Dynamics of a spheroidal gas bubble near a rigid surface

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Abstract. Non-spherical dynamics of a bubble during its axisymmetric collapse near a planar rigid wall is considered. The liquid surrounding the bubble is ideal incompressible, its flow is potential. The pressure in the bubble is uniform, and varying with respect to adiabatic law. At the beginning of collapse, the bubble is spheroidal, the liquid is motionless. A numerical technique is applied, which is based on a stepwise method in time for calculating the bubble surface movement and on the boundary element method. The initial stage of the bubble collapse till the moment of contact between some parts of the bubble surface (i.e., till losing its one-connectedness) is considered. The effect of the ratio of the semi-axes of the spheroidal bubble and the initial thickness of the liquid layer between the bubble and the wall on the bubble shape, the pressure and velocity fields in liquid are investigated. The region of the problem parameters (the distance from the wall, the bubble non-sphericity) is outlined, in which the bubble collapses with formation of a cumulative liquid jet impacting on the bubble surface part nearest to the wall. The liquid pressures resulting from the impact are estimated.

1. Introduction
Features of dynamics of bubbles in liquid near the surfaces of rigid bodies are of importance for understanding the phenomenon of cavitation damage. This topic is a subject of many theoretical and experimental works investigating the influence of the wall curvature and compliance [1, 2], the bubble surface tension [3], the neighboring bubbles [4], and etc. The issue of effect of the initial thickness of the liquid layer between the bubble and the wall was studied quite completely [5, 6]. A much smaller number of publications consider the influence of the initial deflection of the bubble shape from the spherical one. And those publications mainly assume that the bubble collapse takes place in the unlimited volume of liquid, i.e., under the absence of a wall [7]. Some results of investigating the influence of the initial non-sphericity of a bubble on its dynamics near a wall are presented in [8, 9]. In particular, a range of the ratio of the semi-axes of a spheroidal bubble attached to a wall, in which a cumulative liquid jet directed to the wall arose, was determined in [9].

In the present paper, the dependences of dynamics of a bubble near a wall on the initial non-sphericity of the bubble and on the initial distance between the bubble and the wall are considered. General features of such dependences were revealed in [10] for quite large intervals of the initial non-sphericity of the bubble and its distance from the wall. The present paper considers those features in more detail but in narrower intervals, namely, in the case of small distances between the bubble and the wall and in the range of bubble non-sphericity with the high velocities of the cumulative jet, which are most interesting for applications.
2. Problem statement and numerical technique

Axisymmetric collapse of a gas bubble in liquid (water) near a planar rigid wall is considered. At the beginning of collapse, the bubble is spheroidal, with the ratio of the semi-axes $e = b/a$ where $a$ is the semi-axis orthogonal the axis of symmetry. The initial volume $V_0$ of the bubble is equal to that of a sphere with radius $R_0$, the initial distance between the bubble and the wall is $h$. The liquid is inviscid incompressible, its flow is potential. The influence of the surface tension is small and therefore it is not taken into account. The pressure in the bubble $p_b$ is uniform and varying according to the adiabatic law

$$p_b = p_{b0} \left( \frac{V_0}{V} \right)^{\kappa},$$

where $V$ is the current volume of the bubble, $\kappa$ is the specific heat ratio, $p_{b0}$ is the initial pressure in the bubble, taken to be equal to the pressure of the saturated vapor of the surrounding liquid. The liquid pressure far from the bubble $p_{\infty}$ is equal to 1 bar.

A numerical technique, proposed in [11] and realized in [10, 12], is applied, which is based on a stepwise method in time for calculating the movement of the bubble surface and variation of its velocity potential and on the boundary element method. The numerical stability is provided by a procedure of smoothing of the function defining the position of the bubble surface and the magnitude of its liquid velocity potential by means of a cubic spline [13].

In presenting the results, the dimensionless values $r^* = r/R_0$, $z^* = z/R_0$, $h^* = h/R_0$ are utilized, where $r$, $z$ are respectively the radial and axial coordinates of a system with the origin on the wall and with the $z$-axis orthogonal to the wall and passing through the bubble center.

The dependence of collapse of a spheroidal bubble near a wall on the bubble non-sphericity and its distance from the wall is studied in the ranges of $0.75 \leq e \leq 1$ and $0 \leq h^* \leq 0.3$. In the range of $e$, the bubble is initially either a sphere ($e = 1$) or an ellipsoid of revolution oblate along the axis of symmetry ($e < 1$).

