5G Network Planning under Service and EMF Constraints: Formulation and Solutions

Luca Chiaraviglio\textsuperscript{(1,2)}, Cristian Di Paolo\textsuperscript{(2)}, Nicola Blefari-Melazzi\textsuperscript{(1,2)}
\textsuperscript{1} Department of Electronic Engineering, University of Rome Tor Vergata, Rome, Italy, email: luca.chiaraviglio@uniroma2.it, crisdp95@gmail.com, belfari@uniroma2.it
\textsuperscript{2} Consorzio Nazionale Interuniversitario per le Telecomunicazioni (CNIT), Italy

Abstract—We target the planning of a 5G cellular network under 5G service and ElectroMagnetic Fields (EMFs) constraints. We initially model the problem with a Mixed Integer Linear Programming (MILP) formulation. The pursued objective is a weighed function of gNB installation costs and 5G service coverage level. In addition, we precisely model restrictive EMF constraints and we integrate scaling parameters to estimate the power radiated by 5G gNBs. Since the considered planning problem is NP-Hard, and therefore very challenging to be solved even for small problem instances, we design an efficient heuristic, called PLANNING ALGORITHM TOWARDS EMF EMISSIONS ASSESSMENT (PLATEA), to practically solve it. Results, obtained over a realistic scenario that includes EMF exposure from pre-5G technologies (e.g., 2G, 3G, 4G), prove that PLATEA retrieves a planning that ensures 5G service and restrictive EMF constraints. However, we demonstrate that the results are strongly affected by: i) the relative weight between gNB installation costs and 5G service coverage level, ii) the scaling parameters to estimate the exposure generated by 5G gNBs, and iii) the amount of exposure from pre-5G technologies.

Index Terms—5G Mobile Networks, 5G Network Planning, Base Station Deployment, Service and EMF constraints, EMF regulations

1 INTRODUCTION

The provisioning of the 5G service inevitably requires the installation of new 5G equipment, called next-generation Node-B Base Station (gNB), over the territory. The task of selecting and configuring the set of sites hosting 5G equipment is often referred as 5G cellular planning\textsuperscript{(1)}, a complex problem that involves costs, service coverage and ElectroMagnetic Field (EMF) constraints. In general, the planning of a cellular network is a critical step that has a huge impact on the CAPlital EXPenditures (CAPEX) costs incurred by the operator\textsuperscript{(2)}, as well as on the Quality of Service (QoS) perceived by users\textsuperscript{(3)–(4)}. From an operator perspective, the network planning should minimize the costs for deploying new 5G sites and installing 5G equipment. In addition, the operator aims at maximizing the performance (e.g., throughput, delay) that is experienced by 5G User Equipment (UE).

Under realistic settings, the planning problem is strongly affected by the regulations governing the EMF levels radiated by 5G equipment\textsuperscript{(5)}. In general, many countries in the world ensure that the EMF levels radiated by 5G gNBs are lower than maximum values (often referred as EMF limits)\textsuperscript{(6)}, which depend on the frequency exploited by the 5G gNB. Traditionally, international/federal bodies like International Commission on Non-Ionizing Radiation Protection (ICNIRP) and Federal Communications Commission (FCC) define EMF limits for all the cellular frequencies, including the ones used by 5G equipment\textsuperscript{(7)–(8)}. Since no adverse health effects have been scientifically proven so far when EMF exposure is lower than the limits defined by international/federal bodies\textsuperscript{(9)–(11)}, a planning that ensures this condition is a mandatory step to preserve public health. As a result, the constraints introduced by EMF regulations have to be carefully taken into account during the installation and then the operation of 5G equipment. Intuitively, the EMF constraints tend to limit the number of 5G sites installed over the territory and/or the amount of radiated power by each 5G gNB. Therefore, the EMF regulations have a large impact on the 5G gNB installation costs and the 5G service received by UE\textsuperscript{(1)–(3), (12)}.

The picture is further complicated in different countries (such as Italy, Poland, and many others)\textsuperscript{(6)}, which introduce EMF regulations more restrictive that the ones defined by ICNIRP and FCC, on the basis of the application of a precautionary principle, in order to preserve the population from (still unknown) long-term health effects triggered by EMF exposure. Additional rules include: i) EMF limits strongly lower that the ones defined by ICNIRP/FCC\textsuperscript{(6)},\textsuperscript{(13)} and/or ii) minimum distances between sensitive places (e.g., schools, hospitals public parks) and the installed 5G sites\textsuperscript{(14)}. For example, both i) and ii) are enforced in the city of Rome, an area of 1287 square kilometers inhabited by...
almost 3 million people. As sketched in Fig. 1, the introduction of strict EMF limits strongly affects the planning of the 5G network. For example, the installation of new 5G sites is prevented within a minimum distance from the center of the sensitive area (e.g., the school in the figure). In addition, the enforcement of very low EMF limits tends to generate EMF saturation areas (e.g., the one shown on top right of the figure), where the EMF levels from pre-5G technologies are already close to the maximum limits. In such zones, therefore, the installation of new 5G sites is denied. As a result, the 5G sites have to be installed locations, thus further impacting the installation costs and the service level offered by the 5G network.

In this context, a natural question emerges: Is it possible to deploy a heterogeneous 5G network, while ensuring 5G service and restrictive EMF constraints? The ambitions goal of the paper is to tackle such interesting - and challenging - problem. Our innovative contributions can be summarized as follows. First, we take into account restrictive EMF regulations affecting the 5G planning phase, i.e., EMF limits stricter than ICNIRP/FCC ones, as well as enforcement of minimum distances between 5G sites and sensitive places. Second, we optimally model the 5G planning problem under service and EMF constraints. The presented problem integrates the widely used model of Marzetta [15] to compute the service level from a Massive Multiple-Input Multiple-Output (MIMO) system, as well as the EMF point source described in International Telecommunication Union (ITU) K.70 recommendation [16], which is enriched by a set of scaling parameters to take into account the temporal and statistical variation of the radiated power from 5G MIMO systems with beamforming capabilities. We also show that the complete formulation falls within the class of Mixed Integer Linear Programming (MILP) problems and it is NP-Hard. Third, we design PLATEA and two reference algorithms in a realistic scenario, whose parameters have been measured on the field (e.g., the EMF levels radiated by pre-5G sites already deployed in the scenario). Results prove that PLATEA outperforms the reference algorithms, by efficiently balancing between 5G gNB installation costs and amount of 5G service coverage. In addition, we demonstrate that the scaling parameters used to compute the power radiated by 5G gNBs play a critical role in determining the selected planning and theEMF levels over the territory. Eventually, we show that the exposure levels generated by pre-5G technologies have an impact on the 5G planning.

To the best of our knowledge, previous works in the literature are focused on orthogonal aspects w.r.t the ones investigated in this paper. For example, Oughton et al. [2] target the solution of the 5G planning problem by means of techno-economic approaches, with little emphasis on the impact of EMF constraints. On the other hand, Matalata et al. [17, 18] design heuristics targeting the reduction of the radiated power, EMF and/or specific absorption rate (SAR), without considering: i) the linearization of the problem constraints, ii) the impact of the variation of the scaling parameters to compute the EMF levels from 5G gNBs, and iii) the introduction of constraints to ensure a minimum distance between sensitive places and 5G gNBs. In this work, we show that both ii) and iii) are fundamental to determine the actual planning. In addition, we propose an innovative formulation with linear constraints, which are also exploited by PLATEA to further reduce the algorithm complexity.

The rest of the paper is organized as follows. Sec. 2 reviews the related work. The main building blocks of the considered 5G framework are highlighted in Sec. 3. Sec. 4 reports the problem formulation. The PLATEA algorithm is thoroughly described in Sec. 5. The scenario under consideration is detailed in Sec. 6. Results are analyzed in Sec. 7. Finally, Sec. 8 concludes our work.

### 2 RELATED WORKS

Tab [1] reports the positioning of this paper w.r.t. other relevant works that are focused on the planning of 5G networks [2, 17, 18]. More in depth, we consider the following features to classify the literature: i) pursued goal(s) (e.g., cost reduction, power consumption reduction), ii) targeted 5G equipment type (e.g., generic gNBs, micro and macro gNBs), iii) modeled 5G service (e.g., SINR, network spectral efficiency, throughput, maximum coverage distance), iv) EMF features (e.g., temporal and statistical models, presence of exclusion zones in proximity to the gNB), v) considered EMF regulations (e.g., ICNIRP-based EMF limits, strict EMF limits, minimum site distance from sensitive places), vi) pursued methodology (e.g., model assessment, optimal formulation, heuristic) and vii) scenario complexity (e.g., number of candidate sites, regular or irregular coverage layout, size of the service area, pixel or user evaluation).

Compared to the literature, our work moves one step further by: i) explicitly targeting a weighed function of installation costs and service coverage, ii) considering a heterogeneous 5G network composed of micro and macro gNBs, iii) precisely modeling multiple service metrics, including throughput, minimum SIR and maximum coverage distance, iv) performing the variation of the scaling parameters to compute the EMF, as well as including exclusion zones in proximity to the gNB, v) integrating EMF regulations more restrictive than ICNIRP/FCC, both in terms of maximum limits and in terms of minimum distance between a 5G site and a sensitive place, vi) defining a linear formulation (MILP) and exploiting the linearized constraints to design the heuristic, vii) analyzing a large scenario composed of dozens of candidate sites with irregular coverage layouts, a service area in the order of different square kilometers, and service/EMF evaluations performed in each pixel of the territory.

### 3 BUILDING BLOCKS

We briefly overview the main building blocks that are integrated in our 5G framework, namely: i) the model to assess
5G performance, ii) the model to estimate EMF radiated by a set of gNBs and iii) the EMF regulations for the installation of 5G sites. In the following, we provide more details about each building block.

### 3.1 5G Performance Model

We adopt the widely known MIMO model of Marzetta [15] to evaluate the 5G performance for a set of installed gNBs. We refer to [15] for the details, while here we report the salient features. In brief, the model assumes that each site is equipped with arrays composed of a very large number of antenna elements. In the work of Marzetta [15], the number of antennas is higher than the number of users. Since the number of antennas is very large, the downlink SINR is dominated by the interference from neighboring gNBs rather than by the noise floor. More formally, the SIR of the \( k \)-th user served by \( l \)-th gNB operating on frequency \( f \) is defined as:

\[
S(k,l,f) = \frac{\beta^2(l,k,l,f)}{\sum_{j \neq 1}^n \beta^2(l,k,j,f)}
\]

In the previous equation, the \( \beta \) terms are expressed as:

\[
\beta(l,k,j,f) = \frac{\gamma_f(1,k,j,f)}{D(l,k,f)}
\]

where \( D(k,l,f) \) is the distance between 5G UE \( k \) and gNB operating on frequency \( f \) and installed at location \( l \), \( \gamma_f \) is the path-loss exponent for frequency \( f \) and \( z(l,k,j,f) \) is a log-normal random variable, i.e., the quantity \( 10 \log_{10}(z(l,k,j,f)) \) is a distributed zero-mean Gaussian with a standard deviation \( \sigma_f^{SHAD} \) [15].

The \( \beta \) terms appearing in Eq. (1) are also sketched in the toy-case scenario of Fig. 2, which is composed by two gNBs and three UE. Intuitively, each 5G UE is served by a single gNB, while the other gNB contributes to the interference experienced by the 5G UE. It is important to remark that, in the downlink direction, the contributions of interference are solely due to the neighboring gNB, and not to the simultaneous transmissions to other UE in the same cell (e.g., terminals \( k1 \) and \( k3 \) in the figure), due to the fact that an Orthogonal Frequency-Division Multiplexing (OFDM) technology is assumed.

