THE KEPLER DICHOTOMY AMONG THE M DWARFS: HALF OF SYSTEMS CONTAIN FIVE OR MORE COPLANAR PLANETS

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Received 2014 October 13; accepted 2015 November 8; published 2016 January 8

ABSTRACT

We present a statistical analysis of the Kepler M dwarf planet hosts, with a particular focus on the fractional number of systems hosting multiple transiting planets. We manufacture synthetic planetary systems within a range of planet multiplicity and mutual inclination for comparison to the Kepler yield. Similarly to studies of Kepler exoplanetary systems around more massive stars, we report that the number of singly transiting planets found by Kepler is too high to be consistent with a single population of multi-planet systems, a finding that cannot be attributed to selection biases. To account for the excess singleton planetary systems we adopt a mixture model and find that 53 ± 10% of planetary systems are either single or contain multiple planets with large mutual inclinations. The other 47 ± 10% of systems contain 7.5±0.5 planets with mutual inclinations of 2°±1°. This mutual inclination range is consistent with studies of transit durations within multiply transiting systems. The mixture model is preferred 8:1 to a model with only one architecture. Thus, we find that the so-called “Kepler dichotomy” holds for planets orbiting M dwarfs as well as Sun-like stars.

Key words: eclipses – planetary systems

1. INTRODUCTION

While NASA’s Kepler Mission was launched to investigate the frequency of planets orbiting Sun-like stars (Borucki et al. 2010), the mission is foundational to our understanding of planet occurrence around the smallest stars (Johnson 2014). Though M dwarfs comprise less than 4% of the Kepler targets (5500 stars of Kepler’s 160,000 total, per Swift et al. 2013), the Kepler planet yield encodes an occurrence rate of at least 1–2 planets per M dwarf, with periods of less than 150 days (Dressing & Charbonneau 2013; Swift et al. 2013; Morton & Swift 2014). Our understanding of the mutual inclinations of exoplanets is also based, in large part, on results from the Kepler mission. Remarkably, 20% of the planet host stars reported by Batalha et al. (2013) host at least two transiting planets (Fabrycky et al. 2014), including the first exoplanetary system discovered to have more than one transiting planet (Kepler-9, Holman et al. 2010).

Despite the wealth of multi-planet systems discovered by other detection methods, the determination of the true mutual inclinations of the planets is limited to special cases: those orbiting pulsars (Wolszczan 2008) and those with very strong dynamical constraints from radial velocity measurements (e.g., Laughlin et al. 2005; Correia et al. 2010). For multi-planet systems discovered by transits, mutual inclinations can be measured on a system-by-system basis for those that transit starspots (Sanchis-Ojeda et al. 2012) and those that transit one another (Ragazzine & Holman 2010; Masuda et al. 2013).

Other studies of multi-transiting systems have relied upon ensemble statistics to deduce the underlying mutual inclination distribution, including those of Lissauer et al. (2011b), Tremaine & Dong (2012), Fang & Margot (2012), and Fabrycky et al. (2014). All conclude that mutual inclinations of less than 3° are consistent with the Kepler multi-planet yield, both in the number of planets detected to transit and in the distribution of their transit durations. Indeed, flat and manifold architectures are necessary to recover the multi-planet statistics from Kepler. Lissauer et al. (2011b) found that systems contain 3.25 planets per star (that is, 3 planets in 75% of systems and 4 planets in 25% of systems), with a mutual inclinations drawn from a Rayleigh distribution with σ = 2°, best reproduces the Kepler yield (excluding the small handful of systems with 5 and 6 transiting planets). Fang & Margot (2012), similarly found that 75%–80% of planetary systems must host 1–2 planets, and 85% of planetary orbits in multiple-planet systems are inclined by less than 3° with respect to one another. Swift et al. (2013), using the 5-planet system Kepler-32 as a template, thereby assuming 5 planets per star, found they could recover the multi-planet yield of Kepler systems orbiting M dwarfs with inclinations of 1°±0°.2.

However, Lissauer et al. (2011b) noted the puzzling finding that the best-fitting models to the Kepler yield underpredict the number of singly transiting systems by a factor of two. Hansen & Murray (2013) reported a similar finding when comparing to their population synthesis model, which involved growing planetary architectures from multiple protoplanetary cores and then observing their final distributions. In both works, the authors posit a separate population of singly transiting planets to explain the discrepancy. This feature of the Kepler multi-planet ensemble is coined “the Kepler dichotomy.” The mechanism responsible for producing an excess of singles is as-yet unclear. Either the primordial circumstellar disks of these stars produced fewer planets, or the resulting planets were scattered to larger mutual inclinations, ejected, or met their end in collisions with other planets or their host star.

