Toward More Realistic Forecasting of Dark Energy Constraints from Galaxy Redshift Surveys

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ABSTRACT

Galaxy redshift surveys are becoming increasingly important as a dark energy probe. We improve the forecasting of dark energy constraints from galaxy redshift surveys by using the “dewiggled” galaxy power spectrum, $P_{dw}(k)$, in the Fisher matrix calculations. Since $P_{dw}(k)$ is a good fit to real galaxy clustering data over most of the scale range of interest, our approach is more realistic compared to previous work in forecasting dark energy constraints from galaxy redshift surveys. We find that our new approach gives results in excellent agreement when compared to the results from the actual data analysis of the clustering of the Sloan Digital Sky Survey DR7 luminous red galaxies. We provide forecasts of the dark energy constraints from a plausible Stage IV galaxy redshift survey.

Key words: cosmology: observations, distance scale, large-scale structure of universe

1 INTRODUCTION

One of the most important discoveries in modern cosmology is the accelerated expansion of the Universe [Riess et al. [1998], Perlmutter et al. [1999]]. The power spectrum or 2-point correlation function measured from galaxy redshift surveys has provided one of the primary probes of cosmic acceleration, both through the broadband measurement of the shape imprinted by matter-radiation equality (e.g. Percival et al. [2001], Tegmark [2004]) and through the baryon-acoustic oscillation (BAO) feature imprinted at recombination (e.g. Eisenstein et al. [2005]). Galaxy clustering also allows us to differentiate smooth dark energy and modified gravity as the cause for cosmic acceleration through the simultaneous measurements of the cosmic expansion history $H(z)$, and the growth rate of cosmic large scale structure, $f_g(z)$ (Guzzo et al. [2008], Wang [2008], Blake et al. [2012]).

The Fisher matrix approach has generally been used in the forecasts of future galaxy redshift surveys. In this paper, we improve the Fisher matrix approach by making it more realistic. This enables its use in cross-checking dark energy and gravity constraints from current galaxy clustering data, as well as in making the forecasts for future galaxy redshift surveys more robust and reliable.

We present our method in Section 2, our results in Section 3, and summarize and conclude in Section 4.

2 METHOD

The redshift-space galaxy power spectrum $P(k, \mu)$ is a rich source of cosmological information. It includes the BAO feature (Blake & Glazebrook [2003], Seo & Eisenstein [2003]), which has received a great deal of attention as a standard ruler that can be used in both the transverse direction (to measure distances) and the radial direction (to measure the Hubble rate). However, the full galaxy power spectrum at large scales is also sensitive to the underlying matter power spectrum, to the growth of structure via redshift-space distortions (Kaiser [1987]), and to standard ruler effects. This additional information requires some work to extract, since one must simultaneously measure the cosmology and the galaxy biasing parameters. Nevertheless, the galaxy power spectrum provides the most powerful constraints on dark energy and gravity. In this paper, we focus on the analysis of the full set of 2-point galaxy statistics, and do not limit ourselves to only the BAO information.

2.1 Formalism

Our Fisher matrix approach is derived from that of Seo & Eisenstein (2003), and based on Wang (2006, 2008), (2010) and Wang et al. (2010). In the limit where the length scale corresponding to the survey volume is much larger than the scale of any features in the observed galaxy power spectrum $P_g(k)$, we can assume that the likelihood function for the band powers of a galaxy redshift survey is Gaussian (Feldman, Kaiser, & Peacock [1994]), with a measurement error in $\ln P(k)$ that is proportional to...
\[ V_{\text{eff}}(k, \mu) = \int dr^1 \left[ \frac{n(r) P_g(k, \mu)}{n(r) P_g(k, \mu) + 1} \right]^2 \]

where the comoving number density \( n \) is assumed to only depend on the redshift (and constant in each redshift slice) for simplicity in measurement errors for the parameters \( (Tegmark 1997) \).

In order to propagate the measurement error in \( \ln P_g(k) \) into measurement errors for the parameters \( p_i \), we use the Fisher matrix \( \text{F}_{ij} \). Our model for the galaxy power spectrum is that of the parameters \( (Tegmark 1997) \), our model for the galaxy power spectrum: the wavenumber \( k_\perp \) in the reference cosmology is related to the wavenumber in the true cosmology via

\[ k_\perp = \frac{D_A(z)}{D_A(0)} k_\perp \text{ and } k_{\parallel} = \frac{H^d(z)}{H(z)} k_{\parallel}. \]

Based on \( \text{Seo & Eisenstein (2003)} \) and \( \text{Chuang & Wang (2012a)} \), our model for \( P_g(k) \) can then be written as

\[ P_g(k, \mu) = \frac{D_A(z)^2}{D_A(0)^2} \frac{H(z)}{H(0)} \frac{k^3}{2(2\pi)^3} \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{dk^3}{2(2\pi)^3}, \]

where \( k_{\text{min}} \) and \( k_{\text{max}} \) are the parameters to be estimated from data, and the derivatives are evaluated at parameter values of the fiducial model. Note that the Fisher matrix \( F_{ij} \) is the inverse of the covariance matrix of the parameters \( p_i \) if the Gaussian distributed.

