Spectral functions in a two-velocities Tomonaga-Luttinger Liquid

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Abstract. We obtain the spectral functions of a two-velocities Tomonaga-Luttinger liquid at $T = 0K$ in terms of the Appell hypergeometric functions $F_1$ and $F_2$. In the case of SU(N) spin symmetry, the spectral functions can be expressed with the Gauss hypergeometric functions. We discuss the singularities and thresholds of the spectral functions for both the SU(N) invariant case and the general case.

1. Introduction
The Tomonaga-Luttinger liquid (TLL) [1] is a key paradigm of the theory of strongly correlated fermions in one dimension. In the case of fermions with spin, the property of spin-charge separation is expected, where fermion excitations separate into spin ($\sigma$) excitations and charge ($\rho$) excitations propagating with respective velocities $u_\sigma$ and $u_\rho$. The technique of angle-resolved photoemission spectroscopy has been used to measure the spectral functions of electrons in quasi-one dimensional conductors such as BaVS$_3$ [2], TTF-TCNQ [3, 4], (TMTSF)$_2$X [5], and LiMO$_6$O$_{17}$ [6]. Self-assembled one-dimensional gold metallic chains of atoms on semiconductor surfaces have also been considered as potential candidates for the observation of TLL and spin-charge separation [7, 8, 9, 10, 11]. Other candidate systems to detect spin-charge separation are ultracold fermionic gases [12] in optical lattices [13] or magnetic traps [14]. In these systems, however, the internal degrees of freedom may possess a higher symmetry than SU(2) [15] or may result from a mixture of spinless fermionic atoms with different masses [16] leading to a reduced $U(1) \times U(1)$ symmetry as if a magnetic field was present. The accurate calculation of fermion spectral functions beyond the case of SU(2) symmetry is thus necessary for testing quantitatively the TLL theory in such systems. Previously, the asymptotic behavior of spectral functions has been predicted [17, 18] for energies $\hbar \omega$ close to $\pm \hbar u_{\rho,\sigma}$. The zero temperature spectral functions have been calculated in Ref. [1] for the SU(2) invariant case, and finite temperature spectral functions have been studied numerically [19].

2. Results
In a TLL with two components (charge $\rho$ and spin $\sigma$) with fully broken $U(1) \times U(1)$ symmetry, calling $\psi_s(x) = \sum_{r=\pm} e^{i r k_F x} \psi_{r,s}(x)$ the fermion annihilation operator with (pseudo)-spin $s$ the
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SU(N) case is given by spin-independent, we redefine zero-temperature fermion Green’s function is obtained from the correlators:

\[ A(k_F, q, \omega) = \prod_{\beta=\rho, \sigma} \left[ \frac{\alpha}{\alpha + i(u\beta t - rx)} \right], \]

where \( u_\beta \) is the velocity of mode \( \beta \) and the exponents \( \nu_{s, \beta}, \nu'_{s, \beta} \) can be obtained from bosonization techniques [20]. The spectral function \( A_s(a, \omega) \) is obtained by taking the Fourier transform of the retarded Green’s function (1). The Fourier transform can be obtained in closed form using Feynman identities [21] following Ref. [22]. Following that route, in Ref. [20], we derived expressions of the spectral functions \( A_s(k_F + q, \omega) \) in terms of Appell \( F_1 \) and \( F_2 \) hypergeometric functions [23]. We also found that \( A_s(k_F + q, \omega) = 0 \) for \( |\omega| < u_\sigma q \). The typical behavior of the spectral function is shown in Fig. 1. We find, in \( A_1(k_F, q, \omega) \), the large weight at \( \omega \simeq u_\rho q \) while small weight at \( \omega \simeq u_\sigma q \). In contrast, large weight can be seen at \( \omega \simeq u_\sigma q \) and small weight at \( \omega \simeq u_\rho q \) in \( A_2(k_F, q, \omega) \).

Next we consider the model with SU(N) symmetry [24, 25]. The Hamiltonian reads:

\[ H = H_\rho + H_\sigma, \]

\[ H_\rho = \int dx \left[ u_\rho K_\rho (\pi \Pi_\rho)^2 + \frac{u_\rho}{K_\rho} (\partial_x \phi_\rho)^2 \right], \]

\[ H_\sigma = \frac{2\pi u_\sigma}{N + 1} \sum_{A=1}^{N^2-1} \int dx \left[ T^A_R(x) T^A_R(x) + T^A_L(x) T^A_L(x) \right], \]

where \( T^A_R, L \) are the SU(N) currents [1] and \( K_\rho \) is the charge Luttinger exponent. The expression of Eq. (1) is still valid and the exponents are given by \( \nu_{s, \rho} = (2\gamma_\rho + 1)/N, \nu_{s, \sigma} = (N - 1)/N, \nu'_{s, \rho} = (2\gamma_\rho)/N, \) and \( \nu'_{s, \sigma} = 0 \) [25], where \( \gamma_\rho = (K_\rho + 1/K_\rho - 2)/8. \) Since these exponents are spin-independent, we redefine \( \nu_\rho \equiv \nu_{s, \rho}, \nu_\sigma \equiv \nu_{s, \sigma}, \) and \( \nu'_{\rho} \equiv \nu'_{s, \rho}. \) The spectral function for the SU(N) case is given by

