Abstract
Efficient parallelization of algorithms on general-purpose GPUs is essential in many areas today. However, it is a non-trivial task for software engineers to utilize GPUs to improve the performance of high-level programs in general. Although many domain-specific approaches are available for GPU acceleration, it is difficult to accelerate existing high-level programs without rewriting parts of the programs using low-level GPU code. We present a compiler implementation using an alternative approach called expression acceleration. This approach marks expressions for acceleration, and the compiler automatically infers which dependent code needs to be accelerated. We design and implement a compiler supporting expression acceleration for a statically typed functional language and evaluate its applicability and performance.

CCS Concepts: • Software and its engineering → Parallel programming languages; Source code generation.

Keywords: GPUs, acceleration, parallelization

1 Introduction
Parallel computing on GPUs has been tremendously successful in many areas, including 3D graphical rendering, machine learning, and blockchains. However, efficient implementations on general-purpose GPUs require deep knowledge of low-level languages and frameworks, such as CUDA and OpenCL. Moreover, modern software systems are often developed in high-level languages, with closures, garbage collection, and user-defined recursive data types. Parallelizing subsets of such systems—if even possible—is typically very time-consuming. In particular, parts of the code need to be manually translated to low-level GPU code, including interactions between the low-level parallel code and the high-level program and low-level data marshaling.

2 Expression Acceleration
In this section, we first present the key idea of expression acceleration, followed by a short outline of a practical approach to programming with expression acceleration. We
Listing 1. Example program making use of expression acceleration.

```haskell
match readInput () with (t1, t2, n) in
  accelerate (let axpy = lam c. lam i.
               let x = tensorGet t1 [i] in
               let y = tensorGet t2 [i] in
               tensorSet t1 [i] (addi x (muli c y)) in
               loop n (lam i. axpy 2 i));
  let t3 = create n (lam i. tensorGet t1 [i]) in
  if geqi n 100 then reduce addi 0 s in
  else foldl addi 0 s in
  let computeSum = lam s. lam n.
    tensorSet t1 [i] (addi x (muli c y)) in
    tensorGet t2 [i] in
    let x = tensorGet t1 [i] in
    let axpy = lam c. lam i.
      tensorGet t1 [i] in
      tensorGet t2 [i] in
      tensorSet t1 [i] (addi x (muli c y)) in
      tensorGet t2 [i] in
    if geqi n 100 then reduce addi 0 s in
    else foldl addi 0 s in
  let sum = accelerate (computeSum t3 n) in
  writeOutput sum
```

implement expression acceleration in a language extended from the intermediate high-level functional language of the Miking framework [2]. The extensions include parallel keywords used to support expression acceleration. The compiler implementation is available as open-source\(^1\).

2.1 Key Idea

Consider the program of Listing 1 using expression acceleration. The program reads input (line 1), runs an accelerated expression on lines 2-7, executes a sequential expression on line 8 before running another accelerated expression on line 12, and finally writes the result to a file (line 13).

The accelerate keyword (lines 2-7 and line 12 in Listing 1) takes an accelerated expression as an argument, indicating that it should execute in parallel. Our compiler validates that this is supported, and automatically translates parallelism within accelerated expressions to execute on the GPU. The compiler has two separate backends; one is a functional backend that outputs Futhark [12] code and uses the Futhark compiler to produce GPU code (used on line 12), and the other is an imperative backend that produces CUDA code directly (used on lines 2-7).

The accelerated expression can have free variables. Consider the use of accelerate on line 12, which refers to three free variables. The function computeSum has to be included in the code we run in the accelerated context, and the values of t3 and n have to be copied to the accelerated context. We outline the steps required to support this in Section 4.

Our extensions to the sequential source language consist of three categories of parallel constructs:

- The accelerate keyword.
- Functional parallel keywords, such as reduce (line 10).
- Imperative parallel keywords, including loop (line 7).

\(^1\)https://github.com/miking-lang/miking

Figure 1. The recommended workflow when using acceleration, from a source program on the left-hand side to an accelerated binary on the right-hand side. Gray boxes are artifacts and blue boxes are processes.

2.2 Recommended Workflow

Figure 1 illustrates our proposed workflow which starts from a source program and ends with an accelerated binary. First, we compile the program in debug mode (1 in the figure). The compiler validates the accelerated expressions but produces sequential CPU code with runtime checks for details that we cannot verify statically. The user executes the debug binary (2) to ensure no runtime errors occur. If the compilation or execution fails, the user must rewrite the source program. This workflow ensures informative error messages, which is challenging for code running on the GPU.

