Observational constraints on a Yang–Mills condensate dark energy model

Z W Fu¹, Y Zhang¹ and M L Tong¹,²

¹ Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei, Anhui, 230026, People’s Republic of China
² Korea Astronomy and Space Science Institute, Daejeon 305-348, Korea

E-mail: fuzhao@mail.ustc.edu.cn, yzh@ustc.edu.cn and mltong@mail.ustc.edu.cn

Received 1 June 2011, in final form 27 September 2011
Published 20 October 2011
Online at stacks.iop.org/CQG/28/225017

Abstract

Using the recently released Union2 compilation with 557 Type Ia supernovae, the shift parameter of the cosmic microwave background given by the WMAP7 observations and the baryon acoustic oscillation measurement from the Sloan Digital Sky Survey, we perform the $\chi^2$ analysis on the 1-loop Yang–Mills condensate (YMC) dark energy model. The analysis has been made for both non-coupling and coupling models with $\Omega_{m0}$ and $w_0$ being treated as free parameters. It is found that $\chi^2_{\text{min}} = 542.870$ at $\Omega_{m0} = 0.2701$ and $w_0 = -0.9945$ for the non-coupling model, and $\chi^2_{\text{min}} = 542.790$ at $\gamma = -0.015$, $\Omega_{m0} = 0.2715$ and $w_0 = -0.9969$ for the coupling model. Comparing with the $\Lambda$CDM model, the YMC model has a smaller $\chi^2_{\text{min}}$, but it has greater values of the Bayesian and Akaike information criteria. Overall, YMC is as robust as $\Lambda$CDM.

PACS numbers: 98.80.−k, 95.36.+x

(Some figures may appear in colour only in the online journal)

1. Introduction

The discovery of the current accelerating expansion of the Universe via the observations of the SNIa [1] with follow-ups [2], further supported by the observations on the cosmic microwave background (CMB) [3, 4] and on a large-scale structure (LSS) [5], has brought a most challenging problem to both cosmology and physics. An abundance of the literature has been devoted to the issue. To interpret the cosmic acceleration, one may either go beyond general relativity (GR) and seek an alternative, like effective gravity [6], or stay within the framework of GR and simply attribute the acceleration to some mysterious dark energy (DE) as the driving source. A positive cosmological constant $\Lambda$ is the simplest candidate of DE, but it is plagued with the coincidence problem [7]. Various dynamic models have been proposed to
address this issue [8]. One class of models is based upon some scalar field, such as quintessence [9], k-essence [10], phantom [11], quintom [12], etc. In our previous works [13, 14–16], we have developed a vector field type of a dynamic DE model, in which the renormalization group improved effective Yang–Mills condensate (YMC) [18] serves as the dynamical DE. This has been highly motivated by the great success of Yang–Mills fields as a cornerstone of particle physics, which mediate interactions between fundamental particles and define the vacuum structure. It has been demonstrated that the YMC DE model has the following properties desired for a dynamical model: being able to solve the coincidence problem naturally, giving an equation of state (EoS) \( w \) that cross \(-1\) smoothly in the coupling case, having the dynamic stability and alleviating the high-redshift cosmic age problem. Moreover, these properties are retained by YMC models even with the increase of order of quantum corrections, i.e. all 1-loop [14], 2-loop [15] and 3-loop [16] models have exhibited the same dynamical behavior. For a detailed description of the 1-loop YMC DE model, including its theoretical basis and the dynamics, see [14]. Other models using spin-1 fields as a candidate for dark energy can be found in [17].

Any DE model has to be confronted with observations. Recently, there have been some significant updates in the observations of SNIa and CMB. The Union SNe Ia compilation [19] enlarged by the CfA3 sample [20] has recently been updated, including a few SNe Ia of high redshifts, and contains 557 SNe Ia, forming the Union2 compilation [21]. Not only the number of SNe is substantially increased, but also the range of redshifts is extended, crucial for determining the evolution of dynamical DE. Besides, the seven-year data of Wilkinson Microwave Anisotropy Probe (WMAP) have given an improved determination of cosmological parameters [22], in combination with the latest distance measurements [23] of the baryon acoustic oscillations (BAO) [24] in the distribution of galaxies of the Sloan Digital Sky Survey (SDSS) and the 2dF Galaxy Redshift Survey (2dFGRS). Motivated by these, it is natural for us to expand our work [16] to further constrain the YMC model with these observational datasets. In this work, we shall carry out a statistical analysis, utilizing the combination of SNIa, CMB and LSS data to constrain the 1-loop YMC model for both non-coupling and coupling cases. The examination is much more refined than the previous work [16] in performing statistics. Furthermore, employing the Bayesian information criterion (BIC) [27] as well as the Akaike information criterion (AIC) [28], we shall also perform extra statistical examinations for models with different number of parameters. The resulting statistics show that, comparing with the \( \Lambda \)CDM model, the YMC model is still a robust dynamical DE model. The unit system with \( c = \hbar = 1 \) is used in this paper.

