Two-photon decays of vector mesons and dilepton decays of scalar mesons in dense matter

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Abstract. Two-photon decays of vector mesons and dilepton decays of scalar mesons which are forbidden in vacuum and can occur in dense baryonic matter due to the explicit violation of Lorentz symmetry are described within a quark model of the Nambu–Jona-Lasinio type. The temperature and chemical potential dependence of these processes is investigated. It is found that their contribution to the production of photons and leptons in heavy-ion collisions is enhanced near the conditions corresponding to the restoration of chiral symmetry. Moreover, in the case of the $a_0$ meson and especially the $\rho$-meson, a resonant behaviour (an additional amplification) is observed due to the degeneration of $\rho$ and $a_0$ masses when a hot hadron matter is approaching a chirally symmetric phase.

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1. Introduction

The medium-induced breaking of Lorentz symmetry for the ground state of hadron matter, while the interaction is Lorentz-invariant, can open some processes that are forbidden in normal conditions (in free space). The mixing of scalar and vector mesons such as $\sigma$-$\omega$, $\rho$-$a_0$ and $\phi$-$f_0(980)$ is an example of this. The role of $\sigma$-$\gamma$ transition in the enhancement of $e^+e^-$ production due to the (forbidden in vacuum) decay $\sigma \rightarrow e^+e^-$ near the two-pion threshold was discussed by Weldon in [1]. (According to vector-meson dominance (VMD), all electromagnetic interactions of mesons should be mediated by vector mesons: a scalar transforms to a vector which then transforms to a photon.) The effect of $\sigma$-$\omega$ and $\rho$-$a_0$ mixing on the pion-pion annihilation in dilepton production was investigated in [2], where it was shown that additional peaks appeared in the dilepton spectrum in a sufficiently dense baryonic matter. On the other hand, the scalar-vector mixing can trigger two-photon decays of vector mesons, where a vector meson first transforms to a neutral scalar meson which then produces two correlated photons. In
the case of $\rho$-$a_0$ mixing, an additional (resonant) amplification can occur because of the degeneration of the $\rho$ and $a_0$ meson masses in particular conditions in dense matter [3].

The electromagnetic processes are considered as well-suitable probes to study in-medium properties of mesons in heavy-ion collisions. Much efforts have been so far directed toward the understanding of meson properties in baryonic matter, because they can strongly modify in hot and dense medium as a precursor phenomenon of chiral symmetry restoration [4, 5, 6, 7]. Insofar as correlated photon pairs are more difficult objects for detection than charged leptons, the most of attention has been paid to observation of dilepton spectra. Some measurements of dilepton spectra in heavy-ion collisions have been carried out at Lawrence Berkeley National Laboratory [8], CERES [9] at CERN, the NA38 and NA50 Collaborations [10, 11]. Experiments are also planned by PHENIX at the Relativistic Heavy Ion Collider at Brookhaven National Laboratory and by HADES at the Gesellschaft für Schwerionenforschung (GSI) [12]. The facility planned to be constructed at GSI can produce more dense baryonic matter, which better suits for the observation of the above mentioned processes as they are more sensitive to density than to temperature. As to low-energy photon-photon correlations, some experimental and theoretical opportunities of their observation at Nuclotron (Dubna) had been envisaged recently during the round table discussion "Searching for the mixed phase of strongly interacting matter at the JINR Nuclotron" in June 2005 (see http://thsun1.jinr.ru/meetings/2005/roundtable/talks.html).

The pion-pion annihilation is usually considered as the main mechanism of two-photon and dilepton production (see e. g. 13), while direct decays of mesons are of low interest. Due to modification of scalar and vector mesons in hot and dense medium, the spectrum of produced particles both in the annihilation and in direct decays can change. Thus, possible enhancement of photon pair production in the process $\pi\pi \rightarrow \gamma\gamma$ due to modification of $\sigma$ was investigated e. g. in [14, 15]. A peculiarity of the pion-pion annihilation via $\sigma$ is that its enhancement is expected in a very narrow region in the phase diagram. It is impossible to maintain a system with fixed temperature and chemical potential in heavy-ion experiments, therefore the produced particles spectra should be averaged over all phases through which the matter evolves during the collision. Unfortunately, the background processes shadow the peak in the two-photon enhancement because the conditions relevant for the enhancement exist for very short time. On the contrary, the increase in two-photon or dilepton production from the processes with $\sigma$-$\omega$, $\rho$-$a_0$ and $\phi$-$f_0$ mixing survives in a more wide range of temperature and baryon density, which improves the chances of their observation.

In our paper, we focus our attention on some electromagnetic and strong decays of scalar and vector mesons that occur due to $\sigma$-$\omega$, $\rho$-$a_0$ and $\phi$-$f_0$ mixing in dense baryonic matter. In difference to [1], where only pion-loop contributions were taken into account, we use a three-flavour version of the Nambu–Jona-Lasinio (NJL) model with ’t Hooft interaction in the mean-field approximation to estimate the contribution of quark loops to the scalar-vector mixing, because, in the large-$N_c$ (number of colors) approximation, quark loops give the leading contribution. The quark-loop contribution
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to the dilepton production in pion-pion annihilation has been already estimated in [16] for deconfined quarks only. We calculate the partial widths corresponding to direct decays of scalar and vector mesons in the hadron phase but near the transition to the phase with restored chiral symmetry. The "real-time" formalism is implemented to introduce medium effects for the quark propagator in quark-loop calculations [17, 18]. The vector-meson dominance is used for the description of dilepton decays of scalar mesons.

