Decomposing User-APP Graph into Subgraphs for Effective APP and User Embedding Learning

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Abstract—APP-installation information is helpful to describe users’ characteristics. Users with similar APPs installed might share common interests and behave similarly in some scenarios. In this work, we learn a user embedding vector based on each user’s APP-installation information. Since the user APP-installation embedding is learnable without dependency on the historical intra-APP behavioral data of the user, it complements the intra-APP embedding learned within each specific APP. Thus, they considerably help improve the effectiveness of the personalized-ads advertising in each APP, and they are particularly beneficial for the cold start of the new users in the APP. In this paper, we formulate the APP-installation user embedding learning into a bipartite graph embedding problem. The main challenge in learning an effective APP-installation user embedding is the imbalanced data distribution, as graph learning tends to be dominated by the popular APPs installed by billions of users. In comparison, niche/specialized APPs might have a marginal influence on graph learning. To effectively exploit the valuable information from niche APPs, we decompose the APP-installation graph into a set of subgraphs, each containing only one APP node and the users who install the APP. For each mini-batch, we only sample the users from the same subgraph in the training process. Thus, each APP can be involved in the training process in a more balanced manner. A considerable increase in CTR, CVR, and revenue has been observed after integrating the learned APP-installation user embedding into our online personal advertising platform. Additionally, the embeddings learned from our design can be efficiently searched and ranked by various embedding-based retrieval techniques, in particular, the fast neural ranking approach using bipartite graphs [24] (also developed at Baidu Cognitive Computing Lab) would be naturally applicable.

This project is part of the paddlepaddle deep learning platform.

I. INTRODUCTION

For different users, a personalized advertising system feeds different ads based on the estimated relevance between the ad and the user’s interest. Normally, the relevance between a user and an ad is measured by the similarity between their embeddings, which are learned jointly from the users’ historical behaviors on the ads. Nevertheless, for new users, there are no historical user-ad behaviors for learning effective user embedding. This issue of modeling new users is normally defined as the cold start problem. To solve the cold start problem, we usually exploit the user’s demographic attributes, such as age, region, and gender. The attribute embedding has been effectively learned based on the ordinary users’ rich experience accumulated in the past and can readily generalize well to the new users. Since the attribute embedding does not rely on the historical user behaviors, they are useful for tackling the cold start problem.

We explore a new type of attribute embedding learned from the APP-installation information. The users who install the same APP might share some common interests and tend to behave similarly. Meanwhile, a user’s installed APP lists might encode much richer fine-grained information about a user than basic demographic information, such as age, gender, and location. Thus, if exploiting the users’ APP-installation information effectively, we might significantly boost the performance of our personalized advertising platform for the new users. In fact, the APP-installation information benefits not only the new users but also the regular users, as the learned user’s APP-installation embedding complements the user’s behavior embedding. It is thus not surprising that incorporating the APP-installation embedding would lead to improvements for the regular users’ personalized advertising performance.

We formulate the APP-installation embedding as a bipartite graph embedding problem. The bipartite graph consists of two types of nodes, including the user nodes and the APP nodes as visualized in Figure 1. An edge exists between a user node and an APP node if the user has installed the APP in his/her

Fig. 1. The visualization of a user-APP undirected bipartite graph. A node with a blue dot denotes a user and a node with a green box denotes an APP. An edge exists between an APP and a user if the user installs the APP.
mobile phone. Straightforwardly, we could utilize any existing graph learning methods such as graph convolutional neural network (GCN) to learn the user node embedding and the APP node embedding. Nevertheless, a serious issue caused by imbalanced data distribution makes the training of the graph learning model challenging. Specifically, for a popular APP\(^1\), it is installed by billions of users, generating billions of edges in the graph. In contrast, a niche APP installed by millions of users only creates millions of edges. In this case, graph learning is dominated by the billions of edges created by the popular APPs, and the edges from the niche APPs might be swamped. But the edges from the popular APPs might not encode useful discriminating information since everyone almost installs them. In contrast, the edges from the niche APPs might be very useful for describing a user’s characteristics, but that useful information might not gain enough attention when training the graph embedding.

