ON PLUMBED L-SPACES

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Abstract. In the quest for L-spaces we consider links of isolated complete intersection surface singularities. We show that if such a manifold is an L-space, then it is a link of a rational singularity. We also prove that if it is not an L-space then it admits a symplectic filling with $b_2^+ > 0$. Based on these results we pin down all integral homology sphere L-spaces in this realm.

1. Introduction

To understand the geometric content of the Ozsváth-Szabó Floer homology better, it is important to investigate the class of three-manifolds with Floer homology of a particularly simple form: L-spaces. A rational homology sphere $Y$ is called an L-space if $\text{rk}(\widehat{HF}(Y)) = |H_1(Y)|$. Every Lens space is an L-space - this is where the name comes from. However, Lens spaces are not the only representatives of this class, rather there is a very big supply of L-spaces. Indeed, for a knot $K$ in $S^3$ such that $S^3_p(K)$ is an L-space with $p > 0$, $S^3_{p+1}(K)$ is also an L-space. For example, letting $K$ to be the pretzel knot $P(-2,3,7)$ one sees that $S^3_{18}(K)$ is an L-space. It follows that for any integer $n \geq 18$, $S^3_n(K)$ is an L-space. It is worth mentioning that the knot $P(-2,3,7)$ is hyperbolic.

In the quest for L-spaces let us consider links of isolated surface singularities. By the work of Némethi [2] it is known that links of rational surface singularities are L-spaces. We prove the converse of this statement for the singularities that are complete intersections.

Theorem 1.1. Let $Y$ be a link of isolated complete intersection surface singularity. If $Y$ is an L-space, then it is the link of a rational surface singularity.

Note that none of the L-spaces constructed as described in the beginning is an integral homology sphere. In fact, we know only a limited number of such L-spaces, and one is lead to the following conjecture.

Conjecture 1.2. (Ozsváth and Szabó) If an irreducible integral homology sphere $Y$ is an L-space, then $Y = S^3$ or $Y = \pm \Sigma(2,3,5)$.

An irreducible manifold is one that cannot be written as a connected sum of two manifolds different from $S^3$. This is why the integral homology sphere L-spaces

\[ n(\Sigma(2,3,5)) \# m(-\Sigma(2,3,5)), \]
where \( n, m \geq 0 \), are not included in the list. We provide more evidence for this conjecture by proving the following theorem.

**Theorem 1.3.** Consider an irreducible manifold \( Y \neq S^3 \) which is a link of an isolated complete intersection surface singularity. If \( Y \) is an \( L \)-space, then \( \pm Y = \Sigma(2, 3, 5) \).

The following theorem from [9] is a good indication of how much Heegaard Floer homology, namely being an \( L \)-space, affects the geometry of a manifold in general.

**Theorem 1.4.** (Ozsváth and Szabó) All symplectic fillings of an \( L \)-space have \( b^+_2 = 0 \).

Restricting to complete intersection surface singularities we can prove the converse statement.

**Theorem 1.5.** If a link of isolated complete intersection surface singularity is not an \( L \)-space, then it bounds a Stein four-manifold with \( b^+_2 > 0 \). In particular, it has a symplectic filling with \( b^+_2 > 0 \).

The paper is organized as follows: We start with the review of the plumbing construction and its relation to surface singularities. Section 4 contains the proofs of theorems 1.1 and 1.5. The last section starts with the review of the combinatorial method for computing the Ozsváth-Szabó Floer homology in the case of almost rational graphs and concludes with the proof of 1.3.

