ON INTERVAL VALUED FUZZY ALMOST \((m,n)\)-BI-IDEAL IN SEMIGROUPS

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Abstract. In this paper, we study the concept of an interval valued fuzzy almost \((m,n)\)-bi-ideals. We investigate properties of an interval valued fuzzy almost \((m,n)\)-bi-ideal in semigroups.

Keywords: almost \((m,n)\)-bi-ideal; bi-ideal; interval valued fuzzy almost \((m,n)\)-bi-ideal.

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1. INTRODUCTION

The theory of fuzzy set was presented in 1965 by Zadeh [12]. The theory of fuzzy semigroups contained by Kuroki in 1979 [8]. Later the theory of interval valued fuzzy sets was introduced in 1975 by Zadeh [13], as a generalization of the notion of fuzzy sets. Interval valued fuzzy sets have various applications in several areas like medical science [3], image processing [2], decision making [14], etc. In 2006, Narayanan and Manikantan [7] developed the theory of interval valued fuzzy subsemigroup and studied types interval valued fuzzy ideals in semigroups. In 1961, Lajos [5] studied the concepts of \((m,n)\)-ideals in semigroups which generalized of ideals of semigroups. The research of \((m,n)\)-ideals of semigroups has interested many such as Akram et al. [1], N. Yaqoob and M. Aslam [10] and many others. In 2020 Ahsan et al. [6] extended

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the ideals of \((m,n)\)-ideals in semigroups to fuzzy sets in semigroup and they characterize the regular semigroup by using fuzzy \((m,n)\)-ideals.

In this paper, we give the concept of an interval valued fuzzy almost \((m,n)\)-bi-ideals. We prove properties of an interval valued fuzzy almost \((m,n)\)-bi-ideal in semigroups.

2. Preliminaries

In this section, we give some definition and theory helpful in later sections.

A non-empty subset \(L\) of a semigroup \(G\) is called

1. a subsemigroup of \(G\) if \(L^2 \subseteq L\),
2. a left (right) ideal of \(G\) if \(GL \subseteq L\) (\(LG \subseteq G\)),
3. an ideal of a semigroup \(G\) we mean a left ideal and a right ideal of \(G\),
4. an almost bi-ideal of \(G\) if \(L\) is a subsemigroup and \(LgL \cap L \neq \emptyset\).

A non-empty subset \(L\) of a semigroup \(G\). We denote the

\[ [L](m,n) = \bigcup_{r=1}^{m+n} L^r \cap L^mGL^n \text{ is principal } (m,n)\text{-ideal,} \]
\[ [L](m,0) = \bigcup_{r=1}^{m} L^r \cap L^mG \text{ is principal } (m,0)\text{-ideal,} \]
\[ [L](0,n) = \bigcup_{r=1}^{n} L^r \cap GL^n \text{ is the principal } (0,n)\text{-ideal,} \]

i.e., the smallest \((m,n)\) ideal, the smallest \((m,0)\) ideal and the smallest \((0,n)\) ideal of \(G\) containing \(L\), respectively.

**Lemma 2.1.** [4] Let \(G\) be a semigroup and \(m,n\) positive integers, \([\pi](m,n)\) the principal \((m,n)\)-ideal generated by the element \(\pi\). Then

1. \(([\pi](m,0))^mG = \pi^mG.\)
2. \(G([\pi](0,n))^n = G\pi^n.\)
3. \(([\pi](m,0))^mG([\pi](0,n))^n = \pi^mG\pi^n.\)

For any \(p_i \in [0,1]\), where \(i \in \mathcal{A}\), define

\[ \lor_{i \in \mathcal{A}} p_i := \sup_{i \in \mathcal{A}} \{p_i\} \quad \text{and} \quad \land_{i \in \mathcal{A}} p_i := \inf_{i \in \mathcal{A}} \{p_i\}. \]

We see that for any \(p,q \in [0,1]\), we have
\[ p \lor q = \max\{p, q\} \quad \text{and} \quad p \land q = \min\{p, q\}. \]

A fuzzy set of a non-empty set \( T \) is a function \( \omega : L \rightarrow [0, 1] \).

