Research on Multi-loop Cross-eye Jamming Optimization Based on Wavefront Phase Distortion

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Abstract. The strict implementation conditions of cross-eye jamming can be effective improved by multi-loop cross-eye jamming, so it has become a stirring research topic. In this paper, the characteristics of wave-front phase distortion in multi-loop cross-eye jamming system are investigated by using two indexes, the degree of center distortion (DCD) and the effective distortion region (EDR). Firstly, the synthetic field model of multi-element jamming signal is derived, and the constraint model is established by taking DCD and EDR as objective functions. Secondly, the parameters such as amplitude ratio, phase difference and the number of loops are optimized by MOEA/D algorithm. Finally, the jamming effect of radar tracking angle is verified by simulation. The results show that, compared with the single loop jamming system, multi-loop cross-eye jamming system can effectively improve DCD and EDR. It is also found that when the number of loops exceeds 4, the phase distortion of the jamming signal decreases obviously, which makes the jamming system more likely to act as a radar indicator.

1. Introduction

The cross-eye jamming technology evolved from angular flicker jamming can effectively fool angular tracking radars towards the wrong target position [1]. With the development of technology, the realization of cross-eye jamming becomes possible gradually, which has become the research hotspot of anti-monopulse goniometer radar.

Starting from the processing of monopulse radar, W. P. du Plessis from South Africa has conducted detailed derivation of cross-eye jamming and established the geometric model of jamming. On this basis, the factors influencing the interference effect, such as tolerance and dry letter ratio, have been analyzed, providing a detailed idea for the research of cross-eye jamming [3-7]. Domestic scholar Cao Fei established a mathematical model of cross-eye jamming and discussed the doppler frequency shift caused by jamming signals [8]; in addition, in order to improve the harsh conditions of the parameters, some domestic scholars such as Liu Tianpeng, Liu Songyang and Wang Caiyun have studied the multi-array reverse cross-eye jamming, and provided a scheme to increase the cross-eye jamming and expanding the jamming conditions [2, 9-12]. For the wave-front phase problem of jamming signal, Yin Hongcheng and Huang Qingdong conducted theoretical derivation of the wave-front phase of the signals in the style of drag-type decoy and scintillation jamming [13]; wave-front phase distortion of traditional cross-eye jamming is studied by N.M. Harwood [14], but the condition setting is relatively simple and there is no detailed mathematical derivation. In the current research of newer multi-
element retrodirective cross-eye jamming, there is no analysis of wave-front phase distortion. When wave-front phase distortion method is used to study the cross-eye jamming, it can be discussed only from the perspective of the jamming signal. There is a great significance to optimize the modulation parameters of cross-eye jamming signals and to design multi-element jamming antennas.

In this paper, a synthetic field model for the arrival of jamming signals at the monopulse radar is deduced, and two parameters, the effective phase distortion region and the degree of center distortion, are proposed to describe and analyze the characteristics of the wave-front phase distortion of the jamming signal. Simultaneously, a constrained model with degree of center distortion and effective distortion region as the goal is established. The MOEA/D algorithm is used to optimize the signal parameters of different jamming systems, such as amplitude ratio and phase difference, so that the jamming can achieve better results. Finally, the phase distortion of the jamming signal in a certain azimuth and distance range is obtained through simulation, and the phase distortion of different loops obtained by the MOEA/D algorithm is verified and analyzed.

2. Phase distortion of multi-loop cross eye jamming

2.1 Multi-loop cross-eye jamming system

The multi-loop cross-eye jamming system adopts Van Atta antenna array to form a reverse antenna structure. The linear array is used to study this system in this paper, as shown in Figure 1. Each loop signal is composed of two receiving and transmitting signals, and one of the signals is modulated in amplitude and phase, thereby obtaining two coherent jamming signal sources to interfere with the single-pulse radar.

