SLIM DISKS WITH TRANSITION REGIONS AND APPLICATIONS TO MICROQUASARS AND NARROW-LINE SEYFERT 1 GALAXIES

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Received 2003 December 18; accepted 2004 June 17

ABSTRACT

Slim disks have been received much attention because of the increasing evidence of supercritical accreting objects. In this paper, we make an attempt to construct a unified model in which the viscosity and the dimensionless accretion rate can span rather wide ranges. We replace blackbody radiation under the diffusion approximation with a bridged formula that accounts for both blackbody radiation and thermal bremsstrahlung in optically thick and thin cases, respectively. Thus this allows us to investigate the structures of and the emergent spectra from slim disks in a wider parameter space covering transition regions from optically thick to optically thin. We show that there is a maximum transition radius, roughly $R_{\text{tr}}/R_g \sim 50$ when $\dot{M}/\dot{M}_C \sim 15$. The emergent spectra from the unified model of the accretion disk have been calculated. A simple model of the hot corona above the slim disk is taken into account for the hard X-ray spectrum in this paper based on the work of Wang and Netzer. We have applied the present model to the microquasar GRS 1915+105 and the narrow-line Seyfert 1 galaxies RE J1034+396 and Ark 564. Our model can explain well the broadband X-ray spectra of narrow-line Seyfert 1 galaxies, microquasars, and possible ultraluminous compact X-ray sources. The present model can be widely applied to candidates for supercritical accreting objects.

Subject headings: accretion, accretion disks — galaxies: active — galaxies: Seyfert — X-rays: galaxies

1. INTRODUCTION

It is believed that accretion disks around black holes are powering many kinds of celestial objects. The most important scale in the description of accretion disks is the critical accretion rate, defined by $\dot{M}_C = L_{\text{Edd}}/\eta c^2 = 2.6 \times 10^{18} M_{\text{BH}}/M_\odot \text{ g s}^{-1}$, where the accretion efficiency $\eta = 1/16$ is for Schwarzschild black holes whose gravitational effects are approximated by the pseudo-Newtonian potential. The structures of accretion disks rely greatly on the mass of the black hole $M_{\text{BH}}$, the accretion rate $\dot{M}$, and the viscosity $\alpha$, and hence the emergent spectrum from the disk. According to the dimensionless accretion rate $\dot{m} = \dot{M}/\dot{M}_C$, the disk structures can be classified as (1) optically thin advection-dominated accretion flows (ADAFs; Narayan & Yi 1994), advection-dominated inflow-outflows (ADIOs; Blandford & Begelman 1999), and convection-dominated accretion flows (CDAFs; Narayan et al. 2000; Quataert & Gruzinov 2000) when $\dot{m} < \alpha^2$, (2) optically thick and geometrically thin disks (Shakura & Sunyaev 1973) if $\alpha^2 < \dot{m} < 0.2$, and (3) slim disks (Muchotrzeb & Paczyński 1982; Abramowicz et al. 1988; Chen & Taam 1993) once $\alpha^2 < \dot{m} < 0.01$. Chen et al. (1995) presented a unified model, but they assumed a Keplerian angular momentum distribution in the disk. The unified model to cover all these regimes has not been sufficiently understood; especially, we still lack the emergent spectrum of such a model for comparison with observations.

There is growing evidence from current observations that the accretion rates of a significant fraction of celestial objects span quite wide ranges from very low to supercritical in X-ray binaries, microquasars, and active galactic nuclei (AGNs). In our Galaxy, some ultraluminous compact X-ray sources (ULXs) and microquasars such as GRS 1915+105 and SS 433 show large bolometric luminosities and high color temperatures (>1 keV) in their high states (Makishima et al. 2000; Watarai et al. 2001; Ebisawa et al. 2003). This provides strong evidence for the appearance of slim disks in these objects. The Eddington ratios ($L/L_{\text{Edd}}$) in AGNs and quasars have been reliably estimated for revealing the structures of the disks in AGNs and quasars, since the black hole masses can be estimated by the reverberation-mapping relation (Kaspi et al. 2000) and the relation with their host galaxies (e.g., McLure & Dunlop 2001). It has been suggested that some AGNs have close to or supercritical accretion rates (Collin et al. 2002; Netzer 2003; Vestergaard 2004; Wang 2003; Szuksziewicz 2004), whose luminosities seemingly exceed $L_{\text{Edd}}$ (if the estimations of black hole masses are reliable). Collin et al. (2002) have shown that about half of the 34 AGNs of which the masses are determined by reverberation mapping have supercritical accretion rates. Vestergaard (2004) extended the empirical relation of reverberation mapping in the sample of Kaspi et al. (2000) to the cases of high-redshift quasars (see also McLure & Jarvis 2002). She found that there are a significant number of quasars with super-Eddington luminosities, in either intermediate redshift range $1.5 < z < 3.5$ or high redshift range $z > 3.5$. Willott et al. (2003) found that the quasar SDSS J1148+5251 at $z = 6.41$ is radiating at the Eddington luminosity with an estimated mass $M_{\text{BH}} = 3 \times 10^9 M_\odot$. Wang (2003) has also discovered a handful of the candidate supercritical accretors using the limit relation between the black hole mass and FWHM(H$\beta$), $M_{\text{BH}} = (2.9-12.6) \times 10^6 (\text{FWHM(H$\beta$)}/10^3)^{0.67} M_\odot$. This relation is a result of combining empirical reverberation mapping and features of the emergent spectra from slim disks. Although the spectra of 110 bright AGNs in the sample of Laor (1990) are well fitted by standard thin disks (Shakura & Sunyaev 1973), those...
super-Eddington objects can never be explained within the framework of the standard disk theory. A wide range of dimensionless accretion rates should exist in various kinds of celestial objects, but there is no unified model of accretion disks to describe the varieties of accretion.

The significance of slim disks has been exhibited when they are applied to narrow-line Seyfert 1 galaxies (NLS1s; Wang et al. 1999; Mineshige et al. 2000; Wang & Netzer 2003; Kawaguchi 2003), Galactic black hole candidates (Watarai et al. 2000; Fukue 2000), and ULXs (Watarai et al. 2001) with strong soft X-ray emission and violent variability. However, slim-disk models in the above literature suffer from the problem that the effective optical depth is smaller than unity in the inner disk region. The slim disks have been thought to work in these objects with high accretion rates, but the emergent spectra from the disks are not satisfactory enough. We list the models and spectra studied so far in Table 1, in which the main assumptions and features in these models are also given. The emergent spectra from accretion disks with a wide range of \( \dot{m} \) remain open so far; only Wang & Netzer (2003) calculated the spectrum from a slim disk with a hot corona based on a self-similar solution. We note that most of the models assume a low viscosity so that the transition to an optically thin region does not appear in these models (see eq. [1]). Szuszkiewicz et al. (1996) calculated the emergent spectrum with modified blackbody radiation and explained the soft X-ray excesses claimed for some AGNs with slim disks. The vertical structures and the continuum spectra have been computed by Wang et al. (1999) and Shimura & Mannmoto (2003). Szuszkiewicz et al. (1996) presented the optically thin region with a small viscosity parameter \( \alpha = 0.001 \) and accretion rates \( 1 < \dot{m} < 50 \). Kawaguchi et al. (2004) also pointed out these regions in the accretion disks around supermassive black holes (SMBHs). But they did not provide a reasonable treatment in the optically thin hot region. In this paper, we show that a much wider region, where the effective optical depth \( \tau_{\text{eff}} \leq 1 \), appears when the viscosity increases to \( \alpha = 0.1 \) for stellar-mass black hole accretion (Fig. 1). It can be seen that the transition radii from optically thick to thin are located at at least several tens of \( R_g (R_g = 2GM/c^2, \) the Schwarzschild radius) for the accretion rates \( 10 \leq \dot{m} \leq 50 \).

