The Observed Growth of Massive Galaxy Clusters IV: Robust Constraints on Neutrino Properties

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ABSTRACT
This is the fourth of a series of papers in which we derive simultaneous constraints on cosmological parameters and X-ray scaling relations using observations of the growth of massive, X-ray flux-selected galaxy clusters. Our data set consists of 238 clusters drawn from the ROSAT All-Sky Survey, and incorporates extensive follow-up observations using the Chandra X-ray Observatory. Here we examine the constraints on neutrino properties that are enabled by the precise and robust constraint on the amplitude of the matter power spectrum at low redshift available from our data. In combination with cluster gas mass fraction, cosmic microwave background, supernova and baryon acoustic oscillation data, and incorporating conservative allowances for systematic uncertainties, we limit the species-summed neutrino mass, $M_\nu$, to $<0.33$ eV at 95.4 per cent confidence in a spatially flat, cosmological constant ($\Lambda$CDM) model. In a flat $\Lambda$CDM model where the effective number of neutrino species, $N_{\text{eff}}$, is allowed to vary, we find $N_{\text{eff}} = 3.4^{+0.6}_{-0.5}$ (68.3 per cent confidence, incorporating a direct constraint on the Hubble parameter from Cepheid and supernova data). We also obtain results with additional degrees of freedom in the cosmological model, in the form of global spatial curvature ($\Omega_k$) and a primordial spectrum of tensor perturbations ($r$ and $n_t$). The results are not immune to these generalizations; however, in the most general case we consider, in which $M_\nu$, $N_{\text{eff}}$, curvature and tensors are all free, we still obtain $M_\nu < 0.70$ eV and $N_{\text{eff}} = 3.7 \pm 0.7$ (at, respectively, the same confidence levels as above). These results agree well with recent work using independent data, and highlight the importance of measuring cosmic structure and expansion at low as well as high ($z \sim 1100$) redshifts. Although our cluster data extend to redshift $z = 0.5$, the direct effect of neutrino mass on the growth of structure at late times is not yet detected at a significant level.

Key words: cosmology: observations – cosmological parameters – large-scale structure of Universe – X-rays: galaxies: clusters.

1 INTRODUCTION
Observations of neutrino flavor oscillation have conclusively shown that the neutrino mass eigenstates are non-degenerate (e.g. Fukuda et al. 1998, 2002; Ahmad et al. 2002; Ahn et al. 2003, 2006; Eguchi et al. 2003; Sanchez et al. 2003; Giacomelli & Margiotta 2004; Aharim et al. 2005). Although, these observations can place tight constraints on the differences in squared mass, measuring the absolute mass scale remains challenging. Current laboratory efforts focus on tritium beta decay (e.g. Lobashev 2003; Kraus et al. 2005) and neutrinoless double beta decay (e.g. Aalseth et al. 1999; Klapdor-Kleingrothaus et al. 2001; Arnaboldi et al. 2005; Arnold et al. 2005); the latter, if observed, would additionally indicate that neutrinos are Majorana rather than Dirac fermions. The squared mass differences measured from flavor oscillations place a lower bound on the sum of the three masses, $M_\nu = \sum_i m_i$, at $\sim 0.056 (0.095)$ eV/c$^2$ in the normal (inverted) hierarchy, while current tritium beta decay results provide an upper bound on the mass of the electron neutrino at $\sim 2$ eV/c$^2$ (thus on $M_\nu$ at $\sim 6$ eV/c$^2$).1

Because neutrinos play a prominent role in the early

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1 Henceforth, we set $c = 1$. 

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Universe, cosmological observations are also sensitive to their properties (for a review, see Lesgourgues & Pastor 2006; see also Section 2). The primary effect of non-zero neutrino mass on cosmological observables is to suppress the formation of cosmic structure on intermediate and small scales. Comparison of the Cosmic Microwave Background (CMB), which reflects large-scale structure at early times, with measurements of the intermediate- or small-scale structure in the local Universe thus provides a way to constrain the absolute mass scale of the neutrino (e.g. Fukugita et al. 2000).

This approach has the disadvantage that any neutrino properties inferred are at some level sensitive to our incomplete understanding of cosmology, in particular dark energy and inflation. It has long been recognized, however, that using complementary measurements of structure, as described above, significantly reduces the sensitivity of the results to such necessary assumptions (e.g. Allen, Schmidt, & Bridle 2003; Tegmark et al. 2004). Nevertheless, it is imperative that any systematic uncertainties affecting the measurements of cosmic structure be properly accounted for in order to obtain robust results.

The amount of structure in the local Universe is generally described by the parameter $\sigma_8$, defined as the present day root-mean-square fluctuation of the linearly evolved matter density field, smoothed by a spherical top-hat window of comoving radius $8h^{-1}$ Mpc; here $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$ is the normalized Hubble parameter. At present, the most robust observation for measuring this quantity is arguably the abundance of massive galaxy clusters; recent advances in cluster simulation (e.g. Nagai, Vikhlinin, & Kravtsov 2007), comparisons of different mass measurement techniques (e.g. Bradač et al. 2008; Mahdavi et al. 2008; Newman et al. 2009), and the availability of robust mass proxies (e.g. Allen et al. 2004, 2008; Kravtsov, Vikhlinin, & Nagai 2006) have significantly reduced systematic uncertainties associated with the measurement of cluster masses. Consequently, recent estimates of $\sigma_8$ based on independent analyses of galaxy cluster data (both optically and X-ray selected clusters) are in very good agreement ($\sigma_8 \sim 0.8$; Mantz et al. 2008, 2009b (hereafter Paper I); Henry et al. 2009; Vikhlinin et al. 2009; Rozo et al. 2010). Recent analyses of cosmic shear data are also in good agreement (Benjamin et al. 2007; Fu et al. 2008), and provide compatible constraints on the neutrino mass to our own (Tereno et al. 2009). Galaxy redshift surveys have also produced comparable results (Thomas, Abdalla, & Lahav 2009).

