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On the magnetohydrodynamics flows in curved coaxial channels

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Abstract. Stationary plasma flows obtained by using numerical solutions of corresponding stationary problems in curved coaxial channels are considered. It is supposed that plasma interacts with own azimuthal magnetic field. The main attention of the paper is focused on accelerating transonic flows, which attract a considerable interest in the theory of plasma accelerators. It was demonstrated that a curvilinear configuration of a channel makes the significant contribution to distributions of basic plasma parameters.

1. Introduction

1.1. Coaxial plasma accelerators
Research and development of plasma plants attract a considerable interest due to wide range of its applications to solve different technical problems. One of the most promising directions in the theory of plasma plants is the design of plasma accelerators due to high velocities and energies of working medium flows emerging from them. In terms of technical applications these devices can be used to design spaceship engines for cosmic flights and for its orbit adjustment.

Mathematical modeling and numerical simulation of magnetohydrodynamics flows in channels play a considerable role in the successful development of several generations of plasma accelerators [1]. It allows to obtain main regularities in plasma flow properties and thereby to reduce costs of expensive experiments.

For an analysis of a plasma motion in coaxial accelerators mathematical models in which plasma flow parameters depends only on cross section of the channel have been used up to the present moment [2]. Further development of the theory and expansion of perspectives of its applications is the construction and calculation of mathematical model of much less studied plasma flows in curved coaxial channels (figure 1). Some issues of the problem were discussed in [3], where, particularly, the first integrals of the stationary system of differential equations describing the process were obtained. In [4] plasma and gas motions in the channel in the form of a nozzle with curved upper and lower electrodes...
were considered. Particularly, a significant impact of the curvilinear configuration of the plant on the working fluid was noticed in this paper.

Figure 1. Scheme of the curved coaxial channel.

1.2. **Objective of the work**

The objective of the present work is the modeling and calculating of the accelerated dense plasma flows in curved coaxial channels in its interaction with own azimuthal magnetic field.

The basic regularities can be obtained by using a quasi-one-dimensional (hydraulic) approximation. The analysis is based on the solving of the corresponding stationary MHD problems subject to the boundary conditions. Neglecting, as usual in this class of physical problems, plasma thermal conductivity and viscosity, and assuming its electric conductivity is infinite, the mathematics of the MHD model is reduced to the algebraic equation – consequence of the first integrals of the stationary system of the differential equations which represent energy conservation laws.

2. **Mathematical model**

2.1. **Quasi-one-dimensional (hydraulic) approximation**

Plasma with specified density $\rho_0$, temperature $T_0$, and hence, with the pressure $P_0 = C_v (\gamma - 1) \rho_0 T_0$, where $\gamma$ – adiabatic index, $C_v$ – thermal capacity at constant volume, enters the channel through its input section. It is natural to assume that the plasma enters the channel without rotation, i.e. its azimuthal velocity $\nu_\phi$ at the channel entrance equals zero. The value of full electric current $J$ in the system which flows through the left end of the accelerator is also assumed to be known. It means that the azimuthal magnetic field $H_\phi$ is set in the channel input section.

In the present paper, the problem is considered in the quasi-one-dimensional (hydraulic) approximation. It is assumed that distance $h(z)$ between the electrodes is small, and all sought-for quantities are averaged over the cross section of the channel $\bar{S}(z) = \pi (R(z) + h(z))^2 - \pi R(z)^2 \approx 2\pi R(z)h(z)$, where $R(z)$ – lower electrode configuration. It will be convenient to introduce $\bar{h}(z) = 2\pi R(z)\bar{h}(z)$, where $\bar{h}(z) = h(z) \cos \alpha = h(z) \left(1 + \tan^2 \alpha \right)^{-1/2}$.

The tangent of angle $\alpha$ is determined as derivative $R'(z)$ at a given place of the lower electrode (figure 1).

The electrodes forming the side walls of the channel are assumed to be impervious and equipotential, and the channel is formed by the trajectories $\nu_n = 0$. In mathematical language it is expressed in the following form:

$\nu_r / \nu_z = r' / z = \tan \alpha$, where $\nu_r$ – radial component of the plasma velocity vector, $\nu_z$ – its longitudinal component.
Plasma velocity along the channel attracts physical interest for the analysis, so it also will be convenient to introduce $\tilde{v} = \left( v_2^2 + v_z^2 \right)^{1/2} = v_z \left( 1 + R^2 \right)^{1/2}$. 

