**W boson production at hadron colliders:**
the lepton charge asymmetry in NNLO QCD

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**Abstract**

We consider the production of $W^\pm$ bosons in hadron collisions, and the subsequent leptonic decay $W \to l\nu_l$. We study the asymmetry between the rapidity distributions of the charged leptons, and we present its computation up to the next-to-next-to-leading order (NNLO) in QCD perturbation theory. Our calculation includes the dependence on the lepton kinematical cuts that are necessarily applied to select $W \to l\nu_l$ events in actual experimental analyses at hadron colliders. We illustrate the main differences between the $W$ and lepton charge asymmetry, and we discuss their physical origin and the effect of the QCD radiative corrections. We show detailed numerical results on the charge asymmetry in $p\bar{p}$ collisions at the Tevatron, and we discuss the comparison with some of the available data. Some illustrative results on the lepton charge asymmetry in $pp$ collisions at LHC energies are presented.

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1 Introduction

The production of lepton pairs with high invariant mass, of \(Z\) bosons and of \(W\) bosons, through the Drell–Yan (DY) mechanism \([1]\), is the most ‘classical’ hard-scattering process in hadron–hadron collisions. The DY process provides important tests of the Standard Model (SM) and precise determinations of SM parameters, and it places stringent constraints on many form of new physics.

At high-energy hadron colliders, the \(W\) and \(Z\) production processes have large production rates. They also offer clean experimental signatures, because of the presence of one high-\(p_T\) lepton and large missing transverse energy (in the case of \(W\) production), or of two high-\(p_T\) leptons of opposite charge (in the case of \(Z\) production) in the final state. Owing to these features, \(W\) and \(Z\) production are key processes for physics studies at the Tevatron and the LHC.

Single \(W\) and \(Z\) boson production is used at the Tevatron to precisely measure the mass and width of the \(W\) boson and to extract the electroweak (EW) mixing angle from measurements of the forward–backward lepton asymmetry.

The production of DY lepton pairs gives important information on the parton densities, or Parton Distribution Functions (PDFs), of the colliding hadrons. DY production in low-energy proton–proton (\(pp\)) and proton–nucleon collisions is more sensitive to the sea quark densities of the proton than the structure functions measured in Deep Inelastic lepton–hadron Scattering (DIS). \(W\) and \(Z\) production in proton–antiproton (\(p\bar{p}\)) collisions is mainly sensitive to the valence quarks of the proton, in combinations that are different from those appearing in DIS structure functions. The shape of the \(W\) and \(Z\) rapidity distributions in \(pp\) collisions at the LHC gives direct information on the quark and antiquark densities of the proton at high scale and small values of parton momentum fractions.

\(W\) and \(Z\) production at the Tevatron and the LHC represents important background to other SM processes and signals of new physics. The production of DY lepton pairs with invariant mass larger than the masses of the \(W\) and \(Z\) bosons gives direct information and constraints on effective interactions between quarks and leptons that can originate from physics beyond the SM.

The importance of the DY process and the high precision achieved and achievable by the experiments at the Tevatron and the LHC demand for theoretical predictions with corresponding accuracy. These theoretical predictions require, in particular, the computation of QCD radiative corrections up to the next-to-next-to-leading order (NNLO) in perturbation theory.

The DY process is one of the few processes for which NNLO QCD corrections are known \([2]–[6]\). The important NNLO calculations of the total cross section and of the rapidity distribution of the DY lepton pair were performed in Refs. \([2]\) and \([3, 4]\), respectively. A limitation of these NNLO calculations is that only the invariant mass and the rapidity of the lepton pair are explicitly retained; the calculations are fully inclusive over the hadronic (partonic) final state, and they are also inclusive over the separate momenta of the two final-state leptons. This limitation is particularly evident in the case of \(W\) production, since the longitudinal momentum of the neutrino from \(W\) decay and, thus, the momentum of the lepton pair are not measurable. More generally, the identification and selection of DY lepton pairs in actual experiments require the use of various kinematical cuts. Moreover, the explicit dependence on the leptonic kinematical variables that are
measurable by experiments gives additional information on the dynamics of the DY process. The limitation of the inclusive calculations of Refs. [2]-[4] is obviated by considering fully-differential calculations of the DY process at the NNLO [5, 6].

The evaluation of higher-order QCD radiative corrections to hard-scattering processes is definitely a hard task. The presence of infrared singularities at intermediate stages of the calculation does not permit a straightforward implementation of numerical techniques. In particular, fully differential calculations at the NNLO involve a substantial amount of conceptual, analytical and technical complications [7]-[11]. In $e^+e^-$ collisions, NNLO differential cross sections are known only for two [12, 13] and three jet production [14, 15]. In hadron–hadron collisions, fully differential cross sections have been computed in the cases of Higgs production by gluon fusion [16, 17] and of the DY process [5, 6].

A significant observable in $W$ hadroproduction is the asymmetry in the rapidity distribution of $W^+$ and $W^-$ bosons (see Sect. 2). In $p \bar{p}$ collisions, the $W^+$ and $W^-$ bosons are produced with equal rates; however, the $W^+$ is mainly produced in the proton direction, whereas the opposite happens for the $W^-$. In $pp$ collisions, $W$ production is forward–backward symmetric; however, the $W^-$ production rate is smaller than the $W^+$ production rate and, moreover, the $W^-$ is mostly produced at central rapidities, while the $W^+$ is mostly produced at larger rapidities.

These $W^+/W^-$ asymmetries are mainly due to the proton content of $u$ and $d$ quarks and, in particular, to the fact that $u$ quarks carry, on average, more proton momentum fraction than $d$ quarks. Therefore, as already pointed out long ago [13], the $W$ boson charge asymmetries provide important quantitative information on the size and momentum fraction distribution of the $u$ and $d$ parton densities of the proton.

In hadron collisions, the produced $W$ bosons are identified by their leptonic decay $W \rightarrow l\nu_l$. Since the longitudinal component of the neutrino momentum is unmeasured in experiments at hadron colliders, what is actually measured is the rapidity of the charged lepton and the corresponding lepton charge asymmetry (rather than the $W$ charge asymmetry).

The first measurement [20] of the lepton charge asymmetry in hadron collisions was carried out by the CDF Collaboration at the Tevatron Run I, using data from $p \bar{p}$ collisions at the centre–of–mass energy $\sqrt{s} = 1.8$ TeV. The final CDF measurement [21] at the Tevatron Run I is available since more than ten years. The recent data [22]-[24] from the CDF and D0 Collaborations at the Tevatron Run II ($\sqrt{s} = 1.96$ TeV) are more precise than Run I data, extend at larger rapidities and give additional information on the dependence on the lepton transverse energy.

The CDF Run I measurement [21] of the lepton charge asymmetry has played (and still plays) a relevant role in testing and constraining the quark parton densities of the proton. In particular, these data are continuously used in the PDF global fits of the MRST and CTEQ Collaborations since their MRST1998 [25] and CTEQ5 [26] analyses, respectively. The most recent PDF analysis of the MRST/MSTW Collaboration [29, 30] already includes Run II measurements [22, 23] in the global fit.

Future LHC measurements [31] will study the lepton charge asymmetry at energies larger than Tevatron energies. The lepton charge asymmetry at the LHC is sensitive to PDFs with parton

\[\text{†A direct measurement of the } W \text{ charge asymmetry has recently been presented by the CDF collaboration [19] (see Sect. 3).}\]
momentum fractions that are smaller (up to about a factor of seven) than those probed at the Tevatron.

The lepton charge asymmetry is a typical observable that depends on various kinematical selection cuts, such as those that are necessary and applied in actual experimental configurations to identify $W \rightarrow l\nu_l$ events. This dependence has to properly be taken into account in the corresponding theoretical results. QCD studies of the lepton charge asymmetry has so far been performed by considering radiative corrections up to next-to-leading order (NLO). For instance, the QCD calculations of Refs. [20]-[26] use the NLO code DYRAD [27] and the RESBOS code [28] (which includes higher-order soft-gluon contributions to the transverse-momentum spectrum of the $W$).

In this paper we present the calculation of the lepton charge asymmetry in NNLO QCD. We use the numerical program of Ref. [6], which encodes the NNLO radiative corrections to the DY process at the fully-differential level. This allows us to compute the lepton charge asymmetry by including the kinematical cuts applied in experimental analyses.

The paper is organized as follows. In Sect. 2 we introduce the charge asymmetry and the setup of our calculation. In Sect. 3 we consider $p\bar{p}$ collisions and present the results of our calculation at the Tevatron Run II. We also discuss the comparison with some of the available Tevatron data. In Sect. 4 we consider the charge asymmetry in $pp$ collisions and present some illustrative results at LHC energies. We briefly summarize our results in Sect. 5.

2 Preliminaries

We consider the process

$$h_1(p_1) + h_2(p_2) \rightarrow W + X \rightarrow l\nu_l + X .$$

(1)

The $W$ boson is produced by the collision of the two incoming hadrons $h_1$ and $h_2$ (with momenta $p_1$ and $p_2$), and then it decays leptonically. The accompanying final state is denoted by $X$.

In the centre–of–mass frame of the colliding hadrons, $y_W$ and $y_l$ denote the rapidities of the $W$ boson and of the charged lepton $l$, respectively. We consider the $W$ decay in electrons or muons ($l = e, \mu$) and we treat the charged leptons in the massless approximation; thus, $y_l$ coincides with the lepton pseudorapidity $\eta_l$, where $\eta_l = -\ln(\tan(\theta_l/2))$ and $\theta_l$ is the lepton scattering angle. The $W$ production cross section at fixed $y_W$ is denoted by $d\sigma_{h_1h_2}(W^+)/dy_W$. The analogous rapidity cross section of the decaying charged lepton is denoted by $d\sigma_{h_1h_2}(l^+)/dy_l$. The $W$ and lepton charge asymmetries are denoted by $A_{h_1h_2}(y_W)$ and $A_{h_1h_2}(y_l)$, respectively. They are defined as

$$A_{h_1h_2}(y_W) = \frac{d\sigma_{h_1h_2}(W^+)/dy_W - d\sigma_{h_1h_2}(W^-)/dy_W}{d\sigma_{h_1h_2}(W^+)/dy_W + d\sigma_{h_1h_2}(W^-)/dy_W} ,$$

(2)

$$A_{h_1h_2}(y_l) = \frac{d\sigma_{h_1h_2}(l^+)/dy_l - d\sigma_{h_1h_2}(l^-)/dy_l}{d\sigma_{h_1h_2}(l^+)/dy_l + d\sigma_{h_1h_2}(l^-)/dy_l} .$$

(3)

We also introduce the following notation for the kinematical variables of the decaying leptons: $E_T$ is the transverse energy of the charged lepton, $E_T^\nu$ is the neutrino transverse energy (which
corresponds to the missing transverse energy, \( E_T \), measured by experiments. \( \phi_{\ell \nu} \) is the azimuthal angle between the lepton and neutrino transverse momenta, and \( M_T = \sqrt{2E_T E_T'(1 - \cos \phi_{\ell \nu})} \) is the leptonic transverse mass.

In actual experimental determinations, the measured rapidity cross sections and charge asymmetries depend on the kinematical cuts (e.g., lepton isolation requirements, minimum \( M_T \)) and the kinematical variables (e.g., \( E_T \)) that are used to identify and select the observed \( W \rightarrow \ell \nu \) events. This dependence, which is not explicitly denoted in Eqs. (2) and (3), is fully taken into account in our calculation, as briefly mentioned below and shown in Sects. 3 and 4.