2. The 1-loop YMC model

We consider a spatially flat (\( k = 0 \)) Robertson–Walker universe, whose expansion is determined by the Friedmann equations
\[
H^2 = \frac{8\pi G}{3} (\rho_y + \rho_m),
\]
\[
\frac{\dot{a}}{a} = -\frac{4\pi G}{3} (\rho_y + 3p_y + \rho_m),
\]
where \( H = \frac{\dot{a}}{a} \), \( \rho_y \) and \( \rho_m \) are the energy density of the YMC and the matter (including both baryons and dark matter), respectively, \( p_y \) is the pressure of the YMC. For simplicity, the

\[3\] For the Supernova Cosmology Project, the numerical data of the full sample are available at http://supernova.lbl.gov/Union.
radiation component is neglected since its contribution is very small in the matter dominated era under consideration. The energy density \( \rho_y \) and the pressure \( p_y \) of the 1-loop YMC are given by [29, 13]

\[
\rho_y = \frac{1}{2} b \kappa^2 (y + 1) e^y, \tag{3}
\]

\[
p_y = \frac{1}{2} b \kappa^2 \left( \frac{1}{2} y - 1 \right) e^y, \tag{4}
\]

where \( \kappa \) is the renormalization scale of dimension of squared mass, \( b = \frac{22}{34\pi^2} \) for the gauge group \( SU(2) \) without fermions, \( y \equiv \ln |E^2/\kappa^2| \), with \( E^2 \) being the squared electric field of YMC [18]. When one requires that \( \rho_y \) be the dynamical DE and its value at \( z = 0 \) be equal to \( \sim 0.73 \rho_c \), one finds \( \kappa^{1/2} \simeq 5 \times 10^{-3} \) eV [14]. The EoS for the YMC is

\[
w = \frac{p_y}{\rho_y} = \frac{y - 3}{3y + 3}. \tag{5}
\]

The dynamical evolutions of the YMC DE and the matter are given by

\[
\dot{\rho}_y + \frac{3}{a} (\rho_y + p_y) = -\Gamma \rho_y, \tag{6}
\]

\[
\dot{\rho}_m + \frac{3}{a} \rho_m = \Gamma \rho_y, \tag{7}
\]

where \( \Gamma \) is a model parameter, representing phenomenologically the coupling between the YMC and the matter components. For \( \Gamma > 0 \), the interaction term \( \Gamma \rho_y \) is the transfer rate of the YMC energy into matter, whereas \( \Gamma < 0 \) means that the matter transfers energy into the YMC. For computing convenience, equations (1), (6) and (7) can be recast into

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{4\pi G b \kappa^2}{3} h^2, \tag{8}
\]

\[
\frac{dx}{dN} = -\gamma \frac{(1 + y) e^y}{h} - 3x, \tag{9}
\]

\[
\frac{dy}{dN} = -\gamma \frac{1 + y}{(2 + y) h} - \frac{4y}{2 + y}, \tag{10}
\]

where \( x \equiv \rho_m/\frac{1}{2} b \kappa^2 \), \( N \equiv \ln a(t) \), \( h \equiv \sqrt{(1 + y)e^y + x} \) and \( \gamma \equiv \Gamma / \sqrt{4\pi G b \kappa^2/3} \).

