$\mathcal{R}$-parity violation in $\mathcal{F}$-theory

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Outline

\( \mathcal{R} \)-Part:

\( \mathcal{R} \)-Parity, RPV-MSSM, Proton Decay, Motivation

\( \mathcal{F} \)-Part:

\( \mathcal{F} \)-theory, \( SO(12) \) point, Plots & Numerics, Summary

M. C. Romão, AK, S. F. King, G.K. Leontaris, A. K. Meadowcroft:

10.1007/\textit{JHEP}11(2016)081
Intro & Motivation
\( \mathcal{R} \)-parity

\( \star \) MSSM -\( \mathcal{R} \) parity violation (RPV or \( \mathcal{R} \)) superpotential:

\[
W_{RPV} = \mu_i H_u L_i + \frac{1}{2} \lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_j d_k^c + \frac{1}{2} \lambda''_{ijk} u_i^c d_j^c d_k^c
\]

\( \star \) Add a new discrete symmetry to eliminate these terms, called "\( \mathcal{R} \)-parity"

(Farrat & Fayet, Phys. Lett. 76B (1978) 575–579.)

\[
P_R = (-1)^{3(B-L)+2s}
\]

- \( P_R = +1 \) for Standard Model (SM) particles.
- \( P_R = -1 \) for SUSY particles.
Matter parity

\[ W_{RPV} = \mu_i H_u L_i + \frac{1}{2} \lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \frac{1}{2} \lambda''_{ijk} u^c_i d^c_j d^c_k \]

\[ \begin{aligned} &\text{B violation} \quad \{ \quad \{ \quad \{ \end{aligned} \]
\[ \begin{aligned} &\text{L violation} \quad \{ \quad \{ \end{aligned} \]

★ An alternative symmetry with the same physical results is ”Matter parity” :

- \( (L_i, e^c_i, Q_i, u^c_i, d^c_i) \rightarrow P_M = -1 \)
- \( (H_u, H_d) \rightarrow P_M = +1 \)

★ This forbids all terms with an odd power of matter fields and thus forbids all the terms in \( W_{RPV} \).
RPV-SUSY

★ Plethora of new couplings (=48), provide a rich phenomenology:

- Single s-particle production is allowed.
- LSP is unstable (decays to leptons or jets)

(H. Dreiner et al: 1205.0557, Review: R.Barbier et al hep-ph/0406039, LHC-Run I Review: A. Redelbach, arXiv:1512.05956)

Figure: Examples of RPV processes: (a) Proton Decay via $\lambda''_{112}$ and $\lambda'_{112}$, (b) Tau decay via two $\lambda_{13k}$ insertions, (c) Neutralino decay via $\lambda'_{111}$. 
Proton decay

- Proton decay (PD) requires both $\mathcal{L}$ and $\mathcal{B}$.

$$\Gamma(p \rightarrow \pi^0 e^+) \sim |\lambda_1\lambda_{12}\lambda_{11}'| \frac{m_{\text{proton}}^5}{\tilde{m}_{sR}^4} < \frac{1}{10^{33}\text{yr}} \Rightarrow$$

$$|\lambda_1\lambda_{12}| < 5 \times 10^{-27} \left(\frac{\tilde{m}_{sR}}{1\text{TeV}}\right)^2.$$  

- Very strict bound $\rightarrow$ at least one of the couplings is zero.

  - Only B conservation $\rightarrow$ $\mathcal{L}$MSSM
  - Only L conservation $\rightarrow$ $\mathcal{B}$MSSM

(Dimopoulos et al, doi:10.1016/0370-2693(88)91418-9)

- Baryon-parity and lepton-parity are two possible solutions to maintain a stable proton and allow for RPV.

