Abstract.

In this Chapter, first we briefly discuss the impact of Population II and Globular Cluster (GCs) stars on the derivation of the age of the Universe, and on the study of the formation and early evolution of galaxies, our own in particular. The long-standing problem of the actual distance scale to Population II stars and GCs is then addressed, and a variety of different methods commonly used to derive distances to Population II stars are briefly reviewed. Emphasis is given to the discussion of distances and ages for GCs derived using Hipparcos parallaxes of local subdwarfs. Results obtained by different authors are slightly different, depending on different assumptions about metallicity scale, reddenings, and corrections for undetected binaries. These and other uncertainties present in the method are discussed. Finally, we outline progress expected in the near future.

1. Introduction

Since the pioneering work of Baade in the 40’s, the study of stellar populations has been widely recognized as one of the main tool in a variety of astrophysical problems. Despite their rather large distances and rarity, GCs can be used as excellent tracers of the oldest stellar populations in
galaxies. Strong constraints can be put on different scenarios proposed for the formation and early evolution of galaxies by comparing GCs spatial, kinematical and chemical distributions with those of the underlying field population.

However, perhaps one of the most powerful uses of GCs is as time indicators. These stellar aggregates are believed to have formed within $\sim 1$ Gyr from the Big Bang (see Sandage 1993a), thus the oldest GCs provide very stringent lower limits to the age of the Universe. It is possible to use GCs as a clock based on stellar evolutionary principles, to compare with the cosmological clock given by the Hubble law (the expansion age), once estimates of the density parameter $\Omega$, and of the cosmological constant $\Lambda$, are given. The uncertain knowledge of these parameters indicates the strong advantage of deriving age estimates from nearby objects, exploiting only properties related to well known stellar evolution theory.

Stars in a GC have all been formed at the same time and, with mainly a few exceptions, with the same chemical composition; therefore, differences seen among them are only due to differences in mass. Indeed, the colour-magnitude diagram (CMD) of a single GC is the observational counterpart of a model stellar isochrone, i.e. the locus populated by stars with the same chemical composition and age, but different mass. Stellar models show that the absolute magnitude of the turn-off (TO) point is a feature that has reasonably large sensitivity to age variations, depends on basic stellar properties (mass, initial chemical composition), but is quite independent of uncertain factors like mass loss, rotation, mixing, etc., which may have a significant impact on later evolutionary stages. Therefore ages for GCs can be derived by simply comparing the observable (absolute magnitude) of the turn-off point with the corresponding theoretical quantity (bolometric luminosity). The real problem, in this rather straightforward approach, is in determining the parameters that enter in the so-called age equation given by stellar models, namely: distance (which has the largest weight in the error budget, Renzini 1991), reddening, overall metallicity, and also the detailed pattern of some elemental abundances, as the $\alpha$-elements. According to all recent studies (see VandenBerg et al. 1996 for a review), the age-determination problem then shifts to the distance-scale problem, i.e. to the problem of deriving distance moduli for GCs with a precision of 0.1 mag or better. This figure translates into an uncertainty of about 15% in the derived absolute ages.

Furthermore, GCs host a rather large number of Population II pulsating RR Lyrae variables. These are generally considered one of the most reliable (i.e. bright, numerous and well known from a physical point of view) standard candle within our galaxy and for galaxies in the Local Group. Calibrating the RR Lyrae distance scale provides an independent estimate
DISTANCES TO GLOBULAR CLUSTERS USING SUBDWARFS

(beside that of classical Pop. I Cepheids) of the distance to the LMC, which is the traditional first step in the extragalactic distance ladder. This is a crucial point, since presently there is a 0.2-0.3 mag difference between the long distance scale derived from the period luminosity relation for Cepheids, and the short distance scale, derived from the absolute magnitude of the RR Lyrae variables (see next section).

Coming back to the age determination problem, it is well known that relative ages are easier to obtain than absolute ages. In fact, the displacement of the colour or magnitude of the TO in different clusters, with respect to a given standard level, allows a relative comparison that by-passes the exact knowledge of parameters such as the distance modulus and the reddening. The final accuracy, however, heavily depends on which coordinate of the TO (horizontal: colour, or vertical: magnitude) is chosen, in the observational plane, for the comparison. The horizontal approach is more uncertain than the vertical one, due to the still poor knowledge of the stellar radii. In the vertical approach, the level of luminosity of the TO is tied to the magnitude level of the horizontal branch, and both directly rest on the nuclear fuel (hydrogen or helium) burned in the star core, which is known with great accuracy from theory (see Stetson et al. 1996 for more details).

Absolute ages with a high degree of precision are more difficult to extract from observations. Indeed, even if evolutionary models were perfectly adequate and physically sound, one has to bear in mind that errors of only 0.01 mag in the observed colours (or of ~ 0.07 mag in magnitude, due to the slope of the TO region) turn into about 1 Gyr uncertainty in the derived age, by itself about a 10% error. For the same reason, and in order to maintain the total error budget below this limit, metal abundances must be known within 0.05 ÷ 0.1 dex, and reddening must be determined with the highest precision (one or two hundredth of mag, if possible). It is easy to understand how, until recent years, such strict requirements have represented a very difficult observational test, defying all past efforts to obtain a reliable set of absolute ages for GCs, at the required level of accuracy.

2. Methods to derive distances for population II and globular cluster stars

The well-known dichotomy existing between short and long distance scale as derived from old, Population II stars is still an unsolved problem, in spite of the enormous improvement in distance determinations achieved with the accomplishment of the Hipparcos mission. The adoption of either of these two scales directly bears upon the derived model for the Universe, and for the formation and evolution of the Galaxy.

The most straightforward way to derive distances to Population II stars
is to measure their trigonometric parallaxes; however, GCs are too distant for present day’s detector capabilities. Several indirect techniques have therefore been devised, in order to determine the Population II distance scale. Some of them are based on standard candles, other exploit alternative approaches. In the following, the methods most commonly used to derive distances to Population II objects, as well as some newly proposed ones, are reviewed, and their virtues and shortcomings briefly discussed.

2.1. DISTANCES VIA STANDARD CANDLES

A number of natural standard candles exist which can be used to determine accurate distances to Population II objects. They populate different regions of the HR diagram thus sampling different evolutionary stages of a Population II star, and include: Red Giant Branch stars, Horizontal Branch stars (constant stars as well as RR Lyrae variables), Main Sequence Subdwarfs and White Dwarfs.

