Entrance and temperature dependent property effects in the laminar forced convection in straight ducts with uniform wall temperature

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Abstract. The paper reports the results of a parametric investigation on the effects of temperature dependent viscosity and thermal conductivity on forced convection in simultaneously developing laminar flows of liquids in straight ducts of constant cross-sections. Uniform temperature boundary conditions are specified at the duct walls. Viscosity is assumed to vary with temperature according to an exponential relation, while a linear dependence of thermal conductivity on temperature is assumed. The other fluid properties are held constant. Two different cross-sectional geometries, namely circular and flat ducts, are considered. A finite element procedure is employed for the solution of the parabolized momentum and energy equations. Computed axial distributions of the local Nusselt number are presented for different values of the entrance Prandtl number and of the viscosity and thermal conductivity Pearson numbers. Moreover, a superposition method is proved to be applicable in order to obtain an approximate value of the local Nusselt number by separately considering the effects of temperature dependent viscosity and those of temperature dependent thermal conductivity.

1. Introduction
Temperature dependence of fluid properties can play an important role in the simultaneous development of the velocity and temperature fields in duct flows [1, 2]. If the fluid is a liquid, the relative variations of viscosity with temperature are the most relevant, while those of thermal conductivity are, in general, much smaller, and those of density and specific heat capacity are almost negligible [3, 4, 5]. As far as velocity distribution and pressure drop are concerned, the main effects of temperature dependent fluid properties can be retained even if only viscosity is allowed to vary with temperature, while the other properties are assumed constant [1, 2]. Instead, heat transfer characteristics, namely the Nusselt number, are also influenced by thermal conductivity [6, 7, 8]. However, while viscosity exhibits a similar behaviour for all liquids, i.e., it can be assumed to decrease with increasing temperature according to a nearly exponential relation, thermal conductivity can both increase or decrease with increasing temperature, depending on the nature of the fluid considered. For this reason, since parametric analyses valid for all liquids are rather difficult, most of the studies reported in the literature are based on the assumption of temperature dependences of viscosity and thermal conductivity given by specific relations of empirical nature, leading to results which cannot be considered
general and applicable to all liquids [2, 5, 9, 10, 11].

The authors have already carried out systematic studies on entrance and temperature dependent viscosity effects in duct flows with different thermal boundary conditions applied at the solid walls [12, 13], but they did not consider the effects of temperature dependent thermal conductivity. More recently, they proposed a simplified approach to the analysis of the effects of temperature dependent viscosity and thermal conductivity on forced convection in simultaneously developing laminar flows of liquids in straight ducts with uniform wall heat flux boundary conditions [14]. A superposition method, where the effects of temperature dependent viscosity and those of temperature dependent thermal conductivity were considered separately, was demonstrated to be applicable in order to obtain accurate values of characteristic heat transfer parameters. A similar approach was also proposed by the authors in the context of forced convection in laminar microchannel flows of liquids with temperature dependent properties and non-negligible viscous dissipation [15].

In this paper, the effects of temperature dependent viscosity and thermal conductivity on forced convection in simultaneously developing laminar flows of liquids in straight ducts with uniform wall temperature boundary conditions are analysed. Also in this case a superposition method, where the effects of temperature dependent viscosity and those of temperature dependent thermal conductivity are considered separately, is demonstrated to be applicable to predict the value of the Nusselt number.

A finite element procedure is employed for the step-by-step solution of the parabolized momentum and energy equations in a domain corresponding to the cross-section of the duct [16]. In most situations of practical interest, because of the high value of the ratio between the total length and the hydraulic diameter, such an approach is very advantageous with respect to that based on the steady-state solution of the elliptic form of the governing equations in a three-dimensional domain corresponding to the whole duct.

2. Statement of the problem
The laminar forced convection in the entrance region of straight ducts of constant cross-sections with uniform wall temperature \( t_w \) is studied parametrically. The hypotheses made here are that viscous dissipation effects are negligible and that liquid heating begins at the duct inlet, where the velocity boundary layer also starts developing. Therefore, at the entrance of the duct, uniform values \( t_e \) and \( u_e = \bar{u} \) of the temperature \( t \) and of the axial velocity component \( u \) are specified, being \( \bar{u} \) the average axial velocity.