We adopt the standard notation that \( k \) can be decomposed into a line-of-sight component \( k_\parallel \) and the transverse or in-the-plane-of-the-sky component \( k_{\perp} \). The cosine of the angle between \( k \) and the line of sight vector is denoted by \( \mu = k_\parallel / |k| \).

### 2.2 The model for the galaxy power spectrum

At cosmological distances, the “true” galaxy power spectrum is not a direct observable, since one can measure a galaxy’s position only in angular and redshift coordinates and not in its true 3D comoving coordinates. This is of course the basis for extraction of the “standard ruler” information, including the \( \text{Alcock & Paczynski (1979)} \) effect. Therefore standard practice is to project the galaxies to their comoving positions assuming some reference cosmology (or fiducial cosmology), and then a power spectrum or correlation function estimator is applied. The observed galaxy power spectrum is then related to the true galaxy power spectrum via a coordinate transformation: the wavenumber \( k_{\text{ref}} \) in the reference cosmology is related to the wavenumber in the true cosmology via

\[ k_\perp = \frac{D_A(z)}{D_A(0)} k_\perp \text{ and } k_\parallel = \frac{H^d(z)}{H(z)} k_\parallel. \]

The bias between galaxy and matter distributions is denoted by \( b(z) \). The linear RSD parameter \( \beta(z) = f_g(z)/b(z) \) \( \text{(Kaiser 1987)} \), where \( f_g(z) \) is the linear growth rate; it is related to the linear growth factor \( G(z) \) (normalized such that \( G(0) = 1 \)) as follows

\[ f_g(z) = \frac{d \ln G(z)}{d \ln a}. \]

We have assume that the peculiar velocities of galaxies can be modeled with a probability distribution

\[ f(v) = 
\]
The true galaxy power spectrum should have \( p_{\text{NL}} = 1 \). Recently, "reconstruction" algorithms have been proposed (Eisenstein et al. 2007) and implemented (Padmanabhan et al. 2013) that reverse some of the flows and move galaxies back closer to their original (Lagrangian) positions. If such an algorithm is applied to data, the nonlinearity percentage can be reduced. BAO reconstruction is a rapidly developing field, but in its early stages and high-z redshift surveys may have to deal with survey geometries that are more complex and bias-weighted galaxy densities \( b^2 \) that are smaller than that of \( g \). BOSS. For the present work, we consider a range of values for \( p_{\text{NL}} \). The optimistic case of \( p_{\text{NL}} = 0.5 \) corresponds to \( k_s \simeq 0.24 \, h/\text{Mpc} \), whereas the most conservative case of \( p_{\text{NL}} = 1 \) (no reconstruction) corresponds to \( k_s \simeq 0.12 \, h/\text{Mpc} \), assuming \( \sigma_8 = 0.8 \).

For an intuitive understanding of the dewiggled power spectrum of Eq. (10), we can rewrite its corresponding transfer function, Eq. (10), as follows:

\[
T_{\text{daw}}(k, z) = T_{\text{nw}}(k) + \left[ T^2(k) - T_{\text{nw}}^2(k) \right] e^{-g_0 k^2/(2k^2 z)}
\]

\[
T_{\text{daw}}(k, z) = T_{\text{nw}}(k) + T_{\text{BAO}}(k) - g_0 k^2/(2k^2 z) \tag{13}
\]

where we have defined \( T_{\text{daw}}(k) = T^2(k) - T_{\text{nw}}^2(k) \), the difference between the linear matter transfer functions with and without baryons. Clearly, the exponential damping due to nonlinear effects is only applied to the transfer function associated with BAO. Angulo et al. (2008) have compared the spherically-averaged form of this model with measurements from simulated data, and found that it works extremely well on the linear and quasilinear scales; the assessment of its accuracy is presently limited by the shot noise of currently available numerical simulations. In future work, we will test this model without spherical averaging using numerical simulations to fully assess it. We do not expect this model to continue working well on the smallest scales, where the nonlinear damping is coupled with RSD (e.g. Jennings, Baugh, & Pascoli 2011; Reid & White 2011; Chuang & Wang 2012).

