Estimation of the image quality correction efficiency in optical measurement systems under Scheimpflug condition

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Abstract. A criterion for evaluating the image quality correction efficiency in optical systems under Scheimpflug conditions is proposed. The limits of applicability of the geometric optics approximation are clarified for the sensor tilt angle calculating. The results of a simulation optical stereoscopic system arm for three cases: normal, oblique system not under the Scheimpflug condition and oblique system under the Scheimpflug condition are presented. The quantitative estimation of the image quality correction efficiency is performed. The ranges of object distances and viewing angles, providing the maximum image quality correction without loss of accuracy due to perspective distortion are defined.

1. Introduction
Optical research methods are widely used in scientific and production aims. These are flow visualization systems; measuring systems, for example, stereo-PIV [1, 2], close-range photogrammetry [3], automated surface shape control production systems and many others. The design features of stereoscopic, tomographic and some photogrammetric measurement systems do not allow placing the image recording equipment normally to the optical axis. In such cases tilt and shift lens or Scheimpflug adapters are used to aligning the image plane and the sensor plane in accordance with the Scheimpflug principle [4]. Despite the fact that perspective distortions caused by oblique view are increased due to Scheimpflug condition, these distortions are successfully corrected by the different calibration methods [5-10]. Although the image quality is improved, it will not be the same as in the case of the classical optical system, when the object plane and the image plane are parallel to each other and perpendicular to the optical axis. Typically the measurements error is estimated like the standard deviation between the measured results and model predictions [1, 2, 11]. This estimation does not allow identifying individual error components. The reconstruction quality factor (the quality factor) dependences upon the particle image diameter, particle image number density, number of independent cameras are presented in [1] and there is no information about the Scheimpflug correction influence to the quality factor. In the present article, the quantitative estimation criterion of the Scheimpflug correction effectiveness in optical measurement systems based on the information about the deviation of the size of the circle of confusion of nominal is proposed.

In the classical optical system, when the object plane and the sensor plane are perpendicular to the optical axis, the circle of confusion size is usually minimal for the center of the field of view and
increases with increasing field of view. The circle of confusion size distribution in the field of view is symmetrical with respect to the optical axis. In the case of images registration at a certain viewing angle to the optical axis, the distribution ceases to be axisymmetric. For the part of the object plane that is removed from the lens when the optical system is oblique, the size of the scattering spots will be larger than it was for classical images and larger than for the part of the object plane that approaches the lens when the optical system is oblique. The rotation of the sensor plane at a certain viewing angle in accordance with the Scheimpflug principle allows combining image plane and sensor one, and the size of the scattering spots will be smaller than for images captured at a viewing angle not under the Scheimpflug condition. The dispersion of the scattering spot radius on the field of view will also decrease, thus the asymmetry of the scattering circles distribution across the field of view is compensated. Figure 1 shows the spot diagrams for three cases: the classical optical system (a), the oblique system without correction (b) and under the Scheimpflug condition (c).

![Figure 1. Examples of scattering spots distribution across the field of view: (a) the classical optical system; (b) the oblique system without correction; (c) the oblique system under the Scheimpflug condition.](image1)

Figure 2 schematically shows the dependence of the scattering circle size across the field of view. The dashed line corresponds to the images obtained by the optical system normally located to the object plane without tilting the sensor plane (normal images). The red line corresponds to the images obtained by the optical system at a certain viewing angle to the object plane, without tilting the sensor plane (uncorrected images). The green line corresponds to the images obtained by the optical system under the Scheimpflug condition (corrected images). The value $\Delta r(y)$ is the difference between the scattering spots radii of the corrected and normal images for the linear field of view $y$. The value $\Delta r_0(y)$ is the difference between the scattering spots radii of uncorrected and normal images for the linear field of view $y$.

![Figure 2. Distributions of scattering spot radii across the field of view for normal, uncorrected and corrected images.](image2)
To estimate the image quality correction effectiveness in optical measurement systems under Scheimpflug condition, a criterion can be introduced that contains information on the spot size RMS distribution deviation for uncorrected and corrected images compared to the normal image. Perform the averaging over the pupil

$$\sigma_R = \left( \frac{1}{n} \sum_{n} (\Delta r(y))^2 \right)^{1/2}, \quad \sigma_{R0} = \left( \frac{1}{n} \sum_{n} (\Delta r_0(y))^2 \right)^{1/2},$$

(1)

