Non linear dynamics of vortices in superconductors with short coherence length.

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Abstract

In superconductors where the coherence length is comparable to the Fermi wavelength, the vortex viscosity depends on the velocity of the vortex, leading to non linear equations of motion. The trajectories of vortices driven by a. c. fields show a variety of behaviors as function of frequency. Finite perturbations give rise to very long lived transients. The relevance of these results to experiments in high-$T_c$ superconductors is discussed.

74.72.Bk, 74.25.Ha, 74.60.Ge
I. INTRODUCTION.

In the copper oxide superconductors, the coherence length, $\xi$, is similar to the Fermi wavelength, $k_F^{-1}$, and to the lattice spacing, $a$. The separation of energy levels within vortex cores, $\delta\epsilon$ can be comparable, or larger, than the width of these levels (ultra clean limit) and the temperature. Using standard parameters, we estimate that $\delta\epsilon \sim 40K$, in agreement with experimental observations \[1\]. The quantization of levels should change drastically dissipation processes at temperatures lower that $\delta\epsilon$. The standard theory of flux flow dissipation \[2\] breaks down, as it is based on the existence of fast relaxation processes within the vortex core. Note that the limit $T \ll \delta\epsilon$ is also different from what is usually defined as the ultra clean limit, in which $\bar{\hbar} \tau \ll \delta\epsilon$, but $\delta\epsilon < T$ \[3\].

Simple arguments show that energy dissipation due to vortex motion should be exponentially reduced at low velocities. The vortex viscosity behaves as $\eta(v) \sim e^{-v_0/v}$ \[4\], where $v_0$ is given by $v_0 \sim \delta\epsilon \xi/\bar{\hbar}$. The existence of this threshold velocity can be understood very simply: it corresponds to the situation for which the inverse of the vortex-impurity collision time, $\tau_{\text{coll}} \sim \xi/v$, becomes comparable to $\delta\epsilon/\bar{\hbar}$. Taking $\xi \sim 10\AA$, we find $v_0 \sim 4 \text{Km/s}$.

The presence of such a term in the energy dissipation of moving complex quantum objects with internal levels (atoms, molecules) is rather common \[5\]. The existence of a gap in the excitation spectrum of the system implies that energy dissipation is exponentially suppressed at low temperatures or velocities.

The resulting equations of motion for the vortices become highly non-linear. In the following, we study the dynamics of vortices, in the pinned regime, driven by a. c. fields. Our aim is to contribute to the understanding of recent experiments which probe the response of vortices at low temperatures \[6,7\]. These experiments show a viscosity systematically lower than the value predicted by the Bardeen-Stephen theory. These experiments have been performed in the range of $10^6$ to $10^9$ Hz. The previous analysis gives $\tau_{\text{coll}}^{-1} \sim 10^{12}$ Hz. The number of collisions per cycle is large enough to justify the theory presented in \[4\].

A velocity dependent viscosity has also been analyzed in \[8\]. In this work, it arises from the influence of the pinning centers on an (almost) freely moving vortex, at relatively high temperatures. The total viscosity is always finite, and its value is comparable to the Bardeen-Stephen prediction. We do not consider that regime in the following.

II. EQUATION OF MOTION.

We consider pinned vortices, with a linear restoring force. There is some controversy on the magnitude of the Magnus force. To keep the number of parameters to a minimum, we assume that vortices are carried by the applied
current, and that Galilean invariance fully determines this contribution to
the motion \([9]\). Finally, we describe the external probe through the superfluid
velocity, with a given time dependence.

Then, the equations of motion, in the plane perpendicular to the vortex
axis, are:

\[
\begin{align*}
\eta(v)v_x &= -k_p x + \alpha v_y \\
\eta(v)v_y &= -k_p y - \alpha [v_x - v_s(t)]
\end{align*}
\]  

and \(\eta(v) = \eta_0 e^{-\nu_0/v}\). We use units such that \(k_p = 1\), \(\eta_0 = 1\) and \(v_0 = 1\).
The equations (1) are determined by the ratio \(\alpha/\eta_0\). The only difference
with the standard theory \([6–8]\) lies in the assumption of a velocity dependent
viscosity. In the presence of a periodic supercurrent, \(v_s(t) = v_0 \cos(\omega t)\), we
need also to specify the dimensionless values of \(\alpha \omega\) and \(v_0\).

We now solve numerically eqs. (1). We consider first the case of a periodic
driving current. We choose \(\alpha = 1\) and \(v_0 = 0.5\). The value of \(\alpha\) is proportional
to \(n_s\), the condensate density, and \(\eta_0 \propto n_{imp}\), the concentration of impurities.
Hence, \(\alpha/\eta_0 \sim n_s/n_{imp}\). Finally, \(v_0\) is a fraction of the bulk depairing
velocity, \(v_0/v_{dp} \sim \delta \epsilon/\Delta\), so that \(v_0\) should not be greater than \(v_0\).