![Figure 1. Multi-loop cross-eye jamming system structure](image)

2.2 Mathematical analysis

The multi-loop reverse cross-eye jamming scenario is shown in Figure 2. The system is composed of $2N$ array elements and constitutes $N$ cross-eye jamming loops. Antenna $i$ and antenna $2N+i$ are a pair of transmit-receive antenna pairs, which is the jamming loop $i$. The multi-loop cross-eye jamming system antenna array elements are equally spaced, wherein the jamming loop 1 has a baseline length of $d_1$ and the adjacent loop distance is $d_2$. The distance from the center of the jamming system center to the monopulse radar location is $r$, the angle between the jamming antenna connection and the center of the jamming system to the radar path is $\theta$, and the distance from the antenna element $i$ arrival P-point is $r_i$. The angle the jamming antenna connection and the jamming antenna array element to the radar path is $\theta_i$. 
In the multi-loop cross-eye jamming system, the amplitude of the transmitted signal of the interfering antenna is $A_i$ and the phase is $\phi_i$. Therefore the signals of different jamming loops are

$$E_i = A_i e^{j(\alpha x_i)},$$

$$E_{2N+1-i} = A_{2N+1-i} e^{j(\alpha x_{2N+1-i})}$$

(1)

At the P-point monopulse radar in space, the signal transmitted by the $i$th ($i = 1, 2, ..., N$) jamming loop wave source at the point of arrival will be:

$$E_i = A_i e^{j(\alpha x_i - \frac{\pi}{2} \theta_i c r_i)}$$

$$E_{2N+1-i} = A_{2N+1-i} e^{j(\alpha x_{2N+1-i} - \frac{\pi}{2} \theta_i c r_{2N+1-i})}$$

(2)

The synthetic wave signal will be:

$$E = \sum_{i=1}^{N} \left( \hat{\theta}_i E_i + \hat{\theta}_{2N+1-i} E_{2N+1-i} \right)$$

$$= \sum_{i=1}^{N} \left( \hat{\theta}_i A_i e^{j(\alpha x_i + \phi_i)} \right) = \hat{\theta} A e^{j(\alpha x + \phi)}$$

(3)

Where

$$\varphi_i = \phi_i - \frac{\alpha r_i}{c}$$

Since $r \gg d$, then $\hat{\theta}_i = \hat{\theta}$. According to the coherent superposition principle of waves, there will be:

$$E = \hat{\theta} A e^{j(\alpha x + \phi)}$$

$$= \hat{\theta} \sum_{i=1}^{N} \left( A_i e^{j(\alpha x_i + \phi_i)} + A_{2N+1-i} e^{j(\alpha x_{2N+1-i} + \phi_{2N+1-i})} \right)$$

(4)

From the geometric relationship in Figure 2, the distance from the two antennas to the P-point on the jamming loop 1 will be:

$$r = \sqrt{r^2 + (d_c - i \cdot d_r)^2 - 2 \left( d_c - i \cdot d_r \right) \cos \theta}$$

$$r_{2N+1-i} = \sqrt{r^2 + (d_c - i \cdot d_r)^2 + 2 \left( d_c - i \cdot d_r \right) \cos \theta}$$

(5)

The amplitude ratio of $E_i$ and $E_{2N+1-i}$ is $a_i$, and the phase difference is $\phi_i$, moreover the amplitude ratio of $E_i$ and $E_j$ is $c_i$, and the phase difference is $\psi_i$, make
$$A_n = [a_1 e^{i\phi_1}, a_2 e^{i\phi_2}, \ldots, a_N e^{i\phi_N}]$$  \hspace{1cm} (6)

$$C_n = [c_1 e^{i\psi_1}, c_2 e^{i\psi_2}, \ldots, c_N e^{i\psi_N}]$$  \hspace{1cm} (7)

$$B_{1n} = [e^{-i\beta_{\phi_1}}, e^{-i\beta_{\phi_2}}, \ldots, e^{-i\beta_{\phi_N}}]$$  \hspace{1cm} (8)

$$B_{2n} = [e^{-i\beta_{\psi_1}}, e^{-i\beta_{\psi_2}}, \ldots, e^{-i\beta_{\psi_N}}]$$  \hspace{1cm} (9)

$$D_n = [d_1 e^{i\phi_1 - i\beta_{\phi_1}}, d_2 e^{i\phi_2 - i\beta_{\phi_2}}, \ldots, d_N e^{i\phi_N - i\beta_{\phi_N}}]$$  \hspace{1cm} (10)

Where, \( D_n \) is the matrix multiplied by \( A_n \) and \( B_{2n} \) elements.

