Quantum phase transition in the Plaquette lattice with anisotropic spin exchange

A. Voigt ∗
Center for Simulational Physics
Department of Physics and Astronomy
University of Georgia, Athens GA 30605, USA

November 21, 2018

Abstract

I study the influence of anisotropic spin exchange on a quantum phase transition in the Plaquette lattice driven by the purely quantum effect of singlet formation. I study the influence of i) a Dzyaloshinskii-Moriya exchange and ii) four spin exchange on the transition point by evaluating spin–spin correlations and the spin gap with exact diagonalization. The results point to a stabilization of the Néel-like long range order when the Dzyaloshinskii-Moriya exchange is added, whereas the four-spin exchange might stabilize the singlet order as well as the Néel-like order depending on its strength.

quantum antiferromagnets, Plaquette lattice, anisotropic spin exchange
PACS: 75.10.Jm, 75.50.Ee, 75.40.Mg

1 Introduction

Low dimensional quantum antiferromagnets show a wide variety of magnetic low-temperature behavior, like magnetic long range order or spin disorder with or without a spin gap. Based on the findings of a recent experimental study of a new compound \( Na_5RbCu_4(AsO_4)_4Cl_2 \) [1] I will examine a quantum spin system which shows a purely quantum phase transition by singlet formation. This and related models have been studied previously [2, 3, 4]. From this results one argues that the observed low-temperature magnetic transition in \( Na_5RbCu_4(AsO_4)_4Cl_2 \) can probably not be explained within a simple Heisenberg-type model approach. Therefore additional anisotropic spin interactions have to be taken into account and in this paper I will study the influence of a Dzyaloshinskii-Moriya (DM) interaction and a four-spin exchange interaction on the quantum phase transition in the Plaquette lattice.

∗e-mail: andreas@physast.uga.edu, Tel. +1-706-542-3884, Fax +1-706-542-2492
2 The Model

I consider an antiferromagnet on a two-dimensional square lattice with different types of interactions:

\[
H = J_P \sum_{\square} \hat{s}_i \hat{s}_j + J_n \sum_{\square \not=} \hat{s}_i \hat{s}_j + \sum_{\square} D_{ij}(\hat{s}_i \times \hat{s}_j) + W_{4S} \sum_{\square} (\hat{s}_i \hat{s}_j \hat{s}_k \hat{s}_l)
\] (1)

The interactions are: \( J_P \) – Heisenberg type between 4 spins building a plaquette \( \square \), \( J_n \) – Heisenberg type between plaquettes on a simple square lattice, \( D_{ij} \) – DM-type between plaquette spins and \( W_{4S} \) – a four-spin interaction between plaquette spin respectively.

Figure 1: The N=16 lattice with periodic boundary conditions and interactions: \( J_P, J_n \) and \( D_{ij} \).

The DM interaction \( D_{ij} \) is a vector the components of which have to be extracted from the lattice symmetries [5]. Here we consider one particular realization \( D_{ij} = (W_{DM}, W_{DM}, 0) \) where the x- and y-components are equal. Due to symmetry considerations the interaction reverse sign on adjacent plaquette bonds (see Fig[1]). The four-spin exchange \( W_{4S} \) emerges from the strong-coupling expansion of the Hubbard model and has been recently discussed in ladders and two-dimensional lattices [6, 7].

3 Results

It has been shown that without anisotropic exchange (i.e. \( W_{DM} = 0 \) and \( W_{4S} = 0 \)) there is a critical \( J_n \approx 0.55 \) where the Plaquette lattice changes from a disordered spin gap state to a long-range ordered Néel-like ground state [4]. I calculate for small finite lattices (N=16,20) the quantum ground and first excited states with exact diagonalization.
By analyzing the differential spin correlations between plaquettes and the spin gap for selected values of $W_{DM}$ and $W_{4S}$ I extract the critical value of $J_n$ where the phase transition takes place.

For the particular chosen DM interaction we see a shift of $J_n^{\text{crit}}$ to smaller values. We argue that this type of DM interaction always destabilizes the plaquette order by disturbing the singlet building on them. Interestingly enough, a related DM interaction on the $J_1$–$J_2$ square lattice studied previously [5] also showed a similar effect.

For small values of $W_{4S}$ we observe a similar behavior as in the case of the DM interaction. But interestingly for larger $W_{4S}$ the tendency to smaller $J_n^{\text{crit}}$ reverses and it seems that at some strength of $W_{4S}$ the corresponding $J_n^{\text{crit}}$ becomes larger then for the isotropic system. That means that the four-spin exchange $W_{4S}$ can have two opposite effect depending in its strength. It either
stabilizes the Néel-like order on the lattice or (if strong enough) can help to build singlets on the plaquettes.

Additional calculations on larger lattices will be carried out in order to confirm these conclusions.

There are several open questions; about the influence of the particular chosen symmetry of the DM interaction, about the result of a mixing of both anisotropic interactions (which might reveal new and interesting order phenomena) and about the nature of the phase transition (first or second order). In a more detailed study I will consider those and other problems and report the results elsewhere.

This work was supported by NSF grant ACI-0081789.

References

[1] J.A. Clayhold, M. Ulutagay-Kartin, S.-J. Hwu, M.-H. Whangbo, and A. Voigt. Phys. Rev. B, 66:052403, 2002.

[2] A. Koga, S. Kumada, and N. Kawakami. J. Phys. Soc. Jpn., 68(2):642, 1999.

[3] R.R.P. Singh, Z. Weihong, C. J. Hamer, and J. Oitmaa. Phys. Rev. B, 60(10):7278, 1999.

[4] A. Voigt. Computer Physics Communication, 146(1):125, 2002.

[5] A. Voigt and J. Richter. J. Phys. C: Cond. Mat., 8:5059, 1996.

[6] M. M. Matsuda, K. Katsumata, R. S. Eccleston, S. Brehmer, and H.-J. Mikeska. Phys. Rev. B, 62:8903, 2000.

[7] R. Coldea, S. M. Hayden, G. Aeppli, T. G. Perring, C. D. Frost, T. E. Mason, S.-W. Cheong, and Z. Fisk. Phys. Rev. Lett., 86:5377, 2001.