Two-dimensional dynamic analysis of seismic slope stability using DEM coupling with strength reduction technique

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Abstract: The seismic stability of earth slopes attracts more attentions. In this study, the two-dimensional dynamic analyses of slope stability subjected to earthquakes are conducted by DEM. In order to prevent boundary effect in dynamic analysis, the quiet boundary is applied in the model, which can fully absorb seismic waves. The progressive failures are clearly observed. The obtained slip surface is in good agreement with that determined by pseudo-static method. In order to calculate the factor of safety using DEM, the strength reduction technique is adopted. A numerical example regarding the instability of the soil slope induced by earthquake shows the validation of the strength reduction technique. In the procedure, several failure criteria are proposed to determine the global instability of the slope. The factor of safety is defined and then compared with the results from other methods. It demonstrates that the discrete element method with strength reduction technique is a promising and feasible method for assess the seismic slope stability.

1. Introduction
According to a large number of surveys after earthquakes, landslide triggered by earthquake is one of the significant geological disasters, especially too many happened in “5.12” Wenchuan Earthquake. The landslides caused enormous damage to buildings, transportation systems and other critical facilities (Zhang and Jin, 2008). Therefore, it is very necessary and significant to perform dynamic analysis of slope stability under earthquake in order to reduce the loss.

The traditional pseudo-static (PS) approach is still widely used as a standard seismic slope stability evaluation, in which earthquake effects are represented by an equivalent static force. However, such a traditional approach is limited by assumptions concerning the analysis method itself and failure mechanism of the slope, and this methodology is considered to be generally conservative. Newmark’s method is mostly used in permanent displacement analysis, but it is not enough to evaluate the slope stability. Dynamic finite element method (FEM) is popularly used by many researchers, which could be applied in many complex slopes with irregular slope inclinations, inhomogeneous soils, and external loadings (Griffiths and Lane, 1999). Discrete element method (DEM) is now rapidly developed and widely accepted as a feasible tool for studying problems of large deformation and explaining meso-mechanism (Cundall and Strack, 1979). Recently, DEM has been also used in the slope stability analysis by some researchers (e.g., Chao et al., 2009; Zhang and Lu, 2010). However, there are few investigations on the application of DEM in the dynamic analysis of slope stability in the literature. This method can simulate the evolution of the mechanism of the slope failure, which is in
good agreement with investigations. And by this way, the slip surface of the slope can be rather precisely searched without specifying its shape and location.

For slopes, the factor of safety (FOS) is traditionally defined as the ratio of the actual soil shear strength to the minimum shear strength required to prevent failure. Duncan (1996) pointed that FOS is the factor by which the soil shear strength must be divided to bring the slope to the verge of failure. An alternative way of computing FOS with a finite element or a finite difference program is simply to reduce the shear strength until collapse occurs. This strength reduction technique (SRT) was proposed by Zienkiewicz et al. (1975) and widely applied to evaluate the slope stability (e.g., Matsui et al., 1992; Zheng et al., 2010). This technique has several advantages compared with other methods (Chen et al., 2007). The widespread use of the SRT should be seriously considered by geotechnical practitioners as a powerful alternative to the traditional limit equilibrium methods (Griffiths and Lane, 1999). Many experiences have been accumulated and extensive comparisons between the SRT and other methods have been made (e.g., Dawson et al., 1999; Chen et al., 2007). However, the success of SRT relies strongly on the definition of the slope failure. There are several definitions of failure such as limiting of the shear stresses on the potential failure surface (Duncan and Dunlop, 1969), non-convergence of the solution and a dramatic increase in the nodal displacements (Zheng et al., 2010). The non-convergence option and increase of displacements is widely taken as an indicator of collapse. Therefore, the accuracy of the FOS would be easily affected by using criteria.

