Hygrothermal effects on the flexural strength of laminated composite cylindrical panels

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Abstract. In this article, the effect of hygrothermal conditions on the bending behaviour of laminated composite cylindrical panels has been investigated by using a nonlinear finite element micromechanical model. The panel model has been developed mathematically by introducing the geometrical nonlinearity in Green-Lagrange sense in the framework of the higher-order shear deformation theory. In order to achieve any general case, all the nonlinear higher order terms have been incorporated in the present formulation and the lamina material property variations due to hygrothermal effect are included via the micromechanical model. The governing equation is obtained using the variational principle and discretised using nonlinear finite element steps. The convergence behaviour of the present numerical model has been established and validated by comparing the responses with the existing results in the literature. The effects of thickness ratio, lamination scheme and support conditions under different sets of hygrothermal conditions are investigated in details.

Keywords: Laminated composite cylindrical panel; Hygrothermal effect; Green-Lagrange nonlinearity; micromechanical model

1. Introduction

Cylindrical panels made up of laminated composites are popularly used as structures/structural components in aerospace and marine applications because of their excellent compression and torsion resisting capabilities owing to high specific strength and stiffness and lightweight properties. However, throughout their service period these structures are subjected to combined hygro-thermo-mechanical loading. It is well known that the combined temperature and moisture environment induce residual stresses in the structural component and alter their thermo-elastic properties considerably. As a result of this not only the strength but also the mechanical responses like vibration, bending and stability behaviour of the structures/structural components are affected adversely and deviate from expected line. It is well known that for large deformation regime the basic geometry of panel is distorted and nonlinearity in geometry is induced. This in turn affects the structural behaviour of laminated structures. Hence, it is not only interesting but also challenging for design engineers to assess the nonlinear responses accurately for laminated composite shells of various geometries under hygrothermal environments.

Due to the expenses and difficulties associated with the experimental analysis of laminated structure under combined hygrothermal loading, many researchers have attempted to compute the
linear and nonlinear flexural behaviour of laminated composites plates/shells under hygrothermal environment numerically/analytically using various existing and refined theories. The first order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) are more popular in comparison to refined/layer wise theories due to simplicity in formulation ([1], [2]). Attempts are also been made to include the degraded composite properties through macroscopic or microscopic approach. Sai Ram and Sinha [3] investigated the effects of temperature and moisture on the bending characteristics of laminated composite plates using finite element method (FEM). Naidu and Sinha [4] developed a FE model based on the FSDT and Green-Lagrange nonlinear kinematics to investigate the large deflection bending behavior of laminated composite shell panel under hygrothermal environments. Kundu et al. [5] studied the bending behaviour of laminated composite shells in hygrothermal environment by using the FSDT based geometrically nonlinear FE model. Lo et al. [6] investigated the effect of temperature dependent material properties and hygrothermal loading on multilayer composites using FEM based on a global–local HSDT mid-plane kinematics. Zenkour [7] studied the static responses of laminated composite plates using a refined (sinusoidal) shear deformation theory under hygrothermal load. Bending behaviour of laminated composite and sandwich shells in hygrothermal environment have been analyzed by Brischetto [8] by using Carrera’s unified formulation. In all the referred cases the lamina material properties are considered through macroscopic model.

