Charge breaking bounds in the Zee model

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Abstract. We study the possibility that charge breaking minima occur in the Zee model. We reach very different conclusions from those attained in simpler, two Higgs doublet models, and the reason for this is traced back to the existence of cubic terms in the potential. A scan of the Zee model's parameter space shows that CB is restricted to a narrow region of values of the parameters.

1 Introduction

In models with several scalar doublets there is the possibility that minima which break charge and/or colour symmetries occur. That happens when scalar fields which are not singlets of charge and/or colour acquire a non-zero vacuum expectation value. If those minima are deeper than the standard ones, which respect the $SU(3)_C \times U(1)_{em}$ gauge symmetries, the theory might tunnel to them, and find itself in a vacuum that breaks charge and/or colour. As such, the combinations of parameters of the theory that give rise to those minima can be excluded, and the predictive power of the theory is increased. These charge and colour breaking bounds were first introduced by Frére et al \cite{1} and have been extensively used in supersymmetric models, to impose limits on the masses of the supersymmetric partners, for instance \cite{2}. Recently \cite{3} the same idea was applied to two Higgs doublet models (2HDM) without explicit CP violation and a very interesting result was deduced: if a minimum that respects electric charge conservation and also the CP symmetry (we will call this minimum the “normal” one, from this point onwards) exists, then it is the global minimum of the theory. In fact, it was shown that only charge breaking saddle points can occur in that case, and the value of the potential in those points is always superior to its value at the normal minimum. In this paper we will consider the simplest generalisation of the 2HDM, in which we add an $SU(2)$ singlet to the theory. This is the Zee model \cite{4}, which was first conceived as a means to explain the very low masses of the neutrinos. The Zee neutrino mass matrix structure can be shown to arise in a more general context, through mixing of dimension five operators \cite{5}. We will show that in the Zee model charge breaking (CB) minima can and do develop in parallel with normal ones, and sometimes those CB minima are the deepest ones. Then, unlike what happens for the 2HDM, tunneling to CB minima is actually possible even if the theory starts up in a normal minimum. We will investigate the values of the potential’s parameters for which CB occurs, and show that CB is mostly circumscribed to a specific region of the parameter space.

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2 The Zee model

The Zee model [4] has the great advantage of providing a natural reason for the smallness of the neutrino masses. Majorana mass terms for the neutrinos are generated as radiative corrections at the one-loop level. In addition, the model can easily be extended [6] to incorporate a sterile neutrino, in such a way that this four-neutrino version could also accommodate the LSND data. Unlike the seesaw mechanism, the Zee model does not require new physics at an extremely large scale. But it does require a Higgs sector larger than the SM’s, i.e., it requires two scalar doublets, $\Phi_a$ ($a=1, 2$) with hypercharge $Y = 1$ and a charged singlet field $\Phi_0$ with hypercharge $Y = -2$. In this work we are interested in determining the possible vacua of the theory, i.e. we want to study the scalar potential of the model. The 2HDM scalar potential is given by a sum of terms quadratic and quartic in the fields $\Phi_1$ and $\Phi_2$. There is an extensive literature about this potential (see, for instance, [7]), so we simply summarise some of its features: (a) the most general $SU(2) \times U(1)$ gauge invariant potential, which explicitly breaks CP conservation, has 14 independent real parameters, that can be reduced to 11 with a judicious choice of basis [8]; (b) requiring that the 2HDM does not explicitly break CP one is left with a 10 parameter potential (though with a choice of basis we can reduce this number to 9 independent parameters); this is the potential studied in reference [3], end it has flavour changing neutral currents (FCNC); (c) if one wishes to prevent the occurrence of FCNC’s - or indeed the possibility of spontaneous breaking of charge or CP symmetries -, it was shown in reference [9] that the potential has to be invariant under a $Z_2$ or $U(1)$ symmetries; one then obtains two different 7-parameter potentials.

