Gravitational-wave detectors as particle-physics laboratories: Constraining scalar interactions with boson-star binaries

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Gravitational-wave (GW) detections of binary neutron star coalescences play a crucial role to constrain the microscopic interaction of matter at ultrahigh density. Similarly, if boson stars exist in the universe their coalescence can be used to constrain the fundamental coupling constants of a scalar field theory. We develop the first coherent waveform model for the inspiral of boson stars with quartic interactions. The waveform includes coherently spin-induced quadrupolar and tidal-deformability contributions in terms of the masses and spins of the binary and of a single coupling constant of the theory. We show that future instruments such as the Einstein Telescope and LISA can provide strong, complementary bounds on bosonic self-interactions, while the constraining power of current detectors is marginal.

I. INTRODUCTION

Gravitational-wave (GW) measurements of the tidal deformability of neutron stars (NSs) have opened a new window to study the properties of matter beyond the nuclear saturation point within stellar cores [1, 2]. Equations of state with different stiffness provide tidal deformabilities which may vary up to an order of magnitude. This effect magnifies the details of the underlying microscopic model, allowing to probe how fundamental interactions behave in extreme regimes [3, 4].

In this paper, we argue that the very same situation can occur if boson stars (BSs) [5, 6] exist in the Universe and form coalescing binaries within the horizon of current and future detectors. BSs are self-gravitating condensates of a bosonic field (see Refs. [7, 8] for some reviews). In their original and simplest proposal, they are solutions to Einstein gravity minimally coupled to a scalar field theory for a complex scalar φ:

\[ \mathcal{L}_{\text{scalar}} = \frac{1}{2} \partial_\mu \phi^* \partial^\mu \phi + V(\phi), \tag{1} \]

where a star denotes complex conjugation and V is the scalar self potential. The latter plays the same role as the equation of state for NSs: different microscopic interactions give rise to macroscopically different properties of the boson stars.

Depending on the mass of the boson field and on the self-interaction terms, BSs can exist in any mass range and can have a compactness comparable to or larger than that of a NS. It is intriguing that current GW measurements cannot exclude the existence of exotic compact objects other than black holes (BHs) and NSs, especially for GW events in the low-mass [9] and high-mass gap, where neither BHs nor NSs are predicted in the standard scenario.

As a case study, in this paper we consider a simple class of quartic interactions [see Eq. (3)] and quantify the accuracy within which a GW detection of a BS coalescence can constrain the fundamental parameters (boson mass and coupling constants) of a given scalar field theory.

We focus on the inspiral phase, which can be accurately modeled with post-Newtonian (PN) theory [10, 11]. Up to 1.5 PN order (see below), the GW signal depends only on the masses and spins of the binary components and is therefore oblivious to the nature of the latter. However, the details of the coalescing bodies appears at higher PN order, notably through the effects of the spin-induced quadrupole moment (if the binary is spinning) [11, 12], a small tidal-heating term (if at least one of the binary components is a BH or can efficiently absorb radiation) [13–15], and most importantly through the tidal deformability contribution (the so-called tidal Love numbers [11, 16, 17]) that becomes increasingly more relevant during the late stages of the inspiral and merger, as in the case of a binary NS coalescence (see, e.g., Refs. [2, 18] for some recent reviews).

Previous work considered the aforementioned effects independently and included in the waveform only a single effect at the time, focusing on the detectability of the tidal Love number [19, 20], or of the spin-induced quadrupole moment [21–24], or of the tidal heating alone [15, 25, 26]. However, this approach neglects a crucial ingredient: for a given scalar field theory (i.e., fixing the potential in the Lagrangian (1)), both the tidal Love number and the spin quadrupole moment depend only on the masses and spins of the binary. Therefore, their concurrent inclusion does not increase the number of waveform parameters and alleviates their degeneracy.

As we shall show, the sensitivity of current detectors such as LIGO and Virgo is not sufficient to place stringent constraints on the coupling constants of the theory. However, future facilities will provide much more stringent measurements. In particular we consider the future Laser Interferometer Space Antenna (LISA) [27], which can potentially detect supermassive binary BSs, and the Einstein Telescope (ET) [28], a proposed third-generation [29] ground-based GW detector. Putative de-
II. BS BINARIES AS GW SOURCES

A. Equilibrium configurations

BSs are self-gravitating configurations of a complex scalar field described by the action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi} - \mathcal{L}_{\text{scalar}} \right]. \]  

The shape of the potential in Eq. (1) determines the properties of the corresponding star. For a simple Klein-Gordon potential \( V = \frac{\mu^2}{2} |\phi|^2 \), initially considered in [5, 6], BSs have a maximum mass which scales with the boson mass \( m_S \equiv \mu \hbar \) as \( M_{\text{max}} \sim \frac{M_p^2}{m_S} \). Unless the scalar field is ultralight \( (m_S \ll 10^{-11} \text{ eV}) \), the maximum mass is much smaller than the Chandrasekhar mass for compact astrophysical bodies, and therefore such objects are often called "mini" BSs. In this work we shall focus on the case of a quartic potential

\[ V(|\phi|) = \frac{\mu^2}{2} |\phi|^2 + \frac{\lambda}{4} |\phi|^4. \]  

In the strong-coupling limit, \( \lambda \gg \mu^2 \), the maximum mass of a static BS scales as [32]

\[ M_{\text{max}} \approx 0.06 \frac{\sqrt{\lambda} \hbar}{m_S} M_p^3 \approx 10^5 M_\odot \sqrt{\lambda \hbar} \left( \frac{\text{MeV}}{m_S} \right)^2. \]  

Note that, if \( \lambda \hbar = 1 \), varying the boson mass from a few percent of a MeV up to a hundred MeV, one can cover the whole spectrum of physical BH masses1. Hereafter we shall focus only on this regime, which allows for stellar-mass BSs (with mass \( M = \mathcal{O}(10 M_\odot) \)) when \( m_S = \mathcal{O}(10^2 \text{ MeV}) \), and for supermassive BSs (with mass \( M = \mathcal{O}(10^5 M_\odot) \)) when \( m_S = \mathcal{O}(\text{MeV}) \).