We note that, the analytical/numerical solutions of the linear/nonlinear free vibration ([9]), buckling ([10]) and flexural ([11], [12], [13], [14], [15] and [16]) behavior of the laminated composite plates and shells under elevated hygrothermal environment in conjunction with micromechanical material model to evaluate degraded composite properties are very rare in open literature. Moreover, in all the aforementioned cases the mathematical models have been developed using HSDT mid-plane kinematics in conjunction with von-Karman nonlinearity. To the best of the authors’ knowledge, no work has been reported in literature on nonlinear bending analysis of laminated composite structure under hygro-thermo-mechanical loading by taking HSDT mid-plane kinematics with Green-Lagrange geometric nonlinearity. In this present work, the authors’ aim to analyze the large deformation bending behavior of the laminated composite shell panel based on the proposed nonlinear kinematic model under combined hygro-thermo-mechanical loading. Here, the composite material properties are considered to be dependent on temperature and moisture and derived explicitly in terms of the properties of the fiber and the matrix and the fiber volume fraction. In addition to this, all the nonlinear higher order terms have been incorporated in the mathematical model to capture the original flexure of the structure. The nonlinear system governing equations are obtained using variational approach and discretised using suitable FEM. A direct iterative method is employed to solve the system governing equation to obtain the bending responses. The efficacy and accuracy of the model has been established by the convergence study and subsequent comparison of the responses with available published results. The effects of hygrothermal conditions, support conditions and curvature ratio on the bending response of laminated composite cylindrical panel are investigated and discussed in details. The outcome from this paper will be useful in the design of laminated structures for real life engineering (specifically in aircraft, aerospace, automobile, naval) applications.

2. Theory and Nonlinear FEM Scheme

In the present analysis, a typical laminated composite cylindrical panel is considered with length $a$, width $b$ and uniform thickness ‘$h$’. The shell panel consists of ‘$N$’ number of uniformly thick orthotropic layers as shown in Fig. 1. Here, $R_1$ and $R_2$ are the principal radii of curvatures of the shell panel along $x$ and $y$ directions at the mid-surface of the panel ($z=0$). The shell panel is subjected to the hygrothermal load in combination with uniform lateral load and the mid-plane displacement of the panel is modeled based on the HSDT kinematics [17]. Now, the displacement variable of any point within the shell panel with respect to the mid-plane along $x$, $y$, and $z$-directions are conceded as:
where, \((\bar{u}, \bar{v}, \bar{w})\) indicate the displacements at any arbitrary point on the plate along the \((x, y, z)\) respectively. Similarly, \((u, v, w)\) represent the corresponding displacements of the points on the mid-plane, \(\phi_1\) and \(\phi_2\) are the rotations of normal to the mid-plane relating to \(y\)-axis and \(x\)-axis, respectively. This displacement field represents the transverse shear strains as quadratic function of thickness coordinate at any point within the shell and also account for the parabolic distribution of shear stress across the thickness represented by \(\psi_1, \psi_2, \Theta_1\) and \(\Theta_2\), which are the higher order terms of Taylor series expansion defined at the mid-plane.

The nonlinear strain displacement relations (Green-Lagrange type) considered for the present analysis are as in Mahapatra and Panda [18]:

\[
\{ \varepsilon \} = \{ \varepsilon_L \} + \{ \varepsilon_{NL} \} 
\]

Substituting Eq. (1) into Eq. (2), the linear and nonlinear strain terms are rearranged as:

\[
\{ \varepsilon \} = [H_L] \{ \varepsilon_L \} + \frac{1}{2} [H_{NL}] \{ \varepsilon_{NL} \} 
\]

where, \([H_L]\), \([H_{NL}]\) are the functions of thickness coordinate and \(\{ \varepsilon_L \}, \{ \varepsilon_{NL} \}\) are the functions of \(x\) and \(y\). The detail terms of \(\{ \varepsilon_L \}, \{ \varepsilon_{NL} \}, [H_L]\) and \([H_{NL}]\) can be seen in [18].

In the present analysis, it is assumed that the deformation occurring due to unlike temperature and moisture change are not coupled. Hence, the constitutive matrix equation of generalized stress tensor for any general \(k^{th}\) orthotropic composite lamina with any fibre orientation angle \(\theta\) is given by

\[
\{ \sigma_{ij} \} = [Q_{ij}]^k \{ \varepsilon_{ij} - \alpha_{ij} \Delta T - \beta_{ij} \Delta C \}^k
\]
where, $\{\sigma_{ij}\}^k = [\sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4]_T$ and $\{\varepsilon_{ij}\}^k = [\varepsilon_1 \ \varepsilon_2 \ \varepsilon_3 \ \varepsilon_4]_T$ are the stress and strain vectors respectively for the $k$th layer. $[\overline{Q}]^k$ is the transferred reduced stiffness matrix for the $k$th layer. $\{\alpha_{ij}\}^k = [\alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4]_T$ is the thermal expansion/contraction coefficient vector and $\{\beta_{ij}\}^k = [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4]_T$ is the moisture expansion/contraction coefficient vector. Here, $\Delta T = T - T_0$ is the temperature difference, where $T$ is applied and $T_0$ is reference temperature, respectively. Similarly, $\Delta C = C - C_0$ is the moisture difference between applied ($C$) and reference ($C_0$) values of weight percentage of moisture.

