Multiple Coulomb scattering in thin silicon

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ABSTRACT: We present a measurement of multiple Coulomb scattering of 1 to 6 GeV/c electrons in thin (50–140 µm) silicon targets. The data were obtained with the EUDET telescope Aconite at DESY and are compared to parametrisations as used in the Geant4 software package. We find good agreement between data and simulation in the scattering distribution width but large deviations in the shape of the distribution. In order to achieve a better description of the shape, a new scattering model based on a Student’s $t$ distribution is developed and compared to the data.

KEYWORDS: Solid state detectors; Interaction of radiation with matter; Particle tracking detectors (Solid-state detectors); Detector modelling and simulations I (interaction of radiation with matter, interaction of photons with matter, interaction of hadrons with matter, etc)

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1 Motivation

A good understanding of multiple Coulomb scattering of relativistic particles in matter is important both for tracking detectors and calorimetry. The theory of multiple scattering was first treated by Wentzel in 1922 [1] and fully developed in the 1940ies by Goudsmit and Saunderson [2, 3] and Molière [4, 5] (summarized in more elegant notation by Bethe [6]). Their approaches differ in the treatment of the screened nuclear potential and the series expansion applied to make the problem analytically tractable. In both calculations however, the path length of the particle in the material is assumed to be independent of the scattering angle; this problem was addressed by Lewis [7], whose improved approach is also the basis of the default multiple scattering model in the Geant4 simulation package [8, 9]. For an extensive review of multiple scattering theory, see [10].

For experimental purposes, very often the parametrisation suggested by Highland [11] and popularized by the Particle Data Group (PDG) [12] is used:

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \left(\frac{x}{X_0}\right)\right), \quad (1.1)$$

where $\theta_0$ is the RMS calculated from the central 98% of the planar scattering angle distribution (henceforth referred to as $RMS_{98}$), $p$, $\beta c$ and $z$ are the momentum, velocity and charge number of the incident particle and $x/X_0$ is the material thickness in radiation lengths.

Whilst there is a wealth of data on multiple scattering measured in silicon of a few 100 µm thickness in solid state tracking devices, there are only few published data for scattering in thin foils. Measurements of the scattering of 15.7 MeV/c electrons in beryllium by Hanson et al. [13] disagreed with the theoretical models available at the time, whilst the measurements with a gold foil were adequately described. They serve as benchmarks for models to this day [9]. Other published datasets range from 2.25 MeV/c electrons [14] via measurements with pions at a few hundred MeV/c [15] to protons with 0.7 and 4.8 MeV/c [16], 600 MeV [17] and 50 to 200 GeV/c [18]. As targets, metal and carbon foils and slabs were used.
With the advent of very thin (50 $\mu$m) silicon tracking detectors [19–26], a good understanding of scattering in thin silicon is important for the design and calibration of experiments employing such sensors, such as the STAR pixel detector [27, 28], the BELLE II silicon tracker [24, 29, 30] or the Mu3e pixel detector [31, 32]. The present measurement is performed in the course of a test beam campaign for the Mu3e experiment.

2 Measurement setup

The data presented here were obtained with the EUDET telescope *Aconite* [33–35] at the DESY test beam line T22. The beam line provides electrons from converted bremsstrahlung beams produced by carbon fibre targets in the electron synchrotron DESY II with momenta from 1 to 6 GeV/c and an energy spread below 5% [36] at rates up to about 1 kHz. The beam divergence is approximately 1 mrad.

The telescope is built from six layers of Mimosa26 [27, 37, 38] monolithic active pixel sensors (MAPS) thinned to 50 $\mu$m. The active area of the MAPS is approximately $2 \times 1 \text{cm}^2$. The data acquisition is triggered by a coincidence of signals in two crossed pairs of scintillators, one before and one after the telescope. Between the third and fourth telescope plane, we placed either one or two 50 $\mu$m thick unprocessed silicon wafers\(^1\) as scattering targets on a rotating stage, see figure 1 for an overview of the set-up. The silicon wafers are much larger than the Mimosa sensors, it is thus ensured that all tracks in the telescope acceptance pass through the target. The sixth telescope plane was out of operation for this measurement.

