THE GRAVITATIONAL WAVE CONTRIBUTION TO THE COSMIC MICROWAVE BACKGROUND ANISOTROPIES

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ABSTRACT

We study the possible contribution of a stochastic gravitational wave background to the anisotropy of the cosmic microwave background (CMB) in cold and mixed dark matter models. We test this contribution against recent detections of CMB anisotropy at large and intermediate angular scales. Our likelihood analysis indicates that models with blue power spectra ($n \approx 1.2$) and a tensor-to-scalar quadrupole ratio of $R = C_2^T / C_2^S \approx 2$ are most consistent with the anisotropy data considered here. We find that by including the possibility of such a background in the CMB data analysis, it can drastically alter the conclusion of the remaining cosmological parameters.

Subject headings: cosmic microwave background — dark matter — gravitation

1. INTRODUCTION

Inflationary theory has had a large impact on cosmology. On the one hand, it resolves some difficulties of the standard big bang model. On the other, it provides a way of producing those density fluctuations that, in the gravitational instability scenario, are the seeds of the large-scale structure of the universe. In fact, one of the most reliable predictions of the inflationary paradigm is the parallel production of scalar and tensor perturbations from quantum fluctuations of the inflaton field $\phi$ (Starobinsky 1979, 1982; Rubakov, Sazhin, & Veryaskin 1982; Abbot & Wise 1984). The amplitude of tensor fluctuations determines the value of the inflationary potential and, together with other inflationary parameters, its first two derivatives (see, e.g., Turner 1997). Thus, a detection of a nearly scale-invariant stochastic gravitational wave (GW) background (the tensor modes) is crucial for confirming any inflationary model and for constraining the physics occurring near the Planck scale, at $\sim 10^{16} \text{GeV}$.

Observations of cosmic microwave background (CMB) anisotropy promise to be unique in this respect (Starobinsky 1985; Crittenden et al. 1993; Turner, White, & Lidsey 1993). Recent numerical simulations (Zaldarriaga, Spergel, & Seljak 1997; Dodelson, Kinney, & Kolb 1997; Bond, Efstathiou, & Tegmark 1997) have shown that inflationary parameters will be measured with an accuracy of a few percent by the MAP (Bennett et al. 1995) and Planck (Bersanelli et al. 1996) space missions, which will image the CMB anisotropy pattern with high sensitivity and at high angular resolution.

Meanwhile, the number of experiments reporting detections of anisotropy has increased to more than 10 (see Table 1). At the moment, the detections that are available seem to be compatible with the predictions of inflationary models, like the cold dark matter (CDM) model, with "blue" power spectra, i.e., $P(k) = A k^{n_s}$ with $n_s \approx 1$ (de Bernardis et al. 1997; Bennett et al. 1996; Bond & Jaffe 1996). As noticed by many authors, there is a substantial rise in the anisotropy angular power spectrum at $l \sim 200$, which appears to be consistent with the expected location of the first Doppler peak in flat models. This small-scale behavior seems to favor a GW contribution. In fact, as is well known, tensor fluctuations induce anisotropy only on large angular scales ($l \lesssim 30$). If there is a sizable contribution from GWs in large-scale detected anisotropies, this would lower the predicted value of $(\Delta T / T)_{\text{rms}}$ on smaller scales.

Moreover, inflationary models that predict $n_s \gtrsim 1$ generally predict vanishingly small tensor fluctuations (Kolb & Vadas 1994). Based on these arguments, a lot of recent CMB data analyses (Lineweaver et al. 1997; Hancock et al. 1998) have not taken into account the possible presence of a GW background, assuming its contribution to be negligible.

In our opinion, there are two points that can alter these conclusions:

1. Tensor modes are compatible with the theory of linear adiabatic perturbations of a homogeneous and isotropic universe. Like scalar perturbations and in contrast with vector perturbations, they can arise from small deviations from the isotropic Friedman universe near the initial singularity. So CMB data should be analyzed without any a priori assumptions: the presence or absence of a tensor component in models with $n_s \approx 1$ can be tested only by observation.

2. Few variations in the still undetermined cosmological parameters (like the baryonic abundance or the Hubble constant) and inflationary parameters (like the spectral index $n_s$) can counterbalance the effect of tensor modes, increasing the predicted value of $(\Delta T / T)_{\text{rms}}$ on small scales.

