Coulomb Displacement Energy and the Low-Energy Astrophysical
$S_{17}$ Factor for the $^{7}\text{Be}(p,\gamma)^{8}\text{B}$ Reaction

B. A. Brown

*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy*

*Michigan State University, East Lansing, MI 48824-1321*

A. Csótó

*National Superconducting Cyclotron Laboratory*

*Michigan State University, East Lansing, MI 48824-1321*

R. Sherr

*Department of Physics, Princeton University, Princeton, New Jersey 08544*

Abstract

The relationship between the Coulomb displacement energy for the $A=8$, $J=2^+$, $T=1$ state and the low-energy astrophysical $S_{17}$ factor for the $^{7}\text{Be}(p,\gamma)^{8}\text{B}$ reaction is discussed. The displacement energy is interpreted in a particle-hole model. The dependence of the particle displacement energy on the potential well geometry is investigated and is used to relate the particle displacement energy to the rms radius and the asymptotic normalization of the valence proton wave function in $^{8}\text{B}$. The asymptotic normalization is used to calculate the astrophysical $S_{17}$ factor for the $^{7}\text{Be}(p,\gamma)$ reaction. The relationship to the $^{7}\text{Li}(n,\gamma)$ reaction, the $^{8}\text{B}$ quadrupole moment, radial density, and break-up momentum distribution are also discussed.
I. INTRODUCTION

Measurements of high energy solar neutrinos in the Homestake\textsuperscript{1} and Kamiokande II and III\textsuperscript{2} experiments have found a significantly smaller number of solar neutrinos compared to that expected from the standard solar model.\textsuperscript{3} The $^8\text{B}$ $\beta^+$ decay is the main source of these high energy solar neutrinos. $^8\text{B}$ is formed by the $^7\text{Be}(p,\gamma)^8\text{B}$ reaction at a center of mass energy of about 20 keV. Its cross section is conventionally expressed in terms of the $S_{17}$ factor.\textsuperscript{3} $S_{17}$ is known only from an extrapolation of data at energies above 100 keV. Also the highest precision data disagree, with those of Parker\textsuperscript{4} and Kavanagh et al.\textsuperscript{5} being about 30 percent higher than those of Filippone et al.\textsuperscript{6} and Vaughn et al..\textsuperscript{7} This has lead to the investigation of other ways to determine the $S_{17}$ factor such as the Coulomb dissociation,\textsuperscript{8} as well as other properties of $^8\text{B}$ which are indirectly related to the $S_{17}$ factor such as its total reaction cross section,\textsuperscript{9} quadrupole moment,\textsuperscript{10} and break-up momentum distribution.\textsuperscript{11}

Much of the theoretical work has been based upon using a potential model to generate the single-particle wave function for the most loosely bound proton, and then combining this with a shell-model calculation of the spectroscopic factor to obtain the asymptotic normalization of the wave function. This current work was initiated by the observation that the potential model parameters (e.g. the radius $R$ and diffuseness $a$ of the Woods-Saxon potential) used in previous calculations are taken from some “standard” sets based upon nucleon-nucleus scattering optical potential analyses, and that these “standard” parameters were not obviously appropriate to the particular case of $^7\text{Be}$ plus protons. We thus investigated the extent to which the potential parameters could be determined from the displacement energy of the $A=8$, $J^\pi=2^+$, $T=1$ state as given by the binding energy difference between the $^8\text{B}$ and $^8\text{Li}$ ground states. In addition, we also examined several quantities of related interest including the $^7\text{Li}(n,\gamma)$ cross section, the $Q$ moment, the density distributions, and the momentum distributions.
II. COULOMB DISPLACEMENT ENERGY

The relationship between the root-mean-square (rms) radius of the valence proton and the displacement energy is well known. Qualitatively, the larger the rms radius the smaller the displacement energy since the valence proton is further from the core protons. This leads to the Thomas-Ehrman effect in which loosely bound valence nucleons have a relatively smaller displacement energy compared to more tightly bound nucleons. In addition, for a fixed binding energy, nucleons in a low $\ell$ state have a smaller displacement energy compared to those in a high $\ell$ state because the centrifugal barrier for high $\ell$ results in a smaller rms radius. For heavier nuclei ($A > 16$) with a relatively simple shell-model configuration, many quantitative calculations of the displacement energy have been carried out. At the beginning of the 0d1s shell ($A=17$, $T=1/2$) and the 0f1p shell ($A=41$, $T=1/2$), the observed displacement energy is found to be about 10 percent larger than that obtained from calculations which involve only the lowest shell-model configuration together with the Coulomb interaction. These cases include those for high $\ell$ values whose Coulomb shift is not very sensitive to the potential-well geometry. This “Nolen-Schiffer” (NS) anomaly appears not to have a simple explanation but is due to a combination of core-polarization, high-order configuration mixing, and charge-asymmetric strong interactions effects. The systematics of the displacement energies of light nuclei can also be semi-quantitatively accounted for with a simple potential-well geometry.

In this paper we apply what has been learned about the displacement energy systematics to constrain the shape of the potential for the $^7$Be plus valence proton system. The $A=8$, $T=1$ displacement energies are more complicated than the $T=1/2$ systems usually considered, since at least two nucleons are involved. However, the $0p$ shell-model wave functions predict a relatively simple structure for $A=8$, $T=1$. The spectroscopic factors ($C^2S$) for $(A=8, 2^+, T=1) \rightarrow (A=7, 3/2^-, T=1/2)$ and $(A=9, 3/2^-, T=1/2) \rightarrow (A=8, 2^+, T=1)$ are experimentally and theoretically both near unity. Thus, $^8$B can be considered as a proton-particle neutron-hole configuration relative to $^7$Be. The mirror nucleus $^8$Li is a
proton-hole neutron-particle configuration. $^8$Be is itself “deformed” and thus exits in both the ground state and $2^+$ first excited state relative to the particle-hole configuration.

