Engineering models for turbine wake velocity deficit and wake deflection. A new proposed approach for onshore and offshore applications.

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Abstract. An engineering wake model based on the Ainslie model is proposed. The eddy viscosity term associated with momentum diffusivity is modified to take into account the effects of atmospheric stability. The parameters used are typically available from high-quality on-site measurement campaigns and the effects of atmospheric stability are based on empirical models for the estimation of the Monin-Obukhov length. The dependence on physical quantities only is particularly advantageous for fast wake modelling, since no further parametrical tuning is needed for each specific case. The proposed wake model is initially compared to wind tunnel data and CFD simulations to test the chosen Obukhov lengths for specific flow conditions. Wind farm production data and concurrent meteorological data at one onshore site are then used to validate the model for specific on-site flow conditions, obtaining good results. Two offshore wind farms are also used to assess the model in a large-scale wind farm scenario: results look promising although some reservations are expressed on the effect of the wake superposition model. Models for the prediction of wake centreline deflection due to yaw are compared at different yaw angles using wind tunnels data and CFD simulations. Although the EPFL model showed some advantage, especially in non-neutral conditions, both models give satisfactory results. Furthermore, it was showed that the wake deflection caused by the rotating wake for non-yawed turbines can have a large impact on the predictions of the centreline deflection.

1. Introduction
The need for fast and increasingly accurate wake models for wind turbines is promoted by the quest for higher accuracy in energy yield predictions, the increased size of wind farms, and by the need to exploit the potential for increasing wind farm energy production through active wake control methods which are currently being developed. The increased interest of the industry towards wind farm optimisation control algorithms, for example wake steering, highlights even more the importance of reliable wake models which can run fast enough for control design optimisation and testing.

Numerous efforts have resulted in various wake models available in literature: from the seminal works of Lissaman [1], Jensen [2] and Ainslie [3, 4], to the more recent works of Bastankhah & Porté-Agel [5] and Niayfair & Porté-Agel [6], Ishihara [7] and Gebraad [8, 9, 10]. These wake models have been developed and validated against wind farm production data, computational simulations and wind tunnel data. Engineering wake models rely, to different extents, on wake measurements and subsequent fitting of empirical parameters, which exposes the models to the risk of over-fitting, potentially producing erroneous results when extrapolated to different configurations of turbine characteristics, wind farm layouts and atmospheric conditions.

More advanced models based on a Computational Fluid Dynamics (CFD) approach have been used in the recent years. From simulations based on Reynolds-Averaged-Navier-Stokes (RANS) equations and related turbulence models, to the more complex use of Large-Eddy-Simulations (LES) using Actuator-Line turbine models [11, 12, 13]. Their main common drawback is the need of large computational resources and time, which does not suit the market needs for a reliable prediction of wake losses during the planning phase of a wind farm, usually including different turbine models and configurations.
In the wind energy industry, it is common to use RANS simulations associated to the $k - \varepsilon$ turbulence model and generally using an Actuator Disk Model (abbreviated as ADM-R or ADM-NR, where the suffix R indicates a rotating disk model and NR indicates a non-rotating disk model) to allow the calculation of loads and wind speed deficits due to the presence of the rotor. It is also noted that amendments to the $k - \varepsilon$ model have been recently proposed for the use in wind turbine applications in [13]. A more computationally expensive approach, used for high-fidelity analysis, is Large Eddy Simulations (LES) in which only the eddies of smaller scales are modelled, whereas the larger eddies scales are numerically resolved using statistical techniques based on the Kolmogorov energy cascade analogy [14]. High fidelity simulations are usually associated with Actuator Line Models (ALM) considered to be more accurate in the calculation of body forces at the rotor [11].

Another important factor influencing turbine wakes is the atmospheric characteristics at the site and their change with time. The most commonly used wake models do not take atmospheric stability into consideration. However, some commercial wake models currently used in the offshore wind market such as FUGA [14] and FarmFlow [15], do use some simplified methods to estimate the effects of stability on the wake development. Deliverable 1.4 [16] of the CL-Windcon [17] project shows a comparison between different engineering wake models against a field test of a utility-scale wind turbine in different atmospheric conditions. The lack of atmospheric stability corrections in common wake models used for long-term energy yield calculations can easily be explained: taking the average of a sufficiently long time of turbine operation, the non-neutral atmospheric conditions average out to give neutral atmospheric conditions, at least for sites where very stable or very unstable conditions are not predominant. What is more, the estimation of the occurrences of different stability characteristics at the site can be a difficult task, since requiring the measurement of specific wind characteristics.

1.1 Objectives and outline

The main objective of this paper is to propose a new approach for the wake modelling of predominantly stable or unstable atmospheric conditions, detailed in Section 3. The proposed model is a modification of the Ainslie model [4], taking the effects of atmospheric stability into account. This model does not introduce any additional experimental parameter hence not requiring additional calibration against experimental data, and only requires an estimate of the Monin-Obukhov length (or MOL) [14]. A validation of this model against a set of experimental and wind farm production data (detailed in Section 2) is also presented, and an overview of the methodology for atmospheric stability classification is provided. In addition to the above, another objective of this study is to increase confidence in the use of existing wake deflection models for the purpose of wind farm control and optimisation.

