Specific Density Of Binding Energy Of Core
In β - Stable Nuclei Is 2.57 MeV/fm³

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Abstract

Recently an α-cluster model based on the pn-pair interactions with using the isospin invariance of nuclear force has been proposed. According to the model the excess neutron pairs fill out the free space in the core determined by the difference in the charge and matter radii of the α-clusters. Then the number of excess neutrons in β-stable nuclei depends on the number of the core α-clusters. In such a representation the specific density of binding energy of core ρ is the only parameter to fit the experimental binding energies of β-stable nuclei and it turned out to be a constant value equal to 2.57 MeV/fm³. Knowing the value ρ allows one to estimate the size of a nucleus from its experimental binding energy.

Key words: nuclear structure; alpha-cluster model; Coulomb energy; surface tension energy, binding energy; charge radius.
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1 Binding Energy Of Core

The idea that the main properties of nuclei can be described from a simple representation (like liquid drop [1] or some regular forms of α-cluster structure [2]) has been popular since the very beginning of nuclear physics. The main features of nuclear force, its strength and the short range, allow one to find some simple formulas to describe size and binding energies of nuclei.

Recently an α-cluster model based on pn-pair interactions with using isospin invariance of nuclear force has been proposed [3,4,5]. In the framework of this model new formulas to calculate radii and binding energies of β-stable nuclei have been found. Also the model provides some reasonable explanation

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of existing excess neutrons in stable nuclei. The spinless \( n\) \( n\)-pairs of excess neutrons fill out the free space in the core which is determined by the difference in the volumes occupied by the charge and by the matter of the \( \alpha\)-clusters. The proton charge radius is bigger than the neutron radius due to isospin invariance of nuclear force [6].

In such a representation the value of the charge radius \( R\) of an even \( Z\) nucleus can be obtained from an estimation of the volumes occupied by the alpha-clusters of the core \( N_{\text{core}a}\) and the peripheral \( \alpha\)-clusters \( N_{p\alpha} = N_\alpha - N_{\text{core}a}\) \((N_{p\alpha} = 1 \div 5)\), \( N_\alpha = Z/2\),

\[
R^3 = r_\alpha^3 N_{\text{core}a} + r_{^4\text{He}}^3 N_{p\alpha} ,
\]

(1)

where the radius of a peripheral \( \alpha\)-cluster equals the experimental radius of the nucleus \(^4\text{He}\) \( r_{^4\text{He}} = 1.71\) fm [7]. In case of \( N_{\text{core}a} = 0\), \( N_\alpha = N_{p\alpha}\), \( R = r_{^4\text{He}} N_\alpha^{1/3} \) [3-5]. The value of the radius of an \( \alpha\) - cluster of core \( r_\alpha = 1.60\) fm is obtained from fitting the experimental radii of the nuclei with \( N_{\text{core}a} >> N_{p\alpha}\) with the formula \( R = r_\alpha N_\alpha^{1/3}\).

For odd \( Z_1 = Z + 1\) the number of \( \alpha\)-clusters \( N_{1\alpha}\) in the nucleus is \( N_{1\alpha} = N_\alpha + 0.5\) and the number of peripheral \( \alpha\)-clusters is \( N_{1p\alpha} = N_{p\alpha} + 0.5\). Then the formula is

\[
R_{1}^3 = r_\alpha^3 N_{\text{core}a} + r_{^4\text{He}}^3 N_{1p\alpha} .
\]

(2)

Whereas the nuclear radii are calculated by means of estimation of the volumes occupied by the \( \alpha\)-clusters of core and by the \( \alpha\)-clusters of periphery, the binding energy is calculated on the total amount of \( \alpha\)-clusters \( N_\alpha\) [5] disregarding to the fact that some of them belong to the core and the others are of the nucleus periphery. In this paper the model [5] is developed to have a consistency between the ways of how the radii and the binding energies are calculated.