2.1.1. The Red Giant Branch Tip

The luminosity in the near-infrared $I$ band of the brightest stars at the tip of the Red Giant Branch (TRGB) in GCs changes by less than 0.1 mag in the metallicity range $-2.2 < [\text{Fe/H}] < -0.7$ (Da Costa & Armandroff 1990, DA) and, at fixed metallicity, the $I$ luminosity of the TRGB varies by less than $\sim 0.1$ mag for ages in the range 7-17 Gyr (Lee, Freedman & Madore, 1993, LFM). The observed (dereddened) $I$ luminosities of the TRGB, transformed to bolometric luminosities are compared to theoretical TRGB bolometric magnitudes, (see DA, LFM, and Salaris & Cassisi, 1997, 1998; SC97, SC98, for details), and the distance moduli are directly estimated. When calibrated against models with updated input physics (see SC98 and references therein), the TRGB method gives distances in good agreement with Hipparcos GCs distance scale obtained via Main Sequence Fitting. See Madore & Freedman (this book) for a thorough discussion of the virtues and shortcomings of this method.

2.1.2. The Horizontal Branch

The absolute magnitude of the HB has often been used to derive distances and ages for GCs. This is achieved through a number of different methods:

a) Theoretical HBs

The absolute magnitude of the HB can be estimated by comparison of the observed globular cluster HB’s to synthetic HB models. This method suffers from several uncertainties. First, the observed HB morphology is affected from the well known ”second parameter effect” (see Stetson et al 1996 and Fusi Pecci & Bellazzini, 1998; for extensive and updated reviews on this problem). Second, the observed HBs have a non-zero extension in
$M_V$ depending on evolution: when stars evolve off the zero-age HB (ZAHB) they become brighter and change colours. Theoretical models usually give the absolute luminosity of the ZAHB. It is therefore crucial to carefully distinguish among the quantities to compare: lower envelope of the HB, $M_V(ZAHB)$, mean magnitude of the HB, $M_V(HB)$, mean absolute magnitudes of the RR Lyrae variables, $M_V(RR)$. Third, the average absolute magnitude of the HB is a function of metallicity, the strength of this dependence is not univocally defined though, since different methods (Baade-Wesselink, Sandage effect, main sequence fitting, etc.) give a value ranging from 0.16 up to 0.39 for the slope of the $M_V(HB) − [Fe/H]$ relation (see Carney et al. 1992, and references therein). From the theoretical side, HB synthetic models from different authors generally lead to similar shallow slopes ($\sim 0.15$-$0.20$) of the $M_V(HB) − [Fe/H]$ relation.

The most recent stellar models that incorporate updated input physics (revised equation of state, new opacities, improvements of the nuclear cross-sections and neutrino emission rates), as well as a revised treatment of the stellar convection and of the helium and/or heavy elements sedimentation (Straniero & Chieffi 1997, Caloi, D’Antona & Mazzitelli 1997, and Cassisi et al. 1998), tend to give brighter horizontal branch luminosities. When used as standard candles these new HBs lead to larger distances in agreement with Hipparcos Main sequence fitting distances. However, other models (VandenBerg 1997) still tend to provide fainter HBs.

b) Parallax of HB stars

Gratton (1998) used Hipparcos parallaxes for a sample of field metal-poor HB stars in order to directly calibrate these standard candles. His sample consisted of 20 stars with $V < 9$, and 2 more stars slightly fainter than this limit. Three of them are RR Lyrae variables. The mean weighted absolute magnitude found by Gratton (1998) with this procedure is $M_V = +0.69 \pm 0.10$ (at average metallicity $[Fe/H]=-1.41$), and brightens to $M_V = +0.60 \pm 0.12$ (at average $[Fe/H]=-1.51$) when HD17072, a suspected first ascent giant branch star, is eliminated from the sample. This new result is in good agreement with the methods that favour the short distance scale, (statistical parallaxes and Baade-Wesselink), but in conflict with distance determinations based on globular cluster HB’s and Main Sequence Fitting.

c) Statistical Parallaxes of RR Lyrae Variables

Statistical parallaxes of galactic field RR Lyraes give a faint zero point of the RR Lyrae luminosity calibration, $M_V = 0.71 \pm 0.12$ at $[Fe/H]=-1.61$ (Layden et al., 1996). This result is confirmed by Popowski & Gould (1997) re-analysis of Layden et al. sample ($M_V = 0.75 \pm 0.13$ at $<[Fe/H]>=-1.61$). Moreover, Fernley et al. (1998a) have used Hipparcos proper motions for 144 field RR Lyrae and the Statistical Parallax method, as well as the Hipparcos trigonometric parallax of RR Lyrae itself to estimate the ab-
solute magnitude of the RR Lyrae stars. They basically confirm Layden et al. results, and derive $M_V = 0.77 \pm 0.15$ at $[\text{Fe/H}]=-1.53$. Similar results are found by the Hipparcos based statistical parallax analyses of RR Lyrae stars by Tsujimoto, Miyamoto & Yoshii (1998) and Gould & Popowski (1998). The corresponding distance modulus for the LMC is $(m-M)_{0,\text{LMC}} = < V >_{0,\text{LMC}} - 0.77 + 0.18([\text{Fe/H}] + 1.53) = 18.26 \pm 0.15$ with $< V >_0 = 18.98$, and $<[\text{Fe/H}]>< -1.8$ for the RR Lyraes in the LMC, and adopting a value of $0.18 \pm 0.03$ (Fernley et al. 1998a), for the slope of the $M_V(HB),[\text{Fe/H}]$ relation. All these values still favour the short distance scale.

d) Baade-Wesselink for field RR Lyrae variables

The Baade-Wesselink method allows the derivation of the absolute magnitude of RR Lyrae stars from their intrinsic properties (luminosity, colour and radial velocity variations). Several groups have used this technique applying it to field and to a few cluster variables (Liu & Janes 1990a,b, Jones et al 1992, Cacciari, Clementini & Fernley 1992, Skillen et al 1993, Storm, Carney & Latham 1994). A general consensus has been reached by these different groups on a relatively mild slope of the $M_V,[\text{Fe/H}]$ relation, as opposite to the steep slope (0.30) found by Sandage (1993b), and on a zero point about 0.3 mag fainter than derived from classical Cepheids and other methods. In an attempt to fix the zero-point of the B-W, Fernley (1994) used a different value for the conversion factor $(p)$ between observed and true pulsation velocity getting a zero-point brighter by 0.07 mag, (see also Clementini et al 1995). However, this only accounts for about 1/3 of the observed discrepancy. More recently, Feast (1998) and McNamara (1997) have explored different solutions of the slope and zero-point discrepancy of the B-W results. Feast (1998), from a compilation of B-W literature data and adopting $M_V$ as independent variable, derived $M_V = 0.37[\text{Fe/H}] + 1.13$. When used with Walker (1992) data, this calibration provides $(m-M)_{0,\text{LMC}} = 18.53$, in agreement with the classical modulus from Cepheids. However, Feast’s paper offers a rather questionable solution for the slope discrepancy (see also Fernley et al. 1998b), based on the attribution of a zero error to one of the basic ingredients of the $M_V(HB),[\text{Fe/H}]$ relation (i.e. $M_V$), which, on the contrary, is by far the most uncertain quantity. On the other side, McNamara (1997) proposes a revision of the B-W results based on the assumption that optical and near-infrared colours are better temperature indicators than the $V-K$ index. He uses these colours and the new Kurucz (1993) model atmospheres to derive the following relation: $M_V = 0.287[\text{Fe/H}] + 0.964$, that when applied to the LMC RR Lyraes gives $(m-M)_{0,\text{LMC}} = 18.53$ in good agreement with the Cepheids scale. Clearly much work on the B-W is needed to definitely assess the correct $M_V(HB),[\text{Fe/H}]$ dependence.
2.1.3. The HB Clump

