OPTIMAL PRICING AND ORDERING POLICY FOR DEFECTIVE ITEMS UNDER TEMPORARY PRICE REDUCTION WITH INSPECTION ERRORS AND PRICE SENSITIVE DEMAND

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ABSTRACT. This paper studies the retailer’s optimal promotional pricing, special order quantity and screening rate for defective items when a temporary price reduction (i.e., TPR) is offered. Although previous studies have examined the similar issue, they assume a constant demand and an error-free screening process. A subversion of these two assumptions differentiates our paper. First, using a price-sensitive demand, we analyze that the original screening rate may be insufficient, and propose the CPD (i.e., control the promotional demand) and the ISR (i.e., increase the screening rate through investment) strategy to handle it. Second, we incorporate both Type I and Type II inspection errors into our model. Then we establish an inventory model aiming to maximize the retailer’s profit under CPD and ISR, respectively. Finally, numerical examples are conducted and several results are obtained: (1) a higher portion of defects makes ISR more profitable; (2) both a higher probability of a Type I error and a Type II error decrease the profit under CPD and ISR, but a Type I error has a more pronounced negative impact; and (3) comparing with the existing studies with a constant demand, our model generates a higher profit especially in markets with a higher price sensitivity.

1. Introduction. Since November 11, 2009, the Chinese leading e-retail platform, Tmall has been successfully promoting “Double eleven” as a shopping festival during which Tmall offers a fifty-percent-off to consumers, so as to boost short-term sales and to drive profits. The Singles Day sales at Tmall snowballed from RMB 50 million in 2009 to RMB 498.2 billion in 2020. Although the sales miracle achieved by Tmall comes from the e-commerce platforms, the corresponding promotion tool—temporary price reduction (TPR) has long been used by traditional brick-and-mortar retailers. Given its economic significance in various practical settings, the TPR problem has received continuing academic research interest in inventory control theory; see [1]. By definition, TPR is a kind of limited time price incentive which aims to encourage retailers to place a special order quantity which is larger than usual and reduce retail prices temporarily to generate more demand [43]. In a price sensitive market, such TPR strategy benefits retailers through allowing them...
to generate more demand and increase the revenue. However, the larger order quantity is also expected to increase the retailers’ inventory holding cost dramatically in the short term. Thus, when determining the optimal combined ordering and pricing policy to maximize profit in the TPR programme, retailers should consider not only the increased revenue, but also the extra operating related cost.

In practice, retailers frequently face the problem of dealing with defective items, during both the TPR programme and their regular selling period. Factors like wear and tear of machinery in the production process, imperfect quality of components, and product damage/pollution during the transportation process, can cause defective items. In fact, defective items are inevitable for any firm, since it is too costly to identify and remove all defects [30]. For instance, the garment industry is operating at a percentage defective of 4.42, suffering from high rate of rejections of their products [29]. If these defective items are sold to end consumers, they would be unhappy and return them to the retailer for exchange or refund, causing defect sales returns [21]. Subsequently, retailers bear high costs involving not only return costs related to reverse logistics but also penalty cost due to goodwill loss from consumers’ quality dissatisfaction. Moreover, in handling these returned defects, like reworking, salvaging and scraping, retailers incur additional operating costs. To reduce the number of defective items being sold to consumers, retailers usually conduct a screening process to inspect the quality of each item and sort out defects. And during the screening process, retailers are able to control the screening capacity (or the screening rate), through manipulating how much resources they put in the screening activity, like the number of workers employed and the number of facilities and devices put into use. But to ensure the normal sales process, it is required that the screening rate should at least satisfy the consumer demand during the screening period. When considering the extra screening cost and the inspection capacity requirement related to the screening process, what is the retailer’s optimal order quantity? Salameh and Jaber [44] answer this question through establishing an EOQ (economic order quantity) model with defective items. Since then, inventory models with defective items have received much research attention; see Khan et al.[28] for an extensive review.

In the TPR situation, when taking defective items into account, the decision maker is faced with a severe problem that the original screening rate during the regular selling period may not be able to satisfy the dramatically increased promotional demand. To solve this problem, the retailer can choose between two alternative strategies: Strategy 1, through determining the sale price, the retailer controls the promotional demand rate (simplified as CPD hereafter) to the extent that the original screening rate can still satisfy, and Strategy 2, by investing in new facilities and devices in the inspection process, the retailer increases the screening rate (simplified as ISR hereafter) to ensure that the new screening rate (a decision variable) can at least satisfy the promotional demand rate. Clearly, the retailer suffers some profit loss by adding a constraint on his promotional demand under CPD, whereas he incurs some investment costs of increasing the screening rate under ISR.

Then, we would ask which one of the two strategies should the retailer choose? How do product related factors (like the percentage of defects) and market conditions (like price discounts received during TPR) influence such strategy choice? How does the value of these main parameters affect the retailer’s profit differently between the two strategies? To answer these research questions, we establish the retailer’s profit function under the two strategies. Specifically, under CPD, the retailer
maximizes his profit by determining the special order quantity and the promotional price, while considering the extra constraint that the original screening rate can satisfy the promotional demand. By contrast, under ISR, the retailer maximizes his profit by deciding not only the special order quantity and the promotional price, but also the new screening rate which can satisfy the promotional demand; and the retailer incurs extra investment cost of increasing the screening rate. Through introducing an additional decision variable—the screening rate, our paper contributes to existing literature by studying the retailer’s decision of screening rate in the TRP situation. Although previous studies, like Vörös [55], have also treated the screening rate as a decision variable, they study it during the regular selling period.

During the screening process, besides considering the screening capacity problem, the retailer is also bothered with the two-way inspection errors. Type I inspection error refers to misclassifying non-defective items as defective, thereby losing an opportunity to make more profit by not being able to sell those non-defects to consumers. Type II inspection error means misclassifying defective items as non-defective, resulting in defect sales returns by passing them on to customers. Both Type I and Type II inspection errors bring costs to the retailer and thus hurt his profit. Moreover, the cost of false acceptance (the Type II error) is much higher than the cost of false rejection, since the Type II error involves both resolution of consumer quality problems and reverse logistics [12]. Specially, in the TPR situation, the negative impacts of the two-way inspection errors on the retailer’s profit are amplified due to a dramatic increase in sales. Incorporating these two-way inspection errors into the retailer’s profit functions under the CPD and the ISR strategy above, how does the retailer choose between the two strategies? How does the percentage of the Type I error/Type II error affect the retailer’s strategy choice, as well as his profit under the two strategies? We answer all these research questions with the derived theoretical results and numerical examples.

To summarize, considering defective items and inspection errors, this study aims to investigate the retailer’s optimal pricing and ordering policy, as well as the screening rate decision under TPR in a price-sensitive market. Although Hsu and Yu [22] have investigated the inventory models with defective items under TPR, they not only ignore the problem of insufficient screening rate by assuming a constant demand rate but also neglect the problem of defect sales returns by assuming an error-free inspection process. Through filling these research gaps, our study contributes to the literature in two ways:

1. Using a price-sensitive market demand, our study addresses the issue that the original screening rate may not satisfy the increased promotional demand under TPR;

2. We propose two strategies to deal with the problem and examine how the retailer’s strategy choice and performance are affected by product/market related factors, like defective probability, the probability of the Type I and Type II errors, the size of price discount, the investment cost of increasing the screening rate, and price sensitivity.

The remainder of this paper is organized as follows. Section 2 reviews the research on inventory models concerning temporary price reduction, defective items and inspection errors, as well as shows how our research contributes to the current literature. Section 3 first lists the notations and assumptions used in this paper and provides a brief review of the model of [20] to determine retailer’s regular order quantity. Then Section 3 develops mathematical models under the CPD and ISR
strategy respectively, and analyzes the properties for the proposed model under each strategy. Sensitivity analysis is conducted and some results are provided in Section 4, and conclusions and managerial insights are summarized in Section 5.

2. Literature review. Both EOQ and news vendor models have proved useful for managing inventory for many years, analyzing tradeoffs among major cost components. EOQ model determines the order quantity that minimizes the total holding costs and ordering costs, for continually available products like milk. By contrast, news vendor model decides the order quantity that resolves the trade-off between the expected cost of having excess inventory and the expected cost of falling short, basically for single-period products like summer dresses. The basic one-period news vendor models are extended to problems of multiple periods, where units left from one period are available to meet demands in subsequent periods. Specifically, Lotfi et al. [35] study the start times and ordering plans for two-period projects using a two-period news vendor model, which is actually a combination of inventory and project management. Results indicate that the proposed model with interdependent demand offers a better solution than independent demand. Lotfi et al. [36] determine the initial inventory level to maximize profit of a two-period inventory model with interdependent demand. The proposed model can be used in applications like the procurement of raw materials in projects.

Our paper focuses on frequently purchased products (e.g., food items, medical devices, children’s toys and electronics) and hence belongs to inventory management based on EOQ formula. Specifically, we consider that each ordered lot contains a portion of defective items and the screening process suffers from two-way inspection errors; for example, in consumer electronics that are vulnerable to defects, Xbox 360 claims that the Sony PlayStation 3 “hovers in the 3 percent range” for defects and failure [8]. Thus, the retailer conducts a screening process to discern the defects. Given such industry backgrounds, we determine the retailer’s promotional policies and screening capacity in the TPR situation. The relevant literature is classified into three categories: inventory models under TPR, those with defective items (and inspection errors), and those joining the first two categories.

Finally, although our paper assumes a deterministic price-sensitive demand rate, future research can consider EOQ models under TPR with demand uncertainty. This demand uncertainty can be modelled by adopting fuzzy sets theory; see [51, 53]. To encounter such demand uncertainty, some researchers utilize a robust counterpart model; see [61, 7]. Thus, in the sense of extension, our paper is also related to studies on robust and fuzzy theory, with the relevant literature being summarized in subsection 2.4. Moreover, our paper addresses the TPR problem from the standpoint of one retailer and assumes that the returned items are sold to the secondary market. It is worthwhile to reconsider this problem from the perspective a supply chain where the returned merchandise can be remanufactured and resold to the market; see [15] as an example for remanufacturing returned items.

2.1. Inventory models under temporary price reduction. Since the early 1970s, temporary price reduction has emerged as an important marketing strategy to stimulate sales and drive profits for firms in the short term (Lin [32]). Given its economic significance, the TPR problem has received continued attention from researchers for over five decades. As early as in 1966, Naddor [40] analyzes the simplest situation where price increase occurs when the inventory on hand was zero, and determines the retailer’s optimal special order quantity to place at the
current lower price. Actually, Grubbström and Kingsman [17] demonstrate that a TPR problem and an announced price increase problem, as analyzed by Naddor [40], are equivalent. Ramasesh [43] provides a review for this topic. Specifically, according to the price-demand relationship, the literature on the optimal lot-sizing problem under TPR can be classified into two streams. One stream of literature assumes a constant demand rate and determines the optimal special order quantity; see [62, 47, 56, 13].