**Definition 2.2.** [6] A fuzzy set \( \omega \) of a semigroup \( G \) is said to be

1. a **fuzzy subsemigroup** of \( G \) if \( \omega(e_1e_2) \geq \omega(e_1) \land \omega(e_2) \) for all \( e_1, e_2 \in G \),
2. a **fuzzy left (right) ideal** of \( G \) if \( \omega(e_1e_2) \geq \omega(e_2) \) (\( \omega(e_1e_2) \geq \omega(e_1) \)) for all \( e_1, e_2 \in G \),
3. a **fuzzy ideal** of \( G \) if it is both a fuzzy left ideal and a fuzzy right ideal of \( G \),
4. a **fuzzy \((m, n)\)-ideal** of \( G \) if \( \omega(e_1e_2...e_md_1d_2...d_n) \geq \omega(e_1) \land \omega(e_2) \land \cdots \land \omega(e_n) \land \omega(d_1) \land \omega(d_2) \land \cdots \land \omega(d_n) \) for all \( e_1, e_2, ..., e_m, d_1, d_2, ..., d_n, b \in G \) and \( m, n \) are positive integers.

**Definition 2.3.** [11] A fuzzy set \( \omega \) of a semigroup \( G \) such that \( \omega \neq 0 \) is called **fuzzy almost bi-ideal** of \( G \) if \( (\omega \circ \chi_G \circ \omega) \cap \omega \neq 0 \).

Let \( \Omega[0, 1] \) be the set of all closed subintervals of \([0, 1]\), i.e.,

\[ \Omega[0, 1] = \{ p = [p^-, p^+] | 0 \leq p^- \leq p^+ \leq 1 \} . \]

We note that \([p, p] = \{p\}\) for all \( p \in [0, 1] \). For \( p = 0 \) or \( 1 \) we shall denote \([0, 0]\) by \( \bar{0} \) and \([1, 1]\) by \( \bar{1} \). Let \( \bar{p} = [p^-, p^+] \) and \( \bar{q} = [q^-, q^+] \in \Omega[0, 1] \). Define the operations \( \preceq, =, \land \) and \( \lor \) as follows:

1. \( \bar{p} \preceq \bar{q} \) if and only if \( p^- \leq q^- \) and \( p^+ \leq q^+ \)
2. \( \bar{p} = \bar{q} \) if and only if \( p^- = q^- \) and \( p^+ = q^+ \)
3. \( \bar{p} \land \bar{q} = [(p^- \land q^-), (p^+ \land q^+)] \)
4. \( \bar{p} \lor \bar{q} = [(p^- \lor q^-), (p^+ \lor q^+)] \).

If \( \bar{p} \succeq \bar{q} \), we mean \( \bar{q} \preceq \bar{p} \).

For each interval \( \bar{p}_i = [p_i^-, p_i^+] \in \Omega[0, 1], i \in \mathcal{I} \) where \( \mathcal{I} \) is an index set, we define

\[ \land_{i \in \mathcal{I}} \bar{p}_i = [\land_{i \in \mathcal{I}} p_i^-, \land_{i \in \mathcal{I}} p_i^+] \quad \text{and} \quad \lor_{i \in \mathcal{I}} \bar{p}_i = [\lor_{i \in \mathcal{I}} p_i^-, \lor_{i \in \mathcal{I}} p_i^+] . \]

**Definition 2.4.** [9] Let \( L \) be a non-empty set. Then the function \( \bar{f} : L \rightarrow \Omega[0, 1] \) is called **interval valued fuzzy set** (shortly, IVF set) of \( T \).
Definition 2.5. [9] Let \( L \) be a subset of a non-empty set \( G \). An \textit{interval valued characteristic function} of \( L \) is defined to be a function \( \chi_L : T \rightarrow \Omega[0,1] \) by

\[
\chi_L(e) = \begin{cases} 
1 & \text{if } e \in L, \\
0 & \text{if } e \notin L 
\end{cases}
\]

for all \( e \in G \).

For two IVF sets \( \omega \) and \( \varnothing \) of a non-empty set \( G \), define

1. \( \omega \subseteq \varnothing \iff \omega(e) \leq \varnothing(e) \) for all \( e \in G \),
2. \( \omega = \varnothing \iff \omega \subseteq \varnothing \text{ and } \varnothing \subseteq \omega \),
3. \( (\omega \cap \varnothing)(e) = \omega(e) \land \varnothing(e) \) for all \( e \in G \),
4. \( (\omega \cup \varnothing)(e) = \omega(e) \lor \varnothing(e) \) for all \( e \in G \).