While the boson mass sets the scale of the system, in the strong-coupling limit all properties of the BS depend only on the following combination of the fundamental constants appearing in the action (2),

\[ M_B \equiv \sqrt{\frac{\lambda}{\mu^2}} = \sqrt{\lambda \hbar} \frac{M_p^3}{m_S^3}, \]

which has the dimension of a mass in our units.

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1 Following Refs. [31, 33], it is also interesting to note that cosmological observations seem to suggest an interacting dark-matter component, with a cross section per unit mass \( 0.1 \text{cm}^2/\text{g} \lesssim \sigma/\text{cm}^2/\text{g} \lesssim 1 \text{cm}^2/\text{g} \), which, for the quartic potential (3), translates to [31]

\[ \lambda \hbar \sim \left( \frac{m_S}{100 \text{ MeV}} \right)^{3/2}. \]

When \( m_S \sim 10^3 \text{ eV} \), this corresponds to supermassive BSs with \( M \sim 10^3 M_\odot \) (see Fig. 1).
B. GW signatures

The structure of a BS differs from that of a BH in several respects, which introduce distinctive features in the gravitational waveforms from coalescing BSs. The main differences can be summarized as follows [34]:

i. BSs are less compact than BHs, as measured by their compactness $C = M/R$, where $R$ is the BS effective radius. Nonspinning BHs have a compactness $C = M/R = 1/2$, while massive BSs have a maximum compactness $C_{\text{max}} \approx 0.158$ [33], which is comparable to the typical compactness of a NS. This also implies that the “contact” frequency of a binary BS is lower than in the binary BH case. For an equal-mass binary, the GW contact frequency in the point-particle limit reads

$$f_{\text{contact}} = \frac{1}{2\sqrt{\pi}M_i}C^{3/2}$$

$$\approx 1 \text{ mHz} \left(\frac{10^6 M_\odot}{M}\right) \left(\frac{C}{0.15}\right)^{3/2},$$

(7)

which for $C \sim 0.15$ is similar to the frequency of the innermost stable circular orbit (ISCO) of a Schwarzschild BH with mass equal to the total mass $M_i = 2M$ of the binary, $f_{\text{ISCO}} = (6^{3/2}\pi M_i)^{-1}$. The latter can be approximately assumed as the transition frequency between the inspiral and the merger in the binary BH case.

ii. The spin $J$ of a BS is quantized\(^2\) in units of its Noether charge, the latter existing due to the $U(1)$ symmetry of the Lagrangian (1). Nonetheless, in the strong-coupling limit the quantization levels are extremely close to each other, and in practice the dimensionless spin $\chi = J/M^2$ can be treated as a continuous parameter [36].

iii. BSs have a nonvanishing tidal deformability, which expresses the tendency of the object to deform under the action of an external tidal field. In contrast, the tidal deformability – as measured by the tidal Love numbers – of a nonspinning BH is zero\(^4\) [41, 42].

\(2\) At variance with NSs, BSs do not have a hard surface, as the scalar field is spread out all over the radial direction. However, it decays exponentially and the configuration is highly localized in a radius $\sim 1/\mu$. It is customary to define the effective radius $R$ as the radius within which the 99% of the total mass is contained.

\(3\) It has been recently shown that spinning mini BSs made of a scalar field are unstable and decay to their nonspinning state [35]. The instability occurs on dynamical time scales, at least for large compactness. It is unknown whether self-interactions can cure this instability or make it phenomenologically irrelevant.

\(4\) While this property holds true also for slowly spinning BHs in the axisymmetric case [37–39], it has been recently argued – using analytical continuation methods – that the tidal Love numbers of a spinning BH are nonzero in the non-axisymmetric case [40]. Whether and how this affects the gravitational waveform is still an open question. At any rate, even if the tidal deformability of spinning BHs affects the waveform, its (small) fixed value can be used as a baseline for null-hypothesis tests, like the case of the spin-induced quadrupole.

iv. The higher multipoles of a BH are uniquely determined by its mass $M$ and its dimensionless spin $\chi$. In particular, the quadrupole moment of a Kerr BH is $Q_{\text{Kerr}} = -M^2 \chi^2$. On the other hand, as we shall review in Sec. II B 2, the multipole moments of a BS are not necessarily quadratic in the spin and also vary with the compactness.

v. GWs interact weakly with the scalar field, so that effectively BSs do not absorb gravitational radiation. Therefore, at variance with BHs, tidal heating is absent [15, 25].

In brief, the GW signatures of a BS are similar to those of a NS (see Refs. [2, 18] for some reviews), despite the fact that BSs can be supermassive (and therefore are potential exotic sources also for space-based detectors) and, in principle, highly spinning.

In the next subsections we shall review the above properties more quantitatively, and provide useful fits for the quantities that constitute the basic ingredients for the PN waveform model that we shall later use.

1. Tidal deformability

The tidal deformability of (static) BSs was computed in [19, 20], which investigated the possibility of using GW measurements of tidal effects to distinguish BSs from BHs and from NSs (see also Ref. [43]). The (dimensionful) tidal deformability parameter $\Lambda_T$ is defined by [16]

$$Q_{ij} = -\lambda T E_{ij}$$

(8)

where $E_{ij}$ is the external tidal field and $Q_{ij}$ is the induced asymptotic quadrupole moment. It is more convenient to work with the dimensionless tidal deformability (hereafter simply tidal deformability) $\Lambda = \lambda T/M^5$, where $M$ is the mass of the object. A fitting formula for $\Lambda$ in terms of the mass $M$ of the BS and of the parameters $\mu$ and $\lambda$ was obtained in Ref. [20]. In the strong coupling limit, the fit reduces to\(^5\)

$$\frac{M}{M_B} = \frac{\sqrt{7}}{8\sqrt{\pi}} \left[-0.828 + \frac{20.99}{\log \Lambda} - \frac{99.1}{(\log \Lambda)^2} + \frac{149.7}{(\log \Lambda)^3}\right].$$

