Probing the $L_\mu - L_\tau$ gauge boson at electron colliders

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We investigate the minimal $U(1)_{L_\mu - L_\tau}$ model with extra heavy vector-like leptons or charged scalars. By studying the kinetic mixing between $U(1)_{L_\mu - L_\tau}$ gauge boson $Z'$ and standard model photon, which is absent at tree level and will arise at one loop level due to $\mu$, $\tau$ and new heavy charged leptons or scalars, the interesting behavior is shown. It can provide possibility for visible signatures of new heavy particles. We propose to search for $Z'$ at electron collider experiments, such as Belle II, BESIII and future Super Tau Charm Factory (STCF), using the monophoton final state. The parameter space of $Z'$ is probed, and scanned by its gauge coupling constant $g_{Z'}$ and mass $m_{Z'}$. We find that electron colliders have sensitivity to the previously unexplored parameter space for $Z'$ with MeV-GeV mass. Future STCF experiments with $\sqrt{s} = 2 - 7$ GeV can exclude the anomalous muon magnetic moment favored area when $m_{Z'} < 5$ GeV with the luminosity of 30 ab$^{-1}$. For $m_{Z'} < 2m_\mu$, $g_{Z'}$ can be down to $4.2 \times 10^{-5}$ at 2 GeV STCF.

INTRODUCTION

The standard model (SM) of particle physics is a successful and highlypredictive theory of fundamental particles and interactions, but fails to explain many phenomena, including neutrino mass, baryon asymmetry of the universe, presence of dark matter (DM) and dark energy, among others. It implies that SM is only a low-energy approximation of the more fundamental theory; extensions of SM are strongly required.

Among various extended scenarios beyond SM, new $U(1)$ gauge symmetries are of particular interest since this is one of the minimal extensions of the SM. In particular, the $U(1)_{L_\mu - L_\tau}$ model [1–3], with a $U(1)_{L_\mu - L_\tau}$ extension of SM, gauges the difference of the leptonic muon and tau number and induces a new vector boson $Z'$. This model has gained a lot of attention, since it can be free from gauge anomaly without any extension of particle content. Moreover, it is potentially able to address important open issues in particle physics, such as the discrepancy in muon anomalous magnetic moment $(g - 2)_\mu$ [4–7], $B$ decay anomalies [8–13] and recent anomalous excess in $K_L \to \pi^0 + \text{INV}$ [14]. Besides, the $U(1)_{L_\mu - L_\tau}$ model has also been discussed in lepton-flavor-violating decay of the Higgs boson [11, 15], the neutrino masses and mixing [6, 16–18], and dark matter [10, 18–26].

Since $Z'$ can directly couple to muon, related searches for $Z'$ have been performed with the production of $\mu^+\mu^-$ $Z'$ at collider experiments, including BaBar [27] and Belle II [28] at electron colliders and CMS [29] at hadron collider. Subsequently, $Z'$ decaying to muon-pair is considered at BaBar and CMS experiments, and invisible decay of $Z'$ is considered at Belle II. Phenomenally, Ref. [30] investigated the sensitivity on $Z'$ at Belle II with the planned target luminosity of 50 ab$^{-1}$ in the channel of $e^+e^- \to \mu^+\mu^- Z'$, $Z' \to \text{INV}$; Refs. [31–34] proposed the search for $Z'$ at Belle II using the monophoton process $e^+e^- \to \gamma Z'$, $Z' \to \text{invisible}$, which depends on the kinetic mixing between the SM photon and $Z'$.

In this work, we investigate the $\gamma - Z'$ kinetic mixing in the minimal $U(1)_{L_\mu - L_\tau}$, with extra heavy vector-like leptons or charged scalars. Then we propose to search for $L_\mu - L_\tau$ gauge boson $Z'$ at electron collider experiments, such as Belle II, BESIII and future Super Tau Charm Factory (STCF), using the monophoton final state. Belle II is an asymmetric detector and located at SuperKEKB which collides 7 GeV electrons with 4 GeV positrons. SuperKEKB has a largest instantaneous luminosity of $8 \times 10^{35}$ cm$^{-2}$ s$^{-1}$ [35]. The ambitious goal of SuperKEKB is to accumulate an integrated luminosity of 50 ab$^{-1}$ with 8-year data takings [35]. The BESIII detector is symmetric and operated on the BEPCII with the beam energy ranging from 1.0 GeV to 2.3 GeV and a peak luminosity of $10^{33}$ cm$^{-2}$ s$^{-1}$ [36]. STCF is a proposed symmetric detector experiment which collides electron with positron in the range of center-of-mass energies from 2.0 to 7.0 GeV with the peak luminosity $O(10^{35})$ cm$^{-2}$ s$^{-1}$ at 4 GeV [37–39].

The rest paper is organized as follows: First, we introduce the $U(1)_{L_\mu - L_\tau}$ models and discuss their phenomenological features. Then, we calculate the cross sections of the signal and the backgrounds and analysis to improve the significance by appropriate event cuts at three different electron colliders operated at the GeV scale: BelleII, BESIII and STCF. The sensitivities for $Z'$ at these experiments are also investigated. Finally, a short summary and discussions are given.

