DAMPING OF TYPE I X-RAY BURST OSCILLATIONS BY CONVECTION
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Submitted to THE ASTROPHYSICAL JOURNAL

ABSTRACT

I construct a simple model of the convective burning layer during a type I X-ray burst to investigate the effects convection has on the stability of the layer to nonradial oscillations. A linear perturbation analysis demonstrates that the region is stable to nonradial oscillations when energy transport is convection-dominated, but it is unstable when energy transport is radiation-dominated. Thus, efficient convection always damps oscillations. These results may explain the nondetection of oscillations during the peak of some X-ray bursts.

Subject headings: accretion, accretion disks — stars: neutron — X-rays: binaries — X-rays: bursts

1. INTRODUCTION

Type I X-ray bursts are thermonuclear explosions that occur on the surfaces of accreting neutron stars. Fast rises and exponential-like decays lasting ~1 and ~10–100 seconds, respectively, characterize burst lightcurves (for reviews, see [Lewin et al. 1995; Strohmayer & Bildsten 2006]). Strohmayer et al. (1996) detected coherent oscillations in the lightcurve of a type I X-ray burst from the low-mass X-ray binary 4U 1728–34. Since then, astronomers have detected oscillations in burst lightcurves from 19 additional sources (although some detections are unconfirmed; see Bhattacharya et al. 2007; Markwardt et al. 2007; Strohmayer et al. 2008; Lamb & Boutloukos 2008, and references therein). Bursts can exhibit oscillations during the rise, the peak, and/or the decay of their lightcurves. The oscillation frequency typically increases by a few Hz during a burst, but it asymptotes to a specific frequency unique to that source to within a few parts in 10^3 (e.g. Giles et al. 2002; Muno et al. 2002). The asymptotic frequency's stability implies that it corresponds to the neutron star spin frequency (e.g. Strohmayer & Markwardt 1999).

It is most likely the rotational modulation of a growing hot spot on the neutron star surface that generates oscillations during the burst rise (Strohmayer et al. 1997b, 1998). The time needed to accrete enough fuel to trigger a thermal instability (a few hours to days) greatly exceeds that needed to burn the fuel (~seconds), which makes simultaneous ignition over the entire stellar surface highly unlikely. Ignition probably occurs at a point (Joss 1978; Shara 1982), and the resulting hot spot grows and engulfs the stellar surface in ~1 s (e.g. Fryxell & Woosley 1982; Bildsten 1995; Zingale et al. 2001; Spitkovsky et al. 2002). The latitude-dependent Coriolis force regulates the thermonuclear flame propagation speed and thereby the time evolution of the hot spot (Spitkovsky et al. 2002; Bhattacharya & Strohmayer 2007). The accreted fuel usually ignites at the equator, and the resulting thermonuclear flame quickly spreads in longitude and generates an axisymmetric belt around the neutron star. No oscillations occur because there is no azimuthal asymmetry (Spitkovsky et al. 2002). Ignition sometimes occurs off the equator (Cooper & Narayan 2007; Maurer & Watts 2008), the thermonuclear flame propagates in longitude much more slowly, generating a long-lived non-axisymmetric hot spot and hence oscillations.

The rotationally modulated hot spot model fails to explain oscillations during the burst decay, however, since the flame has engulfed the neutron star surface by this time. It is thought that excited surface modes generate nonradial oscillations during the burst decay (McDermott & Taam 1987; Cumming & Bildsten 2003; Lee 2004; Heyl 2004, 2005; Lee & Strohmayer 2005; Piro & Bildsten 2006). In particular, Heyl (2004) suggests buoyant r-modes as the most promising candidate. r-Modes travel backward in the corotating frame, so the observed oscillation frequency is less than the neutron star spin frequency. The r-mode frequency decreases as the surface cools during the burst decay, which explains the observed increase in the oscillation frequency. Narayan & Cooper (2007) propose that the ϵ-mechanism drives surface modes during the burst decay: if heating via nuclear burning is sufficiently strong relative to cooling via radiative diffusion and emission, some nuclear energy converts to mechanical energy and drives the oscillations (see also McDermott & Taam 1987; Strohmayer & Lee 1996; Piro & Bildsten 2004). They predict that short, powerful bursts preferentially exhibit oscillations, which is generally in accord with observations.

