Three-parton contribution to the $B \to \pi$ form factors in $k_T$ factorization

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We calculate the three-parton twist-3 contribution to the $B \to \pi$ transition form factors in the $k_T$ factorization theorem. Since different mesons are involved in the initial and final states, two(three)-parton-to-three(two)-parton amplitudes do not vanish. It is found that the dominant contribution arises from the diagrams with the additional valence gluon attaching to the leading-order hard gluon. Employing the three-parton meson distribution amplitudes from QCD sum rules, we show that this subleading piece amounts only up to few percents of the form factors at large recoil of the pion. The framework for analyzing three-parton contributions to $B$ meson decays in the $k_T$ factorization is established.

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I. INTRODUCTION

The $k_T$ factorization theorem is a theoretical framework appropriate for QCD processes dominated by dynamics at small parton momenta \cite{1-6}. With continuous efforts, progress has been made in the application of the $k_T$ factorization to exclusive processes at subleading level: two-parton twist-3 contributions to the pion form factor and to the $B$ meson transition form factors have been analyzed in \cite{7} and in \cite{8-11}, respectively. These contributions are formally power-suppressed, but numerically crucial for accommodating experimental data or lattice QCD results. Corrections to the pion transition form factor and to the pion form factor at next-to-leading order (NLO) in the coupling constant were calculated in \cite{12} and in \cite{13}, respectively. These NLO pieces can be minimized by choosing a factorization scale close to virtuality of internal particles, and found to be few percents in the former \cite{14} and about 30% in the latter \cite{13}. The three-parton twist-3 contribution to the pion form factor was first formulated and evaluated in the $k_T$ factorization in \cite{15}, and the smallness of this subleading piece (about few percents) was confirmed.

In this paper we shall compute the three-parton twist-3 contribution to the $B \to \pi$ transition form factors, which is down by a power of $1/m_B$, $m_B$ being the $B$ meson mass. We stress that there are many sources of $1/m_B$ corrections. The two-parton twist-3 one mentioned above is suppressed by $m_0/m_B$ with the chiral scale $m_0 \approx 1.4$ GeV. It is the reason why this piece is numerically important, namely, of the same order as the leading-twist one. Another sizable source arises from the difference between the two leading-twist $B$ meson distribution amplitudes \cite{9,16,17}, which can contribute about 30% of the form factors. Other power-suppressed pieces are of order $\Lambda_{QCD}/m_B$, $\Lambda_{QCD}$ being the QCD scale, and should be negligible. The $B$ meson distribution amplitudes from higher-twist spin projectors and associated with the three-parton Fock states belong to this category. We shall demonstrate that the three-parton contribution is only few percents of the $B \to \pi$ transition form factors, consistent with the observation made in the light-cone QCD sum rules \cite{18}.

II. GAUGE INVARIANCE

Compared to the collinear factorization \cite{19}, the construction of the $k_T$ factorization is subtler. For example, the gauge invariance of the $k_T$ factorization, in which parton transverse momenta are retained, becomes an issue \cite{20}. The gauge invariance of the $k_T$ factorization for the $B \to \pi$ transition form factors at the three-parton twist-3 level can be proved in a way similar to the case of the pion form factor \cite{15}. We display in Fig. 1 the leading-order (LO) diagrams, and in Fig. 2 the attachments of an additional valence gluon from the pion to all the lines in the LO diagrams, except the valence quark lines in the pion. There are two sources of gauge dependence \cite{21}, which arise from the patron transverse momentum in Fig. 1 and from the three-parton Fock state in Fig. 2. We shall show in this section that

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FIG. 1: Leading-order diagrams for the $B \to \pi$ transition form factors, where the symbol $\times$ represents the weak decay vertex.

\[
\begin{array}{cccc}
B & A & \times & C \\
D & & & \\
\end{array}
\]

\[
\begin{array}{cccc}
E & F & \times & G \\
H & & & \\
\end{array}
\]

FIG. 2: Attachments of an additional valence gluon in the pion to lines in Fig. 1.