To avoid the direct measurement of the unknown galaxy bias \( b(z) \), we rewrite our model for the measured galaxy power spectrum as (Wang 2012):

\[
P_g(k_{\text{ref}}, k_{\text{ref}}) \equiv P_g(k_{\text{ref}}, k_{\text{ref}})/h^{-3}(\text{Mpc})^3 = \frac{[\sigma_g(z) + f(z) \sigma_m(z) \mu^2]^2}{\sigma_g(z) + f(z) \sigma_m(z) \mu^2 \sigma_{b,\text{obs}}^2 \ln k_{\text{ref}} + k^2 \mu^2 \sigma_{b,\text{obs}}^2 / \sigma_m(z) \mu^2} + P_{\text{shot}} \tag{14}
\]

where we have defined \( \sigma_g(z) \equiv b(z) G(z) \tilde{P}_0^{1/2} \) and \( \sigma_m(z) \equiv G(z) \tilde{P}_0^{1/2} \).

The dimensionless power spectrum normalization constant \( P_0 \) is just \( P_0 \) in Eq. (9) in appropriate units:

\[
P_0 \equiv \frac{P_0}{(\text{Mpc}/h)^3 (\text{Mpc})^3} = \frac{\sigma_8^2}{I_0 h^{n_s}}. \tag{16}
\]

The second part of Eq. (16) is relevant if \( \sigma_8 \) is used to normalize the power spectrum. Note that

\[
I_0 = \int_0^\infty \frac{d\bar{k}}{2\pi^2} T^2(\bar{k} \cdot h \text{Mpc}^{-1}) \left[ \frac{3j_1(8\bar{k})}{8\bar{k}} \right]^2,
\]

where \( \bar{k} \equiv k/\text{Mpc}^{-1} \), and \( j_1(\bar{k} r) \) is spherical Bessel function. Note that \( I_0 = I_0(\omega_m, \omega_b, n_s, h) \). Since \( k_s \) and \( k_{\text{ref}} \) scale as \( H(z) \) and \( 1/D_A(z) \) respectively, \( P_g(k_{\text{ref}}) \) in Eq. (14) does not depend on \( h \).

Eq. (14) is the model we will use in this paper. Its absorption of the bias factor is analogous to the approach of Song & Percival (2009), who proposed the use of \( f_g(z) \sigma_g(z) \) to probe growth of large scale structure. The difference is that Eq. (14) uses \( f_g(z) \sigma_g(z) \equiv f_g(z) G(z) \tilde{P}_0^{1/2} \), which does not introduce an explicit dependence on \( h \) (as in the case of using \( f_g(z) \sigma_m(z) \)).

2.3 Parameters and assumptions

In our method, the full set of parameters that describe the observed \( P_g(k) \) are: \{\( H(z) \), \( D_A(z) \), \( \ln[f_g(z) \sigma_m(z)] \), \( \ln[\sigma_g(z) \sigma_{b,\text{obs}}^2 \ln k_{\text{ref}} + k^2 \mu^2 \sigma_{b,\text{obs}}^2 \sigma_m(z) \mu^2] \}, \) where \( i \) indicates the \( i \)-th redshift slice, and \( \omega_m \equiv \Omega_m h^2 \) and \( \omega_b \equiv \Omega_b h^2 \). We marginalize over \( \ln[\sigma_g(z) \sigma_{b,\text{obs}}^2 \ln k_{\text{ref}} + k^2 \mu^2 \sigma_{b,\text{obs}}^2 \sigma_m(z) \mu^2] \) in each redshift slice, as well as \( k_s \) and \( \sigma_{b,\text{obs}} / (1 + z) \), to obtain a Fisher matrix for \{\( H(z) \), \( D_A(z) \), \( \ln[f_g(z) \sigma_m(z)] \), \( \omega_m \), \( \omega_b \)\}, this full Fisher matrix, or a smaller marginalized over parameters, is projected into the standard set of cosmological parameters \( \{\omega_c, \omega_b, \Omega_X, \Omega_b, \omega_m, n_s, h \} \). There are four different ways of utilizing the information from \( P(k) \) (Wang 2012).

It is important to note that when evaluating the derivatives of \( P_g(k_{\text{ref}}) \) with respect to the parameters described above (required to calculate the Fisher matrix), we should not extract information from the damping factors due to systematic uncertainties, in order to adhere to a conservative and robust approach. These damping factors are only included to represent the loss of information at small scales due to nonlinear effects (and, if applicable, redshift uncertainties). We treat these damping factors as follows when derivatives are taken.

The \( g_0 \) in the nonlinear damping factor, \( e^{-g_0 k^2/(2k^2 z)} \), is fixed at fiducial model values when derivatives are taken, to avoid deriving cosmological information from the NL damping itself. Note that \( k \) is scaled as we vary \( T_{\text{daw}}(k) \) for consistency, and we marginalize over \( k_s \) to allow for the significant uncertainty in the NL damping.

