Photon generation from vacuum in non-stationary circuit QED

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We study theoretically the non-stationary circuit QED system in which the artificial atom transition frequency has a small periodic modulation in time, prescribed externally. We show that, in the dispersive regime, when the modulation periodicity meets the ‘resonances’, the dynamics may be described by the dynamical Casimir effect, Jaynes-Cummings or Anti-Jaynes-Cummings effective Hamiltonians. In the resonant atom-cavity regime, under the modulation ‘resonance’ the dynamics resembles the behavior of the dynamical Casimir effect in a vibrating cavity containing a resonant two-level atom, and entangled states with two photons can be created from vacuum. Thus, an analog of the dynamical Casimir effect may be simulated in circuit QED, and several photons, as well as entangled states, can be generated from vacuum due to the anti-rotating term in the Rabi Hamiltonian.

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I. INTRODUCTION

Over the last few decades nonstationary processes in cavities have received considerable attention. One such process is the nonstationary or dynamical Casimir Effect (DCE) – in particular, the creation of photons from vacuum, or another initial field state, in cavity whose geometry or material properties have a periodic time dependence, with modulation frequency equal to twice the unperturbed field eigenfrequency. Nowadays, DCE in cavities is a well studied problem, with a variety of theoretical predictions concerning the number and the statistics of created photons, as well as the influence of detuning, dissipation, boundary conditions, geometry and non-periodicity of the modulation (see [10, 11, 12] for an extensive list of references). To date, DCE has not been observed in laboratory, however several concrete proposals have appeared over the last years [13, 14, 15, 16, 17, 18, 19], with some of them being currently implemented experimentally [20].

The interest in nonstationary processes in cavities reappeared over the last 5 years due to the progress in the field of Cavity Quantum Electro-dynamics (cavity QED) in the condensed matter systems, e.g. semiconductive quantum dots [22, 23], polar molecules [24] and superconducting circuits [25, 26, 27] coupled to resonators, the latest architecture known as circuit QED [26, 28]. In cavity QED, the effective two-level atom is coupled to the field inside the resonator via the dipole interaction, allowing for observation of the light-matter interaction at the level of single photons and single atoms. A new ingredient in the solid state cavity QED is the possibility of engineering and manipulating the properties of the artificial atom and the resonator [26, 29, 30], as well as the interaction strength between them, either during the fabrication or in situ.

Recently, the strong resonant and the strong dispersive coupling limits between the artificial atom and a single cavity mode were observed experimentally in circuit QED [31] and other solid state architectures [22, 23]. Moreover, the single photon source [32], single artificial-atom maser [33], multiphoton Fock states [34] and interaction between two artificial atoms (qubits) [30, 32] were implemented experimentally, among many other important achievements. Besides, the circuit QED architecture benefits from robust read-out schemes of the atomic internal state and the resonator field state [31, 36, 37, 38, 39], relatively low dissipative losses [31], state preparation techniques [40] and real time manipulation of the atomic transition frequency via electric and magnetic fields [26, 30, 34] or non-resonant microwave fields [35, 39].

Harnessing the tunability of the atomic transition frequency, the realization of the Landau-Zener sweeps, with the atomic frequency increasing linearly in time, was proposed in circuit QED [41], allowing for generation of single photons and entangled states. In a similar direction, in [26, 28, 34] the implementation of DCE in superconductor cavity QED was considered, using the periodic time-dependence of the atom-cavity coupling parameter (the vacuum Rabi frequency). It was shown that there is a substantial photon production from vacuum when the vacuum Rabi frequency is modulated in time with certain ‘resonant’ frequencies [12]. A preliminary theoretical study of the feasibility of realizing the DCE with a quantum flux qubit in superconducting quantum nanocircuits, as well as the detection of the generated photons, was reported in [43]. On the other hand, the possibility of controlling the atomic frequency and detecting the atomic internal state is currently being used to couple/decouple one or several qubits to/from the cavity mode in order to implement quantum logic operations [30, 35, 39].

