Transfer to the continuum of $^{11}$Be with the application of ab-initio S-matrix

D Carbone$^1$, A Bonaccorso$^2$, F Cappuzzello$^{1,3}$, M Cavallaro$^1$, G Hupin$^4$, P Navrátil$^5$ and S Quaglioni$^6$

1 Istituto Nazionale di Fisica Nucleare - Laboratori Nazionali del Sud, Catania, Italy
2 Istituto Nazionale di Fisica Nucleare – Sezione di Pisa, Italy
3 Dipartimento di Fisica ed Astronomia “Ettore Majorana”, Università degli Studi di Catania, Italy
4 Institut de Physique Nucléaire, IN2P3, CNRS, Université Paris-Sud, Université Paris-Saclay, Orsay, France
5 TRIUMF, Vancouver, British Columbia, Canada
6 Lawrence Livermore National Laboratory, Livermore, California, USA

carboned@lns.infn.it

Abstract. The $^{9}$Be($^{18}$O,$^{16}$O)$^{11}$Be two-neutron transfer reaction at 84 MeV is measured at forward angles to explore the continuum of $^{11}$Be nucleus. Transfer-to-the-continuum (TC) reaction calculations are applied using for the first time an ab-initio S-matrix. It is derived from the ab-initio no-core shell model with continuum (NCSMC) method, recently used to describe the bound and continuum spectrum of $^{11}$Be. Previously known levels are confirmed and theoretical and experimental evidence for a $9/2^+$ state at $E_x = 5.8$ MeV is given, whose configuration is thought to be $^{10}$Be($2^+)+n(d_{5/2})$.

1. Introduction
In this paper we discuss the first application of an ab-initio structure model for continuum states [1] to the calculation of a transfer reaction to resonant states [2-6]. Traditionally, a reduction of the complicated many-body problem to a one-body problem via a phenomenological optical potential to treat the continuum neutron-target final state is used, as previously done for $^{10}$Be [6,7] for example. This complication is avoided here thanks to the use of the $S$-matrix obtained within the no-core shell model with continuum (NCSMC) method [1].

The study of neutron transfer reactions on a $^9$Be target is particularly interesting because both $^{10}$Be and $^{11}$Be residual nuclei can be accessed. In particular, $^{11}$Be is a weakly bound nucleus with peculiar phenomenology [8,9], e.g. the role of correlations between core excited states and neutron orbitals [1,10-12], neutron-neutron pairing and even molecular clustering [13].

In our investigation we use an $^{18}$O projectile to induce one- and two-neutron transfer reactions, populating in this way states of neutron-rich nuclei starting from stable targets [6,14-20]. The observed selectivity of $^{18}$O-induced transfer reactions allows for a consistent exploration of both single-particle features via the ($^{18}$O,$^{17}$O) reaction and two-neutron correlations via the ($^{18}$O,$^{16}$O) reaction. In particular, the ($^{18}$O,$^{16}$O) reaction proceeds mainly via two mechanisms: (i) a two-step single-neutron transfer, where the two neutrons are independently transferred and (ii) a one-step transfer of a correlated pair of
neutrons, populating mainly two-neutron configurations in the residual nucleus. In the following we use a theoretical model corresponding to mechanism (i).

By studying the \(^{18}\text{O},^{17}\text{O}\) one-neutron transfer reaction we explored both bound and continuum states of \(^{10}\text{Be}\) [6]. A very accurate description of the spectrum was achieved using an \(S\)-matrix built from two different \(n^{9}\text{Be}\) optical potentials, fitted to scattering data over a very large energy range [7]. The \(S\)-matrix is one of the fundamental ingredients of the transfer-to-the-continuum model [2-4], which allows to reproduce the excitation energy spectrum of the target [5,6]. The \(^{9}\text{Be}(^{18}\text{O},^{17}\text{O})^{10}\text{Be}\) experiment and relative theoretical description [6] constituted the first step in the study of the \(^{9}\text{Be}(^{18}\text{O},^{17}\text{O})^{11}\text{Be}\) reaction that we show in this paper.

2. Experimental setup and results

The experiment was performed at Istituto Nazionale di Fisica Nucleare–Laboratori Nazionali del Sud using an \(^{18}\text{O}\)\(^{6}\) beam at 84 MeV incident energy. It was delivered by the Tandem Van de Graaff facility. A self-supporting 200 \(\mu\)g/cm\(^2\) thick \(^{9}\text{Be}\) target, 50 \(\mu\)g/cm\(^2\) self-supporting \(^{12}\text{C}\) target and 260 \(\mu\)g/cm\(^2\) WO\(_3\) target were used. The last two were necessary to record supplementary runs in order to estimate the background in the energy spectra from \(^{12}\text{C}\) and \(^{16}\text{O}\) impurities in the \(^{9}\text{Be}\) target.

