Examination of Logic Operations with Silent Phase Qubit

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Abstract. Reduced control Hamiltonian of the silent phase qubit has been found and
dependence of the Hamiltonian matrix elements on applied magnetic field has been analyzed.
The obtained results yield in concept of qubit logic operations accomplished by means of
magnetic field application. One can propose a set of the qubits, each designed for specific logic
operation. The qubit logic specialization may be fixed by both the junction current-phase
relation and the characteristic Josephson-to-Coulomb energy ratio $s$. As current-phase relation
is assigned, the qubit specialization can be tuned by the ratio $s$ which varies with Josephson
junction area. Examples of some logic operations are considered. Estimation of both the
decohherence time and the quality factor $Q$ has been obtained for such a qubit based on high-$T_c$
superconductors, which could provide the required current-phase relation.

1. Introduction
Quantum bit (qubit) is a two-level quantum-mechanical system with macroscopically distinguished
quantum states. Hence, the qubit Hamiltonian can be written as follows:
$$H = -S_0 - S \sigma,$$
where $\sigma = (\sigma_3; \sigma_1; \sigma_2)$, $S = (S_3; S_1; S_2)$; $\sigma_3; \sigma_1; \sigma_2$ – Pauli matrices, $S_0$, $S$ - two-dimensional matrices,
and the state of the qubit can be presented as a vector on the Bloch sphere \cite{1}. All logic operations
with single qubit can be expressed by superposition of two elementary logic gates corresponding to
two different axe-rotations during time $\tau$. One of the rotations is $x$- rotation resulting in the following
evolution operator:
$$U_x(\alpha) = \exp\left(i \frac{\alpha \sigma_3}{2h} \right) = \begin{pmatrix} \cos \alpha / 2 & i \sin \alpha / 2 \\ i \sin \alpha / 2 & \cos \alpha / 2 \end{pmatrix},$$
where $\alpha = \tau S_3 / \hbar$. The second one is $z$- rotation with the corresponding evolution operator:
$$U_z(\beta) = \exp\left(i \frac{\beta \sigma_3}{2h} \right) = \begin{pmatrix} e^{i\beta/2} & 0 \\ 0 & e^{-i\beta/2} \end{pmatrix},$$
where $\beta = \tau S_3 / \hbar$. If $S_3(t) \cdot \tau = \pi$ the $x$- rotation gives the well-known NOT operation, while the
$z$- rotation on a fixed angle $\beta$ gives so-called “controlled phase shift” quantum logic operation.

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Hence the problem of logic operations is associated with analysis of the time-dependent \( S_X \) and \( S_Z \) coefficients. The goal of the paper is to consider elementary logic operations with the silent phase qubit.

2. Silent Phase Qubit

Recently promising type of the phase qubit was suggested \([2], [3]\). The device is just a low-inductive dc interferometer with nonsinusoidal Josephson-junction current-phase relation (CPR) and does not require application of half a flux quantum. Such a qubit was called “silent” because of both the high protection against external magnetic field impact and the absence of any state-dependent spontaneous circular currents.

Current-phase relation of the junctions can be written as follows:

\[
I_j(\phi) = A_j \sin \phi_j - B_j \sin 2\phi_j, \quad j = 1, 2
\]  

The potential energy of the quantum-mechanical system is the following:

\[
U(\theta, \phi) = -\frac{\Phi_0}{2\pi} \left[ A_1 \cos \left( \frac{\phi}{2} + \theta \right) - \frac{1}{2} B_1 \cos(\phi + 2\theta) - A_2 \cos \left( \frac{\phi}{2} - \theta \right) - \frac{1}{2} B_2 \cos(\phi - 2\theta) \right] + \ldots,
\]

where, \( \phi = \phi_1 - \phi_2 \) – difference of the Josephson junction phases, \( \theta = (\phi_1 + \phi_2)/2 \), \( \phi_e \equiv 2\pi \Phi_e/\Phi_0 \) – normalized external flux. In the absence of external magnetic field one can easily come to the conditions responsible for the double-well energy potential formation. If only second harmonic is taken into account we have the following condition:

\[
|2(B_1 + B_2)| > |(A_1 + A_2)|, \quad (A_1 + A_2)/2(B_1 + B_2) > 0.
\]  