If the debug executable produces no errors, we compile the program in accelerate mode (3), where parallel keywords in accelerated expressions execute on the GPU. We omit the line from this step to the source program because compilation errors are caught in (1). For efficiency reasons, we omit the runtime checks in the accelerated binary. This workflow enables early and direct feedback and produces a high-performance executable without unnecessary runtime overheads in the accelerated code.

3 Compilation Pipeline

Figure 2 depicts the accelerate compiler pipeline, starting from a source program on the left and ending with an accelerated binary on the right. First, the compiler performs parsing and type-checking (1 and 2 in Figure 2). This step is followed by accelerate extraction in the extractor (3). In this step, we extract the accelerated expressions and the functions they depend on into a separate program (Section 4.1).

Following the extraction, the program consists of two intermediate artifacts. The first part is the sequential code, which represents the parts of the program that execute sequentially. The second part is the accelerated code. This code is classified (Section 4.2) into backend-specific programs using a classifier (4). Our compiler supports two accelerate backends. The first emits Futhark [12] code, a pure functional array language. We use the Futhark compiler to produce efficient CUDA code. This backend supports a purely functional subset of the source language, operating primarily on immutable sequences. Futhark does not support imperative programs or operations with side effects. For this reason, we define another backend that directly emits CUDA code.
This backend supports an imperative subset of the source language, primarily operating on mutable tensors.

The compiler must ensure these programs are supported by the respective accelerate backend. Thus, we define a set of well-formedness rules (Section 4.4) for each backend that the compiler verifies statically (5 & 6). Given that the code is well-formed, each backend generates GPU code (7 & 8). Finally, the backend compiler (9) glues together the sequential code with the backend-specific programs by generating marshaling code based on the types of accelerated expressions (Section 4.3). The resulting binary, which we refer to as the accelerated binary (right side of Figure 2), runs the sequential parts on the CPU while accelerated expressions are offloaded to the GPU.

4 Analysis and Extraction

In this section, we present important analyses in the compilation pipeline. We focus on the key step in supporting expression acceleration—the extraction (Section 4.1). We also discuss the high-level ideas of classification (Section 4.2), data marshaling (Section 4.3), and well-formedness (Section 4.4). For more details, see the technical paper (we will add a reference if this paper is accepted).

4.1 Extraction

We need to extract the parts of a program marked for acceleration into a separate AST. To make acceleration convenient, we allow an expression `accelerate e` to contain free variables. A naive approach to the extraction is to include all bound functions in the accelerated code. However, this includes more code than necessary, leading to increased compilation times. Also, it prevents sequential code from using features unsupported by the target accelerate backend. Therefore, our extraction includes only functions used in an accelerated expression. This enables sequential code to use all features available in the source language. It is also efficient as it avoids duplicating functions used in multiple accelerated expressions.

Extraction of accelerated expressions takes place in two steps. First, we rewrite accelerated bindings and apply lambda lifting to the source program. The second and key step is to emit the parts of the program used in accelerated expressions. In the extraction, we assume the names used in binding expressions (e.g., let-bindings) are unique. This can be achieved through a renaming pass. We refer to the unique name of a variable `x` as an identifier.

As an illustration of the extraction, consider the programs of Listing 3. The input program, which contains an accelerated expression, is shown in Listing 3a. In the first step of the accelerate extraction, we rewrite expressions `accelerate e` as an accelerated binding with identifier `a`, such that `let a = e in a`. We apply lambda lifting [14] modified to treat accelerated bindings `a` as functions. The lambda lifting captures free variables of non-function type in `e` and adds them as parameters of `a`. Lambda lifting also lifts nested bindings to the top of the program, which is required by targets that do not support nested functions (e.g., CUDA).

Listing 3b shows the program after applying the first step. Note that the accelerated expression is replaced with an accelerated binding `a` and that the nested function `r` is lifted outside of `a`. Further, observe that the definitions of `s` and `c` are not included in the accelerated code. Instead, they are captured as arguments of `a` by the lambda lifting (line 5), and passed to the function in the application on line 8. This approach allows us to compute values in the sequential code, without the restrictions imposed on the accelerated code (e.g., the `readSequence` function may perform I/O, which is not supported in accelerated code) and then pass them to the accelerated code.