The set of dynamic equations is completely determined by the initial values \( x_i \) and \( y_i \) for the non-coupling case, plus \( \gamma \) in the coupling case. In actual computation, the initial condition can be taken at a redshift \( z_i = 1092 \). For a given \( \gamma \), each set of \( (x_i, y_i) \) yields a corresponding set \( (x_0, y_0) \) at \( z = 0 \) as the outcome from solving the dynamic equations, and hence EoS \( w_0 = (y_0 - 3)/(3y_0 + 3) \) and the matter fraction \( \Omega_{m0} = x_0/(x_0 + (y_0 + 1)e^{y_0}) \). This point of our treatment differs from those models like XCDM [30], where \( w_X \) or \( \Omega_{m0} \) in some models was put in by hand as parameters, instead of following from dynamic equations. It should also be mentioned that we will not perform statistical examination with the Hubble parameter \( H_0 = 100 \) h km s\(^{-1}\) Mpc\(^{-1}\) in this paper, \( (x_0, y_0) \) can be taken as two independently adjusted parameters for the statistical examination.
3. Constraints from SNIa, BAO and CMB

Below, we confront the YMC model with the latest observational distance modulus $\mu_{\text{obs}}(z_i)$ data of 557 SNIa [21], the BAO measurement from the SDSS [23] and the shift parameter of CMB updated by the seven-year WMAP observations [22]. The theoretical distance modulus is defined as

$$\mu_{\text{th}}(z) \equiv 5 \log_{10} D_L(z) + \mu_0,$$

where

$$\mu_0 = 42.38 - 5 \log_{10} h,$$

and

$$D_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z')}$$

is the luminosity distance in a spatially flat universe, actually independent of the Hubble constant $H_0$. For our model,

$$E(z) = \sqrt{\Omega_{m0} \frac{\rho_m(z)}{\rho_m(0)} + (1 - \Omega_{m0}) \frac{\rho_b(z)}{\rho_b(0)}},$$

which depends upon the adjusted parameters ($\Omega_{m0}$, $w_0$ and $\gamma$). Here, $E(z)$ is an implicit function for YMC through $\rho_m(z)$ and $\rho_b(z)$ as the solution from the dynamical equations (8)–(10). In performing the statistical examination in the following, an ensemble of the solutions of the equations is generated, each of which corresponds to a point in the grid of ($\Omega_{m0}$, $w_0$, $\gamma$). This requires much more computing time than the models like XCDM with an expression for $E(z)$ containing parameters explicitly [30, 31].

Note that, to reveal the possible dynamical property of DE, we choose the EoS $w_0$ as a free parameter, which is of dynamical nature and reflects the second-order time derivative $\ddot{a}(t)$. This is pertinent since the Union2 dataset provides SNIa with higher redshifts, better for constraining the evolutional property of a DE model. For examining a dynamical DE model, this has the advantage to the choice of the Hubble parameter $h$ in [16]. More importantly, the constraints on cosmological parameters by the BAO measurement from the SDSS and 2dFGRS have an intrinsic degeneracy with ($\Omega_{m0}$, $h$). Therefore, one cannot arrive at a reliable estimate for both $\Omega_{m0}$ and $h$ simultaneously [23, 24]. As a matter of fact, if one would want to determine $h$ with sufficient accuracy, one has to go beyond and employ some extra data, such as the Hubble Space Telescope’s key project [25] and the SDSS for the history of the Hubble parameter [26], etc.

For the SNIa data, the corresponding $\chi^2$ estimator is constructed as

$$\chi^2_{\text{SN}}(p; \mu_0) = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_i^2},$$

where $p$ stands for a set of parameters, such as ($\Omega_{m0}$, $w_0$ and $\gamma$), $\mu_{\text{obs}}(z_i)$ is the observed value of distance modulus for the $i$th supernova and $\sigma_i$ is the corresponding 1$\sigma$ error that can be found via [21]. The nuisance parameter $\mu_0$ can be analytically marginalized over [31], so that one actually minimizes $\chi^2_{\text{SN}}(p)$ instead of $\chi^2_{\text{SN}}(p; \mu_0)$. The minimization with respect to $\mu_0$ can be made simply by expanding the $\chi^2$ of equation (15) with respect to $\mu_0$ as

$$\chi^2_{\text{SN}}(p) = A(p) - 2 \mu_0 B(p) + \mu_0^2 C,$$

where

$$A(p) - 2 \mu_0 B(p) + \mu_0^2 C = \sum_{i=1}^{557} [\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2.$$
where
\[ A(p) = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, p)]^2}{\sigma_i^2}, \tag{17} \]
\[ B(p) = \sum_{i=1}^{557} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, p)]^2}{\sigma_i^2}, \tag{18} \]
\[ C = \sum_{i=1}^{557} \frac{1}{\sigma_i^2}. \tag{19} \]