If both junctions (with the same CPR) are of the same size the energy potential remains always symmetric and any state-dependent current is impossible even if an external magnetic field is applied. However, at different sizes of the junction (different critical currents) the external magnetic field always breaks the potential symmetry and produces a state-dependent current in the loop. Expanding the potential energy (5) near its minima one can obtain the following equation for potential asymmetry:

\[
e(\phi_e) = E_j(\theta_e') - E_j(\theta_e) = \frac{\Phi_0 I_0}{2\pi} \frac{A_2 A_1 (A_1 - A_2)}{4 (A_1 + A_2)^3 (B_1 + B_2)} \sqrt{4(B_2 + B_1)^2 - (A_2 + A_1)^2 \phi^2_e},
\]  

**Figure 1.** Qubit potential and qubit wave-functions in the absence of external magnetic flux. The CPRs of the junctions are identical: \( A_1 = A_2 = 1 \); \( B_1 = B_2 = 0.8 \).
The induced small current allows both the read-out process and the qubit coupling. In such a way, the applied magnetic field can be used to “switch on” this qubit to provide both the read-out procedure and coupling with the other qubits.

2.1. Level Splitting

In the frame of quantum mechanical oscillator model with negligible inductance the ground energy level of the system in the absence of external magnetic field goes down to zero with increase of the amplitude of the second harmonic, until the threshold condition (6) becomes feasible. Since then, the potential (5) should be approximated near new points of local minima by two symmetric parabolic wells, and we come to the following expressions for energy levels:

$$E_0 = \frac{(2\pi)^2}{2}, \quad E_i = 3\pi e\sigma \omega = \frac{2E_{Q0}}{e} \sqrt{s(B_1 + B_2)[1 - (A_1 + A_2)^2 / 4(B_1 + B_2)^2]} / I_0$$

(8)

where \(s = E_C/E_{Q0}\) – ratio of characteristic Josephson energy to Coulomb energy. This ratio depends on junction size and therefore plays role of scaling parameter.

Phase tunneling between potential wells (see Fig. 1) causes splitting of the energy levels. In particular, the ground level splitting in the absence of external magnetic field is as follows [3]:

$$\Delta = V \times \exp\left(-2a(E_0)\sqrt{(V - E_0)/E_{Q0}}\right)$$

(9)

where \(V\) – the height of potential barriers between the potential wells, \(a(E_0)\) – the barrier width measured at level \(E_0\). The equations (8) and (9) allow estimating of both the decoherence time and the quality factor of the qubit.

Fig. 3 shows splitting \(\Delta\) versus second harmonic amplitude. When the double-well potential is only just formed the barrier \(V \ll E_0\). The difference \((E_0 - V)\) decreases with harmonic amplitude. The decrease in the difference provides increase of splitting gap which peaks at \(E_0 = V\). At \(V > E_0\) the gap exponentially falls. One can see that decrease of ratio \(s\) shifts the peak position to the higher value of harmonic amplitude and increases the peak magnitude.

We can use the split ground level as a qubit basis well-decoupled from higher energy levels, when the splitting \(\Delta\) exceeds the level of quantum noise \(E_N \approx \pi h / 4R_C\) [4]. The high enough value \(\Delta\) can be achieved with increase of scaling parameter \(s\), even if the charge energy \(E_{Q0}\) goes down the level of quantum noise.

Figure 2. Ground energy level \(E_0\) and the barrier height \(V\) dependences on second harmonic amplitude in single-well \((\alpha = (B_1 + B_2)/2I_0 < 0.5)\) and double-well \((\alpha = (B_1 + B_2)/2I_0 > 0.5)\) potential.
3. Logic Operations with Silent Qubit

Influence of applied magnetic field results in coming together $E_0$ and $V$. Therefore the equality $E_0 = V$ providing maximum gap value is achieved at less harmonic amplitude. Fig. 3 shows the changes in $\Delta$ caused by magnetic field. It is seen, that the point showing maximum gap value just moves down the left side line of the dependence. The specific impact of magnetic field on the splitting gap can be used as a base of logic operation technique.

In our case, the time-dependent coefficient $S_x(t)$ $S_z(t)$ in (1) is $\delta(t)$ changing of the splitting gap $\Delta$, and $\epsilon(t)$ - the caused by magnetic field difference between local minima of the potential (5). Therefore we may associate realization of the basic logic operations with two different types of qubits specialized for concrete logic operation.

