The authors dedicate this article to one of mathematical and physical giants of XX-th century - academician Prof. Nikolai N. Bogolubov on the memory of his 100th Birthday with great appreciation to his brilliant talent and impressive impact to the modern nonlinear mathematics and quantum physics.

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Abstract

The main fundamental principles characterizing the vacuum field structure are formulated, the modeling of the related vacuum medium and charged point particle dynamics by means of devised field theoretic tools are analyzed. The Maxwell electrodynamic theory is revisited and newly derived from the suggested vacuum field structure principles, the classical special relativity theory relationship between the energy and the corresponding point particle mass is revisited and newly obtained. The Lorentz force expression with respect to arbitrary non-inertial reference frames is revisited and discussed in detail, some new interpretations of relations between the special relativity theory and quantum mechanics are presented. The famous quantum-mechanical Schrödinger type equation for a relativistic point particle in the external potential field within the quasiclassical approximation as the Plank constant $\hbar \to 0$ is obtained.
1. Introduction

It is a generally accepted statement that no one physical theory can pretend on the absolutely true picture of Nature, assuming that there always exist boundaries of its application, which are approved by experiments and new experience data. All that said concerns, evidently, the relativity theory which, as it was pondered by A. Einstein, should logically arise from both the light velocity constance principle with respect to the inertial reference frames and the generalized equivalence principle with respect to gravity and inertial masses. This theory, namely the special relativity theory, proved to be very thoroughly confirmed by many experiments delivered by nuclear physics, which simultaneously became the nuclear energetics backgrounds, widely used today worldwide.

Nevertheless, the nature of space-time and surrounding matter objects was and persists to be a one of the most intriguing and challenging problems facing the mankind and natural scientists especially. As we know one of the most brilliant inventions in physics of XIX-th century was combining of electricity and magnetism within the Faraday-Maxwell electromagnetism theory. This theory explained the main physical laws of light propagation in space-time and posed new questions concerning the nature of vacuum. Nonetheless, almost all attempts aiming to unveil the real state of art of the vacuum problem appeared to be unsuccessful in spite of new ideas suggested by Mach, Lorentz, Poincaré, Einstein and some others physicists. Moreover, the non-usual way of treating the space-time devised by Einstein, in reality, favored to eclipsing both its nature and the related physical vacuum origin problems, reducing them to some physically unmotivated formal mathematical principles and recipes, combined in the well known special relativity theory (SRT). The SRT appeared to be adapted to the only inertial reference systems and faced with hard problems of the electromagnetic Lorentz forces explanation and relationships between inertial and gravity forces. The latter was artificially “dissolved” by means of the well known ”equivalence” principle owing to which the ”inertial” mass of a material object was postulated to coincide with its ”gravity” mass.

Simultaneously, the vacuum origin as a problem almost completely disappeared from the Einstein theory being replaced by the geometrization of the space-time nature and all related physical phenomena. Meanwhile, the impressive success of XX-th century quantum physics, especially of quantum electrodynamics, have demonstrated clearly enough that the vacuum polarization and electron-positron annihilation phenomena make it possible to pose new questions about the space-time and vacuum structures, and further to revisit the existing points of view on them.

As it is well known, the classical mechanics uses the notion of ”potential” energy, being a scalar function of spatial variables, as very important for formulating dynamical equations, in spite of that it is determined up to arbitrary constant. A similar situation one also observes in the classical electrodynamic theory, which effectively uses the notions of scalar and potentials related to each other via the well known Lorentz compatibility gauge constraint and defined up to suitable gauge transformations. These, in some sense, ”different” potential functions were later deeply reanalyzed within the classical Einstein relativity theory by many physicists that gave rise to understanding their fundamental role on the way of combining two great theories - the electromagnetism and the gravity.
Moreover, new important problems arose owing to the famous Einstein relationship between the internal energy and the velocity dependent mass, belonging to a material particle. But, as it was mentioned by L. Brillouin [8], the relationship between the particle mass and its internal energy takes into account the only kinetic energy, describing no mass pertaining to the potential energy, what makes the classical relativity theory a not completely closed and physically not compatible theoretical construction. And as it was written by P.W. Bridgeman in [21], "... at construction of his general relativity theory A. Einstein did not make use of those lessons, which he had us taught himself, and of his deep penetration, which he demonstrated us in his special relativity theory".

Below we try to unveil some nontrivial aspects of the real space-time and vacuum origin problems, deeply related with the relativity theory and electrodynamics, to derive from the natural field theory principles all of the well known Maxwell electromagnetism and special relativity theories results, to show their relative or only visible coincidence with real physical phenomena and to feature new perspectives facing the modern fundamental physics.

Moreover, having further developed the field approach to the microscopic vacuum structure, before suggested in [11] and accounted for in [27], we obtained, within the quasi-classical approximation, a new derivation of main quantum mechanical relationships describing evolution of microscopic particle systems, coinciding as $\hbar \to 0$ with those devised at the beginning of the XX-th century by the great physicists Schrödinger, Heisenberg and Dirac.

