Discussion on massive gravitons and propagating torsion in arbitrary dimensions

C. A. Hernaski*, A. A. Vargas-Paredes†, J. A. Helayël-Neto‡
Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud 150, Urca,
Rio de Janeiro, Brazil, CEP 22290-180

In this paper, we reassess a particular $R^2$-type gravity action in $D$ dimensions, recently studied by Nakasone and Oda, now taking torsion effects into account. Considering that the vielbein and the spin connection carry independent propagating degrees of freedom, we conclude that ghosts and tachyons are absent only if torsion is nonpropagating, and we also conclude that there is no room for massive gravitons. To include these excitations, we understand how to enlarge Nakasone-Oda’s model by means of explicit torsion terms in the action and we discuss the unitarity of the enlarged model for arbitrary dimensions.

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I. INTRODUCTION

Massive gravity has been an issue of particular interest since the early days of quantum gravity. More recently, in connection with models based on brane-world scenarios, the discussion of massive gravitons is drawing a great deal of attention, in view of the possibility of their production at LHC and the feasibility of detection of quantum gravity effects at the TeV scale [1-5]. In the framework of branes, the graviton acquires mass via a spontaneous breakdown of general coordinate reparametrization symmetry [6]. However, as it is usual in all Higgs-type mechanisms, a nonvanishing vacuum expectation value for an extra scalar field is needed in the description. There is also an alternative way to generate mass in three dimensions, as proposed by Jackiw, Deser and Templeton [7]. There, a topological parity-violating term is added to the Einstein-Hilbert gravity Lagrangian in order to describe a massive graviton. The final theory is also unitary.

In this context, we asked if it is possible to build up a unitarity and parity-preserving model that generates mass for the graviton without the need of an extra field. Bergshoeff, Hohm and Townsend obtain such a model for $D=3$ [8] by considering a nonlinear theory that is equivalent to the Pauli-Fierz model at the linear level.

In a very recent paper, M. Nakasone and I. Oda [9] have shown that a particular $R^2$-type action in three dimensions is equivalent to the massive Pauli-Fierz gravity at the linear level, as it has been proposed in [8]; moreover, they also describe how, only in three dimensions, there is no ghost, so that the model preserves unitarity. In fact, the question of unitarity in massive gravity theories is a topic of special relevance in the literature [7-11].

Besides these considerations, massive gravity is of interest by itself. For example, the work of Ref. [12] has pointed out the relevance of three-dimensional gravity in connection with Conformal Field Theories (CFT) theories [12]. Three-dimensional gravity has no local degrees of freedom. The Riemann tensor has the same number of components as the Ricci tensor, which means that all solutions in these theories are trivial, with the exception of those that consider topological effects. However, the situation might change if we consider massive spin-2 propagating modes in three dimensions. This is because the Poincaré group representations of massive particles in three dimensions and massless particles in four dimensions are described by the same little group, $SO(2)$, having both two types of helicities $\pm 2$ [8]. We do not however discuss these interesting points in the present paper.

Specifically, we investigate if there is a possible generalization of the results of [8], whenever we have propagating torsion in any dimension. We work with the vielbein and the spin connection as independent fields. Our viewpoint is that this is a more fundamental approach to gravitation, since it is based on the fundamental ideas of the Yang-Mills approach [14], [15]. As it shall become clear in the sequel, we conclude that explicit terms in the torsion field are needed in order to describe a propagating massive graviton.

We also analyze the unitarity of the model, and for this we consider the most general parity-preserving Lagrangian without higher derivatives in $D$ dimensions. We obtain a certain number of unitary Lagrangians that yield a propagating massive graviton and compare them with those Lagrangians found by Sezgin and Nieuwenhuizen [16], in case we reduce our results to $D=4$. As we consider only quadratic terms in the curvature and torsion in the Lagrangian, by virtue of the Gauss-Bonnet theorem, there is a redundant term among the possibilities for $D=4$. But, for $D \neq 4$, this term must be considered, since the Gauss-Bonnet theorem does not involve quadratic terms for $D \neq 4$. So, structurally, this is the important difference from the Lagrangian considered in [16]. The outcome is that we find a set of Lagrangians with a massive graviton that, in the particular case of $D=4$, reproduce those studied in [16]. However, we should mention that we have no intention to reproduce and repeat all the re-
sults of \[16], where there is an exhaustive and complete analysis. We carry out our discussion in D dimensions and, for \( D = 4 \), we shall be pointing out the cases that correspond to intersections with situations contemplated in \[16\].

Our paper is organized according to the following outline: in Sec. II, we present the model, our conventions and obtain the propagators of the corresponding modes. Next, Sec. III tackles the question of how to introduce massive gravitons by enlarging our initial model. In Sec. IV our aim is to analyze the existence of tachyons or ghost modes in the massive and nonmassive sectors. Finally, in Sec. V, we set up our concluding remarks. In the Appendix, we collect the whole set of spin operators that appear in our treatment.

II. DESCRIPTION OF THE MODEL

In order to investigate the changes that occur when torsion propagates, we start off by considering the same Lagrangian as the one analyzed by Nakasone and Oda in Ref. \[6\], with the exception that we consider here the right sign of the Einstein-Hilbert term. In the models of Refs. \[8\], \[9\], the opposite sign is essential for the reduction of the Lagrangian to the Pauli-Fierz model. However, as shown in \[9\], this reduction is possible only in three dimensions. This can be seen by noticing that, in three dimensions, the Einstein-Hilbert Lagrangian does not have any propagation mode, whereas in dimensions higher than three it does propagate a unitary massless mode. With the "wrong" sign, the model necessarily displays ghosts in the spectrum. Therefore, our starting point is the Lagrangian below:

\[
L_R = e \left( -\frac{1}{\kappa} R + \alpha R^2 + \beta R_{\mu a} R^{\mu a} \right) + \epsilon^\gamma \left( R_{\mu \nu a b} R^{\mu \nu a b} - 4 R_{\mu a} R^{\mu a} + R^2 \right),
\]

where \( \alpha, \beta, \gamma \), are arbitrary dimensionless constants and \( e \) is the determinant of the vielbein. In the work of Ref. \[3\], their values are set to be

\[
\alpha = -\frac{D}{4 - D} \beta, \quad \gamma = 0.
\]

We do not adopt these choices here, because now the Lagrangian (1) contains \( R, R^\alpha, R_{\mu a} \), with the vielbein, \( \epsilon^\mu \), and the spin connection, \( \omega_{\mu a} \), taken as independent fields. We must analyze if this yields a consistent quantum theory as far as unitarity and causality are concerned. Our conventions are

\[
\begin{align*}
R_{\mu \nu a b} &= \partial_\mu \omega^b_{\nu} - \partial_\nu \omega^b_{\mu} + \omega^a_{\mu} \omega^c_{\nu} - \omega^a_{\nu} \omega^c_{\mu}, \\
R^a_{\mu} &= \epsilon^a_{\nu} R^\nu_{\mu}, \\
R &= \epsilon^a_{\mu} \epsilon^b_{\nu} R_{\mu \nu a b}, \\
\eta_{a b} &= (1, -1, -1, -1),
\end{align*}
\]

where the Greek indices refer to the world manifold and the Latin ones stand for the frame indices.

