A New Design Framework on Device-to-Device Coded Caching with Optimal Rate and Significantly Less Subpacketizations

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Abstract—In this paper, we propose a new design framework on Device-to-Device (D2D) coded caching networks with optimal rate but significantly less file subpacketizations compared to that of the well-known D2D coded caching scheme proposed by Ji, Caire and Molisch (JCM). The proposed design framework is referred to as the Packet Type-based (PTB) design, where D2D users are first partitioned into multiple groups, which leads to a so-called raw packet saving gain. Then the corresponding multicasting group types and packet types are specified based on the prescribed node partition. By a careful selection of transmitters within each multicasting group, a so-called further splitting ratio gain can also be achieved. By the joint effect of the raw packet saving gain and the further splitting ratio gain, an order-wise subpacketization reduction can be achieved compared to the JCM scheme while preserving the optimal rate for large system parameter regimes. In addition, as the first time presented in the literature according to our knowledge, we find that unequal subpacketization is a key to achieve a subpacketization gain when the number of users is odd. As a by-product, instead of directly translating shared link caching schemes to D2D caching schemes, at least for the sake of subpacketizations, a new design framework is indeed needed.

I. INTRODUCTION

Coded caching has been shown to be an efficient approach to handle dramatically increased traffic in current Internet. In [1], Maddah-Ali and Niesen introduced a centralized shared-link caching network model, where a central controller serves $K$ users, each of which is equipped with a cache of size $M$ files from a library of $N$ files, via an errorless broadcast link (shared link). In order to achieve the optimal rate, the required number of packets where $t = KM/N$. Later, [2] shows that this file subpacketization level is necessary to achieve the optimal rate under uncoded cache placement and a coded delivery schemes were proposed [1] and required to partition each file into $(\binom{N}{t})$ packets where $t = KM/N$. Later, [2] shows that this file subpacketization level is necessary to achieve the optimal rate under uncoded cache placement. Ji, Molisch and Caire (JCM) extended the shared link caching model to Device-to-Device (D2D) coded caching networks, where no central controller is present and all users serve each other via individual shared links [3], and proposed a caching scheme referred to as the JCM scheme that achieves the optimal rate of $R(M) = \frac{N}{K} (1 - \frac{N}{K})$, which surprisingly is not a function of $K$ and hence $R(M)$ is scalable. In order to achieve this rate, the required number of subpackets (subpacketization level) is $F_{\text{JCM}} = t(\binom{N}{t})$, which can be impractical for large $K$. Efforts have been made in reducing the subpacketization levels for D2D coded caching problem [4]–[6]. For example, a design approach named D2D placement delivery array (DPDA) was introduced in [5], which designed new DPDA schemes when $t = 2, t = K - 2$, for which the JCM scheme is actually not optimal in terms of subpacketizations although it achieves the optimal rate.

In this paper, we propose a new design framework called Packet Type-based (PTB) design tailored for subpacketization reduction in D2D coded caching while preserving the optimal rate. In particular, in the PTB design, D2D users are first partitioned into multiple groups, which will lead to a so-called raw packet saving gain. Then the corresponding multicasting group types and packet types are specified based on the prescribed node partition. Based on a careful transmitter selection process within each multicasting group, a so-called further splitting ratio gain can also be achieved. The joint effect of the raw packet saving gain and the further splitting ratio gain can lead to an order-wise subpacketization reduction compared to the JCM scheme while preserving the optimal rate. In fact, the PTB design problem can be cast into an integer optimization problem subject to node cache constraints and the design variables are the choices of possible transmitters within each multicasting group type. Moreover, according to our knowledge, it is the first time in the literature showing that unequal subpacketization is a key to achieve a subpacketization gain when $K$ is odd.

