ΩNN and ΩΩN states

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(Dated: January 18, 2019)

Abstract

The lattice QCD analysis of the HAL QCD Collaboration has recently derived ΩN and ΩΩ interacting potentials with nearly physical quark masses ($m_\pi \simeq 146$ MeV and $m_K \simeq 525$ MeV). They found an attractive interaction in the ΩN $^5S_2$ channel which supports a bound state with a central binding energy of 1.54 MeV. The ΩΩ $^1S_0$ channel shows an overall attraction with a bound state with a central binding energy of 1.6 MeV. In this paper we looked closely at the ΩNN and ΩΩN three-body systems making use of the latest HAL QCD Collaboration ΩN and ΩΩ interactions. Our results show that the Ωd system in the state with maximal spin $(I)J^P = (0)5/2^+$ is bound with a binding energy of about 20 MeV. The $(I)J^P = (1)3/2^+$ Ωnn state presents a resonance decaying to ΛΞn and ΣΞn, with a separation energy of $\sim 1$ MeV. The $(I)J^P = (1/2)1/2^+$ ΩΩN state also exhibits a resonance decaying to ΛΞΩ and ΣΞΩ, with a separation energy of $\sim 4.6$ MeV. We have calculated the contribution of the Coulomb potential to differentiate among the different charged states.

PACS numbers: 21.45.+v,25.10.+s,12.39.Jh

Keywords: baryon-baryon interactions, Faddeev equations

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I. INTRODUCTION

Few-body systems containing nucleons may enhance the binding of two-body resonances or bound states. We have simple examples in nature. Whereas there is no evidence for strangeness $-1$ dibaryon states, the hypertriton $^3\Lambda H$, $(I)J^P = (0)1/2^+$, is bound with a separation energy of $130 \pm 50$ keV, and the $^4\Lambda H$, $(I)J^P = (0)0^+$, is bound with a separation energy of $2.04 \pm 0.04$ MeV. Similarly, in the nonstrange sector the binding per nucleon, $B/A$, increases from $1:3:7$ for increasing number of nucleons, $A = 2, 3, 4$. Thus, the study of few-body systems could help in the search for signals of two-body bound states or resonances in the strange sector.

The possible existence of dibaryons is a challenge since a long time ago [1 –3]. Their occurrence would clearly help us to better understand Quantum Chromodynamics in the nonperturbative regime. Unfortunately, despite innumerable experimental searches, the deuteron has been the only known dibaryon state up to very recently. The increasing quality of experimental data together with exclusive measurements in hadronic reactions [4], direct measurements in relativistic heavy-ion collisions [5], or the development of new methods for nuclear emulsion experiments [6], has renewed the interest in dibaryons. Thus, the so-called WASA dibaryon, a resonance $90$ MeV below the nominal $\Delta\Delta$ threshold with a width of $70$ MeV, is now firmly established although its nature is still under debate [7]. Moreover, the so-called KISO event represents the first clear evidence of a deeply bound state of $\Xi^- - ^{14}N$ [6], which combined with other indications of emulsion data suggest an average attractive $\Xi N$ interaction [8]. In this latter case, the increase in the binding energy per baryon within few-body systems with additional nucleons has already been noticed. If the $(I)J^P = (1)1^+ \Xi N$ state would be bound by $1.56$ MeV [6,8], the $\Xi d$ $(I)J^P = (1/2)3/2^+$ state becomes bound by $17.2$ MeV [9,10].

In addition to improvements in the experimental data and techniques, the theoretical efforts of the lattice HAL QCD Collaboration [11] have reached the point of deriving baryon-baryon interactions near the physical pion mass. Their most recent results hint to the existence of shallow bound states in the $\Omega N$ and $\Omega\Omega$ systems [12,13]. Two-body systems containing an $\Omega$ baryon seem to be specially suited to lodge a two-body bound state or resonance. Thus, the $\Omega N$ interaction is expected to lack a repulsive core since the quark flavors of the nucleon are different from those of the $\Omega$, so that the Pauli exclusion principle can
Moreover, the color magnetic term of the one-gluon exchange interaction is attractive in some particular channels, one example being the $^5S_2$, what led to the prediction of an $\Omega N$ dibaryon in constituent quark models [15, 16]. The recent lattice QCD results by the HAL QCD Collaboration hint toward the existence of such $\Omega N$ bound state [13]. Another two-body system containing $\Omega$ baryons, $\Omega\Omega$, is also interesting because it would be the only possible strong interaction stable state made of two decuplet baryons. The adequacy of such two-body system for having an overall attractive interaction has become apparent on phenomenological quark models predicting an $\Omega\Omega$ $^1S_0$ bound state [17, 18]. The recent near to physical pion mass results of the lattice HAL QCD Collaboration corroborate such finding [12]. It is worth to notice that the theoretical analysis [19] of the first measurement of the proton-$\Omega$ correlation function in heavy-ion collisions by the STAR experiment [20] at the Relativistic Heavy-Ion Collider (RHIC) favors the proton-$\Omega$ bound state hypothesis.

