On the Cosmic Ray Muon Hypothesis for DAMA

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The DAMA dark matter search experiment observes a statistically significant percent-level variation of its low-energy count rate with a period of one year. In this note we recall some of the arguments which challenge the hypothesis that the cosmic ray induced underground muon flux can be the origin of the modulation. In addition, we provide new comments on recent works on this subject.

1 Introduction

In a recent publication\(^1\) henceforth referred to as CPY we examine recent claims that the cosmic ray muon flux can be responsible for generating the modulation signals seen by the DAMA\(^2\) and, more recently, by the CoGeNT\(^4\) Dark Matter (DM) direct detection experiments. By examining the time series of the reported DAMA and CoGeNT signals we find the data sets differ in phase, likely in amplitude, and potentially in their power spectrum from measurements of the underground muon flux. A correlation analysis in CPY reveals that the data sets are incompatible and that the muon flux as the sole source of the reported signals is excluded at high confidence.

In this note we restrict the discussion of the muon hypothesis to DAMA which is the only direct detection experiment which claims detection of a firm DM signal. It is situated at Gran Sasso, Italy, in the underground LNGS laboratory. The collected data encompasses two major runs, DAMA/NaI (Dec 1995 - July 2002) and DAMA/LIBRA (Sept 2003 - Sept 2009). The residual count rate reported by the collaboration exhibits annual modulation compatible with what is expected from generic DM models.

TeV-scale cosmic ray muons produced at stratospheric altitude levels can reach deep underground and induce spallation reactions in the detector and nearby. The seasonal variations of the underground muon flux have been observed on the northern and southern hemisphere alike. For DAMA, the relevant measurement at the LNGS site is the one from LVD\(^5\) which overlaps in time with DAMA/LIBRA cycles 1–5. The integral muon intensity underground reads \(\langle I_\mu \rangle \approx 3 \times 10^{-4} \text{ m}^{-2} \text{ sec}^{-1}\) and exhibits an annual variation of \(\sim 2\%\) in amplitude. In Fig. 1 we show the percent residuals of the muon flux when binned in concordance with DAMA with the annual mean count rate subtracted. In addition, the (2–4) keVee bin of the DAMA/LIBRA experiment are shown; a baseline rate of \(\bar{s} = 1.15 \text{ cpd/kg/keV}\) has been assumed. The seeming similarity in time and amplitude is tantalizing and has lead to the notion that both signals may in fact be measurements of the very same cosmic ray phenomenon\(^6\)\(^7\)\(^8\). In the following we recap and further develop on the arguments which challenge this hypothesis.

2 The Null Hypothesis: periods, binning, and subtraction of means

A spectral analysis will most readily reveal periodic behavior in a time series \(\{t_i, X_i\}_{i=1,...,N}\). In case the times \(t_i\) are unevenly spaced, it is most convenient to make use of the Lomb-Scargle (LS) periodogram\(^9\),

\[
P_X(\omega) = \frac{1}{2\sigma_X^2} \left\{ \frac{\left[ \sum_i (X_i - \bar{X}) \cos \omega (t_i - \tau) \right]^2}{\sum_i \cos^2 \omega (t_i - \tau)} + \frac{\left[ \sum_i (X_i - \bar{X}) \sin \omega (t_i - \tau) \right]^2}{\sum_i \sin^2 \omega (t_i - \tau)} \right\}, \tag{1}
\]

Here \(\tau\) is obtained from \(\tan(2\omega \tau) = \sum_i \sin 2\omega t_i / \sum_i \cos 2\omega t_i\), and it ensures that \(P_X(\omega)\) is invariant to shifts in the time origin; \(\bar{X}\) and \(\sigma_X^2\) are the arithmetic mean and the variance of
the data \( \{X_i\} \), respectively. A Bayesian derivation and generalization of (11) can be found in CPY. Noise is rejected at the \( 1 - \alpha \) confidence level by demanding that the power at a sampled frequency is in excess of

\[
    z_\alpha = -\ln \left[ 1 - (1 - \alpha)^{1/M} \right].
\]

(2)

