An attempt to resolve the cosmological constant problem in the modified Yang’s noncommutative quantized space-time

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Abstract

We attempt to resolve the cosmological constant problem through the key concept of the quantized number of spatial degrees of freedom in the modified Yang’s quantized space-time, \( n_{dof}(V_{3}^{R(\tau)}) \).

Keywords: cosmological constant, noncommutative quantized space-time, big bang, arrow of cosmological time

1. Introduction

In our preceding papers, ‘Where does black-hole entropy lie?’ [1] and ‘A short essay on quantum black holes and underlying noncommutative quantized space-time’ [2], hereafter referred as I and II, respectively, we emphasized the importance of the underlying noncommutative geometry such as Snyder’s and Yang’s noncommutative quantized space-time [3–6] towards the ultimate theory of quantum gravity or Planck scale physics. In the present paper, we more specifically notice the modified Yang’s noncommutative quantized space-time.

In fact, at the end of section 5. Concluding arguments and further outlook in II, we stated: ‘Before closing this short essay, let us note another interesting possibility of Yang’s quantized space-time algebra (YSTA, see appendix A). Indeed, one should notice that YSTA is intrinsically equipped with the long scale parameter \( R \), together with the short scale parameter \( a \) which has been identified with Planck length \( l_{P} \) in our present research so far. On the other hand, as was preliminarily pointed out in [25], \( R \) might be promisingly related to a fundamental cosmological constant in connection with the recent dark-energy problem, under the further idea that YSTA subject to the \( SO(D + 1, 1) \) algebra might be understood in terms of

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1 Emeritus Professor of Kyoto University.
2 In our present research, we have described so far the short scale parameter in YSTA by \( \lambda \) instead of \( a \) which Yang [5–6] used in accord with Snyder [3, 4], as early footnoted inside appendix A in II.
3 See, ‘6. Concluding Remarks’ in 2006 Found. Phys. Lett. 19 (November 2006).
some kind of local reference frame in the ultimate theory of quantum gravity, on the analogy of the familiar local Lorentz frame in Einstein’s general theory of relativity.

The above statement must be the subject of the present paper. In this connection, it might be appropriate here in advance to mention the modified version of Yang’s original quantized space-time (see appendix A).

Indeed, as will be shown in appendix A, we intend in its modified version (MYST) to replace the long scale parameter $R$ in the original version (YST) to be some kind of time $\tau$-dependent: let us say it is the cosmological time dependent, $R(\tau)$ (see the beginning of section 3). In fact, by this treatment, we intend that the series of MYST characterized by $(\lambda, R(\tau))$ becomes able to describe the historical development of our universe beginning with the so-called big bang.

### 2. The essence of the cosmological constant problem

Today, we know that the observed value of cosmological constant $\Lambda_{\text{obs}}$ is over one hundred and twenty orders of magnitude smaller than the Planck energy density. That is,

$$\Lambda_{\text{obs}} \sim 10^{-122}G^{-2}, \quad (2.1)$$

where $G$ denotes the Newton constant and $G^{-2}$ is known to be the so-called Planck energy density, see for instance [7].

At this point, we are almost uniquely led to the following theoretical expression $\Lambda(\tau)_{\text{theor}}$ at the present cosmological time $\tau$

$$\Lambda(\tau)_{\text{theor}} = n_{\text{dof}}(V^R_3(\tau))^{-1}G^{-2} \quad (2.2)$$

on the basis of $n_{\text{dof}}(V^R_3(\tau))$, which is the key concept in the modified Yang’s quantized space-time (see appendix A), and most importantly describes the quantized number of spatial degrees of freedom inside $V^R_3(\tau)$, i.e. $d = 3$—dimensional volume with radius $R(\tau)$:

$$n_{\text{dof}}(V^R_3(\tau)) = ([R(\tau)/\lambda] + 1)^2, \quad (2.3)$$

(see (3.1) in II), where $[R(\tau)/\lambda]$ means the nearest integer of $R(\tau)/\lambda$.

In fact, one can easily confirm that the factor $n_{\text{dof}}(V^R_3(\tau))$ in (2.2) just presents the extremely small factor $10^{-122}$ in (2.1) under $R(\tau) \sim 10^{28}$ cm and $\lambda (= l_P) \sim 10^{-33}$ cm.