THE $U(1)_{L_\mu - L_\tau}$ MODELS

The minimal $U(1)_{L_\mu - L_\tau}$ model

We extend the SM with a new $U(1)$ gauge symmetry, $U(1)_{L_\mu - L_\tau}$, where leptons of the second and third generation couple to the additional $U(1)_{L_\mu - L_\tau}$ gauge boson $Z'$, where leptons of the second and third generation couple to the additional $U(1)_{L_\mu - L_\tau}$ gauge boson $Z'$. A Higgs field $\phi$ is introduced to break the $U(1)_{L_\mu - L_\tau}$ symmetry and the scalar field $\phi$ is coupled to $Z'$. The Lagrangian for the model can be written as:

$$
L = \bar{L}_i \gamma^\mu D_\mu L_i + \frac{1}{2} \sum_{i\neq j} m_{\nu_i}^2 \nu_i^2 + \sum_{i=1}^3 m_{\mu_i} \nu_i^2 \tau_i^2 + \frac{1}{2} m_{Z'}^2 Z'^2 - \frac{1}{2} \frac{g_{Z'}}{\sqrt{2}} \phi Z' \phi^* - \frac{1}{4} f_{\phi \phi Z' Z'} \phi^2 Z'^2 - \frac{1}{2} f_{\phi \phi \phi \phi} \phi^4,
$$

where $L_i$ is the left-handed lepton doublet, $\nu_i$ is the right-handed neutrino, $\tau_i$ is the right-handed tau lepton, $m_{\nu_i}$ and $m_{\mu_i}$ are the neutrino and tau mass, respectively, $m_{Z'}$ is the mass of $Z'$, $f_{\phi \phi Z' Z'}$ and $f_{\phi \phi \phi \phi}$ are the quartic couplings of the Higgs field $\phi$, and $g_{Z'}$ is the coupling constant of $Z'$.

The kinetic mixing term $-\frac{1}{2} \frac{g_{Z'}}{\sqrt{2}} \phi Z' \phi^*$ mixes the SM photon with $Z'$, which is absent at tree level and will arise at one loop level due to $\mu$, $\tau$ and new heavy charged leptons or scalars. The parameter space of $Z'$ is probed, and scanned by its gauge coupling constant $g_{Z'}$ and mass $m_{Z'}$. We find that electron colliders have sensitivity to the previously unexplored parameter space for $Z'$ with MeV-GeV mass. Future STCF experiments with $\sqrt{s} = 2 - 7$ GeV can exclude the anomalous muon magnetic moment favored area when $m_{Z'} < 5$ GeV with the luminosity of 30 ab$^{-1}$. For $m_{Z'} < 2m_\mu$, $g_{Z'}$ can be down to $4.2 \times 10^{-5}$ at 2 GeV STCF.
with equal and opposite charge. The new leptonic gauge interactions can be given as

$$\mathcal{L}_{\text{int}} = g_{Z'} (\bar{\nu}_\mu \gamma^\mu \mu - \bar{\nu}_\tau \gamma^\mu \tau + \bar{\nu}_\mu \gamma^\mu \nu_\mu - \bar{\nu}_\tau \gamma^\mu \nu_\tau) Z'_\mu,$$

where $g_{Z'}$ is gauge coupling constant.

In the minimal $U(1)_{L_\mu - L_\tau}$ model, the kinetic mixing between the $Z'$ and photon is absent at the tree level. Nevertheless, because $\mu$ and $\tau$ are both charged under the electromagnetic $U(1)$ and $U(1)_{L_\mu - L_\tau}$, there exists an unavoidable kinetic mixing at one loop level, which can appear as [32]

$$\varepsilon^{\text{min}}(q^2) = \Pi(q^2) = \gamma \rightarrow q \rightarrow Z' \rightarrow q,$$

$$= \gamma \rightarrow q \rightarrow \mu/\tau \rightarrow q \rightarrow Z' \rightarrow q,$$

$$= \frac{8 e g_{Z'}}{(4\pi)^2} \int_0^1 x (1-x) \ln \frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2} \, dx. \quad (2)$$

Here $e$ is the electromagnetic charge, $m_\tau$ and $m_\mu$ are the masses of tau and muon leptons, $q$ is the transferred momentum.

For large momentum transfer $q^2 \gg m_\tau^2$, this mixing is power suppressed by $1/q^4$, whereas for low momentum transfer $q^2 \sim 0 \ll m_\mu^2$, the mixing tends to be a constant

$$\varepsilon^{\text{min}}(0) = \Pi(0) = \frac{e g_{Z'}}{6\pi^2} \ln \frac{m_\tau}{m_\mu}, \quad (3)$$

which seems like the dark photon model.

The $U(1)_{L_\mu - L_\tau}$ model with extra heavy charged scalars

We add two extra singlet vectorlike leptons ($L_1, L_2$) in the $U(1)_{L_\mu - L_\tau}$ extension of the SM, which are charged under $U(1)_{L_\mu - L_\tau}$, opposite in sign similar as the $\mu$ and $\tau$, and have electric charge of $e$ [33]. Since we mainly focus on the gauge kinetic mixing, we would not provide much details of the model here. In this model, due to the leptons inside the loop, the kinetic mixing of $\gamma$ and $Z'$ can be derived as

$$\varepsilon^{\text{HVL}}(q^2) = \Pi(q^2) = \gamma \rightarrow q \rightarrow Z' \rightarrow q,$$

$$= \gamma \rightarrow q \rightarrow \mu/\tau \rightarrow q \rightarrow Z' \rightarrow q,$$

$$= \frac{8 e g_{Z'}}{(4\pi)^2} \int_0^1 x (1-x) \ln \frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2} \, dx. \quad (4)$$

Here $m_{L_1}$, $m_{L_2}$ are the masses of $L_1$ and $L_2$. When the momentum transfer $q^2 \ll m_{L_1}/m_{L_2}$, which is considered in this work, the mixing can be simplified as

$$\varepsilon^{\text{HVL}}(q^2, r) = \varepsilon^{\text{min}}(q^2) + \frac{e g_{Z'}}{6\pi^2} \ln r, \quad (5)$$

where $r = m_{L_2}/m_{L_1}$ is the mass ratio of $L_1$ and $L_2$.