The Zee model usually considered in the literature is based on the 2HDM described in point (c) above. A priori this seems like a good idea, since in this manner we prevent potentially dangerous FCNC’s and limit the number of free parameters of the theory. The problem is that recent analysis [10, 11] of experimental results in solar and atmospheric neutrinos, namely from the SNO experiment, seems to indicate that the minimal Zee model cannot account for the data. It has been suggested that extensions of the Zee model with extra scalars might do a better job of describing the neutrino results (for instance, see [11, 12]). It has also been shown [13] that a Zee model based on the 2HDM potential described in point (b) above could accurately reproduce the experiments, and that is the case we will study here. An obvious problem with this approach is the fact that we have increased the number of free parameters in the theory. So, any tool that might be useful in reducing the parameter space of the model will be welcome. For instance, the authors of refs. [13] derived mass bounds on the Higgs boson using the triviality condition and (standard) vacuum stability, both for the Zee and two higgs doublet models. If we can show that for certain values of the parameters deeper CB minima occur, we will be able to impose limits on those parameters and improve our understanding of the theory. To achieve this, we must look at the minimisation conditions for the normal and CB minima.

The fields \{\Phi_0, \Phi_1, \Phi_2\} have ten real component fields, which we number as

$$\Phi_1 = \left( \varphi_1 + i\varphi_2, \varphi_5 + i\varphi_7 \right), \quad \Phi_2 = \left( \varphi_3 + i\varphi_4, \varphi_6 + i\varphi_8 \right), \quad \Phi_0 = \left( \varphi_9 + i\varphi_{10} \right).$$ \hspace{1cm} (1)

This convention might seem odd, but it is very convenient for writing the mass matrices of the scalar particles of the theory. We can build five $SU(2) \times U(1)$ quadratic invariants with these fields, which are:

$$x_0 \equiv |\Phi_0|^2 = \varphi_5^2 + \varphi_{10}^2$$
$$x_1 \equiv |\Phi_1|^2 = \varphi_1^2 + \varphi_2^2 + \varphi_5^2 + \varphi_7^2$$
$$x_2 \equiv |\Phi_2|^2 = \varphi_3^2 + \varphi_4^2 + \varphi_6^2 + \varphi_8^2$$

\(^1\)Our convention for the electric charge operator is $Q = T_3 + Y/2$. 


\[ x_3 \equiv \text{Re}(\Phi_1^T \Phi_2) = \varphi_1 \varphi_3 + \varphi_2 \varphi_4 + \varphi_5 \varphi_6 + \varphi_7 \varphi_8 \]
\[ x_4 \equiv \text{Im}(\Phi_1^T \Phi_2) = \varphi_1 \varphi_4 - \varphi_2 \varphi_3 + \varphi_5 \varphi_8 - \varphi_6 \varphi_7 . \]  

(2)

The construction of the scalar potential is thus very simple, one must simply include all terms linear and quadratic in the variables \( x_i \). We define the charge conjugated states as \( \Phi_i^c = \Phi_i^* \). Under this transformation \( x_4 \rightarrow -x_4 \) and all the remaining \( x_i \) are invariant, hence a CP conserving potential is

\[
V_M = a_0 x_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + \sum_{ij} b_{ij} x_i x_j ,
\]

where \( a_i \) parameters have dimensions of mass squared, the \( b_{ij} \) are dimensionless and all the parameters are real. For future convenience let us introduce here a matrix notation similar to the one used in ref. [3]. Let \( X \) be the vector with components \( (x_0, x_1, x_2, x_3, x_4) \) and \( A \) and \( B \) the vector and symmetric matrix given by

\[
A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2b_{00} & b_{01} & b_{02} & b_{03} & 0 \\ b_{01} & 2b_{11} & b_{12} & b_{13} & 0 \\ b_{02} & b_{12} & 2b_{22} & b_{23} & 0 \\ b_{03} & b_{13} & b_{23} & 2b_{33} & 0 \\ 0 & 0 & 0 & 0 & 2b_{44} \end{bmatrix} .
\]

(4)

Therefore, the potential \( V_M \) is given by

\[ V_M = A^T X + \frac{1}{2} X^T B X . \]

(5)