Since the fluids considered here are liquids, the dynamic viscosity \( \mu \) is assumed to decrease with increasing temperature, and \( \mu_e \) is its value at \( t_e \). Exponential type relations are usually employed to represent the temperature dependence of viscosity. The one adopted in this paper is the widely used exponential formula [2, 10, 11, 17, 18, 19]

\[
\mu = \mu_e \exp\left[-\beta (t-t_e)\right] \tag{1}
\]

where \( \beta = -(d\mu/dt)/\mu = \text{const} \) is positive. The thermal conductivity \( k \) is also assumed to vary with temperature and \( k_e \) is its value at \( t_e \). However, while viscosity exhibits a similar behaviour for all liquids, thermal conductivity can both increase or decrease with increasing temperature, depending on the fluid considered. Therefore, the following linear relation [4, 7, 8, 9] is assumed in this paper

\[
k = k_e \left[1 + \alpha (t-t_e)\right] \tag{2}
\]

where \( \alpha = (dk/dt)/k_e = \text{const} \) can be both positive or negative. By means of simple manipulations, equation (1) can be cast in the following dimensionless form

\[
\frac{\mu}{\mu_e} = \exp\left(-P_n \mu T\right) = \left(\frac{\mu_w}{\mu_e}\right)^T \tag{3}
\]
where \( T = (t - t_e)/(t_w - t_e) \) is the dimensionless temperature, \( \mu_w \) is the value of the dynamic viscosity at \( t_w \), while \( Pn_\mu = \beta(t_w - t_e) = -\ln(\mu_w/\mu_e) \) is the viscosity Pearson number, which represents the ratio of the characteristic process temperature difference \((t_w - t_e)\) to the characteristic temperature difference \(1/\beta\) that can produce appreciable viscosity variations. Similarly, equation (2) can be cast in the following dimensionless form

\[
\frac{k}{k_e} = 1 + Pn_k T = 1 + \left(\frac{k_w}{k_e} - 1\right) T
\]

(4)

where \( k_w \) is the value of the thermal conductivity at \( t_w \) and \( Pn_k = \alpha(t_w - t_e) = k_w/k_e - 1 \) is the thermal conductivity Pearson number, which represents the ratio of the characteristic process temperature difference \((t_w - t_e)\) to the characteristic temperature difference \(1/\alpha\) that can produce appreciable thermal conductivity variations.

It is worth noting that the local Reynolds number \( Re = \rho \pi D_h/\mu \), the local Prandtl number \( Pr = \mu c/k \) and the local Péclet number \( Pe = Re Pr = \rho e u D_h/k \) all vary with temperature because of the variations of \( \mu \), of the ratio \( \mu/k \) and of \( k \), respectively. In these expressions \( D_h \) is the hydraulic diameter of the duct, while \( \rho \) and \( c \) are the density and the specific heat of the liquid. Therefore, since \( \rho \) and \( c \) are assumed to be constant, for a given duct flow we have \( \mu/\mu_e = Re_e/Re \) and \( k/k_e = Pe_e/Pe \), where \( Re \) and \( Pe \) are the Reynolds and Péclet numbers evaluated at the entrance temperature \( t_e \). Moreover, since the viscosity of liquids decreases with increasing temperature \((\beta > 0)\), we have \( Pn_\mu > 0 \) and \( Re_e/Re < 1 \) in the case of fluid heating \((t_w > t_e)\) and \( Pn_\mu < 0 \) and \( Re_e/Re > 1 \) in the case of fluid cooling \((t_w < t_e)\), while \( Pn_\mu = 0 \) and \( Re_e/Re = 1 \) refer to constant viscosity fluids \((\beta = 0)\). Instead, since the thermal conductivity can either increase \((\alpha > 0)\) or decrease \((\alpha < 0)\) with increasing temperature, we have \( Pn_k > 0 \) and \( Pe_e/Pe > 1 \) with \( t_w > t_e \) and \( Pn_k < 0 \) and \( Pe_e/Pe < 1 \) with \( t_w < t_e \) in the first case, and \( Pn_k < 0 \) and \( Pe_e/Pe < 1 \) with \( t_w > t_e \) and \( Pn_k > 0 \) and \( Pe_e/Pe > 1 \) with \( t_w < t_e \) in the second case, while \( Pn_k = 0 \) and \( Pe_e/Pe = 1 \) refer to constant thermal conductivity fluids \((\alpha = 0)\).

