Defect Size Evaluation of Cylindrical Roller Bearings with Compound Faults on the Inner and Outer Races

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Abstract

Faults in cylindrical roller bearings are one of the main contributors to major faults in rotating machinery. The development of bearing fault diagnosis technologies is key to measuring the performance, status, and risk of failure of rolling-element bearings and has attracted extensive attention from industry and academia. When faults arise, they are often not a single fault but a compound fault, such as the simultaneous failure of the inner and outer races. In this paper, a method for evaluating the size of compound faults on the inner and outer races of cylindrical roller bearings is proposed. The dynamic modeling method developed by Gupta is employed to create a dynamic model for compound faults on the inner and outer rings of rolling bearings that allows the time domain signal of the vibration responses of compound faults on the inner and outer races to be obtained. Adopting an improved continuous harmonic wavelet packet decomposition method for the decomposition and reconstruction of the compound fault signal, we arrive at the corresponding single-point fault signal. The relationship between defect size and key metrics of the vibration, such as root mean square acceleration (RMS), peak, crest factor (CF), kurtosis, and level crossing rate (LCR), is investigated. The results show that there is a strong linear correlation between LCR and defect size, which can be used to evaluate the size of the defect. Experimental data for cylindrical roller bearings with compound faults on the inner and outer races are examined to verify the results.

1. Introduction

Cylindrical roller bearings are widely used in fields such as aerospace, high-speed railways, and robotics due to their small friction coefficient and superior high-speed properties. However, localized defects caused by fatigue, abrasion, or skidding scratches easily form on the raceways, severely affecting the system performance. To decrease the possible damage caused by localized defects of the bearings to the system, precise and early recognition is vital. Many different signal processing technologies have been applied in identifying the characteristic frequencies of localized defects according to the impulse characteristics of the vibration signals in defective bearings [1], including wavelet and correlation filtering [2, 3], time-frequency ridge enhancement [4], intrinsic time-scale decomposition [5], integrated spectral coherence [6], spectral kurtosis [7], and the sparsity-based algorithm [8]. These methods and relevant variants can inspect faults of rolling-element bearings (REBs), but details concerning defect size are inaccessible. Defect size estimation plays a crucial role in the prognosis and estimation of the remaining useful life (RUL) of REBs [9–12]. However, in practice, the faulty bearing usually contains not just one type of fault, but many kinds, such as the simultaneous spalling of the inner and outer races. Therefore, it is very important to carry out a quantitative evaluation of the compound fault of the bearing.

Currently, two methods are employed for defect size estimation. The first detects entry and exit impulses of rolling elements in the defective area in the time domain. However, as the vibration signal is very noisy, these entry and exit impulses are hard to detect. Thus, the use of a signal...
processing technique such as pre-whitening or minimum entropy deconvolution (MED) is required [13, 14]. Due to the low amplitude at the entry point, such weak impulses are hard to detect, which has led to the development of machine-learning approaches to defect detection. This technique has been successfully applied to the detection of artificially seeded defects. However, the technique will likely be harder to apply to the detection of natural defects. Some researchers have adopted a wavelet transform approach for entry and exit point detection [15–17]. However, while wavelet coefficients exhibit discontinuities at certain decomposition levels more accurately, there are difficulties with using them to identify entry and exit points. Variational mode decomposition (VMD) has been applied to the evaluation of the double pulse signals after pre-whitening. The components, including the step component and the impact component, can be resolved according to the kurtosis value and the maximum mutual information. The double pulse characteristics are used to determine the time at which the rolling elements enter and exit the defect, which allows the defect size to be estimated [18]. Through band-pass filtering, demodulation, low-pass filtering, and ensemble averaging of the LAITs, a reinforced signature called the envelope ensemble average (EEA) is acquired for every rolling element, the characteristics of which allows the defective elements to be identified [19].