In order to obtain the realistic behaviour of laminated structure, the degraded hygro-thermo-elastic composite material properties (to be used in Eq. 4) are evaluated using a micromechanical model as presented in [14].

Here, a nine noded isoparametric quadrilateral Lagrangian element with nine degrees of freedom per node has been taken to discretize the present laminate model. The field displacement vector corresponding to any nodal point of the element can be expressed as

$$\{\delta^i\} = [u \ v \ w \ \phi_1 \ \phi_2 \ \psi_1 \ \psi_2 \ \theta_1 \ \theta_2]^T = \sum_{i=1}^{9} [N_i] \{\delta^i\}$$

(5)

where, $[N_i]$ and $\{\delta^i\}$ are the interpolation function and displacement vector for the $i$th node, respectively and the details of interpolation functions can be seen in [19].

Now, the mid-plane strain vector can be written as

$$\{\varepsilon_i\} = [B_i] \{\delta^i\}$$

(6)

where, $[B_i]$ is the strain displacement relation matrix.

The strain energy of the panel can be expressed as

$$U = \frac{1}{2} \int \{\varepsilon_i\}^T \{\sigma_i\} dV$$

(7)

Using the expression of strain vectors and resultant stress from Eq. (3) and Eq. (4) and putting into Eq. (7) the strain energy can be expressed as

$$U = \frac{1}{2} \int \{\varepsilon_i\}^T \{\overline{Q}\} \{\varepsilon_i\} dV = \frac{1}{2} \iint \{\varepsilon_i + \varepsilon_{NL}\}^T \{\overline{Q}\} \{\varepsilon_i + \varepsilon_{NL}\} \ dx \ dy \ dz$$

$$= \frac{1}{2} \int_A \left( \{\varepsilon_i\}^T \{D_1\} \{\varepsilon_i\} + \frac{1}{2} \{\varepsilon_{NL}\}^T \{D_2\} \{\varepsilon_{NL}\} + \frac{1}{2} \{\varepsilon_{NL}\}^T \{D_1\} \{\varepsilon_i\} + \frac{1}{4} \{\varepsilon_{NL}\}^T \{D_3\} \{\varepsilon_{NL}\} \right) dA$$

(8)

where, $[D_1] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \{H\}^T \{\overline{Q}\} \{H\} \ dz$, $[D_2] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \{H\}^T \{\overline{Q}\} \{H\}_{NL} \ dz$ and $[D_3] = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \{H\}_{NL}^T \{\overline{Q}\} \{H\}_{NL} \ dz$

Substituting Eq. (6) into Eq. (8) the expression for strain energy becomes

$$U = \frac{1}{2} \int_A \left( \{\delta^i\}^T \{B_i\} \{\varepsilon_i\} \{\delta^i\} + \frac{1}{2} \{\delta^i\}^T \{B_i\} \{A\} \{\varepsilon_i\} \{\delta^i\} \right) dA$$

$$+ \frac{1}{2} \{\delta^i\}^T \{G\}^T \{A\} \{D_3\} \{\delta^i\} + \frac{1}{2} \{\delta^i\}^T \{G\}^T \{A\} \{D_4\} \{\delta^i\} \right) dA$$

(9)

where, $\{\varepsilon_{NL}\}_i = [B_{NL}] \{\delta^*\}$ and $\{\varepsilon_{NL}\}_i = \frac{1}{2} \left[[B_{NL}] \{\delta^*\}\right]_i \{\delta^*\} = \frac{1}{2} \left[\Lambda(\delta)_i\right]_i \{\delta^*\}$. 

