Scattering effects on nuclear thermodynamic observables

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The nuclear thermodynamic observables like the temperature, freeze-out volume and the specific heat as obtained from isotopic ratios in hot disassembled nuclear matter are examined in the light of the S-matrix approach to the nuclear equation of state. The values of the observables, as extracted without inclusion of scattering effects are found to be modified appreciably in some cases when scattering between the fragment species is taken care of.

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Understanding the nuclear equation of state (EOS) is central in clarifying issues related to properties in compact stars, supernova dynamics or explosive nucleosynthesis [1, 2, 3, 4]. Laboratory experiments involving energetic nuclear collisions are helpful to this end. The hot compressed nuclear system formed in these collisions subsequently decompresses and then becomes unstable against nuclear multifragmentation. It is generally believed that this disassembly occurs at a density \( \rho \) considerably lower than the saturation density of normal nuclear matter. It is called the freeze-out density where the fragmenting system, \( V/2\pi \exp(\lambda_3/\lambda) \), is the freeze-out volume, \( \lambda_3/\lambda ) \), is the neutron density \( \rho_\text{n} \), a measure of the volume of the fragmenting system is determined from the single isotope yield ratio \( R_1 = Y(A + 1, Z)/Y(A, Z) \) as

\[
\rho_\text{n} = R_1 \left( \frac{A}{A + 1} \right)^{3/2} \frac{g_A}{g_{A+1}} \frac{2}{\lambda^3} \times \exp\left\{\frac{B(A, Z) - B(A + 1, Z)}{T}\right\}. \tag{4}
\]

The difference in the binding energies of the selected fragments \( \Delta B \) is given as

\[
\Delta B = B(A_1', Z_1') - B(A_1, Z_1) + B(A_2, Z_2) - B(A_2', Z_2'). \tag{3}
\]

Generally, the fragment yields \( Y(A, Z) \) are selected in such a way that \( N_1' = N + 1, N_2' = N_2 + 1, Z_1' = Z_1, Z_2' = Z_2 \). As \( T \) is the only unknown quantity in Eq.(2), it can be obtained in terms of the known quantities.

Once the temperature \( T \) is thus known, the free neutron density \( \rho_\text{n} \), a measure of the volume of the fragmenting system is determined from the single isotope yield ratio \( R_1 = Y(A + 1, Z)/Y(A, Z) \) as

\[
\rho_\text{n} = R_1 \left( \frac{A}{A + 1} \right)^{3/2} \frac{g_A}{g_{A+1}} \frac{2}{\lambda^3} \times \exp\left\{\frac{B(A, Z) - B(A + 1, Z)}{T}\right\}. \tag{4}
\]

The temperature obtained from Eq. (2) would yield the real temperature of the disassembling system provided the assumptions made in deriving the NSE model are valid. This may not be so always. One important assumption in this model is that the fragment species produced are taken to be noninteracting within the freeze-out volume. This is not true except for extremely dilute systems, strong interaction corrections may modify
the fragment multiplicities and then the extracted temperature from the analysis as prescribed in Eq. (2) differs from the real temperature of the system. Likewise, the free neutron density extracted through Eq. (4) using this calculated temperature is not a true measure of the freeze-out volume. A number of other assumptions like the production of the fragments only in their ground states and the absence of any collective flow velocity have been made in deriving Eq. (1). The effects of these approximations have already been studied earlier \[8,9\] and would not be considered here.

