Comparison Study of Iterative Algorithms for Center-of-Sets Type-Reduction of Takagi Sugeno Kang Type General Type-2 Fuzzy Logic Systems

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This work was supported in part by the National Natural Science Foundation of China under Grant 61973146 and Grant 61773188, in part by the Doctoral Start-up Foundation of Liaoning Province under Grant 2021-BS-258, and in part by the Youth Fund of Education Department of Liaoning Province under Grant LJKQZ2021143.

ABSTRACT Studying the block of center-of-sets (COS) type-reduction (TR) for Takagi Sugeno Kang (TSK) inference-based general type-2 fuzzy logic systems (GT2 FLSs) is meaningful for applying the systems. Blocks of fuzzy reasoning, COS type-reduction and defuzzification for TSK type GT2 FLSs are first given. According to three kinds of iterative algorithms for computing the centroids of interval type-2 fuzzy sets (IT2 FSs), the paper extends these types of algorithms for studying the COS type-reduction of TSK type GT2 FLSs. Six computer simulation experiments show the computational costs of proposed three kinds of iterative algorithms by computing the outputs of GT2 FLSs, which affords the potential guidance value designers and users of T2 FLSs.

INDEX TERMS Iterative algorithms, Takagi Sugeno Kang inference, general type-2 fuzzy logic systems, computational cost, simulation.

I. INTRODUCTION
In contrast to the traditional type-1 fuzzy sets (T1 FSs), interval or general type-2 fuzzy sets (IT2 or GT2 FSs) can better model and cope with uncertainties by adjusting the additional parameters of footprint of uncertainty (FOU [1]). Therefore, T2 fuzzy logic systems (FLSs) based on T2 FSs [2], [3], [4], [5], [6], [7], [8] have become an emerging technology which has been successfully applied in many fields affected by high uncertainties, time-varying and nonlinearities. As shown in Fig. 1, the type-reduction (TR) plays a key role for T2 FLSs, which mainly maps the T2 FS to the T1 FS. Then the defuzzification transforms the T1 FS to the crisp output. While a T1 FLS does not have the TR block. Therefore, the computations in the former are much complicated than the latter, which makes the calculations in T2 FLSs more challenged.

As the secondary membership grades of IT2 FSs are all equal to one, the computational relatively simple IT2 FLSs [4], [9], [10], [11], [12] are currently most widely used T2 FLSs. However, since the alpha-planes or z-Slices representation [3], [8], [13], [14], [15] of GT2 FSs were proposed by a few different research groups, the computational
complexity of GT2 FLSs were greatly reduced. That is due to the fact that a GT2 FS can be decomposed into IT2 FSs based on alpha-planes or z-_slices. Then the design and applications of GT2 FLSs [7], [16], [17], [18], [19], [20], [21], [22], [23] based on GT2 FSs have gained rapidly development in the past decades.

In early days, the Karnik-Mendel (KM) algorithms [24], [25], [26] were put forward to compute the centroids of IT2 FSs. However, this type of computationally intensive algorithm is time consuming. Wu proposed the enhanced KM (EKM) algorithms [27] to save the calculation time. Compared with the KM algorithms, the EKM algorithms can save about two iterations on average for computing the centroids. Another well-known type of non-KM iterative algorithm was also proposed, which was called as the enhanced iterative algorithms with stopping condition (EIASC [28]). Professor Jerry Mendel [3], [25] proposed that these three types of centroid type-reduction (TR) algorithms were good approaches for calculating the centroids. Despite so, the centroid TR methods focus on pure theoretical research. While the center-of-sets (COS) TR doesn’t require that a complete description of a general type-2 fuzzy set be available as the centroid TR. Actually, studying the COS TR [29], [30], [31] is more helpful for applying T2 FLSs [32], [33], [34].

We arrange the rest of the paper as below. Section two briefly gives the background of Takagi Sugeno Kang (TSK) type GT2 FLSs. Three kinds of iterative algorithms for performing the COS TR of TSK type GT2 FLSs are provided in Section three. Section four shows six computer simulation examples to illustrate the performances of three types of iterative algorithms. Finally the conclusions are given in Section five.

II. TAKAGI SUGENO KANG TYPE GT2 FLSs

GT2 FLSs can usually be divided into Mamdani type and TSK type from the viewpoint of inference (fuzzy reasoning). Here we focus on the TSK type. Take into account a TSK type from the viewpoint of inference (fuzzy reasoning). GT2 FLSs can usually be divided into Mamdani type and extended to complete the COS type-reduction of TSK inference structure-based GT2 FLSs.

