Prediction of stability against subsonic flutter for axial turbine machine compressor blade assemblies

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Abstract. The developed method for prediction of their dynamic stability limit is described for the first flexural and torsional modes of vibrations against subsonic flutter. It involves the use of the database of the critical reduced frequencies of the blade vibrations as a dynamic stability limit within the wide range of variations of the following characteristics: phase angle of flexural, torsional and flexural-torsional vibrations of the adjacent blades; attack angle; reduced vibration frequency; geometric parameters for the airfoil and cascade; law of motion for blade airfoils. The algorithm realized as the numerical software assumes the determination of the critical reduced frequencies in the most loaded section of the full-scale blade assembly via the analysis of the distribution of the relative flow rate, attack angle, amplitudes of vibrations over the blade height and the subsequent comparison with the functional defining its critical value for different attack angles. The results of practical implementation of the developed procedure are given for the assessment of the stability of the blade assemblies of axial compressors in some modern aircraft gas-turbine engine at the first flexural mode of vibrations. It is implied that the proposed method allows one to select the optimal reduced frequency of vibrations of the blade assembly for the specified geometry of its peripheral sections and attack angle of the mainstream flow upon the condition of subsonic flutter at the engine design state.

1. Introduction

The modern design of axial turbine machines tends to ensure higher specific power and best performance with simultaneous reduction of their dimensions and weight that are responsible for a high level and wide spectrum of active loads. This results in growing dynamic load levels of axial-flow compressor blades, and self-excited vibrations (flutter) of single blades and their cascades become most probable. Therefore, the development of a computation procedure to assess the flutter stability of compressor blades, in particular, aircraft gas-turbine engines (AGTE), under different operating conditions at the stage of their design becomes currently topical. The solution of this problem allows one to eliminate considerable material and time costs in the development of engines.

The following methods are known to assess the subsonic cascade flutter stability of GTE compressor blades: statistical, numerical and experimentally-computational.

The statistical methods are based on empiric [1–4] or semiempirical [4–6] approaches. The former is built upon the analysis of statistical bench and full-scale test data for engines and their compressors, the latter is based on the results [7] obtained with experimental unsteady aerodynamic characteristics of a single wing profile [8] or the direct cascade of blade profiles for large angles of attack [9]. These methods are used for the determination of the blades’ stability with the initial geometry and similar
aerodynamic flow conditions. Therefore, in many cases, they cannot be directly applied to determining the stability limits of new engine blades. Moreover, data extrapolation would be required, which in some cases leads to inadequate results [6].

The numerical methods are based on the solution of the Euler and Navier–Stokes equations with different turbulence models [10–16]. Computerization permits numerical methods to be considered as priority ones since they can replace expensive tests and solve 2D [10–12] and 3D [13–16] problems independently on the flow conditions. However, those methods used for solving the problem of dynamic blade stability in the steady statement cannot satisfactorily describe the current phenomena, and simulation of the blade flow is mainly based on the Navier–Stokes equation normalized by the Reynolds number [13]. The solution to this system of equations in the closed-form makes use of different turbulence models. Their great number implies that there is no such one that could adequately describe the areas of continuous and separated blade flows at the moment. It should be noted that the reliability of numerical results is mainly dependent on the choice of the computation mesh determining the area of blade-flow interaction. At present, orthogonal and immobile adaptive meshes, as well as the complex of immobile and mobile ones, are mainly used. However, such approaches cannot characterize the curvilinear limits adequately or the reconstruction and coordination of flow and blade meshes bring about great difficulties [17, 18].

The experimental-computational method of the determination of the stability limit of the blades to prevent subsonic flutter is as following. The aerodynamic loads on the cascades of blade profiles are evaluated based on the experimental aerodynamic influence coefficients (AIC) [19]. Then the calculation of the stability limit for the blade assembly is performed, for instance, via the solution to the eigenvalue problem [8]. Its main drawback lies in the necessity to conduct the tests for the blade cascades at different angles of attack.