Another stream of literature assumes a price-sensitive demand, and simultaneously determines the optimal sale price and the special order quantity. Typically, the retailer aims to maximize the difference between the special quantity profit and the regular EOQ profit during the special sale period [1]. In particular, Abad [1] models the forward-buying behavior (i.e., purchasing additional stock at the reduced price provided by the supplier for later sale at the regular selling price) of the retailer in such TPR situation. The work of Abad [1] is then generalized by Abad [2] who assumes that the discount is offered to the retailer over a time interval. Under a general price-demand relationship, Ardalan [6] compares the difference between the average profit and the present value (where all economic consequences of ordering, holding, and selling decisions are treated as cash flows) approaches in solving the retailer’s pricing and ordering polices under TPR. The results indicate that although the present value approach offers a more accurate solution and should be applied to solve problems involving changes in price, the average profit method ought to be used when the discount is low. Incorporating the retailer’s forward-buying behavior, Arcelus and Srinivasan [5] further extend this work using a linear price-demand relationship. Arcelus et al. [4] compare the advantages of short-term price discounts with those of trade-credit terms. Recently, assuming time-price dependent demand, Shah and Naik [49] establish an inventory control model for deteriorating items under the constraint of full pre-payment based on circumstances like no shortages, full backordering shortages, partial lost sale, and with and without preservation. Whereas the above studies explore price promotions from the perspective of one retailer, other papers investigate promotional policies of competing retailers (Meng and Song [39]), and those within the supply chain framework (Su and Geunes [50]).

Focusing on deciding one retailer’s promotional pricing and ordering policies with price-sensitive demand, our work is more closely related to the second stream of literature. However, differing from the studies in this stream, our inventory model incorporates the impact of defective items and inspection errors; and the relevant literature on defective items and inspection errors is summarized below.

2.2. Inventory models with defective items (and inspection errors). It is common to all industries (e.g., children’s toys and electronics) that a certain percent of produced/ordered items are of imperfect quality [45]. In reality, it is found that a product with the defective rate of 2% or less can be deemed to possess a high quality [30]. In order to deliver high-quality items to consumers, the retailer has an incentive to identify all defects. As Wal-Mart’s recent report indicates, the largest retailer in the world has allocated considerable human resources for its quality assurance program [30]. In this regard, Salameh and Jaber [44] develop an EOQ model in which each ordered lot contains a random proportion of defects. They assume that upon receiving a lot, the retailer inspects the quality of each item in a lot at a screening rate faster than the demand rate to ensure the normal sales process. Under this assumption, they suggest that the optimal order quantity
increases as defective rate increases. The work of Salameh and Jaber [44] has been extended by several researchers, who deal with learning in screening (Jaber et al. [23]), quality improvement (Yoo et al. [59]), backlog and delay of payments (Li et al. [31]), economic production quantity model (Lin et al. [33]), and supply chain coordination (Treviño-Garza et al. [54]).

Whereas the screening rate is considered as a parameter in all the above studies, it is treated as a decision variable by the following scholars. Hauck and Vörös [18] consider that the retailer can improve the screening rate by incurring an extra investment cost, which is related to investing in new facilities and devices used in the inspection process or enlarging the staff in the quality control department. For general and specific investment cost functions, two models are developed to determine not only the optimal lot sizes, but also the screening speed. Assuming that the screening rate is an arbitrary function of time and further extending that it follows learning and forgetting curves with allowed shortages, Alamri et al. [3] indicate that the presence of varying screening rate significantly affects the optimal order quantity. Whereas Hauck and Vörös [18] propose that increasing the screening rate in the regular selling process can reduce the inventory holding cost by discarding defects more quickly, we provide a new perspective that in the TPR situation where the demand increases sharply with a lower price, enlarging the screening rate becomes necessary not only to reduce the rapidly rising inventory, but also to avoid the potential demand loss. Through examining the TPR situation with a sharply increased demand, we identify an important issue that the original screening rate may become insufficient to satisfy the new demand.

Furthermore, the screening process in reality is typically not perfect. That is, during the screening activity, the retailer may either misclassify non-defective items as defective (a Type I inspection error) or misclassify defective items as non-defective (a Type II error). Both inspection errors bring cost to the retailer, but the cost of accepting a defective item is typically higher than the cost of rejecting a non-defective item, since a Type II error involves a penalty cost due to goodwill loss from consumers’ quality dissatisfaction. Then how do the two-way inspection errors affect the retailer’s optimal order quantity and profit? Through analyzing an EOQ model with defective items and two-way inspection errors, Khan et al. [27] derive that comparing with the results of [44], the optimal order quantity is almost the same but the annual profit is much smaller, reflecting the negative impacts of inspection errors on the retailer’s profit. Hsu [19] points out a contradiction between the cycle length and the holding cost per cycle in [27] and fixes the flaw by developing a corrected EOQ. Hsu and Hsu [20] develop one EOQ model considering shortage backlogging, and another applying the overlapping approach if the shortages are not allowed. Considering the impact of trade credit, Chang et al. [11] indicate that a higher defective rate causes longer inspection times and larger order quantity, but lower annual profit and replenishment cycle length. Öztürk [41] assumes that a sub-lot inspection policy which contains inspection errors is adopted to screen defective items and that each shipment is sent by a distant supplier. They determine the optimal order size and sample size, and indicate that the local purchase of replacements for defective items tends to be more profitable than reworking them. Other studies incorporate the impact of two-way inspection errors into EPQ (economic production quantity) models; see [58, 21]. Furthermore, some recent papers identify the issue within the supply chain framework. For example, incorporating both the buyer’s inspection errors and the vendor’s learning in production, Khan
et al. [26] suggest that Type I error has a more pronounced effect on the costs of the integrated vendor-buyer supply chain compared to Type II error. Considering stochastic lead time and freight cost, Wangsa and Wee [57] also derive that Type I error affects the vendor-buyer supply chain cost more significantly and that lead time can be shortened if the vendor speeds up the shipment.

All these studies have examined how inspection errors influence the lot-sizing decisions during a regular ordering process when the screening rate can always satisfy the market demand. However, in the TPR situation where the demand increases sharply due to a lower retail price, the original screening rate may become insufficient. Moreover, this issue of insufficient screening rate becomes even more severe when incorporating the two-way inspection errors. Our study explicitly addresses this issue and proposes two strategies to handle it: (Strategy 1) the retailer controls the promotional demand rate (CPD), where the new demand can still be satisfied by (or is constrained by) the original screening rate, and (Strategy 2) the retailer increases the screening rate (ISR), where the increased screening rate can always satisfy the new demand. Then, how does a higher percentage of Type I error/Type II error affect not only the promotional pricing and special order quantity, but also the retailer’s screening rate choice (i.e., strategy choice)? Through identifying the issue of possible insufficient screening rate in the TPR situation, we contribute to the existing literature by treating the retailer’s screening rate during promotion as a decision variable.

2.3. **Inventory models under TPR with defective items.** Joining the above inventory models on TPR (or equivalently, an announced price increase) and those on defective items, a growing literature studies the retailer’s optimal special order quantity and identifies how the portion of defective items affects the retailer’s profit during promotion; see [22, 60, 62]. All these three studies assume a constant demand rate, implying that the screening rate which satisfies the demand rate during the regular selling period, can also satisfy the demand rate during the special sale period. These studies also assume that the screening process is perfect without any inspection errors. However, in a price-sensitive market in practice, the promotional demand that typically increases sharply with a lower price, may cause an issue that the original screening rate may become insufficient to inspect enough good items to satisfy this promotional demand. Moreover, the screening process, which is generally imperfect with two-way inspection errors, makes the issue of insufficient screening rate even more severe.

Then how would the retailer react to solve this problem? Our paper exclusively addresses this issue and proposes two strategies to handle it, namely, the CPD and the ISR strategy mentioned in subsection 2.2. Specifically, the CPD strategy helps save an additional investment cost of increasing the screening rate, but it makes the retailer bear a higher inventory holding cost due to a constraint on the promotional demand (i.e., a lower promotional demand). By contrast, the ISR strategy effectively reduces the inventory holding cost through enabling a higher demand, but it introduces an additional investment cost. These different advantages make the retailer’s trade-off, and hence promotional decisions and profits diverse between the two strategies. Then how does an increase in the value of some relevant parameters, like the portion of defective items, the probability of Type I and Type II errors, the discount size, the investment cost of increasing the screening rate, and price sensitivity, affect the retailer’s optimal pricing, special order quantity and
screening rate differently between CPD and ISR? Under what conditions will the retailer choose CPD over ISR?

To answer the above questions, we establish the retailer’s profit function under each strategy, considering that each ordered lot contains a portion of defective items and the screening process suffers from two-way inspection errors. Specifically, the retailer determines the optimal promotional price and special order quantity under CPD, whereas under ISR, he additionally decides the improved new screening rate. Then, based on the derived optimal solution, we conduct sensitivity analysis which illustrates how an increase in the value of relevant parameters affects the decisions and profits of the retailer under the CPD and the ISR strategy. Finally, through comparing the retailer’s profit between the two strategies, we analyze how the value of each main parameter affects the retailer’s strategy choice and the mechanism behind the choice. In this way, we not only answer the proposed research questions, but also provide managerial insights to relevant industries.

2.4. Models on robust and fuzzy. Lotfi et al. [34] study a closed-loop supply chain by incorporating sustainability, resilience, robustness, and risk aversion for the first time. The results indicate that the robust counterpart, which is used to handle uncertainties, provides a better estimation of the total cost, pollution, energy consumption, and employment level compared to the basic model. Zare Mehrjerdi and Lotfi [61] also explore a closed-loop supply chain taking into account the same issues as in [34], where sustainability goals include minimizing the costs, CO₂ emission, and energy, and maximizing the employment. Using a robust approach, Lotfi et al. [37] explore the time-cost-quality-energy-environment problem in executing projects, and model the problem by applying robust nonlinear programming (NLP) involving the objectives of cost, quality, energy, and pollution level with resource constrained. The results show that as uncertainty increases, the cost and quality will improve, and pollution and energy will decrease. Recent studies apply robust optimization to other interesting domains. Goli et al. [14] show the effect of robustness in optimizing the just-in-time flow shop scheduling problem with uncertain processing time. Goli et al. [16] apply a hybrid improved artificial intelligence and robust optimization to address the product portfolio problem under return uncertainty. Sangaiah et al. [46] propose a robust mixed-integer linear programming model for liquefied natural gas sales planning.

Other studies apply fuzzy sets theory to the EOQ models. For example, Chang [10] proposes two fuzzy inventory models with defective items. The first model is developed with fuzzy defective rate, while the second is established with both fuzzy defective rate and fuzzy annual demand. Patro et al. [42] establish both crisp and fuzzy EOQ models with proportionate discount (discount increases when the number of defects decreases in each lot) for items with imperfect quality; in the fuzzy models, the fuzzy numbers are used for defective rate, demand rate and/or costs. Also, the impacts of the two-way inspection errors are incorporated into the models. For more research, readers are suggested to refer to [25, 38].

To clearly distinguish our model from the existing literature, we list some of the above-mentioned studies in Table 1.