Example $\rightarrow$
**Motivation**

( AK, S.F.King, G.K.Leontaris, A.K.Meadowcroft, doi : 10.1007/JHEP10(2015)041 )

| Low Energy Spectrum          | $D_4$ rep | $U(1)_{t_5}$ | $Z_2$ |
|------------------------------|-----------|--------------|-------|
| $Q_3, u_3^c, e_3^c$         | 1$^+-$    | 0            | $-$   |
| $u_2^c$                     | 1$^{++}$  | 1            | $+$   |
| $u_1^c$                     | 1$^{++}$  | 0            | $+$   |
| $Q_{1,2}, e_{1,2}^c$        | 2         | 0            | $-$   |
| $L_i, d_i^c$                | 1$^+-$    | 0            | $-$   |
| $\nu_3^c$                   | 1$^+-$    | 0            | $-$   |
| $\nu_{1,2}^c$               | 2         | 0            | $-$   |
| $H_u$                       | 1$^{++}$  | $-1$         | $+$   |
| $H_d$                       | 1$^{++}$  |              |       |

Table: Low energy spectrum of a $SU(5) \times D_4 \times U(1)$ F-theory inspired model with a geometric parity. The fields $u_{1,2}^c$ have different assignment in comparison with the conventional matter parity. As a result $\bar{B}$ terms: $u_1^c d_j^c d_k^c \rightarrow$ neutron-antineutron oscillations (Goity & Sher)
RPV in F-theory?

★ So far in F-theory... plethora of works on $SU(5)$ Yukawa couplings
(Vafa et al, Ibanez & Font, Hayashi et al, Leontaris & Ross, Palti et al, Marchesano et al....)

\[
10 \times 10 \times 5_H \rightarrow y_{top} \quad \checkmark
\]

\[
10 \times \bar{5}_M \times \bar{5}_H \rightarrow y_{bottom}, \quad y_{tau} \quad \checkmark
\]

★ What about RPV couplings...

\[
10 \times \bar{5}_M \times \bar{5}'_M \rightarrow y_{RPV} \quad \ldots?\]

★ A first estimation: we expect a similar behavior to $y_{bottom}$ coupling.
$\mathcal{F}$-theory, the $SO(12)$ point and RPV

M. C. Romão, AK, S. F. King, G.K. Leontaris, A. K. Meadowcroft: 10.1007/JHEP11(2016)081
\textbf{\(\mathcal{F}\)-theory (Basic)}

★ \textbf{Geometrisation of Type II-B superstring} \hfill (Vafa 1996)

II-B: closed string spectrum obtained by combining left and right moving open strings with NS and R-boundary conditions.

★ \textbf{Geometrical Picture}:

- Take the 6-d compact space to be CY 3-fold base \(B_3\).

- Associate a torus \(\tau = C_0 + \frac{\nu}{g_s}\) at each point of \(B_3\).

\[ \Rightarrow \text{Internal space elliptically fibered CY 4-fold } \mathcal{X} \text{ over } B_3 \]

\[ \hookrightarrow \text{F-theory defined on the background } \mathcal{R}^{3,1} \times \mathcal{X} \]
Red points: pinched torus $\mapsto 7$-branes $\perp B_3$. 
Singularities

★ Fibration is described by the Weierstraß Equation

\[ y^2 = x^3 + f(z)x + g(z) \]  \hspace{1cm} (1)

★ ★ The fiber degenerates at the zeros of the discriminant

\[ \Delta = 4f^3 + 27g^2 \]

★ ★ ★ For each point of \( B_3 \), eq(1) describes a torus labeled by \( z \).

★ ★ ★ ★ The fiber degenerates at the zeros of the discriminant

\[ \Delta = 0 \implies \text{singularity of internal mainfold} \]
Singularities & Gauge Symmetry

* Type of Manifold *singularity* is specified by the vanishing order of $f(z)$, $g(z)$ polynomials

** Singularities are classified in terms of *ADE* Lie groups. \textsuperscript{(Kodaira 1968)}

\[ \chi\text{-Singularities} \leftrightarrow \text{Gauge Symmetry} \]

*** The maximum symmetry enhancement is $E_8$,

\[
E_8 \rightarrow G_{GUT} \times SU(n)_{\perp}
\]

with $G_{GUT} = E_6, \ SO(10), \ SU(5)$ for $n = 3, 4, 5$. 
in F-theory: 7-branes wrap certain class of 'internal' 2-complex dim. surface $S$ associated to gauge group $G_S$ (here taken to be $SU(5)$).
Matter resides along intersections with other 7-branes...

Along a matter curve $\Sigma$ gauge symmetry is enhanced...
Yukawa couplings at Triple intersections...