The number of neutrino species participating in weak interactions is known to be three to high precision (Amsler et al. 2008). However, the possibility remains that additional “sterile” species exist. Results consistent with the existence of a fourth neutrino were reported by the Liquid Scintillator Neutrino Detector (LSND; Aguilar 2001); these results are disfavored by the MiniBooNE experiment (Aguilar-Arevalo et al. 2009a,b), although the interpretation remains somewhat ambiguous at present. This is another question that cosmological observations can address. In particular, the synthesis of light elements is sensitive to the number of relativistic species present in the early Universe, since these determine the expansion rate at that time; however, observations of primordial deuterium and helium abundances are challenging and are subject to large systematic uncertainties. An independent probe is provided by the CMB, in combination with other cosmological data, as described in Section 2. In this work, we consider only the case where the neutrino species (whatever their number) have approximately degenerate mass; in this case, $M_\nu$ and the effective number of neutrino species, $N_{\text{eff}}$, are sufficient to describe the cosmological effect of neutrinos. More general mass splittings, and in particular the case favored by the initial LSND results, in which a sterile neutrino is significantly more massive than the other species, require a more complete treatment (as in, e.g., Crotty et al. 2004).

In this paper, we apply the statistically rigorous analysis method of Paper I and the X-ray flux-limited cluster samples and follow-up observations described in Mantz et al. (2009a, hereafter Paper II) to the problem of inferring neutrino properties from cosmological data. These data (collectively referred to here as the cluster X-ray Luminosity Function, or XLF) provide a robust means to measure $\sigma_8$; our analysis method includes generous systematic allowances and accounts fully for all parameter degeneracies. In obtaining our results, we also incorporate CMB data and measurements of cosmic distance in the form of cluster gas mass fractions ($f_{\text{gas}}$), type Ia supernovae (SNIa) fluxes and Baryon Acoustic Oscillation (BAO) data (see Section 3 and references therein). We note that Reid et al. (2010) obtain very similar results to our own by importance sampling 5-year Wilkinson Microwave Anisotropy Probe (WMAP) results using a prior based on the analysis of optically selected clusters by Rozo et al. (2010).

The basic cosmology that we consider in this paper is the spatially flat, cosmological constant ($\Lambda$CDM) model parametrized by the mean baryon density, $\Omega_b$; the mean total matter density including baryons, neutrinos and cold dark matter (CDM), $\Omega_m$; the Hubble parameter, $H_0$; the normalization of the matter power spectrum, $\sigma_8$; the spectral index of the primordial scalar power spectrum, $n_s$; and the optical depth to reionization, $\tau$. Here the mean densities refer to the present day (redshift $z = 0$), since their values at other times are then determined by the Friedmann equation, and $\sigma_8^2$ is the $z = 0$ variance in the matter density field at scales of $8h^{-1}$ Mpc, as defined above. This simple and commonly used model assumes $M_\nu = 0$ and $N_{\text{eff}} = 3.046$, the predicted value for the three weakly interacting neutrinos.

In addition to freeing $M_\nu$ and $N_{\text{eff}}$, we will consider the effect of marginalizing over other parameters with which they are degenerate. However, since the flat $\Lambda$CDM model is known to provide a good fit to currently available cosmological data, we are conservative in incorporating these additional degrees of freedom. In particular, we consider generalizing the description of dark energy, either by allowing global spatial curvature or by varying the dark energy equation of state in a flat universe, and including the effects of primordial tensor perturbations. The former case is parameterized by the effective curvature energy density, $\Omega_k$, or the
equation of state parameter, \( w \), while in the latter case we simultaneously marginalize over the tensor-to-scalar ratio, \( r \) (defined with respect to wavenumber \( k = 0.002h \text{ Mpc}^{-1} \), as in, e.g., Komatsu et al. 2009), and the tensor power spectral index, \( n_t \).

In our results, we will consistently quote one-sided limits (upper bounds) on \( M_\nu \) and \( r \) at the 95.4 per cent confidence level and two-sided constraints on all other parameters at 68.3 per cent confidence.

### 2 BACKGROUND

Although the purpose of this work is to explore the constraints enabled by the XLF data, it is instructive to begin by considering the cosmological constraints on \( M_\nu \) that have been obtained without using measurements of local cosmic structure. As described by Ichikawa, Fukugita, & Kawasaki (2005), Dunkley et al. (2009) and Komatsu et al. (2009), CMB observations alone provide a limit that, while weak (\( M_\nu < 1.3 \text{ eV} \) at 95.4 per cent confidence), is relatively robust to assumptions about dark energy and primordial tensors.\(^3\) While we do not review the details here, the constraint arises because the effect of neutrino mass on the CMB temperature anisotropy spectrum is qualitatively different when neutrinos are relativistic compared with non-relativistic during the decoupling epoch. In particular, for \( M_\nu < 1.5 \text{ eV} \), changes in \( M_\nu \) can be easily mimicked by a corresponding change in the Hubble parameter, \( H_0 \). As a result, \( M_\nu \) is strongly degenerate with \( H_0 \) in analyses using only CMB data, which does not itself constrain \( H_0 \). The combination of CMB data with distance measurements in the form of SNIa and BAO data can place a constraint on \( H_0 \), improving the limits on \( M_\nu \) by roughly a factor of two (\( M_\nu < 0.67 \text{ eV} \) for a spatially flat \( \Lambda \text{CDM} \) background; Komatsu et al. 2009).

As we will show, the inclusion of a measurement of local cosmic structure in the form of galaxy cluster abundance can improve these limits by another factor of two or greater, and furthermore greatly increase the robustness of the results to assumptions about the nature of dark energy and inflation. This improvement is possible because the combination of large-scale, high-redshift and intermediate-scale, low-redshift measurements of cosmic structure exploits the defining characteristic of light, weakly interacting particles: they are relativistic during the earliest stages of cosmic structure formation but non-relativistic at the present day.\(^4\) The imprint of neutrinos on cosmic structure, and in particular on the growth of structure from the surface of last scattering to the present, thus differs from that of the photon background (relativistic at all times) and CDM (assumed to be non-relativistic at all times). In particular, a non-zero mass in neutrinos results in a net suppression of the growth of structure on scales smaller than their free-streaming length (approximately corresponding to wavenumber \( 10^{-2}h \text{ Mpc}^{-1} \) today) relative to an equal mass in CDM (Bond et al. 1980).

