Nucleon Transversity Properties Through $ep$ Scattering

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The $T$-odd distribution functions contributing to transversity properties of the nucleon and their role in fueling nontrivial contributions to azimuthal asymmetries in semi-inclusive deep inelastic scattering are investigated. We use a dynamical model to evaluate these quantities in terms of HERMES kinematics. We point out how the measurements of $\cos 2\phi$ asymmetry may indicate the presence of $T$-odd structures in unpolarized $ep$ scattering.

1 Introduction

It is widely recognized that the distributions in the azimuthal angle $\phi$ of the detected hadron in hard scattering processes provide interesting variables to study in both perturbative and non-perturbative regimes. They are of great interest since they test perturbative quantum chromodynamics (QCD) predictions for the short-distance part of strong interactions and yield important information on the long-distance internal structure of hadrons computed in QCD by non-perturbative methods. These nonperturbative contributions are parameterized effectively by introducing transverse (so-called “intrinsic” transverse motion)
degrees of freedom in the parton distribution and fragmentation functions. The
different combinations of the transverse momentum and spin results, in a rich
variety of information on the hadron spin structure. In particular, time re-
versal odd (T-odd) structures [1, 2, 3, 4, 5, 6] can appear – they filter the
novel transversity properties of quarks in hadrons. Such structures are acces-
sible through the azimuthal asymmetries in semi-inclusive and polarized spin
processes. Beyond the T-odd properties, the existence of these distributions
is a signal of the essential role played by the intrinsic transverse quark mo-
mementum and the corresponding angular momentum of quarks inside the target
and fragmenting hadrons in these hard scattering processes. In this paper we
analyze the T-odd transversity properties of quarks in hadrons which emerge
in semi-inclusive deep inelastic scattering (SIDIS). We apply these results to
predict cos 2φ [6] and sin(φ − φS) Sivers [1] asymmetries in terms of HERMES
kinematics. Also, we discuss how the measurements of cos 2φ asymmetry may
indicate the existence of T-odd structures in spin-independent semi-inclusive ep scattering.

2 T-Odd Distributions in Semi-Inclusive Reactions

Recently rescattering was considered as a mechanism for SSAs in pion electro-
production from transversely polarized nucleons. Using the QCD motivated
quark-diquark model of the nucleon [7, 8], the T-odd distribution function,
f_{1T}^T(x, k_{⊥}) \) (representing the number density of unpolarized quarks in trans-
versely polarized nucleons) and the corresponding analyzing power for the az-
mithal asymmetry in the fragmenting hadron’s momentum and spin distribu-
tions resulted in a leading twist nonzero Sivers asymmetry [9, 3]. Using the
approach in Ref. [9], we investigated the rescattering in terms of initial/final
state interactions to the T-odd function h_{1T}^T(x) and corresponding azimuthal
asymmetry in SIDIS [10, 11, 12]. The asymmetry involves the convolution with
the T-odd fragmentation function, h_{1T}^T(x) \otimes H_{1T}^T(z) [6]. The function h_{1T}^T(x, k_{⊥})
(representing the number density of transversely polarized quarks in unpolar-
ized nucleons) is complimentary to the Sivers function and is of great interest
theoretically, since it vanishes at tree level, and experimentally, since its deter-
mination does not involve polarized nucleons [6, 3, 10, 11, 12, 13].

The T-odd distributions are readily defined from the transverse momen-
tum dependent quark distributions [15, 3] where the well known identities for manipulating the limits of an ordered exponential lead to the expression

\[ \Phi^{[\Gamma]}(x, k_{\perp}) = \frac{1}{2} \sum_n \int \frac{d\xi^+ d^2 \xi_{\perp}}{(2\pi)^3} e^{-i(\xi^- + \xi_{\perp} \cdot \vec{k}_{\perp})} \times \langle P|\bar{\psi}(\xi^-, \xi_{\perp})G^\dagger(\infty, \xi)|n\rangle \Gamma(\infty, 0)|\bar{\psi}(0)|P\rangle \big|_{\xi^+ = 0} \]

(1)

and the path ordered exponential is

\[ G(\infty, \xi) = \mathcal{P} \exp \left( -ig \int_{\xi^-}^{\infty} d\xi^- A^+(\xi) \right) , \]

and \( \{|n\} \) are a complete set of states. While the path ordered light-cone link operator is necessary to maintain gauge invariance and appears to respect factorization [3, 9, 14] when transverse momentum distributions are considered, in non-singular gauges [9, 14], it also provides a mechanism to generate interactions between an eikonalized struck quark and the remaining target. These final state interactions in turn give rise to leading twist contributions to the distribution functions that fuel the novel SSAs that have been reported in the literature [8, 3, 9, 14, 10, 13, 11, 12]. It is worth mentioning that the \( T \)-odd distribution functions in SIDIS and Drell-Yan are not equal and have opposite signs [3, 16]. New experimental results will test the issue of universality of distribution functions.

