Intermittent dynamic bursting in vertically vibrated liquid drops

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A previously unreported regime of type III intermittency is observed in a vertically vibrated milliliter-sized liquid drop submerged in a more viscous and less dense immiscible fluid layer supported by a hydrophobic solid plate. As the vibration amplitude is gradually increased, subharmonic Faraday waves are excited at the upper surface of the drop. We find a narrow window of vibration amplitudes slightly above the Faraday threshold, where the drop exhibits an irregular sequence of large amplitude bursting events alternating with intervals of low amplitude activity. The triggering physical mechanism is linked to the competition between surface Faraday waves and the shape deformation mode of the drop.

Intermittency is an intricate dynamical regime that is observed in many driven dissipative systems. It is broadly associated with irregular alternation between intervals of a low-disorder quiescence phase and a high-disorder bursting phase. Theoretically described in the context of dynamical systems in\cite{1}, intermittency presents itself as an initial instability of the low-disorder phase (usually a periodic regime). Instabilities associated with local bifurcations of periodic solutions—saddle-node, torus or period-doubling—give rise to type I, II and III intermittency, respectively. Different types of intermittency have been experimentally observed, for example, in Rayleigh-Bénard convection\cite{2,3}, Taylor-Couette flow\cite{4}, brain activity\cite{5}, electro-chemical reactions\cite{6,7}, chemical reaction systems with two slow parametric excitations\cite{8} and in thermoacoustic oscillations of flame dynamics\cite{9,10}.

An example of a system with a rich bifurcation scenario is a vertically vibrated liquid drop surrounded by a layer of a more viscous immiscible fluid. At a certain critical vibration amplitude, the drop undergoes a period-doubling bifurcation that corresponds to the onset of subharmonic Faraday waves on its surface\cite{12,13}. If the drop is floating in a more dense fluid, the period-doubling bifurcation gives rise to a new stable configuration in which the drop elongates to form a worm-like structure\cite{12,13}. However, when the drop is immersed in a fluid of similar density (i.e. an almost neutrally buoyant drop), the variety of dynamic states born as the result of the primary period-doubling bifurcation becomes remarkably rich. This includes walking drops driven by the surface Faraday waves, rotating drops and stationary drops with irregular polygonal shapes\cite{14}. However, intermittent dynamic regimes have been thus far not reported in such systems.

Here, we report for the first time on experimental observations of a type III intermittency in a vertically vibrated milliliter-sized drop submerged in a more viscous and less dense immiscible carrier fluid. The experiments are performed using a water or Soybean sauce drop immersed in vegetable or olive oil. Typical oil dynamic viscosity, density and oil-air surface tension are $\mu = 84 \text{ mPa s}$, $\rho = 916 \text{ kg/m}^3$ and $\sigma = 0.035 \text{ N/m}$, respectively. The water viscosity and density are $\mu_w = 1 \text{ mPa s}$ and $\rho_w = 1000 \text{ kg/m}^3$, respectively. Soybean sauce is heavier and more viscous than water with density $1080 \text{ kg/m}^3$ and viscosity between 1.6 and 3.6 mPa s\cite{15}. The newly-found intermittent regime, when the drop transits between a quiescent phase of small-amplitude oscillations and violent bursts of large amplitude waves on its upper surface, is very delicate. It exists only when the drop height is less than, but almost equal to, the thickness of the oil layer, i.e. the drop is fully submerged with its highest point almost touching the oil surface. In the experiments with a 30–60 ml oil bath we found that changing the overall volume of the oil by as little as 1 ml could trigger or destroy the bursting regime. If the drop is covered by the oil layer that is thicker than the capillary length $\sqrt{\sigma/\rho g} \approx 2 \text{ mm}$ of oil, the bursting regime is no longer found.

Such high sensitivity of the bursting regime to the thickness of oil film cap above the drop is not surprising: it is known since the pioneering work of Benjamin Franklin\cite{16} that oil has a “calming” effect on water ripples. Recent studies demonstrate that even a monolayer of oil ($\sim 10 \text{ nm}$) dramatically suppresses capillary waves on water due to the Gibbs-Marangoni effect\cite{17}.

Experiments were conducted with water or Soybean Sauce drops with a horizontal diameter 10–15 mm supported by a hydrophobic Teflon substrate placed on the bottom of a 90 mm glass Petri dish. The drop was submerged in an 5–10 mm deep oil layer, which was sufficient to just cover the drop, see Fig.\cite{1}a).

The dish was mounted on a 6.5” 45 W RMS audio speaker (Sony, Japan) powered by a 30W stereo amplifier (Yamaha TSS-15, China) of a sinusoidal signal produced by a digital tone generator. The dish was vertically vibrated at frequencies in the range 40–70 Hz. The vibration amplitudes were
sufficiently large to excite subharmonic Faraday waves on the surface of the drop but not strong enough to excite Faraday waves on the free surface of the oil layer.