The structure of our paper is as follows. In section 2, the model Lagrangian introduced and parameters are given. Two-photon and some strong decays of vector mesons \( \rho, \omega \) as well as two-photon decays of \( \phi \) are described in section 3. In section 4, dilepton decays of scalar mesons \( \sigma, a_0 \) and \( f_0(980) \) are calculated. In the last section, we draw our conclusion and give discussion of the obtained results.

2. Model and parameters

For the description of meson properties, we use a \( U(3) \times U(3) \) NJL model with 't Hooft interaction [19, 20, 21, 22, 23, 24]. The Lagrangian of the model consists of two parts: a \( U(3) \times U(3) \) symmetric four-quark interaction and 't Hooft determinant [25] with six-quark vertices

\[
\mathcal{L} = \bar{q}(i\not\partial - m^0)q + \frac{G_1}{2} \sum_{i=0}^{8} [\bar{q}\lambda_i q]^2 + [\bar{q}i\gamma_5\lambda_i q]^2 - \frac{G_2}{2} \sum_{i=0}^{8} (\bar{q}\gamma_{\mu}\lambda_i q)^2 - K \left\{ \det[\bar{q}(1 + \gamma_5)q] + \det[\bar{q}(1 - \gamma_5)q] \right\},
\]

(1)

where \( \lambda_i \) (i=1,...,8) are the Gell-Mann matrices and \( \lambda^0 = \sqrt{\frac{2}{3}} \mathbf{1} \), with \( \mathbf{1} \) being the unit matrix; \( q = \{u, d, s\} \) and \( \bar{q} \) are quark and antiquark fields; \( m^0 = \text{diag}(m^0_u, m^0_d, m^0_s) \approx \text{diag}(m_u, m_s) \) is the current quark mass matrix; \( G_1 \) and \( G_2 \) are the four-quark interaction constants in the scalar-pseudoscalar and vector channels; \( K \) is the six-quark interaction constant.

Chiral symmetry is known to be spontaneously broken in vacuum, which leads to the shift of the ground (vacuum) state to one with a nontrivial chiral quark condensate. The latter entails the formation of constituent masses of quarks related to the current quark masses by gap equations. For three flavors with isotopic symmetry, the gap equations are

\[
m_d = m_u = m^0_u + 8m_u G_1 I_1^A(m_u) + 32m_u m_s K I_1^A(m_u) I_1^A(m_s),
\]

(2)

\[
m_s = m^0_s + 8m_s G_1 I_1^A(m_s) + 32K (m_u I_1^A(m_u))^2,
\]

(3)

where \( m_u \) and \( m_s \) are the constituent quark masses; \( I_1^A(m) \) is a quadratically divergent integral that comes from the tadpole diagram and depends on the corresponding constituent quark mass \( m = \{m_u, m_s\} \) and the ultraviolet cut-off parameter \( \Lambda \). In the three-dimensional regularization scheme it equals

\[
I_1^{A\lambda}(m) = \frac{N_c}{(2\pi)^2} \int_0^{\Lambda^2} \frac{p^2}{E} dp.
\]

(4)
The subscript 3 at $\Lambda_3$ means that the cut-off is implemented in three-dimensional momentum space.

The model parameters in vacuum are fixed in a way that allows one to reproduce the masses of $\pi$, $K$ and $\rho$ mesons, the pion weak-decay constant $f_\pi = 92.4$ MeV, the strong decay width of the $\rho$-meson and the mass difference of $\eta$ and $\eta'$ mesons (see [26]). We do not describe here the details of the parameterization scheme (the reader will find it e. g. in [19]) but give the following result $m_u = 280$ MeV, $m_u^0 = 2.1$ MeV, $m_s = 416$ MeV, $m_s^0 = 51$ MeV, $G_1 = 3.2$ GeV$^{-2}$, $G_2 = 16$ GeV$^{-2}$, $K = 4.6$ GeV$^{-5}$, $\Lambda_3 = 1.03$ GeV.

In hot and dense hadron gas, medium effects should be taken into account. In the mean-field approximation, where only one-loop diagrams are calculated, one can use the ”real time” formalism [17, 18], where the quark propagator is replaced by its extended form

$$S(p, T, \mu) = (\hat{p} + m) \left[ \frac{1}{p^2 - m^2 + i\epsilon} ight],$$

with all medium effects being contained in the Fermi-Dirac function for quarks

$$n(p; T, \mu) = \left( 1 + \exp \frac{E - \mu}{T} \right)^{-1}.$$  \hspace{1cm} (6)

Here, $E = \sqrt{p^2 + m^2}$, $p$ and $p$ are four- and three-momentum, respectively; $T$ is the temperature and $\mu$ is the baryonic chemical potential. The hat over $\hat{p}$ stands for the contraction with $\gamma$-matrices: $\hat{p} = p_\alpha \gamma^\alpha$.