Evidently, equation (16) as well as (15) has a minimum for \( \mu_0 = B/C \) at
\[ \tilde{\chi}^2_{\text{SN}}(\mathbf{p}) = A(p) - \frac{B(p)^2}{C}. \tag{20} \]

Since \( \chi^2_{\text{SN}, \min} = \tilde{\chi}^2_{\text{SN}} \), instead of minimizing \( \chi^2_{\text{SN}} \) one can minimize \( \tilde{\chi}^2_{\text{SN}} \), which is now independent of the nuisance parameter \( \mu_0 \). Note that this analytical marginalization over \( \mu_0 \) means that the Hubble parameter \( h \) is effectively marginalized over.

Next, the distance parameter \( A \) of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies is defined as \[ A \equiv \Omega_1^{1/2} E(z) \int_0^{z_b} \frac{dz}{E(z)} [\frac{1}{z_b} \int_0^{z_b} \frac{dz}{E(z)}]^{2/3} \tag{21} \]
with \( z_b = 0.35 \), the redshift at which the acoustic scale has been measured. The observations [24] give
\[ A_{\text{obs}} = 0.469(n_s/0.98)^{-0.35} \pm 0.017, \tag{22} \]
where \( n_s \) is the primordial spectral index and WMAP7 data [22] yield the updated value \( n_s = 0.963 \), while it was \( n_s = 0.960 \) by WMAP5 [4]. The corresponding \( \chi^2 \) of the BAO is given by
\[ \chi^2_{\text{BAO}} = \frac{(A - A_{\text{obs}})^2}{\sigma_A^2}, \tag{23} \]
where \( \sigma_A = 0.017 \).

Finally, the shift parameter \( R \) from the CMB is defined as
\[ R \equiv \Omega_0^{1/2} \int_0^{z_{\text{rec}}} \frac{dz}{E(z)}, \tag{24} \]
with \( z_{\text{rec}} \) being the redshift of recombination. WMAP7 [22] gives \( z_{\text{rec}} = 1091.3 \pm 0.91 \) and \( R_{\text{obs}} = 1.725 \pm 0.018 \), while it was \( z_{\text{rec}} = 1090.0 \pm 0.93 \) and \( R_{\text{obs}} = 1.710 \pm 0.019 \) by WMAP5 [4]. The \( \chi^2 \) of the CMB is
\[ \chi^2_{\text{CMB}} = \frac{(R - R_{\text{obs}})^2}{\sigma_R^2}, \tag{25} \]
with \( \sigma_R = 0.018 \). As usual, assuming these three datasets of observations are mutually independent, and the measurement errors for each set are Gaussian with the likelihood function of the form
\[ L \propto e^{-\chi^2/2}. \tag{26} \]
The three datasets are combined by multiplying the likelihoods, and the combined \( \chi^2 \) is given by
\[ \chi^2 = \tilde{\chi}^2_{\text{SN}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}}. \tag{27} \]
Figure 1. Non-coupling YMC: the CL curves in $\Omega_m-\omega_0$ plane based upon the joint study of SNIa, BAO and CMB.

Variations of values of the parameters ($\Omega_m$, $\omega_0$ and $\gamma$) yield respective values of $\chi^2$. A minimal $\chi^2$ has been found, corresponding to a maximal $\mathcal{L}$, which would be favored by the observations.