However, there is also an \( SU(2) \times U(1) \) invariant cubic term that gives an extra contribution to the potential, namely

\[
V_C = M (\Phi_1^T i \sigma_2 \Phi_2) \Phi_0 + \text{h.c.}
\]

\[ = 2 M \left[ (\varphi_1 \varphi_6 - \varphi_2 \varphi_8 - \varphi_3 \varphi_5 + \varphi_4 \varphi_7) \varphi_9 - (\varphi_1 \varphi_8 + \varphi_2 \varphi_6 - \varphi_3 \varphi_7 - \varphi_4 \varphi_5) \varphi_{10} \right] ,
\]

(6)

where \( M \) is a real coupling with dimensions of mass. The scalar potential of the Zee model is thus \( V = V_M + V_C \).

The “normal” minimum, which preserves charge and colour conservation, occurs when the fields \( \varphi_3 \) and \( \varphi_6 \) have non-zero vevs, \( v_1 \) and \( v_2 \) respectively, such that \( v_1^2 + v_2^2 = (246 \text{ GeV})^2 \). At this minimum, then, the vector \( X \) takes the values \( X_N = (0, v_1^2, v_2^2, v_1 v_2, 0) \), and the value of the potential is given by [3]

\[ V_N = \frac{1}{2} A^T X_N = \frac{1}{2} (a_1 v_1^4 + a_2 v_2^4 + a_3 v_1 v_2) < 0 . \]

(7)

The inclusion of the charged field \( \Phi_0 \) introduces some changes in the theory’s scalar mass spectrum. If one computes the second derivatives of the Zee potential with respect to the fields \( \varphi_i \) at the normal minimum, we see that the neutral sector of this model is identical to that of the 2HDM. The reason for this is quite simple: \( \Phi_0 \) is an electrically charged field and so there are no mixing terms with the neutral components of \( \Phi_1 \) and \( \Phi_2 \). A different thing happens in the charged sector, where there is a mixing among the charged fields caused by the cubic terms \( V_C \). The charged higgs mass matrix is now given by (from the left to the right, the columns of
this matrix correspond to the fields $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_9$ and $\varphi_{10}$)

\[
M_{H^\pm}^2 = \begin{bmatrix}
2V'_1 & 0 & V'_3 & V'_4 & 2Mv_2 & 0 \\
0 & 2V'_1 & -V'_4 & V'_3 & 0 & -2Mv_2 \\
V'_3 & -V'_4 & 2V'_2 & 0 & -2Mv_1 & 0 \\
V'_4 & V'_3 & 0 & 2V'_2 & 0 & 2Mv_1 \\
2Mv_2 & 0 & -2Mv_1 & 0 & 2V'_0 & 0 \\
0 & -2Mv_2 & 0 & 2Mv_1 & 0 & 2V'_0 \\
\end{bmatrix}
\] ,

(8)

where we have introduced the notation $V'_i = \partial V_M/\partial x_i$. The $V'_i$, of course, are evaluated at the normal minimum and one has

\[
V'_1 = -\frac{v_2}{2v_1}V'_3 , \quad V'_2 = -\frac{v_1}{2v_2}V'_3 , \quad V'_4 = 0 .
\]

(9)

The matrix (8) is clearly a generalization of the analogous matrix obtained in reference [3] for the 2HDM. This matrix has two zero eigenvalues (corresponding to the charged Goldstone bosons) and two doubly degenerate non-zero ones, given by

\[
M_{H_{1,2}^\pm}^2 = \frac{1}{2} \left[ V'_0 + V'_1 + V'_2 \pm \sqrt{(V'_1 + V'_2 - V'_0)^2 + 4M^2(v_1^2 + v_2^2)} \right] .
\]

(10)

