D-Branes at angle in pp-wave Background

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Abstract

We show the existence of classical solutions of a system of $D3$-branes oriented at an arbitrary angle with respect to each other, in a six dimensional pp-wave background obtained from $AdS_3 \times S^3 \times R^4$, with $NS - NS$ and $R - R$ 3-form field strength. These $D$-brane bound states are shown to preserve 1/16 of the supersymmetries. We also present more $D$-brane bound state solutions by applying T-duality symmetries. Finally, the probe analysis is discussed along with a brief outline of the open string construction.
Recently, the study of string theory in PP-wave background has been a subject of intense discussion. It is known that “Penrose” limit plays an important role, in obtaining such solutions, as any Einstein gravity admits a plane wave background through these limits [1]. The supergravity realization of string theory in pp-wave background through these limits are studied as well [2–7]. String theory in this background is easy to handle due to the presence of a natural light cone gauge and are exactly solvable in Green-Schwarz formalism [8–10]. The exact solvability of string theory in these backgrounds, provide a powerful tool to investigate the AdS/CFT correspondence in a better way [11–13]. The maximally supersymmetric type IIB pp-wave background was found in [4], on which string theory is exactly solvable and has been used to show the duality between string and gauge theories, by means of an one to one mapping of the stringy modes to the operators in the gauge theory.

D-branes, known as non-perturbative and extended objects, also survive in this limit. The study of $D$-branes in pp-wave background, arising from a geometry of $\text{AdS}_p \times S^q$ type, has been a subject of wide interest [6]. Explicit supergravity solutions of these objects and their open string spectrum, have been discussed at length in the literature [14–24]. These objects play an important role in understanding the duality between string and gauge theories. BPS (supersymmetric) bound of these objects have also been useful in understanding the physics of black holes in string theory. It is now known that, there are also supersymmetric bound states where the component branes are at relative angles, with the angles being restricted to lie in an $SU(N)$ subgroup of rotations [25]. Explicit classical solution, to the supergravity equations coresponding to the configurations of branes at angle, are already discussed in the literature [26–30]. These bound states are of wide applications, in understanding various dualities of superstring theories as well as the study of black holes in string theories.

In veiw of the importance of these bound states, in this paper we show the existence of the classical solution of a system of $D$-branes oriented at angle, belongs to $SU(2)$ subgroup of rotations, in pp-wave background with constant $NS − NS$ 3-form field strengths. In particular, we present two supergravity solutions, one coreponds to a
single D3-brane and the other one is a system of D3-branes which are oriented by certain $SU(2)$ angle, as described above, in $NS – NS$ plane wave backgrounds. Some other D-brane bound states are also obtained by applying $T$-duality and $S$-duality symmetries in the above supergravity solution. It has been explicitly shown that, these bound states preserve 1/16 of the supersymmetries. These brane configurations are also analyzed from the point of view of probe branes. Finally, we have discussed the open string construction briefly.

We now begin by writing down the classical solution of a D3-brane rotated by certain $SU(2)$ angle in the pp-wave background of $AdS_3 \times S^3 \times R^4$ [10, 22, 23] with constant $NS – NS$ 3-form field strength:

$$
\begin{align*}
&ds^2 = \sqrt{1 + X_1} \left\{ \frac{1}{1 + X_1} \left[ 2dx^+ dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^i)^2 \right] \\
&+ \left[ 1 + X_1 \cos^2 \alpha \right] [(dx^5 + (dx^7)^2] + [1 + X_1 \sin^2 \alpha][(dx^6)^2 + (dx^8)^2] \\
&+ 2X_1 \sin \alpha \cos \alpha (dx^7 dx^8 - dx^5 dx^6) + \sum_{i=1}^{4} (dx^i)^2 \right\} \\
&H_{+12} = H_{+34} = 2\mu, \\
&F_{+68}^{(5)} = -\frac{\partial_i X_1}{(1 + X_1)^2} \cos^2 \alpha, \quad F_{+67}^{(5)} = -\frac{\partial_i X_1}{(1 + X_1)^2} \cos \alpha \sin \alpha, \\
&F_{+57}^{(5)} = \frac{\partial_i X_1}{(1 + X_1)^2} \sin^2 \alpha, \quad F_{+58}^{(5)} = -\frac{\partial_i X_1}{(1 + X_1)^2} \cos \alpha \sin \alpha, \\
e^{2\phi_b} = 1.
\end{align*}
$$

(1)

and $X_1$ is given by

$$
X_1(\vec{r}) = \frac{1}{2} \left( \frac{\ell_1}{|\vec{r} - \vec{r}_1|} \right)^2.
$$

(2)

Where $r$ is the radius vector in the transverse space, defined as $r^2 = \sum_{i=1}^{4} (x_i)^2$, $r_1$ is the location of D3-brane in transvers space and $X_1$ is the Harmonic function, which
satisfies the Green function equation in the transverse space.