4. Results

For the non-coupling $\gamma = 0$ case, there are two parameters $\Omega_m$ and $\omega_0$. In our present computation of the confidence level (CL) of $\Omega_m$, the procedure goes as follows. First we compute $\chi^2(\Omega_m, \omega_0)$ and the likelihood $\mathcal{L} = \mathcal{L}(\Omega_m, \omega_0)$ as the functions of both $\Omega_m$ and $\omega_0$, from which, by a standard searching procedure, follows the resulting minimum $\chi^2_{\min} = 542.870$ at $\Omega_m = 0.2701$ and $\omega_0 = -0.9945$. Next, we integrate $\mathcal{L}(\Omega_m, \omega_0)$ over $\omega_0$ and derive the likelihood $\mathcal{L}(\Omega_m)$ as a function of only $\Omega_m$. The prior cutoff $\omega_0 = (-0.9999, -0.90115)$ is assumed here. Then, we compute the CL of $\Omega_m$ based upon the function $\mathcal{L}(\Omega_m)$. The result is that $\Omega_m = 0.2701^{+0.0151}_{-0.0121}$ at 68.3% CL, $\Omega_m = 0.2701^{+0.0297}_{-0.0250}$ at 95.4% CL. Since the non-coupling YMC model always has $\omega_0 \geq -1$ [14], it is found that the likelihood function $\mathcal{L}(\omega_0)$ is mainly distributed in a narrow range close to $\omega_0 = -1$, so that the computation of the CL can only be done for an upper bound of $\omega_0$. The result is that $\omega_0 < -0.9773$ at 68.3% CL and $\omega_0 < -0.9579$ at 95.4% CL. The prior cutoff $\Omega_m = (0.2001, 0.3501)$ is assumed here. Figure 1 shows the details of the CL curves in the $(\Omega_m, \omega_0)$ plane. We remark that, in obtaining the values of CL for a $\Omega_m$ (or $\omega_0$) at the maximum of $\mathcal{L}$, the integration over $\omega_0$ (or $\Omega_m$, respectively) has been carried out.

To make a comparison with the $\Lambda$CDM model as a type of referring model, we have also done the computing and fitting of the $\Lambda$CDM model to the same observational data as above, yielding $\chi^2_{\min} = 542.919$ at $\Omega_m = 0.2701$. The CLs are as follows: $\Omega_m = 0.2701^{+0.0135}_{-0.0140}$ at 68.3% CL, $\Omega_m = 0.2701^{+0.0283}_{-0.0264}$ at 95.4% CL. We present the corresponding $\chi^2$ and likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ in figure 3.
Based on the $\chi^2$ estimator only, one would draw a conclusion that the non-coupling YMC model is more favored in confronting this set of observations, as it has a slightly smaller $\chi^2_{\text{min}}$ than the $\Lambda$CDM model. A similar result was obtained for the non-coupling YMC model at 3-loop for an earlier dataset of the 182 SNIa +CMB+BAO [16]. This kind of better performance of YMC is due to its dynamical nature, say, its $w(z)$ being a function of $z$, instead of a constant $-1$ as in $\Lambda$CDM. In this regard, more observational data at high redshifts are desired to distinguish various DE models with different evolutionary behavior, by using certain diagnosis, such as the Statefinder and Om [32].

For the coupling model, there are three independently adjusted parameters $\gamma$, $\Omega_{m0}$ and $w_0$. In order to search for possible DE models in an extended domain, in particular, we will allow the coupling $\gamma$ to take negative values. For each fixed value of $\gamma$, we minimize $\chi^2$ with $\Omega_{m0}$ and $w_0$. We have done this for ten different values of $\gamma$, and the resulting values of $\chi^2_{\text{min}}$ for the corresponding sets of $(\gamma, \Omega_{m0} \text{ and } w_0)$ are listed in Table 1. It is seen that, as a function of $\gamma$, the minimum $\chi^2_{\text{min}} = 542.790$ is attained at $\gamma = -0.015$, $\Omega_{m0} = 0.2715$ and $w_0 = -0.9969$. This $\chi^2_{\text{min}}$ is slightly smaller than that of the non-coupling and of the $\Lambda$CDM models. Table 1 also shows that, although the $\chi^2_{\text{min}}$ depends on $\gamma$, the dependence is not very strong.

Our previous work [16] did not search for the minimal $\chi^2$ with respect to the parameter $\gamma$ since it calculated only three different values of $\gamma$. Figure 2 shows the CL curves with $\gamma = -0.015$ in the $(\Omega_{m0}, w_0)$ plane.

Thus, the YMC model with a negative coupling $\gamma < 0$ does a bit better than the latter two models in confronting the updated observations. We remark that this conclusion was partly hinted in the 3-loop YMC model, where the consideration was confined only to the limited region of $\gamma > 0$ [16]. Note that $\gamma < 0$ means a situation in which the matter is transferring energy into the YMC, and EoS $w_0$ will not cross over $-1$ [14]. Besides, in the far future when the scale factor is hundred times the present one, the matter density $\rho_m$ will turn into negative, which would be a non-physical region. Thus, the model of a negative coupling is phenomenological and cannot be infinitely extended into far future. In regard to the issue of coupling, we note that a positive interaction is preferred in a study of dynamics of galaxy clusters with a coupling between DE and dark matter [33].