Here we study the nonstationary cavity QED archi-
tecture, in which a single cavity mode is coupled to a single artificial atom whose transition frequency has a small periodic modulation in time prescribed externally. Such a control over the transition frequency, with compatible modulation periodicity, may be achieved in circuit QED with present or near-future technology [30, 31, 39]. We show that, in the dispersive regime, under the ‘resonance’ conditions one obtains completely different effective regimes for the system dynamics, which may be approximately described by the Anti-Jaynes-Cummings (AJC), Jaynes-Cummings (JC) or the dynamical Casimir effect (DCE) Hamiltonians with adjustable parameters. Moreover, in the resonant atom-cavity regime, the dynamics resembles the behavior of the DCE in a cavity with oscillating boundaries containing a resonant two-level atom (detector) [41], and entangled states with up to two photons can be generated from vacuum.

Thus, we demonstrate the possibility of simulating the DCE in circuit QED using a single non-stationary atom, instead of a macroscopic dielectric medium as in [42]. As applications, it may be possible to create excitations, either photonic or atomic, from the initial vacuum state $|g, 0\rangle$, generate nonclassical states of light and realize transitions between the states $\{|g, m\rangle, |e, m \pm 1\rangle\}$ in the dispersive regime. Here $|g\rangle$ and $|e\rangle$ stand for the atomic ground and excited states, respectively, and $|m\rangle$ for the Fock state of the cavity field. A related problem was recently studied in [43], where it was suggested that laser behavior and the creation of a highly non-thermal Fock state of the cavity field. A related problem was recently studied in [43], where it was suggested that laser behavior and the creation of a highly non-thermal Fock state of the cavity field.

II. NONSTATIONARY CIRCUIT QED

We assume that the atomic transition frequency $\Omega(t)$ may be described as the sum of two terms. The first term $\Omega_0$ describes the bare atomic frequency and the second term represents a small modulation amplitude $\varepsilon \ll \Omega_0$ multiplied by a periodic function of time $f_t$ prescribed externally

$$\Omega(t) = \Omega_0 + \varepsilon f_t, \quad f_t = \sum_{k=0}^{\infty} (s_k \sin k\eta t + c_k \cos k\eta t).$$  \hspace{1cm} (1)

Here $\eta$ is the modulation frequency and $\{s_k, c_k\}$ form a set of coefficients describing an arbitrary periodic time-dependence of $f_t$. We suppose that the cavity frequency $\omega$ and the atom-cavity coupling parameter $g_0$ are constant, so at the ‘sweet spot’ in solid state cavity QED the system is described by the Rabi Hamiltonian (RH) [30]

$$H = H_0(t) + g_0 \left( a + a^\dagger \right) \left( \sigma_+ + \sigma_- \right),$$  \hspace{1cm} (2)

where $a$ ($a^\dagger$) is the cavity annihilation (creation) operator, $\sigma_+ = |e\rangle\langle g| \text{ and } \sigma_- = |g\rangle\langle e|$. The free Hamiltonian is

$$H_0(t) = \omega n + \frac{\Omega(t)}{2} \sigma_z,$$

where $n = a^\dagger a$, $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ and we assume $h = 1$. In the stationary case, $\varepsilon = 0$, one may perform the Rotating Wave Approximation (RWA) and obtain the standard Jaynes-Cummings (JC) Hamiltonian [46], which has been verified in several experiments over the last few years [26, 31, 47, 48, 49]. However, in the nonstationary case, as well as under strong dephasing noise [50], the anti-rotating term $\{a\sigma_- + a^\dagger \sigma_+ \}$ cannot always be eliminated. Moreover, it is responsible for producing an analog of the DCE and creating photonic and atomic excitations from vacuum under modulation ‘resonance’ conditions, as shown below.