3. Results

Figure 1 shows the evolution of the bubble shape at collapse in the cases of $e = 0.75, 0.95, 1.0$. In all the cases, the bubbles at the initial time moment $t = 0$ are equally distant from the wall ($h^* = 0.1$).

![Figure 1](image)

**Figure 1.** Evolution of the shape of bubbles during their collapse near a wall till the moment of losing their one-connectedness (contours 2) depending on the initial shape of the bubbles (contours 1) under the initial distance between the bubbles and the wall $h^* = 0.1$.

Contours 1 and 2 respectively correspond to the shapes of the bubbles at $t = 0$ and the moment $t_c$ at which some parts of the bubble surface contact with one another, *i.e.*, the bubble loses its one-connectedness. One can see that in the case of $e = 0.75$, this contact is realized by means of formation of a thin neck near the axis of symmetry leading to the consequent division of the bubble in two, with the more distant bubble being smaller in size [8]. In the case of $e=0.95$, a cumulative liquid jet is formed, directed orthogonally to the wall and, at $t = t_c$, hitting the liquid layer between the bubble and
the wall at a point of the axis of symmetry. In the case of the spherical shape (e=1.0), a similar scenario is implemented but with a wider jet.

Figure 2 presents the liquid velocity and pressure fields at \( t = t_c \) in the cases of the bubbles given in Figure 1 (contours 2). It is seen that the maximal velocity in the case of \( e = 0.75 \) is reached near the neck where the liquid mainly flows to the axis of symmetry. The maximal pressure is also attained in the vicinity of the neck. In the cases of larger values of \( e \) (\( e = 0.95, 1.0 \)), a cumulative jet directed to the wall occurs. The thickness of the jet grows with increasing \( e \). In the case of a thinner jet (\( e = 0.95 \)), the maximal velocity of the jet end is about 155 m/s whereas in the case of a thicker jet (\( e = 1.0 \)) it is approximately equal to 135 m/s. Computations show that the jet end velocity maximum decreases with increasing \( e \).

![Figure 2. The liquid velocity and pressure (bar) fields at the moments of losing one-connectedness of the bubbles, corresponding to the results shown in Fig. 1.](image)

Figure 3 shows the contours of the bubbles in their axial section at the moment of losing their one-connectedness for several values of \( e \) and \( h^* \). One can see that the shapes of the presented bubbles at a fixed \( e \) are nearly similar to one another, whereas their volumes decrease with increasing \( h^* \). The latter may be explained by the fact that with increasing the distance between the bubble and the wall, the influence of the wall becomes weaker so that the liquid pressure near the bubble gets more uniform, i.e., the bubble collapse becomes more spherical.

![Figure 3. The shapes of the bubbles for \( e = 0.75, 0.95, 1.0 \) at about the moment of losing their one-connectedness, depending on the initial distance between the bubble and the wall: \( h^* = 0 \) (black lines), \( h^* = 0.1 \) (blue lines), \( h^* = 0.3 \) (red lines).](image)

Figure 4 demonstrates the liquid pressure distribution on the wall (Figure 4a) and along the axis of symmetry (Figure 4b) at \( t = t_c \) for \( e = 0.95, 1.0 \) and three values of \( h^* \). It follows from Figure 4a that in the cases of larger distances between the bubble and the wall (\( h^* = 0.1, 0.3 \)), the liquid pressure on the wall reaches its maximum on the axis of symmetry, and the wall pressure monotonically decreases
with the distance from the symmetry axis. In the case of a bubble attached to a wall \((h^*=0)\), the wall pressure maximum is attained at some distance from the symmetry axis, where the radially divergent flow of the displaced liquid under the bubble is decelerated by the external slowly-moving liquid. It should be noted that the wall pressure maxima in the case of \(e=0.95\) is lower than in the case of \(e=1.0\), although the velocity of the cumulative jet in the first case is higher than in the second one. Within the presented results, the wall pressure maximum increases with the distance between the bubble and the wall, which seems to take place at relatively small values of \(h^*\).

**Figure 4.** Profiles of the pressure along the wall \((a)\) and the axis of symmetry \((b)\) for \(e=0.95\) (dotted lines) and \(e=1.0\) (solid lines) under three values of \(h^*\). Black, blue, and red lines correspond to \(h^*=0\), \(h^*=0.1\), and \(h^*=0.3\), respectively.

