Magnetic warping in topological insulators

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We analyze the electronic structure of topological surface states in the family of magnetic topological insulators MnBi2nTe3n+1. We show that, at natural-cleavage surfaces, the Dirac cone warping changes its symmetry from hexagonal to trigonal at the magnetic ordering temperature. In particular, an energy splitting develops between the surface states of same band index but opposite surface momenta upon formation of the long-range magnetic order. As a consequence, measurements of such energy splittings constitute a simple protocol to detect the magnetic ordering via the surface electronic structure, alternative to the detection of the surface magnetic gap. Interestingly, while the latter signals a nonzero surface magnetization, the trigonal warping predicted here is, in addition, sensitive to the direction of the surface magnetic flux. Our results may be particularly useful when the Dirac point is buried in the projection of the bulk states, caused by certain terminations of the crystal or in hole-doped systems, since in both situations the surface magnetic gap itself is not accessible in photoemission experiments.

I. INTRODUCTION

Antiferromagnetic topological insulators (AFM-TIs) are topological insulators that spontaneously break time-reversal symmetry (Θ) while preserving the symmetry S = ΘT1/2, where T1/2 is a lattice translation by half of a unit cell [1]. The manifestations of the non-trivial topology on a given surface of such a system depend on whether or not the surface is symmetric under S. In the family MnBi2nTe3n+1, the crystal structure consists of septuple layers (SLs) of MnBi2Te4 separated by n − 1 quintuple layers (QLs) of Bi2Te3 [2, 3]. The Mn ions order in an uniaxial antiferromagnetic structure, with the Néel vector parallel to the stacking axis. The usually studied surfaces are terminated either on a QL or on a SL. As a result, such surfaces are S-broken and in the magnetically ordered phase a gap Δ is allowed in the topological surface electronic spectrum.

Δ is expected to govern the low-energy physics when the chemical potential lies inside the gap, enabling phenomena such as the quantum anomalous Hall effect [4–14]. While the surface electronic structure has been extensively studied [2, 3, 15–32], the observation and the temperature evolution of Δ remain as challenging and heavily debated issues. In the simplest picture, Δ should vanish at T > TN (above the Néel temperature) due to the restoration of Θ in a statistical sense. Perhaps the clearest experimental data available today showing compelling evidence for this is found in the ferromagnetic compounds MnBi2Te13 [31] and MnSB2Te4 [33] and in heterostructures [34–36]. In MnBi2Te4, several photoemission experiments find within their resolution a gapless surface spectrum at all temperatures [16–18]. In these cases, it has been argued that the surface magnetic structure might differ from the bulk one. On the other hand, experiments that do show a finite Δ at low temperatures do not always find it closes above TN [2]. Different possible mechanisms that might prevent the observation of the gap closing in photoemission experiments have been discussed, including short-range magnetic fields generated by chiral spin fluctuations [37], anisotropy of the Mn-spin fluctuations [38], and fermionic fluctuations [39]. In addition, the hybridization between the Dirac cone and other trivial bands might further complicate the direct experimental study of Δ. This has been recognized as a problem at QL-terminated surfaces, where the Dirac cone has been theoretically and experimentally found to be buried in the surface projection of bulk bands, leaving at low energy the so-called hybridization gap [29].

Importantly, doping is ubiquitous in as-grown samples of MnBi2nTe3n+1, triggering the question of further consequences of breaking Θ associated with deviations from the low-energy Dirac limit. In this paper, we analyze this issue and focus on the warping of the surface Dirac cone. In non-magnetic TIs, soon after their discovery it was observed that for usual levels of doping, the surface Fermi contour presents a pronounced hexagonal warping [40–42] and the importance of Θ being preserved for the resulting hexagonal symmetry was recognized early on [43]. Here, we show that the opening of a magnetic gap Δ in S-broken surfaces is generically accompanied by a reduction in the symmetry of the Dirac cone warping from hexagonal to trigonal. As we will show, Δ also plays a role in the low-temperature trigonal warping of
the Dirac cone.

From these results, a simple protocol emerges to detect the long-range magnetic order via the surface electronic structure: measurements of the energy difference between states of same band but of opposite surface momenta should vanish in the paramagnetic phase and become finite in the ordered phase. This protocol may prove particularly useful on terminations of a crystal where the Dirac point is known to be buried in the projection of the bulk states or in $p$-doped systems, since both conditions make the magnetic gap $\Delta$ inaccessible to photoemission experiments. In addition, the trigonal distortion exhibited by the surface Dirac cone warping in the low-temperature phase is sensitive not only to the existence of a net surface magnetic flux but also to its direction.

This article is organized as follows. In Sec. II, we analyze the problem based on general symmetry considerations for a $2 \times 2$ model Hamiltonian of the (001) surface Dirac cone. In Sec. III, we construct a tight-binding model Hamiltonian for AFM-TIs having three-fold rotational symmetry with respect to the Néel propagation vector of the AFM phase together with reflection symmetries with respect to planes containing such axis. This model allows us to continuously vary the sublattice magnetization mimicking the evolution of temperature in uniaxial antiferromagnets. In Sec. IV, we present ab-initio results of finite slabs of MnBi$_2$Te$_7$. In Sec. V we discuss experimental aspects of our predictions. Finally, Sec. VI provides concluding remarks.

II. EFFECTIVE $2 \times 2$ HAMILTONIAN FOR THE SURFACE DIRAC CONE

We start by exploring the symmetries and surface electronic properties of AFM-TIs based on an effective two-band Hamiltonian for their topological surface states.

A. Symmetries

The natural-cleavage planes of compounds in the family MnBi$_{2n}$Te$_{3n+1}$ are positioned at by Te-Te van der Waals gaps, here considered to be perpendicular to the $\hat{z}$ axis. The case $n = 2$ is illustrated in Fig. 1. The crystal structure is minimally described either with the space group $R3\bar{m}$ (when $n$ is even) or with $P3m1$ (odd $n$). In both cases, the surface crystal point symmetry is $C_{3v}$, which can be generated by a three-fold rotation $C_{3z}$ and a reflection symmetry $M : y \rightarrow -y$.

When including spin-orbit coupling, the magnetic point symmetry depends on the direction of the Mn magnetic moments. We will consider the case typically observed in bulk magnetometry experiments, where Mn magnetic moments point along the trigonal axis [44–46]. In this case, the electronic structure is symmetric under $C_{3z}$ and under the antunitary reflection symmetry $M' = M\Theta$. Notice that the momenta invariant under $M$ obey $k_y = 0$ while those invariant under $M'$ satisfy $k_x = 0$.

