About a plausible explanation of $\Upsilon(10860)$

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Abstract. The experimental resonance $\Upsilon(10860)$ presents some typical properties of conventional bottomonium states as well as apparently unconventional features. In this talk it is shown that some of the properties of $\Upsilon(10860)$, namely its mass, dilepton decay width and $\pi\pi\Upsilon$ production rates, can be explained from its interpretation as $\Upsilon(5s)$ bottomonium state, given that one has a consistent description of $b\bar{b}$ hybrid states. It is also pointed out that other puzzling properties (e.g. the dipion transition rate to $h_b$) can be understood under the assumption that $\Upsilon(10860)$ is a mixing of the $\Upsilon(5s)$ with the ground $b\bar{b}$ hybrid state.

1. Introduction

The explanation of $\Upsilon(10860)$ has now been a long standing problem in heavy quark physics. This experimental resonance was first discovered as a peak in the $e^+e^-$ cross section above the $B\bar{B}$ threshold and was promptly identified with the excited spin-triplet $1^{--}$ bottomonium state $\Upsilon(5s)$, since its mass and coupling to $e^+e^-$ were successfully described by quarkonium potential models [1, 2]. The situation changed completely with the discovery of a large production of $\Upsilon\pi\pi$ states from $e^+e^-$ annihilation processes at c.o.m. energy near the $\Upsilon(5s)$ mass, exceeding by two orders of magnitude the corresponding production rates for lower-lying $\Upsilon$ states [3, 4]. This shed some doubt on the interpretation of that resonance as conventional bottomonium state, so that the PDG adopted the notation $\Upsilon(10860)$ instead of $\Upsilon(5s)$ ever since [5]. This “unconventional” nature was further confirmed when dipion widths of $\Upsilon(10860)$ to spin-singlet bottomonium states $\Gamma(\Upsilon(10860) \to h_b(np)\pi\pi)$ were observed to be of the same order of magnitude that for dipion decays to spin-triplet states $\Gamma(\Upsilon(10860) \to \Upsilon(ns)\pi\pi)$ [6], thus making a quarkonium interpretation of $\Upsilon(10860)$ untenable.

In this talk, the nature of $\Upsilon(10860)$ is investigated in a simple, quark model picture. With the idea of evaluating the discrepancy between quark potential model predictions and experimental results, the starting point of this presentation is the historical identification of $\Upsilon(10860)$ with the standard $\Upsilon(5s)$ bottomonium state. Following this interpretation the mass, dilepton decay width and dipion transition rates to lower-lying $\Upsilon$ states are calculated. Remarkably, it is found that the enhancement in the $\Upsilon\pi\pi$ production rates near the $\Upsilon(5s)$ mass can be understood when a realistic description of the $b\bar{b}$ hybrid spectrum is introduced in the calculations. As for the explanation of other “anomalous” properties of $\Upsilon(10860)$, including (but not exclusively) the width of the Heavy Quark Spin Symmetry violating decay $\Upsilon(10860) \to h_b(np)\pi\pi$, it is shown that they can be accounted for interpreting $\Upsilon(10860)$ as a mixing of the bottomonium $\Upsilon(5s)$ state with the ground $b\bar{b}$ hybrid state.
Specifically, the content of this presentation is organized as follows: Section 2 describes the nonrelativistic quark potential model of quarkonium and hybrid states, and recaps the calculation of the dipion transition rates. The results obtained and the mixing picture of $\Upsilon(10860)$ are discussed in Section 3.