In a previous communication, it was shown that strong interaction corrections to the NSE model \[10\] can be appropriately taken up in the \(S\)-matrix approach \[11\] to the grand partition function of the dilute nuclear system, where in addition to all the stable mass particles, the two-body scattering channels between them can be included systematically, in principle, to all orders. This is the virial equation of state in terms of the \(S\)-matrix elements. Assuming the dilute nuclear matter to be composed only of neutrons, protons and \(\alpha\)-particles, Horowitz and Schwenk \[12\] arrived at the nuclear EOS applying this methodology, it was extended later to include \(^3\)H and \(^3\)He \[13\]. Further extension was achieved \[10\] by including all the higher mass particles and their excited states as well as the scattering channels formed by any number of these species, estimated in the resonance approximation. Inclusion of these heavier species is found to have a significant role in the evaluation of the nuclear EOS and other quantities \[14\] like the nuclear symmetry energy and fragment multiplicities. Since the extraction of the thermodynamic state functions like the temperature or the volume depends on the fragment multiplicities, study of the sensitivity of these extracted thermodynamic variables on the strong interaction effects in the nuclear medium is in order. The present communication aims at this study. A somewhat related study in a different approach has been reported \[15\] recently.

The details of the \(S\)-matrix (SM) approach, as applied to the dilute nuclear system is given in Ref. \[10\]. For completeness, a few relevant equations are presented below highlighting the approximations. All the dynamical information concerning the microscopic interaction in the system in thermodynamic equilibrium is incorporated in the grand partition function \(Z\) of the system, which separates out as \[11\]

\[
\ln Z = \ln Z_{\text{part}} + \ln Z_{\text{scat}}. \tag{5}
\]

The first term corresponds to contributions from all the nuclear species in their particle stable states behaving like an ideal quantum gas. The second term corresponds to contributions from scattering states which are absent in the NSE model. The first term in Eq. (5) can further be split into contributions from the ground and the excited states of the bound nucleon clusters, so that

\[
\ln Z_{\text{part}} = \ln Z_{\text{gr}} + \ln Z_{\text{ex}}. \tag{6}
\]

The ground state contribution is given by

\[
\ln Z_{\text{gr}} = -V \sum_{Z,N} g \int \frac{d^3 p}{(2\pi)^3} \times \ln \left(1 + \xi_{Z,N} e^{-\beta (p^2/2Am)}\right) . \tag{7}
\]

Here, the sum is over all the possible fragments including neutrons and protons, \(V\) is the volume of the system and \(g\) the ground state degeneracy of the fragments. For \(A \leq 8\), the experimental values of the degeneracies are taken, for heavier nuclei they are taken to be 1 for the even \(A\) and 2 for odd \(A\) systems. The fragment momentum is \(p\) and \(\beta\) is the inverse temperature \(1/T\). The effective fugacity is given by \(\xi_{Z,N} = e^{\beta (\mu Z,N + B_{Z,N})}\), \(B_{Z,N}\) \((\equiv B(A, Z))\) is the binding energy of the fragment and \(\mu_{Z,N}(\equiv \mu_A)\) is its chemical potential. A nucleus in a particular excited state is taken as a distinctly separate species and can be treated in the same footing as with the ground state. Their contributions can be obtained by convoluting the r.h.s. in Eq. (7) with their density of states in an energy interval \(E_0\) to \(E_s\) where \(E_0\) is the first excited state taken as 2 MeV and \(E_s\) is the particle emission threshold taken as 8 MeV.

The scattering term in Eq. (5) can be formally written for nuclear matter as \[10\]

\[
\ln Z_{\text{scat}} = V \sum_{Z_1, N_1} \frac{e^{\beta \mu_{Z_1, N_1}}}{\lambda_{A_1}} \sum_{\sigma} e^{\beta B_{Z_1, N_1, \sigma}} \times \int_0^{\infty} dc e^{-\beta c} \frac{1}{2\pi i} T_{\text{scat}}(c, \sigma) \left(AS^{-1}(c) \frac{\partial}{\partial c} S(c)\right)_c. \tag{8}
\]

Here, the double sum refers to the sum over all possible scattering channels, formed by taking any number of particles from any of the stable species (excited states included) and the trace is over all plane wave states for each of these channels. \(S\) is the scattering operator, \(A\) the boson symmetrization or the fermion antisymmetrization operator. The subscript \(c\) implies only the connected parts of the diagrammatics of the expression in the parentheses and \(\sigma\) denotes all other labels required to fix a channel in this set with proton and neutron numbers \(Z_t\) and \(N_t\), respectively. \(B_{Z_1, N_1, \sigma}\) is the sum of the individual binding energies of all the particles in the channel and \(\epsilon\) is the total kinetic energy in the c.m. frame of the scattering partners. Only two-particle scattering channels are considered as they are expected, from binding energy considerations, to be more important than multi-particle channels with the same \(Z_t\) and \(N_t\).