Generally, there are two major coupled tasks in the slope stability analysis: the computation of the factor of safety and the location of the failure surface. This study aims at developing a new and feasible approach to locate the failure surface and calculate the factor of safety in the two-dimensional dynamic analysis of slope stability with the DEM. Application of the dynamic boundary in the model could well absorb seismic waves to prevent the waves from being reflected at the boundaries. The progressive failure of a slope under the earthquake is simulated in a homogeneous soil. The numerical analysis can locate the failure surface by the failure evolution. The strength reduction technique is applied in DEM to obtain the factor of safety. The discrete element method with strength reduction technique (DEM-SRT) used in dynamic analysis of slope stability offers a number of advantages over the other methods. The essence of DEM-SRT is the reduction of soil strength parameters until the slope fails. To define the FOS from the DEM-SRT, several failure criteria are proposed and discussed in the following sections. This method is verified in comparison with the other methods.

2. Two-dimensional discrete element modelling of a homogeneous soil slope

DEM is extensively used to study the behavior of granular material. Numerical simulations by DEM require appropriate micro-parameters by means of calibration processes in which the response of the numerical modeling as compared to the observed results of actual material (Potyondy and Cundall, 2004). The microscopic properties of materials are usually calibrated or determined using known macroscopic responses. We performed a series of biaxial numerical tests on granular samples to derive the soil macroscopic parameters of the granular assembly. The biaxial numerical tests have been commonly used to evaluate the microscopic properties of geo-materials. The model preparation and the determination of the microscopic parameters for the model used in this study are discussed below.

The microscopic parameters and macroscopic parameters of the soil are listed in table 1 based on a series of biaxial numerical tests by PFC2D (Itasca, 2004). A profile of slope with a slope of 20 m high and of a slope angle 45° was first built by “closed wall elements”. A large number of particles with artificially small radii were randomly generated inside the zone enclosed by the walls, and then the particles were not expanded until the desired porosity ($n = 0.16$). The microscopic parameters in table 1 were introduced to simulate a homogeneous soil slope. The assembly under the gravity loading was cycled to meet the equilibrium of forces leading to displacements of the particles at the top of the slope (figure 1). Obviously, such a small settlement would have a negligible effect on the numerical results.
Table 1. Microscopic parameters and macroscopic parameters of the soil.

| Parameter                          | Value          | Parameter                          | Value          |
|------------------------------------|----------------|------------------------------------|----------------|
| Friction angle (°)                 | 20             | Normal stiffness (N/m)              | 4.0×10^7       |
| Cohesion (kN)                      | 40             | Shear stiffness (N/m)               | 3.0×10^7       |
| Initial porosity                   | 0.16           | Density (kg/m^3)                    | 2000           |
| Particle friction coefficient      | 0.35           | Normal contact bond strength (N)    | 1×10^4         |
| Shear contact bond strength (N)    | 2×10^4         |

3. Dynamic analysis under earthquake by DEM

3.1 Application of quiet boundary conditions

Quiet boundary conditions can be achieved by the acoustic impedance. Giese (2003) introduced the “quiet boundary” in DEM to prevent vibrations from being reflected at the boundaries. Thus, waves induced by the vibrator are fully absorbed by the quiet boundaries. Therefore, the quiet boundaries can be applied in the slope model to absorb the seismic waves. Three strings of particles bonded together replace the rigid walls (the green particles in figure 1). And then normal force and shear force are exerted on the particles of the boundaries.

The normal force is:

\[ F_n = v_n \left( \frac{\pi}{2} r^2 k_n \rho \right)^{1/2} \]  

The shear force is:

\[ F_s = v_s \left( \frac{\pi}{2} r^2 k_s \rho \right)^{1/2} \]  

where \( v_n \) and \( v_s \) denote the normal and shear velocity of a particle; \( k_n \) and \( k_s \) denote the normal and shear stiffness of the particle; \( r \) and \( \rho \) are the radius and density of the particle, respectively.

