A VOF based multi-material method to study impact and penetration problems

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Abstract. A 2D axisymmetric Eulerian method is developed for studying multi-material elastic-plastic problems involving large material deformations. It consists of a Lagrangian plus remap strategy using a volume-of-fluid (VOF) based material interface tracking. The multi-material formulation used in this method allows one to update the energy and stress components of each material in a mixed cell independently. This assumes common strain-rates to all materials present in a mixed cell. The equivalent pressure and stress in a mixed cell are determined using volume weighted average. The present scheme, therefore, eliminates the commonly used pressure relaxation method, mixed EOS evaluation and energy partition schemes. The stress components of each material are transported using a second-order monotonic upwind scheme (MUSCL) due to van Leer. The capability of the proposed method is demonstrated by applying to various impact and penetration problems involving large material deformations. Reasonable agreement with experimental results are observed.

1. Introduction
The difficulties associated with mesh based pure Lagrangian methods for problems involving large material deformations can be minimized using an Eulerian or ALE method with a specialized technique called interface reconstruction [1], where the materials are assumed to be separated by a sharp macroscopic material interface in a mixed cell. These interfaces are constructed and tracked by using VOF method [2]. The objective of this work is to develop a simple and robust algorithm using VOF method for multi-material elastic-plastic problems involving large material deformations and strong shock waves, e.g. high velocity impact and penetration problems. Also, to include a relatively simple multi-material strength model which avoids commonly used stress/pressure iteration procedures in mixed computational cells.

2. Physical model
Most of the multi-material hydrodynamic algorithms assume a common velocity field and pressure for all materials present in a mixed cell. This pressure can be determined in different ways. One method is the pressure relaxation method, where a common pressure to all materials in a computational cell is found by using an iteration procedure. In each iteration step, the volume fraction or mass fraction of each material is adjusted until the pressures of those materials are converged to a common pressure. Another method (the simplest approach) is to use the volume weighted average of pressures in a mixed cell. The assumption of local pressure equilibrium in multi-material cells and associated iteration procedures may lead to
pressure oscillations near material interfaces [3]. Also, physically, the pressure equilibrium in a computational cell cannot take place in a time shorter than that required for a sound wave to propagate through the cell at least once. For numerical stability, most of the hydrodynamic calculations are performed with a numerical time-step much smaller than this time (Courant time-step condition). Similarly, the assumption of same temperature and pressure to all materials in a mixed cell may lead to unrealistic energy flow from a hot material to a cold material after iteration procedure.

In this work, therefore, we have used the ideas given in Refs. [3, 4] for basic multi-material hydrodynamic formulation in which individual materials can possess different energies and temperatures. This leads to an evaluation of $m$ different energy equations in each mixed cell; where $m$ is the total number of materials in a mixed cell. The mean pressure in a multi-material cell is determined using a simple volume weighted average and thus avoids the iteration procedure. The same approach is extended to determine the equivalent stress in a mixed cell, i.e., by using volume weighted average of individual stress components, where the individual stress components for each material in a mixed cell is updated with an assumption of common strain rates to all materials present in a cell [5]. The governing equations can be found in Ref. [3, 4].

3. Numerical method
The physical variables are updated using a ‘directional split’ method. A fixed Eulerian mesh is used with a virtual Lagrangian calculation. The effect of Lagrangian deformations are remapped on to the original mesh during Eulerian advection and remap step at each time-step. All the solution variables including the material stress components for each material are advected in a conservative manner with the help of VOF algorithm [2] and a MUSCL scheme due to van Leer. However, there is no guarantee that the individual materials will satisfy the yield criterion after the advection step. This discrepancy is removed by applying Von-Mises yield criterion prior to next time-step calculations.

The details of VOF method used in this work can be found in Ref. [2]. The interface line is represented by $\vec{n} \cdot \vec{x} - \alpha = 0$; where, $\vec{n}$ is the exterior normal to the line and $\alpha$ is the perpendicular distance from a local origin in a cell. The local origin in a cell for each material is decided by checking the slope of the interface line [2]. The slope of a material interface line and hence the normal vector are determined using the gradient of volume fractions from neighboring cells. The material interfaces in a mixed cell are constructed using ‘onion-skin’ model with the help of a ‘material order list’ determined dynamically [2].

4. Results and discussions
The numerical experiments described in following sections are performed with a time-step calculated using an expression given in Ref. [6] with CFL coefficient equal to 0.67.

4.1. Impact and penetration of metal sphere on thin metal plate
First, we examine the case of 2D-axisymmetric high velocity impact of metal sphere on thin metal plate [8]. A detailed numerical analysis of the same problem have been reported in Ref. [9] by Mehra et. al using their 2-D and 3-D SPH codes. Two different cases from Ref. [8] are considered here. The initial parameters for both cases are summarized in Table 1. The material strength is modeled using parameters given in Ref. [10] and a Mie-Gruneisen EOS [10] is used. The computational domain consists of $600 \times 240$ cells in $z$ and $r$ directions with a mesh-size of 0.1667 mm in each direction. The background material (vacuum) is modeled with an extremely low density ideal gas with $\rho_0 = 10^{-12} \text{ kg/m}^3$ and $\gamma = 1.4$. Flux limited artificial viscosity (quadratic plus linear) is used.

