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Electromagnetic Wave Propagation in Multilayered Structures with Negative Index Material

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1. Introduction

The emergence of artificially designed subwavelength electromagnetic materials, known as metamaterials, catches an increasing interest of researchers. Metamaterials are artificially structured materials featuring properties that do not or may normally take place and cannot be acquired in nature (Engheta & Ziolkowski, 2006). In recent years greater attention has been paid to the metamaterials with negative index of refraction which have quite uncommon electromagnetic properties. The new type of materials with the negative index of refraction were theoretically predicted in 1968 by Veselago (Veselago, 1967). In these materials both the permittivity and the permeability take on simultaneously negative values at certain frequencies. In materials with the negative refractive index the direction of the Pointing vector is antiparallel to the one of the phase velocity, as contrasted to the case of plane wave propagation in conventional media.

The complex refractive index of a medium is defined as the ratio between the speed of an electromagnetic wave in medium and that in vacuum and can thus be expressed as $n^2 = \mu \varepsilon$, where $\mu$ is relative magnetic permeability and $\varepsilon$ relative dielectric permittivity. If we change simultaneously the signs of $\varepsilon$ and $\mu$, the ratio $n^2 = \mu \varepsilon$ will not change. If both $\varepsilon$ and $\mu$ are positive, this means that $n = \sqrt{\varepsilon \mu}$, if $\varepsilon$ and $\mu$ are negative in a given wavelength range, this means that $n = -\sqrt{\varepsilon \mu}$. In negative index metamaterials the Poynting vector $S = [\mathbf{E} \times \mathbf{H}]$ and vectors $\mathbf{E}$ and $\mathbf{H}$ form the left-hand triple, which leads to opposite directions of the group and phase velocity of plane waves propagating in the material. Consequently, metamaterials with simultaneously negative permittivity and permeability are named as "left-handed" metamaterials, "backward-wave media", double negative materials.

Smith, Shelby et al. were the first to demonstrate by means of experiment the existence of metamaterials with simultaneously negative permittivity and permeability at microwave frequencies (Smith et al., 2000), (Shelby et al., 2001). After the experimental demonstration of such materials, the properties and possible applications of various metamaterials with negative index of refraction gained a rapidly increasing interest. Now the negative refractive index metamaterials are demonstrated for near infrared and optical range (Falcone et al. 2004), (Iyer & Eleftheriades, 2002), (Caloz & Itoh, 2002). In metamaterials with negative refractive index many interesting phenomena that do not appear in natural media can be...
observed. Among them there are the modification of the Snell’s law (Ramakrishna, 2005), reversal of the Doppler effect and Cerenkov radiation (Lu et al., 2003), reformulation of the Fermat principle (Veselago, 2002).

One of the most interesting directions of analysis of the metamaterials unusual properties is the study of periodic structures or photonic crystals composed of metamaterials. Photonic crystals are artificial materials with periodically modulation of the refractive index (Joannopoulos et al., 1997). The simplest one-dimensional photonic crystal consists of alternating layers with different dielectric constant (Yeh, 1988). Such a structure may include partial bandgaps for certain ranges of propagation directions. Photonic crystals containing metamaterials demonstrate a variety of new physical effects, and possess a number of new potentials (Wu et al., 2003 b). In one-dimensional photonic crystals with negative index metamaterial the new zero-index bangaps can be observed (Shadrivov et al., 2003), (Wu et al., 2003 a), (Li et al., 2003). Such novel periodic structures unveil many unusual properties, including substantial suppression of the Anderson localization and long-wavelength resonances (Asatryan et al., 2007). Specially designed one-dimensional structures with negative refraction may include a complete three-dimensional bandgap (Shadrivov et al., 2005). The result is sharply contrasted to the periodic structures with usual dielectrics which lack the complete gap.

In this chapter we study the propagation of electromagnetic waves in one-dimensional periodic structures composed of alternating layers of negative index metamaterial, conventional material (dielectric or semiconductor) and the thin superconducting film in resistive state. If we put the structure with thin superconducting film in external magnetic field, the latter can penetrate into the thickness of superconductor in the form of Abricosov vortex lattice. The presence of the thin superconducting film in the structure leads to attenuation of electromagnetic wave. However, the interaction of electromagnetic waves with moving Abricosov vortex lattice can lead to its amplification (Popkov, 1989), (Glushchenko & Golovkina, 1998). The important feature is that parameters of attenuation and amplification of electromagnetic wave considerably depend on the value of magnetic field. That is why the thin superconducting film can be a control element in considered periodic structures.