The acceleration amplitude \( a \) of the Petri dish was measured with the accuracy of \( \pm 0.08 \) g using an ADXL 326 accelerometer (Analog Devices, USA) connected to a digital oscilloscope (Tektronix TDS 210, USA). The motion of the drop was recorded using an in-house photodetector measuring the intensity of light reflected by the drop surface. In case of the water drop, a small amount of red food dye was added to increase contrast with the yellow oil and a DC LED lamp was used as a light source to avoid flickering in the range of the investigated driving frequencies.

Our main result is the discovery of a narrow window of vibration amplitudes, slightly above the period-doubling bifurcation, where the drop exhibits a random sequence of large-amplitude bursts alternating with long intervals of small-amplitude oscillations. During a bursting event (Fig. 1a.1−a.6) a localised wave develops on the upper surface of the drop. Its wavelength is comparable with the horizontal drop size. The wave crest develops within the first 1−2 s in the centre of the drop and reaches a magnitude of surface deflection of up to 5 mm. After about 1−2 s the initial circular wave is replaced by an irregular polygonal surface wave that eventually dies out and the drop re-enters another interval of small-amplitude harmonic oscillations at the forcing frequency with no visible change in shape of the drop. We note that the dynamics of the surface waves during the burst is not unique and generally depends on the drop volume and the driving frequency.

Fig. 1b shows the results of scanning the \((f, a)\) plane by gradually increasing or decreasing the vibration acceleration amplitude \( a \) by the smallest possible amount of 0.08 g that corresponds to the measurement error at fixed frequency for a 1.5 mL water drop.

Below the dashed line connecting the black squares, the drop behaves as a forced oscillator: it slightly wobbles at the forcing frequency without any visible change in shape. As \( a \) is increased above the dashed line, subharmonic Faraday waves are excited on the upper surface of the drop. Significantly, we found that the dynamics of the drop close to the dashed line is multi-stable with the harmonic and subharmonic regimes coexisting. Thus, when \( a \) is gradually increased, the subharmonic waves set in at the upper end of the error bars associated with the black squares. Alternatively, when \( a \) is gradually decreased, harmonic oscillations are reinstated at the lower end of the error bars associated with the black squares.

We find that the intermittent bursts exist in the frequency range between 40 and 65 Hz at amplitudes larger than those corresponding to a subharmonic transition shown by the thick red vertical bars in the figure. Note that an overlap between the intermittency and multi-stability amplitude ranges exists only at the bottom of an intermittency range. Therefore, the intermittency is not driven by the interaction of periodic and subharmonic oscillation states.

As the amplitude is increased further, the intermittency is destroyed and the drop enters a new dynamic regime characterized by persistent subharmonic surface waves and a horizontal drop elongation. These post-intermittency states are similar to those found in water drops swimming in silicone oil [14] and in a floating liquid lens [12][13].

To validate our experimental results we compare the onset of Faraday waves in a 9 mm-thick oil layer without the drop with the exact theoretical threshold for an infinitely extended horizontal layer on a no-slip substrate [11]. The corresponding experimental (theoretical) threshold is shown by the triangles (dashed-dotted line) in Fig. 1b.

Fig. 2a shows a representative signal recorded by the photodetector. The occurrence of a burst is identified by locating the largest peak in the amplitude (shown by the filled circles). The burst duration is well-defined and is approximately 5 s while the inter-burst intervals (IBI) are highly irregular with the average of 13.9 s and the inter-quartile range of 15.3−13 = 2.3 s, see Fig. 2d. We emphasise that the statistics of the duration of the quiescent phases are similar to that of the inter-burst intervals because the duration of bursts is approximately 5 s.

We recorded the signal over 10 minutes and identified the quiescent phases (low disorder intervals) as times when the oscillation amplitude is below 10% of the maximum spike am-
The dominant harmonic peak at 50 Hz is indicated by the term \( Y_n^m \). The shape deformation mode can be associated with the oscillation between a prolate and oblate spheroid, i.e., \( l = 2 \) and \( n = 0 \). As the bursting phase develops further, the deformation of the drop increases and competes with Faraday waves to eventually suppress them and push the drop back into the quiescent phase.

Note that such a mode competition mechanism has been observed to trigger chaos in earlier experiments with nonlinear Faraday waves excited by vibration in a circular fluid layer\(^\text{[18]}\). More recently, in the experiments with floating liquid drops, the interplay between Faraday waves and deformability of the drop boundaries was shown to lead to their mutual adaptation, when a stable elongated shape of the drop is reached\(^\text{[12, 13]}\). Here we observed that a similar mode competition leads to a new dynamical regime of repeating bursting events in a heavier drop surrounded by a more viscous and less dense fluid layer.

