Extrasolar Planet Transit Light Curves and a Method to Select the Best Planet Candidates for Mass Follow-up

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Abstract. A unique analytical solution of planet and star parameters can be derived from an extrasolar planet transit light curve under a number of assumptions. This analytical solution can be used to choose the best planet transit candidates for radial velocity follow-up measurements, with or without a known spectral type. In practice, high photometric precision (< 0.005 mag) and high time sampling (< 5 minutes) are needed for this method. See Seager & Mallén-Ornelas (2002) for full details.

1. Assumptions

The following assumptions and conditions are necessary for a light curve to yield a unique solution of planet and star parameters:
- The planet orbit is circular (valid for tidally-circularized extrasolar planets);
- \( M_p \ll M_\star \) and the companion is dark compared to the central star;
- The stellar mass-radius relation is known;
- The light comes from a single star, rather than from 2 or more blended stars;
- The eclipses have flat bottoms. This implies that the companion is fully superimposed on the central star’s disk and requires that the data is in a band pass where limb darkening is negligible;
- The period can be derived from the light curve (e.g., the two observed eclipses are consecutive).

In this article \( M \) is mass, \( R \) is radius, \( \rho \) is density, \( P \) is period, \( a \) is orbital semi-major axis, \( i \) is the orbital inclination, and \( G \) is the Gravitational constant. Where required the subscript \( p \) is for planet, \( \star \) for stellar, and \( \odot \) for solar.

2. The Simplified Equations

Five equations are used to uniquely solve for \( M_\star, R_\star, a, i, \) and \( R_p \). The simplified equations presented below require the additional assumption that \( R_\star \ll a \).
Transit depth

\[ \Delta F \equiv \frac{F_{\text{no transit}} - F_{\text{transit}}}{F_{\text{no transit}}} = \left( \frac{R_p}{R_*} \right)^2. \]  

Total transit duration

\[ t_T = \frac{P R_*}{\pi a} \sqrt{\left( 1 + \frac{R_p}{R_*} \right)^2 - \left( \frac{a}{R_*} \cos i \right)^2}. \]  

Transit shape \((t_F = \text{flat part of transit and } t_T = \text{total transit duration})\)

\[ \left( \frac{t_F}{t_T} \right)^2 = \left( \frac{1 - R_p}{R_*} \right)^2 - \left( \frac{a}{R_*} \cos i \right)^2 \left( 1 + \frac{R_p}{R_*} \right)^2 - \left( \frac{a}{R_*} \cos i \right)^2. \]  

Kepler’s Third Law

\[ P^2 = \frac{4\pi^2 a^3}{GM_*}. \]  

Stellar mass-radius relation

\[ R_* = k M_x^x. \]  

Here \(k\) is a constant coefficient for each stellar sequence (main sequence, giants, etc.) and \(x\) describes the power law of the sequence (e.g., \(k = 1\) and \(x \approx 0.8\) for F–K main sequence stars (Cox 2000)). Note that Kepler’s Third Law and the stellar mass-radius relation set a physical scale to two disks passing in front of each other. This breaks the geometrical degeneracy and allows a unique solution.

### 3. The Simplified Solution

The five parameters \(M_*, R_*, a, i,\) and \(R_p\) can be solved for uniquely from the above five equations. Moreover, the impact parameter \(b \equiv a \cos i / R_*\) and stellar density \(\rho_*\) can be solved for uniquely without the stellar mass-radius relation.

\[ b = \left[ \frac{(1 - \sqrt{\Delta F})^2 - \left( \frac{t_F}{t_T} \right)^2 \left( 1 + \sqrt{\Delta F} \right)^2}{1 - \left( \frac{t_F}{t_T} \right)^2} \right]^{1/2}. \]  

\[ \frac{\rho_*}{\rho_\odot} = \frac{32}{G\pi} P \frac{\Delta F^{3/4}}{(t_T^2 - t_F^2)^{3/2}}. \]  

\[ \frac{M_*}{M_\odot} = \left[ k^3 \frac{\rho_*}{\rho_\odot} \right]^{1/x}. \]
Method to Select the Best Planet Candidates for Follow-up

\[
\frac{R_\ast}{R_\odot} = k \left( \frac{M_\ast}{M_\odot} \right)^x = \left[ k^{1/x} \frac{\rho_\ast}{\rho_\odot} \right]^{\frac{x}{1-3x}}. \tag{9}
\]

\[
a = \left[ \frac{P^2 G M_\ast}{4 \pi^2} \right]^{1/3}. \tag{10}
\]

\[
i = \cos^{-1} \left( \frac{b R_\ast}{a} \right). \tag{11}
\]

\[
\frac{R_p}{R_\odot} = \frac{R_\ast}{R_\odot} \sqrt{\Delta F} = \left[ k^{1/x} \frac{\rho_\ast}{\rho_\odot} \right]^{\frac{x}{1-3x}} \sqrt{\Delta F}. \tag{12}
\]

![Figure 1](image)

**Figure 1.** Stellar density \( \rho_\ast \) vs. stellar mass \( M_\ast \) (\( M_\ast \) is used as a proxy for stellar spectral type). See text for details. The box MOV to F0V shows the main sequence stars which are most appropriate for finding transiting planets. See Seager & Mallén-Ornelas (2002) for a discussion of errors.

4. **Application**

The above analytical solution has many applications, all related to selecting the best transit candidates for radial velocity mass follow-up. Here we only have room to describe one application; for others see Seager & Mallén-Ornelas (2002).

The stellar density \( \rho_\ast \) can be uniquely determined from the light curve alone without using the stellar mass-radius relation, as seen from equation (7).
A measured $\rho_*$ can be used in three ways. (1) From the light curve alone a main sequence star and a giant star can be distinguished because main sequence stars occupy a unique position in a $\rho_*$ vs. spectral type diagram (Figure 1). Hence a giant star with an eclipsing stellar companion can be ruled out. (2) From the light curve and the stellar mass-radius relation $R_p$ can be estimated (equation (12)). Even for slightly evolved stars an upper limit on $R_*$ and hence $R_p$ can be derived. (3) A common false positive planet transit can be ruled out by comparing $\rho_*$ derived from the light curve with $\rho_*$ derived from a spectral type. If the two $\rho_*$ differ then something is amiss with the assumptions in §1. The common case is the situation where a binary star system has its eclipse depth reduced to a planet-size eclipse due to the light from a third, contaminating, star (Figure 2). The contaminating star may be a chance alignment of a foreground or background star, or a third star as part of a triple star system. For a real example of this “blended star” situation, see Mallén-Ornelas et al. (2002).

![Diagram of binary star system and planet transit](image)

**Figure 2.** A deep binary star eclipse (dotted line) can mimic a planet transit (solid line) when extra light from a third, contaminating star (not shown) is present.

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**References**

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