Impact of grid reduction on modelling accuracy of line usage rates

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Abstract Grid constraints have a strong influence on the results of energy system optimization models. Since energy system optimization is a problem of high complexity and data e.g. power production potential is not necessarily available for all grid nodes grid reduction is one method to overcome both. It reduces the computational burden and it may fit grid structure to given data. This paper addresses the influence of grid reduction on line usage rates. Line usage rates are indicators to determine necessary investments for grid reinforcement. Three reduction methodologies, preserving original grid parameters, hence allowing to calculate which line of the original grid needs to be upgraded, are tested. Statistical error measures show that those reduction methodologies lead to high errors for usage rate calculation, due to the influence of the intra-zonal network. The high error rates could be reasoned and approaches were identified which might overcome them.

1. Introduction
Due to the “Energiewende” in Germany an increased use of distributed generation systems will lead to challenging power flow scenarios in the existing grid, which was (initially) optimized for a centrally dominated structure. The need for grid transformation has to be evaluated, analysed and optimized to obtain cost-effective solutions. The ENTIGRIS energy system optimizer therefore is able to quantify such solutions including optimal grid expansion and operation in conjunction with the optimal technology mix, regarding energy sources and storage systems [1]. Due to the complexity of such systems, the simplification of high-dimensional grid and power flow models is necessary to ensure time-efficient overall optimization processing. Therefore this paper evaluates the effect of spatial grid reduction on power flows. In the base case, a full transmission grid model is calculated. The base case is taken as reference for the comparison of three different grid reduction methodologies. Grid calculation is based on a DC load flow approach. Goals of grid reduction can be summarized as done by [2]:

- the reduced model should reproduce the physical nature of the power system to be replaced as close as possible
- the equivalent should be flexible enough to handle power system status changes and to be used in a wide range of applications
the equivalent should be compatible with the computational procedures used to solve subsequent problems
- the equivalent should insure that feasible mathematical solutions are obtained
- the reduced network should contain as small a number of nodes as possible

For energy systems it is additionally important to know which lines have to be reinforced due to overloads.

The remainder of this paper is organized as follows. The second chapter starts with a short overview on grid representations in energy system optimizers and grid reduction approaches. Grid reduction approaches are surveyed from the energy system optimizing perspective as well as the electro-technical point of view. Chapter three explains the construction of the use case and chosen grid reduction methodologies. Grid reduction methodologies are the focus of research in this paper. The impacts of grid reduction on grid representation in energy-system optimizers are evaluated by statistical measures. In chapter four results are presented. A critical discussion on the approach and on the limits of the models follows in chapter five.

2. State of the art

This chapter gives an overview on possibilities to represent grids within energy system models and grid reduction methods. Firstly, grid representations in energy system optimizers are discussed. After that, methods on the reduction of electricity grids are shown.

2.1. Grid representation in energy system models

[3] gives a general overview on grid representations in energy system optimizers. First of all, there is the possibility to model the grid as AC power flow model. This is the most accurate possibility to model electrical grids. Therefore requirements regarding grid data and computational complexity are the highest. In order to correctly solve AC power flows, apart from line and transformer data, additional information of flexible AC transmission systems (FACTS) has to be available. To calculate systems states via AC power flows, quadratic problems need to be solved [4]. If the AC power flow problem is integrated into an energy system model, allowing additional discrete grid expansions, this results in a mixed integer non-linear program (MINLP). The conclusion is that for high temporal and spatial resolution data the complexity of the problem is not suitable for real world problems.

Another possibility to model transmission systems with high X/R ratios is the DC load flow model. Due to assuming a constant voltage magnitude and neglecting line resistances, the AC power flow model can be substituted by a linearized model [4]. Combining an energy system optimization model with a DC load flow model transforms the resulting problem again in a MINLP, which is not convex.

