Stratification model of seawater mass structure at the estuaries of Jeneberang River and Tallo River and the influences to current pattern in Makassar coastal areas

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Abstract. The influences of periodically continuous tides and river discharges flowing from upstream will be responded by basic morphological form and geometry of estuaries and resulting in current pattern in estuaries. The current induced by tidal effect creates turbulences in estuaries which influence the change of configuration of stratification of seawater mass structure and suspended sediment. The intrusion of seawater from estuaries upstream in Jeneberang River and Tallo River will affect the pattern of water mass mixing which later generate patterns of current and sediments on the river. Mathematical model is a useful method to predict structural stratification of seawater mass flowing from estuaries upstream. The stages were to develop mathematical model of tidal current and mathematical model of water flow downstream.

1. Introduction
The tidal event is one of the factors that can affect the stratification of water mass structure in estuaries. The effect of periodically continuous tides and river flow discharge from upstream downward to the coast will be responded to by the basic morphological form and geometry of estuaries and resulting in current pattern in estuaries. The currents arising from these tides cause turbulence in the estuary which can affect changes in the configuration of the water mass structure and suspended sediments. The hydraulic condition occurring in the estuary is an important problem to understand in relation to the planning of raw water supply, water supply for pond irrigation, and drainage engineering in coastal areas. Tidal influences are very dominant in estuaries regardless of scales of discharge resulting in the flow becoming unsteady.

In regard with the design of channels functioning whether as a part of pond irrigation system, drainage channel or raw water intake, the characteristics of flow and seawater intrusion due to tides are important to understand. Flow analysis in open channels can be achieved through a mathematical model. In this paper mathematical models of the flow of water in an open channel influenced by tides are developed.

2. Research objectives
The objectives of this research are to create mathematical models of the hydrodynamic conditions of the flow in the river estuary which are the flow model due to the discharge from the upstream and the flow model due to tides.

3. Literature review

3.1 Flow circulation pattern in river estuary
The circulation pattern of the flow in the estuary is influenced by the characteristics of river morphology, tides and river discharge. Flow circulation includes the propagation of tidal waves, mixing of fresh and salt water, sediment motion, and pollutant materials.

3.2 Tides
Tides are sea level fluctuations due to the attraction of celestial objects, especially the sun and moon against the water mass on earth. Seawater intrusion to estuary is accompanied by transport of salt mass. The distance of saltwater intrusion into the estuary depends on the characteristics of the estuary, tides and river discharge where the larger the tidal height and the smaller the river discharge, the further the saltwater intrusion is. Salt transport in estuaries occurs by convection and diffusion. By convection salt water is transported along with the flow of water, because of the influence of flow velocity. Diffusion transport occurs due to turbulences and differences in salinity at a point with points around it, so that the salt content will spread to the point with a lower concentration.

3.3 River discharge
River discharge is one of the important parameters in circulation in estuaries. River discharge depends on hydrological and watershed characteristics. In the watershed where the forest is still conserved a relatively constant flow rate throughout the year can be provided, but if the vegetation condition of the river basin is deteriorated, the discharge discrepancy between the rainy and dry seasons would be very significant. In the rainy season, the discharge will be significantly enormous and in the dry season, the discharge will be low instead. At the event of flooding, the river discharge pushes pollutants (salt, sediments, etc.) into the sea, so that saltwater intrusion and turbidity are pushed further downstream, while at small discharge the pollutants move upstream.

The interface of salt water from the sea and fresh water from the river occurs at the estuary. The location of the interface and the level of mixing of salt water and fresh water significantly vary depending on the magnitudes of tides and river discharge.

![Figure 1. The process of mixing seawater in the estuary.](image_url)

4 Research methodology
The research methodology is by literature study and by development of mathematical models of water flow from upstream and mathematical models of tides derived from the conservation of mass and the law of conservation of momentum.