The downlink throughput received by user \( k \) from gNB installed at location \( l \) and operating on frequency \( f \) is then expressed as:

\[
T(k,l,f) = \frac{B_f \cdot \Gamma_f}{\epsilon^{SEC}_f} \cdot \log_2 \left( 1 + S(k,l,f) \right)
\]

where: \( B_f \) is the gNB bandwidth, \( \epsilon^{SEC}_f \leq 1 \) is a parameter governing the sectorization over frequency \( f \) (equal to 1

| Work | Goal | 5G Equipment Type | 5G Service Metrics | EMF Features | EMF Regulations | Methodology | Scenario Complexity |
|------|------|-------------------|--------------------|--------------|----------------|-------------|---------------------|
| [2]  | Capacity and costs assessment | Micro & macro 5G gNBs | Signal-to-interference plus noise ratio (SINR), network spectral efficiency | -            | -              | Model assessment | Hexagonal coverage layout with 7 candidate sites, service area of 37 [square kilometers], evaluation done on pixels. |
| [3]  | Power consumption reduction, EMF exposure reduction | Generic gNBs operating at 3.7 [GHz] | Throughput, SINR | Presence of exclusion zones | Strict EMF limits | Heuristic | Dozens of candidate sites with irregular coverage layout, service area of several square kilometers, evaluation done on users (not on a pixel base). |
| [4]  | Power consumption reduction, EMF exposure reduction, SAR exposure reduction, dose exposure reduction | Generic gNBs operating at 3.7 [GHz] | Throughput, SINR | Statistical models (with fixed parameters), presence of exclusion zones | ICNIRP-based EMF limits | Mixed Integer Non-Linear Programming (MINLP) optimization model, heuristic | Dozens of candidate sites with irregular coverage layout, service area of several square kilometers, evaluation done on users (not on a pixel base). |
| This work | Installation costs reduction, maximization of the number of served pixels | Micro & macro 5G gNBs | Throughput, minimum signal-to-interference ratio (SIR) threshold, maximum coverage distance | Temporal and statistical models (with variation of parameters), presence of exclusion zones | Strict EMF limits, minimum site distance from sensitive places | MILP optimization model, heuristic | Dozens of candidate sites with irregular coverage layout, service area of different square kilometers, evaluation done on pixels. |
when sectorization is not exploited), \( \Gamma_f \) is a shaping factor, formally expressed as:

\[
\Gamma_f = \left( \frac{\tau_f^{\text{SLOT}} - \tau_f^{\text{Pilot}}} {\tau_f^{\text{SLOT}}} \right) \cdot \tau_f^{\text{Useful}}
\]  

where \( \tau_f^{\text{SLOT}} \) is the slot duration over \( f \), \( \tau_f^{\text{Pilot}} \) is the pilot duration over \( f \), \( \tau_f^{\text{Symbol}} \) is the symbol interval over \( f \) and \( \tau_f^{\text{Useful}} \) is the useful symbol duration over \( f \). The expressions for \( \tau_f^{\text{SLOT}} \), \( \tau_f^{\text{Pilot}} \), \( \tau_f^{\text{Symbol}} \) and \( \tau_f^{\text{Useful}} \) are reported in Table 2 where \( N_{\text{OFDM}} \) is the number of OFDM symbols over \( f \), \( \Delta_f \) is the subcarrier spacing over \( f \), \( \delta_f \) is the cyclic prefix duration over \( f \), \( N_f^{\text{OFDM-Pilot}} \) is the number of OFDM symbols used for pilots over \( f \) and \( \tau_f^{\text{Coherence}} \) is the coherence time over \( f \).

Summarizing, the model of Marzetta [15] allows to compute the SIR and the maximum throughput for each UE, given the set of installed 5G gNBs and the UE-gNB association.

### 3.2 5G EMF Model

The second building block that is instrumental to our framework is the computation of the EMF that is received by a 5G UE from a 5G gNB. In the literature, different EMF models have been proposed to this purpose. We refer the interested reader to ITU-T K.70 recommendation [16] for an overview about the EMF models. In brief, the available options include point source models, synthetic models and full-wave models. In this work, we select the point source model, due to the following key properties:

- the actual EMF levels that are measured over the territory in the far-field region are typically lower than the ones estimated through the point source model of [16]. Therefore, when the EMF is computed through this model and the obtained level is below the maximum limit, the adherence to the limit is always guaranteed;
- a linear set of constraints to compute the total EMF levels can be built when the model is integrated in our framework. Ensuring the linearity of the constraints is a desirable property, which, in fact, allows to reduce the complexity of both the optimal formulation and the designed heuristic.

In more detail, the point source model allows to compute the power density \( P_{(k,l,f)} \) that is received by UE \( k \) from a gNB located at site \( l \) and operating on frequency \( f \). Clearly, the distance \( D_{(k,l,f)} \) between gNB \( l \) and UE \( k \) is assumed to be in the far-field region [16]. More formally, \( P_{(k,l,f)} \) is expressed as:

\[
P_{(k,l,f)} = \frac{\text{EIRP}_{(l,f)}} {4\pi \cdot D_{(k,l,f)}^2} \cdot F_{(k,l,f)}
\]

where \( \text{EIRP}_{(l,f)} \) is the Equivalent Isotropically Radiated Power (EIRP) from gNB operating on frequency \( f \) and located at site \( l \) and \( F_{(k,l,f)} \leq 1 \) is the normalized numeric gain over user \( k \) from an antenna installed at location \( l \) operating on frequency \( f \). More in depth, \( \text{EIRP}_{(l,f)} \) is formally expressed as:

\[
\text{EIRP}_{(l,f)} = \frac{\eta_f^{\text{Gain}} \cdot \eta_f^{\text{Loss}} \cdot O_{\text{MAX}}^{\text{Gain}} \cdot f} {\eta_f^{\text{Gain}}} \cdot f\text{CH}^{\text{Loss}}
\]

where \( O_{\text{MAX}}^{\text{Gain}} \) is the maximum output power for a gNB operating on frequency \( f \) and located at site \( l \), \( \eta_f^{\text{Gain}} \) is the transmission gain on frequency \( f \), and \( \eta_f^{\text{Loss}} \) is the transmission loss on frequency \( f \).

By assuming the maximum achievable numeric gain \( F_{(k,l,f)} = 1 \) (as in [16]), Eq. (5) is simplified into:

\[
P_{(k,l,f)} = \frac{\text{EIRP}_{(l,f)}} {4\pi \cdot D_{(k,l,f)}^2}
\]

Given \( P_{(k,l,f)} \), the electric field value \( E_{(k,l,f)} \) is then expressed as:

\[
E_{(k,l,f)} = \sqrt{P_{(k,l,f)} \cdot Z_0}
\]

where \( Z_0 \) denotes the free space wave impedance.

3. The power density metric is commonly used to characterize the level of exposure. Other exposure metrics include electric field, magnetic field and SAR. We refer the interested reader to [7] for an overview and comparison of the different exposure metrics.
Fig. 3 reports the power density terms $P_{(k,l,f)}$ in the same toy-case scenario of Fig. 2 which is composed by two gNBs operating at the same frequency $f$ and three UEs. Actually, the total power density that is received by each UE is a linear combination of the single terms radiated by the two gNBs. For example, the total power density radiated over 5G UE $k1$ is simply equal to $P_{(k1,j1,f1)} + P_{(k1,j2,f2)}$. Note that, when considering the total electric field from multiple gNBs, a root mean square operator has to be applied. By considering the previous example, the total electric field that is radiated over UE $k1$ is equal to $\sqrt{E_{(k1,j1,f1)}^2 + E_{(k1,j2,f2)}^2}$. This operation introduces a non-linearity in the computation of the total exposure, which also generates non-linear constraints when binary variables are employed to select the set of sites that have to be installed out of the candidate ones. To overcome this issue, in this work we consider the computation of the total exposure through the power density metric, which instead allows to preserve the linearity of the constraints.

When computing the exposure from a 5G gNB, a key role is played by the EIRP value appearing in Eq. 7. Clearly, the higher is the EIRP, the larger will be also the received power density $P_{(k,l,f)}$. This fact imposes to precisely estimate EIRP values that match real 5G gNB exposure patterns. In this context, two important elements that affect the EIRP values of a 5G gNB are the temporal variation and the statistical variation of the radiated power. We refer the interested reader to the International Electrotechnical Commission (IEC) standards [21], [22] for an overview about these aspects. In brief, the temporal variation is due to the fact that the number of 5G UE (and their traffic over the cellular network) exhibits a day-night pattern. Actually, the actual output power levels of the 5G gNB match this variation, with radiated power higher during the day and clearly lower during the night. On the other hand, the application of MIMO with beamforming features introduces strong variations in the radiated power over the territory, resulting in an exposure that is concentrated only to the zones where the served users are located. This issue is typically taken into account by solving statistical exposure models, which allow to compute spatially averaged radiated power values, as a consequence of the actual user distribution over the territory.

In this work, we take into account the temporal and the statistical variability of the EIRP from 5G gNB by introducing two scaling parameters, denoted as $R_{\text{TIME}}(l,f)$ and $R_{\text{STAT}}(l,f)$, respectively. Actually, the introduction of parameters to scale the maximum EIRP is in line with other works that investigate the exposure modeling from 5G gNBs [22]–[25]. More formally, the scaled EIRP from gNB installed at site $l$ and operating on frequency $f$ is computed as:

$$EIRP_{(l,f)} = EIRP_{(l,f)} \cdot R_{\text{TIME}}(l,f) \cdot R_{\text{STAT}}(l,f)$$  (9)

In addition, let us introduce the power density $P_{(k,l,f)}$ computed from $EIRP_{(l,f)}$. By adopting Eq. 9 and the left-hand side of Eq. 7, $P_{(k,l,f)}$ is formally expressed as:

$$P_{(k,l,f)} = P_{(k,l,f)} \cdot R_{\text{TIME}}(l,f) \cdot R_{\text{STAT}}(l,f)$$  (10)

In this work, we use Eq. 7, 9, 10 to characterize the level of exposure from a 5G gNB located at site $l$, operating on frequency $f$ and radiating over UE $k$. Moreover, we demonstrate that the actual values of $R_{\text{TIME}}(l,f)$ and $R_{\text{STAT}}(l,f)$ have a crucial role in determining the level of exposure and consequently the set of gNBs that are installed over the territory.

### 3.3 5G EMF Constraints

We then consider the third building block of our framework, i.e., the integration of the 5G EMF constraints defined in the regulations. To this aim, Tab. 3 reports the set of regulations (R1)-R6), which include ICNIRP guidelines (R1-R2), Italian national regulations (R3)-R4), and the local EMF regulations enforced in the city of Rome (R5)-R6). For each regulation/guideline, the table reports: i) the frequency range relevant to 5G, ii) the maximum electric field limit for each frequency $f$, iii) the maximum power density limit $L_f$ for each $f$, iv) the time interval to compute the average EMF that has to be compared against the limit value and v) the (eventual) minimum distance constraints that have to be ensured between the 5G installations and the sensitive places. As a side comment, we include in Tab. 3 the ICNIRP 1998 guidelines [19] and ICNIRP 2020 ones [7], due to the fact that the formers are still adopted in many countries in the
world, while the latters are the up-to-date regulations which are going to be adopted in the forthcoming month/years, and hence in parallel with the deployment of 5G networks.

Several considerations hold by analyzing Tab. 3. First of all, R2 defines a power density limit and not a limit based on electric field strength, for all the frequencies between 2 [GHz] and 300 [GHz]. This fact further corroborates our choice for selecting the power density as the reference metric when performing the compliance assessment against the maximum limits. Second, the Italian regulations R3-R4 are in general stricter than R1-R2), both in terms of electric field and in terms of power density. Third, the Italian regulations in R3 and R4 further differentiate between general public areas (e.g., zones of the territory where the population is not continuously living) and residential areas (e.g., zones where people tend to live and/or work), respectively. Interestingly, R4 regulations are more restrictive than R3). Fourth, the city of Rome applies a minimum distance \( D_{MIN} \) from sensitive places in addition to the strict EMF limits defined in R3-R4. Therefore, the regulations R5-R6) further restrict R3-R4). Fifth, the averaging time interval strongly varies across the different regulations, ranging from values of few minutes to 24 hours. This interval plays a crucial role in estimating the average EMF that has to be compared against the limit thresholds. Clearly, the lower is the time interval, the higher will be the influence of (possible) spikes on the average EMF. On the other hand, the higher is the time interval, the lower will be the impact of spikes on the average EMF. As a side comment, the instantaneous EMF field can be higher than the thresholds reported in Tab. 3. The actual metric that is meaningful for comparison against the limit is in fact the average EMF over the time interval defined in each regulation.