In this work we study the rapidity cross sections and the charge asymmetries in QCD perturbation theory. A generic differential cross section \( d\sigma \) for the process in Eq. (1) is given as

\[
d\sigma(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \ f_{a/h_1}(x_1, \mu_F^2) \ f_{b/h_2}(x_2, \mu_F^2) \ d\hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2),
\]

where \( f_{a/h}(x, \mu_F^2) \) \((a = q, \bar{q}, g)\) are the parton distributions of the colliding hadron \( h \), \( \mu_F \) is the corresponding factorization scale, and we use the \( \overline{\text{MS}} \) factorization scheme. The partonic cross section \( d\hat{\sigma} \) is computed up to include its NNLO contribution in perturbative QCD. We have:

\[
d\hat{\sigma}(p_1, p_2; \mu_F^2) = d\hat{\sigma}^{(0)}(p_1, p_2) + \alpha_S(\mu_R^2) d\hat{\sigma}^{(1)}(p_1, p_2; \mu_F^2) + \alpha_S^2(\mu_R^2) d\hat{\sigma}^{(2)}(p_1, p_2; \mu_F^2) + \mathcal{O}(\alpha_S^3),
\]

where \( \alpha_S(\mu_R^2) \) is the QCD running coupling, \( \mu_R \) is the renormalization scale, and we use the \( \overline{\text{MS}} \) renormalization scheme. The leading order (LO) partonic cross section is \( d\hat{\sigma}^{(0)} \), while \( d\hat{\sigma}^{(1)} \) and \( d\hat{\sigma}^{(2)} \) are the NLO and NNLO corrections, respectively.

Our computation of the partonic cross section is carried out by using the NNLO numerical program of Ref. [6]. This NNLO QCD calculation is organised at the fully differential level, and it is encoded in a partonic Monte Carlo program. It allows the user to compute differential cross sections and observables with arbitrary kinematical requirements and acceptance cuts on the produced \( W \), leptons and accompanying final state \( X \). The only essential restriction is that the constraints applied to the final state have to be IR safe\(^1\) at the partonic level.

The hadronic cross sections at the LO, NLO and NNLO are evaluated according to Eq. (1). The N\(n\)LO (with \( n = 0, 1, 2 \)) partonic cross sections in Eq. (5) use the expression of \( \alpha_S(\mu_R^2) \) at the \( n \)-th order (i.e., we use the \( \mu_R \) dependence at the level of \( (n + 1) \) loops), and they are consistently convoluted with parton densities at each corresponding order. The reference value of \( \alpha_S(M_Z) \) is fixed at the actual value used in the corresponding set of parton densities. We consider \( N_f = 5 \) flavours of (effectively) massless quarks, and thus we have two ‘up-type’ quarks \((u, c)\) and three ‘down-type’ quarks \((d, s, b)\).

Recent sets of parton densities, which are obtained by analyses of various collaborations, are presented in Refs. [29, 30, 32, 33, 34, 35, 36, 37, 38]. Among these sets, only those of Refs. [29, 35] and [37] include NNLO parton densities with \( N_f = 5 \) (effectively) massless quarks. Since the main purpose of our work is the study of rapidity cross sections and asymmetries at the NNLO, we consider only the parton density sets of Refs. [29, 35] and [37]. Moreover, to avoid multiple presentations of similar results, we mostly use the parton densities of Ref. [29]. The global fit of

\(^1\)The observable must be independent of the presence of arbitrarily-soft partons and independent of the individual-parton momenta of a bunch of collinear partons.
Ref. [29] includes also some data on the lepton charge asymmetry at the Tevatron: the ensuing parton densities are thus expected to produce better agreement with available measurements of charge asymmetries.

In our calculation, the \( W \) boson is treated off shell, thus including finite-width effects, and its leptonic decay retains the corresponding spin correlations. The values of the mass and total width of the \( W \) boson are \( M_W = 80.398 \) GeV and \( \Gamma_W = 2.141 \) GeV. The Fermi constant is set to the value \( G_F = 1.16637 \times 10^{-5} \) GeV\(^{-2} \), and we use the following (unitarity constrained) values of the CKM matrix elements: \( V_{ud} = 0.97419 \), \( V_{us} = 0.2257 \), \( V_{ub} = 0.00359 \), \( V_{cd} = 0.2256 \), \( V_{cs} = 0.97334 \), \( V_{cb} = 0.0415 \). All these values of EW parameters are taken from the PDG 2008 [39]. The \( W \) boson EW couplings to quarks and leptons are treated at the tree level, so that the above parameters are sufficient to fully specify the EW content of our calculation. In particular, the tree-level leptonic width of the \( W \) boson implies the value \( BR(W \to l\nu) = 10.62\% \) of the leptonic branching ratio.

We note that our calculation is invariant under CP transformations. Therefore, in \( p\bar{p} \) collisions, the rapidity cross sections \( d\sigma_{pp}/dy \) of \( W^+(l^+) \) and \( W^-(l^-) \) are simply related by the replacement \( y_W(y_l) \leftrightarrow -y_W(-y_l) \), and the charge asymmetry fulfils \( A_{pp}(y) = -A_{pp}(-y) \). Analogously, in \( pp \) collisions, the rapidity cross sections \( d\sigma_{pp}/dy \) of \( W^\pm(l^\pm) \) and the charge asymmetry \( A_{pp}(y) \) are invariant with respect to the replacement \( y \leftrightarrow -y \).

### 3 Rapidity cross section and asymmetry at the Tevatron

In this section we consider \( p\bar{p} \) collisions. We recall the main features of the production mechanism of the \( W^\pm \) bosons and their decaying leptons. Then, we present the results of our QCD calculation at the centre–of–mass energy \( \sqrt{s} = 1.96 \) TeV. Unless otherwise stated, throughout the paper we use the MSTW2008 sets [29] of parton densities, and we fix the renormalization and factorization scales at the value \( \mu_R = \mu_F = M_W \).

#### 3.1 \( W \) rapidity distribution and asymmetry

The LO cross section is controlled by the partonic subprocesses
\[
U + \bar{D} \rightarrow W^+ \rightarrow l^+ \nu_l , \tag{6}
\]
\[
D + \bar{U} \rightarrow W^- \rightarrow l^- \bar{\nu}_l , \tag{7}
\]
where \( U(D) \) generically denote an ‘up-type’ ('down-type') quark. Most of the proton momentum is carried by \( u \) quarks (and gluons), while most of the antiproton momentum is carried by \( \bar{u} \) antiquark (and gluons). Therefore, owing to the flavour structure of the processes in Eqs. (6) and (7), the produced \( W^+ \) tends to follow the direction of the colliding proton, while the \( W^- \) tends to follows the direction of the colliding antiproton.

We define the sign of the rapidity \( y \) so that the forward region \( (y > 0) \) corresponds to the direction of the momentum of the incoming proton. In Fig. 1 we present the rapidity distribution

\footnote{For comparison, the electron and muon branching ratios of the PDG [39] are \( BR(W \to e\nu_e) = (10.75 \pm 0.13)\% \) and \( BR(W \to \mu\nu_\mu) = (10.57 \pm 0.15)\% \), respectively.}
Figure 1: Rapidity distribution of an on-shell $W^+$ boson at the Tevatron Run II in LO (black dotted), NLO (red dashed) and NNLO (blue solid) QCD. No cuts are applied on the leptons and on their accompanying final state. The height of each histogram bin gives the value (in pb) of the cross section in the corresponding rapidity bin. The lower panel shows the ratios NLO/LO (red dashed) and NNLO/NLO (blue solid) of the cross section results in the upper panel.

for the inclusive production of an on-shell $W^+$. The $W^+$ bosons are mostly produced forward, and the rapidity distribution is peaked at $y_{W} \sim 1$. The three histograms give the results of our LO, NLO and NNLO calculation of the cross section $\sigma = \sigma(W^+) BR(W \rightarrow l\nu)$. The height of each histogram bin gives the value of the cross section in the corresponding rapidity bin. The error bars reported in the histograms refer to an estimate of the numerical error in the Monte Carlo integration carried out by our program. These error bars are small and hardly visible in the plot of Fig. 1 (see Fig. 2 and related comments).

Having computed the rapidity cross section at the first three orders in QCD perturbation theory, we can examine the quantitative convergence of the perturbative expansion. To this purpose, we introduce $y$ dependent ratios of the results at two successive perturbative orders. More precisely, we define the following NLO and NNLO ‘K factors’:

$$K_{NLO}(y) = \frac{[d\sigma/dy]_{NLO}}{[d\sigma/dy]_{LO}}, \quad K_{NNLO}(y) = \frac{[d\sigma/dy]_{NNLO}}{[d\sigma/dy]_{NLO}},$$

where $[d\sigma/dy]_{LO}$, $[d\sigma/dy]_{NLO}$ and $[d\sigma/dy]_{NNLO}$ are the LO, NLO and NNLO cross sections.

†The average value $(d\sigma/dy)_i$ of the rapidity cross section in the $i$-th bin is obtained by rescaling the corresponding bin cross section $\sigma_i$ by the bin size $\Delta y = 0.2$, i.e. $(d\sigma/dy)_i = \sigma_i/0.2 = 5 \sigma_i$.

‡Unless otherwise stated, hereafter the error bars in the histograms of our QCD computations always refer to the numerical error in the Monte Carlo integration carried out by our program.
The K factors for on-shell $W^+$ production are shown in the lower panel of Fig. 1. The NLO effects are bigger than the NNLO effects on both the normalization and the shape of the rapidity cross section. In the rapidity interval $|y_W| \lesssim 2$, the NLO K factor varies in the range $K_{NLO}(y_W) \sim 1.3$–1.4, while the NNLO K factor varies in the range $K_{NNLO}(y_W) \sim 1.02$–1.04. We recall [29] that the K factors computed from the ratio of the total (i.e. integrated over $y_W$) cross sections are $K_{NLO} = 1.35$ and $K_{NNLO} = 1.03$. The fact that $K_{NNLO}(y_W)$ is much closer to unity than $K_{NLO}(y_W)$ indicates a very good quantitative convergence of the truncated perturbative expansion, as first found in Ref. [4]. In particular, the value of $K_{NNLO} - 1$ can consistently be used as a measure of the theoretical uncertainty due to the uncalculated contributions from higher orders (i.e., beyond NNLO).

We have repeated our QCD calculation of the $W^+$ rapidity cross section by using the parton density sets of the ABKM Collaboration [34, 35] and of the Dortmund Group [36, 37]. Here we limit ourselves to presenting the results at the NNLO. In Fig. 2 we show the NNLO ratios $(d\sigma/dy_W)_{PDF}/(d\sigma/dy_W)_{MSTW}$, where the cross section in the numerator is computed by using either the ABKM09 set [35] or the JR09VF set [37], while the cross section in the denominator uses the MSTW2008 set. We see that the NNLO ratios are different from unity and depend on $y_W$; varying $y_W$ in the rapidity interval $|y_W| \lesssim 2$, the differences can reach the level of about 5%. The $W^+$ bosons from the ABKM09 (JR09VF) partons are produced slightly more (less) forward than those from the MSTW2008 partons. Considering the NNLO total cross sections, the ABKM09 (JR09VF) result is about 3% higher (1% lower) than the MSTW2008 result: we find the values $\sigma_{NNLO} = 1.391 \pm 0.002$ nb, $1.349 \pm 0.002$ nb and $1.338 \pm 0.002$ nb, which agree with the corresponding results in Refs. [35, 29] and [37]. This quantitative agreement is a numerical check of our calculation, since the results of Refs. [29, 35, 37] are obtained by using the direct calculation of the NNLO total cross section [2].