(9)

The above expression can be inverted to find $\Lambda = \Lambda(M_B, M)$. Note that Eq. (9) has a stationary point for

\(5\) Notice that the scalar Lagrangian of [20] differs from our Eq. (2) by a factor of 2, which we took into account when rewriting Eq. (9).
\( \Lambda \approx 289 \), corresponding to \( M/M_B \approx 0.0611 \), which is in agreement with Eq. (4) for the maximum mass. Therefore, the tidal deformability of a massive BS is bounded from below by \( \Lambda \gtrsim 289 \). Notice that \( \Lambda \) spans many orders of magnitude as the mass deviates from its maximum value: for example, when \( M/M_B = 0.02 \), \( \Lambda \approx 1.7 \times 10^6 \). As we show in Appendix A, this is related to the fact that \( \Lambda \) can be expressed in the form \( \Lambda \sim k_2/C^3 \), where \( k_2 = \mathcal{O}(0.01-0.1) \) is a numerical factor [19], and the compactness \( C \) can be as low as 0.03 when \( M/M_B = 0.02 \).

2. Spin effects and quadrupole moment

While \( Q_{ij} \) in Eq. (8) is a tidal-induced quadrupole moment, a spinning self-gravitating body has a spin-induced quadrupole moment [44]. An analysis of spinning BSs in the strong-coupling limit was carried out in Ref. [36]. The latter contains several important results, which we briefly summarize:

i. The maximum mass of a rotating BS is higher than in the nonspinning case. In Appendix A we show that \( M_{\text{max}} \) as a function of the spin is very well approximated by

\[
M_{\text{max}} \approx 0.06 \left(1 + 0.76 \chi^2 \right) M_B. \tag{10}
\]

ii. In the axisymmetric case, the spin-induced quadrupole moment \( Q_{ij} \) can be written in terms of a single scalar quantity \( Q \), which for a BS can be parametrized as

\[
Q = -\kappa(\chi, M/M_B) \chi^2 M^3. \tag{11}
\]

At variance with the BH case, the function \( \kappa \) is not a constant (\( \kappa_{\text{BH}} = 1 \)), but it depends on the spin and on the BS mass through the ratio \( M/M_B \). The quantity \( \kappa \) is shown in Fig. 2 as a function of the dimensionless spin \( \chi \) for some representative values of \( M/M_B \). It is worth noticing that \( \kappa \) depends on the parameters of the potential only through the combination \( M_B \) defined in (6), as it is also the case for \( \Lambda \). Hereafter we will refer to \( \kappa \) as the “reduced quadrupole moment”. A similar behaviour is exhibited by the spin-induced octupole moment [36]. In the following, we will neglect this effect since it affects the GW waveform to higher PN order.

For a BH, \( \Lambda = 0 \) and \( \kappa = 1 \), while, for a compact BS, \( \Lambda \sim (10^2, 10^6) \) and \( \kappa(\chi \approx 0) \sim (10, 150) \). Therefore there is a discontinuity gap between BHs and BSs, and the effects of these parameters on the waveform can be potentially large. Crucially, both \( \Lambda \) and \( \kappa \) depend only on the object mass (and spin) and on the coupling constant (6). Therefore, for a given BS model \( \Lambda \) and \( \kappa \) are not independent quantities.

As shown in Fig. 2 the behavior of \( \kappa \) as a function of \( \chi \) for fixed values of \( M/M_B \) is nonmonotonous at large spins, whereas the behavior of \( \kappa_0 \equiv \kappa(\chi \approx 0) \) as a function of \( M/M_B \) is simpler. As shown in Appendix A, after mapping the dependence of \( M/M_B \) to \( \Lambda \) one finds a linear fit for the logarithmic quantities,

\[
\log \kappa_0 \approx 0.61 + 0.3 \log \Lambda. \tag{12}
\]

In practice, this fit relates the reduced quadrupole moment of a slowly-spinning massive BS to its Love number. The full quadrupole moment is corrected by \( \mathcal{O}(\chi^4) \) terms. It is interesting that the \( (\log \kappa_0)-(\log \Lambda) \) relation is approximately linear. This is reminiscent of the case of NSs, for which the “Love-Q” relations have a similar form [45, 46]. In that case the coefficients of the fit are nearly universal for different equations of state of the star. In a future work, we will explore this issue for BSs with different scalar potentials [47].

In the following, we will make use of Eqs. (12) and (9) to express all finite-size effects in the waveform in terms of the single parameter \( M_B \). In the next section we review how these effects (as well as the tidal heating) enter in the PN approximation of the GW waveform.

III. PN CORRECTIONS TO THE WAVEFORM

We focus on the inspiral phase of the signal emitted by a compact binary coalescence, adopting the PN expanded TaylorF template in the frequency domain [48–50]

\[
h(f) = A(f)e^{i\psi(f)}. \tag{13}
\]

The amplitude \( A \) and the phase \( \psi \) are expanded as power series in the parameter \( v = (\pi M_t f)^{1/3} \), where \( M_t = M_1 + M_2 \) is the total mass of the binary, being \( M_i \) the mass of the \( i \)-th binary component (we assume \( M_1 \geq M_2 \)). A term proportional to \( v^n \) corresponds to the \( n/2 \)-PN order of the approximation. In our analysis we retain only the dominant (Newtonian) term in amplitude, also averaging over the orientation and the polarization angles which specify the source’s position with respect to the
where $A$ is the luminosity distance and $\eta = M_1 M_2 / M_1^2$ is the symmetric mass ratio. For LISA, Eq. (14) must also be multiplied by an additional factor of $\sqrt{3}/2$, in order to account for the triangular geometry of the detector [51].