In the second step, we extract the parts of the program used within accelerated expressions into a separate program. The extracted program for the example consists of the highlighted parts of Listing 3b. To arrive at this result, we go through the program in a bottom-up fashion, starting from the accelerated binding `a` on lines 5-7 in Listing 3b. We refer to the expression assigned to an identifier of a let-expression as the body. For instance, the body of `a` is the expression starting after the equals sign on line 5 until the `in` keyword at the end of line 7.

Because we perform lambda lifting to capture free variables of non-function type, we know any free variables in the body of `a` must be functions. These functions are what...
When generating accelerated code, the target backend is typically more restrictive than the default sequential compiler (which, for instance, uses a garbage collector). We perform well-formedness checks to ensure the accelerated code adheres to our defined rules for the CUDA and Futhark backends. The well-formedness includes both assumptions we check at runtime (in debug mode) and static rules we verify at compile-time (the well-formedness checker). For instance, our well-formedness rules ensure that the accelerated code does not pass higher-order functions as arguments, as this is not supported by either accelerate backend.

## 5 Evaluation

In this section, we evaluate the compiler. For the Futhark backend, we implement a small Futhark benchmark suite in the parallel language (Section 5.1). For the CUDA backend, we implement an ODE solver and a neural network (Section 5.2). These benchmarks show that our compiler for expression acceleration can solve non-trivial problems and significantly improve performance over sequential code.

We run the experiments on an Intel Xeon 656 Gold 6136 CPU and a Titan RTX GPU using Ubuntu 18.04. We use OCaml version 4.14.0, CUDA version 11.7, and Futhark version 0.25.16. The compilation times for all benchmarks presented in this section are below 5 seconds on this machine.

### 5.1 Futhark

To show that our compiler can generate efficient code, we implement the Parboil suite of the Futhark benchmark suite\(^2\) in our parallel language and compare the Futhark output from our compiler with existing code using the built-in benchmarking tool in the Futhark compiler (Futhark bench) using Futhark’s CUDA backend, measuring execution time and validating the output for each dataset.

To be able to make a comparison using futhark bench, we post-process the Futhark output from our compiler. We add metadata used by the benchmarking tool, and for compatibility in a few of the benchmarks, we add a custom entry function to convert integer types as our compiler always

\(^2\)https://github.com/diku-dk/futhark-benchmarks
We present execution-time results for the ODE solver and the neural networks in Table 2. For these benchmarks, we accelerate using the CUDA backend and compare the performance of the code when compiled without and with acceleration enabled (labeled Sequential and Accelerated, respectively).

| Dataset          | Accelerate | Futhark |
|------------------|------------|---------|
| histo-default    | 271.5 ± 3.4| 240.7 ± 2.9|
| histo-large      | 215.6 ± 3.3| 202.0 ± 3.4|
| mri-q-large      | 2695.3 ± 36.4| 2739.9 ± 39.4|
| mri-q-small      | 581.7 ± 4.2| 591.4 ± 4.3|
| sgemm-medium     | 964.6 ± 1.9| 978.7 ± 2.2|
| sgemm-small      | 48.6 ± 1.7 | 49.7 ± 1.6 |
| sgemm-tiny       | 22.8 ± 4.3 | 25.4 ± 3.2 |
| stencil-default  | 31838.0 ± 317.0| 36995.0 ± 154.6|
| stencil-small    | 987.6 ± 10.3| 1041.8 ± 9.7|
| tpacf-large       | 758541.1 ± 1833.8| 745027.7 ± 2219.1|
| tpacf-medium      | 126135.6 ± 588.4| 124062.4 ± 6.2 |
| tpacf-small       | 1795.3 ± 2.8 | 1802.0 ± 31.3|

Table 1. Execution times (in microseconds) as reported by Futhark. Each dataset is identified by the benchmark name and the name of the input data used for that run.

We measure the wall time of executing the accelerated expressions. This includes overheads due to marshaling and copying between the CPU and the GPU but excludes I/O. We make two key observations when comparing the sequential and the accelerated results. First, unsurprisingly, the accelerated versions have better performance overall. We also observe that they scale better when the problem size increases; this is because the smaller benchmarks are not fully utilizing the GPU. Second, we see that the overhead of marshaling and copying data between the CPU and the GPU is negligible compared to the performance gains, in these cases. Expression acceleration works best when most work is focused on a single or few uses of `accelerate`, as this reduces the overheads of marshaling and copying.

### 6 Related Work

Our approach is similar to dynamic approaches in Python [3, 8, 16, 24, 26, 27], in particular Numba [16] and JAX [8], where functions are annotated to indicate acceleration. Expression acceleration is a static approach, which requires static analyses to extract code to different backends and ensure well-formedness.

