Higgs Portal to Visible Supersymmetry Breaking

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Abstract

We propose a supersymmetric extension of the standard model whose Higgs sector induces a spontaneous supersymmetry breaking by itself. Unlike the minimal extension, the current Higgs mass bound can be evaded even at the tree-level without the help of the soft breaking terms due to the usual hidden sector, as is reminiscent of the next to minimal case. We also have a possibly light pseudo-goldstino in our visible sector in addition to extra Higgs particles, both of which stem from supersymmetry breaking dynamics. In such a setup of visible supersymmetry breaking, we may see a part of supersymmetry breaking dynamics rather directly in future experiments.
1 Introduction

The electroweak (EW) symmetry, an $SU(2) \times U(1)$ gauge symmetry, plays a major role in the standard model of particle physics. In the standard model, the gauge symmetry is spontaneously broken by the vacuum expectation value of a Higgs scalar field. Although the quark-lepton and gauge sectors are well established, the structure of the Higgs sector is largely uncertain because the Higgs particle has not yet been discovered directly. In addition, there is a naturalness problem about the Higgs scalar mass such that the mass of a scalar field receives large quantum corrections unlike fermion masses. Supersymmetry (SUSY) can provide a possible solution to this problem by introducing the corresponding superpartners into the model with soft SUSY breaking [1]. The minimal supersymmetric extension of the standard model (MSSM) has two Higgs doublets [2] in order to accommodate the anomaly cancellation and the holomorphicity of the superpotential.

The soft SUSY breaking serves to make the introduced superpartners heavy enough so that they have not been observed experimentally. To obtain appropriate soft breaking terms, the SUSY breaking dynamics is usually put in the so-called hidden sector that is somehow separated from the visible standard model sector. Namely, the original SUSY breaking in the hidden sector is mediated to the visible sector by (flavor-blind) interactions such as gravity or the standard model gauge interactions. In the MSSM, the EW symmetry breaking is tied to the resultant SUSY breaking in the visible sector. It is possibly generated radiatively through the SUSY breaking mediated from the hidden sector. If the hidden sector SUSY breaking occurs dynamically with its breaking scale given by dimensional transmutation, then the hierarchy between the Planck/GUT scale and the EW scale may be naturally explained.

Unfortunately, this simple scenario is spoiled by the need for the supersymmetric Higgs mass term called $\mu$-term [3]. The supersymmetric mass scale must be tuned to about the same size as the EW scale for the correct symmetry breaking. Moreover, even if we assume an appropriate order of magnitude for the supersymmetric scale, in the MSSM, additional fine-tuning of a few percent is required as follows. The lightest CP-even Higgs mass $m_h$ is smaller than the $Z$ boson mass at the tree-level in the MSSM. Thus the current experimental bound $m_h > 114$ GeV requires large radiative corrections from the (s)top loops [6] with the stop mass of at least 1 TeV, which in turn affects radiatively on the soft scalar mass of the up-type Higgs field through the Yukawa coupling. The soft mass implied by the renormalization

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1 One approach to this problem is to add a singlet superfield whose scalar component leads to the effective $\mu$-term, which amounts to the Next to Minimal Supersymmetric Standard Model (NMSSM) [4]. Note that when we combine the NMSSM with SUSY-breaking mediation such as gauge mediation, it is not so easy to obtain the correct EW symmetry breaking [3].
is comparable to the stop mass with a negative sign. Then, fine-tuning is needed up to a few percent between the $\mu$-term and the soft scalar mass of the Higgs field in order to obtain the correct $Z$ boson mass. Although many solutions to the above problems have been proposed so far, we do not have any compelling reasons to stick to the minimal (or next to minimal) Higgs sector like the (N)MSSM and the radiative EW symmetry breaking driven by the soft SUSY breaking terms.

On the contrary, in this paper, we regard the Higgs sector as a window to unknown physics beyond the MSSM, in particular, SUSY breaking dynamics. Historically, visible sector SUSY breaking was abandoned due to phenomenological difficulties such as the prediction of light superpartners, and in turn, hidden sector SUSY breaking has been adopted. However, in the presence of the hidden sector, additional visible SUSY breaking is not forbidden phenomenologically. Namely, we may consider that SUSY breaking is ubiquitous not only in the hidden sector but also in the visible sector.

By visible SUSY breaking, we mean the existence of SUSY breaking in the standard model sector even in the absence of the soft breaking terms stemming from the usual hidden sector. The SUSY breaking scale of the hidden sector tends to be too high to observe its dynamics directly in the foreseeable future. In contrast, if visible SUSY breaking exists, we may see a part of SUSY breaking dynamics rather directly in near future experiments. Concretely, as advocated above, we seek visible SUSY breaking in the Higgs sector, which has large uncertainty at present. The simplest possibility may be a model that has a singlet field $S$ like the NMSSM with its superpotential coupling to Higgs fields given by $S H_u H_d$, where $H_u$ and $H_d$ are the up-type and down-type Higgs superfields. Then, the vacuum expectation values of the scalar component and the $F$-term of a visible SUSY breaking field $S$ lead to the effective $\mu$-term and $B\mu$-term, respectively. These vacuum values are possibly generated spontaneously by some low-scale dynamics different from that of the usual SUSY breaking hidden sector. Such a low-scale dynamics is hopefully within the reach of direct experiments.

It is interesting that we are able to consider even more direct SUSY breaking dynamics in the visible sector: the up-type and down-type Higgs fields can participate in the dynamics of visible SUSY breaking as well as EW symmetry breaking. That is, if we turn off the standard model gauge interactions and the soft breaking terms, our Higgs sector reduces to just an O’Raifeartaigh model with global $SU(2) \times U(1)$ symmetry breaking. We concentrate on this possibility below as a concrete example of visible SUSY breaking, since this model seems

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2 For example, see [7] for relieving the tension between generation of the $\mu$-term and gauge mediation. See also [8] for solving the little hierarchy problem from the viewpoint of General Gauge Mediation [9].

3 Even multiple kinds of extended SUSY breaking might be observable rather directly. The presence of extra superpartners such as multiple kinds of gravitinos could open up such a possibility [10].

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Table 1: The charge assignments of the Higgs sector fields under the EW symmetry and $U(1)_R$ symmetry.

|       | $SU(2)_L$ | $U(1)_Y$ | $U(1)_R$ |
|-------|-----------|-----------|-----------|
| $X_0$ | 1         | 0         | 2         |
| $X_1$ | 2         | $-1/2$    | 2         |
| $X_2$ | 2         | 1/2       | 2         |
| $H_u$ | 2         | 1/2       | 0         |
| $H_d$ | 2         | $-1/2$    | 0         |

advantageous from a perspective of direct experimental detection.