2. Periodic structure negative index metamaterial - dielectric

Let us consider a one-dimensional periodic structure shown schematically in Fig. 1, where the layers of usual dielectric material with width $d_1$ are separated by the layers of the negative index metamaterial with the width $d_2$. We will describe the variation of the refractive index in the pair of layers in the following way:

\[
\begin{align*}
n(z) = \begin{cases} 
  n_1 = \sqrt{\varepsilon_1\mu_1}, & z \in (z_m, z_m + d_1) \\
n_2 = -\sqrt{\varepsilon_2\mu_2}, & z \in (z_m + d_1, z_m + d) \end{cases}
\end{align*}
\]

where $n_1$ and $n_2$ are the refractive indices of dielectric and metamaterial layers, respectively, $d = d_1 + d_2$ is period of the structure. The structure is uniform in the $y$ direction ($\partial/\partial y = 0$). We consider TE-polarized waves propagating in the $(x, z)$ plane. The electric field can be expressed as $\exp\left(i\omega t - ik_xx - ik_{z1}z\right)$. The field is assumed to be monochromatic with the frequency $\omega$. 

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The eigenmodes of the problem can be adopted by applying the Bloch theorem for periodic systems and the corresponding Bloch-wave formalism (Yeh, 1988). The Bloch wavenumber \( K \) for TE-polarized waves is defined from the dispersion relation:

\[
\cos Kd = \frac{1}{2} \text{Tr}(M) = \cos k_y d_1 \cos k_y d_2 - \frac{1}{2} \left( \frac{k_{y1}\mu_2}{k_{y2}\mu_1} + \frac{k_{y2}\mu_1}{k_{y1}\mu_2} \right) \sin k_y d_1 \sin k_y d_2 ,
\]

where \( \text{Tr}(M) \) is trace of the transfer matrix \( M \) characterizing the wave scattering in a periodic structure, \( k_{y1,2} = \frac{\omega^2}{c^2} n_{1,2}^2 - k_x^2 \) are the longitudinal wave vector components, \( k_x \) is the wave vector component along the layers, while the indexes 1, 2 indicate the layer. The value \( \cos Kd \) determines the band structure. It should take real values for lossless media and real \( k_y \). Regimes where \( |\cos Kd| < 1 \) correspond to real \( K \) and thus to propagating Bloch waves. In regimes where \( |\cos Kd| > 1 \), \( K \) has an imaginary part; therefore, the Bloch waves are evanescent, and this regime corresponds to forbidden bands of the periodic medium. If the Bloch wavenumber \( K \) remains complex for all real \( k_x \) the bandgap is complete. The dispersion relation of the TM polarized waves is obtained by replacing \( \epsilon \) and \( \mu \) in Eq. (2):

\[
\cos Kd = \cos k_y d_1 \cos k_y d_2 - \frac{1}{2} \left( \frac{k_{y1}\epsilon_2}{k_{y2}\epsilon_1} + \frac{k_{y2}\epsilon_1}{k_{y1}\epsilon_2} \right) \sin k_y d_1 \sin k_y d_2 . \tag{3}
\]

In periodic structures containing alternating layers of negative refractive metamaterials and usual dielectrics arise the novel partial bandgaps (Nefedov & Tretyakov, 2002), (Li, 2003). These bandgaps appear when the condition \( k_{y1}d_1 + k_{y2}d_2 = 0 \) is satisfied for certain angles of propagating. This is possible because \( k_y \) is positive in usual dielectric and negative in negative refractive metamaterial.

Moreover the one-dimensional layered structure with layers of negative index materials can exhibit a complete two-dimensional bandgap (Shadrivov et al., 2005). This bandgap arise only for specified set of material parameters which eliminates the guided modes and
transmission resonance. The more complicated one-dimensional periodic structure may possess the absolute bandgap. This structure contain three different kinds of layers in order to suppress the conditions for the existence of the Brewster angle. The dispersion relation for the structure with the parameters \( \varepsilon_1 = \mu_1 = 1, \varepsilon_2 = \mu_3, \varepsilon_3 = \mu_3 \) and \( d_2 = d_3 \) is written in the following way (Shadrivov et al., 2005):

\[
\cos Kd = \cos k_{y1}d_1 \cos k_{y2}d_2 - \frac{1}{2} \left( \frac{\varepsilon_2}{\mu_2} + \frac{\mu_2}{\varepsilon_2} \right) \cos k_{y1}d_1 \sin^2 k_{y2}d_2 - \frac{1}{4} \left( \varepsilon_2 + \mu_2 \right) \frac{k_{y1}}{k_{y2}} \frac{k_{y2}}{k_{y1}} \sin(k_{y1}d_1)\sin(2k_{y2}d_2).
\]