Similar approaches include standalone languages [5, 12, 17], domain-specific languages embedded in general-purpose programming languages [4, 9, 19, 21–23], library APIs [6, 7, 13, 20], and annotation-based approaches in low-level languages [18, 25]. Our work is distinguished by the simplicity of `accelerate`, which allows us to seamlessly move parts of a program to the GPU without having to modify other parts. For instance, extraction ensures all dependencies are part of the GPU code, and the compiler automatically handles the data marshaling. Furthermore, as this approach is implemented in the compiler, many classes of errors can be detected at compile-time rather than at runtime.

### 7 Conclusions

This paper presents our compiler implementation of expression acceleration for a high-level typed programming language. The key ideas include expression extraction, classification, well-formedness checking, and compilation targeting...
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Michel Steuwer, Toomas Remmelg, and Christophe Dubach. 2017. Lift: a functional data-parallel IR for high-performance GPU code generation. In Proceedings of the 2017 International Symposium on Code Generation and Optimization, CGO 2017, Austin, TX, USA, February 4-8, 2017, Vijay Janapa Reddi, Aaron Smith, and Lingjia Tang (Eds.).
We use the Miking framework’s intermediate language called MExpr as a basis of our source language. MExpr is a simple yet complete typed functional core language that is easily extensible. We use a subset of the core language (Figure 4). The language consists of standard functional language expressions, such as lambdas, let-expressions, applications, and recursive bindings.

Records are represented as \( \{ l_i = e_i \}_{1 \leq i \leq n} \) where \( l_i \) is a label, and \( e_i \) is an expression bound to that label. Sequence literals are represented as \( [ e_i ]_{1 \leq i \leq n} \), where each expression \( e_i \) represents an element in the sequence. Patterns \( p \) consist of variables \( x \) and constants \( c \), as well as record patterns \( \{ l_i = e_i \}_{1 \leq i \leq n} \). In the language of Figure 4, sequences are immutable containers, while tensors are mutable containers with a dynamic number of dimensions. Sequences are constructed using a literal value, but there is no literal for tensors. Tensors are instead constructed from built-in functions. Types are defined closely following the same style as expressions and patterns. For example, a sequence containing elements of type \( T \) has type \( [T] \) and a tensor with elements of type \( T \) has type \( \text{Tensor}[T] \).

Literal values of ground types \( T_i \), such as integers and floating-point numbers, are included in the constants \( c \). Constants also include built-in curried functions such as

\[
c \in \{ \text{addi}, \text{muli}, \text{create}, \text{tensorCreate}, \ldots \}
\]

The MExpr language of Figure 4 does not include the parallel operations we need for acceleration. We present a subset of the parallel extensions we make in Figure 5. In (1), we define the accelerate keyword for controlling which expressions are accelerated. We also need expressions that evaluate in parallel when used in accelerated code. Therefore, we also define extensions for the Futhark and CUDA backends.

We define the key parallel expressions used in the Futhark backend in (2) of Figure 5. The map expression takes a function expression \( e_1 \) and applies it to the elements of a sequence \( e_2 \). Similarly, map2 applies a function \( e_1 \) to the elements of two sequences \( e_2 \) and \( e_3 \). For example, we compute the elementwise sum of two sequences in parallel as

\[
\text{accelerate (map2 addi s1 s2)}
\]

The reduce expression takes a function expression \( e_1 \), the initial value of the accumulator \( e_2 \), and a sequence \( e_3 \) to operate on. For example, we can use it to compute the product of a sequence \( s \) of integers in parallel as

\[
\text{accelerate (reduce multi 1 s)}
\]

The flatten expression translates a two-dimensional sequence to a one-dimensional sequence by concatenating the inner sequences. In addition to the parallel extensions for the Futhark backend, we also include a keyword for writing inline Futhark code. This is used in the evaluation (Section 5.1) to speed up development.
In (3) of Figure 5, we show an extension used for the CUDA backend. The \texttt{loop} expression corresponds to a for-loop, executed in parallel when used within accelerated code. It takes an integer argument \( e_0 \), denoting the iteration count and an iteration function \( e_2 \) corresponding to the loop body. The function \( e_2 \) is invoked once for each integer in the range \([0, e_0]\), in an undefined order. As tensors are mutable, we can encode operations over multiple tensors using a loop. Given tensors \( a \) and \( b \) of length \( n \), we use a loop to compute the elementwise product \( b = 2 \cdot a \) in parallel as

\[
\text{let } f = \text{lam } i.
\quad \text{tensorSet } b \ [i] \ \text{(muli} \ 2 \ \text{tensorGet} \ a \ i) \ \text{in}
\quad \text{accelerate } (\text{loop } n \ f)
\]

The constant \texttt{muli} performs integer multiplication, while \texttt{tensorGet} and \texttt{tensorSet} read from and write to tensors.

