Abstract. In the recent literature on dark energy (DE) model building we have learnt that cosmologies with variable cosmological parameters can mimic more traditional DE pictures exclusively based on scalar fields (e.g. quintessence and phantom). In a previous work we have illustrated this situation within the context of a renormalization group running cosmological term, $\Lambda$. Here we analyze the possibility that both the cosmological term and the gravitational coupling, $G$, are running parameters within a more general framework (a variant of the so-called “$\Lambda X$CDM models”) in which the DE fluid can be a mixture of a running $\Lambda$ and another dynamical entity $X$ (the “cosmon”) which may behave quintessence-like or phantom-like. We compute the effective EOS parameter, $\omega_e$, of this composite fluid and show that the $\Lambda X$CDM can mimic to a large extent the standard $\Lambda$CDM model while retaining features hinting at its potential composite nature (such as the smooth crossing of the cosmological constant boundary $\omega_e = -1$). We further argue that the $\Lambda X$CDM models can cure the cosmological coincidence problem. All in all we suggest that future experimental studies on precision cosmology should take seriously the possibility that the DE fluid can be a composite medium whose dynamical features are partially caused and renormalized by the quantum running of the cosmological parameters.
1 Introduction

One of the most alluring aspects of modern cosmology is that it has become an accurately testable phenomenological branch of Physics. This has granted the field a fairly respectable status of empirical science, which certainly did not possess some two decades ago. Undoubtedly the most prominent accomplishment of cosmology to date has been to provide strong indirect support for the existence of both dark matter (DM) and dark energy (DE) from independent data sets derived from the observation of distant supernovae [1], the anisotropies of the CMB [2], the lensing effects on the propagation of light through weak gravitational fields [3], and the inventory of cosmic matter from the large scale structures (LSS) of the Universe [4]. But in spite of these outstanding achievements, modern cosmology still fails to fulfill the most important of its tasks, to wit: unraveling the ultimate physical nature of the components that build up the mysterious dark side of the Universe.

Ignoring for the moment the puzzling DM sector (which is recently receiving spectacular empirical support [5]) and whose final identity might hopefully be elucidated by directly detecting some day an exotic form (e.g. supersymmetric) of elementary particles, the DE component is nonetheless the most intriguing, unnerving and distressing of all. It turns out to be constant or slowly varying with time, it does not cluster, it fills smoothly all corners of our patch of accessible Universe and – to pitch even higher our level of perplexity – it constitutes by far the dominant form of energy density at the cosmological scale. What is it? Is it vacuum energy in the form of a cosmological constant? Is it the value of a slowly evolving homogeneous and isotropic scalar field? Perhaps a remnant of higher order gravitational terms in the effective action? Or some hint of a modified gravitational theory?... We don’t know. But we do know at least (as of the early work of Zeldovich in 1967 [6]) that the typical size of the vacuum energy in QFT is exceedingly large to be reconciled with the characteristic energy densities at the cosmic scale, such as the cosmological matter-radiation density, the critical density, and of course also the cosmological energy density \(\rho = \Lambda / 8\pi G\) associated to \(\Lambda\). All of them are of order of \((10^{-3} \text{eV})^4\), far too small compared to the average energy densities in particle physics, say in the standard model of strong and electroweak interactions. Unfortunately, moving from QFT to string theory does not seem to help much, for after the process of compactification from 11 dimensions down to 3 we are left with a vastly complex “landscape” consisting of some \(10^{1000}\) metastable (non-supersymmetric) vacua where to entertain our choice of the ground state [7]. All in all it looks very hard to identify \(\rho = \Lambda\) with a vacuum energy density. If we insist on it, we stumble once again upon the excruciating cosmological constant (CC) problem whose only known “technical” solution at present (though certainly not a very natural one!) is to postulate the existence of an ad hoc series of extremely fine-tuned cancellations among the various contributions [8, 9, 2].

In view of this situation it is perhaps wiser to avoid subscribing for the moment to a too strong opinion on the ultimate nature of the \(\Lambda\) term in Einstein’s equation. Instead, we may treat it phenomenologically as a parameter in QFT in curved space-time, therefore acquiring the status of an effective charge similar to any running quantity in, say, QED or QCD. If one takes this practical point of view [10], we may get some guidance from the powerful renormalization group

\[\text{\footnotesize{\textsuperscript{2}See e.g. [10] for a summarized presentation.}}\]
(RG) methods and use them to extract testable information on the possible evolution of $\Lambda$ from its presently measured value – see e.g. Ref. [12] for a devoted attempt in this direction. The RG evolution of the CC is not primarily a time evolution, it stems rather from it being a function $\Lambda = \Lambda(\mu)$ of the typical energy scale $\mu$ of the Universe at any given stage of the cosmic evolution. Following [10] we identify this energy scale with the expansion rate or Hubble function, $\mu = H$, which is of course time dependent. Therefore $\Lambda = \Lambda(H)$ becomes also time dependent. The resulting model is a FLRW type cosmological model with running cosmological parameters. This point of view has been amply exploited in different ways in references [10]-[15] and constitutes an alternate candidate to quintessence [16]. Both pictures (quintessence and RG cosmology) have in common the idea of a dynamical DE. This can be useful in that a time-evolving DE may help to understand another aspect of the CC problem which is also rather intriguing, the so-called “coincidence problem” [9] or the problem of why the presently measured value of the CC (or DE) is so close to the matter density, i.e. both being at present of order of $(10^{-3}\,eV)^4$. This problem has been addressed from different perspectives within modified quintessence models [17]. Remarkably, RG models can also tackle efficiently the coincidence problem [18], a point that will be reexamined here too.