It is also highlighted that this paper will only focus on simplified engineering models and steady-state turbine wakes, therefore not considering dynamic affects such as wake meandering. All the tests presented in this paper have been performed using LongSim, a software created and used internally in DNV GL for advanced studies on wind farm steady-state and dynamic simulation, set-point optimisation and wake steering optimisation.

2. Experimental and numerical data used for the calibration of models

A set of different datasets have been used to perform wake model calibrations and validations. These are detailed in the following sections.

2.1 Wind tunnel measurements and setup

Wind tunnel experiments were carried out at the Politecnico di Milano between 2017 and 2018, as part of the CL-Windcon project [17]. The wind tunnel is a closed loop, boundary-layer type and different turbulence intensities and wind shear profiles can be reproduced. A turbine model with a rotor diameter of 1.1 m, named G1, was designed and built (a detailed description of the turbine model can be found in the public Deliverable no. 3.1 [18]). Experiments were carried out for an isolated turbine and for groups of turbines, but only the single-turbine results have been used in this paper. Free flow wind speed
at the turbine’s hub height (0.825 m or 0.75 rotor diameters) is measured using a Pitot tube located 3 rotor diameters upstream of the turbine, and a full inflow-plane investigation was previously carried out to characterise the free flow (available in public Deliverable 3.4 [19]). Hot-wire probes were used to record flow measurements (with a sampling frequency of 2 kHz) which were averaged over a minute to obtain steady-state effects at 5, 7.5 and 10 rotor diameters behind the turbine location. All the tests were carried out with a mean wind speed of 5.5 m/s.

Two different flow conditions were reproduced in the wind tunnel experiments: the offshore condition is characterised by an averaged wind shear profile corresponding to a power law exponent of 0.08 and lower flow-wise turbulence, whereas the onshore wind shear condition corresponds to a power law exponent of 0.20 and higher flow-wise turbulence (a full description and characterisation of the wind tunnel experiments is presented in the public Deliverables [18, 19]). The standard deviation of the flow-wise free-stream wind speed component \( \sigma_u \) is shown in Figure 1, measured at the wind tunnel test section with no turbine models installed and measured at the location of the turbine rotor plane. The measured \( \sigma_u \) remains almost constant with height for the offshore case whereas it shows a noticeable decrease (for increasing height) for the onshore case across the rotor area. Turbulence stochastic models [20] generally assume that \( \sigma_u \) is constant with height. Although it is suggested to use the value of turbulence intensity averaged across the rotor, it is anticipated that this might cause some discrepancies between the wake models and the measured wake profiles.

Flow measurements were carried out both for a zero-yaw configuration and for different yaw angles, spanning from -40° to 40° with a 10° step. Detailed investigations have also been carried out on the measured time-series at different locations relative to the wind turbine, in particular to analyse the energy spectra, the estimated integral length scale and the measurement scattering. The spectral analysis (performed using a Welch approach with the help of a Hanning filter and a de-trending filter) has not shown unexpected features and the turbulent dissipation rate of the inertial subrange seems to agree well with the Kolmogorov law. These results are not shown here, but indicate that the turbulent flow development within the wake behaves as expected and shows no unexpected artefacts in the measurements provided, which are deemed of good quality.

![Figure 1. Variation with height of the standard deviation of the horizontal wind speed component for both offshore and onshore case, measured for the characterisation of the free-stream conditions in the wind tunnel at Politecnico di Milano.](image-url)

### 2.2 CFD simulations

A set of time-averaged results from the open-source SOWFA computational solver [12] was provided by the National Renewable Energy Laboratory (NREL). SOWFA uses a detailed Actuator Line Model (ALM) to produce high-fidelity unsteady simulations of wakes and turbine loads. The simulation entailed a single prototype 5MW turbine with a rotor diameter of 126 m and a hub height of 90 m, with averaged free-stream wind shear profile with power law exponent of approximately 0.18, averaged inlet
wind speed of 8 m/s and turbulence intensity estimated to be approximately 10 % (therefore characterised by overall neutral flow conditions). The inflow conditions are generated in a precursor simulation run using a one-equation eddy-viscosity model for the definition of the subgrid scale, therefore the averaged wind characteristics can show variations in the free-flow.

2.3 Wind farms production data

For the purpose of comparing the proposed wake model to other wake models and test its applicability to a real case scenario, production data from one small onshore wind farm, Wieringermeer, and two large offshore wind farms, Horns Rev I and Nysted, were utilised.

The Wieringermeer Wind Farm site consists of 5 wind turbines installed in a test site in Netherlands. These are variable speed and pitch turbines of 80 m diameter, 2.45 MW rated power and hub height of 80 m. TNO provided the data as part of the UPWIND project and gave permission to DNV GL to use mast measurements and power turbine production data (for a total of approximately 3.2 years of available data) at a single turbine, namely T6: this is the second turbine from the west, being waked by turbine T5 for westerly winds. The atmospheric conditions at the site were measured at a mast located in the vicinity of the turbines and were made available for wind directions ranging approximately between 230° and 320°. The instruments installed on the mast include a thermocouple measuring the temperature difference between 37 m and 10 m heights, 3 sonic anemometers and 3 cup anemometers mounted at heights of 108 m, 80 m and 52 m, and two wind vanes at 79.2 m and 51.2 m. Further details about the on-site measurements are provided in [21]. As the individual pressure and temperature signals were not provided, these were retrieved from the ERA5 [25] reanalysis dataset at the node centred approximately 4 km to the south of the mast. Although the uncertainty given by such dataset are considered larger than the ones associated to direct site measurements, these are considered acceptable for the purpose of the estimation of the atmospheric stability at the site. The results presented in this paper show binned mast and turbine data using 2° wind direction bins, as well as filtering out wind speed outside the range 6-8 m/s. Also, as the purpose of this analysis is to compare the model to measured data where highly stable and unstable atmospheric conditions are present, only the summer months (June to August) were considered, and the dataset has been subdivided into daytime and night-time periods, respectively ranging between 7-18 hours and 20-5 hours.

