Cosmic ray primary mass composition above the knee: deduction from lateral distribution of electrons

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Influence of shower fluctuations on the shape of lateral distribution of electrons in EAS of fixed size measured by scintillation counters is analyzed in framework of scaling formalism. Correction factors for the mean square radius of electrons are calculated for the experimental conditions of KASCADE array. Possible improvement of the primary mass discrimination by analysis of lateral distribution of EAS electrons is discussed in detail.

1. INTRODUCTION

The determination of the primary cosmic ray chemical composition from extensive air showers (EAS) observations is an open problem. Various techniques based on different EAS characteristics measured by different experiments including multi-component methods do not exhibit consistent results in estimation neither primary mass in the case of individual showers nor the mean mass composition at a certain energy.

One of the key EAS quantities necessary for basic shower parameters reconstruction is the lateral distribution of charged particles at fixed observation depth. The exact form of lateral distribution function (LDF) is still uncertain. The majority of analytical parameterizations of LDF of different EAS components is traditionally based on the well known Nishimura-Kamata-Greizen (NKG) function [1]:

\[
\rho(r; E_0, s) = \frac{N(E_0, s)}{R_0^2} \frac{\Gamma(4.5 - s)}{2\pi \Gamma(s) \Gamma(4.5 - 2s)} \times \\
\times \left( \frac{r}{R_0} \right)^{s-2} \left( 1 + \frac{r}{R_0} \right)^{s-4.5}.
\]

(1)

Here \(\rho(r; E_0, s)\) is the particle density at radial distance \(r\) from the core position in shower with primary energy \(E_0\) and the age parameter \(s\), \(N(E_0, s)\) – total number of particles at the observation depth, \(R_0\) – shower scale radius, which does not depend on primary particle type and energy (originally – the Möllere unit). Various modifications of NKG form, such as introducing an additional fixed or age-dependent scale coefficient or a local age parameter \(s(r)\) and also generalizations of the function by using third power-law term were suggested.

A different theoretically motivated approach (scaling formalism) was proposed in our papers [23]:

\[
\rho(r; E_0, t) = \frac{N(E_0, t)}{R_0^2(E_0, t)} F \left( \frac{r}{R_0(E_0, t)} \right),
\]

where \(t\) is the observation depth. The scaling function \(F(X)\) can be described as follows:

\[
F(X) = CX^{-\alpha}(1 + X)^{-\beta} \times \\
\times \left( 1 + (X/10^\delta) \right)^{-\delta}.
\]

(3)

For electron densities we find \(C = 0.28, \alpha = 1.2, \beta = 4.53, \delta = 0.6, R_0 = R_{ms}.\) – root mean square radius of electrons:

\[
R_{ms}(E_0, t) = \frac{2\pi}{N(E_0, t)} \int_0^\infty r^3 \rho_e(r; E_0, t) dr.
\]

(4)

According to our calculations, the scaling formalism allows to reproduce electron LDF with 10% uncertainty for \(E_0 = (10^{14} - 10^{20})\) eV, \(t = (600 - 1030)\) g/cm², \(X = (0.05 - 25)\). The last condition corresponds to the radial distance range from \(r \sim (5 - 10)\) m to \(r \sim (2.5 - 4)\) km depending on the shower age. This limitation makes scaling approach inadequate for shower size and core position estimation, but accurate enough for description of the shape of LDF measured by ground-based shower arrays far from the core.
The method for mean primary mass deduction based on scaling formalism was developed in [4, 5]. The advantage of this method is its applicability to the experimental data of both compact and giant air shower arrays in wide primary energy range and also relatively weak sensitivity of the conclusions to variations of basic parameters of hadronic interaction model implemented in calculations. Unfortunately, the shape of charged particle LDF measured experimentally is affected by the experimental method of shower classification. This effect can also be described in the framework of scaling formalism, but the variation of the root mean square radius compared with the data obtained theoretically for the fixed primary energy should be taken into account in case of relatively low primary energies, when shower selection is made by the total number of electrons, e.g. for example for KASCADE and Moscow State University air shower arrays.

In this paper we examine thoroughly the influence of shower fluctuations on the shape of lateral distribution of electrons in EAS of fixed size under the experimental conditions of KASCADE array in order to improve the reliability of mean primary mass deduction from the experimentally measured LDFs.

2. CALCULATION METHODS

We made simulations of extensive air showers initiated by protons and iron nuclei of vertical incidence assuming power-law differential energy spectrum of primaries with exponent $\alpha_1 = 2.62$, $\alpha_2 = 3.02$ and also with sharp knee from $\alpha_1$ to $\alpha_2$ at $E_0 = 10^{6.5}$ GeV. We used the semi-analytical code [5] with full Monte-Carlo treatment of hadronic part of cascade based on quark-gluon string model and analytical expressions of pure electromagnetic sub-showers keeping all the basic sources of fluctuations. The fluctuations of different EAS components calculated by our code are in good agreement with CORSIKA/QGSJet results.

According to [6] we simulated shower classification procedure used at KASCADE array and evaluated lateral distributions and root mean square radiuses of electrons in eight bins of shower size. The number of showers in each bin amounts from $\sim 5000$ for lower energies ($\lg N_e = 3.9 - 4.3$) to $\sim 1000$ for higher energies ($\lg N_e = 6.7 - 7.1$).