2. The Maxwell electromagnetism theory: new look and interpretation

We start from the following field theoretical model [27] of the microscopic vacuum structure, considered as some physical reality imbedded into the standard three-dimensional Euclidean space reference system marked with three spatial coordinates $r \in \mathbb{R}^3$, endowed with the standard scalar product $< \cdot, \cdot >$, and parameterized by means of the scalar temporal parameter $t \in \mathbb{R}$. We will describe the physical vacuum matter endowing it with an everywhere enough smooth four-vector potential function $(W, A) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$, naturally related with light propagation properties. The material objects, imbedded into the vacuum, we will model (classically here) by means of the scalar charge density function $\rho : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ and the vector current density $J : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$, being also everywhere enough smooth functions.

(i) The first field theory principle about the vacuum we accept sounds as follows: the four-vector function $(W, A) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ satisfies the standard continuity relationship

\begin{equation}
\frac{1}{c} \frac{\partial W}{\partial t} + < \nabla, A > = 0,
\end{equation}

where, by definition, $\nabla := \partial/\partial r$ is the usual gradient operator.

(ii) The second field theory principle we accept is a dynamical relationship on the scalar potential component $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$:

\begin{equation}
\frac{1}{c^2} \frac{\partial^2 W}{\partial t^2} - \nabla^2 W = \rho,
\end{equation}
meaning the linear law of the small vacuum uniform and isotropic perturbations propagation in the space-time, understood here, evidently, as a first (linear) approximation in the case of weak enough fields.

(iii) The third principle is similar to the first one and means simply the natural continuity condition for the density and current density functions:

\[
\frac{1}{c} \frac{\partial \rho}{\partial t} + < \nabla, J > = 0. 
\]

We need to note here that the vacuum field perturbations velocity parameter \( c > 0 \), used above, coincides with the vacuum light velocity as we are trying to derive successfully from these first principles the well known Maxwell electromagnetism field equations, to analyze the related Lorentz forces and special relativity relationships. To do this, we first combine equations (2.1) and (2.2):

\[
\frac{1}{c^2} \frac{\partial^2 W}{\partial t^2} = - < \nabla, \frac{1}{c} \frac{\partial A}{\partial t} > = < \nabla, \nabla W > + \rho, 
\]

whence

\[
< \nabla, - \frac{1}{c} \frac{\partial A}{\partial t} - \nabla W > = \rho. 
\]

Having put, by definition,

\[
E := - \frac{1}{c} \frac{\partial A}{\partial t} - \nabla W, 
\]

we obtain the first material Maxwell equation

\[
< \nabla, E > = \rho 
\]

for the electric field \( E : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \). Having now applied the rotor-operation \( \nabla \times \) to expression (2.5) we obtain the first Maxwell field equation

\[
\frac{1}{c} \frac{\partial B}{\partial t} - \nabla \times E = 0 
\]

on the magnetic field vector function \( B : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \), defined as and

\[
B := \nabla \times A. 
\]

To derive the second Maxwell field equation we will make use of (2.8), (2.1) and (2.3):

\[
\nabla \times B = \nabla \times (\nabla \times A) = \nabla < \nabla, A > - \nabla^2 A = \\
= \nabla(- \frac{1}{c} \frac{\partial W}{\partial t}) - \nabla^2 A = \frac{1}{c} \frac{\partial}{\partial t}(-\nabla W - \frac{1}{c} \frac{\partial A}{\partial t} + \frac{1}{c} \frac{\partial A}{\partial t}) - \nabla^2 A = \\
= \frac{1}{c} \frac{\partial E}{\partial t} + (\frac{1}{c} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A). 
\]

We have from (2.5), (2.6) and (2.3) that

\[
< \nabla, \frac{1}{c} \frac{\partial E}{\partial t} > = \frac{1}{c} \frac{\partial \rho}{\partial t} = - < \nabla, J >, 
\]

or

\[
< \nabla, - \frac{1}{c} \frac{\partial^2 A}{\partial t^2} \nabla(\frac{1}{c} \frac{\partial W}{\partial t}) + J > = 0. 
\]
Making now use of (2.1), from (2.10) we obtain that
\[
< \nabla, -\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla \left( \frac{1}{c} \frac{\partial W}{\partial t} \right) + \nabla^2 A + \nabla \times (\nabla \times A) + J > = \\
< \nabla, -\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} + \nabla^2 A + J > = 0.
\]  
(2.11)

Thereby, equation (2.11) yields
\[
\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A = J + \nabla \times S
\]
or for some smooth vector function \( S : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \). Here we need to note that continuity equation (2.3) is defined, concerning the current density vector \( J : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \), up to a vorticity expression, that is \( J \simeq J + \nabla \times S \) and equation (2.12) finally can be written down as
\[
\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A = J.
\]  
(2.13)

Having substituted now (2.13) into (2.9) we obtain the second Maxwell field equation
\[
\nabla \times B - \frac{1}{c} \frac{\partial E}{\partial t} = J.
\]  
(2.14)

In addition, from (2.8) one also finds the no-magnetic charge relationship
\[
< \nabla, B > = 0.
\]  
(2.15)

Thus, we have derived all the Maxwell electromagnetic field equations from our three main principles (2.1), (2.2) and (2.3). The success of our undertaking will be more impressive if we adapt our results to those following from the well known relativity theory in the case of point charges or masses. Below we will try to demonstrate the corresponding derivations based on some completely new physical conceptions of the vacuum medium first discussed in [11, 27].

