Heavy ion collisions: Correlations and Fluctuations in particle production.

Sergei A. Voloshin
Department of Physics and Astronomy, Wayne State University,
666 W. Hancock, Detroit, Michigan 48201

Abstract. Correlations and fluctuations (the latter are directly related to the 2-particle correlations) is one of the important directions in analysis of heavy ion collisions. At the current stage of RHIC exploration, when the details matter, basically any physics question is addressed with help of correlation techniques. In this talk I start with a general introduction to the correlation and fluctuation formalism and discuss weak and strong sides of different type of observables. In more detail, I discuss the two-particle $p_t$ correlations/$\langle p_t \rangle$ fluctuations. In spite of not observing any dramatic changes in the event-by-event fluctuations with energy, which would indicate a possible phase transition, such correlations measurements remain an interesting and important subject, bringing valuable information. Lastly, I show how radial flow can generate characteristic azimuthal, transverse momentum and rapidity correlations, which could qualitatively explain many of recently observed phenomena in nuclear collisions.

1. Introduction
The physics of high energy heavy ion collisions attracts strong attention of the physics community as creation of a new type of matter, the Quark-Gluon Plasma, is expected in such collisions. During just a few years of the BNL RHIC operation many new phenomena has been observed, such as strong elliptic flow [1], and suppression of the high transverse momentum two particle back-to-back correlations [2]. Note that both of the above mentioned discoveries at RHIC have been made with help of correlations techniques: in this case analyzing azimuthal correlations. Further understanding of the physics processes responsible for both phenomena, such as what happens with the energy of the away-side jet or whether the observed constituent quark scaling [3, 4, 5] in elliptic flow reflects the hadronization via quark coalescence, require further dedicated correlation studies. In general, as we try to understand the details of system evolution we will rely more and more on correlation measurements.

Many collaborations recently have performed measurements of either multiplicity (such as net charge, $K/\pi$ ratio) or mean transverse momentum fluctuations [6, 7]. The interest was driven by a possibility to observe a dramatic change in fluctuation pattern with collision energy and/or centrality indicating the phase transition. There exist theoretical prediction both for an increase of fluctuations [8] as well as for decrease in fluctuations [9, 10] (the latter is due to an increase in number of effective degrees of freedom in the QGP). At this time many new observables were introduced in the analysis. Unfortunately, the various observables used by experiments almost render a valid comparison of the results difficult or even impossible. The reason for that is a persistent confusion about properties of different observables used to measure fluctuations. Due to the importance of inter experiment comparison of the results, in the next section I try to
give a short summary of the question along with my own recommendations for future analyzes. This part is somewhat technical, and a reader more interested in physical results may skip it and start with section II.

2. Correlation functions and fluctuations

In a multi-particle process the correlations in particle production are described via two and many particle densities [11, 12]. In this talk I use only single and two-particle densities; those are normalized respectively by the mean multiplicity and the mean number of pairs in a given region of momentum space:

$$\int_{\Delta X} dx \rho^{(1)}(x) = \langle n \rangle; \quad \int_{\Delta X} dx_1 \int_{\Delta X} dx_2 \rho^{(2)}(x_1, x_2) = \langle n(n-1) \rangle. \quad (1)$$

Particle densities in general depend on a particle three momentum, e.g. rapidity, transverse momentum, and azimuthal angle, some of the arguments can be integrated out; for brevity I use just $x$. From $\rho^{(1)}(x)$ and $\rho^{(2)}(x_1, x_2)$ one constructs different types of correlation functions:

$$C(x_1, x_2) = \rho^{(2)}(x_1, x_2) - \rho^{(1)}(x_1) \rho^{(1)}(x_2); \quad B(x_1, x_2) = \frac{C(x_1, x_2)}{\rho^{(1)}(x_1)}; \quad R(x_1, x_2) = \frac{C(x_1, x_2)}{\rho^{(1)}(x_1) \rho^{(1)}(x_2)} \quad (2)$$

Roughly speaking $C(x_1, x_2)$ has a meaning of a distribution of correlated pairs. The correlation function $B(x_a, x_b)$ corresponds to the distribution of particles $b$ under condition that particle $a$ is found at $x_a$. Often, a particle of type $a$ is called a “trigger” particle and particle of type $b$ is called an “associated” particle. When particles $a$ and $b$ carry opposite charges, such as electric, baryon, or strangeness, $\int_{x_a} B(x_a, x_b) dx_b = 1$, as there must be one associated particle somewhere in the momentum space that “balances” the charge of the trigger particle. Then $B(x_a, x_b)$ is often referred to as a balance function. The correlation function $R(x_1, x_2)$ may be interpreted as the “probability” that a given pair is correlated. This type of the correlation function is used most often, as it is easiest to measure. Defined as a ratio, it is one of the so-called robust quantities, which do not depend on single particle reconstruction efficiencies.

