Visualization of superparamagnetic dynamics in magnetic topological insulators

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Quantized Hall conductance is a generic feature of two-dimensional electronic systems with broken time reversal symmetry. In the quantum anomalous Hall state recently discovered in magnetic topological insulators, time reversal symmetry is believed to be broken by long-range ferromagnetic order, with quantized resistance observed even at zero external magnetic field. We use scanning nanoSQUID (nano–superconducting quantum interference device) magnetic imaging to provide a direct visualization of the dynamics of the quantum phase transition between the two anomalous Hall plateaus in a Cr-doped (Bi,Sb)₂Te₃ thin film. Contrary to naive expectations based on macroscopic magnetometry, our measurements reveal a superparamagnetic state formed by weakly interacting magnetic domains with a characteristic size of a few tens of nanometers. The magnetic phase transition occurs through random reversals of these local moments, which drive the electronic Hall plateau transition. Surprisingly, we find that the electronic system can, in turn, drive the dynamics of the magnetic system, revealing a subtle interplay between the two coupled quantum phase transitions.

INTRODUCTION

The integer quantum Hall effect (QHE), first observed in clean two-dimensional electronic systems at high magnetic fields (1), is the paradigmatic example of a topological phase: different integer QH states are characterized by identical symmetries but different integer topological quantum numbers η, with the quantized Hall resistance given by \( R_{\text{xy}} = \frac{h}{4\pi e^2} \) (2). However, Hall quantization may also occur in the absence of an external field as long as time reversal symmetry (TRS) is broken (3). This quantum anomalous Hall (QAH) state was recently realized experimentally (4–9), following theoretical proposals based on combining strong spin-orbit coupling with long-range ferromagnetic (FM) order in magnetically doped topological insulators (TIs) (10–18).

The QH plateau transition represents the canonical example of a topological phase transition, described by the divergence of the localization length and a universal critical scaling of transport coefficients (19). Recent theoretical calculations show that under certain assumptions, the QAH plateau transition can be mapped onto the same network model used to describe the integer QH plateau transition, leading to the same scaling laws (20).

Although the electronic topological transition in QH and QAH systems, taken in isolation, can be viewed as identical, the two experimental systems differ in several key aspects. The QH plateau transition is a purely electronic effect, in which delocalization occurs against a background of quenched electronic disorder. In contrast, the QAH plateau transition results from two coupled quantum phase transitions: the field-driven magnetic transition of the FM order and the electronic transition that is driven by TRS breaking of the FM transition. The dynamics of the FM reversal can endow the QAH electronic transition with features that do not have an analog in QHE systems. Exactly at the transition, both systems can be understood as a network of domain walls—magnetic domain walls for the QAH and domain walls of a different filling factor in the QHE—that host counterpropagating chiral edge states. Crucially, however, in contrast to the QH plateau transition, which describes the phase diagram of the system in thermodynamic equilibrium, the FM domain structure in the QAH is metastable, leading to hysteresis and relaxation dynamics that can directly affect the electronic system. Moreover, the energy scale of the electronic delocalization transition and its critical scaling may depend on the details of the microscopic FM structure and on the scaling of the magnetic phase transition. Finally, the electronic system, which commonly mediates FM interactions in dilute magnetic systems, can in turn modify the phase transition of the magnetic system. Most previous studies of QAH systems have used electronic transport measurements to probe the combined effect of magnetic and electronic evolutions, making it difficult to disentangle the individual roles of the two phase transitions. Thus, exploring the reciprocal coupling of the magnetic and electronic quantum phase transitions requires selective measurement tools that can address the two systems independently.