After analyzing the EMF regulations, a natural question emerges: How to perform the compliance assessment w.r.t. the maximum limits when multiple Base Stations operating at different frequencies radiate the same area of territory? To answer this question, let us denote with \( \sum_{f \in \mathcal{F}} P_{k,l,f}(k,l) \) the composite power density that is radiated over UE \( k \) by all the Base Stations operating on frequency \( f \in \mathcal{F} \), where \( \mathcal{F} \) is the set of frequencies in use. The compliance w.r.t. the limits is ensured over \( k \) if the following condition holds:

\[
\sum_{f \in \mathcal{F}} \frac{\sum_{l \in \mathcal{L}} P_{k,l,f}}{L_f} \leq 1 \tag{11}
\]

Clearly, the power density terms \( P_{k,l,f} \) of Eq. (11) have to be computed as average values over the time intervals reported in Tab. 3. In addition, the actual EMF metric that is measured under practical conditions is the electric field strength \( E_{k,l,f}(k,l) \), which is then translated into power density \( P_{k,l,f} \) by applying Eq. (8).

3.4 Summary and Next Steps

The model of Marzetta [15] is used to control the SIR and consequently the maximum downlink throughput provided to the UE. The point source model of ITU-T K.70 [16], integrated with scaling parameters that characterize the exposure from 5G gNB, is instead used to compute an over-estimation of the power density. Finally, the limits defined by international/national bodies and local municipalities are used to ensure that the composite power density is lower than the thresholds. In addition, a minimum distance rule from sensitive places is ensured in accordance to the local regulation. In the next section, we join together these building blocks, in order to build an innovative formulation able to balance between gNBs installation costs and 5G service coverage level, while ensuring QoS and strict EMF constraints.

4 Optimal 5G Planning Formulation

We divide our formulation in the following steps: i) preliminaries, ii) set definition, iii) constraint, variables and input parameters, iv) overall formulation.

4.1 Preliminaries

In the previous section we have provided the models to compute the service coverage and the power density for each UE in the scenario under consideration. In this section, we generalize these models by extending the evaluations from a sparse set of UE to a tessellation of non-overlapping squared pixels that fully cover the area under interest. By applying the pixel tessellation, three important goals can be met, namely:

1) the power density terms are computed over the whole area under consideration. More in depth, we evaluate the power density that is received over the pixel center from all the installed gNBs. In this way, we are able to extend the EMF compliance assessment over the whole territory. In addition, we model the presence of exclusion zones in proximity to the installed gNBs, i.e., zones of the territory that are not accessed by users and therefore in such zones the EMF compliance assessment is not required for the general public;
2) a dense scenario where users are located in each pixel (and not in few locations) is introduced. This assumption appears to be meaningful in the context of 5G, especially for the Enhanced Mobile Broadband (eMBB) scenario. In this way, it is possible to control the amount of throughput provided to each pixel, and consequently to the UE that are located in the pixel;
3) a minimum throughput requirement may be introduced for each pixel rather than for single users. In this way, it is possible to (indirectly) take into account also the effects of UE densification and/or UE mobility. For example, by assigning different values of required throughput, it is possible to model high density zones, where the throughput requirements are high, compared to other zones, which instead are not visited by users. In a similar way, it is possible to vary the throughput requirements based on the UE mobility, e.g., by increasing the throughput for the zones that are subject to high UE mobility, in order to take into account the effect of handovers and/or possible traffic spikes.

4. The evaluation of the UE throughput given the pixel throughput will be done in a future work.
Focusing then on the modelling of the EMF regulations, we assume to enforce the most restrictive ones, namely R5-R6 of Tab. 3. Therefore, we distinguish between general public areas, residential areas and zones within the minimum distance from sensitive places. However, we point out that the other guidelines presented in Tab. 3 can be easily implemented in our framework by applying different limit thresholds and/or by setting the minimum distance $D_{\text{MIN}}$ to zero.

### 4.2 Set Definition

Let us denote with $\mathcal{P}$ the set of pixels under consideration. $\mathcal{P}_{\text{RES}} \subset \mathcal{P}$ and $\mathcal{P}_{\text{GEN}} \subset \mathcal{P}$ are the subsets of pixels in residential areas and in general public areas, respectively. Moreover, $\mathcal{P}_{\text{SENS}} \subset \mathcal{P}$ is the subset of pixels in sensitive areas. In addition, let $\mathcal{L}$ be the set of candidate locations (sites) that can host 5G gNB equipment. Eventually, let $\mathcal{F}$ be the set of frequencies that can be exploited by 5G gNBs.

### 4.3 Constraints, Variables and Input Parameters

We then detail constraints, variables and input parameters to our problem by adopting a step-by-step approach. We also refer the reader to Tab. 4 for the main notation that is adopted throughout the section.

#### 5G Coverage and Service Constraints

We initially model the constraint that a pixel $p \in \mathcal{P}$ can be covered by a 5G gNB located in $l$ only if the distance $D_{l,p,f}$ between the pixel and the installed gNB is lower than a maximum one, denoted with $D_{\text{MAX}}$, where $f$ is the operating frequency of the 5G gNB installed in $l$. More formally, we have:

$$D_{l,p,f} \cdot x(p,l,f) \leq D_{\text{MAX}} \cdot y(l,f), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F}$$

(12)

where $x(p,l,f)$ is a binary variable, set to 1 if $p$ is served by gNB operating on frequency $f$ and located at $l$ (0 otherwise). Moreover, $y(l,f)$ is another binary variable, set to 1 if 5G gNB operating on frequency $f$ is installed at location $l$ (0 otherwise).

We then impose the constraint that each pixel $p$ can be served by at most $N_{\text{SER}} \geq 1$ gNBs at the same time:

$$\sum_{l \in \mathcal{L}} \sum_{f \in \mathcal{F}} x(p,l,f) \leq N_{\text{SER}}, \quad \forall p \in \mathcal{P}$$

(13)

In the following, we impose that the SIR value in each pixel $p$ that is served by a 5G gNB operating on frequency $f$ has to be higher than a minimum value $S_{\text{MIN}}$. By adopting the SIR computation already introduced in Eq. (1), we have:

$$\frac{\beta^2_{l,p,f} \cdot y(l,f)}{S(p,l,f)} \sum_{i \neq l \in \mathcal{L}} \beta^2_{i,p,l,f} \cdot y(i,f)} \geq S_{\text{MIN}} \cdot x(p,l,f), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F}$$

(14)

5. Although multiple coverage from different gNB is a desirable condition, the increase in the number of gNBs covering the same pixel may introduce side effects, like an increase in the handover rates for UE, which may dramatically decrease the perceived QoS. Therefore, we introduce a constraint to control the number of gNBs serving the same pixel.

6. Since we have extended the evaluation from the single UE to the whole set of pixels, the $k$ index of Eq. (1) is replaced with $p \in \mathcal{P}$.

#### 4.4 Objective Function

Clearly, the previous constraint is not linear, due to the optimization variables $y(l,f)$ that appear on both the numerator and the denominator of the left-hand side, coupled with the presence of the $x(p,l,f)$ variables on the right-hand side of the constraint. In order to linearize Eq. (14), we initially exploit the following equivalence:

$$\sum_{l \neq i \in \mathcal{L}} \beta^2_{i,p,l,f} \cdot y(i,f) \sum_{l \neq i \in \mathcal{L}} \beta^2_{i,p,l,f} \cdot y(i,f)$$

(15)

By assuming that the right-hand side of Eq. (15) is greater than or equal to 0, we then replace the denominator of
Eq. [14] with Eq. [15], thus obtaining:
\[
\beta_{l,p,f}^{2} \cdot y(l,f) \geq S_{f}^{\text{MIN}} \cdot x(p,l,f),
\]
\[
\sum_{l_{2} \in L} \beta_{l(p,l_{2},f)}^{2} \cdot y(l_{2},f) - \beta_{l(p,l_{2},f)}^{2} \cdot y(l,f) \right),
\]
\[
\forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{16}
\]

We then divide both sides of the constraint by the left hand side term, thus obtaining:
\[
S_{f}^{\text{MIN}} \cdot x(p,l,f) \cdot \left[ \frac{\sum_{l_{2} \in L} \beta_{l(p,l_{2},f)}^{2} \cdot y(l_{2},f)}{\beta_{l(p,l_{2},f)}^{2}} \cdot y(l,f) - 1 \right] \leq 1,
\]
\[
\forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{17}
\]

The previous constraint is equivalent to the following one:
\[
S_{f}^{\text{MIN}} \left[ \sum_{l_{2} \in L} \beta_{l(p,l_{2},f)}^{2} \cdot y(l_{2},f) \cdot x(p,l_{2},f) - x(p,l,f) \right] \leq 1,
\]
\[
\forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{18}
\]

By recalling constraint [12], we know that \( x(p,l,f) = 1 \) only if \( y(l,f) = 1 \) (and both Eq. [12] and Eq. [13] are ensured). In other words, \( x(p,l,f) \) can not be set to 1 if the gNB operating on \( f \) is not installed in \( l \), i.e., \( y(l,f) = 0 \). As a result, the ratio \( x(p,l,f)/y(l,f) \) can be simply expressed as \( x(p,l,f) \). Consequently, constraint [15] can be rewritten in the following equivalent form:
\[
S_{f}^{\text{MIN}} \left[ \sum_{l_{2} \in L} \beta_{l(p,l_{2},f)}^{2} \cdot y(l_{2},f) \cdot x(p,l_{2},f) - x(p,l,f) \right] \leq 1,
\]
\[
\forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{19}
\]

The previous constraint can be easily linearized by: i) introducing the binary auxiliary variable \( v_{l(p,l_{2},f)} \in \{0, 1\} \), ii) replacing [19] with the following set of constraints:
\[
v_{l(p,l_{2},f)} \leq x(p,l_{2},f), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, l_{2} \in \mathcal{L}, f \in \mathcal{F} \tag{20}
\]
\[
v_{l(p,l_{2},f)} \leq y(l_{2},f), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, l_{2} \in \mathcal{L}, f \in \mathcal{F} \tag{21}
\]
\[
v_{l(p,l_{2},f)} \geq x(p,l_{2},f) + y(l_{2},f) - 1, \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, l_{2} \in \mathcal{L}, f \in \mathcal{F} \tag{22}
\]
\[
S_{f}^{\text{MIN}} \left[ \sum_{l_{2} \in L} \beta_{l(p,l_{2},f)}^{2} \cdot v_{l(p,l_{2},f)} - x(p,l_{2},f) \right] \leq 1
\]
\[
\forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{23}
\]

**Power Density Limits.** We initially select the pixels that fall in the exclusion zones of the installed 5G gNBs and therefore are not subject to the EMF limits defined for the general public. More formally, we introduce the binary variable \( w_{p} \), set to 1 if \( p \) is located inside an exclusion zone of an installed gNB (0 otherwise). In addition, input parameter \( E_{p,l,f} \) takes value 1 if pixel \( p \) is inside the exclusion zone of gNB operating on frequency \( f \) and located at \( l \) (0 otherwise).