Studying the correlations in nuclear collisions one usually is interested if those are different from the correlations in elementary collision. If one superimposes a few, $N_{\text{coll}}$, nucleon-nucleon collisions together:

$$\rho^{(1), AA}(x) = N_{\text{coll}} \rho^{(1), NN}(x) \quad (3)$$

$$\rho^{(2), AA}(x_1, x_2) = N_{\text{coll}} \rho^{(2), NN}(x_1, x_2) + N_{\text{coll}} (N_{\text{coll}} - 1) \rho^{(1), NN}(x_1) \rho^{(1), NN}(x_2). \quad (4)$$

Then, the balance function, $B(x_1, x_2)$, does not change at all, and the signal in $R(x_1, x_2)$ gets diluted: $R^{AA}(x_1, x_2) = R^{NN}(x_1, x_2)/N_{\text{coll}}$.

All of the above correlation functions can be calculated at fixed multiplicities. In this case they usually carry subscript $n$, e.g. $C_n(x_1, x_2)$, and called semi-inclusive. In nuclear collision analyses it is not possible to fix the multiplicity of individual nucleon-nucleon collisions, and one should compare the results with inclusive NN correlation function, although one should be careful comparing very peripheral collisions selected on the basis of total multiplicity. In this case the approximation of correlations in individual NN collision by inclusive correlation function can break down.

2.1. Fluctuations

Many different quantities have been suggested for fluctuation analyses. Before further discussion, let us formulate in general terms what constitutes a good observable. It should be (i) sensitive to the physics under study, (ii) it should be defined at the “theoretical” level, be apparatus/experiment independent, (iii) have clear physical meaning, and (iv) it should not be
limited in scope, provide new venues for further study. If most of the proposed observables do satisfy the first requirement, very few satisfy (ii)–(iv). It made basically impossible to compare many of the results, even published, one to another. Comparing different observables I argue that the “old fashion” correlation functions, and, similar the two particle transverse momentum correlations, \( \langle \Delta p_{t,1} \Delta p_{t,2} \rangle \), are the best in many respects.

2.1.1. Multiplicity fluctuation measures How the two, correlations and fluctuations, are related to each other? Let us look at the so-called reduced variance:

\[
\omega_n \equiv \frac{\sigma_n^2}{\langle n \rangle} = 1 + \langle n \rangle \bar{R}_{\Delta X}; \quad \bar{R}_{\Delta X} = \frac{\int_{\Delta X} dx_1 \int_{\Delta X} dx_2 \rho_2(x_1, x_2)}{\int_{\Delta X} dx_1 \int_{\Delta X} dx_2 \rho_1(x_1)\rho_1(x_2)},
\]

where the last equation shows the relation to two particle density. If \( \rho_1(x_1) \) does not vary much over the region \( \Delta X \), \( \bar{R}_{\Delta X} \) gives an average of the correlation function \( R(x_1, x_2) \) over the momentum region used in the analysis. It follows that the reduced variance deviates from unity (which is interpreted as the value for statistical fluctuations) if and only if the correlation term \( \bar{R} \) equals zero.

Note that \( \omega \) depends on the product of \( \langle n \rangle \) and \( \bar{R} \). Given \( \bar{R} \) is a robust quantity, the mean event multiplicity is not, it depends on detector efficiency, track quality cuts, etc. Another feature of \( \omega \) is that it can be different even if the underlying correlations are the same. Let us compare fluctuations of charged particles with fluctuations of only negative (or only positive) particles. Even if the correlations between particles have no any dependence on the particle charge, the reduced variance would show different values simply because only “half” of the particles are being considered. Lastly note that the reduced variance can not be independent of the size of the rapidity region \( \Delta X \) as it sometimes assumed (it is possible only if \( R(x_1, x_2) \propto \delta(x_1 - x_2) \) which is not physical) as often people compare its values at different \( \Delta X \).