The conclusions of Morton & Winn (2014) favor the latter possibilities. They reported that that obliquity of planet host stars is larger for the singly transiting planets, indicating a separate population of “dynamically hotter” systems. Johansen et al. (2012) considered planetary instability as the responsible mechanism. They reported that planet–planet collisions in the typical packed Kepler architecture would occur with higher frequency than ejections or collisions with the host star.

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number of planets per star and their mutual inclination distribution, to generate these synthetic planetary systems to fit for the number of planets per star. In Section 4, we summarize our conclusions and motivate future work.

2. DATA SELECTION

We draw our sample of KOIs from the publicly available NASA Exoplanet Science Institute (NExScI) database. We select our sample of M dwarf hosts from those characterized by Muirhead et al. (2014), which we supplement with those characterized by Mann et al. (2014) that are hotter than 3950 K and cooler than 4200 K. From this list, we eliminate two eclipsing binaries identified in McQuillan et al. (2013a, 2013b) and Muirhead et al. (2013): KOIs 1459 and 256. We also remove KOIs 249 and 1725, which Muirhead et al. (2014) determined to be blends, KOI 2626, which resides in a blended triple star system (Cartier et al. 2015), and KOIs 531 and 3263, which Morton & Swift (2014) found to have false positive probabilities (FPFs) >0.95. In addition, one KOI present in Muirhead et al. (2014), KOI 1902.01, is now identified on the NExScI database to be a false positive, so we remove it. Our sample comprises 167 individual KOIs orbiting 106 stars: 71 host one KOI, 17 host two KOIs, 12 host three KOIs, 4 host 4 KOIs, and 2 host 5 KOIs (each of these has its own discovery paper: Kepler-32, described in Swift et al. 2013 and Kepler-186, described in Quintana et al. 2014).

Of the 109 stars included in our study, 84 are drawn from Muirhead et al. (2014), and 22 from Mann et al. (2014). Dressing & Charbonneau (2013) also characterized the M dwarf sample of Kepler target stars with their broadband colors. In addition to target stars, there are 21 KOIs with properties reported in Dressing & Charbonneau (2013) that are not present in Muirhead et al. (2014) or Mann et al. (2014): of these, 9 are reported to be false positives in the NExScI database, and the remaining 12 have not received a planet candidate disposition. Finally, there are four KOIs that are dispositioned as candidates and likely cooler than 4200 K (from the broadband colors reported by Dressing & Charbonneau 2013), but were not characterized in either manuscript: these are KOIs 2992, 3140, 3414, and 4087. All of the stars in the sample reside between the M4V–K7V spectral types.

2.1. Investigation of Selection Biases

We investigate the possibility that incompleteness or selection bias affects the observed multiplicity of KOIs. We direct the reader toward other sophisticated analyses (Morton & Swift 2014 and Dressing & Charbonneau 2015 in particular) of the completeness of the Kepler pipeline to M dwarf planets as a function of radius and period. For the purpose of this paper, we aim only to show that this completeness distribution is not different between hosts with a single transiting planet versus multiply transiting systems (although detection rates exceed 80% for 2 $R_{\oplus}$ planets even at the highest end of the period range we consider, per Dressing & Charbonneau 2015). We consider several explanations that are not astrophysical in

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4. http://exoplanetarchive.ipac.caltech.edu/, accessed on 2013 July 3.
5. The Mann et al. sample contains stars hotter than 3950 K. Beyond this temperature threshold, the H$_{2}$O–K2 metric has saturated, per Rojas-Ayala et al. (2012), which is why they are not present in the Muirhead et al. sample.
6. The KOI with the latest spectral type, KOI 4290.01, has an effective temperature of 3200 K, per Muirhead et al. (2014).
nature, but rather produced by observational biases in the sample. First, the sample of \textit{Kepler} M dwarfs is heavily weighted toward the largest and hottest of that spectral type, with most stars having spectral types M0–M1. Their larger size renders a given planet size less detectable because of the less favorable planet/star radius ratio. We have reason to be suspicious of our finding if single transit systems are simply likelier to reside around larger stars, since the smaller $R_p/R_*$ means that more small planets do transit, but are eluding detection. Likewise, if noisier stars are overwhelmingly the likeliest hosts of single transit systems, we would need to consider the same possibility that noise is swamping transits that do exist, but do not lie above the detection threshold. In Figure 1, we depict the cumulative distribution for the single and multi-KOI systems for properties predictive of possible selection bias. These include the 3-hr Combined Differential Photometric Precision (CDPP), the stellar radius, and the planetary radius. Finally, we compare the total transit SNR for each planet, making the simplifying assumption of constant phase coverage over four years for each target. We overplot the Anderson–Darling statistic for each parameter.

