Nonlinear finite element analysis of stress-strain state of plate-rod reinforced concrete structures under the action of bending and torsion

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Abstract. In this study, loadings of plate-rod structures were considered, causing a complex stress state, which are exposed to significant external forces applied outside the plane, causing torsion of the structure as a whole. An analysis of the stress-strain state of a plate-rod reinforced concrete truss with regard to concrete creep is given. The core and volumetric models and the calculation of the plate-rod trusses in the ANSYS software package taking into account the creep of concrete are created, the results of computer calculations and the comparative characteristics of the results obtained are presented. The forces in the truss elements from the load located in its plane are determined by any method known from structural mechanics for the calculation of plate-rod systems. External torques that act on the structure are balanced in the amount of reactive support torques. The calculation of the strength of elements of reinforced concrete plate-rod trusses is carried out for sections that are normal to their longitudinal axis, and also for sections of the most dangerous direction that are inclined to it. Calculation of the plate-rod trusses under the action of bending and torque is associated with the determination of the twist angle of the truss sections or with the determination of its torsional stiffness.

1. Introduction

Compared to the traditional rod reinforced concrete structures the plate-rod reinforced concrete structures are able to perceive loads acting outside their plane and have their own stability. The use of plate-rod structures is advisable where loads are applied outside the plane of the structure. In such loading conditions, flat lattice structures are adopted as structural formations that consist entirely of plate elements that can work on bending from the plane of the structure, or in combination with linear rods that receive axial forces.

2. Methodology

2.1. Determination of internal forces and bending moments in elements of plate-rod trusses

The calculation of the strength of elements of reinforced concrete plate-rod trusses is carried out for sections that are normal to their longitudinal axis, and also for sections of the most dangerous direction that are inclined to it. Since the lower belt is experiencing only axial stretching, its strength, in the general case having pre-tensioned reinforcement in the cross section, with cross-sectional area \( A_s \) and unstressed with cross-sectional area \( A_{sp} \), is calculated by the condition [1]:

\[ N \leq \gamma_{sp} \cdot R_s \cdot A_{sp} + R_y A_y \]  

(1)

2.2. Calculation of movements in the stage without cracks

Calculation of the plate-rod trusses, which are under the action of bending and torque, is associated with determining the angle of twist of sections of the truss \( \varphi \) or determining its torsional stiffness.
The angle of twisting can be determined from the condition of equality to zero of the sum of the work of external forces and the internal forces and bending moments of the structure caused by them [2]. By analogy with the formula for displacements of flat rod systems, the twist angle for plate-rod trusses can be determined from the expression:

$$\phi = \sum_{i=1}^{n} \int_{0}^{l_i} \frac{M_{pi} \cdot \overline{M}_i}{E \cdot J_i} \, dx,$$

(2)

where $M_{pi}$ – bending moment in the i-th member of the truss from external torque; $M_i$ – the bending moment in the i-th element of the truss from a single torque applied in a section for which the twist angle is determined; $l$ – the length of each element.

For the upper belt, the product $E \cdot J_i$ in the case of no cracks is constant, therefore the fraction of the twisting angle of the deformations of the upper belt [3]:

$$\phi^b_m = \frac{l^b}{3EJ^b} \sum_{i=1}^{n} M_{pi max}^b \cdot \overline{M}_i,$$

(3)

For diagonals, the proportion of the twisting angle is given by the following formula:

$$\phi^g_m = B \cdot \sum_{i=1}^{n} M_{pi max}^g \cdot \overline{M}_i,$$

(4)

where

$$B = \frac{12 \cdot l^g}{E \cdot b \cdot \left(h^g - h^i\right)^3} \left[ \ln \left(\frac{h^g}{h^i}\right) + \frac{h^i \left(4h^g - h^i\right)}{2 \left(h^g\right)^2} - 1.5 \right].$$

(5)

The total twisting angle of the plate-rod truss from the action of external torques applied at the nodes of the upper belt is

$$\phi_m = \frac{l^b}{3EJ^b} \sum_{i=1}^{n} M_{pi max}^b \cdot \overline{M}_i + B \cdot \sum_{i=1}^{n} M_{pi max}^g \cdot \overline{M}_i,$$

(6)

Farm torsional stiffness in the crack-free stage:

$$C = \frac{T}{\phi_m},$$

(7)

where $l^b$ – top bar length; $J^b$ – the moment of inertia of the cross section of the upper belt relative to the vertical axis of symmetry; $M_{pi max}^b$ and $M_{pi max}^g$ – maximum bending moments respectively in the panels of the upper belt and bracing from the action of external torque; $\overline{M}_i$ – maximum bending moments, respectively, in the panels of the upper belt and diagonals of the action of a single torque applied in the section for which the twist angle is determined.