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2. The Plumbing Construction

Let \( G \) be a weighed forest, i.e. not necessarily connected graph without any cycles, to each vertex of which an integer is assigned. In what follows, the term ”graph” will be used for this specific class of objects. Let \( m(v) \) and \( d(v) \) be respectively the weight and degree of the vertex \( v \). To each vertex \( v \) associate the disk bundle on \( S^2 \) with Euler number equal to \( m(v) \). For any edge \( vw \) of the graph, choose a small disk on the spheres corresponding to \( v \) and \( w \). When the bundles are restricted to these disks, (topologically) we have \( D^2 \times D^2 \). Glue the two bundles along this two \( D^2 \times D^2 \)'s, by exchanging the base and fiber coordinates – i.e. plumb these two bundles. We denote the resulting four-manifold with boundary by \( X(G) \). It is said to be obtained by plumbing on \( G \). We let \( Y(G) \) be the oriented boundary of \( X(G) \). Three-manifolds obtained so are called plumbed manifolds.

Note that \( X = X(G) \) is simply-connected, and has a very natural representation for its second homology group furnished by the zero sections of the bundles. In fact, the group \( H_2(X; \mathbb{Z}) \) is the lattice freely spanned by vertices of \( G \). Denoting by \([v]\) the homology class in \( H_2(X; \mathbb{Z}) \) corresponding to a vertex \( v \) of \( G \), the values of the
intersection form of $X$ on the basis are given by $[v] \cdot [v] = m(v); [v] \cdot [w] = 1$ if $vw$ is an edge of $G$ and $[v] \cdot [w] = 0$ otherwise. $G$ is called negative-definite if the form is negative-definite.

The plumbed manifold $Y(G)$ has an alternative description in terms of surgery on links. To each vertex $v$ of $G$ one associates an unknot in $S^3$, framed by $m(v)$. Two unknots are linked geometrically once if there is an edge between the corresponding vertices. Since our graph does not have any cycles, this can be done unambiguously. Making surgery on this link gives us $Y(G)$. The four-manifold $X(G)$ can be obtained by attaching 2-handles to the 4-ball according to this link. It is clear that we can describe the same manifolds by different graphs, and Kirby moves would allow to move us between any two plumbing representations. These Kirby moves can be conveniently described as some combinatorial changes of the plumbing graph, see for example [12].

A graph $G$ will be called minimal if it cannot be made smaller (less vertices and/or edges) using just one Kirby move.

3. Plumbed manifolds and links of singularities

Given $m$ analytic functions $f_1, ..., f_m : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ one can consider their common zero locus $Z = \{f_1 = 0, ..., f_m = 0\}$. For each point $p \in Z$ consider the matrix

$$J(p) = \left( \frac{\partial f_i}{\partial z_j} \right)(p).$$

We will be interested in the case when the rank of $J(p)$ is equal to $n - 2$ for every $p \in Z - \{0\}$. If the rank of $J(0)$ is also equal to $n - 2$, $Z$ is said to be smooth and is analytically isomorphic to $(\mathbb{C}^2, 0)$. Otherwise, $(Z, 0)$ is called an isolated surface singularity.

Assuming that $(Z, 0)$ is irreducible and normal, one can associate to it a three-manifold as follows. Given an embedding of $(Z, 0)$ into $(\mathbb{C}^n, 0)$, one can find $\epsilon_0$ such that for any $0 < \epsilon \leq \epsilon_0$, the $2n - 1$ dimensional sphere of radius $\epsilon$ centered at the origin intersects $(Z, 0)$ transversally. This intersection is an oriented three-manifold independent of the arbitrary choices. It is called the the link of singularity of $(Z, 0)$. If $(Z, 0)$ is smooth the link is $S^3$.

Fix a good resolution $\pi : (\tilde{Z}, E) \to (Z, 0)$. Then the exceptional set $E = \pi^{-1}(0)$ is a union of smooth curves intersecting transversally. One can consider the resolution dual graph $\Gamma$ of $E$, and it turns out that the link of singularity of $(Z, 0)$ can be identified with the with the plumbed three-manifold $Y(\Gamma)$. The link is a rational homology sphere if and only if the components of $E$ are rational, and the graph $\Gamma$ does not have any loops. As a result, all links of singularities which are rational homology spheres are plumbed manifolds. Conversely, a plumbed manifold can be realized as a link of some normal surface singularity if and only if it can be obtained as a plumbing on a negative definite tree.
The geometric genus \( p_g(Z,0) \) of a singularity is defined as the dimension of \( H^1(\tilde{Z}, \mathcal{O}_{\tilde{Z}}) \).

