The Cardy-Verlinde Formula and Asymptotically de Sitter Brane Universe

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Abstract

We consider the brane universe in the bulk background of the topological AdS-Schwarzschild black holes, where the brane tension takes larger value than the fine-tuned value. The resulting universe is radiation dominated and has positive cosmological constant. We obtain the associated cosmological Cardy formula and the Cardy-Verlinde formula. We also derive the Hubble and the Bekenstein entropy bounds from the conjectured holography bound on the Casimir entropy.

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In Ref. [1], Verlinde made an interesting proposal that the Cardy formula [2] for the two-dimensional conformal field theory (CFT) can be generalized to arbitrary spacetime dimensions. Such generalized formula, called the Cardy-Verlinde formula, is shown to coincide with the Friedmann equation at the moment when the conjectured holographic entropy bound is saturated. Within the context of the radiation dominated universe corresponding to the brane moving in the bulk background of the AdS-Schwarzschild black hole, it was shown [3] that such a special moment corresponds to the moment when the brane crosses the black hole horizon. This result is later generalized [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] to brane universes in the bulks of various AdS black holes. It was also shown [18] that the Cardy-Verlinde formula should hold generically in thermodynamical systems with a first order phase transition. Verlinde [1] further proposed a holographic bound on the Casimir energy which unifies the Hubble and the Bekenstein bounds in an elegant way. In particular, in our previous works [10, 16], we have shown that these results, originally proposed [1, 3] for the closed universe case, continues to hold even for the flat and the open universes.

Recently, attempts [19, 20, 21, 22, 23, 24] have been made to generalize the Verlinde’s results to the case of the dS black holes. Such attempts were motivated by the observational evidence that our universe has positive cosmological constant. Study of holographic principle for dS spacetime is complicated by subtle points that AdS spacetime does not have, such as nonexistence of globally defined timelike killing vector and spacelike infinity, which make it difficult to define conserved charges of a gravitating system in the asymptotically dS spacetime. Following the Brown-York prescription [25], the authors of Ref. [26] proposed a way to define conserved charges of asymptotically dS spacetimes from data at the timelike past or future infinity. A surprising result is that black holes in dS spacetime is less massive than the dS spacetime itself. So, unlike the black holes in the AdS spacetime, the mass of a black hole in the AdS spacetime is proportional to the minus of the mass parameter of the black hole solution, excluding the Casimir contribution from the background dS spacetime. (These results were explicitly checked and explicit expressions for thermodynamic quantities of the dS black holes in arbitrary spacetime dimensions were obtained in Ref. [27].) Because of this fact, the holography bound on the Casimir energy, conjectured by Verlinde [1], does not lead to the Hubble and the Bekenstein entropy bounds [1], although thermodynamic quantities (including entropy) of the dual CFT take usual special forms expressed in terms of the Hubble parameter and its time derivative at the moment when the brane crosses the horizon. Also, difficulty of expressing entropy associated with the black hole horizon, rather than the cosmological horizon, in the form of the

\footnote{The recent work [24] claims to have derived such bounds from bound on the Casimir quantity, however the author considers the dS black hole solution with the mass term having the opposite sign from that of the conventional solution.}
Cardy-Verlinde formula was observed \[22\]. In this paper, we instead study the brane world cosmology in the bulk background of the AdS black hole, where the brane tension takes a value giving rise to a positive cosmological constant in the Friedmann equations \[3\]. (In fact, the astronomical data indicates that the Friedmann equations have the positive cosmological constant term, not necessarily that bulk spacetime of the brane universe is asymptotically dS if our universe is described by the brane world scenario.) We obtain the associated Cardy-Verlinde formula. We find that the Hubble and the Bekenstein entropy bounds can be derived from the conjectured holographic bound on the Casimir entropy, even when the Friedmann equations have the cosmological constant term. It turns out that the matching of the first Friedmann equation and the entropy density of the dual CFT when the brane crosses the black hole horizon does not hold when the cosmological constant in the Friedmann equation is nonzero. The Bekenstein entropy bound turns out to depend on the cosmological constant.