The liquid pressure profiles along the axis of symmetry (Figure 4b) are not monotonic. In all the cases, there is a local maximum at the base of the jet, which significantly exceeds the corresponding pressure maximum on the wall. And in the case of initially spherical bubble \((e=1.0)\), the magnitude of the maximum is higher than it is in the case of the oblate bubble \((e=0.95)\). Similar to the wall pressure, this maximum grows with increasing \(h^*\).

**Figure 5.** Influence of \(e\) on the impact pressure (determined from the one-dimensional approximation) resulting from the collision of the cumulative liquid jet with the wall \((h^*=0)\) or the liquid layer between the bubble and the wall \((h^*>0)\) in a small vicinity of the point of contact of the jet tip with the wall or the liquid layer, respectively. Black, blue, and red lines correspond to \(h^*=0\), \(h^*=0.1\), and \(h^*=0.3\), respectively.

The results presented in Figure 4 allow one to estimate the liquid pressure on the wall and on the axis of symmetry just prior to the cumulative jet impact. One-dimensional approximation of liquid-wall or liquid-liquid collision makes it possible to approximately determine the impact pressure \(P^*\) occurring in a small vicinity of the point of contact of the jet tip with the wall \((h^*=0)\) or the liquid layer between the bubble and the wall \((h^*>0)\). The impact pressure is found as \(P^*=\rho CV[14]\) in the case of impact on the wall, and as \(P^*=\rho CV/2\) in the case of impact on the liquid layer. Here \(\rho\) is the liquid...
density, $C$ is the speed of sound in liquid, $V$ is the jet tip velocity relative to the surface of the wall or the liquid layer in the vicinity of the impact point.

Figure 5 shows the dependences of the impact pressure $P^*$ on $e$ for three values of $h^*$. It is seen that the impact pressure levels are here much higher than those in Figure 4. At any $h^*$, the impact pressure monotonically decreases with increasing $e$. It should be noted that the formulas of $P^*$ presented above take into account the liquid compressibility. In the case of incompressible liquid ($C \to \infty$), the impact pressure $P^*$ would be infinitely large.

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Figure 6 presents, in terms of coordinates $e$, $h^*$, two regions in which the bubble loses its one-connectedness because of the formation of the neck near the axis of symmetry (scenario I) and owing to the impact of the cumulative jet on the liquid layer between the bubble and the wall at a point of the axis of symmetry (scenario II). It is obvious that the potential of the scenario II with respect to cavitation damage is greater than that of scenario I since a part of kinetic energy in scenario I is related to liquid flow radially converging to the axis of symmetry.

![Figure 6](image)

**Figure 6.** The regions of the plane $(e, h^*)$ without (I, shaded) and with (II, unshaded) formation of a cumulative liquid jet during collapse of the bubbles.

**References**

[1] Klaseboer E, Khoo B C An oscillating bubble near an elastic material 2004 Journal Applied Physics 96 No 10 5808
[2] Tomita Y, Robinson P B, Tong R P, Blake J R 2002 Journal of Fluid Mechanics 466 259
[3] Zhang Z Y, Zhang H S 2004 Physical Review E 70 056310
[4] Blake J R, Robinson P B, Shima A and Tomita Y J 1993 Journal of Fluid Mechanics 255 707
[5] Philipp A, Lauterborn W J 1998 Journal of Fluid Mechanics 361 75
[6] Tong R P, Schiffers W P, Shaw S J, Blake J R, Emmony D C 1999 Journal of Fluid Mechanics 380 339
[7] Tsiglifis K, Pelekasis N 2005 Physics of fluids 17 102101
[8] Voinov O V, Voinov V V 1976 Soviet Physics Doklady 21 133
[9] Voinov O V 1979 Journal of Applied Mechanics and Technical Physics 20 Is. 3 333
[10] Aganin A V, Ilgamon M A, Kosolapova L A, Malakhov V G 2016 Thermophysics and Aeromechanics 23 Is.2 211
[11] Voinov O V, Voinov V V 1975 Soviet Physics Doklady 20 179
[12] Blake J R, Taib B B, Doherty G J. 1986 Journal of Fluid Mechanics 170 479
[13] Aganin A A, Kosolapova L A and Malakhov V G 2018 Mathematical Models and Computer Simulations 10 No. 1 189
[14] Heymann F J 1969 Journal of Applied Physics 40 5113