Prediction of possible exotic states from $\eta K^*$ system

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We investigate the $\eta K^*$ three body system in order to look for possible $I^G(J^{PC}) = 0^+ (1^{-+})$ exotic states within the framework of the fixed center approximation to the Faddeev equation. The study is made assuming scattering of a $\eta$ on a clustered system $\bar{K}K^*$, which has known to generate the $f_1(1285)$ or a $\bar{K}$ on a clustered system $\eta K^*$, which is shown to generate the $K_1(1270)$. In the case of the $\eta K^*$ scattering, we find evidence of a bound state $I^G(J^{PC}) = 0^+ (1^{-+})$ below the $\eta f_1(1285)$ threshold with mass around 1700 MeV and width about 180 MeV. On the other hand, considering the $K^-(\eta K^*) K_1(1270)$ scattering, we obtain a bound state $I^G(J^{PC}) = 0(1^-)$ just below the $\bar{K}K_1(1270)$ threshold with mass around 1680 MeV and width about 160 MeV.

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I. INTRODUCTION

Exotic states cannot be described by the traditional quark model and may be of more complex structure allowed in QCD such as glueballs, hybrid mesons and multiquark states. The discovery of exotic states and the study of their structure will apparently extend our knowledge of the strong interaction dynamics [1–3]. A meson with quantum numbers $J^{PC} = 1^{-+}$ which is excluded by the traditional quark model with $qq$ picture is an exotic state [4]. Interestingly, three isovector $J^{PC} = 1^{-+}$ exotic candidates, namely $\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2015)$ have been reported experimentally [5]. The $\pi_1(1400)$ was first observed by the E852 Collaboration in the reaction $\pi^-p \rightarrow \pi^-\eta N$ [6], and by the Crystal Barrel experiments in the reaction $\bar{p}n \rightarrow \pi^-\pi^0\eta$ [7] and $\bar{p}p \rightarrow 2\pi^0\eta$ [6]. The $\pi_1(1600)$ was observed by the E852 Collaboration in the reaction $\pi^-N \rightarrow \pi^-\eta N$ [7], and by the Crystal Barrel in the reaction $\bar{p}n \rightarrow \pi^-\pi^0\eta$ [8] and $\bar{p}p \rightarrow 2\pi^0\eta$ [6]. The $\pi_1(1600)$ was observed by the E852 Collaboration in the reaction $\rho^-N \rightarrow \pi^-\eta N$ [7], and by the Crystal Barrel in the reaction $\bar{p}n \rightarrow \pi^-\pi^0\eta$ [8] and $\bar{p}p \rightarrow 2\pi^0\eta$ [6]. The $\pi_1(1600)$ was observed by the E852 Collaboration in the reaction $\rho^-N \rightarrow \pi^-\eta N$ [7], and by the Crystal Barrel in the reaction $\bar{p}n \rightarrow \pi^-\pi^0\eta$ [8] and $\bar{p}p \rightarrow 2\pi^0\eta$ [6].

In this paper we study the $\eta K^*$ three body system in order to look for possible $0^+ (1^{-+})$ exotic states within the FCA approach, which has been used to investigate the interaction of $K^-d$ at threshold [33, 35]. Within the FCA approach, the $\rho(1700)$ and $\eta(1475)$ were studied in Refs. [36, 37]. The interactions of multi-vectors are studied in Refs. [38, 39]. The $\pi_2(1670)$, $\eta_2(1645)$, and $K_2^*(1700)$ were proposed as molecules made of a pseudoscalar and a tensor meson, where the latter is itself made of two vector mesons [40]. Besides, a possible state in three body system $K^-pp$ according to the calculation done within the framework of FCA approach [41, 42], has supported by the recent J-PARC experiments [43]. In Ref. [44] the $\Delta_{5/2}^+(2000)$ puzzle is solved in the study of the $\eta^-(\Delta^{++})$ interaction, and in Ref. [45], it is found a peak around 1920 MeV indicating a $NKK$ state with $I = 1/2$ around that energy, which support the existence of a $N^*$ resonance with $J^P = 1/2^+$ around 1920 MeV [46, 47]. Recently, the predictions of several heavy flavor resonance states in three body system have been carried out within the framework of FCA approach like $K^{(*)}B^{(*)}B^{(*)}$ [48], $D^{(*)}B^{(*)}B^{(*)}$ [49], $\rho B^* B^*$ [50], $\rho^* D^* D^*$ [51], $\rho D^* D^*$ [52], $\rho D^* D^*$ [53, 54], $D KK$ (DKK) [55], and $BDD$ (BDD) [56].
There are two possible scattering cases for the \( \eta \bar{K}K^* \) three body system since the \( \bar{K}K^* \) and \( \eta K^* \) system lead to the formation of two dynamically generated resonances, \( f_1(1285) \) and \( K_1(1270) \), respectively. Based on the two body \( \eta \bar{K}, \eta K^* \), and \( K^*-\bar{K} \) scattering amplitudes obtained from the chiral unitary approach \([30, 57, 58]\), we perform an analysis of the \( \eta \bar{K}K^* \) \( f_1(1285) \) and \( K^-\bar{K} \) \( K_1(1270) \) scattering amplitude, which will allow us to predict the possible exotic states with quantum numbers \( J^P = 0^+ \) in particular.