Simulation of the lamellar rod truss was performed in the certified ANSYS software package.

2.3. Calculation by the first group of limit states

The initial data for the calculation (Figure 1) are the data in [4]:

- Applied load $P = 20$ kN with eccentricity $e_0 = 0.4$ m;
- longitudinal reinforcement class A400;
- reinforcement transverse class B500;
- Concrete is heavy on small aggregate of class B40.

The design scheme of the truss and the diagram of the moment vectors are shown in figure 2. The posts on the diagram are not indicated, since no effort arises in them at this load. Axial forces are found by constructing a Cremona-Maxwell diagram.

2.4. Construction of the core and volume model of a plate-rod truss in the ANSYS software package

A truss model built in a software package corresponds to a manually calculated truss: $l = 3100$ mm, $h^a = 200$ mm, $h^{u,n} = 70$ mm, $b = 50$ mm.
Figure 1. General view of the plate-rod truss.

Figure 2. Design scheme plate-rod truss
a) diagrams of bending moments; b) moment vector diagram.

Variable cross sections in the core model can only be defined by splitting the cross section into several parts and specifying each part of a different cross section size, as shown in figure 3.

Figure 3. General view of the truss rod model.

Fastening the hinge truss, the load is applied to the middle node with eccentricity \( e = 0.4 \text{ m} \), \( P = 20 \text{ kN} \). The results of the linear calculation of the core model are shown in figure 4.

Figure 4. Graph of the deformed model with display of total displacements.

Considering the efforts arising in the elements of the frame are shown in figure 5-7.
Figure 5. The diagram of bending moments relative to the axis Y (My).

Figure 6. The diagram of transverse forces along the axis Z (Qz).

Figure 7. The angle of twist farm relative to the axis X (Ux).

3. Results

3.1. Results with plots and isofields for convenience

The results with plots and isofields for convenience are summarized in table 1.
Table 1. Summary table of the calculation results.

|                        | Upper belt | Lower belt | Stretched brace | Compressed brace |
|------------------------|------------|------------|-----------------|------------------|
| Longitudinal force     | 23.8 kN    | 42.8 kN    | 25.3 kN         | 21.2 kN          |
| Bending moment (max)   | 8.54 kNm   | -          | 9.45 kNm        | 9.45 kNm         |
| Lateral force          | 11.42 kN   | -          | 11.42 kN        | 11.42 kN         |
| Total displacement (max)| 4 mm       | -          | 11.42 kN        | 11.42 kN         |
| Torsion angle (max)    | 0.06139 rad| -          | -               | -                |

3.2. Volume model

The volume model consists of bulk finite elements. First, the volume of the truss is created in the Ansys Space Claim subroutine, then using the triangulation network, the model is transformed into a set of three-dimensional elements [5]. All properties of materials are set the same as in the manual calculation. Fastening the hinge truss, as shown in figure 8.

![Figure 8. General view of the volume model of the truss.](image)

The volumetric model does not show a plot of effort in braces and belts. But one can compare the total displacements and the general scheme of deformation of the volume model of a plate-rod truss with a rod model (Figure 9).

![Figure 9. Comparative diagram of the core and volume models.](image)
3.3. Accounting for the creep of concrete in the software package

The creep of concrete in the software package is taken into account according to the laws of nonlinear deformation. To take into account creep, it is necessary to calculate the coefficients $\varphi_0$ and $\beta_H$ according to [6]:

Conditional creep factor:

$$
\varphi_0 = \varphi_{RH} \cdot \beta_{(f_{cm})} \cdot \beta_{(h_0)}
$$

(8)

For $f_{cm} \leq 35$ MPa:

$$
\varphi_{RH} = 1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot \sqrt{h_0}}
$$

(9)

$$
h_0 = \frac{2A_c}{u}
$$

(10)

$$
\beta_{(f_{cm})} = \frac{16.8}{\sqrt{f_{cm}}}
$$

(11)

$$
\beta_{(h_0)} = \frac{1}{0.1 + (t_0)^{0.2}}
$$

(12)

$$
\beta_H = 1.5 \cdot [1 + (0.012 \cdot RH)^{18}] \cdot h_0 + 250 \leq 1500
$$

(13)

where $A_c$ – total cross-sectional area of concrete; $u$ – the perimeter of the element in contact environment; $RH$ – relative air humidity of the environment, %; $f_{cm}$ – average compressive strength of concrete at the age of 28 days, MPa; $t_0$ – concrete age at the time of application of the load, days; $\beta_H$ – coefficient taking into account the relative humidity ($RH$, %) and the conditional size of the element ($h_{0,mm}$).

