Effects of four-body breakup on $^6$Li elastic scattering near the Coulomb barrier

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We investigate projectile breakup effects on $^6$Li+$^{209}$Bi elastic scattering near the Coulomb barrier with the four-body version of the continuum-discretized coupled-channels method (four-body CDCC). This is the first application of four-body CDCC to $^6$Li elastic scattering. The elastic scattering is well described by the $p+n+^4$He+$^{209}$Bi four-body model. We propose a reasonable three-body model for describing the four-body scattering, clarifying four-body dynamics of the elastic scattering.

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Introduction. Plenty of nuclei are considered to have two-cluster or three-cluster configurations as their main components. Three-cluster dynamics is, however, nontrivial compared with two-cluster dynamics. Systematic understanding of three-cluster dynamics is hence important. There are many nuclei that can be described by three-cluster models. For example, low-lying states of $^6$He and $^6$Li are explained by $N + N + ^4$He three-body models [1,2], where $N$ stands for a nucleon. The comparison of the two nuclei is important to see the difference between dineutron and proton-neutron correlations. Two-neutron halo nuclei such as $^{11}$Li, $^{14}$Be, and $^{22}$C are reasonably described by an $n + n + X$ three-cluster model, where $X$ is a core nucleus. Properties of these three-cluster configurations should be confirmed by measuring scattering of the nuclei and analyzing the measured cross sections with accurate reaction theories. The reactions are essentially four-body scattering composed of three constituents of projectile and a target nucleus. Accurate theoretical description of four-body scattering is thus an important subject in nuclear physics.

The continuum-discretized coupled-channels method (CDCC) is a fully quantum-mechanical method of describing not only three-body scattering but also four-body scattering [7–9]. CDCC has succeeded in reproducing experimental data on both three- and four-body scattering. The theoretical foundation of CDCC is shown with the distorted Faddeev equation [10–12]. CDCC for four-body (three-body) scattering is often called four-body (three-body) CDCC; see Refs [13–25] and references therein for four-body CDCC. So far four-body CDCC was applied to only $^6$He scattering.

For $^6$He+$^{209}$Bi scattering at 19 and 22.5 MeV near the Coulomb barrier, the measured total reaction cross sections are largely enhanced in comparison with that for $^6$Li+$^{209}$Bi scattering at 29.9 MeV near the Coulomb barrier [26,27]. Keeley et al. [28] analyzed the $^6$He+$^{209}$Bi scattering with three-body CDCC in which the $^6$He+$^{209}$Bi system was assumed to be a $^2n + ^4$He+$^{209}$Bi three-body system, i.e., a pair of extra neutrons in $^6$He was treated as a single particle, dineutron ($^2n$). The enhancement of the total reaction cross section of the $^6$He+$^{209}$Bi scattering is found to be due to the electric dipole ($E1$) excitation of $^6$He to its continuum states [29], i.e., Coulomb breakup of $^6$He, which is almost absent in the $^6$Li+$^{209}$Bi scattering. The three-body CDCC calculation, however, does not reproduce the angular distribution of the measured elastic cross section and overestimates the measured total reaction cross section by a factor of 2.5. This problem is solved by four-body CDCC [19] in which the total system is assumed to be an $n + n + ^4$He+$^{209}$Bi four-body system.

The $^6$Li+$^{209}$Bi scattering near the Coulomb barrier was, meanwhile, analyzed with three-body CDCC by assuming a $d + ^4$He+$^{209}$Bi three-body model [28]. The three-body CDCC calculation could not reproduce the data without normalization factors for the potentials between $^6$Li and $^{209}$Bi. This result indicates that four-body CDCC should be applied to the $^6$Li+$^{209}$Bi scattering.

In this paper, we analyze $^6$Li+$^{209}$Bi elastic scattering at 29.9 and 32.8 MeV with four-body CDCC by assuming the $p + n + ^4$He+$^{209}$Bi four-body model. This is the first application of four-body CDCC to $^6$Li scattering. The four-body CDCC calculation reproduces the measured elastic cross sections, whereas the previous three-body CDCC calculation does not. Four-body dynamics of the elastic scattering is investigated, and it is discussed what causes the failure of the previous three-body CDCC calculation. Finally we propose a reasonable three-body model for describing the four-body scattering.