In [3], in order to achieve the optimal rate, the JCM scheme proposed a direct translation from Maddah-ali and Niesen’s scheme by partitioning each packet further into $t$ subpackets. It turns out that when the cache placement is uncoded and the delivery scheme is one-shot [3], this translation holds in general and it seems that the design procedure for the D2D coded caching scheme is 1) design a shared link coded caching scheme; 2) translate it into D2D coded caching scheme. As a by-product of the PTB design, we show that the above design methodology is not optimal in terms of subpacketizations in general. Hence, in order to achieve good subpacketizations in D2D coded caching networks, a new

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1 Rate is defined as the total number of file transmissions in the network.

2 One-shot delivery scheme means that for each coded transmission, every user can successfully one request packet.
B. Packet Type-based (PTB) Design Framework

In order to achieve the above goal, we propose a new D2D coded caching design framework called Packet Type-based (PTB) design, which classifies packets and multicasting groups in multiple types. We will present the PTB design framework by decomposing it into the concepts including Node Grouping, Packet Type, Multicasting Group Type, Further Splitting Ratio (FSR), Further Splitting Ratio Table (FSRT), Memory Constraint Table (MCT) and PTB Design as an Integer Optimization Problem. For each part, we will present a corresponding example.

1) Node Grouping: $U$ is partitioned into $m \in \mathbb{Z}^+$ non-empty groups denoted as $Q_i$, where $i \in [K]$ and $|Q_i| = q_i$. Let the partition vector be $\mathbf{q} := (q_1, q_2, \ldots, q_K)$ satisfying $\sum_{i=1}^K q_i = K$ and $q_1 \geq q_2 \geq \cdots \geq q_m > q_{m+1} = \cdots = q_K = 0, m$ and $\mathbf{q}$ are design parameters. Let $N_d$ be the number of distinct elements/parts in $\mathbf{q}$. We define a unique group as the union of non-empty groups/parts which contain the same number of nodes. The $i$-th, $i \in [N_d]$ unique group, denoted by $U_i$, contains $\psi_i$ node groups which all contain $\beta_i > 0$ nodes, where \( \sum_{i=1}^{N_d} \beta_i \psi_i = K, \sum_{i=1}^{N_d} \psi_i = m \). For example, let $K = 7$ and $U = [K]$. $\mathbf{q} = (3, 2, 1, 1, 0)$ is a partition of $K = 7$ and we

$\text{Fig. 1. An illustration of packet types under node grouping } q = (3, 3).$

partition $U$ into $m = 4$ groups, which are $\{1, 2, 3\}$, $\{4, 5\}$, $\{6\}$ and $\{7\}$. In this case, $N_d = 3$, $(\beta_1, \psi_1) = (3, 1), (\beta_2, \psi_2) = (2, 1), (\beta_3, \psi_3) = (1, 2)$. $U_1 = Q_1 = \{1, 2, 3\}, U_2 = Q_2 = \{4, 5\}, U_3 = \{6, 7\}$ and $Q_4 = \{7\}$. We call a node grouping an equal grouping if all the groups contain the same number of nodes, i.e., $q_1 = q_2 = \cdots = q_m = \frac{K}{m}$. Otherwise, it is called an unequal grouping. Clearly, the example above is an unequal grouping.

2) Packet Type: A packet type refers to a partition of $t := \frac{KM}{N} \in \mathbb{Z}^+$ nodes and is represented by a partition vector $\mathbf{v} := (v_1, v_2, \ldots, v_t)$ satisfying $\sum_{i=1}^t v_i = t$, where $v_i \in \mathbb{Z}^+$, $\forall i \in [t]$ and $v_1 \geq v_2 \geq \cdots \geq v_t \geq 0$. Different partitions of $t$ correspond to different packet types. Let $V$ be the number of different packet types that can be valid in a given node grouping realization $q$. A raw packet $W_{n,T}$, for some $T \subset U$, $|T| = t$ refers to a packet that is cached exclusively in a set of nodes in $T$. Each packet type may contain multiple raw packets. Since some of $T \subset U$ with $|T| = t$ may not be included in the raw packet partitions, we may need a lesser number of raw packets compared to the JCM scheme. This is called raw packets saving gain. In the delivery phase, raw packets might be further split into multiple subpackets, i.e., $W_{n,T} = \{W_{n,T}^{(1)}, \ldots, W_{n,T}^{(\alpha_v)}\}$ where $\mathbf{v}$ is the packet type and $\alpha_v(\mathbf{v})$ is called further splitting ratio. Raw packets of the same type must have the same further splitting ratio. Note that all raw packets have the same further splitting ratio $\alpha_v(\mathbf{v}) = t, \forall \mathbf{v}$ in the JCM scheme. The following example illustrates the concept of packet types and raw packets.

