Accidental Approximate Generation Universality

and its Possible Verification

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Abstract

The universality of $e - \mu - \tau$ interactions may only be an accidental approximate symmetry analogous to that of flavor $SU(2)$ and $SU(3)$. This was specifically realized by an extension of the standard model proposed in 1981. Two key predictions are that the $\tau$ lifetime should be longer and that the $\rho$ parameter measured at the $Z$ peak should have an additional negative contribution. These are consistent with present precision electroweak measurements. A future decisive test of this model would be the discovery of new $W$ and $Z$ bosons with nearly degenerate masses of a few $TeV$.

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1 Introduction

Historically, flavor $SU(2)$ and $SU(3)$ were thought to be fundamental, albeit approximate, symmetries. We now know that the real fundamental symmetry is color $SU(3)$, and that the former are merely accidental symmetries: flavor $SU(2)$ is the result of the hierarchy $m_u,m_d << \Lambda_{QCD}$, and flavor $SU(3)$ is even less exact because $m_s$ is not as negligible.

Consider now generation universality. In the standard model, the implicit assumption is that it is a fundamental symmetry resulting from the existence of 3 (or more) generations of quarks and leptons which have identical gauge interactions. On the other hand, this may be the wrong way of looking at it. Generations could be fundamentally different and their interactions at low energies may be only accidentally and approximately universal, in analogy with flavor $SU(2)$ and $SU(3)$. This alternative viewpoint was specifically realized in a model by X. Li and myself, published in 1981.[1]

The 2 main predictions of this model relevant to present electroweak measurements are: (1) the $\tau$ lifetime should be longer than predicted by the standard model; and (2) the $\rho$ parameter measured in the decay of $Z$ into leptons should have an additional negative contribution. Both trends are now noticeable in the data, although one certainly cannot claim that definite deviations from the standard model have been observed.

2 The Model

Consider the gauge group $U(1) \times SU(2)_1 \times SU(2)_2 \times SU(2)_3$ with couplings $g_0,g_1,g_2,g_3$, respectively. The left-handed fermions are doublets under $U(1) \times SU(2)_i$, with each generation coupling to a separate $SU(2)$. The right-handed fermions are singlets coupling only to $U(1)$. This structure is anomaly-free and has no redundancies. The Higgs sector consists of 3 $SU(2)_i$ doublets with vacuum expectation values $v_{0i}$ and 3 $SU(2)_j \times SU(2)_k$ self-dual bidoublets with vacuum expectation values $v_{jk}$. This then guarantees the equality of the effective weak-interaction coupling matrix $(G_F)_{ij}$ for charged and neutral currents, as in the standard model. If $v_{01}^2 + v_{02}^2 + v_{03}^2 << v_{12}^2$, then $(G_F)_{ij} = G_F$ for $i \neq 3$ and $j \neq 3$, and $(G_F)_{ij} = \xi^{-1}G_F$ for $i = 3$ or $j = 3$ but not both, where $\xi \equiv 1 + v_{03}^2/(v_{13}^2 + v_{23}^2)$. Hence $e - \mu$ universality is valid, but $e - \mu - \tau$ universality is only approximate. In particular, the $\tau$ lifetime should be longer than the standard-model prediction by the factor $\xi^2$. Experimentally, from all the data available up to several months ago, $\xi - 1$ was determined to be $0.027 \pm 0.012$, which was a 2.3$\sigma$ effect. However, mainly because of the new measurement
of $m_e = 1776.9 \pm 0.4 \pm 0.3$ MeV by the BES collaboration at the BEPC $e^+e^-$ collider in Beijing, it has now been reduced to $0.015 \pm 0.008$, which is only a $1.8\sigma$ effect in support of our prediction.

Let us also define $r \equiv (v_{01}^2 + v_{02}^2)/v_{03}^2$ and $y \equiv g_{123}^2/g_3^2$, where $g_{123}^2 = g_1^{-2} + g_2^{-2} + g_3^{-2}$. Then $e^{-2} = g_0^{-2} + g_{123}^2$, and for the first 2 generations,

$$H_{NC}^{eff} = \frac{4G_F}{\sqrt{2}}\left[(j^{(3)})^2 - s^2 j^{em})^2 + C(j^{em})^2\right],$$

(1)

where

$$s^2 = 1 - e^2/g_0^2 - (1 - \xi^{-1})e^2/g_3^2,$$

(2)

and

$$C = (e^4/g_3^4)(1 - \xi^{-1})(\xi^{-1} + r) \approx (\xi - 1) s^4 y^2(1 + r).$$

(3)

The above effective interaction is the result of the virtual exchange of all 3 $Z$ bosons of this model, but at LEP, only one of them (the lightest) is produced and it should certainly not be identical to the standard-model $Z$. In fact, its mass squared is given by $1 + (\xi - 1)y((1 - s^2)^{-1} - (1 + r)y)$ times that of the standard model, as predicted from the value of $s^2$ in Eq. 1. The corresponding factor for $M_W^2$ is $1 + (\xi - 1)y(1 - (1 + r)y)$.