Theoretical framework. One of the most natural frameworks to describe $^6$Li+$^{209}$Bi scattering is the $p + n + ^4$He+$^{209}$Bi four-body model. Dynamics of the scattering is governed by the Schrödinger equation

$$ (H - E)\Psi = 0 \quad (1) $$

for the total wave function $\Psi$, where $E$ is a total energy of the system. The total Hamiltonian $H$ is defined by

$$ H = K_R + U + h \quad (2) $$

with

$$ U = U_n(R_n) + U_p(R_p) + U_\alpha(R_\alpha) + \frac{e^2Z_{Li}Z_{Bi}}{R}, \quad (3) $$

where $K_R$ is the kinetic energy, $U$ is the potential energy, and $h$ is the regularization term.
where $h$ denotes the internal Hamiltonian of $^6\text{Li}$, $R$ is the center-of-mass coordinate of $^6\text{Li}$ relative to $^{209}\text{Bi}$, $K_R$ stands for the kinetic energy operator associated with $R$, and $U_x$ describes the nuclear part of the optical potential between $x$ and $^{209}\text{Bi}$ as a function of the relative coordinate $R_x$. As $U_n$, we adopt the optical potential of Barnett and Lilley [30]. Parameters of $U_n$ are fitted to reproduce experimental data [31] on $n$ + $^{209}\text{Bi}$ elastic scattering at 5 MeV, where only the central interaction is taken for simplicity. As shown in Fig. 1 the neutron optical potential $U_{n^{OP}}$ thus fitted is consistent with the data. The resultant parameter set is the same as that in the global optical potential of Koning and Delaroche [32, except parameters $\alpha_V$, $W_V$ and $W_D$ are changed into 0.55 fm, 0 MeV and 4.0 MeV, respectively. The proton optical potential $U_p$ is assumed to be the same as $U_n$. Coulomb interactions in the $p$-$^{209}\text{Bi}$ and $\alpha$-$^{209}\text{Bi}$ subsystems are approximated into $e^2Z_{\text{Li}}Z_{\text{Bi}}/R$, because Coulomb breakup effects are negligibly small in the present scattering.

FIG. 1: Angular distribution of elastic cross section for $n$ + $^{209}\text{Bi}$ scattering at 5 MeV. The solid line is the result of the neutron optical potential $U_{n^{OP}}$. The experimental data is taken from Ref. [31].

The internal Hamiltonian $h$ of $^6\text{Li}$ is described by the $p + n + ^4\text{He}$ orthogonality condition model [33]. The Hamiltonian of $^6\text{Li}$ agrees with that of $^4\text{He}$ in Ref. [19], when the Coulomb interaction between $p$ and $^4\text{He}$ is neglected. Namely, the Bonn-A interaction [34] is taken in the $p$-$n$ subsystem and the so-called KKNN interaction [35] is used in the $p$-$\alpha$ and $n$-$\alpha$ subsystems, where the KKNN interaction is determined from experimental data on low-energy nucleon-$\alpha$ scattering.

Eigenstates of $h$ consist of finite number of discrete states with negative energies and continuum states with positive energies. In four-body CDCC, the continuum states of projectile are discretized into a finite number of pseudostates by either the pseudostate method [13, 21, 23, 25] or the momentum-bin method [22]. The Schrödinger equation (1) is solved in a modelspace $\mathcal{P}$ spanned by the discrete and discretized-continuum states:

$$\mathcal{P}(H - E)\mathcal{P}\Psi_{\text{CDCC}} = 0,$$

In the pseudostate method, the discrete and discretized continuum states are obtained by diagonalizing $h$ in a space spanned by $L^2$-type basis functions. As the basis function, the Gaussian [14, 16, 19, 23, 25] or the transformed Harmonic Oscillator function [13, 17, 18, 20, 21] is usually taken. In this paper, we use the Gaussian function. The modelspace $\mathcal{P}$ is then described by

$$\mathcal{P} = \sum_{nI} |\Phi_{nIm}\rangle\langle\Phi_{nIm}|,$$

where $\Phi_{nIm}$ is the $n$th eigenstate of $^6\text{Li}$ with an energy $\epsilon_{nI}$, a total spin $I$ and its projection on the $z$-axis $m$.

