Polarized Photon Structure: $g_1^\gamma$ and $g_2^\gamma$ *)

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We investigate the polarized photon structure functions $g_1^\gamma$ and $g_2^\gamma$ which can be studied in the future polarized version of ep or $e^+e^-$ colliders. The NLO QCD calculations of $g_1^\gamma$ and the possible twist-3 effects in $g_2^\gamma$ are discussed.

1 Introduction

In the last ten years, the spin-dependent structure functions $g_1$ and $g_2$ of the nucleon have been intensively studied in the polarized deep-inelastic scattering of polarized lepton on polarized nucleon targets. Based on the next-to-leading order (NLO) QCD analysis of the experimental data, the polarized parton distributions inside the nucleon have been extracted. The gluon polarization $\Delta G$ is now one of the central issues in spin physics.

On the other hand, there has been growing interest in the polarized photon structure functions. Especially the first moment of a photon structure function $g_1^\gamma$ has attracted much attention in the literature in connection with the axial anomaly which is also relevant in the nucleon spin structure function $g_1^{p(n)}$. The polarized structure function $g_1^\gamma$ could be experimentally studied in the polarized version of ep collider HERA, or more directly measured by the polarized $e^+e^-$ collision in the future linear collider. And the next-to-leading order QCD analysis of $g_1^\gamma$ has been performed in the literature.

Now there exists another structure function $g_2^\gamma$ for the virtual photon target, where the twist-3 effect is also relevant in addition to the usual twist-2 effect. In this talk, we first briefly summarize our results for the $g_1^\gamma$ for the virtual photon target, and then investigate the twist-3 effects in $g_2^\gamma$.

*) Presented by T. Uematsu at the Advanced Study Institute on Symmetries and Spin, Praha-SPIN 2001, Prague, July 15-28, 2001. KUCP-203, YNU-HEPTh-02-103, to appear in the Proceedings.

Czechoslovak Journal of Physics, Vol. 52 (2002), Suppl. A
2 $g_1^\gamma (x, Q^2, P^2)$ and QCD Sum Rule

We consider the polarized deep inelastic scattering on a polarized virtual photon target and study the virtual photon structure functions for the kinematical region:

$$\Lambda^2 \ll P^2 \ll Q^2,$$

where $-Q^2 (-P^2)$ is the mass squared of the probe (target) photon, and $\Lambda$ is the QCD scale parameter.

The same framework used in the analysis of nucleon spin structure functions can be applied to the present case. Namely, we can either base our argument on the operator product expansion (OPE) with the use of renormalization group method (RG) or on the DGLAP parton evolution equations.

The $n$-th moment of $g_1^\gamma (x, Q^2, P^2)$ turns out to be given by [9]:

$$\int_0^1 dx x^{n-1} g_1^\gamma (x, Q^2, P^2) = \frac{\alpha_s}{2\pi} \lambda^n \sum_{i=1}^{n_f} e_i \alpha_s (Q^2) \frac{\lambda_i^{\gamma_2/2\beta_0+1}}{\alpha_s (P^2)} + C^n + O(\alpha_s),$$

(2)

where $L_i^n, A_i^n, B_i^n$ and $C^n$ are computed from the NLO QCD perturbation theory. $\lambda_i^n (i = +, -, NS)$ are eigenvalues of one-loop anomalous dimensions. Note that the same formula with the different coefficients holds for the unpolarized structure function $F_2^\gamma (x, Q^2, P^2)$ [12]. We also note that the real photon’s $g_1^\gamma$ was studied to the LO in [13] and to the NLO in [8].

One of the remarkable consequences from the above results is the non-vanishing first moment of $g_1^\gamma (x, Q^2, P^2)$, for which we have

$$\int_0^1 dx g_1^\gamma (x, Q^2, P^2) = -\frac{3\alpha_s}{\pi} \sum_{i=1}^{n_f} e_i \alpha_s (Q^2),$$

(3)

in contrast to the vanishing first moment for the real photon case ($P^2 = 0$):

$$\int_0^1 dx g_1^\gamma (x, Q^2) = 0,$$

(4)

which holds to all orders of $\alpha_s (Q^2)$ in QCD [8]. We have also computed the $O(\alpha_s)$ QCD corrections in (3) [9], which coincides with the result obtained in [8].
3 Spin-Dependent Parton Distributions

