Dipolar Drag in Bilayer Harmonically Trapped Gases

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We consider two separated pancake-shaped trapped gases interacting with a dipolar (either magnetic or electric) force. We study how the center of mass motion propagates from one cloud to the other as a consequence of the long-range nature of the interaction. The corresponding dynamics is fixed by the frequency difference between the in-phase and the out-of-phase center of mass modes of the two clouds, whose dependence on the dipolar interaction strength and the cloud separation is explicitly investigated. We discuss Fermi gases in the degenerate as well as in the classical limit and comment on the case of Bose-Einstein condensed gases.

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I. INTRODUCTION

In recent years atomic and molecular dipolar gases have attracted a lot of interest since the long-range and the anisotropic nature of the interaction is expected to give rise to new important features at both the microscopic and macroscopic level. These include, among others, novel effects in the mechanism of the expansion and of the collective modes \(^1\) and in the structure of the superfluid \(^2\) and normal \(^3\) phase, new exotic phases of crystalline nature (see e.g., the recent work \(^4\)) and new schemes for quantum computation in the presence of optical lattices \(^5\). Some of these effects, in particular those concerning the expansion and the collective modes, have been already experimentally observed in magnetic dipolar atomic gases\(^6\). The recent progress in the realization of gases of electric polar molecules, where the effect of the dipolar force is particularly strong, is expected to open new challenging frontiers in this area of research (see \(^7\)–\(^9\) and references therein).

The aim of the present work is to propose a drag experiment induced by the long range nature of the dipolar interaction. We consider an atomic or molecular gas harmonically trapped in a double well configuration such that the overlap between the two clouds and the corresponding tunneling effect can be neglected (see Fig. 1). The only force acting between the two gases is of long-range nature (here and in the following we assume that dipoles are oriented in the direction orthogonal to the discs, i.e along the \(z\)-th axis of Figure 1) and we study how the out-of-phase transverse dipole mode is affected by the long-range interaction. Displacing one of the two clouds out of its equilibrium position and releasing it, will excite both the in-phase (center of mass) and the out-of-phase dipole modes. On a time scale fixed by the inverse of the frequency difference between the two modes, the center of mass motion of the first cloud will be transferred to the second one. We call this effect “dipolar drag” in analogy to the well known Coulomb drag (see e.g., \(^10\)) exhibited by electrons in uniform bilayer systems\(^11\).

![FIG. 1: Scheme of two not overlapping pancake shaped clouds of a dipolar gas. The distance between the centers of mass of the clouds is \(2L\). The clouds are harmonically confined in the transverse directions \(x, y\).](image)

II. DIPOLAR DRAG OF THE CENTER-OF-MASS MOTION

We consider a gas confined by a cylindrically harmonic potential:

\[
V_{\text{trap}}^{1,2}(x, y, z) = \frac{1}{2} m \omega_{\perp}^2 [x^2 + y^2 + \lambda^2 (z + z_0)^2].
\]

where \(2z_0\) is the distance between the minima of the potential along \(z\), \(\lambda = \omega_z / \omega_{\perp}\) is the ratio between the transverse and longitudinal trapping frequencies and we consider pancake configurations, i.e., \(\lambda \gg 1\). Let \(x_i\) being the center of mass coordinate along \(x\) of the \(i\)-th cloud. The equations of motion can be written as

\[
\frac{dx_1}{dt} = \omega_{\perp}^2 x_1 + \alpha(x_1 - x_2),
\]

\[
\frac{dx_2}{dt} = \omega_{\perp}^2 x_2 - \alpha(x_1 - x_2),
\]

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The interaction potential between two dipoles \( \vec{d}_1 = \vec{d}_2 = \vec{d} \) has the standard form:

\[
V_D(\vec{r}_1, \vec{r}_2, \theta) = \frac{d^2 (1 - 3 \cos^2 \theta)}{|\vec{r}_1 - \vec{r}_2|^3},
\]

where \( \theta \) is the angle between \( \vec{d} \), i.e., the \( z \) direction, and \( \vec{r}_1 - \vec{r}_2 \).