3. Model.

3.1. Notations, assumptions and specialized problem definition. Generalized problem definition. To derive the retailer’s promotional decisions in the
Table 1. Brief literature on inventory models under limited-time price incentives and/or with defective items

| Authors                        | Limited-time price incentives | Constant demand | Price-sensitive demand | Imperfect quality | Increase the screening rate | Inspection errors | Fuzzy | Robust |
|--------------------------------|-------------------------------|-----------------|------------------------|-------------------|-----------------------------|-------------------|-------|--------|
| Ardalan [6]                    | ✓                             | ✓               |                        |                   |                             |                   |       |        |
| Abad [2]                       | ✓                             | ✓               |                        |                   |                             |                   |       |        |
| Su and Geunes [50]             | ✓                             | ✓               |                        |                   |                             |                   |       |        |
| Shah and Naik [49]             | ✓                             | ✓               |                        |                   |                             |                   |       |        |
| Salameh and Jaber [44]         |                               | ✓               |                        |                   |                             |                   |       |        |
| Hauck and Vörös [18]           | ✓                             | ✓               | ✓                      |                   |                             |                   |       |        |
| Khan et al. [27]               | ✓                             | ✓               |                        | ✓                 |                             |                   |       |        |
| Wangsa and Wee [57]            | ✓                             | ✓               |                        |                   |                             |                   |       |        |
| Hsu and Hsu [20]               | ✓                             | ✓               |                        |                   | ✓                           |                   |       |        |
| Hsu and Yu [22]                | ✓                             | ✓               |                        |                   |                             |                   | ✓     |        |
| Zhou et al. [62]               | ✓                             | ✓               |                        |                   |                             |                   |       |        |
| Lotfi et al. [34]              |                               |                 |                        |                   |                             | ✓                 |       |        |
| Zare                           |                               |                 |                        |                   |                             | ✓                 |       |        |
| Mehrjerdi and Lotfi [61]       |                               |                 |                        |                   |                             |                   |       |        |
| Lotfi et al. [37]              |                               |                 |                        |                   |                             |                   |       |        |
| Chang [10]                     | ✓                             | ✓               |                        |                   |                             | ✓                 |       |        |
| Patro et al. [42]              | ✓                             | ✓               | ✓                      |                   | ✓                           |                   |       |        |
| Present paper                  | ✓                             | ✓               | ✓                      | ✓                 | ✓                           | ✓                 |       |        |
TPR situation with price-sensitive demand, three steps are required. The three steps provide a standard procedure to establish the retailer’s objective function under TPR, as previously stated by Taleizadeh et al. [52]. Firstly, we determine the retailer’s optimal regular order quantity $Q^*_0$ to maximize his annual profit. Secondly, when the unit purchasing cost is reduced from $c$ to $c - k$, the retailer can choose from two ordering and pricing policies. (1) The retailer continues on the regular ordering and pricing policy (denoted as the regular order policy). That is, the retailer orders the regular order quantity $Q^*_0$ and sells the items at the regular unit selling price. It should be noted that the retailer can obtain extra profit for the first order at time 0 due to the reduced purchasing cost $c - k$. (2) The retailer adopts the special order policy. That is, the retailer orders a special quantity $Q_1$ to take advantage of the reduced purchasing cost $c - k$ and charges a reduced retail price $s_1$ temporarily to increase the demand. Thirdly, the retailer determines $s_1$ and $Q_1$ by maximizing the extra profit earned from the special order policy, i.e., the profit difference between the special and the regular order policy.

Notations. Referring to [36, 18, 20], The following notations and assumptions are used to model the problem.

Assumptions. We employ the following assumptions to develop the retailer’s profit functions under both the regular situation and the TPR situation.

(1) A single product is considered.

(2) The demand rate (per day) is a linearly decreasing function of the selling price. Let $D_0 = a - bs_0$ denote the demand rate at the regular unit selling price $s_0$, and $D_1 = a - bs_1$ denote the demand rate at the special sale price $s_1$, where $a > 0$ is the scaling factor and $b > 0$ is the price-elasticity coefficient.

(3) Since the inventory carrying cost is $r$, the holding cost rate during the regular selling period is $rc$, whereas the holding cost rate during the special sale period is $r(c - k)$.

(4) Under the regular order policy of an inventory system with defective items and inspection errors, the screening activity and the demand satisfaction proceed simultaneously, while it is required that during the screening time, the number of items screened as good can at least satisfy the demand, i.e., $(1 - p_e)x_0 \geq (D_0)$; see [20].

(5) Consumers who receive the defective items can detect the quality problems and return them immediately for full price refunds.

(6) Referring to [1], we assume that temporary price reduction is offered at time 0 and coincides with a regular replenishment time. Moreover, there is only one chance for the retailer to place an order at the reduced wholesale price.

(7) Shortages are not permitted.

Specialized Problem definition. When defective items and inspection errors are taken into account, the retailer who determines the optimal promotional decisions needs to not only adjust the profit function (in both the regular and the special situations), but also consider the issue of possible insufficient screening rate. Given these changes, the specialized problem definition is clarified with three steps. Firstly, as a benchmark for comparison, we determine the retailer’s optimal regular order quantity $Q^*_0$ by maximizing the total profit per cycle, considering an inventory model with defective items and inspection errors; see [20], except that we do not allow shortages. Specifically, the presence of defective items and inspection errors changes the retailer’s objective function through three forces: (1) changing his revenue structure, (2) altering his inventory holding cost (via changing the behavior
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Parameters:

- $A$: the ordering cost per order
- $r$: the inventory carrying cost
- $c$: the purchasing cost per unit
- $x_0$: the screening rate
- $d$: the unit screening cost
- $v$: the unit selling price of defective items, $v < c$
- $s_0$: the regular unit selling price of items of good quality
- $T_0$: retailer’s regular replenishment cycle length
- $Q_0$: the retailer’s regular order quantity
- $p$: percentage of defective items for each order
- $m_1$: the probability of Type I error (classifying a non-defective item as defective)
- $m_2$: the probability of Type II error (classifying a defective item as non-defective)
- $p_e$: percentage of defective items observed by the retailer through screening, $p_e = (1 - p)m_1 + p(1 - m_2)$
- $c_a$: the cost of accepting a defective item
- $c_r$: the cost of rejecting a non-defective item
- $B_{01}$: the number of items that are classified as defective in one cycle
- $B_{02}$: the number of defective items returned from the market in one cycle
- $N$: the number of working days in a year
- $k$: supplier’s unit price discount to the retailer
- $B_1$: the number of items classified as defects during the special sale time
- $B_2$: the number of defective items that are returned from the market during the special sale time

Decision variables:

- $s_1$: the retailer’s special sale price of items of good quality
- $Q_1$: the retailer’s special order quantity
- $z$: the ratio of the promotional demand rate to the new screening rate, i.e., $z = \frac{D_1}{x}$, and $z_{max} = \frac{D_1}{x_0}$, where $x$ denotes the screening rate after the retailer’s investment in a sales promotion (the new screening rate). And $x = x_0$ strands for the CPD strategy, while $x > x_0$ strands for the ISR strategy. We also note that $g(z)$ represents the cost of increasing the screening rate from the current $x_0$ to $x$ per unit time.

Objectives:

- $G^{(1)}(s_1, Q_1)$: The retailer’s additional profit under the CPD strategy
- $G^{(2)}(s_1, Q_1, z)$: The retailer’s additional profit under the ISR strategy

of inventory level over time) and the expression of the regular replenishment cycle length, and (3) adding a screening cost including the cost of both inspection and misclassifications. We analyze the revenue and cost functions in subsection 3.2, while in this part, we only illustrate how inventory level changes over time, as depicted in Figure 1. From Figure 1, the retailer orders $Q_0$ unit at time 0 when he
conducts a screening activity at rate $x_0$ to identify the defective items. The screening period is $Q_0$, and at time $Q_0$, the batch classified as defective $B_{01}$ is sold at a reduced unit price $v$. Due to Type II error, items classified as non-defective contain a portion of defects, which are sold to and then returned by final consumers, with the size being $B_{02}$. Based on the behavior of inventory level, we can compute the inventory holding cost in the regular situation. Specifically, to ensure a normal selling process during $[0, Q_0]$, it is assumed that the number of items screened as good can at least satisfy the demand $D_0$. Given the expression of the revenue and the cost, we establish the retailer’s profit function and determine the optimal regular order quantity $Q_0^\star$.

![Figure 1. Retailer’s regular order policy](image)

Secondly, without loss of generality, consider that at time 0 in Figure 2, TPR is available and there is only one chance that the purchasing cost per unit $c$ is reduced to $c - k$. In a price-sensitive market, the retailer can choose from one of the two ordering and pricing policies. (1) The retailer adopts the regular policy. That is, the retailer orders the regular order quantity $Q_0^\star$ and sells the items at the regular unit selling price (inventory variation is same with that in Figure 1). We note that only for the first order at time 0 can the retailer obtain extra profit due to the reduced purchasing cost $c - k$. (2) The retailer adopts the special order policy, whose behavior of the inventory level is shown in Figure 2. The retailer orders a special quantity $Q_1$ to take advantage of the reduced purchasing cost $c - k$ and charges a lower retail price $s_1$ temporarily to increase the demand to $D_1$ from time 0 to $T_1$. Then at time $T_1$, the model switches back to the original EOQ model. The behavior of the inventory level under the special order policy is similar with that under the regular order policy except some differences in parameters, thus we omit the description.

Then, the retailer’s profit function during $T_1$ under the regular and the special order policy is built respectively. Under the regular policy, the retailer’s profit function during $T_1$ is established and denoted as $TP_r$. By contrast, under the special policy, the issue of possible insufficient screening rate makes the retailer choose either the CPD or the ISR strategy; under each strategy, we develop the retailer’s profit function, denoted as $TP_s^{(i)}$, where $i = 1, 2$ represents different strategies.
Thirdly, the retailer’s objective function under the CPD and the ISR strategy is developed respectively, by maximizing the difference between the special and the regular order policy (i.e., $\max G^{(i)} = \text{TP}_s^{(i)} - \text{TP}_r^{(i)}$).

### 3.2. Model formulation.

#### 3.2.1. Retailer’s optimal order quantity under the regular situation.

As mentioned previously, we first determine the retailer’s optimal order quantity under the regular selling situation as the benchmark for the subsequent analysis of his decision problem in the TPR situation. Referring to the model analyzed in [20] (see Figure 1), the total profit per cycle $\text{TP}(Q_0)$, consisting of sales of good-quality items, sales of salvaging items, procurement cost, screening cost (which is the sum of the cost of inspection and misclassifications) and holding cost, is established as

$$\text{TP}(Q_0) = s_0(1-p)(1-m_1)Q_0 + v[p + (1-p)m_1]Q_0 - (A + cQ_0)$$

$$- [d + c_a pm_2 + c_r (1-p)m_1]Q_0 - rc\frac{pe}{x_0} + \frac{(1-p_e)^2}{2D_0} + \frac{pm_2(1-p_e)}{2D_0}Q_0^2,$$

(1)

Since the retailer’s regular replenishment cycle length is $T_0 = \frac{1-p_e)Q_0}{D_0}$ and the number of working days in a year is $N$, the number of orders per year is $\frac{N}{T_0} = \frac{ND_0}{(1-p_e)Q_0}$. Thus, the annual profit is written as

$$\text{TPU}(Q_0) = \frac{ND_0}{(1-p_e)Q_0} \text{TP}(Q_0),$$

(2)

The optimal order size that maximizes Equation (2) is

$$Q_0^* = \sqrt{\frac{2AD_0x_0}{rc[2p_eD_0 + (1-p_e)^2x_0 + pm_2(1-p_e)x_0]}},$$

(3)
Substituting Equation (3) into Equation (2), the retailer’s optimal annual profit is obtained and denoted as $TPU(Q_0^*)$.

