Unsteady Aerodynamic Parameter Estimation for Multirotor Helicopters*

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Today, multirotor helicopters (MRHs) play an important role in a broad range of applications such as transportation, observation and construction, and the safety of MRH flight is a matter of great concern. This study contributes to clarifying an aerodynamic aspect that enables the prediction of MRH behavior in unsteady conditions through flight tests. In working towards a comprehensive mathematical model that determines unsteady aerodynamics, a quadcopter is equipped with a data acquisition system to gather flight data including acceleration, angular rates, flow angles, airspeed and rotational speed. Based on the data collected, the combined blade element momentum theory is utilized to calculate steady and unsteady aerodynamic parameters. It is found that the experimental aerodynamic coefficients agree well with the theoretical results for steady forward flight. However, the conventional theory was insufficient to model the aerodynamic parameters under unsteady conditions. A new model to predict aerodynamic parameters under unsteady flight is proposed and validated on the basis of the flight data.

Key Words: Multirotor Helicopter, Unsteady Aerodynamics, Flight Experiment, Load Factor

Nomenclature

- \(a\): 2-D lift curve slope
- \(a_x\): acceleration along \(x_b\) axis
- \(a_y\): acceleration along \(y_b\) axis
- \(a_z\): acceleration along \(z_b\) axis
- \(A\): rotor disk area
- \(B\): tip loss factor
- \(c\): blade chord
- \(C_D\): drag coefficient
- \(C_H\): horizontal force coefficient
- \(C_T\): thrust coefficient
- \(h\): altitude
- \(H\): horizontal force
- \(n\): load factor
- \(N\): number of blades
- \(p\): roll rate
- \(q\): pitch rate
- \(r\): yaw rate
- \(r_l\): fraction of blade span from axis (\(= l/R\))
- \(R\): rotor radius
- \(T\): thrust
- \(u\): translational velocity along \(x_b\) axis
- \(v\): translational velocity along \(y_b\) axis
- \(v_i\): induced velocity
- \(V\): total velocity
- \(V_{\infty}\): free-stream velocity
- \(w\): translational velocity along \(z_b\) axis
- \(\alpha\): angle of attack
- \(\beta\): sideslip angle
- \(\gamma\): lock number
- \(\lambda\): inflow ratio
- \(\lambda_p\): bias of roll rate
- \(\lambda_q\): bias of pitch rate
- \(\lambda_r\): bias of yaw rate
- \(\lambda_x\): bias of acceleration \(a_x\)
- \(\lambda_y\): bias of acceleration \(a_y\)
- \(\lambda_z\): bias of acceleration \(a_z\)
- \(\mu\): advance ratio
- \(\rho\): air density
- \(\sigma\): solidity
- \(\phi\): roll angle
- \(\theta\): pitch angle
- \(\theta_{avg}\): rotor average pitch angle
- \(\psi\): yaw angle
- \(\omega\): rotational speed
- \(i\): inertial frame
- \(\exp\): experiment

1. Introduction

Today, multirotor helicopters (MRHs) play an important role in a broad range of applications such as transportation, observation and construction.\(^1,2\) For the safety of MRH flight, unsteady the aerodynamic response of MRHs to abrupt steering and/or wind gusts is a matter of great concern.

In literature, a few studies have been devoted to MRH aerodynamics.\(^3,5\) The interference of the front rotor on operation of the rear rotor due to the wake generated by the front rotor was examined in Hung et al.\(^3\) The effect of rotor blade flapping on attitude control was explored by Hoffmann et al.\(^5\) A comparison of fixed and variable pitch actuators was presented through a series of experiments conducted by Cutler et al.\(^5\) However, the unsteady aerodynamic characteristics of MRHs are still not completely understood.

Achieving a precise model of external forces acting on
MRHs under maneuverable flight conditions leads to the requirement that considerable flight experiments need to be carried out. In our previous study, a wind-tunnel was used to investigate the steady aerodynamics of a quadrotor. However, that was not sufficient. Unsteady conditions with time-varying parameters occur frequently in actual flight due to pilot operation and/or wind gusts. The load factor, $n$, has been used conventionally to characterize the unsteady aerodynamic load in structural design and maneuverable capability of conventional helicopters. For safety when designing the airframe and automatic control, it is indispensable to study the unsteady aerodynamic response of MRHs.

