A novel distributed event-triggered control for reactive power sharing based on hierarchical structure in islanded microgrid

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Abstract
Due to line impedance mismatches, nonlinear loads and other reasons, the traditional droop control algorithms have great limitations on the control of reactive power sharing. Distributed control algorithms based on hierarchical structure have become an effective approach for reactive power sharing compared with traditional centralized control methods. In this paper, an event-triggered control algorithm based on stability analysis of Lyapunov method is put forward in order to satisfy the demand of low-bandwidth communication for distributed generator in islanded microgrid. Subsequently, a distributed hierarchical control scheme adopting proposed event-triggered strategy is designed to achieve proportional reactive power sharing in an islanded microgrid. Finally, the feasibility and validity of the proposed algorithm are further verified in MATLAB/Simulink environment.

Keywords
Consensus algorithm, event-triggered condition, hierarchical control, reactive power sharing

Introduction
The concept of microgrid refers to a small-scale low-voltage power system which usually contains renewable energy resources, storage systems, distributed generators (DGs) and various loads. In the case of islanded microgrid with inductive line impedance, the droop control methods including the active power to frequency regulation and reactive power to voltage amplitude regulation are usually employed to realize power sharing. The droop control has good robust characteristic and works autonomously. However, the control of DGs is facing some challenging issues such as the diversity of rated power, output impedance, hardware filters, feeder impedance and so on, which may worsen the power sharing and cause the faults of overcurrent and overload. Generally, the active power sharing and frequency control can be regulated well by some amended droop control algorithms, but the performance of reactive power sharing is still poor under unmatched feeder impedance condition. Accordingly, how to ensure the proportional distribution of reactive power is a key factor to improve the stability of islanded microgrid. Basically, the centralized control and distributed coordinated control are the two main control ways in hierarchical control structure. But the most obvious shortcoming of the centralized control is the low reliability, for a single communication failure may cause the islanded microgrid to crash in small-scale DG group. In contrast to the centralized control structure, the distributed coordinated control approach based on hierarchical control structure is more suitable for islanded microgrid and has been widely applied. The block diagram of distributed hierarchical coordination control structure for DG units in the islanded microgrid can be basically described in Figure 1.

In Figure 1, the main function of primary control layer keeps the output voltage amplitude and angular frequency of DG stable according to voltage and angular frequency reference based on droop control algorithm. In the secondary control layer, individual DG is usually regarded as an independent intelligent agent node. Each node only communicates with its
neighboring nodes, in this way a sparse communication network with n nodes is formed. Distributed consensus algorithms are designed to rely on the sparse communication network to acquire global average reactive power and regulate reactive power sharing by modifying the voltage reference in primary control layer.

It has been generally acknowledged that periodic communication may cause excessive consumption of communication resources of DGs, particularly when the distributed control system approaches equilibrium point.14 Therefore, the traditional consensus algorithms based on periodic trigger mode are difficult to implement in practice, especially when the communication channels and communication bandwidths of DG are limited. Distributed event-triggered consensus control algorithms can reduce the occupation of communication resources and decrease the communication bandwidth effectively.12–20 These event-triggered algorithms are all implemented in a predefined event-triggered situation related to system output or states. For example, if the estimation error of specified DG is beyond the set range, the event-triggered situations will be activated and the DG will trigger communication with its neighbors’ DGs.

In the last few years, the event-triggered consensus control algorithms have been wildly applied in isolated microgrids.12–17 21,22 For example, different event-triggering algorithms are proposed and used to control voltage stability or reactive power in Chen et al.,13 Lai et al.14 and Zhang et al.,15 but these proposed schemes are droop-based and are unable to work normally when communication failure occurs. In Lai et al.14 and Zhang et al.,15 two distributed event-triggering schemes based on droop control are put forward to control frequency and active power sharing, respectively. In addition, a need-based aperiodic information communication scheme is presented to control load sharing in Meng et al.,16 but the formula derivation based on event-triggering control is inaccurate, and the threshold of trigger condition is not optimal. According to the Lyapunov stability theory, Gao et al.17 analyzed the triggered condition in detail, but an important term in its triggered condition has been ignored.