If we could take \(\eta = 0\), the solution of eqs. (1) depends only on the
dimensionless ratio \(k_p/(\alpha \omega)\).

The dynamics of the vortex depend strongly on the value of the driving
frequency. To a first approximation, we can neglect the friction term in eqs.
(1). Then, the motion of the vortex is purely inductive, and the Hall angle
changes from 0 (\(\omega \ll 1\)) to \(\pi/2\) (\(\omega \gg 1\)). This picture is qualitatively correct,
as seen in fig.(1). The unit of length is \(\eta_0 v_0/k_p\). Taking typical experimental
values (at low temperatures) for \(k_p \sim 2 \times 10^5 \text{ N } / \text{ m}^2\) and \(\eta_0 \sim 10^{-6} \text{ N s } / \text{ m}^2\) \([6,7]\), our unit of length is \(\sim 40\text{Å}\).

The vortex orbit, however, contains many higher harmonics (see next
section). At sufficiently high frequencies, these higher harmonics dominate
the trajectory, and the role of the dissipation needs to be taken into account.

The corresponding velocities are plotted in fig.(2). In order to interpret
this result, we need to consider the influence of the dissipation. Firstly, it
limits the maximum values of the vortex velocity, which never becomes much
greater than 1, even close to resonance. Above the resonant frequency, the
motion becomes highly irregular. Note that the number of degrees of freedom
in eqs.(1) suffices to generate chaotic behavior in the presence of a periodic
driving force. We have checked that eqs.(1) do, indeed, show chaotic behavior,
for \(\alpha \ll \eta_0\) and high frequencies.

We have also analyzed the response to a finite pulse, as in the experiments
reported in \([7]\). We find very long lived transients after the pulse has been
switched off. This effect can be understood analytically. In the absence of a
driving force, we can take the time derivative of eqs. (1), to obtain:

\[
\begin{align*}
\eta(v) \left(1 + \frac{v_0}{v}\right) (v_x \dot{v}_x + v_y \dot{v}_y) &= -k_p v^2 + \alpha (\dot{v}_y v_x - \dot{v}_x v_y) \\
\eta(v) (\dot{v}_x v_y - \dot{v}_y v_x) &= \alpha (\dot{v}_x v_y + \dot{v}_y v_x)
\end{align*}
\]  

(2)
which gives:

\[ \dot{v} = \frac{k_p v \eta(v)}{\alpha^2 + \eta^2(v) (1 + \frac{\alpha v_0}{v})} \]  

(3)

At long times, \( v \ll v_0 \) and \( \eta(v) \ll \alpha \). Then:

\[ \dot{v} \approx -\frac{k_p v \eta_0}{\alpha^2} e^{-\frac{v_0}{v}} \]  

(4)

which, to logarithmic accuracy, yields:

\[ \lim_{t \to \infty} v \sim \frac{v_0}{\log \left( \frac{k_p \eta_0 t}{\alpha^2} \right)} \]  

(5)

Our numerical results are consistent with this asymptotic behavior. Note that, if the viscosity was independent of velocity, transients decay exponentially, with a well defined relaxation time, \( \tau^{-1} \propto \eta/(\eta^2 + \alpha^2) \).

### III. NON LINEAR RESISTIVITY.

From the vortex velocities, plotted in fig. (2), we infer the induced voltage, by means of the Josephson relation, \( V \sim \vec{v} \times \vec{B} \). Hence, \( \rho_{xx} \propto v_y \) and \( \rho_{xy} \propto v_x \). In order to analyze numerically the generation of higher harmonics, solve eqs. (1) in the presence of a periodic current, and perform a Fourier transform.

The linear resistivities are plotted in fig. (3). The two components which are finite in the limit of vanishing dissipation, \( \text{Im} \rho_{xx} \) and \( \text{Re} \rho_{xy} \) are approximately well described by the standard solution of eqs. (1). We find a sharp resonance at \( \omega \sim k_p/\alpha \). The resonance, however, is very asymmetric, with a sharp rise at low frequencies, and a slow decay at high frequencies.