The quark-nucleon-spectator model used in previous rescattering calculations assumed a point-like nucleon-quark-diquark vertex, which leads to logarithmically divergent, \( x \)-dependent distributions. To address the log divergence [8, 9, 10, 13, 11] we assume the transverse momentum dependence of the quark-nucleon-spectator vertex can be approximated by a Gaussian distribution in \( k_{\perp}^2 \) [12]. Performing the loop integration, and projecting the unpolarized piece from \( \Phi^{i(\sigma_{\perp} + \gamma_5)} \) results in the leading twist, \( T \)-odd, unpolarized contribution

\[ \Phi^{i(\sigma_{\perp} + \gamma_5)}_{[\Lambda_{\perp}]} = \frac{e_{+(-)} k_{\perp}^2}{M} h_{\perp}(x, k_{\perp}) \]

\[ = \frac{e_1 e_2 g^2 b^2}{2(2\pi)^4} \frac{m + x M}{\Lambda(k_{\perp}^2)} \frac{1 - x}{k_{\perp}^2} \times e^{-b(k_{\perp}^2 - \Lambda(0))}
\]

\[ \times \left[ \Gamma(0, b\Lambda(0)) - \Gamma(0, b\Lambda(k_{\perp}^2)) \right] . \]

(2)
Here, $e_1$ ($e_2$) is the charge of the struck quark (gluon-scalar diquark coupling), and $\Lambda(k^2_\perp) = k^2_\perp + (1 - x)m^2 + x\lambda^2 - x(1 - x)M^2$, where $M$, $m$, and $\lambda$ are the nucleon, quark, and diquark masses respectively. Also, $b = \frac{1}{\langle k^2_\perp \rangle}$, where $\langle k^2_\perp \rangle$ is fixed below. The average $k^2_\perp$ is a regulating scale which we fit to the expression for the integrated unpolarized structure function

$$f(x) = \frac{g^2}{(2\pi)^2} \frac{b^2}{\pi^2} (1 - x) \times \left\{ \begin{array}{c} (m + xM)^2 - \Lambda(0) \\ \Lambda(0) \end{array} \right\}$$

which multiplied by $x$ at $\langle k^2_\perp \rangle = (0.4)^2$ GeV$^2$ is in good agreement with the valence distribution of Ref. [17].

### 3 Azimuthal Asymmetries

We discuss the explicit results and numerical evaluation of the spin-independent double $T$-odd $\cos 2\phi$ and single transverse-spin $\sin(\phi - \phi_s)$ asymmetries for $\pi^+$ production in SIDIS. We use the conventions established in [6] for the asymmetries. Being $T$-odd, $h^\perp_1$ appears with the $H^\perp_1$, the $T$-odd fragmentation function in observable quantities. In particular, the following weighted SIDIS cross section projects out a leading double $T$-odd $\cos 2\phi$ asymmetry,

$$\frac{\langle P_{h^\perp_1,\perp} \rangle}{MM_h} \cos 2\phi\right|_{UU} = \frac{\int d^2 P_{h^\perp_1,\perp} \frac{P^2_{h^\perp_1,\perp}}{MM_h} \cos 2\phi \, d\sigma}{\int d^2 P_{h^\perp_1,\perp} \, d\sigma} = \frac{8(1 - y) \sum_q e_q^2 h^\perp_1(x)z^2 H^\perp_1(z)}{(1 + (1 - y)^2) \sum_q e^2_q f_1(x)D_1(z)}$$

where the subscript $UU$ indicates unpolarized beam and target. The non-vanishing $\cos 2\phi$ asymmetry originating from $T$-even distribution and fragmentation function appears at order $1/Q^2$ [18, 5, 19]. This consideration will be discussed in the next section.
Additionally, the SSA characterizing the so-called Sivers effect is

\[
\frac{\langle |P_{h\perp}| M \sin(\phi - \phi_S) \rangle_{UT}}{\int d^2 P_{h\perp} \frac{|P_{h\perp}| M \sin(\phi - \phi_S)}{d\sigma}} = \frac{(1 + (1 - y)^2) \sum_q e_q^2 f^{(1)}_{1T}(x) z D^q_1(z)}{(1 + (1 - y)^2) \sum_q e_q^2 f_1(x) D_1(z)},
\]

where the subscript \( UT \) indicates unpolarized beam and transversely polarized target. The functions \( h_i^{(1)}(x) \), \( f^{(1)}_{1T}(x) \), and \( H_i^{(1)}(z) \) are the weighted moments of the distribution and fragmentation functions [20].

