Spacetime Quantization of BPFTW Action: Spacelike Plasmon Cut & New Phase of the Thermal Vacuum

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Sept 24, 1994

Abstract

The properties of quark propagation through a hot medium are summarized by the BPFTW effective action. The fermion thermal propagator shows a pseudo-Lorentz invariant particle pole as well as a spacelike cut. In an earlier paper, we have performed the explicit quantization of the action in momentum space, and showed how a canonical Dirac field of mass $T'$ arises.

In this paper, we perform the quantization in coordinate space. In the process, we relate the spacelike plasmon cut in the propagator to the homogeneous solutions of the local equation of motion for auxiliary fields.

Our quantization shows how the spacelike cut produces a 90° phase factor in the thermal vacuum at high $T$. This phase factor is responsible for the vanishing of $< \bar{\psi} \psi >$ at high $T$.

PACS: 12.38 Aw, 11.10.wx, 11.15.Ex, 11.30.Rd

1 Introduction

In an earlier paper[1], we have studied the canonical quantization of the BPFTW[2] action for the propagation of a massless fermion through the hot environment[3]. The propagator $< T (\psi(\vec{x}, t) \bar{\psi}(\vec{y}, t')) >_{\beta}$ in spite of its apparent chiral symmetry shows a pseudo-Lorentz invariant particle pole[4] at $p_o = \omega = \sqrt{\vec{p}^2 + T'^2}$. This pole is described by the canonical Dirac field,

$$\Psi(\vec{x}, t) = \frac{1}{\sqrt{V}} \sum_{p,s} e^{i\vec{p} \cdot \vec{x}} \left\{ U_{p,s} A_{p,s} e^{-i\omega t} + V_{-p,s} B^+_s e^{i\omega t} \right\}$$ (1)

where $A_{p,s}, B_{p,s}$ are the annihilation operators for a particle of mass $T'$, and $U_{p,s}, V_{-p,s}$ are the massive Dirac spinors[5]. In addition, the thermal fermion propagator shows a pair of parallel spacelike cuts, just above and below the real axis and running from $p_o = -|\vec{p}|$ to $p_o = |\vec{p}|$.

This work has been supported in part by a grant from PSC-BHE of CUNY.
This pair of spacelike cuts is attributed to the hot plasma state. But how exactly is it related to the thermal vacuum in the plasma state? In this paper we investigate the origin of the spacelike cut in terms of the space-time quantization of the BPFTW action.

2 BPFTW Action

The BPFTW action\[2\] for the propagation of the fermion through a hot environ takes the form

$$L_{\text{eff}} = -\bar{\psi} \gamma_\mu \partial^\mu \psi - \frac{T'^2}{2} \bar{\psi} \left( \frac{\gamma_o - \hat{\gamma} \cdot \hat{n}}{D_o + \hat{n} \cdot \hat{D}} \right) \psi \quad (2)$$

where

$$T'^2 = \frac{g^2 C_f T^2}{4} \quad (3)$$

and $C_f$ is the Casimir invariant equal to $(N^2 - 1)/(2N)$ for $SU(N)$ group. The angular bracket denotes an average over the orientation $\hat{n}$.

Eq.(2) is a nonlocal action in spacetime. If we suppress the gluon fields and concentrate on the fermion sector of the effective action, then the nonlocality is a lightlike separation between the two fermion fields

$$L_{\text{eff}} = -\bar{\psi} \gamma_\mu \partial^\mu \psi + \frac{T'^2}{8} \int dt' \epsilon(t-t') \left\{ \begin{array}{c} < \bar{\psi}(x) (\hat{\gamma} \cdot \hat{n} - \gamma_o) \psi(x_P) > - < \bar{\psi}(x) (\hat{\gamma} \cdot \hat{n} + \gamma_o) \psi(x_{P'}) > \end{array} \right\} \quad (4)$$

where $x_P$ and $x_{P'}$ refer to the pair of conjugate points lightlike separated from $(\vec{x}, t)$

$$x_P = (\vec{x} - \hat{n}(t-t'), t'), \quad x_{P'} = (\vec{x} + \hat{n}(t-t'), t') \quad (5)$$