The second method used to estimate defect size is based on statistical methods. There is a relationship between the parameters (vibration metrics) of the vibration signal and the defect size. For example, the linear relationship between root-mean-square acceleration and defect size has been applied to RUL estimation [20]. Al-Ghamd and Mba [21] experimentally explored the vibration and acoustic emission parameters for seeded defects in different sizes of an REB. Larizza et al. [22] developed a new defect size estimation method to improve the accuracy in estimating the size of a defect that has sloping leading and trailing edges. Rohani and Vahid [23] carried out a comprehensive study of the effect of defect size in the outer race, inner race, and rolling elements on metrics such as root-mean-square acceleration, peak acceleration, crest factor, kurtosis, level crossing rate, and presented simple connections between defect size and vibration characteristics. A Levenberg–Marquardt back-propagation neural network (BPNN) has also been used to predict inner race, outer race, and roller defect sizes [24]. A high-efficient assessment method using a deep neural network (DNN) was presented to correctly predict bearing degradation levels with a range of crack sizes [25]. Lu et al. [26] proposed an innovative diagnosis model based on a deep convolutional network with the Bayesian optimization to recognize bearing defect severity. While Nguyen et al. [27] presented a high-efficient method to evaluate bearing severity degree using the discrete wavelet packet transform (DWPT) and envelope analysis.

However, statistical methods cannot accurately detect entry and exit events, and defect size can be measured without the need for machine-learning-based approaches. The key advantage of this approach is that the measurement of vibration metrics requires none of the sophisticated and time-consuming computation required by some other approaches. Most studies in the literature also focus only on the quantitative evaluation of single faults. There has been very little quantitative evaluation of compound faults published. Given this, a systematic survey of the relationship between the metrics of the time-domain signal and the defect size of a cylindrical roller bearing with compound faults on the outer and inner races is performed in the current research. The analysis process of this paper is shown in Figure 1:

2. Dynamic Modeling of Compound Faults on the Inner and Outer Races

For the dynamic modeling of cylindrical roller bearing without any faults, we have adopted the method described in Chen et al. [28] and Cao et al. [29]. In the case of the dynamic modeling of compound faults on the inner and outer races, to facilitate effective analysis and simplify the calculation, we assume the following conditions:

(i) The axial motion of all parts can be neglected
(ii) The influence of temperature can be neglected
(iii) The centroid of each element of the bearing is in line with the geometric centre

In Figure 2, \( \omega_{di}, \omega_{do} \), and \( \theta_{id}, \theta_{od} \) denote the width of the damaged area and the depths of the inner and outer races, respectively, while hod. \( \theta_i, \theta_o \) are half the centre angles in line with the damaged areas of the inner and outer races, respectively. \( \theta_{id} \) and \( \theta_{od} \) are the angles of rotation of the centre of the damaged area of the inner and outer races, respectively, with \( \theta_{ib}, \theta_{ib} \) being the angles of revolution of the ith and \( i + 1 \)th rollers, respectively.

To identify whether the roller enters the damaged area, it is necessary to calculate the centre angle consistent with the damage and the centre of the roller:

\[
\begin{align*}
\theta_{bdi} &= \text{mod} (\theta_{i1b} \cdot 2\pi) - \text{mod} (\theta_{id}, 2\pi), \\
\theta_{bdo} &= \text{mod} (\theta_{i0b} \cdot 2\pi) - \text{mod} (\theta_{od}, 2\pi).
\end{align*}
\]

(1)

If \( \theta_{bdo} < \theta_{ib} \), the roller enters the damaged area of the inner race and otherwise does not. While if \( \theta_{bdo} < \theta_{oe} \), the roller enters the damaged area of the outer race and otherwise does not.

If the roller does not enter the damaged area, the contact deformations of the roller and the race can be computed following Chen et al. [7]. If it does enter the damaged area, the contact deformations need to be computed, considering the additional deformation resulting from the damage. The calculation of the total deformations is presented as follows:

\[
\begin{align*}
\delta_{di} &= \begin{cases} 
\delta_i - h_{id}, & |\theta_{bdi}| < \theta_{oe}, \\
\delta_i, & \text{else}.
\end{cases} \\
\delta_{do} &= \begin{cases} 
\delta_o - h_{od}, & |\theta_{bdo}| < \theta_{oe}, \\
\delta_o, & \text{else}.
\end{cases}
\end{align*}
\]

(2)

Furthermore, as Figure 3 shows, the direction of the contact load alters if the roller enters the damaged area. \( G_i \) and \( G_o \) are the contact points in the inner and outer races, respectively. In the normal bearing model, the direction of
the contact load denotes the centre of curvature of the race, as depicted in the contact coordinate system. However, when damage has been encountered, the contact load indicates the centre of mass of the roller via $G_i$ or $G_o$.