This paper is organized as follows: the basic formulations for the calculations of structures and spectra are given in § 2, and the numerical results are presented in § 3. We find that the transonic location changes because of the presence of a transition region. So does the structure in the inner disk region. We pay attention to how the emergent spectrum gradually changes

| Reference                        | Model         | \( \alpha \) | Vertical | \( \tau_{\text{eff}} \) | Corona | Spectrum |
|---------------------------------|---------------|-------------|----------|-----------------|--------|----------|
| Muchotrzeb & Paczyński 1982.....| Global        | Low         | No       | \( \gg 1 \)     | No     | No       |
| Abramowicz et al. 1988          | Global        | Low         | No       | \( \gg 1 \)     | No     | No       |
| Chen & Taam 1993                | Global        | Low         | No       | \( \gg 1 \)     | No     | No       |
| Chen et al. 1995                | Kepler        | ...         | No       | ...             | No     | No       |
| Honma 1996                      | Global        | ...         | No       | ...             | No     | No       |
| Szuszkiewicz et al. 1996        | Global        | Low         | No       | \( \gg 1 \)     | No     | Yes\(^b\) |
| Beloborodov 1998                | Global        | ...         | No       | ...             | No     | No       |
| Wang et al. 1999                | Global        | Low         | No       | \( \gg 1 \)     | No     | Yes\(^d\) |
| Wang & Zhou 1999                | Self-similar  | Low         | No       | \( \gg 1 \)     | No     | No       |
| Fukue 2000                      | Self-similar  | Low         | No       | \( \gg 1 \)     | No     | Yes\(^e\) |
| Mineshige et al. 2000            | Global        | Low         | No       | \( \gg 1 \)     | No     | Yes\(^e\) |
| Watarai et al. 2000             | Global        | Low         | No       | \( \gg 1 \)     | No     | Yes\(^e\) |
| Artemova et al. 2001            | Global        | Low         | No       | \( \gg 1 \)     | No     | No       |
| Zampieri et al. 2001            | Global        | ...         | Yes      | ...             | No     | Yes\(^b\) |
| Kawaguchi 2003                  | Global        | Low         | No       | \( \gg 1 \)     | No     | Yes\(^e\) |
| Shimura & Mannmoto 2003         | Global        | Low         | Yes      | \( \gg 1 \)     | No     | Yes\(^e\) |
| Wang & Netzer 2003              | Self-similar  | Low         | No       | \( \gg 1 \)     | Yes    | Yes\(^e\) |
| Lu et al. 2004                  | Global        | High        | No       | ...             | No     | No       |
| This work                       | Global        | ...         | No       | ...             | Yes    | Yes\(^e\) |

\(^{a}\) Modified blackbody.  
\(^{b}\) Solving radiative transfer equation.  
\(^{c}\) Local blackbody approximation.  
\(^{d}\) Modified blackbody including Comptonization.

![Fig. 1.—Effective optical depth (\( \tau_{\text{eff}} \)) of the classical slim disk (see § 2.1). The corresponding accretion rate is labeled. The effective optically thin region (\( \tau_{\text{eff}} \leq 1 \)) should be noted for \( 1 < \dot{m} \leq 50 \).](image-url)
with the accretion rate, increasing from sub- to supercritical. The transition to an optically thin region modifies the emergent spectrum, especially in the EUV and soft X-ray bands. We apply the present slim-disk model with a transition region to Galactic black hole candidates, one microquasar, and two NLS1 sources in § 4. The conclusions are given in the final section.

2. BASIC MODEL

According to the model of Shakura & Sunyaev (1973), the effective optical depth will be less than unity at the radius

$$ R_{tr} / R_g \approx 25 \alpha_{vis}^{34/93} H_0^{64/93} m_4^{2} , $$

(1)

where we have neglected the factor $1 - r^{-1/2}$ (r = R/R_g and m = M/M_\odot). This formula shows that the transition covers the region where most of the gravitational energy is released. Within this region the radiation diffusion approximation does not work. In addition, the transonic point is also located in this region. The transition radius $R_{tr}$ is very sensitive to the accretion rate and the viscosity. We will focus on the global structure and the emergent spectrum from the slim disk with transition region. The equations are described as below.

2.1. Equations for Slim Disks with Transition Regions

The basic equations for slim disks are taken from Muchotrzeb & Paczyński (1982) and Abramowicz et al. (1988), but we deal with radiative cooling more carefully. Vertical static equilibrium is assumed, namely, $H = B_2 c_s / \Omega_K$, where $H$ is the half-height of the disk at the radius $R$, $c_s = (P/\rho)^{1/2}$ is the sound speed, $\Omega_K = (GM/R)^{1/2}/(1 - R_g/R)$ is the Keplerian angular velocity, $P$ is the total pressure, and $\rho$ is the density. Unless stated, all physical quantities refer to their equatorial plane values. We use the Shakura-Sunyaev viscosity prescription $\tau_{vis} = -\alpha P$, where $\tau_{vis}$ is the viscous torque tensor and $\alpha$ is a phenomenological viscosity parameter. The pseudo-Newtonian potential (Paczynski & Witt 1980) is adopted in our calculations for Schwarzschild black holes.

The four equations of mass, radial momentum, angular momentum, and energy conservation control the structure of the slim disk. The momentum equation in the radial direction is

$$ \frac{1}{\rho} \frac{dP}{dR} - \left( \Omega^2 - \Omega_K^2 \right) R + v_R \frac{dv_R}{dR} = 0, $$

(2)

where $v_R$ is the radial drift velocity of the accreting gas and $\Omega$ is the angular velocity. The specific angular momentum and the Keplerian angular momentum are defined by $l$ and $\Omega_K = \Omega R^2$. The angular momentum momentum conservation equation reads

$$ M(l - l_m) = 4\pi R^2 H \alpha P, $$

(3)

where $l_m$ is the eigenvalue of angular momentum at the inner disk edge. The mass accretion rate $\dot{M}$ is given by

$$ \dot{M} = B_2 2\pi R \Sigma v_R, $$

(4)

where $\Sigma = 2 \rho H$ is the surface density.