For convenience, the channels are divided into light and heavy ones, consisting of low mass particles \((A \leq 8\), say) and heavier masses, respectively when

\[
\ln Z_{\text{scat}} = \ln Z_{\text{light}} + \ln Z_{\text{heavy}}. \tag{9}
\]

The scattering of relatively heavier nuclei is dominated by a multitude of resonances near the threshold; the \(S\)-matrix elements are then approximated by resonances
which, like the excited states, are again treated as ideal gas terms \([16, 17]\). Then, \(\ln Z_{\text{heavy}}\) can be written in the same form as \(\ln Z_{\text{exc}}\), assuming the level density of the resonances to have the same functional dependence on energy as the excited states. We consider resonances up to excitations of 12 MeV. The damping of the integral from resonances to have the same functional dependence on \(Z\) as same form as \(\ln Z\).

For the evaluation of \(\ln Z_{\text{light}}\), only the elastic scattering channels for the pairs NN, Nt, N ³He, N α (N refers to the nucleon) and αα have been considered. These calculations involve only experimental inputs, namely, the phase shifts and binding energies. Details of these calculations can be seen in \([12, 14]\).

Once the partition function for nuclear matter is obtained, the pressure \(P\), the number density of the \(i\)th species \(\rho_i\), the free energy per baryon \(f\) or the entropy per baryon \(s\) are evaluated from the relations,

\[
P = T \ln \frac{Z}{V}, \quad \rho_i = \zeta_i \left( \frac{\partial}{\partial \rho_i} \ln \frac{Z}{V} \right)_{V,T},
\]

\[
f = \frac{1}{\rho} \left( \sum_i \mu_i \rho_i - P \right), \quad s = \frac{1}{\rho} \left( \frac{\partial P}{\partial T} \right)_{\mu_i}.
\]

The total neutron and proton density are given by

\[
\rho_n^{\text{tot}} = \sum_i N_i \rho_i, \quad \rho_p^{\text{tot}} = \sum_i Z_i \rho_i,
\]

with \(\rho = \rho_n^{\text{tot}} + \rho_p^{\text{tot}}\). The energy per baryon \(\epsilon(= f + Ts)\) and the heat capacity per baryon \(C_V(= (\partial\epsilon/\partial T)_V)\) can then be obtained.

It is evident from the above that the fragment multiplicities obtained from the NSE model are rather approximate, the presence of the scattering among the different fragment species modifies their multiplicities. We specifically study the influence of scattering on the extracted observables, namely, the temperature and the freeze-out volume using double and single isotope ratios and also on the heat capacity. For this study, we consider nuclear matter where Coulomb is absent and only strong interaction is operative. The sums in Eqs. (7) and (8) run up to infinity in principle; in practice a finite sum with a maximum mass number \(A_{\text{max}}=1000\) is taken for computational facilitation. The results are found to be not very sensitive to further increase of \(A_{\text{max}}\). The binding energies of these nuclei are obtained using the liquid-drop type mass formula \([13]\) with Coulomb switched off. To obtain the effects of scattering on the aforesaid observables, the fragment multiplicities are calculated at a chosen temperature \(T\), baryon density \(\rho\) and asymmetry \(X(= (\rho_n^{\text{tot}} - \rho_p^{\text{tot}})/\rho)\) in both the NSE model and the SM approach. One can then infer an output temperature and free nucleon density from the cluster composition of the system in the two models as prescribed in Eqs. (2) and (4). One easily sees that this way in the NSE model, one gets back \(T^{\text{NSE}} = T\), the chosen temperature. The ratio \(T^{\text{NSE}}/T^{\text{SM}}\) and \(\rho_n^{\text{NSE}}/\rho_n^{\text{SM}} = V^{\text{SM}}/V^{\text{NSE}}\) then represent measure of the corrections due to scattering effects on the extraction of temperature and freeze-out volume \(V\) of the system with given \(\rho, T\) and \(X\). The superscripts NSE and SM refer to quantities obtained in the NSE and SM models, respectively. The difference in the inferred temperatures and energies \(e\) in the NSE and the SM model affects the heat capacity. Reflection of this is shown in the difference between the specific heats at constant volume \(C_V^{\text{SM}} = (\partial e/\partial T)_V^{\text{SM}}\) and \(C_V^{\text{NSE}} = (\partial e/\partial T)_V^{\text{NSE}}\).