The nonlocal Euler-Lagrange equation of motion for the action is

$$\gamma_\mu \partial^\mu \psi(\vec{x}, t) = \frac{T'^2}{8} \int dt' \epsilon(t-t') \left( (\hat{\gamma} \cdot \hat{n} - \gamma_o) \psi(\vec{x} - \hat{n}(t-t'), t') > - \frac{T'^2}{8} \int dt' \epsilon(t-t') \left( (\hat{\gamma} \cdot \hat{n} + \gamma_o) \psi(\vec{x} + \hat{n}(t-t'), t') > \right) \right. \quad (6)$$

Following Weldon\[6\], we may replace the nonlocal action by a local one involving auxiliary fields\[7\]

$$\gamma_\mu \partial^\mu \psi(\vec{x}, t) = \frac{T'}{2} < \gamma \cdot n_+ \chi_+ > - \frac{T'}{2} < \gamma \cdot n_- \chi_- > \quad (7)$$

$$n_+ \partial^\mu \chi_+ = \frac{T'}{2} \psi(\vec{x}, t) \quad (8)$$

$$n_- \partial^\mu \chi_- = \frac{T'}{2} \psi(\vec{x}, t) \quad (9)$$
where we have introduced the two conjugate lightlike 4-vectors

\[ n_{+,\mu} \equiv (\hat{n}, 1) \]  \hspace{1cm} (10) \\
\[ n_{-,\mu} \equiv (\hat{n}, -1) \]  \hspace{1cm} (11)

so that

\[ \gamma \cdot n_+ = (\vec{\gamma} \cdot \hat{n} - \gamma_0) \]  \hspace{1cm} (12) \\
\[ \gamma \cdot n_- = (\vec{\gamma} \cdot \hat{n} + \gamma_0) \]  \hspace{1cm} (13)

and the new local effective Lagrangian takes the form

\[
\mathcal{L}_{\text{eff}} = -\bar{\psi} \gamma_{\mu} \partial^\mu \psi + \frac{T'}{2} \bar{\psi} \langle \gamma \cdot n_+ \chi_+ \rangle - \frac{T'}{2} \bar{\psi} \langle \gamma \cdot n_- \chi_- \rangle - \frac{T'}{2} \langle \bar{\chi}_+ \gamma \cdot n_+ \psi \rangle + \frac{T'}{2} \langle \bar{\chi}_- \gamma \cdot n_- \psi \rangle + \langle \bar{\chi}_+ \gamma \cdot n_+ n_+ \cdot \partial \chi_+ \rangle - \langle \bar{\chi}_- \gamma \cdot n_- n_- \cdot \partial \chi_- \rangle
\]  \hspace{1cm} (14)

Note that apart from the equations of motion, the auxiliary fields, \( \chi_\pm \), do not satisfy any other constraints. This is unlike in the case of Weldon\cite{6}, where his auxiliary field, \( \phi_{\text{Weldon}} \), satisfies the constraint

\[ \gamma \cdot n_+ \gamma_0 \phi_{\text{Weldon}} = 2 \phi_{\text{Weldon}} \]  \hspace{1cm} (15)

beyond the equation of motion

\[ i n_{+,\mu} \partial^\mu \phi_{\text{Weldon}}(x) = \frac{T'}{2} \gamma \cdot n_+ \psi(x) \]  \hspace{1cm} (16)

In addition, Weldon does not distinguish between the two light-cones \( n_{+,\mu} \) and \( n_{-,\mu} \). The relation between our auxiliary field \( \chi_+ \) and that of Weldon is

\[ \phi_{\text{Weldon}}(x) = -i \gamma \cdot n_+ \chi_+(x) \]  \hspace{1cm} (17)

Written in this form, the reason for the constraint on Weldon’s field becomes clear. It comes about because of the nilpotent operator \((\vec{\gamma} \cdot \hat{n} - \gamma_0)\) acting on \( \chi_+ \),

\[ (\gamma \cdot n_+)^2 = 0 \]  \hspace{1cm} (18)

The advantage of the coupled set of local equations of motion\cite{3,4} is that it gives us an insight into the role of the homogeneous solutions to eq.(8) and eq.(9), which otherwise would not be transparent with the nonlocal equation of motion\cite{3}. These homogeneous solutions turn out to be related to the spacelike cuts of the thermal fermion propagator.