5 Results and discussion

5.1 Flow equation on the river
5.1.1 Conservation of mass
The law of conservation of mass states that the change in the average mass coming out through the control volume (CV) is equal to the amount of mass generated from the source. According to the Reynold’s transport theorem.
\[
\frac{\partial}{\partial t} = \int_{cy} \rho d\vec{v} + \sum_{cv} \rho A(\vec{v} \cdot \vec{n}) = 0
\]

for two-dimensional control volume the above equation can be written

\[
\frac{d}{dt} (\rho \vec{v}) + \sum_{cv} \rho A \vec{u} = 0
\]

\[
\frac{d}{dt} (\rho \vec{v})_e - (\rho \vec{v})_w + (\rho \vec{v})_n - (\rho \vec{v})_s + (\rho \vec{v})_b - (\rho \vec{v})_t
\]

if \( \forall = \Delta x \Delta y \Delta z \), \( A_e = A_w = \Delta y \Delta z \), \( A_n = A_s = \Delta x \Delta z \), dan \( A_t = A_b = \Delta x \Delta y \)

Then equation (1) can be written as:

\[
\frac{d}{dt} (\rho \Delta x \Delta y \Delta z) + \rho (u_e - u_w) \Delta y \Delta z + \rho (v_n - v_s) \Delta x \Delta z + \rho (w_b - w_t) \Delta x \Delta y = 0
\]

(2)

Because the control volume is fixed and non-deforming, equation (2) can be expressed by:

\[
\frac{\partial \rho}{\partial t} + \frac{\rho (u_e - u_w)}{\partial x} + \frac{\rho (v_n - v_s)}{\partial y} + \frac{\rho (w_b - w_t)}{\partial z} = 0
\]

(3)

By using the finite volume method for \( \Delta x \rightarrow 0, \Delta y \rightarrow 0, \Delta z \rightarrow 0 \), and the flow is considered incompressible, equation (3) can be stated as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(4)

If integration is conducted towards the z axis (depth, \( \Delta h (x, y, t) \)), the equation becomes:

\[
\int_{z_0}^{z_h} \frac{\partial u}{\partial x} dz + \int_{z_0}^{z_h} \frac{\partial v}{\partial y} dz + \int_{z_0}^{z_h} \frac{\partial w}{\partial z} dz = 0
\]

(5)

Using Leibnitz’s rules, the equation becomes:

\[
\frac{\partial}{\partial x} \int_{z_0}^{z_h} u \ dz - v(x,y,z_h) \frac{\partial z_h}{\partial x} + u(x,y,z_0) \frac{\partial z_0}{\partial x} + \frac{\partial}{\partial y} \int_{z_0}^{z_h} v \ dz - v(x,y,z_h) \frac{\partial z_h}{\partial y} + v(x,y,z_0) \frac{\partial z_0}{\partial y} +
\]

\[
w(x,y,z_h) - w(x,y,z_0) = 0
\]

(6)

By deriving the total \( z = z(x,y,t) \) to time \( t \), for \( z = z_h \), the equation (6) becomes

\[
w(x,y,z_h) = u(x,y,z_h) \frac{\partial z_h}{\partial x} + v(x,y,z_h) \frac{\partial z_h}{\partial y} + \frac{\partial z_h}{\partial t}
\]

(6a)

For \( z = z_{oh} \), equation (6) becomes:

\[
w(x,y,z_0) = u(x,y,z_0) \frac{\partial z_0}{\partial x} + v(x,y,z_0) \frac{\partial z_0}{\partial y} + \frac{\partial z_0}{\partial t}
\]

(6b)

By substituting eq. (6a) and eq. (6b) to equation (6), it will be obtained:

\[
\frac{\partial}{\partial x} \int_{z_0}^{z_h} u \ dz - u(x,y,z_h) \frac{\partial z_h}{\partial x} + u(x,y,z_0) \frac{\partial z_0}{\partial x} + \frac{\partial}{\partial y} \int_{z_0}^{z_h} v \ dz - v(x,y,z_h) \frac{\partial z_h}{\partial y} + v(x,y,z_0) \frac{\partial z_0}{\partial y} +
\]

\[
u(x,y,z_h) \frac{\partial z_h}{\partial x} + v(x,y,z_h) \frac{\partial z_h}{\partial y} - \left( u(x,y,z_h) \frac{\partial z_0}{\partial x} + v(x,y,z_0) \frac{\partial z_0}{\partial y} + \frac{\partial z_0}{\partial t} \right) = 0
\]