In the representation with dividing nucleus for core and periphery the binding energy of a nucleus is to be the sum of the binding energy of the core \( E_{\text{core}}\) and the internal binding energy of the compound peripheral cluster \( E_{N_{p\alpha}}\) consisting of \( N_{p\alpha}\) \( \alpha\)-clusters minus the Coulomb energy of the compound cluster interaction with the core \( \alpha\)-clusters \( E_{N_{p\alpha}N_{\text{core}a}}^C = 2N_{p\alpha}2N_{\text{core}a}e^2/R_p\), where \( R_p\) is the radius of the last alpha-cluster position in the nucleus (in the center of core mass system)[5]\((R_{p1}\) is the radius of the single pn-pair’s position in the odd \( Z_1 = Z + 1\) nuclei),

\[
R_p = 2.168(N_\alpha - 4)^{1/3}; R_{p1} = 2.168(N_\alpha + 0.5 - 4)^{1/3}
\]

(3)
So the new formula to calculate binding energy is to be as follows

\[ E = E_{\text{core}} + E_{N_{\text{pa}}} - E_{N_{\text{pa}}N_{\text{core}}}^C. \]  

(4)

The energy of the peripheral compound cluster \( E_{N_{\text{pa}}} \) consisting of \( N_{\text{pa}} \) \( \alpha \)-clusters is taken equal to the experimental binding energy of the nucleus \(^8\text{Be} \) \( (N_{\text{pa}} = 2) \) \( E_{\text{8Be}} = 56.5 \text{ MeV} \), \(^{12}\text{C} \) \( (N_{\text{pa}} = 3) \) \( E_{\text{12C}} = 92.2 \text{ MeV} \), \(^{16}\text{O} \) \( (N_{\text{pa}} = 4) \) \( E_{\text{16O}} = 127.6 \text{ MeV} \), \(^{20}\text{Ne} \) \( (N_{\text{pa}} = 5) \) \( E_{\text{20Ne}} = 160.6 \text{ MeV} \).

In this representation a nucleus \( A(Z, N + \Delta N) \) with even \( Z, N = Z \) and \( \Delta N \) is an even number of excess neutrons, has the same core as the nucleus \( A_1(Z_1, N_1 + \Delta N + 1) \) with \( Z_1 = Z + 1, N_1 = Z_1 \). Then \( A_1 = A + 3 \). In case of the odd \( Z_1 \) one excess neutron is glued to the single pn-pair, which is bound with the three nearest peripheral clusters \([4, 5]\). The long range Coulomb energy of the single pn-pair interaction with the core is compensated with its contribution to the surface tension energy (see section 2). So for the odd-odd nuclei the binding energy is calculated as follows

\[ E_1 = E_{\text{core}} + E_{N_{\text{1pa}}} - E_{N_{\text{1pa}}N_{\text{core}}}^C, \]  

(5)

where \( E_{N_{\text{1pa}}} \) is the experimental binding energy of the nucleus \(^7\text{Li} \) \( (N_{\text{1pa}} = 1.5) \) \( E_{\text{7Li}} = 39.2 \text{ MeV} \), \(^{11}\text{B} \) \( (N_{\text{1pa}} = 2.5) \) \( E_{\text{11B}} = 76.2 \text{ MeV} \), \(^{15}\text{N} \) \( (N_{\text{1pa}} = 3.5) \) \( E_{\text{15N}} = 115.5 \text{ MeV} \), \(^{19}\text{F} \) \( (N_{\text{1pa}} = 4.5) \) \( E_{\text{19F}} = 147.8 \text{ MeV} \) and \(^{23}\text{Na} \) \( (N_{\text{1pa}} = 5.5) \) \( E_{\text{23Na}} = 186.6 \text{ MeV} \).

