A QUASINONEXPANSIVE EXTENSION OF A MAPPING WITH AN ATTRACTIVE POINT IN A HILBERT SPACE

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Abstract. In this paper, we show that, under appropriate conditions, there exists a quasinonexpansive extension of a mapping with an attractive point in the sense of Takahashi and Takeuchi [17] such that the fixed point set of the extension equals the attractive point set of the given mapping. Then using the quasinonexpansive extension, we establish some convergence theorems for approximating attractive points of a generalized hybrid mapping in the sense of Kocourek, Takahashi, and Yao [12].

1. Introduction

Let $H$ be a Hilbert space, $C$ a subset of $H$, and $T: C \to H$ a mapping. Takahashi and Takeuchi [17] introduced the notion of an attractive point of $T$; see §2 for the definition of an attractive point. It is easy to verify that if $T$ is quasinonexpansive, then every fixed point of $T$ is an attractive point of $T$. Thus an attractive point is regarded as a generalization of a fixed point for a quasinonexpansive mapping.

Takahashi and Takeuchi [17] also established a mean convergence theorem for an attractive point of a generalized hybrid mapping in the sense of Kocourek et al. [12]; see §2 for the definition of a generalized hybrid mapping. Such a mapping originates from a $\lambda$-hybrid mapping introduced in Aoyama et al. [2]; see also [4, 5]. We know some existence and convergence results for attractive points of a generalized hybrid mapping and its variants; see, for example, [1, 11, 18, 19].

In this paper, we prove that, under appropriate conditions, if a mapping $T: C \to H$ has an attractive point, then there exists a quasinonexpansive extension $\tilde{T}: H \to H$ of $T$ such that the set of fixed points (or asymptotic fixed points) of $\tilde{T}$ equals that of attractive points of $T$. Then using the quasinonexpansive extension, we derive convergence theorems for attractive points from those for fixed points of quasinonexpansive mappings. Moreover, we also obtain convergence results for attractive points of a generalized hybrid mapping.

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2. Preliminaries

Throughout the present paper, $H$ denotes a real Hilbert space, $\langle \cdot, \cdot \rangle$ the inner product of $H$, $\| \cdot \|$ the norm of $H$, $C$ a nonempty subset of $H$, $I$ the identity mapping on $H$, and $\mathbb{N}$ the set of positive integers. Strong convergence of a sequence $\{x_n\}$ in $H$ to $z \in H$ is denoted by $x_n \to z$ and weak convergence by $x_n \rightharpoonup z$.

Let $T: C \to H$ be a mapping. Then the set of fixed points of $T$ is denoted by $F(T)$, that is, $F(T) = \{z \in C: Tz = z\}$. A point $z \in H$ is said to be an asymptotic fixed point of $T$ if there exists a sequence $\{x_n\}$ in $C$ such that $x_n - Tx_n \to 0$ and $x_n \rightharpoonup z$. The set of asymptotic fixed points of $T$ is denoted by $\hat{F}(T)$. It is clear that $F(T) \subset \hat{F}(T)$. A point $z \in H$ is said to be an attractive point of $T$ if $\|Tx - z\| \leq \|x - z\|$ for all $x \in C$. The set of attractive points of $T$ is denoted by $A(T)$, that is,

$$A(T) = \bigcap_{x \in C} \{z \in H: \|Tx - z\| \leq \|x - z\|\}.$$ 

It is clear that $C \cap A(T) \subset F(T)$, and that $A(T)$ is closed and convex.

Let $T: C \to H$ be a mapping and $F$ a nonempty subset of $H$. Then $T$ is said to be quasinonexpansive with respect to $F$ if $\|Tx - z\| \leq \|x - z\|$ for all $x \in C$ and $z \in F$; $T$ is said to be quasinonexpansive if $F(T) \neq \emptyset$ and $\|Tx - z\| \leq \|x - z\|$ for all $x \in C$ and $z \in F(T)$; $T$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$; $T$ is said to be a generalized hybrid if there exist $\alpha, \beta \in \mathbb{R}$ such that

$$\alpha \|Tx - Ty\|^2 + (1 - \alpha) \|x - Ty\|^2 \leq \beta \|Tx - y\|^2 + (1 - \beta) \|x - y\|^2$$

for all $x, y \in C$; $T$ is said to be demiclosed at 0 if $Tz = 0$ whenever $\{x_n\}$ is a sequence in $C$ such that $x_n \rightharpoonup z$ and $Tx_n \to 0$; see, for example, [9]. It is clear that