The gauge-dependent amplitudes cancel in each of the two sources. The proof for the amplitudes with three partons from the $B$ meson side is the same.

The $B$ meson momentum $P_1$ and the pion momentum $P_2$ are parameterized as

\[
P_1 = (P_1^+, P_1^-, 0_T) = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad P_2 = (0, P_2^-, 0_T) = \frac{m_B}{\sqrt{2}}(0, \eta, 0_T),
\]

where the energy fraction $\eta = 1 - q^2/m_B^2$ carried by the pion ranges between 0 and 1. The momenta of the antiquarks in the $B$ meson and in the pion, represented by the lower fermion line, are parameterized as

\[
k_1 = (x_1 P_1^+, 0, k_{1T}), \quad k_2 = (0, x_2 P_2^-, k_{2T}),
\]

respectively, $x_1$ and $x_2$ being the momentum fractions. It is understood that the components $k_1^-$ and $k_2^+$ have been dropped in hard kernels, and integrated out of the $B$ meson and pion wave functions, respectively. The gluon propagator of momentum $l$ is written as

\[
-\frac{i}{l^2} \left( g^{\sigma\nu} - \lambda^{\sigma\nu} \frac{l^2}{l^2} \right),
\]

in the covariant gauge, where the parameter $\lambda$ is used to identify sources of gauge dependence.

We sandwich Fig. 1(a) with the spin projectors

\[
\frac{1}{4N_c} (P_1 + m_B)\gamma_5, \quad \frac{1}{4N_c} \gamma_5 \gamma_\beta,
\]

from the initial and final states, respectively, where $N_c = 3$ is the number of colors, $\gamma_5 \gamma_\beta$ is a higher-twist projector selected for the proof below, and the subscript $\beta$ takes the transverse components. The resultant hard kernel contains the gauge-dependent piece

\[
H^{a\lambda} = \frac{1}{16} g^2 C_F \frac{1}{N_c} \lambda tr[\gamma^\sigma \gamma_5 \gamma_\beta \gamma_\mu (1 - \gamma_5)(P_1 - k_2 + m_b)\gamma^\nu (P_1 + m_B)\gamma_5] \frac{(k_1 - k_2)_{\sigma}(k_1 - k_2)_{\nu}}{(k_1 - k_2)^2},
\]

with the $b$ quark mass $m_b$. In the small $x$ region where the $k_T$ factorization applies, we keep the transverse momentum dependence in the denominator [22]. The transverse momentum dependence in the numerator belongs to the twist-3.
contribution. Inserting the identity $k_1 - k_2 = (P_1 - k_2 - m_b) - (P_1 - k_1 - m_b)$ for the gluon vertex on the $b$ quark line, we find that the second term vanishes at leading-twist accuracy on the $B$ meson side, as it is multiplied by $P_1 + m_B$. The derivative of the numerator with respect to $k_{2T}^\beta$ gives

$$H_T^{2\lambda} = -\frac{g^2 C_F}{16 N_c} (k_1 - k_2)^4 \left[ g_\lambda^{\beta\gamma^\lambda} (1 - \gamma_5) (P_1 + m_B) \gamma_5 \right].$$

(6)

The LO hard kernel from Fig. 1(b) contains the gauge-dependent amplitude

$$H_T^b = \frac{1}{16} g^2 C_F \lambda \left[ g_\lambda^{\beta\gamma^\lambda} (P_2 - k_2) \gamma_5 (1 - \gamma_5) (P_1 + m_B) \gamma_5 \right] \frac{(k_2 - k_1)^2 (k_1 - k_2)^2}{(k_1 - k_2)^4}.$$

(7)

The similar differentiation with respect to $k_{2T}^\beta$ leads to

$$H_T^{2\lambda} = -\frac{1}{16} g^2 C_F \lambda \left[ g_\lambda^{\beta\gamma^\lambda} (P_2 - k_2) \gamma_5 (1 - \gamma_5) (P_1 + m_B) \gamma_5 \right] \frac{(k_2 - k_1)^2 (k_1 - k_2)^2}{(k_1 - k_2)^4}.$$

(8)