There exists no evidence that the singly transiting KOI systems are drawn from a different distribution than that of the multiply transiting KOI systems in CDPP, stellar radius, planetary radius, or average transit SNR. In all four cases, the test statistic is not in excess of the critical significance. We note that the three Saturn-sized and larger planet candidates ($>6\ R_\oplus$) all orbit in singly transiting systems: one of these systems is the only known Hot Jupiter orbiting an M dwarf: KOI 254 (Johnson et al. 2012). The phenomenon of the singly transiting systems hosting a population of larger planets is described in Johansen et al. (2012) and Morton & Winn (2014). But if we consider planets smaller than 6 $R_\oplus$ (that is, if we remove only these three planets), the planets in multiply transiting systems are slightly larger than those in singly transiting systems, which is the opposite of the predicted trend for a selection bias. This finding is apparent in the second panel of Figure 1. We report no evidence for selection bias producing an overly large sample of singly transiting KOIs.

Another important source of selection bias is the false-positive rate. We consider all KOIs in multiply transiting systems to be authentic, following the reasoning described in detail in Lissauer et al. (2012), Rowe et al. (2014), and Lissauer et al. (2014). Therefore, false positives will appear as an excess of singly transiting systems. Fressin et al. (2013) reported an overall false-positive rate of $8.8 \pm 1.9\%$ for planets between 1.25 and 2 $R_\oplus$. Planets of this size comprise 40% of our sample. Another 30% of the sample is smaller than 1.25 $R_\oplus$, which have a false positive rate of 12.3 ± 3%, and the remaining KOIs (nearly all between 2 and 4 $R_\oplus$) have a 6.7 ± 1.1% false positive rate (Fressin et al. 2013). However, this sample of M dwarf KOIs is unusually pristine: we have already discarded 5 single KOIs as potential false positives because they are blends or eclipsing binaries, which is 6.3% of the original sample of singles. Morton & Swift (2014) individually calculated FPPs for 115 of the 167 KOIs in our sample. Of these, 87 have false positive properties <0.05, and 104 have FPPs below 0.10. Of the 11 remaining KOIs with false positive probability greater than 0.1, 6 reside in multiply transiting systems: in this work, we consider these to be authentic, as stated above. An additional KOI with FPP > 0.1 is the bona fide hot Jupiter KOI 254.01 (Johnson et al. 2012). In the event that the remaining four KOIs are all false positives, we adopt the conservative value of 5% false positives among KOIs.

![Figure 1](image_url). Cumulative distribution in four parameters, from top to bottom: \textit{Kepler} CDPP, planet radius, stellar radius, and total transit SNR (making simplifying assumption of constant phase coverage over four years). The red curve depicts the distribution for multiples, and the black for the singly transiting KOI systems. The value of the A-D test and the critical value associated with a $p = 0.01$ are overplotted in each panel. There is a low probability in that the multiple systems are drawn from a different distribution than the singles.
the single-KOI systems in hand. The findings we report in this work could only be falsified by false positive rates of 50%, which we describe in the next section.

3. ANALYSIS

3.1. Methodology

We compare the observed sample of KOIs to synthetic populations of exoplanets, which we manufacture across a grid of multiplicity and orbital mutual inclination, as we describe in detail in this section. We first assume a single type of planetary system architecture, in which each star hosts N planets, with mutual inclinations drawn from a Rayleigh distribution with scatter σ. For the sake of comparison to the Kepler sample, we ask how many stars, μ, would be observed to host n transiting planets, if the mean orbital plane of planetary systems is distributed isotropically. We define our model $\mu \equiv \mu_n(N, \sigma)$, which describes the expected number of stars with n transiting planets, given a parent population characterized by N and σ. In our analysis we consider indices n running from 1 to 8, and compared to the Kepler observations $M_n$ (where M is the number of M dwarfs hosting n transiting planets). For the Kepler M dwarf sample, the largest observed n is 5, for which $M_n = 2$ (for Kepler-32, per Swift et al. 2013, and Kepler-186, per Quintana et al. 2014).