Based on the generalization on the experimental investigations on aerodamping and aeroexcitation of straight cascades of the compressor blade assemblies performed at the G.S. Pisarenko Institute for Problems of Strength of the NAS of Ukraine within the rather broad range of the variations in the angle of attack, reduced frequency and phase shifts for their flexural, torsional and flexural-torsional vibrations of the blade profiles, as well as geometric cascade characteristics, the basic tenets of the procedure for prediction of the stability of the blade assemblies against subsonic flutter [20] have been developed. The purpose of the paper lies in the system description of the basic statements of the developed procedure and results of its practical implementation for the stability analysis to prevent the subsonic flutter of the blade assemblies of the number of stages of some AGTE compressors.

2. Physical fundamentals of prediction of subsonic flutter of blade assemblies

The authors determined the subsonic flutter initiation conditions using the data on aerodamping of AGTE compressor blade vibrations obtained in the experimental investigations on the straight cascades of blade profiles in a subsonic flow under their flexural [20–22], torsional [24, 25] and flexural-torsional vibrations at the phase shift of 180° between translational and angular displacements [25]. The model of the straight cascade is given in Fig. 1. The authors obtained the two-parameter relations of the aerodynamic decrement (increment) $\delta_a$ from the phase shift $\mu$ of the vibrations of the profiles $K = \omega b/V_1$ at the specified attack angle $i$ and the angle of phase shift $\mu$ – the angle of attack $i$ at the specified reduced frequency $K$, where $\omega$ is the circular frequency of blade vibrations. As an example, Fig. 2 illustrates the relations $\delta_a(\mu, K)$ for $i = 13^\circ$, while Fig. 3 – $\delta_i(\mu, i)$ for $K = 0.4$ with the flexural-torsional coupling coefficient $\psi = (a_p - a_t)/(a_p + a_t) = 0.26$, where $a_p$ and $a_t$ are the leading and trailing blade edge displacement amplitudes, respectively. Analysis of those relations allows one to formulate the mechanisms of the influence of the reduced frequency and angle of attack on the aerodamping of vibrations and subsonic flutter initiation in compressor blade cascades, namely:

1. The relations of the aerodynamic decrement against the phase shift of blade vibrations are of smooth periodic nature with its well-defined minimum and maximum values.
2. With a decrease in the reduced vibration frequency at fixed angles of attack and geometric cascade parameters (spacing and stagger angle), the minimum vibration decrement value comes down and the maximum one grows (Fig. 2). In a qualitative sense, a similar phenomenon is observed at the growing angles of attack (Fig. 3).

3. Such combinations of reduced frequency and angle of attack values are possible when the decrement equals zero (plane A, Figs. 2 and 3). This state is consistent with the critical reduced vibration frequency values, which defines the dynamic stability limit of the blade assembly to prevent subsonic flutter.

4. A decrease in the reduced vibration frequency below the critical level and an increase in the angles of attack lead to the phase shift interval extension wherein negative aerodamping, i.e., aeroexcitation of blade vibrations, occurs with an increase in its level (region of vibration decrement values below the plane A, Figs. 2 and 3).

Moreover, the results of the performed tests of the straight cascades of the blade assemblies implied the following:

— variation of geometric parameters of the blade profile, such as curvature, wall thickness, and coordinates of their maximum relative value, as well as of the relative spacing and stagger angle, does not qualitatively influence the relation of the aerodynamic decrement against the phase shift of blade vibrations, it only changes the critical reduced vibration frequency value;

— for flow conditions of the blades, which are consistent with the aeroexcitation of vibrations even in a narrow range of the angle of the phase shift, blade displacements become unsteady;

— relation of the critical reduced vibration frequency against the angle of attack for its positive and negative values is monotone increasing. The profile and cascade geometry, as well as blade vibration modes, cause only the change in the intensity of critical reduced vibration frequency growth;

— flutter initiation conditions become most favorable in the case of a uniform cascade when blade vibrations exhibit the same phase shift. Detuning of blade vibration frequencies determined its difference, as a result, at the given reduced vibration frequency and angle of attack, the dynamic cascade stability increases. With a decrease in the reduced vibration frequency and an increase in the angle of attack, the effect of detuning on an increase in the dynamic cascade stability comes down.

![Figure 1. Model of straight cascade of blade airfoils.](image)

\(b\) – airfoil chord, \(t\) – cascade spacing, \(\beta\) – stagger angle, \(i\) – angle of attack, 
\(V_1\) – in-flow velocity
3. Basic principles of the rapid method of predicting the dynamic blade assembly stability

The blade vibration aerodamping mechanisms are the basis for the rapid method of predicting the dynamic compressor blade cascade stability against flutter (hereinafter referred as rapid method).