Inspired by the previous results, this paper further studies and optimizes the triggered condition which only depends on local state information including neighbors’ nodes is established, and the corresponding analysis is provided. Secondary, the plug-and-play function is a basic request for DGs in parallel in island microgrid; with this function, the reliability of power supply can be greatly improved. Therefore, a distributed hierarchical control approach supporting plug-and-play to reduce communication bandwidth is designed for reactive power sharing in islanded microgrid.

The reminders of this paper are organized as follows. The consensus algorithm analysis and event-triggered condition are studied in section “Consensus algorithm analysis and event-triggered condition.” The distributed hierarchical control scheme based on the recommended algorithm is discussed in section “Distributed hierarchical control for reactive power sharing.” The two different simulation cases are studied and the simulation results are discussed in section “Simulation analyses.” The conclusions and the proof of theorem are given in section “Conclusion.”

**Consensus algorithm analysis and event-triggered condition**

**Preliminaries of algebraic graph theory**

The general approach of graph theory usually regards the communication network formed by n intelligent agent nodes as an undirected graph. Let $G = \{N, E\}$ represents the undirected graph, in which $N = \{1, 2, \ldots, n\}$ is the set of nodes and $E \subseteq N \times N$ is the set of edges. If $(i, j) \in E$, $e_{ij} = e_{ji} = 1$, otherwise, $e_{ij} = e_{ji} = 0$. A Laplace matrix $L = l_{ij} \in \mathbb{R}^{N \times N}$ closely related to the undirected graph is set by $l_{ij} = \sum_{k=1}^{n} \delta_{ij} e_{ik}$, and $l_{ii} = -\sum_{k=1}^{n} e_{ik}$. In addition, let $A = I - e L = (a_{ij}) \in \mathbb{R}^{N \times N}$. Finally, the neighbor set of node $i$ is indicated by $N_i = \{j \in N | e_{ij} \in E\}$.

For any undirected graph $G = \{N, E\}$, the adjacency matrix $E$, the Laplace matrix $L$ and the matrix $A$ are all symmetrical. Taking a bidirectional communication framework with four nodes as an example is given in Figure 2. Its set of nodes is $N = \{1, 2, 3, 4\}$, then corresponding matrix $A$ can be represented as
Analysis of consensus algorithm with non-periodic trigger

With the development of distributed consensus cooperation with event-triggering function, some related research works have been studied. The general first-order discrete protocol is usually defined as follows (see, for example, Meng et al.\textsuperscript{16} and Gao et al.\textsuperscript{17})

$$x_i(k + 1) = x_i(k) + u_i(k)$$

where $x_i(k) \in \mathbb{R}$ is the state variable of node $i$, and $u_i(k)$ is the control protocol of node $i$ at $k$th step.

A usual practice in solving the problem of average consensus of node $i$ is defined by Olfati-Saber et al.\textsuperscript{18}

$$u_i(k) = e \sum_{j \in N_i} e_{ij} [x_j(k) - x_i(k)]$$

where $0 < \epsilon < 1/\Delta$ is the step size with $\Delta = \max_{i,j \in V} \min_{1 \leq k \leq n} e_{ij}$. For instance, $\Delta$ should be 2 for the communication framework shown in Figure 2.

As mentioned earlier, the traditional consensus algorithm exchanges the communication information based on fixed time-triggered fashion, that is, each node sends its latest state to its neighbors in a fixed cycle. However, this periodic control fashion is difficult to apply in complex microgrid condition, because it requires high communication bandwidth, which may cause excessive consumption and waste of communication resources.

In order to reduce the occupation of communication resources and improve the applicability of consensus algorithm, one of the hotspots of current research works is concentrated on non-periodic distributed consensus algorithm by which the communication transmission can be triggered only when the specified trigger conditions are satisfied. In this way, any node $i$ will send its latest state to its neighbors only when the event is triggered at some time instants, which can be indicated in ascending order by $t_{i1}^{e}, t_{i2}^{e}, \ldots, t_{ik}^{e}, \ldots$. Once a pre-design event condition inequality is satisfied, the time instant $t_{ik}^{e}$ will be determined, at the same time node $i$ updates its control protocol by implementing its own state estimation and transmits its latest state to all its neighbors. Therefore, as an amendment to the event-trigger algorithm, the control protocol $u_i(k)$ is modified to calculate based on own last state estimation information and the information received from its neighbors which can be represented as node $j \in N_i$ as follows

$$u_i(t) = e \sum_{j \in N_i} e_{ij} \left[ \hat{x}_j(t_{jk}^{e}) - \hat{x}_i(t_{ki}^{e}) \right] , \quad t \in [t_{ki}^{e}, t_{ki}^{e} + \epsilon)$$

where $t_{ik}^{e}$ and $t_{jk}^{e}$ mean the time instant of the $k$th event for node $i$ and node $j$, respectively, and $\hat{x}_j(t_{jk}^{e})$ and $\hat{x}_i(t_{ki}^{e})$ represent the estimating values of node $i$.