- if $A(T) \neq \emptyset$, then $T$ is quasinonexpansive with respect to $A(T)$;
- if $T$ is a generalized hybrid mapping, then $F(T) \subset A(T)$;
- $I - T$ is demiclosed at 0 if and only if $\hat{F}(T) = F(T)$.

Moreover, under the assumption that $C$ is closed and convex, we know the following:

- If $T$ is quasinonexpansive, then $F(T)$ is closed and convex; see [8, Theorem 1];
- if $T$ is nonexpansive, then $I - T$ is demiclosed at 0; see [9].

A generalized hybrid mapping has the following property:

**Lemma 2.1** ([19] Lemma 3.1). Let $H$ be a Hilbert space, $C$ a nonempty subset of $H$, $T: C \to H$ a generalized hybrid mapping, and $\{x_n\}$ a sequence in $C$ such that $x_n - Tx_n \to 0$ and $x_n \rightharpoonup z$. Then $z \in A(T)$, that is, $\hat{F}(T) \subset A(T)$. 
Let $D$ be a nonempty closed convex subset of $H$. It is known that, for each $x \in H$, there exists a unique point $x_0 \in D$ such that

$$\|x - x_0\| = \min \{\|x - y\| : y \in D\}.$$  

Such a point $x_0$ is denoted by $P_D(x)$ and $P_D$ is called the metric projection of $H$ onto $D$. It is known that the metric projection is nonexpansive; see [16] for more details.

The following theorem is a direct consequence of [3, Theorem 5.5]; see also [14, Theorem 3.4].

**Theorem 2.2.** Let $H$ be a Hilbert space, $T : H \to H$ a quasinonexpansive mapping, $\{\alpha_n\}$ a sequence in $(0, 1]$, $\{\beta_n\}$ a sequence in $[0, 1]$, and $\{x_n\}$ a sequence defined by $u, x_1 \in H$ and

$$x_{n+1} = \alpha_n u + (1 - \alpha_n) [\beta_n x_n + (1 - \beta_n)Tx_n]$$

for $n \in \mathbb{N}$. Suppose that $\hat{F}(T) = F(T)$, $\alpha_n \to 0$, $\sum_n \alpha_n = \infty$, and $\liminf_n \beta_n (1 - \beta_n) > 0$. Then $\{x_n\}$ converges strongly to $P_{\hat{F}(T)}(u)$.

**Remark 2.3.** In Theorem 2.2, the condition $\liminf_n \beta_n (1 - \beta_n) > 0$ is equivalent to the following: $\liminf_n \beta_n > 0$ and $\limsup_n \beta_n < 1$.

The following theorem is a direct consequence of [13, Theorem 3.2]; see also [7].

**Theorem 2.4.** Let $H$ be a Hilbert space, $T : H \to H$ a quasinonexpansive mapping, $\{\alpha_n\}$ a sequence in $[0, 1]$, and $\{x_n\}$ a sequence defined by $x_1 \in H$ and

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n$$

for $n \in \mathbb{N}$. Suppose that $\hat{F}(T) = F(T)$ and $\liminf_n \alpha_n (1 - \alpha_n) > 0$. Then $\{x_n\}$ converges weakly to some point $w \in F(T)$.

### 3. Quasinonexpansive Extensions

In this section, we prove that, under appropriate assumptions, a mapping with an attractive point has a quasinonexpansive extension such that the set of fixed points (or asymptotic fixed points) is equal to that of attractive points (Lemma 3.4). We begin with the following:

**Lemma 3.1.** Let $H$ be a Hilbert space, $C$ a nonempty subset of $H$, $T : C \to H$ a mapping with an attractive point, and $\hat{T} : H \to H$ a mapping defined by

$$\hat{T}x = \begin{cases} 
Tx, & x \in C; \\
P_{A(T)}(x), & \text{otherwise}. 
\end{cases}$$  

Then $\hat{T}$ is an extension of $T$ and quasinonexpansive with respect to $A(T)$. Moreover, $A(T) \subset F(\hat{T})$. 