The $U(1)_{L_\mu - L_\tau}$ model with extra heavy vector-like leptons

We add two extra singlet vectorlike leptons ($L_1, L_2$) in the $U(1)_{L_\mu - L_\tau}$ extension of the SM, which are charged under $U(1)_{L_\mu - L_\tau}$, opposite in sign similar as the $\mu$ and $\tau$, and have electric charge of $e$ [33]. Since we mainly focus on the gauge kinetic mixing, we would not provide much details of the model here. In this model, due to the leptons inside the loop, the kinetic mixing of $\gamma$ and $Z'$ can be derived as
Those of the minimal U(Ube significant, and the results are distinctly different from that, the additional lepton or scalar contributions could contributions to the kinetic mixing.

In Fig. 1, we present the square of the kinetic mixing \( |\varepsilon|^2 \) as a function of the momentum transfer \( |q| \) with \( r = 0.1, 1 \) and 10. The horizontal dotted lines are the same situations but for the case of \( \varepsilon = 0 \), which are shown as a comparison. When \( r = 1 \), the contribution for the kinetic mixing due to additional leptons or scalars vanishes, and the results will become same as those in the minimal \( U(1)_{L^{-}L^{+}} \) model, i.e., \( \varepsilon_{\text{HVL}}(q^2, 1) = \varepsilon_{\text{HCS}}(q^2, 1) = \varepsilon_{\text{min}}(q^2) \). In the minimal \( U(1)_{L^{-}L^{+}} \) model, \( |\varepsilon/g_Z|^2 \) has two peaks at the position of \( |q| = m_\mu \) and \( |q| = m_\tau \), and drops quickly with the increment of \( |q| \) when \( |q| > m_\tau \). This feature distinguishes the phenomenology of the \( U(1)_{L^{-}L^{+}} \) model from the dark photon models with a constant value of the kinetic mixing.

We also present the dependence of the kinetic mixing ratio \( R = |\varepsilon_{\text{HVL}}/\varepsilon_{\text{HCS}}|^2/|\varepsilon_{\text{min}}|^2 \) between the \( U(1)_{L^{-}L^{+}} \) model with two singlet vectorlike leptons or with two charged scalars and the minimal \( U(1)_{L^{-}L^{+}} \) model on the mass ratio \( r \) in Fig. 2. There we consider five typical momentum transfers \( |q| = 0.1 \text{ GeV}, 1 \text{ GeV}, 10 \text{ GeV}, 2m_\mu, 2m_\tau \). It can be seen that, the additional lepton or scalar contributions could be significant, and the results are distinctly different from those of the minimal \( U(1)_{L^{-}L^{+}} \) model. Though the additional leptons and scalars cannot be detected directly due to their heavy mass, they can provide significant contributions to the kinetic mixing.

### Decay modes of Z'

Since the \( Z' \) direct couples with the leptons of second and third generation, it can decay into a pair of neutrinos, and also may decay into muon and tau leptons if kinematic allowed. In addition, since \( Z' \) provides possible scenarios of dark matter, there can be the channel \( Z' \rightarrow \chi \bar{\chi} \). The decay widths of \( Z' \) are given by,

\[
\Gamma(Z' \rightarrow \nu\bar{\nu}) = \frac{g_D^2}{24\pi} m_{Z'},
\]

\[
\Gamma(Z' \rightarrow e^+e^-) = \frac{g_D^2}{12\pi} m_{Z'} \left[ 1 + \frac{2m_e^2}{m_{Z'}^2} \right] \sqrt{1 - \frac{4m_e^2}{m_{Z'}^2}},
\]

\[
\Gamma(Z' \rightarrow \tau^+\tau^-) = \frac{g_D^2}{12\pi} m_{Z'} \left[ 1 + \frac{2m_\tau^2}{m_{Z'}^2} \right] \sqrt{1 - \frac{4m_\tau^2}{m_{Z'}^2}},
\]

where \( \ell = \{\mu, \tau\} \), \( g_D \) is the coupling constant of the \( Z' \) with dark matter, and \( g_D \gg g_Z \) is assumed. We ignore the channel \( Z' \rightarrow e^+e^- \) since it is suppressed by the kinetic mixing. Since neutrinos and dark matter are invisible at particle detectors, we take the \( Z' \) invisible decay as \( \Gamma(Z' \rightarrow \text{INV}) = \Gamma(Z' \rightarrow \nu\bar{\nu}) + \Gamma(Z' \rightarrow \chi \bar{\chi}) \), whose decay ratio can be expressed as

\[
\text{Br}(Z' \rightarrow \text{INV}) = \frac{1}{\sum_{f=\nu,\mu,\tau} \Gamma(Z' \rightarrow ff)} \left[ \Gamma(Z' \rightarrow \nu\bar{\nu}) \right] \left[ \sum_{f=\nu,\mu,\tau} \frac{\Gamma(Z' \rightarrow ff)}{\text{Br}(Z' \rightarrow \text{INV})} \right]
\]

### EXISTING CONSTRAINTS

In this section, we summarize the existing constraints relevant to the parameter regions we are interested for the minimal \( U(1)_{L^{-}L^{+}} \) model from various experiments as follows:

- **Muon anomalous magnetic moment.** The significant discrepancy between the experimental measurement and the SM prediction in the magnetic moment of the muon remains one of the largest anomalies in particle physics [40]:

\[
\Delta a_\mu^Z = a_\mu^\text{exp} - a_\mu^\text{SM} = (261 \pm 61_{\text{exp}} \pm 48_{\text{the}}) \times 10^{-11},
\]

where the errors are from experiment and theory prediction, respectively. We require the contribution in Eq. (12) to be within 2\( \sigma \) that leads to

\[103 \lesssim \Delta a_\mu^Z \times 10^{-11} \lesssim 420.\]

The minimal \( U(1)_{L^{-}L^{+}} \) model, was first introduced to address the discrepancy, which can provide a new interaction with muons. An extra contribution to \( a_\mu \) arises solely from a one-loop diagram involving \( Z' \), which can be given by

\[
a_\mu^{Z'} = \frac{g_D^2}{8\pi^2} \int_0^1 \frac{2m_e^2 x^2 (1-x)}{x^2 m_\mu^2 + (1-x)m_{Z'}^2} \mathrm{d}x.
\]

The parameter region on which the \( Z' \) contribution in the minimal \( L_H - L_T \) model resolves the discrepancy in the muon anomalous magnetic moment at 2\( \sigma \) is indicated with the red band in Fig. 3.