For this study, outdoor flight experiments were conducted to measure the aerodynamic forces and incoming flow vector using on-board sensors and a data acquisition system so that unsteady aerodynamics could be examined. For investigating the aerodynamic characteristics of a quadrotor helicopter in actual flight, this study addresses two primary interests: Steady forward flight and transition from forward flight to descent. Moreover, the thrust coefficient in transient flight is formulated as a function of the load factor, and is validated based on the experimental data collected. The methodology proposed will have a strong impact on the future studies of flight dynamics, flight simulation and adaptive control system design of MRHs.

2. Mathematical Model

2.1. Coordinate system

It is necessary to define the different coordinate systems for the following reasons:

- Aerodynamic forces act on a quadrotor, such as the drag force described in the wind axes system and the thrust force described in the body axes system.
- On-board sensors such as accelerometers and gyroscopes measure acceleration and angular rates with respect to the sensory axes system.

The coordinate systems are defined as: Body axes system, $O_{B}x_B y_B z_B$; wind axes system, $O_{W}x_W y_W z_W$; and sensory axes system, $O_{S}x_S y_S z_S$; as shown in Fig. 1.

2.2. Aerodynamic modeling for forward flight

This model is based on the momentum theory and blade element theory. In the momentum theory, the flow is assumed to be incompressible and inviscid, and blade-loading is assumed to be distributed uniformly over the blade. The blade element theory is based on the lifting-line assumption, neglecting stall. The advance ratio $\mu$, inflow ratio $\lambda$, and solidity $\sigma$ are defined as

\[
\mu = \frac{V_{\infty} \cos \alpha}{\omega R}
\]

\[
\lambda = \frac{V_{\infty} \sin \alpha + v_i}{\omega R}
\]

\[
\sigma = \frac{Nc}{\pi R}.
\]

In the conventional theory, the thrust coefficient of a rotor in forward flight with a linearly twisted pitch angle is $\theta = \theta_0 + \theta_{tw} r_i$,

\[
C_T = \frac{1}{2} \sigma \left( \frac{1}{3} B^3 \theta_0 + \frac{1}{4} B^4 \theta_{tw} + \frac{1}{2} B \mu^2 \theta_0 \right)
\]

\[
+ \frac{1}{4} B^2 \mu^2 \theta_{tw} - \frac{1}{2} B^2 \lambda \right)
\]

or, constant twist, $\theta = \theta_{avg}$

\[
C_T = \frac{1}{2} \sigma \left( \frac{1}{3} B^3 \theta_{avg} + \frac{1}{2} B \mu^2 \theta_{avg} - \frac{1}{2} B^2 \lambda \right)
\]

and thrust,

\[
T = C_T \rho(\omega R)^2 \frac{4 \pi R^2}{2}.
\]

The thrust coefficient based on the momentum theory is:

\[
C_T = 2(\lambda - \mu \tan \alpha) \sqrt{\mu^2 + \lambda^2}.
\]

By equating the right-hand sides of Eqs. (5) and (7), the formula of the inflow ratio can be derived as:

\[
\lambda = \frac{2}{B^2} \left( \frac{1}{3} B^3 \theta_{avg} + \frac{1}{2} B \mu^2 \theta_{avg} \right)
\]

\[
- \frac{8}{B^2 \sigma \lambda} (\lambda - \mu \tan \alpha) \sqrt{\mu^2 + \lambda^2}.
\]

The Newton-Raphson procedure can be used to solve for $\lambda$ iteratively.

The drag force can be split into three components: induced drag, profile drag and parasite drag. The horizontal force coefficient is:

\[
C_H = \frac{\sigma C_{d_{H}}}{4} \mu + \frac{\sigma}{2} \left[ \frac{\theta_0}{3} \left(-\beta_{1c} + \frac{3}{2} \mu \lambda \right) \right]
\]

\[
+ \frac{3}{4} \lambda \beta_{1c} + \frac{1}{4} \beta_{0} \beta_{1s} + \frac{1}{4} \mu \left( \beta_{0}^2 + \beta_{1s}^2 \right)
\]

The first term in Eq. (9) is the profile drag and the second term is the induced drag.