In this work, we propose a novel sampling approach to tackle the imbalanced data distribution issue for learning effective APP-installation user embeddings. Specifically, we decompose the user-APP graph into a set of sub-graphs. Each subgraph contains only a single APP and the users who install the APP. In the training process, we sample a subgraph for each iteration and construct training triplets based on users within the subgraph for embedding learning. In this manner, the popular APPs and the niche APPs will be involved in the training process in a fair manner. The offline and online experiments demonstrate the excellence of our method.

II. RELATED WORK

Factorization based methods. Factorization-based methods rely on an affinity matrix encoding the connections between nodes in the graph. They factorize the affinity matrix to obtain the embedding vectors for nodes. A pioneering work, Laplacian Eigenmaps [3] aims to keep the embedding of two nodes close when the weight of the edge connecting these two nodes is high. It seeks to minimize the weighted summation of squares of distance between nodes while the weight of the edge connecting these two nodes close when the weight of the edge connecting these two nodes is high. It conducts a trade-off between breadth-first searches (BFS) and depth-first searches (DFS) on the graph to generate a more effective graph embedding than DeepWalk. Walklets [20] additionally incorporates explicit modeling in random walks. Hierarchical Representation Learning for Networks (HARP) [7] proposes a better initialization strategy to avoid the local optima in optimization.

Neural network based methods. SDNE [26] stacks multiple layers of non-linear functions to preserve highly non-linear network structure. It adopts an auto-encoder structure which uses the embedding to reconstruct its neighbors. DNGR [6] feeds the positive point-wise mutual information matrix into a stacked denoising autoencoder to capture higher-order proximity in the learned graph embedding. Nevertheless, SDNE and DNGR consider the whole graph and take as input the global neighborhood of each node, which are not efficient for large-scale graphs. Recently, graph convolution neural network (GCN) provides an effective and efficient solution by adopting a configuration with local constraints. These methods can be categorized into spatial-based methods [2], [8], [12], [17], [25], [27] and spectral-based methods [4], [9], [13]–[16]. Spatial-based methods directly conduct convolution on the original graph. In contrast, spectral-based methods conduct convolution on the spectrum of the adjacent matrix of the graph. In [21], [22], it was shown that the incorporation of the node representation vectors computed by a random walk based method in GCN can effectively boost the performance of the GCN.

Embedding-based retrieval (EBR). The learned embeddings can be subsequently used as features for enhancing recommendation and/or advertising models. They can also be directly used for retrieval. Embedding-based retrievals have been widely used in Baidu’s many applications in search, recommendation, and advertising [10], [28]–[30]. In particular, the bipartite graph designed of the proposed method is naturally suitable for the fast neural ranking algorithm based on bipartite graphs [24].

III. METHOD

In this section, we introduce graph-based embedding learning for modeling the APP-installation information of users.

A. Graph Decomposition

Definition. We denote the set of APPs used for building the graph by \( \{s_i\}_{i=1}^N \), and denote the set of users by \( \{t_j\}_{j=1}^M \). They constitute the node set \( V = \{p_1, \ldots, p_N, u_1, \ldots, u_M\} \). Meanwhile, the edge set \( E \) contains all edges connecting two nodes \( \{e_{s_i, t_j}\}_{i,j} \), where \( s_i \) denotes the index of the user and \( t_j \) denotes the index of the user in the \( i \)-th edge, \( e_{s_i, t_j} \). That

\(^1\)https://www.businessofapps.com/data/most-popular-apps/
To make the contributions from different APPs balanced, we need to ensure that the number of users and the contributions from some niche APPs do not dominate the training process. This can be achieved by decomposing the graph into subgraphs, where each subgraph $G_i$ contains only one APP node. The user-APP graph $G$ is constructed based on the node set $V = \{u_i\}_{i=1}^{n} \cup \{a_i\}_{i=1}^{n}$, where $p_i$ denotes an APP node and $u_i$ is a user node. $G$ is decomposed into $N$ subgraphs $\{G_i\}_{i=1}^{N}$, where $G_i$ contains an APP node and the users installing the APP. We visualize the process of decomposing a graph into a set of subgraphs in Figure 2.