In the following, we shall consider fluctuations of the fundamental fields in order to set up the quantum theory:

\[
e^a_{\mu} = \delta^a_{\mu} + \tilde{\epsilon}^a_{\mu}, \quad \omega^a_{\mu} = \tilde{\omega}^a_{\mu}.
\]

We also define the \( \phi \) and \( \chi \) fields as

\[
\begin{align*}
\phi_{a b} &= \frac{1}{2} (\tilde{e}_{a b} + \tilde{e}_{b a}), \\
\chi_{a b} &= \frac{1}{2} (\tilde{e}_{a b} - \tilde{e}_{b a}).
\end{align*}
\]

The Lagrangian, up to second-order terms in the quantum fluctuations, can be written as

\[
(L_R)_2 = \sum_{\alpha, \beta} \psi_\alpha O_{\alpha \beta} \lambda_\beta,
\]

where \( \psi_\alpha \), \( \lambda_\beta \), carry the 40 components (\( \phi_{a b}, \chi_{a b}, \omega_{a b c} \)).

In order to investigate the spectrum of our model, we work with a complete set of spin projector operators for a conserved parity model describing a rank-3 antisymmetric tensor in two indices and a rank-2 tensor. With the help of these operators, given in the Appendix we split the bilinear piece of the Lagrangian as:

\[
(L_R)_2 = \sum_{\alpha, \beta, i, j, P} \psi_\alpha \gamma^i \delta_{ij} (J^P) P^\gamma_{ij} (J^P) \alpha \beta \lambda_\beta.
\]

Here, we adopt the conventions of Ref. \[16\]. The diagonal operators, \( P^\gamma_{ii} (J^P) \), are projectors in the spin \( (J) \) and parity \( (P) \) sectors of the field \( \Psi \). The off-diagonal operators \( (i \neq j) \) implement mappings inside the spin/parity subspace. These operators form a basis with a completeness relationship:

\[
P^{\Sigma i j} (J^P)_{\alpha \beta} P^\Lambda_{kl} (J^Q)_{\beta \gamma} = \delta^P Q \delta^P \delta^\Pi \delta^i \delta^j \delta^k \delta^l \sum_{\alpha, \beta, J^P} (J^P)_{\alpha \beta} \lambda_\beta.
\]

The \( a_{ij} (J^P) \) coefficient matrices, representing the contribution to the spin \( (J) \) and parity \( (P) \), are given by
\begin{align}
a_{ij} (2^+) &= \begin{pmatrix} \omega & \phi \\ -\frac{1}{2\kappa^2} + \frac{1}{2} \beta p^2 & \frac{i}{\sqrt{p^2} \sqrt{2\kappa^2}} \end{pmatrix}, \\
a (2^-) &= -\frac{1}{2\kappa^2} + 2\gamma p^2, \\
a_{ij} (1^+) &= \begin{pmatrix} \omega & \phi & \chi \\ -\frac{1}{\sqrt{2} \kappa^2} + \frac{1}{2} \beta p^2 & -\frac{1}{\sqrt{2} \kappa^2} & 0 \\ \frac{i}{\sqrt{p^2} \sqrt{2\kappa^2}} & 0 & 0 \end{pmatrix},
\end{align}

As it can be readily seen, the matrices for the spins \( J^P = (1^\pm, 0^+) \) are degenerate; this reflects the fact that there are some local invariances in our Lagrangian. We already expected this, since our model is invariant under linearized general coordinates and local Lorentz transformations. If these matrices were invertible, propagators saturated with the external sources could be written as

\begin{align}
a_{ij} (1^-) &= \begin{pmatrix} \omega & \phi & \chi \\ \frac{i}{\sqrt{2} \kappa} \sqrt{p^2 (D-2)^{1/2}} & 0 & 0 \\ \frac{1}{2\kappa^2} i \sqrt{p^2 (D-2)^{1/2}} & 0 & 0 \end{pmatrix}, \\
a_{ij} (0^+) &= \begin{pmatrix} \frac{\beta}{2} p^2 - \frac{1}{\kappa^2} (1 - \frac{D}{2}) + 2 (D-1) \alpha p^2 & 0 & 0 \\ \frac{\beta}{2} p^2 - \frac{1}{\kappa^2} (1 - \frac{D}{2}) + 2 (D-1) \alpha p^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
a (0^-) &= \frac{1}{\kappa^2} + 2\gamma p^2.
\end{align}

As in addition to the symmetries shared by the sources and fields, they are conserved. They then satisfy

\( \partial_{\alpha} \tau^{abc} \equiv \partial_{\alpha} \Sigma^{(ab)} \equiv \partial_{\alpha} \Sigma^{[ab]} = 0. \)

In the present case of degenerate matrices, the correct propagator is obtained by taking the inverse of the largest nondegenerate submatrix and saturating it with the conserved sources. Since these sources are conserved, the resulting propagator is gauge invariant, as shown in [17]. The nondegenerate matrices are given by

\begin{align}
a_{ij} (2^+) &= \begin{pmatrix} -\frac{1}{2\kappa^2} + \frac{1}{2} \beta p^2 & i \sqrt{p^2} \frac{1}{\sqrt{2\kappa^2}} \\ -i \sqrt{p^2} \frac{1}{\sqrt{2\kappa^2}} & 0 \end{pmatrix},
\end{align}
\[ b_{ij} (1^+) = \left( \begin{array}{c} \frac{1}{2\kappa^2} + \frac{1}{2} \beta p^2 - \frac{1}{\sqrt{2} \kappa^2} \\ -\frac{1}{\sqrt{2} \kappa^2} \\ 0 \end{array} \right), \] (18)