Different from most conventional helicopters, stiff fixed-pitch rotor blades are used on MRHs. In this study, a thin air-
foil is utilized and the approximate calculation of rotor blade flapping is based on the formula derived for helicopter rotor blades:

\[ \beta_0 = \frac{1}{2} \gamma' \left( \frac{\theta_0.5}{4} (1 + \mu^2) - \frac{\lambda}{3} \right) - \frac{M_w}{I_0 \omega^2} \]

(10)

\[ \beta_{1x} = 2 \mu \left( \lambda - \frac{4}{3} \theta_0.5 \right) \]

(11)

\[ \beta_{1z} = -\frac{4}{3} \beta_0 \mu \]

(12)

where, \( \gamma \) is the lock number, \( M_w = m_{blade} g R^2 / 2 \) is the moment caused by the rotor blade and \( I = m_{blade} R^3 / 3 \) is the inertia moment of the rotor blade. More precise calculations of the flapping coefficients for stiff, fixed-pitch rotor blades were provided by Johnson. Then, the horizontal force \( H \) becomes:

\[ H = C_H \rho (\omega R)^2 4\pi R^2. \]

(13)

The measurement of drag force is necessary to know the maximum forward speed that the quadcopter can achieve for a specific angle of attack and rotational speed (RPM). For this purpose, the total drag force of the quadcopter is expressed as:

\[ D_{\text{total}} = T \sin \alpha - H \cos \alpha - D_{\text{parasite}}. \]

(14)

The total drag coefficient of the quadcopter is finally formulated as:

\[ C_D = \frac{D_{\text{total}}}{\rho (\omega R)^2 4\pi R^2}. \]

(15)

At a sufficiently high advanced ratio, reverse flow is generated in a small circular region on the rotordisk. The diameter of the region of reverse flow corresponds to \( \mu \), and it is known that Eq. (9) is valid for \( \mu \) less than 0.4 where the effect of reverse flow is negligible.

### 2.3. Parameter estimation from flight data

Thrust is represented along the \( z \) direction of the body axes and points up, and is therefore directly measured by

\[ T^{\exp} = m a_z'. \]

(16)

Total drag force is aligned and opposite the velocity vector, therefore the acceleration measured should be first transformed to body axes \( a_x = a_x', a_y = -a_x', \) and \( a_z = -a_z' \), and then transformed to wind axes using the directional cosine matrix \( H_B^W \):

\[ H_B^W = \begin{bmatrix} \cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\ -\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}. \]

(17)

As a result,

\[ D_{\text{total}}^{\exp} = m \left( a_x' \cos \alpha \cos \beta - a_y' \sin \beta - a_z' \sin \alpha \cos \beta \right), \]

(18)

and thrust coefficient, \( C_T^{\exp} \), and drag coefficient, \( C_D^{\exp} \), are:

\[ C_T^{\exp} = \frac{T^{\exp}}{\rho (\omega R)^2 4\pi R^2}, \]

(19)

\[ C_D^{\exp} = \frac{D_{\text{total}}^{\exp}}{\rho (\omega R)^2 4\pi R^2} \cdot \]

(20)

It should be noted that Eqs. (5) and (15) are respectively theoretical thrust coefficient and drag coefficient in steady forward flight, whereas Eqs. (19) and (20) are respectively experimental thrust coefficient and drag coefficient applied for all flight regimes including hovering, forward, descent and transient flight.

### 2.4. Unsteady aerodynamic parameter estimation using the recursive least squares (RLS) method

On the basis of Eq. (5), the thrust coefficient in steady forward flight can be written in polynomial form as the sum of the static term and derivatives of inflow ratio and advance ratio as:

\[ C_T = C_{T_0} + C_{T_1} \lambda + C_{T_2} \mu^2. \]

(21)

By equating the right-hand sides of Eq. (5) and Eq. (21), the expressions of parameters in steady forward flight are:

\[ C_{T_0} = \frac{1}{6} \sigma a_b V T_{\text{avg}} \]

(22)

\[ C_{T_1} = -\frac{1}{4} \sigma b \]

(23)

\[ C_{T_2} = \frac{1}{4} \sigma a \]

(24)

There are some aspects that should be noticed. First, the final form of the thrust coefficient depends on the theories applied, as blade element theory (Eq. (5)) or momentum theory (Eq. (7)), in which several approximations are made to derive those equations; such as approximating the lift coefficient of the blade section, approximating blade flapping angle, and approximating the twist angle of the blade section. If the reference twist angle is taken to be included in the azimuth angle term, then the first-order term of \( \mu \) will appear in thrust coefficient expressions Eqs. (5) and (21).