### B. Graph Learning

**Initialization.** We denote the embedding of the user $u_i$ by $\mathbf{u}_i$ and the embedding of an APP $a_j$ by $\mathbf{p}_j$. We denote the indices of users installing the APP $u_i$ by $I_i$. The user embeddings are randomly initialized. In parallel, an APP embedding $\mathbf{p}_j$ is initialized by averaging the embeddings of users installing the APP:

$$\mathbf{p}_j = \frac{\sum_{k \in I_i} \mathbf{u}_k}{|I_i|},$$

where $|I_i|$ denotes the cardinality of the set $I_i$, i.e., the number of users installing the APP $p_i$.

**Subgraph sampling.** As we mentioned, for a subgraph $G_i$, it contains an APP node ($p_i$) and the users installing the APP $p_i$. Let us denote the probability of sampling the subgraph $G_i$ as $P(G_i)$. A naive sampling approach is to sample the subgraph with a probability proportional to the number of user nodes in the subgraph. That is,

$$P(G_i) = \frac{|I_i|}{\sum_{j=1}^{N} |I_j|}, \forall i \in [1, N],$$

where $N$ is the total number of APPs and $|I_i|$ denotes the number of users in the subgraph $G_i$. In this case, each edge connecting a user and an APP will be involved in the training process with an equal probability. Nevertheless, this strategy will make the embedding learning dominated by the popular APPs with a huge number of users and the contributions from some niche APPs with a small number of users will be underestimated. To make the contributions from different APPs balanced, we can devise that the sampling probability of each sub-graph to be equal. That is,

$$P(G_i) = \frac{1}{N}, \forall i \in [1, N].$$

In this case, the edges based on niche APPs will be over-sampled, and the edges based on the popular APPs will be under-sampled. Nevertheless, it might lead to repeatedly sampling for edges from niche APPs, and some edges from the popular APPs might have little chance to be involved in the training process. It tends to make the learned embedding prone to over-fitting due to a lack of diversity in the training samples. To achieve a balanced sampling and meanwhile suppress over-fitting, we adopt a trade-off sampling approach. It devises the probability as

$$P(G_i) = \frac{|I_i|^\tau}{\sum_{j=1}^{N} |I_j|^\tau}, \forall i \in [1, N],$$

where $\tau$ is a pre-defined positive constant. Normally, we set $0 < \tau < 1$. It assigns a higher sampling probability to the subgraph containing more nodes for suppressing over-fitting and meanwhile achieving a good balance among different APPs. When $\tau = 1$, it degenerates to the naive sampling approach defined in Eq. (2). On the other hand, when $\tau = 0$, it degrades to the balanced sampling approach defined in Eq. (3). By default, we set $\tau = 0.5$ in our experiments.

**Embedding learning within a subgraph.** Let denote the APP embedding with a subgraph by $\mathbf{p}$, the embedding of a user installing the APP by $\mathbf{u}_i^+$ and that of a user who does not install the APP by $\mathbf{u}_i^-$. The user and APP embedding learning seeks to keep a large similarity between $\mathbf{p}$ and $\mathbf{u}_i^+$. At the same time, it seeks to maintain a small similarity between $\mathbf{p}$ and $\mathbf{u}_i^-$. Straightforwardly, we can learn the user and the APP embedding through a pairwise loss:

$$\mathcal{L}_{\text{pair}} = \frac{1}{n^+} \sum_{i=1}^{n^+} \log(1 + e^{-\beta s(\mathbf{p}, \mathbf{u}_i^+)}) - \frac{1}{n^-} \sum_{i=1}^{n^-} \log(1 + e^{-\beta s(\mathbf{p}, \mathbf{u}_i^-)}),$$

where $n^+$ denotes the number of users installing the APP and $n^-$ denotes the number of users who do not install, $\beta$ is a pre-defined positive constant controlling the softness, and $s(\cdot, \cdot)$ measures the cosine similarity between two vectors, as visualized in Figure 3.