2.2. Stationary equations

In the quasi-one-dimensional approximation in the dimensionless form, neglecting aforementioned dissipative effects, stationary plasma flows in curved coaxial channels in its interaction with own azimuthal magnetic field and in the absence of the rotation at the channel entrance are described by the following ordinary differential equations resolved relatively to derivatives:

$$
\begin{align*}
\frac{d\rho}{dz} &= -\frac{\rho}{\tilde{v}^2 - C_f^2} \left[ \tilde{v}^2 - C_\phi^2 - 2C_\phi^2 \frac{R'}{R} \right] = -\frac{\rho}{\tilde{v}^2 - C_f^2} \left[ \tilde{v}^2 \frac{\tilde{h}'}{h} + \left( \tilde{v}^2 - 2C_\phi^2 \right) \frac{R'}{R} \right] \\
\frac{dH_\phi}{dz} &= -\frac{H_\phi}{\tilde{v}^2 - C_f^2} \left[ \tilde{v}^2 \frac{\tilde{S}'}{S} + \left( C_\phi^2 - C_\phi^2 - \tilde{v}^2 \right) \frac{R'}{R} \right] = -\frac{H_\phi}{\tilde{v}^2 - C_f^2} \left[ \tilde{v}^2 \frac{\tilde{h}'}{h} + \left( C_\phi^2 - C_\phi^2 \right) \frac{R'}{R} \right] \\
\frac{d\tilde{v}}{dz} &= \frac{\tilde{v}}{\tilde{v}^2 - C_f^2} \left[ \left( C_\phi^2 + C_\phi^2 \right) \frac{\tilde{S}'}{S} - 2C_\phi^2 \frac{R'}{R} \right] = \frac{\tilde{v}}{\tilde{v}^2 - C_f^2} \left[ \left( C_\phi^2 + C_\phi^2 \right) \frac{\tilde{h}'}{h} + \left( C_\phi^2 - C_\phi^2 \right) \frac{R'}{R} \right]
\end{align*}
$$

where

- $C_\phi = H_\phi \rho^{1/2}$ – Alfvén velocity calculated by an azimuthal magnetic field;
- $C_T = \left( \gamma P / \rho \right)^{1/2}$ – gas-dynamic (thermal) sound velocity;
- $\tilde{C}_f^2 = C_f^2 + C_\phi^2$ – fast magnetosonic velocity square.

The units of measurement are the dimensional quantities participating in the formulation of the problem:

$$
\rho_U = \rho_0, \, z_U = L, \, S_U = S(0), \, T_U = T_0, \, H_U = H_{\phi 0}, \, \tilde{v}_U = H_{\phi 0}^2 / 4\pi \rho_0, \, P_U = H_{\phi 0}^2 / 4\pi
$$

where $L$ – the length of the channel.

The problem also includes the dimensionless parameter $\beta = 8\pi P_0 / H_{\phi 0}^2$ traditionally participating in MHD. It shows, how many times a magnetic pressure differs from a gas-dynamic pressure.

The analysis of the equations (1) allows to make the conclusion that the curvilinear configuration of the channel enables the influence of the correlation between gas-dynamic (defined by $C_T$) and electromagnetic (defined by $C_\phi$) flow properties. In terms of mathematics this assertion is manifested in the existence of the additional summands in the nominators of the basic plasma flow parameters.

2.3. The first integrals

Stationary system of MHD equations (1) has full set of the first integrals – laws of conservation:

$$
C_1 = \rho v_z S; \, C_2 = \frac{P}{\rho^\gamma}; \, C_3 = C_1 v_\phi R; \, C_4 = \frac{C_1 H_\phi}{R \rho}; \, C_5 = \frac{\tilde{v}^2 + v_\phi^2}{2} + \frac{\gamma - 1}{\gamma - 1} \frac{P}{\rho} + \frac{C_4 H_\phi R}{C_1} \tag{3}
$$

Constant $C_1$ is proportional to the longitudinal plasma velocity in the input section of the channel $v_{z0}$, which is defined based on the condition of a transonic accelerated flow regime receiving. Values of other constants are determined by the boundary conditions in the input section of the channel.

We can express from the equations (3) all plasma parameters via its density $\rho$ and obtain one algebraic equation which defines $\rho$ as a function of $z$ and $C_1$: 

$\text{Here comes the equation}$
\[ \Phi(\rho, z, C_i) = \rho^2 \left( C_5 - \frac{\gamma}{\gamma-1} C_2 \rho^{\gamma-1} - \frac{C_i^2 \rho R^2(z)}{C_i} \right) - \frac{C_i^2}{2S^2(z)} = 0 \] (4)

The solution of the equation (4) completely describes a quasi-one-dimensional stationary MHD flow in case of infinite plasma conductivity.

2.4. Method of calculation

The following algorithm is used as a numerical method of calculation of stationary transonic accelerated plasma flows in curved coaxial channels:

1. In order to determine plasma density in the transition point in the center of the channel and the plasma stream inflow velocity in which the studied transonic flow regime is obtained, the following system of equations is numerically solved:

\[ \begin{align*}
\Phi(\rho_{0.5}, 0.5, \nu_{z_0}) &= 0 \\
\frac{\partial \Phi}{\partial \rho}(\rho_{0.5}, 0.5, \nu_{z_0}) &= 0
\end{align*} \] (5)

2. By defining inflow velocity, the problem of forming the plasma flow density distribution \( \rho(z) \) along the channel is numerically solved by using the equation \( \Phi(\rho, z, \nu_{z_0}) = 0 \).