2. The model

It is a very well known fact that the nonperturbative nature of QCD at low energies makes ab-initio calculations prohibitively difficult. Fortunately, because of the nonrelativistic nature of heavy quarkonium systems it is possible to integrate out the gluon and light quarks degrees of freedom, so that effective quark models can be used to successfully describe the spectrum and many decay properties. Nonetheless there are some interesting hadronic decays of quarkonium states, like dipion emission processes, that involve low energy QCD transitions that are difficult to calculate. When the size of the quarkonium system is smaller than the scale of the gluonic field fluctuations there is a viable way to deal with these transitions, that is to expand the gluon field in multipole moments analogously to how is done in electrodynamics [7]. This is known as the QCD Multipole Expansion (QCDME), whose Hamiltonian at leading order reads

$$H_{\text{eff}} = H^{(0)} + Q_a A_0^a - d_a \cdot E^a - m_a \cdot B^a,$$

where $H^{(0)}$ is the quark model Hamiltonian, $Q_a$, $d_a$ and $m_a$ are the color charge, color electric dipole moment and color magnetic dipole moment, while $A_0^a$, $E^a$ and $B^a$ are the color static potential, the chromoelectric field and chromomagnetic field, respectively [8]. In the QCDME the zeroth order Hamiltonian $H^{(0)}$ determines the spectrum, while the multipole expansion of the gluonic field allows transitions between different bound states via gluon emissions. In this framework, the bound states correspond to solutions of the radial, nonrelativistic Schrödinger equation

$$H^{(0)}\psi_n(r) = \left[ -\frac{\hbar^2}{m_Q} \frac{d}{dr} \right]^2 + \frac{L_Q^2}{m_Q r^2} + V_\Gamma(r) \right] \psi_n(r) = E_n \psi_n(r)$$

with mass $M_n = E_n + 2m_Q$, where $m_Q$ is the quark mass, $L_Q$ is the angular momentum of the quark-antiquark pair, and $V_\Gamma(r)$ is the quark potential, specified by the Born-Oppenheimer (BO) quantum numbers $\Gamma$ [9].

For quarkonium states (corresponding to vacuum-like BO quantum numbers $\Sigma^+_g$) the quark-antiquark potential $V(r)$ is given by the ground state energy for static gluonic field configurations in presence of a color-anticolor source at a fixed distance $r$, which can be calculated in quenched lattice QCD [10]. Those energy levels are perfectly well described by a Cornell-like potential

$$V_{\Sigma^+_g}(r) = \sigma r - \frac{\zeta}{r},$$

approaching at short distances the well known attractive Coulomb law, and behaving as a string confining potential in the faraway region. Following the philosophy of the quark model, the Coulomb strength $\zeta$, the string tension $\sigma$ and the quark mass $m_Q$ are to be treated as effective parameters, whose value is chosen as to get a reasonably good fit to the $1^{--}$ bottomonium spectrum.

At leading order in the QCDME, the dipion transitions between $s$-wave bottomonium states are described as the emission of two gluons via a double electric dipole ($E_1E_1$) transition with the gluons then hadronizing into pions. This process is drawn schematically in Figure 1. The associated transition matrix element is given by

$$\mathcal{M} = i g_E^2 \langle \pi^+\pi^- | E_E | 0 \rangle \sum_{np} \frac{\langle n_f s | r | n_p \rangle \langle n_p | r | n_i s \rangle}{M_{n_i s} - M_{n_p}},$$
where we can recognize the hadronization vertex in the first term, and the $E1E1$ emission in the term inside the summation [8]. The evaluation of the hadronization vertex proceeds using soft pion techniques, yielding a phase space factor and an unknown constant that will be fixed phenomenologically. On the other hand, the calculation of the second term requires a description for the $p$--wave intermediate states between the first and second gluon emission. Note that this description should give account of the wavefunctions $|np\rangle$ as well as the mass spectrum $M_{np}$ consistently with the model used for $Υ(n_i f s)$ states. This is needed in order to get meaningful values for the matrix elements $⟨n_i f s | r | np⟩$ and the mass difference $M_{n_i s} - M_{np}$.