For the first term in the above expression, $k_1$ ($k_2$) implies one more derivative of the spectator field on the $B$ meson (pion) side, so it is neglected. The second term, after employing $k_1 - k_2 = (P_2 - k_2) - (P_2 - k_1)$, gives

$$H_T^{2\lambda} = \frac{1}{16} g^2 C_F \lambda \left[ g_\lambda^{\beta\gamma^\lambda} (1 - \gamma_5) (P_1 + m_B) \gamma_5 \right] \frac{(k_2 - k_1)^2 (k_1 - k_2)^2}{(k_1 - k_2)^4}.$$

(9)

The corresponding amplitude is written as

$$H_A^b = \frac{1}{16} g^2 C_F \lambda \left[ g_\lambda^{\beta\gamma^\lambda} (1 - \gamma_5) (P_1 - k_2 - k_2 - m_b) \gamma_5 (P_1 + m_B) \gamma_5 \right] \frac{(k_1 - k_2 - l_2)^2 - m_b^2 (k_1 - k_2)^4}{(k_1 - k_2)^4}.$$

(10)

According to the above explanation, if the vertex on the spectator line contains $k_1 - k_2$, the associated term comes from the derivative of the spectator field, and should be dropped. Hence, the gauge dependence can appear only in the first term linear in $\lambda$, which leads to

$$H_C^\lambda = \frac{1}{32} g^2 C_F \lambda \left[ g_\lambda^{\beta\gamma^\lambda} (1 - \gamma_5) (P_1 + m_B) \gamma_5 \right] \frac{(k_1 - k_2 - l_2)^2 - m_b^2 (k_1 - k_2)^4}{(k_1 - k_2)^4}.$$

(11)
The evaluation of the gauge-dependent pieces for the rest of attachments is similar. With the color factor for the attachment $D$, $\text{tr}[T^k T^\alpha T^\beta T^\gamma] = -\delta^{\alpha\beta}/(4N_c)$, the corresponding amplitude is written as
\begin{equation}
H^\lambda_D = \frac{1}{32} g^2 \frac{1}{N_c^2} \lambda \frac{\text{tr}[\gamma_\beta \gamma_\gamma \gamma_\mu (1 - \gamma_5)(P_1 + m_B)\gamma_5]}{(k_1 - k_2 - l_2)^4}.
\end{equation}

The gauge-dependent amplitudes from the attachments $E$ and $F$ diminish. The attachments $G$ and $H$ give
\begin{align}
H^\lambda_G &= \frac{1}{32} g^2 \frac{1}{N_c^2} \lambda \frac{\text{tr}[\gamma_\beta \gamma_\gamma \gamma_\mu (1 - \gamma_5)(P_1 + m_B)\gamma_5]}{(k_1 - k_2 - l_2)^4}, \\
H^\lambda_H &= \frac{1}{32} g^2 \frac{1}{N_c^2} \lambda \frac{\text{tr}[\gamma_\beta \gamma_\gamma \gamma_\mu (1 - \gamma_5)(P_1 + m_B)\gamma_5]}{(k_1 - k_2 - l_2)^4},
\end{align}
respectively. The cancellation between Eqs. (11) and (13), and between Eqs. (12) and (14) is observed. That is, the gauge dependence from the three-parton Fock state also disappears. This completes the proof of the gauge invariance of the $k_T$ factorization for the $B \to \pi$ transition form factors at leading order in $\alpha_s$ and at three-parton twist-3 level.