The BS signatures described in Sec. II B affect the signal’s phase at different PN orders, reflecting the frequency content of each effect. At small frequencies, where lower PN terms play a more significant role, sources behave as point-particles and the details on their internal structure are effaced [10, 52]. For larger frequencies, however, finite-size terms induced by spin-quadrupole and tidal effects become relevant. Changes in the waveform due to the BS structure add linearly to the BH phase, namely:

$$\psi(f) = \psi_{BH}(f) + \psi_\kappa(f) + \psi_\Lambda(f),$$

where $\psi_\kappa$ and $\psi_\Lambda$ identify the modifications induced by the spin-quadrupole and by the tidal terms, respectively. In our analysis we consider a 3.5PN expanded phase $\psi_{BH}$ [53], which includes spin-orbit, spin-spin (up to 3PN) and cubic spin corrections [54, 55]. Furthermore, in the BH waveform $\psi_{BH}$ we ignore the tidal heating term which enters the phase with a $v^4 \log v$ (resp., $v^8 \log v$) correction for a spinning (nonspinning) BH.

The dominant tidal correction enters the waveform at 5PN order, hence it is suppressed by a factor $v^4$ with respect to the 3.5PN phase $\psi_{BH}$. However, the potentially large values of $\Lambda$ render tidal effects comparable with the rest of the point-particle expansion. The leading tidal correction to the phase is given by [16, 56]

$$\psi_\Lambda = \frac{117}{256\eta} \tilde{\Lambda} (\pi M_1 f)^{5/3}$$

where

$$\tilde{\Lambda} = \frac{16}{13} \left[ \left( 1 + \frac{12}{q} \right) \frac{M_1^5}{M_1^2 M_2^2} \Lambda_1 + (1 + 12q) \frac{M_2^5}{M_2^2 M_1^2} \Lambda_2 \right]$$

is an effective total tidal deformability, and $q = M_1 / M_2 \geq 1$ is the binary mass ratio; the normalization of $\tilde{\Lambda}$ is chosen so that $\tilde{\Lambda} = \Lambda_1 = \Lambda_2$ for an equal mass binary. Indeed, note that $\Lambda_1$ and $\Lambda_2$ are not independent, since $\Lambda_1 = \Lambda_2(M_1, M_2)$, so for a given theory they are fixed once the masses are known\(^6\). The next-to-leading tidal correction enters at 6PN order and depends on both $\tilde{\Lambda}$ and on a second combination $\delta \tilde{\Lambda}$ of $\Lambda_1$ and $\Lambda_2$ [57], and also on the magnetic tidal Love numbers [58–61]. The correction $\delta \tilde{\Lambda}$ has been in general neglected within previous studies on BS waveforms, since it is subdominant in the PN expansion and, if $\Lambda_1$ and $\Lambda_2$ are treated as independent quantities, it would introduce an extra waveform parameter. However, since in our model $\Lambda_1 = \Lambda_2(M_B, M_t)$, including 6PN corrections does not introduce any extra parameter in the waveform, and at the same may actually lead to an overall improvement on the constraints that characterize the BS’s structure. On the other hand, the effect of the magnetic tidal deformability is typically negligible, since the magnetic tidal Love numbers are much smaller than the standard (electric) ones [19, 61]. For this reason, we shall neglect the magnetic-tidal contribution to the 6PN waveform.

The spin-induced quadrupole moments affect the GW phase already at 2PN order [21, 22, 43, 62]. For aligned spin binaries, the dominant quadrupole contribution reads

$$\psi_\kappa = \frac{75}{64} \frac{\kappa_1 M_1^5 \chi_1^2 + \kappa_2 M_2^5 \chi_2^2}{M_1 M_2} \frac{1}{v},$$

where $\kappa_i$ is the spin-induced reduced quadrupole of the $i$-th body (the binary BH case corresponds to $\kappa_1 = \kappa_2 = 1$). We incorporate also the first two subdominant corrections (appearing, respectively, at 3PN and 3.5PN) as given in Ref. [22]. We neglect the contributions from the spin-induced octupolar corrections, which are subleading relative to the quadrupolar corrections.

Finally, we restrict our analysis to non-precessing binaries, i.e., we assume that the individual spins are aligned with the binary orbital angular momentum.

### IV. MEASURABILITY OF THE BINARY PARAMETERS

In this section we briefly review the basic properties of the Fisher information matrix formalism that we use to infer the uncertainties on the waveform parameters [63, 64], and we summarize the results of previous applications of such formalism to the problem of distinguishing BS binaries from their BH counterparts.

We assume that a detection criterion for a GW signal $h(t, \tilde{\theta})$ has been met, providing us with the best estimates for the source parameters $\tilde{\theta}_0$, (as masses, spins, distance, orientation angles, etc.). Let $h(f, \tilde{\theta}_0)$ be the waveform in the frequency domain, evaluated at $\tilde{\theta} = \tilde{\theta}_0$. The signal-to-noise ratio (SNR) associated to the detection is given by

$$(\text{SNR})^2 = \langle h | \ h \rangle$$

where

$$\langle h_1 | h_2 \rangle = 4 \text{Re} \int_{f_{\text{min}}}^{f_{\text{max}}} df \frac{h_1(f)^* h_2(f)}{S_n(f)}$$

The same situation occurs for NSs once the equation of state is fixed. We remind that in the BS case the role of the equation of state is played by the potential $\psi(|\psi|)$.\(^6\)
is the waveform inner product defined over the detector’s noise spectral density $S_n(f)$. For LISA, we choose the range of integration in the frequency domain as follows:

$$f_{\text{min}} = \max (10^{-4} \text{ Hz}, f_{\text{obs}})$$  \hspace{1cm} (21a)
$$f_{\text{max}} = \min (1 \text{ Hz}, f_{\text{ISCO}})$$  \hspace{1cm} (21b)

where we remind $f_{\text{ISCO}} = (6^{3/2}M_s \pi^{-1})^{-1}$, whereas $f_{\text{obs}}$ is determined by requiring that the observation lasted $T_{\text{obs}} = 1 \text{ year}$ before the binary reached the ISCO frequency, namely (using a Newtonian approximation [65])

$$f_{\text{obs}} = 4.149 \times 10^{-5} \left( \frac{M}{10^6 M_\odot} \right)^{-5/8} \left( \frac{T_{\text{obs}}}{\text{1 yr}} \right)^{-3/8} \text{ Hz}.$$  \hspace{1cm} (22)