Preliminary results by the HAL QCD Collaboration [21] had already shown the attractive character of the $^5S_2 \Omega N$ interaction. These exploratory results were parametrized in Ref. [22] by means of an equivalent local potential reproducing the $\Omega N$ $^5S_2$ scattering amplitude of Ref. [21]. Making use of the $\Omega N$ equivalent local potential of Ref. [22] together with the Malfliet-Tjon $^3S_1 NN$ potential of Ref. [23], we studied the $\Omega d$ system in the maximal spin channel, ($I J^P = (0)5/2^+$). We concluded that the system is bound, with a binding energy of about 17 MeV [24]. The latest results of the HAL QCD Collaboration nearly physical quark masses ($m_\pi \simeq 146$ MeV and $m_K \simeq 525$ MeV) have been recently made public [13], validating the attractive nature of the $\Omega N$ $^5S_2$ state and providing their own parametrization of the lattice QCD interaction. Moreover, the HAL QCD Collaboration has also recently published the $\Omega\Omega$ $^1S_0$ interaction nearly physical pion mass, $m_\pi = 146$ MeV, showing its overall attractive character [12]. Thus, we have all the necessary ingredients to explore possible implications of the attractive nature of the $\Omega N$ and $\Omega\Omega$ interactions on few-body systems on the basis of first-principle lattice QCD-based interactions.

Our purpose in this work is to study the three-body systems $\Omega NN$ and $\Omega\Omega N$ looking for deeply bound states or resonances which may be sought experimentally. The paper is organized as follows. In Sec. II we summarize the two-body interactions. In Sec. III we

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1 Note, however, that the first measurement of the proton-$\Omega$ correlation function drives to a proton-$\Omega$ bound state with a binding energy of $\sim 27$ MeV [20], far from the present lattice QCD results of about 1.54 MeV [13] that will be used in this work.
TABLE I: Low-energy data and parameters of the local central Yukawa-type potential given by Eq. (1) for the $^1S_0$ and $^3S_1$ $NN$ interactions [23].

| $(I, J)$ | $a_0$ (fm) | $r_{\text{eff}}$ (fm) | $C_1$ (MeV fm) | $\mu_1$ (fm$^{-1}$) | $C_2$ (MeV fm) | $\mu_2$ (fm$^{-1}$) |
|---------|------------|------------------------|----------------|-------------------|----------------|-------------------|
| $(1, 0)$ | $-23.56$   | $2.88$                 | $-513.968$     | $1.55$            | $1438.72$     | $3.11$            |
| $(0, 1)$ | $5.51$     | $1.89$                 | $-626.885$     | $1.55$            | $1438.72$     | $3.11$            |

outline the solution of the three-body bound-state Faddeev equations for the case of two identical particles. In Sec. IV we present and discuss our results. The most important conclusions of our work are summarized in Sec. V.

II. TWO-BODY INTERACTIONS

In the case of the $NN$ subsystem the two-body potential in configuration space is given by [23],

$$V_{NN}(r) = \sum_{n=1}^{2} C_n e^{-\mu_n r},$$

(1)

with the parameters specified in Table II. The predicted binding energy of the deuteron is $B = 2.2307$ MeV.