The displacement energy of the particle-hole state is simply the sum of the particle and hole displacement energies (the Coulomb particle-hole interaction is zero):

$$\Delta_8 = \Delta_h + \Delta_p,$$

where $\Delta_8$ is the $A=8, T=1$ displacement energy (the binding energy difference between $^8$Li and $^8$B). In the simplest approximation $\Delta_h$ is the $A=7, 3/2^-, T=1/2$ displacement energy and $\Delta_p$ is the $A=9, 3/2^-, T=1/2$ displacement energy. The experimental values are $\Delta_7=1.645$ MeV and $\Delta_9=1.851$ MeV which gives $\Delta_7 + \Delta_9=3.496$ MeV compared to the experimental A=8 value of $\Delta_8=3.540$ MeV. So the simplest model works rather well. There are however several reasons why this is not exact. One is that the actual shell-model configuration is a little more complicated than just particle-hole. Also the Thomas-Ehrman effects may be significant since the proton is unbound by by 186 keV in $^9$B and bound by 138 keV in $^8$B.

Since we are particularly interested in the properties of the valence proton in $^8$B we will proceed as follows. First Eq. (1) will be modified to read

$$\Delta_8 = \Delta_h + \Delta_p' + \Delta_{sm},$$

where $\Delta_{sm}$ will take into account the 0p shell-model structure beyond particle-hole. Since the hole state is relatively tightly bound, the Thomas-Ehrman shift will be small and we will take $\Delta_h = \Delta_7 = 1.645$ MeV. $\Delta_p'$ indicates the displacement energy for the particle in $^8$B which may differ from $\Delta_p = \Delta_9$ because of the Thomas-Ehrman shift.

The shell-model correction $\Delta_{sm}$ was calculated by using the Coulomb plus charge asymmetric interaction of Ormand and Brown$^{17}$ within a full 0p shell-model basis. The matrix elements were calculated with harmonic-oscillator radial wave functions. The results for the displacement energies are: $\Delta_h = \Delta_7 = 1.719$ MeV, $\Delta_8 = 3.656$ MeV and $\Delta_p = \Delta_p' = \Delta_9 = 1.873$ MeV. This gives $\Delta_{sm} = 64$ keV. Since harmonic-oscillator radial wave
functions are used the shell-model calculation should not necessarily be in good agreement with experiment, however, it should provide an estimate for the $\Delta_{sm}$ correction. Thus the displacement energy of the $A=8$ particle state is

$$\Delta'_p = \Delta_8 - \Delta_h - \Delta_{sm} = 1.831 \text{ MeV}.\quad (3)$$

We are interested on how $\Delta'_p$ depends upon the potential geometry, and on how the potential geometry effects the rms radius and the astrophysical $S_{17}$ factor. To investigate this we will calculate the direct part of the Coulomb shift in a Woods-Saxon geometry. This means that we calculate the single-particle binding energy with and without the one-body Coulomb potential and take the difference to obtain the Coulomb displacement energy. The Woods-Saxons potential has the usual form of central plus spin-orbit plus Coulomb terms. The central potential has the form

$$V(r) = V_o\{1 + \exp[r - R]/a\}^{-1},\quad (4)$$

where $R$ is the radius, $a$ is the diffuseness. We use the reduced mass in the kinetic energy operator. The spin-orbit potential is the usual derivative form with the same geometry as the central (we will show below that the spin-orbit potential is not important for our analysis). The Coulomb potential is obtained from the density distribution of four tightly-bound protons (two in 0s and two in 0p) obtained with another Woods-Saxon potential which is constrained to reproduce the experimental rms charge radius$^{18}$ of 2.52 fm for $^9$Be.

We thus investigate the dependence of the Coulomb energy, the rms radius and the astrophysical $S_{17}$ factor on the radius $R$ and the diffuseness $a$ of the central potential. In all cases we vary $R$ and $a$ and fix $V_o$ in order to reproduce the proton separation energy of 138 keV. We calculate the direct Coulomb displacement energy $\Delta_o$ and relate it to $\Delta'_p$ by making several additional corrections, all of which are relatively small:

$$\Delta'_p = \Delta_o + \Delta_{ex} + \Delta_{so} + \Delta_{vp} + \Delta_{np} + \Delta_{NS}.\quad (5)$$
\( \Delta_{ex} \) is the exchange correction. A value of \( \Delta_{ex} = -125 \text{ keV} \) was obtained from the harmonic-oscillator shell-model calculation for \( \Delta_9 \) by comparing the results with and without the Coulomb exchange terms. The next three terms are the relativistic spin-orbit correction (Eq. 21 in Ref 12), the vacuum polarization correction (Eq. 4.23 in Ref 13) and the proton-neutron mass difference correction (Eq. 4.29 in Ref 13), respectively. Our estimates for these are \(-30 \text{ keV}, 12 \text{ keV} \) and \(14 \text{ keV} \), respectively. The Nolen-Schiffer correction, \( \Delta_{NS} \) is perhaps the most uncertain. For \( A=17 \) and \( A=41 \), \( \Delta_{NS}/\Delta_9 \) is about 0.10 and we see no reason why it should differ much from this for the beginning of the 0p shell. Since \( \Delta_9 \) is about 1.8 MeV we will take \( \Delta_{NS}=180 \text{ keV} \). Thus we arrive at the empirical value of

\[
\Delta_9(\text{empirical}) = 1.780 \text{ MeV},
\]

with which we will compare our calculations for the direct Coulomb shift of the valence particle.