The binding energy of a core occupying the volume \( V_{\text{core}} = 4/3\pi r_\alpha^3N_{\text{core}} \) can be expressed by a formula with using the specific density of binding energy \( \rho \) [MeV/fm\(^3\)]

\[ E_{\text{core}} = V_{\text{core}}\rho. \]  

(6)

The binding energy of \( N_{\text{core}} \) \( \alpha \)-clusters \( E_{N_{\text{core}}} \) is easily calculated in the framework of the \( \alpha \)-cluster model (see section 2), where the energy of short range nuclear force \( E^\text{nuc} \), the energy of surface tension \( E^{\text{ST}} \) and the Coulomb energy \( E^C \) are calculated on the number of \( \alpha \)-clusters. Then

\[ E_{\Delta N} = E_{\text{core}} - E_{N_{\text{core}}} \]  

(7)

The binding energy of excess neutrons \( E_{\Delta N} \) depends only on the number of the excess nn - pairs \( N_{nn} \) \([5]\) (see also section 2), which means that only some particular number of \( \Delta N = 2N_{nn} \) can have place in the core. This allows one to find the correspondence between \( \Delta N \) and \( N_{\text{pa}} \). It brings a result that the specific density of core binding energy is a constant value for all nuclei of the \( \beta \)-stability valley and its vicinity.
2 Binding Energy Of Core $\alpha$ - Clusters

The $\alpha$-cluster model [3,4,5] has been developed on the basis of the fact that the radii of the most abundant isotopes are determined by the number $N_\alpha$ [8] and that the binding energies of symmetrical even $Z$ nuclei are calculated by the formula [9]

$$E = N_\alpha \epsilon_\alpha + 3(N_\alpha - 2)\epsilon_{\alpha\alpha}, \quad (8)$$

where $\epsilon_\alpha = 28.296$ MeV is the experimental energy of the nucleus $^4\text{He}$. The value $3(N_\alpha - 2)$ is considered as the number of short range bonds between nearby alpha-clusters and $\epsilon_{\alpha\alpha} = 2.425$ MeV. In case of an odd $Z_1 = Z + 1$ symmetrical nucleus the single pn-pair is glued to the three nearby peripheral clusters with 6 bonds with their six pn-pairs. Thus, for the symmetrical odd nuclei [9] the binding energy $E_1$ is calculated as follows

$$E_1 = E + \epsilon_{pm} + 6\epsilon_{pmn}, \quad (9)$$

where $\epsilon_{pm} = 1.659$ MeV and $\epsilon_{pmn} = 2.037$ MeV. What was remarkable in [9] that in (8) and (9) the energy portions $\epsilon_\alpha$, $\epsilon_{\alpha\alpha}$, $\epsilon_{pm}$ and $\epsilon_{pmn}$ were obtained from analysis of the lightest nuclei with $Z \leq 6$. The binding energy of nuclear force of an $\alpha$-cluster $\epsilon_{\alpha\alpha}^{\text{nuc}} = 29.060$ MeV was found from the relation $\epsilon_\alpha = \epsilon_{\alpha\alpha}^{\text{nuc}} - \epsilon_\alpha^C$ where $\epsilon_\alpha^C = \Delta E_{np} = 0.764$ MeV, $\Delta E_{np}$ is the difference between the binding energies of the last neutron and the last proton in the nucleus $^4\text{He}$. The energy of nuclear force interaction $\epsilon_{\alpha\alpha}^{\text{nuc}} = 4.350$ MeV and the Coulomb energy $\epsilon_{\alpha\alpha}^C = 1.925$ MeV in a short range bond between two nearby $\alpha$-clusters were obtained from the analysis of the experimental binding energies of the lightest nuclei using the relation $\epsilon_{\alpha\alpha} = \epsilon_{\alpha\alpha}^{\text{nuc}} - \epsilon_{\alpha\alpha}^C$ [5].