We study the system by means of the following energy functional of the cloud densities \( n_i, i = 1, 2 \)

\[
E[n_1, n_2] = \sum_{i=1,2} (E_{kin}[n_i] + E_{trap}[n_i] + E_{dd}[n_i]) + E_{dd}^{12}[n_1, n_2],
\]

where we introduce the kinetic energy \( E_{kin} = \hbar^2 / (2m \pi^2) \int d\vec{r} [\hat{n}_i^2(\vec{r})]^{5/3} \) calculated within local density approximation, the potential energy \( E_{trap} = \int d\vec{r} n_i(\vec{r})V_{trap} \) and the intra- and inter-cloud dipolar energies

\[
E_{dd} = \frac{1}{2} \int n_i(\vec{r})n_j(\vec{r})V_D(\vec{r} - \vec{r})d\vec{r}d\vec{r}'.
\]

In the energy functional Eq. (4) we have not included the intra-cloud exchange energy. We safely neglect it, since for the pancake-like configurations, which we are mainly interested in, the direct term is the dominant effect (see, e.g., [14]).

In order to study the center-of-mass oscillations in the transverse direction (see Fig. 1) we consider a scaling transformation for the densities of the type \( n_i(x, y, z) \rightarrow n_i(x + \epsilon_i, y, z) \). From the variation of the energy functional [5] we get as expected two modes. The in-phase mode is not affected by the dipolar interaction and has a frequency \( \omega \) equal to the harmonic trapping one. Conversely the out-of-phase mode is affected by the dipole interaction and is characterized by the frequency

\[
\frac{\omega_{out}}{\omega} = \sqrt{1 - \frac{2}{m\omega^2_\perp} \frac{\int d\vec{r}_1 d\vec{r}_2 V_D(\vec{r}_1 - \vec{r}_2) \frac{\partial n_1(\vec{r}_1)}{\partial x} \frac{\partial n_2(\vec{r}_2)}{\partial x}}{N}},
\]

where \( N \) is the total atom/molecule number.

Assuming the two clouds are identical we can get the densities of the clouds by means of an easy variational gaussian ansatz

\[
n_{1,2}(x, y, z) = \frac{N \kappa}{W_{\perp}^3 \pi^2} \exp \left( -\frac{x^2 + y^2 + \kappa^2(z + L)^2}{W_{\perp}^2} \right),
\]

where \( W_{\perp} \) and \( W_z = W_{\perp} / \kappa \) are the widths of single cloud and \( 2L \) is the distance between the clouds, which, for our parameters, namely a strong confinement in the \( z \) direction, is very close to the distance \( 2z_0 \).

Inserting the gaussian ansatz Eq. (5) into the single
TABLE I: The cloud size for $N = 2200^{40\text{K}}{^{87}\text{Rb}}$ molecules with $\omega_x/2\pi = 10$ kHz (corresponding to $a_x = 8.89 \times 10^{-6}$ cm) and dipole moment $d = 0.56$ D.

| $\lambda$ | $W_\perp$, 10^{-3}$cm | $W_z$, 10^{-3}$cm |
|----------|------------------------|-------------------|
| 10       | 1.8                    | 2.05              |
| 20       | 2.9                    | 1.54              |

be easily shown that considering spherical clouds with $W_\perp = W_z = W$ in Eq. 7 the frequency of out-of-phase dipole mode reads

$$\omega_{out} = \omega_\perp \left(1 - \frac{\sqrt{2}N d^2 h(L/W)}{3\sqrt{\pi}m\omega^2 L^3}\right)^{1/2},$$  

where $h(y) = e^{-y^2}(4y^5 + 6y^3 + 9/2y) - 9/2 \sqrt{\pi} \text{Erf}(\sqrt{2}y)$, which approaches a constant for large values of $y$.

The results reported in Fig. 3 show how the frequency shift can be large enough, for the chosen parameters, to be experimentally measurable. Moreover we checked that the results are the essentially the same using Thomas-Fermi profiles, instead of the gaussian ansatz Eq. 3.

It is useful to compare the above predictions for the frequency shifts calculated for a zero temperature Fermi gas with the ones holding for a Bose-Einstein condensed gas or for a classical thermal configuration. To this purpose one can still use Eq. 7 with the proper density profiles (an inverted parabola for a BEC gas and a Boltzmann distribution for a classical gas). As emerges from Eq. 7 the effect is amplified for smaller radial sizes where the gradient of the density is larger. One then understands that a Bose gas interacting with a moderate value of the scattering length will provide larger shifts with respect to both a Fermi gas and a thermal configuration. The shifts for a Bose gas were investigated in 12 where, however, magnetic dipolar atomic gases were considered, which are characterized by a significantly small value of the dipolar coupling constant $d^2$ in Eq. 4. At present the more promising perspectives for realizing electric dipolar molecules, where the effect of the dipolar interaction is particularly strong, concern the fermionic species which ensure better stability conditions and which are already available in the thermal regime.