To help illustrate this, the damage contact coordinate systems of $G_i$ and $G_o$ have formed at contact points $G_i$ and $G_o$ separately. Then, the contact loads can be presented as follows:

$$F_{dc}^i = K \delta_{di},$$
$$F_{dc}^o = K \delta_{do}. \tag{3}$$

To introduce a mathematical expression of damage into the dynamic bearing model, it is necessary to transform the damage contact coordinate system into the normal bearing contact coordinate system. Thus, the transformation matrix between the coordinate systems is required. In addition, the radius of the roller and the radius of curvature of the races remain distinct. The angle $\Psi_i (o)$ between the $z_{ic}$-axis and the $z_{i(o)dc}$-axis can be decided per the geometric relationship. The transformation matrix can thus be denoted as follows:

$$A_{i(o)dc} = A(\Psi_i(o), 0, 0). \tag{4}$$

Figure 4 shows the process flow of the simulation program of the dynamic model of compound faults on the inner and outer races.

The structural parameters and operating conditions of the model bearing used in the present study are given in Tables 1 and 2.

3. Decomposition and Reconstruction of the Compound Fault Signal

3.1. Improved Continuous Harmonic Wavelet Packet Decomposition. In harmonic wavelet decomposition, the
scaling functions $m$ and $n$ determine the number of wavelet decomposition layers. With an increase in decomposition layers, the thinning ability of the low-frequency band improves, but the resolution of the high-frequency band decreases. Therefore, many studies use dyadic harmonic wavelet packet decomposition to realize an arbitrary refinement of the signal frequency band.

$$B = \frac{f_h}{2^j},$$

where $B$ is the analysis bandwidth and $f_h$ is the highest analysis frequency of the signal. The upper and lower limits of the corresponding analysis frequency bandwidth are as follows:

$$B = \frac{f_h}{2^j},$$

$$m = sB$$

$$n = (s + 1)B$$

where $s = 0, 1, 2, \ldots, 2^j - 1$. (6)

Figure 7 shows the decomposition process of this method [30]. With an increase in decomposition layers, the number of subbands and the bandwidth range after harmonic wavelet packet decomposition will be limited by this use of a binary decomposition method and the bandwidth range of interest cannot be selected arbitrarily.
To eliminate the limitations of binary decomposition on the number of subbands and bandwidth range, the arbitrary subdivision of the signal frequency band can be realized by changing the values of the scaling functions \( m \) and \( n \); hence, a useful frequency band range can be obtained. Figure 8 shows the decomposition process under this scheme.

The decomposition steps are as follows:

According to the frequency range of the fault, the number of layers \( k \) of signal \( x(t) \) to be decomposed is determined and the corresponding subband width is as follows:

\[
B = \frac{f_s}{k}. \tag{7}
\]

The upper and lower limits corresponding to the subband width are as follows:

\[
\begin{align*}
\{ & n = sB \\
& m = (s + 1)B \\
\} \\
& s = 0, 1, 2, \cdots, k - 1.
\tag{8}
\end{align*}
\]

The frequency-domain expression for the harmonic wavelet of each subband is as follows:

\[
W_{m,n}(\omega) = \begin{cases} 
\frac{1}{[2\pi(n-m)]}, & \omega \in \left[ \frac{mN}{f_s}, \frac{nN}{f_x} \right], \\
0, & \text{else,}
\end{cases}
\tag{9}
\]

where \( N \) is the signal length and \( f_x \) is the sampling frequency.

To ensure that the decomposed spectrum amplitude is the same as that of the original spectrum, an amplitude correction coefficient \( 4\pi (n-m) \) is defined, and the decomposed spectrum is as follows:

\[
X'(f) = 4\pi(n-m)X(f)W_{m,n}(\omega). \tag{10}
\]

The inverse Fourier transform is applied to equation (10) to obtain the decomposed subband signal of the \( k \)-layer of the continuous harmonic wavelet packet.

3.2. Compound Fault Decomposition and Single Point Fault Reconstruction. The compound fault signal of a roller bearing can be depicted as follows:

\[
x(t) = \sum_{i=1}^{I} p_i s_i(t), \tag{11}
\]

where \( I \) is the number of bearing faults, \( s_i(t) \) is the \( i \)th fault source signal, and \( p_i \) is the weight coefficient of the \( i \)th fault source signal in the composite fault signal. The steps of harmonic wavelet packet decomposition of a bearing composite fault then become:

1. The characteristic frequency of a single point fault of the inner and outer race of the roller bearing is calculated: \( f_i (i = 1, 2, \cdots, I) \).

