Little Rip dark energy and cyclic cosmology from the Maxwell symmetry

Salih Kibaroğlu¹/²*

¹) Maltepe University, Faculty of Engineering and Natural Sciences, 34857, Istanbul, Turkey and
²) Institute of Space Sciences (IEEC-CSIC) C. Can Magrans s/n, 08193 Barcelona, Spain
(Dated: August 16, 2022)

In this study, we consider finding a cosmological model for the Maxwell gauge theory of gravity. In analogy with the Einstein-Yang-Mills theory which involves the cosmological consequences of the gauge fields, we express the Maxwell gauge field in terms of two time-dependent scalar fields. For a cosmological scenario, we use a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker universe and present the generalized Friedmann equations together with new contributions. Then we show that the Little Rip dark energy and cyclic cosmology can be unified in terms of this model in a certain condition.

PACS numbers: 02.20.Sv, 11.15.-q, 98.80.-k, 95.36.+x

I. INTRODUCTION

Einstein’s general theory of relativity is known as the most successful theory which describes the gravitational phenomena in a large range of scales. It has been tested through many experiments and observations. Despite its observational success, there are many unsolved puzzles. For example, the nature of dark energy, which is thought to be responsible for the accelerated expansion of the universe still remains unknown. In addition to this, the theory is not useful at very high energies where quantum effects are expected to become important. These are some of the motivations for studying generalized theories of gravity.

There exists an interesting extended gravitation theory that comes from the gauge theory of the Maxwell algebra [1, 2]. The Maxwell algebra can be interpreted as a modification of the Poincaré algebra by six additional tensorial abelian symmetry generators that make the four-momenta non-commutative \([P_a, P_b] = iZ_{ab}\) [3]. When one constructs a gauge theory of gravity based on this algebra, it leads to a generalized theory of gravity that includes the cosmological constant and additional term to the energy-momentum tensor [4–15]. Up to now, this energy-momentum term has not been extensively analyzed yet, but it is known that such an additional term may be related to dark energy [16, 17]. In the cosmological framework, a minimal cosmological model related to this symmetry can be found in [18]. Besides, the gauge fields of the Maxwell symmetry may provide a geometric background to describe vector inlatons in cosmological models [19]. For the non-gravitational case, this symmetry group is used to describe a particle moving in a Minkowski spacetime filled with a constant electromagnetic background field which is formed by the additional degrees of freedom related to \(Z_{ab}\). From this idea, Maxwell symmetry is considered as considered the symmetry group of a particle moving in a constant electromagnetic field [20]. It is also used to describe planar dynamics of the Landau problem [21], higher spin fields [22, 23], and applied to the string theory as an internal symmetry of the matter gauge fields [24].

The gauge field configurations in the context of the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology has been discussed in the literature before [25–32]. For example, in the Yang-Mills (YM) theories in which the non-Abelian YM field couples to the scalar curvature, one examines the cosmological consequences of the nonminimal gravitational coupling of the YM field. YM theory is also widely used in the inflationary cosmology [31, 33–35] (for a complete review see [36]) and used in the models for dark energy [29, 30, 37–41] and dark matter [42–44]. In this paper, inspired by the YM theory and gauge field construction proposed in [19], we consider to find a cosmological model using the gauge theory of gravity for the Maxwell algebra [4].

This paper is organized as follows. In Section II, we give a brief review of the Maxwell symmetry and its gauge theory of gravity. In Section III, we consider finding a cosmological model for the Maxwell gravity by using the idea of the Yang-Mills type cosmological model in which the gauge fields are defined by time-dependent scalar fields. Then we analyze this model for two conditions and we show that one can obtain the little rip dark energy and cyclic cosmology by using the proposed model. Finally, the last section concludes the paper by giving some discussion.