Example 1: (Packet Type) For $(K, t) = (6, 3)$ and $U = [K]$, consider the node grouping $q = (3, 3)$ with a specific node-group assignment $Q_1 = \{1, 2, 3\}, Q_2 = \{4, 5, 6\}$, which is shown in Fig. 1. There are two different types of packets, i.e., $v_1 = (3, 0)$, meaning picking three nodes from one of the two groups, and $v_2 = (2, 1)$, meaning picking two nodes from one group and one node from the remaining group. For example, the packet $W_{n,4,5,6}$ which is cached in nodes 4, 5, 6 is a type-$v_1$ packet. The packet $W_{n,3,5,6}$ which is cached in nodes 3, 5, 6 is a type-$v_2$ packet. It can be seen that there are in total $2{\binom{t}{1}}_0 = 2$ type-$v_1$ packets and $2{\binom{t}{2}}_1 = 18$ type-$v_2$ packets. These two packets are called raw packets since they have not been further split into subpackets for the purpose of multicast transmissions in the delivery phase.

3) Multicasting Group Type: A multicasting group is a set of $t+1$ nodes among which each node broadcasts some packets needed by the remaining $t$ nodes. A multicasting group type refers to a specific partition of $t + 1$ which is represented by $s := (s_1, s_2, \ldots, s_{t+1})$ satisfying $\sum_{i=1}^{t+1} s_i = t + 1$, where
A unique group of s of $U_i^r$, $i \in |N_s^n|$ refers to the union of parts of s that contain the same number of nodes, where $N_s^n$ is the number of distinct parts of s. For a specific multicasting group $S$ of type s, the set of unique groups of s are represented as $\{U_i^n\}_{i \in |N_s^n|}$ and we have $S = \bigcup_{i \in |N_s^n|} U_i^n$. We define a involved packet type set, denoted by $\rho_i$, corresponding to a specific multicasting group type $s_i$, $i \in |S|$, as the set of packet types that can appear in the transmission process within that multicasting group type. We also denote the indices of the packet types contained in $\rho_i$ as $J_i := \{j | v_j \in \rho_i\}$.

We illustrate the above concepts using Example 1. Note that a multicasting group contains l + 1 nodes. There are two different multicasting group types, i.e., $s_1 = (3,1)$ and $s_2 = (2,2)$. Type-v1 multicasting group is composed of three nodes from one group and one node from the other group. Two type-v2 multicasting group is composed of two nodes from $Q_1$ and $Q_2$. For example, the multicasting group $S_1 = \{3,4,5,6\}$ is a type-s1 multicasting group which contains one type-v1 packet $W_{d_4,4,5,6}$ and three type-v2 packets $W_{d_4,4,6}, W_{d_4,4,6}, W_{d_4,4,6}$ and $W_{d_4,4,6}$. Hence, the involved packet type set associated with $s_1$ is $\rho_1 = \{v_1, v_2\}$ and $J_1 = \{1, 2\}$ since both packet types will be used in the delivery within type-s1 multicasting groups. There are $N_s^n = 2$ unique groups in $S_1$ which are $U_1^n = \{3\}$ and $U_2^n = Q_2 = \{4,5,6\}$. Moreover, $S_2 = \{1, 2, 4, 5\}$ is a type-s2 multicasting group containing four type-v2 packets $\{W_{d_4,4,6}, k \in S_2\}$. Hence, $\rho_2 = \{v_2\}$, $J_2 = \{2\}$. There is only $N_s^n = 1$ unique group in $S_2$ which is $U_2^n = S_2$.

