Abstract—Amdahl’s Law is generating debates in the community of experts dealing with parallel processing since it was suggested. The paper explains why Amdahl’s Law shall be interpreted specifically for distributed parallel systems and why it generated so many debates, discussions and abuses. A general model is set up where many of the terms affecting parallel processing are listed and the validity of neglecting certain terms in different approximations are scrutinized, with special emphasis on the famous scaling laws of parallel processing. It is clarified that using the right interpretation of terms, Amdahl’s Law is the governing law of all kinds of parallel processing. Amdahl’s Law describes among others the history of supercomputing, the inherent performance limitation of the different kinds of parallel processing and it is the basic Law of the ‘modern computing’ paradigm, that is desperately needed for the computing systems working under extreme computing conditions.

I. INTRODUCTION

AMDAHL in his famous paper [1], even in the title, wanted to draw the attention to that (as he has coined out) the Single Processor Approach (SPA) seriously will limit the achievable computing performance, given that “the organization of a single computer has reached its limits” and attempted to explain why it was so. Unfortunately, his intention was nearly completely misunderstood by his successors. Rather than developing “interconnection of a multiplicity of computers in such a manner as to permit cooperative solution” his idea was only used to derive the limitations of computing systems built from components manufactured for the SPA. His famous formula was constructed later, and unfortunately, attributed an unfortunate meaning to its terms. The quick technical development suppressed the real intention and meaning of the Law, and when the computing needs and possibilities reached the point where the precise meaning of the Law matters, the incorrect interpretation attributed to its terms did not describe the experiences, giving way to different other ‘laws’ and scaling modes. With the proper interpretation, however, it can be shown “that Amdahl’s Law is one of the few, fundamental laws of computing” [2]. Not only of computing, but of all – even computing unrelated – partly parallelized otherwise sequential activities. In this paper only the consequences of the idea on scaling of computer systems built in SPA are discussed; for the idea he was really thinking about, see [3], [4], [5].

In section [1] the considered scaling methods are shortly reviewed and some of their consequences discussed. In section [II-A] Amdahl’s idea is shortly described: his famous formula using our notations is introduced. In section [II-B] the basic idea of the massively parallel processing, Gustafson’s idea is scrutinized. The recently introduced scaling law [6], based on the idea of “modern computing” [7], essentially Amdahl’s idea applied to the modern computing systems, is presented in section [II-C].

Section [III] introduces a (by intention strongly simplified and non-technical) model of the parallelized sequential processing. The model visualizes the meaning of the “parallelizable portion” and enables to draw the region of validity of the “strong” and “weak” scaling methods. As those scaling methods and principles are relevant on all fields of parallel and distributed processing, the application of the presented formalism for different tasks is demonstrated in section [IV].

II. THE SCALING METHODS

The scaling methods used in the field are essentially approximations to the general model presented in section [III]. The nature and validity of those approximations will be discussed here, and this section also introduces the notations and the formalism.

A. Amdahl’s Law

Amdahl’s Law is usually formulated with an equation such as

\[
S^{-1} = (1 - \alpha) + \alpha/N
\]  

where \(N\) is the number of parallelized code fragments, \(\alpha\) is the ratio of the parallelizable fraction to the total (so \((1-\alpha)\) is the “serial percentage”), \(S\) is the measurable speedup. That is, Amdahl’s Law considers a fixed-size problem, and the \(\alpha\) portion of the task is distributed to the fellow processors; this approximation is called also ‘weak scaling’.

When calculating the speedup, one actually calculates

\[
S = \frac{(1 - \alpha) + \alpha}{(1 - \alpha) + \alpha/N} = \frac{N}{N \cdot (1 - \alpha) + \alpha} \tag{2}
\]

However, as expressed in [8]: “Even though Amdahl’s Law is theoretically correct, the serial percentage is not practically obtainable.” That is, concerning \(S\) there is no doubt that it is derived as the ratio of the measured execution times, for the non-parallelized and the parallelized case, respectively. But, what is the exact interpretation of \(\alpha\), and how can it be used?