\[ b_{ij} (1^-) = \left( \begin{array}{c} \frac{(D-2)}{2} \beta p^2 + \frac{D-3}{2} \gamma p^2 \\ 2 (D-3) \frac{\gamma p^2}{2} \\ -\frac{(D-2)^{1/2}}{2} \frac{1}{\kappa^2} \\ 0 \end{array} \right), \] (19)

\[ b_{ij} (0^+) = \left( \begin{array}{c} \frac{D}{2} \beta p^2 - \frac{1}{\kappa^2} (1 - \frac{D}{2}) + 2 (D-1) \alpha p^2 - i \sqrt{p^2 (D-2)} \frac{1}{\sqrt{2} \kappa^2} \\ i \sqrt{p^2 (D-2)} \frac{1}{\sqrt{2} \kappa^2} \\ 0 \end{array} \right), \] (20)

\[ a (2^-) = -\frac{1}{2\kappa^2} + 2 \gamma p^2; \] (21)

\[ a (0^-) = \frac{1}{\kappa^2} + 2 \gamma p^2; \] (22)

their respective inverses are listed in the sequel:

\[ a_{ij}^{-1} (2^+) = -\frac{2\kappa^4}{p^2} \left( \begin{array}{c} 0 \\ \frac{1}{\sqrt{2} \kappa^2} \\ -\frac{1}{\sqrt{2} \kappa^2} + \frac{1}{2} \beta p^2 \end{array} \right), \] (23)

\[ b^{-1} (1^+) = -2\kappa^4 \left( \begin{array}{c} 0 \\ \frac{1}{\sqrt{2} \kappa^2} \\ -\frac{1}{\sqrt{2} \kappa^2} + \frac{1}{2} \beta p^2 \end{array} \right), \] (24)

\[ b_{ij}^{-1} (1^-) = -\frac{4\kappa^4}{(D-2)} \left( \begin{array}{c} 0 \\ \frac{(D-2)^{1/2}}{2} \frac{1}{\kappa^2} \\ \frac{(D-2)^{1/2}}{2} \beta p^2 + \frac{(D-2)^{1/2}}{2} \frac{1}{\kappa^2} (D-3) \gamma p^2 \end{array} \right), \] (25)

\[ b_{ij}^{-1} (0^+) = -\frac{2\kappa^4}{(D-2)^2 \kappa^2} \left( \begin{array}{c} 0 \\ -i \sqrt{p^2 (D-2)} \frac{1}{\sqrt{2} \kappa^2} \\ D \beta p^2 - \frac{1}{\kappa^2} (1 - \frac{D}{2}) + 2 (D-1) \alpha p^2 \end{array} \right), \] (26)

\[ a^{-1} (2^-) = \frac{1}{2\gamma \left( p^2 - \frac{1}{4\gamma \kappa^2} \right)}, \] (27)

\[ a^{-1} (0^-) = \frac{1}{2\gamma \left( p^2 + \frac{1}{2\gamma \kappa^2} \right)}. \] (28)

We immediately get that there are two nonmassive poles in the \( 2^+, 0^+ \) sectors and two massive poles in the \( 2^-, 0^- \) sectors. These results highlight a remarkable difference with respect to [3], because we do not have spin-2 massive propagation for the vielbein; so, we do not expect spin-2 massive graviton in any dimension. Actually, as it will be shown in the next section, if we impose unitarity, the model becomes trivial, in the sense that none of the modes can propagate.
III. TOWARD A MASSIVE GRAVITON

Our initial motivation was to investigate the role of a propagating torsion in the description of massive gravity. Ever since, our results are not encouraging in the sense that, as seen from the previous analysis, there is no room for the propagation of a massive graviton in our model.

From the inspection of the structure of the matrices \(T_{\mu\nu}^\alpha T_{\mu\nu}^\alpha, T_{\alpha}^\mu T_{\mu}^\alpha, T_{\nu}^\mu T_{\mu}^\nu\), we can understand how to cure this problem. In the curvature terms, we have only contributions of the form \(\omega\phi, \phi\omega\) propagators. Once the structure of the \(\omega\phi, \phi\omega\) contributions are always of the form \(\sqrt{p^2}(\text{function of the constants } \kappa, \alpha, \beta, \text{ and } D)\), we do not expect that the determinant may exhibit zeroes at \(p^2 = \mu^2 \neq 0\), which would correspond to massive poles. So, we claim that the only way to get a massive pole is to insert a \(\phi\phi\) contribution into these matrices. But, this is possible only if we enrich our initial Lagrangian with explicit torsion terms.

Within all possible quadratic terms that we can form with torsion, the independent contributions turn out to be: \(T_{\mu\nu}^\alpha T_{\mu\nu}^\alpha, T_{\mu}^\alpha T_{\mu}^\alpha, T_{\nu}^\mu T_{\mu}^\nu\). For an initial attempt, we could take a representative case and check that it does the job we have in mind, namely, to introduce a massive pole. But, as we wish to find possible unitary Lagrangians that describe massive gravitons, we have aside our initial model and consider the most general parity-preserving Lagrangian without higher derivatives, that is

\[
\mathcal{L} = -\lambda R + \xi R^2 + (s + t) R_{ab} R^{ab} + (s - t) R_{ab} R^{ab} + \frac{1}{6} (2d + q) R_{abcd} R^{abcd} + \frac{1}{6} (2d + q - 6r) R_{abcd} R^{abcd} + \frac{2}{3} (d - q) R_{abcd} R^{abcd} + \frac{1}{12} (4u + v + 3\lambda) T_{ab} T^{ab} + \frac{1}{D - 1} (-u + 2w - (D - 1) \lambda) T_{ab} b T^{ac} c. \tag{29}
\]

The constant factors are chosen in this cumbersome way in order to simplify the analysis of the conditions for unitarity. By linearizing \(\mathcal{L}\) and, using the results of the Appendix, we write \(\mathcal{L}_2\) in terms of the spin operators. The total linearized Lagrangian can be written again as

\[
(\mathcal{L})_2 = \sum_{\alpha, \beta, ij, J^P} \psi_\alpha a_{ij}^\psi (J^P) p_{ij}^{\psi\lambda} (J^P) a_{ij}^\lambda \lambda. \tag{30}
\]