4) Further Splitting Ratio (FSR): For a multicasting group $S$ of type s, a set of nodes $T_x$ is selected to serve as the transmitters. We can select $T_x$ in such a way that it can be expressed as a union of $|D_T|$ different unique groups where, in the multicasting group $S$ of type s, $D_T$ is defined as the set of the indices of the unique groups from which the transmitters are selected, i.e., $T_x = \bigcup_{i \in D_T} U_i^n$. Denote $g_i := |U_i^n|$, then we have $|T_x| = \sum_{i \in D_T} g_i$. There are $N_s^n$ different packet types associated with s, where the packet type $v_i$ corresponds to the packets including $W_{n,s_i^n(k_i), n \in |N|}$ in which $k_i$ is a node belonging to the i-th unique group, i.e., $k_i \in U_i^n$. Under this selection of transmitters, the further splitting ratios for the involved packet types are

$$\alpha(v_i) = \begin{cases} \sum_{i \in D_T} g_i - 1 & \text{if } i \in D_T \\ \sum_{i \in D_T} g_i & \text{if } i \notin D_T \end{cases}$$

which means that the raw packets $W_{n,s_i^n(k_i)}$ associated to $v_i$ need to be further split into $\alpha(v_i)$ subpackets in the delivery phase when only considering the transmission process of multicasting groups $S$ of type s. Since one packet type can possibly be contained in multiple involved packet type sets and the above further splitting ratios are derived when only one multicasting group type is considered, we refer to this further splitting ratio as local further splitting ratio, which is illustrated in the following using Example 1.

Consider multicasting group $S_1 = \{3,4,5,6\}$, composed of two unique groups $U_1^n = \{3\}$, $U_2^n = \{4,5,6\}$. We have three different choices for transmitters, which will lead to different further splitting ratios for raw packets in $S_1$. For example, if we choose $T_x = U_1^n = \{3\}$ ($D_T = \{1\}$), meaning that only node 3 will transmit while nodes 4, 5, 6 only receive. In this case, the type-v1 packet $W_{d_4,4,5,6}$ needed by node 3 is not transmitted by any other node and the type-v2 packet $W_{d_4,4,6}$ needed by node 5 is transmitted by node 3. Since each one of the three type-v2 packets is only transmitted by one node, there is no need for further splitting or $\alpha(v_2) = |U_2^n| = 1$. However, no nodes are transmitting packet $W_{d_4,4,5,6}$ meaning that packet $W_{d_4,4,5,6}$ can be excluded from the caching scheme, i.e., type-v1 packets should be excluded, leading to a raw packet saving gain.

Note that in the JCM scheme, all the nodes in each multicasting group $S$ are selected as transmitters, i.e., $T_x = S$, $\forall S \in |K|$ such that $|S| = t + 1$. As a result, the further splitting ratio for any packets is equal to $\alpha = |S| - 1 = t$. This explains why each packet is further split into $t$ smaller subpackets in the JCM scheme. However, as mentioned above, it is not always necessary to select all $t + 1$ nodes within S to serve as transmitters. A selection of one or more (not all) unique groups within S as transmitters will lead to smaller further splitting ratios of packets as indicated in Eq. (1), providing an opportunity to reduce the subpacketization. This gain is called further splitting ratio gain. The overall subpacketization reduction of the proposed design framework is the result of both raw packet saving gain and further splitting ratio gain.

5) Further Splitting Ratio Table (FSRT): A further splitting ratio table is a matrix $A = [a_{ij}]_{S \times V}$ which specifies the local further splitting ratios derived from all the S multicasting types. More specifically, the i-th, $i \in |S|$ row of the FSRT, which is referred to as the local further splitting ratio vector $\alpha_i$, consists of entries equal to $\alpha(v_j), \forall v_j \in J_i$, specified by (1) and all the other entries are left empty. Note that a further splitting ratio of $\alpha = 0$ is not the same as an empty entry. To determine the overall further splitting ratio for all the $V$ types of packets, we need to derive the Least Common Multiple (LCM) vector $\alpha_{LCM}$ (defined below) of the S different local further splitting ratio vectors, which determines the overall further splitting ratio for each packet type.