We construct the parallel source language PMExpr by combining the source language of Figure 4 with the extensions of Figure 5. This enables each accelerated expression to use either backend, meaning both backends may be used in the same program. For example, the program of Listing 1 uses the CUDA backend on lines 2-7, and the Futhark backend on line 12.

## B Free variables

We present the complete definition of free variables for all PMExpr expressions in Figure 6. In Figure 6a, we define the supporting function \( B(p) \), which computes the set of variables bound in a pattern \( p \). Based on this definition, we present the complete definition of \( \text{FV}(e) \) computing the set of free variables in an expression \( e \) in Figure 6b. This definition is used in the definitions of classification in Section D and in the well-formedness rules in Section E.

## C Extraction

In this section, we give a formal presentation of the extraction in terms of the \texttt{Extract} function of Algorithm 1. In the algorithm, we use typewriter font to refer to AST nodes. The function takes a set of identifiers \( I \) which correspond to the identifiers of accelerated bindings that are to be extracted. It also takes an expression \( e \) to extract from. We use the term \textit{binding} to refer to a let-expression or a binding in a recursive let-expression. The result consists of two parts. The first is an updated set \( I' \), containing the identifiers of all extracted bindings. The second part is an expression \( e' \), which corresponds to the extracted program. For instance, when Algorithm 1 is applied to Listing 3b (giving identifiers \( I_{\text{acc}} = \{ a \} \) and the whole program expression as input), the returned identifiers are \( I' = \{ a, f, g \} \), and the returned expression \( e' \) is equal to the highlighted parts of Listing 3b.

The \texttt{Extract} function in Algorithm 1 consists of three cases, by matching on the shape of the input expression \( e \). The first case handles recursive let-expressions (line 4) and the second case handles let-expressions (line 13). Note that both these cases start with a recursive call to the \texttt{Extract} function on the in-expression of the recursive let- and let-expressions, respectively. That is, the algorithm traverses the program in a bottom-up fashion. The final case, which is also the base case of the function, matches other kinds of expressions (line 18).

Consider the program of Listing 3b presented in Section 4.1. In this program, we only have one accelerated binding \( a \) to extract, on lines 5-7. Therefore the input set \( I_{\text{acc}} \) is \( \{ a \} \). For a program with multiple accelerated expressions, the input set \( I_{\text{acc}} \) consists of multiple identifiers. The complete program of Listing 3b corresponds to the argument \( e \) to \texttt{Extract}.

As we noted, the \texttt{Extract} function traverses the program in a bottom-up fashion, as the let- and recursive let-expression cases start with a self-recursive call. Thus, we reach the base case with the \texttt{printSum sum} expression on line 9. For this expression, we return \( I \) as is, and an empty set \( \{ \} \) as the extracted expression. The next expression to consider is that of line 8 in Listing 3b. This is a let-expression, so we are in the second case of \texttt{Extract} (line 13). In the recursive call on line 14, we get the returned values from the base case. On line 15, we check if the identifier \( \text{sum} \) is in \( I' \). As \( I \) was returned in the base case, the value of \( I' \) is \( \{ a \} \). Therefore the condition evaluates to false. Thus, we return the \( I' \) without modifying it, and \( \{ \} \) from the base case.

Next, we consider the let-expression on lines 5-7 in Listing 3b. We enter the let-expression case of the \texttt{Extract} function once more. The recursive call results in \( I' \) equal to \( I \) and \( e' \) equal to \( \{ \} \), as was the result of the previous case.

### Algorithm 1 Extraction of accelerated code given an input program and a set of identifiers corresponding to \textit{accelerate} expressions.