At the time interval $[t_{ki}^{e}, t_{ki}^{e} + \epsilon)$, the control protocol of node $i$ will remain unchanged until the next trigger condition is satisfied again or any neighbor triggers an event. The relationship between control protocol and node trigger based on Figure 2 is shown in Figure 3 to illustrate the difference between event-triggered consensus algorithm and traditional periodic consensus algorithm. As shown in Figure 2, the node 1’s neighbors are node 2 and node 4. Setting trigger time of node 1 is at 0, 4Ts, and the trigger time of node 2 and node 4 are 0, 5Ts and 0, 2Ts, respectively. According to formula (4), it is easy to find that the earliest control protocol $u_1(t_{11}^{e})$ of node 1 is $e[x_4(0) + x_4(0) - 2x_4(0)]$ and it keeps unchanged until the node 4 triggers at 2Ts, which leads to its use of $x_4(2)$ instead of $x_4(0)$. Then, $x_4(2)$ is transferred from node 4 to node 1 and causes $u_1(t_{12}^{e})$ to become $e[x_4(2) + x_4(2) - 2x_4(2)]$. At 4Ts, the state $x_1(0)$ is replaced by its latest state $x_1(4)$, which causes $u_1(t_{13}^{e})$ to become $e[x_1(4) + x_4(2) - 2x_4(4)]$. $u_1(t_{14}^{e})$ keeps unchanged until another event in node 2 is triggered at 5Ts. Then, $x_2(5)$ is transferred from node 2 to node 1 and causes $u_1(t_{14}^{e})$ to become $e[x_2(5) + x_4(2) - 2x_4(4)]$ and its value still remains constant until any event in nodes 1, 2 and 4 is triggered again.

The design of event-triggered condition

Event-triggered condition should be designed reasonably to achieve fast convergence of average consensus algorithms as well as reduction in usage of communication resources. Generally, the trigger condition is determined by detecting measurement error which is defined as the degree that the estimated state deviates from its actual state. A variable named estimation error for node $i$ is defined as the difference between the estimated value $\hat{x}_i(t_{ik}^{e})$ and the actual value $x_i(t)$

$$\hat{e}_i(t) = \hat{x}_i(t_{ik}^{e}) - x_i(t), \quad t \in [t_{ki}^{e}, t_{ki}^{e} + \epsilon)$$
Once the estimation error exceeds a predetermined threshold value, the communication event will be triggered instantly and then the triggered node will update its local measurement state and broadcast its latest state information to neighboring nodes. This paper proposes the following event-triggering condition

\[

e_i^2(t) \geq \frac{a_i^2}{4(1 - a_i)} \sum_{j \in N_i} a_{ij} (\hat{x}_i(t_{kj}) - \hat{x}_j(t_{kj}))^2 + \frac{a_i}{2(1 - a_i)} \sum_{j \in N_i, j > i} a_{ij} a_{ji} (\hat{x}_i(t_{kj}) - \hat{x}_j(t_{kj}))^2
\]

for every node \(i \in \mathbb{N} \) and time instants \( t \in [t_{kl}, t_{kl+1}) \).

**Theorem 1.** Provided under the condition (6), the consensus of the communication network meeting the iteration equation (2) with the control protocol (4) can be achieved, then all nodes will converge to the average value of the initial values of all agent nodes, that is

\[
\lim_{t \to \infty} x_i(t) = \frac{1}{n} \sum_{j=1}^{n} x_j(0)
\]

The proof of Theorem 1 is presented in Appendix 1.

