Energy Dependence of the Contribution of Pion Exchange to Large-Rapidity-Gap Events in Deep Inelastic Scattering

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Abstract

We study the energy dependence of the contribution of pion exchange to large-rapidity-gap events in deep inelastic scattering. The results show that this contribution can be quite significant at low energy and that the LRG events observed by E665 collaboration in $\mu Xe$ and $\mu D$ interactions at 490 $GeV$ can be reasonably well described in terms of meson exchange. We also show that the distribution of the maximum rapidity for all hadrons is quite different from that for charged hadrons only and that the former exhibits also shoulder-like structure for events at 490 $GeV$ similar to that at HERA.
Large-rapidity-gap (LRG) events in deep inelastic scattering have been studied extensively at HERA both experimentally\textsuperscript{1-3} and theoretically \textsuperscript{\textsuperscript{1,4-7}}. It has been observed that there exist a class of events in which there are no particles or energy depositions in the forward part of the detector, i.e., there is a large rapidity gap in the final state. Such events take about 10\% of the total deep inelastic events in the kinematic range $10^{-4} < x < 10^{-2}$ and $5 < Q^2 < 120 \text{ GeV}^2$. (Here $x$ is the Bjorken-$x$, $Q^2 = -q^2$ and $q$ is the four-momentum transfer carried by the virtual photon.) The experimental data obtained by ZEUS\textsuperscript{2} and H1\textsuperscript{3} collaborations show that the distribution of the maximum pseudo-rapidity $\eta_{\text{max}}$ for the produced hadrons has a clear shoulder-like structure which signifies the occurrence of the LRG events.

The existence of the LRG shows that, in such events, the exchanged object between the virtual photon and the incoming proton must be colorless (c.f. Fig. 1a given later in this paper). It has been shown that different features of LRG events can be well described\textsuperscript{3} in terms of pomeron ($IP$) exchange in Regge phenomenology. At the same time, it was also well known that one-meson exchange such as one-pion exchange process contributes significantly\textsuperscript{8} to deep inelastic scattering $ep \rightarrow eX$. One-pion exchange itself takes about 10\% of the whole $ep \rightarrow eX$ events in the abovementioned kinematic region\textsuperscript{9}. Therefore, it was expected\textsuperscript{9} that meson exchange might also contribute to the LRG events. This can be studied using Monte Carlo event generators. Such study showed that the contribution is negligible\textsuperscript{10} at HERA energy. It has completely no contribution to the characteristic shoulder-like structure in $\eta_{\text{max}}$ distribution.

It is interesting to note that LRG events have also been observed\textsuperscript{11} by E665 collaboration at FNAL in $\mu Xe$ and $\mu D$ fixed target experiments at 490 GeV. In these experiments, LRG events were defined as the events with a rapidity gap $\Delta y^* > 2$, where $\Delta y^*$ is the difference between the rapidity of the target nucleon before the scattering and the lowest rapidity of the charged hadrons in the event in $\gamma^*$-nucleon c.m. frame. The distribution for the probability of $\Delta y^*$ greater than a given value i.e., the probability distribution $P(\Delta y^*)$, rather than the probability density, has been given. It is interesting to note that the $P(\Delta y^*)$ obtained
by E665 does not have the same shoulder-like structure as that for $\eta_{\text{max}}$ distribution at HERA. Having in mind that $\Delta y^*$ and $\eta_{\text{max}}$ are essentially the same except for the small difference in reference system, we are naturally led to the following questions: Why is there such a large difference between the $\Delta y^*$ distribution obtained by E665 collaboration and the $\eta_{\text{max}}$ distribution obtained by H1 and ZEUS collaborations? How is the energy dependence of the rapidity distribution in events where $IP$ or $\pi$ is exchanged? Can meson exchange give a significant contribution to the LRG events observed by E665 collaboration?

