Impact of the evanescent waves on the backflow of power in the near field

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Abstract. For an elliptically polarized optical vortex with an arbitrary integer topological charge, using the expressions for all six components of the electric and magnetic field strength vectors, we obtain an expression for the longitudinal component of the Poynting vector in the initial plane. For the particular case of a narrow angular spectrum of plane waves (Bessel beam) and for the circular polarization, it is shown that in the presence of the inhomogeneous evanescent waves in the initial light field, a reverse flux of light energy can occur near the optical axis. It is shown that this reverse energy flux is due to toroidal vortices in the longitudinal plane.

1. Introduction
In optics, optical "tractor" beams are actively studied [1-4], which allow attracting microscopic objects toward the light source. At the same time, another interesting optical effect is being investigated - reverse propagation [5] or reverse energy flow [6, 7]. This effect occurs, for instance, in the focus of a plane wave [8, 9] or of a bounded paraxial [10] and unbounded nonparaxial [11] scalar Gaussian beams, as well as in the focus of nonparaxial Laguerre-Gauss beams [12]. It also appears in foci of nonparaxial vector beams with linear [13] or circular polarizations, in various vector mode beams (Bessel [14, 15], Airy [16], X-waves [6]), at focusing onto the interface between media [17], or near nanostructured surfaces [18, 19]. In this paper, in contrast to [8], which uses the Richards-Wolf formalism, and in contrast to [6, 13–16], which uses the well-known exact solutions of Maxwell's equations (X-waves, Bessel beams, Airy, quasi-Gaussian [5], vectorial nonparaxial Gaussian beams), we obtain general expressions for the components of the Poynting vector for arbitrary elliptically polarized vortex electromagnetic field by using the plane waves expansion. We obtain a condition, when a reverse energy flow occurs in the initial plane of the beam (in its waist). It follows from this condition that the reverse energy flux can occur only in the presence of inhomogeneous evanescent waves with large transverse components of the wave vector in the waist plane. Moreover, the greater the contribution of evanescent waves to the total amplitude of the beam, the greater the magnitude of the reverse energy flux, which in this case is comparable in magnitude with the direct flux. We also shown numerically that the reverse flow occurs because of toroidal vortices in the longitudinal plane.
2. Distributions of intensity and of the longitudinal component of the Poynting vector

It is easy to show that the following light field is a solution of the Maxwell's equations:

\[ E_x(r, \varphi, z) = i^{n+1} e^{i\sigma \varphi} I_{1,n+1} \]
\[ E_y(r, \varphi, z) = -i^{n-1} \sigma e^{i\varphi} I_{1,n-1} \]
\[ E_z(r, \varphi, z) = i^n [\gamma_n e^{i(n+1)\varphi} I_{2,n+1} - \gamma_n e^{i(n-1)\varphi} I_{2,n-1}] \]
\[ H_x(r, \varphi, z) = \frac{1}{2} i^n [\gamma_n e^{i(n+1)\varphi} \bar{T}_{2,n+1} - \gamma_n e^{i(n-1)\varphi} \bar{T}_{2,n-1} + \sigma e^{i\varphi} (2I_{1,n} - \bar{T}_{2,n})] \]
\[ H_y(r, \varphi, z) = \frac{1}{2} i^{n+1} [\gamma_n e^{i(n+1)\varphi} \bar{T}_{2,n+1} + \gamma_n e^{i(n-1)\varphi} \bar{T}_{2,n-1} - e^{i\varphi} (2I_{1,n} - \bar{T}_{2,n})] \]
\[ H_z(r, \varphi, z) = -i^{n+1} [\gamma_n e^{i(n+1)\varphi} I_{4,n+1} + \gamma_n e^{i(n-1)\varphi} I_{4,n-1}] \]

where \( \sigma = \pm 1 \) is the spin index determining left or right circular polarization (\( \sigma = +1 \) for right polarization and \( \sigma = -1 \) for left polarization, whereas for other values of \( \sigma \) Eqs. (1)-(5) describe an elliptically polarized field), \( \gamma_n = (1 \pm \sigma)/2 \),

