Neutron spin-dependent structure function, Bjorken sum rule, and first evidence for singlet contribution at low $x$

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Abstract

We perform the isospin decomposition of proton and neutron SLAC data for moderately low $x$ in the region $0.01 \leq x \leq 0.1$. The isovector part is well described by a power behaviour $x^\alpha$, where $\alpha$ does not correspond to the intercept of single $a_1$ Regge trajectory, as expected. However, the observed power leads to the validity of Bjorken sum rule and it is consistent with the power extracted from all previous data using NLO evolution. At the same time, the isoscalar part behaviour may be interpreted as a partial cancellation between a positive non-singlet contribution and a strongly negative singlet one. Further experimental consequences are mentioned.

I. INTRODUCTION

The very accurate measurement of the neutron spin-dependent structure function $g_1^n$, whose results have been presented recently, possess two remarkable properties. First, the values of neutron structure function at $Q^2 = 5GeV^2$ are rather large and negative in the region of moderately low $x$. Second, the data can be rather accurately fitted by the power
function $x^{-0.8}$ and this power seems to be unexpectedly large. It is not obvious, where this large number, for which there is no indication in the proton data, is coming from, so it may affect the extrapolation to $x = 0$ and cast some doubts on the validity of Bjorken sum rule.

In the present paper we perform the isospin decomposition of the data for both proton and neutron. As a result, we conclude that the isovector contribution is well approximated by the power behaviour found earlier by an elaborate method based on NLO evolution [2]. It may be interpreted either as the manifestation of $ln^2 x$ terms [3] or as the contribution of the Regge cut, produced by the $a_1$ meson and a BFKL pomeron which has a high intercept. At the same time, for the isoscalar part one has the signature for a rather singular singlet contribution, compatible with the similar behaviour predicted by QCD [4], although statistical errors are still rather large. This may imply a significant gluon polarization in the nucleon in this range of $x$, which could also clearly show up in double helicity asymmetries $A_{LL}$ at RHIC, if they turn out to be larger than estimated earlier [5].

II. BJORKEN SUM RULE VALIDITY

The main feature of the new neutron data [1] is large and negative $g_1^n \sim -g_1^p$, measured with a good accuracy up to small $x$, say $x \sim 0.01$. The SLAC proton data [6] at $Q^2 = 3 GeV^2$ are positive and of roughly the same magnitude, but on the contrary, rather flat in this region. This could be understood, qualitatively, as a result of the interplay of a negative contribution at low $x$, responsible for the singular behaviour of the neutron, and a positive contribution at larger $x$. To check this assumption it is instructive to consider the isovector contribution. Since there is no clear evidence of scaling violations between $Q^2 = 3 GeV^2$ and $Q^2 = 5 GeV^2$ in any polarized deep inelastic scattering experiment, we may neglect, for the time being, the effects of QCD evolution, because we are only interested in the results, provided by the data at the present level of statistical accuracy.

It is then possible, by combining the SLAC neutron data with the SLAC proton data, to determine the quantity $g_1^{p-n} \equiv g_1^p - g_1^n$, entering the Bjorken sum rule, with a higher
accuracy, than for proton alone. This is because the neutron data are fortunately negative, so the difference is larger in magnitude than $g_1^p$, while the errors are practically the same for $g_1^{p-n}$ and $g_1^p$, given the better accuracy of the neutron data.

Performing this analysis of the data, we see immediately a behaviour less sharp than that of neutron. Since phase space effects, as powers of $(1-x)$, are not so important in the region under consideration, we were looking for a simple power parametrization of the data, implied by Regge pole behaviour, namely

$$g_1^{p-n}(x) = g_1^{p-n}(x_0) \left( \frac{x}{x_0} \right)^{-a},$$

(1)

and we found that this works rather well, for $0.016 \leq x \leq 0.125$ with the following choice of parameters:

$$g_1^{p-n}(x) = 0.147x^{-0.45},$$

(2)

as shown in Fig.1. The power we obtain is significantly smaller than the expected contribution of the $a_1(1260)$ meson trajectory ($\sim 0.14$). As a result, the contribution to the Bjorken integral from the region $0 \leq x \leq 0.125$ is large

$$\int_0^{0.125} dx g_1^{p-n}(x) = 0.085.$$  

(3)

The region of higher $x$ corresponds to a neutron contribution much smaller than that of the proton, the latter also providing a large contribution to the integral, and we have

$$\int_{0.125}^{1} dx g_1^{p-n}(x) \approx \int_{0.125}^{1} dx g_1^p(x) = 0.09.$$  

(4)

The total contribution to the Bjorken sum rule,

$$\int_0^{1} dx g_1^{p-n}(x) = 0.175$$

(5)

appears to be in the fair agreement with the theoretical value. Note that when the final neutron data will be available, as well as more precise proton data, it will allow one a more serious analysis, taking also into account the effects of QCD evolution.
At the present moment we would like to stress, that by combining the current neutron and proton data, one is led to good agreement with Bjorken sum rule. Let us stress that this power is compatible with the one obtained using next-to-leading order (NLO) fit to all previous data ($-0.56 \pm 0.21$). It is consistent with the result of $\ln^2 x$ summation. We may also understand the observed power $\alpha = -0.45$ by considering the intercept of the Regge cut associated to the $a_1$ meson and a Pomeron, namely

$$\alpha = 1 - \alpha_P - \alpha_{a_1}$$

provided we use the famous BFKL Pomeron with the intercept $\alpha_P \sim 1.6$ and $\alpha_{a_1} = -0.14$.