Actual flight experiments often consist of steady and unsteady flights, and it is therefore necessary to extend conventional models by accounting for the influence of vertical flow (\( w \)) and horizontal flow (\( v \)) or inflow angles (\( \alpha, \beta \)) explicitly for some large amplitude maneuvers, transient flights or rapid excursions from the steady flight conditions. Therefore, we propose new models for thrust coefficient and drag coefficient that can be applied to all flight regimes:

\[ C_T^{\exp} = C_{T_0} + C_{T_1} \lambda + C_{T_2} \mu + C_{T_3} v + C_{T_4} w + C_{T_5} \mu^2 \]

(25)

\[ C_D^{\exp} = C_{D_0} + C_{D_1} \lambda + C_{D_2} \mu + C_{D_3} \alpha + C_{D_4} \beta + C_{D_5} \lambda^2 + C_{D_6} \mu^2 + C_{D_7} \alpha^2 \]

(26)

For the purpose of calculating aerodynamic forces in real-time, it is better to directly use Eqs. (19) and (20), whereas Eqs. (25) and (26) are useful for modeling aerodynamic forces acting on a quadcopter, which will be used in computing the torques of roll, pitch and yaw motions for flight controller design.
When applying to all flight regimes, parameters $C_{T_0}$, $C_{T_1}$, $C_{T_2}$, etc. in Eqs. (25) and (26) are time-varying and can be specified using the RLS method. Equation (25) can be written in the general form using the vector and matrix notation

$$y = x^T \theta$$

(27)

where,

$$y = C^\text{exp}_T , \quad x = \begin{bmatrix} 1 & \lambda & \mu & v & w & \mu^2 \end{bmatrix}^T$$

$$\theta = \begin{bmatrix} C_{T_0} & C_{T_1} & C_{T_2} & C_{T_3} & C_{T_4} & C_{T_5} & C_{T_6} \end{bmatrix}^T .$$

A similar way can be applied for $C^\text{exp}_D$. The estimated value of $\theta$ is denoted by $\hat{\theta}$. Using RLS, the time variance of aerodynamic derivative parameters can be examined so that the influence of each parameter to the aerodynamic characteristics of the quadcopter in transient mode can be investigated.

In general, $\hat{\theta}_k$ is obtained each time by adding a correction term to $\hat{\theta}_{k-1}$. The correction term depends on the current measurement and a predicted value, $\hat{y}_k = x^T \hat{\theta}_{k-1}$. The derivation of RLS can be found in the study by Young. \(^{10}\)

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k (y_k - x^T \hat{\theta}_{k-1})$$

(28)

$$K_k = P_k x_k = P_{k-1} x_k (I + x_k^T P_{k-1} x_k)^{-1}$$

(29)

$$P_k = P_{k-1} - P_{k-1} x_k (I + x_k^T P_{k-1} x_k)^{-1} x_k^T P_{k-1}$$

(30)

The RLS algorithm requires an initial value of parameters $\theta$. $\theta_0$ is chosen based on the least squares (LS) algorithm.

3. Experimental Method

3.1. Facilities

The quadcopter platform utilized in this experiment has a 0.45-m diameter and a mass of 1.3 kg. The characteristic parameters of the rotor, such as rotor radius, $R$, blade number, $N$, blade chord, $c$, and solidity, $\sigma$, were directly measured as tabulated in Table 1. The 2-D lift slope factor, $a$, is denoted by $C_{a_{\text{avg}}}$, and profile drag coefficient, $C_{d_{\text{p}}}$, were determined on the basis of a wind tunnel experiment. The rotor possesses a typical static thrust coefficient of $C_T = 0.013$, measured when hovering.

A five-hole pitot tube was used to measure the velocity vector of the inflow to the rotor. The probe was calibrated to determine its sensitivity to the angle of attack and the sideslip angle during a uniform flow in the wind tunnel.