\[ I_{1,n} = \int_0^\infty A_n(\rho) e^{i\sqrt{\rho^2 - \rho^2}} J_n(k \rho \rho) \rho d \rho \]  
\[ I_{3,n} = \int_0^\infty \sqrt{1 - \rho^2} A_n(\rho) e^{i\sqrt{\rho^2 - \rho^2}} J_n(k \rho \rho) \rho d \rho \]  
\[ I_{2,n} = \int_0^\infty A_n(\rho) e^{i\sqrt{\rho^2 - \rho^2}} J_n(k \rho \rho) \rho^2 d \rho \]  
\[ \bar{T}_{2,n} = \int_0^\infty \rho^2 A_n(\rho) e^{i\sqrt{\rho^2 - \rho^2}} J_n(k \rho \rho) \rho d \rho \]  
\[ I_{4,n} = \int_0^\infty \sqrt{1 - \rho^2} A_n(\rho) e^{i\sqrt{\rho^2 - \rho^2}} J_n(k \rho \rho) \rho^2 d \rho \]

where \( n \) is the topological charge of the optical vortex, \( A_n(\rho) \) is an arbitrary generally complex-valued function of the amplitude of the plane waves spectrum, \( J_n(x) \) is the \( n \)th-order Bessel function of the first kind.

Using the transverse components of the electric (1) and magnetic (3), (4) vectors of the light wave, an expression can be derived for the longitudinal component of the Poynting vector (energy flow), according to the formula [15] \( S = c \text{Re}[\mathbf{E} \times \mathbf{H}] / (8\pi) \):

\[ S_z = \frac{c}{8\pi} \text{Re} \left\{ E_x H_y - E_y H_x \right\} \]

where \( \text{Re} \) is the real part of a complex number. Substituting Eqs. (1), (3), and (4) into Eq. (10), for circular polarization (\( \sigma = \pm 1 \)) we get (omitting the constant \( c/(8\pi) \)):

\[ S_z = \text{Re} \left\{ I_{3,n} \left( 2I_{1,n} - \bar{T}_{2,n} \right) \right\} \]

3. Reverse energy flux in the initial plane

From Eq. (11) follows that the longitudinal energy flow is rotationally symmetric and is the same for left and right circular polarization, in contrast to the intensity distribution.
Let, for simplicity, the amplitude of plane waves spectrum is described by the real function $A_n(\rho)$. Then, substituting Eqs. (6)-(9) into Eq. (11), instead of Eq. (11), we obtain for $z=0$:

$$S_\epsilon = \left[ \sqrt{1-\rho^2} A_n(\rho) J_n(k\rho) \rho d\rho \right] \left[ \sqrt{2-\rho^2} A_n(\rho) J_n(k\rho) \rho d\rho \right].$$

(12)

For simplicity, we consider a hypothetical amplitude function of the plane waves spectrum as a linear combination of two Dirac delta functions. Physically, this means that some portion of light field energy is concentrated in a narrow region of the spectrum of propagating plane waves, whereas the other portion is in the region of evanescent waves:

$$A_n(\rho) = A_\sigma(\rho - \rho_1) + B_\sigma(\rho - \rho_2),$$

(13)

where $A > 0, B > 0, 0 < \rho_1 < 1, 1 < \rho_2 < \infty$.

Then, instead of Eq. (12), we get:

$$S_\epsilon = A\sqrt{1-\rho_1^2} J_n(k\rho_1) \rho_1 \left[ A(2-\rho_1^2) J_n(k\rho_1) \rho_1 - B(\rho_2^2-2) J_n(k\rho_2) \rho_2 \right].$$

(14)

We note that the Bessel beam of any order is a solution of the nonparaxial Helmholtz equation for any scale ($k\rho_1 < 1$ is for propagating Bessel beams, and $k\rho_2 > 1$ is for evanescent Bessel beams). Evanescent Bessel beams are used to overcome the diffraction limit in the near field [20, 21]. Near the optical axis ($kr_1 < kr_2 << 1$), we leave in the Taylor expansion of the Bessel function only the first term and instead of Eq. (14) we derive:

$$S_\epsilon \approx \left[ A(k^2/2^{n+1}) \rho_1^{\nu+1} - B(\rho_2^2-2) \rho_2^{\nu+1} \right] < 0.$$  

(15)

From Eq. (15) follows that $S_\epsilon < 0$ when

$$\frac{A}{B} < \left( \frac{\rho_2}{\rho_1} \right)^{\nu+1} \left( \frac{\rho_1^2-2}{2-\rho_1^2} \right),$$

(16)

i.e. near the optical axis in the initial plane the energy flow is reverse. The condition (16) is possible only if $\rho_2 > 2^{1/2}$, i.e. the evanescent waves should have large decay ratio. It is seen in Eq. (15) that at $n=0$ the maximal reverse flux is on the optical axis.