We show in Table I the results for \( \Delta_9 \), the rms proton radius and the density of the valence proton at \( r = 10 \text{ fm} \), \( \rho(10 \text{ fm}) \), as a function of \( R \) and \( a \). Beyond the influence of the strong interaction (about 6 fm) the shape of the radial wave function is entirely and uniquely determined by the Coulomb plus centrifugal potentials. The only quantity which depends on the potential is the asymptotic normalization as represented, for example, by the value of \( \rho \) at \( r = 10 \text{ fm} \). Our \( \rho(r) \) is defined by the normalization:

\[
4\pi \int \rho(r)r^2 \, dr = 1.
\]

In order to show the correlation between \( \Delta_9 \), rms and \( \rho(10 \text{ fm}) \) we plot values obtained for these quantities in pairs in Figs. 1 (\( \Delta_9 \) vs rms), 2 (\( \Delta_9 \) vs \( \rho(10 \text{ fm}) \)) and 3 (rms vs \( \rho(10 \text{ fm}) \)). We find that there is strong correlation between them, which implies that from a knowledge of any one of them (in particular \( \Delta_9 \)) we can infer a range of values for the other two.

The results in Table I were obtained for a 0p\( _{3/2} \) valence particle. For \( R=2.4 \text{ fm} \) we show the results with and without the spin-orbit potential, and from this comparison it is
clear that spin-orbit potential is not important (as long as $V_o$ is fixed from the 138 keV separation energy). Thus to a good approximation, our results apply to both $0p_{3/2}$ and $0p_{1/2}$.

The results shown in Figs. 1–3 show that there is a rather narrow band of points which relate the quantities of interest in the framework of the Woods-Saxon potential model. From Fig. 1 one finds that the value of $\Delta_o=1.780$ MeV corresponds to a narrow range of rms valence radii from 4.5 to 4.7 fm, and from Fig. 2 one finds that the same $\Delta_o$ corresponds to $\rho(10$ fm) values in the range 7.2 to 8.0 fm$^{-3}$. The correlation between the rms radius and $\rho$ shown in Fig. 3 is even more parameter independent.

The potential parameter values required fall into three sets with $(R,a)$ values of about (2.8, 0.51 fm) (set A), (2.4, 0.69 fm) (set B), and (2.0, 0.81 fm) (set C). Or analysis is consistent with any of these sets or any interpolation between them. These are compared in Table II to the parameters used in a number of other potential model calculations for $S_{17}$. The results for $\Delta_o$, rms and $\rho(10$ fm) for the potentials in Table II are shown by the labeled crosses in Figs. 1-3. The results with Tombrello parameters (T) are close to ours. They were obtained from the optical model analysis of 180 MeV proton scattering on Li and Be by Johansson et al..\textsuperscript{19} The Johansson parameters were subsequently used by Aurdal\textsuperscript{20} and Robertson.\textsuperscript{21} Parameter set I from Barker is also close to ours. But his modifications of the parameters needed to reproduce the $^7\text{Li}(n,\gamma)$ cross section (see the discussion below) are clearly outside of our range.

### III. RESULTS FOR THE ASTROPHYSICAL $S_{17}$ FACTOR

The astrophysical $S_{17}$ factor is related to $\rho(10$ fm) in the following way. At very low energies ($< 50$ keV) the $^7\text{Be}(p,\gamma)^8\text{B}$ capture cross section is dominated by the E1 transition between the $^7\text{Be}+p$ scattering states and the $^8\text{B}$ ground state,\textsuperscript{22} and it is almost exclusively determined by contributions coming from the external part of the scattering and bound state wave functions. The asymptotic behavior of the scattering states is uniquely defined (the
phase shifts being the hard sphere phase shifts – practically zero), and the asymptotic part of the wave function, which describes the $^7\text{Be}+p$ relative motion in the bound state $^8\text{B}$, is proportional to the fixed Whittaker function,

$$
\psi_{^8\text{B}}^{I}(r) = \bar{c}_I \frac{W_{\eta,\ell}^+(kr)}{r}, \quad r \to \infty,
$$

(8)

where $I = 1, 2$ is the channel spin, $r$ is the radial distance between $^7\text{Be}$ and $p$, and $k$ is the wave number corresponding to the $^8\text{B}$ binding energy relative to the $^7\text{Be}+p$ threshold. $\bar{c}_I$ are the constants which are required to normalize the Whittaker function to the asymptotic $^8\text{B}$ wave function. Thus, at low energies the astrophysical $S_{17}$ factor of the capture reaction,

$$
S_{17}(E) = \sigma(E)E \exp \left[2\pi \eta(E) \right],
$$

(9)

depends only on $\bar{c}$.\textsuperscript{23,24} (here $\eta = e^2 Z_1 Z_7 / \hbar \nu$ with $Z_1 = 1$, and $Z_7 = 4$ is the Sommerfeld parameter). From the hard sphere scattering states one obtains:

$$
S_{17}(20\text{keV}) = 36.5(\bar{c}_1^2 + \bar{c}_2^2).
$$

(10)

