Composite fault diagnosis of gearbox based on empirical mode decomposition and improved variational mode decomposition

Jingyue Wang¹,²,³, Jiangang Li¹, Haotian Wang⁴ and Lixin Guo⁵

Abstract
In order to identify the nonlinear nonstationary pitting-wear fault signal of gears in gearbox, a new method of composite fault diagnosis for gearbox is proposed, which combines empirical mode decomposition with improved variational mode decomposition. Aiming at the problem of false modes in the results of empirical mode decomposition when processing signals, the energy method is used to eliminate and update the mode components, and the correlation coefficient method is used to calculate the correlation between the updated mode components and the original signal. The components with strong correlation are selected to form the combined mode components to weaken the noise and improve the signal-to-noise ratio. Aiming at the problem that variational mode decomposition method needs to manually determine the number of mode components $k$ and the penalty factor $\alpha$ during signal decomposition, a combination of envelope spectral entropy and waveform method is proposed to determine the optimal parameter combination. By analyzing the pitting-wear composite fault vibration signals of gears in the gearbox and the normal signals of the gearbox, the effectiveness of the proposed method is verified, and a comparative analysis with the empirical mode decomposition method is performed to highlight the superiority of the proposed method.

Keywords
Composite fault, gearbox, empirical mode decomposition, improved variational mode decomposition, envelope spectral entropy, nonlinear

Introduction
Gearbox is one of the most important transmission mechanisms. The condition of the gearbox directly affects the normal operation of the mechanical equipment. If the fault characteristics and location of gearbox can be identified and determined quickly and effectively, the huge economic losses and casualties can be avoided. However, when mechanical failure occurs, it is often not single. One kind of fault usually causes other faults, which leads to the occurrence of complex faults. At present, the research on the gearbox composite fault diagnosis method is still a hot and difficult point. How to effectively separate the coupling fault of gearbox and extract the correct fault feature is particularly important in the composite fault diagnosis of gearbox.¹–³
At present, scholars have made some contributions in the complex fault diagnosis of gearbox. Huang et al. proposed an adaptive signal decomposition method—empirical mode decomposition (EMD) in 1998. Then, combining EMD with Hilbert transform proposed by Liu et al., they proposed Hilbert Huang transform (HHT). EMD can decompose the signal into a series of intrinsic mode functions (IMFs) arranged from high frequency to low frequency. By performing Hilbert transform on the IMFs, the fault characteristic frequency in the signal can be identified. Tollis et al. and Mohanty et al. have used the HHT method to diagnose the failure of rolling bearings. However, EMD has endpoint effect and mode aliasing problem, and sampling frequency has a greater impact on it, which makes the diagnosis results prone to misdiagnosis. Aiming at the problems of EMD, a noise-assisted analysis method—ensemble empirical mode decomposition (EEMD) is proposed. In this method, the white noise amplitude is added to the original signal for continuous screening, and the average value is used as the final result, so that the signal is disturbed in the true solution neighborhood, and the adaptive separation of signals at different scales is achieved. Yang et al. combined multi-point optimal minimum entropy deconvolution with EEMD to separate and extract the composite fault features of gears and bearings in a gearbox. However, the white noise level added to the signal has a large impact on the resolution accuracy of the EEMD. If the level of white noise is too high, it will lead to over-decomposition. If the level is too small, the signal separation will not be thorough. At present, how to choose the level of white noise adaptively has not been well solved. Smith proposed a new adaptive signal decomposition method based on the theory of EMD—local mean decomposition (LMD). Compared with EMD method, LMD can effectively remove the endpoint effect and reduce the mode aliasing to a certain extent. Li et al. introduced Hermite interpolation algorithm to eliminate the problem of moving average algorithm in LMD method and Hermite-LMD method is proposed and the fault diagnosis of gears is realized. However, the LMD method is greatly affected by noise and needs to be further improved. In response to the shortcomings of the aforementioned methods, Dragomiretskiy et al. proposed a noniterative signal decomposition method—variational mode decomposition (VMD) in 2014. Compared with EMD, EEMD, LMD and other methods, VMD has more outstanding advantages. It not only has stronger mathematical theory but also has faster convergence speed and good robustness to noise. However, before VMD decomposes signals, it is necessary to choose the appropriate number of components $k$ and penalty factor $\alpha$ artificially. To solve this problem, a method based on the combination of envelope spectral entropy and wavefront method is proposed to determine the best combination of $k$ and $\alpha$.

