PRICING AND MODULARITY DECISIONS UNDER COMPETITION

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(Communicated by Stefan Wolfgang Pickl)

ABSTRACT. This paper considers price and modularity of competition between two firms with deterministic demand, in which demand is dependent on both the prices and the modularity levels determined by two firms. Bertrand competition and Stackelberg competition are formulated to derive the equilibrium solutions analytically. Because of the complexity, an intensive numerical study is conducted to investigate the impact of the sensitive parameters on equilibrium prices and modularity levels, as well as optimal profits of the two firms. An important and interesting finding is that optimal profits of the two firms under both types of competition are decreasing with the modularity cost when the price and modularity sensitivities are low, where both firms are worse-off due to decrease of the modularity levels; but they are increasing when the price and modularity sensitivities are high, where both firms are better-off at the expense of modular design. Our research reveals that Stackelberg game improves the modularity levels in most of the cases, though both firms perform better in Bertrand competition in these cases when jointly deciding the prices and modularity levels in the two firms.

1. Introduction. One of the most important development trends in marketing is that customers are paying more and more attention to the variety of the products in the last two decades. Manufacturers are encouraged to provide customized products to meet the demand by means of various production approaches, such as flexibility, built-to-order system, etc. Among these tools, one of the effective ways is to modularize the product [15], [30]. Modular design means the degree to which a product can be segregated and disassembled into a series of standardized components which can be recombined into different products [5]. It is an effective tool to respond to the diversity of customer demand [16].

2010 Mathematics Subject Classification. Primary: 90B50, 90B30; Secondary: 49K20.

Key words and phrases. Pricing, manufacturing, game theory, optimization.

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Modularization is considered as an important way to improve the product quality in both academic and practice areas [28], [1], [29] and thus, it helps the firm gaining competitive advantage in the market [24], [3]. To enhance the capability, modularity design is integrated with many production theories and applications, such as Engineer-To-Order (ETO) industries at the beginning of product design [17], the customer quality [29], the performance of manufacturing and product [7], the developing cycle of new product [26], production flexibility coping with supply interruption [9], and a return policy in a two echelon supply chain [5]. Apparently, modularity has become a key competing factor.

Another important competitive factor in the market is the pricing which has been recognized as a widely occurring issue in practice and a hot research topic in the past several decades [25], [2], [23], to name a few. In terms of price competition, Bertrand competition is the well-known competition type where competitors determine the optimal prices at the same time [19], [4], [11]. However, a leader-follower price competition is more practical in the market, in which the leader determines the price to maximize its profit and after that the follower chooses the response price to optimize its own profit [6], [20].

The joint competition in pricing and quality has been investigated in detail [2], [14], [22], [27], [13], however, as one of the key elements that represents the quality, the interaction between pricing and modularity, especially considering the different power structures in decision sequence has not been revealed in literature. Therefore, in this paper, the pricing and modularity decisions are optimized and compared by considering two types of competing mechanisms including Stackelberg and Nash games.

Many researchers have plunged themselves into modularity decision. Most of them are empirical or case studies [8], [18]. Salvador et al. [21] characterized the appropriate modularity that could be embedded into a product family by examining six product families that belong to six European companies. Agard and Bassetto [1] proposed a single-level modular design to optimize the quality and cost by an algorithm based on a selected case. Gualandris and Kalchschmidt [9] proposed a framework with postponement enablers to reduce the negative effect of a supply failure risk caused by process modularity, which directly improves production flexibility. Most recently, by empirically analyzing the European manufacturing production networks, Lampon et al. [12] pointed out that modular platforms increase mobility and thrift ability of manufacturing systems, which leads to improved coordination of the system. On the other hand, some researchers have focused on quantitative analysis with modular design by formulating mathematical models to investigate the impact of modularity on price and order quantities in a return policy [5], [10]. Mukhopadhyay and Setoputro [15] developed a newsvendor model with deterministic demand to jointly determine the optimal return policy and modularity levels. Further, Mukhopadhyay and Setaputra [16] extended the work by assuming a price-dependent stochastic demand to characterize the return policy.

The most related works to our paper are done by Roy et al. [20], Mukhopadhyay and Setoputro [15] and Chang and Yeh [5], hereafter RHR, MS, and CY respectively. In RHR, they propose an estimation procedure to examine the pricing rule in the leader-follower market practice using sales data in the automobile market. In MS, a return policy is considered as the key factor to investigate the optimal return policy and the corresponding modularity level for a build-to-order production system within a single firm. In CY, the centralized and decentralized two-echelon
supply chains are developed separately to determine the optimal modularity levels as well as the production quantities. Our model is somewhat similar to these papers, however, we distinguish our work in the following aspects: 1) pricing and modularity competition are jointly considered in our model, however, in RHR, only pricing competition is estimated; 2) two firms are competing on both the price and modularity by considering sequentially and mutually decision mechanisms in our paper, but a single firm is studied in MS and an independent supply chain is modeled in CY to derive the optimal modularity levels; 3) more importantly, the relationship between the price and the modularity in two competing firms are investigated analytically and numerically in this paper to derive some managerial insights. Specifically, we consider a scenario where two firms produce substitutable products for a market, in which the demand is price as well as modularity sensitive. The Bertrand competition and Stackelberg competition are developed to derive the equilibrium solutions on the pricing strategies and modularity levels, respectively, where both firms determine the optimal prices simultaneously followed by modularity levels decisions in the former competition case and the leader firm decides the optimal price and modularity level of its own in the first step and the follower firm determines the corresponding price and modularity level in the second step in the latter competition case. Notwithstanding the complexity of deriving the analytical solutions, we conclude from the numerical study that when two firms jointly compete on prices and modularity levels, compared to the prices only competition, the dominant advantage of Stackelberg competition over Bertrand competition and outperformance of the follower in Stackelberg competition may get diluted if modular design is included in the competition. Both firms may be worse off due to the increased modularity cost when the price and modularity sensitivities are low; but both may be better off at the expense of modularity when the corresponding sensitivities are high.