On the other hand, for the analysis with the ET we use

$$f_{\text{min}} = 3 \text{ Hz} , \quad f_{\text{max}} = f_{\text{ISCO}}.$$  \hspace{1cm} (23)

In the limit of large SNR, the best estimates $\bar{\theta}_0$ are unbiased, meaning that they approach the true values. If we also assume that the instrumental noise is Gaussian, then the posterior distribution of the waveform parameters $\bar{\theta}$ is described by a normal distribution peaked around $\bar{\theta}_0$ [64]:

$$p(\theta | s) \propto e^{-\frac{1}{2}(\theta - \theta_0)^T \Gamma_{\mu \nu} (\theta - \theta_0)_\nu}$$  \hspace{1cm} (24)

where $\Gamma$ is the Fisher information matrix, defined by

$$\Gamma_{\mu \nu} = \left( \frac{\partial h}{\partial \theta^\mu} \bigg| \frac{\partial h}{\partial \theta^\nu} \right).$$  \hspace{1cm} (25)

The 1-$\sigma$ uncertainty on $\Delta \bar{\theta} = \bar{\theta} - \bar{\theta}_0$ is given by

$$\sigma_j \equiv \sqrt{\langle (\Delta \theta^j)^2 \rangle} = \sqrt{(\Gamma^{-1})_{jj}}.$$  \hspace{1cm} (26)

The result (26) is formally valid only in the limit of infinite SNR, while for large SNR it is understood to provide only an order-of-magnitude estimate [64]. A rigorous approach would require a Bayesian analysis (see, e.g., [66]). However, Bayesian simulations are computationally costly, while the Fisher formalism is extremely cheap; therefore, the latter is useful to understand if the problem is promising enough to be addressed with a more rigorous Bayesian approach.

Previous studies focused on waveform’s changes induced by tidal interactions [19, 20, 67], horizon absorption effects [15, 25], and spin-induced multipole moments [21–24, 68]. In particular, previous work considered only a single effect at the time, focusing for instance on the detectability of $\Lambda$ or $\kappa$. However, given a BS model, both $\Lambda$ and $\kappa$ (as well as all other inspiral waveform parameters) depend only on the masses and spins of the binary. Therefore, including all effects at once does not increase the number of waveform parameters, on the contrary, it might help breaking degeneracy and improving the accuracy of the template.

V. RESULTS

Here we present the results of our Fisher-matrix analysis aimed to assess the ability of measuring the fundamental coupling constants of a scalar field theory using GW detections of binary BSs by LISA and ET. For concreteness, we restrict our attention to slightly unequal-mass binaries, such that $M_1 \approx 0.06M_B$ is approximately the maximum mass for nonspinning BSs and $M_2 = 0.05M_B$, with the mass ratio equal to $q = 1.2$. We proceed by increasing levels of complexity:

i. First, we consider the case in which the waveform has corrections only from the spin quadrupole moments, ignoring tidal deformabilities. We refer to this case as the “3.5PN test”, because the waveform is expanded only up to 3.5PN order.

ii. Then, we consider the case in which the waveform has corrections only from the tidal deformabilities, ignoring spin-quadrupole moment terms. We refer to this case as the “5PN test”, because we include only the leading 5PN tidal correction, and the $\dagger$ signals the absence of spin-quadrupole corrections.

iii. In the third case we incorporate both quadrupole and tidal corrections, the latter only at the leading 5PN order. We refer to this case as the “5PN test”.

iv. Finally, we also include the 6PN tidal correction (though we neglect the 6PN contribution from the smaller magnetic tidal deformabilities). We refer to this case as the “6PN test”.

For all cases, the independent parameters of the Fisher matrix are

$$\bar{\theta} = (A, t_c, \phi_c, \log M, \log \eta, \chi_s, \chi_a, M_B) ,$$  \hspace{1cm} (27)

where $t_c$ and $\phi_c$ are the coalescence time and phase, $M = (M_1 M_2)^{3/5}/M_1^{1/5}$ is the chirp mass, and $\chi_{s,a} = (\chi_1 \pm \chi_2)/2$ are the symmetric and antisymmetric spin

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7 Technically speaking, one also has to assume that the priors over $\theta^0$ are flat, which is approximately valid if the scale over which the priors change is smaller than the scale over which (24) changes.

8 From Eq. (9), $\Lambda$ has an extremum at $M = M_{\text{max}}$, so its derivative with respect to $M/M_B$ diverges at that point, thus rendering the Fisher matrix ill-defined. To overcome this problem, we choose $(M_{\text{max}} - M)/M_{\text{max}} = 10^{-5}$ and check that the results do not depend on $\epsilon < 1$. 

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FIG. 3. Relative percentage errors $\sigma_{M_B}/M_B[\%]$ for binary BSs with a coupling parameter in the range $M_B \in [1.7 \times 10^5, 10^7]M_\odot$, located at a luminosity distance $D_L = 1$ Gpc and observed with LISA. From left to right, panels refer to different terms included in the PN waveform model, see text for details.

FIG. 4. Same as in Fig. 3 but for binary BSs with coupling parameter in the range $M_B \in [1.7 \times 10^5, 10^7]M_\odot$, located at a luminosity distance $D_L = 500$ Mpc and observed with ET.

components. By considering the amplitude $A$ as an independent parameter, it decouples from the other variables [63], such that the Fisher matrix becomes block-diagonal $\chi_{iA} = \delta_{iA}$; therefore we will not report the uncertainties for $A$.