Thus, in this paper, we will discuss what kind of constraint the present CMB anisotropy data can provide on the tensor contribution, allowing all the remaining parameters to vary freely in their acceptable ranges. We will extend our previous CMB data analysis (de Bernardis et al. 1997) by including new CMB detections and by analyzing a larger set of models. We restrict ourselves to critical universes ($\Omega_{\text{matter}} = 1$), since a recent analysis of CMB anisotropies and galaxy surveys (Gawiser & Silk 1998) has shown that pure scalar mixed dark matter (MDM) models are in good agreement with the data set. We will address the importance...
of a cosmological constant, reported by Riess et al. (1998) and Perlmutter et al. (1998), in a forthcoming paper.

Since we treat the GW contribution as a free parameter, we will not test any specific inflationary model. So our approach will be mainly phenomenological: we assume that GWs are created in the early universe by some process during or immediately after inflation, which we do not want to specify any further here. Nonetheless, since the amplitude of the GW spectrum provides a test for inflation (see next section), in our conclusions, we will discuss whether or not the results are compatible with this paradigm.

Since any possible GW signal will affect the matter power spectrum normalization inferred from COBE, we will test the models that best fitted the CMB data with the normalization of the matter fluctuation in $8h^{-1}$ spheres and with the shape of the spectrum from the Peacock & Dodds (1994) analysis.

The plan of this paper is as follows. In § 2, we write the set of equations that are necessary for describing the inflationary process in the slow-roll approximation. In § 3, we briefly discuss the analysis of the current degree-scale CMB experiments. In § 4, we test the best-fit models with the large-scale structure (LSS) data. Finally, in § 5, we present and discuss our conclusions.

2. EARLY UNIVERSE

Inflation in the early universe is determined by the potential $V(\phi)$, where $\phi$ can be a multiplet of scalar fields. Here we restrict ourselves to the case of a single, minimally coupled scalar field $\phi$ with potential $V$ and equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

(as usual, the dot and prime indicate derivatives with respect to physical time $t$ and to the scalar field $\phi$, respectively). The expansion rate in the early universe can be written as

$$H^2 = \frac{8\pi}{3m^2_{pl}} \left[ \frac{1}{2} \phi^2 + V(\phi) \right],$$

where $m_{pl} = 1.2 \times 10^{19}$ GeV is the Planck mass (we use natural units, i.e., $\hbar = c = 1$).

The slow-roll approximation holds in most of the inflationary models. This condition is valid if (Copeland et al.

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**TABLE 1**

| Experiment          | $\Delta T^2$ (µK$^2$) | $\geq 68\%$ (µK$^2$) | $\leq 68\%$ (µK$^2$) | Sky Coverage | $l_{\ell}$ | Reference |
|---------------------|------------------------|-----------------------|-----------------------|--------------|-----------|-----------|
| COBE1 .............. | 25.2                   | 183                   | 25.2                  | 0.65         | 2.5       | 1         |
| COBE2 .............. | 212                    | 126                   | 128                   | 0.65         | 3.3       | 1         |
| COBE3 .............. | 256                    | 96.5                  | 96.9                  | 0.65         | 4.1       | 1         |
| COBE4 .............. | 105.5                  | 48.3                  | 48.2                  | 0.65         | 5.5       | 1         |
| COBE5 .............. | 101.9                  | 26.5                  | 26.4                  | 0.65         | 8.1       | 1         |
| COBE6 .............. | 63.4                   | 19.11                 | 18.9                  | 0.65         | 11.6      | 1         |
| COBE7 .............. | 39.6                   | 14.5                  | 14.5                  | 0.65         | 16.7      | 1         |
| COBE8 .............. | 42.5                   | 12.7                  | 12.8                  | 0.65         | 25.1      | 1         |
| Tenerife ..........  | 1770                   | 840                   | 25.2                  | 0.65         | 20.1      | 2         |
| South Pole Q ....... | 3975                   | 2855                  | 1807                  | 0.0124       | 20.1      | 3         |
| South Pole K ....... | 480                    | 470                   | 160                   | 0.005        | 49.4      | 4         |
| Python ............. | 2040                   | 2330                  | 25.2                  | 0.65         | 20.1      | 4         |
| ARGO Hercules ...... | 360                    | 170                   | 140                   | 0.0024       | 118.9     | 5         |
| ARGO Aries .......... | 580                    | 150                   | 130                   | 0.0024       | 118.9     | 6         |
| MAX HR ............. | 2430                   | 1850                  | 25.2                  | 0.65         | 20.1      | 7         |
| MAX PH ............. | 5960                   | 5080                  | 25.2                  | 0.65         | 20.1      | 8         |
| MAX GUM ............ | 6580                   | 4450                  | 25.2                  | 0.65         | 20.1      | 9         |
| MAX ID ............. | 4960                   | 2690                  | 25.2                  | 0.65         | 20.1      | 10        |
| MAX SH ............. | 360                    | 170                   | 140                   | 0.0024       | 118.9     | 11        |
| MSAM93 ............. | 8698                   | 6457                  | 3406                  | 0.007        | 179       | 12        |
| MSAM94 ............. | 3975                   | 2855                  | 1807                  | 0.0124       | 20.1      | 13        |
| MSAM95 ............. | 3975                   | 2855                  | 1807                  | 0.0124       | 20.1      | 14        |
| Saskatoon .......... | 1990                   | 950                   | 630                   | 0.0037       | 99.9      | 15        |
| CAT1 ................ | 1180                   | 720                   | 520                   | 0.0001       | 414       | 16        |
| CAT2 ................ | 760                    | 760                   | 360                   | 0.0001       | 579       | 17        |
| CAT1 ................ | 934                    | 403                   | 232                   | 0.0001       | 414       | 18        |
| CAT2 ................ | 577                    | 416                   | 238                   | 0.0001       | 579       | 19        |