Proof. By the definition of $\tilde{T}$, it is clear that $\tilde{T}$ is an extension of $T$. We show that $\tilde{T}$ is quasinonexpansive with respect to $A(T)$. Let $x \in H$ and $z \in A(T)$. Suppose that $z \in C$. Since $z$ is an attractive point of $T$, we have $\|\tilde{T}x - z\| = \|Tx - z\| \leq \|x - z\|$. On the other hand, suppose that $z \notin C$. Since $P_{A(T)}$ is nonexpansive and $z = P_{A(T)}(z)$, we have

$$\|\tilde{T}x - z\| = \|P_{A(T)}(x) - P_{A(T)}(z)\| \leq \|x - z\|.$$  

Therefore, $\tilde{T}$ is quasinonexpansive with respect to $A(T)$.

We next show that $A(T) \subset F(\tilde{T})$. Let $z \in A(T)$. Suppose that $z \in C$. Since $\tilde{T}$ is quasinonexpansive with respect to $A(T)$, we have $\|\tilde{T}z - z\| \leq \|z - z\| = 0$, and hence $z \in F(\tilde{T})$. On the other hand, suppose that $z \notin C$. Then we have $\tilde{T}z = P_{A(T)}(z) = z$, and hence $z \in F(\tilde{T})$. As a result, we conclude that $A(T) \subset F(\tilde{T})$. \hfill $\square$

Remark 3.2. In Lemma 3.1, one can verify that $A(\tilde{T}) = A(T)$.

The following example shows that $A(T) \neq F(\tilde{T})$ in Lemma 3.1.

Example 3.3. Let $H = \mathbb{R}$ and $C = \mathbb{R} \setminus \{0\}$. Let $T : C \to C$ be a mapping defined by

$$Tx = \begin{cases} 1, & x = 1; \\ -x, & \text{otherwise.} \end{cases}$$

Then $F(T) = \{1\}$ and $A(T) = \{0\}$. Moreover, let $\tilde{T} : H \to H$ be a mapping defined by (3.1), that is,

$$\tilde{T}x = \begin{cases} 0, & x = 0; \\ Tx, & \text{otherwise.} \end{cases}$$

Then $F(\tilde{T}) = \{0, 1\}$. Therefore, $A(T) \neq F(\tilde{T})$.

Proof. The equality $F(T) = \{1\}$ is obvious. We first show that $A(T) = \{0\}$. Let $x \in C$. If $x = 1$, then $|Tx - 0| = |1| = |x - 0|$; otherwise $|Tx - 0| = |-x| = |x - 0|$. Thus $0 \in A(T)$. On the other hand, suppose that $z \in A(T)$ and $z \neq 0$. Then $z \in C$. As a result, we have $z \in C \cap A(T) \subset F(T)$, and hence $z = 1$. However, since

$$|T(1/2) - 1| = |-1/2 - 1| = 3/2 > 1/2 = |1/2 - 1|,$$

we have $z \notin A(T)$, which is a contradiction. Therefore we conclude that $A(T) = \{0\}$.

We next show that $F(\tilde{T}) = \{0, 1\}$. By definition, $\tilde{T}0 = 0$ and $\tilde{T}1 = T1 = 1$. Thus $\{0, 1\} \subset F(\tilde{T})$. If $z \notin \{0, 1\}$, then we have $\tilde{T}z = Tz = -z \neq z$. This means that $\{0, 1\} \supset F(\tilde{T})$. \hfill $\square$

Lemma 3.4. Let $H$ be a Hilbert space, $C$ a nonempty subset of $H$, $T : C \to H$ a mapping with an attractive point, and $\tilde{T} : H \to H$ a mapping defined by (3.1). Then the following hold:
(1) If $F(T) \subset A(T)$, then $A(T) = F(\hat{T})$ and $\hat{T}$ is quasinonexpansive; 
(2) if $\hat{F}(T) \subset A(T)$, then $\hat{F}(T) = F(\hat{T})$, that is, $I - \hat{T}$ is demiclosed at 0.