These are the questions that we would like to study in this note. For explicitness, we take one-pion exchange as an example. Similar effects should also exist for other mesons. We now start with a qualitative analysis. In fact, from the following simple qualitative analysis, we already expect that the contribution of $IP$ or $\pi$ exchange to LRG events at E665 energy may be quite different from that at HERA energy. We use the notations as those shown in Fig.1a and recall that the invariant mass of the total hadronic system and that of the hadronic subsystem $X$ are given by,

$$W^2 \equiv (P + q)^2 = Q^2(1/x - 1) + M^2,$$

$$M_X^2 \equiv (q + q_\kappa)^2 = Q^2(\xi/x - 1) + t,$$

respectively. Here $t \equiv q_\kappa^2$ is the square of the four-momentum transfer between the proton and the virtual photon; $\xi \equiv (q \cdot q_\kappa)/(q \cdot P)$ can be interpreted as the fraction of the momentum carried by the exchanged object $\kappa$ (which represents $IP$ or $\pi$) from the incident proton in the infinite momentum frame. Independent of the reference frame one uses, the width of the rapidity distribution of the hadronic subsystem $X$ is proportional to $\ln M_X^2$, i.e., $\Delta y_X \sim \ln M_X^2$; and the width of the rapidity distribution of the total hadronic system is proportional to $\ln W^2$. This is illustrated in Fig.1b. We see that, for a given $W$, the smaller $M_X$ is, the larger the rapidity gap between the hadronic subsystem $X$ and the scattered nucleon is. Hence, $M_X \ll W$ is a necessary condition for LRG to appear. From Eq. (2), we see that $M_X$ increases with increasing $\xi$ and decreases with increasing $|t|$ at fixed $Q^2$ and $x$. 
This implies that $\xi$ can not be large to ensure $M_X \ll W$. In the experiments at HERA, the kinematic range is $5 < Q^2 < 120 \text{ GeV}^2$ and $0.0001 < x < 0.01$, and the typical value for $M_X$ is several ($1 \sim 7$, say) GeV. The corresponding $\xi$ value is of the order of 0.01 or less. This means that only events where $\xi$ is very small contribute to the LRG events at HERA. Hence, the contribution of $\pi$ or $IP$ exchange to LRG events at HERA is determined by its contribution to deep inelastic scattering at small $\xi$.

The $\xi$-dependences of the contributions of $IP$ and $\pi$ exchange to deep inelastic scattering $ep \to eX$ at given $x$ and $Q^2$ are given by their contributions to the “diffractive structure function” $F_2^{D(4)}(x, Q^2, \xi, t)$. To make a qualitative comparison of the two contributions, we use the approximately valid factorization theorem, i.e. take $F_2^{D(4)}(x, Q^2, \xi, t) = f_{\kappa/p}(\xi, t)F_2^{\kappa}(x/\xi, Q^2)$. Here, $f_{\kappa/p}$ is the corresponding flux factor; $F_2^{\kappa}$ is the structure function of $\kappa$ ($IP$ or $\pi$). In the case of $IP$ exchange, $f_{IP/p}(\xi, t)$ can be parameterized as:

$$f_{IP/p}(\xi, t) = \frac{9\delta^2}{4\pi^2}[F(t)]^2 \xi^{1-2\alpha_{IP}(t)},$$

where $\delta^2 = 3.24 \text{ GeV}^{-2}$, $\alpha_{IP}(t) = 1 + \epsilon + \alpha't$, $\epsilon \approx 0.085$, $\alpha' = 0.25$ and the elastic form factor $F(t)$ is given by:

$$F(t) = \frac{4m_p^2 - 2.8t}{4m_p^2 - t}(1 - t/0.7)^2,$$

where $m_p$ is the mass of proton. The pion flux factor can be derived from the pion cloud model. For the case $p \to \pi^*N$, we have:

$$f_{\pi/p}(\xi, t) = 3.257\frac{-t}{(m_\pi^2 - t)^2} \exp\left(-\frac{m_\pi^2 - t}{1.21\xi}\right),$$

where both $m_\pi^2$ and $t$ are taken in unit of GeV$^2$. Take,

$$F_2^{IP}(x, Q^2) = 3x(1 - x)/2,$$

and the SMRS-P2-parameterization of $F_2^\pi(x, Q^2)$, we obtained their contributions to $F_2^{D(4)}(x, Q^2, \xi, t)$ at $t = -|t|_{\min} \approx m_p^2\xi^2/(1 - \xi)$ in Fig.2. From the figure, we explicitly see that $F_2^{D(4)}(x, Q^2, \xi, -|t|_{\min}) \gg F_2^{D(4)}(x, Q^2, \xi, |t|_{\min})$ in the small $\xi$ (say, $\xi < 0.05$).
region. Since the LRG events at HERA come mainly from this small $\xi$ region, we reach the conclusion that the contribution of IP exchange to LRG events at HERA is dominant and that the contribution of $\pi$ exchange can be neglected.

