Charge modulation, spin response, and dual Hofstadter butterfly in high-$T_c$ cuprates

Zlatko Tešanović
Department of Physics and Astronomy,
Johns Hopkins University, Baltimore, MD 21218, USA
(March 22, 2022)

The modulated density of states observed in recent STM experiments in underdoped cuprates is argued to be a manifestation of the charge density wave of Cooper pairs (CPCDW). CPCDW formation is due to superconducting phase fluctuations enhanced by Mott-Hubbard correlations near half-filling. The physics behind the CPCDW is related to a Hofstadter problem in a dual superconductor. It is shown that CPCDW does not impact nodal fermions at the leading order. An experiment is proposed to probe coupling of the CPCDW to the spin carried by nodal quasiparticles.

Recent STM experiments have reinvigorated the debate on the nature of the pseudogap state in underdoped cuprates. The central issue is whether the pseudogap state is a phase disordered superconductor or some other, entirely different form of competing order, originating from the particle-hole channel. The observed modulation in the local density of states (DOS), which breaks the lattice translational symmetry of CuO$_2$ planes, is conceivably attributable to both.

Within the phase-fluctuating superconductor scenario a natural temptation is to ascribe this modulation to the “helium physics”: a system of bosons (Cooper pairs) with short range repulsion is superfluid in its ground state as long as it is compressible – the only alternative to superfluidity is an incompressible state. In cuprates, as doping $x$ is reduced toward half-filling, $x \to 0$, strong onsite repulsion suppresses phase density fluctuations and reduces compressibility. This leads to enhanced phase fluctuations and reduced superfluid density $\rho_s$, courtesy of the uncertainty relation $\Delta N \Delta \varphi \gtrsim 1$. At $x = x_c$, a compressible superfluid turns into an incompressible Mott insulator. Such insulator tends to maintain a fixed number of particles in a given area and, at some doping $x < x_c$, the CuO$_2$ lattice symmetry typically will be broken in favor of a superlattice with a large unit cell, tied to $1/x \gg 1$. In this Letter I succumb to this temptation and examine several of its experimental consequences.

The first step is to recognize that the pseudogap physics differs in an essential way from the above $^4$He analogy: cuprates are $d$-wave superconductors and, in contrast to $^4$He or $s$-wave systems, any useful description must contain not only the bosons (Cooper pairs) but also fermionic excitations in the form of nodal Bogoliubov-deGennes (BdG) quasiparticles. The quasiparticles carry well-defined spin $S = \frac{1}{2}$ and their coupling to the charge sector, dominated by the $S = 0$ Cooper pairs, is arguably the crucial element of quantum dynamics of cuprates. This spin-charge interaction is topological in origin and peculiur for fluctuating spin-singlet superconductors.

The nodal fermions convey another fundamental information: Cooper pairs in cuprates are inherently the momentum-space objects in contrast to the real-space pairs behaving as “elementary” bosons, like $^4$He or the SO(5) hard-core plaquette bosons. Thus one encounters in cuprates an echo of the historical debate on Blatt-Schafroth versus BCS pairs. This is an important issue – while certain long-distance features of the two descriptions are equivalent, many crucial physical properties are not. In particular, the observed charged modulation is a finite wavevector, non-universal phenomenon. As shown in this Letter, the modulation patterns and stable states arising from the two descriptions are essentially different.

To appreciate this difference, note that Cooper pairs in nodal $d$-wave superconductors are highly non-local objects in real space and any description in terms of their center-of-mass coordinates will reflect this non-locality through complicated intrinsically multi-body, extended-range interactions. Such complexity haunts any attempt at constructing a theory using Cooper pairs as “elementary” bosons. The basic idea advanced in this Letter is that, under these circumstances, the role of “elementary” bosons should be accorded to vortices instead of Cooper pairs. Vortices in cuprates, with their small cores, are simple real space objects and the effective theory of quantum fluctuating vortex-antivortex pairs can be written in the form that is local and simple to analyze.