**Definition 1:** (Least Common Multiple (LCM) Vector) For a set of $n$ vectors $A = \{a_i\}_{i \in |n|}$ in which $|a_i| = V$ and $a_i$ may contain ‘empty’ entries. The LCM vector of $A$, denoted by $\alpha_{LCM} := \text{LCM}(A)$, is defined as: $\exists z_1, z_2, \cdots, z_n \in \mathbb{Z}^+$ such that: (1) $z_1 a_1 = z_2 a_2 = \cdots = z_n a_n$ and (2) $\alpha_{LCM} = \min_{z_1, z_2, \cdots, z_n} \|a_i\|_2 = \min_{z_1, z_2, \cdots, z_n} \|\text{combine} \{z_1 a_i\}_{i \in |n|}\|_2$ in which the combine operation means that the j-th entry of $\alpha_{LCM}$ takes the value of the non-zero and non-empty value among the j-th entries all the n vectors $z_i a_i, i \in |n|$. We assume that: 1) the product of any integer and an empty entry is still an empty entry; 2) entry ‘0’ is equal to any other entries, including non-zero entries and empty entries; 3) empty entry
is equal to any other zero/non-zero entries. △

Note that the LCM vector may not always exist. If it exists, it must be unique. In a specific PTB design, the overall splitting ratio vector, denoted by $\alpha^{\text{LCM}}$, is obtained via deriving the LCM vector of the set of local splitting ratio vectors $\{\alpha_i\}_{i \in [S]}$, i.e., $\alpha^{\text{LCM}} := \text{LCM}\{\alpha_i\}_{i \in [S]}$.

Consider Example 4 where there are $S = 2$ multicasting group types $s_1 = (3, 1)$ and $s_2 = (2, 2)$. There are also $V = 2$ packets types $v_1 = (3, 0)$ and $v_2 = (2, 1)$. The corresponding involved packet type sets are $\rho_1 = \{v_1, v_2\}$ and $\rho_2 = \{v_2\}$. The selection of transmitters is as follows. For $s_1$, we choose the second unique group as transmitters. We use the superscript * to mark the transmitters within a multicasting group type, i.e., $s_1^* = (3, 1^*)$. For example, in a specific multicasting group $S_1 = \{3, 4, 5, 6\}$, the transmitters is node 3. This selection will result in a local further splitting ratio vector $\alpha_1 = (\alpha(v_1), \alpha(v_2)) = (0, 1)$. For $s_2$, the only choice is to select all $t + 1 = 4$ nodes as transmitters since there is only $N_{s_2}^t = 1$ unique group which has to be selected. This results in a further splitting ratio vector $\alpha_2 = (\ast, \alpha(v_2)) = (\ast, t)$ in which the symbol $\ast$ denotes an empty entry since type-$v_1$ packets do not appear in type-$s_2$ multicasting groups. As a result, the FSRT is

\[
\begin{array}{c|cc|}
| & v_1 & v_2 \\
\hline
\alpha_1 & 0 & 1 \\
\alpha_2 & \ast & 3 \\
\end{array}
\]

form which we can easily obtain $\alpha^{\text{LCM}} = \text{LCM}(\alpha_1, \alpha_2) = (0, 3)$, implying that in the PTB design, type-$v_1$ raw packets are excluded while each type-$v_2$ raw packet is further split into 3 subpackets.

