On the birthrates of Galactic neutron stars

E. F. Keane* and M. Kramer

University of Manchester, Jodrell Bank Centre for Astrophysics, Alan-Turing Building, Oxford Road, Manchester M13 9PL

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ABSTRACT

In light of the recently discovered neutron star populations, we discuss the various estimates for the birthrates of these populations. We revisit the question as to whether the Galactic supernova rate can account for all of the known groups of isolated neutron stars. After reviewing the rates and population estimates, we find that if the estimates are in fact accurate, the current birthrate and population estimates are not consistent with the Galactic supernova rate. We discuss possible solutions to this problem including whether or not some of the birthrates are hugely overestimated. We also consider a possible evolutionary scenario between some of the known neutron star classes which could solve this potential birthrate problem.

Key words: stars: neutron – pulsars: general – supernovae: general – Galaxy: stellar content.

1 INTRODUCTION

In the standard scenario, neutron stars (NSs) are formed during the core collapse of massive stars which links their number in the Galaxy to the Galactic supernova rate. The number of Galactic NSs can be inferred from observations, taking the various manifestations of NSs into account. In recent years, new and different observational manifestations of NSs have been discovered, so that it is warranted to study the impact, if any, of these discoveries on to the birthrates that is required to sustain this increased NS population.

The new manifestations of NSs include Rotating Radio Transients (RRATs; McLaughlin et al. 2006) and X-Ray Dim Isolated Neutron Stars (XDINS; see Haberl 2007 and references therein). These objects join the ~1800 known radio pulsars and the small group of magnetars (Woods & Thompson 2004). Do these previously unknown types of observable NSs increase the overall population by an amount that it is difficult to reconcile the formation rates with those predicted by theory? The basic requirement we make to answer this question is that the individual birthrates of the different NS populations should not exceed the Galactic core-collapse supernova (CCSN) rate, that is

\[ \beta_{\text{CCSN}} \geq \beta_{\text{total}} = \beta_{\text{PSR}} + \beta_{\text{XDINS}} + \beta_{\text{RRAT}} + \beta_{\text{magnetar}}, \]

where \( \beta_X \) is the birthrate (per century) of a NS of type X.

Recently, this question has also been addressed by Popov, Turolla & Possenti (2006), where it was concluded that this requirement can be met if we assume that XDINSs are in fact nearby RRATs. However, as we detail below, the pulsar birthrate considered is a lower limit which has since been superceded. In addition, the recent non-detection of any radio RRAT-like bursts from the XDINSs (Kondratiev et al. 2008) means that the identification of these two populations is not certain. Furthermore, recent work suggests that the heretofore neglected magnetar contribution may not be negligible so that the question as to whether the CCSN rate requirement is satisfied is reinstated.

The aim of this paper is to study the posed question by investigating the most recent knowledge about each contributing NS population and its Galactic birthrate. After introducing each manifestation of NS in turn, we will revisit the estimates for all terms in equation (1). The results are then discussed in detail before conclusions are drawn.

2 DIFFERENT MANIFESTATIONS OF NEUTRON STARS

2.1 Radio pulsars

Radio pulsars are rapidly rotating, highly magnetized NSs. Coherent radio emission is produced by a pair plasma above the magnetic polar caps of the NS, believed to originate from particle cascades after an acceleration of electrons and positrons in the strong electric and magnetic fields (e.g. Lorimer & Kramer 2005). The spectra for this emission typically increases with decreasing radio frequency with mean spectral index of ~1.8 (Maron et al. 2000) before peaking in the range 100–300 MHz (Malofeev et al. 1994). Pulsar periods range from 1.4 ms to 8.5 s with two distinct distributions – the ‘normal’ radio pulsars which have periods of ~500 ms and the so-called ‘millisecond pulsars’ with typical periods of ~5 ms. Fig. 1 shows a \( P-P' \) diagram, a standard pulsar classification tool, where these two populations are easily identified. The standard model of pulsar physics assumes that pulsars have dipolar magnetic fields and that the loss of rotational energy powers the pulsar. With these we can determine the ‘characteristic surface magnetic field’ of the pulsar to be

\[ B_S = 3.2 \times 10^{19} G \sqrt{P P'}, \]
For a pure dipole, the ‘braking index’ \( n = 3 \), resulting in the ‘characteristic age’ \( \tau_c = P/2P \). We can also define the pulsar ‘spin-down luminosity’ \( \dot{E} = -d\dot{P}/dt[(1/2)I\Omega^2] \). Assuming again the canonical NS values, we find

\[
\dot{E} = 3.95 \times 10^{31} \text{ erg s}^{-1} \left( \frac{P}{10^{-15}} \right) \left( \frac{P}{s} \right)^{-3}.
\]

Lines of constant \( \dot{E} \), \( B_3 \) and \( \tau_c \) are shown in Fig. 1 along with different evolutionary paths for different braking indices. The lower right area of the diagram devoid of any pulsars is known as the pulsar ‘death valley’ (Chen & Ruderman 1993). It is here that it is believed that the electric potential at the polar caps is insufficient for ripping particles from the NS surface, hence failing to provide the plasma needed for radio emission.