If the ϵ-mechanism does indeed drive oscillations, then one would naively predict that oscillations occur preferentially when nuclear burning is strongest, i.e. near the lightcurve’s peak. This is not true; bursts exhibiting oscillations show them during the peak only about half the time (Galloway et al. 2008). Some bursts are so powerful that the nuclear luminosity temporarily exceeds the local Eddington luminosity; the extra thermal energy converts to both kinetic energy and gravitational potential energy and expands the outermost layers of the neutron star. Astronomers often detect oscillations from these so-called photospheric radius expansion (PRE) bursts (Muno et al. 2000, 2001), but they rarely detect oscillations during the PRE phase itself (Smith et al. 1997, 1998; Hanawa & Sugimoto 1982; Wallace et al. 1982; Paczyński 1983; Woosley et al. 2004; Fisker et al. 2008). Why, then, do bursts often fail to exhibit oscillations during the peak phase?

To address this question, one must consider the mechanisms by which the burning layer transports energy. Highly efficient convection is the primary energy transport mechanism during the initial phase of a powerful burst (e.g. Joss 1977, 1978; Hanawa & Sugimoto 1982; Wallace et al. 1982; Paczyński 1983; Woosley et al. 2004; Fisker et al. 2008). It is well known that convection can either drive or dampen pulsations in variable stars (e.g. Cox 1980; Stellingwerf...
I proceed as follows. First, I write down the hydrodynamic equations. These equations describe the aggregate behavior of both the oscillatory and convective motions. The convective motions have short wavelengths of order the mixing length $l$, which is much smaller than the oscillation wavelengths by assumption (II). The disparity of the two lengthscales allows me to separate the equations governing the oscillatory motions from those governing the convective motions. In this work, I investigate only the stability of the oscillations to nonadiabatic perturbations, not the properties of the oscillations themselves (for a discussion of the latter, see, e.g., Longuet-Higgins 1968, Bildsten et al. 1996, Narayan & Cooper 2007). Therefore, I conduct a linear stability analysis on only the entropy equation governing the oscillatory motion. The perturbed entropy equation contains perturbations of quantities describing the convective motions. I solve for the perturbed convective quantities in terms of quantities describing the oscillatory motions by perturbing the convective equations.

2.1. Fundamental Hydrodynamic Equations

The fundamental continuity, momentum, and entropy equations of the model are

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + 2\Omega \times \mathbf{v} = -g \hat{\mathbf{z}} - \frac{1}{\rho} \nabla p, \tag{2}
\]

\[
\rho T \left( \frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s \right) = \rho \epsilon - \nabla \cdot \mathbf{F}_R, \tag{3}
\]

where $\rho$ is the density, $\mathbf{v}$ is the velocity, $g$ is the gravitational acceleration, $p$ is the pressure, $T$ is the temperature, $s$ is the entropy per unit mass, $\epsilon$ is the nuclear energy generation rate,

\[
\mathbf{F}_R = -\frac{ac}{3k\rho} \nabla T^4 \tag{4}
\]

is the radiative flux, $a$ is the radiation constant, and $\kappa$ is the radiative opacity. $g$ is presumed constant since the vertical thickness of the convective layer is small relative to the stellar radius. Equations (1)–(3) govern the aggregate behavior of the burning layer during a type I X-ray bursts. Specifically, they describe both the oscillatory and convective motions of a matter element. To proceed further I need to derive both the equations that govern only the long wavelength, oscillatory motions and the equations that govern only the short wavelength, convective motions. I do so in the following subsection.

2.2. Oscillatory and Convective Equations

Motivated by the work of Unno (1967) and Gabriel et al. (1975), I decompose the physical quantities into their oscillatory and convective parts:

\[
\rho = \bar{\rho} + \Delta \rho,
\]

\[
p = \bar{p} + \Delta p,
\]

\[
T = \bar{T} + \Delta T,
\]

\[
s = \bar{s} + \Delta s,
\]

\[
\mathbf{v} = \mathbf{u} + \mathbf{V},
\]

where $\bar{\rho}$, $\bar{p}$, $\bar{T}$, $\bar{s}$, and $\mathbf{u}$ are values averaged over a horizontal area with dimensions much larger than the mixing length but much smaller than the oscillation wavelength, and $\Delta \rho$, $\Delta p$, $\Delta T$, $\Delta s$, and $\mathbf{V}$ are the local quantities that describe the
convective fluctuations. The convective quantities are much smaller than their respective oscillatory quantities except for the velocity, in which case $|\mathbf{V}| \gg |\mathbf{u}|$. In fact, $\mathbf{u} = 0$ in the unperturbed configuration. Note that $\Delta x = 0$ for all quantities $x$. Furthermore, I define the Lagrangian derivative following the oscillatory equation from the fundamental equation.