Data were taken at electron momenta between 1 and 6 GeV/c in 1 GeV/c increments. For every momentum point, we measured scattering angles with a 50 or 100 $\mu$m thick target oriented at beam incidence angles of 0°, 15°, 30° and 45° resulting in a projected thickness $d_{\text{eff}}$ between 50 and 141 $\mu$m. For a determination of the contribution of the telescope, measurements without the silicon target were performed. For each data point we collected about one million triggers, resulting in approximately 300'000 tracks after selection cuts.

3 Data analysis

The telescope planes are aligned using reconstructed tracks in the configuration without silicon target using the EUTelescope software framework [39]. Track residuals after alignment are below

\(^1\)The manufacturer specifies the wafer thickness as 40-60 $\mu$m; we measured 50 $\mu$m within an uncertainty of 5 $\mu$m for all samples.
Figure 2. Horizontal scattering angle distribution for 1, 3 and 6 GeV/c electrons with no scattering target. As fit function (red line), the sum of a Gaussian and a Student’s $t$-distribution is used as described in the text.

2 $\mu$m. The distance of the target scattering plane to the third and fourth telescope planes is known to about 1 mm.

For the scattering analysis, tracks are reconstructed separately in the up- and downstream parts of the telescope and extrapolated to the silicon target plane. If an up- and a downstream track intersect within 150 $\mu$m on that plane and there are no matching ambiguities, the scattering angles $\theta$ between the tracks are calculated in both the horizontal and vertical projections.

The effect of multiple scattering in the telescope including the air surrounding the target together is larger than the scattering in the target. The measured distribution of planar scattering angles $f(\theta)$ can be described by convoluting the telescope scattering contributions with the scattering distribution in the target:

$$f(\theta) = f_{\text{telescope, upstream}} \otimes f_{\text{target}} \otimes f_{\text{telescope, downstream}}$$  \hspace{1cm} (3.1)

In order to determine the effect of the telescope, we first study the scattering angle distribution $f_{\text{telescope}} = f_{\text{telescope, upstream}} \otimes f_{\text{telescope, downstream}}$ for datasets without silicon target in the beam. As an ansatz we use the sum of a Gaussian and a Student’s $t$ distribution [40]; the core of the scattering distribution is expected to be Gaussian and the Student’s $t$ distribution can account for the tails caused by the few large angle scattering events for which the law of large numbers does not apply.
Figure 3. Horizontal scattering angle distribution for 1, 3 and 6 GeV/c electrons with 50 µm silicon target and an incidence angle to the beam of 15° in the device-under-test position. As fit function, a convolution of the shape obtained from a fit to the angular distribution without target and a Student’s \( t \)-distribution is used as described in the text.

Empirically we found that the measured distributions are well described by the this sum:

\[
 f_{\text{telescope}}(\theta) = N \cdot \left( 1 - a \right) \cdot \frac{1}{\sigma_G \sqrt{2\pi}} \left( \frac{(\theta - \mu)^2}{\sigma_G^2} \right) + a \cdot \frac{\Gamma\left( \frac{\nu+1}{2}\right)}{\sqrt{\nu\pi\sigma^2} \Gamma\left( \frac{\nu}{2}\right)} \left( 1 + \frac{(\theta - \mu)^2}{\nu \sigma^2} \right)^{-\frac{\nu+1}{2}}. \tag{3.2} \]

A binned likelihood fit with six free parameters, namely overall normalization \( N \), relative fraction \( a \) of the Student’s \( t \) distribution, a common mean \( \mu \), the width of the Gaussian \( \sigma_G \) and the width \( \sigma \) and tail parameter \( \nu \) of the \( t \) distribution is used. For \( \nu \to \infty \), the Student’s \( t \) distribution turns into a Gaussian, whereas for \( \nu \to 1 \), the tails get more pronounced. At \( \nu = 1 \), a Lorentzian distribution is obtained. We obtain good fits at all electron momenta. Figure 2 shows the fitted horizontal scattering angle distributions at 1, 3 and 6 GeV/c electron momentum. The fits for the horizontal and vertical scattering angles give results that are compatible within statistical uncertainties, thus reassuring us that there are no large residual effects of telescope misalignment or acceptance.