1. \( I \): identifiers to be included in the extracted expression
2. \( e \): the lambda lifted source expression to extract from
3. \textbf{function} \texttt{Extract} \((I, e)\)
4. \texttt{match} \( e \) \texttt{with} \texttt{recursive} \((\text{let } x_i = e_i)_{i=1}^n \) \texttt{in} \( e_0 \) \texttt{then}
5. \( (I', e'_0) \leftarrow \texttt{Extract} (I, e_0) \)
6. \( B \leftarrow \{ x_i \mid i \in 1 \ldots n \} \)
7. \( I_B \leftarrow \{ x_i \mid x_i \in B \cap I' \} \)
8. \( I_B \leftarrow \texttt{FIX}_B (I_B \cup \{ (\bigcup_{x_i \in I_B} \text{FV}(e_i)) \cap B \}) \)
9. \( I'' \leftarrow I' \cup I_B \cup \{ (\bigcup_{x_i \in I_B} \text{FV}(e_i)) \}
10. \( N \leftarrow \{ i \mid x_i \in I_B \} \)
11. \( e' \leftarrow \texttt{recursive} ((\text{let } x_j = e_j)_{j \in N} \text{ in } e'_0) \)
12. \texttt{return} \((I'', e')\)
13. \texttt{else match} \( e \) \texttt{with} \texttt{let} \( x = e_1 \) \texttt{in} \( e_2 \) \texttt{then}
14. \( (I', e'_2) \leftarrow \texttt{Extract} (I, e_2) \)
15. \texttt{if} \( x \in I' \) \texttt{then}
16. \texttt{return} \((I' \cup \text{FV}(e_1), \text{let } x = e_1 \text{ in } e'_2)\)
17. \texttt{else return} \((I', e'_2)\)
18. \texttt{else return} \((I, \{ \})\)
The accelerated binding on lines 5–7 uses the expression. These are extensions for the Futhark backend, so we use that backend to compile the binding.

The input to the classification is the accelerated code $e_{\text{acc}}$ produced by the extraction. The output is two programs, one for the Futhark backend and another for the CUDA backend.

$$\text{FV}(x) = \{x\}
\text{FV}(c) = \emptyset
\text{FV}(\lambda x : T, e) = \text{FV}(e) \setminus \{x\}
\text{FV}(e_1, e_2) = \text{FV}(e_1) \cup \text{FV}(e_2)
\text{FV}(\text{let } x : T = e_1 \text{ in } e_2) = \text{FV}(e_1) \cup (\text{FV}(e_2) \setminus \{x\})$$

$$\text{FV}(\text{recursive}(\text{let } x_i : T_i = e_i \mid i \in 1 \ldots n \text{ in } e)) = \left( \bigcup_{i \in 1 \ldots n} \text{FV}(e_i) \right) \setminus \{x_i \mid i \in 1 \ldots n\}
\text{FV}(\text{match } e_1 \text{ with } p \text{ then } e_2 \text{ else } e_3) = \text{FV}(e_1) \cup (\text{FV}(e_2) \setminus \text{B}(p)) \cup \text{FV}(e_3)
\text{FV}(\text{never}) = \emptyset
\text{FV}(\{l_i = e_i \mid i \in 1 \ldots n\}) = \bigcup_{i \in 1 \ldots n} \text{FV}(e_i)
\text{FV}(\{e_i \mid i \in 1 \ldots n\}) = \bigcup_{i \in 1 \ldots n} \text{FV}(e_i)
\text{FV}(\text{accelerate } e) = \text{FV}(e)
\text{FV}(\text{map } e_1, e_2) = \text{FV}(e_1) \cup \text{FV}(e_2)
\text{FV}(\text{map2 } e_1, e_2, e_3) = \text{FV}(e_1) \cup \text{FV}(e_2) \cup \text{FV}(e_3)
\text{FV}(\text{reduce } e_1, e_2, e_3) = \text{FV}(e_1) \cup \text{FV}(e_2) \cup \text{FV}(e_3)
\text{FV}(\text{flatten } e_1) = \text{FV}(e_1)
\text{FV}(\text{loop } e_1, e_2) = \text{FV}(e_1) \cup \text{FV}(e_2)$$

(b) Definition of the $\text{FV}(e)$ function for computing the free variables of an expression $e$.

This time, the condition on line 15 of Extract evaluates to true, as $I' = I = \{a\}$. The result on line 16 is computed as follows. We update the set of identifiers $I$ by including the free variables in the body of the let-expression, $e_1$. We find that $\text{FV}(e_1) = \{f\}$ in this case, and thus we return $I = \{a, f\}$.

The end result of applying Extract on Listing 3b is the set of identifiers $\{a, f, g\}$. The extracted expression includes the highlighted expressions of Listing 3b, with a trailing $\emptyset$ as the final in-expression at the end of line 7.

### D Classification

The extraction produces the accelerated code, containing the accelerated bindings and the bindings they depend on. By classifying the accelerated code, the compiler decides which backend to use for each accelerated binding.