The topological AdS black hole solution in \((n+2)\)-dimensions has the form:

\[
\begin{align*}
\frac{ds^2_{n+2}}{a^2} &= -h(a)dt^2 + \frac{1}{h(a)}da^2 + a^2\gamma_{ij}(x)dx^idx^j, \\
h(a) &= k - \frac{w_{n+1}m}{a^{n-1}} + \frac{a^2}{L^2}, \quad w_{n+1} = \frac{16\pi G_{n+2}}{n\text{Vol}(M_n)},
\end{align*}
\tag{1}
\]

where \(\gamma_{ij}\) is the horizon metric for a constant curvature manifold \(M_n\) with the volume \(\text{Vol}(M_n) = \int d^n x \sqrt{\gamma}\), \(G_{n+2}\) is the \((n + 2)\)-dimensional Newton’s constant, \(m\) is the ADM mass of the black hole, and \(L\) is the curvature radius of the AdS spacetime. The horizon geometry of the black hole is elliptic, flat and hyperbolic for \(k = 1, 0, -1\), respectively. The Bekenstein-Hawking entropy \(S\) and the Hawking temperature \(T\) of the black hole are

\[
S = \frac{a_H^n \text{Vol}(M_n)}{4G_{n+2}}, \quad T = \frac{h'(a_H)}{4\pi} = \frac{(n + 1)a_H^2 + (n - 1)KL^2}{4\pi L^2 a_H},
\tag{2}
\]

where \(a_H\) is the black hole horizon, defined as the largest root of \(h(a_H) = 0\).

According to the holographic principle, thermodynamic quantities of the holographic dual CFT theory at high temperature are identified with those of the bulk AdS black hole \[30\]. The metric for the CFT is given by

\[
\frac{ds^2_{\text{CFT}}}{a^2} = \lim_{a \to \infty} \left[ \frac{L^2}{a^2} ds^2_{n+2} \right] = -dt^2 + L^2\gamma_{ij}dx^idx^j.
\tag{3}
\]

Since the CFT time is rescaled by the factor of \(L/a\) w.r.t. the AdS time, the energy \(E\) and the temperature \(T\) of the CFT are rescaled by the same factor \(L/a\) w.r.t. those

\[\footnote{Some aspects of the brane world cosmology in the AdS black hole background, where the brane tension does not take the fine-tuned value, were also previously studied in Ref. \[28, 29\].}
of the bulk AdS-Schwarzschild black hole. This fact appears to hold even when the brane tension is not fine-tuned, since argument leading to this fact does not involve the brane tension. This position is taken, for example, in Refs. [27, 28]. However, it is argued in Ref. [29] that the scale factor between energy and temperature of the AdS black hole and those of the CFT is rather given by

$$\lim_{a \to \infty} \frac{dt}{d\tau} = \frac{\kappa L^2}{a},$$

where $\tau$ is the time coordinate for the induced metric (6) on the brane, given in the below, and $t$ is the time coordinate for the bulk metric (1). Furthermore, it is argued [29] that energy of the AdS-Schwarzschild black hole is rather given by

$$E_{bh} = \frac{m}{\kappa L^2},$$

These modified scale factor and the black hole energy take the usual forms $L/a$ and $m$, when the brane tension takes the fine-tuned value such that $\kappa = 1/L$. The energy and the temperature of the CFT are therefore given by

$$E = E_{bh} \frac{\kappa L^2}{a} = \frac{m}{\kappa a}, \quad T = \frac{\kappa L^2}{4\pi a} \left[ (n + 1) \frac{a_H}{L} + (n - 1) \frac{k L}{a_H} \right], \quad (4)$$

whereas the entropy $S$ of the CFT is still given by the Bekenstein-Hawking entropy (3) of the black hole without rescaling.

The dynamics of the brane is determined by the Israel junction conditions [31], which for our case take the form (Cf. Ref. [32]):

$$\mathcal{K}_{\mu \nu} = \frac{\kappa}{n} h_{\mu \nu}, \quad (5)$$

where $\mathcal{K}_{\mu \nu}$ is the extrinsic curvature, $\kappa$ is related to the brane tension, and $h_{\mu \nu}$ is the induced metric on the brane. The induced metric can be brought to the following form of the Robertson-Walker metric:

$$h_{\mu \nu} dx^\mu dx^\nu = -d\tau^2 + a^2(\tau) \gamma_{ij} dx^i dx^j, \quad (6)$$

by introducing a new time coordinate $\tau$ satisfying

$$\frac{1}{h(a)} \left( \frac{da}{d\tau} \right)^2 - h(a) \left( \frac{dt}{d\tau} \right)^2 = -1. \quad (7)$$