The so called transshipment models are the simplest way to take grids in energy system models into account. The only binding relation between real power system operation and the transshipment model is the balance of power at each node of the grid. According to [3], only continuous grid expansion is taken into account in such a model. If the energy system optimizer is linear, the combination of transshipment model and energy system model holds linearity. In general, grid expansion may also be taken as integer variables into account. Then, the overall model is a MILP as shown in [5]. In [5] an entire energy system of a district is modelled using MILP. Grid constraints are taken into account as thermal limitations. This is possible due to the short line lengths, which imply that the first overload in the grid is always thermal. Hence if there is no thermal overload there is no voltage overloads as well. Grid expansion measures are integrated into the model via binary decision variables. Due to the tree-like grid structure the transshipment model finally is a sufficient representation of the real grid. Lastly, the main advantage of this model is the comparatively low computational burden. Additionally, combinations of the upper methods are possible.
The simplest approach to represent a grid is to neglect it. So does the so called copper plate approach used for example in the REMod-D model [6]. This model focuses on the coupling of all sectors. Taking the sector coupling potential into account it aims at answering the question what a 100 % renewable energy supply for German energy infrastructure implies. Within Germany no restrictions for the electrical grid are taken into account. There is only a maximum value for the electricity import defined as a constraint.

2.2. Grid reduction for grid simulations

This subchapter gives an overview on grid reduction methodologies that do not aim at integrating grids into energy system optimizers, but at maintaining grid behavior as good as possible for grid simulations.

2.2.1. Wards method

One of the first important publications on grid reduction or grid equivalencing are [7] and [8]. Grid analyzers at that time had a constraint on the maximum number of nodes and since power systems were growing faster than grid analyzers’ capacity, a method to reduce the complexity of the grid had to be found. From a grid operator’s point of view he only needs to know what happens in the zone that he is responsible for. Therefore they developed a method based on AC load flows to reduce the behavior of the external zones on a lower amount of characteristic nodes and lines. For that the interaction with the grid operator’s zone does not change significantly.

2.2.2. REI-DIMO

Another method to reduce complexity of grid structures is REI-DIMO’s method. It is an improvement of REI’s method to carry out steady state stability analysis. Similar to Wards method, it is based on AC load flow. In order to reduce the amount of necessary nodes for grid calculation, only manually defined essential nodes are kept within the original grid model. Non-essential nodes are combined to a set of virtual nodes whose behavior regarding power injection is linearized depending on voltage. Updating of grid reductions is necessary depending on system state changes in order to have a high accuracy of system reaction [2].

2.2.3. Zonal Power Transfer Distribution Factors

Power Transfer Distribution Factors (PTDFs) represent line usage rate changes for varying node power injections. Zonal-PTDFs are aggregated PTDFs which represent the distribution of power flow when a certain amount of power is transferred from one zone into another. By doing this, parallel flows are taken into account. The accuracy of this method strongly depends on the distance to the working point, therefore PTDFs need to be updated regularly [9]. According to [10] there is even a high influence between day and night operating conditions for certain elements of the PTDF matrix.

2.2.4. Equivalent impedances method

In [11] the equivalent impedances method is developed. This method minimizes the squared differences between the flows in the original grid and the reduced grid. The reduced grid is represented as well as DC load flow model. In order to minimize squared flow differences, the admittance matrix of the resulting grid is modified. In order to guarantee a broad operating range, optimization is done for a large set of working points. The resulting grid does not take the possibility of more than one connection between reduced grid nodes into account.

2.3. The role of clustering for grid reduction methods

In many cases the grid will be reduced according to clusters. Although the choice of clusters may have great influence on the result after grid reduction, clusters are often predetermined by regions or
countries like in [10], [11] and [12]. Adjustments of line parameters or alternative forms of load flow expressions like Zonal-PTDFs are necessary to obtain load flow behaviour. [13] and [14] combine the clustering with the grid reduction procedure to obtain clusters which are suitable for their reduction methods. In our work we will use predefined regions as clusters which are based on areas of transmission system operators. We don’t adapt line parameters arbitrarily to reproduce the original load flow like in the equivalent impedances method. Instead we use methodologies which intend to keep important properties of removed lines and integrate them into those lines which are kept after the reduction.

3. Methodology
As seen in the previous section there is a wide range of possibilities to reduce grids. Each of them has their own drawbacks. This is the reason on why we decided to test new possibilities in this paper.
An overview on the methodology of this paper is given in Figure 1. The ENTIGRIS energy system optimization model creates nodal powers with its highest possible spatial resolution. Those nodal powers are transferred to the DC load flow solver. The DC load flow algorithm calculates the power flows within the grid based on this detailed input. Then grid reduction methodologies, introduced in subchapter 3.5., are applied to reduce the grid to predefined regions. Each region is seen as a virtual node uniting all nodes within that region. All virtual nodes are connected to each other following the specific grid reduction methodology.

