Theory of Helimagnons in Itinerant Quantum Systems III: Quasiparticle Description

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In two previous papers we studied the problem of electronic properties in a system with long-ranged helimagnetic order caused by itinerant electrons. A standard many-fermion formalism was used. The calculations were quite tedious because different spin projections were coupled in the action, and because of the inhomogeneous nature of a system with long-ranged helimagnetic order. Here we introduce a canonical transformation that diagonalizes the action in spin space, and maps the problem onto a homogeneous fermion problem. This transformation to quasiparticle degrees of freedom greatly simplifies the calculations. We use the quasiparticle action to calculate single-particle properties, in particular the single-particle relaxation rate. We first reproduce our previous results for clean systems in a simpler fashion, and then study the much more complicated problem of three-dimensional itinerant helimagnets in the presence of an elastic relaxation rate $1/\tau$ due to nonmagnetic quenched disorder. Our most important result involves the temperature dependence of the single-particle relaxation rate in the ballistic limit, $\tau^2 T \epsilon_F > 1$, for which we find a linear temperature dependence. We show how this result is related to a similar result found in nonmagnetic two-dimensional disordered metals.

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I. INTRODUCTION

In two previous papers, hereafter denoted by I and II,\textsuperscript{1,2} we considered various properties of clean itinerant helimagnets in their ordered phase at low temperatures. These papers considered and discussed in some detail the nature of the ordered state, and in particular the Goldstone mode that results from the spontaneously broken symmetry. We also calculated a variety of electronic properties in the ordered state that are influenced by the Goldstone mode or helimagnon, which physically amounts to fluctuations of the helical magnetization. For various observables, we found that couplings between electronic degrees of freedom and helimagnon fluctuations lead to a nonanalytic (i.e., non-Fermi-liquid-like) temperature dependence at low temperature. For most quantities, this takes the form of corrections to Fermi-liquid behavior, but in some cases, e.g., for the single-particle relaxation rate, the nonanalytic dependence constitutes the leading low-temperature behavior.

A prototypical itinerant helimagnet is MnSi. At low temperatures and ambient pressure, the ground state of MnSi has helical or spiral order, where the magnetization is ferromagnetically ordered in the planes perpendicular to some direction $\mathbf{q}$, with a helical modulation of wavelength $2\pi/|\mathbf{q}|$ along the $\mathbf{q}$ axis.\textsuperscript{2} MnSi displays helical order below a temperature $T_c \approx 30$K at ambient pressure, with $2\pi/|\mathbf{q}| \approx 180\text{Å}$. That is, the pitch length scale is large compared to microscopic length scales. Application of hydrostatic pressure $p$ suppresses $T_c$, which goes to zero at a critical pressure $p_c \approx 14$ kbar.\textsuperscript{2} Physically, the helimagnetism is caused by the spin-orbit interaction, which breaks lattice inversion symmetry. In a Landau theory this effect leads to a term of the form $\mathbf{m} \cdot (\nabla \times \mathbf{m})$ in the Landau free energy, with $\mathbf{m}$ the local magnetization, and a prefactor proportional to $|\mathbf{q}|$.\textsuperscript{5,6}

In I and II we used a technical description based on itinerant electrons subject to an effective inhomogeneous external field that represents helimagnetic order, with helimagnon fluctuations coupled to the remaining electronic degrees of freedom. We emphasize that for the theory developed in either papers I and II, or in the current paper and a forthcoming paper IV, it is irrelevant whether the helimagnetism is caused by the conduction electrons, or whether the conduction electrons experience a background of helimagnetic order caused by electrons in a different band. The Gaussian or ‘non-interacting’ part of the action was not diagonal in either spin or wave number space, and the latter property reflected the fact that the system is inhomogeneous. These features substantially complicated the explicit calculations performed in II. In order to make progress beyond the discussion in II, and to discuss the effects of quenched disorder in particular, it therefore is desirable to find a technically simpler description. In the current paper, our first main result is the construction of a canonical transformation that diagonalizes the action in spin space and simultaneously makes the Gaussian action diagonal in wave number space. The new transformed action makes our previous calculations much simpler than before. It also enables us to extend our previous work in a number of ways. In particular, we will treat the much more complicated problem of the quasiparticle properties of helimagnets in the presence of non-magnetic quenched disorder.

The study of the electronic properties of disordered
metals has produced a variety of surprises over the past thirty years. The initial work on this subject was mostly related to diffusive electrons and the phenomena known as weak-localization and/or Altshuler-Aronov (AA) effects (for reviews see, e.g., Refs. [5,8]). In the clean limit, mode-mode coupling effects analogous to the AA effects have been shown to lead to a nonanalytic wave number dependence of the spin susceptibility at $T = 0.2$. More recently, disordered interacting (via a Coulomb interaction) electron systems have been studied in the ballistic limit, $T \tau > 1$, but still at temperatures low compared to all energy scales other than $1/\tau$, with $\tau$ the elastic scattering rate. Interestingly, in this limit it has been shown that for two-dimensional (2D) systems, the temperature correction to the elastic scattering rate is proportional to $T$, i.e., shows non-Fermi liquid behavior. In contrast, in 3D systems the corresponding correction is proportional to $T^2 \ln(1/T)$, i.e., the behavior is marginally Fermi-liquid-like, with a logarithmic correction. The second main result of the current paper is that in the ballistic limit, the low-temperature correction to the single-particle relaxation rate in ordered helimagnets is linear in $T$. The technical reason for why a 3D disordered itinerant helimagnet behaves in certain ways in close analogy to a 2D nonmagnetic disordered metal will be discussed in detail below. Transport properties, in particular the electrical conductivity, will be considered in a separate paper, which we will refer to as paper IV.

The organization of this paper is as follows. In Sec. II we introduce a canonical transformation that vastly simplifies the calculation of electronic properties in the helimagnet state. In Sec. III we calculate various single-particle and quasiparticle properties at low temperatures in both clean and disordered helimagnetic systems. In the latter case we focus on the ballistic limit (which is slightly differently defined than in nonmagnetic materials), where the various effects are most interesting, and which is likely of most experimental interest, given the levels of disorder in the samples used in previous experiments. The paper is concluded in Sec. IV with a summary and a discussion. Throughout this paper we will occasionally refer to results obtained in I and II, and will refer to equations in these papers in the format (I.x.y) and (II.x.y), respectively.

