Mixed Convection Peristaltic Flow of Third Order Nanofluid with an Induced Magnetic Field

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Abstract

This research is concerned with the peristaltic flow of third order nanofluid in an asymmetric channel. The governing equations of third order nanofluid are modelled in wave frame of reference. Effect of induced magnetic field is considered. Long wavelength and low Reynolds number situation is tackled. Numerical solutions of the governing problem are computed and analyzed. The effects of Brownian motion and thermophoretic diffusion of nano particles are particularly emphasized. Physical quantities such as velocity, pressure rise, temperature, induced magnetic field and concentration distributions are discussed.

Introduction

Peristaltic motion is now an important research topic due to its immense applications in engineering and physiology. This type of rhythmic contraction is the basis of peristaltic pumps that move fluids through tubes without direct contact with pump components. This is a particular advantage in biological/medical applications where the pumped material need not to contact any surface except the interior of the tube. The word “peristalsis” comes from a Greek word “Peristaltikos” which means clasp ing and compressing. The peristaltic flow has specific involvement in the transport of urine from kidney to the bladder, chyme movement in gastrointestinal tract, movement of ovum in the female fallopian tubes, blood circulation in the small blood vessels, roller and finger pumps, sanitary fluid transport and many others. Latham [1] and Shapiro et al. [2] reported initial studies for the peristaltic flow of viscous fluid. Since then ample attempts have been made for peristalsis in symmetric flow geometry (see references [3–8]). Recently, physiologists argued that the intra-uterine fluid flow (because of myometrical contractions) represents peristaltic mechanism and the myometrical contractions may appear in both asymmetric and symmetric channels [9]. Hence some researchers [10–15] discussed the peristaltic transport in an asymmetric channel with regard to an application of intra-uterine fluid flow in a nonpregnant uterus.

Heat transfer in cooling processes is quite popular area of industrial research. Conventional methods for increasing cooling rates include the extended surfaces such as fins and enhancing flow rates. These conventional methods have their own limitations such as undesirable increase in the thermal management system’s size and increasing pumping power respectively. The thermal conductivity characteristics of ordinary heat transfer fluids like oil, water and ethylene glycol mixture are not adequate to meet today’s requirements. The thermal conductivity of these fluids have key role in heat transfer coefficient between the heat transfer medium and heat transfer surface. Hence many techniques have been proposed for improvement in thermal conductivity of ordinary fluids by suspending nano particles in liquids. The term “nano” introduced by Choi [16] describes a liquid suspension containing ultra-fine particles (diameter less than 50 nm). The nanoparticle can be made of metal, metal oxides, carbide, nitride and even immiscible nano scale liquid droplets. Although the literature on flow of viscous nanofluid has grown during the last few years (see [17–32] and many refs. therein) but the information regarding peristaltic flow of nano fluids is yet scant. To our information, Akbar and Nadeem [33] studied the peristaltic flow of viscous nanofluid with an endoscope. Influence of partial slip in peristaltic flow of viscous fluid is explained by Akbar et al. [34].

The aim of present study is to venture further in the regime of peristalsis for fluids with nanoparticles. Therefore we examine here the mixed convective peristaltic transport of third order nanofluid in an asymmetric channel. Channel asymmetry is produced by peristaltic waves of different amplitude and phases. Mathematical modelling involves the consideration of induced magnetic field, Brownian motion and thermophoresis effects. Numerical solution of nonlinear problem is obtained using shooting method. Limiting case for viscous nanofluid in symmetric channel is also analyzed. Detailed analysis for the quantities of interest is seen.

Physical Model

Extra stress tensor $\mathbf{S}$ for third order fluid model is given by

\[ \mathbf{T} = -p\mathbf{I} + \mathbf{S}, \]
S = \mu A_1 + \epsilon_1 A_2 + \epsilon_2 A_3^2 + \beta_1 A_3 + \beta_2 (A_2 A_1 + A_1 A_2) + \beta_3 (tr A_1^2) A_1, \\
A_1 = L + L^T, \quad A_2 = \frac{dA_1}{dt} + A_1 L + L^T A_1, \quad L = \text{grad} V, \\
\text{in which I, } p, \mu, S \text{ and } A_1, A_2 \text{ respectively stand for the identity tensor, the pressure, the fluid dynamic viscosity, the extra stress tensor and the first and second Rivlin Ericksin tensors in which the material parameters } \epsilon_i; i(i = 1 - 2) \text{ and } \beta_i; i(i = 1 - 3) \text{ must satisfy}

\[ \mu \geq 0, \quad \epsilon_1 \geq 0, |\epsilon_1 + \epsilon_2| \leq \sqrt{24\mu \beta_3}, \beta_1 + \beta_2 = 0, \beta_2 \geq 0. \]