Red clump stars are intermediate age (2-10 Gyr) helium core burning stars, i.e. the metal-rich counterpart of the horizontal branch stars. Red clump stars were observed in the Galactic Bulge by the OGLE microlensing experiment (Udalsky et al 1992), and in M31 with the Hubble Space Telescope (Holland, Fahlman & Richer 1996, Rich et al 1996). The CMD of the data obtained from the Hipparcos satellite shows a well determined red clump of solar-neighbourhood stars at an absolute magnitude of $M_V \sim 0.8$ mag (Jimenez, Flynn & Kotoneva 1998). Paczynski & Stanek, (1998) and Stanek & Garnavich (1998), have used the mean absolute magnitude at $I$ of red clump stars with parallaxes measured by Hipparcos to derive the distance to the Galactic Bulge and M31, respectively, and Udalski (1998) has recently derived a distance modulus $(m - M)_{0,LMC} = 18.07 \pm 0.12$ by applying the red clump method to the LMC. Udalski (1998) result, however, is based on a reddening value for the LMC bar, larger than the commonly adopted value of $E(B-V) = 0.10$ (Bessel 1991).

The reliability of the red clump method strongly relies on the assumption that its absolute magnitude is independent of age and chemical composition, and that the stellar populations in the various systems do not significantly differ from the solar neighborhood red clump population. These assumptions have recently been questioned by the model calculations of Girardi et al (1998), and by Cole (1998) reanalysis of Udalski et al (1998) results for the LMC. (See Girardi et al, 1998, and Cole, 1998, for a more thorough discussion of these issues).

2.1.4. Main Sequence

a) Main Sequence Fitting to Isochrones

Distances to GCs may be derived by fitting model isochrones to the observed cluster main sequences. The method is model dependent, and distances determined by this procedure suffer from the uncertainties still existing in the equation of state, in the treatment of convection, and in the transformations from the theoretical $\log L/L_\odot - \log T_{\text{eff}}$ plane to the observational $M_V - \text{colour}$ plane. Additional concerns are also represented by the possible existence of phenomena such as the He-diffusion, core rotation, or problems connected with unconventional physics like WIMPS (see VandenBerg et al. 1996, for a very extensive discussion of all these issues).

b) Subdwarf Fitting

The simplest technique to derive a distance to a GC is to compare their main sequence (MS) with a suitable template formed by local, metal-poor subdwarfs with known trigonometric parallaxes (Sandage 1970). Until the release of the Hipparcos catalog this procedure mainly suffered due to the the lack of metal-poor dwarfs in the solar neighborhood with accurate paral-
laxes (see VandenBerg et al 1996). With Hipparcos the number of subdwarfs suitable for main sequence fitting has enormously increased. Moreover, Hipparcos parallaxes for the local subdwarfs are systematically smaller than the corresponding ground-based measurements, thus leading, by itself, to longer distance moduli. In the last year, three different groups employed field subdwarfs with parallaxes from the Hipparcos mission in order to perform fitting of globular cluster main sequences: Gratton et al. (1997: G97; 9 clusters), Reid (1997: R97, 1998: R98; 11 clusters) and Pont et al. (1998: P98; 1 cluster). In total, twelve clusters were analyzed, from the most metal-poor (M15, M92, and M68), to the most metal-rich ones (M71 and 47 Tuc). Of the three different studies, P98 essentially confirm the distance, and hence age, determinations based on subdwarfs parallaxes from ground-based observations; in contrast, R97, R98, and G97 derive higher distances and hence younger ages, in overall agreement with each other, and with the cosmological age for the Universe. This point is further discussed in Section 3 of this Chapter.

2.1.5. The White Dwarfs Cooling Sequence

The cooling sequence of white dwarfs provides a faint but theoretically secure distance ladder, since it is independent of metallicity and age. The comparison of a globular cluster white dwarf cooling sequence with a template sequence formed by local white dwarfs with known parallaxes and masses allows to derive the distance to the cluster. White dwarfs have recently been observed with HST in 3 GCs: M4 (Richer et al 1995), NGC 6752 (Renzini et al 1996) and 47 Tuc (Zoccali et al 1998, Z98); the two last set of observations were used to get results favoring a short distance scale. The method is independent of metallicity and details of the convection theory, but depends on the observational assumptions related to colours. Furthermore, it assumes that the calibrating white dwarfs have the same mass of the GC white dwarfs, an assumption that may be criticized in view of the differences in the age of the parent populations.

2.2. THE DISTANCE TO THE MAGELLANIC CLOUDS

Walker (this book), gives a detailed review of the methods to derive distances to the Magellanic Clouds. Here we simply remind that: (i) Feast & Catchpole (1997) from a new Period-Luminosity relation for classical Cepheids, based on the Hipparcos parallaxes of a sample of Galactic Cepheids, derive \((m - M)_{0, \text{LMC}} = 18.70 \pm 0.10\). However, Madore & Freedman (1998), find that depending on reddening and metallicity effects on the selected sample of Galactic Cepheids with Hipparcos parallaxes, the LMC distance modulus may range from 18.44 to 18.57. (ii) van Leeuwen et al (1997) used
Hipparcos parallaxes and infrared photometry of a sample of Mira variables to establish the zero point of the Mira period-luminosity relation. They derive a distance modulus for the LMC of: \((m - M)_{0, \text{LMC}} = 18.54 \pm 0.18\).