For two IVF sets \( \omega \) and \( \varnothing \) in a semigroup \( G \), define the product \( \omega \circ \varnothing \) as follows: for all \( e \in G \),

\[
(\omega \circ \varnothing)(e) = \begin{cases} 
\bigwedge_{(t,h) \in F_e} \{ \bar{I}(t) \land \varnothing(h) \} & \text{if } F_e \neq \emptyset, \\
\emptyset & \text{if } F_u = \emptyset,
\end{cases}
\]

where \( F_e := \{(t,h) \in G \times G \mid e = th\} \).

Next, we shall give definitions of various types of IVF subsemigroups.

**Definition 2.6.** [7] An IVF set \( \omega \) of a semigroup \( G \) is said to be an \textit{IVF subsemigroup} of \( G \) if \( \omega(e_1e_2) \geq \omega(e_1) \land \omega(e_2) \) for all \( e_1, e_2 \in G \).

**Definition 2.7.** [7] An IVF set \( \omega \) of a semigroup \( G \) is said to be an \textit{IVF left (right) ideal} of \( G \) if \( \omega(e_1e_2) \geq \omega(e_2) \) \( (\omega(e_1e_2) \geq \omega(e_1)) \) for all \( e_1, e_2 \in G \). An IVF subset \( \omega \) of \( G \) is called an \textit{IVF ideal} of \( G \) if it is both an IVF left ideal and an IVF right ideal of \( G \).

**Theorem 2.8.** [7] Let \( L \) be a non-empty subset of a semigroup \( G \). Then \( \chi_L \) is an IVF subsemigroup of \( G \) if and only if \( L \) is a subsemigroup of \( G \).

**Theorem 2.9.** Let \( \omega \) be an IVF set of a semigroup \( G \). Then \( \omega \) is a subsemigroup of \( G \) if and only if \( \text{sup}(\omega) \) is an IVF subsemigroup of \( G \).

Let \( \omega \) be an IVF set of a semigroup \( G \) and \( m \in \mathbb{Z} \). Then
Theorem 2.10. Let $\omega, \varpi$ and $\kappa$ be IVF set of a semigroup $G$. Then the following statements hold:

1. If $\omega \subseteq \varpi$ then $\omega^m \subseteq \varpi^m$ for all $m \in \mathbb{Z}$.
2. If $\omega \subseteq \varpi$ then $\omega \circ \kappa \subseteq \varpi \circ \kappa$.
3. If $\omega \subseteq \varpi$ then $\omega \circ \kappa \cap \varpi \circ \kappa$.

Definition 2.11. An IVF set $\omega$ of a semigroup $G$ is called an IVF almost $(m,n)$-ideal of $G$ if $(\omega^m \circ G \circ \omega^n) \cap \omega \neq \emptyset$ for all $m,n \in \mathbb{Z}$.

3. ON INTERVAL VALUED FUZZY ALMOST $(m,n)$-BI-IDEAL IN SEMIGROUPS

In this section, we give the concept of an interval valued fuzzy almost $(m,n)$-bi-ideals and investigate properties of an interval valued fuzzy almost $(m,n)$-bi-ideal in semigroups.

Definition 3.1. An IVF subsemigroup $\omega$ of a semigroup $G$ is called an IVF almost $(m,n)$-bi-ideal of $G$ if $(\omega^m \circ G \circ \omega^n) \cap \omega \neq \emptyset$ for all $m,n \in \mathbb{Z}$.

Theorem 3.2. Suppose that $\omega$ is an IVF almost $(m,n)$-bi-ideal and $\varpi$ is an IVF subsemigroup of a semigroup $G$ and $m,n \in \mathbb{Z}$. Then the following statements hold:

1. If $\omega \subseteq \varpi$, then $\varpi$ is an IVF almost $(m,n)$-bi-ideal of $G$.
2. $\omega \cup \varpi$ is an IVF almost $(m,n)$-bi-ideal of $G$.

Proof. (1) Suppose that $\omega \subseteq \varpi$. Then $\emptyset \neq (\omega^m \circ G \circ \omega^n) \cap \omega \subseteq (\varpi^m \circ G \circ \varpi^n) \cap \varpi$. Thus $\varpi$ is an IVF almost $(m,n)$-bi-ideal of $G$.

(2) Clearly $\omega \subseteq \varpi \cup \varpi$. By (1) we have $\varpi \cup \varpi$ is an IVF almost $(m,n)$-bi-ideal of $G$.

