Composite gauge field models with broken symmetries

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ABSTRACT

We present a generalization of the non-Abelian version of the $CP^{N-1}$ models (also known as Grassmannian models) that involve composite gauge fields to accommodate partial breaking of the non-Abelian gauge symmetry. For this to be possible, in most cases, the constituent fields need to belong to an anomaly free complex representation. Symmetry is broken dynamically for large $N$ primarily by a naturally generated composite scalar which simulates Higgs mechanism. In the example studied in some detail, the gauge group SO(10) gets broken down to subgroups like SU(5) or SU(5)×U(1).

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The concept of composite gauge fields is attractive because of a possible reduction in
the number of fields, economy in the number of parameters and the expectation of softer
ultraviolet divergences. They have a long history[1,2,3,4]. They appear in $CP^{N-1}$ models[2]
and their non-Abelian generalizations called Grassmannian models(GMs)[3] as composites
of bosonic or fermionic[4] constituents. These models have been studied in the large $N$ limit,
and it is found that the non-Abelian symmetry is either exact or completely broken. If such
composite models are to be candidates for a physical theory, we do need a version in which
the symmetry is partially broken. The purpose of this note is to present one such version. It
is a generalization of the GMs made possible due to a modification of the constraint equation
by a natural inclusion of a composite scalar in the adjoint representation of the gauge group
simulating Higgs mechanism. It is interesting to note here that the agent of symmetry
breaking in grand unified theories is usually a Higgs scalar in the adjoint representation. In
the following, we confine ourselves to bosonic constituent fields. In the case of the gauge
group SO(10) studied in some detail, there exists a rich phase structure with symmetry
breaking to subgroups like SU(5) or SU(5)×U(1).

The Grassmannian model[3] is based on $N$ scalars in the fundamental representation of
the gauge group, represented collectively by a $M \times N$ matrix $Z$ with the elements $Z_{\alpha i}$. The
column index $i$ is an internal index or a flavor index. The row index $\alpha$ is the gauge index
associated with a non-Abelian gauge symmetry which in the present case is $U(M)$. In other
words, the action of $U(M)$ on $Z$ is from the left, $Z \to UZ$. The case of the $CP^{N-1}$ model
can be recovered from GM by setting $M = 1$. The model is defined under the constraint
$ZZ^\dagger = I_M$ where $I_M$ is an identity matrix of order $M$. The Lagrangian written in terms of
an auxiliary field $A_\mu = iZ\partial_\mu Z^\dagger$ reads
\[
L = \beta N \text{tr} \left[ D_\mu Z (D_\mu Z)^\dagger \right] ,
\]
where $D_\mu Z$ is the covariant derivative $(\partial_\mu - iA_\mu)Z$; $\beta$ is an inverse coupling constant. An
overall multiplicative factor $N$ is introduced for convenience in the $1/N$ expansion. It is easy
to verify that the composite field $A_\mu$ transforms as a gauge field under the local transformation
$Z \to UZ$, thanks to the constraint. It is expected to become dynamical and hence a
genuine gauge field after quantum corrections. This can be seen in the $1/N$ expansion, for
instance.

The phase structure of these models has been studied in the literature for large $N$[3]. It
is determined by a critical coupling $\beta_c$ given by

$$\beta_c = \frac{1}{\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}}, \quad (2)$$

where, as in the rest of this note, we have suppressed the dependence on the momentum cutoff. There are two phases. For $\beta > \beta_c$, there is the broken phase where $Z$ has an expectation value. This breaks $U(M)$ completely and all the gauge bosons become massive. For $\beta < \beta_c$, we have the unbroken phase where the gauge symmetry is unbroken and the gauge bosons are massless. In other words, the gauge group is either completely broken or not broken at all. Apparently there are no phases, at least for large $N$, where a partial breaking of the gauge group is possible. These features follow as a special case of our general discussion below. To obtain a richer phase structure, we invoke a generalization of these models and study them for large $N$ in the following.

