Heliospheric magnetic field structures: Predictions & Implications

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Abstract. The observational implication of braided solar magnetic fields opens a new venue for an interpretation of various solar and interplanetary phenomena. Direct imaging of the coronal fields at frequencies corresponding to atomic transition of hot (~1MK) plasma pinpoints to their braiding structure, while solar wind measurements of magnetized plasma parcels with distinct orientation, large field deviation and intermittent fading of energetic flare ions suggest that coronal braided field may have been carried by the solar wind to 1AU. The interconnection between the mathematical braids and knots is applied to the topologically non-trivial magnetized structures and their dynamics, from solar corona to the interplanetary medium. The analysis of braided magnetic configurations results in conjectures regarding (i) their stability under large oscillations of magnetic loops, (ii) the structure and stability of the coronal field through successive emergence/decay of heated magnetic braids, and (ii) their evolution into the solar wind. It is anticipated that the resulting conjectures will add to the understanding of the physical processes in the magnetosphere (Van Allen, Cluster satellites) and particularly will attain confirmation with the launch of the Parker Solar Probe.

1. Introduction
Most of the heliospheric magnetized plasmas evolve slowly over typical plasma time-scales, coupling currents due to the motion of charged particles with magnetic fields, in a variety of geometrical configurations. The heliospheric field is formed mainly due to convection in solar and planetary interiors via the dynamo process through conversion of kinetic into magnetic energy. Some of the newly emerging solar fields with a large curl appear in geometrical forms compatible with current sheets. Small-scale magnetic features hold the key to various facets of solar magnetism on larger scales: (a) formation of small-scale helicity in dynamo process and its subsequent transformation into large-scale fields (1-2), (b) local Alfvén waves (3-4) or nanoflares (5-6) heating coronal and chromospheric plasma over extended spatial domain, (3) emergence of magnetic flux from the solar interior with the appearance of small-scale events (compact flare, plasmoid, X-ray brightening) leading to a large-scale CME eruption. Generally, stable, topologically non-standard small-scale magnetic coronal configuration may be carried into the heliosphere expanding into large-scale, entangled magnetic structure.

Recent improvements in the spatial resolution of solar magnetic fields allows one to advance the high resolution analysis of small-scale structures in the coronal field, inferring their evolution as they are fed by the solar dynamo and eventually carried into the interplanetary medium by the solar wind. This ability resulted in remote observations of inhomogeneous bundles of arched solar fields with braiding features, i.e., axial bending of strands of enhanced field with twisting and over/under crossing of adjacent field line strands, forming shapes resembling braids. The bundling and braiding of the fields indicate that the solar dynamo may result in configurations with topological intricacies which, when extended into the interplanetary medium could modify, among others, (a) the cascade rates and
the electromagnetic turbulent wave spectra and (b) the connectivity to flaring sites during active solar times, both leading to measurable results. Hence, nontrivial magnetic topologies with large field deviation within the solar corona or solar wind (7) and repeated drop-outs and reappearance of energetic ion fluxes from flaring sites (8) are consistent with an emergence of magnetized structures in the form of braids or knots. The observation of braided magnetic fields allows us also to cast them into a framework of mathematical braids and their equivalent knots, facilitating a new description of magnetic fields as they emerge from the photosphere and are transported into the interplanetary medium. Combining theoretical considerations, remote images and soon to be realized in situ satellite observations at solar vicinity, one may construct new characteristics of the braided/knotted magnetic structures to constrain their detectable oscillations and specify the conditions of solar magnetic fields stability, presenting an additional perspective into their topological evolution.

2. Braids in coronal fields

Solar braids were coined as such due to the inferred geometry of the observed solar emissions, which were deduced as concentrated along segments of the arched magnetic field lines anchored at the photospheric foot-points, indicating adjacent strands passing over or under, consistent with a 2-D projection of a braid (9). The braiding of the field lines is partly due to the ubiquitous small-scale motion at the photospheric leg(s) of the magnetic field. Part of the motional energy is transmitted to coronal loops through up-flows and propagating Alfvén waves whose damping along the field lines heats up the coronal plasma (10-11); additionally, small-scale reconfigurations may form new active magnetic field lines, such that braid formation may be inherently connected to heating processes. The outflows or jets from an active region observed via motion of braided field may be used to assess the necessary additional coronal heating through the nano-flare reconnection processes (12-14). Since transition line intensity of the coronal ions at given equilibrium temperature are inherently related to their charging states, filters at specific wavelength band indicate the approximate temperature to which the particular charge state was heated. Hence, the “flaring” of loops and the intensity of the emission, captured at a specific frequency band are indicators of the heating process.

