Collapse dynamics of a $^{176}$Yb-$^{174}$Yb Bose-Einstein condensate

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In this paper, we present a theoretical study of a two-component Bose-Einstein condensate composed of Ytterbium (Yb) isotopes in a three dimensional anisotropic harmonic potential. The condensate consists of a mixture of $^{176}$Yb atoms which have a negative s-wave scattering length and $^{174}$Yb atoms having a positive s-wave scattering length. We study the ground state as well as dynamic properties of this two-component condensate. Due to the attractive interactions between $^{176}$Yb atoms, the condensate of $^{176}$Yb undergo a collapse when the particle number exceed a critical value. The critical number and the collapse dynamics are modified due to the presence of $^{174}$Yb atoms. We use coupled two-component Gross-Pitaevskii equations to study the collapse dynamics. The theoretical results obtained are in reasonable agreement with the experimental results of Fukuhara et al. [PRA 79, 021601(R) (2009)].

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I. INTRODUCTION

The first experimental observation of Bose-Einstein condensate (BEC)\(^{1,3}\) in bose atom vapors have initiated an exciting field of research, both theoretically and experimentally. One of the most interesting developments in this field is the formation of multi-component condensates. Multi-component BECs have been observed experimentally by Myatt et al.\(^{4}\) and Hall et al.\(^{5}\) for two different hyperfine spin states of $^{87}$Rb, by Modugno et al.\(^{6}\) for different atoms ($^{41}$K and $^{87}$Rb), and Papp et al.\(^{7}\) for different isotopes of the same atom ($^{85}$Rb and $^{87}$Rb). A rich variety of various interesting effects exhibited by these two-component BECs have inspired a number of theoretical studies covering various aspects of these systems\(^{8-13}\). The common feature of the Bose systems in these experiments is that the intra-component and the inter-component boson-boson interactions are all repulsive. It rises the curiosity about the properties of a multi-component condensate in which one kind of atoms have repulsive interactions while another kind of atoms have attractive interactions. Recently, Fukuhara et al.\(^{14,15}\) observed BEC of spin-zero Yb isotopes by implementing an all-optical cooling protocol. The Bose-bose mixture in these experiments contain $^{174}$Yb atoms having a positive s-wave scattering length and $^{176}$Yb atoms having a negative s-wave scattering length\(^{14,15}\). The s-wave scattering length between $^{174}$Yb and $^{176}$Yb is also positive\(^{15}\). Such a two-component condensate can be expected to show dynamical properties far more complex than a one-component condensate of attractively interacting bosons, which becomes unstable when the number of atoms exceed a critical value\(^{2,10,17}\).

For a two-component BEC with attractive interactions between bosons in one component and repulsive interactions in the second component, two most basic questions are of that of its stability and that of the collapse dynamics. In this paper, we theoretically study the ground state as well as dynamic properties of such a two-component condensate in a three dimensional (3D) anisotropic harmonic potential. We specifically consider the $^{174}$Yb-$^{176}$Yb Bose-Bose mixture in the anisotropic harmonic confining potential as in the experiment of Fukuhara et al.\(^{14,15}\). The paper is arranged as follows. In Sec. II, we describe the theoretical model for the study of a two-component BEC. In Sec. III, we discuss the ground state profile of a stable two-component BEC. In Sec. IV, we present the collapse dynamics of the system and compare it with the experimental results\(^{14}\). The conclusions are given in Sec. V.

II. TWO-COMPONENT BEC: THEORETICAL MODEL

The ground state and dynamic properties of a two component Bose-Einstein condensate (BEC) is well described by a set of coupled Gross-Pitaevskii equations\(^{15,20}\) (GPE) given by

$$i\hbar \frac{\partial \psi_i(r,t)}{\partial t} = \left(-\frac{\hbar^2}{2m_i} \nabla^2 + v_i(r) + g_{ii} |\psi_i(r,t)|^2 + g_{ij} |\psi_j(r,t)|^2 \right) \psi_i(r,t),$$  \hspace{1cm} (1)

where $i = 1, 2$ are indices for the two components (1 for $^{174}$Yb and 2 for $^{176}$Yb), $j = 3 - i$, $r = (x, y, z)^T$ is the spatial coordinate vector, $v_i(r) = (1/2)m_i(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)$ is the trapping potential, $g_{ii} = 4\pi \hbar^2 a_{ii}/m_i$ is the intra-species interaction and $g_{ij} = 2\pi \hbar^2 a_{ij}/m_{ij}$ the inter-species interaction strength between atoms in the condensed state, $a_{ii}$ is intra-species and $a_{ij}$ is inter-species s-wave scattering length, $m_{ij} = m_i m_j/(m_i + m_j)$ is the reduced mass in which
\[ m_i \text{ and } m_j \text{ are atomic masses. The normalization condition for each component is } \int |\psi_i(r)|^2 dr = N_i, \text{ where } N_i \text{ is number of atoms in each component.} \]