A straightforward approach is to retain the constraint $ZZ^\dagger = I_M$ as such, but to gauge only a subgroup of $U(M)$. This, however, does not alter the symmetry breaking patterns. A more interesting generalization occurs when the constraint itself is modified. Here we take the $Z$ fields to be in any representation of the gauge group, say $R$, of dimension $M$ and multiplicity $N$. The transformation matrix $U$ is now in $R$ acting on the matrix $Z$ as before, $Z \rightarrow UZ$. We look for a Lagrangian that is of the form (1). Note that the part of the Lagrangian quadratic in $A_{\mu} = A^a_{\mu}T_a$ is proportional to $A^a_{\mu}A^b_{\mu} \text{tr}(T_a T_b ZZ^\dagger)$, where $T_a$'s are the generators of the Lie algebra being gauged. In the GM, the constraint $ZZ^\dagger = I_M$ is responsible for rendering it quadratic in $A_{\mu}$ alone. Now, more generally, we achieve the same goal by imposing the following constraint instead:

$$\text{tr}(T_{ab}ZZ^\dagger) = I_\delta_{ab}. \quad (3)$$

Here $T_{ab} = (T_a T_b + T_b T_a)/2$ and $l$ is the index of the representation $R$ defined by $\text{tr}(T_a T_b) = l\delta_{ab}$. Note that this new constraint also respects the gauge symmetry. Using it, it is easy to obtain an expression for the composite field $A_{\mu}$,

$$A^a_{\mu} = \frac{i}{2l} \text{tr} \left[ T_a (Z \partial_{\mu} Z^\dagger - \partial_{\mu} ZZ^\dagger) \right], \quad (4)$$

and an expression for the Lagrangian in terms of the $Z$ fields alone,

$$L = \beta N \text{tr}(\partial_{\mu} Z \partial_{\mu} Z^\dagger) + \frac{1}{4lN} \beta N \left\{ \text{tr} \left[ T_a (Z \partial_{\mu} Z^\dagger - \partial_{\mu} ZZ^\dagger) \right] \right\}^2. \quad (5)$$
This derivation should ensure gauge invariance and this is easily seen to be the case.

Our earlier constraint \( ZZ^\dagger = I_M \) clearly solves (3). But, there are cases where this is not the only solution. In other words, the constraint (3) is in some cases weaker than the earlier one. To see this, introduce a hermitian matrix \( W \) by \( ZZ^\dagger = I_M + W \) and observe that the constraint is equivalent to looking for a solution of \( \text{tr}(T_{ab}W) = 0 \). Given a \( W \) that leads to a positive semidefinite \( ZZ^\dagger \), \( Z \) is solvable generally as \( Z = (I_M + W)^{1/2}Z_0 \) for some \( Z_0 \) obeying \( Z_0Z_0^\dagger = I_M \). The earlier constraint corresponds to the trivial solution \( W = 0 \). That there exist cases where \( W \) is nontrivial can be seen as follows. Make the ansatz that \( W \) is in the Lie algebra itself, i.e., \( W = W_aT_a \). Now, the constraint \( \text{tr}(T_{ab}W) = 0 \) simply states that the ABJ anomaly associated with the representation \( R \) should vanish. That there exist anomaly free representations is well known. It is easy to observe that the representation \( R \) should be complex for an ansatz of this type to solve the constraint (3). An example is the spinor representation 16 of the gauge group SO(10) that is well known to be anomaly free. Our ansatz gives a solution for the composite scalar \( W \) that is in the adjoint representation 45. This is interesting, for it is known that an adjoint scalar is a promising candidate to break SO(10) to SU(5). Its appearance here is quite unexpected.