In the classical slim-disk model of Abramowicz et al. (1988), the viscosity parameter $\alpha$ is very low. This ensures the validity of the radiation diffusion approximation for radiative cooling. However, the low viscosity only covers a very small parameter space. When the viscosity increases, the density decreases, so the effective optical depth will be less than unity. In such case the radiation diffusion approximation breaks down. In the slim-disk regime we adopt a bridged formula,

$$ Q_{rad} = B_3 \frac{4\pi T^4}{3(\tau_{es} + \tau_{abs})} \frac{(1 - e^{-\tau_{rad}}) \tau_{eff}}{e^{-\tau_{rad}} + (1 - e^{-\tau_{rad}}) \tau_{eff}}, $$

(5)

for the vertical radiation transport (see also Wandel & Liang 1991; Chen et al. 1995). Here $\sigma$ is the Stefan-Boltzmann constant, $T$ is the midplane temperature, and the effective optical depth $\tau_{eff} = (\tau_{es} + \tau_{abs})^{1/2}$, where we take the electron scattering depth $\tau_{es} = 0.34\rho H$, the absorption depth $\tau_{abs} = 6.4 \times 10^{22} \rho^3 T^{-3.5} H$ for stellar-mass black holes, and $\tau_{abs} = 1.92 \times 10^{23} \rho^3 T^{-3.5} H$ for SMBHs due to bound-free processes (Czerny & Elvis 1987). This bridged formula works for a transition from optically thick to optically thin. The Comptonization amplification factor $\Lambda$ is adopted from Wandel & Liang (1991) for simplicity. Compared with another type of bridged formula (Suszukiewicz & Miller 1998; Narayan & Yi 1995), this bridged formula and the definition of $\tau_{eff}$ are consistent with those in spectral calculations described below. Heating due to viscosity, $Q_{vis}$, is cooled by local radiation $Q_{rad}$ and global radial advection $Q_{adv}$, so the energy equation can be symbolically written as

$$ Q_{vis} = Q_{rad} + Q_{adv}, $$

(6)

where $Q_{adv} = B_3 (\dot{M}/4\pi R^2) (P/\rho) \xi; \xi = -12 - 10.5 \beta d \ln T / d \ln R + (4 - 3\beta) d \ln \rho / d \ln R$ is the dimensionless advection factor, where $\beta$ is the ratio of gas to total pressure. The radial energy transport is neglected. The equation of state is

$$ P = \frac{k_B}{\mu m_H} \rho T + \frac{Q_{rad}(\tau_{es} + \tau_{abs})}{c}, $$

(7)

where $\mu = 0.617$ is the mean molecular weight, $k_B$ is the Boltzmann constant, and $m_H$ is the mass of hydrogen. In this paper, we adopt $B_1 = 0.67, B_2^3 = B_3 = B_4 = 6$, and $B_5 = 0.5$, just as in Paczyński & Bisnovatyi-Kogan (1981), Muchotrzeb & Paczyński (1982), and Muchotrzeb (1983).

Two-temperature plasma is used in Wandel & Liang (1991); we assume that the accreting gas is of a single temperature over the entire disk, i.e., that the energy transfer between the electrons and protons is very efficient. This assumption can be justified by comparing the timescales of free-free cooling and Coulomb interaction between electrons and protons. The free-free cooling timescale is $t_{ff} = 2 \times 10^2 \rho_{10}^2 T_{6.5}^{1.5}$ s, and the timescale of the Coulomb interaction between electrons and protons is $t_{Coul} \approx 2 \times 10^{-4} \rho_{10}^2 T_{6.5}^{1.5}$ s, where $\rho_{10} = \rho / 10^{-10}$ and $T_6 = T/10^6$. When $t_{Coul} \geq t_{ff}$, i.e., $\rho \leq 10^{-16}$ g cm$^{-3}$, the plasma has two temperatures. We find that all the calculated $\rho$-values in this paper are consistent with the single-temperature assumption.

2.2. Formulations for Emergent Spectra

We employ the method described by Czerny & Elvis (1987) to calculate the emergent spectra from accretion disks, considering the effect of opacity due to elastic electron scatterings and the effect of energy exchange in nonelastic scatterings (i.e., Comptonization). The photons are scattered before they escape from the surface of the disk (Rybicki & Lightman 1979), and the emergent spectra are thus modified. However, Laor & Netzer (1989) pointed out that Comptonization is not important for disks around $10^8 M_\odot$ black holes and $L/L_{Edd} < 0.3$. In slim disks, the transition region appears, where the free-free absorption is not important but the scattering becomes essential.
Wang & Netzer (2003) have shown this process is very important in slim disks, as confirmed by Kawaguchi (2003); see also § 3.3 of this paper. The local radiation intensity

\[ I_s(r) = B_s(T_s) f_s(T_s, r), \]

where \( B_s \) is the Planck function, \( T_s \) is the temperature on the disk surface, and \( f_s \), describing the departure from blackbody radiation including Comptonization, is

\[ f_s(T_s, r) = f_{tt}(1 - f_{th}) + C. \]

Here \( f_{tt} \) and the fraction of thermalized photons \( f_{th} \) are given as

\[ f_{tt}(T_s, r) = \frac{2(1 - e^{-2\tau_{es}})}{1 + \sqrt{1 + \tau_{es}/\tau_{abs, \nu}}}, \]

and

\[ f_{th}(T_s, r) = \exp \left[ -\frac{\ln (k_B T_s / \hbar \nu)}{\tau_{es}^2 \ln (1 + \Xi_e + 16 \Xi_e^2)} \right], \]

respectively, where the electron's dimensionless temperature \( \Xi_e = 4k_B T_e / m_e c^2 \) and \( \tau_{es} = (\tau_{es} + \tau_{abs, \nu})/(1 + \tau_{es}) \); \( f_{tt} \) restores the modified blackbody radiation for \( \tau_{eff} \gg 1 \) and describes the thermal bremsstrahlung from an optically thin medium for low \( \tau_{eff} \). The Comptonization normalization constant is calculated by

\[ C = \frac{3k_B T_s}{\pi^2} \int_0^\infty f_{th}(\pi I_s(\nu) / \hbar \nu) d\nu. \]

The surface temperature \( T_s \) is reached by the balance between the radiated energy and the dissipated gravitational energy:

\[ Q_{\text{rad}} = \int_0^\infty \pi I_s(R, T_s) d\nu. \]

This simple method for calculating \( T_s \) allows us to avoid the calculations of the vertical structure of the slim disk, and the accuracy of the spectrum is enough for the present aims. The total emergent spectrum is derived by integrating \( I_s(r) \) over the radius,

\[ I_s = 2 \int_{R_{in}}^{R_{out}} \pi I_s(R) 2\pi R dR. \]

The hard X-ray emission from the hot corona above the slim disk is also calculated based on the detailed description in Wang & Netzer (2003). The most important characterized parameter, \( f \), the fraction of the gravitational energy released in the hot corona, is assumed in our calculation. However, this factor \( f \) is confined by the fact that the hot corona should remain, at least as an optically thin medium; otherwise it will be rapidly cooled. This limit can be roughly estimated by \( \tau_{es} < 1 \) at the last stable radius with the assumption that the radial velocity is the speed of light. Wang & Netzer (2003) showed that

\[ f < 0.0375 \left( \frac{\dot{m}}{10} \right)^{-1}. \]

This limit shows that the hot corona becomes weak with an increase in the accretion rate. Indeed, the hard X-ray spectra from microquasars and NLS1s show such a property. In our calculations, we thus neglect the feedback of the hot corona to the slim disk. This is justified because the factor \( f \) is small for a high accretion rate. However, a detailed treatment including this feedback for the case of an intermediate accretion rate will be carried out in a separate paper.

