Evaporation transition in vibro-fluidized granular matter

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Abstract – Using laser facility, we investigate the speed distribution in the gas phase of a vibrated 3D granular matter bed. Invoking a volume-exclusion principle in a dissipative theory we present a kinetic model to explain the non-Maxwellian distribution, the mean free path and its associated granular-matter global temperature. We reveal that the fluidized gap is a lattice-gas with a constant packing factor.

Granular matter (GM), a paradigm system for investigating various complexities of dissipative non-equilibrium systems, has been studied extensively [1–3]. When a sand box is shaken, the grains in the upper layer start jumping resembling “evaporating”, and the transition to gas phase begins at the upper surface of the GM. One of the interesting aspect of this gas phase is the deviation from Maxwell velocity distribution [1]. Furthermore, the grains in the bulk exhibit cooperative non-Brownian behavior. In recent studies [4], it was shown that the system obeys a Fermi-like statistic in the real space, a result that showed to fit the 2D measurement of the GM mass density [5]. This analogy to quantum particles is a consequence of the volume exclusion principle and is the key element for describing a weakly vibrated GM under gravity. As demonstrated in our previous studies [6], this led to a successful definition of a GM global temperature, a quantity associated with the Lagrange parameter in a maximum entropy assumption. Our perturbation analysis resulted in a generalized equipartition law for GM. We postulated, found and measured a fundamental dissipative parameter, written in terms of the pumping and gravitational energies, linking the configurational entropy to the response of the system, i.e., the expansion of the center of mass (cm) of the GM bed.

In our previous work [6] we carried out an experiment in a 3D GM bed; that experiment was carefully set up in order to adjust the theoretical assumptions of Hayakawa and Hong [4]. The results of our experiments drove us to generalize the equipartition law by considering the number of active particles in the fluidized gap, and to take into account the dissipation of the system in a new parameter \( \delta \), as we stated in [6]. It is in a GM system that inelastic collisions are very important to resolve the problem, thus we introduced \( \delta \) into the generalized equipartition law [6]; this is an approach based in the Fermi-like statistics but is not the same as proposed in [4]. In the present work we are measuring the speed distribution in the fluidized gap of a vibrated 3D GM bed and introduce a kinetic model to understand this experiment, showing that what happens in the gap is correlated to the expansion of the whole GM bed, i.e., there is a one-to-one relation between the velocity fluctuations and the GM mass density.

It should be noted that below a critical vibration the GM bed behaves as a crystal [7] where the particles move in “thermally” activated harmonic oscillations, a regime where a Brownian description is suitable [8]. Nevertheless, in a previous work, we have studied the frequency spectrum and shown that in the weakly excited regime the dynamics of these evaporated particles cannot be described as Brownian [9]. We showed that the frequency spectrum follows a power law with an exponent that depends on the driven parameter of the GM bed. This led to a description of the chaotic dynamics in terms of non-Markovian particles representing the collective effects of the excited GM [9]. Recent reports concerning the GM non-Brownian behavior of the velocity fluctuations can also be found in resistivity experiments, which showed that the electric noise exhibits scale invariance and also intermittence, which arises from thermal expansion locally creating or destroying electrical contacts [10].
In this paper, we report on measurement of the kinetic transport and present a “jumping” model describing the velocity distribution of the GM gas. In the experiments done at constant acceleration with the scanning from high to low frequencies (beginning from a GM bed with packing factor associated to a fcc or hcp with a few defects), the evaporation transition is found at a critical frequency, \( f_{\text{onset}} \) where the grains of the first layer begin to jump to the available positions. Surprisingly, in the whole range of weak vibration that we have studied, this description reveals a lattice-gas in the fluidized gap leading to a packing factor which is independent of the driving parameters. In addition we also measured the \textit{mean free path} as a function of the driving frequency and corroborate our theoretical prediction. For frequencies larger than \( f_{\text{onset}} \), where there is no jumping of grains, the cooperative behavior of the bulk leads to the expansion of the cm due to the motion of the grains around their equilibrium positions.