The rest of the paper goes as follows. In section 2, we present our model and analyze its vacuum structure. Then, in section 3, we show the mass spectrum of the Higgs sector in the visible SUSY and EW symmetry breaking vacuum. It turns out that the lightest CP-even Higgs mass can evade its current bound even at the tree-level, as is reminiscent of the next to minimal case. In section 4, we discuss a possible connection between the mass parameters in our Higgs sector and the mass scales of the hidden sector. Finally, in section 5, we conclude our discussion and provide possible directions for future works.

2 Visible SUSY & EW breaking

Let us first present our model of visible SUSY breaking to provide the scalar potential. Then, we identify our vacuum in which both of the visible SUSY and the EW symmetry are spontaneously broken before analyzing the mass spectrum of the Higgs sector in the vacuum in the next section.

2.1 The model

As mentioned in the Introduction, we consider an O’Raifeartaigh model as a mechanism of visible SUSY breaking, in which an $F$-term of a superfield is non-vanishing. The minimal extension for this purpose is to introduce a gauge singlet $X_0$ under $SU(2)_L \times U(1)_Y$, and a vector-like pair $X_1, X_2$ of $SU(2)_L$ doublets in addition to the usual up-type and down-type Higgs fields $H_{u,d}$ of the MSSM. For simplicity, we assume that the model has $U(1)_R$

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4We can also consider an O’Raifeartaigh model with a vector-like pair of $SU(2)_L$ triplets instead of the doublets, which we regard as the next to minimal extension and only study the minimal case in this paper.

5The doubling of the Higgs doublets might be a manifestation of hidden partial extended SUSY (see also footnote 3). We note that one of the advantages in the minimal (or next to minimal) Higgs sector like the (N)MSSM may be the gauge coupling unification. See [15] for discussions on the gauge coupling unification in the case with four Higgs doublets like the present setup.
symmetry except for Majorana gaugino masses.\footnote{R-symmetric supersymmetric standard model was studied in [14], whose authors assume that the gauge sector also respects R-symmetry, so that Majorana gaugino masses are forbidden. In order to give non-zero masses for the gauginos, they introduce new fields of adjoint representations under the standard model gauge symmetries, and form the Dirac gaugino mass terms. Here, just for simplicity of the presentation, we assume that the gauge sector does not respect R-symmetry, and hence Majorana gaugino mass terms are allowed. It is straightforward to extend our model to include the Dirac mass terms to preserve $U(1)_R$ symmetry by introducing additional fields of the adjoint representations under the standard model gauge group. Then, the supersymmetric flavor problems may be ameliorated, as pointed out in [14].} The charge assignments of the Higgs sector fields under the EW symmetry and $U(1)_R$ symmetry are summarized in Table \ref{tab:charge_assignments}. We assign $R$-charge 1 for all the matter superfields,\footnote{This assignment allows Majorana neutrino mass terms $H_u L H_u L$.} so that it forbids renormalizable superpotential terms such as $QLd + LL\bar{e} + LH_u + \bar{d}\bar{d}u$, which violate the lepton or baryon number. Apart from the usual Yukawa couplings of Higgs fields $H_{u,d}$ with matters, the symmetries allow our superpotential to have the following terms:

$$W_{\text{Higgs}} = X_0 (f + \lambda H_u H_d) + m_1 X_1 H_u + m_2 X_2 H_d,$$

(2.1)

where a coupling $f$ has mass dimension 2, and $m_1, m_2$ have mass dimension 1. We can take all these couplings real without loss of generality. All the mass scales are assumed to be of order the EW scale.

With the canonical Kähler potential of all the fields, the superpotential and the gauge interactions determine the scalar potential of the Higgs sector. The entire scalar potential of the Higgs sector consists of $F$-terms, $D$-terms and the soft SUSY breaking terms:

$$V_{\text{Higgs}} = V_F + V_D + V_{\text{soft}},$$

(2.2)

From (2.1), the $F$-term contribution to the scalar potential is given by

$$V_F = |f + \lambda H_u H_d - \lambda H_u H_d|^2
\quad + m_1^2 (|H_u|^2 + |H_d|^2) + m_2^2 (|H_u|^2 + |H_d|^2)
\quad + |\lambda X_0 H_d - m_1 X_1|^2
\quad + |\lambda X_0 H_u + m_2 X_2|^2,$$

(2.3)

where the superscripts of the fields denote the electric charges. On the other hand, from Table \ref{tab:charge_assignments} we can derive the following $D$-term contribution of the Higgs sector:

$$V_D = \frac{1}{2} D_2^a D_2^a + \frac{1}{2} D_1 D_1,$$

(2.4)

where $D_2^a (a = 1, 2, 3)$ and $D_1$ represent the contributions of the Higgs sector to the $D$-terms of $SU(2)_L$ and $U(1)_Y$ vector superfields, and the summation over $a$ should be understood.
\[ D_2 = -g_2 (H_u^+ \tau^a H_u + H_d^+ \tau^a H_d + X_1^+ \tau^a X_1 + X_2^+ \tau^a X_2), \]
\[ D_1 = -\frac{g_1}{2} (|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2 - |X_1^0|^2 - |X_1^-|^2 + |X_2^0|^2 + |X_2^+|^2). \] (2.5)

where \( g_2 \) and \( g_1 \) are the gauge couplings of \( SU(2)_L \) and \( U(1)_Y \), and \( \tau^a \) denote \( SU(2)_L \) generators. The soft SUSY breaking terms for the Higgs fields are the usual ones mediated from the hidden sector. The soft terms which respect the symmetries are given as follows:

\[ V_{soft} = m_{H_u}^2 (|H_u^0|^2 + |H_u^+|^2) + m_{H_d}^2 (|H_d^0|^2 + |H_d^-|^2) + m_{X_0}^2 |X_0|^2 \]
\[ + m_{X_1}^2 (|X_1^0|^2 + |X_1^-|^2) + m_{X_2}^2 (|X_2^0|^2 + |X_2^+|^2) \]
\[ + b(H_u^+ H_d^- - H_u^0 H_d^0) + c.c., \] (2.6)

where \( m_i^2 (i = H_u, H_d, X_0, X_1, X_2) \) are soft scalar masses of the fields and \( b \) is a bilinear coupling for the Higgs fields.