In the interaction picture with respect to $H_0(t)$ the interaction Hamiltonian reads

$$H_I = g_0 \left( e^{i\varepsilon t} a \sigma_+ + e^{i\varepsilon t} a^\dagger \sigma_+ + h.c. \right),$$  \hspace{1cm} (3)

where h.c. stands for the Hermitian conjugate and $\Xi_{\pm} \equiv \int_0^{\infty} d\tau \left[ \Omega(\tau) \pm \omega \right]$. All the information about the dynamics of the system is contained in the time-dependent coefficients $\exp(i\Xi_{\pm})$, which may be significantly simplified by adjusting the modulation frequency $\eta$ in order to achieve the ‘resonances’. We have explicitly

$$g_0 e^{i\Xi_+} = g e^{i\Delta_{\pm} t} \sum_{l=0}^{\infty} \frac{1}{l!} \left[ \varepsilon \sum_{k=1}^{\infty} (\Lambda_k e^{-ik\eta t} - \Lambda_k^* e^{ik\eta t}) \right]^l,$$  \hspace{1cm} (4)

where we defined a complex coupling constant $g \equiv g_0 \exp \left[ i (\varepsilon/\eta) \sum_{k=1}^{\infty} k^{-1}s_k \right]$ and parameters

$$\Lambda_k \equiv - \frac{c_k + is_k}{2k}, \quad \Delta_{\pm} \equiv \Omega_0 + \varepsilon c_0 \pm \omega.$$

III. AJC AND JC RESONANCES

The ‘Anti-Jaynes-Cummings’ (AJC) resonance occurs for

$$\eta = \eta_{AJC} \equiv \Delta_+ - \xi,$$

where $|\xi| \ll \eta$ is a small ‘resonance shift’. Assuming a reasonable experimental condition $\varepsilon/\eta \ll 1$ we expand $\exp(\Xi)$ to the first order in $\varepsilon/\eta$ and make the RWA in (3), obtaining

$$H_I \simeq g \left( \theta e^{i\xi t} a^\dagger \sigma_+ + e^{i\Delta_- t} a \sigma_+ \right) + h.c.,$$  \hspace{1cm} (5)

where the AJC dimensionless coupling is

$$\theta \equiv \Lambda_1 \varepsilon/\eta.$$  \hspace{1cm} (6)

In the resonant regime, $|\Delta_-|/g_0 \ll 1$, we apply the method of slowly varying amplitudes to the Hamiltonian [44], repeating the procedure employed originally for
studying the photon generation from vacuum due to the DCE in a vibrating cavity containing a stationary two-level atom. We find that for the initial state \( |g, 0\rangle \) the photon generation occurs for two values of the resonance shift

\[
\xi = \xi_{\pm} = \Delta_{-}/2 \pm \sqrt{2}g_{0}
\]

and one gets the following non-zero probabilities \( P_{e,m} \), with \( x = \{e, g\} \) denoting the atomic state and \( m \) the photon number

\[
P_{g,0} \approx \cos^2 (\chi t), \quad P_{e,1} \approx \sin^2 (y + q) \sin^2 (\chi t), \quad P_{g,2} \approx \cos^2 (y - q) \sin^2 (\chi t).
\]

(7)

Here

\[
\chi \approx g_{0} |\theta| \sin (y + q), \quad \tan y \approx \left[ 2\sqrt{2}g_{0} + \Delta_{-} / 2\sqrt{2}g_{0} - \Delta_{-} \right]^{1/2}
\]

and \( q = 0 \) \((\pi/2)\) for \( \xi_{-} \) \((\xi_{+})\). Thus, in the resonant regime, when the atomic transition frequency is modulated with the frequency \( \Delta_{+} - \xi_{\pm} \), a superposition of states \( |g, 0\rangle, |e, 1\rangle \) and \( |g, 2\rangle \) is created from the initial vacuum state \( |g, 0\rangle \), and the probability of exciting the atom is limited by \( \sin^2 y \approx 1/2 \).