### 3 Analytic Properties of Thermal Fermion Propagator

The BPFTW action leads to the thermal fermion propagator\cite{3}

\[
< T(\psi(x)\bar{\psi}(y)) >_\beta = \frac{1}{i} \int \frac{d^4 p}{(2\pi)^4} \frac{-i\vec{\gamma} \cdot \vec{p}(1 - \frac{T'^2}{2}a) + i\gamma_0 p_0(1 - \frac{T'^2}{2}b)}{p^2 - p_0^2 + T'^2 - i\epsilon + \frac{T'^4}{4}(p^2a^2 - p_0^2b^2)}
\]  \hspace{1cm} (19)
where $a, b$ are the functions
\begin{align}
a &= \frac{p_0}{2p^3} \ln \left| \frac{p_0 + p}{p_0 - p} \right| - \frac{1}{p^2} \quad (20) \\
b &= \frac{1}{2p_0 p} \ln \left| \frac{p_0 + p}{p_0 - p} \right| \quad (21)
\end{align}
chosen to be real along the entire real $p_0$ axis, and $p$ denotes the magnitude of $\vec{p}$.

For $t > 0$, we have
\begin{align}
<T(\psi(x)\bar{\psi}(0))>_{\beta} &= <\psi(x)\bar{\psi}(0)>
\end{align}
\begin{align}
&= \int \frac{d^3 p}{(2\pi)^3} e^{i\vec{p} \cdot \vec{x}} \left\{ \frac{Z_p - i\vec{\gamma} \cdot \vec{p}}{2\omega} e^{-i\omega t} \right. \\
&\quad \left. - \frac{T'^2}{8} \int_{-p}^p \frac{dp'_0}{p^3} \frac{i\vec{\gamma} \cdot \vec{p}'_0 - i\gamma_0 p'^2}{p^2 - p_0^2 + T'^2} e^{-ip'_0 t} \right\} + O(T'^4) \quad (24)
\end{align}

Here, we have performed the contour integration in $p_0$-plane and isolated the pole as well as the cut contributions for $t > 0$. In doing so, we have dropped an $O(T'^4)$ term along the spacelike cut.

Note that the spinor structure of the massive particle pole term has the (unusual) feature of being manifestly chiral invariant. The wave function renormalization constant $Z_p$ at the pole is given by
\begin{align}
Z_p &= 1 - \frac{T'^2}{4p^2} \left( \ln \frac{4p^2}{T'^2} - 1 \right) \quad (25)
\end{align}

## 4 Relation between $\psi$ and $\Psi$

Based on the earlier work[1], the field $\psi$ is related to the canonical $\Psi$ by the expansion
\begin{align}
\psi(x) &= \Psi(x) + \frac{T'}{8} \int dt' \epsilon(t - t') \left( <\gamma \cdot n_+ \Psi(x_p) > - <\gamma \cdot n_- \Psi(x_{p'}) > \right) + \ldots \quad (26)
\end{align}
This expansion correctly describes the thermal fermion propagator accurately to order $T'$.

To go beyond the $O(T')$ term in the expansion, we go back to eq.(8,9) and note the homogeneous solutions
\begin{align}
\chi^s_+ (\vec{x},t) &= \frac{1}{\sqrt{V}} \sum_{p,s} \left\{ F_+ (\hat{n}, p) U_{p,s} A_{p,s} + G_+ (\hat{n}, p) V_{-p,s} B^\dagger_{-p,s} \right\} e^{i\vec{p} \cdot \vec{x} - i\hat{n} \cdot \vec{p}} \\
\chi^s_- (\vec{x},t) &= \frac{1}{\sqrt{V}} \sum_{p,s} \left\{ F_- (\hat{n}, p) U_{p,s} A_{p,s} + G_- (\hat{n}, p) V_{-p,s} B^\dagger_{-p,s} \right\} e^{i\vec{p} \cdot \vec{x} + i\hat{n} \cdot \vec{p}} \quad (27, 28)
\end{align}
where $A_{p,s}$ and $B_{p,s}$ are the particle annihilation operators of the massive canonical Dirac field of eq.(8) and the $F_\pm$ and $G_\pm$ are Dirac matrix functions of $\hat{n}, \vec{p}, p_0$. They are to be determined so that they reproduce the location, magnitude and phase of the spacelike cuts in the thermal fermion propagator.
These homogeneous solutions in turn contribute to the expansion for $\psi$ through eq. (7), so that eq. (26) now becomes