The $\chi^2$ analysis is effective in searching for the best-fit values of parameters within a given model. But for models with a different number of parameters, one would expect $\chi^2_{\text{min}}$ decreases as the number of free model parameters increases. For model comparisons in this case, one can use other kinds of criteria for model selection, such as BIC [27], AIC [28] and $\chi^2_{\text{min}}/\text{dof}$ with the degree of freedom $\text{dof} = N - k$, whereas $N$ and $k$ are the number of data points and the dimension (number of independently adjusted parameters) of the statistical model,

### Table 1

For each given coupling $\gamma$, the values of $\chi^2_{\text{min}}$ and the corresponding best-fit parameters $\Omega_{m0}$ and $w_0$ are listed.

| $\chi^2_{\text{min}}$ | $\gamma$ | $\Omega_{m0}$ | $w_0$ |
|-----------------------|---------|---------------|-------|
| 543.419               | -0.05   | 0.2755        | -0.9898 |
| 543.119               | -0.04   | 0.2743        | -0.9918 |
| 542.912               | -0.03   | 0.2731        | -0.9938 |
| 542.804               | -0.02   | 0.2720        | -0.9959 |
| 542.790               | -0.015  | 0.2715        | -0.9969 |
| 542.804               | -0.01   | 0.2710        | -0.9979 |
| 542.870               | 0       | 0.2701        | -0.9945 |
| 543.202               | 0.05    | 0.2669        | -0.9797 |
| 543.496               | 0.1     | 0.2651        | -0.9692 |
| 544.511               | 0.5     | 0.2636        | -0.9313 |
Figure 2. Coupling YMC with $\gamma = -0.015$: the CL curves in $\Omega_m - w_0$ plane based upon the joint of SNIa, BAO and CMB.

Figure 3. The $\chi^2$ and $\mathcal{L}$ of $\Lambda$CDM based upon the joint of SNIa, BAO and CMB.

respectively. The BIC is defined as $\text{BIC} \equiv -2 \ln \mathcal{L}_{\text{max}} + k \ln N$, where $\mathcal{L}_{\text{max}}$ is the maximum likelihood. In the Gaussian case, $\chi^2_{\text{min}} = -2 \ln \mathcal{L}_{\text{max}} + \text{constant}$, so that the difference in the BIC is given by $\Delta \text{BIC} = \Delta \chi^2_{\text{min}} + \Delta k \ln N$. The AIC is defined as $\text{AIC} \equiv -2 \ln \mathcal{L}_{\text{max}} + 2k$, and the difference in the AIC is given by $\Delta \text{AIC} = \Delta \chi^2_{\text{min}} + 2\Delta k$. We present the results for these three criteria in table 2, which imply that the $\Lambda$CDM is still favored due to its simplicity, whereas the coupling model is ranked last as it has most parameters.

In summary, using the $\chi^2$ analysis alone in confronting the updated observational data of 557 SNIa+CMB+BAO, we find that the non-coupling YMC is favored over $\Lambda$CDM, and the coupling YMC with a negative $\gamma$ is the most favored. Taking into account the dimension of a statistical model, $\Lambda$CDM is still simplest. Overall, YMC is as robust as $\Lambda$CDM. More
Table 2. Comparison of models.

| Model       | ΛCDM   | Non-coupling | Coupling |
|-------------|--------|--------------|----------|
| $\chi^2_{\text{min}}$ | 542.919 | 542.870       | 542.790  |
| $k$         | 1      | 2            | 3        |
| $\chi^2_{\text{min}}$/dof | 0.973  | 0.975        | 0.976    |
| $\Delta\text{BIC}$  | 0      | 6.277        | 12.523   |
| $\Delta\text{AIC}$  | 0      | 1.951        | 3.871    |
| Rank        | 1      | 2            | 3        |

observational data at higher redshifts in the future are much desired to shed light on the issue of DE.

Acknowledgments

The research work of YZ has been supported by the CNSF no 11073018, SRFDP and CAS. We would like to thank W Zhao for many helpful discussions.