RESULTS

Here, we combine electronic transport with a scanning superconducting quantum interference device (SQUID) of 200-nm diameter that resides on the apex of a quartz tip [SQUID-on-Tip (SOT)] (21, 22) to simultaneously probe the magnetic and electronic transitions at ⁴He temperatures in a 7–quintuple layer (QL)–thick Cr₈₀₁(Bi₀₉Sb₀₂)₁ₓTe₃ film grown by molecular beam epitaxy (MBE) on an SrTiO₃ substrate (Fig. 1). The large dielectric constant of the substrate allows effective control of the chemical potential through back gating. The hysteretic \( R_{\text{xy}} \) transition occurs at a coercive field of \( H_c = 130 \text{ mT} \) at 250 mK (Fig. 1A). At elevated fields, the longitudinal resistance \( R_{\text{xx}} \) shows a pronounced dip as a function of the back-gate voltage at \( V_b \approx 6 \text{ V} \).
due to an incipient QAH state (Fig. 1B). At the lowest measurement temperature (250 mK) used here, the sample we discuss does not yet show the fully developed QAH. This is similar to previous experiments on Cr-doped TI films (4–9) where quantized Hall resistance appears only at dilution refrigerator temperatures, far below the onset temperature of hysteretic magnetic behavior (4, 23). Figure 1 (E to H) shows images of the local distribution of the magnetic field $B_z(x,y)$ in the sample at various points along the magnetization loop. We find $B_z(x,y)$ to be highly inhomogeneous at all fields, with peak contrast at $H_c$, where the average magnetization vanishes (Fig. 1, F and G). Submicrometer structure is evident even at fields corresponding to saturation of transport coefficients (Fig. 1, E and H), and images corresponding to opposite magnetization are highly anticorrelated on microscopic scales, including at full saturation. This suggests an inhomogeneous distribution of magnetic moments due to a random spatial distribution of the Cr dopants. Although high-resolution transmission electron microscopy measurements on our samples have yet to show any obvious evidence of Cr clustering (see fig. S1), we cannot preclude inhomogeneity at the nanoscale, akin to that found in other magnetically doped semiconductors such as Cr-doped ZnTe (24). Clear evidence of phase separation has only been seen in Cr-doped Bi$_2$Se$_3$ thin-film (25) samples that do not show the QAH state, whereas large nanoscale fluctuations in the local Cr density have been observed in Te-based samples (26) because of random doping that introduces strong disorder in the material (27).

In metallic FM thin films with out-of-plane magnetization, it is well established that magnetization reversal develops via the nucleation and propagation of domain walls separating regions of opposite magnetization. Such domain wall–mediated magnetization reversal has also been imaged in FM semiconductor films (28, 29) at magnetic dopant concentrations comparable to those in our magnetic TI films. However, scanning SOT microscopy reveals a very different picture of the magnetization reversal process in magnetic TIs. Figure 2A shows a sequence of $B_z(x,y)$ images acquired for increasing values of $\mu_0H$ near $H_c$. The five images appear almost identical; however, numerical subtraction of successive image data $|\Delta B_z(x,y)|$, see Fig. 2B] reveals the underlying dynamic process. Instead of the anticipated domain wall motion, magnetization reversal occurs through a series of random events in which isolated nanoscale islands undergo a reversal of their out-of-plane magnetic moment (see movie S1). These islands are a system of small, weakly interacting magnetic moments (potentially FM single domains) that constitutes a superparamagnet. As we discuss below, our findings establish a direct microscopic observation of superparamagnetism in magnetically doped TI films. Our observations caution against drawing conclusions about the FM state solely from macroscopic magnetization probes (SQUID magnetometry, magneto-optical Kerr effect) that show square hysteresis loops with robust zero field remanence (30).

To quantify the superparamagnetic dynamics across the Hall plateau transition, we fit each of the local features in the $\Delta B_z(x,y)$ maps with a point-like out-of-plane magnetic moment $m$ (fig. S2). Figure 2C summarizes ~1700 such fits, accumulated over four different ranges of magnetic field. Throughout the measured range, the spatial distribution of reversal events is random (Fig. 2D, inset, and fig. S3), suggesting weak interactions between neighboring islands and supporting the conclusion that superparamagnetism in Cr-doped BiSbTe films is characterized by aggregations of dopant atoms. The deviation of the Hall resistance from its saturation value begins around $\mu_0H = 0$ and is accompanied by the reversal of nanoscale moments with average $\langle m \rangle = 3 \times 10^5 \mu_2$ (Fig. 2, C and D). Given an average saturation magnetization of $\sim 3 \mu_2$/Cr atom as obtained from global magnetization measurements (fig. S4), the estimated average diameter of these flipping islands is $d = 51$ nm for our 7-nm-thick film, considerably below our spatial resolution of ~300 nm. However, as the field is increased toward $H_c$, a pronounced change in the