The damping factor due to redshift uncertainty, \( e^{-k^2 \mu^2 \sigma_m^2} \), is computed with \( \partial \ln z/\partial z \) from the fiducial model, to avoid deriving cosmological information from the damping itself. We marginalize over \( \sigma_m^2 / (1 + z) \) to allow for the uncertainty in our knowledge of redshift accuracy.

The RSD factor due to small scale random motion of galaxies, \( 1/[1 + k^2 \mu^2 \sigma_{r,p}^2 / 2] \), is fixed at fiducial model values when derivatives are taken. Note that when we vary \( H(z) \), this RSD factor remains unchanged, since \( k \mu = k_1 \propto H(z) \), while \( \sigma_{r,p} \propto 1/H(z) \). The RSD factor is included here to represent the suppression of power due to galaxy peculiar velocities, and not to provide an accurate modeling of RSD on all scales.

Since our model fits real data well on these scales (Chuang & Wang 2012a), it represents a step forward in making Fisher matrix forecasting for galaxy redshift surveys more realistic.

3 RESULTS

We will present results on

\[
x_b(z) \equiv H(z) s/c \quad \text{and} \quad x_d(z) \equiv D_A(z)/s,
\]

where \( k \) is the redshift.
where $s \equiv r_s(z_d)$ is the sound horizon at the drag epoch, which is the characteristic scale of BAO.

We assume the fiducial model adopted by the FoMSWG (Albrecht et al. 2009), itself based on the 5-year Wilkinson Microwave Anisotropy Probe (WMAP) results (Dunkley et al. 2009): $\omega_m \equiv \Omega_m h^2 = 0.1326$, $\omega_b \equiv \Omega_b h^2 = 0.0227$, $h = 0.719$, $\Omega_k = 0$, $w = -1.0$, $n_s = 0.963$, and $\sigma_8 = 0.798$.

We will first present results for SDSS DR7 LRGs, in order to compare with the results from actual data analysis. The analysis of current GC data require the assumption of cosmological priors; we impose the same broad priors as Chuang & Wang (2012a).

Next, we will present results for StageIV+BOSS spectroscopic galaxy redshift surveys, and compare these with those from the previously widely adopted approach derived from Seo & Eisenstein (2007) and developed in detail in Wang (2012).

No priors are used in deriving $\{x_h(z), x_d(z), f_g(z)\sigma_m(z)/s^2\}$ constraints for future surveys, since these provide model-independent constraints on the cosmic expansion history and the growth rate of cosmic large scale structure. These allow the detection of dark energy evolution, and the differentiation between an unknown energy component and modified gravity as the causes for the observed cosmic acceleration.

In order to derive dark energy figure of merit (FoM), as defined by the DETF (Albrecht et al. 2006), we project our Fisher matrices into the standard set of dark energy and cosmological parameters ($\omega_b, \omega_c, \Omega_k, \omega_m, \omega_b, n_s, ln A_s$). To include Planck priors [4] we convert the Planck Fisher matrix for 44 parameters (including 36 parameters that parametrize the dark energy equation of state in redshift bins) from the FoMSWG into a Planck Fisher matrix for this set of dark energy and cosmological parameters.

### 3.1 Comparison with analysis of data

To gauge the accuracy of our forecasting methodology compared to the full analysis of real data, we present our forecasts for the SDSS DR7 set of 87,000 LRGs in the redshift range 0.16–0.44 analyzed by Chuang & Wang (2012a) in Table I and compare them with the actual measurements performed as part of this work, using both the SDSS DR7 LRGs and the SDSS LRG mocks from LasDamas [4].

The scale range analyzed by Chuang & Wang (2012a) is $r = 40-120$ h$^{-1}$Mpc, which corresponds to the $k = 2\pi/r$ range of 0.0524–0.157 h Mpc$^{-1}$. Chuang & Wang (2012a) used flat priors on $\omega_b$ and $n_s$ that have widths of $\pm 7\sigma_{W_{MAP}}$ (with $\sigma_{W_{MAP}}$ given by the WMAP seven year results from Komatsu et al. (2011)). In addition, Chuang & Wang (2012a) imposed flat priors of $0.1 < \beta < 0.6$, $0 < \sigma_p < 500$ km/s, and $0.09 < k_s(z = 0.35)/h$ Mpc$^{-1} < 0.13$. We use Gaussian priors on the same parameters with the same widths for the priors as Chuang & Wang (2012a), and with means of $\sigma_p = 250$ km/s, and $k_s/h$ Mpc$^{-1} = 0.11 G(z = 0.33) = 0.0939$ (the width of $k_s/h$ Mpc$^{-1}$ is 0.02 $G(z = 0.33) = 0.01707$). Note that our definition of $k_s$ is independent of redshift, thus it is divided by the growth factor at the effective redshift of the data set used by Chuang & Wang (2012a).