Throughout this section, in order to construct $2 \times 2$ Hamiltonians for the surface Dirac cone, we use the representations

$$C_{3z} = \exp(-i\sigma_z \pi/3),$$  

$$M = i\sigma_y, \quad \Theta = i\sigma_y K, \quad M' = -K,$$

where $\sigma_i$ are the Pauli matrices and $K$ is complex conjugation. When a Hamiltonian $H$ is said to be symmetric under $C_{3z}$, $M$, $\Theta$, or $M'$, it satisfies

$$C_{3z}^{-1}H(k_x, k_y)C_{3z} = H(-k_x + \sqrt{3}k_y, -\sqrt{3}k_x - k_y),$$

$$M^{-1}H(k_x, k_y)M = H(k_x, -k_y),$$

$$\Theta^{-1}H(k_x, k_y)\Theta = H(-k_x, -k_y),$$

$$M'^{-1}H(k_x, k_y)M' = H(-k_x, k_y),$$

respectively.
the energies of the bands are
\[ \theta \]
Defining the azimuth angle \( \theta \) with respect to the \( \hat{x} \) axis, the energies of the bands are
\[ \varepsilon_{\pm}(k) = \pm \sqrt{\nu^2 k^2 + \lambda^2 k^6 \sin^2(\theta)}. \] (4)

Warping effects originate from the term proportional to \( \lambda \) and the Fermi contour has a hexagonal symmetry, \( \theta \to \theta + 2\pi/6 \), with the vertices of the hexagon pointing along the \( \Gamma M \).

### C. Hexagonal warping as a helical correction to the magnetic gap

Consider adding to Eq. (3) the simplest \( \Theta \)-breaking term
\[ H_{\Delta}(k) = \varepsilon(k) + \Delta \sigma_z. \] (5)

This Hamiltonian obeys Eq. (2a) and breaks both Eqs. (2b) and (2c), but satisfies their combination, Eq. (2d). The energies are
\[ \varepsilon_{\pm}(k) = \pm \sqrt{\Delta^2 + \nu^2 k^2 - 2\lambda \Delta k^3 \sin(3\theta) + \lambda^2 k^6 \sin^2(3\theta)}. \] (6)

In this case, the spectrum acquires a gap \( \Delta \) and the symmetry of the Fermi contour is lowered to trigonal \( \theta \to \theta + 2\pi/3 \) due to the term proportional to \( \sin(3\theta) \).

Figure 2(a-c) shows the Fermi contour for Fermi energies \( E_F/E^* = 0.6, 0.9 \), and 1.2, respectively, where we have used the units of energy \( E^* = \nu/k^* \) and of momentum \( k^* = \sqrt{\nu/\lambda} \), as introduced in Ref. [43]. The latter is a natural unit to account for the length scale associated with the hexagonal warping. For each Fermi energy, the Fermi contours corresponding to \( \Delta = 0 \) and \( 0.4 \) are shown. As a reference, notice that for the case \( \Delta = 0 \), the parameters used in panel (c) match those found in Ref. [43] to reproduce experimental results in \( \text{Sb}_2\text{Te}_3 \) \((v_0 = 2.55 \text{ eVA}, \lambda = 250 \text{ eVA}^3, E^* = 0.23 \text{ eV} \) and \( E_F = 1.2E^* \)). For these parameters, \( \Delta = 0.4E^* \sim 0.1 \text{ eV} \). While the trigonal distortion is apparent in all cases, it becomes less pronounced for relatively large doping because of the larger power in \( k \) associated with the hexagonal warping term in Eq. (6).

Within this model, the product of \( \Delta \) and \( \lambda \) characterizes the strength of the trigonal distortion. This can be traced back to the fact that two terms associated with these parameters are proportional to the Pauli matrix \( \sigma_z \). Therefore, one can define an effective gap function
\[ \Delta_{\text{eff}}(k, \theta) = \Delta + \lambda k^3 \sin(3\theta). \] (7)

While the first term induces a net spin moment and breaks \( \Theta \), the second term induces a helical, momentum-dependent spin texture compatible with \( \Theta \). Figure 3 shows this term using the value of \( \lambda = 250 \text{ eVA}^3 \), which was found in Ref. [43] to appropriately describe the hexagonal warping in \( \text{Sb}_2\text{Te}_3 \). This term cannot by itself open a gap at \( \Gamma \), where it vanishes. For finite \( \Delta \), however, it does affect the energy dispersion of the gapped Dirac cone in a quite distinctive manner. Taking, for example, momenta along the \( M' \)-invariant line \( k_y = 0 \) \((\Gamma K)\), for \( \nu k_y \gg \Delta \) the eigenenergies of the Hamiltonian Eq. (5), linearized in \( \Delta \) and \( \lambda \), read
\[ \pm \nu |k_y| \left( 1 + \frac{\Delta \lambda}{\nu^2 k_y} \right). \] (8)

Thus, for finite \( \Delta \), the hexagonal warping term bends the Dirac cone. This bending is forbidden along \( \Gamma M \) due to Eq. (2d) (see Fig. 4).
transition of the Dirac cone warping, but have a fundamental shortcoming that various parameters associated with the breaking of $\Theta$ are generally allowed. Their relative importance is not obvious without additional information, e.g., from experiment. In this section, we analyze the bulk electronic structure and topological surface states based on tight-binding models that obey the point symmetries of different compounds in the MnBi$_{2n}$Te$_{3m+1}$ family.

We first consider a model for a strong topological insulator (STI) on a triangular lattice with Bravais vectors:

$$H_{TI}(k) = \Gamma_1 [\mu + f(k_1, k_2) - \cos(k_3)] + \lambda [\Gamma_2 \sin(k_1) + \Gamma_{2,1} \sin(k_2) - \Gamma_{2,2} \sin(k_1 + k_2) + \Gamma_3 \sin(k_3)].$$

The scalar function $f(k_1, k_2)$ is:

$$f(k_1, k_2) = -\sum_{j=0}^{2} \cos(C_j k_1),$$

and $\Gamma_1 = \tau_z \sigma_0$, $\Gamma_2 = \tau_z \sigma_x$, and $\Gamma_3 = \tau_y \sigma_0$, where Pauli matrices $\tau, \sigma$ encode the degree of freedom associated with two orbitals per site. $C_3$ denotes a threefold rotation applied to the momentum, such that $k_1 \rightarrow k_2$, $k_2 \rightarrow -k_1 - k_2$, and $-k_1 - k_2 \rightarrow k_1$. The matrices $\Gamma_{2,1}$ and $\Gamma_{2,2}$ are related to $\Gamma_2$ by a threefold rotation $C_3$:

$$\Gamma_{2,j} = C_3 \Gamma_2 C_3^{-j}, \quad C_3 = \tau_0 \exp \left( i \frac{\pi}{3} \sigma_z \right).$$

The Hamiltonian in Eq. (15) has inversion symmetry $I = \tau_z \sigma_0$, time-reversal symmetry $\Theta = i \tau_0 \sigma_y K$, threefold rotation symmetry $C_3$ and reflection symmetry $M$ with respect to the $k_1 = -2k_2$ plane (and analog planes related by $C_3$). Setting $\mu = 3$ and $\lambda = 1$, we obtain an STI with $Z_2$ indices $(\nu_0; \nu_1 \nu_2 \nu_3) = (1; 000)$ and a mirror-protected topological crystalline insulator [48].