$n$ is number of points, $y$ is linear field of view, $\sigma_R$ is the RMS across the field of view of the spot radii difference for corrected and normal images, $\mu m$; $\sigma_{R0}$ is the RMS across the field of view of the spot radii difference for uncorrected and normal images, $\mu m$. The criterion for evaluating the image quality correction effectiveness is determined as

$$\eta = \frac{\sigma_{R0} - \sigma_R}{\sigma_{R0}}.$$ 

(2)

If no correction is made $\sigma_R = \sigma_{R0}$ and $\eta = 0$, in case of full correction $\sigma_R = 0$ and $\eta = 1$. To determine the dependence of the image correction efficiency on such parameters of the optical measuring system as the relative object distance and the viewing angle, the optical system with oblique image plane was simulated.

2. Simulation of the optical stereoscopic system arm with image plane tilt

A photographic lens with a focal length of 35 mm was chosen for modeling. The aperture diaphragm located between the second and third lenses, the modelling was carried out for the aperture diaphragm size $f'/22$. The simulation was performed for three wavelengths: 450 nm, 550 nm and 650 nm; primary wavelength is 550 nm. Figure 3 shows the optical system layout with tilted image plane. Object distance $A = 350$ mm, it is corresponds to 10 focal lengths.

![Figure 3. Rays layout for the following optical system parameters: $\theta = 40^\circ, \alpha = 5.2^\circ$.](image)

In the course of the preliminary work the limits of applicability of the geometric optics approximation were clarified, and it was shown that the equation [12]

$$\tan \alpha = \frac{f'}{A} \tan \theta,$$

(3)

gives a result that is in good agreement with the results of numerical simulation for the values of the relative object distance $A \geq 10 f'$. In the equation (3) are indicates: $\theta$ is the viewing angle, $\alpha$ is the angle of the image plane inclination. Figure 4 shows graphs of the image plane inclination on the viewing angle for the values of the object distance $Af' = 2, 4, 10, 20, 30$. 
Figure 4. Functions $\alpha(\theta)$ obtained by MTF analysis (curves with markers) and analytically using the equation (3) (curves without markers).

The modulation transfer function of the optical system with tilted image plane analysis showed that each viewing angle corresponds to the image plane tilting angle in which the MTF of the top and bottom pupil zones are the same. Pairs of values $\theta$ and $\alpha$, determined by the coincidence of MTF for two symmetrical relative to the object rotation axis points, are marked in figure 3. It should be noted that for the viewing angles from $0^\circ$ to $40^\circ$, the functions $\alpha(\theta)$ are almost linear. For the object distance over 10 focal ones requires precision of sensor positioning not worse than 1° for every 10° the object plane tilt. For each pair of the viewing angle and the object distance by the equation (3) the corresponding image plane tilting angle was calculated, which provides the maximum image correction for this configuration. The calculation results are shown in table 1.

| Viewing angle $\theta$ | 10° | 20° | 30° | 40° | 50° | 60° |
|------------------------|-----|-----|-----|-----|-----|-----|
| 1/30                   | 0.3 | 0.7 | 1.1 | 1.6 | 2.3 | 3.3 |
| 1/25                   | 0.4 | 0.8 | 1.3 | 1.9 | 2.7 | 3.7 |
| 1/20                   | 0.5 | 1.0 | 1.7 | 2.4 | 3.4 | 4.9 |
| 1/15                   | 0.7 | 1.4 | 2.2 | 3.2 | 4.4 | 6.6 |
| 1/10                   | 1.0 | 2.1 | 3.3 | 4.8 | 6.8 | 9.8 |

Table 1. Image plane tilting angle $\alpha^\circ$.

Thus, three series of images were obtained:
1) the object plane and the image plane are perpendicular to the optical axis (normal images);
2) the tilt of object plane is not corrected (uncorrected images);
3) the tilt of object plane is corrected according to Scheimpflug principle (corrected images).
The first and second images series are reference and allow estimating the corrected images deviation from the normal and uncorrected ones at the same object distance. It is also possible to estimate the deviation of the corrected image from the normal, i.e. the image quality reduction. Spot diagrams are shown on the figure 5. In the bottom field displays the RMS scattering spot radii for each field of view (inside the red outline).

![Figure 5. Spot diagrams analysis window.](image)

3. Simulation results
To analyze the dependence of the standard deviation of the scattering spot size of the corrected and normal images on the object distance and the viewing angle by the equation (1) the values $\sigma R$ were calculated and functions graphs were plotted (figures 6 - 7).

