Spatial bunching of same-charge polarization singularities in two-dimensional random vector waves

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(Dated: July 4, 2018)

Topological singularities are ubiquitous in many areas of physics. Polarization singularities are locations at which an aspect of the polarization ellipse of light becomes undetermined or degenerate. At C points the orientation of the ellipse becomes degenerate and lights electric field vector describes a perfect circle in time. In 2D slices of 3D random fields the distribution in space of the C points is reminiscent of that of interacting particles. With near-field experiments we show that when light becomes truly 2D, this has severe consequences for the distribution of C points in space. The most notable change is that the probability of finding two C points with the same topological charge at a vanishing distance is enhanced in a 2D field. This is an unusual finding for any system which exhibits topological singularities as same-charge repulsion is typically observed. All our experimental findings are supported with theory and excellent agreement is found between theory and experiment.

I. INTRODUCTION

Light-based technology has transformed today’s society and will continue to do so, with applications which range from energy harvesting to telecommunications and quantum informatics [1–3]. Increasing control over light’s polarization is one key capability inspiring new developments. For instance, optical fields near nanostructures can be engineered to exhibit locations of circular polarization [4–6], allowing applications such as spin-dependent directional coupling [7], also with local solid-state spin into optical information conversion [8]. Interestingly, points of circular polarization are singularities of the light field, also known as C points [9, 10], widely studied in structured light beams [11–13] and representative of the transverse spin momentum of light [14–17].

More in general, C points are topological defects of the vector field which describes light’s polarization. Knowledge and study of topological defects goes way beyond optics. Currently, dislocations of the local magnetization known as skirnions are being intensively investigated [18–20]. In nematic systems, topological defects continuously attract interest for their fascinating behavior [21, 22]. Besides fascination, it was shown how this kind of defects can even govern the physics of biological system [23], and that their spatial arrangement is representative of intrinsic properties of the system in which they are found [24].

Interestingly, also the large ensemble of C points which naturally arises in random light fields exhibits an emblematic and rigorous spatial distribution [25–28], which resembles that of particles in a simple liquid and only scales with the wavelength of the interfering waves [29]. However, a random wave field can be realized in several ways [29, 30]. The work so far has centered on the investigation of polarization singularities in two-dimensional (2D) slices through random three-dimensional (3D) fields in the paraxial limit. The question now arises how limiting the propagation of light to a truly 2D situation, e.g., by confining it on a flat optical chip, would affect the spatial distribution of its polarization singularities. In such a case, transverse propagation would set a one-to-one relation between the wave propagation direction and the direction of the electric field. Moreover, this would create correlations between right-handed and left-handed polarization that are absent in the three-dimensional fields.

By means of near-field experiments we investigate the spatial distribution of C points in a planar random light field, and reveal crucial differences with respect to existing paraxial theory [20]. We demonstrate that confining light propagation in two dimensions leads to a large increase in the probability of finding C points with the same topological charge at close proximity. This is an exotic behavior for topological singularities, which usually exhibit same-charge repulsion. We relate our experimental findings to light’s handedness and perfectly describe them with a new theoretical model developed for the two-dimensional case.