For each choice of $M_B$, we use Eq. (9) to compute the individual tidal deformabilities, and Eq. (12) to compute the corresponding reduced quadrupole moments. This is not entirely consistent, because Eq. (12) is valid only at small spins, while for generic spins $\kappa$ has the complicate dependence shown in Fig. 2. However, it is difficult to extract a $M_B$ dependence from Fig. 2, especially due to the fact that data are not dense enough. Work in this direction is in progress and will be reported elsewhere [47]. Here for simplicity we assumed $\kappa \approx \kappa_0$, which amounts to neglect $O(\chi^4)$ corrections in the expression (11) for the quadrupole moment. This approximation is valid for moderately small spins.

For LISA we consider a coupling in the range $M_B \in [1.7 \times 10^5, 10^7]M_\odot$, which corresponds to a maximum mass for nonspinning BSs $M_{\text{max}} \in [10^4, 10^6]M_\odot$, at a fixed luminosity distance $D_L = 1$ Gpc. For ET we consider $M_B \in [1.7 \times 10^5, 10^7]M_\odot$, corresponding to $M_{\text{max}} \in [1, 100]M_\odot$, at a fixed luminosity distance $D_L = 500$ Mpc. In both cases, we consider three different spin combinations $(\chi_1, \chi_2) = (0.1, 0), (0.6, 0.3), (0.9, 0.8)$. Since we consider relatively small distances, the effect of the cosmological redshift is small, therefore in the Fisher matrix we will use the values of the masses in the source-frame.

Figures 3 and 4 show the relative uncertainties on $M_B$ in the entire mass ranges considered, respectively, for LISA and ET (the lower $x$-axis shows the range for $M_{\text{max}}$, while the upper $x$-axis shows the corresponding range in $M_B$). We can understand the importance of the various contributions to the waveform as follows. First, tidal corrections have a larger impact than spin-quadrupole ones on the accuracy in $\sigma_{M_B}$: this can be seen by comparing the 3.5PN and 5PN$^*$ panels, from which it is evident that for the 5PN$^*$ model the uncertainties are smaller by 1-2 orders of magnitude. The results of the 5PN$^*$ model are in agreement with those of Ref. [19], as expected. Second, including the spin-induced quadrupole moment
helps to break the degeneracy with the spin among different PN terms in the waveform, leading to an improvement in the overall measurability at moderate and high spins: indeed, by contrasting the full 5PN results with the reduced 5PN, we can observe that the relative uncertainty in $M_B$ improves by an order of magnitude when $(\chi_1, \chi_2) = (0.6, 0.3), (0.9, 0.8)$. Finally, the inclusion of 6PN tidal corrections leads to a moderate improvement around 30% on $\sigma_{M_B}$, showing that the values inferred from the 5PN case are robust against the inclusion of the next-order tidal correction. This also confirms that the PN series is converging well also at $f \approx f_{\text{ISCO}}$, suggesting that a PN waveform approximant is sufficiently accurate up to those frequencies.

Crucially, when all corrections are included, the uncertainties on $M_B$ lie significantly below 100%, being at sub-percent and percent level in the most optimistic configurations for LISA and ET, respectively. This means that, in the considered mass range, a putative binary BS detection can be used to measure the coupling $M_B = \sqrt{\lambda/\mu}$ with great precision. We remind that the errors coming from the Fisher analysis scales linearly with the luminosity distance, so assuming (say) a distance $D_L = 10$ Gpc for LISA binaries would decrease the SNR and increase the errors by a factor 10. Given the small errors in Figs. 3 and 4, even in the more conservative case in which the event is at $\mathcal{O}(10)$ Gpc (for LISA) or at $\mathcal{O}(1)$ Gpc (for ET), the measurement of $M_B$ would still be accurate at $\mathcal{O}(10)$% level or better.

In Table I we also show the results of the Fisher analysis for LISA, for a fiducial binary BS system with $M_{\text{max}} = 0.06M_\odot = 5 \times 10^5 M_\odot$, corresponding to SNR $\approx 1.5 \times 10^4$. Similarly, Table II shows the results for ET, for a fiducial system with $M_{\text{max}} = 50M_\odot$ and corresponding SNR $\approx 620$. From the tabulated values, we observe that the inclusion of all the corrections greatly improves the precision in the measurements of the component spins.

We have also checked whether, in the mass range considered for ET, a successful measurement can be achieved with the advanced LIGO detector at design sensitivity. However, we find that the relative uncertainties in $M_B$, as measured with aLIGO, always lie above 100%, with the exception of some high spins configurations which are only marginally detectable. This is not only due to the globally lower SNR relative to ET, but more importantly to LIGO’s low-frequency cutoff, which is higher than ET’s. Higher sensitivity at low frequency helps significantly to measure the low-PN parameters, and in turn to improve the measurement of high-PN parameters at higher frequency. We confirmed this expectation by artificially truncating the ET noise curve at higher low-frequency cutoff, finding that the constraints on $M_B$ quickly deteriorate as the low-frequency cutoff increases. To summarize, our analysis suggests that future-generation GW observatories are necessary to make precise measurement of the fundamental couplings of a BS model.

VI. CONCLUDING REMARKS

We have estimated the ability of measuring the fundamental coupling constants of a scalar field theory through GW measurements of binary BSs. If such binaries exist and merge, future detectors such as LISA and the ET have the potential to measure the properties of the component BSs with exquisite precision. Such measurements can be mapped into constraints on the fundamental coupling constants of the scalar field theory. As depicted in Fig. 1, stellar-mass and supermassive BSs probe different regions of the parameter space, so future LISA and ET measurements will be complementary.

Our estimates are obtained through a Fisher matrix analysis, which is only an approximate treatment (although, given that LISA will reach SNR $\gg 100$, the approximation should be less dramatic than in the case of, e.g., aLIGO). A more accurate parameter estimation would require using Bayesian techniques. The latter can also be used to compute odd factors and perform model selection between different hypotheses for the nature of the binary: BS-BS, BH-BH, BH-BS, or to perform model selection among different BS models. Another source of approximation is that we excluded the merger-ringdown part of the spectrum from our waveform model, and only included the PN corrections to the inspiral. Therefore, in the future, it would be important to incorporate phenomenological completions of the waveform to the merger-inspiral region, possibly informed by numerical simulations (see, e.g., Refs. [69–71]). The merger of two BSs shows some distinctive features compared to its BH counterpart, which can be used to confidently determine the nature of the coalescing binaries, thus subsequently focus on a precision analysis of the inspiral signal to extract information about the model parameters.

