Improving Metrological Reliability of Information-Measuring Systems Using Mathematical Modeling of Their Metrological Characteristics

R Yu Kurnosov, T I Chernyshova, V N Chernyshov
Tambov State Technical University, 106, Sovetskaya St., Tambov, 392000, Russia
E-mail: romankurnosov@rambler.ru, energo@nnn.tstu.ru

Abstract: The algorithms for improving the metrological reliability of analogue blocks of measuring channels and information-measuring systems are developed. The proposed algorithms ensure the optimum values of their metrological reliability indices for a given analogue circuit block solution.

1. Introduction
One of the most important characteristics of the quality of measuring equipment (ME), including information-measuring systems (IMSs), is metrological reliability (MR) considered as the ability of the measuring equipment to keep metrological characteristics (MCs) in time within the established norms for specified operating modes of maintenance, storage and transportation. As is known [1, 2], the most important IMS blocks, which determine the MR of the measuring equipment in general, are analogue blocks that perform various transformations of the input signal. Therefore, the metrological reliability of the measuring equipment is determined mainly by the metrological reliability of the analogue blocks, which make up the measuring channel of the measuring equipment under consideration. The MR indices of the IMS analogue blocks are metrological resource \( t_r \), determined by the intersection time of the realizations of the nonstationary random process of the MC time variation limits and the probability of maintaining metrological accuracy \( P_{\text{acc}}(t) \) of the IMS analogue blocks at arbitrary times of operation. When designing the IMS, the problems of increasing the MR indices are often set. The most expedient way to increase the MR during the design stage of electronic measuring systems, including IMSs, is to increase MR indices by replacing unreliable analogue block elements with the elements having higher metrological stability indices to achieve optimal values for both metrological resource and probability of maintaining metrological accuracy at any time during the forthcoming operation.

2. Methods
The problem of increasing MR can be considered as the optimization problem of one of the MR indices, solved by varying the parameters of the analogue block elements. At the same time, optimization of parameters aims to ensure the best quality. In addition, when solving such problems, it is necessary to take into account the preservation of the functional purpose of the designed analogue block, i.e., it is necessary to meet the requirements for the output characteristics of analogue blocks and the IMS in general. To carry out the optimization, one needs a mathematical model of the
analogue block, the target function and the optimization algorithm (Figure 1). The objective function formalizes the requirements imposed on the analogue block. In the problem under consideration, this is an increase in the MR indices of the designed IMS. The target functions are the MR indices, i.e. \( t_p \) and \( P_{\text{acc}}(t) \). At the same time, the solution of the optimization problem can be considered when \( t_p \), and \( P_{\text{acc}}(t) \) reach the maximum at a certain point in time of the forthcoming operation.

The optimization algorithm seeks the extremum of the objective function. Figure 1 shows the structure of the interaction of the named components, which are necessary for solving the problem of increasing the MR of the IMS.

![Figure 1. Block diagram of optimization process](image)

\( \xi = \{\xi_1, \xi_2, ..., \xi_n\} \) is parameter vector of block elements; \( \xi_{\text{init}} \) is initial values of element parameters; \( \xi_{\text{opt}} \) are optimal values of element parameters.

The problem of improving the metrological resource of the IMS at the design stage is formulated as follows: it is necessary to select the element base so that for this circuit solution, the metrological resource has reached the maximum value for the designed analogue block of the IMS measuring channel, provided it retains its functional purpose. The specified problem can be written in the form:

\[
\begin{align*}
 t_p^* &= \max \left\{ \min_{i=1,...,p} \left\{ S_i(t, \varphi) \right\} \right\} \text{при } S(t, \varphi) = \text{var}, \varphi \in \Phi, y(t, \varphi) \in A,
\end{align*}
\]

(1)

where \( t_p^* \) is the metrological resource of the IMS in general; \( t_{pi} \) is the value of the metrological resource of the \( i \)-th analogue block; \( i = 1,...,p \) is the number of analogue blocks in the measuring channel; \( t \) is time; \( S \) is a combination of metrological of the IMS; \( \varphi \) is vector of external disturbances; \( \Phi \) is the region of external disturbances; \( y(t, \varphi) \) is a combination of output characteristics of the IMS; \( A \) is the operability region.

The algorithm for improving the metrological reliability of the IMS analogue blocks using the criterion of the maximum probability of maintaining metrological accuracy by selecting the nominal values of elements that ensure the maximum value of the selected metrological reliability of the IMS analogue blocks has the form:

\[
\begin{align*}
 P_{\text{acc}}(t) &= P\{0 \leq S(t) \leq S_{\text{per}}\} = \max \text{ if } S(t, \varphi) = \text{var}, \text{\varphi} \in \Phi, y(t, \varphi) \in A, \quad (2)
\end{align*}
\]

where \( P_{\text{acc}}(t) \) is probability of maintaining metrological accuracy of analogue blocks; \( S_{\text{per}} \) is permissible value of metrological characteristic of analogue blocks; \( t \) is time; \( y(t, \varphi) \) is output.
characteristic of IMS analogue blocks; \( \vec{\phi} \) is vector of external disturbances; \( \Phi \) is the region of external disturbances.