III. THREE-PARTON CONTRIBUTIONS

In this section we calculate the $B \to \pi$ transition form factors $F_+$ and $F_0$ involved in the semileptonic decay $B(P_1) \to \pi(P_2)\ell\nu$,
\begin{equation}
\langle \pi(P_2)|\bar{b}(0)\gamma_\mu u(0)|B(P_1)\rangle = F_+(q^2) \left[(P_1 + P_2)_\mu - \frac{m_B^2}{q^2}q_\mu\right] + F_0(q^2)\frac{m_B^2}{q^2}q_\mu,
\end{equation}
where $q = P_1 - P_2$ is the lepton-pair momentum. Another equivalent definition is given by
\begin{equation}
\langle \pi(P_2)|\bar{b}(0)\gamma_\mu u(0)|B(P_1)\rangle = f_1(q^2)P_{1\mu} + f_2(q^2)P_{2\mu},
\end{equation}
in which the form factors $f_1$ and $f_2$ are related to $F_+$ and $F_0$ via
\begin{equation}
F_+ = \frac{f_1 + f_2}{2}, \quad F_0 = \frac{f_1}{2} \left(1 + \frac{q^2}{m_B^2}\right) + \frac{f_2}{2} \left(1 - \frac{q^2}{m_B^2}\right).
\end{equation}

We start with the hard kernels from the two-parton-to-three-parton diagrams in the Feynman gauge ($\lambda = 0$). The following matrix element 23 defines the three-parton twist-3 pion wave function $T(z, z')$,
\begin{equation}
(0|\bar{q}(z)\gamma_\alpha \gamma_5 q G^+_{\alpha}(z')q(0)|\pi(P_1)) = i f_2 m_0 (P_1^+)\gamma_\alpha T(z, z'),
\end{equation}
with the chiral scale $m_0 = m^2_\pi/(m_u + m_d)$, $m_\pi$, $m_\nu$, and $m_d$ being the pion, $u$, and $d$ quark masses, respectively. The three momenta $P_1 - k_2 - l_2$, $k_2$, and $l_2$ are assigned to the final-state quark, antiquark, and gluon, respectively.

For the calculation, we replace the projector for the pion in Eq. (1) by $\gamma_5 P_2 \gamma_3 m_0/(4y_2)$ 13, where the valence gluon momentum fraction is defined by $y_2 = l_2^2/P_2^2$, the gamma matrix $\gamma^T$ contains only transverse components, and the pion decay constant has been absorbed into the wave function $T(z, z')$.

The amplitudes from the attachments $A$, $B$, $\cdots$, $H$ in Fig. 2 are collected as follows:
\begin{align}
H^\lambda_A &= \frac{g^2 C_F}{2N_c y_2} \left[\frac{1}{(P_1 - k_2 - l_2)^2 - m^2_B} + \frac{1}{(P_1 - k_2)^2 - m^2_B}\right] m_B m_0 P_{2\mu}, \\
H^\lambda_B &= 0, \\
H^\lambda_C &= \frac{g^2 \eta(x_2 - y_2)}{8\eta y_2(x_2 + y_2)} \frac{m_B m_0 P_{2\mu}}{(k_1 - k_2)^2(k_1 - k_2 - l_2)^2}, \\
H^\lambda_D &= -\frac{g^2}{4N_c^2} \frac{1}{x_2 + y_2 (k_1 - l_2)^2(k_1 - k_2 - l_2)^2} m_B m_0 P_{2\mu}, \\
H^\lambda_E &= H^\lambda_E, \quad H^\lambda_F = H^\lambda_F = 0, \\
H^\lambda_G &= -\frac{g^2}{8y_2} \frac{m_B m_0 (P_{2\mu} + k_{1\mu})}{(k_1 - k_2)^2(k_1 - k_2 - l_2)^2}. 
\end{align}
The denominators of Eqs. (19) and (21) indicate that the contribution from the former is down by a power of $k_1^2/m_B \sim \Lambda_{\text{QCD}}/m_B$. That is, the attachments to the $b$ quark line and to the energetic parton line of the pion give power-suppressed contributions in the domain region with soft spectator momenta. This observation is similar to that obtained in the study of the three-parton twist-3 contribution to the pion form factor \[15\]. Equation (20) vanishes, since the $\gamma$ matrix associated with the valence gluon attachment takes only the transverse components. One can then flip the $b$ quark propagator and this $\gamma$ matrix, and apply $(P_1 - k_1 - m_b)(P_1 + m_B) \approx 0$ at leading-twist accuracy on the $B$ meson side. The attachments $E, F$, and $H$ do not contribute as shown in Eq. (23), simply because of $\gamma_\nu \gamma_5 P_2^\gamma \gamma^\nu = 0$. The $k_{1\mu}$ term in Eq. (24) is of higher-power and negligible.

