Sliding motion of a two-dimensional Wigner crystal in a strong magnetic field

Xuejun Zhu, P. B. Littlewood, and A. J. Millis

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(January 1, 2018)

Abstract

We study the sliding state of a two-dimensional Wigner crystal in a strong magnetic field and a random impurity potential. Using a high-velocity perturbation theory, we compute the nonlinear conductivity, various correlation functions, and the interference effects arising in combined AC + DC electric effects, including the Shapiro anomaly and the linear response to an AC field. Disorder is found to induce mainly transverse distortions in the sliding state of the lattice. The Hall resistivity retains its classical value. We find that, within the large velocity perturbation theory, free carriers which affect the longitudinal phonon modes of the Wigner crystal do not change the form of the nonlinear conductivity. We compare the present sliding Wigner crystal in a strong magnetic field to the conventional sliding charge-density wave systems. Our result for the nonlinear conductivity agrees well with the $I - V$ characteristics measured in some experiments at low temperatures or large depinning fields, for the insulating phases near filling factor $\nu = 1/5$. We summarize the available experimental data, and point out the differences...
among them.

PACS: 73.40.Kp, 73.20.Mf, 71.45.Lr
I. INTRODUCTION

The $T = 0$ phase diagram of a two-dimensional (2D) electron gas in a strong magnetic field is very rich. Two of the possible competing phases are the Wigner crystal phase, which is believed to occur at sufficiently strong magnetic field or low density, and the fractionally quantized Hall effect (FQHE) liquid phase, which occurs if the field is not too strong and the filling factor $\nu$ has an odd denominator \[1,2\]. The two-dimensional electron gas has been studied experimentally in modulation doped semiconductor heterojunctions. Currently, the FQHE state is firmly established down to filling factor $\nu = 1/5$ \[3\] for samples with $n$-type dopants and down to $\nu = 1/3$ for samples with $p$-type dopants. For $\nu$ slightly greater than $1/5$, but less than $2/9$, and for $\nu$ smaller than $1/5$, insulating phases have been observed in the cleanest $n$-type samples currently available \[3–7\]. There is also increasing experimental evidence that similar reentrant insulating phases appear in $p$-type samples for $\nu$ slightly greater and for $\nu$ less than $1/3$ \[8\]. Most of the experimental data on the insulating phases in this regime are interpreted as due to the Wigner crystallization \[3–9\].

While the insulating nature of the states around $\nu = 1/5$ has been clearly demonstrated, it remains controversial if the insulating behavior is primarily due to interaction-induced Wigner crystallization or primarily due to disorder. Nonlinearities in the conductivity have been argued by several groups \[4–7\] to imply that the insulating phases are due to the formation of the Wigner crystal which is then pinned by disorder. However, there exist some puzzling discrepancies among experiments under seemingly similar experimental conditions. For example, the apparent depinning field (see Eq. 3) differs by almost three orders of magnitude between two experiments \[4,5\] on samples with similar zero field mobility. In another experiment \[7\], nonlinearity in the conductivity was observed in both of these field ranges; but the nonlinearity at larger fields was attributed to electron heating effects at larger currents. Even within the small field ($\sim 0.2$ mV across samples with typical size $\sim 3$ mm) nonlinear conductivity studies, some significant differences exist among experiments \[4–7\]. We shall discuss these and other related experiments in more detail later in the
paper.

In view of the experimental situation, it is desirable to provide a theoretical framework for describing the properties of a sliding Wigner crystal in a strong magnetic field [10]. The principle theoretical issue is the proper treatment of the disorder which distorts the Wigner crystal during sliding. In this paper, we apply techniques originally developed for studying sliding charge-density wave (CDW) systems [11–13] to the present system. We use an elastic model for pinning and for the interaction of the sliding Wigner crystal with disorder, similar to that used in the Fukuyama-Lee-Rice (FLR) model for CDW’s [11], and the high-velocity perturbation theory [12,13] to calculate various properties of a sliding Wigner crystal in the presence of a strong magnetic field and disorder potential. This approach is expected to be valid as long as the sliding velocity is large, in comparison to disorder effects, but not so large as to invalidate the elastic medium theory. In the context of CDW systems, similar perturbation theory [12,13] was compared to numerical simulations, and was found to be quantitatively reliable for fields greater than $\sim 2$ times the threshold field.

The physics of a Wigner crystal in a strong magnetic field differs that of a conventional CDW because: 1) the presence of a large magnetic field gives rise to a large Hall component of the motion; 2) the motion is two-dimensional whereas it is one-dimensional in a conventional CDW system in arbitrary spatial dimensions, and the related issue of the shear modulus of a Wigner crystal being much smaller than the bulk modulus; 3) the presence of a weakly screened, two-dimensional long-range Coulomb interaction; 4) the order-parameter, i.e., the Fourier-transformed charge density, is non-zero on a two dimensional grid, and does not decay very fast with increasing reciprocal lattice vector $\vec{G}$. The first point does not affect the static properties of the Wigner crystal. Points 2 and 3 have been shown to affect the static properties, and together with point 1, the $k$-dependence of the collective excitations of a Wigner crystal in a significant way [17]. In the sliding regime where we are interested in the dynamic response of the Wigner crystal, they will all affect the results.

Some recent experiments done at relatively high temperatures [7] have also indicated the possible involvement of the thermally excited free carriers in the sliding Wigner crystal.
For example, the differential conductivity continues to be thermally activated above the sliding threshold in Ref. [7], up to a second threshold field, thus raising the possibility that the thermally excited free carriers play an important role in determining the conductivity of the sliding state. We have therefore considered, within the same perturbation theory framework, the effect of coupling to a sliding Wigner crystal a set of free carriers which are phenomenologically characterized by a conductivity tensor. We find that the present perturbation theory is unable to account for the experimental results in Ref. [7] between the two threshold fields. This suggests that the starting point of our theory, i.e., the assumption that we can perturb around a uniformly sliding state, is probably not valid in this regime.

We have also considered the effects of combined AC + DC fields, where interference between the internal “washboard” frequency of the sliding motion and the AC driving frequency can be used to further identify the nature of the sliding state of the insulating phases observed experimentally. These include the Shapiro anomaly in the DC conductivity in the presence of an AC field, and the AC response in the presence of a large DC current. These effects are also studied using the same perturbation theory, carried out to the same order in disorder potential as that for the DC nonlinear conductivity.

It has also been claimed recently that a pinned Wigner crystal in zero external magnetic field exists at Si/SiO$_2$ interfaces [18]. For completeness and convenience of reference, we also give our results for zero magnetic field, although they are similar to those for the flux lattice sliding motion in a type-II superconductor as studied by Schmid and Hauger and by Larkin and Ovchinnicov in Ref. [15]. The main difference here is that for the present Wigner crystal case the long-range Coulomb interaction yields a $k^{1/2}$-dispersive longitudinal phonon mode, and this dispersion could be affected by the presence of free-carriers in ways that are described in [17] and also discussed later in this paper. As we will show, however, the main distortions in a sliding Wigner crystal are transverse; and many properties of a sliding Wigner crystal are not affected by the longitudinal mode spectrum.

The balance of the paper is as follows. In Sec. II, we introduce our model, make some general remarks regarding the Hall resistivity, and summarize recent experiments concern-
ing the Hall resistivity for the insulating phases around $\nu = 1/5$. In Sec. III, we study the nonlinear conductivity of the Wigner crystal in the sliding regime in the absence of free carriers, treating the scattering by impurities as a perturbation. Various correlation functions of the sliding motion are studied in Sec. IV. The effects of the disparity of the bulk modulus and the shear modulus are illustrated. In Sec. V, we consider the effects of free carriers on the sliding dynamics of the Wigner crystal. In Sec. VI, we consider the two AC + DC interference effects mentioned above. Results for the zero magnetic field case are given in Sec. VII. All of our main results for the transport properties of a sliding Wigner crystal are summarized in Sec. VIII. In Sec. IX, we discuss in some detail the current experimental situation and attempt to make contact between our theory and these experiments. Sec. X is the conclusion. Readers who are only interested in the results and the current experimental status may go to Secs. VIII, IX and X directly.

II. EQUATION OF MOTION AND THE HALL RESISTIVITY

In Sec. IIA, we first establish the equation of motion. We remark briefly on its similarities to and differences from the equations of motion of related models. In Sec. IIB, we give results for the Hall resistivity that can be obtained immediately by examination of the equation of motion. The steady state solution to the equation of motion in the absence of random potential scattering is discussed in Sec. IIC.

A. Model and Equation of Motion

We begin by considering the various forces acting on an element of the Wigner crystal at $(\vec{r}, t)$ in two dimensions, which we treat classically as an elastic medium. We denote the two-dimensional displacement of the Wigner crystal at $(\vec{r}, t)$ as $\vec{u}(\vec{r}, t)$, average mass and charge densities as $\rho_m$ and $\rho_c$, lattice constant of the Wigner crystal as $a$, and background dielectric constant $\epsilon$. The following terms enter the equation of motion:

1) inertial force: $\rho_m \ddot{\vec{u}}$;
2) dissipation: $\lambda \rho_m \dot{u}$. $\lambda$ is a phenomenological constant describing damping due to everything other than the disorder potential that is treated explicitly below;

3) Lorentz force: $\rho_m \omega_c \vec{e}_z \times \dot{u}$, where $\omega_c = \rho_c B / \rho_m$ is the cyclotron frequency, $\vec{e}_z$ is the unit vector in the direction of the perpendicular magnetic field;

4) elastic restoring force: $\int D(\vec{r} - \vec{r}') \cdot (\ddot{u}(\vec{r}', t) - \ddot{v}_t) d^2 \vec{r}'$. Here $D$ is the real-space dynamic matrix tensor, and $\ddot{v}_t$ is the uniform component of $\ddot{u}(\vec{r}, t)$ in the sliding state of the Wigner crystal defined as $\ddot{v}_t = \lim_{T \to \infty} \lim_{V \to \infty} \int \frac{dt}{T} \int d\vec{r} V \ddot{u}(\vec{r}, t)$. Upon Fourier transformation, the component transverse to $\vec{k}$ is $D^T(\vec{k}) / \rho_m = c^T k^2$ and the longitudinal component $D^L(\vec{k}) / \rho_m = c^L k$. These are valid in the long wavelength limit, i.e., for $|\ddot{u}(\vec{r}) - \ddot{u}(\vec{r}')|$ much less than $|\vec{r} - \vec{r}'|$. Following Ref. [17], we have: $c^T = \Omega^2 a^2 \alpha$, and the $k$-dependent $c^L = \Omega^2 (2\pi a + (1 + \alpha)a^2 k)$, where $\alpha \sim 0.02$ is the ratio of the shear modulus with the bulk modulus, and $\Omega^2 = \rho_c^2 / \rho_m a \epsilon$ is a characteristic frequency of the Wigner crystal, on the order of the zone-boundary phonon frequency at zero magnetic field. The first term in $c^L$ is due to the long-range Coulomb interaction. Here we view the Wigner crystal as a deformable classical solid characterized by its dynamic matrix $D(\vec{k})$, given by, for example, Bonsall and Maradudin [19]. Quantum mechanical corrections have been studied within the time-dependent Hartree-Fock theory and are found to be on the order of 10% for the long wavelength phonons that we are interested in [20];

5) disorder potential which couples to the $\vec{r}$-dependent charge density $\rho(\vec{r})$: $\rho(\vec{r}) \nabla_r V_{dis}(\vec{r} + \ddot{u}(\vec{r}, t))$. Here $V_{dis}(\vec{r})$ is the disorder potential at $\vec{r}$. Only the correlation function of $V_{dis}$ will enter our results, so we will not specify its form. In cases where $V_{dis}$ is due to the remote dopants at a distance $d$ away from the plane of the 2D electron gas, the Fourier transformation of $V_{dis}(\vec{r})$, $V_{dis}(\vec{q})$ decays as $e^{-qd}$ from simple electrostatic considerations;

6) and finally, the external driving force on the total charge density $\rho_c \vec{E}^{ext}$.

Combining the six terms, we obtain the equation of motion in real space:
\[ \rho_m \ddot{u} + \lambda \rho_m \dot{u} + \rho_m \omega_c \vec{e}_z \times \dot{u} + \int \mathbf{D}(\vec{r} - \vec{r}') \cdot \vec{u}(\vec{r}', t)d^2\vec{r}' = \rho_c \vec{E}^{\text{ext}} - \rho(\vec{r}) \nabla \varphi V_{\text{dis}}(\vec{r} + \vec{u}(\vec{r}, t)). \tag{1} \]

In a perfectly ordered Wigner crystal, the charge density is determined by its Fourier components \( \rho(\vec{G}) \) at the reciprocal lattice vectors \( \vec{G} \), via, \( \rho(\vec{r}) = \sum_{\vec{G}} \rho(\vec{G}) e^{i\vec{G} \cdot \vec{r}} \). In the systems of interest, the magnetic field is so strong that the magnetic length \( l_B = \sqrt{\hbar c/eH} \) is smaller than the lattice constant, and \( \rho(\vec{G}) \) decays as \( \rho(\vec{G}) = e^{-G^2 l_B^2/2} \), so many reciprocal lattice vectors will be important, in contrast to the CDW case in which only the lowest \( \vec{G} \) is important.

For later reference, we write down here also the equation of motion in the FLR model for a CDW. Apart from the absence of a magnetic field, the most important difference is that because of the quasi-one dimensional crystal structure of CDW systems, the electrons are free to move only in one direction, which we take to be \( z \). Therefore:

\[ \rho_m \ddot{u}_z + \lambda \rho_m \dot{u}_z + \int \mathbf{D}_{zz}(\vec{r} - \vec{r}') u_z(\vec{r}', t)d\vec{r}' = \rho_c E_z^{\text{ext}} - \rho(\vec{r}) \frac{\partial}{\partial z} V_{\text{dis}}(\vec{r} + \vec{e}_z u_z(\vec{r}, t)). \tag{2} \]

Dimensionality, which is crucial in determining the functional form of the nonlinear conductivity of a sliding CDW \cite{12}, enters through the elastic term in the above equation.

In the absence of the external potential, very weak disorder can pin the overall phase of the charge-density wave \cite{16}. In this case, the phase fluctuations diverge beyond a length \( \xi^T \) which defines a typical domain size in a pinned CDW. Similar considerations apply here to the Wigner crystal case as well \cite{17}. As a result of pinning, the linear response conductivity vanishes. But when \( E^{\text{ext}} \) is greater than some threshold field \( E_{Th} \), the pinned Wigner crystal will begin to slide. The pinning-depinning transition between the static state and the sliding state in a CDW system has been argued to be second order, exhibiting the critical phenomena \cite{21}.

In the weak pinning limit, following the Fukuyama-Lee-Rice argument \cite{16,17}, one can relate \( E_{Th} \) to the static correlation length \( \xi^T \) of the pinned Wigner crystal as follows. At the threshold field \( E_{Th} \), the total force acting on a pinned domain of Wigner crystal of linear
size $\xi^T$ is $E_{Th}\rho_c\pi(\xi^T)^2$. This should be balanced by the total elastic force acting on the same domain. Since by definition, the displacement over the length $\xi^T$ is $\sim a$, on the order of the lattice constant, then the total elastic restoring force must be $\rho_m\Omega^2a^2\alpha \times a$. Therefore,

$$E_{Th} \approx \frac{\rho_m\Omega^2a^2\alpha}{\rho_c(\xi^T)^2\pi/a}.$$  \hspace{1cm} (3)

We can also define a longitudinal correlation length $\xi^L$ beyond which the Coulomb interaction energy is smaller than the pinning energy: $c^L/\xi^L = c^T/(\xi^T)^2$, or $\xi^L = 2\pi(\xi^T)^2/(a\alpha)$. Thus there are two length scales involved in the present problem, related to the shear modulus and the long range Coulomb interaction respectively. In the case where the latter is screened by free carriers, $\xi^L$ will be larger than $\xi^T$ by at least a factor of $1/\sqrt{\alpha}$.