Based on the above research background, this paper proposes a composite fault diagnosis method for gearbox based on EMD-IVMD. The original signal is decomposed by EMD method, and the false IMF is judged by energy principle. If the false IMF occurs, the energy method is used to eliminate the false modes and update the IMF components. After calculating the correlation coefficients between the updated IMF components and the original signal, the highly correlated IMF components are selected to form the combined intrinsic mode function (CIMF) to reduce noise and improve the signal-to-noise ratio. Then the CIMF is decomposed by IVMD, and the decomposed components are analyzed by envelope spectrum. Finally, the effectiveness of the proposed method is verified by analyzing the vibration signals of pitting-wear compound faults of gears in the gearbox.

**Fundamental theory**

**Empirical mode decomposition**

EMD uses a method of linearization and smoothing to decompose the signal into a limited set of oscillatory functions, called IMF. Each IMF must satisfy two conditions:

1. The number of extreme points and zero points are equal or not more than one.
2. The average value of upper and lower envelopes defined by the local maximum and local minimum must be zero at any time of the signal, that is, the local symmetry of the signal with respect to time.

IMF represents the time scale embedded in the signal and is defined as the time interval between two continuous extremes. IMF is not necessarily a sine function, it can be nonstationary, and its amplitude and frequency can also be modulated. The EMD method decomposes a signal $x(t)$ into a set of IMFs as follows:

1. Finding out all local maximum and local minimum points of signal $x(t)$, and using cubic spline interpolation method to connect local maximum points into upper envelope $u(t)$, local minimum values into lower envelope $l(t)$, and calculate the average $m(t)$ through upper and lower envelopes.
2. The first component \( h_1(t) \) is obtained by subtracting the average \( m(t) \) from the original signal \( x(t) \), and checking whether \( h_1(t) \) satisfies the two conditions of IMF. If not, repeat above steps with \( h_1(t) \) as the original signal until the obtained \( h_{1k}(t) \) meets the two conditions of the IMF. \( h_{1k}(t) \) is regarded as the first IMF of signal \( x(t) \) and is recorded as \( c_1(t) \).

3. The first IMF is subtracted from the original signal and the residual signal \( r_1(t) \) is obtained.

4. The residual signal \( r_1(t) \) is used as a new original signal to repeat the above steps (1) to (3) to obtain several intrinsic mode functions IMFs \( (c_j(t)) \), until the final residual signal \( r_N(t) \) is a constant or monotonic function, and the screening process ends. The signal \( x(t) \) can be expressed as

\[
x(t) = \sum_{j=1}^{N} c_j(t) + r_N(t)
\]  

(1)

In most cases, even very complex signals can be represented by several IMFs. The residual signal \( r_N(t) \) usually does not contain useful information of signals and can be ignored.

**Variational mode decomposition**

VMD can nonrecursively decompose a multi-component signal into \( k \) band-limited intrinsic mode functions (BLIMFs) with FM and AM characteristics. Each mode component \( u_k \) is concentrated at the central frequency \( w_k \). Its bandwidth can be estimated by \( H^1 \) Gaussian smoothness, which is the square of the second norm of the gradient.

The essence of VMD algorithm is a constrained variational problem. The estimation of all mode bandwidth problems can be expressed as follows:

\[
\begin{align*}
\min_{\{u_k\}, \{w_k\}} & \left\{ \sum_k \left\| \partial_t \left[ \frac{\delta(t) + j}{\pi t} \right] * u_k(t) e^{-jw_k t} \right\|_2^2 \right\} \\
\text{s.t.} & \sum_k u_k = f
\end{align*}
\]  

(2)

where, \( \{u_k\} = \{u_1, \ldots, u_K\} \) represents \( k \) BLIMF components obtained by VMD decomposition of the signal; \( \{w_k\} = \{w_1, \ldots, w_K\} \) represents the central frequency of \( k \) BLIMF components; \( f \) represents the input signal. By introducing the quadratic penalty factor \( \alpha \) and Lagrange multiplier \( \lambda(t) \), the constrained variational problem of objective function in formula (2) is transformed into an unconstrained problem. The constructed augmented Lagrange multiplier \( L \) is as follows

\[
L(\{u_k\}, \{w_k\}, \lambda) = \alpha \sum_k \left\| \partial_t \left[ \frac{\delta(t) + j}{\pi t} \right] * u_k(t) e^{-jw_k t} \right\|_2^2 + \left\| f(t) - \sum_k u_k(t) \right\|_2^2 + (\lambda(t), f(t) - \sum_k u_k(t))
\]  