We illustrate this behavior in Fig. 1a, where we show the exact dynamics of \( P_{g,0}, P_{e,1} \) and \( P_{g,2} \) vs. time \( (t) \) for the AJC resonance, \( \theta = \sin \eta_{AJC} t \) with \( \xi = \xi_{-} \), using the parameters \( g_{0}/\omega = 4 \cdot 10^{-2}, \Delta_{-} = g_{0}/10 \) and \( \varepsilon/\omega = 10^{-1} \). This dynamics resembles the one occurring in the context of DCE, where a resonant (stationary) two-level atom or detector is fixed inside a cavity whose boundary is oscillating with the frequency close to \( 2\omega \). In both cases not more than two photons can be created from the vacuum state \( |g, 0\rangle \) and the probability of exciting the atom is limited by the value \( 1/2 \). This similarity is not surprising, since in this case the modulation frequency \( \Delta_{+} \approx 2\omega \), and the atom plays the role of the two-level detector and the cavity modulating mechanism (via the atom-cavity coupling) at the same time.

In the dispersive regime, \( g_{0}\sqrt{\langle n \rangle}/|\Delta_{-}| \ll 1 \), where \( \langle n \rangle \) is the mean photon number, the Hamiltonian may be approximated by

\[
H_{J}^{(1)} \approx \left(g\theta e^{i\xi t} a^{\dagger} \sigma_{+} + h.c.\right) + \delta (n + 1/2) \sigma_{z},
\]

where

\[
\delta = g_{0}^{2}/\Delta_{-}
\]

is the dispersive shift. In a rotating frame we obtain the effective AJC Hamiltonian

\[
H_{AJC} \approx \left[\xi + \delta (1 + 2n)\right] \sigma_{z} / 2 + \left(g\theta a^{\dagger} \sigma_{+} + h.c.\right).
\]

(8)

By adjusting the resonance shift \( \xi \) in order to make \( |\xi + \delta (1 + 2n)| \) small compared to \( |g\theta| \), one obtains the resonant AJC Hamiltonian. From the physical point of view, the external modulation supplies the energy \( \omega + \Omega_{0} \) necessary to create one photon and one atomic excitation simultaneously. Thus, one can create the superposition of states \( |e, 1\rangle \) and \( |g, 0\rangle \) starting from the initial state \( |g, 0\rangle \), as illustrated in Fig. 1b, where we show the exact dynamics of \( \langle n \rangle \) and \( P_{g} \) for the AJC resonance, \( \theta = \sin \eta_{AJC} t \) with \( \xi = -2\delta \), using the experimental circuit QED parameters \( \Omega_{0}/\omega = 1.4, g_{0}/\omega = 2 \cdot 10^{-2} \) and assuming \( \varepsilon/\omega = 0.2 \).

Also in the dispersive regime, the ‘Jaynes-Cummings’ (JC) resonance occurs for

\[
\eta = \eta_{JC} \equiv |\Delta_{-}| - \xi.
\]

For positive \( \Delta_{-} \) we get the effective JC Hamiltonian

\[
H_{JC} \approx |\xi + \delta (1 + 2n)| \sigma_{z} / 2 + \left(g\theta a^{\dagger} \sigma_{+} + h.c.\right).
\]

(9)

If \( \Delta_{-} \) is negative, we obtain the same effective Hamiltonian upon replacements \( \theta \to -\theta^{*} \) and \( \xi \to -\xi \) in Eq. (9). Thus, by employing the JC resonance and adjusting the resonance shift \( \xi \), one may couple the subspaces \( \{|g, m\}, |e, m - 1\rangle \) when the atom and the field are far off-resonant, since the external modulation supplies the energy difference \( |\omega - \Omega_{0}| \) necessary to couple the atom and the cavity field. This behavior is illustrated in Fig. 1c, where we show the exact dynamics of \( \langle n \rangle \) and \( P_{e} \) for the JC resonance, \( \theta = \sin \eta_{JC} t \) with \( \xi = -2\delta \), for the initial state \( |g, 1\rangle \) and the parameters of Fig. 1b. Moreover, one could engineer entangled states with several photons from the initial vacuum state \( |g, 0\rangle \) by alternating between the AJC and JC resonances and controlling the time interval and the resonance shift of each resonance.
IV. DCE RESONANCE

