Strong CP Violation in External Magnetic Fields

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We study the response of the QCD vacuum to an external magnetic field, in the presence of strong CP violation. Using chiral perturbation theory and large $N_c$ expansion, we show that the external field would polarize quantum fluctuations and induce an electric dipole moment of the vacuum, along the direction of the magnetic field. We estimate the magnitude of this effect in different physical scenarios. In particular, we find that the polarization induced by the magnetic field of a magnetar could accelerate electric charges up to energies of the order $\sim 10^3$ TeV. We also suggest a connection with the possible existence of "hot-spots" on the surface of neutron stars.

The request of a gauge-invariant definition of the vacuum gives rise to the so-called $\theta$-terms in the QCD Lagrangian:

$$S_\theta = \frac{\theta}{32\pi^2} \int d^4x \, G_{\mu\nu} \tilde{G}^{\mu\nu},$$

where $\theta$ includes also a contribution from the weak sector, i.e. $\theta = \theta_0 + \arg \det[M]$, where $M$ is the quark complex, non-hermitian mass matrix.

The interaction term in Eq. (1) is a source of CP-violation in the non-perturbative strong sector of the Standard Model [1]. At the moment, the most constraining bound $\theta \lesssim 3 \times 10^{-10}$ comes from the measurement of the neutron electric dipole moment. In the context of the search for new physics, it is very important to quantify the amount of CP violation which has to be attributed to the Standard Model, and in particular to QCD. This motivates further research to provide better estimates of $\theta$.

The main idea of the present work is to consider a CP-violating process in which the smallness of $\theta$ is compensated by the coupling to some other very large scale. To this end, we analyze the CP-odd response of the QCD vacuum to a uniform external magnetic field and we show that the vacuum develops an induced electric dipole moment along the direction of the external magnetic field. This effect vanishes for $\theta \to 0$ and is qualitatively different to the ordinary (i.e. CP-even) vacuum polarization, which always occurs in the direction parallel to the external electric field.

At zero temperature, the energy which can be delivered via such a mechanism to a charged particle in a magnetic domain with field strength $B$ extending for a distance $L$ is found to be proportional to $\theta B^2 L$. Finite temperature corrections scale like $\theta B L^2$, for small $T$. Hence, it is natural to consider three different scenarios: (i) a configuration in which the field is coherent over extremely large distance scales (ii) a configuration in which the region permeated by the field is hot and (iii) a configuration in which the external field is extremely intense.

In Nature, the last scenario is realized in the vicinity compact objects such as e.g. magnetars, where the magnetic field can reach strengths as high as $10^{15} \div 10^{16}$ G [2]. The first scenario is realized in regions of the observed Universe permeated by the so-called Large-Scale Magnetic Fields (LSMF). These are $\mu G$ fields with correlation lengths as large as the size of galaxy clusters, $\sim 10^2$ kpc [2]. According to the so-called primordial hypothesis, such fields are the result of the evolution of “seed” fields, which were formed in the early stages of the Big Bang, when the temperature was large and therefore the second scenario may apply (for a review see e.g. [3]).

The starting point of our discussion is to express the electric-dipole density distribution in the vacuum in terms of a QCD matrix element:

$$p(t) = \frac{1}{V} \int d^3x \,(\theta J_{\mu}^{e/m}(x,t)|\theta) A_\mu,$$

where $|\theta \rangle_{A_\mu}$ represents the $\theta$-vacuum state in the presence of the external field $A_\mu$ and $eJ_{\mu}^{e/m} = e \sum_f Q_f \gamma_\mu q_f$, is the electro-magnetic current operator. We stress the fact that, in the absence of CP violation, an external electromagnetic field cannot induce electric polarization along the direction of the $\mathbf{B}$ vector, hence $\mathbf{p} \cdot \mathbf{B} = 0$.