One may describe RG cosmological models from the viewpoint of an effective scalar field parametrization of the DE and compute e.g. the effective equation of state (EOS) for the running $\Lambda$. This was done in detail in [19] for a particular case with fixed gravitational coupling $G$ and with non-conserved CC and matter densities. In a subsequent work a general algorithm was established to compute the effective EOS for general cosmological models with variable cosmological parameters [20]. A further generalization (the so-called $\Lambda X$CDM models [18]) is to admit the possibility that the DE is a composite medium made out of a running $\Lambda$ and a dynamical component $X$ that we called the “cosmon” in [18]. In the present work we elaborate further along this line but we allow the gravitational coupling $G$ being variable. In this way we obtain a picture in which matter and cosmon densities are separately conserved, whereas $\Lambda$ evolves together with the gravitational coupling $G$ according to the Bianchi identity – thus preserving general covariance. Apart from $\Lambda = \Lambda(H)$ we explicitly obtain the function $G = G(H)$. The outcome is a variant of the $\Lambda X$CDM model in which the change of $\Lambda$ is compensated for by the evolution of $G$.

2 Cosmon models with running $\Lambda$ and $G$

Consider Einstein’s equations with a CC term, $\Lambda$, and move it onto the r.h.s of the field equations to form the combined quantity $8\pi G \tilde{T}_{\mu\nu}$. Here $\tilde{T}_{\mu\nu}$ is the effective energy-momentum tensor $\tilde{T}_{\mu\nu} = T_{\mu\nu} + g_{\mu\nu} \rho_\Lambda$, where $T_{\mu\nu}$ is the ordinary contribution from matter-radiation, and $\rho_\Lambda = \Lambda/(8\pi G)$. Next let us impose the Bianchi identity, $\nabla^\mu (G \tilde{T}_{\mu\nu}) = 0$. Upon evaluating it in the FLRW metric one finds

$$\frac{d}{dt} [G (\rho_m + \rho_\Lambda)] + G H \alpha_m \rho_m = 0, \quad \alpha_m \equiv 3(1 + \omega_m), \quad (2.1)$$

3The name “cosmon” was originally introduced in [21] to represent a dynamically adjusting DE field. Here we borrow the name to mean any (effective or fundamental) dynamical component of the DE (not necessarily quintessence-like) other than the cosmological term.
where $\rho_m$ and $\rho_\Lambda$ are the matter-radiation and CC densities, and $\omega_m = 0, 1/3$ ($\alpha_m = 3, 4$) for matter and radiation respectively. Notice that we contemplate $\dot{\rho}_\Lambda \equiv d\rho_\Lambda/dt \neq 0$ and moreover we do not drop $G$ from (2.1) because we also admit the possibility that $\dot{G} \neq 0$. Let us next assume that the DE fluid is a composite medium consisting of the CC term and of another dynamical quantity $X$ (“the cosmon”), with energy density $\rho_X$, characterized by an EOS parameter $\omega_X$. Then it is easy to see that Eq. (2.1) generalizes into

$$\frac{d}{dt}[G (\rho_m + \rho_D)] + G H (\alpha_m \rho_m + \alpha_X \rho_X) = 0, \quad \alpha_X \equiv 3(1 + \omega_X), \quad (2.2)$$

where $\rho_D = \rho_\Lambda + \rho_X$ is the total DE density. Following [18] we call the resulting composite DE model the $\Lambda X$CDM model. There are, however, different possible implementations of the $\Lambda X$CDM model: if we assume that $\rho_D$ and $\rho_m$ are separately conserved, then $G$ must be constant and we obtain precisely the sort of $\Lambda X$CDM framework studied in detail in Ref. [18], which will be referred hereafter as type I cosmon models. Alternatively, if we take as conserved quantities $\rho_X$ and $\rho_m$, then we must have $\dot{\rho}_i + \alpha_i \rho_i H = 0$ for both $\rho_i = \rho_m, \rho_X$, and in this case (2.2) implies that $G$ must vary in time in combination with $\rho_\Lambda$ according to the equation

$$\dot{G} (\rho_\Lambda + \rho_X + \rho_m) + G \dot{\rho}_\Lambda = 0. \quad (2.3)$$

The class of $\Lambda X$CDM models based on these equations will be referred to as type II cosmon models and its study is the main aim of this work. We remark that the type II models constitute a generalization of the $G$-running cosmological model presented in Ref. [13] where the new ingredient here is the introduction of the self-conserved cosmon entity $X$ which, as we will see, can play an important distinctive role. We should stress from the beginning that the cosmon need not be a fundamental field. In particular, $X$ is not necessarily a quintessence scalar field [16], not even a “standard” phantom field [22]. In fact, the barotropic index $\omega_X$ of $X$ can be above or below $-1$, and moreover the energy density $\rho_X$ can be positive or negative. Thus e.g. if $\omega_X < -1$ and $\rho_X < 0$ we obtain a kind of unusual fluid which has been called “phantom matter” (see Fig. 1 of [18]). Phantom matter does preserve – in contrast to usual phantom energy – the strong energy condition, as ordinary matter does. Clearly the cosmon plays the role of a very general entity ranging from the overall representation of a mixture of dynamical fields of various sorts, to the effective behavior of higher order curvature terms in the effective action. In spite of the phenomenological nature of this approach at the present stage, we cannot exclude the existence of a consistent Lagrangian formulation for some of these models. In the remainder of this paper we focus on the interesting phenomenological implications of the type II models. Still, it is important to say that the type II models are motivated by the models with a well defined Lagrangian formulation such as QFT in curved space-time [10], to be discussed in the following paragraphs.

To solve Eq. (2.3) for $G$ we need some input on the evolution of $\rho_\Lambda$, otherwise we are led to the trivial solution $G = \text{const.}$ Following [10][12] we shall adopt the RG inspired evolution equation

$$\frac{d\rho_\Lambda}{d\ln \mu} = \frac{3 \nu}{4 \pi} M_P^2 \mu^2, \quad (2.4)$$

where $\nu$ is a free parameter: it essentially provides the ratio squared of the heavy masses contributing to the $\beta$-function of $\Lambda$ versus the Planck mass, $M_P$. We naturally expect $\nu \ll 1$. As mentioned
in the introduction, we will choose $\mu = H$ as the typical RG scale in cosmology [10]. Assuming a FLRW type of Universe with flat geometry we finally meet the following set of equations for the type II cosmon models:

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda + \rho_X),$$
$$\rho_\Lambda = C_0 + C_1 H^2,$$
$$(\rho_m + \rho_\Lambda + \rho_X) dG + G d\rho_\Lambda = 0. \tag{2.5}$$

