Dielectric constant of plasma rotating in crossed electric and magnetic fields

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Abstract. One-dimensional plasma motion is calculated in the crossed radial electric and axial magnetic fields in the non-stationary case. A physical model of the penetration of a radial electric field into plasma is proposed. The dependence of the azimuthal plasma velocity on time is determined. The concept of a non-stationary dielectric constant of plasma is introduced.

1. Introduction

Interest to study of rotating plasma is due to development of an effective plasma centrifuge designed to (i) separation of isotopes of elements that do not have gaseous compounds with high vapor pressure [1], (ii) reprocessing spent nuclear fuel [2, 3], as well as (iii) a number of astrophysical problems [4].

Let us dwell on the question how to achieve high plasma rotation speeds in laboratory conditions. The most common method of exciting rotating plasma is associated with the action of the crossed radial electric and axial magnetic fields [5]. Exactly in this case, it is the interaction of the radial electric current with the transverse magnetic field leads to the appearance of the azimuthal component of the Ampere force \( \mathbf{j} \cdot \mathbf{B} \), which accelerates the plasma. In the case of magnetized plasma, the characteristic collision frequencies of charged particles are much lower than their cyclotron frequencies. Often there are doubts about the possibility to accelerate such plasma to high speeds of rotation, related to penetration of an electric field into plasma under the conditions of the presence of a transverse magnetic field.

As is known, the corresponding component of the dielectric constant tensor of collisionless fully ionized plasma in the uniform longitudinal magnetic field is described by the following expression

\[
\varepsilon_{rr} \approx 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} = 1 + \frac{\rho \varepsilon_o}{\varepsilon_o B^2},
\]

where \( \omega_{pi} \) is the ion plasma frequency, \( \omega_{ci} \) is the ion cyclotron frequency, \( \rho \approx m_i n_i \) is the plasma density, \( \varepsilon_o \) is the electric constant, \( B \) is the longitudinal field induction. With the conventional
lithium plasma parameters, i.e. the density $n_i = 10^{18} \text{ m}^{-3}$, induction of the longitudinal axial magnetic field $B_z = 0.1 \text{ T}$, the ion mass $m_i = 10^{-26} \text{ kg}$, the $\varepsilon_{rr}$ value is about $10^5$.

At first glance, it follows that it is practically impossible to introduce a sufficiently strong external electric field into the plasma in the presence of a transverse magnetic field, as a result of which problems should arise with achieving high rotation speeds. However, when considering the behavior of plasma in crossed radial electric and axial magnetic fields, one should pay attention to a number of circumstances.

2. Physical and Mathematical Model

First, equation (1) is obtained by neglecting any particle collisions. In addition, it was assumed that the charge on the cylindrical electrodes of the device during the plasma spinning process is constant, as a result of which the electric field in the plasma decreases with increasing dielectric constant in accordance with the law $E_x = E_0 / \varepsilon_{rr}$, where $E_0$ is the vacuum field. In practice, however, there is always the possibility of using an external source to increase the field to a $E_0$ value. Secondly, for the process of azimuthal acceleration of a conducting medium, as already noted, it is necessary, generally speaking, not an electric field, but a current flowing across the magnetic field. It is the interaction of the electric current and the magnetic field that creates the torque that untwists the plasma.

Note that in solving the problem we will not take into account two circumstances. Namely, we will not touch on the issue of creating plasma. This can be either a non-self-sustaining discharge or a discharge with preliminary ionization of the working substance, for example, using an electron beam (the so-called “semi-self-sustaining” discharge). In addition, we note that the second type of discharge is preferable, since it significantly reduces the problems associated with the breakdown of the gas-discharge gap by an external electric field. Besides, we will not consider sufficiently complex near-electrode phenomena.

Thus, the physical and mathematical model of the process under investigation will look as follows.

1. Let the plasma be between two concentric cylindrical electrodes to which a potential difference is supplied from an external source. In the case of fairly dense partially ionized plasma, using a constant voltage source and a closed external circuit, the electric field penetrates the plasma “together with the current”, which is the conduction current. If we neglect the effects of magnetization, the radial current is determined by the usual conductivity and electric field. Since the forces of viscosity in a dense medium cannot be neglected, stationary rotation of the plasma is if it is available a certain radial electric current to overcome the azimuthal force of viscous friction.

2. Consider the situation when the medium is sufficiently rarefied and highly ionized, and its viscosity and other dissipative processes can be neglected. This case is characteristic for many experimental facilities. Let the plasma be inside the dielectric capacitor, the plates of which are charged with the opposite sign by an external voltage source, so that a potential difference $U_0$ arises between the plates.

3. First, let the external circuit be closed and electric charges may come from the source. Even if the magnetization of electrons is large, this does not mean that the parameter characterizing the magnetization is infinite. In this situation, polarization charges do not have time to accumulate near the electrodes, and the electric field completely penetrates the plasma in accordance with the voltage supplied from the source and the Ohmic resistance of the gap between the electrodes. At the same time, a radial electric current runs during the plasma rotation, and the plasma conductivity is quite strongly dependent on the magnetic field. However, it ceases when the plasma reaches the drift velocity
Consider the case when the external circuit is open. At the first moment of time after the appearance of a vacuum field in plasma, plasma ions and electrons under the action of this field tend to shift respectively along the field and against it, causing polarization and a short-term electric current. In this case, the external electric field created by the charges on the electrodes is destroyed by the internal polarization field. However, the rotational moment obtained by the plasma as a result of leakage of the polarization current is preserved due to neglect of the viscous friction forces and the entire plasma begins to drift in the azimuthal direction transversely to the directions of the electric and magnetic fields. Due to inertia, a certain finite time is required for the plasma to reach a stationary state. Since the external circuit is open, when a stationary state is reached, the radial electric current in the plasma ceases, and it rotates with a constant in time drift velocity in crossed fields (2).