Horns Rev I and Nysted are large offshore wind farms situated respectively 20 km off the west coast of Denmark and 15 km off the south coast of Lolland Island, Denmark. Details on the turbine model, hub height and inter-turbine spacing are detailed in Table 1. Both wind farms have an oblique rectangle layout as shown in Figure 2. The atmospheric stability conditions at the sites have been classified based on 12 months of data recorded at the Horns Rev I site masts and based on 3 years of data recorded at the Nysted site mast: these were for the UPWIND project [22] from the calculation of the Richardson number based on the temperature difference and wind speed difference at two heights. The stability classification for Horns Rev I is subdivided into seven categories (very unstable, unstable, neutral/unstable, neutral/stable, stable, very stable), whereas in Nysted six categories were used (very unstable, unstable, neutral/unstable, neutral/stable, stable, very stable): these are shown in Figure 2. The production data and the averaged wind conditions (mean wind speed, wind direction and

| Turbine model            | Horns Rev I       | Nysted          |
|--------------------------|-------------------|-----------------|
| Vestas V80 2.0 MW        | Bonus 2.3 MW      |                 |
| Diameter [m]             | 80                | 82.4            |
| Hub height [m]           | 70                | 68.8            |
| Inter-turbine spacing [D]| 7                 | 10.3 east-west  |
| Available SCADA data     | 2 hours of data for each wind | 18 hours of data |
| Directional bins         | 255° - 285°, 5° step | 263° - 293°, 5° step |
| Reference turbine        | T07               | A5              |

| Table 1. Summary of characteristics of Horns Rev I and Nysted wind farms. |
mean turbulence intensity) recorded at the sites have been provided for three wind speed bins (6±1 m/s, 8±1 m/s and 10±1 m/s) and 7 direction bins as detailed in Table 1. The power production data for each turbine was provided as normalised to the reference upwind turbine for each respective site. Details on the data used for the modelling of the wind farms’ production are given in Section 4. Production and wind condition data for both Horns Rev I and Nysted wind farms have been provided as part of the UPWIND project [22, 23, 24].
3. Wake models

3.1 Wake deficit model

The Ainslie model is part of that class of turbine wake models called Gaussian models and it is sometimes referred to as eddy viscosity model (or EVM). It is obtained solving the Reynolds-Averaged-Navier-Stokes (RANS) equations in two dimensions and formulated in cylindrical coordinates, using the assumptions for incompressible, stationary and axisymmetric flow and neglecting gravity forces and pressure gradients. By using the axisymmetric flow and the Boussinesq assumptions, the viscous forces can be simplified to:

\[ -u_x u_r = \frac{\mu}{\rho} \frac{\partial u_x}{\partial r} = \nu \frac{\partial u_x}{\partial r} \]  

(1)

In equation (1), \( \rho \) is the air density, \( \mu \) is the dynamic viscosity of the air, \( x \) and \( r \) are respectively the axial and radial coordinates, and \( u_x \) and \( u_r \) are respectively the axial and radial velocity components. Also, the kinematic viscosity (or momentum diffusivity) indicated as \( \nu \) in the equation above can be expressed as:

\[ \nu = \nu_{amb} + \nu_t, \]

where the first is the ambient kinematic viscosity (small for air in standard conditions, in the order of \( 1.5 \times 10^{-5} \) m\(^2\)/s and usually neglected), and where \( \nu_t \) is the eddy viscosity term. Based on dimensional analysis, this term can be rewritten as:

\[ \nu_t = k_1 b_w \left( \frac{U}{D} \right) \left( 1 - \frac{U_c}{U_{HH}} \right) + \frac{\kappa^2 \left( \frac{U_{HH}}{D} \right) \ln \left( \frac{U_{HH}}{z_0} + \Psi_m \left( \frac{Z}{L} \right) \Phi_m \left( \frac{Z}{L} \right) \right)}{\nu_t} \]

(6)
In the formula above, $b_w$ is the wake width and $U_c$ is the wake centreline velocity. Also, note that no assumption that $HH \approx D$ is used and the stability correction term $\Phi_m$ is defined in the next section. Also, the filter function used by Ainslie for near-wake regions is set to $F=1$, hence this formula is valid only in the far-wake. Furthermore, using the approximation $\frac{u_k}{\kappa} \approx \sigma_u [23, 28]$, the logarithmic function above can be approximated to $1/\text{Tl}$, where TI indicated the turbulence intensity. The proposed model will be referred in this paper as the \textit{Stratified-EVM} wake model.

