Towards precise asteroseismology of solar-like stars

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Abstract  Adiabatic modeling of solar-like oscillations cannot exceed a certain level of precision for fitting individual frequencies. This is known as the problem of near-surface effects on the mode physics. We present a theoretical study which addresses the problem of frequency precision in non-adiabatic models using a time-dependent convection treatment. We find that the number of acceptable model solutions is significantly reduced and more precise constraints can be imposed on the models. Results obtained for a specific star (β Hydri) lead to very good agreement with both global and local seismic observables. This indicates that the accuracy of model fitting to seismic data is greatly improved when a more complete description of the interaction between convection and pulsation is taken into account.

1 Introduction

Two conditions are necessary for precise probing of the physics of stellar interiors with asteroseismology. One condition concerns the quality of the observed data. Space missions have resulted in data of unprecedented quality and improved data analysis. The other condition is that models should be able to reproduce the complex...
Table 1: Some properties of the best-fitting models.

| Model   | M (M_☉) | R (R_☉) | Age (Gyr) | T eff (K) | Z/X | αMLT | ⟨Δν⟩ (µ Hz) | χ^2  | χ^2 ν | χ^2 P |
|---------|---------|---------|-----------|-----------|-----|-------|-------------|-------|-------|-------|
| TDC     | 1.072   | 1.8211  | 8.1957    | 5858      | 0.0194 | 2.300 | 57.43       | 0.14  | 0.26  |
| Adiab.  | 1.070   | 1.8223  | 8.2923    | 5856      | 0.0194 | 2.500 | 57.67       | 0.58  | 0.29  |
| Corr.   | 1.080   | 1.8207  | 7.8221    | 5856      | 0.0194 | 1.900 | 57.63       | 0.40  | 0.31  |

The treatment of convection is one of the largest sources of uncertainty in stellar astrophysics. For cool pulsating stars, the convective envelope introduces severe complications for modelling the oscillations. In these stars, the interaction between pulsation and convection is very important for proper modeling of the oscillations. For example, this interaction has been found to be a major source of damping in δ Scuti stars [5] and in driving of oscillations in γ Doradus stars [4].

For stars with solar-like pulsations, it turns out that the transition region where the thermal relaxation time is of the same order as the pulsation period is located inside the superficial convective zone. Since these near-surface layers have not been properly modeled, they give rise to discrepancies between the computed and observed frequencies [6].

Gabriel’s formalism of time-dependent convection (TDC) [7, 8], as implemented in the MAD code [2], leads to very important results for models of δ Sct and γ Dor stars [4, 5] and also for the solar damping rates [3].

In this work, we present TDC results for models of β Hydri, a G2 IV sub-giant showing solar-like oscillations. A recent list of observed pulsation frequencies in this star is given in Brandão et al. [1]. We compare these frequencies with calculations from adiabatic models with and without the near-surface corrections of Kjeldsen et al. [10].

2 TDC modeling of β Hydri

We constructed a grid of TDC models with equilibrium models and non-adiabatic non-radial TDC oscillations calculated as described in Dupret et al. [5]. We then searched for the best-fitting model by maximizing the likelihood:

\[ \mathcal{L} = \prod_{j} \frac{1}{\sqrt{2\pi}\sigma_j} \times \exp(-\chi^2 / 2), \]

where

\[ \chi^2 = \chi^2_\nu + \chi^2_P = \frac{1}{N_{\theta}} \sum_{j=1}^{N_{\theta}} \left( \frac{\theta_{\text{theo}} - \theta_{\text{obs}}}{\sigma_j} \right)^2, \]

describes the match between the N_{\theta} parameters of the theoretical model θ_{\text{theo}} and the observed values θ_{\text{obs}}. These
parameters are the effective temperature, $T_{\text{eff}}$, the mass, $M$, the gravity, $\log g$, the radius, $R$, and the chemical composition, $(X,Z)$ as well as the frequencies of the radial modes. The quantities $\chi^2_{\nu}$ and $\chi^2_{P}$ give the partial values of $\chi^2$ associated with the frequencies and the fundamental parameters respectively.

The difference between the theoretical and observed frequencies is shown in Fig. 1 and the resulting values of $\chi^2_{\nu}$ listed in column 9 of Table 1. We note that the adiabatic models give the poorest agreement with observations. The TDC frequencies agree best with observations as shown by $\chi^2_{\nu}$. The fact that $\chi^2_{P}$ for models

![Fig. 1](image1.png)

**Fig. 1** The difference between theoretical and observed frequencies for radial modes using TDC (squares), pure adiabatic (asterisks) and adiabatic with near-surface corrections (triangles).

![Fig. 2](image2.png)

**Fig. 2** The relative probability distribution as a function of mass, $M$, and age.
with near-surface corrections is much larger than for TDC models shows that near-
surface corrections only apply to stars very similar to the Sun. This is, of course, to
be expected since the Sun is used to obtain these corrections.

In Fig. 2 we give the probability distribution as a function of mass and age. In
Table 1, columns 2–7 give the derived global parameters. Column 10 lists values of
$\chi^2_{O}$. The adiabatic models give the largest radii and the lowest masses. TDC models
and models with near-surface corrections give different masses. Models with near-
surface corrections give the lowest ages because they use the smallest values of the
mixing-length parameter, $\alpha_{MLT}$. The pure adiabatic models give the highest ages for
the same reason: they use the highest values of $\alpha_{MLT}$. Column 8 in Table 1 gives
the average large separations, $\langle \Delta \nu \rangle$. Note that these are all very similar and differ
by less than 1 $\mu$Hz from the observed value of 57.48 $\mu$Hz. The large separation
obtained using TDC agrees best with the observed value.

On the other hand the mixing length, $\alpha_{MLT}$, which is a free parameter, differs
considerably for the different models and is higher than the calibrated value for the
Sun ($\alpha_{MLT} \sim 1.8$). This agrees with the expected variability of $\alpha_{MLT}$ across the HR
diagram.

3 Conclusions

The main results of this work are as follows. (i) TDC leads to a significant improve-
ment in the agreement between calculated and observed frequencies of solar-like
oscillations as compared to the pure adiabatic frequencies. (ii) A physically more
robust modeling of the mode physics near the surface provides better constraints
on the global and free stellar parameters. (iii) Near-surface corrections are probably
only valid for stars most closely resembling the Sun.

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