But now, the coefficient matrices are given by

\[
a (0^+) = \begin{pmatrix}
[D_s + 2d - 2r + 2(1 - \xi) p^2 + w - i\sqrt{2} \sqrt{p^2} w] & 0 & 0 \\
i\sqrt{2} \sqrt{p^2} w & 2[w - (\frac{D - 2}{D - 4})] & 0 \\
0 & 0 & 0
\end{pmatrix}, \tag{31}
\]

\[
a (1^-) = \begin{pmatrix}
\frac{[D_s + 2d - 2r + (s + t) + d]}{2} p^2 & -\frac{(D - 2)}{2} \sqrt{\frac{2}{p^2}} & -i\sqrt{p^2} \frac{(D - 2)}{2} \sqrt{\frac{2}{p^2}} & -i\sqrt{p^2} \frac{(D - 2)}{2} \sqrt{\frac{2}{p^2}} \\
-i\sqrt{p^2} \frac{(D - 2)}{2} \sqrt{\frac{2}{p^2}} & \frac{[w + u + (D - 2)]}{2} p^2 & \frac{[w + u + (D - 2)]}{2} p^2 & \frac{[w + u + (D - 2)]}{2} p^2 \\
i\sqrt{p^2} \frac{(D - 2)}{2} \sqrt{\frac{2}{p^2}} & \frac{[w + u + (D - 2)]}{2} p^2 & \frac{[w + u + (D - 2)]}{2} p^2 & \frac{[w + u + (D - 2)]}{2} p^2 \\
i\sqrt{p^2} \frac{(D - 2)}{2} \sqrt{\frac{2}{p^2}} & \frac{[w + u + (D - 2)]}{2} p^2 & \frac{[w + u + (D - 2)]}{2} p^2 & \frac{[w + u + (D - 2)]}{2} p^2
\end{pmatrix}, \tag{32}
\]

\[
a (2^+) = \begin{pmatrix}
[s + 2d - 2r] p^2 & \frac{w}{2} & -i\sqrt{p^2} \frac{1}{2} \sqrt{\frac{2}{p^2}} u & u + \lambda
\end{pmatrix}, \tag{33}
\]

\[
a (2^-) = dp^2 + \frac{u}{2}, \tag{34}
\]

\[
a (1^+) = \begin{pmatrix}
(t - 2r) p^2 + \frac{(w + 4u)}{6} & -\frac{(2u - w)}{3} \sqrt{\frac{2}{p^2}} & -i\sqrt{p^2} \frac{(2u - w)}{3} & \frac{(w + 4u)}{3} \sqrt{\frac{2}{p^2}} \\
-i\sqrt{p^2} \frac{(2u - w)}{3} & i\sqrt{p^2} \frac{(u + v)}{3} & \frac{(u + v)}{3} \sqrt{\frac{2}{p^2}} & -i\sqrt{p^2} \frac{(2u - w)}{3}
\end{pmatrix}, \tag{35}
\]

\[
a (0^-) = qp^2 + v. \tag{36}
\]
Again, we have degeneracies and, in order to obtain the saturated propagator, we must pick out the nondegenerate submatrices. We quote their inverses below:

\[ a^{-1} (2^+) = p^{-2} \left( (s + 2d - 2r) (u + \lambda) p^2 + \frac{u}{2} \lambda \right)^{-1} \left( \begin{pmatrix} (u + \lambda) p^2 & \frac{i}{\sqrt{p^2}} \lambda \sqrt{2} u \\ -i \sqrt{p^2} \lambda \sqrt{2} u & (s + 2d - 2r) p^2 + \frac{u}{2} \right), \] (37)

\[ a^{-1} (0^-) = (q p^2 + v)^{-1}, \] (38)

\[ a^{-1} (2^-) = (d p^2 + \frac{u}{2})^{-1}, \] (39)

\[ b^{-1} (1^+) = \left[ \frac{1}{3} (t - 2r) (u + v) p^2 + \frac{w v}{2} \right]^{-1} \left( \frac{(w + v) p^2}{3} + \frac{w v}{2} \right) \] (40)

\[ \times \left( \frac{(s + t) + p}{2} \right) \left( \frac{2}{3} \right) \left( \frac{2^{(2) \frac{1}{2}}}{2} \right) \left( \frac{D - 2}{3} \right) \left( \frac{D - 2}{3} (s + t) + p \right) \left( p^2 + \frac{2 (D - 2) w + u}{2 (D - 1)} \right) \] (41)

\[ b^{-1} (0^+) = p^{-2} \left[ \frac{1}{2} (D + 2d - 2r + 2 (D - 1) \xi) \left( w - \frac{D - 2}{2} \lambda \right) p^2 - w (D - 2) \lambda \right]^{-1} \] (42)

\[ \times \left( \frac{2}{\sqrt{2}} \sqrt{p^2} w \right) \left[ D (D + 2d - 2r + 2 (D - 1) \xi) p^2 + w \right) \]

We can now realize that our chances to describe a massive graviton have enhanced. At the same time, the introduction of the new term endowed the other spin sector with dynamics. So, apparently, we could obtain a Pauli-Fierz analogue, that is, with only a spin-2 massive particle propagating, if we prevent the extra modes from propagating by imposing relations among the parameters of the Lagrangian, so that the extra poles vanish. But we do not investigate it. Our aim is only to give some insights on how to describe a massive graviton whenever we have propagating torsion.

### IV. ANALYSIS OF TACHYONIC AND GHOSTS MODES IN THE EXTENDED MODEL

Now, that we have obtained the inverses of the nondegenerate submatrices, we can write the saturated propagator with an external current, \( S_n \):

\[ \Pi = i \sum_{j} S_{n} A_{ij} \left( J^P \right) \left( P_{ij} \right) = \frac{1}{\alpha \beta} S_{\alpha \beta} \left( p^2 - m^2 \right)^{-1}, \] (43)

where \( A_{ij} \) are the matrices given above with the massive pole extracted. So, these are \( 2 \times 2 \) (or \( 1 \times 1 \) for \( 1^- \) and \( 2^- \) sectors) matrices, which are degenerate at the pole. According to Ref. \[ 14 \], for a massive propagating particle not to be a tachyon or a ghost, we must require that

\[ m^2 > 0 \quad \text{and} \quad \text{Im Res} (\Pi) \big|_{p^2 = m^2} > 0, \] (44)

which implies that, for each \( J^P \), we must have

\[ (-1)^P trA \left( J^P \right) \big|_{p^2 = m^2} > 0. \] (45)

The \((-1)^P\) factor comes from the evaluation of the spin operators at the pole. The even (odd) operators have an even (odd) number of \( \theta \) in their structure and each \( \theta \) contributes with a \((-1)^P\) factor. For each spin we have the conditions:

\[ 2^+ : 2d - 2r + s > 0; \quad u \lambda (u + \lambda) < 0. \]
\[ 2^- : d < 0; \quad u > 0. \]
\[ 1^+ : 2r + t > 0; \quad uv (u + v) < 0. \]
\[ 1^- : \left[ \frac{(D - 2)}{2} (s + t) + d \right] < 0; \quad wu \left[ w + u \left( \frac{D - 2}{2} \right) \right] > 0. \]
$0^+ : |D s + 2 d - 2 r + 2 (D - 1) \xi| > 0; \ w \lambda \left[w - \left(\frac{D - 2}{2}\right) \lambda\right] > 0.$

$0^- : q < 0; \ v > 0.$

For the case of massless poles, the analysis requires extra care, because there are new singularities, coming from the operators themselves, when we evaluate them at the pole $p^2 = 0$. For this reason, we proceed in a somewhat different form.