(3)

The problem in formula (3) can be solved by introducing alternating direction multiplier method (ADMM). The estimated mode \( u_k \) and its corresponding central frequency \( w_k \) are updated to

\[
\begin{align*}
\hat{u}_{k+1}(w) &= \frac{\hat{f}(w) - \sum_{i \neq k} \hat{u}_i(w) + \frac{\hat{\lambda}(w)}{2}}{1 + 2\alpha |w - w_k|^2} \\
\hat{w}_{k+1} &= \frac{\int_0^\infty w |\hat{u}_k(w)|^2 dw}{\int_0^\infty |\hat{u}_k(w)|^2 dw}
\end{align*}
\]  

(4)

(5)
where the center frequency $w_k^n$ is the center of gravity of the corresponding BLIMF power spectrum $\hat{u}_k^{n+1}(w)$. Mode $u_k(t)$ in time domain is the real part obtained by inverse Fourier transform of $\hat{u}_k(w)$ Wiener filtered signal.

After each update of the estimated mode $u_k$ and the center frequency $w_k$, the Lagrangian multiplier $\lambda(t)$ will also be updated as follows

$$\lambda(t) = \lambda(t) + \tau \left( \hat{f}(t) - \sum_k \hat{u}_k^{n+1} \right)$$

(6)

Update through repeated iterations until the iteration stopping condition is satisfied

$$\sum_k \left\| \hat{u}_k^{n+1} - \hat{u}_k^n \right\|_2^2 / \left\| \hat{u}_k^n \right\|_2^2 < \varepsilon$$

(7)

According to the basic principle of VMD mentioned above, it can be concluded that the decomposition process of VMD is to divide the frequency band according to the frequency characteristics of the signal, and the mode $u_k$ and the corresponding central frequency $w_k$ are updated continuously in the frequency domain, and finally the decomposition of the signal is achieved.

**Envelope spectral entropy**

As a linear transformation, Hilbert transformation can transform one signal into another in the same domain, revealing the relationship between the real part and the imaginary part of the signal. When the gear fails, the vibration signal of gearbox usually shows the modulation of gear meshing frequency by the rotation frequency of fault gear shaft. The envelope spectrum analysis of the fault vibration signal using Hilbert transform can demodulate the low-frequency fault information from the high-frequency signal, which can effectively avoid confusion with other interference signals and is very suitable for the fault diagnosis of the gearbox.

The concept of information entropy was put forward by CE Shannon in 1948, and then it was expanded by the methods of probability theory and mathematical statistics. Information entropy is a quantitative measurement of the average state of the overall data of a signal. It can indicate the degree of uncertainty of the output data of a signal. Assuming that the random variable $X$ contains $n$ results $\{x_1, x_2, \ldots, x_n\}$, the information entropy represented by $H(X)$ is defined as

$$H(X) = - \sum_{i=1}^{n} P(X_i) \log_2 P(X_i) = - \sum_{i=1}^{n} P_i \log_2 P_i$$

(8)

where $P(X_i)$ is the probability density function of the random variable $X$.

This paper combines the envelope spectrum with information entropy, that is, envelope spectral entropy to determine the parameters $k$ and $\alpha$ needed for VMD decomposition. After the fault signal passes through VMD, $k$ BLIMF components are obtained. The corresponding envelope spectrum is obtained by Hilbert transform of each component, and then the information entropy of each envelope spectrum is calculated. If the envelope spectrum contains a large number of shock signals and the fault cycle is obvious, the signal is more sparse and the envelope spectral entropy value is small. If the envelope spectrum contains more noise and the fault period characteristic is not obvious, the sparsity of the signal is weak and the envelope spectral entropy value is larger. The steps of calculating envelope spectral entropy for BLIMF components are as follows:

1. Hilbert transform is applied to BLIMF components $u_k(t)$

$$H[u_k(t)] = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{u_k(t)}{t-\tau} d\tau$$

(9)

2. Finding envelope signal

$$z(t) = \sqrt{[u_k(t)]^2 + \{H[u_k(t)]\}^2}$$

(10)
3. Finding envelope spectrum $Q$ from envelope signal

$$H(w) = FFT(z(t))$$

$$Q = |H(w)|$$

1. The concept of entropy is applied to envelope spectrum $Q$. Assuming the envelope spectrum contains $n$ sequences, that is, $Q = \{Q_1, Q_2, \ldots, Q_n\}$, the envelope spectrum entropy $R$ is calculated as

$$R = -\sum_{i=1}^{n} q_i \log_2 q_i$$

In the formula, $q_i$ represents the envelope spectrum $Q_i$ of the $i$-th sequence as a percentage of the entire envelope spectrum $Q$.

