The application of the statistical method for calculating the area of the areal object

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Abstract. This article introduces a new concept and a new type of modelling – granular information modelling. This modelling is based on the granular computing method and a systematic approach. It is shown that a number of information field objects are comparable to the concept of information granules. The concept of information granules is used in computing technologies. The article develops the concept of information granules as information field objects. The hierarchy of information granules in the information field is shown. The similarity and difference between a cluster and an information granule are shown. The article proposes the interpretation of information granules as a mechanism of information modelling. It also proposes the information modelling method that allows building a structure of granules and relations between them. This method uses a linguistic and set-theoretic approach.

1. Introduction
The ideas of granular computing (GrC) have been closely studied in many fields [1,2]. The ideas of granular modelling have been investigated to a lesser extent. Granular computing can be treated as an approach to finding data based on regularities that occur at different levels of data detailing, just as various functions do in these data. Therefore, granular information computing modelling can be treated as an approach to finding models based on regularities that occur at different levels of data detailing. The goal of granular modelling is to develop additional modelling methods and reasoning techniques. Granular modelling can be correlated to an approach that is structural and cognitive as opposed to strictly formal. Granular information modelling is close to cognitive modelling and fuzzy computing. Fuzzy information granularity was first proposed by L.A. Zadeh. The theory of granular information modelling uses the theory of fuzzy information granularity (TFIG). This theory is based on the cognitive capability of generalisation.

In this granular modelling, we use information models that are built based on generalisation and are called ‘granules’. Granules are built based on the concept of generalised constraints. This refers them to the area of fuzzy sets. Relations between granules can be represented in terms of fuzzy graphs and rules [3]. Information factors should be taken into account when performing the granular information modelling. The most important information factor is information field [4]. This field is a source used to obtain data and information about the laws of the surrounding world. The field elements are information units [5] that set a limit of detail for granular information modelling. Information situation models [6]
localise the field area where the modelling is performed. Information constructions [7] are generalised models for the field and granular information modelling. In addition to fuzzy relations, granular information modelling requires the use of information compliance relations [8]. This relation ensures that information field objects are adequately displayed for information models. The presence of information compliance creates semantic compliance. Semantic compliance allows applying the semantics of information units [9] and including semantic modelling in granular information modelling. A number of new concepts for granular computing and granular information modelling are not new for informatics. Their synthesis with the concepts of granular computing extends the capabilities of granular information modelling.

2. Methods
The basis of this study includes systematic analysis, structural analysis, linguistic approach and set-theoretic approach. Publications concerning information modelling and granular computing are used as materials.

3. Research results

3.1. Characteristics of granules
It is possible to analyse the contents of granules in terms of structural analysis. Granular information modelling is a part of the modelling theory for the effective use of cluster- and subset-type granule models in the information field. A separate granule can be treated as a composite complex model or an elementary model, i.e. an information unit. Together, granules represent models with different levels of detail.

Granules are the main model of granular information modelling. Descriptions of granules as models become clearer when dealing with specific problems. Subsets with fuzzy boundaries can be treated as set granules. Information field elements can be treated as subset granules. A cluster is an illustrative example for a granule. As a homogeneous entity, a cluster corresponds to a granule. However, a cluster does not cover the entire field and is a heterogeneous selective field coverage. A granule covers the entire field due to the hierarchy. The size of a cluster characterises an area, and the size of a granule determines its level. The size of a granule can be interpreted as the degree of abstraction, specificity or detail. Clusters usually describe one level and represent a collection of one-level objects. Granules belong to different levels and represent a connected granule system. A cluster is more often a crisp object. A granule is always a fuzzy object. In a set-theoretic setting, the size of a granule can describe the potency of a set.

Binary relations serve as the basis for describing connections and relations between granules of one or different levels. In a granule system, a level of relations can be interpreted as membership, union, intersection and proportionality. For example, granules of the same level are most often proportional. A composite granule typically has a higher level of abstraction than its components. Connections and relations between granules enable the determination of order relation for granules. Depending on particular circumstances, this relation can be interpreted as a comparative characteristic. An order relation for granules may be reflexive and transitive, but not symmetrical. An order relation is important when studying granules at different levels.

Main modelling operations with granules are aimed at creating a new granule or decomposing a granule into a granule set. These operations are based on cognitive analysis because granules are fuzzy objects. Operations with granules are most often a dichotomous division [10]. Operations with granules enable the performance of granular information modelling.

Modelling uses the concept of modelling level. There are different modelling levels, including conceptual, formal and realisational. The conceptual level defines the concept of modelling. The formal level describes the steps that should be taken to solve the problem and defines granules-related requirements. The realisational level specifies the actual parameters of a model. The determination of levels, the number of levels and relations between them are currently under discussion. The levels are
also used for descriptions and explanations. In granular modelling, the number of levels depends on the purpose of modelling. Granular modelling requires the performance of a structural analysis.

3.2. Analysing an information granule
Granular information modelling is based on structural analysis and the use of information granules. This concept also exists in granular computing, but with an accent on computing. Regardless of computing, the concept of granularity appeared in a number of areas: fuzzy data sets, cluster analysis [11]. Information granules are formed on the basis of information description according to their similarity, proximity and functionality [12]. A granule transmits a certain aspect of a problem simulated [13]. Traditional information models are based on the principle of information certainty, so they do not give a solution in case of uncertainty. Information granules provide a solution under conditions of uncertainty. Granular models are complementary to information uncertainty. Crisp information models are closer to formulas. Granular information models are closer to relations. The hierarchical structure of granular systems in combination with cognitive methods of human thinking makes it a convenient tool for abstraction at different levels of modelling.

