Characterization and Uncertainty Analysis of the Interlaminar Inelastic Properties of Unidirectional Fibre-reinforced Composites

Tiren He*, Liu Liu* and Jifeng Xu*

*Beijing Key Laboratory of Commercial Aircraft Structures and Composite Materials, BASTRI, COMAC

School of Aerospace Engineering, Beijing Institute of Technology, Beijing, China

*Email: liuliu@bit.edu.cn

Abstract. The compression failure mode has been achieved through changing the SBS (Short Beam Shear) experiment parameters, and the nonlinear constitutive relationship along thickness direction for the typical specimens loaded in 1-3 principal material plane has been computed iterative by FEMU with the reconstruction strain field and the numerical simulated stress data of FEM model. The uncertainly of the constitutive parameters is induced by the measurement system noise in the DIC technique and the approximation error in the displacements and strains smoothing algorithm. The covariance matrix of the extracted material constitutive parameters has been given explicitly. Three material constitutive parameters were identified from a customized short-shear experiment simultaneously using an estimated optimal reconstruction mesh size as an illustration. Sensitivity of measurement noise and reconstruction parameter on extracted material properties has been investigated. The effects of region of interest (ROI) and DIC image number on uncertainties of extracted material properties have been addressed. It is suggested that there exist an appropriate ROI and the number of images, from which reliable material parameters can be identified, but much more data used in identification process always lead to smaller standard deviation and COV. It is observed that the material constants used to characterize the interlaminar stress-strain behavior show strong robustness to the measurement noise. Another key finding is that the reconstruction parameter in the global finite-element based approximation approach is critical for reliable material properties identification. Its value has to stay close to optimum for guaranteeing reliable identification of material properties.

1. Introduction

The mechanical properties of composite laminates under interlaminar loads have an important influence on the overall mechanical properties and failure modes of composite laminates[1,2]. Lateral failure often occurs first in the loading process and may lead to further damage of other layers of laminates, such as interlaminar delamination failure. With the increasing complexity of composite components under actual working conditions, the analysis of strain distribution and deformation mechanism of composite materials under interlaminar loads is the basis for further study of mechanical properties of composite materials[3-6]. In the short beam shear test[7], the material under the loading nose has a large displacement and strain gradient, and the deformation is more complex[8-11]. Therefore, in the study of identifying the mechanical constitutive parameters of materials by...
combining the short beam shear test with digital image correlation technology, the ROI far from the loading nose and the supports is mainly used. According to the previous study[12], the reliable constitutive parameters of materials can be obtained by a local region, so the complex high gradient strain region captured by digital image correlation directly under the loading nose is adopted. In this paper, the constitutive parameters of carbon fiber reinforced resin matrix (IM7/8552) unidirectional composite laminates representing stress-strain relationship along thickness direction is identified by short beam shear test and finite element model updating method. Just like the previous study, the displacement data and strain data under the loading nose are reconstructed by a full-field finite element approximation method and the constitutive parameters describing the non-linear stress-strain relationship along the thickness direction of the composite material are identified by minimizing the variance between the reconstructed strain field data and the numerical computation strain field data of FEM model. The effects of the random error level and the reconstructed parameters on the identified constitutive parameters are discussed.

2. Experiment

The test parameters and equipment are mostly as the same as that in the previous study [12] expect the loading nose diameter, which is 50.8 mm to ensure the compression-shear combined failure mode. The test equipment setup and the material direction are shown in figure 1. The deformation images is captured by the VIC-2D setup in every 5 seconds and a total number of 48 images are acquired for material constitutive constant extraction consequently.

![Figure 1](image1.png)

**Figure 1.** (a) Short beam shear (SBS) experiment setup [12]. (b) Specimen configuration of the SBS test.

The location of the ROI area on the specimen surface is showed in figure 2(a). ROI-2 supplies the stress-strain data for material parameters characterization. Considering edge effect of the reconstruction process, deformation data in a larger area ROI-1 is reconstructed to make sure the explicit of the data in ROI-2. Figure 2(b) shows the shear-compression failure mode of specimen.