For the $\Omega N$ system the HAL QCD Collaboration has recently published the $S$-wave and spin 2 interaction with nearly physical quark masses [13]. The lattice results are fitted by an analytic function composed of an attractive Gaussian core plus a long range (Yukawa)$^2$ attraction with a form factor [14],

$$V_{\Omega N}(r) = b_1 e^{-b_2 r^2} + b_3 \left(1 - e^{-b_4 r^2}\right) \left(\frac{e^{-m_\pi r}}{r}\right)^2,$$

(2)

The (Yukawa)$^2$ form at long distance is motivated by the picture of two-pion exchange between $N$ and $\Omega$ with an OZI violating vertex [22]. The pion mass in Eq. (2) is taken from the simulation, $m_\pi = 146$ MeV [13]. The lattice results are fitted reasonably well, $\chi^2$/d.o.f $\simeq 1$, with four different set of parameters given in Table III. The above interactions drive to the following central values of the low-energy data: a scattering length $a_{0}^{\Omega N} = 5.30$ fm, an effective range $r_{\text{eff}}^{\Omega N} = 1.26$ fm, and a binding energy $B_{\Omega N} = 1.54$ MeV.

Finally, the $\Omega\Omega$ interaction in the $^1S_0$ channel has also been recently studied by the HAL QCD Collaboration with a large volume and nearly the physical pion mass [12].
TABLE II: Fitting parameters in Eq. (2) for different models, $P_i$, of the $^5S_2$ $\Omega N$ interaction [13].

|       | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|-------|-------|-------|-------|-------|
| $b_1$ (MeV) | -306.5 | -313.0 | -316.7 | -296 |
| $b_2$ (fm$^{-2}$) | 73.9 | 81.7 | 81.9 | 64 |
| $b_3$ (MeV fm$^2$) | -266 | -252 | -237 | -272 |
| $b_4$ (fm$^{-2}$) | 0.78 | 0.85 | 0.91 | 0.76 |

results indicate that the $\Omega\Omega$ interaction has an overall attraction. In particular, the potential derived has been fitted by means of an analytic function composed of three Gaussians,

$$V_{\Omega\Omega}(r) = \sum_{j=1}^{3} c_je^{-(r/d_j)^2},$$

where $(c_1, c_2, c_3) = (914, 305, -112)$ in MeV and $(d_1, d_2, d_3) = (0.143, 0.305, 0.949)$ in fm with $\chi^2$/d.o.f $\simeq 1.3$. A functional form similar to the one used for the $\Omega N$ interaction, i.e., two Gaussians plus a long range (Yukawa function)$^2$ provides an equally good fit and does not affect the results. The low-energy data obtained with this interaction are the following: a scattering length $a_{0\Omega}^\Omega = 4.6$ fm, an effective range $r_{\text{eff}}^{\Omega\Omega} = 1.27$ fm, and a binding energy $B_{\Omega\Omega} = 1.6$ MeV.

III. THE THREE-BODY BOUND-STATE FADDEEV EQUATIONS

The $\Omega NN$ and $\Omega\Omega N$ three-body problems are studied using the method of Ref. [25], expanding the two-body amplitudes in terms of Legendre polynomials. We restrict ourselves to configurations where all three particles are in $S$-wave states so that the Faddeev equations for the bound-state problem in the case of three baryons with total isospin $I$ and spin $J$ are,

$$T_{i;i,j}^{i;i,j}(p_iq_i) = \sum_{j\neq i} \sum_{i,j} h_{i;j;ij}^{i;j;ij} \frac{1}{2} \int_0^\infty q_j^2dq_j \int_{-1}^1 d\cos\theta \; t_{i;i,j}^{i;i,j}(p_i; p'_i; E - q_i^2/2\nu)$$

$$\times \frac{1}{E - p_j^2/2\mu_j - q_j^2/2\nu_j} T_{j;j,j}^{i;i,j}(p_jq_j),$$

where $t_{i;i,j}$ stands for the two-body amplitudes with isospin $i$ and spin $j$. $p_i$ is the momentum of the pair $jk$ (with $ijk$ an even permutation of 123) and $q_i$ the momentum of particle
i with respect to the pair \(jk\). \(\mu_i\) and \(\nu_i\) are the corresponding reduced masses,

\[
\mu_i = \frac{m_j m_k}{m_j + m_k}, \\
\nu_i = \frac{m_i (m_j + m_k)}{m_i + m_j + m_k},
\]

and the momenta \(p'_i\) and \(p_j\) in Eq. (4) are given by,

\[
p'_i = \sqrt{q_j^2 + \frac{\mu^2}{m_k} q_i^2 + 2 \frac{\mu_i}{m_k} q_i q_j \cos \theta}, \\
p_j = \sqrt{q_i^2 + \frac{\mu^2}{m_k} q_j^2 + 2 \frac{\mu_j}{m_k} q_i q_j \cos \theta}.
\]