(The uncertainty coming mainly from the fact that the nucleon mass is not well defined in nonrelativistic quantum mechanics, is 1–2 percent. The value of the $S$ factor at zero energy is roughly 0.4 eV-barn higher than at 20 keV.) Using this formula we can express $S_{17}$ (in units of eV-barns) in terms of the valence proton density at any given asymptotic radius, e.g. at 10 fm,

$$
S_{17}(20\text{keV}) = 2.99 \times 10^6 \rho_{3/2}(10 \text{fm}) \ S_{3/2} \ [\left(\alpha_{1,3/2} + \gamma \alpha_{1,1/2}\right)^2 + \left(\alpha_{2,3/2} + \gamma \alpha_{2,1/2}\right)^2],
$$

(11)

were the $\alpha$ coefficients are determined from the transformation between the $^7\text{Be}(J_i) + \ell_j = ^8\text{B}(J_f)$ coupling and the channel-spin coupling:

$$
[J_i \otimes \ell_j]^{J_f} = \sum_I \alpha_{I,J} [(J_i \otimes 1/2)^I \otimes \ell]^{J_f},
$$

(12)
and $\gamma$ is given by the ratio:

$$
\gamma = \frac{\theta_{1/2} \psi_{1/2}(10 \text{ fm})}{\theta_{3/2} \psi_{3/2}(10 \text{ fm})},
$$

(13)

In Eq. (13) $\theta_j$ is the $n = 0$, $\ell = 1$ spectroscopic amplitude and in Eqs. (11) $S_j$ is the spectroscopic factor. The amplitudes $\theta_j$ are given by the reduced matrix elements of the creation operator, $a^+$:

$$
\theta_j(J_i, J_f) = \frac{\langle ^8\text{B}, J_i | a^+_{\text{proton}} || ^7\text{Be}, J_f \rangle}{\sqrt{2J_i + 1}},
$$

(14)

When $\theta_j$ is given without its $J_i, J_f$ arguments as in Eq. (13), it corresponds to the $^8\text{B}$ ground state ($J_i = 2$) to $^7\text{Be}$ ground state ($J_f = 3/2$) value. The spectroscopic factors take into account the additional center of mass correction factor:

$$
S_j = \frac{A_i}{A_i - 1} \theta_j^2,
$$

(15)

In Eq. (13), $\psi(10 \text{ fm})$ is the radial amplitude at $r = 10 \text{ fm}$:

$$
\rho_j(10 \text{ fm}) = \psi_j^2(10 \text{ fm}).
$$

(16)

In our case where $\ell = 1$, $J_i = 3/2$, $J_f = 2$ and $I = 1, 2$, the transformation coefficients are $\alpha_{1,3/2} = \alpha_{2,3/2} = \alpha_{2,1/2} = 1/\sqrt{2}$ and $\alpha_{1,1/2} = -1/\sqrt{2}$. In addition, to a good approximation, $\psi_{1/2}(10 \text{ fm}) = \psi_{3/2}(10 \text{ fm}) = \psi(10 \text{ fm})$. Hence Eq. (11) simplifies to:

$$
S_{17}(20 \text{ kev}) = 2.99 \times 10^6 \rho(10 \text{ fm}) S,
$$

(17)

where $S = S_{3/2} + S_{1/2}$.

In Table III we give the values of $\theta_j$ obtained from 0p shell model calculations with a variety of interactions which are appropriate for the lower part of the 0p shell. These amplitudes are quite stable with respect to a reasonable range of interactions.
In Table IV we compare the theoretical spectroscopic factors with those obtained from reaction data\textsuperscript{15} for states of \(^{8}\)Li as well as those extracted from the observed widths of unbound states in \(^{8}\)Li and \(^{8}\)B.\textsuperscript{27} The spectroscopic factors for the unbound states are obtained from \(\Gamma_{\text{exp}} = S \Gamma_{\text{sp}}\) where \(\Gamma_{\text{sp}}\) is the single-particle width for a resonance at the experimental separation energy in the potential geometry B \((R = 2.4\ \text{fm} \text{ and } a = 0.69\ \text{fm})\). The decay data are observed to be in excellent agreement with theory. The reaction spectroscopic factor for the \(2^+\) state is low with respect to theory, however, it depends upon the potential parameters used for the DWBA calculations (we have not attempted to repeat the DWBA calculations). The reaction spectroscopic factor for the \(1^+\) state is also low with respect to the decay spectroscopic factor of the mirror state. The data in Table IV support the 0p shell-model calculation for \(S\) and the present potential parameters.

Combining with our results of \(S = 1.15 \pm 0.05\) and \(\rho(10\ \text{fm}) = (7.7 \pm 0.4) \times 10^{-6}\ \text{fm}^{-3}\) we obtain \(S_{17}(20\ \text{keV}) = 26.5 \pm 2.0\ \text{eV-barns}\).

**IV. RELATIONSHIP TO THE \(^{7}\text{Li}(n,\gamma)^{8}\text{Li}\) CROSS SECTION**

Barker\textsuperscript{28} has pointed out that standard potential models tend to overestimate the experimentally well-determined low-energy \(^{7}\text{Li}(n,\gamma)^{8}\text{Li}\) cross section. He argued, that one should modify either the potential parameters or the spectroscopic factor to get agreement with experiment. To study this issue, we performed calculations for the \(^{7}\text{Li}(n,\gamma)^{8}\text{Li}\) reaction.