That is, the essence of the cosmological constant problem is now clearly understood or resolved in the fundamental structure of $\Lambda(\tau)_{\text{theor}}$ derived in (2.2) where the most puzzling factor $10^{-122}$ in the $\Lambda_{\text{obs}}$ in (2.1) is theoretically understood in terms of the key concept $n_{\text{dof}}(V^R_3(\tau))$.

By the way, $n_{\text{dof}}(V^R_3(\tau))$ and the relation (2.3) itself as a whole were derived according to the general consideration of the kinematical holographic relation (KHR) given by (3.1) in II, through the proper irreducible representation of Yang’s quantized space-time algebra YSTA (see appendix A in II), while it is now developed on the modified Yang’s quantized space-time algebra (MYSTA) (see appendix A).

Furthermore, one should notice the fact that $n_{\text{dof}}(V^R_3(\tau))$ is nothing but the whole number of quantized spatial degrees of freedom inside of $V^R_3(\tau)$, that is, the whole spatial volume of our universe at the cosmological time $\tau$, as seen from the consideration given in I, II and originally in [8].
3. Concluding arguments and further outlook

First, let us notice that the theoretical cosmological constant \( \Lambda(\tau)_{\text{theor.}} = n_{\text{dof}}(V_{3}^{R(\tau)})^{-1} G^{-2} \) derived in equation (2.2) becomes the cosmological time \( \tau \) dependent through the term \( n_{\text{dof}}(V_{3}^{R(\tau)}) \), as noted in appendix A in general. In this connection, it is quite important to notice that the term \( n_{\text{dof}}(V_{3}^{R(\tau)}) \) is directly related to the entropy of the static and equilibrium system in the following way

\[
S(V_{3}^{R(\tau)}) = -\text{Tr}[W(V_{3}^{R(\tau)})\ln W(V_{3}^{R(\tau)})] = n_{\text{dof}}(V_{3}^{R(\tau)}) S(\text{site})
\]

\[
= ([R(\tau)/\lambda] + 1)^2 S(\text{site}),
\]

as seen in equation (3.10) in II with \( L \) replaced by \( R(\tau) \). In the above expression \( S(\text{site}) \) denotes the entropy assumed to be commonly realized in every (site), that is, every basis vector in Hilbert space II (see, section 2 in II). This fact guarantees that the cosmological time \( \tau \) acquires the so-called arrow of time, beyond a simple parameter (see appendix A).

Second, let us briefly consider the big bang stage. We assume that it starts at \( \tau \sim 0 \) with the uncertainty \( \lambda \). Then, one finds in equation (2.2) \( n_{\text{dof}}(V_{3}^{R(\lambda)}) \sim 4 \) and

\[
\Lambda(\lambda)_{\text{theor.}} \sim \frac{1}{4} G^{-2}
\]

at the very beginning stage of the big bang, in a sharp contrast to equation (2.1). It reminds us the extremely microscopic black hole system considered in the section 4.3 in II. It is our great interest to further investigate the later various stages of the big bang in accordance with the consideration of the black holes ranging over from macroscopic to extremely microscopic scales given also in section 4.3 in II.

It should be furthermore noted here that the above consideration of the big bang might be fulfilled without suffering from the so-called ‘Hawking–Penrose singularity theorems’, because of the fact that our present research based on the noncommutative geometry or quantized space-time is well associated with Heisenberg’s uncertainty principle, so as to be free from singularities (see appendix B in II, ‘Historical background of noncommutative quantized space and time’).

In this connection, it might possibly be claimed that the above derivation of \( \Lambda(\tau)_{\text{theor.}} = n_{\text{dof}}(V_{3}^{R(\tau)})^{-1} G^{-2} \) in equation (2.2) seems a bit ad hoc and a certain numerical fine tuning is required to explain the puzzling factor \( 10^{-122} \) in the equation (2.1) accidentally through \( n_{\text{dof}}(V_{3}^{R(\tau)}) \). However, the present author should like to notice that on the back of the key concept \( n_{\text{dof}}(V_{3}^{R(\tau)}) \), clearly lies the idea of Yukawa’s atomism of quantized space-time or elementary domain beginning from the preceding idea of Dirac’s generalized transformation function (‘g.t.f.’ see appendix B in II), which governs the whole space of the universe, in association with the present cosmological constant problem or the dark energy problem. We anticipate that our \( \Lambda(\tau)_{\text{theor.}} \) given by equation (2.2) will widely show the predictive power in the later various stages of the big bang over the present single prediction: \( \Lambda(\lambda)_{\text{theor.}} \sim \frac{1}{4} G^{-2} \) derived in equation (3.2) for its very beginning stage.