### 3 Z Leptonic Widths

From precision measurements of the widths and forward-backward asymmetries of $Z \to \ell\ell$ ($l=e,\mu,\tau$) at LEP, the parameters $\rho_l$ and $\sin^2\theta_l$ are extracted.

$$\Gamma_l = \frac{G_F M_Z^3}{24\sqrt{2}\pi} \left(1 + \frac{3\alpha}{4\pi}\right) \rho_l \left[1 + (1 - 4\sin^2\theta_l)^2\right],$$

(4)

and

$$A_{FB}(M_Z^2) \approx 3 \left(1 - 4\sin^2\theta_l\right)^2.$$ 

(5)

In the nonuniversality model $\rho_{e,\mu} = 1 - (\xi - 1)y^2(1 + r) + \rho_{rad}$, where $\rho_{rad}$ is dominated by the standard-model $m_t^2$ contribution. [The Higgs sector of this model has an automatic $SU(2)$ custodial symmetry which eliminates all the quadratic mass terms of the scalar bosons to one-loop order.] Since $\xi > 1$, $r > 0$, and $0 < y < 1$ are required by their definitions, this model has a necessarily negative contribution to $\rho_{e,\mu}$, in addition to the necessarily positive contribution of $\rho_{rad} \approx 3\sqrt{2}G_F m_t^2/16\pi^2$ of the standard model.
Let \( x \equiv y(1 + r) \), then \( \rho_{e,\mu} = \rho_{e,\mu} - 2(\xi - 1)(1 - x) \) and

\[
\frac{\Gamma_{\tau}}{\Gamma_{e,\mu}} \approx 1 - \frac{2(\xi - 1)(1 - 2s^2)}{1 - 4s^2 + 8s^4}(1 - x).
\]  

(6)

This means that universality in \( Z \to \bar{t}t \) would still hold if \( x = 1 \). [In our previous papers, it was assumed that \( v_{01}^2 + v_{02}^2 << v_{03}^2 \), i.e. \( x << 1 \), in which case \( \Gamma_{\tau} < \Gamma_{e,\mu} \) would be required.]

As for \( \sin^2\theta_l \), if we take the average over \( l = e, \mu, \tau \), then

\[
\sin^2\theta_{\text{eff}} \approx s^2_0 \left[ 1 - \left( \frac{1 - s^2_0}{1 - 2s^2_0} \right) \rho_{\text{rad}} + (\xi - 1) \left[ \frac{1 - x}{3} + \frac{y(1 - s^2_0x)}{1 - 2s^2_0} \right] \right],
\]

(7)

where \( s^2_0(1 - s^2_0) \equiv \pi\alpha(M_Z^2)/\sqrt{2}G_FM_Z^2 \). The corresponding average value of \( \rho_l \) is

\[
\rho_{\text{eff}} \approx 1 + \rho_{\text{rad}} - (\xi - 1) \left[ \frac{2}{3}(1 - x) + xy \right].
\]

(8)

4 Comparison with Data

As mentioned already, the \( \tau \)-lifetime discrepancy implies that \( \xi - 1 = 0.015 \pm 0.008 \). To pin down \( x \) and \( y \), we use Eqs. 6,7,8 and compare with the present LEP data. We also use \( s^2 = 0.231 \pm 0.006 \) (in the on-shell renormalization scheme) from neutrino data and compare what it predicts for \( M_Z \) as a function of \( m_t \) in the standard model to the observed value \( M_Z = 91.175 \pm 0.021 \text{ GeV} \), which gives us another constraint. The combined 1989 and 1990 data of all 4 LEP collaborations are now published. 4 The preliminary results of the 1991 run have also recently become available. 5 We put these together and find \( \rho_{\text{eff}} = 0.9990 \pm 0.0032 \), \( \sin^2\theta_{\text{eff}} = 0.2322 \pm 0.0015 \), \( \Gamma_{e,\mu} = 83.44 \pm 0.27 \text{ MeV} \), \( \Gamma_{\tau} = 83.38 \pm 0.60 \text{ MeV} \), and \( s^2 = 0.2338 \pm 0.0005 \), where \( \alpha^{-1}(M_Z^2) = 127.9 \pm 0.2 \) has been used. Taking into account the 0.19 \( \text{MeV} \) reduction of \( \Gamma_{\tau} \) due to \( m_\tau \), we obtain from Eq. 6 the following restriction on \( x \):

\[
1 - \frac{0.0030}{\xi - 1} < x < 1 + \frac{0.0045}{\xi - 1}.
\]