If execute coefficient calculation:

$$
\varphi_{RH} = 1 + \frac{1 - \frac{73}{100}}{0.1 \cdot \sqrt{40}} = 1 + \frac{0.27}{0.342} = 1.789
$$

$$
h_0 = \frac{2 \cdot 10,000}{500} = 40 \text{ mm}
$$

$$
\beta_{(f_{cm})} = \frac{16.8}{\sqrt{19.27}} = 3.827
$$

$$
\beta_{(h_0)} = \frac{1}{0.1 + (28)^{0.2}} = \frac{1}{2.047} = 0.4885
$$

$$
\varphi_0 = 1.789 \cdot 3.827 \cdot 0.4885 = 3.34
$$

$$
\beta_H = 1.5 \cdot [1 + (0.012 \cdot 73)^{18}] \cdot 40 + 250 = 1.5 \cdot 1.092 \cdot 40 + 250 = 315.54
$$

3.4. Modeling and non-linear calculation of plate-rod trusses, taking into account the concrete creep of compressed elements in the ANSYS software package by the finite element method

For analyzing the physically and geometrically nonlinear behavior of concrete in ANSYS, special volumetric hexagonal (eight-node) elements of the SOLID 65 type with three degrees of freedom in the node were used.

A concrete material model, CONCRETE, was used to model the concrete, which supports the consideration of cracking and crushing (compression damage). Before the onset of cracking, the tensile behavior is assumed to be linearly elastic. To take into account the nonlinear behavior in compression before the onset of failure, a material diagram is additionally introduced (exponential law of deformation) [7, 8].
The volume-stressed state of the SOLID 65 elements with a non-linear calculation is estimated in three orthogonal directions. Upon reaching any major surface stress fracture for stretching, a crack appears. A crack may occur in one or several directions [9].

For reinforcement, a plasticity diagram is introduced (exponential law of deformation). Additionally, creep is taken into account by specifying the dependence of additional creep deformations on time and on the existing stress-strain state (using the power law of creep) [10].

Consider the resulting farm model. In order to prevent local destruction of the truss when setting the load, it is necessary to provide steel holders on the trusses of the truss and in the center, simulating an experimental installation with a jack (Figure 10).

Figure 10. Fragment of a volume model for non-linear calculation.

Since nonlinear calculation is more complicated and time consuming than linear, in order to accelerate the calculation can be used the symmetry, thereby reducing the number of bulk finite elements by half [11-14].

Consider the results obtained after the calculation (Figure 11).

Figure 11. Graph of the deformed model with the display of the total displacements when taking into account creep.

By taking into account the creep of concrete in a non-linear calculation, we obtain maximum deformations of 0.5 cm. (what is a valid value \( f_{\text{max}} = \frac{f}{l} = \frac{300}{150} = 2.0 \text{ cm} \)). In the linear calculation, a value of 0.38 cm was obtained. Consequently, when taking into account the creep of concrete when performing non-linear calculation, we obtain greater deformations than in the linear calculation.

Also, after conducting a non-linear calculation, it is possible to estimate the stresses in the reinforcement and the appearance of cracks in the truss elements (Figure 12).

Figure 12. Equivalent stresses in reinforcement, MPa.
3.5. Comparison and analysis of the results

The obtained results for ease of comparison are summarized in Table 2.

| Stress | ANSYS | Manual account | Experiment |
|--------|-------|----------------|------------|
| $N^b$  | 23.8 kN | 21.4 kN | 20.49 kN |
| $N^g$  | 21.2 kN | 23.6 kN | 22.79 kN |
| $N^{nm}$ | 42.8 kN | 42.8 kN | |
| $M^b_{max}$ | 8.54 kNm | 8.56 kNm | |
| $M^g_{max}$ | 9.45 kNm | 9.44 kN | |
| $Q$    | 11.42 kN | 11.42 kN | |
| $\phi$ | 0.06139 rad | 0.0614 rad | 0.0617 rad |

The results in the table clearly show good convergence. The error between the manual count, the experiment and the software complex is minimal. This suggests that the method of determining the angle of twist is correct and it is convenient to design such structures as lamellar-rod trusses in a software package.

4. Conclusions

1. Analyzing the data obtained after updating the calculation for the second group of limit states, the calculation of the Ansys software package and the experiment, we obtain good convergence of the obtained results with an accuracy of 0.5%.

2. The core and volume models of a plate-rod reinforced concrete truss after carrying out a linear calculation give a good convergence of the obtained results.

3. In a lamellar rod farm under the action of a torsional load, the total deformations increase with consideration of concrete creep.

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