In the dispersive regime, the ‘dynamical Casimir effect’ (DCE) resonance occurs for

\[ \eta = \eta_{\text{DCE}} = 2\omega - 2\xi. \]

Performing the RWA in the interaction Hamiltonian \( H^{(1)} \), in a rotating frame we obtain the time-independent Hamiltonian consisting of the JC Hamiltonian plus the AJC term multiplied by the adjustable coupling \( g\theta \)

\[ H^{(1)}_I \simeq \xi a + \frac{\Delta - \xi}{2} \sigma_z + (g\sigma_+ + g\theta a^\dagger \sigma_+ + h.c.). \quad (10) \]

We may obtain an effective Hamiltonian by applying a sequence of small unitary transformations \([46]\) on \((10)\) and performing the Hausdorff expansion after each step. Assuming that \( \theta \sim O(|g/\Delta_-|) \) we apply the ‘rotating’ unitary transformation

\[ U_r = \exp \left[ (g\sigma_+ - h.c.)/|\Delta_-| \right] \]

followed by the ‘anti-rotating’ one

\[ U_a = \exp \left[ (g\theta a^\dagger \sigma_+ - h.c.)/ (\Delta_- + 2\xi) \right] \]

to obtain the effective Hamiltonian \( H_{\text{eff}} = U_a U_r H^{(1)}_I U^\dagger_a U^\dagger_r \), which in a rotating frame reads

\[ H_{\text{eff}} \simeq \left( \xi + \delta \sigma_z \right) n + \delta \sigma_z (\theta^* a^2 + h.c.) \]
\[ - \frac{2\delta}{\Delta_-} (ge^{i\Delta_- t} an\sigma_+ + h.c.) + O(|g/\Delta_-|^3). \quad (11) \]

The first two terms of the effective Hamiltonian \((11)\) form the DCE part and the remaining terms represent the corrections, whose leading term (oscillating with high frequency \( \sim |\Delta_-| \)) describes the nonresonant photon absorption by the atom. These corrections become relevant when the third term becomes large, so for initial times (roughly for \( g\theta \sqrt{\langle n \rangle}/|\Delta_-| \ll 1 \)) their contribution is relatively small and \( \sigma_z \) becomes approximately a constant. If the atom is initially in the ground state, the Eq. \((11)\) becomes the DCE Hamiltonian

\[ H_{\text{DCE}} \simeq (\xi - \delta) n - \delta (\theta^* a^2 + h.c.). \quad (12) \]

For the atom initially in the excited state, a similar effective Hamiltonian is obtained under substitution \( \delta \to -\delta \). Therefore, by adjusting the frequency shift to \( \xi = \pm \delta \), depending on the initial atomic state \([53]\), we have photon pairs creation from vacuum and field amplification due to an analog of the DCE.

Here the DCE is simulated by the atomic transition frequency modulation through the atom-cavity coupling. However, the photon generation process is not steady because after several photons have been created the third and further terms in \((11)\) become important, and the photon generation is interrupted. Nevertheless, the Hamiltonian \((11)\) shows that it is possible to simulate DCE and generate several photons from vacuum using a single artificial atom. This is illustrated in Fig. 2, where we show the exact dynamics of \( \langle n \rangle \) and \( P_\varepsilon \) vs. time for the DCE resonance, \( f_t = \sin \eta_{\text{DCE}} t \) with \( \xi = \delta \), for the initial state \( |g, 0\rangle \) and the parameters \( \Omega_0/\omega = 1.4, g_0/\omega = 2 \cdot 10^{-2}, \varepsilon/\omega = 0.4 \). We also show the curve \( \langle n \rangle_{\text{DCE}} = \sin^2 (2|\delta/\theta| t) \), which gives the expected mean photon number for the DCE Hamiltonian \([12]\), demonstrating that for initial times the exact dynamics can be described by the DCE Hamiltonian. However, after the creation of a few photons the atom acquires a finite probability of being excited, and the dynamics starts to deviate from the DCE Hamiltonian. The photon generation is interrupted and restarts again as time goes on, respecting the limit \( g\theta \sqrt{\langle n \rangle}/|\Delta_-| \ll 1 \), and the behavior of \( P_\varepsilon \) resembles the one of \( \langle n \rangle \).