### 2.2 Millisecond pulsars and X-ray binaries

The standard evolutionary picture for millisecond pulsars (e.g. Alpar et al. 1982) is that they are born in supernovae with periods of 10s of milliseconds, then evolve along a line of approximately constant magnetic field strength (i.e. \( n = 3 \)) on the \( P-P \) diagram, slowing down until eventually radio emission ceases once they pass into the pulsar death valley. Here, those ‘dead’ pulsars which happen to be in binary systems can undergo accretion from their binary companion. This accretion can heat areas of the NS surface (‘hotspots’) which emit strongly in X-rays – the system is now an X-ray binary system. As well as heating the star the accretion can spin up the star to periods of a few milliseconds. The pulsar is now reborn as a millisecond pulsar and once again is seen to emit as a radio pulsar. In what follows we do not consider the NSs which are millisecond pulsars or those seen in X-ray binaries as this standard evolutionary picture sees these two populations as originating from ‘normal’ radio pulsars. Their birthrates should thus be accounted for in the pulsar birthrate. However, we note that if some NSs in X-ray binaries did not originate from the normal radio pulsars the problem outlined below may be emphasized even further.

### 2.3 RRATs

The discovery of 11 RRAT sources was made (McLaughlin et al. 2006) from single pulse searches of the 1.4-GHz Parkes Multibeam Pulsar Survey (PMP; Manchester et al. 2001). These are distant (~2–7 kpc), transient sources which emit single radio bursts. The burst arrival times are stochastic in some sources with others showing distinct on-off states (McLaughlin, private communication). The times between bursts range from 4 min to 3 h with typical burst rates \( \dot{\chi} \sim 1\text{ h}^{-1} \). The bursts are narrow (2–30 ms) and rather bright (with peak flux densities of 0.1–3.6 Jy) and thus have high brightness temperatures of \( 10^{22}–10^{23} \text{ K} \). These values are higher for the RRATs than all known sources except for the giant radio pulses and nanogiant pulses emitted by the Crab pulsar which have \( T_B \approx 10^{11} \text{ K} \) (Hankins & Rickett 1975) and \( T_B \approx 10^{18} \text{ K} \) (Hankins et al. 2003), respectively.

Continued observations of these sources have enabled underlying periodicities to be determined in all 11 sources and period derivatives to be determined for three sources. Periods in the range 0.7–7 s are seen which suggests a NS origin. This seems to be confirmed from X-ray observations of the most prolific source –RRAT J1819—1458. Using first Chandra (Reynolds et al. 2006) and then XMM–Newton (McLaughlin et al. 2007), a thermal X-ray spectrum (characteristic of a cooling NS) with \( kT \sim 140 \text{ eV} \) was found, as well as X-ray pulsations at the identical period as determined from the radio observations.

We can place the three sources with known period derivative on the \( P-P \) diagram and estimate their surface magnetic field strength using equation (2). The magnetic field strengths are in the range of the normal radio pulsars (\( \sim\text{few } \times 10^{15} \text{ G} \)) except for J1819—1458 which lies between the normal pulsars and the magnetars with \( B_3 = 5 \times 10^{15} \text{ G} \). The discovery of a spectral feature in the X-ray spectrum of J1819—1458 (which may be due to proton cyclotron resonant scattering) seems to support this estimate (McLaughlin et al. 2007).

### 2.4 XDINSs

The XDINSs are a small group of radio-quiet, close by (~100 pc) X-ray pulsars situated in the Gould Belt, a local, partial ring of stars which includes the Sun (Poppel 1997). XDINSs were originally discovered over a decade ago (Walter, Wolk & Neuhauser 1996) with seven sources now known (sometimes referred to as ‘The Magnificent Seven’). XDINSs have thermal X-ray spectra with \( kT = 50–100 \text{ eV} \) and show X-ray pulsations with periods in the range \( \sim 3–11 \text{ s} \) (Haberl 2004). All seven sources have determined periods with three well-known period derivatives and upper limits for three more (see tables 1 and 3 in Haberl 2007; Tiengo & Mereghetti 2001; van Kerkwijk & Kaplan 2008). However, the upper limits determined are one to two orders of magnitude higher.

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than the three well-known $P$ values so may not be very constraining. We can place the three sources with known $P$ on the $P$–$P$ diagram and can determine $B_3 \sim 10^{13}$ G in the standard way. These three sources lie just below the magnetars.

The X-ray spectra of XDINSs can be fit well with a single blackbody and interestingly do not require a power-law component which suggests that XDINSs do not have magnetospheres. Also, as for RRATs, there are observed spectral features which may be due to proton-cyclotron lines in a strong magnetic field (Haberl 2007). We note that there is much current work underway, searching for RRAT-like bursty emission from XDINSs. However, no emission has been found above a flux density limit of $\sim 10$ mJy (Kondratiev et al. 2008) from 820-MHz observations with the Green-Bank Telescope. In addition, there has been no detection with GMRT at 320 MHz (B. C. Joshi, private communication). Searches are also underway using the Parkes telescope at 1.4 GHz (A. Possenti, private communication). These non-detections suggest that the identification of XDINSs as nearby RRATs (Popov et al. 2006) might be incorrect.