The value of \( w_{p} \), is then set through the following set of constraints:
\[
w_{p} \geq E_{p,l,f} \cdot y(l,f), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{24}
\]
\[
w_{p} \leq \sum_{l \in \mathcal{L}} \sum_{f \in \mathcal{F}} E_{p,l,f} \cdot y(l,f), \quad \forall p \in \mathcal{P} \tag{25}
\]

More in depth, constraint [24] activates \( w_{p} \) if \( p \) is inside at least one exclusion zone of an installed gNB. On the other hand, constraint [25] forces \( w_{p} \) to 0 if \( p \) is outside the exclusion zones for all the installed gNBs.

In the following, we introduce the constraints to compute the power density received by pixel \( p \) over frequency \( f \). Let us denote with input parameter \( P_{ADD}^{(p,f)} \) the additional power density that is received by pixel \( p \) when a gNB operating on frequency \( f \) is installed in \( l \). Let us denote with \( P_{ADD}^{TS} \) the variable storing the additional power density for pixel \( p \) over \( f \), which is computed from \( P_{ADD}^{(p,f)} \) by applying the scaling factors \( R_{l,f}^{\text{TIME}} \) and \( R_{l,f}^{\text{STAT}} \). More formally, we include Eq. [10] to our problem, thus yielding:
\[
P_{ADD}^{TS} \underset{p,f}{=} (1 - w_{p}) \sum_{l \in \mathcal{L}} P_{ADD}^{(p,l,f)} \cdot R_{l,f}^{\text{TIME}} \cdot R_{l,f}^{\text{STAT}} \cdot y(l,f)
\]
\[
\forall p \in \mathcal{P}, f \in \mathcal{F} \tag{26}
\]

In the previous constraint, the term \((1 - w_{p}) \) ensures that a pixel falling inside the exclusion zone of an installed 5G gNB is not considered when the power density is evaluated against the limits.

Since constraint [25] is not linear, we linearize it by: i) introducing the auxiliary variable \( z_{p,l,f} \in \{0, 1\} \), and ii) replacing Eq. [25] with the following set of constraints:
\[
z_{p,l,f} \leq (1 - w_{p}), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{27}
\]
\[
z_{p,l,f} \leq y(l,f), \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{28}
\]
\[
z_{p,l,f} \geq y(l,f) - w_{p}, \quad \forall p \in \mathcal{P}, l \in \mathcal{L}, f \in \mathcal{F} \tag{29}
\]

\[
P_{ADD}^{TS} \underset{p,f}{=} \sum_{l \in \mathcal{L}} P_{ADD}^{(p,l,f)} \cdot R_{l,f}^{\text{TIME}} \cdot R_{l,f}^{\text{STAT}} \cdot z_{p,l,f}
\]
\[
\forall p \in \mathcal{P}, f \in \mathcal{F} \tag{30}
\]

In a similar way, we compute the additional total power density \( P_{ADD}^{TS} \) that is received by pixel \( p \) on frequency \( f \), without applying the scaling factors \( R_{l,f}^{\text{TIME}} \) and \( R_{l,f}^{\text{STAT}} \). \( P_{ADD}^{TS} \) is meaningful when \( p \) belongs to a general public area (e.g., R5 of Tab. 3). In this case, in fact, the scaling parameters are not applied. Therefore, we have:
\[
P_{ADD}^{\text{NOTS}} \underset{p,f}{=} \sum_{l \in \mathcal{L}} P_{ADD}^{(p,l,f)} \cdot z_{p,l,f}, \forall p \in \mathcal{P}, f \in \mathcal{F} \tag{31}
\]

We then impose the power density limit on residential areas, which has to be ensured for each pixel \( p \in \mathcal{P}_{\text{RES}} \). More technically, we include the compliance assessment model of Eq. [11] in our problem, thus obtaining:
\[
\sum_{f \in \mathcal{F}} \frac{P_{BASE}^{\text{RES}} \cdot (1 - w_{p}) + P_{ADD}^{\text{TS}}}{R_{f}^{\text{RES}}} \leq 1, \quad \forall p \in \mathcal{P}_{\text{RES}} \tag{32}
\]
where \( P_{BASE}(p,f) \) is the baseline power density over \( p \) from all the radio-frequency sources operating of frequency \( f \) and already installed in the scenario under consideration.

In a similar way, we impose the power density limit on general public areas, which has to be ensured for each pixel \( p \in P^{GEN} \), by introducing the following constraint:

\[
\sum_{f \in F} P_{BASE}(p,f) \cdot (1 - w_p) + P_{ADD-NOTS}(p,f) \leq 1, \quad \forall p \in P^{GEN} \quad (33)
\]

In order to clarify how the computation of the power density is governed by the optimization variables modelling the exclusion zones, Fig. 4 shows a graphical representation of \( P_{ADD}(p,f) \cdot z(p,f) \) terms that appear in Eq. (30)-(31) as well as \( P_{BASE}(p,f) \cdot (1 - w_p) \) that are included in Eq. (32)-(33). More in depth, the considered toy-case scenario includes one legacy 4G Node-B already installed over the territory and two newly installed gNBs. Fig. 4(a) focuses on a pixel \( p1 \) outside the exclusion zones of the gNBs. By applying Eq. (24)-(25), it holds that \( w_{p1} = 1 \). Then, by applying constraints (27)-(29), it holds that: \( z_{p1,l1,f2} = 1, z_{p1,l2,f2} = 1 \). Consequently, the computation of the total power density in (32)-(33) will include the contributions from the newly installed gNBs as well as the already installed 4G Node-B. Therefore, the installation of the newly installed gNB is possible only if the EMF compliance assessment constraints (32)-(33) are ensured. On the other hand, Fig. 4(b) reports the power density terms when the considered pixel \( p2 \) falls inside the exclusion zone of a gNB. By applying Eq. (24)-(25), it holds that \( w_{p2} = 0 \), \( z_{p2,l1,f2} = 0 \), and \( z_{p2,l2,f2} = 0 \). As a result, the power density terms \( P_{ADD}(p2,l,f) \cdot z(p2,l,f) \) and \( P_{BASE}(p2,f) \cdot (1 - w_{p2}) \) are now set to zero. Therefore, the EMF compliance assessment constraints (32)-(33) are always ensured for \( p2 \).

Minimum Distance from Sensitive Places. We then introduce the distance constraints that are included in regulations R5-R6 of Tab. 3. We remind that these constraints define a minimum distance between each installed 5G gNB and a sensitive place. More formally, we have:

\[
D_{(p,l,f)} \cdot y(l,f) \geq D_{MIN}, \quad \forall p \in P^{SENS}, l \in L, f \in F \quad (34)
\]

Site Constraints. In the following, we impose that each site location can host up to \( N^{MAX} \) gNB types operating at different frequencies. More formally, we have:

\[
\sum_{f \in F} y(l,f) \leq N^{MAX}, \quad \forall l \in L \quad (35)
\]

In addition, we introduce the indicator parameter \( I_{(l,f)} \), taking value 1 if gNB of type \( f \) can be hosted at location \( l \), 0 otherwise. Clearly, a gNB operating on frequency \( f \) can be installed at \( l \) only if the indicator parameter is 1. More formally, we have:

\[
y(l,f) \leq I_{(l,f)}, \quad \forall l \in L, f \in F \quad (36)
\]

Total Cost Computation. Finally, we compute the total costs for installing the 5G gNBs. To this aim, let us denote with \( C^{EQUIP} \) the monetary costs of a 5G gNB equipment operating on frequency \( f \). In addition, let us denote with \( C^{SITE}(l,f) \) the site installation cost for a 5G gNB operating on frequency \( f \) and installed at location \( l \). The total costs \( C^{TOT} \) for installing the new 5G gNBs are formally expressed as:

\[
C^{TOT} = \sum_{l \in L} \sum_{f \in F} \left( C^{EQUIP}_f + C^{SITE}(l,f) \right) \cdot y(l,f) \quad (37)
\]

4.4 Objective Function and Overall Formulation

The considered objective function targets the minimization of the total costs for installing the 5G gNBs and the maximization of the number of pixels that are served by the installed 5G gNBs. The two terms are properly taken into account by the weight factor \( \alpha_{(l,f)} \), which depends on frequency \( f \) and on gNB location \( l \). By tuning \( \alpha_{(l,f)} \), the operator can easily control the CAPEX costs and the level of service coverage over the territory.

The complete OPTIMAL PLANNING for 5G NETWORKS UNDER SERVICE AND STRICT EMF CONSTRAINTS (OPTPLAN-5G) is formally expressed as:

\[
\min \left( C^{TOT} - \sum_{p \in P} \sum_{l \in L} \sum_{f \in F} \alpha_{(l,f)} \cdot x_{(p,l,f)} \right) \quad (38)
\]

8. Other alternative formulations commonly adopted during the cellular planning phase include the minimization of the CAPEX costs under a given percentage of service coverage. However, this goal does not always guarantee problem feasibility. To overcome this issue, in this work we keep the service coverage in the objective function. As a result, our choice allows to preserve the problem feasibility on one side and to explore the impact of \( \alpha_{(l,f)} \) on the obtained planning on the other one.
Algorithm 1 Pseudo-Code of PLATEA algorithm

Input: Parameters and sets defined in Tab.1 assumed to be available through global variables
Output: Variables \( y, x, P_{ADD-\text{TS}}, P_{ADD-\text{NOTS}}, w, C_{\text{TOT}} \) for the best solution found

1: // Step 1: Initialization
2: \( \text{num}_f_1 \_\text{max} = \sum_{f \in F} I_{(f, l)} \); // Max. number of installable \( f_1 \) gNBs
3: \( \text{num}_f_2 \_\text{max} = \sum_{f \in F} I_{(f, l)} \); // Max. number of installable \( f_2 \) gNBs
4: \( \text{best}_\text{obj}=\text{inf} \); // Best objective initialization
5: \([y, x, P_{ADD-\text{TS}}, P_{ADD-\text{NOTS}}, w, C_{\text{TOT}}]=\text{INITIAL\_SOL}(); // Solution variables initialization
6: // Step 2: iteration over candidate deployments with frequency \( f_1 \)
7: \text{for} \( \text{num}_f_1 = \text{num}_f_1 \_\text{max} \) \text{do}
8: \([\text{flag}_\text{end}, x\_\text{curr}, y\_\text{curr}, pd\_\text{curr}] = \text{SELECT\_BEST\_SET\_F1(\text{num}_f_1)}(); // Selection of the best deployment with \( \text{num}_f_1 \) gNBs
9: \text{if} \( \text{flag}_\text{end} = \text{false} \) \text{then}
10: \text{sites}_f_2\_\text{perm} = \text{EXTRACT\_SITES(\text{num}_f_2, f_2)}(); // Extraction of the candidate deployments with frequency \( f_2 \)
11: \text{for} \( \text{sites}_f_2 \) in \( \text{sites}_f_2\_\text{perm} \) \text{do}
12: \([\text{flag}_\text{check}, y\_\text{curr}, pd\_\text{curr}] = \text{INSTALL\_CHECK(\text{sites}_f_2, y\_\text{curr}, pd\_\text{curr})}(); // Based on Eq. \( 24 \), \( 25 \), \( 26 \)
13: \text{if} \( \text{flag}_\text{check} = \text{true} \) \text{then}
14: \([x\_\text{curr}], y\_\text{curr}, pd\_\text{curr}] = \text{ASSOCIATE\_PIXELS}(y\_\text{curr}, x\_\text{curr})(); // Based on Eq. \( 12 \), \( 13 \), \( 20 \) - \( 23 \)
15: \text{if} \( \text{curr}_\text{obj} < \text{best}_\text{obj} \text{then}
16: \text{best}_\text{obj} = \text{curr}_\text{obj};
17: \( [y, x, P_{ADD-\text{TS}}, P_{ADD-\text{NOTS}}, w, C_{\text{TOT}}]=\text{SAVE\_SOL(y\_\text{curr}, x\_\text{curr}, pd\_\text{curr})}; // Best Solution Saving
18: \text{end if}
19: \text{end if}
20: \text{flag}_\text{end} = \text{true}; // All pixels served
21: \text{end if}
22: \text{end for}
23: \text{end if}
24: \text{end for}
25: \text{end if}
26: \text{end for}
27: \text{end if}
28: \text{end if}
29: \text{UNINSTALL}(f_2, x\_\text{curr}, y\_\text{curr}, pd\_\text{curr}); // Revert changes for \( f_2 \)
30: \text{UNINSTALL}(f_1, x\_\text{curr}, y\_\text{curr}, pd\_\text{curr}); // Revert changes for \( f_1 \)
31: \text{end for}

subject to:

\begin{align*}
& 5G \text{ Coverage and Service: Eq. } \{12, 13\}, \{20\} - \{23\} \\
& \text{Power Density Limits: Eq. } \{24\}, \{26\}, \{27\} - \{33\} \\
& \text{Min. Distance Constraint: Eq. } \{34\} \\
& \text{Site Constraints: Eq. } \{35\}, \{36\}, \{37\} \\
& \text{Total Cost Computation: Eq. } \{39\} \\
& C_{\text{TOT}} \geq 0, x_{(p,i,f)} \in \{0,1\}, y_{(l,f)} \in \{0,1\}, w_p \in \{0,1\}, v_{(p,l,i,f)} \in \{0,1\}, z_{(p,l,f)} \in \{0,1\}.
\end{align*}

Proposition 1. The OPTPLAN-5G problem is NP-Hard.