\(h_{ij;ij}^{\mu \nu} \) are the spin–isospin coefficients,

\[
h_{ij;ij}^{\mu \nu} = (-)^{t_j + \tau_j - I} \sqrt{(2i_i + 1)(2i_j + 1) W(\tau_j \tau_k I \tau_i; i_i i_j)} \times (-)^{t_j + \sigma_j - J} \sqrt{(2j_i + 1)(2j_j + 1) W(\sigma_j \sigma_k J \sigma_i; j_i j_j)},
\]

where \(W\) is the Racah coefficient and \(\tau_i, i_i,\) and \(I (\sigma_i, j_i,\) and \(J\) are the isospins (spins) of particle \(i\), of the pair \(jk\), and of the three–body system.

Since the variable \(p_i\) in Eq. (4) runs from 0 to \(\infty\), it is convenient to make the transformation

\[
x_i = \frac{p_i - b}{p_i + b},
\]

where the new variable \(x_i\) runs from \(-1\) to \(1\) and \(b\) is a scale parameter that has no effect on the solution. With this transformation Eq. (4) takes the form,

\[
T_{i;ij}^{ii} (x_i q_i) = \sum_{j \neq i} \sum_{ij, j} h_{ij;ij}^{ii} \frac{1}{2} \int_0^\infty q_j^2 dq_j \int_{-1}^1 d\cos \theta t_{i;ij} (x_i, x'_i; E - q_j^2/2\nu_j) \\
\times \frac{1}{E - p_j^2/2\mu_j - q_j^2/2\nu_j} T_{ij;ij} (x_j q_j).
\]

Since in the amplitude \(t_{i;ij} (x_i, x'_i; e)\) the variables \(x_i\) and \(x'_i\) run from \(-1\) to \(1\), one can expand this amplitude in terms of Legendre polynomials as,

\[
t_{i;ij} (x_i, x'_i; e) = \sum_{nr} P_n (x_i) \tau_{i;ij}^{nr} (e) P_r (x'_i),
\]

where the expansion coefficients are given by,

\[
\tau_{i;ij}^{nr} (e) = \frac{2n + 1}{2} \frac{2r + 1}{2} \int_{-1}^1 dx_i \int_{-1}^1 dx'_i P_n (x_i) t_{i;ij} (x_i, x'_i; e) P_r (x'_i).
\]
Applying expansion (10) in Eq. (9) one gets,

\[ T_{i;IJ}^{ij}(x, q_i) = \sum_n P_n(x_i) T_{i;IJ}^{ni}_{ij}(q_i), \tag{12} \]

where \( T_{i;IJ}^{ni}_{ij}(q_i) \) satisfies the one-dimensional integral equation,

\[ T_{i;IJ}^{ni_{ij}}(q_i) = \sum_{j \neq i} \sum_{m_{ij}r} \int_0^\infty dq_j A_{ij;IJ}^{ni_{ij};ni_{ij}}(q_i, q_j; E) T_{j;IJ}^{ni_{ij}}(q_j), \tag{13} \]

with \n
\[ A_{ij;IJ}^{ni_{ij};ni_{ij}}(q_i, q_j; E) = h_{ij;IJ}^{ni_{ij};ni_{ij}} \sum_r T_{r;ij}^{nr} (E - q_i^2/2\mu_i) q_j^2/2 \]
\[ \times \int_{-1}^{1} d\cos \theta \frac{P_r(x_i') P_m(x_j)}{E - p_j^2/2\mu_j - q_j^2/2\nu_j}. \tag{14} \]