The temperature of a non-equilibrium GM is a controversial quantity presenting anisotropy and inhomogeneous characteristics, and its behavior as a function of the external excitation is still an unsolved question. Here we give a clear evidence that the Lagrange parameter \( \beta \) appearing in [6] can be associated with the GM \textit{global temperature} \( \beta^{-1} \). We have succeeded in linking the kinetic distribution of the particles, agitated at the surface, to the \( \beta \) appearing in the mass profile obtained from a maximum-entropy assumption. In an earlier study, Hayakawa and Hong [4] assumed the existence of a maximum entropy to obtain the non-equilibrium steady-state GM density profile

\[
\phi(\epsilon) = \frac{\Omega}{mgD} [1 + Q \exp(\beta \epsilon)]^{-1}, \quad 1 + Q^{-1} = e^{\beta \mu},
\]

where \( \Omega \) is the degeneracy (number of boxes in which to put the maximum amount of balls in a constant gravitational layer), \( \beta \) and \( \mu \) (compactivity) are Lagrange parameters with \( Q^{-1} = \exp(\beta \mu) \) the fugacity, and \( \epsilon = mgz \); \( z = Ds \) with \( s = 0, 1, 2, 3 \ldots \). The \textit{zero-point “chemical potential”} is \( \mu_0 = mgDN/\Omega \), so that \( N/\Omega \) is the number of balls in an elementary \textit{column} of diameter \( D \). Consequently, without pumping of energy the cm is given by \( z_{cm} = \mu_0/2mg \). To dominant \( \mathcal{O}(\beta^{-2}) \), by generalizing the equipartition law we have shown that \( \beta \) is a non-standard function of \( \sigma^2_z \) which can be related to \( f_e \) [6]. It is non-trivial to link the speed distribution of the particles evaporating at the surface of the bed with \( \beta \), which will be our goal in the present work.

The GM bed was set up with \( N = 5437 \) balls of ZrO\(_2\)-Y\(_2\)O\(_3\), in a glass container of \( D_e = 50 \) mm diameter compacted in a lattice with an initial packing factor of almost 0.74. The system is confined to an air chamber at 1 atm under low water-vapor content conditions (5.8 ± 0.2 g/m\(^3\)). The diameter of the balls was \( D = 1.99 \) mm and their mass \( m = 26.8 ± 0.1 \) mg.

Under weak vertical vibration, it is possible to get the balls at the lower gravitational position of the container in a crystalline state (the bulk as an expanded fcc lattice), and those at the upper one in a gas state, called the fluidized gap [6]. This gas does not expand to fill the whole volume of the container, and the number of \textit{active} particles in the gas phase is a function of the pumping energy.

Details of the experimental set up are described in ref. [6]. A sinusoidal vibration (with intensity \( \Gamma = A\omega^2/g \), where \( A \) is the amplitude, \( g \) is the acceleration of gravity, and \( \omega = 2\pi f_e \)) is driven at a frequency \( f_e \) by a vibration plate on the GM bed of height \( h = \mu_0/mg \). The vertical stochastic trajectory \( z(t) \) of one particle was followed in a 12 mm window with a laser device by using a triangulation method. A laser emitter with a spot of 70 \( \mu \)m and a linear image sensor (CCD-like array) enables a high-speed measurement with 100 \( \mu \)s sampling with a resolution of 1 \( \mu \)m. The shaker and the laser displacement sensor were placed on vibration-isolated tables to decouple them from external perturbations. The vertical velocity \( V(t) = dz/dt \) of the particles was calculated numerically for \( \Delta t = 100 \) \( \mu \)s from \( z(t) \) registers; the lateral \( (x, y) \) translation does not play a part in our 1D degenerated Fermi-like approach [6]. The amplitude dispersion \( \sigma_z = \sqrt{\langle z(t)^2 \rangle - \langle z(t) \rangle^2} \) and velocity variance \( \sigma_v^2 \) were obtained from a window of 2 seconds for each pair of registers \( \{z(t), V(t)\} \). In fig. 1(a) we report \( \sigma_z \) against \( f_e \) for fixed \( \Gamma = 10 \), for a GM bed of \( h = 18 \) mm. Figure 1(b) shows the relation between \( \sigma_z \) and \( \sigma_v^2 \).