Experimental verification

In order to verify the effectiveness of the composite fault diagnosis method proposed in this paper, the vibration analysis and fault diagnosis test platform of QPZZ-II rotating machinery is used to simulate the pitting-wear composite fault of the gear in the gearbox, and the vibration signal of the composite fault is obtained by the acceleration sensor installed on the bearing seat. The schematic diagram of the test bench structure and sensor installation position is shown in Figure 2.

It is known that the gearbox is a single-stage transmission, in which the pinion gear is the driving gear with the number of teeth $z_1 = 55$, and the wear fault is simulated by grinding a certain tooth of the pinion; the large gear is the driven gear, with the number of teeth $z_2 = 75$, and the pitting fault is simulated by machining a circular groove on a certain tooth of the big gear. The module of the big and small gears is both 2, and the material is S45C. The drive motor drives the input shaft of the gearbox through a belt. The number of teeth of the pulley is $z_3 = 32$.

During the experiment, the sampling frequency is 5120 Hz, sampling point is 7680, the current in the electric magnetic powder brake is 0.1 A, input shaft speed measured by photoelectric sensor is $n = 849$ r/min. According to the formula in Table 1, we can calculate $f_{r1} = 14.15$ Hz, $f_{r2} = 10.38$ Hz, $f_{m1} = 778.25$ Hz, and $f_{m2} = 452.8$ Hz.

Then the gearbox compound fault vibration signal waveform when the small gear is worn and the large gear is pitted is shown in Figure 3. As can be seen from the time domain diagram of Figure 3, there is an obvious fault shock component in the signal, but the fault characteristic period is not obvious; in the frequency domain diagram, the gearbox meshing frequency $f_{m1}$ and the half-fold frequency of the pulley meshing
Input composite fault signal \( x \)

EMD

Is there a false IMF?

No

Selection of components with larger correlation coefficient with the original signal to compose CIMF

Initialize parameters \( k \) and \( \alpha \) and determine the range of \( k \) according to the waveform method

VMD for CIMF

Computing envelope entropy of each BLIMF

\( \alpha = \alpha + 1000 \)

Yes

\( \alpha \leq 5000 \) ?

No

Selecting \( k \) and \( \alpha \) corresponding to minimum envelope entropy and IVMD for CIMF

Fault feature extraction for BLIMF

End

Figure 1. EMD-IVMD composite fault diagnosis process. EMD: empirical mode decomposition; IVMD: improved variational mode decomposition; IMF: intrinsic mode function; CIMF: combined intrinsic mode function; BLIMF: band-limited intrinsic mode function.

Figure 2. QPZZ-II: Schematic diagram of vibration analysis and fault diagnosis test platform for rotating machinery.

Table 1. Calculation formula of frequency and meshing frequency.

|                      | Input shaft frequency \( f_{i1}(Hz) \) | Output shaft frequency \( f_{i2}(Hz) \) | Gear meshing frequency \( f_{m1}(Hz) \) | Pulley meshing frequency \( f_{m2}(Hz) \) |
|----------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| \( n/60 \)           | \( z_1 \times f_{i1}/z_2 \)             | \( z_2 \times f_{i2} \)                | \( z_3 \times f_{i1} \)                |
frequency $f_{m2}$ are summed. The peak at the integer multiple is more obvious, but there is no obvious fault characteristic frequency modulation on both sides of the meshing frequency. Due to the influence of the actual background noise, the fault types cannot be effectively identified in both the time domain and frequency domain diagrams. Therefore, in order to realize the composite fault diagnosis of gearbox, the diagnosis method based on EMD-IVMD proposed in this paper is used to further analyze the composite fault vibration signal.