II. EXPERIMENT AND METHODS

A. Near-field Optical Measurements

In our experiments we map the near field of light waves propagating in the planar chaotic cavity sketched in Fig. 1(a). This is a photonic crystal cavity realized in a silicon-on-insulator platform (220 nm silicon slab) and designed to provide random waves propagation [30]. With a monochromatic laser at the telecom frequencies (λ0 ≃ 1550 nm) we excite a transverse electric (TE) slab mode which results in a random superposition of monochromatic TE waves inside the cavity [31, 32]. With a custom-built near-field scanning optical microscope (NSOM) we probe the light field approximately 20
A comprehensive description of light’s polarization is provided by its Stokes parameters \[ \text{[39]}. \] These parameters are often used to characterize the polarization state of light in the far field, ranging from a simple laser beam to the polarized emission of exotic structures \[ \text{[40]}, \] but they can be used for a local analysis of the near field as well. Figures [1](c)-(f) present the near-field maps of the Stokes parameters for the optical random field inside the chaotic cavity. As result of vector light waves randomly interfering, these patterns are quite difficult to interpret. However, we can spot a few specific features in the morphology of each different map. \( S_1 \) exhibits patterns of spatial modulation approximately half a wavelength wide and several wavelengths long. Depending on their color (sign), these stripy patterns are either oriented along the \( x \) or \( y \) axis. The same observation is valid for \( S_2 \), but here the modulations are oriented at \( \pm 45 \) deg with respect to the horizontal axis. No clear preferential direction stands out from the map of \( S_3 \). In fact, \( S_3 \) is representative of circular polarization.
A more concise yet comprehensive summary on the complex polarization pattern illustrated in Fig. 1 can be obtained from the analysis of its singularities [28]. In general, light’s polarization is elliptical, thus parametrized with the orientation $\psi$ of the polarization ellipse, the ellipticity angle $\chi$ and the handedness $h$ [Fig. 1(b)]. However, there are special cases in which the polarization ellipse degenerates into a circle or a line, and some of these parameters are not well defined anymore. In two dimensions, such singularities of the vector field are respectively points of circular polarization (C points) and lines of linearly polarized light (L lines) [41].

Figure 2 is a map of the orientation of the polarization ellipse for a small subsection of the measurement presented in Fig. 1. The position of C points is highlighted by circles and triangles, whose color represents their topological charge. This is defined as the half-integer number of times that the axis of the polarization ellipse rotates around the singularity, clockwise (positive charge) or anticlockwise (negative charge). In Fig. 2 we only observe topological charges of $\pm 1/2$.

Strictly related to their topological charge is the so-called line classification of C points, which differentiates them in three types: lemons, stars and monstars [42, 43]. The line classification can be understood by looking at the orientation of the polarization ellipse around the singularity, highlighted by the black directors in Fig. 2 and in the zoomed-in images of Fig. 3. For lemon-type singularities (lemons) there is only one direction along which the orientation of the polarization ellipse is directed towards the singularity, whereas the possible directions are always three for star-type singularities (stars and monstars). To determine the line classification of all the C points in our dataset in a deterministic way, we apply the method illustrated by Dennis for computing the number of directors pointing towards each singularity [42]. In our figures, we indicate stars and monstars with triangles, lemons with circles.

Already a quick glance at Fig. 2 illustrates the clear relation between topological charge (markers color) and line classification (markers shape) of C points. In fact, negative-charge singularities are always stars, whereas both lemons and monstars are characterized by a positive charge, as expected in general for C points [42]. Table I lists the fraction of C points for each of the kinds observed in our experimental dataset. 50% of the total number of C points are stars, and they all carry a negative topological charge. Approximately 45% of the singularities are lemons, and only 5% monstars, both types being positively charged. In the same table, we directly compare our experimental outcome with the results from previous paraxial theory [26] and experiments [27]. All these examined statistics are perfectly consistent with each other. In summary, the abundance of C points with a particular line classification is the same for C points in truly two-dimensional light and two-dimensional slices through a three-dimensional field.

![Polarization singularities in random waves](image)

**FIG. 2.** False-color map for the orientation of the major axis of the polarization ellipse. The black directors indicate the orientation of such axis too. The plot is representative of a subsection of the measured optical random field. Circles and triangles are C points. The color of the symbols, white or black, denotes a positive or negative topological charge, respectively. The shape of the symbols, triangles or circles, denotes the a star-type or lemon-type classification, respectively.

**FIG. 3.** An overview of the three kinds of C points based on their line classification [42]. The lines are the orientation of the polarization ellipse at each pixel around the C-point (circle or triangle), as determined from experimental data.

| Singularity | 2D Field | 2D Slice of a 3D Field |
|-------------|----------|-----------------------|
| Type        | Experiment | Experiment | Theory |
| Star        | 0.4997 ± 0.0002 | 0.506 ± 0.003 | 0.500 |
| Lemon       | 0.4493 ± 0.0013 | 0.443 ± 0.002 | 0.447 |
| Monstar     | 0.0503 ± 0.0013 | 0.050 ± 0.003 | 0.053 |