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[B_2] is the product form of the differential operator and nodal interpolation function in the linear strain terms. [A] is function of the displacements and [G] is the product form of differential operator and shape function in the nonlinear strain terms. The expressions of [A] and [G] arising due to the Green-Lagrange nonlinearity in the nonlinear stiffness matrices can be seen in [20].

The work done due to the external applied distributed transverse static load “q” can be expressed as

\[ W = \int_A \{ \delta \}^T q dA \]  

where, the intensity of transverse static load is expressed in terms of the applied uniform lateral pressure as

\[ q = \frac{Qh^4 E_{22}}{a^4} \]  

### 3. System governing Equation

The governing equation of nonlinear bending for laminated composite flat panel is obtained by minimizing the total energy expression. This result in

\[ \delta \Pi = 0 \]  

where, \( \Pi = (U-W) \)

Using, Eq. (9) and Eq. (10) in Eq. (12) and applying finite element approximation, the system governing expression can be obtained as:

\[ [K] \{ \delta \} = \{ q \} \text{ or } [K_L + K_{NL}] \{ \delta \} = \{ q \} \]  

where, \([K_L]\) and \([K_{NL}]\) are the global linear and nonlinear stiffness matrices. The nonlinear stiffness matrix depends on the displacement vector linearly and quadratically, respectively.

Now, the numerical solutions are obtained using a direct iterative method and solutions scheme are depicted in [20].

### 4. Results and Discussion

A nonlinear finite element computer code have been developed in MATLAB 7.10.0 and based on the present formulation in order to obtain the hygro-thermo-mechanical linear/nonlinear bending responses of laminated composite shell panel. As a first step, the validation behaviour of the present model has been established. Subsequently, different parametric studies are also been carried out and their significance have been discussed in details. For computation, the material properties of the composite material are considered to be dependent on hygrothermal conditions. Their values corresponding to reference temperature 21ºC and moisture concentration 0% are considered as in Upadhyay et al. [14] and remain unchanged for each case if not specified otherwise.

\[ E_{f1} = 220 \text{ GPa}, \ E_{f2} = 13.79 \text{ GPa}, \ E_m = 3.45 \text{ GPa}, \ G_{12} = 8.97 \text{ GPa}, \ \nu_{12} = 0.2, \ \nu_m = 0.35, \ \alpha_{f1} = -0.99 \times 10^{-6}/\text{ºC}, \ \alpha_{f2} = 10.08 \times 10^{-6}/\text{ºC}, \ \alpha_m = 72 \times 10^{-6}/\text{ºC}, \ \beta_m = 0.33, \ T_{r0} = 216\text{ºC}. \]

The boundary conditions used for analysis are given below:

(a) All edges simply support (SSSS): 
\[ v = w = \Phi_1 = \Psi_1 = \theta_1 = 0 \text{ at } x = 0, a \text{ and } u = w = \Phi_1 = \Psi_1 = \theta_1 = 0 \text{ at } y = 0, b. \]

(b) All edges clamped (CCCC):
\[ u = v = w = \Phi_2 = \Psi_2 = \theta_2 = 0 \text{ for both } x = 0, a \text{ and } y = 0, b. \]

(c) All edges hinged (HHHH):
\[ u = v = w = \Phi_2 = \Psi_2 = \theta_2 = 0 \text{ at } x = 0, a \text{ and } u = v = w = \Phi_2 = \Psi_1 = \theta_2 = 0 \text{ at } y = 0, b. \]

The linear/nonlinear transverse central deflections are non-dimensionalized using the relations

\[ Q = (q / E_j) * (a / h)^4 \]  

and \( w_{\text{central}} = w_{\text{max}} / h \), respectively. It is important to mention here that the value of \( E_j \) is corresponding to actual (instantaneous) hygrothermal conditions.
4.1. Convergence and Comparison Study