Unfortunately, Amdahl used \(\alpha\) with the meaning “the fraction of the number of instructions which permit parallelism” in
Fig. 1 he used as illustration in his paper. The illustration refers to the case when "around a point corresponding to 25% data management overhead and 10% of the problem operations forced to be sequential". At that "point" there is no place to discuss finer details of the performance affecting factors (otherwise mentioned by Amdahl, such as "boundaries are likely to be irregular; interiors are inhomogeneous; computations required may be dependent on the states of the variables at each point; propagation rates of different physical effects may be quite different; the rate of convergence, or convergence at all may be strongly dependent on sweeping through the array along different axes on succeeding passes, etc"). It is worth to notice that Amdahl has foreseen issues with "sparse" calculations (or in general: the role of data transfer) as well as that the physical size of the computer (especially in the case of distributed systems) also matters.

However, at that time (unlike today [9], [10]) the execution time was strictly determined by the number of the executed instructions. What he actually wanted to say was "the fraction of the time spent with executing the instructions which permit parallelism" (at other places the correct expression "the fraction of the computational load" was used). This unfortunately formulated phrase "has caused nearly three decades of confusion in the parallel processing community. This confusion disappears when processing times are used in the formulations" [8]. On one side, it was guessed that Amdahl's Law is valid only for software (for the executed instructions) and on the other side the other affecting factors, he mentioned but did not discuss in details, were forgotten.

As expressed correctly in [8]: "For example, if the serial percentage is to be derived from computational experiments, i.e., recording the total parallel elapsed time and the parallel-only elapsed time, then it can contain all overheads, such as communication, synchronization, input/output and memory access. The law offers no help to separate these factors. On the other hand, if we obtain the serial percentage by counting the number of total serial and parallel instructions in a program, then all other overheads are excluded. However, in this case the predicted speedup may never agree with the experiments."

Really, one can express \( \alpha \) from Eq. (1) in terms measurable experimentally as
\[
\alpha = \frac{N}{N-1} \cdot \frac{S-1}{S} \quad (3)
\]
That is this \( \alpha_{eff} \) value, the effective parallel portion, can be derived from the experimental data for the individual cases. Also, it is useful to express the efficiency with the pseudo-experimentally measurable
\[
E(N, \alpha) = \frac{S}{N} = \frac{1}{N \cdot (1-\alpha) + \alpha} = \frac{R_{Peak}}{R_{Max}} \quad (4)
\]
data, because for many parallelized sequential systems (including the TOP500 supercomputers) the efficiency (as \( \frac{R_{Max}}{R_{Peak}} \)) and the number of processors \( N \) are provided. Reversing the relation, the value of \( \alpha_{eff} \) can be calculated as
\[
\alpha(E, N) = \frac{E \cdot N - 1}{E \cdot (N - 1)} \quad (5)
\]

As seen, the efficiency is a two-parameter function (the corresponding surface is shown in Fig. 1), demonstratively underpinning that "This decay in performance is not a fault of the architecture, but is dictated by the limited parallelism" [11] and that its dependence on its variables can be perfectly described by the properly interpreted Amdahl’s Law. This also means that Amdahl’s Law (after the pinpointing given in section II-C) shall describe the behavior of systems using a variety of parallelism, see section IV.

B. Gustafson’s Law

Partly because of the outstanding achievements of the parallelization technology, partly because of the issues around the practical utilization of Amdahl’s Law, the 'strong scaling' (called also Gustafson’s Law [12]) was also introduced, meaning that the computing resources grow proportionally with the task size. It was formulated as (using our notations)
\[
S = (1 - \alpha) + \alpha \cdot N \quad (6)
\]

Similarly to the Amdahl’s Law, the efficiency can be derived for the Gustafson’s Law as (compare to Eq. (4))
\[
E(N, \alpha) = \frac{S}{N} = \frac{\alpha}{1 + (1 - \alpha)} \quad (7)
\]
From these equations immediately follows that the speedup (the parallelization gain) increases linearly with the number of processors, without limitation; a conclusion that was launched amid much fanfare. They imply, however, some more immediate conclusions, such as

- the speedup can be measured even if no processor is present
- the efficiency slightly increases with the number of processors \( N \) (the more processors, the better efficacy)
- the non-parallelizable portion of the job either shrinks as the number of processors grows, or despite that it is non-parallelizable, the portion \( 1 - \alpha \) is distributed between the \( N \) processors
- executing the extra instructions needed by the first processor to organize the joint work need no time
- all non-payload computing contributions such as communication (including network transfer), synchronization, input/output and memory access take no time