First, EMD is applied to the fault signal, and the first 12 IMF components are obtained as shown in Figure 4. The energy principle in Huang is used to determine whether there are false components. The total energy $E_1$ and $E_2$ of the composite fault signal $x(t)$ and the IMF components are calculated as follows

$$E_1 = 137.7363$$  \(14\)

$$E_2 = \sum_{i} IMF_i = 154.4446$$  \(15\)

Then the energy discrimination is $E_1 < E_2$. According to the properties of false modes, the decomposition of EMD does not obey the conservation of energy, and there are false modes in IMF components.

There is a decomposition error in the decomposition of EMD due to the presence of false components in the IMF. Generally, the error component exists in the mode component with a relatively low sampling rate. The error

**Figure 3.** Time-domain and frequency-domain diagrams of compound faults in gearbox: (a) time-domain diagram and (b) frequency-domain diagram.

**Figure 4.** EMD decomposition results of composite fault signals. IMF: intrinsic mode function.
is caused by the lowest sampling rate of the first-order mode component. The component usually exists in the first-order component. The first-order eigenmode function and other higher order eigenmode functions are added one by one to judge the increase or decrease of their energy, so as to judge whether they are false modes, and to eliminate the error component by adding the false mode and the first-order mode component. Through calculation, the results of judgment of each mode and the updated mode components are shown in Table 2.

A series of new IMF components obtained after eliminating false modes are shown in Figure 5. In order to highlight the effectiveness of the EMD-IVMD gearbox composite fault diagnosis method proposed in this paper, EMD and EMD-IVMD are compared and analyzed. Also, select the IMF components with strong correlation with the original composite fault signal for envelope spectrum analysis. The correlation of each new IMF component is shown in Table 3.

The IMF1 and IMF2 components with strong correlation with the original signal are selected for envelope spectrum analysis, and the envelope spectrum is shown in Figure 6. From Figure 6(a), it can be seen that the

| Energy index | Component test | Judge | Mode updating |
|--------------|----------------|-------|---------------|
| $E_{x_1}$    | $98.4318$      | Real mode | IMF1=IMF1+IMF6+IMF7+IMF8+ |
| $E_{x_2}$    | $28.6028$      | Real mode | IMF9+IMF11+IMF12+IMF13 |
| $E_{x_3}$    | $14.9124$      | Real mode | IMF2=IMF2 |
| $E_{x_4}$    | $9.7795$       | Real mode | IMF3=IMF3 |
| $E_{x_5}$    | $1.6660$       | Real mode | IMF4=IMF4 |
| $E_{x_6}$    | $0.5305$       | Spurious mode | IMF5=IMF5 |
| $E_{x_7}$    | $0.2479$       | Spurious mode | IMF6=IMF10 |
| $E_{x_8}$    | $0.1328$       | Spurious mode | IMF7=IMF7 |
| $E_{x_9}$    | $0.0526$       | Spurious mode | IMF8=IMF8 |

IMF: intrinsic mode function.
characteristic frequency $f_{r2}$ of big gear fault is more obvious in envelope spectrum, and its $2f_{r2}$ appears, but the $2f_{r1}$ and $3f_{r1}$ of small gear fault characteristic frequency are also more prominent. From Figure 6(b), it can be seen that although the characteristic frequency $f_{r2}$ of big gear fault and its $9f_{r2}$ and $14f_{r2}$ have been extracted, there are also $3f_{r1}$ of small gear fault frequency. Therefore, the results of the composite fault diagnosis of the gearbox by the EMD method show that the method cannot effectively separate fault characteristic components with similar frequencies in the composite fault signal, which makes the results appear mode aliasing, and even cause misdiagnosis.

The EMD-IVMD gearbox composite fault diagnosis method proposed in this paper is used to analyze the fault signal. After EMD decomposition and energy method to eliminate the false modes, the IMF components obtained are shown in Figure 5. The IMF1 and IMF2 components which are highly correlated with the original signal are selected to form the combined mode function CIMF, as shown in Figure 7. Before VMD decomposition of CIMF, it is necessary to determine the range of the number of BLIMF components $k$ by waveform method, and the range of penalty factor $\alpha$ is 1000–5000 according to literature. The initial values $\alpha = 1000$, $k = 1$, $k = k + 1$ are selected to increase $k$ value continuously. By waveform method, when the center frequency of BLIMFs decomposed by VMD appears aliasing phenomenon, $k$ value stops. The central frequency of BLIMFs corresponding to different $k$ values is shown in Figure 8.