For each of the three fundamental equations (1-3), I first derive the oscillatory equations by taking the horizontal average of the corresponding fundamental equation. I then derive each convective equation by subtracting the corresponding oscillatory equation from the fundamental equation.

### 2.2.1. Continuity Equation

Implementing equations (5) and (6), I write the continuity equation (1) as

$$\frac{d(\bar{\rho} + \Delta \rho)}{dt} + (\bar{\rho} + \Delta \rho) \nabla \cdot \mathbf{u} + \nabla \cdot (\bar{\rho} \mathbf{V}) = 0. \quad (7)$$

Setting $\Delta \rho = 0$ by assumption (I) and noting that $\bar{\rho} \mathbf{V} = 0$ (since convection involves no bulk motion, e.g. Gabriel 1996), the space-averaged continuity equation becomes

$$\frac{d\bar{\rho}}{dt} + \bar{\rho} \nabla \cdot \mathbf{u} = 0. \quad (8)$$

Subtracting equation (8) from (1) and again implementing assumption (I) ($\Delta \rho = 0$, $\bar{\rho} = \text{constant}$) gives the continuity equation describing the convective motions,

$$\nabla \cdot \mathbf{V} = 0. \quad (9)$$

### 2.2.2. Momentum Equation

Taking the horizontal average of equation (2) gives

$$\frac{d\mathbf{u}}{dt} + (\nabla \cdot \mathbf{V}) \mathbf{u} + (\mathbf{V} \cdot \nabla) \mathbf{u} + 2\Omega \times \mathbf{u} = -\frac{1}{\bar{\rho}} \nabla \bar{p} - g \mathbf{z}. \quad (10)$$

Subtracting equation (10) from (2) gives the momentum equation for the convective motions,

$$\frac{d\mathbf{V}}{dt} + (\mathbf{V} \cdot \nabla) \mathbf{V} - (\nabla \cdot \mathbf{V}) \mathbf{V} + (\mathbf{V} \cdot \nabla) \mathbf{V} - (\mathbf{V} \cdot \nabla) \mathbf{u} - (\mathbf{V} \cdot \nabla) \mathbf{u} + 2\Omega \times \mathbf{V} = -\frac{1}{\bar{\rho}} \nabla (\Delta \rho) + \frac{\Delta \rho}{\bar{\rho}^2} \nabla \bar{p}. \quad (11)$$

To be consistent with the mixing length theory, I set

$$(\mathbf{V} \cdot \nabla) \mathbf{V} - (\nabla \cdot \mathbf{V}) \mathbf{V} = \Lambda \frac{\mathbf{V}}{\tau_c},$$

$$(\mathbf{V} \cdot \nabla) \mathbf{u} - (\mathbf{V} \cdot \nabla) \mathbf{u} = \Lambda \frac{\mathbf{u}}{\tau_c}, \quad (12)$$

where

$$\tau_c = \frac{l}{|\mathbf{V}|} \quad (13)$$

is the lifetime of a convective element, $V_c = \mathbf{V} \cdot \mathbf{z}$, and $\Lambda$ is a number of order unity (Böhm-Vitense 1958, Unno 1967, Grigahcène et al. 2005). The momentum equation for convection becomes

$$\frac{d\mathbf{V}}{dt} + 2\Omega \times \mathbf{V} = -\nabla (\Delta \rho) + \frac{\Delta \rho}{\bar{\rho}^2} \nabla \bar{p} - \Lambda \frac{\mathbf{V}}{\tau_c} - \Lambda \frac{\mathbf{V}}{\tau_c}. \quad (14)$$

### 2.2.3. Entropy Equation

Taking the horizontal average of equation (3) gives

$$\bar{\rho} \frac{d\mathbf{s}}{dt} = \bar{p} \nabla \cdot \mathbf{F_R} - \bar{\rho} TV \nabla \cdot \mathbf{V}s. \quad (15)$$

I need to express the last term in a more useful form. Using the thermodynamic identity $Td\mathbf{s} = dH - d\mathbf{p}/\bar{\rho}$, where $H$ is the enthalpy per unit mass, I find

$$\bar{p} TV \nabla \cdot \mathbf{V}s = \nabla \cdot (\mathbf{F_R} - \bar{F}_C). \quad (16)$$

$\mathbf{F}_C$ is the horizontally-averaged convective flux. I use equations (13) and (19) in $\S3$. Subtracting equation (15) from (3) gives the convective entropy equation

$$(\Delta \rho \bar{T} + \bar{p} \Delta T) \frac{d\mathbf{s}}{dt} + \bar{p} TV \frac{d\mathbf{s}}{dt} + \rho TV \cdot \nabla s - \bar{p} TV \cdot \nabla s = \rho \mathbf{c} - \bar{\rho} \nabla \cdot (\mathbf{F_R} - \bar{F}_R). \quad (20)$$