Numerical results are provided to establish the convergence behavior of the present model under unlike environmental conditions. The linear and nonlinear flexural responses are computed using the present model by considering available flat panel examples in [13] and [14] and plotted with respect to mesh divisions in Fig. 2 (a) and (b), respectively. The geometrical, material properties, lamination scheme and support conditions are taken same as [13] and [14]. Analytical results of the individual reference are also shown for the comparison purpose. The results show very good convergence rate with respect to mesh refinement and also agree with existing results in the literature. Based on the convergence study, it is concluded that a (6x6) mesh is adequate to obtain desired responses throughout the analysis. For validation purpose the nondimensional linear and nonlinear central deflections of simply supported square anti-symmetric angle-ply ([\(\pm 45^\circ\)]_2) laminated spherical panel (\(R/a=5, \ a/h=15\)) is computed and compared with the numerical results available reference [15]. The responses are computed for load parameter Q=150 by using the material (\(V_f=0.6\)) and geometrical parameters same as the reference and presented in Table 1. It is evident that the present results are showing excellent agreement with the available analytical and numerical results. It is worthy to mention that the difference in results exists because the present model is developed based on the HSDT and Green-Lagrange nonlinearity whereas all the references are modelled using HSDT and von-Karman nonlinearity. In addition to that, the present model have included all the nonlinear higher order terms in the formulation which makes the model more realistic in nature for laminated structures.

![Fig. 2: Convergence study of linear/nonlinear flexural responses of laminated composite flat panel.](image)

**Table 1.** Comparison of linear and nonlinear flexural responses of simply supported laminated composite spherical panel ([\(\pm 45^\circ\)]_2, \(R/a=5, \ a/h=15\), \(V_f=0.6\))

| Hygrothermal Conditions | Present Linear [15] Linear | Difference (%) | Present Nonlinear [15] Nonlinear | Difference (%) |
|-------------------------|---------------------------|---------------|-------------------------------|---------------|
| \(\Delta T=50, \Delta C=1\%\) | 0.9279 | 0.9576 | -3.2008 | 0.6231 | 0.5144 | 17.4450 |
| \(\Delta T=50, \Delta C=1.5\%\) | 0.9365 | 0.9583 | -2.3278 | 0.6262 | 0.5299 | 15.3785 |
| \(\Delta T=100, \Delta C=1\%\) | 0.9878 | 1.028 | -4.0697 | 0.6434 | 0.5659 | 12.0454 |
| \(\Delta T=100, \Delta C=1.5\%\) | 1.0052 | 1.0287 | -2.3378 | 0.6489 | 0.5869 | 9.5546 |

4.2. Numerical illustrations

Now, in this section some more numerical experimentation have been presented to demonstrate the applicability of the proposed nonlinear finite element micromechanical shell panel model and the
The effect of parameters (hygrothermal conditions, curvature ratio and support conditions) on the hygrothermo-mechanical bending behavior of laminated (cylindrical) structural components. If not stated otherwise, all the illustrations discussed here are computed by setting the fiber volume fraction ($V_f$) as 0.6 and for four sets of hygro-thermal loading conditions ($\Delta T=0$, $\Delta C=0\%$, $\Delta T=100$, $\Delta C=1\%$, $\Delta T=200$, $\Delta C=2\%$ and $\Delta T=300$, $\Delta C=3\%$). The transversely applied load parameter are considered as ($Q=100$, 200, 300, 400 and 500).

4.2.1. Illustration 1. It is well known that the lamination schemes are one of the deciding factors for the strength of fiber reinforced composite structure. Hence, in this example the effect of lamination scheme on the nonlinear flexural response of laminated composite cylindrical panel under elevated hygrothermal environment is investigated and presented in Fig. 3 (a) and (b). The figure shows the load-deflection responses of a square clamped symmetric/anti-symmetric cross-ply/angle-ply laminated composite cylindrical shell panels ($a/h=10$, $R/a=10$) under four sets of the hygrothermal load as mentioned earlier. It is observed that the cylindrical shells with symmetric lamination are less affected under hygrothermal loads in comparison to anti-symmetric laminations. It is worthy to note the hard spring type of behaviour for all lamination schemes except the anti-symmetric angle-ply lamination, which shows soft spring type of behaviour with increasing hygro-thermo-mechanical loads. Nonlinear central deflections are decreasing with increasing in hygrothermal load for all sorts of lamination schemes. However, sudden increase in deflection values are noticed for $\Delta T=300$ºC and $\Delta C=3\%$. It is because the applied temperature is shifted beyond the glass transition temperature of the material and this may lead to such variation in the responses. It is also understood that, the original geometry of the laminated structure distorted considerably due to the high strain rate loading under the combined elevated environment and this in turn affect the final responses of the structural component due to the induction of the high residual stresses and degraded material properties.