This paper is organized as follows. In Sec. II we present the FCA formalism and ingredients to analyze the \( \eta \bar{K}K^* \) \( f_1(1285) \) and \( K^-\bar{K} \) \( K_1(1270) \) systems. In Sec. III, numerical results and discussions are shown. Finally, a short summary is given in Sec. IV.

II. FORMALISM AND INGREDIENTS

Within the framework of FCA, we consider \( \bar{K}K^* (\eta K^*) \) as a cluster and \( \eta (\bar{K}) \) interacts with the components of the cluster. The total three body scattering amplitude \( T \) can be simplified as the summation of the two partition functions \( T_1 \) and \( T_2 \). \( T_1 \) \( (T_2) \) accounts for all the diagrams of Fig. 1 starting with the interaction of particle 3 with particle 1(2) of the cluster. Then the FCA equations are

\[
T_1 = t_1 + t_1 G_0 T_2, \quad (1) \\
T_2 = t_2 + t_2 G_0 T_1, \quad (2) \\
T = T_1 + T_2, \quad (3)
\]

where the amplitudes \( t_1 \) and \( t_2 \) represent the unitary scattering amplitudes with coupled channels for the interactions of particle 3 with particle 1 and 2, respectively.

On the other hand, it is worth noting that the argument of the total scattering amplitude \( T \) is regarded as a function of the total invariant mass \( \sqrt{s} \) of the three body system, while the arguments of two body scattering amplitudes \( t_1 \) and \( t_2 \) depend on the two body invariant mass \( s_1 \) and \( s_2 \), respectively, which are given by

\[
s_1 = m_3^2 + m_1^2 + \frac{(s - m_3^2 - m_{\text{cls}}^2)(m_{\text{cls}}^2 + m_1^2 - m_2^2)}{2m_{\text{cls}}^2}, \\
s_2 = m_3^2 + m_2^2 + \frac{(s - m_3^2 - m_{\text{cls}}^2)(m_{\text{cls}}^2 + m_2^2 - m_1^2)}{2m_{\text{cls}}^2},
\]

where \( m_l \) (\( l = 1, 2, 3 \)) are the masses of the corresponding particles in the three body system and \( m_{\text{cls}} \) the mass of the cluster.

Following the field normalization of Refs. \([38, 39]\), we can write down the \( S \)-matrix for the single scattering term [Fig. 1(a) and 1(e)] as

\[
S^{(1)} = S_1^{(1)} + S_2^{(1)} = \frac{(2\pi)^4}{\sqrt{2}} \delta^4(k_3 + k_{\text{cls}} - k'_3 - k'_{\text{cls}}) \times \\
\frac{1}{\sqrt{2w_3}} \frac{1}{\sqrt{2w_3'}} \left(-\frac{it_1}{\sqrt{2w_1}} + \frac{-it_2}{\sqrt{2w_2}}\right), \quad (4)
\]

where \( V \) stands for the volume of a box in which the states are normalized to unity, while momentum \( k'(k) \) and the on-shell energy \( w(w') \) refer to the initial (final) particles, respectively.

The double scattering contributions are from Fig. 1(b) and 1(f). The expression for the \( S \)-matrix for the double scattering \( [S_2^{(2)} = S_1^{(2)}] \) is given by

\[
S^{(2)} = -it_1 t_2 \frac{(2\pi)^4}{\sqrt{2}} \delta^4(k_3 + k_{\text{cls}} - k'_3 - k'_{\text{cls}}) \times \\
\frac{1}{\sqrt{2w_3}} \frac{1}{\sqrt{2w_3'}} \frac{1}{\sqrt{2w_1}} \frac{1}{\sqrt{2w_1'}} \frac{1}{\sqrt{2w_2}} \frac{1}{\sqrt{2w_2'}},
\]

\[
\times \int \frac{d^3q}{(2\pi)^3} F_{\text{cls}}(q) \left( \frac{1}{q_0^2 - |\vec{q}|^2 - m_{\text{cls}}^2 + i\epsilon} \right), \quad (5)
\]

where the \( F_{\text{cls}}(q) \) is the form factor of the cluster of particles 1 and 2, and the variable \( q_0 \) is the energy carried by the particle 3 that is given by

\[
q_0(s) = \frac{s + m_3^2 - m_{\text{cls}}^2}{2\sqrt{s}}. \quad (6)
\]