In actual calculations, the $\Phi_{nIm}$ are obtained for $I^e = 1^+$, $2^+$ and $3^+$ by diagonalizing $h$ with 10 Gaussian functions for each coordinate in which the range parameters are taken form 0.1 fm to 12 fm in geometric series. The $\Phi_{nIm}$ with $\epsilon_{nI} \leq 20$ MeV are excluded from $\mathcal{P}$, since they do not affect cross sections of $^6\text{Li} + ^{209}\text{Bi}$ scattering. The resulting numbers of the discrete states are 65 (including the ground state of $^6\text{Li}$), 57 and 63 for $1^+$, $2^+$, and $3^+$ states, respectively.

The CDCC wave function $\Psi_{\text{CDCC}}^M$ with the total angular momentum $J$ and its projection on the $z$-axis $M$, are expressed as

$$\Psi_{\text{CDCC}}^M = \sum_{nIL}^{J} \chi_{nIL}(P_{nI}, R) / R \gamma_{nIL}^M$$

with

$$\gamma_{nIL}^M = [\Phi_{nI}(Y_L(R)) \otimes Y_L(R)]_M$$

for the orbital angular momentum $L$ regarding $R$. The expansion-coefficient $\chi_{nIL}^M$, where $\gamma = (n, I, L)$, describes a motion of $^6\text{Li}$ in its $(n, I)$ state with linear and angular relative momenta $P_{nI}$ and $L$. Multiplying the four-body Schrödinger equation (1) by $\gamma_{nIL}^M$ from the left and integrating it over all variables except $R$, one can obtain a set of coupled differential equations for $\chi_{nIL}^M$:

$$[\frac{d^2}{dR^2} - \frac{L(L + 1)}{R^2} - \frac{2\mu}{\hbar^2}U_{\gamma\gamma}(R) + P_{nI}^2] \chi_{nIL}^M(P_{nI}, R) = \frac{2\mu}{\hbar^2} \sum_{\gamma' \neq \gamma} U_{\gamma'\gamma}(R) \chi_{nIL}^M(P_{nI'}, R)$$

with the coupling potentials

$$U_{\gamma'\gamma}(R) = \langle \gamma_{nIL}^M | \gamma_{nIL}^M \rangle = \langle \gamma_{nIL}^M | U_n(R_n) + U_p(R_p) + U_\alpha(R_\alpha) \rangle \gamma_{nIL}^M,$$

where $\mu$ is the reduced mass between $^6\text{He}$ and $^{209}\text{Bi}$. The elastic and discrete breakup $S$-matrix elements are obtained by solving Eq. (8) under the standard asymptotic boundary condition [17, 36].

We also do three-body CDCC calculations by assuming a $d + ^4\text{He} + ^{209}\text{Bi}$ model, following Refs. [28, 29]. As an interaction between $d$ and $^4\text{He}$, we take the potential of Ref. [37], which are determined from experimental data on the ground-state energy ($-1.47$ MeV) and the $3^+$-resonance state energy (0.71 MeV) of $^6\text{Li}$ and low-energy $d$-$\alpha$ scattering phase shifts.
The continuum states between $d$ and $^4$He are discretized with the pseudostate method [14] and are truncated at 20 MeV in the excitation energy of $^6$Li from the $d$-$^4$He threshold. The $d$-$^209$Bi (type-a) optical potential ($U_d^{OP}$) [38] is taken as $U_d$, whereas $U_n$ is common between three- and four-body CDCC calculations.

**Results.** Figure 2 shows the angular distribution of elastic cross section for $^6$Li + $^209$Bi scattering at 29.9 MeV. The dotted line shows the result of three-body CDCC calculation with $U_d^{OP}$ as $U_d$. This result, which is consistent with the previous result of Ref. [28], underestimates the measured cross section [26, 27]. The solid (dashed) line, meanwhile, stands for the result of four-body CDCC calculation with (without) projectile breakup effects. In CDCC calculations without $^6$Li-breakup, the modelspace $\mathcal{P}$ is composed only of the $^6$Li ground state. The solid line reproduces the experimental cross section, but the dashed line does not. The projectile breakup effects are thus significant and the present $^6$Li scattering is well described by the $p + n + ^4$He + $^209$Bi four-body model. This conclusion is true also for $^6$Li + $^209$Bi scattering at 32.8 MeV shown in Fig. 3.