**TABLE I.** Fraction of C points with different line classifications. The results of our 2D experiment are compared with a previous experiments [27] and theory [26].
III. SPATIAL DISTRIBUTION OF C POINTS

A. Pair and Charge Correlation Function

Having established that there is no difference between the abundances of the various types of singularities observed in 2D slices of 3D light fields and truly 2D fields, the question now arises whether their distribution in space is also the same. The natural way of investigating the spatial distribution of point-like singularities, is determining their pair correlation function \( g(r) \). Given a C point, this function describes how the density of the surrounding C points varies as a function of distance. This method is widely used to describe the physics of discrete systems \([43,49]\), it can be directly related to the structure factor \([50]\), and it represents a spatial analogous of the degree of second-order coherence \( g^{(2)}(r) \), commonly used to determine photon bunching and antibunching \([51]\).

Figure 4 presents the pair correlation function for C points in two-dimensional random light, as obtained from our experimental data. With the position of each singularity known, we can compute their pairwise distances \(|r_i - r_j|\), and eventually the pair correlation function

\[
g(r) = \frac{1}{N\rho} \sum_{i \neq j} \delta(|r_i - r_j|),
\]

where \( N \) is the total number of singularities, \( \rho \) is the average density of surrounding singularities and \( \delta \) the Dirac function. We compute the average and uncertainty of such a correlation function by combining the outcome of 20 near-field measurements of the optical random field under investigation. In each of these maps we precisely pinpoint the location and topological charge of approximately 6500 C points, with a spatial accuracy which is limited by the pixel size of the experiment (\( \approx 20 \) nm).

\( g(r) \) is not flat, indicating that C points in random light exhibit spatial correlation. At first glance, this \( g(r) \) seems similar to the one of phase singularities in scalar random waves \([37,52]\), and therefore also reminiscent of that of particles in a simple liquid. In fact, \( g(r) \) displays a damped oscillatory behavior around unity as a function of \( r \), with a maximum, representative of a surplus of singularities, at approximately half a wavelength of distance. Surprisingly, the pair correlation of C points in 2D actually increases as \( r \) approaches 0. While the zero dimensionality of optical singularities would in principle allow for a finite probability of having two at the same location, an increase of \( g(r) \) towards zero has never been observed, neither for phase singularities in scalar/vector random waves \([37,52]\) nor for C points in a 2D slice of a 3D random field \([26]\) and gray lines in Fig. 4.

To understand the unexpected behavior at small distances and to obtain an overview of the spatial distribution of the C points, it is useful to also consider the charge correlation function \( g_Q(r) \): a more general expression of the pair correlation function in which each singularity is weighted with its topological charge \([52]\).

The orange data points in Fig. 4 display our experimental results for \( g_Q(r) \). The most striking observation here is that the charge correlation function is positive near \( r = 0 \). This means that when singularities are found at a close distance from each other they most often carry the same topological charge. Then, at \( r \approx \lambda/4 \) the charge correlation function flips sign, indicating the beginning of a displacement range where two singularities are more likely to have opposite sign. The zero crossing roughly coincides with the distance at which \( g(r) \) exhibited the unexpected increase towards small \( r \). This increase can therefore be attributed to the surplus of same-sign singularities in such a displacement range.

The reason why C points in 2D tend to rearrange so to form closely spaced pairs with the same topological charge is at this stage still unclear. However, the topological charge is not the only intrinsic property carried by C points. More insight could come by analyzing their behavior with respect to light’s handedness.

B. C points and Light’s Handedness

The correlation functions displayed in Fig. 4 provide an extensive description of the distribution of C points, but still not the full picture. This is because the information carried by C points is not limited to their topological charge. In fact, light’s polarization is purely circular at every C point, however it can be left- or right-handed, independent of the topological charge. In Fig. 5 we show a spatial map of the degree of circular polarization \( s_3 = S_3/S_0 \), together with the position, topological charge and handedness of the C points therein. We notice how C points fall in domains of a given handedness. Of course, \( s_3 \) equals exactly +1 or −1 at every C point, with a sign which determines the handedness of the C-point.
FIG. 5. False-color map for the degree of circular polarization $s_3 = S_3/S_0$, as obtained from our experimental data. The plot corresponds to the same subsection of the measured optical random field displayed in Fig. 2. The black directors indicate the orientation of the polarization ellipse. Circles and triangles are C points, and their filling color (purple or green) represents their handedness (left or right, respectively).