Note that for a subset $L$ of $G$, define $L^0 := G$. 

\[ \omega^0 := \overline{\chi}_G \quad \text{and} \quad \omega^0 \circ G \circ \omega^0 := \overline{\chi}_G, \]

\[ \overline{\omega} := \overline{\omega} \circ \overline{\omega} \circ \cdots \circ \overline{\omega}, \quad m \text{-times} \]

\[ \overline{\omega} \circ G \circ \omega^0 := \overline{\omega} \circ \overline{\chi}_G, \]

\[ \overline{\omega} \circ G \circ \omega^m \circ \omega^0 := \overline{\chi}_G \circ \overline{\omega}^m. \]
Lemma 3.3. Let $L$ be a non-empty subset of a semigroup $G$ and $m \in \mathbb{Z}$. Then $(\chi_L)^m = \chi_L^m$.

Theorem 3.4. Let $L$ be a non-empty subset of a semigroup $G$. Then $L$ is an almost $(m,n)$-bi-ideal of $G$ if and only if the characteristic function $\chi_L$ is an IVF almost $(m,n)$-bi-ideal of $G$ for all $m,n \in \mathbb{Z}$.

Proof. Suppose that $L$ is an almost $(m,n)$-bi-ideal of $G$. Then $L$ is a subsemigroup of $G$. Thus by Theorem 2.11, $\chi_L$ is an IVF subsemigroup of $G$. Let $d \in G$. Then by assumption, there exists $e \in (L^mGL^n) \cap L$ such that $[((\chi_L)^m \circ \overline{G} \circ (\chi_L)^n) \cap \chi_L](e) \neq 0$. By Lemma 3.3,

$$[((\chi_L)^m \circ \overline{G} \circ (\chi_L)^n) \cap \chi_L](e) \neq 0.$$

Thus $\chi_L$ is an IVF almost $(m,n)$-bi-ideal of $G$.

Conversely, suppose that $\chi_L$ is an IVF almost $(m,n)$-bi-ideal of $G$. Then $\chi_L$ is an IVF subsemigroup. Thus by Theorem 2.11, $L$ is a subsemigroup of $G$. Let $d \in G$. Then

$$[((\chi_L)^m \circ \overline{G} \circ (\chi_L)^n) \cap \chi_L](e) \neq 0.$$

Thus there exists $e \in G$ such that $[((\chi_L)^m \circ \overline{G} \circ (\chi_L)^n) \cap \chi_L](e) \neq 0$. By Lemma 3.3,

$$[\chi_L^m \circ \overline{G} \circ (\chi_L)^n] \cap \chi_L](e) \neq 0.$$

Thus $e \in L^mGL^n \cap L$. Hence $L^mGL^n \cap L \neq \emptyset$.

We conclude that $L$ is an almost $(m,n)$-bi-ideal of $G$. \qed

For IVF set $\overline{\omega}$ of a semigroup $G$, defined $\text{supp}(\overline{\omega}) := \{e \in G \mid \overline{\omega}(e) \neq 0\}$.

Theorem 3.5. Let $\overline{\omega}$ be an IVF set of a semigroup $G$. Then $\overline{\omega}$ is an almost $(m,n)$-bi-ideal of $G$ if and only if $\sup(\overline{\omega})$ is an IVF almost $(m,n)$-bi-ideal of $G$ for all $m,n \in \mathbb{Z}$.

Proof. Suppose that $\overline{\omega}$ is an almost $(m,n)$-bi-ideal of $G$. Then $\overline{\omega}$ is a subsemigroup of $G$. Thus by Theorem 2.9, $\sup(\overline{\omega})$ is an IVF subsemigroup of $G$. Let $d \in G$. Then there exists $e \in G$ such that $[\overline{\omega}^m \circ \overline{G} \circ \overline{\omega}^n] \cap \overline{\omega}(e) \neq 0$. Thus $\overline{\omega}(r) \neq 0$ and $e = c_1c_2 \ldots c_mE_1e_2 \ldots e_n$ for some $c_1, c_2, \ldots, c_m, e_1, e_2, \ldots, e_n \in G$ such that

$$\overline{\omega}(c_1) \neq 0, \overline{\omega}(c_2) \neq 0, \ldots, \overline{\omega}(c_m) \neq 0, \overline{\omega}(e_1) \neq 0, \overline{\omega}(e_2) \neq 0, \ldots, \overline{\omega}(e_n) \neq 0.$$
So \(c_1, c_2, \ldots, c_m, e_1, e_2, \ldots, e_n, d \in \text{supp}(\overline{oc}).\) It implies that

\[
[(\mathcal{X}_{\text{supp}(oc)})^m \circ \overline{G} \circ (\mathcal{X}_{\text{supp}(oc)})^n](e) \neq 0 \text{ and } \mathcal{X}_{\text{supp}(f)}(e) \neq 0.
\]