Various aspects of the FLR model have been studied in the $B = 0$ charge-density wave context. The linear response of the pinned state for $B \neq 0$ has also been determined. In Ref. [17], the large difference between the bulk modulus and the shear modulus was taken into account. One consequence is the occurrence of a $k^{1/2}$-dispersing mode for $k$ lying between $1/\xi^L$ and $1/\xi^T$. In the absence of disorder, or for $k$ greater than $1/\xi^T$ in a statically disordered lattice, there exists a $k^{3/2}$-dispersing mode which is characteristic of a 2D Wigner crystal in a strong magnetic field; it results from the mixing due to the magnetic field of the transverse and the longitudinal phonon modes which disperse as $k$ and $k^{1/2}$ respectively when there is no magnetic field, as one can see from the small $k$-behavior of the dynamic matrix given earlier. The present study focuses on the sliding regime, treating the disorder potential as a perturbation relative to the external driving force.

We note that although the present problem bears some resemblance to the sliding motion of the vortex lattice in a type-II superconductor, in that there is a Lorentz force, and that the shear modulus is much smaller than the bulk modulus, there is a very important difference. The sliding motion of the vortex lattice is driven by the Lorentz force due to the superconducting current. The moving vortex lattice then in turn induces a voltage drop in the direction of the superconducting current. In the present problem, the depinning electric
force is supplied externally and the magnetic field enters explicitly the equation of motion.

The present model may be used to study the sliding of a 2D Wigner crystal with no external magnetic field by simply setting the Lorentz force to zero. Such a system has been shown to exist for electrons on a helium film [23], and more recently, has been argued for doped electrons at Si/SiO$_2$ interfaces [18]. We will also give our results for cases where there is no external magnetic field. When expressed in terms of the shear modulus, they are basically the same as those for a sliding vortex lattice [15]. As we mentioned already, the difference in the longitudinal phonon spectrum between the two systems does not appear in the large velocity limit.

**B. The Hall Resistivity**

Before we embark upon a detailed study of Eq. 1, it is worth noting that one can obtain the Hall resistivity $\rho_{xy}$ immediately from Eq. 1 for a Wigner crystal in the sliding state. With a constant current density $j_x$ in the $x$-direction, the Hall resistivity relates $E_{y}^{\text{ext}}$ to $j_x$ via $E_{y}^{\text{ext}} = \rho_{yx}j_x$. We disorder-average the Fourier transformed Eq. 1 under the steady state condition $\dot{u}_x \neq 0$. Assuming isotropy, we obtain (using $j_x = \rho_c \omega_c \dot{u}_x$):

$$\rho_c E_{y}^{\text{ext}} = \rho_m \omega_c \dot{u}_x = \rho_c \left( \frac{B}{\rho_c} \right) j_x,$$

so $\rho_{yx} = B/\rho_c e$. This argument is independent of the form of the disorder potential (aside from isotropy), and requires only that the current be transported by a $k = 0$ mode of the crystal. We note however that these arguments are strictly only valid at $T = 0$, where there are no thermally activated excitations that carry current. Effects of nonzero temperatures on the Wigner crystal state below the sliding threshold are not well understood, and in general will depend on the nature of the possible charged thermal excitations in the system.

The issue of Hall resistivity for the insulating phases in the FQHE regime was raised in how to best characterize the nature of the insulating phase [24]. Kivelson, Zhang, and Lee [24] have suggested that it be better described as a “Hall insulator”, which they characterize
by a diverging longitudinal resistivity and a constant Hall resistivity at zero temperature. A
more concrete description of the “Hall insulator” in the FQHE regime has not been given.
Several authors have recently argued that an Anderson insulator is in fact also a “Hall
insulator” with Hall resistivity given approximately by \( B/\rho_c e \), \( \rho_c \) being the total localized
carrier density [24,25].

Experimental results for \( \rho_{xy} \) in the insulating phases have now become available [26].
They are obtained by measuring the differential Hall resistance at a constant current. The
mixture of \( \rho_{xx} \) into \( \rho_{xy} \), which can be rather large at low temperatures, can be subtracted
by an average over \( \rho_{xy}(\vec{B}) \) and \( \rho_{xy}(-\vec{B}) \) [26]. The experimental findings can be summarized
as follows:

1). At low temperatures and well below the sliding threshold, the Hall resistivity is given
by \( \rho_{yx} \sim B/\rho_c \), where \( \rho_c \) is the total electron density that was introduced into the sample
during doping, and therefore \( \rho_{xy} \) is insensitive to temperature, in sharp contrast to \( \rho_{xx} \) in the
same regime which is often thermally activated. When it was measured, the low-frequency
response was always found to be resistive, with very small out-of-phase component in these
systems [26].

2). The Hall resistivity is found to have little temperature or field dependence above and
below the so-called critical temperature which is marked by the absence of an apparent
depinning transition. The longitudinal resistivity changes behavior: it may or may not show
thermally-activated temperature dependence; also it may or may not be ohmic [26].

If the applied field is above the sliding threshold for the pinned Wigner crystal, our
argument above can be used to argue for a normal Hall resistivity. It is not clear at this
point if the experimental data below the apparent sliding threshold could be understood
within the Wigner crystal picture.

It is perhaps worth commenting that at \( T = 0 \) the Hall resistivity of a pinned Wigner
crystal within the linear response regime, \( i.e., \) below sliding threshold can also be shown to
be simply \( B/\rho_c \), following earlier work by Fukuyama and Lee on the magneto-conductivity
of a pinned CDW [22]. This conclusion can be generalized at a finite frequency below sliding
threshold, by repeating the argument given at the beginning of this subsection leading to Eq. 4, thus avoiding the need to resort to the particular form of the pinned phonon spectrum as was done in [22]. These arguments, as well as those for Anderson insulators [24], are made at $T = 0$ where there are no thermal excitations in the insulator. However, this mechanism necessarily requires that the conduction be carried by the polarization current excited at a finite frequency, implying a capacitive response. It then follows that the measured current and the voltage must be out of phase by $90^\circ$. This scenario is disfavored by the current experiments [26]. More careful measurements geared specifically toward this issue would certainly be very useful, however.

A final comment is in order about the distinction between resistivity and conductivity in a disordered system, when the response functions become non-local and local field effects are important. In particular, the conductivity tensor is defined by $\vec{J}(\vec{q}) = \sum_{\vec{q}'} \vec{\sigma}(\vec{q}, \vec{q}') \cdot \vec{E}(\vec{q}')$; $\sigma$ becomes diagonal only after averaging over disorder. The resistivity is the inverse of the conductivity matrix in $\vec{q}$-space. The general arguments we have given in this section relate to $\rho_{xy}$ defined as $\lim_{\vec{q}, \vec{q}' \to 0} \sigma_{xy}^{-1}(\vec{q}, \vec{q}') \neq [\lim_{\vec{q}, \vec{q}' \to 0} \sigma_{xy}(\vec{q}, \vec{q}')]^{-1}$. This is the resistivity measured when the current flows entirely in the uniform mode, and may not correspond to the simple inversion of a measurement of the conductance.

C. Steady State Solution with No Disorder

For $\nabla r V_{\text{dis}} = 0$, the equation of motion Eq. 4, can be solved trivially. In steady state:

$$\frac{\rho_c}{\rho_m} E_{\text{ext}} = \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \vec{v}, \quad \text{and} \quad \vec{u}(\vec{r}, t) = \vec{v} t,$$

with $v = \frac{\rho_c}{\rho_m \sqrt{\lambda^2 + \omega_c^2}} E_{\text{ext}}$. The resistivity tensor $\rho$ is simply:

$$\rho = \frac{\rho_m}{\rho_c^2} \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix},$$

and the conductivity tensor $\sigma$ is:

$$\sigma = \frac{\rho_c^2}{\rho_m (\lambda^2 + \omega_c^2)} \begin{pmatrix} \lambda & \omega_c \\ -\omega_c & \lambda \end{pmatrix}.$$
If there is an additional physical dissipative process which changes $\lambda$ by a small amount $\delta \lambda$, we will have for the components of the resistivity tensor

$$
\frac{\delta \rho_{xx}}{\delta \lambda} = \frac{\rho_m}{\rho_c^2},
$$

(8)

and,

$$
\frac{\delta \rho_{xy}}{\delta \lambda} = 0;
$$

(9)

and for the conductivity tensor

$$
\frac{\delta \sigma_{xx}}{\delta \lambda} = \frac{\rho_c^2}{\rho_m} \frac{\omega_c^2 - \lambda^2}{(\omega_c^2 + \lambda^2)^2},
$$

(10)

and,

$$
\frac{\delta \sigma_{xy}}{\delta \lambda} = \frac{\rho_c^2}{\rho_m} \frac{-2\omega_c \lambda}{(\omega_c^2 + \lambda^2)^2}.
$$

(11)

One sees that for $\lambda < \omega_c$, $\sigma_{xx}$ increases with $\lambda$, in contrast to the zero-magnetic-field case. In addition, $|\delta \sigma_{xx}| \gg |\delta \sigma_{xy}|$ while $|\sigma_{xx}| \ll |\sigma_{xy}|$ for $\lambda \ll \omega_c$.

And finally,

$$
\frac{\delta v}{v} = -\frac{\lambda \delta \lambda}{\lambda^2 + \omega_c^2}.
$$

(12)

If one views the disorder scattering as an additional source of dissipation which renormalizes $\lambda$, many of these observations can be carried over. The problem therefore is to find how disorder scattering modifies $\lambda$. To this end, a perturbative analysis is carried out in Sec. III.

### III. PERTURBATION THEORY: NONLINEAR CONDUCTIVITY

We give here the detailed results for the nonlinear conductivity of a sliding Wigner crystal in the presence of a strong perpendicular magnetic field with disorder scattering. Results here are also used in the next section for studying the correlation functions of the sliding Wigner crystal. The dependence of the Hall angle on the sliding current is calculated. In
Sec. IIIA, we introduce the response function for the present system, and we outline the perturbation theory we use. In Sec. IIIB, we solve for the velocity to the second order in disorder scattering, thereby obtaining the nonlinear conductivity. We show in Sec. IIIC that the large-velocity perturbation theory also gives a measure of the depinning field. The Hall angle is studied in Sec. IIID.

A. The Perturbation Series and the Green’s Function

We shall in this section restrict ourselves to a DC electric field where most of the experiments are carried out. The extension to the more general DC + AC field case will be discussed in Sec. VI. It is convenient in the present problem to write the DC external field in terms of a velocity \( \vec{v}_0 \) as:

\[
\frac{\rho_c}{\rho_m} \vec{E}_{\text{ext}} = \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \vec{v}_0, \tag{13}
\]

and the displacement as:

\[
\vec{u}(\vec{r}, t) = \vec{v}_0 t + \vec{x}(\vec{r}, t), \tag{14}
\]

where the first term would be the correct solution to Eq. 1 for the field in Eq. 13 if there was no disorder (see Eqs. 6 and 7). The last term, \( \vec{x} \), represents the additional displacement due to disorder scattering. By substituting Eq. 14 into Eq. 1 and using Eq. 13, we find that \( \vec{x}(\vec{r}, t) \) satisfies the following equation of motion:

\[
\rho_m \ddot{\vec{x}} + \rho_m \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \dot{\vec{x}} + \int D(\vec{r} - \vec{r}') \cdot \ddot{\vec{x}}(\vec{r}', t) d\vec{r}' = -\rho(\vec{r}) \nabla \rho V_{\text{dis}}(\vec{r} + \vec{u}(\vec{r}, t)). \tag{15}
\]

Formally, the solution of the above equation can be written in terms of tensor Green’s function \( \mathbf{G} \) as:

\[
\vec{x}(\vec{r}, t) = \int d\vec{r}' dt' \mathbf{G}(\vec{r} - \vec{r}', t - t') \cdot \left[ -\rho(\vec{r}') \nabla \rho V_{\text{dis}}(\vec{r}' + \vec{u}(\vec{r}', t')) \right], \tag{16}
\]

where \( \mathbf{G} \) is the solution of:
\[ \rho_m \ddot{\mathbf{G}}(\mathbf{r}, t) + \rho_m \left( \begin{array}{cc} \lambda & -\omega_c \\ \omega_c & \lambda \end{array} \right) \cdot \dot{\mathbf{G}}(\mathbf{r}, t) + \int \mathbf{D}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{G}(\mathbf{r}', t) \, d\mathbf{r}' = \delta(t) \delta(\mathbf{r}) \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right). \tag{17} \]

Fourier transforming Eq. (17), using the long-wavelength limit expression for \( \mathbf{D} \) which we gave in Sec. IIA, and assuming \( \omega \ll \lambda \), we have:

\[ \mathbf{G}^{-1}(\mathbf{k}, \omega) / \rho_m = c^L k / k^2 + c^T k^2 (1 - k^2 / k^2) - i \omega \left( \begin{array}{cc} \lambda & -\omega_c \\ \omega_c & \lambda \end{array} \right), \tag{18} \]

the inverse of which can be taken easily:

\[ \mathbf{G}(\mathbf{k}, \omega) = \frac{1}{P(\mathbf{k}, \omega)} \left[ c^L k (1 - k^2 / k^2) + c^T k^2 k^2 - i \omega \left( \begin{array}{cc} \lambda & \omega_c \\ -\omega_c & \lambda \end{array} \right) \right]. \tag{19} \]

The poles in \( \mathbf{G} \) are given by the zeroes of \( P(\mathbf{k}, \omega) \) which is:

\[ P(\mathbf{k}, \omega) = 2\pi \Omega^4 \alpha \rho_m \left[ (ka)^3 - \kappa_\omega^3 - i(ka)\kappa_\lambda^2 \text{sgn}(\omega) \right]. \tag{20} \]

In the above equation, we have defined two dimensionless quantities:

\[ \kappa_\omega^3 = \frac{\omega^2(\omega_c^2 + \lambda^2)}{2\pi \Omega^4 \alpha}, \]

\[ \kappa_\lambda^2 = \frac{\omega \lambda}{\Omega^2 \alpha}. \tag{21} \]

We have also ignored the \( \alpha(ka)^2 \) term whenever it appears together with \( (ka) \) term since \( \alpha \) is small and the relevant \( (ka) \)'s are also small. \( \kappa_\omega / a \) is the wavevector at which the \( k^{3/2} \)-mode is at frequency \( \omega \), and \( \kappa_\lambda \) measures the strength of dissipation at the frequency \( \omega \). Their physical meaning for the sliding state will become more clear later in the paper.

Eq. (14) can be used to generate a perturbation series for \( \tilde{x}(\mathbf{r}, t) \) in terms of the disorder potential: to first order, \( \tilde{x}_1 \) is given by Eq. (10) with \( \tilde{u}(\mathbf{r}', t') \) in \( V_{\text{dis}} \) on the right-hand-side replaced by \( \tilde{v}t' \); and the second correction \( \tilde{x}_2 \) is given by replacing \( V_{\text{dis}}(\mathbf{r} + \tilde{u}(\mathbf{r}', t')) \) by \( \tilde{x}_1(\mathbf{r}', t') \cdot \nabla \nabla V_{\text{dis}}(\mathbf{r}' + \tilde{v}t') \); etc.