While the solar atmosphere has been observed from ground in the visible/infrared frequencies down to coherent structures of a fraction of arcsec, the ultraviolet/X ray spectra require observations above the terrestrial atmosphere. The NASA Atmospheric Imaging Assembly (AIA) instrument on the Solar Dynamics Observatory (SDO), with a continuous imaging of the full Sun in extreme ultraviolet (EUV), ultraviolet and visible channels, incurred a resolution in spatial scales of <1.0 arcsec (11,15), while the High resolution Coronal Imager (Hi-C) reached a resolution of 0.2 arcsec (~150 km) at wavelength centred at 193A (16-17) indicating coronal plasma at a temperature of 1.5MK with transition lines of Fe XII. Both measurements revealed braided magnetic structures, with the appearance and decay of field lines without affecting the stability of the whole arched structure. High resolution observations at various frequencies are instrumental to an understanding of the intricacies of coronal magnetic field evolution.

Following the observations (16) and considering a small section of the coronal field line (i.e., ignoring the large scale field curvature), magnetic braid may be depicted as a disjoint collection of crossings among a number of strands attached to foot-points at two photospheric (parallel) planes (“top” and “bottom”), with one-to-one correspondence between the foot-points. Large number of these magnetic field braids can be envisioned as a small scale braid-like topology (18). The observed loops are undergoing constant modifications due to the dynamic motion of the underlying photosphere, as new loops light up, fade out or connect into more extended compact structures (11,15,19).

3. Magnetized plasma as a knot

Both mathematical knots and field lines of magnetized plasma in the magnetohydrodynamics (MHD) approximation are described as closed loops or bundles in three-dimensional (3D) space, transformed dynamically via continuous deformation of 3D upon itself, pushed smoothly in the surrounding
viscous (plasma) fluid, respectively, without self-intersection. Since these magnetic bundles of enhanced magnetic field usually form the most active sites for plasma processes, the dynamics of the magnetic configuration can be approximated as the evolution of finite width knots. The physical “frozen in” condition for magnetic knots is controlled by the three Reidemeister diagrammatic moves (e.g., 20), as shown on Figure 1: R1 - twist (green), R2 - poke (orange) and R3 - slide (yellow or blue). Trivial deformations like (wave) field line wiggling, which clearly preserve the topology, are denoted by R0. Rj moves reduce then the complicated topological problem to a simpler diagrammatic one, relating the changes in the observed projection to the relations between the crossings in the magnetic configuration. The invariance under Rj assures that any quantity which characterizes the magnetic field knot must preserve its value while undergoing the Rj transformations, becoming a topological invariant which assigns uniqueness to each knotted magnetic configuration. Helicity is one of these invariants. Non-equivalent magnetized knots are characterized by many additional invariants in the form of various polynomials (e.g., Alexander, Conway, Jones).

Figure 1. Reidemeister moves Rj, j=1,3

4. Braid Structure – Stable Coronal Field Oscillations

Mathematical (Artin) braid is a set of n disjoint strings in 3-space attached to two (horizontal) bars at the top and at the bottom (21). In many aspects they are similar to the solar magnetic braids observed in high resolution. Conventionally, the braids (strings) head downwards as one moves from the top to the bottom bar (footpoints), indicating the direction of the magnetic vector field, with a condition that each string never "turns back"; no string intersects any horizontal plane more than once. Pictorially, in this vertical configuration, the path of each string in a braid could be traced out by a falling object if acted upon only by gravity and horizontal forces (like ExB). Projecting the configuration on 2D and aligning the foot-points into a line results in a form shown in Figure 2 for two examples of simple braids; these forms have remarkable similarity to the remotely observed magnetic solar braids. Perturbations of the photospheric legs may wiggle the braids as Alfvénic-like oscillations without changing their topological characteristics, similarly to the knot R0, R2, R3 moves. In more general moves, magnetic or mathematical braid with the same number of strings may change its configuration to an equivalent braid if the strings of one can be continuously deformed (without intersections) into the strings of the other braid. Additionally, large successive braid reconfiguration due to interaction with new strings occurs when plasma processes heat adjacent magnetic fields lines.