Therefore, from the equation (4), the functional relationship of the monopulse radar synthesis field with respect to the variables \( \theta, N, A_n, C_n \) and \( d_e \) can be obtained.

\[
E(\theta, r, A_n, C_n, d_e, N) = E C_n (B_{1n} + D_n)^T
\]

According to the literature [14][15], the ranges of \( \theta \) and \( d_e \) in the formula are as follows

\[
\theta \in [90' - \Delta \theta, 90' + \Delta \theta]
\]

\[
d_e = \left[ 0, \frac{d_e}{2N} \right]
\]

2.3 Phase distortion parameter characterization

According to the principle of cross-eye jamming, the formation of angle cheating by monopulse radar is caused by the phase distortion of the synthesis field of the jamming signal, and the size of the distortion area directly affects the use and effect of cross-eye jamming. The phase distortion is the phase change in the same distance from the signal wave source, as shown in Figure 3. And the phase change rate can reflect the degree of phase distortion. Therefore, the phase gradient \( \kappa \) on the circumference is used to indicate the degree of phase distortion.

\[
\kappa = \frac{\partial \text{phase}(E)}{\partial \theta}
\]

\[
\rho = \frac{\partial \text{phase}(E)_{\text{value}} - \theta_0}{\partial \theta_{\text{value}}}
\]

Where, \( \text{phase}(E) \) is the phase function of the resultant field and \( \text{value} \) is the phase gradient value of the boundary of the effective distortion region.
3. Phase distortion-based constrained optimization

3.1 Constraint model

According to the characterization of the phase distortion of the interfering signal in Section 2, the central distortion degree of the composite field should satisfy \( \kappa_{\text{min}} \geq \kappa \), where \( \kappa_{\text{min}} \) is the minimum requirement for center distortion when the deception effect is achieved, and on the other hand, the effective distortion region boundary should be greater than the fixed-value gradient \( \kappa_{\text{min}} \). In order to achieve a good jamming effect, the central distortion degree and the size of the effective distortion area are larger, the better, the objective function is obtained with the central distortion degree and the effective distortion area as indexes.

\[
\begin{align*}
\text{obj} &: \begin{cases}
\max \kappa = \frac{\partial \text{phase}(E)}{\partial \theta} \bigg|_{\theta = \theta/2} \\
\max \rho = \left| \theta \frac{\partial \text{phase}(E)}{\partial \theta} \right|_{\theta = \theta_{\text{min}}} - \theta \frac{\partial \text{phase}(E)}{\partial \theta} \bigg|_{\theta = \theta_{\text{min}}} 
\end{cases}
\end{align*}
\]

Simultaneous equations (11), \( N \), \( A_n \), \( C_n \) and \( d_e \) are decision variables. According to the requirement of amplitude ratio and phase difference between two antennas in each loop of \( A_n \) in multi-loop cross-eye jamming in the literature [7], The variable constraint range is as follows

\[
\begin{align*}
\alpha_i &\in (0.5,1.5) \\
\phi_i &\in (90^\circ, 270^\circ)
\end{align*}
\]

\( \kappa_{\text{min}} \) is the minimum degree of central distortion required to achieve the purpose of deception, ie, when the jamming system is aligned with the radar, such that the radar indication angle deviates by 0.5 times the beam width to reach the unlocked state.

In order to simplify the analysis, the signal modulation between different loops is not considered in this paper, so \( C_n = [1,1,\cdots,1] \).

