HYDRAULIC FRACTURING OF AN ARENITIC RESERVOIR BASED IN THE PERKINS-KERN MODEL USING A STIMPLAN SIMULATOR

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ABSTRACT

Hydraulic fracturing consists of a technique capable of stimulating oil wells that have suffered a decline in production over time. It also allows the production in reservoirs that have low permeability through the creation of a network of channels in the rock. In this context, this article aims to numerically simulate the hydraulic fracturing applied in a sandstone reservoir according to data extracted from an oil well located in the Aracaju City field of the Sergipe-Alagoas Basin. To complete this study, a geological model of the reservoir was generated. Subsequently, a fracture was created in the rock-reservoir in a controlled manner using the Perkins and Kern fracture model. Results show that the fracture takes a satisfactory proportion in the reservoir rock, reaching a depth of penetration equivalent to 695.7 meters.

KEYWORDS

oil reservoir; hydraulic fracking; Perkins & Kern model

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1. INTRODUCTION

Over the years, studies have enabled the development of techniques to allow the production in an oil field. Among these techniques is hydraulic fracturing, characterized by a stimulation operation, and which is applied frequently in low permeability reservoirs to ensure their production. It is also an operation that can be applied in mature fields with the goal of increasing well production significantly.

According to Silva (2017), investing in the stimulation of existing wells, through the hydraulic fracturing method, for example, is more profitable than drilling of new wells because this is a costly operation in the oil industry.

In a hydraulic fracturing operation, a fracture is created. It propagates in the rock formation at great depths from the injection of a given volume of fracturing fluid, which has in its composition a support agent (proppant) that allows that the fracture does not close when the pumping ceases this fluid. The fracture is created in the reservoir rock, precisely due to the differential pressure generated between the well (injected fluid) and the formation (rock strength).

Some fracture models have been created to better understand how the fracture's geometry develops in the subsurface environment when induced hydraulically. Among these models are, for example, the Perkins-Kern-Nordgren (PKN), Khristianovic-Geertsma-de Klerk (KGD), pseudo-3D, and 3D-planar. Such fracture models provide a close approximation to real scenarios. However, PKN and Pseudo-3D models are applied to typical Tight Gas 2 formation (Rahman & Rahman, 2010).

Through this model, it is possible to have good ratings for the width and length of the fracture created within the rock formation. Figure 1 illustrates the model presented by Adachi et al. (2007).

2. THEORETICAL FOUNDATION

2.1 Perkins and Kern fracture model

Perkins and Kern fracture model is used for situations where a long fracture and, at the same time, limited height is desired. This fracture has an elliptical vertical cross-section.

The PKN fracture model was developed in 1961 by Perkins and Kern. This model considers the fracture propagation in a vertical plane along with other assumptions such as (Pitombo, 1987; Andrade, 2016): constant height (H); ratio of length (L) / height (h) >> 1; fracturing fluid pressure ($P_f$) included in the sections of the vertical plane; elliptical fracture width (w) variable; constant injection flow; and minimum horizontal voltage ($\sigma h$) equal to the fluid pressure ($P_f$) at the end of the fracture.

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Production wells and facilitate the development of oil fields.

In this article, the numerical simulation for the study of hydraulic fracturing was done through the use of software StimPlan 3. Our goal was to increase our understanding of how a fracture is created and propagated through the rocky hydraulically-controlled manner.

In this context, this article aims to simulate hydraulic fracturing applied in a sandstone reservoir numerically, according to the data extracted from an oil well that has suffered production decline over time. This choice was selected to ensure an increased production from this well after the application of the hydraulic fracturing technique.

Software developed by NSI Technologies. The academic license was issued to the Federal University of Paraíba (UFPB) to conduct the numerical simulation of hydraulic fracturing.
According to Lucci (2015), the fracture mechanics are formalized as a mechanical fracture of the area applied since 1921, with the first work in this area made by Griffith (1921).

Considering a vertical open well, wherein it is subjected to horizontal tensions in situ $\sigma_{\text{min}}$ and $\sigma_{\text{max}}$, also taking into account that the rock consists of an elastic medium and has a tensile strength $\sigma_T$ (Voltage failure), there is a breaking pressure $P_b$ needed to induce a hydraulic fracture in the surface of a well, that is given by Equation (1) (Yew, 2008).

$$P_b = 3\sigma_{\text{min}} - \sigma_{\text{max}} + \sigma_T$$

Equation (1) is independent from the elastic moduli of the rock and not on the pit dimensions. According to Yew (2008), at an equivalent depth of 10000 ft, the typical values of maximum and minimum voltage in situ rock formation correspond, respectively, to 6000 and 7000 psi.

