Cuvée: Blending SMT-LIB with Programs and Weakest Preconditions

Gidon Ernst

LMU Munich, Germany, gidon.ernst@lmu.de

Abstract. Cuvée is a program verification tool that reads SMT-LIB-like input files where terms may additionally contain weakest precondition operators over abstract programs. Cuvée translates such inputs into first-order SMT-LIB by symbolically executing these programs. The input format used by Cuvée is intended to achieve a similar unification of tools for that for example synthesize loop summaries. A notable technical aspect of Cuvée itself is the consequent use of loop pre-/postconditions instead of invariants, and we demonstrate how this lowers the annotation burden on some simple while programs.

Keywords: Program Verification, SMT-LIB, Weakest Precondition

1 Introduction

Intermediate verification languages and tools such as Boogie [7], Why3 [5], and Viper [10] have had a significant impact on the state-of-the-art of (deductive) program verification. At the annual competition on interactive program verification VerifyThis [6], tools like these are put to practice on small but intricate verification problems. Currently, VerifyThis lacks a common input format. This is fine for hard challenges where efficiency of the tool and seamless interaction are required. SMT-LIB [2] is a standardized interchange format for verification tasks in first-order logic that is widely used in many different application domains such as constraint-solving, program verification, and model-checking. Many mature tools are available, and the annual SMT-COMP evaluates and compares their performance on benchmark problems. Part of the success of the SMT-LIB format is its regular syntax and its precise standardization. SV-COMP [4] is a competition where the participating tools analyze C code fully automatically. It is therefore restricted to less complex properties that cannot be solved without the guidance of a proof engineer.

The goal of this work is to occupy the spot between these communities, as well as the different technologies involved [3]. How to make use, e.g., of tools that can infer invariants? The input format that is described here and implemented in Cuvée takes deliberate trade-offs to position itself between the highly expressive logic of e.g. Dafny, and stressing the interoperability that made SMT-LIB successful. As such one design goal, for example, is to re-use the existing logical data types of that format, even if it might be less convenient as a front-end for humans.
\(\langle \text{var\_binding} \rangle ::= (\langle \text{symbol} \rangle \langle \text{term} \rangle)\)

\(\langle \text{term} \rangle ::= (\text{wp} \langle \text{program} \rangle \langle \text{term} \rangle)\)
| \(\text{box} \langle \text{program} \rangle \langle \text{term} \rangle\)
| \(\text{dia} \langle \text{program} \rangle \langle \text{term} \rangle\)
| \(\text{old} \langle \text{term} \rangle\)
| ...

\(\langle \text{program} \rangle ::= (\text{assign} (\langle \text{var\_binding} \rangle^+)\)
| \(\text{spec} (\langle \text{symbol} \rangle^+) \langle \text{term} \rangle \langle \text{term} \rangle\)
| \(\text{block} \langle \text{program} \rangle^+)\)
| \(\text{if} \langle \text{term} \rangle \langle \text{term} \rangle \langle \text{term} \rangle\)
| \(\text{while} \langle \text{term} \rangle \langle \text{term} \rangle \langle \text{attribute} \rangle^+\)

\(\langle \text{command} \rangle ::= (\text{assert-counterexample} \langle \text{term} \rangle \langle \text{program} \rangle \langle \text{term} \rangle)\)
| ...

**Fig. 1.** Extension of the SMT-LIB supported by Cuvée: Weakest-precondition operators, simple nondeterministic While programs, and a top-level command to specify Hoare triples (\(\langle \text{var\_binding} \rangle\) is from the SMT-LIB grammar).

### 2 Syntax

The new constructs accepted by Cuvée are shown in Fig. 1.

Terms \(\langle \text{term} \rangle\) of SMT-LIB are extended by three weakest-precondition operators, \(\text{wp} \langle \text{program} \rangle \langle \text{term} \rangle\), \(\text{box} \langle \text{program} \rangle \langle \text{term} \rangle\), \(\text{dia} \langle \text{program} \rangle \langle \text{term} \rangle\) for programs \(p\) (see below) and postconditions \(t\) (terms of sort \(\text{Bool}\), which state slightly different correctness criteria with respect to termination and nondeterministic choices.

The first one, \(\text{wp}\) denotes Dijkstra’s well-known weakest precondition: All executions of \(p\), when started in the current state, terminate and lead to a state that satisfies \(t\). For instance, a Hoare triple for total correctness \([\phi]p[\psi]\) can be written as the implication \((\Rightarrow \phi (\text{wp} p \psi))\)

Similarly, \(\text{box}\) does not require termination (i.e., it expresses the weakest liberal precondition), i.e., \([\phi]p[\psi]\) can be written as \((\Rightarrow \phi (\text{box} p \psi))\).

The operator \(\text{dia}\) reflects angelic execution instead of demonic execution: For \(\langle \text{dia} p t \rangle\) it is required that there is at least one execution of \(p\) that terminates and leads to a state that satisfies \(t\). The names of the latter two operators is taken from Dynamic Logic.