3.2 Experimental laws for atmospheric stability

Different experiments were carried out to describe the wind and temperature profiles in the atmospheric boundary layer (ABL), most notably the Kansas study [20], from which the Businger-Dyer experimental model was obtained [26], and some tall mast experiments [29] from which the Högström [27] and the Gryning [30] models originate. The Businger-Dyer relationship (3,4) is used in this study with the revised parameters by Högström. This defines the correction term $\Phi_m$ as follows:

- Stable conditions:  
  \begin{equation}  
  \Phi_m = 1 + \frac{5z}{L}  \tag{7}  
  \end{equation}

- Unstable conditions:  
  \begin{equation}  
  \Phi_m = \left(1 - \frac{19.3z}{L}\right)^{-0.25} \tag{8}  
  \end{equation}

The set of equations above depend on the Monin-Obukhov length, $L$ (or the Monin-Obukhov stability parameter $z/L$). This parameter can be difficult to estimate, as it changes throughout the day (and therefore should be seen rather as a distribution or a time-series) especially for sites with extreme weather conditions. Another common parameter used for the classification of stability in the atmosphere is the bulk Richardson number, which can be defined as:

\begin{equation}  
Ri_B = \frac{g(\bar{\theta} - T_s)\Delta z}{T_s \Delta U^2} \tag{9}  
\end{equation}

where $g$ is the gravitational acceleration constant, $\bar{\theta}$ is the potential temperature, which can be defined as $\theta = T \left(\frac{P}{P_{ref}}\right)^{\left(\frac{R_d}{C_p}\right)}$, where $R_d$ is the gas constant for dry air and $C_p$ is the specific heat constant for air, $P$ is the atmospheric pressure, $P_{ref}$ the pressure reference and $T_s$ the temperature near the ground expressed in Kelvin.

The ranges of MOL, Obukhov stability parameter and bulk Richardson number typically used in the industry are summarised in the following table.

| Stability class     | Pasquill $^{[31]}$ | $L$ [m]$^{[14]}$ | $z/L$ [-]$^{[32]}$ | $\text{RiB}$$^{[33]}$ |
|---------------------|--------------------|------------------|------------------|-----------------|
| Very unstable       | A                  | -100 < L < -50   | -1.4 < $z/L$ < -0.35 | $\text{RiB} \leq -0.023$ |
| Unstable            | B                  | -200 < L < -100  |                  | $-0.023 \leq \text{RiB} < -0.011$ |
| Neutral/Unstable    | C                  | -500 < L < -200  |                  | $-0.011 \leq \text{RiB} < -0.0036$ |
| Neutral             | D                  | $|L| > 500$      | $|z/L| < 0.35$    | $-0.0036 \leq \text{RiB} < 0.0072$ |
| Neutral/Stable      | E                  | 200 < L < 500    | $0.35 \leq z/L < 7.0$ | $0.0072 \leq \text{RiB} < 0.042$ |
| Stable              | F                  | 50 < L < 200     |                  | $0.042 \leq \text{RiB} < 0.084$ |
| Very stable         | G                  | 10 < L < 50      |                  | $\text{RiB} \geq 0.84$ |

\textbf{Table 2.} Summary of atmospheric stability classes used in literature, expressed using different metrics.

3.3 Turbine interaction

Every turbine affected by the wakes of a turbine upstream will experience inlet flow characteristics which cannot be approximated to the ones experienced in the free, undisturbed flow. Two main
characteristics are taken into consideration when modelling the wake at a turbine downstream: the added turbulence model and the superposition model.

The added turbulence model allows to estimate the turbulence intensity seen by the affected turbine. Two approaches are typically used: the Quarton-Ainslie model [34] and the Crespo-Hernandez model [35]. They model the turbulence intensity at the affected turbine as the combination of the free-stream turbulence intensity and an added turbulence, which is modelled as a function of turbulence intensity and thrust coefficient (in the case of the first model) or as a function of turbulence intensity and axial induced velocity (in the case of the second model). Moreover, it is noted that the Crespo-Hernandez model is considered valid only in the far-wake, assumed to start at 5 rotors diameters downstream of the turbine, whereas this limitation is not considered in the Quarton-Ainslie model.

Different superposition models are commonly used in order to obtain combined effects from superposition of turbine wakes, hence predicting combined wake velocity deficits and wake added turbulence. Examples of these models are dominant wake and sum-of-deficits models, however none of it has been shown to have a clear advantage in the literature (see [36, 37] for more details), and certainly none of these are based on proper physical laws, but rather mathematical or pragmatic laws.

The models for added turbulence intensity and turbine wake superposition are not discussed and validated in depth in this publication. However, it is worthwhile to stress how these models are fundamental for the overall correct modelling of wind-farm-wide wake effects. For instance, when additional deep-array effects are introduced, such as the large wind farm corrections used in Windfarmer [38], materially different predictions can be obtained when different superposition models are used. It is also underlined that different assumptions on added turbulence affect the wake dissipation of downstream turbines, which is important to keep in mind when comparing different models: for instance, the increased wake dissipation downstream due to the added turbulence is not considered in the model from Bastankhah & Porté-Agel [5], contrarily to the wake model used in Windfarmer.