### 2.2 Our vacuum

We now specify our vacuum to minimize the above potential. In order to demonstrate the idea of visible SUSY breaking (in the Higgs sector), we first analyze the limit of turning off the standard model gauge interactions and the soft breaking terms \( (\text{2.6}) \). Then, the model \( (\text{2.1}) \) just reduces to an O’Raifeartaigh model with global \( SU(2) \times U(1) \) symmetry spontaneously broken\(^8\) so that it is enough to deal with the \( F \)-term contribution \( (\text{2.3}) \). We assume that the vacuum expectation values of all the electrically charged fields are vanishing, which will be justified retrospectively by the mass spectrum around the vacuum. Then, the scalar potential is written as

\[ V_F = |f - \lambda H_u^0 H_d^0|^2 + m_1^2 |H_u^0|^2 + m_2^2 |H_d^0|^2 \]
\[ + |\lambda X_0 H_d^0 - m_1 X_1^0|^2 + |\lambda X_0 H_u^0 + m_2 X_2^0|^2. \] (2.7)

We emphasize here that the \( F \)-terms of all the neutral fields cannot be simultaneously taken to be zero in the vacuum, and hence SUSY is spontaneously broken in the Higgs sector. Since the soft SUSY breaking terms have been turned off, SUSY is broken in the visible sector by itself. This is, what we call, the visible SUSY breaking in the present scenario.

First, let us consider the minimization of the above scalar potential with respect to \( X_0 \),

\[ \frac{\partial V}{\partial X_0} = \lambda H_d^{0*} (\lambda X_0 H_d^0 - m_1 X_1^0) + \lambda H_u^{0*} (\lambda X_0 H_u^0 + m_2 X_2^0) = 0. \] (2.8)

\(^8\)In particular, the \( U(1)_R \) symmetry makes \( A \)-terms vanishing.

\(^9\)We temporarily require a coupling relation \( \lambda f > m_1 m_2 \) in this limit. In contrast, this kind of O’Raifeartaigh models as a hidden sector \( \text{[10]} \) requires \( \lambda f < m_1 m_2 \) in order to obtain a SUSY breaking vacuum without the gauge symmetry breaking.
We can choose $X_0 = X_1^0 = X_2^0 = 0$ as a solution to this equation, which also satisfies the similar minimization conditions about $X_1^0$ and $X_2^0$. Next, we proceed to the minimization about the ordinary Higgs fields $H_u^0, H_d^0$. The vacuum conditions are given by

\[
\frac{\partial V}{\partial H_u^{0*}} = -\lambda H_d^{0*} \left( f - \lambda H_u^0 H_d^0 \right) + m_u^2 H_u^0 = 0,
\]

\[
\frac{\partial V}{\partial H_d^{0*}} = -\lambda H_u^{0*} \left( f - \lambda H_u^0 H_d^0 \right) + m_d^2 H_d^0 = 0,
\]

where we have used the solution $X_0 = X_1^0 = X_2^0 = 0$. Up to symmetry rotation, the expectation values of the fields $H_u^0, H_d^0$ can be taken to be real. Then, the above conditions can be solved as follows:

\[
H_u^0 = \frac{1}{\lambda} \sqrt{\frac{m_2}{m_1}} (\lambda f - m_1 m_2), \quad H_d^0 = \frac{1}{\lambda} \sqrt{\frac{m_1}{m_2}} (\lambda f - m_1 m_2),
\]

where the global $SU(2) \times U(1)$ symmetry is broken to the remaining $U(1)$ symmetry. When the global symmetry is gauged as is done in the standard model, this corresponds to the EW symmetry breaking.

We are now in a position to analyze the full scalar potential \cite[p.22]{2.2} and specify our vacuum in which SUSY and the EW symmetry are broken. As described above, we here assume that the vacuum values of all the electrically charged fields are vanishing. Then, the relevant scalar potential is given by

\[
V = \left| f - \lambda H_u^0 H_d^0 \right|^2 + \left| \lambda X_0 H_d^0 - m_1 X_1^0 \right|^2 + \left| \lambda X_0 H_u^0 + m_2 X_2^0 \right|^2
\]

\[
+ \mu_1^2 |H_u|^2 + \mu_2^2 |H_d|^2 - (b H_u^0 H_d^0 + c.c.)
\]

\[
+ m_{X_0}^2 |X_0|^2 + m_{X_1}^2 |X_1^0|^2 + m_{X_2}^2 |X_2^0|^2
\]

\[
+ \frac{1}{8} g^2 \left( |H_u|^2 - |H_d|^2 - |X_1^0|^2 + |X_2^0|^2 \right),
\]

where we have defined mass parameters $\mu_1^2 = m_1^2 + m_{H_u}^2$, $\mu_2^2 = m_2^2 + m_{H_d}^2$, and a coupling $g^2 = g_1^2 + g_2^2$ to simplify the expression. Although the minimization condition about $X_0$ is slightly changed from \cite[p.28]{2.8} by the soft scalar mass term of $X_0$, we can keep choosing $X_0 = X_1^0 = X_2^0 = 0$ as a solution which simultaneously satisfies the minimization conditions about $X_1^0$ and $X_2^0$. Next, we consider the minimization conditions about the Higgs fields $H_u^0$ and $H_d^0$. We can again take the expectation values of these fields real without loss of generality, and express them as $H_u^0 = \frac{1}{\sqrt{2}} v \sin \beta$ and $H_d^0 = \frac{1}{\sqrt{2}} v \cos \beta$, as is done in the case of the MSSM. These vacuum values break the EW gauge symmetry to produce masses for the $W$ bosons and the $Z$ boson,

\[
m_W^2 = \frac{1}{4} g_2^2 v^2, \quad m_Z^2 = \frac{1}{4} g_2^2 v^2,
\]

\(7\)
where $v^2 \simeq (246 \text{ GeV})^2$ is required in order to obtain the observed values of the masses. Then, the minimization conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ result in the following expressions:

\begin{align*}
\mu_1^2 + \frac{1}{2} \lambda^2 v^2 \cos^2 \beta &= (\lambda f + b) \cot \beta + \frac{m_Z^2}{2} \cos 2\beta, \\
\mu_2^2 + \frac{1}{2} \lambda^2 v^2 \sin^2 \beta &= (\lambda f + b) \tan \beta - \frac{m_Z^2}{2} \cos 2\beta.
\end{align*}

(2.13)

Note that these conditions are very similar to the ones in the case of the MSSM. In fact, if we take the limit $\lambda \to 0$, the conditions appear the same as the corresponding equations of the MSSM. In this limit or in the MSSM, the soft SUSY breaking terms are essential for the correct EW symmetry breaking \[17\]. On the other hand, in our model, the correct symmetry breaking is realized even in the absence of the soft breaking terms for nonzero $\lambda$, since the effects of the soft SUSY breaking terms are solely contained in the expressions through the forms $\mu_1^2 = m_1^2 + m_{H_u}^2$, $\mu_2^2 = m_2^2 + m_{H_d}^2$ and $\lambda f + b$.