Because there is no Coulomb barrier in this reaction, the inner parts of the wave functions have the same importance as the asymptotic parts. Thus the cross section does not depend solely on the asymptotic normalization of the bound state wave function. For the \(^{8}\text{Li}\) bound state we used the same potential parameters as for \(^{8}\text{B}\), except a change in the potential depth to get the exact neutron separation energy of 2.033 MeV. For the \(^{7}\text{Li}+n\) scattering states, we modify the potentials to reproduce the experimental scattering lengths of the \(I = 1\), and \(I = 2\) channel spin states, respectively.\textsuperscript{28}

The thermal \(^{7}\text{Li}(n,\gamma)^{8}\text{Li}\) cross section in our model, using potentials A, B, and C, are
78, 80 and 83 mb, respectively. The experimental thermal cross section is 45.4±3.0 mb.\(^2\) Thus we also obtain an overestimation of the \(^7\)Li(\(n,\gamma\))^8Li cross section as did Barker. He concluded that either the potential parameters or the spectroscopic factor has to be changed in order to agree with the experiment, and that these changes would bring the \(^7\)Be(\(p,\gamma\))^8B \(S_{17}(20\text{ keV})\) factor down to 16-17 eV-barns. The modifications to the potential are very large (\(R\) is changed from 2.39 to 0.53 fm or \(a\) is change from 0.65 fm to 0.27 fm) and we can see from Figs. 1−2 that these large changes are inconsistent with the Coulomb displacement energy. Thus we can exclude the possibility of radically changing the potential parameters. In the spirit of Ref 28, the only remaining possibility would be the reduction of the spectroscopic factor to about 0.71. But given the general agreement we obtain for the decay widths in Table IV, such a large change in the spectroscopic factor seems unreasonable. In fact such a drastic change would question the adequacy of the potential model itself.

We would like to point out that the discrepancy in the \(^7\)Li(\(n,\gamma\))^8Li cross section could be resolved in a way which does not affect the \(^7\)Be(\(p,\gamma\))^8B cross section. As mentioned, contrary to \(^7\)Be(\(p,\gamma\))^8B, the inner part of the wave functions are important in the case of \(^7\)Li(\(n,\gamma\))^8Li. Although the reproduction of the scattering lengths fixes the external part of the scattering wave functions, the internal, off-shell, part is not well-constrained. For instance, if the inner node of the wave function were somewhat further outside than in the potential model, this would bring the \(^7\)Li(\(n,\gamma\))^8Li cross section down. To illustrate that the node position is not well-defined, we show in Fig. 4 the inner part of the \(I = 2\) scattering wave function of the standard potential (BI) of Barker\(^2\) (solid line) together with the scattering state obtained from the cluster model of Ref 29 (dashed line) at \(E_{CM}=10\text{ keV}\). This change in the off-shell behavior is enough to reduce the \(^7\)Li(\(n,\gamma\))^8Li cross section considerably. In fact, the dashed line of Fig. 4, together with the bound state of the standard potential of Barker\(^2\) results (after a \(1/v\) extrapolation from 10 keV) in a thermal cross section of 46.3 mb, which is close to the experimental value. We emphasize again, that this modification in the off-shell behavior of the scattering wave functions has no effect on the \(^7\)Be(\(p,\gamma\))^8B cross
V. RELATION TO THE $^8$B QUADRUPOLE MOMENT

The quadrupole moment, $Q$, of $^8$B is related to the above calculations in the following way. $Q$ is proportional to the matrix element of the $E2 = r^2 Y^{(2)}$ one-body operator whose reduce matrix element is given by a summation over products of many-body matrix elements times single-particle matrix elements\(^{30}\):

$$\langle J_i \| E2 \| J_i \rangle = \sum_{j,j',t_z} \frac{\langle J_i \| [a^+_{j,t_z} \otimes \tilde{a}_{j',t_z}]^{(\lambda)} \| J_i \rangle}{\sqrt{2\lambda + 1}} \langle j, t_z \| E2 \| j', t_z \rangle,$$

where $\lambda = 2$. The $t_z$ indicates a sum over protons and neutrons. By inserting a complete set of states\(^{31}\) ($J_f$) of the $A = 7$ system between the $a^+$ and $\tilde{a}$, one can rewrite this as a sum over all 0p shell states of the $A = 7$ system:

$$\langle J_i \| E2 \| J_i \rangle = (2J_i + 1) \sum_{j,j',t_z,J_f} (-1)^{J_i + J_f} \theta_{j,t_z}(J_i, J_f) \theta_{j',t_z}(J_i, J_f) \times \left\{ \begin{array}{c} J_i \ J_i \ 2 \\ j \ J_f \ \ j' \ \end{array} \right\} \langle j, t_z \| E2 \| j', t_z \rangle,$$

where the states $J_f$ are in $^7$Be for $t_z$=proton and are in $^7$B for $t_z$=neutron. The sum over $J_f$ can be broken down into the term coming from the ground state of $^7$Be (referred to as the valence term, $vp$) and all other terms coming from excited states in $^7$Be and all states in $^7$B (referred to as 0p core proton and neutron terms, $pcp$ and $pcn$, respectively). The single-particle matrix elements are given by a geometrical term times the single-particle mean-square radius, $\langle r^2 \rangle$.\(^{30}\) The $Q$ moment can thus be expressed in the form

$$Q(^8B) = -0.80 \theta_{3/2} \theta_{1/2} \langle r^2 \rangle_{vp} e_p + 0.203 \langle r^2 \rangle_{pcp} e_p + 0.183 \langle r^2 \rangle_{pcn} e_n$$