### 3. Numerical Results

Equations (2)–(7) can be reduced to two first-order differential equations about two variables, \( \rho \) and \( T \). We use the shooting method to solve these equations with appropriate boundary conditions. We assume an outer boundary at \( 10^4 R_g \); where Shakura-Sunyaev solutions (gas pressure dominated and \( \kappa_{abs} \gg \kappa_{es} \) ) are available. At this boundary, we first choose the derivatives of \( \rho \) and \( T \) obtained from Shakura-Sunyaev disk solutions, substitute them into the two differential equations, and then calculate the values of \( \rho \) and \( T \) by iterations. Although they are very close to Shakura-Sunyaev values, the new values of \( \rho \) and \( T \) allow the inward integration to run on the right way. The general relativistic effects are simulated by the pseudo-Newtonian potential near the horizon of black holes. The fourth-order Runge-Kutta method is employed to integrate the two differential equations to the inner disk edge \( R_{in} \), where \( R_{in} \) is taken as a free parameter, as has been done by other authors (e.g., Mineshige et al. 2000; Wang & Netzer 2003). We use the torque-free condition at the inner boundary. The shooting method determines the eigen-angular momentum \( l_{in} \), provided
the solution satisfies the regularity condition at the critical point close to the transonic radius.

After we obtain the global transonic solutions, the basic physical quantities in the accretion flow and its emergent spectrum are given for stellar black holes (10 M) and SMBHs (10\(^6\) M) in this section. We show in detail how the transition to an optically thin region appears with the change of accretion rate.

3.1. Transonic Solutions

The detailed transonic situations are analyzed here, and they are compared with the cases of slim disks without the correct treatment of transition regions. Figure 2 presents the exact distributions of the angular momentum \(J\), total pressure \(P\), and radial velocity \(v_r\) in the vicinity of transonic position, with the viscosity parameter \(\alpha = 0.1\) and \(m = 10\). The cases of \(\dot{m} = 1\) and \(10\) are both exhibited. From this figure, it can be seen that the curves of \(\dot{m} = 1\) of our model are almost the same as those of the classical slim-disk model. Compared to the classical slim disk, in the slim disks with transition regions, the angular momentums shift upward, the total pressures decrease, and the ratios of radial velocity to sound speed increase at the innermost radius, because of the effects of the optically thin region.

In the inner disk region where the effective optical depth is smaller than unity, the diffusion approximation for blackbody radiation is invalid and radiative cooling becomes inefficient. Thus the temperature is higher than that of the classical slim disk, where local blackbody radiation is assumed in the entire disk. At the same time, the lower total pressure resulting from the decreased radiation pressure due to \(\tau_{\text{eff}} < 1\) leads to the smaller viscous tensor \((\tau_{\text{eff}} \propto P)\). Therefore, the curve of angular momentum is flatter (see eq. [3]) and the flow becomes more dense. The accretion flow reaches the sonic point earlier than the classical slim disk, since the sound speed \(c_s = (P/\rho)^{0.5}\) is much smaller and the ratio \(v_r/c_s\) becomes larger. This explains why the transonic positions shift outward, compared with those in Abramowicz et al. (1988).

As is well known, the accretion flow has four types of transonic cases, according to which kind of pressure (gas or radiation) is dominant at the sonic point and the magnitude of the viscosity parameter. For low \(\alpha\), it is total pressure that pushes the flow through the sonic point, while for large \(\alpha\), the action of viscosity drives the flow across the transonic point. Figure 3 shows the dependency of the transonic radius on the accretion rate and the viscosity parameter for a black hole of 10 M. It is apparent that the transition to an optically thin region moves the transonic point out, when the accretion rate is in the range 1 < \(\dot{m}\) < 100 and the viscosity parameter is \(\alpha \geq 0.01\). In other parameter space comprising \(\dot{m}\) and \(\alpha\), the transonic location in our result is almost the same as that of the classical slim disk.

The transonic flows with transition regions of high viscosity can be compared with the flows of low viscosity. First, they are both radiation pressure dominated for supercritical accretion rates. Second, they are different in some properties. One of them is manifested in the optical depth. The accretion flow is almost entirely optically thick for a small viscosity parameter, e.g., \(\alpha = 0.0001\). While the transition region appears for a large \(\alpha\) (if \(\alpha\) is large enough, the inner flow can be purely optically thin and gas pressure dominated). Another distinct feature lies in the position of the sonic point. For low \(\alpha\), the flow cannot acquire enough velocity to be transonic until it is near the marginal bound orbit (2R\(_g\)), since the sound speed is relatively high and the viscosity-driven process is not efficient. On the contrary, the sound speed of the flow with large \(\alpha\) is small and the sonic point is located far outside the last stable orbit (3R\(_g\)). Therefore, we conclude that the transonic situation of the accretion flow with transition region is, to some extent, between the case of low viscosity and that of high viscosity in the classical slim disk.

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**Fig. 3.** Location of the transonic point \(R_o\) as a function of the accretion rate \(\dot{m}\) and the viscosity parameter \(\alpha\) (labeled for each line).

**Fig. 4.** S-shaped curves for accretion flows of stellar black holes and SMBHs. The solid, dashed, dot-dashed, and dotted lines are at the radii 5R\(_g\), 10R\(_g\), 50R\(_g\), and 100R\(_g\), respectively.
The S-shaped curves for different parameters are shown in Figure 4. It is found that they are strongly affected by the transition region when the viscosity is high. For the case of stellar-mass black holes, the upper ADAF and the middle unstable branches change much. The upper branch due to advection is modified, and the unstable branch is compressed. The appearance of the transition to an optically thin region lowers the radiation efficiency, and hence the advection becomes more important. In the accretion flows of SMBHs, the S-shaped curves are less altered, as shown in Figures 4c and 4d. It is easy to understand that the curves are not seriously influenced in the outer region, since there the optical depth is large and tends toward the classical slim disk. The distorted S-shaped curves may have observational features in stellar black hole accretions. The transition timescale from the upper to the lower branches, $t_\#$, will be shorter than that of the classical slim disk. For example, with the same parameters the difference at $R = 5R_g$ is a factor of 2–4. But the transition timescale from the lower branch to the upper, $t_\$, is unchanged and greater than $t_\#$. This would be of interest in the explanation of X-ray transient objects. The variation of state-transition timescales in stellar black holes is also one of the crucial characteristics of the transition to optically thin regions in slim disks.

3.2. Global Disk Structure

Figure 5 provides the physical quantities, such as the half-thickness, density, midplane temperature (including surface temperature), effective optical depth, and ratio of gas pressure to total pressure, for black holes of (a) $m = 10$ and (b) $m = 10^6$ with $\alpha = 0.1$. The solid, dashed, dot-dashed, dotted, and triple-dot-dashed lines are for $m = 1, 3, 10, 20, \text{ and } 50$, respectively. The surface temperatures $T_s$ are shifted down by 0.5 logarithmic units for clarity.

![Graphs showing various physical quantities for black holes of different masses.](image)
especially for the accretion flow around a stellar-mass black hole.

First is the half-thickness. For all the supercritical accretion rates $1 \leq \dot{m} \leq 50$, $H \leq 1$, indicating that the flow is geometrically slim. The thickness increases with the accretion rate, which is consistent with the result of the classical slim disk. At the outer boundary radius, the disk is always geometrically thin, and the solutions can be approximated by Shakura-Sunyaev solutions. Because of $H = B_{c2} \Omega / \Omega_K$, the lower sound speed results in the reduction of half-thickness, in comparison with the classical slim-disk model. The reduced height favors the trapped photons escaping from the disk. This shape of the accretion flow is somewhat analogous to the funnels in thick disks.