4. Numerical simulation

Now we consider a light field with the angular spectrum (13). This field is a superposition of two Bessel modes – one propagating and one evanescent:

$$E_l(x, y, z) = e^{i\sigma} \left[ A e^{i\nu\sqrt{1-\rho^2}} J_n(k\rho) + i Be^{i\nu\sqrt{2-\rho^2}} J_n(k\rho) \right].$$

(17)

According to Eq. (16), if

$$\frac{A}{B} < \left( \frac{\rho_2}{\rho_1} \right)^{\nu+1} \left( \frac{\rho_1^2-2}{2-\rho_1^2} \right) \left( \frac{1-\rho_1^2}{\rho_2^2-1} \right),$$

(18)

then near the optical axis in the initial plane a reverse energy flow appears. Conditions (18) can be achieved by increasing the contribution $B$ of the evanescent waves.

We chose the following parameters for numerical simulation: wavelength $\lambda = 532$ nm, $\rho_1 = 0.8, \rho_2 = 2.4, A = 1$, topological charge $n = 3$, polarization – left circular ($\sigma = -1$) and right circular ($\sigma = +1$), calculation area $z=0, -R \leq x, y \leq R (R = 2.5\lambda)$. According to Eq. (18) for the reverse energy flow it is
necessary that $A/B < 20.53$. We chose $B = 1$ and $B = 10$. Figure 1 shows distributions of the intensity and of the power flow of a superposition of the Bessel modes (17) in the initial plane for the different contribution of the evanescent mode $B = 1$ [Fig. 1(a,b)] and $B = 10$ [Fig. 1(c,d)], as well as for the different handedness of circular polarization.

Figure 1. 2D intensity distributions combined with 1D distributions of power flow of superpositions of two Bessel modes (17) in the initial plane ($z = 0$) for different contribution of the evanescent mode $B = 1$ (a, b) and $B = 10$ (c, d), for left (a, c) and right circular polarization (b, d).

According to Fig. 1, increasing contribution of evanescent waves leads to increasing power of the negative light flux. For example, at $B = 1$, the maximal (in modulus) power of the negative flow is 7.4% of the maximal power of the positive flow, while at $B = 10$, this ratio is 75%. It can also be seen that the negative flux occurs in the first light ring, since the ring near the evanescent Bessel mode has a smaller diameter.

Next, we consider generation of the negative flow for a vortex-free field ($n = 0$). Figure 2 shows the intensity distribution and power flow of the superposition of Bessel modes (17) in the initial plane. The contribution of the evanescent mode is $B = 10$, polarization is left circular, the remaining parameters are the same as in Fig. 1.

Figure 2. 2D intensity distribution combined with 1D distribution of power flow of a superposition of two Bessel modes (17) in the initial plane ($z = 0$).

According to Fig. 2, in this case the maximal reverse flow occurs in the centre of the diffraction pattern.

It is rather difficult to find and register the reverse energy flux experimentally, since it occurs in the region of light rings in the intensity distribution (Fig. 1). Therefore, to detect the reverse flow, it is necessary to simultaneously measure the intensity distribution in the focal plane and the distribution of the modulus of the energy flow. If on the same radius in the focal plane the intensity is nonzero (bright ring) while the modulus of the Poynting vector is zero (dark ring), then in this place the energy propagates in the reverse direction. Note that in the reverse propagation of the considered vortex, the patterns of intensity and energy flow similar to Fig. 1 occur in the focal plane.

5. Toroidal vortices

Figure 3(a) shows the distribution of intensity combined with the distribution of power flow from Fig. 1(d). Figure 3(b) shows a zoomed fragment from Fig. 3(a) (shown in red dotted frame) and Fig. 3(c) shows distribution of the Poynting vector in the longitudinal plane $xz$ (in the area $0 \leq x \leq 1$, $0 \leq z \leq 0.1\lambda$). It is seen in Fig. 3 that the areas with positive and negative power flux in the plane $z = 0$ are due to the toroidal power flux in the plane $xz$. 
6. Conclusion
Based on the expressions for six components of the electric and magnetic field strength vectors for a circularly polarized optical vortex with an arbitrary topological charge and with an arbitrary rotationally symmetric real amplitude function of the plane-wave spectrum, an expression is obtained for the longitudinal component of the Poynting vector in the initial plane. In the particular case of a narrow angular spectrum of plane waves (Bessel beam), it was shown that, in the presence of inhomogeneous evanescent waves in the initial light field, a reverse flow of light energy can occur on the optical axis. It is shown numerically that this reverse flow occurs because of toroidal vortices in the longitudinal plane. This reverse flow can be used to “pull” the microparticle into the centre of the annular waist of the vortex beam, i.e. to demonstrate the “optical tractor” effect.

7. References
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