If \( p_g = 0 \), the singularity is called rational. If a negative definite plumbed manifold is the link of a rational singularity, then the plumbed manifold and the plumbing graph are said to be rational. A graph \( G \) is called almost-rational if by decreasing just one of the weighs, the graph can be made rational, see [2].

4. Proofs of theorems 1.1 and 1.5

Recall that a smooth compact oriented 2\( n \)-dimensional manifold with possibly non-empty boundary is said to be symplectic if it admits a closed 2-form \( \omega \) such that \( \omega^n \) does not vanish. A complex manifold \( M \) (possibly with boundary) is a Stein manifold if it admits a positive, proper, and strictly plurisubharmonic (Morse) function for which \( M \) is a regular level set. By a theorem of Grauert, a complex manifold is Stein if and only if it embeds analytically and properly into \( \mathbb{C}^N \) for some \( N \). Note that Stein manifolds are symplectic.

A 1-form \( \alpha \) on an oriented closed three-manifold \( Y \) is called a contact form if \( \alpha \wedge d\alpha > 0 \). Given a contact manifold, i.e. a pair \( (Y, \alpha) \), it said to admit a symplectic filling if there is a symplectic four-manifold \( (X, \omega) \) such that \( \partial X = Y \) and \( \omega|_{\ker \alpha} \neq 0 \).

Let \( Y \) be the link of an isolated complete intersection surface singularity, say \( (Z,0) \), given by \( f = (f_1, \ldots, f_m) \). To complete intersection surface singularities one can associate a four-manifold \( F \) called Milnor fiber. It is defined as the manifold given by intersection \( f^{-1}(\delta) \cap B(\epsilon) \), where \( B(\epsilon) \) is a sufficiently small ball about the origin, \( \delta \) is a general point of \( C^m \) very close to 0. \( F \) is a smooth simply-connected four-manifold. It is oriented by the complex structure on its interior. Furthermore, its boundary is diffeomorphic to the link of singularity of \( (Z,0) \), i.e. \( Y \). Note that \( F \) is Stein, because it is a piece of complex surface in a ball.

There is a connection between the geometric genus of singularity \( (Z,0) \) and the Betti number of its Milnor fiber \( F \). In fact, we have the following formula due to Wahl, Durfee and Steenbrink:

\[
p_g(Z,0) = b_2(F)/2.
\]

The proof of this formula in its whole generality uses mixed Hodge theory.

We will use the following theorem.

**Theorem 4.1.** ([9]) Let \( Y \) be an L-space. If \( X \) is any Stein four-manifold with boundary \( \pm Y \), then \( b_2^+(X) = 0 \).

To prove the theorem, suppose that \( Y \) is an L-space. Consider the Milnor fiber \( F \) of the corresponding singularity \( (Z,0) \). Since \( F \) is a Stein manifold and \( \partial F = Y \), we have that \( b_2^+(F) = 0 \). By the formula of Wahl, Durfee and Steenbrink we get \( p_g(Z,0) = b_2^+(F)/2 = 0 \), thus \( (Z,0) \) is a rational singularity. This concludes the proof of the statement that if a link of a complete intersection surface singularity is an L-space, then is is a link of rational singularity.
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Now we turn to the theorem 1.5. If \( Y \) is not an \( L \)-space then by the previous argument \((Z, 0)\) is not rational. As a result \( b_2^+(F) = 2p_g(Z, 0) > 0 \). By a result of Varchenko [14], \( Y(G) \) admits a special contact structure \( \alpha \) which can be shown to be the contact boundary of the Milnor fiber. Thus \( F \) provides us with a Stein filling of \((Y, \alpha)\) with \( b_2^+ > 0 \). This finishes the proof.