Now the factorization theorem tells us that the physically observable quantities like cross sections or structure functions can be factored into the long-distance part (distribution function) and short-distance part (coefficient function). Thus the polarized photon structure function can be schematically written as

\[ g_1^\gamma = \Delta q^\gamma \otimes \Delta C^\gamma, \]  

(5)

where spin-dependent parton distributions \( \Delta q^\gamma \):

\[ \Delta q^\gamma(x, Q^2, P^2) = (\Delta q^\gamma_S, \Delta q^\gamma_G, \Delta q^\gamma_{NS}, \Delta q^\gamma_\gamma) \]  

(6)

are polarized flavor-singlet quark, gluon, non-singlet quark and photon distribution functions in the virtual photon (we put the symbol \( \Delta \) for polarized quantities), and \( \Delta C^\gamma_T \):

\[ \Delta C^\gamma_T = (\Delta C^\gamma_S, \Delta C^\gamma_G, \Delta C^\gamma_{NS}, \Delta C^\gamma_\gamma) \]  

(7)

are the corresponding coefficient functions. In the leading order in QED coupling \( \alpha = \frac{e^2}{4\pi} \), the photon distribution function can be taken as \( \Delta q^\gamma(x, Q^2, P^2) = \delta(1-x) \). Therefore we have the following inhomogeneous DGLAP evolution equation for \( \Delta q^\gamma = (\Delta q^\gamma_S, \Delta q^\gamma_G, \Delta q^\gamma_{NS}) \):

\[ \frac{d\Delta q^\gamma(x, Q^2, P^2)}{d\ln Q^2} = \Delta K(x, Q^2) + \int_x^1 \frac{dy}{y} \Delta q^\gamma(y, Q^2, P^2) \times \Delta P\left(\frac{x}{y}, Q^2\right), \]  

(8)

where \( \Delta K(x, Q^2) \) is the splitting function of the photon into quark and gluon, whereas \( \Delta P(x/y, Q^2) \) is the 3\( \times \)3 splitting function matrix. The solution to the DGLAP evolution equation can be given by

\[ \Delta q^\gamma(t) = \Delta q^\gamma(0) + \Delta q^\gamma(1)(t), \quad t = \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(Q^2)}, \]  

(9)

where the first (second) term corresponds to LO (NLO) approximation. The initial condition we impose is the following,

\[ \Delta q^\gamma(0) = 0, \quad \Delta q^\gamma(1) = \frac{\alpha}{4\pi} \tilde{A}_n, \]  

(10)

where \( \tilde{A}_n \) is the constant which depends on the factorization scheme to be used. Or equivalently in the language of OPE, this constant appears as a finite matrix element of the operators, \( \tilde{O}_n \) renormalized at \( \mu^2 = P^2 \) between the photon states:

\[ \langle \gamma(p) \mid \tilde{O}_n(\mu) \mid \gamma(p) \rangle|_{\mu^2=P^2} = \frac{\alpha}{4\pi} \tilde{A}_n. \]  

(11)

This scheme dependence arises from the freedom of multiplying the arbitrary finite renormalization constant \( Z_a \) and its inverse \( Z_a^{-1} \) in the n-th moment of [1]:

\[ g_1^\gamma(a, Q^2, P^2) = \Delta q^\gamma \cdot \Delta C^\gamma = \Delta q^\gamma Z_a \cdot Z_a^{-1} \Delta C^\gamma = \Delta q^\gamma a \cdot \Delta C^\gamma a, \]  

(12)

where the resulting \( \Delta q^\gamma a \) and \( \Delta C^\gamma a \) are the distribution function and the coefficient function in the a-scheme. For the parton distributions in various schemes, see ref. [10].
4 $g_2^\gamma(x, Q^2, P^2)$ and Twist-3 Effects

The antisymmetric part of the structure tensor, $W_{\mu\nu\rho\tau}^A(p, q)$ for the target (probe) photon with momentum $p$ ($q$), relevant for the polarized structure, can be written in terms of the structure functions, $g_1^\gamma$ and $g_2^\gamma$, as 

$$W_{\mu\nu\rho\tau}^A = \frac{1}{(p \cdot q)^2} [ (I_-)_{\mu\nu\rho\tau} g_1^\gamma - (J_-)_{\mu\nu\rho\tau} g_2^\gamma ],$$  