All divergent parts of the one-loop diagrams necessary for our calculations can be represented by two basic integrals with quadratic and logarithmic ultraviolet divergencies

$$I_1^{\Lambda_3}(m, T, \mu) = \frac{N_c}{(2\pi)^2} \int_0^{\Lambda_3} dp \frac{p^2}{E} (1 - n(p; T, \mu) - n(p; T, -\mu)),$$

$$I_2^{\Lambda_3}(m, T, \mu) = \frac{N_c}{2(2\pi)^2} \int_0^{\Lambda_3} dp \frac{p^2}{E^3} (1 - n(p; T, \mu) - n(p; T, -\mu)).$$  \hspace{1cm} (7)

In (7), the reader will recognize the thermalized form of the integral $I_1^{\Lambda_3}(m)$ introduced in (4).

Following the usual procedure of finite $T$ and $\mu$ calculations in the NJL model, we make an assumption here that the model parameters $G_1$, $G_2$, $K$, $m_u^0$, $m_s^0$ and $\Lambda_3$ do not depend on $T$ and $\mu$ [18]. The dependence of the constituent quark masses $m_u$, $m_s$ on $T$ and $\mu$ is found from gap equations (2) and (3). One can see the behaviour of $m_u$ and $m_s$ vs. $\mu$ in figures 1 and 2 at $T = 20$ and 100 MeV. As one can see from these plots, the mass of the strange quark changes little with $\mu$ in the range of interest (in the hadron phase), therefore $m_s$ can be considered as being approximately a constant fixed to its vacuum value.
3. Vector-meson decays

3.1. The decay $\rho \to \gamma \gamma$

The direct decay of the neutral $\rho$-meson to two photons is forbidden in vacuum, but in dense medium the transition of $\rho$ to the $a_0$-meson becomes possible because of the explicit breaking of Lorentz symmetry for the ground state. The created thereby state with quantum numbers of a scalar meson decays then to a couple of photons. Schematically, the process is represented by the diagram drawn in figure 3. First,
a quark loop gives vector-scalar mixing, which is followed by an intermediate scalar resonance and ends up by the triangle quark loop describing a decay of the scalar meson to photons. The amplitude of this process has the form

\[ A_{\rho \to \gamma \gamma}^{\alpha \mu \nu} = J_{\rho \to a_0}^\alpha D_{a_0}^{\mu \nu} T_{a_0 \to \gamma \gamma}^{\alpha \mu \nu}, \]

where \( J_{\rho \to a_0}^\alpha \) describes \( \rho - a_0 \) mixing, \( D_{a_0} \) is the \( a_0 \)-meson propagator, and \( T_{a_0 \to \gamma \gamma}^{\alpha \mu \nu} \) is the amplitude of the \( a_0 \to \gamma \gamma \) decay (see figure 3).

Let us consider the right hand side of (9) in detail. The \( \rho - a_0 \) mixing is given by (see [10])

\[ J_{\rho \to a_0}^{\alpha} = \frac{C_{\rho \to a_0}}{4} \int \frac{d^4k}{(2\pi)^4} \text{tr} [S(k_-, T, \mu) \gamma^\alpha S(k_+, T, \mu)] \]

\[ = 2m_u C_{\rho \to a_0} \int \frac{d^4k}{(2\pi)^4} \delta(k_+^2 - m^2) \frac{n(k_+; T, \mu) - n(k_+; T, -\mu)}{k_+^2 - m^2} \times (\theta(k_+^0) - \theta(-k_+^0)) \]

\[ C_{\rho \to a_0} = 4N_c g_{a_0} g_{\rho}, \quad k_\pm = k \pm \frac{p}{2}, \]

\[ g_{a_0} = (4I_2^{\Lambda}(m_u, T, \mu))^{-1/2}, \quad g_{\rho} = \sqrt{6} g_{a_0}, \]
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where \( g_{a_0} \) and \( g_{\rho} \) are the \( a_0 \) and \( \rho \) meson-quark coupling constants, \( k \) is the internal quark momentum, and \( p \) is the four-momentum of the decaying meson.

In vacuum, at \( \mu = T = 0 \), \( J_{a_0 \to a_0}^\alpha = 0 \), whereas in medium it is nontrivial and vanishes only if the particle is at rest in the heat-bath frame. Moreover, \( J_{a_0 \to a_0}^\alpha \) satisfies gauge invariance: \( p_\alpha J_{a_0 \to a_0}^\alpha = 0 \), which allows simplify the following expressions, using the relation

\[
|J|^2 = (J_{a_0 \to a_0}^\alpha)^* J_{a_0 \to a_0}^\alpha = -\frac{p^2}{p^2} |J_{a_0 \to a_0}^0|^2, \tag{13}
\]

where \( J_{a_0 \to a_0}^0 \) is computed by integration over angles in the heat-bath frame

\[
J_{\rho \to a_0}^0 = \frac{C_{\rho \to a_0} m_u}{(4\pi)^2 |p|} \int_{m_u}^{\infty} dE \delta(E) \left[ (2E + p_0^0) \ln (F_+) + (2E - p_0^0) \ln (F_-) \right], \tag{14}
\]

\[
\delta(E) = \frac{\sinh(\mu/T)}{\cosh(\mu/T) + \cosh(E/T)}, \tag{15}
\]

\[
F_{\pm} = \frac{p_0^2 \pm 2p_0E + 2|p|q}{p_0^2 \pm 2p_0E - 2|p|q}, \tag{16}
\]

with \( q = \sqrt{E^2 - m^2} \).