The three amplitudes \( T_{1;IJ}^{r_{1}ij}(q_1) \), \( T_{2;IJ}^{r_{2}ij}(q_2) \), and \( T_{3;IJ}^{r_{3}ij}(q_3) \) in Eq. (13) are coupled together. The number of coupled equations can be reduced, however, when two of the particles are identical, which is currently the case, \( \Omega N\bar{N} \) and \( \Omega N \). The procedure for the case of two identical fermions has been described before \[26, 27\] and will not be repeated here. With the assumption that particles 2 and 3 are identical and particle 1 is the different one, only the amplitudes \( T_{1;IJ}^{r_{1}ij}(q_1) \) and \( T_{2;IJ}^{r_{2}ij}(q_2) \) are independent from each other and they satisfy the coupled integral equations,

\[ T_{1;IJ}^{r_{1}ij}(q_1) = 2 \sum_{m_{ij}r} \int_0^\infty dq_3 A_{13;IJ}^{r_{1}ij;mi_{2}j}(q_1, q_3; E) T_{2;IJ}^{mi_{2}j}(q_3), \tag{15} \]

\[ T_{2;IJ}^{mi_{2}j}(q_2) = \sum_{m_{ij}r} g \int_0^\infty dq_3 A_{23;IJ}^{mi_{2}j;mi_{3}j}(q_2, q_3; E) T_{2;IJ}^{mi_{3}j}(q_3) \]
\[ + \sum_{r_{1}ij} \int_0^\infty dq_1 A_{31;IJ}^{r_{1}ij;mi_{2}j}(q_2, q_1; E) T_{1;IJ}^{r_{1}ij}(q_1), \tag{16} \]

with the identical–particle factor

\[ g = (-1)^{1+\sigma_1+\sigma_3 - j_2 + \tau_1 + \tau_3 - i_2}, \tag{17} \]

where \( \sigma_1 (\tau_1) \) stand for the spin (isospin) of the different particle and \( \sigma_3 (\tau_3) \) for those of the identical ones.

Substitution of Eq. (15) into Eq. (16) yields an equation with only the amplitude \( T_2 \),

\[ T_{2;IJ}^{mi_{2}j}(q_2) = \sum_{m_{ij}r} \int_0^\infty dq_3 K_{IJ}^{mi_{2}j;mi_{3}j}(q_2, q_3; E) T_{2;IJ}^{mi_{3}j}(q_3), \tag{18} \]
where
\[ K_{IJ}^{\alpha_1\beta_1;\mu_1\nu_1}(q_2, q_3; E) = gA_{23;IJ}^{\alpha_1\beta_1;\mu_1\nu_1}(q_2, q_3; E) + 2 \sum_{r_1;1} \int_0^\infty dq_1 A_{31;IJ}^{\alpha_1\beta_1;r_1;1}(q_2, q_1; E) A_{13;IJ}^{r_1;1;\mu_1\nu_1}(q_1, q_3; E). \] (19)

The off-shell two-body \( t \)–matrices are obtained by solving the Lippmann-Schwinger equation,
\[ t_i(p_i, p'_i; e) = V_i(p_i, p'_i) + \int_0^\infty p_i^{\mu_2}dp_i^{\mu_2}V_i(p_i, p''_i) \frac{1}{e - p''_i^2/2\eta_i + i\epsilon}t_i(p''_i, p'_i; e). \] (20)
with the two-body interactions \( V_l, i = NN, \Omega N, \Omega \Omega \), described in section II.

Finally, in order to separate the binding energies of the different charged states we included the Coulomb interaction as,
\[ V_C(r) = \pm \alpha \frac{e^{-r/r_0}}{r}, \] (21)
where \( \alpha \) is the fine structure constant and \( r_0 \) a screening radius taken to be \( r_0 = 50 \) fm.

IV. RESULTS

Let us first of all present and discuss our results for the corresponding \( \Omega NN \) states\(^2\). We have first studied the \( \Omega d \) state. We show in Table III the binding energies of the state with maximal spin, \( (I)J^P = (0)5/2^+ \), for the different models of the \( \Omega N \) interaction reported in Ref. [13] and summarized in Table II. For completeness we also include the binding energy of the \( 5S_2 \) \( \Omega N \) state. As indicated in the caption, the numbers between parenthesis correspond to the results using the N and \( \Omega \) masses derived by the HAL QCD Collaboration [12, 13], that are somewhat larger than the experimental masses. If the masses increase, the repulsive kinetic energy contribution decreases, resulting in slightly larger binding energies. As it can be seen, the binding energy of the \( \Omega N \) system is larger than in the preliminary results of the \( \Omega N \) interaction presented in Ref. [21] and parametrized in Ref. [22], where the \( \Omega N \) binding energy was 0.3 MeV [22, 24] for the \( N \) and \( \Omega \) physical masses. As a consequence, the binding of the \( \Omega d \) state with maximal spin \( (I)J^P = (0)5/2^+ \) also increases from the 16.34 MeV measured with respect to the \( \Omega np \) threshold [24] to about 20 MeV. Let us emphasize