The remainder of this paper is organized as follows. The demand function and the cost function are defined in Section 2. In Section 3, we characterize the equilibrium solutions on prices and modularity levels of two firms under Bertrand and Stackelberg competition. Considering the complexity of the problem, an intensive numerical study is conducted to examine the effects of the parameters on the equilibrium solutions in Section 4. Finally, we conclude the paper and point out possible directions for future research in Section 5.

2. Preliminaries and notations. We consider two firms competing on prices and modularity levels in a market, where modularity of the product is denoted by $m$. A higher $m$ represents a higher modularity level. Modular design of a product is attractive as it can satisfy the demand for customized products and reduce the lead time, though additional effort is required to re-design the product [15].

2.1. The demand function. The market demand for the product with modular design is assumed to be deterministic and sensitive to price and modularity level during the selling season. Following Banker et al. [2] and Mukhopadhyay and Setoputro [15], we model firm $i$ demand as a linear function of selling price $p_i$ and the modularity level $m_i$, which takes the following form:

$$D(p_i, m_i) = a - \alpha \cdot p_i + \beta \cdot p_j + \gamma \cdot m_i - \phi \cdot m_j$$  \hspace{1cm} (1)

where $i, j = 1, 2, i \neq j$, and $a, \alpha, \beta, \gamma, \phi > 0$. The parameters are explained as follows. $a$ measures the potential market when the prices and the modularity levels are zero, which is determined by the functionality and quality of the product.
itself. $\alpha$ indicates sensitivity of demand to the own price. $\beta$ is the demand substitution parameter that represents the cross price-sensitivity of the demand and the level of firm differentiation. $\gamma$ denotes the demand responsiveness to the firms own modularity level, while $\phi$ stands for demand responsiveness to the competitors modularity level. Apparently, since the modular design improves the attractiveness of the product, modularity level has positive impact on the firms own demand and negative effect on the competitors demand. Additionally, we assume $\alpha > \beta$ and $\gamma > \phi$ to ensure that the demand for each firm is more sensitive to its own price and modularity than those of the competitors, which is a widely used assumption in extant literature [15], [2].

2.2. The cost function. Generally, the production cost in our model is divided into two types. (1) The fixed cost associated with the modularity of the design, denoted by $C^f_i$. This cost arises due to the investment in improving the design skills and the relevant equipment. We assume that the fixed cost is increasing and convex when the modularity level is $m_i$, which takes the following form:

$$C^f_i = \frac{1}{2}e \cdot m_i^2, i = 1, 2$$

(2) The variable cost related to the modularity level in the production is denoted by $C^v_i$. Specifically, let $c$ denote the variable production cost per unit exclusive of the modularity level. Given a modularity level $m_i$ determined by the firm, the variable cost increases by $b$, where $b > 0$. Thus, $C^v_i$ takes the following form:

$$C^v_i = c + b \cdot m_i, i = 1, 2$$

3. Model and analysis. Competition between the two firms is considered in two formats: Bertrand competition and Stackelberg competition. In the former competition, following Banker et al. [2], both firms decide the modularity levels simultaneously in the first place, and then determine the corresponding prices at the same time, before demand is realized; in the latter, the Stackelberg leader determines its modularity level and selling price in the first step, and the follower firm chooses its own modularity level and selling price in the second step.

3.1. Bertrand competition. In Bertrand game, two competitive firms sequentially decide their respective modularity levels and prices, but both firms make the decisions simultaneously. We use backward induction to obtain the solutions.

In the second step, given the modularity levels, both firms decide the pricing strategies simultaneously by maximizing the objective profit functions as shown below:

$$\pi^B_i(p_i, m_i) = (p_i - c - b \cdot m_i) \cdot (a - \alpha \cdot p_i + \beta \cdot p_j + \gamma \cdot m_i - \phi \cdot m_j) - \frac{e}{2} \cdot m_i^2.$$

(4)

where the superscript $B$ represents the Bertrand competition. For any given modularity levels determined by the two firms, it is easy to verify that the objective function is concave with respect to the price $p_i, i = 1, 2$. Hence, the first derivative of Eq. (4) with respect to $p_i$ is

$$\frac{\partial \pi^B_i(p_i, m_i)}{\partial p_i} = (a - \alpha \cdot p_i + \beta \cdot p_j + \gamma \cdot m_i - \phi \cdot m_j) - \alpha \cdot (p_i - c - b \cdot m_i).$$

(5)

We can also get the first derivative of the objective function with respect to $p_j$, which is similar to Eq. (5). With the first order condition, solving $p_i$ and
Define the demand with respect to $b$ are zero, it is necessary to assume $a$ functions as:

$$p_i^*(m_i, m_j) = B_1 + B_2 \cdot m_i + B_3 \cdot m_j,$$

where $B_1 = \frac{(\alpha + \alpha \phi)(\beta + 2\alpha)}{4\alpha^2 - \beta \phi}$, $B_2 = \frac{2\alpha(\gamma + \beta \phi)}{4\alpha^2 - \beta \phi}$, $B_3 = \frac{\beta(\gamma + \beta \phi) - 2\alpha \phi}{4\alpha^2 - \beta \phi}$.