I start by proposing that the modulation observed in reflects the Copper pair charge-density wave (CPCDW) in a fluctuating nodal $d$-wave superconductor. I then show that the physics behind CPCDW relates to an Abrikosov-Hofstadter problem for a dual type-II lattice superconductor with a flux per unit cell $f = (1-x)/2$. This mapping allows one to identify stable states as function of $x$ and to extract the periodicity and orientation of CPCDW relative to the CuO$_2$ lattice. I elucidate the origin of stable fractions and contrast the results with those for the real-space pairs. The two differ in a fundamental way, akin to the difference between strongly type-II and strongly type-I superconductors. Next, I argue that the formation of CPCDW is irrelevant for the physics of nodal fermions – CPCDW is a “high-energy” phenomenon in the parlance of the effective theory. Consequently, the leading behavior of nodal fermions remains undisturbed. Finally, I suggest an observable effect of CPCDW which probes an essential element of the theory: the presence of a gauge field which frustrates the propagation of spin, exclusively carried...
by nodal quasiparticles. The effect is an enhanced modulation, with the periodicity related to CPCDW, of the subleading, $T^2$, term in the spin susceptibility $\chi$. This effect takes place in the “supersolid” state, where superconductivity and CPCDW coexist, and its experimental observation would provide direct evidence of the topological coupling between the fluctuating vortex-antivortex pairs responsible for CPCDW and the electronic spin.

The effective theory of a quantum fluctuating $d_x^2 - y^2$-wave superconductor was developed in Eq. (6) and represents the interactions of fermions with $\hbar c/2e$ vortex-antivortex excitations in terms of two gauge fields, $\nu_\mu$ and $\alpha_\mu$:

$$L = \bar{\Psi} \left[ D_0 + i\nu_\mu + \frac{(D + i\nu)^2}{2m} - \mu \right] \Psi - i\Delta \Psi^T \sigma_2 \tilde{\eta} \Psi + c.c. + L_0,$$

where $\Psi = (\tilde{\psi}_\uparrow, \tilde{\psi}_\downarrow)$, $\nu = (\tau, x, y)$, $D_\mu = \partial_\mu + i\alpha_\mu \sigma_3$, $\sigma_i$'s are the Pauli matrices, and $\tilde{\eta} = D_2^2 - D_3^2$. $L_0[v, a]$ is the Jacobian of the transformation from discrete (anti)vortex coordinates to continuous fields $v$ and $a$:

$$\int \mathcal{L}_d[\Phi, A_d, \kappa] e^{-\int d^4x (\frac{1}{2} \langle A_d^2 \rangle + |\partial_\phi - 2\pi \kappa|^2)},$$

and $C||\Phi||$ is a normalization factor

$$C = \int \mathcal{L}_d[a, \phi, \kappa] e^{-\int d^4x e^{i\phi} |\phi|^2}.$$

The physics behind Eq. (1) is simple: The fermionic part of $\mathcal{L}$ is just the BdG action for a nodal $d$-wave superconductor, the awkward phase factor $\exp(i\varphi(x))$ having been removed from $\Delta$ by a gauge transformation. This transformation generates gauge fields $\nu$ and $a$, which mimic the effect of vortex-antivortex pair fluctuations on the BdG quasiparticles $v$ in the charge and $a$ in the spin channel. Finally, a bosonic field $\Phi$ describes quantum vortex-antivortex pairs: vortices/antivortices can be thought of as particles/antiparticles created and annihilated by dual field $\Phi$.

The dual “normal” state ($\langle \Phi \rangle = 0$) is a physical superconductor while dual condensate ($\langle \Phi \rangle \neq 0$) describes the pseudogap state. The purpose behind the mathematics is to reformulate the problem in terms of the BdG action for fermions (1) and the local Lagrangian of vortex bosons $L_d$ [3], kept in mutual communication via two pairs of gauge fields $(v, a)$ and $(A_d, \kappa)$.