6) Memory Constraint Table (MCT): A memory constraint table is a matrix $\Omega = [\omega_{ij}]_{N_d \times V}$ with $\omega_{ij} := F_i(v_j)$ where $F_i(v_j)$ denotes the number of packets of type $v_j$ cached by a node in the $i$-th unique group. Furthermore, $\forall i \in [N_d - 1]$, we define the node cache difference vector as $\Delta F_i := (f_{i1}, f_{i2}, \ldots, f_{iV}) = F_i(v_j) - F_i(v_j), \forall j \in [V]$ is the difference of the number of type-$v_j$ raw packets cached by nodes in the $i$-th and $(i + 1)$-th unique group $U_i$ and $U_i+1$. Let all the subpackets have the same size, the memory constraint can be represented as $\alpha^{\text{LCM}} \Delta F_i = \text{LCM}(F_{i+1}(v_j) - F_i(v_j)), \forall i \in [1 : N_d - 1], \text{i.e., } \alpha^{\text{LCM}} F_1 = \cdots = \alpha^{\text{LCM}} F_{N_d}, \text{ implying nodes in the all the } N_d \text{ unique groups have cached the same number of subpackets.}$

Consider Example 5 since the node grouping $q = (3, 3)$ is symmetric, the node caching memory constraint is automatically satisfied. The number of subpackets required is equal to $F = \alpha^{\text{LCM}} F^T = (0, 3)(F(v_1), F(v_2))^T = 54$, which is less than $F^{\text{JCM}} = 60$. Note that the subpacketization reduction comes from excluding type-$v_1$, raw packets. Hence, only raw packet saving gain is available in this example.

7) PTB Design As An Integer Optimization: With all the above definitions, under the condition of equal subpacketizations, we introduce the following integer optimization problem that determines the optimal LCM vector which results in the minimum $F$.

\[
\begin{align*}
\min \quad & F := \alpha^{\text{LCM}} F^T \\
\text{s.t.} \quad & \alpha^{\text{LCM}} \in \Phi, \quad \alpha^{\text{LCM}} \Delta F_i^T = 0, \forall i \in [N_d - 1].
\end{align*}
\]

where $\Phi$ represents the set of possible LCM vectors derived from the $S$ local further splitting vectors based on the set of all possible node grouping $q$ and the set of all possible selections of transmitters within each multicasting type under each $q$. Although each feasible solution of the above optimization problem corresponds to a valid PTB design which may or may not yield less number of subpackets than the JCM scheme, in this paper, we present several PTB designs with order or constant reduction on the subpacketization levels compared to the JCM scheme, implying that the JCM scheme is far from optimal in terms of subpacketization in general.

Moreover, we can extend the optimization problem to the case of unequal subpacketizations as follows.

\[
\begin{align*}
\min \quad & F := \left( \sum_{h=1}^{H} \alpha^{(h)} \right) F^T \\
\text{s.t.} \quad & \alpha^{(h)} \in \Phi, \quad \forall h \in [H], \quad \sum_{h=1}^{H} \gamma_h \alpha^{(h)} \Delta F_i^T = 0, \forall i \in [N_d - 1],
\end{align*}
\]

where $H$ denotes the number of different coupled groups $G_h, h \in [H]$, which is defined as a set of packet types with the same subpacket size $\ell_h$ bits. $\gamma_h$ denotes the subpacket size ratio of $G_h$ over $G_1$, i.e., $\gamma_h := \frac{\ell_h}{\ell_1}, \forall h \in [H]$. Moreover, $\alpha^{(h)}$ denotes the LCM vector derived based on the coupled group $G_h$ and $\Delta F_i^T$ represents the node cache difference vector based on the coupled group $G_h, h \in [H]$. 

III. AN EXAMPLE

In this section, we illustrate the above combinatorial design framework using an example with parameters $(K, t) = (3m, 3)$ with $m \geq 3$ and node grouping $q = (3, 3, \ldots, 3)$. In this case, $(S, V) = (2, 3)$. The packet types, multicasting group types and involved packet type sets are

\[
\begin{align*}
\nu_1 &= (3^{(m - 3)}, 2^{(3)}), \quad \nu_2 = (3^{(m - 2)}, 2^{(2)}), \quad \nu_3 = (3^{(m - 1)}, 1^{(1)}), \\
\nu_4 &= (3^{(m - 2)}, 2^{(2)}, 1^{(1)}), \quad \nu_5 = (3^{(m - 1)}, 1^{(1)}), \quad \nu_6 = (3^{(m - 2)}, 2^{(2)}), \quad \nu_7 = (3^{(m - 1)}, 1^{(1)}).
\end{align*}
\]