* salihkibaroglu@maltepe.edu.tr
II. MAXWELL SYMMETRY

In this section, we shortly review the Maxwell symmetry and its gauge theory of gravity construction. Maxwell symmetry is a non-central extension of the Poincaré symmetry and has noncommutative momentum generators. The corresponding non-zero commutation relationships of this extended symmetry can be given as follows,

\[ [M_{ab}, M_{cd}] = i (\eta_{ad}M_{bc} + \eta_{bd}M_{ac} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac}), \]
\[ [P_a, P_b] = i \lambda Z_{ab}, \]
\[ [M_{ab}, P_c] = i (\eta_{ac}P_b - \eta_{bc}P_a), \]
\[ [M_{ab}, Z_{cd}] = i (\eta_{ad}Z_{bc} + \eta_{bd}Z_{ac} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac}), \]

(1)

where \( \eta_{ab} \) is the Minkowski metric which have \( \text{diag} (\eta_{ab}) = (+, -, -, -) \) and the indices \( a, b, \ldots = 0, \ldots, 3 \). Here, the constant \( \lambda \) has the unit of \( L^{-2} \), and it will be related to the cosmological constant where \( L \) is considered as the unit of length. In addition to the Poincare algebra, this algebra contains six new additional tensorial generators \( Z_{ab} \). To construct gauge theory of gravity based on the Maxwell algebra, we firstly introduce the following one-form gauge field,

\[ A = e^a P_a + B^{ab} Z_{ab} - \frac{1}{2} \omega^{ab} M_{ab}, \]

(2)

where \( e^a (x) \), \( B^{ab} (x) \), and \( \omega^{ab} (x) \) are the one form gauge fields of corresponding generators. Also, the unit dimension of all gauge fields have zero other than \( [e^a] = L \). Using the structure equation \( \mathcal{F} = dA + \frac{1}{2} [A, A] \) and defining the curvatures as \( \mathcal{F} = F^a P_a + F^{ab} Z_{ab} - \frac{1}{2} R^{ab} M_{ab} \) we find the associated two-form curvatures as,

\[ F^a = de^a + \omega^a \land e^b, \]
\[ F^{ab} = dB^{ab} + \omega^{[a} \land B^{c|b]} - \frac{1}{2} \omega^a \land e^b, \]
\[ R^{ab} = d\omega^{ab} + \omega^a \land \omega^{cb}. \]

(3)

The infinitesimal gauge transformation of the curvatures can be found by introducing the Lie algebra valued parameters,

\[ \zeta (x) = y^a (x) P_a + \varphi^{ab} (x) Z_{ab} - \frac{1}{2} \tau^{ab} (x) M_{ab}, \]

(4)

and using the equation \( \delta \mathcal{F} = i [\zeta, \mathcal{F}] \). Here \( y^a (x) \), \( \varphi^{ab} (x) \) and \( \tau^{ab} (x) \) are space-time translations, translation in tensorial space, and the Lorentz transformation parameters respectively. Thus, the transformations of the curvatures under \( \mathcal{M} \) algebra are found as follows,

\[ \delta F^a = \omega^a \land e^b, \]
\[ \delta F^{ab} = \omega^c \land B^{c|b]} + \frac{\lambda}{2} F^{a|b] - R^{[a}_{c} F^{c|b]},} \]
\[ \delta R^{ab} = \omega^c \land e^b. \]

(5)

Taking the covariant derivative of given curvatures, the corresponding Bianchi identities can be found as

\[ \mathcal{D} F^a = R^a \land e^b, \]
\[ \mathcal{D} F^{ab} = R^{[a}_{c} \land B^{c|b]} - \frac{\lambda}{2} F^{[a}_{c} \land e^b,} \]
\[ \mathcal{D} R^{ab} = 0, \]

(6)

where \( \mathcal{D} \Phi = [d + \omega] \Phi \) is the Lorentz covariant derivative. From this point of view, by defining a shifted curvature as \( \mathcal{F}^{ab} = R^{ab} - \mu F^{ab} \) one can write the following action,
\[ S = \int \frac{1}{2\kappa \mu} J \wedge * J, \]  
(7)

where the asterisk represents Hodge duality operation, \( \kappa \) and \( \mu \) are constants. The equations of motion can be found by varying over the gauge fields \( \omega^{ab}(x) \), \( e^a(x) \) and \( B^{ab}(x) \), respectively,

\[ DF^{ab} - J_{[a} \wedge B^{b]} = 0, \]  
(8)

\[ \varepsilon_{abcd} J^{ab} \wedge e^c = 0, \]  
(9)

\[ \mathcal{D} J^{ab} = 0. \]  
(10)