Because of the singularities of spin operators, even the matrices with massive poles can contribute to the residue of the massless poles. The $p^{-6}$ and $p^{-4}$ singularities cancel out when we use the source constraints. It can be shown that from all $p^{-2}$ singularities, only those associated with the Einstein-Hilbert survives. The final result to the residue of massless poles is

$$\text{Im Res} (\Pi)_{p^2=0} = \frac{\lambda^{-1}}{p^2} \left( \tau^{ab} \Sigma^{ab} \right) \left( \begin{array}{cc} 4 & 2i \\ -2i & 1 \end{array} \right) \times \left[ P 2^+, \eta \right] - \frac{1}{D - 2} P 0^+, \eta$$

$$\times \left( \tau^{cd} \Sigma^{cd} \right). \quad (46)$$

As the matrix that appears in this equation is Hermitian, it can be diagonalized by a suitable unitary matrix. Making this change of variables, we can rewrite this expression as

$$\text{Im Res} (\Pi)_{p^2=0} = \frac{\lambda^{-1}}{p^2} \left( \tau^{(ab)} \tau_{(ab)} \right) - \frac{1}{D - 2} \tau^{a\ast b} \tau^{b \ast a}. \quad (47)$$

Furthermore, choosing a suitable basis in the D-dimensional Minkowski space, we can expand the source $\tau^{(ab)}$ as

$$\tau^{(ab)} = c_1 p^a p^b + c_{2a} (p^b \epsilon^{a\alpha} + p^b \epsilon^{a\alpha})$$

$$+ c_{3a\beta} (\epsilon^{a\alpha} \epsilon^{b\beta}) \quad (48)$$

where,

$$p^a = (p_0, \vec{p}), \quad (49a)$$

$$q^a = (p_0, -\vec{p}), \quad (49b)$$

$$\epsilon^{a\alpha}, \quad \alpha = 1, ..., D - 2 \quad (49c)$$

with,

$$p^2 = q^2 = 0, \quad (49d)$$

$$p . q = (p_0)^2 + (\vec{p})^2 \neq 0, \quad (49e)$$

$$p . \epsilon_{\alpha} = q . \epsilon_{\alpha} = 0, \quad (49f)$$

$$\epsilon_{\alpha} . \epsilon_{\beta} = -\delta_{\alpha\beta}. \quad (49g)$$

These vectors span the D-dimensional Minkowski space. Plugging (48) in (46), we obtain

$$\text{Im Res} (\Pi)_{p^2=0} = \frac{\lambda^{-1}}{p^2} \left( \epsilon_{3a\beta} \epsilon_{3\beta} - \frac{1}{D - 2} \epsilon_{3a\alpha} \epsilon_{3\beta} \right). \quad (50)$$

Let us relabel the $(D - 2)$ $c_i$’s by $c_i$, with $i = 1, ..., D - 2 = N$. So, this expression can be written as:

$$\text{Im Res} (\Pi)_{p^2=0} = \frac{\lambda^{-1}}{p^2} \sum_{i, j = 1}^N \frac{1}{N} (|c_i|^2 + |c_j|^2 - (c_i c_j + c_j c_i)). \quad (51)$$

This expression vanishes for $D = 3 \ (N = 1)$ and is positive-definite for $D > 3 \ (N > 1)$ if the condition $\lambda = \frac{1}{D - 2} > 0$ is chosen. As the above matrix is degenerate, there is only one propagating mode.

We are now in a position to analyze the spectrum of the initial model of Sec. II. Comparing the two Lagrangians, (11) and (29), we see that both agree if we identify

$$\lambda = \frac{1}{k^2}; \quad \beta - 4 \gamma = s + t;$$

$$\alpha + \gamma = \xi; \quad \gamma = \frac{d}{2} = \frac{q}{2} = r$$

$$u = - v = - \lambda; \quad D - 2 = u + w = 0. \quad (52)$$

We see, from the matrices (47)-(48), that these relations confirm our previous result that there are only massive poles in the $2^-$ and $0^-$ sectors. Furthermore, the conditions in the parameters for the full Lagrangian, derived above, tell us that the propagation of these modes are incompatible, since $d = q$. Therefore, the initial model has no propagation mode for $D = 3$ and has only a massless graviton propagating for $D > 3$.

Now, we must search for the possible unitary Lagrangians resulting from the possible intersections of the conditions above for the extended Lagrangian. However, there is a net conflict among these relations. Namely, the conditions for the $2^-$ sector requires $u > 0$, whereas the conditions for $2^+$ impose $u < 0$. Therefore, for arbitrary values of the parameters in the Lagrangian (29), we have a nonunitary model. In order to achieve unitarity for a propagating massive graviton, we must assume that some modes do not propagate. In so doing, the conditions related to these modes do not need to be satisfied. The conditions for the nonpropagation of a mode are readily seen from the matrices (47)-(48). They are obtained by requiring the absence of a pole related to the mode. In the sequel, we present the conditions that must be satisfied in order to get a unitary model with a propagating massive graviton:

i) $d = 0; \ 2r + t < 0; \ s = -t; \ u = -v;$

$$\frac{v}{D - 2} > v > 0; \ q < 0; \ w (w - \frac{D - 2}{2}) > 0$$

$$\frac{\tilde{D} t - 2 r + 2 (D - 1) \xi > 0.}$$
This corresponds to the following Lagrangian:

\[
\mathcal{L} = -\frac{1}{\kappa^2} R + \xi R^2 - 2t R_{ab} R^{ba} + \frac{1}{6} q R_{abcd} R^{abcd} + \frac{1}{6} (q - 6r) R_{abcd} R^{cdab} - \frac{2}{3} q R_{abcd} R^{acbd} + \frac{1}{4} \left( -v + \frac{1}{\kappa^2} \right) (T_{abc} T^{abc} - 2 T_{abc} T^{bca}) + \frac{1}{D - 1} \left( v + 2w - \frac{(D - 1)}{\kappa^2} \right) T_{ab} T^{bca},
\]