We non-dimensionalize Eq. (1) through a set of linear transformations: \( \tilde{t} = \omega_x t, \tilde{r} = r/l, \tilde{\psi}_i(r) = N_i^{-1/2} \psi_i(r). \)

After dropping the wiggles on the symbols, we obtain

\[ i \frac{\partial \psi_i(r,t)}{\partial t} = -\frac{1}{2} \nabla^2 \psi_i(r) + v_i(r) + \lambda_{ii} |\psi_i(r,t)|^2 + \lambda_{ij} |\psi_j(r,t)|^2 \psi_i(r,t), \tag{2} \]

where

\[
\begin{align*}
\lambda_{ii} &= \frac{4 \pi a_{ii}}{l}, \\
\lambda_{ij} &= \frac{4 \pi a_{ij}}{l}, \\
v_i(r) &= \frac{1}{2} (x^2 + \kappa y^2 + \gamma z^2), \\
\kappa &= \frac{\omega_y}{\omega_x}, \\
\gamma &= \frac{\omega_z}{\omega_x}
\end{align*}
\tag{3}
\]

Since the masses of the two isotopes are nearly equal, we have taken \( m_1 = m_2 = m. \) In order to find a stationary solution of Eq. (2), we do a separation of variables \( \psi_i(r,t) = \psi_i(r) \times \exp[-i(\mu_i/\hbar \omega_x)t], \) where \( \mu_i \) is the chemical potential of the \( i \)th component. Starting from Eq. (2), we obtain

\[ (-\frac{1}{2} \nabla^2 + v_i(r) + \lambda_{ii} |\psi_i(r)|^2 + \lambda_{ij} |\psi_j(r)|^2) \psi_i(r) = \frac{\mu_i}{m \omega_x} \psi_i(r). \tag{4} \]

### III. GROUND STATE PROFILES OF A TWO-COMPONENT BEC OF YB ATOMS

In this section, we discuss the ground state properties of a two-component BEC of Yb isotopes by numerically solving the coupled GPE (Eq. (4)). The ground state solution of the GPE is found by the imaginary time propagation method. In this method, the time dependent GPE is evolved in imaginary time starting from an initial guess using a finite difference Crank-Nicholson (FDCN) scheme. In imaginary time propagation we have taken the space step as \( \delta x = \delta y = \delta z = 0.1 \) and the time step as \( \delta t = 0.00005. \) We have used a set of parameters corresponding to the \(^{174}\text{Yb}-^{176}\text{Yb} \) system in the experiment: \( m = 2.8734238 \times 10^{-25} \text{ Kg}, \) \( a_{11} = 5.55 \times 10^{-9} \text{ m}, \) \( a_{22} = -1.28 \times 10^{-9} \text{ m}, \) \( a_{12} = a_{21} = 2.85 \times 10^{-9} \text{ m}, \) \( \nu_x (= \omega_x/2\pi) = 45 \text{ Hz}, \) \( \nu_y (= \omega_y/2\pi) = 200 \text{ Hz}, \) \( \nu_z (= \omega_z/2\pi) = 300 \text{ Hz}. \)

Due to the attractive interaction between \(^{176}\text{Yb} \) atoms, the condensate of \(^{176}\text{Yb} \) undergo a collapse if the particle number exceeds a critical value \( N_c. \) For a single component \(^{176}\text{Yb} \) this value is given by \( N_c \approx 0.5 L/|a|, \) where
they make negligible contribution in this case.

We time evolve the coupled time-dependent GP equations Eq. (6) using the finite difference Crank-Nicholson (FDCN) scheme with a known initial condition. In real time propagation, we have taken the space step as $\delta x = \delta y = \delta z = 0.1$ and the time step as $\delta t = 0.005$. The time evolution of the number of $^{176}$Yb and $^{174}$Yb is shown in Fig. 2 along with the experimental data of Fukuhara et al.\(^{14}\). Considering the complex dynamics of the

IV. COLLAPSE DYNAMICS OF A TWO-COMPONENT BEC OF YB ATOMS

In this section, we study the collapse dynamics of a two component BEC composed of $^{176}$Yb and $^{174}$Yb atoms using the coupled time-dependent GP\'s. Due to the negative s-wave scattering length, the $^{176}$Yb condensate becomes unstable if the number of atoms becomes greater than a critical value $N_{2c}$. This condensate collapses by emitting atoms out of it. Due to high density of atoms in the attractive condensate, the loss of atoms from the condensate occurs through three-body collisions. To model this collapse we add a imaginary three body quintic loss term\(^{22–25}\) to the RHS of GPE Eq. (1) given by

$$K_i \psi_i = -\frac{1}{12}iK_i^3|\psi_i|^4 \psi_i,$$

(5)

where $K_i^3$ is the three-body loss coefficient for each component. We have neglected the two-body dipolar loss term as they make negligible contribution in this case\(^{14,23–25}\). We have also left out the loss terms for $^{174}$Yb – $^{174}$Yb, $^{174}$Yb – $^{176}$Yb, $^{176}$Yb – $^{176}$Yb, $^{176}$Yb – $^{174}$Yb, and $^{176}$Yb – $^{174}$Yb collisions, since the losses due to these are comparatively small\(^{14}\). So, Eq. (2) becomes

$$i\frac{\partial \psi_i(r,t)}{\partial t} = \left(-\frac{1}{2}\nabla^2 + v_i(r) + \lambda_{ii} |\psi_i(r,t)|^2 + \lambda_{ij} |\psi_j(r,t)|^2 - i\xi_i |\psi_i(r,t)|^4\right) \psi_i(r,t),$$

(6)

where $\xi_i = (1/12)N^2K_i^3t^{-6} \omega_i^{-1}$.