To simplify the equation and keep it consistent with the mixing length theory, I set in analogy with equations (12) see Grigahcène et al. 2005.

$$\nabla \cdot (\rho TV \nabla s) = \nabla \cdot (\rho TV \nabla s) = \rho \bar{T} \frac{\Delta s}{\tau_c}. \quad (21)$$

Setting $\bar{\rho} (\mathbf{c}) = 0$ and $\bar{F}_R = 0$ by assumption (III) gives the convective entropy equation

$$\left(\frac{\Delta \rho}{\rho} + \frac{\Delta T}{T}\right) \frac{d\mathbf{s}}{dt} + \frac{d\mathbf{s}}{dt} + \rho TV \cdot \nabla s = \frac{\Delta s}{\tau_c}. \quad (22)$$

### 3. Perturbation of the Height-Integrated Equations

For simplicity, I construct a one-zone model for the vertical structure of the accreted layer (e.g. Fujimoto et al. 1981; Bildsten 1998; Narayan & Cooper 2007). I assume that the horizontally-averaged quantities describing the accreted matter are constant throughout the layer. By assumption (V), I write the height of the accreted layer as

$$\bar{\rho} = \frac{\bar{p}}{\bar{\rho} g}. \quad (23)$$

The equation of state is

$$p = \frac{\rho k_B T}{\mu m_p} + \frac{a T^4}{3}, \quad (24)$$

$$s = \frac{k_B}{\mu m_p} \left[ \ln \left( \frac{T^{3/2}}{\rho} \right) + \frac{4}{3} - 4 \right] + \text{constant}, \quad (25)$$

where $k_B$ is Boltzmann’s constant, $\mu$ is the mean molecular weight, $m_p$ is the proton mass, and $\beta = \rho k_B T / \mu m_p p$ is the
ratio of gas to total pressure. Perturbing the equation of state gives
\[ \delta \frac{\delta T}{\delta \rho} = \frac{1}{\chi} \delta \frac{\rho}{\rho} - 1 \frac{\delta \rho}{\rho}, \]
where \( \chi = 4 - 3\beta \) and
\[ \nu_T \equiv -\left( \frac{\partial \ln \rho}{\partial \ln T} \right)_p = \frac{4 - 3\beta}{\beta} \]
is minus the coefficient of thermal expansion. From equations (4), (18), and (19), the height-integrated entropy equation is
\[ \bar{p} \frac{d\bar{s}}{dt} = \bar{p} c - g(\hat{F}_R + F_C), \]
\[ F_R = \frac{acg T^4}{3\kappa T}, \]
\[ F_C = \rho TV_c \Delta s. \]
The goal of this work is to perturb equation (28) and determine the stability of the burning layer to nonradial oscillations. However, the expression for the perturbed convective flux \( \delta F_C \) includes perturbations of convective quantities. To proceed, I first need to express the perturbed convective quantities \( \delta V_c \) and \( \Delta s \) in terms of the perturbed oscillatory quantities \( \delta \rho, \delta T, \) and \( \delta \bar{p} \). The following subsection is devoted to this.

3.1. Perturbed Convective Equations

I conduct a standard WKB perturbation analysis on the convective equations. I assume that all dynamical quantities vary as \( \exp(-i\omega t + k \cdot x) \), where \( k = k_x x + k_y y + k_z z \) and \( k h \ll 1 \). Since the lifetime of a convective element \( \tau_c \) is assumed to be much smaller than the mode period \( 2\pi/\omega \), I presume that all unperturbed convective quantities are in quasi-steady state.

Deriving the perturbed continuity equation for convection is simple. From equation (29), the continuity equation for convective motions is
\[ k \cdot V = 0, \]
so the perturbed continuity equation for convection is
\[ k \cdot \delta V = 0. \]