This dispersion relation remains the same for both TE and TM polarizations. Though, the bandgaps will appear for both polarizations simultaneously. The presence of the bandgaps for both polarizations indicates the existence of an absolute bandgap. The one-dimensional periodic structure, under present consideration, can trap light in three dimensions as opposed to conventional photonic crystals which require the presence of two- and three-dimensional periodicity.

3. Periodic structure negative index metamaterial - semiconductor

The main difference between the periodic structures with semiconductor layers and structures with dielectric layers consists in the frequency dependence of permittivity in semiconductor layers. The dependence of permittivity on frequency leads to emergence of new types of waves and formation of various instabilities. We will now dwell upon the case of absence of losses in the layers. In order to find out the basic features of behaviour of eigenmodes in structures with semiconductors layers we should assume that the permittivity and permeability of negative index material layers are constant.

We will use the hydrodynamic model to describe the movement of charge carriers in the semiconductor (Yu & Cardona, 2005). The application of hydrodynamical model is reasonable for frequencies \( \omega >> \nu \), where \( \nu \) is effective collision frequency, and for frequencies \( \omega << \nu \). The typical values of the effective collision frequency for semiconductor make it possible to draw a conclusion that hydrodynamical model is applicable for the description of wave processes with the frequencies \( \omega > 10^{10} \text{ s}^{-1} \). In the considered periodic structure the hydrodynamical approach can be used if the period of structure \( d = d_1 + d_2 \) is much larger than the wavelength of electromagnetic wave. An individual semiconductor layer of thickness \( d_1 \) is described in hydrodynamic approach by dielectric permittivity

\[
\varepsilon_1 = \varepsilon_{01} \left( 1 - \omega_{p1}^2 / \omega^2 \right),
\]

where \( \varepsilon_{01} \) is the high-frequency dielectric constant, \( \omega_{p1} = \left( Ne_0^2 / m \varepsilon_0 \varepsilon_{01} \right)^{1/2} \) is plasma frequency; \( e, m \) and \( N \) are charge, effective mass and density of the free charge carriers; \( \varepsilon_0 \) is the permittivity of the free space.

Here, let us turn to the propagation of TM polarized waves. The dispersion relation for TM waves (3) is same as for the structure negative index material - dielectric. We will compare the position of the bandgaps in the structures semiconductor - negative index metamaterial...
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and semiconductor - dielectric (Golovkina, 2007). Let the left part of the equation (3) be marked as $F(k_x, \omega)$. Fig. 2 shows a graph of function $\cos K d = F(k_x, \omega)$ for the structure semiconductor - negative index metamaterial for arbitrary fixed value of frequency $\omega$. Comparatively, Fig.3 shows the graph of function $F$ for the periodic structure semiconductor - usual dielectric, which has the same magnitudes of permittivity and permeability as the negative index metamaterial.

![Graph of function $F(k_x, \omega)$ for the structure semiconductor - negative index metamaterial.](image)

Fig. 2. The graphic detection of bandgaps for structure semiconductor - negative index metamaterial corresponding to the equation (3).

![Graph of function $F(k_x, \omega)$ for the periodic structure semiconductor - dielectric.](image)

Fig. 3. The graphic detection of bandgaps for structure semiconductor - dielectric corresponding to the equation (3).

We will consider the frequency $\omega$ which corresponds to the positive values of $\varepsilon_1$. The existence of a propagative Bloch mode requires $|\cos K d| \leq 1$. If $|\cos K d| > 1$, the Bloch wavenumber $K$ becomes complex and the wave propagation is prohibited. These areas in the figures are hatched. We can observe that the number of bangaps and their position are different in the structures negative index metamaterial - semiconductor and negative index metamaterial - dielectric.