Figure 2: On-shell $W^+$ boson production at the Tevatron Run II in NNLO QCD. The rapidity cross sections computed with the ABKM09 (blue solid) and JR09VF (red dashed) parton densities are rescaled by the corresponding MSTW2008 result.

The error bars reported in Fig. 2 are the numerical errors of our Monte Carlo computations:

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*The errors on the values of the cross sections are those from the Monte Carlo integration in our calculation.

The authors of Ref. [29] use the leptonic branching ratio $B_{\ell\nu} = 10.80\%$, which is the PDG 2008 value that is obtained by averaging the electron, muon and tau branching ratios.
they are the relative errors of the MSTW08 result presented in Fig. 1 and the analogous errors of the corresponding ABKM09 and JR09VF results. In the rapidity range $|y_W| \lesssim 2$, these errors are smaller than $\pm 1\%$ and, typically, at the level of about $\pm 5\%$.

The PDF analyses of Refs. [29, 35, 37] include estimates of the PDF uncertainties that originate from the experimental errors of the data used in the corresponding global fits. These PDF uncertainties can be used to evaluate the ensuing errors on the theoretical computation of physical observables. We do not explicitly show the PDF errors on the $W^+$ rapidity distribution. Considering the region of small and medium rapidities (say, $|y_W| \lesssim 2$), these errors depend slightly on $y_W$ and, therefore, their sizes can be argued from the PDF error on the corresponding total cross sections. The quoted PDF errors at the one-sigma level (68\% C.L.) on the NNLO total cross section are about $\pm 1.7\%$ (MSTW2008) [29], $\pm 1\%$ (ABKM09) [35] and $\pm 1.2\%$ (JR09VF) [37]. The differences between the values of the NNLO total cross section obtained by the three PDF sets are (almost) covered by these PDF errors, but these PDF errors do not fully cover the PDF differences shown by the $y_W$ dependent ratios in Fig. 2.

The CDF Collaboration has recently presented the first direct measurement of the $W$ asymmetry [19]. The experimental difficulty in the determination of the neutrino’s longitudinal momentum is resolved [40] by means of a Monte Carlo driven extrapolation of the lepton rapidity distribution, and the final CDF results refer to the charge asymmetry of inclusive production of on-shell $W$ bosons.

In Fig. 3 we present our perturbative results for the asymmetry of an on-shell $W$ boson at LO (dotted histogram), NLO (dashed histogram) and NNLO (solid histogram), and we compare them with the CDF data. The results in Fig. 3 confirm previous findings [4]: the perturbative corrections to the $W$ asymmetry are small. In particular, the corrections to the asymmetry are smaller than the corresponding corrections to the rapidity cross section (see Fig. 1): this is somehow expected since the asymmetry involves ratio of cross sections. More precisely, the smallness of the QCD radiative corrections to the asymmetry could have been argued by the direct inspection of the K factors in Fig. 1, since they have a high degree of symmetry with respect to the exchange $y_W \leftrightarrow -y_W$ (note that a forward–backward symmetric K factor for the rapidity cross section would imply no radiative corrections to the asymmetry).

The effect of the radiative corrections is quantified by the NLO (dashed histogram) and NNLO (solid histogram) asymmetry K factors, which are shown in the lower panel of Fig. 3. These K factors are computed analogously to Eq. (3) by simply replacing $d\sigma/dy$ with the asymmetry $A(y)$. Both K factors are close to unity, with small deviations in the region of small to medium $y_W$. The K factors vary in the ranges $K_{NLO}(y_W) \sim 0.98–1.08$ and $K_{NNLO}(y_W) \sim 0.94–1.02$, and the NNLO effects tend to be smaller than the NLO effects. To understand the origin of these effects, we have recomputed the NLO and NNLO asymmetry K factors by always using the LO partonic cross sections, though still using the PDFs at LO, NLO and NNLO. This procedure removes the effect of the radiative corrections in the partonic cross sections, so that the NLO (NNLO) K factor is directly sensitive to the NLO/LO (NNLO/NLO) ratio of PDFs. These ‘PDF driven’ NLO (dotted histogram) and NNLO (dot-dashed histogram) K factors are also displayed in the lower panel of Fig. 3. Since each ‘PDF driven’ K factor closely follows the quantitative behaviour

\footnote{\footnotesize Here and in the following calculations of rapidity distributions at the Tevatron Run II, the sizes of the rapidity bins are always fixed to be equal to those used in the corresponding measurements of the CDF and D0 Collaborations.}
Figure 3: The charge asymmetry of on-shell $W$ production at the Tevatron Run II. Upper panel: the CDF data [19] are compared with the QCD calculations at the LO (black dotted), NLO (red dashed) and NNLO (blue solid). Lower panel: the ratios NLO/LO (red dashed) and NNLO/NLO (blue solid) of the QCD results in the upper panel, and the corresponding ratios (NLO/LO)$_{LO}$ (magenta dotted) and (NNLO/NLO)$_{LO}$ (magenta dot-dashed) computed by using LO partonic cross sections.

of the corresponding K factor in the upper part of the lower panel, we conclude that the bulk of the radiative effects is produced by the variation of the PDFs in going from LO to NLO and to NNLO.

We have studied the factorization and renormalization scale dependence of the QCD results by varying $\mu_F = \mu_R$ between $M_W/2$ and $2M_W$. We find that the scale dependence of the $W$ asymmetry at NNLO is at the level of the numerical errors from our NNLO Monte Carlo computation (i.e. the error bars of the NNLO result in the upper panel of Fig. 3).

The theoretical predictions in Fig. 3 agree well with the CDF data, except in the last (i.e. highest-rapidity) bin. To measure the consistency between the theoretical and experimental results, we consider the $\chi^2$ probability function:

$$\frac{\chi^2}{N_{\text{pts.}}} = \frac{1}{N_{\text{pts.}}} \sum_{i=1}^{N_{\text{pts.}}} \frac{(th_i - exp_i)^2}{\Delta^2_{i,\text{exp}}},$$

where $N_{\text{pts.}}$ is the number of data points (rapidity bins), $th_i$ and $exp_i$ denote the theoretical and experimental values of the $i$-th data point (i.e. in the $i$-th rapidity bin), respectively, and
$\Delta_{i,\text{exp}}$ is the corresponding experimental error. The values of $\chi^2/N_{\text{pts}}$ at each perturbative order are reported in Fig. 3 and are dominated by the contribution of the last bin (removing the contribution of the last bin from the computation of $\chi^2$, at the NNLO we obtain $\chi^2/12 = 1.6$).

Figure 4: The charge asymmetry of on-shell $W$ production at the Tevatron Run II. The CDF data [19] are compared with the NNLO MSTW2008 result, including the corresponding PDF errors at the 1$\sigma$ level [29].

The NNLO result (solid histogram) of Fig. 3 is reported in Fig. 4 by including the PDF errors from the MSTW2008 parton densities [29]. The inclusion of the PDF errors increases the consistency between the CDF data and the MSTW2008 prediction. We have repeated the NNLO calculation of $W$ asymmetry at the Tevatron Run II by using the ABKM09 [35] and JR09VF [37] partons: the results (solid histograms), with the corresponding PDF errors, are presented in Fig. 5. Considering the different slopes of the cross section ratios in Fig. 2 we expect that the $W$ charge asymmetry from the ABKM09 (JR09VF) partons is typically larger (smaller) than the asymmetry from the MSTW2008 partons. This expectation is confirmed by the results in Figs. 4 and 5. We see that the ABKM09 prediction tends to overshoot the CDF data, while the JR09VF prediction tends to undershoot the CDF data in the region of small and medium rapidities. The values of $\chi^2/N_{\text{pts}}$ computed from the ABKM09 and JR09VF partons are reported in the corresponding plots of Fig. 5. Unlike the case of the MSTW2008 partons, the contribution of the highest-rapidity bin to the values of $\chi^2$ is not dominant. Removing the contribution of the last bin from the computation of $\chi^2$, we obtain $\chi^2/12 = 8.9$ (ABKM09 partons) and $\chi^2/12 = 7.4$ (JR09VF partons).

We add some comments on the CDF measurement of the $W$ asymmetry. Since the longitudinal momentum of the neutrino from $W$ decay is not determined experimentally, the rapidity of the $W$ and its asymmetry cannot directly be measured. The results of Ref. [19] regard the first ‘direct determination’, rather than ‘direct measurement’, of the $W$ asymmetry.
The CDF Collaboration \cite{19} selects $W \rightarrow e\nu_e$ events and measures the momentum of the electrons and positrons and the missing (i.e., neutrino) transverse momentum. Assuming that the decaying $W$ is on-shell (or, more generally, that the invariant mass of the $W$ is fixed), this kinematical information determines the neutrino’s longitudinal momentum (and, thus, $y_W$) within a twofold ambiguity (see, e.g., Eqs. (13) or (17)). This twofold ambiguity is resolved on a statistical (event–by–event) basis by using information of theoretical calculations. This information regards, for instance, the rapidity distributions of the $W$ boson (Ref. \cite{19} uses the NNLO calculation of Ref. \cite{4}) and of the charged lepton and the related boson–lepton momentum correlations (Ref. \cite{19} uses results from MC@NLO \cite{41}). The results of the theoretical calculations depend on the input PDFs (Ref. \cite{19} explicitly considers the MRST2006 NNLO set \cite{42} and the CTEQ6.1 NLO set \cite{43}). The use of this amount of theoretical information requires a consistent (and reliable) estimate of the theoretical uncertainty that eventually affects the determination of the $W$ asymmetry. For instance, Ref. \cite{19} considers the effect of the PDF errors of the CTEQ6.1 NLO set (though these PDF errors are known not to always match the differences between various available sets of PDFs). Another theoretically-driven corrections to be taken into account \cite{19} regards the off-shell $W$ effects in the experimentally selected $W \rightarrow e\nu_e$ events. These effects depend on the finite width of the $W$ and on the experimentally accepted $W$ mass range, and they are more important at high rapidities.

Reference \cite{19} reports the result of the $W$ asymmetry in each $y_W$ bin, and it also presents the value, $\langle y_W \rangle$, of the ‘average’ bin center in each bin. The ‘average’ bin centers include the estimated corrections (assuming a fixed $W$ mass of 80.403 GeV) \cite{19} to the $W$ asymmetry from off-shell $W$ effects. In our computation of $\chi^2$ (whose values are reported in Figs. 3–5), we have compared the theoretical calculations of the $W$ asymmetry with the CDF data at the level of histogram bins. As pointed out to us by the MSTW Group \cite{44}, the value of $\chi^2$ can vary if the comparison

**The relation between the rapidity distributions of the \textit{W} and the charged lepton is discussed in Sect. \textit{3.2}.**
between the theoretical calculations and the CDF data is performed by considering the values of the $W$ asymmetry at the ‘average’ bin centers. The NNLO calculation of the asymmetry at the ‘average’ bin centers can be carried out by using the numerical program of Ref. [4] (we could also use our Monte Carlo program and compute the asymmetry in a small rapidity bin around each ‘average’ bin center). We have constructed an approximated functional form of $A(y_W)$ by fitting our histogram results, and we have used this approximation to compute $\chi^2$ from the comparison with the CDF data at the ‘average’ bin centers. In the case of the MSTW2008 partons, we find values of $\chi^2/N_{\text{pts}}$ that are relatively close to unity; the main difference with respect to the values of $\chi^2/N_{\text{pts}}$ in Fig. 3 and 4 is due to the reduced contribution of the last (i.e. highest-rapidity) bin. In the cases of the ABKM09 and JR09VF partons, we find values of $\chi^2/N_{\text{pts}}$ that are slightly reduced (about 10%) with respect to those reported in Fig. 5.