Itself. Each domain is delimited by L lines (white lines), where polarization is purely linear ($s_3 = 0$), and light’s handedness is undetermined. L lines have to separate C points of opposite handedness. Contrarily, several co-handed singularities can occur within the same domain. Furthermore, from Fig. 5 one immediately realizes how handedness and topological charge of a C point are not directly related, as every combination of these quantities is possible.

The handedness of C points provides an additional degree of freedom to be accounted for in their spatial distribution. It is illuminating to include this degree of freedom in the computation of a new set of pair correlation functions. In general, $g(r)$ can be expressed as the average of all the possible partial correlation functions for C points with the same or opposite handedness and the same or opposite topological charge:

$$g(r) = \frac{1}{16} \sum_{i,j} \sum_{\alpha,\beta} g_{i,j}^{\alpha,\beta}(r),$$

(2)

where $i, j \in \{+, -\}$ are indices for topological charge and $\alpha, \beta \in \{l, r\}$ indicate handedness. Following the notation of Dennis [26], Eq. (2) can be simplified with the definition of

$$g_{\text{same}}^C = g_{i,i}^{\alpha,\alpha} \quad \text{and} \quad g_{\text{opp}}^C = g_{i,-i}^{\alpha,\alpha},$$

(3)

both correspond to co-handed singularities, for the cases of same and opposite topological charge, respectively. Analogously, for anti-handed C points we have

$$g_{\text{same}}^A = g_{i,i}^{\alpha,\alpha} \quad \text{and} \quad g_{\text{opp}}^A = g_{i,-i}^{\alpha,\alpha}.$$  

(4)

Thus, we can express Eq. (2) as a function of these four correlation functions:

$$g(r) = \frac{1}{4} \left[ g_{\text{same}}^C + g_{\text{opp}}^C + g_{\text{same}}^A + g_{\text{opp}}^A \right].$$

(5)

Figure 6 presents our experimental results for the four pair correlation functions of the decomposition in Eq. 5, taking both topological charge and handedness of the C points into account. In the distribution functions depicted in Fig. 6(a) we only consider co-handed C points, either with same (green) or opposite (purple) topological charge. In this cases, the experimentally determined functions describe the standard characteristic properties exhibited by phase singularities in random waves. In fact, $g_{\text{same}}(r \to 0) = 0$ for singularities with the same topological charge, and a monotone decrease towards finite value at $r \to 0$ in $g_{\text{opp}}^C$. The experimental results displayed in Fig. 6(a) perfectly match the prediction of the model for polarization singularities in a 2D slice of a 3D field in the paraxial regime [26], which is equivalent to the model for phase singularities in scalar random waves [52].

In fact, we can interpret C points as phase singularities in either in the left- or right-handed circular components of $\mathbf{E}$:

$$\psi_l = E_x + iE_y, \quad \psi_r = E_x - iE_y.$$

(6)

This is because a phase singularity in $\psi_l$ corresponds to a zero in $\psi_r$, resulting in a point where $\mathbf{E}$ has only contribution from its circular-right component $\psi_r$, i.e., a right-handed C point. And vice versa. Therefore, the spatial distribution of co-handed C points is exactly equivalent to that of phase singularities arising in a single circular field component $\psi_l/r$, i.e., of phase singularities in a scalar random wave field [26].