A test is performed on the slope model in order to verify the reliability of the boundaries. A seismic wave (figure 2) is input along the base. The velocity of the particles both on the boundary and near the boundary is monitored in figure 3. The velocity of the particles on the boundary is close to zero. Thus, the seismic wave is fully absorbed by the quiet boundaries.

3.2 Results for the slope under earthquake

The failure of the slope under earthquake is progressive (figure 4). At the beginning of the earthquake, small deformations occur below the toe of slope in figure 4a. Then permanent sliding displacements occur surrounding the toe of the slope in figure 4b. Some cracks begin to develop in the upper part of the slope at 12s and some slides are caused from the crest to the toe on the surface in figure 4c. After the seismic loading, a destructive slide is induced along a failure surface in figure 4d.
The results of simulation after the completion of the seismic loading are shown in figure 5. Large permanent displacements are caused by the earthquake and develop a slip surface from the toe to the top (figure 5a). As shown in figure 5b, the slides are still happened. So the slope can be regarded as failure.

3.3 Analysis of the slip surface

The shape and location of the slip surface are widely researched in slope stability analysis all the time, which is also significant in practice. The critical slip surfaces are usually determined approximately through some technical measures, such as, the mesh deformation plots, the collection of some failed elements in the critical equilibrium state and other visualization techniques (Zheng et al., 2009). Although the critical slip surface can’t be directly calculated by DEM, it could be observed clearly through the displacement or velocity vectorgraphs. Once a soil slope reaches the limit equilibrium state under earthquake by DEM, a slip surface will go through the slope from the toe to the top. Figure 6 is the difference between proposed slip line by DEM and that by the pseudo-static method. It can be seen that locations of the slip surfaces from the two methods are very close to each other, particularly near the toe and the top of the slope. However, the slip surface is a little deeper in the middle of the slope than that by the pseudo-static method. Therefore, the potential failure volume of the slope may be extended slightly due to the development of the progressive failure. Despite some minor differences of results between the DEM and the pseudo-static method, the results from these two methods are generally in good agreement which suggests that the use of the DEM is satisfactory in general.

Figure 4. Simulation results of the slope at different time.

Figure 5. Results of the slope after the earthquake: (a) the displacement vectorgraph; (b) the velocity vectorgraph.

Figure 6. Slip surfaces corresponding to the pseudo-static method and DEM.
4. Evaluation of the slope stability by the strength reduction technique

From the previous analysis, DEM can be well used to locate the slip surface. Unfortunately, it can’t directly evaluate the slope stability by a factor of safety like pseudo-static method or limit equilibrium method. The strength reduction technique in combination with the limit equilibrium method, FEM or finite difference method has been widely used to analyze the slope stability (Chen et al., 2007; Huang and Jia, 2009; Dawson et al., 1999). According to the dynamic analysis above, DEM with the strength reduction technique (DEM-SRT) is a new and feasible tool to evaluate the slope stability. However, the criteria to determine the global instability of the slope are still not exactly defined and have many controversies. In this study several criteria will be proposed to evaluate the global instability of the slope.

In order to perform the slope stability analysis with the strength reduction technique, simulations are run for a series of trial factors of safety $F_s$ with cohesion and friction angle reduced by the same reduction factor. However, the values of the cohesion and friction angle after reduction are not directly input in DEM and the microscopic parameters are not reduced by the reduction factor due to the cross relationships between the microscopic parameters and the macroscopic parameters. The microscopic parameters according to the reduction of macroscopic parameters can be obtained by calibration processes.

In order to validate DEM-SRT, a numerical test is conducted on the homogeneous soil slope in the earlier section, as shown in figure 1. In the dynamic computation, horizontal displacements of three key points are recorded with time history. Notice that the locations of characteristic points A below the toe of the slope, B on the potential slip surface and C near the boundary are indicated in figure 1.

After a series of calculations in different strength reduction factors, a sudden change occurs when the strength reduction factor is from 0.87. As shown in figure 7, the horizontal displacements of the key points are stable after 16s of the seismic loading ($F_s < 0.87$), but the displacements of the points A and B are not stable and continue to increase after the earthquake ($F_s > 0.87$). It indicates that the slope is safe when the strength reduction factor is less than 0.87.