The second multiplier in (13) is the \( a_0 \)-meson propagator calculated on the mass-shell of the \( \rho \)-meson in medium,

\[
D_{a_0} = (M_{a_0}^2 - M_{\rho}^2 - i\Gamma_{a_0}(M_{\rho})M_{a_0})^{-1}. \tag{17}
\]

As is known from various investigations [18] (see also figure 4), the \( \rho \)-meson mass is a smooth function of \( T \) and \( \mu \) and slightly decreases when the conditions of chiral symmetry restoration are being approached. Therefore, for rough estimates it can be assumed to be constant in the hadron phase.

As it follows from the NJL model calculations [18], the \( \sigma \)-meson mass drops down significantly with growing temperature and chemical potential, until it becomes almost degenerate with the pion mass when chiral symmetry is restored. The same is expected for the \( a_0 \)-meson. According to the NJL model, the \( a_0 \) mass is to be found from the expression [27]

\[
M_{a_0}^2 = g_{a_0}^2 \left[ \frac{1}{G_{a_0}} - 8I_{a_0}^\Lambda(m_u) \right] + 4m_u^2,
\]

\[
G_{a_0} = G_1 - 4K m_s I_{a_0}^\Lambda(m_s), \tag{18}
\]

which gives an underestimated value \( M_{a_0} \approx 800 \) MeV, comparing to the measured mass \( M_{a_0}^{\exp} = 984.7 \pm 1.2 \) MeV [26].

A possible explanation to the mass deficit in the NJL calculations is the assumption of the existence of a four-quark component in the \( a_0 \)-meson [23, 29, 30], which is usually ignored in the NJL-like models. To take into account the four-quark component, we introduce an additional term \( \Delta \) into the mass formula for the \( a_0 \)-meson

\[
M_{a_0}^2 + \Delta = M_{a_0}^2. \tag{19}
\]
The model part of the mass of the \( a_0 \)-meson as well as the order parameter \( m_u \) decrease with growing \( T \) and \( \mu \), as expected. It is natural to assume that \( \Delta \) has a similar behaviour. We take \( \Delta \) in the simplest form and consider two cases: i) \( \Delta = \Delta_u \) and ii) \( \Delta = \Delta_{us} \) where \( \Delta_u = 4m_u^2 \) and \( \Delta_{us} = 2.75m_u m_s \) which is enough to reproduce the measured \( a_0 \) mass in vacuum. After the phase transition, \( \Delta \) vanishes, which is required by restoration of chiral symmetry.

The second quantity in the propagator of the \( a_0 \)-meson is its width. The partial decay mode \( a_0 \to \eta \pi \) determines almost 100% of the total width, for which we obtain from the NJL model

\[
\Gamma_{a_0}(M_\rho) \approx \Gamma_{a_0\eta\pi}(M_\rho) = \frac{g^{a_0\eta\pi}_2}{16\pi M_\rho} \sqrt{1 - \left[ \frac{M_\eta + M_\pi}{M_\rho} \right]^2} \\
\times \sqrt{1 - \left[ \frac{M_\eta - M_\pi}{M_\rho} \right]^2},
\]

(20)

\[
g_{a_0\eta\pi} = 2m_u g_{a_0} \sin \bar{\theta}_p, \quad \bar{\theta}_p = \theta_P - \theta_0,
\]

(21)

where \( \theta_0 \approx 35.3^\circ \) is the ideal mixing angle (ctg \( \theta_0 = \sqrt{2} \)) and \( \theta_P \) is the singlet-octet mixing angle for pseudoscalar mesons. The dependence of the mixing angle on the chemical potential \( \mu \) is given in figure \( \ref{fig:mixing_angles} \) for \( T = 20 \) and \( T = 120 \) MeV. The behaviour of \( \Gamma_{a_0\eta\pi}(M_\rho) \) vs. \( \mu \) is shown in figures \( \ref{fig:2photon_decay} \) and \( \ref{fig:2photon_decay} \).

The last multiplier in (9), \( T^{\mu\nu}_{a_0 \to \gamma\gamma} \), is the amplitude describing the two-photon decay of the \( a_0 \)-meson. The expression for it is well known [14, 31]

\[
T^{\mu\nu}_{a_0 \to \gamma\gamma} = C_{a_0 \to \gamma\gamma} f_1(T, \mu) F^{\mu\nu}, \quad C_{a_0 \to \gamma\gamma} = \frac{2\alpha g_{a_0}}{3\pi m_u},
\]

(22)
where $\alpha = 1/137$, $F^{\mu \nu}$ is the electromagnetic tensor, and function $f_1$ is defined as

$$f_1(T, \mu) = 1 - \frac{3}{2} m_0^2(T, \mu) \int_0^{\infty} \frac{dk}{E_0(k)} \ln \left( \frac{E(k) + k}{E(k) - k} \right) \times [n(k; T, \mu) + n(k; T, -\mu)]. \tag{23}$$