By means of (2.13), we obtain the following expression of the $Z$ boson mass in terms of the mass parameters $\mu_1$ and $\mu_2$:

\[ m_Z^2 = - \left( \frac{\mu_2^2 - \mu_1^2}{\cos 2\beta} + \mu_2^2 + \mu_1^2 \right). \]

(2.14)

As will be shown in the next section, in this model, we can obtain the lightest CP-even Higgs mass $m_h$ so as to evade the current mass bound $m_h > 114 \text{ GeV}^{10}$ even at the tree-level, as is reminiscent of the NMSSM. Thus, we do not need large soft scalar masses beyond 1 TeV to get large radiative corrections. Namely, the mass parameters $\mu_1, \mu_2$ can be near the EW scale, so that lesser fine-tuning is required to obtain the correct $Z$ boson mass in the above equation.

3 Mass spectrum

In this section, we show the mass spectrum of the Higgs sector fields in the visible SUSY and EW symmetry breaking vacuum discussed above. We first analyze the scalar masses. It turns out that the lightest CP-even Higgs mass can be above the current mass bound even at the tree-level, as is reminiscent of the NMSSM case. Then, we move to the discussion of the fermion masses. One of the neutralinos is massless at the tree-level, which would correspond to the goldstino in the visible SUSY breaking without soft SUSY breaking terms.

\[ ^{10}\text{We simply adopt this value for the Higgs boson in the standard model as a point of reference also in our estimate, though it does not necessarily apply in our case.} \]
3.1 The scalar masses

The scalar fields of the Higgs sector consist of 18 real field degrees of freedom. When the EW symmetry is broken, three of them are the would-be Nambu-Goldstone bosons which are eaten by the $Z$ and the $W^{\pm}$. The remaining 15 of them are the physical modes. We now expand the Higgs fields around their vacuum expectation values as

$$H_u^0 \rightarrow \frac{1}{\sqrt{2}} v \sin \beta + H_u^0,$$

$$H_d^0 \rightarrow \frac{1}{\sqrt{2}} v \cos \beta + H_d^0,$$

where the dynamical parts are further decomposed into CP-even and odd ones as follows:

$$H_u^0 = \frac{1}{\sqrt{2}} (\eta_1 + i\xi_1), \quad H_d^0 = \frac{1}{\sqrt{2}} (\eta_2 + i\xi_2).$$

Here, $\eta_{1,2}$ are CP-even scalar fields and $\xi_{1,2}$ are CP-odd ones.

First, we analyze the masses of the CP-odd parts. From (2.2), we can read the mass terms of the corresponding fields. The mass matrix for $\xi_1$ and $\xi_2$ is given by

$$M_{\xi}^2 = \frac{1}{2} \begin{pmatrix} \xi_1 & \xi_2 \end{pmatrix} \begin{pmatrix} (\lambda_f + b) \cot \beta & \lambda_f + b \\ \lambda_f + b & (\lambda_f + b) \tan \beta \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix},$$

which takes the same form as that of the MSSM except for the $\lambda_f$ terms. Diagonalizing this matrix, the eigenvalues turn out to be

$$m_{\chi^0}^2 = 0, \quad m_{A^0}^2 = \mu_1^2 + \mu_2^2 + \frac{1}{2} \lambda^2 v^2 = \frac{2(\lambda_f + b)}{\sin 2\beta}. \quad (3.4)$$

The massless field is the would-be Nambu-Goldstone mode eaten by the $Z$ boson. The corresponding mass eigenstates are expressed as

$$\chi^0 = \xi_1 \sin \beta - \xi_2 \cos \beta, \quad A^0 = \xi_1 \cos \beta + \xi_2 \sin \beta. \quad (3.5)$$

Next, we investigate the masses of the CP-even parts $\eta_1$ and $\eta_2$ of the neutral Higgs fields. The analysis of the mass terms proceeds in the same way as above. The mass matrix is given by

$$M_{\eta}^2 = \frac{1}{2} \begin{pmatrix} \eta_1 & \eta_2 \end{pmatrix} \begin{pmatrix} m_{\eta_1}^2 \cos^2 \beta + m_{\eta_2}^2 \sin^2 \beta & (\lambda^2 v^2 - m_{\eta_1}^2 - m_{\eta_2}^2) \sin \beta \cos \beta \\ (\lambda^2 v^2 - m_{\eta_1}^2 - m_{\eta_2}^2) \sin \beta \cos \beta & m_{\eta_1}^2 \sin^2 \beta + m_{\eta_2}^2 \cos^2 \beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}. \quad (3.6)$$

11Their expressions are summarized in the Appendix.
Figure 1: The mass of the lighter Higgs, $m_h$, in our model (red (solid) curves). The horizontal axis is the mass of $A^0$. The left panel is plotted with $\tan \beta = 3$, while the right panel is done with $\tan \beta = 10$. The horizontal (dotted) line denotes the current Higgs mass bound. The green (dashed) curve represents the case of the MSSM.

Then, the eigenvalues of this mass matrix are given by

$$m^2_{h,H} = \frac{1}{2} \left( m^2_{A^0} + m^2_Z \mp \sqrt{(m^2_{A^0} - m^2_Z)^2 + 4 \left( m^2_{A^0} - \frac{1}{2} \lambda^2 v^2 \right) \left( m^2_Z - \frac{1}{2} \lambda^2 v^2 \right) \sin^2 2\beta} \right),$$

which also take the same forms as those in the MSSM except for the terms dependent on $\lambda$. Note that this slight difference is, nonetheless, crucial for the lighter CP-even Higgs mass to evade the current experimental bound, as is the case for the NMSSM. In fact, in the limit of large $m_{A^0}$, the lighter Higgs mass can be written as

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{1}{2} \lambda^2 v^2 \sin^2 2\beta,$$

which is lifted up by the second term in the right-hand side for large $\lambda$ and small $\tan \beta$.

Let us now analyze the masses of the charged Higgs fields. The analysis of the mass terms again proceeds in the same way. The mass matrix for the charged Higgs fields is given by

$$\mathcal{M}_{H^\pm}^2 = (\mu_1^2 + \mu_2^2 + M_W^2) \begin{pmatrix} H_u^+ & H_d^- \end{pmatrix} \begin{pmatrix} \cos^2 \beta & \sin \beta \cos \beta \\ \sin \beta \cos \beta & \sin^2 \beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^- \end{pmatrix}.$$

Then, the eigenvalues of this mass matrix are obtained as

$$m_{\chi^\pm}^2 = 0, \quad m_{H^\pm}^2 = \mu_1^2 + \mu_2^2 + M_W^2 = m_{A^0}^2 + M_W^2 - \frac{1}{2} \lambda^2 v^2,$$

where $\chi^- = \chi^{+*}$ and $H^- = H^{+*}$. The massless modes $\chi^\pm$ are would-be Nambu-Goldstone modes eaten by the $W$ boson. We also note that the mass relation between the $A^0$ mass and
the masses of the $H^\pm$ coincides with that of the MSSM except for the term dependent on the coupling $\lambda$. The mass eigenstates are given by

$$
\chi^+ = H_u^+ \sin \beta - H_d^- \cos \beta, \quad H^+ = H_u^+ \cos \beta + H_d^- \sin \beta.
$$