In some instances, the above ansatz gives the most general solution. This is the case for the SO(10) example mentioned above. This can be seen from representation theory, viewing \( W \) to belong to \( 16 \times 16 = 1 + 45 + 210 \). It can be shown that the constraint, involving a symmetric product of the generators of the gauge group, suppresses the representations 1 and 210 from \( W \) leaving only the 45 as claimed. Note, however, that the ansatz does not always give the most general solution. For instance, if one were to pick a sufficiently large representation of the gauge group for \( R \), one will easily end up with more representations that remain unsuppressed in \( W \). Representation theory should still be applicable to solve for \( W \) in general.

Let us call the models as type one models when \( W \) is identically zero and our new constraint reduces to the old one. They are closer to the GMs discussed in the beginning, or rather to their generalizations involving subgroups of U(M). The other models where \( W \) can be nonzero is referred to as type two models. Models based on a reducible \( R \) are quite generally of type two. This is because the constraint does not determine some components of \( W \), for instance those connecting different subrepresentations. Type one models are thus necessarily based on irreducible \( R \)’s.
The constraint (3) can be incorporated into the Lagrangian with the help of a Lagrangian multiplier $\Sigma = \Sigma_{ab}^T$, a $M \times M$ matrix. The result is

$$L = \beta N \text{tr} \left[ D_\mu Z (D_\mu Z) + \Sigma ZZ^\dagger - \Sigma \right].$$

(6)

To understand symmetry breaking, and hence to identify the various phases, we need to obtain the effective potential. We will do this for large $N$. Because $A_\mu$ is not expected to pick up any expectation value, it can be set to zero. The contribution to the effective potential coming from the Lagrangian (3) is obtained by dropping the derivative terms. Because $1/N$ appears in the Lagrangian like the Planck’s constant, the quantum corrections to this contribution are expected to be suppressed by a factor $1/N$. But there are $N$ representations contributing equally and this can offset the $1/N$ suppression. The result is that for large $N$ the effective potential for the $Z$ and $\Sigma$ fields obtained by integrating away the $Z$ fluctuations carries a correction

$$N \int \frac{d^4k}{(2\pi)^4} \text{tr} \ln \left( k^2 I_M + \Sigma \right).$$

(7)

The total effective potential is thus

$$V_{\text{eff}} = \beta N \text{tr} \left( \Sigma ZZ^\dagger - \Sigma \right) + N \int \frac{d^4k}{(2\pi)^4} \text{tr} \ln \left( k^2 I_M + \Sigma \right).$$

(8)

To determine the various phases, we need to extremize this potential. The resulting saddle point equations are $\Sigma Z = 0$ obtained by varying $Z^\dagger$ and

$$\beta \text{tr} \left[ T_{ab}(ZZ^\dagger - I_M) \right] + \int \frac{d^4k}{(2\pi)^4} \text{tr} \left( T_{ab} \frac{1}{k^2 I_M + \Sigma} \right) = 0$$

coming from varying $\Sigma$. We now look for solutions to this system of equations.

For type one models, the traces and the $T_{ab}$’s can be dropped and Eq. (3) becomes equivalent to that of the GM. The solutions lead to two phases [3]. There is the broken phase for $\beta > \beta_c$ with $ZZ^\dagger = (1 - \beta_c/\beta)I_M$ and $\Sigma = 0$, and the unbroken phase for $\beta < \beta_c$ with $Z = 0$ and $\Sigma = \sigma I_M$. Here $\sigma$ is given by

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + \sigma} = \beta.$$ (10)

We have to ensure that the solution for $\Sigma$ is consistent with its definition $\Sigma = \Sigma_{ab} T_{ab}$. $\Sigma = \sigma I_M$ is acceptable since it is equivalent to $\Sigma_{ab} = \sigma \delta_{ab}/C_2(R)$, where $C_2(R)$ is the second Casimir invariant of the irreducible representation $R$ defined by $T_a T_a = C_2(R) I_M$. 