Proof. Let us consider a special case of the problem, where a single pixel is evaluated. Moreover, let us assume that this single pixel is covered if the gNB operating on frequency \( f \) is installed in \( l \), i.e., \( x_{(p,i,f)} = y_{(l,f)}, \forall l \in L, f \in F \). Let us also consider the possibility to install up to one gNB in each site, i.e., \( N^{\text{MAX}} = 1 \). Consequently, constraint (33) can be rewritten as:

\[
\sum_{f \in F} y_{(l,f)} \leq 1, \quad \forall l \in L
\] (40)

Moreover, let us assume that the considered pixel is outside the exclusion zone of each installed gNB, i.e., \( w_p = 0 \).

9. In this way, we assume that the pixel is within \( D^{\text{MAX}} \) distance from all the gNBs and that the minimum value of service throughput is equal to 0.

Consequently, \( z_{(p,l,f)} = y_{(l,f)}, \forall l \in L, f \in F \). Moreover, we consider: i) the application of general public limits, i.e., the scaling parameters are not applied and ii) a relaxation of the power density constraints in (33) with no background power density (i.e., \( \rho_{(p,f)} = 0, \forall f \in F \)) and the limit verification for each frequency in isolation w.r.t. the other frequencies. More formally, constraint (33) is replaced with the following one:

\[
\sum_{l \in L} P_{\text{ADD}}(x_{(p,i,f)} \cdot y_{(l,f)} \leq L_{f}^{\text{GEN}}, \quad \forall f \in F
\] (41)

We then assume the maximization of the service coverage, which in our problem is equivalent to the maximization of the number of installed gNBs, weighted by \( \alpha_{(l,f)} \). More formally, we have:

\[
\max \sum_{l \in L} \sum_{f \in F} \alpha_{(l,f)} \cdot y_{(l,f)}
\] (42)

subject to: Eq. (40), Eq. (41); under variables: \( y_{(l,f)} \in \{0,1\} \). It is therefore trivial to note that the aforementioned formulation is the well-known Generalized Assignment Problem (GAP), which is NP-Hard [27]. Since GAP is a special case of our problem, we can conclude that also OPTPLAN-5G is NP-Hard. \(\Box\)
5 PLATEA Algorithm

Since the OPTPLAN-5G is NP-Hard, and therefore very challenging to be solved even for small problem instances, we design an efficient algorithm, called PLATEA (PlANNING ALGoRITHM TOWARDS EMF EMISSION ASSESSMENT), to practically solve it. We base our solution on the following intuitions:

1) we apply a divide et impera approach, in which the complex planning problem is split into sets of subproblems. More in depth, since the different frequencies used in 5G have in general different goals (e.g., throughput maximization and/or coverage maximization), we exploit the gNB operating frequency as the main metric to split the original problem into smaller subproblems;

2) we restrict the exploration of the solution space by evaluating controlled sets of candidate deployments. However, we introduce a parameter to control the exploration level of the candidate deployments;

3) we exploit the linear constraints introduced in the previous section to limit the computational complexity of PLATEA.

Alg. 1 reports the high-level pseudo-code of PLATEA. The presented algorithm is tailored to the case in which two frequencies $f_1$ and $f_2$ are exploited, with $f_1$ targeting throughput maximization and $f_2$ targeting coverage maximization. In order to ease the presented pseudo-codes, we adopt the following guidelines: i) the input parameters and sets defined in Tab. 1 are assumed to be available through global variables, and ii) the subscripts appearing in the parameters/variables are hindered. The algorithm then produces as output the selected deployment $y$, the pixel to gNB association $x$, the power density variables $P_{ADD-TS}$, $P_{ADD-NOTS}$, the exclusion zone variable $w$ and the total installation costs $C_{TOT}$ for the selected deployment.

We then describe the operations performed by PLATEA. Initially, the maximum number of installable gNB is computed (lines 2-3). In the following, all the variables are initialized to zero values by the INITIALIZE function (line 5). PLATEA then iterates over the possible candidate deployments with frequency $f_1$ (lines 7-31). In particular, the SELECT_BEST_SET_F1 function in line 8 retrieves the best solution found for each number of installable $f_1$ gNB, starting from one up to the maximum number (line 7).

In the following, we provide more details about the SELECT_BEST_SET_F1 function, which is expanded in Alg. 2.

The function requires as input the number of targeted gNBs to be installed, denoted as num_f1. Then, the function produces as output a flag (indicating if a feasible deployment has been found), as well as temporary variables storing the current set of installed $f_1$ gNBs, the current pixel to gNB association, and the current power density over the set of pixels. After an initialization step to setup the routine variables (line 1-2), the function retrieves the possible permutations of $f_1$ gNBs, by running the EXTRACT_SITES routine (line 3). Since enumerating all the possible permutations is a challenging step in terms of computational requirements, we control the amount of generated permutations by assuming that up to num_f1 candidate deployments are randomly generated. Intuitively, when num_f1 is low, it is not meaningful to explore the whole space of permutations, since the number of served pixels will be in any case rather limited. On the other hand, we consider more permutations as num_f1 increases, since the impact on service coverage may be not negligible in this case.

In the following (lines 4-18), the SELECT_BEST_SET_F1 function iterates over the set of selected permutations. In particular, the constraints about power density limits in Eq. (24), (25), minimum distance from sensitive places in Eq. (34) and site constraints in Eq. (35)-(36) are checked over frequency $f_1$. If the previous constraints are all met, the pixels are associated to the installed gNBs (line 9) and the objective function is evaluated (line 10). More in depth, the association of pixels in line 9 is performed by sequentially analyzing the set of installed gNBs and by associating each pixel while ensuring the 5G coverage and service constraints of Eq. (12)-(13), (21)-(23) with frequency $f_1$. Clearly, if the previous constraints are not met, the current pixel is not associated to the gNB under consideration.

In addition, the computation of the objective function in line 10 exploits constraints Eq. (37), (38) with frequency $f_1$. Eventually, the best solution is updated in lines 11-16.

When SELECT_BEST_SET_F1 is terminated, PLATEA per-
forms lines 9-31 of Alg. 1. In particular, the algorithm iterates over the candidate deployments on frequency $f_2$ (line 10). Clearly, this step is performed only if a feasible candidate deployment over frequency $f_1$ has been found (line 11). In the following, the algorithm generates $\text{num}_f^\mathcal{F}_2$ permutations of $f_2$ gNBs (line 12), and then iterates over each candidate deployment (lines 13-27) in order to verify the constraints (line 14) and eventually to perform the gNB-pixel association (lines 15-16). The functions used in these steps are exactly the same adopted in Alg. 2 except from the adopted frequencies, which now also include $f_2$ gNBs. In the following, the objective function is evaluated (line 17), and the best solution is eventually updated (lines 18-21). The algorithm then stops evaluating further deployments if all the pixels have been served (line 22-24). Clearly, when passing between the evaluation of one deployment to the following one, the changes operated on the temporary variables are reverted to the previous state (lines 25,30).

**Computational Complexity.** Tab. 5 reports the computational complexity of the routines, functions and the whole PLATEA algorithm. Several considerations hold by analyzing the table. First, we denote with $N^\mathcal{P}$ the number of permutations that are generated by the EXTRACT_SITES routine. Second, the computational complexity of INSTALL_CHECK, ASSOCIATE_PIXELS and COMPARE_OBJ are derived from the implementation of constraints Eq. (24), (25), (26), (27), (28), (29), (30), (31), (32). Third, the whole computational complexity of PLATEA grows linearly with the number of pixels and with the number of frequencies. Fourth, although the complexity of PLATEA may appear substantially large at a first glance, due to the term $N^\mathcal{P} \times |\mathcal{L}|^4$, we remind that the number of candidate sites $|\mathcal{L}|$ is rather limited in practice, due to the intrinsic difficulty in finding suitable locations that can host gNB equipment. In addition, in our work we constrain $N^\mathcal{P}$ to be in the same order of magnitude of $|\mathcal{L}|$. Therefore, overall complexity of PLATEA is in the order of $O(|\mathcal{P}| \times |\mathcal{L}|^5 \times |\mathcal{F}|)$.

### Table 5

| Procedure | Complexity |
|-----------|------------|
| INITIAL_SOL | $O(|\mathcal{P}| \times |\mathcal{L}| \times |\mathcal{F}|)$ |
| EXTRACT_SITES | $O(N^\mathcal{P})$ |
| INSTALL_CHECK | $O(|\mathcal{P}| \times |\mathcal{L}| \times |\mathcal{F}|)$ |
| ASSOCIATE_PIXELS | $O(|\mathcal{P}| \times |\mathcal{L}|^2 \times |\mathcal{F}|)$ |
| COMPARE_OBJ | $O(|\mathcal{P}| \times |\mathcal{L}| \times |\mathcal{F}|)$ |
| SAVE_SOL | $O(|\mathcal{P}| \times |\mathcal{L}| \times |\mathcal{F}|)$ |
| INITIAL | $O(|\mathcal{P}| \times |\mathcal{L}| \times |\mathcal{F}|)$ |
| ALL_SERVED | $O(|\mathcal{P}| \times |\mathcal{L}| \times |\mathcal{F}|)$ |
| SELECT_BEST_SET_F1 | $O(N^\mathcal{P} \times |\mathcal{P}| \times |\mathcal{L}|^2 \times |\mathcal{F}|)$ |
| PLATEA | $O(N^\mathcal{P} \times |\mathcal{P}| \times |\mathcal{L}|^4 \times |\mathcal{F}|)$ |

### 6 Scenario Description

We consider as reference scenario the Torrino MezzoCammino (TMC) neighborhood in Rome, Italy. The area under consideration, spanning over 2.47 [km$^2$], is actually populated by more than 10000 inhabitants. We select the TMC neighborhood due to the following reasons: i) TMC includes residential areas and sensitive places (i.e., public parks, schools, churches, recreation centers); therefore, its territory is subject to very stringent EMF regulations (i.e., R6 regulation of Tab. 3). ii) The terrain is almost plain, i.e., there are not steep hills and/or large obstacles (apart from the buildings), which would otherwise affect the propagation conditions. iii) 5G coverage is not actually provided in the neighborhood, iv) pre-5G base stations are installed only outside the neighborhood, v) background information about pre-5G coverage and QoS levels experienced in TMC is already available in [5].