Numerical results concerning axial distributions of the local Nusselt number

\[
Nu = \frac{h D_h}{k_e}
\]

(5)

are presented in the following. In this expression \( h \) is the local convective heat transfer coefficient defined as

\[
h = \frac{q''_w}{t_w - t_b}
\]

(6)

where \( q''_w \) is the wall heat flux (with \( q''_w > 0 \) in fluid heating and and \( q''_w < 0 \) in fluid cooling) and \( t_b \) is the bulk temperature. It is worth noting that in the above definition of the Nusselt number we use the value of the thermal conductivity at the duct entrance, with the purpose of showing how the heat transfer coefficient is actually affected by the temperature dependence of thermophysical properties.

3. Mathematical model

If the effects of axial diffusion can be neglected and there is no recirculation in the longitudinal direction, steady-state flow and heat transfer in straight ducts of constant cross-sections are governed by the continuity and the parabolized Navier-Stokes and energy equations [20, 21]. Since the inverse of the Reynolds number is representative of the relative importance of diffusive and advective components of the axial momentum flow rate, while the inverse of the Péclet number is representative of the relative importance of conductive and advective components of the axial heat flow rate, the parabolic approximation of the Navier-Stokes and energy equations can be considered adequate, except for the immediate neighbourhood of the inlet, for values
of the Reynolds and Péclet numbers larger than 50 [1, 22]. According to the assumption of parabolic flows, all the derivatives in the axial direction are neglected in the diffusive terms of momentum and energy equations [21]. Moreover, in the axial momentum equation, reference can be made to the gradient of the cross-section averaged pressure. In fact, except for very close to the entrance, the hydrodynamically developing flow problem is similar to a boundary layer problem and, therefore, the pressure in the axial momentum equation can be considered a function only of the axial coordinate $x$ [1]. With reference to incompressible fluids with temperature dependent viscosity and thermal conductivity, in the hypotheses of negligible body forces and viscous dissipation, the above mentioned equations can be written in the following form, valid for axisymmetric geometries

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (rv) = 0$$

(7)

$$\rho u \frac{\partial u}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( \mu r \frac{\partial u}{\partial r} \right) - \rho v \frac{\partial u}{\partial r} - \frac{dp}{dx}$$

(8)

$$\rho u \frac{\partial v}{\partial x} = 2 \left[ \frac{\partial}{\partial r} \left( \mu r \frac{\partial v}{\partial r} \right) - \mu \frac{v}{r} \right] - \rho v \frac{\partial v}{\partial r} - \frac{dp}{dr}$$

(9)

$$\rho c u \frac{\partial t}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial t}{\partial r} \right) - \rho c v \frac{\partial t}{\partial r}$$

(10)

In the above equations, $r$ is the radial coordinate, $v$ represents the radial components of velocity, $p$ is the deviation from the hydrostatic pressure and $\overline{p}$ is its average value over the cross-section.

At the entrance of the duct, the inlet conditions $u = u_e = \pi, v = 0$ and $t = t_e$ are imposed. The solution domain can be bounded by rigid walls or by symmetry axes. On rigid boundaries, the no-slip conditions, that is, $u = v = 0$, are imposed together with the temperature (Dirichlet) boundary condition $t = t_w$. Instead, symmetry conditions are $\partial u/\partial r = 0, v = 0$ and $\partial t/\partial r = 0$ at the symmetry axis.