$$= 0.187 \langle r^2 \rangle_{vp} e_p + 0.203 \langle r^2 \rangle_{pcp} e_p + 0.183 \langle r^2 \rangle_{pcn} e_n,$$  \hspace{1cm} (20)
where the CKI interaction was used for the spectroscopic factors. The numerical coefficient $-0.80$ is purely geometrical. The effective charges $e_p$ and $e_n$ take into account the non-0p parts of the wave functions which include 0s to 0p and 0p to 0d1s proton excitations. For the remaining discussion we will use values of $e_p = 1.35e$ and $e_n = 0.35e$, although we realize that these are approximate values and that they may depend upon the binding energy.\textsuperscript{32} Our calculations for the radial matrix elements (with potential B) give $\langle r^2 \rangle_{vp} = 21.6 \text{ fm}^2$, $\langle r^2 \rangle_{pcp} = 8.1 \text{ fm}^2$, and $\langle r^2 \rangle_{pcn} = 7.8 \text{ fm}^2$, and hence we obtain $Q(\textsf{8B}) = 8.2 \text{ e fm}^2$, in reasonable agreement with the experimental value\textsuperscript{10} of $6.83\pm0.21 \text{ e fm}^2$. The $Q$ moment for $^8\text{Li}$ can be obtained by interchanging the labels for $p$ and $n$ in Eq. (20), and using our value of $\langle r^2 \rangle_{vn} = 14.0 \text{ fm}^2$, to obtain $Q(\textsf{8Li}) = 3.47 \text{ e fm}^2$ which is close to the experimental value\textsuperscript{10} of $3.27\pm0.06 \text{ e fm}^2$. The effective charges we use are approximate and it would very difficult to estimate the them more quantitatively. Even though our calculationed values are close to the experimental values, it is clear that the relationship to the valence rms radius and asymptotic normalization is quite complicated. In addition, we note that the $Q$ moment depends upon the interference term $\theta_{3/2}\theta_{1/2}$ whereas the tail density is determined by the combination $A_{3/2}^2 + A_{1/2}^2$ which is dominated by $\theta_{3/2}$. The term in Eq. (19) which is proportional to $\theta_{3/2}\theta_{3/2}$ is zero because the 6j symbol vanishes (physically it is related to the vanishing of the $Q$ moments in the middle of a single $j$ shell).

VI. RADIAL DENSITIES AND MOMENTUM DISTRIBUTIONS

The radial densities obtained in our potential B calculations are shown in various ways in Figs. 5-7. For the sake of simplification it will be assumed in this section that there is a single valence proton with a binding energy of 0.138 MeV, four tightly bound core protons (two in the 0p shell with a separation energy of about 6 MeV and two in the 0s shell with a separation energy of about 16 MeV) and three tightly bound neutrons (one on the 0p shell with a separation energy of about 8 MeV and two in the 0s shell with a separation energy of about 18 MeV). The actual situation is a little more complicated than this because $S = 1.15$
and because there is also some parentage of the protons in $^8$B to the first excited state in $^7$Be which will result in some leakage of the core protons to larger radii. The results here are thus more qualitative than those given above for $S_{17}$.

The normal density $\rho(r)$ is shown in Fig. 5, the probability density $P(r) = 4\pi r^2 \rho(r)$ on a log scale in is shown Fig. 6, and the probability density on a linear scale is shown Fig. 7. In all figures the neutron density is shown by the dashed line, the core proton density with crosses, and the valence proton density with a solid line. Note in Figs. 6–7 that the areas are equal to three, four and one, respectively. The valence proton clearly has a large extension, but whether or not it constitutes a “halo” or a “skin” is a question of semantics.

The valence proton ($vp$), core proton ($cp$) and neutron ($n$) rms radii are:

$$\sqrt{\langle r^2 \rangle_{vp}} = 4.60 \text{ fm},$$

(21)

$$\sqrt{\langle r^2 \rangle_{cp}} = 2.39 \text{ fm},$$

(22)

and

$$\sqrt{\langle r^2 \rangle_{n}} = 2.21 \text{ fm}.$$  

(23)

The total proton rms radius is given by

$$\sqrt{\langle r^2 \rangle_p} = \sqrt{[4\langle r^2 \rangle_{cp} + \langle r^2 \rangle_{vp}]/5} = 2.97 \text{ fm}.$$  

(24)

The rms charge radius includes the rms radius of 0.80 fm for the proton:

$$\sqrt{\langle r^2 \rangle_{ch}} = \sqrt{[\langle r^2 \rangle_p + 0.64 \text{ fm}^2]} = 3.05 \text{ fm},$$  

(25)

and the matter radius is given by:

$$\sqrt{\langle r^2 \rangle_m} = \sqrt{[5\langle r^2 \rangle_p + 3\langle r^2 \rangle_n]/8} = 2.71 \text{ fm}.$$  

(26)
Our results can be compared to those of other theoretical calculations. The results of $\sqrt{\langle r^2 \rangle_{vp}} = 3.75$ fm obtained by Riisager and Jensen\textsuperscript{33} is much smaller than ours but they use an arbitrary potential shape. The reason for their small $S_{17}$ is obvious from Fig. 3. The calculations presented in Ref 10 for the $Q$ moment appear to be very close to our results.