A statistical penalty has been included for examining \( M \) independent frequencies. In order to obtain a meaningful statistical interpretation we have to choose \( M \) carefully. In CPY we chose to sample the data at frequencies \( \omega_n = n\omega_F \) with \( n = 1, \ldots, N, \ M = N \), and with the fundamental frequency \( \omega_F = 2\pi/T \) where \( T \) is roughly the range of years covered by the data set. In this note we instead oversample the data using a finer grid of frequencies. Since \( X_i \) are not grossly unevenly spaced in \( t_i \), \( M \approx N \) holds as long as we only sample \( X_i \) above the average Nyquist frequency, \( \nu_c = 1/2\Delta \) where \( \Delta \) denotes the average spacing between \( t_i \). We checked that both approaches agree on coinciding frequencies.

The left panel of Fig. 2 shows the DAMA/LIBRA power spectrum for the residual count rates as reported in [3] and which are obtained by subtracting the mean count rate in each cycle 11 for the various energy bins as labeled. Above the horizontal line, noise is rejected at 99% C.L. With an approximate power \( P_X(2\pi/1\ yr) \sim 12 \) at a period of 1 yr, the null hypothesis is disfavored by at least 4.5\( \sigma \) at a single frequency and at 3.6\( \sigma \) according to (2). Notably, there is significant power at a period at 480 days in the \( (2 - 4) \) keV_{ee} and \( (2 - 6) \) keV_{ee} bins and a consistent power in all three energy bins with period 1/3 yr corresponding to a triannual mode; for the latter only the \( (2 - 5) \) keV_{ee} bin exceeds the 99% C.L. level.

The right panel of Fig. 2 shows the LS periodogram obtained from LVD data where only those data points were included which have actual temporal overlap with the bins of the DAMA/LIBRA cycles, namely, full cycles 1–5. We only show a fraction of the full power spectrum with smaller periods omitted. The horizontal line is calculated according to (2) from all sampled independent frequencies. Since \( N_{LVD} \gg N_{DAMA} \) the required power to reject the null hypothesis is larger when compared to the right plot —despite the similar period intervals shown. The solid line is obtained from the unbinned LVD data set. Aside from the significant yearly modulation, the spectrum exhibits temporal variations on time-scales smaller and greater than one year. At first sight this is in stark contrast to the DAMA spectra which barely exhibit power at periods other than at \( T = 1 \) yr. However, in order to better compare these data sets one should (1) bin the muon data over the same time spans DAMA/LIBRA bins their signal and (2) subtract the yearly mean of muon flux. The latter mimics the procedure employed by the DAMA collaboration when they present their residual count rates. To isolate the effects of (1) and (2) we carry out each procedure separately and show its effect on the power spectrum.
We first observe that binning reduces the number of data points from $N \sim 1.3k$ to $N = 38$ when considering DAMA/LIBRA cycles 1–5. For a given significance $\alpha$ this changes the condition on the minimum power in Eq. (2). Therefore, we first bin the LVD data and subsequently scale the resulting power spectrum in such a way that for a given $y$-value the significance agrees with the one from the unbinned data. The result is shown by the dotted line in Fig. 2. Though the dotted line is not a LS anymore, one can now directly compare the dotted with solid line. We observe that the generic effect of binning is to mask power on all time scales. The effect of subtracting the mean intensity from the data on a yearly basis is shown by the dashed line instead. This time the power spectrum is not scaled and the dashed line is indeed a LS periodogram. In contrast to binning, this procedure largely preserves power on periods $T \leq 1$ yr. However, for larger periods we again observe that variations are masked due to the particular treatment of the data.

In contrast to the LVD spectrum, the Fourier transformed DAMA residuals in the left panel of Fig. 2 show much less significant power at periods smaller and greater than a year. As shown above, this may simply be an artifact of the way the DAMA/LIBRA modulation signal is obtained. It is unfortunate since a Fourier analysis could otherwise be used to discriminate between background induced effects and real signal. It is not clear how to assign a quantitative significance to this difference without knowledge of the full, unsubtracted time series. This calls for a release of the yearly baseline count rates by the DAMA collaboration.