The Eqs. (8) and (9) mean that the long range Coulomb interactions between alpha-clusters in the nuclei with $N_\alpha \geq 5$ (for the nuclei with $N_\alpha \leq 4$ there are only short range interactions) must be compensated by the surface tension energy. From this assumption some formulas to calculate the radius of the last $\alpha$-cluster position in the nucleus (3), the Coulomb radius of the nucleus $R_1^C = 1.869N_\alpha^{1/3}$ and the formula to calculate the Coulomb energy of the charge sphere of the radius $R_1^C$ were obtained [5].

$$E_1^C = 1.849(N_\alpha + 0.5)^{5/3}. \quad (10)$$

The surface tension energy $E_1^{ST}$ is the sum of the square radii of the $N_\alpha - 4$ alpha-clusters’ positions ( Eq. (9) in [5]).
A phenomenological formula to calculate the binding energy of the excess neutron pairs $E_{\Delta N}$ was found from fitting the experimental separation energies of 27 nn-pairs, see Eq. (13) in Ref. [5].

Thus, the formula to calculate binding energy is [5]

$$E_{th} = E^{nuc} + E^{ST} - E^C + E_{\Delta N}. \quad (11)$$

The Eq. (11), obtained from analysis of a reduced amount of nuclei (the symmetrical nuclei with $Z \leq 22$ for which the experimental values of $\Delta E_{np}$ are known), turned out to be good for all $\beta$-stable nuclei. The accuracy is a few MeV, which is the same as that of Weizsäcker formula, although the ways of calculations are different.

One of the important conclusions of the model is that the energy of excess neutrons $E_{\Delta N}$ depends only on the number of excess neutron pairs $N_{nn} = \Delta N/2$. To make the calculations easy we propose a simple approximation to the phenomenological formula (13) in Ref. [5]

$$E_{\Delta N} = (21.93 - 0.762N_{nn}^{2/3})N_{nn}. \quad (12)$$

The values of $E_{\Delta N}$ are given in Fig. 1 in comparison with the values calculated by the Eq. (13) of Ref. [5]. In the figure some empirical values obtained from the Eq. $E_{exp} - (E^{nuc} - E^C + E^{ST})$ where the values ($E^{nuc} - E^C + E^{ST}$) are calculated in the framework of the model are given too. One can see from the figure that the empirical values of $E_{\Delta N}$ depend only on the number of $N_{nn}$ disregarding to $Z$.

The binding energy $E_{N_{core\alpha}}$ of core alpha-clusters $N_{core\alpha}$ must include the energy of nuclear force of the core $\alpha$-clusters $E_{N_{core\alpha}}^{nuc}$, the Coulomb energy $E_{N_{core\alpha}}^C$ and surface tension energy $E^{ST}$. The energy of nuclear force of the core $\alpha$-clusters includes $N_{core\alpha}^{nuc} + 3(N_{core\alpha} - 2)\epsilon_{\alpha\alpha}^{nuc}$. Also the energy of the number of bonds $\Delta$ between the core alpha-clusters and the peripheral compound cluster consisting of $N_{pa}$ alpha-clusters must be taken into account

$$\Delta = 3(N_{\alpha} - 2) - 3(N_{core\alpha} - 2) - 3(N_{pa} - 2). \quad (13)$$

One can see that for $N_{pa} \geq 2$ $\Delta_{bonds} = 6$. So the nuclear force energy $E_{N_{core\alpha}}^{nuc}$ is calculated as follows

$$E_{N_{core\alpha}}^{nuc} = N_{core\alpha}\epsilon_{\alpha\alpha}^{nuc} + 3(N_{core\alpha} - 2)\epsilon_{\alpha\alpha}^{nuc} + \Delta^{nuc}_{\alpha\alpha}. \quad (14)$$

The Coulomb energy of core is equal to the Coulomb energy of $N_{core\alpha}$ $\alpha$-clusters, which is calculated as $1.849N_{core\alpha}^{5/3}$ due to (10) plus the Coulomb
Fig. 1. The values $E_{\text{exp}} - (E^{\text{nuc}} - E^C + E^{\text{ST}})$ for all $\beta$-stable even-even nuclei with $Z = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ (41 square points), where $E^{\text{nuc}} - E^C + E^{\text{ST}}$ is calculated on the $\alpha$-cluster model [5]. The solid line indicates the values $E_{\Delta N}$ calculated on the Eq. (13) in [5]. The dashed line denotes the values $E_{\Delta N}$ (12) of this paper.