II. CANONICAL TRANSFORMATION TO QUASIPARTICLE DEGREES OF FREEDOM

In this section we start with an electron action that takes into account helical magnetic order and helical magnetic fluctuations. The fundamental variables in this description are the usual fermionic (i.e., Grassmann-valued) fields $\bar{\psi}_\alpha(x)$ and $\psi_\alpha(x)$. Here $x = (x, \tau)$ is a four-vector that comprises real space position $x$ and imaginary time $\tau$, and $\alpha$ is the spin index. Due to the helical magnetic order, the quadratic part of this action is not diagonal in either the spin indices or in wave number space. We will see that there is a canonical transformation which leads, in terms of new Grassmann variables, to an action that is diagonal in both spin and wave number space. This transformed action enormously simplifies calculations of the electronic properties of both clean and dirty helical magnetic metals.

A. The action in terms of canonical variables

In II we derived an effective action for clean itinerant electrons in the presence of long-range helical magnetic order, and helical magnetic fluctuations interacting with the electronic degrees of freedom. This action can be written (see Eq. (II.3.13)),

$$S_{\text{eff}}[\bar{\psi}, \psi] = S_0[\bar{\psi}, \psi] + \frac{\Gamma_0^2}{2} \int dx dy \, \delta n_s^2(x) \chi^{ij}_s(x, y) \delta n_s^2(y),$$

(2.1)

where $n_s^2(x) = \bar{\psi}_\alpha(x) \sigma^i_{\alpha\beta} \psi_\alpha(x)$ is the electronic spin density, $\sigma^i = (i, 1, 2, 3)$ denotes the Pauli matrices, $\delta n_s^2 = n_s^2 - \langle n_s^2 \rangle$ is the spin density fluctuation, $\Gamma_0$ is the spin-triplet interaction amplitude, and $\int dx = \int dx_0^{1/T}$. Here and it what follows we use units such that $\hbar = k_B = 1$. In Eq. (2.1), $S_0$ denotes an action,

$$S_0[\bar{\psi}, \psi] = \hat{S}_0[\bar{\psi}, \psi] + \int dx \, H_0(x) \cdot n_s(x),$$

(2.2a)

where,

$$H_0(x) = \Gamma_1 \langle n_s(x) \rangle = \Gamma_1 m(x)$$

(2.2b)

is proportional to the average magnetization $m(x) = \langle n_s(x) \rangle$. In the helimagnetic state,

$$H_0(x) = \lambda \langle \cos(q \cdot z), \sin(q \cdot z), 0 \rangle,$$

(2.2c)

where $q$ is the pitch vector of the helix, which we will take to point in the $z$-direction, $q = q \hat{z}$, and $\lambda = \Gamma_1 m_0$ is the Stoner gap, with $m_0$ the magnetization amplitude. $\hat{S}_0$ in Eq. (2.2a) contains the action for non-interacting band electrons plus, possibly, an interaction in the spin-singlet channel. Finally, fluctuations of the helimagnetic order are taken into account by generalizing $H_0$ to a fluctuating classical field $H(x) = \Gamma_1 M(x) = H_0 + \Gamma_1 \delta M(x)$, where $M(x)$ represents the spin density averaged over the quantum mechanical degrees of freedom. $\chi^{ij}_s(x, y) = \langle \delta M_i(x) \delta M_j(y) \rangle$ in Eq. (2.1) is the magnetic susceptibility in the helimagnetic state, and the action (2.1) has been obtained by integrating out the fluctuations $\delta M$. The susceptibility $\chi_s$ was calculated before, see Sec. IV.E in I. The part of $\chi_s$ that gives the dominant low-temperature contributions to the various thermodynamic and transport quantities is the helimagnon or Goldstone mode contribution. In I it was shown that the helimagnon is a propagating mode with a qualitatively anisotropic dispersion relation. For the geometry given...
above, the helimagnon is given in terms of magnetization fluctuations that can be parameterized by (see (1.3.4a))

\[
\begin{align*}
\delta M_x(x) &= -m_0 \phi(x) \sin qz \quad (2.3a) \\
\delta M_y(x) &= m_0 \phi(x) \cos qz. \quad (2.3b)
\end{align*}
\]

\(\delta M_z = 0\) in an approximation that suffices to determine the leading behavior of observables. In Eqs. (2.3), \(\phi\) is a phase variable. In Fourier space, the phase-phase correlation function in the long-wavelength and low-frequency limit is,

\[
\chi(k) \equiv \langle \phi(k) \phi(-k) \rangle = \frac{1}{2N_F} \frac{q^2}{3k_F} \frac{1}{\omega_n^2(k) - (i\Omega)^2}, \tag{2.4a}
\]

with \(N_F\) the electronic density of states per spin at the Fermi surface, \(k_F\) the Fermi wave number, \(i\Omega \equiv \Omega_n = i2 \pi T n\) \((n = 0, \pm 1, \pm 2, \ldots)\) a bosonic Matsubara frequency, and \(k = (k_\perp, i\Omega)\). If we write \(k = (k_\perp, k_z)\), with \(k_\perp = (k_x, k_y)\), the pole frequency is

\[
\omega_0(k) = \sqrt{c_z k_\perp^2 + c_\perp k_z^4}. \tag{2.4b}
\]

Note the anisotropic nature of this dispersion relation, which implies that \(k_z\) scales as \(k_\perp^2\), which in turn scales as the frequency or temperature, \(k_z \sim k_\perp^2 \sim T\). This feature will play a fundamental role in our explicit calculations in Sec. II.\(\text{III}\) that relate 3D helimagnetic metals to 2D nonmagnetic metals, at least in the ballistic limit. In a weak-coupling calculation the elastic constants \(c_z\) and \(c_\perp\) are given by, see Eqs. (II.3.8),

\[
\begin{align*}
\frac{c_z}{\lambda} &= \frac{\lambda^2 q^2}{4k_F^4} \\
\frac{c_\perp}{\lambda} &= \frac{\lambda^2}{96k_F^4}. \tag{2.4c}
\end{align*}
\]