Indeed, we have focused on massive BSs, but a natural extension of our work is to consider different classes of the scalar potential $V$, for example mini BSs [5, 6], solitonic BSs [72], or the recently studied model of axion BSs [73, 74]. The latter two models allow for very compact BSs, which are expected to have properties closer to those of a BH. When spinning, mini BSs are unstable and decay to their ground, nonspinning case, at least for certain compactnesses [35]. Strong self-interactions might cure this instability or make it phenomenologically irrelevant.

Likewise, BSs exist also for other integer-spin fundamental fields, such as Proca [75, 76] and massive spin-2 [77] fields. Interestingly enough, at variance with their scalar counterpart, spinning Proca stars are not unstable, due to the spherical topology of the Proca field profile in the ground state as opposed to the toroidal topology of a scalar field [35]. Our methods can be directly applied also to Proca stars or more generic BSs, provided a theoretical estimate of the tidal deformability and spin quadrupole is available for those models.

We also highlighted the existence of a simple phenomenological relation between the tidal deformability and the spin-induced quadrupole moment of static BSs.
| Test | $(\chi_1, \chi_2)$ | $\sigma_{Lc}[\text{sec}]$ | $\sigma_{\phi_c}[\text{rad}]$ | $\sigma_{\log M_\text{M}}[\%]$ | $\sigma_{\log M_\text{B}}[\%]$ | $\sigma_{\chi_1}$ | $\sigma_{\chi_2}$ | $\sigma_{M_B/M_\text{M}}[\%]$ |
|------|------------------|-----------------|----------------|------------------|----------------|----------------|----------------|------------------|
| 3.5PN | (0.1, 0) | 25.5 | 2.61 | $5.41 \times 10^{-3}$ | $5.21 \times 10^{-2}$ | 0.270 | 2.40 | 12.0 |
| 5PN$^1$ | (0.1, 0) | 44.5 | 1.13 | $2.83 \times 10^{-3}$ | 0.128 | 9.72 $\times 10^{-2}$ | 0.917 | 0.595 |
| 5PN | (0.1, 0) | 20.0 | 1.74 | $3.67 \times 10^{-3}$ | 0.175 | 2.35 $\times 10^{-2}$ | 0.151 | 0.817 |
| 6PN | (0.1, 0) | 11.1 | 1.24 | $3.27 \times 10^{-3}$ | $7.92 \times 10^{-2}$ | 1.61 $\times 10^{-2}$ | 9.79 $\times 10^{-2}$ | 0.371 |
| 3.5PN | (0.6, 0.3) | 27.9 | 4.25 | $3.46 \times 10^{-3}$ | 1.60 | 4.13 $\times 10^{-2}$ | 0.225 | 8.97 |
| 5PN$^1$ | (0.6, 0.3) | 42.1 | 1.19 | $2.90 \times 10^{-3}$ | 0.150 | 8.62 $\times 10^{-2}$ | 0.834 | 0.702 |
| 5PN | (0.6, 0.3) | 27.2 | 1.45 | $3.62 \times 10^{-3}$ | $3.64 \times 10^{-2}$ | 6.52 $\times 10^{-3}$ | 1.05 $\times 10^{-2}$ | 0.178 |
| 6PN | (0.6, 0.3) | 20.7 | 1.27 | $3.43 \times 10^{-3}$ | $3.04 \times 10^{-2}$ | 5.54 $\times 10^{-3}$ | 6.02 $\times 10^{-3}$ | 0.148 |
| 3.5PN | (0.9, 0.8) | 27.5 | 4.10 | $3.64 \times 10^{-3}$ | $1.50 \times 10^{-2}$ | 1.70 $\times 10^{-2}$ | 5.29 $\times 10^{-2}$ | 8.25 |
| 5PN$^1$ | (0.9, 0.8) | 42.1 | 1.17 | $2.74 \times 10^{-3}$ | 0.345 | 8.90 $\times 10^{-2}$ | 0.869 | 1.62 |
| 5PN | (0.9, 0.8) | 25.3 | 1.57 | $3.67 \times 10^{-3}$ | $2.83 \times 10^{-2}$ | 5.81 $\times 10^{-3}$ | 4.21 $\times 10^{-3}$ | 0.141 |
| 6PN | (0.9, 0.8) | 18.9 | 1.35 | $3.44 \times 10^{-3}$ | $2.08 \times 10^{-2}$ | 4.93 $\times 10^{-3}$ | 4.85 $\times 10^{-3}$ | 0.103 |