2.5 Magnetars

It is thought that both Soft Gamma Repeater (SGRs) and Anomalous X-ray Pulsars (AXPs) belong to the magnetar class of NSs (Woods & Thompson 2004). Magnetars are believed to be isolated X-ray pulsars with strong magnetic fields ($10^{14}$–$10^{15}$ G) and periods in the range 2–12 s and were, until recently, thought to be radio-silent. However transient radio emission has been detected from two AXPs – XTE J1810–197 (Camilo et al. 2006) and 1E 1547.0–5408 (Camilo et al. 2008) with both sources showing flat radio spectra. This is different to what is seen in normal radio pulsars (see Section 2.1). Magnetic fields inferred again from the observed spin and spin-down rates are shown for magnetars in the same $P$–$P$ diagram in Fig. 1.

3 BIRTHRATES

3.1 The core-collapse supernova rate

Recently, the CCSN rate (for Type Ib, Ic and Type II SNe) has been determined from measurements of $\gamma$-ray radiation from $^{26}$Al in the Galaxy (Diehl et al. 2006). Quantifying this $\gamma$-ray emission allowed the authors to weigh the amount of $^{26}$Al in the Galaxy, as each CCSN expels a well-known yield of $^{26}$Al. Assuming an initial mass function1 (IMF), a Scalo IMF ($d \log \Phi / d \log m = -2.7$ for high masses), they inferred the Galactic CCSN rate to be

$$\beta_{\text{CCSN}} = 1.9 \pm 1.1 \text{ century}^{-1}. \quad (5)$$

As a consistency check, we integrate the IMF to compute

$$\beta_{\text{CCSN}} = \frac{\text{SFR}}{(m)} f_{\text{CCSN}}, \quad (6)$$

where $(m)$ is the mass expectation value and $f_{\text{CCSN}}$ is the fraction of stars which end their lives in a CCSN, i.e. those with initial masses in the range $(11 \pm 1) - 25 M_\odot$ (Heger et al. 2003; Podsiadlowski et al. 2004). Using the ‘standard’ IMF, considered to be that of Kroupa (2001, 2002) and McKee & Ostriker (2007) and defined between 0.1 and $120 M_\odot$, and assuming a star formation rate of

$$\text{SFR} = 4 M_\odot \text{ yr}^{-1} \quad (\text{Stahler & Palla 2004; Diehl et al. 2006}),$$

we determine a CCSN rate of as high as $\approx 1.9 \pm 0.9 \text{ century}^{-1}$ if we use a Salpeter-like high-mass IMF ($d \log \Phi / d \log m = -2.3$). While this is consistent with the rate of Diehl et al., it has been suggested that the observed Salpeter-like IMF may be artificially large at high masses due to the effect of unresolved binaries (Kroupa 2002), so that a steeper Scalo-like IMF is more realistic in the high mass range of interest here. In this case, we obtain a lower CCSN rate of just $\approx 0.8 \pm 0.4 \text{ century}^{-1}$. In both cases, here, we have assumed the error to be dominated by the uncertainty of the SFR of up to $\sim 50$ per cent (Stahler & Palla 2004).

3.2 Radio pulsars

The most thoroughly studied NS population is that of radio pulsars. A recent estimate of the birthrate and the number of radio pulsars in the Galaxy was performed by Lorimer et al. (2006) (L+06 from herein) using 1008 non-recycled pulsars detected in 1.4-GHz surveys using the Parkes telescope (the PMPS and the Parkes High-Latitude Survey). Using a pulsar current analysis, they determined population details for sources above a 1.4-GHz radio luminosity threshold of $0.1 \text{ mJy kpc}^2$. Pulsar current analyses use the fact that the typical age of pulsars is much shorter than the age of the Galaxy so that the pulsar population can be considered to be in a steady state. The flow of pulsars across the $P$–$P$ diagram can thus be considered a ‘current’ obeying a continuity equation (Phinney & Blandford 1981; Vivekanand & Narayan 1981). This current, $J(P)$, equals the pulsar birthrate minus the pulsar death rate in the period range $0–P$. Thus, the maximum value of $J(P)$ equals $\beta_{\text{PSR}}$, but as we have a flux limited sample of the pulsar population, the maximum value of $J(P)$ provides only a lower limit to the pulsar birthrate.

The results obtained have model dependencies on the Galactic electron density model and on the pulsar beaming fraction model. The current best model for the electron density is the NE2001 model (Cordes & Lazio 2002), and this was adopted as well as the Taurus & Manchester (1998) beaming model. L+06 determine a birthrate of $\beta_{\text{PSR}} = 1.38 \pm 0.21 \text{ century}^{-1}$ and $N_{\text{PSR}} = 155 000 \pm 6000$. This result is consistent with the earlier work of van den Heuvel et al. (2004) ($V+04$ from herein) which used 815 non-recycled PMPS pulsars to determine $\beta_{\text{PSR}} = 1.58 \pm 0.33 \text{ century}^{-1}$ and $N_{\text{PSR}} = 106 600 \pm 11700$ in this case determined above a higher threshold of $1 \text{ mJy kpc}^2$. In both cases, the now superceded TC93 (Taylor & Cordes 1993) electron density model was also used and for both analyses, this produces lower birthrate estimates.