In parallel to the pairwise loss defined above, we devise an additional centroid loss to further enhance the effectiveness of the learned embedding. To be specific, we first compute the centroid of the embeddings of users installing the APP:

$$\mathbf{u}_c = \frac{1}{n^+} \sum_{i=1}^{n^+} \mathbf{u}_i^+.$$ 

Then the centroid loss is computed by

$$\mathcal{L}_{\text{centroid}} = \log(1 + e^{-\beta s(\mathbf{u}_c, \mathbf{u}_i^+)}).$$

Fig. 2. The visualization of decomposing a user-APP graph into subgraphs. The user-APP graph $G$ consists of the node set $V = \{p_i\}_{i=1}^{n} \cup \{u_i\}_{i=1}^{n}$ where $p_i$ denotes an APP node and $u_i$ is a user node. $G$ is decomposed into three subgraphs $\{G_i\}_{i=1}^{3}$. $G_i$ consists of the node set $V_i$. In this example, $V_1 = \{u_1, u_2, u_3, u_4\}$, $V_2 = \{p_1, u_2, u_2, u_3\}$, and $V_3 = \{p_3, u_7, u_8, u_1\}$. Note that, the user $u_1$ is connected with two APPs $p_2$ and $p_3$. Thus, $u_1$ is included in two subgraphs, $G_1$ and $G_2$. 

\[ \begin{align*} 
\mathbf{p}_j &= \frac{\sum_{k \in I_i} \mathbf{u}_k}{|I_i|}, \\
\mathcal{L}_{\text{pair}} &= \frac{1}{n^+} \sum_{i=1}^{n^+} \log(1 + e^{-\beta s(\mathbf{p}, \mathbf{u}_i^+)}) \\
&\quad - \frac{1}{n^-} \sum_{i=1}^{n^-} \log(1 + e^{-\beta s(\mathbf{p}, \mathbf{u}_i^-)}), \\
\mathcal{L}_{\text{centroid}} &= \log(1 + e^{-\beta s(\mathbf{u}_c, \mathbf{u}_i^+)}). 
\end{align*} \]
Fig. 3. The visualization of user and APP embedding learning within a subgraph. In this example, the APP embedding is \( \mathbf{p} \) (green box). There are four users installing the APP, \( \{u^+_i\}_{i=1}^{N_u} \) (blue dots) with the centroid \( \mathbf{u}^c \) (yellow dot) and four users not installing the APP, \( \{u^-_i\}_{i=1}^{N_u} \) (red dots).

To stabilize the training, we update the user embedding and the APP embedding in an alternating manner:

1) Fix user embedding \( \{u^+_i\}_{i=1}^{N_u} \) and \( \{u^-_i\}_{i=1}^{N_u} \), and update the APP embedding \( \mathbf{p} \) using the centroid loss \( L_{\text{centroid}} \).

2) Fix the APP embedding \( \mathbf{p} \), and update the positive user embedding \( \{u^+_i\}_{i=1}^{N_u} \) using the pairwise loss \( L_{\text{pair}} \).

To improve the training efficiency, we achieve this iterative training manner in a parallel way by utilizing the stop-gradient trick. That is, we devise the final loss \( L = L_{\text{centroid}} + L_{\text{pair}} \). In the meanwhile, we stop the gradient derived by \( L_{\text{centoid}} \) back-propagating to \( \{u^+_i\}_{i=1}^{N_u} \) and \( \{u^-_i\}_{i=1}^{N_u} \) and meanwhile stop the gradient from \( L_{\text{pair}} \) back-propagating to \( \mathbf{p} \) and \( \{u^-_i\}_{i=1}^{N_u} \).

IV. EXPERIMENTS

Dataset. We collected the information of 80 million users and 50 thousand APPs. On average, each user installs around 30 APPs. We build a graph consisting of 0.6 billion user nodes and 34 thousand APPs. Meanwhile, it contains 12 billion edges connecting user nodes and APP nodes.