The parallel expressions we introduce may be used in either one of the backends. Therefore, we classify expressions based on which parallel expressions are used. This enables efficient production of backend-specific programs from accelerated code. For example, the program of Listing 3b. The accelerated binding on lines 5–7 uses the map and reduce expressions. These are extensions for the Futhark backend, so we use that backend to compile the binding.

The input to the classification is the accelerated code $e_{\text{acc}}$ produced by the extraction. The output is two programs, one for the Futhark backend and another for the CUDA backend.

Figure 7 contains the formal definitions we use for the classification. Also, we define a function $S$ for computing the set of all subexpressions of an expression $e$, including $e$. For brevity, we define $S$ in terms of an auxiliary function $S'$ which excludes $e$ (i.e., $S(e) = S'(e) \cup \{e\}$) in Figure 8. Note that this approach to classification is for presentation purposes only — in the implementation, we perform classification by traversing the AST more efficiently.

In Figure 7a, we define the $P_F$ and $P_C$ functions. These take an expression $e$ as input and compute the number of occurrences of parallel expressions for Futhark or CUDA, respectively. We use $e' \equiv p$ to denote pattern matching. This evaluates to true if $e'$ matches the pattern expression $p$. For instance, if $e' = \text{map } f \ s$, then $e' \equiv \text{map } e_1 \ e_2$ is true.

We define similar functions $P_{F'}$ and $P_{C'}$ on bindings in Figure 7b. A binding has an identifier $x$ and it is bound to an expression $e_x$ (e.g., $\text{let } x = e_x \text{ in } \ldots$). For a given identifier of a binding $x$, we assume that we know its bound expression $e_x$. We compute the number of occurrences of parallel expressions in a binding in two steps. First, we consider the occurrences in its body $e_x$, using $P_F$ or $P_C$. Second, we include occurrences in expressions of bindings that $e_x$ depends on, i.e., given by the free variables in $e_x$. We keep track of the visited bindings in the set $V$ of identifiers to prevent bindings from being counted more than once.

(a) Definition of the $B(p)$ function, computing the set of variables bound by patterns.

Figure 6. Definition of functions $B$ and $FV$. 
This function takes a binding as input and categorizes it as an element of the set \( S \) function \( C \) an element of the set \( S \).

We define the binding classification function \( C \) in Figure 7c. This function takes a binding as input and categorizes it as an element of the set \{Any, Futhark, CUDA, Invalid\}. The function \( C \) is used to construct two subsets of identifiers, one for each backend, as

\[
I_F = \{ a \mid a \in I_{\text{acc}} \land C(a) = \text{Futhark} \} \\
I_C = \{ a \mid a \in I_{\text{acc}} \land C(a) = \text{CUDA} \}
\]

Recall that \( I_{\text{acc}} \) is the set of identifiers of all accelerated bindings in the program. The compilation fails if an accelerated binding \( a \) is classified as Any or Invalid. If the first case, \( a \) does not use any parallel expressions, which goes against the purpose of using acceleration. In the second case, \( a \) uses parallel expressions of both backends, which is unsupported.

We have now classified the identifiers used for the Futhark and CUDA backend, \( I_F \) and \( I_C \), respectively. However, at the end of this phase, we need the backend-specific programs \( e_F \) and \( e_C \). To produce these two backend-specific programs, we reuse the Extract function of Algorithm 1 as follows

\[
(I_F, e_F) \leftarrow \text{Extract}(I_F, e_{\text{acc}}) \\
(I_C, e_C) \leftarrow \text{Extract}(I_C, e_{\text{acc}})
\]

We use \( e_{\text{acc}} \) to denote the accelerated code produced by the accelerate extraction in Section 4.1.

### E Well-formedness

The compiler has limitations on what expressions can be accelerated. These limitations also depend on which accelerate backend we use. First, we define a set of assumptions on accelerated code involving properties we need to check dynamically (Section E.1). Second, we present the static well-formedness rules for the CUDA and Futhark backends (Section E.2).

#### E.1 Dynamic Assumptions

We define a set of assumptions that must hold at runtime in accelerated code.

We use \(|x|\) to denote the length of a sequence \( x \). In Futhark arrays must be regular, meaning that for an array of arrays \([x_1, \ldots, x_n]\) we must have \(|x_1| = \ldots = |x_n|\). Sequences in PM-Expr, which we translate to arrays, do not have this restriction. Thus, we assume all sequences are regular in accelerated code targeting Futhark. Further, we assume constraints on sizes are annotated as required by Futhark.