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For type two models, Eq. (9) can still be reduced to resemble that of the GM, but with a matrix $W$ satisfying $\text{tr}(T_{ab}W) = 0$ on the r.h.s,
\begin{equation}
\beta \left(ZZ^\dagger - I_M \right) + \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 I_M + \Sigma} = \beta W. \tag{11}
\end{equation}
The factor $\beta$ on the r.h.s makes this equation agree with $ZZ^\dagger = I_M + W$ at the level of expectation values. For the unbroken case of $\beta < \beta_c$, we have the previous solution $\Sigma = \sigma I_M$, $Z$ and $W$ being zero. $\Sigma = \sigma I_M$ is acceptable for irreducible $R$’s as it follows from $\Sigma_{ab} = \sigma \delta_{ab}/C^2(R)$. The case of reducible $R$’s can be handled starting with the ansatz $\Sigma_{ab} = \sigma \delta_{ab}$ suitably choosing $W$. For the completely broken case of $\beta > \beta_c$, the solution is $ZZ^\dagger = (1 - \beta_c/\beta)I_M + W$ where $W$ is arbitrary to the extent that $ZZ^\dagger$ is positive semidefinite.

These solutions are not a priori the most general ones. We use our SO(10) example to look for more solutions. The solution we are looking for is intended to break SO(10) to SU(5) or to SU(5)$\times$U(1). Our following attempt is not successful, but illustrates the way more solutions could arise. Besides, it is an exercise that will be useful to our discussion later.

Note that SO(10) has a maximal subgroup SU(5)$\times$U(1). The representation 16 of SO(10) transforms under SU(5) as $10 + \bar{5} + 1$. The U(1) charges for $10, \bar{5}$ and 1 are proportional to $1, -3$ and 5 respectively. Consider first the case $Z = 0$. In order to have symmetry breaking to SU(5)$\times$U(1), we now take $\Sigma_{ab} = \sigma \delta_{ab}$ along the SU(5) directions, $\rho$ along the U(1) and zero otherwise.\footnote{Note that $a$ or $b$ index runs over the adjoint representation 45 of SO(10) that under SU(5) decomposes to $24 + 10 + \bar{10} + 1$. Our ansatz for $\Sigma_{ab}$ corresponds to having it nonzero for $(a,b)$ along $(24,24)$ and $(1,1)$. One could have it nonzero along $(10, \bar{10})$ and $(\bar{10},10)$ as well, but it turns out that this can be absorbed into $\sigma$ and $\rho$. This means that the $\Sigma$ matrix is diagonal with values $C_2(10)\sigma + \rho, C_2(\bar{5})\sigma + 9\rho$ and $25\rho$ along the representations 10, $\bar{5}$ and 1 respectively. With $C_2(10)/C_2(\bar{5}) = 3/2$ and a suitable scaling of $\sigma$, we may take them to $3\sigma + \rho, 2\sigma + 9\rho$ and $25\rho$. The $W$ matrix is taken to be along the U(1) direction; in other words, it is diagonal with values $w, -3w$ and $5w$. It is now straightforward to write down the saddle point equations,}

\begin{align*}
-\beta + \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 3\sigma + \rho} &= \beta w, \\
-\beta + \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 2\sigma + 9\rho} &= -3\beta w, \\
-\beta + \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 25\rho} &= 5\beta w. \tag{12}
\end{align*}
We find no solutions to these equations other than the one already discussed wherein $\Sigma$ is proportional to identity and $W = 0$. Note that the $\Sigma$ eigenvalues $3\sigma + \rho$, $2\sigma + 9\rho$ and $25\rho$ are either in the ascending order or in the descending order (or equal), and this makes it difficult to obtain the alternating signs on the r.h.s of the above equations. Allowing for nonvanishing $Z$ does not improve the situation. This is due to the requirement that $ZZ^\dagger$ and $\Sigma$ have nonnegative eigenvalues, and due to the equation $\Sigma ZZ^\dagger = 0$ that requires at least $3\sigma + \rho$ or $25\rho$ to vanish to allow for a nonzero eigenvalue of $ZZ^\dagger$. We have also examined the case of $E_6$ where $R$ stands for the representation 27 for possible breaking to SO(10) or SO(10) $\times$ U(1) and find no solutions.