In addition, we assume a redshift accuracy of $\sigma \ln (1 + z) = 5 \times 10^{-4}$, and a bias of $b = 2.2$ for the SDSS DR7 LRGs. These additional assumptions are not needed for the analysis of real or mock data, since the overall amplitude is marginalized over (Chuang & Wang 2012a).

Since the Fisher matrix results depend on the fiducial model assumed, we give our Fisher matrix forecasts for two different fiducial models in Table I: the FoMSWG fiducial model (Albrecht et al. 2009) with with $\omega_m \equiv \Omega_m h^2 = 0.1326$, $\omega_b \equiv \Omega_b h^2 = 0.0227$, $h = 0.719$, $\Omega_k = 0$, $w = -1.0$, $n_s = 0.963$, and $\sigma_8 = 0.798$, and the Euclid Red Book fiducial model (Laureijs et al. 2011) with with $\omega_m \equiv \Omega_m h^2 = 0.1225$, $\omega_b \equiv \Omega_b h^2 = 0.021805$, $h = 0.7$, $\Omega_k = 0$, $w = -0.95$, $n_s = 0.963$, and $\sigma_8 = 0.8$. These fiducial models lead to $\sigma \ln x_h(z)$ and $\sigma \ln x_d(z)$ that differ by 3.1% and 6.8% respectively.

The model used here, Eq. (14), differs somewhat from that used by Chuang & Wang (2012a). Our new model, Eq. (15), uses anisotropic dewiggling whereas Chuang & Wang (2012a) used isotropic dewiggling, which neglects the additional damping along the line of sight due to the enhanced Lagrangian displacement in redshift space. Using our Fisher matrix method, we find that assuming isotropic dewiggling leads to an underestimate of $\sigma_m x_h(z)$ and $\sigma_m x_d(z)$ of $\sim 23 - 24\%$ and $\sim 12 - 13\%$ respectively.

In order to make an accurate comparison, we have repeated the analysis of the SDSS DR7 LRG sample used by Chuang & Wang (2012a) using Eq. (14) as part of this work. We find that the data and mocks give $\sigma_m x_h(z)$ and $\sigma_m x_d(z)$ that differ by 13% and 30% respectively, with the mock results agreeing with our Fisher matrix forecasts at a level of 10% or better, given the dependence of the Fisher matrix forecasts on the assumed fiducial model.

Our Fisher matrix forecasts for the measurement uncertainty on $f_g(z)\sigma_m(z)/s^2$ are in excellent agreement with the results from the MCMC analysis of the LasDamas SDSS LRG mocks, while the results from the MCMC analysis of SDSS DR7 LRG data give significantly smaller uncertainty on $f_g(z)\sigma_m(z)/s^2$. This is likely due to the apparent excess clustering of SDSS DR7 LRGs along the line of sight (this is apparent from comparing the mock and data panels of Fig.1 in Chuang & Wang (2012a), which is likely also responsible for the smaller than expected measurement uncertainty of $x_h(z)$ and $x_d(z)$ from this data sample. This excess power along the line of sight explains the widely noted excess power on large scales for the spherically-averaged galaxy correlation function (see, e.g., Cabre & Gaztanaga 2009, Kazin et al. 2010, Chuang, Wang, & Hemantha (2012)). Since the BOSS CMASS galaxies do not have the excess clustering on the same large scales (Reid et al. 2012), the excess large scale clustering of SDSS DR7 LRGs must be due to sample variance or unknown systematic effects.

Taking all the factors discussed above into consideration, our Fisher matrix forecasts are in excellent agreement with the results from actual data analysis. It is reassuring that our Fisher matrix method gives very similar results compared to actual data analysis, making it a reliable tool for parameter forecasting for future surveys.

### 3.2 Forecasts for a Stage IV galaxy redshift survey

We perform forecasts for a Stage IV galaxy redshift survey covering an area of 15,000 deg$^2$, using slitless grism spectroscopy to detect the H$\alpha$ emission line. A wavelength range of 1.1–2.0 $\mu$m, corresponding to $0.7 < z < 2.0$, was assumed. The depth of the...
survey was computed using instrument parameters (throughput, exposure time, etc.) similar to those provided for the Euclid mission in Laureijs et al. (2011); it is thus representative of a next-generation space-based galaxy survey, though it may not correspond precisely to the final Euclid numbers.