More recent work by Faucher-Giguere & Kaspi (2006) (FK06 from herein) yields a much higher pulsar birthrate of $\beta_{\text{PSR}} = 2.8 \pm 0.5 \text{ century}^{-1}$. The approach of this analysis is different – the authors model the birth properties of pulsars (velocity distributions, magnetic fields and detectability in the PMPS and Swinburne Multibeam surveys) from the observational data before performing Monte Carlo simulations to evolve the initial population to obtain the observed pulsar sample. The quoted birthrate is the average of 50 runs of their simulations. While it is twice as large as that provided by the pulsar current analyses the results are entirely consistent as the pulsar current analysis is to be interpreted as providing a lower limit to the pulsar birthrate.

3.3 RRATs

The estimated number of RRATs is $N_{\text{RRAT}} \gtrsim 2 \times 10^5$ (McLaughlin et al. 2006) and therefore even higher than that of the radio pulsar population (see Section 5 for discussion of the
using the same population synthesis methods as for the XDINSs, obtaining $\beta_{\text{AXP}} \approx 0.2 \pm 0.2 \text{ century}^{-1}$. Another very recent determination of $\beta_{\text{magnetar}} = 0.15 \pm 0.3 \text{ century}^{-1}$ has also just been reported by Ferrario & Wickramasinghe (2008). The second means by which magnetar age estimates can be obtained, involves using ages of supernova remnant associations of SGRs and AXPs. These have yielded slightly smaller estimates (van Paradijs, Taam & van den Heuvel 1995) as the supernova remnant ages tend to be longer than the spin-down ages resulting in a smaller birthrate.

Due to these small magnetar birthrate estimates relative to the other populations of NSs, one might think that we can safely neglect the magnetar contribution to equation (1). However, we note the possibility that if, for example, magnetars experience magnetic field decay (as considered by Arras, Cumming & Thompson 2004 and by Colpi, Geppert & Page 2000), the true age is smaller than the characteristic age. This would imply a higher birthrate, possibly as high as $\sim 2 \text{ century}^{-1}$ for AXPs (Gill & Heyl 2007). In addition, larger magnetar birthrate estimates have been reported recently by Muno et al. (2008). These authors studied 947 archival observations from XMM–Newton and Chandra. From the seven magnetars detected they determine the most likely number of Galactic magnetars considering the small fraction of the sky covered in these observations. They obtain, separately, birthrates for persistent AXPs, transient AXPs as well as a small contribution from SGRs yielding a large magnetar birthrate of $\beta_{\text{magnetar}} = 2.6_{-1.5}^{+5.0} \text{ century}^{-1}$. This, however, assumes lifetimes of $10^7 \text{ yr}$ (see Fig. 1) for each of these subpopulations. As the lifetime for transient AXPs is very uncertain, it is possible that their lifetime is larger by an order of magnitude. In this case, the persistent AXPs give the most reliable magnetar birthrate of $\beta_{\text{magnetar}} = 0.6_{-0.3}^{+0.9} \text{ century}^{-1}$.

It is not clear if the question of beaming has been considered in the estimates of Muno et al., but we should not necessarily expect magnetar emission to be isotropic. In this case, only magnetars beamed towards us will have been detected. However, judging from the observed pulse shapes (e.g. Woods & Thompson 2004), we assume that the beaming fraction is larger than for radio pulsars, so that the effect of beaming may not be quite as significant as for pulsars. Nevertheless, noting that the derived values may represent a lower limit, we proceed by neglecting this extra beaming factor and considering all the estimates reviewed here. We adopt a conservative magnetar birthrate of $\beta_{\text{magnetar}} \approx 0.3_{-0.2}^{+1.2} \text{ century}^{-1}$, where our extended error bars allow for the potentially much higher values suggested by the Muno et al. study.

## 4 Too Many Neutron Stars?

The birthrates for each of the NS populations are summarized in Table 1 and Fig. 2. It appears that the CCSN rate cannot sustain all the separate NS populations. In a previous consideration of this question by Popov et al. (2006), XDINSs and RRATs were identified as a single NS population, so that only one birthrate contribution was taken, i.e. that of the XDINSs. Moreover, the magnetar contribution was neglected and an XDINS birthrate was assumed such that $\beta_{\text{XDINS}} \approx \beta_{\text{PSR}}$. In addition, Popov et al. used the lower limit pulsar birthrate of $V+04$ to be the pulsar birthrate, a result since superseded by the work of FK06. In this picture, where XDINSs are identified with nearby RRATs, the total birthrate is $\beta_{\text{tot}} = 2-4 \text{ century}^{-1}$ which is barely consistent with the CCSN rate. However, including the magnetars, allowing for separate RRAT and XDINS contributions and using the more accurate pulsar birthrate of FK06, equation (1) cannot be satisfied with the estimates from Table 1. This seems to be the case even if we assume the highest CCSN

various parameters on which this estimate depends). However, the determination of $N_{\text{RRAT}}$ is obviously based on a very small sample of sources. In order to account for this uncertainty, we will use the following parametrization in our computations, that is $N_{\text{RRAT}} = \gamma N_{\text{PSR}}$ where we take $\gamma \sim 1–3$.