References

[1] Riess A G et al 1998 Astron. J. 116 1009
Riess A G et al 1999 Astron. J. 117 707
Riess A G et al 2004 Astrophys. J. 607 665
Perlmutter S et al 1998 Nature 391 51
Perlmutter S et al 1999 Astrophys. J. 517 565
Peter M G et al 1998 Astrophys. J. 509 74
Schmidt B P et al 1998 Astrophys. J. 507 46

[2] Tonry J L et al 2003 Astrophys. J. 594 1
Knop R A et al 2003 Astrophys. J. 598 102
Brian J B et al 2004 Astrophys. J. 602 571
Nobili S et al 2009 Astrophys. J. 700 1415
Astier P et al 2006 Astron. Astrophys. 447 31
Wood-Vasey W M et al 2007 Astrophys. J. 666 694
Guy J et al 2010 Astron. Astrophys. 523 A7
Jon A H et al 2008 Astron. J. 136 2806
Richard K et al 2009 Astrophys. J. Suppl. Ser. 185 32
Riess A G et al 2007 Astrophys. J. 659 98

[3] Bennett C L et al 2003 Astrophys. J. Suppl. Ser. 148 1
Spergel D N et al 2007 Astrophys. J. Suppl. Ser. 170 377

[4] Dunkley J et al 2009 Astrophys. J. Suppl. Ser. 180 306
Komatsu E et al 2009 Astrophys. J. Suppl. Ser. 180 330

[5] Bahcall N A, Ostriker J P, Perlmutter S and Steinhardt P J 1999 Science 284 1481
Tegmark M et al 2004 Phys. Rev. D 69 103501
Tegmark M et al 2006 Phys. Rev. D 74 123507
Adrian C P et al 2004 Astrophys. J. 607 655
Percival W J et al 2007 Astrophys. J. 657 51
Reid B A et al 2010 Mon. Not. R. Astron. Soc. 404 60

[6] Parker L and Raval A 1999 Phys. Rev. D 60 063512
Parker L and Vanzella D A T 2004 Phys. Rev. D 69 104009

[7] Weinberg S 2000 arXiv:astro-ph/0005265
Carroll S M 2001 Living Rev. Rel. 4 1 (available at http://www.livingreviews.org/lrr-2001-1)
Peebles P J E and Ratra B 2003 Rev. Mod. Phys. 75 559

[8] Padmanabhan T 2003 Phys. Rep. 380 235
Copeland E J, Sami M and Tsujikawa S 2006 Int. J. Mod. Phys. D 15 1753
[9] Ratra B and Peebles P J E 1988 Phys. Rev. D 37 3406
Peebles P J E and Ratra B 1988 Astrophys. J. 325 L17
Wetterich C 1988 Nucl. Phys. B 302 668
Wetterich C 1995 Astron. Astrophys. 301 321 (arXiv:hep-th/9408025)
Caldwell R R, Dave R and Steinhardt P J 1998 Phys. Rev. Lett. 80 1582
Zlatev I, Wang L and Steinhardt P J 1999 Phys. Rev. Lett. 82 896

[10] Armendariz-Picon C, Mukhanov V and Steinhardt P J 2001 Phys. Rev. D 63 103510
Armendariz-Picon C, Mukhanov V and Steinhardt P J 2000 Phys. Rev. Lett. 85 4438
Chiba T, Okabe T and Yamaguchi M 2000 Phys. Rev. D 62 023511

[11] Caldwell R R 2002 Phys. Lett. B 545 23
Carroll S M, Hoffman M and Trodden M 2003 Phys. Rev. D 68 023509
Caldwell R R, Kamionkowski M and Weinberg N N 2003 Phys. Rev. Lett. 91 071301

[12] Hu W 2005 Phys. Rev. D 71 047301
Armendariz-Picon C, Mukhanov V and Steinhardt P J 2001 Phys. Rev. D 63 103510
Armendariz-Picon C, Mukhanov V and Steinhardt P J 2000 Phys. Rev. Lett. 85 4438
Chiba T, Okabe T and Yamaguchi M 2000 Phys. Rev. D 62 023511

[13] Caldwell R R 2002 Phys. Lett. B 545 23
Carroll S M, Hoffman M and Trodden M 2003 Phys. Rev. D 68 023509
Caldwell R R, Kamionkowski M and Weinberg N N 2003 Phys. Rev. Lett. 91 071301

