw angle \( \psi^* \) are given in Fig. 5. It should be noted that the pitch angle curves and AoA curves for three scenarios are overlapped respectively. AoAs rapidly increase to 90° within about 1 s from the beginning of the maneuver and then becomes steady. \( \phi^* \) and \( \psi^* \) fluctuate near 0° and their absolute values are always kept within 0.8°. Certainly, \( \phi^* \) and \( \psi^* \) cannot be kept at 0° invariably because of the real-time change in aerodynamic disturbances. Huge drag and unavailability of the main engine thrust in the direction of velocity lead to a distinct drop in velocity magnitude, as shown in Fig. 6. The velocity curves are also overlapped.

Estimation errors for uncertainty \( f_{a1}, f_{p1}, f_{q1} \) and \( f_{r1} \), representing aerodynamic information, are shown in Fig. 7, which illustrates the ESO’s ability to obtain approximations of uncertainties. All ESOs used in the proposed control method take the same design parameters of \( g_1 = g_2 = 1, \alpha = 0.8 \) and \( \epsilon = 0.01 \). Furthermore, it is observed that estimation errors change vigorously in the early stages of simulation. At the beginning, the estimation error is big while \( \varepsilon \) is small, which leads to large quantities on the right-hand sides.

![Fig. 4. Time histories of rolling and yawing moment coefficients.](image1)

![Fig. 5. Time histories of responses.](image2)

![Fig. 6. Velocity curves.](image3)
of the equalities in ESO dynamics Eq. (6). Consequently, big derivatives of state variables result in dramatic changes. Furthermore, estimation errors for \( f_{q1} \) and \( f_r \) appear suddenly and change the subsequent process due to the aerodynamic discontinuous step disturbance for Scenario 2.

Figure 8 shows the RCS control inputs of the yaw channel with the sinusoidal changing moment coefficients (Scenario 3). If the control law Eq. (17) is selected as the control input, the NTSM approach is used alone to stabilize the missile attitude. At this rate, the control input appears to have a serious chattering phenomenon. If the compound control law Eq. (19) based on NTSM and ESO is applied, the chattering is lessened significantly and the need for thrust is also reduced. The advantage associated with the introduction of the ESO system is the chattering amplitude decay depending on the values of switch gains in the control input. However, eliminating the chattering completely, depending on perfect performance of disturbance rejection, is unpractical and unnecessary. Figure 9 illustrates the time histories of responses in three kinds of scenarios when the contribution of ESO to the control system is artificially reduced. Here \( \hat{f}_{al} \) and \( \hat{f}_{q1} \) are replaced with \( \hat{f}_{al}/2 \) and \( \hat{f}_{q1}/2 \) in pitch control law Eqs. (10) and (13), respectively, while the standard NTSM control law Eq. (17) without ESO is adopted for roll and yaw control. Compared with the simulation results shown in Fig. 5, the pitch angles and AoAs increase more slowly; meanwhile, the quasi roll angles and quasi yaw angles fluctuate far more out of equilibrium.

In order to observe the robustness of ESO to time delay, a simulation of the longitudinal control is performed by replacing \( y(t) = x_1(t) \) with \( y(t) = x_1(t + \tau) \) for \( x_1 = \alpha \) (AoA control loop), \( x_1 = q \) (pitch rate control loop) and \( \tau = 0.03 \) in Scenario 3. The result plotted in Fig. 10 shows that the ESO can tolerate a small output time delay. In addition, simulations of the ESO system used in the pitch autopilot with additive white Gaussian noises are performed for the purpose of evaluating the robustness against high-frequency disturbances. One of the two kinds of disturbances that we are concerned with is measurement noises in the state variable \( q \) for mean value = 0 and variance = 0.001. The signal with noises and the estimated result of uncertainty \( f_{q1} \) are shown in Fig. 11. The other disturbance is the high-frequency aerodynamic disturbance included in pitch rate dynamics Eq. (7) for mean value = 0 and variance = 0.1. The result is plotted in Fig. 12. It is seen that ESO is qualified to be used in the agile missile control system with measurement noises and high-frequency disturbances.

Finally, AoAs and pitch angles of a 180° turn under a given 90° AoA command are plotted in Fig. 13 for three kinds
of scenarios. Then Fig. 14 illustrates missile trajectories accordingly. The missile inevitably flies out of expected maneuvering plane due to the effect of rolling and yawing moments. However, the missile is able to roughly keep flying in the vertical plane when under control. In addition, the horizontal flight path angles given by numerical calculation at the end of the simulation are 179.79\degree, 178.62\degree\textsuperscript{a} and 179.73\degree\textsuperscript{a} for the three scenarios, respectively, closing to the ideal value, 180\degree.

5. Conclusion

A nonlinear missile autopilot based on the ESO method has been synthesized to fulfill a fast 180\degree turn maneuver. High AoA aerodynamic data is so hard to predict precisely that we treat the aerodynamic forces as uncertainties in the control system. To achieve decoupling control in three channels, an ESO technique is used to estimate the total system uncertainty including aerodynamic data online. After the approximations of uncertainties are determined, the pitching dynamics can be linearized, and consequently high AoA control is achieved. According to the proposed quasi BTT steering technique, the attitude stabilization for the roll and yaw channels has been addressed successfully using a compound method based on NTSM control and the ESO technique. The performance of the proposed autopilot was proven by numerical simulation. In addition, the control input chattering caused by unmodeled dynamics can be mitigated effectively through the approach of combining SMC and ESO. Significantly, the proposed method achieves high-precision effective control for low-precision modeling of missile dynamics without using an aerodynamics database. This is an outstanding feature when plant dynamics are difficult or impossible to model accurately for the purpose of control design and implementation.

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