It is important to realise the following caveat when interpreting this estimate for the total number of RRATs. The fact that it appears to be larger than that of pulsars does not necessarily imply that a NS is more likely to be a RRAT than a pulsar. This would assume that the physical mechanisms for the emission of the RRAT radio bursts are identical to that of regular pulsars. There is no reason to assume this, especially as RRAT spectra are as yet unknown. Emission criteria (which may represent certain ‘active’ areas on the $P–\dot{P}$ diagram) for RRAT and pulsar emission may be different and the respective ‘death lines’ may also be different, so that the duration of RRAT and pulsar emitting phases would not be the same either. Here lies the advantage of considering birthrates (e.g. pulsar current analyses) rather than absolute population numbers (Popov et al. 2006).

However, as we do not have a reliable age estimator for RRATs, we are forced to assume similar active lifetimes for RRATs and pulsars to work out birthrates from population estimates. We could conceivably use temperature as a measure age (see Section 5) but as there is just one RRAT with known temperature, we follow Popov et al. (2006) who have argued that if RRATs are rotating NSs with pulsar-scale magnetic fields then the active lifetime of pulsars $\tau_{\text{PSR}} \approx N_{\text{PSR}}/\beta_{\text{PSR}} \sim 5 \times 10^4 \text{ yr}$ would be similar to that of RRATs, $\tau_{\text{RRAT}}$. This holds provided the initial spin periods of RRATs and pulsars are within a factor of a few of each other. With the conclusion of approximately equal timescales and $N_{\text{RRAT}} = \gamma N_{\text{PSR}}$, this implies a RRAT birthrate of $\beta_{\text{RRAT}} \approx \gamma \beta_{\text{PSR}}$ (Popov et al. 2006). Thus, if we take $\gamma \sim 2$, then we have an indicative RRAT birthrate of $\beta_{\text{RRAT}} \approx 2.8 \pm 1 \text{ century}^{-1}$ considering the pulsar current analyses or as large as $\beta_{\text{RRAT}} \sim 5.6 \pm 1 \text{ century}^{-1}$ considering the FK06 result.

### 3.4 XDINSs

The birthrate for XDINSs has recently been estimated by Gill & Heyl (2007). These authors performed a population synthesis for XDINSs based on the seven XDINSs detected in the ROSAT All-Sky Survey (Voges et al. 1999). A limiting volume for OB progenitor stars was determined and then compared to the actual number of OB stars detected in this volume in the survey for the relevant scalings. The authors then use an age estimate to find birthrates from the simulated number of sources and determine $\beta_{\text{XDINS}} \sim 2.1 \pm 1 \text{ century}^{-1}$. The age estimate is arrived at simply from averaging the characteristic age for the two XDINSs which then had well-known $P$’s ($\approx 1.5 \pm 1.9 \text{ Myr}$) and earlier estimates for their NS cooling ages ($\sim 0.5 \text{ Myr}$). The result is consistent with a recent lower estimate of $\beta_{\text{XDINS}} \sim 1 \text{ century}^{-1}$ made by Popov et al. (2006) which used a NS cooling age of $\tau_{\text{XDINS}} \approx 1 \text{ Myr}$.

### 3.5 Magnetars

Magnetars birthrates have typically been determined using two different methods of age estimation, required to convert simulated populations to birthrates. The first method uses spin-down age estimates for magnetars, as used by Kouveliotou et al. (1998) to determine a SGR birthrate of $\beta_{\text{SGR}} \approx 0.1 \text{ century}^{-1}$ which we consider as a lower limit for the magnetar birthrate (Woods & Thompson 2004). An AXP birthrate was calculated similarly by Gill & Heyl (2007)
rate allowable within the uncertainties (i.e. $\beta_{\text{CCSN}} = 3$ century$^{-1}$), while at the same time allowing for the lowest required total required NS birthrate, $\beta_{\text{total}} = 5.8$ century$^{-1}$. It seems that the number of NSs produced via CCSNe is not sufficient.

We can just about reconcile the rates if we choose the highest allowable CCSN rate and the lowest allowable total NS birthrate from the L+06 result using the TC93 electron density model (see Table 1). However, as we discussed earlier, the pulsar current results are lower limits and the NE2001 model is often considered to be a more accurate model than TC93 (Cordes & Lazio 2002, 2003).

From looking at Fig. 2, we are left to conclude that either the individual NS birthrates are over-estimated or the uncertainties in these values are under-estimated. To reconcile the values within the errors would require the RRAT and XDINSs errors (which recall are the most uncertain) to be under-estimated by a factor of 2. If this is not the case then it would seem that equation (1) is not satisfied. Taking this at face value implies that there are too many NSs in the Galaxy. We will discuss the nature of this potential NS ‘birthrate problem’ in the following.

5 DISCUSSION

In trying to determine some possible solutions to the birthrate problem, we consider in the following the possibility that the various birthrates are incorrect or that there is an evolutionary answer. Some possible conclusions include the following.

(1) The Pulsar Birthrate is wrong: the pulsar birthrate is the most crucial component of our discussion as pulsars are the most well-studied population and the RRAT birthrate depends on that of the pulsars. Thankfully, the pulsar birthrates are by far the most accurate. The pulsar current analyses make no assumptions and are ‘model free’ even though they depend on the Galactic electron density distribution and the beaming fraction. The lower limits obtained from them are thus quite secure. In order to compensate for the flux limited nature of these studies, we would need to choose a functional form for the luminosity (depending on $P$ and $P$) but the inclusion of such a correction can only increase the determined birthrate.

