Algorithm for the formation of a linguistic integrated assessment of resource sustainability of an enterprise

T V Azarnova¹, N G Asnina¹, Yu V Bondarenko¹ and S A Barkalov²

¹Department of Applied Mathematics, Informatics and Mechanics, Voronezh State University, 1, University Square, Voronezh, 394018, Russia
²Department of Economics, Management and Information Technology, Voronezh State Technical University, 84, 20 let Oktyabrya st., Voronezh, 394006, Russia

E-mail: ivdas92@mail.ru

Abstract. The paper proposes an algorithm for the formation of a linguistic integrated assessment of the resource sustainability of an enterprise, based on the theory of extensive and intensive use of resources and methods of aggregating linguistic information using linguistic ordinal operators that formalize sequential averaging strategies for multicriteria estimation. The assessment of the extent of extensive and intensive use of resources in the work is carried out through groups of models that reflect the dependence of the resulting indicators of the enterprise on the quantity and quality of resources of various types. Quantitative indicators of the intensity of resource use are estimated by the method of chain substitutions; the Harrington function is used to switch to linguistic values. The integrated linguistic assessment obtained as a result of the work of the considered algorithm aggregates the linguistic values of the resource use intensity by all models and is interpreted as an integral indicator of resource stability, which is achieved due to intensive production factors.

1. Introduction

Sustainable development of enterprises involves a process of expedient continuous irreversible directed regular changes in time, characterized by a transition to a qualitatively new, more perfect state. To assess the appropriateness of changes in the functioning of enterprises and other business entities, formalized tools are required, presented in the form of models, methods, algorithms and information-analytical systems, which allow to carry out: measurement (assessment) of the direct or indirect manifestation of indicators of ongoing changes; aggregate information on various indicators in the form of a single integrated indicator; to carry out a cognitive analysis of the relationships between ongoing changes in various fields of activity and simulation modeling of the impact of ongoing changes on the future development of the studied business entity; form promising corrective action strategies that are optimal from the perspective of sustainable development in the future.

Formalized models containing certain prerequisites for describing the problems of sustainable development and their management were built and investigated by V. I. Gurman, F. I. Ereshko, I. Z. Kaganovich, V. F. Krapivin, V. V. Mazalov, G. M. Mkrtchyan, N. N. Moiseev, A. M. Molchanov, V. G. Pryazhinskaya, E. V. Ryumina, Yu. M. Svirezhev, A. M. Tarko, A. B. Gorstko. A great contribution to the mathematical theory of hierarchical management of sustainable development was made by G.A. Ugolnitsky [1]. In his works, a static game-
theoretic model of hierarchical management of sustainable development is built. Methods of hierarchical management (coercion, motivation, persuasion) are formalized as solutions to a hierarchical game, the necessary and sufficient conditions for their existence are found. Dynamic models of hierarchical management of sustainable development in full and reduced form are built. Actual unsolved problem today is to obtain integrated integrated indicators of sustainable development of business entities. In domestic and foreign literature, a number of approaches to the construction of mathematical models for the integral assessment of the effectiveness of complex systems are proposed (E. Kveyd, R. Kulikovskiy, A. S. Rykov, M. Mesarovich, I. Takahara, I. B. Russmann, R. R. Yager, D. I. Batishchev, Y. E. L’vovich, A. V. Ereemeev, V. N. Burkov, D. A. Novikov).

Within the framework of this article, one of the algorithmic approaches to the formation of integral indicators of enterprise resource sustainability is also proposed.

2. Description of the algorithm for the formation of the linguistic integral indicator of resource sustainability

The systems of diverse indicators used to assess the sustainable development of enterprises of various types of activities do not always allow a comprehensive idea of the direction and depth of manifestation of emerging trends to be formed. In such a situation, it is advisable to use integral indicators, with the help of which it becomes possible to present a qualitative and quantitative assessment of the state of the system under study with one indicator and identify the most significant factors and conditions that affect the process of sustainable development [1].

The article [2] proposes a methodology for the formation of a comprehensive assessment of resource sustainability of an enterprise, based on the theory of intensive and extensive use of resources. In the framework of this work, an algorithm for calculating the linguistic integral assessment of resource stability based on the indicated technique is considered (figure 1).

![Algorithm diagram]

1. One of the models is selected for the dependence of revenue (sales) on the quantitative (intensive) and qualitative (intensive factors) [1].

2. For the analyzed model, the Harrington function is constructed by the expert to evaluate the level of desirability (linguistic value) of the influence of the quality factor from the position of intensive (sustainable) development.