We compare the model-predicted population $\mu_n$ to the observed number $M_n$ of multiples with N true planets per star. Poisson counting statistics describe integer numbers of transiting planets, so we evaluate the likelihood of N and σ with a Poisson likelihood function, which is conditioned on the observed number of stars hosting n planets, $M_n$, with the ensemble of bins given by $\{M\}$. Therefore, we can describe the likelihood $L \equiv P(\{M\} | N, \sigma)$ of observing the distribution $\{M\}$, given some N and some σ, by

$$L \propto \prod_{n} \frac{\mu_n(N, \sigma)^{M_n} e^{-\mu_n(N, \sigma)}}{M_n!}.$$  

(1)

It remains for us to find the values of N and σ that maximize the likelihood of observing $\{M\}$. We employ the Bayesian sampler MultiNest (Feroz & Hobson 2008; Feroz et al. 2009, 2013) to evaluate these likelihoods and posterior distributions. In practice, MultiNest calculates the log of the likelihood $L$ defined in Equation (1).

3.2. Modeling a Single Population of Multi-planet Systems

We initially make the additional assumption of circular orbits for our simplest scenario. In order to evaluate how many planets we expect to transit, we use a Monte Carlo method to generate a synthetic transiting planet sample. We generate $10^5$ planetary systems for each value of N and σ, allowing N to vary from 1 to 8 planets in increments of one planet, and σ to vary from 0° to 10° in increments of 0.1°. In each scenario, we draw N periods randomly from a flat distribution in log P space, ranging from 1 to 200 days. While other studies predict complex structure in the period probability distribution with several peaks (Hansen & Murray 2013 from simulation work, for example), we assume the approximate flatness that Foreman-Mackey et al. (2014) reports from fitting to the Kepler data set. This probability does fall off slightly for periods >50 days for planets <2 $R_\odot$, but is still consistent with flatness. We note that this is a simplifying assumption: Dressing & Charbonneau (2015) found that M dwarf planet occurrence with period in fact rises toward periods as long as 200 days, and varies with planet radius.

We then test each synthetic planetary system for Hill stability, using Equation (3) from Fabrycky et al. (2014). This expression defines the mutual Hill radius to be:

$$R_H = \left[ \frac{M_m + M_out}{3M_e} \right]^{1/3} \frac{(a_in + a_out)}{2},$$  

(2)

where the stability criterion is satisfied if

$$\frac{a_in + a_out}{R_H} > \Delta_{crit},$$  

(3)

where $a_in$ and $a_out$ are the semimajor axes of the inner and outer planets, respectively, measured in AU. The critical separation is $\Delta_{crit} = 2\sqrt{3}$ for adjacent planets. For systems with more than three planets, Fabrycky et al. (2014) require that $\Delta_{inner} + \Delta_{outer} > 18$ for adjacent inner and outer pairs of planets. If the set of planetary orbits violate these criteria, we reject that iteration and draw a new set of periods.

We assign mutual inclinations from a Rayleigh distribution of angles with scatter σ (Fabrycky & Winn 2009; Lissauer et al. 2011a; Fang & Margot 2012; Figueira et al. 2012; Fabrycky et al. 2014). For each set of planets, we calculate the impact parameter b of the planet in units of stellar radii, where the planet “transits” if b < 1. We tested a more sophisticated detection criterion for impact parameter. We forced consistency between the known impact parameter distribution of the planets detected around Kepler’s M dwarfs from Swift et al. (2015) (flat out to b = 0.8, and declining thereafter) and the impact parameters of the “detected” planets in our simulation. After applying the more sophisticated detection criterion in impact parameter, we reexamined the marginalized posterior distributions in N and σ. Both of these quantities shifted by less than 1σ of their uncertainties (slightly favoring higher N and lower σ). In the interest of computational time, we employed the b < 1 criterion. We record the number of planets that transit for each synthetic exoplanetary system. After $10^5$ iterations, we produce the model histogram $\mu$ of the number of systems hosting N planets, of which n transit. We then interpolate $\mu$ between integer values of N (at constant σ) in intervals of 0.1 planets. In this way, we evaluate the likelihoods of non-integer numbers of planets. For the interpretation of a non-integer average number of planets, we reference Lissauer et al. (2011a), who quoted a value of 3.25 planets star$^{-1}$. The 3.25 planets figure describes a population of planet host stars possessing 3 planets 75% of the time and 4 planets 25% of the time.