![Image](image.jpg)

**Fig. 1.** Electrical transport and scanning magnetic imaging of 7-QL-thick Cr$_{0.1}$(Bi$_{0.5}$Sb$_{0.5}$)$_{1.9}$Te$_3$ sample at $T = 250$ mK. (A and B) Transport measurements showing magnetic field dependence of $R_{xx}$ (red) and $R_{xy}$ (black) at $V_g = 6$ V (A) and the $V_g$ dependence at 1 T (B). The dip in $R_{xx}$ marked by the arrow shows the incipient QAH state. (C) Optical image of the sample and SOT showing the electrical contacts and the SOT reflection from the sample surface. (D) Electron micrograph of the SOT used for the magnetic imaging. (E to H) Scanning SOT images ($5 \times 5 \mu$m$^2$) of the out-of-plane magnetic field $B_z(x,y)$ at ~300 nm above the sample surface at four antisymmetric locations along the magnetization loop marked in (A). Note strong anticorrelation between (E) and (H), and (F) and (G). Pixel size, 50 nm; pixel integration time, 10 ms.
moment distribution is observed with a shift to higher \( m \) values and appearance of a long tail of large moments with \( m \geq 10^{5} \mu_{B} \) (Fig. 2, C and D). The microscopic reversal moments \( m \) can be summed to obtain the net change in magnetization \( M \) over a continuous field range (Fig. 2E). Comparison with simultaneously acquired \( R_{xy} \) shows a qualitative match, implying that the behavior of the transport coefficients through the transition is mainly determined by the underlying change in magnetization. Superparamagnetic behavior and a similar relationship between measured magnetization and transport coefficients were found in a second sample as well as in an Mn-doped Bi₂Te₃ film (see movie S2 and figs. S6 to S9).

Hysteretic magnetic transitions are a signature of metastability and typically display temporal relaxation via thermal activation or quantum tunneling. We probe magnetic relaxation in real time by polarizing the system at \( \mu_{0}H = -1 \) T and then ramping the field to a positive set point \( \mu_{0}H_{set} \). We then acquire repeated images of \( B_{z}(x,y) \) while simultaneously monitoring electronic transport. No relaxation is observed on laboratory time scales for \( \mu_{0}H_{set} < 0 \) in either magnetization or transport coefficients. Spontaneous relaxation begins to be evident at small positive fields \( \mu_{0}H_{set} = 63 \) mT, manifesting as magnetic reversal events \( \Delta B_{z}(x,y) \) and a slow upward drift of \( R_{xy} \). The frequency and number of these reversals significantly increase near the coercive field at \( \mu_{0}H_{set} = 126 \) mT (Fig. 3, A and C).

The temporal relaxation measurements further corroborate the superparamagnetic behavior: at temperatures well below the blocking temperature, the magnetization of a superparamagnet is hysteretic, showing minimal relaxation at low fields. On approaching \( H_{c} \), the magnetic anisotropy barrier \( U \) is reduced, leading to relaxation when \( U \approx k_{B}T \). Because \( U \) is proportional to the volume of the superparamagnetic particles, smaller islands undergo thermal activation at a lower field. Simultaneous transport measurements (Fig. 3D) indicate that the electronic transition closely tracks the magnetic relaxation, with transport coefficient relaxation evident at \( \mu_{0}H_{set} = 63 \) mT and pronounced at 126 mT, in accord with the total temporal change in magnetization extracted from the SOT data (Fig. 3C).