[14] Hu W 2005 Phys. Rev. D 71 047301
Armendariz-Picon C, Mukhanov V and Steinhardt P J 2001 Phys. Rev. D 63 103510
Armendariz-Picon C, Mukhanov V and Steinhardt P J 2000 Phys. Rev. Lett. 85 4438
Chiba T, Okabe T and Yamaguchi M 2000 Phys. Rev. D 62 023511

[15] Zhang Y 2002 Gen. Rel. Grav. 34 2155
Zhang Y 2003 Gen. Rel. Grav. 35 689
Zhang Y 2003 Chin. Phys. Lett. 20 1899
Zhang Y 2004 Chin. Phys. Lett. 21 1183
Zhou W and Zhang Y 2006 Phys. Lett. B 640 69
Zhou W and Zhang Y 2006 Class. Quantum Grav. 23 3405

[16] Zhang Y, Xia T Y and Zhao W 2007 Phys. Lett. B 656 19
Zhou W 2009 Int. J. Mod. Phys. D 18 1331
Xia T Y and Zhang Y 2007 Phys. Lett. B 656 19

[17] Elizalde E, Lidsey J, Norjiri S and Ordintsov S 2003 Phys. Lett. B 574 L 1
Boehmer C G and Harko T 2007 Eur. Phys. J. C 50 423
Membiela F A and Bellini M 2008 Nuovo Cimento B 123 241
Jimenez J B and Maroto A L 2009 J. Cosmol. Astropart. Phys. JCAP03(2009)016
Gal’usov D V 2009 arXiv:0901.0115

[18] Adler S L 1981 Phys. Rev. D 23 2905
Adler S L 1983 Nucl. Phys. B 217 381
Adler S L and Piran T 1984 Rev. Mod. Phys. 56 1

[19] Komatsu E et al 2011 Astrophys. J. 734 706
Komatsu E et al 2010 Astrophys. J. 726 168
Komatsu E et al 2011 Astrophys. J. Suppl. Ser. 192 18

[20] Percival W J et al 2010 Mon. Not. R. Astron. Soc. 401 2148
Percival W J et al 2007 Mon. Not. R. Astron. Soc. 381 1053
Percival W J et al 2002 Mon. Not. R. Astron. Soc. 337 1068

[21] Eisenstein D J et al 2005 Astrophys. J. 633 560
Percival W J 2007 Astrophys. J. 657 51

[22] Riess A G et al 2009 Astrophys. J. Suppl. Ser. 183 109
Riess A G et al 2009 Astrophys. J. 699 539
Gaztanaga E, Cabré A and Hui L 2009 Mon. Not. R. Astron. Soc. 399 1663
Schwarz G 1997 Ann. Stat. 6 461
Akaikae H 1978 IEEE Trans. Autom. Control 19 716

[23] Zhang J 1994 Phys. Lett. B 340 18
Zhang Y 1996 Class. Quantum Grav. 13 2145
Zhang Y 1997 Chin. Phys. Lett. 14 237
Zhang Y 1998 Chin. Phys. Lett. 15 622
Zhang Y 1998 Commun. Theor. Phys. 30 237
Zhao W 2002 Chin. Phys. Lett. 19 1569

[24] Hu W 2010 J. Cosmol. Astropart. Phys. JCAP08(2010)020
Nesseris S and Perivolaropoulos L 2005 Phys. Rev. D 72 123519
Sahni V, Saini T, Starobinsky A and Alam U 2003 JETP Lett. 77 201
Alam U, Sahni V, Deep Saini T and Starobinsky A A 2003 Mon. Not. R. Astron. Soc. 344 1057
Sahni V, Shafieloo A and Starobinsky A A 2008 Phys. Rev. D 78 103502
Tong M L, Zhang Y and Xia T Y 2009 Int. J. Mod. Phys. D 18 797
Tong M L and Zhang Y 2009 Phys. Rev. D 80 023503
Zhao W 2008 Int. J. Mod. Phys. D 17 1245
Tong M and Noh H 2011 Eur. Phys. J. C 71 1586

[33] Abdalla E, Abramo L R and de Souza J C C 2010 Phys. Rev. D 82 023508