Substituting Eq. (6) into Eq. (1) and (4), we have the demand and profit functions as:

$$D_i^*(m_i, m_j) = a - (\alpha - \beta) \cdot B_1 + (\gamma - \alpha \cdot B_2 + \beta \cdot B_3) \cdot m_i - (\phi - \beta \cdot B_2 + \alpha \cdot B_3) \cdot m_j,$$

and

$$\pi_i^*(m_i, m_j) = (B_1 - c + (B_2 - b) \cdot m_i + B_3 \cdot m_j) \cdot D_i^*(m_i, m_j) - \frac{e}{2} \cdot m_i^2.$$

To make sure that the demand is positive when modularity levels of both firms are zero, it is necessary to assume $a - (\alpha - \beta) \cdot B_1 > 0$, which implies the following assumption:

$$c < \frac{a}{\alpha - \beta}. \quad (9)$$

Additionally, the demand function reveals that when $\phi - \beta \cdot B_2 + \alpha \cdot B_3 < 0$, the demand of firm $i$ will increase even if it does nothing to enhance its modularity, which is unreasonable. Therefore, another necessary condition is $\phi - \beta \cdot B_2 + \alpha \cdot B_3 < 0$, which requires that

$$0 < b < \frac{2\alpha \phi - \beta \gamma}{\alpha \beta} \text{ and } 2\alpha \phi - \beta \gamma > 0. \quad (10)$$

Furthermore, when $\gamma - \alpha \cdot B_2 + \beta \cdot B_3 < \phi - \beta \cdot B_2 + \alpha \cdot B_3$, the demand of firm $i$ will increase when both firms decide to reduce the modularity levels by the same value. Again, it is impractical. Thus, we obtain the following condition:

$$0 < b < \frac{\gamma - \phi}{\alpha - \beta}. \quad (11)$$

Before presenting the optimal modularity levels decisions, another required condition with respect to $b$ is $0 < b < \frac{2\alpha \gamma - \beta \phi}{2\alpha^2 - \beta \phi}$, discussed in the following. Since $\alpha \beta (2\alpha \gamma - \beta \phi) - (2\alpha \phi - \beta \gamma)(2\alpha^2 - \beta^2) = -\alpha \phi(4\alpha^2 - \beta^2) - \beta^3 \gamma$, one can verify that $\frac{2\alpha \gamma - \beta \phi}{2\alpha^2 - \beta \phi} \leq \frac{2\alpha \phi - \beta \gamma}{\alpha \beta}$. Therefore, together with Eq. (10) and (11), to meet the requirements, we present the following assumption:

$$0 < b < \min \left\{ \frac{2\alpha \gamma - \beta \phi}{2\alpha^2 - \beta^2}, \frac{\gamma - \phi}{\alpha - \beta} \right\}. \quad (12)$$

Now, we derive the equilibrium modularity levels of the two competing firms. Define $B_4 = a - (\alpha - \beta) \cdot B_1$, $B_5 = \gamma - \alpha \cdot B_2$, and $B_6 = \phi - \beta \cdot B_2 + \alpha \cdot B_3$, Eq. (8) can be rearranged as

$$\pi_i^*(m_i, m_j) = (B_1 - c + (B_2 - b) \cdot m_i + B_3 \cdot m_j) \cdot (B_4 + B_5 \cdot m_i - B_6 \cdot m_j) - \frac{e}{2} \cdot m_i^2. \quad (13)$$

**Proposition 1.** When $e > 2B_5 \cdot (B_2 + \max(B_3, 0) - b)$, the equilibrium modularity levels of the two firms are

$$m_i^B = \frac{(B_1 - c) \cdot B_5 + (B_2 - b) \cdot B_4}{e - 2B_5 \cdot (B_2 + B_3 - b)}, i = 1, 2. \quad (14)$$

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1For ease of exposition, we define some constant parameters in the analysis when necessary to simplify the calculation.
Proof. Clearly, we can get that \( \frac{\partial^2 \pi_i^*}{\partial m_i \partial m_j} = 2 (B_2 - b) B_5 - e \), and the cross partial derivatives are \( \frac{\partial^2 \pi_i^*}{\partial m_i \partial m_j} = B_3 B_5 - (B_2 - b) B_6 \). Therefore, when \( e > 2B_5 (B_2 + \max (B_3, 0) - b) \), it is easy to verify that \( 2 (B_2 - b) B_5 - e < 0 \) and \( \left| \frac{\partial^2 \pi_i^*}{\partial m_i \partial m_j} \right| > \left| \frac{\partial^2 \pi_i^*}{\partial m_i \partial m_j} \right| \). Consequently, the Hessian matrix is negative definite as \( H_1 < 0 \) and \( H_2 > 0 \), where \( H_1 \) and \( H_2 \) are the principal minors such that \( H_1 = 2 (B_2 - b) B_5 - e \), and \( H_2 = (2 (B_2 - b) B_5 - e)^2 - (2B_5 (B_2 - b) B_3)^2 \). We know that the objective function of Eq. (13) is strictly concave such that the first derivatives of Eq. (13) with respect to \( m_i \) and \( m_j \) are

\[
\frac{\partial \pi_i^*}{\partial m_i} = (B_2 - b) B_4 + (B_1 - c) B_5 + (2 (B_2 - b) B_5 - e) m_i
+ (B_3 B_5 - (B_2 - b) B_6) m_j
\]

\[
\frac{\partial \pi_i^*}{\partial m_j} = (B_2 - b) B_4 + (B_1 - c) B_5 + (2 (B_2 - b) B_5 - e) m_j
+ (B_3 B_5 - (B_2 - b) B_6) m_i
\]