In the absence of displacement current, the Maxwell’s equations are

\[ \nabla \cdot E = 0, \quad \nabla \times E = -\mu \frac{\partial H}{\partial t}, \quad \nabla \times H = J, \quad \nabla \cdot H = 0. \]
Mathematical Formulation

Consider third order nanofluid in an asymmetric channel of width \(d_1 + d_2\). Let \(c\) be the speed by which sinusoidal wavetrains propagate along the channel walls. The \(X\) and \(Y\)-axes in the rectangular coordinates \((X,Y)\) system are taken parallel and transverse to the direction of wave propagation, respectively. A constant magnetic field of strength \(H_0\) acts in the transverse direction resulting in an induced magnetic field

\[
H \left( h_1(X,Y,t), h_2(X,Y,t), 0 \right). \quad \text{The total magnetic field is} \quad H^+ \left( h'_1(X,Y,t), H_0 + h'_2(X,Y,t), 0 \right).
\]

Further the lower wall is maintained at temperature \(T_0\) and nano particles concentration \(C_0\) while the temperature and nanoparticles concentration at the upper wall are \(T_1\) and \(C_1\) respectively. The wall surfaces satisfy

\[
h_1(X,t) = d_1 + a_1 \cos \left( \frac{2\pi}{\lambda} (X - c t) \right) \quad \text{.........upper wall,}
\]

\[
h_2(X,t) = d_2 + a_2 \cos \left( \frac{2\pi}{\lambda} (X - c t) + \phi \right) \quad \text{.........lower wall,}
\]
where $a_1, a_2$ are the wave amplitudes and the phase difference $\phi'$ varies in the range $0 \leq \phi' \leq \pi$. The case $\phi' = 0$ is subjected to the symmetric channel with waves out of phase and the waves are in phase for $\phi = \pi$. Further $\lambda$ is the wavelength, $t$ the time and $a_1, a_2, d_1, d_2$ and $\phi$ satisfy $a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi \leq (d_1 + d_2)^2$.

Denoting the velocity components $U$ and $V$ along the $X$ and $Y$—directions in the fixed frame, one can write $V$ as

$$V = [U(X,Y,t), V(X,Y,t), 0].$$

The fundamental equations governing the flow of an incompressible fluid are

$$\nabla \cdot V = 0.$$  

$$\rho \frac{dV}{dt} = \text{div} \mathbf{T} + \mu_s (\nabla \times \mathbf{H}^+) \times \mathbf{H}^+ + \rho g (T - T_0)$$

$$+ \rho g (C - C_0),$$

Figure 5. Influence of $Gc$ on $\Delta P_\lambda$

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Figure 6. Influence of $M$ on $\Delta P_\lambda$

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\[ \rho \left[ \frac{T}{t} + U \frac{T}{X} + V \frac{T}{Y} \right] T = \kappa \left[ \frac{T^2}{X^2} + \frac{T^2}{Y^2} \right] + \left[ \frac{\hat{C} C T}{\hat{C}} + \frac{\hat{C} C T}{\hat{C}} \right] + \frac{D_B}{T_m} \left[ \frac{T^2}{X^2} + \frac{T^2}{Y^2} \right], \]

\[ \frac{\partial H^+}{\partial t} = \nabla \times \left( \nabla \times H^+ \right) + \frac{1}{\varsigma} \nabla^2 H^+, \]

in which \( \rho \) denotes the density of fluid, \( D_T \) the thermophoretic diffusion coefficient, \( T \) the temperature, \( \hat{C} \) the concentration, \( \ell \) the thermal conductivity, \( g \) the acceleration due to gravity, \( \mathbf{p} \) is the pressure, \( D_B \) is Brownian diffusion coefficient, \( \tau = (\rho C) / (\rho C)_T \) the ratio of the specific heat capacity of the nanoparticle material and heat capacity of the fluid, \( \kappa \) the thermal diffusivity, \( \varsigma \) is the volumetric volume expansion coefficient, and \( \mu_p \) is the density of the particle, \( S_{XX}, S_{XY}, S_{YY} \) are components of extra stress tensor and \( \varsigma = \sigma \).