In this article we have omitted the important arguments related to the so-called standard model of high energy physics. It is our important task to reconstruct the M-theory [9] in terms of the present modified Yang’s Lorentz covariant quantized space-time towards the ultimate theory of quantum gravity and quantum cosmology.
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Appendix. Modified Yang’s Lorentz covariant quantized space-time (MYST)

Even in the MYST, as in the original Yang’s quantized space-time (YST) (see appendix A in II) it is equipped with the so-called Inonu–Wigner’s two contraction parameters, long $R$ and short $\lambda$. The MYST is simply given by replacing the long scale parameter $R$ in YST to be the cosmological time $\tau$–dependent $R(\tau)$, as noted at the end of our introduction.

In addition, we now intend to introduce this cosmological time $\tau$ so as to describe the observation time of $\Lambda_{\text{obs.}} (\sim 10^{-122}G^{-2})$ in (2.1) on the one hand, and on the other hand, the set of long and short scale parameters $(R(\tau), \lambda)$ at $\tau$ to specify the modified Yang’s quantized space-time as the local reference frame at the cosmological time $\tau$, as was mentioned in our introduction, according to the idea of the local Lorentz frame in the general theory of relativity.

First, (A.1) holds as it stands

$$\hat{\Sigma}_{MN} \equiv i(q_M \partial / \partial q_N - q_N \partial / \partial q_M) \quad \text{(A.1)}$$

and (A.2) clearly tends into

$$-q_0^2 + q_1^2 + \cdots + q_{D-1}^2 + q_a^2 + q_b^2 = R(\tau)^2. \quad \text{(A.2)}$$

Now, $D$-dimensional space-time and momentum operators, $\hat{X}_\mu$ and $\hat{P}_\mu$, with $\mu = 1, 2, \cdots, D$, are defined by

$$\hat{X}_\mu \equiv \lambda \hat{\Sigma}_{\mu a} \quad \text{(A.3)}$$

$$\hat{P}_\mu(\tau) \equiv \hbar / R(\tau) \hat{\Sigma}_{\mu b}, \quad \text{(A.4)}$$

together with $D$–dimensional angular momentum $\hat{M}_{\mu\nu}$

$$\hat{M}_{\mu\nu} \equiv \hbar \hat{\Sigma}_{\mu\nu} \quad \text{(A.5)}$$

and the so-called reciprocity operator

$$\hat{N}(\tau) \equiv \lambda / R(\tau) \hat{\Sigma}_{ab}. \quad \text{(A.6)}$$

Here and hereafter, one should notice that in MYST the momentum operator or the reciprocity operator and so on become the cosmological constant $\tau$–dependent in general.

Finally, one finds that the following relations are as a whole to hold

$$[\hat{X}_\mu, \hat{X}_\nu] = -i(\lambda^2 / \hbar)\hat{M}_{\mu\nu} \quad \text{(A.7)}$$

$$[\hat{P}_\mu, (\tau), \hat{P}_\nu(\tau)] = -i\hbar / R(\tau)^2 \hat{M}_{\mu\nu} \quad \text{(A.8)}$$

$$[\hat{X}_\mu, \hat{P}_\nu(\tau)] = -i\hbar \hat{N}(\tau)\hat{M}_{\mu\nu}. \quad \text{(A.9)}$$

$$[\hat{N}(\tau), \hat{X}_\mu] = -i\lambda^2 / \hbar \hat{P}_\mu(\tau) \quad \text{(A.10)}$$
\[ [\hat{N}(\tau), \hat{P}_\mu] = i\hbar/R(\tau)^2 \hat{X}_\mu, \] (A.11)

with other familiar relations concerning $\hat{M}_{\mu\nu}$’s omitted.

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