Yukawa Coupling

\[ \Sigma_1 \quad \Sigma_2 \quad \Sigma_3 \]

\[ \text{SU}(5) \quad \mathbb{Z}_2 \quad \mathbb{Z}_1 \]

---

gauge symmetry ... further ... enhanced!
SU(5) : Singularity enhancement

★ Matter curves accommodating $\bar{5}$ are associated with $SU(6)$

\[
\Sigma_{\bar{5}} = S \cap S_{\bar{5}} \Rightarrow \quad SU(5) \rightarrow SU(6)
\]

\[
ad_{SU_6} = 35 \rightarrow \quad 24_0 + 1_0 + 5_6 + \bar{5}_{-6}
\]

★ Matter curves accommodating $10$ are associated with $SO(10)$

\[
\Sigma_{10} = S \cap S_{10} \Rightarrow \quad SU(5) \rightarrow SO(10)
\]

\[
ad_{SO_{10}} = 45 \rightarrow \quad 24_0 + 1_0 + 10_4 + \bar{10}_{-4}
\]

★ Further enhancement in triple intersections → Yukawas:

\[
SO(10) \equiv E_5 \Rightarrow \quad E_6 \rightarrow 10 \times 10 \times 5
\]

\[
SU(6) \Rightarrow \quad SO(12) \rightarrow 10 \times \bar{5} \times \bar{5}
\]

⇒ RPV couplings → SO(12) point enhancement
\( S \) 

- \( \mu \)-term
- \( E6 \) point (top Yukawa)
- \( SO(12) \) point (bottom Yukawa)

- \( 5H \)
- \( 10 \)
- \( 5M \)
- \( 5M' \)
Effective theory

★ The 4-d theory can be obtained by integrating out the 8-d theory over $S$

$$W = m_*^4 \int_S \text{Tr}(F \wedge \Phi)$$

- $F = dA - iA \wedge A$ is the field-strength of the gauge vector boson $A$.
- $\Phi$ is $(2,0)$-form on $S$.
- $m_* : F$-theory characteristic scale

★★ Away from the enh. point $\Phi$ breaks $SO(12) \rightarrow$ GUT group $SU(5)$:

$$SO(12) \rightarrow SU(5) \times U(1) \times U(1)$$
Fluxes

★ We also need fluxes

- \( \langle F \rangle \rightarrow \) chirality on the matter curves
- \( \langle F_Y \rangle \rightarrow \) breaks the GUT down to SM

★★ Collectively the total flux is:

\[
\langle F_{\text{total}} \rangle = i(dz_2 \wedge d\bar{z}_2 - dz_1 \wedge d\bar{z}_1)Q_P \\
+ i(dz_1 \wedge d\bar{z}_2 + dz_2 \wedge d\bar{z}_1)Q_S \\
+ i(dz_2 \wedge d\bar{z}_2 + dz_1 \wedge d\bar{z}_1)M_{z_1z_2}Q_F
\]  

(2)

with the definitions

\[
Q_P = MQ_F + \tilde{N}_Y Q_Y
\]  

(3)

\[
Q_S = N_a Q_{z_1} + N_b Q_{z_2} + N_Y Q_Y
\]  

(4)
| Sector | SM | $q_F$ | $q_{z_1}$ | $q_{z_2}$ | $q_S$ | $q_P$ |
|--------|----|-------|-----------|-----------|-------|-------|
| $a_1$  | $(\bar{3}, 1)_{-\frac{1}{3}}$ | 1     | -1        | 0         | $-N_a - \frac{1}{3} N_Y$ | $M - \frac{1}{3} \tilde{N}_Y$ |
| $a_2$  | $(1, 2)_{\frac{1}{2}}$     | 1     | -1        | 0         | $-N_a + \frac{1}{2} N_Y$ | $M + \frac{1}{2} \tilde{N}_Y$ |
| $b_1$  | $(\bar{3}, 1)_{\frac{2}{3}}$ | -1    | 0         | 1         | $N_b + \frac{2}{3} N_Y$ | $-M + \frac{2}{3} \tilde{N}_Y$ |
| $b_2$  | $(3, 2)_{-\frac{1}{6}}$    | -1    | 0         | 1         | $N_b - \frac{1}{6} N_Y$ | $-M - \frac{1}{6} \tilde{N}_Y$ |
| $b_3$  | $(1, 1)_{-1}$              | -1    | 0         | 1         | $N_b - N_Y$             | $-M - \tilde{N}_Y$              |
| $c_1$  | $(\bar{3}, 1)_{-\frac{1}{3}}$ | 0     | 1         | -1        | $N_a - N_b - \frac{1}{3} N_Y$ | $-\frac{1}{3} \tilde{N}_Y$ |
| $c_2$  | $(1, 2)_{\frac{1}{2}}$     | 0     | 1         | -1        | $N_a - N_b + \frac{1}{2} N_Y$ | $\frac{1}{2} \tilde{N}_Y$ |