However, an error was made in deriving Eq. (6): the \( N - 1 \) processors are idle waiting while the first one is executing the sequential-only portion. Because of this, the time that serves as the base for calculating the speedup in the case of using \( N \) processors
\[
T_N = (1 - \alpha)_{\text{processing}} + \alpha \cdot N + (1 - \alpha) \cdot (N - 1)_{\text{idle}}
\]
\[
= (1 - \alpha) \cdot N + \alpha \cdot N
\]
\[
= N
\]

That is, before fixing the arithmetic error, impossible conclusions follow, after fixing it, the conceptual error comes
to the light: the strong scaling assumes that the efficiency of the single-processor efficiency can be transferred to the parallelized sequential subsystems without loss, i.e. that the efficacy of a system comprising \(N\) single-thread processors remains the same than that of a single-thread processor; a fact that strongly contradicts the experienced ‘efficiency’ of the parallelized systems, not speaking about the ‘different efficiencies’ [6], see also Fig. 1.

That is, the Gustafson’s Law is simply a misinterpretation of the argument \(\alpha\): a simple function form transforms Gustafson’ Law to Amdahl’s Law [8]. After making that transformation, the two (apparently very different) laws are identical. However, as suspected by [8]: “Gustafson’s formulation gives an illusion that as if \(N\) can increase indefinitely”. This illusion led to the moon-shot of targeting to build supercomputers with computing performance well above the feasible (and reasonable) size and may lead to false conclusion in the case of using clouds. The ‘modern scaling’ explains why this illusion could not be revealed for decades and provoked decades-long debates in the community.

C. Modern scaling

The role of \(\alpha\) was theoretically established [13] and the phenomenon itself, that the efficiency (in contrast with Eq. (7)) decreases as the number of the processing units increases, is known since decades [11] (although it was not formulated in the functional form given by Eq. (4)). In the past decades, however, the theory was somewhat faded mainly due to the quick development of the parallelization technology and the increase of the single-processor performance; and finally, because the ‘strong scaling’ approximation was used to calculate the expected performance values, in many cases outside its range of validity. The ‘gold rush’ for building exa-scale computers finally made obvious that under the extreme conditions represented by the need of millions of processors the mostly used ‘strong scaling’ leads to false conclusions: it “can be seen in our current situation where the historical ten-year cadence between the attainment of megaflops, teraflops, and petaflops has not been the case for exaflops”[14]. It looks like, however, that in the feasibility studies of supercomputing using parallelized sequential systems an analysis whether building computers of such size is feasible (and reasonable) remained out of sight either in USA [15, 16] or in Japan [18] or in China [19].

Figure 1 depicts the two-parameter efficiency surface stemming out from Amdahl’s law (see Eq. (2)). On the surface some measured efficiencies of the present top supercomputers are also depicted, just to illustrate some general rules. To validate the model described in section III the data of the rigorously verified supercomputer database [20] was used, as described in [6]. The High Performance Linpack (HPL) efficiencies are sitting on the surface, while the corresponding High Performance Conjugate Gradients (HPCG) values are much below those values. The conclusion drawn here was that “the supercomputers have two different efficiencies” [21].

![Figure 1](https://www.epcc.ed.ac.uk/blog/2015/07/30/hpcg)

Dependence of \(E_{HPL}\) and \(E_{HPCG}\) on \((1 - \alpha_{HPL}^{eff})\) and \(N\)

Fig. 1. The 2-parameter efficiency surface (in function of the parallelization efficiency measured by benchmark HPL and the number of the processing elements) as concluded from Amdahl’s Law (see Eq. (4)), in the first order approximation. Some sample efficiency values for some selected supercomputers are shown, measured with benchmarks HPL and HPCG, respectively.

Because the experience cannot be explained in the frame of the “classic computing paradigm”.

The Taihulight and K computer stand out from the “millions core” middle group. Thanks to its 0.3M cores, K computer has the best efficiency for the HPCG benchmark, while Taihulight with its 10M cores the worst one. The middle group follows the rules [6]. For HPL benchmark: the more cores, the lower efficiency. For HPCG benchmark: the “roofline” [22] of that communication intensity was already reached, all computers have about the same efficiency.

According to Eq. (4) the efficiency can be interpreted in terms of \(\alpha\) and \(N\), and the payload performanc of a parallelized sequential computing system can be calculated as

\[
P(N, \alpha) = \frac{N \cdot P_{single}}{N \cdot (1 - \alpha) + \alpha}
\]

This simple formula explains why the payload performance is not a linear function of the nominal performance and why in the case of very good parallelization \((1 - \alpha) \ll 1\) and low \(N\) this nonlinearity cannot be noticed. The functional form of the dependence discovers a surprising analogy shown in details in Table 3 and Fig 2.