3. RESULTS

The primary energy distributions in four from eight bins of shower size is shown in fig. One
Figure 2. Correction factors $K = (R_{N_e}^{ms}/R_{E}^{ms})$ for proton initiated vertical EAS at sea level assuming different primary energy spectrum exponents: $\alpha_1 = 2.62$ (triangles with dotted approximation curve), $\alpha_2 = 3.02$ (squares with dashed curve), spectrum with the knee at $E_0 = 10^{6.5}$ GeV (solid circles with solid curve). See text for details.

can see that the energies largely overlap in different bins though the selected bins are not neighboring.

We evaluated the correction factors defined as the ratio of root mean square radius calculated for certain shower size bin to that for corresponding average primary energy: $K = (R_{N_e}^{ms}/R_{E}^{ms})$. These correction factors for vertical proton initiated showers at sea level calculated with above mentioned assumptions about primary energy spectrum are shown in fig. 2. It is clear, that values of $K$ approach to 1 with energy as shower fluctuations decrease. The same effect takes place for a heavier primary nuclear or for smaller shower size bins. At the same time the correction, which should be made for an adequate comparison of theoretical and experimentally estimated root mean square radiiess is essential for all considered shower size bins.

Assuming the validity of scaling approach we made one-parametric fitting of experimental lateral distribution of electrons obtained by KASCADE array [6] using formula (2) with three different scaling functions:

1) theoretically proved function (3) with mean square radius of electrons as radial scale parameter;

2) modified NKG-function [6] with fixed shower age parameter ($s=1.65$) and variable $R_0$;

3) polinomial function:

$$F(X) = \tilde{C} \exp \left\{ \sum_{i=0}^{n} a_i (\ln X)^i \right\},$$  \hspace{1cm} (5)

with $n = 4$ and parameters $a_i$ being fixed for all bins independently from $R_0$. Some discrimination of experimental data at small radial distances was done in order to eliminate points with $r < 0.05R_{ms}$.

All the fitting functions give satisfactory overall fit of experimental data of KASCADE with residuals not exceeding 10% for polynomial and modified NKG functions and 15% for function (3). Though polynomial and NKG functions give considerably better accuracy in considered radial distance range, they both lead to incorrect predic-
For primary spectrum with the knee. As 

the rate of change of electron mass above the knee increases with energy, that is in reasonable agreement with the large number of experimental observations (see e.g. [7]) and also with recent results of the anomalous diffusion model [8].

4. CONCLUSIONS

1. Correction by a factor \( K = \left( \frac{R_{\text{ex}}^N}{R_{\text{ex}}^E} \right) \) should be made when comparison of lateral distributions of electrons measured by ground-based experimental arrays to LDFs calculated theoretically for fixed primary energy is carried out.

2. Absolute values of radial scale factor \( R_0 \) contain systematic errors and could be biased depending on the form of lateral distribution function chosen for experimental data processing and final fitting. However, if one concerns the rate of change of \( R_0 \) with primary energy, which is a good measure for primary mass composition variation, then different scaling functions used in the framework of scaling formalism do not contradict each other.

3. Basic analysis using different assumptions about the form of scaling LDF leads to consistent model insensitive conclusion that average primary particle mass above the knee increases with energy. The values of radial scale parameters \( R_0 \) obtained by fitting the experimental data are summarized in Table 1. It is not surprising, that different fitting functions correspond to significantly different values of \( R_0 \). An additional bias in \( R_0 \) can be related with the insufficiency of radial distance range well covered by the array or some other systematic errors in data processing. So it is worth to compare the rate of change of radial scale factors with energy \( \partial R_0 / \partial \log E_0 \) which obviously reflects the rate of change of mean primary mass. In fig. 3 values of \( R_0 \) superposed with each other by the appropriate factors are shown in comparison with root mean square radius calculated for vertical proton and iron initiated showers at sea level taking into account the correction factor \( K \) for primary spectrum with the knee. As it is seen from the figure the rate of change of \( R_0 \) obtained using different functions is consistent.

### Table 1

Radial scale factors \((R_0 \pm \delta R_0)\) obtained by fitting of experimental LDF [5] by different scaling functions.

| \( \lg N_e \) | Function [3] | Modified NKG \((s=1.65)\) | Polynomial [5] |
|-------------|-------------|-----------------------------|----------------|
| 3.9 - 4.3   | 146.8 ± 2.5 | 29.79 ± 3.0 \( \cdot 10^{-1} \) | 174.4 ± 1.6 |
| 4.3 - 4.7   | 134.3 ± 2.2 | 26.93 ± 2.1 \( \cdot 10^{-1} \) | 156.6 ± 1.2 |
| 4.7 - 5.1   | 125.0 ± 1.5 | 25.17 ± 1.9 \( \cdot 10^{-1} \) | 146.0 ± 1.2 |
| 5.1 - 5.5   | 122.6 ± 1.3 | 23.92 ± 1.3 \( \cdot 10^{-1} \) | 138.6 ± 0.9 |
| 5.5 - 5.9   | 122.8 ± 1.6 | 23.65 ± 1.1 \( \cdot 10^{-1} \) | 137.8 ± 0.5 |
| 5.9 - 6.3   | 122.6 ± 2.1 | 24.03 ± 1.6 \( \cdot 10^{-1} \) | 139.8 ± 0.9 |
| 6.3 - 6.7   | 125.4 ± 2.3 | 24.70 ± 2.1 \( \cdot 10^{-1} \) | 141.0 ± 1.2 |
| 6.7 - 7.1   | 130.6 ± 1.8 | 25.55 ± 3.5 \( \cdot 10^{-1} \) | 145.1 ± 2.1 |

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Cosmic ray primary mass composition above the knee

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