Invoking the WKB approximation, integrating over \( z \), and simplifying, the momentum equation for convection (14) becomes
\[ \frac{dV}{dt} + 2\Omega \times V = -i(k \Delta p/\rho) + \frac{\Delta \rho}{\rho} g z - \Lambda u/\tau_c - \frac{\Delta V}{\tau_c}, \]
where I have assumed that \( \nabla \bar{p} \approx (\partial \bar{p}/\partial z) z \) in the unperturbed state. Perturbing equation (33), setting both \( d/dt \to 0 \) and \( \Omega \tau_c \to 0 \) by assumption (III), and ignoring the term \( \Lambda u/\tau_c \) because \( \|V\| \gg \|u\| \) gives the perturbed convective momentum equation
\[ ik \delta \left( \frac{\Delta p}{\rho g} \right) + \Lambda g \left( \delta \frac{\Delta \bar{p}}{\rho} \right) = \delta \left( \frac{\Delta \rho}{\rho} \right) z. \]
(34)

Equation (34) contains the perturbed convective quantities \( \delta \Delta \rho, \delta \Delta \rho, \delta V, \) and \( \delta \tau_c \), whereas the desired perturbed quantities are \( \delta \Delta s \) and \( \delta V_c \). To remedy this, I proceed as follows. Taking the dot product of equation (34) and \( k \) and using equations (31) and (33), I find
\[ \delta \left( \frac{\Delta \rho}{\rho g} \right) = \delta \left( \frac{\Delta \rho}{\rho} \right) \frac{k_z}{ik^2}. \]

Taking the dot product of equation (34) and \( z \) and using equation (35) then gives
\[ \frac{\Delta V_c}{g \tau_c} \left( \frac{\delta V_c - \delta \tau_c}{\tau_c} \right) = \delta \left( \frac{\Delta \rho}{\rho} \right) \left( 1 - \frac{k_z^2}{k^2} \right). \]
(36)

Conducting a similar procedure on the unperturbed, steady-state convective momentum equation, I find
\[ \frac{\Delta V_c}{g \tau_c} = \left( \frac{\Delta \rho}{\rho} \right) \left( 1 - \frac{k_z^2}{k^2} \right). \]
(37)

It follows that
\[ \frac{\delta V_c}{V_c} = \frac{\delta \tau_c}{\tau_c} \]
(38)

To proceed further, I need expressions for the two terms on the right hand side of the above equation. From equation (13),
\[ \frac{\delta V_c}{\tau_c} = \frac{\delta l}{l} \frac{\delta V_c}{V_c}. \]
(39)

I make the standard assumption that the mixing length \( l \) is proportional to the pressure scale height \( \bar{h} \) (e.g., Cox & Giuli 1968, Hansen et al. 2004). The lifetime of a convective element is negligible compared to the oscillation period (i.e. \( \omega \tau_c \ll 1 \)) by assumption (III). Therefore, I assume the mixing length instantaneously adjusts to changes in the scale height. It follows that
\[ \frac{\delta l}{l} = \frac{\delta \bar{h}}{\bar{h}} \approx \frac{\delta \bar{p}}{\bar{p}} \frac{\delta \bar{p}}{\bar{p}}, \]
(40)

where the last equality follows from equation (23). Note that equation (40) would not apply if, for example, \( \omega \tau_c \gg 1 \); in that case, \( \delta l/l = 0 \). Equations (39) and (40) then give
\[ \frac{\delta \tau_c}{\tau_c} = \frac{\delta \rho}{\rho} \frac{\delta \rho}{\rho} \frac{\delta V_c}{V_c}. \]
(41)

Writing the convective entropy as \( T \Delta s = c_p [\Delta T - (\partial T/\partial p) \Delta \rho] \), setting \( \Delta \rho = 0 \) by assumption (I), and noting that \( \Delta T/T = -(1/\tau_c) \bar{\Delta \rho}/\bar{\rho} \) from the equation of state, I get
\[ \frac{\Delta \rho}{\rho} = -\frac{\Delta s}{c_p}. \]
(42)

where
\[ c_p = \frac{5k_B}{2\mu c_p} \left( \frac{32 - 24\beta - 3\beta^2}{5\beta^2} \right) \]
(43)
is the specific heat at constant pressure. Thus, I find that
\[ \frac{\delta V_c}{V_c} = \frac{1}{2} \left( \frac{\delta \bar{p}}{\rho} \frac{\delta \bar{p}}{\rho} + \frac{\delta \Delta s}{\rho} + \frac{\delta \bar{\rho}}{\bar{\rho}} \frac{\delta \bar{\rho}}{\bar{\rho}} \right). \]
(44)

All I need now is an expression for \( \delta \Delta s \) in terms of the oscillatory variables.