The first one is Friedmann’s equation for a spatially flat Universe, the second equation is the integrated form of (2.4) with the boundary condition $\rho_\Lambda = \rho_\Lambda^0$ (the measured value of the CC density) for $H = H_0$ (Hubble’s parameter at present), hence

$$C_0 = \rho_\Lambda^0 - \frac{3\nu}{8\pi} M_P^2 H_0^2, \quad C_1 = \frac{3\nu}{8\pi} M_P^2; \tag{2.6}$$

and the third equation is an equivalent differential form of (2.3). The basic set (2.5) can be analytically solved to determine $G$ as a function of the scale $\mu = H$, with the following result:

$$G(H) = \frac{G_0}{1 + \nu \ln \frac{H^2}{H_0^2}}, \tag{2.7}$$

where $G_0 = 1/M_P^2$. This logarithmic running law for $G$ formally coincides with the one obtained in the framework of [13]. Of course the time/redshift dependence of $G$ will be different here because of the cosmon contribution. The form of (2.7) suggests that $\nu$ acts also as the $\beta$-function for the RG running of $G$. To obtain $G = G(z)$ and $\rho_\Lambda = \rho_\Lambda(z)$ as functions of the cosmological redshift $z$ requires more work on solving differential equations. We limit ourselves to quote the final results. The gravitational coupling can only be expressed as an implicit function of $z$:

$$\frac{1}{g(z)} - 1 + \nu \ln \left( \frac{1}{g(z)} - \nu \right) = \nu \ln \left[ \Omega_m^0 (1 + z)^{\alpha_m} + \Omega_X^0 (1 + z)^{\alpha_X} + \Omega_\Lambda^0 - \nu \right], \tag{2.8}$$

where we have defined $g(z) \equiv G(z)/G_0$ and $\Omega_i^0 = \rho_i^0/\rho_c^0$ for the various components, with $\rho_c^0$ the critical density at present. Equation (2.8) defines implicitly the function $g = g(z)$, and once this is known the CC density as a function of the redshift is obtained from

$$\rho_\Lambda(z) = \frac{\rho_\Lambda^0 + \nu \left( \rho_m(z) + \rho_X(z) \right) g(z) - \nu \rho_c^0}{1 - \nu g(z)}. \tag{2.9}$$

With these equations the type II class of cosmon models is solved. However, a highly convenient next step to do is to characterize this model with an effective equation of state (EOS) for its DE density. This is most useful in order to better compare the effective behavior of this model with alternative models of the DE (e.g. quintessence type models).

### 3 Effective equation of state for the DE in type II cosmon models

The effective EOS for type I cosmon models was studied in great detail in [18]. Let us thus concentrate here on type II models only. The general procedure to obtain the effective EOS
parameter $\omega_e$ for models with variable cosmological parameters was thoroughly explained in [19] and [20] and we refer the reader to these and other useful references on this subject [23]. The formula for $\omega_e$ as a function of the cosmological redshift is

$$\omega_e(z) = -1 + \frac{1 + z}{3} \frac{1}{\tilde{\rho}_D(z)} \frac{d\tilde{\rho}_D(z)}{dz}.$$  (3.1)

The density $\tilde{\rho}_D$ (not to be confused with $\rho_D$) is the total DE density in the effective DE picture [20]. The relation between $\tilde{\rho}_D$ and $\rho_D$ can be obtained from matching (i.e. equating) the expansion rates $H$ in the two pictures (in this case the $\Lambda$XCDM model and the effective DE picture):

$$G(\rho_m + \rho_D) = G_0 (\tilde{\rho}_m + \tilde{\rho}_D).$$  (3.2)

By definition, the effective DE picture [20] is characterized by self-conserved DE and matter densities ($\tilde{\rho}_D$, $\tilde{\rho}_m$) and by a constant gravitational coupling: $G = G_0$. Generalizing the procedure of [20] we can show that $\tilde{\rho}_D(z)$ satisfies the following differential equation (stemming from the Bianchi identity)

$$\frac{d\tilde{\rho}_D}{dz} = \alpha_m \tilde{\rho}_D(z) - \xi(z) \left( 1 + z \right),$$  (3.3)

where

$$\xi(z) = g(z) \left( \rho_\Lambda(z) + \frac{\alpha_m - \alpha_X}{\alpha_m} \rho_X^0 (1 + z)^{\alpha_X} \right).$$  (3.4)

The solution of (3.3) satisfying $\tilde{\rho}_D(0) = \tilde{\rho}_D^0$ reads

$$\tilde{\rho}_D(z) = (1 + z)^{\alpha_m} \left[ \tilde{\rho}_D^0 - \alpha_m \int_0^z \frac{dz'}{(1 + z')^{(\alpha_m+1)}} \xi(z') \right].$$  (3.5)

Again following the methods of [20] it is convenient to rewrite Eq. (3.2) as follows:

$$\Omega_m^0 f_m(z) (1 + z)^{\alpha_m} + \Omega_\Lambda^0 f_\Lambda(z) + \Omega_X^0 f_X(z) (1 + z)^{\alpha_X} = \tilde{\Omega}_m^0 (1 + z)^{\alpha_m} + \tilde{\Omega}_D(z).$$  (3.6)

Here $\tilde{\Omega}_D(z) = \tilde{\rho}_D(z)/\rho_D^0$ with $\tilde{\rho}_D(z)$ given by (3.5). The $f_i$ functions above, whatever it be their form, must satisfy $f_m(0) = f_\Lambda(0) = f_X(0) = 1$ in order to preserve the cosmic sum rule of the $\Lambda$XCDM models, which reads (in the flat case)

$$\Omega_m^0 + \Omega_D^0 = \Omega_m^0 + \Omega_\Lambda^0 + \Omega_X^0 = 1.$$  (3.7)

At the same time the parameters $\tilde{\Omega}_m^0$ and $\tilde{\Omega}_D^0$ in the effective DE picture fulfill their own cosmic sum rule $\tilde{\Omega}_m^0 + \tilde{\Omega}_D^0 = 1$. In general the quantities $\Delta \Omega_m^0 \equiv \Omega_m^0 - \tilde{\Omega}_m^0$, $\Delta \Omega_D^0 \equiv \Omega_D^0 - \tilde{\Omega}_D^0$ will be non-vanishing because they correspond to different parametrizations of the same data [19, 20]. Of course it only makes sense to distinguish between $\tilde{\Omega}_i^0$ and $\Omega_i^0$ when we fit densities of matter and of DE e.g. from distant supernovae data. However, for the radiation component (which is very well determined by CMB measurements) we set these differences to zero (see Section 4).