\[ A_s(k_F, q, \omega)|_{u_\sigma q < \omega < u_\rho q} = \frac{(\alpha/\Delta \mu)^{\nu_{\rho}+\nu'_{\rho}-1}}{\Gamma(\nu_{\rho}+\nu'_{\rho}) \Gamma(\nu_\sigma)} \left[ (\omega - u_\sigma q)^{\nu_{\rho}+\nu'_{\rho}-1}(-\omega + u_\rho q)^{\nu_{\sigma}} \right]^{\nu_{\rho}} \]
As \((k_F+q, w)\) (arb. unit) for the spectral function goes to zero as the spectral function as a function of \(w\) for \(\gamma_\rho > 1/2\) power-law divergences are obtained for both \(\omega = u_\rho q\) and \(\omega = u_\sigma q\). For \(\gamma_\rho < 1/2\) power-law divergences are replaced by cusps. The dots represent the finite values at \(\omega = u_\sigma q\).

\[
\times 2F_1 \left( \nu - 1, \nu'_\rho; \nu_\rho + \nu'_\rho; \frac{2u_\rho (\omega - u_\sigma q)}{\Delta u (\omega + u_\rho q)} \right),
\]

for \(u_\sigma q < \omega < u_\rho q\), and

\[
A_s(k_F + q, \omega)|_{|\omega| > u_\rho q} = \frac{(\alpha/2u_\rho)^{\nu_\rho - 1}}{\Gamma(\nu_\rho + \nu_\sigma)\Gamma(\nu'_\rho)} \frac{|\omega - u_\rho q|^{\nu_\rho + \nu'_\rho - 1} |\omega + u_\rho q|^{\nu_\rho + \nu_\sigma - 1}}{|\omega - u_\sigma q|^{\nu_\sigma}} 
\times 2F_1 \left( \nu - 1, \nu_\sigma; \nu_\rho + \nu'_\rho; \frac{\Delta u |\omega + u_\rho q|}{2u_\rho |\omega - u_\sigma q|} \right),
\]

for \(|\omega| > u_\rho q\). Here \(2F_1(\alpha, \beta; \gamma; x)\) is the Gauss hypergeometric function, and \(\nu \equiv (\nu_\rho + \nu_\sigma + \nu'_\rho)\).

For \(-u_\rho q < \omega < +u_\sigma q\) the spectral function vanishes. It is worthwhile to note that Eqs. (3) and (4) can also be derived from the results of Ref. [20] using known limits of Appell hypergeometric functions in terms of Gauss hypergeometric functions. We also note that by taking \(N = 2\), these expressions reproduce the formula for the SU(2) case given in Eq. (19.27) in Ref. [1]. From Eqs. (3)–(4), a power-law divergence \(A_s(k_F + q, \omega) \sim |\omega - u_\rho q|^{\nu_\rho + \nu'_\rho - 1} = |\omega - u_\rho q|^{(2\gamma_\rho - 1)/N}\) of the spectral function for \(\omega = u_\rho q\) is expected when \(\gamma_\rho < 1/2\). For \(\gamma_\rho > 1/2\) a cusp is obtained instead of a divergence. When \(\omega \to u_\rho q\), Eq. (3) predicts a power-law divergence \(A_s(k_F + q, \omega) \sim (\omega - u_\rho q)^{\nu_\rho + \nu'_\rho - 1} = (\omega - u_\rho q)^{(4\gamma_\rho - 1)/N}\). For \(\gamma_\rho > (N - 1)/4\) instead, the spectral function goes to zero as \(\omega \to u_\sigma q\). In contrast with the \(N = 2\) case, for \(N \geq 4\), the power-law divergence at \(\omega = u_\sigma q\) is the last to disappear. For \(N = 3\), both divergences disappear for \(\gamma_\rho = 1/2\). Finally, for \(\omega \to -u_\rho q\), the spectral function vanishes as \(A_s(k_F + q, \omega) \sim (u_\rho q + \omega)^{\nu_\rho + \nu'_\rho - 1} = (u_\rho q + \omega)^{2\gamma_\rho/N}\). We have represented the evolution of the spectral function as a function of \(\gamma_\rho\) for \(N = 3\) on Fig. 2.

3. Summary and discussions

In the present paper, we have derived expressions of the spectral functions of a two-velocities TLL in terms of Appell hypergeometric functions. For the model with SU(N) spin symmetry,
the spectral functions can be expressed with the Gauss hypergeometric functions. We have analyzed the power-law singularities of the spectral functions for both cases.

Here we briefly discuss the experimental findings by the angle-resolved photoemission spectroscopy (ARPES) measurements for gold atom chains on silicon surfaces [7, 8, 9, 10, 11]. Initially, the ARPES measurement revealed a double-band structure and the spin-charge separation was suggested for its origin [7, 8, 9]. However, recent spin-resolved ARPES experiments on this system have shown that the double-band nature originates in the spin splitting caused by the Rashba effect [11]. The present analysis shown in Fig. 1 is relevant to the spin-resolved ARPES measurements. The results of Fig. 1 suggest that the spectral functions $A_1$ and $A_2$ exhibit strongest peaks at different frequencies. This behavior is consistent with the spin-splitting situation observed in Ref. [11]. From the present analysis, we infer that, in addition to the largest peak at $\omega = u_\sigma q + k_F \cdot (q + \omega)$ in $A_1$, another weaker cusp or shoulder structure can also be seen at $\omega = u_\sigma q (\omega = u_\sigma q)$ due to the correlation effects in one dimension.

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