In practical terms, the constraint on \( M_\nu \) arising from the combination of CMB and cluster data can be understood as follows. Even when \( M_\nu \neq 0 \), the CMB data provide a good constraint on the high-redshift amplitude of the power spectrum, since the direct effect of neutrino mass on the observed temperature anisotropies is relatively small. However, the late-time, intermediate-scale power spectrum amplitude (the value of \( \sigma_8 \)) that would be predicted based on that high-redshift constraint is very sensitive to \( M_\nu \), since neutrino mass suppresses the development of structure on those scales. This strong degeneracy is apparent in the blue confidence regions in Figure 1. By constraining \( \sigma_8 \) independently of \( M_\nu \), clusters (or other low-redshift structure measurements) break this degeneracy, improving the constraints on neutrino mass (gold contours in the figure).

Even more power is available if the growth of structure can be measured, exploiting the time-dependent nature of the effects of neutrino mass on cosmic structure development (e.g., Lesgourgues & Pastor 2006). However, we show in Section 5.1 that the growth of structure in current cluster data, while able to constrain models of dark energy (Paper I: Vikhlinin et al. 2009) and modified gravity (Rapetti et al. 2009a,b, Schmidt, Vikhlinin, & Hu 2009), are not yet sufficient to contribute to constraints on \( M_\nu \).

In the case of the effective number of relativistic species, \( N_{\text{eff}} \), CMB data alone can provide a lower bound, due to the gravitational effect of their anisotropic stress. Apart from providing this bound, however, the CMB data are hampered by a near-perfect degeneracy between \( N_{\text{eff}} \) and \( \Omega_m h^2 \), the physical matter density (Dunkley et al. 2009). This degeneracy arises because CMB observations do not constrain either \( N_{\text{eff}} \) or \( \Omega_m h^2 \) directly, only the redshift of matter-radiation equality, \( z_{\text{eq}} \), which is a function of both parameters. Again, the addition of cosmic distance measurements can reduce the effect of this degeneracy by placing an independent constraint on \( \Omega_m h^2 \); Komatsu et al. (2009) obtained \( N_{\text{eff}} = 4.4 \pm 1.5 \) (68 per cent confidence).

Measurements of local structure can also contribute to improved constraints on \( N_{\text{eff}} \), as we show in Section 6. The contribution of clusters to this problem is less direct than in the case of \( M_\nu \): the constraint on \( \sigma_8 \) and \( \Omega_m \) from cluster data improves the constraint on \( \Omega_m h^2 \) from the CMB+distance measurements (e.g. Figure 1 of Paper I), indirectly improving the constraint on \( N_{\text{eff}} \) through the degeneracy described above.

### 3 DATA

The galaxy cluster data used in this work, as well as their selection and reduction, are discussed in detail in Paper II. Using three wide-area cluster samples drawn from the ROSAT All-Sky Survey (RASS: Trümper 1993) – the ROSAT Brightest Cluster Sample (BCS: Ebeling et al. 1998), the ROSAT-ESO Flux-Limited X-ray sample (REFLEX: Böhringer et al. 2004), and the bright sub-sample of the Massive Cluster Survey (Bright MACS: Ebeling, Edge, & Henry 2001; Ebeling et al. 2010) – we select a statistically complete sample of 238 X-ray luminous clusters covering the redshift range \( z < 0.5 \). Of these 238

\(^3\) We note that this constraint is not entirely robust, and can be significantly degraded when multiple additional degrees of freedom are included in the cosmological model. In several of the models we consider, the combination of CMB data and cosmic distance measurements produces weaker constraints; however, in the worst case, the upper limit is still \( \sim 2.5 \text{ eV} \) (Tables 1 and 2).

\(^4\) The latter condition follows from the lower bound \( M_\nu > 0.056 \text{ eV} \) obtained from flavor oscillation measurements.
clusters, 94 have follow-up Chandra or ROSAT observations that we incorporate into the analysis. From the follow-up observations, we measure X-ray luminosity, average temperature, and gas mass within $r_{500}$. The gas mass is used as a proxy for total mass, using the finding of Allen et al. (2008) that the gas mass fraction, $f_{\text{gas}} = M_{\text{gas}}/M_{\text{tot}}$, is a constant for hot, massive clusters (see also Section 4). Cluster $f_{\text{gas}}$ measurements additionally provide a precise measure of cosmic distance, so we include the full data set and analysis of Allen et al. in this work.

In addition to these cluster data sets, we incorporate CMB, SNIa and BAO data. Our analysis of the CMB anisotropies uses 5-year WMAP data (Hinshaw et al. 2009; Hill et al. 2009; Nolta et al. 2009) with the March 2008 version of the WMAP likelihood code (Dunkley et al. 2009), as well as Arcminute Cosmology Bolometer Array Receiver (ACBAR) data at smaller angular scales (Reichardt et al. 2009) and measurements of the CMB polarization from the Background Imaging of Cosmic Extragalactic Polarization (BICEP) instrument (Chiang et al. 2010). The SNIa results are derived from the Union compilation (Kowalski et al. 2008), which includes data from a variety of sources (307 SNIa in total; see references in Paper I). Our analysis of BAO data uses the constraints on the ratio of the sound horizon to the distance scale at $z = 0.25$ and $z = 0.35$ derived by Percival et al. (2007) from the galaxy correlation function in 2dF (Colless et al. 2001, 2003) and Sloan Digital Sky Survey (Adelman-McCarthy et al. 2007) data. Finally, we employ in some cases a Gaussian prior on the Hubble parameter, $h = 0.742 \pm 0.036$, based on the results of Riess et al. (2009).