As long as the inequality of condition (6) is established, the event at node \( i \) will be triggered. It can be seen that the state information needed for the trigger condition is all local, and is entirely supported by distributed control occasion where the global information is not available. In addition, the first term in the proposed event-triggered condition indicates the relationship between the node \( i \) and its neighbor named node \( j \), while the second term indicates the relationship among different neighbors of node \( i \).

What needs special explanation is that because \( a_{ij} \) is mainly determined by the Laplace matrix \( L \), if node \( i \) (\( i \in \mathbb{N} \)) rejoins and leaves the communication network, condition (6) can support the function of plug-and-play. Otherwise, a node \( i \) (\( i \notin \mathbb{N} \)) joins the communication network, and the function of plug-and-play in condition (6) will not work.

**Distributed hierarchical control for reactive power sharing**

In the primary layer, considering the power difference of different DGs, according to Han et al.\(^6\) the droop control based on the vector control for DG\(_i\) can be mathematically designed as

\[
\begin{align*}
\omega^{*}_i &= \omega_0 - k_{pu} \frac{p_0}{X_i} \\
V^{*}_{di} &= V_0 - k_{pq} \frac{q_i}{X_i} \\
V^{*}_{qi} &= 0
\end{align*}
\]

where \( \omega^{*}_i \) and \( \omega_0 \) are the angular frequency reference and the rated angular frequency, respectively. \( V^{*}_{di} \) and \( V^{*}_{qi} \) are the voltage reference in \( d \)-axis and \( q \)-axis, respectively. \( V_0 \) is the rated voltage. \( k_{pu} \) and \( k_{pq} \) are the dropping gain terms. \( \chi_i \) is the weight coefficient, generally speaking, an actual setting for \( \chi_i \) is to make it equal to \( S_i \), which is the nominal power of DG\(_i\).

In the distributed secondary layer, each DG is considered as an agent node to exchange the information of respective reactive power with neighboring nodes, and process the data to acquire global average value for proportional reactive power sharing. Taking the possible nominal power differences of DGs into account, for any two nodes \( i \) and \( j \), the proportional reactive power sharing will be achieved if the equation of

\[
\frac{Q_i}{\chi_i} = \frac{Q_j}{\chi_j}
\]

is satisfied.\(^1^2\) Accordingly, the state variables of \( x(k) = Q_i/\chi_i \) are defined at any node \( i \) for reactive power sharing. Inspired by reactive compensation approach in Schiffer et al.,\(^7\) the reactive power correction term \( \Delta \hat{V}_i \) in voltage reference based on event-triggered algorithm can be written as

\[
\Delta \hat{V}_i = e \sum_{j \in N_i} e_{ij} [\hat{x}_j(t_{kj}) - \hat{x}_i(t_{ki})]
\]

According to equations (8) and (9), the reference of output voltage and the reference of angular frequency in primary control layer combining with the output of the secondary layer with event-triggered control can be written as

\[
\begin{align*}
\omega_i^* &= \omega_0 - k_{pu} \frac{p_0}{X_i} \\
V_{di}^* &= V_0 - k_{pq} \frac{q_i}{X_i} + e \sum_{j \in N_i} e_{ij} [\hat{x}_j(t_{kj}) - \hat{x}_i(t_{ki})] \\
V_{qi}^* &= 0
\end{align*}
\]

It should be pointed out that in the above formula, the updating of \( e \) and \( e_{ij} \) depends on the communication topology. Once the communication changes, their variable updates need to be synchronized. Such design denotes that the traditional droop control has been modified by equivalent term of reactive power to voltage derivative, which can enhance dynamic performance and reactive power allocation accuracy.

The block diagram of distributed hierarchical control based on proposed event-triggered consensus algorithm for reactive power sharing is illustrated in Figure 4, in which \( G_i(s) \) and \( G_{qs}(s) \) are the voltage feedback controller and the current feedback controller, respectively. Distributed secondary control layer includes proposed event-triggered consensus algorithm and exchanges information with neighboring DGs and acquires the global average value of the ratio of reactive power to rated nominal power capacity. Based on the proposed event-triggered algorithm, the compensation value for the regulation of voltage reference is transferred to primary control layer to regulate the reactive power allocation in real time.
Simulation analyses

To demonstrate the performance of the proposed control strategy, a simulation model with s-function module is built in the MATLAB/Simulink environment. The communication data can be exchanged according to Figure 2. Considering the influence of $\chi$, the effective droop gain for both $k_{pv}$ and $k_{pq}$ is set to 0.05, and $\varepsilon$ is set to 1/3. The main parameters of the simulation model are listed in Table 1.