As can be seen from Figure 8 that when $k$ is 2, 3 or 4, the central frequencies of BLIMF components decomposed by VMD are divided into different central frequency bands, respectively, and without mode aliasing. In (a), (b) and (c), 809.5 Hz, 804.5 Hz, and 815.8 Hz are similar to meshing frequencies $f_{m1}$, 438.8 Hz, 1327 Hz (1306 Hz, 1307 Hz) are similar to meshing frequencies $f_{m2}$ and its triple, quadruple frequencies 1836 Hz (1840 Hz). When $k$ is 5, (d) although 813.1 Hz, 432.9 Hz and 1297 Hz are also decomposed in the graph, which are similar to the central frequencies $f_{m1}$, $f_{m2}$ and its triple frequencies. However, in the same central frequency band from 1500

| Table 3. Correlation between the updated IMF component and the original signal. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| IMF1 | IMF2 | IMF3 | IMF4 | IMF5 | IMF6 |
| 0.7740 | 0.4296 | 0.2468 | 0.2028 | 0.0398 | 0.0004 |

IMF: intrinsic mode function.

Figure 6. IMF component envelope spectrum: (a) IMF1 component envelope spectrum and (b) IMF2 component envelope spectrum. IMF: intrinsic mode function.

Figure 7. Combination mode function CIMF.
to 2000 Hz, similar central frequencies of 1693 Hz and 1956 Hz appear, which indicates that the same central frequency is decomposed into BLIMF4 and BLIMF5, respectively, and the phenomenon of mode aliasing appears, so the value of $k$ should be less than 5.

To further determine $k$ value and penalty factor $\alpha$, envelope spectral entropy of each BLIMF component decomposed by VMD is calculated, and the best combination of $k$ value and $\alpha$ is selected when envelope spectral entropy is minimum. Select the initial value $k_1=1$, $k_2=k_1+1$, $\alpha = 1000$, $\alpha = \alpha + 1000$. The trend of envelope spectral entropy of each BLIMF corresponding to different $k$ values and $\alpha$ is shown in Figure 9.

It can be seen from Figure 9 that the envelope spectral entropy of each BLIMF component corresponding to different $k$ values and $\alpha$ changes obviously. When $k = 4$, $\alpha = 1000$, the envelope spectral entropy is the smallest. Therefore, we choose this group of parameters is the best parameter combination for IVMD. After the parameters are determined, the CIMF is decomposed by IVMD, and the BLIMF component and its corresponding spectrum are shown in Figure 10. From the spectrum of each component in Figure 10(b), it can be seen that the

---

**Figure 8.** The central frequency of BLIMF corresponding to different $k$ values: (a) $k = 2$, (b) $k = 3$, (c) $k = 4$, and (d) $k = 5$.

**Figure 9.** Envelope spectral entropy corresponding to different $k$ values and $\alpha$. 
corresponding central frequency of each component is better decomposed into different frequency bands without frequency aliasing.

The envelope spectra of the four BLIMF components decomposed by IVMD are analyzed, and the results are shown in Figure 11. It can be seen from Figure 11(a) and (d) that the characteristic frequency $f_{r2}$ of big gear failure is more obvious, and the peak value of $2f_{r2}$ of double frequency in (a) is more obvious. In (d), not only the $2f_{r2}$ is demodulated, but also the peaks of $3f_{r2}, 4f_{r2}, 5f_{r2}$ and $7f_{r2}$ are more prominent. In Figure 11(b) and (c), the pinion fault characteristic frequency $f_{r1}$ and its $3f_{r1}$ peak value are very prominent, indicating that the composite fault signal can effectively separate and extract different fault characteristic frequencies in the fault signal after being processed by the EMD-IVMD method.