Figure 2. Examples of mathematical braids.
The classification of braids is done in a projection diagram given by elementary crossings of strands, similarly to coronal observations. The i-th strand passing from above over the (i-1)-th strand defines the crossing generator $s_i$, while passing below it forms $s_i^{-1}$, as shown for a braid with $n=4$ strings at Figure 3. The examples include also two unperturbed, straight strings $I_2$. These mathematical generators constitute the basis of the physical magnetic braid structure and its oscillations.

![Braid Generators]

**Figure 3.** Elementary generators of braid oscillations for 4 strings.

Following this description, the braid can be characterized by its successive crossing generators; moving from the top down the braids of Figure 2 can be described as $s_1s_2^{-1}$ and $s_2^{-1}s_2^{-1}$, respectively. More generally, any (magnetic or mathematical) braid can be formulated as a combination of a set of elementary generators $s_i, s_i^{-1}$, forming a braid word. These words translate the braiding images into an algebraic entity, and each mathematical braid is characterized by its own word. Similarly, each magnetic braid in high resolution can be classified by its crossings, forming magnetic braid word which undergoes temporal modifications due to the motion of its strands. The present resolution of coronal braids is able to observe these structures and their oscillations imprecisely; an improved resolution will be able to assign to each braid fully its magnetic braid word, allowing to validate the braid operations in physical environment.

The connection between the mathematical and the magnetic braids facilitates to conjecture the conditions for coronal field moves, which form equivalent braids. The equivalency of two braids indicate stability of the magnetic configuration which could be seen by an observer. These moves can be then translated into constrains on its generators to satisfy the following relations:

\[ a) \quad s_1s_1^{-1} = 1. \]

![Braid Relations]

**Figure 4a.** Two adjacent magnetic strands affected by a local ExB or large Alfvénic force.

This relation describes a local braid motion resembling the $R_2$ knot move, when two strands interchange positions (Figure 4a) at some segment of the braid. In the observed solar braids this may happen when part of one (or both) of adjacent strand(s) is (are) subjected to (different) local cross ExB motion. The resulting braid word acquires $s_1s_1^{-1}$ at the observed level, while the braid preserves its characteristics; Figure 4a shows its equivalency for $n=4$.

\[ b) \quad s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}. \]

This relation resembles the $R_3$ knot move when one strand moves with respect to a pair of crossed strands, e.g., the middle one in Figure 4b. In the corona it may happen when an active magnetic strand is anchored at different foot-point of its neighbours, and its perpendicular motion is independent of the motions of other strands.
Figure 4b. Braid equivalency under separate motion of a strand vs two intertwined strands \((i=1, j=2)\).

\[ s_1 s_2 s_1 = s_2 s_1 s_2 \]

c) \[ s_i s_j = s_j s_i \quad (|i-j| > 1) \]

This braid relation occurs when motion along one part of the braid does not affect the other on a non-adjacent field line. This can happen at solar corona when a small scale (e.g., Alfvénic) pulse is applied only to a segment of the magnetic field braid (Fig. 4c), forming local commutative operation.

Figure 4c. Pulse along a segment of the braid, \((i=1, j=3)\).

Clearly, the above moves which could become observable in very fine resolution, can be repeated numerous times. They preserve the topological structure of the braids, hence their successive observation at the corona is consistent with the stable magnetic braid activity. Therefore, time dependent braid oscillations due to photospheric footpoint moves, Alfvénic pulses or local ExB forces become natural eigenmodes of the magnetic braid. In mathematical terms these conditions, with the existence of the generators \(s_i\) and \(s_i^{-1}\) and unit strings \(I_k\) (k straight lines) create the (Artin) braid group \(B_n\) of \(n\) strings: the composite field lines of magnetic braids satisfy associativity – do not change when split into segments in different orders.

The changes in brightness with an emergence or decay of stable solar braids indicate that the system is dynamically active with significant structural braid modifications. The local repeated braid moves of the Artin group form the stable modes which should be observed as basic solar oscillations at fine resolution. At times the emergence of new braids and major modifications in the braid configuration well beyond the Artin group still may preserve its basic topological structure and stability, while other modifications may indicate global, unstable magnetic reconfigurations. The mathematical braids can shed light on these aspects of magnetic braid evolution.