![Graph](image1)

Fig. 3:- Effect of lamination scheme and hygrothermal conditions on the nonlinear central deflection of laminated composite cylindrical shell panel.

4.2.2. Illustration 2. It is true that the membrane strength of the shell panel structure greatly depends on its curvature. In order to understand the bending strength of laminated composite cylindrical panel under combined hygro-thermo-mechanical loading, this numerical example has been solved for different curvature ratios ($R/a=10$, 40, 70 and 100). The responses are computed for the simply supported square thin ($a/h=100$) anti-symmetric cross-ply laminated composite cylindrical panels and presented in Fig. 4 (a) and (b). It is observed that the nonlinear transverse central deflections increase

![Graph](image2)
as the curvature ratio increases, irrespective of the hygrothermal loading conditions. This is because of the fact that as the curvature ratio increases the panel becomes flat and the membrane stiffness decreases subsequently.

Fig. 4:- Effect of curvature ratio on the nonlinear central deflection of laminated composite cylindrical shell panel.

Fig. 5:- Effect of support conditions on the nonlinear central deflection of laminated composite cylindrical shell panel.

4.2.2. Illustration 3. The strength and stiffness behavior of the laminated structures are largely dependent on the support conditions which in turn affects the structural responses largely. In this
example the effect of different support conditions (SSSS, CCCC, HHHH and SCSC) on the nonlinear central deflection of anti-symmetric angle-ply \([\pm45]\) laminated square \((a/b=1)\) cylindrical panel \((R/a=40)\) under hygro-thermo-mechanical load have been investigated and the results are shown in Fig. 5. It is noted that the non-dimensional nonlinear central deflections are lowest for clamped and highest for simply-supported case. This is due to the fact that as the number of constraints increase or decrease, the stiffness of the structure decreases and/or increases and the deflection values decrease or increase monotonically. However, it is interesting to note that in all the cases the panels are showing soft spring type of behavior irrespective of hygrothermal load and the support conditions, except the HHHH support condition, for which the nonlinearity is observed to be more pronounced.

5. Conclusion

The effect of the hygrothermal conditions, curvature and constraint conditions on the flexural behaviour of laminated composite cylindrical shell panels have been investigated in the present study. A general mathematical model based on the HSDT mid-plane kinematics and Green-Lagrange geometric nonlinearity is developed to count the exact in-plane and out-of-plane shear stresses and strains. The effective material properties of the composite lamina due to temperature and moisture variation are incorporated via micromechanical model. In addition all the nonlinear higher order terms are included in the present formulation to capture the actual flexure of the shell structure. The system governing equations are obtained using the variational principle and discretised using isoparametric FE steps. The convergence and validation of the present model has been established. The significance of the present HSDT based model for the flexural analysis of laminated structures under combined hygrothermal loading condition is observed. The subsequent numerical illustrations indicate that considering the effect of hygrothermal conditions is inevitable for laminated composite structures, particularly when they are exposed to a severe environmental condition. Nonlinearity due to moisture absorption at high temperature is severe for anti-symmetric angle-ply in comparison to symmetric laminations. Irrespective of the hygrothermal load the nondimensional central deflections increase with curvature ratios. It worthy to note that as the number of restraints increase the nonlinear central deflections decrease irrespective of hygrothermal conditions. The outcome from this present work can be utilized in the optimum design of real structures.