We may now compute $d\tilde{\rho}_D(z)/dz$ from (3.6) and insert the result in (3.1). In the process we use the Bianchi identity derived in the previous section and the conservation laws in the $\Lambda$XCDM models. For the type II cosmon model it is easy to see that these conservation laws actually entail
\( f_m(z) = f_X(z) = g(z) \). And moreover the Bianchi identity insures that the following differential expression vanishes identically:

\[
\Omega_m^0 \frac{dg}{dz} (1 + z)^{\alpha_m} + \Omega_\Lambda^0 \frac{df_\Lambda}{dz} + \Omega_X^0 \frac{dg}{dz} (1 + z)^{\alpha_X} = 0. \tag{3.8}
\]

Using these relations we can obtain, after a straightforward calculation, the final result. Let us present it in compact form as follows:

\[
\omega_e(z) = -1 + \frac{\delta(z)}{3 \Omega_D(z)}, \tag{3.9}
\]

where

\[
\delta(z) = \alpha_m \left( g(z) \Omega_m^0 - \tilde{\Omega}_m^0 \right) (1 + z)^{\alpha_m} + \alpha_X \, g(z) \Omega_X^0 (1 + z)^{\alpha_X}. \tag{3.10}
\]

As expected, for the special case characterized by \( \nu = 0 \) (no running \( \Lambda \)), \( \Omega_X^0 = 0 \) (no cosmon) and \( \Delta \Omega_m^0 = 0 \) (no difference between the parameters of the two pictures) we obtain \( \delta(z) = 0 \), for all \( z \), and we retrieve the EOS parameter of the CC: \( \omega_e = -1 \). Recall that the function \( g(z) = G(z)/G_0 \) has been obtained implicitly in Eq. (2.8) and that it takes the simple explicit form (2.7) only when written in terms of \( H \). The latter (log) form suggests that \( G(z) \) evolves very little with \( z \) and therefore we expect that the departure of the effective EOS parameter (3.9) from the CC boundary will be mainly controlled by the parametrization difference \( \Delta \Omega_m^0 \) and by the cosmon contribution – the last term on the r.h.s. of (3.10). We shall illustrate the behavior of (3.9) with some non-trivial numerical examples in Section 5. Let us now study the bounds on \( \nu \), the parameter that regulates the evolution of both \( \rho_\Lambda(z) \) and \( G(z) \).

4 Nucleosynthesis bound on \( \nu \). Potential implications for cosmology and astrophysics

As mentioned above, the type II cosmon models furnish a logarithmic law \( G = G(H) \), Eq. (2.7), which is formally identical to the one obtained in Ref. [13] in which the cosmon was absent. In the last reference a bound was obtained on the parameter \( \nu \) by considering the experimental limits placed by nucleosynthesis on the variation of \( G \), with the result \( |\nu| \lesssim 10^{-2} \). Here we are going to obtain a more stringent bound on \( \nu \) from the experimental restriction on the ratio of total DE to matter-radiation densities in the effective DE picture \( \tilde{\rho}_D/\tilde{\rho}_m = \tilde{\Omega}_D/\tilde{\Omega}_m \). This ratio evolves with cosmic time or redshift. Particularly, in the standard ΛCDM model this ratio is given by \( \rho_\Lambda^0/\rho_m^0 \) and increases without end as time passes by because \( \rho_m \rightarrow 0 \). At present its value is \( \tilde{\rho}_0 \equiv \rho_\Lambda^0/\rho_m^0 \simeq 7/3 \), i.e. it is now “coincidentally” of order 1. This is in essence the cosmic coincidence problem mentioned in the introduction: why is this ratio of \( O(1) \) right now? Let us denote the value of \( \tilde{\rho} \) at the nucleosynthesis epoch by \( \tilde{\rho}_N = (\tilde{\rho}_D/\tilde{\rho}_m)_N \). As in previous works we will use the upper bound \( |\tilde{\rho}_N| \lesssim 10\% \) [18] [24]. For higher values of \( \tilde{\rho}_N \) the expansion rate at nucleosynthesis would be too large and the amount of primordial helium synthesized would overshoot the experimental limits. To convert the upper bound on \( |\tilde{\rho}_N| \) into an upper bound on \( |\nu| \) we substitute (2.7) in (2.8) and solve the resulting equation for \( \nu \). The final (exact) analytical
\begin{equation}
\nu = \frac{\Omega_m^0 (1 + z)^{\alpha_m} + \Omega_X^0 (1 + z)^{\alpha_X} + \Omega_\Lambda^0 - H^2(z)/H^2_0}{1 + H^2(z)/H^2_0 \left[ \ln \left( \frac{H^2(z)/H^2_0}{1} \right) - 1 \right]}.
\end{equation}

Let us use the formula for the expansion rate in the effective DE picture to eliminate $H^2(z)/H^2_0$ from (4.1) in terms of the ratio $\tilde{r}$ defined above:

\begin{equation}
\frac{H^2(z)}{H^2_0} = \tilde{\Omega}_m(z) + \tilde{\Omega}_D(z) = \tilde{\Omega}_m^0 (1 + z)^{\alpha_m} (1 + \tilde{r}).
\end{equation}