$$\psi(x) = \Psi(x) + \frac{T'}{8} \int dt' \epsilon(t-t') \left( <\gamma \cdot n_+ \Psi(x_p) > - <\gamma \cdot n_- \Psi(x_p) > \right) + \Psi^s(\vec{x}, t)$$

with

$$\Psi^s(x) = i \frac{T'}{2} \int d^4y \left\{ <S^{-1}(x-y)\gamma \cdot n_+ \chi^s(y) > - <S^{-1}(x-y)\gamma \cdot n_- \chi^s(y) > \right\}$$

where $S^{-1}$ is the inverse propagator satisfying the property

$$\left( \gamma \cdot \frac{\partial}{\partial x} + T' \right) S^{-1}(x-y) = -i \delta^4(x-y)$$

The Dirac matrix functions, $F_\pm$ and $G_\pm$, have the representation

$$F_\pm(\hat{n}, \vec{p}, p_0) = \pm (\vec{\gamma} \cdot \hat{n} \pm \gamma_o) f_1 \pm (\vec{\gamma} \cdot \vec{p} \mp \hat{n} \cdot \vec{p} \gamma_o) f_2$$

$$G_\pm(\hat{n}, \vec{p}, p_0) = \pm (\vec{\gamma} \cdot \hat{n} \pm \gamma_o) g_1 \pm (\vec{\gamma} \cdot \vec{p} \mp \hat{n} \cdot \vec{p} \gamma_o) g_2$$

where the scalar functions $f, g$ are even under the exchange $\hat{n} \rightarrow -\hat{n}$. This representation ensures that the thermal fermion propagator is spacetime translation invariant. This arises because according to eq. (29), the thermal fermion Green function may be written in terms of the canonical field $\Psi$ and the new spacelike $\Psi^s$

$$<\psi(x)\bar{\psi}(y)> = <\Psi(\vec{x}, t)\overline{\Psi}(\vec{y}, y_o) >$$

$$- \frac{T'}{4} \int dt' \epsilon(y_o - t') <\Psi(\vec{x}, t)\overline{\Psi}(\vec{y} - \hat{n}(y_o - t'), t') \left( \vec{\gamma} \cdot \hat{n} - \gamma_o \right) >$$

$$+ \frac{T'}{4} \int dt' \epsilon(t - t') <(\vec{\gamma} \cdot \hat{n} - \gamma_o) \Psi(\vec{x} - \hat{n}(t - t'), t') \overline{\Psi}(\vec{y}, y_o) >$$

$$+ <\Psi(\vec{x}, t)\overline{\Psi}(\vec{y}, y_o) > + <\Psi^s(\vec{x}, t)\overline{\Psi}(\vec{y}, y_o) > + <\Psi^s(\vec{x}, t)\overline{\Psi}^s(\vec{y}, y_o) >$$

(34)

Here, for simplicity, we have identified the two lightcone averages and written them as a single average over the orientation, $\hat{n}$. The first three terms in the expansion involve only the canonical Dirac field $\Psi$, and, as pointed out in the earlier work \[1\], they together correctly reproduce the spinor as well as the pole structure of the thermal Green function. The chiral flip part of the canonical massive Green function $<\Psi\Psi>$ is cancelled by the other two terms, thus agreeing with the apparent chiral symmetry of the thermal Green function $<\psi\bar{\psi}>$.