![Figure 7. Horizontal displacements of the key points versus time in different strength reduction factors.](image)

From the analysis above, the deformations begin to cause below the toe of the slope and the local failure plays an important role on the progressive failure of the slope. It can also be found in figure 8 that a sudden increase of the maximal horizontal displacement of the point A occurs when the strength reduction factor increases from 0.87.
Development of a slip surface is a significant criterion to determine the failure of the slope. Figure 9 shows the displacement vectorgraphs in different strength reduction factors. When the strength reduction factor is 0.87, the slope is safe and has no obvious slip surface (figure 9a). Slides begin to be triggered when the strength reduction factor is 0.90. A potential slip surface can be clearly observed, but not absolutely develop from the toe to the top of the slope (figure 9b). The slip surface is obviously shown in figure 9c and it has gone though the slope from the toe to the top.

![Figure 9](image9.png)

The numerical non-convergence is often taken as the indication of the global instability of the slope. In order to diagnose the state of the model, both the maximum unbalanced force and the average unbalanced force are calculated automatically for the entire assembly of balls during time stepping in DEM. If the unbalanced forces approach a nonzero value, this indicates that continuous movement of particles is occurring within the model (Itasca, 2004). When the unbalanced forces are not convergent in the soil slope model, the slope will be regarded as instability. As shown in figure 10, the maximum unbalanced force and the average unbalanced force still fluctuate seriously after the earthquake ($F_S = 0.90$). On the contrary, they respectively tend to a stable value ($F_S = 0.87$).

When all of the previous criteria are satisfied, the slope can be regarded as failure. The corresponding reduction factor can be taken as the FOS of the slope. The FOS determined from DEM-SRT analysis is 0.94. The result is compared with those obtained using the conventional pseudo-static method (FOS=1.08) and the FLAC with strength reduction technique by Zheng et al. (2010) (FOS=1.02). It can be found that the FOS by DEM-SRT is less than the others. As we known, the pseudo-static method is conservative because seismic effects are represented by an equivalent static force. The method by Zheng and DEM-SRT are absolutely dynamic analysis considering the seismic effects. Although the criteria to determine the global instability of the slope are similar, the FOS by DEM is less than that predicted by FLAC. Their close agreement verifies that DEM-SRT is reasonably effective in slope stability analysis. Although satisfactory results have been obtained with present DEM-SRT analysis, it should be mentioned that the accuracy of the DEM-SRT analysis may be influenced by the criteria. Therefore, the determination of the global instability of the slope in the DEM-SRT analysis should be also studied further.
5. Conclusions
Two-dimensional dynamic analysis of slope stability subjected to earthquake was performed by DEM. A number of interesting features of the DEM were highlighted, which are important for a proper analysis of the slope stability. In order to consider boundary effect in dynamic analysis, the quiet boundary was applied in the model, which could fully absorb seismic waves. In a homogeneous soil slope, the slope failure could be clearly defined and progressive failure development was also observed. Although the DEM could not directly calculate a critical slip surface, the slip surface was obtained through the displacement or velocity vectorgraphs. The location of the slip surface by DEM is similar to that by pseudo-static method but only a little deeper in the middle of the surface. An approach using strength reduction technique based on DEM, with the provision of a failure criterion and incorporation of graphical output, was used in order to evaluate the slope stability and obtain the corresponding factor of safety. Several failure criteria were proposed to determine the global instability of the slope: continuous displacements increment of some key points after earthquake, a sudden increase of the maximal horizontal displacement below the toe of the slope, and development of a slip surface and serious fluctuation of the unbalanced forces after earthquake. The factor of safety was obtained by the proposed procedure and then compared with the other methods. The result has demonstrated that the discrete element method with strength reduction technique is an effective and feasible method for assess the factor of safety of soil slope under earthquake.

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