Perhaps this is illustrative of a generic phenomenon or suggestive of the need to look at larger representations that might lead to more solutions. These examples, though not successful in a partial breaking of the gauge symmetry, will be useful to us below where a potential is introduced leading to a rich phase structure. The adjoint scalar $W$, that has not played any significant role so far, is going to play a major one in the presence of a potential.

One could investigate models with potential terms that are polynomials in $Z$ and $Z^\dagger$. There is no simple way to do this in the canonical GM without spoiling the local and global symmetries and the constraint equation. However, the unexpected appearance of an adjoint scalar $W$ helps us to construct suitable potentials, and this allows for a partial breaking of the gauge symmetry. Let us keep the potential quite general to begin with, $\beta N \text{tr} V(ZZ^\dagger)$, where $V(\cdot)$ is some polynomial in its argument. We expect this to be a nontrivial extension only in the case of type two models, since for type one the constraint $ZZ^\dagger = I_M$ reduces it to the addition of a constant. It is convenient to introduce a composite field variable $X$ for $ZZ^\dagger$ and write the potential as $\beta N \text{tr} V(X)$. The requirement $X = ZZ^\dagger$ can be incorporated with the help of a Lagrange multiplier $Y$, adding a term $\beta N \text{tr}(Y ZZ^\dagger - Y X)$ to the potential. As before, the constraint $\text{tr}(T_{ab} ZZ^\dagger) = l\delta_{ab}$ can be accommodated with the help of a Lagrange multiplier $\Sigma = \Sigma_{ab} T_{ab}$. Its effect is, as we know, to add a term $\beta N \text{tr}(\Sigma ZZ^\dagger - \Sigma)$ to the potential. After translating $Y$ to $Y - \Sigma$ for convenience, the total Lagrangian looks like

$$L = \beta N \text{tr} \left[ D_\mu Z (D_\mu Z)^\dagger + V(X) + Y ZZ^\dagger - Y X + \Sigma X - \Sigma \right]. \quad (13)$$

The large $N$ effective potential is now computable,

$$V_{\text{eff}} = \beta N \text{tr} \left[ V(X) + Y ZZ^\dagger - Y X + \Sigma X - \Sigma \right] + N \int \frac{d^4k}{(2\pi)^4} \text{tr} \ln \left(k^2 I_M + Y\right). \quad (14)$$
The saddle point equations are obtained by extremizing this potential. Varying $X$ gives $Y = \Sigma + V'(X)$ where a prime denotes differentiation with respect to the argument. Varying $\Sigma$ gives $\operatorname{tr}[T_{ab}(X - I_M)] = 0$. As before, one may look for a solution to this in the form $X = I_M + W$ where $W$ satisfies $\operatorname{tr}(T_{ab}W) = 0$. These solutions simply determine $X$ and $Y$. Varying $Y$ and using these results one gets

$$\beta \left( ZZ^\dagger - I_M \right) + \int \frac{d^4k}{(2\pi)^4 k^2} \frac{1}{k^2 + \Sigma + V'(I_M + W)} = \beta W.$$  \hspace{1cm} (15)

This is to be supplemented with $YZ = [\Sigma + V'(I_M + W)] Z = 0$ obtained by varying $Z^\dagger$. This system of equations resembles the one obtained earlier (see Eq. (11)) with $\Sigma$ replaced by $\Sigma + V'(I_M + W)$. The presence of $V'(I_M + W)$, however, is suggestive of a different phase structure.

For type one models $W = 0$, and $V'(I_M + W)$ just adds a constant to $\Sigma$. This can be absorbed into $\Sigma$ because these models, being based on an irreducible $R$, allow for the addition of a term proportional to identity to $\Sigma$. As expected earlier, this is a trivial extension. However, this is not the case for type two models and we expect a rich phase structure.