We then focus on the set of frequencies $\mathcal{F}$ that are employed by 5G gNBs. More in detail, we consider the exploitation of two distinct frequencies, namely $f_1 = 3700$ [MHz] and $f_2 = 700$ [MHz]. Both $f_1$ and $f_2$ have been recently auctioned to 5G operators in Italy [29]. Therefore, we expect that both $f_1$ and $f_2$ will be used by 5G equipment in the forthcoming years. In this work, we assume that $f_1$ is exploited by micro gNBs, while the $f_2$ is employed by macro gNBs. We believe that this choice is meaningful, as $f_1$ will be mainly used to provide capacity, while $f_2$ will ensure coverage.

In the following, we provide more details about the area under consideration, the set of pixels, the set of candidate gNBs and the set of sensitive places. To this aim, Fig. 5 reports: i) the TMC neighborhood (transparent blue area), ii) the set of locations $\mathcal{L}$ that can host 5G gNBs (i.e., the union of blue and orange pins), iii) the subset of locations that can host $f_1$ gNBs (orange pins), and iv) the subset of locations that can host $f_2$ gNBs (blue pins). The selection of locations in iii) and iv) is driven by the following principles: a) installation of $f_2$ gNBs mainly on top of buildings, in order
to maximize the coverage over the territory, b) installation of $f_1$ gNBs close to the zones where capacity is needed (along the roads and in proximity to the residential buildings). As a result, a total of $|\mathcal{L}| = 69$ candidate locations are taken into account in this work. In the following, we move our attention to the set of pixels $\mathcal{P}$. More in detail, we assume a pixel tessellation over the TMC area, with a pixel granularity equal to $10 \times 10$ [m$^2$]. The total number of pixels $|\mathcal{P}|$ is then equal to 24318. Focusing then on the sensitive places, Fig. 6 highlights in yellow the areas hosting public parks, schools and/or churches. By adopting R6 regulation from Tab. 3 the installation of gNBs is prohibited within a minimum distance of $D_{\text{MIN}} = 100$ [m] from the external perimeter of these sensitive places.

We then analyze the setting of the remaining frequency-dependent parameters that are required as input. To this aim, Tab. 6 reports: i) $I_{l,f}$ parameter (obtained from Fig. 5 by assuming $N_{\text{MAX}} = 1$), ii) distance-based parameters, iii) 5G EMF parameters, iv) 5G performance parameters, and v) 5G costs parameters. We now focus on the setting of the key parameters, while we refer the reader to the references reported in Tab. 6 for more information about the setting of each single parameter. More in depth, we assume that the maximum power $O_{l,f}^{\text{MAX}}$ that is radiated by a gNB operating over $f_1$ is actually higher than the one radiated by a gNB operating over $f_2$. Although this setting may appear quite counter-intuitive at a first glance, since a micro gNB is expected to radiate less power than a macro gNB, we remind that $O_{l,f}^{\text{MAX}}$ refers to the maximum power, which can clearly differ w.r.t. the actual one that is received over the territory. When considering micro gNBs, $O_{l,f}^{\text{MAX}}$ is split across the radiating elements, and thus the actual power that is received over the territory is clearly lower compared to the maximum one. This effect is taken into account when setting the statistical scaling factor $R_{l,f}^{\text{STAT}}$. More in detail, we assume a strong statistical scaling factor that is applied to micro gNBs operating on $f_1$, while no statistical scaling factor is applied to macro gNBs operating on $f_2$. This choice is also motivated by the different goals of two gNB types, i.e., maximizing throughput for $f_1$ (and hence large spatial power variability) vs. ensuring coverage for $f_2$ (and hence less spatial power variability). Focusing then on the exclusion zones, we assume again two distinct values for $f_1$ and $f_2$, which are set in accordance to the minimum distance guaranteeing a EMF level below the limit of 6 [V/m] for a single gNB. Eventually, $P_{\text{BASE}}^{\text{Obs}}(p,f)$ is retrieved from real measurements over the real scenario, with the methodology described in Appendix A. Moreover, we assume that the minimum SIR is set in order to ensure a minimum pixel throughput of 30 [Mbps] for $f_1$. In addition, we do not constrain the SIR over $f_2$, since the goal of gNBs operating over this frequency is mainly to provide coverage. Therefore, even low throughput values may be admitted.
TABLE 7
Breakdown of the evaluation metrics.

| Metric | Notation/Expression | Reference Equations |
|--------|---------------------|---------------------|
| Total Installation Costs | $C_{TOT}^{\Sigma}$ | Eq. (57) (total costs computation). |
| Number of Installations | $N_{f1} = \sum_{l \in L} y_{l(f1)}$, $N_{f2} = \sum_{l \in L} y_{l(f2)}$ | - |
| Served Pixels | $X_{SERVED}^{f1} = \sum_{p \in P} \sum_{l \in L} x_{p(l,f1)}$, $X_{SERVED}^{f2} = \sum_{p \in P} \sum_{l \in L} x_{p(l,f2)}$ | - |
| Unserved pixels [%] | $X_{NOT-SERVED}^{\Sigma} = 100 \cdot \frac{\sum_{p \in P} \sum_{l \in L} z_{p(l,f)} = 1 - \left(1 - \sum_{l \in L} \sum_{p \in P} x_{p(l,f)} \right)}{\sum_{p \in P} \sum_{l \in L} x_{p(l,f)}}$ | - |
| Pixel Throughput | $T_{p} = \sum_{l \in L} \sum_{f \in F} (1 + S_{(p,l,f)}) \cdot x_{(p,l,f)}$ | Eq. (19) (SIR computation on left hand side), Eq. (20) (throughput computation). |
| Average Pixel Throughput | $T_{AVG} = \frac{\sum_{p \in P} T_{p}}{|P| \cdot \left(1 - \frac{x_{p,l,f}}{100}\right)}$ | See computation of $T_{p}$ and $X_{NOT-SERVED}^{\Sigma}$. |
| Pixel EMF | $E_{p} = \sqrt{\sum_{f \in F} \left( p_{BASE}(f) \cdot \left(1 - w_{p}\right) + p_{ADD-TERMS}(f) \right) \cdot Z_{0}}$ | Eq. (22) (total power density computation in the numerator on left-hand side), Eq. (58) (electric field computation). |
| Average Pixel EMF | $E_{AVG} = \sqrt{\sum_{p \in P} \sum_{l \in L} p_{BASE}(f) \cdot \left(1 - w_{p}\right) + p_{ADD-TERMS}(f) \cdot Z_{0}}$ | See computation of $E_{p}$. |

Although these throughput settings may appear relatively loose at first glance, we will show that the actual throughput levels experienced over the served pixels are not negligible and in line with the 5G service requirements. Finally, the $\alpha_{(l,f)}$ parameter, which acts as a weight for the number of served pixels in the objective function, is varied over a huge range, in order to test its sensitivity on the obtained planning.

Eventually, we provide the setting for the remaining parameters, namely $Z_{0}$ and $N^{\text{SER}}$. In particular, we set $Z_{0} = 377$ [Ohm] (in accordance to [16]). In addition, we impose $N^{\text{SER}} = 2$. In this way, we consider a conservative case in which each pixel is served by at most two gNBs.

7 RESULTS

We code PLATEA algorithm in MATLAB R2019b and we run it on a Dell PowerEdge R230 equipped with Intel Xeon E3-1230 v6 3.5 [GHz] processors and 64 [GB] of RAM. We then describe the following steps: i) introduction of two reference algorithms as terms of comparison, ii) definition of evaluation metrics, iii) tuning of PLATEA parameters, iv) comparison of PLATEA vs. the reference algorithms, v) impact of planning parameters, and vi) impact of pre-5G exposure levels.

Reference Algorithms. In order to introduce a term of comparison, we code two reference algorithms, named Evaluation Algorithm (EA) and Maximum Coverage Macro Algorithm (MCMA). We refer the reader to Appendix B for a detailed description of EA and MCMA, while here we briefly summarize their salient features. In brief, EA and MCMA evaluate the feasibility constraints of a random set of installed gNBs, without exploring the possible site permutations (which are instead analyzed by PLATEA). More in depth, EA explores a single possible deployment (which is generated from a fixed number of gNBs, passed as input to the algorithm). On the other hand, MCMA goes one step further, by selecting the set of macro gNBs operating on $f_2$ maximizing the service coverage, given an integer number of $f_1$ gNBs that have to be installed over the territory. We refer the reader to Appendix B for the pseudo-code of the two solutions, including also the evaluation of their computational complexity.

Evaluation Metrics. We then formally introduce the metrics to evaluate the performance of PLATEA, EA and MCMA. To this aim, Tab. 7 reports: i) total installation costs $C_{TOT}$, ii) number $N_{f1}$ ($N_{f2}$) of $f_1$ ($f_2$) gNBs installations, iii) number of pixels $X_{SERVED}$ ($X_{SERVED}^{\Sigma}$) served by $f_1$ ($f_2$) gNBs, iv) percentage of unserved pixels $X_{NOT-SERVED}$, v) pixel throughput $T_p$, vi) average pixel throughput $T_{AVG}$, computed over the pixels that are served by gNBs, vii) pixel EMF $E_p$, viii) average pixel EMF $E_{AVG}$, computed over the whole set of pixels in the scenario. For each metric, the table reports the metric name, the mathematical notation, and the reference equation(s) used to compute the metric.

Tuning of PLATEA Parameters. We initially concentrate on the impact of the $\alpha_{(l,f)}$ terms that are included in the objective function of PLATEA. As reported in Tab. 6, we consider a wide range of values for both $\alpha_{(l,f1)}$ and $\alpha_{(l,f2)}$. For the sake of simplicity, we impose the same weights applied for all the candidate gNBs working at the same frequency. In addition, we initially assume the exclusion of legacy pre-5G Base Stations that radiate over TMC, in order to evaluate the performance of PLATEA in a clean-slate condition. Therefore, we set $p_{BASE}(f) = 0 \forall f \in F, p \in P$. We then run PLATEA over the selected ranges of $\alpha_{(l,f1)}$ and $\alpha_{(l,f2)}$, by picking values on logarithmic scales. Fig. 7 highlights the obtained results in terms of: i) total installation costs $C_{TOT}$ (Fig. 7(a)), ii) number $N_{f1}$ of $f_1$ gNBs (Fig. 7(b)), iii) number $N_{f2}$ of $f_2$ gNBs (Fig. 7(c)), iv) percentage of not served pixels $X_{NOT-SERVED}$ (Fig. 7(d)), v) average pixel throughput $T_{AVG}$ (Fig. 7(e)) and vi) average electric field $E_{AVG}$ (Fig. 7(f)).

Several considerations hold by analyzing in detail Fig. 7. First, $C_{TOT}$ is proportional to $\alpha_{(l,f1)}$ and $\alpha_{(l,f2)}$ (Fig. 7(a)).

13. We remind that, in any case, pixels beyond the maximum distance coverage $D_{MAX}^l$ from a given gNB cannot be served by the gNB.
14. The evaluation of $N^{\text{SER}}$ greater than 2 is left for future work.
due to the fact that the weights play a major role in determining the objective function, e.g., cost minimization, service maximization or a mixture between them. Clearly, \( \alpha_{(l,f_1)} \) affects \( N_{f_1} \) (Fig. 7(b)), as shown in Fig. 7(c). In addition, \( X_{\text{NOT-SERVED}} \) is inversely proportional to \( \alpha_{(l,f_2)} \) (Fig. 7(d)). For example, when \( \alpha_{(l,f_2)} \approx 10 \), more than 10% of pixels are not served by any gNB. This is due to the fact that the number of gNBs that are installed passes from 3 to 1 (see Fig. 7(c)), thus creating coverage holes. On the other hand, the variation of \( \alpha_{(l,f_1)} \) has a clear impact on \( X_{\text{NOT-SERVED}} \) only when \( \alpha_{(l,f_2)} \approx 10 \), i.e., when \( f_2 \) gNBs are not able to cover the whole territory. Moreover, Fig. 7(e) reveals that \( T_{\text{AVG}} \) has a complex trend, which results from the combination of: i) the coverage provided by \( f_1 \) and \( f_2 \) gNBs installed over the territory, ii) the amount of interference, which tends to be impacted by the number of neighboring gNBs operating at the same frequency, and iii) the percentage of served pixels, since \( T_{\text{AVG}} \) is computed over the pixels that receive service coverage from at least one gNB. As a consequence, \( T_{\text{AVG}} \) is not always proportional or inversely proportional with \( \alpha_{(l,f_1)} \). For example, the maximum value of \( T_{\text{AVG}} \) is achieved when \( \alpha_{(l,f_2)} = 10 \), which however leads to a huge number of unserved pixels (i.e., more than 10%). Finally, \( E_{\text{AVG}} \) is proportional to \( \alpha_{(l,f_2)} \), due to the variation in the number of radiating sources that contribute to the EMF exposure. However, we point out that the average EMF level is almost one order of magnitude lower than the 6 [V/m] restrictive limit.