This results in a further splitting ratio table is

\[
\begin{array}{c|cc|}
|   & v_1 & v_2 \\
\hline
\nu_1 & 1 & 1 \\
\nu_2 & 0 & 3 \\
\nu_3 & 1 & 3 \\
\nu_4 & 0 & 3 \\
\nu_5 & 1 & 3 \\
\nu_6 & 0 & 3 \\
\nu_7 & 1 & 3 \\
\end{array}
\]

from which we obtain $\alpha^{\text{LCM}} = (4, 3, 0)$, implying that $v_3$ is excluded (raw packet saving gain) and $v_1, v_2$ have further splitting ratios of 4, 3 respectively (further splitting ratio gain). Hence, the number of subpackets is equal to

\[
F = (4, 3, 0)(F(v_1), F(v_2), F(v_3))^T = \frac{K(K - 3)(2K - 3)}{3}
\]

where $F(v_1) = \left(\frac{m}{3}\right)^3 = \frac{K(K - 1)(K - 2)}{6}, F(v_2) = \left(\frac{m}{2}\right)^2(\frac{m}{3})(\frac{m}{1}) = K(K - 3)$ and $F(v_3) = \left(\frac{m}{3}\right)^3 = \frac{K}{3}$. It
can be seen that when $K \geq 9$, $F'(F^\text{JCM}) = 4/K + o(4/K)$, which means an order gain of subpacketization level is obtained using the proposed scheme compared to the JCM scheme.

Next we consider the detailed delivery procedure for a system with $K = 9, N = 3, M = 2$ and $t = 6$. Consider a specific equal-grouping assignment $Q_2 = \{1, 2, 3\}, Q_3 = \{4, 5, 6\}$ and $Q_3 = \{7, 8, 9\}$. Since $\alpha^\text{JCM} = (4, 3, 0)$, type-$v_1$ packets are excluded and type-$v_1$ and type-$v_2$ packets need to be further split into 4 and 3 subpackets respectively. In this case $F = 270$ while $F^\text{JCM} = 504$. The cache placement is that node $k$ stores any subpacket $W^{(j)}_{j,k}$ if $k \in T$. Note that in this example the subpacketization reduction gain compared to the JCM scheme consists of two parts: 1) raw packet saving gain: $tF(v_1) = 18$ subpackets and 2) further splitting ratio gain: $(t - 4)F(v_1) + (t - 3)F(v_2) = 216$ subpackets. We can see that the reduction is mainly due to smaller further splitting ratios of type-$v_1$ and type-$v_2$ packets.

For a type-$s_1$ multicasting group $S_1 = \{7, 8\}$, node 7 is the only transmitter and it transmits three coded multicast messages $W^{(1)}_{4,7}, W^{(1,2)}_{5,7}$, $j = 1, 3$ to other nodes in $S_1$. Each node $k$ receives its desired subpackets $W^{(j)}_{i,j,k}$ with the help of the cached content while node 7 itself only transmits but receives nothing. For a type-$s_2$ multicasting group $S_2 = \{9\}, \{6, 9\}$, the set of type-$v_1$ and type-$v_2$ subpackets involved are $W^{(j)}_{i,j,k}$: $j \in \{4, 5\}, k \in \{3\}$ and $W^{(j)}_{i,j,k}$: $j \in \{3\}, k \in S_2 \setminus Q_1$ respectively. Denote $W^{(j)} := \bigoplus_{i \in 3} W^{(j)}_{i,j,k} : j \in \{4, 5\}, 7, 8$ each sends a coded multicast message as follows.