The sequence of material deformation (for case 1 in Table 1) at different times are depicted in Fig. 1. The Al sphere completely penetrates through the thin Al slab. Both the impactor and
Table 1. Parameters used for impact problem. In the table, Y and $\mu$ are the yield and shear modulus respectively (in GPa). The velocity $v_c$ is given in km/s. $R_c$ is the sphere radius (cm) and $t_{\text{slab}}$ the target thickness (cm).

| Case | $R_c$ | $v_c$ | $t_{\text{slab}}$ | Impactor | Slab | $Y_{\text{Al}}, Y_{\text{Cu}}$ | $\mu_{\text{Al}}, \mu_{\text{Cu}}$ |
|------|-------|-------|-------------------|----------|------|------------------|------------------|
| 1    | 0.5   | 6.18  | 0.4               | Al       | Al   | 0.29, 0.12       | 27.6, 47.7       |
| 2    | 0.5   | 5.75  | 0.15              | Al       | Cu   | 0.29, 0.12       | 27.6, 47.7       |

Figure 1. The left side plot show the material interfaces at different times for sphere impact problem described in Sec. 4.1. Sub-plots (b) and (c) are conveniently shifted along the axis by 2 and 5 cm, respectively. The right side plot show the projectile shape and depth-of-penetration at 390 $\mu$s for penetration problem described in Sec. 4.2.

Table 2. Comparison of crater diameter, $d_{cr}$, with experimental [8] and simulation values given in [8, 9] with our results for impact problem at 20 $\mu$s. The values are given in cm.

| Set | Our Sim. | Expt. [8] | Sim. [8] | 2D-Sim. [9] | 3D-Sim. [9] |
|-----|----------|-----------|----------|-------------|-------------|
| 1   | 2.7      | 2.75-3.45 | 3.5      | 3.2–3.3     | 2.4–3.2     |
| 2   | 2.1      | 2.12      | 2.3      | 2.1–2.5     | 2.0–2.3     |

target undergo severe deformation, See Fig. 1. The nature of debris cloud expansion and the deformation of both the target and the impactor are in good agreement with the results given in Refs. [8, 9].

A comparison of simulated crater diameter ($d_{cr}$) and length to width ($l/w$) ratio of the debris cloud against experimental results is summarized in Table 2 and Table 3. The comparisons are made 20 $\mu$s after the impact, the same time point where experimental results are reported in Ref. [8]. The crater size from simulation show reasonable agreement with experimentally observed values. Inspecting the values given in Table 3, it is clear that our simulation results are close to the 3-D SPH simulation results. The reason for the difference in our simulation results with experimental values may be due to the assumption of a constant yield value for materials in our simulations. Perhaps, a rate dependent material strength model may give better match with experimental results. However, the present algorithm is capable of handling high material deformations with fairly accurate prediction of overall dynamics.
Table 3. Comparison of debris length to width ratio, $l/w$, with experimental [8] and simulation values given in [8, 9] with our results for impact problem.

| Set | Our Sim. | Expt. [8] | Sim. [8] | 2D-Sim. [9] | 3D-Sim. [9] |
|-----|----------|-----------|----------|--------------|--------------|
| 1   | 1.58     | 1.39      | 1.33     | 1.31−1.44    | 1.53−1.9     |
| 2   | 1.51     | 1.39      | 1.11     | 1.41−1.51    | 1.7−2.3      |

4.2. Impact and penetration of an ogive nosed metal projectile in to a metal slab

Next, we have studied the penetration of metal targets by ogive nose metal projectile [11]. The experiments in Ref. [11] were conducted for a range of velocities from 0.5 to 3 km/s. We have compared the depth-of-penetration for a particular case reported in Ref. [11] with our simulation result. The dimensions of the projectile and target are taken from Ref. [11]. A steel projectile is used with an ogive shaped nose with an axial length of the nose equal to 11.8 mm. The projectile diameter and total length (including ogive nose) are 7.11 mm and 71.1 mm respectively. The aluminum target is 250 mm in diameter. The calculations were performed with a variable mesh consists of 550 × 75 cells. The material parameters are taken from Ref. [11]. Also, both the projectile and target were modeled with Mie-Gruneisen EOS. The initial projectile velocity was 570 m/s. The right side plot in Fig. 1 shows the shape of the projectile and target. The penetration has stopped at about 390 µs. The projectile shows no permanent deformation (globally), which is consistent with the experiment [11]. However, a small amount of local material erosion is observed (towards the end of penetration for both the projectile and target, which was not reported in [11]. A contact algorithm such as boundary-layer or free-slip algorithm may mitigate this problem. The depth-of-penetration from simulation, 52 mm, matches well with the experimental value of 55 mm [11].

5. Summary and conclusions

We have presented an axisymmetric multi-material algorithm with material strength model and volume of fluid (VOF) method for material interface tracking. The present formulation allows one to have different EOS for each material in a mixed cell and eliminates the pressure equilibration method and mixed EOS evaluation. Numerical experiments show that the approximation of equivalent stress and pressure in a multi-material mixed cell using a simple volume-weighted average works well to approximate physical solutions reasonably. Also, the proposed algorithm can handle multi-material penetration problems with large material distortions.

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