Our experiment confirms that also in 2D the distribution of co-handed C points is the same as that of phase singularities in a scalar random field. Therefore, the origin of the unusual behavior of the global distribution of C points must necessarily lie in anti-handed singularities. Figure 6(b) presents the correlation functions for singularities with opposite handedness. $g_{\text{same}}^A(r)$ reaches its maximum values at $r \approx 0$. Singularities of opposite handedness and same topological charge are often found at close distances from each other, confined in an extremely subwavelength regime. Regarding pairs of C points with opposite topological charge, the distribution $g_{\text{opp}}^A$ exhibits a behavior that is qualitatively highly similar to that of $g_{\text{opp}}^C$. This creates two clearly distinct behaviors for the four combinations of charge and handedness. On the one hand, the impact of the handedness of C points on their spatial correlations seems to be only minor for singularities with opposite topological charge, for which we do not observe big qualitative differences between $g_{\text{opp}}^C$ and $g_{\text{opp}}^A$ (purple data in Fig. 6). On the other hand, considering the same or opposite handedness is crucial in the same-charge case, in which the behavior $g_{\text{same}}$ and $g_{\text{same}}^A$ is evidently different, eventually with an opposite gradient for $r \to 0$ (green data in Fig. 6).
As a matter of fact, the data displayed in Fig. 6(b) offer a clear illustration of the novel behavior registered for C points in 2D random light compared to the case of a 2D slice of a 3D field. Especially, it clarifies that in the 2D case C points of opposite handedness are far from being independent, and so must be for the left- and right-handed field projections from which they arise.

IV. CORRELATION AMONG LIGHT’S VECTOR COMPONENTS

The overall spatial correlation of C points in 2D random light (Fig. 4) and more specifically the correlation of singularities with opposite handedness [Fig. 6(b)], exhibit a number of features that were not accounted for in a previous paraxial theory [26]. In that theory, an assumption was made, consisting of the absence of any correlation between oppositely handed C points, i.e., $g_{\text{same}}^A = g_{\text{opp}}^A = 1$. This assumption corresponds to a situation in which $\psi_l$ and $\psi_r$ are completely uncorrelated.

In fact, in three-dimensions there are no restrictions that would imply a correlation among the circular components $\psi_l$ and $\psi_r$ of a paraxial random field. The same holds true for a two-dimensional slice of such a three-dimensional field [27]. In this circumstance, transversality can be fulfilled out of the plane in which the field is observed, meaning that the vector components of such a field can even be independently generated. Contrarily, in a truly two-dimensional vector field transverse propagation must be fulfilled in the same plane in which the waves are actually propagating. Dismissing the third dimension while obeying transversality then results in a correlation among the vector components of the field, eventually its left- and right-handed projections.

We will now adapt the paraxial model of Dennis [26] in order to account for the correlations intrinsic to a 2D light field. The key for explaining our results is that in our system the electric field can be modeled as a superposition of TE waves only. Note that we would find completely equivalent results considering the in-plane component of a field composed only of TM waves [53]. A TE mode in 2D can be expressed starting from a scalar field $H_z$:

$$
\begin{align*}
E_x &= k_y H_z \\
E_y &= -k_x H_z,
\end{align*}
$$

which by default satisfies the transverse condition.

For a random wave field, we follow Berry’s hypothesis and assume $H_z$ to be an isotropic superposition of monochromatic plane waves, each of them with a random phase $\delta_k$ [52],

$$
H_z = \sum_{|k| = k_0} \exp(i k \cdot r + i \delta_k),
$$

where $\delta_k$ is a random variable uniformly distributed in $[0, 2\pi]$. The autocorrelation of such a scalar random wave field is well known [52]: this is a Bessel function of order zero,

$$
C_{zz}(r) = \int dr_0 H_z^*(r_0) H_z(r_0 + r) = J_0(k_0 r).
$$

The autocorrelation of $E_x$ and $E_y$ are also known [37], the main difference with $C_{zz}(r)$ being an anisotropic term dependent on the orientation $\varphi$ of $r$:

$$
\begin{align*}
C_{xx}(r) &= \frac{1}{2} [J_0(k_0 r) + \cos(2\varphi) J_2(k_0 r)], \\
C_{yy}(r) &= \frac{1}{2} [J_0(k_0 r) - \cos(2\varphi) J_2(k_0 r)].
\end{align*}
$$

FIG. 6. Pair correlation function $g(r)$ for C points with same (a) or opposite (b) handedness, and same ($g_{\text{same}}$) or opposite ($g_{\text{opp}}$) topological charge. Data points represent our experimental results, colored solid lines are our model for isotropic 2D random field, solid gray lines are the 3D paraxial model [26]. The solid gray lines in (a) overlap exactly with the colored solid lines.