4 ANALYSIS METHOD AND SYSTEMATICS

Our results are obtained via Markov Chain Monte Carlo (MCMC), employing the Metropolis sampler embedded in the COSMOMC code of Lewis & Bridle (2002). The May 2008 COSMOMC release includes the 5-year WMAP and Union supernova data and analysis codes; an additional module implementing the $f_{\text{gas}}$ analysis has also been publicly released (Rapetti, Allen, & Weller 2005; Allen et al. 2008). Further modifications were made to include the likelihood codes for the XLF and BAO data. The CMB and matter power spectrum calculations were performed using the CAMB package of Lewis, Challinor, & Lasenby (2000).

When analyzing the CMB data, we marginalize over a plausible range in the amplitude of the Sunyaev-Zel’dovich signal due to galaxy clusters ($0 < A_{SZ} < 2$; introduced by Spergel et al. 2007). Our analysis of the Union supernova sample of Kowalski et al. (2008) includes their treatment of systematic uncertainties, which accounts for the effects of Malmquist bias and uncertainties in lightcurve fitting and photometry (among others).

The method used to analyze cluster $f_{\text{gas}}$ data is described in full by Allen et al. (2008). It incorporates generous systematic allowances for instrument calibration (10 per cent), non-thermal pressure support (10 per cent, Nagai et al. 2007), the depletion of baryons in clusters with respect to the cosmic mean (20 per cent), and evolution with redshift of the baryonic and stellar content of clusters (10 and 20 per cent).

The analysis of the XLF data is detailed in Paper I. The method combines cluster survey data with follow-up observations in an internally consistent way, rigorously accounting for the effects of Malmquist and Eddington biases and parameter degeneracies. Conservative systematic allowances are included to account for uncertainty in the predicted cluster mass function, the overall cluster survey completeness and purity, and instrument calibration.

As discussed in Section 2 and illustrated in Sections 5 and 6, the XLF data contribute to constraints on $M_{\nu}$ through their measurement of $\sigma_8$. The posterior uncertainty on this measurement (from the XLF alone) is $\sim 6$ per cent, and is dominated by systematic uncertainty, as described in Paper I. Specifically, the uncertainty in $\sigma_8$ is determined by the accuracy with which cluster masses can be measured. As detailed in Paper II, we do not directly infer cluster masses at $r_{500}$ by assuming hydrostatic equilibrium of the intracluster medium, a procedure which, when applied to a typical cluster, introduces a large and highly variable bias (Faltenbacher et al. 2005; Rasia et al. 2006; Nagai et al. 2007). Instead, we use the gas mass at $r_{500}$, which can be measured without significant bias, as a proxy for total mass. The proxy relation is calibrated using the $f_{\text{gas}}$ data of Allen et al. (2008), which effectively consist of gas mass and total mass measurements for the subset of clusters where the hydrostatic assumption is applicable (the most massive, dynamically relaxed clusters). Of the systematic allowances described above, the most relevant for this procedure are the allowances for non-thermal support and instrument calibration in the $f_{\text{gas}}$ analysis. In addition, we account for uncertainty in the difference in $f_{\text{gas}}$ between $r_{2500}$ (as measured by Allen et al.) and $r_{500}$, as well as possible scatter in $f_{\text{gas}}$ from cluster to cluster (see Paper II). Ultimately, both our estimates of individual masses and the mean cluster mass scale include a systematic error budget of $\sim 15$ per cent.

5 LIMITS ON $M_{\nu}$

5.1 Simple models

We first consider the case of a spatially flat $\Lambda$CDM cosmology with non-zero neutrino mass. For this model, the joint 68.3 and 95.4 per cent confidence regions in the $M_{\nu}-\sigma_8$ plane from the combination of CMB, $f_{\text{gas}}$, SNIa and BAO data appear in the left panel of Figure 1 (blue contours). The 95.4 per cent confidence upper bound on $M_{\nu}$ from these data (including the systematic allowances described in Section 4) is $M_{\nu} < 0.61$ eV (Table 1), a marginal improvement over that
obtained by Komatsu et al. (2009) using only WMAP, SNIa and BAO data.

However, it is clear in the figure that a tight constraint on \( \sigma_8 \) can improve the limits on \( M_\nu \). The upper limit obtained by including the XLF data, which provide such a constraint, is \( M_\nu < 0.33 \) eV (corresponding to the gold contours in Figure 1), nearly a factor of two improvement.

We note that when the results excluding the XLF data are combined with the Gaussian prior \( \sigma_8 = 0.82 \pm 0.05 \), based on the XLF results for a flat \( \Lambda \)CDM cosmology with \( n_s = 0.95 \) (Paper I), the limits on \( M_\nu \) obtained are virtually identical to those from the full combination of data, \( M_\nu < 0.33 \) eV. From this we conclude that the XLF data currently contribute to the bound on \( M_\nu \) only through their ability to constrain \( \sigma_8 \); the effect of non-zero neutrino mass on the (time-dependent) growth of structure is not detected in the present data.

5.2 Extended models

Next, we consider how the neutrino mass limits are affected when additional degrees of freedom are introduced in the cosmological model. Several possibilities are worth considering here. To begin with, the model for dark energy may be generalized; the simplest ways to accomplish this by introducing a single additional free parameter are through the spatial curvature (i.e. by not linking the dark energy density directly to the matter density) and through the equation of state, retaining the assumption of spatial flatness in the latter case. The flat \( \Lambda \)CDM case corresponds to effective curvature density \( \Omega_k = 0 \) and equation of state \( w = -1 \). Since current data do not support departures from the flat \( \Lambda \)CDM model either through \( \Omega_k \neq 0 \) or \( w \neq -1 \), we consider these cases separately rather than introducing both parameters simultaneously. Secondly, we consider the effect of primordial tensor modes, parametrized through the tensor-to-scalar ratio, \( r \). We additionally marginalize over the tensor spectral index, \( n_t \), rather than assuming a particular model of inflation where \( n_t \) would be linked to \( r \) via a consistency relation.

Finally, we consider the effect of varying \( N_{\text{eff}} \) on the bounds obtained for \( M_\nu \), leaving a discussion of the constraints on \( N_{\text{eff}} \) itself for Section 6.