(13)

where the two tensor structures are explicitly given by

$$(I_-)_{\mu\nu\rho\tau} \equiv p \cdot q \epsilon_{\mu\nu\lambda\sigma} \epsilon_{\rho\tau}^{\quad\sigma\beta} q^\lambda p_\beta,$$

(14)

$$(J_-)_{\mu\nu\rho\tau} \equiv \epsilon_{\mu\nu\lambda\sigma} \epsilon_{\rho\tau}^{\quad\alpha\beta} q^\lambda p^\alpha - p \cdot q \epsilon_{\mu\nu\lambda\sigma} \epsilon_{\rho\tau}^{\quad\sigma\beta} q^\lambda p_\beta.$$

(15)

Here we note that we have the structure function $g_2^\gamma$ only for non-zero $P^2$, i.e. virtual photon. For the nucleon, the twist-3 contribution to $g_2$ is negligibly small in the experimental data so far obtained [14, 15].

Now let us decompose $g_2$ into twist-2 and twist-3 contributions:

$$g_2 = g_{2}^{\text{tw.2}} + g_{2}^{\text{tw.3}}.$$  

(16)

Experimental data for nucleons show

$$g_2 \approx g_{2}^{\text{tw.2}} = g_{2}^{\text{WW}},$$

(17)

where $g_{2}^{\text{WW}}$ is Wandzura-Wilczek relation [16]:

$$g_{2}^{\text{WW}}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2).$$

(18)

Now we ask what about the photon structure, especially virtual photon case?

First we consider the operator product expansion (OPE) relevant for the photon structure functions. The OPE can be decomposed into the twist-2 and twist-3 contributions as follows:

$$\int d^4x e^{iq \cdot x} J_\mu(x) J_\nu(0)^{[A]} \sim \sum_n R_n^{(2)}(0) E_n^{(2)}(Q^2) + \sum_n R_n^{(3)}(0) E_n^{(3)}(Q^2),$$

(19)

where $R_n^{(2)}$ and $R_n^{(3)}$ denote the twist-2 and twist-3 operators, respectively. For the nucleon the matrix element:

$$\langle N(p, s)|R_n^{(3)}|N(p, s)\rangle$$

is small. For the photon, as we will see below, the matrix element is non-vanishing:

$$\langle \gamma(p, s)|R_n^{(3)}|\gamma(p, s)\rangle \neq 0,$$

(21)
which contributes to $g_2^\gamma$ for the virtual photon. Namely we have

$$\int d^4x e^{i\mathbf{q} \cdot \mathbf{x}} \langle \gamma(p, s) | J_\mu(x) J_\nu(0) \rangle \langle A | \gamma(p, s) \rangle \sim \sum_n E_n^{(2)}(Q^2) \langle \gamma(p, s) | R_n^{(2)}(p) \rangle + \sum_n E_n^{(3)}(Q^2) \langle \gamma(p, s) | R_n^{(3)}(p) \rangle. \quad (22)$$

Now we calculate the virtual photon-photon forward scattering amplitude arising from the so-called Box diagram, with $\langle e^4 \rangle = \sum_{i=1}^{N_f} \epsilon_i^4 / N_f$, and $N_f$ being the number of active flavors.

$$g_1^{\gamma(\text{Box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[(2x - 1) \ln \frac{Q^2}{P^2} - 2(2x - 1) \ln x + (1)\right]$$

$$g_2^{\gamma(\text{Box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[-(2x - 1) \ln \frac{Q^2}{P^2} + 2(2x - 1) \ln x + 6x - 4\right]. \quad (23)$$

Remarkably, the $g_2^{\gamma(\text{Box})}$ satisfies the Burkhardt-Cottingham sum rule \[17\]:

$$\int \frac{1}{0} dx g_2^{\gamma(\text{Box})}(x, Q^2, P^2) = 0. \quad (24)$$

While it turns out that the twist-3 contribution to $g_2^\gamma$ is actually non-vanishing:

$$g_2^\gamma = g_2^\gamma - g_2^{\gamma(\text{WW})} = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \left[(2x - 2 - \ln x) \ln \frac{Q^2}{P^2} - 2(2x - 1) \ln x + 2(x - 1) + \ln^2 x\right]. \quad (25)$$

We have plotted the $g_1^\gamma$, $g_2^\gamma$ and $\bar{g}_2^\gamma$ as a function of $x$ for the virtual photon target, where $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$ for $N_f = 3$ in Fig.1.