Why one want to study \( \langle \langle n \rangle R \rangle \) instead of just \( R \)? The reason is that this product about should stay constant as function of centrality in a case when \( AA \) collision is considered as a superposition of independent nucleon-nucleon collisions. In this case the correlation functions scales as \( 1/N_{\text{coll}} \), \( \langle n \rangle \propto N_{\text{coll}} \), and the product remains constant. The deviation from constant is easy to observe. Experimentally, the centrality is often obtained by measuring multiplicity, and it was wrongly assumed that such quantities as reduced variance do not depend on multiplicity in general. It would be desirable to report both quantities separately. Instead of \( \langle n \rangle \) it would be better to use \( dn/dy \), a quantity totally corrected for efficiency and only weakly dependent on \( \Delta X \). Besides, if \( \bar{R} \) is known, one can check how it scales also other quantities, such as number of participating nucleons, \( N_{\text{part}} \), or the number of binary collisions, or something else that might be related to the number of sources of particle production.

To avoid the so called “volume fluctuations” (the effect of mixture of different impact parameter collisions in one event sample) the fluctuations in particle ratios are often used, e.g. \( K/\pi \), where \( K \) and \( \pi \) stand for the particle respective multiplicities in a given event. Again, many “details” often forgotten. First, even in the original paper [9], the fluctuations of particle ratios are “reduced” (approximated) to the following

\[
\frac{\sigma_{K/\pi}^2}{(K/\pi)^2} \approx \frac{\sigma_K^2}{(K)^2} + \frac{\sigma_{\pi}^2}{(\pi)^2} - 2 \frac{\langle K \pi \rangle - \langle K \rangle \langle \pi \rangle}{(K) \langle \pi \rangle} = \frac{1}{(K)} + \frac{1}{(\pi)} + (\bar{R}_{KK} + \bar{R}_{\pi\pi} - 2\bar{R}_{K\pi}).
\]

Note, that we started with statistically rather bad defined quantity as any ratio of two random numbers is (what do you do if denominator is zero?) and ended up with a much better behaved quantity defined via the correlation function. The question is why one would want to deal with ratios when even theoretical predictions are based on a really different quantity, which only approximately corresponds to fluctuations in ratios, namely only in the case when multiplicities
of particles used in the denominator is large? It is much better to report directly the correlation of the type $(\tilde{R}_{KK} + \tilde{R}_{\pi\pi} - 2\tilde{R}_{K\pi})$ [15]. The latter is easier to measure (robust quantity) and can be analyzed for particles with very low mean multiplicities (for example, $p$ and $\bar{p}$).

2.2. Mean $p_t$ fluctuation measures

Mean $p_t$ event by event fluctuation analyses are often performed by calculating the event-wise mean transverse momentum, an overall event average, and the variance:

$$\langle p_t \rangle = \frac{1}{n} \sum_i p_{t,i}; \quad \langle \langle p_t \rangle \rangle = \frac{1}{N_{ev}} \sum_j \langle p_t \rangle ; \quad \sigma_{\langle p_t \rangle}^2 = \langle \langle p_t \rangle \rangle^2 - \langle \langle p_t \rangle \rangle \langle \langle p_t \rangle \rangle = \frac{\sigma_{\langle p_t, incl \rangle}^2}{n} + \frac{n - 1}{n} \langle \Delta p_{t,1} \Delta p_{t,2} \rangle,$$

where the first term in $\sigma_{\langle p_t \rangle}^2$ is defined by a single particle $p_t$ spectrum and is attributed to statistical fluctuations (the fluctuations in a case of independent particle production with the same single-particle distributions). The non-trivial fluctuations (non-statistical, sometimes called dynamical) are included in the second term, $\langle \Delta p_{t,1} \Delta p_{t,2} \rangle$ [14]. All the measures of fluctuations used by different experiments are related to this second term (two particle transverse momentum correlation (covariance)), although sometimes in a rather complicated way. In many cases such a relation involves tracking and analysis efficiencies, and if those are not reported separately, the final results become not comparable to other experiments/measurements.