It is found that all the above amplitudes are proportional to $m_B$, namely, diminish as $m_B \rightarrow 0$. This must be the case, since the two(three)-parton-to-three(two)-parton diagrams do not contribute to the pion form factor \[15\]. In the numerical analysis below we shall not differentiate $m_B$ and $m_b$, whose difference gives an additional power of $1/m_B$.

Ignoring Eq. (19) and the second term in Eq. (24), the two-parton-to-three-parton amplitudes are summed into

$$H^{2\rightarrow 3} = -\frac{g^2}{8(x_2 + y_2)} \left[ \frac{2ny_2 + x_1}{\eta y_2} \frac{1}{(k_1 - k_2)^2} + \frac{1}{N_c^2 (k_1 - l_2)^2} \right] \frac{m_B m_0 P_{2\mu}}{(k_1 - k_2 - l_2)^2}. \quad (25)$$

To derive the above expression, we have followed the hierarchy among the relevant scales $x m_B^2 \gg k_1^2$. \[13\] Under which the $k_T$-dependent terms in the denominators of the $b$ quark and energetic quark propagators are dropped.

For the three-parton-to-two-parton amplitudes, we need to introduce the three-parton $B$ meson distribution amplitude. Consider the following matrix elements associated with the $B$ meson \[24\]

$$\langle 0|\bar{q}\sigma \cdot g_E \gamma_\nu h_v|B(v)\rangle = F(\mu)\lambda^E_2(\mu),$$
$$\langle 0|\bar{q}\sigma \cdot g_H \gamma_\nu h_v|B(v)\rangle = iF(\mu)\lambda^H_2(\mu), \quad (26)$$

where $E^i = G^{a\mu}$ and $H^i = (-1/2)e^{ijk}G^{jk}$ are the chromoelectric and chromomagnetic fields, respectively, $h_v$ is the effective heavy quark field, $v = (1,0)$ is the $B$ meson velocity, and $\mu$ is the renormalization scale. The normalization $F(\mu) = f_B\sqrt{m_B} + O(\alpha_s, 1/m_B)$, $f_B$ being the $B$ meson decay constant, is defined via $\langle 0|\bar{q}\gamma_\nu \gamma_5 h_v|B(v)\rangle = iF(\mu)v_\nu$. The analysis based on QCD sum rules in \[24\] and \[22\] led to the values

$$\lambda^E_2(1 \text{ GeV}) = (0.11 \pm 0.06) \text{ GeV}^2, \quad \lambda^H_2(1 \text{ GeV}) = (0.18 \pm 0.07) \text{ GeV}^2,$$  \quad \lambda^E_2(1 \text{ GeV}) = (0.03 \pm 0.02) \text{ GeV}^2, \quad \lambda^H_2(1 \text{ GeV}) = (0.06 \pm 0.03) \text{ GeV}^2, \quad (27)$$

respectively. The two matrix elements in Eq. (26) can be reexpressed as

$$\langle 0|\bar{q}\sigma^+_{\alpha_1} \gamma_5 g G^{+\alpha_2} h_v|B(v)\rangle = iF(\mu)\lambda^T_2(\mu)g^T_{\alpha_1\alpha_2},$$  \quad (29)$$

with the normalization factors $\lambda^T_2 \equiv (\lambda^E_2 + \lambda^H_2)/2 \approx 0.145 (0.045) \text{ GeV}^2$ and $\lambda^T_2 \equiv (\lambda^E_2 - \lambda^H_2)/2 \approx -0.035 (-0.015) \text{ GeV}^2$ from Eq. (27) [Eq. (28)].