**Remark 2.1.** It is interesting to analyze a partial case of the first field theory vacuum principle (2.1) when the following conservation law for the scalar potential field function \( W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R} \) holds:
\[
\frac{d}{dt} \int_{\Omega_t} W \, d^3r = 0,
\]
where \( \Omega_t \subset \mathbb{R}^3 \) is an open domain in space \( \mathbb{R}^3 \) with the smooth boundary \( \partial \Omega_t \) for all \( t \in \mathbb{R} \) and \( d^3r \) is the standard volume measure in \( \mathbb{R}^3 \) in a vicinity of the point \( r \in \Omega_t \).

Having calculated expression (2.16) we obtain the following equivalent continuity equation
\[
\frac{1}{c} \frac{\partial W}{\partial t} + < \nabla, \frac{v}{c} W > = 0,
\]  
(2.17)

where \( \nabla := \nabla_r \) and \( v := \frac{dr}{dt} \) is the velocity vector of a vacuum medium perturbation at point \( r \in \mathbb{R}^3 \) carrying the field potential quantity \( W \). Comparing now equations (2.1), (2.17) and using equation (2.3) we can make the suitable identifications:
\[
A = \frac{v}{c} W, \quad J = \rho v,
\]
well known from the classical superconductivity theory [7]. Thus, we face with a new physical interpretation of the conservative electromagnetic field theory when the vector potential \( A : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \) completely determines via expression (2.18) by the scalar field potential function \( W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R} \). It is also evident
that all of the Maxwell electromagnetism filed equations derived above hold too in the case (2.18), as it was first demonstrated in [11] (but with some mathematical inaccuracies) and in [27].

Consider now the conservation equation (2.16) jointly with the related integral momentum conservation relationship

\begin{equation}
\frac{d}{dt} \int_{\Omega_t} \left( \frac{vW}{c^2} \right) d^3 r = 0, \quad \Omega_t|_{t=0} = \Omega_0,
\end{equation}

where as above \( \Omega_t \subset \mathbb{R}^3 \) is for any time \( t \in \mathbb{R} \) an open domain with the smooth boundary \( \partial \Omega_t \), whose evolution is governed by the equation

\begin{equation}
\frac{dr}{dt} = v(r, t)
\end{equation}

for all \( x \in \Omega_t \) and \( t \in \mathbb{R} \), as well as by the initial state of the boundary \( \partial \Omega_0 \). As a result of relationship (2.19) one obtains the new continuity equation

\begin{equation}
\frac{d(vW)}{dt} + vW < \nabla, v > = 0.
\end{equation}

Making now use of (2.17) in the equivalent form

\begin{equation}
\frac{dW}{dt} + W < \nabla, v > = 0,
\end{equation}

we obtain finally a very interesting local conservation relationship

\begin{equation}
\frac{dv}{dt} = 0
\end{equation}

on the vacuum matter perturbations velocity \( v = \frac{dr}{dt} \), holding for all values of the time parameter \( t \in \mathbb{R} \). As it is easy to observe, the obtained relationship completely coincides with the well known hydrodynamic equation [25] of ideal compressible liquid without any external exertion, that is any external forces and field ”pressure” are equal identically to zero. We received a natural enough result that the propagation velocity of the vacuum field matter is constant and equals exactly \( v = c \), that is the light velocity in the vacuum, if to recall the starting wave equation (2.2) owing to which the small vacuum field matter perturbations propagate in the space with the light velocity.

### 3. Special Relativity Theory and Dynamical Field Equations

From classical electrodynamics we know that the main dynamical relationship relates the particle mass acceleration with the Lorentz force which strongly depends on the absolute charge velocity. For the electrodynamics to be independent on the reference system physicists were forced to reject the Galilean transformations and replace them with the artificially postulated Lorentz transformations. This resulted later in the Einstein relativity theory which has partly reconciled the problems concerned with deriving true dynamical equations for a charged point particle.

We will now start from the scalar field vacuum medium function \( W : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R} \) in the conservation condition case (2.16) discussed above. This means, obviously, that the vacuum medium field vector potential \( A : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3 \), charge and current densities \( (\rho, J) : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3 \times \mathbb{R} \) are related owing to expressions (2.18).
Consider now jointly vacuum field medium conservation equations (2.17) and (2.22) at the density $\rho = 0$:

$$
-\frac{1}{c^2} \frac{\partial^2 W}{\partial t^2} = \frac{1}{c^2} \frac{\partial}{\partial t} \left( -\frac{\partial W}{\partial t} \right) = \frac{1}{c^2} \frac{\partial}{\partial t} \left( <\nabla, v W> \right) = <\nabla, \frac{\partial}{\partial t} \left( \frac{W_v}{c^2} \right) > = -<\nabla, \nabla W>.
$$