5. Integral homology spheres among links of complete intersection surface singularities

5.1. Heegaard Floer Homology of Almost Rational Plumbings. Here we review the results of [10] and [2]. Let \( \mathcal{T}_0^+ \) denote the \( \mathbb{Z}[U] \)-module which is the quotient of \( \mathbb{Z}[U, U^{-1}] \) by the submodule \( U \cdot \mathbb{Z}[U] \), graded so that the element \( U^{-d} \) (for \( d \geq 0 \)) has degree \( 2d \). Let \( \mathcal{T}_d^+ = \mathcal{T}_0^+[d] \), i.e. the grading is shifted by \( d \).

Denoting by \( \text{Char}(G) \) the set of characteristic vectors for the intersection form of \( X(G) \), define

\[
\mathbb{H}^+(G) \subset \text{Hom}(\text{Char}(G), \mathcal{T}_0^+)
\]

to be the set of finitely supported functions satisfying the following relations for all characteristic vectors \( K \) and vertices \( v \):

\[
U^n \cdot \phi(K + 2\text{PD}[v]) = \phi(K),
\]

if \( 2n = \langle K, v \rangle + v \cdot v \geq 0 \); and

\[
\phi(K + 2\text{PD}[v]) = U^{-n} \cdot \phi(K)
\]

for \( n < 0 \).

We can decompose \( \mathbb{H}^+(G) \) according to Spin\(^c\) structures over \( Y \). Note that the first Chern class gives an identification of the set of Spin\(^c\) structures over \( X = X(G) \) with the set of characteristic vectors \( \text{Char}(G) \). Observe that the image of \( H^2(X, \partial X; \mathbb{Z}) \) in \( H^2(X; \mathbb{Z}) \) is spanned by the Poincaré duals of the spheres corresponding to the vertices. Using the restriction to boundary, it is easy to see that the set of Spin\(^c\) structures over \( Y \) is identified with the set of \( 2H^2(X, \partial X; \mathbb{Z}) \)-orbits in \( \text{Char}(G) \).

Fix a Spin\(^c\) structure \( t \) over \( Y \). Let \( \text{Char}_t(G) \) denote the set of characteristic vectors for \( X \) that can be realized as the first Chern classes of Spin\(^c\) structures whose restriction to the boundary is \( t \). Similarly, we let

\[
\mathbb{H}^+(G, t) \subset \mathbb{H}^+(G)
\]

be the subset of maps supported on \( \text{Char}_t(G) \subset \text{Char}(G) \). We have a direct sum splitting:

\[
\mathbb{H}^+(G) \cong \bigoplus_{t \in \text{Spin}^c(Y)} \mathbb{H}^+(G, t).
\]
The grading on $\mathbb{H}^+(G)$ is introduced as follows: we say that an element $\phi \in \mathbb{H}^+(G)$ is homogeneous of degree $d$, if for each characteristic vector $K$ with $\phi(K) \neq 0$, $\phi(K) \in T_0^+$ is a homogeneous element with:

$$\deg(\phi(K)) - \frac{K^2 + |G|}{4} = d. \quad (3)$$

The following theorem is proved in [2]. It is a generalization of a result from [10].

**Theorem 5.1.** Let $G$ be an almost rational negative-definite weighted forest. Then, for each Spin$^c$ structure $t$ over $-Y(G)$, there is an isomorphism of graded $\mathbb{Z}[U]$-modules,

$$HF^+(-Y(G), t) \cong \mathbb{H}^+(G, t).$$

For calculational purposes it is helpful to adopt the dual point of view. Let $K^+(G)$ be the set of equivalence classes of elements of $\mathbb{Z}_{\geq 0} \times \text{Char}(G)$ (and we write $U^m \otimes K$ for the pair $(m, K)$) under the following equivalence relation. For any vertex $v$, let

$$2n = \langle K, v \rangle + v \cdot v.$$  

If $n \geq 0$, then

$$U^{n+m} \otimes (K + 2PD[v]) \sim U^m \otimes K, \quad (4)$$

while if $n \leq 0$, then

$$U^m \otimes (K + 2PD[v]) \sim U^{m-n} \otimes K. \quad (5)$$

Starting with a map

$$\phi : \text{Char}(G) \longrightarrow T_0^+,$$

consider an induced map

$$\tilde{\phi} : \mathbb{Z}_{\geq 0} \times \text{Char}(G) \longrightarrow T_0^+$$