3. The method of chain substitutions calculates the percentage increase (decrease) compared with the previous (base) period of sales due to changes in the qualitative and quantitative factors.

4. According to the constructed function of Harrington’s desirability, a linguistic assessment of the level of desirability is determined from the position of sustainable development of the current percentage change in revenue due to the qualitative factor in the model.

5. Steps 1–4 are repeated in a cycle for all models of the dependence of revenue on quantitative and qualitative factors.

6. A weight vector is formed (aggregation strategy) for convolution of the linguistic values of resource stability obtained according to individual models using the LOWA-operator.
Let us dwell on each step of this algorithm.

When assessing the resource sustainability of an enterprise, it is customary to use groups of models [2] that characterize the dependence of the organization’s revenue on the efficiency of resource use: models of the dependence of revenue on the efficiency of the use of elements (groups) of costs (expenses); models of the dependence of revenue on asset utilization; models of the dependence of revenue on the efficiency of use of funds and financial investments; models of the dependence of revenue on the efficiency of use of labor resources (table 1).

### Table 1. The main models of resource sustainability of the enterprise.

| Model name                                           | Formula                                      | Designations |
|------------------------------------------------------|----------------------------------------------|--------------|
| 1. Models of the dependence of revenue on the efficiency of the use of elements (groups) of costs (expenses) |                                             |              |
| The model of dependence of revenue on cost of sales   | $N = S^N \times \lambda^{S,N}$, $\lambda^{S,N} = \frac{N}{S^N}$ | $N$ — revenue (sales volume), rub.; $S^N$ — total costs (cost of sales), rub.; $\lambda^{S,N}$ — return on sales, rub/rub. |
| Model of revenue versus material costs                | $N = S^M \times \lambda^{S,M}$, $\lambda^{S,M} = \frac{N}{S^M}$ | $N$ — revenue (sales volume), rub.; $S^M$ — material costs, rub.; $\lambda^{S,M}$ — material output, rub/rub. |
| The model of the dependence of revenue on labor costs  | $N = S^U \times \lambda^{S,U}$, $\lambda^{S,U} = \frac{N}{S^U}$ | $N$ — revenue (sales volume), rub.; $S^U$ — labor costs, rub.; $\lambda^{S,U}$ — payroll, rub/rub. |
| Model of dependence of revenue on depreciation        | $N = S^A \times \lambda^{S,A}$, $\lambda^{S,A} = \frac{N}{S^A}$ | $N$ — revenue (sales volume), rub.; $S^A$ — depreciation deductions, rub.; $\lambda^{S,A}$ — depreciation, rub/rub. |
| The model of revenue from management expenses         | $N = S^Y \times \lambda^{S,Y}$, $\lambda^{S,Y} = \frac{N}{S^Y}$ | $N$ — revenue (sales volume), rub.; $S^Y$ — administrative expenses, rub.; $\lambda^{S,Y}$ — return on management expenses, rub/rub. |
| 2. Models of revenue versus asset utilization         |                                             |              |
| Asset dependence model                                | $N = \frac{\bar{A}}{A} \times \lambda^A$, $\lambda^A = \frac{N}{\bar{A}}$ | $N$ — revenue (sales volume), rub.; $\bar{A}$ — average asset value, rub.; $\lambda^A$ — asset turnover, rub/rub. |
The model of the dependence of revenue from fixed assets
\[ N = F \cdot \lambda^F, \]
\[ \lambda^F = \frac{N}{F}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( F \) — average cost of fixed assets, rub.;
- \( \lambda^F \) — capital productivity of fixed assets, rub/rub.

The model of the dependence of revenue on current assets
\[ N = E \cdot \lambda^E, \]
\[ \lambda^E = \frac{N}{E}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( E \) — average current value of current assets, rub.;
- \( \lambda^E \) — turnover of current assets, rub/rub.

Income revenue model
\[ N = E^S \cdot \lambda^{E,S}, \]
\[ \lambda^{E,S} = \frac{N}{E^S}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( E^S \) — average current value of stocks, rub.;
- \( \lambda^{E,S} \) — inventory turnover, rub/rub.

3. Models of the dependence of revenue on the efficiency of use of funds and financial investments

The model of revenue dependence on accounts receivable
\[ N = E^R \cdot \lambda^{E,R}, \]
\[ \lambda^{E,R} = \frac{N}{E^R}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( E^R \) — average current value of receivables, rub.;
- \( \lambda^{E,R} \) — receivables turnover, rub/rub.

