Testing dynamic stabilization in complex Langevin simulations

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Phase diagram for QCD

$T$

$\mu$

Quark-Gluon Plasma

Hadrons
Phase diagram for QCD

Hadrons

Quark-Gluon Plasma

Nuclear matter

Colour Superconductor?

Critical point?
Phase diagram for QCD

- Sign problem $\leftrightarrow \det D(\mu \neq 0) \in \mathbb{C}$
Complex Langevin simulations

- Complexify degrees of freedom $\text{SU}(3) \rightarrow \text{SL}(3, \mathbb{C})$
  $$U_{x,\nu} = \exp \left[ i a \lambda^c (A^c_{x,\nu} + i B^c_{x,\nu}) \right]$$

- Evolve links according (1st order) Langevin equation
  $$U_{x,\nu}(\theta + \varepsilon) = \exp \left[ i \lambda^a (\varepsilon D^a_{x,\nu} S + \sqrt{\varepsilon} \eta^a_{x,\nu}) \right] U_{x,\nu}(\theta)$$

- Gauge cooling is essential, but sometimes not sufficient.
  $$U_{x,\nu} = \Omega_x U_{x,\nu} \Omega^{-1}_{x+\nu}$$
• For small $\beta$ Complex Langevin results differ from reweighting.
Gauge cooling

HDQCD: $10^3 \times 4, \mu = 0.7, \beta = 5.8, \kappa = 0.04, N_f = 2$

- Tunneling to wrong results.
Gauge cooling

Tunneling to wrong results, when unitnorm grows too large.
Dynamic stabilization

- Adding a trivial force to the Langevin dynamics

\[ U_{x,\nu}(\theta + \varepsilon) = \exp \left[ i \lambda^a (\varepsilon K_{x,\nu}^a - \varepsilon \alpha_{DS} M_{x}^a + \sqrt{\varepsilon} \eta_{x,\nu}^a) \right] U_{x,\nu}(\theta) \]

where

\[ M_{x}^a = b_{x}^a \left( \sum_c b_{x}^c b_{x}^c \right)^3 \quad \text{and} \quad b_{x}^a = \text{Tr} \left[ \lambda^a \sum_{\nu} U_{x,\nu} U_{x,\nu}^\dagger \right]. \]

- Expanding the force in terms of gauge fields \( A \) and \( B \)

\[ M_{x}^a \sim a^7 \left( \overline{B}_y^c \overline{B}_y^c \right)^3 \overline{B}_x^a + \mathcal{O}(a^8). \]

- Dynamic stabilization is numerically cheap and can be combined with gauge cooling (Here: 1 step)
Dynamic stablization

- Start with random $\text{SL}(3, \mathbb{C})$ configuration.
- Only Dynamic stablization, no action and noise
Dynamic stabilization

- Strength of dynamic stabilization $\rightarrow$ Control of unitnorm
Dynamic stablization

For sufficient large $\alpha_{DS}$ we obtain the correct result.
The correct transition is obtained, even for SL(3, \mathbb{C}) start.
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Heavy Dense QCD

HDQCD: $10^3 \times 4, \mu = 0.7, \beta = 5.8, \kappa = 0.04, N_f = 2$

- Improved stability using dynamic stabilization
Correct values recovered for all gauge couplings.
Heavy Dense QCD

HDQCD: $\beta = 5.8, \kappa = 0.04, 8^3, N_f = 2$

- Comparison of with capped (unitnorm $< 0.03$) gauge cooling
Comparison of with capped (unitnorm < 0.03) gauge cooling
Last but not least - Full QCD

Full QCD: $8^3 \times 8$, $\beta = 6.4$, $m = 0.05$, $N_f = 2$

Chiral condensate

- Unimproved Staggered quarks
Last but not least - Full QCD

Full QCD: $8^3 \times 8$, $\beta = 6.4$, $m = 0.05$, $N_f = 2$

Chiral condensate susceptibility $[\times 10^{-3}]$

- Unimproved Staggered quarks
Last but not least - Full QCD

- Unimproved Staggered quarks
Conclusion
- Dynamical stabilisation allows to control the excursion into imaginary directions of $\text{SL}(3, \mathbb{C})$
- Dynamical stabilisation improves convergence

Future work
- More tests to check correct behaviour
- Start Full QCD simulations to identify phase structure of QCD.
- Extend simulations to real time QCD ($e^{iS_{QCD}}$).
Thank you for your attention!