defined by

$$\tilde{\phi}(U^n \otimes K) = U^n \cdot \phi(K).$$

Clearly, the set of finitely-supported functions $\phi : \text{Char}(G) \longrightarrow T_0^+$ whose induced map $\tilde{\phi}$ descends to $K^+(G)$ is precisely $\mathbb{H}^+(G)$.

A basic element of $K^+(G)$ is one whose equivalence class does not contain any element of the form $U^m \otimes K$ with $m > 0$. Given two non-equivalent basic elements $K_1 = U^0 \otimes K_1$ and $K_2 = U^0 \otimes K_2$ in the same Spin$^c$ structure, one can find positive integers $n$ and $m$ such that

$$U^n \otimes K_1 \sim U^m \otimes K_2.$$  

If, moreover, the numbers $n$ and $m$ are as small as possible, then this relation will be called the minimal relationship between $K_1$ and $K_2$. One can see that $K^+(G)$ is specified as soon as one finds its basic elements and the minimal relationships between each pair of them.
Remark 5.2. The manifold under consideration is an \( L \)-space if and only if the number of basic vectors is equal to the number of the Spin\(^c\) structures. Of course, in this case there will be one basic vectors in each Spin\(^c\) structure, and the basic vectors will be unrelated.

Now we review the algorithm given in \([10]\) for calculating the basic elements. Let \( K \) satisfy
\[
m(v) + 2 \leq \langle K, v \rangle \leq -m(v) \tag{6}
\]
Construct a sequence of vectors \( K = K_0, K_1, \ldots, K_n \), where \( K_{i+1} \) is obtained from \( K_i \) by choosing any vertex \( v_{i+1} \) with
\[
\langle K_i, v_{i+1} \rangle = -m(v_{i+1}),
\]
and then letting
\[
K_{i+1} = K_i + 2PD[v_{i+1}].
\]
Note that any two vectors in this sequence are equivalent.

This sequence can terminate in one of two ways: either

- the final vector \( L = K_n \) satisfies the inequality,
  \[
  m(v) \leq \langle L, v \rangle \leq -m(v) - 2 \tag{7}
  \]
at each vertex \( v \) or

- there is some vertex \( v \) for which
  \[
  \langle K_n, v \rangle > -m(v). \tag{8}
  \]

It turns out that the basic elements of \( \mathbb{K}^+(G) \) are in one-to-one correspondence with initial vectors \( K \) satisfying inequality \([6]\) for which the algorithm above terminates in a characteristic vector \( L \) satisfying inequality \([7]\).

5.2. L-spaces and rational singularities. Nemethi proves in \([2]\) the following theorem.

Theorem 5.3. (Némethi) For a negative definite plumbing graph \( G \) the following are equivalent:

(i) \( G \) is rational;
(ii) \( \mathbb{H}^+(G, t_{\text{can}}) = T_d^+ \) for some \( d \);
(iii) For every \( t \), \( \mathbb{H}^+(G, t) = T_d^+ \) for some \( d = d(t) \).

Here \( t_{\text{can}} \) is the canonical Spin\(^c\) structure on \( Y(G) \). In our description it can be specified as follows. Let \( K_{\text{can}} \) be the canonical characteristic vector, i.e. the vector in \( \text{Char}(G) \) that satisfies
\[
\langle K, v \rangle = m(v) + 2,
\]
for every vertex \( v \) of the plumbing graph \( G \). There is a unique Spin\(^c\) structure, which we will denote by \( t_{\text{can}} \), such that the inclusion \( K_{\text{can}} \in \text{Char}(G, t_{\text{can}}) \) is true.
Recall that for each Spin\(^c\) structure \(t\), we have the map of \(\mathbb{Z}[U]\)-modules
\[
T^+: \text{HF}^+(-Y(G), t) \longrightarrow \mathbb{H}^+(G, t).
\]
Némethi proves that if \(G\) is rational then \(T^+\) is an isomorphism. This proves the following theorem.