Based on the geometry of the fracture presented, the opening of the fracture ($w$) is given by (Yew, 2008):

$$w = \frac{1 - v}{G} p \sqrt{H^2 - 4z^2}$$

where $w = w_{\text{max}}$ is the maximum aperture of the fracture, $v$ is the Poisson's modulus, $G$ is the shear modulus, $p$ is the liquid pressure, and $H$ is the height of the fracture.

In Equation 2, one can observe that the fracture opening (elliptical) is directly proportional to the net pressure.

Nordgren (1972) described the continuity equation for the case of an incompressible fluid flow through the fracture (Equation 3).

$$\frac{\partial q}{\partial x} + q + \frac{\partial A}{\partial t} = 0$$

where $q(x, t)$ is the cross-sectional flow of the fracture.

According to Yew (2008), the fluid flow "$q$" can be related to the pressure gradient through the solution for the laminar flow of a Newtonian fluid in an elliptical tube, according to Equation 4.

$$q = \frac{\pi W^3 h}{64 \mu} \frac{\partial p}{\partial x}$$

where $\mu$ is the viscosity of the Newtonian fluid and $q_1(x, t)$ is the filtration rate (loss) per unit length of the fracture, given by Equation (5).

$$q_1 = \frac{2c_i H}{\sqrt{t - \tau(x)}}$$

where $c_i$ is the filter coefficient, $t$ is time, $\tau(x)$ is the time when the filtration occurs at point $x$, and $A(x, t)$ is the cross-sectional area of the fracture represented by Equation (6).

$$\int_{-h/2}^{h/2} w dz = \frac{\pi}{4} WH$$

Making a substitution of Equations (4), (5), and (6) at the continuity equation, Equation (3), we have the equation that governs the propagation of a hydraulically induced fracture, given by (Yew, 2008):

$$\frac{G}{64(1 - v)\mu H} = \frac{\partial W^4}{\partial x^2} = \frac{8c_i}{\pi \sqrt{t - \tau(x)}} + \frac{\partial W}{\partial t}$$

For Equation (7) the initial condition is $w(x, 0) = 0$ and the boundary conditions are $W(x, t) = 0, x \geq L(t)$.

$$\left[ \frac{\partial W^4}{\partial x} \right]_{x=0} = \frac{256(1 - v)\mu}{\pi G} Q$$

where $Q$ is the flow of injection fluid.
It is important to note that, in the Perkins and Kern fracture model, well pressure increases with the increase in fracture length.

3. MATERIALS AND METHODS

To perform the hydraulic fracturing technique, we used data from an existing well (1-CAU-3-SE) drilled in the Aracaju City field, which is part of Sergipe-Alagoas basin. This oil well is located in the area known as SSEQEL-T5. It was auctioned by the ANP (Brazilian National Agency of Petroleum, Natural Gas and Biofuels) in its 7th bidding round held in 2005, as shown in Figure 2 (ANP, 2005).

Figure 3 shows the well log 1-CAU-3-SE up to a end depth of 2155m (7070.21ft).

For this pit log, one can observe that the reservoir consists of interleaved shales and sandstones. However, the reservoir was considered as sandstone type having its greatest thickness from 2005 to 2065 meters, where the hydraulic fracturing technique was applied.

Table 1 shows the reservoir properties under study and some parameters that are necessary as input data in StimPlan software.

Table 2 presents some properties of the reservoir fluid in the separator used in the StimPlan software.

For numerical simulation of hydraulic fracturing in the geological formation under study, with a permeability of 30 mD, a fracturing fluid with the addition of the propellant Ottawa Sand with a density of 2.65 g/cm³ was used (Campos et al., 2018). The Perkins and Kern fracture model was used to create extensive and, at the same time,

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3Sandstone reservoirs consist of sandstone sedimentary rocks that were formed due to compression process and lithification of sandy sediments. The Campos Basin, responsible for a considerable portion of the Brazilian oil production, for example, has a geological structure composed mainly of sandstone reservoirs.
narrow fractures in the reservoir rock. All the data presented in Tables 1 and 2 were inserted in the Stimplan 3D software prior to the simulation.

4. RESULTS AND DISCUSSIONS

The study of methods capable of predicting the geometry of a fracture has been of great importance in the area of fracture mechanics. Variables, such as injection pressure of the fracturing fluid, must be considered carefully to ensure good results in a hydraulic fracturing operation.