The work of FK06 models this luminosity evolution across the $P-P$ diagram as well as many other birth properties (modelled from the observed pulsar population). The analysis did assume magnetic dipole spin down of pulsars but allowed for magnetic field decay as well as drawing braking indices from a uniform distribution in the range $n \in [1.4, 3.0]$ (note that the few measured braking indices are found to lie in the range 1.4–2.9, see Lyne & Graham-Smith 2004; Livingstone et al. 2007 and references therein).

Another uncertainty for pulsars is the beaming fraction. An indication of this may be the recent discovery of a pulsar with an extremely small duty cycle (Keith et al. 2008). This pulsar has a beaming fraction of just 0.04 per cent or only 0.14 of longitude. Usually, we would expect the minimum pulse width (for an orthogonal rotator) for this pulsar period of $P = 91$ ms to be given by $\Delta P_{\text{min}}(h) \sim 8/\mu h/(10\text{ km})^{1/2}$, where $h$ is the emission height and we have assumed $\beta$, the impact parameter, to be small. What is observed is a pulsar that is narrower by a factor of $\sim 10$. It is possible that the pulse represents a cut at the very edge of the conical beam but this seems to be at odds with the two observed distinct components in the pulse profile (Keith et al. 2008). Pulsars with pulse widths this narrow therefore raise the question whether or not our beaming fraction estimates are accurate. If they are in fact overestimates then there may be many more pulsars which we do not see.

In summary, taking the pulsar current analysis to provide a reliable lower limit, it seems indeed reasonable to take a pulsar birthrate of $\beta_{\text{PSR}} = 2$ century$^{-1}$ as being quite conservative when the many low luminosity pulsars are included.

(2) The RRAT birthrate is wrong and hugely over-estimated: the RRAT birthrate depends on the RRAT population estimate being correct. This is based on assumptions that the Galactic distribution of RRATs follows that of pulsars, on assumptions about the impact of man-made Radio Frequency Interference (RFI) during the analysis of the PMPS data as well as on beaming and Galactic electron distribution models used and on RRAT burst rate estimates. The full expression for the number of RRATs includes a factor for each of

| $\beta_{\text{PSR}}$, $n_{\text{c}}$ | PSRs | RRATs | XDINSs | Magnetars | Total | CCSN rate |
|-----------------|------|-------|--------|-----------|-------|-----------|
| FK06, NE2001    | 2.8 ± 0.5 | 5.6 ± 1.3 | 2.1 ± 1.0 | 0.3 ± 0.2 | 10.8 ± 0.2 | 1.9 ± 1.1 |
| L+06, NE2001    | 1.4 ± 0.2 | 2.8 ± 1.6 | 2.1 ± 1.0 | 0.3 ± 0.2 | 6.6 ± 0.2 | 1.9 ± 1.1 |
| L+06, TC93      | 1.1 ± 0.2 | 2.2 ± 1.7 | 2.1 ± 1.0 | 0.3 ± 0.2 | 5.7 ± 0.2 | 1.9 ± 1.1 |
| V+04, NE2001    | 1.6 ± 0.3 | 3.2 ± 2.5 | 2.1 ± 1.0 | 0.3 ± 0.2 | 7.2 ± 0.2 | 1.9 ± 1.1 |
| V+04, TC93      | 1.1 ± 0.2 | 2.2 ± 1.7 | 2.1 ± 1.0 | 0.3 ± 0.2 | 5.7 ± 0.2 | 1.9 ± 1.1 |
these input assumptions and is given by (McLaughlin et al. 2006)
\[
N_{\text{RRAT}} \approx 2 \times 10^5 \left( \frac{100 \text{ mJy kpc}^2}{L_{\text{min}}} \right) \left( \frac{0.5}{f_{\text{on}}} \right) \left( \frac{0.5}{f_{\text{RFI}}} \right) \left( \frac{0.1}{f_{\text{beam}}} \right),
\]
where \(f_{\text{on}}\) is the fraction of RRATs which were ‘on’ during a 35-min PMPS observation, \(f_{\text{RFI}}\) is the fraction of RRATs not missed due to RFI, \(f_{\text{beam}}\) is the beaming fraction where, due to recent studies, a modification to the equation has been made (Lorimer, private communication). All of these effects are treated conservatively but to really improve the accuracy of the estimate, we need to discover many more sources. This will enable us to accurately determine the factors in this equation. The ‘on’ factor can be better constrained with accurate RRAT burst rates for a larger population of RRATs. The RFI factor is more difficult to quantify but recently we have made progress in this regard with the development of a new RFI removal scheme which is capable of removing the vast majority of terrestrial RFI (Eatough et al., in preparation). The RFI removal means that we will be able to essentially ignore the \(f_{\text{RFI}}\)-factor (i.e. \(f_{\text{RFI}} \approx 1\)). A reprocessing of the PMPS data applying this superior RFI scheme is underway and will eventually enable us to detect all RRATs that were originally overlooked due to RFI contamination. This will yield a more accurate value of \(N_{\text{RRAT}}\) and the results of this will be reported later.