- **Neutrino trident production.** The neutrino trident production is a muon neutrino scattering off the Coulomb field of a target nucleus \( (N) \), producing two muons in the final state, \( \nu N \rightarrow \nu N \mu^+\mu^- \). Besides the SM Z boson, in the \( U(1)_{L^{-}L^{+}} \) model, the \( Z' \) boson can also contribute to this process, which can offer a sensitive search for the light \( Z' \).
are the same situations but for the case of \( \varepsilon \) and 10 for \( U(1)_{L_\mu - L_\tau} \) model with extra heavy vector-like leptons (Left) or charged scalars (Right). The horizontal dotted lines are the same situations but for the case of \( \varepsilon(q^2 = 0) \), which are shown as a comparison.

Figure 1: The square of the kinetic mixing \( |\varepsilon/g_\text{Z'}|^2 \) as a function of the momentum transfer \( |q| \) with the mass ratio \( r = 0.1, 1 \) and 10 for \( U(1)_{L_\mu - L_\tau} \) model with extra heavy vector-like leptons (Left) or charged scalars (Right). The most stringent constraints come from the Borexino solar neutrino experiment. Lim-its for \( U(1)_{L_\mu - L_\tau} \) gauge boson have been derived in Refs.[32, 43] by converting existing bounds on \( U(1)_{B-L} \) models [44] using earlier Borexino \(^7\)Be data [45]. The bounds are updated in Ref.[46] using the recently-released Borexino measurement of \(^7\)Be neutrinos[47]. We show them in Fig. 3.

- Neutrino-electron scattering. The neutrino-electron elastic scattering processes can probe \( U(1)_{L_\mu - L_\tau} \) gauge boson, since \( U(1)_{L_\mu - L_\tau} \) gauge boson can contribute through the kinetic mixing. The most stringent constraints come from the Borexino solar neutrino experiment. Limits for \( U(1)_{L_\mu - L_\tau} \) gauge boson have been derived in Refs.[32, 43] by converting existing bounds on \( U(1)_{B-L} \) models [44] using earlier Borexino \(^7\)Be data [45]. The bounds are updated in Ref.[46] using the recently-released Borexino measurement of \(^7\)Be neutrinos[47]. We show them in Fig. 3.

- \( Z' \) production associated with muon pair. Via the direct coupling to \( \mu, Z' \) can be produced at \( e^+e^- \) colliders in the process \( e^+e^- \rightarrow \mu^+\mu^-Z' \). Babar experiment has reported the bounds using 514 fb\(^{-1}\) data collected in the reaction \( e^+e^- \rightarrow \mu^+\mu^-Z', Z' \rightarrow \mu^+\mu^- \) for \( m_{Z'} > 2m_\mu \) [27]. Recently, Belle II experiment perform the first searches for the invisible decay of a \( Z' \) in the process \( e^+e^- \rightarrow \mu^+\mu^-Z', Z' \rightarrow \text{INV} \) using 276 pb\(^{-1}\) collected [28], which can touch the region of \( m_{Z'} < 2m_\mu \).
• Z’ production associated with SM photon. At e^+e^- colliders, the Z’ boson can also be produced associated with SM photon via the kinetic mixing in the process e^+e^- → γZ’ [48]. The search for invisible decays of dark photon has been performed at BaBar experiment using the single-photon events with 53 fb^-1 data. We translate the constraints for dark photon to U(1)_{L_{\mu}-L_{\tau}} gauge boson Z’ using

\[ \varepsilon_{\text{DP}}^2 \rightarrow |\varepsilon|^2 \text{Br}(Z' \rightarrow \text{INV}), \tag{15} \]

where \( \varepsilon_{\text{DP}} \) is the photon and dark photon kinetic mixing parameter in the dark photon model, and \( \varepsilon \) is the \( \gamma-Z' \) kinetic mixing in the U(1)_{L_{\mu}-L_{\tau}} model.

In Fig. 3, we assume Z’ does not decay into dark sector, i.e., \( \Gamma(Z' \rightarrow \text{INV}) = \Gamma(Z' \rightarrow \nu \bar{\nu}) \). The BR(Z’ \rightarrow \text{INV}) \approx 1 \) cases are also shown as dotted line for a visual display. Taking the constraints above into account, a narrow window of the \( m_{Z'} - g_{Z'} \) parameter region in the minimal U(1)_{L_{\mu}-L_{\tau}} model desired by the muon anomalous magnetic moment,

\[ 10 \text{ MeV} \lesssim M_{Z'} \lesssim 210 \text{ MeV}, \quad 4 \times 10^{-4} \lesssim g_{\chi} \lesssim 10^{-3}, \tag{16} \]

is still allowed.

**SEARCHING FOR U(1)_{L_{\mu}-L_{\tau}} GAUGE BOSON AT ELECTRON COLLIDERS**

At the electron colliders, the production of Z’ can be associated with a SM photon through the kinetic mixing in the process e^+e^- → γZ’, whose diagrams are shown in Fig.4. Subsequently, the produced Z’ boson can decay into charged leptons, a pair of neutrinos or light dark matter. In this paper, we focus on the Z’ invisible decay channel Z’ → γINV, including Z’ → νν, and Z’ → X, to probe Z’ boson via the monophoton searches e^+e^- → γZ’ → γ + INV at electron colliders. We assume that the decay width of the Z’ is negligible compared to the experimental resolution, which justifies the use of the narrow width approximation.