The plateau transition observed in electronic transport appears to be mainly dictated by the underlying magnetic reversal. Surprisingly, however, we find that the dynamics of the magnetic system can, in turn, be influenced by the electronic system. To explore this effect, we perform a

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Fig. 2. Magnetization reversal dynamics. (A) Sequence of SOT magnetic images \( B_{z}(x,y) \) taken at consecutive magnetic fields in 0.5 mT steps at \( T = 250 \) mK. (B) Differential images \( \Delta B_{z}(x,y) \) obtained by subtracting pairs of consecutive \( B_{z}(x,y) \) images in (A) showing the isolated magnetic reversal events (red) of the superparamagnetic moments (see movie S1). (C) Statistical analysis of 1690 reversal events attained over ranges of magnetic fields centered around four \( \mu_{0}H \) values: total number of moment reversals \( N_{mv} \), average magnetic moment \( \bar{m} \), average superparamagnetic island diameter \( \bar{d} \), and rate of the magnetization change \( dM/d(\mu_{0}H) \) over the given range. (D) Chart of relative contribution of different moment sizes \( m \) to the total magnetization change \( M \) within two field ranges centered at \( \mu_{0}H = -15 \) mT (yellow) and \( \mu_{0}H = 154 \) mT (blue). Inset: Location of the moment reversals within the field range around \( \mu_{0}H = 154 \) mT. (E) Cumulative magnetization change \( M \) due to moment reversals \( m \) over four field ranges (left axis, colored symbols) and the simultaneously acquired \( R_{xy} \) (right axis, black). The total magnetization in each range is offset by an arbitrary constant.
sequence of magnetic imaging scans at 126 mT interspersing consecutive scans with small excursions of the back-gate voltage $\Delta V_g$ (Fig. 3B). All imaging scans are taken at a gate voltage of 6 V. Between scans, the gate is ramped at 500 mV/s to a voltage 6 V + $\Delta V_g$ and back to 6 V, with a wait time of a few seconds at the extremal value. Remarkably, even small $\Delta V_g$ ∼ 1 V excursions significantly enhance the relaxation of the superparamagnetic islands, as is evident from the statistics of the observed moment reversals (Fig. 3C). This enhancement, in turn, is evident in the transport coefficients, as shown in Fig. 3D.

A full comparison of the effects of applied magnetic field, gate voltage, and time on transport coefficients is presented in Fig. 4C, which shows a unified parametric plot of the transport coefficient vector ($R_{xx}$, $R_{xy}$). On this plot, the large magnetic hysteresis evident in Fig. 4 (A and B) is absent, demonstrating that the relation between $R_{xx}$ and $R_{xy}$ (at a given $V_g$) is a universal function determined by the magnetization. Within a single constant-$V_g$ arc-shaped plot, zero net magnetization corresponds to the maximum of the arc at $R_{xy}$ = 0, whereas the varying Hall angle along the arc reflects the varying sample magnetization. Variable $V_g$ traces over $\Delta V_g = \pm 30$ V at $\mu_0 H = \pm 1$ T, marked in black, show contours of variable Fermi level at constant saturated magnetization. Note that at full saturation, variable $V_g$ data retrace the same path upon repetition.

On this plot, the temporal relaxation of transport coefficients (fig. S13) is seen to track constant $V_g$ arcs, implying that temporal magnetic relaxation is the dominant mechanism and consistent with the scanning magnetometry results (Fig. 4, C and D, gray dots along $V_g = 6$ V arc). Again, consistent with magnetometry data, $V_g$ excursions markedly enhance the magnetization relaxation in the metastable regime (Fig. 4, C and D). The blue trace shows the evolution of the transport coefficients during six successive $V_g$ excursions, with $\Delta V_g$ increasing from $\pm 1$ to $\pm 3$ V at a constant field of 126 mT. Successive $V_g$ sweeps do not retrace each other, consistent with an irreversible change in the net magnetization. Repeating the same experiment for larger excursion of $\Delta V_g = \pm 30$ V, the magnetization is observed to rapidly relax, nearly reaching complete saturation (cyan in Fig. 4, C and D). As Fig. 4C makes clear, the gate-induced magnetic relaxation is strongly dependent on maximum extent of the voltage excursion, although we find it to be independent of sweep direction and rate.