(3.11)

Figure 1 shows the mass of the lighter CP-even Higgs, $m_h$, by varying the $A^0$ (red (solid) curves) for $\lambda = 1$ in the cases with $\tan \beta = 3$ (the left panel) and with $\tan \beta = 10$ (the right panel). The horizontal (dotted) lines represent the current experimental bound on the Higgs mass, $m_h > 114 \text{ GeV}$. For comparison, the mass of the lighter Higgs in the MSSM is shown (green (dashed) curves), in which $\lambda$ is taken to be zero. In the left panel, we see that the lighter CP-even Higgs mass can reach above the current experimental bound for $m_{A^0} > 220 \text{ GeV}$ in our model with $\tan \beta = 3$ unlike the MSSM case. We also see in the right panel that the Higgs mass in our model approaches that of the MSSM as $\tan \beta$ is increased. This behavior can be understood by means of (3.8).

In figure 2, we show the behavior of the masses of the lighter Higgs, $m_h$ (red (solid) curves), the heavier Higgs, $m_H$ (green (dashed) curves), and the charged Higgs, $m_{H^\pm}$ (blue (dotted) curves), in terms of $m_{A^0}$ for $\lambda = 1$. Here, $\tan \beta$ is fixed to $3$ (the left panel) and $10$ (the right panel), respectively. Both the panels imply that the masses of the charged Higgs are tachyonic for $m_{A^0}$ smaller than $150 \text{ GeV}$. This is due to the term dependent on $\lambda$ in (3.10). Similarly, one sees that the mass of the lighter Higgs becomes tachyonic for $\tan \beta = 3$ (the left panel) when $m_{A^0}$ is smaller than $130 \text{ GeV}$, while it does not for $\tan \beta = 10$ (the right panel). This is because the terms dependent on $\lambda$ in (3.7) are proportional to $\sin 2\beta$, which become smaller
Figure 3: The lighter CP-even Higgs mass $m_h$. The horizontal axis is the mass of $A^0$ and the vertical axis is the coupling $\lambda$. The left panel is plotted with $\tan \beta = 3$, while the right panel is done with $\tan \beta = 10$.

Figure 4: The heavier Higgs mass $m_H$. The left panel is plotted with $\tan \beta = 3$, while the right panel is done with $\tan \beta = 10$.

Figure 5: The charged Higgs masses $m_{H^\pm}$. The left panel is plotted with $\tan \beta = 3$, while the right panel is done with $\tan \beta = 10$. 
as tan \( \beta \) becomes larger. Their negative contribution to the lighter Higgs mass is small for larger tan \( \beta \), and hence the mass is positive for \( m_{A^0} < 130 \text{ GeV} \) with tan \( \beta = 10 \).

To see the dependence of the masses on the coupling \( \lambda \), we show the contour plots of the masses, \( m_h \) (figure 3), \( m_H \) (figure 4), and \( m_{H^\pm} \) (figure 5). In each figure, tan \( \beta = 3 \) for the left panel, tan \( \beta = 10 \) for the right panel, and values of the masses are indicated by a color bar aside. In figure 3, we see that the lighter Higgs mass becomes tachyonic (white region) for large \( \lambda \) and small \( m_{A^0} \). This is because the second term in the right-hand side of (3.7) becomes larger than the first term as \( \lambda \) is large. The region of tachyonic mass for tan \( \beta = 10 \) is smaller than that for tan \( \beta = 3 \), since the \( \lambda \) dependent terms are suppressed by sin\(^2\) 2\( \beta \). In the left panel of figure 3, it is seen that the lighter Higgs mass exceeds the current experimental bound in a large region of \( \lambda > 0.6 \). In figure 4, we can see that the heavier Higgs mass is less sensitive to \( \lambda \) and mainly determined by \( m_{A^0} \). The terms dependent on \( \lambda \) are significant only in a region of small \( m_{A^0} \) and large \( \lambda \). However, such a region is excluded by the lighter Higgs mass to be smaller than the experimental bound (or even tachyonic). In figure 5, similarly to the lighter Higgs mass, one sees that the charged Higgs mass becomes tachyonic for large \( \lambda \) (white region). To avoid the tachyonic mass, one can obtain an upper bound on \( \lambda \) from (3.10) as

\[
\lambda < g \sqrt{\frac{m_{A^0}^2 + m_W^2}{2m_Z^2}}. \tag{3.12}
\]

This bound is independent of tan \( \beta \) and therefore gives a stronger constraint on \( \lambda \) than that by the lighter Higgs mass\(^{12}\).

We now express the mass eigenstates of the neutral CP-even Higgs fields in terms of the mixing angle \( \alpha \) in the same way as is often done in the analysis of the MSSM as follows:

\[
h = \eta_1 \cos \alpha - \eta_2 \sin \alpha, \quad H = \eta_1 \sin \alpha + \eta_2 \cos \alpha. \tag{3.13}
\]

Here, \( h \) corresponds to the lighter mass eigenstate, while \( H \) corresponds to the heavier one. The mixing angle \( \alpha \) is given by

\[
\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{A^0}^2 + m_Z^2 - \lambda^2 v^2}{m_H^2 - m_h^2}, \quad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_{A^0}^2 - m_Z^2}{m_H^2 - m_h^2}, \tag{3.14}
\]

where the first relation takes the same form as that of the MSSM except for the \( \lambda^2 v^2 \) term, and the second one exactly coincides with that of the MSSM. In order to identify which Higgs boson is the standard-model-like one, we need to know which Higgs boson is more

\(^{12}\)This bound is conservative because we only require that \( m_{H^\pm} \) is positive. When we take into account the current experimental bound on \( m_{H^\pm} \), a more stringent constraint is obtained.
Figure 6: The neutral CP-even Higgs boson couplings with the standard model gauge fields. The left graph is plotted with \( \tan \beta = 3 \), while the right graph is done with \( \tan \beta = 10 \).

strongly coupled with the standard model gauge bosons and the matter fields. Higgs–gauge boson–gauge boson couplings are expressed as follows:

\[
\mathcal{L}_{hgg} = -g_2 m_W \sin(\alpha - \beta) h W^\dagger W^\mu - \frac{1}{2} g m_Z \sin(\alpha - \beta) h Z^\mu Z^\mu + g_2 m_W \cos(\alpha - \beta) H W^\dagger W^\mu + \frac{1}{2} g m_Z \cos(\alpha - \beta) H Z^\mu Z^\mu.
\]

Figure 6 shows the strength of these couplings for \( \lambda = 1 \) by varying the mass of the \( A^0 \) in the case with \( \tan \beta = 3 \) (the left panel) and \( \tan \beta = 10 \) (the right panel). The red (solid) curve represents the coupling of the lighter Higgs \( h \) with the gauge fields (by \( \sin(\alpha - \beta) \)), while the blue (dotted) curve represents that of the heavier Higgs \( H \) (by \( \cos(\alpha - \beta) \)). For comparison, we also show the same couplings in the case of the MSSM. The green (dashed) curve represents the coupling of the lighter Higgs in the MSSM, while the pink (dashed-dotted) curve denotes that of the heavier Higgs field. From the figure, we see that the coupling of the lighter Higgs in our model is smaller than that of the MSSM for small \( m_{A^0} \), and can be in anti-decoupling region for \( m_{A^0} < 180 \text{ GeV} \) in the \( \tan \beta = 3 \) case. We also see that the lighter CP-even Higgs field \( h \) is more strongly coupled with the standard model gauge fields than the heavier one \( H \) in the region of large \( A^0 \) mass, which is the same as in the MSSM.