### 3.2 Charged lepton rapidity distribution and asymmetry

We now move to our study of the rapidity distributions of the charged lepton from $W$ decay. These differences originate from the underlying short-distance dynamics and kinematics, as briefly described below.

**EW dynamics correlations.**

Owing to the spin 1 nature of the $W$ boson, its production and decay mechanisms are correlated. The actual correlation depends on the $V - A$ coupling of the $W$ boson to both the annihilating $q\bar{q}$ pair and the decaying lepton pair. The main effect of the correlation can be understood by simply considering the LO partonic subprocesses in Eqs. (6) and (7). The corresponding angular distribution of the charged lepton is (see, e.g., Ref. [45])

$$
\frac{1}{\hat{\sigma}^{(0)}_{DD}} \frac{d\hat{\sigma}^{(0)}_{UD}}{d \cos \theta^{*}_{ID}} = \frac{1}{\hat{\sigma}^{(0)}_{DD}} \frac{d\hat{\sigma}^{(0)}_{D\bar{D}}}{d \cos \theta^{*}_{ID}} = \frac{3}{8} (1 + \cos \theta^{*}_{ID})^2 \right.,
$$

where $\theta^*$ is the scattering angle of the charged lepton in the centre–of–mass system of the colliding quark and antiquark. More precisely, $\theta^*_{ID}$ is the lepton scattering angle with respect to the direction of the ‘down-type’ quark or antiquark. Therefore, the form of the angular distribution on the right-hand side of Eq. (10) implies that at the partonic level the charged lepton tends to follow the direction of the colliding $D$ (see Eq. (6)) or $\bar{D}$ (see Eq. (7)).

The rapidity (angular) distribution of the charged lepton at the hadronic level results from the combined effect of Eq. (10) and of the parton densities of the colliding hadrons. More precisely, since the $W$ and lepton rapidities are related by

$$
y_l = y_W + \frac{1}{2} \ln \frac{1 + \cos \theta^*}{1 - \cos \theta^*} \right.,
$$

the lepton rapidity distribution arises from the convolution of Eq. (10) with the $W$ rapidity distribution.

In the case of $p\bar{p}$ collisions, the $W$ boson tends to follow the direction of the colliding ‘up-type’ quark or antiquark and, therefore, the forward–backward asymmetry produced by the EW
distribution in Eq. (10) exactly acts in the opposite direction. As a consequence, the rapidity distribution of the positive (negative) charged lepton is shifted backward (forward) with respect to the distribution of the parent $W^+$ ($W^-$). The relative weight of the two competitive effects depends on the detailed kinematical correlations between the lepton and the boson, and it can be controlled by varying, for instance, the lepton $E_T$.

**Kinematics correlations.**

In hadron collisions $W$ boson events are selected by requiring a lower limit on the invariant mass (or, typically, on the leptonic transverse mass $M_T$) of the $W$ boson. Provided this lower limit is close to $M_W$ (though smaller than $M_W$) the kinematics of the $W$ and its decaying leptons is well described by using the narrow-width approximation (NWA), i.e. by assuming that the $W$ is on-shell. At the LO in perturbative QCD, the $W$ boson is produced with a vanishing transverse momentum $q_T$; therefore, within the NWA the kinematical variables of the $W$ and charged lepton fulfil the relation (the symbol ‘$\sim$’ denotes the use of the NWA)

\[ M_W \sim 2E_T \cosh(y_W - y_l) , \quad E_T \lesssim M_W/2 \ , \quad (12) \]

or, equivalently, \(1 - \cos^2 \theta^* = 4E_T^2/M_W^2\). At fixed $E_T$, the rapidities of the $W$ and the charged lepton are thus directly correlated:

\[ |y_W - y_l| \sim \ln \left[ \frac{M_W}{2E_T} + \sqrt{\left( \frac{M_W}{2E_T} \right)^2 - 1} \right] . \quad (13) \]

In particular, by increasing $E_T$, $y_l$ is forced to be close to $y_W$ and the rapidity distribution of the charged lepton tends to follow the rapidity distribution of the $W$, thus minimizing the impact of the rapidity asymmetry produced by the EW dynamics (i.e. by Eq. (10)).

Incidentally, we also note that, at the LO and within the NWA, the leptonic variables $E_T^\nu$, $M_T$ and $E_T$ are not independent. We have

\[ E_T^\nu \sim E_T , \quad M_T \sim 2E_T \ , \quad (14) \]

so that fixing, for instance, $E_T$ fully specifies both $E_T^\nu$ and $M_T$.

Having discussed the main differences between the rapidity distributions of the $W$ boson and of the charged lepton $l$, we add a comment on a direct consequence of thses differences. The $W$ and $l$ rapidity distributions have a different dependence on the parton densities of the colliding hadrons.

In $pp\bar{p}$ collisions, for instance, owing to the effect of the angular distribution in Eq. (10), the $y_l$ distribution of the $l^+$ at positive (negative) rapidity is more (less) sensitive to the antiquark densities of the proton than the $y_W$ distribution of the $W^+$ at positive (negative) rapidity. This point is quantitatively illustrated in Sect. 11.1 of Ref. [29].

Moreover, the boson and lepton rapidity distributions probe the parton densities at different typical values of parton momentum fractions $x_1$ and $x_2$ (see Eq. (11)). In LO QCD and using the NWA, the production of $W^\pm$ bosons (and their decaying $l^\pm$ leptons) is mostly sensitive to the region of parton momentum fractions with $x_1 x_2 \simeq M_W^2/s$. The typical value of the ratio $x_1/x_2$ is instead controlled by the boson or lepton rapidity. Fixing the rapidity of the $W$, we have

\[ \frac{1}{2} \ln(x_1/x_2) \simeq \pm y_W , \quad (15) \]
and different values of the ratio $x_1/x_2$ are explored by varying $y_W$. Owing to the kinematical relation \( 12 \ln(x_1/x_2) \approx |y_l - \ln(y_l) + \ln(y_l) - (M_W^2/E_T^2 - 1) \mid \).

We see that equal values of $y_W$ and $y_l$ actually probe different values of the ratio $x_1/x_2$, the difference being controlled by the value of the lepton $E_T$.

The differences between the rapidity distributions of the $W$ boson of the charged lepton $l$ have a direct impact on any observables that are measured, or computed, by applying restrictions on the kinematics of the leptons from $W$ decay. This comment is valid also for observables that directly refer to kinematical variables of the $W$ boson. For instance, it is interesting to study how the results for the charge asymmetry of fully-inclusive $W$ production (Fig. 3) are affected by typical selection cuts that are used in experiments at the Tevatron Run II.

For illustrative purpose, we consider the lepton selection cuts used by the CDF Collaboration in Ref. \cite{22}: the observed charged lepton must be produced in the rapidity region $|y_l| \leq y_{l,\text{MAX}}$ with $y_{l,\text{MAX}} = 2.45$, and it has to be isolated from hadronic activity (the CDF isolation criterion is described later in this subsection); the selected $W \to l\nu$ events are required to have $E_T > 25 \text{ GeV}$ and $50 \text{ GeV} < M_T < 100 \text{ GeV}$. In Fig. 6 we show the QCD results on the $W$ charge asymmetry after the implementation of the lepton selection cuts. Following the experimental analysis \cite{22}, we consider two bins of the charged lepton $E_T$: 25 GeV < $E_T$ < 35 GeV (left panels in Fig. 6) and 35 GeV < $E_T$ < 45 GeV (right panels in Fig. 6).

Comparing the histogram in Fig. 3 with those in Fig. 6, we see that the shape (also the size) of the $W$ charge asymmetry is changed by the introduction of the lepton selection cuts.

As in Fig. 3 the lower panels in Fig. 6 show the asymmetry K factors at NLO (dashed) and NNLO (solid), and the K factors computed with the corresponding NLO (dotted) and NNLO (dot-dashed) PDFs by using the LO partonic cross sections. The NLO and NNLO K factors in Fig. 6 are similar to those in Fig. 3: the radiative corrections to the asymmetry remain small after introducing the selection cuts, and the main effect of these corrections acts in the region where $y_W \lesssim 1.5$. As in the case of the inclusive-$W$ asymmetry, the effect of the radiative corrections is mostly produced by the variation of the PDFs in going from the LO PDF set to the NLO set and to NNLO set.

To explain the origin of the differences between the charge asymmetry results in Figs. 3 and 6 we have to understand which are the most significant lepton selection cuts. The charge asymmetry in Figs. 3 refers to on-shell $W$ production. The results in Fig. 6 are obtained without fixing the $W$ invariant mass to its on-shell value; the off-shell effects have, however, a little impact on the results since the minimum value of $M_T$ is smaller than $M_W$ and close to it. The charged lepton isolation has also a minor impact, since it is not effective at the LO and the radiative corrections are small. At the LO and within the NWA, the constraints $E_T > 25 \text{ GeV}$ and $50 \text{ GeV} < M_T < 100 \text{ GeV}$ are superseded by the constraints on $E_T$: using Eq. (14), the cut 25 GeV < $E_T$ < 35 GeV implies $E_T > 25 \text{ GeV}$ and $50 \text{ GeV} \lesssim M_T \lesssim 70 \text{ GeV}$, and the cut 35 GeV < $E_T$ < 45 GeV implies $E_T > 35 \text{ GeV}$ and 70 GeV $\lesssim M_T \lesssim 90 \text{ GeV}$. We can conclude that the mainly-relevant parameters that control the results in Fig. 6 are the maximum rapidity $y_{l,\text{MAX}}$ and the $E_T$ of the charged lepton.
Figure 6: $W$ production at the Tevatron Run II with lepton selection cuts. The rapidity of the charged lepton is constrained to be in the region $|y_l| \leq 2.45$, and its transverse energy is in the intervals $25 \text{ GeV} < E_T < 35 \text{ GeV}$ (left) and $35 \text{ GeV} < E_T < 45 \text{ GeV}$ (right). Upper panels: the $W$ charge asymmetry at the LO (black dotted), NLO (red dashed) and NNLO (blue solid). Lower panels: the ratios NLO/LO (red dashed) and NNLO/NLO (blue solid) of the QCD results in the upper panels, and the corresponding ratios $(\text{NLO/LO})_{\text{LO}}$ (magenta dotted) and $(\text{NNLO/NLO})_{\text{LO}}$ (magenta dot-dashed) computed by using LO partonic cross sections.

The cut on the charged lepton rapidity selects a fraction of the inclusively-produced $W$, and this fraction is not uniform with respect to $y_W$. Owing to the EW dynamics correlations discussed at the beginning of this subsection, the $l^+(l^-)$ originates from a parent $W^+(W^-)$ that is preferably produced at $y_W > y_l(y_W < y_l)$. Therefore, the forward–backward symmetric cut, $-y_{l,\text{MAX}} < y_l < y_{l,\text{MAX}}$, on $y_l$ selects an event sample with an enriched component of forward-produced $W^+$ (backward-produced $W^-$): the $y_W$ charge asymmetry of this sample is thus higher than the asymmetry of the fully-inclusive sample. The increase of the (absolute value of the) asymmetry is larger in the region of high values of $|y_W|$, where the selection cut $|y_l| < y_{l,\text{MAX}}$ is more effective (the constraint $|y_l| < y_{l,\text{MAX}}$ has a relatively-small effect on the $W$ bosons at $y_W \sim 0$). Moreover, the region of high-$|y_W|$ values where the asymmetry increases depends on $E_T$. Increasing $E_T$ this region moves to larger values of $|y_W|$ since, owing to the lepton–boson kinematic correlations, by increasing $E_T$ the lepton and boson rapidities get closer (see Eq. (13)) and, consequently, the cut $|y_l| < y_{l,\text{MAX}}$ selects the parent $W$ bosons more uniformly over an extended region of $|y_W|$.