The situation is, however, quite different in the E665 experiment. Here, events in the kinematic range $1 < Q^2 < 100 \text{ GeV}^2$ and $0.002 < x < 0.3$ were selected. Compared with those at HERA, $Q^2$ is much smaller and $x$ is larger. Thus, to obtain the same $M_X$ as that at HERA, $\xi$ should be much larger. In fact, for typical $M_X$ around $1 \sim 7 \text{ GeV}$, $\xi$ is of the order of $10^{-1} \sim 10^{-2}$ and can even be significantly larger than 0.1. From Fig.2, we see that $F_{2(p)}^{D(4)}(x, Q^2, \xi, -|t|_{\text{min}})$ decreases with increasing $\xi$, but $F_{2(\pi)}^{D(4)}(x, Q^2, \xi, -|t|_{\text{min}})$ increases very rapidly with increasing $\xi$. As a result, their difference becomes very small in the region of $\xi \sim 0.05$, and the latter can even be larger than the former for large $\xi$ ($>0.1$, say). Thus the contribution of pion exchange to the LRG events obtained in the E665 experiments should be quite significant compared with that from Pomeron.

Now, we explicitly calculate the rapidity distribution of final hadrons in events where IP or $\pi$ exchange takes place. Presently this can only be carried out using Monte Carlo events generator. There exist a number of Monte Carlo event generators, such as POMPYT and RAPGAP, which simulate the processes shown in Fig.1a. Here, it is envisaged that the incoming proton ‘emits’ a IP or a $\pi^*$, and the IP or $\pi^*$ then collides with the virtual photon emitted by incident lepton to produce the sub-hadronic-system $X$ shown in Fig.1a. Various options for the effective IP flux and the parton densities in IP are available in the programs. Using such Monte Carlo program we can easily calculate the contribution of IP or $\pi$ exchange to the LRG events at different energies or in different reference frames. Both POMPYT and RAPGAP are slave systems, which must be called by our own steering program. Thus we first simulate the events at HERA to check our steering program then apply it to E665 energy.

We simulate the events where the abovementioned IP or $\pi$ exchange takes place using POMPYT or RAPGAP and the usual DIS events using LEPTO and obtain the $n_{max}$ distribution for each class of events respectively. We calculated them using different options for IP structure functions. We found out that both the results from POMPYT and those from
RAPGAP with different options for IP structure function are essentially the same. Adding the different contributions together with the corresponding weights which measure the relative contribution of each type of process to the inclusive process $ep \rightarrow eX$, we can obtain the $\eta_{\text{max}}$ distribution for final hadrons in deep inelastic scattering at HERA energy. The contribution of $\pi$ exchange to deep inelastic scattering $ep \rightarrow eX$ in the HERA kinematic region was estimated in [3]. The results depend on the parameterization of pion structure function, but they are all of the order of 10% of all DIS events at HERA energy. In Fig.3a, we show the results that we obtained by adding 12% from IP or $\pi$ exchange with 88% usual DIS events from LEPTO (the lower solid and dotted lines). Here, in obtaining these results, all hadrons that can be observed by the H1 detector, i.e., those with $-3.8 < \eta < 3.65$ and energy higher than 400 $MeV$, are taken into account. Since our purpose is to study the energy dependence of the contribution of IP or $\pi$ exchange to LRG events, we do not simulate the detector effects at HERA. The results show that the usual DIS events can give a reasonable account of the shape of the $\eta_{\text{max}}$ distribution for values above 1.5 and that the usual DIS and IP exchange together can well describe the $\eta_{\text{max}}$ distribution at HERA. The results also show that the $\pi$ exchange has a very small contribution to the LRG events at HERA.