The other components of the resistivity, \( \text{Re} \rho_{xx} \) and \( \text{Im} \rho_{xy} \) display more unusual behavior. Both are very small at low frequencies. They become comparable to the other components near the resonance frequency, and slowly decrease at high frequencies. Note that, if the effective viscosity remains finite as \( \omega \to \infty \), these functions should tend to a constant. At low frequencies, we find the scaling law \( \rho_{xy}(\omega) \sim \rho_{xx}^2(\omega) \) as function of frequency. The effective viscosity near the resonance can be understood from the standard, linear, solution to eqs. (1). Near the resonance, the vortex velocity behaves as \( v \sim \alpha v_0^0 / \eta(v) \). As \( v \) cannot exceed 1, we find that the value of \( \eta(v) \) at resonance should be similar to the value of \( \alpha \).

If the results shown in fig.(3) were analyzed in terms of an effective, frequency dependent viscosity, we conclude that \( \eta_{eff} \sim 0 \) for \( \omega \ll k_p/\alpha \), \( \eta_{eff} \sim \min(\alpha, \eta_0) \) at resonance, \( \omega \sim k_p/\alpha \), and that \( \eta \to 0 \) at high frequencies.

The resistivities also show a significant number of higher harmonics. At low frequencies, this effect is more pronounced in \( \text{Re} \rho_{xx} \) and \( \text{Im} \rho_{xy} \), where the standard analysis indicates that they vanish as \( \eta \to 0 \). It is unclear to us the contribution to this effect of the long lived transients discussed in the
The value of these harmonics is shown in fig (4) (note that the scales are different from those in fig (3)). The harmonics show also a maximum near the resonance frequency. Note that the third harmonic is greater than the second. In general, we find the odd harmonics to be larger than the even ones.

IV. CONCLUSIONS.

We have studied the motion of pinned vortices in the limit where the spacing between quasiparticle states within the core is greater than the inverse scattering time and the temperature. In this regime, the conventional analysis of flux flow dissipation ceases to be valid. The dependence of the energy dissipation on the vortex velocity is highly non linear. There is a crossover velocity, above which dissipation can be regarded as “conventional”, and below which dissipation is exponentially suppressed.

The non linear effects dominate the behavior of Re$\rho_{xx}$ and Im$\rho_{xy}$ at low frequencies. There is an asymmetric resonance around $\omega_{res} \approx k_p/\alpha$, where $k_p$ is the pinning constant, $\alpha = h n_s/2$ is the magnitude of the Magnus force ($n_s$ is the condensate density). In terms of a frequency dependent viscosity, we find that $\eta(\omega) \sim 0$ below the resonance, $\eta(\omega_{res}) \sim \min(\eta_0, \alpha)$, where $\eta_0$ is comparable to the Bardeen-Stephen value, and that $\eta(\omega) \rightarrow 0$ as $\omega \rightarrow \infty$.

We have not attempted to average over a distribution of pinning constants. To a first approximation, it is equivalent to an average over frequencies, in a plot like the one shown in fig. (3). The most noticeable effect will be the smoothing of the resonance shown there. We have also not analyzed the interaction between vortices pinned in different regimes. The voltages induced by the vortex motion tend to be higher for those vortices close to to resonance, and it is likely that they will entrain the others. Finally, we do not consider here additional sources of dissipation, like the existence of subgap states in a d-wave superconductor [7]. These effects will be analyzed elsewhere.

The experiments reported so far focus mainly on the temperature dependence of the vortex viscosity and pinning constant [6,7]. Pulse experiments, like those presented in [3,4], may be difficult to interpret, as the exponential suppression of the vortex viscosity at low velocities implies the existence of long lived transients (which may be observed, however, as echo signals). It would be interesting if the non linear effects discussed here can be confirmed experimentally.
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FIGURES

FIG. 1. Trajectories followed by vortices driven at different frequencies. a) \( \omega = 0.05 \). b) \( \omega = 0.15 \). c) \( \omega = 0.5 \). d) \( \omega = 5 \). The “ideal” resonance frequency described in the text is, in these units, \( \omega_{\text{res}} = \frac{k_p}{2\pi \alpha} = 0.159 \). For typical experimental parameters, \( \omega_{\text{res}} \sim 2 \times 10^{11} \) Hz. As mentioned in the text, a reasonable unit of length is \( \sim 40\AA \).

FIG. 2. Time dependence of the vortex velocities for the four cases shown in fig. 1. The unit of velocity is \( v_0 \sim 4 \) Km/s. Full line, \( v_y \). Dashed line \( v_x \). \( T \) is the period.

FIG. 3. Frequency dependence of the vortex resistivities as function of frequency. The lowest harmonic of the resistivity is shown here. Full line, \( \text{Re } \rho \). Dotted line, \( \text{Im } \rho \). Resistivity units are arbitrary.

FIG. 4. Second (upper panels) and third (lower panels) harmonics of the resistivity. The conventions are as in fig. 3. The units are the same as the ones used in fig. 3.