A. Offline experiments

Memory. For each APP, we randomly sample 96 users who have already installed the APP and 96 users not installing the APP. Note that these APP installation has been involved in the training process. For each APP-user pair, we compute the cosine similarity between their embeddings. Then we threshold the cosine similarity to 0 or 1 to predict whether the user has installed the APP or not. In Table I, we show the experimental result. As shown in the table, in the training data, the learned embedding can achieve a 0.953 precision and 0.981 AUC, which demonstrates the powerful fitting capability of the learned embeddings.

Inference. To evaluate the inference performance of the learned user and APP embedding, we report the classification AUC on the user side and that on the APP side. The user-side AUC is averaged over users. For each user, we test the prediction accuracy using several APPs the user has installed and several APPs the user does not install. The APP-side AUC is measured in a similar manner but is averaged over APPs. Note that the testing cases for inference are not involved in the training process. To be specific, our whole data is collected during \( N \) days. We use the data in the first \( N - 5 \) days for training and that from the last 5 days for testing. Meanwhile, we also report the AUC without excluding APPs with a huge number of users. To be specific, we report \( \text{AUC}^* \), which excludes APPs with more than 2.5% users. We also report \( \text{AUC}^+ \), which excludes that with more than 8% users. As shown in Table II, the AUC achieved in the inference is lower than that in Table I. In the meanwhile, by excluding some APPs with a huge number of users, \( \text{AUC}^+ \) and \( \text{AUC}^* \) are larger than AUC.

Ablation study Here, we investigate the influence of removing \( L_{\text{centroid}} \) or the stop-gradient strategy through ablation study. As shown in Table III, when removing \( L_{\text{centroid}} \), the AUC drops from 0.981 to 0.977 and the precision decreases from 0.953 to 0.948. Meanwhile, without the stop-gradient strategy, both the AUC and the precision decrease considerably.

B. Online experiments

We have integrated the user embedding learned from the APP-installation information as a feature which complements the existing user embedding learned from historical behaviors. After launching it in our online personalized advertising platform, we achieved a +1.1% CTR improvement, a +1.7% CVR boost, a +2.6% increase in revenue as shown in Table IV. The online experiments were conducted from May 1st, 2021 to May 7th, 2021.

| TABLE I | THE MEMORY PERFORMANCE OF THE LEARNED USER AND APP EMBEDDINGS. WE REPORT THE PRECISION PREDICTION AND AREA-UNDER-CURVE (AUC) FOR THE APP INSTALLATION. |
|----------------|----------------|----------------|
| Precision      | Ours           | 0.953          |
|                | w/o \( L_{\text{centroid}} \) | 0.948          |
| AUC            | Ours           | 0.981          |
|                | w/o \( L_{\text{centroid}} \) | 0.977          |
|                | w/o stop-gradient | 0.973          |

| TABLE II | THE INFERENCE PERFORMANCE OF THE LEARNED USER AND APP EMBEDDINGS. \( \text{AUC}^+ \) DENOTES THE AUC EXCLUDING APPS WITH MORE THAN 8% USERS AND \( \text{AUC}^* \) DENOTES THE AUC EXCLUDING APPS WITH MORE THAN 2.5% USERS. |
|----------------|----------------|----------------|----------------|----------------|
|                | APP-side       | 0.797          | 0.840          | 0.834          |
|                | User-side      | 0.846          | 0.829          | 0.844          |

| TABLE III | THE INFLUENCE OF REMOVING \( L_{\text{centroid}} \) OR THE STOP-GRADIENT STRATEGY IN INFERENCE. |
|----------------|----------------|----------------|----------------|----------------|
|                | Ours           | w/o \( L_{\text{centroid}} \) | w/o stop-gradient |
| Precision      | 0.953          | 0.948          | 0.942          |
| AUC            | 0.981          | 0.977          | 0.973          |
In this paper, we demonstrate that APP-installation information can be helpful to describe users’ characteristics, because users with similar APPs installed might share common interests and behave similarly. In our implementation, we exploit the APP-installation information to assist in modeling the user’s characteristics for personalized advertising. To this end, we build a user-APP bipartite graph and adopt a graph convolution network to learn the user embedding. We use the learned user embedding from our user-APP graph as the complementary information to the existing user representation learned from the user profile and the user’s historical behaviors. After deploying it in our advertising platform, both CTR and CVR improve considerably.