The most loaded blade assembly sections are taken as those controlling its stability. The selection of the most loaded sections is performed by analyzing the distributions of the relative flow velocity, angle of attack, vibration amplitudes throughout the height of the airfoil. For the first flexural and torsional modes of vibrations of the blades, the most loaded sections are cylindrical sections located as 0.75-0.95 height $h$ of their airfoil. This is explained by the fact that under these modes of vibrations 80% of flow energy (approximately) falls on the peripheral part of the airfoil, which is about 20% of its height. Therefore, the critical parameters of cascade profile vibrations corresponding to a chosen peripheral section of the airfoil would also determine the stability of blade vibrations by the first flexural and torsional modes with a high probability.

Thus, the dynamic stability criterion of the blade assembly is the accomplishment of the following inequality for the most loaded sections of the blade airfoil under different modes of AGTE operation:

$$K^B \geq K_{cr},$$

where $K^B$ is the reduced vibration frequency of the blade assembly under consideration; $K_{cr}$ is the critical value for the blade airfoil corresponding to the considered section at various angles of attack.

Based on the generalization of the known data of the investigations on the influence of the geometric parameters of the blade assemblies and flow conditions on their stability limit to prevent subsonic flutter, let us introduce the critical value of the reduced vibration frequency as the functional:

$$K_{cr} = K(i, \beta, \bar{T}, \psi)K(\bar{\varepsilon}, \bar{\tau}, \bar{\tau}_0, \bar{\tau}_c)K_MK_fK_\delta,$$

where $K(i, \beta, \bar{T}, \psi)$ is the functional representing the effect of the angle attack and geometry of cascade (stagger angle and relative spacing); $K(\bar{\varepsilon}, \bar{\tau}, \bar{\tau}_0, \bar{\tau}_c)$ is the corrective functional considering the effect of the geometric profile parameters: curvature $\bar{\varepsilon}$, relative wall thickness $\bar{\tau}$, coordinates of the maximum curvature $\bar{\tau}_0$ and thickness $\bar{\tau}_c$; $K_M$, $K_f$, $K_\delta$ are the coefficients considering the effect of the Mach number, blade frequency mistuning, and mechanical damping of blade vibrations due to energy dissipation in the material and lock joint, respectively.

The functional $K(i, \beta, \bar{T}, \psi)$ is selected using the critical reduced vibration frequency from the database of the critical reduced vibration frequency obtained for different relative spacing, stagger angle and angle of attack, as well as flexural-torsional coupling coefficient.
4. Critical reduced vibration frequency database

The critical reduced vibration frequency database is presented as a table with the specified flexural-torsional coupling coefficient for the selected values of the relative cascade spacing $\bar{t}$, angle of stagger $\beta$ and angle of attack $i$. The example of the table is illustrated in Fig. 4 for the first flexural mode of the blade vibrations $\psi = 0.48$.

| $\beta$, deg | $\bar{t}$ | $i$, deg |
|--------------|----------|----------|
| 60           |          |          |
| 0.7          | 0.676    | 0.435    | 0.402    | 0.484    | 0.594    | 0.735    | 0.864    | 1.056    | 1.222    |
| 1.0          | 0.56    | 0.324    | 0.281    | 0.361    | 0.460    | 0.592    | 0.729    | 0.930    | 1.074    |
| 1.3          | 0.430    | 0.249    | 0.184    | 0.257    | 0.358    | 0.507    | 0.630    | 0.854    | 1.058    |
| 45           |          |          |
| 0.7          | 0.567    | 0.405    | 0.356    | 0.438    | 0.520    | 0.653    | 0.763    | 0.915    | 1.063    |
| 1.0          | 0.456    | 0.301    | 0.244    | 0.304    | 0.394    | 0.534    | 0.626    | 0.778    | 0.902    |
| 1.3          | 0.331    | 0.212    | 0.144    | 0.204    | 0.289    | 0.410    | 0.529    | 0.707    | 0.861    |
| 30           |          |          |
| 0.7          | 0.481    | 0.353    | 0.298    | 0.379    | 0.462    | 0.574    | 0.672    | 0.787    | 0.913    |
| 1.0          | 0.392    | 0.260    | 0.205    | 0.257    | 0.339    | 0.437    | 0.523    | 0.648    | 0.735    |
| 1.3          | 0.274    | 0.164    | 0.120    | 0.182    | 0.251    | 0.348    | 0.440    | 0.579    | 0.702    |
| 15           |          |          |
| 0.7          | 0.410    | 0.305    | 0.249    | 0.323    | 0.41    | 0.495    | 0.587    | 0.684    | 0.784    |
| 1.0          | 0.298    | 0.212    | 0.152    | 0.209    | 0.282    | 0.369    | 0.441    | 0.539    | 0.629    |
| 1.3          | 0.210    | 0.130    | 0.092    | 0.151    | 0.270    | 0.295    | 0.369    | 0.476    | 0.574    |
| 0            |          |          |
| 0.7          | 0.325    | 0.239    | 0.210    | 0.269    | 0.341    | 0.426    | 0.510    | 0.594    | 0.664    |
| 1.0          | 0.242    | 0.156    | 0.116    | 0.169    | 0.234    | 0.314    | 0.378    | 0.456    | 0.518    |
| 1.3          | 0.167    | 0.095    | 0.069    | 0.121    | 0.180    | 0.254    | 0.308    | 0.389    | 0.459    |