With the first order condition and the equation \( B_3 B_5 + (B_2 - b) B_6 = 0 \), solving the above two equations simultaneously, we can obtain the equilibrium modularity levels of the two firms determined by Eq. (14). We complete the proof. \( \square \)

It is worth pointing out that the numerator of Eq. (14) is positive according to the condition specified in Eq. (12). One may argue that when the numerator and denominator of the modularity levels determined by Eq. (14) are both negative, it can also ensure that \( m_i > 0 \), where \( i = 1, 2 \). However, when the numerator is negative, the condition for variable modularity cost is \( b > \frac{2 \alpha \gamma - \beta \phi}{2 \alpha^2 - \beta^2} \). Combining this with the condition determined by Eq. (11), the new requirement is \( \frac{2 \alpha \gamma - \beta \phi}{2 \alpha^2 - \beta^2} < \frac{\alpha \phi}{\beta^2} \). Unfortunately, only with the condition \( \alpha > \beta \) and \( \gamma > \phi \), it is intractable to determine the sign of the following term \( (2 \alpha \gamma - \beta \phi) (\alpha - \beta) - (2 \alpha^2 - \beta^2) (\gamma - \phi) = (2 \alpha - \beta) (\alpha \phi - \beta \gamma) \), which depends on \( \alpha \phi \) and \( \beta \gamma \). In this case, the problems solution may be infeasible if we consider the negative scenario. Therefore, we only consider the positive case.

From Eq. (14), we obtain \( \frac{\partial m_i}{\partial e} = - \frac{(B_1 - e) B_6 + (B_2 + B_3 - b) B_4}{(e - 2B_5 (B_2 + B_3 - b))^2} < 0 \), the modularity levels are strictly decreasing with sensitivity to the fixed modular cost. Due to the complexity, however, it is not easy to determine the monotonicity of the modularity levels with respect to the variable modular cost, which is examined in the numerical study instead.

Since the equilibrium modularity levels of the two firms satisfy \( m_i^B = m_j^B \), substituting them into price functions Eq. (6), we obtain the equilibrium pricing strategies for the two firms as

\[
p_i^B = B_1 + (B_2 + B_3) \cdot m_i^B, \quad i = 1, 2.
\]

Since \( B_2 + B_3 = \frac{(2 \alpha + \beta) (\gamma + \beta \phi - \phi)}{2 \alpha^2 - \beta^2} > 0 \), we have \( \frac{\partial p_i}{\partial e} = (B_2 + B_3) \cdot \frac{\partial m_i}{\partial e} < 0 \), which implies that the equilibrium prices and the equilibrium modularity levels of the two firms in Bertrand competition have a similar shape with respect to the fixed modular cost.
Eq. (14) and (15) indicate that in Bertrand competition scenario, the equilibrium pricing strategies and the modularity levels of the two firms are exactly the same\(^2\). Therefore, neither firm has the incentive to improve the modularity levels and there are no differences between the two products provided by the two firms.

3.2. Stackelberg competition. In this subsection, we consider the Stackelberg competition between the two firms and we assume, without loss of generality, firm \(i\) is the Stackelberg leader in the market and announces the optimal pricing and modularity level to maximize its own profit. Then the follower firm \(j\) optimizes its own profit by determining its own price and modularity level after observing the leaders (firm \(i\)’s) decisions. Again, we use backward induction to obtain the equilibrium solutions.

At the second step, given the price and modularity level determined by the leader firm \(i\), firm \(j\) optimizes its profit by the following expression:

\[
\pi_j^S (p_j, m_j) = (p_j - c - b \cdot m_j) \cdot (a - \alpha \cdot p_j + \beta \cdot p_i + \gamma \cdot m_j - \phi \cdot m_i) - \frac{e}{2} \cdot m_j^2 \tag{16}
\]

where the superscript \(S\) indicates the Stackelberg competition.

Proposition 2. When \(e > \frac{(\gamma - ba)^2}{2a}\), the optimal pricing and modularity level determined by the objective function are:

\[
p_j^* = S_1 + S_2 \cdot p_i + S_3 \cdot m_i, \tag{17}
\]

and

\[
m_j^* = S_4 + S_5 \cdot p_i + S_6 \cdot m_i, \tag{18}
\]

where \(S_1 = \frac{(a+\alpha \cdot c+b \cdot c) \cdot (\gamma - ba)}{2e\cdot (\gamma - ba)}, S_2 = \frac{(e+b (\gamma - ba))}{2e\cdot (\gamma - ba)}, S_3 = \frac{(c+b (\gamma - ba))}{2e\cdot (\gamma - ba)}, S_4 = \frac{(a-c \cdot (\gamma - ba))}{2e\cdot (\gamma - ba)}, S_5 = \frac{(\gamma - ba)}{2e\cdot (\gamma - ba)}, S_6 = \frac{(\gamma - ba)}{2e\cdot (\gamma - ba)}.

Proof. It is easy to get \(\frac{\partial^2 \pi_j^S}{\partial p_j^2} = -2a, \frac{\partial^2 \pi_j^S}{\partial m_j^2} = -2b\gamma - e, \) and \(\frac{\partial^2 \pi_j^S}{\partial p_j \partial m_j} = \frac{\partial^2 \pi_j^S}{\partial m_j \partial p_j} = \gamma + ba\).