From relationship (3.1) one follows that

$$
\frac{\partial}{\partial t} \left( \frac{W_v}{c^2} \right) + \nabla W = \nabla \times F,
$$

where $F: \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3$ is some smooth function, which we put, by definition, to be zero owing to the a priori assumed vortexless vacuum medium dynamics. So, our dynamical equation on the vacuum medium scalar field function $W: \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$ looks like

$$
\frac{\partial}{\partial t} \left( \frac{W_v}{c^2} \right) + \nabla W = 0.
$$

Consider now a charged point particle $q$ in a space point $r = R(t) \in \mathbb{R}^3$, depending on time parameter $t \in \mathbb{R}$. Since the vacuum medium field is described by means of the potential field function $W: \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$, which is naturally disturbed by the charged particle $q$, we will model this fact approximately as the following resulting functional relationship:

$$
W(r, t) = \tilde{W}(r, R(t))
$$

for some scalar function $\tilde{W}: \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$. This function must satisfy equation (2.17), that is

$$
<\frac{\partial \tilde{W}}{\partial R} >, u > + <\nabla, \tilde{W} v > = 0.
$$

As we are interested in the function $\tilde{W} := \tilde{W}|_{r\rightarrow R(t)}$ as $r \rightarrow R(t) \in \mathbb{R}^3$, where the charged point particle is located, we obtain from (3.6) that

$$
<\frac{\partial \tilde{W}}{\partial R} + \frac{\partial \tilde{W}}{\partial r}, u > |_{r\rightarrow R(t)} + \tilde{W} <\nabla, v > |_{r\rightarrow R(t)} = 0,
$$

giving rise to the relationship

$$
\frac{\partial \tilde{W}}{\partial R} = -\frac{\partial \tilde{W}}{\partial r},
$$

as $r \rightarrow R(t)$, since $v|_{r\rightarrow R(t)} \rightarrow dR(t)/dt := u(t)$ and $<\nabla, v > |_{r\rightarrow R(t)} \rightarrow <\nabla, u(t) >= 0$ for all $t \in \mathbb{R}$.

Returning now to equation (3.3) we can write down, owing to (3.6), that

$$
\frac{1}{c^2} \left( \frac{\partial \tilde{W} v}{\partial t} + \tilde{W} \frac{\partial v}{\partial t} \right)|_{r\rightarrow R(t)} = \frac{1}{c^2} \left( -<\frac{\partial \tilde{W}}{\partial r}, v > + v + \tilde{W} \frac{\partial u}{\partial t} \right)|_{r\rightarrow R(t)} = \frac{1}{c^2} \left( <\frac{\partial \tilde{W}}{\partial R}, u > + u + \tilde{W} \frac{\partial u}{\partial t} \right)|_{r\rightarrow R(t)} = \frac{1}{c^2} \frac{d}{dt}(\tilde{W} u) = -\frac{\partial \tilde{W}}{\partial R} |_{r\rightarrow R(t)} = \frac{\partial \tilde{W}}{\partial R},
$$

where we put, by definition, $\tilde{W} := \tilde{W}(r, R(t))|_{r\rightarrow R(t)}$. Thus, we obtained from (3.7) that the function $\tilde{W}: \mathbb{R}^3 \rightarrow \mathbb{R}$ satisfies the determining dynamical equation

$$
\frac{d}{dt}(\frac{\tilde{W}}{c^2} u) = -\frac{\partial \tilde{W}}{\partial R}
$$

at point $R(t) \in \mathbb{R}^3$ of the point charge $q$ location.

Now we need to proceed with our calculations ahead and would like make the following very important assumption: we will interpret the quantity $\frac{\tilde{W}}{c^2}$ as the real "dynamical" mass of our point charge $q$ at point $R(t) \in \mathbb{R}^3$, that is

$$
m := \frac{\tilde{W}}{c^2}.
$$
This, in part, means that the whole observed particle mass \( m \) depends only on the vacuum field potential \( \bar{W} \) owing to its both gravitational and electromagnetic interactions with long distant and closely ambient it material particles! This statement, evidently, in many points coincides with the well known Mach principle [8, 11, 27] and formalizes it concerning the real field structure of vacuum. We need here to mention that this idea was also earlier claimed, but not realized practically, by L. Brillouin in [8]. We press here that no assumption about the equivalence of the inertial mass and gravitational mass is made and, moreover, such a kind statement is completely alien within the theory devised here and in [11, 27].

Then, using (3.9) we can rewrite equation (3.8) as

\[
\frac{dp}{dt} = -\frac{\partial \bar{W}}{\partial R},
\]

where the quantity \( p := mu \) has the natural momentum interpretation.

The obtained equation (3.10) is very interesting from the dynamical point of view. Really, from equation (3.10) we obtain that

\[
< u, \frac{d}{dt}(mu) > = c^2 < \frac{\partial m}{\partial R}, u > = c^2 \frac{dm}{dt}.
\]

As a result of (3.11) we easily derive, following [11], the conservative relationship

\[
\frac{d}{dt} \left( m \sqrt{1 - \frac{u^2}{c^2}} \right) = 0
\]

for all \( t \in \mathbb{R} \). Thereby, the quantity

\[
m \sqrt{1 - \frac{u^2}{c^2}} = m_0
\]

is constant for all \( t \in \mathbb{R} \), giving rise to the well known relativistic expression for the mass of a point particle:

\[
m = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}}
\]

