Stochastic Dynamics of Infrared Fluctuations in Accelerating Universe

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Based on arXiv: 1508.07877
Introduction

• In the presence of massless and minimally coupled scalar fields in accelerating universes, quantum fluctuations at super-horizon scales make vacuum expectation values of field operators growing with time (Infrared effects).

• From a semiclassical viewpoint, it was proposed that such infrared effects are well-described by a Langevin equation.

  ’94 J. Yokoyama, A. A. Starobinsky

• In de Sitter space, the stochastic approach has been proved to be equivalent to the leading power resummation of the growing time dependences.

  ’05 N. C. Tsamis, R. P. Woodard

• We extend these investigations in a general accelerating universe.
Free scalar field
in Accelerating universe

\[ ds^2 = -dt^2 + a^2(t)dx^2 \]

\[ H \equiv \frac{\dot{a}}{a} > 0 : \text{expanding era} \quad \Rightarrow \partial_t \]

\[ 0 \leq \epsilon \equiv \frac{-\dot{H}}{H^2} < 1 : \text{acceleration} \]

massless, minimally coupled

\[ S_2 = -\frac{1}{2} \int \sqrt{-g}d^4x \left[ g^\mu\nu \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 + \xi R \phi^2 \right] \]

\[ \varphi_0(x) = \int \frac{d^3p}{(2\pi)^3} \left[ a_p \phi_p(x) + a_p^* \phi_p^*(x) \right] \]

At \( P \equiv p/a \ll H \),

\[ \phi_p(x) \simeq -i \frac{2^{\nu-1} \Gamma(\nu^*)}{\sqrt{\pi}} \left\{ (1 - \epsilon^*) H^0 a^{\epsilon^*} \right\}^{1-\epsilon^*} \frac{1}{p^{\nu^*}} e^{+i p \cdot x} \]

freezing

\[ \nu = \frac{3}{2} + \frac{\epsilon}{1 - \epsilon} \]

*: horizon crossing
Free propagator

Propagator acquires a growing time dependence through the increase of d. o. f. at super-horizon scales

$$\langle \varphi_0^2(x) \rangle \simeq \int_{p < H_0} \frac{d^3 p}{(2\pi)^3} \frac{2^{2\nu_* - 2} \Gamma^2(\nu_*)}{\pi} \frac{(1 - \epsilon_*) H_* a_\epsilon^*}{p^{2\nu_*}} \frac{1}{p^{2\nu_* - 2}}$$

$$\nu \geq 3/2$$

Changing variables: $p = H' a' \Rightarrow dp = (1 - \epsilon') p H' dt'$,

$$\langle \varphi_0^2(x) \rangle = \int_{t_0}^{t} dt' (1 - \epsilon') \times \frac{(2 - 2\epsilon')^{2\nu' - 3} \Gamma^2(\nu')}{\pi^3} (1 - \epsilon')^2 H'^3$$

$$> 0$$

$$\rightarrow \frac{H^3}{4\pi^2} (t - t_0) = \frac{H^2}{4\pi^2} \log(a/a_0) \text{ at dS limit}$$
In interacting field theories

With each increase of \# of vertices, additional propagators appear

\[
\begin{align*}
\text{one of them is } \mathcal{G}^R(x, x') &= \theta(t - t')[\varphi_0(x), \varphi_0(x')] \\
\text{the others are } \langle \varphi_0(x)\varphi_0(x') \rangle \text{ and } \langle \varphi_0(x')\varphi_0(x) \rangle
\end{align*}
\]

Causality

- Secular growths of the Wightman functions originate in

\[
\varphi_0(x) \simeq \int \frac{d^3p}{(2\pi)^3} \theta(Ha - p) \left[ -i \frac{2\nu_*^{-1} \Gamma(\nu_*)}{\sqrt{\pi}} \left\{ \left( 1 - \epsilon_* \right) \epsilon_* \right\} \frac{1}{p^{\nu_*}} e^{+ip \cdot x} a_p + \text{(h.c.)} \right]
\]

const.

- Each vertex integral induces a secular growth

\[
\int \sqrt{-g'} d^4x' \ G^R(x, x') \simeq -i \int^t dt' \left( a'^3 \int_{t'}^\infty dt'' \ a''^{-3} \right)
\]

\[
\rightarrow -\frac{i}{3H^2} \int^a d(log a') \text{ at dS limit}
\]
Leading IR effects

For example $V = \frac{\lambda}{4!} \varphi^4$, with each increase in the loop level, quantum corrections are multiplied by up to the factor:

$$
\lambda \left[ \int_{t'}^t dt' (a'^3 \int_{t'}^\infty dt''' a'''^{\nu_3-3}) \times \int_{t''}^{t'} dt'' \frac{(2 - 2\epsilon'')^{2\nu_3-3} \Gamma^2(\nu'')}{\pi^3} (1 - \epsilon'')^3 H''^3 \right]
$$

Wightman function

$$
\int \sqrt{-g} d^4x' \ G^R(x, x') \rightarrow \lambda[ \log^2(a/a_0) ] \text{ at dS limit}
$$

Even if $\lambda \ll 1$, the perturbation theory is eventually broken after an enough time: $\lambda[ \cdots ] \sim 1$ passed

↓

Resummation formula for the leading powers of the growing time dependence is necessary to evaluate them nonperturbatively
Resummation formula

Yang-Feldman formalism is reduced to the Langevin eq. up to the leading IR effects

\[ \varphi(x) = \varphi_0(x) - i \int \sqrt{-g'} d^4x' \ G^R(x, x') \frac{\partial}{\partial \varphi} V(\varphi(x')) \]