A finite element procedure for the analysis of the forced convection of fluids with temperature dependent properties in the entrance region of straight ducts and microchannels [12, 13, 16, 23, 24, 25] is used to solve the model equations. The adopted procedure is based on a segregated approach which implies the sequential solution of the momentum and energy equations on a one-dimensional domain corresponding to the cross-section of the duct. A marching method is then used to move forward in the axial direction. The pressure-velocity coupling is dealt with using an improved projection algorithm already employed by one of the authors (C.N.) for the solution of the Navier-Stokes equations in their elliptic form [26]. Since the description reported in Reference [16] can be easily adapted to the case of a fluid with temperature dependent properties considered here, only the details concerning the estimation of the gradient of the cross-section averaged pressure $d\overline{p}/dx$, which is necessary to solve the momentum equation in the axial direction, are reported in this paper. With reference to the flow in straight ducts, integration of the axial momentum equation over the cross-section $A$ gives [1, 16]

$$-\frac{d\overline{p}}{dx} = \frac{d}{dx} \left( \frac{1}{A} \int_A \rho u^2 dA \right) - \frac{1}{A} \int_C \mu \frac{\partial u}{\partial n} dC$$

(11)

where $C$ is the perimeter of the cross-section and $n$ is the outward normal.

The adopted procedure has already been validated, with reference to both constant and temperature dependent property fluids, by comparing heat transfer and pressure drop results with existing literature data for simultaneously developing laminar flows in straight ducts and microchannels [12, 16, 23, 24].
4. Numerical results and approximations
In this study, two cross-sectional geometries, namely circular ducts with outer radius $r_o$ and flat ducts with plate spacing $a$, are considered, representing the limiting situations corresponding to the minimum and maximum values of the ratio between the perimeter and the area of the cross-section. Both these cross-sectional geometries are axisymmetric, since the latter can be considered the limiting case of a concentric annular duct with finite spacing $a = r_o - r_i$ and the ratio $r_o/r_i$ between the outer and the inner radiuses tending to one, which implies $r_i, r_o \to \infty$. However, in practice, reference is made to equations (7) to (10) for the solution of circular duct problems, while to solve parallel plate problems it is convenient to refer to the equations valid for one-dimensional plane problems, i.e., to the limiting forms of equations (7) to (10) corresponding to $r \to \infty$. The computational domains have been defined taking into account existing symmetries. Therefore, the circular and flat cross-sections correspond to one-dimensional domains of lengths equal to the external radius $r_o$ and to the plate spacing $a$, respectively.

The one-dimensional domains are discretized by means of three-node Lagrangian parabolic elements whose sizes gradually increase with increasing distance from the walls. A total of 40 elements and 81 nodal points are used in the discretization of the domain corresponding to the circular cross-section, and a total of 80 elements and 161 nodal points in that of the domain corresponding to the cross-section of the parallel plate channel. The adopted meshes are finer near the walls to allow an accurate representation of the steep velocity and temperature gradients taking place in that region as the flow develops. Several preliminary test were carried out to verify that these discretizations are fine enough to give mesh-independent results. In all the computations, the axial step is gradually increased from the starting value $\Delta x = 0.0001 D_h$ to the maximum value $\Delta x = 0.05 D_h$. As the initial value of the axial step is very small, the strong variations of the axial pressure gradient arising near the entrance of the duct can be adequately captured.

The same value $Re_e = 450$ of the entrance Reynolds number is assumed in all the computations. Instead, the values $Pr_e = 5$ and 100 of the Prandtl number at $t_e$ are selected to take into account the behaviours of different liquids. The corresponding values of the Pécel number are $Pe_e = 2250$ and 45000. The values of the dynamic viscosity Pearson number $Pn_\mu = 0, \pm 0.75$ and $\pm 1.5$ are considered to account for reasonable viscosity temperature dependences. Thus, for the assumed $Re_e$, the minimum and maximum values $Re = Re_w$ of the local Reynolds number are 100 (fluid cooling) and 2017 (fluid heating), respectively. For each value of $Pn_k$ four nonzero values of $Pn_k$ are selected (two positive and two negative), giving the corresponding values of the ratio $Pn_\mu/Pn_k = \beta/\alpha = \pm 10$ and $\pm 20$. Therefore, on the whole, the six values $Pn_k = \pm 0.375, \pm 0.75$ and $\pm 1.5$ are considered, besides the value $Pn_k = 0$. Therefore, the local Pécel number $Pe = Pe_w$ varies between 1957 ($Pr_e = 5$ and $Pn_k = 0.15$) and 52941 ($Pr_e = 100$ and $Pn_k = -0.15$). Finally, for the considered values of $Pr_e$, $Pn_\mu$ and $Pn_k$, the minimum and maximum values $Pr = Pr_w$ of the local Prandtl number are 0.97 and 527, respectively.