B. Nonlinear Conductivity

Following the last subsection, we obtain for the displacement to first order in the disorder potential \( V_{\text{dis}} \):

15
\[ \bar{x}(\vec{r}, t) = \int \frac{d^2 \vec{k}}{(2\pi)^2} \frac{d\omega}{2\pi} \frac{d^2 \vec{q}}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r} - i\omega t} \tilde{F}_1(\vec{k}, \vec{q}, \omega), \]  

(22)

where \( \tilde{F}_1(\vec{k}, \vec{q}, \omega) \) is given by

\[ \tilde{F}_1(\vec{k}, \vec{q}, \omega) = -\rho(\vec{k} - \vec{q}) \left[ \mathbf{G}(\vec{k}, \omega) \cdot (i\vec{q}) \right] V_{\text{dis}}(\vec{q}) 2\pi \delta(\omega + \vec{q} \cdot \vec{v}); \]  

(23)

and for the velocity to second order in \( V_{\text{dis}} \):

\[ \rho_m \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \langle \dot{\vec{x}} \rangle = \sum_{\vec{G}} |\rho(\vec{G})|^2 \int \frac{d^2 \vec{k}}{(2\pi)^2} \tilde{F}_2(\vec{k}, \vec{G}), \]  

(24)

where \( \tilde{F}_2(\vec{k}, \vec{G}) \) is:

\[ \tilde{F}_2(\vec{k}) = \vec{k} \Gamma(\vec{k}) \left[ \vec{k} \cdot i\mathbf{G}(-\vec{k} + \vec{G}, \vec{k} \cdot \vec{v}) \cdot \vec{k} \right]. \]  

(25)

In Eq. 24, we have averaged over disorder and integrated over the \( \delta \)-function. \( V_{\text{dis}}(\vec{q}) = \int d^2 \vec{r} e^{-i\vec{q} \cdot \vec{r}} V_{\text{dis}}(\vec{r}) \), and \( \Gamma(\vec{k}) \) is the disorder correlation function: \( < V_{\text{dis}}(\vec{k}) V_{\text{dis}}(\vec{q}) > = \delta(\vec{k} + \vec{q}) \Gamma(\vec{k}) \), where \( < ... > \) indicates averaging over disorder. We have also used the fact that for an undistorted Wigner crystal:

\[ \rho(\vec{q}) = (2\pi)^2 \sum_{\vec{G}} \rho(\vec{G}) \delta(\vec{q} - \vec{G}), \]  

(26)

where \( \vec{G} \)'s are reciprocal lattice vectors.

Using Eqs. 23 and 24 in Eqs. 22 and 24, we have:

\[ \bar{x}(\vec{r}, t) = \int \frac{d^2 \vec{k}}{(2\pi)^2} \frac{d^2 \vec{q}}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r} + i(\vec{q} \cdot \vec{v})t} \left[ -\rho(\vec{k} - \vec{q}) \left[ \mathbf{G}(\vec{k}, -\vec{q} \cdot \vec{v}) \right] V_{\text{dis}}(\vec{q}) \right], \]  

(27)

and,

\[ \rho_m \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \langle \dot{\vec{x}} \rangle = \sum_{\vec{G}} |\rho(\vec{G})|^2 \int \frac{d^2 \vec{q}}{(2\pi)^2} \vec{q} \Gamma(\vec{q}) \left[ \vec{q} \cdot (-Im \mathbf{G}(-\vec{q} + \vec{G}, \vec{q} \cdot \vec{v})) \cdot \vec{q} \right]. \]  

(28)

In both Eq. 27 and Eq. 28, the integration over \( \vec{k} \) is over the first Brillouin zone, and that over \( \vec{q} \) is over the whole reciprocal lattice space. Eq. 27 is used later for studying various correlation functions in a sliding Wigner crystal. We shall now focus on Eq. 28.
which gives the lowest order correction to the conductivity. It is most easily studied using the representation of $G$ in Eq. [19].

Before we proceed further, we should mention that an alternative view of the same perturbation theory is to ask: In order to sustain the current flow $\rho_c \vec{v}$ in the presence of $V_{\text{dis}}$, what additional external field one would need above that given by Eq. [13]. The answer is that to second order in disorder potential the additional field $\vec{E}^p$ is given by:

$$\rho_c \vec{E}^p = \sum_G |\rho(\vec{G})|^2 \int \frac{d^2 q}{(2\pi)^2} \vec{q} \Gamma(q) \left[ \vec{q} \cdot Im G(-\vec{q} + \vec{G}, \vec{q} \cdot \vec{v}) \cdot \vec{q} \right].$$  \hspace{1cm} (29)

The integral in Eq. [28] or Eq. [29] is dominated by two infrared terms where the elastic medium theory is expected to be valid. They are respectively related to $\kappa_\omega$ and $\kappa_\lambda$ in Eq. [20]. For the convenience of the reader, we rewrite them here at $\omega = \omega_v = |\vec{G} \cdot \vec{v}|$:

$$\kappa_\omega^3 = \frac{\omega_v^2(\omega_v^2 + \lambda^2)}{2\pi \Omega^4 \alpha},$$

$$\kappa_\lambda^2 = \frac{\omega_v \lambda}{\Omega^2 \alpha}. \hspace{1cm} (30)$$

We have suppressed the dependence of $\omega_v$, $\kappa_\omega$ and $\kappa_\lambda$ on $\vec{G}$ for ease of writing. Both $\kappa_\omega$ and $\kappa_\lambda$ must be much less than unity in order for the elastic medium theory to be valid, and their ratio $\kappa_\omega/\kappa_\lambda \sim (\omega_v^4 \alpha/4\pi^2 \lambda^3 \Omega^2)^{1/6}$ depends on the sliding velocity $\vec{v}$ through $\omega_v$. The dependence is not very strong, $\sim \nu^{1/6}$.

The result of the integral in Eq. [28] depends on the relative size of $\kappa_\omega$ versus $\kappa_\lambda$. We find that in general:

$$\left( \begin{array}{cc} \lambda & -\omega_v \\ -\omega_v & \lambda \end{array} \right) \cdot \langle \vec{x} \rangle = -\alpha_0 \frac{1}{\rho_m} \sum_G A(\vec{G}) \vec{G} \text{sgn}(\vec{G} \cdot \vec{v})$$

$$= -\alpha_{\text{dis}} \frac{\vec{v}}{\nu}. \hspace{1cm} (31)$$

where

$$A(\vec{G}) = \frac{1}{4\pi \rho_m c^T} |\rho(\vec{G})|^2 \Gamma(\vec{G}) |\vec{G}|^2. \hspace{1cm} (32)$$

The constant $\alpha_0$ in Eq. [31] is $\pi/3$ if $\kappa_\omega \gg \kappa_\lambda$ and is $\pi/4$ if $\kappa_\omega \ll \kappa_\lambda$, and changes monotonically between these limits. $\alpha_{\text{dis}}$ is uncertain up to a numerical prefactor of order unity that
depends on the direction of $\vec{v}$ relative to the lattice orientation. The relative stability of various sliding directions was discussed in [15] in the sliding vortex lattice context from a minimum-entropy-generation argument. In this work, we simply assume that the Wigner crystal is sliding along one of the stable configurations which may be defined with a vector $\vec{G}$.

Therefore, if we use the alternative approach given by Eq. 29, we find that for a given average sliding velocity $\vec{v}$, the total applied field $\vec{E}^{ext}$ must be:

$$\vec{E}^{ext} = \frac{\rho_m}{\rho_c} \left[ \begin{array}{c} \lambda \\ -\omega_c \\ \omega_c \\ \lambda \end{array} \right] \cdot \vec{v} + \alpha_{dis} \frac{\vec{v}}{v^2}.$$  (33)

It is important to note that the right-hand-side of Eq. 31 does not depend on the magnitude of $\vec{v}$, only on its direction. This feature is common to all the 2D systems studied within the second order Born approximation, and has been noted in the CDW context in, e.g., Ref. [12]. Eq. 31 also shows that the net effect of disorder is to increase the $\lambda$ in the equation of motion by an amount of $\alpha_{dis}/v$.

It is straightforward to obtain the total velocity $\vec{v} + \langle \dot{\vec{x}} \rangle$, for a given external field $\vec{E}^{ext}$ in Eq. 13. The resultant conductivity tensor has components given by:

$$\sigma_{xx} = \frac{\rho_c^2}{\rho_m} \frac{1}{\lambda^2 + \omega_c^2} \left[ \lambda + \alpha_{dis} \frac{\omega_c^2 - \lambda^2}{\omega_c^2 + \lambda^2} \right],$$  (34)

and

$$\sigma_{xy} = -\sigma_{yx} = \frac{\rho_c^2}{\rho_m} \frac{1}{\lambda^2 + \omega_c^2} \left[ \omega_c - \frac{\alpha_{dis}}{v} \frac{2\omega_c \lambda}{\omega_c^2 + \lambda^2} \right].$$  (35)

Or equivalently in terms of the resistivity to order $\alpha_{dis}$:

$$\rho_{xx} = \frac{\lambda \rho_m}{\rho_c^2} \left[ 1 + \alpha_{dis}/(v\lambda) \right],$$  (36)

and

$$\rho_{xy} = \frac{\omega_c \rho_m}{\rho_c^2} = B/\rho_c.$$  (37)

Notice that $\rho_{xy}$ is not modified to order $\alpha_{dis}$, as was expected from the general analysis given earlier. The above equations are the same as Eqs. 8 to 11 if $\delta \lambda = \alpha_{dis}/v$. 

18
The $v$ appearing in Eq. 31 is the magnitude of the average sliding velocity. If one does not require self-consistency in the solution for the sliding velocity in the presence of disorder potential, one then must take $\vec{v} = \vec{v}_0$ from Eq. 13. However, Eq. 31 can also be solved self-consistently for the velocity, giving a self-consistent $\vec{v}$. These two procedures give the same result to order $\alpha_{\text{dis}}$, but differ in the regime where the external field is not much larger than the sliding threshold. The alternative approach given by Eq. 29 starts off with the self-consistent velocity, and thus automatically gives results that are self-consistent, and is in fact identical to what one obtains by solving Eq. 31 self-consistently. However at fields not too far from the threshold field, \textit{a priori}, it is not clear which one gives the better solution. We shall come back to this when we discuss the Hall angle, and give more details for results at fields not too larger than the threshold field where these two treatments differ. Our results for the conductivity and resistivity above were given to order $\alpha_{\text{dis}}$, at which level the self-consistent theory and the non-self-consistent theory agree.

Our results for the conductivity and resistivity can be used to compare with experiments. In Fig. 1, we show the calculated $I - V$ characteristics of a sliding Wigner crystal in the presence of disorder scattering. We assume a Hall bar geometry of measurement, \textit{i.e.}, we assume a fixed $I$, and plot the longitudinal ($V_L$) and Hall ($V_H$) voltages as a function of $I$. In Fig. 2, the differential resistivity is depicted as a function of the applied voltage, also in the Hall bar geometry. Our results are not expected to be reliable close to depinning threshold. In particular, the exponent of the $I - V$ curve given by the present perturbation theory is incorrect near threshold. The dotted portions of the lines are meant to convey the approximate nature of our results in this regime. We shall compare these with the experimental findings \cite{4,7} later in this paper.

C. Relation to the Static Depinning Field

We here show that the quantity $\rho_m \alpha_{\text{dis}}/\rho_c$ appeared in the last subsection is also a measure of the threshold field which has been defined in Eq. 2 in terms of properties of the
static Wigner crystal. The static $E_{Th}$, from Eq. 2, is [17]:

$$E_{Th} \sim \frac{\rho_m c^T a}{\rho_c \xi_T^2},$$

$$\xi_T \sim \frac{\rho_m c^T a^2}{\sqrt{n_i V}}, \quad (38)$$

where $n_i$ and $V = \langle V^2 \rangle^{1/2}$ are the average impurity density and strength. Combining Eqs. 38 and putting $c^T = \alpha \Omega^2 a^2$, we have

$$E_{Th} \sim \frac{n_i V^2 / \rho_c a^3}{\rho_m \alpha \Omega^2 a^2}. \quad (39)$$

We may also estimate $E_{Th}$ as the field at which the second-order correction to velocity becomes comparable to the zeroth-order term. From Eq. 31, we estimate:

$$E_{Th} \sim \frac{\alpha_{\text{dis}} \rho_m}{\rho_c} \sum_{\vec{G}} \frac{|\rho(\vec{G})|^2 \Gamma(\vec{G})/\rho_c a^3}{\rho_m \alpha \Omega^2 a^2}. \quad (40)$$

Indeed the above two expressions are equivalent. Therefore the high-velocity perturbation theory in fact gives an estimate for the threshold field which can be used to estimate other quantities such as the correlation length as was done in Ref. [17]. Using this, one may also relate the quantity $A(\vec{G})$ defined earlier to the depinning threshold field. We then have:

$$\frac{2}{3} \alpha_0 \sum_{\vec{G}} |\vec{G}| A(|\vec{G}|) \sim \rho_c E_{Th}. \quad (41)$$

D. Dependence of Hall Angle on Sliding Current

As we mentioned already, the relationship between the external field and the total sliding current depends on whether one requires self-consistency in the velocity for fields not too large compared to the depinning field.

In the case where we do require self-consistency in Eq. 31, i.e., the velocity that we use on its right-hand-side is not $\vec{v}_0$ in Eq. 13, but the self-consistent $\vec{v}$, we find:

$$v = -\alpha_{\text{dis}} \lambda + \sqrt{\alpha_{\text{dis}}^2 \lambda^2 + (\omega_c^2 + \lambda^2)(\rho_c^2 E^2 / \rho_m^3 - \alpha_{\text{dis}}^2)} \frac{\omega_c^2 + \lambda^2}{\omega_c^2 + \lambda^2}. \quad (42)$$
For $E \gg \alpha_{\text{dis}} \rho_m / \rho_c$, it reduces to $\rho_c E / (\rho_m \sqrt{\omega_c^2 + \lambda^2})$; and for $E$ only slightly greater than $\alpha_{\text{dis}} \rho_m / \rho_c$, it is $(\rho_c E / \rho_m - \alpha_{\text{dis}})/\lambda$. The Hall angle at an arbitrary velocity $v$ is:

$$
\tan \theta_H = \frac{\omega_c v}{\lambda v + \rho_m \alpha_{\text{dis}} / \rho_c}.
$$

(43)

It becomes $45^\circ$, \textit{i.e.}, the field along the current becomes equal to the field perpendicular to it, when:

$$
\lambda + \alpha_{\text{dis}} / v = \omega_c.
$$

(44)

Putting in the velocity in Eq. (12), substituting $\alpha_{\text{dis}}$ with $\rho_c E_{Th} / \rho_m$ as we argued in the previous subsection, and solving for $E$, we find:

$$
\frac{E_{45^\circ}}{E_{Th}} = \sqrt{2} \frac{\omega_c}{\omega_c - \lambda}.
$$

(45)

For $E$ less than that given in Eq. (13), the Hall angle is smaller than $45^\circ$ and the field and the current are largely parallel. It becomes greater than $45^\circ$ for greater $E$. For $\omega_c$ less than $45^\circ$, the Hall angle can never reach $45^\circ$.

The results are different if one does not require self-consistency in the velocity, \textit{i.e.}, if one uses $\vec{v}_0$ from Eq. (13) in the right-hand-side of Eq. (31). In this case, the velocity is related to the external field as:

$$
v = \sqrt{\frac{E^2 + E_{Th}^2 - 2EE_{Th} \lambda / \sqrt{\lambda^2 + \omega_c^2}}{\lambda^2 + \omega_c^2}},
$$

(46)

where we have used $E_{Th}$ for $\rho_m \alpha_{dis} / \rho_c$. The large velocity limit remains the same, but in the limit of $E$ only slightly greater than $E_{Th}$, it becomes,

$$
v = E_{Th} \sqrt{\frac{2(1 - \lambda / \sqrt{\lambda^2 + \omega_c^2})}{\lambda^2 + \omega_c^2}}.
$$

(47)

Now the Hall angle is:

$$
\tan \theta_H = \frac{\omega_c E - 2 \lambda \omega_c E_{Th} / \sqrt{\lambda^2 + \omega_c^2}}{\lambda E - (\lambda^2 - \omega_c^2) E_{Th} / \sqrt{\lambda^2 + \omega_c^2}}.
$$

(48)
It becomes 45° when the total velocity is:

\[ v_{45°} = \sqrt{2} \frac{\omega_c E_{Th}}{\sqrt{\omega_c - \lambda} \sqrt{\lambda^2 + \omega_c^2}}. \]  

(49)

We should add that these results must be treated with caution at fields just above the threshold. Both approaches given above are extrapolations of the perturbation theory to a regime where perturbative corrections are no longer small compared to the unperturbed term. However, if we identify \( E_{Th} = \alpha_{dis} \rho_m / \rho_c \), then the self-consistent solution is more sensible for applied fields not much greater than \( E_{Th} = \alpha_{dis} \rho_m / \rho_c \), in that it gives a sliding velocity \( v \propto (E - E_{Th}) \). Although the exponent relating the velocity to the field is incorrect, \( v \) does approach zero at the depinning threshold field \( E_{Th} \).