Thus, the corresponding Hessian matrix is negative definite as \(H_1 < 0\) and \(H_2 > 0\) when \(e > \frac{(\gamma - ba)^2}{2a}\), where \(H_1 < 0\) and \(H_2 > 0\) are principal minors. We know that the objective function is strictly concave such that the first derivatives of Eq. (16) with respect to \(p_j\) and \(m_j\) are:

\[
\frac{\partial \pi_j^S}{\partial p_j} = a + \alpha c - 2a \cdot p_j + \beta p_i + (\gamma + ba) \cdot m_j - \phi m_i
\]

\[
\frac{\partial \pi_j^S}{\partial m_j} = -ab - c \gamma + (\gamma + ba) \cdot p_j - b \beta p_i - (2b \gamma + e) \cdot m_j + b \phi m_i
\]

With the first order condition and solving the two equations simultaneously, we obtain the response price and modularity level for firm determined by Eq. (17) and (18). We complete the proof. \(\square\)

At the first step, anticipating how the follower firm \(j\) will respond to the price and modularity level, the leader firm \(i\) maximizes its profit by:

\[
\pi_i^S (p_i, m_i) = (p_i - c - b \cdot m_i) \cdot (a - \alpha \cdot p_i + \beta \cdot p_j^* + \gamma \cdot m_i - \phi \cdot m_j^*) - \frac{e}{2} \cdot m_i^2 \tag{19}
\]

\(^2\)We also formulate the problem as complete competition such that both firms decide the pricing strategies and modularity levels simultaneously. The prices and the modularity levels of the two firms are also the same.
Plugging Eq. (17) and (18) into Eq. (19), with some algebra, we get the demand function of firm $i$ as

$$D_i^S(p_i, m_i) = a + \beta S_1 - \phi S_4 - (\alpha - \beta S_2 + \phi S_5) p_i + (\gamma + \beta S_3 - \phi S_6) m_i$$  \hspace{1cm} (20)

and the profit function as

$$\pi_i^S(p_i, m_i) = -\frac{e}{2} \cdot m_i^2 + (p_i - c - bm_i) \cdot D_i^S$$  \hspace{1cm} (21)

Define $S_7 = a + \beta S_1 - \phi S_4$, $S_8 = \alpha - \beta S_2 + \phi S_5$, $S_9 = \gamma + \beta S_3 - \phi S_6$, Eq. (21) can be rewritten as

$$\pi_i^S(p_i, m_i) = (p_i - c - bm_i) \cdot (S_7 - S_8 p_i + S_9 m_i) - \frac{e}{2} \cdot m_i^2.$$  \hspace{1cm} (22)

Analogously, to ensure that the demand is positive when both the price and the modularity level of the firm $i$ are zero, it requires $S_7 > 0$, which implies:

$$c < \frac{a \left((\gamma - b\alpha)(b\beta - \phi) + e\beta\right) + \alpha \left(2e\alpha - (\gamma - b\alpha)^2\right)}{\gamma (b\alpha - \gamma) (\phi - \beta) - \alpha b e}.$$  \hspace{1cm} (23)

Here, $(a \left((\gamma - b\alpha)(b\beta - \phi) + e\beta\right) + \alpha \left(2e\alpha - (\gamma - b\alpha)^2\right)(\gamma (b\alpha - \gamma) (\phi - \beta) - \alpha b e) > 0$ is an implicit assumption to ensure the production cost is positive. Because the numerator and denominator of Eq. (23) are complicated, it is difficult to further characterize the required conditions regarding the related sensitivity parameters, which directly result in carefully selection of the parameters in the numerical study section. Additionally, we require $S_8 > 0$ to ensure that the demand is decreasing in its own price and $S_9 > 0$ to make sure that the demand is increasing in its own modularity level. Mathematically, these two conditions imply that

$$e > \max \left\{ \frac{(\gamma - b\alpha)(\alpha (\gamma - b\alpha) - \beta (\phi - b\beta))}{2\alpha^2 - \beta^2}, \frac{(\gamma - b\alpha)(\gamma - b\alpha) - \phi (\phi - b\beta))}{2\alpha^2 - \beta^2}, 0 \right\}.$$  \hspace{1cm} (24)

We require $e > 0$ to indicate that the fixed modular cost is monotonically increasing in the modularity level; however, the first two terms in the maximum operator are not necessarily larger than zero without any additional conditions derived by the parameters. In addition, because of the complexity, it is infeasible to characterize the required condition regarding the variable modularity cost $b$ as we did in the Bertrand competition. The decision sequence contributes to the intractability. However, conditions Eq. (23), (24), together with $2ae > (\gamma - b\alpha)^2$ preserve a feasible region of the problem. Therefore, we have the following conclusion in terms of the equilibrium solutions.

**Proposition 3.** When $b$ and $e$ satisfy $S_8 > 0$, $S_9 > 0$, and $2aeS_8 - (bS_8 - S_9)^2 > 0$, the equilibrium pricing and the modularity level determined by Eq. (19) are

$$p_i^S = \frac{(S_7 - cS_8)(S_9 - bS_8)}{2eS_8 - (S_9 - bS_8)^2},$$  \hspace{1cm} (25)

and

$$m_i^S = \frac{S_7 (e + bS_9) + S_8 (e - b^2 S_7) - cS_9 (S_9 - bS_8)}{2eS_8 - (S_9 - bS_8)^2}.$$  \hspace{1cm} (26)