To facilitate the analysis, we introduce the following transformations between fixed and wave frames

\[ x = X - c t, \quad y = Y, \quad \mathbf{p}(x,y) = \mathbf{p}(X,Y,t) \]

\[ u(x,y) = \bar{U} - c, \quad \mathbf{v}(x,y) = \mathbf{V}. \]

Equations (1)–(12) in terms of above transformations give

\[ \frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} = \bar{U}, \quad \mathbf{p}(x,y) = \mathbf{p}(X,Y,t) \]

\[ \rho \left( \frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} \right) \mathbf{u} + \frac{\partial p_u}{\partial X} = \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \rho g \mathbf{C} - T_0 \]

\[ + \rho g \mathbf{C} - T_0 = \frac{\partial H^+}{\partial c} + \frac{1}{\varsigma} \nabla^2 H^+ \]

\[ \mathbf{p}(X,Y,t) = \mathbf{p}(x,y) \]

\[ \frac{\partial \mathbf{p}}{\partial x} + \frac{\partial \mathbf{p}}{\partial y} = \frac{\partial S_{XX}}{\partial x} + \frac{\partial S_{XY}}{\partial y} + \mu_p \left( \frac{\partial \mathbf{H}^+}{\partial X} + \frac{1}{\varsigma} \nabla^2 \mathbf{H}^+ \right) \]

\[ \mathbf{p}(x,y) = \mathbf{p}(X,Y,t) \]

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial X} + \frac{\partial \mathbf{v}}{\partial Y} \right) \mathbf{u} + \frac{\partial \mathbf{p}_m}{\partial X} = \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \mu_p \left( \frac{\partial \mathbf{H}^+}{\partial X} + \frac{1}{\varsigma} \nabla^2 \mathbf{H}^+ \right) \]

\[ \mathbf{p}(x,y) = \mathbf{p}(X,Y,t) \]

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial X} + \frac{\partial \mathbf{v}}{\partial Y} \right) \mathbf{u} + \frac{\partial \mathbf{p}_m}{\partial X} = \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \mu_p \left( \frac{\partial \mathbf{H}^+}{\partial X} + \frac{1}{\varsigma} \nabla^2 \mathbf{H}^+ \right) \]

\[ \mathbf{p}(x,y) = \mathbf{p}(X,Y,t) \]

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\[ \mathbf{p}(x,y) = \mathbf{p}(X,Y,t) \]
Defining $G_c, Pr, Gt, Nb, Nt, R_m, M$ as mass Grashof number, Prandtl number, local temperature Grashof number, Brownian number, respectively.

$$\frac{\partial E}{\partial y} = \frac{\partial}{\partial y} (u + \alpha h_y - \beta h_z) + \frac{1}{R_m} \left( \delta^2 \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) h_z.$$  

Figure 8. Influence of $G_t$ on $u$.
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Figure 9. Influence of $G_c$ on $u$.
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motion parameter, thermophoresis parameter, magnetic Reynolds number and Hartman number

\[ \lambda_l = \frac{c_1 c}{\mu d_1}, \quad x = \frac{\bar{x}}{\lambda}, \quad y = \frac{\bar{y}}{d_1}, \quad \bar{t} = \frac{\bar{t}}{\lambda}, \quad p_m = p + \frac{1}{2} \text{Re} \delta \frac{\mu_s (H^+)^2}{\rho c^2}, \]

\[ d = \frac{d_2}{d_1}, \]

\[ \gamma = \frac{T - T_0}{T_1 - T_0}, \quad u = \frac{\bar{u}}{c}, \quad \delta = \frac{d_1}{\lambda}, \quad S_y = \frac{d_1 S_{\theta}}{\mu c} \quad (\text{for } i, j = 1, 2, 3), \quad v = \frac{\bar{v}}{c}, \]

\[ G_c = \frac{g x d_1^2 (C_1 - C_0)}{\nu^2}, \quad G_l = \frac{g x d_1^2 (T_1 - T_0)}{\nu^2}, \quad S = \frac{H_0}{c} \sqrt{\frac{\mu_i}{\rho}}, \]

\[ E = \frac{E}{c H_0 \mu_i}, \]

Figure 10. Influence of \( \theta \) on \( u \).
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Figure 11. Influence of \( N_t \) on \( u \).
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\[
N_b = \frac{\tau D_\phi (C_1 - C_0)}{\alpha}, \quad R_m = \sigma \mu_n d_1 c, \quad Re = \frac{cd_1 \rho}{\mu}, \quad b = \frac{b_1}{d_1}, \quad h_2 = \frac{h_2}{d_1}, \quad \Omega = \frac{C - C_0}{C_1 - C_0}, \quad a = \frac{a_1}{d_1}, \quad h_y = -\frac{\phi_x}{\mu_1} \quad \phi = \frac{\bar{\phi}}{H_\phi d_1}.
\]

\[
\psi = -\frac{\partial \psi}{\partial y}, \quad u = \frac{\partial \psi}{\partial y}, \quad \psi = -\frac{\partial \psi}{\partial x}, \quad \zeta_3 = \frac{\beta_3 c^2}{\mu d_1}, \quad h_1 = \frac{h_1}{d_1}, \quad Pr = \frac{\nu}{\alpha}, \quad \zeta_2 = \frac{\beta_2 c^2}{\mu d_1}, \quad \zeta_1 = \frac{\beta_1 c^2}{\mu d_1}
\]