The qualitative features that we have just discussed are confirmed by the quantitative results in Fig. 6. In the low-$E_T$ bin, the $W$ charge asymmetry (left panel in Fig. 6) closely follows the inclusive-$W$ asymmetry (Fig. 3) in the region $y_W \lesssim 1.5$, and it sharply increases at larger values of $y_W$. In the high-$E_T$ bin, the $W$ charge asymmetry (right panel in Fig. 6) closely follows the inclusive-$W$ asymmetry (Fig. 3) up to $y_W \sim 2$; then, it increases and, at large $y_W (y_W \sim 3)$, it reaches the values of the charge asymmetry in the low-$E_T$ bin.

In summary, the differences between the charge asymmetry results in Figs. 3 and 6 are due
to the fact that the rapidity distribution of the lepton is, on average, more forward–backward symmetric than the rapidity distribution of the parent $W$. More precisely, as discussed at the beginning of this subsection, the rapidity distribution of the $l^+(l^-)$ lepton is slightly bent backward (forward) with respect to the distribution of the $W^+$ ($W^-$) boson.

![Figure 7](image.png)

**Figure 7:** Rapidity distribution of the charged lepton from $W^+$ decay up to NNLO QCD: low-$E_T$ bin (left), high-$E_T$ bin (right). In the lower plots the NLO/LO (dashed) and NNLO/NLO (solid) ratios are shown.

The typical behaviour of the $l^+$ rapidity$^\dagger$ cross section is shown in Fig. 7. As in Fig. 6 we consider the lepton selection cuts of Ref. [22] and two $E_T$ bins. The three histograms in each upper panel of Fig. 7 give the results of our LO, NLO and NNLO calculation of the lepton rapidity cross section in each corresponding $E_T$ bin. As in the case of the $W$ rapidity distribution (see Fig. 1), the height of each histogram bin gives the value of the cross section in the corresponding rapidity bin.

In the lower panels of Fig. 7, we present the NLO (dashed) and NNLO (solid) K factors, computed from the rapidity cross sections in the upper panels (see Eq. (8)). The NLO effects are bigger than the NNLO effects on both the normalization and the shape of the lepton rapidity cross section, analogously to the case of the $W$ rapidity cross section (see Fig. 1). In the low-$E_T$ bin, the NLO K factor varies in the range $K_{NLO}(y_l) \sim 1.08$–1.28 (considering $|y_l| \lesssim 2$, the range is $K_{NLO}(y_l) \sim 1.10$–1.22), while the NNLO K factor varies in the range $K_{NNLO}(y_l) \sim 0.94$–1.04. In the high-$E_T$ bin, the NLO K factor varies in the range $K_{NLO}(y_l) \sim 1.12$–1.26 (considering $|y_l| \lesssim 2$, the range is $K_{NLO}(y_l) \sim 1.16$–1.26), while the NNLO K factor varies in the range $K_{NNLO}(y_l) \sim 0.96$–1.04. These lepton K factors tend to be closer to unity than the corresponding K factors for inclusive $W$ production. The same tendency is observed by considering the K factors computed from the ratio of the total accepted cross sections (i.e. integrated over the rapidity range $|y_l| \leq 2.45$): in the low-$E_T$ bin, we find $K_{NLO} = 1.16$ and $K_{NNLO} = 0.99$; in the high-$E_T$

$^\dagger$Hadron collider experiments actually measure the lepton pseudorapidity $\eta_l$, and we use the label $\eta_l$ in the figures of the paper. Since in our QCD calculations with massless leptons we have $\eta_l = y_l$, in the text we always refer to the lepton rapidity and we equivalently use the labels $\eta_l$ and $y_l$. 

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bin, we find $K_{NLO} = 1.21$ and $K_{NNLO} = 0.99$.

Although the QCD radiative corrections to the lepton rapidity cross section tend, on average, to be smaller than those to the $W$ rapidity cross section, the lepton K factors in Fig. 7 and the inclusive K factors in Fig. 1 have definitely different shapes. The difference is particularly evident at NLO. In particular, the lepton K factors are certainly less forward–backward symmetric than the inclusive K factors. Therefore, we can expect that the effect of the radiative corrections on the lepton charge asymmetry is larger than the effect on the $W$ charge asymmetry, especially a high values of $y_l$.

In Figs. 1 and 7 we see that both the $W^+$ and the $l^+$ are mainly produced in the forward region. Comparing the shapes of the $W^+$ and $l^+$ rapidity distributions, we can also see the effect of the $V - A$ structure of the EW interactions: in going from the $W^+$ to the $l^+$, the peak of the distribution is shifted and the overall distribution is bent toward the backward direction. In agreement with Eqs. (12) and (13), the effect is less evident when higher values of $E_T$ are selected. These features are expected from a basic LO analysis of boson–lepton correlations (as discussed at the beginning of this subsection). The impact of higher-order QCD corrections on this LO picture can be seen by a careful inspection of the K factors in Fig. 7: the QCD corrections partly compensate for the boson–lepton differences that occur at the LO. To a good approximation, the K factors in the low-$E_T$ (high-$E_T$) bin monotonically increase (decrease) as $y_l$ increases. Thus, the effect of the NLO (and NNLO) corrections in the lower-$E_T$ bin is to (slightly) shift the lepton rapidity distribution forward, whereas the opposite happens in the higher-$E_T$ bin. As a consequence, in the lower (higher) $E_T$ bin we expect the effect of QCD corrections on the lepton charge asymmetry to be positive (negative). Moreover, owing to the monotonic behaviour of the K factors, the absolute size of this effect is expected to increase at high $|y_l|$.

The qualitative effect of the QCD radiative corrections deserves some comments. Our physical interpretation is that QCD corrections produce dynamical and kinematical decorrelation effects of the LO correlations between the rapidity distributions of the $W$ boson and of its decaying lepton.

The QCD corrections to the LO partonic cross section in Eq. (10) are produced by multiparton radiation from the initial-state (colliding) partons. Since the QCD couplings are purely vector-like and flavour blind (insensitive to the difference between 'up-type' and 'down-type' quarks and antiquarks), QCD radiation dynamically dilutes the forward–backward asymmetry (see Eq. (10)) that is produced at the LO by the $V - A$ structure of the EW couplings. After convolution with the PDFs of the initial-state partons, this partonic diluted asymmetry reduces the impact of the lepton decay on the shape of the (PDF-driven) rapidity distribution of the parent $W$.

The QCD corrections also affect the kinematics of the $W$. In particular, the $W$ boson is no longer produced with a vanishing transverse momentum $q_T$ and, thus, Eq. (12) can be modified as follows:

$$M_W \simeq 2E_T \left[ \cosh(y_W - y_l) + O(q_T/M_W) \right]. \quad (17)$$

Since the typical values of the transverse momentum are $q_T \lesssim \alpha_S M_W$, the correction term $O(q_T/M_W)$ is not large. Nonetheless, the effect of this correction term reduces the LO correlation between the rapidities of the $W$ and the charged lepton (see Eq. (13)). In particular, the right-hand side of Eq. (17) shows that the kinematical decorrelation produced by the correction term becomes more important by increasing $E_T$ (i.e. when $y_l$ is closer to $y_W$).
In the following we concentrate our attention on the lepton charge asymmetry. We present our perturbative QCD calculations up to NNLO, and their comparison with some of the published Tevatron data.

At the Tevatron Run II, the CDF and D0 Collaborations have performed measurements of the lepton charge asymmetry by analyzing data samples with increasing integrated luminosity $L$ (statistics), namely $L = 170$ pb$^{-1}$ [22], 300 pb$^{-1}$ [23] and 750 pb$^{-1}$ [24]. We list the lepton selection cuts that are used in these measurements and implemented in our corresponding QCD calculations. As already mentioned, the electron (and positron) event cuts of the CDF Collaboration [22] are $E_T^e > 25$ GeV and $50$ GeV $< M_T < 100$ GeV, and the electron charge asymmetry is measured in three different $E_T$ bins: $E_T > 25$ GeV, $25$ GeV $< E_T < 35$ GeV and $35$ GeV $< E_T < 45$ GeV. The D0 muon charge asymmetry [23] is measured in the region where $E_T > 20$ GeV, with the selection cuts $E_T^\mu > 20$ GeV and $M_T > 40$ GeV. The electron (and positron) event cuts of the D0 Collaboration [24] are $E_T^e > 25$ GeV and $M_T > 50$ GeV, and the electron charge asymmetry is measured in three different $E_T$ bins: $E_T > 25$ GeV, $25$ GeV $< E_T < 35$ GeV and $E_T > 35$ GeV. In all these measurements the charged leptons are required to be isolated.

The charged lepton isolation is defined by considering a cone along the direction $\{\eta_l, \phi_l\}$ of the lepton momentum in pseudorapidity–azimuth ($\eta$–$\phi$) space. The cone radius is $R = \sqrt{(\eta - \eta_l)^2 + (\phi - \phi_l)^2}$ and the hadronic (partonic) transverse energy in the cone is denoted by $E_{T,iso}^l$. The CDF and D0 isolation criterion for electrons and positrons fixes $R = 0.4$: the CDF Collaboration requires $E_{T,iso}^e/E_T < 0.1$ [22], while D0 requires $E_{T,iso}^e/E_T < 0.15$ [24]. The D0 isolation criterion for $\mu^\pm$ [23] requires $E_{T,iso}^\mu < 2.5$ GeV, where $E_{T,iso}^\mu$ is the hadronic transverse energy in a hollow cone of inner radius $R = 0.1$ and outer radius $R = 0.4$.

The MSTW Group has analyzed the Tevatron Run II data on the lepton charge asymmetry in the context of his global fit of PDFs [29]. Their QCD calculation of the lepton charge asymmetry uses the NNLO code FEWZ [31], although the partonic cross sections are computed only up to NLO (see Sect. 11.1 in Ref. [29]). The main conclusions of the MSTW study are as follows [29]. The CDF electron asymmetry (low-$E_T$ and high-$E_T$ bins) [22] and D0 muon asymmetry [23] data are reasonably well fitted at NNLO (at high $y_l$, some evidence for a systematic discrepancy between the NNLO fit and the data is reported). The D0 electron asymmetry data [24] are not included in the MSTW PDF fit: their inclusion in the global analysis does not permit to obtain a good quality NNLO fit, with significant tension between the D0 electron asymmetry data and other data (DIS structure functions and low-mass DY production).

We first consider the electron charge asymmetry in the experimental configuration of the CDF data [22], which cover the rapidity region $|\eta_e| \leq 2.45$. We examine the low-$E_T$ and the high-$E_T$ bins (the same bins as in Figs. 6 and 7), whose data are included in the MSTW2008 fit. In Fig. 8 we report the CDF data and present the results of our calculations.