**Proof.** The second derivatives of Eq. (22) with respect to $p_i$ and $m_i$ are $\frac{\partial^2 \pi_i^S}{\partial p_i^2} = -2S_8$, $\frac{\partial^2 \pi_i^S}{\partial m_i^2} = -2bS_9 - e$, and $\frac{\partial^2 \pi_i^S}{\partial m_i \partial p_i} = \frac{\partial^2 \pi_i^S}{\partial p_i \partial m_i} = S_9 + bS_8$, such that the corresponding Hessian matrix is negative definite as $H_1 < 0$ and $H_2 > 0$, where $H_1 < 0$
and \( H_2 > 0 \) are principal minors. Thus, Eq. (22) is strictly concave function with respect to \( p_i \) and \( m_i \). The corresponding first derivatives are

\[
\frac{\partial \pi_i^S}{\partial p_i} = S_7 - cS_8 + 2S_8p_i + (S_9 - bS_8)m_i, \\
\frac{\partial \pi_i^S}{\partial m_i} = -bS_7 - cS_9 + (S_9 - bS_8)p_i - (2bS_9 + e)m_i.
\]

Solving the above two equations according to the first order condition, we obtain the equilibrium solutions determined by Eq. (25) and (26). We complete the proof.

Substituting Eq. (25) and (26) into Eq. (17) and (18), the equilibrium pricing and modularity level for the follower firm \( j \) are

\[
p_j^S = S_1 + S_2 \cdot p_i^S + S_3 \cdot m_i^S, \tag{27}
\]

and

\[
m_j^S = S_4 + S_5 \cdot p_i^S + S_6 \cdot m_i^S. \tag{28}
\]

In Stackelberg competition scenario, the leading firm holds a modularity strategy as well as a pricing strategy different from those determined by the follower firm. Though it is intractable to compare the corresponding pricing strategies and the modularity strategies analytically because of the complexity, we conduct an intensive numerical study on the parameters to reveal some implications in the next section.

4. **Numerical study.** In this section, we conduct an intensive numerical study on pricing strategies and modularity level strategies of the two firms under the Bertrand competition case and the Stackelberg competition case, respectively. In Stackelberg game, without loss of generality, we assume firm 2 is the Stackelberg leader and firm 1 is the follower. The potential market demand volume is set to be \( a = 100 \) and the variable production cost is \( c = 5 \). The other parameters are defined in each case accordingly to meet the necessary conditions (Section 3).

4.1. **On the price sensitivity.** To conduct the analysis of the price sensitivity, we let \( \gamma = 2, \phi = 1.5, b = 0.5, e = 1 \) and the results are summarized in Figures 1 and 2.

In Figure 1\(^3\), we choose \( \beta = 1 \) and \( \alpha \sim U(1.5, 2.5) \) with 21 discrete values to investigate the impact of firms own price sensitivity on the prices, modularity levels, as well as the maximum profits. We can see that the prices and modularity levels both are decreasing with the increasing of firms own price sensitivity. In most of the cases, prices and modularity levels in Stackelberg game are larger than the corresponding values in the Bertrand competition such that the unbalanced power in competition increases the prices and the modularity levels at the same time. On the other hand, compared to the follower, the leader may be worse-off if its own price sensitivity is high (see the third figure of Fig. 1, the blue line), which means the first-mover advantage is lost. The possible reason is that when the demand is more sensitive to the firms own selling price, it is necessary to reduce the price to attract more demand. However, due to the cost of modularity, firms would decrease the modularity levels at the same time to maintain profit. We can see that when the demand is less sensitive to the own price, the leader (firm 2) would keep high

\[3\]In practice, the modular units of a product may be improved from 4 to 5, the corresponding modularity levels here may be from 400 to 500. Otherwise, the values calculated here may not make any sense.
price and modularity level to use the first mover advantage. However, this may not be true when the demand is more sensitive to its own price. The follower (firm 1) may earn a slightly higher profit than the leader. Nonetheless, both firms perform better in Bertrand competition though they provide less modular products to the market.

We choose $\alpha = 1.5$ and $\beta \sim U(0.1, 2.4)$ with 21 discrete values to examine the impact of competitors price sensitivity on the prices, modularity levels and optimal profits, which is shown in Figure 2. The followers (firm 1) price and modularity level are both increasing with the competitors price sensitivity. However, both price and modularity level of the Stackelberg leader decreases first and then increases gradually (the price decreases so slightly that it is not so apparent when compared to the modularity level). From Eq. (27) and (28), with some algebra, we can get that the price and modularity level of the leader are both quadratic functions with respect to the competitors price sensitivity ($\beta$). The reason is that when the products are less substitutable (small $\beta$), the leaders modularity level is larger than that of the followers. Therefore, the leader can take advantage of first-mover and decrease its price and modularity simultaneously to take more profit. In this condition, to keep competence in the market, the follower needs to increase the modularity level as well as price to cover the modularity cost. When the products are getting more and more substitutable, the follower benefits more from improving the modularity level. To be profitable, the leader (firm 2) needs to increase modularity level to attract more demand and increase the price at the same time to keep the profit. However, the first-mover advantage is lost, and the follower is overwhelmed by keeping on increasing the modularity to attract more demand and increasing the price subsequently to take more profit.
4.2. On the modularity sensitivity. To illustrate the impact of the modularity sensitivity on the equilibrium decisions, we choose $\alpha = 2$, $beta = 1.5$, $b = 0.5$, $e = 0.5$ in this subcase. 

First, we examine the effect of the own modular sensitivity and choose $\phi U(1.0, 1.8)$ with 21 discrete values. We can see that the prices and the modularity levels, as well as the profits are all increasing in the firms own modularity sensitivity ($\gamma$). Generally, from Eq. (1), when $\gamma$ increases, the demand increases accordingly. To capture more demand, firms are willing to improve the modularity levels to increase the sales. At the same time, prices are increased to cover the modularity cost. Clearly, the own modularity sensitivity benefits both firms; however, the Stackelberg leader is hardly likely to take the first-mover advantage.