Figure 12. Influence of \( Nb \) on \( u \).
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Figure 13. Influence of \( Pr \) on \( u \).
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\[
N_t = \frac{\tau D_f (T_1 - T_0)}{s T_m}, \quad b = \frac{a_5}{d_1}, \quad h_s = \ddot{\phi}_s, \quad M^2 = Re S^2 R_m,
\]

\[
\dot{\lambda}_2 = \frac{\epsilon_2 c}{\mu d_1}.
\]

and then employing long wavelength and low Reynolds number approximation, the dimensionless forms of above equations in terms of stream function \( \Psi \) and magnetic force function \( \phi \) can be expressed as

\[
\frac{\partial^4 \Psi}{\partial y^4} + 2\Gamma \frac{\partial^2 \Psi}{\partial y^2} \left( \frac{\partial^2 \Psi}{\partial y^2} \right)^3 - M^2 \frac{\partial^2 \Psi}{\partial y^2} = -Gt \frac{\partial \gamma}{\partial y} - Gr \frac{\partial \Omega}{\partial y} \quad (21)
\]

\[
\frac{\partial p}{\partial x} = \frac{\partial^3 \Psi}{\partial y^3} + 2\Gamma \frac{\partial \Psi}{\partial y} \left( \frac{\partial^2 \Psi}{\partial y^2} \right)^2 - M^2 \left( \frac{\partial \Psi}{\partial y} + 1 \right) + Gt \gamma + Gr \Omega, \quad (22)
\]
The dimensionless boundary conditions are given by

\[ \Psi = \frac{F}{2}, \quad \frac{\partial \Psi}{\partial y} = -1, \quad \gamma = 0, \quad \Omega = 0, \quad \phi = 0 \quad \text{at} \]

\[ y = h_1 = 1 + a \cos(2\pi x), \]

\[ \Psi = -\frac{F}{2}, \quad \frac{\partial \Psi}{\partial y} = -1, \quad \gamma = 1, \Omega = 1, \quad \phi = 0 \quad \text{at} \]

\[ y = h_2 = -d - b \cos(2\pi x + \phi'), \]

with \( a^2 + b^2 + 2ab \cos \phi' \leq (1 + d)^2 \). The dimensionless time mean flow rate \( F \) in the wave frame is related to the dimensionless time
mean flow rate $\theta$ in the laboratory frame by the following expressions

$$\theta = F + 1 + d, \quad F = \int_{\frac{1}{2}}^{\frac{1}{2}} \frac{\partial \Psi}{\partial y} \, dy.$$  \hfill (27)

### Results and Discussion

Our main interest in this section is to examine the velocity ($u$), temperature ($\gamma$), concentration ($C$), pressure rise per wavelength ($\Delta P_l$), induced magnetic field ($h_v$) for the influence of local Grashof number ($Gt$), Deborah number ($\Gamma$), mass Grashof number ($Gc$), Prandtl number ($Pr$), Brownian motion parameter ($Nb$), Hartman number ($M$), magnetic Reynolds number ($R_m$) and thermophoresis parameter ($Nt$).

#### 4.1. Pumping characteristics

This subsection illustrates the behavior of emerging parameters $Nt$, $Nb$, $Gt$, $Gc$, and $M$ on pressure rise per wavelength $\Delta P_l$. The dimensionless pressure rise per wavelength versus time-averaged flux $\theta$ has been plotted in the Figs. 1–6. Here the upper right-hand quadrant (I) denotes the region of peristalsis pumping, where $\theta > 0$ (positive pumping) and $\Delta P_l > 0$ (adverse pressure gradient). Quadrant (II), where $\Delta P_l < 0$ (favorable pressure gradient) and $\theta > 0$ (positive pumping), is designated as augmented flow (copumping region). Quadrant (IV), such that $\Delta P_l > 0$ (adverse pressure gradient) and $\theta < 0$, is called retrograde or backward pumping. The flow is opposite to the direction of the

![Figure 18. Influence of Nb on $\Omega$.](doi:10.1371/journal.pone.0078770.g018)