***REFERENCES***

(1) Tegmark & Hamilton 1997; (2) Hancock et al. 1998; (3) Gutierrez et al. 1997; (4) Gundersen et al. 1993; (5) Dragovan et al. 1993; (6) de Bernardis et al. 1994; (7) Masi et al. 1993; (8) Tanaka et al. 1996; (9) Cheng et al. 1994; (10) Cheng et al. 1996; (11) Cheng et al. 1997; (12) Nettlesfield et al. 1993; (13) Scott et al. 1996; and (14) Baker et al. 1997.
The Hubble parameter conditions, and where is a constant, to be derived from the boundary where is the wavenumber of a fluctuation that reenters the horizon at the present time, and the field can be written as a function of the wavelength:

\[ A_\phi(\phi) = \sqrt{\frac{2}{\pi}} \frac{1}{H_0} \frac{H^2}{m_{\text{pl}}^2 |H'|}, \]

and

\[ A_T(\phi) = \frac{1}{\sqrt{2\pi}} \frac{H}{m_{\text{pl}}} . \]

We can relate the wavelength, \( \lambda \), and the Hubble parameter during inflation, \( H(\phi) \), with the scalar field by writing

\[ \frac{d \ln \lambda}{d \phi} = \frac{\sqrt{4\pi} A_S}{m_{\text{pl}} A_T} \]

and

\[ \frac{\partial \ln H}{\partial \phi} = \frac{\sqrt{4\pi} A_T}{m_{\text{pl}} A_S} , \]

respectively.

Let us define the spectral equations for scalar and tensor components as follows:

\[ A^2_S(k) = A^2 \left( \frac{k}{k_0} \right)^{ns - 1} , \]

and

\[ A^2_T(k) = B^2 \left( \frac{k}{k_0} \right)^{nr} , \]

where \( k_0 = H_0 \) is the wavenumber of a fluctuation that reenters the horizon at the present time, and \( A \) and \( B \) are constants. It is easy to see that \( n_T = -2 \epsilon(k) \) if \( \lambda = \lambda_0 \) and that \( n_T = 0 \) if \( \lambda \rightarrow 0 \) (Lidsey et al. 1997).

We define the ratio of amplitudes of the scalar and tensor modes by

\[ r = \sqrt{\epsilon(k_0)} = \frac{B}{A} . \]

By solving equation (7) and assuming \( n_T \sim 0 \), the scalar field can be written as a function of the wavelength:

\[ \phi(\lambda) = \phi_0 + \phi_1 \left( \frac{\lambda}{\lambda_0} \right)^{(ns - 1)/2} - 1 , \]

where \( \phi_0 \) is a constant, to be derived from the boundary conditions, and \( \phi_1 = [r/(n_0 - 1)]m_{\text{pl}}/\pi^{1/2} \).