Why is this reformulation useful? The CPCDW, an intractably non-local problem in the language of electrons, has a simple local expression in the language of vortex field $\Phi$. To recognize this, observe that the phase $\varphi(x)$ couples in $\mathcal{L}$ (12) to the overall electron density as $\bar{\psi} \sigma_3 \partial_\varphi \psi$, where $\bar{n} = \bar{n}_\uparrow + \bar{n}_\downarrow$. This translates to a dual “magnetic field” $\nabla \times A_d = \frac{1}{2} \bar{n}$ [17] seen by vortex-antivortex pairs [18]. Physically, this effect gives dual voice to the “helium physics” discussed earlier: to prevent superfluid ground state the system turns into an incompressible solid, a dual Abrikosov lattice [7, 16]. Therefore, the quantum vortex-antivortex unbinding leads to the breaking of (lattice) translational symmetry. When the pattern of symmetry breaking is determined by the local dual problem, the results are “communicated” back to the fermionic part of $\mathcal{L}$ [1] via the gauge fields $(v, a)$ and $(A_d, \kappa)$—hence CPCDW.

The above arguments are explicit in the dual mean-field approximation, combined with the linearization of the spectrum near the nodes. The linearization splits the fermions in (1) into low-energy nodal spin-$\frac{1}{2}$ Dirac-like particles $\psi_{\nu,\alpha}$, where $\alpha = 1, 1, 2$, and high-energy anti-nodal fermions combined into spin-singlet Cooper pairs, $\psi_{\nu,(\alpha\beta)}$, where $\langle (\alpha\beta) \rangle = \langle 12, 21, 12 \rangle$, and $\langle 12 \rangle$. Nodal Dirac fermions have no overall charge density — the overall charge is carried by $\psi_{\nu,(\alpha\beta)}$ (Cooper pairs). Furthermore, $\psi_{\nu,(\alpha\beta)}$ form spin-singlets and do not couple to $\kappa$. This enables us to separate the mean-field equations for the spin sector from those for charge:

$$\begin{align*}
\pi(n_\uparrow(r, \tau) + n_\downarrow(r, \tau)) &= \nabla \times A_d(r), \\
\pm \delta_{y(\nu)} \delta v_0(r) &= \bar{\pi}_x^B(r), \\
m^2\phi - (\nabla - iA_d)^2\phi + g|\phi|^2\phi &= 0, \\
\langle \frac{\delta L}{\delta \Delta(r)} \rangle &= (2/\lambda_{\text{eff}})\Delta(r),
\end{align*}$$

where $n_\nu(x) = \bar{\psi}_{\nu,(\alpha\beta)}(x)\psi_{\nu,(\alpha\beta)}(x) + \bar{\psi}_{\nu,(\alpha\beta)}\psi_{\nu,(\alpha\beta)}$. $\pm$ is a dual current, $\bar{\pi}_x^B = -i\delta v_0 \partial_\phi + c.c. + A_d[\hat{\phi}]^2$, and $\lambda_{\text{eff}}$ is the effective coupling constant (the last equation is the BdG self-consistency condition for the pseudogap).

The first of Eqs. (5) is an implicit expression for $\delta v_0(r)$. In cuprates, the loss of superconductivity through underdoping to a pseudo-gap state is small for low $\kappa$.

In cuprates, the loss of superconductivity through underdoping to a pseudo-gap state is small for low $\kappa$. The solution of (6,7) in continuum is just the celebrated triangular vortex lattice of Abrikosov [16]. In our dual problem, however, the effect of the underlying CuO$_2$ lattice must be considered. This is so since, for dopings of interest, we are quite close to half-filling and $x = 0$. Translated to dual language this means we are close to $f = 1/2$, the maximally frustrated case of (7). The pinning to the atomic lattice is significant and we should expect a near-square symmetry for the resulting CPCDW.

Eqs. (6,7) are solved as follows: (13): first, various derivatives in (6,7) are replaced by their CuO$_2$ lattice counterparts.
We then consider \( \Phi_i \) with a uniform dual field \( H_d = \frac{1}{2} n \) and initially set \( g \to 0 \). This is a variant of the Hofstadter problem for dual bosons \( \Phi_i \), with a fractional flux \( f = p/q = (1 - x)/2 \) through a plaquette of a dual lattice. The solution is a “Hofstadter butterfly” spectrum with deep energy minima for major fractions. The ground state is \( q \)-fold degenerate and one must choose the linear combination of states for dual bosons to condense into. The degeneracy is lifted by finite \( g \) in \( \Phi_i \).