The right side of Fig. 2 reveals why the nonlinearity of the dependence of the payload performance on the nominal performance was not noticeable earlier: in the age of 1K processors the effect was thousand times smaller than in the age of 1M processors and the increase really seemed to be linear. But anyhow: the ‘strong scaling’ that could be safely used up to around up to a few PFlops is surely not valid any more. How much the nonlinearity manifests, depends on the type of the workload of the computing system [6]. That is, according to the ‘modern scaling’ Eq. 5 defines the scaling. The linear approximation (according to the ‘strong scaling’)

1http://www.netlib.org/benchmark/hpl/
2https://www.epcc.ed.ac.uk/blog/2015/07/30/hpcg
is not valid any more, although it was a good approximation at lower performance values and for shorter extrapolation distances.

Notice that in this section it was assumed that $\alpha$ does not depend on $N$. This assumption is surely valid for low number of processors, and surely not valid for the cutting-edge supercomputers. That is, as discussed below, the bad news is that the increase of the payload performance is not linear in function of nominal performance (as would be expected on the basis of ‘strong scaling’), but has a performance limit at which it saturates (according to the first order approximation) or starts to decrease (according to the second order approximation).

The parallelized sequential processing has different rules of game [11]. [7]: the performance gain ("the speedup") has its inherent bounds [23].

III. A NON-TECHNICAL MODEL OF PARALLELIZED SEQUENTIAL OPERATION

To understand why the different ‘scaling’ methods are actually approximations with limited range of validity, a simple non-technical model is set up.

The speedup measurements are simple time measurements\(^3\) (although they need careful handling and proper interpretation, see good textbooks such as [24]): a standardized set of machine instructions is executed (a large number of times) and the known number of operations is divided by the measurement time; for both the single-processor and the distributed parallelized sequential systems. In the latter case, however, the joint work must also be organized, implemented with extra machine instructions and extra execution time, forming an overhead. This is the origin of the inherent efficiency of the parallelized sequential systems: one of the processors orchestrates the joint operation, the others are idle waiting. At this point the “dark performance” appears: the processing units are ready to operate, consume power, but do not make any payload work.

A closer analysis reveals, that one of the essential prerequisites to applying Amdahl’s Law is not strictly fulfilled even by the Amdahl’s Law because “It requires the serial algorithm to retain its structure such that the same number of instructions are processed by both the serial and the parallel implementations for the same input” [8]. Because of this, Amdahl’s Law itself is an approximation. In its original form it is called as the first order approximation to Amdahl’s Law, i.e. the approximation takes that compared to the payload work, organizing the joint work is negligible. The validity of this assumption is limited to very low number of cores and relatively high ratio of overhead. Recall that in the age of Amdahl the non-payload workload ratio was in the range of dozens or percents, so some extra work really did not make a considerable difference. Today, as will be discussed below, the ratio of the overhead is by orders of magnitude lower, while the number of cores is by orders of magnitude higher (see also the the parameters of the different configuration in Fig. 3); this aspect is considered by the second order approximation to Amdahl’s Law.

Amdahl’s major idea is to put everything that cannot be parallelized, i.e. distributed between the fellow processing units, into the sequential-only fraction. In the spirit of this, for describing the parallel operation of sequentially working units the model depicted in Figure 3 was prepared. The technical implementations of the different parallelization methods show up virtually infinite variety [25], so here a (by intention)

\(^3\)Sometimes also secondary merits, such as GFlops/Watt or GFlops/USD are also derived
\[
\alpha = \frac{\text{Payload}}{\text{Total}}
\]

\[
1993 \quad \alpha = 1 - 1 \cdot 10^{-3}
\]

\[
N_{\text{cores}} = 10^3
\]

\[
\frac{R_{\text{Max}}}{R_{\text{Peak}}} = \frac{1}{N \cdot (1-\alpha) + \alpha}
\]

\[
= \frac{1}{10^3 \cdot 10^{-3} + 1}
\]

\[
= 0.5
\]

\[
\alpha = 1 - 3.3 \cdot 10^{-8}
\]

\[
\text{Total} = 10^{13} \text{ clocks}
\]

\[
N_{\text{cores}} = 10^7
\]

\[
\frac{R_{\text{Max}}}{R_{\text{Peak}}} = \frac{1}{N \cdot (1-\alpha) + \alpha}
\]

\[
= 0.74
\]

Fig. 3. A non-technical, simplified model of parallelized sequential computing operations. The contributions of the model component \(XXX\) to \(\alpha\) (sometimes used as \(\alpha_{\text{eff}}\) to emphasize that it is an effective, empirical value) will be denoted by \(\alpha_{\text{eff}}^\text{XXX}\) in the text. Notice the different nature of those contributions. They have only one common feature: they all consume time. The vertical scale displays the actual activity for processing units shown on the horizontal scale.