The information on the bound state is encoded in the form factor \( F_{\text{cls}}(q) \) appearing in Eq. (5), which represents essentially the Fourier transform of the cluster wave function. We will use the following form factor only for s-wave bound states, as it was discussed in Refs. \([38, 39]\):

\[
F_{\text{cls}}(q) = \frac{1}{N} \int_{|p| < \Lambda, |p - q| < \Lambda} d^3\vec{p} \frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \times \\
\frac{1}{m_{\text{cls}} - w_1(\vec{p}) - w_2(\vec{p})} \frac{1}{2w_1(\vec{p} - q)} \frac{1}{2w_2(\vec{p} - q)} \times \\
\frac{1}{m_{\text{cls}} - w_1(\vec{p} - q') - w_2(\vec{p} - q')}, \quad (7)
\]

where the normalization factor \( N \) is

\[
N = \int_{|p| < \Lambda} d^3\vec{p} \frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{m_{\text{cls}} - w_1(\vec{p}) - w_2(\vec{p})}^2.
\]

In this work we take \( \Lambda = 990 \text{ MeV} \) such that the \( f_1(1285) \) is obtained in Refs. \([53, 60]\), while for \( K_1(1270) \) we take \( \Lambda = 1000 \text{ MeV} \). The cut-off is turned to get a pole at \( 1288 - i74 \) of \( t \) matrix for the \( K_1(1270) \) state. In Fig. 2 we show these form factors of \( f_1(1285) \) and \( K_1(1270) \), respectively. We take \( m_{\text{cls}} = 1281.3 \text{ MeV} \) for \( f_1(1285) \) and 1284 MeV for \( K_1(1270) \) as obtained in Ref. \([57]\).

Similarly, the full \( S \)-matrix for the scattering of particle 3 with the cluster will be given by

\[
S = -iT \frac{(2\pi)^4}{\sqrt{2}} \delta^4(k_3 + k_{\text{cls}} - k'_3 - k'_{\text{cls}}) \times \\
\frac{1}{\sqrt{2w_3}} \frac{1}{\sqrt{2w_3'}} \frac{1}{\sqrt{2w_1}} \frac{1}{\sqrt{2w_1'}} \frac{1}{\sqrt{2w_2}} \frac{1}{\sqrt{2w_2'}}, \quad (8)
\]

By comparing Eqs. (4), (5), and (8), we can introduce suitable factors in the elementary amplitudes,
\[ \tilde{t}_1 = t_1 \sqrt{\frac{2w_{\text{cls}}}{2w_1}}, \quad \tilde{t}_2 = t_2 \sqrt{\frac{2w_{\text{cls}}}{2w_1}} \]

In Fig. 1, we show the real (solid curves) and imaginary (dashed curves) parts of the \( G_0 \) function for the \( \eta(\bar{K}K^*)_{f_1(1285)} \) system. In Fig. 2, we show the real (solid curves) and imaginary (dashed curves) parts of the \( G_0 \) function for the \( \bar{K}(\eta K^*)_{K_1(1270)} \) systems.

Then, one can quickly solve Eqs. (3) and obtain

\[ T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}. \]

The function \( G_0 \) in the above equation is the meson exchange propagator

\[ G_0(s) = \frac{1}{2m_{\text{cls}}} \int \frac{d^3q}{(2\pi)^3} F_{\text{cls}}(q) \frac{q^2 - |\vec{q}|^2 - m_3^2 + i\epsilon}{s - m_3^2 + i\epsilon}. \]

Note also that in order to evaluate the two body amplitudes \( t_1 \) and \( t_2 \), the isospin of the cluster should be considered. For the case of \( \eta(\bar{K}K^*)_{f_1(1285)} \) system, the
cluster of $\bar{K}K^*$ has isospin $I_{\bar{K}K^*} = 0$. Therefore, we have
\begin{equation}
|\bar{K}K^*\rangle_{t=0} = \frac{1}{\sqrt{2}} \left( \begin{array}{c}
\left( \frac{1}{2} , -\frac{1}{2} \right)
\end{array} \right) - \frac{1}{\sqrt{2}} \left( \begin{array}{c}
\left( -\frac{1}{2} , \frac{1}{2} \right)
\end{array} \right),
\end{equation}

where the notation followed in the last term for the states is $|I_{\bar{K}K^*}, I_{\bar{K}K^*}^z\rangle$ for $t_{31}$, while $|I_{\bar{K}K^*}, I_{\bar{K}K^*}^z, I_{\bar{K}K^*}^+\rangle$ for $t_{32}$. This leads to the following amplitudes for the single scattering contribution [Fig. 1(a) and 1(c)] in the $\eta-(\bar{K}K^*)_{f_1(1285)}$ system,
\begin{equation}
t_1 = t_{\eta\bar{K}}^{I=1/2}, \quad t_2 = t_{\eta\bar{K}}^{I=1/2}.
\end{equation}