As is obvious from Fig. 2, 3, with increasing $k_x$, the bandgaps become narrower. Finally, following further increase of $k_x$, the permitted bands become so narrow that forbidden bands merge with each other. Though, all this considered, one of the layers is getting opaque. At last, since certain value of $k_x$ both of a layer become opaque and the continuous forbidden band is formed. This value of $k_x$ is the same for the structures semiconductor - negative index metamaterial and semiconductor - dielectric. In the permitted band the Bloch
wavenumber varies from \(2m\pi/d\) up to \((2m+1)\pi/d\). On each border a direct and reflected waves appear in the equal phase. We can see that the position and quantity of the forbidden bands for structures semiconductor - negative index metamaterial and semiconductor - dielectric is various because the Bloch wavenumbers for these structures have different values.

Now we will try to examine the case when \(k^2_x >> \omega^2 n_{1,2}^2/c^2\). Then, accordingly,

\[
k_{x1,2} = \pm i k_x. \tag{6}\]

The described above inequality imposes constraints on the thickness of layers. The deepest influence of periodicity occurs at \(k_x d \approx 1\). Thus, the following restriction should be satisfied

\[
d^2 << c^2/\omega^2 \max(\varepsilon_1, \varepsilon_2). \tag{7}\]

The dispersion equation for TM polarized waves can be transformed to the following form (Golovkina, 2009 a):

\[
\frac{\varepsilon_1^2(\omega) + \varepsilon_2^2(\omega)}{\varepsilon_1(\omega)\varepsilon_2(\omega)} = -2 \frac{\cos Kd - \cosh k_x d_1 \cosh k_x d_2}{\sinh k_x d_1 \sinh k_x d_2}. \tag{7}
\]

Fig. 4 gives a sketchy idea of the corresponding dispersion curves. As is known, the spectrum of the semiconductor-dielectric structure consists of two different branches. The acoustic branch starts at \(k_x = 0\) and \(\omega = 0\) and tends to \(\omega_{ps}\), where

\[
\omega_{ps} = \left[4\pi e^2 n_{01}/m(\varepsilon_{01} + \varepsilon_2)\right]^{1/2}
\]

is the frequency of surface plasmon, \(n_{01}\) is the concentration of carriers. The optic branch starts at \(\omega = \omega_{p1}\) and \(k_x \rightarrow 0\) and tends to \(\omega_{ps}\) (Fig 1, a).

### 4. Periodic structure with thin superconducting film

When in the 1986 high temperature superconductors were discovered the problems of their practical use in microelectronics became actual. New techniques have been developed in
recent years to produce superconducting layered systems. Multilayer stackings of different superconducting materials with very small layer thickness intercalated with dielectric layers are obtainable (Silva et al., 1996). Rapid progress in such techniques as molecular beam epitaxy or chemical vapour deposition enables us to grow systems with pre-determined film thickness. Such systems seem to provide a new type of material which does not exist naturally (Wua, 2005). One can create materials with properties distinct from those of any single constituent. The successes of technology have lead to constant increase of use of multilaered structures based on high temperature superconductors in microelectronics and computer hardware (Lancaster, 2006). In most of the experimental and theoretical work on artificially superconducting layered systems, researches have dealt with a study electromagnetic wave propagation along the interfaces (Ghamsari & Majedi, 2007). Nevertheless the problem of electromagnetic wave propagation in such structures in arbitrary directions is practically investigated.

In this section we present the theoretical development for calculating the dispersion relations for electromagnetic wave propagation in an infinite periodic structure. The periodic structure we consider is shown in Fig. 5.

![Fig. 5. Geometry of problem. One-dimensional structure dielectric - superconductor - negative index metamaterial.](image)

The structure consists of alternating layers of dielectric with thickness $d_1$, type-II superconductor with thickness $t \ll \lambda$, where $\lambda$ is a microwave penetration depth and layers of negative index metamaterial with thickness $d_2$. We let the interfaces of the superlattice lie parallel to the $x$-$z$ plane, while the $y$ axis points into the structure. A static magnetic field $B_{y0}$ is applied antiparallel the $y$ axis, perpendicular to the interfaces of the superlattice. The value of magnetic field does not exceed the second critical field for a superconductor. The magnetic field penetrate in the depth of the superconductor in the form of Abrikosov vortex lattice. Under the impact of transport current directed perpendicularly to magnetic field $B_{y0}$ along the $0z$ axis, the flux-line lattice in the superconductor layers starts to move along the $0x$ axis. Let’s consider the propagation in the given structure p-polarized wave being incident with angle $\theta$ in the $x0y$ plane. It can be assumed that $\partial / \partial z = 0$.