(iii) the "light echo" of SN1987A leads to distances to LMC ranging from \((m - M)_{0, \text{LMC}} < 18.37 \pm 0.04\) (possibly increased to 18.44 for an elliptical shape of the supernova expansion ring: Gould & Uza 1998), to 18.58 \pm 0.03\) (Panagia, Gilmozzi & Kirshner 1997), and 18.67 \pm 0.08, Lundqvist & Sonneborn (1997).

2.3. ECLIPSING BINARIES

Detached eclipsing double-lined spectroscopic binaries have recently been discovered near the main sequence turn-off of a number of GCs by Kaluzny et al. (1996,1997). They can be used to directly measure distances to clusters via a procedure which is similar but simpler than the Baade-Wesselink for RR Lyrae variables. In practice, the orbital parameters, the luminosity-ratios, the size of the orbit, and the linear radii of the two components are derived from the light and radial velocity curves, on a purely geometrical approach. The latter are combined with the surface brightness of each component inferred from the observed colours, and distances are so derived. A preliminary estimate of the LMC distance modulus using detached eclipsing binaries gives \((m - M)_{0, \text{LMC}} = 18.6 \pm 0.2\) (Guinan et al. 1995). The major shortcoming of this very promising technique is how to properly derive the surface brightness from the observed quantities (colours, line ratios etc.).

2.4. DYNAMICAL MODELS FOR GLOBULAR CLUSTERS

Distances to GCs may be derived by comparing proper motion and radial velocity dispersions within each cluster using King-Michie type dynamical models. While results for individual clusters derived by this procedure are affected by large error bars, and depend on cluster dynamical models, they do not make use of any standard candles and are totally independent from stellar evolution models. Rees (1996) gives updated distances based on this technique for ten GCs. According to Chaboyer et al. (1998) revision of Rees’ results, this method leads to \(M_V(\text{RR}) = 0.59 \pm 0.11\) (average on 6 clusters). The astrometric distances to GCs might be somewhat shorter than those derived from subdwarf fittings. Data for a larger number of clusters would be required to increase the accuracy of the method.

3. Fitting globular cluster main sequences with local subdwarfs

In principle, fitting the main sequence of GCs with sequences of local subdwarfs is a standard one-step distance calibration which uses local (primary)
distance indicators observed in farther objects. As usual in such distance calibrations, the critical issues are related to the supposed identity of local calibrators and farther ladders, since it is implicitly assumed that (i) the two samples are extracted from the same parent population (that is: the local subdwarf population is supposed to be identical to the main sequence of GCs); and (ii) the same selection criteria were applied in selecting objects in the field and in clusters. While, naively, the first assumption sounds reasonable in the present case, it should be critically reviewed to assess its validity. On the other side, the selection criteria used for local subdwarfs are clearly very different from those which apply to stars in GCs; hence some systematic correction must be applied.

3.1. TEMPLATE MAIN SEQUENCES

There are two basic difficulties in the fitting procedure. First, the main sequence is a rather steep relation between colour and magnitude \(4 < \frac{dV}{d(B-V)} < 7\) in the colour range of interest). Very accurate photometric data are then required, since any error in the intrinsic colours of either sequences will cause a corresponding (large) error in the derived magnitudes. Second, due to the sensitivity of the main sequence colour to the chemical composition the comparison should be made over a restricted range in metal abundances. Unfortunately, local subdwarfs are rare objects. The most metal-poor (and likely oldest) GCs have \([\text{Fe/H}] < -2\). There are no subdwarfs with parallax error below 10% in this metallicity bin, in the Hipparcos catalogue, and less than 10 with \(-2 < [\text{Fe/H}] < -1.5\). Furthermore, the distribution of stars with metallicity is strongly skewed toward high abundances, and large errors may be done by misidentifying the mean value within a bin with the value at mid bin (see discussion in P98).

Since unevolved main sequences for systems of different metallicity are closely parallel to each other in the magnitude range \(5.5 < M_V < 7\) (G97; this feature is predicted by models and confirmed by observations of GCs although the observed slope of the main sequence is slightly shallower than predicted by most recent models) it is however possible to correct the colour of each local subdwarf to that the star would have if it had a given absolute magnitude, and derive a relation between the colour at this absolute magnitude and metallicity. Once this relation is determined (see Figure 1), colours of subdwarfs within the magnitude range \(5.5 < M_V < 7\) can be (empirically) corrected for the metallicity effect, providing template main sequences for any metallicity within the range covered by the calibrators (essentially \([\text{Fe/H}] > -2\)). As shown in Figure 1, various sets of isochrones (Straniero & Chieffi 1991; D’Antona, Caloi & Mazzitelli 1997; Bertelli et al. 1997; VandenBerg 1997) well reproduce the observed slope of the colour-
Figure 1. Relation between the colour of the main sequence \((B - V)_{Mv=6}\) and the metallicity \([Fe/H]\) from local subdwarfs. Filled symbols are bona fide single stars; open symbols are known or suspected binaries. Overimposed are predictions from different isochrone sets (SC91=Straniero & Chieffi 1991; VDB97=VandenBerg 1997; DCM97=D’Antona, Caloi & Mazzitelli 1997; B97=Bertelli et al. 1997)

metallicity relation (the helium content for all these sets of isochrones is close to the cosmological value of \(Y = 0.24\)).

3.2. PHYSICAL BIASES

Systematic differences between local subdwarfs and cluster stars might be due to a different formation mechanism, and/or to dynamical effects of the environment. For instance, evidences of extra-mixing along the red giant branch causing anomalous abundances of some elements (C, N, O, Na, Mg, Al) are found in globular cluster giants, while are much less extreme amongst field stars (see Kraft 1994 for a review). While the origin of these abundance anomalies is still unclear, they show that some structural parameter may be significantly different for stars in different environments. It is generally assumed that the impact of these differences on the main sequence colours and magnitudes is small, but they likely have a significant impact in later evolutionary phases (see e.g. Sweigart & Catelan 1998).
3.3. BINARITY

Further concern arises from the contamination due to binary systems. The presence of a secondary component brightens and reddens the light from the primary. Hence, main sequences contaminated by binaries would appear brighter (by as much as 0.75 mag), causing a too long distance scale whether contamination is in the local subdwarf sample, or a too short distance scale if the contamination is in the GCs. Binary incidence is likely much higher amongst field stars than amongst globular cluster stars, since a large fraction of primordial binaries originally present in GCs are destroyed by close interactions with other cluster stars during the cluster lifetime. A large fraction of new binary systems may form in the very dense core of some GCs; however these regions are not sampled by the accurate and deep colour-magnitude diagrams considered here. The extensive binary search by Carney et al. (1994) provided a binary fraction for field metal poor stars roughly similar to that obtained for Population I stars: likely \( \sim \frac{1}{2} \) of stars are binaries. Analogous searches in GCs lead to much lower values (\( \leq 20\% \): Pryor et al. 1989; Kaluzny et al. 1998; Rubenstein & Bailyn 1997; Ferraro et al. 1997) at least in the outer cluster regions. However, larger values have been obtained from extensive radial velocity surveys using Fabry-Perot spectrographs (Fischer et al. 1994; Cote et al. 1994). On the other side, random star blending (a rather frequent occurrence in the crowded field of GCs) has the same photometric consequences of physical binarity.