Furthermore, equations (12) and (8) allow us to find the Hubble parameter \( H(\phi) \) during inflation:

\[ H(\phi) = H_i \exp \left( \frac{r^2}{n_5 - 1} \xi^2 \right) , \]

where \( \xi = (\phi + \phi_1 - \phi_0)/\phi_1 \) and \( H_i \) is a constant.

The potential can be written in terms of the Hubble parameter:

\[ V(\phi) = \frac{3m_{\text{pl}}^2}{8\pi} H^2(\phi) , \]

At this point, we can define the relation between the quadrupole multipoles of the CMB anisotropy generated by scalar and tensor perturbations: \( C^S_2 \) and \( C^T_2 \), respectively. To do this, we will follow the calculations done by Souradeep & Sahni (1992) in which both \( C^S_2 \) and \( C^T_2 \) were found as a function of \( H(\phi) \) at \( \lambda = H^{-1} \). So we have

\[ C^S_2 = \frac{2\pi^2}{25} f(n_s) \frac{1}{m_{\text{pl}}^2} \frac{H^2}{(H')^2} , \]

and

\[ C^T_2 = \frac{29}{5\pi m_{\text{pl}}^2} H^2 , \]

where

\[ f(n_s) = \frac{\Gamma(3 - n_s) \Gamma(3 + n_s)/2}{\Gamma^2(4 - n_s)/2 \Gamma(9 - n_s)/2} . \]

Using equation (13), we can write

\[ \frac{\partial \ln H(\phi)}{\partial \phi} = \frac{2\sqrt{\pi}}{m_{\text{pl}}} r \xi . \]

Therefore, at \( \phi = \phi_0 \) (i.e., \( \xi = 1 \)), we have

\[ R(n_s) \equiv \frac{C^T_2}{C^S_2} = \frac{29r^2}{n^2 f(n_s)} . \]

As we can see from the equation above, the tensor-to-scalar quadrupole ratio \( R \) is related to the slow-roll parameter \( \eta \). Equation (20) identifies a region in the \( (n_s, R) \)-space of values where the slow-roll condition is satisfied. Furthermore, since \( \epsilon < 1 \) only if the universe has undergone a period of accelerated expansion, one can use this equation to test the inflationary scenario.

In the same way, we can use equation (19) in equation (3) in order to find

\[ 2\eta = n_s - 1 + 2r^2 , \]

so the slow-roll condition (4) implies equation (3) if \( n_s \sim 1 \).

Using equation (14), we can now write the potential as

\[ V(\phi_0) = \frac{15}{23.2} C^T_2 m_{\text{pl}}^4 . \]

Therefore, the measurement of the contribution to the quadrupole anisotropy of tensor fluctuation, \( C^T_2 \), allows us to estimate the size of the potential that is responsible for the inflation.

### 3. CMB ANISOTROPY

#### 3.1. Method

We use a set of the most recent CMB anisotropy detections, on both large and degree angular scales, in order to estimate the amplitude of tensor fluctuations. The likelihood of the assumed independent CMB anisotropy data is
(see de Bernardis et al. 1997)

\[ \mathcal{L} = \prod_j \frac{1}{\sqrt{2\pi(\Sigma_j^{\text{th}})^2 + (\Sigma_j^{\text{exp}})^2}} \times \exp \left[ -\frac{1}{2} \frac{(\Delta \exp - \Delta \text{th})^2}{(\Sigma_j^{\text{th}})^2 + (\Sigma_j^{\text{exp}})^2} \right], \tag{23} \]

where \( \Delta^{\text{th}} \) and \( \Delta^{\text{exp}} \) are the experimentally detected and the theoretically expected mean square anisotropy, respectively. The \( (\Sigma_j^{\text{th}})^2 \) and \( (\Sigma_j^{\text{exp}})^2 \) are the respective cosmic and experimental variances. Obviously, the likelihood depends on the parameters of the cosmological model. Although a complete analysis should cover all the parameter space, here we restrict ourselves to flat models (\( \Omega_0 = 1 \)) composed of baryons (0.01 \( \leq \Omega_b \leq 0.14 \)), cold dark matter (\( \Omega_{\text{CDM}} \geq 0.7 \)), hot dark matter (\( \Omega_h \leq 0.3 \)), photons, and massless neutrinos. As shown in de Bernardis et al. (1997) Ma & Bertschinger (1995), and Dodelson, Gates, & Stebbins (1996), the angular power spectrum of MDM models differs from pure CDM by less than 10% in the angular scales of interest. Given the poor sensitivity of the available CMB anisotropy detections at degree angular scales, we restrict ourselves to pure CDM models, keeping in mind that basically the same power spectrum is also expected for MDM models. The predictions of CDM and MDM models for the matter power spectrum obviously differ, and in a substantial way: we will discuss this point in more detail below.