The results of the LO, NLO and NNLO calculations in the low-$E_T$ and high-$E_T$ bins are shown in Figs. 8(a) and 8(b), respectively. At small values of $\eta_e$, the radiative corrections lead to little effects. In the low-$E_T$ bin (Fig. 8(a)), as $\eta_e$ increases, both the NLO and NNLO effects

\[\text{‡}\] The D0 measurement uses $p_T > 20$ GeV, where $p_T$ is the transverse momentum of the muon. In our QCD calculations with massless leptons, $p_T$ and $E_T$ are equivalent.

\[§\] The size of the rapidity bins is not explicitly reported in Table I of Ref. [22]. The bin size is (from correspondence with C. Issever) $\Delta \eta_e = 0.2$ in the central region ($|\eta_e| < 1.2$) and $\Delta \eta_e = 0.25$ in the forward region ($1.2 < |\eta_e| < 2.45$).
slightly increase the value of the asymmetry; the NNLO effect is smaller than the experimental uncertainties. These small and positive radiative corrections are consistent with the corresponding shift of the lepton rapidity distribution observed in the left-side plot of Fig. 7. In the high-$E_T$ bin (Fig. 8(b)), as $\eta_e$ increases, the effect of the QCD radiative corrections is negative and the asymmetry slightly decreases; the NNLO effect is definitely smaller than the experimental uncertainties. These negative contributions of the radiative corrections are also consistent with the effects already seen in the right-side plot of Fig. 7.

In Figs. 8(c) and 8(d) we present the results obtained by convolution of the LO, NLO and NNLO PDFs with the LO partonic cross sections. Comparing the histograms in Fig. 8(a) with the corresponding (in the sense of corresponding PDFs) histograms in Fig. 8(c), we observe a very different pattern of perturbative corrections: in the low-$E_T$ bin, the radiative corrections to the lepton asymmetry are definitely dominated by the radiative contribution to the partonic cross section.
sections. In the high-$E_T$ bin we observe a different behaviour. The histograms in Fig. 8(b) are qualitatively similar to the corresponding histograms in Fig. 8(d), and the NLO and NNLO effects on the asymmetry are thus mostly driven by the corresponding PDFs.

We add some overall qualitative comments on the results in Fig. 8 and on their comparison with the corresponding results for the $W$ charge asymmetry (see Fig. 3). Owing to the invariance under CP transformations, in the following comments we always refer to the rapidity region $y > 0$. In $p\bar{p}$ collisions, the lepton asymmetry is definitely different from the $W$ asymmetry. The lepton asymmetry is smaller than the $W$ asymmetry and the difference increases as the rapidity increases. The difference is mostly due to the LO effect produced by the $V - A$ structure of the EW couplings in the underlying partonic process (see Eq. (10)). By increasing the lepton $E_T$, the difference is reduced because of the increased LO kinematical correlation between the $W$ and the lepton (see Eq. (13)). The QCD radiative corrections to the charge asymmetry are small in both the $W$ and lepton cases. Their effect is relatively larger in the lepton case. Since the lepton charge asymmetry depends on the EW asymmetry generated by the partonic subprocesses, it is more sensitive to the QCD radiative corrections to these partonic subprocesses. As previously mentioned, at low values of $E_T$, QCD radiation dynamically reduces the partonic EW asymmetry and, therefore, the value of the lepton charge asymmetry slightly increases with respect to its LO result (i.e. the difference between the lepton and $W$ asymmetry tends to be slightly reduced). This dynamical effect is compensated by a QCD kinematical effect (see Eq. (17)) that weakens the LO kinematical correlation between the $W$ and the lepton at high values of $E_T$. By increasing $E_T$, the dynamical effect is eventually over-compensated by the kinematical effect and, therefore, the value of the lepton charge asymmetry slightly decreases with respect to its LO result (i.e. the difference between the lepton and $W$ asymmetry tends to be slightly increased).

In Fig. 9 we consider the scale dependence of the electron charge asymmetry at NLO and NNLO. The scale dependence bands are obtained by fixing $\mu_F = \mu_R = \mu$ and considering the values $\mu = M_W/2$ and $2M_W$ in the calculations at NLO and NNLO. The scale dependence is
small at both NLO and NNLO (at NNLO, the scale dependence is comparable to the numerical errors of our NNLO Monte Carlo computation). In particular, the NLO and NNLO bands tend to overlap in both the low-$E_T$ and high-$E_T$ bins.

The values of $\chi^2$ from the comparison of the CDF electron data with the QCD calculations are reported in Figs. S(a) and S(b). The data are well fitted by the MRSTW2008 NNLO partons, especially in the low-$E_T$ bin. Considering the high-$E_T$ bin, we see that the QCD results tend to overshoot the data in the intermediate region of lepton rapidities; the value of $\chi^2$ is dominated by the contributions in the region $1.2 \lesssim \eta_e \lesssim 1.9$. The NNLO effects reduce the lepton asymmetry at intermediate and large rapidities, but their impact is not yet quantitatively relevant in comparison with the size of the experimental uncertainties.

We now move to briefly consider the lepton selection cuts used in the D0 measurement of the muon charge asymmetry [23]. The D0 data cover the rapidity region where $|\eta_\mu| \leq 2$. In Fig. 10 we display the data and present the results of our corresponding calculation at LO, NLO and NNLO. Note that the lepton $E_T$ is selected in the entire region where $E_T > 20$ GeV. This region includes both the low-$E_T$ and high-$E_T$ regions that are examined separately in Fig. S. The main features of the results in Fig. 10 are thus intermediate between those of Figs. S(a) and S(b). The QCD radiative corrections are small. In particular, at high $\eta_\mu$, the small NNLO effect is positive and slightly increases the NLO charge asymmetry.

In Fig. 10 we can see that the size of the QCD radiative corrections is certainly smaller than the experimental errors of the D0 data. The values of $\chi^2$ from our QCD calculations are reported.
in the figure. The D0 data are reasonably well fitted at NNLO by the MSTW2008 PDFs, and the conclusions of the MSTW analysis [29] are unchanged by the effect of the NNLO corrections to the partonic cross sections. A systematic difference between the data and the QCD result appears starting from the region of intermediate rapidities ($\eta_\mu > 0.9$).

![Figure 11: Electron charge asymmetry up to NNLO QCD compared to the D0 data of Ref. [24]: $E_T > 25$ GeV (left), $E_T > 35$ GeV (right).](image)

We finally consider the D0 electron charge asymmetry [24]. The D0 measurement extends to high rapidities, $|\eta_e| \leq 3.2$. We recall that the D0 data are not included in the PDF fit of the MSTW Group [29]. We examine two regions of $E_T$: $E_T > 25$ GeV (wide $E_T$ region) and $E_T > 35$ GeV (high-$E_T$ region).

In the left panel of Fig. 11, we consider the wide $E_T$ region: we display the D0 electron data and present the corresponding QCD results at LO, NLO and NNLO. The QCD results behave similarly to those in Fig. 10. This is not unexpected since, at the LO and within the NWA (see Eq. (14)), the relevant lepton selection cuts used in these two cases are quite similar: we have $E_T \sim > 25$ GeV and $E_T \sim > 20$ GeV in Fig. 11 (left) and Fig. 10 respectively. At high values of the electron rapidity, the effect of the NLO and NNLO corrections is positive and increases the deviation of the QCD results from the D0 data. We see that (consistently with the comments in Ref. [29]) the agreement between the QCD calculations and the data is poor. This is quantitatively confirmed by the values of $\chi^2$, which are reported in Fig. 11 (left). We note that the value of $\chi^2$ increases in going from LO to NLO and to NNLO.

In the right panel of Fig. 11, we consider the high-$E_T$ region. In this region (as in the case of the high-$E_T$ bin in Fig. 8) the effect of the NLO and NNLO corrections is negative as the lepton rapidity increases toward high values. This effect reduces the difference between the QCD calculations and the D0 data, although a substantial disagreement between them still persists at the NNLO. The value of $\chi^2$ is reported in Fig. 11 (right); it decreases in going from LO to NLO and to NNLO.

In Fig. 12 we present the result of the QCD calculation performed by convoluting the NLO
partonic cross sections with the NNLO PDFs. This result is compared with the customary NLO and NNLO results (the same results as in Fig. 11). The comparison shows that the NNLO corrections to the partonic cross sections are not negligible. Their effect is quantified by the variation of the $\chi^2$ values, which are reported in the plots of Fig. 12. In the region $E_T > 25$ GeV (plot on the left) the NNLO partonic corrections tend to increase the value of the asymmetry, while in the region $E_T > 35$ GeV (plot on the right) the corrections tend to decrease the asymmetry. This different behaviour in different $E_T$ regions is consistent with our physical expectation about the $E_T$ dependence of the QCD radiative corrections (see, e.g., our related comments on the CDF electron asymmetry).

We have computed the scale dependence of the NLO and NNLO results presented in Fig. 11. This scale dependence is very similar to that documented in Fig. 9. In particular, the scale variations produce effects that are quantitatively smaller than the size of the PDF errors, which are considered below.

The NNLO results (solid histograms) of Fig. 11 are reported in Fig. 13 by including the PDF errors (68% C.L.) from the MSTW2008 parton densities [29]. In most of the rapidity bins, the PDF errors are comparable to (or, larger than) the D0 experimental errors. The inclusion of the PDF errors thus reduces the differences between the NNLO MSTW2008 results and the D0 electron data, although a substantial disagreement still remains.

In Fig. 14 we present the NNLO electron asymmetry computed with the ABKM09 set of PDFs [35], and we include the corresponding PDF errors. These PDF errors are slightly smaller than the PDF errors of the MSTW2008 set. In the case of the inclusive-$W$ asymmetry, the ABKM09 result tends to overshoot the MSTW2008 result (see Figs. 5 and 4) in the rapidity region where $0.5 \lesssim y_W \lesssim 2.3$. This effect appears also in the case of the electron asymmetry, as can be seen by comparing the results in Fig. 14 with those in Fig. 13. In the region $E_T > 25$ GeV (plots on the
The ABKM09 result evidently overshoots the MSTW2008 result at large rapidities ($|\eta_e| \gtrsim 1$). The overshooting effect is even more evident (with the exception of the last rapidity bin at high $|\eta_e|$) in the region where $E_T > 35$ GeV (plots on the right). The use of the ABKM09 parton densities does not reduce the disagreement with the D0 data, and the corresponding values of $\chi^2$ (see Fig. 14) are definitely larger than the MSTW2008 values.

In Fig. 15 we present the NNLO electron asymmetry computed with the JR09VF set of PDFs [37], and we include the corresponding PDF errors. These PDF errors are larger than the PDF errors of the MSTW2008 set. In the case of the inclusive-$W$ asymmetry, the JR09VF result tends to undershoot the MSTW2008 result (see Figs. 5 and 4) in the rapidity region where $0.2 \lesssim y_W \lesssim 1.8$. Comparing the electron asymmetry results in Fig. 15 with those in Fig. 13 we see that the JR09VF result undershoots the MSTW2008 result even at high rapidities. In the region where $E_T > 25$ GeV (Fig. 15 left), the JR09VF result is (relatively) consistent with the D0 data. In the region where $E_T > 35$ GeV (Fig. 15 right), the $\chi^2$ value of the MSTW2008 result is reduced by considering the JR09VF parton densities; the main deviations of the JR09VF result from the D0 data occur in the rapidity region around $\eta_e \sim 0.8$ and at high rapidities ($\eta_e \gtrsim 2.2$).
Figure 14: Electron charge asymmetry at NNLO with the PDFs of Ref. [35]: $E_T > 25$ GeV (left), $E_T > 35$ GeV (right).

Figure 15: Electron charge asymmetry at NNLO with the PDFs of Ref. [37]: $E_T > 25$ GeV (left), $E_T > 35$ GeV (right).
4 Rapidity cross section and asymmetry at the LHC

In this section we consider \( pp \) collisions at LHC energies. We recall the main features of \( W^\pm \) (and \( l^\pm \)) production, and we present results of our QCD calculations at the centre–of–mass energies \( \sqrt{s} = 10 \) TeV and 7 TeV.