We then employ this empirical $\mu(N, \sigma)$ to evaluate Equation (1). We depict the joint posterior distribution for N and σ in Figure 2, with contours enclosing the 1σ and 2σ confidence intervals (as reflected by the density of MCMC realizations). We also plot the Kepler yield and the range of best-fitting models, i.e., those that maximize the likelihood, in the 68% and 95% confidence intervals. A comparison of the best-fitting family of models to the Kepler sample shows that no set of $\{N, \sigma\}$ furnish a multi-transiting yield similar to Kepler’s. The best-fitting models, which combine high multiplicity and a high degree of scatter in mutual inclinations, significantly underpredict the number of singly transiting systems and overpredict the number of multiply transiting
systems. The best-fitting models have $N = 6 \pm 2$ planets and $\sigma = 7.4^{+2.5}_{-4.6}$ degrees.

However, when we compare the predicted Kepler yields against only the set of multiply transiting systems (i.e., excluding singly transiting systems, similarly to Fabrycky et al. 2014) our results change significantly. First, we find good agreement between model predictions and the data, which is clear in Figure 3. Second, this posterior distribution is very dissimilar from the one derived from the full set of transiting planets (that is, including the singles). In order to reproduce Kepler’s yield for systems with two or more KOIs, there must exist $>5$ planets per planet-hosting star, with a mutual inclination scatter of $\sigma = 4.6^{+1.7}_{-3.0}$ degrees. This range is consistent, though less constraining, than the findings of Fabrycky et al. (2014). These parameter values also enclose the solar system, with its $N = 8$ planets and $\sigma = 2.1^{+0.3}_{-0.21}$. The latter value is obtained by fitting a Rayleigh distribution to the mutual inclinations of the planets orbiting the Sun. Of course, the solar system planets span a much broader range of periods and masses than the typical system around an M dwarf. In this sense the M dwarf planetary systems can be viewed as scaled-down versions of the solar system.

We repeat the experiment with a modified and more physically motivated assumption for eccentricity. Limbach & Turner (2014) employ the RV sample of exoplanets to study the eccentricity distribution of planets as a function of system multiplicity. They find that a mean eccentricity of 0.27 is representative for the sample of single-planet systems, but that this mean value decreases to 0.1 for systems with five or six planets (the mean eccentricity of the solar system, as-yet the only known system with eight planets, is 0.06). They provide the probability densities functions for eccentricity for multiplicities ranging from 1 to 8 planets per star, which they fit from the observed cumulative distributions in eccentricity of the known RV-detected exoplanets.

We fold these density functions (M.-A. Limbach 2014, private communication) into our machinery to simulate planetary systems, where the eccentricity of a planet in our simulated sample is drawn from the distribution corresponding to its number of neighboring planets. We assign the longitude of periastron from a uniform distribution from 0° to 360° for each planet. We apply the Hill stability criterion using the same inequality in Equation (3), but conservatively define $d_{in}$ to be the apastron distance of the inner planet, and $d_{out}$ to be the periastron distance of the outer planet. That is, we mandate that the closest possible approach of the two planets still satisfies the stability criterion. Pu & Wu (2015) found that latter criterion roughly approximates the stability outcomes of their N-body analyses. As in our analysis with circular orbits, we reject the synthetic system if the criterion is not satisfied for any one pair of adjacent planets, and draw a new set of periods.

We consider dynamical equipartition between inclinations and eccentricities as follows. If the Rayleigh distribution from which we draw inclination is defined by $\sigma_{\text{inc}}$ (with $\sigma$ expressed in radians), then the Rayleigh distribution from which we draw eccentricity should scale as $\sigma_e = \eta \sigma_{\text{inc}}$. Other investigations, such as Fabrycky et al. (2014), fit for $\eta$ from Kepler observables. From the set of observed transit durations, they concluded that a value of $\eta = 2$ fit the distribution best, but that values from $0 > \eta > 7$ were acceptable fits to the data. For this
reason, we employ a flat prior on $\eta$. We implicitly allow $\eta$ to vary as we hold $\sigma$ constant and increase $N$. For example, to produce $\mu(N = 4, \sigma = 4^\circ)$ we employ the fact that systems with four planets are the likeliest to have an average eccentricity of 0.11 (Limbach & Turner 2014). Therefore, we assume $\eta = 1.6$ for systems of four planets mutually inclined by an average of $4^\circ$. For systems of eight planets, with the likeliest eccentricity of 0.048, an average mutual inclination of $4^\circ$ translates to $\eta = 0.7$.