A. KM ALGORITHMS

For the TSK type GT2 FLSs [3], [6], [19], let the COS type-reduced set under the related alpha-level be an interval, i.e.,
Y. Chen: Comparison Study of Iterative Algorithms for COS TR of Takagi Sugeno Kang Type General Type-2 Fuzzy Logic Systems

| TABLE 1. Calculational steps for KM algorithms to perform the COS type-reduction of GT2 FLSs. |
|---|
| Step | KM algorithms to calculate $y_{i,a}$ |
| 1 | 1. Initialize $\theta_{a,n}$ set $\theta_{a,n} = (\alpha_n^L + \alpha_n^R)/2$, $n = 1, L, N$.  
   compute $c' = \frac{\sum_n^{N} \beta_n^a \theta_{a,n}}{\sum_n^{N} \beta_n^a}$  |
| 2 | 2. Find $L(1 \leq L \leq N - 1)$, which satisfies $\alpha_n^L \leq c' \leq \beta_n^a$  |
| 3 | 3. When $n \leq L$, set $\theta_{a,n} = \alpha_n^L$, when $n \geq L + 1$, set $\theta_{a,n} = \alpha_n^R$.  
   compute $y_{i,a}(n) = \frac{\sum_n^{L} \beta_n^a \alpha_n^L + \sum_n^{R} \beta_n^a \alpha_n^R}{\sum_n^{L} \alpha_n^L + \sum_n^{R} \alpha_n^R}$  |
| 4 | 4. Check if $y_{i,a}(n) = c'$, if yes, stop and set $y_{i,a}(n) = y_{i,a}$, $n = L$  |
| 5 | 5. Set $c' = y_{i,a}(n)$ and return to step 2 |

$Y_{TSK,a} = [y_{i,a}, y_{r,a}]$. Then the two end points $y_{i,a}$ and $y_{r,a}$ can be calculated in a non-closed form as:

$$y_{i,a}(L) = \frac{\sum_{n=1}^{L} \beta_n^a \alpha_n^L + \sum_{n=L+1}^{N} \beta_n^a \alpha_n^L}{\sum_{n=1}^{L} \alpha_n^L + \sum_{n=L+1}^{N} \alpha_n^L} \approx y_{i,a}$$  \hspace{1cm} (7)

and

$$y_{r,a}(R) = \frac{\sum_{n=1}^{R} \beta_n^a \alpha_n^R + \sum_{n=R+1}^{N} \beta_n^a \alpha_n^R}{\sum_{n=1}^{R} \alpha_n^R + \sum_{n=R+1}^{N} \alpha_n^R} \approx y_{r,a}$$  \hspace{1cm} (8)

where $L$ and $R$ are the left and right switching points.

Table 1 provides the specific calculational steps for KM algorithms to perform the COS type-reduction of GT2 FLSs. As for the equation (7), when $n = L + 1$, $\theta_{a,n}$ starts to change from the upper firing degree $\alpha_n^L$ to the lower firing degree $\alpha_n^R$; for the equation (8), when $n = R + 1$, $\theta_{a,n}$ starts to change from the lower firing degree $\alpha_n^R$ to the upper firing degree $\alpha_n^L$.

Then the COS defuzzified value at the related alpha-level can be computed as:

$$Y_{KM,a} = (y_{i,a} + y_{r,a})/2.$$  \hspace{1cm} (9)

Aggregating all the $Y_{KM,a}$ to obtain the final type-reduced set $Y_{KM}$, i.e.,

$$Y_{KM} = \sup_{\forall \alpha \in [0,1]} \alpha / Y_{KM,a}.$$  \hspace{1cm} (10)

Finally the output of GT2 FLSs is as:

$$Y_{KM} = \frac{\sum_{i=1}^{k} \alpha_i [(y_{i,a}(x) + y_{r,a}(x))]/2}{\sum_{i=1}^{k} \alpha_i}.$$  \hspace{1cm} (11)

B. EKM ALGORITHMS

In fact, the EKM algorithms [25], [27], [32], [33] generate from the KM algorithms. Despite so, the EKM algorithms improve the KM algorithms in three points as:

1) a better initialization approach is provided for saving the iterations; 2) according to the stopping condition, the unnecessary iteration is cancelled; 3) in terms of a smart calculation technique, the computational cost is reduced.

Table 2 provides the specific calculational steps for EKM algorithms to perform the COS type-reduction of GT2 FLSs.