If all events in the event sample have the same multiplicity, then the equations above provide semi-inclusive mean transverse momentum and correspondingly $\langle \Delta p_{t,1} \Delta p_{t,2} \rangle$. If the multiplicity vary within the event sample, there are two possibilities: one can average the semi-inclusive quantities over all events (what is used most often, I would call it event-by-event (EbyE) average) or calculate true inclusive quantities

$$\bar{p_t} \equiv \frac{\sum_{\text{events}} \sum_i^n p_{t,i}}{N_{\text{events}} \langle n \rangle}; \quad \Delta p_{t,1,2} \equiv \frac{\sum_{\text{events}} \sum_{i<j} (p_{t,i} - \bar{p_t})(p_{t,j} - \bar{p_t})}{N_{\text{events}} \langle n(n-1) \rangle}.$$  

(8)

The latter can be written directly using two particle density

$$\frac{\Delta p_{t,1} \Delta p_{t,2}}{\langle n(n-1) \rangle} = \int_{\Delta \eta} d\eta_1 \int_{\Delta \eta} d\eta_2 \rho^{(2)}(x_1, x_2) \Delta p_{t,1} \Delta p_{t,2}$$

(9)
Although quantitatively both definitions (EbyE and inclusive) would give almost indistinguishable values (unless the multiplicity is too low and event sample include events with very different multiplicities), theoretically, it is easier to analyze inclusive correlations. Therefore my preference would be to “migrate” from the EbyE to inclusive observables.

Note also that \( \langle \Delta \eta_1 \Delta \eta_2 \rangle \) can be easily generalized for study of correlation between particles of different type and for differential measurements such as \( \Delta \eta_1 (\eta_1, \phi_1) \Delta \eta_2 (\eta_2, \phi_2) \). Similar to the multiplicity correlation, its centrality dependence can be tested against different hypothesis such scaling with rapidity density, number of participants, etc.

3. Mean \( p_t \) fluctuations/correlations at RHIC

Recently STAR Collaboration has reported results on integrated two particle \( p_t \) correlations for Au+Au collisions at different collisions energies [18]. Fig. 2 shows the dependence of the two particle \( p_t \) correlations on \( \Delta \eta = 2|\eta_{\text{max}}| \) the size of the pseudorapidity window taken around midrapidity. The results for different centralities are shown starting from the most peripheral (on the top, the strongest correlations) to the most central. The dependence on the size of the pseudorapidity region is rather weak. At higher collision energy \( (\sqrt{s_{NN}} = 200 \text{ GeV}) \) the dependence is even weaker. Under assumption that spectra dependence on transverse momentum and pseudorapidity factorize, according to Eq. 9, the pseudorapidity dependence of \( \langle \Delta \eta_1 \Delta \eta_2 \rangle \) should follow the one of correlation function \( R(\eta_1, \text{eta}_2) \). Then \( \langle \Delta \eta_1 \Delta \eta_2 \rangle \propto \tilde{R} \). For a simple estimate one can fit the correlation function by the form \( R(\Delta \eta) \propto 1 - \alpha|\Delta \eta| \), which translates into \( \tilde{R} \propto 1 - 4/3\alpha Y \), where \( Y = (\Delta \eta)_{\text{max}}/2 \). The blue lines shown in Figures 1 and 2 correspond to the same \( \alpha = 0.16 \) and describe the data well.

Note that the slope of \( \langle \Delta \eta_1 \Delta \eta_2 \rangle \) dependence on \( Y \) changes with centrality. It is noticeably smaller for central collisions. Taking into account the relation to the correlation function, it means that the correlations in more central collision widens in \( \eta \). The same conclusion can be derived directly from the differential measurements of the two particle \( p_t \) correlation on \( \langle \Delta \eta, \Delta \phi \rangle \) presented at this conference (see Fig. 6 in [16]). I argue below that such correlations can be due to radial transverse flow.

Fig. 3 shows STAR results on the centrality and incident energy dependence of \( \sqrt{\langle \Delta \eta_1 \Delta \eta_2 \rangle / \langle p_t \rangle} \). Remarkably the results in this form (two particle relative transverse momentum correlations) exhibit almost no dependence on the collision energy. The insert shows the results for the most central collision where \( \sqrt{\langle \Delta \eta_1 \Delta \eta_2 \rangle / \langle p_t \rangle} \approx 1.2\% \) from lower SPS (Pb+Pb collisions, CERES Collaboration [7]) to the top RHIC energy. The centrality dependence of \( \langle \Delta \eta_1 \Delta \eta_2 \rangle \) roughly follows the expectations for a dilution of the correlations as more and more independent nucleon-nucleon collisions mixed up. For more detailed analysis of the centrality dependence note that in Eq. 9 both numerator and denominator depend on centrality. Taking this into account one finds

\[
\langle \Delta \eta_1 \Delta \eta_2 \rangle_{\text{AA}} \approx D_{\text{coll}} \langle \Delta \eta_1 \Delta \eta_2 \rangle_{\text{NN}} ; \quad D_{\text{coll}} = \frac{(n(n-1))_{\text{NN}}}{(N_{\text{coll}} - 1)(n)_{\text{NN}}^2 + (n(n-1))_{\text{NN}}}. \tag{10}
\]