(53)

where the parameters satisfy the conditions of item i. The propagating modes are: a spin-2\textsuperscript{+} massless (for \( D \geq 4 \)), and massive spin-2\textsuperscript{-}, spin-0\textsuperscript{+}, spin-0\textsuperscript{-}. There are several particular cases of (53) corresponding to inhibition of the propagation of the modes 0\textsuperscript{+} and 0\textsuperscript{-}.

ii) \( d = 0; \ -2r + s > 0; \ t = -2r; \ w = -u \left( \frac{D-2}{2} \right); \ -\frac{1}{\kappa^2} \ < \ u < \ 0; \ Ds - 2r + 2(D - 1) \xi = 0; \ q < 0; \ v > 0 \).

With these parameters, we have the second unitary Lagrangian:

\[
\mathcal{L} = -\frac{1}{\kappa^2} R + \frac{2r - Ds}{2(D - 1)} R^2 + (s - 2r) R_{ab} R^{ab} + \frac{1}{6} q R_{abcd} R^{abcd} + \frac{1}{6} (q - 6r) R_{abcd} R^{cdab} - \frac{2}{3} q R_{abcd} R^{acbd} + \frac{1}{12} \left( 4u + v + \frac{3}{\kappa^2} \right) T_{abc} T^{abc} + \frac{1}{6} \left( -2u + v - \frac{3}{\kappa^2} \right) T_{abc} T^{bca} - \left( u + \frac{1}{\kappa^2} \right) T_{ab} T^{bca}.
\]

(54)

In addition to the massive graviton, this model carries the massless graviton (for \( D \geq 4 \)) and a 0\textsuperscript{-} massive particle.

iii) \( d = 0; \ -2r + s > 0; \ u = -v; \ s + t < 0; \ 2r + t < 0; \ w \left( w - v \left( \frac{D-2}{2} \right) \right) < 0; \ \frac{1}{\kappa^2} \ > \ v > 0; \ q < 0; \ Ds - 2r + 2(D - 1) \xi = 0 \).

The related Lagrangian is

\[
\mathcal{L} = -\lambda R + \frac{2r - Ds}{2(D - 1)} R^2 + (s + t) R_{ab} R^{ab} + \frac{1}{6} q R_{abcd} R^{abcd} + \frac{1}{2} \left( v - \frac{1}{\kappa^2} \right) T_{abc} T^{bca} + \frac{1}{6} (q - 6r) R_{abcd} R^{cdab} - \frac{2}{3} q R_{abcd} R^{acbd} + \frac{1}{4} \left( -v + \frac{1}{\kappa^2} \right) T_{abc} T^{abc} + (s - t) R_{ab} R^{ba} + \frac{1}{12} \left( 4u + v + \frac{3}{\kappa^2} \right) T_{abc} T^{abc} + \frac{1}{6} \left( -2u + v - \frac{3}{\kappa^2} \right) T_{abc} T^{bca}.
\]

(55)

This model propagates the massless (for \( D \geq 4 \)) and the massive graviton, along with massive 1\textsuperscript{-} and 0\textsuperscript{-} particles.

iv) \( d = 0; \ 2r + t > 0; \ -2r + s > 0; \ w = -u \left( \frac{D-2}{2} \right); \ -\frac{1}{\kappa^2} \ < \ u < \ 0; \ xi = \frac{2r - Ds}{2(D - 1)}; \ q < 0; \ v > 0; \ u > -v \).

These conditions exhaust our possibilities of describing a massive graviton in a unitary way. The Lagrangian associated is given by

\[
\mathcal{L} = -\lambda R + \frac{2r - Ds}{2(D - 1)} R^2 + (s + t) R_{ab} R^{ab} - \left( u - \frac{1}{\kappa^2} \right) T_{ab} T^{bca} + \frac{1}{6} q R_{abcd} R^{abcd} + \frac{1}{6} (q - 6r) R_{abcd} R^{cdab} - \frac{2}{3} q R_{abcd} R^{acbd} + \frac{1}{12} \left( 4u + v + \frac{3}{\kappa^2} \right) T_{abc} T^{abc} + \frac{1}{6} \left( -2u + v - \frac{3}{\kappa^2} \right) T_{abc} T^{bca} + (s - t) R_{ab} R^{ba}.
\]

(56)

In addition to the massive and massless graviton (for \( D \geq 4 \)), there are massive 1\textsuperscript{+} and 0\textsuperscript{-} dynamical particles. As we have pointed out in the Introduction, there is considerable interest in \( D = 3 \) gravity. So, it is worthy to stress that our results are valid for this dimension too. With the exception that in this dimension there is no propagating massless spin-2\textsuperscript{+}, the other analyzed features are essentially the same. In order to compare with the work of Ref. [10], we investigate these conditions for \( D = 4 \). In this case, we have the following unitary Lagrangians:
the parameters, due to the extra parameter conditions compatible with the unitarity constraints on than two curvature-type tensors, there are many more verify that these Lagrangians agree with those listed in we find a set of unitary Lagrangians in D dimensions without higher derivatives, and we inves-
sider the most general parity-preserving Lagrangian in as we are interested in the analysis of unitarity, we con-
explicit torsion terms in the Lagrangian. Furthermore, wish to introduce massive gravitons, we should include
Some considerations guide us to the conclusion that, if we
sufficient to describe massive gravitons, and the requirement
of unitarity is so severe that the model becomes trivial. Some considerations guide us to the conclusion that, if we
wish to introduce massive gravitons, we should include explicit torsion terms in the Lagrangian. Furthermore, as we are interested in the analysis of unitarity, we con-
sider the most general parity-preserving Lagrangian in D dimensions without higher derivatives, and we inves-
tigate the constraints on the parameters so as to ensure the unitarity. We find a set of unitary Lagrangians in D dimensions that propagate a massive graviton, and we verify that these Lagrangians agree with those listed in in the particular case of $D = 4$. But, for $D \neq 4$, as the Gauss-Bonet theorem includes products of more than two curvature-type tensors, there are many more conditions compatible with the unitarity constraints on the parameters, due to the extra parameter $\alpha$.