This specifies the action given in Eq. (2.1). In Fourier space, and neglecting any spin-singlet interaction, it can be written,

\[
\begin{align*}
S_{\text{eff}}[\bar{\psi}, \psi] &= S_0[\bar{\psi}, \psi] + S_{\text{int}}[\bar{\psi}, \psi], \\
S_0[\bar{\psi}, \psi] &= \sum_p (i\omega - \xi_p) \sum_{\sigma} \bar{\psi}_\sigma(p) \psi_\sigma(p) + \lambda \sum_p [\bar{\psi}_1(p) \psi_1(p + q) + \bar{\psi}_1(p) \psi_1(p - q)], \\
S_{\text{int}}[\bar{\psi}, \psi] &= -\frac{\lambda^2 T}{2V} \sum_\alpha \sum_k (2.5b)
\end{align*}
\]

where \(V\) is the system volume, and \(i\omega \equiv i\omega_n = i2 \pi T (n + 1/2)\) \((n = 0, \pm 1, \pm 2, \ldots)\) is a fermionic Matsubara frequency,

\[
n_{\sigma_1, \sigma_2}(k) = \sum_p \bar{\psi}_{\sigma_1}(p) \psi_{\sigma_2}(p - k), \tag{2.5d}
\]

and

\[
\delta n_{\sigma_1, \sigma_2}(k) = n_{\sigma_1, \sigma_2}(k) - \langle n_{\sigma_1, \sigma_2}(k) \rangle. \tag{2.5e}
\]

Here \(p = (p, i\omega)\), and \(q\) denotes the four-vector \((q, 0)\). Elsewhere in this paper we use the notation \(q = |q|\), which should not lead to any confusion. In Eq. (2.5b), \(\xi_p = \epsilon_p - \epsilon_F\), with \(\epsilon_F\) the Fermi energy, and \(\epsilon_p\) the single-particle energy-momentum relation. The latter we will specify in Eq. (2.10) below.

In the above effective action, \(S_0\) represents noninteracting electrons on the background of helimagnetic order that has been taken into account in a mean-field or Stoner approximation. Fluctuations of the helimagnetic order lead to an effective interaction between the electrons via an exchange of helimagnetic fluctuations or helimagnons. This is reflected by the term \(S_{\text{int}}\), and the effective potential is proportional to the susceptibility \(\chi\).

B. Canonical transformation to quasiparticle variables

The action \(S_0\) in Eq. (2.5b) above is not diagonal in either the spin index or the wave number. A cursory inspection shows that by a suitable combination of the fermion fields it is possible to diagonalize \(S_0\) in spin space. It is much less obvious that it is possible to find a transformation that simultaneously diagonalizes \(S_0\) in wave number space. In what follows we construct such a transformation, i.e. we map the electronic helimagnon problem onto an equivalent problem in which space is homogeneous.

Let us tentatively define a canonical transformation of the electronic Grassmann fields \(\bar{\psi}\) and \(\psi\) to new quasi-particle fields \(\bar{\varphi}\) and \(\varphi\), which also are Grassmann-valued, by

\[
\begin{align*}
\bar{\psi}_1(p) &= \bar{\varphi}_1(p) + \alpha_p^* \bar{\varphi}_1(p) \tag{2.6a} \\
\bar{\psi}_1(p) &= \bar{\varphi}_1(p - q) + \beta_p^* \bar{\varphi}_1(p - q) \tag{2.6b} \\
\psi_1(p) &= \varphi_1(p) + \alpha_p \varphi_1(p) \tag{2.6c} \\
\psi_1(p) &= \varphi_1(p - q) + \beta_p \varphi_1(p - q). \tag{2.6d}
\end{align*}
\]
The coefficients $\alpha$ and $\beta$ are determined by inserting the Eqs. (2.3) into Eq. (2.5) and requiring this noninteracting part of that action to be diagonal in the spin labels. This requirement can be fulfilled by choosing them to be real and frequency independent, and given by

$$
\alpha_p = \alpha_p^* = -\beta_{p+q} = \alpha_p
$$

$$
= \frac{1}{2\lambda} \left[ \xi_{p+q} - \xi_p + \sqrt{(\xi_{p+q} - \xi_p)^2 + 4\lambda^2} \right].
$$

The noninteracting part of the action in terms of these new Grassmann fields is readily seen to be diagonal in both spin space and wave number space.

To fully take into account the effect of the change of variables from the fields $\bar{\psi}(p)$ and $\psi(p)$ to the fields $\bar{\varphi}(p)$ and $\varphi(p)$ we also need to consider the functional integration that obtains the partition function $Z$ from the action via

$$
Z = \int D[\bar{\psi}, \psi] e^{S_{\text{int}}[\bar{\psi}, \psi]}.
$$

The transformation of variables changes the integration measure as follows:

$$
D[\bar{\psi}, \psi] = \prod_{p,\sigma} d\bar{\psi}_\sigma(p) d\psi_\sigma(p)
$$

$$
= \prod_{p,\sigma} J(p) d\bar{\varphi}_\sigma(p) d\varphi_\sigma(p),
$$

with a Jacobian

$$
J(p) = (1 + \alpha_p^2)^2.
$$

We can thus normalize the transformation by defining final quasiparticle variables $\bar{\eta}$ and $\eta$ by

$$
\bar{\psi}_\uparrow(p) = \left[ \bar{\eta}_\uparrow(p) + \alpha_p \bar{\eta}_\downarrow(p) \right] / \sqrt{1 + \alpha_p^2},
$$

$$
\bar{\psi}_\downarrow(p) = \left[ \bar{\eta}_\downarrow(p - q) - \alpha_{p-q} \bar{\eta}_\downarrow(p - q) \right] / \sqrt{1 + \alpha_{p-q}^2},
$$

$$
\psi_\uparrow(p) = \left[ \eta_\uparrow(p + q) - \alpha_{p+q} \eta_\downarrow(p + q) \right] / \sqrt{1 + \alpha_{p+q}^2},
$$

$$
\psi_\downarrow(p) = \left[ \eta_\downarrow(p - q) - \alpha_{p-q} \eta_\downarrow(p - q) \right] / \sqrt{1 + \alpha_{p-q}^2}.
$$