Using Eq.(9) and transforming the tangent indices to world space-time indices, one can show that the generalized Einstein field equations can be written as follows,

\[ R^\mu_\alpha - \frac{1}{2} \delta^\mu_\alpha R - 3 \Lambda \delta^\mu_\alpha = \mu \left( e^\mu_a e^\beta_b \mathcal{D}_\alpha B^{ab} - \delta^\mu_\alpha (e^\rho_a e^\sigma_b \mathcal{D}_{\rho} B^{ab}) \right), \]  
(11)

where \( \Lambda = \lambda \mu \) is the cosmological constant and the Greek indices \( \mu, \nu, ..., \) = 0, ..., 3. In the light of this result, we can say that the standard gravitational equation is extended to include a cosmological constant and an additional term to the energy-momentum tensor in terms of the Maxwell symmetry. These are the main characteristics of the Maxwell extended (super)-gravitational theories. Now we will analyze the cosmological behaviour of this gravitational equation.

### III. COSMOLOGICAL SETUP

Inspired by the Einstein-Yang-Mills cosmology which employed for non-abelian gauge fields to obtain inflationary universe or late time acceleration, we consider to find potential cosmological consequences of the Maxwell fields \( B^{ab}_\mu \) which present in gravitational field equation in Eq.(11). First of all, we define the spatially flat FLRW universe

\[ ds^2 = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2). \]  
(12)

In this space, the space-time metric can be defined in terms of vierbein fields

\[ g_{\mu\nu}(x) = \eta_{ab} e^a_{\mu} e^b_{\nu}, \]  
(13)

where \( \eta_{ab} \) is the tangent space metric which is

\[ \eta_{ab} = g_{\mu\nu}(x) e^a_{\mu} e^b_{\nu}. \]  
(14)

Let us identify the vierbein field. The vierbein field \( e^a_{\mu}(x) \) can be written\([45, 46]\)

\[ e^a_{\mu}(x) = (e^a_0, e^a_i) = (\delta^a_0, -\delta^a_i a(t)), \]  
(15)

and satisfy the following relations

\[ e^a_{\mu}(x) e^b_{\nu}(x) = \delta^a_b, \quad e^a_{\mu}(x) e^a_{\nu}(x) = \delta^a_{\nu}, \]  
(16)

with

\[ e = \text{det} (e^a_\mu) = \sqrt{-\text{det} (g_{\mu\nu})}. \]  
(17)
For the spatially flat FLRW universe (12) by taking account of the torsion-free condition [47], the spin connection $\omega^{ab}_{\mu}$ can be expressed in terms of vierbein and the Levi Civita connection as

$$
\omega^{ab}_{\mu} = e^{a}_{\nu} \partial_{\mu} e^{b}_{\nu} + e^{a}_{\nu} \Gamma^{b}_{\mu \sigma} e^{b}_{\sigma}.
$$

By the help of the vierbein in Eq.(15), the non-zero components of the spin connection can be found as

$$
\omega^{ab}_{\mu} = \{(0, 1, 1) = (0, 2, 2) = (0, 3, 3) = \dot{a} (t) , (1, 0, 1) = (2, 0, 2) = (3, 0, 3) = -\dot{a} (t) \},
$$

where the index structure of spin connection is $\omega^{ab}_{\mu} = (a, b, \mu)$. To find the effect of the Maxwell gauge fields that contain six vector fields in the cosmological framework, we assume that these gauge fields can be defined by one dimensional fields $\psi(t)$ and $\zeta(t)$ [19],

$$
B^{0s}_{\mu} (x) \rightarrow B^{0s}_{\mu} (t) = (0, \delta_{i}^{s} a (t) \psi (t)) ,
$$

$$
B^{rs}_{\mu} (x) \rightarrow B^{rs}_{\mu} (t) = (0, \epsilon_{i}^{rs} \zeta (t)) ,
$$