The \(^{16}\text{O}\) ejectiles were momentum analyzed by the MAGNEX magnetic spectrometer [21] and detected by its focal plane detector [22], covering and angular range \(3^\circ < \theta_{lab} < 10^\circ\). Details about the particle identification and the trajectory reconstruction techniques used for the data reduction are described in Refs. [23-26]. An overall energy and angular resolutions of about 500 keV (FWHM) and 0.3\(^\circ\) were obtained. The absolute cross section was estimated according to Ref. [24], with a total error of about 10\% induced by the uncertainties in the target thickness and beam current integration.

The obtained \(^{11}\text{Be}\) absolute cross-section spectrum, after the background subtraction, is shown in Figure 1. Several peaks signal the population of bound and resonant states of \(^{11}\text{Be}\) in this transfer reaction. The \(1/2^+\) g.s. and the \(1/2^+\) first excited state at 0.320 MeV are not resolved. Above the one-neutron separation energy \((S_n = 0.502 \text{ MeV})\), a strong excitation of the \(5/2^+\) state at 1.783 MeV is observed, while the states at 2.654 and 3.400 MeV are less populated.

Beyond the \(^{10}\text{Be}\) core excitation threshold we observe a peak at 3.9 MeV, where we cannot separate the transitions to the known states at 3.889 MeV \((5/2^-)\) and 3.955 MeV \((3/2^-)\). The two states in a two-neutron transfer reaction were separated only in Ref. [27], showing that they are both strongly excited by such a probe. Intense transitions are observed to the \(5/2^-\) state at 5.255 MeV and the state at 6.705 MeV, resembling the situation observed in all the reported two-neutron transfer reaction data [28–30], which describe such states by \(^{9}\text{Be} \otimes (sd)^2\) configurations.

Debated are the states at 5.8 and 8.813 MeV, which are both present in all reported two-neutron transfer experiments, even though the centroid energy and width are slightly different in the various studies [28-33]. Recently, for the former state, a 9/2\(^+\) spin parity was found for a state with a \(^{10}\text{Be}(2,1^+) \otimes (1d)^{5/2}(9/2^+)\) stretched configuration using the \textit{ab-initio} NCSMC approach [1]. This result is in agreement with the present finding. For the state at 8.813 MeV, our data just confirm the population of it in two-neutron transfer reactions.
Figure 1. $^{11}$Be experimental inclusive excitation-energy spectrum from the $^{9}$Be($^{18}$O, $^{16}$O)$^{11}$Be reaction at 84 MeV incident energy for $3^\circ < \theta_{lab} < 10^\circ$ (black histogram). Total 2n transfer calculations resulting from the use of the $S$-matrix from Ref. [1] folded with the experimental resolution (500 keV) and renormalized by a factor of 3.2 (green curve). The values of one- and two-neutron separation energies are also indicated (grey dashed lines).

3. Theoretical approach and data analysis

The description of the $n + ^{10}$Be in the continuum (above $S_n$) is very challenging from the theoretical point of view, because no experimental data exist on the free $n + ^{10}$Be scattering. Here it was possible to overcome this limitation thanks to the recent ab-initio calculation of the $^{11}$Be spectrum [1], which described both bound and continuum states. From the continuum part the $S$-matrix was extracted to be used in the TC method.

We describe the reaction in two independent steps: in the first step the $^{9}$Be($^{18}$O, $^{17}$O)$^{10}$Be reaction occurs and neutron $n1$ is transferred from $^{18}$O to populate the bound states of $^{10}$Be ($^{10}$Be$_{gs}$, $^{10}$Be(2$^+_1$)) at 3.37 MeV, $^{10}$Be(2$^+_2$) at 5.96 MeV). Then the second neutron ($n2$) is transferred from $^{17}$O to $^{10}$Be. Since we want to describe only the continuum part of the spectrum (above $S_n$) we use the following formula for the two-neutron transfer description:

$$\frac{d\sigma_{2n}}{dE_{f2}} = C^2 S^{\text{phen}}_{n1}(R_s) \int_{b_{\text{min}}}^{b_{\text{max}}} db_c |S_{CT}(b_c)|^2 \frac{dP_{n2}(b_c)}{db_c}$$

in which $P_{n1}^{\text{phen}}$ is the bound state transfer probability for neutron $n1$ from $^{18}$O to $^{9}$Be, $n2$ is the second neutron transferred to a continuum state from $^{17}$O and $C^2S$ is its initial wave function spectroscopic amplitude. $S_{CT}(b_c)$ is calculated according to Ref. [34].