It must be pointed out that, even if the numerical results reported in the following were obtained for $Re_e = 450$, they are much more general than they appear, since they are valid in the laminar regime for any value of the entrance Reynolds number larger than 50. In fact, for given values of the entrance Prandtl number $Pr_e$ and of the Pearson numbers $Pn_\mu$ and $Pn_k$, the axial distributions of Nu are independent of the entrance Reynolds number $Re_e$ provided that the appropriate dimensionless axial coordinate $X^* = x/(D_h Pe_e)$ is employed. The validity of the above statement, which is well established for constant property flows ($Pn_\mu = Pn_k = 0$) with negligible axial diffusion of momentum and negligible axial heat conduction in the absence of recirculation in the longitudinal direction [1, pp. 44 and 58], was verified by means of sample numerical tests under the variable viscosity and thermal conductivity assumptions in the ranges.
Figure 1. Axial distributions of the Nusselt number $Nu_c$ for constant property flows with $Pr_e = 5$ and $Pr_e = 100$ in the entrance region of (a) circular and (b) flat ducts.

$50 \leq Re_e \leq 1500, \ 5 \leq Pr_e \leq 100, \ -1.5 \leq Pn_\mu \leq 1.5$ and $-0.15 \leq Pn_k \leq 0.15$.

To illustrate the effects on heat transfer of temperature dependent properties (viscosity and thermal conductivity), the local Nusselt number $Nu_{\mu k}$, obtained for given nonzero values of $Pn_\mu$ and $Pn_k$, is compared with the corresponding local Nusselt number $Nu_c$ computed for simultaneously developing constant property flows ($Pn_\mu = Pn_k = 0$). Axial distributions of $Nu_c$ for both cross-sectional geometries considered here are presented in figure 1, while axial distributions of the ratio $Nu_{\mu k}/Nu_c$ with different values of $Pn_\mu$ and $Pn_k$ are presented in the following.

As can be seen in figure 1, for a given cross-sectional geometry $Pr_e$ significantly affects $Nu_c$ for low values of $X^*$ while, as expected, the same asymptotic value of $Nu_c$ is reached for fully developed conditions. The computed asymptotic values 3.65680 and 7.54075 for circular and flat ducts, respectively, match very well the corresponding literature values 3.65679 and 7.54070 [1].

Axial distributions of the ratio $Nu_{\mu k}/Nu_c$ for circular and flat ducts with different values of $Pr_e$, $Pn_\mu$ and $Pn_k$ are presented in figures 2 and 3 (solid lines). As expected, the effects of $Pr_e$ are significant only for low values of $X^*$, resulting in larger differences from 1 of the value of $Nu_{\mu k}/Nu_c$ for larger values of $Pr_e$. Near the entrance the effects of temperature dependent viscosity prevail over those due to the temperature dependence of thermal conductivity. In fact, the values of $Nu_{\mu k}/Nu_c$ are always higher than 1 for $Pn_\mu > 0$ (fluid heating) and lower than 1 for $Pn_\mu < 0$ (fluid cooling), no matter whether $Pn_k$ is positive or negative. Instead, when fully developed conditions are approached, the effects of temperature dependence of viscosity vanish since $t_b$ tends to $t_w$. Thus, the asymptotic value of $Nu_{\mu k}/Nu_c$ is higher than 1 for $Pn_k > 0$ and lower than 1 for $Pn_k < 0$, being equal to $k_w/k_e = 1 + Pn_k$. As can be seen, the effects of temperature dependent thermal conductivity are comparable with those of temperature dependent viscosity ($Pn_k = 0$). Thus, if thermal conductivity significantly varies with temperature, heat transfer results obtained under the assumptions of temperature dependent viscosity ($Pn_\mu \neq 0$) and constant thermal conductivity ($Pn_k = 0$) are inadequate to predict actual heat transfer rates. In other words, the Nusselt number $Nu_\mu$, computed under such assumptions, can be considerably different from $Nu_{\mu k}$.