To start with, the $D3$-brane is along $x^+, x^-, x^6$ and $x^8$. By applying a rotation between $x^5 - x^6$ and $x^7 - x^8$ planes following [28], with rotation angles $(\alpha_1, \alpha_2) = (0, \alpha)$, we get the configuration where the original $D3$-brane is tilted by an angle $\alpha$. In the limit $\mu = 0$, the above solution reduces to the flat space $D3$-brane solution rotated by $SU(2)$ angle, which can be obtained by applying a $T$-duality transformation, along one of the transverse coordinate of the rotated $D2$-membrane solution as given in [29]. One can also check that in the limit $\alpha = 0$, the above solution exactly matches with the $D3$-brane solution given in [23]. Type IIB field equations (see e.g. [31]) are being satisfied by the solution given in eqn.(1). For example, the constant 3-form field strengths along the pp-wave directions are in fact needed to cancel the $\mu$-dependent part of $R_{++}$ equation. One can also find out the rotated $D3$-brane solution in constant $R - R$ three form pp-wave background by applying $S$-duality symmetry in the above solution. Under $S$-duality transformation the above solution will remain as it is with the constant $NS - NS$ 3-form field strength being replaced to $R - R$ 3-form field strength.

Now we present the low energy classical solution of a system of two $D3$ branes, when both are placed at a relative $SU(2)$ angle with respect to each other. The supergravity solution of such a system in pp-wave background with constant $NS - NS$ 3-form field strength is given by:

\[
\begin{align*}
\mathcal{ds}^2 & = \sqrt{1 + X} \left\{ \frac{1}{1 + X} \left( 2 dx^+ dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^i)^2 \right) \\
& + (1 + X_2) [(dx^5)^2 + (dx^7)^2] + (dx^6)^2 + (dx^8)^2 \\
& + X_1 \left[ (\cos \alpha dx^5 - \sin \alpha dx^6)^2 + (\cos \alpha dx^7 + \sin \alpha dx^8)^2 \right] \right\} + \sum_{i=1}^{4} (dx^i)^2, \\
H_{+12} & = H_{+34} = 2\mu,
\end{align*}
\]
\[ F^{(5)}_{+-68i} = \partial_i \left\{ \frac{X_2 + X_1 \cos^2 \alpha + X_1 X_2 \sin^2 \alpha}{(1 + X)} \right\}, \]

\[ F^{(5)}_{+-58i} = -F^{(5)}_{+-67i} = \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{(1 + X)} \right\}, \]

\[ F^{(5)}_{+-57i} = -\partial_i \left\{ \frac{(X_1 + X_1 X_2) \sin^2 \alpha}{(1 + X)} \right\}, \quad e^{2\phi_b} = 1. \] (3)

and \( X \) is given by

\[ X = X_1 + X_2 + X_1 X_2 \sin^2 \alpha, \] (4)

where as defined earlier, \( X_{1,2} = \frac{1}{2} \left( \frac{\ell_{1,2}}{||r_1 - r_2||} \right)^2 \).

In this case, to start with the two \( D3 \)-branes are parallel to each other and are lying along \( x^+, x^-, x^6, x^8 \). By applying an \( SU(2) \) rotation, as described in previous case, the second brane rotated by an angle \( \alpha \), now lies in \( x^+, x^-, x^5 \) and \( x^7 \). At the same time the branes are delocalized along \( x^5 - x^7 \) and \( x^6 - x^8 \) planes respectively. One can check that the solution given in eqn.(3), solves type IIB field equations. In the limit \( \mu = 0 \), the above solution goes back to that given in [28]. By setting \( X_2 = 0 \), the above solution reduces to the one given in eqn.(1). We would like to point out that at \( \alpha = \pi/2 \), the solution given in eqn.(3) represents the supergravity solution of two \( D3 \)-branes intersecting orthogonally along a string. We emphasise that the possibility of this configuration is already discussed in [23], after applying successive T-dualities along the \( D1 - D5 \) solution [23]. Therefore the solutions given in eqn.(1) and eqn.(3), gives a pp-wave generalization of the ones given in [28, 29].