In the monophoton signature at electron colliders, the major backgrounds (BGs) from SM contain two types: irreducible and reducible BG. The irreducible monophoton BG comes from the process e^+e^- → ννγ, where \( \nu \) is the three neutrinos. The reducible monophoton BG arises from the electromagnetic processes e^+e^- → γ + X, where X denotes other visible particles but undetected due to the limitations of the detector acceptance. We discuss the reducible BG in detail later for each experiment, since it strongly depends on the angular coverage of the detectors.

The differential cross section for an on-shell Z’ and a photon

\[ \frac{d\sigma_{\gamma Z'}}{dz_\gamma} = \frac{2\pi\alpha^2|\varepsilon|2(m_{Z'}^2)|^2}{s} \left( 1 - \frac{m_{Z'}^2}{s} \right) \frac{1 + z_\gamma^2 + \frac{4s m_{\gamma}^2}{(s-m_{Z'}^2)^2}}{(1 + z_\gamma)(1 - z_\gamma)}, \tag{17} \]

where \( \alpha \) is the fine structure constant, \( z_\gamma \equiv \cos \theta_\gamma \) with \( \theta_\gamma \) being the relative angle between the electron beam axis and the photon momentum in the center-of-mass (CM) frame, \( s \) is the square of the CM energy, \( m_{Z'} \) is the mass of the U(1)_{L_{\mu}-L_{\tau}} gauge boson. The photon energy \( E_\gamma \) in the CM frame is related to the Z’ mass as

\[ E_\gamma = \frac{s - m_{Z'}^2}{2\sqrt{s}}. \tag{18} \]

The cross section after integrating the polar angle \( \theta_\gamma \) is

![Figure 3: Summary for the m_{Z'} - g_{Z'} parameter space of the minimal U(1)_{L_{\mu}-L_{\tau}} model, where Z’ has no additional decay channel to dark sector. The shaded regions show the existing bounds excluded by CCFR experiment in neutrino-trident production [41], by Borexino detector in neutrino-electron scattering [46], by BaBar in the reactions e^+e^- → \mu^+\mu^- Z’, Z’ → \mu^+\mu^- with 514 fb^-1 data [27] and e^+e^- → γZ’, Z’ → INV with 53 fb^-1 data [48], and by Belle II in the process e^+e^- → \mu^+\mu^- Z’, Z’ → INV with 276 pb^-1 data [28]. The dotted lines indicate BR(Z’ → INV) \approx 1 \) cases. The red band indicate the allowed region at 2\sigma from the experimental measurements of muon magnetic momentum.](image-url)

![Figure 4: The Feynman diagrams for the production of an on-shell Z’ and a photon.](image-url)
detector. In order to remove the above bBG, we use the polar angles corresponding to the edges of the ECL

\[
\sigma_{\gamma Z'} = \frac{2\pi\alpha^2|\varepsilon(m^2_{Z'})|^2}{s} \left( 1 - \frac{m^2_{Z'}}{s} \right) \times \left[ 1 + \frac{2sm^2_{Z'}}{(s - m^2_{Z'})^2} \right] Z - \zeta^\text{max} - \zeta^\text{min}, \tag{19}
\]

where

\[
Z = \ln \frac{(1 + \zeta^\text{max})(1 - \zeta^\text{min})}{(1 - \zeta^\text{max})(1 + \zeta^\text{min})}. \tag{20}
\]

**Belle II**

At Belle II, photons and electrons can be detected in the Electromagnetic Calorimeter (ECL), which is made up of three segments: forward endcap with 12.4° < θ < 31.4°, barrel with 32.2° < θ < 128.7°, and backward endcap 130.7° < θ < 155.1° in the lab frame [35]. At Belle II, the reducible BG for monophoton signature consists of two major parts: one is mainly due to the lack of polar angle coverage of the ECL near the beam directions, which is referred to as the “bBG”; the other one is mainly due to the gap between the three segments in the ECL detector, which is referred to as the “gBG”.

The bBG comes from the electromagnetic processes \(e^+e^- \rightarrow \gamma + X\), mainly including \(e^+e^- \rightarrow \gamma\gamma\gamma\) and \(e^+e^- \rightarrow \not\!\!\!p\not\!\!\!p\gamma\), where all the other final state particles except the detected photon are emitted along the beam directions with \(\theta > 155.1°\) or \(\theta < 12.4°\) in the lab frame. At Belle II, we adopt the detector cuts for the final detected photon (hereafter the “pre-selection cuts”): 12.4° < θ < 155.1° in the lab frame.

In Fig.3, we show the production rates of the process \(e^+e^- \rightarrow \gamma Z'\) in \(U(1)_{L_\mu-L_\tau}\) models after the “pre-selection cuts” for the photon at Belle II with \(\sqrt{s} = 10.58\) GeV. The dotted lines correspond to the case of constant \(\varepsilon(q^2 = 0)\), which are shown as a comparison. We can see that, with constant \(\varepsilon(q^2 = 0)\), the cross sections all increase with the increment of the mass of \(Z'\). In the minimal \(U(1)_{L_\mu-L_\tau}\) model, the production rates for the process \(e^+e^- \rightarrow \gamma Z'\) at Belle II generally drop but exist two peaks at the positions of \(m_\mu\) and \(m_\tau\) when \(m_{Z'} < 8.5\) GeV, while raise at the tail of the plotted region.

For the Belle II detector, which is asymmetric, the maximum energy of the monophoton events in the bBG in the CM frame, \(E^m_\gamma\), is given by [50] (if not exceeding \(\sqrt{5}/2\))

\[
E^m_\gamma(\theta_\gamma) = \frac{\sqrt{5}(A\cos\theta_1 - \sin\theta_1)}{A(\cos\theta_1 - \cos\theta_\gamma) - (\sin\theta_\gamma + \sin\theta_1)}, \tag{21}
\]

where all angles are given in the CM frame, and \(A = (\sin\theta_1 - \sin\theta_2)/(\cos\theta_1 - \cos\theta_2)\), with \(\theta_1\) and \(\theta_2\) being the polar angles corresponding to the edges of the ECL detector. In order to remove the above bBG, we use the detector cut \(E_\gamma > E^m_\gamma\) (hereafter the “bBG cuts”) for the final monophoton.