Account payable revenue model
\[ N = C^{PR} \cdot \lambda^{C,PR}, \]
\[ \lambda^{C,PR} = \frac{N}{C^{PR}}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( C^{PR} \) — average current value of accounts payable, rub.;
- \( \lambda^{C,PR} \) — accounts payable turnover, rub/rub.

The model of revenue dependence on financial investments
\[ N = E^{FI} \cdot \lambda^{E,FI}, \]
\[ \lambda^{E,FI} = \frac{N}{E^{FI}}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( E^{FI} \) — average current value of financial investments, rub.;
- \( \lambda^{E,FI} \) — return on financial investments, rub/rub.

4. Models of the dependence of revenue on labor efficiency

The model of the dependence of revenue on the average number of employees
\[ N = R \cdot \lambda^R, \]
\[ \lambda^R = \frac{N}{R}, \]
where:
- \( N \) — revenue (sales volume), rub.;
- \( R \) — average number of employees, rub.;
- \( \lambda^R \) — average number of employees, rub/person.

Each of the resource sustainability assessment models presented in this table is characterized by the presence of a quantitative and qualitative factor. For example, for the model of dependence of revenue on the average number of employees, the role of the quantitative factor is played by \( R \) — the average number of employees, and the role of the qualitative factor is played by \( \lambda^R \) — labor productivity (production) of workers. At the third step of the algorithm, the method of chain substitutions calculates the percentage increase (decrease) compared with the previous (base) period of sales due to changes in the qualitative and quantitative factors. The scheme of the chain substitution method is shown in figure 2.
The degree of influence of a qualitative factor is an indicator of intensive development. Depending on the field of activity and the model of resource sustainability under consideration, experts can establish certain requirements for the degree of influence of the quality factor. It is possible to formalize the requirements and preferences of experts by constructing the Harrington desirability function [3].

**Figure 2.** The scheme of the method of chain substitutions.

Harrington’s desirability scale is given in table 2.

| Linguistic system of preferences (desirability) | Numerical preference system |
|-----------------------------------------------|-----------------------------|
| Very good stability                           | 1.00–0.80                   |
| Good stability                                | 0.80–0.63                   |
| Satisfactory stability                        | 0.63–0.3                    |
| Poor stability                                | 0.3–0                        |

The desirability function $d(y)$ varies in the range from 0 to 1. Its values reflect the desirability of a certain value of resource sustainability for an expert or decision maker.

Two types of Harrington preference functions are distinguished (figure 3).
– with one-sided restrictions (during construction, the minimum \( y_{\text{min}} \) or maximum value \( y_{\text{max}} \)):
\[
d(y) = e^{e^{-y'}},
\]
– with bilateral restrictions (when constructing, the minimum \( y_{\text{min}} \) and maximum value \( y_{\text{max}} \)):
\[
d(y) = e^{-|y'|^n},
\]
where \( y' \) — coded value \( y \), and \( n \) — degree value.

For a Harrington function with one-way constraint, the encoded value \( y' \) is calculated as follows:
1. The expert indicates two pairs of values \((y_1, d_1), (y_2, d_2)\) of resource stability and the corresponding values of desirability.
2. The encoded values are calculated:
\[
y'_1 = \ln \ln \left( \frac{1}{d_1} \right), \quad y'_2 = \ln \ln \left( \frac{1}{d_2} \right).
\]
3. Using pairs \((y_1, d_1), (y_2, d_2)\) and the corresponding coded values, by solving a system of linear equations, the linear transformation parameters \( a, b \) are calculated
\[
y' = a y + b.
\]

For the Harrington function with two-sided constraints, the parameter \( n \) and the encoded value \( y' \) are calculated as follows:
1. The expert indicates a pair of values \((y, d)\) of resource sustainability and the corresponding value of desirability.
2. The parameter \( n \) and the encoded value \( y' \) are calculated:
\[
y' = \frac{(2y - (y_{\text{max}} + y_{\text{min}}))}{(y_{\text{max}} - y_{\text{min}})}, \quad n = \frac{\ln \ln \left( \frac{1}{d} \right)}{\ln |y'|}.
\]

As a result of calculating the Harrington function for each model listed in table 1, a set of linguistic stability values is formed for the resources included in the model.

To obtain an integrated assessment of stability over the totality of resources, it is proposed to use a linguistic LOWA-operator [4]. The linguistic scale \( A = \{A_1, A_2, A_3, A_4\} \) for working with the linguistic operator corresponds to the linguistic scale of the Harrington function values given in the table 2: \( A_1 \) — a poor stability; \( A_2 \) — a satisfactory stability; \( A_3 \) — a good stability; \( A_4 \) — a very good stability.