Integrating the convective entropy equation (22) over \( z \) gives
\[ \left( \frac{\Delta \rho}{\rho} + \frac{\Delta T}{T} \right) \frac{d\bar{s}}{dt} + \frac{d\Delta s}{dt} + \frac{V_c \bar{s} \bar{p} g}{\tau_c} = -\frac{\Delta \bar{p}}{\rho}. \]
(45)

Perturbing this equation and setting \( \omega \tau_c \to 0 \) by assumption (III), I find
\[ \frac{\delta V_c}{V_c} + \frac{\delta \bar{s}}{\bar{s}} + \frac{\delta \bar{p}}{\rho} + \frac{\delta \bar{p}}{\rho} = \frac{\delta \Delta s}{\Delta s} - \frac{\delta \tau_c}{\tau_c}. \]
(46)

Using equations (41) and (44), the above relation simplifies to
\[ \frac{\delta \Delta s}{\Delta s} = \frac{\delta \bar{s}}{\bar{s}}. \]
(47)
3.2. Perturbed Entropy Equation

Perturbing the left side of the horizontally-averaged entropy equation (18) and using equation (26) to eliminate $\delta T/T$, I find
\[
\delta \left( \frac{\rho T d\delta s}{dt} \right) = -i\omega \frac{\rho h}{(T_3 - 1)} \left( \frac{\delta \bar{\rho}}{\bar{\rho}} - \frac{\Gamma_1}{T_3} \frac{\delta \bar{T}}{T_3} - \frac{\Gamma_3}{T_3} \frac{\delta \bar{T}}{T_3} \right),
\]
(48)
where $\Gamma_1 = (\partial \ln \rho / \partial \ln p)$, and $\Gamma_3 = 1 - \Gamma_1 (\partial \ln T / \partial \ln p)$, are the usual adiabatic exponents (e.g. Hansen et al. 2004). The nuclear energy generation rate $\epsilon$ is a function of both density and temperature. For small perturbations about the initial configuration, I write
\[
\delta \epsilon = \Omega_{\hbar} \hbar \left( \frac{\nu \delta \bar{\rho}}{\chi T} + \eta \frac{\delta \bar{T}}{\bar{T}} \right), \nu \equiv \left( \frac{\partial \ln \epsilon}{\partial \ln \rho} \right)_T, \eta \equiv \left( \frac{\partial \ln \epsilon}{\partial \ln T} \right)_\rho.
\]
(49)
where $\Omega_{\hbar}^{-1}$ is the characteristic heating time of the layer via nuclear burning. Using equation (26) to replace $\delta T/T$ in favor of $\delta \bar{\rho}/\bar{\rho}$, the perturbed heating term becomes
\[
\delta \epsilon = \Omega_{\hbar} \hbar \left( \frac{\nu \delta \bar{\rho}}{\chi T} + \eta \frac{\delta \bar{T}}{\bar{T}} \right).
\]
(50)
The layer cools via both radiative diffusion and convection. The radiative flux $F_R$ depends on the radiative opacity $\kappa$, which is a function of both density and temperature. For small perturbations about the initial configuration, I write
\[
\delta \kappa = \frac{\delta \bar{T}}{T} \bar{\rho} + \frac{\delta \bar{T}}{\bar{T}} \bar{\kappa}, \quad \xi \equiv \left( \frac{\partial \ln \kappa}{\partial \ln \rho} \right)_T, \quad \xi \equiv \left( \frac{\partial \ln \kappa}{\partial \ln T} \right)_\rho.
\]
(51)
Note that equations (48, 51) are simply generalizations of their analogous equations of Narayan & Cooper (2007). The magnitude of the unperturbed convective flux $F_C$ depends on the convective quantities $V_c$ and $\Delta s$. I cannot determine their values in terms of the oscillatory quantities $\bar{\rho}$, $\bar{T}$, and $T$ from first principles, so the magnitude of $F_C$ is uncertain. I parameterize this uncertainty by defining the dimensionless quantity
\[
f_c \equiv \frac{F_C}{F_R + F_C},
\]
(52)
the ratio of the unperturbed convective flux to the unperturbed total flux. Using equations (29, 50, 51, and 52), I find
\[
\delta (F_R + F_C) = \Omega_{\hbar} \hbar \left\{ (1 - f_c) \left[ \frac{4 - \zeta}{\chi_T} - \frac{\delta \bar{\rho}}{\bar{\rho}} - \frac{\delta \bar{T}}{\bar{T}} \right] + f_c \left[ \frac{\delta \bar{T}}{\bar{T}} - \frac{\delta V_c}{\bar{V}_c} - \frac{\delta \Delta s}{\bar{\Delta} s} \right] \right\},
\]
(53)
where $\Omega_{\hbar}^{-1}$ is the characteristic cooling time of the layer. Finally, using equations (44) and (44) to eliminate the perturbative convective terms and using equation (26) to eliminate $\delta T/T$, the perturbed cooling term becomes
\[
\delta (F_R + F_C) = \Omega_{\hbar} \hbar \left\{ \left[ \frac{4 - \zeta}{\chi_T} - 1 - f_c \left( \frac{3}{2} - \frac{3 - \zeta}{\chi_T} \right) \right] \frac{\delta \bar{\rho}}{\bar{\rho}} - \left[ \frac{4 - \zeta}{\nu_T} + \xi - f_c \left( \frac{1}{2} - \frac{3 - \zeta}{\nu_T} + \xi \right) \right] \frac{\delta \bar{T}}{\bar{T}} + f_c \left( \frac{\delta \bar{T}}{\bar{T}} - \frac{\delta \bar{\rho}}{\bar{\rho}} \right) \right\}.
\]
(54)
Substituting equations (48, 50), and (54) into the perturbed entropy equation
\[
\delta \left( \rho T d\delta s / dt \right) = \delta (\rho \epsilon) - g \delta (F_R + F_C)
\]
(55)
and simplifying gives expression for $\delta \bar{\rho}$ in terms of $\delta \bar{\rho}$,
\[
\frac{\delta \bar{\rho}}{\bar{\rho}} = A \frac{\delta \bar{\rho}}{\bar{\rho}},
\]
(56)
where $A$ is a dimensionless complex quantity. For purely adiabatic perturbations, $A = \Gamma_1$. In the limit where the convective layer is gas pressure dominated ($\beta \to 1$)
\[
A = \frac{5}{3} \left( \frac{1 + (2i/3)(\Omega_{\hbar}/\Omega_c)[(\omega - c_f)/\Omega_c] - (\Omega_{\hbar}/\Omega_c)(\nu - \eta)}{1 + (2i/3)(\Omega_{\hbar}/\Omega_c)[(\omega - c_f)/\Omega_c] - (\Omega_{\hbar}/\Omega_c)(\nu - \eta)} \right),
\]
(57)
and where the layer is radiation pressure dominated ($\beta \to 0$)
\[
A = \frac{4}{3} \left( \frac{1 + (i/4)(\Omega_{\hbar}/\Omega_c)[(\omega - c_f)/\Omega_c] - (\Omega_{\hbar}/\Omega_c)(\nu - \eta)}{1 + (i/4)(\Omega_{\hbar}/\Omega_c)[(\omega - c_f)/\Omega_c] - (\Omega_{\hbar}/\Omega_c)(\nu - \eta)} \right).
\]
(58)
I work exclusively in these two limits hereafter for simplicity.