![Figure 2](image2.png)

**Figure 2.** (a) The ROI on the specimen surface; (b) The shear-compression failure mode in SBS test

3. Finite element model updating

The finite element model updating method is applied to identify the interlaminar compressive nonlinear stress-strain relation. The initial approximation of the constitutive parameters are supplied by the material manufacturer. The convergence criteria of the identification process is that the relative value of the parameters updates is less than 0.5% and the variance ratio of the objective function value
is less than 1%. The flowchart in figure 3 illustrated the identification process. The objective function and the identification process are as similar as the previous study[11,12].

![Flowchart of the identification process](image)

**Figure 3.** Flowchart of the identification process

### 4. Finite element sub model of the local high-scale area

#### 4.1. Global model of finite element method

As shown in figure 4(a), a half 3D numerical finite element model with 42000 elements and 48081 nodes is established by ABAQUS. The model involves geometric nonlinearity, material nonlinearity and contact interaction. The symmetric boundary condition is defined at middle width (z=0). The FEM model is meshed with first-order continuum rectangular shape element (C3D8I) and consists of 48 steps to match the number of the images which captured by DIC in SBS test. In order to simulate the material nonlinearity, user subroutine UMAT is applied to the FEM model. The nonlinearly shear properties and interlaminar constitutive relationship are illustrated by Ramberg-Osgood expression.

#### 4.2. Finite element sub model of the specimen

Considering the high level of the stress state and the large gratitude of the strain data under the loading nose of a specimen in SBS test, in this study, a sub model is established to improve the precision of stress calculation and enhance the efficiency of iterative calculation. Only the stress-strain data in a narrow band area (ROI-2), which is shown in figure 4(b), right under the loading nose is considered in the identification process to avoid the uncertain influence of shear stress on the interlaminar stress-strain relationship. The driving variable of the sub model is the displacement of global model. The boundary condition and the loading process of the sub model is the same as the global model[11,12].

![Global modal and sub model of the specimen in SBS test by finite element method](image)

**Figure 4.** Global modal and sub model of the specimen in SBS test by finite element method

### 5. Uncertainly analysis

#### 5.1. Displacement measurement and reconstruction

The reconstruction process of full-field displacement and strain field data is followed by the previous study[12]. The standard deviation of the Gaussian white noise is \( \mu=5 \times 10^{-5} \) mm, which is determined by the average confidence margin given by VIC-2D software refer to the analysis between \( \eta_{exp} \) and \( \eta \), the analysis process is as the same as the previous study[12].
5.2. Results
A least-square regression is used to extract the stress-strain relationship through the FEM-calculated stress and the DIC-reconstructed strain data of all images in the loading process. The convergence expectation, standard deviation and the coefficient variation (COV) of each identified material parameter are listed in Table 1. Figure 5 shows the typical nonlinear converged results.

Table 1. Exception and standard deviation of identified material parameter

|          | $1/E_{33}$ (1/MPa) | $1/K_{33}$ (1/MPa) | $n_{33}$ |
|----------|--------------------|--------------------|----------|
| Exception| 1/8603.36          | 1/786.15           | 3.59     |
| Standard Deviation (SD) | 1/(8603.36×84.75) | 1/(786.15×68.97)   | 0.016    |
| COV      | 1.18%              | 1.45%              | 0.45%    |

Figure 5. Converged result for material constitutive relationship derived through least-square regression coupling of reconstruction strain and FEM stress

Figure 6 shows the displacement field obtained from DIC and FEM model with the converged identification results respectively.

Figure 6. (a) Displacement data obtained from DIC; (b) Displacement data obtained from FEM

The identification process achieved the convergence criteria after 5 iterations, the stress-strain relationship and the objective function value of each iteration are indicated in Figure 7(a) and Figure 7(b), respectively. The A curve in figure 7(a) shows a linear estimation of the constitutive relationship and it changes to three parameters Ramberg-Osgood relationship after one iteration through nonlinear least-square regression between FEM-calculated stress and reconstruction strain data. It is clearly indicated in Figure 7(b) that the identification procedure achieved the convergence criteria after 5 iterations, the objective function value downward rapidly after one iteration and converges gradually.