As we can see, the point particle mass \( m \) depends, in reality, not on the coordinate \( R(t) \in \mathbb{R}^3 \) of the point particle \( q \), but on its velocity \( u := dR(t)/dt \). Since the field potential \( \bar{W} : \mathbb{R}^3 \rightarrow \mathbb{R} \) consists of two parts

\[
\bar{W} = \bar{W}_0 + \Delta \bar{w},
\]

where \( \bar{W}_0 : \mathbb{R}^3 \rightarrow \mathbb{R} \) is constant and responsible for the external influence of all long distant objects in the Universe upon the point particle \( q \) and \( \Delta \bar{w} : \mathbb{R}^3 \rightarrow \mathbb{R} \) is responsible for the local field potential perturbation by the point charge \( q \) and its closest ambient neighborhood. Then, obviously,

\[
\Delta m := m - m_0 = -\frac{\Delta \bar{w}}{c^2}
\]

is the strictly dynamical mass component belonging to the point particle \( q \). Moreover, since the full momentum \( p = mu \) satisfies equation (3.10), one can easily obtain that the quantity

\[
\bar{W}^2 - p^2 c^2 = E_0^2
\]

is not depending on time \( t \in \mathbb{R} \), that is \( dE_0/dt = 0 \), where \( E_0 := m_0 c^2 \). The result (3.17) demonstrates us the important property of the energy essence: the point particle \( q \) is, in reality, endowed with the only dynamical energy \( \Delta E := \Delta mc^2 \). Concerning the so called "internal" particle energy \( E_0 = m_0 c^2 \) we see that it has nothing to do with the real particle energy, since its origin is determined completely owing
to the long distant objects of the Universe and could not be used for any physical processes, contrary to the known Einstein theory statements about a "huge" internal energy stored inside the particle mass. Equivalently, the Einstein theory statement about the "equivalence" of the mass and the "internal" energy of particle appears to be senseless, since the main part of the field potential function $\bar{W} : \mathbb{R}^3 \rightarrow \mathbb{R}$ at the location point of the particle $q$ is constant and owes to the long distant objects in the Universe, which obviously can not be used for so called "practical applications".

Nonetheless, we have observed above, as a by-product, the well known "relativistic" effect of the particle mass growth depending on the particle velocity in the form (3.14). As it was already mentioned in [11, 27] this "mass growth" is, in reality, completely of dynamical nature and is not a consequence of the Lorentz transformations, as it was stated within the Einstein SRT. Moreover, we can state that all of so called "relativistic" effects have also nothing to do with both the mentioned above Lorentz transformations and with such artificial "effects" as length "shortening" and time "slowing". There is also no reasonable cause to identify the particle mass with its real energy and vice versa. Concerning the interesting physical effect called particles "annihilation" we need here to stress that it has also nothing to do with the transformation of particles masses into energy. The field theoretical explanation of this phenomenon consists in creating their very special bonding state, whose interaction with ambient objects is vanishing. As a result the visible inertial or dynamical mass of this bound state is also zero, what the experiment shows, and nothing else. Inversely, if an intensive enough gamma-quant meets such a bound state of two particles, it can break them back into two separate particles, what the experiment also shows to happen. Here we can recall a similar analogy borrowed from the modern quantum physics of infinite particle systems described by means of the second quantization scenario [16, 30, 28, 29] suggested in 1932 by V. Fock. Within this scenario there also realize creating-annihilation effects which are present owing to the inter-particle interaction forces. Moreover, as we know from the modern superfluidity and superconductivity theories within this picture one can describe special bound states of particles, so called "Cooper pairs", whose interaction to each other completely vanishes and whose combined mass strongly differs from the sum of the suitable components and equals the so called "effective" compound mass, depending strongly on the potential field intensity inside the superfluid or superconductor matter.

Proceed now to discussing the derived above relationship (3.17) where the conserved quantity $E_0$ is naturally interpreted as the total energy of a particle moving with velocity $u := dR(t)/dt$ in vacuum endowed with the field potential $\bar{W}$ in a vicinity of the particle located at point $R(t) \in \mathbb{R}^3$. We see that the total particle energy $E_0$ strictly depends both on the field potential $\bar{W}$ and on its velocity $u$, as the particle momentum $p = m_0 u / \sqrt{1 - \frac{u^2}{c^2}}$ depends strictly relativistically on its velocity $u$.

As it was still mentioned by L. Brillouin in [8] the Einstein SRT postulates that the total energy $E_0$ of a moving particle in a potential field $U$ equals $E_0 = m_0 c^2 / \sqrt{1 - \frac{u^2}{c^2}} + U$. He writes: "This means that any possibility of existing the particle mass related with the external potential energy is completely excluded... Thereby, the strange situation appears: the internal (particle) energy is endowed with the mass but the external- is not". (the citation is taken from [8]). Contrary to this inference from the Einstein SRT, the obtained above relationship (3.17) naturally takes into account both the kinetic energy of the
particle motion with velocity $u$ and the field potential energy $\tilde{W}$ in a vicinity of the particle located at point $R(t) \in \mathbb{R}^3$.

Moreover, L. Brillouin in [8] writes: "...Einstein tends by any way to reduce gravity to geometry by means of changing the newtonian gravitational potential by a tensor potential of second order, realizing the joint description of gravity and geometry: this is achieved owing to the appearance of a huge gap between gravity and electromagnetism.... The (Einstein) article - a genuine mathematical work but its application to the physical reality remains to be open.” Later V. Fock in his famous book [15] tried to rescue the situation that has arisen with gravity and electromagnetism but his approach was also based on the Einstein geometrization ideas and no real success was achieved too. Evaluating the gravity and electromagnetism theories state of art L. Brillouin in [8] says that "In general, the necessity of considering a curved space-time Universe is still not proved; the physical meaning of the general relativity is still very vague”.