As an illustration of the principle of granular modelling, let us consider cluster analysis. Here, objects are grouped based on their similarity and difference. If a crisp boundary between similarity and difference cannot be drawn, the boundary becomes fuzzy. Clusters with fuzzy boundaries can be treated as granules [14]. Information granules can be combined into larger collections that are perceived as new information granules. Information granules can separate small parts that are also perceived as information granules. Dividing downwards is possible up to information field elements, i.e. information units. Combining upwards is possible up to the level of a problem. It, therefore, follows that hierarchy is an attribute of granular information models.

Due to the fuzziness of some characteristics, information granules must comply with the principle of justifiable granularity (PJG) [15]. In a broad sense, information granules must conform to the principle of semantic information compliance [8]. PJG is a conditional empirical rule for building information granules under the condition of justifiability and specificity. PJG requires that constructed granules cover a part of the information field, but not be highly dispersed throughout the field. This condition is different from clustering because clusters typically occupy a small part of the information field. It can be achieved by forming information granules taking into account the semantics of the information field.

Granular modelling allows a better understanding of the subject area by including semantic aspects, crisp parameters and uncertainty parameters. Granular modelling is used to analyse fuzzy time series and spatial data, to perform conceptual-cognitive learning based on big and multiple data in the formal analysis of concepts [16]. The principles of granular modelling are used to cluster data taking into account the distance between granules and the density of newly created ones. The data thus prepared are input into the Adaptive Neuro-Fuzzy Inference System (ANFIS). The procedure has been tested and confirmed its suitability [17].

The use of granular models allows the recognition of faces that have been surgically altered. Several levels of granules are created, some granules contain information about an entire face, some of them about specific areas, and others about small details of certain functions. These granularity levels are then sent to classifiers [18] for integral and local evaluations.

3.3. Modelling methods
The basis of granular information modelling is a set-theoretic and linguistic approach where a linguistic description of an information granule is made. The intersection of two crisp sets \( G_1, G_4 \) is called a set \( X_3 \)

\[
X_3 = G_1 \cap G_4
\]  

The intersection of two fuzzy sets \( G_1^*, G_4^* \) is called a fuzzy set \( X_3^* \) specified at the same universe \( U \) whose membership function is defined as

\[
\mu_{X_3^*}(x) = \min\{\mu_{G_1^*}(x), \mu_{G_4^*}(x)\} \quad \forall x \in U
\]
The intersection of fuzzy sets $X^*3$ is the largest fuzzy set $G1^* \cap G4^*$ contained simultaneously in $G1^*$ and $G4^*$ with the membership function specified by the expression (2). Figure 1 (related to figure 2) shows the intersection results of crisp (solid lines) and fuzzy (dashed) sets ($X3 < X^*3$).

Figure 2 shows a granular modelling chart, the purpose of which is to determine relations between granules of different levels on the universe $U$ (information field). The fuzziness of sets describing granules is shown by double boundaries. To simplify, asterisks showing fuzzy sets are removed.

Granules of higher level are based on membership and union.

$$G0 \subset U$$

$$G0 = G1 \cup G2 \cup G3 \cup G4 \cup G5 \cup G6$$

The expression (3) shows that a granule of higher level belongs to the universe. For all levels, there is a linguistic description of the type.

$$G0 =< G1, G2, G3, G4, G5, G6 >$$

The expression (5) describes the nesting of parts, the expression (4) describes the relation of granule parts. Granules of the next lower level are based on the relation (6) of membership and union

$$G1 \subset G0, G2 \subset G0, G3 \subset G0, G4 \subset G0, G5 \subset G0, G6 \subset G0$$

$$G1 = X2 \cup Z1 \cup Y1; \ G2 = X1 \cup Z2 \cup Y3;$$
For example, only two granules are described in expression (7). As an example, granules of the next lower level (figure 2) are based on the membership relation (8) and those of the second level on the intersection relation (9)

\[ X_1 \in G_1, X_1 \in G_4, X_5 \in G_3, X_5 \in G_5, X_1 \in G_2, X_1 \in G_2 \]  

\[ X_3 = G_1 \cap G_4; X_5 = G_3 \cap G_5; X_1 = G_2 \cap G_5; Y_1 = G_1 \cap G_6; Y_2 = G_2 \cap G_6; Y_5 = G_3 \cap G_6; \]  

Unlike the expression (6) for subsets, the expression (8) describes information field elements. Such elements are called 'information units'. Granules of the next lower level \( X_2, X_4, Y_2, Y_4 \) are based on the membership relation and the difference relation of granules of higher levels. For this example, as a result of modelling, we obtain two elementary levels with information units. According to this method, the information structure of granules is formed. In this example, we construct a system of granules of four levels covering the information field.

4. Conclusion
For the time being, granular modelling is a theoretical opportunity rather than a coherent set of methods or principles. Information modelling is not aimed at computing. Information modelling is intended to create a method of describing an object in the information field. In this method, information modelling allows us to build a granule structure in the information field. The main idea of this method is to compare granules and sets and build granules based on different set relations. The number of levels can be four or more. Fundamentally, the method is applicable not only to two-dimensional sets but also to sets with a larger number of dimensions. From a philosophical point of view, granular modelling can be described as a way of thinking that relies on a person's ability to perceive the structure of the real world at various levels of detail. Granular modelling plays an important role in describing structures and affects the development and implementation of cognitive and intelligent systems.

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