The initial purpose was partly reached, once we have
found unitary Lagrangians with propagating torsion that describes at least a massive graviton. However, what we have done is not quite a generalization of the results of since there, the linearized Lagrangian corresponds to the Pauli-Fierz Lagrangian, which is intrinsically defined in the second-order formalism for gravitation. We could try to define such a model in case the torsion propagates by inhibiting the propagation of all the other modes, but the massive graviton. However, we are here more interested on the considerations that should be made to shed some light on models for massive gravitons, whenever we consider a more fundamental approach to gravitation (in the sense of gauge theories). The lesson we draw is that torsion actually plays a crucial role in the discussion, confirming previous results we have referred to in the course of this paper.

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APPENDIX: SPIN PROJECTORS
1. $P_{21}^{2\omega} (0^+)_{abcdf} = \frac{\sqrt{7}}{2(D-1)} (\theta_{ab}\theta_{dc}pf - \theta_{ab}\theta_{df}pc)$

2. $P_{31}^{2\omega} (0^+)_{abcdf} = \frac{\sqrt{7}}{2(D-1)^{1/2}} (\omega_{ab}\theta_{de}pf - \omega_{ab}\theta_{df}pe)$

3. $P_{12}^{\phi\phi} (0^+)_{abcdf} = \frac{\sqrt{7}}{2(D-1)^{1/2}} (\theta_{ab}\theta_{df}pe - \theta_{ac}\theta_{df}pb)$

4. $P_{13}^{\phi\phi} (0^+)_{abcdf} = \frac{\sqrt{7}}{2(D-1)^{1/2}} (\theta_{ab}pc - \theta_{ac}pb) \omega df$

5. $P_{22}^{\phi\phi} (0^+)_{abcd} = \frac{1}{\sqrt{7}} \theta_{ab} \theta_{cd}$

6. $P_{33}^{\phi\phi} (0^+)_{abcd} = \omega_{ab} \omega_{cd}$

7. $P_{23}^{\phi\phi} (0^+)_{abcd} = \frac{1}{\sqrt{D-1}} \theta_{ab} \omega_{cd}$

8. $P_{32}^{\phi\phi} (0^+)_{abcd} = \frac{1}{\sqrt{D-1}} \omega_{ab} \theta_{cd}$

9. $P_{11}^{\omega\omega} (0^+)_{abcde} = \frac{1}{2(D-1)^{1/2}} \left[ \theta_{ab} \left( \theta_{de}\omega_{cf} - \theta_{df}\omega_{ce} \right) - \theta_{ac} \left( \theta_{de}\omega_{bf} - \theta_{df}\omega_{be} \right) \right]$

10. $P_{11}^{\omega\omega} (1^-)_{abcde} = \frac{1}{2(D-2)^{1/2}} \left[ \theta_{ab} \left( \theta_{de}\omega_{cf} - \theta_{df}\omega_{ce} \right) - \theta_{ac} \left( \theta_{de}\omega_{bf} - \theta_{df}\omega_{be} \right) \right]$

11. $P_{31}^{\omega\omega} (1^-)_{abcde} = -\frac{1}{2(D-2)^{1/2}} \left( \theta_{dc}\theta_{af}pb + \theta_{dc}\theta_{bf}pa - \theta_{df}\theta_{ac}pb - \theta_{df}\theta_{bc}pa \right)$

12. $P_{32}^{\omega\omega} (1^-)_{abcde} = \frac{1}{2} \left[ \omega_{ad} \left( \theta_{bf}pf - \theta_{bc}pf \right) + \omega_{bd} \left( \theta_{bf}pf - \theta_{ac}pf \right) \right]$

13. $P_{13}^{\omega\phi} (1^-)_{abcde} = \frac{1}{2(D-2)^{1/2}} \left( \theta_{ac}\theta_{bd}pf + \theta_{ac}\theta_{bf}pd - \theta_{ab}\theta_{cd}pf - \theta_{ab}\theta_{cf}pd \right)$

14. $P_{23}^{\omega\phi} (1^-)_{abcde} = \frac{1}{2} \left[ \omega_{da} \left( \theta_{fc}pb - \theta_{fb}pc \right) + \omega_{fa} \left( \theta_{dc}pb - \theta_{db}pc \right) \right]$

15. $P_{41}^{\omega\phi} (1^-)_{abcde} = \frac{1}{2(D-2)^{1/2}} \left( \theta_{ac}\theta_{df}pb - \theta_{bc}\theta_{bf}pd - \theta_{af}\theta_{dc}pb + \theta_{bf}\theta_{de}pd \right)$

16. $P_{32}^{\omega\phi} (1^-)_{abcde} = \frac{1}{2} \left[ \omega_{ad} \left( \theta_{be}pf - \theta_{bf}pc \right) + \omega_{bd} \left( \theta_{af}pc - \theta_{ac}pf \right) \right]$

17. $P_{14}^{\omega\phi} (1^-)_{abcde} = \frac{1}{2(D-2)^{1/2}} \left( \theta_{ac}\theta_{bd}pf - \theta_{ab}\theta_{cd}pf - \theta_{ac}\theta_{bf}pd + \theta_{ab}\theta_{cf}pd \right)$

18. $P_{24}^{\omega\phi} (1^-)_{abcde} = \frac{1}{2} \left[ \omega_{da} \left( \theta_{fc}pb - \theta_{fb}pc \right) + \omega_{fa} \left( \theta_{dc}pb - \theta_{db}pc \right) \right]$

19. $P_{33}^{\phi\phi} (1^-)_{abcd} = \frac{1}{2} \left( \theta_{ac}\omega_{bd} + \theta_{bc}\omega_{ad} + \theta_{ad}\omega_{bc} + \theta_{bd}\omega_{ac} \right)$

20. $P_{44}^{\phi\phi} (1^-)_{abcd} = \frac{1}{2} \left( \theta_{ac}\omega_{bd} - \theta_{ad}\omega_{bc} - \theta_{bc}\omega_{ad} + \theta_{bd}\omega_{ac} \right)$

21. $P_{34}^{\phi\phi} (1^-)_{abcd} = \frac{1}{2} \left( \theta_{ac}\omega_{bd} - \theta_{ad}\omega_{bc} + \theta_{bc}\omega_{ad} - \theta_{bd}\omega_{ac} \right)$

