Cooperation Prevails When Individuals Adjust Their Social Ties

NetSci 2007, NY

Francisco C. Santos
Université Libre de Bruxelles, Belgium

In collaboration with:

Jorge M. Pacheco
Universidade de Lisboa, Portugal

and

Tom Lenaerts
Vrije Universiteit Brussel, Belgium
Cooperation is essential for the evolution of reproductive entities

genes *cooperate* to form the genome
cells *cooperate* to form multi-cellular organisms
Individuals *cooperate* to form groups and societies
Human culture is a *cooperative* process.

[ Nowak, *Evolutionary Dynamics: Exploring the Equations of Life*, HUP06 ]

---

*Cooperation is on the basis of some of the major transitions In evolution*

[ Maynard-Smith & Szathmáry, *The major transitions in evolution*, OUP95 ]
The benefits and costs of cooperation

Following Hamilton, Trivers and Wilson:

Cooperation requires at least \textit{two} individuals:
- \textbf{A}: the one providing cooperation (DONOR)
- \textbf{B}: the one benefiting from cooperation (RECEIVER)

Cooperation involves a cost $c$ to \textbf{A}

cooperation confers a benefit $b$ to \textbf{B}

with $b > c$ \rightarrow \textit{prisoner’s dilemma}

\begin{array}{c|cc}
\text{C} & \text{D} \\
\hline
\text{C} & b - c & -c \\
\text{D} & b & 0 \\
\end{array}
Generalizing...

Payoff matrix:

- **R (REWARD)**: mutual cooperation
- **P (PUNISHMENT)**: mutual defection
- **S**: sucker’s payoff
- **T**: temptation to defect

|     | C   | D   |
|-----|-----|-----|
| C   | b - c | -c  |
| D   | b    | 0   |
Generalizing...

Payoff matrix:

- **R** (REWARD): mutual cooperation
- **P** (PUNISHMENT): mutual defection
- **S**: sucker’s payoff
- **T**: temptation to defect

|     | C  | D   |
|-----|----|-----|
| C   | R  | -c  |
| D   | b  | 0   |
Generalizing...

Payoff matrix:

\[ \begin{array}{cc}
   & C & D \\
C & R & -c \\
D & b & P \\
\end{array} \]

- **R** (REWARD): mutual cooperation
- **P** (PUNISHMENT): mutual defection
- **S**: sucker’s payoff
- **T**: temptation to defect

Without loss of generality we can normalize the difference between mutual cooperation and mutual defection by assuming \( P=0 \) and \( R=1 \).
Generalizing...

Payoff matrix:

- $R$ (REWARD): mutual cooperation
- $P$ (PUNISHMENT): mutual defection
- $S$: sucker’s payoff
- $T$: temptation to defect

... leaving two parameters $0 \leq T \leq 2$ and $-1 \leq S \leq 1$
Generalizing...

Payoff matrix:

- **R** (REWARD): mutual cooperation
- **P** (PUNISHMENT): mutual defection
- **S**: sucker’s payoff
- **T**: temptation to defect

Depending on the ranking of the payoffs we can have 3 well-known dilemmas...
Different ordering -> Different tensions

SG: snowdrift game: $T > 1 > S > 0$
SH: stag-hunt game: $1 > T > 0 > S$
PD: prisoner’s dilemma: $T > 1 > 0 > S$

Macy & Flache, PNAS 99(3), 7229 (2002)
Formally, these dilemmas span the parameter space of

\( T \) (temptation to defect = greed) and 

\( S \) (sucker’s payoff = fear).

FcS, Pacheco, Lenaerts, 
PNAS 103 (2006) 3490-3494
The paradox of cooperation

Ex: prisoner’s dilemma -> cooperation never survives evolution

however, cooperation is widespread in nature

what’s wrong? Evolutionary game theory?
The paradox of cooperation

Ex: prisoner’s dilemma -> cooperation never survives evolution

however, cooperation is widespread in nature

what’s wrong? Evolutionary game theory?

ways out of the conundrum:

- kin-selection  
  *all in the family...*

- multi-level/group selection  
  *successful (cooperative) groups grow faster and divide more often...*

- direct reciprocity  
  *I scratch your back & you scratch mine...*

- indirect reciprocity  
  *I scratch your back & someone else scratches mine...*

- > structured populations  
  *Well-mixed assumption is unrealistic -> Evolution on graphs*
Evolution on graphs *pairwise comparison*

Imitate a random neighbor with a probability proportional to the payoff difference.

- Vertex \( x \) plays \( k_x \) times per generation and accumulates payoff \( f_x \).
- Choose a random neighbor \( y \) with payoff \( f_y \).
- Replace strategy \( s_x \) by \( s_y \) with probability:

\[
p = \frac{1}{1 + e^{-\beta(f_x - f_y)}}
\]

Traulsen, Nowak, Pacheco, Phys. Rev. E 74 (2006) 011909
Some recent conclusions: static networks

In more realistic networks of interactions, cooperation is much easier to sustain than in homogeneous populations.