**Table**: Complete data of sectors present in the three curves crossing in an SO(12) enhancement point considering the effects of non-vanishing fluxes.
Coupling coefficients

★ Matter fields arise as fluctuations of the 8-dim fields

$$\Psi_{8D} = \phi_{4D} \times \psi_{\text{int}}$$

★★ Operator coefficients arise as overlaps of wavefunctions

$$\int_{8D} \Psi_1 \Psi_2 \Psi_3 = \int_{4D} \phi_1 \phi_2 \phi_3 (\int_S \psi_1 \psi_2 \psi_3)$$

★★★ Solve the eom for the zero mode wavefunctions (Font et al, 2012)

(Heckman et al, 2008)
Wavefunctions

★ Wavefunctions (WF) in holomorphic gauge:

\[ \tilde{\psi}_{10M}^{(b) \text{hol}} = \tilde{v}(b) \chi_{10M}^{(b) \text{hol}} = \tilde{v}(b) \kappa_{10M}^{(b)} e^{\lambda_b \bar{z}_2 (\bar{z}_2 - \zeta_b \bar{z}_1)} \]

\[ \tilde{\psi}_{5M}^{(a) \text{hol}} = \tilde{v}(a) \chi_{5M}^{(a) \text{hol}} = \tilde{v}(a) \kappa_{5M}^{(a)} e^{\lambda_a z_1 (\bar{z}_1 - \zeta_a \bar{z}_2)} \]

\[ \tilde{\psi}_{5H}^{(c) \text{hol}} = \tilde{v}(c) \chi_{5H}^{(c) \text{hol}} = \tilde{v}(c) \kappa_{5H}^{(c)} e^{(z_1 - z_2)(\zeta_c \bar{z}_1 - (\lambda_c - \zeta_c) \bar{z}_2)} \]

where \( \lambda_\rho \) is the smallest eigenvalue of the matrix

\[
\begin{pmatrix}
-\rho_P & q_S & im^2q_{z_1} \\
q_S & -\rho_P & im^2q_{z_2} \\
-im^2q_{z_1} & im^2q_{z_2} & 0
\end{pmatrix}
\]

\( m_\rho \) is the smallest eigenvalue of the matrix

\[
\begin{pmatrix}
-\rho_P & q_S & im^2q_{z_1} \\
q_S & -\rho_P & im^2q_{z_2} \\
-im^2q_{z_1} & im^2q_{z_2} & 0
\end{pmatrix}
\]
\( b, \tau \) and RPV couplings

\[ y_{b, \tau} = \pi^2 \left( \frac{m_*}{m} \right)^4 t_{abc} \kappa_1^{(b)} \kappa_5^{(a)} \kappa_5^{(c)} \]

\[ y_{RPV} = \pi^2 \left( \frac{m_*}{m} \right)^4 t_{abc} \kappa_1^{(b)} \kappa_5^{(a)} \kappa_5^{(c)} \]

As we observe the flux dependence is hidden on the normalization factors.
Normalization factors

star fixed by imposing canonical kinetic terms

\[ 1 = 2m^4 \|v^{(e)}\|^2 \int (\chi^{(e)})^*_i \chi^{(e)}_i d\text{Vol}_S \]

double star partial results...