In between the first and the second gluon emission the quark-antiquark pair is in a color-octet configuration that interacts strongly with the emitted gluon. Conventionally these intermediate states are regarded as a strongly interacting bound systems made of a $Q\bar{Q}$ color-octet and a gluon, called hybrid states. Following the same philosophy used for quarkonium, hybrid states are to be described as solutions to the Schrödinger equation (2) with some potential chosen suitably as to take into account the presence of the gluonic excitation. In the BO approximation this potential is given by the excited energy level for static gluonic field configuration with quantum numbers of the $1^+ -$ glue lump [10, 11]. This potential is plotted in Figure 2. In the perturbative region it presents a repulsive coulombic interaction, as expected for a quark-antiquark pair in color-octet configuration, while for larger $Q\bar{Q}$ distances it takes the form of the excited string potential:

$$V_{Π_u}(r) \approx \sqrt{\sigma^2 r^2 + σh} \quad \text{if} \quad r \gtrsim 0.5 \text{ fm}.$$  

Since the $p$--wave states are expected to be little affected by the details of the potential at short distances because of the centrifugal barrier, in the calculation of the intermediate hybrid states spectrum the string potential has been used to effectively describe the BO hybrid potential for all values of $r$. In this way it is not necessary to introduce additional parameters (the string tension is fixed from the $Υ$ spectrum) and the model does not lose predictive power. Note that in terms of the BO hybrid potential this choice would correspond to a mass of the $1^+ -$ glue lump of approximately $990 \text{ MeV}$, in agreement with lattice QCD estimates [12].

Intuitively, bound states correspond roughly to the spectrum of a relativistic string with two endpoints representing the $Q\bar{Q}$ degrees of freedom, while the eventual presence of gluon excitations is associated with a stationary vibration mode of the string [13]. The dipion transition between $s$-wave $Q\bar{Q}$ states is dominated by double electric dipole emission and the states in between the first and second gluon emission are associated with $Q\bar{Q}$ hybrid states.
Table 1. Calculated dipion decay width of excited Υ states to lower lying ones and \(\pi^+\pi^-\), compared to PDG average values [5].

| Process \(\gamma\) | \(\Gamma(\text{keV})\) | PDG (\(\text{keV}\)) |
|------------------|----------------|-------------------|
| \(2s \rightarrow 1s\) | 5.7       | 5.7 ± 0.6         |
| \(3s \rightarrow 1s\) | 0.94     | 0.89 ± 0.10      |
| \(3s \rightarrow 2s\) | 0.58     | 0.57 ± 0.09      |
| \(4s \rightarrow 1s\) | 6.9      | 1.7 ± 0.3        |
| \(4s \rightarrow 2s\) | 4.0      | 1.7 ± 0.4        |
| \(5s \rightarrow 1s\) | 660      | 270 ± 70         |
| \(5s \rightarrow 2s\) | 120      | 400 ± 120        |
| \(5s \rightarrow 3s\) | 20       | 240 ± 130        |

3. Results and discussion

The dipion transition rates calculated within the QCDME are reported in Table 1. The experimental value of the width \(\Gamma(\Upsilon(2s) \rightarrow \Upsilon(1s)\pi^+\pi^-)\) has been used to fix the value of the unknown coupling constant of the \(gg \rightarrow \pi^+\pi^-\) vertex. Then the calculated transition rates of \(\Upsilon(3s)\) are perfectly reproduced, while the dipion widths of \(\Upsilon(4s)\) are bigger but still in the correct order of magnitude. It should be said though that the \(\Upsilon(4s)\) is expected to have some mixing with the \(\Upsilon(3d)\) state, and this mixing could reduce the dipion decay width of the experimental state (dipion decays of \(d^-\)-wave quarkonium are expected to be suppressed [14]).