The effect of scattering on temperature from the double isotopic ratio is displayed in Fig. 1. Here the ratio of \(T^{\text{NSE}}/T^{\text{SM}}\) is shown as a function of the given temperature \(T\) at different densities, both for symmetric \((X = 0.0)\) and asymmetric \((X = 0.2)\) nuclear matter. The thermometer used for this calculation is the double isotopic ratio \((t/d)/(^{4}\text{He}/^{3}\text{He})\) denoted as the t-He thermometer. The temperature \(T^{\text{SM}}\) is always found to be lower than the NSE temperature; the scattering effect on the inferred temperature \(T^{\text{SM}}\) increases with density. In general, at a given density, this effect has a peaked structure which becomes more pronounced at lower densities. Isospin asymmetry is seen to be relatively more important only at lower temperatures, it does not affect the results qualitatively. So, following results are presented only for symmetric nuclear matter.

Scattering effects on the extracted temperature with the choice of the double-ratio thermometer are presented in Fig. 2 at different densities as a function of temperature. Three thermometers are chosen, namely, t-He, \((d/p)/(^{4}\text{He}/^{3}\text{He})\) (denoted d-He) and \((^{4}\text{He}/^{3}\text{He})/(^{7}\text{Li}/^{6}\text{Li})\) (denoted He-Li). Choice of thermometers is seen to have a perceptible role on the extracted temperature, particularly at relatively higher densities and temperatures. Unless otherwise mentioned, the subsequent calculations refer to t-He thermometer; among the three thermometers, it has the least scattering effect.

For very dilute warm nuclear matter, it is generally perceived that the matter consists besides neutrons and protons a few light species, namely, deuterons, tritons, \(^{3}\text{He}\) and \(^{4}\text{He}\) \([12, 13]\). Importance of heavier clusters surface with increasing density and at lower temperature \([14]\). In the density and temperature range we explore, it is then expected that the departure of the temperature \(T^{\text{SM}}\) from the NSE temperature \(T^{\text{NSE}}\) would depend on the selection of fragment species in the model calculation. In Fig. 3, we compare the ratio \(T^{\text{NSE}}/T^{\text{SM}}\) as a function of temperature at different densities with the choice of light species upto \(^{4}\text{He}\) (the light species model or LSM) and also with inclusion of the heavy species (the heavy species model or HSM). At high temperatures, the two results tend to merge as matter breaks up into lighter clusters. Significant difference between the two model calculations are evident at lower temperatures and relatively higher densities where heavier clusters are more abundant. These may be attributed to the fact that in the LSM, heavier species are absent by definition, conservation of baryon density increases the population of the
light clusters there in comparison to those in the HSM which introduces larger scattering effects in the LSM.