The aim of the flight test program was to identify aerodynamic parameters such as thrust coefficient and drag coefficient. To achieve this purpose, the experimental quadcopter is equipped with a data acquisition system (DAQ) for gathering flight data, including acceleration, angular rates, attitudes, inflow angles, airspeed, altitude, and rotational speed. The architecture of the DAQ is depicted in Fig. 2.

The center piece of the DAQ is a Mbed microcontroller hosting an ARM Cortex-M3 processor with multiple I/O protocol and an onboard sensor system including:

- 6 DOF inertia measurement unit (IMU) 6050MPU to measure linear acceleration ($a_x$, $a_y$, $a_z$) and angular rates ($p$, $q$, $r$).
- Altimeter for measuring quadcopter altitude ($h$).
- Five-hole pitot tube for airspeed ($V$) and angles relative to the flow ($\alpha$, $\beta$) information.
- Four hall-effect sensors to record the rotational speeds (RPM) of the four rotors.
- Additional global positioning unit (GPS) necessary for tracking the flight path.

The data obtained was stored on a SD card at an update rate of 30 Hz.

3.2. Flight test procedure

The general procedure for flight tests is described as follows. Prior to each flight test, the onboard sensor system was calibrated and the whole system was checked first to avoid failure. The quadcopter then took-off from the ground and climbed to a height of 4–5 m. Before performing maneuver flight, hovering was carried out for 5–10 s. Data was collected during maneuver flight. The pilot commanded the quadcopter to land after the flight test scenario was completed. The flight test trajectory is depicted in Fig. 3.

The raw acceleration and angular rates obtained from the IMU were noisy, and therefore had to be filtered to get appropriate data. The time history of the data collected after filtering is shown in Fig. 4.

4. Results and Discussion

4.1. Phase of flight

The maneuver flight was comprised of three main phases: Forward (F), transition (T) and descent (D). The forward
phase was from the time of beginning to $t = 20\text{s}$. The maximum altitude reached at $t = 20\text{s}$ corresponds to $w = 0$. Transition from the forward phase to the descent phase was during the time $t = 20–22\text{s}$. The transient behavior was as decelerating climb at which time the thrust vector changed from a forward tendency to a rearward tendency, as described in Fig. 5. From $t = 22\text{s}$, the quadcopter performed a descent represented by a positive value of $w$.

A quadcopter may be required to perform maneuvers consisting of high-load factor turns and climbing to avoid obstacles. The experimental quadcopter performed a climbing maneuver during the transient phase as demonstrated in Fig. 6. The experimental blade loading, $C_{T}/\sigma$, on the quadcopter is shown in Fig. 7. The load factor is defined as the ratio of blade load during maneuverable (transient) flight and when hovering (i.e., hovering corresponds to 1-g flight). Flight test results indicate that the decelerating climbing transient maneuver happens at low translational kinetic energy and the transient load factors is $n = 1.5$.

### 4.2 Results of state estimation

The state and output estimation using the Kalman filter\textsuperscript{7} is presented in Fig. 8. Since the quadcopter performed nearly straight-forward flight, the velocity component along the $x_{b}$ axis, $u$, is dominant. The negative value for vertical velocity, $w$, manifests the climbing motion. The flight data are subject to noise and random measurement errors. Noise is suppressed by the Kalman filter. The smooth values of roll attitude, $\phi$, pitch attitude, $\theta$, yaw attitude, $\psi$, airspeed, $V$, altitude, $h$, angle of attack, $\alpha$, and sideslip angle, $\beta$, after removing the noise will be used to calculate the aerodynamic parameters.

The inflow ratio and advance ratio flight data recorded are shown in Fig. 9 and Fig. 10, respectively.

### 4.3 Results of parameter estimation

The comparison between experimental (red dotted line) and theoretical (black solid line) thrust coefficient values is shown in Fig. 11. These values can be validated by comparing the thrust coefficient when hovering, which is 0.013. The large discrepancy between theoretical result (Eq. (5)) and experimental result (Eq. (19)) happens in the transient phase ("T" region) and descent phase ("D" region). For the descent phase, this is because of the velocity induced, and therefore, the inflow ratio needs to be recomputed.