**IV. Correlation Functions and Local Strain**

To better understand the state of a sliding Wigner crystal, we consider in this section some correlation functions. In charge-density waves, examination of the strain has proven useful in understanding the qualitative features of the Fukuyama-Lee-Rice model [14] and determining the limit of validity of the elastic model. We use the perturbation theoretical analysis in Sec. III to study the correlation functions of the magnetically-induced Wigner crystal under the sliding condition. We show our results for the velocity-velocity correlation function in Sec. IVA. A more detailed study of the correlation length in the sliding state is given in Sec. IVB. The results for the strain are given in Sec. IVC.

**A. Correlation Functions**

In the sliding state, only the fluctuating parts of the displacement and velocity need to be considered in the correlation functions. We begin with the solution to Eq. [13] for the displacement in the DC limit:

\[ \vec{x} (\vec{k}, t) = \int \frac{d^2 \vec{q}}{(2\pi)^2} e^{i\vec{q} \cdot \vec{v} t} \rho (\vec{k} - \vec{q}) \mathbf{G} (\vec{k}, -\vec{q}, -\vec{v}) \cdot (i\vec{q}) V_{dis} (\vec{q}). \]  

(50)
\( \vec{x}(k, t) \) is the Fourier transformation of \( \vec{x}(r, t) \).

After averaging over disorder, one has for \( k \ll G \) for equal-time correlation function:

\[
\langle \vec{x}(k, t)\vec{x}(-k, t) \rangle = \sum G |\rho(G)|^2 \Gamma(G) \left[ G(k, -\vec{G} \cdot \vec{v}) \cdot \vec{G} \right] \left[ G(-k, \vec{G} \cdot \vec{v}) \cdot \vec{G} \right],
\]

(51)

and similarly for the velocity,

\[
\langle \vec{v}(k, t)\vec{v}(-k, t) \rangle = \sum \vec{G} |\vec{G} \cdot \vec{v}|^2 |\rho(G)|^2 \Gamma(G) \left[ G(k, -\vec{G} \cdot \vec{v}) \cdot \vec{G} \right] \left[ G(-k, \vec{G} \cdot \vec{v}) \cdot \vec{G} \right].
\]

(52)

In perturbation theory on the FLR model, Matsukawa and Takayama [13] and Fisher [21] have argued that the expression for the positional correlation function contains divergent terms at 4-th order in the perturbation series, but the expression for the velocity correlation function does not up to the highest order terms examined. For this reason, we view our velocity-velocity correlation function as being more reliable. However, at the present level of perturbation theory, the two correlation functions behave in almost exactly the same way and are described by the same correlation length that we discuss in the next subsection.

The correlations at large distance can be obtained by the Fourier transformation of the above equations. The \( r \)-dependence of the real-space correlation functions is given by the \( k \)-dependent part of the \( k \)-space correlation functions:

\[
g(r) \sim \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r}} |G(k, -\vec{G} \cdot \vec{v})|^2.
\]

(53)

The imaginary part of the poles in the Green’s function \( G \) determines the characteristic lengths in these correlation functions. For these dynamic quantities, the magnetic field necessarily enters in contrast to the static case. We discuss these lengths in the next subsection.

**B. Dynamic Correlation Length**

Before we describe our results for the correlation length which is contained in Eq. (53), it is useful to review the results for a CDW system treated within the FLR model. The corresponding equation of motion was given in Eq. (4). In the CDW case, the Green’s function takes a simpler form:
\[ G_{CDW}(\vec{k}, \omega) = \frac{1}{\Delta k^2 - i \rho_m \lambda \omega}. \] (54)

Here \( \Delta \) is the CDW elastic constant.

The correlation functions for the FLR model in 3D have been calculated by Matsukawa and Takayama \[13\]. One can easily extend their result to 1D and 2D. We find that the velocity-velocity correlation as defined in the last subsection in the large \( r \)-limit becomes

\[
\left( \frac{1}{\sqrt{r}} \right)^{D-1} e^{-\frac{\xi_v \sqrt{2}}{r}} \times \text{a purely oscillatory factor} \sin\left( \frac{r}{\xi_v \sqrt{2}} - \phi_0 \right),
\]

where \( \phi_0 \) is a phase factor which depends on dimensionality \( D \), and the correlation length \( \xi_v \) is given by

\[
\xi_v = \sqrt{\frac{\Delta \xi^2 \rho_m \lambda G |v_z|}{\rho_m \lambda G |v_z|}}.
\] (55)

This expression for the correlation length holds as long as \( \omega \ll \lambda \), i.e., as long as the \( \omega^2 \)-term can be ignored in the Green’s function so that the oscillation at \( \omega \) is purely diffusive. \( \xi_v \) is nothing but the decay length of an oscillation at frequency \( \omega = |\vec{G} \cdot \vec{v}| \). It describes the distance over which a disturbance at frequency \( \omega_v \) will propagate.

This “dynamic correlation length” can be related to the static Fukuyama-Lee-Rice length \( \xi \) in the CDW case as:

\[
\xi_v = \xi \sqrt{\frac{\Delta}{\xi^2 \rho_m \lambda G |v_z|}} \sim \xi \sqrt{\frac{E_{Th}}{E - E_{Th}}}. \] (56)

So for fields \( E \) not too large compared to the sliding threshold field \( E_{Th} \), the dynamic correlation length is comparable to the static correlation length in the \( B = 0 \) CDW case.

With this picture in mind, we examine the present situation which is more complex. Since it is clear from the above arguments that the dynamic correlation length is determined by the excitations of a Wigner crystal at the washboard frequency \( \omega_v \), we examine the excitation spectra which appear as poles in the Green’s function, given by Eq. 20 and Eq. 21. Setting the denominator of the Green’s function to zero, one has simply in terms of \( \kappa_\omega \) and \( \kappa_\lambda \):

\[
(ka)^3 - \kappa_\omega^3 + i(ka)\kappa_\lambda^2 = 0. \] (57)

We are interested in the case where \( \kappa_\omega \) and \( \kappa_\lambda \) are given by those at \( \omega = \omega_v = |\vec{G} \cdot \vec{v}| \).
Note that Eq. 57 is a cubic equation for $k$. So for each given $\omega$, one finds generally three poles in the complex $k$-plane. If $\omega_v$ is very small, then the modes are once again diffusive. In this frequency range, there is no propagating $q^{3/2}$-dispersive mode. More quantitatively, at very small average sliding velocity, for $\kappa_\omega \ll \kappa_\lambda$, the three roots of Eq. 57 are approximately:

$$k_{1,2}^2 a^2 + i\kappa_\lambda^2 = 0,$$

(58)

and,

$$-\kappa_\omega^3 + i k_3 a \kappa_\lambda^2 = 0.$$

(59)

These poles give rise to two lengths, as:

$$\xi_{1,2} \sim \frac{1}{k_{1,2}} \sim \frac{a}{\kappa_\lambda},$$

(60)

and,

$$\xi_3 \sim \frac{1}{k_3} \sim a \frac{\kappa_\lambda^2}{\kappa_\omega^3},$$

(61)

The largest one of the three lengths, $\xi_3$, will determine the large-distance behavior of the velocity-velocity correlation function, although stronger correlations between the velocities of different elements will set in when their distance becomes comparable to $\xi_{1,2}$. The first one of the two lengths ($\xi_{1,2}$) in this over-damped limit is similar to the CDW case, since:

$$\frac{a}{\kappa_\lambda} = \sqrt{c^T/(\rho_m \lambda |G \cdot \bar{v}|)}.$$

(62)

In the limit of $E$ only slightly greater than $E_{Th}$, it can be rewritten using the small $v$-limit of Eq. 42 as:

$$\frac{a}{\kappa_\lambda} = \xi_T \sqrt{\frac{E_{Th}}{E - E_{Th}}}.$$

(63)

But in the regime where $E$ is much larger than $E_{Th}$ while $\kappa_\omega$ is still much less than $\kappa_\lambda$, we have instead:

$$\frac{a}{\kappa_\lambda} = \xi_T \sqrt{\frac{\omega_c E_{Th}}{\lambda E}}.$$

(64)
which differs from the CDW result in Eq. [54].

As the sliding velocity increases, we get into the regime where $\kappa_\omega \gg \kappa_\lambda$. The three roots are now approximately:

$$k_1a = \kappa_\omega + i\frac{\kappa_\lambda^2}{3\kappa_\omega},$$

and, $$k_{2,3}a = \kappa_\omega e^{\pm i\pi/3}.$$ 

In this case, the $k_1$-mode is almost propagating. The corresponding lengths are related to the static correlation lengths as:

$$\xi_1 \sim a\frac{\kappa_\omega}{\kappa_\lambda^2},$$

and, $$\xi_{2,3} \sim a\frac{1}{\kappa_\omega}.$$ 

In this limit, $\xi_1$ still describes the decay length of an oscillation at frequency $\omega_v$, but the other two describe the dispersive propagation within a length that is smaller than the decay length. This is due to the particular dispersion of the $k^{3/2}$-mode. This phenomenon does not appear in the usual CDW systems where excitations at $\omega_v$ are always overdamped as long as $\omega_v \ll \lambda$. In the present case, the magnetic field makes it possible that even for $\omega_v \ll \lambda$ in the sliding state, there is still an almost propagating mode. It also has interesting effects on the strain in the sliding Wigner crystal, which is the subject of the next subsection.

For a discussion of the crossover between the two regimes in a typical experimental system, we refer the reader to Sec. VIIIB. Under most experimental conditions where the DC current is a few $nA$, $\kappa_\omega$ and $\kappa_\lambda$ are comparable. Since their ratio has only a weak dependence on the sliding velocity ($\sim v^{1/6}$) for all cases of practical interest, neither limit that we discussed above for the velocity-velocity correlation length is strictly applicable.
C. Strain

One can also calculate from Eq. 50 the local longitudinal \( (L) \) and transverse \( (T) \) strain in the sliding Wigner crystal, defined as:

\[
E^s_L = \langle (\nabla \cdot \vec{x}(\vec{r}, t))^2 \rangle,
\]

(69)

and,

\[
E^s_T = \langle (\nabla \times \vec{x}(\vec{r}, t))^2 \rangle.
\]

(70)

Eq. 70 gives the local elastic energy associated with the shear modulus; and Eq. 69 would be the local elastic energy associated with the bulk modulus, if there were no long-range Coulomb interaction.

From Eq. 51, we find that:

\[
E^s_L = \sum_{\vec{G}} |\rho(\vec{G})| \Gamma(\vec{G}) \int \frac{d^2\vec{k}}{(2\pi)^2} \vec{k} \cdot \mathbf{G}(\vec{k}, -\vec{G} \cdot \vec{v}) \cdot \vec{G} |^2,
\]

(71)

and,

\[
E^s_T = \sum_{\vec{G}} |\rho(\vec{G})| \Gamma(\vec{G}) \int \frac{d^2\vec{k}}{(2\pi)^2} \vec{k} \times (\mathbf{G}(\vec{k}, -\vec{G} \cdot \vec{v}) \cdot \vec{G}) |^2.
\]

(72)

\( E^s_T \) can be transformed into

\[
E^s_T = \sum_{\vec{G}} |\rho(\vec{G})| \Gamma(\vec{G}) \Gamma(\vec{G}) \int \frac{d^2\vec{k}}{(2\pi)^2} \vec{k} \cdot \mathbf{G}(\vec{k}, -\vec{G} \cdot \vec{v}) \cdot \vec{G} |^2 \frac{1}{4\pi \rho_m^2 a^4 \Omega^2 \alpha^2} \frac{1}{(ka)^5} \frac{1}{((ka)^3 - \kappa_\omega^3)^2 + (ka)^2 \kappa_\lambda^4},
\]

(73)

Once again, the result of this integral depends on the relative size of \( \kappa_\omega \) and \( \kappa_\lambda \), or, more physically, depends on if there is an almost propagating mode at the washboard frequency. If not, \( i.e., \) for \( \kappa_\omega \ll \kappa_\lambda \),

\[
E^\text{strain}_T = \sum_{\vec{G}} |\rho(\vec{G})| \Gamma(\vec{G}) \Gamma(\vec{G}) \int \frac{d^2\vec{k}}{(2\pi)^2} \vec{k} \cdot \mathbf{G}(\vec{k}, -\vec{G} \cdot \vec{v}) \cdot \vec{G} |^2 \frac{1}{4\pi \rho_m^2 (c^T)^2} \frac{1}{ln \frac{1}{\kappa_\lambda}} \frac{ln \frac{1}{\kappa_\omega}}{\Omega^2 \alpha a} = \beta \frac{\alpha_{\Omega \lambda}}{\Omega^2 \alpha a} \frac{ln \frac{1}{\kappa_\lambda}}{\kappa_\omega},
\]

(74)

while for \( \kappa_\omega \gg \kappa_\lambda \),

\[
E^\text{strain}_T = \sum_{\vec{G}} |\rho(\vec{G})| \Gamma(\vec{G}) \Gamma(\vec{G}) \int \frac{d^2\vec{k}}{(2\pi)^2} \vec{k} \cdot \mathbf{G}(\vec{k}, -\vec{G} \cdot \vec{v}) \cdot \vec{G} |^2 \frac{1}{4\pi \rho_m^2 (c^T)^2} \left( ln \frac{1}{\kappa_\omega} + \frac{\pi \kappa_\omega^2}{3 \kappa_\lambda^2} \right) = \beta \frac{\alpha_{\Omega \lambda}}{\Omega^2 \alpha a} \left( ln \frac{1}{\kappa_\omega} + \frac{\pi \kappa_\omega^2}{3 \kappa_\lambda^2} \right).
\]

(75)
Here $\beta$ is a dimensionless constant of order unity, and $\alpha_{\text{dis}}$ is $\sim \rho_c E_{Th}/\rho_m$, as was given in Eq. [31]. As we have mentioned already and will discuss further in Sec. VIIIB, the second term on the right-hand-side in Eq. [75] is smaller than the first in the experimental systems.

The first term in Eqs. [74] and [75] diverges as $v \to 0$ as $ln [E - E_{Th}/E_{Th}]$. The second term in Eq. [75], which is due to the $k_1$-mode in Eq. [65], has no counterpart in the usual charge-density wave cases. This is due to the fact that in the CDW cases, one only considers the strongly diffusive mode as long as the washboard frequency is smaller than $\lambda$. In the present case, however, under the same condition, there is the possibility of having a mode like $k_1$ in Eq. [65] whose real part is much larger than its imaginary part. It directly reflects the fact that the $k_1$-mode is only weakly damped, causing an almost resonant absorption at $(\omega_v, k_1)$.

Within our high velocity perturbation theory, the local transverse strain diverges weakly as $v \to 0$, suggesting that the elastic medium theory used here will break down before the pinning transition is reached. However, it is possible that higher order terms may change the form of the divergence found in Eq. [74] and Eq. [75], or eliminate it entirely. In all cases $E_{\text{strain}}^L$ is smaller than $E_{\text{strain}}^T$ by at least a factor of $(2\pi/\alpha)^2$, and does not contain the divergences that we discussed above if the Coulomb force is unscreened.

The possibility that the elastic model breaks down at small velocity has been discussed by Coppersmith and Coppersmith and Millis [14] for the Fukuyama-Lee-Rice model without a magnetic field. Using Eq. [74] or Eq. [75] and typical material parameters for a FQHE device outlined later in this paper, we find that the transverse strain at a typical transport current of $1nA$ is slightly less than 10%; the longitudinal strain is completely negligible.

The static correlation length is determined by the shear modulus alone, because the shear modulus is much smaller than the bulk modulus [17]. We see that in the sliding state, disorder scattering also induces principally transverse distortions. This is confirmed in both the study of the nonlinear conductivity in the previous section, and here by examination of the strain distribution. However for the velocity-velocity correlation length, the magnetic field and the bulk modulus both enter.
V. FREE CARRIER EFFECTS: LARGE VELOCITY LIMIT

In the previous sections, we have studied some aspects of the sliding motion of a Wigner crystal in the fractional quantum Hall regime. We now consider the effects of free carriers, which may be characterized by a separate conductivity tensor, on the sliding motion of the Wigner crystal. These free carriers may be thermally excited, or due to sample inhomogeneities. We treat them phenomenologically.