In order to systematically account for the non-perturbative QCD dynamics in [2], we adopt a chiral effective field theory description to $\mathcal{O}(p^4)$, with 2 degenerate flavors, in which topological effects are accounted to leading-order in the $1/N_c$ expansion. Our generating functional is therefore

$$Z_{\text{EFT}}[a_\mu, \theta] = \int D\mathcal{U} e^{i \int d^4x \mathcal{L}_{\text{EFT}}[a_\mu, \theta]},$$

where the effective Lagrangian is

$$\mathcal{L}_{\text{EFT}}[a_\mu, \theta] = \mathcal{L}_{\chi PT}^{(2)}[a_\mu] + \mathcal{L}_{\chi PT}^{(4)}[a_\mu] + \mathcal{L}_{\text{anom.}}[\theta].$$

$\mathcal{L}_{\chi PT}^{(2)}[a_\mu]$ and $\mathcal{L}_{\chi PT}^{(4)}[a_\mu]$ are the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ chiral perturbation theory Lagrangians respectively, including the mass term and a gauge-invariant coupling to the vector potential source term, $a_\mu$. We recall that the $\mathcal{L}_{\chi PT}^{(4)}[a_\mu]$
the meson fields. The expansion of the electro-magnetic field contains also the anomalous electro-magnetic Wess-Zumino coupling \[6\].

\[ \mathcal{L}_{\text{anom}}[\theta] = \frac{f^2 a}{4 N_c} \left[ \bar{\theta}^2 - \frac{1}{4} \left( \frac{\log \left( \left| \det U \right| \right)}{\det U^\dagger} \right)^2 + i \mathcal{P} \left( \text{Tr}(U - U^\dagger) - \frac{\log \left( \left| \det U \right| \right)}{\det U^\dagger} \right) \right], \tag{5} \]

where \( \frac{\bar{\theta}}{N_c} \) is identified with the mass of the iso-singlet pseudo-scalar meson (which for \( N_f = 2 \) we shall denote with \( \eta \)) and \( \frac{\bar{\theta}}{N_c} \) is aligned with the external magnetic field. Eq. (9) can be written as \[7, 8\]:

\[ a \exp(\alpha \hat{B}) \]

In chiral perturbation theory, the matrix element of the charge operator in \[2\] can be computed by functionally differentiating the generating functional with respect to the external vector-potential source \( a_\mu(x) \):

\[ \langle \theta | J_\mu(x) | \theta \rangle_{A_\mu} \propto \left( \frac{\delta}{\delta e a_\mu(x)} \log Z_{\text{EFT}} \right)_{a_\mu = A_\mu} \tag{6} \]

A useful topological property of the CP violating diagrams contributing to the matrix element \[6\] is revealed, when the relevant operators are expanded in powers of the meson fields. The expansion of the electro-magnetic current operator

\[ J_\mu(x) = \frac{\delta}{\delta e a_\mu(x)} \int d^4z \mathcal{L}_{\text{EFT}}[a_\mu] \tag{7} \]

contains terms with both odd and even powers of the fields. In particular, the terms coming from functionally differentiating \( \mathcal{L}_{X}^{(2)} \) and the non-anomalous pieces of \( \mathcal{L}_{X,\text{PT}}^{(4)} \) display even powers of meson field operators, while those coming from the anomalous Wess-Zumino term in \( \mathcal{L}_{X}^{(4)} \) display odd powers of meson field operators. On the other hand, the CP-violating interaction \( \mathcal{L}_{\text{anom}} \) leads to a three-meson field vertex. As a consequence, all CP-violating diagrams contributing to \[6\] must contain the Wess-Zumino piece of the electro-magnetic current operator. This condition holds to any chiral orders, since it simply follows from the requirement that propagator lines in the diagrams have to connect in a specific way in order to form closed loops. It is a manifestation of the fact that strong CP-violation is a purely quantum, anomaly-mediated process.

In particular, the lowest-order diagrams (in the combined chiral and \( 1/N_c \) counting and for small \( \theta \) are those reported in Fig.1 and contain the Wess-Zumino electromagnetic operator,

\[ e J_{WZ}^\mu = \frac{e^2 N_c}{48 \pi^2 f_\pi} e^{\mu\nu\alpha\beta} F_{\nu\alpha} \partial_\beta \left( \frac{5}{3} \eta + \pi_0 \right), \tag{8} \]

along with one CP-odd three-meson interaction and with the non-anomalous \( \mathcal{O}(p^2) \) electro-magnetic vertexes.