Using the relation $\epsilon_{aa} = \epsilon_{aa}^{\text{nuc}} - \epsilon_{aa}^C$ the formula to calculate the binding energy of the $N_{\text{core}}\alpha$ clusters is as follows

$$E_{N_{\text{core}}} = N_{\text{core}}\epsilon_{\alpha} + 3(N_{\text{core}} - 2)\epsilon_{aa}^{\text{nuc}} - 1.849N_{\text{core}}^{5/3} + \Delta\epsilon_{aa} + E^{\text{ST}}. \hspace{1cm} (16)$$

We use here the simple function proposed in [5], which provides a good approximation to the phenomenological formula of calculation of the surface tension energy $E^{\text{ST}}$

$$E^{\text{ST}} = (N_{\alpha} + 1.7)(N_{\alpha} - 4)^{2/3}; \hspace{0.5cm} E_{1}^{\text{ST}} = E^{\text{ST}} + 1.1(N_{\alpha} - 4)^{2/3}. \hspace{1cm} (17)$$

The value of $1.1(N_{\alpha} - 4)^{2/3}$ is the contribution of the single pn-pair into the surface tension energy. But the energy of the Coulomb interaction of the pn-pair with the $\alpha$-clusters of the core $2N_{\text{core}}e^2/R_{p1}$ almost compensates it. For $N_{\alpha} < 59$ the difference is within few MeV. Therefore to calculate the binding energy of the odd-odd nuclei in Eq. (5) the Coulomb energy $E_{N_{\alpha}N_{\text{core}}}^{\text{C}}$ is used.

To fit experimental data for heavy nuclei one has to use a representation that the peripheral compound cluster consists of two compound clusters of smaller size. In the formulas the values of $\Delta$ is changed. For example, for two peripheral compound clusters $N_{p1} = 2$ and $N_{p2} = 3$ with the total amount of alpha-clusters in them $N_{p} = 5$ the total number of bonds is $1 + 3 = 4$. In
(13) the number 3 (5 - 2) = 9 is changed for 4, which leads to $\Delta = 6 + 5 = 11$. In (4) $E_{N_{pa}} = E_{N_{pa1}} + E_{N_{pa2}}$ where $E_{N_{pa1}} = E_{8\text{Be}}$ and $E_{N_{pa2}} = E_{12C}$.

In calculation of the energy $E_{N_{pa},N_{core\alpha}}^C$ one has to take into account the long range interactions between $N_{pa1}$ and $N_{pa2}$ clusters. The algorithm looks simple if one uses a two dimensional matrix $E_{N_{\alpha}, N_{pa}}^C$ for $N_{\alpha} = 9 \div 59$ and $N_{ap} = 1 \div 8$, which is

$$E_{N_{\alpha}, N_{pa}}^C = 2N_{pa}(2N_{\alpha} - 2N_{pa})e^2/(2.168((N_{\alpha} - 4)^{1/3}). \quad (18)$$

Then the Coulomb energy for the case is easily expressed as

$$E_{N_{pa}, N_{core\alpha}}^C = E_{N_{\alpha}, 2}^C + E_{N_{\alpha} - 2, 3}. \quad (19)$$

So, the binding energy of the nucleus $A(Z, N + \Delta N)$ is calculated by the following way. For $\Delta N$ the value $N_{pa}$ is calculated from Eq. (7) and Eq. (12) taking into account that the number of nn-pairs in core $N_{nn}$ is integer. Then the binding energy is calculated by (4) and (5). The radii are calculated by (1) and (2). In this approach the expected accuracy is a few MeV, because the single particle effects (shell effects) are not taken into account. The value $\rho = 2.57\text{MeV/fm}^3$ fits the binding energies of all $\beta$-stable isotopes and not stable isotopes of the vicinity. In Table 1 the results of calculation by (4) and (5) are given for some nuclei. The values of $E_{th}$ (11) are also presented.