A strongly simplified model is presented. The non-parallelizable contributions are virtually classified (sometimes contracted) and shown as general contribution terms in the figure. The model in this way is general enough to discuss qualitatively some case studies of parallelly working systems, neglecting different contributions as possible. The model can easily be converted to a technical (quantitative) one via interpreting the contributions in technical terms; although with some obvious limitations.

As Figure 3 shows, in the parallel operating mode (in addition to the calculation, furthermore the communication of data between the processing units) both the software (in this sense: computation and communication, including data access) and the hardware (interconnection, accelerator latency, etc.) contribute to the execution time, i.e. both must be considered in Amdahl’s Law. This is not new, again: see [1], [8].

The non-parallelizable (i.e. apparently sequential) part comprises contributions from Hardware (HW), Operating System (OS), Software (SW) and Propagation Delay (PD) (the “propagation rates of different physical effects”), and also some access time is needed for reaching the parallelized system. This separation is rather conceptual than strict, although dedicated measurements can reveal their role, at least approximately. Some features can be implemented in either SW or HW, or shared between them, and also some apparently sequential activities may happen partly parallel with each other. The relative weights of the contributions are very different for different parallelized systems, and even within those cases depend on many specific factors, so in every single parallelization case a careful analysis is required. The SW activity represents what was assumed by Amdahl as the total sequential fraction. What did not yet exist in the age of Amdahl, the non-determinism of the modern HW systems [9], [10] also contributes to the non parallelizable portion of the task: the resulting execution time of the parallelly working processing elements is defined by the slowest unit. Also notice that optimization possibilities are present in the system; for an example see in Fig. 3 how the contribution of class propagation delay and looping delay can be combined to achieve better timing.

Our model assumes no interaction between the processes running on the parallelized systems in addition to the absolutely necessary minimum: starting and terminating the otherwise independent processes, which take parameters at
the beginning and return result at the end. It can, however, be trivially extended to the more general case when processes must share some resource (like a database, which shall provide different records for the different processes), either implicitly or explicitly. The concurrent objects have their inherent sequentiality [26], and the synchronization and communication between those objects considerably increase [27] the non-parallelizable portion (i.e. contribution to \((1 - \alpha_{eff}^{SW})\) or \((1 - \alpha_{eff}^{OS})\)), so in the case of extremely large number of processors special attention must be devoted to their role on the efficiency of the application on the parallelized system.

In the case of distributed systems the physical size of the computing system also matters: the processor, connected to the first one with a cable of length of dozens of meters, must spend several hundreds clock cycles with waiting, only because of the finite speed of propagation of light, topped by the latency time and hoppings of the interconnection (not mentioning geographically distributed computer systems, such as some clouds, connected through general-purpose networks). This aspect is completely neglected in the 'strong scaling' approximation. Detailed calculations are given in [28].

After reaching a certain number of processors there is no more increase in the payload fraction when adding more processors: the first fellow processor already finished the task and is idle waiting, while the last one is still idle waiting for the start command. This limiting number can be increased by organizing the processors into clusters: the first computer must speak directly only to the head of the cluster. Another way is to distribute the job near to the processing units, either inside the processor [29] or using processors to let do the job by the processing units of a GPGPU.

This looping contribution is not considerable (and so: not noticeable) at low number of processing units, but can be a dominating factor at high number of processing units. This "high number" was a few dozens at the time of writing the paper [11], today it may be in the order of a few millions. Considering the effect of the looping contribution is the border line between the first and second order approximations in modeling the performance: the housekeeping keeps growing with the growing number of processors, while the resulting performance does not increase any more. Even, the housekeeping gradually becomes the dominating factor of the performance limitation, and leads to a decrease in the payload performance: "there comes a point when using more processors ... actually increases the execution time rather than reducing it" [11]. That is, the first order approximation results only in saturated performance, the second order approximation leads to reaching an inflexion point followed by decreasing performance and efficiency.