The three momenta $P_1 - k_1 - l_1$, $k_1$ and $l_1$ are assigned to the initial-state $b$ quark, antiquark, and gluon, respectively. Equation (28) corresponds to the spin projector $\gamma^+ \gamma^- H_0^+ \gamma \sqrt{2}\lambda^T_2/(4y_1)$ from the $B$ meson side, where the decay constant $f_B$ has been absorbed into the three-parton $B$ meson distribution amplitude, and the valence gluon momentum fraction is defined by $y_1 = l_1^+/P_1^+$. For the attachments of the valence gluon in the $B$ meson to the lines in Fig. (11a), only the one to the hard gluon contributes, because of $\gamma^\nu \gamma^+ \gamma^- \gamma_5 \gamma_\nu = 0$. All attachments to the lines in Fig. (11b) do not contribute, since the corresponding Feynman rules have no $m_B$ dependence. As explained before, the three-parton-to-two-parton amplitudes must be proportional to $m_B$. Assuming the same three-parton distribution amplitudes associated with the normalization constants $\lambda^T_2$, we derive

$$H^{3\rightarrow 2} = -\frac{g^2}{32y_1}\sqrt{2}\lambda^T_2 \left[ \frac{\text{tr}[P_1 - k_2)^2 - m_B^2][(k_1 - k_2)^2]}{(k_1 + l_1 - k_2)^2} \right] m_B$$
$$= \frac{g^2}{32y_1} \sqrt{2}\lambda^T_2 \left[ \frac{\text{tr}[P_2 \gamma_\mu \gamma_5 \gamma_\nu] m_B}{(k_1 - k_2)^2} \right] \frac{P_{2\mu}}{(k_1 + l_1 - k_2)^2}. \quad (31)$$

It has been known that the form factor $f_1$ is suppressed by $m_0/m_B$ compared to $f_2 \quad \[8\]$. Therefore, it is natural that the three-parton contribution corrects only $f_2$ at the accuracy considered here, which is summarized as

$$f_2^{3p}(q^2) = f_2^{2\rightarrow 3}(q^2) + f_2^{2\rightarrow 2}(q^2), \quad (32)$$
with the factorization formulas

\[ f_2^{2\to3} = \int dx_1 dx_2 dy_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \Phi_\pi(x_2, y_2) \exp[-s(P_2^-, b_2)] \]

\[ \times [h_1^{2\to3}(x_1, x_2, y_2, b_1, b_2) + h_2^{2\to3}(x_1, x_2, y_2, b_1, b_2)], \]

\[ f_3^{2\to2} = \int dx_1 dy_1 dx_2 \int b_1 db_1 b_2 db_2 \Phi_B(x_1, y_1, b_1, b_2) \phi_\pi(x_2) \exp[-s(P_2^-, b_1)] h_3^{2\to2}(x_1, y_1, x_2, b_1, b_2). \] (33)

We have neglected the intrinsic \( b \) dependence of the pion distribution amplitudes, because the suppression of the Sudakov factor \( \exp[-s(P_2^-, b)] \) is strong enough in the large \( b \) region [9, 29]. On the contrary, the Sudakov effect associated with the \( B \) meson is weak, since it is dominated by soft dynamics. For the \( B \) meson distribution amplitudes, the intrinsic \( b \) dependence is more effective. The hard kernels are written as

\[ h_1^{2\to3}(x_1, x_2, y_2, b_1, b_2) = -\frac{\pi}{2} \frac{2u_2 + x_1}{\eta y_2 (x_2 + y_2)} m_B m_0 \alpha_s K_0 \left( \sqrt{x_1 (x_2 + y_2) \eta y_2 b_2} \right) \]

\[ \times [\theta(b_1 - b_2) K_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_1}) I_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_2}) + \theta(b_2 - b_1) K_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_2}) I_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_1})], \]

\[ h_2^{2\to3}(x_1, x_2, y_2, b_1, b_2) = -\frac{\pi}{N_c} \frac{m_B m_0}{x_2 + y_2} \alpha_s K_0 \left( \sqrt{x_1 (x_2 + y_2) \eta y_2 b_2} \right) \]

\[ \times [\theta(b_1 - b_2) K_0 (\sqrt{x_1 y_2 \eta y_2 m_B b_1}) I_0 (\sqrt{x_1 y_2 \eta y_2 m_B b_2}) + \theta(b_2 - b_1) K_0 (\sqrt{x_1 y_2 \eta y_2 m_B b_2}) I_0 (\sqrt{x_1 y_2 \eta y_2 m_B b_1})], \]