Since the $^{9}$Be($^{18}$O, $^{17}$O)$^{10}$Be reaction was measured in the same experimental conditions and the corresponding experimental cross section extracted [6], we used that experimental data to deduce $P_{n1}^{\text{phen}}$. To this purpose we used the transfer cross section between bound state formula from Ref. [35].

The advantage of this method is that we assume that all the structure information, such as the spectroscopic factors of both projectile and target are included in the measured cross sections.
Then the second neutron $n2$ transfer probability to the continuum states of $^{11}\text{Be}$ is given by

$$\frac{dP_{n2}}{d\varepsilon_{f2}} = K \sum_{j_f,\nu} \left( |1 - S_{j_f,\nu}|^2 + 1 - |S_{j_f,\nu}|^2 \right) B(j_f, j_i)$$ (2)

The definition of the various quantities beside the $n$-target $S$-matrix in equation (2) can be found in Ref. [3], in which many details of the TC method are given. In equation (2) for each continuum energy the sum is over all possible $n + ^{10}\text{Be}$ total angular momenta. Above the thresholds for the first and second $2^+$ excited states in $^{10}\text{Be}$, for each angular momentum there is also an incoherent sum over all channels contributing to it. The $S$-matrix appearing in equation (2) is taken from the ab-initio calculations of Ref. [1].

In Figure 1 we show the resulting theoretical calculations. A folding with the experimental resolution (500 keV) and a renormalization factor of 3.2 to fit the data are applied. The first three continuum states are well reproduced as far as the position is concerned, indicating that the TC model contains the correct dynamics and the $S$-matrix corresponds to accurate structure information. Note that also the order of magnitude is reproduced reasonably well within the uncertainties due to the description of the two-neutron transfer reaction mechanism. The state at 2.654 MeV is depleted in our model calculation because of the unfavorable one-neutron transfer reaction matching [36]. However, it is well seen in the present data as well as in all previous two-neutron transfer experiments [30], confirming for this state a relevant $2n + ^{9}\text{Be}$ configuration and a population via the simultaneous $2n$-transfer reaction mechanism. On the other hand, the calculated peak at $E_x = 3.400$ MeV has such a small width (~ 0.02 MeV) from [1] that its presence is hardly visible as a structure in the experiment. As far as the absolute total cross sections are concerned the only comparable values are those for the 1.783 MeV $d_{5/2}$ state, for which we get $\sigma_{\text{exp}} = 359 \pm 35 \mu b$ and $\sigma_{\text{th}} = 175 \mu b$.

The cross section of the resonance at 5.8 MeV is reasonably well reproduced by our model, thus strengthening the interpretation of a $9/2^+$ stretched configuration, discussed above and in Ref. [1]. It is the only state that can have a definite single-particle nature in the high-energy part of the spectrum, because it has a high spin which does not mix with underlying components of low angular momenta. It can be seen only in a reaction with heavy ions where the matching conditions allow the reaching of a high resonance energy and high angular momentum. This state can be reproduced by the TC, which contains explicitly the spin couplings between initial and final states besides the angular momentum couplings. Therefore, the present work provides the first evidence for the existence of such a state.

Finally, the free $n + ^{10}\text{Be}$ cross section (elastic plus inelastic), calculated with the ab-initio $S$-matrix is shown in Figure 2 together with the partial wave decomposition in terms of total angular momentum $L$. This is helpful in order to understand the origin of the various state strength distribution in the spectrum. For instance, the discussed resonance at 5.8 MeV gets contributions only from $L = 4$. 


Figure 2. Dominant contributions to the partial wave decomposition of the free $n + ^{10}\text{Be}$ theoretical energy spectrum (cyan solid curve). The legend indicates the total angular momentum of each individual strength distribution. The lines starting at $E_x = 0.501$ MeV refer to the $^{10}\text{Be}_{g.s.} + n$ system, those starting at 3.9 MeV and 6.5 MeV to the $^{10}\text{Be}(2_1^+) + ^{10}\text{Be}(2_2^+)$, respectively (see text).