$W_4 = W^{(1)} \oplus W^{(1)}_{4,7} \oplus W^{(1)}_{5,7} \oplus W^{(1)}_{6,7}$

$W_5 = W^{(2)} \oplus W^{(2)}_{4,7} \oplus W^{(2)}_{5,7} \oplus W^{(2)}_{6,7}$

$W_7 = W^{(3)} \oplus W^{(3)}_{4,7} \oplus W^{(3)}_{5,7} \oplus W^{(3)}_{6,7}$

$W_8 = W^{(4)} \oplus W^{(4)}_{4,7} \oplus W^{(4)}_{5,7} \oplus W^{(4)}_{6,7}$

from which we can see that all nodes can recover their desired subpackets. Since each coded message is simultaneously useful for $t = 6$ nodes, the rate is optimal. The transmission procedure for other multicasting groups is similar.

IV. MAIN RESULTS AND DISCUSSIONS

Theorem 1: For even $\ell := K - t$, where $K = 2m$, using the PTB design framework, the optimal rate of D2D caching networks is achievable and

$$F' = \Theta \left( \frac{f(\ell)}{K - t} \right)$$

where $f(\ell) := \prod_{i=1}^{\ell} (2i - 1)$ is a function which depends only on $\ell$. Moreover, $\forall K \geq 2\ell$, and $\ell = O(\log\log K)$, $\frac{F}{F'}$ vanishes as $K$ goes to infinity.

From Theorem 1 it can be seen that when $K \leq 2\ell$ is even and $t$ is large enough (i.e., $t = K - O(\log\log K)$), an order gain in terms of subpacketization can be obtained using the PTB design compared to the JCM scheme while preserving the optimal rate. However, it can be seen that for small $t$, the PTB design achieving Theorem 1 may result in an even worse subpacketization compared to the JCM scheme. In the following theorem, we provide a general result for even $K$ and $t$ based on a specific PTB design and provide subpacketization gains compared to the JCM scheme when $t$ is small.

Theorem 2: Let $\zeta(t) := \lim_{K \to \infty} \frac{F}{F'}$, $t \in [2 : K]$. For $(K, t) = (2q, 2r)$ with $q \geq t + 1$ and $r \geq 1$, using PTB design framework with the two-group equal grouping, i.e., $q = \left( \frac{K + 1}{2}, \frac{K - 1}{2} \right)$, the optimal rate of D2D caching networks is achievable by the further splitting ratio vector $\alpha^\text{JCM} = (0, 1, 2, \ldots, r)$. Further, when $r \geq 2$, $\zeta(t) < \frac{1}{t}(1 - \frac{1}{1})$. From Theorem 2 it can be seen that under the conditions of even $K$ and $t$, based on a 2-group equal grouping PTB design, a constant gain of a factor of about 0.5 can be always achieved compared to the JCM scheme. This automatically applies to the case of small $t$ compared to $K$. When $K$ is odd, it is surprisingly more difficult than the case of even $K$. In Section III we provided an example showing that when $K = 3m$ and $t = K - 3$, $m \in \mathbb{Z}^+$, it is possible to exploit an equal user grouping to achieve an order gain of subpacketization level compared to the JCM scheme. However, in general, we may need to use the general PTB design framework in (4) to (6) that exploits the heterogeneous subpacket size.

Theorem 3: Let $\zeta(t) := \lim_{K \to \infty} \frac{F}{F'}$, $t \in [2 : K]$. For $(K, t) = (2q, 2r)$ with $q \geq 2r + 1, r \geq 1$, using the two-group unequal grouping $q = \left( \frac{K + 1}{2}, \frac{K - 1}{2} \right)$, the optimal rate of D2D caching networks is achievable by the further splitting ratio vector $\alpha^\text{JCM} = (0, 2, 4, \ldots, t - 2, t, t, \ldots, t)$. Moreover, $\zeta(t) \leq \frac{1}{t}(\frac{t}{2r} - 1) + 1$.

From Theorem 3 it can be seen that by using the general PTB design framework in (4) to (6) with the consideration of heterogeneous subpacket size, when $K$ is odd, a constant gain in terms of subpacketization compared to the JCM scheme can be achieved while preserving the optimal rate when $t$ small.

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