To reduce concern related to binary contamination, the usual approach is to remove known or suspected binaries from the field stars (see e.g. R97), and to use modal rather than mean values to identify the main sequence mean loci for globular clusters (see e.g. Sandquist et al. 1996). However, some residual undetected binary may still be present among the field stars, and modal colours may still be somewhat affected by binaries. Unfortunately, determination of accurate binary corrections is difficult, since they depend on both the actual incidence of binaries, and on the distribution of secondary masses (or luminosities). A large rough estimate of 0.18 mag was estimated by P98 for the correction to apply to field stars, on the assumption that half the stars are binaries, and that the correction for each binary is on average half the maximum value. This correction was applied to known and suspected binaries, as well as \textit{bona fide} single stars. A much more in depth analysis of this problem was presented by G97. These authors considered both the average offset of the colours and the scatter around it, when comparing binaries and \textit{bona fide} single stars. From this comparison, they derived an average correction for each binary of 0.16 \pm 0.05 mag, and a fraction \( p \leq 0.16 \) of undetected binaries in their subdwarf sample (to be added to the 41\% of stars which are known or suspected binaries). The av-
average binary correction (in the sense to decrease distances) derived by G97 is of $0.02 \pm 0.01$ mag; this correction must however be applied only to *bona fide* single stars (correction for the whole sample would be $\sim 0.08$ mag). Carretta et al. (1998, C98) considered the effect of the binary contamination on the mean loci of GCs. They found that in typical ground-based c-m diagrams, the mean loci should be $\sim 0.005$ mag redder (with an uncertainty of about 50%) than the real single star main sequence, even after the iterative mean procedure has been adopted. The correction on the cluster distance moduli should then be $\sim 0.03 \pm 0.02$ mag, in the sense to increase distances.

3.4. STATISTICAL BIASES

The statistical correction to parallaxes obviously depend on the sample selection criteria. We consider here three effects.

Parallaxes are systematically overestimated (and then stars are placed too close, on average) if stars are selected according to the parallax value ($\pi > \pi_{\text{threshold}}$) and/or if weights are given according to the ratio between the measured parallax and its error ($\pi/\sigma_{\pi}$). The corresponding corrections are called Lutz-Kelker corrections from the authors who first proposed a solution to this problem (Lutz & Kelker 1973) for a sample extracted from a population uniformly distributed in space. Lutz-Kelker corrections are strongly dependent on $\pi/\sigma_{\pi}$, and are a function of the parallax distribution of the original population, which is generally not known. To solve this problem, Hanson (1979) proposed to use the proper motion ($\mu$), since proper motions distribute as $\sim \mu^{1-n}$ when parallaxes distribute as $\sim \pi^{-n}$. However, considerable care should be exerted when Lutz-Kelker corrections are large, because the distribution of parallaxes may well be different from a simple power law. Best procedure is to use only stars with good parallaxes ($\sigma_{\pi}/\pi < 0.12$, or similar), because in this case Lutz-Kelker corrections are small ($< 0.2$ mag). On the whole, Hipparcos provided enough accurate parallaxes for subdwarfs to reduce concern related to the Lutz-Kelker correction to $< 0.02$ mag for the weighted average of the sample.

If stars have a range of luminosities, a magnitude limited sample would be biased toward luminous objects (Malmquist bias). The corrections for Malmquist bias have opposite sign with respect to Lutz-Kelker corrections. Malmquist bias should be unimportant for unevolved main sequence stars ($M_V > 5.5$), where the intrinsic scatter in luminosity is negligible; while it may significantly affect the region around the turn-off, where a rather large intrinsic magnitude range exists, biasing the population toward more evolved objects. On the whole, the effect should be negligible in the sample considered by G97 (only unevolved main sequence stars), while it should
be considered (although it is small) for the samples considered by R97 and P98, who included turn-off stars in their distance determinations. Indeed, P98 derived systematic corrections for Malmquist bias using MonteCarlo simulations.

To enlarge the sample of metal-poor stars with good parallaxes, P98 extracted stars from the whole Hipparcos catalogue, using selection criteria based on colours, since most metal-poor stars are expected to be bluer (at a given absolute magnitude). While these selection criteria added only a few stars (see C98), the properties of the resulting sample are strongly affected by the metallicity distribution of the stars in the Hipparcos catalogue, and by the distribution of errors in colours and magnitudes. Both these quantities are poorly known. Results obtained from such samples are thus likely to be biased by stars with colours measured too blue (and hence too faint luminosities). This problem is overcome by using samples selected according to different criteria (e.g. proper motions or metal abundances: R97; G97; C98)

3.5. ERRORS IN STELLAR PARAMETERS

The largest source of uncertainty in distances to GCs derived via subdwarf fitting arises from the determination of the stellar parameters. Here, we are mainly concerned with systematic differences between field subdwarfs and GCs stars.

3.5.1. Colours

Due to the steepness of the main sequence, systematic errors in the photometry may cause large errors in the derived distance moduli. Ideally, both samples should be observed on the same photometric system, and, possibly, with the same instrumental set-up. However, this is made difficult by the large difference in luminosity (about 10 mag). Indeed, magnitudes and colours for field stars were obtained with photomultipliers, while GCs are observed with CCD’s. Furthermore, $V - R$ and $V - I$ colours in the Johnson-Cousins system are not available for most field subdwarfs, and colours obtained transforming data originally taken in other photometric systems (Kron and Johnson $V - R$ and $V - I$) have a large scatter (Clementini et al. 1998). Since infrared deep colour magnitude diagrams are not available for most clusters, the only colour well suited for main sequence fitting is, at present, the $B - V$. Data for GCs were obtained with CCD’s, and transformed to Johnson-Cousins system by observation of Landolt’s standards (Landolt 1983). Albeit considerable care was devoted to these calibrations, some uncertainty still exists on the derived transformations, and results for individual clusters may well have rather large errors (from
3.5.2. Reddening

Reddening is a very crucial issue. Again, here we are mainly interested in the derivation of a uniform scale for both field subdwarfs and GCs. Unfortunately, a direct comparison of the excitation temperatures of field subdwarfs and globular cluster main sequence stars is not feasible yet, due to the limits of the present day’s instrumentation (advances are expected with UVES at the VLT). Nevertheless, some constraints can be derived by comparing the adopted reddenings with a cosecant law.