The brane equation of motion (5) is translated into

$$\frac{dt}{d\tau} = \frac{\kappa a}{h(a)}. \quad (8)$$

Making use of Eqs. (7,8), we obtain the following Friedmann equation:

$$H^2 = \kappa^2 - \frac{h}{a^2} = \frac{w_{n+1} m}{a^{n+1}} - \frac{k}{a^2} + \kappa^2 - \frac{1}{L^2}, \quad (9)$$

describing the evolution of the $(n+1)$-dimensional universe on the brane, where $H \equiv \dot{a}/a$ is the Hubble parameter. From this equation, we see that the brane motion in
the bulk background (1) of the AdS black hole induces radiation matter \( \sim m/a^{n+1} \). In this paper, we assume that \( \kappa^2 > 1/L^2 \) so that the effective cosmological constant \( \Lambda = \frac{n(n-1)}{2} \left( \kappa^2 - \frac{1}{L^2} \right) \) of the brane universe is positive. Taking the \( \tau \)-derivative of Eq. (9), we obtain the second Friedmann equation

\[
\dot{H} = -\frac{n+1}{2} \frac{w_{n+1} m}{a^{n+1}} + \frac{n^2 w_{n+1}^2}{8(n-1)} + \frac{k}{a^2}.
\] (10)

The energy density of the CFT within the volume \( V = a^n \text{Vol}(M_n) \) is given by \( \rho = \frac{E}{V} = \frac{n w_{n+1} m}{16\pi G n^{n+2} a^{n+1}} \) and the induced CFT matter, being a radiation matter, satisfies the equation of state of the form \( p = \rho/n \), where \( p \) is the pressure of the CFT matter. So, the Friedmann equations (9,10) can be brought to the following standard forms of the Friedmann equations:

\[
H^2 = \frac{16\pi G}{n(n-1)} \rho + \frac{2}{n(n-1)} \Lambda - \frac{k}{a^2},
\] (11)

\[
\dot{H} = -\frac{8\pi G}{n-1} (\rho + p) + \frac{k}{a^2},
\] (12)

where \( G = (n-1)\kappa G_{n+2} \) is the modified \((n+1)\)-dimensional Newton’s constant on the brane proposed in Ref. [29] and once again \( \Lambda = \frac{n(n-1)}{2} \left( \kappa^2 - \frac{1}{L^2} \right) \) is the effective cosmological constant on the brane. From these Friedmann equations, we obtain the following energy conservation equation:

\[
\dot{\rho} + n(\rho + p) \frac{\dot{a}}{a} = 0.
\] (13)

The Friedmann equations (11,12) can be respectively put into the following forms resembling thermodynamic formulas of the CFT:

\[
S_H = \frac{2\pi}{n} a \sqrt{E_{BH}[2(E + E_\Lambda) - kE_{BH}]},
\] (14)

\[
kE_{BH} = n(E + pV - T_H S_H),
\] (15)

in terms of the Hubble entropy \( S_H \) and the Bekenstein-Hawking energy \( E_{BH} \), where

\[
S_H \equiv (n - 1) \frac{HV}{4G}, \quad E_{BH} \equiv n(n - 1) \frac{V}{8\pi Ga^2}, \quad T_H \equiv -\frac{\dot{H}}{2\pi H}, \quad E_\Lambda \equiv \frac{\Lambda V}{8\pi G}.
\] (16)