Strikingly, despite the marked effect of gate voltage variations on magnetic relaxation, field sweeps at different values of $V_g$ show only small deviations in the coercive field (Fig. 4, A and B), indicating that carrier density has little effect on the average magnetization. These seemingly contradictory observations can be qualitatively understood by invoking the strongly disordered nature of the superparamagnetic state. In our strongly disordered system, the global gap in the density of states is bridged by a proliferation of subgap states induced by the random polarization of the superparamagnetic islands, possibly leading to electron-hole puddles (27) rather than a hard gap. These localized states can, in turn, mediate local FM interactions. $V_g$ excursions modify the magnetic anisotropy energy of individual islands through the strong dependence of density of states on position and energy, randomly changing the magnetic potential landscape without significantly changing the average properties of the magnetic disorder potential (and thus the coercive field measured at fixed $V_g$). However, unlike in QH systems, the magnetic disorder is not quenched: the weakly interacting superparamagnetic islands are metastable and subject to irreversible relaxation. When the applied field is oriented opposite to the net magnetization, some local configurations are separated from the ground state by a small energy barrier. Any local perturbation, including a modulation of local density of states, can cause an irreversible flip, leading to rapid stimulated relaxation upon repeated excursions of $V_g$. Our statistical analysis of temporal relaxation (fig. S15) shows that gate voltage does modulate the average size of the flipping moments for the same applied magnetic field. This again suggests that carriers mediate magnetic interactions.

**DISCUSSION**

In conclusion, we have imaged the magnetic structure of Cr-doped thin films of (Bi,Sb)$_2$Te$_3$ and found it to be strongly disordered. The magnetic reversal transition and its temporal relaxation are mediated by isolated nanoscale island reversals rather than domain wall motion, pointing to a superparamagnetic state composed of nearly noninteracting magnetic domains. Quantitative analysis shows that the observed dynamics account for most of the total magnetic moment, excluding the possibility of an underlying uniform FM state. Unexpectedly, gate voltage modulation strongly enhances the magnetic relaxation. Note that at full saturation, variable $V_g$ data retrace the same path upon repetition.

Repeating the same experiment for larger excursion of $\Delta V_g = \pm 30$ V, the magnetization is observed to rapidly relax, nearly reaching complete saturation (cyan in Fig. 4, C and D). As Fig. 4C makes clear, the gate-induced magnetic relaxation is strongly dependent on maximum extent of the voltage excursion, although we find it to be independent of sweep direction and rate.

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relaxation, suggesting that the charge carriers are involved in the mediation of magnetic interactions. Our results suggest that the quantum phase transition between QAH states can be strongly affected by the nature of the underlying magnetic transition, leading to deviations from the expected universal scaling of the plateau transition as indicated by recent transport studies (31). In particular, the magnetic disorder potential lowers the local QAH gap, lowering energy barriers for hopping conductivity across the sample bulk. Moreover, the large distribution in the flipping energy barrier $U$ guarantees that, for zero magnetic field and finite temperature, some magnetic reversals will occur because of thermal fluctuations, further decreasing the average bulk gap. Although topological states are robust to disorder in the low temperature limit, as the temperature increases toward the blocking temperature $T_B$ of the superparamagnetic islands, the enhanced dissipation due to thermally excited magnetization reversals is bound to destroy the protected state. Thus, nanoscale superparamagnetism may be one of the sources responsible for the fragility of the QAH state at elevated temperatures.

MATERIALS AND METHODS

Samples were grown by MBE on commercial SrTiO3 substrates with dimensions of $5 \times 5 \times 0.5$ mm$^3$. Substrates were typically annealed in oxygen at 925°C for 2.5 hours and were screened using atomic force microscopy (AFM) to make sure that the surface was atomically ordered with a root mean square roughness of less than 0.3 nm (fig.S16). Substrates were indium-mounted for growth in the MBE chamber and outgassed at a substrate temperature of 550°C for about 1 hour to remove any residual contamination before growth. The TI films were grown using elemental materials of at least 5N purity evaporated from Knudsen-type thermal cells at a growth rate of about 0.4 QL per minute at a temperature of 250°C, as measured by a pyrometer. The Cr, Bi, and Sb compositions were estimated from secondary ion mass spectrometry (SIMS) measurements on a similarly grown sample. The SIMS atomic percentages were calibrated using another sample of known composition that was measured at the same time.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/1/10/e1500740/DC1

Sample characterization

Fig. S1. STEM imaging and EDS elemental mapping of a Cr-doped (Bi,Sb)2Te3 film on SrTiO3.