### IV. Application fields

According to the model, \((1 - \alpha_{eff})\) is expected to describe the fraction of the total (even unintended or only apparently) sequential part in any HW/SW system, and it is a sensitive measure of disturbances and inefficiencies of parallelization [28]. This value can be used as the merit of \(\alpha\) to compare setups, computers manufactured in different ages with different technologies, conditions of network operation, algorithm communication within a closed chip, SW load balancing, etc.

In this section (except section IV-D) we assume that the parallelized computing system is accessible in negligible time, and that the parallelized system under study is properly defined. We do not care whether the one-time contributions (such as initiating the data structures and starting the calculations) are done by the user SW or by the OS; furthermore we assume that the payload calculation is repeated so many times that the one-time contributions can be neglected.

#### A. Load balancing compiler

Today, mainly because of the more and more widespread utilization of multi-core processors, more and more applications are considered to be re-implemented in multi-core aware form. Because it is a serious (and expensive!) effort, before deciding to start such a re-implementation, one needs to guess the speed gain that can be hoped. After finishing re-implementation, it would be desirable to measure whether the goal was achieved. A method which would enable to find out during development, whether further parallelization can still be achieved using reasonable amount of development work, would be highly desirable. Since the achievable speed gain depends on both the structure of the code and the hardware architecture, all those aspects must be scrutinized.

A compiler making load balancing of an originally sequential code for different number of cores is described and validated in paper [30], by running the executable code on platforms having different number of cores. In terms of efficiency, the results they presented have common features and can be discussed together.

The left subfigure of Fig. 4 (Fig 8 in [30]) displays their results in function of the number of cores, using the figure of merit the authors used, the efficiency \(E\) (see Equ. [11]). The data displayed in the figures are derived simply through reading back diagram values from the mentioned figures in [30], so they may not be accurate. However, they are accurate enough to support our conclusions.

| TABLE I |
| --- |
| **The analogy of adding speeds in physics and adding performances in computing, in the classic and modern paradigm, respectively. In both cases a correction term is introduced, that provides noticeable effect only at extremely large values.** |

| Physics | Computing |
| --- | --- |
| Adding of speeds | Adding of performance |
| \(v(t) = t \cdot a\) | Performance of the payload \(P_{total}(N) = N \cdot P_{single}\) |
| \(t\) = time | \(N\) = number of cores |
| \(a\) = acceleration | \(P_{single}\) = Single performance |
| \(n\) = optical density | \(V\) = communication |
| \(c\) = Light Speed | \(V\) = parameter |
| **Note** (multivariate) | **Modern (Amdahl-aware) [7] see Eq. (4)\)** |
| \(v(t) = \sqrt{1 + \left(\frac{t \cdot a}{c/n}\right)^2}\) | \(P(N) = N \cdot P_{single} \cdot (1 - \alpha) + \alpha\) |

---

**Note:** (multivariate) **Modern (Amdahl-aware) [7] see Eq. (4)**
Their first example shows results of implementing parallelized processing of an audio stream manually, with an initial (first attempt), and more careful (having already experienced programmers) implementation. For the two different processings of audio streams, using efficiency $E$ as merit enables only to claim a qualitative statement about load balancing, that “The higher number of parallel processes in Audio-2 gives better results”, because the Audio-2 diagram decreases less steeply, than Audio-1. In the first implementation, where the programmer had no previous experience with parallelization, the efficiency quickly drops with the increasing number of cores. In the second round, with experiences from the first implementation, the loss is much less, so $1-E$ rises less speedily.

Their second example is processing radar signals. Without switching the load balancing optimization on, the slope of the curve $1-E$ is much bigger. It seems to be unavoidable, that as number of cores increases, the efficiency (according to Eq. (4)) decreases, even at such low number of cores. Both examples leave the question open whether further improvements are possible or whether the parallelization is uniform in function of the number of cores.

In the right subfigure of Fig. 4 (Fig. 10 in [30]) the diagrams show the $(1 - \alpha_{eff})$ values, derived from the same data. In contrast with the left side, these values are nearly constant (at least within the measurement data readback error) which means that the derived parameter is really characteristic to the system. By recalling Eq. (1) one can identify this parameter as the resulting non-parallelizable part of the activity, which – even with careful balancing – cannot be distributed among the cores, and cannot be reduced.