For the surface states to have a pronounced hexagonal warping in the $\Theta$-symmetric phase, as usually observed experimentally, we add a next-nearest-neighbor intralayer hopping term

$$H_{s}(k) = t \Gamma_1 [\sin(k_1 + 2k_2) + \sin(k_1 - k_2) - \sin(2k_1 + k_2)],$$

with $\Gamma_1 = \tau_z \sigma_z$. This term obeys time-reversal $\Theta$, threefold rotation $C_3$ and the mirror symmetries. Figure 5(a) shows the surface Fermi contour obtained by solving the Hamiltonian $H_{TI} + H_s$ in a slab geometry consisting of a finite number of layers in the $z$ direction. For the calculations in this section we use the Kwant code [49].

III. EFFECTIVE LATTICE HAMILTONIAN FOR MAGNETIC TIs WITH $C_{3v}$

The models studied in the previous section contain the basic ingredients to describe the hexagonal to trigonal
FIG. 5. (a) Surface Fermi contour for a tight-binding Hamiltonian of a strong topological insulator with $C_{3v}$ symmetry. The iso-energetic contour has a sixfold rotational symmetry. (b) Surface Fermi contour in the presence of a Zeeman field. The Fermi contour of the top and bottom surfaces are rotated by $\pi/3$ with respect to each other and both exhibit a trigonal symmetry. Each band has only a trigonal symmetry due to the time-reversal symmetry breaking. Bands associated with states at the bottom and top surfaces are colored in red and blue respectively. (c) Surface Fermi contour for an antiferromagnetic topological insulator. For an even number of magnetic layers, the surface magnetic flux at opposite surfaces is identical and so are the trigonally-distorted Fermi contours. Parameters were chosen as $\mu = 3$, $\lambda = 1$, $t = 10$, $b = 0.2$, and Fermi energy $E = 0.7$ for a system with 20 layers stacked along the $z$ direction.

We now consider the breaking of the time-reversal symmetry via a Zeeman field:

$$H_b = bS_z,$$

where $S_z = \tau_0 \sigma_z$. This term breaks both the time-reversal and mirror symmetry but preserves their product, $\Theta M$. Figure 5(b) shows the surface Fermi contour with this added perturbation. Compared to the $\Theta$-symmetric case, the warping exhibits a trigonal symmetry. In addition, the Fermi contours at opposite surfaces of the slab (shown in red and blue) are rotated with respect to each other. This rotation originates from the fact that, at opposite surfaces of the finite slab, the field $H_b$ points in opposite directions relative to the surface normal. Thus, the surface states at the two surfaces experience an opposite magnetic flux.

We now build a tight-binding model appropriate for an AFM-TI. We follow the procedure introduced in Ref. [1], which consists in adding a staggered $\Theta$-breaking term to a Hamiltonian that describes an STI. To do this, we double the unit-cell long the $\hat{z}$ direction and add a sublattice degree of freedom represented by the Pauli matrices $\gamma_i$ ($i = 0, \ldots, 3$).

We introduce the Hamiltonian

$$H_{AFM} = [H_{TI}(k) + H_{s}(k)]\gamma_0 + bS_z \gamma_z.$$  (20)

In this model the Zeeman coupling is opposite for adjacent layers. Importantly, this staggered field is the only term associated with the breaking $\Theta$, and it related to the average Mn sublattice magnetization in a transparent way.

Figure 5(c) shows the corresponding surface Fermi contour. For an even number of magnetic layers, the surface magnetic flux at opposite surfaces points along the same direction relative to the surface normal. Accordingly, in this case the Fermi contour at both surfaces exhibits a trigonal distortion of identical shape.

Both the results for the FM and for the AFM models show that, on a given surface, the relative orientation of the trigonal distortion is essentially set by the local direction of the surface magnetization with respect to the surface normal. As a consequence, if there exist pieces of a given surface where the surface magnetization points in opposite directions, our results suggest that the trigonal distortions observed in both cases should be oriented differently and related by a $\pi/3$ rotation. This phenomenology can be directly appreciated in the simplest model Hamiltonian Eq. (5). Without hexagonal warping ($\lambda = 0$), the band structure [Eq. (6)] depends only on $\Delta^2$ so that information on the phase of $\Delta$ is lost. This is no longer the case for finite $\lambda$, for which the energy bands depend on the sign of $\Delta \lambda$. As a consequence, assuming a value of $\lambda$ independent of temperature and $\Delta \propto M_s$, with $M_s$ the surface magnetization, the trigonal distortion in the warping yields information on the sign of $M_s$ (or, equivalently, of $\Delta$).

IV. AB INITIO RESULTS

In this section, we present density functional theory (DFT) calculations based on finite slabs of MnBi$_4$Te$_7$. We use a structural model consisting of four unit cells and of a vacuum of 30 Bohr radii. We use the experimental
FIG. 6. (a) MnBi$_4$Te$_7$ band structure projected on the outermost quintuple layer (orange) and on the outermost septuple layer (blue). Energies are measured with respect to the Fermi energy of the full slab. (b) Constant energy contours of quintuple layer-projected band-structure at energies 100 and 150 meV. (c) Same as panel (b) for the septuple layer termination.

V. DISCUSSION

The results presented above follow from the observation that long-range magnetic order will generally reduce the symmetry of the topological surface states, effectively introducing a warping in their Fermi contours. While we have focused on the magnetic structure that has been experimentally determined from several bulk magnetometry techniques, it is always conceivable that other surface phenomena induce modifications in the surface magnetic structure. For example, in-plane magnetic moments at the surface have been suggested to play a role in the thermal evolution of the surface gap (e.g. Ref. [16]). In such cases, the magnetic warping of the surface states could also reveal useful information. For example, if the magnetic moments lie in-plane and remain collinear, the Fermi contours should reflect the breaking of the three-fold rotational symmetry. If, additionally, the magnetic moments point perpendicular to one of the three crystal reflection-symmetry planes, the Fermi contours should exhibit a reflection symmetry with respect to that plane.