\(^2\) We have not considered the channel coupling to lower inelastic channels and, therefore, we have not estimated the width of the states having a lower decay channel different from those made of \( N \)'s and \( \Omega \)'s.
TABLE III: Binding energy of the \(^5S_2\) \(\Omega N\) state, \(B_{\Omega N}\), and the \((I)J^P = (0)5/2^+\) \(\Omega d\) state, \(B_{\Omega d}\), for the different models of the \(\Omega N\) interaction given in Table II \[13\]. The results have been obtained with the experimental masses of the \(N\) and \(\Omega\), 938.9 MeV/c\(^2\) and 1672.45 MeV/c\(^2\) respectively. We have indicated between parenthesis the results corresponding to the \(N\) and \(\Omega\) masses derived by the HAL QCD Collaboration, 954.7 MeV/c\(^2\) and 1711.5 MeV/c\(^2\) respectively \[12, 13\]. All energies are in MeV.

|       | \(P_1\)          | \(P_2\)          | \(P_3\)          | \(P_4\)          |
|-------|------------------|------------------|------------------|------------------|
| \(B_{\Omega N}\) | 1.29 (1.52)      | 1.38 (1.61)      | 1.29 (1.44)      | 1.37 (1.60)      |
| \(B_{\Omega d}\)  | 19.6 (20.6)      | 20.0 (21.1)      | 19.6 (20.5)      | 19.9 (20.9)      |

that the \(\Omega d\) in the maximal spin channel \((I)J^P = (0)5/2^+\) cannot couple to the lower channels \(\Lambda \Xi N\) and \(\Sigma \Xi N\) with the \(\Lambda \Xi\) and \(\Sigma \Xi\) subsystems in \(S\) waves, so that the width of a \(\Omega d\) bound state is expected to be small.

Regarding the \(\Omega nn\) (\(\Omega pp\)) system, with the \(\Omega N\) interaction derived in Ref. \[13\] one can construct a \(\Omega nn\) (\(\Omega pp\)) state with quantum numbers \((I)J^P = (1)3/2^+\). The results obtained for this state with the different models of the \(\Omega N\) interaction given in Table II are shown in Table IV. Such \(\Omega nn\) state could decay to \(\Lambda \Xi n\) and \(\Sigma \Xi n\), thus it would appear as a resonance.

We present now the results obtained for the \(\Omega \Omega N\) system using the \(\Omega N\) potential derived in Ref. \[13\] and the \(\Omega \Omega\) interaction reported in Ref. \[12\], both for the same pion mass. The two partial waves analyzed by the HAL QCD Collaboration generate an \(\Omega \Omega N\) three-body system with quantum numbers \((I)J^P = (1/2)1/2^+\). This state is bound for all parametrizations of the \(\Omega N\) interaction, with the binding energy, \(B_{\Omega \Omega N}\), and separation energy, \(S_{\Omega \Omega N}\),

TABLE IV: Binding energy, \(B_{\Omega nn}\), and separation energy, \(S_{\Omega nn}\), of the the \((I)J^P = (1)3/2^+\) state for the different models of the \(\Omega N\) interaction given in Table II \[13\]. All energies are in MeV. The numbers between parenthesis have the same meaning as in Table III.

|       | \(P_1\)          | \(P_2\)          | \(P_3\)          | \(P_4\)          |
|-------|------------------|------------------|------------------|------------------|
| \(B_{\Omega nn}\) | 2.25 (2.60)      | 2.35 (2.72)      | 2.14 (2.50)      | 2.34 (2.71)      |
| \(S_{\Omega nn}\)  | 0.96 (1.08)      | 0.97 (1.11)      | 0.85 (1.06)      | 0.97 (1.11)      |
TABLE V: Binding energy, $B_{\Omega\Omega N}$, and separation energy, $S_{\Omega\Omega N}$, of the the $(I)J^P = (1/2)1/2^+$ $\Omega\Omega N$ state for the different models of the $\Omega N$ interaction given in Table II [13] and the $\Omega\Omega$ interaction of Ref. [12]. All energies are in MeV. The numbers between parenthesis have the same meaning as in Table III.