The effects of adding each of these degrees of freedom individually to the \( \Lambda \)CDM+\( M_\nu \) model are displayed in Figure 2 (95.4 per cent confidence regions only). The left panel shows results from the combination of CMB, SNIa, BAO and \( f_{\text{gas}} \) data. Here it is clear that degeneracies exist between \( M_\nu \) and some of the additional parameters. In particular, the presence of primordial tensor modes (\( M_\nu < 0.92 \) eV) or curvature (\( M_\nu < 1.44 \) eV) significantly degrades the limits on neutrino mass relative to the simpler model (\( M_\nu < 0.61 \) eV) by lengthening the major axis of the confidence region. In contrast, while \( N_{\text{eff}} \) has a small effect on the \( M_\nu \) limit (\( M_\nu < 0.67 \) eV), it significantly degrades the constraints on other cosmological parameters, including \( \sigma_8 \), resulting in the much thicker confidence region in the figure (see also Section 6.1). The equation of state of dark energy is an intermediate case which increases the bound on the neutrino mass somewhat (\( M_\nu < 0.75 \) eV), while also degrading the constraints on other parameters slightly. Marginalized confidence intervals on parameters of interest for these cases can be found in Table 1.

The right panel of Figure 2 shows the equivalent tests when the XLF data are included in the analysis. In this case, the weakest constraint occurs when \( N_{\text{eff}} \) is free (\( M_\nu < 0.54 \) eV); this is sensible, since it had the smallest correlation with \( \sigma_8 \) a priori (left panel). In the other cases shown, limits on \( M_\nu \) are improved by a factor of 2–3. In the most general model considered, where curvature, tensors and additional relativistic species are all marginalized over, the full combination of data yields \( M_\nu < 1.31 \) eV, compared with \( M_\nu < 2.34 \) eV when the XLF is not used (Table 1).

Figure 1. Joint 68.3 and 95.4 per cent confidence regions in the \( M_\nu-\sigma_8 \) plane from the combination of CMB, \( f_{\text{gas}} \), SNIa and BAO data (blue), and the combination of those data with the XLF (gold). The XLF data provide a tight constraint on \( \sigma_8 \), breaking the degeneracy in this plane. To demonstrate the robustness of constraints on \( M_\nu \) obtained using the XLF data, we compare (left panel) results for a simple \( \Lambda \)CDM+\( M_\nu \) model with (right panel) results obtained when nuisance parameters are included in the model (in this case, \( \Omega_k \), \( r \) and \( n_t \); note the difference in scale). Note that conservative systematic uncertainties are included here (and in all subsequent figures).
Figure 2. [preprint note: a monochrome version of this figure appears after the references, as Figure 6] Joint 95.4 per cent confidence regions for $M_\nu$ and $\sigma_8$ for various cosmological models. Yellow contours correspond to the basic $\Lambda$CDM+$M_\nu$ model, blue contours are marginalized over $\Omega_m$, red contours over $w$, green over $r$ and $n_t$, and purple over $N_{\text{eff}}$. The left panel shows constraints obtained from the combination of CMB, $f_{\text{gas}}$, SNIa and BAO data; the right panel shows results that include the XLF in addition to those data. No external prior on $H_0$ is used.

Table 1. Constraints on the species-summed neutrino mass and other model parameters, including conservative systematic allowances. Note that limits on $M_\nu$ and $r$ are listed at 95.4 per cent confidence, while all others are 68.3 per cent confidence. Parameters with a single value listed were fixed at that value. Constraints on $\Omega_m$ do not appear in the table, as they were extremely similar for all models, consistent with $\Omega_m = 0.26 \pm 0.015$. The first column indicates whether the XLF data were included in the fit; the combination of CMB, $f_{\text{gas}}$, SNIa and BAO data is used in all cases. The results in this table do not use an external prior on $H_0$. The use of such a prior, in addition to the XLF data, greatly improves constraints on both $N_{\text{eff}}$ and $M_\nu$ when $N_{\text{eff}}$ is a free parameter (see Table 2).

| XLF | $\sigma_8$   | $\Omega_m$ | $w$ | $r$ | $n_t$ | $N_{\text{eff}}$ | $M_\nu$ (eV) |
|-----|--------------|------------|-----|-----|-------|-----------------|--------------|
| $\sqrt{\text{XLF}}$ | 0.77 ± 0.05  | 0          | $-1$| 0   | 0     | 3.046           | < 0.61        |
| $\checkmark$ | 0.79 ± 0.03  | 0          | $-1$| 0   | 0     | 3.046           | < 0.33        |
| $\sqrt{\text{XLF}}$ | 0.72 ± 0.08  | 0.009 ± 0.008 | $-1$| 0   | 0     | 3.046           | < 1.44        |
| $\checkmark$ | 0.79 ± 0.03  | 0.004 ± 0.006 | $-1$| 0   | 0     | 3.046           | < 0.50        |
| $\sqrt{\text{XLF}}$ | 0.76 ± 0.05  | 0          | $-1.02 \pm 0.08$| 0   | 0     | 3.046           | < 0.72        |
| $\checkmark$ | 0.79 ± 0.03  | 0          | $-1.03 \pm 0.07$| 0   | 0     | 3.046           | < 0.43        |
| $\sqrt{\text{XLF}}$ | 0.76 ± 0.06  | 0          | $-1$| < 0.33| 0.3$^{+0.8}_{-0.7}$| 3.046           | < 0.92        |
| $\checkmark$ | 0.79 ± 0.03  | 0          | $-1$| < 0.22| 0.5$^{+0.7}_{-0.8}$| 3.046           | < 0.38        |
| $\sqrt{\text{XLF}}$ | 0.80 ± 0.09  | 0          | $-1$| 0   | 0     | 3.4$^{+2.0}_{-1.3}$| < 0.67        |
| $\checkmark$ | 0.80 ± 0.04  | 0          | $-1$| 0   | 0     | 3.6$^{+1.4}_{-1.0}$| < 0.54        |
| $\sqrt{\text{XLF}}$ | 0.69 ± 0.08  | 0.008 ± 0.009 | $-1$| < 0.29| 0.5$^{+0.8}_{-0.7}$| 3.046           | < 1.60        |
| $\checkmark$ | 0.79 ± 0.03  | 0.003 ± 0.006 | $-1$| < 0.19| 0.1$^{+0.8}_{-0.7}$| 3.046           | < 0.49        |
| $\sqrt{\text{XLF}}$ | 0.72 ± 0.10  | 0.010 ± 0.009 | $-1$| 0   | 0     | 3.4$^{+2.0}_{-1.2}$| < 1.84        |
| $\checkmark$ | 0.79 ± 0.04  | 0.007 ± 0.007 | $-1$| 0   | 0     | 3.6$^{+1.7}_{-0.9}$| < 1.14        |
| $\sqrt{\text{XLF}}$ | 0.79 ± 0.10  | 0          | $-1$| < 0.43| 0.2$^{+0.6}_{-0.7}$| 4.5$^{+2.2}_{-2.0}$| < 1.46        |
| $\checkmark$ | 0.81 ± 0.05  | 0          | $-1$| < 0.33| 0.2$^{+0.7}_{-0.8}$| 4.5$^{+1.4}_{-1.6}$| < 0.95        |
| $\sqrt{\text{XLF}}$ | 0.71 ± 0.10  | 0.008 ± 0.009 | $-1$| < 0.36| 0.4$^{+0.6}_{-0.7}$| 4.5$^{+2.2}_{-2.1}$| < 2.34        |
| $\checkmark$ | 0.80 ± 0.04  | 0.004 ± 0.008 | $-1$| < 0.30| 0.1$^{+0.8}_{-0.7}$| 4.0$^{+1.9}_{-1.1}$| < 1.31        |