Eq. (14) is referred to as the cosmological Cardy formula, due to its resemblance to the Cardy formula for the two-dimensional CFT. Note, Eq. (15) resembles the Smarr’s formula for a thermodynamic system having the Casimir contribution. The first Friedmann equation (11) can be expressed also as the following relation among the
Bekenstein entropy \( S_B \equiv \frac{2\pi a}{n} E \), the Bekenstein-Hawking entropy \( S_{BH} \equiv (n - 1) \frac{V}{4G a^n} \),
the Hubble entropy \( S_H \), and \( S_{\Lambda} \equiv \frac{2\pi a}{n} \cdot \frac{\Lambda V}{8\pi G} \):
\[
S_H^2 = 2(S_B + S_{\Lambda})S_{BH} - kS_{BH}^2. \tag{17}
\]
For the \( k = 1 \) case, this can be expressed as the following quadratic relation:
\[
S_H^2 + (S_B + S_{\Lambda} - S_{BH})^2 = (S_B + S_{\Lambda})^2. \tag{18}
\]
Unlike the \( \Lambda = 0 \) case considered in Ref. \[1\], \( S_B + S_{\Lambda} \) does not remain constant during
the cosmological evolution, as can be seen by applying the energy conservation equation
(13) along with the equation of state \( \rho = p/n \). Nevertheless, we can extract inequalities
among entropies from this relation. First of all, it is trivial to see that \( S_H \leq S_B + S_{\Lambda} \)
all the time. When \( S_H \geq S_{BH} \) \([S_H \leq S_{BH}]\), namely when \( Ha \geq 1 \) \([Ha \leq 1]\), we have
\( S_{BH} \leq S_B + S_{\Lambda} \) \([S_{BH} \geq S_B + S_{\Lambda}]\). These inequalities are nothing but the second criteria
(38) for the weakly or the strongly self-gravitating universe given in the below.
We study thermodynamics of the CFT at the moment when the brane crosses the
black hole horizon \( a = a_H \). Since the black hole horizon \( a_H \) is a root of \( h(a_H) = 0 \), we see from Eq. (9) that
\[
H^2 = \kappa^2 \quad \text{at} \quad a = a_H. \tag{19}
\]
The total entropy \( S \) of the CFT remains constant, but the entropy density,
\[
s \equiv \frac{S}{V} = (n - 1) \frac{\kappa a_H^n}{4Ga^n}, \tag{20}
\]
varies with time. When the brane crosses the black hole horizon, \( s \) can be expressed
in terms of \( H \) in the following form:
\[
s = (n - 1) \frac{H}{4G} \quad \text{at} \quad a = a_H, \tag{21}
\]
which implies
\[
S = S_H \quad \text{at} \quad a = a_H. \tag{22}
\]
Making use of Eqs. (9,19), we see that the CFT temperature \( T = h'(a_H)L/(4\pi a_H) \) at
\( a = a_H \) can be expressed in terms of \( H \) and \( \dot{H} \) as
\[
T = -\frac{\dot{H}}{2\pi H} = T_H \quad \text{at} \quad a = a_H. \tag{23}
\]
Note, entropy and temperature of the CFT take the same forms regardless of the value
of the brane tension, when \( a = a_H \). From Eq. (15) along with Eqs. (22,23), we we see that
\[
E_C = kE_{BH} \quad \text{at} \quad a = a_H, \tag{24}
\]
where $E_C$ is the Casimir energy defined as

$$E_C \equiv n(E + pV - TS). \quad (25)$$

We now study thermodynamics of the CFT for an arbitrary value of $a$. Thermodynamic quantities of the CFT satisfy the first law of thermodynamics:

$$TdS = dE + pdV, \quad (26)$$

which takes the following form in terms of the densities:

$$TdS = d\rho + n(\rho + p - Ts) \frac{da}{a}. \quad (27)$$

Here, the combination $\rho + p - Ts$ measures the subextensive contribution in the thermodynamic system. Making use of the following expression for the energy density of the CFT, obtained from $h(a_H) = 0$,

$$\rho = \frac{na_H^n}{16\pi G n+2a^{n+1}} \left( \frac{a_H}{L} + k \frac{L}{a_H} \right), \quad (28)$$

together with the equation of state $p = \rho/n$, we have

$$\frac{n}{2}(\rho + p - Ts) = k \frac{\gamma}{a^2}, \quad (29)$$

where $\gamma$ is the Casimir quantity given by

$$\gamma = \frac{n(n-1)a_H^{n-1}}{16\pi Ga^{n-1}}. \quad (30)$$

By multiplying Eq. (29) by $2V$, we obtain the following explicit expression for the Casimir energy, defined by Eq. (25), in terms of quantities of the black hole solution:

$$E_C = \frac{kn(n-1)a_H^{n-1}Vol(M_n)}{8\pi Ga}. \quad (31)$$

The entropy density (20) of the CFT can be expressed in terms of other thermodynamic quantities of the CFT as

$$s^2 = \left( \frac{4\pi \kappa L}{n} \right)^2 \gamma \left( \rho - k \frac{\gamma}{a^2} \right). \quad (32)$$

For a general value of $\kappa$, this entropy density expression does not reproduce the first Friedmann equation (11) when the brane crosses the horizon. However, when the brane tension takes the fine-tunned value giving rise to $\Lambda = 0$, i.e., when $\kappa = 1/L$, making use of Eq. (21) we see that Eq. (32) reduces to the first Friedmann equation when
\( a = a_H \). The matching of the first Friedmann equation and the entropy density of the CFT for \( a = a_H \) therefore turns out to be actually accidental for the \( \kappa = 1/L \) case. On the other hand, Eq. (29) reduces to the second Friedmann equation (12) when \( a = a_H \), for any values of \( \kappa \), perhaps because the second Friedmann equation is independent of the cosmological constant.