\[ |\kappa_{10M}^{(b)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{q_P(b)(-2\lambda_b + q_P(b)(1 + \zeta_b^2))}{\lambda_b(1 + \zeta_b^2) + m^4} \]

\[ |\kappa_{5H}^{(c)}|^2 = -4\pi g_s \sigma^2 \cdot \frac{2(q_P(c) + \zeta_c)(q_P(c) + 2\zeta_c - 2\lambda_c) + (q_S(c) + \lambda_c)^2}{\zeta_c^2 + (\lambda_c - \zeta_c)^2 + m^4} \]

where \( \sigma = (m/m_{st})^2 \), with \( m_{st} \) the string scale.
Numerical analysis

★ The couplings can be written as:

\[
y_{b,\tau} = 2g_s^{1/2} \sigma \ y'_{b,\tau}
\]

\[
y_{RPV} = 2g_s^{1/2} \sigma \ y'_{RPV}
\]

★★ five parameters - \(N_a, N_b, M, N_Y\) and \(\tilde{N}_Y\)

★★★ constraint: elimination of Higgs colour triplets \(\rightarrow\) (Font & Ibanez et al.)

\[N_b = N_a - \frac{1}{3} N_Y\]

★★★★ At the GUT scale \(Y_{\tau}/Y_b = 1.37 \pm 0.1 \pm 0.2\) (G.Ross & M. Serna 2008)
$Y_\tau / Y_b$

**Figure**: Ratio between bottom Yukawa and tau Yukawa couplings, shown as contours in the plane of local fluxes. The requirement for chiral matter and absence of coloured Higgs triplets fixes $N_b = N_a - \frac{1}{3} N_Y$
**RPV (in absence of \( N_Y \) and \( \tilde{N}_Y \))**

Figure: Dependency of the RPV coupling (in units of \( 2g_s^{1/2}\sigma \)) on the \((N_a, N_b)\)-plane, in absence of hypercharge fluxes and for different values of \( M \).

Top: left \( M = 0.5 \), right \( M = 1.0 \). Bottom: left \( M = 2.0 \), right \( M = 3.0 \).
RPV allowed regions

$N_{\tilde{Y}} = +1$ and $N_Y = +1$

$N_{\tilde{Y}} = +1$ and $N_Y = -1$

$N_{\tilde{Y}} = -1$ and $N_Y = +1$

$N_{\tilde{Y}} = -1$ and $N_Y = -1$
Figure: Allowed regions in the parameter space for different RPV couplings with $\tilde{N}_Y = -N_Y = 1$. We have also include the corresponding contours for the $u^c d^c d^c$ operator (left) and $LL e^c$ (right).
Figure : Allowed regions in the parameter space for different RPV couplings with $N_Y = -\tilde{N}_Y = 1$. We have also include the corresponding contours for the $u^c d^c d^c$ operator (left) and $QLd^c$ (middle and right). The scripts a, b and c refer to which sector each state 'lives'.

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Bounds

* partial results in (Allanach, Dedes & Dreiner 1999)

| \(ijk\) | \(\lambda_{ijk}\) | \(\lambda'_{ijk}\) | \(\lambda''_{ijk}\) |
|--------|-----------------|-----------------|------------------|
| 111    | -               | \(1.5 \times 10^{-4}\) | -                |
| 112    | -               | \(6.7 \times 10^{-4}\) | \(4.1 \times 10^{-10}\) |
| 113    | -               | 0.0059          | \(1.1 \times 10^{-8}\) |
| 121    | 0.032           | 0.0015          | \(4.1 \times 10^{-10}\) |
| 122    | 0.032           | 0.0015          | -                |
| 123    | 0.032           | 0.012           | \(1.3 \times 10^{-7}\) |
| 131    | 0.041           | 0.0027          | \(1.1 \times 10^{-8}\) |
| 132    | 0.041           | 0.0027          | \(1.3 \times 10^{-7}\) |
| 133    | 0.0039          | \(4.4 \times 10^{-4}\) | -                |
| 211    | 0.032           | 0.0015          | -                |
| 212    | 0.032           | 0.0015          | (1.23)           |
| 213    | 0.032           | 0.016           | (1.23)           |
| 231    | 0.046           | 0.0027          | (1.23)           |
| 232    | 0.046           | 0.0028          | (1.23)           |
| 233    | 0.046           | 0.048           | -                |
| 311    | 0.041           | 0.0015          | -                |
| 312    | 0.041           | 0.0015          | 0.099            |
| 313    | 0.0039          | 0.0031          | 0.015            |
| 321    | 0.046           | 0.0015          | 0.099            |
| 322    | 0.046           | 0.0015          | -                |
| 333    | -               | 0.091           | -                |
For $\tan\beta = 5$, $y_b(M_{GUT}) \simeq 0.03$.