The DAMA collaboration has recently responded to the criticism that baselines are subtracted per cycle. Instead of providing the actual values, the LS power spectrum of the joint DAMA/NaI and DAMA/LIBRA baselines is presented. No significant power which would point towards long-term behavior is observed. However, the baselines are a mere set of 13 numbers.
It is not surprising that no statistically significant power is observed from such a small data set. Similarly, we find that the LVD baseline spectrum does not yield any significant power when calculated according to (1), too. This casts doubt that a presentation of the baseline-LS precludes the existence of variations in DAMA with periods larger than one year, similarly to what is observed in muons; see also [13,14].

3 The phase of muons: a potentially stretched concept

Given an isotropic DM velocity distribution $f(v)$ in the earth’s vicinity, the differential recoil rate $dR/dE_R$ of DM scattering in the detector is predicted to peak on June 2nd, corresponding to a phase of $t_0 = 152.5$ days after January 1st,

$$\frac{dR}{dE_R} \propto \int_{v_{\min}}^{\infty} \frac{f(v)}{v} dv \approx c_0 + c_1 \cos[\omega(t - t_0)].$$

(3)

Here, $v_{\min}$ is the minimum required relative velocity between DM and target which produces a nuclear recoil of energy $E_R$. In the canonical case, the annual mode is at the percent level, $c_1/c_0 = \mathcal{O}(10^{-2})$; higher harmonic corrections to (3) have first been pointed out in CPY. The clear-cut prediction on $t_0$ arises from the geometric setup of the earth’s rotation around the sun in conjunction with the movement of the solar system relative to the Galactic coordinate system. Hence, apart from the period, the second most important characteristic of the oscillations observed by the DAMA experiment is the phase of the signal.

On the other hand, when considering environmental factors as potential background, the variation in time will in general not be modulated with a strict period of a year. Additionally, we cannot expect a sinusoidal behavior similar to (3). Both assertions can be tested on the LVD and DAMA data sets. For this we model the data by a sinusoid, minimize the usual $\chi^2$ function, and assess the goodness-of-fit. Confidence regions in $t_0$ and $T$ can be inferred from a $\Delta\chi^2$-method which is equivalent to constructing a profile likelihood. Fixing the period to one year we find that the DAMA/LIBRA ($2 - 4$) keV$_{ee}$ and the full LVD data do not agree with respective values

$$\begin{align*}
\text{DAMA/LIBRA} : & \quad t_0 = (130 \pm 8) \text{ days}, \quad \chi^2/dof = 37.5/41, \\
\text{full LVD} : & \quad t_0 = (185 \pm 1.5) \text{ days}, \quad \chi^2/dof = 3450/1971.
\end{align*}$$

(4)

(5)

The uncertainty quoted is statistical. An additional uncertainty of $\pm 2$ days may be attributed to digitization. A few comments are in order: 1) The sinusoid provides a very poor description of the LVD data with prohibitively large $\chi^2$. This supports our expectation that a complex phenomenon such as the underground muon flux cannot be adequately described by this simple model. 2) Even if we scale up the error bars for LVD to the point where $\chi^2/dof \simeq 1$, the stringent error on the phase remains below $\pm 2$ days. 3) An error of $\pm 15$ days on $t_0$ as quoted by LVD is certainly incorrect. 4) Once $T$ is allowed to float, the confidence regions in $t_0$ and $T$ become sensitive to the time-origin; for further details we refer the reader to CPY. These results have recently been confirmed in [15]. One major shortcoming of this approach is that the data has to be subjected to the simple model of sinusoidal variation, before any conclusions can be drawn. Indeed, the power-spectrum of the LVD data makes it clear that significant power exists in modes with periods larger and smaller than one year. The notion of a single phase must therefore be treated with caution.

4 A correlation analysis: the conclusive approach

A glance at Fig. 1 suggests that the DAMA/LIBRA and LVD data sets are correlated. Indeed, when evaluating Pearson’s coefficient of correlation, $r \in [-1, 1]$, one finds for the DAMA and
LVD sets as presented in Fig. 1

\[ r_{LVD, DAMA} = 0.44. \]  

(6)

This value excludes the no-correlation hypothesis with a confidence level greater than 99%. However, just because the data sets are correlated does not imply that they are causally connected. This can only be answered on a model-by-model basis.