In photoemission experiments, an important aspect which should be taken into account is that the photoemission intensity is not solely determined by the single-particle spectral function $A(k, \omega)$, but it also depends on the photoemission matrix elements which incorporate the transition between the initial and final states [52]. These can provide information on the orbital characters of electronic states [53, 54], but may also obscure the characterization of anisotropies of the band energy dispersion [55–57]. In particular, matrix elements can introduce an asymmetry in the measured spectral weight of states with opposite in-plane momenta. Nevertheless, using a variable photon energy and also changing the orientation of the setup with respect to the crystal can make it possible to discern bulk from surface states and probe the inherent warping of the latter [40, 58].

Another possibly relevant technical aspect is the presence of twin domains, which can in principle mix Fermi contours with their $\pi/3$-rotated partners. While their presence can be detrimental to the experimental detection of the trigonal warping, their density can be reduced with adjusted growth methods [59, 60].

The magnetic warping could also be probed by using scanning tunneling microscopy to measure quasiparticle interference (QPI) patterns around surface impurities [61, 62]. As already shown for time-reversal symmetric TIs, the almost parallel segments of hexagonally warped isoenergy lines provide a nesting condition, producing highly focused QPI patterns with similar six-fold symmetric Fourier transforms [63, 64]. For magnetic topological insulators with trigonally warped energy contours, we expect the nesting to be reduced, leading to the appearance of a three-fold symmetric QPI pattern. Lastly, the change in the symmetry of the warping may become an important ingredient in various surface phenomena associated with Dirac physics, such us surface magnetic textures [65–67] and anomalous surface transport prop-

bulk lattice parameters and atomic positions. The calculations are based on the GGA+$U$ method with the generalized gradient approximation [50] as implemented in the FPLO code version 48.00-52 [51]. We fix parameters $U = 5.34$ eV and $J = 0$, as in Ref. [2], and use the atomic limit flavor for the double counting correction. The spin-orbit interaction is considered in the fully-relativistic, four-component formalism. Numerical $k$-space integrations are performed with a triangle method with a mesh of $12 \times 12 \times 1$ subdivisions in the Brillouin zone.

Figure 6(a) shows the band-structure of MnBi$_4$Te$_7$ projected on the outermost SL and on the outermost QL of the slab. The dependence on the surface termination has been studied in detail experimentally and theoretically in Ref. [29]. One important point is that for the SL termination the apparent gap is actually a magnetic gap, while for QL-termination the magnetic gap is buried into the projection of bulk bands. Figures 6(b) and (c) show constant-energy contours which evidence the trigonal symmetry of the warping, regardless of the surface termination.

While here we only present results for MnBi$_4$Te$_7$, theoretically-obtained surface Fermi contours with a substantial trigonal warping is also for MnBi$_2$Te$_4$, e.g. in Ref. [22].
VI. CONCLUSIONS

We have analyzed how long-range magnetic order affects the symmetry of the surface Dirac cone in the family of magnetic topological insulators MnBi$_2$Te$_3n+1$. With this aim, we have introduced effective continuum as well as tight-binding models obeying the relevant symmetries of the problem. Based on these models and on density-functional calculations in the ordered phase, we have shown that, in addition to opening a gap $\Delta$ in the surface electronic structure, the magnetic order lowers the symmetry of the Dirac cone warping from hexagonal to trigonal. As a consequence, surface states of opposite momenta (or, in general, of states of surface momenta related by a $n\pi/3$ rotation, with $n$ odd) become energy-split below the ordering temperature, as recently noticed in related preprints [71, 72]. This observation may provide a practical protocol to detect the magnetic transition via the surface electronic structure, particularly useful in cases where the magnetic gap $\Delta$ is inaccessible to photoemission experiments either because the Dirac cone is buried into the projection of bulk bands or because the system is hole-doped. Furthermore, as opposed to the surface magnetic gap, the trigonal distortion is sensitive to the sign of the surface magnetization.