We then record which planets transit for each iteration to establish a new grid of $\mu(N, \sigma)$, where the eccentricity of planets $e \equiv e(N)$. Our grid is 10 times coarser for this analysis than for our initial test with $e = 0$ in order to decrease computational time. We evaluate $\mu$ over a range in $N$ from 1 to 8 (in increments of 1 planet), and a range in $\sigma$ from $0^\circ$ to $9^\circ$, in increments of one degree. As expected, the sample of transiting planets is slightly biased toward higher eccentricities than the true underlying distribution, because the closer approach to the star at perihelion increases the transit probability. While this bias is clear for systems with one or two stars, for which eccentricities are on average larger than $e = 0.2$, it is negligible for systems with higher multiplicity because the average eccentricity is too low to furnish a much higher likelihood of transit at perihelion. We find that our results are similar whether we allow non-zero orbital eccentricity, or fix eccentricity to zero. In Figure 4, similar to Figures 2 and 3, we show the best-fitting models to the Kepler yield, and contour for the posterior distribution on number of planets and their mutual inclination.

We conclude that our results are robust from zero to modest eccentricities.

### 3.3. Employing a Mixture Model for a Dual Population

We next compare the KOI sample to a two-mode model for planetary architectures. Hansen & Murray (2013), in their comparison of the Kepler multiple planet yield to their population synthesis models, considered a similar question. They invoked an unknown process that uniformly produced singly transiting systems to compensate for their overabundance. In this case, we define a $\mu'(N, \sigma)$ that is the normalized linear combination of two populations. This suite of models is defined by five free parameters: $N_1$, $\sigma_1$, $N_2$, $\sigma_2$, and $f$: the second mode of planetary system occurs a fraction $f$ of the time.

In this mixture scenario, we rewrite Equation (1) as follows, now with five free parameters:

$$L = \prod_n \mu'(N_1, \sigma_1, N_2, \sigma_2, f) \times \ln \frac{\mu(N_1, \sigma_1)}{M_0}$$

where

$$\mu'(N_1, \sigma_1, N_2, \sigma_2, f) = (1 - f) \mu(N_1, \sigma_1) + f \mu(N_2, \sigma_2).$$

We show in Figure 5 the posterior distributions on $N_1$, $\sigma_1$, $N_2$, $\sigma_2$, and $f$. We find that the two likeliest modes of planet occurrence are well-separated in parameter space. The first type of system contains $7.4^{+0.6}_{-1.2}$ planets, drawn from a mutual inclination distribution with $\sigma = 15^{\circ} 9 \pm 1^{\circ} 1$. The second population of systems is well-modeled by a single planet per star. For this second population, $N = 2$ is only allowable with 95% confidence for mutual inclinations $>8^{\circ}$. The ratio of Bayesian evidences between the simple (forcing $N = 1$ for the second population, as in Hansen & Murray 2013) and more complex mixture models (allowing $N_2$ and $\sigma_2$ to vary) is 2, indicating no significant preference. In order to reproduce the observed distribution in multiplicity, this unknown mechanism is operating in a fraction $f = 0.61 \pm 0.08$ of systems.

If we relax the assumption of circular orbits and apply the eccentricity distributions of Limbach & Turner (2014), we find similar results. We depict the posterior contours for the two populations of $N$ and $\sigma$ in Figure 6. For the more multiplicity population, we find that the data favor $N = 7.5^{+0.5}_{-0.3}$ planets per star, mutually included by $2^{\circ} 0 \pm 1^{\circ} 3$. A separate fraction $f = 0.53 \pm 0.11$ of systems host fewer planets, or they reside in more highly mutually inclined orbits. We note that when we include the effects of eccentricity in our simulations, more planets per star characterize the second population of planets. This feature reflects our stability criterion: systems with higher eccentricity must possess larger orbital spacing between planets, and this effect is strongest for systems with $N = 1$, 2, or 3 planets, where average eccentricity is highest. The larger spacing produces the same singly transiting population that we observe, because of diminishing transit probability with distance.