We now apply T-duality transformation along the world volume directions of the configuration given in eqn. (3) to generate more interesting \( D \)-brane bound states in pp-wave background. For example, applying T-duality along \( x^8 \), one gets:

\[
\begin{align*}
    ds^2 &= \sqrt{1 + X} \left\{ \frac{1}{1 + X} \left( 2 dx^+ dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2 \right) \\
    &\quad + (1 + X_2) (dx^5)^2 + (dx^6)^2 + X_1 (\cos \alpha dx^5 - \sin \alpha dx^6)^2 \right\}
\end{align*}
\]
\[
\sum_{i=1}^{4} (dx^i)^2
\]

\[
H_{+12} = H_{+34} = 2\mu,
\]

\[
F_{+6i}^{(4)} = \partial_i \left\{ \frac{X_2 + X_1 \cos^2 \alpha + X_1 X_2 \sin^2 \alpha}{1 + X} \right\}, \quad F_{+5i}^{(4)} = \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{1 + X} \right\},
\]

\[
B_{78} = -\frac{X_1 \cos \alpha \sin \alpha}{1 + X_1 \sin^2 \alpha}, \quad e^{2\phi_a} = \frac{\sqrt{1 + X}}{1 + X_1 \sin^2 \alpha}.
\]

One notices that setting \(X_1 = 0\), one gets a \(D2\)-brane lying along the coordinates, \(x^+, x^-\) and \(x^6\) and at the same time ‘delocalized’ along \(x^5, x^7\) and \(x^8\). Setting \(X_2 = 0\), the solution given in eqn. (5), reduces to the \(D2 - D4\) bound state in pp-wave background. By applying \(S\)-duality in the solution given in eqn. (3), one can get classical solution of a system of \(D3\)-branes at an angle in \(R - R\) pp-wave background. Again by applying \(T\)-duality transformation along the directions \(x_6, x_7, x_8\), one gets:

\[
ds^2 = \sqrt{1 + X} \left\{ \frac{1}{1 + X} \left( 2dx^+ dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2 \right) + \frac{(dx^5) + (dx^6) + (dx^7)^2 + (dx^8)^2}{1 + X_1 \sin^2 \alpha} + \sum_{i=1}^{4} (dx^i)^2 \right\}
\]

\[
F_{+1268} = F_{+3468} = 2\mu,
\]

\[
F_{+i}^{(3)} = -\partial_i \left\{ \frac{X_2 + X_1 \cos^2 \alpha + X_1 X_2 \sin^2 \alpha}{1 + X} \right\}, \quad F_{+78i}^{(5)} = \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{1 + X} \right\},
\]

\[
B_{56} = -B_{78} = -\frac{X_1 \cos \alpha \sin \alpha}{1 + X_1 \sin^2 \alpha}, \quad e^{2\phi_a} = \frac{1 + X}{1 + X_1 \sin^2 \alpha}.
\]

Now one can see that by setting \(X_1 = 0\) the above solution reduces to that of a \(D\)-string lying along \(x^+, x^-\) and delocalised along \(x^5, x^7, x^8\) directions. Setting \(X_2 = 0\) the above supergravity solution corresponds to a \(D(5, 3, 3, 1)\) bound state in pp-wave background with constant \(R - R\) 5-form field strengths. One can see that at \(\mu = 0\)
limit this solution reduces to one given in [28], which can be obtained by applying $T$-duality transformation along one of the transverse directions. Similarly, by applying $S$-duality and $T$-duality transformation in eqn.(3), one can generate more bound states of various $D$-branes in $NS - NS$ and $R - R$ plane-wave backgrounds. We however, skip the details here. We again emphasize that the bound state solutions presented here also gives the pp-wave generalization of the ones given in flat space [28,32,33].