The departure of the derived value of the freeze-out volume $V_{SM}$ from the corresponding NSE value $V_{NSE}$ due to strong interaction effects is shown as a function of temperature in Fig. 4 through the ratio $V_{SM}/V_{NSE}$ at two relatively higher densities where the effects are found to be more pronounced. Model calculations have been performed in both HSM (upper panel) and in LSM (lower panel). The single isotope ratios used are (d/p) and (t/d) and the corresponding temperatures are obtained from the thermometers (d-He) and (t-He), respectively. The values of the $V_{SM}/V_{NSE}$ are found to be sensitive to the choice of the single and double ratios. In LSM, the extracted volume $V_{SM}$ differs largely from the NSE value at low temperatures, but falls sharply with increase in temperature. With inclusion of heavier species, a qualitative change in the temperature dependence of $V_{SM}/V_{NSE}$ is observed with a marked peak at $T \sim 6.0$ MeV for the chosen densities. In HSM, the estimation of the freeze-out volume could be maximally uncertain by a factor of $\sim 2$ for the cases studied.

Comparison is made between the values of the specific heat $C_V^{SM}$ (full lines) and $C_V^{NSE}$ (dashed lines) in Fig. 5, where they are presented at a few densities as a function of temperature inferred from the respective models. The calculations at $\rho = 0.0001, 0.001, 0.01$ and $0.02$ fm$^{-3}$ are displayed in colors red, black, magenta and cyan, respectively. The two sets of specific heats do not show any qualitative difference. In the S-matrix approach, because of scattering, the peaks in specific heat are shifted at a somewhat lower temperature, but their magnitudes are a bit higher. At a particular density, the temperature corresponding to the peak in the heat capacity reflects the sudden onset of condensation in isochoric cooling. This is the condensation temperature [10]. The scattering effects on the extracted thermodynamic observables are found to be maximum here as also can be seen from Figs. 1 and 4(a).

The ACCR method, in the context of the NSE model, gives an estimate of the thermodynamic parameters like the temperature and the freeze-out density of the disassembling system. They are, however, very approximate. Various corrections are called for to improve upon these model values. We have here explored only the effect of strong interactions among the different species produced in the nuclear disassembly on these thermodynamic observables. They involve model-independent entities. Significant deviations from the ACCR values are obtained, particularly around the condensation temperature. The extracted temperature and the freeze-out density are always underestimated, the scattering-corrected values can, however, be reconstructed either through iteration or by preparing a table.

In Ref. [14], modifications of the ACCR values of the temperature and the freeze-out density are arrived at assuming that the binding energies of the fragments in the freeze-out configuration are dressed-up due to the presence of the medium. They considered only light fragments and their results are qualitatively the same as ours in the LSM. They have also incorporated effects due to strong interactions (they use explicit model interactions as opposed to ours), but the connection between the two approaches is not immediately obvious.

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FIG. 1: The ratio of the temperature obtained in NSE to that in SM model using (t-He) thermometer at different densities $\rho$ (in fm$^{-3}$) as a function of $T$ for symmetric ($X=0.0$, full lines) and asymmetric ($X=0.2$, dashed lines) nuclear matter.

FIG. 2: Comparison of $T_{NSE}/T_{SM}$ at different densities $\rho$ (in fm$^{-3}$) as a function of temperature using (t-He), (d-He) and (He-Li) thermometers shown with full, dashed and dot-dashed lines, respectively.
FIG. 3: The same as in Fig. 1 for symmetric nuclear matter in HSM (full lines) and LSM (dashed lines).

FIG. 4: The ratio of the freeze-out volumes obtained in SM ($V^{SM}$) and in NSE model ($V^{NSE}$) as a function of temperature at $\rho=0.01$ and 0.02 fm$^{-3}$ using the single ratios (d/p) and (t/d), the corresponding thermometers are (d-He) and (t-He).
FIG. 5: (color online) Heat capacities at constant volume calculated in SM ($C^\text{SM}_V$) and in NSE ($C^\text{NSE}_V$) as a function of $T^\text{SM}$ and $T^\text{NSE}$, respectively at different densities. The red, black, magenta and cyan lines correspond to $\rho = 0.0001$, 0.001, 0.01 and 0.02 fm$^{-3}$, respectively.