At the nucleosynthesis epoch ($z_N \sim 10^3$) we have $\alpha_m = 4$ because radiation dominates. Being $z$ very large we can neglect $\Omega_\Lambda^0$ in the numerator of (4.1). Furthermore, given the fact that $X$ is assumed to be a DE component ($-1 - \delta < \omega_X < -1/3$, with $\delta > 0$ small), we have $\alpha_X = 3(1 + \omega_X) < 2$ (hence $\alpha_X < \alpha_m$) and the cosmon contribution can also be neglected at $z = z_N$. Finally, as mentioned in Section 3, we set $\Delta \Omega_m^0 = 0$ for radiation. All in all at the nucleosynthesis epoch Eq. (4.1) boils down to

\begin{equation}
\nu \approx \frac{-\tilde{r}_N \Omega_R^0 (1 + z_N)^4}{1 + \Omega_R^0 (1 + z_N)^4 (1 + \tilde{r}_N)} \frac{\ln \left( \frac{\Omega_R^0 (1 + z_N)^4 (1 + \tilde{r}_N)}{1} \right) - 1}{(1 + \tilde{r}_N) \ln \left[ \Omega_R^0 z_N^4 \right]}.
\end{equation}

Here $\Omega_R^0$ is the total radiation density (from photons and neutrinos) normalized to the critical density at present; $\Omega_R^0$ is given by 1.68 times the standard value of the normalized photon density $\Omega_\gamma^0 \approx 4.6 \times 10^{-5}$ (for $h = 0.73$) [2]. From these equations the desired nucleosynthesis bound on $\nu$ ensues immediately:

\begin{equation}
|\nu| \lesssim 10^{-3},
\end{equation}

which is an order of magnitude more restrictive than the previous bound obtained in [13]. Since the bound (4.3) essentially does not depend on the X (cosmon) component, it also applies to the models studied in [13]. The fact that $\nu \ll 1$ is indeed a natural theoretical expectation from the interpretation of this parameter within effective field theory (cf. Ref. [12] for details). An obvious implication of this bound on type II cosmon models is that the corresponding running of $\Lambda$ is in principle rather hampered as compared to that in type I models [13]. In the latter, the nucleosynthesis bound affected a combined parameter involving both the $\Lambda$ and cosmon dynamics: $\epsilon \equiv \nu(1 + \omega_X) \lesssim 0.1$. However, for type II cosmon models nucleosynthesis does not place any bound on the cosmon barotropic index $\omega_X$ and in this sense we have more freedom to play with it. We point out that the ratio $r$ in the $\Lambda$XCDM picture, i.e. $r = \rho_D/\rho_m$, is related to $\tilde{r}$ as $r = g(z)^{-1} (\tilde{\Omega}_m^0/\Omega_m^0) (1 + \tilde{r}) - 1$, where we have used (3.2). Let us next consider this expression at the nucleosynthesis time, where $H(z_N) \equiv H_N$. Setting $\Delta \Omega_R^0 = 0$, as previously discussed, and using (2.7) we find

\begin{equation}
r_N = \tilde{r}_N + \nu(1 + \tilde{r}_N) \ln \frac{H_N^2}{H_0^2} \approx \tilde{r}_N + \nu(1 + \tilde{r}_N) \ln \left[ \Omega_R^0 z_N^4 \right].
\end{equation}

In view of (4.4) the second term on the r.h.s. of the above expression remains below 10%. Therefore we find that at the nucleosynthesis time the bound on $r$ lies within the same order of magnitude as the bound on $\tilde{r}$, i.e. $|r_N| < 10\%$. Taking this into account and making the natural assumption that the value of $\Delta \Omega_m^0$ is small (big differences between the two pictures are not expected), we see
that in practice we need not distinguish between $\tilde{r}$ and $r$. This is explicitly corroborated in the numerical analysis performed in the next section.\footnote{We point out that the bound on the relative abundance of DE in the nucleosynthesis epoch \cite{24} applies originally to $\tilde{r}$ because in that analysis it is assumed that $G$ is constant. Since, however, this implies a tight constraint on $\nu$, \textit{Eq. (4.4)}, it follows that the bound on $\tilde{r}$ essentially applies to $r$ as well, at least within order of magnitude.}

The tighter upper limit (4.4) on type II models from nucleosynthesis as compared to type I does not necessarily go to the detriment of the potentially important role played by $\nu$ when considering other implications. For example, for sufficiently small $\nu > 0$ (specifically for $\nu \sim 10^{-6}$), the type II cosmon models can offer an explanation for the flat rotation curves of galaxies exactly as the RG model of Ref. [13]. This is because the type II models preserve the running logarithmic law (2.7) wherein at the astrophysics level $H^{-1}$ is replaced by the radial coordinate of the galactic system [13]. Remarkably, we see that even for very small (but positive) values of $\nu$ – far below the limits placed by cosmology considerations (4.4) – we may have dramatic implications at the astrophysical scale.

For $\nu < 0$, however, these astrophysical implications are not possible. But in compensation we can get a handle on the cosmological coincidence problem mentioned in the introduction. The line of argumentation closely follows the approach of Ref. [18] for type I models. Namely, if the Universe’s evolution has a stopping (or, more properly, a turning) point at some time in its future, then the ratio $r$ will be bounded from above for the entire lifetime of the Universe, and therefore our present time cannot be considered special; that is to say, in a $\Lambda$CDM model $r$ can be of order 1 not because – as argued within the standard $\Lambda$CDM model – we just happen to live at a time near the transition from deceleration into acceleration, but simply because $r$ can never increase beyond a fixed number. Moreover, in a large portion of the parameter space $r$ remains below $O(1 - 10)$. We refer the reader to [18] for an expanded exposition of this approach to the coincidence problem, here we limit ourselves to apply the same arguments within the context of type II models. For these models the turning point associated to the possible resolution of the coincidence problem exists only for $\nu < 0$. This can be seen immediately from (2.7): $H$ will vanish in the future at the very same point where $G$ also vanishes. However, the stopping redshift value (call it $z_S$) can be approached with the correct sign of the gravitational coupling ($G > 0$), if and only if $\nu < 0$. For $\nu > 0$ we would have $G < 0$ before reaching $z_S$. One can also show that the existence of the turning point can be realized both for the cosmon barotropic index $\omega_X$ above or below $-1$, i.e. for quintessence-like or phantom-like cosmon. To study the precise conditions for the existence of the turning point it proves convenient to use Eq. (4.1) and set $H(z) = 0$ in it. The root $z = z_S$ satisfying the resulting equation will obviously lie in the matter dominated epoch, so we can set $\alpha_m = 3$ in (4.1). Introducing the (continuous) function

$$f(z) = \Omega_m^0 (1 + z)^3 + \Omega_X^0 (1 + z)^3(1 + \omega_X) + (1 - \Omega_m^0 - \Omega_X^0 - \nu)$$

we find, equivalently, that $z_S$ is obtained as a root of it: $f(z_S) = 0$. Next we apply Bolzano’s theorem for continuous functions, and to this end we explore the change of sign of $f(z)$ at present