Here we keep as free parameters \( \Omega_b, \Omega_h \), and \( h \). Both parameters affect the positions and amplitudes of the so-called Doppler peaks of the angular power spectrum. In fact, changing \( \Omega_h \) at fixed \( h \) changes the pressure of the baryon-photino fluid before recombination, increasing its oscillations below its Jeans length. A larger baryon-to-photino ratio will increase the compressions (which produce the even peaks in \( C_l \) for inflationary models) and decrease the rarefaction (odd peaks for inflationary models). Lowering \( h \) at fixed \( \Omega_b \) changes the epoch of matter-radiation equality: potentials inside the horizon decay in a radiation-dominated era but not in a fully matter-dominated one. The combination \( \Omega_b h^2 \), which actually appears in the calculations, is also constrained by primordial nucleosynthesis arguments (Copi, Schramm, & Turner 1995): 0.01 \( \leq \Omega_b h^2 \leq 0.026 \). Moreover, from globular cluster ages, 0.4 \( \leq h \leq 0.65 \) (Kolb & Turner 1990).

We will also explore variations in the spectral index of the (scalar) primordial power spectrum \( n_s \). We restrict ourselves to values of \( n_s \leq 1.5 \), to be consistent with the absence of spectral distortions in the COBE/Far-Infrared Absolute Spectrometer data (Hu, Scott, & Sil 1994). A parameter-independent normalization for the power spectrum can be expressed in terms of the amplitude of the multipole \( C_{10} \). We define the parameter \( \mathcal{A} \equiv A_{\text{COBE}} / A_{\text{COBE}}^0 \) as the amplitude \( \mathcal{A} \) of the power spectrum (considered as a free parameter) in units of \( A_{\text{COBE}} \), the amplitude needed to reproduce \( C_{10} \sim 47.6 \mu K^2 \), as observed on the COBE/Differential Microwave Radiometer (DMR) 4 yr maps (Bunn & White 1997).

Finally, for tensor fluctuations, we will assume \( n_T = 0 \). In fact, variations in the tensor spectral index in the range \(-1 \leq n_T \leq 0 \) do not produce appreciable changes in the structure of the \( C_l \)’s, given the cosmic variance and the current experimental sensitivity. We parameterize the amplitude of these tensor fluctuations with \( R \), defined in equation (22). So, in the end, we will consider only five quantities as free parameters: \( \mathcal{A}, n_s, R, \Omega_b, \) and \( h \).

We have computed the angular power spectrum of CMB anisotropy by solving the Boltzmann equation for fluctuations in CMB brightness (Peebles & Yu 1970; Hu et al. 1995). Our code is described in de Bernardis et al. (1997) and Melchiorri & Vittorio (1997) and allows for the study of CMB anisotropy in both cold and mixed dark matter models. Our \( C_l \)’s match to better than 0.5% for \( l \leq 1500 \) compared with those of other codes (Seljak & Zaldarriaga 1996; Ma & Bertschinger 1995). In Figure 1, we show the \( C_l \)’s for different parameter choices.

The data we consider are listed in Table 1 and shown in Figure 1. We have updated the data presented in our previous paper (de Bernardis et al. 1997) to include the new results from the Tenerife, MSAM, and CAT experiments. For the COBE data, we use the eight data points from Tegmark & Hamilton (1997), which have the advantage of uncorrelated error bars.

### 3.2. Results

The best-fit parameters (i.e., those that maximize the likelihood) are (with 95% confidence) \( n_s = 1.23^{+0.15}_{-0.17} \) and \( R = 2.4^{+0.7}_{-0.8} \), with \( \mathcal{A} = 0.92, \Omega_b = 0.07, \) and \( h = 0.46 \). We can only put the following upper limits (at 68%) on these last two best-fit values: \( \Omega_h < 0.11 \) and \( h < 0.58 \).