3.2.2. The CPD strategy. When TPR is offered, the retailer can choose either the regular policy or the special policy. The retailer’s profit functions during the length of special sale time $T_1$ are established under the two policies.

Under the regular policy (see Figure 1), the retailer orders the regular EOQ model’s order quantity $Q_0^*$ at the reduced purchasing cost $c - k$ for the first order at time 0, and his profit in a cycle length, denoted as $TP_1(Q_0^*)$, is given by

$$
TP_1(Q_0^*) = s_0(1 - p)(1 - m_1)Q_0^* + v[p + (1 - p)m_1]Q_0^* - [A + (c - k)Q_0^*] \\
- [d + c_a pm_2 + c_r (1 - p)m_1]Q_0^* \\
- r(c - k)[\frac{pm_2(1 - p_e)}{2D_0} + \frac{pm_2(1 - p_e)}{2D_0}]Q_0^*.
$$

(4)

For the next orders, the retailer orders $Q_0^*$ at the regular purchasing cost $c$, and his profit each day is $\frac{TPU(Q_0^*)}{N}$. Therefore, $TP_r^{(1)}$ is written as

$$
TP_r^{(1)} = TP_1(Q_0^*) + (T_1 - T_0^*) \frac{TPU(Q_0^*)}{N}.
$$

(5)

Under the special policy, a special order of size $Q_1$ is placed at time 0 with a discounted purchasing cost $c - k$. Being faced with the insufficient screening rate problem, the retailer can control the promotional demand rate (i.e., the CPD strategy) to the extent that the original screening rate $x_0$ can still satisfy, namely, $D_1$ is restricted by a constraint $(1 - p_e)x_0 \geq D_1$. Figure 2 illustrates this CPD strategy: the screening and consumption of the inventory continue until time $\frac{Q_1}{x_0}$, after which all the items classified as defective ($B_1$ in Figure 2) are withdrawn from the inventory as a single batch and are sold to the secondary market at the unit selling price $v$; then the consumption process continues at the promotional demand rate $D_1$ until the end of the special sale time $T_1$.

Since the imperfect screening process generates both Type I and Type II inspection errors, it is necessary to first calculate the number of items related to the two-way inspection errors. The number of non-defective items misclassified as defective from Type I error is $(1 - p)m_1Q_1$. Adding this result with the number of defective items classified as defective, $p(1 - m_2)Q_1$, we can obtain the total number of items classified as defective ($B_1$ in Figure 2) as

$$
B_1 = (1 - p)m_1Q_1 + p(1 - m_2)Q_1 = p_c Q_1,
$$

(6)

The number of defective items returned from the market due to Type II error ($B_2$ in Figure 2) is

$$
B_2 = pm_2Q_1.
$$

(7)

From Figure 2, the length of the special sale time is

$$
T_1 = \frac{Q_1 - B_1}{D_1} = \frac{(1 - p_c)Q_1}{D_1}.
$$

(8)

Thus, under the special policy, the relevant revenue and costs are given below.

1. The total revenue during $T_1$ is $s_1(1 - p)(1 - m_1)Q_1 + v[p + (1 - p)m_1]Q_1$.

2. The procurement cost is $A + (c - k)Q_1$.

3. The screening cost, which is the sum of the cost of inspection and misclassifications, is $dQ_1 + c_a pm_2Q_1 + c_r (1 - p)m_1Q_1 = scQ_1$, where

$$
sc = d + c_a pm_2 + c_r (1 - p)m_1.
$$

(9)
denotes the average screening cost of each item.

(4) The holding cost is the cost of carrying the (i) non-defective lot, (ii) defective lot and (iii) returned lot. So, from Figure 2 the holding cost is \( r(c - k)(\frac{\Delta s_1 A_1}{2} + \frac{D_1 T_{11}^2}{2} + \frac{B_x T_1^2}{2}) = r(c - k)[\frac{p_c}{x_0} + \frac{(1-p_c)^2}{2D_1} + \frac{pm_2(1-p_c)}{2D_1}]Q_1^2 \). Synthesizing the above revenue and costs, under the special policy, the retailer’s total profit during \( T_1 \) is

\[
TP_s^{(1)}(s_1, Q_1) = s_1(1 - p)(1 - m_1)Q_1 + v[p + (1 - p)m_1]Q_1 - [A + (c - k)Q_1] - scQ_1 - r(c - k)[\frac{p_c}{x_0} + \frac{(1-p_c)^2}{2D_1} + \frac{pm_2(1-p_c)}{2D_1}]Q_1^2.
\]

(10)

Based on Equations (5) and (10), the retailer’s objective function under CPD is

\[
G^{(1)}(s_1, Q_1) = TP_{s_1}^{(1)}(s_1, Q_1) - TP_{r_1}^{(1)}(s_1, Q_1)
= s_1(1 - p)(1 - m_1)Q_1 + v[p + (1 - p)m_1]Q_1 - [A + (c - k)Q_1] - scQ_1 - r(c - k)[\frac{p_c}{x_0} + \frac{(1-p_c)^2}{2D_1} + \frac{pm_2(1-p_c)}{2D_1}]Q_1^2
- TP_1(Q_0^*) - \left[ \frac{(1-p_c)Q_1}{D_1} - \frac{(1-p_c)Q_0^*}{D_0} \right] \frac{TPU(Q_0^*)}{N}
\]

subject to

\[
D_1 = a - bs_1,
\]

\[
(1 - p_c)x_0 \geq D_1.
\]

3.2.3. The ISR strategy. When TPR is offered, the retailer can choose from either the regular policy or the special policy. Under the regular policy, the inventory diagram is the same with Figure 1; and consistent with the analysis process in 3.2.2, the retailer’s profit function is obtained as

\[
TP_r^{(2)} = TP_1(Q_0^*) + (T_1 - T_0^*) \frac{TPU(Q_0^*)}{N},
\]

(12)

Under the special policy, the retailer places a special order quantity \( Q_1 \) at time \( 0 \) with a discounted purchasing cost \( c - k \) and sells the product at a lower price \( s_1 \), which generates an increased demand \( D_1 \). Being faced with the insufficient screening rate problem, the retailer improves the screening rate from the original \( x_0 \) to \( x \) (i.e., the ISR strategy) to the degree that can satisfy whatever level of the increased demand rate, that is, \((1 - p_c)x \geq D_1 \). Figure 3 illustrates this ISR strategy: the screening and consumption of the inventory continue until time \( \frac{Q_1}{x} \), after which all the items classified as defective (\( B_1 \) in Figure 3) are withdrawn from the inventory as a single batch and sold at the unit price \( v \); then the consumption process continues at the promotional demand rate \( D_1 \) until the end of the special sale time \( T_1 \).

Due to a looser constraint on demand under ISR, the promotional demand under ISR is higher than that under CPD, which makes Figure 3 differ from Figure 2 in two ways. (1) The special sale time under ISR is shorter than that under CPD; and (2) the inventory holding cost per unit under ISR is lower than that under CPD. Also, due to an increased screening rate under ISR, the screening time under ISR (i.e., \( \frac{Q_1}{x} \) in Figure 3) is shorter than that under CPD (i.e., \( \frac{Q_1}{x_0} \) in Figure 2).
Figure 3. Retailer’s special order policy with TPR at time 0 (corresponding to ISR)

The expression of $B_1$ and $B_2$ under the ISR strategy are the same with those under the CPD strategy, so we give the results: $B_1 = p_1 Q_1$, $B_2 = p m_2 Q_1$. The special sale time under ISR is $T_1 = (1 - p_e) Q_1$.

Under the special policy, whereas the expressions of the revenue, the procurement cost and the screening cost under ISR are the same with those under CPD, the holding cost is different, which is given below. In addition, the retailer incurs an extra investment cost of increasing the screening rate.

(1) The holding cost is $r(c - k) \left( \frac{p_e}{2D_1} + \frac{(1-p_e)^2}{2D_1} + \frac{p m_2 (1-p_e)}{2D_1} \right) Q_1^2$.

(2) The retailer incurs an investment cost of increasing the screening rate from $x_0$ to $x$. Since the cost per unit time is $g(z) = \frac{M z}{x}$, where $z = \frac{D_1}{x}$, and the screening period is $Q_1$, the investment cost is $\frac{Q_1 g(z)}{x} = \frac{MQ_1}{D_1 z}$.

Therefore, under ISR, the retailer’s total profit within $T_1$ is

$$TP_s^{(2)}(s_1, Q_1, z) = s_1 (1 - p) (1 - m_1) Q_1 + v [p + (1 - p) m_1] Q_1 - [A + (c - k) Q_1]
- sc Q_1 - r(c - k) \left[ \frac{p_e z}{D_1} + \frac{(1-p_e)^2}{2D_1} + \frac{p m_2 (1-p_e)}{2D_1} \right] Q_1^2 - \frac{MQ_1}{D_1 z}. \quad \text{(13)}$$

According to Equations (12) and (13), we have

$$G^{(2)}(s_1, Q_1, z) = TP_s^{(2)}(s_1, Q_1, z) - TP_r^{(2)}(s_1, Q_1)
= s_1 (1 - p) (1 - m_1) Q_1 + v [p + (1 - p) m_1] Q_1 - [A + (c - k) Q_1]
- sc Q_1 - r(c - k) \left[ \frac{p_e z}{D_1} + \frac{(1-p_e)^2}{2D_1} + \frac{p m_2 (1-p_e)}{2D_1} \right] Q_1^2 - \frac{MQ_1}{D_1 z}.$$
subject to

\[ D_1 = a - bs_1, \]

\[ (1 - p_e)x_0 < D_1, \]

\[ z = \frac{D_1}{x} < 1 - p_e. \]

### 3.3. Theoretical results.

**Proposition 1.** \( G^{(1)}(s_1, Q_1) \) has at most three stationary points which simultaneously satisfy \( \frac{\partial G^{(1)}(s_1, Q_1)}{\partial Q_1} = 0 \) and \( \frac{\partial G^{(1)}(s_1, Q_1)}{\partial s_1} = 0 \).