Up to the leading IR effects

\[ \varphi_0(x) \simeq \bar{\varphi}_0(x) = \int \frac{d^3p}{(2\pi)^3} \ \theta(Ha - p) [(\text{const. spectrum})] \]

\[ \int \sqrt{-g'} d^4x \ G^R(x, x') \simeq -i \int^t dt' \ (a'^3 \int_{t'}^\infty dt'' \ a''^{-3}) \]

Langevin eq.: \[ \dot{\varphi}(x) = \dot{\bar{\varphi}}_0(x) - (a^3 \int_t^\infty dt' \ a'^{-3}) \frac{\partial}{\partial \varphi} V(\varphi(x)) \]

\[ \rightarrow 1/3H \text{ at dS limit} \]

\[ \langle \dot{\varphi}_0(t, x) \dot{\bar{\varphi}}_0(t', x) \rangle = \frac{(2 - 2\epsilon)^{2\nu-3} \Gamma^2(\nu)}{\pi^3} \{(1 - \epsilon)H\}^3 \delta(t - t') \]

White noise
Fokker-Planck equation

Langevin eq. is translated to the equation of the probability density $\rho$:

$$
\dot{\rho}(t, \phi) = \frac{AH^3}{2} \frac{\partial^2}{\partial \phi^2} \rho(t, \phi) + \left( a^3 \int_t^\infty dt' \ a'^{-3} \right) \frac{\partial}{\partial \phi} \left( \rho(t, \phi) \frac{\partial}{\partial \phi} V(\phi) \right)
$$

$$
A = \frac{(2 - 2\epsilon)^{2\nu-3} \Gamma^2(\nu)}{\pi^3} (1 - \epsilon)^3 > 0
$$

$$
\langle F(\varphi(x)) \rangle = \int_{-\infty}^\infty d\phi \ \rho(t, \phi) F(\phi), \quad F: \text{arbitrary function}
$$

If the variations of $H, \epsilon$ are negligible during the IR effects grow, eventually,

$$
\rho(t, \phi) \rightarrow N^{-1} \exp \left( - \frac{2}{AH^3} \left( a^3 \int_t^\infty dt' \ a'^{-3} \right) V(\phi) \right)
$$

e.g. $V = \frac{\lambda}{4!} \varphi^4$, \quad $\langle V(\varphi(x)) \rangle \rightarrow \frac{AH^3}{8} \left( a^3 \int_t^\infty dt' \ a'^{-3} \right)^{-1}$

Not suppressed by $\lambda \ll 1$
Naive semiclassical description

\[
\left( \frac{\partial^2}{\partial t^2} + 3H \frac{\partial}{\partial t} - \frac{1}{a^2} \frac{\partial^2}{\partial x^2} \right) \varphi(x) = -\frac{\partial}{\partial \varphi} V(\varphi(x))
\]

Extracting IR dynamics: \( \varphi = \tilde{\varphi} + \varphi_{UV} \)

Neglecting \( \partial_t^2, \partial_x^2 \)

Identifying \( \varphi_{UV} \) as a source of \( \tilde{\varphi} \)

\[
3H \frac{\partial}{\partial t} \left\{ \tilde{\varphi}(x) + \varphi_{UV}(x) \right\} = -\frac{\partial}{\partial \tilde{\varphi}} V(\tilde{\varphi}(x))
\]

\[
\varphi_{UV}(x) = \int \frac{d^3p}{(2\pi)^3} \theta(p - Ha) \text{[(const. spectrum)]}
\]

Since \( \varphi_{UV} = -\dot{\varphi}_0 \),

\[
\dot{\varphi}(x) = \dot{\varphi}_0(x) - \frac{1}{3H} \frac{\partial}{\partial \tilde{\varphi}} V(\tilde{\varphi}(x))
\]

Inconsistent with Resummation formula except in dS space
Improved semiclassical description

For a general choice of time coordinate: $dT = \mathcal{H} dt$,

$$\frac{\partial}{\partial t^2} + 3H \frac{\partial}{\partial t} = \mathcal{H}^2 \left( \frac{\partial^2}{\partial T^2} + \frac{\dot{\mathcal{H}} + 3H\mathcal{H}}{\mathcal{H}^2} \frac{\partial}{\partial T} \right)$$

To compare $\frac{\partial}{\partial T} \varphi$ with $\frac{\partial^2}{\partial T^2} \varphi$ directly, we choose $T$ as its friction coefficient is constant

$$\frac{\dot{\mathcal{H}} + 3H\mathcal{H}}{\mathcal{H}^2} = \mu_0 : \text{const.}$$

$$\rightarrow \mathcal{H} = \frac{1}{\mu_0} \left( a^3 \int_t^\infty dt' a'^{-3} \right)^{-1}$$

Neglecting $\partial^2_T$, $\partial^2_x$ rather than $\partial^2_t$, $\partial^2_x$,

$$\mu_0 \mathcal{H}^2 \frac{\partial}{\partial T} \{ \tilde{\varphi}(x) + \varphi_{UV}(x) \} = \left( a^3 \int_t^\infty dt' a'^{-3} \right)^{-1} \frac{\partial}{\partial t} \{ \tilde{\varphi}(x) + \varphi_{UV}(x) \}$$

Consistent with Resummation formula
Summary

• In accelerating universes, the increase of d. o. f. at super-horizon scales makes vevs of field operators growing with time through the propagator of a massless and minimally coupled scalar field

• In order to evaluate the IR effects nonperturbatively, we extended the resummation formula of the leading IR effects in a general accelerating universe

• The resulting equation is given by a Langevin eq. with a white noise, and the coefficient of each term is modified by the slow-roll parameter

• We can derive the same stochastic equation also by the semiclassical description of the scalar field, as far as we choose the time coordinate as its friction coefficient is constant