As an example, consider a sixth order potential in $Z, Z^\dagger$ that leads to $V'(I_M + W) = aI_M + bw + cW^2$ for some parameters $a, b$ and $c$. For a model based on an irreducible $R$, the term $aI_M$ can be absorbed into $\Sigma$ as we have already noted. When $W$ is an adjoint variable, the term $cW^2$ can also be absorbed into $\Sigma$. In the SO(10) example, these correspond to translating $\sigma$ to $\sigma - 8a/25$ and $\rho$ to $\rho - a/25 - cw^2$. With this taken care of, the set of equations to be solved in our SO(10) example is (for $Z = 0$)

$$-\beta + \int \frac{d^4k}{(2\pi)^4 k^2} \frac{1}{k^2 + 3\sigma + \rho + bw} = \beta w,$$

$$-\beta + \int \frac{d^4k}{(2\pi)^4 k^2} \frac{1}{k^2 + 2\sigma + 9\rho - 3bw} = -3\beta w,$$

$$-\beta + \int \frac{d^4k}{(2\pi)^4 k^2} \frac{1}{k^2 + 25\rho + 5bw} = 5\beta w.$$  \hspace{1cm} (16)

Note the addition of $bw$ terms in the denominators compared to Eq. (12). These equations do have solutions for a range of parameters. This can be seen by treating $x = 3\sigma + \rho + bw$ and $y = 2\sigma + 9\rho - 3bw$ as independent variables to determine $\beta$ and $w$ from the first two equations, and $b$ from the last one. Both $x$ and $y$ should remain positive (or zero) to keep the momentum integrals well defined. Solutions are noted to exist in a certain domain of $x$ and $y$ giving rise to a range for the parameters. This happens for $\beta < \beta_c$ and $b < 0$. The
results of our numerical investigation is presented in Fig. 1. There are in fact two solutions for a given $\beta$ and $b$ in the region between the curves (a) and (b), and one solution between the curves (b) and (c). In other words, one of the solutions extends from curve (a) to curve (b) while the other from curve (a) to curve (c). The symmetry breaking involved here is from SO(10) to SU(5)×U(1) as noted before.

One also finds solutions for a nonzero $Z$. Consider giving an expectation value $v^2$ for $ZZ^\dagger$ along the singlet in the decomposition $16 = 10 + \overline{5} + 1$. In this case, the last equation above is modified to

$$\beta v^2 - \beta + \beta_c = 5\beta w,$$  \hspace{1cm} (17)

where we have set $z = 25\rho + 5bw$ to zero to satisfy $YZ = 0$. That there exist solutions yielding a positive $v^2$ for a range of parameters can again be seen by treating $x$ and $y$ as independent variables to determine the others. These solutions also require $\beta < \beta_c$ and a negative $b$. The region of the parameter space covered by them (one solution for a given $\beta$ and $b$) is that in between the dashed curves (b) and (d) of Fig. 1. The symmetry is broken from SO(10) to SU(5). There are other possibilities. Giving an expectation value for $ZZ^\dagger$ along $\overline{5}$ (instead of the singlet) also leads to a solution which falls above the curve (c). Solutions are also noted to exist for a nonzero $ZZ^\dagger$ along $10$ and $1$ (extending above curve (d)), or $10$ and $\overline{5}$ (extending beyond that of $\overline{5}$). All of these, however, break the gauge group completely.

What we have in the end is a two sheeted cover of the parameter space above the critical curve (a) in Fig. 1. One of them (call it the upper sheet) is through the solid curves while the other one (call it the lower sheet) is through the dashed curves. They meet along curve (a). There is of course one more sheet (call it the top sheet) for the solutions of our earlier case of the unbroken gauge group covering all of the parameter space for $\beta < \beta_c$. This too meets the other two sheets along curve (a). For every point on any one of the sheets, there is a solution.

There could be more solutions. For instance, there is the possibility that a solution breaking SO(10) to SU(3)×SU(2)×U(1) (perhaps, with an additional U(1)) exists. The number of variables and the number of equations at least matches, each being six. This suggests the need to look at all the possibilities, including SO(10)→SU(4), SU(4)×U(1), etc. The system of equations are complicated to handle, and we defer their study to the future.