22. $P_{43}^{\phi\phi} (1^-)_{abcd} = \frac{1}{2} \left( \theta_{ac}\omega_{bd} + \theta_{ad}\omega_{bc} - \theta_{bc}\omega_{ad} - \theta_{bd}\omega_{ac} \right)$

23. $P_{22}^{\phi\phi} (1^-)_{abcde} = \frac{1}{2} \omega_{ad} \left[ \theta_{bc}\omega_{cf} - \theta_{bf}\omega_{ce} - \theta_{ce}\omega_{bf} + \theta_{cf}\omega_{be} \right]$

24. $P_{12}^{\omega\omega} (1^-)_{abcde} = \frac{1}{2(D-2)^{1/2}} \left\{ \theta_{ab} \left[ \theta_{ce}\omega_{df} - \theta_{cf}\omega_{de} \right] - \theta_{ac} \left[ \theta_{bc}\omega_{df} - \theta_{bf}\omega_{de} \right] \right\}$

25. $P_{21}^{\omega\omega} (1^-)_{abcde} = \frac{1}{2(D-2)^{1/2}} \left\{ \omega_{ab} \left[ \theta_{df}\theta_{ce} - \theta_{de}\theta_{cf} \right] - \omega_{ab} \left[ \theta_{df}\theta_{bc} - \theta_{dc}\theta_{bf} \right] \right\}$

26. $P_{21}^{2\omega} (2^+)_{abcde} = \frac{\sqrt{7}}{4} \left[ \left( \theta_{ad}\theta_{be} + \theta_{ac}\theta_{bd} - \frac{2(D-1)}{\theta_{ab}\theta_{de}} \right) p_f - \left( \theta_{ad}\theta_{bf} + \theta_{af}\theta_{bd} - \frac{2}{(D-1)} \theta_{ab}\theta_{df} \right) p_c \right]$

27. $P_{12}^{2\omega} (2^+)_{abcde} = \frac{\sqrt{7}}{4} \left[ \left( \theta_{ad}\theta_{bf} + \theta_{af}\theta_{bd} - \frac{2(D-1)}{\theta_{ab}\theta_{df}} \right) p_c - \left( \theta_{ad}\theta_{cf} + \theta_{af}\theta_{cd} - \frac{2}{(D-1)} \theta_{ac}\theta_{df} \right) p_b \right]$

28. $P_{22}^{2\omega} (2^+)_{abcd} = \frac{1}{2} \left( \theta_{ac}\theta_{bd} + \theta_{ad}\theta_{bc} \right) - \frac{1}{(D-1)} \theta_{ab}\theta_{cd}$
29. $P_{11}^{αω}(2^+)_{abcdef} = \frac{1}{4}θ_{ad}[θ_{beωcf} - θ_{bfωce} - θ_{ceωbf} + θ_{cfωbe}] - \frac{1}{4}θ_{ad}[θ_{beωcf} - θ_{bfωce} - θ_{ceωbf} + θ_{cfωbe}]
+ \frac{1}{4}θ_{de}[θ_{dfωbe} - θ_{dbωce} - θ_{cdωbe} - \frac{1}{2(D-2)}] [θ_{ab}(θ_{dcωcf} - θ_{dfωce}) - θ_{ac}(θ_{deωbf} - θ_{dfωbe})]

30. $P_{31}^{αω}(1^+)_{abcdef} = -\frac{1}{4}θ_{ad}(θ_{baf}θ_{be}p_e - θ_{bad}θ_{af}p_f - θ_{abd}θ_{af}p_e - θ_{abe}θ_{af}p_c)$

31. $P_{32}^{αω}(1^+)_{abcdef} = -\frac{1}{2}θ_{af}θ_{ae} + θ_{af}θ_{be} p_d$

32. $P_{13}^{αω}(1^+)_{abcdef} = \frac{1}{4}θ_{af}θ_{ad}p_b - θ_{bad}θ_{af}p_c + θ_{ad}θ_{bf}p_c - θ_{af}θ_{af}p_b)

33. $P_{23}^{αω}(1^+)_{abcdef} = -\frac{1}{2}θ_{bd}θ_{cf} - θ_{cd}θ_{df} p_a$

34. $P_{33}^{αω}(1^+)_{abcdef} = \frac{1}{2}θ_{ac}θ_{bd} - θ_{ad}θ_{bc}$

35. $P_{22}^{αω}(1^+)_{abcdef} = \frac{1}{2}ω_{ad}[θ_{be}θ_{cf} - θ_{bf}θ_{ce}]

36. $P_{12}^{αω}(1^+)_{abcdef} = -\frac{1}{4}θ_{ae}(θ_{bfωcd} - θ_{cfωbd}) - θ_{af}(θ_{beωcd} - θ_{ceωbd})]

37. $P_{21}^{αω}(1^+)_{abcdef} = -\frac{1}{2}θ_{cd}(θ_{bfωac} - θ_{bcωaf}) + θ_{bd}(θ_{ccωaf} - θ_{cfωac})]

38. $P_{11}^{αω}(1^+)_{abcdef} = \frac{1}{4}θ_{ad}[θ_{beωcf} - θ_{bfωce} - θ_{ceωbf} + θ_{cfωbe}] + \frac{1}{4}θ_{ae}[θ_{cdωcf} - θ_{bfωce}] + \frac{1}{4}θ_{af}[θ_{bdωce} - θ_{cdωbe}]

39. $P_{01}^{αω}(0^-)_{abcdef} = \frac{1}{6}θ_{ad}[θ_{be}θ_{cf} - θ_{bf}θ_{ce}] + θ_{ac}[θ_{bf}θ_{cf} - θ_{bd}θ_{cf}] + θ_{af}[θ_{bd}θ_{ce} - θ_{be}θ_{cd}]

40. $P_{20}^{αω}(2^-)_{abcdef} = \frac{1}{2}θ_{ad}[θ_{be}θ_{cf} - θ_{bf}θ_{ce}] - \frac{1}{2(D-2)} [θ_{ab}(θ_{de}θ_{cf} - θ_{df}θ_{ce}) - θ_{ac}(θ_{de}θ_{bf} - θ_{df}θ_{be})]
+ \frac{1}{6}θ_{ad}[θ_{be}θ_{cf} - θ_{bf}θ_{ce}] + θ_{ae}[θ_{bf}θ_{cd} - θ_{bd}θ_{cf}] + θ_{af}[θ_{bd}θ_{ce} - θ_{be}θ_{cd}]

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