In the descent phase, a quadcopter operated in the vortex ring state corresponds to the condition $0 < w < 2v_{h}$, where $w$ descent rate (Fig. 8) and velocity $v_{h}$ is induced when hovering. It is known that the momentum theory is not applicable and velocity induced in the vortex ring state is calculated using an empirical model\textsuperscript{4}:

$$\frac{v_{i}}{v_{h}} = k_{0} + k_{1}\frac{w}{v_{h}}.$$  \hspace{1cm} (31)

Equation (31) and corresponding coefficients $k_{0} = -0.709$ and $k_{1} = 1.871$ were obtained by fitting thrust coefficient theoretical results to experimental results. The coefficients achieved can only be used for the MRH used in our experiment. However, Eq. (31) can be expanded to a higher-
order polynomial (i.e., in the current study, it is the first-order of \(w/v_h\)), thus it can be implemented for MRHs of different configurations and sizes under different flight conditions.

The thrust coefficient in the descent phase is calculated using Eq. (5) with the following modification to inflow ratio:

\[
\lambda = \frac{w - v_i}{\omega R} .
\]  

(32)
Using Eqs. (5), (31) and (32), calculating the thrust coefficient during descent (D) has been improved, as seen in Fig. 11 (green solid line).

The experimental and theoretical drag coefficients are shown in Fig. 12. $C_D$ is positive in forward flight, faster in movement the large $C_D$ is. $C_D$ is negative in transition from forward flight to descent because of the change in direction of the thrust vector.

Equations (5) and (21) are insufficient to model the thrust coefficient in the transient phase ("T" region). The reason is Eqs. (5) and (21) do not account for the influence of vertical flow, $w$, in the transient phase. Since the quadcopter also performed sideways flight, both $v$ and $w$ are taken into account. Therefore, to apply for all flight regimes including transient phase, Eq. (25) should be used. There exists the difference in comparing LS and analytical equations when calculating the parameters of Eq. (25), as tabulated in Table 2. Figure 13 shows the experimental results for the thrust coefficient and its reconstruction from Eq. (25) based on the LS method. The correlation coefficient, or 'goodness of fit,' of the LS model is 0.7. Equation (25) and its parameters are determined by minimizing the error, and therefore, these parameters do not reflect the aerodynamic sense. Nevertheless, the approximation equations (Eqs. (25) and (26)) are useful for flight controller design.

The time variance in parameters in Eq. (25) are also estimated using the RLS method, and therefore, investigating what the influential variables might be in the transient mode is required. The static and derivative parameters of the thrust coefficient using the RLS method are shown in Fig. 14. $C_{T_D}$ decreases. From $t = 18–20$ s, $C_T$ increases due to the decrease in $\lambda$. Advance ratio, $\mu$, contributes a positive increment of $C_T$, represented by a positive value of $C_{T_{\mu}}$. The vertical flow, $w$, has an influence on $C_T$, especially in the transient phase represented by an increase in the value of $C_{T_{w}}$ after $t = 20$ s.

Validation of the RLS method for calculating parameters $C_{T_D}$ and $C_{T_{\mu}}$ in the transient phase, based on load factor, $n$, is as follows:

$$n \left( C_{T,D} \right)_{\text{forward}} = \left( C_{T,D} \right)_{\text{trans}}$$

$$n \left( C_{T,\mu} \right)_{\text{forward}} = \left( C_{T,\mu} \right)_{\text{trans}}.$$  

As a result,

$$C_{T,D} \left| \text{trans} = n \left( C_{T,D} \right)_{\text{forward}} \frac{\lambda_{\text{trans}}}{\lambda} \right.$$  

$$C_{T,\mu} \left| \text{trans} = n \left( C_{T,\mu} \right)_{\text{forward}} \frac{\mu_{\text{trans}}}{\mu} \right.$$  

By substituting the average values of $n = 1.5$, $C_{T_D} = -0.01$, $\lambda = 0.09$, $C_{T_{\mu}} = 0.02$, $\mu = 0.125$ for forward; $\lambda = 0.05$, and $\mu = 0.075$ for transition gives $C_{T,D} \left| \text{trans} = -0.03$, compared to $-0.03$ using RLS; $C_{T_{\mu}} \left| \text{trans} = 0.05$, in comparison with 0.07 using RLS (see Fig. 14).