\[ h_3^{2\to2}(x_1, y_1, x_2, b_1, b_2) = \frac{\pi}{y_1} \left( \frac{x_1 + 2y_1}{\eta y_2} \right) \alpha_s K_0 \left( \sqrt{(x_1 + y_1) x_2 y_2 \eta y_2 b_2} \right) \]

\[ \times [\theta(b_1 - b_2) K_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_1}) I_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_2}) + \theta(b_2 - b_1) K_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_2}) I_0 (\sqrt{x_1 x_2 \eta y_2 m_B b_1})]. \] (35)

The functional form of the three-parton \( B \) meson distribution amplitude is still unknown in the literature, though there are already studies of its relation to the two-parton ones [9, 29]. Below we shall postulate a simple form for an order-of-magnitude estimate. The involved two-parton and three-parton meson distribution amplitudes are chosen as

\[ \phi_B(x_1, b_1) = N_B f_B x_1^2 (1-x_1)^2 \exp \left[ -\frac{1}{2} \left( \frac{x_1 m_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b_1^2}{2} \right], \] (36)

\[ \Phi_B(x_1, y_1, b_1, b_2) = N_B' f_B x_1^2 (1-x_1-x_1) y_1^2 \exp \left[ -\frac{\omega_B^2}{2} (b_1^2 + b_2^2) \right], \] (37)

\[ \phi_\pi(x_2) = 6 f_\pi x_2 (1-x_2) \left[ 1 + 0.44 C_2^{3/2} (2x_2 - 1) + 0.25 C_4^{3/2} (2x_2 - 1) \right], \] (38)

\[ \Phi_\pi(x_2, y_2) = 360 \eta_\pi f_\pi x_2 (1-x_2-y_2) y_2^2 \left[ 1 - \frac{3}{2} (7 y_2 - 3) \right], \] (39)

with the parameters \( \omega_B = 0.4 \) GeV [27] and \( \eta_\pi = 0.015 \) [30], and the Gegenbauer polynomials

\[ C_2^{3/2}(t) = \frac{3}{2} (5 t^2 - 1), \]

\[ C_4^{3/2}(t) = \frac{15}{8} (21 t^4 - 14 t^2 + 1). \] (40)

The normalization constants \( N_B \) and \( N_B' \) are determined through the relations \( \int dx_1 \phi_B(x_1,0) = \int dy_1 \phi_B(x_1, y_1,0,0) = f_B \). The two-parton \( B \) meson and pion distribution amplitudes have been chosen as in [8] in order to have an appropriate comparison of numerical outcomes.

Equation (32) represents the three-parton contribution to the form factor \( f_2 \), which then corrects the form factors \( F_+ \) and \( F_0 \) via Eq. (17). The numerical results derived from Eq. (32) for \( f_B = 0.2 \) GeV, \( f_\pi = 0.13 \) GeV, \( m_B = 5.28 \) GeV, \( m_0 = 1.4 \) GeV and \( \alpha_s = 0.5 \) are listed in Table I, which confirm the ratio of the three-parton-to-two-parton contribution over the two-parton-to-three-parton one, \( 2 A_2^{3/2} / (m_B m_0) \eta_\pi \approx 2.6 \) (0.8) from Eq. (27) [Eq. (28)]. The dominant contribution arises from the diagrams with the additional valence gluon attaching to the leading-order hard gluon, i.e., from Eqs (21) and (23). Figure 8 shows that the three-parton contribution amounts only up to few percent of the \( B \to \pi \) transition form factors \( F_+(0) = F_0(0) \approx 0.3 \) at large recoil of the pion. The relative importance is obvious from the order-of-magnitude estimate \( \eta_\pi m_0 / t \sim \lambda_1^2 / (m_B t) \sim 1\% \), in which the scale \( \eta_\pi m_0 \) \( \left( \lambda_1^2 / m_B \right) \) is associated with the spin projector of the three-parton pion (\( B \) meson) distribution amplitude, and \( t \sim 1.7 \) GeV denotes
TABLE I: Two-parton-to-three-parton and three-parton-to-two-parton contributions to $f_2(q^2)$ corresponding to Eq. (27) (upper half) and to Eq. (28) (lower half).