A probability confidence level contour in the five-dimensional volume of parameters is obtained by cutting the \( \mathcal{L} \) distribution with the isosurface \( \mathcal{L} = D \) and by requiring that the volume inside \( \mathcal{L} = D \) is a fraction \( P \) of the total volume. The projections of the \( \mathcal{L} = D \) and \( \mathcal{L} = 3P \) surfaces on the \( n_s-R \) plane are shown in Figure 2.

As we can see from Figure 2, the likelihood contours are very broad, and models with spectral index \( n_s \approx 1 \) and \( R = 0 \) are statistically indistinguishable from models with \( n_s \approx 1.4 \) and \( R \approx 4 \).

The quite large values of \( R \) for \( n_s \approx 1 \) are due to a parameter-degeneracy problem that the present CMB anisotropy detections are not able to solve (see Fig. 1). In fact, increasing the contribution of tensor modes boosts the anisotropy on large scales (\( > 2 \Omega_b h^2 \)). As the theoretical predictions are normalized to the COBE/DMR, adding tensor fluctuations while keeping all the other parameters fixed actually suppresses the level of degree-scale anisotropy. To counterbalance this effect, it is necessary to postulate blue primordial spectra, i.e., \( n_s \approx 1 \). The shape of the confidence level region in the \( n_s-R \) plane reflects this correlation. This degeneracy in the model prediction is actually broken at a higher angular resolution, \( l \approx 300 \) say, where present experiments are particularly affected by cosmic variance, because of the very small region of the sky sampled (see Table 1). We have the following 95% confidence level (c.l.) upper limits on \( R: 0.3, 1.3, 2.5, 4.5, 7.8, \) and 12.5 for \( n_s = 0.8, 0.9, 1.0, 1.1, 1.2, \) and 1.3, respectively. At \( n_s = 1.4 \) and 1.5, we can put 95% c.l. lower limits of 1.0 and 2.8 on \( R \). A quadratic fit to the maxima distribution gives

\[ R = 34.3 - 70.8 n_s + 36.5 n_s^2 \tag{24} \]

for \( 1.1 \leq n_s \leq 1.5 \). With the above equation, we find that the tensor component can have an rms amplitude value of \( \approx 28 \mu K \) for \( n_s = 1.1 \) and \( \approx 49 \mu K \) for \( n_s = 1.5 \), while the scalar component remains at \( \approx 100 \mu K \).

It is interesting to see (Fig. 1) that models with \( n_s \approx 1.4 \) and \( R \approx 3 \), which are very compatible with our analysis, seem to have a greater Hubble constant, \( h \approx 0.6 \). So the gravitational wave contribution also seems to moderate the
Fig. 1.—Power spectra of CMB anisotropies for different combinations of inflationary and cosmological parameters. The data points are derived from the experiments listed in Table 1.

Fig. 2.—Confidence level (68% and 95%) regions for the spectral index $n_S$ and the tensor-to-scalar quadrupole ratio $R = C_T^2/C_S^2$. The region below the black line is where the slow-roll approximation is valid.
discrepancy between the value $h \sim 0.7$ (Freedman 1997), inferred by several different methods, and the value $h \sim 0.4$ (Lineweaver et al. 1997), inferred by scalar-only CMB analysis. We found that inside the 95% contour, the overall normalization amplitude, in units of $A_{\text{COBE}}$, is $A = 1 \pm 0.2$; i.e., all the models considered therein correspond well with COBE/DMR normalization.

The simple analysis carried out here does not take into account the correlation that is due to overlapping sky coverage (e.g., Tenerife and COBE and/or MSAM and Saskatoon). We check the stability of our analysis with a jackknife test, i.e., removing one set of experimental data each time. We have a maximum variation of 3%–4% in our limits in the plane, except with the removal of COBE data that modiﬁes our results by $\pm 10\%$. So neglecting this correlation does not signiﬁcantly change the results of our analysis. We also repeated the analysis, including the possible $\pm 14\%$ calibration error to the five Saskatoon points (Netterﬁeld et al. 1997), and we did not ﬁnd signiﬁcant variations. In the limited cases where a comparison was possible, our analysis produced results similar to those of Bond & Jaffe (1996), Lineweaver et al. (1997), and Hancock et al. (1998).