After calculating the square of the absolute value of the amplitude, summing over the polarization of the final state photons and integrating over the solid angle, one obtains the width

$$\Gamma_{\rho \rightarrow \gamma \gamma} = \frac{\alpha^2 M_\rho^6 g_{a_0}^2}{432 \pi^3 m_0^2 E_\rho |P|^2} \cdot \frac{|J_{\rho \rightarrow a_0}|^2}{(M_{a_0}^2 - M_\rho^2)^2 + \Gamma_{a_0}^2 (M_\rho M_{a_0}^2)} \tag{24}$$

where $E_\rho = \sqrt{|P|^2 + M_\rho^2}$. Numerical estimates for the width of the decay $\rho \rightarrow \gamma \gamma$ as of a function of three-momentum $p$ and chemical potential $\mu$ at temperatures $T = 20$ and 120 MeV are shown in figures 8–11 with $\Delta = \{\Delta_u, \Delta_w\}$. It is easy to see that at some values of $T$ and $\mu$ the two-photon decay width of the $\rho$-meson can be comparable to the two-photon decay width of scalar and pseudoscalar mesons in vacuum (see table 1).

Analogously, the two-photon decay of the $\omega$-meson is given by the same diagram as for $\rho$. The difference is that $\omega$ mixes with the isoscalar scalar meson $\sigma$. According to the NJL model, the mass of the $\sigma$-meson is to be found from the equation

$$M_\sigma^2 \approx g_{a_0}^2 \left( \frac{1}{G_\sigma} - 8 I_1^{\Lambda_3}(m_u) \right) + 4 m_u^2, \tag{25}$$

$$G_\sigma = G_1 + 4 m_u I_1^{\Lambda_3}(m_u). \tag{26}$$

The width of $\sigma$ is mostly determined by its strong decay to two pions and is predicted by the NJL model (on the mass-shell of the $\omega$-meson) to be of the form

$$\Gamma_{\sigma}(M_\omega) \approx \Gamma_{\sigma \pi \pi}(M_\omega) = \frac{3 g_{\sigma \pi \pi}^2}{32 \pi M_\omega} \left( 1 - \frac{4 M_\pi^2}{M_\omega^2} \right). \tag{27}$$

$$g_{\sigma \pi \pi} = 2 m_u g_{a_0} \cos \theta_S. \tag{28}$$

The amplitude for the decay $\omega \rightarrow \gamma \gamma$ can be obtained from (13) simply by replacing the factor $C_{a_0 \rightarrow \gamma \gamma}$ in $T_{a_0 \rightarrow \gamma \gamma}^{\mu \nu}$ (see (22)) by $C_{\sigma \rightarrow \gamma \gamma} = 10 \alpha g_{a_0}/(9 \pi m_u)$ and by using the $\sigma$-meson propagator instead of $a_0$ ($D_{a_0} \rightarrow D_{\sigma}$)

$$D_{\sigma} = (M_{\sigma}^2 - M_{\omega}^2 - i \Gamma_{\sigma}(M_\omega) M_\sigma)^{-1}. \tag{29}$$

The angle $\theta_S$ is defined as the difference between the mixing angle for scalar mesons $\theta_S$ and the ideal mixing angle $\theta_0$: $\tilde{\theta}_S = \theta_S - \theta_0$. Its behaviour with respect to $\mu$ is given in figure 24 from which one can see that the singlet-octet mixing among scalar mesons is almost ideal and we put $\tilde{\theta}_S = 0$ in further calculations.

A peculiarity of the decay $\rho \rightarrow \gamma \gamma$ is that the $a_0$-meson mass is larger than the $\rho$-meson mass in vacuum and significantly drops down when approaching the transition to the phase with restored chiral symmetry. One can conclude from this that in particular conditions the masses of $a_0$ and $\rho$ become degenerate, which can be followed by a resonant amplification of the decay $\rho \rightarrow \gamma \gamma$ and noticeable enhancement near the $\rho$-meson mass in the two-photon spectrum. As to the $\sigma$-meson, it is always lighter than...
Two-photon decays of vector mesons and dilepton decays of scalar mesons... Moreover, their mass difference grows with \( \mu \), and, as a consequence, no resonant phenomenon occurs in this decay. Therefore, the number of two-photon events related to decays of vector meson will be dominated by those coming from \( \rho \to \gamma \gamma \) rather than from \( \omega \to \gamma \gamma \), as one can see from figures 12 and 13.

3.2. The decay \( \phi \to \gamma \gamma \)

The decay of the \( \phi \)-meson occurs due to \( \phi-f_0 \) mixing (\( \phi \to f_0(980) \to \gamma \gamma \)), where the photon pair is produced by the decay of the intermediate \( f_0(980) \)-meson. As one can see from figure 5, the mixing among scalar mesons is almost ideal, and one can consider the \( f_0(980) \)-meson as composed of \( s \)- and \( \bar{s} \)-quarks only. The \( \phi \)-meson also consists mostly of strange quarks. Using this approximation together with the fact that the mass of the strange quark is almost constant (see figure 1 and 2) in the hadron phase, one can roughly estimate the decay of \( \phi \) to \( \gamma \gamma \).