Having analyzed from this point of view the results formulated above we see that all of the Einstein SRT statements were obtained within the classical Euclidean space-time scenario and no four-dimensional space-time geometry, like the invariance of the four-dimensional metric interval with respect to the Lorentz transformations, was needed to be involved. Going this way ahead we will show below that within the approaches devised above and in [11, 27] we will derive the next very important result of the SRT and Maxwell electromagnetism theory, namely that related with the nature of the Lorentz force acting on a moving in space charged particle. As it is well known [8, 15, 12, 11], wishing to make the Lorentz force expression compatible with the postulated relativity principles appeared to be decisive in Einstein’s endeavors to construct his SRT and later the general relativity theory.

4. THE LORENTZ FORCE AND THE RELATIVITY THEORY PRINCIPLES REVISITING

It is a well known fact that the Einstein special relativity theory is applicable only for physical processes related to each other by means of the inertial reference systems, moving with constant velocities. In this case one can make use of the Lorentz transformations and calculate the components of suitable four-vectors and the resulting mass growth of particles owing to formula (3.11). A nontrivial problem arises when we wish to analyze these quantities with respect to non-inertial reference systems moving with some nonzero acceleration. Below we will revisit this problem from the devised above vacuum field theory scenario and show that the whole "special" relativity theory emerges as its partial case or by-product and is free of the artificial "inertial reference systems" problems mentioned above.

Really, our vacuum field theory structure is described by dynamical equation (3.3), which we would like to investigate in a neighborhood of two interacting to each other point particles $q_1$ at point $R_1(t) \in \mathbb{R}^3$ and $q_2$ at point $R_2(t) \in \mathbb{R}^3$, respectively. As it was already done in Section 2 we assume that the vacuum potential field function $W : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$ can be representable as $W = \tilde{W}(r; R_1(t), R_2(t))$ for some function $\tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$ and all $t \in \mathbb{R}$. Then, based on continuity equation (2.17) we obtain

$$(4.1) \quad < \frac{\partial \tilde{W}}{\partial R_1}, u_1 > + < \frac{\partial \tilde{W}}{\partial R_2}, u_2 > + < \frac{\partial \tilde{W}}{\partial r}, v > + \tilde{W} < \nabla, v > = 0.$$
We will now be interested in the potential field function $\tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ in a vicinity of the relative vector $R(t) := R_2(t) - R_1(t) \in \mathbb{R}^3$, keeping in mind that the interaction between particles $q_1$ and $q_2$ depends on this relative interparticle distance $R(t) \in \mathbb{R}^3$. From [4.1] as the relative distance $r \to R(t) \in \mathbb{R}^3$ we derive easily that

$$
\left. \frac{\partial \tilde{W}}{\partial R_1} + \frac{\partial \tilde{W}}{\partial R_2} \right|_{r \to R(t)} = 0, \quad \left. \frac{\partial \tilde{W}}{\partial R} + \frac{\partial \tilde{W}}{\partial r} \right|_{r \to R(t)} = 0.
$$

Combining relationships [4.2] with dynamical field equations [4.3] we obtain that

$$
\left. \frac{1}{c^2} \frac{\partial}{\partial t} (\tilde{W} \nu) \right|_{r \to R(t)} = \frac{1}{c^2} \left( < \frac{\partial \tilde{W}}{\partial R_1}, u_1 > + v + < \frac{\partial \tilde{W}}{\partial R_2}, u_2 > + v + \tilde{W} \frac{\partial \nu}{\partial t} \right) \right|_{r \to R(t)} =

= \frac{1}{c^2} \left( < \frac{\partial \tilde{W}}{\partial R}, u_2 - u_1 > (u_2 - u_1) \right) \right|_{r \to R(t)} = - \left. \frac{\partial \tilde{W}}{\partial R} \right|_{r \to R(t)} = \left. \frac{\partial \tilde{W}}{\partial R} \right|_{r \to R(t)},
$$

whence one derives the new dynamical equation

$$
\frac{d}{dt} \left( \frac{\tilde{W}}{c^2} (u_2 - u_1) \right) = - \frac{\partial \tilde{W}}{\partial R}
$$
on the resulting function $\tilde{W} := \tilde{W}|_{r \to R(t)}$.

Equation [4.3] possesses a very important feature of depending on the only relative quantities not depending on the reference system. Moreover, we have not on the whole met the necessity to use other transformations of coordinates different from the Galilean transformations. We mention here that dynamical equation [4.3] was also derived in [11] making use of some not completely true relationships and mathematical manipulations. But the main corollary of [11] and of our derivation [27], saying that equation [4.3] fits for all reference systems, both inertial and accelerated, appears to be fundamental and gives rise to new unexpected results in the modern electrodynamics and gravity theory. Below we will proceed to one of very important relativity physics aspect, concerned with the well known Lorentz force expression measuring the action exerted by external electromagnetic field on a charged point particle $q$ at space point $R_2(t) \in \mathbb{R}^3$ for any time moment $t \in \mathbb{R}$.