In terms of these new Grassmann fields the Jacobian is unity, and the noninteracting part of the action reads

$$
S_{0}[\bar{\eta}, \eta] = \sum_{p,\sigma} \left[ \bar{\eta}_\sigma(p) \right] \bar{\eta}_\sigma(p) \eta_\sigma(p).
$$

Here $\sigma = (1, 1) \equiv (1, 2)$, and

$$
\omega_{1,2}(p) = \frac{1}{2} \left[ \xi_{p+q} + \xi_p \pm \sqrt{(\xi_{p+q} - \xi_p)^2 + 4\lambda^2} \right].
$$

The noninteracting quasiparticle Green function thus is

$$
G_{0,\sigma}(p) = \frac{1}{i\omega - \omega_{\sigma}(p)}.
$$

Physically, the Eqs. (2.10) represent soft fermionic excitations about the two Fermi surfaces that result from the helimagnetism splitting the original band. The resonance frequencies $\omega_{1,2}$ are the same as those obtained in II, see Eq. (II.3.19). We stress again that this Gaussian action is diagonal in wave number space, so the quasiparticle system is homogeneous.

The interacting part of the action consist of two pieces. One contains terms that couple the two Fermi surfaces. Because there is an energy gap, namely, the Stoner gap $\lambda$, between these surfaces, these terms always lead to exponentially small contributions to the electronic properties at low temperatures, and will be neglected here. The second piece is, in terms of the quasiparticle fields,

$$
S_{\text{int}}[\bar{\eta}, \eta] = -\frac{\lambda^2 q^2 T}{8m_e^2} \sum_k \chi(k) \delta\rho(k) \delta\rho(-k).
$$

Here we have defined

$$
\rho(k) = \sum_p \gamma(k, p) \sum_\sigma \bar{\eta}_\sigma(p) \eta_\sigma(p - k),
$$

with

$$
\gamma(k, p) = \frac{2m_e}{q} \frac{\alpha_p - \alpha_{p-k}}{\sqrt{1 + \alpha_p^2 \sqrt{1 + \alpha_{p-k}^2}}},
$$

where $m_e$ is the electron effective mass, and

$$
\delta\rho_\sigma(k) = \rho_\sigma(k) - \langle \rho_\sigma(k) \rangle.
$$

An important feature of this result is the vertex function $\gamma(k, p)$, which is proportional to $k$ for $k \to 0$. The physical significance is that $\phi$ is a phase, and hence only the gradient of $\phi$ is physically meaningful. Therefore, the $\phi$-susceptibility $\chi$ must occur with a gradient squared in Eq. (2.11a). In the formalism of II this feature became apparent only after complicated cancellations; in the current formalism it is automatically built in. Also note the wave number structure of the fermion fields in Eq. (2.11b), it the same as in a homogeneous problem.

### C. Nonmagnetic disorder

In the presence of nonmagnetic disorder there is an additional term in the action. In terms of the original Grassmann variables, it reads

$$
S_{\text{dis}}[\bar{\psi}, \psi] = \int dx \, u(x) \sum_\sigma \bar{\psi}_\sigma(x) \psi_\sigma(x).
$$
Here \( u(x) \) is a random potential that we assume to be governed by a Gaussian distribution with a variance given by

\[
\{u(x)u(y)\}_{\text{dis}} = \frac{1}{2\pi N_F \tau} \delta(x - y). \tag{2.13}
\]

Here \( \{\ldots\}_{\text{dis}} \) denotes an average with respect to the Gaussian probability distribution function, and \( \tau \) is the (bare) elastic mean-free time. Inserting the Eqs. (2.9) into Eq. (2.12) yields \( S_{\text{dis}}[\bar{n}, \eta] \). Ignoring terms that couple the two Fermi surfaces (which lead to exponentially small effects at low temperatures) yields

\[
S_{\text{dis}}[\bar{n}, \eta] = \sum_{k,p} \sum_{i\omega} \sum_{\sigma} \frac{1 + \alpha_k \alpha_p}{\sqrt{(1 + \alpha_k^2)(1 + \alpha_p^2)}} u(k - p) \times \bar{n}_{\sigma}(k, i\omega) \eta_{\sigma}(p, i\omega). \tag{2.14}
\]

**D. Explicit quasiparticle action**

So far we have been very general in our discussion. In order to perform explicit calculations, we need to specify certain aspects of our model. First of all, we make the following simplification. In most of our calculations below we will work in the limit where \( \lambda \gg v_F q = 2v_F q/k_F \) with \( v_F \) the Fermi velocity; i.e., the Stoner splitting of the Fermi surfaces is large compared to the Fermi energy times the ratio of the pitch wave number to the Fermi momentum. Since the dominant contributions to the observables will come from wave vectors on the Fermi surface, this implies that we can replace the transformation coefficients \( \alpha_p \), Eq. (2.7), by unity in Eq. (2.14), and in the denominator of Eq. (2.14) \( \lambda \). In particular, this means that the disorder potential in Eq. (2.14) couples to the quasiparticle density:

\[
S_{\text{dis}}[\bar{n}, \eta] = \sum_{k,p} u(k - p) \sum_{i\omega} \sum_{\sigma} \bar{n}_{\sigma}(k, i\omega) \eta_{\sigma}(p, i\omega). \tag{2.15}
\]

Second, we must specify the electronic energy-momentum relation \( \epsilon_p \). For reasons already discussed in II, many of the electronic effects in metallic helimagnets are stronger when the underlying lattice and the resulting anisotropic energy-momentum relation is taken into account, as opposed to working within a nearly-free electron model. We will assume a cubic lattice, as appropriate for MnSi, so any terms consistent with cubic symmetry are allowed. To quartic order in \( p \) the most general \( \epsilon_p \) consistent with a cubic symmetry can be written

\[
\epsilon_p = \frac{p^2}{2m_e} + \frac{\nu}{2m_e k_F^2} (p_x^2 p_y^2 + p_y^2 p_z^2 + p_z^2 p_x^2), \tag{2.16}
\]

with \( \nu \) a dimensionless measure of deviations from a nearly-free electron model. Generically one expects \( \nu = O(1) \).