The \( \phi-f_0(980) \) mixing is represented by an integral of the form (14), where one should replace the \( u \)-quark mass by the mass of the strange quark (\( m_u \to m_s \), \( J_{\rho \to \omega} \to J_{\phi \to f_0} \)). Qualitatively, the decay of the \( \phi \) meson to photons has the behaviour with respect to \( \mu \) similar to that of the lighter vector mesons (\( \rho \) and \( \omega \)), so we omit here the plots for the \( \phi \to \gamma \gamma \) decay rate. We should like only to note here that the width for the decay \( \phi \to \gamma \gamma \) can be as large as 2 keV at appropriate conditions in the hadron phase at the three-momentum about 1 GeV. The width is greater for larger momenta: at \( |p| \sim 5 \) GeV the width can reach 8 keV, but such momenta are less probable at temperatures of order 100–200 MeV.

3.3. Strong decays \( \rho \to \pi \eta \) and \( \omega \to \pi \pi \)

The strong decays \( \rho \to \pi \eta \) and \( \omega \to \pi \pi \) can be considered in the same manner as the electromagnetic ones which were discussed above. The difference is that an intermediate scalar meson decays to hadrons instead of photons and one needs only to replace the third multiplier in (9) by an appropriate amplitude for the corresponding strong decay. For these decay widths at temperature \( T = 20 \) MeV, we give numerical estimates in figures 14 and 15. The maximal value of the width of the decay \( \rho \to \gamma \gamma \) is about 40 MeV, which is comparable with the main decay of the \( \rho \)-meson — the decay to two pions (150 MeV). The decay \( \omega \to \pi \pi \) can occur in vacuum due to the broken isotopic invariance (\( m_u \neq m_d \)) and gives \( \Gamma_{\omega \to \pi \pi} = 145 \pm 25 \) keV; in medium, its decay rate grows significantly and reaches 1.5 MeV. One should note here that in dense medium these strong decays become comparable with regular electromagnetic decays of scalar and pseudoscalar mesons whose widths are shown in table 1.

‡ The in-medium decay width \( \omega \to \pi \pi \) is calculated for nuclear matter within a hadronic model including mesons, nucleons and \( \Delta \)-isobars in [32].
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Figure 6. The widths of decays $a_0 \to \eta \pi$ and $\sigma \to \pi \pi$ in the NJL model for $T = 20$ MeV as functions of chemical potential $\mu$.

![Graph showing the widths of decays $a_0 \to \eta \pi$ and $\sigma \to \pi \pi$ in the NJL model for $T = 20$ MeV.]

Figure 7. The widths of decays $a_0 \to \eta \pi$ and $\sigma \to \pi \pi$ in the NJL model for $T = 120$ MeV as functions of chemical potential $\mu$.

![Graph showing the widths of decays $a_0 \to \eta \pi$ and $\sigma \to \pi \pi$ in the NJL model for $T = 120$ MeV.]

Table 1. Two-photon decays of scalar and pseudoscalar mesons in vacuum [26].

| Particle | Mass, MeV | $\Gamma_{\gamma \gamma}$, keV |
|----------|-----------|-------------------------------|
| $\pi^0$  | $134.9766 \pm 0.0006$ | $(7.8 \pm 0.5) \cdot 10^{-3}$ |
| $\eta$   | $547.75 \pm 0.12$     | $1.29 \pm 0.07$              |
| $\eta'$  | $957.78 \pm 0.14$     | $4.29 \pm 0.15$              |
| $\sigma$ | $400 - 1200$          | $\sim 1$                     |
| $a_0$    | $984.7 \pm 1.2$       | $0.3 \pm 0.1$                |
| $f_0$    | $980 \pm 10$          | $0.39^{+0.1}_{-0.13}$        |
4. Scalar meson decays

4.1. The decay $\sigma \to e^+e^-$

The direct decay of the $\sigma$-meson to dileptons is also forbidden in vacuum and is allowed in dense medium due to $\sigma$-$\omega$ mixing. After transition of $\sigma$ to $\omega$, the latter transforms to a photon (according to VMD) which then produces an electron and a positron (see
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**Figure 10.** Two-photon decay width of the $\rho$-meson with the $a_0$ mass corrected by $\Delta = \Delta_u$ as a function of $\mu$ and $|p|$ for $T = 120$ MeV.

![Graph 10](image1.png)

**Figure 11.** Two-photon decay width of the $\rho$-meson with the $a_0$ mass corrected by $\Delta = \Delta_u$ as a function of $\mu$ and $|p|$ for $T = 120$ MeV.