To do this we put, owing to the vacuum field theory, that the resulting potential field function $\tilde{W} : \mathbb{R}^3 \to \mathbb{R}$ can be representable in a vicinity of the charged point particle $q$ as

$$
\tilde{W} = \tilde{W}_0 + q \varphi,
$$

where $\varphi : \mathbb{R}^3 \to \mathbb{R}$ is a suitable local electromagnetic field potential and $\tilde{W}_0 : \mathbb{R}^3 \to \mathbb{R}$ is a constant vacuum field potential owing to the particle interaction with the external distant Universe objects. Then, having substituted [4.4] into main dynamical field equation [4.3] we obtain that

$$
\frac{d}{dt} \left( \frac{\tilde{W}}{c^2} u \right) = \frac{d}{dt} \left( \frac{\tilde{W}}{c^2} v \right) - \nabla \tilde{W} = -\nabla \tilde{W} + \frac{\partial}{\partial t} \left( \frac{\tilde{W}}{c^2} v \right) + < u, \nabla > \left( \frac{\tilde{W}}{c^2} v \right) =

= -\nabla \tilde{W} + \frac{1}{c} \frac{\partial}{\partial t} \left( \frac{\tilde{W}}{c} \right) - u \times (v \times \nabla \tilde{W}) - \frac{1}{c^2} u \times (v \times \nabla \tilde{W}) =

= -q \nabla \varphi \left( 1 + \frac{< u, v >}{c^2} \right) - \frac{q}{c} \frac{\partial}{\partial t} \left( \frac{\varphi}{c} \right) + \frac{q}{c} u \times (\nabla \times \varphi) =

= -q \nabla \varphi \left( 1 + \frac{< u, v >}{c^2} \right) - \frac{q}{c} \frac{\partial A}{\partial t} + \frac{q}{c} u \times (\nabla \times A),
$$

(4.5)
where we denoted \( u := u_2, \ v := u_1, \ \nabla := \partial/\partial R_2 = \partial/\partial R \) and \( A := \varphi v/c, \) being the related magnetic potential. Since we have already shown that the Lorentz force

\[
F \ := \ \frac{d}{dt} \left( \frac{m_0}{\sqrt{1 - u^2}} u \right) = \frac{d}{dt} \left( \frac{m_0}{\sqrt{1 - u^2}} \right)
\]

is given by expression (4.5), it can be rewritten down in the form

\[
(4.6) \quad F \ = \ \frac{d}{dt} \left( \frac{m_0}{\sqrt{1 - u^2}} \right) = qE + \frac{q}{c} u \times B - \frac{q}{c^2} \nabla \phi < u, v >
\]

which was derived also in [11] and where we put, by definition, \( E := -\nabla \phi - \frac{1}{c} \frac{\partial A}{\partial t}, \ B := \nabla \times A, \) being respectively the suitable electric and magnetic vector fields.

The resulting expression (4.6) is almost completely equivalent to the well known [5] classical Lorentz force expression \( F \) up to the additional "inertial" term

\[
(4.7) \quad F_c := -\frac{q}{c} \nabla < u, A >,
\]

which is absent in the relativistic theory. Namely, owing to the absence of term (4.7) the classical relativistic Lorentz force expression in the four-vector form was not invariant with respect to any reference frame transformations, except inertial ones. And, as it was noticed in [11, 8], owing only to this crucial fact the relativistic physics faced with many difficulties during the past century and the physicists were forced to use the artificial Lorentz transformations and related with them visible length shortening and time slowing effects. Moreover, they gave rise to such strange enough and non-adequate notions as non-Euclidean time-spaces [18, 22, 23, 8], black holes [31, 32, 18, 26] and some other singular and nonphysical objects.

Concerning the results described above we could state that the vacuum field theory approach of [11, 27] to fundamental physical phenomena is really a powerful tool in hands of researchers, who wish to penetrate into the hidden properties of the surrounding us Universe. As the microscopical quantum level of describing the vacuum field matter structure is, with no doubt, very important, we see the next challenging steps both in understanding the backgrounds of quantum processes within the approach devised in [11, 27] and in this work and in deriving new physical relationships, which will help us to explain the Nature more deeply and adequately.

5. Quantum mechanics backgrounds revisited

We will start from relationship (3.17) rewritten in the following form:

\[
(5.1) \quad E_0^2 = \bar{W} - p^2 c^2, \quad dE_0/dt = 0,
\]

and make the following canonical quantization replacements:

\[
(5.2) \quad p \rightarrow \hat{p} := \frac{\hbar}{i} \nabla, \quad E_0 \rightarrow \hat{E}_0 := -\frac{\hbar}{i} \frac{\partial}{\partial t}, \quad \bar{W} \rightarrow \hat{W} \circ, \quad \hat{W} \circ \quad \text{acting in the Hilbert space} \ \mathcal{H} := L_2(\mathbb{R}^3; \mathbb{C}) \text{ with the standard scalar product}
\]
\( (\cdot, \cdot) \). As an elementary result of this replacement we can easily write down, following the quantization recipes similar to those from \([2, 3, 4]\), that the observable squared energy \( E_0^2 \) is the average value

\[
E_0^2 := (\bar{E}_0 \psi, \bar{E}_0 \psi) = (\psi, (\bar{W}^2 + \hbar^2 c^2 \nabla^2) \psi),
\]