Then the COS defuzzified value for EKM algorithms at the related alpha-level can be computed as:

$$Y_{EKM,a} = (y_{i,a} + y_{r,a})/2.$$  \hspace{1cm} (12)

Aggregating all the $Y_{EKM,a}$ to get the final type-reduced set $Y_{EKM}$, i.e.,

$$Y_{EKM} = \sup_{\forall \alpha \in [0,1]} \alpha / Y_{EKM,a}.$$  \hspace{1cm} (13)

Finally the output of GT2 FLSs for EKM algorithms is as:

$$Y_{EKM} = \frac{\sum_{i=1}^{k} \alpha_i [(y_{i,a}(x) + y_{r,a}(x))]/2}{\sum_{i=1}^{k} \alpha_i}.$$  \hspace{1cm} (14)

C. EIASC

The non-KM type of EIASC is a type of iterative algorithm which originates from the monotoncity and shapes of IT2 FSs is comparatively easy to understand. For the equation (7), here we let $y_{i,a}(L)$ be first monotone decreasing as $L$ increases, then it should be monotone increasing as $L$ increases. And for the equation (8), here we let $y_{r,a}(R)$ be first monotone increasing as $R$ increases, then it should be monotone decreasing as $R$ increases. Table 3 provides the specific calculational steps for EIASC to perform the COS type-reduction of GT2 FLSs.
TABLE 2. Calculational steps for EKM algorithms to perform the COS type-reduction of GT2 FLSs.

| Step | EKM algorithms to compute $Y_{i,a}$ |
|------|-----------------------------------|
| 1.   | Set $s = \lceil N / 2.4 \rceil$ (the closest integer to $N / 2.4$) and calculate $a = \sum_{i=1}^{L} \sum_{a_i} \mu_i a_i$, $b = \sum_{i=1}^{L} \sum_{a_i} \alpha_i a_i$. $c' = a / b$ |
| 2.   | Find $x' \in [1, N - 1]$, which satisfies $\beta_x' \leq c' \leq \beta_x$ |
| 3.   | Check if $x' = s$, if so, stop and set $Y_{i,a} = 1$. otherwise, go to step 4 |
| 4.   | Compute $1 = \text{sign}(s' - s)$, $a' = a + 1 \sum_{x=\text{sign}(s' - s)}^{N-1} \beta_x a_x$, $b' = b + 1 \sum_{x=\text{sign}(s' - s)}^{N-1} \beta_x a_x$. $c'(s') = a' / b'$ |
| 5.   | Set $c'(s), a = a'$, and $b = b'$ and return to step 3 |

Then the COS defuzzified value for EIASC at the related alpha-level can be computed as:

$$Y_{EIASC,\alpha} = (Y_{1,\alpha} + Y_{r,\alpha}) / 2.$$  (15)

Aggregating all $Y_{EIASC,\alpha}$ to obtain the final type-reduced set $Y_{EIASC}$, i.e.,

$$Y_{EIASC} = \sup_{\forall \alpha \in [0,1]} \alpha / Y_{EIASC,\alpha}.$$  (16)

Finally the output of GT2 FLSs can be as:

$$Y_{EIASC} = \frac{\sum_{i=1}^{k} \alpha_i [(Y_{1,\alpha_i}(x) + Y_{r,\alpha_i}(x)]) / 2}{\sum_{i=1}^{k} \alpha_i}.$$  (17)

IV. SIMULATION EXPERIMENTS

Here six simulation instances are used to show how to adopt the three kinds of iterative algorithms to perform the COS type-reduction and defuzzification of TSK inference based GT2 FLSs. In the simulations, the value of $\alpha$ is equally divided into $\Delta$ effective values as: $\alpha = 0, 1/\Delta, \cdots, 1$. In addition, let the $\Delta$ be changed from 1 to the maximum number 100 with the stepsize of 1. Here the COS type-reduced set as $\Delta = 100$ and the COS defuzzified values as $\Delta = 1 : 1 : 100$ computed by three types of iterative algorithms are investigated.

In the simulations example 1 and example 2, let each fuzzy rule be described by 4 antecedents and 1 consequent. Furthermore, suppose that 2 GT2 FSs are used for characterizing each antecedent. So that, there are totally $2^4$, i.e., six, fuzzy rules for the TSK inference based GT2 FLSs. For the $n$th fuzzy rule, whose form can be as:

If $x_1$ is $F_{1}^{n}$ and ... and $x_4$ is $F_{4}^{n}$,

then $Y^n = C_{0}^{n} + \sum_{i=1}^{4} C_{i}^{n} x_i$  (18)

where $F_{l}^{n}(l = 1, \cdots, 4; n = 1, \cdots, 16)$ denotes the antecedent GT2 FS, $C_{i}^{n}(i = 0, \cdots, 4; n = 1, \cdots, 16)$ represents the consequent, and $C_{i}^{n} = [c_{i}^{n} - s_{i}^{n}, c_{i}^{n} + s_{i}^{n}]$.