Specifically, we have the following results.

\[
\frac{\partial G^{(1)}(s_1, Q_1)}{\partial Q_1} = s_1(1 - p)(1 - m_1) + v[p + (1 - p)m_1] - (c - k) - \frac{(1 - p_e)TPU(Q_1^*)}{D_1N} - sc - r(c - k)e^{\frac{2pe}{x_0} + \frac{(1 - p_e)^2}{D_1} + \frac{pm_2(1 - p_e)}{D_1}} Q_1 = 0,
\]

\[
\frac{\partial G^{(1)}(s_1, Q_1)}{\partial s_1} = (1 - p)(1 - m_1)Q_1 - \frac{b(1 - p_e)TPU(Q_1^*)Q_1}{ND_1^2} - r(c - k)b\left[\frac{(1 - p_e)^2}{2D_1^2} + \frac{pm_2(1 - p_e)}{2D_1}\right]Q_1^2 = 0.
\]

Solving Equation (15), the following result is obtained

\[
\bar{Q}_1 = s_1(1 - p)(1 - m_1) + v[p + (1 - p)m_1] - (c - k) - \frac{(1 - p_e)TPU(Q_1^*)}{D_1N} - sc - r(c - k)e^{\frac{2pe}{x_0} + \frac{(1 - p_e)^2}{D_1} + \frac{pm_2(1 - p_e)}{D_1}}.
\]

Then, substituting Equation (17) into Equation (16) yields a cubic equation of the variable \( s_1 \), which has at most three roots.

Given the above analysis, we denote the three stationary points of \( G^{(1)}(s_1, Q_1) \) as \((\bar{s}_1^{(j)}, Q_1^{(j)})\), where \( j = 1, 2, 3 \) stands for different stationary points. If \((\bar{s}_1^{(j)}, Q_1^{(j)})\) satisfies the boundary conditions \( \bar{s}_1^{(j)} \in \left[ \frac{a - (1 - p_e)x_0}{b}, s_0 \right) \) and \( Q_1^{(j)} \in (0, +\infty) \), we substitute \((\bar{s}_1^{(h)}, Q_1^{(h)})\), where \( h \) stands for the stationary points satisfying the boundary conditions, into Equation (11) and obtain the resulting profit \( G^{(1)}(\bar{s}_1^{(h)}, Q_1^{(h)}) \). Then the largest of \( G^{(1)}(\bar{s}_1^{(h)}, Q_1^{(h)}) \) is obtained as the largest value of the stationary points.

Besides, the four boundary values are obtained as follows.

\[
(1) \quad \lim_{s_1 \to \frac{a - (1 - p_e)x_0}{b}, Q_1 \to 0} G^{(1)}(s_1, Q_1) = -A - TP_1(Q_0^*) + \frac{Q_0^*(1 - p_e)TPU(Q_0^*)}{D_0N};
\]

\[
(2) \quad \lim_{s_1 \to \frac{a - (1 - p_e)x_0}{b}, Q_1 \to +\infty} G^{(1)}(s_1, Q_1) = -\infty;
\]

\[
(3) \quad \lim_{s_1 \to 0, Q_1 \to 0} G^{(1)}(s_1, Q_1) = -A - TP_1(Q_0^*) + \frac{Q_0^*(1 - p_e)TPU(Q_0^*)}{D_0N};
\]

\[
(4) \quad \lim_{s_1 \to 0, Q_1 \to +\infty} G^{(1)}(s_1, Q_1) = -\infty.
\]

Since all the four boundary values are negative, the global maximum value of \( G^{(1)}(s_1, Q_1) \) is the largest value of the stationary points.
\[ G^{(2)}(s_1, Q_1, z) \] is a function of \( s_1, Q_1, \) and \( z \). The necessary conditions for the \( G^{(2)}(s_1, Q_1, z) \) to be maximized are \( \frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial q_1} = 0, \) \( \frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial z} = 0, \) and \( \frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial s_1} = 0 \) simultaneously. That is,

\[
\frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial q_1} = s_1(1 - p)(1 - m_1) + v[p + (1 - p)m_1] - (c - k) - \frac{(1 - p_e)TPU(Q_0^*)}{D_1N} - sc - \frac{M}{D_1z} - r(c - k)\frac{2p_c z}{D_1} + \frac{(1 - p_c)^2}{D_1} + \frac{pm_2(1 - p_c)}{D_1}Q_1 = 0,
\]

(18)

\[
\frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial z} = -r(c - k)p_cQ_1^2 + \frac{MQ_1}{D_1z^2} = 0,
\]

(19)

\[
\frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial s_1} = (1 - p)(1 - m_1)Q_1 - \frac{bMQ_1}{zD_1^2} - \frac{b(1 - p_e)TPU(Q_0^*)Q_1}{ND_1^2} - r(c - k)b\frac{p_c z}{D_1^2} + \frac{(1 - p_c)^2}{2D_1^2} + \frac{pm_2(1 - p_c)}{2D_1^2}Q_1^2 = 0.
\]

(20)

**Proposition 2.** For any given \( s_1 \), the solution of \( G^{(2)}(s_1, Q_1, z) \) which satisfies Equations (18) and (19) and \( z > 0 \), not only exists but also is unique.

**Proposition 3.** For any given \((Q_1, z)\), \( G^{(2)}(s_1, Q_1, z) \) is concave with respect to \( s_1 \), and \( G^{(2)}(s_1, Q_1, z) \) has at most one maximum point which is denoted as \( s_1^* \), where \( s_1^* \in (c - k, s_0) \) and satisfies Equation (B2).

Moreover, the decision variables of \( G^{(2)}(s_1, Q_1, z) \) are subject to \( s_1 \in (c - k, \frac{a - (1 - p_e)x_0}{b}), Q_1 \in (0, +\infty) \), and \( z \in (0, 1 - p_e] \). Thus, we can obtain the eight boundary values of \( G^{(2)}(s_1, Q_1, z) \), which are omitted since the calculation method is similar with that under \( G^{(1)}(s_1, Q_1) \).

Since all the derived eight boundary values are negative, the global maximum value of \( G^{(2)}(s_1, Q_1, z) \) would be the largest value of the stationary points.

Based on the above results, now we propose a solution procedure to obtain the optimal solution \( s_1^*, Q_1^*, z^* \) of the problem.

**3.4 Solution procedure.**

**Step 1.** Determine the optimal regular order quantity \( Q_0^* \) and the corresponding annual profit \( TPU(Q_0^*) \) by substituting Equation (3) into Equation (2).

**Step 2.** Begin with \( j = 0 \) and the initial trial value of \( s_1^j = s_1 \).

**Step 3.** Calculate \( \Delta(s_1^j) = (1 - p_e)x_0 - (a - bs_1^j) \) for a given \( s_1^j \).

**Step 3.1.** If \( \Delta(s_1^j) \geq 0 \), find the optimal \( Q_1 \) using Equation (17). Substitute \( Q_1 \) into Equation (16) and solve this equation to obtain \( s_1^{j+1} \); go to step 3.2.

**Step 3.2.** If \( \Delta(s_1^j) < 0 \), find the optimal \((Q_1^*, z^*)\) which satisfies \( z^* > 0 \) using Equations (18) and (19).

**Step 3.2.1.** If \( z^* < 1 - p_e \), substitute \((Q_1^*, z^*)\) into Equation (20) and solve this equation to obtain \( s_1^{j+1} \); go to step 4.

**Step 3.2.2.** If \( z^* > 1 - p_e \), let \( z^* = 1 - p_e \) and substitute \( z^* = 1 - p_e \) into Equation (19) and solve the corresponding value \( Q_1^* \). Then substitute \((Q_1^*, z^*)\) into Equation (20) and solve this equation to obtain \( s_1^{j+1} \); go to step 4.
Step 4. If the difference between $s_j^1$ and $s_j^{j+1}$ is sufficiently small (i.e., $s_j^1 - s_j^{j+1} < 10^{-5}$), set $s^*_1 = s_j^{j+1}$; $s^*_1$, $Q^*_1$, $z^*$ is the optimal solution and stop. Otherwise, set $j = j + 1$ and return to Step 3.

4. Sensitivity analysis and managerial insights.

4.1. Impact of main parameters on promotional policies and profits. In manufacturing, quality control is a process that ensures that consumers receive products free from defects and meet their demands. Despite the quality control activity conducted by firms, defective items still widely exist in many categories, such as medical devices, electronics, toys, and apparel. And the tolerance level of defective components can vary from industry to industry. To describe this tolerance level, acceptable quality limit (AQL) is used, referring to the maximum percent defective that can be considered satisfactory. Investopedia indicates that in practice, AQL for three categories of defects is defined as below [9]. (1) Critical Defects: Defects, when accepted could harm users. Such defects are unacceptable. Critical defects are defined as 0% AQL. For example, medical products have stringent AQLs because defective products are a health risk. (2) Major Defects: Defects usually not acceptable by the end-users, as they are likely to result in failure. The AQL for major defects is 2.5%. Examples are earphones, whose defects may cause uncomfortable use or show imperfect appearance. (3) Minor Defects: Defects not likely to reduce materially the usability of the product for its intended purpose but that differ from specified standards; some end users will still buy such products. The AQL for minor defects is 4%. For instance, minor defects of garments include untrimmed threads, missing stitches, and dirt material that can be washed off.

Our numerical examples focus on categories with major defects. Consumers who receive and discover the defects would return them for full refund. These categories include electronics, and toys and games. For example, PowerLocus, an international brand founded in 2017 who focuses on developing and producing consumer electronics like headphones. With good quality control, the company shares on its official website that it maintains products with 1% defect rate [24]. By contrast, the defect rate of other earphone brands may reach at most 2.5%, but are still considered to be acceptable. Focusing on categories with major defects like earphones and allowing a wider range of $p$, our numerical examples assume that $p$ varies between 0.001 and 0.025. We also surveyed a certain Bluetooth over-ear headphone and found that it is sold at $49.99 in the franchise store on a commercial street. And according to the sales data, the demand per day is nearly 200. Since it is hard to obtain the probability of two-way inspection errors, we use the value referring to existing papers and assume that the value of $m_1$ and $m_2$ varies between 0.01 and 0.05; see [20, 48].

Based on the above practical data and academic studies, the value of each parameter is estimated below.

Moreover, let $g(z) = \frac{Mz^2}{2}$, where $M = 32$. The demand curve is assumed below: let $D_j = a - bs_j$, $j = 0, 1$, $a = 800$, $b = 12$, then $D_0 = 200\ units/day$ and $D_1 = 800 - 12s_1\ units/day$.