Hence \([(\mathcal{X}_{\text{supp}(oc)})^m \circ \overline{G} \circ (\mathcal{X}_{\text{supp}(oc)})^n \cap \mathcal{X}_{\text{supp}(oc)}](e) \neq 0\). Thus \(\mathcal{X}_{\text{supp}(oc)}\) is an IVF almost \((m,n)\)-bi-ideal of \(G\). By Theorem 3.4, \(\text{supp}(\overline{oc})\) is an almost \((m,n)\)-bi-ideal of \(G\).

Conversely, suppose that \(\text{supp}(\overline{oc})\) is an IVF almost \((m,n)\)-bi-ideal of \(G\). Then \(\overline{oc}\) is an IVF subsemigroup of \(G\). Thus by Theorem 2.9, \(\overline{oc}\) is a subsemigroup of \(G\). Let \(r \in G\). Then by Theorem 3.4, \(\mathcal{X}_{\text{supp}(oc)}\) is an IVF almost \((m,n)\)-bi-ideal of \(G\).

Thus

\[
[(\mathcal{X}_{\text{supp}(oc)})^m \circ \overline{G} \circ (\mathcal{X}_{\text{supp}(oc)})^n \cap \mathcal{X}_{\text{supp}(oc)}](e) \neq 0.
\]

So there exists \(e \in S\) such that \([(\mathcal{X}_{\text{supp}(oc)})^m \circ \overline{G} \circ (\mathcal{X}_{\text{supp}(oc)})^n \cap \mathcal{X}_{\text{supp}(oc)}](r) \neq 0\).

Hence \((\mathcal{X}_{\text{supp}(oc)})^m \circ \overline{G} \circ (\mathcal{X}_{\text{supp}(oc)})^n (e) \neq 0\) and \(\mathcal{X}_{\text{supp}(oc)}(e) \neq 0\).

Thus there exist \(c_1, c_2, \ldots, c_m, e_1, e_2, \ldots, e_n, d \in G\) \(\text{supp}(\overline{oc})\) and \(e = c_1 c_2 \ldots c_m d e_1 e_2 \ldots e_n\). So \(\overline{oc}(c_1) \neq 0\), \(\overline{oc}(c_2) \neq 0\), \(\ldots\), \(\overline{oc}(c_m) \neq 0\), \(\overline{oc}(e_1) \neq 0\), \(\overline{oc}(e_2) \neq 0\), \(\ldots\), \(\overline{oc}(e_n) \neq 0\).

Hence \([(\overline{oc}^m \circ \overline{G} \circ \overline{oc}^n)](e) \neq 0\) implies \([(\overline{oc}^m \circ \overline{G} \circ \overline{oc}^n) \cap \overline{oc}](e) \neq 0\).

Therefore \(\overline{oc}\) is an almost \((m,n)\)-bi-ideal of \(G\). \(\square\)

**Definition 3.6.** An IVF almost bi-ideal \(\overline{oc}\) is called **minimal** if for all nonzero IVF almost bi-ideals \(\overline{oc}\) of a semigroup \(G\) such that \(\overline{oc} \subseteq \overline{oc}\) implies \(\text{supp}(\overline{oc}) = \text{supp}(\overline{oc})\).

**Definition 3.7.** An IVF almost \((m,n)\)-bi-ideal \(\overline{oc}\) is called **minimal** if for all nonzero IVF almost \((m,n)\)-bi-ideals \(\overline{oc}\) of a semigroup \(G\) such that \(\overline{oc} \subseteq \overline{oc}\) implies \(\text{supp}(\overline{oc}) = \text{supp}(\overline{oc})\).

**Theorem 3.8.** Let \(L\) be a non-empty subset of a semigroup \(G\). Then \(L\) is a minimal almost \((m,n)\)-bi-ideal of \(G\) if and only if \(\overline{X}_L\) is a minimal IVF almost \((m,n)\)-bi-ideal of \(G\).