This phenomenon may be qualitatively understood as follows. In the dispersive regime the atom acts as an effective non-linear capacitance \([36]\), pulling the cavity frequency to

\[ \dot{\omega} (t) \approx \omega + \frac{\sigma_z g_0^2}{\Delta_- + \varepsilon f_t} \approx (\omega + \sigma_z \delta) - \sigma_z \varepsilon \frac{\delta}{\Delta_-} f_t. \]

Consequently, one expects that the periodic modulation of \( f_t \) with the modulation frequency close to \( \eta \approx 2 (\omega \pm \delta) \) would lead to DCE \([11, 44]\), for which the photons are generated as long as the modulation is present. The energy \( 2\omega \) necessary to create pairs of photons is provided through the atomic frequency modulation and the resonance shift \( \xi \) must be adjusted \([52]\) in order to get a constructive interference on the cavity field \([11, 12]\). However, as time goes on the atom gets entangled with the field, acquiring a finite probability of being excited...
through photon absorption, and the photon generation cannot continue steadily due to the loss of constructive interference. This is different from the usual DCE situation, where the properties of the macroscopic linear, lossless and nondispersive dielectric medium inside the cavity are modulated \[1\ \pm \|\] , and the field does not get entangled with individual atoms.

V. DISCUSSION AND CONCLUSIONS

In the previous sections we have considered only the first order resonances. In general, the \( K \)-th order resonances occur for an integer \( K \) when

\[ \eta = \eta_i^{(K)} \equiv K^{-1} \eta_i, \]

where \( \eta_i \) stands for the AJC, JC and DCE resonances. In this case one recovers the previous result upon substitutions \( \delta \rightarrow \delta_K \approx \delta + \epsilon^2/\Lambda_{+} \) and \( \theta \rightarrow \theta_K \), as follows from the expressions \[3\] and \[4\]. Now the effective dispersive shift \( \delta_K \) contains the contribution of many terms, among them the Bloch-Zieger \[14\] shift \( \epsilon^2/\Lambda_{+} \) and powers of \( (\epsilon/\eta) \). \( \theta_K \) contains contributions due to the nonharmonic shape of the pulse, \( \Lambda_{k>1} \), as well as due to the powers of \( (\epsilon/\eta) \)

\[ \theta_K = \Lambda_{k} \epsilon/\eta + \ldots + (A_{1} \epsilon/\eta)^{K}/K! + \ldots . \]

However, to employ the higher order resonances the effective dispersive shift \( \delta_K \) should be carefully evaluated, otherwise there is a risk of missing the exact modulation resonance, since \( \theta_K \) becomes smaller and there is less freedom in committing small errors in the resonance shift \( \xi \).

Our results may be easily translated to the situation where \( \Omega(t) = \Omega_0 \) is constant and \( g_0(t) = g_0 + \epsilon f_t \) has a periodic time-dependence. In this case the interaction Hamiltonian is

\[ H_I = g_0 (t) \left( e^{i(\Omega_0 - \omega)t} a \sigma_+ + e^{i(\Omega_0 + \omega)t} a^\dagger \sigma_+ + h.c. \right) . \] (13)

If one expands \( e^{i\Xi_\pm} \) in Eq. (4) to the first order in \( \epsilon/\eta \), the Hamiltonian \[3\] becomes equivalent to the Hamiltonian \[13\], so the results obtained above for \( \Omega(t) \) also hold for \( g_0(t) \) after making appropriate substitutions. The main difference is that in the \( \Omega(t) \) case the higher order resonances occur due to the powers of \( \epsilon/\eta \) and non-zero coefficients \( \Lambda_K \), while in the \( g_0(t) \) case they are due to non-zero coefficients \( \Lambda_K \) only. Finally, if the cavity frequency is modulated periodically, with constant \( \Omega \) and \( g_0 \), the AJC and JC resonances also occur, besides the well known DCE resonance \[44\].