![Graph 11](image2.png)

the diagram in figure 16. The corresponding amplitude is

$$A_{\sigma \rightarrow e^+e^-} = J_{\sigma \rightarrow \omega}^{\alpha}D_{\omega \alpha \beta}T_{\omega \rightarrow e^+e^-}^{\beta \sigma \epsilon_2},$$

(30)

where $J_{\sigma \rightarrow \omega}^{\alpha}$ describes the $\sigma$-$\omega$ mixing, $D_{\omega \alpha \beta}$ is the $\omega$-meson propagator, and $T_{\omega \rightarrow e^+e^-}^{\beta \sigma \epsilon_2}$ is the amplitude of the decay $\omega \rightarrow e^+e^-$. Fortunately, the $\sigma$-$\omega$ mixing coincides with $\rho$-$a_0$ mixing

$$J_{\sigma \rightarrow \omega}^{\alpha} = J_{\rho \rightarrow a_0}^{\alpha},$$

(31)
and the results of previous section can be used here. One then needs the \( \omega \)-meson propagator on the mass of the \( \sigma \)-meson in medium \( (p^2 = M^2_\sigma) \):

\[
D_{\omega \alpha \beta} = \frac{g_{\alpha \beta} - P_\alpha P_\beta / M^2_\omega}{M^2_\omega - M^2_\sigma - i\Gamma_\omega (M_\sigma) M_\omega} \equiv \left( g_{\alpha \beta} - \frac{P_\alpha P_\beta}{M^2_\omega} \right) D_\omega. \tag{32}
\]

The width of the \( \omega \)-meson is negligibly small and can be neglected here because the mass difference for the \( \omega \) and \( \sigma \) mesons increases while approaching the phase transition and dominates in the denominator in \( (32) \):

\[
D_\omega \approx \frac{1}{M^2_\omega - M^2_\sigma}. \tag{33}
\]

**Figure 12.** Two-photon decay width of the \( \omega \)-meson as a function of \( \mu \) and \( |p| \) for \( T = 20 \).

**Figure 13.** Two-photon decay width of the \( \omega \)-meson as a function of \( \mu \) and \( |p| \) for \( T = 120 \) MeV.
Figure 14. Strong decay width $\rho \rightarrow \pi \eta$ at $T = 20$ MeV.

Figure 15. Strong decay width $\omega \rightarrow \pi \pi$ at $T = 20$ MeV.

Figure 16. The diagram that describes dilepton decays of scalar meson in medium via scalar-vector mixing
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Figure 17. The decay $\sigma \rightarrow e^+e^-$ at $T = 20$ MeV

For the same reason that was discussed in previous section, the $\omega$-meson mass can be assumed to be constant. The $\sigma$-meson mass (in the NJL model) has been already given in (25) and is estimated to be about $M_\sigma \approx 550$ MeV in vacuum (see figure 4).

The last multiplier in the amplitude, $T_{\omega \rightarrow s_1s_2}^{s_1s_2}$, is the amplitude describing the decay $\omega \rightarrow \gamma^* \rightarrow e^+e^-$ in the VMD model:

$$T_{\omega \rightarrow e^+e^-}^{s_1s_2} = \frac{4\pi\alpha}{3g_\omega} M_\omega^2 \bar{v}(k_2|s_2)\gamma^\mu u(k_1|s_1),$$

(34)

where $u(k_1|s_1)$ and $\bar{v}(k_2|s_2)$ are spinors corresponding to the produced electron and positron, with $s_i$ ($i = 1, 2$) being their spin projections and $k_i$ their four-momenta.

To calculate the partial width for the dilepton decay of a scalar meson, one needs then to take the square of the absolute value of the amplitude (30), sum over the spin projections (because the polarization of leptons is not measured) and integrate over the solid angle in the final state. The result is

$$\Gamma_{\sigma \rightarrow e^+e^-}(M_\sigma, p) \approx \frac{4\alpha^2\pi^3 M_\omega^4}{27g_\omega^2 M_\sigma|p|^2} \frac{|J^0_{\rho \rightarrow a_0}|^2}{(M_\sigma^2 - M_\rho^2)^2},$$

(35)

Numerically (see figures 17 and 18), the decay $\sigma \rightarrow e^+e^-$ reaches 1.5 keV in maximum.

4.2. The decay $a_0 \rightarrow e^+e^-$

The decay $a_0 \rightarrow e^+e^-$ is qualitatively the same as $\sigma \rightarrow e^+e^-$. The width is as follows

$$\Gamma_{a_0 \rightarrow e^+e^-}(M_{a_0}, p) \approx \frac{4\alpha^2\pi^3 M_\rho^4}{3g_\omega^2 M_{a_0}|p|^2} \frac{|J^0_{\rho \rightarrow a_0}|^2}{(M_{a_0}^2 - M_{\rho}^2)^2 + M_{\rho}^2 \Gamma_\rho(M_{a_0})^2}$$

(36)
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Figure 18. The decay $\sigma \to e^+e^-$ at $T = 120$ MeV

The width of the $\rho$-meson is given by its decay to pions:

$$\Gamma_{\rho}(M_{a_0}) \approx \Gamma_{\rho \to \pi \pi}(M_{a_0}) = \frac{g_{\rho}^2}{48\pi M_{a_0}^2}(M_{a_0}^2 - 4M_{\pi}^2)^{3/2}. \quad (37)$$

The numerical estimates for the decay $a_0 \to \gamma\gamma$ are given in figures 19 and 20. Due to the mass degeneration of $\rho$ and $a_0$ near the phase transition, a resonant enhancement is to be observed in the decay $a_0 \to e^+e^-$. However, because of the large width of the $\rho$-meson, there is no sharp resonance. Nevertheless, insofar as the $\rho-\gamma$ transition is three times greater than the $\omega-\gamma$ transition (according to VMD) the rate of the decay $a_0 \to e^+e^-$ turns to be larger almost by an order ($\sim 10$ keV at the maximum), comparing to the decay $\sigma \to e^+e^-$.  