This leads to the following amplitudes for the single scattering contribution in the $\bar{K}-(\eta K^*)_{K_1(1270)}$ system,
\begin{equation}
t_1 = t_{\bar{K}\eta}^{I=1/2}, \quad t_2 = t_{\bar{K}\eta}^{I=0}.
\end{equation}

III. NUMERICAL RESULTS AND DISCUSSION

For the numerical evaluation of the three body amplitudes, we shall need the calculation of two body interaction amplitudes of $\eta\bar{K}$, $\eta K^*$, and $\bar{K}K^*$, which were investigated by the chiral dynamics and unitary couple channels approach in Refs. [30, 57, 58]. These two body scattering amplitudes depend on the subtraction constants $a_{\eta\bar{K}}$, $a_{\eta K^*}$, and $a_{\bar{K}K^*}$. We take them as used in Refs. [30, 57, 58]: $a_{\eta\bar{K}} = -1.38$ and $\mu = m_\bar{K}$ for $I_{\bar{K}K^*} = 1/2$; $a_{\eta K^*} = -1.85$ and $\mu = 1000$ MeV for $I_{\eta K^*} = 1/2$; $a_{\bar{K}K^*} = -1.85$ and $\mu = 1000$ MeV for $I_{\bar{K}K^*} = 0$. With those parameters, we can get the mass of $f_1(1285)$ and $K_1(1270)$ at their estimated values. Then we calculate the total scattering amplitude $T$ and associate the peaks/bumps in the modulus squared $|T|^2$ to resonance states.

In Fig. 1(a), we find evidence of a bound state in the modulus squared of $\eta-(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude, which is below the $\eta f_1(1285)$ threshold with mass around 1700 MeV and width about 180 MeV. Furthermore, taking $\sqrt{s} = 1700$ MeV, we get $\sqrt{s_{\eta\bar{K}}}/2 = 927$ MeV and $\sqrt{s_{\eta K^*}} = 1315$ MeV. At this energy point, the interactions of $\eta\bar{K}$ and $\eta K^*$ are strong.

In Fig. 1(b), we show the results of the $|T|^2$ for the $\bar{K}-(\eta K^*)_{K_1(1270)}$ system. A strong resonant structure around 1680 MeV with a width about 160 MeV shows
up, which has a standard Breit-Wigner form, and suggest that a $K-(\eta K^*)_{K_1(1270)}$ state can be formed. The mass of the state is below the $K$ and $K_1(1270)$ mass threshold.

Note that the $\eta-(\bar{K}K^*)_{f_1(1285)}$ and $K-(\eta K^*)_{K_1(1270)}$ system peak positions and width are quite stable with small variation of the parameters of $a_{\eta K}$, $a_{\eta K^*}$ and $a_{KK^*}$ in the ranges of values to reproduce the results of Refs. [31, 57, 58] within uncertainties. This gives us confidence that the $\eta-(\bar{K}K^*)_{f_1(1285)}$ and $K-(\eta K^*)_{K_1(1270)}$ bound states can be formed.

IV. SUMMARY

In this work, we have used the FCA to the Faddeev equations in order to look for possible $f^G(J^{PC}) = 0^+(1-\pm)$ exotic states generated from $\eta\bar{K}K^*$ three body interactions. We first select a cluster of $\bar{K}K^*$, which has known to generated the $f_1(1285)$ in $I = 0$, and then let the $\eta$ meson interact with $\bar{K}$ and $K^*$. In the modulus squared of $\eta-(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude, we find evidence of a bound state below the $\eta f_1(1285)$ threshold with mass around 1700 MeV and width about 100 MeV. In the case of $\bar{K}$ scattering with the cluster $\eta K^*$, which is shown to generated the $K_1(1270)$ in $J = 1/2$, we obtain a bound state $I(J^P) = 0^-$ just below the $KK_1(1270)$ threshold with mass around 1680 MeV and width about 160 MeV.

The predictions of existence of possible exotic states have been made within the framework of flux tube model [19], Lattice QCD [24] and QCD sum rule [32]. The results obtained here provide a different theoretical approach for a devoted investigated of these possible exotic states.

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