3.1. Calculation of nodal powers
The ENTIGRIS\(^1\) energy system optimizer calculates power exchanges for each region. Since runtime is long only 120 load cases are calculated. Those load cases include all combinations of high and low PV and wind productions as well as high and low demand and local effects.
The highest possible regional resolution for ENTIGRIS is NUTS3 level due to lack of data for further underlying regions. In a first step the grid needs to be slightly adapted for the ENTIGRIS model to have exactly one grid node in each model region. The following cases make this necessary: NUTS3 regions having more than one node or none at all. Since every region is connected to the transmission grid via underlying grid levels, we combine regions having no nodes to the region with lowest centroid distances. NUTS3 regions with more than one node are defined as one node. This necessitates a post-processing of nodal powers within the DC load flow model. Power of regions having more than one grid node is divided equally to inner nodes. Due to the lack of more precise datasets, a more accurate distribution of power is not possible.

\(^1\)https://www.ise.fraunhofer.de/en/business-areas/energy-system-technology/energy-system-analysis/energy-system-models-at-fraunhofer-ise.html
3.2. Grid data

The ELMOD-DE model is taken as the grid dataset in this paper [12]. According to [12] there is one type of a line trace that may possibly carry two lines. Hence, if there is only one line it may be expanded to two lines. As there are two types of traces there are two different voltage levels 220 kV and 380 kV. 220 kV traces may be upgraded to 380 kV.

3.3. Line parameters, expansion and upgrade costs.

The German grid consists of 436 nodes and 697 lines. For the sake of simplicity we constrain the area to a region including and surrounding Baden-Württemberg. Resulting number of nodes and lines is 69 and 127 respectively. Since there are no security calculations performed, maximum line capacity is reduced to 70% of its original capacity.

3.4. Load flow calculation and grid reinforcement

Load flows are calculated via a DC load flow model that is integrated into an optimization problem as done in [12]. Due to the formulation as an optimization problem, necessary calculations of line reinforcements due to overloads are integrated within the solution of the load flow. The optimizer finds an angle $\varphi_i$ for each node $i$ in the grid which ensures balance of power $P_i$ at each node. The angles depend on the lines’ susceptance matrix $B_{lines}$ that is constructed from lines between nodes.

$$P_i = \sum_{k=1}^{N} B_{ik} (\varphi_i - \varphi_k)$$

Power flows over a line linearly depend on the difference of the angle from one side to the other. Due to the linear dependence it can be easily integrated into a linear optimization model. Overloaded lines get reinforced by the minimum costs so that capacity constrains are not violated provided that angle differences $\varphi$ remain the same. In order to hold linearity, changes in line properties due to line upgrades may not be integrated in the optimization problem. Nevertheless the load flow will be recalculated after reinforcement measures, so that the resulting optimization is a sequential linear program.

3.5. Grid reduction methodologies

Overall goal of this paper is to find a grid reduction methodology with the least loss of accuracy. The most important goal is to calculate line reinforcement costs as exact as possible. Grid reduction methods described in section 2 are not suitable since they either necessitate AC power flows as basis for grid reduction, line reinforcements may not be referred to existing lines or have other drawbacks.

3.5.1. Cross-border

The first method reduces every region to one node. Only lines connecting areas are taken into account.

![Figure 2. Principle of cross-border methodology. Line lengths of inter-regional lines stay the same.](image)

The advantage of this method is the simplicity of implementation. The disadvantage of this method is the high loss of information from conducting the reduction step. All intra-regional expansions will be lost and expansion costs only take the short lengths of connecting lines into account. This will lead to an underestimation of expansion lengths and costs.
3.5.2. Equivalent lengths

The equivalent lengths method overcomes the problem of the cross-border method by distributing the intra-regional line lengths linearly to the cross-border lines according to their lengths.

![Figure 3](image)

**Figure 3.** Principle of equivalents lengths methodology. Line lengths of inter-regional lines will be increased artificially.

Consequently if a line needs to be reinforced, costs for expansion rise since longer line lengths are taken into account. In order to reflect the intra-regional behaviour, normalized reactances of intra-regional lines are added to reactances of cross-border lines according to their reactance.

3.5.3. Shortest paths

The shortest paths methodology chooses the grid node from the original grid which has the shortest distance to the geographical centre of the combined region.

![Figure 4](image)

**Figure 4.** Principle of shortest paths methodology. Line lengths of inter-regional lines will be increased artificially.