3 Charge breaking minima

For charge breaking to occur, some charged scalar field has to acquire a non-vanishing vev. One might think that the case \{ $\varphi_9 = \gamma \neq 0$, $\varphi_i = 0$, $\forall i \neq 3$ \} (or others, equivalent to this one, where a single charged field of the doublet has a non-zero vev) would give rise to a CB minimum. In fact, it is simple to show that such a choice of vevs for any $SU(2)$ doublets does not actually break charge conservation [15]. However, giving a vev to the $\Phi_0$ singlet definitively gives the photon a mass. Let $\varphi_9 = \gamma \neq 0$ (or $\varphi_{10} \neq 0$, which is equivalent) and all the other fields zero. In this case only $x_9 = \gamma^2$ is different from zero, and a trivial minimisation of the Zee potential shows that $\gamma^2 = -a_0/2b_{00}$. For the potential to be bounded from below we must have $b_{00} > 0$ and so this minimum only exists if $a_0 < 0$. In that case the value of the potential is

\[
V_{CB_1} = -\frac{a_0^2}{4b_{00}} .
\]

(11)

Then, charge breaking does not occur if $V_{CB_1} > V_N$, which implies that the theory’s parameters must obey the inequality

\[
b_{00} > -\frac{1}{2} \frac{a_0^2}{a_1 v_1^2 + a_2 v_2^2 + a_3 v_1 v_2} > 0 .
\]

(12)

Given \{ $a_0, a_1, a_2, a_3, v_1, v_2$ \}, we have thus a restriction on how small the parameter $b_{00}$ may be. This CB minimum is rather trivial, and it seems to impose constraints on only a few of the theory’s 16 free parameters. In ref. [3] we considered a CB minimum of the form $\varphi_5 = v'_1$, $\varphi_6 = v'_2$ and $\varphi_3 = \alpha$. It is easy to see, though, that for the most general choice of parameters of the potential this CB minimum is not always possible. The minimisation conditions are $\partial V/\partial \varphi_i = 0$, $\forall i = 1, \ldots, 10$ and we wish to find if a solution with only $\{ \varphi_3, \varphi_5, \varphi_6 \} \neq 0$ is possible. From $\partial V/\partial \varphi_9 = 0$ we obtain $2Mv'_9 \alpha = 0$, which, for $\alpha \neq 0$, implies either $M = 0$ (a case of
limited interest, as the model almost reduces to the 2HDM) or \( v'_1 = 0 \). From \( \partial V/\partial \varphi_{5,6} = 0 \), with \( \varphi_5 = v'_1 = 0 \) and \( \varphi_6 = v'_2 \neq 0 \), we obtain

\[
\begin{align*}
\begin{cases}
a_3 v'_2 + b_{23} v'_2^3 &= 0 \\
2 a_2 v'_2 + 4 b_{22} v'_2^3 &= 0
\end{cases}
\end{align*}
\]

which can only be satisfied for particular combinations of parameters, namely \( a_3/b_{23} = a_2/2 b_{22} \), with \( a_2 < 0 \).

Let us study the general case and take the parameters of the theory as being uncorrelated. The conclusions we will reach should therefore encompass all the special cases, such as the examples we have just discussed. A simple investigation of the minimisation conditions quickly tells us that the simplest CB choice of vevs, besides the trivial one considered above, is \( \varphi_1 = \rho \), \( \varphi_3 = \alpha \), \( \varphi_5 = v'_1 \), \( \varphi_6 = v'_2 \) and \( \varphi_9 = \gamma \). This gives \( \hat{x}_0 = \gamma^2 \), \( \hat{x}_1 = v'_1^2 + \rho^2 \), \( \hat{x}_2 = v'_2^2 + \alpha^2 \), \( \hat{x}_3 = v'_1 v'_2 + \rho \alpha \) and \( \hat{x}_4 = 0 \) (we use the hat to distinguish these \( x_i \) from their counterparts at the normal minimum). In reference [3] a very useful relation between the values of the 2HDM potential at the normal and CB stationary points was derived, namely

\[
V_{CB} - V_N = \left( -\frac{V'_3}{4 x_3} \right) [(v'_1 v_2 - v'_2 v_1)^2 + \alpha^2 v_1^2] + \frac{M_{H^\pm}^2}{2 (v'_1 + v'_2)} [(v'_1 v_2 - v'_2 v_1)^2 + \alpha^2 v_1^2] ,
\]

where \( \{v'_1, v'_2, \alpha\} \) are the CB vevs and \( V'_3 = \partial V/\partial x_3 \) is evaluated at the normal stationary point. \( M_{H^\pm}^2 \) is the squared mass of the charged scalars at the normal stationary point. The second line in the equation above makes it plain that if the normal minimum exists then we will necessarily have \( V_{CB} - V_N > 0 \), and tunneling to a deeper CB minimum is impossible. Let us now generalise this result for the present case. As a first step we define the vector \( V' \), with components \( V'_i = \partial V_M/\partial x_i \) evaluated at the normal minimum, i.e.,

\[
V' = A + B X_N .
\]  