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[1] Roger S. K. Mong, Andrew M. Essin, and Joel E. Moore, “Antiferromagnetic topological insulators,” Phys. Rev. B 81, 245209 (2010).
[2] M. M. Otrokov, I. I. Klimovskikh, H. Bentmann, D. Estynin, A. Zeugner, Z. S. Aliev, S. Gaß, A. U. B. Wolter, A. V. Koroleva, A. M. Shikin, M. Blanconi-Rey, M. Hoffmann, I. P. Rusinov, A. Yu Vyazovskaya, S. V. Ere-meev, Yu M. Koroteev, V. M. Kuznetsov, F. Freyse, J. Sánchez-Barriga, I. R. Amiraslavlin, M. B. Babanly, N. T. Mamedov, N. A. Abdullayev, V. N. Zverev, A. Al-fonsov, V. Kataev, B. Bürchner, E. F. Schwier, S. Ku-mar, A. Kimura, L. Petaccia, G. Di Santo, R. C. Vidal, S. Schatz, K. Kiffler, M. Ünzelmam, C. H. Min, Simon Moser, T. R. F. Peixoto, F. Reiner, A. Ernst, P. M. Echenique, A. Isaeva, and E. V. Chulkov, “Prediction and observation of an antiferromagnetic topological insulator,” Nature 576, 416–422 (2019).
[3] Yan Gong, Jingwen Guo, Jiaheng Li, Kejing Zhu, Meng-han Liao, Xiaozhi Liu, Qinghua Zhang, Lin Gu, Lin Tang, Xiao Feng, et al., “Experimental realization of an intrinsic magnetic topological insulator,” Chin. Phys. Lett. 36, 076801 (2019).
[4] Xiao-Liang Qi, Taylor L. Hughes, and Shou-Cheng Zhang, “Topological field theory of time-reversal invariant insulators,” Phys. Rev. B 78, 195424 (2008).
[5] Andrew M. Essin, Joel E. Moore, and David Vanderbilt, “Magneto-electric polarizability and Axion Electrodynamics in Crystalline Insulators,” Phys. Rev. Lett. 102, 146805 (2009).
[6] Jing Wang, Biao Lian, Xiao-Liang Qi, and Shou-Cheng Zhang, “Quantized topological magneto-electric effect of the zero-plateau quantum anomalous Hall state,” Phys. Rev. B 92, 081107 (2015).
[7] Jing Wang, Biao Lian, and Shou-Cheng Zhang, “Dynamical axion field in a magnetic topological insulator superlattice,” Phys. Rev. B 93, 045115 (2016).
[8] Di Xiao, Jue Jiang, Jie-Ho Shin, Wenbo Wang, Fei Wang, Yi-Fan Zhao, Chaoxing Liu, Wei da Wu, Moses H. W. Chan, Nitin Samarth, and Cui-Zu Chang, “Realization of the Axion Insulator State in Quantum Anomalous Hall Sandwich Heterostructures,” Phys. Rev. Lett. 120, 056801 (2018).
[9] M Mogi, M Kawamura, R Yoshimi, A Tsukazaki, Y Kozuka, N Shirakawa, KS Takahashi, M Kawasaki, and Y Tokura, “A magnetic heterostructure of topological insulators as a candidate for an axion insulator,” Nat. Mater. 16, 516–521 (2017).
[10] Monica Allen, Yongtao Cui, Eric Yue Ma, Masataka Mogi, Minoru Kawamura, Ion Cosma Fulga, David Goldhaber-Gordon, Yoshinori Tokura, and Zhi-Xun Shen, “Visualization of an axion insulating state at the transition between 2 chiral quantum anomalous Hall states,” Proc. Natl. Acad. Sci. U.S.A. 116, 14511–14515 (2019).
[11] Chang Liu, Yongchao Wang, Hao Li, Yang Wu, Yaoxin Li, Jiaheng Li, Ke He, Yong Xu, Jinsong Zhang, and Yuyu Wang, “Robust axion insulator and Chern insulator phases in a two-dimensional antiferromagnetic topological insulator,” Nat. Mater. 19, 522–527 (2020).
[12] M. M. Otrokov, I. P. Rusinov, M. Blanconi-Rey, M. Hoffmann, A. Yu. Vyazovskaya, S. V. Ere-meev, A. Ernst, P. M. Echenique, A. Arnau, and E. V. Chulkov, “Unique Thickness-Dependent Properties of the van der Waals Interlayer Antiferromagnet MnBi$_2$Te$_4$ Films,” Phys. Rev. Lett. 122, 107202 (2019).
[13] Chao Lei, Shu Chen, and Allan H MacDonald, “Magnitized topological insulator multilayers,” Proc. Natl. Acad. Sci. 117, 27224–27230 (2020).
[14] K. M. Fijalkowski, N. Liu, M. Hartl, M. Winnerlein, P. Mandal, A. Coschizza, A. Fothergill, S. Grauer, S. Schreyeck, K. Brunner, M. Greiter, R. Thomale, C. Gould, and L. W. Molenkamp, “Any axion insulator...
must be a bulk three-dimensional topological insulator,” Phys. Rev. B 103, 235111 (2021).

[15] R. C. Vidal, H. Bentmann, T. R. F. Peixoto, A. Zeugner, S. Moser, C.-H. Min, S. Schatz, K. Kühn, M. Ünzelmans, C. I. Fornari, H. B. Vasili, M. Valdivares, K. Sakamoto, D. Mondal, J. Fujii, I. Vobornik, S. Jung, C. Cacho, T. K. Kim, R. J. Koch, C. Jozwiak, A. Bostwick, J. D. Denlinger, E. Rotenberg, J. Buck, M. Hoesch, F. Diekmann, S. Rohlf, M. Källäne, K. Rossnagel, M. M. Otrokov, E. V. Chulkov, M. Ruck, A. Isaeva, and F. Reinert, “Surface states and Rashba-type spin polarization in antiferromagnetic MnBi₂Te₄(0001),” Phys. Rev. B 100, 121104 (2019).

[16] Yu-Jie Hao, Pengfei Liu, Yue Feng, Xiao-Ming Ma, Eike F. Schwier, Masarita Arita, Shiv Kumar, Chaowei Hu, Rui’e Lu, Meng Zeng, Yuan Wang, Zhanyang Hao, Hong-Yi Sun, Ke Zhang, Jiawei Mei, Ni Ni, Liusuo Wu, Kenya Shimada, Chaoyu Chen, Qihang Liu, and Chang Liu, “Gapless Surface Dirac Cone in Antiferromagnetic Topological Insulator MnBi₂Te₄,” Phys. Rev. X 9, 041038 (2019).

[17] Hang Li, Shun-Ye Gao, Shao-Feng Duan, Yuan-Feng Xu, Ke-Jia Zhu, Shang-Jie Tian, Jia-Cheng Gao, Wen-Hui Fan, Zhi-Cheng Hao, Jie-Rui Huang, Jia-Jun Li, Da-Yu Yan, Zheng-Tai Liu, Wan-Ling Liu, Yao-Bo Lu, Yu-Liang Li, Yi Liu, Guo-Bin Zhang, Peng Zhang, Takeshi Kondo, Shik Shin, He-Chang Lei, You-Guo Shi, Wen-Tao Zhang, Hong-Ming Weng, Tian Qian, and Hong Ding, “Dirac Surface States in Intrinsic Magnetic Topological Insulators EuSb₂As₂ and MnBi₂₈Te₃₀₋₁,” Phys. Rev. X 9, 041039 (2019).

[18] Y. J. Chen, L. X. Xu, J. H. Li, Y. W. Li, H. Y. Wang, C. F. Zhang, H. Li, Y. Wu, A. J. Liang, C. Chen, S. W. Jung, C. Cacho, Y. H. Mao, S. Liu, M. X. Wang, Y. F. Guo, Y. Xu, Z. K. Liu, L. X. Yang, and Y. L. Chen, “Topological Electronic Structure and Its Temperature Evolution in Antiferromagnetic Topological Insulator MnBi₂Te₄,” Phys. Rev. X 9, 041040 (2019).

[19] Jiazheng Wu, Fucai Liu, Masato Sassa, Koichiro Ienaga, Yukiko Obata, Ryu Yukawa, Koji Horiba, Hiroshi Kunita, Satoshi Okuma, Takeshi Inoshita, and Hideo Yukawa, “Gapless Surface Dirac cone in Antiferromagnetic topological insulator MnBi₂Te₄,” Phys. Rev. B 96, eaba4275 (2019).