3.4 Models for yawed rotors

The theory behind the flow characteristics for a yawed rotor has roots in the helicopter industry since used to calculate the rotor loads during forward flight. Glauert’s correction for yawed rotors to the Rankine-Freude actuator disk model [39] and Coleman’s study on skew deflection angle in the rotor near-wake (based on a vortex line approach) [40] are the main examples. More recently, two theories have been extensively used in literature to model the wake past yawed rotors: the Jimenez model [41] and the model proposed by Bastankhah & Porté-Agel (commonly referred to as the EPFL model, also in this paper) [5,6]. It is worth mentioning that other, more recent studies have appeared in literature, trying to study the wake deflection effect from a vorticity point of view [42, 43], but these lie outside the scope of this paper. The Jimenez model is based on a momentum balance approach and does not take into account the tangential losses caused by the three-dimensional characteristics of the wake flow, as well as any effect from atmospheric stability; it is therefore considered to overestimate the wake deflection. Moreover, it uses one empirical parameter which needs to be calibrated against measurements, even if a link between this parameter, $K_d$, and the turbulence intensity has been suggested in [28].

The EPFL model uses two Gaussian distributions aligned to the vertical and lateral axes (with respect to the plane normal to the flow direction) to allow for different velocity deficit distributions in the two axes. After approximating the wake growth rate within the far-wake region as linear and estimating the length of the potential core, used to define the inlet boundary conditions for the far-wake region (based on Coleman’s approximations for the wake skew angle), it is possible to calculate the lateral wake deflection due to a given yaw angle, depending on both the incoming turbulence intensity and the thrust coefficient. The EPFL model needs two empirical parameters for the wake velocity deficit calculations and two more for the lateral deflection calculations: these were estimated from wind tunnel experiments and LES simulations [5]. However, more validation on real-scale turbines would be beneficial to obtain reliable sets of empirical parameters to be used for different classes of turbines. Furthermore, it is worth commenting on the importance of the definition of the velocity and thrust vectors in the different models,
which might differ from one another. For example, in the Ainslie model, the wind speed is defined as perpendicular to the rotor plane, and this will need to be taken into account when calculating the thrust coefficient for a yawed rotor, as the final formula will depend on which direction the velocity and the thrust force vectors are projected onto.

Additionally, the effects of wake centreline displacement due to the combined effect of wake rotation and non-uniform inflow shear have been described in [44] and these are discussed in the test cases used for this study.

4. Results and discussion

4.1 Wind tunnel tests

4.1.1 Wake velocity deficit comparisons (zero yaw) for an isolated turbine.

Comparison between the measured, steady-state wake velocity deficit plots and the wake model predictions are performed at a horizontal plane at hub height (corresponding to 0.75 rotor diameters). The standard Ainslie model, the EPFL model and the Stratified-EVM model are compared to the measurements at the three downstream planes, as shown in Figures 3 to 5.

In Figure 3, the standard Ainslie model is shown not to fit the experimental data. The EPFL model is compared to the wind tunnel measurements in Figure 4. A considerable difference between the plots labelled (a,b) and the ones labelled (c,d) is visible. By using the EPFL parameters originally obtained by Bastankhah & Porté-Argel [5], the wake model manages to fit reasonably well the measurements with low turbulence (a), whereas it performs poorly in the other test case with higher turbulence intensity (b). A new set of parameters was optimised from the measurements using a cost-minimising function approach during the CL-Windcon project [16]. As it can be seen in plots (c, d) this new set of parameters improves the model fit noticeably. Although the optimisation of the set of parameters could constitute a good mathematical approach to calibrate the wake model for any specific case, it could be unfeasible when used for wake assessments where wake measurements are not available. The two sets of parameters are shown in Table 3.

| Proposed in [5] | α     | β     | ka    | kb    |
|-----------------|-------|-------|-------|-------|
|                 | 2.32  | 0.154 | 0.3837| 0.003678|
| Optimised in [16] | 1.2973| 0.2375| 0.1177| 0.0194  |

Table 3. Sets of parameters used in the EPFL model, used for the wind tunnel comparisons

Figure 3. Standard Ainslie model applied to the wind tunnel tests, both for the offshore (a) and onshore (b) flow case.

The Stratified-EVM wake model is compared to the same test case, as shown in Figure 5. Based on the broad analogies between the inflow conditions and the different stability classes in the literature (see
Table 2), different Obukhov lengths were used to fit the measurements. For the offshore case, characterised by a low turbulence intensity and predominantly stable conditions, a MOL of 200 m has been found to give the best fit with the measurements at the most upstream plane. An Obukhov length characteristic of stable conditions, however, results in the overprediction of the wind speed deficits at the two most downstream measurement planes, for which neutral characteristics were imposed to obtain a reasonable fit with the data, using a MOL of 2000 m. For the onshore case, instead, using a MOL of 450 m representative of slightly stable conditions allows to obtain a reasonable fit to the experimental data at the three measurement planes, with a small wind speed deficit underprediction for the most upstream measurement plane and a larger wind speed deficit overprediction for the most downstream measurement plane. As it can be observed comparing the onshore and the offshore test cases, the wake centreline wind speeds at the two more downstream planes are similar. This might suggests that the wake in the offshore case tends to recover quickly to the same wind speeds observed in the onshore case, which might explain the need to use different Obukhov lengths for different measurement planes in order to obtain a good fit for this test case. This might also explain the not satisfactory fit found using the standard Ainslie model for both the offshore and the onshore cases.