where $r, s = 1, 2, 3$ are the tangent indices and $i, j = 1, 2, 3$ are the space-time indices. Here, $\delta_{i}^{s}$ and $\epsilon_{i}^{rs}$ are three dimensional $so(3)$ tensors. Let us consider the Friedmann equations. The first equation can be derived from the (0,0) component of Eq.(11)

$$
H^{2} = 2 \mu H \psi (t) + \Lambda,
$$

where $H(t) = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter and the dot denotes derivative with respect to the comoving time t. The second Friedmann equation is derived from the $(i, i)$ components of the field equation,

$$
2 \dot{H} + 3 H^{2} = 2 \mu \left[ \dot{\psi} (t) + 6 H (t) \psi (t) \right] + 3 \Lambda,
$$

then substituting Eq.(22) to (23), we get

$$
\dot{H} = \mu \left[ \dot{\psi} (t) + 6 H (t) \psi (t) \right].
$$

Now if we solve Eq.(22) with respect to the Hubble parameter, we find

$$
H = \mu \psi (t) \pm \sqrt{\mu^{2} \psi (t)^{2} + \Lambda},
$$

here if the scalar function $\psi(t)$ goes to zero, this equation takes the form $H = \pm \sqrt{\Lambda}$ which is known as the de Sitter universe (for positive sign). Besides, if we consider $\mu \psi (t)$ is close to zero, we can neglect the square term under the square root then the equation goes to $H \approx \mu \psi (t) \pm \sqrt{\Lambda}$ thus this assumption behaves like the quasi-de Sitter evolution because of $\dot{H} \approx 0$. Note that the second scalar field $\zeta(t)$ does not contribute to the Friedmann equations for this model. To find the evolution of the scale factor, we can solve Eq.(25) with respect to $a(t)$,

$$
a(t) = a_{0} e^{\int (\mu \psi(t) \pm \sqrt{\mu^{2} \psi(t)^{2} + \Lambda})dt},
$$

where $a_{0}$ is the integration constant.

### A. Little Rip universe

We can define the time-dependent function as $\psi(t) = e^{\mu t}$ then the scale factor and the Hubble parameter take the following form in which we considered Eq.(26) with positive sign,

$$
a(t) = a_{0} \exp \left( \frac{\mu e^{\mu t} + \sqrt{e^{2 \mu t} \mu^{2} + \Lambda} - \sqrt{\Lambda} \ln \left( \frac{2 \Lambda + 2 \sqrt{\Lambda} e^{2 \mu t} \mu^{2} + \Lambda}{e^{\mu t} \mu^{2} + \Lambda} \right)}{h} \right),
$$

$$
(27)
$$
\[ H = \mu e^{ht} + \sqrt{e^{2ht} \mu^2 + \Lambda}, \]  

(28)

where \(a_0\) and \(h\) are constants. Then the effective equation of state (EoS) parameter can be written as follows \([48, 49]\),

\[
w_{\text{eff}} = -\frac{2\dot{H}}{3H^2} = -\frac{e^{2ht}\mu^2 + \mu e^{ht} \left( \sqrt{e^{2ht}\mu^2 + \Lambda} + \frac{2h}{3} \right) + \Lambda}{\sqrt{e^{2ht}\mu^2 + \Lambda} \left( \mu e^{ht} + \sqrt{e^{2ht}\mu^2 + \Lambda} \right)}. \]

(29)

In Fig.(1), we present the evolution of the scale factor, the Hubble parameter and the effective EoS with respect to \(t\) for a certain condition. In this graph, \(a(t) \to \infty\) and \(H(t) \to \infty\) with exponential characteristic and \(w_{\text{eff}}(t)\) less than \(-1\) and asymptotically approaches to \(-1\), thus one can say that this model lead to the well-known Little Rip universe \([50–55]\). Moreover, it is easy to show that when setting the function as \(\psi(t) = ht\) it leads to the new type Little Rip scenario.