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6 CONSTRAINTS ON $N_{\text{eff}}$

6.1 Simple models

Joint constraints on $N_{\text{eff}}$ and $\Omega_m h^2$ are shown in Figure 3 for a flat $\Lambda$CDM cosmology. The strong correlation in this plane is due to the fact that the CMB data constrain directly $\omega_{\text{m}}$, which is a degenerate combination of $N_{\text{eff}}$ and $\Omega_m h^2$, as discussed previously in Section 2. The combination of data that we use places tight constraints on $\Omega_m$, so significant improvement can be obtained by incorporating a direct measurement of the Hubble constant; throughout this section we use a Gaussian prior, $h = 0.742 \pm 0.036$, based on the results of Riess et al. (2009). The constraints thus obtained are listed in Table 2.

We note that, ordinarily, the combination of $f_{\text{gas}}$ and CMB data provides a tight constraint on $H_0$ (Allen et al. 2008); however, this is not the case when $N_{\text{eff}}$ is free. The reason why can be seen in Figure 4. The green contours in the figure show constraints obtained from combining the CMB, $f_{\text{gas}}$, SNIa and BAO data (without a prior on $H_0$). A correlation in this plane occurs naturally in the $f_{\text{gas}}$ analysis, since X-ray observations of clusters measure a degenerate combination of the cosmic baryon fraction, $\Omega_b/\Omega_m$, and distance. When $N_{\text{eff}}$ is fixed, CMB data place tight constraints on the baryon fraction, though not on the Hubble parameter, and so the combination of CMB and $f_{\text{gas}}$ data produces tight constraints on both $H_0$ and $\Omega_b/\Omega_m$. When $N_{\text{eff}}$ is free, however, the CMB constraints in this plane are degenerate along nearly the same axis as the $f_{\text{gas}}$ constraints,\(^{11}\) and so the inclusion of additional, independent distance measurements is necessary to place a constraint on $H_0$. The combination with a direct measurement of $H_0$ (blue contours) significantly improves matters.

As Figure 3 shows, the addition of the XLF data improves the constraint on $N_{\text{eff}}$ somewhat, from $N_{\text{eff}} = 3.6^{+0.7}_{-0.6}$ to $N_{\text{eff}} = 3.4^{+0.6}_{-0.5}$ (68.3 per cent confidence). The mechanism for this improvement is a degeneracy between $\Omega_m h^2$ and $\sigma_8$, shown in Figure 5, which the XLF data reduce through their constraint on $\sigma_8$.

\(^{11}\) Respectively, the quantities $\Omega_m h^2$ and $\Omega_m h^{1.5}$ are degenerate in the analysis of CMB and $f_{\text{gas}}$ data.
Table 2. Constraints on the neutrino effective number and other model parameters. Note that limits on \( M_\nu \) and \( r \) are listed at 95.4 per cent confidence, while all others are 68.3 per cent confidence. Parameters with a single value listed were fixed at that value. The first two columns respectively indicate whether the prior on \( H_0 \) and the XLF data were included in the fit; the combination of CMB, \( f_{\text{gas}} \), SNIa and BAO data is used in all cases.