Based on the above considerations, we select \( \alpha_{(l,f_1)} = 50 \) [€] and \( \alpha_{(l,f_2)} = 500 \) [€] henceforth. In this way, we balance between: i) increasing \( T_{\text{AVG}} \), ii) reducing \( C_{\text{TOT}} \), iii) minimizing \( X_{\text{NOT-SERVED}} \), iv) reducing \( E_{\text{AVG}} \). To give more insights, Fig. 8 shows a run of the planning selected by PLATEA with the aforementioned setting. Interestingly, only a subset of the candidate gNBs, i.e., 11 \( f_1 \) gNBs and 3 \( f_2 \) gNBs, is deployed over the TMC scenario (Fig. 8(a)). On the other hand, the resulting EMF levels are always pretty low (see Fig. 8(b)), with an electric field close to the 6 [V/m] limit only in proximity to the \( f_1 \) gNBs.

**Algorithms Comparison.** In the following, we compare the performance of PLATEA against EA and MCMA. Unless otherwise specified, we compute each metric by averaging the results over 10 independent runs. Focusing on the number of \( f_1 \) and \( f_2 \) gNBs selected by PLATEA, we have found that our solution requires on average \( N_{f_1} = 10.7 \) and \( N_{f_2} = 3 \), respectively. Consequently, we have passed to EA and MCMA a number of \( f_1 \) gNBs equal to 11. In addition, EA requires the number of \( f_2 \) gNB, which is set to 3. Tab. 8 reports the performance of the algorithms over the different metrics. More in detail, the total installation
The EMF exposure is very low (i.e., lower than 0.4 [V/m]), as shown at the bottom left corner of Fig. 9(a). In this region, PLATEA installs more than 10 f1 gNBs (Fig. 9(b)) and 3 f2 gNBs (Fig. 9(c)). In addition, a large throughput is achieved (Fig. 9(d)) and (almost) all the pixels are served Fig. 9(e).

Impact of planning parameters. We then focus on the impact of the planning parameters, namely: i) the scaling parameters $R_{\text{TIME}}^{(l,f)}$ and $R_{\text{STAT}}^{(l,f)}$, which affect the EIRP and consequently the EMF exposure generated by gNBs and ii) the minimum distance from sensitive places $D_{\text{MIN}}$ which influences the subset of sites that can host gNBs. Focusing on i) we perform a sensitivity analysis by running PLATEA over a wide range of $R_{\text{TIME}}^{(l,f)}$ and $R_{\text{STAT}}^{(l,f)}$ values. For the sake of simplicity, we impose $R_{\text{TIME}}^{(l,f)} \in [0.1-0.6]$ and $R_{\text{STAT}}^{(l,f)} \in [0.1-0.6]$ and $R_{\text{TIME}}^{(l,f)} = R_{\text{STAT}}^{(l,f)} = 1$ for all $l \in L$. In this way, f1 gNBs are subject to temporal and statistical scaling factors, while f2 gNBs are affected only by temporal scaling factors.

Fig. 9 reports the obtained results in terms of: i) average EMF $E_{\text{AVG}}$, ii) number $N_{f1}$ of f1 gNBs, iii) number $N_{f2}$ of f2 gNBs, iv) average throughput $T_{\text{AVG}}$, and v) percentage of not served pixels $X_{\text{NOT-SERVED}}$.

On the other hand, when $R_{\text{TIME}}^{(l,f)}$ and $R_{\text{STAT}}^{(l,f)}$ are increased, the average EMF tends to increase and therefore it is challenging to ensure the strict EMF constraints in the proximity of the installed gNBs. Therefore, the number of installed gNBs is reduced, the throughput is decreased and the percentage of not served pixels abruptly increases. Eventually, for large values of $R_{\text{TIME}}^{(l,f)}$ and $R_{\text{STAT}}^{(l,f)}$ (top right corner of subfigures), it is not possible to install any gNB and therefore all the pixels are unserved. In addition, we can note that a frontier region emerges for intermediate values of the scaling parameters. Interestingly, for most of $R_{\text{TIME}}^{(l,f)}$ and $R_{\text{STAT}}^{(l,f)}$ combinations laying on the frontier, a huge amount of pixels is served by f2 gNBs (Fig. 9(f)).

The obtained results demonstrate that the setting of the scaling parameters will play a major role in the planning of 5G networks, especially for countries adopting strict EMF limits. As a side comment, we believe that the methodology to estimate the values of $R_{\text{TIME}}^{(l,f)}$ and $R_{\text{STAT}}^{(l,f)}$ should be integrated in the national EMF regulations. Clearly, the exact settings of the scaling parameters depend on the considered scenario.

We then evaluate the impact of varying $D_{\text{MIN}}$, which we remind is an additional restriction imposed by the munici-

| Metric | EA | MCMA | PLATEA |
|-------|----|------|--------|
| $C_{\text{TOT}}$ | 391.4 | 588.7 | 386.1 |
| $N_{f1}$ | 11 | 11 | 10.7 |
| $N_{f2}$ | 3 | 6 | 3 |
| $X_{\text{SERVED}}^{(f1)}$ | 7783 | 7784 | 9156 |
| $X_{\text{SERVED}}^{(f2)}$ | 14078 | 16534 | 15148 |
| $X_{\text{NOT-SERVED}}^{(f1)}$ | 10.11 | 0 | 0.06 |
| $T_{\text{AVG}}$ [Mbps] | 241.1 | 257.1 | 353.9 |
| $E_{\text{AVG}}$ [V/m] | 0.57 | 0.64 | 0.57 |
Impact of pre-5G exposure levels. In the last part of our work, we study the impact of adding the pre-5G exposure term on the 5G planning. As detailed in Appendix A, the actual electric field strength in TMC hardly exceeds 1 [V/m], even for the pixels that are at the shortest distance and Line-of-Sight (LOS) conditions w.r.t. the serving gNB. On the other hand, the electric field rapidly decreases to negligible values (below 1 [V/m]) as the distance between the pixel and the radiating gNBs increases. However, in order to introduce a set of conservative (and worst case) scenarios, we assume: i) three different settings of pre-5G exposure, namely 1 [V/m], 1.5 [V/m] and 2 [V/m], and ii) a uniform term of pre-5G exposure for all the pixels in the TMC scenario. As a consequence, the cumulative pre-5G power density is set as \( \sum_{f \in F} I_{(p,f)} = \{0.00265, 0.00597, 0.0106\} \) [W/m²], \( \forall p \in P \), respectively. In addition, since the same restrictive limit is applied for all the pixels of TMC, Eq. (32) is rewritten as:

\[
(1-w_{p}) \cdot \sum_{f \in F} P_{BASE}^{(p,f)} + \sum_{f \in F} P_{ADD-TS}^{(p,f)} \leq L_{RES}, \quad \forall p \in P_{RES}
\]

where \( L_{RES} = 0.1 \) [W/m²] (in accordance to R6 of Tab. 3). Intuitively, the introduction of the pre-5G exposure term may limit the amount of 5G gNBs that are installed over the territory, since it is more challenging to ensure Eq. (43) compared to the case in which the pre-5G technologies are dismissed.

Tab. 5 reports the performance metrics of PLATEA (averaged over 10 runs) vs. the different values of pre-5G exposure. When the pre-5G exposure is increased, we can note: i) a reduction in the number of 5G gNBs, and consequently of total costs, ii) an increase in the number of pixels served by 5G gNBs, iii) a slight throughput decrease, and iv) an EMF increase, mainly due to the pre-5G exposure term. Overall, these results prove that the performance metrics are impacted by the level of background exposure. However, PLATEA is always able to retrieve a feasible planning, with a percentage of unserved pixels at most equal to 0.16%.

8 Conclusions and Future Works

We have focused on the problem of planning a 5G network under service and EMF constraints. To this aim, we have targeted an objective function that balances between gNB installation costs and 5G service coverage level. After providing the OPTPLAN-5G MILP formulation, we have demonstrated that the considered problem is NP-Hard, and therefore very challenging to be solved even for small problem instances. To face this issue, we have designed the PLATEA algorithm, which is able to select a 5G planning by iterating over the set of candidate gNBs. In addition, PLATEA exploits the linear constraints that have been defined for OPTPLAN-5G. We have then considered the TMC scenario, which is subject to very strict EMF regulations that include minimum distances from sensitive places and very stringent EMF limits.

Results, obtained by running PLATEA over the considered scenario, prove that our solution outperforms EA and MCMA. In addition, we have demonstrated that the 5G planning is overall feasible, i.e., it is possible to serve a huge amount of pixels while limiting the installation costs and while ensuring the EMF constraints outside the exclusion zones of the installed gNBs. However, our work points out an important aspect: the scaling parameters that are used to estimate the exposure level from 5G gNBs play a fundamental role in determining the problem feasibility and consequently the set of installed gNBs. Eventually, when pre-5G exposure is considered, PLATEA is still able to retrieve an admissible planning, with a moderate impact on the pixel throughput.

We believe that this work could be the first step towards a more comprehensive approach. First of all, the integration of gNBs operating on mm-Waves may be an interesting future work. In addition, the optimization of the scaling parameters may be another research avenue. This step could include e.g., the optimal setting of the scaling parameters based on the chosen location and the selected frequency, as well as the evaluation of the chosen set of values during the management phase (e.g., when the 5G network is under operation). Eventually, we plan to integrate detailed propagation models (e.g., including indoor evaluation) and more complex EMF models in the planning phase.

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APPENDIX A
EMF MEASUREMENTS IN THE TMC SCENARIO

We describe here the methodology adopted to perform the EMF measurements in the TMC scenario, as well as the main outcomes from the measurements. Actually, the TMC neighborhood does not host any installation of legacy pre-5G Base Stations, mainly due to the following reasons: i) TMC is a relatively new neighborhood, which was build during the last decade and ii) all the requests done by operators to install Base Stations in the neighborhood have been denied by the municipality, since the selected locations did not ensure the minimum distance from the sensitive places. As a result, the cellular service over TMC is provided by a set of Base Stations installed in other neighborhoods. We refer the interested reader to Fig. 8 of [1] for the maps reporting the localization of pre-5G Base Stations serving TMC. In brief, these Base Stations include pole mounted and roof mounted installations (with some examples reported in Fig. 10), mainly close to the north-east and east borders of the neighborhood. Therefore, rather than measuring the EMF levels in each pixel of TMC neighborhood (which would require a huge amount of time), we concentrate on the TMC zones in close proximity to the Base Stations installed in the other neighborhoods, since these zones are expected to receive the highest EMF exposure levels. To this aim, Fig. 11 reports the considered measurement locations, each of them labelled with a unique ID. In addition, we place the EMF meter in outdoor locations and in general in zones not covered by obstacles. To this aim, Fig. 12 reports three examples of measurement locations, by differentiating between: roadside positioning (Fig 12(a)), countryside positioning (Fig. 12(b)), and positioning in the main square (Fig. 12(c)).