**Figure 4.** Diagram of the critical reduced vibration frequencies for the flexural-torsional coefficient $\psi = 0.48$.

There are two approaches for the generation of the database of the critical reduced frequency values. The first one involves the dependence of the aerodynamic vibration decrement on the angle of phase shift of vibrations of the adjacent blades $\delta_a = f(\mu)$, from the results of the experimental investigations of aerodamping of vibrations of the straight cascades of the blade airfoils. Based on them, the relations $\delta_{amin} = f(\mu)$ are determined which points of intersection with the axis of abscissa define $K_{cr}$ of the reduced vibration frequency [23, 26]. As an example, Fig. 5 illustrated the specified dependencies at the various angles of attacks for the cascade with the relative spacing $\bar{t} = 1.0$ and stagger angle $\beta = 45^\circ$.

**Figure 5.** Minimum aerodynamic vibration decrement vs reduced frequency at different angles of attack.
The second approach is based on the energy method \cite{27}. It assumes the calculation of the coefficient of work $C_A$ of unsteady aerodynamic loads by varying the phase shift $\mu$ of the adjacent blades as follows:

$$
C_A = \sum_{n=1}^{1} \lim \left[ \left( l_{ny} y_n^2 + m_{ny} \right) + \sum_{n=1}^{1} \left( \alpha_n \left( \frac{t_{na} e^{i\omega_n \mu} + m_{na} e^{-i\omega_n \mu}}{2} \right) \right) \right].
$$

(3)

Here $j$ is the imaginary unit; $y_n$ and $\alpha_n$ are the translational and angular displacements of the $n$th blade airfoil; $y_n = y_n / b$; $l_{na}$, $m_{na}$, $m_{ny}$ are the aerodynamic coefficients of influence defined by the formula \cite{27}:

$$
l_{na} = \frac{L_{na}}{2 \rho V_1^2 \cdot b \cdot \alpha_n \cdot h}; \quad m_{ny} = \frac{M_{ny}}{2 \rho V_1^2 \cdot b \cdot y_n \cdot h};
$$

$$
l_{ny} = \frac{L_{ny}}{2 \rho V_1^2 \cdot b \cdot y_n \cdot h}; \quad m_{na} = \frac{M_{na}}{2 \rho V_1^2 \cdot b \cdot \alpha_n \cdot h},
$$

where $L_{ny}$, $L_{na}$ и $M_{ny}$, $M_{na}$ are the components of unsteady aerodynamic forces and momenta acting on vibrating blade airfoil; $\rho V_1^2 / 2$ is the dynamic head.

The dynamic stability of the blade assembly is determined by the coefficient sign $C_A$. If $C_A > 0$, the blade cascade is unsteady, and at $C_A < 0$ it is steady.

In the calculations, the experimental values of $l_{ny}$, $l_{na}$, $m_{ny}$, $m_{na}$, were used, which were obtained for three adjacent blade airfoils of the cascade $n = -1, 0, 1$. The phase shift between the translational and angular displacements $\theta_{na}$ was taken to be 180°.