In the light of this, one can conclude that both the programmer in the case of audio stream and the compiler in the case of radial signals correctly identified and reduced the amount of non-parallelizable activity: $\alpha_{eff}$ is practically constant in function of the number of cores, nearly all optimization possibilities found and they hit the wall due to the unavoidable contribution of non-parallelizable software contributions. Better parallelization leads to lower $(1 - \alpha_{eff})$ values, and less scatter in function of the number of cores.

The uniformity of the values make also highly probable, that in the case of audio streams further optimization can be done, at least for 6-core and 8-core systems, while processing of radar signals reached its bounds.

Note that the absolute values for analyzing different programs must not be compared: they represent the sequential only part of two programs, which may be different. It looks like that the $(1 - \alpha_{eff})$ imperfectness can be reduced to about $10^{-1}$ with software methods of parallelization.

B. The history of supercomputing

The TOP500 database [20] provides all needed data to calculate $\alpha$, independently from the date of manufacturing, technology, manufacturer, number and kind of processors, etc: the parallelization efficiency can be used to study (among others) the history of supercomputing.

During the past quarter of century, the proportion of the contributions changed considerably: today the number of processors is thousands of times higher than it was a quarter of century ago. The growing physical size and the higher processing speed increased the role of the propagation overhead, furthermore the large number of processing units strongly amplified the role of the looping overhead. As a result of the technical development, the phenomenon on the performance limitation returned in a technically different form at much higher number of processors.

As will be discussed below, with the exception of extremely high number of processors, it can be assumed that $\alpha$ is independent from the number of processors. Equ. (5) can be used to derive quickly the value of $\alpha$ from the values of parameters $R_{Max}/R_{Peak}$ and the number of cores $N$.

C. Measuring the efficiency of the on-chip networking

It is not a trivial task to find out the fine points of on-chip networking, because both the limited accessibility and of the low number of processing units. The merit developed here, however, can help also in that case: although the available non-dedicated measurements enable to draw only conclusions of limited accuracy.

![Figure 4. Relative speedup (left side) and $(1 - \alpha_{eff})$ (right side) values, measured running the audio and radar processing on different number of cores. [30]](image-url)
Fig. 5. Comparing efficiency, efficiency slope and $\alpha_{eff}$ for different communication strategies when running two minimization task on SoC by [31]
In [31] the authors compare different communication strategies their Particle Swarm Optimization (PSO) uses when minimizing Rosenbrock’s function and Rastrigin’s functions, respectively. From their data the corresponding \( \alpha_{eff} \) values have been calculated and displayed in Fig. [5] The fluctuations seen in the figure show the limitations of the (otherwise excellent) measurement precision; for this type of investigations much longer measurement times would be needed.

The contribution of the OS cannot be separated, again, from SW contribution. Although the precision of the available data does not enable to make a detailed analysis of the behavior of the scaling and to fully qualify the communication method, some observations can be made. When utilizing only two cores, the way to communicate is very limited. For this case the same \( \alpha_{eff} \) value is delivered by all communication methods; this also proves self-consistency of the model. Values of \( \alpha_{eff} \) (5\( \times \)10\(^{-3} \) and 2\( \times \)10\(^{-4} \)) deviate considerably for the two minimization methods; this can be attributed to the different structure (\( \alpha_{eff}^{SW} \)) of the two applications. As can be seen from the diagrams of the Rastrigin method, propagation delay can be in the order of 1\( \times \)10\(^{-3} \); which is considerable for the Rastrigin method, but not for the Rosenbrock method. This is why for higher core numbers, \( \alpha_{eff} \) is nearly constant in the case of the first two communication methods: one of the contributions dominates; although for the Rastrigin method \( \alpha_{eff}^{SW} \), while for the Rastrigin method \( \alpha_{communication} \) is the dominating term.

A bit different is the case for the broadcast-type communication, for both types of minimization methods: the resulting \( (1 - \alpha_{eff}) \) increases with the increasing number of cores. Here the reason is that the number of collisions (and so the time spent with waiting for repeating) increases with the number of cores. This contribution increasingly dominates for the Rastrigin case, and increases moderately the already high \( (1 - \alpha_{eff}) \) at high number of cores, while at low number of cores \( (1 - \alpha_{eff}^{SW}) \) persists to dominate for the Rosenbrock case.