4. PULSATIONAL STABILITY

I invoke the quasi-adiabatic approximation to determine the linear stability of the layer. That is, I assume the fractional entropy change during one oscillation period is small. An equivalent statement is that the heating and cooling rates are small relative to the oscillation frequency, i.e. $\Omega_{\hbar}/\omega \ll 1$ and $\Omega_c/\omega \ll 1$. The typical heating and cooling times during a type I X-ray burst are of order one second, so $\Omega_{\hbar}$ and $\Omega_c \sim 1 \text{s}^{-1}$, whereas the modes have angular frequencies $\sim 100 \text{radians s}^{-1}$ (e.g. Heyl 2004), so the quasi-adiabatic approximation is valid.

I use the following criterion to determine the pulsational stability of the convective layer: it is unstable if the work done on the layer during one oscillation period is positive, and it is stable if the work done on the layer during one oscillation period is negative (Cox 1980, Unno et al. 1989). The pressure and density perturbations both vary in time as $\exp(-i\omega t)$, but they are in general out of phase because $A$ is complex. If $\text{Im}(A) > 0$, $\delta \bar{\rho}$ lags behind $\delta \bar{\rho}$ and the area traced in the $P-V$ diagram during one oscillation period is positive; the $PdV$ work done on the layer is positive and the layer is unstable. Conversely, if $\text{Im}(A) < 0$, $\delta \bar{\rho}$ leads $\delta \bar{\rho}$ and the area traced in the $P-V$ diagram is negative; the $PdV$ work done on the layer is negative and the layer is stable. For purely adiabatic perturbations, $A$ is real and $\delta \bar{\rho}$ and $\delta \bar{\rho}$ are in phase. The time evolution of $\delta \bar{\rho}$ and $\delta \bar{\rho}$ traces a curve of zero area in the $P-V$ diagram, so the $PdV$ work done in one cycle is zero; the modes neither grow nor decay. Thus the sign of $\text{Im}(A)$ determines the stability of the layer.