In the physical scenarios we are presently interested in, the external field can be considered static and uniform, as compared to the typical QCD scales. It is nevertheless instructive to analyze the case of a uniform oscillating field, \( B(t) = B_0 \cos \omega t \), which leads to a time-dependent vacuum polarization:

\[ \tilde{p}_z(t) = \dot{\tilde{z}} \theta \alpha^2 \frac{5 B_0^3}{6 m_\eta^5 (4 \pi f_\pi)^2} \times \left\{ \begin{array}{l} 1 + \frac{\tilde{\varpi}^2}{m_\eta^2} + \frac{1}{10} \frac{\tilde{\varpi}^2}{m_\pi^2} \cos(3 \varpi t) \\ + \frac{1}{2} - \frac{\tilde{\varpi}^2}{3 m_\eta^2} + \frac{11}{45} \frac{\tilde{\varpi}^2}{m_\pi^2} \cos(\varpi t) \end{array} \right\}, \tag{9} \]

where \( \alpha \approx 1/137 \) is the electromagnetic fine structure constant and we have chosen a frame in which the versor \( \tilde{z} \) is aligned with the external magnetic field. Eq. (9) holds in the limit \( B_0 < m_\pi^2 \) and for \( \varpi \ll m_\eta \).

Interestingly, we find that the induced Vacuum Electric Dipole Moment (VEDM) displays two characteristic modes of oscillation, with frequencies \( \varpi \) and \( 3 \varpi \). The origin of such modes is connected with the fact that, to lowest order in our chiral counting, the vacuum fluctuations interact with three external electro-magnetic field lines (see Fig.1). The mode with frequency \( 3 \varpi \) corresponds to processes in which virtual states in vacuum fluctuations absorb an energy quantum \( \varpi \) from each of the three external lines. On the other hand, the mode with frequency \( \varpi \) corresponds to events in which virtual states release one quantum of energy to one or two external field lines. We expect higher order electro-magnetic interactions to give raise to a numerable infinity of additional characteristic oscillation modes, with strengths suppressed by higher powers of fine structure constant \( \alpha \).

The induced VEDM in the presence of an static external field is readily obtained from \[6\] by taking the limit \( \varpi \rightarrow 0 \) and reads

\[ \tilde{p}_z(t) = \dot{\tilde{z}} \theta \alpha^2 \frac{5 B_0^3}{9 m_\eta^5 (4 \pi f_\pi)^2} \tag{10} \]
It should be stressed that this formula has been obtained in a model-independent and parameter-free way. On the other hand, it corresponds to the lowest-order in \( \theta \) and in the combined chiral and \( 1/N_c \) expansion.

The microscopic dynamical mechanism underlying the anomalous CP-odd electric polarization has been explored using an instanton liquid model in [2]. It was shown that the instanton-mediated correlations provided by the \( \theta \)-term lead to a flavor- and spin-dependent repulsion between quarks and antiquarks. As a result of such an interaction, \( u(d) \) quarks (antiquarks) are pushed in the direction parallel (anti-parallel) to their spin. The external magnetic field considered here generates a polarization of the quark and antiquark magnetic moments and therefore selects the direction for the electric polarization of vacuum quantum fluctuations.

Let us now discuss the corrections to Eq. (10) which arise at finite temperature. Since we are relying on a low-energy effective description, in the present work we can only consider temperatures much below the deconfinement temperature, \( T \ll T_c \approx 160 \text{ MeV} \). At low temperatures, the heat-bath consists primarily of soft pions and we can use the approximation

\[
\langle J_\theta \rangle_T \simeq \langle J_\theta \rangle_0 + \sum_n \int \frac{d^3p}{(2\pi)^32\omega_n(p)} \frac{\langle \pi_n(p) | J_\theta | \pi_n(p) \rangle}{2\omega_n(p)} - 1.
\]

We find that, for a static magnetic field along the \( \hat{z} \) axis, the leading chiral-order expression for the induced electric polarization is

\[
p_z \simeq \frac{5}{9(4\pi f_\pi)^2} \frac{\theta \alpha B_0}{m_\pi^2} \left( \alpha B_0^2 + \frac{5\pi m_\pi^2}{16} T^2 + \cdots \right)
\]

We note that the finite temperature correction displays qualitative differences with respect to the \( T = 0 \) contribution: it is of order \( \alpha \) (rather than \( \alpha^2 \)), and grows linearly (rather than cubically) with the external magnetic field. From Eq. (11) it follows that the energy transferred to a charged particle propagating through the uniformly polarized vacuum for a distance \( L \) is proportional to \( \alpha^2 \theta B_0^3 L \), with temperature corrections of the order \( \alpha \theta B L T^2 \).