The presence of a thin superconductor layer with the thickness of $t \ll \lambda$ is reasonable to be accounted by introduction of a special boundary condition because of a small amount of thickness. Let’s consider the superconductor layer at the boundary $y=0$. At the inertia-free
approximation and without account of elasticity of fluxon lattice (the presence of elastic forces in the fluxon lattice at its deformation results in non-linear relation of the wave to the lattice, that is insignificant at the given linear approximation) the boundary condition is written in the following way (Popkov, 1989):

\[
\frac{\partial B_y}{\partial t}(y = t) + \frac{j_{z0}\Phi_0}{\eta} \frac{\partial B_y}{\partial t}(y = t) = \frac{B_{y0}\Phi_0}{\eta} \frac{\partial}{\partial t}[H_x(y = t) - H_x(y = 0)],
\]

(8)

where \( j_{z0} \) is the current density in the superconducting layer and \( \eta \) is the vortex viscosity.

Let's write the boundary condition (8) in the form of matrix \( M_s \), binding fields at the boundaries \( y=d_1 \) and \( y=d_1+t \):

\[
\begin{pmatrix}
E_z(d_1 + t) \\
H_x(d_1 + t)
\end{pmatrix}
= M_s \begin{pmatrix}
E_z(d_1) \\
H_x(d_1)
\end{pmatrix},
\]

(9)

where \( k_y \) is the projection of the passing wave vector onto the 0x axis and \( \omega \) is the frequency of the passing wave.

Let us apply the transfer matrix method. This method has been extensively applied to band structure calculations of photonic crystals containing absorptive and frequency dispersive materials. In considered periodic structure the presence of thin superconducting film leads to the dissipation of energy. By usage of matrix method we expressed dispersion relation for TE-wave in the following way:

\[
\cos Kd = \cos k_{y_1}d_1 \cos k_{y_2}d_2 - \frac{1}{2} \left( \frac{k_{y_1}\mu_2}{k_{y_2}\mu_1} + \frac{k_{y_2}\mu_1}{k_{y_1}\mu_2} \right) \sin k_{y_1}d_1 \sin k_{y_2}d_2 -
\]

\[
- \frac{1}{2} i \omega \mu_0 \left( \frac{j_{z0}k_x}{\Phi_0} \right) \left( \frac{\mu_1}{k_{y_1}} \sin k_{y_1}d_1 \cos k_{y_2}d_2 + \frac{\mu_2}{k_{y_2}} \cos k_{y_1}d_1 \sin k_{y_2}d_2 \right)
\]

(10)

where \( K = K' + iK^* \) is the Bloch wave number and \( k_y \) is the projection of passing wave vector onto the 0y axis. The interaction of electromagnetic wave with thin superconducting film leads to emergence of the imaginary unit in the dispersion equation. The presence of imaginary part of the Bloch wave number indicates that electromagnetic wave damps exponentially when passing through the periodic system even if the negative index metamaterial and dielectric layers are lossless. However, when one of the conditions

\[
\frac{\mu_1}{k_{y_1}} \sin k_{y_1}d_1 \cos k_{y_2}d_2 + \frac{\mu_2}{k_{y_2}} \cos k_{y_1}d_1 \sin k_{y_2}d_2 = 0,
\]

(11)

\[
\frac{j_{z0}k_x}{\omega} - \frac{\eta}{\Phi_0} = 0
\]

(12)
is executed, the Bloch wave vector becomes purely real and electromagnetic wave may penetrate into the periodic structure (Golovkina, 2009 c). The implementation of condition (11) depends on the relation between the parameters of layers and the frequency of electromagnetic wave, while the implementation of condition (12) depends – on parameters of superconducting film, namely on current density $j_{z0}$. Still, we are able to manage the attenuation and propagation of electromagnetic waves by changing the value of $j_{z0}$. Moreover, the electromagnetic wave can implement the amplification in such a structure. It is well-known that electromagnetic wave can be amplified in structures with electron flow if the velocity of electromagnetic wave is equal to the velocity of electrons. Such amplification is observed in the traveling-wave tube and backward-wave tube at the interaction between the electromagnetic wave and the electron flow. As is shown in papers (Glushchenko & Golovkina, 1998), (Glushchenko & Golovkina, 2007), the electromagnetic wave can be amplified in layered structures with thin superconducting film in resistive state by means of interaction with moving Abrikosov vortex lattice. Nonetheless, when the layered structure includes the combination of two layers with positive and negative refractive index, the amplification occurs at a lesser velocity of vortex lattice.