However, it will be shown that the values of $Nu_\mu$, together with those of the Nusselt number $Nu_k$, computed under the assumptions of temperature dependent thermal conductivity ($Pn_k \neq 0$) and constant viscosity ($Pn_\mu = 0$), can be used to predict the corresponding values
of $\text{Nu}_{\mu k}$ with an accuracy which can be considered satisfactory in most situations. In fact, if the effects of temperature dependence of viscosity and thermal conductivity are considered separately to compute $\text{Nu}_{\mu}$ and $\text{Nu}_{k}$, respectively, a superposition method is applicable in order to obtain approximate values of the Nusselt number $\text{Nu}_{\mu k}$ according to the relation

$$\frac{\text{Nu}_{\mu k} - \text{Nu}_c}{\text{Nu}_c} \approx \frac{\text{Nu}_{\mu} - \text{Nu}_c}{\text{Nu}_c} + \frac{\text{Nu}_k - \text{Nu}_c}{\text{Nu}_c}$$  \hspace{1cm} (12)$$

The applicability of such a superposition method can be justified by considering that temperature dependent viscosity effects are mostly due to the temperature variations over the duct cross-section, while those of temperature dependent thermal conductivity mainly come from the temperature variations between the inlet section and the considered axial location. With reference to the ratios $\text{Nu}_{\mu k}/\text{Nu}_c$, $\text{Nu}_{\mu}/\text{Nu}_c$ and $\text{Nu}_k/\text{Nu}_c$, equation (12) can be recast in the form

$$\frac{\text{Nu}_{\mu k}}{\text{Nu}_c} \approx \left( \frac{\text{Nu}_{\mu k}}{\text{Nu}_c} \right)' = \frac{\text{Nu}_{\mu}}{\text{Nu}_c} + \frac{\text{Nu}_k}{\text{Nu}_c} - 1$$  \hspace{1cm} (13)$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Axial distributions of the ratios $\text{Nu}_{\mu k}/\text{Nu}_c$ and $(\text{Nu}_{\mu k}/\text{Nu}_c)'$ for simultaneously developing laminar flows in circular ducts with different values of $\text{Pn}_k$ and: $\text{Pr}_e = 5$ and (a) $|\text{Pn}_\mu| = 0.75$, (b) $|\text{Pn}_\mu| = 1.5$; $\text{Pr}_e = 100$ and (c) $|\text{Pn}_\mu| = 0.75$, (d) $|\text{Pn}_\mu| = 1.5$.}
\end{figure}
Figure 3. Axial distributions of the ratios $\frac{Nu_{uk}}{Nu_c}$ and $(\frac{Nu_{uk}}{Nu_c})'$ for simultaneously developing laminar flows in flat ducts with different values of $Pn_k$ and: $Pr_e = 5$ and (a) $|Pn_\mu| = 0.75$, (b) $|Pn_\mu| = 1.5$; $Pr_e = 100$ and (c) $|Pn_\mu| = 0.75$, (d) $|Pn_\mu| = 1.5$.

where $(\frac{Nu_{uk}}{Nu_c})'$ is the approximate value of $\frac{Nu_{uk}}{Nu_c}$ given by the superposition method. To prove the validity of equation (13), axial distributions of the ratio $(\frac{Nu_{uk}}{Nu_c})'$ for circular and flat ducts for different values of $Pn_\mu$ and $Pn_k$, are compared in figures 2 and 3 with the corresponding distributions of $\frac{Nu_{uk}}{Nu_c}$. As can be seen, the dashed curves, representing axial distributions of the ratio $(\frac{Nu_{uk}}{Nu_c})'$, are very close, even for the highest/lowest values of $Pn_\mu$ and $Pn_k$ considered here, to the solid ones representing the computed axial distributions of $\frac{Nu_{uk}}{Nu_c}$. Therefore, the validity of the approximation given by equation (13) can be considered confirmed.