V. CONCLUSION

In this paper, we demonstrate that APP-installation information can be helpful to describe users’ characteristics, because users with similar APPs installed might share common interests and behave similarly. In our implementation, we exploit the APP-installation information to assist in modeling the user’s characteristics for personalized advertising. To this end, we build a user-APP bipartite graph and adopt a graph convolution network to learn the user embedding. We use the learned user embedding from our user-APP graph as the complementary information to the existing user representation learned from the user profile and the user’s historical behaviors. After deploying it in our advertising platform, both CTR and CVR improve considerably.

REFERENCES

[1] Amr Ahmed, Nino Shervashidze, Shrvan M. Narayanamurthy, Vanja Josifovski, and Alexander J. Smola. Distributed large-scale natural graph factorization. In Proceedings of the 22nd International World Wide Web Conference (WWW), pages 37–48, Rio de Janeiro, Brazil, 2013.

[2] James Atwood and Don Towsley. Diffusion-convolutional neural networks. In Advances in Neural Information Processing Systems (NIPS), pages 1993–2001, Barcelona, Spain, 2016.

[3] Mikhail Belkin and Partha Niyogi. Laplacian eigenmaps and spectral techniques for embedding and clustering. In Advances in Neural Information Processing Systems (NIPS), pages 585–591, Vancouver, Canada, 2001.

[4] Joan Bruna, Wojciech Zaremba, Arthur Szlam, and Yann LeCun. Spectral networks and locally connected networks on graphs. In Proceedings of the 2nd International Conference on Learning Representations (ICLR), Banff, Canada, 2014.

[5] Shaoqing Cao, Wei Lu, and Qiongkai Xu. GraRep: Learning graph representations with global structural information. In Proceedings of the 24th ACM International Conference on Information and Knowledge Management (CIKM), pages 891–900, Melbourne, Australia, 2015.

[6] Yue Cao, Mingsheng Long, Jianming Wang, Qiang Yang, and Philip S. Yu. Deep visual-semantic hashing for cross-modal retrieval. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), pages 1445–1454, San Francisco, CA, 2016.

[7] Haochen Chen, Bryan Perozzi, Yifan Hu, and Steven Skiena. HARP: hierarchical representation learning for networks. In Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence (AAAI), pages 2127–2134, New Orleans, LA, 2018.

[8] Wei-Lin Chiang, Xuangxing Liu, Si Si, Yang Li, Samy Bengio, and Cho-Jui Hsieh. Cluster-GCN: An efficient algorithm for training deep and large graph convolutional networks. In Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining (KDD), pages 257–266, Anchorage, AK, 2019.

[9] Michael Defferrard, Xavier Bresson, and Pierre Vandergheynst. Convolutional neural networks on graphs with fast localized spectral filtering. In Advances in Neural Information Processing Systems (NIPS), pages 3837–3845, Barcelona, Spain, 2016.

[10] Miao Fan, Jiacheng Guo, Shuai Zhu, Shuo Miao, Mingming Sun, and Ping Li. MOBIBUS: towards the next generation of query-ad matching in baidu’s sponsored search. In Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining (KDD), pages 2509–2517, Anchorage, AK, 2019.

[11] Aditya Grover and Jure Leskovec. node2vec: Scalable feature learning for networks. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), pages 855–864, San Francisco, CA, 2016.

[12] William L. Hamilton, Zhitao Ying, and Jure Leskovec. Inductive representation learning on large graphs. In Advances in Neural Information Processing Systems (NIPS), pages 1024–1034, Long Beach, CA, 2017.

[13] Mikael Henaff, Joan Bruna, and Yann LeCun. Deep convolutional networks on graph-structured data. arXiv preprint arXiv:1506.05163, 2015.