This problem is a typical multi-objective optimization problem. In order to solve this kind of problem, this paper uses a decomposition-based MOEA/D evolutionary algorithm [15]. Because the target subproblem in the MOEA/D evolution algorithm can be optimized by the optimization information of the neighboring subproblems. Meanwhile, when the target increases, there is no obvious decrease in the solution performance, which is suitable for solving multi-element cross-eye jamming system optimization.

3.2 MOEA/D Evolutionary Algorithm

The MOEA/D evolutionary algorithm is a multi-objective evolutionary algorithm based on decomposition technology. The steps of the algorithm are divided into initialization, evolution and shutdown. The input parameters and significance of the algorithm are shown in Table 1. Combined with the above constraint model, the specific steps are as follows [16].

| Entry | Meaning |
|-------|---------|
| \( F = \{f_1, f_2, \ldots, f_m\} \) | The objective function, \( m \) is the number of target |
| \( S \) | Decision variables |
| \( N_p \) | The number of sub-issues, ie the size of the population |
| \( N_{\text{vec}} \) | The size of the set of neighbors for each weight vector \( W_j \) |
| \( T_{\text{max}} \) | The maximum number of iterations |

Where, \( j = 1, 2, \ldots, N_p \)

The algorithm flow is shown in Figure 4.
According to the constraint model obtained in 3.1, the decision variable is the amplitude ratio $a_i$ and the phase difference $\phi_i$ between loops. A random method is used to generate $n \times m$ matrix within the value range of the decision variable, where $n$ is the number of decision variables and $m$ is the size of the population.

4. Simulation verification

4.1 Typical simulation parameters

In the cross-eye jamming process, the main parameters include the jamming loop antenna baseline length, jamming signal amplitude ratio, phase difference, number of jamming loops, etc. The indicators to be considered are mainly the range of effective distortion regions and the degree of distortion where the jamming signal reaches the monopulse radar. In order to maximize the range of the effective distortion region and the degree of center distortion, the MOEA/D algorithm was used to optimize the design and better jamming parameter settings were obtained. In the simulation, typical multi-element cross eye jamming scene parameters are shown in Table 2, and the MOEA/D parameter settings are shown in Table 3.

**Table 2. Typical parameters of multi-loop cross-eye jamming scene**

| Parameter                  | Typical value |
|----------------------------|---------------|
| Radar frequency $f$ / GHz  | 10            |
| Beam width / (°)           | 10            |
| Jamming distance / m       | 1000          |
| Jamming system loops $N$   | 2,3,4,5       |
| Baseline length of jamming loop 1 $d_c$ / m | 10 |
| Jammer indication angle $\theta_c$ / (°) | 0 |
Radar antenna array spacing $d_r = 2.54\lambda$
Adjacent loop distance $d_{e}/m = 1$

| Algorithm parameter settings |
|------------------------------|
| total group number: 100     |
| Neighbor scale: 20          |
| Evolutionary algebra: 700   |
| The number of goals: 2      |
| Problem dimension: Determined by the number of jamming loops|

4.2 Calculation analysis
The amplitude ratio and the phase difference of jamming signals on different loops are used as decision variables, and the effective distortion region and degree of center distortion are calculated as objective functions to obtain the Pareto frontal solution under the different loops of the cross-eye jamming system, as shown in Figure 5. and Figure 6. The left end of the figure is the optimal solution for the effective distortion region in the Pareto optimal solution, and the right end is the solution with the better degree of central distortion.

Figure 5. Pareto front solution of single loop jamming system

(a) Two-loop
(b) Three-loop
Table 4. The leftmost value of Pareto front solution under different loops

| Number of loops | 1    | 2    | 3    | 4    | 5    |
|-----------------|------|------|------|------|------|
| effective distortion region $\rho$ | 0.0032° | 0.0032° | 0.0032° | 0.0031° | 0.0032° |
| Degree of distortion center (Phase gradient) $\kappa$ | $4.38 \times 10^4$ | $4.46 \times 10^4$ | $4.48 \times 10^4$ | $5.25 \times 10^4$ | $4.52 \times 10^4$ |
| Center point amplitude | 0.045 | 0.087 | 0.104 | 0.093 | 0.097 |