![Graphs showing correlation function g(r) for C points with same and opposite handedness, and same and opposite topological charge.](image-url)
Highly relevant to our study is also the cross term among $E_x$ and $E_y$, which exhibit the following correlation:

$$C_{xy}(r) = \int dr_0 E_x^*(r_0) E_y(r_0 + r) = \frac{1}{2} \sin(2\varphi) J_2(k_0r).$$

(11)

This equation can be easily proven by carrying out the integral in Fourier space and substituting the relations $E_x(k) \propto \sin(\theta_k) \delta(|k| - k_0)$ and $E_y(k) \propto -\cos(\theta_k) \delta(|k| - k_0)$ \[22\]. It is interesting to note that $E_x$ and $E_y$ only exhibit correlation when displaced, since $C_{xy}(r)$ lacks the term proportional to $J_0$, and $J_2(0) = 0$.

With these correlation functions known, and given the expression of $\psi_l$ and $\psi_r$ [Eq. (9)], we have all the ingredients to compute the correlations among the circular components of a TE random vector field. The autocorrelation of the left-handed component is

$$C_{ll}(r) = \int dr_0 \psi_l^*(r_0) \psi_l(r_0 + r) = C_{xx}(r) + C_{yy}(r) = J_0(k_0r),$$

and the same for $C_{rr}(r)$. The result of Eq. (12) is also identical to what obtained in Eq. (9) for $H_z$, proving that each separate circular component behaves as a random scalar field. Similarly to Eq. (12), we can finally determine the correlation among left and right circular components:

$$C_{lr}(r) = [\cos(2\varphi) - i \sin(2\varphi)] J_2(k_0r),$$

(13)

and

$$C_{rl}(r) = [\cos(2\varphi) + i \sin(2\varphi)] J_2(k_0r).$$

(14)

As elegantly explained by Berry and Dennis \[52\], the autocorrelation function of a complex field contains all the information needed to retrieve the pair/charge correlation function of its phase singularities. In the case of C points, i.e., phase singularities in the right- or left-handed field component, also the cross-terms ($C_{rl}$ and $C_{lr}$) are necessary. Following the same procedure of Berry and Dennis, we first calculate the point density of singularities in a scalar complex field, $\eta_{\psi} \equiv \psi_l + i\psi_r$, which is defined as

$$\eta[\mathbf{u}] = \delta(\psi_l)\delta(\psi_r) \left| \frac{\partial \psi_l}{\partial x} \frac{\partial \psi_r}{\partial y} - \frac{\partial \psi_l}{\partial y} \frac{\partial \psi_r}{\partial x} \right|,$$

(15)

where $\delta$ indicates the one-dimensional Dirac’s delta function, and where for compactness we have introduced the real vector $\mathbf{u} = [\psi_l, \psi_r, \partial_x \psi_l, \partial_y \psi_l, \partial_x \psi_r, \partial_y \psi_r]^T$. An analogous density can be defined for $\psi_r$.

The pair correlation function between C points at two different space points $\mathbf{r}_A$ and $\mathbf{r}_B$ and with opposite handedness can now be written in a straightforward way as

$$g^A(\mathbf{r}_B - \mathbf{r}_A) = \frac{\langle \rho[\mathbf{u}(\mathbf{r}_A)] \rho[\mathbf{u}(\mathbf{r}_B)] \rangle}{\langle \rho[\mathbf{u}(\mathbf{r}_A)] \rho[\mathbf{u}(\mathbf{r}_B)] \rangle}.$$  

(16)

In this equation, the notation $\langle f[\mathbf{u}(\mathbf{r}_A), \mathbf{u}(\mathbf{r}_B)] \rangle$ indicates the statistical average of a generic $f$, functional of the field components and of their derivatives at different points in space. Introducing the combined vector $\mathbf{u} = [\mathbf{u}(\mathbf{r}_A), \mathbf{u}(\mathbf{r}_B)]^T$, the average can be explicitly written in the form:

$$\langle f[\mathbf{u}] \rangle = \frac{1}{(2\pi)^D/2\sqrt{\det M}} \int d^D \mathbf{u} \rho[\mathbf{u}] \exp(-\frac{1}{2} \mathbf{u}^T M^{-1} \mathbf{u}),$$