The interaction cross sections for $^8$B and $^8$Li on $^{12}$C have been calculated with the method of Ref 34. The results with the finite-range interaction are $\sigma = 843$ mb for $^8$B and $\sigma = 820$ mb for $^8$Li. These can be compared to the experimental values of $\sigma = 784(14)$ mb\textsuperscript{35} for $^8$B and $\sigma = 768(9)$ mb\textsuperscript{36} for $^8$Li. The agreement for the magnitudes is as good as can be expected from the uncertainties in the calculation. However, the $^8$B/$^8$Li ratio (in which some of the reaction uncertainties may cancell) is in excellent agreement between theory and experiment. The effects of the proton “halo” in $^8$B and the neutron “halo” in $^8$Li are not large compared with the classic cases\textsuperscript{34,36} of $^{11}$Li, $^{11}$Be and $^{14}$Be.

Recent radioactive beam experiments\textsuperscript{37,38} have looked at the momentum distribution of the $^7$Be fragments which result from the break-up of a beam $^8$B on various targets. From these experiments one expects to measure the momentum distribution of the most loosely bound protons. The longitudinal momentum $P(k_z)$ obtained for the valence proton is shown in Fig. 8. This is obtained by the Fourier transform of the spacial wave function:

$$
\bar{\Psi}(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int \Psi(\vec{r}) e^{i\vec{k} \cdot \vec{r}} d^3r. \tag{27}
$$

where $\Psi(\vec{r}) = \psi(r)Y_\ell(\hat{r})$, $\bar{\Psi}(\vec{k}) = \tilde{\psi}(k)Y_\ell(\hat{k})$, and where the radial momentum distribution is given by:

$$
\bar{\tilde{y}}(k) = \sqrt{\frac{2}{\pi}} \frac{i^{\ell}}{k} \int \psi(r) j_\ell(kr) r^2 dr. \tag{28}
$$

The longitudinal momentum distribution is given by

$$
P(k_z) = \frac{1}{2} \int |\bar{\tilde{y}}(k)|^2 k_r dk_r, \tag{29}
$$

where $k^2 = k_r^2 + k_z^2$.  


The calculated momentum distribution has a width of about 150 MeV/c compared to the experimental value of 81±6 MeV/c. Given the good agreement generally found for calculated and observed neutron halos this disagreement is puzzling. There is some discussion in the literature about ways to improve the above calculation to take into account the peripheral nature of the reaction. In the peripheral direct reaction model one puts in an additional cut-off in Eq. (29) to exclude the interior part of the radial distribution which presumably does not contribute because the cross section coming from that part is dominated by a more violent reaction where the core (^7Be in this case) is destroyed. We have phenomenologically modeled this effect by putting a Fermi shaped cut-off factor in Eq. (28) which has the effect of excluding the interior out to a radius \( R_{\text{cut}} \) and with a diffuseness \( a_{\text{cut}} \). We take \( a_{\text{cut}} = 0.65 \) fm and vary \( R_{\text{cut}} \) to get about the observed momentum distribution. This requires \( R_{\text{cut}} = 5 \) fm and the results are shown by the dashed line in Fig. 8. The cut-off results in a reduction of \( P(k_z) \) at small momenta by a factor of 2.5 and we have renormalized the cut-off distribution by this factor in order to show the change in width. It is already known that the cut-off factor does not have much effect on the neutron halo momentum distributions, and we have demonstrated that even a value as large as \( R_{\text{cut}} = 5 \) fm has little effect on the width of the ^11Be neutron halo momentum distribution. We do not know why we should need \( R_{\text{cut}} = 5 \) fm, but we note that this corresponds to the point in Fig. 5–7 where the valence proton density falls below the core density. More work needs to be done to understand these results.

Schwab et al. present an RPA calculation which goes beyond the 0p shell and which agrees with the shape of the observed momentum distribution. However, the shape of the wave function beyond about 6 fm as shown in Fig. 4 in their paper appears unrealistic to us. The shape beyond about 6 fm is entirely determined by the Coulomb and centrifugal barriers, and their shape is differs from this expectation. Our own calculation can of course be criticized for staying within the 0p shell. However, a very recent “no-core” calculation along the lines of those given in Ref 42 which takes into account the lowest six major shells.
(21 shell-model orbitals) and up to $4\hbar\omega$ in excitation gives spectroscopic factors which are close to the present 0p shell results.

VII. CONCLUSIONS

Our calculated value of $S_{17}(20\text{ keV}) = 26.5 \pm 2.0$ eV-barns is agreement with the higher values of 25-27 eV-barns inferred from the $(p,\gamma)$ data of Parker$^4$ and Kavanagh et al.$^5$ and is higher than the value obtained from the weighted average of all experimental data, $S_{17} = 22.2 \pm 2.3$ eV-barns,$^{43}$ which is the value currently adopted in most solar models.$^{44}$ As far as other theoretical predictions are concerned, our current result for $S_{17}$ is roughly near the highest of these ($\sim 30$ eV-barns)$^{45}$ and and much larger than the smallest ($\sim 17$ eV-barns)$^{24,46}$

We note, that in contrast to the common belief (e.g. Ref 46), a small value of $S_{17}$ does not make the solar neutrino problem less severe. If one takes standard nuclear and solar physics, and standard neutrino properties, then the best fit$^{47}$ of the neutrino fluxes indicates a suppression in both the $^7\text{Be}$ ($\phi_7$) and the $^8\text{B}$ ($\phi_8$) neutrino fluxes, but the suppression is much stronger in $\phi_7$. However, a smaller $S_{17}$ value alone would make the predicted $\phi_7/\phi_8$ ratio larger, and hence would exaggerate the solar neutrino problem.