4.1 \( W \) rapidity distribution and asymmetry

![Figure 16: Rapidity distribution of an on-shell \( W^- \) (left panel) and \( W^+ \) (right panel) at the LHC \( (\sqrt{s} = 10 \) TeV), in LO (black dotted), NLO (red dashed) and NNLO (blue solid) QCD. No cuts are applied on the leptons and on their accompanying final state. The lower panel shows the ratios NLO/LO (red dashed) and NNLO/NLO (blue solid) of the cross section results in the upper panel.](image)

All the numerical results presented in this subsection refer to \( W \) production at the centre–of–mass energy \( \sqrt{s} = 10 \) TeV. In Fig. 16 we present the rapidity cross sections of on-shell \( W^- \) (left panel) and \( W^+ \) (right panel) bosons. The histograms in the upper panels give the values of our LO, NLO and NNLO calculation of the cross sections \( \sigma = \sigma(W^\pm)BR(W \rightarrow l\nu) \) in each rapidity bin. Owing to CP invariance of the QCD cross section, both the \( W^- \) and \( W^+ \) rapidity
distributions are forward–backward symmetric in $pp$ collisions, and thus only half of the rapidity range needs be shown.

The LO cross sections are controlled by the partonic subprocesses in Eqs. (6) and (7). The $W^+$ boson is mainly produced by $u\bar{d}$ collisions, while the $W^-$ boson is mainly produced by $d\bar{u}$ collisions. The antiquark parton densities $\bar{d}(x)$ and $\bar{u}(x)$ of the proton are relatively similar, especially at small values of the parton momentum fraction $x$ (at LHC energies the typical values of $x$ are smaller than at Tevatron energies). The $u$ quarks carry, on average, more proton momentum fraction than $d$ quarks and, moreover $u(x)$ is larger than $d(x)$. As a consequence, the $W^+$ cross section is larger than the $W^-$ cross section, and the $W^+$ tends to be produced at larger rapidities with respect to the $W^-$. In Fig. 16 we see that the $W^+$ and $W^-$ rapidity distributions are (smoothly) peaked at $|y_W| \sim 2.5$ and $y_W \sim 0$, respectively.

As in the case of $W$ production at the Tevatron, having computed the rapidity cross section up to NNLO, we can examine the quantitative convergence of the perturbative expansion. We use the K factors defined in Eq. (8); their values for $W^+$ and $W^-$ production at the LHC are shown in the lower panels of Fig. 16. As already found at the Tevatron, the NLO effects are bigger than the NNLO effects on both the normalization and the shape of the rapidity cross section. We note that the K factors for $W^+$ and $W^-$ production are quantitatively very similar and they also have a very similar dependence on $y_W$. In the rapidity interval $|y_W| \lesssim 3$, the NLO K factors vary in the range $K_{NLO}(y_W) \sim 1.1–1.3$, while the NNLO K factors vary in the range $K_{NNLO}(y_W) \sim 1.01–1.05$. At NLO the K factors computed from the ratio of the total (i.e. integrated over $y_W$) cross sections are $K_{NLO}^{W^+} = 1.17$ and $K_{NLO}^{W^-} = 1.21$, respectively, whereas at NNLO we find $K_{NNLO}^{W^+} = 1.03$ for both $W^+$ and $W^-$ production. Our value of $K_{NNLO}^{W^+}$ is consistent with the one given in Ref. [29]. We note that, at NLO, the K factors at the LHC are slightly smaller than those at the Tevatron; at NNLO, the K factors are comparable. The smallness of $K_{NNLO} - 1$ indicates the good quality of the truncated perturbative expansion.

Figure 17: On-shell $W^-$ (left panel) and $W^+$ (right panel) production at the LHC ($\sqrt{s} = 10$ TeV) in NNLO QCD. The rapidity cross sections computed with the ABKM09 (blue solid) and JR09VF (red dashed) parton densities are rescaled by the corresponding MSTW2008 result.
As in Sect. 3.1, we have repeated our QCD calculation of the $W^+$ and $W^-$ rapidity cross sections by using the PDFs of the ABKM Collaboration [34, 35] and of the Dortmund Group [36, 37]. In Fig. 17 we show the NNLO ratios \( (\sigma/dy_W)/PDF/(\sigma/dy_W)_{MSTW} \), where the cross section in the numerator is computed by using either the ABKM09 set [35] or the JR09VF set [37], while the cross section in the denominator uses the MSTW2008 set. In the case of $W^-$ production (left panel of Fig. 17) and considering the rapidity region where \(|y_W| \lesssim 2\), we see that the ABKM09 result is consistent with the MSTW result, within the numerical uncertainties of our NNLO computation; the JR09VF result is instead smaller than the MSTW result, the difference ranging between about 6 and 11%. In the case of $W^+$ production (right panel of Fig. 17), the ABKM09 result is larger than the MSTW result, the difference ranging from about 1 to 4% in the region where \(|y_W| \leq 2\); the JR09VF result is instead smaller than the MSTW result, the difference ranging between about 6 and 9%. At fixed values of $y_W$, the typical values of parton momentum fractions probed by the cross section ratios in Fig. 17 are about a factor of five (e.g., using $(10 \text{ TeV})/(1.96 \text{ TeV}) \simeq 5.1$) smaller than those in Fig. 2; the differences between the ratios in Figs. 2 and 17 are due to the underlying differences between the ABKM09, MSTW2008 and JR09VF partons in different regions of parton momentum fraction. Considering the NNLO total cross sections at the LHC, in the case of $W^+$ production, the ABKM09 (JR09VF) result is about 4% higher (lower) than the MSTW2008 result. In the case of $W^-$ production, the ABKM09 and MSTW2008 results are comparable, whereas the JR09VF result is smaller than the others by about 7%. For $W^+$ production, we find the values \( \sigma_{\text{NNLO}}^{W^+} = 9.15 \pm 0.03 \text{ nb}, \ 8.77 \pm 0.03 \text{ nb} \) and \( 8.38 \pm 0.03 \text{ nb} \); for $W^-$ production, we find \( \sigma_{\text{NNLO}}^{W^-} = 6.40 \pm 0.02 \text{ nb}, \ 6.40 \pm 0.02 \text{ nb} \) and \( 5.94 \pm 0.02 \text{ nb} \). These values are consistent with the results of Refs. [35, 29] and [37], once the leptonic branching ratios are taken into account.

We now consider the $W$ charge asymmetry. In Fig. 18 we present our results for the asymmetry of on-shell $W$ bosons. The charge asymmetry is computed up to NNLO in QCD perturbation theory. From the histograms in the upper panel, we see that the perturbative result for the $W$ asymmetry at the LHC is very stable against the effect of QCD radiative corrections. This is confirmed by the lower panel in the figure, where we show the asymmetry K factors at NLO and NNLO (they are computed analogously to Eq. (33)). The values of the K factors are consistent with unity within the numerical uncertainties of the Monte Carlo computation that we have carried out.

Since the $W^+$ and $W^-$ cross sections are different, the asymmetry does not vanish at $y_W = 0$ where its value is about 0.04. As $y_W$ increases, the asymmetry increases and reaches the values $A(y_W) \sim 0.08$ at $y_W \sim 1.5$, $A(y_W) \sim 0.3$ at $y_W \sim 3$ and $A(y_W) \sim 0.5$ at $y_W \sim 4$. It is interesting to compare this result with the corresponding result for the $W$ asymmetry at the Tevatron (see Fig. 3). At $y_W = 0$ the Tevatron asymmetry is constrained to vanish by CP invariance. However, as $y_W$ increases the Tevatron asymmetry increases much faster than the LHC asymmetry. The $W$ asymmetry at the Tevatron reaches the values $A(y_W) \sim 0.08$ in the region around $y_W \sim 0.5$, and $A(y_W) \sim 0.3$ in the region around $y_W \sim 1.5$; the value $A(y_W) \sim 0.5$ is already reached at $y_W \lesssim 2.5$. This different behaviour of the $W$ asymmetry at the Tevatron and at the LHC is essentially due to the sensitivity to parton densities with different values of parton momentum fraction. The increase of the $W$ charge asymmetry at large rapidities is mainly driven by the difference (actually, by the ratio) of the parton densities, $u(x)$ and $d(x)$, of $u$ and $d$ quarks. Since $u(x)$ and $d(x)$ tend to become similar as their momentum fraction $x$ decreases, at large rapidities

\footnote{The errors on the values of the cross sections are those from the Monte Carlo integration in our calculation.}
Figure 18: The charge asymmetry for on-shell $W$ production at the LHC ($\sqrt{s} = 10$ TeV). Upper panel: LO (dotted), NLO (red dashes), NNLO (blue solid) predictions. The lower panel shows the ratios NLO/LO (red dashed) and NNLO/NLO (blue solid) of the results in the upper panel.

The main features of $W$ production at the LHC depend on the PDFs of the proton. Increasing the centre–of–mass energy of the colliding protons, $W$ production probes smaller values of parton momentum fractions $x$. At smaller values of $x$, all the parton densities (especially the gluon density) are larger, and all the quark and antiquark (independently of their flavour) parton densities tend to become similar. Therefore, the main features of $W$ production at $\sqrt{s} = 10$ TeV (which we have illustrated in this subsection) are intermediate between those at $\sqrt{s} = 7$ TeV and those at $\sqrt{s} = 14$ TeV. Increasing $\sqrt{s}$, both the $W^+$ and $W^-$ cross sections increase; the difference between the absolute values of the $W^+$ and $W^-$ rapidity cross sections, at fixed $y_W$, is reduced; the shapes of the $W^+$ and $W^-$rapidity distributions become more similar (e.g., the peak of the $W^+$ distribution moves toward $y_W \sim 0$); the $W$ charge asymmetry, at fixed $y_W$, decreases.

the $W$ charge asymmetry tends to become smaller as $\sqrt{s}$ increases (e.g. going from the Tevatron to the LHC).
4.2 Charged lepton rapidity distribution and asymmetry

In this subsection we study the rapidity distributions of the charged lepton from $W$ decay. Owing to EW dynamical correlations (see Eq. (10) and the discussion at the beginning of Sect. 3.2), at the partonic level the charged lepton tends to be produced in the direction of the colliding ‘down-type’ quark or antiquark (or, equivalently, in the opposite direction with respect to the ‘up-type’ quark or antiquark). As discussed in Sect. 4.1, in $pp$ collisions, the shape of the rapidity distribution of the $W^+$ boson is mainly controlled by $u\bar{d}$ collisions, and the peak of the distribution is due to the larger momentum fraction carried by $u$ quarks. Therefore, the parton level EW correlations shift the rapidity distribution of the positively-charged lepton to be more central than the distribution of the parent $W^+$. On the contrary, the $W^-$ is mostly produced at central rapidities by $d\bar{u}$ collisions, and, therefore the EW correlations shift the rapidity distribution of the negatively-charged lepton at higher rapidities. We thus anticipate that, in comparison with the $W$ charge asymmetry, the lepton charge asymmetry is larger at low rapidities and smaller at high rapidities. Eventually (as in $p\bar{p}$ collisions at the Tevatron), at large values of the rapidity the partonic EW asymmetry dominates over the asymmetry produced by the PDFs, and the lepton charge asymmetry becomes negative.

To the purpose of presenting some quantitative, though illustrative, results on the lepton rapidity distributions and asymmetry at the LHC, we refer to the framework considered in a recent ’Physics Analysis’ [31] of the CMS Collaboration. The study of Ref. [31], based on data sets from Monte Carlo simulations, regards the muon rapidity cross sections and charge asymmetry that can be measured with the CMS detector at the LHC.