Thus, a unique state \( \Phi_i^{(0)} \) is selected, the only remaining degeneracy associated with discrete lattice symmetries. Once \( \Phi_i^{(0)} \) is known, one computes the current \( \Phi_i^{(0)} \) and uses Maxwell equation \( \mathcal{L} \) to find the modulation in dual induction \( \delta B_d = \mathbf{B}_d - \mathbf{H}_d = \nabla \times \delta \mathbf{A}_d(\mathbf{r}) \). This procedure is then iterated to convergence.

The major fractions and their modulation patterns are primarily determined by the Abrikosov-Hofstadter problem \( \mathcal{L} \), the magnetic energy being a small correction in a type-II system. The interactions among vortices in \( \Phi_i^{(0)} \) responsible for these patterns are intrinsically multi-body and of extended range – they are the interactions among the center-of-mass coordinates of Cooper pairs. This is in contrast to the real-space pairs with pairwise short-ranged interactions \( V(r - r') \). The pair density-wave is determined not by \( \mathcal{L} \) but by the Wigner crystallization, encoded in \( \mathcal{R} \), which in this limit turns to the minimization of: \( \frac{1}{2} \int d^2r d^2r' B_d(\mathbf{r}) V(r - r') B_d(\mathbf{r}') \), where \( B_d(\mathbf{r}) = \sum_i \delta(r - r_i) \) and \( \{r_i\} \) are the pairs’ positions. Thus, the Cooper and the real-space pairs correspond to the two opposite limits of \( \mathcal{L} \), that of the type-II and the type-I regime of a dual superconductor, respectively.

While the analysis of \( \mathcal{L} \) is given in \( \mathcal{L} \), I outline here general features of the solution. \( \Phi_i^{(0)} \) and \( \nabla \times \delta \mathbf{A}_d(\mathbf{r}) \) break the translational symmetry of the dual and the CuO\(_2\) lattice. The new superlattice is characterized by the set of reciprocal vectors \( \{\mathbf{G}_i\} \). The major fractions \( f \), i.e. the energetically most favored states, are those with \( q \) being a small integer, (integer)\(^2\) or a multiple of 2, reflecting the square symmetry of the CuO\(_2\) planes. In the window of \( x \) relevant to cuprates, these are \( f = 7/16, 4/9, 3/7, 6/13, 11/24, 15/32, 13/32, 29/64, 27/64, . . . \), \( x = 0.125 \) (1/8), 0.111 (1/9), 0.143 (1/7), 0.077 (1/13), 0.083 (1/12), 0.0625 (1/16), 0.1875 (3/16), 0.09375 (3/32), 0.15625 (5/32) etc. Other potentially prominent \( f \), like 1/4, 1/3, 2/5, or 3/8, correspond to \( x \) outside the regime of vortex-antivortex fluctuations.

The above information allows insight into \( \{\mathbf{G}_i\} \)’s of major fractions. The non-linear term in \( \Phi_i^{(0)} \) favors the smallest unit cell containing an integer number of flux quanta and a homogeneous modulation in \( \Phi_i^{(0)} \). These conditions single out doping \( x = 0.125 \) \((f = 7/16)\) as a particularly prominent fraction. At \( x = 0.125 \) \((q = 16)\), the modulation in \( \nabla \times \mathbf{A}_d(\mathbf{r}) \) can take advantage of a 4 \( \times \) 4 elementary checkerboard block which, when oriented along the \( x \)-direction, fits neatly into plaquettes of the dual lattice. Near \( f = 1/2 \), however, a large number of vortices \( (p = 7) \) per such a block leads to a redistribution and a larger, rhombic unit cell \( \mathcal{R}_2 \) – the energy gap relative to the 4 \( \times \) 4 checkerboard, however, is extremely small. The modulation in \( \delta \mathbf{n}(\mathbf{r}) \) (and \( \Delta \mathcal{R}_2 \)) still retains a memory of the 4 \( \times \) 4 block and is characterized by wavevectors \( \mathbf{G}_1 = (\pm 2\pi/4a, 0) \), \( \mathbf{G}_2 = (\pm 2\pi/8a, \pm 2\pi/4a) \), and \( \mathbf{G}_3 = (\pm 2\pi/(8a/3), \pm 2\pi/4a) \), with \( \mathbf{G}_1 \) oriented along the antinodal (either \( x \) or \( y \)) directions of the CuO\(_2\) lattice. The domains of the above modulation pattern offer a natural explanation for the observations in Ref. \[2\].