(a) $\lambda L \bar{L} e^c$ region with $N_Y = 10$, $\tilde{N}_Y = 0.1$

(b) $\lambda L \bar{L} e^c$ region with $N_Y = -10$, $\tilde{N}_Y = 0.1$
(c) $\lambda' Q L d^c$ region with $N_Y = 0.1$, $\tilde{N}_Y = (d) \lambda'' u^c d^c d^c$ region with $N_Y = -0.1$, $\tilde{N}_Y = -10$
Summary

* $\mathcal{R}$-parity violation (RPV) in semi-local $\mathcal{F}$-theory SU(5) models is a generic feature without Proton Decay.

* RPV couplings in local $\mathcal{F}$-theory SU(5) models can be study in a $SO(12)$ point of enhancement.

* At the GUT scale may be naturally suppressed over large regions of the parameter space.

* $LLe^c$ and $u^c d^c d^c$ (especially with the heaviest generations) type of RPV interactions from $\mathcal{F}$-theory are expected to be within current bounds.

* Study of other cases where generations resides in the same matter curve or mixed (2+1) cases.
Thank you!
For the bottom and tau coupling we have:

\[ y_{b,\tau} = m_*^4 \, t_{abc} \int_S \det(\bar{\psi}^{(b)\text{hol}}_{10M}, \psi^{(a)\text{hol}}_{5M}, \psi^{(c)\text{hol}}_{5H}) d\text{Vol}_S \]

\[ = m_*^4 \, t_{abc} \, \det(\bar{\nu}^{(b)}, \bar{\nu}^{(a)}, \bar{\nu}^{(c)}) \int_S \chi^{(b)\text{hol}}_{10M} \chi^{(a)\text{hol}}_{5M} \chi^{(c)\text{hol}}_{5H} d\text{Vol}_S. \]

A similar formula can be written down for the RPV coupling:

\[ y_{\text{RPV}} = m_*^4 \, t_{abc} \int_S \det(\bar{\psi}^{(b)\text{hol}}_{10M}, \psi^{(a)\text{hol}}_{5M}, \psi^{(c)\text{hol}}_{5'M}) d\text{Vol}_S \]

\[ = m_*^4 \, t_{abc} \, \det(\bar{\nu}^{(b)}, \bar{\nu}^{(a)}, \bar{\nu}^{(c)}) \int_S \chi^{(b)\text{hol}}_{10M} \chi^{(a)\text{hol}}_{5M} \chi^{(c)\text{hol}}_{5'M} d\text{Vol}_S. \]

\[ t_{abc} \] is a group factor. Computing the Integral we have
The presence of a chiral state in a sector with root $\rho$ is given if $\det m_\rho > 0$. Depending on the sign of $N_Y$, the above conditions define different regions of the flux density parameter space.

| Region                      | $M < \frac{\tilde{N}_Y}{3}$ | $\frac{\tilde{N}_Y}{3} < M < \frac{-\tilde{N}_Y}{6}$ | $\frac{-\tilde{N}_Y}{6} < M < -\tilde{N}_Y$ | $-\tilde{N}_Y < M$ |
|-----------------------------|-----------------------------|--------------------------------------------------|---------------------------------|------------------|
| $(N_a - N_b) < -\frac{N_Y}{2}$ | None                       | None                                            | None                           | None             |
| $-\frac{N_Y}{2} < (N_a - N_b) < \frac{N_Y}{3}$ | None                       | None                                            | $QLd^c$                         | $QLd^c$, $LLe^c$ |
| $\frac{N_Y}{3} < (N_a - N_b)$ | None                       | $u^c d^c d^c$                                    | $QLd^c$, $u^c d^c d^c$          | All              |

**Table**: Regions of the parameter space and the respective RPV operators supported for $\tilde{N}_Y \leq 0$, $N_Y > 0$