III. ISOSCALAR CHANNEL AND THE SINGLET CONTRIBUTION

Since we found, that the sharp neutron structure function is not seen in the difference between proton and neutron, it should be attributed to the isoscalar channel. Also, the partial cancellation, we suspect to be at the origin of a flat proton structure function, should be manifested in this channel as well. To check this, we calculated the quantity $g_1^{p+n} = g_1^p + g_1^n$. It really shows a rather flat structure for $x \geq 0.035$. The relative errors are much larger in this case because $g_1^{p+n}$ is small due to the fact that $g_1^p$ and $g_1^n$ have opposite signs. Of course this fact also implies a small value of the deuteron structure function in this kinematic region, which is barely consistent with the existing data.

This flat structure should be related to the interplay of the negative sharp contribution, showing itself in the fit $x^{-0.8}$ to the neutron data, and a positive contribution with a smaller power, dominating at larger $x$. Since there is no counterpart for such a sharp behaviour in the conventional Regge analysis, we make a strong, but natural assumption. Namely, we suggest that it is manifested in the $SU(3)$-singlet channel.

It is in this channel that a strong mixing between polarized quarks and gluons provides the anomalous gluon contribution to the first moment of $g_1$. Recent studies show that
generalized anomalous gluon contribution appears also in all moments \cite{11,12}. At low \( x \) the quark-gluon mixing provides a strong correction to the subleading behaviour \cite{4}, producing a power close to 1. One might expect that a similar effect is also present in the non-perturbative region where it can give rise to a strong \( x \) dependence in gluon distribution at low \( Q^2 \), which is an initial condition in the approach just mentioned above. An example of such a non-perturbative contribution is given by instanton effects \cite{13} which, however, do not provide yet a reliable quantitative estimate.

Moreover, the \( SU(3) \)- nonsinglet part receives the contributions from the \( f_1(1285) \)- and \( \eta(547) \)-mesons trajectories. The first one has an intercept close to that of \( a_1 \) occurring in the isovector channel, while the second one produces a smoother behaviour like \( x^{0.3} \). For the first estimate we neglect the latter and find that the data are well described by the formula:

\[
g_1^{p+n}(x) = 0.145x^{-0.45} - 0.03x^{-0.87},
\]

as seen in Fig.2. This formula is suggesting that the isoscalar contribution is approximately equal to the isovector one, which is not so surprising, in order to have the neutron structure function, dominated by the most singular power only. This would mean, that in this region of \( x \), say between 0.01 and 0.1, one has

\[
\Delta u(x) = \Delta d(x) \sim \Delta d(x) - \Delta s(x),
\]

requiring a strong negative \( s \)-quark polarization. Apart from the difficulties of incorporating this result to current models of nucleon structure, it could also conflict with the Bjorken sum rule for the decay of strange baryons, implying that the integrals of both sides of this equation should be of opposite signs. Although the suggested above equality may be valid only in a limited region of \( x \), and violations of \( SU(3) \) symmetry may be possible, it is more likely, that the \( \eta \) contribution makes the \( x \) dependence of isovector and isoscalar combinations, different in the region under consideration, so we will have

\[
g_1^{p+n}(x) = C_{ns}x^{-0.45} + C_{\eta}x^{0.3} - C_sx^{-a_s}. \]
The numerical analysis shows, that the present inaccurate data, mainly for \( x \leq 0.035 \) are equally well described with a wide range of coefficients \( C_f \) and \( C_\eta \). Bearing in mind the problem with the second Bjorken sum rule mentioned above, one may suspect a negative value for \( C_\eta \) in order to reduce the isoscalar integral at larger values of \( x \). On the other hand, the parameters of the singlet contribution are rather stable, \( C_s \sim 0.03, a_s \sim 1 \), which is again related to the good accuracy of neutron data.

Note that, if this neutron behaviour is really an isoscalar phenomenon, as suggested by our analysis, one should observe a decrease and sign change for the proton structure function not too far from \( x \sim 0.005 \) at low \( Q^2 \). This is the first major check of this result. However, negative proton structure function for much smaller values of \( x \) and low \( Q^2 \) have been considered in the literature and a sign change may also come at large \( Q^2 \) from the effect of QCD evolution [9].

Note that the obtained power is also compatible with the NLO fit [2] result, but the accuracy of the neutron data would allow to reduce the error.