Two reddening scales have been used for subdwarfs. R97, G97 and C98 used reddening estimates from Carney et al. (1994), Schuster & Nissen (1989), and Ryan & Norris (1991). A star-by-star comparison shows that these reddening estimates are on a uniform scale. The reddenings used by P98 (from Arenou et al. 1992) are on average 0.016 mag larger (for the 17 stars of P98 Table 2), leading to a shorter distance scale for globular clusters. On the other side, all authors used the same scale (from Zinn 1980) for globular cluster reddenings. (To reduce errors for individual objects, G97 averaged Zinn’s reddening estimates with other most recent values, but these were first put on the same uniform scale). When compared to cosecant-laws for reddening (like e.g. that adopted by Bond 1980), we find that GCs and subdwarfs are on a uniform reddening scale if the height scale of the galactic dust disk is 100 pc when G97, C98 and R97 reddenings are considered; and 40 pc if the reddenings used by P98 are considered. While the former value is in the middle of current determinations of the galactic dust scale-height, (50-150 pc: Lynga 1982, Pandey & Mahra 1987, Scheffler & Elsässer 1987, Spitzer 1978, Salomon et al. 1979, Burton 1992, Chen 1998), the latter is at the lower extreme of the admitted range. On the whole, the most appropriate reddening scale of subdwarfs is still quite uncertain (±0.015 mag), with the value used by G97, C98 and R97 at mid of the admitted range.

3.5.3. Metal abundances

A systematic difference of 0.1 dex in the adopted metallicity scale for subdwarfs and GCs results into an error of ~ 0.07 mag (0.03 mag at [Fe/H]=−2, and 0.11 mag at [Fe/H]=−1.0) on the distances when main sequence stars are used, and it is much larger (but of opposite sign) when subgiants are considered. C98 discussed this point at some length. Clearly, a uniform abundance analysis should be used (see G97); however, some differential effect (at ~ 0.1 dex) may still be present, since abundances for globular clusters are derived using giants rather than dwarfs. A substantial progress is expected in the near future, when UVES at VLT will allow observations
of dwarfs and turn-off stars in GCs.

Metal abundance is not only defined by the Fe abundance, since the element-to-element ratios may be different from star-to-star. The most important elements other than Fe are here O and \(\alpha\)-elements. These element are overabundant in most metal-poor stars (Wheeler et al. 1989; see however King 1997, and Carney et al. 1997 for examples of metal-poor stars with no excess of O- and \(\alpha\)-elements) and in globular clusters (Gratton and Ortolani 1989). While overabundances in the two samples appear to be quite similar (Carney 1996; Clementini et al. 1998), a slightly larger excess in cluster stars cannot be excluded. If this were true, the distance scale would be somewhat underestimated.

4. Conclusion

In this Chapter we have discussed the results provided by the subdwarf-main sequence fitting method to derive distances and ages for GCs using Hipparcos trigonometrical parallaxes. Figure 2 illustrates some examples of fitting taken from C98, along with the distance moduli for a number of well observed clusters. Table 1 lists the values derived for the cluster distance moduli (including corrections for undetected binaries) by these authors, as well as the ages provided by the Straniero & Chieffi (1991) isochrones. These are in the middle of the determinations obtained with various isochrone sets.

The distance scale obtained by C98 is summarized by the following relation between the ZAHB absolute magnitude and metallicity:

\[
M_V(ZAHB) = (0.18 \pm 0.09)([\text{Fe/H}] + 1.5) + (0.53 \pm 0.04) \tag{1}
\]
Figure 2. Fits of the fiducial mean loci of the Globular Clusters considered in C98 with the position of the subdwarfs. Only \textit{bona fide} single stars with $5 < M_V < 8$ are used in the fits. Values of the parameters adopted in the analysis (without binary correction) are shown in each panel.

Table 2 lists the main sources of errors in this distance scale. The largest contributions come from uncertainties in the reddening scale and from the adopted metallicity scale. A reasonable guess of the overall uncertainty ($2 \sigma$) is given by a quadratic sum of the various sources of errors, this adds to about 0.12 mag, by itself implying a 15% uncertainty in derived ages.

Error bars on the ages should also include uncertainties in the stellar models. Some of the error sources should be interpreted as maximum errors, while others are standard deviations; furthermore, various error bars
are asymmetric. A Monte Carlo procedure may be devised to give statistically significative error bars (Chaboyer et al. 1996; G97). Using a similar approach, C98 best value for the absolute age of the oldest globular cluster is:

$$\text{Age} = 12.3^{+2.1}_{-2.0} \text{ Gyr}$$

where the error bars refer to the 95% confidence range.

In order to compare the distance moduli derived from subdwarf fitting with those from other methods, we list in Table 3 the distance modulus of the LMC obtained by various techniques. Whenever possible, we list distances originally estimated by the various authors. For some of the methods based on galactic RR Lyraes where direct estimates were not available we used $V_0 = 18.94 \pm 0.04$ and [Fe/H]$=-1.9$ for the variables in LMC GCs (Walker 1992). For the HB clump method, we give the original distance by Udalski (1998), as well as the values obtained after correcting for the metallicity dependence of the clump $M_1$ magnitude (Girardi et al. 1998), and using a lower reddening of $E(B-V) = 0.10$ (Bessell 1991). Finally, when considering the various determinations listed in Table 3, it should be reminded that the optical depth of the LMC, while small, is not negligible. While some of the distance estimates refer to the bar (like e.g. those for the Cepheids or for the HB clump), others are for a few individual objects (SN1987a, GCs with RR Lyrae) that may well be a few kpc (i.e. up to 10%) in front or behind the plane of the LMC.