![Figure 1. The scale factor \(a(t)\) (red line) in Eq.(27), the Hubble parameter \(H(t)\) (blue dashed line) in Eq.(28) and the effective equation of state \(w_{\text{eff}}(t)\) (green dash-dot line) in Eq.(29) with the parameters \(a_0 = 0.05, h = 0.6, \mu = 0.05\) and \(\Lambda = 10^{-52}\).](image)

B. **Cyclic universe**

For \(\psi(t) = A\sin(ht)\), the scale factor \(a(t)\) and the Hubble parameter \(H(t)\) given in Eqs.(26) and (25) take the following form,

\[
a(t) = a_0 \exp \left( \frac{A \sqrt{\cos(ht)^2 \sqrt{\frac{A^2 \sin^2(ht) \mu^2 + \Lambda}{\Lambda}}}}{\cos(ht) \sqrt{A^2 \sin^2(ht)^2 \mu^2 + \Lambda h}} - \frac{A \mu \cos(ht)}{h} \right), \]

(30)

\[
H = \mu A \sin(ht) + \sqrt{\mu^2 A^2 \sin^2(ht)^2 + \Lambda},
\]

(31)

where \(a_0, A, h\) are constants and the function \(\Upsilon(z, k)\) is defined as

\[
\Upsilon(z, k) = \int_0^z \sqrt{\frac{1 - k^2 t^2}{1 - t^2}} dt.
\]

(32)
Figure 2. The scale factor $a(t)$ (red line) in Eq.(30), the Hubble parameter $H(t)$ (blue dashed line) in Eq.(31) and the effective equation of state $w_{\text{eff}}(t)$ (green dash-dot line) in Eq.(33) with the parameters $A = 10, a_0 = 0.75, h = 1, \mu = 0.1$ and $\Lambda = 10^{-52}$.

Then, the effective EoS becomes,

$$w_{\text{eff}} = -\left(\frac{2B + 2B^2 + \Lambda}{B + \sqrt{B^2 + \Lambda}} + \Lambda B\right) \sqrt{B^2 + \Lambda (B + \sqrt{B^2 + \Lambda})^2},$$ \hspace{1cm} (33)

where we’ve used a time-dependent function as $B(t) = \mu A \sin(ht)$ for simplicity. According to Fig.(2), we see that the scale factor, the Hubble parameter, and the effective EoS oscillate periodically with time, respectively. Thus we derive a specific example for the dynamics of the universe and it may corresponds to a sub-class of the cyclic model of the universe in which the universe has an endless periodic sequence of expansion and contraction [56–59] (for more detail see [60]).

IV. CONCLUSION

In this work, we examined to find a cosmological model for the gauge theory of gravity from Maxwell algebra. For this purpose, in analogy with the Einstein-Yang-Mills cosmology, we described the Maxwell gauge field as a function of time in terms of two time-dependent functions $\psi(t)$ and $\zeta(t)$ in Eqs.(20) and (21). We see that the only $\psi(t)$ function contributed to the field equations for this minimal model but we can easily say that if one considers a different gravitational action or choose a different kind of Maxwell algebras there may be some contributions that come from $\zeta(t)$ function. Then we found the Friedmann equations involving contributions of the Maxwell gauge field. We also derive the scale factor as a function of $\psi(t)$ and the cosmological constant by solving the first Friedmann equation Eq.(22). We note that, when we consider a special limit as $\psi(t) \to 0$, this model reduces the ordinary de Sitter cosmology.

Furthermore, we analyzed this model under two conditions. In the first one, we assume that $\psi(t)$ evolves as an exponential function of $t$. Then we found the scale factor, the Hubble parameter and the effective EoS, respectively. According to these equations, we obtained the so-called Little Rip universe which satisfy the conditions $a(t) \to \infty$, $H(t) \to \infty$ at future infinity, $w_{\text{eff}} < -1$ and $w_{\text{eff}}$ asymptotically approaches to $-1$. In addition to this result, one can say that the phantom ($w_{\text{eff}} < -1$) or the quintessence ($-1/3 < w_{\text{eff}} < -1$) dark energy models may be obtained as a solution to the Maxwell gauge theory of gravity for a different set of parameters (for detailed information about rip models see [61]). In the second model, we used the function $\psi(t) = A \sin(ht)$. Then we again obtained the scale factor, the Hubble parameter and the effective EoS which have periodic evolution with respect to time for a certain condition. These results correspond to the cyclic universe.
In summary, there are a lot of interpretations in the literature about the physical meaning of the Maxwell symmetry that we discussed before but these interpretations have not been analyzed in detail yet. In this paper, we analyzed this gravitation model in the context of cosmology. Then we showed that one can derive the Little Rip dark energy cosmology and the cyclic universe by using the Maxwell extended gravitation theory. Thus, we can say that the gauge theory of gravity based on the Maxwell symmetry provides an alternative way that may help the explanation of dark energy and the evolution of our universe. We also know that there is a wide variety of the Maxwell algebras, so it is easy to say that it can be derived different results by gauging these extended algebras. Moreover, we know that the Einstein-Yang-Mills cosmology provides a useful background to study the inflationary universe (more detail see [36]) so the inflationary behaviour of the resulting model will be discussed elsewhere.