An uncertainty of the beaming fraction of radio pulsars also affects RRATs as we have, as the best available assumption, adopted a beaming fraction as observed for long period pulsars. If the beams are narrower then this increases the estimate for the number of RRATs. The RRAT lifetime is another uncertain parameter. If one was to propose a much larger active lifetime than that proposed by Popov et al. (2006) then this would reduce the implied RRAT birthrate. However, even if this was an order of magnitude larger (i.e. \(\sim 50\) Myr) then the birthrate problem would remain although less emphatic. Conversely, we note that the RRATs seem to have higher \(P\) values than most pulsars and thus may evolve to higher periods (i.e. towards the death valley) more quickly than pulsars which would then imply a shorter lifetime than that assumed above. Noting all of these caveats, it does not seem unreasonable to take, as a conservative estimate, \(\beta_{\text{RRAT}} = 2 - 6\) century\(^{-1}\) for \(\gamma = 1 - 3\) as before.

(3) The XDINS birthrate is wrong and hugely over-estimated: like in the case of RRATs this could be due to the small sample size of this population. Furthermore, some of the XDINS birthrate estimates assume spin-down ages to be an accurate age estimate and there are only three XDINSs with well-known \(P\). Furthermore, the estimate of Gill & Heyl (2007) also depends on the used Galactic \(N_{\text{HI}}\) model, i.e. the authors applied an exponential model (ignoring any warp or spiral arm components). Long-term monitoring of the known XDINSs can yield exact period derivatives which in turn will give us accurate characteristic ages which, along with NS cooling ages, may enable us to determine the true age of XDINSs. Certainly, however, as for RRATs, the best way to improve the used estimates is to increase the known population of XDINSs. Recent work to determine where best to search for these elusive sources, has pointed towards the Cygnus–Cepheus region behind the Gould Belt (Posselt et al. 2008).

(4) A possible evolution from different types of NSs into others: we consider here the possibility that pulsars, RRATs and XDINSs might be different evolutionary stages of a single class of object. If this is the case, we need only to take one birthrate for these populations into balancing the CCSN rate, that is the birthrate of the earliest stage in this cycle. To determine the direction of this evolution requires a reliable age estimator. As all of the considered NSs are isolated objects, we expect that they simply slow down as they age, so that the longer periods of XDINSs (3–11 s), compared with those of the RRATs (0.7–7 s), imply RRATs to be younger than XDINSs. Similarly, the even lower periods of isolated pulsars (0.03–8.5 s) might imply that the evolutionary track in question is pulsar → RRAT → XDINS. To investigate this possibility, we performed Kolmogorov–Smirnov (K–S) tests comparing the cumulative period probability distributions (see Fig. 3). The probability that the pulsar and RRAT distributions are drawn from the same parent distribution is found to be 0.02 per cent. However, we believe the comparison to be unfair due to the large difference in distribution sizes (1500 and 11). To test this, we randomly selected 20 pulsar periods from their period distribution and compared these with the RRAT distribution over many iterations. The resulting probabilities vary largely with \(\sim 17\) per cent of iterations showing probabilities below 1 per cent, but \(\sim 18\) per cent of iterations showing probabilities above 20 per cent. Next, we compared the median periods of the randomly selected pulsar samples to that of the RRATs. This value is stable with the median pulsar period being 615 ms over 10 000 iterations. The RRAT median period is \(\sim 7\sigma\) above this value considering 16 known RRATs (the 11 original sources plus five newly detected sources, M. McLaughlin, private communication) and \(\sim 20\sigma\) considering only the published sources. From this, we conclude that RRAT periods are intrinsically longer than those of the pulsars.

Comparing the RRAT and XDINS distributions using the K–S test gives a probability that these two distributions are drawn from the same parent distribution of only 2 per cent but not low enough to reject this possibility given the small numbers in each category (\(N_{\text{RRAT}} = 11, N_{\text{XDINS}} = 7\)). If we, nevertheless, assume these distributions to be different, then we can estimate the time needed for the RRAT period distribution to evolve to the XDINS period distribution by comparing the average period and the period derivatives. The average periods are, respectively, \(\left\langle P_{\text{RRAT}} \right\rangle = 3.6\) s and \(\left\langle P_{\text{XDINS}} \right\rangle = 8.1\) s. Using the average RRAT period derivative of \(\left\langle P \right\rangle = 2 \times 10^{-13}\) and assuming that it is constant with time, we estimate an evolutionary time of

\[
t = \frac{\left( P_{\text{XDINS}} \right) - \left( P_{\text{RRAT}} \right)}{\left( P_{\text{RRAT}} \right) / \left( P_{\text{XDINS}} \right)} \sim 0.7\ \text{Myr.}
\]

Another indicator of age is temperature, assuming that no significant heating occurs during the life of any of these sources.
Assuming NSs are born with the same temperature and then cool along a NS cooling curve, we can determine the age if the surface temperature can be measured\(^2\). However, NS cooling curves are not very well constrained (as they often depend on the unknown NS equation-of-state, e.g. Yakovlev & Pethick 2004) so that the exact age–temperature relationship is not known. Still, we can safely expect an older star to be cooler than a young NS, so that we might suppose XDINSs to be cooler than RRATs as XDINSs have \(T \approx 0.7 \times 10^6 \text{ K}\) (Yakovlev & Pethick 2004) and RRAT J1819−1458 has \(T \approx 1.6 \times 10^6 \text{ K}\), lending tentative support to our conclusion based upon periods.