4.1.1. How does a higher $p$ affect the retailer’s strategy choice? Table 3 shows how an even minor increase in $p$ changes the retailer’s strategy choice. We further use Figure 4 to illustrate the changing trends of the results in Table 3.
Table 2. Model parameters

| Parameters | Value |
|------------|-------|
| A         | $500/\text{order}$ |
| $r$        | 0.1$/\text{$/day}$ |
| $c$        | $25$/\text{unit}$ |
| $x_0$      | 242\text{units/day}$ |
| $d$        | $0.5$/\text{unit}$ |
| $v$        | $20$/\text{unit}$ |
| $s_0$      | $50$/\text{unit}$ |
| $p$        | $p$ varies between 0.001 and 0.025; for example, $p = 0.001$ |
| $m_1$      | $m_1$ varies between 0.01 to 0.05; for example, $m_1 = 0.01$ |
| $m_2$      | $m_2$ varies between 0.01 to 0.05; for example, $m_2 = 0.01$ |
| $p_e$      | $p_e = (1 - p)m_1 + p(1 - m_2)$; for example, when $p = 0.001$, $m_1 = 0.01$, and $m_2 = 0.01$, we have $p_e = 0.01098$ |
| $c_a$      | $500$/\text{unit}$ |
| $c_r$      | $100$/\text{unit}$ |
| $N$        | 365 |
| $k$        | $7.5$/\text{unit}$ |

Table 3. Effects of an increase in defect proportion $p$ on the solutions and profits under the CPD and ISR strategy ($m_1 = 0.02$, and $m_2 = 0.02$). $G^{(1)*}$ represents $G^{(1)}(s_1^*, Q_1^*)$, and $G^{(2)*}$ represents $G^{(2)}(s_1^*, Q_1^*, z^*)$. Tables 4-8 follow this marking method.

| $p$   | $Q_0$      | $s_1^*$ | $Q_1^*$ | $G^{(1)*}$ | $s_1^*$ | $Q_1^*$ | $z^*$ | $x^*$ | $G^{(2)*}$ |
|-------|------------|---------|---------|------------|---------|---------|-------|-------|------------|
| 0.001 | 283.81     | 46.92   | 1472.15 | 5221.87    | 46.02   | 1499.90 | 0.76  | 324.92| 5089.82    |
| 0.003 | 283.89     | 46.96   | 1469.48 | 5207.56    | 46.03   | 1500.85 | 0.73  | 339.41| 5084.93    |
| 0.005 | 283.97     | 47.00   | 1466.81 | 5193.30    | 46.04   | 1501.93 | 0.70  | 353.31| 5089.96    |
| 0.007 | 284.04     | 47.04   | 1464.16 | 5179.07    | 46.05   | 1503.12 | 0.67  | 366.69| 5077.82    |
| 0.009 | 284.12     | 47.08   | 1461.51 | 5164.89    | 46.06   | 1504.42 | 0.65  | 379.60| 5075.43    |
| 0.011 | 284.19     | 47.12   | 1458.87 | 5150.74    | 46.07   | 1505.81 | 0.63  | 392.09| 5073.71    |
| 0.013 | 284.26     | 47.16   | 1456.24 | 5136.64    | 46.08   | 1507.29 | 0.61  | 404.20| 5072.60    |
| 0.015 | 284.33     | 47.19   | 1453.62 | 5122.58    | 46.09   | 1508.85 | 0.59  | 415.97| 5072.06    |
| 0.017 | 284.41     | 47.23   | 1451.00 | 5108.56    | 46.10   | 1510.48 | 0.58  | 427.42| 5072.05    |
| 0.019 | 284.48     | 47.27   | 1448.40 | 5094.59    | 46.11   | 1512.18 | 0.56  | 438.58| 5072.51    |
| 0.021 | 284.54     | 47.31   | 1445.80 | 5080.66    | 46.13   | 1513.94 | 0.55  | 449.46| 5073.43    |
| 0.023 | 284.61     | 47.35   | 1443.21 | 5066.78    | 46.14   | 1515.76 | 0.54  | 460.11| 5074.77    |
| 0.025 | 284.68     | 47.39   | 1440.63 | 5052.96    | 46.15   | 1517.64 | 0.52  | 470.51| 5076.52    |

We observe several interesting results from Table 3 (or Figure 4). First, we derive that a higher $p$ affects the changing trends of the retailer’s optimal solutions and profits differently between the two strategies. Under CPD, a higher $p$ increases the optimal price, but decreases both the special order quantity and profit of the retailer. The rationale is explained below. (1) A larger unit price discount ($k = 7.5$) makes the interior point violate the inequality constraint $(1 - p_e)x_0 \geq D_1$. Thus,
the sale price is taken at the boundary: \( s^*_1 = \frac{a-(1-p_0) x_0}{b} \). A higher \( p \), which means a lower portion of good items to satisfy the demand (i.e., a lower \( 1-p_0 \)), inflates the price which is taken at the boundary condition. (2) How does a higher \( p \) affect the special order quantity, which is a trade-off between the profit margin and the inventory holding cost per unit? On one hand, although a higher \( p \) increases the special sale price, it decreases the profit margin in two ways: it not only increases the portion of items inspected as defective (whose unit price is even lower than the unit purchasing cost, i.e., \( v < c \)), but also increases the screening cost. On the other, a higher \( p \), through inflating the special sale price, decreases the promotional demand and thus increases the inventory holding cost per unit. Given that a higher \( p \) decreases the profit margin and increases the inventory holding cost per unit, it prompts the retailer to reduce the order quantity. (3) The profit drops as \( p \) increases due to not only a reduced profit margin and a decreased order quantity, but also a sharply increased inventory holding cost.

By contrast, under ISR, as \( p \) increases, the special sale price also increases, whereas the special order quantity and the screening rate increase, and the profit first decreases slightly and then increases. However, the rationale behind the rise

\[ \text{Figure 4. Comparison of the solutions and profits under the CPD and ISR strategy as } p \text{ increases} \]
in the sale price under ISR differs from that under CPD. Under ISR, a higher \( p \), through increasing the screening cost of each item, which is \( sc = d + (c_a m_2 - c_r m_1) p + c_r m_1 \), where \( c_a m_2 - c_r m_1 \) is positive, prompts the retailer to charge a higher sale price to cover this increased operational cost. Given the different rationale, the sale price under ISR rises slower than that under CPD. A higher \( p \), which means a lower portion of good items to satisfy the market demand, prompts the retailer to increase the screening rate under ISR. This increased screening rate, which not only drives up the investment cost of increasing the screening rate, but also cuts down the inventory holding cost of items screened as defects due to a reduced screening time and an insignificant decrease in demand, induces the retailer to increase the order quantity (see Equation (A1)). Finally, a higher \( p \) affects the retailer’s profit through two opposing impacts: a negative impact of rising the investment cost and inventory holding cost and a positive impact of increasing the optimal price and order quantity. Since the negative impact modestly dominates the positive impact for lower values of \( p \), whereas the positive impact becomes more dominant as \( p \) is higher than a threshold, a higher \( p \) first slightly drives down and then drives up the profit.

Second, through comparing the changing trends of profits as \( p \) increases between the two strategies, we obtain that the CPD strategy dominates the ISR strategy for smaller values of \( p \), whereas the ISR strategy becomes more profitable when the value of \( p \) is higher than a threshold. Figure 4 (4) directly reflects this result: as \( p \) increases, whereas the retailer’s profit falls rapidly under CPD, it drops slightly and then increases under ISR, and the profit under ISR is higher once \( p \) is higher than a threshold. The rationale is explained in aid of Figure 4 (1) and Figure 4 (2). From Figure 4 (1), as \( p \) increases, the sale price under CPD increases faster than that under ISR, since the former grows due to a tighter constraint (which is a more direct effect), while the latter increases due to a higher screening cost. Under CPD, this higher sale price drives down the retailer’s profit rapidly through two forces: it not only increases the inventory holding cost per unit, but also reduces the special order quantity; the latter force can be seen in Figure 4 (2). By contrast, through increasing the screening rate with an additional investment cost, the ISR strategy not only makes the retailer adjust the sale price more flexible, but also enables him to control the suddenly-increased inventory holding cost more effectively and place a larger order quantity. Thus, the ISR strategy dominates the CPD strategy for higher values of \( p \).

4.1.2. How does the impact of a higher \( m_1 \) differ from that of a higher \( m_2 \)? Below, Table 4 describes how an increase in \( m_1 \) affects the solutions and profits under the CPD and ISR strategy, whereas Table 5 shows how an increase in \( m_2 \) affects these results. To compare the impacts of a higher \( m_1 \) with those of a higher \( m_2 \), while the results in Table 4 are illustrated with Figure 5 (1), (2), (3), the results in Table 5 are visualized with Figure 5 (4), (5), (6).

Comparing Figure 5 (1) with Figure 5 (4), two results are obtained regarding the changing trends of the sale price. First, under CPD, whereas the sale price increases with a higher \( m_1 \), it slightly decreases with a higher \( m_2 \). This is because although the sale price takes at the boundary \( s_1^* = \frac{a - (1 - p_e) x_0}{b} \) for both cases, a higher \( m_1 \) inflates the sale price by decreasing the portion of items screened as “good” (i.e., by decreasing \( 1 - p_e \)), while a higher \( m_2 \) reduces the sale price by increasing the portion of items inspected as “good” (i.e., by enlarging \( 1 - p_e \)). Second, under
Figure 5. Comparison between the changing trends of the results as \( m_1 \) increases and those as \( m_2 \) increases.
Table 4. Effects of an increase in the proportion of a Type I error $m_1$ on the solutions and profits under the CPD and ISR strategy ($p = 0.02$, and $m_2 = 0.02$)

| $m_1$ | $Q^*_0$ | the CPD strategy | the ISR strategy |
|-------|---------|-----------------|-----------------|
|       | $s_1^*$ | $Q_1^*$ | $G^{(1)*}$ | $s_t^*$ | $Q_t^*$ | $z^*$ | $x^*$ | $G^{(2)*}$ |
| 0.010 | 284.12  | 47.09 | 1478.09 | 5353.91 | 45.62 | 1547.53 | 0.6340 | 398.43 | 5346.96 |
| 0.015 | 284.32  | 47.19 | 1462.26 | 5217.06 | 45.87 | 1529.80 | 0.5903 | 422.82 | 5203.96 |
| 0.020 | 284.51  | 47.30 | 1447.10 | 5087.62 | 46.12 | 1513.05 | 0.5552 | 444.05 | 5072.92 |
| 0.025 | 284.70  | 47.39 | 1432.59 | 4965.51 | 46.38 | 1497.22 | 0.5263 | 462.63 | 4953.37 |
| 0.030 | 284.88  | 47.49 | 1418.73 | 4850.64 | 46.64 | 1482.26 | 0.5018 | 444.05 | 4845.05 |
| 0.035 | 285.05  | 47.59 | 1405.54 | 4742.93 | 46.91 | 1468.16 | 0.4807 | 422.82 | 4747.85 |
| 0.040 | 285.22  | 47.69 | 1393.00 | 4642.31 | 47.18 | 1454.91 | 0.4623 | 405.74 | 4661.78 |
| 0.045 | 285.38  | 47.91 | 1377.04 | 4549.81 | 47.46 | 1442.51 | 0.4461 | 393.23 | 4586.92 |
| 0.050 | 285.53  | 48.23 | 1359.58 | 4469.73 | 47.74 | 1430.96 | 0.4316 | 380.93 | 4523.48 |