ACKNOWLEDGMENTS

The author wishes to thank Sergei D. Odintsov for useful discussions and comments regarding the results presented in this work. This study is supported by the Scientific and Technological Research Council of Turkey (TUBİTAK) under grant number 2219-A.

[1] H. Bacry, P. Combe, and J.-L. Richard, Il Nuovo Cimento A (1965-1970) 67, 267 (1970).
[2] R. Schrader, Fortschr. Phys. 20, 701 (1972).
[3] D. V. Soroka and V. A. Soroka, Phys. Lett. B 607, 302 (2005).
[4] J. A. de Azcárraga, K. Kamimura, and J. Lukierski, Phys. Rev. D 83, 124036 (2011).
[5] R. Durka, J. Kowalski-Glikman, and M. Szczachor, Mod. Phys. Lett. A 26, 2689 (2011).
[6] D. V. Soroka and V. A. Soroka, Phys. Lett. B 707, 160 (2012).
[7] J. de Azcarraga and J. Izquierdo, Nucl. Phys. B 885, 34 (2014).
[8] O. Cebecioglu and S. Kibaroğlu, Phys. Rev. D 90, 084053 (2014).
[9] O. Cebecioglu and S. Kibaroğlu, Phys. Lett. B 751, 131 (2015).
[10] P. Concha, E. Rodriguez, and P. Salgado, J. High Energy Phys. 2015 (8), 1.
[11] S. Kibaroğlu, M. Senay, and O. Cebecioglu, Mod. Phys. Lett. A 34, 1950016 (2019).
[12] S. Kibaroğlu and O. Cebecioglu, Eur. Phys. J. C 79, 1 (2019).
[13] S. Kibaroğlu and O. Cebecioglu, Phys. Lett. B 803, 135295 (2020).
[14] S. Kibaroğlu and O. Cebecioglu, Int. J. Mod. Phys. D 30, 2150075 (2021).
[15] O. Cebecioglu and S. Kibaroğlu, Eur. Phys. J. C 81, 1 (2021).
[16] J. A. Frieman, M. S. Turner, and D. Huterer, Annu. Rev. Astron. Astrophys. 46, 385 (2008).
[17] T. Padmanabhan, Adv. Sci. Lett. 2, 174 (2009).
[18] R. Durka and J. Kowalski-Glikman, arXiv preprint arXiv:1110.6812 (2011).
[19] J. A. de Azcárraga, K. Kamimura, and J. Lukierski, in Int. J. Mod. Phys. Conf. Ser., Vol. 23 (World Scientific, 2013) pp. 350–356.
[20] S. Bonanos and J. Gomis, J. Phys. A 43, 015201 (2009).
[21] S. Fedoruk and J. Lukierski, Phys. Lett. B 718, 646 (2012).
[22] S. Fedoruk and J. Lukierski, in J. Phys.: Conf. Ser., Vol. 474 (IOP Publishing, 2013) p. 012016.
[23] S. Fedoruk and J. Lukierski, J. High Energy Phys. 2013 (2), 1.
[24] S. Hoseinzadeh and A. Rezaei-Aghdam, Phys. Rev. D 90, 084008 (2014).
[25] J. Cervero and L. Jacobs, Phys. Lett. B 78, 427 (1978).
[26] M. Henneaux, J. Math. Phys. 23, 830 (1982).
[27] D. Galtsov and M. Volkov, Phys. Lett. B 256, 17 (1991).
[28] P. Moniz, J. Mourao, and P. Sá, Class. Quantum Gravity 10, 517 (1993).
[29] K. Bamba, S. Nojiri, and S. D. Odintsov, Phys. Rev. D 77, 123532 (2008).
[30] D. V. Gal’tsov, arXiv preprint arXiv:0901.0115 (2008).