Gently shaking of a box of GM is an example of a non-equilibrium system that can be characterized by a global dissipative parameter \( \delta \), which is given as the ratio of the

![Fig. 1: (a) \( \sigma_z \) as a function of \( f_e \) shows the evaporation transition at \( f_{\text{onset}} \approx 85 \) Hz. (b) \( \sigma_z \) vs. \( \sigma_v^2 \) and the corresponding theoretical fittings \( \sigma_z \approx \sqrt{\frac{\Theta}{m^2}} \sigma_v \Theta \) from [6]. \( f_e \) was varied in the range 50–100 Hz.](image-url)
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Two important energies of the system [6]:

\[ \delta = \frac{1}{\mu_0} mg \Delta z_{cm}/ \left( \frac{A \omega}{g \rho} \right)^2. \] (2)

Since the motion of GM particles, at the surface, can be studied in terms of the cumulative probability \( \phi(\epsilon = mg z)/N \), we have shown that at low GM global temperature, \( \beta \mu_0 \gg 1 \), we get \( \sigma_z \approx 2 \Theta (mg z)^2 \), where \( \Theta = 1 + \ln(1 + \sqrt{1 - e^{-\epsilon/2}}) \) [6]. This expression gives the amplitude dispersion \( \sigma_z = \epsilon_0/mg \) as a function of \( \beta \).

The connection between the fluctuation in the velocity \( \sigma_{V,0}^2 \) and \( \beta \) is, to \( O(\beta^{-2}) \), given by our generalized equipartition law: \( \frac{1}{2} m^* \sigma_{V,0}^2 = \frac{1}{2} \Delta N/ \beta^{-1} \), where \( m^* \) is the mean number of particles per unit volume. The mean free path is \( L = \tau \delta \). Assuming the relative speed \( V = c_1 \sigma_V \) and the mean speed \( \bar{v} = c_2 \sigma_V \) (actually \( \bar{V} \) should be somewhat larger than \( \bar{v} \)) and using the fact that \( n \) at the fluidized gap is given by \( n = 4 c_3 \Delta N/ \pi D^2 2 \sigma_z \) and finally assuming all constants \( c_1 = O(1) \) and \( \beta \mu_0 \gg 1 \) we obtain \( L \sim \frac{1}{2} (D/\rho)^2 \sigma_z / \Delta N. \)

The function \( \sigma_{V,0}^2 \) is depicted in fig. 2(a). In fig. 2(b) we show \( \beta \) against \( f_e \) to note the evaporation transition point at \( f_e^{\text{sett}} \). The linear fit of fig. 2(a) we obtain \( L \approx 2.28 \text{ mm}. \)

With a video camera microscope we photographed the surface of the GM bed from the top of the container. Our analysis of the time-averaged images show that \( L \sim D \), a result which agrees with the measurements of correlation in the gas phase [11]. By estimating the degeneracy from \( \pi (D/2)^2 \sim \Omega D^2 \), we get \( L \sim \Omega D/\pi \ln 2 \sim 1.35 D \). This important result shows that one can build up a kinetic theory from a “jumping” model and thus calculate the speed distribution for the particles in the fluidized gap.

We emphasize that fig. 2(a) supports the application of a jumping model to study the kinetics of this non-Maxwellian gas because \( L \sim D \), i.e., we shall associate a jump distribution \( \rho(r) \) (proportional to the derivative of the mass profile) with the transport mechanism of a set of particles (at the surface of the bed) in a background of a well defined GM global temperature \( \beta^{-1} \).

Assuming a constant velocity \( V \) in between collisions, a jump of length \( r \) corresponds to a velocity \( V = r/\tau > 0 \). With the kinetic energy as \( K = mV^2/2 \) and using the transformation of random variables, we obtain \( P(K) dK = \rho(r) dr \). Thus the jumping model gives us a closed expression for the distribution in terms of the \( \beta \),

\[ P(K) \propto g \sqrt{\frac{m}{2K}} \psi(\mu_0 + g \tau \sqrt{2mK}), \] (3)

where we have used the mass profile (1) to get \( \psi(\epsilon) = -d \phi / d \epsilon \), and \( \rho(r) \propto \psi(\mu_0 + \epsilon) \mid d \epsilon \mid \). Therefore from (3) we obtain an analytic formula for the Kinetic Distribution Function (KDF), which predicts at medium- and high-energy ranges a larger population than the Maxwellian. In the asymptotic regime \( \epsilon \gg \mu_0 \), \( \psi(e) \rightarrow \beta e^{-\beta z} \), so the KDF for high energy goes like \( \propto g \tau \sqrt{\psi(\mu_0 + g \tau \sqrt{2mK})} \).