Finally, we analyze the masses of the \( X \) scalar fields. Their mass terms derived from (2.2) are also summarized in the Appendix. The masses of the charged fields are given by

\[
m^2_{X^-} = m^2_1 + m^2_{X_1} - \frac{1}{2} m^2_Z \cos 2\theta_W \cos 2\beta,
\]

\[
m^2_{X^+} = m^2_2 + m^2_{X_2} + \frac{1}{2} m^2_Z \cos 2\theta_W \cos 2\beta,
\]

where \( \theta_W \) denotes the Weinberg angle, \( \sin^2 \theta_W \simeq 0.23 \). In order to analyze the masses of the
neutral fields, we define
\[ X_0 = \frac{1}{\sqrt{2}} (\sigma_0 + i\rho_0), \]
\[ X_1^0 = \frac{1}{\sqrt{2}} (\sigma_1 + i\rho_1), \]
\[ X_2^0 = \frac{1}{\sqrt{2}} (\sigma_2 + i\rho_2). \]  

Then, the mass matrix of the real parts \( \sigma \) is expressed as
\[ M^2_{\sigma} = \frac{1}{2} \begin{pmatrix} \sigma_0 & \sigma_1 & \sigma_2 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \lambda^2 v^2 + m_{X_0}^2 & -\frac{\lambda}{\sqrt{2}} m_1 v c_\beta & \frac{\lambda}{\sqrt{2}} m_2 v s_\beta \\ -\frac{\lambda}{\sqrt{2}} m_1 v c_\beta & m_1^2 + m_{X_1}^2 + \frac{1}{2} m_{Z_2}^2 c_2 \beta & 0 \\ \frac{\lambda}{\sqrt{2}} m_2 v s_\beta & 0 & m_2^2 + m_{X_2}^2 - \frac{1}{2} m_{Z_2}^2 c_2 \beta \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_1 \\ \sigma_2 \end{pmatrix}, \]

where \( s_\beta, c_\beta \) and \( c_{2\beta} \) denote \( \sin \beta, \cos \beta \) and \( \cos 2\beta \), respectively. As for the imaginary parts \( \rho \), their mass matrix takes the same form as the real part mass matrix (3.18). We present sample spectra of the \( X \) scalar masses in table 2 where the following four cases with \( \tan \beta = 3 \) (the left panel) and \( \tan \beta = 10 \) (the right panel) for \( \lambda = 1 \) are shown:

\[
\begin{align*}
(i) & \quad m_1 = m_2 = 300 \text{ GeV} \\
& \quad M_1 = M_2 = 300 \text{ GeV} \\
& \quad m_{X_0} = m_{X_1} = m_{X_2} = 300 \text{ GeV}, \\
& \quad m_1 = m_2 = 500 \text{ GeV} \\
& \quad M_1 = M_2 = 300 \text{ GeV} \\
& \quad m_{X_0} = m_{X_1} = m_{X_2} = 300 \text{ GeV}, \\
(ii) & \quad m_1 = m_2 = 300 \text{ GeV} \\
& \quad M_1 = M_2 = 300 \text{ GeV} \\
& \quad m_{X_0} = m_{X_1} = m_{X_2} = 500 \text{ GeV}, \\
& \quad m_1 = m_2 = 300 \text{ GeV} \\
& \quad M_1 = M_2 = 300 \text{ GeV} \\
& \quad m_{X_0} = 0, \ m_{X_1} = m_{X_2} = 300 \text{ GeV}. \\
\end{align*}
\]

Here, \( M_1 \) and \( M_2 \) are the soft SUSY breaking masses of the Bino and the Winos discussed in the next subsection (see (3.20)). In the table, \( m_{\sigma_1}, m_{\sigma_2}, m_{\sigma_3} \) denote (the square roots of) the three eigenvalues of the mass matrix (3.18), while \( m_{\rho_1}, m_{\rho_2}, m_{\rho_3} \) are those of the imaginary parts \( \rho \).
Table 2: The chargino masses, the neutralino masses, and the scalar masses of the $X$ fields in the four cases (i), (ii), (iii), (iv) explained in the main text. The left panel is plotted with $\tan \beta = 3$, while the right panel is done with $\tan \beta = 10$.

3.2 The fermion masses

Let us analyze the fermion masses of the Higgs sector. We assume that the gauginos have the following Majorana mass terms, which break $U(1)_R$ symmetry softly (presumably due to hidden sector dynamics):

\begin{align}
\mathcal{L}_{\text{gaugino}} &= -\frac{1}{2} M_2 \left( \tilde{W}_+ \tilde{W}_- + \tilde{W}_- \tilde{W}_+ \right) \\
&\quad - \frac{1}{2} M_2 \left( \tilde{W}^3 \tilde{W}^3 + \text{c.c.} \right) \\
&\quad - \frac{1}{2} M_1 \left( \tilde{B} \tilde{B} + \text{c.c.} \right),
\end{align}

where we have omitted the Gluino mass terms with no need for the present purposes. Certain modes in the Higgs sector have the mass mixings with the Winos and the Bino. These mixings come from the Yukawa-type coupling of a gaugino, a fermion, and its scalar superpartner which has a nonzero vacuum expectation value. The mixing terms of the Winos and the Higgsinos are given by

\begin{align}
\mathcal{L}_{SU(2)} &= -\sqrt{2} m_W \sin \beta \tilde{H}_u^+ \tilde{W}^- - \sqrt{2} m_W \cos \beta \tilde{H}_d^- \tilde{W}^+ \\
&\quad + m_Z \sin \beta \cos \theta_W \tilde{H}_u^0 \tilde{W}^3 - m_Z \cos \beta \cos \theta_W \tilde{H}_d^0 \tilde{W}^3 + \text{c.c.},
\end{align}
while the mixing terms of the Bino and the Higgsinos are given by

\[ \mathcal{L}_{U(1)} = -m_Z \sin \beta \sin \theta_W \tilde{H}_u^0 \tilde{B} + m_Z \cos \beta \sin \theta_W \tilde{H}_d^0 \tilde{B} + c.c. \tag{3.22} \]