We assume that the effects of the free carriers can be represented by a conductivity tensor whose diagonal and off-diagonal elements are \((\sigma_{xx}^F, \pm \sigma_{xy}^F)\). Under the assumptions that 1) there is no interconversion between the free carriers and the Wigner crystal, and, 2) the coupling between the free carriers and the Wigner crystal is purely electrostatic, it can be shown \cite{17} that longitudinal component \(c^L\) in the Green’s function becomes:

\[
c^L k \rightarrow \Omega^2 \left[ \frac{2\pi ka}{1 + ika \frac{\sigma_{xx}^F \Omega}{\sigma_0 \omega}} + (1 + \alpha)(ka)^2 \right]. \tag{76}
\]

Here \(\sigma_0 = a\epsilon\Omega/2\pi\), slightly smaller than \(e^2/h\) for a typical GaAs/AlGaAs modulation doped FQHE sample.

It is therefore clear that the importance of free carriers is measured by the magnitude of the quantity \(M_F = k a \sigma_{xx}^F \Omega/\sigma_0 \omega\). We are interested in the range of \((\vec{k}, \omega)\) which corresponds to that of the disorder scattering potential. The characteristic frequency of the disorder potential is \(\omega_v \sim v/a\) in the sliding state of sliding velocity \(v\). For a current of a few nA, which is typical of most experiments, \(v\) is \(\sim 10^{-2} m/s\). This yields \(\omega_v \sim 10^6\) Hz, much smaller than \(\Omega\). This has two consequences. The first is that for such frequencies \(G(\vec{k}, \omega)\) can be approximated by that given by the elastic medium theory, which is true with or without the free carriers. Secondly, combining this with the phonon dispersion which determines the characteristic value of \(\vec{k}\), we conclude that \(M_F \gg 1\) as long as \(\sigma_{xx}^F/\sigma_0\) of the free carriers is greater than \(10^{-3}\), as a conservative estimate \cite{27}.

This range of \(\sigma_{xx}^F\) in fact covers most of the experimentally measurable range. We therefore conclude that for our purposes, we only need to consider the case \(M_F \gg 1\). We
then have:

\[ c^r k = -i\omega \lambda_1 + \Omega^2 (ka)^2, \]  

(77)

with

\[ \lambda_1 = 2\pi \Omega \frac{\sigma_0}{\sigma_{xx}}, \]  

(78)

and the corresponding Green’s function is given by:

\[
G^{-1}(\vec{k}, \omega) / \rho_m = \Omega^2 a^2 k^2 \frac{\vec{k} \cdot \vec{k}}{k^2} + \alpha \Omega^2 a^2 k^2 (1 - \frac{\vec{k} \cdot \vec{k}}{k^2}) - i\omega \lambda_1 \frac{\vec{k} \cdot \vec{k}}{k^2} - i\omega \left( \begin{array}{cc} \lambda & -\omega_c \\ \omega_c & \lambda \end{array} \right). \]  

(79)

Its determinant \( P(\vec{k}, \omega) \) is

\[ P(\vec{k}, \omega) = \alpha \Omega^4 (ka)^4 - \omega^2 \left[ \omega_c^2 + \lambda (\lambda_1 + \lambda) \right] - i\omega \Omega^2 (ka)^2 \left[ \alpha \lambda_1 + (1 + \alpha) \lambda \right]. \]  

(80)

From here most of the results from the previous sections follow without major changes. We therefore only list the results in the rest of this section.

**A. Nonlinear Conductivity**

The perturbative calculation proceeds as in Sec. III. We find it convenient to define in the present case three dimensionless quantities:

\[ \kappa_1^2 = \frac{|\omega_v| \omega_c}{\sqrt{\alpha \Omega^2}}, \]  

(81)

\[ \kappa_2^2 = \frac{|\omega_v|}{\Omega^2} (\lambda_1 + \lambda / \alpha), \]  

(82)

\[ \kappa_3^2 = \frac{|\omega_v|}{\Omega^2} (\lambda_1 + 2 \lambda), \]  

(83)

where \( \omega_v = \vec{G} \cdot \vec{v} \) is the sliding washboard frequency, and \( \kappa_3 \) is less than \( \kappa_2 \). \( \kappa_1 / a \) is the wavenumber at which the lower hybrid phonon mode, which disperses like \( \omega^2 = \alpha \Omega^4 (ka)^4 / \omega_c^2 \) in the present case, matches the washboard frequency, and \( \kappa_2 \) and \( \kappa_3 \) measure the strength of dissipation in the present context.
In the limit of \( \omega_c \gg \lambda \) and \( \lambda_1 \gg \lambda/\alpha \), we find that the spatially averaged velocity correction to second order in disorder scattering \( \vec{v}_2 \) is:

\[
\left( \begin{array}{cc}
\lambda & -\omega_c \\
\omega_c & \lambda
\end{array} \right) \cdot \vec{v}_2 = -\frac{1}{16\alpha\Omega^2a^2\rho_m^2} \sum_{\vec{G}} |\rho(\vec{G})|^2 \Gamma(\vec{G})|\vec{G}|^2 \vec{G} \ sgn(\vec{G} \cdot \vec{v}) = -\alpha_{\text{dis}} \frac{\vec{v}}{v}.
\]

(84)

This yields a resistivity tensor which is identical to that in the absence of free carriers. This is because the leading corrections depend solely on transverse distortions, and only the longitudinal modes are affected by free carrier screening. The results were given in Sec. III, and will not be repeated here. At higher orders of the perturbation theory, the longitudinal channel affects the results; and the conductivities with or without the screening effects will be different.

**B. Correlation Functions**

As in Sec. IVB, the correlation length of the sliding state for all of the correlation functions will be determined by the poles of the Green’s function \( \mathbf{G}(\vec{k}, \omega) \), given by:

\[
(ka)^4 - \kappa_1^4 + i(ka)^2\kappa_2^2 = 0.
\]

(85)

For simplicity, we assume \( \kappa_1 \gg \kappa_2 \), and obtain:

\[
\xi = 4a\kappa_1/\kappa_2^2 \sim 1/\sqrt{|\omega_v|},
\]

(86)

which approaches infinity as \( |\omega_v| \to 0 \).

The evaluation for the strain also proceeds as before. We here only give the results:

\[
E^S_T = |\nabla \times \vec{x}|^2 = \sum_{\vec{G}} |\rho(\vec{G})|^2 \Gamma(\vec{G}) \frac{|\vec{G}|^2}{4\pi\alpha^2\Omega^4a^4\rho_m^2} \int (ka)^3d(ka) \frac{(ka)^4 + \alpha\kappa_1^4(1 + \lambda_1^2/\omega_c^2)}{(ka)^4 - \kappa_1^4)^2 + (ka)^4\kappa_2^4},
\]

(87)

and

\[
E^S_L = (\nabla \cdot \vec{x})^2 = \sum_{\vec{G}} |\rho(\vec{G})|^2 \Gamma(\vec{G}) \frac{|\vec{G}|^2}{4\pi\alpha^2\Omega^4a^4\rho_m^2} \int (ka)^3d(ka) \frac{\alpha^2(ka)^4 + \alpha\kappa_1^4}{(ka)^4 - \kappa_1^4)^2 + (ka)^4\kappa_2^4}.
\]

(88)

We see once again that \( E^S_T \gg E^S_L \), although the actual ratio depends on the details of the relevant parameters. For \( \kappa_1 \gg \kappa_2 \), we have:
Here $\beta$ is once again a constant of order unity. The logarithmic divergence has the same origin as that for the transverse strain distribution function previously. It is the result of the dimensionality of the present problem.

VI. AC + DC INTERFERENCE EFFECTS

We now consider the possible interference effects due to the presence of both a DC field and an AC field. It is more convenient here to adopt the alternative approach mentioned in Sec. IIIB and determine the electric field $\vec{E}_{\text{tot}}$ required to sustain a current density $\vec{j}$ which has both DC and AC components, given by:

$$\vec{j} = \rho_c \vec{v}_{DC} + \rho_c \vec{v}_{AC} \cos \omega_{AC} t,$$

within the perturbation theory,

$$\vec{E}_{\text{tot}} = \vec{E} + \vec{E}^p,$$

where

$$\rho_c \vec{E} = \rho_m \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} (\vec{v}_{DC} + \vec{v}_{AC} \cos \omega_{AC} t),$$

and to second order in disorder potential

$$\rho_c \vec{E}^p = -i \sum_{\vec{G}, n_1, n_2} |\rho(\vec{G})|^2 \int \frac{d^2 \vec{q}}{(2\pi)^2} \vec{q} \Gamma(\vec{q}) J_{n_1}(\vec{q} \cdot \vec{v}_{AC} / \omega_{AC}) J_{n_2}(\vec{q} \cdot \vec{v}_{AC} / \omega_{AC})$$

$$e^{i \omega_{AC}(n_2 - n_1) t} \left( \vec{q} \cdot \vec{G}(-\vec{q} + \vec{G}, n_1 \omega_{AC} + \vec{q} \cdot \vec{v}_{DC}) \cdot \vec{q} \right).$$

The notations here are the same as before. $J_n(x)$ is the $n$-th order integer Bessel function. The sums over $n_1, n_2$ include both positive and negative integers. The derivation of Eq. (93) is the same as its counterpart without the AC field and essentially follows that in e.g. Ref. [12]. Therefore it will not be repeated here.
There are two interference effects:

1). For \( n_1 = n_2 \), the resultant DC field is modified by the AC current (Shapiro anomaly);

2). The linear AC-response, i.e., the \( \omega_{AC} \)-component of \( \vec{E}' \), will be affected by the DC current.

Both effects enter through the frequency dependence of the Green’s function \( G \) where \( n\omega_{AC} + \vec{q} \cdot \vec{v}_{DC} \) appears. We discuss them separately in the next two subsections.

**A. Shapiro Anomaly**

Setting \( n_1 = n_2 \) in Eq. 93, we have:

\[
\rho_c \vec{E}' = -i \sum_{\vec{G}, n} |\rho(\vec{G})|^2 \int \frac{d^2 \vec{q}}{(2\pi)^2} \mathcal{T}(\vec{q})(J_n(\vec{q} \cdot \vec{v}_{AC}/\omega_{AC}))^2 \left( \vec{q} \cdot G(-\vec{q} + \vec{G}, n\omega_{AC} + \vec{q} \cdot \vec{v}_{DC}) \cdot \vec{q} \right). 
\]

(94)

The \( n = 0 \) term in the above equation gives us the nonlinear conductivity in the absence of the AC field, which was studied in previous sections. If \( n \neq 0 \), under the condition that \( \frac{|\vec{G} \cdot \vec{v}_{DC} + n\omega_{AC}|}{|\vec{G} \cdot \vec{v}_{DC}|} \gg \max(\kappa_\lambda, \kappa_\omega) \), i.e., we are somewhat away from the resonance, we find:

\[
\rho_c \vec{E}' = \alpha_0 \sum_{\vec{G}, n} \vec{G} \text{sgn}(\vec{G} \cdot \vec{v}_{DC} - n\omega_{AC})J_n^2(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC})A(\vec{G}). 
\]

(95)

\( A(\vec{G}) \) here is the same as that given in Eq. 32.

We here fix \( \omega_{AC} \) to be positive, and have used the fact that in summing over \( \vec{G}, \pm \vec{G} \) enter together. \( \alpha_0 \) in Eq. 33 is once again \( \pi/3 \) if \( \kappa_\omega \gg \kappa_\lambda \) and \( \pi/4 \) if \( \kappa_\omega \ll \kappa_\lambda \).

If we further relate \( A(\vec{G}) \) to the threshold field \( E_{Th} \) through \( \alpha_{dis} \) as was done earlier in the paper in Eq. 32 and Eq. 41, we finally have for a DC current \( \vec{j} = \rho_c \vec{v}_{DC} \), the total external DC field is:

\[
\rho_c \vec{E} = \rho_m \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \vec{v} + \rho_c E_{Th} \vec{v}_{DC}/v_{DC} \\
+ \frac{\alpha_0}{4} \sum_{\vec{G}, n} |\vec{G}|A(\vec{G})J_n(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC})\text{sgn}(|\vec{G} \cdot \vec{v}_{DC}| - n\omega_{AC})\vec{v}_{DC}/v_{DC}. 
\]

(96)
In cases when $\vec{G}$ is not parallel to $\vec{v}_{DC}$, as long as $\vec{v}_{DC}$ is along one of the six nearest neighbor or the six next nearest neighbor $\vec{G}$-vectors, summing over all $\vec{G}$’s with the same $\vec{G} \cdot \vec{v}_{DC}$ would yield a correction to the field in the direction of $\vec{v}_{DC}$.

We therefore see that the Hall resistivity is once again not altered. The longitudinal resistivity shows upward jumps with increasing current, or downward jumps with increasing frequency of the AC field. If one is instead interested in the conductivity, the direction of the jumps will depend on whether or not the Hall angle is greater than 45°. If not, downward jumps in resistivity will translate into upward jumps in conductivity; otherwise, downward jumps in resistivity will become a downward jump in conductivity. This is familiar in experiments in the FQHE regime \[1\] where one often encounters a situation in which the off-diagonal elements of the conductivity/resistivity tensor are greater than the diagonal elements. The maximum value of the $\vec{G}$-vectors that can give rise to the jump is cutoff by either the $\vec{G}$-dependence of disorder correlation function $\Gamma(\vec{G})$ or by the density $\rho(\vec{G}) \sim e^{-|\vec{G}|^2l_B^2/2}$. Both enter through the expression for $A(\vec{G})$.

This is the Shapiro step in the DC voltage, on the order of the depinning field but smaller in magnitude, appearing when we vary either the frequency of the AC field or the DC current. ($\rho_c \vec{E}^p$ is proportional to the DC voltage.) The main experimental signatures are therefore:

1). If one sweeps the frequency while keeping the DC current constant, as $n\omega_{AC}$ crosses $|\vec{G} \cdot \vec{v}_{DC}|$ from below, the DC voltage decreases, leading to a decrease in the DC resistivity. The jump in DC voltage should be proportional to the threshold field, with the proportionality coefficient being $J_n^2(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC})$, which gives its dependence on the amplitude and the frequency (or the DC current) of the AC field.

2). If one instead gradually increases the DC current at a fixed AC frequency, one observes an increase in resistivity.

3). In usual CDW systems, one often measures the differential resistivity and looks for peaks at resonance. The width of the peak gives additional information on the correlation

34
length. In the presence case, the situation is more complicated and the width will depend on the velocity-velocity correlation length which we discussed extensively earlier.

Overall, neither $\rho(\vec{G})$ nor $\Gamma(\vec{G})$ decays very fast with $\vec{G}$ for small filling factors with small magnetic length. There will be many more peaks than in the traditional CDW systems where $\rho(\vec{G})$ is finite at a single $\vec{G}$ and its inversion. The features, if indeed observable, are expected to be much broader because the contributions come from many $\vec{G}$’s. Perhaps a more severe hindrance is that the correlation length in the Wigner crystals around $\nu = 1/5$ appears short [17]. Traditionally, it has been easier to observe the interference effect in the AC response. We turn to it in the next subsection.