The results of numerical calculation concerning the module and the phase of the reflection coefficient $R$ as a function of external magnetic field $B_{y0}$ are given in Fig. 6 and Fig. 7. The structure consists of the layer MgO with thickness $d_1 = 0.1 \mu m$, the layer of negative index metamaterial with thickness $d_2 = 0.1 \mu m$ ($\epsilon_2 = -9 - 0.09i$, $\mu_2 = -1 - 0.01i$), thin superconducting film YBa$_2$Cu$_3$O$_y$ on the substrate of SrTiO$_3$. This calculations touched upon the losses in the layers. For the given structure the reflection coefficient is greatly dependent on incident angle $\theta$, thickness of superconducting film $t$ and magnitudes of external magnetic field $B_{y0}$. We can observe from Fig. 6 that the module of the reflection coefficient reaches the value more than unit at the specified magnitude of the magnetic field. The amplification appears out of energy from moving flux line lattice.

![Fig. 6. The module of the reflection coefficient R versus external magnetic field for two layer structure with thin superconducting film YBa$_2$Cu$_3$O$_y$ on the substrate of SrTiO$_3$. The parameters are as follows: $\eta = 10^{-5} N/(s \cdot m^2)$, $j_{z0} = 10^8 A/m^2$, $\omega = 10^{13} rad/s$. The solid line: $t=40$ nm, the short dashed line: $t=43$ nm, the long dashed line: $t=46$ nm, where $t$ is thickness of superconducting film.](image-url)
A high electromagnetic wave amplification coefficient during the reflection from the layered structure with superconducting film by the interaction with a fluxon structure enables to create wide-ranged amplifying devices using the structures analyzed above. Heavy dependence of reflection coefficient on incident wave frequency, incident angle and external magnetic field magnitude makes it possible to create new devices and filters with high parameter selectivity controlled by magnetic field by means using the considered structures.

5. Waveguide structure with thin superconducting film

We have examined the wave propagation in two-layered waveguide divided by a thin superconducting film. On layer of thickness $d_1$ is a negative index material ($\varepsilon_1 < 0$, $\mu_1 < 0$) and the other one of thickness $d_2$ is a usual dielectric ($\varepsilon_2 > 0$, $\mu_2 > 0$) (Fig. 8). The thin film of type-II superconductor with thickness $t$ is placed in the plane $y_0z$. The thickness of superconductor $t << \lambda$, where $\lambda$ is magnetic field penetration depth. A static magnetic field $B_0$ is applied parallel to the $x$ axis, perpendicular to the plane of the film. The value of magnetic field does not exceed the second critical field for a superconductor. Under the impact of transport current directed along the $0y$ axis, the flux-line lattice in the superconductor film moves along the $0z$ axis. We have considered the $TE_{0n}$ modes which effectively interact with flux-line lattice in superconductor.

We have received the dispersion relation for the $TE_{0n}$ modes in the following way (Golovkina, 2009 b):
\[
\frac{k_{x1}}{\mu_1} \cot k_{x1} d_1 + \frac{k_{x2}}{\mu_2} \cot k_{x2} d_2 = -\frac{i \mu_0 \eta t}{\Phi_0} \left( \frac{j\omega \Phi_0 \beta}{\eta} - \omega \right),
\]

(13)

where \( jy_0 \) is transport current density, \( \beta \) is propagation constant, \( \omega \) is angular frequency. The time dependence is \( \exp(\text{i}\omega t) \). One important difference from the two-layered waveguide without superconducting film is a presence of imaginary part of the propagation constant \( \beta = \beta' + \text{i}\beta'' \). The negative values of imaginary part \( \beta \) correspond to the attenuation of electromagnetic waves, the positive values of \( \beta \) correspond to amplification.