| \( H_0 \) | XLF | \( \sigma_8 \) | \( \Omega_k \) | \( r \) | \( n_t \) | \( N_{\text{eff}} \) | \( M_\nu \) (eV) |
|---|---|---|---|---|---|---|---|
| 0.86 ± 0.07 | 0 | 0 | 0 | 3.8^{+1.9}_{-1.5} | 0 |
| √ | 0.83 ± 0.04 | 0 | 0 | 3.6^{+0.7}_{-0.6} | 0 |
| √ | 0.82 ± 0.03 | 0 | 0 | 3.4^{+0.6}_{-0.5} | 0 |
| 0.85 ± 0.07 | 0.002 ± 0.006 | 0 | 0 | 3.4^{+1.8}_{-1.3} | 0 |
| √ | 0.83 ± 0.04 | 0.001 ± 0.005 | 0 | 3.4^{+0.8}_{-0.7} | 0 |
| √ | 0.88 ± 0.08 | 0 | < 0.25 | 0.1^{+0.9}_{-0.7} | 4.2^{+2.6}_{-1.7} |
| √ | 0.83 ± 0.04 | 0 | < 0.20 | 0.3^{+0.9}_{-0.9} | 3.6^{+0.8}_{-0.8} |
| 0.80 ± 0.09 | 0 | 0 | 0 | 3.4^{+2.0}_{-1.3} | < 0.67 |
| √ | 0.79 ± 0.05 | 0 | 0 | 3.5^{+0.7}_{-0.4} | < 0.64 |
| √ | 0.80 ± 0.03 | 0 | 0 | 3.5^{+0.7}_{-0.5} | < 0.48 |
| 0.72 ± 0.10 | 0.010 ± 0.009 | 0 | 0 | 3.4^{+1.2}_{-1.2} | < 1.84 |
| √ | 0.73 ± 0.08 | 0.008 ± 0.009 | 0 | 3.4^{+0.9}_{-0.6} | < 1.58 |
| √ | 0.79 ± 0.03 | 0.005 ± 0.006 | 0 | 3.5^{+0.8}_{-0.5} | < 0.72 |
| 0.79 ± 0.10 | 0 | < 0.43 | 0.2^{+0.6}_{-0.7} | 4.5^{+2.2}_{-2.0} | < 1.46 |
| √ | 0.76 ± 0.07 | 0 | < 0.34 | 0.2^{+0.7}_{-0.7} | 3.7^{+0.7}_{-0.9} | < 1.14 |
| √ | 0.80 ± 0.04 | 0 | < 0.23 | 0.3^{+0.9}_{-0.8} | 3.6^{+0.8}_{-0.6} | < 0.55 |
| 0.71 ± 0.10 | 0.008 ± 0.009 | 0 | < 0.36 | 0.4^{+0.6}_{-0.7} | 4.5^{+2.2}_{-2.1} | < 2.34 |
| √ | 0.71 ± 0.09 | 0.006 ± 0.009 | 0 | < 0.33 | 0.4^{+0.7}_{-0.7} | 3.7^{+0.7}_{-0.8} | < 1.75 |
| √ | 0.80 ± 0.04 | 0.002 ± 0.007 | 0 | < 0.29 | 0.2^{+0.8}_{-0.7} | 3.7^{+0.7}_{-0.7} | < 0.70 |

6.2 Extended models

The addition of nuisance parameters in the form of curvature, tensors, and non-zero neutrino mass tends to weaken the constraints on \( N_{\text{eff}} \) and \( H_0 \) along their primary degeneracy axis, similarly to what was observed in Section 5.2. Table 2 lists the constraints obtained with and without the \( H_0 \) prior when various nuisance parameters are marginalized over, as well as when the XLF data are included in the fit. As the table shows, the inclusion of the prior on \( H_0 \) effectively eliminates the degeneracies between \( N_{\text{eff}} \) and the nuisance parameters; in particular, the constraint on \( N_{\text{eff}} \) obtained when marginalizing over curvature, tensors and neutrino mass simultaneously, \( N_{\text{eff}} = 3.7^{+0.7}_{-0.8} \), is very similar to those obtained when the nuisance parameters are fixed, \( N_{\text{eff}} = 3.6^{+0.7}_{-0.6} \). With so much extra freedom in the model, the inclusion of the XLF data produces only a small improvement in \( N_{\text{eff}} \) in that case.

7 PROSPECTS FOR IMPROVEMENT

An interesting question is where future improvements in the determination of \( M_\nu \) and \( N_{\text{eff}} \) from cosmological data will come from. One simple way of addressing this is to see what other cosmological parameters are most correlated with the parameters of interest in the current results. We consider the simple cases of a ΛCDM model where either \( M_\nu \) or \( N_{\text{eff}} \) is additionally free, as well as the more general case with both parameters free in addition to curvature and tensor modes.

7.1 Improvements on \( M_\nu \)

When only \( M_\nu \) is additionally free, it remains most degenerate with \( \sigma_8 \), despite the tight constraints placed on this parameter by current data, with a correlation coefficient of \( \rho = -0.63 \). In the more general model (with \( M_\nu \), \( N_{\text{eff}} \), \( \Omega_k \), \( r \) and \( n_t \) free), the degeneracy with \( \sigma_8 \) is also relatively important \( \rho = -0.51 \), but degeneracies with \( \Omega_m h^2 \) \( (\rho = 0.60) \) and \( \Omega_b \) \( (\rho = 0.48) \) appear at approximately the same level.

The situation will change, however, with the availability of improved CMB data from Planck, and ultimately from future high-resolution CMB polarization and lensing observations. These new data will tighten constraints on a number of parameters that contribute to the width of the degenerate region in the \( M_\nu - \sigma_8 \) plane,\(^{12}\) (Figures 1 and 2; Kaplinghat, Knox, & Song 2009; Bashinsky & Seljak 2004; Colombo, Pierpaoli, & Pritchard 2009; de Putter, Zahn, & Linder 2009) resulting in a stronger correlation of \( M_\nu \) with \( \sigma_8 \).

\(^{12}\) In addition to the curvature and tensor parameters discussed in this paper, standard cosmological parameters such as \( \Omega_m h^2 \), \( \tau \), \( n_s \) and the high-redshift power spectrum amplitude all contribute to the width.
Using a simulated Planck data set produced using the FutuR CMB code of Perotto et al. (2006), we project that such data, combined with a 2 per cent constraint on $\sigma_8$, will produce a limit $M_\nu < 0.18$ eV for minimal neutrino mass in the normal hierarchy ($M_\nu = 0.056$ eV). Alternatively, $M_\nu = 0$ would be ruled out at approximately $1\sigma$ for minimal neutrino mass in the inverted hierarchy ($M_\nu = 0.095$ eV). A constraint on $\sigma_8$ at this level may be possible in the near term from galaxy cluster observations, primarily by better understanding cluster mass measurements (e.g. Mahdavi et al. 2008; Zhang et al. 2010; von der Linden et al., in preparation), as well as from improved measurements of the CMB power spectrum at multipoles $\sim 2000$ (Lueker et al. 2009; Fowler et al. 2010; Komatsu et al. 2010).