Linguistic LOWA-operator has the following form:
\[
Z_V(B) = C^n \{ (v_k, b_k), k = \overline{1, n} \} = v_1 \otimes b_1 \oplus (1 - v_1) \otimes C^{n-1} \{ (\alpha_h, b_h), h = \overline{2, n} \},
\]
where \( V = (v_1, v_2, \ldots, v_n) \) — a weight vector \( \sum_{i=1}^n v_i = 1 \).

In the framework of the application of the linguistic operator considered in the article, the vector \( B = (b_1, b_2, \ldots, b_{13}) \) is an ordered vector of resource stability values for all thirteen models shown in table 1.

In the process of calculating the value of the linguistic LOWA — operator during the transition from iteration to iteration \((h = 2, \ldots, n)\) the weighting factors are recalculated by the formula:
\[
\alpha_h = \frac{v_h}{\sum_{k=2}^n v_k}, \quad (h = 2, \ldots, n)
\]
When calculating the convolution operators $C^n$ and $C^{n-1}$ the dimension decreases. For $n = 2$ the convolution operator has the form:

$$C^2\{v_i, b_i, i = 1, 2\} = v_1 \otimes Z_j \oplus (1 - v_1)Z_i = Z_k, \quad j \geq i, \quad b_1 = j, \quad b_2 = i$$

$$k = \min\{n, i + \text{round}(v_1(j - i))\},$$

where \text{round} — means rounding to the nearest integer.

Using various sets of weights, the LOWA operator allows you to:
1) focus on high estimates of resource sustainability obtained by models;
2) focus on low estimates of resource sustainability obtained by models;
3) focus on average estimates of resource sustainability obtained by models;
4) focus on high and low estimates of resource sustainability obtained by models;
5) make other accents by constructing a vector of weights using quantifiers.

3. Testing the algorithm for the formation of an integrated linguistic assessment of enterprise resource sustainability

The algorithm presented in this article was tested on the data of JSC "Eletsky" building materials plant. The calculation results are shown in table 3.

Summarizing the results of the calculations, we can draw the following conclusions:
1. "very good stability" was obtained using three models: the model of the dependence of revenue on managerial expenses, the model of the dependence of revenue on financial investments and the model of the dependence of revenue on the average number of employees.
Table 3. Algorithm test results.

| Model name | Elements of the model for the previous period | Elements of the model for the current period | The value of the intensive factor | Harrington function settings | Point and Linguistic Meaning of the Harrington Function |
|------------|----------------------------------------------|---------------------------------------------|----------------------------------|------------------------------|-----------------------------------------------------|
| 1. Models of the dependence of revenue on the efficiency of the use of elements (groups) of costs (expenses) |
| The model of dependence of revenue on cost of sales | \( N = 184798 \) | \( N = 194833 \) | \(-106,033\) | \((-120;0,01)\) \( (60;0,8) \) | "Poor stability" |
| Model of revenue versus material costs | \( N = 184798 \) | \( N = 194833 \) | \(174,119\) | \((-50;0,1)\) \( (200;0,8) \) | "Good stability" |
| The model of the dependence of revenue on labor costs | \( N = 184798 \) | \( N = 194833 \) | \(-5,888\) | \((-30;0,1)\) \( (80;0,9) \) | "Satisfactory stability" |
| The model of the dependence of revenue from depreciation deductions | \( N = 184798 \) | \( N = 194833 \) | \(-613,006\) | \((-300;0,01)\) \( (10;0,7) \) | "Poor stability" |
| The model of revenue from management expenses | \( N = 184798 \) | \( N = 194833 \) | \(399,565\) | \((-10;0,01)\) \( (250;0,7) \) | "Very good stability" |
| 2. Models of revenue versus asset utilization |
| Asset dependence model | \( N = 184798 \) | \( N = 194833 \) | \(45,003\) | \((-50;0,05)\) \( (75;0,8) \) | "Good stability" |
| The model of the dependence of revenue from fixed assets | \( N = 184798 \) | \( N = 194833 \) | \(209,787\) | \((10;0,06)\) \( (280;0,85) \) | "Good stability" |
The model of the dependence of revenue from current assets

\[ N = 184798 \quad E = 95131 \]
\[ N = 194833 \quad E = 81333 \]
\[ E = 1 \quad 943 \quad 774 \]
\[ R = 19530 \quad R = 21631 \]
\[ R = 9 \quad 007 \quad 218 \]
\[ E = 109840 \quad E = 81333 \]
\[ E = 1 \quad 774 \quad 395 \]
\[ E = 184,735 \quad E = 148,579 \]
\[ (–180;0,1) \quad (130;0,8) \]
\[ 0,092 \quad 0,042 \]