Table 5. Effects of an increase in the proportion of a Type II error $m_2$ on the solutions and profits under the CPD and ISR strategy ($p = 0.02$, and $m_1 = 0.02$)

| $m_2$ | $Q^*_0$ | the CPD strategy | the ISR strategy |
|-------|---------|-----------------|-----------------|
|       | $s_1^*$ | $Q_1^*$ | $G^{(1)*}$ | $s_t^*$ | $Q_t^*$ | $z^*$ | $x^*$ | $G^{(2)*}$ |
| 0.010 | 284.55  | 47.30 | 1449.00 | 5108.07 | 46.07 | 1521.57 | 0.5347 | 462.17 | 5126.40 |
| 0.015 | 284.53  | 47.30 | 1448.05 | 5097.85 | 46.10 | 1518.91 | 0.5359 | 460.61 | 5110.85 |
| 0.020 | 284.51  | 47.30 | 1447.10 | 5087.62 | 46.12 | 1513.05 | 0.5552 | 444.05 | 5072.92 |
| 0.025 | 284.49  | 47.29 | 1446.14 | 5077.39 | 46.15 | 1510.42 | 0.5564 | 442.55 | 5057.65 |
| 0.030 | 284.47  | 47.29 | 1445.19 | 5067.15 | 46.17 | 1507.80 | 0.5576 | 441.06 | 5042.48 |
| 0.035 | 284.46  | 47.29 | 1444.23 | 5056.90 | 46.20 | 1505.19 | 0.5588 | 439.56 | 5027.42 |
| 0.040 | 284.44  | 47.29 | 1443.28 | 5046.65 | 46.22 | 1502.59 | 0.5600 | 438.07 | 5012.46 |
| 0.045 | 284.42  | 47.29 | 1442.32 | 5036.39 | 46.25 | 1500.00 | 0.5612 | 436.58 | 4997.62 |
| 0.050 | 284.40  | 47.28 | 1441.36 | 5026.12 | 46.27 | 1497.42 | 0.5625 | 435.10 | 4982.86 |

ISR, a higher $m_1$ drives up the sale price more significantly than a higher $m_2$. Because although a higher value of both $m_1$ and $m_2$ inflates the sale price through increasing the screening cost, the former raises this cost through misclassifying the overwhelming majority of non-defects (see $sc = d + c_a p m_2 + c_r (1 - p) m_1$), whereas the latter raises this cost by misclassifying the tiny minority of defects (see $sc = d + c_a p m_2 + c_r (1 - p) m_1$). Since $c_r (1 - p) > c_a p$ always holds, the sale price increases more significantly with a higher $m_1$ than with a higher $m_2$.

Based on the above results, we analyze how a higher $m_1$ ($m_2$) affects the order quantity. Comparing Figure 5 (2) with Figure 5 (5), it is obtained that a higher $m_1$ drives down the order quantity more significantly than a higher $m_2$, under both CPD and ISR. The rationale is that under CPD, the order quantity is a trade-off between the profit margin and the inventory holding cost per unit. Since a higher $m_1$ drives up the inventory holding cost per unit more significantly than a higher $m_2$ (though increasing the price more sharply and hence reducing demand more significantly), a higher $m_1$ decreases the order quantity more quickly. Two factors lead to the comparison result of the order quantity under ISR (which is expressed as $\frac{M}{(c - k) p z^*}$):
(1) a higher $m_1$ drives down the order quantity by increasing $p_e$, whereas a higher $m_2$ drives up the order quantity by decreasing $p_e$; see $p_e = (1 - p)m_1 + p(1 - m_2)$; and (2) a higher $m_1$ drives up the order quantity by reducing $z$, whereas a higher $m_2$ drives down the order quantity by increasing $z$ a little; see Table 4 and Table 5. Since the first factor plays a more dominant role, a higher $m_1$ drives down the order quantity more significantly than a higher $m_2$.

Comparing the changing trend of the screening rate under ISR between Table 4 and Table 5, a higher $m_1$ significantly increases the screening rate by reducing the portion of screened good items, whereas a higher $m_2$ slightly decreases the screening rate by mistakenly classifying defects as non-defects and hence rising the portion of screened “good” items.

Given the above analysis that under both CPD and ISR, a higher $m_1$ exerts a more significant impact on the retailer’s solutions than a higher $m_2$. Thus, a higher $m_1$ drives down the retailer’s profit faster than a higher $m_2$ under both strategies; compare Figure 5 (3) with Figure 5 (6).

Finally, we derive the retailer’s strategy choice for different values of $m_1$ and $m_2$. (1) Observing Figure 5 (3), the ISR strategy slightly dominates the CPD strategy when $m_1$ is higher than a threshold. This is because whereas the increasing (decreasing) rate of the sale price (the order quantity) under ISR as $m_1$ grows is rather stable, the increasing (decreasing) rate of the sale price (the order quantity) under CPD as $m_1$ grows becomes larger when $m_1$ is higher than a threshold. Thus, as $m_1$ is higher than a threshold, the sharply decreased special order quantity significantly hurts the retailer’s profit under CPD and makes CPD inferior to ISR. (2) Observing Figure 5 (6), the CPD strategy dominates the ISR strategy when $m_2$ is higher than a threshold. The rationale is that although a higher $m_2$ hurts the retailer’s profit under both strategies, it actually makes the sale price and order quantity remain relative stable by relaxing the constraint and also helps the retailer save extra investment cost in the CPD strategy, and hence makes the CPD strategy better than the ISR strategy.

### 4.1.3. How do marketing related parameters affect the retailer’s strategy choice?

Below, we will show how the retailer’s strategy choice is affected by some marketing related parameters, like unit price discount $k$, and the investment cost related parameter $M$.

| $k$  | $Q_0^*$ | $Q_1^*$ | $G_{(1)^*}$ | $s_1^*$ | $Q_1^*$ | $z^*$ | $x^*$ | $G_{(2)^*}$ |
|------|---------|---------|-------------|---------|---------|------|------|-------------|
| 2.5  | 284.51  | 47.68   | 603.27      | -       | -       | -    | -    | -           |
| 5    | 284.51  | 47.30   | 975.23      | 2063.16 | 46.82   | 987.63| 0.6429| 370.45      |
| 7.5  | 284.51  | 47.30   | 1447.10     | 5087.62 | 46.12   | 1513.05| 0.5552| 444.05      |
| 10   | 284.51  | 47.30   | 2076.25     | 10343.92| 45.41   | 2237.55| 0.4932| 517.20      |
| 12.5 | 284.51  | 47.30   | 2957.06     | 19171.15| 44.69   | 3280.39| 0.4462| 590.97      |
| 15   | 284.51  | 47.30   | 4278.28     | 34247.53| 43.97   | 4880.25| 0.4090| 665.97      |
| 17.5 | 284.51  | 47.30   | 531.03      | 32178.79| 43.28   | 5528.39| 0.3780| 741.32      |
| 20   | 284.51  | 47.30   | 677.27      | 40788.79| 42.58   | 6178.39| 0.3480| 816.32      |
| 22.5 | 284.51  | 47.30   | 863.27      | 50164.79| 41.88   | 6828.39| 0.3180| 891.32      |

Table 6. Effects of an increase in the unit price discount $k$ on the solutions and profits under the CPD and ISR strategy ($p = 0.02$, $m_1 = 0.02$, and $m_2 = 0.02$)
Table 6 reveals that as \( k \) increases, under both strategies, the optimal price decreases, but the special order quantity and the profit increase, and the screening rate increases in ISR. The results are rather intuitive, so we omit the explanation. Specifically, the ISR strategy becomes more dominant for higher values of \( k \). Because the original lower screening rate, through imposing a very tight constraint on the promotional demand, limits the optimization of the special price under CPD and hence worsens the CPD strategy.

Table 7. Effects of an increase in \( M \) on the solutions and profits under the ISR strategy \((p = 0.02, m_1 = 0.02, \text{ and } m_2 = 0.02)\)

| \( M \) | \( Q_0^* \) | \( Q_1^* \) | \( G^{(1)*} \) | \( s_1^* \) | \( Q_1^* \) | \( z^* \) | \( x^* \) | \( G^{(2)*} \) |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 20    | 284.51 | 47.30 | 1447.10 | 5087.62 | 46.13 | 1534.35 | 0.4359 | 565.26 | 5222.67 |
| 25    | 284.51 | 47.30 | 1447.10 | 5087.62 | 46.13 | 1524.82 | 0.4889 | 504.16 | 5155.54 |
| 30    | 284.51 | 47.30 | 1447.10 | 5087.62 | 46.12 | 1516.26 | 0.5370 | 459.06 | 5095.41 |
| 35    | 284.51 | 47.30 | 1447.10 | 5087.62 | 46.12 | 1508.43 | 0.5816 | 424.01 | 5040.58 |
| 40    | 284.51 | 47.30 | 1447.10 | 5087.62 | 46.11 | 1501.18 | 0.6232 | 395.76 | 4989.93 |
| 45    | 284.51 | 47.30 | 1447.10 | 5087.62 | 46.11 | 1494.40 | 0.6625 | 372.35 | 4942.71 |

We also conduct numerical examples that examine the impact of a higher \( M \) on the retailer’s strategy choice in Table 7 and derive a rather intuitive result that the ISR strategy becomes inferior to the CPD strategy when \( M \) is higher than a threshold. That is, when the investment cost of increasing the screening rate is higher for the retailer (e.g., in a market with a high cost of employing labor), it is wiser for the retailer to choose the CPD strategy that keeps the screening rate at the original level.

4.2. A comparison of the proposed model against the similar EOQ models. Through comparing our model with existing similar EOQ models (see [22, 60, 62]) that also explore the retailer’s TPR problem for defective items, we derive that two points show the superiority and complexity of our model. First, in the TPR situation, whereas the above previous studies assume a constant demand rate, our model employs a price-sensitive demand, considering that TPR aims to generate a higher demand in the short-term in practice. This increased promotional demand causes an issue that the original screening rate may become insufficient. Our study explicitly addresses this issue that is ignored by existing studies, and proposes the CPD and ISR strategy to deal with it. In other words, whereas previous similar EOQ models only determine the special order quantity \( Q_1 \) by assuming a constant demand, our marketing-operations interface model, through examining a price-sensitive market, decides not only the special order quantity \( Q_1 \) and the special sale price \( s_1 \), but also the screening rate. The optimization of \( s_1 \), through boosting the demand in the short-term, benefits the retailer in two ways: it not only enables him to better control the sharply increased inventory holding cost during promotion, but also allows him to place a larger special order quantity. Moreover, the optimization of the screening rate helps retailer determine under what market conditions will generating a higher demand be more profitable. Second, whereas previous studies assume an error-free screening process, our study incorporates the
two-way inspection errors into the model under both CPD and ISR. Thus, our model can better reflect the realistic operations.