It is noted that the MOL values mentioned above are kept as an approximated round number, and a best fit value is not calculated using optimisation techniques in order to avoid overfitting. Due to the nature of the wind tunnel tests, and hence the lack of proper atmospheric stability classification, it is not possible to obtain a precise value for the MOL. Moreover, as the atmospheric boundary layer and the turbine model are scaled down in this experimental setup, the MOL has been also scaled down by a factor of 100, under the assumption that the turbine model in question is the (approximately) 1:100 scale reproduction of a utility-scale turbine.

In conclusion, the uncertainties associated with the behaviour of the flow and the atmospheric stratification characteristics recreated in the wind tunnel do not allow to use this data to fully validate the model. Nevertheless these tests show that when an Obukhov length in line with the indicative atmospheric characteristics recreated in the wind tunnel is chosen to modify the eddy viscosity term in the Ainslie model, this can be used to obtain a satisfactory prediction of the turbine wake. In order to gain more confidence in the proposed model, this will be compared against additional test cases in this paper, including utility-scale operational wind turbines.

4.1.2 Wake centreline deflection due to rotor yaw, for an isolated turbine

The measurements for the non-zero yaw cases were analysed and compared to the Jimenez and EPFL models, as detailed in Section 2. As described above, it is known in the literature that the combination of a rotating turbine wake and a non-uniform wind shear profile deflect the wake centreline even when the turbine’s yaw angle is zero. Analysing the wind tunnel measurements, no clear indication of such deflection was found and therefore it is not considered in the results presented in this section.

Different $k_d$ parameters were tested in order to obtain a closer fit from the Jimenez model to the wind tunnel measurements. A value of 0.1 was deemed to be the optimal value. This value is lower than the other ones found in literature for utility-scale wind turbines [8, 41], which has the effect of increasing the centreline deflection. When using the EPFL model, the four optimised parameters for this wind turbine model test will be used for these comparisons. The measurements are compared to the Jimenez and the EPFL models in Figure 6 for negative yaw angles. The Jimenez model better fits the measurements for the onshore test case, whereas the largest discrepancy is obtained for the largest absolute yaw angles in the offshore test case. A possible explanation to this could be the lack of any relationship to the variation of atmospheric stability within this model: as the flow becomes more stable, the overall wake deflection will be larger due to the reduced effect of flow mixing and entrainment along the wake. This indicates that in order to obtain a better fit for specific atmospheric conditions using the Jimenez model, specific $k_d$ parameter should be chosen for each case, which might be unpractical. The EPFL model shows smaller deflection discrepancies compared to the Jimenez model: this is particularly true for the offshore test case. Although a large discrepancy is noticeable for the onshore test case at a
Figure 4. EPFL model applied to the wind tunnel tests, both for the offshore (left) and onshore (right) flow case. The set of parameters proposed in [5] have been used in (a-b), whereas the set of parameters proposed in [16] are used in (c-d).

Figure 5. The Stratified-EVM wake model applied to the wind tunnel tests, both for the offshore (a) and onshore (b) flow case.
yaw angle of -40°, this is considered to be of lesser importance at least as far as wake steering applications are concerned, as smaller angles are usually used due to the risk of large increases in loads.

Figure 6. Wake lateral deflection due to yawed rotor for the wind tunnel test case, comparing Jimenez and EPFL models. (a,b,c,d) are for the offshore case, respectively at -10°, -20°, -30°, -40°; (e,f,g,h) are for the onshore case, respectively at -10°, -20°, -30°, -40°.

Although both models compare reasonably well to the measurements, it is highlighted that additional validations are needed to increase confidence on the parameters to be used for real-world applications, in both models.
4.2 CFD simulations

In this section, data from CFD simulations at different yaw angles are compared against the two Jimenez and EPFL wake deflection models. It is worth underlying these CFD results are not representative of any field test and are not validated against actual field measurements. Therefore, the CFD data analysed here are used assuming they realistically represent the wake development behind the turbine.

4.2.1 Wake velocity deficit comparisons

The simulated wake deficit profiles at four different downstream planes are initially compared to the proposed wake model (Figure 7a) and the standard Ainslie model (Figure 7b), for the turbine simulation with no rotor yaw. Based on the comparisons at four downstream planes (4, 6, 8 and 10 rotor diameters), using a MOL of -1000 m (characteristic of neutral conditions, see Table 2) allowed to obtain a good fit between the proposed model and the simulated wake deficits. The centreline wake deficits from the simulation and the two wake models are shown in Figure 7b: both models show to predict the wake deficits reasonably well.

![Figure 7](image)

**Figure 7.** Wake velocity deficit comparisons for the CFD test case, shown at 4, 6, 8 and 10 rotor diameters downstream: (a) Stratified-EVM wake model with MOL=-1000 m, (b) Standard Ainslie model and Stratified-EVM wake model.

It is visible from Figure 7 how the centreline of the velocity deficits tends to shift towards the left for increasing distance downstream: this is caused by the combined effect of the rotating turbine wake and the non-uniform inflow wind shear profile. In the next section, it is shown how the knowledge of this component of wake displacement allows to obtain better wake deflection predictions.

4.2.2 Wake centreline deflection due to rotor yaw, for an isolated turbine

Both the Jimenez and the EPFL models were used to predict lateral wake displacement due to yawed rotor for this CFD test case in neutral flow. Simulations of the same turbine analysed above have been carried out at the following yaw angles: -25°, -20°, -15°, 10°, 15°, 20°, 25°.