Figure 7. (a) Iteration results for the interlaminar nonlinear constitutive relationship; (b) variance of the normalized objective function value with iterations

5.3. Uncertainly of the initial approximation
The converged nonlinear interlaminar stress-strain curves obtained for a unidirectional SBS specimen from linear initial approximation (curve A) and over nonlinear initial approximation (curve B) of the material parameters are illustrated in figure 8. The agreement between the convergence results illustrated that the identification procedure is not sensitive to the initial approximation.

Figure 8. Converged stress-strain curves obtained from two different initial approximation A and B. A uses $E_{33}=8000\text{MPa}$ with an assumption of linear shear behavior in the all three material planes; B uses $E_{33}=8000\text{MPa}$, $K_{33}=100\text{MPa}$, $n_{33}=2.5$ with an assumption of over-nonlinear behavior in interlaminar direction.

5.4. Sensitivity to the measurement error and the reconstruction error

The uncertainly of identification results to the random error and reconstruction element mesh are conducted as the same as the previous study[12].

The influence of the displacement measurement error on the identified constitutive parameter are listed in figure 9. The synthetic data is combined with the FEM numerical displacement data and a Gaussian white noise with a standard deviation of $\mu=5\times10^{-5}$ mm, $\mu=1.5\times10^{-4}$ mm, $\mu=2.5\times10^{-4}$ mm, $\mu=3.5\times10^{-4}$ mm, $\mu=5\times10^{-4}$ mm, respectively. The reconstruction mesh size remains $h=0.18\text{mm}$ in the reconstruction process. It is clearly illustrated that all the three material parameters show strong robustness with the displacement measurement error and keeps very agreement with the nominal value with the $\mu<10\mu_0$.

Figure 9. Variations of the normalized identified material constant and the standard deviation with the standard deviation of the measurement error. (a) Mean value; (b) Standard deviation.

Besides, the effects of the reconstruction mesh size on the identified material parameter has been investigated and presented in figure 10. The synthetic data is combined with the FEM numerical displacement data and a Gaussian white noise with a standard deviation of $\mu_0=5\times10^{-5}$ mm. The results indicated that the identified material parameter are sensitive to reconstruction mesh size. The standard deviation of the identified material parameter decreases as the enlargement of the reconstruction mesh size. It has been pointed out that no parameter can be extracted reliably if an inappropriate mesh size is used for reconstruction. Thus an estimation of an appropriate reconstruction parameter, based on minimization of average strain reconstruction error using synthetic data or other complementary prior information is necessary for improving reliability of material constant identification procedures.

Figure 10. Variations of the normalized identified material constant and its standard deviation with the reconstruction mesh size. (a) Mean value; (b) Standard deviation.
6. Concluding remarks
The constitutive parameters of carbon fiber epoxy resin based unidirectional composite laminates along the thickness direction are obtained by the non-linear least squares regression through the reconstructed strain field data and the numerical stress calculated by finite element model. The Ramberg-Osgood expression is used to assume the constitutive relation along the thickness direction directly below the loading nose. Finally, the mean, standard deviation and variation coefficients of the constitutive parameters are calculated. The mean of identification results is insensitive to random error, and the nonlinear coefficient $n_{33}$ has the best robustness to random error. The standard deviation of identification results increases linearly with the level of random error. The mean of recognition results is very sensitive to the reconstruction parameters. Only when the optimal reconstruction parameters are obtained, the accurate identification results can be obtained. In the process of identification, the simulation data should be obtained from the initial constitutive values and then the approximate optimal size of reconstructed elements should be estimated. This step is indispensable in the process of identifying constitutive parameters by using finite element model updating method and digital image correlation technology.

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