With the emergence of the transition region, the temperature and density of the disk are both larger than those in the classical slim disks. There are still considerable radiation fluxes in the innermost region after the flow passes through the sonic point, which was pointed out by Watarai et al. (2000). The effective optical depth is also shown in Figure 5. It should be noted that there is a large zone of $\tau_{\text{eff}} \leq 1$ when $\dot{m} \geq 3$, e.g., more than $30R_g$ or $15R_g$ for the disk of a black hole of $10$ or $10^6 M_\odot$, respectively. The lowest effective optical depth reaches 0.03. More important is that, in the accretion flows of both stellar-mass and supermassive black holes, there are wider regions of at least $100R_g$ showing $\tau_{\text{eff}} \sim 1$ for a moderate supercritical accretion rate $10 \leq \dot{m} \leq 50$. In such large transition regions, blackbody radiation is not adequate to describe the radiative transfer. Accordingly, a new radiation formula valid for any optical depth, as adopted in the present work, is necessary.

When the accretion rate tends to be very large, for example $\dot{m} > 10$, in the inner disk region the advected energy dominates over the surface cooling, i.e., $Q_{\text{adv}} \gg Q_{\text{rad}}$, and the flow will have self-similar behavior (Wang & Zhou 1999). We find that the global solution indeed shows self-similar signatures: the radial velocity $v_r \propto r^{-1/2}$, the total pressure $P \propto r^{-5/2}$, the density $\rho \propto r^{-3/2}$, the temperature $T \propto r^{-5/8}$, and $\tau_{\text{eff}} \propto r^3/16$ weakly depend on the radius, except in the vicinity of the inner disk edge. Watarai & Mineshige (2003) have also shown these properties.

Figure 6 shows the dependence of the transition radius on the accretion rate. We find the relation can be well fitted by an analytical formula:

$$R_{\text{tr}}/R_g = 95.38 \alpha^{0.91} \dot{m}^{0.06} \exp (-0.1 \dot{m}^{0.9}).$$

There is a maximum transition radius $R_{\text{tr, max}}/R_g \approx 50$ at $\dot{m} \sim 15$. This is clearly different from that in the standard disk model, i.e., $R_{\text{tr}} \propto \dot{m}^{0.75}$ (see eq. [1]). Our calculations show that there is no transition radius when $\dot{m} < 0.5$, and it drops quickly if $\dot{m} > 30$. The maximum radius of the transition region can be understood from the S-shaped curves in Figure 4. Near the turning point between the upper and the middle branch, the surface density is the lowest and at the same time the very high temperature makes the absorption opacity small. The farther from this area, the larger the effective optical depth because of the increase of either the surface density or the absorption opacity, or of both. This is an interesting result since the intermediate supercritical accretion has the largest transition radius, where the gravitational energy is mostly released.

We briefly discuss the viscosity effect on the disk structure. Figure 7 is the $\alpha$-dependence of the global disk structure. In the slim disks, $\alpha$ has only a little effect on the disk half-thickness. In our model, a large $\alpha$-value such as 0.3 considerably reduces the height and increases the density and temperature in the inner region, which differs from the result of Kawaguchi (2003). At the outer radius, the situation is similar to the result of Nakaguchi (2003): the larger the viscosity, the lower the density, midplane temperature, and optical depth, but the higher the surface temperature. This is because of more efficient angular momentum transfer outward and more trapped advection energy in the flow.

Figure 8 is the dependence of the transition radius on the viscosity $\alpha$. It can be found that the results from our calculations are not in agreement with the Shakura-Sunyaev disk, even at a small viscosity ($\alpha < 0.01$). This is because the Shakura-Sunyaev disk breaks down when $\dot{m} > 0.2$. The dependence of the transition radius on $\alpha$ becomes very strong, roughly $R_{\text{tr}} \propto \alpha^{0.91}$ for $\alpha > 0.01$ and $R_{\text{tr}} \propto \alpha^{0.22}$ for $\alpha < 0.01$. While $R_{\text{tr}} \propto \alpha^{3/4}$ in the Shakura-Sunyaev disk. The high viscosity drives the accreting gas to fall into the black hole more rapidly, leading to a low-density region. This region then becomes absorption-weak, enlarging the transition region. Since the radial motion and the advection effect are neglected in the standard disk, its prediction does not hold when either the accretion rate or the viscosity is large.

In the end, we should pay attention that the properties of the effective optically thin region discussed here are completely different from optically thin ADAFs. First, the flow is only effectively optically thin. The volume density and the surface density are still larger than those of optically thin ADAFs. The Thomson scattering is very important and the Comptonization is significant. That the optical depth becomes effectively thin is because of the transparency to photon absorption at a high temperature. In optically thick ADAFs, the plasma density is very low and the Thomson scattering optical depth is rather small. Second, the temperature in the present slim-disk model is not as high as that of optically thin ADAFs ($10^8-10^9$ K). Third, the pressure is still radiation pressure dominated, while for ADAFs, there is merely gas pressure and radiation pressure can be neglected. However, we believe that slim disks (optically thick ADAFs) are able to transit to optically thin ADAFs in the inner region, once the viscosity is large enough. We leave this topic for a later paper.
3.3. Emergent Spectra

We have shown that the global structures of the slim disks with transition regions are quite different from those slim disks in Abramowicz et al. (1988). With the presence of the transition region, only the theoretically predicted spectrum based on the global structure is of great significance to observations. The emergent spectra of slim disks have been extensively studied. Szuszkiewicz et al. (1996) were the first to calculate the spectrum with modified blackbody radiation, and they explained the soft-X excesses of quasars. Wang et al. (1999) calculated the vertical structures and emergent spectra from slim disks. Two prominent features appear: the saturated total luminosity and the constant cutoff energy. Mineshige et al. (2000) and Watarai et al. (2001) used a purely blackbody radiation approximation, although it is not adequate in the inner disk region. Wang & Netzer (2003) and Kawaguchi (2003) employed the method of Czerny & Elvis (1987) to consider Thompson electron scattering and Comptonization in accretion disks. However, since they did not treat the phenomena as optically thin, their results are only available for low viscosity parameters. As for the case of high viscosity parameters, the local radiation in spectral calculations tends toward thermal bremsstrahlung when the optical depth is small, while local blackbody radiation that holds only for an optically thick medium is assumed to determine the disk structure. This inconsistent treatment between structure and spectral calculations leads to the problem of temperature inversion (Czerny & Elvis 1987); namely, the calculated surface temperature is even higher than the midplane temperature in the accretion flow (see the gray lines in the middle panels of Fig. 5; in the classical slim disk, $T_s$ is much larger than $T$ at several $R_g$). The reason is that the blackbody approximation predicts too much energy loss compared to realistic radiation in optically thin flows. Thus, the radiated flux is heavily overestimated at high energies.

We have solved the problem in this paper. By adopting the bridged formula that can describe the radiation energy for both optically thick and thin cases, the calculated surface temperature is not larger than the midplane temperature, as shown in Figures 5 and 7, but Thompson electron scattering modifies the radiation to deviate from blackbody and inelastic scattering also exchanges the energy between photons and electrons. The Comptonization effect has been stressed in Wang & Netzer (2003) and Kawaguchi (2003). We can give a more detailed estimation of the importance of this process. The effective Compton parameter

$$y_c = \frac{\tau'_c}{\tau_c} \max(\tau'_s, \tau'_{es}),$$

(17)

where $\tau'_c = \tau_c / \max(\tau_{eff}, 1)$ and the absorption opacity is taken as $\kappa_{abs, \nu} = 1.4 \times 10^{25} \rho T^{-3/2} x^{-3} (1 - e^{-x})(x = h\nu/k_B T)$, and 30 times this value for SMBHs. Figure 9 shows the parameter $y_c$ for different photon energies, accretion rates, and black hole masses. When $y_c > 1$, Comptonization cannot be neglected and the emergent spectrum will be strongly modified. Figure 10 clearly shows how important the Comptonization process is for disks with different parameters. We can draw the conclusion that the Comptonization effect is very important in the inner region of supercritical accretion flows. The saturated Comptonization process is inevitable for the disk of less
massive black holes ($10^6 - 10^7 M_\odot$). This confirms the conclusion of Wang & Netzer (2003). Comptonization is less important for a $10^8 M_\odot$ black hole disk, unless the disk has a highly supercritical accretion rate, which agrees with Laor & Netzer (1989). The higher the accretion rate, the more important the Comptonization process. We adopt the scheme described in §2.2 to calculate the spectra from slim disks, taking into account the Comptonization effect by a simple but practical method.

