The Influence of Multiplicity Distribution on the Erraticity Behavior of Multiparticle Production

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Abstract

The origin of the erraticity behaviour observed recently in the experiment is studied in some detail. The negative-binominal distribution is used to fit the experimental multiplicity distribution. It is shown that, with the multiplicity distribution taken into account, the experimentally observed erraticity behaviour can be well reproduced using a flat probability distribution. The dependence of erraticity behaviour on the width of multiplicity distribution is studied.

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Since the finding of unexpectedly large local fluctuations in a high multiplicity event recorded by the JACEE collaboration [1], the investigation of non-linear phenomena in high energy collisions has attracted much attention [2]. The anomalous scaling of factorial moments, defined as

$$F_q = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m - 1) \cdots (n_m - q + 1) \rangle}{\langle n_m \rangle^q}$$

at diminishing phase space scale or increasing division number $M$ of phase space [3]:

$$F_q \propto M^{-\phi_q},$$

called intermittency (or fractal) has been proposed for this purpose. The average $\langle \cdots \rangle$ in Eq.(1) is over the whole event sample and $n_m$ is the number of particle falling in the $m$th bin. This kind of anomalous scaling has been observed successfully in various experiments [4][5].

A recent new development along this direction is the event-by-event analysis [6][7]. An important step in this kind of analysis was made by Cao and Hwa [8], who first pointed out the importance of the fluctuation in event space of the event factorial moments defined as

$$F_{q}^{(e)} = \frac{1}{M} \sum_{m=1}^{M} \frac{n_m(n_m - 1) \cdots (n_m - q + 1)}{\left( \frac{1}{M} \sum_{m=1}^{M} n_m \right)^q}.$$

Its fluctuations from event to event can be quantified by its normalized moments as:

$$C_{p,q} = \langle \Phi_q^p \rangle, \quad \Phi_q = \frac{F_{q}^{(e)}}{\langle F_{q}^{(e)} \rangle},$$

and by $dC_{p,q}/dp$ at $p = 1$:

$$\Sigma_q = \langle \Phi_q \ln \Phi_q \rangle.$$

If there is a power law behavior of the fluctuation as division number goes to infinity, or as resolution $\delta = \Delta/M$ goes to very small, i.e.,

$$C_{p,q}(M) \propto M^{\psi_q(p)},$$

then the phenomenon is referred to as erraticity [9]. The derivative of exponent $\psi_q(p)$ at $p = 1$

$$\mu_q = \left. \frac{d}{dp} \psi_q(p) \right|_{p=1} = \frac{\partial \Sigma_q}{\partial \ln M}.$$

describes the anomalous scaling property of fluctuation-width and is called entropy index.
The erraticity behaviour of multiparticle final states as described above has been observed in the experimental data of 400 GeV/c pp collisions from NA27 [10]. However, it has been shown [11] that the single event factorial moment as defined in Eq.(3), using only the horizontal average over bins, cannot eliminate the statistical fluctuations well, especially when the multiplicity is low. A preliminary study shows that the experimentally observed phenomenon [10] can be reproduced by using a flat probability distribution with only statistical fluctuations [11]. This result is preliminary in the sense that it has fixed the multiplicity to 9 while the multiplicity is fluctuating in the experiment and has an average of $\langle n_{ch} \rangle = 9.84$ [12]. Since the erraticity phenomenon is a kind of fluctuation in event space and depends strongly on the multiplicity, the fluctuation in event space of the multiplicity is expected to have important influence on this phenomenon.

In this letter this problem is discussed in some detail. The negative binomial distribution [13] will be used to fit the experimental multiplicity distribution [12]. Putting the resulting multiplicity distribution into a flat-probability-distribution model, the erraticity behaviour is obtained and compared with the experimental data. The consistency of these two shows that the erraticity behaviour observed in the 400 GeV/c pp collision data from NA27 is mainly due to statistical fluctuations.

The negative-binomial distribution is defined as [13]

$$P_n = \binom{n+k-1}{n} \left( \frac{\bar{n}/k}{1+\bar{n}/k} \right)^n \frac{1}{(1+\bar{n}/k)^k}, \quad (8)$$

where $n$ is the multiplicity, $\bar{n}$ is its average over event sample, $k$ is a parameter related to the second order scaled moment $C_2 \equiv \langle n^2 \rangle / \langle n \rangle^2$ through [13]

$$C_2 - 1 = \frac{1}{\bar{n}} + \frac{1}{k}. \quad (9)$$

Using Eq.(8) to fit the multiplicity distribution of 400 GeV/c pp collision data from NA27, we get the parameter $k = 12.76$. The result of fitting is shown in Fig.1. It can be seen that the fitting is good.