Let us illustrate the plasma acceleration mechanism based on a one-component magnetohydrodynamic analysis. We will use the picture of the unsteady plasma spinning process, first proposed, justified and experimentally confirmed in the papers published by the University of Berkeley staff [6, 7] using the so-called “hydromagnetic capacitor” as an example.

Following [7], we consider the unsteady process of plasma spinning, when the time to establish the azimuthal velocity significantly exceeds the period of the Larmor rotation of ions. As the plasma unwinds, such an unusual “capacitor” is gradually charged by the source with a constant difference in the potentials of the plates $U$ supported by the voltage source. In this case, a radial current with a density $j_r$ runs in the plasma, resembling the polarization current in a conventional capacitor with an insulator when it is charged. Only in a dielectric does this happen quickly and energy is stored in the elastic bonds of the molecules, and in the “hydromagnetic capacitor” it is comparatively slow converted to the kinetic energy of ion rotation. We emphasize that the electric field in plasma that penetrates very quickly into a conducting medium does not decrease due to polarization, but remains constant throughout the time and corresponds to the voltage of the source.

In addition to the radial electric current $j_r$, an azimuthal current can be excited in the plasma, the nature of which is due to both plasma diamagnetism and the Hall phenomena. Consider the case when the external radial electric field is directed to the internal electrode. In this situation, the radial electromagnetic force $[jB]_r$ is directed toward the center and partially or fully balances the centrifugal forces. For this reason, in this consideration, we will neglect the radial plasma pressure gradient. In the hydrodynamic approximation, we have the equation of unsteady plasma rotation in the projection onto the axis $\varphi$ [6]

$$\rho \frac{dV_\varphi}{dt} = -j_z B_z,$$

(3)

describing the acceleration of a unit volume of a conducting medium by ampere force (we direct the axial magnetic field along the positive direction of the axis). In this case, $j_z$ represents the current density, which changes in time, since the plasma rotation velocity changes in time. Using the Ohm’s law in the usual form, we obtain

$$\rho \frac{dV_\varphi}{dt} = -\sigma_r \left( E_{r0} + V_\varphi B_z \right) B_z,$$

(4)

where $E_{r0} < 0$, since this is the projection of the external vacuum electric field onto the radial direction, $\sigma_r$ is the radial conductivity of the plasma.
3. Results of Calculation
We emphasize once again in order to avoid misunderstandings that we consider the case of negative polarity of the voltage applied between concentric cylinders when the external electrode is positively charged and the radial projection of the electric field is negative. At the initial moment of time \( V_\phi = 0 \), the density of the radial electric current is maximum. As the plasma unwinds, it decreases due to the action of the Lorentz force on charged particles and under the condition \( t \to \infty \) tends to zero. The solution of the equation (4) for \( V_\phi(0) = 0 \) has the following form

\[
V_\phi(t) = -\frac{E_{r0}}{B_z} \left( 1 - \exp\left( -\frac{\sigma_r B_z^2}{\rho} t \right) \right).
\]

(5)

It is quite obvious that under the condition \( t \to \infty \) the azimuthal velocity component will tend to the value \( V_{\phi0} = -E_{r0} / B_z \). The current also changes in time in accordance with the expression

\[
j_r = \sigma_r E_{r0} \exp\left( -\frac{\sigma_r B_z^2}{\rho} t \right)
\]

(6)

As expected, the current density is negative, decreases in absolute value with time, tending to zero in the steady state. You can enter the “effective dielectric constant” of the plasma by means of the relation

\[
\varepsilon_\rho^* = \frac{\varepsilon_0 E_r^2}{2} + \rho V_\phi^2 / 2
\]

(7)

From the equation (7) it follows that the “effective dielectric constant” is equal to

\[
\varepsilon_\rho^* = 1 + \frac{\rho}{\varepsilon_0 B_z^2} \left( 1 - \exp\left( -\frac{\sigma_r B_z^2}{\rho} t \right) \right)^2
\]

(8)

Now we estimate the necessary time to reach the steady-state for the case of the completely ionized lithium plasma under the assumption that the “reconstructed” radial conductivity is determined by electrons [8]: \( \sigma_r \approx \sigma_{par} / \omega_e \tau_e^2 \) is conductivity across the magnetic field; \( \sigma_{par} \) is conductivity along the magnetic field; \( \omega_e \) is cyclotron frequency of electrons; \( \tau_e \) is time of electrons-ions collisions. Assuming that the plasma density \( n = 10^{18} \) m\(^{-3}\), temperature \( T_e \approx 1 \) eV, magnetic induction \( B_z = 0.1 \) T, one can obtain the following results: \( \tau_e \approx 3.5 \cdot 10^{-8} \) s, \( \sigma_{par} \approx 10^3 \) m\(^{-1}\)s\(^{-1}\), the magnetization parameter \( \omega_e \tau_e \approx 6 \cdot 10^2 \), \( \sigma_r \approx 2.8 \cdot 10^{-3} \) 1/m\(\cdot\)s, \( \tau \approx 3 \cdot 10^{-4} \) s.

4. Conclusions
In this study, we calculated the non-stationary process of acceleration for fully ionized plasma in crossed radial electric and axial magnetic fields. It is shown that the new concept of unsteady effective “dielectric constant” of plasma introduced during this study makes it possible to monitor the transient process for the azimuthal velocity overclocking for a fully ionized medium.
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