We time evolve the coupled time-dependent GP equations Eq. (6) using the finite difference Crank-Nicholson (FDCN) scheme with a known initial condition. In real time propagation, we have taken the space step as $\delta x = \delta y = \delta z = 0.1$ and the time step as $\delta t = 0.005$. The time evolution of the number of $^{176}$Yb and $^{174}$Yb is shown in Fig. 2 along with the experimental data of Fukuhara et al.\(^{14}\). Considering the complex dynamics of the
the two-component system with mixed interactions, the theoretical results may be said to be in reasonable agreement with the experimental results. We observe that there is a significant loss of $^{176}$Yb atoms. We also see that the decay of $^{176}$Yb is very rapid. It is due to the collapse of the $^{176}$Yb condensate. The number of $^{174}$Yb atoms does show a very small decrease, which is not visible on the scale of this figure. To understand the details of the decay process, we study the condensate profiles of each component at different times. The results are shown in the Fig. 3 for $^{176}$Yb and in Fig. 4 for $^{174}$Yb. At $t = 0$, the $^{176}$Yb are at the center of the trap (top left panel of Fig. 3) surrounded by $^{174}$Yb (top left panel of Fig. 4). Since the number of atoms in the attractive component is higher than the critical number for stability, the system is unstable. When the system evolves in time, the attractive component explodes as is evident from the spreading of this component with time, in real space, as shown in Fig. 3. The spiky structures in these figures represent the inhomogeneities produced due to the on-going explosion process. As mentioned earlier, the explosion also leads to a spread of the ground state profile. Due to the coupling between the attractive and repulsive components, the condensate of $^{174}$Yb is also redistributed in real space during the time evolution, as shown in Fig. 4. The numbers of remaining atoms in each condensate component during the time evolution, for a longer period of time, is shown in Fig. 5. We note that the best agreement with the experimental results (Fig. 2) is obtained for the measured values of the $K_1^3$ and $K_2^3$. The disagreement at later times is likely to be originating from the neglect of atomic loss due to collisions involving $^{176}$Yb and $^{174}$Yb. During the initial stages of the time evolution, the bosons distribution gets heavily mixed due to the explosion and due to the coupling between the two components. Then, at later times, the inter-component collisions is likely to affect the atom loss. We are unable to include these loss terms since their values are not known at present.

V. CONCLUSIONS

In this paper, we presented a study of some static and dynamic properties of a two-component bose condensate consisting of repulsively interacting $^{174}$Yb atoms and attractively interacting $^{176}$Yb atoms in an anisotropic harmonic confinement. In the stable state, the ground state has $^{176}$Yb atoms in the center of the trap surrounded by $^{176}$Yb atoms. When the number of atoms in the attractive component exceed a critical value, the the system undergo a collapse. We analyzed the time evolution of this collapse process for the specific system parameters of the $^{176}$Yb-$^{174}$Yb system studied in the experiment of Fukuhara et al.. The details of the collapse dynamics are found to be in reasonable agreement with experimental results. The critical number for stability of the attractive condensate is reduced by it’s interaction with the repulsive condensate.
FIG. 4: Ground state profile of $^{174}\text{Yb}$ at different times. The four figures correspond to $t(\text{seconds}) = 0.0$ (top left panel), 0.2 (top right) 0.4 (bottom left), and 2 (bottom right). The values of the three body recombination terms are: $K_1^3 = 4.2 \times 10^{-29} \text{cm}^6 \text{s}^{-1}$, $K_2^3 = 3.0 \times 10^{-28} \text{cm}^6 \text{s}^{-1}$. Here, $N_1 = 6 \times 10^4$ and $N_2 = 2 \times 10^4$.

FIG. 5: Time evolution of number of $^{174}\text{Yb}$ (top curve), $^{176}\text{Yb}$ (solid line, dashed, dash-dot lines). The values of three body recombination terms are: $K_1^3 = 4.2 \times 10^{-29} \text{cm}^6 \text{s}^{-1}$ (for all lines), $K_2^3 = 3.0 \times 10^{-28} \text{cm}^6 \text{s}^{-1}$ (solid line), $K_3^3 = 3.0 \times 10^{-27} \text{cm}^6 \text{s}^{-1}$ (dashed line), $K_4^3 = 3.0 \times 10^{-26} \text{cm}^6 \text{s}^{-1}$ (dash-dot line). The time is in units of seconds.

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