It is important to check the consistency of the value of \( \beta \) obtained from two different experiments. While in the experiment shown in figs. 1 and 2 we scan over \( f_e \) at a given \( \Gamma \) to investigate the behavior of \( \beta \), in a second experiment we carried out an isothermal-like measurement during a long enough time to get the \( P(K) \) and the corresponding \( \beta \). In addition we have also verified the values of \( \beta \) obtained from the vertical (KDF profile) and horizontal (mass profile) laser devices shown in fig. 1(a,b) of ref. [6]. From the vertical measurement of \( z(t) \) we were able to calculate the kinetic energy flux in the measurement of a 1D window, from which we obtained the KDF. At a given \( f_e \) and \( \Gamma \) we measured 5000 windows of 2 seconds to get a total of 25 million events from which we built up the KDF. In fig. 3 we fit our experimental data using formulae (3); the experiment was carried out for fixed \( f_e = 80 \text{ Hz} \) and \( \Gamma = 10 \). Medium- and high-energy particles, which are at the surface of the vibrated GM bed, are well described by formulae (3), while at low energy the behavior is Maxwellian. We also fit the whole run with a Maxwellian in kinetic energy, \( \exp(-K \beta) / \sqrt{\pi K \beta} \), as well as with a stretched exponential for the asymptotic large velocity distribution \( \propto \exp[-(V/V_0)^\alpha] \), with \( \alpha = 3/2 \), where \( V_0 \) is the thermal rms velocity [12]; this last fitting shows that the agreement is also good, as reported by [11,13]. A tricky obstacle to the analytical solution of this problem is that the supply of energy to the
GM bed *compensates* the dissipation caused by inelastic collisions, thus the kinetic theory [12] is only applicable for the asymptotically large velocities, an aspect which was clarified using simulations of molecular dynamics [14]. Let us remark that in [12] the loss of momentum when the grains hit the bottom plate, and gravitational effects indeed a family of distributions, with apparent exponents $\alpha < 2$, which are governed by the ratio between the numbers of heating events and inelastic collisions [14]. Both under uniform heating or boundary heating of the GM, if dissipative collisions dominate, the heating process produces a liquid-like cluster surrounded by a volume exclusion and the clustering of mass and then the number of active particles $\Delta N$ schematically with a Fermi mass profile (1) as shown in ref. [6]. Thus we conclude that the fluidized gap is indeed a lattice gas with $p_f$ according to eq. (4). The fluidized gap our theory and make it possible to give the KDF in terms of $\mu$, $\Gamma$, and also by the measurement of $\delta$ and $\tau$ in the experiment. We would like to stress that our approach was based on the values of the variables $\{f_e, \Gamma, \mu_0, D, N, \Omega\}$, and also by the measurement of $\delta$ and $\tau$ in the experiment. We would like to stress that our theory has no free parameters. The fitting for the curve in fig. 3 gives $\beta = (1.7 \pm 0.5)(\mu J)^{-1}$ and for the corresponding value from fig. 2(b) we get $(5 \pm 1)(\mu J)^{-1}$, which is in fact a good agreement considering that in the first experiment we are scanning in $f_e$ at constant rate, and in the second experiment (time-average) both $f_e$ and $\Gamma$ are fixed. The high-energy tail corresponds to beads near the interface (bulk-gas), where jumping combined with collisions dominate the process.