At high temperatures achieved during a burst, the opacity scales according to the approximate formula given by Paczyński (1983), which give $-0.1 < \zeta < 0$, $-0.5 < \zeta \leq 0$. I follow Narayan & Cooper (2007) and set $\zeta = 0$ and $\zeta = -0.25$ for simplicity. Solving for the imaginary part of $A$ in equations (57) and (58), I find the instability criterion for nonradial oscillations to be
\[
\frac{\Omega_c}{\Omega_{\hbar}} > \frac{7 + 5 f_c}{10 + 6 \eta + 4 \nu}
\]
(59)
when the layer is gas pressure dominated ($\beta \to 1$), and
\[
\frac{\Omega_c}{\Omega_{\hbar}} > \frac{1 + 17 f_c}{16 + 12 \eta + 4 \nu}
\]
(60)
when the layer is radiation pressure dominated ($\beta \rightarrow 0$). Note that equations (59) and (60) reduce to equations (76) and (77) of Narayan & Cooper (2007) when $f_c = 0$, as they should. If the heating rate due to thermonuclear burning during a burst is sufficiently large relative to the cooling rate, some of the nuclear energy converts to mechanical energy and drives the surface modes. It is clear from equations (59) and (60) that convection dampens oscillations in both cases. For $\beta = 1$, the effect is fairly modest; the minimum heating rate needed for instability when cooling is convection-dominated is less than a factor of 2 greater than that needed when cooling is radiation-dominated. However, the temperature of the layer during a burst is often $\gtrsim 10^9 K$, especially when a sizable convective layer develops (Joss 1977; Woosley et al. 2004), so the case $\beta = 0$ is more relevant for this work. For $\beta = 0$, the effect is huge; the minimum heating rate needed for instability when cooling is convection-dominated is 18 times greater than that needed when cooling is radiation-dominated. Therefore, oscillations during type I X-ray bursts are unlikely when convection dominates the cooling.

5. DISCUSSION

In this investigation, I have found that efficient convection dampens oscillations during type I X-ray bursts. This may explain the nondetection of oscillations near the peak of some bursts. The basic physics of this effect is simple. Consider the lateral compression of a column of matter during a burst. When the matter is compressed, the temperature and density rise and generally increase the nuclear energy generation rate; the cooling rate may either increase or decrease. If the marginal increase in the heating rate is sufficiently large relative to the cooling rate at maximum compression, the pressure will continue to rise and thereby drive the oscillations. Energy transport by convection is more effective than by that of radiative diffusion when the layer is compressed, so the cooling rate is larger at maximum compression if the layer is convective. In this case, the pressure is more likely to decrease at maximum compression and hence damp the oscillations.

It is unclear if convection alone explains the paucity of burst oscillation detections during PRE. Strong convection is guaranteed during a PRE burst since the nuclear burning timescale greatly exceeds the radiative cooling timescale, but the heating rate is probably large enough relative to the convective cooling rate to satisfy the instability criterion. It may be that, although convection cannot damp the oscillations entirely, it lowers the growth rate enough to keep the oscillation amplitudes below the detectability threshold. Alternatively, perhaps the surface modes are unstable during PRE, but the bursting layer’s vertical extent is sufficiently large relative to the mode wavelength to smear out the nonaxisymmetry (Cumming & Bildsten 2000).

I have presented a simplified, analytical, one-zone model of the convective burning layer as a first attempt to study the effect convection has on the driving of burst oscillations. There are several important issues I am unable to address. First, convection alters both the energy transport mechanism and the temperature gradient. I have investigated only the former in this work. Second, the model assumes the convective zone’s extent is constant in time. During an actual burst, the convective zone first grows to lower pressures and then recedes to the ignition region (e.g., Joss 1978; Woosley et al. 2004; Weinberg et al. 2006; Fisker et al. 2008). Presumably, the convective zone in a given column of matter will expand when the column is compressed and thereby further dampen the oscillations, but this is merely a speculation. Third, I cannot determine the actual values of the critical parameters $\Omega_0/\Omega_c$, $f_c$, $\nu$, and $\eta$ of the stability criterion. One can determine them only with detailed, multi-zone calculations such as those of Woosley et al. (2004) and Fisker et al. (2008).

I thank Ramesh Narayan for his advice and encouragement, Duncan Galloway and Mike Muno for answering my questions about the latest observations of burst oscillations, and Nevin Weinberg for reviewing the manuscript. This work was supported by NASA grant NNG04GL38G.

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