Clearly, a muon background model for DAMA must encompass the stochastic nature of the underlying process. Even if the timing between muons and DAMA are incommensurate at first sight, could it be that such a Poisson smearing alleviates the observed tension in the annual phase? That this is indeed the case has been shown in\(^8\) where a simple, linear model for how a muon-sourced background may be realized is employed. Taking from this that the muon hypothesis for DAMA would become viable again is however not correct.

In \textit{CPY} we generate \(10^4\) realizations for DAMA based on the generic model presented in\(^8\) which connects the muon flux measured in LVD with the count rate observed in DAMA/LIBRA. The question which has to be asked is now the following: How likely is it that one realization out of the mock data indeed induces the signal observed in DAMA. To answer this question we can again look at the correlation coefficient. It has the merit that we do not impose any functional form on the data. As expected, the generated mock data exhibits a high degree of correlation with LVD. In fact, the correlation with the actual muon data is substantially higher than the one which is actually observed in (6). This rules out the model at 99% C.L.. Given the generality of the model, it also strongly disfavors the hypothesis that muons can be the origin for the count rate variation seen in the DAMA detectors.

5 Conclusions

In \textit{CPY} we lay out a detailed time-series analysis of DM direct detection data as well as related datasets. In particular, we find no evidence to support the claim that atmospheric muons are responsible for the signal that the DAMA collaboration observes. We identify difficulties in phase, amplitude, power spectrum and degree of implied correlation between the data sets. Here we offer some additional comments:

- When binning data, fine-grained information on the time-series is lost. In addition, we find that when the LVD measured muon flux is binned in time in the same way as DAMA/LIBRA presents their data, significant power is lost on all time scales. This has the consequence that the LVD and DAMA data sets become more alike in the frequency domain with a dominating peak at a period of one year. Thus binning seems to diminish discriminating potential between the data sets. This calls for an unbinned analysis of the DAMA data set.

- The uncertainty on the phase of the underground muon flux reported by LVD, \(t_0 = (185 \pm 15)\) days is erroneous. In \textit{CPY}, an uncertainty of \(\pm 2\) days was obtained, but the discrepancy had not been emphasized. An independent analysis in\(^15\) confirms our value.

- A recent paper by the DAMA collaboration presents the LS spectra of the average count rates in the DAMA/NaI and DAMA/LIBRA cycles. No significant power indicating potential long-term variation (\(T > 1\) yr) is observed. Following this procedure for LVD data, we can reach analogous conclusions—in seeming contradiction with the power observed in the right panel of Fig. 2. One cannot expect statistical significance from a LS-transform of \(O(10)\) numbers without further insight into the normalization of (1).

- The same paper confirms our previous findings in \textit{CPY} that a putative cosmic ray activation can only lead to a maximum phase shift of a quarter year in the subsequently modulated
decay. Hence it is not possible to overcome the ∼11 month discrepancy between muons and DAMA.

- A recent paper\textsuperscript{15} incorporates additional data on the Gran Sasso underground muon flux. Frequentist fits to period and phase improve on the already significantly disjoint LVD and DAMA regions presented in \textsuperscript{CPY}. This corroborates the statement that both data sets are seemingly incompatible. However, as argued in Sec.\textsuperscript{4} and at great length in \textsuperscript{CPY} reliance on the inferred phase can be misleading. Indeed in \textsuperscript{8} and \textsuperscript{CPY} it was shown that Poisson smearing can sufficiently alleviate the tension between the phases of the muon and DAMA data sets. It is the correlation analysis which reveals that the data sets remain nevertheless incompatible. However, the latter conclusion depends on the model which connects the muon flux with the DAMA count rate and it can only be achieved considering data sets which have actual temporal overlap. From this perspective, no further gain of information with respect to DAMA is obtained by enlarging the set of muon measurements as done in \textsuperscript{15}.

As in the case of DAMA, in \textsuperscript{CPY}—employing a similar line of analyses—we find no significant correlation between the CoGeNT data set and the yearly variation of the muon flux. In summary, cosmic ray muons cannot be the sole source for the observed modulation signals in these experiments.

Acknowledgments

It is a pleasure to thank S. Chang and I. Yavin for collaboration on this subject. The author also acknowledges useful discussions with K. Blum and E. Fernandez-Martinez.

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