B. Linear AC Response Function

To obtain the linear AC response in the presence of a DC current, we assume small $\vec{v}_{AC}$ and only keep terms linear in $\vec{v}_{AC}$ in Eq. 93. We have:

$$\rho_c E^p = -\frac{i}{(2\pi)^2} e^{-i\omega_{AC}t} \sum_\vec{G} |\rho(\vec{G})|^2$$

$$\int d^2 \vec{q} \Gamma(\vec{q})(\vec{q} \cdot \vec{v}_{AC}/2\omega_{AC}) \left( \vec{q} \cdot (-\vec{G}(\vec{q} + \vec{G}, \vec{q} \cdot \vec{v}_{DC}) + \vec{G}(\vec{q} + \vec{G}, \omega_{AC} + \vec{q} \cdot \vec{v}_{DC}) \cdot \vec{q} \right)$$

$$+ c.c.,$$

(97)

from which we can determine the in-phase and the out-of-phase component of the AC resistivity. For the in-phase part that is proportional to $\cos\omega_{AC}t$, we find:

$$(\rho_c E^p)^{in} = \alpha_0 \sum_\vec{G} \vec{G} \text{sgn}(\omega_{AC} - \vec{G} \cdot \vec{v}_{DC})(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC}) A(\vec{G}).$$

(98)

And for the out of phase part that is proportional to $\sin\omega_{AC}t$, we have:

$$(\rho_c E^p)^{out} = \sum_\vec{G} \vec{G} A(\vec{G})(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC}) \ln \frac{\max(\kappa_\omega, \kappa_\lambda)}{\max(\kappa_\omega, \kappa_\lambda)}. $$

(99)

where $\kappa_\omega$ and $\kappa_\lambda$ are defined as before and the primed quantities are obtained by replacing $\vec{G} \cdot \vec{v}_{DC}$ with $\omega_{AC} - \vec{G} \cdot \vec{v}_{DC}$ in the respective unprimed quantities. We have in terms of the threshold field:
\[(\rho_c \vec{E}^p)_{in} = \rho_c E_{Th} \sum_{\vec{G}} \vec{G}/|\vec{G}| \text{sgn}(\omega_{AC} - \vec{G} \cdot \vec{v}_{DC})(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC}), \quad (100)\]

and

\[(\rho_c \vec{E}^p)_{out} = \rho_c E_{Th}/4\alpha_0 \sum_{\vec{G}} \vec{G}/|\vec{G}|(\vec{G} \cdot \vec{v}_{AC}/\omega_{AC}) \ln\frac{\max(\kappa_\varphi, \kappa_\lambda)}{\max(\kappa_\omega, \kappa_\lambda)}, \quad (101)\]

where the sums are over those that give the same \(\vec{G} \cdot \vec{v}\) among the six smallest \(\vec{G}\)'s only. The above equations also hold for larger \(\vec{G}\)'s, with the necessary replacement of \(E_{Th}\) in Eq. \(100\) by an expression in which the \(|\vec{G}|\)-dependence of various terms in \(A(\vec{G})\) is restored.

The out-of-phase part of the AC resistivity should show an inductive to capacitive transition as \(\omega_{AC}\) goes from below \(\sim \vec{G} \cdot \vec{v}_{DC}\) to above. For large \(\omega_{AC}\), it becomes \(\frac{1}{\omega_{AC}} \ln \omega_{AC}\). We also note that it has only a component along the direction of \(\vec{v}_{DC}\), that is, it only enters the longitudinal resistivity.

The in-phase term involves now three vectors: \(\vec{G}\), the DC current \(\vec{v}_{DC}\), and the AC current \(\vec{v}_{AC}\). In cases when the latter two are not aligned, it will enter both the diagonal and off-diagonal AC resistivity. In particular, if the relevant \(\vec{G}\) satisfies \(\vec{G} \cdot \vec{v}_{AC} = 0\), the jump in the real part of the AC resistivity disappears, although the imaginary part will still show an inductive anomaly. If \(\vec{v}_{AC}\) is parallel to \(\vec{v}_{DC}\), then

\[(\rho_c \vec{E}^p)_{in} = \rho_c E_{Th} \mathrm{sgn}(\omega_{AC} - |\vec{G} \cdot \vec{v}_{DC}|)/(a\omega_{AC})\vec{v}_{AC}. \quad (102)\]

The equation is uncertain up to a numerical factor smaller than unity. The diagonal resistivity changes from roughly \(\frac{e\mu}{\rho_c} (\lambda - \alpha_{dis}/a\omega_{AC})\) to \(\frac{\mu}{\rho_c} (\lambda + \alpha_{dis}/a\omega_{AC})\) as \(\omega_{AC}\) increases through \(\vec{G} \cdot \vec{v}_{DC}\). Approximately, for the out-of-phase part, we find:

\[(\rho_c \vec{E})_{out} = \sum_{\vec{G}} A(\vec{G}) \frac{\vec{v}_{AC}}{a\omega_{AC}} \ln\frac{|\omega_{AC} - \vec{G} \cdot \vec{v}_{DC}|}{\vec{G} \cdot \vec{v}_{DC}}. \quad (103)\]

We also note that when the DC current \(\vec{v}_{DC}\) and the AC current \(\vec{v}_{AC}\) are not parallel, the system no longer exhibits the rotational invariance that we invoked in arguing for a classical Hall resistivity. Indeed, in this case, we find that the Hall component of the in-phase AC resistivity will not be \(B/\rho_c\), with corrections occurring first in second order in disorder potential.
Care must be exercised when converting these statements about the changes in the AC resistivity to those in conductivity, as in the case of the Shapino anomaly that we discussed in the last subsection. When the Hall angle for the AC current is smaller than 45°, upward jumps in the real part of the diagonal resistivity translates into downward jumps in the real part of the diagonal conductivity, and the inductive to capacitive transition remains. But for Hall angles greater than 45°, the situation will be reversed. we therefore choose to give us results in terms of resistivity.

VII. RESULTS FOR ZERO MAGNETIC FIELD

In the case of a zero magnetic field, the Green’s function has the same form, with \( \omega_c \) now equal to 0, and \( P(\vec{k}, \omega) \) replaced by:

\[
P(\vec{k}, \omega) = 2\pi \alpha \Omega^4 \rho_m [(ka)^3 - \kappa_\omega^3 - i\kappa_\lambda^2 (ka) \text{sgn}(\omega)],
\]

where as before, we have ignored the \( \alpha (ka)^2 / \alpha \pi \) term in the last term on the right-hand-side, and

\[
\kappa_\lambda^2 = \frac{|\omega| \lambda}{\alpha \Omega^2}, \tag{105}
\]

and,

\[
\kappa_\omega^3 = \kappa_\lambda^2 \frac{|\omega| \lambda}{2\pi \Omega^2} = \frac{\omega^2 \lambda^2}{2\pi \alpha \Omega^4}. \tag{106}
\]

All of the results for the resistivity are in exactly the same form as those of the diagonal resistivity with the magnetic field, if we set \( \omega_c = 0 \) and put in the new \( \kappa_\lambda \) and \( \kappa_\omega \). We will not repeat them here. The only exceptions are the velocity-velocity correlation length. We give the new results here:

1). For \( \kappa_\omega \ll \kappa_\lambda \), i.e., the overdamped case,

\[
\xi_{1,2} = \xi^T \sqrt{\frac{E_{th}}{E - E_{th}}}, \quad \xi_3 = \xi^L \frac{E_{th}}{E - E_{th}}; \tag{107}
\]

2). For \( \kappa_\omega \gg \kappa_\lambda \),
\[ \xi_{2,3} = \xi_T \left( \frac{\xi_L}{\xi_T} \right)^{1/3} \left( \frac{E_{Th}}{E - E_{Th}} \right)^{2/3}, \quad \xi_1 = \xi_T \left( \frac{\xi_L}{\xi_T} \right)^{1/3} \left( \frac{E_{th}}{E - E_{Th}} \right)^{1/3}. \] (108)

In the overdamped region as specified in case 1, the equations are the same as for the CDWs. Case 2 is where the differences lie.

\[ \begin{aligned} \xi_2 &= \xi_T \left( \frac{\xi_L}{\xi_T} \right)^{1/3} \left( \frac{E_{Th}}{E - E_{Th}} \right)^{2/3}, \\ \xi_1 &= \xi_T \left( \frac{\xi_L}{\xi_T} \right)^{1/3} \left( \frac{E_{th}}{E - E_{Th}} \right)^{1/3}. \end{aligned} \] (108)

\[ \begin{aligned} \bar{E}_x &= \frac{\lambda \rho_m}{\rho_c^2} j_x + E_{Th}, \\ \bar{E}_y &= \frac{\omega_c \rho_m}{\rho_c^2} j_x. \end{aligned} \] (109) (110)

We notice that the functional form of Eq. (109) is in very good agreement with the experiments by Li et al. \[ 10 \] and those by Williams et al. \[ 11 \]. It also agrees with the large

\[ \xi_{2,3} = \xi_T \left( \frac{\xi_L}{\xi_T} \right)^{1/3} \left( \frac{E_{Th}}{E - E_{Th}} \right)^{2/3}, \quad \xi_1 = \xi_T \left( \frac{\xi_L}{\xi_T} \right)^{1/3} \left( \frac{E_{th}}{E - E_{Th}} \right)^{1/3}. \] (108)

In the overdamped region as specified in case 1, the equations are the same as for the CDWs. Case 2 is where the differences lie.

**VIII. SUMMARY OF OUR MAIN RESULTS**

In Sec. VIIIA, we summarize the main theoretical results regarding the transport properties of a sliding Wigner crystal obtained in this work. In Sec. VIIIB, we estimate various characteristic lengths and frequencies that have appeared in our theory using parameters appropriate for a typical high mobility modulation doped GaAs/AlGaAs heterojunction. Our calculations should be strictly valid only in the very low temperature limit, while experiments are done at a variety of temperatures. Therefore in Sec. VIIIA, we discuss the possible temperature effects on the nonlinear \( I - V \) curves.

**A. Summary of the Theoretical Transport Properties**

We summarize our main results in the form of predicted \( I - V \) characteristics of a sliding Wigner crystal.

A). Hall bar geometry

In the Hall bar geometry, one controls the current that runs through the sample. If \( \bar{j} = \rho_c \bar{v} \) along the \( x \)-direction, then,

\[ \begin{aligned} \bar{E}_x &= \frac{\lambda \rho_m}{\rho_c^2} j_x + E_{Th}, \\ \bar{E}_y &= \frac{\omega_c \rho_m}{\rho_c^2} j_x. \end{aligned} \] (109) (110)

We notice that the functional form of Eq. (109) is in very good agreement with the experiments by Li et al. \[ 10 \] and those by Williams et al. \[ 11 \]. It also agrees with the large
depinning field data of Jiang et al. ($V \sim 50\text{mV}$) [4]. (See Fig. 8 in their paper.) We discuss the comparison in more detail in Sec. IX.

The first equation holds also for the case of a sliding Wigner crystal without a magnetic field, in which case there is simply no Hall component. The same equations also hold when there is coupling to free carriers as long as we are in the large velocity regime. Figs. 1 and 2 given previously in this paper depict the theoretical results in the Hall bar geometry.

B). Corbino geometry

In Corbino geometry, the electric field is controlled by virtue of symmetry of the experimental set-up. If a constant DC field $\vec{E}$ is along the $x$-direction, i.e., $E_y = 0$, then for $\omega_c \gg \lambda$,

$$j_x = \frac{\rho_c^2}{\rho_m} \frac{\lambda}{\omega_c^2} (E_x + \alpha_{\text{dis}} \frac{\rho_m \omega_c}{\rho_c} \lambda),$$ \hspace{1cm} (111)

and

$$j_y = \frac{\rho_c^2}{\rho_m} \frac{1}{\omega_c} (E_x - 2\alpha_{\text{dis}} \frac{\rho_m \omega_c}{\rho_c} \lambda).$$ \hspace{1cm} (112)

And the total current $j$ is given to order $\alpha_{\text{dis}}$ by:

$$j = \frac{\rho_c^2}{\rho_m} \frac{1}{\omega_c} (E - E_{\text{Th}} \frac{\lambda}{\omega_c}).$$ \hspace{1cm} (113)

C). Hall Angle

The Hall angle, i.e., the angle between the current and the external field, increases with the external field. In the theory in which the velocity on the right-hand-side of Eq. [3] is the self-consistent velocity, taking into account the disorder effects on the velocity, rather than the bare $\vec{v}_0$ in Eq. [13], the Hall angle becomes $45^\circ$ when $E/E_{\text{Th}} \approx \sqrt{2\omega_c/(\omega_c - \lambda)}$.

D). Shapiro anomaly
The longitudinal DC field changes by \( \sim E_{Th} J_n (\vec{G} \cdot \vec{v}_{AC} / \omega_{AC}) sgn(|\vec{G} \cdot \vec{v}_{DC}| - n \omega_{AC}) / \omega_{AC} \) across the resonance as the DC current or the AC frequency is swept.

E). AC response

The out-of-phase AC response only occurs in the longitudinal resistivity. It changes from inductive to capacitive as the frequency increases through resonance.

The in-phase AC field will change by \( \sim E_{th} sgn(\omega_{AC} - |\vec{G} \cdot \vec{v}_{DC}|)|\vec{G} \cdot \vec{v}_{AC}| / \omega_{AC} \) across the resonance. If the DC current and the AC current are not in the same direction, it will have both diagonal and off-diagonal components.

B. Characteristic Lengths and Frequencies

The elastic medium theory that we adopted to describe the sliding Wigner crystal is only valid in the long wavelength limit. In this subsection, we estimate various characteristic length scales and frequency scales that enter our previous analysis. We use numbers that are typical of a high mobility modulation-doped GaAs/AlGaAs heterojunction. Therefore \( m = 0.067 m_e, \epsilon = 13, \) electron density \( n = 10^{11} \text{ cm}^{-2}, \) magnetic field \( B = 20 \text{ Tesla}, \) mobility \( \mu = 10^7 \text{ cm}^2/\text{V} \cdot \text{s}, \) and when needed, the size of the sample is assumed to be \( 3 \text{ mm} \times 3 \text{ mm}. \) The ratio of the shear modulus versus the bulk modulus \( \alpha \) is taken to be its classical zero-temperature value of 0.02 \cite{19}. Of these parameters, \( \alpha \) has the largest uncertainty, with corrections from both quantum and thermal effects, the former being perhaps more important near the zero-temperature Wigner solid/FQHE liquid boundary, the latter more important near the thermal melting transition of the Wigner solid.

With these numbers, the cyclotron frequency \( \omega_c \sim 10^{13} \text{s}^{-1}, \Omega \sim 10^{12} \text{s}^{-1}. \) At \( qa = 1/3, \) the zero-field phonon frequencies are: \( \omega_L \sim 10^{12} \text{s}^{-1}, \omega_T \sim 10^{11} \text{s}^{-1}, \) and the acoustic magnetophonon frequency is \( \sim 10^{10} \text{s}^{-1}. \) The collision frequency deduced from the zero-field mobility is \( \tau^{-1} \sim 10^9 \text{s}^{-1}. \) How this number should change in a strong magnetic field is an open question, as is the relation between \( \tau^{-1} \) and the parameter \( \lambda \) in Eq. \cite{4}. Values of \( \lambda \)
inferred from the high field \( I - V \) measurements are much larger, \( \lambda \sim 10^{12} - 10^{13} \text{s}^{-1} \).

Typically, measurements are done with a current \( I \sim 1 \text{nA} \) in the sliding regime. Assuming a large fraction of the Wigner crystal is sliding, we have \( v \sim 10^{-2} \text{ms}^{-1} \). This gives a washboard frequency of \( \omega_v \) between \( 10^5 \) to \( 10^6 \text{s}^{-1} \). We see that this is much smaller than the characteristic phonon frequencies involved. Therefore, the elastic medium theory should be valid. We also have \( \kappa_\omega \sim 10^{-4} \), implying a typical wavelength \( 10^4 a \), which approaches the typical size of a sample. It has been shown that within the perturbation theory for a CDW system [12], the broad band noise vanishes upon average over volume in an infinite system, but does not vanish if the system size is not infinite. The fact that \( \kappa_\omega \) is comparable to system size in the present case may be responsible for the sizable broad band noise observed experimentally [4\&6]. It would be very interesting to see if experimentally the narrow band noise around \( \omega_v \) could be observed. \( \kappa_\lambda \) is between \( 10^{-3} \) to \( 10^{-4} \), depending on the size of \( \lambda \) chosen.

We note that if one converts the resistivity in the sliding state as measured by the experiments [4\&7] to a scattering frequency using Eq. 33 in the high field limit, the scattering frequency \( \lambda \) is rather large. In fact, it is comparable to \( \omega_c \), the largest frequency scale in the problem. It is much larger than the scattering rate deduced from the zero-field mobility, and much larger than that deduced from the metallic state resistivity at larger filling factors than \( \nu = 1/5 \) [3]. At present, it is unclear what mechanism is responsible for such a high resistivity in the sliding regime.