The gBG for the monophoton signature have been simulated in the Ref. [35] to search for dark photons decaying into light dark matter. The projected upper limits on the kinetic mixing of dark photon and SM photon \(\varepsilon\) for a 20 fb\(^{-1}\) Belle II dataset are present there. The constraint for the kinetic mixing between \(U(1)_{L_\mu-L_\tau}\) gauge boson and SM photon \(\varepsilon\) can be translated from the dark photon using Eq. (15). We scale the expected sensitivity \(S(gZ')\) to the planned full of integrated luminosity of 50 ab\(^{-1}\) at Belle II using \(S(gZ') \propto \sqrt{Z}\). Then the corresponding constraint based on the simulation in Ref. [35] from 20 fb\(^{-1}\) to 50 ab\(^{-1}\) can be simply projected by a factor of \(\sqrt{50/20}\), which is present in Fig.6 and the invisible decay ratio \(Br(Z' \rightarrow \text{INV}) \simeq 1\) is assumed. It is shown that the sensitivity for \(gZ'\) at Belle-II experiment with 50 ab\(^{-1}\) via monophoton searches is expected to be worse in the minimal \(U(1)_{L_\mu-L_\tau}\) model with the increment of \(m_{Z'}\), while become better with extra heavy vector-like leptons (charged scalars) in the case of \(r = 0.1\) when \(m_{Z'} < 7\) GeV. With \(r = 10\) in the \(U(1)_{L_\mu-L_\tau}\) model with extra heavy leptons (scalars), expected \(gZ'\) sensitivity gets improved when \(m_{Z'} > 4\) GeV and then gets worse.

We further carry out an analysis without gBG taking into account, to compare with other experiments in which detailed simulations with gBG are not available. We use the “bBG cuts” to remove the reducible BG events; this moment the BG events survived the “bBG cuts” come from irreducible BG without gBG considered. Since the energy of the final photon in the signal process is related to \(m_{Z'}\), in addition to the “bBG cuts”, we select final photon in the energy window of \(E_\gamma - (s - m^2_{Z'})/(2\sqrt{s}) < \sigma_E/2\) (hereafter the “optimized cut”) to enhance the discovery sensitivity, where \(\sigma_E\) is detector energy resolution for the photon. At Belle II, \(\sigma_E = 4\%\) (1.6\%) at 0.1 (8) GeV [35] and we take \(\sigma_E = 128\) MeV conservatively. In Fig. 6, we present the expected 95% confidence level (C.L.) exclusion limits on \(gZ'\) by considering the irreducible BG only after “optimized cut”, which is labeled as Belle-II’. We define \(\chi^2(\varepsilon) \equiv S^2/(S + B)\) [51], where \(S (B)\) is the number of events in the signal (BG) processes. The 95% C.L. upper bound on \(gZ'\) is obtained by solving \(\chi^2(\varepsilon) - \chi^2(0) = 2.71\), and assuming photon detection efficiency as 95\% [35]. One can see that if we don’t consider the “gBG” and apply the “optimized cut”, the Belle II experiment with 50 ab\(^{-1}\) via monophoton searches is expected to be sensitive to the parameter region with \(m_{Z'} \lesssim 1.2\) GeV and \(gZ' \gtrsim 4 \times 10^{-4}\) in the minimal \(L_\mu-L_\tau\) model, which can be improved by almost 1 order of magnitude comparing with considering the “gBG”.

\[
\chi^2 = S^2/(S + B) \approx 2.71. \tag{21}
\]
Figure 5: The cross sections of the process $e^+e^- \rightarrow \gamma Z'$ at Belle II with $\sqrt{s} = 10.58$ GeV after the "pre-selection cuts" for $L_\mu - L_\tau$ model with extra heavy vector-like leptons (Left) or charged scalars (Right). The horizontal dotted lines are the same situations but for the case of $\varepsilon(q^2 = 0)$, which are shown as a comparison.

Figure 6: Sensitivity limit for $g_{Z'}$ at Belle-II experiment with 50 ab$^{-1}$ to search for dark photon decaying into light dark matter based on the simulation in Ref. [35], labeled as “Belle II”, red color. The expected 95% C.L. exclusion limits on $g_{Z'}$ for monophoton searches at 50 ab$^{-1}$ Belle-II with gBG omitted after “optimized cut”, labeled as Belle-II’, black color. For $L_\mu - L_\tau$ model with extra heavy vector-like leptons (Left) or charged scalars (Right) in the cases of $r = 0.1$ (dashed), 1 (solid) and 10 (dotted).

**BESIII AND STCF**

At BESIII and STCF, for the final state photons, we adopt the “preselection cuts” by BESIII Collaboration [52]: $E_\gamma > 25$ MeV with $|\cos \theta| < 0.8$ or $E_\gamma > 50$ MeV with $0.86 < |\cos \theta| < 0.92$. In Fig. 7, we present the cross section of the process $e^+e^- \rightarrow \gamma Z'$ at BESIII and STCF with $\sqrt{s} = 4$ GeV in $U(1)_{L_\mu - L_\tau}$ models after the “pre-selection cuts". The dotted lines correspond to the case of constant $\varepsilon(q^2 = 0)$, which are shown as a comparison. One can see that, the cross section always increases for larger $m_{Z'}$ in $U(1)_{L_\mu - L_\tau}$ models with extra heavy leptons or scalars in the case of $r = 0.1$, while there is a twist near $m_{Z'} = 2m_\tau$ in the case of $r = 1$ and $r = 10$. At BESIII and STCF, which are symmetric, the maximum energy of the monophoton events in the bBG in the CM frame, $E_m^\gamma$, is given by [53]

$$E_m^\gamma(\theta_\gamma) = \sqrt{s} \left( \frac{\sin \theta_\gamma}{\sin \theta_\theta} \right)^{-1},$$