\footnote{We omit details here. See section 7 of Ref. [13] for a detailed discussion of the possible astrophysical implications of a running $G$ on the flat rotation curves of galaxies. Type II cosmon models lead to the same kind of picture.}
(z = 0) as compared to the remote future (z = -1). We find

$$\lim_{z \to -1} f(z) = \begin{cases} 
1 - \Omega_m^0 - \Omega_X^0 - \nu & \text{if } \omega_X > -1 \\
\text{sign}(\Omega_X^0) \cdot \infty & \text{if } \omega_X < -1
\end{cases} \quad (4.7)$$

$$f(0) = 1 - \nu \quad (4.8)$$

Being $1 - \nu$ obviously positive (because $\nu < 0$) we can guarantee stopping point $z_S$ in the future under the following conditions:

$$\Omega_X^0 > 1 - \Omega_m^0 - \nu \quad \text{for} \quad \omega_X > -1, \quad (4.9)$$

$$\Omega_X^0 < 0 \quad \text{for} \quad \omega_X < -1. \quad (4.10)$$

Notice that $-1 < \omega_X < -1/3$ corresponds to a quintessence-like cosmon. Given the approximate prior $\Omega_m^0 = 0.3$ (e.g. from LSS [4] and supernovae data [11]) and using the bound (4.4) and the sum rule (3.7), we learn that the corresponding stopping condition (4.9) is essentially equivalent to require $\Omega_\Lambda^0 < 0$. This should not be considered as a drawback; within our composite DE framework the generalized sum rule (3.7) makes allowance for any sign of $\Omega_\Lambda^0$, provided the total DE density $\Omega_D^0 = \Omega_\Lambda^0 + \Omega_X^0$ is positive and of the order of the experimentally measured value $\Omega_D^0 \simeq 0.7$. In other words, this essentially implies $\Omega_X^0 > 0$ and it confirms that the condition (4.9) is realized by a quintessence-like cosmon whose density is, therefore, decreasing with time. Being the running of $\Lambda$ very mild it is not surprising that the stopping condition is almost equivalent to $\rho_\Lambda^0 < 0$. On the other hand, for $\nu < 0$ and $-1 - \delta < \omega_X < -1$ we must necessarily have $\Omega_X^0 < 0$, Eq. (4.10), and in this case the cosmon behaves like “phantom matter” rather than as usual phantom energy (cf. Fig. 1 of [18]). Finally, let us consider the simplest case $\nu = 0$ (i.e. cosmon models with fixed $\Lambda$ and $G$). In this particular instance the total normalized DE density is the same as in the type I cosmon models for $\nu = 0$ [18]:

$$\Omega_D(z) = \Omega_\Lambda^0 + \Omega_X^0 (1 + z)^{3(1 + \omega_X)}. \quad (4.11)$$

For $\omega_X > -1$ (resp. $\omega_X < -1$) the stopping point exists if $\Omega_\Lambda^0 < 0$ (resp. $\Omega_X^0 < 0$). This reflects the continuity of our analysis with respect to the variable $\nu$ as this situation corresponds to the $\nu \to 0$ limit of the $\nu \neq 0$ case studied above. Remarkably, for any value of $\nu$ the existence of a turning point for $\omega_X < -1$ does not correspond to $X$ behaving as phantom DE but rather as “phantom matter” [18]. For this reason there is no “Big Rip” singularity [22] in this kind of models. To summarize, the existence of a turning point is guaranteed in many circumstances within the framework of $\Lambda$XCDM cosmologies, and it appears to be a very desirable ingredient to help quenching the severity of the cosmological coincidence problem and at the same time to eschew future cosmic singularities of the “Big Rip” type. In its absence (namely of a turning point) the Universe’s expansion would be eternal and the ratio $r = \rho_D/\rho_m$ would tend to infinity as in the standard $\Lambda$CDM model. In the next section we present some numerical examples illustrating the various possibilities described above.
Figure 1: Some examples of the behavior of the effective equation of state function $\omega_e$ for the type II cosmon models: (a) $\omega_X = -1.65$, $\nu = +0.001$, $\Omega^0_\Lambda = 0.67$, $\Delta \Omega^0_m = 0.01$. In this case, the EOS presents an effective transition from quintessence to phantom regime in the recent past; (b) $\omega_X = -0.85$, $\nu = -0.001$, $\Omega^0_\Lambda = 0.3$, $\Delta \Omega^0_m = -0.01$. Here the transition is just the other way around; (c) $\omega_X = -0.95$, $\nu = 0.001$, $\Omega^0_\Lambda = 0.75$, $\Delta \Omega^0_m = 0$, the EOS mimics the behavior of a cosmological constant; (d) $\omega_X = -1.15$, $\nu = -0.001$, $\Delta \Omega^0_m = 0$ and $\Omega^0_\Lambda = 0.4$ (solid) or $\Omega^0_\Lambda = 0.8$ (dashed). In the last two examples the EOS parameter remains in the phantom or quintessence regime for all the redshifts attainable by present and scheduled supernovae experiments.