The influence of vertical flow on thrust coefficient during the deceleration climbing transition is formulated using Eq. (39). For a steady vertical rate, $\delta C_T$ and mass flux, $\dot{m}_u$ are computed as:

$$\delta C_T = \frac{\dot{m}_uw}{\rho 4\pi R^2 (\omega R)^2}$$

$$\dot{m}_u = \rho 4\pi R^2 (w + v_i).$$

As a result,

$$C_{T,D} \left| \text{trans} = \frac{\partial C_T}{\partial w} \right. = \frac{w + v_i}{(\omega R)^2}.$$
Similarly, by substituting for the average values of \( w = 1 \text{ m/s} \), \( v_i = 2 \text{ m/s} \), \( \omega = 440 \text{ rad/s} \), and \( R = 0.12 \text{ m} \) gives \( C_{T_l, \text{trans}} = 0.001 \), in a comparison with a difference of 0.001 using RLS.

Several flight tests were performed at different load factor values, \( n \). Table 3 presents the results of estimating the parameters in transient flight by applying two methods: RLS and analytical Eqs. (35), (36), and (39).

Although the aerodynamic parameters listed in Table 2 can only be true for the multicopter used in our experiment, the method proposed in this study can be applied for MRHs of different configurations and sizes under different flight conditions.

In addition, the flight test vehicle used in this study is a small-scale quadrotor. The Reynolds number of flow passing through the rotor is \( 10^3 \) to \( 10^4 \), while it is around \( 10^6 \) for conventional helicopters. It is well known that flows of such low-Reynolds numbers are different from flows of higher Reynolds numbers since the effects of viscous forces are dominant in low Reynolds number flow regimes, which may cause the laminar flow to separate. Under certain circumstances, a separated flow causes negative effects on aerodynamic performance. These negative effects may increase drag and decrease lift. In our previous study, \( 3^9 \) flow separation caused a discontinuity in the mass flow through the rotor disk, which happens during high climb rate conditions and high angle of attack, in which the assumption of inviscid flow is invalid.

5. Conclusions

On the basis of the flight tests of a quadrotor helicopter, the following conclusions are made:

1. In steady forward flight, the experimental results agree well with the conventional theory throughout the investigated inflow ratio range: from 0 to 0.12, advance ratio less than 0.2 and angle of attack from \( 0^\circ \) to \( -30^\circ \).

2. The external forces measured based on acceleration data were interpreted as a linear combination of aerodynamic parameters manifests the correlation of the data collected.

3. The effects of inflow ratio, advance ratio and vertical flow on thrust coefficient are represented by influential parameters \( C_{T_r}, C_{T_w}, \) and \( C_{T_l} \). In transient flight, those parameters are larger in magnitude when compared to those of steady forward flight. Vertical flow should be taken into account explicitly in transient flight.

4. Conventional theory was insufficient to model the thrust coefficient during a decelerating climbing transient maneuver. The influential parameters in transition were formulated using the method proposed in this study and validated using the RLS method. The method proposed provides results that are similar to those of the RLS method. The time variance in aerodynamic parameters, in the sense of flight mode changing, is useful for the adaptive control system design of multirotor helicopters.

| Data set     | Parameter | Proposed Eqs. (35), (36), (39) | RLS (Fig. 14) |
|--------------|-----------|--------------------------------|---------------|
| Set 1 (\( n = 1.5 \)) | \( C_{T_r, \text{trans}} \) | ~0.03                          | 0.07          |
|              | \( C_{T_w, \text{trans}} \) | 0.05                           | 0.07          |
|              | \( C_{T_l, \text{trans}} \) | 0.001                          | 0.001         |
| Set 2 (\( n = 1.3 \)) | \( C_{T_r, \text{trans}} \) | ~0.013                         | ~0.015        |
|              | \( C_{T_w, \text{trans}} \) | 0.041                          | 0.045         |
|              | \( C_{T_l, \text{trans}} \) | 0.0015                         | 0.001         |
| Set 3 (\( n = 1.2 \)) | \( C_{T_r, \text{trans}} \) | ~0.013                         | ~0.015        |
|              | \( C_{T_w, \text{trans}} \) | 0.128                          | 0.110         |
|              | \( C_{T_l, \text{trans}} \) | 0.0015                         | 0.0013        |

Fig. 14. Estimation of aerodynamic derivative of thrust coefficient using RLS.

Table 3. Validation of parameter estimation during transient flight.
Acknowledgments

A sincere thanks to Mr. Yang Sida for his support in conducting the flight tests.

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