(22)

where $\cos \theta_\theta$ is the polar angle corresponding to the edge of the detector. Taking into account the coverage of MDC, EMC, and TOF, we have $\cos \theta_\theta = 0.95$ at the BESIII [54]. We further demand $E_\gamma > E_m^\gamma$ for the final monophoton to remove the reducible BG (hereafter the
At BESIII, the photon energy resolution of the EMC $\sigma_E/E = 2.3%/\sqrt{E/\text{GeV}} \pm 1%$ [36], and we take $\sigma_E = 40$ MeV for all energy conservatively. At the BESIII, photon reconstruction efficiencies are all more than 99% [55], we assume them to be 100% in our paper. For the EMC at STCF, we assume the same energy resolution and reconstruction efficiencies with BESIII to present a preliminary projection limit, because of the similarity of the two experiments. We take $\sigma_E = 25$ (40, 50) MeV for $\sqrt{s} = 2, (4, 7)$ GeV. In addition to the “bBG cuts”, we select final photon in the energy window of $|E_\gamma - (s - m_{Z'})/(2\sqrt{s})| < \sigma_E/2$ (hereafter the “optimized cut”) to enhance the discovery sensitivity.

At BESIII, since 2012 monophoton trigger has been implemented and the corresponding data luminosity reach about 14 fb$^{-1}$ with the CM energy from 2.125 GeV to 4.6 GeV [56]. We define $\chi^2_{tot}(\varepsilon) = \sum_i \chi^2_i(\varepsilon)$, where $\chi^2_i(\varepsilon) \equiv S^2_i/(S_i + B_i)$ for each BESIII colliding energy. The 95% C.L. upper bound on $g_{Z'}$ from BESIII is obtained by demanding $\chi^2_{tot}(\varepsilon_{95}) - \chi^2_{tot}(0) = 2.71$. In Fig. 8, we present the corresponding results for the $U(1)_{\mu^- L_\tau}$ models with extra vector-like leptons and charged scalars in cases of $r = 0.1, 1, 10$ via monophoton searches at BESIII with 14 fb$^{-1}$ and at 4 GeV STCF with 30 ab$^{-1}$, respectively. The invisible decay ratio of $Z'$ is assumed to be 1. The constraints on $g_{Z'}$ get looser with the increment of $m_{Z'}$ for both two considered models in cases of $r = 1, 10$ at BESIII and 4 GeV STCF, while tighter in cases of $r = 0.1$ for the $U(1)_{\mu^- L_\tau}$ models with extra leptons (scalars) when $m_{Z'} \lesssim 2.7$ GeV (1.0 GeV $\lesssim m_{Z'} \lesssim 2.7$ GeV) at 4 GeV STCF.

Fig. 9 summarizes the sensitivity on gauge coupling $g_{Z'}$ in the minimal $L_\mu - L_\tau$ model from electron colliders, including Belle II, BESII, and STCF. The solid lines indicate the case that $Z'$ cannot decay into dark matter, i.e., $\text{Br}(Z' \rightarrow \text{INV}) = \text{Br}(Z' \rightarrow \nu \bar{\nu})$, and the dotted lines indicate $\text{Br}(Z' \rightarrow \text{INV}) \approx 1$ case. The existing constraints are also presented in the shaded region, and the summary for these limits from different experiments can be found in Fig.3. The red band shows the region that could explain the muon anomalous magnetic moment $(g - 2)_\mu \pm 2\sigma$. We present three expected limits with different experiments at Belle II.

1. $\gamma + \text{INV}$ channel with bBG and gBG considered. We translate the constraints on the dark photon from the search of invisible decay at Belle II assuming a 20 fb$^{-1}$ dataset [35], where the bBG and gBG are all considered, to $L_\mu - L_\tau$ gauge boson using the relation of Eq.(15). Then we scale the constraints to 50 ab$^{-1}$ by a factor of $(50 \text{ ab}^{-1})^{1/4}$. This case is labeled as “Belle II $\gamma + \text{INV}$” in Fig. (9).

2. $\gamma + \text{INV}$ channel with only bBG considered. We compute the limits without gBG taking into account as mentioned above. The “bBG cuts” are applied to remove the reducible BG events and only the irreducible BG contribute to the BG events if gBG is not considered. After the “optimized cut”, we show the 95% C.L. upper bound on $g_{Z'}$ at Belle II with the integrated luminosity of 50 ab$^{-1}$ in Fig.9, which is labeled as “Belle II’ $\gamma + \text{INV}$”.

Figure 7: The cross sections of the process $e^+e^- \to \gamma Z'$ at BESIII or STCF with $\sqrt{s} = 10.58$ GeV after the “pre-selection cuts” for $L_\mu - L_\tau$ model with extra heavy vector-like leptons (Left) or charged scalars (Right). The horizontal dotted lines are the same situations but for the case of $\varepsilon(q^2 = 0)$, which are shown as a comparison.

"bBG cuts")
are presented at SIII are excluded by CCFR experiment. The STCF limits collected during 2012–2018 [56] the upper limits from BE-
−
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in the future. With about 14 fb
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sensitivity on the \( E(1)_\mu = L_\mu \) gauge boson \( Z' \) with \( e^+e^- \rightarrow \mu^+\mu^- Z', Z' \rightarrow \text{INV} \) channel in 50 ab\(^{-1}\)
Belle II experiment, we simply scale the recent 276 pb
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the gauge coupling constant \( g_{Z'} \) down to about \( 4.2 \times 10^{-4} \) when \( m_{Z'} < 2 m_\mu \), which still left a thin slice of mass region \( \sim (0.01 - 0.03) \) GeV to explain the moun \( (g - 2) \) anomaly. The one order of magnitude difference in sensitivity between the two Belle II limits via the monophoton search, shows that the control on gBG is very important in probing the \( Z' \) parameter space.