Table 1. Reservoir properties studied.

| Property | Value   |
|----------|---------|
| $R_w$ - Well radius (ft) | 0.4     |
| $A$ - Drainage area (acres) | 160.0  |
| $P_i$ - Initial Pressure (psi) | 0.0     |
| $P_e$ - Current pressure (psi) | 2500.0  |
| $P_{hfp}$ - Flow Pressure (psi) | 250.0   |
| $T_{res}$ - Temperature (°F) | 215.0   |
| $T_{wh}$ - Temperature at the wellhead (°F) | 70.0    |
| $T$ - Downhole temperature (°F) | 215.0   |
| Top (ft) | 6562.0  |
| Base (ft) | 7070.0 |
| Porosity reservoir (%) | 9.2     |
| Reservoir permeability (mD) | 30.0    |
Figure 4 shows the geological model of the reservoir generated in StimPlan software from well data of Aracaju City field, part of the Sergipe-Alagoas basin.

Table 3 shows some data acquired after simulation. Based on the data, it is possible to observe pressure, proppant volume, treatment time, and fracture length created and propagated in the rock formation.

In this case, the hydraulically induced fracture in the rock, which consists of sandstone rock, had a length of 695.7 meters. Length which housed proppant in the fracture after the pumping of fluid ceased.

The maximum pressure necessary to fracture the rock, which has a permeability of 30 mD, and reach a length of 695.7 meters, was 1226.8 psi. The volume of total fracturing fluid was 4442.1 BBL, equivalent to approximately 706.24 m³.

Table 3. Results of simulation in StimPlan generated according to the entered data.

|                      | Pumping programming - Real | Geological Model - Real |
|----------------------|---------------------------|-------------------------|
| **Frac-1 Drilling**  |                           | License to: 3D StimPlan 7:23 - Network License UFPB |
|                      |                           |                          |
| **Half the length**  |                           |                          |
|                      | Length 'hydraulic' (m)    | 824.5                   |
|                      | Assigned Length (m)      | 695.7                   |
| **Pressure**         | Network Maximum Pressure (psi) | 674.7             |
|                      | Liquid Final Pressure (psi) | 622.1             |
|                      | Maximum surface pressure (psi) | 1226.8            |
| **Time**             | Maximum exposure time for training (min) | 163.0             |
| **Rate**             | Fluid loss rate for the cushion (BPM) | 0.03             |
| **Efficiency**       | At the end of the pumping schedule | 0.98             |
| **Proppant**         | Mean Concentration in situ (lb / ft²) | 1.02             |
|                      | Conductivity Medium (mD-ft) | 4548.2            |
|                      | Dimensionless fracture conductivity - Cf (kfw / KXF) | 19.93            |
| **Height**           | Maximum fracture height (m) | 84.2              |
| **Width**            | Average width at the end of pumping (in) | 0.27              |
| **Volume**           | Total volume of fluid (BBL) | 4442.1            |
|                      | Total volume of proppant (M-Lbs) | 1395.8           |
The fracture reached a maximum height equivalent to 84.2 meters according to the input data in StimPlan software.

For the simulation conditions presented in Tables 1 and 2, seven phases of pumping of the fracturing fluid occurred with a constant flow of 30 BPM until the fracture reached the hydraulic length of 824.5 m, as shown in Table 3. The first pumping stage is given by an injection pad consisting of the water volume required to create and propagate fractures in the reservoir rock. Subsequently, there was an addition of the supporting agent (propellant) to the fracturing fluid to prevent the closure of the hydraulic fracture after pressure reduction. In this case, the fluid is called slurry. Throughout this fracturing process, a variation of pressure is recorded (Figure 5).

Variations between surface pressure and fracture fluid injection pressure are shown in Figure 5. In Figures 6 and 7, it is possible to observe, in more detail, the pressure distribution over the time that the entire fracture opening process occurs in the rocky environment. The injection pressure should be significantly greater than the pressure within the well for the opening and propagation of the fracture to occur.

Figure 6 shows that the surface pressure predicted for the first 7 minutes reaches the maximum pressure equivalent to 1225 psi.
Subsequently, this pressure begins to decrease due to the stages of hydraulic fracturing, in which the supporting agent (propellant) is added to the fluid.

The increase in pressure up to a certain point occurs due to the rock’s resistance to traction. When the maximum pressure peak is reached, it starts to decrease considerably, showing that the fracture was created in the formation and it will propagate in the rocky environment until the pressure stabilizes.