Proof. We first show (1). We know from Lemma 3.1 that $\hat{T}$ is quasinonexpansive with respect to $A(T)$, and that $A(T) \subset F(T)$. Thus it is enough to show that $A(T) \supset F(T)$. Let $z \in F(\hat{T})$. If $z \in C$, then $z = \hat{T}z = Tz$, and hence $z \in F(T) \subset A(T)$. If $z \notin C$, then $z = P_{A(T)}(z) \in A(T)$. Consequently, it turns out that $A(T) \supset F(\hat{T})$.

We next show (2). Since $F(T) \subset \hat{F}(T) \subset A(T)$, it follows from (1) that $A(T) = F(\hat{T})$. Thus it is enough to prove that $\hat{F}(T) \subset A(T)$. Let $z \in \hat{F}(T)$. Then there exists a sequence $\{x_n\}$ in $H$ such that $x_n - \hat{T}x_n \to 0$ and $x_n \to z$. We consider two cases, which might not be exclusive. (i) Suppose that there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \in C$ for all $i \in \mathbb{N}$. Then it follows that $x_{n_i} - Tx_{n_i} = x_{n_i} - \hat{T}x_{n_i} \to 0$ and $x_{n_i} \to z$. Thus, by assumption, we deduce that $z \in F(T) \subset A(T)$. (ii) Suppose that there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \notin C$ for all $i \in \mathbb{N}$. Then $x_{n_i} - P_{A(T)}(x_{n_i}) = x_{n_i} - \hat{T}x_{n_i} \to 0$ and $x_{n_i} \to z$. Since $P_{A(T)}$ is a nonexpansive mapping on $H$, $I - P_{A(T)}$ is demiclosed at 0. Hence $z \in F(P_{A(T)}) = A(T)$. This completes the proof. \qed

4. APPROXIMATION OF ATTRACTIVE POINTS

In this section, using lemmas in the previous section (Lemmas 3.1 and 3.4) and convergence theorems for quasinonexpansive mappings (Theorems 2.2 and 2.4), we obtain two convergence theorems for attractive points of a mapping satisfying the condition that every asymptotic fixed point is an attractive point, and as corollaries of them, we also obtain convergence results for attractive points of generalized hybrid mappings.

Theorem 4.1. Let $H$ be a Hilbert space, $C$ a nonempty convex subset of $H$, $T : C \to C$ a mapping with an attractive point, $\{\alpha_n\}$ a sequence in $(0, 1]$, $\{\beta_n\}$ a sequence in $[0, 1]$, and $\{x_n\}$ a sequence in $C$ defined by $u, x_1 \in C$ and

$$x_{n+1} = \alpha_n u + (1 - \alpha_n)[\beta_n x_n + (1 - \beta_n)Tx_n]$$

for $n \in \mathbb{N}$. Suppose that $\sum_n \alpha_n = \infty$, $\lim_n \alpha_n = 0$, and $\liminf_n \beta_n(1 - \beta_n) > 0$. If $\hat{F}(T) \subset A(T)$, then $\{x_n\}$ converges strongly to $P_{A(T)}(u)$.

Proof. Let $\hat{T} : H \to H$ be an extension of $T$ defined by (3.1). By the assumption that $F(T) \subset A(T)$, we see that $F(T) \subset \hat{F}(T) \subset A(T)$. Thus Lemma 3.3 implies that $\hat{F}(T) = F(\hat{T}) = A(T)$ and $\hat{T}$ is quasinonexpansive. Moreover, since $C$ is convex and $\hat{T}$ is an extension of $T$, it follows that

$$x_{n+1} = \alpha_n u + (1 - \alpha_n)[\beta_n x_n + (1 - \beta_n)\hat{T}x_n]$$
for all \( n \in \mathbb{N} \). Therefore we deduce from Theorem 2.2 that \( x_n \to P_{\hat{F}(\tilde{T})}(u) = P_{A(T)}(u) \).

Using Theorem 4.1 and Lemma 2.1 we obtain the following corollary; see Takahashi, Wong, and Yao [19, Theorem 3.2].