3. The other sought-for plasma parameters distributions along the channel, namely, \( \nu(z) \) and \( H_{\phi}(z) \), are built using the first integrals equations (3) and plasma density distribution \( \rho(z) \) obtained.

3. Results

In order to analyze plasma flows in real plasma plants without using two-dimensional calculations, it is expedient to subdivide the channel area into thin tubes. The quasi-one-dimensional mathematical model of the plasma flow motion in curved thin tubes was created in the present paper. The tubes configuration into which the channel is subdivided is completely defined by the configurations of upper and lower electrodes of the plant.

It is important to note that the boundary condition on the azimuthal magnetic field and the value of the dimensionless parameter \( \beta \) are individual for each tube because of the condition \( H_{\phi}r = const \) in the two-dimensional problem. At the time boundary conditions in the input section of each tube have the following form:

\[ \rho = 1, T = 1, \nu_{\rho_0} = 0, H_{\phi_0} = \frac{r_0}{R(0)}, \] (6)

where \( r_0 \) – plant characteristic radius and \( R(0) \) – configuration of the lower boundary of the tube at the entrance of the channel. At the same time we choose \( \beta = 0.1 \left( R(0)/r_0 \right)^2 \), because electromagnetic properties exceed gas-dynamic properties in real plasma plants.

3.1. Plasma accelerators with curved upper electrode

We consider plasma accelerator with the following configuration:

\[ electrode_{upper} = 1.2(z - 0.5)^2 + 0.4; electrode_{lower} = 0.3; r_0 = 0.5 \] (7)

In order to analyze transonic accelerated plasma flows, the accelerator area is subdivided into thin tubes corresponding to the flow streams (figure 2). Parameter \( a \) defines a tube curvature and its location in the channel area.
In figure 3, the distributions of the basic plasma parameters in the transonic accelerated flow regime in the thin tubes corresponding to the different parts of plasma accelerator with curved upper electrode are represented.

Plasma compression zone is formed and then is amplified during the transfer from lower electrode to curved upper electrode: the maximum of the plasma density distribution occurs and then increases moving to the channel center. The outflow plasma density is also increases. Accelerating characteristics of the channel get worse: outflow plasma velocity decreases. During the transfer from lower electrode to curved upper electrode inverse currents in plasma are generated because of forming regions of intensity rise. At the same time the full electric current quantity defined by the difference between output and input azimuthal magnetic field intensity values decreases.

3.2. Plasma accelerators with curved lower electrode
We consider plasma accelerator with the following configuration (figure 4):

\[
electrode_{\text{upper}} = 0.5, \quad electrode_{\text{lower}} = -0.8(z - 0.5)^2 + 0.3, \quad r_0 = 0.3
\]  

Plasma compression zone is formed and then is amplified during the transfer from lower electrode to curved upper electrode: the maximum of the plasma density distribution occurs and then increases moving to the channel center. The outflow plasma density is also increases. Accelerating characteristics of the channel get worse: outflow plasma velocity decreases. During the transfer from lower electrode to curved upper electrode inverse currents in plasma are generated because of forming regions of intensity rise. At the same time the full electric current quantity defined by the difference between output and input azimuthal magnetic field intensity values decreases.
In figure 5, the distributions of the basic plasma parameters in the transonic accelerated flow regime in the thin tubes corresponding to the different parts of plasma accelerator with curved lower electrode are given.

Figure 5. Distributions of basic plasma parameters in the set of thin tubes in plasma accelerator with curved lower electrode.

Plasma density decreasing, and hence, thermal energy consumption become more intensive during the transfer from upper electrode to curved lower electrode. At the same time ill-defined zone of plasma rarefaction occurs near the lower electrode. Azimuthal magnetic field intensity drop, and hence, electromagnetic energy consumption increase because of input magnetic field intensity augmentation. Along with the increasing of electromagnetic pressure influence, the considered regularities improve accelerating characteristics of the channel: plasma is accelerated significantly more intensive near the lower electrode.

4. Conclusion

- In this work, a quasi-one-dimensional stationary MHD model of a plasma flow in curved narrow coaxial channels in its interaction with own azimuthal magnetic field has been considered and realized.
- It has been established that a curvilinear geometry of a channel enables the influence of the correlation between gas-dynamic and electromagnetic properties of a process on the flow parameters.
- By using method of the subdivision of a plasma accelerator area into thin tubes, it has been demonstrated that plants with a curved lower electrode is preferable to plants with a curved upper electrode in terms of the acceleration effectiveness.
- It has been established that in curved channels in contrast to channels with constant average radius, depending on a channel geometry and a correlation between gas-dynamic and electromagnetic pressures, areas of plasma compression and rarefaction, areas of the electric current generation and inverse currents can occur in a transonic accelerated plasma flow regime.

Acknowledgments

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