4. COMPARISON WITH LSS

As we have seen, blue models with a substantial tensor component agree well with CMB data. Tensor modes have dramatic effects on the matter power spectrum, reducing its normalization by a factor of $(1 + R)^{-1}$. Using the above fit formula, the tensor contribution to the CMB correlation function on the COBE/DMR scales can be between 54% for $n_s = 1.1$ and 91% for $n_s = 1.5$. In this section, we want to test these models with large-scale matter distribution. As is well known, CDM blue models predict a universe that is too inhomogeneous on scales $\leq 10 h^{-1}$ Mpc. Nonetheless, the excess power on these scales can be reduced by considering a mixture of cold and hot dark matter, i.e., MDM models. The difference in the $C_l$ behavior between a pure CDM and an MDM ($\Omega_c \leq 0.3$) model is very tiny, $\leq 2\%$ up to $l \sim 300$ and $\leq 8\%$ up to $l \sim 800$ (see, e.g., de Gasperis, Muciaccia, & Vittorio 1995). Therefore, the results of our CMB analysis are the same in this kind of model. In Figure 3, MDM matter power spectra from models that agree with CMB data are shown. The data points are an estimate of the linear power spectrum from Peacock & Doods (1994), assuming a CDM ﬂat universe and bias values between Abell, radio, optical, and IRAS catalogs ($b_A:b_R:b_O:b_I = 4.5:1.9:1.3:1.0$ with $b_I = 1.0$). As shown in Smith et al. (1998), the recovered linear power spectra of CDM and MDM models are nearly the same in the region $0.01 < l < 0.15 h^{-1}$ Mpc but diverge from this spectrum at higher $k$, so we restrict ourselves to this range. The $\chi^2$ (with 11 degrees of freedom) are 15, 10, 21, 9, 37, and 53 for models in Figure 3 with $(1.4, 3.3), (1.3, 3.9), (1.2, 1.3), (1.1, 0.6), (1.0, 0.1),$ and $(0.9, 0)$, respectively, in the $(n_s, R)$-space. So models with a large tensor contribution on COBE scales and a blue spectral index seem to agree well also with the shape of matter distribution on a large scale. The values for the $\sigma_8$, computed with CMBFAST (Seljak & Zaldarriaga 1996), are 0.69, 0.61, 0.66, 0.63, 0.63, and 0.74, in very reasonable agreement with the value of $\sigma_8^{\text{IRAS}} = 0.69 \pm 0.05$ (Fisher et al. 1994) derived from the IRAS catalog.

Whether IRAS galaxies are biased is still under debate. Analysis from cluster data (Eke et al. 1996; Pen 1998; Bryan & Norman 1998) shows a preferred value of $\sigma_8 \sim 0.5$–0.6 with error bars of few percent. Analysis from peculiar
velocities (Zehavi 1998) results in a larger value of $\sigma_8 = 0.85 \pm 0.2$, which seems to be in severe conflict with the cluster data. Thus, the theoretical values of $\sigma_8$ for blue MDM models with a relic gravitational wave background are between the $\sigma_8$ values derived from the cluster abundance and peculiar velocities. In any case, the likelihood of the CMB data is quite flat around its maximum. So it is easy to find models, statistically indistinguishable from the best-fit models, with $\sigma_8$ nearer to either 0.5 or 0.8. Because of statistical and/or systematic uncertainties, we do not consider it appropriate to put more than qualitative conclusions on these results, but still one can say that the lower matter normalization that is due to the tensor component helps us match the blue MDM models to the LSS data.

5. CONCLUSIONS

Our main conclusions are as follows:

1. The conditional likelihood shows a maximum at $n_g = 1.23^{+0.19}_{-0.15}$, $R = 2.4^{+2.2}_{-2.2}$, with $\alpha = 0.92$, $\Omega_b = 0.07$, and $h = 0.46$. Thus, there is some evidence that a tensor component can be present, and in a substantial way, in models with $n_g > 1$. Inflaton models of this type have been investigated by Copeland et al. (1993), Lukash & Mikheeva (1996), Lucchin et al. (1996), Borgani et al. (1996), and Bonometto & Pierpaoli (1998) and thus belong to the class of hybrid inflationary models (Kinney, Dodelson, & Kolb 1998; Linde 1994). The general form of the potential can be written as $V(\phi) = V_0 + \frac{1}{2}m^2\phi^2$. At the end of inflation, the inflationary potential $V(\phi_0)$ is not equal to zero, being $V_0$ on the order of $\sim (6\times10^{16} \text{ GeV})^4$. In order to be consistent with the present vacuum energy $\eta \lesssim (10^{-30} \text{ GeV})^4$, one additional field is necessary to finish inflation. The inclusion of this field does not change the conclusion of our analysis, since it affects only the high-frequency region of the GW spectrum ($\sim 100 \text{ MHz}$). For models on the best-fit curve (eq. [24]), $V(\phi_0)$ belongs to the interval $4.3 \times 10^{-11} m^4 < V_0 < 1.3 \times 10^{-10} m^4$. In Figure 2, we plot equation (24) with the condition $\epsilon = 1$. The region below this curve in the $n_g$-$R$ plane is where the slow-roll approximation is valid. As we can see, models on our best-fit curve satisfy this condition, even if models with $\epsilon \geq 1$ are compatible with observations. Approaching the limiting region $\epsilon = 1$, higher order terms in the slow-roll approximation became valuable. This leads to changes in our conclusions on the potential by a factor $1 - \epsilon/3 \sim 30\%$ (Kinney et al. 1998; Linde 1994).

2. The 95% region on the $n_g$-$R$ plane includes a wide range of parameters. This means that the presently available data set is not sensitive enough to produce precise determinations for $n_g$ and $R$. Systematic and statistical errors in the different experiments are still significant, but, as we have shown, the difficulties involved in such determinations are mainly due to a degeneracy in these parameters. So the $(n_g, R)$-degeneracy has important consequences for tests of the inflationary theory: increasing the scalar spectral index and the tensor component leads to a break in the slow-roll approximation, but it also produces CMB power spectra that are near to the scale-invariant spectrum. Therefore, it is difficult with the present CMB data to see if the slow-roll condition is correct. Furthermore, current CMB results on the normalization of the matter power spectrum and/or its spectral index can be biased and/or antibiased by a huge tensor contribution. As we can see from Figure 1, this degeneration also has an effect on the constraints of the remaining cosmological parameters, being a model with $h \sim 0.6$ that is statistically indistinguishable from a model with $h \sim 0.4$. The inflationary background of primordial gravitational waves is assumed to be detectable mainly through CMB experiments. The local energy density of this background is, in the most optimistic situation, extremely low, with $d\Omega_{GW} h^2 / d \log k \sim 10^{-16}$ at frequencies $10^{-15}$ Hz $< f < 10^{15}$ Hz. The tenuity of this signal makes the degeneracy in the $n_g$ and $R$ parameters much more worrying than similar degeneracies in other parameters (e.g., $h$ and $\Omega_b$) that could be constrained through other measurements.

3. Blue MDM models with a tensor contribution are in reasonable agreement with the present values of $\sigma_8$, and with the shape of the matter power spectrum inferred by the Peacock & Dodds (1994) analysis. A tensor contribution could also be a viable mechanism for reconciling these models with a low value for the $\sigma_8$ around $\sim 0.5$ (Henry & Arnaud 1991). This being the situation, a measure of the structure of the secondary peaks becomes a crucial test for the presence of tensor perturbations. Using the above best-fit equation, we can make some predictions regarding future detections. We found that an experiment with a window function probing the multipoles 500 $\leq l \leq 680$ will measure a total rms anisotropy of 23.5 $\mu$K for $n_g = 1.1$ and 34.4 $\mu$K with $n_g = 1.5$. An $\sim 20\%$ difference that could be proved, when the sensitivity of these experiments is within a few $\mu$K, with an improved sky coverage. Polarization measurements at intermediate angular scales can also be helpful (Sazhin 1984; Polnarev 1985; Sazhin & Benitez 1995; Sazhin & Shulga 1996a). The possibility of a direct separation of scalar perturbations from tensor perturbations by the method of the decomposition of Stokes parameters in sets of $\pm 2$ spherical harmonics seems extremely promising (Kamionkovsky, Kosowsky, & Stebbins 1997; Seljak & Zaldarriaga 1997; Sazhin & Shulga 1996b). Possibly a definitive answer will come when future CMB experiments provide a clear and robust picture of subdegree angular scale anisotropy and polarization.

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