![Figure 19. Influence of Pr on $\Omega$.](doi:10.1371/journal.pone.0078770.g019)
peristaltic motion and there is no flow in the last (Quadrant (III)). There is an inverse linear relation between $\Delta P_l$ and $h$. It is noticed from Figs. 1–2 and 4–5 that $\Delta P_l$ increases with $N_t$, $N_b$, $G_l$ and $G_c$ in all the pumping regions. Fig. 3 shows that pumping rate increases by increasing $\Gamma$ in pumping region. There are specific values of $h$ for which there is no difference between viscous and third order nanofluids. On the other hand, in the copumping region the pumping rate decreases with the increase in Deborah number. Fig. 6 shows that $\Delta P_l$ decreases with $M$ in copumping region.

4.2 Flow characteristics
The variations of $\Gamma$, $G_c$, $G_l$, $N_t$, $N_b$, and $Pr$ on the velocity have been plotted in this subsection. Fig. 7 shows that there is an increase in velocity at the centre of the channel when $\Gamma$ increases. We see a little influence of Deborah number on velocity near the

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**Figure 20. Influence of $\Gamma$ on $h_x$.**
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**Figure 21. Influence of $R_m$ on $h_x$.**
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walls of channel. However, magnitude of the velocity of third order nanofluid is more than viscous nanofluid. Figs. 8 and 9 depict the influence of local and mass Grashof number. Clearly the velocity increases near the lower wall. Increase in \( \theta \) supports the motion in the channel which is shown in Fig. 10. Fig. 11 shows the influence of \( N_t \) on velocity distribution. Interestingly an increasing thermophoresis leads to an increase in the fluid velocity at the lower wall of channel. There is a considerable variation near the walls \( y = h_1 \) and \( y = h_2 \) for \( N_t \) and \( Pr \) (Figs. 12–13).

4.3 Heat transfer characteristics

Effect of heat transfer on peristalsis is shown in the Figs. 14–16. Figs. 14 and 15 depict the effects of Brownian motion parameter (\( Nb \)) and thermophoresis parameter (\( N_t \)) on the temperature profile. One can observe that the temperature profile is an increasing function of \( Nb \) and \( N_t \) between the walls \( y = h_1 \) and \( y = h_2 \). In Fig. 16, we observed the effects of \( Pr \) on the temperature profile \( \gamma \) by fixing the other parameters. This Fig. indicates that the temperature increases with the increase of \( Pr \).

4.4 Mass transfer characteristics

Influence of mass transfer on peristalsis is shown in the Figs. 17–19. Figs. 17 and 18 depict that the concentration distribution increases at the upper and lower walls of channel when \( N_t \) and \( Nb \) are increased. Fig. 19 shows the effect of \( Pr \) on the concentration when the other parameters are fixed. It shows increasing behavior of \( Pr \) on concentration distribution \( \Omega \) near the walls \( h_1 \) and \( h_2 \).

4.5 Induced magnetic field characteristics

The variations of \( C, R_m \) and \( M \) on the induced magnetic field have been plotted in the Figs. 20–22. Fig. 20 shows that there is an increase in \( h_x \) when \( C \) increases. We see that magnitude of the induced magnetic field in third order nanofluid is more than...
viscous nano fluid. Figs. 21 and 22 depict the influence of $R_m$ and $M$. Clearly the $h_s$ increases near the lower half of channel.

4.6 Trapping

Trapping phenomenon is shown in the Figs. 23 and 24 for different values of $G_t$ and $N_t$ respectively. Trapping is an interesting aspect of peristaltic motion. It is the formation of a bolus of fluid by the closed streamlines. Fig. 23 is made for increasing values of $G_t$. We note that trapping exists for $G_t=0,0.1,0.2$ in the upper part of channel. It is observed that number of closed streamlines circulating the bolus reduce in number as we increase the values of local Grashof number. Meanwhile size of trapped bolus increases. Streamlines are plotted in Fig. 24 to see the effects of thermophoresis parameter ($N_t$). Clearly, the size of trapped bolus increases when $N_t$ increases from 0 to 11. An upper shift and flatness of bolus along with reduced closed streamlines is observed.

Conclusions

A detailed analysis is presented for peristaltic transport of third order nano fluid in an asymmetric channel with an induced magnetic field and mixed convection. The main findings of the presented study are listed below.

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Author Contributions

Conceived and designed the experiments: SN. Performed the experiments: SN. Analyzed the data: SN. Contributed reagents/materials/analysis tools: SN. Wrote the paper: SN. Design of problem: SN. Mathematical formulation: SN.
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