The factor \( D \) takes into account the dilution of the correlations due to a mixture of particles from \( N_{\text{coll}} \) uncorrelated \( NN \) collisions, and that in an individual \( NN \) collision the mean number of particle pairs, \( (n(n-1))_{\text{NN}} \), on average is larger than \( (n)_{\text{NN}}^2 \). At ISR energies in central rapidity region \( (n(n-1))_{\text{NN}} \approx 1.66(n)_{\text{NN}}^2 \) [12] (see Fig. 1). Taking into account the dilution factor \( D_{\text{coll}} \) one find that the correlations increase for about 50% with centrality relative to the expectations based on the superposition of independent \( NN \) collisions (see Fig. 6 below).
4. Transverse radial flow and two-particle correlations

At the first stage of a AA collision many individual nucleon-nucleon collision happen. Parton re-interactions lead to pressure build-up and the system undergoes longitudinal and transverse expansion. Transverse flow in the system creates strong position-momentum correlations in the transverse plane: further from the center axis of the system a particle is produced initially, on average the larger push it gets from other particles during the system evolution. As all particles produced in the same NN collision have initially the same spatial position in the transverse plane, they get on average the same push and thus become correlated. This picture leads to many distinctive phenomena, most of which can be studied by means of two (and many-) particle correlations [25].

The single particle spectra are affected by radial flow such that the mean transverse momentum is mostly sensitive to the average expansion velocity squared $\langle p_t \rangle_{AA} \approx \langle p_t \rangle_{NN} + \alpha \langle v^2 \rangle$, (see Fig. 5 below) and to much lesser extend to the actual velocity profile (dependence of the expansion velocity on the radial distance from the center axis of the system). The two-particle transverse momentum correlations [14], $\langle \Delta p_{t,1} \Delta p_{t,2} \rangle$, measure the variance in collective transverse expansion velocity, and thus are more sensitive to the actual velocity profile. We employ a thermal model [13] for further calculations. In this model particles are produced by freeze-out of the thermalized matter at temperature $T$, approximated by a boosted Boltzmann distribution. Assuming boost-invariant longitudinal expansion and freeze-out at constant proper time, one finds

$$\frac{dn}{dp_t} \sim \int d\rho_t d\phi_t \rho_t^{2(n-1)} J(p_t; T, \rho_t, \phi_b); J(p_t; T, \rho_t, \phi_b) \equiv m_t K_1(\beta_t) e^{\alpha_t \cos(\phi_b - \phi)},$$

(11)

where $\rho_t$ is the transverse flow rapidity, $\phi_b$ is the boost direction, $\alpha_t = (p_t/T) \sinh(\rho_t)$, and $\beta_t = (m_t/T) \cosh(\rho_t)$. It also assumes a uniform matter density within a cylinder, $r < R$, and a power law transverse rapidity flow profile $\rho_t \propto r^n$. Two particle spectrum for particles originating from the same NN collision in this picture can be written as

$$\frac{dn_{pair}}{dp_{t,1} dp_{t,2}} \sim \int d\rho_t d\phi_t \rho_t^{2(n-1)} J(p_{t,1}; T, \rho_t, \phi_b) J(p_{t,2}; T, \rho_t, \phi_b).$$

(12)
It additionally assumes that during the expansion time (before the freeze-out) the particles produced originally at the same spatial position do not diffuse far one from another compared to the system size. The results of the numerical calculations based on the above equations are presented in Fig. 5 as function of $\langle \rho_t^2 \rangle = \langle \rho_t \rangle^2 (4n + 4)/(2 + n)^2$. The results are shown for two different velocity (transverse rapidity) profiles, $n = 2$, and $n = 0.5$. One observes that indeed for all the particle types presented, $\langle p_t \rangle$ depends very weakly on the actual profile. On opposite, the correlations are drastically different for two cases studied.