The following generalized Cardy-Verlinde formula can be obtained by multiplying Eq. (32) by \( V^2 \) and then taking square root:

\[
S = \kappa L \sqrt{\frac{2\pi a}{n} S_C [2E - E_C]},
\]

where the Casimir entropy \( S_C \) and the Casimir energy \( E_C \) are defined as

\[
S_C \equiv (n-1) a_H^{n-1} \text{Vol}(M_n) \frac{4G}{a}, \\
E_C \equiv \frac{kn}{2\pi a} S_C = \frac{kn(n-1) a_H^{n-1} \text{Vol}(M_n)}{8\pi Ga}. \tag{34}
\]

Note, the generalized Cardy-Verlinde formula (33) has apparent dependence on the brane tension, although \( S \) is nothing but the Bekenstein-Hawking entropy (2) of the AdS black hole (as prescribed by the holographic principle), which has nothing to do with the brane tension. This is due to the fact that the modified definition for \( G \), proposed in Ref. [29], involves the brane tension. On the other hand, cosmological Cardy formula (14) has explicit dependence on \( \Lambda \), namely on the brane tension, since it describes the dynamics of the brane in the black hole bulk spacetime. The matching of the generalized Cardy-Verlinde formula and the cosmological Cardy formula when the brane crosses the horizon is actually accidental for the \( \Lambda = 0 \) case, for which the cosmological Cardy formula happens to be independent of the brane tension. The generalized Cardy-Verlinde formula can be rewritten as the following relation among various entropies:

\[
S^2 = \kappa^2 L^2 (2S_B S_C - kS_C^2). \tag{35}
\]

This relation coincides with the relation (17) among the cosmological entropy bounds when \( a = a_H \), only for the \( \Lambda = 0 \), i.e., \( \kappa = 1/L \), case.

We derive the cosmological holographic bounds from the conjectured holographic bound on the Casimir entropy \( S_C \), proposed by Verlinde [1]. Since the explicit expressions for \( S_C \) and \( S_{BH} \) obtained in the above depend explicitly on neither \( k \) nor \( \Lambda \), we conjecture that the cosmological bound on \( S_C \) proposed in Ref. [1] for the \( k = 1 \) and \( \Lambda = 0 \) case continues to hold even for the \( k \neq 1 \) and \( \Lambda \neq 0 \) cases without modification:

\[
S_C \leq S_{BH}. \tag{36}
\]

Using the explicit expressions for \( S_C \) and \( S_{BH} \), we see that this conjectured holographic bound is equivalent to \( a \geq a_H \), namely that the brane has to be outside of the horizon.
In other words, cosmological holographic bounds that follow from Eq. (36) do not hold after the brane falls into the horizon. And the conjectured bound (36) is saturated at the moment when the brane crosses the horizon (i.e., $a = a_H$), at which moment the thermodynamic quantities of the CFT take special forms given by Eqs. (22-24). As for the criteria for the weakly and the strongly self-gravitating universes, we consider the two possibilities. First, if we choose to define the weakly and the strongly self-gravitating universes by comparing the energy $E$ to the Bekenstein-Hawking energy $E_{BH}$, defined as the energy for which $S_B = S_{BH}$ and interpreted as the energy required to form a black hole with the size of the universe, then the criteria in terms of the Hubble parameter $H$ should be modified. Namely, the criteria for the weakly and the strongly self-gravitating universes are respectively

$$E \leq E_{BH} \iff S_B \leq S_{BH} \quad \text{for} \quad H^2 \leq \frac{2 - k}{a^2} + \frac{2}{n(n-1)}\Lambda$$

$$E \geq E_{BH} \iff S_B \geq S_{BH} \quad \text{for} \quad H^2 \geq \frac{2 - k}{a^2} + \frac{2}{n(n-1)}\Lambda. \quad (37)$$