It has already been discussed in the literature [25] that a system of $D$-branes oriented at certain angle, lying inside the $SU(N)$ subgroup of rotations, with respect to each other, preserve certain amount of unbroken supersymmetries. We now confirm this fact, in pp-wave background through the examples discussed in this paper by analyzing the type IIB supersymmetry variations explicitly. The supersymmetry variation of dilatino and gravitino fields of type IIB supergravity in ten dimension, in string frame, is given by [34,35]:

\[
\delta \lambda_\pm = \frac{1}{2}(\Gamma^\mu \partial_\mu \phi \mp \frac{1}{12} \Gamma^{\mu \rho} H_{\mu \rho}) \epsilon_\pm + \frac{1}{2} e^\phi (\pm \Gamma^M F^{(1)}_M + \frac{1}{12} \Gamma^{\mu \rho} F^{(3)}_{\mu \rho}) \epsilon_\mp, \tag{7}
\]

\[
\delta \Psi_\mu^\pm = \left[ \partial_\mu + \frac{1}{4} (w_{\mu \hat{a} \hat{b}} \mp \frac{1}{2} H_{\mu \hat{a} \hat{b}}) \Gamma^{\hat{a} \hat{b}} \right] \epsilon_\pm + \frac{1}{8} e^\phi \left[ \mp \Gamma^\mu F^{(1)}_\mu - \frac{1}{3!} \Gamma^{\mu \rho} F^{(3)}_{\mu \rho} \mp \frac{1}{2.5!} \Gamma^{\mu \rho \alpha \beta} F^{(5)}_{\mu \rho \alpha \beta} \right] \Gamma_\mu \epsilon_\mp, \tag{8}
\]

where we have used $(\mu, \nu, \rho)$ to describe the ten dimensional space-time indices, and hat’s represent the corresponding tangent space indices. Solving the above two equations for the solution describing the system of two $D3$-branes as given in eqn. (3), we get several conditions on the spinors. First the dilatino variation gives:

\[
\Gamma^+ (\Gamma^{12} + \Gamma^{34}) \epsilon_\pm = 0. \tag{9}
\]

Gravitino variations gives the following conditions on the spinors:

\[
\delta \psi^\pm_+ = \partial_+ \epsilon_\pm - \frac{\partial_+ X}{8(1 + X)^{3/2}} \Gamma^\pm_+ \epsilon_\pm \mp \frac{\mu}{2 \sqrt{1 + X}} (\Gamma^{12} + \Gamma^{34}) \epsilon_\pm
\]
\[\begin{aligned}
\mp \frac{1}{8} \Gamma^{\hat{5}\hat{8}\hat{1}} & \left( (1 + X_1 \sin^2 \alpha)^2 \partial_i X_2 + \cos^2 \alpha \, \partial_i X_1 \right) \\
\mp \frac{1}{8} \Gamma^{\hat{5}\hat{7}\hat{6}} & \left( \frac{1}{(1 + X)^{5/2}} (1 + X_1 \sin^2 \alpha) \right)
\times (X_1^2 \cos^2 \alpha \sin^2 \alpha \, \partial_i X_2 + (1 + X_2)^2 \sin^2 \alpha \, \partial_i X_1) \\
- \frac{1}{(1 + X)^{5/2}} \left( (1 + X_1 \sin^2 \alpha)^2 \partial_i X_2 + \cos^2 \alpha \, \partial_i X_1 \right)
\mp \frac{1}{(1 + X)^{5/2}} \left( 2X_1^2 \cos^2 \alpha \sin^2 \alpha \, \partial_i X_2 + 2X_1^3 \cos^2 \alpha \sin^4 \alpha \, \partial_i X_1 \\
- (X_1^2 (1 + X_2) \cos \alpha \sin \alpha \, \partial_i X_1) \right) \Gamma^\mp \epsilon_\mp
\mp \frac{1}{8} \left\{ \frac{\Gamma^{\hat{5}\hat{5}\hat{8}\hat{1}} - \Gamma^{\hat{6}\hat{7}\hat{8}}}{(1 + X)^2 (1 + X_1 \sin^2 \alpha)} \right\}
\left[ -X_1 \cos \alpha \sin \alpha \, \partial_i X_2 \\
+ (1 + X_2) \sin \alpha \cos \alpha \, \partial_i X_1 - X_1^2 \sin^3 \alpha \cos \alpha \, \partial_i X_1 \right] \Gamma^\mp \epsilon_\mp = 0,
\end{aligned}\]

In writing down the above variations we have made use of a necessary condition:

\[\Gamma^\mp \epsilon_\pm = 0,\] (12)