With this model, and assuming \( \lambda \gg q v_F \), which is typically satisfied, given the weakness of the spin-orbit interaction, we obtain for the interaction part of the action from Eqs. (2.11)

\[
S_{\text{int}} = \frac{T}{V} \sum_{k,p_1,p_2} V(k;p_1,p_2) \sum_{\sigma_1} \left[ \bar{n}_{\sigma_1}(p_1 + k) \eta_{\sigma_1}(p_1) - \langle \bar{n}_{\sigma_1}(p_1 + k) \eta_{\sigma_1}(p_1) \rangle \right] \\
\times \sum_{\sigma_2} \left[ \bar{n}_{\sigma_2}(p_2 - k) \eta_{\sigma_2}(p_2) - \langle \bar{n}_{\sigma_2}(p_2 - k) \eta_{\sigma_2}(p_2) \rangle \right], \tag{2.17}
\]

where the effective potential is

\[
V(k;p_1,p_2) = V_0 \chi(k) \gamma(k,p_1) \gamma(-k,p_2). \tag{2.18a}
\]

Here,

\[
V_0 = \frac{\lambda^2 q^2}{8m_e^2}, \tag{2.18b}
\]

and,

\[
\gamma(k,p) = \frac{1}{2\lambda} \left[ k_z + \frac{\nu}{k_F^2} (k_z p_{\perp}^2 + 2(k_\perp \cdot p_\perp) p_z) \right] + O(k^2). \tag{2.18c}
\]

The effective interaction is depicted graphically in Fig. [1].

Examining the Eqs. (2.17) and (2.18) we see three important features. First, the effective potential is indeed proportional to \( k^2 \chi(k) \). As was mentioned after Eq. (2.11a), this is required for a phase fluctuation effect. Second, the presence of the lattice, as reflected by the term proportional to \( \nu \) in Eq. (2.18c), allows for a term proportional to \( k^2 \chi(k) \) in the potential, which by power counting is large compared to \( k^2 \chi \), for reasons pointed out in the context of Eq. (2.1a). It is this part of the potential that results in the leading, and most interesting, low-temperature effects that will be discussed in the next section of this paper, and in paper IV. Also as a result of this feature, the dominant interaction between the quasiparticles is not a density interaction, but rather an interaction between stress fluctuations, due to the bilinear dependence on \( p \) of the dominant term in \( \gamma(k,p) \). Third, the effective interaction is long-ranged, due to the...
Inserting \( \text{Eq. (2.21)} \) in \( \text{Eq. (2.20)} \) leads to an algebraic consequence of the soft mode, the helimagnon, that mediates the interaction.

In summary, we now have the following quasiparticle action:

\[
S_{\text{QP}}[\tilde{\eta}, \eta] = S_0[\tilde{\eta}, \eta] + S_{\text{int}}[\tilde{\eta}, \eta] + S_{\text{dis}}[\tilde{\eta}, \eta], \tag{2.19a}
\]

with \( S_0 \) from Eqs. (2.10), \( S_{\text{int}} \) from Eqs. (2.17, 2.18), and \( S_{\text{dis}} \) given by Eq. (2.15). The partition function is given by

\[
Z = \int D[\tilde{\eta}, \eta] e^{S_{\text{QP}}[\tilde{\eta}, \eta]}, \tag{2.19b}
\]

with a canonical measure

\[
D[\tilde{\eta}, \eta] = \prod_p d\tilde{\eta}_p(p) d\eta_\sigma(p). \tag{2.19c}
\]

E. Screening of the quasiparticle interaction

The quasiparticle interaction potential shown in Eqs. (2.18) and Fig. 1 must be screened, and an important question is whether this will change its long-ranged nature. In the usual ladder or random-phase approximation the screened potential \( V_{\text{sc}} \) is determined by an integral equation that is shown graphically in Fig. 2 and analytically given by

\[
V_{\text{sc}}(k; p_1, p_2) = V(k; p_1, p_2) + \frac{T}{V} \sum_{p_3} V(k; p_1, p_3)
\]

\[
\times \sum_\sigma G_{0,\sigma}(p_3 - k) G_{0,\sigma}(p_3) V_{\text{sc}}(k; p_3, p_2).
\] \( \tag{2.20} \)

It is convenient to define a screening factor \( f_{\text{sc}} \) by writing

\[
V_{\text{sc}}(k; p_1, p_2) = V(k; p_1, p_2) f_{\text{sc}}(k; p_1, p_2). \tag{2.21} \]

Inserting Eq. (2.21) in Eq. (2.20) leads to an algebraic equation for \( f_{\text{sc}} \) with a solution

\[
f_{\text{sc}}(k; p_1, p_2) = \frac{1}{1 - V_0 f_{\text{sc}} \sum_p \gamma(k, p) \gamma(-k, p) \chi_L(p, i\Omega)}, \tag{2.22a}
\]

where

\[
\chi_L(p, i\Omega) = -T \sum_\sigma \sum_{i\omega} G_{0,\sigma}(p, i\omega) G_{0,\sigma}(p, i\omega - i\Omega).
\] \( \tag{2.22b} \)

The most interesting effect of the screening is at \( k \to 0 \), and therefore we need to consider only \( \chi_L(p, i\Omega = 0) \equiv \chi_L(p) \). This is essentially the Lindhard function, and we use the approximation \((1/V) \sum_p |p|^n \chi_L(p) \approx k_0^2 N_F \). Neglecting prefactors of \( O(1) \) this yields

\[
\frac{1}{V} \sum_p \gamma(k, p) \gamma(-k, p) \chi_L(p) \approx \frac{N_F}{4\alpha^2} \left[ (1 + \nu)^2 k_0^2 + \nu^2 k_1^2 \right].
\]

We finally obtain

\[
V_{\text{sc}}(k; p_1, p_2) = V_0 \chi_{\text{sc}}(k) \gamma(k, p_1) \gamma(-k, p_2), \tag{2.23a}
\]

where

\[
\chi_{\text{sc}}(k) = \frac{1}{2N_F} \frac{q^2}{3k_0^2} \frac{1}{\omega_0^2(k) - (\Omega)^2}, \tag{2.23b}
\]

Here

\[
\omega_0^2(k) = c_z k_0^2 + V_0 \frac{\nu^2}{24 k_F^2} \frac{q^2}{k_F^2} k_1^2 + c_\perp k_1^4, \tag{2.23c}
\]

with

\[
c_z = c_z \left[ 1 + \frac{q^2}{k_F^2} \left( \frac{c_\perp}{\lambda} \right)^2 \right]. \tag{2.23d}
\]