(7)

If simplified Equation (7) can be written into:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial (z_h-z_0)}{\partial t} = 0
\]

(8)

With \( u = \frac{1}{h} \int_{z_0}^{z_h} u \ dz \), dan \( v = \frac{1}{h} \int_{z_0}^{z_h} v \ dz \),

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and because $z_h - z_0 = h$, equation (8) becomes

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0 \tag{9}$$

### 5.1.2 Conservation of momentum

The law of momentum conservation that applies to the main river flow according to the continuum principle states that the law of conservation of momentum is the average change in momentum in Control Volume (CV) added by the momentum flow coming out through the Control Surface (CS) and is equal to the rate of change of momentum over time.

$$\frac{d}{dt}\int_{CV} \rho v \, dV + \sum \rho \vec{A} \vec{v} \cdot \vec{n} = \sum F_{working \ on \ the \ system}$$

$$\frac{d}{dt} (\rho \vec{v} u) + (\rho \vec{u} u)_e - (\rho \vec{u} u)_w + (\rho \vec{v} u)_n - (\rho \vec{v} u)_s + (\rho w \vec{u})_t - (\rho w \vec{u})_b = (F_s A)_e - (F_s A)_w + (F_s A)_n - (F_s A)_s - (F_s A)_t - (F_s A)_b + F_b \vec{v} \tag{10}$$

If $\forall = \Delta x \Delta y \Delta z$, $A_e = A_w = \Delta y \Delta z$, $A_n = A_s = \Delta x \Delta z$, dan $A_t = A_b = \Delta x \Delta y$ If $y$ then equation (10) can be written:

$$\frac{d}{dt} (\rho u \Delta x \Delta y \Delta z) + ((\rho u^2)_e - (\rho u^2)_w) \Delta y \Delta z + ((\rho uv)_n - (\rho uv)_s) \Delta x \Delta z + ((\rho uw)_t - (\rho uw)_b) \Delta x \Delta y = (F_s)_e - (F_s)_w \Delta y \Delta z + ((F_s)_n - (F_s)_s) \Delta x \Delta z + ((F_s)_t - (F_s)_b) \Delta x \Delta y + F_b \Delta x \Delta y \Delta z \tag{11}$$

Because the control volume is fixed and non-deforming, Equation (11) can be expressed by the equation:

$$\Delta x \Delta y \Delta z \frac{d}{dt} (\rho u) + ((\rho u^2)_e - (\rho u^2)_w) \Delta y \Delta z + ((\rho uv)_n - (\rho uv)_s) \Delta x \Delta z + ((\rho uw)_t - (\rho uw)_b) \Delta x \Delta y = (F_s)_e - (F_s)_w \Delta y \Delta z + ((F_s)_n - (F_s)_s) \Delta x \Delta z + ((F_s)_t - (F_s)_b) \Delta x \Delta y + F_b \Delta x \Delta y \Delta z \tag{12}$$

If equation (12) is divided by $\Delta x \Delta y \Delta z$ and by using the finite volume method, equation (13) will be obtained:

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} + \frac{\partial (\rho uw)}{\partial z} = \frac{\partial (F_{sx})}{\partial x} + \frac{\partial (F_{sy})}{\partial y} + \frac{\partial (F_{sz})}{\partial z} + F_b \tag{13}$$

For 2-dimensional models

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} = \frac{\partial (F_{sx})}{\partial x} + \frac{\partial (F_{sy})}{\partial y} + F_b \tag{14}$$

If equation (14) is integrated to the $z$ axis, then the equation for conservation of momentum becomes

$$\frac{\partial (\rho hu)}{\partial t} + \frac{\partial (\rho hu^2)}{\partial x} + \frac{\partial (\rho huw)}{\partial y} = \frac{\partial (hF_{sx})}{\partial x} + \frac{\partial (hF_{sy})}{\partial y} + hF_b \tag{15}$$

Assuming the flow to be incompressible, then

$$\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{\partial (huv)}{\partial y} = \frac{1}{\rho} \left( \frac{\partial (hF_{sx})}{\partial x} + \frac{\partial (hF_{sy})}{\partial y} + hF_b \right) \tag{16}$$