Now the OPE reads in more details

$$i \int d^4x e^{i\mathbf{q} \cdot \mathbf{x}} T(J_\mu(x) J_\nu(0)) = -i\epsilon_{\mu\nu\lambda\sigma} q^\lambda \sum_{n=1,3,\ldots} \left(\frac{2}{Q^2}\right)^n q_{\mu_1} \cdots q_{\mu_{n-1}}$$

$$\times \left\{ \sum_i E_{n(i)}^{(2)} R_{n(i)}^{\mu_1 \cdots \mu_{n-1}} + \sum_i E_{n(i)}^{(3)} R_{n(i)}^{\sigma \mu_1 \cdots \mu_{n-1}} \right\}. \quad (26)$$

For the twist-2 quark operator we have

$$R_{q(2)}^{\sigma \mu_1 \cdots \mu_{n-1}} = i^{n-1} \bar{\psi} \gamma_5 \gamma^\sigma D_{\mu_1} \cdots D_{\mu_{n-1}} \psi - \text{traces} \quad (27)$$

where $\{ \}$ is the total symmetrization with respect to the Lorentz indices, and the twist-2 photon operator turns out to be

$$R_{\gamma(2)}^{\sigma \mu_1 \cdots \mu_{n-1}} = \frac{1}{4} \sum_{\alpha, \beta} \epsilon_{\alpha, \beta, \gamma} F^\alpha_{\mu_1 \cdots \mu_2} F^\beta_{\mu_2 \cdots \mu_{n-1}} F^\gamma - \text{traces} \quad (28)$$

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The matrix elements for these operators read

\[
\langle \gamma(p,s) | R^{\sigma_1 \cdots \sigma_{n-1}}_{\gamma(2)} | \gamma(p,s) \rangle = a_{i(2)}^{n} s^{\sigma} p^{\mu_1} \cdots p^{\mu_{n-1}}
\]

\[
s^{\sigma} p^{\mu_1} \cdots p^{\mu_{n-1}} = \frac{1}{n} \left[ s^{\sigma} p^{\mu_1} \cdots p^{\mu_{n-1}} + s^{\sigma} p^{\mu_2} \cdots p^{\mu_{n-1}} + \cdots + s^{\sigma} p^{\mu_2} \cdots p^{\mu_{n-1}} \right] (29)
\]

For the photon operator, \( a_{\gamma(2)}^{n} = 1 \). While for the twist-3 quark operators we find

\[
R^{\sigma_1 \cdots \sigma_{n-1}}_{\gamma(3)} = \frac{1}{4} \int n^{-1} \bar{\psi} \gamma^5 \gamma^{\sigma} D^\alpha \cdots D^\mu_{n-1} \psi - \text{traces} (30)
\]

where \([ , ]\) denotes anti-symmetrization with respect to the indices and the twist-3 photon operators are given by

\[
R^{\sigma_1 \cdots \sigma_{n-1}}_{\gamma(3)} = \frac{1}{4} \int n^{-1} \epsilon^{\sigma \alpha \beta \gamma} F^\alpha \cdots D^\mu_{n-1} F^\beta \gamma - \text{traces} (31)
\]

The resulting matrix elements are

\[
\langle \gamma(p,s) | R^{\sigma_1 \cdots \sigma_{n-1}}_{\gamma(3)} | \gamma(p,s) \rangle = a_{i(3)}^{n} s^{\sigma} p^{\mu_1} p^{\mu_2} \cdots p^{\mu_{n-1}}
\]

\[
s^{\sigma} p^{\mu_1} p^{\mu_2} \cdots p^{\mu_{n-1}} = \left[ \frac{n-1}{n} s^{\sigma} p^{\mu_1} \cdots p^{\mu_{n-1}} - \frac{1}{n} \sum_{j=1}^{n-1} s^{\sigma} p^{\mu_1} \cdots p^{\mu_j} \cdots p^{\mu_{n-1}} \right] (32)
\]
For photon operator, we also have $a^n_{1(3)} = 1$.