The next leading fractions are \( x = 0.077 \) \((1/13)\) \((f = 6/13)\) and \( x = 0.111 \) \((1/9)\) \((f = 4/9)\). The modulation patterns are now more complicated and do not fit easily into the underlying CuO\(_2\) lattice. \( \delta \mathbf{n}(\mathbf{r}) \) (and \( \Delta \mathcal{R}_2 \)) exhibits a rhombic unit cell with \( \{\mathbf{G}_i\} \)’s oriented closer to the lattice diagonals, i.e. the nodal directions. Thus, as \( x \) decreases away from 1/8 there will be a tendency to reorient the superlattice away from antinodal directions and align closer to the CuO\(_2\) lattice diagonals. Such reorientation effects of the CPCDW, if observed, would provide support for the physics described in this Letter.

The above considerations include dopings like \( x = 1/8 \) or 1/9 for which cuprates are typically still superconducting. In such cases the mean-field \( \mathcal{R}_2 \) is inadequate and one must include fluctuations in \( \Phi \) and \( A_d \). The fluctuations act to depopulate the mean-field ground state \( \Phi^{(0)} \) and transfer some of the dual bosons to the states nearby in energy. As \( x \) increases toward \( x_c \), \( \Phi^{(0)} \) eventually ceases to be macroscopically occupied \((\Phi \to 0)\) and the system returns to the superconducting state. However, as long as the transition is not strongly first-order, dual bosons still preferentially occupy the states close to \( \Phi^{(0)} \) on the “Hofstadter butterfly” energy landscape. This results in \((\Phi(\mathbf{r})^2)\) which is finite and still modulated. Only for yet higher \( x \) will the translational symmetry of the superconducting state be finally restored.

The above is an example of the “supersolid state”, in which superconductivity coexists with the CPCDW. The modulation is dominated by \( \Phi^{(0)} \) and thus our mean-field symmetry analysis of major fractions still goes through. The fluctuations that produce the “supersolid” state consist of a liquid of vacancies and interstitials superimposed on the original mean-field dual vortex lattice. This leads to low \( \rho_s \) and tends to shift the periodicity away from the mean-field set of \( \{\mathbf{G}_i\} \)’s associated with major fractions, particularly as a function of \( T \), since the self-energies of vacancies and interstitials are generically different. Such fluctuation-induced incommensurability could be behind the difference between the CPCDW periodocities observed in \( \mathcal{H} \) (high \( T\)) and \( \mathcal{E} \) (very low \( T\)).

The preceding discussion of the charge sector sets the stage for the question of what happens to spin, carried by nodal quasiparticles (for convenience, I now rotate a \( d_{x^2-y^2} \)-wave superconductor into a \( d_{xy} \)-wave one). The CPCDW affects low-energy fermions in two ways: first, \( \delta \nu_{\mathbf{r}} \) couples to \( \psi_{\sigma, \mathbf{r}} \) as a periodically modulated chemical potential and can be absorbed into a locally varying Fermi wavevector: \( k_F \to k_F + \delta k_F(\mathbf{r}) \), where \( \delta k_F(\mathbf{r}) = \delta \nu_{\mathbf{r}}(\mathbf{r})/v_F \). Such shift leaves the nodal point in the energy space undisturbed. Similarly, there also is a modulation in the size of...
the pseudogap: $\Delta \rightarrow \Delta + \delta \Delta(r)$, arising from the BdG self-consistency equation \[9\]. Near the nodes $\Delta(P; k) \rightarrow \Delta(k_x^2 - k_y^2) + \delta\Delta(G_i; k)$, where $P$ is related to the center-of-mass momentum of Cooper pairs. Assuming that the pseudogap retains the overall $d_{x^2-y^2}$-wave symmetry throughout the underdoped regime, one finds $\delta\Delta(G_i; k) \sim k_x^2 - k_y^2$.