A point of special interest is that the *packing fraction* for this fluidized gap is a constant. For the whole range of low GM global temperatures $\beta^{-1}$ we get

$$ p_f \equiv \frac{2}{3} \left( \frac{D}{D_e} \right)^2 \frac{D \cdot \Delta N}{\sigma_z} \sim \frac{\tau \ln 2}{2 \Theta} \sim 0.25. $$

At constant cooling rate, by increasing $f_e$ at constant $\Gamma$, this gas condenses into a $fcc$ crystal with a $p_f < 0.74$ due to the expansion of the cm, and vice versa from the $fcc$ lattice under constant heating rate the expansion of the cm entails the expansion of this $fcc$. This expansion under gravity creates fluctuations in the number of particles around $\epsilon = \mu_0$, inducing the transport of matter at the gap. The time integration (3 hour duration of this experiment) of such fluctuations reveals that the gas has a structure with a Fermi mass profile (1) as shown in ref. [6]. Thus we conclude that the fluidized gap is indeed a lattice gas with $p_f$ according to eq. (4). The picture we get for this lattice gas should be understood as a “jammed” state of motion, since the $p_f$ does not change for the range of $\beta$ we measured.

D’Anna et al. [8] showed that a vibrated GM bed exhibits a formal analogy with a *thermal bath*, where it is possible to apply a fluctuation-dissipation theorem. In that experiment, a torsion pendulum is coupled to the particles in the GM bulk. Thus the torsion pendulum acts as a thermometer coupled to a Brownian ensemble. Nevertheless if evaporation starts the viscosity should be related to the volume exclusion, since the jumping of the particles is conditioned by the availability of free volume. In [8], equipartition means that $2K = \beta^{-1}$, and by measuring long enough, a GM temperature was found for the bed, the range of temperatures tested being lower than 0.1 $\mu J$. Our set up explores beyond the Brownian regime of the GM bed, that is, at the onset of the jumping events (evaporation) and the appearance of the fluidized gap. Our “thermometer” is a *test particle* from the fluidized gap which occupies the position tested by the laser. Measurement of $\beta^{-1}$ as a function of $f_e$ exhibits at 10PK the critical point that characterizes the evaporation transition. For $\beta^{-1} < 10$ PK $(\beta^{-1} < 0.1 \mu J)$ the system is condensed and the motion corresponds to Brownian oscillators. If evaporation starts the following picture is possible: when the ensemble of particles is disturbed, energy is dissipated by the mechanisms of jumping and random inelastic collisions between them, but conditioned by the fact that these events are self-organized in time, keeping $p_f$ constant. The relation between the transport of mass and the number of active particles $\Delta N$ (at the gap) links the two Lagrange parameters $\beta$ and $\mu$, and therefore suggests the existence of a lattice gas. The coupling between $\beta$ and $\mu$ implies that, this ensemble of Fermi-like particles, dissipate energy keeping $p_f$ constant. The difference with a Markovian motion is that “Brownons” can dissipate energy through viscosity independently of any gas structure.

Our approach focuses on the gravitational energy which is a functional of the configuration entropy and is related to the kinetic energy per active particle at the fluidized gap. In other words, while in the Brownian situation we can describe the system in terms of energy fluctuations and viscosity, beyond the Markovian approximation, the important events are the fluctuations in the number of
particles according to the mass profile (the Fermi profile held by the dissipative parameter $\delta$). What we find here is a rather curious relationship between the fluctuations in the number of active particles at the gap and the GM global temperature $\beta^{-1}$. A fluctuation in the number $\Delta N$ will be compensated in time in such a way that the gas keeps $p_f$ constant; this is a fluctuation-compensation relation. Thus a stochastic formalism is still valid but equations should describe a fluid where the key difficulty is articulating the description of the lattice gas, the jumping phenomena and dissipative collisions in a space framed by the volume exclusion.

Our approach agrees with the discussion in [15] related to the idea that a glass far from equilibrium can be described with more than one temperature. Such temperatures are related to time constants of the different configurational structures. In GM experiments, the structure of the gas is only measurable if we wait long enough. This structure is related to transport of mass at the fluidized gap. The characteristic time we must wait to measure $\beta$ in this system is that needed to integrate the mass profile $\phi(\epsilon)$. However, unlike glasses, GM vibrated under gravity is far away from equilibrium but stationary: the mass profile persists while $\delta$ is held at a fixed value, and the fluctuation-compensation plays a role stabilizing the steady state, i.e., the precise balance of fluctuations lets us know the temperature $\beta^{-1}$ without waiting too long.

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