Extracting Low-Lying Lambda Resonances Using Correlation Matrix Techniques

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Abstract. The lowest-lying negative-parity state of the Lambda is investigated in \((2 + 1)\)-flavour full-QCD on the PACS-CS configurations made available through the ILDG. We show that a variational analysis using multiple source and sink smearings can extract a state lying lower than that obtained by using a standard fixed smeared source and sink operator alone.

Keywords: Lambda, \(\Lambda(1405)\), Lattice QCD, resonances, negative parity

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INTRODUCTION

The 1405 MeV resonance of the Lambda baryon has puzzled researchers for many years. It is the lowest-lying excited state of the Lambda, and yet it has negative parity (a property associated with angular momentum). Moreover, it lies lower than the lowest negative-parity state of the nucleon, even though it has valence strange quarks. It also lacks a nearby spin-orbit partner, with the lowest spin-\(3/2^-\) state being the \(\Lambda(1520)\). The internal structure of this resonance has remained a mystery for many years. On one hand, it is regarded as a conventional three-quark state, while on the other it is interpreted as an antikaon-nucleon bound state.

There have so far been several Lattice QCD studies of this resonance [1–5], however most of these have used the quenched approximation, and very few have managed to identify the mass-suppression associated with the \(\Lambda(1405)\). Recent work by the CSSM Lattice Collaboration [6–8] has had significant success in isolating the Roper resonance (\(P_{11}(1440)\)) using correlation matrix techniques together with source and sink smearing, and so we use this method here in an attempt to isolate the otherwise-elusive \(\Lambda(1405)\). We use the \((2 + 1)\)-flavour, full-QCD configurations from the PACS-CS collaboration [9], made available through the ILDG. In particular, we focus on the configurations with light-quark Hopping parameter \(\kappa_{u,d} = 0.13770\).

VARIATIONAL ANALYSIS

To isolate individual excited states, we use the variational method [10, 11], by considering the cross-correlation of operators and diagonalising the operator space. To access \(N\) states of the spectrum, we require at least \(N\) operators.
The parity-projected, two-point correlation function matrix for \( \mathbf{p} = \mathbf{0} \) can be written as
\[
G_{ij}^{\pm}(t) = \sum_{x} \text{tr}_{\mathbf{S}}(\Gamma_{\pm} \langle \chi_{i}(x) \overline{\chi}_{j}(0) \rangle) = \sum_{\alpha=0}^{N-1} \lambda_{i}^{\alpha} \lambda_{j}^{\alpha} e^{-m_{\alpha}t},
\]
(1)
where \( \Gamma_{\pm} \) are the parity-projection operators and \( \lambda_{i}^{\alpha} \) and \( \lambda_{j}^{\alpha} \) are, respectively, the couplings of interpolators \( \chi_{i} \) and \( \overline{\chi}_{j} \) at the sink and source to eigenstates \( \alpha = 0, \ldots, N-1 \) of mass \( m_{\alpha} \). The idea now is to construct \( N \) independent operators \( \phi_{i} \) that isolate \( N \) baryon states \( |B_{\alpha}\rangle \); that is, to find operators \( \overline{\phi}_{\alpha} = \sum_{i=1}^{N} u_{i}^{\alpha} \chi_{i} \) and \( \phi_{\alpha} = \sum_{i=1}^{N} v_{i}^{\alpha} \chi_{i} \) such that
\[
\langle B_{\beta}, p, s | \overline{\phi}_{\alpha} | \Omega \rangle = \delta_{\alpha\beta} \overline{\varepsilon}_{\alpha} u(\alpha, p, s), \quad \text{and}
\langle \Omega | \phi_{\alpha} | B_{\beta}, p, s \rangle = \delta_{\alpha\beta} \varepsilon_{\alpha} u(\alpha, p, s),
\]
(2)
where \( \varepsilon_{\alpha} \) and \( \overline{\varepsilon}_{\alpha} \) are the coupling strengths of \( \phi_{\alpha} \) and \( \overline{\phi}_{\alpha} \) to the state \( |B_{\alpha}\rangle \). It follows that
\[
G_{ij}^{\pm}(t) u_{j}^{\alpha} = \lambda_{i}^{\alpha} \overline{\varepsilon}_{\alpha} e^{-m_{\alpha}t},
\]
(3)
where, for notational convenience, we take the repeated Latin indices to be summed over while repeated Greek indices are not.

The only \( t \) dependence in Eq. (3) is in the exponential term, so we immediately construct the recurrence relation \( G_{ij}^{\pm}(t) u_{j}^{\alpha} = e^{m_{\alpha}t} G_{ik}^{\pm}(t+\Delta t) u_{k}^{\alpha} \), which can be written as
\[
(G_{ij}^{\pm}(t+\Delta t))^{-1} G_{ij}^{\pm}(t) \mathbf{u}^{\alpha} = e^{m_{\alpha}t} \mathbf{u}^{\alpha}.
\]
(4)
This is an eigensystem equation for the matrix \( (G_{ij}^{\pm}(t+\Delta t))^{-1} G_{ij}^{\pm}(t) \), with eigenvectors \( \mathbf{u}^{\alpha} \) and eigenvalues \( e^{m_{\alpha}t} \).

Similarly, we can construct the associated right-eigensystem equation \( \mathbf{v}^{\alpha \dagger} G_{ij}^{\pm}(t) G_{ij}^{\pm}(t+\Delta t))^{-1} = e^{m_{\alpha}t} \mathbf{v}^{\alpha \dagger} \), and then Eq. (2) implies that
\[
G_{\alpha}^{\pm}(t) := \mathbf{v}^{\alpha \dagger} G_{ij}^{\pm}(t) \mathbf{u}^{\alpha} = e^{m_{\alpha}t} \varepsilon_{\alpha} \overline{\varepsilon}_{\alpha} e^{-m_{\alpha}t}.
\]
(5)
Thus, the only state present in \( G_{\alpha}^{\pm}(t) \) is \( |B_{\alpha}\rangle \) of mass \( m_{\alpha} \).

We test each of the four possible operator bases and find that the projected correlation functions agree to within statistical errors. As such, we select the basis \( (16, 100, 200) \) as it contains the broadest and most evenly spaced set of smearings.

In order to determine if this approach will help in isolating the low-lying \( \Lambda(1405) \), we compare the effective mass of the lowest projected state obtained from \( G_{ij}^{1}(t) \) with that obtained from using a single smeared source and sink without a variational analysis. As
we can see from Fig. 1, the lowest projected state lies lower than any of the diagonal elements. Consequently, we conclude that this method has the potential to enable us to isolate the low-lying $\Lambda(1405)$, and further investigation at the other values of $\kappa$ is warranted in order to calculate a projection of this state to the physical value of $m_\pi^2$.

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