2. \( B(B > \bar{f})_i \) is the frequency domain bandwidth of each decomposed subband calculated using equation (7).

3. The scale parameters \( m \) and \( n \) of the harmonic wavelet for \( k \)-layer decomposition are adjusted to obtain the \( k \) subband signal expressed as follows:

\[
\{ x_1(t), x_2(t), \cdots, x_k(t) \}. \tag{12}
\]

After harmonic wavelet packet decomposition, the \( j \)-th subband signal \( x_j(t) \) can be regarded as a modulated signal of the form:

\[
x_j(t) = a_j(t)\cos[\phi_j(t)]. \tag{13}
\]

Then, the energy operator of \( x_j(t) \) is expressed as follows:

\[
\varphi[x_j(t)] = [x_j(t)]^2 - x_j(t)\dot{x}_j(t). \tag{14}
\]

Moreover, the instantaneous amplitude of the signal \( x_j(t) \) is as follows:

\[
|a_j(t)| = \frac{\varphi[x_j(t)]}{\sqrt{\varphi[x_j(t)]}}. \tag{15}
\]

The instantaneous amplitude spectrum of \( x_j(t) \) can be obtained by spectral analysis of the instantaneous amplitude. Once the subband signal is demodulated by the energy operator, the fault frequency of each single-point fault pulse can be obtained. The proportion of the peak amplitude corresponding to the characteristic frequency of each single point fault in the whole envelope spectrum of the subband in the compound fault of the roller bearing is the weight coefficient of the single point fault signal in the subband signal, which can be expressed as follows:

\[
r_{ai} = \frac{W_a(T_f)}{\sum W_a(T_f)} (a = 1, 2, \cdots, k \quad i = 1, 2, \cdots, I), \tag{16}
\]

where \( r_{ai} \) is the weight coefficient of the \( i \)th single point fault in the \( a \)th subband.

The single-point fault signal in each subband can be expressed as follows:

\[
s_{ai}(t) = r_{ai} x_a(t). \tag{17}
\]

By adding the signals representing the same single point fault source in each subband, the single point fault signal of the roller bearing can be obtained:

\[
s_i(t) = \sum_{a=1}^{k} s_{ai}(t) (i = 1, 2, \cdots, I). \tag{18}
\]
As Figures 9 and 10 show, by using the above method, the compound fault signal of the cylindrical roller bearing is decomposed and reconstructed to obtain the vibration signal corresponding to the single fault on the outer and inner races, and the periodicity of the signals is clear.

Figures 11 and 12 show the spectral analysis of vibration signals in the case of a single fault in the inner and outer races. The frequency components in the figures are clear of noise, with the characteristic fault frequency being prominent. These results are consistent with the theoretical calculations, verifying the approach.

4. Quantitative Evaluation of the Defect Size

In this section, we explore the evolution of defects in the outer and inner races as the diameter of the defect is enlarged from 1 mm (representing a tiny defect) to 25 mm (the outer race is completely defective). Acceleration signals have been obtained by dynamic modeling of compound faults for every diameter of the defect. In contrast, their single-point fault signals are obtained by the method described in Sections 2 and 3. The values of the root mean square acceleration (RMS), CF, kurtosis, peak of acceleration, and LCR metrics are investigated to determine how they change with the changing size of the defect.

Figure 13 shows a plot of RMS (the most used vibration metric) versus the defect diameters of the outer and inner races. RMS has a good linear relationship with fault size to a large extent. For the outer race fault, the transition from a linear to a nonlinear relationship occurs when the defect reaches 17 mm in diameter. For the inner race fault, the deviation occurs at 11 mm.

Figure 14 shows a plot of peak (another popular vibration metric) versus defect diameters of the outer and inner races. A fast rise in the value of peak is seen at the commencement of defect evolution, and the fluctuation is very violent, the linear relationship is not significant. For the outer ring fault, when the defect size is greater than 17 mm, peak approaches a constant value. For the inner ring fault, peak plateaus once the fault reaches 11 mm in diameter.
In REB fault monitoring, crest factor (CF)—the ratio of peak to RMS acceleration—is a commonly used metric, which is valued for being dimensionless. As Figure 15 shows, the values of CF/defect proportion are high in the case of smaller defects, then drop to a constant value as the outer race, and inner race defect sizes increase to 17 mm and 11 mm, respectively. Moreover, the linear relationship is not significant.