The $n$-th moments of the photon structure functions $g_1^\gamma$ and $g_2^\gamma$ are given by

$$
\int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) = \sum_i a^n_{i(2)} E^n_{i(2)}(Q^2) \tag{33}
$$

$$
\int_0^1 dx x^{n-1} g_2^\gamma(x, Q^2, P^2) = \frac{n-1}{n} \left[ -\sum_i a^n_{i(2)} E^n_{i(2)}(Q^2) + \sum_i a^n_{i(3)} E^n_{i(3)}(Q^2) \right] \tag{34}
$$

Therefore in the general framework of the OPE we conclude that the Burkhardt-Cottingham sum rule holds [17]:

$$
\int_0^1 dx x^2 g_2^\gamma(x, Q^2, P^2) = 0 \tag{35}
$$

The mixing of quark gluon twist-3 operators are very complicated. Thus $g_2^\gamma$ with QCD effects can be treated with operator mixing problem of twist-3 operators.

Pure QED effects can be studied through operator mixing between quark and photon operators. This mixing arises from the triangular diagrams.

$$
\langle \gamma(p, s) | \mathcal{P}_{q(2)}^{\mu_1, \cdots, \mu_{n-1}} | \gamma(p, s) \rangle = \frac{\alpha}{4\pi} \left( -\frac{1}{2} K_n^{(2)} \ln \frac{P^2}{\mu^2} + A_n^{q(2)} \right) s^{\{\mu_1 \cdots \mu_{n-1}\}} \tag{36}
$$

where the mixing anomalous dimensions are given by

$$
K_n^{(2)} = 24 N_f \langle e^4 \rangle \frac{n-1}{n(n+1)}, \quad K_n^{(3)} = -24 N_f \langle e^4 \rangle \frac{1}{n(n+1)} \tag{37}
$$

The coefficient functions are given by

$$
E_n^{q(2,3)}(1, \bar{g}(Q)) = 1 + \cdots, \quad E_{\gamma(2,3)}(1, \bar{g}(Q)) = \frac{\alpha}{4\pi} \delta_{\gamma} B_n^{\gamma(2,3)} \tag{38}
$$

Thus we obtain

$$
\int_0^1 dx x^{n-1} g_1^{\gamma(\text{Box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \frac{n-1}{n(n+1)} \ln \frac{Q^2}{P^2} + \frac{\alpha}{4\pi} (A_n^{q(2)} + \delta_{\gamma} B_n^{q(2)})
$$

$$
\int_0^1 dx x^{n-1} g_2^{\gamma(\text{Box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} N_f \langle e^4 \rangle \frac{n-1}{n(n+1)} \ln \frac{Q^2}{P^2} + \frac{n-1}{n} \frac{\alpha}{4\pi} (-A_n^{q(2)} - \delta_{\gamma} B_n^{q(2)} + A_n^{q(3)} + \delta_{\gamma} B_n^{q(3)}) \tag{39}
$$

This is nothing but the Box-diagram contribution. Here we note that there exists the twist-3 effects even at the pure QED level. Here we also note that $g_1^\gamma + g_2^\gamma$ does not have $\ln Q^2 / P^2$ behavior.
5 Concluding Remarks

We have investigated the virtual photon’s spin structure functions $g_1^\gamma(x, Q^2, P^2)$ and $g_2^\gamma(x, Q^2, P^2)$ in the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$. The 1st moment of $g_1^\gamma$ is non-vanishing which leads to a sum rule. The polarized quark and gluon distributions are computed to the NLO by the perturbative method, but these distributions are factorization-scheme dependent. The twist-3 effects do exist in $g_2^\gamma$ for the virtual photon target. Here we have discussed the OPE analysis for the pure QED effects corresponding to the Box-diagrams. The full QCD analysis is now under investigation. Finally we can also discuss the positivity constraints on the polarized and unpolarized photon structure functions [8].

One of the authors (T.U.) would like to express his sincere thanks to the Organizers of the ASI, especially to Prof. Finger for the wonderful organization and the hospitality during the School.

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