The parameters originally proposed by Bastankhah & Porté-Agel [5] have been used in these tests for the EPFL model, whereas different values of $k_d$ have been used in the comparisons in Figure 8 and Figure 9. In order to take into account the wake centreline deflection for non-yawed rotor visible in Figure 7a, the centreline deflections from the LES data were linearly summed to the deflection component obtained for the no-yaw case, an approach which is also used in the software FLORIS [9, 10]. The plots in Figure 8 and 9 show both the deflections obtained from the LES data (indicated simply as ‘LES’ in the plots) and the summed deflection components (indicated with ‘LES + Linear deflection’).
Figure 8. Wake lateral deflection for the CFD test case, shown for yaw angles -25° (a), -20° (b), -15° (c) and including the EPFL model and the Jimenez model (with different $k_d$ parameters).

The plots clearly show that if the no-yaw component of centreline deflection is not taken into account, the EPFL model largely overpredicts (for negative yaw angles) and underpredicts (for positive yaw angles) the wake deflection. The Jimenez model is shown to be able to better approximate the wake deflection downstream of the turbine, however very different values would need to be used for the $k_d$ parameter for positive and negative angles.

When the effect of the non-yaw deflection component is removed via linear sum, it can be seen how both the EPFL model and the Jimenez model better approximate the data from the simulations. Particularly for positive yaw angles, both the EPFL model and the Jimenez model (with a $k_d$ parameter of 0.15) predict well the wake deflection downstream of the turbine for all the angles used in this investigations. When the yaw angles are negative, an acceptable fit of the two models with the data is visible only for the -25° and -20° yaw angle cases and up to approximately 5 rotor diameters downstream of the turbine.

These comparisons show how both the models used for this test case can predict reasonably well the wake deflection in neutral conditions, although the centreline deflection for no-yaw needs to be estimated to give reliable results. To the best knowledge of the authors, no low-fidelity or medium-fidelity models are available in the literature to help predict this deflections effect.

4.3 Wind farms production data

In order to further increase confidence in the described methodology, wind farm production data is used in this section to perform comparisons with the proposed model.
Figure 9. Wake lateral deflection for the CFD test case, shown for yaw angles 25°(a), 20° (b) 15° (c) 10° (d), including the EPFL model and the Jimenez model (with different k_d parameters).

4.3.1 Wieringermeer onshore wind farm

This dataset was used to test the Stratified-EVM wake model against on-site meteorological mast data and SCADA data at a site where the effects of atmospheric stability and instability are not negligible. The power production data for Turbines T06 are normalised by the production of the leading turbine for westerly winds, T05. The data is binned for the specific wind speed range 6-8 m/s and was also binned for wind directions every 2°, between 230° and 320°. As a first test, the whole production dataset was compared to the proposed wake model using a range of MOL values characteristic of neutral atmospheric conditions: as it can be seen in Figure 11(a), a good fit is obtained using an MOL ranging from 750 m to 1000 m, and the results from the Windfarmer model are also superimposed for reference.

In order to compare the model to measured data where highly stable and unstable atmospheric conditions are present, only the summer months (June to August) were considered, and the dataset has been subdivided into daytime and night-time periods (respectively ranging between 7-18 hours and 20-5 hours). It is expected that the summer-daytime atmospheric characteristics exhibit signs of unstable conditions, whereas the summer-night-time atmospheric characteristics exhibit signs of stable conditions. In order to estimate the on-site atmospheric stability, the bulk Richardson number was calculated for each 10-minute averaged record from the mast signals and its distribution is shown for both daytime and night-time in Figure 10. Binned wind speeds, binned wind directions and binned turbulence intensities were used as model inputs for each directional bin and were compared to the measured, filtered data. The comparison shows how the daytime and night-time filtered production data at Turbine T06 exhibit different wake effects due to the predominance of unstable and stable
atmospheric effects, respectively. The median bulk Richardson number for each distribution has been calculated and empirical formulas have been used to relate them to the correspondent Obukhov lengths, as described in Holtslag [29]. The results obtained from the Stratified-EVM wake model are shown in Figure 11(b); the model appears to be able to correctly predict the power deficit due to wake of the preceding turbine for both predominantly stable and unstable conditions.

![Distribution of bulk Richardson number](image)

**Figure 10.** Distribution of bulk Richardson number for daytime(a) and night-time (b), both for the summer period, at the Wieringermeer site.

4.3.2 Horns Rev I and Nysted offshore wind farms

Turbine production data at the Horns Rev I offshore wind farm, provided as part of the UPWIND project [22], have been used to further validate the proposed Stratified-EVM wake model against a real-world scenario. In order to model the production for all the wind turbines of the wind farm, the choice of the added turbulence model and superposition model required particular attention. The Crespo-Hernandez model [35] was shown to perform well on previous wake model validations based on a large offshore wind farm, like in [6]. Since preliminary tests indicated this added turbulence model performed better than the Quarton-Ainslie model for the specific test setup, and since the relatively large inter-turbine spacing at the Horns Rev I wind farm falls within the indicated range [35], this model was selected for this test case. Both the dominant wake and the sum-of-deficits superposition models have been tested on this wind farm validation exercise and the latter has shown to have a noticeable improvement in the turbine row power predictions (this is also in accordance to what is shown by Niayifar & Porté-Agel [6]). Based on the available information on the stability classification at the site and the occurrence of each stability class, the Stratified-EVM wake model was run using three different MOL setting and the results were weight-averaged according to the neutral, stable and unstable occurrences at the site, as shown in Figure 2. The measured power production for each turbine row is compared to the modelled power production in Figure 12(a). The comparison is shown for the specific 10±1 m/s wind speed bin. The same added turbulence model and wake superposition model used for the Horns Rev I wind farm above, were applied to the Nysted offshore wind farm too. Measured power production for each turbine row and representative of the 10±1 m/s wind speed bin is compared to the modelled power production in Figure 12(b). For this specific test case, the results are the weight-averaged of two runs of the proposed model, to better follow the provided stability classification at the site (i.e. one run using an Obukhov length representative of all the stable and neutral/stable subclasses, and another using an Obukhov length representative of all the unstable and neutral/unstable subclasses).