The $K_{cr}$ values were defined by interpolation and extrapolation of minimum $C_A$ calculated by varying the phase shift $\mu$ of the adjacent blades for different reduced frequencies and angles of attack.

5. Mathematical method implementation

For the mathematical description of the database the equation of multiple regression has been developed, which depends on the four ($r = 4$) parameters: relative spacing $\tilde{\gamma}$, stagger angle $\beta$, angle of attack $\alpha$ and flexural-torsional coupling coefficient $\psi$ based on the system of equations \cite{28}:

$$
\{K_{cr}\} = [F] \{S\},
$$

(4)

Here

$$
[F] = \begin{bmatrix}
    f_{00} & f_{01} & \cdots & f_{0k} \\
    f_{10} & f_{11} & \cdots & f_{1k} \\
    \vdots & \vdots & \ddots & \vdots \\
    f_{0n} & f_{1n} & \cdots & f_{kn}
\end{bmatrix}, \quad \{S\} = \begin{bmatrix}
    x_0 \\
    x_1 \\
    \vdots \\
    x_k
\end{bmatrix}, \quad \{K_{cr}\} = \begin{bmatrix}
    K_{cr1} \\
    K_{cr2} \\
    \vdots
\end{bmatrix},
$$

where $[F]$ is the regression matrix, $f_{pq}$ ($p = 0 \ldots k; q = 1 \ldots \sigma$) are the regressors selected using the orthogonal polynomial; $\{S\}$ is the column-vector $k$ of the determining regression factors $s_q$; $\{K_{cr}\}$ is the column-vector plotted using the database of the considered $\sigma$ critical values of the reduced vibration frequency $cr$.

The column-vector $\{S\}$ of the unknown regression factors is defined by the expression obtained from (4):

$$
\{S\} = \left( \begin{bmatrix} F^{T} \\ F \end{bmatrix} \right)^{-1} \begin{bmatrix} F^{T} \\ F \end{bmatrix} \{K_{cr}\},
$$

(5)

where the upper indexes (T) and (-1) denote the transposed and reciprocal matrices.
To compose the regression matrix \([F]\) the Chebyshev orthogonal polynomials were used. Their linear \(X_{uq}\), quadratic \(Z_{uq}\) and cubic \(W_{uq}\) contrasts were written as the functions of variation of factors \(x_{uq}\) [29]:

\[
X_{uq} = a_{1u}(x_{uq} + a_{0u}), \quad Z_{uq} = a_{22u}(X_{uq}^2 + a_{21u}X_{uq} + a_{20u}),
\]

\[
W_{uq} = a_{33u}(X_{uq}^3 + a_{32u}X_{uq}^2 + a_{31u}X_{uq} + a_{30u}) \quad u = 1, \ldots, r,
\]

where

\[
a_{10u} = -\frac{\sum_{q=1}^\sigma X_{uq}}{\sigma}, \quad a_{20u} = -\frac{\sum_{q=1}^\sigma X_{uq}^2}{\sigma}, \quad a_{20u} = -\frac{\sum_{q=1}^\sigma X_{uq}^3}{\sigma}.
\]

The factors \(a_{10u}, a_{22u}, a_{33u}\) are selected to have their final values as the minimum ones after the determination of the contrast values of \(X_{uq}\), \(Z_{uq}\), \(W_{uq}\) by the absolute magnitude of integral numbers.

The factors \(a_{30u}, a_{31u}\) and \(a_{32u}\) of the cubic contrast ratio are determined via the solution to the system of three equations:

\[
\begin{align*}
\sum_{q=0}^{\sigma} W_{uq} &= \sum_{q=0}^{\sigma} X_{uq}^3 + a_{32u} \sum_{q=0}^{\sigma} X_{uq}^2 + a_{31u} \sum_{q=0}^{\sigma} X_{uq} + a_{30u} = 0; \\
\sum_{q=0}^{\sigma} W_{uq} \cdot X_{uq} &= \sum_{q=0}^{\sigma} X_{uq}^4 + a_{32u} \sum_{q=0}^{\sigma} X_{uq}^3 + a_{31u} \sum_{q=0}^{\sigma} X_{uq}^2 + a_{30u} \sum_{q=0}^{\sigma} X_{uq} = 0; \\
\sum_{q=0}^{\sigma} W_{uq} \cdot Z_{uq} &= \sum_{q=0}^{\sigma} (X_{uq}^3 + a_{32u} X_{uq}^2 + a_{31u} X_{uq} + a_{30u}) \cdot (X_{uq}^2 + a_{21u} X_{uq} + a_{20u}) = 0.
\end{align*}
\]