**Corollary 4.2.** Let \( H, C, \{\alpha_n\}, \) and \( \{\beta_n\} \) be the same as in Theorem 4.1. Let \( T: C \to C \) be a generalized hybrid mapping with an attractive point and \( \{x_n\} \) a sequence in \( C \) defined by \( u, x_1 \in C \) and (4.1) for \( n \in \mathbb{N} \). Then \( \{x_n\} \) converges strongly to \( P_{A(T)}(u) \).

**Proof.** Lemma 2.1 shows that \( \hat{F}(\tilde{T}) \subset A(T) \). Thus Theorem 4.1 implies the conclusion. \( \square \)

**Remark 4.3.** Corollary 4.2 is almost the same as [19, Theorem 3.2], except that \( \{\alpha_n\} \) and \( \{\beta_n\} \) are assumed to be sequences in \((0, 1)\) in [19, Theorem 3.2].

**Theorem 4.4.** Let \( H \) be a Hilbert space, \( C \) a nonempty convex subset of \( H \), \( T: C \to C \) a mapping with an attractive point, \( \{\alpha_n\} \) a sequence in \([0, 1]\), and \( \{x_n\} \) a sequence in \( C \) defined by \( u, x_1 \in C \) and

\[
(4.2) \quad x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T x_n
\]

for \( n \in \mathbb{N} \). Suppose that \( \lim \inf_n \alpha_n(1 - \alpha_n) > 0 \). If \( \hat{F}(T) \subset A(T) \), then \( \{x_n\} \) converges weakly to some point in \( A(T) \).

**Proof.** Let \( \tilde{T}: H \to H \) be an extension of \( T \) defined by (3.1). As in the proof of Theorem 4.1, Lemma 3.4 shows that a mapping \( \tilde{T} \) is a quasinonexpansive extension of \( T \), and that \( \hat{F}(\tilde{T}) = F(\tilde{T}) = A(T) \). We can also check that

\[
(4.3) \quad x_{n+1} = \alpha_n x_n + (1 - \alpha_n)\tilde{T} x_n
\]

for all \( n \in \mathbb{N} \). Therefore Theorem 2.4 implies the conclusion. \( \square \)

Finally, we obtain a weak convergence result for a widely more generalized hybrid mapping in the sense of [11] as a corollary of Theorem 4.4.

Let \( C \) be a nonempty subset of a Hilbert space \( H \) and \( T: C \to H \) a mapping. Recall that \( T \) is widely more generalized hybrid [11] if there exist \( \alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta \in \mathbb{R} \) such that

\[
(4.3) \quad \alpha \|Tx - Ty\|^2 + \beta \|x - Ty\|^2 + \gamma \|Tx - y\|^2 + \delta \|x - y\|^2
+ \epsilon \|x - Tx\|^2 + \zeta \|y - Ty\|^2 + \eta \|x - Tx - (y - Ty)\|^2 \leq 0
\]

for all \( x, y \in C \). Such a mapping \( T \) is called an \((\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)\)-widely more generalized hybrid mapping.

Using Theorem 4.4 and [10, Lemma 11], we obtain the following corollary; see [10, Theorem 14].
Corollary 4.5. Let $H$, $C$, and $\{\alpha_n\}$ be the same as in Theorem 4.4. Let $T: C \to C$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping with an attractive point and $\{x_n\}$ a sequence in $C$ defined by $x_1 \in C$ and (4.2) for $n \in \mathbb{N}$. Suppose that

$$\alpha + \beta + \gamma + \delta \geq 0, \alpha + \gamma > 0, \text{ and } \epsilon + \eta \geq 0$$

hold. Then $\{x_n\}$ converges weakly to some point in $A(T)$.

Proof. [10, Lemma 11] shows that $\hat{F}(T) \subset A(T)$. Thus Theorem 4.4 implies the conclusion.

Remark 4.6. Corollary 4.5 is almost the same as [10, Theorem 14], except that $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$, and $\eta$ are assumed to satisfy (4.4) or

$$\alpha + \beta + \gamma + \delta \geq 0, \alpha + \beta > 0, \text{ and } \zeta + \eta \geq 0$$

in [10, Theorem 14]. We can confirm that the conditions (4.4) and (4.5) are equivalent for an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping.

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