If the value of the specific density of binding energy is known, it gives an opportunity to estimate the size of the isotopes from their experimental binding energies. Then the value $N_{pa}$ is calculated by (4) where instead of $E$ the value $E_{exp}$ is used. For example, the nucleus $^{142}\text{Nd}_{60}$ has $E_{exp} = 1185$ MeV. The value $N_{pa} = 2 \div 5$ are tried and only $N_{pa} = 4$ corresponds to the $\rho = 2.57$ MeV/fm$^3$. So the radius (1) $R = 5.02$ fm and for $^{145}\text{Pm}_{61}$ (2) $R = 5.05$ fm.

Table 1. Binding energies and radii calculated for $\beta$-stable isotopes. For stable nuclei the most abundant isotopes have been selected. Also the results are presented for the corresponding nucleus having the same core (if it is a $\beta$-unstable nucleus, it is marked by *).
| \( Z \) | \( \Delta N \) | \( N_{pp} \) | \( A \) | \( E_{pp}^{[10]} \) | \( E_{th}^{(4,5)} \) | \( R_{pp}^{[11,12,13]} \) | \( R^{(1,2)} \) |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 10  | 0   | 20  | 5   | 161 | 161 | 158 | 2.992(8) | 2.92 |
| 11  | 1   | 23  | 5.5 | 187 | 187 | 185 | 2.94(6)  | 3.02 |
| 12  | 0   | 24  | 4   | 198 | 199 | 196 | 3.075(15) | 3.04 |
| 13  | 1   | 27  | 4.5 | 225 | 219 | 223 | 3.069(9) | 3.13 |
| 14  | 0   | 28  | 4   | 237 | 238 | 234 | 3.144(4) | 3.18 |
| 15  | 1   | 31  | 4.5 | 263 | 258 | 260 | 3.193(3) | 3.26 |
| 16  | 0   | 32  | 4   | 272 | 277 | 271 | 3.240(11) | 3.31 |
| 17  | 1   | 35  | 4.5 | 298 | 297 | 297 | 3.388(17) | 3.39 |
| 18  | 0   | 36  | 5   | 307 | 306 | 307 | 3.327(15) | 3.43 |
| 19  | 1   | 39  | 5.5 | 334 | 332 | 333 | 3.408(27) | 3.53 |
| 18  | 4   | 40  | 2   | 344 | 343 | 349 | 3.393(15) | 3.38 |
| 19  | 5   | 43* | 2.5 | 369 | 363 | 369 | 3.482(25) | 3.57 |
| 20  | 1   | 43* | 5.5 | 367 | 371 | 369 | 3.635(3)  | 3.63 |
| 22  | 2   | 42  | 3   | 362 | 370 | 364 | 3.505(2)  | 3.52 |
| 21  | 3   | 45  | 3.5 | 388 | 394 | 390 | 3.550(5)  | 3.59 |
| 24  | 4   | 52  | 3   | 456 | 453 | 452 | 3.645(5)  | 3.73 |
| 25  | 5   | 55  | 3.5 | 482 | 477 | 477 | 3.680(11) | 3.79 |
| 28  | 2   | 58  | 5   | 506 | 502 | 502 | 3.760(10) | 3.96 |
| 29  | 3   | 61* | 5.5 | 532 | 528 | 529 | 4.015(3)  | 4.01 |
| 28  | 4   | 60  | 4   | 527 | 520 | 524 | 3.812(30) | 3.94 |
| 29  | 5   | 63  | 4.5 | 551 | 540 | 548 | 3.888(5)  | 3.99 |
| 30  | 4   | 64  | 4   | 559 | 560 | 557 | 3.918(11) | 4.02 |
| 31  | 5   | 67* | 4.5 | 583 | 580 | 583 | 4.072(30) | 4.07 |
| 30  | 6   | 66  | 3   | 578 | 578 | 577 | 3.977(20) | 4.00 |
| 31  | 7   | 69  | 3.5 | 602 | 602 | 602 | 4.051(20) | 4.05 |
Table 1. Continued.