If some/all RRATs do evolve into XDINSs it means the XDINS birthrate can be removed from consideration in balancing equation (1). We note that a NS evolution from RRAT to XDINS suggests a path on the \(P−P\) diagram (along, say, an \(n = 3\) line) which starts in the region of the high-B radio pulsars. Such an evolution where RRATs and XDINSs are evolutionary states reached by some of the normal radio pulsars means both of these birthrate contributions can be removed from equation (1) and the birthrate problem is much less severe and is solved within the errors of \(\beta_{\text{PSR}}\) and \(\beta_{\text{CCSN}}\).

This of course does not include the magnetar contribution. If the magnetar birthrate is in fact small, it is not necessary to include magnetars in an evolutionary scenario to solve the problem but none the less we consider it. Comparing the magnetar period distribution to that of RRATs and XDINSs suggests that the XDINSs and magnetar distributions are quite similar. A K–S test finds their distributions to be the same with a 92 per cent probability. This might suggest that the XDINS and magnetar evolutionary end states come from the same initial population. Such a scenario with some high-B radio pulsars evolving towards the magnetar region of the \(P−P\) diagram involves increasing \(B_0\). While at first this may seem counterintuitive, we note that equation (2) describes the surface magnetic field which could indeed increase with time. In fact, this is suggested by long-term observations of the Crab pulsar and PSR B1737−30 (Lyne & Smith 2004; Lyne 2004), and by the measured braking indices from young pulsars as \(n < 3\).

(5) An unknown NS formation process: another (even more drastic) solution might be that there is some other unknown mechanism of forming NSs besides CCSNe. This might well be required even if there is an evolutionary answer to the birthrate problem because, as we have noted, the number of pulsars alone is pushing the allowed limit. The only other known mechanisms for forming NSs are electron-capture SNe (Nomoto 1984, 1987) and an accretion induced collapse (AIC) (Grindlay 1987).

While stellar modelling results (Eldridge & Tout 2004; Podsiadlowski et al. 2004) had seen electron-capture SNe only possible in close binary systems,\(^3\) some very recent work has examined this process in isolated stars (Prelarends et al 2008). The results suggest that electron-capture SNe likely account for just ~4 per cent of all SNe (i.e. not just CC SNe) which increases the number of Galactic NSs but only by a small amount.

AIC is a process naturally only occurring in binary systems, but here we consider only isolated NSs so this process cannot resolve the problem. It would seem that another as yet unknown NS formation mechanism would be required.

\(^2\)We note the extra downward emission above the surfaces of pulsars will act as a heating mechanism of the polar cap. This scenario does not apply for RRATs and XDINSs which are "off" most of all the time

\(^3\)It is thought that the lower mass pulsar in the double pulsar system, J0737−3039B, was formed in this way.

6 CONCLUSIONS

In this work, we have presented a detailed and critical review of the current state of birthrate calculations for a number of NS manifestations. Based on this review, we suggest that the current CCSN rate cannot explain the birthrates of the various NS populations. Unless new birthrate estimates emerge which differ vastly from those discussed here, we have a birthrate problem. If this is the case we favour an evolutionary interpretation, where radio pulsars RRATs and XDINSs (and possibly also magnetars) are different evolutionary-ary stages of the same object. Another possible, more exotic solution would be the existence of some, as yet unknown, mechanism for forming NSs. While the determined birthrates are uncertain for XDINSs, RRATs and magnetars, we consider the currently claimed uncertainties, if in fact they are correct, not to be large enough to convincingly remove the described problem.

To truly advance in answering these questions, it is essential to find a significantly larger number of sources to increase the known populations of RRATs, XDINSs and magnetars. Searches to this end are underway and many more are planned. There are prospects for discovering XDINSs and magnetars from future high-energy observatories. These include the International X-Ray Observatory [which has now superceded the planned XEUS (X-Ray Evolving Universe Spectroscopy) and Con-X (Constellation-X) missions] as well as the recently launched Fermi Gamma-Ray Space Observatory. Radio surveys include the ongoing P-ALFA survey at Arecibo (Cordes et al. 2006), the planned transients and pulsar observations with Low Frequency Array (LOFAR), and the new Parkes and Effelsberg Multibeam All-Sky Surveys (beginning 2008 Autumn). Ultimately, a Galactic census of pulsars with the Square Kilometre Array (SKA) should improve our knowledge of Galactic NSs phenomenally with the expected detection of ~20,000 new pulsars (Cordes et al. 2004). Together, the SKA and LOFAR will monitor the Northern and Southern hemisphere essentially continuously. Even though the spectrum of RRATs and hence their discovery potential at low frequencies still has to be assessed, both telescopes should allow us to potentially observe most RRATs in the Galaxy that are beaming towards the Earth. In other words, SKA observations will, with certainty, establish the relative population numbers for RRATs and pulsars, confirming or rejecting the results of this study. Extremely valuable information will, however, be already available much sooner with the help of LOFAR (van Leeuwen & Stappers 2008).

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