In our calculations, we use the lepton selection cuts applied in Ref. [31]. The transverse mass and the missing transverse energy are required to be $M_T > 50$ GeV and $E_T^\nu > 20$ GeV, respectively. The muons are isolated: the hadronic (partonic) transverse energy $E_{iso}^T$ in a cone of radius $R = 0.3$ is required to fulfil the constraints $\frac{E_{iso}^T}{E_T} < z/(1 - z)$ with $z = 0.05$, and $E_T + E_{iso}^T > E_T^{\max} = 25$ GeV.

The rapidity distributions of the charged leptons at the centre–of-mass energy $\sqrt{s} = 10$ TeV are shown in Fig. 19. We present the results of our QCD calculation up to NNLO. As in the case of Fig. 16, we show only half of the rapidity range, since the lepton rapidity distributions are invariant under the replacement $\eta \leftrightarrow -\eta$. The qualitative expectations (discussed at the beginning of this subsection) about the differences between the $W$ and lepton rapidity distributions are confirmed by the comparison of Fig. 16 with Fig. 19. The rapidity distribution of the $l^+$ is peaked in the rapidity region around $\eta_l \sim 2$, and the rapidity distribution of the $l^-$, though still peaked at $\eta_l \sim 0$, is broader and flatter than the $W^-$ rapidity distribution. Moreover, the difference of the rapidity cross sections in the region around $\eta_l \sim 0$ is larger in Fig. 19 than in Fig. 16.

We comment on the impact of the QCD radiative corrections. The rapidity dependent K factors, defined as in Eq. (8), are shown in the lower panels of Fig. 19. In the rapidity region where $|\eta_l| \lesssim 3$, the NLO K factor of the $l^+$ ($l^-$) rapidity cross section decreases from about 1.16 (1.21) to about 1.01 (1.03). The NNLO K factor is very close to unity for both the $l^+$ and $l^-$ rapidity cross sections. We recall that an equal (rapidity dependent) K factor for $l^+$ and $l^-$ production would

\[^{\dagger}\text{We note that the two constraints also imply a lower bound on the lepton transverse energy, namely } E_T > E_T^{\max}(1 - z) = 23.75 \text{ GeV.}\]
imply no radiative correction to the charge asymmetry. Since the NLO K factor for $l^-$ production is slightly larger than the other, we anticipate a reduction of the lepton asymmetry in going from LO to NLO. Owing to the similarity of the NNLO K factor, we also anticipate very little NNLO effects on the lepton asymmetry.

The results for the lepton charge asymmetry are presented in Fig. 20. The three histograms in the upper panel show the LO (dotted), NLO (dashed) and NNLO (solid) results of our calculation. Comparing Fig. 20 with Fig. 19, we see that, as expected, the lepton asymmetry is larger (slightly smaller) than the $W$ asymmetry at small (large, i.e. $\eta \sim 2.5$) rapidities. Considering the effect of the QCD radiative corrections, we see that, in going from LO to NLO, the QCD corrections tend to decrease the asymmetry, as expected from the results in Fig. 19. The NNLO corrections do not change the asymmetry in a significant way. The impact of the radiative corrections is quantified by the NLO (dashed histogram) and NNLO (solid histogram) asymmetry K factors, which are shown in the lower panel of Fig. 20. In the panel we also show the K factors computed with the corresponding NLO (dotted) and NNLO (dot-dashed) PDFs by using the LO partonic

![Figure 19: Rapidity distribution of the $l^-$ (left panel) and $l^+$ (right panel) at the LHC ($\sqrt{s} = 10$ TeV), in LO (black dotted), NLO (red dashed) and NNLO (blue solid) QCD. The cuts applied are described in the text. The lower panel shows the ratios NLO/LO (red dashed) and NNLO/NLO (blue solid) of the cross section results in the upper panel.](image-url)

The three histograms in the upper panel show the LO (dotted), NLO (dashed) and NNLO (solid) results of our calculation. Comparing Fig. 20 with Fig. 19 we see that, as expected, the lepton asymmetry is larger (slightly smaller) than the $W$ asymmetry at small (large, i.e. $\eta \sim 2.5$) rapidities. Considering the effect of the QCD radiative corrections, we see that, in going from LO to NLO, the QCD corrections tend to decrease the asymmetry, as expected from the results in Fig. 19. The NNLO corrections do not change the asymmetry in a significant way. The impact of the radiative corrections is quantified by the NLO (dashed histogram) and NNLO (solid histogram) asymmetry K factors, which are shown in the lower panel of Fig. 20. In the panel we also show the K factors computed with the corresponding NLO (dotted) and NNLO (dot-dashed) PDFs by using the LO partonic
cross sections. The NLO and NNLO K factors vary in the range $K_{NLO}(\eta_{l}) \sim 0.80$–0.94 and $K_{NNLO}(\eta_{l}) \sim 0.96$–1.07. Comparing the NLO K factor (dashed histogram) with the ‘PDF driven’ NLO K factor (dotted histogram), we see that the variation of the PDFs (in going from the LO to the NLO set) produce a sizeable contribution to the radiative corrections. However, a careful inspection of the differences between the dashed and dotted histograms shows that the NLO corrections to the partonic cross sections are not negligible. For instance, the value of the ‘PDF driven’ NLO K factor is $K_{NLO}(\eta_{l}) \sim 0.88$ at both $\eta_{l} \simeq 0$ and $\eta_{l} \simeq 3$. This implies that the NLO corrections to the partonic cross sections are negative (i.e. they decrease the lepton asymmetry) at small rapidities and positive at large rapidities ($\eta_{l} \sim 2.5$). Therefore, these corrections tend to slightly reduce the differences between the lepton asymmetry and the asymmetry of the parent $W$. This effect is qualitatively consistent with the analogous effect observed at the Tevatron, for instance, in the low-$E_T$ bin of the CDF electron asymmetry (see the results in the left panels of Fig. 8 and the related comments).

Figure 20: Muon charge asymmetry at the LHC: $\sqrt{s} = 10$ TeV.

We have repeated our calculations of $W$ and lepton rapidity cross sections and asymmetries at the centre–of–mass energy $\sqrt{s} = 7$ TeV. The overall features of our results do not change significantly, apart from the quantitative differences that are expected from the decreased value of $\sqrt{s}$. Here we limit ourselves to presenting the results for the lepton charge asymmetry, considering the same lepton kinematical configuration [31] as in Fig. 20.

The lepton asymmetry at $\sqrt{s} = 7$ TeV is shown in Fig. 21. We first note that, at low and medium rapidities ($\eta_{l} \lesssim 2.5$), the asymmetry is larger at $\sqrt{s} = 7$ TeV than at $\sqrt{s} = 10$ TeV. This is not unexpected, since at smaller energies larger values of parton momentum fraction $x$ are probed, where flavour asymmetries of the PDFs are more sizeable. At high rapidities ($\eta_{l} \gtrsim 2.5$), the shape
of the asymmetry changes: $A(\eta_l)$ decreases as $\eta_l$ increases. This behaviour (as we have already seen at the Tevatron and commented about in Sect. 3.2) is a distinctive effect of the impact of the parton level EW asymmetry on the flavour asymmetries of the PDFs. The effect takes place when the flavour asymmetries are more sizeable, i.e. when larger parton momentum fractions are probed. Therefore, at fixed $\sqrt{s}$, the effect sets in in the high-rapidity region; correspondingly, at higher $\sqrt{s}$, the effect sets in at higher rapidities. Indeed, at $\sqrt{s} = 10$ TeV, the decreasing behaviour of the lepton asymmetry starts to be visible at $\eta_l \sim 3$ (Fig. 20).

As for the impact of the QCD radiative corrections on the lepton charge asymmetry at $\sqrt{s} = 7$ TeV, the effects are very similar to those in Fig. 20: the NLO corrections tend to decrease the asymmetry, and the NNLO corrections do not significantly change the NLO result. Analogously to Fig. 20 in the lower panel of Fig. 21 we present the asymmetry K factors at NLO and NNLO, and the corresponding ‘PDF driven’ K factors. The similarity between the K factors and their ‘PDF driven’ versions shows that a sizeable part of the radiative corrections to the asymmetry is due to the variation of the PDFs (in particular, going from the LO to the NLO set). The contribution of the radiative corrections to the partonic cross sections is nonetheless clearly visible. Comparing the NLO K factor (dashed histogram) with its PDF driven’ version (dotted histogram), we can see that the NLO corrections to the partonic cross sections are negative at small rapidities and positive at large rapidities. The same effect has been observed in the lepton asymmetry at $\sqrt{s} = 10$ TeV (see the lower panel in Fig. 20 and related comments).

![Figure 21: Muon charge asymmetry at the LHC: $\sqrt{s} = 7$ TeV.](image-url)
5 Summary

In this paper we have considered $W$ boson production at hadron colliders, and the subsequent leptonic decay $W \to l\nu_l$. We have performed QCD studies of the boson and lepton rapidity distributions, by including the contributions of higher-order radiative corrections. The main part of our work regards the study of the lepton charge asymmetry. We have presented the results of the calculation of the lepton charge asymmetry up to NNLO in QCD perturbation theory. Our calculation is carried out by using the parton level Monte Carlo program of Ref. [6], which allows us to take into account the lepton kinematical cuts that are typically applied to perform measurements of the charge asymmetry in experiments at hadron colliders.

The lepton charge asymmetry is the result of two different physical mechanisms: the asymmetry generated by the parton content of the colliding protons and/or antiprotons and the asymmetry of the partonic (hard-scattering) production mechanism. The effect of the latter is almost completely cancelled by calculations or measurements that are highly inclusive over the kinematics of the leptons from $W$ decay. Therefore, the inclusive $W$ asymmetry is mostly sensitive to the PDFs of the colliding hadrons, and, in particular, to the differences between the momentum fraction distributions of $u$ and $d$ quarks in the proton. What is primarily and directly measured is, however, the lepton charge asymmetry. The lepton charge asymmetry has an additional sensitivity to the underlying partonic scattering, whose asymmetry mostly originates from EW dynamics. Moreover, the study the lepton asymmetry at different values of the lepton transverse energy gives a finer probe of the momentum fraction dependence of the PDFs of the colliding hadrons.

In the paper we have first considered the case of $p\bar{p}$ collisions at the Tevatron Run II. We have examined the charge asymmetry of inclusive $W$ production and confirmed earlier findings [4]: the QCD radiative corrections to this quantity are very small. We have also shown the $W$ asymmetry results obtained by using different sets of NNLO PDFs. Then, we have considered the lepton charge asymmetry and computed this observable in kinematical configurations with various lepton selection cuts, as those applied by the CDF and D0 Collaborations. We have shown that the QCD radiative corrections to the lepton asymmetry are small, but they are not as small as those to the $W$ asymmetry. This difference is not fully unexpected. Since the lepton asymmetry also depends on the EW asymmetry generated by the partonic subprocesses, it is more sensitive to the QCD radiative corrections to these partonic subprocesses. Moreover, the effect of the QCD corrections to the lepton charge asymmetry has an impact that qualitatively and quantitatively depends on the lepton kinematics and the specific lepton selection cuts. We have performed NNLO calculations of the lepton charge asymmetry by using different sets of PDFs and presented the comparison with some of the available Tevatron data.

We have finally considered the charge asymmetry in $pp$ collisions. At LHC energies, the lepton charge asymmetry is sensitive to PDFs with momentum fractions that are smaller than those probed at the Tevatron. We have presented some illustrative results at the centre–of–mass energies $\sqrt{s} = 10$ TeV and 7 TeV. These specific results shown that (as in $p\bar{p}$ collisions at the Tevatron) the QCD radiative corrections to the lepton charge asymmetry are small, particularly at NNLO, though larger than those to the $W$ charge asymmetry.

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