Now, gravitino variation (10) is solved by imposing:

\[\begin{aligned}
\Gamma^\hat{5}\hat{8} \epsilon_\mp = 0, \quad \Gamma^\hat{6}\hat{7} \epsilon_\mp = 0,
(\Gamma^{\hat{5}\hat{7}} + \Gamma^{\hat{6}\hat{8}}) \epsilon_\mp = 0,
\end{aligned}\] (13)

\[\Gamma^{\hat{6}\hat{8}} \epsilon_\mp = \epsilon_\pm, \quad \Gamma^{\hat{5}\hat{7}} \epsilon_\mp = \epsilon_\pm,\] (14)
in addition to the condition:

\[(1 - \Gamma^{1234})\epsilon_x = 0.\]  \hspace{1cm} (15)

To explain it further, below, we state the cancellations clearly. Imposing the condition (13), the last term of the gravitino variation (10) vanishes and the fourth and fifth terms add up to give the coefficient of the second term. Imposition of the brane conditions (14) along with (15), solves gravitino variation (10) fully. The expressions for the other two gravitino variations: \(\delta \psi^+_{a} (a = 5, ..., 8)\) and \(\delta \psi^+_{i} (i = 1, ..., 4)\), are of similar type and they are also solved by following the same arguments described above. Therefore, the configuration of two \(D3\)-branes with a relative \(SU(2)\) angle between the two, as given in eqn.(3), preserves 1/16 supersymmetry. However the supersymmetry obtained here is consistent with the \(D3\)-brane supersymmetry given in [23], when two of them are parallel to each other. The above supersymmetry can also be found out by doing an analysis following [25, 36, 37], by choosing the ‘rotation matrix’ appropriately. For convenience, the expressions for the non vanishing Christoffels and some of the spin connections are presented in Appendix A.

We now consider the situation from worldvolume point of view, where we will use a close relation between \(\kappa\)-symmetry and supersymmetry to determine the fraction of supersymmetry preserved by the above described configuration. We will use the \(\kappa\)-symmetry formulation of \(Dp\)-brane [38–42] to show this. To do this computation, we will treat the branes, involved in the intersection, as probes, propagating in \(D = 10\) space-time of the type \(AdS_3 \times S^3 \times R^4\) with a six dimensional pp-wave. As described in [41], we have a system of two \(D3\)-branes intersecting at arbitrary angle with a vanishing 2-form BI field. For simplicity, each brane is identified with a subspace of a ten dimensional spacetime.

Given a \(D\)-brane probe the surviving supersymmetries satisfy the condition:

\[(1 - \Gamma)\xi = 0,\]  \hspace{1cm} (16)

where \(\Gamma\) is a projector that depends on the details of the brane configuration. For
vanishing BI field, $\Gamma$ will take a form as defined in [41]:

$$\Gamma = \Gamma'(0),$$  \hspace{1cm} (17)

where $\Gamma'(0)$ and $\Gamma(0)$ are as follows:

$$\Gamma'(0) = (\sigma_3) \frac{2\pi}{i} \sigma_2 \otimes \Gamma(0), \quad \text{(for type IIB)}$$  \hspace{1cm} (18)

with

$$\Gamma(0) = \frac{1}{(p+1)!} \sqrt{-\det G} \epsilon^{i_1 \ldots i_{p+1}} \partial_{i_1} X^{M_1} \ldots \partial_{i_{p+1}} X^{M_{p+1}} \Gamma'_{M_1 \ldots \ldots M_{p+1}}.$$  \hspace{1cm} (19)

$\Gamma'_M$ are the ten dimensional $\Gamma$-matrices in a coordinate basis defined by

$$\Gamma'_M = E_A^M \Gamma_A,$$  \hspace{1cm} (20)

with $\Gamma_A$ being the flat space matrices. The induced world volume metric $G_{ij}$ is:

$$G_{ij} = \partial_i X^M \partial_j X^N g_{MN}.$$  \hspace{1cm} (21)

For our case, the first $D3$-brane is lying along $(x^+, x^-, x^6, x^8)$ and stuck at the origin. Imposing the light cone gauge and physical gauge conditions: $X^+(\tau, \xi) = p^+ \tau$, $X^- (\tau, \xi) = \xi^1$, and $X^k(\tau, \xi) = \xi^k$, as explained in [42], and denoting the world volume coordinates of the $D$-brane by $(\tau, \xi^k)$, $k = 1, 6, 8$, we have:

$$\Gamma(0) = \Gamma^{\perp = \hat{\delta} \delta 8}.$$  \hspace{1cm} (22)