"Poor stability"

Revenue versus inventory model

\[ N = 184798 \quad E = 71660 \]
\[ N = 194833 \quad E = 81333 \]
\[ E = 2 \quad 579 \quad 395 \]
\[ E = 109840 \quad E = 81333 \]
\[ E = 1 \quad 774 \quad 395 \]
\[ E = 184,735 \quad E = 148,579 \]
\[ (–120;0,09) \quad (130;0,8) \]
\[ 0,042 \quad 0,092 \]

"Poor stability"

3. Models of the dependence of revenue on the efficiency of use of funds and financial investments

The model of revenue from receivables

\[ N = 184798 \quad E = 3928 \]
\[ N = 194833 \quad E = 5000 \]
\[ E = 1 \quad 046 \quad 967 \]
\[ E = 184,735 \quad E = 402,578 \]
\[ (–300;0,1) \quad (50;0,7) \]
\[ 0,019 \quad 0,019 \]

"Poor stability"

Revenue versus accounts payable

\[ N = 184798 \quad E = 28560 \]
\[ N = 194833 \quad E = 26994 \]
\[ E = 6 \quad 471 \quad 218 \]
\[ E = 230;0,8 \]
\[ 0,737 \quad 0,737 \]

"Good stability"

The model of the dependence of revenue on financial investments

\[ N = 184798 \quad E = 237 \]
\[ N = 194833 \quad E = 208 \]
\[ E = 779;738 \quad E = 936,697 \]
\[ E = 10;08 \quad E = 85 \]
\[ 0,822 \quad 0,822 \]

"Very good stability"

4. Models of the dependence of revenue on labor efficiency

The model of the dependence of revenue on the average number of employees

\[ N = 184798 \quad E = 779,738 \]
\[ N = 194833 \quad E = 936,697 \]
\[ E = 10;08 \quad E = 85 \]
\[ 0,822 \quad 0,822 \]

"Very good stability"

2. "good stability" was obtained according to four models: the model of the dependence of revenue on material costs, the model of dependence of revenue on assets, the model of dependence of revenue on fixed assets and the model of dependence of revenue on accounts payable.

3. the model of the dependence of revenue on labor costs showed "satisfactory" stability.

4. for all other models received "bad" stability.

With the "averaged" strategy of forming an integrated linguistic assessment of resource stability with the same ordinal weights \(v_i = \frac{1}{13}, i = 1, \ldots, 13\) for an ordered vector of linguistic values, the value "poor stability" is obtained.

In the case of using the "emphasis on high performance" strategy \(v_1 = \frac{1}{3}, v_2 = \frac{1}{3}, v_3 = \frac{1}{3}, v_4 = 0, i = 4, \ldots, 13\) generalized integral linguistic assessment resource sustainability has taken the value of "very good sustainability".
When choosing the strategy "focus on average indicators" \((v_6 = \frac{1}{4}, v_7 = \frac{1}{4}, v_8 = \frac{1}{4}, v_9 = \frac{1}{4}, v_i = 0, i = 1, 2, 3, 10, 11, 12, 13)\) the integrated linguistic assessment of resource sustainability turned out to be "satisfactory stability".

With different strategies for the formation of an integrated assessment, different results are obtained, the choice of strategy reflects the subjective preferences of the decision maker.

4. Conclusion
The algorithm proposed in this article for constructing an integrated indicator of the resource sustainability of an enterprise, based on the aggregation of linguistic information from different assessment models, reproduces well the specifics and logic of the considered assessment problem:

- the quantitative indicators of the intensity of resource use obtained by the method of chain substitutions do not have a clear interpretation, the algorithm allows, using the Harrington function, to reproduce the expert preferences that are not formulated clearly regarding gradations of the values of these factors;
- an integral indicator of resource sustainability should take into account the strategy of averaging linguistic estimates of resource sustainability for individual models and groups of models, the proposed algorithm allows you to form a whole range of strategies through the use of linguistic quantifiers.

Practical testing, conducted on specially developed software, showed that the proposed algorithm can serve as an effective means of supporting decision-making in multicriteria assessment tasks. A constructive tool for organizational change can be not only an integrated assessment, but also estimates for individual models that reflect the situation for various types of strategic resources.

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