To test the preference of the data for the different models, we calculate the Bayes factor: the ratio of the Bayesian evidence for the single mode case $\mu(N, \sigma)$ to the evidence for the mixture model $\mu'(N_1, N_2, \sigma_1, \sigma_2, f)$ case. The evidence is calculated automatically with the MultiNest machinery (Feroz & Hobson 2008; Feroz et al. 2009, 2013), one of the advantages to employing this package. We list the natural log evidence values in Table 1 for both treatments of eccentricity.
We report a Bayes factor of 9.7 for the scenario with eccentricity forced to zero: that is, the data prefer the mixture model by a factor of 9.7 over the single mode model. With non-zero orbital eccentricities, we find a Bayes factor of 8 with the same interpretation.

4. CONCLUSIONS

We have investigated the Kepler multiplicity yield of planets transiting M dwarfs. By comparing to predicted yields, given M dwarfs with \( N \) planets star\(^{-1} \) and an average mutual inclination \( \sigma \), we make the following conclusions:

1. The multiplicity statistics of the Kepler exoplanet sample orbiting M dwarfs cannot be reproduced by assuming a single planetary system architecture. The best-fitting models, which have large multiplicities (\( N > 5 \)) and large mutual inclinations (7°), do not resemble the typical Kepler multi-planet systems, which are mutually inclined by 1°–2° (Fabrycky et al. 2014).

2. However, the multi-planet yield (that is, the sample of KOIs hosting 2 or more transiting planets) is well-modeled by a well-populated and approximately coplanar architecture. The best-fitting planetary model has 7 planets, and typical mutual inclination of 2°. Because M dwarf stars are the most populous in the Galaxy, we consider this architecture a “typical” galactic multi-planet system (c.f. also Swift et al. 2013).

3. These conclusions are robust to assumptions of eccentricity. We report approximately the same findings whether we force \( e = 0 \) for all planets, or whether we draw eccentricities from the distributions of Limbach & Turner (2014).

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4. If we invoke an additional population of planets, this mixture model is preferred by 8:1 to that with only one type of planetary architecture. The secondary population with fewer planets at higher mutual inclination are $0.53 \pm 0.11$ of the total number of systems; half of systems are in well-aligned multis, and half are either singletons or in systems with large mutual inclinations.

The fraction $f$ near 50% is consistent with the estimate of Hansen & Murray (2013). They found that if the mechanism generated excess singly transiting systems operates half the time, with the scenario described in their simulations operating the other half the time, they recover the ratio of doubly transiting-to-singly transiting systems of 0.2. We note that our findings are also consistent with Morton & Swift (2014), who considered the ensemble of planets orbiting the Kepler M dwarfs. They assumed a single model for M dwarf planetary systems and concluded that each system has $2.0 \pm 0.45$ planets star$^{-1}$. This result is in moderate agreement with our value for the average number of planets per host star, which is 3.0 (taking the average after combining both the $N_1$ and $N_2$ posteriors).

We consider two explanations for the bimodal nature of planetary systems orbiting M dwarfs. First, the dichotomy may originate primarily from formation conditions. Alternatively, it may reflect a dynamical disruption history over longer timescales. In this latter hypothesis, flat and multiplicity systems of planets are subsequently dynamically disrupted, leaving planets scattered to higher mutual inclinations that we observed as “singles.” Both Volk & Gladman (2015) and Pu & Wu (2015) examined dynamical simulations of tightly packed systems of planets, for which our best-fit model system of $>5$ planets with periods $<200$ days qualifies. They concluded that these systems are only metastable, with disruption timescales of Gyr. In this scenario, we interpret the $53 \pm 10\%$ fraction of low-multiplicity/high inclination systems to be dynamically hotter remnants of primordial flat systems. If the disruption timescale is indeed Gyr, we might expect the observed 50/50 fraction given the age of the Milky Way; in another few Gyr, we would expect to see a much smaller fraction of multiply transiting systems. The fact that systems hosting only one transiting planet tentatively tend to be more misaligned with the spin axes of their host stars (Morton & Winn 2014) also hints at a history of scattering encoded in the singly transiting systems.