4.3. The decay $f_0(980) \to e^+e^-$

As it has been already mentioned in section 3 the mixing among scalar mesons is almost ideal near the phase transition, and the $f_0(980)$-meson is considered as composed of $s$- and $\bar{s}$-quarks only. For the $f_0(980)$ and $\phi$ masses assumed to be constant in the hadron phase, one can roughly estimate the decay of $f_0(980)$ to $e^+e^-$. This decay is mediated by the $\phi$-meson. The $\phi$-$f_0(980)$ mixing is again represented by the integral of the form (14) with the $u$-quark mass replaced by the mass of the strange quark (see section 3). The other difference is that the transition of the $\phi$-meson to a photon is larger that that for the $\omega$-meson by factor $\sqrt{2}$ [19]. Taking into account all the written above, one obtains for $f_0 \to e^+e^-$:

$$\Gamma_{f_0 \to e^+e^-}(M_{f_0}, p) \approx \frac{8\alpha^2\pi^3M_{\phi}^4}{27g_{\phi}^2M_{f_0}|p|^2} \cdot \frac{|J_{\phi \to f_0}|^2}{(M_{f_0}^2 - M_{\phi}^2)^2 + M_{\phi}^2\Gamma_{\phi}(M_{f_0})^2}. \quad (38)$$

Qualitatively, the behaviour of this width with respect to temperature and chemical potential is similar to the decay $\sigma(a_0) \to e^+e^-$, and we do not show plots for this decay.
We should like to note here only that below $\mu = 350$ MeV, the maximal width is about 10 keV.

5. Conclusion

Two-photon decays of vector mesons and dilepton decays of scalar mesons are forbidden in free space but in medium they give an additional contribution to the corresponding two-photon and dilepton spectra in heavy-ion collisions. These decays are determined by $\sigma$-$\omega$, $\rho$-$a_0$, and $\phi$-$f_0(980)$ mixing which strongly depend on temperature, chemical potential and on the momentum of the decaying particle in the medium. The mixing
Two-photon decays of vector mesons and dilepton decays of scalar mesons... is mostly density-driven, and the effect is to be observable for relatively high chemical potentials and not too large temperatures. The maximum of the mixing is reached for the those particles whose momentum corresponds to the maximum in the momentum distribution at fixed $T$ and $\mu$. The rate of each decay vanishes in free space and is maximal near the transition from the hadron phase to the phase with restored chiral symmetry. The conditions in which the effect is noticeable correspond to a wide range in the phase diagram. The latter is important for heavy-ion collisions because one can thereby hope that the evolution of a fireball will lie mostly in this range. Additional enhancement is produced by the resonant effect if the $\rho$ in involved. This enhancement is similar to that in the process $\pi\pi \rightarrow \gamma\gamma$ \cite{14,15} and $\pi\pi \rightarrow \pi\pi$ \cite{33}.

The case of $\rho$-$a_0$ mixing seems more intriguing than the others because there are points in the phase diagram that correspond to conditions at which the masses of $\rho$ and $a_0$ are close to each other. This leads to additional amplification of the two-photon decay of the $\rho$-meson because the intermediate $a_0$ meson becomes a sharp resonance. As a result, the forbidden in vacuum decay $\rho \rightarrow \gamma\gamma$ turns out to be comparable (about 2.5 keV) to regular decays $a_0 \rightarrow \gamma\gamma$ (0.3 keV, see table\cite{1}), $\sigma \rightarrow \gamma\gamma$ ($1 \div 10$ keV) and $\eta \rightarrow \gamma\gamma$ ($\sim 1.3$ keV). Similar amplification will also be observed in the (strong) decay $\rho \rightarrow \eta\pi$, which reaches the maximum value about 40 MeV. As to the $\omega$-meson, its decay to pions significantly increases near the phase transition, comparing to its value in vacuum.

Despite the degeneration of $\rho$ and $a_0$ masses, there is no enhancement in the decay $a_0 \rightarrow \gamma\gamma$ similar to that in the decay $\rho \rightarrow \gamma\gamma$ because of the large width of the $\rho$-meson. Concerning the decay $\sigma \rightarrow e^+e^-$, insofar as $\sigma$-$\omega$ mass difference is relatively large and grows near the phase transition, no resonance is observed here.

All the effects discussed here take place in the case of the decays $\phi \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow e^+e^-$. In addition, we would like to remark that, according to VMD, $\rho$-$\gamma$ mixing is three times larger than $\omega$-$\gamma$. As a consequence, the number of dileptons produced by $a_0$ exceeds that produced by $\sigma$ by an order.

In this paper we did not consider diagrams excluding intermediate resonances, such as the decay of $\rho$-meson into a photon pair through the triangle quark diagram (see \cite{34,35}). Concerning the pion-loop contribution, we do not consider them here because they give next-to-leading order in the $1/N_c$ expansion. Concluding our paper, we would like to emphasize that the processes discussed here occur only in dense matter and can serve as indicators of approaching the quark-gluon plasma in heavy-ion collision experiments.

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