We see that the screening has two effects on the frequency \( \omega_0 \) that enters the screened potential instead of the helimagnon frequency \( \omega_0 \). First, it renormalizes the elastic constant \( c_z \) by a term of order \( (q/k_F)^2 (\epsilon_F/\lambda)^2 \). This is a small effect as long as \( qv_F \ll \lambda \). Second, it leads to a term proportional to \( k_1^2 \) in \( \omega_0^2 \). Such a term also exists in the helimagnon frequency proper, since the cubic lattice in conjunction with spin-orbit effects breaks the rotational symmetry that is responsible for the absence of a \( k_1^2 \) term in \( \omega_0 \), see Eq. (I.2.23) or (II.4.8), and it is of order \( b_c q^2 k^2 / k_F^4 \), with \( b = 0(1) \). The complete expression for \( \omega_0^2 \) is thus given by

\[
\omega_0^2(k) = c_z k_0^2 + \tilde{b} c_z (q/k_F)^2 k_1^2 + c_\perp k_1^4, \tag{2.24a}
\]

with

\[
\tilde{b} = b + (\epsilon_F/\lambda)^2. \tag{2.24b}
\]

As was shown in paper II, this puts a lower limit on the temperature range where the isotropic helimagnon description is valid. In the absence of screening, this lower limit is given by Eq. (II.4.9),

\[
T > T_{so} = b\lambda (q/k_F)^4. \tag{2.25a}
\]

This lower limit reflects the spin-orbit interaction effects that break the rotational symmetry, and it is small of order \( (q/k_F)^4 \). Screening changes this condition to

\[
T > \tilde{T}_{so} = \tilde{b}\lambda (q/k_F)^4, \tag{2.25b}
\]
which is still small provided \(qv_F \ll \lambda\). We will therefore ignore the screening in the remainder of this paper (as well as the spin-orbit term in \(\omega_0\)), and return to a semi-quantitative discussion of its effects in paper IV.

III. QUASIPARTICLE PROPERTIES

In this Section we use the effective quasiparticle action derived in Sec. II to discuss the single-particle properties of an itinerant helimagnet in the ordered phase. In Sec. III A we consider the elastic scattering time in the helimagnetic state, in Sec. III B we consider the effects of interactions on the single-particle relaxation rate for both clean and disordered helimagnet, and in Sec. III C we consider the effects of interactions on the single-particle density of states for both clean and disordered helimagnets.

A. Elastic relaxation time

Helimagnetism modifies the elastic scattering rate, even in the absence of interaction effects. To see this we calculate the quasiparticle self energy from the action \(S_0 + S_{\text{dis}}\) from Eqs. (2.10a, 2.14). To first order in the disorder the relevant diagram is given in Fig. 3. Analytically it is given by,

\[
\Sigma^{(3)}_{\sigma}(p,i\omega) = \frac{-1}{8\pi N_F} \frac{1}{V} \sum_k [1 + \sigma_p \alpha_k]^2 G_{0,\sigma}(k,i\omega),
\]

with \(G_0\) the noninteracting Green function from Eq. (2.10a). For simplicity we put \(\nu = 0\) in Eq. (2.10a), i.e., we consider nearly free electrons. In the limit \(qv_F \ll \lambda\) we obtain for the elastic scattering rate, \(1/\tau_{el} = -2\text{Im} \Sigma_{\sigma}(p,i0)\),

\[
\frac{1}{\tau_{el}} = \frac{1}{\tau} \sqrt{1 - \lambda/\epsilon_F},
\]

(3.2a)

In the opposite limit, \(qv_F \gg \lambda\), we find

\[
\frac{1}{\tau_{el}} = \frac{1}{4\tau} \left[ 1 - q/2k_F + O((q/k_F)^2) \right].
\]

(3.2b)

To first order in the disorder and to zeroth order in interactions, the disorder averaged Green function is

\[
G_0(p) = \frac{1}{i\omega - \omega_{\sigma}(p) + \frac{1}{2\tau_{el}} \text{sgn}(\omega)}.
\]

(3.3)

B. Interacting single-particle relaxation rate

In this subsection we determine the single-particle relaxation rate due to interactions, and its modification due to disorder in the ballistic limit.

1. Clean helimagnets

We first reproduce the results of paper II for the interaction-induced single-particle relaxation rate. This serves as a check on our formalism, and to demonstrate the technical ease of calculations within the quasiparticle model compared to the formalism in papers I and II. To this end we calculate the quasiparticle self energy for an action \(S_0 + S_{\text{int}}\) from Eqs. (2.10c, 2.17, 2.18). To first order in the interaction there are two self-energy diagrams that are shown in Fig. 4. The direct or Hartree contribution, Fig. 4(a), is purely real and hence does not contribute to the scattering rate. The exchange or Fock contribution, Fig. 4(b), is given by

\[
\Sigma^{(4b)}_{\sigma}(p) = -\frac{T}{V} \sum_k V(k;p - k,p) G_{0,\sigma}(k - p).
\]

(3.4)

In order to compare with the results given in II, we consider the Fermi surface given by \(\omega_1(p) = 0\). The single-particle relaxation rate is given by \(1/\tau(k,\epsilon) = -2\text{Im} \Sigma_1(k,\epsilon + i0)\). With Eqs. (2.10c) and (2.18) in Eq. (3.4), we find

\[
\frac{1}{\tau(k,\epsilon)} = 2 \int_{-\infty}^{\infty} du \left[ n_B \left( \frac{u}{T} \right) + n_F \left( \frac{\epsilon + u}{T} \right) \right]
\times V''(p - k; k, p; u) \delta(\epsilon + u - \omega_1(p)).
\]