As growth of structure data become available at higher redshifts, the time-dependent effects of neutrino mass on structure formation will also provide a powerful and more direct constraint on $M_\nu$. Future cluster surveys may provide limits an order of magnitude better than current results (Wang et al. 2005). Eventually, other cosmological observables, e.g. the cross-correlation of Lyman-alpha absorption with CMB convergence (Vallinotto et al. 2009), may extend measurements of structure formation to even higher redshifts ($z \sim 2$).

7.2 Improvements on $N_{\text{eff}}$

Unsurprisingly, the correlation of $N_{\text{eff}}$ with $\Omega_m h^2$ is dominant in both the simple (ACDM+$N_{\text{eff}}$) and more general ($M_\nu$, $N_{\text{eff}}$, $\Omega_k$, $r$ and $n_t$ free) models, respectively with $\rho = 0.94$ and 0.87, although we note that in the simple case there is also a strong degeneracy with $\sigma_8$ ($\rho = 0.73$). Thus, improved measurements of the Hubble parameter and/or mean matter density will be very significant for cosmological constraints on $N_{\text{eff}}$.

In particular, we consider the possibility of a direct constraint on $H_0$ at the 2 per cent level. Such a constraint could come from several avenues, including Spitzer (and ultimately the James Webb Space Telescope) observations of Cepheids and SNIa (Freedman 2009; Riess et al. 2009), gravitational lensing time delays (e.g. Saha et al. 2006; Oguri 2007; Cole & Moustakas 2009; Dobke et al. 2009), or the comparison of the Compton-$y$ parameter inferred from Sunyaev-Zel’dovich and X-ray observations of clusters (Schmidt, Allen, & Fabian 2004; Bongente et al. 2006; Rapetti, Allen, & Mantz 2008). The latter two naturally complement efforts to extend measurements of the growth of structure using clusters to larger samples and higher redshifts. Even with no other improvements in cosmological data, such a measurement (we take $h = 0.72 \pm 0.0144$, for concreteness) would provide a constraint $N_{\text{eff}} = 3.04 \pm 0.35$ in the simple ΛCDM+$N_{\text{eff}}$ model, or $N_{\text{eff}} = 3.14 \pm 0.48$ in the general model (parameters listed above; 68.3 per cent confidence). Since $N_{\text{eff}}$ and $M_\nu$ are degenerate in this model, the upper limit on $M_\nu$ would also be improved, from 0.70 to 0.55 eV.

8 CONCLUSION

We have applied measurements of the growth of massive galaxy clusters (detailed in Papers I and II), in combination with other cosmological data, to the problem of constraining the species-summed neutrino mass, $M_\nu$, and effective number, $N_{\text{eff}}$.

Our results show that a robust measurement of $\sigma_8$ from clusters significantly improves limits on $M_\nu$ from current data, and reduces their sensitivity to assumptions about the cosmological model. In a simple ACMD+$M_\nu$ cosmology, the addition of the XLF data improves the 95.4 per cent confidence upper limit on $M_\nu$ to 0.33 eV, compared with 0.61 eV from the combination of CMB, $f_{\text{aux}}$, SNIa and BAO data. In a more general model, marginalized over spatial curvature, primordial tensors and the effective number of neutrinos, and incorporating a prior on the Hubble parameter, the XLF data improve the limit from 1.75 eV to 0.7 eV. The results indicate that this improvement is due entirely to the ability of the cluster data to constrain $\sigma_8$ (at $z = 0$); while measurements of the (z-dependent) growth of structure in principle contain additional information about $M_\nu$, current data are not sufficient to exploit it.

The cluster data additionally improve constraints on the effective number of relativistic species, from $N_{\text{eff}} = 3.6^{+0.7}_{-0.6}$ to $N_{\text{eff}} = 3.4^{+0.9}_{-0.5}$ in a simple ACMD+$N_{\text{eff}}$ model (68.3 per cent confidence). For the more general model, including curvature, tensors and neutrino mass, only a marginal improvement is observed.

The results obtained here are compatible with other recent estimates based on galaxy clusters (Reid et al. 2010; Vikhlinin et al. 2009), reflecting the good agreement in recent $\sigma_8$ constraints based on X-ray and optically selected clusters (Paper I; Henry et al. 2009; Rozo et al. 2010). Although current cosmological data are not sufficient to rule out the existence of sterile neutrino species or distinguish between the normal and inverted mass hierarchies, they continue to provide some of the tightest constraints available, in particular on the mass scale.

We also consider the prospects for further improvement, finding that a few per cent level measurement of the Hubble parameter will significantly improve the constraints on models where $N_{\text{eff}}$ is free. A similarly improved determination of $\sigma_8$, combined with improved CMB data from Planck, will further tighten limits on the neutrino mass scale, perhaps providing the first hints of non-zero mass from cosmological data. More precise measurements of the growth of structure at late times, and the extension of such data to higher redshifts, will provide a powerful, new mechanism to constrain neutrino mass.

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13 We note that the 7-year WMAP results, which were released while this work was in revision, already offer some improvement. Specifically, Komatsu et al. (2010) find $M_\nu < 0.58$ eV by combining 7-year WMAP data with the Riess et al. (2009) $H_0$ prior and the BAO results of Percival et al. (2010), a noticeable improvement over the limit $M_\nu < 0.66$ eV obtained by Sekiguchi et al. (2010) from the same auxiliary data sets combined with WMAP5.
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Figure 6. (Black and white version of Figure 2.) Joint 95.4 per cent confidence regions for $M_\nu$ and $\sigma_8$ for various cosmological models. Solid contours correspond to the basic ΛCDM+M$_\nu$ model, dotted contours are marginalized over $\Omega_k$, dashed contours over $w$, dot-dashed over $\Omega_b$ and $\Omega_c$, and 3-dot-dashed over $N_{\text{eff}}$. The left panel shows constraints obtained from the combination of CMB, $f_{\text{gas}}$, SNIa and BAO data; the right panel shows results that include the XLF in addition to those data. No external prior on $H_0$ is used.

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