We conclude by giving a few indications for future observations aimed at refining the present distance determinations. Current available instrumentation (both ground based and the HST) may be used to improve the photometric calibrations, that in various cases may be uncertain. It is par-

### TABLE 2. Systematic errors in GC distance moduli from subdwarf fitting

| Error Source       | Value       | $\Delta(m - M)$ |
|--------------------|-------------|-----------------|
| random errors      | $\pm 0.04$  |                 |
| Lutz-Kelker corrections | $\pm 0.02$  |                 |
| binaries           | $\pm 0.02$  |                 |
| [Fe/H]-scale       | $\pm 0.1$ dex| $\pm 0.08$     |
| $E(B - V)$-scale   | $\pm 0.015$ mag | $\pm 0.07$   |
| colour calibration | $\pm 0.01$ mag | $\pm 0.04$   |
| total              | $\pm 0.12$  |                 |
| Indicator                  | $ (m - M)_{0, LMC} $ | Authors                                      |
|----------------------------|---------------------|----------------------------------------------|
| Subdwarf fitting           | 18.54 ± 0.12        | Carretta et al. 1998                        |
| Cepheids                   | 18.70 ± 0.10        | Feast & Catchpole 1997                      |
|                            | 18.50 ± 0.07        | Madore & Freedman 1998                      |
| Miras                      | 18.54 ± 0.18        | van Leeuwen et al. 1997                     |
| SN1987a ring               | 18.37 ± 0.04        | Gould & Uza 1998 (circular)                 |
|                            | 18.44               | Gould & Uza 1998 (elliptical)               |
|                            | 18.59 ± 0.03        | Panagia et al. 1997                        |
|                            | 18.67 ± 0.08        | Lundqvist & Sonneborn 1997                  |
| Eclipsing binaries         | 18.6 ± 0.2          | Guinan et al. 1995                         |
| HB clump                   | 18.07 ± 0.12        | Udalski 1998                                |
|                            | 18.31 ± 0.12        | revised by Girardi et al. 1998              |
|                            | 18.43 ± 0.12        | E(B−V)=0.10                                |
| HB trig. parallax          | 18.37 ± 0.10        | Gratton 1998                                |
|                            | 18.42 ± 0.12        | eliminating HD17072                        |
| Stat. parallax             | 18.29 ± 0.12        | Layden et al. 1996                         |
|                            | 18.26 ± 0.15        | Fernley et al. 1998a                       |
| Baade-Wesselink            | 18.26 ± 0.04        | Clementini et al. 1995                     |
|                            | 18.34 ± 0.04        | Clementini et al. 1995 (p=1.38)             |
|                            | 18.53 ± 0.05        | McNamara 1997                              |
| GC Dynamical models        | 18.44 ± 0.11        | Rees 1996, revised by Chaboyer et al 1998   |

particularly important to repeat the observations for M92, which yields by far the largest age, and it is then the best candidate as the oldest GC. Determination of consistent reddening and metal abundance scales for GCs and local subdwarfs is a crucial step. Significant progresses in this respect are expected using UVES at VLT2, which will get high resolution, high $S/N$ spectra of cluster main sequence stars. High precision (error $\sim$ 1 mas) ground-based parallaxes on the Hipparcos absolute scale may be obtained for some very metal-poor ([Fe/H] $< -2$) subdwarfs with $6 < M_V < 8$ and $V > 11$. Finally, trigonometric parallaxes of GC stars will be directly measurable when GAIA will be launched.