Figures 10 and 11 show the details of the emergent spectra from the slim disks with transition regions. We would like to stress that the present calculations have two advantages: (1) we include the transition region, and (2) the model covers wide ranges of the accretion rate and viscosity. Thus the present model involves all the known disk models (except for the optically thin ADIOs and CDAFs). Figure 10 clearly shows the spectra from the accretion disks of which the central black hole masses are $10^6$ and $10^8 M_\odot$. We start the accretion rate from subcritical ($\dot{m} = 0.3$) to supercritical ($\dot{m} = 50$). We find that the spectrum shifts toward high energy with the increasing accretion rate. This is due to the increases of temperature and the importance of Comptonization with the accretion rate. The spectra show very strong humps at high energies for both stellar and supermassive black holes. The spectral flux is proportional to the accretion rate for the subcritical accretion flow. The distinct spectral characteristic of supercritical disks is their broadness and flatness. The cutoff energies of the spectra almost remain constant when the accretion rate $\dot{m}$ is above 10, and the total radiated luminosity from the disk tends to be saturated. The constant cutoff energy was first found in Wang et al. (1999), which is caused by advection. The feature of saturated luminosity has also been found in Abramowicz et al. (1988), which is caused by the advection effect: most of the dissipated energy is swallowed by the central black hole. The maximum cutoff energy is about 100 or 4 keV for a black hole of $10^6$ or $10^8 M_\odot$. The saturated luminosity is about several Eddington limits ($\sim 10^{53}$ and $10^{44}$ erg s$^{-1}$ for $10^6$ and $10^8 M_\odot$ black holes, respectively). This extreme slim disk with a very high accretion rate can be explained by the behavior of a
self-similar solution (Wang & Zhou 1999; Wang & Netzer 2003). The features of the broad and flat spectrum, constant cutoff energy, and saturated luminosity are very useful in explaining the X-ray spectra of microquasars, ULXs, and NLS1s.

For comparison with spectra from classical slim disks, we use gray lines in Figure 10. We use the same computation scheme for the spectra from the classical slim disks. We find that the emergent spectra are quite different from those based on the present model with transition region, when the accretion rate is above the critical value. The cutoff energy in the classical slim disk is unrealistically higher than the present model. Thus the present model provides an improved version of the slim disk.

We investigate the effect of viscosity on the spectrum. For all the accretion rates, the larger viscosity parameter causes the spectrum to extend to higher frequency. This is the contribution of the more significant elastic electron scatterings and Comptonization. From Czerny & Elvis (1987), the critical temperature is \( T_{\text{es}} \propto m^{-4/11} \alpha^{1/11} \), above which the modification due to electron scattering is important. If \( \alpha \) increases, the transition region will enlarge and the surface temperature will increase. In such a flow the Comptonization effect becomes stronger. Thus the emergent spectrum is broader and flatter with the higher viscosity parameter. Usually the cutoff energy strongly relies on both the accretion rate and the viscosity parameter. But in a saturated accretion system, it is only affected by \( \alpha \). This may be utilized to determine the unknown viscosity parameter through X-ray observations of ultraluminous objects.

Finally, with detailed calculations, we verify the importance of photon trapping in supercritical flows. If the diffusion timescale

\[
  t_{\text{diff}} = \frac{H(r_{\text{es}} + \tau_{\text{abs}})}{c} 
\]

is longer than the accretion timescale

\[
  t_{\text{acc}} = \frac{R}{v_R} 
\]

in a flow, the photons cannot escape from the disk at the same radius. This causes the so-called photon trapping. Such a case

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![Figure 10](image1.png)

**Fig. 10.**—Emergent spectra from slim disks with transition regions for (a) \( m = 10 \) and (b) \( m = 10^6 \). From bottom to top, the accretion rates are 0.3, 1, 3, 10, 20, and 50, respectively. The gray lines are the results for the classical slim disks, of which the calculated surface temperature is higher than the midplane value in the optically thin region. Thus the radiation fluxes at high energies are overestimated.

![Figure 11](image2.png)

**Fig. 11.**—The \( \alpha \)-dependency of the emergent spectra from slim disks with transition regions. The solid, dashed, dot-dashed, and dotted lines are for \( \alpha = 0.001, 0.01, 0.1, \) and 0.3, respectively. The accretion rates are labeled in each panel.
was recognized early on (Katz 1977; Begelman 1978), but the quantitative features have been discussed by Wang & Zhou (1999), Ohsuga et al. (2002, 2003), and Shimura & Mannmoto (2003).

Let us compare the two timescales. Figure 12 shows \( t_{\text{acc}} \gg t_{\text{dyn}} \) in the entire \( r \leq 10^4 \) region for any \( \dot{m} \leq 50 \). This means that the viscosity works for the disk, transporting the angular momentum and dissipating the gravitational energy. When \( \dot{m} > 1 \) in the inner region \( t_{\text{diff}} > t_{\text{acc}} \), the photons are hardly radiated out, so they are trapped in the accretion flow. In this trapped region, energy transport in the form of advection is the main cooling mechanism. In this sense, the slim disk is often called an optically thick ADAF. After comparison, we conclude that the photon-trapping effect is still significant, but it is not as severe as that in the classical slim disk. This is mainly because of the greatly reduced disk height. The present model does not treat photon trapping carefully, but see Ohsuga et al. (2002, 2003), who treated photon trapping with the given disk structure. Wang & Zhou (1999) have shown that the radiated luminosity is \( 4 \times 10^{39} (M_{\text{BH}}/10M_\odot) \) ergs s\(^{-1}\) for an extreme supercritical disk, which is as small as one-third of the Eddington luminosity. In fact, in the physics the photon-trapping effect makes it difficult to estimate the real accretion rate from the saturated observed spectrum when the black hole has a supercritical accretion rate. One of the goals of this paper is to investigate the physics of the transition region. For a moderate supercritical rate \( \dot{m} < 10 \), the photon-trapping effect is not as strong, and our present model does work for our applications in the next section. The model coupling the photon-trapping effect is only in preparation. Considering the photon-trapping effect, we believe that the supercritical accretors in microquasars and NLS1s are more numerous than we know at present. Kawaguchi (2003) realized the role of the self-gravitation in the optical spectrum of NLS1s. We do not consider self-gravitation in the present work. The general relativistic effects are not included either. Future work will solve the equations of the general relativistic disk with transition region, as well as calculate the spectrum from the disk.