5. Braid Closure - Large Stable Coronal Field Modifications

Both braids and knots and their physical analogues at the solar corona and in the interplanetary medium form entities which may be related topologically. An important relation between braids and knots/links was given through the Alexander (22) theorem, stating that any knot may be obtained from a closure of some braid, joining orderly through simple arcs the endpoints at the upper loose ends of a braid with the lower ones. Conversely, there exist many braids which through closure become the same knot. There exists simple physical interpretation of this theorem: since the magnetic braid consists of a bundle of coronal magnetic field lines anchored at the photospheric foot points, these field lines must be connected through the (turbulent) convection region such that the closure has physical validity, irrespectively of the details of the current system in the photosphere or convective region. Therefore, each magnetic braid is related to some knot or link – two or more intertwined knots. Figure 5 shows the closure of the Borromean ring braid into Borromean ring knot (5a) and through regular planar \(R_o\) isotopy movements, into its standard form of three mutually intertwined circles (5b). Hence, if a closure of two different braids results in the same knot, magnetic deformation leading from one braid to another describes magnetic reconstructions preserving the knot invariant. Since knot
invariant is a robust characteristic quantity these deformations may be used as a stability criterion for significant modifications of magnetic braids, forming a broad range of stable coronal deformations.

Figure 5. (a) Closure of a braid into a Borromean ring. (b) Final form of Borromean ring – three mutually intertwined circles.

The closure of a braid into a knot is not a one-on-one transformation. The convergence of multiple (infinite) braids into the same knot is based on Markov theorem (23), stating that two braids under closure become the same knot if they are related by a sequence of Markov moves (below). Physically it indicates that a large number of magnetic braids may undergo significant modifications, changing their braid word well beyond the above group Bn moves and still converge by closure to the same knot. This redundancy offers an important observational criterion, indicating that large changes in the structure of the magnetic braid, with an excitation (or fading) of magnetic strands, including nanoflare/microflare disruptions, are constrained by the preservation of knot invariant, keeping the whole active field structurally stable. Therefore, long term coronal heating may modify significantly the active coronal fields while keeping them stable over very long times, as often observed. Conversely, modification of a braid into another one with a different knot closure violates the knot invariants, indicating configuration instability, which may occur over a very short time scale.

The two Markov moves include:

1) Conjugation: combination of a braid with two complementary mirror parts of another braid. Dynamic fluctuations of a braid with straight magnetic lines (i.e., unit n-braid I) may undergo eigenmode oscillations with a series of generators $s_i, s_i^{-1}$. When reconfiguration process splits this braid into its mirror components $B$ and $B^{-1}$ (with $I=BB^{-1}$) which later join another braid $A$ on both sides, one obtains a new equivalent braid $C=BAB^{-1}$; this braid (Figure 6a) is related to the same knot as $A$ at braid closure. Hence, sudden observation of an elongated solar magnetic braid at both ends by symmetric mirror pieces of another braid indicates a major stable braid reconfiguration which preserves the whole magnetic structure. This operation may be repeated numerous times, indicating that significant observable modifications in magnetic braids may preserve the stable configuration.

Figure 6a. (First) Markov conjugation move.

2) Stabilization: addition of a new strand with crossing. When a new, single active magnetic field line reconnects with n-th string of a braid $A$ adding a new $s_{n+1}$ crossing string, the resulting knot of $n+1$
strings formed under closure is isotopic to the closure of A – the new braid converges to the same knot under a closure (Figure 6b). The heating of magnetic field lines and addition of string generators \( s \) to \( A \) may be repeated many times, forming new braids which do not change the equivalent knot structure. Similar result applies to reconfiguration adding multiple times the generator \((s_{n+1})^4\).

![Figure 6b](attachment:image.png) (Second) Markov stabilization move.

As a corollary of the Markov moves, a sequence of the above operations which modify significantly the structure of magnetic braid due to coronal heating and emergence of new active field lines, converge after braid closure into an identical knot preserving the stability of the magnetic structure. In contrast, those large braid moves which do not satisfy these conditions are candidates for the dissolution of the magnetic configuration.