The enhancement of cooperation results from the interplay between heterogeneity, degree-degree correlations and long-range connections.

Topological constraints provide a new route to the emergence of cooperation

FcS, Pacheco, Phys. Rev. Lett. 95 (2005) 098104
FcS, Pacheco, Lenaerts, PNAS 103 (2006) 3490-3494
Some recent conclusions: static networks

However...

-Specific population structures work as promoters of cooperation but... only in the case of sparse graphs.

-A simple relation has been obtained (for the PD & weak selection) stating that $b/c > z$ to cooperation to thrive.

Ohtsuki, Hauert, Lieberman, Nowak, *Nature* 411: 502-505 (2006)

...observed $z_{social} = 2$ to 170 !!!
Evolving social ties

Social networks are not static!!
- Unsatisfied individuals will tend to rewire/remove their connections.

FcS, Pacheco, Lenaerts, PLoS CB (2006) 2 (12):e178
Evolving social ties

Social networks *are not static!!*
- Unsatisfied individuals will tend to rewire/remove their connections.

What is the result of a co-evolution between strategy and structure?
- Will this produce the topologies that are observed in reality?
- What is the effect on the frequency of cooperators?
- Does it depend on the type of social tension?

FcS, Pacheco, Lenaerts, PLoS CB (2006) 2 (12):e178
Topological evolution

if you have a well-defined behaviour: C or D

what is your best (most convenient) partner?

for ALL social dilemmas

the best partner for any strategy is always a Cooperator

consequently, irrespective of the dilemma:

Cs look for Cs to cooperate with
Ds look for Cs to exploit

--> potential conflicts
Resolving potential conflicts

- choose randomly an edge (A<->B)
- B is satisfied (A is C); A is NOT satisfied (B is D);
- B wants to keep link; A wants to change;
- with probability $p_A$ rewires to a 2nd neighbour;
- with probability $(1-p)$ B keeps link;

$$p = \frac{1}{1 + e^{-\beta(\Pi_A - \Pi_B)}}$$
Co-evolution of strategy and structure

**features of the model:**
- rewiring is a self-motivated decision
- population is constant
- average connectivity (z) remains constant
- graph remains connected

now we have *two different* time-scales:

- strategy evolution $T_S$
- structural evolution $T_A$

we define the ratio $W = \frac{T_S}{T_A}$

*and study co-evolution as a function of* $W$

*for ALL social dilemmas*
Results: cooperation

Depending on the topological update rate $W$, cooperation is now able to survive in the entire range of dilemmas.

Strategy evolution favors defection while topological evolution favors cooperation.
Results: cooperation

Depending on the topological update rate $W$, cooperation is now able to survive in the entire range of dilemmas.

Strategy evolution tend to favor defection while topological evolution favors cooperation.

The competition among this two factors create a critical edge of cooperation.
Results: critical network update rate ($W_c$)

PD: $\frac{\text{benefit}}{\text{cost}} = 2$

Since there are more links to be rewired, the larger $z$, the harder it gets.
Results: heterogeneity

PD: heterogeneity is maximal @ $W_{\text{crit}}$...

$D(k)$

$W_{\text{crit}}$

$W < W_{\text{crit}}$

$W > W_{\text{crit}}$

$\frac{\text{benefit}}{\text{cost}} = 2$

Games on graphs
June 8, 2007
Pag. 24
2007
Results: Heterogeneity

Greed-only
- small and highly connected core of nodes -> large difference between the richest and the poorest

Fear-only
- low heterogeneity and clustering

Fear+Greed
- broad-scale distributions

Depending on the underlying social tension and on the rate of update $W$, the network will end up in $single-scale$, $broad-scale$ or $scale-free$ networks.

Each real-world topologies emerge as a result of a $specific social tension$. 
Conclusions

Co-evolution of topology and strategies results in another route to cooperators’ survival.

Structures resulting from this coupled dynamics depend on the social tension and correspond to those found in empirical data.

The more individuals interact the more they must be able to promptly decide future partnerships.

Highly connected social networks can promote high levels of cooperation even for small benefit/cost ratios.
Results: weak vs strong selection

$\frac{\text{benefit}}{\text{cost}} = 2$

$\beta_e = \beta_a = \beta$

The less important is the game (weak selection), the less alert individuals need to be.
Understanding the evolution of cooperation remains a fundamental challenge, for scientists from fields like economics, evolutionary biology, political science, anthropology, etc.

**Common tools:**

*Mathematical Framework*: (Evolutionary) Game Theory

*Metaphors*: Prisoner’s dilemma, etc.