The power production predicted at the two sites using the DNV GL’s Windfarmer wake model is also shown for comparison in Figure 12, which fits the production data better than the proposed model.
Figure 11. Wieringermeer: power production at Turbine T06, normalised by power at turbine T05, shown for filtered and binned data. (a) neutral case, (b) stable and unstable cases.

Nevertheless, the proposed model is deemed to perform reasonably well especially for the first turbine rows. The largest discrepancies between the production data and the proposed model are found in the last turbine rows in both sites. It is noted that the turbine production data and the atmospheric classification are provided as averaged values, therefore it was not possible to make a time-series-based comparison by filtering and isolating stable and unstable occurrences to be compared against the model (as it was done for the Wieringermeer test case). As the greatest discrepancies are visible in the last wind farms’ rows, it is suspected the choice of a specific wake superposition model can heavily influence the results, and it is therefore considered by the authors as an area where further investigations are needed.

5. Conclusions
A wake model based on a formulation of the Ainslie model for stratified flow has been proposed in this work: the eddy viscosity parameter characterising the Ainslie model was modified to take into account the effects of atmospheric stability. This model has been initially calibrated against wind tunnel tests carried out for two different inflow conditions (characteristic of neutral onshore and stable offshore
scenarios), and against the results from an LES simulation of a utility-scale wind turbine in neutral atmospheric conditions. The proposed model showed to be able to predict reasonably well the wake deficits for these test cases, once an appropriate Obukhov lengths characteristic of specific atmospheric stability conditions is selected. Due to the nature of the wind tunnel tests and the LES simulations, it was not possible to obtain an estimate of the Obukhov length from the data. Nevertheless, it is shown that when a value of Obukhov length is chosen in line with the indicative atmospheric characteristics of each test case, hence modifying accordingly the eddy viscosity term in the Ainslie model, this can be used to obtain a satisfactory prediction of the turbine wake.

![Figure 12](image)

Figure 12. Measured and modelled turbine power averaged for each turbine row, normalised by reference power. (a) Horns Rev I wind farm, (b) Nysted wind farm.

Production data from three wind farms have also been used to increase confidence in the validity of this model. The Wieringermeer wind farm was used to further validate the model for neutral and non-neutral atmospheric conditions. The proposed model was calibrated against the whole production dataset characterised by neutral atmospheric conditions, and a MOL ranging between 750 m and 1000 m allowed to predict well the production at the test turbine. Furthermore, the measured production data and the meteorological data were further filtered to isolate daytime and night-time data during the summer months. The bulk Richardson number was estimated using the filtered data and a correspondent Obukhov length was obtained for both time periods. The use of a characteristic Obukhov length on meteorological measurements for both time periods allowed to obtain a good agreement between modelled and measured power production.

In order to test the proposed model against large wind farms, production data from two large offshore wind farms (Horns Rev I and Nysted) were used. The proposed model was used alongside the Crespo-Hernandez model for added turbulence and the sum-of-deficits wake superposition model. The stability classification at these sites was available as probability of occurrence for each considered wind speed and direction bin, therefore a weighted average approach was used to combine simulations representative of neutral, stable and unstable conditions, using representative Obukhov lengths. The proposed model is deemed to fit reasonably well to production data especially for the initial turbine rows, although the wake model used in DNV GL’s Windfarmer is shown to better fit the production data. As the greatest discrepancies are visible in the last wind farms’ rows, it is suspected the choice of a specific wake superposition model can heavily influence the results, and it is therefore considered by the authors as an area where further investigations are needed.

Additionally, the EPFL and the Jimenez models for wake deflection have been compared using both the wind tunnel and the high-fidelity CFD simulation test cases. For the wind tunnel experiments, which
were carried out for two different flow conditions, the EPFL model was found to better predict the wake deflections compared to the Jimenez model, especially for the experiments carried out with low ambient turbulence. However, it is found that both models heavily rely on numerical parameters and additional validation is required to increase the confidence in their tuning. The data from the LES simulations exhibited a clear wake centreline deflection for non-yawed rotor, an effect which is caused by the rotating wake and the impinging non-uniform wind flow. It was shown in this study that when the effect of this deflection component is not addressed, both the EPFL and the Jimenez models do not predict well the deflections downstream of the turbine. After the non-yaw deflection component is linearly added to the overall deflection obtained from the simulation, it was noticed how both the EPFL model and the Jimenez model (using a \( k_a \) parameter of 0.15) were able to fit reasonably well with the obtained deflections.

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