Fig. 9. The real and imaginary part of propagating constant \( \beta \) versus normalized frequency.
Parameters: \( B_0 = 0.1 \, \text{T} \), \( jz_0 = 10^6 \, \text{A/m}^2 \), \( \varepsilon_1 = -2 \), \( \mu_1 = -1 \), \( \varepsilon_2 = 1 \), \( \mu_2 = 1 \), \( d_1/d_2 = 3/2 \).
Curve 1: the real part \( \beta' \), curve 2: the imaginary part \( \beta'' \).

Fig. 10. The real and imaginary part of propagating constant \( \beta \) versus normalized frequency.
Parameters: \( B_0 = 0.1 \, \text{T} \), \( jz_0 = 10^6 \, \text{A/m}^2 \), \( \varepsilon_1 = -2 \), \( \mu_1 = -1 \), \( \varepsilon_2 = 1 \), \( \mu_2 = 1 \), \( d_1/d_2 = 1/4 \).
Curve 1: the real part \( \beta' \), curve 2: the imaginary part \( \beta'' \).
Numerical calculations have resulted in the following. If the velocity of Abricosov vortex lattice is small, the imaginary part of $\beta$ is negative (Fig. 9). When the frequency is less than cutoff frequency of usual two-layered waveguide, the imaginary part $\beta''$ is large, and the electromagnetic wave attenuates. When the frequency is greater than cutoff frequency of usual two-layered waveguide, the imaginary part $\beta''$ becomes smaller and the real part $\beta'$ increases. This means that the electromagnetic wave propagates with smaller attenuation. If the condition of equality of electromagnetic wave velocity and vortex velocity is executed, the imaginary part of propagation constant becomes positive (Fig. 10). The positive values of $\beta''$ indicates the presence of amplification. The amplification depends on frequency, parameters of superconducting film and magnetic field magnitude. It is essential to note that the significant amplification is observed below from cutoff frequency of the two-layered waveguide. In this frequency domain the growing evanescent waves increase as well because of coupling to the moving vortex in superconductor. The dependence of coefficient of amplification (or attenuation) on the external magnetic field allows to create the amplifiers and filters controlled by magnetic field on the basis of the considered above waveguide structure.

6. Conclusion

In this chapter we examined some of the most interesting features of one-dimensional layered structure containing the negative index metamaterials. The greater number of features of these structures has resulted from the point that the waves in the negative index material are backward, since the phase and group velocities are antiparallel. In periodic structures with negative index metamaterials the number and position of bandgaps changes, new type of bandgaps appears. The presence of thin superconducting film in resistive state in one-dimensional periodic structure can cause the attenuation or amplification of electromagnetic waves by means of moving Abricosov vortex lattice in superconducting layer. The heavy dependence of amplification parameters on the value of external magnetic field allows managing the electromagnetic waves propagation in structures with superconducting films. On the basis of periodical structures with negative index material layers and thin superconducting film it is highly possible to create new devices such as filters and amplifiers controlled by magnetic field.

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Yu, P.Y. & Cardona, M. (2005). *Fundamentals of semiconductors: physics and material properties*, Springer, ISBN 3540254706.
In the recent decades, there has been a growing interest in micro- and nanotechnology. The advances in nanotechnology give rise to new applications and new types of materials with unique electromagnetic and mechanical properties. This book is devoted to the modern methods in electrodynamics and acoustics, which have been developed to describe wave propagation in these modern materials and nanodevices. The book consists of original works of leading scientists in the field of wave propagation who produced new theoretical and experimental methods in the research field and obtained new and important results. The first part of the book consists of chapters with general mathematical methods and approaches to the problem of wave propagation. A special attention is attracted to the advanced numerical methods fruitfully applied in the field of wave propagation. The second part of the book is devoted to the problems of wave propagation in newly developed metamaterials, micro- and nanostructures and porous media. In this part the interested reader will find important and fundamental results on electromagnetic wave propagation in media with negative refraction index and electromagnetic imaging in devices based on the materials. The third part of the book is devoted to the problems of wave propagation in elastic and piezoelectric media. In the fourth part, the works on the problems of wave propagation in plasma are collected. The fifth, sixth and seventh parts are devoted to the problems of wave propagation in media with chemical reactions, in nonlinear and disperse media, respectively. And finally, in the eighth part of the book some experimental methods in wave propagations are considered. It is necessary to emphasize that this book is not a textbook. It is important that the results combined in it are taken "from the desks of researchers". Therefore, I am sure that in this book the interested and actively working readers (scientists, engineers and students) will find many interesting results and new ideas.

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