Both qubit types should answer condition $V < E_0$. Qubits of the first type have the splitting gap value close to maximum one. In this case, magnetic field sufficiently decreases the gap and provide time-dependent coefficient $\delta(t)$. This allows execution of $x$- rotation according to (3), i. e. NOT operation, which is one of two elementary logic operations with single qubit. Assuming $E_{Q0} \approx 2$ GHz and $\delta(t) \approx E_{Q0}$, that seems to be rather realistic [7], one can come to the estimation $\tau \approx 0.5$ ns for the time span required for NOT-operation. In the qubits of the first type the $z$- rotation during this time span is negligible small due to cubic dependence of the small external flux $\phi_e$ (see equation 7).

Qubits of the second type should have splitting gap which is less enough than the maximum one. In such a case, magnetic field does not change the gap (i. e. $\delta(t) = 0$) and provides only $\epsilon(t)$. This means that we execute controlled phase shift operation (2), which is the other elementary logic operation with single qubit.

In summary we can conclude that the use of the two specialized types of qubits completely solves the problem of single-qubit logic operations. The necessary qubit tuning (gap value setting) can be achieved by scaling of the junction sizes that result in changes of ratio $s$.

Figure 3. The ground energy level splitting $\Delta$ versus the total second harmonic amplitude $(B_1 + B_2)$ at $s = 160$ (solid lines) and $s = 40$ (dashed lines) when magnetic flux is not applied (regular lines) and flux of about $0.05\Phi_0$ is applied (thin lines). The segment between two solid circles is the gap change $\delta(t)$ at $s = 160$ while the open circle shows the unchanged gap value at $s = 40$, both at the same harmonic amplitude $(B_1 + B_2)/2I_0 = 0.62$. 

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4. Qubit realization

It has been recently reported that the high second harmonic, sufficient for the double-well potential formation, is observed for high-Tc YBCO junctions [5]. In case of the qubit based on high-Tc Josephson junctions, the dominant contribution to decoherence processes comes from nodal quasiparticle current [6]. This leads to the following estimations of the decoherence time $\tau^*$ and quality factor $Q$:

$$\tau^* = \frac{4e}{D^2 I(\Delta/e)},$$

$$Q = \tau^* \Delta/2h,$$

where $I(\Delta/e)$ is the nodal quasi-particle current, $D$ – distance between the potential minima. Using the experimental data given in [7], we have calculated $\Delta$ and have come to the following figures: $\tau^* \approx 0.1 \mu s$, $Q \approx 10^4$. These values are accepted as high enough for logic operations and error-correction algorithms.

To demonstrate the possibility of the silent phase qubit construction without exploiting of the high-Tc superconductors, we have made the proper calculations for the other candidates – SFS (Superconductor-Ferromagnet-Superconductor) structures based on the published experimental data. The calculations were made in the framework of the model based on Gor’kov equations taking into account s-d scattering in the ferromagnetic layer[8]. It is well known that the scattering of semi-free s-p electrons into d-band plays main role in the transport phenomena for the transition metals and their alloys [9]. This is a consequence of the dominant contribution of d-state density to the total density at Fermi energy level. The s-d scattering in the ferromagnetic layers effectively destroys the BCS correlation resulting from proximity effect. This model has allowed to describe quite satisfactorily the experimental data for SFS Josephson junctions consisting of niobium banks and a spacer of weak ferromagnetic alloy Cu$_{0.47}$Ni$_{0.53}$ [8]. The calculations show a region of the F-layer thickness where the condition for the double-well potential (6) could be satisfied. The dependence of harmonic amplitudes $A$, $B$, and $C$ on the ferromagnetic layer thickness is presented in Fig.4. In such a way, one can conclude, that the silent phase qubit may be also implemented on the base of a low-inductance dc-SQUID with two SFS Josephson junctions.

Figure 4. The absolute values of harmonic amplitudes $A$, $B$, and $C$ versus the ferromagnetic layer thickness of Nb- Cu$_{0.47}$Ni$_{0.53}$ – Nb Josephson junction. On the right plot presented the behaviour of the harmonic amplitudes in the region of the F-layer thickness where the condition for the double-well potential existence could be satisfied.
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