![Figure 6. Dependency $\sigma R(\alpha f')$ graphs for different viewing angles.](image)

![Figure 7. Dependency $\sigma R(\theta)$ graphs for different relative object distance.](image)
It is possible to distinguish the range of values $A f'$ from 20 to 25, within which the value $\sigma R$ does not change significantly in the entire range of the investigated viewing angles. As the viewing angle increases, the value $\sigma R$ increases for all distance values. Moreover, the smaller object distances, the greater gradient of the function $\sigma R(\theta)$ and its nonlinearity. Using the obtained results, in accordance with the proposed criterion for estimating the image quality correction effectiveness in optical measurement systems under the Scheimpflug condition, the values of the parameter $\eta$ were calculated and graphs $\eta$ as a function of the object distances and viewing angles were constructed (figure 8).

![Image](image_url)

**Figure 8.** Image correction efficiency as a function of viewing angle.

The correction effectiveness increases nonlinearly with increasing object distance and decreases slightly with increasing viewing angle (not more than 1%). In the manual [12] it is shown that while the viewing angle increases to $45^\circ$, the error of the normal velocity component determined by the stereo-PIV method decreases nonlinearly; the viewing angle values more than $45^\circ$ is also undesirable because of the considerable calibration coefficients inhomogeneity. The accuracy of measurements by optical systems under the Scheimpflug condition is determined by many factors, including the lens and camera parameters. With increasing object distance and viewing angle the resultant error may increase, despite the high correction of optical image quality, by increasing contributions to the overall error of constituents due to other mechanisms.

4. **Conclusion**

The criterion for the estimating image quality correction in optical measurement systems under the Scheimpflug condition based on the analysis of scattering spot size dynamics is proposed. It is shown that the results of calculating the image plane angle in the approximation of geometric optics and obtained by ray tracing coincide for the object distances greater than 10 focal ones. Using the optical system model with the image tilt is obtained that the Scheimpflug correction efficiency increases from 55% to 63% for the relative distances range from 10 to 30 focal lengths in the range of viewing angles from $10^\circ$ to $60^\circ$. The high correction in which there is no loss of measurement accuracy due to perspective distortions is 61-62% and is achieved at a relative object distance of 20-25 focus ones for the viewing angles is in the range of $45^\circ$. 


The results are obtained for a computed model in which the inclination angles of the object plane and the image one are precisely matched in accordance with the Scheimpflug principle. In real measurements systems, taking into account the fact that the required accuracy of the image tilt is about 0.1° at the object distances of more than 10 focal lengths, we can expect an image quality correction efficiency decrease due to inaccurate positioning of the sensor plane.

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References
[1] Raffel M, Willert C E, Scarano F, Kähler C, Wereley S T and Kompenhans J 2018 Particle Image Velocimetry. A Practical Guide (Springer International Publishing AG, part of Springer Nature) 669
[2] Scarano F 2013 Meas. Sci. Technol. 24(1) 012001
[3] Nocerino E, Menna F, Remondino F, Beraldin J A, Cournoyer L and Reain G 2016 The Int. Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences XL1-B5 99–105
[4] Merklinger H M 2010 Focusing the view camera: A scientific way to focus the view camera and estimate depth of field Internet edition 1.6.1 (Canada) 146
[5] Fournel T, Louhichi H, Barat C and Menudet J F 2006 In The 12th Int. Symp. on Flow Visualization (Göttingen Germany) 1–10
[6] Louhichi H, Fournel T, Lavest J M and Aissia H B 2007 Meas. Sci. Technol. 18(8) 2616–22
[7] Albers O, Poesch A and Reithmeier E 2015 Optics Express 23(23) 29592
[8] Fasogbon P, Duvieubourg L, Lacaze P A and Macaire L 2015 Proc. SPIE 9534, Twelfth International Conference on Quality Control by Artificial Vision (Le Creusot France) 953416
[9] Fasogbon P, Duvieubourg L and Macaire L 2016 Proc. of Int. Conf. on Computer Vision and Image Processing, Advances in Intelligent Systems and Computing 1 159–169
[10] Cong S, Haibo L, Mengna J and Shengyi C 2018 J. of Sensors 2018 3901431
[11] Sutton M A, Orteu J-I and Schreier H W 2009 Image Correlation for Shape, Motion and Deformation Measurements: Basic Concepts, Theory and Applications (New York: Springer Science+Business Media) 321
[12] User manual software ActualFlow 2004-2016 LTD SigmaPRO