Subsequently, we apply the method to the $\mu Xe$ and $\mu D$ fixed target scattering at 490 $GeV$ beam energy [11], and calculate the rapidity distribution of final hadrons in events where IP or $\pi$ exchange takes place. Since our purpose is to study the energy dependence of the contribution of IP or $\pi$ exchange to the LRG events, we do not take the nuclear effects into account. This means that we simply regard $\mu D$ or $\mu Xe$ scattering in the E665 kinematic region as $\mu N$ scattering in the same kinematic region. We select the events in the same kinematic range as that chosen by E665 collaboration [11] and obtain the probability distributions of $\Delta y^*$ in events where IP (upper solid line) or $\pi$ (upper dotted line) exchange takes place compared with the E665 $\mu D$ or $\mu Xe$ data [11] shown in Fig.4a. From the results we see clearly that, as we expected in the abovementioned qualitative analysis, $\pi$ exchange can have a very significant contribution [14] to the LRG events observed by E665 collaboration. We found out
also, to reproduce the E665 data\[1\], we need a rather large contribution of \( IP \) and/or \( \pi \) exchange. We estimated the contribution of pion-exchange to \( \mu p \rightarrow \mu X \) using the pion flux factor given in Eq.(5) and different parameterizations of pion structure functions. We found out that the results are slightly different if different parameterizations are used. But they are all of the order of 10 ~ 20% in the E665 kinematic regions. In Fig.4a, we show the results obtained by adding 20% \( IP \) exchange with 80% from \textsc{Lepto} (lower solid line) and those obtained by adding 20% \( \pi \) exchange with 80% from \textsc{Lepto} (lower dotted line). We see in particular that pion exchange contributes significantly to the LRG events but cannot account for all of them. However, what we discussed till now is only the contribution from the case \( p \rightarrow \pi^*N \), which is an explicit and calculable example of different meson exchange processes. Similar contributions should be expected from other mesons which cannot be calculated presently because of the lack of the corresponding flux factors and the structure functions. To show what we may expect if all different meson exchange processes are taken into account, we simply add more contributions from \( \pi \) exchange to the whole events. Hence, in Fig.4a, we show also the results obtained by adding 40% from \( \pi \) exchange with 60% from \textsc{Lepto} (dash-dotted line). We see that the results agree reasonably well with the data.

From Fig.4a, we also see that the shape of the \( \Delta y^* \) distribution does not have a clear shoulder-like structure at the E665 energy as that in \( \eta_{\text{max}} \) distribution at HERA (see Fig.3a). We are therefore led to the question about the reason of the disappearance of such characteristic structure for LRG events. We note that, besides the energy is lower, only the charged hadrons are taken into account by E665 whereas all the hadrons are taken into account at HERA. Thus we take also all the hadrons with energy higher than 400 MeV into account and re-calculate \( \Delta y^* \) distribution at E665 energy. The obtained results are shown in Fig.4b. It is interesting to see that, compared with that for charged hadrons only, the \( \Delta y^* \) distribution obtained from \textsc{Lepto} is much narrower whereas those obtained from \textsc{Rapgap} for \( \pi \) or \( IP \) exchange remain essentially the same. Their difference becomes much larger. Adding them together with the corresponding weights mentioned above, we obtain the total \( \Delta y^* \) distribution, which has now a significant shoulder-like structure in the case of
$IP$ exchange. But the shoulder structure is not clear if only $\pi$ exchange is involved. This is similar to that at HERA. These results clearly show that it is much more efficient to distinguish events where $IP$ or $\pi$ is exchanged from the usual DIS events by studying the $\Delta y^*$ distribution for all the hadrons than that for charged hadrons only. To check whether this is also true at other energies, we re-calculate the $\eta_{\text{max}}$ distribution for the charged hadrons only at HERA energy. The obtained result is shown in Fig.3b. The results show similar effect, i.e., compared with those for all the hadrons (see Fig.3a), the $\eta_{\text{max}}$ distribution for the charged hadrons from usual DIS or $\pi$ exchange events at HERA is much wider whereas that for $IP$ exchange remains essentially the same. The contamination from the fluctuation in usual DIS to LRG events would be much higher if one would study charged hadrons only.