(3.5)
Here \( n_B(x) = 1/(e^x - 1) \) and \( n_F(x) = 1/(e^x + 1) \) are the Bose and Fermi distribution functions, respectively, and \( V''(k; p_1, p_2; u) = \text{Im } V(k = (k, i\Omega \rightarrow u + i0); p_1, p_2) \) is the spectrum of the potential. On the Fermi surface, \( \epsilon = 0 \) and \( \omega_1(k) = 0 \), we find for the relaxation rate

\[
1/\tau(k) \equiv 1/\tau(k, \epsilon = 0),
\]

\[
\frac{1}{\tau(k)} = C_k \frac{k_z^2 k_y^2 (k_x^2 - k_y^2)^2}{(k_x^2 A_x^2 + k_y^2 A_y^2)^{3/2}} \left( \frac{T}{\lambda} \right)^{3/2}. \tag{3.6a}
\]

The quantities \( A_{x,y} \) and \( C_k \) are defined as

\[
A_{x,y} = 1 + \frac{\nu}{k_F} (k_{y,x}^2 + k_z^2), \tag{3.6b}
\]

and

\[
C_k = \frac{B
\nu^4}{8\lambda k_F^2} \frac{k_z^2}{k_F^2} \frac{q^3 k_F}{m_e^2}, \tag{3.6c}
\]

with

\[
B = \frac{48}{6^{1/4}} \int_0^\infty dx \, dz \, \frac{x^2}{\sqrt{z^2 + x^4}} \frac{1}{\sinh \sqrt{z^2 + x^4}}. \tag{3.6d}
\]

They are identical with the objects defined in Eqs. (II.3.29), provided the latter are evaluated to lowest order in \( q/k_F \). The temperature dependence for generic (i.e., \( k_x \neq k_y \)) directions in wave number space is thus

\[
\frac{1}{\tau(k)} \propto \nu^4 \lambda \left( \frac{q}{k_F} \right)^6 \left( \frac{\nu}{\lambda} \right)^2 \left( \frac{T}{T_q} \right)^{3/2}, \tag{3.7}
\]

in agreement with Eq. (II.3.29d). \( T_q \) is a temperature related to the length scale where the helimagnon dispersion relation is valid, \( |k| < q \). Explicitly, in a weakly coupling approximation, it is given by

\[
T_q = \lambda q^2 / 6k_F, \tag{3.8}
\]

see the definition after Eq. (II.3.9). \( T_q \) also gives the energy or frequency scale where the helimagnon crosses over to the usual ferromagnetic magnon, see the discussion in Sec. IV.A of paper II.

The most interesting aspect of this result is that at low temperatures it is stronger than the usual Fermi-liquid \( T^2 \) dependence, and nonanalytic in \( T^2 \). Also note the strong angular dependence of the prefactor of the \( T^{3/2} \) in Eq. (3.6a). The experimental implications of this result have been discussed in paper II.

2. Disordered helimagnets in the ballistic limit

We now consider effects to linear order in the quenched disorder. These can be considered disorder corrections to the clean relaxation rate derived in the previous subsection, or temperature corrections to the elastic relaxation rate. The small parameter for the disorder expansion turns out to be

\[
\delta = 1/\sqrt{(\epsilon_F \tau)^2 T/\lambda} \ll 1. \tag{3.9}
\]

That is, the results derived below are valid at weak disorder, \( \epsilon_F \tau \gg \sqrt{\lambda / T} \), or at intermediate temperature, \( T \gg \lambda/(\epsilon_F \tau)^2 \). This can be seen from an inspection of the relevant integrals in the disorder expansion, and will be discussed in more detail in paper IV. For stronger disorder, or lower temperature, the behavior of the quasiparticles is diffusive and will be discussed elsewhere.15 The ballistic regime in a helimagnet is different from that in a system of electrons interacting via a Coulomb interaction, where the condition corresponding to Eq. (3.9) reads \( T \tau \gg 1 \).10

To first order in the disorder there are two types of diagrammatic contributions to the single-particle relaxation rate: (A) diagrams that are formally the same as those shown in Fig. 4 except that the solid lines represent the disorder-averaged Green function given by Eq. (3.25), and (B) diagrams that have one explicit impurity line. The latter are shown in Fig. 5. It is easy to show that the various Hartree diagrams do not contribute. The class (A) Fock contribution to the self energy is given by Eq. (3.25), with \( G_{0,\sigma} \) replaced by \( G_{\sigma} \) from Eq. (3.23).

Power counting shows that, (1) the leading contribution to the single-particle relaxation rate in the ballistic limit is proportional to \( T \), (2) the diagrams of class (A) do not contribute to this leading term, and (3) of the diagrams of class (B) only diagram (a) in Fig. 5 contributes. Analytically, the contribution of this diagram to the self energy is

\[
\Sigma^{(5a)}_{\sigma}(p, i\omega) = \Sigma^{(5a)}_{\sigma}(i\omega)
\]

\[
= \frac{-1}{2\pi N_F \tau} \sum_{k, i\Omega} \sum_{p'} \frac{V(k, i\Omega; p' - k, p')}{V(k, i\Omega; p' - k, p')} \times G_{\sigma}^2(p', i\omega) G_{\sigma}(p' - k, i\omega - i\Omega). \tag{3.10}
\]

Notice that \( \Sigma^{(5a)} \) does not depend on the wave vector. This leads to the following leading disorder correction to
Here the clean single-particle rate in Eqs. (3.6):
\[
\delta(1/\tau(p)) \equiv \delta(1/\tau)
\]
\[
= \frac{V_0}{2\pi N_F T} \sum_k \int_{-\infty}^{\infty} \frac{du}{\pi} \frac{1}{n_F(u/T)} \chi''(k, u)
\]
\[
\times \text{Im} L^{++,-}(k). \quad (3.11a)
\]
Here \(\chi''\) is the spectral function in the spectral function of the susceptibility in Eq. (2.4a),
\[
\chi''(k, u) = \text{Im} \chi(k, i\Omega \rightarrow u + i0)
\]
\[
= \frac{\pi}{12N_F} \frac{q^2}{k_F^2} \frac{1}{\omega_0(k)} \left[ \delta(u - \omega_0(k)) - \delta(u + \omega_0(k)) \right], \quad (3.11b)
\]
and \(L^{++,-}\) is an integral that will also appear in the calculation of the conductivity in paper IV,
\[
L^{++,-}(k) = \frac{1}{V} \sum_p \gamma(k, p) \gamma(k, p - k) G_{R_\sigma}(p) G_{A}(p - k)
\]
\[
= i\nu^2 \frac{2\pi N_F m_e^2}{3} \lambda^2 k_F^2 + O(1/\tau, k_F^2), \quad (3.11c)
\]
with \(G_{R_\sigma}(p) = G_1(p, i\omega \rightarrow \pm i0)\) the retarded and advanced Green functions.