In this case study an indirect proof is shown that \( \alpha \) does not change considerably in this range of number of cores. As Eq. (3) suggests, the inverse of the efficiency is a linear function of \( N \) and the slope value is \( (1 - \alpha) \). This an independent method to calculate \( (1 - \alpha) \), to calculate the slope at the individual measured points. The two methods show reasonably good agreement.

D. The effect of not considering the access time

In the case of using cloud services the parallelized system and the one which interfaces user to its application are physically different. These systems differ from the ones discussed above at least in two essential points: the access and the inter-node connections are provided through using Internet, and the architecture is not necessarily optimized to offer the best possible parallelization. As discussed in section [III] the time needed to access the parallelized computing system must be properly corrected for.

In [32] the authors benchmarked some commercially available cloud services, fortunately using HPL benchmark. Their results are shown in Fig. [6] On the left side the efficiency (i.e. \( \frac{R_{Max}}{R_{Peak}} \)) on the right side the \( (1 - \alpha) \) values are displayed in function of the number of processors in the used configuration. One can immediately notice on one side that the values of \( \frac{R_{Max}}{R_{Peak}} \) are considerably lower than unity, even for very low number of cores; on the other side, that the \((1 - \alpha) \) values steeply decrease as number of cores increases, although the model contains only contributions which may only increase as number of cores increases.

As discussed above, HPL characterizes the setup, so the benchmark is chosen correctly. When acquiring measurement data, in the case of clouds, also the access time must be considered, see Fig. [3] If the time is measured on client’s computer (and this is what is possible using those services), the time Extended is utilized in the calculation of place of Total, that is the ‘device under test’ is chosen improperly.

This artifact is responsible for both mentioned differences. The efficiency measured in this way would not achieve 100% even on a system comprising only one single processor. Since \( \alpha \) measures the average utilization of processors, this foreign contribution is divided by the number of processors, so with increasing the number of processors the relative weight of this foreign contribution decreases, causing to decrease the calculated value of \((1 - \alpha) \). Since the access is provided through the Internet where the operation is stochastic, the measurements cannot be as accurate as in purpose-built systems. Some qualitative conclusions of limited validity, however, can be drawn even from those data.

At such low number of processors neither of the contributions depending on the processor number is considerable, so one can expect that in the case of correct measurement \((1 - \alpha) \) would be constant. So, extrapolating the diagram lines of \((1 - \alpha) \) to the value corresponding to a one-processor system, one can see that both for Edison supercomputer and Azure A series grid (and maybe also Rackspace) the expected value is approaching unity (but obviously below it). From the slope of the curve (increasing the denominator 1000 times, \((1 - \alpha) \) reduces to 10\(^{-3} \)) and one can even find out that \((1 - \alpha) \) should be around 10\(^{-3} \). Based on these data, one can agree with the conclusion that –on a good grid– benchmark HPCG can run as effectively as on the supercomputer used in the work. One should note, however, that \((1 - \alpha) \) is about 3 orders of magnitude better for TOP500 class supercomputers, but this makes a difference only for HPL class benchmarks and only at large number of processors. This conclusion can be misleading: whether a high performance cloud can replace a supercomputer in solving a task, strongly depends on the number of cores, because of the largely different \( \alpha \) values.

Note that in the case of AWS grids and Azure F series the \( \alpha_{eff}^{SW} \) starts at about 10\(^{-3} \), and this is reflected by the fact that their efficiency drops quickly as the number of the cores increases. Interesting to note that ranking based on \( \alpha \) is just the opposite of ranking based on efficiency (and strongly correlates with the price of the service).

One can extrapolate also the efficiency values to the point corresponding to one core only. In the case of measurement with no such artifact, the backprojected value should be around

\[ 4 \text{ A long term systematic study [33] derived the results that measured data show dozens of percents of variation in long term run, and also unexpected variation in short term run.} \]
Fig. 6. The effect of neglecting the access time when measuring efficiency of some cloud services
V. Conclusion

The scaling methods, mainly due to their simplicity, can be useful when applied in the range of their validity. Given that they are approximations, the validity of the omissions must be periodically scrutinized. The approximations to the performance of parallelized sequential systems routinely deployed the 'strong scaling' method to estimate the payload performance of future, ever larger scale system; without scrutinizing the validity of the method under the current technical situation. However, using this approximation (the incremental development) led to unexpected phenomena, failed supercomputers and unexpectedly low efficiency of the systems. The 'modern scaling' is in complete agreement with the experiences and measured values.

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