We suggest that it is, at least at moderately high temperatures, due to the damping caused by the backflow of free carriers to screen out the charge density fluctuations in a sliding Wigner crystal [28]. It is clear from Eqs. 76 and 77 and the ensuing discussion that the free carriers lead to a longitudinal mode dissipation proportional to the free carrier resistivity. Although in the present problem, this does not enter the nonlinear conductivity to second order in the randomness, it presumably will in higher orders, as in the CDW problem [28].
C. Nonzero Temperatures

The calculations reported in this paper did not include thermal effects explicitly, while at least in some experiments such effects are quite apparent. In this subsection, we outline the effects that we expect to occur at $T > 0$.

We believe the four important effects occurring as $T$ increases are: 1) an increase in the number of free carriers; 2) a decrease in the strength of the interaction of the Wigner crystal with the impurity potential; 3) an increase in the number as well as in the size of dislocation pairs; and 4) the subsequent melting of the Wigner crystal, presumably via a Kosterlitz-Thouless transition.

The first point is straightforward: presumably, creation of free carriers involves excitations over an energy gap, $\Delta g$, and so the number of free carriers and thus the free carrier conductivity should have a roughly activated temperature dependence. One therefore expects the conductivity at $E < E_{Th}$ to be activated. Interestingly, in at least some experiments the conductivity at $E > E_{Th}$ is also activated, with the same gap, suggesting that the conductivity at $E > E_{Th}$ is also controlled by the free carriers. An additional point, inferred from the experiments in [7], is that there is one single excitation gap at temperatures below $\sim 140mK$. In other words, the low-energy charged excitations in the Wigner crystal appear to be well-defined, and well-separated from higher energy excitations.

Free carriers not only provide conductivity and dissipation, they may also screen the impurity potential. Also thermal fluctuations of the Wigner crystal may tend to average out the impurity potential. For these reasons, we expect that the effective strength of the random potential will get weaker as the temperature is raised.

The third effect of finite temperatures, the presence and the proliferation of dislocation pairs, requires mode discussion. The conventional Fukuyama-Lee-Rice theory of pinning and nonlinear conductivity is based on a “phase-only” model in which the amplitude fluctuations of the order-parameter or the dislocations are not allowed. One important consequence of forbidding the formation of dislocations is the existence of a well-defined threshold field
below which the whole solid is pinned and above which the whole solid moves collectively. In a model in which dislocations are allowed, it is possible for some portions of the solid to move while other portions remain pinned at any given moment; and the depinning transition is no longer sharp nor is it unambiguously defined \[14\]. In a two-dimensional ordered solid, the elastic energy of an isolated dislocation is logarithmically divergent in system size so at low temperatures the dislocations are created via thermal activation of bound pairs whereby the divergent elastic energy due to each one cancels out. As \(T\) increases, the number of pairs increases as does the typical size of a pair, \(\xi_{KT}\). Dislocation pairs proliferate and \(\xi_{KT}\) diverges as the Kosterlitz-Thouless melting temperature is approached from below \[29\].

Another way to see this is as follows. At zero or very low temperatures, disorder pinning dominates over the thermal effects and the Fukuyama-Lee-Rice theory \[16\] gives the disorder-induced correlation length \(\xi_T\) beyond which the positional order disappears. As temperature is raised, \(\xi_T\) decreases according to \(\xi_T \propto c^T\), proportional to the (decreasing) shear modulus, assuming that the change in disorder potential itself is smooth and featureless. However the increasing thermal fluctuations also give rise to a thermally-induced correlation length \(\xi_{KT}\) on the order of the size of a typical dislocation pair in the Wigner crystal. \(\xi_{KT}\) increases very sharply as the melting transition is approached and diverges at the transition. Therefore it must become larger than the decreasing \(\xi_T\) before the melting transition occurs, thus invalidating the Fukuyama-Lee-Rice description which considers only phase-fluctuations \[16\]. We expect that as \(T\) increases and dislocation pairs proliferate and increase their size, the depinning transition will become rounded and a more appropriate physical picture of the system at \(E > 0\) would involve a combination of pinned and moving regions. The effects of dislocations on the nonlinear conductivity have been studied in numerical simulations by Shi and Berlinsky for vortex lattices in type-II superconductors at zero temperature \[15\].

Finally, the issue of the melting of the Wigner crystal is also straightforward. The melting temperature may be roughly estimated from the Kosterlitz-Thouless theory and the theoretical shear modulus \[29\]. It is found to be of order \(1K\) by Fisher \[29\], much higher than the temperature at which the observed \(E_{TH}\) vanishes. It is in fact much higher than
the temperatures at which these nonlinear transport measurements were typically carried out [7]. Combining this with the observations that we made in the last two paragraphs, we conclude that it is unlikely that the vanishing of $E_{Th}$ observed in the experiments [4–7] was due to the Wigner crystal melting transition.

IX. SUMMARY OF RECENT EXPERIMENTS AND COMPARISON TO THEORY

So far we have focused on the theoretical aspects of the problem of a sliding Wigner crystal in a strong magnetic field when the sliding velocity is large. In what follows, we give a brief review of the relationship of our theoretical results to the nonlinear conductivity measurements reported in the literature. Seemingly similar experiments obtain apparently inconsistent results. We discuss these apparent inconsistencies in detail. Experiments regarding the Hall resistivity were discussed earlier in this paper; so here we will focus on the longitudinal conductivity only. We review and compare the DC transport experiments in Sec. IXA, and we compare the theory and the experiments in Sec. IXB. AC + DC interference effects are briefly discussed in Sec. IXC.

A. Nonlinear DC Transport Experiments

The basic idea of these experiments is quite simple [4–7,11,16]: a large enough DC electric field will dislodge the pinned Wigner crystal and cause it to slide. Therefore, by measuring the conductivity as a function of the applied DC field, one expects to observe a sudden increase of conductivity, and the accompanying noise generated by the disorder scattering off the sliding Wigner crystal.

There are to our knowledge four sets of published results probing the nonlinear conductivity of the insulating phases around the filling factor $\nu = 1/5$ fractional quantum Hall state: Goldman et al., Phys. Rev. Lett. 65, 2189 (1990) (Ref. [4]); Williams et al., Phys. Rev. Lett. 66, 3285 (1991) (Ref. [5]); Li et al., Phys. Rev. Lett. 67, 1630 (1991) (Ref. [6]);
and, Jiang et al., Phys. Rev. B 44, 8107 (1991) (Ref. [7]). Among them, work by Jiang et al. [7] is done at somewhat higher temperatures and covers a broader range of applied field.

In the first half of this subsection, we shall discuss and compare the first three experiments; in the second half, we summarize the findings from the last one.

A1

A. Sample quality

The mobility of the samples used in Ref. [4] was not given in the paper. However, a rising resistivity at $\nu = 1/5$ as the temperature is lowered is observed, although at temperatures reported in the paper the resistivity at $\nu = 1/5$ is a well-defined local minimum. The lowest temperature state at $\nu = 1/5$ is therefore insulating in these samples. What impact, if any, it may have on the insulating phases around $\nu = 1/5$ is not clear.

The mobilities quoted in Ref. [5] for various samples used range from $4 \times 10^6$ to $9 \times 10^6$ cm$^2$V$^{-1}$s$^{-1}$, and those quoted in Ref. [6] were somewhat lower $\sim 1 \times 10^6$ Hall liquid state was observed. The size of the FQHE gap at $\nu = 1/5$ was reported to be $\sim 0.1K$ in Ref. [6]; it was not given in Ref. [5].

B. Threshold field, broad band noise, and their temperature dependence

The observed apparent depinning threshold field is $\sim 1mVcm^{-1}$ in Ref. [4], which is similar to that in Ref. [5], found to be $\sim 1.2mVcm^{-1}$, but differs greatly from the much larger value of $\sim 250mVcm^{-1}$ found in Ref. [5]. It is certainly worth checking if in Ref. [5] there is another threshold at fields similar to those in Refs. [4,6] which may have escaped observation. We also note here that the conventional four-terminal measurement was done in Ref. [4], but a two-terminal geometry was adopted in the nonlinear conductivity measurement in Ref. [5]. In Ref. [6], both the four-terminal and the two-terminal measurements were carried
out in the low field (less than $10mVcm^{-1}$) regime, and were found to give essentially the same results for the depinning threshold field and the nonlinear conductivity.

In addition, at fields above the depinning threshold, broad band noise (BBN) in the frequency range of $\sim 1KHz$ was observed and its power spectrum measured in Refs. [4,5]. But in Ref. [4], the external field at which the BBN was observed is greater by a factor of $\sim 70$ than that at which the nonlinear conductivity set in. No explanation was given for this rather large discrepancy [4]. In Ref. [4], the threshold field for the onset of BBN and that for the nonlinear conductivity were found to agree with each other.

Despite the wide range of the observed depinning threshold field itself, its temperature dependence was remarkably similar in Refs. [4,5]. It decreases smoothly and continuously as temperature rises, and becomes unobservable at about $\sim 120mK$. It was suggested in Refs. [4,5] that the disappearance of an apparent depinning field could be due to the melting of the Wigner crystal at this temperature. The BBN in Refs. [4,5] had roughly the same temperature dependence as the depinning threshold field. In particular, it disappears at about the same temperature as the threshold field in Ref. [4], but in Ref. [5], it was found the disappear earlier, at $\sim 60mK$. It was therefore cautioned in Ref. [4] that it remains unclear what is the significance of these temperatures.

We should also note that although Refs. [4] and [5] reported similar threshold fields, the details of the depinning transition appear to be rather different: In Ref. [5], a step-function-like increase in differential conductivity was observed at the depinning transition while a gradual and smooth one was found in Ref. [4] at and above the depinning field. The transition was the sharpest for the insulating phase at $\nu = 0.214$, slightly above $\nu = 1/5$ in Ref. [4]. In Ref. [5] with much larger depinning fields, the transition also appears very sharp for all filling factors reported, down to $\nu \sim 0.15$.

C. Magnetic field dependence of the threshold field

In Refs. [4] and [5], studies were carried out for the magnetic field, or the filling factor.
dependence of the depinning threshold field at temperatures low enough so that the threshold field does not vary with temperature. This information is useful as such dependences near the transition from the insulating phase into the FQHE liquid phase at and immediately near $\nu = 1/5$ may yield information about the nature of the phase transition \cite{17}.

In Ref. \cite{5}, the threshold field was found to decrease rapidly and monotonically approaching the FQHE liquid phase at $\nu = 1/5$ on both sides of the magnetic field. This differs from the observations in Ref. \cite{6} in which the threshold field was found to increase rapidly on both sides of $\nu = 1/5$ as the (reentrant) insulating-FQHE transition was approached. For large enough field, or small enough filling factor, as the insulator-FQHE transition was far enough, the depinning threshold field was found to increase with the magnetic field in Ref. \cite{5} as was in Ref. \cite{4}.

D. Temperature dependence of the differential resistivity above the apparent threshold field

In Ref. \cite{6}, only the experiments done at the lowest temperatures ($\sim 22mK$) were reported, and it was found that the differential resistivity is $T$-independent above the threshold field at this temperature. In Ref. \cite{6} with a much larger depinning threshold field, a wider temperature range (between $20mK$ and $100mK$) was examined. It was found also that the differential resistivity is $T$-independent above the depinning transition, but strongly $T$-dependent below it. From these results, we may conclude that above the threshold field, conduction is no longer activated. We may also conclude that the main damping mechanism above the threshold field should be the same for all temperatures in \cite{6}.

In Ref. \cite{4}, the differential resistivity remains field-dependent above the depinning transition and the temperature dependence is difficult to establish clearly with the data given there.

E. Size of the insulating gap

Below the depinning threshold field, the conduction in the insulating phase is presumably
due to thermally excited charged defects/excitations. If there is only a single mechanism for the thermal excitations, the measurement of the temperature dependence of the conductivity yields the size of the gap for such excitations.

The size of the insulating gap was not reported in Refs. [5,6]. In Ref. [4], the temperature dependence of the longitudinal resistivity in the insulating phases shows a complicated behavior, apparently involving two regimes with different temperature dependence in the temperature range of 30 — 100 mK. The insulating transport gap was reported to be $\sim 0.5K$. We note that this gap is much larger than the critical temperature ($\sim 0.12K$) at which the threshold field disappears, postulated to be related to the melting of the Wigner crystal [4]. Furthermore, the insulating gap is well-defined at temperatures above the so-called melting temperature. This may be taken as a sign that the amplitude of the charge density does not vary too much in this temperature range, since the size of the insulating gap is presumably strongly dependent on the charge-density wave amplitude.

**A2** Summary of findings from Ref. [7]

We believe that the samples used in [7] are probably of higher quality than those used in Refs. [4-6], because the FQHE gap at $\nu = 1/5$ in the samples of Ref. [7] was 1.1K, much larger than that in the others. Interestingly, the activation gap in the insulating phases is larger by a factor of 3 — 4, than that in [4] where the $\nu = 1/5$ state is insulating at the lowest temperatures, for the same filling factors. If the size of the insulating gap can be used as a measure of the strength of the insulating phases, we then must conclude that as sample quality improves, or as disorder becomes weaker, the insulating phases become stronger. We believe this is strong evidence supporting the notion that these insulating phases are primarily due to interaction, and not disorder.

Threshold nonlinearity in conductivity was also observed in [7] for applied voltages
\[ \sim 0.1 \text{ mV across samples of typical dimensions } 3\text{mm} \times 3\text{mm}, \text{ giving a depinning field } \sim 1.3\text{mV cm}^{-1}, \text{ similar to those in [1] and [3]. However, at and above the threshold field, there is no clear step-wise change in the resistivity. This is similar to [4], but differs from [3] in which lower temperatures were reached.} \]

Although the threshold field diminishes at a temperature about 100 mK, below and well above the threshold field (within a factor of 10 of \( E_{Th} \)), \( \rho_{xx} \) remains thermally activated. This differs qualitatively from the observations in [3] which are done at generally lower temperatures. Furthermore, the conduction activation gap remains the same below and well above the threshold field. In addition, not only the gap itself is an order of magnitude larger than the so-called critical temperature, but also the activated behavior extends above the critical temperature by at least a factor of 3 — 4. Two related conclusions that one can draw from this important observation are: 1). The dominant damping mechanism giving rise to resistivity above and below the low-field threshold field ought to be the same; and 2). The solid retains its identity up to temperatures higher than the critical temperature by at least a factor of 3 – 4. The first point has a natural explanation from the coupled motion of a sliding Wigner crystal with thermally activated carriers. The second point supports the notion that the critical \( T \) at which the apparent threshold disappears does not signify the melting temperature of the Wigner crystal. We offer a speculation for this experimental observation in the next subsection.

A voltage breakdown is observed in the high field limit around 50 mV in [7], in the same range as where the work in [3] was carried out. It is suggested by the authors of [7] that the power dissipation observed in [7] is consistent with an excessive electron heating interpretation.

In addition, it is also found in [4] that the activation gap in the insulating phases may change upon thermal cycling to room temperature by a factor of two, as does the conductivity threshold. Upon thermal cycling to room temperature, the disorder configuration must change due to diffusion or other thermally activated processes. This suggests that disorder also plays an important role in determining the insulating gap. There is also a correlation
between the density and the activation gap, which may be taken as evidence for the role of interaction. The insulating gap decreases toward \( \nu = 1/5 \) in its vicinity, similar to the work in [4]. Although the activation gap itself changes upon thermal cycling, its variation with filling factor is the same within a cycle. Therefore the filling-factor dependence of the insulating gap around the \( \nu = 1/5 \) fractional quantum Hall liquid state may be intrinsic to a clean system.

We see from the above discussion that the experimental situation is not very clear, and different experiments often contradict each other in important details. In the next subsection, we attempt to interpret some of these experiments related to the nonlinear conductivity in light of our theoretical results.