Inserting Eqs. (3.11a), (3.11c) into Eq. (3.11a) and performing the integrals yields, for the leading temperature-dependent contribution to \(\delta(1/\tau)\),
\[
\delta(1/\tau) = \frac{\nu^2 \pi \ln 2}{12\sqrt{6} \tau} \left( \frac{q}{k_F} \right)^5 \frac{\tau}{T}. \quad (3.12)
\]
Notice that \(\delta(1/\tau)\) has none of the complicated angular dependence seen in the clean relaxation rate, Eq. (3.10). While quenched disorder is expected to make the scattering process more isotropic in general, it is quite remarkable that there is no angular dependence whatsoever in this contribution to \(\delta(1/\tau)\).

C. The single-particle density of states

The single-particle density of states, as a function of the temperature and the energy distance \(e\) from the Fermi surface, can be defined in terms of the Green functions by
\[
N(\epsilon, T) = \frac{1}{\pi V} \sum_p \sum_{\sigma} \text{Im} G_{\sigma}(p, i\Omega \rightarrow \epsilon + i0). \quad (3.13)
\]
Here \(G\) is the fully dressed Green function. The interaction correction to \(N\), to first order in the interaction, can be written
\[
\delta N(\epsilon) = -\frac{1}{\pi V} \sum_p \sum_{\sigma} \text{Im} \left[ G_{\sigma}(p, i\Omega) \Sigma_{\sigma}(p, i\Omega) \right]_{\Omega \rightarrow \epsilon + i0}, \quad (3.14)
\]
with the dominant contribution to the self-energy \(\Sigma\) given by Eq. (3.10). From the calculation in Sec. III B 2 we know that the leading contribution to \(\Sigma\) is of order \(T/\tau\), and the integral over the Green functions is of \(O(T^0)\), so \(\delta N\) potentially has a contribution of \(O(\tau^0 T)\). However, an inspection of the integrals shows that this term has a zero prefactor. Hence, to this order in the interaction, there is no interesting contribution to the temperature-dependent density of states.

IV. DISCUSSION AND CONCLUSION

In summary, there have been two important results in this paper. First, we have shown that there is a canonical transformation that diagonalizes the action for helimagnets in the ordered state in spin space, and in the clean limit maps the problem onto a homogenous fermion action. This transformation enormously simplifies the calculations of electronic properties in an itinerant electron system with long-ranged helimagnetic order. As was mentioned in the Introduction, our model and conclusions are valid whether or not the helimagnetism is due to the conduction electrons. We have also discussed the effect of screening on the effective interaction that was first derived in paper II. We have found that screening makes the interaction less long-ranged, as is the case for a Coulomb potential. However, in contrast to the latter, screening does not introduce a true mass in the effective electron-electron interaction in a helimagnet. Rather, it removes the qualitative anisotropy characteristic of the unscreened potential in a rotationally invariant model, and introduces a term similar to one that is also generated by the spin-orbit interaction in a lattice model.

We have used the transformed action to compute a number of the low-temperature quasiparticle properties in a helimagnet. Some of the results derived here reproduce previous results that were obtained with more cumbersome methods in paper II. We then added quenched nonmagnetic disorder to the action, and considered various single-particle observables in the ballistic limit. All
of these results are new. The second important result in this paper is our calculation of the single-particle relaxation rate in systems with quenched disorder in the ballistic limit, \( \tau^2 T^2/\lambda > 1 \), where we find a linear temperature dependence. This non-Fermi-liquid result is to be contrasted with the previously derived \( T^{3/2} \) leading term in clean helimagnets, and the usual \( T^2 \) behavior in clean Fermi liquids.

In paper IV of this series we will treat the interesting problem of transport in clean and weakly disordered electron systems with long-ranged helimagnetic order. Specifically, we will use the canonical transformation introduced here to compute the electrical conductivity. In the clean limit we will recover the result derived previously in paper II, while in the ballistic regime we find a leading temperature dependence proportional to \( T \). This linear term is directly related to the \( T \)-term found above for the single-particle relaxation rate. For the case of the electrical conductivity, the \( T \) term is much stronger than either the Fermi liquid contribution (\( T^2 \)) or the contribution from the helimagnon scattering in the clean limit (\( T^{3/2} \)).

A detailed discussion of the experimental consequences of these results will be given in paper IV. There we will also give a complete discussion of the limitations of our results, and in particular of the various temperature scales in the problem, including the one introduced by screening the effective potential.

The linear temperature terms found here for the various relaxation times in bulk helimagnets is closely related to the linear \( T \) terms found in two-dimensional nonmagnetic metals, also in a ballistic limit.\(^{10}\) The analogy between 3D helimagnets and 2D nonmagnetic materials is a consequence of the anisotropic dispersion relation of the helical Goldstone mode or helimagnons. Technically, a typical integral that appears in the bulk helimagnet case is of the form

\[
\int d k_z \int d k_\perp k_\perp^2 \delta (\Omega^2 - k_z^2 - k_\perp^2) f(k_z, k_\perp) \propto \int d k_\perp k_\perp^2 \frac{\Theta (\Omega^2 - k_\perp^4)}{\sqrt{\Omega^2 - k_\perp^4}} f(k_z = 0, k_\perp),
\]

and the dependence of \( f \) on \( k_z \) can be dropped since it does not contribute to the leading temperature scaling. The prefactor of the \( k_\perp \) dependence of \( f \) is of \( O(1) \) in a scaling sense. As a result, the 3D integral over \( k \) behaves effectively like the integral in the 2D nonmagnetic case. Physically the slow relaxation in the plane perpendicular to the pitch vector makes the physics two-dimensional.

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14. Strictly speaking, this is true only in systems with rotational symmetry. In an actual helimagnet on a lattice, the spin-orbit interaction leads to a term proportional to \( c_z (q/k_F)^2 k_\perp^4 \) \( \omega_0^2(k) \). We will discuss this in the context of screening the phase-phase susceptibility in Sec. 14 below.
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