In order to further verify the effectiveness of the EMD-IVMD method proposed in this paper, the vibration signals of the gearbox under normal conditions were analyzed. The experimental process is the same as for the composite fault, that is, the sampling frequency is 5120 Hz, the number of sampling points is 7680, the current in
the electric magnetic powder brake is 0.1 A, and the measured input shaft speed is \( n = 820 \text{ r/min} \). The gear meshing frequency \( f_{m1} = 751.67 \text{Hz} \) and the pulley meshing frequency \( f_{m2} = 437.33 \text{Hz} \) can be calculated from Table 1. The vibration signal waveform when the gearbox is normal is shown in Figure 12. Compared with the composite fault vibration signal in Figure 3, the amplitude of the normal signal is smaller and there is no obvious impact characteristic in the time domain, but the amplitude at the meshing frequencies \( f_{m1} \), \( f_{m2} \) and its half-fold frequencies is more obvious in the frequency domain. And the frequency crossover phenomenon of \( f_{m1} + 2f_{m2} \) appeared, indicating that the noise effect was more serious during this experiment.

EMD decomposition of the normal signal, the IMF component obtained is shown in Figure 13. According to calculations, the energy of the normal signal is \( E_1 = 11.9031 \), and the sum of the energy of each IMF is \( E_2 = 8.4546 \). According to the energy discrimination method, it is known that there is no false mode. The first three layers of IMF components that have strong correlation with the original signal are selected to form CIMF. The results are shown in Figure 14.

IVMD analysis was performed on CIMF. The steps are the same as in the case of composite fault analysis. First, the value range of \( k \) is determined by the wave method. Then, the center frequency of BLIMFs under different \( k \) values is shown in Figure 15. It can be seen that when \( k = 5 \), modal aliasing occurs in 0–500 Hz and 1000–1500 Hz frequency bands so the value of \( k \) should be less than 5. In order to determine the specific \( k \) and \( x \), let \( x = 1000, x = x + 1000, k = 1, k = k + 1 \), find the minimum envelope spectral entropy of the BLIMFs obtained by VMD decomposition of the signal when different \( k \) and \( x \) are combined. The results are shown in Figure 16. It can be seen that when \( k = 2 \) and \( x = 1000 \), the corresponding envelope spectral entropy is the smallest, so this group of parameters is selected as the optimal parameter combination of IVMD.

The IVMD decomposition of CIMF, the resulting BLIMF component, and its corresponding spectrum are shown in Figure 17, and it can be seen from the spectrum that different meshing frequencies are decomposed into
different frequency bands, and the effect is obvious. The envelope spectrum analysis is performed on each BLIMF component, and the results are shown in Figure 18. It can be seen that there are no obvious peak frequencies in the envelope spectra of BLIMF1 and BLIMF2, indicating that there is no frequency modulation in the signal, that is, the signal is normal to the gearbox. The vibration signal further validates the effectiveness of the EMD-IVMD method proposed in this paper.

Figure 14. The CIMF made up of IMF components.

Figure 15. The center frequency of BLIMF corresponding to different $k$ values: (a) $k = 2$, (b) $k = 3$, (c) $k = 4$, and (d) $k = 5$.

Figure 16. Envelope spectral entropy corresponding to different values of $k$ and $\alpha$. 
Conclusion

Aiming at the problem that it is difficult to separate and extract the fault characteristic frequency of the gearbox composite fault diagnosis, a method of compound fault diagnosis of gearbox based on EMD-IVMD is proposed. In this method, EMD is used to weaken the influence of background noise in the fault signal and highlight the fault characteristic frequency. In view of the false modes in EMD decomposition process, energy method is proposed to eliminate them and the signal-to-noise ratio of the signal is improved through CIMF. For the problem that the parameters $k$ and $a$ need to be determined artificially in the signal processing of VMD, the combination of waveform method and envelope spectral entropy is proposed to determine the best parameter combination. Compared with VMD method, IVMD method has a certain degree of adaptability in fault diagnosis, which avoids the problem of mode aliasing in decomposition results caused by artificial selection of parameters. Through EMD-IVMD analysis of gearbox composite fault vibration signal, it is verified that this method can effectively separate and extract the two similar fault characteristic frequencies of gear pitting and wear. At the same time, the normal signal of gearbox is analyzed to further verify the effectiveness of this method. Finally, through comparison with EMD method, the method proposed in this paper is highlighted superiority.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors gratefully acknowledge the support of project funded by the Science and Technology Research Projects of Education Department of Liaoning Province of China (LG201921), State Key Laboratory of Mechanical Transmissions (SKLMT-KFKT-201605) and Natural Science Foundation of Liaoning Province of China (20170540786), and China Postdoctoral Science Foundation (2017M610496).
References