With the aid of these mass terms and the superpotential (2.1) of the model, we can derive the chargino mass terms, which are expressed as

\[ \mathcal{L}_{\text{chargino}} = -\frac{1}{2} \psi^T \mathcal{M}_\tilde{C} \psi + c.c., \tag{3.23} \]

where \( \psi = (\tilde{W}^+, \tilde{H}^+_u, \tilde{X}^+_2, \tilde{W}^-, \tilde{H}^-_d, \tilde{X}^-_1) \) and the mass matrix \( \mathcal{M}_\tilde{C} \) is given by

\[ \mathcal{M}_\tilde{C} = \begin{pmatrix} 0 & M^T \\ M & 0 \end{pmatrix}, \quad M = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta & 0 \\ \sqrt{2}m_W \cos \beta & 0 & m_2 \\ 0 & m_1 & 0 \end{pmatrix}. \tag{3.24} \]

The mass matrix \( M \) can be diagonalized as

\[ L^* M R^+ = \begin{pmatrix} m_{\tilde{C}_1} & 0 & 0 \\ 0 & m_{\tilde{C}_2} & 0 \\ 0 & 0 & m_{\tilde{C}_3} \end{pmatrix}, \tag{3.25} \]

with the corresponding mass eigenstates given by

\[ \begin{pmatrix} \tilde{C}_1^+ \\ \tilde{C}_2^+ \\ \tilde{C}_3^+ \end{pmatrix} = R \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+_u \\ \tilde{X}^+_2 \end{pmatrix}, \quad \begin{pmatrix} \tilde{C}_1^- \\ \tilde{C}_2^- \\ \tilde{C}_3^- \end{pmatrix} = L \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^-_d \\ \tilde{X}^-_1 \end{pmatrix}, \tag{3.26} \]

where \( L \) and \( R \) are unitary matrices. We present sample mass spectra of the charginos in table 2 where we show four cases (i), (ii), (iii), (iv) given above with \( \tan \beta = 3 \) (the left panel) and \( \tan \beta = 10 \) (the right panel).

Finally, we analyze the masses of the neutralinos \( \psi^0 = (\tilde{B}, \tilde{W}^3, \tilde{H}^0, \tilde{H}^0_u, \tilde{X}^0_1, \tilde{X}^0_2, \tilde{X}^0_0) \). The neutralino mass terms are expressed as

\[ \mathcal{L}_{\text{neutralino}} = -\frac{1}{2} (\psi^0)^T \mathcal{M}_\tilde{N} \psi^0 + c.c., \tag{3.27} \]

where the mass matrix \( \mathcal{M}_\tilde{N} \) is given by

\[ \mathcal{M}_\tilde{N} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W & 0 & 0 & 0 \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W & 0 & 0 & 0 \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & 0 & 0 & -m_2 & -\frac{\lambda}{\sqrt{2}} s_\beta \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & 0 & 0 & -m_1 & 0 & -\frac{\lambda}{\sqrt{2}} c_\beta \\ 0 & 0 & -m_2 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\lambda}{\sqrt{2}} s_\beta & -\frac{\lambda}{\sqrt{2}} c_\beta & 0 & 0 & 0 \\ 0 & 0 & -\frac{\lambda}{\sqrt{2}} c_\beta & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.28} \]
with $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$. We also present sample mass spectra of the neutralinos in table 2, where we show four cases (i), (ii), (iii), (iv) given above with $\tan \beta = 3$ (the left panel) and $\tan \beta = 10$ (the right panel) for $\lambda = 1$. The table implies that the lightest neutralino $\tilde{N}_1$ is massless at the tree-level. This originates from the fact that the determinant of the mass matrix (3.28) is vanishing:

$$\det \mathcal{M}_{\tilde{N}} = 0.$$  

(3.29)

The massless mode would correspond to the goldstino mode in the visible SUSY breaking without soft SUSY breaking terms. In the full setup with hidden sector SUSY breaking in supergravity, it turns out to be a massive pseudo-goldstino. If the hidden sector was sequestered from our visible sector, the pseudo-goldstino mass would be twice the gravitino mass in accord with [19], whereas it may be orders of magnitude different from the gravitino mass in general due to higher order effects beyond the simple tree-level analysis. Anyhow, the visible sector pseudo-goldstino might be seen as a remarkable feature in the visible SUSY breaking scenario. Of course, we have more to investigate on higher order effects. For instance, the visible SUSY breaking in the Higgs sector also affects soft SUSY breaking pattern due to gauge mediation effects with the Higgses as messengers [21]. These and other features depend crucially on the hidden sector SUSY breaking and its mediation to the visible sector, in particular, its connection with Higgs interactions thereof.

## 4 A connection with hidden sector SUSY breaking

As an illustrative example of connecting the visible SUSY breaking in the Higgs sector to the higher-scale SUSY breaking in the hidden sector, we here present Giudice-Masiero-like effective operators [22] in our setup. This also serves as a sample case that the visible SUSY breaking is a cascade phenomenon induced by the hidden sector SUSY breaking. Let us consider both $F$-type and $D$-type SUSY breaking spurions representing the hidden sector effects:

$$S = \theta^2 F, \quad W_\alpha = \theta_\alpha D,$$

(4.1)

whose R-charges are 2 and 1, respectively.

Then the superpotential (2.1) comes from

$$\int d^4 \theta \left[ a_1 \frac{S^\dagger}{M} X_1 H_u + a_2 \frac{S^\dagger}{M} X_2 H_d + a_0 \frac{W_{\dot{\alpha}} W^{\dot{\alpha}}}{M^2} X_0 + c.c. \right],$$

(4.2)

13In this case, sleptons have tachyonic masses due to the anomaly mediation [18].

14Such a low-scale pseudo-goldstino, as well as gravitinos (see also footnote 3), might constitute extra radiation [20] in the early universe.
where $M$ is the mediation scale of the hidden sector SUSY breaking to the Higgs sector with $a_0, a_1, a_2$ as coupling constants. These terms result in the parameters

$$m_1 = a_1 \frac{F^+}{M}, \quad m_2 = a_2 \frac{F^+}{M}, \quad f = a_0 \frac{D^2}{M^2}.$$  \hspace{1cm} (4.3)$$

We can also obtain the soft SUSY breaking terms with the aid of terms like

$$\int d^4\theta \left[ \left( \frac{S^\dagger S}{M^2} H_u H_d + \text{c.c.} \right) + \frac{S^\dagger S}{M^2} \left( H_u^\dagger H_u + H_d^\dagger H_d + X_0^\dagger X_0 + X_1^\dagger X_1 + X_2^\dagger X_2 \right) \right],$$  \hspace{1cm} (4.4)$$

where the first term gives the $B_{\mu}$-term and the rest gives soft scalar masses of the Higgs sector in our model.