In Fig. 4 we report the predictions for the frequency shifts exhibited by a thermal configuration calculated at the temperature $T = T_F$, where $T_F = \hbar \omega_\perp (6N\lambda)^{1/3}/k_B$ is the Fermi energy and $k_B$ the Boltzmann’s constant. We used simply the gaussian density profiles of Eq. 3, but with the radii given by the Boltzmann expression $W_\perp^2 = 2k_BT/(m\omega^2)$ and $\kappa = \lambda$. The corresponding parameters are given in Table I. Comparison with the predictions reported in Fig. 3 shows that the effect, for the same trapping conditions and number of particles, is indeed smaller than for a degenerate Fermi gas, since the thermal radii are larger and the densities smaller than the ones of the degenerate configuration.
IV. CONCLUSIONS

We have proposed a drag experiment between two non-overlapping atomic/molecular clouds (see Fig. 1 and Fig. 2) to test the long-range nature of the dipolar potential. The method is independent of quantum statistics and holds for both degenerate and thermal gases. This effect corresponds to the trapped version of the famous Coulomb drag exhibited by electrons in uniform bilayer systems. The realization of such a drag experiment would provide a direct and easy signature of the long-range nature of the dipole interaction.

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[1] T Sogo, L He, T Miyakawa, S Yi, H Lu and H Pu, New J. Phys. 11, 055017 (2009); R. M. W. van Bijnen, N. G. Parker, S. J. J. M. F. Kokkelmans, A. M. Martin, and D. H. J. ODell, Phys. Rev. A 82, 033612 (2010).
[2] M. A. Baranov, M. S. Marenko, Val. S. Rychikov, and G. V. Shlyapnikov, Phys. Rev. A 66, 013606 (2002).
[3] C.-K. Chan, C. Wu, W.-C. Lee, and S. Das Sarma, Phys. Rev. A 81, 023602 (2010); S. Ronen and J. L. Bohn, Phys. Rev A 81, 033601 (2010).
[4] B. Capogrosso-Sansone, C. Treffzger, M. Lewenstein, P. Zoller, and G. Pupillo, Phys. Rev. Lett. 104, 125301 (2010).
[5] D. DeMille, Phys. Rev. Lett. 88, 067901 (2002).
[6] J. Stuhler, A. Griesmaier, T. Koch, M. Fattori, T. Pfau, S. Giovanazzi, P. Pedri, and L. Santos, Phys. Rev. Lett. 95, 150406 (2005) G. Bismut, B. Pasquion, E. Maréchal, P. Pedri, L. Vernac, O. Gorceix and B. Laburthe-Tolra, Phys. Rev. Lett. 105, 040404 (2010).
[7] L. D Carr, D. DeMille, R. V. Krems and J. Ye, New J. Phys. 11, 055049 (2009).
[8] K.-K. Ni, S. Ospelkaus, D. J. Nesbitt, J. Ye and D. S. Jin, Phys. Chem. Chem. Phys. 11, 9626 (2009); K.-K. Ni, S. Ospelkaus, D. Wang, G. Quéméner, B. Neyenhuis, M. H. G. de Miranda, J. L. Bohn, J. Ye and D. S. Jin, Nature 464, 1324 (2010).
[9] E. S. Shuman, J. F. Barry and D. DeMille, Nature 467, 820 (2010).
[10] A. G. Rojo, J. Phys.: Condens. Matter 11, R31 (1999).
[11] The in-phase and out-of-phase “plasmon” mode of a homogeneous bilayer dipolar configuration have been recently investigated by Q. Li, E. H. Hwang, and S. Das Sarma, Phys. Rev. B 82, 235126 (2010).
[12] C.-C. Huang and W.-C. Wu, Phys. Rev. A 82, 053612 (2010).
[13] M. H. G. de Miranda, A. Chotia, B. Neyenhuis, D. Wang, G. Quéméner, S. Ospelkaus, J. L. Bohn, J. Ye and D. S. Jin, Nat. Phys. DOI: 10.1038/NPHYS1939.
[14] T. Miyakawa, T. Sogo, and H. Pu, Phys. Rev. A 77, 061603(R) (2008).
[15] T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, T. Pfau, Rep. Progr. Phys. 72, 126401 (2009).