References
Arenou, F., Grenon, M., & Gomez, A. 1992, A&A 258, 104
Bertelli, P., Girardi, L., Bressan, A., Chiosi, C., & Nasi, E. 1997, in preparation
Bessel, M.S. 1991, A&A, 242, L17
Bond, H.E. 1980, ApJS, 44, 517
Burton, W.B. 1992, in The Galactic Interstellar Medium, eds. D. Pfenniger & P. Bartholdi, Spriger-Verlag, Berlin, 1
Cacciari, C., Clementini, G. & Fernley, J.A.1992, ApJ, 396, 219
Caloi, V., D’Antona, F. & Mazzitelli, I. 1997, A&A, 320, 823
Carney, B.W. 1996, PASP, 108, 900
Carney, B.W., Storm, J. & Jones, R.W. 1992, ApJ, 386, 663
Carney, B.W., Latham, D.W., Laird, J.B., & Aguilar, L.A. 1994, AJ, 107, 2240
Carney, B.W., Wright, J.S., Sneden, C., Laird, J.B., & Aguilar, L.A. 1997, AJ, 114, 363
Carretta, E., Gratton, R.G., Fusi Pecci, F., & Clementini, G. 1998, in preparation, (C98)
Cassisi, S., Castellani, V., DeGriessenti, S., Weiss, A. 1998, A&AS, 129, 267
Chaboyer, B., Demarque, P., Kernan, P. J., & Krauss, L.M. 1996, Science, 271, 957
Chaboyer, B., Demarque, P., Kernan, P. J., Krauss, L.M. 1998, ApJ, 494, 96
Chen, B., 1998 private communication
Clementini, G., Carretta, E., Gratton, R. G., Merighi, R., Mould, J. R., McCarthy, J. K. 1995, AJ, 110, 2319
Clementini, G., Gratton, R.G., Carretta, E., & Sneden, C. 1998, submitted to MNRAS
Cole, A.A. 1998, ApJ, 500, L137
Cote, P., Welch, D.L., Fischer, P., Da Costa, G.S., Tumblyn, P., Seitzer, P., Irwin, M.J. 1994, ApJS, 90, 83
Da Costa, G.S. & Armandroff, T.E. 1990, AJ, 100, 162, (DA)
D’Antona, F., Caloi, V., & Mazzitelli, A. 1997, ApJ, 477, 519
Feast, M.W. 1998, MNRAS, 293, L27
Feast, M. W. & Catchpole, R. M. 1997, MNRAS, 286, L1
Fernley, J., 1994, A&A 284, L16
Fernley, J., Barnes, T.G., Skillen, I., Hawley, S.L., Hanley, C.J., Evans, D.W., Solano, E., Garrido, R. 1998a, A&A, 332, 875
Fernley, J., Carney, B.W., Skillen, I., Cacciari, C., Janes, K. 1998b, MNRAS, 293, L61
Ferraro, R., Carretta E., Fusi Pecci, F., Zamboni, A. 1997, A&A, 327, 526
Fischer, P., Welch, D.L., Mateo, M., Cote, P. 1994, AJ, 106, 1508
Fusi Pecci, F., & Bellazzini, M. 1998, in II Conference on Faint Blue Stars, A.G.D. Philips Ed., in press
Girardi, L., Groenewegen, M.A.T., Weiss, A., Salaris, M. 1998, submitted to MNRAS, astro-ph/9805127
Gould, A., & Popowski, P. 1998, astro-ph/9805176
Gould, A. & Uza, O. 1998, ApJ, 494, 118
Gratton, R.G. 1998, MNRAS, 296, 739
Gratton, R.G. & Ortolani, S. 1989, A&AS, 211, 41
Gratton, R.G., Fusi Pecci, F., Carretta, E., Clementini, G., Corsi, C.E., & Lattanzi, M. 1997, ApJ, 491, 771, (G97)
Guinan, E.F., Bradstreet, D.H., & DeWarf, L.E. 1995, in The Origins, Evolution, and Destinies of Binary Stars in Clusters, E.F. Milone & J.-C. Mermilliod, Eds., ASP Conf. Ser 90, p. 196
Hanson, R.B. 1979, MNRAS, 186, 675
Holland, S., Fahman, G.G., Richer, H.B. 1996, AJ, 112, 1035
Jimenez, R., Flynn, C., & Kotoneva, E. 1998, MNRAS, in press
Jones, R. V., Carney, B. W., Storm, J., Latham, B. W. 1992, ApJ, 386, 646
Kaluzny, J., Kubiak, M., Szymansi, M., Udalski, A., Krzeminski, W., Mateo, M. 1996, A&AS, 120, 139
Kaluzny, J., Kubiak, M., Szymansi, M., Udalski, A., Krzeminski, W., Mateo, M., Stanek, K. 1997, A&AS, 122, 471
Kaluzny, J., Hilditch, R.W., Clement, C., Rucinski, S.M. 1998, MNRAS, 296, 347
King, J.R. 1997, AJ, 113, 2302
Kraft, R.P. 1994, PASP, 106, 553
Kurucz, R.L. 1993, CD-ROM 13, ATLAS9 Stellar Atmosphere Program and 2 km/s Grid
(Cambridge: Smithsonian Astrophys. Obs.)
Layden, A.C., Hanson, R.B., Hawley, S.L., Klemola, A.R., Hanley, C.J. 1996, AJ, 112, 2110
Landolt, A.U. 1983, AJ, 88, 439
Lee, M.G., Freedman, W.L., & Madore, B.F. 1993, ApJ, 417, 553, (LFM)
Liu, T., & Janes, K.A. 1990a, ApJ, 360, 561
Liu, T., & Janes, K.A. 1990b, ApJ, 354, 273
Lundqvist, P., & Sonneborn. G. 1997, in SN1987A: Ten Years After, M. Phillips & N. Suntzeff, Eds., ASP Conf. Ser., in press
Lutz, T.E., & Kelker, D.H. 1973, PASP, 85, 573
Lynga, G., 1982, A&A, 109, 213
Madore, B. & Freedman, W. 1998, ApJ, 492, 110
McNamara, D.H. 1997, PASP, 109, 857
Panagia, N., Gilmozzi, R., & Kirshner, R.P. 1997, in SN 1987A: Ten Years After, eds. M. Phillips and N. Suntzeff, ASP Conf. Ser., in press
Paczynski, B., & Stanek, K.Z. 1998, ApJ, 494, L219
Panek, A.K., & Mahra, H.S., 1987, MNRAS, 226, 635
Pont, F., Mayor, M., Turon, C., & VandenBerg, D.A. 1998, A&A, 329, 87, (P98)
Popowski, P., & Gould, A. 1997, astro-ph/9703140
Pryor, C., McClure, R.D., Hesser, J.E., Fletcher, J.M. 1989, in Dynamics of Dense Stellar Systems (Cambridge: Cambridge University Press) p. 175
Rees, Jr. R.F. 1996, in Formation of the Galactic Halo... Inside and Out, H. Morrison, & A. Sarajedini eds., ASP Conf. Ser. 92, p. 289
Reid, I.N. 1997, AJ, 114, 161, (R97)
Reid, I.N. 1998, AJ, 115, 204, (R98)
Renzini, A. 1991, in Observational Tests of Inflation, ed. T. Banday & T. Shanks (Dordrecht: Kluwer) 131
Renzini, A. et al. 1996, ApJ, 465, L23
Rich, R.M., Mighell, K.J., Freedman, W.L., & Neill, J.D. 1996, AJ, 111, 768
Richer, H.B., et al 1995, ApJL, 451, L17
Rubenstein, E.P. & Baylin, C.D. 1997, ApJ, 474, 701
Ryan, S.G., & Norris, J.E. 1991, AJ, 101, 1835
Salaris, M. & Cassisi, S. 1997, MNRAS, 289, 406, (SC97)
Salaris, M. & Cassisi, S. 1998, astro-ph/9803103. (SC98)
Salomon, P.M., Sanders, D.B., & Scoville, N.Z. 1979, in Large-Scale Characteristics of the Galaxy, IAU Symp. no. 84, Burton, W.B. ed., Reidel, Dordrecht, p. 35
Sandage, A. 1970, ApJ, 162, 841
Sandage, A. 1993a, AJ, 106, 719
Sandage, A. 1993b, AJ, 106, 703
Sandquist, E.L., Bolte, M., Stetson, P.B., & Hesser, J.E., 1996, ApJ, 470, 910
Scheffler, H., & Elsässer, H., 1987, Physics of the Galaxy and Interstellar Medium, Springer-Verlag, Berlin
Schuster, W.J., & Nissen, P.E. 1989, A&A, 221, 65
Skillen, I., Fernely, J.A., Stobie, R.S., & Jameson, R.F., 1993, MNRAS, 265, 301
Spitzer, L. 1978, Physical Processes in the Interstellar Medium, Wiley, New York
Stanek, K.Z., & Garnavich, P.M. 1998, astro-ph/9802121
Stetson, P.B., VandenBerg, D.A., Bolte, M. 1996, PASP, 108, 560
Storm, J., Carney, B. W., & Latham, D. W. 1994, A&A, 290, 443
Straniero, O. & Chieffi, A. 1991, ApJS, 76, 525
Straniero, O. & Chieffi, A. 1997, private communication
Sweigart, A.W. & Catelan 1998, submitted to ApJL, preprint
Tsujimoto, T., Miyamoto, M., & Yoshii, Y. 1998, ApJ, 492, L79
Udalsky, A., et al 1992, AcA, 42, 253
Udalsky, A. 1998, astro-ph/9805221
Udalsky, A., Szynański, M., Kubacki, M., Pietrzyński, G., Woźniak, P., & Zebruń, K.,
1998, AcA 48,1 (astro-ph/9803035)
VandenBerg, D.A. 1997, private communication
VandenBerg, D.A., Stetson, P.B., Bolte, M. 1996, ARAA, 34, 461
van Leeuwen, F., Feast M. W., Whitelock, P. A. & Yudin, B. 1997, MNRAS, 287, 955
Walker, A. R. 1992, ApJ, 390, L81
Wheeler, J.C., Sneden, C., & Truran, J.W. 1989, ARAA, 27, 279
Zinn, R. 1980, ApJS, 42, 19
Zoccali, et al 1998, in STScI Symposium on Unsolved Problems in Stellar Evolution, Baltimore, May 1998, M.Livio Ed., in press (Z98)