Nuclear deformation and the two-neutrino double-β decay in 124,126Xe, 128,130Te, 130,132Ba and 150Nd isotopes

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Abstract. The two-neutrino double-beta decay of 124,126Xe, 128,130Te, 130,132Ba and 150Nd isotopes is studied in the Projected Hartree-Fock-Bogoliubov (PHFB) model. Theoretical 2ν β−β− half-lives of 128,130Te, and 150Nd isotopes, and 2ν β+β+, 2ν β+EC and 2ν ECEC for 124,126Xe and 130,132Ba nuclei are presented. Calculated quadrupolar transition probabilities B(E2; 0+ → 2+) static quadrupole moments and g-factors in the parent and daughter nuclei reproduce the experimental information, validating the reliability of the model wave functions. The anticorrelation between nuclear deformation and the nuclear transition matrix element M2ν is confirmed.

PACS. 23.40.Hc Relation with nuclear matrix elements and nuclear structure – 21.60.Jz Hartree-Fock and random-phase approximations – 23.20.-g Electromagnetic transitions – 27.60. +j 90 \leq A \leq 149

1 Introduction

The nuclear double-beta decay (ββ) is a weak process in which two neutrons (protons) inside a nucleus decay into two protons (neutrons), emitting two electrons (positrons) [1,2]. In the lepton-number-conserving process, namely, two-neutrino double-beta (2ν ββ) decay two antineutrinos (neutrinos) are also emitted [3–8]. It has been observed in several nuclei [9–13]. Given that lepton number is not a gauge symmetry, several extensions of the standard model predict its violation. In this neutrinoless double-beta (0ν ββ) decay no neutrinos are emitted. This 0ν ββ decay has never been observed, although controversial evidence has been published [14]. If positively detected, it would provide unique information about the Majorana mass of the neutrino, and for the absolute scale of neutrino masses [15–20].

Theoretical calculations of ββ decay represent one of the hardest challenges ever faced in nuclear physics. The process is definitively non-collective, and proceed through strongly suppressed channels, which are very sensitive to details of the wave functions of the parent and daughter nuclei. While specific models have been built which are able to adjust (postdict) most of the observed 2ν ββ decay half-lives, they usually have parameters which allow different results as well, making their predictions unreliable.

A case-by-case analysis, studying each nuclei with the best available model, and describing as much nuclear observables as possible within the same theoretical scheme, seems to be a sensible approach [21–23].

The 2ν ββ decay can occur in four different modes, namely double-electron (β+β−) emission, double-positron (β+β+) emission, electron-positron conversion (β+EC) and double-electron capture (ECEC). The later three processes are energetically competing and we shall refer to them all generically as positron double-beta decay (e+DBD) modes. Nuclear matrix elements (NMEs) M2ν associated with the 2ν ββ decay can be extracted directly from the observed half-lives of eleven nuclei undergoing 2ν β−β− decay out of 35 possible candidates [13]. In case of 2ν e+DBD modes, experimental limits on half-lives have already been given for 24 out of 34 possible isotopes [13].

In all ββ decay emitters, which are even-Z and even-N nuclei, the pairing of like nucleons plays a fundamental role, energetically inhibiting the decay to the intermediate odd-odd nuclei. The quadrupole-quadrupole interaction drives nuclei to deformed shapes when both proton and neutron shells are open. The interplay between this two major components of the effective nuclear interaction can generate quite complex energy spectra. In the mass region A ≈ 130 it can be seen that, while Te isotopes have a vibrational excitation spectra, Xe and Ba isotopes develop rotational bands. The mass region A ~ 150 of-
fers an example of shape transition, i.e. the sudden onset of deformation at neutron number $N = 90$. Nuclei range from spherical to well deformed, with large static quadrupole moments. Thus, it is expected that pairing and deformation degrees of freedom will play some crucial role in the structure of $^{124,126,128,130,132}$Xe, $^{124,126,128,130}$Te, $^{130,132}$Ba, $^{150}$Nd and $^{150}$Sn nuclei. The $\beta\beta$ decay can be studied in the same framework as many other nuclear properties and decays. A vast amount of data has been collected over the past years concerning the level energies as well as electromagnetic properties through experimental studies involving in-beam $\gamma$-ray spectroscopy. The availability of data permits a rigorous and detailed critique of the ingredients of the microscopic model that seeks to provide a description of nuclear $\beta\beta$ decay.

Theoretical studies predict that deformation plays a crucial role in case of $2\nu\beta^{-}\beta^{-}$ decay of $^{100}$Mo and $^{150}$Nd [24,25]. Auerbach et al. [26] and Troletien et al. [27] have already shown that there exists an inverse correlation between the Gamow-Teller (GT) strength and quadrupole moment. The effect of deformation on the distribution of the Gamow-Teller and $\beta$-decay properties has been studied using a quasiparticle Tamm-Dancoff approximation (TDA) based on deformed Hartree-Fock (DHF) calculations with Skyrme interactions [28], a deformed self-consistent HF+RPA method with Skyrme-type interactions [29]. The comparison of the experimental GT strength distribution $B(\text{GT})$ from its decay with the results of QRPA calculations was employed as a novel method of deducing the deformation of the $N = Z$ nucleus $^{76}$Sr [30]. The effect of deformation on the $2\nu\beta^{-}\beta^{-}$ decay for ground-state transition $^{76}$Ge $\rightarrow$ $^{76}$Se was studied in the framework of the deformed QRPA with separable GT residual interaction [31]. A deformed QRPA formalism to describe simultaneously the energy distributions of the single-beta GT strength and the $2\nu\beta$ decay matrix elements, using deformed Woods-Saxon potentials and deformed Skyrme-Hartree-Fock mean fields was developed [32].