The following two simulation examples are shown to test and verify the performance of the recommended reactive power sharing control protocol.

Case 1: load variation confirmation

In this case, the ratio of rated power capacity for DG1, DG2, DG3 and DG4 is set to 2:2:1:1. The whole simulation process is divided into four states. Stage 1 (0–2 s), only the primary control level works; the secondary control layer does not work; stage 2 (2–2.2 s), the four DGs all operate with recommended secondary control layer; stage 3 (2.2–2.4 s), an additional nonlinear load suddenly is connected to the simulated island microgrid, meanwhile the proposed secondary control layer works normally; and stage 4 (2.4–2.6 s), the additional nonlinear load suddenly detaches from the simulated island microgrid, meanwhile the proposed secondary control layer still works normally.

Figure 5 shows reactive power sharing performance with proposed control protocol under load variation condition. According to the simulation results, the proposed control protocol in secondary layer can carry out proportional reactive power distribution well and can also achieve good dynamic performance under the condition of load variation. For comparison, Figures 6 and 7 denote the event time instants of four nodes using proposed event-triggered approach and traditional time-triggered approach, respectively. It should be noted that for the conventional periodic-triggered algorithms, a 5 ms interval is considered. Comparing the simulation results of Figures 6 and 7, the number of triggers proposed in this

Table 1. Main simulation parameters.

| Parameter | Value |
|-----------|-------|
| $G_v(s)$  | $0.24\,s + 0.1\,s$ |
| $G_i(s)$  | $12.5\,s + 0.8\,s$ |
| Voltage reference | 380 V |
| Feeder impedance of DG1 | 1.2 + j2.5 $\Omega$ |
| Feeder impedance of DG2 | 0.8 + j1 $\Omega$ |
| Feeder impedance of DG3 | 0.5 + j1 $\Omega$ |
| Feeder impedance of DG4 | 0.3 + j0.6 $\Omega$ |
| Load | $17.3 - j6.89\,\Omega$ |

DG: distributed generator.
paper is much smaller than that of traditional periodic-triggered algorithms.

**Case 2: DG variation confirmation**

In this case, the four DGs are set to have the same rated power capacity. The whole simulation process is also divided into four states. Stage 1 (0–2 s), the four DGs still only use droop algorithm to supply power to the load jointly, while the secondary control layer does not work; stage 2 (2–2.2 s), the four DGs all operate with recommended secondary control layer; stage 3 (2.2–2.4 s), DG4 is disconnected from the islanded system, meanwhile the remaining three DGs still adopt proposed secondary control layer; stage 4 (2.4–2.6 s), DG4 is connected to the islanded system again, and the four DGs all use proposed secondary control layer.

Figure 8 shows reactive power sharing performance with proposed control protocol under DG variation condition. Based on the simulation results, the proposed event-triggered approach in secondary layer can still implement the task of reactive power sharing well under the condition of DG increases or decreases. In addition, Figure 9 represents the event time instants of the four DGs in this case. When the proposed control protocol begins to work or when the number of work DGs changes, the number of communication events is also increased relative to steady state, but the trigger number is still small compared with Figure 7.

According to above simulation results, it is worthy to point out that the proposed control protocol can achieve the satisfactory reactive power sharing performance and reduce the communication times effectively.

**Conclusion**

In this paper, an event-triggered consensus algorithm based on the Lyapunov stability analysis approach is proposed to control the reactive power sharing. It has proved that the proposed control method can not only save the communication resources but also support the plug-and-play function within the established communication topology. Finally, the feasibility and validity of the proposed algorithm for reactive power sharing in islanded microgrid are further verified by the simulation of MATLAB.