being realized on the one-particle quantum mechanical state vector \( \psi \in \mathcal{H} \). Taking into account representation \((5.3)\) and assuming that the Plank constant \( \hbar \to 0 \), we will try to factorize the linear differential operator \((\bar{W}^2 + \hbar^2 c^2 \nabla^2)\), well defined on a suitable dense linear subset \( D(\bar{E}_0) \subset \mathcal{H} \), in the following \textit{a priori} nonnegative canonical form:

\[
\bar{W}^2 + \hbar^2 c^2 \nabla^2 = \bar{P}^+ \bar{P},
\]

where the sign “+” means the standard conjugation operation in the Hilbert space \( \mathcal{H} \). It is easy to find from \((5.4)\) that

\[
\bar{P} := \hat{S}(1 + \hbar^2 c^2 \bar{W}^{-1} \circ \nabla^2 \circ \bar{W}^{-1})^{1/2} \bar{W} \circ \bar{O}
\]

with an arbitrary unitary operator \( \hat{S} : \mathcal{H} \to \mathcal{H} \), satisfying the usual condition \( \hat{S}^+ \hat{S} = 1 \). As a result, we obtain from \((5.3)\) and \((5.4)\) that

\[
(\bar{E}_0 \psi, \bar{E}_0 \psi) = (\psi, \bar{P}^+ \bar{P} \psi) = (\bar{P} \psi, \bar{P} \psi).
\]

Thereby, based on expression \((5.6)\), we can derive the following Schrödinger type linear evolution equation:

\[
\bar{E}_0 \psi = i \hbar \frac{\partial \psi}{\partial t} = \bar{U} \bar{P} \psi
\]

for all \( t \in \mathbb{R} \), where \( \bar{U} : \mathcal{H} \to \mathcal{H} \) is some still not determined unitary operator.

Let us now symbolically calculate operator expression \((5.5)\) up to the operator accuracy \( O(\hbar^4) \):

\[
\bar{P} = \hat{S}(1 + \hbar^2 c^2 \bar{W}^{-1} \circ \nabla^2 \circ \bar{W}^{-1})^{1/2} \bar{W} = \hat{S}(1 + \frac{\hbar^2 c^2}{2 \bar{W}} \nabla^2 \circ \bar{W}^{-1}) \bar{W} + O(\hbar^4).
\]

Having substituted result \((5.8)\) into \((5.7)\) one obtains (up to the operator accuracy \( O(\hbar^4) \)) the next evolution equation:

\[
i \hbar \frac{\partial \psi}{\partial t} = \bar{U} \hat{S}(-\frac{\hbar^2}{2(-\bar{W}/c^2)} \nabla^2 + \bar{W}) \psi.
\]

Taking into account now that the energy operator \( \bar{E}_0 = -\frac{\hbar^2}{\bar{m}} \frac{\partial^2}{\partial t^2} \) is formally selfadjoint, from physical point of view we need to suppose in \((5.9)\) that \( \bar{U} := \hat{S}^{-1} \), as the operator \( \hat{H} := -\frac{\hbar^2}{2(-\bar{W}/c^2)} \nabla^2 + \bar{W} \) is formally selfadjoint.

Recalling now that owing to \((3.9)\) and \((3.14)\) the expression \( -\bar{W}/c^2 := \bar{m}(u) = m_0/\sqrt{1 - c^2} \) is the dynamical relativistic mass of a particle, whose quantum motion is under study, equation \((5.9)\) can be finally rewritten down in the following classical selfadjoint Schrödinger type form:

\[
i \hbar \frac{\partial \psi}{\partial t} = \hat{\bar{H}} \psi := -\frac{\hbar^2}{2\bar{m}(u)} \nabla^2 \psi + \bar{W} \psi,
\]

where \( \hat{\bar{H}} : \mathcal{H} \to \mathcal{H} \) is the corresponding selfadjoint Hamiltonian operator for a relativistic point particle under an external potential field. It is easy to observe that equation \((5.10)\) in the classical non-relativistic case when the velocity \( c \to \infty \) reduces to the well known classical Schrödinger equation

\[
i \hbar \frac{\partial \psi}{\partial t} = \hat{H}_0 \psi := -\frac{\hbar^2}{2m_0} \nabla^2 \psi + \bar{W} \psi,
\]
derived at the beginning of the past century from completely different points of view by the great physicists Schrödinger, Heisenberg and Dirac.

The result obtained above says very eloquently that our quasiclassical approach to the description of the vacuum field structure as the Plank constant $\hbar \to 0$ gives rise to the related classical quantum Schrödinger dynamics very naturally, being simultaneously deeply physically motivated. It is also worthy to mention that in all of our derivations we have used no Lorentz invariance in spite of the fact that the dynamical mass $m = m_0 / \sqrt{1 - \frac{u^2}{c^2}}$ is expressed by means of a suitable Lorentz factor. It is, evidently, a byproduct result of the second field theory principle equation (2.2), which is a priori Lorentz invariant.

To finish the discussion we make here a plausible claim that the suitable completely relativistic quantum field theoretic analysis of our dynamical vacuum field model via the well known Dirac type factorization approach will necessary shade new light on the so complicated and sophisticated nature of the quantum world of elementary particles.

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