In all these works calculations are performed in the intrinsic basis, where angular momentum is not a good quantum number. The Projected Hartree-Fock-Bogoliubov (PHFB) model offers, in this sense, a competitive alternative. On one hand, the PHFB model restores the rotational symmetry, providing very reliable wave functions for the parent and daughter $\beta\beta$ decay emitters. On the other hand, in its present version the PHFB model is unable to provide information about the structure of the intermediate odd-odd nuclei, and, in particular, on the single-$\beta$ decay rates and the distribution of Gamow-Teller strength. Notwithstanding, the PHFB model has been successfully applied to the $2\nu\beta^{-}\beta^{-}$ decay of many emitters in the mass region $A \sim 100$, where it was possible to describe, in the same context, the lowest excited states of the parent and daughter nuclei, as well as their electromagnetic transition strengths, and to reproduce their measured $\beta\beta$ decay rates on the other [33].

A quantitative description of the $2\nu\beta^{-}\beta^{-}$ decay [33] and $2\nu e^{+}$DBD modes [34] of nuclei in the mass region $A \sim 100$ for the $0^{+} \rightarrow 0^{+}$ transition has been obtained. The same formalism allowed the analysis of other observed nuclear properties, including the yrast spectra, the reduced transition probabilities $B(E2: 0^{+} \rightarrow 2^{+})$, the static quadrupole moments $Q(2^{+})$ and the $g$-factors $g(2^{+})$ of both parent and daughter nuclei. The study was performed using the PHFB model. Its application, in conjunction with the summation method, has motivated the present study of the $2\nu\beta^{-}\beta^{-}$ decay of $^{128,130}$Te, $^{150}$Nd and $2\nu e^{+}$DBD modes of $^{124,126}$Xe and $^{130,132}$Ba isotopes. The $2\nu e^{+}$DBD modes of $^{124,126}$Xe and $^{130,132}$Ba isotopes were studied in the earlier work of Shukla et al. [35]. In the present work, the HFB wave functions are generated with higher accuracy and in these nuclei, it is observed that the results are very much dependent on the later.

By using the pairing plus quadrupole-quadrupole $PPQQ$ interaction [36] the interplay between sphericity and deformation can be studied. In this way, the PHFB formalism, employed in conjunction with the $PPQQ$ interaction, is a convenient choice to examine the explicit role of deformation on the NTME $M_{2\nu}$. A strong dependence of the $M_{2\nu}$ on the quadrupole deformation was found varying the strength of the quadrupole-quadrupole interaction, for the $2\nu\beta^{-}\beta^{-}$ decay [33] and $2\nu e^{+}$DBD modes [34] of nuclei in the mass region $A \sim 100$. The anticorrelation between the quadrupolar deformation and the $\beta\beta$ decay transition amplitude is analyzed in detail.

The present paper is organized as follows. The theoretical formalism to calculate the half-lives of $2\nu\beta\beta$ modes has been given in a number of reviews [15,18]. Details of the PHFB study were presented in previous publications, for $2\nu\beta^{-}\beta^{-}$ decay [33] and $2\nu e^{+}$DBD modes [34] of nuclei in the mass region $A \sim 100$. Details of the mathematical expressions used to calculate the spectroscopic properties of nuclei in the PHFB model have been given by Dixit et al. [37]. Here, we briefly outline the relevant results in sect. 2. In sect. 3, we present a detailed study of the wave functions of $^{124,126,128,130,132}$Xe, $^{124,126,128,130}$Te, $^{130,132}$Ba, $^{150}$Nd and $^{150}$Sm nuclei, calculating the yrast spectra, reduced $B(E2: 0^{+} \rightarrow 2^{+})$ transition probabilities, static quadrupole moments $Q(2^{+})$ and $g$-factors $g(2^{+})$ and comparing them with the available experimental data. The half-lives $T_{1/2}^{2\nu}$ for the $2\nu\beta^{-}\beta^{-}$ decay of $^{128,130}$Te, $^{150}$Nd and $2\nu e^{+}$DBD modes of $^{124,126}$Xe and $^{130,132}$Ba isotopes for the $0^{+} \rightarrow 0^{+}$ transition are calculated. The role of deformation on NTMEs $M_{2\nu}$ is also studied. We present some concluding remarks in sect. 4.

2 Theoretical framework

The inverse half-life of the $2\nu\beta\beta$ decay modes for the $0^{+} \rightarrow 0^{+}$ transition is given by

$$T_{1/2}^{2\nu}(k)^{-1} = G_{2\nu}(k)M_{2\nu}(k)^{2},$$

where $k$ denotes the $\beta^{-}\beta^{-}$ and $e^{+}$DBD modes. The integrated kinematical factor $G_{2\nu}(k)$ can be calculated with