Now, consider all the solutions for $\beta < \beta_c$, i.e., both unbroken and partially broken ones.
Which solution is preferred is of course determined by the effective potential. For this, one needs to compute $V_{\text{eff}}$ for all the solutions and pick the one (or more) that has the the lowest value. This is not an easy task given the number of possibilities involved, and hence we will be content with doing this numerically for the solutions found above. Our purpose here is to show that there exists a range of the parameter space that prefers a partial breaking of the gauge group.

It is straightforward to obtain the following expression for the effective potential at a saddle point:

$$
V_{\text{eff}} = -\beta N \text{tr} \left( \Sigma + bW^2/2 \right) + N \int \frac{d^4 k}{(2\pi)^4} \text{tr} \ln \left( k^2 I_M + \Sigma + bW \right).
$$

(18)

As usual, we are concerned with an adjoint $W$ for an irreducible $R$, with the SO(10) example in mind. We have again absorbed the terms $aI$ and $cW^2$ into $\Sigma$ for convenience. Fig. 2 is a plot of this effective potential for the three sheets involved (we have chosen a path in the plane of Fig. 1 crossing all the curves suitably fixing $z$ in the upper sheet and $y$ in the lower sheet). The uppermost curve is for the top sheet, the middle one is for the upper sheet and the lowermost one is for the lower sheet. Note that the lower sheet ends up always having the lowest potential. In other words, for $\beta < \beta_c$ but not close to it, a partial breaking of the gauge group is preferred over the unbroken case.

What is remarkable of our exercise in SO(10) is that a set of equations governed by only two parameters ($\beta$ and $b$) gives rise to a rich set of solutions with interesting symmetry breaking patterns. Apparently, there exist regions of the parameter space where SO(10) breaking to SU(5), or to SU(5)\times U(1), or perhaps to some other groups is possible. This example achieves our goal of constructing an induced gauge theory with composite gauge bosons having partial symmetry breaking.

There are some issues that have not been addressed here. The kinetic terms for the gauge fields need to be computed to determine the gauge coupling constant and the gauge boson masses. The kinetic term for the adjoint scalar when computed will determine its properties. The fate of the global symmetry and the number of Goldstone bosons that could emerge from its breaking need to be worked out. Also, the possible existence of other breakings of SO(10) needs to be explored. Results of these investigations will be published elsewhere. There is of course the issue of renormalizability. It is interesting to note in this connection that renormalizable models of composite gauge fields have been constructed in
ref. [4].

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[5] Interestingly, for a consistent chiral theory relevant to nature, the fermions are required to belong to an anomaly free complex representation. Here we encounter the same requirement, though in the bosonic version.

[6] The case of subrepresentations being all alike can be viewed as type one with all its multiplicity dumped to \( N \).

[7] Note that for \( Z = 0 \), the unbroken symmetry corresponds to those generators that commute with the ansatz for \( \Sigma \), and this happens to be \( SU(5) \times U(1) \). A nonvanishing \( Z \) along the singlet in the decomposition \( 16 = 10 + \overline{5} + 1 \) would break the \( U(1) \) as well leaving only \( SU(5) \) unbroken.
[8] A negative $b$ corresponds to the adjoint scalar $W$ having a negative mass squared in its potential, as in the Higgs mechanism.

Figure captions

Fig. 1: The regions of parameter space of $\beta$ and $b$ that yield solutions to Eq. (16), $\Lambda$ being the momentum cutoff. See the explanations after Eqs. (16) and (17) in the text.

Fig. 2: A plot of the effective potential (with its zero appropriately chosen) versus $\beta$ for some path that crosses all the curves in Fig. 1. The crossings are denoted by (a), (b), (c) and (d). The uppermost curve corresponds to the unbroken case and the lower two correspond to symmetry breaking as discussed in the text after Eq. (18).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-th/9310094v3