In Figs. 1 and 2 the results from Ref. [12] for the \( \langle \cos 2\phi \rangle_{UU} \) and \( A_{UT}^{\sin(\phi - \phi_S)} \) for \( \pi^+ \) production on a proton target, evaluated for HERMES kinematics, are presented as a function of \( x \) and \( z \), respectively.
4 The phenomenology of $\cos 2\phi$ asymmetry

The effects that vanish as $M^2/Q^2$ are important at small and moderate values of $Q^2$. Such effects can arise in the Feynman-Bjorken model. The best known of these is the ratio of the longitudinal to transverse cross section [21]:

$$R = \frac{\sigma_S}{\sigma_T} = \frac{4((m^2 + (k_{\perp}^2 \pm \Delta^2))/Q^2),}{(6)}$$

where $m$ is the parton mass, and $\Delta$ an unknown correction due to possible parton-parton interactions. The ratio is zero in the standard limit of $Q^2 \to \infty$, but cannot be ignored for moderate $Q^2$.

The differential cross section for the process $ep \to e'hX$ involves four structure functions [22]:

$$d\sigma \propto W^{++} + (1-y)W^{00} + (1-y)W^{+-} \cos 2\phi + \sqrt{(1-y)(2-y)}Re(W^+) \cos \phi,$$

where $+,-,0$ correspond to photons with positive, negative, and zero helicity.
Figure 3: The $z$-dependence of the $\cos 2\phi$ asymmetry. The full and dotted curves correspond to the $T$-even and $T$-odd terms of asymmetry, respectively. The dot-dashed and dashed curves are the sum and the difference of those terms, respectively.

With spin 1/2 partons one gets [23]:

$$\frac{W^{00}}{W^{++}} = \frac{4(m^2 + \langle k^2 \rangle \pm \Delta^2)}{Q^2} \Rightarrow R, \quad (8)$$

$$\frac{W^{+-}}{W^{++}} = \frac{2(\langle k^2 \rangle_x - \langle k^2 \rangle_y \pm \Delta^2)}{Q^2} \Rightarrow \langle \cos 2\phi \rangle, \quad (9)$$

$$\text{Re} \frac{W^{0+}}{W^{++}} = \frac{\sqrt{2}(\langle k^2 \rangle \pm \Delta)}{Q} \Rightarrow \langle \cos \phi \rangle. \quad (10)$$

The result (8) corresponds to the ratio $R$ in Eq.(6). The predictions that the $\cos \phi$- and $\cos 2\phi$-dependences in the cross section behave like $Q^{-1}$ and $Q^{-2}$ should, in practice, be easier to test than the ratio in Eq.(8) or Eq.(6).
The $\langle \cos 2\phi \rangle$ from ordinary sub-sub-leading $T$-even and leading double $T$-odd (up to a sign) effects to order $1/Q^2$ can be written in the form

$$\langle \cos 2\phi \rangle_{UU} = \frac{2(\langle k^2 \rangle)(1-y)f_1(x)D_1(z) \pm 8(1-y)\tilde{h}_1^{+(1)}(x)H_1^{+(1)}(z)}{1 + (1-y)^2 + 2(\langle k^2 \rangle)(1-y)} f_1(x)D_1(z).$$

The $z$-dependences of this asymmetry are shown in Fig. 3. The full and dotted curves correspond to the $T$-even and $T$-odd terms in the asymmetry, respectively. The dot-dashed and dashed curves are the sum and the difference of those terms, respectively. From Fig.3 one can see that the double $T$-odd asymmetry behaves like $z^2$, while the $T$-even asymmetry is flat in the whole range of $z$. Therefore, aside from the competing $T$-even $\cos 2\phi$ effect, the experimental observation of a strong $z$-dependence (especially at high $z$ region) would indicate the presence of $T$-odd structures in unpolarized SIDIS implying that novel transversity properties of the nucleon can be accessed without involving spin polarization.

5 Conclusion

The interdependence of intrinsic transverse quark momentum and angular momentum conservation are intimately connected with studies of transversity. This was demonstrated previously from analyses of the tensor charge in the context of the axial-vector dominance approach to exclusive meson photoproduction [24]. We have analyzed the $T$-odd transversity properties in hadrons that emerged in SIDIS. We have considered the $\cos 2\phi$ asymmetry that appears in unpolarized $ep$ scattering and have pointed out how its measurements may indicate the presence of the asymmetric distributions of transversely polarized quarks inside an unpolarized nucleon. Furthermore, we have predicted a sizeable Sivers asymmetry at HERMES energies.

Beyond these model calculations it is clear that final state interactions can account for SSAs. In addition, it has been shown that other mechanisms, ranging from initial state interactions to the non-trivial phases of light-cone wave functions [8] can account for SSAs. These various mechanisms can be understood in the context of gauge fixing as it impacts the gauge link operator in the transverse momentum quark distribution functions [9, 14]. Thus using rescat-
tering as a mechanism to generate $T$-odd distribution functions opens a new window into the theory and phenomenology of transversity in hard processes.

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