![Figure 2](image1.png)

**FIG. 2:** Angular distribution of the elastic cross section for $^6$Li + $^209$Bi scattering at 29.9 MeV. The cross section is normalized by the Rutherford cross section. The dotted (dot-dashed) line stands for the result of three-body CDCC calculation in which $U_d^{OP}$ ($U_d^{SF}$) is taken as $U_d$. The solid (dashed) line represents the results of four-body CDCC calculations with (without) breakup effects. The experimental data are taken from Ref. [26, 27].

Now we consider $d$-breakup in the $^6$Li scattering in order to understand four-body dynamics of the scattering. In the limit of no $d$-breakup, the interaction between $d$ and $^209$Bi can be obtained by folding $U_n$ and $U_p$ with the deuteron density. This potential is referred to as the single-folding potential $U_d^{SF}$. In Figs. 2 and 3 the dot-dashed lines show the results of three-body CDCC calculations with $U_d^{SF}$ as $U_d$. The results well simulate those of four-body CDCC calculations, i.e., the solid lines. This indicates that $d$-breakup is suppressed in the $^6$Li scattering. Intuitive understanding of this property is as follows. As a characteristic of the present $^6$Li scattering, it is quite peripheral in virtue of the Coulomb barrier. The scattering is dominated by the configuration in which $\alpha$ is located between $d$ and the target, because $U_n$ is more attractive than $U_d$. In this configuration, $d$ is out of the range of $U_n$ and $U_p$, so that $d$-breakup is suppressed. The $^6$Li elastic scattering near the Coulomb barrier is thus well described by the $d + \alpha + ^209$Bi three-body model, if $U_d^{SF}$ is taken as $U_d$.

Figure 4 shows the angular distribution of elastic cross section for $d + ^209$Bi scattering at 12.8 MeV. The solid (dashed) line stands for the result of three-body CDCC calculation with (without) deuteron breakup, whereas the dotted line is the result of the deuteron optical potential $U_d^{OP}$. The experimental data are taken from Ref. [26, 27].

![Figure 4](image2.png)

**FIG. 4:** Angular distribution of the elastic cross section for $d + ^209$Bi scattering at 12.8 MeV. The solid (dashed) line stands for the result of three-body CDCC calculation with (without) deuteron breakup, whereas the dotted line is the result of the deuteron optical potential $U_d^{OP}$. The experimental data are taken from Ref. [26, 27].
produces the data fairly well, but the dashed line does not. Thus \( d \)-breakup is significant for the deuteron scattering. The deuteron optical potential \( U^{\text{OP}}_d \) (dotted line) yields fairly good agreement with the data, but the radius of \( U^{\text{OP}}_d \) is larger than that of \( U^{\text{SF}}_d \). This is the reason why three-body CDCC calculations with \( U^{\text{OP}}_d \) as \( U_d \) can not reproduce the measured elastic cross section for \( ^6\text{Li} + ^{209}\text{Bi} \) scattering. The difference between \( U^{\text{SF}}_d \) and \( U^{\text{OP}}_d \) mainly comes from the fact that \( U^{\text{OP}}_d \) includes \( d \)-breakup effects, whereas \( U^{\text{SF}}_d \) does not.

Summary. The \( ^4\text{He} + ^{209}\text{Bi} \) scattering at 29.9 MeV and 32.8 MeV near the Coulomb barrier are well described by four-body CDCC based on the \( p + n + ^{5}\text{He} + ^{209}\text{Bi} \) model. This is the first application of four-body CDCC to \( ^6\text{Li} \) scattering. In the \( ^6\text{Li} \) scattering, \( d \)-breakup is strongly suppressed, suggesting that the \( d + ^4\text{He} + ^{209}\text{Bi} \) model becomes good, if the single-folding potential \( U^{\text{SF}}_d \) with no \( d \)-breakup is taken as an interaction between \( d \) and the target. For \( d + ^{209}\text{Bi} \) scattering at 12.8 MeV, meanwhile, \( d \)-breakup is significant, so that the deuteron optical potential \( U^{\text{OP}}_d \) includes \( d \)-breakup effects.

Four-body CDCC is applicable also for \( n + ^6\text{Li} \) scattering that is a key reaction in nuclear engineering. In the scattering, \( ^6\text{Li} \) breakup into \( n + p + \alpha \) is considered to be not negligible for emitted neutron spectra [39]. We will discuss this point in a forthcoming paper.

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