In Fig. 6 we compare our estimates with STAR preliminary data [18] on two particle $p_t$ correlations (taking into account the dilution factor, Eq. 10). We use $\langle \rho_t \rangle$ and $T$ parameters from [23] and assume $N_{coll} = N_{part}/2$ and $(\Delta p_{t,1} \Delta p_{t,2}) / \langle p_t \rangle^2 = 0.011$ (about 10% smaller than measured at ISR [17] (Fig. 4). It is observed that the transverse flow with $n = 1$ produces too strong correlations.

While transverse flow generates in general an elongation of the pseudorapidity correlations (along with narrowing in azimuthal angle) it should lead to narrowing of the charge balance function [19] due to the increase in mean $p_t$ as [20]: $\Delta p_z = m_t \sinh(\Delta y) \approx m_t \Delta y \approx const.
Quantitatively, the effect is consistent with experimentally observed narrowing for about 15 – 20% of the balance function width with centrality [21] and with centrality dependence of the net charge fluctuations [22]. As all particles from the same \( NN \) collisions are pushed in the same direction they become correlated in azimuthal space. The correlations can become really strong for large transverse flow as shown in Fig. 7 (for particles originated from the same \( NN \) collision). Our estimates show that the azimuthal correlations generated by transverse expansion could be a major contributor to the non-flow azimuthal correlations [24].

The above described picture of \( AA \) collisions has many interesting observable effects, only a few mentioned here. The picture become even richer if one looks at the identified particle correlations. Many questions require a detailed model study, but the approach opens a potentially very interesting possibility to address the initial conditions and the subsequent evolution of the system created in an \( AA \) collision.

5. Conclusion

The correlation techniques are a very powerful tool in our quest to understand multiparticle production in nuclear collision. They have been proven to lead to many discoveries in the past and promise even more to come.

Acknowledgments. Discussions with R. Bellwied, S. Gavin, C. Pruneau, S. Pratt, and U. Heinz are gratefully acknowledged. This work was supported in part by the U.S. Department of Energy Grant No. DE-FG02-92ER40713.

[1] STAR Collaboration, Ackermann K H et al. 2001 Phys. Rev. Lett. 86 402
[2] STAR Collaboration, Adler C et al. 2003 Phys. Rev. Lett. 90 082302
[3] Voloshin S A 2003 Nucl. Phys. A 715, 379.
[4] STAR Collaboration, Adams J et al. 2004 Phys. Rev. Lett. 92, 052302
[5] Molnar D and Voloshin S A 2003 Phys. Rev. Lett. 91, 092301
[6] Mitchell J T 2004 J. Phys. G 30, S819
[7] Appelshauser H 2004 J. Phys. G 30, S935
[8] Stephanov M A, Rajagopal K and Shuryak E V 1999 Phys. Rev. D 60 114028
[9] Baym G and Heiselberg H 1999 Phys. Lett. B 469 7; Jeon S and Koch V 2000 Phys. Rev. Lett. 85 2076
[10] Asakawa M, Heinz U W and Muller B 2000 Phys. Rev. Lett. 85 2072
[11] Whitmore J 1976 Phys. Rept. 27 187
[12] Foa L 1975 Phys. Rept. 22 1
[13] Schnedermann E, Sollfrank J and Heinz U 1993 Phys. Rev. C 48, 2462
[14] Voloshin S A, Koch V and Ritter H G 1999 Phys. Rev. C 60, 024901
[15] Pruneau C, Gavin S and Voloshin S 2002 Phys. Rev. C 66, 044904
[16] Kopytine M [STAR Collaboration] this proceedings, arXive:nucl-ex/0504006
[17] Braune K et al. 1983 Phys. Lett. 123B 467
[18] Westfall G D [STAR collaboration] 2004 J. Phys. G 30, S1389; Adams J et al. arXive:nucl-ex/0504031
[19] Bass S A, Danielewicz P and Pratt S 2000 Phys. Rev. Lett. 85, 2689
[20] Scott Pratt and Sen Cheng 2003 Phys. Rev. C 68, 014907
[21] STAR Collaboration, Adams J et al. 2003 Phys. Rev. Lett. 90, 172301
[22] STAR Collaboration, Adams J et al. 2003 Phys. Rev. C 68, 044905
[23] STAR Collaboration, Adams J et al. 2004 Phys. Rev. C 92, 112301
[24] STAR Collaboration, Adler C et al. 2002 Phys. Rev. C 66, 034904
[25] Voloshin S A 2003 arXiv:nucl-th/0312065