It should be also pointed out from Table 1 that the dipion transition rates of \(\Upsilon(5s)\) are two orders of magnitude bigger than the corresponding rates for \(\Upsilon(2s,3s,4s)\), as observed in \(\Upsilon(10860)\). The only exception is \(\Upsilon(5s) \rightarrow \Upsilon(3s)\pi^+\pi^-\), but this can still be qualitatively understood. In fact, resonant contributions (that are not taken into account here) are observed to be much more important for dipion decay to \(\Upsilon(3s)\) than to \(\Upsilon(1s,2s)\) [15]. Although there are uncontrolled sources of error in the calculation (e.g. resonant contributions, higher multipole contributions, gluon hadronization vertex) becoming more important for higher bottomonium excitation, these results show that the enhancement in the \(\pi\pi\) production rates at the mass of \(\Upsilon(5s)\) can be understood. So, should the bottomonium interpretation of \(\Upsilon(10860)\) be restored? Experimental data on dipion transitions to spin singlet states, \(\Upsilon(10860) \rightarrow h_b(np)\pi^+\pi^-\), suggests that this is not the case [6]. Although it is not possible to quantify precisely the deviation of observations from quark model predictions, semi-quantitative calculations within the QCDME yield a result that is at least two orders of magnitude smaller than data [16].

If \(\Upsilon(10860)\) cannot be interpreted as a quarkonium state, then the most straightforward alternative is to consider a possible hybrid component of this resonance. Indeed the hybrid states used to calculate the dipion transition rates are physical bound states of the present model, and as such undergo dynamical mixing with quarkonium bound states with same quantum numbers and similar mass. A look at the bottomonium and \(b\bar{b}\) hybrid spectrum reported in Table 2 reveals that the \(1^-\)-wave hybrid mass is similar to the calculated \(\Upsilon(5s)\) mass and to the experimental \(\Upsilon(10860)\). Therefore a mixed nature of \(\Upsilon(10860)\) is feasible. In this mixing picture the state vector of \(\Upsilon(10860)\) would be written as

\[
|\Upsilon(10860)\rangle = \cos \theta |\Upsilon(5s)\rangle + \sin \theta |H_b(1p)\rangle,
\]

where \(\theta\) is the mixing angle and \(H_b(1p)\) is the spin-singlet, ground hybrid state with \(J^{PC} = 1^{-+}\).

At present there is no way to reliably calculate the mixing angle \(\theta\) nor the hybrid decay properties, but it is possible to give some phenomenological constraints [16]. First, since the experimental branching fractions \(\mathcal{B}(\Upsilon(10860) \rightarrow e^+e^-)\) are well reproduced by the calculated
Table 2. Calculated $b\bar{b}$ hybrid spectrum compared to calculated $1^{-+}$ bottomonium spectrum. Values of the experimental $\Upsilon$ resonances from [5] are also quoted for comparison.

| Hybrid $nl$ | $M_{nl}$ (MeV) | Bottomonium $nl$ | $M_{nl}$ (MeV) | PDG $M$ (MeV) |
|------------|----------------|-----------------|----------------|--------------|
| 1s         | 9463.2         | 9460.3          |                |              |
| 2s         | 10022.9        | 10023.3         |                |              |
| 1d         | 10168.9        | 10163.7         |                |              |
| 3s         | 10357.8        | 10355.2         |                |              |
| 2d         | 10454.7        |                |                |              |
| 4s         | 10628.4        | 10579.4         |                |              |
| 3d         | 10702.6        |                |                |              |
| 1p         | 10887.9        | 5s              | 10856.4        | 10889.9      |
| 2p         | 11082.2        | 6s              | 11080.9        | 10992.9      |

$\mathcal{B}(\Upsilon(5s) \rightarrow e^+ e^-)$, an upper bound on the mixing amplitude of at most some percent is expected. Second, in order to justify the big dipion width to $h_b$ with such a small mixing angle, the hybrid state is supposed to have relatively large decay widths. Note that the upper bound on the mixing amplitude can be qualitatively justified from approximate Heavy Quark Spin Symmetry, which forbids the mixing between spin-singlet and spin-triplet states. On the other hand, the lower bounds on the hybrid decay widths are in agreement with theoretical estimates from constituent gluon models for the so-called ‘quark-excited’ hybrid states [17].

To summarize, a small mixing between the $\Upsilon(5s)$ quarkonium state and the $H_b(1p)$ hybrid state might justify all the known properties of $\Upsilon(10860)$.

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