Below, we conduct numerical examples to compare our EOQ model (under either CPD or ISR) with the EOQ model for Situation 1 (that assumes a constant demand) developed by [22]. It is noted that the EOQ model in [22] is adjusted to incorporate the two-way inspection errors, to make the comparison meaningful. Table 8 shows how a higher price elasticity \( b \) makes our model more profitable for the retailer than the model in [22]. The value of each parameter in Table 8 is same with that in subsection 4.1, except that the value of \( b \) changes, and that the value of \( a \) is also adjusted following the change in \( b \) to ensure that the daily demand remains at 200.

| \( b \) | Hsu and Yu [22] | the CPD strategy | the ISR strategy |
|-------|----------------|-----------------|-----------------|
|       |  \( Q_1^* \) | \( D^{(1)}(Q_1^*) \) | \( s_1^* \) \( Q_1^* \) \( G^{(1)*} \) | \( s_1^* \) \( Q_1^* \) \( z^* \) \( G^{(2)*} \) |
| 7     | 1271.04 | 4221.78 | 50 | 1271.04 | 4221.78 | 50.02 | 1190.03 | - | 2491.92 |
| 8     | 1271.04 | 4221.78 | 50 | 1271.04 | 4221.78 | 50.05 | 1271.08 | - | 3234.62 |
| 9     | 1271.04 | 4221.78 | 49.09 | 1299.50 | 4265.23 | 46.13 | 1334.12 | - | 3846.09 |
| 10    | 1271.04 | 4221.78 | 47.95 | 1354.46 | 4468.04 | 46.52 | 1384.54 | - | 4356.57 |
| 11    | 1271.04 | 4221.78 | 47.04 | 1419.59 | 4790.88 | 46.81 | 1424.99 | 0.66 | 4633.18 |
| 12    | 1271.04 | 4221.78 | 47.10 | 1460.19 | 5157.81 | 46.06 | 1505.11 | 0.64 | 5074.49 |
| 13    | 1271.04 | 4221.78 | 47.32 | 1489.28 | 5477.32 | 45.44 | 1591.45 | 0.62 | 5585.67 |
| 14    | 1271.04 | 4221.78 | 47.51 | 1514.22 | 5756.21 | 44.92 | 1682.64 | 0.61 | 6153.24 |
| 15    | 1271.04 | 4221.78 | 47.68 | 1535.83 | 6001.65 | 44.48 | 1777.69 | 0.59 | 6767.56 |
| 16    | 1271.04 | 4221.78 | 47.82 | 1554.74 | 6219.27 | 44.09 | 1875.85 | 0.57 | 7421.47 |

Figure 6 is illustrated to reflect the results of profits in Table 8. Table 8 shows that when consumers are price insensitive (i.e., for lower values of \( b \)), the EOQ
model in Hsu and Yu [22] and the CPD strategy in our model generate the same level of profit. Because for lower values of $b$, a reduced retail price only slightly decreases the retailer’s holding cost per unit by mildly driving up the demand, but it significantly hurts the retailer’s profit through reducing his profit margin. Thus, maintaining the retail price during promotion is the best strategy for the retailer. However, for moderate values of $b$, the CPD strategy chosen by the retailer performs better than the model in Hsu and Yu [22]. Under this market condition, offering a reduced retail price, through effectively reducing the inventory holding cost per unit, enables the retailer to place a larger order quantity than the model in Hsu and Yu [22]. Since the profit gain from both a larger quantity and a well-controlled inventory holding cost can offset the profit loss from the margin erosion, the retailer earns a higher profit under CPD than in Hsu and Yu [22]. Finally, for higher values of $b$, the ISR strategy chosen by the retailer in our model performs much better than the model in Hsu and Yu [22]. The ISR strategy under this market condition allows the retailer to lower down the retail price to a larger degree and earn a significantly increased promotional demand. This sharply increased demand, through enabling the retailer not only to place a larger quantity but also to decrease the inventory holding cost per unit, makes him obtain a higher profit (even though he incurs an additional investment cost of increasing the screening rate) than the model in Hsu and Yu [22].

5. Conclusion. In a price-sensitive market, this paper explores the retailer’s optimal promotional pricing, special order quantity and screening rate decision, when he is offered with a temporary reduction in wholesale price (i.e., TPR) for a lot containing a certain portion of defective items. Specifically, two points differentiate our model from previous similar EOQ models that examine the retailer’s TPR problem for defective items. First, whereas previous studies generally assume a constant demand, our model applies to a price-sensitive market where the demand increases sharply when the TPR is passed to end consumers. This increased promotional demand generates an issue that the original screening rate may become insufficient during promotion, which has been ignored by previous studies. We explicitly address this issue and propose that the retailer can choose either CPD (i.e., the retailer controls the promotional demand to ensure that the original screening rate can still satisfy) or ISR (i.e., the retailer incurs an additional investment cost to increase the screening rate to satisfy whatever the promotional demand) to deal with it. Second, we incorporate the two-way inspection errors (i.e., a Type I error of rejecting a non-defective item and a Type II error of accepting a defective item) into our model under both CPD and ISR, whereas previous studies assume an error-free screening process. Based on these two unique points, we contribute to the relevant literature by deriving several results on how an increase in the value of certain parameters (i.e., the portion of defects, the probability of the two-way inspection errors, the discount size, the investment cost parameter, and the price sensitivity) affects the retailer’s strategy choice and performance.

(1) For larger values of $p$, the retailer prefers ISR to CPD. Because whereas a higher $p$ reduces the special order quantity under CPD through driving up the inventory holding cost per unit with a tighter constraint on the demand, it increases the special order quantity under ISR through cutting down the inventory holding cost per unit of defects with a higher screening rate. For higher values of $p$, the larger order quantity under ISR makes this strategy more profitable than CPD.
(2) A higher $m_1$ drives down the retailer’s profit more significantly than a higher $m_2$ under both CPD and ISR. This is because, for example, under CPD where the special sale price is taken at the boundary, a higher $m_1$ drives up the sale price significantly through imposing a much tighter constraint on demand, while a higher $m_2$ drives down the sale price slightly through making the constraint looser. Thus, a higher $m_1$, by increasing the inventory holding cost per unit more sharply, reduces the special order quantity more significantly than a higher $m_2$. Through leading to a more drastic change in both the sale price and the special order quantity, a higher $m_1$ reduces the retailer’s profit more significantly than a higher $m_2$ under CPD.

(3) For higher values of $m_1$, the retailer prefers ISR to CPD. Because when $m_1$ is higher than a threshold, through drastically driving up the sale price with a tighter constraint and hence the driving down the special order quantity under CPD, a higher $m_1$ makes CPD inferior to ISR. By contrast, for higher values of $m_2$, the retailer prefers CPD to ISR. Because when $m_2$ is higher than a threshold, a higher $m_2$ actually relaxes the constraint and makes CPD superior.

(4) For higher values of the wholesale price discount, the retailer chooses ISR over CPD, whereas for higher values of the investment cost of increasing the screening rate (i.e., when it is expensive to hire employees to inspect the product), the retailer chooses CPD over ISR.

(5) One salient feature of our model is that we jointly determine the promotional pricing and inventory policies in a price-sensitive market. Thus, comparing with the similar EOQ models which deal with the TPR problem for defective items but assume a constant demand, our model, through optimizing the special sale price and hence allowing the retailer to place a larger order quantity with well-controlled inventory holding cost, generates a higher profit for the retailer. Moreover, a higher price elasticity especially makes our model more profitable.

The TPR problem in the inventory management has captured the researchers’ attention since the mid 1960-1970 (see the first work by Naddor [40] in 1966). Yet, this classic problem is enjoying a revival in the last two decades when product categories are becoming more abundant, when distribution channels are becoming more diversified, and when some new business models are emerging. For example, some recent studies have analyzed the special order quantity considering the nature of the inventoried item, like deteriorated items, and items with a portion of defects. Following this line of thought, we revisit the TPR problem by considering not only defective items, but also the two-way inspection errors and the issue of insufficient screening rate in a price-sensitive market. There are still other valuable and interesting issues when we incorporate new business models into the TPR problem. In a channel where one manufacturer sells one product through both its online channel and an independent traditional retailer, when the manufacturer offers TPR to the retailer, what’s the retailer’s optimal order quantity considering consumers’ channel choice behavior? Moreover, recent business news report that more and more online firms are beginning to offer consumers the option to buy online and pick up in store (i.e., BOPS). In a dual-channel framework, how does BOPS affect the retailer’s pricing and ordering decision considering that BOPS affects consumers’ purchase behavior?

Moreover, although our study assumes a deterministic price-sensitive demand, retailers may face demand uncertainties in other markets, e.g., in the fashion goods industry. When making promotional decisions in markets with uncertainties, it is better for retailers to apply fuzzy theory and robust optimization to their models.
Thus, it is worthwhile for future research to consider demand uncertainty in the TPR situation, and apply fuzzy theory and robust optimization to develop models and obtain managerial insights.

Future research can develop models based on the above ideas and derive insights for relevant firms in the new, constantly changing and even uncertain marketing environments.

Appendix.

Proof of Proposition 2. First, by solving Equation (19), the solution $\tilde{Q}_1$ is obtained:

$$\tilde{Q}_1 = \frac{M}{r(c-k)p_cz^2}$$

(A1)

Substituting Equation (A1) into Equation (18) yields the following result:

$$\frac{\partial G^{(2)}(s_1, Q_1, z)}{\partial Q_1} = s_1(1-p)(1-m_1) + v[p + (1-p)m_1] - (c-k) - sc - \frac{M}{D_1z} \nonumber$$

$$- 2\left[\frac{p_z}{D_1} + \frac{(1-p)^2}{2D_1} + \frac{pm_2(1-p_c)}{2D_1} \right] - \frac{(1-p_c)TPU(Q^*_0)}{D_1N} = 0$$

(A2)

Solving $z$ in Equation (A2) equals to solving $z$ in Equation (A3), which is given by

$$B_1z^2 - B_2z - B_3 = 0,$$

(A3)

where $B_1 = s_1(1-p)(1-m_1) + v[p + (1-p)m_1] - (c-k) - sc - \frac{(1-p_c)TPU(Q^*_0)}{D_1N} > 0$ denotes the average profit per item under the special policy compared with that under the regular policy, $B_2 = \frac{3M}{D_1} > 0$, and $B_3 = 2\left(\frac{(1-p_c)^2}{2D_1} + \frac{pm_2(1-p_c)}{2D_1}\right) > 0$.

According to the properties of the quadratic equation, the solution of $z \in (0, +\infty)$ which satisfies Equation (A3) not only exists but also is unique.\qed

Proof of Proposition 3. By taking the second-order derivative of Equation (14) with respect to $s_1$, we have

$$\frac{\partial^2 G^{(2)}(s_1, Q_1, z)}{\partial s_1^2} = -\frac{2b^2MQ_1}{zD_1^3} - \frac{2b^2(1-p_c)TPU(Q^*_0)Q_1}{ND_1^3} \nonumber$$

$$- 2r(c-k)b^2\left[\frac{p_cz}{D_1^2} + \frac{(1-p_c)^2}{2D_1^2} + \frac{pm_2(1-p_c)}{2D_1^2}\right]Q_1^2.$$ 

(B1)

Thus, for any given $(Q_1, z)$, Equation (14) is concave with respect to $s_1$ for all $s_1 > 0$. Then by solving Equation (20), we have:

$$s_1 = \frac{a - \sqrt{\frac{L_2}{(1-p)(1-m_1)}}}{b},$$

(B2)

where $L_2 = \frac{b(1-p_c)TPU(Q^*_0)}{N} + r(c-k)b[p_cz + \frac{(1-p_c)^2+pm_2(1-p_c)}{2}]Q_1 + \frac{bM}{z}$. \qed

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