The injection pressure at the bottom of the well is high, initially, and this is responsible for breaking the rock, see Figure 7. After decreasing, it undergoes a considerable increase caused by the addition of a certain concentration of proppant to the fracturing fluid. This sudden increase is responsible for propagating the cracks and, at the same time, allowing the displacement of the proppant therethrough.

When the fracturing operation occurs for up to 160 minutes, the injection pressure drops considerably, implying that the propellant has stopped being added to the fluid. At this point, the fracturing operation is completed, and the well is closed so that the pressure is in equilibrium with the pressure of the reservoir, known as pore pressure.

Figure 7. Injection pressure in the deep versus fracturing operation time.

Figure 8. Maximum width of the generated 3D Stimplam fracture.
Figure 8 shows the maximum width of the hydraulically induced fracture in the reservoir. The fracture developed extensively in the rock allows a great connectivity between the pores of the formation, presenting a geometry with desirable dimensions, with great length and small width. The hydraulically induced fracture had, thus, a typical geometry of fractures generated by the Perkins and Kern fracture model, as expected.

When in large proportions, the fracture allows a higher connection between the pores of the rock and, consequently, promotes a significant increase in permeability, creating a larger drainage area. This implies that, when applied, the hydraulic fracturing technique favors a significant increase in well production, enabling the operation.

Figure 9 shows the geometry of both the fracture induced and the one hydraulically induced. 

![Figure 9](image1.png)

**Figure 9.** The fracture closure.

![Figure 10](image2.png)

**Figure 10.** Fracture propagation in the subsurface environment.
generated in the sandstone formation. The final length of the fracture, after the pumping the fracturing fluid ceases, is represented by the gray color, which consists of the accommodation of the granular material (proppant) inside the rock.

The continuous growth of the fracture over the time of operation in which the hydraulic fracturing occurs is shown in Figure 10 for times of 5.3, 30.9, 75.4, and 3135.7 minutes.

The fracture model used for this treatment is the Perkins and Kern. The crack has a constant height and increasing growth to the design engineer's desired size. This study must be done prior to starting the hydraulic fracturing operation. The objective is this process is that the fracture approaches the actual maximum according to the input data in Stimplan software.

The stimulation of this well through this operation shows how the production can be increased significantly considering that reservoir rock fissures allow the connection between the

Figure 11. Geometry of the fracture.

Figure 12. Net pressure versus fracturing operation time.
pores of the rock, thus, allowing accessibility of reservoir fluids into the well.

Figure 11 shows the dimensions of the crack, i.e., the height, width, and length of the final fracture according to the pumping time. In this case, one can see a narrowing at the tip of the corresponding fracture to its closure.

The liquid pressure, shown in Figure 12, undergoes a slight increase at the beginning of treatment. The fracture is followed by a decrease in attenuated form. This pressure variation occurs precisely due to the stages of the hydraulic fracturing operation.

Through Figure 12, it is possible to observe the change in pressure in the well, as expected by the Perkins and Kern fracture model. As the fluid pump continues, there is a gradual increase in pressure as the proppant is being injected to the fluid to move and settle within the fracture.

One of the important factors to be considered in a hydraulic fracturing project is the fracture's conductivity. Figure 13 shows a conductivity decrease as the fracture length increase in the formation. This is explained precisely because the width of the hydraulically induced fractures in the reservoir is greater in the region around the well. The conductivity of the fracture is a function of its width. Therefore, the larger the aperture of the fracture is, the greater its conductivity will be.

5. CONCLUSIONS

The simulation showed that, for the application of the hydraulic fracturing technique in the reservoir rock under study with a permeability of 30mD, seven phases of pumping of the fracturing fluid were necessary with the increasing addition of propellant Ottawa Sand, with a constant flow of 30 BPM, allowing the fracture to develop inside the rock reaching a depth of penetration equivalent to 695.6 meters.

There was a good distribution of the granular material and one can see accommodation within the fissure. The delimitation corresponds to the fracture's final geometry.

The injection pressure behaved as expected during treatment. Initially, it was high and, subsequently, it suffered a decline caused by the addition of certain concentrations of proppant.

The fracture's conductivity showed high values close to the well region, decreasing to the point of fracture, which implies that the larger the aperture, the greater the conductivity of the fracture.

However, the fracture created in the rock formation, characterized as a sandstone reservoir, behaved according to the Perkins and Kern fracture model, in which the fracture geometry has a small width with great length.
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