Considering the mentioned above, the equation of multiple regression can be expressed as:

\[
K_{cr} = s_0 + s_1 f_1(x_1) + \ldots + s_1 f_1^{(A-1)}(x_1) + s_u f_u^{(1)}(x_u) + \ldots + s_u f_u^{(A-1)}(x_u) + \Pi,
\]

where \(s_1 f_1^{(1)}(x_1), \ldots, s_u f_u^{(1)}(x_u)\) is the first order polynom in the variation of factors \(x_{uq}\), \(s_1 f_u^{(A-1)}(x_u)\) is the polynom of \((A-1)\) order in the variation of factors \(x_{uq}\); \(\Pi\) is the symbolic notation of the multiplication of functions \(f_\lambda\) by \(f_g\) (\(\lambda = 1 \ldots u, g = 1 \ldots u, \lambda \neq g\)).

To simplify the type of equation (7) the Student–criterion is used [29] by the following formula:

\[
T_q = \frac{s_q \eta (\eta - 1) \sum_{q=1}^{\sigma} (x_{uq})^2}{\sum_{q=1}^{\sigma} (K_{crq} - \bar{K}_{cr})^2},
\]

where \(\bar{K}_{cr} = \sum_{q=1}^{\sigma} K_{crq} / \sigma\), \(\eta\) is the number of the obtained data of investigations at the same cascade parameters.

At the Student’s parameters \(T_q < T_{tab}\) the \(s_q\) factors are statistically insignificant, that is why they can be neglected.

The substantiation of the obtained equation of multiple regression for the solution to the task is evaluated by the correlation factor:
where $K_{eq}$ is the arithmetical average of the critical reduced value of vibrations for $\eta$; $\hat{K}_{eq}$ is the value of $q$-th critical reduced frequency of vibrations obtained from the equation of multiple regression.

In the derivation of the equation of multiple regression showing the stability limit of the blade assembly to prevent the subsonic flutter for the first flexural mode of vibrations of blades, the following factors were considered: relative spacing $t_x = x_{1q}$, stagger angle $\beta = x_{2q}$, angle of attack $i_a = x_{3q}$ and coefficient of flexural-torsional coupling between blades $\psi = x_{4q}$.

Based on the developed database of the critical reduced frequencies of vibrations, the Chebyshev orthogonal polynomials were obtained as follows:

$$X_1(\tilde{r}) = \frac{(\tilde{r} - 1)}{0.3}, \quad X_2(\beta) = \frac{(\beta - 30)}{15}, \quad X_3(i) = \frac{(i - 5)}{5},$$

$$X_4(\psi) = -1 + 3.4673\psi + 1.4568\psi^2;$$

$$Z_1(\tilde{r}) = 3\left(X_1^2 - \frac{2}{3}\right), \quad Z_2(\beta) = X_2^2 - 2, \quad Z_3(i) = X_3^2 - 4, \quad Z_4(\psi) = 3\left(X_4^2 - \frac{2}{3}\right);$$

$$W_1(\tilde{r}) = 0, \quad W_2(\beta) = \frac{5X_1^2 - 3.4X_2}{6}, \quad W_3(i) = \frac{X_3^2 - 7X_3}{6}, \quad W_4(\psi) = 0.$$

The statistical significance of the selected factors is determined using the Student $t$-criterion by formula (10) and is compared to $T_{tab}$ [29]. Since 106 coefficients in the equation of multiple regression are statistically insignificant and they can be neglected, the final equation of regression will have only 38 coefficients. Here the correlation factor $R$ is within the range of 0.92 - 0.98 of the variation in the considered parameters, which implies the adequacy of the equation of multiple regression. The largest discrepancy between the calculated and experimental data is observed for the angles of attack close to zero, which is not essential in the assessment of the stability of the blade assemblies to prevent the subsonic flutter within the off-design mode.

Based on the obtained equation of multiple regression (8), the program developed in Microsoft Office Excel makes it possible to determine the stability limit of the blade assembly against subsonic flutter for the first flexural mode of vibrations.