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Appendix I

The proof of Theorem 1

Under the condition of setting the Lyapunov candidate $V(t) = x^T(t)x(t)$, referring to the derivation of Gao et al., the difference between $V(t + 1)$ and $V(t)$ can be written as

$$\Delta V(t) = \Delta V(t + 1) - \Delta V(t) = -2\sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} a_{ij} \hat{e}_{ij}(t) \left( \hat{x}_{ij}(t_{ij}) - \hat{x}_{ij}(t_{ij}) \right)$$

(11)

Turn the left side of the above formula into a plus sign and get the following inequality

$$\Delta V(t) \leq 2\sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} a_{ij} \hat{e}_{ij}(t) \left( \hat{x}_{ij}(t_{ij}) - \hat{x}_{ij}(t_{ij}) \right) + \frac{1}{\beta_{ij}} \left( \hat{x}_{ij}(t_{ij}) - \hat{x}_{ij}(t_{ij}) \right)^2$$

(12)

According to the Young’s inequality $2xy \leq \beta_1 \|x\|^2 + \frac{1}{\beta_2} \|y\|^2$, where any $\beta_1 > 0$, then in above inequality, one can be written as

$$2\sum_{j=1,j\neq i}^{n} a_{ij} \hat{e}_{ij}(t) \left( \hat{x}_{ij}(t_{ij}) - \hat{x}_{ij}(t_{ij}) \right) \leq \sum_{j=1,j\neq i}^{n} a_{ij} \left( \beta_1 \hat{e}_{ij}^2(t) \right) + \frac{1}{\beta_1} \left( \hat{x}_{ij}(t_{ij}) - \hat{x}_{ij}(t_{ij}) \right)^2$$

(13)

Notice that $\sum_{j=1,j\neq i}^{n} a_{ij} = 1$ is satisfied, and subsequently, the following equations can be obtained

$$\sum_{j=1,j\neq i}^{n} a_{ij} \beta_1 \hat{e}_{ij}^2(t) = (1 - a_{ij}) \beta_1 \hat{e}_{ij}^2(t)$$

(14)

From (13) and (14), the simplified inequality (12) can be written as

$$\Delta V(t) \leq \sum_{i=1}^{n} (1 - a_{ij}) \beta_1 \hat{e}_{ij}^2(t) + \sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} a_{ij} \left( \frac{1}{\beta_{ij}} - a_{ij} \right) \left( \hat{x}_{ij}(t_{ij}) - \hat{x}_{ij}(t_{ij}) \right)^2$$

(15)

In accordance with LaSalle’s invariance theorem, the inequality $\Delta V(t) < 0$ must be satisfied, and then the following inequality is obtained
Consider improving the efficiency of event-triggered algorithm and saving communication resources, it can be proved that only when $\beta_i = 2/\alpha_{ii}$, the algebraic formula $(\alpha_{ii} - 1/\beta_i) / (1 - \alpha_{ii})$ will take the maximum value. The proof is completed.

In order to guarantee the stability condition of equality (16) is satisfied, the event-triggered condition is designed as follows

$$
\dot{e}_i(t) < \frac{a_{ii} - 1/\beta_i}{(1 - a_{ii})\beta_i} \sum_{j=1, i\neq j}^{n} a_{ij} \left( \dot{x}_i(t_j) - \dot{x}_i(t_{ki}) \right)^2
+ \frac{1}{(1 - a_{ii})\beta_i} \sum_{j=1, j \neq i}^{n} \sum_{l=1, l \neq i}^{n} a_{ij} a_{il} \left( \dot{x}_j(t_j) - \dot{x}_i(t_{kl}) \right)^2
$$

(16)

Consider improving the efficiency of event-triggered algorithm and saving communication resources, it can be proved that only when $\beta_i = 2/\alpha_{ii}$, the algebraic formula $(\alpha_{ii} - 1/\beta_i) / (1 - \alpha_{ii})$ will take the maximum value. The proof is completed.

In order to guarantee the stability condition of equality (16) is satisfied, the event-triggered condition is designed as follows

$$
\dot{e}_i(t) > \frac{a_{ii} - 1/\beta_i}{(1 - a_{ii})\beta_i} \sum_{j=1, i\neq j}^{n} a_{ij} \left( \dot{x}_i(t_j) - \dot{x}_i(t_{ki}) \right)^2
+ \frac{1}{(1 - a_{ii})\beta_i} \sum_{j=1, j \neq i}^{n} \sum_{l=1, l \neq i}^{n} a_{ij} a_{il} \left( \dot{x}_j(t_j) - \dot{x}_i(t_{kl}) \right)^2
$$

(17)