[20] Raphael C. Vidal, Alexander Zeugner, Jorge I. Facio, Rajayvardhan R. Vaid, M. Hossein Laghiphagi, Anja U. B. Wolter, Laura T. Corredor Bohorquez, Federico Caglieris, Simon Moser, Tim Figgemeier, Thiago R. F. Peixoto, Hari Babu Vasili, Manuel Valdivares, Sungwon Jung, Cephise Cacho, Alexey Alfonsov, Kavita Mehlawat, Vladislav Kataev, Christian Hess, Manuel Richter, Bernd Büchner, Jeroen van den Brink, Michael Ruck, Friedrich Reinert, Hendrik Bentmann, and Anna Isaeva, “Topological Electronic Structure and Intrinsic Magnetization in MnBi₂Te₄: A Bi₂Te₃ Derivative with a Periodic Mn Sublattice,” Phys. Rev. X 9, 041065 (2019).

[21] Chaowei Hu, Kyle N Gordon, Pengfei Liu, Jinyu Liu, Xiaoying Zhou, Peipei Hao, Dushyan Naranjan, Eve Emmanouilidou, Hongyi Sun, Yuntian Liu, et al., “A van der Waals antiferromagnetic topological insulator with weak interlayer magnetic coupling,” Nat. Commun. 11, 1–8 (2020).

[22] Przemyslaw Swatek, Yun Wu, Lin-Lin Wang, Kyungchan Lee, Benjamin Schrank, Jiaqiang Yan, and Adam Kaminski, “Gapless Dirac surface states in the antiferromagnetic topological insulator MnBi₂Te₄,” Phys. Rev. B 101, 161109 (2020).

[23] Yong Hu, Lixuan Xu, Mengzhu Shi, Ayun Luo, Shuteng Peng, Z. Y. Wang, J. J. Ying, T. Wu, Z. K. Liu, C. F. Zhang, Y. L. Chen, G. Xu, X.-H. Chen, and J.-F. He, “Universal gapless Dirac cone and tunable topological states in (MnBi₂Te₄)ₓ(Bi₂Te₃)ₙ heterostructures,” Phys. Rev. B 101, 161113 (2020).

[24] Kyle N Gordon, Hongyi Sun, Chaowei Hu, A Garrison Linn, Haoxian Li, Yuntian Liu, Pengfei Liu, Scott Mackey, Qihang Liu, Ni Ni, et al., “Strongly Gapped Topological Surface States on Protected Surfaces of Antiferromagnetic MnBi₂Te₇ and MnBi₂Te₁₀,” arXiv preprint arXiv:1910.13943 (2019).

[25] Xiao-Ming Ma, Zhongjia Chen, Eike F. Schwier, Yang Zhang, Yu-Jie Hao, Shiv Kumar, Ruie Lu, Jifeng Shao, Yuanjun Jin, Meng Zeng, Xiang-Rui Liu, Zhanyang Hao, Ke Zhang, Wumiti Mansuer, Chunjiao Song, Yuan Wang, Boyan Zhao, Cai Liu, Ke Deng, Jiawei Mei, Kenya Shimada, Yue Zhao, Xiangjing Zhou, Bing Shen, Wen Huang, Chang Liu, Hu Xu, and Chaoyu Chen, “Hybridization-induced gapped and gapless states on the surface of magnetic topological insulators,” Phys. Rev. B 102, 245136 (2020).

[26] Ilya I. Klimovskikh, Mikhail M. Otrokov, Dmitry Eshtyumin, Sergey V. Eremeev, Sergey O. Filnov, Alexander Koroleva, Evgeny Shevchenko, Vladimir Voroshnin, Artem G. Rybkin, Igor P. Rusinov, Maria Blanco-Rey, Martin Hoffmann, Ziya S. Aliyev, Mahammad B. Babanly, Imamaddin R. Amiraslanov, Nadir A. Abdullayev, Vladimir N. Zverev, Akio Kimura, Oleg E. Tereshchenko, Konstantin A. Kohk, Luca Petaccia, Giovanni Di Santo, Arthur Ernst, Pedro M. Echenique, Nazim T. Mamedov, Alexander M. Shikin, and Evgenie V. Chulkov, “Tunable 3D/2D magnetism in the (MnBi₂Te₄)(Bi₂Te₃)ₘ topological insulators family,” npj Quantum Mater. 5, 54 (2020).

[27] LX Xu, YH Mao, HY Wang, JH Li, YJ Chen, YYY Xia, YW Li, J Zhang, HJ Zheng, K Huang, et al., “Persistent gapless surface states in MnBi₂Te₄/Bi₂Te₃ superlattice antiferromagnetic topological insulator,” arXiv preprint arXiv:1910.11014 (2019).

[28] Xuefeng Wu, Jiayu Li, Xiao-Ming Ma, Yu Zhang, Yuntian Liu, Chun-Sheng Zhou, Jingfeng Shao, Qiaoqing Wang, Yu-Jie Hao, Yue Feng, Eike F. Schwier, Shiv Kumar, Hongyi Sun, Pengfei Liu, Kenya Shimada, Koji Miyamoto, Taichi Okuda, Kedong Wang, Maohai Xie, Chaoyu Chen, Qihang Liu, Chang Liu, and Yue Zhao, “Distinct topological surface states on the two terminations of MnBi₂Te₇,” Phys. Rev. X 10, 031013 (2020).

[29] R. C. Vidal, H. Bentmann, J. I. Facio, T. Heider, P. Kagerer, C. I. Fornari, T. R. F. Peixoto, T. Figgemeier, S. Jung, C. Cacho, B. Büchner, J. van den Brink, C. M. Schneider, L. Plucinski, E. F. Schwier, K. Shimada, M. Richter, A. Isaeva, and F. Reinert, “Orbital Complexity in Intrinsic Magnetic Topological Insulators MnBi₂Te₇ and MnBi₂Te₁₀,” Phys. Rev. Lett. 126, 176403 (2021).

[30] Chaowei Hu, Lei Ding, Kyle N Gordon, Barun Ghosh, Hung-Ju Tien, Haoxian Li, A Garrison Linn, Shang-Wei Lien, Cheng-Yi Huang, Scott Mackey, et al., “Realization of an intrinsic ferromagnetic topological state in MnBi₂Te₁₃,” Sci. Adv. 6, eaba4275 (2020).
[31] Ruie Lu, Hongyi Sun, Shiv Kumar, Yuan Wang, Mingqiang Gu, Meng Zeng, Yu-Jie Hao, Jiayu Li, Jifeng Shao, Xiao-Ming Ma, Zhanyang He, Ke Zhang, Wumiti Mansuer, Jiawei Mei, Yue Zhao, Cai Liu, Ke Deng, Wen Huang, Bing Shen, Kenya Shimada, Eike F. Schwier, Chang Liu, Qihang Liu, and Chaoyu Chen, “Half-Magnetic Topological Insulator with Magnetization-Induced Dirac Gap at a Selected Surface,” Phys. Rev. X 11, 011039 (2021).