|     | $P_1$  | $P_2$  | $P_3$  | $P_4$  |
|-----|--------|--------|--------|--------|
| $B_{\Omega\Omega N}$ | 6.0 (6.7) | 6.2 (6.9) | 5.9 (6.6) | 6.1 (6.8) |
| $S_{\Omega\Omega N}$ | 4.6 (5.1) | 4.8 (5.3) | 4.5 (5.0) | 4.7 (5.2) |

given in Table V. Note that the binding energy of the $^1S_0$ $\Omega\Omega$ state is 1.4 MeV for the physical mass of the $\Omega$ baryon, and 1.6 MeV for the HAL QCD mass of the $\Omega$ baryon. This state would also appear as a resonance decaying to $\Lambda\Xi\Omega$ and $\Sigma\Xi\Omega$.

We have calculated the contribution of the Coulomb potential exactly as dictated by Eq. (21), in order to differentiate among the different charged states of the $\Omega N$, $\Omega NN$ and $\Omega\Omega N$ systems. The Coulomb increases the binding for systems containing a proton compared to those with a neutron, due to the attractive $\Omega^- p$ interaction. The Coulomb effects on the binding energy of the $^5S_2$ $\Omega N$ state were estimated in Ref. [13], obtaining a difference of 0.92 MeV between the $\Omega p$ and $\Omega n$ states. We have checked this result for the different models, $P_i$, of the $\Omega N$ interaction by means of the potential given in Eq. (21). We have obtained corrections of: 0.90, 0.92, 0.90 and 0.91 MeV respectively, in agreement with the lattice QCD results. In the $(I)J^P = (1)3/2^+$ $\Omega NN$ system the Coulomb potential induces an extra binding of 0.7 MeV for the $\Omega pp$ state as compared to the $\Omega nn$ one. In the $(I)J^P = (0)5/2^+$ $\Omega d$ state the Coulomb interaction generates an additional binding of about 1.3 MeV. For the $\Omega\Omega$ $^1S_0$ state the Coulomb interaction reduces the binding energy from the 1.6 MeV reported in Ref. [12] to 0.9 MeV. Finally, in the $\Omega\Omega N$ system the Coulomb potential induces an extra binding of 0.3 MeV for the $\Omega\Omega p$ state whereas it penalizes the $\Omega\Omega n$ state by 1 MeV, generating a splitting of 1.3 MeV between these two states. The final results are summarized in Table VI.

Let us finally note that to draw definite conclusion about the width of states with a lower decay channel, one should have done a coupled-channel calculation. However, for this purpose one would need the transition potentials to the inelastic channels that have still not been derived from lattice QCD and thus the conclusions would be speculative.
TABLE VI: Binding energy of the different $\Omega N$, $\Omega NN$ and $\Omega\Omega N$ charged states including the Coulomb potential given by Eq. (21). All energies are in MeV. The numbers between parenthesis have the same meaning as in Table III.

| $(I)J^P$ | System | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|-----------|--------|-------|-------|-------|-------|
| $(1/2)2^+$ | $\Omega n$ | 1.29 (1.52) | 1.38 (1.61) | 1.29 (1.44) | 1.37 (1.60) |
|           | $\Omega p$ | 2.19 (2.42) | 2.30 (2.53) | 2.19 (2.34) | 2.28 (2.51) |
| $(1)3/2^+$ | $\Omega nn$ | 2.25 (2.60) | 2.35 (2.72) | 2.14 (2.50) | 2.34 (2.71) |
|           | $\Omega pp$ | 2.91 (3.30) | 3.04 (3.43) | 2.81 (3.20) | 3.02 (3.41) |
| $(0)5/2^+$ | $\Omega d$ | 20.9 (22.0) | 21.3 (22.4) | 20.7 (21.8) | 21.2 (22.3) |
| $(1/2)1/2^+$ | $\Omega\Omega n$ | 5.0 (5.6) | 5.1 (5.8) | 4.9 (5.5) | 5.1 (5.8) |
|           | $\Omega\Omega p$ | 6.3 (7.0) | 6.5 (7.2) | 6.2 (6.9) | 6.4 (7.2) |