All our forecast results are shown for Stage IV plus BOSS. The BOSS survey is assumed to cover 10,000 (deg²), a redshift range of 0.1 < z < 0.7, with a fixed galaxy number density of $n = 3 \times 10^{-4} h^3$Mpc⁻³, and a fixed linear bias of $b = 1.7$.

We discuss the galaxy yields from a Stage IV Euclid galaxy redshift survey in detail in Sec. 3.2.1. For clustering analysis, we also require the galaxy bias. We use the galaxy bias function for emission line galaxies given by Orsi et al. (2010), which increases with redshift reaching $b = 1.7$ at $z = 2$. Again, we note that this is likely to be conservative: the recent bias determination of Geach et al. (2012) for Hα emitters is $b = 2.4^{+0.1}_{-0.2}$ at $z = 2.23$. We assume a redshift accuracy of $\sigma_z/(1+z) = 0.001$, and a peculiar velocity dispersion of $\sigma_v = 290$ km/s.

We consider two different cutoffs in scale: $k_{max} = 0.2 h$/Mpc and $k_{max} = 0.2 h$/Mpc, in order to include the quasilinear regime only in our forecasts. The choice of $k_{max} = 0.2 h$/Mpc is conservative, and represents the lower bound of the scale range in which our model works well in analyzing real data. The choice of $k_{max} = 0.3 h$/Mpc is more optimistic, but represents a feasible goal for the lower bound of the scale range in which future studies will enable robust and accurate modeling.

### 3.2.1 Galaxy yields for a Stage IV redshift survey

Galaxy yields were computed using the exposure time calculator described in Hirata et al. (2012). Two exposures on each field in each grism bandpass were assumed. The zodiacal background was set to that at 45° ecliptic latitude and 90° away from the Sun at the mean of the annual cycle, and we include a foreground dust column of $E(B-V) = 0.05$ magnitudes; these values vary over any realistic survey but are representative. Standard read noise assumptions for the $2k \times 2k$ Teledyne HgCdTe detectors were used (32 channel readout, 1 frame per 1.3 s, 20 electrons rms per correlated double sample, with a noise floor of 5 electron rms for many reads). The galaxy survey was assumed to be 70 per cent complete down to the flux limit for a 7σ significance matched-filter detection [1]. The extinction-corrected Hα flux limit varies with redshift and galaxy size, but is in the range of $(2.2 - 3.6) \times 10^{-16}$ erg s⁻¹ cm⁻² for a source half-light radius of 0.3 arcsec.

As a point of comparison, we ran the Hirata et al. (2012) code on the Hubble Space Telescope Wide Field Camera 3 (HST/WFC3) G141 grism (1.1 < $\lambda$ < 1.7 µm), using parameters from the Instrument Handbook (Dressel 2011). We find that for an exposure time of 2700 s, the 5σ sensitivity should be $(4.6 - 8.2) \times 10^{-17}$ erg s⁻¹ cm⁻², with the lower (better) numbers at the red end of the bandpass. This is in good agreement (~20 per cent) with the median sensitivity actually achieved by WFC3 observations – see e.g. Figure 5 of Atek et al. (2010).

The line flux sensitivity and completeness are only part of determining the number of redshifts obtained by a survey – one also needs a luminosity function. In the past decade of space-based redshift survey mission planning, the Hα luminosity function (HαLF) has been a matter of vigorous debate; direct measurements have suffered from small-number statistics, while indirect methods (based on scaling from rest-frame ultraviolet or [O II] luminosities) have had difficult-to-quantify systematic errors. Indeed, the estimates used for space mission planning (e.g. Yan et al. 1999, Hopkins et al. 2000, Reddy et al. 2008, Jouvel et al. 2009, Geach et al. 2010, Sobral et al. 2012) have spanned a factor of ~3 in number density, even accounting for the different cosmologies assumed. Fortunately, empirical measures of the HαLF across the relevant range of redshifts with large-number statistics (dozens of objects in the relevant flux range, in multiple fields) are now available.

We use the HαLF of Sobral et al. (2013), with conversions described in Hirata et al. (2012) §3E to ensure consistency with the exposure time calculator inputs. This is based on blind narrow-band surveys, and updates the previous estimate by Geach et al. (2010). The new HαLF is lower than the previous estimate; in approximate decreasing order of importance, the main differences are:

1. Consistent treatment of internal (host galaxy) extinction corrections, which are applied to some HαLF results and must be undone to predict redshift survey yields.
2. Improved statistics and addition of data at new redshifts.
3. Redshift-averaging effects in some of the grism luminosity functions (this does not occur in narrowband surveys).
4. Aperture corrections.
5. Conversion to the WMAP-5/ FoMSWG cosmology.