The cross-terms involving $\Psi$ with $\Psi^s$ would, however, give rise to Fourier integrals of the type

$$e^{i\vec{p} \cdot (\vec{x} - \vec{y})} e^{-ip\cdot t \mp i\hat{n} \cdot \vec{p} y_o}$$

which are manifestly not invariant under time translations. The Dirac matrix representation in eq. (32, 33) ensures the absence of these terms. The last term in the expansion reproduces correctly
the spacelike cuts of the thermal Green function, when

\[ F_+ = \frac{\eta}{2} \left\{ \frac{(\vec{\gamma} \cdot \hat{n} + \gamma_0)}{2} + i \frac{\vec{\gamma} \cdot \vec{p} - \hat{n} \cdot \vec{p} \gamma_0}{\sqrt{p^2 - (\hat{n} \cdot \vec{p})^2}} \right\} \]

\[ F_- = \frac{\eta}{2} \left\{ -\frac{(\vec{\gamma} \cdot \hat{n} - \gamma_0)}{2} - i \frac{\vec{\gamma} \cdot \vec{p} + \hat{n} \cdot \vec{p} \gamma_0}{\sqrt{p^2 - (\hat{n} \cdot \vec{p})^2}} \right\} \]

\[ G_+ = \frac{\eta'}{2} \left\{ \frac{(\vec{\gamma} \cdot \hat{n} + \gamma_0)}{2} + i \frac{\vec{\gamma} \cdot \vec{p} - \hat{n} \cdot \vec{p} \gamma_0}{\sqrt{p^2 - (\hat{n} \cdot \vec{p})^2}} \right\} \]

\[ G_- = \frac{\eta'}{2} \left\{ -\frac{(\vec{\gamma} \cdot \hat{n} - \gamma_0)}{2} - i \frac{\vec{\gamma} \cdot \vec{p} + \hat{n} \cdot \vec{p} \gamma_0}{\sqrt{p^2 - (\hat{n} \cdot \vec{p})^2}} \right\} \]

(35) \hspace{1cm} (36) \hspace{1cm} (37) \hspace{1cm} (38)

Here \( \eta, \eta' \) are overall phase factors that remain arbitrary. The requirement that the \( \langle \Psi^s \Psi^s \rangle \) reproduce the cut structure exhibited in eq.(24) only demands an internal 90° phase difference between \( f_1 \) and \( f_2 \), and likewise between \( g_1 \) and \( g_2 \). The choice for the overall phase factors in \( F_\pm \) and \( G_\pm \) will become apparent only when we turn to the relation between \( \psi \) and the canonical Dirac field \( \Psi \) at time \( t = 0 \) and study their connection with the thermal vacuum at high \( T \).

5 Relation to Thermal Vacuum

Eq.(29) provides the expansion of \( \psi(x) \) in terms of the canonical field, \( \Psi(x) \). In this section, we shall use it to relate the \( \psi \) field at \( t = 0 \) to the canonical operators. Let

\[ \psi(\vec{x}, 0) = \frac{1}{\sqrt{V}} \sum_p e^{i\vec{p} \cdot \vec{x}} \left\{ \begin{array}{c} \chi_L a_{p,L} \\ \chi_R a_{p,R} \end{array} \right\} + \left\{ \begin{array}{c} \chi_R b_{p,R}^\dagger \\ -\chi_L b_{p,L}^\dagger \end{array} \right\} \]

(39)

where \( \chi_{L,R} \) is an eigenfunction of the helicity operator \( \vec{\sigma} \cdot \hat{p} \) with eigenvalue \pm 1 respectively. A straightforward evaluation of the gamma matrix algebra implied in eq.(29) shows that

\[ a_{p,s} = A_{p,s} + \eta' s \frac{T'}{2p} B^\dagger_{-p,s} + O(T'^2/p^2) \]

(40)

\[ b_{p,s} = B_{p,s} - \eta^* s \frac{T'}{2p} A^\dagger_{-p,s} + O(T'^2/p^2) \]

(41)

At this stage, the rule that \( a_{p,s} \) and \( b_{p,s} \) should anticommute, places the requirement

\[ \eta' = \eta^* \]

(42)

leaving us with still an overall phase factor \( \eta \) to be determined.