1. Yang F, Shen X and Wang Z. Multi-fault diagnosis of gearbox based on improved multipoint optimal minimum entropy deconvolution. *Entropy* 2018; 20: 611.
2. Singh A and Parey A. Gearbox fault diagnosis under fluctuating load conditions with independent angular re-sampling technique, continuous wavelet transform and multilayer perception neural network. *IET Sci Measure Technol* 2017; 11: 220–225.
3. Zhong J, Wong P and Yang Z. Simultaneous-fault diagnosis of gearboxes using probabilistic committee machine. *Sensors (Basel)* 2016; 16: 185.
4. Liu B, Riemschneidera S and Xu Y. Gearbox fault diagnosis using empirical mode decomposition and Hilbert spectrum. *Mech Syst Signal Process* 2006; 20: 718–734.
5. Huang NE, Shen Z, Long SR, et al. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc R Soc Lond A* 1998; 454: 903–995.
6. Tollis G, Chiariotti P and Martarelli M. Rolling bearing diagnostics by means of EMD-based independent component analysis on vibration and acoustic data. *Shock Vibat* 2017; 9: 293–300.
7. Mohanty S, Gupta KK and Raju KS. Hurst based vibro-acoustic feature extraction of bearing using EMD and VMD. *Measurement* 2018; 117: 200–220.
8. Wand T, Zhang H, Lu D, et al. Dynamic weighing of full-size wood composite panels based on wavelet denoising and empirical mode decomposition algorithm. *Acta Metrol Sinica* 2017; 38: 300–303.
9. Huang NE. Introduction to the Hilbert-Huang transform and its related mathematical problems. *Interdiscip Math Sci* 2005; 5: 1–26.
10. Rilling G and Flandrin P. On the influence of sampling on the empirical mode decomposition. In: *IEEE international conference on acoustics*, Toulouse, France, 14-19 May 2006, III-III. Piscataway, NJ: IEEE.
11. Wu Z and Huang NE. Ensemble empirical mode decomposition: a noise-assisted data analysis method. *Adv Adapt Data Anal* 2009; 1: 1–41.
12. Yang F, Shen X and Wang Z. Multi-fault diagnosis of gearbox based on improved multipoint optimal minimum entropy deconvolution. *Entropy* 2018; 20: 611.
13. Jia R, Ma F, Dang J, et al. Research on multidomain fault diagnosis of large wind turbines under complex environment. *Complexity* 2018; 2018: 1–13.
14. Smith JS. The local mean decomposition and its application to EEG perception data. *J R Soc Interface* 2005; 2: 443–454.
15. Li Y, Xu M, Yang Z, et al. A new rotating machinery fault diagnosis method based on improved local mean decomposition. *Digital Signal Process* 2015; 46: 201–214.
16. Jiang J, Liu W, Hou Y, et al. Bearing fault diagnosis based on integral waveform extension LMD and SVM. *J Vibat Shock* 2016; 35: 104–108.
17. Dragomiretskiy K and Zosso D. Variational mode decomposition. *IEEE Trans Signal Process* 2014; 62: 531–544.
18. Huang D. Elimination of false mode components in empirical mode decomposition. *J Vibat Measure Diagn* 2011; 31: 381–384.
19. Wang J, Wang H, Guo L, et al. Rolling bearing fault detection using autocorrelation based morphological filtering and empirical mode decomposition. *Mechanika* 2018; 24: 817–823.
20. Feng Z, Zhang D and Zuo MJ. Adaptive mode decomposition methods and their applications in signal analysis for machinery fault diagnosis: a review with examples. *IEEE Access* 2017; 5: 24301–24331.
21. Li Z, Chen J, Zi Y, et al. Independence-oriented VMD to identify fault feature for wheel set bearing fault diagnosis of high speed locomotive. *Mech Syst Signal Process* 2017; 85: 512–529.
22. Jiang X, Li S and Cheng C. A novel method for adaptive multiresonance bands detection based on VMD and using MTEO to enhance rolling element bearing fault diagnosis. *Shock Vibat* 2016; 2016: 1–20.
23. Yan X, Jia M and Xiang L. Compound fault diagnosis of rotating machinery based on OVMD and a 1.5-dimension envelope spectrum. *Measure Sci Technol* 2016; 27: 075002.
24. Sun J, Xiao Q, Wen J, et al. Natural gas pipeline small leakage feature extraction and recognition based on LMD envelope spectrum entropy and SVM. *Measurement* 2014; 55: 434–443.