4. APPLICATIONS

The most prominent characteristics of the spectra from slim disks are the strong humps and the constant cutoff frequency. When \( L/L_{\text{Edd}} \geq 0.3 \), the standard accretion-disk model breaks down (Laor & Netzer 1989) and the slim disk is available. Our present model covers a rather wide range of accretion rates. The distinguished soft X-ray humps in some objects are explained by the self-consistent emergent spectra based on the corresponding global disk structures. In this section, we apply our model of slim disks to the microquasar GRS 1915+105 and two NLS1s: RE J1034+396 and Ark 564.

4.1. Microquasars: GRS 1915+105

Some Galactic black hole candidates, especially microquasars and ULXs, are regarded as supercritical accretors (Watarai et al. 2000, 2001). The broadband X-ray spectra of microquasars and ULXs have been observed extensively with RXTE and ASCA (Makishima et al. 2000; Mizuno et al. 2001; Ueda et al. 2002). The observed spectra are well fitted with multicolor blackbody spectra in the framework of a standard optically thick accretion disk. However, the multicolor disk (MCD) spectrum has some shortcomings. The spectral hardening factor due to Comptonization in the disk atmosphere is assumed to be a constant (usually 1.7; Shimura & Mannmoto 2003) over the entire disk. Our calculations show that Comptonization is only crucial in the inner region. In addition, the MCD model is based on the standard disk theory and the viscous dissipation is thought to be radiated efficiently locally. This gives an upper luminosity limit (\( L < 0.3L_{\text{Edd}} \); Laor & Netzer 1989) to its scope of use. Obviously, the luminosities of ULXs and other bright objects are beyond this limit. Therefore, the fitting parameters of the MCD model have difficulties in connecting with physics. First, the X-ray–estimated black hole mass with MCD is smaller than the precise dynamic measurement by optical observations. Second, the fitted color temperature is too hot to be produced by the standard disk; i.e., this is the problem of too hot a disk (e.g., King & Puchnarewicz 2002; Ebisawa et al. 2003). Finally, the MCD model cannot account for spectral variations. The five periods of spectral fitting of IC 342 source 1 (a ULX) show that the black hole mass varies by a factor of 2 (Ebisawa et al. 2003). This is unreasonable.

Ebisawa et al. (2003) found that the spectra of Galactic superluminal jet sources can be explained with the standard Kerr disk model while ULXs need a slim-disk model. Sobolewska & Życki (2003) stated that the relativistic effects of the Kerr model can only partially account for the apparently complex soft X-ray spectra of GRS 1915+105. In this subsection, we show that the emergent spectrum from our slim-disk model, considering the effects of deviated blackbody radiation, Comptonization, and the transition to optically thin, is quite adequate for describing the soft spectral component of GRS 1915+105 when it is in the high state.

The measurements of the black hole mass and the distance to GRS 1915+105 are quite reliable, so we are able to apply the present model to this object. The black hole mass \( M_{\text{BH}} = 14 \pm 4 M_\odot \) is deduced from direct measurement of the orbital period and mass function of GRS 1915+105 (Greiner et al. 2001). The inclination is \( i = 70^\circ \), and the distance is \( D = 12.5 \) kpc (Mirabel & Rodriguez 1994). We adopt the recent values \( i = 66^\circ \) and \( D = 11 \) kpc (Fender et al. 1999).

GRS 1915+105 stays almost permanently in a soft spectral state, although it has strong variability of both flux and energy spectra. RXTE data have been extensively studied by Belloni...
et al. (2000). The light curves are divided into 12 classes according to color-color diagrams and count rates. Each class can be decomposed into just three basic spectral states: A, B, and C. Sobolewska & Życki (2003) attempted the modeling of these three states, using the data of the PCA and HEXTE on board RXTE. In the framework of the MCD model, they found that the X-ray spectra of A and B (corresponding to relatively higher luminosities) are complex: only one Comptonized disk plus another model such as a Comptonized disk, blackbody, or Kerr disk can give an acceptable fitting. They derived the accretion rate $M = 7.5 \times 10^{10}$ g s$^{-1}$, namely, $m \sim 0.3$, for the spectral state B. We choose the same observation data to do fitting with the present model. For comparison, we employ the absorption cross section given by Morrison & McCammon (1983) to correct the spectrum from our slim disk. The effective hydrogen column density is taken as $6 \times 10^{22}$ cm$^{-2}$ (Sobolewska & Życki 2003). The spectral energy distribution is shown in Figure 13. The dotted line is the spectrum from the slim disk described in this paper. The Kerr MCD spectral component of Sobolewska & Życki (2003) is also shown by the gray dashed line. The hard tail is modeled by the thermal Comptonized spectrum (dashed line) from the corona above the slim disk.

With our slim-disk model, the best-fit parameters are $\alpha = 0.01$, $m = 2.1$, and $f = 0.03$. It can be seen that the theoretical spectrum coincides with the observation very well in the entire X-ray band. With these parameters, the transition radius is $\sim 3.4R_g$. In this case, the difference from the classical slim disk is not distinguishable, but it is clear that our model is more self-consistent in the inner disk region.

Since our slim disk covers rather wide ranges of accretion rate and viscosity parameter, it can also be applied to other stellar black holes, such as Galactic black hole binaries, superluminal jet sources (microquasars), and ULXs. We have also successfully fitted the X-ray spectra of GRO J1655–40 (Tomskick et al. 1999) and LMC X-1 (Gierlinski et al. 2001). Their accretion rates are $\sim 0.5M_\odot$. Ebisawa et al. (2003) applied the slim-disk spectra to IC 342 source 1 (see also Watari et al. 2001 for other ULXs). Since the basic quantities of ULXs, e.g., the mass and distance, are unknown, spectral fitting is a little difficult. Once we implement the present model into XSPEC, it will be possible to model many other sources, especially XTE J1550–56 and IC 342 source 1, suspected of having supercritical accretion rates. This will help us understand more physics in ULXs. Making use of the wide validity of the present model, we can understand the process of state transition, of which the transition to an optically thin region is an example.

4.2. Narrow-Line Seyfert 1 Galaxies

NLS1s, as a special subgroup of Seyfert 1 galaxies, are characterized by their unusual properties: the H$\beta$ line is narrow [FWHM(H$\beta$) < 2000 km s$^{-1}$] and relatively weak (less than a third of the intensity of the [O iii] $\lambda 5007$ line), and the optical Fe ii lines are very strong. It is revealed that the excess and the tendency of the spectrum to flatten at low energy have been found in several NLS1s, e.g., PKS 0558–504 (O’Brien et al. 2001), Mrk 766 (Boller et al. 2001), PG 1244+026 (Fiore et al. 1998), PG 1404+226, PG 1440+356, and PG 1211+143 (George et al. 2000), IRAS 13224–3809 (Vaughan et al. 1999), 1H 0707–495 (Boller et al. 2002; Dewangan et al. 2002), RX J1702.5+3247 (Gliozzi et al. 2001), and Ton S180 (Turner et al. 2001a, 2002). BeppoSAX and ASCA observations of Ark 564 and RE J1034+396 clearly show these extreme properties, including a soft X-ray hump with a very flat spectrum ($\nu F_\nu = \text{const}$; Comastri et al. 2001; Puchnarewicz et al. 2001; Turner et al. 2001b; Vignali et al. 2004; Romano et al. 2004). Other unusual X-ray properties are discussed in Brandt et al. (1994), Comastri et al. (2001), Pounds et al. (1995), and Collinge et al. (2001). Earlier suggestions attributing the soft excess to blends of emission lines are not supported by recent ASCA, XMM-Newton, and Chandra observations (e.g., Turner et al. 1999, 2001a, 2002; Puchnarewicz et al. 2001; Collinge et al. 2001). Chandra shows that there are no features of emission lines in the soft X-ray band (Collier et al. 2001). The model of a less massive black hole with high accretion rate is preferred for its very prominent soft X-ray hump (Boller et al. 1998; Wang & Netzer 2003; Kawaguchi 2003). In fact, the feature of a soft X-ray hump, $\nu F_\nu = \text{const}$, is a natural explanation of supercritical accretion disks (Wang & Netzer 2003). With the black hole mass estimated by the reverberation method, Wang & Netzer (2003) found that half of NLS1s have $L/L_{\text{Edd}} > 1$ in the sample of Véron-Cetty et al. (2001). We believe that NLS1s are good candidates for the application of the present model.