Since the dependence of $K_c$ on the considered parameters (as established in [3]) is of monotonous changing nature, the program can be used for the assessment of the stability of the first flexural mode of vibrations of the compressor blades and fans in the following range of parameters of cascade and flow conditions: relative spacing – $\tilde{r} = 0.6...1.4$; stagger angle – $\beta = 0...65^\circ$; flexural-torsional coupling coefficient – $\psi = 0...0.5$; angle of attack – $i = -10...20^\circ$.

Figure 6 illustrates the results of the comparison of the stability limit obtained using the developed program with the experimental data for the models of compressor cascades of the first stages of high and low pressure, respectively. The models for blade airfoils were performed for the peripheral section of 0.85 of the blade height and had the following geometric parameters: high pressure compressor – $\beta = 45^\circ$, $\tilde{r} = 0.78$, $\psi = 0.263$; low pressure compressor – $\beta = 60^\circ$, $\tilde{r} = 0.78$, $\psi = 0.2857$. 
Figure 6. Calculated dynamic stability limits to prevent the subsonic flutter (solid lines) and experimentally determined critical values of the reduced frequency (▲) of the first stage high-pressure (a) and low-pressure (b) compressors.

As is seen from the data there is a rather good correlation between the results of experimental investigations and stability limit obtained employing the developed program for its calculation. The higher experimental values of $K_{cr}$ at angles of attack 10° and 12° regarding the specified limit for the cascade of the first stage (Fig. 6, a), are responsible for the higher curvature of the blade airfoils ($\varepsilon = 23.95°$) as compared with that one introduced in the program ($\varepsilon = 15°$).

6. Application of program of prediction of the dynamic stability for the flexural mode of vibrations

To illustrate the developed program, some examples of its application are given for the explanation of the phenomena occurring in the development and operation of the AGTE compressor.

Let us compare the calculated dynamic stability limit of the blades of the first stage of AGTE low-pressure compressor with the results of bench tests for full-scale blade assembly where the subsonic cascade flutter was observed (Fig. 7).

The geometric cascade parameters are the following: $\bar{t} = 1.044; \beta = 48.1°$ and $\bar{t} = 1.123; \beta = 52.7°$, and $\psi = 0.255$.

Figure 7 illustrates the operation engine modes. On the right, the figure shows the numerical critical values of the reduced oscillation frequency for the cross sections of 0.95 ($K_{cr}^{0.95}$) and 0.75 ($K_{cr}^{0.75}$) at different angles of attack.

Figure 7. Calculated dependencies of the critical values of the reduced frequency on the angle of attack of the first stage low-pressure compressor determined from the test data of the straight cascade airfoils: solid line – section 0.95 of the blade height and dashed line – section 0.75 of the blade height; symbols – operation engine modes.
Now let us consider the possibility to determine the nature of excitation of vibrations of the axial compressor blade assemblies using the developed method.

The engine development of AGTE was accompanied by the regular occurrence of cracks in the blade root. They are likely to occur due to the loss of dynamic stability of the blade assembly within the transition modes of the engine operation. To prevent possible self-excited vibrations, the structural alterations were introduced.

The results of calculations of the stability limit of the blade assembly in the initial state and after additional twist are demonstrated in Fig. 8. It is seen that the operation modes of the blade assembly are within the stability region, while the increase of the stagger angle does not practically affect its stability to prevent the subsonic flutter but reduces the operational efficiency of the high-pressure compressor due to the decrease of the maximum angle of attack. Therefore, initiation of the crack in the blade root is not caused by the occurrence of the cascade flutter but their forced blade vibrations due to the wakes in the field of velocities.

![Figure 8](image_url)

**Figure 8.** Calculated dependencies of the critical values of the reduced vibration frequency of the initial blade assembly ($\beta = 40^\circ$, solid line) and after additional twist ($\beta = 45^\circ$, dashed line) on the angle of attack. Dark and open circles – operation engine modes with the specified assemblies, respectively.

7. Conclusions

From the results of investigations, the following conclusions were drawn:

1. The description of the method that allows one to predict the stability limit of the axial compressor blade assemblies to prevent the subsonic cascade flutter at different operation modes of GTE was presented.
2. The mathematical implementation of the proposed method and some applications of the practical approval of the developed software were provided. The results of the comparison between the calculated and experimental data imply the method efficiency.

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