**B. Comparison of Theory and Experiments in the DC limit**

According to the behavior of the conductivity at fields above threshold, the current experiments reviewed above fall into two categories:

1). The *differential* conductivity is voltage- and temperature-independent above threshold. This is reported in Ref. [5] which finds a very large threshold field (\( \sim 50 \mathrm{mV} \)) at the lowest temperatures \( \sim 36 \mathrm{mK} \), and in Ref. [6] which finds a much smaller threshold field (\( \sim 0.2 \mathrm{mV} \)) and is done at very low temperatures \( \sim 20 \mathrm{mK} \).

2). The differential conductivity above the threshold field continues to depend on voltage and temperature, as reported in Ref. [4] and Ref. [7]. The threshold field is found to be small (\( \sim 0.2 \mathrm{mV} \)) [4,7]. The temperature dependence of the differential resistivity is found in Ref. [4] to be thermally activated with an activation gap identical to that below threshold, for temperatures between \( \sim 80 \ \mathrm{mK} \) and \( \sim 200 \ \mathrm{mK} \). However, it is also reported in Ref. [7] that there exists a second threshold field at \( \sim 30 \mathrm{mV} \) above which the differential resistivity is no longer field- or temperature-dependent, and its value is consistent with those in Ref. [4,6].

We first compare our theory with experiments done at the lowest temperatures where
thermally activated free carriers are unimportant, i.e., the experiments in the first category described above. The predicted \( I - V \) curve from Eq. 109 agrees very well with the low temperature experiments (Compare Figs. 1 and 2 from this work to Fig. 2 in Ref. [3] and Fig. 2 in Ref. [4]) – that is, the differential DC resistivity does not depend on the driving field, or equivalently on the resulting current above the sliding threshold. In addition, the intercept of the \( I - V \) curve from the high velocity perturbation theory gives also a measure of the sliding threshold field. From Eqs. 109 and 111, one sees that this statement can only be true when, in plotting the \( I - V \), the field \( V \) is measured along the direction of the current flow.

We now consider the large field (> 30mV), therefore high velocity, data reported in Ref. [7] in which electron heating was invoked to explain the data. We wish to propose an alternative possibility. The differential conductivity in the \( V > 30mV \) regime is field and temperature independent, and has roughly the same value (\( \sim 10^5 ohms \)) as the lowest temperature data in Ref. [3]. This experimental fact [31] is consistent with our result that the large-velocity differential resistivity is \( \sigma_{xx}^F \)-independent. We propose that the second apparent threshold field in fact signifies a crossover from a regime where disorder effects enter non-perturbatively to one where they can be treated perturbatively as was done in this paper. As the velocity decreases, it is expected that our second-order perturbation theory becomes no longer valid, and that \( \sigma_{xx}^F \) affects the differential conductivity. In this limit, the effective damping constant may be of order \( \lambda_1 \propto (\sigma_{xx}^F)^{-1} \), and would lead to an activated temperature dependence as observed. This is, however, beyond the range we can treat by the present perturbation theory.

Our predicted \( V - I \) characteristic along the direction of the current \( \vec{j} \) for a sliding Wigner crystal: \( E = \rho_m \lambda j/\rho_F^2 + E_{Th} \) agrees very well with the experiments in III done at the lowest temperatures and the high-field results in II and [6]. The details of the pinning-depinning transition are not accessible from the present perturbation theory and there is no clear picture from the experiments. The strong temperature dependence of the differential conductivity above the small threshold field observed in [6] at somewhat
elevated temperatures indicates the possible role of thermally excited free carriers in the sliding regime beyond that revealed by our perturbation theory.

We finally speculate on the possible cause of the disappearance of the apparent threshold field at temperatures around $100 - 200 \text{ mK}$ observed in several recent experiments [4–7]. It was suggested that it may signify the melting of the Wigner crystal in Refs. [4,5], but this suggestion was called into question in Refs. [6,7]. We here wish to echo the cautions that have already been raised. First of all, one could estimate roughly the classical melting temperature from the shear modulus, assuming a Kosterlitz-Thouless (KT) melting mechanism [29]. We find a temperature $T_{KT}$ around $400 - 600 \text{ mK}$. Secondly, the magneto-optical measurements [30] indicate the possibility of a still higher melting temperature of $1 - 2 \text{ K}$. Thirdly, above the so-determined “melting temperature”, the measured $I - V$ curves continue to be highly nonlinear [32,33], which is also inconsistent with the notion that the Wigner crystal has already melted into a liquid. A natural alternative to this interpretation is to assume what has been observed is in fact the thermal depinning of the Wigner crystal. At temperatures above $100 - 200 \text{ mK}$ in these experiments, we propose that thermal fluctuations destroy the pinning due to disorder potential, and the subsequent sliding motion is one of a solid with large thermal fluctuations and subjected to disorder.

C. Theory and Experiments for AC + DC Interferences

Finally, we briefly remark on some recent experimental work [32,33] on the AC + DC interference effects in a sliding Wigner crystal around filling factor $\nu = 1/5$. In the absence of a DC current, the AC experiments by Li et al., in the frequency range of $\sim 30 - 100 \text{ MHz}$ have established the capacitive response of the pinned Wigner crystal [32]. A detailed analysis and the possible implications for the nature of the insulating phases from the pure AC measurements were given in their original paper [32].

So far, there have been no reports on the observation of the Shapiro anomaly in the DC response due to an AC field. However, preliminary data have established the interference
effects in the AC response in the presence of a large DC current. This situation is not
surprising since from the experience with the CDW systems [11], the requirements on the
sample quality are usually less stringent for the observation of the latter effect.

In Fig. 3, we give a qualitative sketch of the calculated inductive anomaly from Eqs. 99
and 101 in Sec. VIB. The dotted portion of the curve indicates that our linear AC response
theory is not reliable at AC frequencies very close to the washboard frequency of the DC
current. Nonetheless, its overall shape is in good agreement with the experimental observa-
tions, where it was found that the capacitive AC response in the absence of the DC current
becomes inductive in its presence at frequencies below 2 MHz [33]. However, a direct tran-
sition from the inductive response to capacitive response as the AC frequency increases has
not been observed due to the limited AC frequency range attainable in the interference effect
measurements [33].

Our theory (see Eq. 99 and Eq. 103) also suggests that at low AC frequencies, the linear
slope of Imρ(ω) in ω at small ω would depend on the DC current as $\sim 1/j_{DC}^2$. To observe
this effect experimentally, care must be taken to ensure that the AC-response remains in the
linear regime, i.e., the AC-component amplitude must be reduced along with its frequency ω
to ensure the accuracy of the small parameter expansion of the Bessel’s function (see Eq. 93
and Eq. 97).

Clearly, further experiments are needed to test other aspects of our theory in terms of the
AC + DC interference effects. We refer the reader to Sec. VI for a more detailed description
of what one might expect from the present perturbation theory.

X. CONCLUSIONS

In conclusion, we have studied the sliding motion of a Wigner crystal in a strong magnetic
field. We obtain the form of the nonlinear resistivity in the regime of large sliding velocity.
The Hall resistance of a sliding Wigner crystal is found not to be changed by disorder
scattering. We predict the AC + DC interference effects and compare the present case to
the CDW systems and point out the differences. We also give a brief summary of the available experiments that measure the nonlinear conductivity of the insulating phases around filling factor $\nu = 1/5$. At low temperatures and/or large depinning fields, the experimentally observed nonlinear $I - V$ curves are in agreement with our theoretical results. In the regime of smaller fields and elevated temperatures, results from various experiments differ in important ways as we discussed in the paper, and a definitive comparison to the present theory is difficult to make.

ACKNOWLEDGMENTS

We thank Drs. Y. P. Li, D. C. Tsui, H. W. Jiang, and H. Stormer for discussions of the current experiments. We are particularly grateful to Y. P. Li and D. C. Tsui for showing us their data prior to publication and we wish to thank Dr. S. N. Coppersmith for many helpful conversations and a careful reading of the manuscript.
REFERENCES

[1] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).

[2] The Quantum Hall Effect, 2nd. ed., eds. R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).

[3] H. W. Jiang, R. L. Willett, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 65, 633 (1990).

[4] V. J. Goldman, M. Santos, M. Shayegan, J. E. Cunningham, Phys. Rev. Lett. 65, 2189 (1990).

[5] F. I. B. Williams, P. A. Wright, R. G. Clark, E. Y. Andrei, G. Deville, D. C. Glattli, O. Probst, B. Etienne, C. Dorin, C. T. Foxon, and J. J. Harris, Phys. Rev. Lett. 66, 3285 (1991).

[6] Y. P. Li, T. Sajoto, L. W. Engel, D. C. Tsui, and M. Shayegan, Phys. Rev. Lett. 67, 1630 (1991); and private communications.

[7] H. W. Jiang, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, Phys. Rev. B 44, 8107 (1991).

[8] M. B. Santos, Y. W. Suen, M. Shayagan, Y. P. Li, L. W. Engel, and D. C. Tsui, Phys. Rev. Lett. 68, 1188 (1992); M. B. Santos, J. Jo, Y. W. Suen, L. W. Engel, and M. Shayegan, Phys. Rev. B 46, 13639 (1992); H. C. Manoharan and M. Shayegan, unpublished.

[9] X. Zhu and S. G. Louie, Phys. Rev. Lett. 70, 335 (1993); and unpublished; R. Price, P. M. Platzman, and S. He, Phys. Rev. Lett. 70, 339 (1993).

[10] A brief account of our work has been given in, X. J. Zhu, P. B. Littlewood, and A. J. Millis, Phys. Rev. Lett. 72, 2255 (1994).

[11] For a review on CDW systems, see, G. Grüner, Rev. Mod. Phys. 60, 1129 (1988).
[12] L. Sneddon, M. C. Cross, and D. S. Fisher, Phys. Rev. Lett. 49, 292 (1982); L. Sneddon, Phys. Rev. B 29, 719, 725 (1984).

[13] H. Matsukawa and H. Takayama, J. Phys. Soc. Jap. 56, 1507, 1522 (1987).

[14] S. N. Coppersmith, Phys. Rev. Lett. 65, 1044 (1990); S. N. Coppersmith and A. J. Millis, Phys. Rev. B 44, 7799 (1991); S. N. Coppersmith, Phys. Rev. B 34, 2073 (1986).

[15] A. Schmid and W. Hauger, J. Low. Temp. Phys. 11, 667 (1973); A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 65, 1704 (1973), 68, 1915 (1975) [Sov. Phys. JETP 38, 854 (1974), 41, 960 (1996)]; for a recent numerical simulation of the effects of dislocations on the nonlinear $I-V$ characteristics of a sliding vortex lattice, see, e.g., A. C. Shi and A. J. Berlinsky, Phys. Rev. Lett. 67, 1926 (1991).

[16] H. Fukuyama and P. A. Lee, Phys. Rev. B 17, 535, (1978); P. A. Lee and H. Fukuyama, Phys. Rev. B 17, 542 (1978); P. A. Lee and T. M. Rice, Phys. Rev. B 19, 3970 (1979); T. M. Rice, P. A. Lee, and M. C. Cross, Phys. Rev. B 20, 1345 (1979).

[17] B. G. A. Normand, P. B. Littlewood, and A. J. Millis, Phys. Rev. B 46, 3920 (1992); and references therein. See also, A. J. Millis and P. B. Littlewood, Phys. Rev. B xx, xxxx (1994).

[18] V. M. Pudalov, M. D’Iorio, S. V. Kravchenko, and J. W. Campbell, Phys. Rev. Lett. 70, 1866 (1993); M. D’Iorio, V. M. Pudalov, and S. G. Semenchinsky, Phys. Rev. B 46, 15992 (1992).

[19] L. Bonsall and A. A. Maradudin, Phys. Rev. B. 15, 1959 (1977).

[20] R. Coté and A. H. MacDonald, Phys. Rev. Lett. 65, 2662 (1990); Phys. Rev. B 44, 8759 (1991).

[21] D. S. Fisher, Phys. Rev. B 31, 1396 (1985).

[22] H. Fukuyama and P. A. Lee, Phys. Rev. B 18, 6245 (1978); H. Fukuyama, J. Phys. Soc.
[23] H. W. Jiang and A. J. Dahm, Phys. Rev. Lett. 62, 1396 (1989).

[24] S. C. Zhang, S. Kivelson, and D. H. Lee, Phys. Rev. Lett. 69, 1252 (1992); S. Kivelson, D. H. Lee, and S. C. Zhang, Phys. Rev. B 46, 2223 (1992).

[25] O. Viehweger and K. B. Efetov, Phys. Rev. B 44, 1168 (1991); Y. Imry, Phys. Rev. Lett. 71, 1868 (1993).

[26] V. J. Goldman, J. K. Wang, B. Su, and M. Shayegan, Phys. Rev. Lett. 70, 647 (1993); T. Sajoto, Y. P. Li, L. W. Engel, D. C. Tsui, and M. Shayegan, Phys. Rev. Lett. 70, 2321 (1993); S. I. Dorozhkin et al., JETP Lett. 57, 58 (1993) [Pis'ma Zh. Eksp. Teor. Fiz. 57, 55 (1993)]; C. E. Johnson and H. W. Jiang, Phys. Rev. B 48, 2823 (1993); H. W. Jiang, C. E. Johnson, K. L. Wang, and S. T. Hannahs, Phys. Rev. Lett. 71, 1439 (1993).

[27] We should also caution that $\sigma_{xx}^F$ is itself $(\vec{k}, \omega)$-dependent. In our discussion, we have implicitly assumed that the $(\vec{k}, \omega)$-range of interest in the sliding Wigner crystal lies within the conductivity limit of $\sigma_{xx}^F$. For more discussion, see Appendix A in Ref. [17].

[28] P. B. Littlewood, Solid State Comm. 65, 1347 (1988); Phys. Rev. B 36, 3108 (1987).

[29] J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973); D. S. Fisher, Phys. Rev. B 26, 5009 (1982).

[30] H. Buhmann, W. Joss, K. v. Klitzing, I. V. Kukushkin, A. S. Plaut, G. Martinez, K. Ploog, and V. B. Timofeev, Phys. Rev. Lett. 66, 926 (1991).

[31] This is despite the fact that their behaviors below the upper threshold field are very different, and dependent on temperature [3][4], often in an activated fashion suggesting the involvement of dissipation in the screened longitudinal channel [7].

[32] Y. P. Li, D. C. Tsui, T. Sajoto, L. W. Engel, L. Xu, M. Santos, and M. Shayegan,
unpublished.

[33] Y. P. Li, D. C. Tsui, L. W. Engel, T. Sajoto, L. Xu, M. Santos, and M. Shayegan, unpublished; and Y. P. Li, Ph. D. thesis, (Princeton University, 1994), unpublished.
Figure Captions

Fig. 1. Theoretical nonlinear $I-V$ properties of a sliding Wigner crystal in a strong magnetic field in the presence of disorder for the Hall bar geometry. $I$ is the total current, $V_L$ is the field component parallel to $I$, and $V_H$ is the field component perpendicular to $I$. The crossing point of the two curves is where the Hall angle becomes 45°. The dotted portion in $V_L$ indicates that the present perturbation theory is not reliable when the external field is only slightly greater than the depinning threshold field, indicated by $V_{Th}$. We have assumed $\omega_c$ greater than $\lambda$. The dashed portion is the conductivity due to free carriers at fields below the threshold field. It is extended to fields slightly above $V_{Th}$ for clarity of illustration. The line of $V_H$ as a function of $I$ is valid and extends all the way to zero $I$ provided that the transport current is due to the sliding of the whole Wigner crystal under a combined total external field $\sqrt{V_H^2 + V_L^2}$ greater than $V_{Th}$.

Fig. 2. The theoretical differential resistivity of a sliding Wigner crystal in a strong magnetic field as a function of the external field for the Hall bar geometry. The depinning threshold field is indicated by $V_{Th}$. We have assumed $\omega_c$ greater than $\lambda$. The dotted portion of the $dV_L/dI$ curve indicates that the theory is not reliable near threshold field. The dashed portion of the $dV_L/dI$ curve indicates the conduction of thermally excited free carriers in this regime where $dV_L/dI = \rho_{xx}^{F}$, the diagonal resistivity of the free carriers.

Fig. 3. The inductive anomaly in the imaginary part of the AC response of a Wigner crystal in the presence of a large DC current. The dotted portion indicates that the linear AC response theory is not reliable near frequency locking.