Second, if we choose to define the weakly and the strongly self-gravitating universes by comparing the radius $a$ of the universe to the Hubble radius $H^{-1}$, then the criteria in terms of energy or entropy have to be modified. Namely, the criteria for the weakly and the strongly self-gravitating universes are respectively

$$E \leq \frac{k + 1}{2}E_{BH} - E_\Lambda \iff S_B \leq \frac{k + 1}{2}S_{BH} - S_\Lambda \quad \text{for} \quad H^2 a^2 \leq 1$$

$$E \geq \frac{k + 1}{2}E_{BH} - E_\Lambda \iff S_B \geq \frac{k + 1}{2}S_{BH} - S_\Lambda \quad \text{for} \quad H^2 a^2 \geq 1. \quad (38)$$

We now derive cosmological holographic bounds from the conjectured holographic bound (36) on the Casimir entropy $S_C$. First, we consider the first criteria (37). For the strongly self-gravitating case, we have from Eqs. (38,37) that $S_C \leq S_{BH} \leq S_B$. Since $S$ is a monotonically increasing function of $S_C$ in the interval $S_C \leq S_B$ (as can be seen from Eq. (35)), $S$ takes the maximum value when $S_C = S_{BH}$, which implies $S_C = S_{BH}$ (because $S_C \leq S_{BH} \leq S_B$ for the strongly self-gravitating case). When $S_C = S_{BH}$, we can see from Eqs. (41,38) that $S^2 = \kappa^2 L^2 (S_B^2 - 2S\Lambda S_{BH}) = S_H^2$, where we made use of Eq. (33), which is valid for $a = a_H$, i.e., when $S_C = S_{BH}$. Therefore, we have the following Hubble entropy bound for the strongly self-gravitating universe:

$$S \leq S_H. \quad (39)$$

Note, the Hubble entropy bound does not depend on the brane tension, therefore on the cosmological constant $\Lambda$. For the weakly self-gravitating case, we have $S_C \leq S_B \leq S_{BH}$ for the $k = 1$ case, because Eq. (33) expressed in the form $S^2 + \kappa^2 L^2 (S_B - S_C)^2 = \kappa^2 L^2 S_B^2$ implies $S_C \leq S_B$. $S$, as a function of $S_C$, takes the maximum value when
$S_C = S_B$, for which $S = \kappa L S_B$. Therefore, for the weakly self-gravitating case the conjectured holographic bound (36) implies the modified Bekenstein bound:

$$S \leq \kappa L S_B = \sqrt{1 + \frac{2A L^2}{n(n-1)}} S_B.$$  \hspace{1cm} (40)

Note, the Bekenstein bound depends on the brane tension, therefore on the cosmological constant $\Lambda$. In the case of $k \neq 1$, if we assume $S_C \leq S_B$ as the holographic bound on the degrees of freedom of the dual CFT, then we have the following generalized Bekenstein bound for the weakly self-gravitating universe:

$$S \leq \sqrt{2 - k\kappa L S_B}.$$  \hspace{1cm} (41)

Next, we comment on the second criteria (38). For the $k = -1$ case, this criteria implies that the universe is always strongly self-gravitating, which appears to not make sense. So, we assume $k \neq -1$ in the following discussion. First, for the strongly self-gravitating case, we have $S_C \leq S_{BH} \leq \frac{2}{k+1} (S_B + S_{\Lambda})$, which implies $S$ takes the maximum value when $S_C = S_{BH}$. We have therefore the same Hubble entropy bound (39) even with the second criteria. Second, for the weakly self-gravitating case, we have $S_C \leq S_B \leq \frac{k+1}{2} S_{BH} - S_{\Lambda}$. So, the maximum of $S$ is achieved when $S_C = S_B$, which implies the same modified Bekenstein bound (11) even with a choice of the second criteria.

Note Added

After the first version of the paper is appeared in the preprint achieve, Ref. [29], which studies the same brane world cosmology, was brought to our attention by R. Gregory. Unlike the first version of this paper and Refs. [4, 28], it is claimed in Ref. [29] that when the brane tension does not take the fine-tuned value the relations between thermodynamic quantities of the CFT and the bulk black hole, and the $(n+1)$-dimensional gravitational constant $G$ have to be modified. Later, Ref. [33] which studies implication of such modification for the results of the first version of this paper appeared in the preprint achieve, before this paper can be revised to take into account such modifications.

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