The expression \(\langle \text{old} t \rangle\) is intended for use in loop annotations. It refers to previous states of the execution at the beginning of loop iterations.

Programs \(\langle \text{program} \rangle\) provide familiar constructs from a simple sequential While language.

Parallel assignments evaluate all right-hand-sides simultaneously, e.g. \(\text{assign} (x \ y) (y \ x)\) swaps the values stored in the variables \(x\) and \(y\). Note that there is no syntactic difference between program variables and logical ones, and the former ones may range over arbitrary SMT-LIB data types.
Specification statements \((\text{spec } (x_1 \cdots x_n) \ \phi \psi)\) (see [9]) encode arbitrary, possibly nondeterministic transitions. The effect of executing such a statement is that the precondition \(\phi\) of the statement is asserted (i.e., needs to hold in the current state), then the variables \(x_1 \cdots x_n\) are given fresh arbitrary values, and the postcondition \(\psi\) is assumed for the remainder of the execution.

Specification statements can encode assertions (\(\text{assert } \phi\) becomes \((\text{spec } () \ \phi \ \text{true})\)), assumptions (\(\text{assume } \psi\) becomes \((\text{spec } () \ \text{true } \ \psi)\)), and the statement \text{havoc } x_1 \cdots x_n from e.g. Boogie (which becomes \((\text{spec } (x_1 \cdots x_n) \ \text{true } \ \text{true})\)).

Within the postcondition of a specification statement, \textsf{old} refers to the pre-state of the statement itself. This admits elegant encoding of transition relations, i.e., \((\text{spec } (x) \ \text{true } (> x \ \text{old } x))\) specifies that the new value of \(x\) is strictly larger than the previous one.

Specification statements are useful internally, too, to encode the inductive hypothesis of the loop rule implemented in \textsc{Cuvée} (see Section 2). Sequential composition is written as \((\text{block } p_1 \cdots p_n)\), where \((\text{block})\) denotes the empty statement. Conditional statements \((\text{if } b p_1 p_2)\) execute either \(p_1\) or \(p_2\) depending of the evaluation of the test \(b\) (a boolean term) in the current state.

While loops
\[(\text{while } b p : \text{termination } t : \text{precondition } \phi : \text{postcondition } \psi)\]
execute \(p\) as long as the test \(b\) holds true. \textsc{Cuvée} supports some attributes that can be used to specify loop annotations, namely, a termination measure \(t\), a loop precondition \(\phi\), and a loop postcondition \(\psi\). All three annotations are optional.

A new top-level command \((\text{assert-counterexample } \phi p \psi)\) asserts that the Hoare triple \([\phi] p [\psi]\) is not valid. This command roughly translates to \((\text{assert } (\text{not } (> (\text{wp } p \psi))))\), however, \textsc{Cuvée} implements special cases when \(p\) is a while loop without a pre-/postcondition annotation to derive the loop specification from such a contract. Moreover, within a \text{assert-counterexample} command, \textsf{old} in the postcondition \(\psi\) refers to the pre-state. Note that this feature is currently not expressible in the expression/program language alone, but can be emulated if needed by introducing additional logical variables capturing this pre-state explicitly.

Consistent with the standard pattern in SMT-LIB, where formulas are proved by searching for satisfying assignments of the negation, an \textsf{unsat} from the underlying SMT solver on the problem translated by \textsc{Cuvée} indicates that there is no counterexample i.e., the program is correct with respect to the specified contract.

## 3 Proof Rules

The predicate transformer semantics \([8]\) of the input language is shown in Fig. 2 for the \text{wp} operator, with the exception of the loop rule, which is shortened to
omit the termination conditions and therefore expressed using box. The rules for box and dia are similar. Note that $Q$ is a term that may again contain weakest-precondition operators.

The first three equivalences are standard. Assignments propagate as a simultaneous substitution into the postcondition $Q$. Sequential execution simply nests the weakest precondition operator. Conditionals produce two branches that evaluate the test positively resp. negatively.

The specification statement binds fresh copies $y$ for the havoc’ed variables $x$, and substitutes old expressions in the postcondition $ψ$. The first premise simply asserts the precondition $φ$.

The loop rule is more involved and proceeds by induction on the number of loop iterations, see Alexandru’s bachelor thesis [1]. It has three premises: The first, abstracts the entire execution of the loop loop with a specification statement, after which the postcondition $Q$ needs to hold.

The second premise corresponds to the base case of the induction and establishes $Q$ upon termination (when $b$ is false). Old values in $Q$ refer to the current state.

The third premise corresponds to the inductive case, where $b$ holds. Then, the condition to show is that after executing the loop body $p$ once, the inductive hypothesis about the remaining iterations is sufficient to establish $ψ$ after the entire loop. This hypothesis is encoded as a specification statement $(\text{spec } (\underline{x}) \ φ \ ψ)$ that abstracts the remaining iterations, similarly as in the first premise.
The interaction between the different occurrences of old is somewhat intricate: Within the specification statement, old refers to the state after the first iteration, according to the corresponding proof rule. Hence, the it encodes that it is possible at that point to turn the precondition \( \phi \) into an arbitrary state after the remaining iterations that satisfies \( \psi \) for fresh copies of the modified variables \( x \) introduced by the specification statement. Ultimately, this knowledge needs to suffice to establish \( \psi \) after the loop with all iterations, where old refers to the state before executing the first iteration \( p \).