5 Numerical analysis

The results of the preceding sections are further illustrated in this section through particular numerical examples. We present plots of the evolution of the dark energy effective EOS parameter $\omega_e$, the Hubble parameter $H$ and the ratio of the energy densities of dark energy and matter components $r$. These plots depict the interesting phenomena that the type II cosmon models exhibit in different parameter regimes. We first focus on the redshift dependence of the DE effective EOS parameter $\omega_e$, illustrated in the plots of Fig. 1. Each of these plots demonstrates how type II models can result in the features of $\omega_e(z)$ which are consistent with the current observational data. The type II cosmon models can exhibit a very interesting phenomenon of the CC boundary crossing, both the crossing from the quintessence regime to the phantom regime with the expansion and the transition in the opposite direction. In Fig. 1, we show an example of the realization
Figure 2: The effective equation of state parameter \( \omega_e = \omega_e(z) \) for the case \( \omega_X = -1.85, \nu = -0.001, \Omega_\Lambda^0 = 0.75 \) and different values of \( \Delta \Omega_m^0 \). We see that this last parameter crucially determines the evolution. All the curves present stopping of the expansion and a ratio between dark energy and matter radiation bounded from above.

The dependence of the form of the function \( \omega_e(z) \) on the value of the parameter \( \Delta \Omega_m^0 \) is given in Fig. 2 for the following values of the remaining parameters: \( \omega_X = -1.65, \nu = +0.001, \Omega_\Lambda^0 = 0.67 \) and \( \Delta \Omega_m^0 = 0.01 \). An example of the transition in the opposite direction for parameter values \( \omega_X = -0.85, \nu = -0.001, \Omega_\Lambda^0 = 0.3, \Delta \Omega_m^0 = -0.01 \) is depicted in Fig. 1b. The plot in Fig. 1c shows that the \( \Lambda X \)CDM models may mimic the behavior of the cosmological constant. The values of the parameters in this case are \( \omega_X = -0.95, \nu = 0.001, \Omega_\Lambda^0 = 0.75 \) and \( \Delta \Omega_m^0 = 0 \). Finally, in the plot given in Fig. 1d we give two examples in which the type II models exhibit either quintessence-like or phantom-like features for redshifts amenable to the SNIa observations. An example of the quintessence-like behavior (the dashed line in Fig. 1d) is obtained for \( \omega_X = -1.15, \nu = -0.001, \Delta \Omega_m^0 = 0 \) and \( \Omega_\Lambda^0 = 0.8 \), whereas an example of the phantom-like behavior (the solid line in Fig. 1d) is realized for \( \omega_X = -1.15, \nu = -0.001, \Delta \Omega_m^0 = 0 \) and \( \Omega_\Lambda^0 = 0.4 \). These examples illustrate the potential of the \( \Lambda X \)CDM models in explaining various types of dynamics for \( \omega_e(z) \).

The dependence of the form of the function \( \omega_e(z) \) on the value of the parameter \( \Delta \Omega_m^0 \) is given in Fig. 2 for the following values of the remaining parameters: \( \omega_X = -1.85, \nu = -0.001, \Omega_\Lambda^0 = 0.75 \). From the figure it is clear that \( \omega_e(z) \) function changes considerably when \( \Delta \Omega_m^0 \) varies at a percent level, i.e. it is rather sensitive to the choice of \( \Delta \Omega_m^0 \). All curves in this figure are characterized by the stopping of the expansion of the universe and with \( r(z) = \rho_D(z)/\rho_m(z) \) bounded from above.

The discussion presented so far indicates that the \( \Lambda X \)CDM models can easily provide the effective dark energy EOS in accordance with the observational data. This feature is certainly expected since dark energy contains multiple components and a correspondingly larger number of parameters. This possible drawback is, however, more than compensated, by the characteristics of the type II models which offer a robust solution to the coincidence problem, as already discussed.
Figure 3: The Hubble function in units of its present value and depending on (a) redshift or (b) cosmic time (measured in Hubble time units, \( H_0^{-1} \)), for the values of the parameters of Fig. 2 and the specific choice \( \Delta \Omega_m^0 = -0.005 \). The stopping and subsequent reversal of the expansion takes place after a time longer than the present age of the Universe, \( t_0 = 13.7 \, \text{Gyr} \sim 0.99 H_0^{-1} \).

in detail in [18] for the type I models. The class of type II models studied in the present paper also offers the solution of the coincidence problem. Namely, it is possible to select the model parameters so that the expansion of the universe stops at some future moment. An example of such a scenario is depicted in Fig. 3 where the evolution of the Hubble parameter is given in terms of (a) redshift and (b) cosmic time. The model parameters used are those from Fig. 2 together with \( \Delta \Omega_m^0 = -0.005 \). The evolution of the universe of this type is further characterized by the specific form of the evolution of the ratio \( r \). Namely, \( r(z) (r(t)) \) is bounded from above, i.e. it grows with the expansion up to some maximal value and then starts to decrease, changes sign at some future moment and attains the value -1 at the stopping point. It is especially interesting that for a considerable volume in the model parametric space \( (\omega_X, \nu, \Omega_0^\Lambda) \) the ratio \( r/r_0 \) (where \( r_0 \) denotes the present value of the ratio \( r \)) remains of the order 1 once the universe starts accelerating. This fact implies that the class of type II models can provide a solution (or a significant alleviation) of the coincidence problem. An example of the described dynamics of \( r \) in terms of (a) redshift and (b) cosmic time is given in Fig. 4 for the same set of parameters as in Fig. 3. A very similar conclusion about the evolution of the ratio \( r \) was reached within the class of type I models studied in [18]. The fact that both type I and type II models provide the solution to the coincidence problem indicates that this solution is not model dependent and gives support to a general claim that \( \Lambda \)CDM models provide a robust explanation of the coincidence problem.