Let us now discuss some of the phenomenological implications of these results. We first analyze the magnitude of the polarization induced on cosmological distance scales by the LSMF. It is immediate to realize that the \( T = 0 \) contribution is extremely small. To see this, we consider a box of side size \( L = 10^3 \) kpc, permeated by a field of \( 1 \mu G = 10^{-10} \) T. This represents a typical configuration for an observed domain permeated by a LSMF. We find that the induced electric potential difference at the opposite sides of the box would be only \( \Delta V \approx \theta 10^{-35} \text{ V} \). Given the smallness of this effect, it is extremely unlikely that present LSMF could provide observable signatures of strong CP violation.

The situation is partially modified at finite temperature. We consider a scenario in which \( T \approx 150 \text{ MeV} \), \( B_0 \approx 0.1 \text{ nG} \) and the magnetic domain is modeled as a box of two astronomical unit side. This configuration represents the most favorable compatible with the primordial hypothesis for the LSMF, at the end of the hadron epoch. We find that the induced VEDM would be \( \Delta V \approx 2 \times 10^{43} \theta \text{ e cm} \) i.e. about \( 10^{45} \) times the electric dipole moment of a single neutron—, corresponding to a induced surface pion density of \( n \approx 2 \times 10^{-9} \text{ m}^{-2} \) and a potential energy difference at the side of the box of \( \Delta V \approx \theta \times 1 \text{ V} \). This result shows that, if \( \theta \neq 0 \) during the hadron epoch, then very small finite separation of electric (and flavor) charge was induced in the regions permeated by the static primordial fields.

We stress the fact that, although the effect is still extremely small, we have found that it is strongly enhanced at finite temperature. Hence, it would be interesting to investigate the magnitude of the polarization at much higher temperatures, which were reached in earlier stages of the Big Bang. Above the QCD critical temperature \( T_c \approx 160 \text{ MeV} \) quarks and gluons are de-confined and the low-energy effective description adopted here breaks down. However, the dominant topological correlations in the large temperature limit can systematically be accounted for by computing the effect of perturbative fluctuations around the small-sized caloron configurations.

In passing, we note that a charge asymmetry in pion momentum distribution has also been discussed in the context of ultra-relativistic heavy-ion collisions [10], assuming the formation of meta-stable CP-odd "false vacua", where \( \theta = 1 \). In heavy ion collisions the direction of the asymmetry is provided by the total orbital angular momentum, which is perpendicular to the collision plane.

Let us now estimate the magnitude of the induced VEDM if the magnetic field is extremely intense. Such a scenario is realized in near the accretion disk of a black hole or in the vicinity of magnetar, where the magnetic field strength can be as high as \( 10^{15} \) G and the magnetic domain is modeled as a box of two astronomical unit side. This represents a typical configuration for an observed domain permeated by a LSMF. We find that the induced electric potential difference at the side of the box would be only \( \Delta V \approx \theta 10^{-35} \text{ V} \). Given the smallness of this effect, it is extremely unlikely that present LSMF could provide observable signatures of strong CP violation.
In view of such a result, it is interesting to take a closer look to the induced VEDM generated by the magnetic field of a magnetar. If we neglect the effects associated to the rotations of the star, then the magnetic field can be assumed to be purely dipolar \[11\]. In this case, the resulting total induced electric dipole moment can be easily computed by performing the integral Eq. (2) over the entire space. For a typical magnetic moment \[m = 10^{37} \text{G m}^3\], we find an induced electric dipole moment of \[D \sim 3 \times 10^{26} \text{e cm}\]. It should be noted that, if the star rotates with angular velocity \(\Omega\) directed along the axis parallel to its magnetic moment, an additional CP-even electric field is developed \[11\]. However, the resulting charge polarization would lead to qualitatively different electric fields. In fact, it can be shown that in this case the induced charge density is in the form \(\rho(x) = -1/(2\pi) \Omega \cdot \mathbf{B}\), hence it only contributes to the electric quadrupole moment of the star.

A detailed investigation of the phenomenological consequences of the CP-odd electric dipole moment of a magnetar is beyond the scope of the present work. Here, we only point out that the anomalous electric currents generated by the VEDM would be most intense near the poles of the star, where they would induce a local enhancement of the temperature. This phenomenon may have astrophysical implications: in fact, the existence of "hot spots" near the surface of neutron stars has been suggested as a possible scheme to explain the structure of "hot-spots" on the surface of neutron stars. This work was motivated by a discussion with A. Zhitnitsky. We thank V. Pascalutsa, A. Steiner, D. Blaschke and E.V. Shuryak for important discussions. Feynman diagrams have been drawn using Jaxodraw\[14\]..

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