As an application of the present model, we make an attempt to explain the observed soft X-ray humps of Ark 564 and RE J1034+396. RE J1034+396 shows a very clear appearance of the flattening of the soft X-ray spectrum from $\log \nu = 16.5$ to 17.2 in the $\nu L_\nu$ plot (Comastri et al. 2001; Puchnarewicz et al. 2001; Collinge et al. 2001; Turner et al. 2001b). Here we suggest that this feature reflects the power of the slim disk in the two objects.

RE J1034+396.—The ultrasoft X-ray radiation of RE J1034+396 was first discovered by Puchnarewicz et al. (1995). The spectrum shown in Figure 14 shows a clear soft X-ray hump extending from $\log \nu = 16.5$ (the lowest observed frequency) to 17.2. The data were taken from Puchnarewicz et al. (2001). They explained the strong soft X-ray spectrum of RE J1034+396 based on the standard accretion-disk model (Czerny & Elvis 1987). The resulting accretion rate is $m \sim 0.3$–0.7, which is beyond the scope of the standard disk model. They also suggested a high inclination $i_{\text{obs}} = 60^\circ$–75$^\circ$ in order to lower the luminosity.

The top panel of Figure 14 shows the comparison of our model with the observation data. The FWHM(H$\beta$) = 2000 km
s^{-1} and the luminosity $L_{5100} = 10^{44}$ ergs s^{-1} at 5100 Å; the mass of the black hole is estimated as $M_{BH} = 2.25 \times 10^6 M_\odot$ by the empirical reverberation-mapping relation (Kaspi et al. 2000). We assume an inclination angle $i = 0^\circ$, since no accurate estimation has been made in the literature. The viscosity $\alpha = 0.2$ is required from the fitting. We obtain the accretion rate $\dot{m} = 1.2$. The factor $f = 0.07$ is found, which is within the limit of equation (15). This result is in rough agreement with that of the self-similar solution in Wang & Netzer (2003).

**Arakelian 564.**—Pounds et al. (2001) estimated a black hole mass in Ark 564 of $10^7 M_\odot$, and an accretion rate $\dot{m} \approx 0.2$–$1$ from the break frequency of the power spectrum of the light curves. The multiwavelength spectra of Ark 564 have been extensively studied by Romano et al. (2004). They found $M_{BH} = 4 \times 10^6 M_\odot$ and $\dot{m} = 1$. Wang & Netzer (2003) provided a simple method to estimate the black hole mass if the accretion rate is supercritical, $M_{BH} = 2.8 \times 10^6 (\nu L_{\nu}/10^{44} \text{ergs s}^{-1}) M_\odot$, which yields a black hole mass $2.0 \times 10^6 M_\odot$ (Romano et al. 2004) for Ark 564. The current data have factors of uncertainty of a few $M_{BH}$. The inclination angle $i = 27.4^\circ$ (Ballantyne et al. 2001), which was derived from fitting ASCA spectra with the ionized reflection disk models of Ross & Fabian (1993). The observed spectral energy distribution of Ark 564 is taken from Comastri et al. (2001). The comparison of the present model with the observations provides $M_{BH} = 10^6 M_\odot$, $\dot{m} = 1.1$, $\alpha = 0.32$, and $f = 0.021$. The hot corona is relatively weaker than that in RE J1034+396.

The fitting results indicate that the two NLS1s are moderate supercritical accreting sources. In this case, the transition region plays a great role in the emergent spectrum. The transition radii, where $\tau_{\text{eff}} = 1$, are $11.5 R_g$ and $15.5 R_g$ for RE J1034+396 and Ark 564, respectively.

This is just the main X-ray emitting zone. Therefore, although the transition region has little influence on the S-shaped curves (Fig. 4), it has an important effect on the spectrum from the disk. Our slim-disk model provides more accurate treatment of this transition region, so it has a wider application field than the previous slim-disk model. As stated by Pounds & Reeves (2004), the soft X-ray humps are still poorly understood since we do not know what drives the shape of the soft X-ray spectrum. We need more observations. Future work will focus on spectral fitting in a sample so that we can discover the primary parameters controlling the shape of the soft X-ray spectrum.

We note that the mass accretion rates for the three objects GRS 1915+105, RE J1034+396, and Ark 564 are of 1~2 critical accretion rates. For this range of accretion rate, slim disks with an $\alpha$ viscous law have large thermally unstable regions. These unstable regions are responsible for the observed rapid and giant variabilities from microquasars and NLS1s. To describe the structures accurately, the time-dependent slim-disk model with transition region is needed to explore the variabilities of these supercritical accretors. We leave this topic to future research.

5. CONCLUSIONS

The detailed structure and spectral calculations of the slim disk with transition region (effective optical depth $\tau_{\text{eff}} \leq 1$) have been presented. We have shown that there is a quite large transition region in the slim disk. This region becomes larger with the increase of viscosity parameter. We get an empirical formula for the transition radius, i.e., $R_{tr}/R_g = 95.38 \alpha^{0.91} \dot{m}^{0.36} \exp(-0.1 \dot{m}^{0.9})$. The maximum transition radius reaches $\sim 50R_g$ at $\dot{m} \sim 15$ for a fixed viscosity $\alpha$. In the slim-disk regime, the transition radius is $R_{tr}/R_g \propto \alpha^{0.91}$ for a fixed accretion rate (e.g., $\dot{m} = 10$) and $\alpha > 0.01$. The transition region affects not only the global disk structure but also the emergent spectrum. The transonic location moves outward and the local radiated spectrum departs from blackbody radiation, because of the presence of the transition region. In comparison with the classical slim disk, we have given a more accurate treatment with the optically thin region, in the calculations of both structure and spectrum. The present model covers a rather wide parameter space. We have also shown how the soft X-ray spectrum changes from the subcritical, to the moderate supercritical, to the extreme supercritical accretion rate. We have calculated the Comptonization parameter for the slim disk and shown that Comptonization is very essential for stellar-mass black holes and less massive black holes ($\lesssim 10^6 M_\odot$), whereas it is not very important for more massive black holes with masses greater than $10^7 M_\odot$. This shows that the Comptonization process plays a key role in the formation of spectra in luminous objects. The present model can be widely applied to microquasars, ultraluminous X-ray sources, and narrow-line Seyfert 1 galaxies.

The authors are grateful for helpful discussions with J. F. Lu and M. Wu. Detailed reading of the manuscript and productive comments from the referee, Ewa Szuszkiewicz, are also appreciated. J. M. W. is grateful for support from the National Science Foundation of China and Special Funds for Major State Basic Research.