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Reference

[1] A. A. Khdeir, J. N. Reddy, D. A. Frederick, Study of bending, vibration and buckling of cross-ply circular cylindrical shells with various shell theories, *International Journal of Engineering Science* 27 (11) (1989) 1337–1351.

[2] R. Khandan, S. Noroozi, P. Sewell, J. Vinney, The Development of Laminated Composite Plate Theories – A Review, *Journal of Material Science* 47(16) (2012) 5901- 5910

[3] K. S. Sai Ram, P. K. Sinha, Hygrothermal effects on the bending characteristics of laminated composite plates, *Computers and Structures* 40(4) (1991) 1009–1015

[4] N. V. S. Naidu, P. K. Sinha, Nonlinear finite element analysis of laminated composite shells in hygrothermal environments, *Composite Structures* 69(4) (2005) 387–395
[5] C. K. Kundu, D. K. Maiti, P. K. Sinha, Nonlinear finite element analysis of laminated composite doubly curved shells in hygrothermal environment, *Journal of Reinforced Plastics and Composites* 26(14) (2007) 1461–1478

[6] S. H. Lo, W. Zhen, Y. K. Cheung, Hygrothermal effects on multilayered composite plates using a refined higher order theory *Composite Structures* 92(3) (2010) 633-646

[7] A. M. Zenkour, Hygrothermal effects on the bending of angle-ply composite plates using a sinusoidal theory *Composite Structures* 94(12) (2012) 3685–3696

[8] S. Brischetto, Hygrothermoelastic analysis of multilayered composite and sandwich Shells, *Journal of Sandwich Structures and Materials* 15(2) (2013) 168–202

[9] X. L. Huang, H. S. Shen and J. J. Zheng, Nonlinear vibration and dynamic response of shear deformable laminated plates in hygrothermal environments, *Composites Science and Technology* 64(10–11) (2004) 1419–1435

[10] H. S. Shen, Hygrothermal effects on the postbuckling of shear deformable laminated plates, *International Journal of Mechanical Sciences* 43(5) (2001) 1259–1281

[11] V. V. S. Rao, P. K. Sinha, Bending Characteristics of Thick Multidirectional Composite Plates under Hygrothermal Environment, *Journal of Reinforced Plastics and Composites* 23(14) (2004) 1481–1495

[12] A. Sharma, A. K. Upadhyay, K. K. Shukla, Flexural response of doubly curved laminated composite shells, *Science China Physics Mechanics Astronomy* 56(4) (2013) 812–817

[13] H. S. Shen, Hygrothermal effects on the nonlinear bending of shear deformable laminated plates, *Journal of Engineering Mechanics* 128(4) (2002) 493–496

[14] A. K. Upadhyay, R. Pande, K. K. Shukla, Nonlinear response of laminated composite plates under hygro-thermo-mechanical loading, *Communications in Nonlinear Science and Numerical Simulation* 15(9) (2010) 2634–2650

[15] A. Lal, B. N. Singh, S. Anand, Nonlinear bending response of laminated composite spherical shell panel with system randomness subjected to hygro-thermo-mechanical loading, *International Journal of Mechanical Science* 53(10) (2011) 855–866

[16] R. Kumar, H. S. Patil, A. Lal, Nonlinear flexural response of laminated composite plates on a nonlinear elastic foundation with uncertain system properties under lateral pressure and hygrothermal loading: micromechanical model, *Journal of Aerospace Engineering* 27(3) (2014) 529–547

[17] Reddy J N 2004 *Mechanics of laminated Composite Plates and Shells* Second Edition CRC Press

[18] T. R. Mahapatra, S. K. Panda, Thermoelastic vibration analysis of laminated doubly curved shallow panels using non-linear FEM, *Journal of Thermal Stresses* 38(1) (2015) 39-68

[19] Cook R D, Malkus D S and Plesha M E 2000 *Concepts and applications of finite element analysis* Third Edition John Willy and Sons (Asia) Pvt. Ltd., Singapore

[20] V. R. Kar, T. R. Mahapatra, S. K. Panda, Nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading *Steel and Composite Structures An International Journal* 19(4) (2015) 1011-1033