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Figure Captions

Fig. 1: The valence proton rms radius as a function of $\Delta_o$. The three lines join the points obtained for different value of the diffuseness $a$ for $R=2.0$ fm (filled circles), $R=2.4$ fm (open circles) and $R=2.8$ fm (squares). The results for the specific potentials in Table II are shown by the labeled crosses.

Fig. 2: The valence proton density at $r=10$ fm as a function of $\Delta_o$ (see caption to Fig. 1).

Fig. 3: The valence proton density at $r=10$ fm as a function of the valence proton rms radius (see caption to Fig. 1).

Fig. 4: The scattering wave function for $^7\text{Li}(n,\gamma)$ obtained with the standard potential (BI) of Barker\textsuperscript{28} (solid line) together with the results obtained with the cluster model\textsuperscript{29} (dashed line).

Fig. 5: The radial density profile for $^8\text{B}$ for the neutrons (dashed line), the core protons (crosses) and the valence proton (solid line).

Fig. 6: The radial probability distribution for $^8\text{B}$ on a log scale (see caption to Fig. 5).

Fig. 7: The radial probability distribution for $^8\text{B}$ on a linear scale (see caption to Fig. 5).

Fig. 8: The momentum distribution for the $^8\text{B}$ valence proton. The solid line corresponds to the full radial wave function and the dashed line corresponds to the radial wave function cut-off at $r=5$ fm.
TABLE I. Values of $\Delta_o$, the rms proton radius and $\rho(10 \text{ fm})$ as a function of $R$ and $a$.

| spin-orbit | $R$ (fm) | $a$ (fm) | $\Delta_o$ (MeV) | rms (fm) | $\rho(10 \text{ fm}) \cdot 10^6$ (fm$^{-3}$) |
|------------|---------|---------|------------------|----------|---------------------------------|
| yes        | 2.0     | 0.4     | 2.108            | 3.70     | 4.25                            |
| yes        | 2.0     | 0.6     | 1.947            | 4.14     | 5.81                            |
| yes        | 2.0     | 0.8     | 1.786            | 4.62     | 8.00                            |
| yes        | 2.4     | 0.4     | 1.969            | 4.02     | 5.27                            |
| yes        | 2.4     | 0.6     | 1.840            | 4.40     | 6.85                            |
| yes        | 2.4     | 0.8     | 1.705            | 4.84     | 9.07                            |
| yes        | 2.8     | 0.4     | 1.836            | 4.35     | 6.48                            |
| yes        | 2.8     | 0.6     | 1.735            | 4.68     | 8.10                            |
| yes        | 2.8     | 0.8     | 1.623            | 5.08     | 10.40                           |
| no         | 2.4     | 0.4     | 1.991            | 3.97     | 5.09                            |
| no         | 2.4     | 0.6     | 1.851            | 4.38     | 6.74                            |
| no         | 2.4     | 0.8     | 1.708            | 4.84     | 9.05                            |
TABLE II. Woods-Saxon potential parameters from the present analysis (sets A, B and C) compared to those used by Tombrello\textsuperscript{48} and Barker I.\textsuperscript{28} Barker II and III correspond to those values Barker needed to reproduce the $^7\text{Li}(n,\gamma)$ cross section.

| Set         | $R$ (fm) | $a$ (fm) |
|-------------|----------|----------|
| A           | 2.8      | 0.51     |
| B           | 2.4      | 0.69     |
| C           | 2.0      | 0.81     |
| Tombrello (T)| 2.95    | 0.52     |
| Barker BI   | 2.39     | 0.65     |
| Barker BII  | 0.53     | 0.65     |
| Barker BIII | 2.39     | 0.27     |
TABLE III. Spectroscopic amplitudes $\theta_j(J_i, J_f = 3/2)$ for $^8$B to $^7$Be from the CKI, $^{49}$Kumar $^{50}$ and PTBME $^{51}$ interactions. The order of the states for a given $J_f^\pi$ is indicated by $n_f$.

| $J_f^\pi$, $n_f$ | $j$   | CKI  | Kumar | PTBME |
|------------------|-------|------|-------|-------|
| $2^+,1$          | 3/2   | 0.988| 0.966 | 0.986 |
|                  | 1/2   | -0.237| -0.259| -0.253|
| $1^+,1$          | 3/2   | 0.567| 0.606 | 0.552 |
|                  | 1/2   | -0.352| -0.244| -0.342|
| $3^+,1$          | 3/2   | 0.581| 0.555 | 0.565 |
| $1^+,2$          | 3/2   | 0.617| 0.574 | 0.525 |
|                  | 1/2   | 0.840| 0.861 | 0.859 |
TABLE IV. Spectroscopic factors $S = S_{1/2} + S_{3/2}$ for $^8\text{Li}$ and $^8\text{B}$.

| $J^\pi$ | $^8\text{B}_{\text{exp}}$ (decay) | $^8\text{Li}_{\text{exp}}$ (decay) | $^8\text{Li}_{\text{exp}}$ (reaction) | CKI | Kumar | PTBME |
|---------|----------------------------------|----------------------------------|----------------------------------|-----|-------|-------|
| 2$^+$   | 0.87(13)                         | 1.17                             | 1.14                             | 1.19|       |       |
| 1$^+$   | 0.49(8)                          | 0.48(7)                          | 0.51                             | 0.49| 0.47  |       |
| 3$^+$   | 0.32(4)                          | 0.38(7)                          | 0.39                             | 0.35| 0.36  |       |
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