4. Conclusions
We have presented experimental data on the $^{11}\text{Be}$ energy spectrum populated by the $^9\text{Be}(^{18}\text{O},^{16}\text{O})$ two-neutron transfer reaction at 84 MeV. We have been able to reproduce the position, widths and magnitude of the cross section of the $n + ^{10}\text{Be}$ components in the experimental excitation energy spectrum of the exotic nucleus $^{11}\text{Be}$ by using an $ab$-initio $S$-matrix coupled for the first time with the TC method. Also there is an evidence, both theoretical and experimental, of the presence of $9/2^+$ state at $E_x = 5.8$ MeV with a $[^{10}\text{Be}(2_1^+) \otimes (1d)^{5/2}]^{9/2^+}$ configuration.

Acknowledgments
D.C and M.C., received funding from the European Research Council under the European Union’s Horizon 2020 research and innovation program (Grant Agreement No. 714625). P.N.’s work was supported in part by NSERC Grant No. SAPIN-2016-00033. TRIUMF received federal funding via a contribution agreement with the National Research Council of Canada. Computing sup- port for G.H., P.N., and S.Q. came from an INCITE Award on the Titan supercomputer of the Oak Ridge Leadership Computing Facility at Oak Ridge National Laboratory, from Compute Canada, and from the Lawrence Livermore National Laboratory (LLNL) institutional Computing Grand Challenge Program. This article was prepared in part by LLNL (S.Q.) under Contract No. DE-AC52-07NA27344. This material is based in part upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Field Work Proposal No. SCW0498.
References

[1] Calci A, Navrátil P, Roth R, Dohet-Eraly J, Quaglioni S and Hupin G 2016 Phys. Rev. Lett. 117 242501.
[2] Bonaccorso A and Brink D M 1988 Phys. Rev. C 38 1776.
[3] Bonaccorso A and Brink D M 1991 Phys. Rev.C 43 299.
[4] Bonaccorso A and Brink D M 1991 Phys. Rev. C 44 1559.
[5] Blanchon G, Bonaccorso A and Vinh Mau N 2004 Nucl. Phys. A 739 259.
[6] Carbone D et al., Phys. Rev. C 90 064621.
[7] Bonaccorso A and Charity R J 2014 Phys. Rev. C 89 024619.
[8] Fortune H T, 2108 Eur. Phys. J. A 54 73.
[9] Kelley J H, Purcell J E and Sheu C G 2017 Nucl. Phys. A 968 71.
[10] Vinh Mau N 1995 Nucl. Phys. A 592 33.
[11] Lenske H et al. 2001 Rep. Prog. Nucl. Part. Phys. 46 187.
[12] Barranco F, Potel G, Broglia R A and Vigezzi E 2017 Phys. Rev. Lett. 119 082501.
[13] von Oertzen, W 1996 Z. Phys. A 354 37.
[14] Cappuzzello F et al. 2012 Phys. Lett. B 711 347.
[15] Cavallaro M et al. 2016 Phys. Rev. C 93 064323.
[16] Carbone D et al. 2017 Phys. Rev. C 95 034603.
[17] Cappuzzello F et al. 2015 Nat. Commun. 6 6743.
[18] Ermamatov M J et al. 2016 Phys. Rev. C 94 024610.
[19] Paes B et al. 2017 Phys. Rev. C 96 044612.
[20] Agodi C et al. 2018 Phys. Rev. C 97 034616.
[21] Cappuzzello F, Agodi C, Carbone D and Cavallaro M 2016 Eur. Phys. J.A 52 167.
[22] Cavallaro M et al. 2012 Eur. Phys. J.A 48 59.
[23] Cappuzzello F et al. 2010 Nucl. Instrum. Methods A 621 419.
[24] Carbone D 2015 Eur. Phys. J. Plus 130 143.
[25] Cavallaro M et al. 2011 Nucl. Instrum. Methods A 637 77.
[26] Cappuzzello F, Carbone D and Cavallaro M 2011 Nucl. Instrum. Methods A 638 74.
[27] Liu G B and Fortune H T 1990 Phys. Rev. C 42 167.
[28] Bohlen H G et al. 2003 Phys. At. Nucl. 66 1494.
[29] Bohlen H G et al. 2003 Nucl. Phys. A 722 C3.
[30] Haigh P J et al. 2009 Phys. Rev. C 79 014302.
[31] Cappuzzello F et al. 2001 Phys. Lett. B 516 21.
[32] Hirayama Y et al. 2005 Phys. Lett. B 611 239.
[33] Fynbo H O U et al. 2004 Nucl. Phys. A 736 39.
[34] Bonaccorso A, Carstoiu F and Charity R J 2016 Phys. Rev. C 94 034604.
[35] Bonaccorso A, Brink, D M and Lo Monaco L 1987 J. Phys. G 13 1407.
[36] Schmitt K T et al. 2013 Phys. Rev. C 88 064612.