| Z  | \( \Delta N \) | A   | \( N_{p\alpha} \) | \( E_{exp}[10] \) | \( E(4,5) \) | \( E_{th}(11) \) | \( R_{exp}[11,12,13] \) | \( R(1,2) \) |
|----|----------------|------|-----------------|---------------|-------------|-------------|-----------------|-------------|
| Z_1| \( \Delta N+1 \) | A_1  | \( N_{1p\alpha} \) | MeV           | MeV         | MeV         | fm              | fm          |
| 40 | 10             | 90   | 3               | 784           | 788         | 778         | 4.28(2)         | 4.39        |
| 41 | 11             | 93   | 3.5             | 806           | 810         | 802         | 4.317(8)        | 4.43        |
| 50 | 18             | 118  | 3               | 1005          | 999         | 1002        | 4.72            |             |
| 51 | 19             | 121  | 3.5             | 1026          | 1022        | 1024        | 4.63(9)         | 4.76        |
| 50 | 20             | 120  | 2               | 1021          | 1026        | 1018        | 4.630(7)        | 4.71        |
| 51 | 21             | 123  | 2.5             | 1042          | 1046        | 1040        | 4.74            |             |
| 60 | 22             | 142  | 4               | 1185          | 1181        | 1177        | 4.993(35)       | 5.02        |
| 61 | 23             | 145  | 4.5             | 1204          | 1201        | 1198        | 5.05            |             |
| 70 | 32             | 172  | 4               | 1393          | 1390        | 1391        | 5.28            |             |
| 71 | 33             | 175  | 4.5             | 1412          | 1410        | 1410        | 5.378(30)       | 5.31        |
| 80 | 42             | 202  | 2+2             | 1595          | 1580        | 1587        | 5.499(17)       | 5.51        |
| 81 | 43             | 205  | 2+2.5           | 1616          | 1615        | 1605        | 5.484(6)        | 5.52        |
| 90 | 50             | 230  | 2+3             | 1755          | 1752        | 1756        | 5.74            |             |
| 91 | 51             | 233* | 2+3.5           | 1772          | 1776        | 1773        | 5.76            |             |
| 100| 52             | 252  | 3+4             | 1879          | 1881        | 1881        | 5.95            |             |
| 101| 53             | 255* | 3+4.5           | 1896          | 1901        | 1900        | 5.98            |             |
| 110| 60             | 281  | 2+3+3           | 2031          | 2030        | 2037        | 6.15            |             |
| 111| 61             | 283* | 2+3+3.5         | 2047          | 2045        | 2045        | 6.17            |             |
| 116| 72             | 304  | 2+3+3           | 2145          | 2147        |             | 6.26            |             |
| 117| 73             | 307(*) | 2+3+3.5       | 2168          | 2157        |             | 6.28            |             |

3 Conclusion

The alpha - cluster model \([3,4,5]\) has been developed to find some formulas for calculation of the binding energies of \( \beta \) - stable nuclei with using the notion of core. This brings to a discovery that the specific density of the binding energy of core for the nuclei of \( \beta \)-stable valley and those which are in its vicinity can be a constant value equal to 2.57 MeV/fm\(^3\). The idea that the number of
excess neutrons is determined by the amount of α-clusters of the core, which has been approved before in terms of charge radii of nuclei [3], is approved now in terms of the binding energies.

In the formulas (1) and (2) to calculate radii the radius of an α-cluster of the core $r_{\alpha} = 1.60$ fm is the only fitting parameter to describe the experimental radii. So is the value $\rho = 2.57$ MeV/fm$^3$ to describe the experimental binding energies of all stable isotopes having core by the formulas (4) and (5). Thus, it is clearly seen here that the radius of one α-cluster, as well as the specific density of binding energy of core, are the constant values. Then one has an opportunity to estimate the size of a nucleus from its experimental binding energy.

In the heavy nuclei the growing Coulomb energy pushes out small compound clusters from the core to the periphery of the nucleus. If for the stable nuclei the total number of peripheral α-clusters is within $2 \div 5$, for the nuclei with $Z \geq 80$ the number is within $5 \div 8$.

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