We choose $\gamma = 2$ and $\phi U(1.0, 1.8)$ with 21 discrete values to demonstrate the effect of competitors modular sensitivity. The conclusion is analogous to the case when considering the competitors price sensitivity except that the price, modularity level and profit of the follower firm (firm 1) are all decreasing in $\phi$. Also, From Eq. (27) and (28), with some algebra, we know that the leaders equilibrium price and modularity level are both quadratic functions with respect to the competitors modularity sensitivity ($\phi$) such that the price and modularity level exhibit a U shape. The reason we believe this is that when the competitors modularity sensitivity is small, both firms are willing to improve the modularity levels to attract the customers. With the increasing of the competitors modularity sensitivity, the
Figure 3. Effect of own modularity sensitivity

Figure 4. Effect of competitor’s modularity sensitivity
attractiveness of the own product decreases such that they reduce the modularity levels. In Stackelberg game, the leader could anticipate the followers response. As such, when the competitors modularity sensitivity is large, if it keeps on reducing the modularity level, the competitor would increase its own modularity levels to take more profit. Therefore, the leader begins to increase the modularity level, which in turn decreases the followers modularity level even further. From this point onward, the leader takes the first-mover advantage.

We can see that prices and modularity of the follower and the leader intersect twice (first and second figure of Fig. 4). The leader benefits from a larger competitors modularity sensitivity; however, in this case, Bertrand competition is more profitable for both firms.

4.3. **On the modularity cost.** The effect of modularity cost is much more complicated than that of price and modularity sensitivity. To clarify the results, this subcase is divided into two types: low price and modularity sensitivities and high price and modularity sensitivities. (1) In low sensitivity case, the parameters are $\alpha = 1$, $\beta = 0.5$, $\gamma = 1$, $\phi = 0.5$. We choose $e = 0.55$, $b U(0.1, 0.9)$ with 21 discrete values to investigate the effect of variable modularity cost on the equilibrium decisions and $b = 0.5$, $e U(0.5, 5.5)$ with 21 discrete values to examine the impact of fixed modularity cost on the pricing strategies and modularity levels. The results are summarized in Figures 5 and 6.

![Figure 5. Effect of competitor’s modularity sensitivity](image)

In Figure 5, the price strategies and the modularity levels in Bertrand competition are decreasing with respect to the variable modularity cost. The leaders decisions on these two strategies have a similar pattern. However, the followers decisions experience an inverted U shape such that there are threshold values of variable...
modularity cost \( b^1 \) (\( b^2 \)) such that the price and modularity level (profit) are increasing when \( b < b^1 \) (\( b < b^2 \)); and they are decreasing when \( b > b^1 \) (\( b > b^2 \)). This reveals that in Stackelberg game, when the variable modularity cost is small, the follower has the incentive to improve the modularity levels. However, when the variable modularity cost is larger, the follower would reduce the modularity level as the leader does. This may probably be because of the low price and modularity sensitivities of the two firms. Although the increasing of variable modularity cost reduces the profit, when the cost is small, a lower sensitivity to price and modularity encourage the follower to increase the modularity level to attract more demand and therefore increase the profitability.

![Figure 6. Effect of competitor’s modularity sensitivity](image)

In Figure 6, intuitively and apparently, all decision strategies in Bertrand competition and Stackelberg game are decreasing with the fixed modularity cost when the sensitivities of pricing strategies and modularity levels are small. The performance of the follower overwhelms that of the leader as shown in the third figure of Figure 6. When the fixed modularity cost is high, the leaders profit is larger than the corresponding value in Bertrand competition.

(2) In high price and modularity sensitivities case, the related parameters are \( \alpha = 2, \beta = 1.5, \gamma = 2, \phi = 1.5 \). We choose \( e = 1.5, b U(0.1, 0.9) \) with 21 discrete values and \( b = 0.5, e U(0.5, 5.5) \) with 21 discrete values to develop the impact of variable and fixed modularity cost on the equilibrium prices and modularity levels, respectively. The results are presented by Figures 7 and 8 as follows.

Comparing Figure 7 and 5, they are a little different from each other in that the pricing strategies now are all increasing in the first step and then decreasing with respect to the variable modular cost, but the modularity levels are all decreasing. More importantly, the profits are all increasing in variable modularity cost and
asymptotically reach to a maximum value as the corresponding modularity levels are decreasing to zero. When the sensitivities of price and modularity are high, if the variable modularity cost is small, it is better to have high modularity to attract customers. When the variable modularity cost increases, it is optimal to reduce the modularity level to save the cost. However, at the beginning, even though increasing the price will directly decrease the demand, the profit increment owing to price increase and the cost reduction because of modularity decreasing can cover the loss of demand as the variable modularity cost is small. Nevertheless, when the variable modularity cost keeps on increasing, which would reduce the demand indirectly according to Eq. (7) and (20), increasing the prices further would lead to lower profit. Thus, both firms choose to reduce the price. In this case, since the sensitivities of price and modularity strategy are high, decreasing the price and the modularity level simultaneously would slow down the decreasing of the demand such that the firms profits increase gradually. Consequently, the profits are increasing, but at the expense of modularity.