Then we take a flat (pseudo)rapidity distribution, i.e. let the probability for a particle to fall into each bin be equal to $p_m = 1/M$ when the (pseudo)rapidity space is divided into $M$ bins. This means that there isn’t any dynamical fluctuation.

Let the number $N$ of particles in an event be a random number distributed according to the negative binomial distribution Eq.(8) with $\bar{n} = 9.84, k = 12.76$. Put these $N$ particles into the $M$ bins according to the Bernoulli distribution
\[
B(n_1, n_2, \cdots, n_M | p_1, p_2, \cdots, p_M) = \frac{N!}{n_1! \cdots n_M!} p_1^{n_1} \cdots p_M^{n_M},
\]

(10)

\[
\sum_{m=1}^{M} n_m = N.
\]

In total 60000 events are simulated in this way and the resulting \( C_{p,q} \) are shown in Fig.2 together with the experimental data of 400 GeV/c pp collisions from NA27. It can be seen from the figures that the model results are consistent with the data, showing that the erraticity phenomenon observed in this experiment is mainly due to statistical fluctuations.

In order to study the relation of erraticity behaviour with the width of multiplicity distribution, the same calculation has been done for the cases: \( \bar{n} = 9, k = 0.1, 0.5, 1.0, 2.25, 4.5, 9, 18 \). These values of \( k \) corresponds to diminishing width of distribution with \( C_2 = 11.1, 3.11, 2.11, 1.56, 1.33, 1.22, 1.17 \) respectively, cf. Fig.3. The resulting \( \ln C_{p,2} \) and \( \Sigma_2 \) as function of \( \ln M \) are shown in Fig.4 and Fig.5.

It can be seen from the figures that the moments \( C_{p,2} \) for different \( p \) separate farther and the characteristic function \( \Sigma_2 \) becomes larger when the value of \( k \) is smaller. This means that the single event factorial moments fluctuate stronger in event space when the width of multiplicity distribution is wider. On the other hand, the straight lines obtained from fitting the last three points of \( \Sigma_2 \) versus \( \ln M \) are almost parallel for different \( k \), and their slopes — the entropy indices \( \mu_2 \), which is the characteristic quantity of erraticity are insensitive to the width of multiplicity distribution.

In summary, the multiplicity distribution of 400 GeV/c pp collision data from NA27 has been fitted to the negative binomial distribution. Taking this multiplicity distribution into account, the erraticity phenomenon in a model without any dynamical fluctuation, i.e. with a flat probability distribution, has been studied. The resulting moments \( C_{p,q} \) turn out to fit the experimental data very well. This shows that the erraticity phenomenon observed in this experiment is mainly due to statistical fluctuations.

The dependence of erraticity phenomenon on the width of multiplicity distribution is examined. It is found that the fluctuation of single event factorial moments in event space becomes stronger — \( C_{p,2} \) and \( \Sigma_2 \) become larger — when the width of multiplicity distribution is wider. On the other hand, the entropy index \( \mu_2 \) depends mainly on the average multiplicity and is insensitive to the width of multiplicity distribution.
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Figure Captions

Fig.1  Fitting of the multiplicity distribution of 400 GeV/c pp collision data to negative binomial distribution. Data taken from Ref.[12].

Fig.2  The moments $C_{p,2}$ from a flat probability distribution model with the multiplicity distribution taken into account, as compared with the 400 GeV/c pp collision data taken from Ref.[12].

Fig.3  The negative binomial distribution with different values of parameter $k$. The average multiplicity is $\bar{n} = 9$.

Fig.4  The dependence of $\ln C_{p,2}$ on $\ln M$ in the flat probability distribution model, taken the negative-binomial type multiplicity distribution into account. The parameter $k$ takes different vaues as shown in the figure. The average multiplicity is $\bar{n} = 9$.

Fig.5  The dependence of $\Sigma_2$ on $\ln M$ in the flat probability distribution model, taken the negative-binomial type multiplicity distribution into account. The parameter $k$ takes different vaues as shown in the figure. The average multiplicity is $\bar{n} = 9$. 
Fig. 1

Fig. 2
\ln P(n_{\text{ch}})

Fig. 3
Negative Binomial Distribution

\(<n>\) = 9

★ k = 0.1
● k = 0.5
□ k = 1.0
▲ k = 2.25
△ k = 4.5
▽ k = 9.0
○ k = 18.0

g = 2.0

Fig. 4
Negative Binomial Distribution

$\langle n \rangle = 9$

$\star k = 0.1$

$\circ k = 0.5$

$\square k = 1.0$

$\triangle k = 2.25$

$\triangle k = 4.5$

$\blacktriangledown k = 9.0$

$\bigcirc k = 18.0$

Fig. 5