5 Conclusion

We have presented a supersymmetric extension of the standard model whose Higgs sector has spontaneous SUSY breaking even in the absence of the soft breaking terms from the usual hidden sector. This extension is along the lines of general perspectives such that the Higgs sector may be a window to some unknown physics and SUSY breaking may be ubiquitous even in the visible sector. The current experimental bound for the lighter CP-even Higgs mass can be evaded even at the tree-level, which is reminiscent of the NMSSM. The pseudo-goldstino lies in the visible sector since the corresponding SUSY breaking is visible in the Higgs sector.

Since the scale of the visible SUSY breaking can be near the EW scale, it might be possible to observe the breaking dynamics rather directly in future experiments. It may be interesting to analyze new decay channels of the Higgs fields in such a model. We have regarded the current experimental bound $m_h > 114$ GeV for the lighter CP-even Higgs field as a point of reference in our consideration. However, this bound might be totally inadequate for our model since, among others, decays of Higgs particles beyond the standard model have not been taken into account. In this connection, the production and detection of the pseudo-goldstino mode in the Higgs sector is another interesting experimental challenge.

We have restricted ourselves to the vacuum in our model that has a desired breaking pattern of the visible SUSY and the EW symmetry in this paper. In the MSSM and its cousins, thorough analyses of the potentially dangerous directions in their field spaces have

\footnote{We have not included a term like $S^\dagger X_0$ without $M$ suppression. If the $S$ has a non-vanishing scalar component, we may do without the $D$-type SUSY breaking spurion by replacing the $W_\alpha$-dependent term with a term like $S^\dagger S X_0/M^2$.}
been carried out [23]. Our extension might have charge and/or color breaking minima in the
landscape of vacua, which is to be further examined.

We have not specified the details of the hidden sector SUSY breaking in the present
analyses mainly at the tree-level, though it is intriguing to study connections between the
visible SUSY breaking in the Higgs sector and the hidden sector SUSY breaking in a variety
of mediation mechanisms. Since the Higgs sector is largely unknown experimentally, and
even theoretically, we often encounter puzzles such as $\mu$ and $B\mu$ problems in the MSSM with
hidden sector SUSY breaking, various possibilities concerning the Higgs sector and its possible
extensions may deserve open-minded investigations.

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Appendix

Here, we summarize the mass terms of the Higgs sector fields derived from the scalar potential
(2.2). We can read the mass matrices presented in the main text from these mass terms.

The charged Higgs mass terms

The mass terms from the $F$-term contribution to the scalar potential $V_F$ are given by

$$m_1^2 |H_u^+|^2 + m_2^2 |H_d^-|^2 + \lambda \left(f - \frac{1}{2} \lambda v^2 \sin \beta \cos \beta \right) (H_u^+ H_d^- + \text{c.c.}),$$

where we have used the vacuum expectation values of the Higgs fields $H_u^0 = \frac{1}{\sqrt{2}} v \sin \beta$, $H_d^0 = \frac{1}{\sqrt{2}} v \cos \beta$, and the redefinitions of the scalar fields (3.1). The $D$-term contribution to the
charged Higgs mass terms is given by

$$-\frac{1}{8} g^2 v^2 \cos 2\beta (|H_u^+|^2 - |H_d^-|^2) + \frac{1}{4} g_2^2 v^2 \cos^2 \beta |H_u^+|^2 + \frac{1}{4} g_2^2 v^2 \sin^2 \beta |H_d^-|^2$$

$$+ \frac{1}{4} g_2^2 v^2 \sin \beta \cos \beta (H_u^+ H_d^- + \text{c.c.}).$$
The contribution from the soft SUSY breaking terms is given by

\[ m_{H_u}^2 |H_u^+|^2 + m_{H_d}^2 |H_d^-|^2 + b (H_u^+ H_d^- + c.c.) . \]

**The neutral Higgs mass terms**

The mass terms from the $F$-term contribution to the scalar potential $V_F$ are given by

\[
\begin{align*}
&\left( m_1^2 + \frac{1}{2} \lambda^2 v^2 \cos^2 \beta \right) |H_u^0|^2 + \left( m_2^2 + \frac{1}{2} \lambda^2 v^2 \sin^2 \beta \right) |H_d^0|^2 \\
&+ \frac{1}{2} \lambda^2 v^2 \sin \beta \cos \beta \left( H_u^0 H_d^0 + c.c. \right) \\
&- \lambda \left( f - \frac{1}{2} \lambda v^2 \sin \beta \cos \beta \right) \left( H_u^0 H_d^0 + c.c. \right).
\end{align*}
\]

The contribution from the $D$-term $V_D$ is given by

\[
\frac{1}{8} g^2 \left[ v^2 \left( \sin^2 \beta - \cos^2 \beta \right) \left( |H_u^0|^2 - |H_d^0|^2 \right) \\
+ \frac{1}{2} \left( \sin \beta \left( H_u^0 + H_u^0 \ast \right) - \cos \beta \left( H_u^0 + H_u^0 \ast \right) \right)^2 \right].
\]

The soft SUSY breaking contribution is given by

\[ m_{H_u}^2 |H_u^0|^2 + m_{H_d}^2 |H_d^0|^2 - b (H_u^0 H_d^0 + c.c.) . \]

**The scalar mass terms of the $X$ fields**

The mass terms of the $X$ scalar fields are given by

\[
\begin{align*}
&\left( \frac{1}{2} \lambda^2 v^2 + m_{X_0}^2 \right) |X_0|^2 \\
&+ \left( m_1^2 + m_{X_1}^2 + \frac{1}{2} m_Z^2 \cos 2 \beta \right) |X_1^0|^2 + \left( m_2^2 + m_{X_2}^2 - \frac{1}{2} m_Z^2 \cos 2 \beta \right) |X_2^0|^2 \\
&+ \left( m_1^2 + m_{X_1}^2 - \frac{1}{2} m_Z^2 \cos 2 \theta_W \cos 2 \beta \right) |X_1^-|^2 \\
&+ \left( m_2^2 + m_{X_2}^2 + \frac{1}{2} m_Z^2 \cos 2 \theta_W \cos 2 \beta \right) |X_2^-|^2 \\
&- \lambda \left( m_1 v \cos \beta \left( X_0^* X_1^0 + X_0 X_1^0 \ast \right) + \lambda \left( m_2 v \sin \beta \left( X_0^* X_2^0 + X_0 X_2^0 \ast \right) \right). \right.
\]
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