Using above projection, eqn. (16) simplifies to:

$$\Gamma^{\perp = \hat{\delta} \delta 8} \epsilon = \epsilon.$$  \hspace{1cm} (23)

The $\kappa$-symmetry projection of two intersecting $Dp$-branes at an arbitrary angle can be obtained by applying certain lorentz transformation ($\Lambda$) as given in [41]:

$$\tilde{\Gamma}_a = \Gamma_b \Lambda^b_a = S^{-1} \Gamma_a S,$$  \hspace{1cm} (24)
where $\tilde{\Gamma}_a$ is the projection that depends on the detailed configuration of the second brane and $S$ is an element in $\text{spin}(1,9)$ that depends on $\Lambda$. In our case, $\tilde{\Gamma}_{(0)}$ is the projection for the rotated $D3$-brane, which can be expressed in terms of $\Gamma_{(0)}$, the unrotated $D3$-brane projection:

$$
\tilde{\Gamma}_{(0)} = \Gamma_{+\pm}(\cos \alpha \, \Gamma_6 + \sin \alpha \, \Gamma_5)(\cos \alpha \, \Gamma_8 + \sin \alpha \, \Gamma_7),
$$

$$
= \Gamma_{+\mp68} \, e^{\alpha(\Gamma_{65}+\Gamma_{87})}.
$$

Using eqn.(16) and (23) in eqn. (25) one gets the constraint :

$$
[ e\alpha (\Gamma_{65}+\Gamma_{87}) - 1 ] \epsilon = 0 = \sin \alpha \Gamma_{68} \, e^{\alpha \Gamma_{65}(1 - \Gamma_{57})} \epsilon.
$$

(26)

For any arbitrary angle ($\alpha$) eqn.(26) can be satisfied only when

$$
\Gamma_{5678} \epsilon = \epsilon = \Gamma_{+\mp57}\Gamma_{+\mp68} \epsilon.
$$

(27)

Using eqn.(23) the above condition simplifies to:

$$
\Gamma_{+\mp57} \epsilon = \epsilon.
$$

(28)

But eqn.(28) is the supersymmetry condition associated with the rotated $D3$-brane along the $(x^+, x^-, x^5, x^7)$ directions. The supersymmetry conditions obtained in eqns. (23) and (28), commute with each other [41], which implies pull back of the 1/4 supersymmetry to the worldvolume of the $D3$-branes. One can check that the probe analysis presented here is also consistent with that of the flat space case [25].

Now we briefly outline the open string construction of the $D3$-brane system, presented from the supergravity point of view and from probe analysis, following [25]. Boundary conditions are:

$$
D3_1: \quad \partial_\sigma x^{+,-6,8} = 0, \quad \partial_\tau x^{i,5,7} = 0, \quad i = 1,...,4.
$$

(29)

$$
D3_2: \quad \partial_\sigma (\cos \alpha \, x^{6,8} \mp \sin \alpha \, x^{5,7}) = 0, \quad \partial_\tau (\pm \sin \alpha \, x^{6,8} + \cos \alpha \, x^{5,7}) = 0,
$$

(30)
with $x^+, x^-$ and $x^i$ satisfying the usual Neumann and Dirichlet boundary conditions respectively. In eqns. (29) and (30), $D3_1$ and $D3_2$ denotes the unrotated and the rotated branes. Referring to the e.o.m. given in [22], with the above boundary conditions, it is straightforward to write down the mode expansion and hence the canonical commutation relation. We, however skip the details here.

We, therefore, have analyzed the existence and stability of a system of $D$-branes oriented by $SU(2)$ rotations in $NS-NS$ and $R-R$ pp-wave background, arising from $AdS_3 \times S^3 \times R^4$, transverse to the branes. We have also shown the presence of more $D$-brane bound states by applying $T$-duality symmetry. They preserve $1/16$ of the supersymmetries. These configurations are also studied as the probe branes in the given background. They are shown to preserve $1/4$ supersymmetry of the world volume. It will be interesting to generalize these to arbitrary $SU(N)$ rotations. As there is an appropriate representation of world volume gauge fields by means of the angle between the rotated branes, it may be interesting to study them further in pp-wave background. By using these configurations one may like to study the physics of black holes as well.