We propose a simple phenomenological model of intermittent bursting, given by system Eqs.\(^\text{(12)}\) that takes into account the coupled dynamics of two oscillation modes: the Faraday surface wave with amplitude \( F \) and a shape deformation mode with amplitude \( v \).

\[
\ddot{F} + (c_1 + dv^2)\dot{F} + (w_0^2 + A\cos(\Omega t))F + F^3 = b_1 \cos(\Omega t), \quad \Omega \nonumber
\]

\[
\ddot{v} + c_2\dot{v} + (\omega_0^2 + F)v + v^3 = 0, \tag{2}
\]

The Faraday mode \( F \) follows a damped and driven Mathieu equation Eq.\(^\text{(1)}\) with cubic nonlinearity that describes the three-wave interaction process known to give rise to triangular broad-band power spectra in nonlinear Faraday waves\(^\text{[19, 21]}\). The frequency of the forcing is \( \Omega = 2 \) in the units of the eigenfrequency of the subharmonic Faraday mode \( w_0 = 1 \). The shape deformation mode \( v \), on the other hand, has a much smaller natural frequency \( \omega_0 \ll 1 \) than the Faraday mode and is not directly excited by forcing, as it can be seen in the slow-motion videos in the supplement, which clearly show that the drop almost “freezes” between bursting events.

To couple the two modes, we assume that the shape deformation mode \( v \) is linearly excited by the Faraday mode \( F \) and that the latter is suppressed by \( v \) via a quadratic nonlinearity, given by the term \( dv^2\dot{F} \). Other model parameters include the intrinsic damping coefficients \( c_1 \) and \( c_2 \), the parametric forcing amplitude \( A \) and the direct forcing amplitude \( b \).

We study periodic solutions of the system\(^\text{(12)}\) using a numerical continuation method\(^\text{[22]}\). The bifurcation diagram obtained by varying the forcing amplitude \( A \) is shown in Fig.\(^\text{3(a)}\). The first period-doubling (pd) bifurcation at \( A = 0.2 \) corresponds to the onset of the Faraday waves at the upper
FIG. 3. (a) Bifurcation diagram of system (1) and (2) with a varied forcing amplitude $A$ for $c_1 = 0.1$, $d = 1$, $b_1 = 0.1$, $c_2 = 0.05$ and $\omega_0^2 = 0.01$. The thick solid (dashed) lines correspond to stable (unstable) solution branches. The green circles are obtained from the direct numerical integration of (1) and (2). The labels pd, sn and tr denote the period-doubling, saddle-node and torus bifurcations, respectively. The insets in (a) show the solutions at the period-doubling bifurcation point and at $A = 0.4$, as indicated by arrows. The solid, dashed and dotted lines correspond to the Faraday mode, the shape deformation mode and forcing, respectively. (b) Power spectrum (psd) of the Faraday mode $F$ at $A = 0.4$.

The surface of the drop without the excitation of the shape deformation mode, as shown in the inset of Fig. 3(a). This point corresponds to the dashed line in Fig. 1(b). Slightly above the primary period-doubling bifurcation, Faraday waves undergo a secondary subcritical period-doubling bifurcation at $A = 0.35$. At this point, the drop enters the regime of irregular pulsations represented by the sequence of amplitude spikes in both the Faraday and volumetric modes (see the inset in Fig. 3(a)). This non-periodic regime can only be found by numerical integration of (1) and (2). The time-averaged $L_2$-norm of the solution is shown by the green circles in Fig. 3(a). The power spectrum of the Faraday mode contains one sharp peak at the driving frequency $\omega = \Omega = 2$ and a broadband background as shown for $A = 0.4$ in Fig. 3(b).

To identify the type of the intermittent bursting regime, we extract all peaks from the time series of the Faraday mode amplitude $F(t)$ at $A = 0.4$. The first and the second return maps of the inter-peak intervals $IPI(n)$ are shown in Fig. 4(a,b). We focus on the quiescent phase that is characterised by the inter-peak intervals close to $\pi$. The first return maps of the quiescent phase appear to cross the bisector line while the second return maps are parallel to it.

This feature is a signature of the type III intermittency [23]. We find a similar feature in the return maps of the inter-peak intervals with the duration larger than 40 ms extracted from the experimental data series shown in Fig. 4(c,d) [24].

To conclude, we have observed experimentally intermittent dynamic bursting in a vertically vibrated liquid drop surrounded by a less dense and more viscous liquid layer and supported by a hydrophobic solid plate. The physical mechanism triggering such a regime is identified as a competition between the Faraday mode and the shape deformation mode that is responsible for the horizontal elongation of the drop. The universal nature of the phenomenon is confirmed by detecting intermittency in organic (Soybean Sauce) and inorganic (water) liquid drops. The occurrence of the intermittent bursting regime is found to be highly sensitive to the variations of the oil layer thickness above the drop that diminishes during bursting events due to the action of large-amplitude Faraday waves mixing of the microscopic quantities of the drop liquid with oil.

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