[32] AM Shikin, DA Estyunin, NL Zaitsev, D Glazkovna, II Klimovskikh, S Filnov, AG Rybkin, EF Schwier, S Kumar, A Kimura, et al., “Sample-dependent Dirac point gap in MnBi₂Te₄ and its response to the applied surface charge: a combined photoemission and ab initio study,” arXiv preprint arXiv:2107.04428 (2021).

[33] Stefan Wimmer, Jaime Sánchez-Barriga, Philipp Küppers, Andreas Ney, Enrico Schierle, Friedrich Freyse, Ondrej Caha, Jan Michalicka, Marcus Liebmann, Daniel Primetzhofe, et al., “Mn-Rich MnSb₂Te₄: A topological insulator with magnetic gap closing at high Curie temperatures of 45–50 K,” Advanced Materials 33, 2102935 (2021).

[34] Qile Li, Chi Xuan Trang, Weikang Wu, Jinwoong Hwang, David Cortie, Nikhil Medhekar, Sung-Kwon Ma, Shengyuan A. Yang, and Mark T Edmonds, “Large Magnetic Gap in a Designer Ferromagnet-Topological Insulator–Ferromagnet Heterostructure,” Advanced Materials 33, 2107520 (2022).

[35] Mengke Liu, Chao Lei, Hyunsue Kim, Yanxing Li, Lisa Frammolino, Jiaqiang Yan, Allan H Macdonald, and Chih-Kang Shih, “Visualizing the interplay of Dirac mass gap and magnetism at nanoscale in intrinsic magnetic topological insulators,” arXiv preprint arXiv:2205.09195 (2022).

[36] P. Kagerer, C. I. Fornari, S. Buchberger, T. Tschier, L. Veyrat, M. Kamp, A. V. Tcakaev, V. Zabolotny, S. L. Morelhão, B. Geldiyev, S. Muller, A. Fedorov, E. Riensk, P. Gargiani, M. Valvidares, L. C. Folkers, A. Isaeva, B. Buechner, V. Hinkov, R. Claessen, H. Bentmann, and F. Reinert, “Two-dimensional ferromagnetic extension of a topological insulator,” arXiv preprint arXiv:2207.14421 (2022).

[37] A. M. Shikin, D. A. Estyunin, I. I. Klimovskikh, S. O. Filnov, E. F. Schwier, S. Kumar, K. Miyamoto, T. Okuda, A. Kimura, K. Kuroda, K. Yaji, S. Shin, Y. Takeda, Y. Saitoh, Z. S. Aliev, N. T. Mamedov, I. R. Ami- rashlano, M. B. Babanly, M. M. Otrokov, S. V. Ereemee, and E. V. Chulkov, “Nature of the Dirac gap modulation and surface magnetic interaction in axion antiferromagnetic topological insulator MnBi₂Te₄,” Sci. Rep. 10, 13226 (2020).

[38] A. Alfonsov, J. I. Facio, K. Mehlawat, A. G. Moghad-dam, R. Ray, A. Zeugner, M. Richter, J. van den Brink, A. Isaeva, B. Büchner, and V. Kataev, “Strongly anisotropic spin dynamics in magnetic topological insulators,” Phys. Rev. B 103, L180403 (2021).

[39] Marius Scholten, Jorge I. Facio, Rajyavardhan Ray, Ilya M. Eremin, Jeroen van den Brink, and Flavio S. Nogueira, “Finite temperature fluctuation-induced order and responses in magnetic topological insulators,” Phys. Rev. Res. 3, L032014 (2021).

[40] YL Chen, James G Analytis, J-H Chu, ZK Liu, S-K Mo, Xiao-Liang Qi, HJ Zhang, DH Lu, Xi Dai, Zhong Fang, et al., “Experimental realization of a three-dimensional topological insulator, Bi₂Te₃,” Science 325, 178–181 (2009).

[41] Yuqi Xia, Dong Qian, David Hsieh, L Wray, Arijiet Pal, Hsin Lin, Arun Bansil, DHYS Grauer, Yew Sun Hor, Robert Joseph Cava, et al., “Observation of a large-gap topological-insulator class with a single Dirac cone on the surface,” Nat. Phys. 5, 398–402 (2009).

[42] D. Hsieh, Y. Xia, D. Qian, L. Wray, F. Meier, J. H. Dil, J. Osterwalder, L. Patthey, A. V. Fedorov, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, “Observation of Time-Reversal-Protected Single-Dirac-Cone Topological-Insulator States in Bi₂Te₃ and Sb₂Te₃,” Phys. Rev. Lett. 103, 146401 (2009).

[43] Liang Fu, “Hexagonal Warping Effects in the Surface States of the Topological Insulator Bi₂Te₃,” Phys. Rev. Lett. 103, 266801 (2009).

[44] Alexander Zeugner, Frederik Nietschke, Anja UB Wolter, Sebastian Gaß, Raphael C Vidal, Thiago RF Peixoto, Darius Pohl, Christine Damm, Axel Lubk, Richard Henrich, et al., “Chemical aspects of the candidate antiferromagnetic topological insulator MnBi₂Te₄,” Chem. Mater. 31, 2795–2806 (2019).

[45] J.-Q. Yan, Q. Zhang, T. Heitmann, Zengle Huang, K. Y. Chen, J.-G. Cheng, Weida Wu, D. Vaknin, B. C. Sales, and R. J. McQueeney, “Crystal growth and magnetic structure of MnBi₂Te₄,” Phys. Rev. Mater. 3, 064202 (2019).

[46] Aoyu Tan, Valentin Labracherie, Narayan Kunchur, Anja U. B. Wolter, Joaquin Cornejo, Joseph Dufouleur, Bernd Büchner, Anna Isaeva, and Romain Giraud, “Metamagnetism of Weakly Coupled Antiferromagnetic Topological Insulators,” Phys. Rev. Lett. 124, 197201 (2020).

[47] Dániel Varjas, Témas Ö Rosdahl, and Anton R Akhmerov, “Qsymm: Algorithmic symmetry finding and symmetric Hamiltonian generation,” New J. Phys. 20, 093026 (2018).

[48] Jorge I. Facio, Sanjib Kumar Das, Yang Zhang, Klaus Koepernik, Jeroen van den Brink, and Ion Cosma Fulga, “Dual topology in jactungita P₄T₂HgSe₃,” Phys. Rev. Mater. 3, 074202 (2019).