(17)

where $D$ is the dimension of the vector $\mathbf{u}$ and $M$ is the matrix of the correlations between the various components of $\mathbf{u}$, i.e., $M_{ij} = \langle u_i u_j \rangle$. These elements correspond to the correlations between the different components of the left- and right-handed fields that we have summarized above, and their spatial derivatives. Similar expressions for different combinations of the fields $\psi_l$ and $\psi_r$, and for specific choices of the charge of the singularities can be obtained from Eq. (16) with intuitive modifications.

In some particular cases \[20, 37, 52\], it is possible to derive a closed analytical expression for averages of the form in Eq. (16) by reducing the integrand to a quadratic form and integrating with standard mathematical techniques \[54\]. However, the specific form of the correlation matrix in our model does not lend itself easily to applying the formalism of Ref. \[54\]. This is due to the additional correlations between the real and imaginary parts of the field components, corresponding to the imaginary terms in $C_{lr}$ and $C_{rl}$ [Eqs. (13) and (14)]. Nevertheless, the average in Eq. (16) is particularly suited to numerical integration with Monte Carlo techniques \[55\]. We therefore calculated the pair correlation functions of C points and polarization vortices in two steps. Firstly, we perform analytically the integral over the terms containing the Dirac’s delta functions in the integrand of Eq. (16). Subsequently, we carry out numerically the integration over the remaining variables, using the multidimensional Monte Carlo method \[55\].

We plot the theoretical expectations for the pair/charge correlation functions in direct comparison with the experimental data. In Fig. 4 we show the pair and charge correlation function for C points in 2D random vector waves and in Fig. 6 the pair correlation functions for C points with the same or opposite handedness, respectively. For each of these curves we find an excellent agreement with the experiment. In particular, the pair correlation functions displayed in Fig. 4(b) for C points with opposite handedness represent the major novelty introduced by the model for 2D light. Among these functions, $g^A_{same}$ exhibits a behavior which is extremely unusual for pair correlations of this kind. Although this behavior is perfectly consistent with the experimental observation, it might conceal further interesting properties of random light confined in 2D.
V. CONCLUSIONS

In this work we investigated the spatial correlation of C points in 2D random light. We compared it to existing theory and experiments for 2D slices through a 3D random field in the paraxial regime. We demonstrated that confining the optical field to propagate in two dimensions induces severe changes in the spatial distribution of its C points. The shortage of degrees of freedom caused by the removal of one dimension results in a correlation among the vector components of the 2D light field. In the circular basis, this results in a correlation among the oppositely-handed optical-spin components of light. One of the key consequences was the observation that the chance of finding C points with same topological charge actually increases as their mutual distance goes to zero. This is an unusual finding for dislocations of any kind. We quantify the correlation between left- and right-handed spin for the case of a TE field and incorporate it in a newly developed theoretical model. Our results are general for in-plane fields, including those of a TM mode as well. The outcome of the 2D model is found to be in perfect agreement with our experimental results. Given the unusual properties of the ensemble of C points in 2D random vector waves, our findings may trigger a re-evaluation of concepts which are considered pillars of singular optics and topological defects, i.e. the sign principle [56] and topological screening [57]. The behavior at short distances might lead to more unexplored features such as polarization vortices and higher-order singularities.

ACKNOWLEDGMENTS

We thank Andrea Di Falco for fabricating the chaotic cavity used in the near-field experiments and Thomas Bauer for useful discussions. This work is part of the research program of the Netherlands Organization for Scientific Research (NWO). The authors acknowledge funding from the European Research Council (ERC Advanced Grant No. 340438-CONSTANS). F. A. acknowledges support from the Marie Sklodowska-Curie individual fellowship BISTRO-LIGHT (Grant No. 748950).

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