(9)

Consider now the standard-model limit: \( \xi - 1 = 0 \). Assuming that \( \rho_{\text{rad}} \) is dominated by \( m_t \), we then find from \( \rho_{\text{eff}} < 1.0022 \) that \( m_t < 84 \text{ GeV} \), which is already ruled out by the CDF result \( m_t > 91 \text{ GeV} \). However, if we allow \( \xi - 1 = 0.015 \pm 0.008 \) as present data indicate, then this particular restriction on \( m_t \) is removed. On the other hand, the constraint from neutrino data which gives \( m_t < 147 \text{ GeV} \) for \( \xi - 1 = 0 \) is only extended a little to \( m_t < 159 \text{ GeV} \), the maximum occurring at \( \xi - 1 = 0.007 \), \( x = 0.57 \), and \( y = 0.91 \). Hence this model would still require \( m_t \) to be small enough to be experimentally accessible in the near future.
In Fig. 1 we show the allowed region in \( y \) versus \( x \) for \( \xi - 1 = 0.01 \) and \( m_t = 150 \text{ GeV} \). It is seen that \( y \) is bounded from below by \( \rho_{\text{eff}} \) and the \( \nu N \) data, and from above by \( \sin^2 \theta_{\text{eff}} \). If \( \xi - 1 \) is increased to 0.02, then \( \rho_{\text{eff}} \) provides both upper and lower bounds on \( y \), and if \( m_t \) is decreased to 120 GeV, the \( \nu N \) data do not restrict \( y \) at all, as shown in Fig. 2. Recall that \( 0 < y < 1 \) is required by definition and \( x \) is restricted by \( \xi - 1 \) according to Eq. 9.

5 Future Implications

The effects due to nonuniversality are naturally very small at present energies. They will probably never be a decisive test of our model. However, we also predict the existence of a second set of \( W \) and \( Z \) bosons with nearly degenerate masses approximately given by

\[
M_{W'}^2 = M_{Z'}^2 = \frac{M_Z^2(1 - s_\theta^2)}{(\xi - 1)(1 - y)x}.
\]

To maximize the denominator, we take \( \xi - 1 = 0.023 \), \( y = 0 \), \( x = 0.974 \), and \( m_t = 91 \text{ GeV} \), whereby we find a lower bound of 533 GeV for \( M_{W'} = M_{Z'} \). There is no upper bound at present, because the \( y = 1 \) limit cannot be ruled out by the data. However, once \( m_t \) is measured, this may become possible. In any case, we expect it to be no more than a few TeV. Hence these new vector gauge bosons are predicted to be discovered at future accelerators such as the SSC or LHC. Their decay rates into the third versus the first or second generations are predicted to be in the ratio \((1 - y)^2 : y^2\), and they only have left-handed interactions.

6 Conclusion

The idea that \( e - \mu - \tau \) universality may be only accidental and approximate as realized in our 1981 model is now being tested by precision electroweak data. The results are certainly not definitive, but they do show the trends as predicted: (1) the \( \tau \) lifetime should be longer by a small amount, and (2) the \( \rho \) parameter measured at the \( Z \) peak should have an additional negative contribution of comparable magnitude to the positive radiative correction proportional to \( m_t^2 \). A future decisive test of this model would be the discovery of new \( W \) and \( Z \) bosons with nearly degenerate masses of a few TeV.

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References

[1] X. Li and E. Ma, Phys. Rev. Lett. 47 (1981) 1788. See also E. Ma, X. Li, and S. F. Tuan, Phys. Rev. Lett. 60 (1988) 495; E. Ma and D. Ng, Phys. Rev. D38 (1988) 304; X. Li and E. Ma, Phys. Rev. D46 (1992) in press.

[2] K. Riles, in Proc. of the Vancouver Meeting - Particles and Fields ’91, ed. D. Axen et al. (World Scientific, Singapore, 1992) p. 272.

[3] The LEP Collaborations: ALEPH, DELPHI, L3 and OPAL, Phys. Lett. B276 (1992) 247.

[4] J. Nash, in Proc. of XXVII Rencontres de Moriond: Electroweak Interactions, March 1992, to be published.