The data with the silicon target in the beam are fitted using a binned likelihood function based on the convolution of a Student’s \( t \) distribution, representing the contribution by the target, and the shape of the scattering angle distribution of the telescope and air as given in equation (3.2):

\[
 f(\theta) = N \cdot \int f_{\text{telescope}}(\theta - \tau) \cdot \frac{\Gamma\left( \frac{\nu+1}{2}\right)}{\sqrt{\nu\pi\sigma^2} \Gamma\left( \frac{\nu}{2}\right)} \left( 1 + \frac{(\tau)^2}{\nu \sigma^2} \right)^{-\frac{\nu+1}{2}} d\tau. \tag{3.3} \]
Figure 4. RMS of the central 98% of the fitted Student’s $t$ distribution versus electron momentum for varying silicon target thickness compared to the Highland-parametrisation (left). Fitted tail parameter $\nu$ of the Student $t$ distribution versus electron momentum for varying silicon target thickness (right). The data points are slightly offset from their horizontal positions at multiples of 1 GeV/$c$ for better visibility. The error bars represent the 1 $\sigma$ uncertainty of the fit. Smaller $\nu$ values correspond to larger tail fractions.

Figure 5. RMS of the central 98% of the fitted Student’s $t$ distribution versus silicon target thickness for varying electron momenta compared to the Highland-parametrisation (left). Fitted tail parameter $\nu$ of the Student $t$ distribution versus silicon target thickness for varying electron momenta (right). The error bars represent the 1 $\sigma$ uncertainty of the fit.

The free parameters in this fit are the overall normalization $N$ and the width $\sigma$ and tail parameter $\nu$ of the $t$ distribution. Again we obtain good fits, see figure 3.

The fits are performed for the horizontal and vertical scattering angles separately; the results are consistent within uncertainties. All the figures shown in the following are based on a combination of the results from the two projections. All fit results and their statistical uncertainties are listed in table 1.

The $RMS_{98}$ as well as the tail parameter $\nu$ of the distributions are shown as a function of electron momentum in figure 4 and effective thickness in figure 5. As can be seen in the left panels of figures 4 and 5, the $RMS$ of the core of the scattering distributions is described by the Highland formula within the 10% uncertainty quoted [12]. The amount of tails increases with momenta, see the right panel of figure 4. This is expected, as higher momentum electrons get closer to the nuclei.
of the scatterer and thus see a less screened nuclear potential leading to larger deflections. The tail fraction also slightly decreases with thickness, see the right panel of figure 5; this seems to indicate that for the thin scatterers used here, the statistical approach to multiple scattering starts to break down as individual large angle scattering events become important.

4 Comparison with simulation models

In order to compare the results with multiple scattering calculations, we simulate one million electron tracks propagating through 50 and 100 μm of silicon at incident angles of $0^\circ$, $15^\circ$, $30^\circ$ and $45^\circ$. The simulated scattering distributions are fitted with a Student’s t distribution. The following models in Geant4 [41, 42] are tested:

| Table 1. Student’s t distribution parameters $v$ and $\sigma$ fitted to the scattering angle distribution of $p = 1-6$ GeV/c electrons on a $d = 50-141$ μm thin silicon target. For comparison, simulation results using the default Urbán model in Geant4 10.0 and our model are shown. In addition, the corresponding RMSσs values are given.

| $p$ (GeV/c) | $d$ (μm) | $\sigma$ | $\nu$ | RMSσ | $\sigma$ | $\nu$ | RMSσ |
|----------|---------|---------|------|-------|---------|------|-------|
| 1-6       | 0.1 rad | 0.1 rad |       |       | 0.1 rad | 0.1 rad |       |

For the Urbán model in Geant4 10.0, the following parameters are used:

| $\sigma$ (μm) | $\nu$ (μm) |
|--------------|------------|
| 0.10         | 0.10       |
| 0.15         | 0.15       |
| 0.20         | 0.20       |

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Figure 6. Comparison of the RMS of the central 98% of the fitted Student’s t distribution versus momentum for various scattering models in Geant4 with our data obtained with a 50 µm silicon target perpendicular to the beam (left) and a 100 µm silicon target tilted by 45° (right). The data points are slightly offset from their horizontal positions at multiples of 1 GeV/c for better visibility. The Highland parametrisation is also shown for reference. The error bars represent the statistical uncertainty of the fit.

- Single scattering: Electrons are propagated from one Coulomb scattering to the next. This procedure should give the most accurate results, assuming the data on scattering lengths and the screened nuclear potential are adequate. However, for any simulation of moderately complex set-ups involving solids, this approach is too computing-intensive.