6. Discussion

Fluid motion in the solar convection region results through dynamo processes in an emergence of magnetic fields in various geometrical forms. The measured fields rely on remote observations of photons which are emitted by the heated plasma in the presence of magnetic fields. These observations depict the magnetic fields as simply connected arched lines and intermittently as magnetic braids. The braids preserve their topology while being carried later by the Solar Wind into the heliosphere in the form of magnetic knots. Temporal changes in the observed braid structure are due to the motion of field line foot-points, local perturbations or formation of newly heated magnetic strands. The present work promotes the idea of correlating high resolution solar observations of small scale coronal magnetic field geometry with the topological properties of mathematical braids and knots. This analogy allows us to extend the realm of magnetized plasma at the Sun and in the interplanetary medium into new, not often explored magnetic structures. Following the mathematical axioms one may impose stability constrains on the dynamics of heliospheric fields and offer predictions of topological configurations in soon to be realized measurements at the inner heliosphere when the Parker Solar Probe satellite, equipped with modern instrumentation, will reach the unexplored regime of the solar system well beyond the region which was intermittently traversed 40 years ago by the Helios satellites. The interconnection between the magnetic and mathematical braids/knots is performed partly as a gedanken experiment in which mathematical properties are transcribed into the anticipated physical measurements.

The formulation of the Artin \((21)\) braid group as a set of disjoint strings with crossings attached to two bars can be uplifted directly to magnetic braids, when the basic coronal braid oscillations transform one braid into an equivalent one. The notion of a (mathematical) group allows operations among its members which keep the braid properties intact and by analogy to magnetic braids, indicate various observable coronal oscillations. Local \(\mathbf{E}\times\mathbf{B}\) drift of magnetic strand looping around another and returning to its previous site forms two crossings resulting in observable small scale oscillations at solar corona. Similarly, local wandering of a strand with respect to two other strands as well as independent Alfvenic pulses of two non-adjacent crossings along their field lines are part of the group definition, and as such become additional observable features. These moves form the basic
eigenmodes which could be measured in high resolution as braid oscillations. Hence, the existence of a mathematical group which transforms one braid to another under its operations can be translated into detectable physical features in coronal excitations.

Magnetic braid stability under large perturbations can be partly addressed through its equivalent conversion into a knot, following its closure according to Alexander theorem (22). Each (magnetic) knot possesses its unique topological invariants which are dynamically preserved. All braids undergoing successive Markov moves are related to the same knot, indicating stability of the braid under these large distortions. These modifications may include major reconfigurations of the magnetic field with cutting and pasting of field lines due to resistivity or plasma kinetic interactions.

Therefore, magnetic reconfiguration through attaching a braid and its mirror to another braid at both ends, respectively, (Markov move I), or successive crossing of newly heated strands with an existing braid (Markov move II) are sources of significant perturbations which preserve the relevant closure knot. Hence, these operations, which can be repeated numerous times, qualify as a wide range of stable, large braid perturbations, observable at high resolution. Heating and nano-flare events fall under this category of magnetic reconfigurations. Alternatively, large magnetic braid modifications which alter (after closure) the knot invariants, form observable conditions of coronal destabilization.

The evolution of the coronal field is envisioned as follows: magnetic field emerges from the photosphere either as an arched non-intersecting strand (trivial braid) or as a braid with multiple crossings. The small-scale braid crossings are supported by current sheets while their oscillations are due to photospheric input or interaction with the hot coronal plasma currents. Any braid is equivalent to some knot, hence magnetic field observations include bundles of unknots (distorted loops) or prime knots – knots which as a result of any cut in a strand disintegrate into an unknot. The stable modifications which are included in the Artin equivalency group (21) or in Markov moves (23) may modify significantly the braids but preserve their knot invariants. If braid modification violates the knot invariants, the magnetic structure may become unstable and cause eruption observed as energy release through X-ray brightening on a small scale or CME on a large scale. Non-erupting knotted configurations may be dragged by the solar wind resulting in very elongated geometrical forms in the interplanetary medium, which at some distance from the Sun may become structurally unstable. Therefore, predictions for the observation of knotted structures is balanced by their formation probability (lower crossing number) vs probability of survival (larger crossing number). A satellite encountering such a structure will measure intermittently different plasmas, especially when the knotted field lines are connected to active coronal region. Due to the geometrical stretching the probability to encounter a topologically non-trivial magnetic structures decreases with the increasing solar distance, hence it is expected that the incursion of Parker Solar Probe into a region inside Mercury orbit and in the vicinity of solar corona will increase the probability of encountering a variety of knotted features in magnetic field. It is believed that the proposed scenario can be extended to magnetospheric plasmas with observations on Van Allen, Cluster and Themis satellites.

7. References.

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