In the absence of \( \Psi^s \), the relation between \( a_{p,s}, b_{p,s} \) and \( A_{p,s}, B_{p,s} \) would have been simply \( a_{p,s} = A_{p,s} \) and \( b_{p,s} = B_{p,s} \). The additional terms of \( B^\dagger_{-p,s} \) and \( A^\dagger_{-p,s} \) arose entirely from the spacelike \( \Psi^s \) field in eq.(29).

Thus the spacelike cut in the fermion propagator signals the presence of Bogoliubov pairing in the thermal vacuum.
Can we determine the precise nature of this Bogoliubov pairing in the new thermal vacuum? The answer is yes. For a study of the order parameter \( < \bar{\psi} \psi \beta > \) will yield information on the properties of the vacuum. Since we now have the relation between \( \psi \) and \( \Psi \) at time \( t = 0 \), we may proceed to obtain the canonical expansion for \( \bar{\psi} \psi \). Let us introduce the order operator \[ R \]

\[
\frac{1}{2} \int d^3x \bar{\psi}(\vec{x}, 0) \psi(\vec{x}, 0) = - \sum_p Y_{1p} \tag{43}
\]

where

\[
Y_{1p} \equiv - \sum_s \frac{s}{2} \left( a_{p,s}^\dagger b_{-p,s}^\dagger + a_{p,s} b_{-p,s} \right) \tag{44}
\]

The order parameter \( < \bar{\psi} \psi \beta > \) is the expectation value of the order operator with respect to the new thermal vacuum \( | \text{vac} > \). Recall that the original Fock space vacuum satisfies the property

\[
a_{p,s} |0 > = b_{p,s} |0 > = 0 \tag{45}
\]

while the new thermal vacuum is the one annihilated by \( A_{p,s}, B_{p,s} \)

\[
A_{p,s} | \text{vac} > = B_{p,s} | \text{vac} > = 0 \tag{46}
\]

Upon substituting the relations eq.(43, 44) into the order operator eq.(44), we find the canonical expansion

\[
\int d^3x \bar{\psi}(\vec{x}, 0) \psi(\vec{x}, 0) = \sum_{p,s} \left\{ s \left( A_{p,s}^\dagger B_{-p,s}^\dagger + B_{p,s} A_{p,s} \right) - (\eta + \eta^*) \frac{T'}{2p} \left( A_{p,s}^\dagger A_{p,s} - B_{p,s} B_{-p,s}^\dagger \right) \right\} \tag{47}
\]

and we have the order parameter at \( t = 0 \)

\[
\int d^3x < \bar{\psi} \psi \beta > = \sum_{p,s} | \text{vac} > \left\{ s \left( A_{p,s}^\dagger B_{-p,s}^\dagger + B_{p,s} A_{p,s} \right) - (\eta + \eta^*) \frac{T'}{2p} \left( A_{p,s}^\dagger A_{p,s} - B_{p,s} B_{-p,s}^\dagger \right) \right\} | \text{vac} > = (\eta + \eta^*) \sum_p \frac{T'}{p} \tag{48}
\]

Since \( < \bar{\psi} \psi \beta > \) vanishes, we finally arrive at the result that\[ R \]

\[
\eta = -\eta' = i \tag{49}
\]

which in turn implies that to order \( O(T'/p) \), we have

\[
a_{p,s} = A_{p,s} - i s \frac{T'}{2p} B_{-p,s} \tag{50}
\]

\[
b_{p,s} = B_{p,s} + i s \frac{T'}{2p} A_{-p,s} \tag{51}
\]

\[ 2 \text{ There is, of course, a conjugate solution with } \eta = -\eta' = -i. \text{ To the extent that they yield the same spacelike cut contribution, it is physically equivalent to the solution we pick.} \]
This relation spells out the precise nature of the Bogoliubov transformation that takes us from the original Fock space vacuum to the new thermal vacuum. The new vacuum is the generalized Nambu-Jona-Lasinio vacuum\[10\]

\[
|\text{vac}\rangle = \prod_{p,s} \left( \cos \theta_p - i s \sin \theta_p a^\dagger_{p,s} b^\dagger_{-p,s} \right) |0\rangle
\]

where

\[
\tan 2\theta_p = \frac{T_p}{p}
\]

The interesting feature of this generalized NJL vacuum is the presence of the phase factor \(i\) in the quark-antiquark pair.