| Test | $(\chi_1, \chi_2)$ | $\sigma_{Lc}[\text{sec}]$ | $\sigma_{\phi_c}[\text{rad}]$ | $\sigma_{\log M_\text{M}}[\%]$ | $\sigma_{\log M_\text{B}}[\%]$ | $\sigma_{\chi_1}$ | $\sigma_{\chi_2}$ | $\sigma_{M_B/M_\text{M}}[\%]$ |
|------|------------------|-----------------|----------------|------------------|----------------|----------------|----------------|------------------|
| 3.5PN | (0.1, 0) | 0.106 | 118 | 0.320 | 2.24 | 12.1 | 105 | 524 |
| 5PN$^1$ | (0.1, 0) | 0.167 | 42.2 | 0.145 | 4.81 | 3.62 | 34.3 | 22.4 |
| 5PN | (0.1, 0) | 8.02 $\times 10^{-2}$ | 69.4 | 0.197 | 6.86 | 0.954 | 6.03 | 32.1 |
| 6PN | (0.1, 0) | 4.29 $\times 10^{-2}$ | 47.3 | 0.167 | 3.05 | 0.629 | 3.75 | 14.2 |
| 3.5PN | (0.6, 0.3) | 0.106 | 170 | 0.200 | 62.8 | 0.188 | 8.86 | 351 |
| 5PN$^1$ | (0.6, 0.3) | 0.159 | 44.6 | 0.150 | 5.57 | 3.24 | 31.4 | 26.0 |
| 5PN | (0.6, 0.3) | 0.107 | 57.3 | 0.191 | 1.47 | 0.271 | 0.382 | 7.27 |
| 6PN | (0.6, 0.3) | 8.07 $\times 10^{-2}$ | 49.2 | 0.177 | 1.21 | 0.227 | 0.210 | 5.95 |
| 3.5PN | (0.9, 0.8) | 8.75 $\times 10^{-2}$ | 137 | 0.177 | 48.9 | 0.577 | 1.78 | 268 |
| 5PN$^1$ | (0.9, 0.8) | 0.159 | 43.9 | 0.143 | 0.129 | 3.35 | 32.8 | 60.7 |
| 5PN | (0.9, 0.8) | 9.96 $\times 10^{-2}$ | 62.2 | 0.194 | 1.24 | 0.243 | 0.129 | 5.73 |
| 6PN | (0.9, 0.8) | 7.34 $\times 10^{-2}$ | 52.6 | 0.178 | 0.837 | 0.204 | 0.175 | 4.18 |

**Table I.** Fisher matrix uncertainties for a binary BS with fundamental couplings such that $0.06M_B = 5 \times 10^5M_\odot$, individual masses $M_1/M_B \approx 0.06$ and $M_2/M_B = 0.05$, located at a luminosity distance $D_L = 1$ Gpc and observed with LISA. The corresponding SNR is $\approx 1.5 \times 10^4$.

**Table II.** Same as Table I but for a binary BS with fundamental couplings such that $0.06M_B = 50M_\odot$, located at a luminosity distance $D_L = 500$ Mpc and observed with ET. In this case the corresponding SNR is $\approx 620$. 
described by the quartic potential (3). The relation is a straight line in log-space, reminiscent of the approximately universal Love-Q relations for NSs [45, 46]. We believe that such a remarkably simple behaviour deserves further investigation. A computation of quadrupole moments for spinning mini BSs, solitonic BSs, or axion BSs would definitely reveal if such a relation is approximately universal also in the BS case, or it is just an accident of the potential (3).

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Appendix A: Properties of BSs

In this appendix we summarize our fits for various BS quantities, obtained by collecting and analyzing the data already existing in the literature. In particular, we will refer to the plots in Ref. [36] for the properties of spinning BSs, and to the fits given in Ref. [20] for the of tidal Love numbers. We will show that several interesting relations emerge.

a. Maximum mass The dependence of the maximum mass on the spin was studied in [36]. As shown in Fig. 5, we found that the numerical data are well fitted by a parabolic formula in \( \chi \),

\[
\frac{M_{\text{max}}}{M_B} = 0.06 \left( 1 + 0.76 \chi^2 \right).
\]

(A1)

In particular, the mean square error of the fit is \( \approx 0.02 \), indicating a very good agreement with the true data points.

b. Compactness Fig. 7 shows the inverse compactness \( C^{-1} = R/M \) as plotted in [36] for a spinning BS. We interpolated the data points with a cubic spline and extracted from the interpolation the values of \( C^{-1} \) at \( \chi = 0 \). We find that, at \( \chi = 0 \), the following quadratic fitting formula is a good approximation of the numerical data:

\[
C^{-1} = 56.3 - 97.7 \left( \frac{M}{M_{\text{max}}} \right) + 48.8 \left( \frac{M}{M_{\text{max}}} \right)^2
\]

\[
\approx 7.5 + 48.8 \left( 1 - \frac{M}{M_{\text{max}}} \right)^2
\]

(A2)

\[\]

9 The data in [36] are not tabulated, therefore we extracted them directly from the figures using a coordinate-location software.

from which we see that \( C \) ranges from \( \sim 0.13 \) when \( M = M_{\text{max}} \) to \( \sim 0.034 \) when \( M = 0.02 M_B \) (recall that \( M_{\text{max}} = 0.06 M_B \)).

c. Tidal deformability We extract the data for the tidal deformability from the fitting formula (9), which is obtained from Eq. (47) in Ref. [20] in the limit \( \sqrt{\lambda/\mu} \gg 1 \). Next, we define a new quantity \( k_2 \) by rescaling \( \Lambda \) with \( C^5 \), \( k_2 \equiv \Lambda C^5 \). Figure 6 shows the profile of \( k_2 \) as a function of \( M/M_B \); we see that approximately \( k_2 \in (0.01, 0.1) \). This, together with the small values that can be attained by the compactness \( C \), explains why \( \Lambda \) can take large values as \( M/M_B \) decreases.

d. Spin-induced quadrupole moment We extracted numerical values for \( \kappa \) from Fig. 4 in Ref. [36] and interpolated them with a cubic spline. Then we extrapolated down to \( \chi = 0 \), thus obtaining the values of \( \kappa_0 = \kappa(\chi = 0) \) corresponding to \( M/M_B = 0.02, 0.03, 0.04, 0.05, 0.06 \). It is intriguing that, as shown in Fig. 8, \( \kappa_0 \) and \( \Lambda \) obey a simple relation of the form

\[
\log \kappa_0 = 0.61 + 0.30 (\log \Lambda).
\]

(A3)
In particular the mean square error is \(\approx 0.01\), indicating an excellent agreement.

![Graph showing inverse compactness \(C^{-1} = R/M\) of a massive BS. The data were extracted and interpolated from Fig. 2 in Ref. [56].](image)

**FIG. 7.** Inverse compactness \(C^{-1} = R/M\) of a massive BS. The data were extracted and interpolated from Fig. 2 in Ref. [56].

**FIG. 8.** Love-Q relation (12): \(\kappa_0\) versus \(\Lambda\) for a slowly rotating BS.

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