Figure 8 looks similar to Figure 7. Again, analogous to the less price and modularity sensitivities case, the equilibrium prices and modularity levels are decreasing in the fixed modular cost. However, the profits are increasing gradually to maximum values, in which the modularity levels decrease to zero gradually. Similarly, due to high sensitivities of the price and modularity level, when fixed modular cost increases, both price and modularity level decrease rapidly in the beginning which alleviates the demand decrease such that the cost reduction can compensate for the price reduction as well as the demand decrease. Therefore, the profits are increasing, but once more at the expense of modularity levels.
4.4. **Discussion.** One can conduct a numerical study on the impact of the price sensitivity ($\alpha$ and $\beta$) without considering the modularity decisions ($m_i = m_j = 0$). The results show that: 1) both firms in Stackelberg game perform better than in Bertrand competition; and 2) in Stackelberg price competition, the first-mover advantage is not observed though the leader determines a higher price than the follower, which means the follower outperforms the leader. However, when the modularity becomes one of the key competing factors, the performance of the two firms will change and the first-mover advantage comes to play an important role for the leader. Specifically, we can see from Figures 1 to 8 that there are critical values of price sensitivities, modularity sensitivities, modularity costs such that when the corresponding parameters are below (or above) the values, both firms prefer Bertrand competition rather than Stackelberg competition and in these cases, the leader can benefit from the first-mover advantage and he is more profitable than the follower in Stackelberg game. Concerning the performance of the leader and the follower, it is also readily seen that there are threshold values of price and modularity sensitivities as well as modularity costs that when the values of the related parameters are larger than the threshold values, the leaders performance in Bertrand competition exceeds that in Stackelberg competition. In some of these cases, see Figures 1, 5, 7 and 8, when the parameters are less than the corresponding threshold values, the follower also benefits more from the Bertrand competition than the Stackelberg competition. Thus, comparing to the cases where there is only price competition between the two firms, we may conclude that the profitable competition type for two firms would change and the leader firm would take the advantage.

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4We did it both in Bertrand competition and Stackelberg competition. Due to the simplicity and also to saving the space, we do not present the results. It could be provided upon request.
of first-mover in the Stackelberg game when pricing and modularity are jointly determined in the market. Despite this, both leader and follower would be better-off in the Bertrand competition when the related parameters are appropriately determined in advance if the modularity competition is additionally introduced between the two firms. Consequently, the dominant competition format for both firms would change from Stackelberg competition to Bertrand competition.

As mentioned above, in Bertrand game, both firms decide the modularity levels simultaneously in the first place and then determine the selling prices at the same time after observing the modularity levels. Therefore, the modularity levels, as well as the selling prices in both firms are equal. But when they compete in Stackelberg game, the leader announces the modularity level and the selling price in the first step and then the follower makes its own decisions. Thus, the leader can benefit from this competition in the following three cases: 1) increasing the selling price and the modularity level when the pricing and modularity sensitivities are small (Figures 1, 2, 3 and 4) or the variable modularity cost is small (Figure 5); 2) increasing the price but decreasing the modularity level when the fixed modularity cost is large (Figure 6); 3) decreasing the price and the modularity level when modularity costs are large (Figure 7 and 8). Apparently, the mutual effects of pricing and modularity give rise to the overwhelming performance of the leader in Stackelberg competition.

5. Conclusions and future research. In this paper, we consider the equilibrium pricing and modularity decisions of two firms who are competing in a market. The Bertrand competition and Stackelberg competition are examined by assuming that the demand is mutually price and modularity dependent. The corresponding analytical equilibrium solutions are derived and an intensive numerical study on the parameters is conducted to investigate the impact of the related parameters on the equilibrium solutions, as well as the maximum profits.

Because of the complexity of the problem, it is intractable to characterize the properties of the equilibrium solutions. Alternatively, by resorting to numerical evidence, we find that, the prices and modularity levels, as well as the maximum profits in Stackelberg game are no longer larger than those in the Bertrand competition. There are critical values of price sensitivities (own and competitors), modular level sensitivities (own and competitors), variable and fixed modularity costs; when the corresponding parameters are larger (smaller) than the critical values, both firms are better-off in Bertrand competition and the leader in Stackelberg game would take the advantage of first-mover if the parameters are appropriately selected in advance. Furthermore, the impact of the variable and fixed modularity costs on the prices, especially on the maximum profits is dependent on the sensitivities of the prices and modularity levels. When the sensitivities are small, the profits are decreasing with the modularity cost, where the firms are worse-off due to lowering of the modular design; however, when the sensitivities are large, the profits are increasing with the modularity cost, where the firms are better-off at the expense of modularity levels. But they both asymptotically approach to the maximum value where the modularity levels are decreasing to zero gradually. Our research also reveals that the modularity levels in Stackelberg competition are mostly improved for both firms.

Finally, the price and modular sensitivity and the modularity costs of both firms in this paper are assumed to be identical to characterize the equilibrium solutions. One can hypothesize that they are different between two firms to investigate the
impact of these parameters on the equilibrium solutions. Future research may consider stochastic demand which depends on price and modularity simultaneously to examine the effect of demand variation on the equilibrium solutions under different types of competitions. The decision on quantities as the classical Stackelberg game does is also worthy of developing, though it must be much more complicated than our model. Last but not least, asymmetric information of the modularity levels between two firms is another interesting topic to follow on.

Acknowledgments. We thank Professor Kok Lay Teo (EIC), and three anonymous reviews for their valuable comments and suggestions that helped improve the model and analysis presented in this paper. The work described in this paper was supported in part by Shanghai Pujiang Program (18PJ025); in part by National Natural Science Foundation of China (71872064, 71603240, 71201059, 71772063, and 71431004); in part by Natural Science Foundation of Shanghai (18ZR1409400).

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Received October 2017; revised May 2018.

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