The STCF and BESIII limits are obtained when the BG due to the gaps in the detectors are neglected, since BESIII did not released any analysis about gBG. We emphasize that more rigorous BESIII and STCF sensitivities could be obtained with such gBG analysis available in the future. With about 14 fb\(^{-1}\) integrated luminosity collected during 2012-2018 [56] the upper limits from BESIII are excluded by CCFR experiment. The STCF limits are presented at \( \sqrt{s} = 4, (4, 7) \) GeV with the integrated luminosity of 30 ab\(^{-1}\). The future monophoton searches at the STCF experiment operated at \( \sqrt{s} = 2 - 7 \) GeV can eliminate the moun \( g - 2 \) favored window when \( m_{Z'} \lesssim 5 \) GeV. In the low mass region, 2 GeV STCF provide best

3. \( \mu^+\mu^- + \text{INV} \) channel. In order to project the
sensitivity on the \( U(1)_\mu = L_\mu \) gauge boson \( Z' \) with \( e^+e^- \rightarrow \mu^+\mu^- Z', Z' \rightarrow \text{INV} \) channel in 50 ab\(^{-1}\)
Belle II experiment, we simply scale the recent 276 pb\(^{-1}\) results by a factor of \( \left( \frac{50 \text{ ab}^{-1}}{276 \text{ pb}^{-1}} \right)^{1/4} \) for the
kinetic mixing, which is labelled as “Belle II \( \mu^+\mu^- + \text{INV} \)”

One observes that on the searches for the invisible decay of \( Z' \), the sensitivity at 50 ab\(^{-1}\) Belle II with \( \mu^+\mu^- + \text{INV} \) channel is slightly better with the \( \gamma + \text{INV} \) channel. It can also be found that these two results are already excluded by current constraints. While without the gBG considered in the \( \gamma + \text{INV} \) channel, the sensitivity can be improved almost 1 order and the gauge coupling constant \( g_{Z'} \) down to about \( 4.2 \times 10^{-4} \) when \( m_{Z'} < 2 m_\mu \), which still left a thin slice of mass region \( \sim (0.01 - 0.03) \) GeV to explain the moun \( (g - 2) \) anomaly. The one order of magnitude difference in sensitivity between the two Belle II limits via the monophoton search, shows that the control on gGB is very important in probing the \( Z' \) parameter space.

The STCF and BESIII limits are obtained when the BG due to the gaps in the detectors are neglected, since BESIII did not released any analysis about gBG. We emphasize that more rigorous BESIII and STCF sensitivities could be obtained with such gBG analysis available in the future. With about 14 fb\(^{-1}\) integrated luminosity collected during 2012-2018 [56] the upper limits from BESIII are excluded by CCFR experiment. The STCF limits are presented at \( \sqrt{s} = 2, (4, 7) \) GeV with the integrated luminosity of 30 ab\(^{-1}\). The future monophoton searches at the STCF experiment operated at \( \sqrt{s} = 2 - 7 \) GeV can eliminate the moun \( g - 2 \) favored window when \( m_{Z'} \lesssim 5 \) GeV. In the low mass region, 2 GeV STCF provide best

### SUMMARY

In this paper, we probe the invisible decay of the \( L_\mu - L_\tau \) gauge boson via monophoton signature at three different electron colliders operated at the GeV scale: Belle II, BESIII, and STCF. In the minimal \( \left( U(1)_\mu = L_\mu \right) \) model with extra heavy \( \mu^+\mu^- + \text{INV} \) channel, we extend the SM with a \( U(1)_\mu = L_\mu \) gauge symmetry and assume that the kinetic mixing term between \( Z' \) and photon is absent at tree level, but can arise at one loop level due to \( \mu \) and \( \tau \) leptons. We also further extend the minimal \( \left( U(1)_\mu = L_\mu \right) \) model with extra heavy \( \mu^+\mu^- + \text{INV} \) channel, where the additional contributions to the kinetic mixing arising from extra particles inside the loop. The exciting nondecoupling behavior of the contribution since the extra heavy \( \mu^+\mu^- + \text{INV} \) channel is also demonstrated. The visible signatures of heavy leptons or charged scalars, too heavy to be directly de-
We translate the sensitivity for dark photon within monophoton signature projected by Belle II to $U(1)_{L_\mu-L_\tau}$ gauge boson taking into account various SM BGs. We also recast the recent invisible search of $Z'$ in the $\mu^+\mu^-Z'$ production at Belle II. It is found that, by ignoring the BG due to the gaps in the detectors, we present the constraints at BESIII with 14 fb$^{-1}$ luminosity and at future 30 ab$^{-1}$ STCF. For comparison, we also compute the limits at 50 ab$^{-1}$ Belle II without gBG taking into account. It is found that the future 2 GeV STCF can further improve the sensitivity to low mass $Z'$ than Belle II via monophoton signature since it is operated at lower energy. The future STCF can exclude the moun $g - 2$ anomaly favored parameter region when $m_{Z'} \lesssim 5$ GeV. And gauge coupling constant $g_{Z'}$ in the minimal $U(1)_{L_\mu-L_\tau}$ model can be probed down to about $4.2 \times 10^{-5}$ when $m_{Z'} < 2m_\mu$ at future 30 ab$^{-1}$ STCF with $\sqrt{s} = 2$ GeV.

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Figure 10: The expected 95% C.L. exclusion limits in the $g_{Z'} - r$ plane for $m_{Z'} = 0.1$ GeV from BESIII with 14 fb$^{-1}$, Belle II with 50 ab$^{-1}$ and 4 GeV STCF with 30 ab$^{-1}$. The shaded grey region is already excluded by CCFR experiments, which is independent on the mass ratio.

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