In the following, we measure the electric field in each measurement point, by adopting a professional EMF meter, whose settings are detailed in Tab. 10. More in depth, the meter provides the total electric field over the set of frequencies used by cellular operators. In addition, all the measurements have been performed during morning/afternoon hours of business days. In this way, the measured electric field is retrieved under moderate/high utilization of the cellular network. In addition, we consider the value of 6 [min] as the reference time interval to compute the average electric field (in accordance to regulation $R1$ of Tab. 3). Although Italian regulations integrate longer time-intervals to compute the average electric field (see e.g., $R3$-$R4$ of Tab. 3), we believe that the considered scenario is rather conservative, since the measurements are performed during moderate/high utilization of the cellular network, and hence during time intervals during which the electric field is higher compared to low traffic conditions (e.g., at night, during holidays, during week-ends).

Fig. 13 reports the average EMF levels over the measurement locations. Several consideration hold by analyzing the figure. First, the average electric field is always pretty low,
In addition, the information about the obtained planning is returned in the $y$, $x$, $P_{ADD-TS}$, $P_{ADD-NOTS}$, $w$, $C_{TOT}$ variables. Initially (line 1), EA generates a random deployment with $\text{num}_{-}f_1$ $f_1$ gNBs and $\text{num}_{-}f_2$ $f_2$ gNBs. In the following, EA initializes a set of internal variables (line 2). Then, EA performs in line 3 the feasibility checks of: $i$) power density limits outside exclusion zones, $ii$) minimum distance from sensitive places, $iii$) maximum number of gNBs installed in each site and $iv$) site installation constraints. If all these constraints are ensured (line 4), EA performs the pixel-gNB association (by first iterating over the $f_1$ gNBs and then over the $f_2$ gNBs) and then the solution is saved (lines 5-6).

Computational Complexity. The `GENERATE_SITES` function has a complexity of $O(|\mathcal{L}|)$. The complexity of `INITIALIZE`, `INSTALL_CHECK`, `ASSOCIATE_PIXELS` and `SAVE_SOL` is reported in Tab. 5. Therefore, the total computational complexity of EA is in the order of $O(|\mathcal{P}| \times |\mathcal{L}|^{2} \times \mathcal{F})$.

B.2 MCMA Description

Alg. 4 reports the pseudo-code of MCMA algorithm. The algorithm requires as input the number of $f_1$ gNBs to be installed. MCMA then produces as output a flag, indicating if a feasible solution has been found, and the problem variables $y$, $x$, $P_{ADD-TS}$, $P_{ADD-NOTS}$, $w$, $C_{TOT}$. The main intuition behind MCMA is to sequentially iterate over the number of $f_2$ gNBs to be installed, i.e. from 1 up to $\text{num}_{-}f_2_{-}max$ (lines 5-21). In particular, MCMA generates the set of candidate gNBs from $\text{num}_{-}f_1$ and $\text{num}_{-}f_2$ (line 6), initializes the variables (line 7), runs the `INSTALL_CHECK` function (line 8) and eventually associates the pixels to the installed gNBs (line 10). The iteration stops if all the pixels have been served or the maximum number of $f_2$ gNBs have been evaluated (line 13). Under this condition, the current solution is eventually saved and the algorithm is ended (lines 14-17). Otherwise, the current number of $f_2$ gNB is increased (line 18) and lines 5-21 are evaluated again.

Computational complexity. The functions employed by MCMA are the ones also used by EA (detailed in Sec. B.1) and PLATEA (detailed in Tab. 5). In addition, the while cycle in line 5 has a complexity of $O(|\mathcal{L}|)$. Therefore, the total complexity of MCMA is in the order of $O(|\mathcal{P}| \times |\mathcal{L}|^{2} \times \mathcal{F})$. 

**APPENDIX B**

**REFERENCE ALGORITHMS**

We describe here in more detail the EA and MCMA algorithms.

B.1 EA Description

Alg. 3 reports the EA pseudo-code. This algorithm takes as input the number of deployed gNBs over the two frequencies, which are stored in the $\text{num}_{-}f_1$ and $\text{num}_{-}f_2$ parameters. EA then returns as output a warning flag, which is set to false if the installation have been unsuccessful (due to the fact the constraints are not ensured), true otherwise.

**TABLE 10**

| Setting/Feature | Value |
|-----------------|-------|
| Measured Frequencies | 200-900 [MHz], 1800-1900 [MHz], 2100 [MHz] |
| Measurable EMF Range | 0.04-65 [V/m] |
| Average Interval | 6 [min] |
| Height from ground | 1.5 [m] |

**Fig. 13.** Average EMF of each measurement in the TMC scenario.

i.e., always lower than 0.9 [V/m]. Second, the electric field generally varies across the locations. Third, very low levels of electric field are measured over locations not in proximity to the TMC border (e.g., locations with ID 24, 25, 26, 10 of Fig. 11). This fact further corroborates our intuition that the electric field from pre-5G Base Stations is almost negligible for most of the pixels in the TMC neighborhood.

In the following part of this step, we analyze in more detail the EMF measurements. To this aim, Fig. 14 reports the electric field vs. the measurement ID. Bar report average electric field values, while error ranges denote the confidence intervals (which are computed by assuming a 95% of confidence levels). Interestingly, we can note that the confidence intervals are reduced when the measured electric field level decreases.

Finally, we remind that all the EMF measurements based on electric field are then converted to power density by applying Eq. 8 with $Z_0=377$ [Ω], in accordance to [16].

**Fig. 14.** Measured electric field vs. measurement ID.
Algorithm 3 Pseudo-Code of the EVALUATION ALGORITHM (EA)

Input: num_f1 of deployed gNBs with frequency f1, num_f2 of deployed gNBs with frequency f2
Output: flag_check flag with installation status (true = installation successful, false = installation unsuccessful), variables y, x, pADD-STS, pADD-NOTS, w, CTOT

1: cand_sites=GENERATE_SITES(num_f1,num_f2); // Random generation of a candidate deployment
2: [x_temp y_temp pd_temp]=INITIALIZE(cand_sites);
3: [flag_check, pd_temp]=INSTALL_CHECK(cand_sites, y_temp, pd_temp); // Based on Eq. (24), (25), (27)-(36)
4: if flag_check==true then
   5:      [x_temp]=ASSOCIATE_PIXELS(y_temp, x_temp); // Based on Eq. (12), (13), (20)-(23)
   6:      [y, x, pADD-STS, pADD-NOTS, w, CTOT]=SAVE_SOL(y_temp, x_temp, pd_temp); // Solution saving
5: end if

Algorithm 4 Pseudo-Code of the MAXIMUM COVERAGE MACRO ALGORITHM (MCMA)

Input: num_f1 of deployed gNBs with frequency f1
Output: flag_sol flag with installation status (true = feasible solution found, false = no feasible solution found), variables y, x, pADD-STS, pADD-NOTS, w, CTOT

1: num_f2_max=\sum_{l \in L} I(f2,l);
2: flag_end=false;
3: flag_sol=false;
4: num_f2=1;
5: while flag_end==false do
6:      cand_sites=GENERATE_SITES(num_f1,num_f2); // Random generation of a candidate deployment
7:      [x_curr y_curr pd_curr]=INITIALIZE(cand_sites);
8:      [flag_check, pd_temp]=INSTALL_CHECK(cand_sites, y_curr, pd_curr); // Based on Eq. (24), (25), (27)-(36)
9:      if flag_check==true then
   10:         [x_curr]=ASSOCIATE_PIXELS(y_curr, x_curr); // Based on Eq. (12), (13), (20)-(23)
   11:         flag_sol=true;
12:     end if
13:     if (ALL_SERVED(x_curr)==true) or (num_f2==num_f2_max) then
14:         if flag_check==true then
15:            [y, x, pADD-STS, pADD-NOTS, w, CTOT]=SAVE_SOL(y_curr, x_curr, pd_curr); // best Solution Saving
16:         end if
17:     end if
18:     else
19:        num_f2++;
20:     end if
21: end while

APPENDIX C
ADDITIONAL RESULTS

In this section, we provide a set of additional results, in order to better position the PLATEA algorithm and the results presented in Sec. 7.

C.1 Throughput Comparison

Tab. 8 in Sec. 7 highlights that PLATEA achieves a better average throughput $T^{AVG}$ compared to EA and MCMA. However, no indication about the throughput of the single pixels (which we remind is denoted with $T_p$) is provided. Therefore, a natural question is: which is the performance of PLATEA when considering $T_p$ and not $T^{AVG}$? To answer this question, Fig. 15 reports the Cumulative Distribution Function (CDF) of the throughput $T_p$ obtained with PLATEA. In addition, the figure shows the CDF obtained by running EA. Three considerations hold by analyzing Fig. 15. First, PLATEA is able to guarantee more than 500 [Mbps] of throughput for more than 20% of pixels. Second, the percentage of pixels receiving very low throughput is overall pretty limited (less than 10%). Third, EA performs consistently worse, being its CDF clearly moved on the left w.r.t. PLATEA.

C.2 Number of $f_2$ gNBs selected by MCMA

According to Tab. 8 in Sec. 7, MCMA requires a consistently higher amount of $f_2$ gNBs compared to PLATEA. However, a natural question is: Does MCMA always install more $f_2$ gNBs w.r.t. PLATEA? To answer this question, we run MCMA by varying num_f1 between 1 and 24 (which corresponds to the maximum number of gNB installed by...
PLATEA for very large values of $\alpha_{(l,f_1)}$. For each value of $\text{num}_{-}f_1$, we then perform 10 executions of MCMA, and then we collect the average number of installed $f_2$ gNBs over the 10 runs. Fig. 16 reports the variation of the average number of installed $f_2$ gNBs vs. $\text{num}_{-}f_1$. For completeness, the figure reports also the number of $f_2$ gNBs that are installed by PLATEA. Interestingly, we can note that the average number of installed $f_2$ gNBs is always higher than the one selected by PLATEA. This difference may be explained in the different planning policies adopted by the two solutions. MCMA, in fact, sequentially evaluates an increasing number of $f_2$ gNB, given the number of $f_1$ gNBs that is passed as input parameter. On the other hand, PLATEA operates a wiser choice, by: i) evaluating a number of permutations that increases with the number of gNBs that need to be installed, and ii) jointly varying the number of $f_1$ gNB and $f_2$ gNB when evaluating the problem constraints and the objective function.

C.3 Impact of minimum distance constraint

The set of regulations taken under consideration in this work (namely R6 of Tab. 3) includes a minimum distance $D_{\text{MIN}}$ between each installed gNBs and each sensitive place. A natural question is then: What is the impact of $D_{\text{MIN}}$ variation on the planning? To answer this question, we have assumed that $D_{\text{MIN}}$ can take different values w.r.t. the ones reported in the regulations. In particular, we have considered the following range of values for $D_{\text{MIN}} = \{25, 50, 75, 100, 125, 150\}$. We have then run PLATEA algorithm for each $D_{\text{MIN}}$ value, and we have collected the performance metrics. Fig. 17 reports the obtained results in terms of: i) average electric field $E_{\text{AVG}}$, ii) average throughput $T_{\text{AVG}}$, iii) number $N_{f_1}$ of $f_1$ gNBs and iv) percentage of not served pixels $X_{\text{NOT-SERVED}}$. We remind that $D_{\text{MIN}} = 100$ [m] is the value currently enforced in the Rome regulations. Interestingly, when $D_{\text{MIN}} < 100$ [m], $E_{\text{AVG}}$, $T_{\text{AVG}}$ and $N_{f_1}$ tend to be increased, due to the fact that it is possible to install more gNBs over the territory while ensuring the minimum distance constraint. On the other hand, the opposite holds $D_{\text{MIN}} > 100$ [m]. In particular, when $D_{\text{MIN}} = 150$ [m], an abrupt decrease of $E_{\text{AVG}}$, $T_{\text{AVG}}$ and $N_{f_1}$ is observed. These results are a direct consequence of the $D_{\text{MIN}}$, which prevents the installation of gNBs in many TMC locations.