The narrowband surveys do not cleanly separate the Hα 6563 Å line from the [N II] doublet at 6548,6583 Å, and the Sobral et al. (2012) HαLF removes the estimated [N II] contribution. Of course, in a grism survey the two lines will be a partial blend, thus we may be underestimating the final detection significance of the galaxies.
For this reason, we expect that our analysis is somewhat conservative.

Table 3 gives our resultant galaxy yields as a function of redshift.

3.2.2 Dark energy figure of merit results

Table 3 shows the dark energy figure-of-merit (FoM) \cite{Wang2008b},

\[
FoM(w_0, w_a) \equiv \frac{1}{\sqrt{\det \text{Cov}(w_0, w_a)}}
\]

(19)

for the four different approaches to utilizing the information from the measured anisotropic galaxy power spectrum \cite{Wang2012}:

1. \( \{x_\perp(z), x_{\parallel}(z)\} \) from \( P(k) \);
2. \( \{x_\perp(z), x_{\parallel}(z), f_g(z)\sigma_m(z)/s^n\} \) from \( P(k) \);
3. \( P(k) \), marginalized over \( f_g(z)\sigma_m(z) \);
4. \( P(k) + f_g(z) : P(k) \) including \( f_g(z)\sigma_m(z) \).

It is clear from Table 3 that for a given cutoff \( k_{\text{max}} \), the FoM for \( (w_0, w_a) \) increases as we increase the dewiggling scale \( k_* \) (i.e., decrease the nonlinear effects). For a fixed level of nonlinearity (i.e., fixed \( k_* \)), the FoM for \( (w_0, w_a) \) increases as we increase the cutoff \( k_{\text{max}} \). The scaling of \( f_g(z)\sigma_m(z) \) with \( s \) depends on the level of nonlinearity assumed: for 50% nonlinearity \( (k_* = 0.24 \text{ h}/\text{Mpc}) \), \( \alpha \approx 4 \), while for 100% nonlinearity \( (k_* = 0.12 \text{ h}/\text{Mpc}) \), \( \alpha \approx 5 \). This is not surprising, since the scaling of \( f_g(z)\sigma_m(z) \) with \( s^4 \) (i.e., \( \alpha = 4 \)) originates from the linear power matter spectrum \cite{Wang2012}. When nonlinear effects are fully included (and not assumed to be reduced due to density field reconstruction), the appropriate model for \( P(k) \) (i.e., Eq. (14)) deviates significantly from the linear power spectrum, leading to modification of the scaling of \( f_g(z)\sigma_m(z) \) with \( s \).

The choice of \( \alpha \) does not affect the FoM\((w_0, w_a)\) from the \{\( x_\perp, x_{\parallel}, f_g\sigma_m/s^n \}\) case, as the correlations between \( f_g(z)\sigma_m(z)/s^n \) and \( \{x_\perp(z), x_{\parallel}(z)\} \) depend on \( \alpha \). Choosing the \( \alpha \) that minimizes the uncertainties in \( f_g(z)\sigma_m(z)/s^n \) does maximize the FoM\((w_0, w_a)\) when Planck priors are included.

3.3 Comparison with previous work

Previously, the forecasts of dark energy constraints from full \( P(k) \) assumed that

\[
P_g^{\text{old}}(k_{\perp}^r, k_{\parallel}^r) = \frac{[D_A(z)^{rref}]^2}{[D_A(z)]^2} \frac{H(z)}{H(z)^{rref}} \times \sigma^2 + \frac{\sigma_g}{c} \times \left[ e^{-\frac{1}{2}\frac{k^2}{2\sigma_m^2}} - \frac{k^2}{2\sigma_m^4} e^{-k^2\sigma_m^2} \right] + P_{\text{shot}}.
\]

(20)

where the linear matter power spectrum \( P_{\text{lin}}(k) \) is \( G^2(z)P_0k^{\mu_\perp}T^2 (k) \) (with \( T(k) \) denoting the linear matter transfer function), and

\[
\sigma^2_{r_{\perp,\parallel,p}} = \left[ \frac{\partial \sigma}{\partial z} \right]^2 \left[ \sigma^2 + \frac{\sigma_g}{c} \right].
\]

(21)

Alternatively, we can write

\[
P_g^{\text{old}}(k_{\perp}^r, k_{\parallel}^r) = P_g(k_{\perp}^{rref}, k_{\parallel}^{rref})/(h^{-1}\text{Mpc})^3
\]

\[
= \frac{[D_A(z)^{rref}]^2}{[D_A(z)]^2} \frac{H(z)}{H(z)^{rref}} \left[ \sigma_g(z) + f_g(z)\sigma_m(z) \mu^2 \right]_2 \times \left( \frac{k}{\text{Mpc}} \right)^{n_k} T^2 (k) e^{-\frac{1}{2}\frac{k^2}{2\sigma_m^2} e^{-k^2\sigma_m^2} \sigma_{r_{\perp,\parallel,p}}^2} + P_{\text{shot}}.
\]