Again, the nodal point is left intact. The semiclassical spectrum is:

$$E(k; r) = \pm \sqrt{v_F^2(r)k_x^2 + v_A^2(r)k_y^2},$$ (8)

where $v_F(r) = v_F + (\delta k_F(r)/m)$ and $v_A(r) = v_A + (\Delta(r)/k_F)$ \[2\]. The local DOS exhibits modulation at wavevectors $\{G_i\}$’s but still vanishes linearly at the nodes. The only exception to this behavior is the situation in which CPCDW is commensurate with the nodes and $\{G_i\}$’s happen to coincide with some of the internodal wave vectors: $Q_{12}$, $Q_{13}$, etc. Such commensuration can only be purely accidental since the dual lattice physics \[6\] that determines $\{G_i\}$’s has no simple relation to the location of nodes in the Brillouin zone.

There is, however, a yet another way by which the CPCDW affects nodal fermions $\psi_{\sigma, \alpha}$. This is through the coupling to a Berry gauge field $a_{\mu}^\sigma$, which describes topological fluctuation of BdG “spinons” moving through space filled with fluctuating $hc/2e$ vortex-antivortex pairs. This non-trivial coupling of charge and spin sector is captured by the effective Lagrangian:

$$L = \bar{\psi}_n(i\gamma_\mu \partial_\mu + \gamma_\mu a_\mu)\psi_n + \mathcal{L}_0^0[a_\mu],$$ (9)

obtained as the low energy ($\ll \Delta$) limit of $L$. In \[9\], $\psi_{\sigma, \alpha}$ have been arranged into four component Dirac-BdG spinors $\psi_n$, following conventions of Ref. \[8\] and the summation over $N = 2$ nodal flavors is understood.

Below the pseudogap energy scale $\Delta$, the spin correlator of physical electrons is \[2\] '

$$\langle S_z(-k)S_z(k) \rangle = \frac{\Pi_F^0(k)\Pi_A^0(k)}{\Pi_A^0(k) + \Pi_A^0(k)} \frac{k^2}{\omega^2_{\text{ni}}},$$ (10)

where $\Pi_A^F(k) \sim |k|$ denotes the fermion current polarization and $\Pi_A^0(k) \sim k^2$ dominates the expression for the static spin susceptibility $\chi$ leading to a non-Fermi liquid behavior of nodal quasiparticles \[2\]. In the superconducting state, $a$ has mass $M^2$, $\Pi_A^0 \rightarrow M^2$, and the leading order behavior is set by $\Pi_A^F$. For $T \ll \Delta$:

$$\chi \sim (2N \ln 2/\pi)T - \frac{(2N \ln 2/\pi)^2}{M^2}T^2 + \ldots.$$ (11)

The leading term ($\sim T$) in \[11\] is just the renormalized $d$-wave Yoshiida function of non-interacting BdG quasiparticles. The subleading term ($\sim T^2$), however, involves $M^2$. In the “supersolid” phase $M^2$ is modulated via the non-uniformity in $\langle|\Psi|^2\rangle$ \[4\] – this modulation carries an imprint of the CPCDW periodicity set by $\{G_i\}$’s, since it reflects the variation of $\langle|\Psi|^2\rangle$ on the lattice dual to the CuO$_2$ one. Furthermore, since $M^2 \sim \xi_d$ \[4\], where $\xi_d$ is the dual superconducting correlation length, the $T^2$ term in \[11\] is $\propto \xi_d$ and consequently strongly enhanced as $x \rightarrow x_c$.