The rank of a tensor is unknown at compile-time. In the CUDA backend, we represent their dimensions using a fixed-size array. We make the maximum rank of tensors configurable as in, for example, cuDNN. Our compiler uses 3 as a default value to cover common use cases while also reducing wasted memory. We assume all tensors have dimensions no more than this maximum value. Our assumptions on regularity and tensor rank are checked at runtime for programs compiled in debug mode.
We also make assumptions about the parallel expressions. In particular, they must yield the same result regardless of execution order. While there are approaches to verify assumptions of deterministic parallelism statically \cite{1, 10, 11, 15}, they either require complex type systems or do not support our input language. Moreover, there are no efficient methods for verifying these assumptions dynamically. Therefore, the user is responsible for verifying them. For example, we assume for a reduced expression \texttt{reduce \textit{f} \ 0 \ 0} that \textit{f} is associative and \texttt{n} is the neutral element of \textit{f}.

## E.2 Static Rules

We define the static well-formedness rules for the CUDA and Futhark backends. Due to the similarities of the backends, we define a set of rules shared by both backends in Figure 9. We define rules specific to the CUDA backend in Figure 10 and the rules for the Futhark backend in Figure 11.

We define rules over expressions (\(E\)), types (\(T\)), and patterns (\(P\)), which are pairwise combined with the two backends CUDA (C) and Futhark (F). That is, there are six different relations marked with EC, TC, PC, EF, TF, and PF. For instance, the relation for expressions in the CUDA backend is marked with EC. In this case, we say that an expression \(e\) with well-formed type \(T\) is well-formed in an environment \(\Gamma\)
iff $\Gamma \vdash_{EC} e : T$, where $\Gamma$ denotes an environment of pairs of identifiers and types, $(x, T)$. We do not need the environment $\Gamma$ for the well-formedness of types, as the language has no type variables. A type is well-formed in backend $X$ iff $\vdash_{TX} T$.

In several cases, the rules for both CUDA and Futhark are the same. Instead of repeating identical rules for both backends, we mark it with an $X$ to indicate that it implicitly represents two rules. For instance, a rule $\Gamma \vdash_{EX} e : T$ represents two rules, both for $EC$ and $EF$.

We discuss select rules to give an intuition of how they work and what we use them for. Consider the rule for match-expressions (WF-EX-Match) at the top center of Figure 9. We may bind variables in the pattern $p$ to the corresponding subexpressions of $e_1$. Thus, we need an updated environment $\Gamma'$ in the then-branch ($e_2$) containing these bound variables. Therefore, we define the well-formedness of a pattern $p$ matching on expression $e$ as well-formed in backend $X$ iff $\Gamma \vdash_{PX} (e, p) : \Gamma'$. Note also how the result of a match expression cannot be of a function type in either backend.

Consider the rules for let-expressions, WF-EC-Let and WF-EF-Let, at the top left of Figure 10 and Figure 11, respectively. The rule for CUDA includes a premise using an auxiliary relation $\triangleright e : T$, defined in WF-BC-1 and WF-BC-2. This premise prevents user-defined functions from using or producing higher-order functions, as this is not supported.

The rule for loop expressions (WF-EC-Loop) uses a different auxiliary relation of the form $\triangleright e : T$, used for higher-order functions (see the defining rules WF-HC-Var and WF-HC-App at the bottom of Figure 10). The rule for applications, WF-HC-App, is a relaxed variant of WF-EC-App, which allows the result to be a higher-order function. These rules are needed to enable the iteration function of a loop to use free variables (they are added as parameters to the iteration function by the lambda lifting). We assume ANF has been applied, but where applications are not lifted out of the higher-order function argument in parallel expressions. Therefore, the iteration function must be a variable or an application.

Assume that $e_C$ and $e_F$ represent the expressions resulting from the classification for the CUDA and Futhark backends. We say that the input program is well-formed if and only if, for well-formed types $T_1$ and $T_2$, it holds that

$$\Gamma \vdash_{EC} e_C : T_1 \land \Gamma \vdash_{EF} e_F : T_2$$

Assume that we have an input program for which the assumptions of Section E.1 are valid. If the program is well-typed and well-formed according to the well-formedness rules, our compiler ensures that all accelerated expressions are observationally equivalent to $\circ$. Our implementation of the well-formedness checks is tested on a suite of example and benchmark programs. It is closely based on the definition of the well-formedness rules.