Example 1: As for the TSK inference based GT2 FLSs, the primary MF of antecedent GT2 FS is chosen as the Gaussian type MF, i.e., see the Figure 2,

$$\mu_{l}^{n}(x_l) = \exp[- \frac{1}{2} \frac{(x_l - m_{l}^{n})^2}{\sigma_{l}^{n}^2}]$$  

($l = 1, \cdots, 4; n = 1, \cdots, 16$)  (19)
in which $\sigma^n_l \in [\sigma^n_{l1}, \sigma^n_{l2}]$, and the secondary membership function is selected as the triangular type MF, that is to say,

$$\text{Apex} = u_1(x) + [u_2(x) - u_1(x)]/5$$  \hspace{1cm} (20)$$

in which $u_2(x)$ and $u_1(x)$ represent the upper bound and lower bound for FOU, respectively.

Then the parameters for antecedents are selected as:

$$\sigma^n_{l1} = 2.2 + \text{rand}(4, 16)$$ \hspace{1cm} (21)

$$\sigma^n_{l2} = \sigma^n_{l1} + \text{rand}(4, 16)$$ \hspace{1cm} (22)

$$m^n_l = 1.8 + 2 * \text{rand}(4, 16).$$ \hspace{1cm} (23)

In addition, the parameters for $C^n_l$ are selected as:

$$c^n_l = \text{rand}(1, 4), \quad s^n_l = \text{rand}(1, 4)(i = 0, 1, \cdots, 4)$$ \hspace{1cm} (24)

The input measurement set is chosen as:

$$x = 7 * \text{rand}(16, 4).$$ \hspace{1cm} (25)

**Example 2:** For the TSK inference based GT2 FLSs, the primary MF of antecedent GT2 FS is chosen as the Gaussian type MF with uncertain mean, i.e., see the Figure 3.

$$\mu^n_l(x_l) = \exp\left\{-\frac{1}{2}\left(\frac{x_l - m^n_l}{\sigma^n_l}\right)^2\right\}$$ \hspace{1cm} (26)

in which $m^n_l \in [m^n_{l1}, m^n_{l2}]$ and the secondary membership function is selected as the trapezoidal type MF, that is to say,

$$L(x) = u_1(x)$$ \hspace{1cm} (27)

$$R(x) = u_2(x) - 3[u_2(x) - u_1(x)]/5.$$ \hspace{1cm} (28)

Then the parameters for antecedents are selected as:

$$m^n_{l1} = 1.7 + \text{rand}(4, 16)$$ \hspace{1cm} (29)

$$m^n_{l2} = m^n_{l1} + 2 * \text{rand}(4, 16)$$ \hspace{1cm} (30)

$$\sigma^n_l = 2.3 + \text{rand}(4, 16).$$ \hspace{1cm} (31)

In addition, the parameters for $C^n_l$ and input measurement are selected the same form as in equation (24) and equation (25), respectively.

As for the example 3 and example 4, the forms of all parameters of TSK inference based GT2 FLSs are selected the same as in 1 examples 1 and example 2, respectively. However, the number of antecedents in each fuzzy rule is chosen as 5. Therefore, the number of fuzzy rules will be $2^5$, that is to say, 32 in these two examples. While for the example 5 and example 6, the number of antecedents in each fuzzy rule is chosen as 6. Therefore, the number of fuzzy rules will be $2^6$, that is to say, 64 in the last two examples. Next we perform both the quantitative and qualitative studies. For these six examples, as $\Delta = 100$, the COS type-reduced sets calculated by three types of iterative algorithms are provided in Figure 4.

As $\Delta$ be changed from 1 to the maximum number 100 with the stepsize of 1, the COS defuzzified outputs calculated by three types of iterative algorithms are provided in Figure 5.

Next the calculational times of three kinds of iterative algorithms are investigated. The hardware and software platforms

| Example | KM | EKM | EIASC | TRR$_{KM}$ | TRR$_{EIASC}$ |
|---------|----|-----|-------|-------------|---------------|
| Example 1 | 0.020 | 0.012 | 0.009 | 36.64 | 52.95 |
| Example 2 | 0.039 | 0.012 | 0.009 | 68.93 | 75.41 |
| Example 3 | 0.112 | 0.045 | 0.031 | 59.42 | 71.77 |
| Example 4 | 0.121 | 0.042 | 0.035 | 65.08 | 70.31 |
| Example 5 | 0.343 | 0.077 | 0.059 | 77.49 | 82.70 |
| Example 6 | 0.213 | 0.055 | 0.049 | 73.84 | 76.79 |
| Average | 0.141 | 0.040 | 0.032 | 71.05 | 76.96 |
are chosen as a dell desktop with dual core CPU and the Mat-\lab 2013a, respectively. In order to measure the efficiencies of these three types of iterative algorithms, the specific calculational times for obtaining the COS type-reduced sets and defuzzified values are provided in the following Table 4 and Table 5, respectively. Here we select the time unit as the second (s). Furthermore, the last two columns denote the time reducing rate (TRR) for the EKM algorithms and EIASC compared with the KM algorithms, respectively, and the last line in Table 1 and Table 2 represents the mean of 6 examples. The TRR is defined as:

$$
TRR_{EKM,EIASC} = \frac{t_{KM} - t_{EKM,EIASC}}{t_{KM}} \times 100\% \quad (32)
$$

where $t$ is the computational time of iterative algorithm.
Observing from the Figures 4-5 and Tables 4-5, the quantitative and qualitative conclusions for the provided six examples can be made:

1) For the left parts of COS type-reduced sets (see the Figure 4), from the first example to the last one, the simulation results of three types of iterative algorithms are almost completely the same; for the right parts of COS type-reduced sets, the results of KM algorithms, EKM algorithms and EIASC are slightly different, while the results two formers are almost completely the same in Examples 1, 3, and 5.

2) For the COS defuzzified outputs (see the Figure 5), the simulation results of KM algorithms, EKM algorithms and EIASC are slightly different, while the results two formers are almost completely the same in Examples 1, 3, and 5.

3) Compared with the KM algorithms, in these six examples, the EKM algorithms and EIASC obtain the maximum TRR values as 77.49%, and 82.70% for getting the COS type-reduced sets; and they obtain the maximum TRR values as 72.78%, and 73.78% for getting the COS defuzzified values.

4) In contrast to the KM algorithms, the EKM algorithms and EIASC obtain the average value of TRRs as 71.05%,
and 76.96% for getting the COS type-reduced sets; and they obtain the average value of TRRs as 71.68%, and 72.88% for getting the COS defuzzified values.

Here we put forward the three kinds of iterative algorithms for performing the COS TR and defuzzification of TSK inference based GT2 FLSs by means of six computer simulation examples. It can be shown that the calculational times of three kinds of iterative algorithms are gradually decreased, that is to say, the computational efficiencies are improved. Therefore, we may improve the iterative types of algorithms to investigate the COS TR of GT2 FLSs.

V. CONCLUSION AND EXPECTATIONS

Three kinds of iterative algorithms for performing the COS TR of TSK inference based GT2 FLSs are proposed in this paper. Furthermore, we provide the blocks of fuzzy reasoning, COS TR and defuzzification for GT2 FLSs. Then six computer simulation examples to used to illustrate the computational values and times of COS type-reduced sets and defuzzified for three types of iterative algorithms. Simulation results show that the efficiencies of three types of iterative algorithms are gradually increased, which may be meaningful for designing GT2 FLSs in fields influenced by the uncertainties.

In the next, the COS TR of both IT2 FLSs and GT2 FLSs based on both noniterative algorithms and iterative algorithms [30], [35], [36], [37], [38], [39], [40], [41] will be further studies. In addition, we will investigate the applications of IT2 and GT2 fuzzy neural networks optimized with the combination of different types of intelligent algorithms [42], [43], [44], [45], [46] in forecasting, fuzzy control, and fuzzy identification and so on.

ACKNOWLEDGMENT

The author is grateful to the well-known Academician Jerry Mendel, who has provided many invaluable advices.

**TABLE 5.** Calculational time for obtaining the defuzzified values.

| Num | KM | EKM | EIAS C | TRR e  | TRR e  |
|-----|----|-----|--------|--------|--------|
| Examp 1 | 1.8509 | 0.621 | 0.536 | 66.40 | 71.00 |
| Examp 2 | 1.8965 | 0.578 | 0.536 | 69.49 | 71.69 |
| Examp 3 | 4.5305 | 1.318 | 1.275 | 70.89 | 71.84 |
| Examp 4 | 7.64 | 0.976 | 0.939 | 72.18 | 73.73 |
| Examp 5 | 10.506 | 2.941 | 2.849 | 72.00 | 72.88 |
| Examp 6 | 10.694 | 2.911 | 2.840 | 72.79 | 73.44 |
| Average | 5.7126 | 1.617 | 1.549 | 71.68 | 72.88 |

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Y. Chen: Comparison Study of Iterative Algorithms for COS TR of Takagi Sugeno Kang Type General Type-2 Fuzzy Logic Systems

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