- Urbán: The standard multiple scattering model in Geant4, based on the theoretical work of Lewis [7]. A recently re-tuned model is available in Geant4 version 10.0.²

- Goudsmit-Saunderson [2, 3]: The model produces a purely Gaussian distribution for our set-up, the ν parameter thus is fitted at very large values above 100 and therefore not shown in the following figures.

- Our model: A model drawing scattering angles from a Student’s t distribution with parameters tuned to our data, details are described in section 5.

Comparisons of the models with our data as a function of electron momentum are presented in figures 6 and 7 and as a function of effective thickness in figures 8 and 9. The RMS₉₈ of the distributions is well described by all models, including the Highland parametrisation; however the data show a markedly higher tail fraction (and a correspondingly narrower core) than all the models. The difference is more pronounced at low momenta.

5 A new multiple Coulomb scattering model

Building on the success of the various models in describing the RMS₉₈ of the scattering distribution, we built a new model for the Geant4 framework with a better description of the shape of the distribution. It is based on the Urbán model and reuses the code for the calculation of the RMS of the

²The older model is taken from Geant4 version 9.6 patch 2.
Figure 7. Comparison of the Student’s $t$ tail parameter $\nu$ versus momentum for various scattering models in Geant4 with our data obtained with a 50 $\mu$m silicon target perpendicular to the beam (left) and a 100 $\mu$m silicon target tilted by 45$^\circ$ (right). The error bars represent the statistical uncertainty of the fit.

Figure 8. Comparison of the RMS of the central 98% of the fitted Student’s $t$ distribution versus projected silicon target thickness for various scattering models in Geant4 with our data obtained at 1 GeV/$c$ (left) and 6 GeV/$c$ (right) electron momentum. The error bars represent the statistical uncertainty of the fit. The Highland parametrisation is also shown for reference.

scattering angle (essentially the Highland parametrisation), but instead of using different parametrisations for core and tail of the distribution, it draws all angles from a Student’s $t$ distribution. The tail parameter $\nu$ of the distribution is obtained from an empirical fit to our data of the form

$$\nu(p,d)_{\text{fit}} = A + B \cdot \frac{1}{p - D} + C \cdot d$$

where $p$ is the electron momentum in GeV/$c$ and $d$ the silicon thickness in radiation lengths. $A$, $B$, $C$ and $D$ are the fit parameters; the numerical values are shown in table 2.

As Geant4 sometimes splits the tracking step through the thin silicon in two (e.g. due to emission of a $\delta$-electron), the approach described above invariably produces too much tails. The input angular distribution in Geant4 is thus different from the scattering distribution in the simulation output. It turns out that forcing the $\nu$ parameter to be at least two,

$$\nu_{\text{Geant}} = \max(\nu(p,d)_{\text{fit}}, 2),$$

where $p$ is the electron momentum in GeV/$c$ and $d$ the silicon thickness in radiation lengths.
Figure 9. Comparison of the Student’s $t$ tail parameter $\nu$ versus projected silicon target thickness for various scattering models in Geant4 with our data obtained at 1 GeV/$c$ (left) and 6 GeV/$c$ (right) electron momentum. The error bars represent the statistical uncertainty of the fit.

Table 2. Numerical values of the parameters in the Student $t$ scattering model.

| Parameter | Value from fit | Uncertainty from fit |
|-----------|----------------|----------------------|
| A         | 1.10           | 0.07                 |
| B         | 4.36           | 0.65                 |
| C         | 1.90           | 0.17                 |
| D         | -2.04          | 0.37                 |

leads to a much improved description of the scattering distribution shape of our data. A comparison of our model to the data and existing Geant4 models can be seen in figures 6 to 9. The small differences in the $RMS_{98}$ between our model and the Urbán model are partly due to the multiple step effect and partly due to the differences between the full $RMS$ and the $RMS_{98}$.

6 Conclusion

We have measured multiple Coulomb scattering of 1-6 GeV/$c$ electrons in a 50-141 $\mu$m thin silicon target. We found a good description of the scattering distribution $RMS_{98}$ width in data by models implemented in Geant4, but large differences in the tail fraction of the distribution. A newly developed model based on drawing scattering angles from a Student’s $t$ distribution with parameters obtained from our data gives a greatly improved shape description for ultra-thin silicon trackers as presently used by many experiments.

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