**Theorem 5.4.** (Némethi) For any negative definite rational plumbing graph \(G\), the three-manifold \(Y(G)\) is an \(L\)-space, i.e. links of rational surface singularities are \(L\)-spaces.

5.3. **Proof of Theorem 5.3** By the theorem 1.1 we have to find all rational graphs for which the plumbed manifold is an integral homology sphere. Note that by the condition of irreducibility we can concentrate on connected graphs, because for disconnected graphs the resulting manifold is the connected sum of plumbed manifolds corresponding to the components of the graph. Without loss of generality we assume that the graph to be minimal. We claim that the only connected rational negative definite plumbing graph which gives rise to an integral homology sphere is (negative) \(E_8\). Obviously, this claim implies the theorem.

We will use the characterization of rational graphs by Nemethi, and the fact that any subgraph of a rational graph is also rational. The same is true about being negative definite. Letting \(G\) be our weighed plumbing graph with weigh at the vertex \(v\) denoted by \(m(v)\), we have several possibilities.

**Case 1** The graph has at least one vertex, say \(w\), with weigh \(-1\). The condition of minimality and the fact that we are not interested in \(S^3\) means that there are at least three vertices adjacent to \(w\). Consider the graph \(G'\) made of the vertex \(w\) and all vertices adjacent to it. We will arrive at a contradiction by assuming that \(G\) is rational. Note that by this assumption, \(G'\) is also rational. Thus \(\pm Y(G')\) is an \(L\)-space. Consider the graph \(-G'\), i.e. the same graph with weighs multiplied by \((-1)\). Using usual transformations that do not change the plumbed manifold, we can modify this graph to get a new graph \(H\) with all of its weighs \(\leq -2\). \(H\) will have a strand of \(-2\)'s of length \(m(v) - 1\) for each vertex \(v \neq w\). These strands will join together at a central vertex with weigh \(m = 1 - \#(\text{vertices adjacent to } w \text{ in } G')\). Thus we have \(Y(G') = -Y(H)\). Consider the four-manifold \(X(H)\). It has \(b_2^+ = 1\) and it is Stein, because it can be obtained by Legendrian surgery on a link in \(S^3\). The last is true because all the weighs of \(H\) are less than \(-1\). Remembering that \(\partial X(H) = Y(H)\), the theorem 5.1 implies that \(\pm Y(H) = \mp Y(G')\) is not an \(L\)-space, thus yielding a contradiction.

**Case 2** The graph has no vertex with weigh \(-1\), and there is at least one vertex \(w\) with \(m(w) \leq -3\). Consider the canonical vector \(K_{can}\). We can modify it by adding 2 to its entry corresponding to the vertex \(w\). Let \(K'\) denote this new vector. Obviously \(K'\) satisfies the inequality 6. Now if we run the algorithm described in the section 5.1 on this vector, the algorithm will stop at the very first step yielding \(L = K'\). Clearly \(L\) satisfies the inequality 7 thus \(K'\) is a basic vector. Since we want \(G\) to be rational, it
follows that $K'$ cannot be in the same Spin$^c$ structure as $K_{con}$, because otherwise for no $d$, the identification $\mathbb{H}^+(G, t_{con}) = \mathcal{T}_d^+$ would be possible. As a result, we have at least two different Spin$^c$ structures on $\pm Y(G)$, which means that $\pm Y(G)$ is not an integral homology sphere.

**Case 3** The only remaining case to consider is when all of the weights on the graph are $-2$. It is a simple exercise in combinatorics to see that the condition of negative definiteness leaves us only one choice – the (negative) $E_8$ plumbing. Indeed, no such graph can have a vertex with degree greater than 3. Neither it can contain two vertices with degrees both greater than 2. The remaining case of the star graph with three emanating strings is easily investigated.

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