We note also that one can use different variables to describe LRG events, such as the $\Delta y^*$ used by E665, $\eta_{\text{max}}$ used at HERA or $\eta_{\text{max}}^*$ and $y_{\text{max}}^*$ in the hadronic c.m. system. We also studied the question of which one is more efficient by calculating the corresponding distributions in the scattering of the 490 $GeV$ lepton beam off the fixed target proton. The obtained results show no significant difference between these variables, all of them can give good description to the occurrence of LRG events.

In summary, using the Monte Carlo event generators POMPYT, RAPGAP and LEPTO, we showed that $\pi$ exchange has a significant contribution to the LRG events in the $\mu Xe$ and $\mu D$ fixed target scattering at 490 $GeV$ beam energy. Taking all the contributions from different meson exchange processes into account, we should obtain a reasonably well description of the corresponding E665 data at that energy. The distribution of the maximum rapidity for all hadrons is quite different from that for charged hadrons only and the former exhibits also shoulder-like structure for events at 490 $GeV$ beam energy similar to that at HERA.

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Fig. 1. (a) Schematic of the semi-inclusive process $ep \rightarrow eNX$ with the exchange of a colorless object $\kappa^*$ which can be a $IP$ or a $\pi^*$; and (b) Diagram illustrating the range of the rapidities of the hadrons in the subsystem $X$ and that for all the hadrons including the outgoing nucleon $N$. Here we see in particular that the necessary condition for LRG to appear is $\ln W^2 \gg \ln M_X^2$.

Fig. 2. $\xi$-dependence of $IP$ or $\pi$ exchange contribution to the “diffractive structure function” $F_{2D(\kappa)}^{D(4)}(x, Q^2, \xi, t)$ at $t = -|t|_{\text{min}}$. 
Fig. 3. Distribution of $\eta_{\text{max}}$ for hadrons in DIS events at HERA. The data were taken from \cite{3}. The total number of the Monte Carlo events is normalized to the data. In obtaining the results in (a), all hadrons which can be observed by H1 detector (i.e., those with $-3.8 < \eta < 3.65$ and energy higher than 400 MeV) are taken into account. In (b), only the charged hadrons are taken into account.
Fig. 4. Probability $P(\Delta y^*)$ of events with rapidity difference between the most backward hadron and the incoming nucleon in the $\gamma^*p$ c.m. frame to be greater than $\Delta y^*$ in the scattering of 490 GeV lepton off fixed proton target. In obtaining the results in (a), only the charged hadrons with energy higher than 400 MeV are taken into account. The data are obtained by E665 collaboration [1] in $\mu Xe$ and $\mu D$ scattering at 490 GeV. In (b), all the hadrons with energy higher than 400 MeV are taken into account.
Figure captions

Fig.1: (a) Schematic of the semi-inclusive process $ep \rightarrow eNX$ with the exchange of a colorless object $\kappa^*$ which can be an $IP$ or a $\pi^*$; and (b) Diagram illustrating the range of the rapidities of the hadrons in the subsystem $X$ and that for all the hadrons including the outgoing nucleon $N$. Here we see in particular that the necessary condition for LRG to appear is $\ln W^2 \gg \ln M^2_X$.

Fig.2: $\xi$-dependence of $IP$ or $\pi$ exchange contribution to the “diffractive structure function” $F_{2(\kappa)}^{D(4)}(x, Q^2, \xi, t)$ at $t = -|t|_{\text{min}}$.

Fig.3: Distribution of $\eta_{\text{max}}$ for hadrons in DIS events at HERA. The data were taken from [3]. The total number of the Monte Carlo events is normalized to the data. In obtaining the results in (a), all hadrons which can be observed by H1 detector (i.e. those with $-3.8 < \eta < 3.65$ and energy higher than 400 MeV) are taken into account. In (b), only the charged hadrons are taken into account.

Fig.4: Probability $P(\Delta y^*)$ of events with rapidity difference between the most backward hadron and the incoming nucleon in the $\gamma^*p$ c.m. frame to be greater than $\Delta y^*$ in the scattering of 490 GeV lepton off fixed proton target. In obtaining the results in (a), only the charged hadrons with energy higher than 400 MeV are taken into account. The data are obtained by E665 collaboration [11] in $\mu X e$ and $\mu D$ scattering at 490 GeV. In (b), all the hadrons with energy higher than 400 MeV are taken into account.