4 Tool Description

CUVÈÈ is implemented in the Scala programming language\(^1\) and relies on SMT solvers as back-ends to solve the first-order verification conditions.

CUVÈÈ is open source under the MIT License at https://github.com/gernst/cuvee. It reads one or more input files, reduces the weakest-precondition operators according to the rules in Fig. 2 using symbolic execution (i.e., the substitutions are delayed and propagated down the term structure).

The tool can be invoked from the command line in different ways as exemplified below:

```
./cuvee # read from stdin, write to stdin
./cuvee <file> -o <out> # read from file, write to out
./cuvee <file1> ... <filen> -- ./z3 -in # invoke SMT solver directly
```

It can either save the generated SMT-LIB script to a file, or invoke an SMT solver, whose command line is appended after --, Z3 in this case. In this mode of operation, it pipes the generated verification task directly to the solver. There are builtin abbreviations -z3 and -cvc4 that pass the necessary arguments, assuming those solvers are present in $PATH$.

5 Example

We demonstrate CUVÈÈ on a simple example, shown in Fig. 3, which is taken from the VerifyThis competition 2012. It is an algorithm that searches the maximum element in an array by elimination. It maintains two indices, \( x \leq y \) and moves the one pointing to the smaller element in each iteration. The postcondition asserts that for any valid index \( z \), the returned index \( x \) contains an element that is in fact at least as large.

CUVÈÈ infers the precondition and postcondition of the loop from the specified contract, such that the example is solved without further interaction.

A few more examples are contained in the examples subfolder on github, including GCD, and mapping an array range by an unspecified function.

\(^1\) https://scala-lang.org
(declare-const x Int)
(declare-const y Int)
(declare-const a (Array Int Int))

(assert-counterexample
 (<= x y)
 (while (not (= x y))
   (if (<= (select a x) (select a y))
       (assign (x (+ x 1)))
       (assign (y (- y 1))))
   :termination (- y x))
(forall ((z Int))
  (=> (and (<= (old x) z)
            (<= z (old y)))
       (<= (select a z) (select a x)))))

(check-sat)

Fig. 3. Finding the maximum in an array by elimination.

6 Conclusion and Outlook

Cuvée is a tool that aims to bridge the gap between fully automated program verification and approaches that have the human in the loop. It does so by extending SMT-LIB to cover weakest precondition statements about abstract programs, which tightly integrate into the existing standard.

There are plenty of opportunities for future development. For one, it is hoped that the format will be taken up by others, supported by standardization efforts. Extension to recursive procedures is planned as well.

Ultimately, following this approach opens up the possibility to evaluate and compare verification tools that can handle complex functional correctness conditions involving quantifiers, arrays, and other data types.
Bibliography

[1] Alexandru, G.: Specifying loops with contracts (2019), Bachelor’s Thesis, LMU Munich
[2] Barrett, C., Stump, A., Tinelli, C., et al.: The smt-lib standard: Version 2.0. In: Proceedings of the 8th International Workshop on Satisfiability Modulo Theories (Edinburgh, England). vol. 13, p. 14 (2010)
[3] Bartocci, E., Beyer, D., Black, P.E., Fedynukovich, G., Garavel, H., Hartmanns, A., Huisman, M., Kordon, F., Nagele, J., Sighireanu, M., et al.: Toolympics 2019: an overview of competitions in formal methods. In: International Conference on Tools and Algorithms for the Construction and Analysis of Systems. pp. 3–24. Springer (2019)
[4] Beyer, D.: Advances in automatic software verification: Sv-comp 2020. In: International Conference on Tools and Algorithms for the Construction and Analysis of Systems. pp. 347–367. Springer (2020)
[5] Bobot, F., Filliâtre, J.C., Marché, C., Paskevich, A.: Why3: Shepherd your herd of provers. In: Boogie 2011: First International Workshop on Intermediate Verification Languages. pp. 53–64 (2011)
[6] Ernst, G., Huisman, M., Mostowski, W., Ulbrich, M.: VerifyThis—verification competition with a human factor. In: International Conference on Tools and Algorithms for the Construction and Analysis of Systems. pp. 176–195. Springer (2019)
[7] Leino, K.R.M.: This is Boogie 2 (2008), microsoft RiSE
[8] Manes, E.G.: Predicate transformer semantics, vol. 33. Cambridge University Press (2004)
[9] Morgan, C.: The specification statement. ACM Transactions on Programming Languages and Systems (TOPLAS) 10(3), 403–419 (1988)
[10] Müller, P., Schwerhoff, M., Summers, A.J.: Viper: A verification infrastructure for permission-based reasoning. In: International Conference on Verification, Model Checking, and Abstract Interpretation. pp. 41–62. Springer (2016)