5. Conclusions
The effects of temperature dependent viscosity and thermal conductivity on forced convection in simultaneously developing laminar flows of liquids in straight ducts of constant cross-sections have been investigated parametrically. Uniform temperature boundary conditions have been specified at the duct walls. Viscosity has been assumed to vary with temperature according to an exponential relation, while a linear dependence of thermal conductivity on temperature
has been considered. The other fluid properties have been held constant. Two different cross-
sectional geometries have been considered, namely circular and flat ducts. An efficient finite
element procedure has been employed for the solution of the parabolized momentum and energy
equations.

In this parametric study, computed axial distributions of the local Nusselt number have
been presented for different values of the entrance Prandtl number and of the viscosity and
thermal conductivity Pearson numbers. Moreover, a superposition method has been proved to
be applicable in order to obtain an approximate value of the local Nusselt number by separately
considering the effects of temperature dependent viscosity and those of temperature dependent
thermal conductivity.

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References
[1] Shah R K and London A L 1978 (New York: Academic Press)
[2] Kakac S 1987 Handbook of Single-Phase Convective Heat Transfer ed S Kakac, R K Shah and W Aung (New
York: Wiley) chapter 18
[3] Herwig H 1985 Int. J. Heat Mass Transfer 28 423
[4] Herwig H, Voigt M and Bauhaus F J 1989 Int. J. Heat Mass Transfer 32 1907
[5] Herwig H and Mahulikar S P 2006 Int. J. Thermal Sci. 45 977
[6] Mahulikar S P and Herwig H 2006 Phys. Fluids 18 073601
[7] Hooman K, Hooman F and Famouri M 2009 Int. Commun. Heat Mass Transfer 36 192
[8] Hooman K and Ejlali A 2010 Int. Commun. Heat Mass Transfer 37 34
[9] Nobrega J M, Pinho F T, Oliveira P J and Carneiro O S 2004 Int. J. Heat Mass Transfer 47 1141
[10] Shannon R L and Depew C A 1969 J. Heat Transfer 91 251
[11] Joshi S D and Bergles A E 1980 AIChE Symp. Ser. 76 270
[12] Nonino C, Del Giudice S and Savino S 2006 Int. J. Heat Mass Transfer 49 4469
[13] Del Giudice S, Savino S and Nonino C 2011 J. Heat Transfer, Trans. ASME 133 101702 p 1-11.
[14] Del Giudice S, Savino S and Nonino C 2010 Proc. 28th UIT Heat Transfer Congress (Brescia, Italy) (UIT)
p 231
[15] Del Giudice S, Savino S and Nonino C 2012 Proc. 10th ICNMM2012 (Puerto Rico, USA) (ASME)
[16] Nonino C, Del Giudice S and Comini G 1988 Numer. Heat Transfer 13 451
[17] Lin C R and Chen C K 1994 J. Phys. D: Appl. Phys. 27 29
[18] Costa A and Macedonio G 2002 Geophysics Research Letters 29 Art. 1402
[19] Costa A and Macedonio G 2003 Nonlinear Processes in Geophysics 10 545
[20] Patankar S V and Spalding D B 1972 Int. J. Heat Mass Transfer 15 1787
[21] Hirsh C 1988 vol 1 (New York: Wiley) p 70
[22] Javeri V 1977 Wärme- und Stoffübertragung 10 137
[23] Del Giudice S, Nonino C and Savino S 2007 Int. J. Heat and Fluid Flow 28 15
[24] Nonino C, Del Giudice S and Savino S 2007 J. Heat Transfer, Trans. ASME 129 1187
[25] Nonino C, Del Giudice S and Savino S 2010 Heat Transfer Eng., 31 682
[26] Nonino C 2003 Numer. Heat Transfer B 44 61