The value of the MR index considered in the problem (2) for the IMS as a whole is determined by the expression:

\[
P_{\text{acc}}(t) = \min \left\{ P_{\text{acc}_i}(t) \right\}_{i=1,\ldots,p},
\]

where \( P_{\text{acc}_i}(t) \) is the probability of maintaining metrological accuracy of the \( i \)-th analogue block; \( i = 1,\ldots,p \) is the number of analogue blocks in the measuring channel.

Obviously, solutions of problems (1) and (2) are preceded by the determination of the maximum values of the corresponding MR indices for each of the analogue blocks in the measuring channel. The accuracy and reliability of the results obtained in this case is determined by the adequacy of the applied mathematical models constructed at the design stage of the analogue block.

The main stages of the developed algorithms improving the MR are as follows.

1. A database for blocks constituting the measuring channels of the IMS is created; it includes structural diagram and electrical circuit, the element base parameters and technical characteristics determining the functional purpose of the blocks constituting the measuring channels.

2. Based on the analysis of the structural diagram and electrical circuit for each analog block included in the measuring channel of the IMS, a mathematical model of its metrological characteristics is constructed:

\[
S = F(x, \vec{\xi}),
\]

characterizing the dependence of the values of metrological characteristic \( S \) normalized for the investigated block on the value of input signal \( x \) and the parameters of element base \( \vec{\xi} = \{\xi_1, \xi_2, \ldots, \xi_n\} \).

3. Statistical modeling of the state of metrological characteristics of the blocks under investigation is performed, which consists in sequential modeling given the aging process of the parameters of the circuit elements in each time section \( t_j \) of control regions \( T_1, t_j \in T_1, j = 0, 1, \ldots, k \) and modeling of the realizations of the metrological characteristics of the blocks in different time sections \( S(t_j), j = 0, 1, \ldots, k \). In this case, the well-known assumption of the normal law of distribution of element parameters.

4. Using the interpolation methods and the values of the parameters for the distribution law of metrological characteristics \( m_s(t_j) \) and \( \sigma_s(t_j), j = 0, \ldots, k \) in control area \( T_1 \), a mathematical model of the process of metrological characteristic variation over time for each block is constructed.

As detailed in [3.5], a mathematical model of the metrological characteristic variation in time is a set of analytical dependencies obtained for the time variation function of the expected value of metrological characteristic \( M_s(t) \) and the functions characterizing the time variation in the limits of variation of metrological characteristic possible values from its expected value defined by the expression:

\[
\Psi_{\pm}(t) = m_s(t) \pm c\sigma_s(t),
\]

where \( c \) is a constant coefficient selected depending on the given level of confidence probability \( P \) and the law of distribution of metrological characteristic \( S \) (in practice \( c=3 \) if confidence level \( P=0.997 \) and the normal distribution law of metrological characteristic); \( \sigma_s(t) \) is the value of the standard variation of metrological characteristic.
5. The MR indices are determined by extrapolating dependencies $M_S(t)$ and $\psi_{sg}(t)$, making a mathematical model of the time variation of the metrological characteristic under study for the area of future operation.

6. In case of non-compliance of values of MR indices with established requirements, one of the algorithms for increasing the MR for indices $t_p$ and $P_{acc}(t)$ in accordance with (1) or (2) is used.

To increase the MR, in the mathematical model of the analogue block one selects the elements, the increase or decrease in the time parameters of which causes the maximum variation in the values of the metrological characteristic. The selection of these elements is carried out according to the value of the normalized partial derivative of the form:

$$G(\xi_i) = \frac{\overline{G}(\xi_i)\sigma_{\xi_i}}{\sqrt{\sum_i \overline{G}^2(\xi_i)\sigma^2_{\xi_i}}} \quad i = 1, \ldots, n,$$

where $\overline{G}(\xi_i) = \left| \frac{\partial S}{\partial \xi_i} \right|$ are the values of the partial derivatives calculated from the nominal values of the parameters of the $i$-th element of the block; $\xi_i$ are parameters of the block elements; $\sigma_{\xi_i}$ is rms deviation of the parameter of the $i$-th element of the block, $i=1,\ldots,n$ is the number of analogue block element parameters.