Starting from this centre node, the shortest paths to the centre nodes of neighbouring regions are calculated. As distance measure for the determination of shortest paths lines normalized reactance is chosen. Expansion costs for the resulting line are determined as if it would consist out of the original lines. This enables to determine upgrade requirements and costs according to the original line pieces and lengths.

4. Results

In Figure 5 a statistical evaluation of the results of discussed reduction technologies is done. Four errors are taken to analyze differences in line usage. The comparison is done between inter-regional lines of the reference scenario and the lines of the reduced grids. Data for the boxplot is always from a comparison of the lines over all 120 load cases. The first error is the maximum absolute error (MAE). The MAE describes the maximum difference between reference case and reduced grid for a time step. Hence it is a measure for the maximum deviation within a single time step. This basically shows the maximum error which is to be expected for a special load case. Cross-border and equivalent lengths methodologies perform comparable with a high maximum error of about 90%. Shortest paths methodology is even worse with an MAE of 250%.

The maximal usage error takes the information into account that is necessary for grid reinforcement. It subtracts for each line the maximum usage rate values of the reference case from the reduced case. Since in the end grid reinforcement is done for the maximal usage, this method shows the most important difference for grid reinforcement decisions, but it gives no information on quality of grid representation. It is interesting to note that some line usage rates are strongly underestimated for all cases. More importantly it shows that bad results seen from MAE are levelled out by this method for cross-border and equivalent lengths methodologies. Shortest paths remain the worst case here as well.
The maximum error ME shows the maximal difference that happens within one load case for a time series. It extends the information given by the MAE on the sign of the maximum error. From the MAE it is not possible to derive if the error is positive or negative. High negative errors basically show which lines are not detected to be overloaded whereas positive errors give a hint if lines are unnecessarily reinforced. In the end it shows that the error within time single load cases is worse for reinforcement evaluation than the maximal usage error but it is better than the MAE for all methodologies but the shortest paths. The reason for the high positive error in the shortest paths method is the loss of many inter-regional lines. The remaining lines basically have to carry the same load as before.

In the end the root-means-square-error (RMSE) shows the overall quality of the usage calculation of reduced cases compared to the reference case. This helps to evaluate, whether the other statistical measures state incidentally good results for reduction methodologies or if the grid is represented well by the reduction in all load cases. The median of all three reinforcement methodologies is at 20 % usage error, the outliers go up to 70 % error and for shortest paths methodology up to 150 %.

5. Discussion and Outlook
The statistical evaluation of the comparison between the reduced grid and the reference grid shows that usage rates calculated on the reduced grid deviate strongly from the reference grid. If the maxima of usage rates are compared, the error is reduced. Since maximal usage rates determine the grid reinforcements, this is an important error measure. A problem of all tested methods is that line characteristics of inter-regional lines are not changed. This leads to an unequal distribution of power flows over lines and to high errors of usage rates. This effect was weakened in the original grid by the intra-regional network. The drawback of unequal power flow distribution is unlikely bigger than the advantage of preserving line characteristics and related reinforcement costs and measures. Hence in future work reactances of parallel inter-regional lines will be equalized. Another reason for the high error rates is that regions to which the original grid is reduced are predefined. Therefore in future work an optimization algorithm will be searched and a sensitivity analysis regarding the definition and number of regions will be carried out.

The replacement of one region with one sole node has been identified as further potential problem, since in some regions nodes are not connected via lines to each other within the region. Combining those nodes to one resulting node can strongly change electrical behaviour of load flows in this grid area. Replacing each unconnected node component inside a region by one node which are not connected to the others in this region can result in a valuable improvement which will be tested in future work.
Although the shortest paths methodology represents the inner grid somehow, representation seems to be that bad, that it leads to a strong over estimation of usage rates. Wards method and the REI-Dimo approach that were described above are pretty good in solving this problem but they necessitate an AC power flow. It needs to be investigated if there is a possibility to integrate them with DC power flows. According to literature Zonal-PTDFs are a method to solve this problem; hence they will be tested against future developed grid reduction methodologies.

The research question of this paper was answered by developing three grid reduction methodologies. For those methodologies error measures were defined to evaluate the effect of grid reduction on energy system optimizations. Their drawbacks can be overcome in future work by modifying the reduction algorithms as described earlier in this section.

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