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**Appendix A**

Here we write down all the non vanishing Christoffels, derived from eqn. (3):

\[
\Gamma_{++}^i = \frac{(\mu x_i^2) \partial_i X}{4(1+X)^2} - \frac{\mu^2 x_i}{(1+X)}, \quad \Gamma_{+-}^i = \frac{\partial_i X}{4(1+X)^2},
\]

\[
\Gamma_{-i}^- = \Gamma_{+i}^+ = -\frac{\partial_i X}{4(1+X)}, \quad \Gamma_{+i}^- = -\frac{\mu_i}{4(1+X)},
\]

\[
\Gamma_{ji}^j = \frac{\partial_i X}{4(1+X)},
\]
\begin{align}
\Gamma_{5i} &= \frac{1 + X_1 \sin^2 \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{1 + X_2 + X_1 \cos^2 \alpha}{\sqrt{1 + X}} \right\} \\
&\quad - \frac{X_1 \cos \alpha \sin \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} = \Gamma_{7i}^7,
\Gamma_{6i} &= \frac{1 + X_2 + X_1 \cos^2 \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{1 + X_1 \sin^2 \alpha}{\sqrt{1 + X}} \right\} \\
&\quad - \frac{X_1 \cos \alpha \sin \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} = \Gamma_{8i}^8,
\Gamma_{5i} &= -\frac{1 + X_1 \sin^2 \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} \\
&\quad + \frac{X_1 \cos \alpha \sin \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{1 + X_1 \sin^2 \alpha}{\sqrt{1 + X}} \right\} = -\Gamma_{7i}^7,
\Gamma_{6i} &= -\frac{1 + X_2 + X_1 \cos^2 \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} \\
&\quad + \frac{X_1 \cos \alpha \sin \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{1 + X_2 + X_1 \cos^2 \alpha}{\sqrt{1 + X}} \right\} = -\Gamma_{8i}^8.
\end{align}

(A1)

Now we present some of the non vanishing spin connections. As the expressions are complicated and long, we mention only some of them:

\begin{align}
\omega^{\hat{i}}_+ &= \frac{(\mu x_i)^2 \partial_i X}{2(1 + X)^{3/2}} - \frac{\mu^2 x_i}{(1 + X)^{1/2}},
\omega^{\hat{i}}_+ &= \omega^{\hat{i}}_-= \frac{-\partial_i X}{4(1 + X)^{3/2}},
\omega^5_{5i} &= \left\{ \frac{1 + X_1 \sin^2 \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{1 + X_2 + X_1 \cos^2 \alpha}{\sqrt{1 + X}} \right\} \right\}
- \frac{X_1 \cos \alpha \sin \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} \left( \frac{1}{\sqrt{1 + X_1 \sin^2 \alpha}} \right),
\omega^6_{6i} &= \left\{ \left( \frac{1 + X_2 + X_1 \cos^2 \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{1 + X_1 \sin^2 \alpha}{\sqrt{1 + X}} \right\} \right) \left( 1 + X_1 \sin^2 \alpha \right) \right\}
- \frac{X_1 \cos \alpha \sin \alpha}{2\sqrt{1 + X}} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} \left( 1 + X_1 \sin^2 \alpha \right).
\end{align}
\[
\begin{align*}
&+ \left( \frac{1 + X_1 \sin^2 \alpha}{2 \sqrt{1 + X}} \right) \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} \\
&- \left( \frac{X_1 \cos \alpha \sin \alpha}{2 \sqrt{1 + X}} \right) \partial_i \left\{ \frac{1 + X_1 \sin^2 \alpha}{\sqrt{1 + X}} \right\} \left( \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right) \\
&\times \left( \frac{1}{\sqrt{1 + X} \sqrt{1 + X_1 \sin^2 \alpha}} \right), \\
\omega_{1}^{5\delta} &= - \frac{1}{2} \partial_i \left\{ \frac{X_1 \cos \alpha \sin \alpha}{\sqrt{1 + X}} \right\} \\
&+ \frac{X_1 \cos \alpha \sin \alpha}{2(1 + X_1 \sin^2 \alpha)} \partial_i \left\{ \frac{1 + X_1 \sin^2 \alpha}{\sqrt{1 + X}} \right\}, \text{ etc... (A2)}
\end{align*}
\]

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