At the CB stationary point we define the vectors \( X_{CB} \), with components \( X_{CB,i} = \hat{x}_i \), and \( \hat{V}' = \hat{V}'_i = \partial V_M/\partial x_i \big|_{CB} \), that is,

\[
\hat{V}' = A + B X_{CB} .
\]  

It is easy to see, from the CB stationarity conditions, that we have the following relations:

\[
\begin{align*}
\hat{V}'_0 &= -\frac{V_C}{2 \hat{x}_0} , \\
\hat{V}'_1 &= -\frac{\hat{x}_2}{2 \hat{x}_3} \hat{V}'_3 , \\
\hat{V}'_2 &= -\frac{\hat{x}_1}{2 \hat{x}_3} \hat{V}'_3 , \\
\hat{V}'_3 &= \frac{4 M^2 \hat{x}_0 \hat{x}_3}{V_C} , \\
\hat{V}'_4 &= 0 .
\end{align*}
\]

\( V_C \) is now evaluated at the CB stationary point and is equal to \( V_C = 2 M \gamma (\rho v'_2 - \alpha v'_1) \). Since the potential is a polynomial with quadratic, cubic and quartic terms, it is trivial to show that its value at any stationary point is given by half of the quadratic terms plus a quarter of the cubic ones. Then, it follows that

\[
V_{CB} = \frac{1}{2} X_{CB}^T A + \frac{1}{4} V_C .
\]

From eqs. 15 and 16 we obtain

\[
\begin{align*}
\begin{cases}
X_{CB}^T V' &= X_{CB}^T A + X_{CB}^T B X_N \\
X_N^T \hat{V}' &= X_N^T A + X_N^T B X_{CB}
\end{cases}
\]  

(19)
Because the matrix \( B \) is symmetric, we have \( X_{CB}^T B X_N = X_N^T B X_{CB} \). Then, subtracting these equations, and using equations (7) and (18) we obtain

\[
X_{CB}^T V' - X_N^T \hat{V}' = X_{CB}^T A - X_N^T A = 2(V_{CB} - V_N) - \frac{1}{2} V_C \ .
\]  

(20)

On the other hand, from eqs. (9), we have

\[
X_{CB}^T V' = \gamma^2 V_0' + (v_1'^2 + \rho^2) V_1' + (v_2'^2 + \alpha^2) V_2' + (v_1' v_2' + \rho \alpha) V_3' = \frac{1}{2x_3} [(v_1' v_2' - v_2' v_1')^2 + (v_1 \rho - v_2 \alpha)^2] \ .
\]

(21)

and (17) give

\[
X_N^T \hat{V}' = v_1^2 \hat{V}_1' + v_2^2 \hat{V}_2' + v_1 v_2 \hat{V}_3' = - \frac{V_3'}{2 \hat{x}_3} [(v_1' v_2' - v_2' v_1')^2 + (v_1 \rho - v_2 \alpha)^2] \ .
\]

(22)

Replacing these expressions in (20) and using (17), we finally obtain

\[
2(V_{CB} - V_N) = \gamma^2 \left( V_0' - \hat{V}_0' \right) + \frac{1}{2} \left( \frac{V_3'}{\hat{x}_3} - \frac{V_3'}{x_3} \right) [(v_1' v_2' - v_2' v_1')^2 + (v_1 \rho - v_2 \alpha)^2] \ .
\]

(23)