The experimental verification of this scheme seems possible in circuit QED architecture with superconducting qubits and coplanar waveguide resonators \[36\], where one may adjust the system parameters \emph{in situ} via electric and magnetic fields, as demonstrated in \[26\ \mp \| \ 37\ 38\]. Moreover, several schemes to read out the cavity and the atomic states are currently available \[31\ \pm \| \ 39\] and under investigation \[39\]. The main issue would be to modulate periodically the atomic transition frequency with a stable modulation frequency \( \eta \sim 10 \) GHz, what is within experimental reach \[30\]. One could also use this scheme to couple \( M \) identical qubits (e.g. superconducting 2-level atoms \[38\ \pm \| \ 39\] or a cloud of polar molecules \[24\]) to the same cavity mode and modulate the frequency of \( M \) atoms simultaneously, since in this case the effective coupling increases to \( \sqrt{M} g_0 \).

One important point we did not analyze here is the dissipation and decoherence of both the artificial atom and the cavity due to the noisy solid state environment \[39\ \pm \| \ 51\]. Recent experiments achieved experimental values \{ \kappa/\omega < 10^{-4}, \gamma/\omega < 10^{-3}, \gamma_{ph}/\omega < 10^{-3} \} \[31\], where \( \kappa \) is the cavity decay rate, \( \gamma \) is the atomic decay rate and \( \gamma_{ph} \) is the atomic pure dephasing rate. To deal with dissipation in a qualitative manner, we compare the rates of the photon production from vacuum for each resonance to the dissipation rates. We take the current experimental value of the coupling constant \( g_0/\omega \approx 2 \cdot 10^{-3} \) \[31\] and assume \( \epsilon/\omega \sim \Delta /\omega \sim 10^{-3} \) to make the estimative. The photon creation rate for the first order DCE resonance is roughly \( |\delta \theta|/\omega \sim 10^{-4} \), and for the first order AJC resonance \( |\theta \theta|/\omega \sim 10^{-3} \). Both these values are larger or of the order of magnitude of the dissipation/decoherence rates, so the photon production due to modulation of \( \Omega(t) \) seems possible in the future.

In conclusion, we analyzed the nonstationary circuit QED system in which the atomic transition frequency has a small periodic modulation in time, prescribed externally. In the dispersive regime, under the modulation ‘resonances’ the dynamics can be effectively described by the Anti-Jaynes-Cummings, Jaynes-Cummings or the dynamical Casimir effect Hamiltonians. Moreover, in the resonant atom-cavity regime, under the corresponding ‘resonance’, an entangled state with two photons can be created from the vacuum state \( |g, 0 \rangle \), analogously to the dynamical Casimir effect in a vibrating cavity containing a two-level atom. This study illustrates the importance of the anti-rotating term in the Rabi Hamiltonian, neglected in the Jaynes-Cummings model – here this term is responsible for photon generation from vacuum and field amplification. As applications, this scheme can be used to verify photon creation from vacuum in nonstationary circuit QED due to an analog of the dynamical Casimir effect using a single atom, as well as off-resonant transitions between the states \{ \( |g, m \rangle, |e, m \pm 1 \rangle \} \} and generation of entangled states with several photons.

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If initially the atom is in the superpositions of states $|g\rangle$ and $|e\rangle$, we may choose any of the signs to obtain photon generation. However, the photon generation is optimized if initially the atom is exactly in $|g\rangle$ or $|e\rangle$. 

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