**Proof.** Suppose that \(L\) is a minimal almost \((m,n)\)-bi-ideal of \(G\). Then by Theorem 3.4, \(\overline{X}_L\) is an IVF almost \((m,n)\)-bi-ideal of \(G\). Let \(\overline{oc}\) be an IVF almost \((m,n)\)-bi-ideal of \(S\) such that \(\overline{oc} \subseteq \overline{X}_L\).

Then \(\text{supp}(\overline{oc}) \subseteq \text{supp}(\overline{X}_L) = L\). By Theorem 3.5, \(\text{supp}(\overline{oc})\) is an almost \((m,n)\)-bi-ideal of \(G\). By supposition, \(\text{supp}(\overline{oc}) = L = \text{supp}(\overline{X}_L)\). Hence \(\overline{X}_L\) is a minimal IVF almost \((m,n)\)-bi-ideal of \(G\).

Conversely, suppose that \(\overline{X}_L\) is a minimal IVF almost \((m,n)\)-bi-ideal of \(G\) and let \(D\) be an almost \((m,n)\)-bi-ideal of \(G\) such that \(D \subseteq L\). Then \(\overline{X}_D\) is an IVF almost \((m,n)\)-bi-ideal of \(G\).
such that $\overline{X}_D \subseteq \overline{X}_L$. Thus $D = \text{supp}(\overline{X}_B) = \text{supp}(\overline{X}_L) = L$. Therefore $L$ is a minimal almost $(m,n)$-bi-ideal of $G$.

**Corollary 3.9.** Let $G$ has no proper almost $(m,n)$-bi-ideal if and only if for all IVF almost $(m,n)$-bi-ideal $\omega$ of $G$, supp($\omega$) = $G$.

**Proof.** It follows from Theorem 3.8.

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**Conflict of Interests**

The author(s) declare that there is no conflict of interests.

**References**

[1] M. Akram, N. Yaqoob and M. Khan, On $(m, n)$-ideals in LA-semigroups, Appl Math Sci. 7(44) (2013), 2187–2191.

[2] A. Jurio, J.A. Sanz, D. Paternain, J. Fernandez, H. Bustince, Interval-Valued Fuzzy Sets for Color Image Super-Resolution, in: J.A. Lozano, J.A. Gámez, J.A. Moreno (Eds.), Advances in Artificial Intelligence, Springer Berlin Heidelberg, Berlin, Heidelberg, 2011: pp. 373–382.

[3] H. Bustince, Indicator of inclusion grade for interval valued fuzzy sets. Application to approximate reasoning based on interval valued fuzzy sets, Int. J. Approx. Reason. 23 (1998), 137-209.

[4] D.N. Krgovic, On $(m, n)$-regular semigroups, Publ. Linst. Math. 18(32) (2008), 107–110.

[5] S. Lajos, Notes on $(m, n)$-ideals I, Proc. Japan Acad. 39 (1963), 419–421.

[6] A. Mahboob, B. Davvaz and N. M. Khan, Fuzzy $(m, n)$-ideals in semigroups, Comput. Appl. Math. 38 (2019), 189.

[7] A.L. Narayanan, T. Manikantan, Interval valued fuzzy ideals generated by an interval valued fuzzy subset in semigroups, J. Appl. Math. Comput. 20(1-2) (2006), 455-464.

[8] N. Kuroki, Fuzzy bi-ideals in semigroup, Comment. Math. Univ. St. Paul, 5 (1979), 128-132.

[9] J.N. Mordeson, D. S. Malik, N. Kuroki, Fuzzy semigroup, Springer Science and Business Media, (2003).

[10] N. Yaqoob and M. Aslam, Prime $(m, n)$ bi-hyperideals in-semihypergroups, Appl. Math. Inform. Sci. 8(5) (2014), 2243-2249.

[11] K. Wattanatripop, R. Chinram and T. Changphas, Fuzzy almost bi-ideals in semigroups. Int. J. Math. Computer Sci. 13(1) (2018), 51-58.
[12] L.A. Zadeh, Fuzzy sets, Inform. Control, 8 (1965), 338-353.

[13] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Inform. Sci. 8 (1975), 199-249.

[14] M. Zulquanain and M. Saeed, A new decision making method on interval value fuzzy soft matrix, Br. J. Math. Computer Sci. 20(5) (2017), 1-17.