There is a striking similarity between this expression and the one found for the 2HDM potential, equation (14). However, whereas (14) establishes unequivocally that \( V_{CB} - V_N > 0 \), for the Zee model this is not possible. Depending on the values of the parameters of the theory, we may well have \( V_0' > \hat{V}_0' \) and \( \hat{V}_3'/\hat{x}_3 > V_3'/x_3 \), or exactly the opposite. Hence, for certain combinations of parameters, it is a priori possible that a CB minimum deeper than the normal minimum exists. It is the existence of the cubic terms in the potential that makes the simultaneous existence of normal and CB minima possible. In fact, if \( M = 0 \) eqs. (17) show that \( \hat{V}_3' = \hat{V}_0' = 0 \), and that \( V_0' > 0 \) and \( -V_3'/x_3 > 0 \) are the values of the two charged higgs masses at the normal minimum. So we trivially obtain, from eq. (23), \( V_{CB} > V_N \). Notice that the presence of the cubic term makes the Zee model more similar to supersymmetric theories than to the 2HDM, and one knows that CB can occur in SUSY theories.

### 4 Numerical results

In view of the result that we have obtained in the previous section, let us try to determine what combinations of parameters \( \{ M, a_i, b_j \} \) could give rise to CB minima deeper than the normal one. It is impossible to solve analytically the CB minimisation conditions. Even numerically, this turns out to be a difficult task. So, we adopt the following strategy: we choose the values of the parameters such that \( V \) has a normal minimum. For the same parameters we now take \( V' \) as a function of \( \{ v_1', v_2', \rho, \alpha, \gamma \} \). Starting from an initial guess for the values of these CB vevs, we numerically sample the value of the potential in its neighborhood and continue along the direction for which \( V \) decreases. At the end of this evaluation, we compare the value of \( V \) that we have reached with \( V_N \). If it is smaller than \( V_N \), then this set of parameters is rejected, because it could give rise to CB. We tried to overcome the dependence of this method on the initial guess by performing multiple runs of the algorithm with different initial conditions.

The potential depends on 16 independent parameters. We randomly generate values for 14 of these parameters, leaving \( a_1 \) and \( a_2 \) out of this procedure. The value of \( \tan \beta = v_2/v_1 \)
is also used as input (with $0.5 < \tan \beta < 100$). We impose some restrictions, though. The $b_i$ parameters are chosen as being of the same order of magnitude, since it would not seem natural to do otherwise. Here we were inspired by SUSY models, in which all $b_{ij}$ are of similar magnitudes. Likewise, we take the $\{M, a_i\}$ parameters to be of the same order. The intervals of variation for the parameters are: $-3 < M < 3$ TeV, $-1 < a_i < 1$ (TeV)$^2$ and $0.01 < b_{ij} < 10$. Having chosen these parameters, we use the values of $v_1 = v \cos \beta$ and $v_2 = v \sin \beta$ (with $v = 246$ GeV) and the minimisation conditions of the normal minimum, eqs. (19), to determine the values of $a_1$ and $a_2$, given by

$$
\begin{align*}
    a_1 &= -\frac{1}{2v_1} \left[ a_3 v_2 + 4 b_{11} v_1^3 + 2 \left( b_{12} + b_{33} \right) v_2^2 v_1 + 3 b_{13} v_1^2 v_2 + b_{23} v_2^3 \right] \\
    a_2 &= -\frac{1}{2v_2} \left[ a_3 v_1 + 4 b_{22} v_2^3 + 2 \left( b_{12} + b_{33} \right) v_1^2 v_2 + 3 b_{23} v_2^2 v_1 + b_{13} v_1^3 \right] .
\end{align*}
$$

With the values of these parameters we compute the masses of the scalar particles, using the expression (5) for the charged scalars and the formulae (37) and (38) of reference [3] for the neutral scalars. We accept only those sets of parameters for which (80) is unbounded from below (UFB) and this combination of $b_{ij}$ should be rejected on physical grounds. In ref. [3] we gave some necessary conditions on the $b$'s to prevent the occurrence of UFB directions in the 2HDM, and similar conditions were deduced and used in this work. The procedure to find them is straightforward: take one or more of the $\varphi_i$ fields to infinity and determine for which values of $b_{ij}$ we might obtain $V \to -\infty$. The use of polar/spherical coordinates is very helpful to study these limits. The most trivial conditions one obtains in this fashion are $\{b_{00}, b_{11}, b_{22}\} \geq 0$. The most stringent UFB bounds were found taking $\{\varphi_1, \varphi_3, \varphi_9\} \to \infty$ and parameterizing these fields as $\varphi_1 = r \sin \alpha \cos \theta$, $\varphi_3 = r \sin \alpha \cos \beta$, $\varphi_9 = r \cos \alpha$, with $r \to +\infty$ and $\{\alpha, \theta\}$ arbitrary angles. Requiring that the limit of the quartic terms of the potential is larger or equal to zero produces the condition $f(\alpha, \theta) \geq g(\alpha, \theta)$, $\forall_{\alpha, \theta}$, with