Another common metric found in REB fault monitoring is kurtosis. Figure 16 shows the variation of the kurtosis/defect proportion with defect size. Kurtosis leaps to a high value as soon as the defect begins to grow but then begins to decline and flatten. For the outer race and inner race faults, kurtosis approaches a constant value once the defect diameters reach 17 mm and 11 mm, respectively. Moreover, the linear relationship is not significant.

As a metric often used to define randomly fluctuating signals, we also investigated the LCR (level crossing rate) in parallel with more conventional vibration metrics. The results showed a strong linear correlation between LCR and defect size. We assign LCR $LCR(t, \alpha)$ as the unit temporal frequency at which the plot of $x(t)$ crosses a designated threshold $\alpha$ while having a positive slope. Figure 17 shows the LCRs of the outer ring fault vibrations under 2 × RMS. A linear correlation between the LCR with the size of the defect is seen. For the outer ring fault, LCR increases linearly with the defect size until 17 mm; afterward, the LCR continues to increase linearly but with a lower slope. For the inner ring fault, the turning point occurs at 11 mm.

5. Discussion and Verification

According to the change law of the five statistical parameters with the size of the defect, the root means square value can better reflect the change of the size of the inner and outer...
Figure 15: CF versus defect diameters of the outer and inner races. (a) Defect on the outer race. (b) Defect on the inner race.

Figure 16: Kurtosis versus defect diameters of the outer and inner races. (a) Defect on the outer race. (b) Defect on the inner race.

Figure 17: LCR versus defect diameters of the outer and inner races. (a) Defect on the outer race. (b) Defect on the inner race.
races defect, but there are certain limitations on the effectiveness. The LCR value can better reflect the size of the defect, showing a good linear relationship, but pay attention to the existence of inflection points. Peak, CP, and kurtosis oscillate violently with the change in fault size, and the linear relationship is not very significant, but it can qualitatively describe the change in defect size.

To test the validity of the simulation results, a series of experimental tests were performed for comparison.

Figure 18 shows the bearing test rig, which comprised a driving system, a mechanical cantilever architecture, and a loading device. The spindle was supported on two bearing sets, and the test bearing was arranged at one end. The spindle rotation was powered by a motor that was driven directly. A hydraulic cylinder, which was capable of imposing both radial and axial loads on the bearing, was utilized to load the loading device. As Figure 19 shows, the cylinder pressure was taken as the measure of the load, and Figure 20 shows the bearing with compound faults.

Taking the inner and outer ring’s fault of 2 mm as an example, as shown in Figure 21, the vibration signal of
Figure 22: Reconstructed single-point fault vibration signal based on the test. (a) Defect on the outer race. (b) Defect on the inner race.

Figure 23: Envelope spectra of single-point fault vibration signal based on the test. (a) Defect on the outer race. (b) Defect on the inner race.

Figure 24: Comparison of experimental data and simulation results of RMS. (a) Defect on the outer race. (b) Defect on the inner race.
A cylindrical roller bearing experiencing a compound fault has been modeled dynamically based on the Gupta principle, and the compound fault vibration signals have been obtained by the fourth-order variable step length Runge–Kutta method. The periodicity of the vibration signal is not significant in the temporal domain and the analysis of the envelope spectrum reveals that the aliasing of the fault characteristic frequency is pronounced. Through an improved continuous harmonic wavelet packet decomposition method, the composite fault signal can be decomposed and reconstructed efficiently, allowing the characteristic frequency and time-domain signal of a single fault in the inner and outer circles to be obtained. Through numerical simulations, the dependencies of five common vibration metrics on the diameter of the defect were derived, which were then validated from experimental data. The results suggest that LCR would be a useful metric for REB condition surveillance. The quantitative analysis of compound faults in the inner and outer races of cylindrical roller bearings presented here offers new insights that could lead to the easier forecasting of defect size and the state of REB degradation from vibration data. In future work, we will continue to study the fault identification methods of three kinds of bearing components that have faults at the same time [31, 32].

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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