The width of a three-body resonance in a coupled two-channel system has been recently estimated in Ref. [28], presenting a plausible argument to explain the small width of a three-body resonance lying close to the upper channel in spite of being open the lower one. The analysis of the width of a two-body resonance in a coupled-channel system has also been recently presented in Ref. [29]. It has been demonstrated how the width does not come only determined by the available phase space for its decay to the detection channel, but it greatly depends on the relative position of the mass of the resonance with respect to the masses of the coupled-channels generating the state. As seen in Fig. 1 of this reference, the resonance may still be narrow being close to the upper threshold if there is a little overlap with the wave function of the lower inelastic channel. Hence, in the region where the dynamics is dominated by the attraction in the upper channel, the resonance could still be narrow and the lower channel would be mainly a tool for the detection. This mechanism is somewhat related to the ‘synchronization of resonances’ proposed by D. Bugg [30]. Thus, our studies about the width of two- and three-body resonances suggest the possibility of the experimental observation of narrow resonances lying well above their lowest decay threshold.
V. OUTLOOK

The lattice QCD analysis of the HAL QCD Collaboration has recently derived $\Omega N$ and $\Omega\Omega$ interacting potentials with nearly physical quark masses ($m_\pi \simeq 146$ MeV and $m_K \simeq 525$ MeV). They found an attractive potential in the $\Omega N ^5S_2$ channel which supports a bound state with a central binding energy of 1.54 MeV. The $\Omega\Omega ^1S_0$ channel shows an overall attraction with a bound state with a central binding energy of 1.6 MeV. On the basis of our current understanding of the nonstrange sector, where the binding energy may increase with the number of baryons, in this paper we have examined carefully the $\Omega N N$ and $\Omega\Omega N$ three-body systems making use of the latest HAL QCD Collaboration $\Omega N$ and $\Omega\Omega$ interactions. We have looked for deeply bound states or resonances which may be sought experimentally. Our results show that the $\Omega d$ system in the state with maximal spin $(I, J^P) = (0, 5/2^+)$ is bound with a binding energy of about 20 MeV. The $\Omega d$ in the maximal spin channel $(I) J^P = (0) 5/2^+$ cannot couple to the lower channels $\Lambda \Xi N$ and $\Sigma \Xi N$ with the $\Lambda \Xi$ and $\Sigma \Xi$ subsystems in $S$ waves, so that the width of a $\Omega d$ bound state is expected to be small. The $(I, J^P) = (1, 3/2^+)$ $\Omega nn$ state presents a resonance decaying to $\Lambda \Xi n$ and $\Sigma \Xi n$, with a separation energy of $\sim 1$ MeV. The $(I, J^P) = (1/2, 1/2^+)$ $\Omega\Omega N$ state also exhibits a resonance decaying to $\Lambda \Xi \Omega$ and $\Sigma \Xi \Omega$, with a separation energy of $\sim 4.6$ MeV. The Coulomb potential increases the binding for systems containing a proton compared to those with a neutron, due to the attractive $\Omega^- p$ interaction. It also penalizes the binding of the $\Omega\Omega$ state. The overall effect is of the order of 1 MeV, as noticed in the lattice QCD calculations of the two-body systems.

The latest baryon-baryon interactions in the strange sector with nearly physical pion mass based on lattice QCD hint toward the existence of bound states or sharp resonances. These states could be observed by hadron beam experiments at J-PARC and FAIR, or by relativistic heavy-ion collisions at RHIC and LHC. In Ref. [19] it has been discussed how the two-particle momentum correlation between the proton and the $\Omega$ baryon in high-energy heavy ion collisions may unveil the existence of these states. The first measurement of the proton-$\Omega$ correlation function in heavy-ion collisions by the STAR experiment [20] at RHIC favors the proton-$\Omega$ bound state hypothesis. We hope our theoretical studies could help to design experiments where these lattice QCD-based predictions could be tested.
Acknowledgments

This work has been partially funded by COFAA-IPN (México) and by Ministerio de Economía, Industria y Competitividad and EU FEDER under Contract No. FPA2016-77177-C2-2-P.

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