[49] Christoph W Groth, Michael Wimmer, Anton R Akhmerov, and Xavier Waintal, “Kwant: a software package for quantum transport,” New J. Phys. 16, 063065 (2014).

[50] John P. Perdew, Kieron Burke, and Matthias Ernzerhof, “Generalized Gradient Approximation Made Simple,” Phys. Rev. Lett. 77, 3865–3868 (1996).

[51] Klaus Koepernik and Helmut Eschrig, “Full-potential nonorthogonal local-orbital minimum-basis band-structure scheme,” Phys. Rev. B 59, 1743–1757 (1999).

[52] Baiqing Lv, Tian Qian, and Hong Ding, “Angle-resolved photoemission spectroscopy and its application to topological insulators,” Phys. Rev. B 97, 195403 (2008).

[53] Ming Yi, Donghui Lu, Jiun-Haw Chu, James G Analytis, Adam P Sorini, Alexander F Kemper, Brian Moritz, Sung-Kwan Mo, Rob G Moore, Makoto Hashimoto, et al., “Symmetry-breaking orbital anisotropy observed for detwinned BaFe₁₋ₓCoₓ₂As₂ above the spin density...
wave transition,” Proc. Natl. Acad. Sci. U.S.A. 108, 6878 (2011).

[55] C. Jozwiak, Y L. Chen, A V. Fedorov, J G. Analytis, C R. Rotundu, A K. Schmid, J D. Denlinger, Y.-D. Chuang, D.-H. Lee, I R. Fisher, R J. Birgeneau, Z.-X. Shen, Z. Hussain, and A. Lanzara, “Widespread spin polarization effects in photoemission from topological insulators,” Phys. Rev. B 84, 165113 (2011).

[56] Sergey V Eremeev, Gabriel Landolt, Tatiana V Menschikova, Bartosz Slomski, Yury M Koroteev, Ziya S Aliev, Mahammad B Babanly, Jurgen Henk, Arthur Ernst, Luc Patthey, et al., “Atom-specific spin mapping and buried topological states in a homologous series of topological insulators,” Nat. Comm. 3, 635 (2012).

[57] A. Herdt, L. Plucinski, G. Bihlmayer, G. Mussler, S. Droing, J. Krumrini, D. Grützmacher, S. Bluigel, and C. M. Schneider, “Spin-polarization limit in Bi₂Te₃ Dirac cone studied by angle- and spin-resolved photoemission experiments and ab initio calculations,” Phys. Rev. B 87, 035127 (2013).

[58] M. Nomura, S. Souma, A. Takayama, T. Sato, T. Takahashi, K. Eto, Kouji Segawa, and Yoichi Ando, “Relationship between Fermi surface warping and out-of-plane spin polarization in topological insulators: A view from spin- and angle-resolved photoemission,” Phys. Rev. B 89, 045134 (2014).

[59] Jörn Kampmeier, Svetlana Borisova, Lukasz Plucinski, Martina Luysberg, Gregor Mussler, and Detlev Grützmacher, “Suppressing Twin Domains in Molecular Beam Epitaxy Grown Bi₂Te₃ Topological Insulator Thin Films,” Cryst. Growth Des. 15, 390–394 (2015).

[60] HD Li, ZY Wang, X Kan, Xinli Guo, HT He, Zichen Wang, JL Wang, TL Wong, Ning Wang, and Mao Hai Xie, “The van der Waals epitaxy of Bi₂Se₃ on the vicinal Si (111) surface: an approach for preparing high-quality thin films of a topological insulator,” New Journal of Physics 12, 103038 (2010).

[61] Wei-Cheng Lee, Congjun Wu, Daniel P. Arovas, and Shou-Cheng Zhang, “Quasiparticle interference on the surface of the topological insulator Bi₂Te₃,” Phys. Rev. B 80, 245439 (2009).

[62] Haim Beidenkopf, Pedram Roushan, Jungpil Seo, Lindsay Gorman, Ilya Drozdov, Yew San Hor, Robert J Cava, and Ali Yazdani, “Spatial fluctuations of helical Dirac fermions on the surface of topological insulators,” Nat. Phys. 7, 939 (2011).

[63] P. Sessi, P. Rümmünn, T. Bathon, A. Barla, K. A. Kokh, O. E. Tereshchenko, K. Fauth, S. K. Mahatha, M. A. Valbuena, S. Godey, F. Glott, A. Mugarza, P. Gargiani, M. Valvidares, N. H. Long, C. Carbone, P. Mavropoulos, S. Blügel, and M. Bode, “Superparamagnetism-induced mesoscopic electron focusing in topological insulators,” Phys. Rev. B 94, 075137 (2016).

[64] Peter Thalmeier and Alireza Akbari, “Gapped Dirac cones and spin texture in thin film topological insulator,” Phys. Rev. Research 2, 033002 (2020).

[65] Chao-Kai Li, Xu-Ping Yao, and Gang Chen, “Twisted magnetic topological insulators,” Phys. Rev. Res. 3, 033156 (2021).

[66] Nisarga Paul and Liang Fu, “Topological magnetic textures in magnetic topological insulators,” Phys. Rev. Res. 3, 033173 (2021).

[67] Stefan Divic, Henry Ting, T. Peregr-Barnea, and Arun Parmeckanti, “Magnetic skyrmion crystal at a topological insulator surface,” Phys. Rev. B 105, 035156 (2022).

[68] Christian Tutschki, Flavio S. Nogueira, Christian Northe, Jeroen van den Brink, and E. M. Hankiewicz, “Temperature and chemical potential dependence of the parity anomaly in quantum anomalous Hall insulators,” Phys. Rev. B 102, 205407 (2020).

[69] Xiao-Qin Yu, Zhen-Gang Zhu, and Gang Su, “Hexagonal warping induced nonlinear planar Nernst effect in nonmagnetic topological insulators,” Phys. Rev. B 103, 035410 (2021).

[70] Michal Papaj and Liang Fu, “Enhanced anomalous Nernst effect in disordered Dirac and Weyl materials,” Phys. Rev. B 103, 075424 (2021).

[71] Hengxin Tan, Daniel Kaplan, and Binghai Yan, “Momentum-inversion symmetry breaking on the fermi surface of magnetic topological insulators,” arXiv preprint arXiv:2205.07343 (2022).

[72] Dinghui Wang, Huaqiang Wang, Dingyu Xing, and Haijun Zhang, “Three-Dirac-fermion approach to unexpected gapless surface states of van der Waals magnetic topological insulators,” arXiv preprint arXiv:2205.08204 (2022).