| $q^2$ (GeV$^2$) | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| $2 \rightarrow 3$ | -0.463 | -1.13 | -1.237 | -1.314 | -1.391 | -1.472 | -1.562 | -1.662 | -1.774 | -1.901 | -2.046 |
| $3 \rightarrow 2$ | 1.223 | 2.885 | 2.911 | 2.982 | 3.091 | 3.233 | 3.407 | 3.614 | 3.858 | 4.142 | 4.476 |
| total(10$^{-2}$) | 0.761 | 1.754 | 1.167 | 1.167 | 1.700 | 1.845 | 1.952 | 2.083 | 2.241 | 2.429 |

| $q^2$ (GeV$^2$) | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| $2 \rightarrow 3$ | -0.463 | -1.13 | -1.237 | -1.314 | -1.391 | -1.472 | -1.562 | -1.662 | -1.774 | -1.901 | -2.046 |
| $3 \rightarrow 2$ | 0.376 | 0.888 | 0.896 | 0.918 | 0.951 | 0.996 | 1.049 | 1.114 | 1.189 | 1.277 | 1.380 |
| total(10$^{-2}$) | -0.086 | -0.243 | -0.341 | -0.396 | -0.439 | -0.477 | -0.512 | -0.549 | -0.585 | -0.624 | -0.667 |

In this letter we have extended the investigation of the $B \rightarrow \pi$ transition form factors in the $k_T$ factorization theorem to the three-parton twist-3 level. It was demonstrated that the gauge-dependent pieces cancel each other in the two(three)-parton-to-three(two)-parton diagrams, so the gauge invariance of this formalism is verified. The contributions from the above diagrams were then calculated, and found to be few percents at most, considering the normalization inputs for the three-parton $B$ meson distribution amplitudes from QCD sum rules. The theoretical framework for analyzing three-parton contributions to $B$ meson decays was established in this work, which can be compared to other approaches, such as light-cone sum rules \cite{33}, the QCD (collinear) factorization \cite{34}, and the soft-collinear effective theory \cite{35}.

\begin{align}
\langle F_1 + m_B \rangle \left[ \phi_B(k_1) - \frac{\hat{m}_+ - \hat{m}_-}{\sqrt{2}} \bar{\phi}_B(k_1) - \Delta(k_1) \gamma^\mu \frac{\partial}{\partial k_{1T}^\mu} \right] \gamma_5, \tag{41}
\end{align}

with the dimensionless vectors $n_+ = (1, 0, 0_T)$ and $n_- = (0, 1, 0_T)$. Collecting the observations obtained in the literature, we summarize the various contributions to the $B \rightarrow \pi$ transition form factors: the first term in the above projector, which has been considered in \cite{8}, gives the leading contribution. The second term $\bar{\phi}_B$, proportional to the difference of the two leading-power $B$ meson wave functions, contributes $30\%$ \cite{32}. The third term, proportional to the integration of $\bar{\phi}_B$ in the momentum fraction, and the three-parton Fock state contribute only few percents.

**IV. CONCLUSION**

In this letter we have extended the investigation of the $B \rightarrow \pi$ transition form factors in the $k_T$ factorization theorem to the three-parton twist-3 level. It was demonstrated that the gauge-dependent pieces cancel each other in the two(three)-parton-to-three(two)-parton diagrams, so the gauge invariance of this formalism is verified. The contributions from the above diagrams were then calculated, and found to be few percents at most, considering the normalization inputs for the three-parton $B$ meson distribution amplitudes from QCD sum rules. The theoretical framework for analyzing three-parton contributions to $B$ meson decays was established in this work, which can be compared to other approaches, such as light-cone sum rules \cite{33}, the QCD (collinear) factorization \cite{34}, and the soft-collinear effective theory \cite{35}.
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