[14] Thomas N. Kipf and Max Welling. Semi-supervised classification with graph convolutional networks. In Proceedings of the 5th International Conference on Learning Representations (ICLR), Toulon, France, 2017.

[15] Ron Levie, Federico Monti, Xavier Bresson, and Michael M. Bronstein. CayleyNets: Graph convolutional neural networks with complex rational spectral filters. IEEE Trans. Signal Process., 67(1):97–109, 2019.

[16] Ruoyu Li, Sheng Wang, Feiyan Zhu, and Junzhou Huang. Adaptive graph convolutional neural networks. In Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence (AAAI), pages 3546–3553, New Orleans, LA, 2018.

[17] Alessio Micheli. Neural network for graphs: A contextual constructive approach. IEEE Trans. Neural Networks, 20(3):498–511, 2009.

[18] Mingdong Ou, Peng Cui, Jian Pei, Ziwei Zhang, and Wenwu Zhu. Asymmetric transitivity preserving graph embedding. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), pages 1105–1114, San Francisco, CA, 2016.

[19] Bryan Perozzi, Rami Al-Rfou, and Steven Skiena. Deepwalk: online learning of social representations. In Proceedings of the 20th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), pages 701–710, New York, NY, 2014.

[20] Bryan Perozzi, Vivek Kulkarni, and Steven Skiena. Walklets: Multiscale graph embeddings for interpretable network classification. arXiv preprint arXiv:1605.02115, 2016.

[21] Mostafa Rahmani and Ping Li. The necessity of geometrical representation for deep graph analysis. In Proceedings of the 20th IEEE International Conference on Data Mining (ICDM), pages 1232–1237, Sorrento, Italy, 2020.

[22] Mostafa Rahmani, Rassoul Shafipour, and Ping Li. Non-local feature aggregation on graphs via latent fixed data structures. In Proceedings of the 55th Asilomar Conference on Signals, Systems, and Computers (Asilomar), pages 1551–1557, Pacific Grove, CA, 2021.

[23] Anshumali Shrivastava and Ping Li. A new space for comparing graphs. In Proceedings of the 2014 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining (ASONAM), pages 62–71, Beijing, China, 2014.

[24] Shulong Tan, Weijie Zhao, and Ping Li. Fast neural ranking on bipartite graph indices. Proc. VLDB Endow., 15(4):794–803, 2021.

[25] Petar Velickovic, Guillem Cucurull, Arantxa Casanova, Adriana Romero, Pietro Liò, and Yoshua Bengio. Graph attention networks. In Proceedings of the 6th International Conference on Learning Representations (ICLR), Vancouver, Canada, 2018.

[26] Daixin Wang, Peng Cui, and Wenwu Zhu. Structural deep network embedding. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), pages 1225–1234, San Francisco, CA, 2016.

[27] Keyulu Xu, Weihua Hu, Jure Leskovec, and Stefanie Jegelka. How powerful are graph neural networks? In Proceedings of the 35th International Conference on Machine Learning (ICML), pages 5365–5374, Stockhol, Sweden, 2018.

[28] Tan Yu, Jie Liu, Yi Yang, Yi Li, Hongliang Fei, and Ping Li. EGM: enhanced graph-based model for large-scale video advertisement search. In Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), pages 4443–4451, Washington, DC, 2022.

[29] Weijie Zhao, Shulong Tan, and Ping Li. SONG: approximate nearest neighbor search on GPU. In Proceedings of the 36th IEEE International Conference on Data Engineering (ICDE), pages 1033–1044, Dallas, TX, 2020.

[30] Zhixin Zhou, Shulong Tan, Zhaozhuo Xu, and Ping Li. Möbius transformation for fast inner product search on graph. In Advances in Neural Information Processing Systems (NeurIPS), pages 8216–8227, Vancouver, Canada, 2019.

### TABLE IV

| CTR | CVR | Revenue |
|-----|-----|---------|
| +1.1% | +1.7% | +2.6% |