Fig. S2. Fitting procedure of magnetic moments.

Fig. S3. Spatial distribution of the magnetization reversal process in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 thin film at $T = 250$ mK.

Fig. S4. SQUID magnetometry measurements of a ~40-QL-thick film.

Fig. S5. Movie snapshot of magnetization reversal process in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 film at $T = 250$ mK.

Fig. S6. Movie snapshot of magnetization reversal process in 10-QL Cr-doped (Bi,Sb)2Te3 film at $T = 250$ mK.

Fig. S7. Magnetization reversal dynamics and transport coefficients in 10-QL-thick Cr-doped (Bi,Sb)2Te3 film at $T = 250$ mK.

Fig. S8. Scaling of the cumulative magnetization change and the transverse resistance in 10-QL Cr-doped (Bi,Sb)2Te3 film at $T = 250$ mK.

Fig. S9. Transport and superparamagnetic dynamics in 10-QL Cr-doped (Bi,Sb)2Te3 film.

Fig. S10. Schematics of transport measurements.

Fig. S11. Temperature dependence of the transport coefficients in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 film.

Fig. S12. Gate voltage dependence of $R_{xx}$ in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 sample.

Fig. S13. Temperature dependence of transport coefficients near the coercive field in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 sample.

Fig. S14. Transport coefficients in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 sample with continuous gate excursions.

Fig. S15. Statistical analysis of temporal moment relaxation for different $V_g$ values in 7-QL Cr0.1(Bi0.5Sb0.5)1.9Te3 film at $T = 250$ mK.

AFM of Cr-doped samples

Fig. S16. AFM topography images of the two studied Cr-doped (Bi,Sb)2Te3 samples.
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Acknowledgments: We thank E. Berg for illuminating discussions. Funding: This work was
supported by the Minerva Foundation with funding from the German Federal Ministry of Edu-
cation and Research and by the Israel Science Foundation (grant no. 132/14), N.S., A.R., A.K.,
and S.K. acknowledge support from DARPA (Defense Advanced Research Projects Agency) Meso-
(Mesodynamic Architectures) (grant no. N66001-11-1-4110), ONR (Office of Naval Research)
(grant no. N00014-12-1-0117), and ARO MUR (Army Research Office Multidisciplinary University
Research Initiative) (grant no. W911NF-12-1-0461), as well as use of the NSF National Nanofab-
ication Users Network Facility at the Pennsylvania State University. A.F.Y. acknowledges the
support of Goldschleger Center for Nanophysics. Author contributions: E.O.L., A.F.Y., and J.C.
performed the magnetic imaging and transport measurements. A.R., A.K., and N.S. fabricated and
characterized the samples. H.R.N. fabricated the SOTs. Y.A. and A.Y.M. developed the nu-
merical data analysis. S.K. performed global magnetization studies. M.E.H. developed the SOT
readout system. Y.M. prepared the samples. E.O.L., A.F.Y., E.Z., A.R., and N.S. wrote the paper
with contributions from other authors. Competing interests: The authors declare that they
have no competing interests. Data and materials availability: All data needed to evaluate the
conclusions in the paper are present in the paper and/or the Supplementary Materials.
Additional data related to this paper may be requested from the authors.

Submitted 8 June 2015
Accepted 26 August 2015
Published 6 November 2015
10.1126/sciadv.1500740

Citation: E. O. Lachman, A. F. Young, A. Richardella, J. Cuppens, H. R. Naren, Y. Anahory,
A. Y. Meltzer, A. Kandala, S. Kempinger, Y. Myasoedov, M. E. Huber, N. Samarth, E. Zeldov,
Visualization of superparamagnetic dynamics in magnetic topological insulators. Sci. Adv. 1,
e1500740 (2015).