Then, the elements are ranked according to absolute value $G(\xi_i)$. The elements having maximum value $G(\xi_i)$, are selected to search for the optimal value of the selected optimization criterion, provided that the analogue block circuit operability is maintained.

7. Further, a procedure to find the optimum values of the parameters of the selected elements ensuring the solution of the stated optimization problems in accordance with conditions (1), (2) is performed. The implementation of the developed algorithms is carried out using the basic provisions of the method of configurations, which involves performing the procedure of trial search and working search until the maximum values of the optimized MR indices are obtained. The structure of the configuration method is presented in [4].

8. The procedure for finding optimal values for the parameters of the element base involves the implementation of mathematical modeling of the metrological characteristic and the calculation of the MR indices at each step of the search.

It should be noted that the operation of the designed analogue block is tested at each stage after the replacement of the corresponding elements.

The described process is carried out until the maximum values of the MR indices of each designed analogue block of the measuring channel and the IMS as a whole are reached.

3. Results

The considered algorithms were implemented using the example of the analog-pulse converter (APC) block, which is a part of the measuring channel of the information measuring systems for the object quality control. The APC block circuit is shown in Figure 2.
The investigated metrological characteristic which makes it possible to evaluate the metrological reliability of the APC block and the IIS as a whole is the basic relative error. The initial data for parametric optimization of APC are the nominal values of the circuit element parameters, data on the aging of the parameters of the APC element base, the mathematical model of the basic relative error of the APC, as well as the values of the time sections in which the statistical modeling of the basic relative error of the APC is performed. The performed procedure of mathematical modeling of this block showed that the probability of metrological accuracy maintenance is $P_{\text{acc}}(t) = 0.77$ at $t = 2700$ h, and the value of metrological resource index is $t_p = 38550$ h, with confidence probability $P=0.997$. The solution of the problem of increasing $P_{\text{acc}}(t)$ and $t_p$ by the selected method showed the need to replace the elements. The choice of the elements to be replaced was carried out by the value of the normalized partial derivative of the form (6). The calculation showed that the nominal values of the two elements of the circuit $R_2=5.6$ kΩ and $C_1=0.022$ µF were replaced. Since the value of resistor $R_2$ is standard, it should be replaced with two series-connected resistors with nominal values of 4.7 kΩ and 1 kΩ, respectively. Repeated mathematical modeling of APC and estimation of MR showed that the value of a metrological resource with confidence probability $P=0.997$ would be 42420 hours, the probability of maintaining metrological accuracy is $P_{\text{acc}}(t) = 0.87$ at $t = 2700$ h.

Table 1 presents summary data on the implementation of algorithms for increasing metrological reliability indices for the analogue block under study.

**Table 1. Results of improving metrological reliability**

| Block type | MR indices by the criterion of maximum MR | Increasing MR by the criterion of maximum $P_{\text{acc}}(t)$ |
|------------|-----------------------------------------|-----------------------------------------------------------|
|            | Values of before optimization, h | Values of MR after optimization, h | Values of $P_{\text{acc}}(t)$ before optimization | Values of $P_{\text{acc}}(t)$ after optimization |
| APC        | 38550 | 42000 | 0.77 | 0.87 |

$t = 2700$ h
The values of MR indices in Table 1 show that the application of the developed algorithms based on the use of mathematical models of the MC of the investigated analogue block makes it possible to increase the MR indices for the block under investigation at the design stage by at least 9%.

4. Conclusion

Thus, algorithms for increasing MR of the IMS analogue block based on the mathematical modeling of the MC of the blocks under investigation have been developed and experimentally tested during their design. The algorithms consist in determining the optimal values of the parameters of the IMS analogue block elemental base, which ensure the maximum value of MR and the probability of maintaining metrological accuracy.

References

[1] Chernyshova T I, Chernyshov V N 2016 Metody i informacionno-izmeritel’nye sistemy nerazrashajushchego kontrolja teplofizicheskih svojstv materialov i izdelij. Nauchnoe izdanie: monografija (SPb: Jekspertnye reshenija)
[2] Pudovkin A P, Danilov S N and Panasjuk Ju N 2014 Perspektivnye metody obrabot-ki informacii v radiotehnicheskikh sistemah (Sankt-Peterburg: Jekspertnye reshenija)
[3] Cvetkov Je. I. 2014 Metrologija. Modeli ob#ektov, procedur i sredstv izmerenij. Metrologicheskij analiz. Metrologicheskij sintez. (SPb.: Izd-vo SPbGJeTU LJeTI)
[4] Bank G, Guddat J, Klatte D, Kummer B, Tammer D 1983 Non-linear Parametric Optimization, Birkhauser Verlag, Basel.
[5] Chernyshova T I, Kamanskaya M A, and Kurnosov R Yu 2017 Vestnik TGTU 23 (2) 209-215