$$
\begin{align*}
    f(\alpha, \theta) &= -\frac{\sin 2\theta}{2} \left[ \frac{b_{03}}{4} \sin^2 2\alpha + (b_{13} \cos^2 \theta + b_{23} \sin^2 \theta) \right] \\
    g(\alpha, \theta) &= b_{00} \cos^4 \alpha + \left[ b_{11} \cos^4 \theta + b_{22} \sin^4 \theta + \frac{1}{4} (b_{12} + b_{33}) \sin^2 2\theta \right] \sin^4 \alpha + \\
    &\quad \frac{1}{4} (b_{01} \cos^2 \theta + b_{02} \sin^2 \theta) \sin^2 2\alpha .
\end{align*}
$$

We checked, for several values of $\{\alpha, \theta\}$, if the values of the $b$'s obeyed the previous inequality. The UFB conditions found in ref. [3] for the 2HDM are still valid for the Zee model and have been used. Furthermore, it is easy to obtain two additional conditions,

$$
\begin{align*}
    b_{01} &\geq -2 \sqrt{b_{00} b_{11}} \\
    b_{02} &\geq -2 \sqrt{b_{00} b_{22}} .
\end{align*}
$$

Although these are only necessary conditions, we found that they eliminate most of the UFB potentials. Nevertheless, we will see that UFB potentials still appear in our analysis, but only for a small subset of parameters.
Having required these conditions on the $b$ parameters and rejected those sets of $\{M, a_i, b_{ij}\}$ that did not produce scalar masses within the given interval, we built a set containing nearly 103000 different combinations of parameters. Although this a small part of the total parameter space, we hope that it will serve to illustrate the general CB properties of the Zee model potential. For each “point” in this parameter space, we computed the value of the potential at the normal minimum, $V_N$, numerically obtained the value of $V_{CB}$ and compared both these values. Despite our effort to enforce the UFB conditions, we have obtained UFB potentials for roughly 2% of the points.

CB was found to occur, mostly limited to a specific region of parameters. In fact, the majority of CB points occurred for large absolute values of the $M$ parameter ($|M| > 500$ GeV), which is easy to understand, given our earlier conclusion that the existence of the cubic terms in the potential is intimately linked to the possibility of CB. The region where CB was most likely to occur has lower values of $\tan \beta$ (inferior to about 10), negative values of the parameters $b_{33}$, $b_{44}$ and $a_3$ (this last one always superior to $-(800$ GeV)$^2$) and positive values for $a_1$ and $b_{12}$. Having varied the $b$ parameters in an interval spanning three orders of magnitude (from 0.01 to 10) we found that CB occurs less frequently for the larger values of the $b$’s. Although these observations are interesting in themselves, looking for the possible impact of CB on physical parameters, such as scalar masses, is of much greater interest. We observed that CB occurred for comparatively smaller values of the lightest neutral (CP even) higgs scalar, mostly below $\sim 350$ GeV with the largest concentration occurring for $90 < M_h < 130$ GeV. This is illustrated in figure 1 where we plot the values of $M_h$ versus $M_H$. The area delimited by the dashed lines shows the entire zone filled out by our $\sim 10^5$ sample of the parameter space. Those points for which CB occurs are marked with a cross. It is easy to see that CB occurs mostly for a narrow region of values of $M_h$. Similar “trends” in the values of the masses are found for the other scalars of the theory. CB is concentrated on the relatively small values of the lightest charged higgs ($M_{H^\pm} < 300$ GeV), meaning larger values of the heaviest charged scalar ($M_{H^\pm} > 500$ GeV), and for values of $M_A$, the pseudoscalar mass, usually larger than 200 GeV.