However, other authors, such as Johansen et al. (2012) and Becker & Adams (2015), concluded that self-excitation is insufficient to scatter planets from a transiting geometry. Johansen et al. (2012) also find that the planetary instability hypothesis is inconsistent with the resulting radius distributions of planets: there ought to be more small planets in the singly transiting systems, and more large planets in the multiply transiting systems, for dynamical instability to be responsible. They go on to posit that the dichotomy must instead have arisen during the formation process itself, due to the effects of massive planets on the protoplanetary disk. In this case, higher metallicity, which is responsible for increased giant planet occurrence (e.g., Fischer & Valenti 2005; Johnson & Appps 2009), and more eccentric planetary orbits (Dawson & Murray-Clay 2013) would be predictive of the final planetary architecture.

We also note that the Kepler dichotomy may result from a single mode of planet occurrence with mixed architecture. In this scenario, the innermost planets reside in a compact and coplanar configuration (aligned with the spin axis of the host star), while one or two outer planets are much more highly mutually inclined. If this were the case, we would be very unlikely to see the inner planets transit if the outer planet transited, and vice versa. The singly transiting systems would appear to be more misaligned with the stellar rotation axis. A careful examination into the period and transit duration distribution of the two populations would effectively test this hypothesis.

Stellar binarity may contribute to the dichotomy in some measure. The disruption of the circumstellar disk by a nearby companion may result in the departure from the compact and coplanar distribution observed in half the sample. Alternatively, such systems may form fewer planets. If binarity were responsible in full for the excess population of singly transiting planets, then we might simplistically require 50% of M dwarfs to reside in binaries. This is a reasonable estimate for solar-type stars, for which 50% exist in binary systems (Duquennoy & Mayor 1991). Fischer & Marcy (1992) measured a multiplicity rate of 42 \pm 9% among M dwarfs over a wide range of orbital separations, while Leinert et al. (1997) and Reid & Gizis (1997) found that the binary frequency for M dwarfs is only 30%. Recent work by Janson et al. (2012) concluded that only 27 \pm 3% of M dwarfs are part of binary systems with separations between 3 and 227 AU. Wang et al. (2014) found that binarity can decrease planetary occurrence by a factor of several, with modest confidence. In that work, the impact on planet formation decreases with increasing orbital separation of the stars (with 10 AU being the closest orbital distance in their investigation). It is therefore not plausible to invoke binarity as the sole explanation for excess singly transiting systems. This is true because binarity rates are $<50\%$ for M dwarfs as a whole, and per Wang et al. (2014), only the closest of these binaries are responsible for disruption in planet occurrence.

We require more exoplanetary systems to robustly test which properties inform the outcomes of M dwarf planetary systems. In the next upcoming years, such a sample will be forthcoming.

### Table 1

| Data Set   | Model  | $N$ (planets) $\pm$ | $\sigma$ (degrees) $\pm$ | $f$          | ln(Evidence) |
|------------|--------|---------------------|--------------------------|-------------|--------------|
| Only multis| One mode, $e = 0$ | $6.4_{-1.6}^{+1.6}$ | $4.0_{-2.0}^{+2.0}$ | \ldots | \ldots |
| All data   | One mode, $e = 0$ | $6.0_{-1.4}^{+1.4}$ | $7.4_{-3.3}^{+3.3}$ | \ldots | $-27.0$ |
| All data   | One mode, $e = 0$ | $7.5_{-0.4}^{+0.4}$ | $8.0_{-0.4}^{+0.4}$ | \ldots | $-23.0$ |
| All data   | Two modes, $e = 0$ | $7.4_{-0.6}^{+0.6}$ | $1.9 \pm 1.1$ | $0.61 \pm 0.08$ | $-17.3$ |
| All data   | Two modes, $e = 0$ | $7.5_{-0.5}^{+0.5}$ | $2.0 \pm 1.3$ | $0.53 \pm 0.11$ | $-15.0$ |
from NASA’s Two-Wheeled extended Kepler Mission, known as K2 (Howell et al. 2014). Accepted GO programs\(^5\) include more than 15,000 M dwarfs in Campaigns 1–5, with each Campaign field receiving 75 days of continuous monitoring. The sample of M dwarf planets is already growing, with one multiply transiting system already uncovered (Crossfield et al. 2015). The addition of a fresh sample of M dwarf planets will allow firmer conclusions about which stellar properties sculpt their planetary architectures.

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