As has been pointed out elsewhere\[1, 11\] this phase factor is responsible for the vanishing of \(\langle \bar{\psi} \psi \rangle\), without however breaking up the quark-antiquark pairs. As a result the generalized NJL vacuum is not chiral-invariant. By this, we mean that under the old (zero temperature) chirality

\[
a_{p,s} \to e^{is\alpha} a_{p,s}
\]

\[
b_{-p,s} \to e^{is\alpha} b_{-p,s}
\]

the thermal vacuum goes over into a a new unitarily inequivalent vacuum.

And yet there is an apparent chiral symmetry of the BPFTW action in eq.(2). The resolution of this paradox comes when we recognize that the new chiral symmetry at high \(T\) is associated with a different Noether charge\[6\] than the original (zero temperature) chirality. This new \(Q_5^\beta\) has the expansion\[1\]

\[
Q_5^\beta = -\frac{1}{2} \sum_{p,s} \left( A^\dagger_{p,s} A_{p,s} + B^\dagger_{-p,s} B_{p,s} \right)
\]

which clearly annihilates the thermal vacuum \(|\text{vac}\rangle\) and explains the apparent chiral symmetry of the high temperature effective action. But this high temperature chirality is not identical to the old chirality, \(Q_5\) given by

\[
Q_5 = -\frac{1}{2} \sum_{p,s} \left( a^\dagger_{p,s} a_{p,s} + b^\dagger_{-p,s} b_{-p,s} \right)
\]

6 Conclusion

In this paper, we have performed the quantization of the BPFTW action in coordinate space. By reformulating it in terms of auxiliary fields, the action may be made local. The auxiliary fields, \(\chi_{\pm}\), satisfy local equations of motion that admit homogeneous solutions, which turn out to be directly related to the spacelike cuts in the thermal propagator.

The homogeneous solutions impact on the relation between \(\psi\) and the canonical \(\Psi\) field at \(t = 0\). These relations spell out the connection between the vacuum of the massless free field \(a_{p,s}, b_{-p,s}\) and the thermal vacuum of the effective action. Our results show how the presence of the spacelike cuts in the fermion propagator signals a new phase in the underlying thermal vacuum.

This thermal vacuum continues to exhibit the rich and complex structure of chiral symmetry violations as the zero temperature case. That \(\langle \bar{\psi} \psi \rangle\) vanishes is no proof of chiral restoration
at high $T$. In a separate communication, we introduce and discuss the $SU(2N_f)_p \otimes SU(2N_f)_p$ chirality algebra of order parameters that probe the state the chiral symmetry breaking.

Many questions remain to be answered. What is the underlying physics origin of the auxiliary field? How do the quark-antiquark pairing at high temperatures generate the intriguing $90^\circ$ phase in the BCS-like ground state? There is clearly an interplay between the new chirality $Q^{5\beta}$ at high $T$ and the old $Q_5$ chirality. How does it impact on the interactions of the pion at high $T$?

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[6] H.A. Weldon, Proceedings of the Winnipeg Workshop on Finite Temperature Field Theory, July 1992, published in Canadian J. Phys (1993).

[7] We use the metric such that $a \cdot b \equiv \vec{a} \cdot \vec{b} - a_\mu b_\mu$. Here $\gamma_\mu = (\vec{\gamma}, \gamma_0)$, with $\vec{\gamma}$ hermitian, while $\gamma_0$ is antihermitian.

[8] Our analyticity conforms to the usual spectral representation in thermal field theory. See H.A. Weldon, Phys. Rev. D 40, 2410 (1989); Physica A 158, 169 (1989) for a general discussion of the spectral properties. Note that his $\mathcal{M}^2 = T'^2/2$ here.

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