This initial foray into the CB analysis of the Zee model parameter space allowed us to identify the areas for which it was more likely to occur. Our next step was to generate a second set of parameters (about 53000, this time), following the same philosophy but with some extra restrictions: namely, $\tan \beta \leq 10$, $0.01 < b_{ij} < 1$ and imposing $M_h < 500$ GeV. Our intent was to narrow the search for CB to those areas where it is more likely to occur, to gauge its importance in terms of percentage of parameter space rejected. We found that CB occurred for about 5% of this second parameter space, and confirmed the “trends” found earlier for the parameters and masses of the theory. As an illustration of this, the reader can see in figure 2 how CB occurs for smaller values of the lightest charged scalar (below 300 GeV, as mentioned above) and mostly for higher values of the pseudoscalar mass. Again, in this figure the area delimited by the dashed lines shows the entire range of allowed values. Unfortunately, these plots do not mean that, after having eliminated all the points of the parameter space for which CB occurs, we would be left with “holes” in this space. If it were so, we could make definite restrictions, for instance on the higgs’ masses. But this is not the case: given the size of the parameter space, there are many combinations of $\{M, a_i, b_{ij}\}$ which produce the same scalar masses. In other words, if in figure 1 we showed that CB occurred for $90 < M_h < 130$ GeV, that does not mean that all Zee model points which produce masses in that interval were eliminated. On the contrary, there are many left which produce such masses. This means that we cannot produce a definite statement about the areas of parameter space for which CB occurs. We can, however, say with a good degree of certainty where CB does not occur: the regions with high values for the masses of the lightest neutral and charged higgses ($M_h > 500$ GeV, $M_{H^\pm} > 300$ GeV) are almost entirely “clean” of CB.
5 Conclusions

We have shown that charge breaking may occur in the Zee model, and have developed here the algorithm for finding the regions of parameter space where that may happen. The values of the lightest neutral higgs mass for which CB is more likely to occur are specially interesting, given that they correspond to the values that are currently being probed by our accelerators. Our analysis was very general, considering large regions of parameter space with randomly generated parameters. As was mentioned in the text, the minimal Zee model does not seem to be compatible with current neutrino data. This work focused on more general forms for the two doublets plus one singlet scalar potential, but extensions of the Zee model, with a larger scalar sector, are also of interest. The possibility of charge breaking would also arise in those models, and it would be interesting to perform CB studies on those theories. The results we obtained here cannot, clearly, be generalised to those other models, and in fact there is the possibility that, given a particular scalar content, the conclusions reached will be very different. For instance, in ref. [16], a model with a hyperchargeless triplet majoron was studied, and it was shown that the vacuum of that theory always broke charge conservation. Extra scalar content of a more “usual” nature - singlets and doublets, charged or not - would make the Zee potential even more similar to the supersymmetric one, and thus we would expect that charge breaking would still occur, as it does in the SUSY case.

Given the results encountered for our study of the non-minimal Zee model, we feel that further studies in this area are warranted: specific embodiments of the Zee model, with values of parameters specially tuned to explain neutrino observational data, should be tested to verify if they are safe from CB. This is not usually done, since in most applications of the Zee model [10, 11, 13] one does not consider in detail the scalar sector of the theory beyond its spectrum of masses and mixing angles. However, such a study might help narrow down the range of variation of the theory’s parameters and, perhaps, shed some light on the generation of neutrino masses.

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Figure 1: Plot of the mass of the lightest CP-even higgs neutral scalar versus the mass of the heaviest; the area delimited by the dashed lines shows the whole region of the parameter space. The crosses denote those “points” for which CB occurs. Most of these are circumscribed to a narrow interval of low values of $M_h$.

Figure 2: Plot of the mass of the lightest charged higgs scalar versus the mass of the pseudoscalar; the area delimited by the dashed lines represents the whole of the parameter space, and the crosses those points for which CB occurs.