It is, of course, too early to relate unambiguously the observed behaviour to the results of [4], because of the limited experimental accuracy and some theoretical problems. In particular, it is not absolutely clear, to what values of \( x \) and \( Q^2 \) the results of [4] should be applicable. Nevertheless, the relative closeness of the experimental and theoretical numbers may be a signal, that the low \( x \) asymptotic behaviour is manifested rather early, especially in the neutron case, where it is not screened by a large nonsinglet contribution, like in the proton case. Note that clear evidence for negative \( g_1^n \) no longer requires the negative gluon polarization, as was guessed in [4], relying on earlier data. Moreover, we found that the formula (3.24) of [4] is not incompatible with the data, if some mean values (like suggested in [4]) of parton distributions are taken as an input. However, this approach does not allow one to extract the \( x \)-dependent parton distribution, and we shall use now the continuity with the region of average \( x \) in order to get an estimate.

It is not clear, to which extent one should attribute a small \( x \) singlet contribution to quarks or to gluons. However, according to [4] the contribution of gluons is dominant,
so we neglect the quark contribution in the present approach. Requiring the qualitative
continuity in transition between low and average $x$, and applying the parton-like formula
for the anomalous gluon contribution it seems natural to expect that the gluon distribution
is behaving like

$$\Delta G(x) \sim x^{-0.87}. \quad (10)$$

Note, that this formula is assuming the simple relation between singlet contribution to $g_1$
and $\Delta G$ at average $x$

$$g_1^s(x) = -\frac{\alpha_s}{6\pi} \Delta G(x), \quad (11)$$

which is, strictly speaking, is valid for the first moment only. More generally, one has \[14\]

$$g_1^s(x) = -\frac{\alpha_s}{6\pi} \int_x^1 \frac{\Delta G(z)}{z} E(\frac{x}{z}) dz, \quad (12)$$

where $E(y)$ is the coefficient function, describing the gluon-photon interaction. Recent
studies, based on the non-local generalization of the axial anomaly \[11\], support the following
choice \[12\]

$$E(z) = 2(1 - z), \quad (13)$$

leading, for the gluon distribution of the type $const \times x^{-a}$, to the relation

$$g_1^s(x) = -\frac{2}{a(a + 1)} \frac{\alpha_s}{6\pi} \Delta G(x). \quad (14)$$

As a result, for $a = 0.87$, $\Delta G(x)$ should be multiplied, for a given $g_1^s(x)$, by a factor
$\sim 0.8$ and the integral of $\Delta G(x)$ in the range $0.01 \leq x \leq 0.1$ is about 1.

The double helicity asymmetry $A_{LL}$ in prompt photon production in $pp$ collisions is
directly related to $\Delta G(x)$ and for the center of mass energy $\sqrt{s} = 500GeV$, which will
be reached at RHIC, one is probing precisely this kinematic region of $x$. So given such a
strong gluon polarization, we anticipate a larger $A_{LL}$ than previously predicted \[3\]. Similar
comments can be made for $A_{LL}$ in inclusive jet production.
IV. DISCUSSION AND CONCLUSIONS

We have performed here an analysis based on the present level of the experimental accuracy, still not enough to see the effects of QCD evolution, but providing interesting information about isospin structure. For this reason we restrict ourself to the SLAC data, and neglect the effects of QCD evolution in our analysis. Note that the SMC data [15] are unfortunately not accurate enough to be used for such a simple isospin decomposition. The results on $g_1^d$ are certainly not incompatible with our $g_1^{p+n}$ but in order to obtain $g_1^{p-n}$, it is necessary to extract $g_1^n$ from $g_1^d$ after subtracting $g_1^p$, which enhances substantially the statistical errors. However, the elaborate statistical analysis using NLO evolution lead to the very similar powers, although the error for the singlet case is still very large.

The presented simple picture of the nucleon structure is based on the two observations.

i) As suggested by the E154 Collaboration, the $g_1^n$ behaviour is well described by $\sim x^{-0.8}$.

ii) From our simultaneous analysis of proton and neutron data, there is no indication of such a behaviour in $g_1^{p-n}$. Instead, it is well described by the $\sim x^{-0.45}$, which leads to a good saturation of Bjorken sum rule.

Consequently, the existence of a strong negative isoscalar contribution is implied by these two facts. It seems rather well established, and leads to predict a negative $g_1^p$ for $x$ below 0.005.

Both the interpretation of the nonsinglet behaviour as a $ln^2x$ terms (or cut produced by the BFKL pomeron), as well as the relation of the sharp behaviour of the singlet contribution at low $x$, and even further, to a strong gluon polarization, can be considered more speculative. It would be an unusual coincidence, that two rather different aspects of small-$x$ physics manifest themselves in the same physical quantity. However, these assumptions seem to us possible, and they will be either supported or disproved by future more accurate data which will allow to elaborate a better analysis of the problem.

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FIGURES

FIG. 1. Comparison for $g_1^{p-n}$ between the curve given by eq. (2) and the SLAC data ref. [1,6]. For $g_1^p$ at $x = 0.0165$ due to the absence of SLAC data we used the SMC data ref. [15].

FIG. 2. Same as Fig.1 for $g_1^{p+n}$ with the curve given by eq. (7)
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