Table 5. The rightmost value of Pareto front solution under different loops

| Number of loops | 1    | 2    | 3    | 4    | 5    |
|-----------------|------|------|------|------|------|
| Degree of distortion center (Phase gradient) $\kappa$ | $8.99 \times 10^5$ | $9.94 \times 10^5$ | $9.87 \times 10^5$ | $9.97 \times 10^5$ | $9.91 \times 10^5$ |
| effective distortion region $\rho$ | 0.0002° | 0.0003° | 0.0004° | 0.0003° | 0.0003° |

The phase distortion of different loops when the effective distortion region $\rho$ is superior is shown in Table 4. The results in the table are the effective distortion area, the degree of center distortion, and the center point of the composite field under the better solution (non-closed solution) for the decision variable (amplitude ratio, phase difference) obtained at this time.

The size of the effective distortion region when the central distortion degree $\kappa$ is superior is shown in Table 5. The results in the table show the effective distortion region and center distortion under the better solution (non-closed solution) for the decision variable (amplitude ratio, phase difference) obtained at this time.

It can be seen from Table 4 that when the effective distortion area is better, the multi-loop system has a great effect on increasing the center point amplitude of the synthesis field. In Table 5, it can also be seen that when the degree of center distortion is better, the degree of central distortion and the effective distortion area of the multi-loop cross-eye jamming system are better than that of the single-loop cross-eye jamming system, and thus multi-element intersection can be judged. The eye jamming system can effectively improve the cross eye jamming performance.

According to the parameters calculated in Table 4, the results of the phase distortion of the jamming antenna at an azimuth of 90±0.005° and a distance of 1000±10 m are obtained, that is, the phase gradient value of each point in the range, as shown in Figure 7.

It can be seen from Figure 7 that the phase distortion of the two-loop jamming changes significantly in the azimuth dimension, there is no significant change in the distance dimension, but the center phase gradient value is small. The phase distortions of the three-loop and four-loop jamming have obvious changes in the azimuth dimension and the distance dimension, which the maximum phase gradient value is concentrated on the 90° and the difference is that the largest distortion degree
of the third loop jamming is far from the center point of 1000m, while the fourth loop jamming is close to the center point of 1000m. The phase distortions of the five-loop jamming and the two-loop jamming are similar, which the difference is that the area where the distortion degree of the five-loop jamming is larger is deviated with a distance away from the center point of 1000m, and is less than the degree of distortion of the three-loop and four-loop. Therefore, when the number of loops is 3 or 4, the degree of phase distortion is better than that of the jamming system with the number of loops of 2 and 5, and at the same time, in the vicinity of the azimuth angle of 90°, the effective distortion region when the number of loops is two is greater than the five-loop. So, when the number of loops exceeds 4, the effect of jamming will be significantly reduced.

From the above analysis, the number of loops is not as good as possible for multi-loop cross-eye jamming. As the number of loops increases, the length of the interfering antenna baseline of loop N will become smaller and smaller, thus reducing the degree of phase distortion. This can make the interfering system act as a radar indicator. At the same time, the increase in the number of loops will also increase the cost and complexity of the system.

5. Conclusion
In this paper, the wavefront phase distortion characteristics of the multi-loop cross-eye jamming system are investigated. The results show that compared with the single-loop cross-eye jamming system, the degree of central distortion and the effective distortion area of the multi-loop cross-eye jamming system are improved. In addition, the analysis of the phase distortion in the region of the multi-loop cross-eye jamming system reveals that the increase in the number of loops does not lead to better results, and the phase distortion of the three-loop and four-loop jamming systems is better than that of the five-loop jamming system. The effective distortion area of the second-loop jamming system
is greater than the phase distortion of the five-loop jamming system. That is, if the number of loops exceeds four, the effect of jamming will be significantly reduced. The above research provides a theoretical basis for the engineering application of the multi-loop cross-eye jamming system.

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