The combination of modulation and enhancement, as the superconductivity is extinguished at $x = x_c$, sets this term apart from other contributions to $\chi$. The observation of such a modulation, in a μSR or an NMR experiment, for instance, would provide a vivid illustration of the subtle interplay between the charge and spin channels which is the hallmark of theory \[1\].

I thank A. Melikyan, J.C. Davis, M. Franz, J.E. Hoffman, A. Sudbø, O. Vafek, and A. Yazdani for useful discussions. This work was supported in part by the NSF grant DMR-00-94981.

[1] M. Vershinin et al., Science 303, 1995 (2004).
[2] K. McElroy et al., cond-mat/0404005; J.E. Hoffman et al., Science 295, 466 (2002).
[3] C. Howald et al., Phys. Rev. B 67, 014533 (2003).
[4] H.D. Chen, O. Vafek, A. Yazdani, and S.C. Zhang, cond-mat/0402323; H.D. Chen et al., cond-mat/0312600; H. D. Chen et al., Phys. Rev. Lett. 89, 137004 (2002).
[5] H.C. Fu, J.C. Davis, and D.H. Lee, cond-mat/0403001
[6] V.I. Emery and S.A. Kivelson, Nature 374, 434 (1995).
[7] L. Balents, M.P.A. Fisher and C. Nayak, Phys. Rev. B 60, 1654 (1999).
[8] M. Franz and Z. Tešanović, Phys. Rev. Lett. 87, 257003 (2001); Z. Tešanović, O. Vafek, and M. Franz, Phys. Rev. B 65, 180511 (2002); M. Franz, Z. Tešanović, and O. Vafek, ibid. 66, 054535 (2002).
[9] I.F. Herbut, Phys. Rev. Lett. 88, 047006 (2002); B. Seradjeh and I.F. Herbut, Phys. Rev. B 66, 184507 (2002).
[10] S. Chakravarty, R.B. Laughlin, D.K. Morr, and C. Nayak, Phys. Rev. B 63, 094503 (2001).
[11] Y. Zhang, E. Demler, and S. Sachdev, Phys. Rev. B 66, 094501 (2002).
[12] J. Zaanen and O. Gunnarsson, Phys. Rev. B 40, 7391 (1989); K. Machida, Physica C 158, 192 (1989); S.A. Kivelson et al., Rev. Mod. Phys. 75, 1201 (2003).
[13] L. Reatto and G.V. Chester, Phys. Rev. 155, 88 (1967).
[14] M.P.A. Fisher et al., Phys. Rev. B 40, 546 (1989).
[15] D.R. Hofstadter, Phys. Rev. B 14, 2239 (1976).
[16] A.A. Abrikosov, Sov. Phys. JETP 5, 1174 (1957).
[17] This argument is borrowed from M.P.A. Fisher and D.H. Lee, Phys. Rev. B 39, 2756 (1989).
[18] There is no analogue “magnetic field” for the spin dual field $\kappa$ since $n_i - n_{i\uparrow}$ = $n_i - n_{i\downarrow}$.
[19] A. Melikyan and Z. Tešanović, cond-mat/0408344
[20] A care is needed here since Cooper pairs reside on bonds. Also, the non-locality in $\chi$, complicates the algebra behind \[4\] while leaving the physics unchanged \[15\].
[21] The reasoning behind \[8\] can be fortified beyond semiclassical -- the modulation $\delta\tau_{i\alpha}(r)$ defines some new “band-structure” of a lattice $d$-wave superconductor with an effective hopping $t_{ij}$ and a bond pseudogap $\Delta_j$. If we fold the original Brillouin zone to accommodate the supercell of charge modulation, the nodal points are still at the new Fermi surface, as long as
\[ \Delta_{ij} \] retains the overall \( d \)-wave symmetry. Near the nodes, the spectrum is linear with perturbative velocity renormalizations 
\[ \sim (\delta \nu_0 (\{ G \}_i))^2. \]

[22] O. Vafek and Z. Tešanović, Phys. Rev. Lett. 91, 237001 (2003).