Before finishing this section the following observation is in order. As we have seen, the type II models with flat FLRW metric and a prior on \( \Omega_D^0 \) (typically \( \Omega_D^0 = 0.7 \), the current value of the DE density) contain three parameters \( (\omega_X, \nu, \Omega_0^\Lambda) \), just as in type I models [18]. Of these three parameters \( \nu \) is tightly constrained by the nucleosynthesis considerations, as described in Section 4. This parameter space can be compared with that of other DE models which also try to solve the coincidence problem. For instance, in interactive quintessence models (IQE) [17] one has a similar 6For a sufficiently small value of \( \Delta \Omega_m^0 \) (which is indeed expected to be so) and \( \nu \) values allowed by (4.4), the plots for \( r \) and \( \tilde{r} \) are practically indistinguishable. For this reason the ones for \( r \) are not included in Fig. 4.
number of parameters, to wit: the EOS parameter of the quintessence field, $\omega_\phi$, the coupling of matter to quintessence (usually in the form of a source function – let us call it $Q$ – which one has to introduce totally *ad hoc* and depends on at least one parameter), and finally one or more parameters related to the (assumed) form of the potential. Moreover, if one wishes to trigger a transition between quintessence and phantom regimes one usually has to resort to more complex, e.g. hybrid, structures \cite{17} in which at least one additional scalar field is necessary, though carrying a “wrong sign” kinetic term (i.e. it must be a ghost field). Similar considerations apply to $k$-essence models \cite{25}, where an *ad hoc* non-linear function of the kinetic terms must be introduced along with additional parameters. In our case we can modulate that transition through the parameter $\Delta \Omega_m^0$, as we have seen above. However, even this parameter is not a new input of the model since it actually belongs to the corresponding effective EOS picture, not to the $\Lambda$XCDM model itself. To summarize, quintessence-like models have in general a similar number of parameters (if not more) as compared to the $\Lambda$XCDM, and may present additional features that one has to ponder carefully. For example, in IQE models the aforesaid coupling source $Q$ (whose presence is essential to tackle the coincidence problem) necessarily entails the non-conservation of matter. This is in contradiction to the $\Lambda$XCDM models, both of type I and II, where matter is strictly conserved.

### 6 Conclusions

We have studied the possibility that the dark energy (DE) of the Universe is a composite medium where certain dynamical components could be entangled with running cosmological parameters. We have modeled this composite structure by assuming that the DE is a mixture of a running cosmological term $\Lambda$ and a dynamical entity $X$ (the cosmon). The latter is not necessarily an elementary scalar field, it could rather be the effective behavior of a multicomponent field system or even the result of higher order terms in the effective action. In fact the generality of $X$ is such

Figure 4: The ratio $\tilde{r}$ between dark energy and matter-radiation in units of its present value and depending on redshift (a) or cosmic time (b), for the values of the parameters of Fig. 2 and the specific choice $\Delta \Omega_m^0 = -0.005$. We see that the ratio presents a maximum that can help to explain or significantly alleviate the cosmic coincidence problem. The plots for $r$ result to be nearly indistinguishable from these due to the small value of $\Delta \Omega_m^0$, and thus are not shown.
that its effective barotropic index $\omega_X$ can be both quintessence-like ($\omega_X \gtrsim -1$) and phantom-like ($\omega_X \lesssim -1$), and the energy density of $X$ can be positive or negative ($\rho_X \lesssim 0$). The kind of composite DE model we have studied is a variant of the previously considered $\Lambda X$CDM model [18]. In all of these models the total DE density, $\rho_D$, is the sum of the cosmon density, $\rho_X$, and the $\Lambda$ density, $\rho_\Lambda$. However, in contradistinction to [18], here the cosmon energy density is conserved (as the matter density itself), and thus the covariance of Einstein’s equations (expressed by the Bianchi identity) requires that the running of $\Lambda$ must be accompanied by a running gravitational coupling $G$. We have computed the effective equation of state (EOS) of this composite DE system and found that under suitable conditions it can mimic to a large extent the behavior of the standard $\Lambda$CDM model, but at the same time the fine details reveal the possibility to observe mild transitions from quintessence-like into phantom-like behavior, and vice versa. If observed in the next generation of high precision cosmology experiments (such as DES, SNAP and PLANCK [26]), it would point at the possible composite structure of the DE. Furthermore, we have revisited the cosmological coincidence problem within this model, and found that in a large portion of the parameter space (similarly to [18]) there is the possibility that the Universe exhibits a turning point in its evolution, a fact that would automatically keep the ratio between the DE density to matter density ($r = \rho_D/\rho_m$) within bounds. Incidentally, it would also free the cosmic evolution from future singularities such as the Big Rip [22]. We have displayed concrete examples where the ratio $r$ stays within a few times its present value $r_0$ for the entire history of the Universe. This feature seems to be independent of the particular implementation of the $\Lambda X$CDM model (whether of type I or of type II as defined in our analysis) and it suggests that this kind of composite models could provide a clue to solving, or at least highly mitigating, the cosmological coincidence problem.

Finally, let us mention that the choice of scale in the RG motivated approach depends on the symmetries of the underlying space-time metric. As an example, in reference [13] it was explored the possibility to apply the RG approach at the astrophysical level, which leads the RG scale to develop a dependence on the geometric coordinates of the system. Furthermore, when we move to the cosmological context we expect that the $\Lambda$ term in an inhomogeneous Universe should develop spatial fluctuations which could affect the growth of the matter structure and even the CMB anisotropies. The detailed estimate of the effect of the $\Lambda$ inhomogeneities on these observables requires an analysis far beyond the scope of this paper. To get a hint of the kind of investigation required we refer the reader to the work of [27], which concentrates on a simpler RG model for $\Lambda$ in which $G$ is constant and there is no cosmon. In this case one finds that the RG running of $\Lambda$ may have a significant effect on the structure growth, i.e. that the large scale structure data may provide strong constraints on the RG parameters. Although the RG effects in $\Lambda X$CDM models of type II are not expected to be so important, owing to the existence of the cosmon component and the severe nucleosynthesis constraints, the LSS and CMB data are important tools for placing further constraints on these models. This kind of considerations will be addressed elsewhere.

To summarize, the $\Lambda X$CDM models seem to offer a viable extension of the standard $\Lambda$CDM model in which some cosmological problems seem to be better under control. At the same time they offer a new approach to modeling the DE behavior without compromising the analysis to a particular field structure of the theory (e.g. a specific scalar field potential). The dynamics of
these models is entirely determined by the RG behavior of the cosmological parameters together with the covariant conservation laws involving the different components of the DE. In our opinion the ΛXCDM models should be considered as serious candidates for describing the potential DE features observed in the next generation of experiments.

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