### 5.2 Tidal equations

There are four types of forces working on seawater masses; forces on pressure gradients, coriolis forces, gravitational forces, and frictional forces per unit of mass $[1],[2],[3]$. The hydrodynamic equation is derived from Newton’s Second Law, called the momentum conservation law which states that the change in momentum over time equals the total force working on the system. This law is derived in the form of two-dimensional Hydrodynamic equations.
component $x$: \[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + f v + k \frac{\partial^2 u}{\partial z^2} + A \Delta u
\] (17)

component $y$: \[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + f u + k \frac{\partial^2 v}{\partial z^2} + A \Delta v
\]  (18)

Seawater is assumed to be an incompressible fluid, hence the continuity equation is added to the system of equations in the form [1]:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  (19)

Tidal hydrodynamic model is assumed as two-dimensional. Other assumptions applied in the equation of the tidal hydrodynamic model include [2].
- Surface atmospheric pressure (Pa) is constant, so the partial derivatives to $x$ and $y$ are equal to zero.
\[
\frac{\partial p_a}{\partial x} = \frac{\partial p_a}{\partial y} = 0
\]  (20)

- The influence of coriolis force on motion in water masses is negligible given the relatively small model domain and the proximity to the equator.
- It is assumed that there is no stratification of sea water density (constant $\rho$).
- There is no source (source) and leakage (seawater) that occurs in the model area, meaning that evaporation and precipitation are ignored and the seabed is impermeable.
- There is no source of momentum (external forces) occurring in the areas, such as ship movements, tsunamis and earthquakes.
- Closed boundaries do not shift with the rise and fall of sea levels.

Therefore, the hydrodynamic equation used based on the assumptions above is [1]:

component $x$: \[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -gH \frac{\partial \zeta}{\partial x} + \lambda W_x \left( \frac{W_x^2}{W_x^2 + W_y^2} - r U \frac{(U^2 + V^2)^{1/2}}{H^2} \right) + A \Delta u
\] (21)

component $y$: \[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -gH \frac{\partial \zeta}{\partial y} + \lambda W_y \left( \frac{W_x^2}{W_x^2 + W_y^2} - r V \frac{(U^2 + V^2)^{1/2}}{H^2} \right) + A \Delta v
\] (22)

where $\zeta$ = elevation (m), $\tau_s =$ surface friction force, and $\tau_b =$ basic friction force.

Equation (7) and Eq (8) are then integrated into depth vertically from the base ($z = -H$) to the surface ($z = 0$) to obtain the equation of mass transport so that the average velocity is obtained in the form of transport in two-dimensional water masses [1].

component $x$: \[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -gH \frac{\partial \zeta}{\partial x} + \lambda W_x \left( \frac{W_x^2}{W_x^2 + W_y^2} - r U \frac{(U^2 + V^2)^{1/2}}{H^2} \right) + A \Delta u
\] (23)

component $y$: \[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -gH \frac{\partial \zeta}{\partial y} + \lambda W_y \left( \frac{W_x^2}{W_x^2 + W_y^2} - r V \frac{(U^2 + V^2)^{1/2}}{H^2} \right) + A \Delta v
\] (24)

If the force of wind pressure is ignored hydrodynamic equations of two dimensions of tides ultimately will be in the form [4]:

component $x$: \[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -gH \frac{\partial \zeta}{\partial x} - r U \frac{(U^2 + V^2)^{1/2}}{H^2} + A \Delta u
\] (25)

component $y$: \[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -gH \frac{\partial \zeta}{\partial y} - r V \frac{(U^2 + V^2)^{1/2}}{H^2} + A \Delta v
\] (26)

Resulting, the equation of continuity will be in the form.
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \zeta}{\partial t} = 0
\] (27)

6 Conclusions
In developing the hydrodynamic models of water flow in the river and tides, a mathematical approach can be conducted. These mathematical models require two equations; mass conversion equation and momentum conservation equation. The law of continuity from unsteady flow can be reduced by assuming that water is incompressible and the density of water to distance and time is fixed.
Continuity of water flow in rivers and estuaries can be obtained by integrating these equations in the wetted sections of rivers and estuaries.

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