[31] A. Maleknejad and M. Sheikh-Jabbari, Phys. Lett. B 723, 224 (2013).
[32] A. Guarnizo, J. B. Orjuela-Quintana, and C. A. Valenzuela-Toledo, Phys. Rev. D 102, 083507 (2020).
[33] A. Maleknejad and M. Sheikh-Jabbari, Phys. Rev. D 84, 043515 (2011).
[34] M. Sheikh-Jabbari, Phys. Lett. B 717, 6 (2012).
[35] J. Biehlefeld and R. R. Caldwell, Phys. Rev. D 91, 124004 (2015).
[36] A. Maleknejad, M. M. Sheikh-Jabbari, and J. Soda, Phys. Rep. 528, 161 (2013).
[37] D. Gal’tsov and E. Davydov, in Int. J. Mod. Phys. Conf. Ser., Vol. 14 (World Scientific, 2012) pp. 316–325.
[38] E. Elizalde, A. Lopez-Revelles, S. Odintsov, and S. Y. Vernov, Physics of Atomic Nuclei 76, 996 (2013).
[39] M. Setare and V. Kamali, Phys. Lett. B 726, 56 (2013).
[40] M. Rinaldi, J. Cosmol. Astropart. Phys. 2015 (10), 023.
[41] A. Mehrabi, A. Maleknejad, and V. Kamali, Astrophys. Space Sci. 362, 1 (2017).
[42] H. Zhang, C. S. Li, Q.-H. Cao, and Z. Li, Phys. Rev. D 82, 075003 (2010).
[43] M. A. Buen-Abad, G. Marques-Tavares, and M. Schmaltz, Phys. Rev. D 92, 023531 (2015).
[44] C. Gross, O. Lebedev, and Y. Mambrini, J. High Energy Phys. 2015 (8), 1.
[45] C. Armendariz-Picon and P. B. Greene, Gen. Relativ. Gravit. 35, 1637 (2003).
[46] R. Ferraro and F. Fiorini, Phys. Lett. B 702, 75 (2011).
[47] T. Verwimp, arXiv preprint arXiv:1006.1614 (2010).
[48] S. Nojiri and S. D. Odintsov, Int. J. Geom. Methods Mod. Phys. 4, 115 (2007).
[49] S. Nojiri and S. D. Odintsov, Phys. Rep. 505, 59 (2011).
[50] P. H. Frampton, K. J. Ludwick, and R. J. Scherrer, Phys. Rev. D 84, 063003 (2011).
[51] P. H. Frampton, K. J. Ludwick, S. Nojiri, S. D. Odintsov, and R. J. Scherrer, Phys. Lett. B 708, 204 (2012).
[52] I. Brevik, E. Elizalde, S. Nojiri, and S. Odintsov, Phys. Rev. D 84, 103508 (2011).
[53] I. Brevik, R. Myrzakulov, S. Nojiri, and S. Odintsov, Phys. Rev. D 86, 063007 (2012).
[54] A. N. Makarenko, V. V. Obukhov, and I. V. Kirnos, Astrophys. Space Sci. 343, 481 (2013).
[55] A. N. Makarenko, S. D. Odintsov, and G. J. Olmo, Phys. Lett. B 734, 36 (2014), arXiv:1404.2850 [gr-qc].
[56] A. Friedmann, Zeitschrift für Physik 10, 377 (1922).
[57] R. C. Tolman, Phys. Rev. 38, 1758 (1931).
[58] P. J. Steinhardt and N. Turok, Science 296, 1436 (2002).
[59] P. J. Steinhardt and N. Turok, Phys. Rev. D 65, 126003 (2002).
[60] J.-L. Lehners, Phys. Rep. 465, 223 (2008).
[61] K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, Astrophys. Space Sci. 342, 155 (2012).