(22)

Table 3 lists the FoM for \((w_0, w_a)\) for the same four cases as listed in Table 1. Each line in Table 3 and its corresponding line in Table 3 assume the same level of nonlinearity and the same cutoff \( k_{\text{max}} \). The only difference between the two tables is the model assumed for \( P(k) \): Eq. (14) (from Eq. (4)) is assumed for Table 3 while Eq. (22) (from Eq. (20)) is assumed for Table 4.

Note that the two assumed models of \( P(k) \) give similar FoM for all the cases that marginalize over the growth information, and for the cases that include growth information but assume only a nonlinearity level of 50% \((P_{\text{NL}} = 0.5 \text{ or } k_* = 0.24 \text{ h}/\text{Mpc}) \), our new model (Eq. (14)) gives significantly larger FoM for the cases that include the growth information. This is because the old model in Eq. (22) simply damps the linear matter power spectrum exponentially, while the new model in Eq. (14) only damps the BAO oscillations, and retain smaller scale information via the “no-wiggle” matter power spectrum \( P_{\text{nw}}(k) = G^2(z)P_0k^{\mu_\perp}T^2 (k) \) (with \( T(k) \) denoting the zero baryon transfer function from \cite{EisensteinHu1998}). This results in significantly smaller uncertainties in \( \ln(\beta(z)) \) (and \( f_g(z)\sigma_m(z)/s^n \)) when our new model is used, which in turn leads to significantly larger FoM\((w_0, w_a)\) when growth information is included.

While the scaling of \( f_g(z)\sigma_m(z) \) with \( s \) depends on the level of nonlinearity assumed in our new model (see discussion in the previous subsection), \( f_g(z)\sigma_m(z) \) scales with \( s^4 \) in the old model \cite{Wang2012}. We find that our FoM results are not sensitive to the exact choice of \( \alpha \); we have chosen \( \alpha = 4 \) for all the FoM tabulated in Tables 3 and 4 when growth information is included.

4 CONCLUSION

We have shown that the forecasting of dark energy constraints from galaxy redshift surveys can be improved in fidelity by using the “dewiggled” galaxy power spectrum, \( P_{\text{nw}}(k) \), in the Fisher matrix calculations. Since \( P_{\text{nw}}(k) \) is a good fit to real galaxy clustering data over most of the scale range of interest, our approach is more realistic compared to previous work in forecasting dark energy constraints from galaxy redshift surveys.

We tested our methodology by comparing our Fisher matrix forecasts with results from actual data analysis, and found excellent agreement (see Table 1). Our Fisher matrix method gives very similar results compared to actual data analysis, making it a reliable tool for parameter forecasting for future surveys.

Using our new approach, we studied a Stage IV galaxy redshift survey, in combination with BOSS, without and with Planck priors. We find that in this new approach, increasing nonlinear effects from 50% (best case) to 100% (most conservative) has a significantly reduced impact on the dark energy figure of merit compared to previous work. This indicates that the erasure of information by nonlinear smearing is having only a modest effect on our ability to constrain cosmology using the “full \( P(k) \)” method in our new realistic approach.

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Table 2. Galaxy yields for a 2-exposure Stage IV galaxy redshift survey as discussed in Sec. [5.2.1] Columns indicate: the redshift; observer-frame wavelength of Hα; the half-light radius (encircled energy 50%, EE50) of the point-spread function; the cosmological volume element \( \frac{dV}{dz \cdot dA} \); the limiting flux for a 0.3 arcsec half-light radius galaxy; the number of observed sources per unit comoving volume; and the number of sources \( dN/dz \cdot dA \) per unit redshift per unit solid angle.

Table 3. Our Fisher matrix forecasts for Stage IV+BOSS galaxy redshift surveys using our new galaxy power spectrum model, Eq.[14], for the four cases discussed in [Wang 2013]. \( \text{FoM}_{GR} \) denoted the FoM assuming general relativity. The parameter \( \gamma \) is defined by \( f_{s}(z) = \gamma \frac{dN}{dz \cdot dA} \).

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Table 4. Our Fisher matrix forecasts for Stage IV+BOSS galaxy redshift surveys using the galaxy power spectrum model from previous work, Eq. (22), for the four cases discussed in Wang (2012). FoM\textsubscript{GR} denoted the FoM assuming general relativity. The parameter $\gamma$ is defined by $f_g(z) = \left[\Omega_m(a)\right]^{\gamma}$.