Weighted Motion Averaging for the Registration of Multi-View Range Scans

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Abstract—Multi-view registration is a fundamental but challenging problem in 3D reconstruction and robot vision. Although the original motion averaging algorithm has been introduced as an effective means to solve the multi-view registration problem, it does not consider the reliability and accuracy of each relative motion. Accordingly, this paper proposes a novel motion averaging algorithm for multi-view registration. Firstly, it utilizes the pair-wise registration algorithm to estimate the relative motion and overlapping percentage of each scan pair with a certain degree of overlap. With the overlapping percentage available, it views the overlapping percentage as the corresponding weight of each scan pair and proposes the weight motion averaging algorithm, which can pay more attention to reliable and accurate relative motions. By treating each relative motion distinctively, more accurate registration can be achieved by applying the weighted motion averaging to multi-view range scans. Experimental results demonstrate the superiority of our proposed approach compared with the state-of-the-art methods in terms of accuracy, robustness and efficiency.

Index Terms—Multi-view registration Iterative closest point algorithm Overlapping percentage Motion averaging

1 INTRODUCTION

RANGE scan registration has attracted extensive attention in recent years, which is widely used in 3D reconstruction\textsuperscript{[1, 2, 3]}, shape recognition\textsuperscript{[4]} and robot mapping\textsuperscript{[5, 6]}. Based on the number of range scans to be registered, the issues of range scan registration can be divided into two categories: Pair-wise registration and Multi-view registration.

The first category has activated a bunch of algorithms to be proposed and improved, among which the most popular ones are the iterative closest point (ICP) algorithm\textsuperscript{[7]} and the trimmed iterative closest point (TrICP) algorithm\textsuperscript{[8]}. ICP adopts distance as metric to match scan correspondences and is widely accepted for its high accuracy and fast speed, however, it usually fails when the two scan pairs are partially overlapping. To overcome this drawback, a straightforward solution is to reject scan pairs including distances greater than the userspecified threshold\textsuperscript{[9]}. Further, Godin\textsuperscript{[10]} proposed the concept of weighted of scan pairs, and assign lower weights to pairs with greater point-to-point distances. This approach is simple but ineffective. Moreover, Chetverikov et al. proposed the trimmed ICP (TrICP) algorithm, which introduced an overlapping percentage into the original ICP to exclude outliers. Although this can achieve partial registration accurately, failing results of low overlapping percentage propose a striking challenge to most practical application areas. Beside these algorithm that we have mentioned before, some probabilistic approaches\textsuperscript{[11, 12, 13, 14]} were also proposed for registration of partially overlapping scans. To obtain the desired global minimum, genetic algorithm (GA)\textsuperscript{[15, 16]} and the particle filter\textsuperscript{[17]} have been adopted to search optimal solution for registration problem.

Most theories of multi-view registration are the variants of the pair-wise registration algorithms. In\textsuperscript{[18]}, the authors assume that the individual range scan pairs are registered using ICP. Given these registration results, an overall error measure is relaxed across all the views with an aim of distributing the errors as well as possible. While paper\textsuperscript{[18]} solve for the registration of multiple scans, they do not directly address the issue of solving for correspondences between range scan pairs during the process of registration.

More recently, Fantoni et al.\textsuperscript{[19]} and Guo et al.\textsuperscript{[20]} proposed two novel approaches for registration of multiple 3-D range scans by extracting and describing the features from range scans. Although these approaches are efficient, perhaps there are not enough features to be extracted from some range scans, resulting in a failed registration. Zhu et al.\textsuperscript{[21]} proposed a coarse-to-fine approach for multi-view registration. In this approach, each range scan should be sequentially registered to a coarse model reconstructed by other registered range scans. By applying the TrICP algorithm, it can obtain good multi-view registration results for each range scan, which can then be immediately utilized to refine the coarse model for registration of other range scans. Since the TrICP algorithm is applied to registration, its accuracy is satisfactory, yet it may fail into local minima due to the poor initial parameters.

Another method of significance in our context is a low-rank and sparse(LRS) matrix decomposition\textsuperscript{[22]}. It’s just a general framework, not a concrete method. In order to make the approach concrete, Federica Arrigoni and his team put forward three LRS algorithms: R-GoDec\textsuperscript{[23]}, Grasta\textsuperscript{[24]} and L1-Alm\textsuperscript{[25]}, showing that they can be profitably applied to address the registration problem. However, when the ratio of missing data is very high, it will lead to the failure of the experiment.

In\textsuperscript{[26]}, Chen and Medioni proposed the primary ap-
proach for multi-view registration. It repeatedly registers
two range scans and integrates them into one range scan
until all the range scans are integrated into the whole model.
However, the error accumulation is an unavoidable problem
in this approach. To address this issue, Govindu and Pooja
[27] proposed the motion averaging algorithm, which can be
applied to a set of relative motions and recover the global
motions for each range scan. With the motion averaging
algorithm, the prerequisite is how to obtain accurate and
reliable relative motions. Subsequently, Li and Zhu [29]
proposed a method to estimate the overlapping percentage
between each range scan pair, so as to apply the pair-wise
registration approach to scan pairs with a certain degree of
overlapping percentage. However, the original motion aver-
gaging algorithm treat each relative motion equally, which
may lead to inaccurate results for multi-view registration.

Accordingly, this paper extends the approach presented in
[27] and proposes the weighted motion averaging algo-
nism for multi-view registration of range scans. The main
differences between the previous work [27] and this one are
described as follows: (1) It presents a method to estimate
the overlapping percentage between scan pair involved in
multi-view registration. (2) For the scan pair with a certain
degree of overlapping percentage, the TrICP algorithm is
utilized to calculate the relative motion with the corre-
sponding weight indicated its accuracy and reliability. (3)
Weighted motion algorithm is proposed to pay more at-
tention to the relative motion with large weight. All these
techniques can improve the performance of multi-
view registration approach.

The remainder of this paper is organized as follows: Section 2 briefly introduces the principle of Motion aver-
ing. After that, our proposed approach is demonstrated
in section 3. Then in section 4, experimental results are
displayed to compare with some related approaches. Finally,
some conclusions are drawn in section 5.

2 MOTION AVERAGING

In motion averaging algorithm, the rigid transformation
(R, ⃗t) can be assembled into one matrix and has the form as
the following:

\[ M = \begin{bmatrix} R & ⃗t \\ O & 1 \end{bmatrix}, \]

where \( M \in SE(3) \), \( R \in SO(3) \) and \( O = [0, 0, 0] \).

In multi-view registration, the above matrix has two forms:
relative motion \( M_{ij} \) and global motion \( M_N \), which
indicate the motion from \( j \)th range scan to \( i \)th range scan
and \( N \)th range scan to the reference range scan, respectively.
The goal of multi-view registration is to acquire a set of
global motions:

\[ M_{\text{global}} = \{I, M_2, ..., M_N\}, \]

from a set of relative motions:

\[ M_{ij} = (M_i)^{-1}M_j. \]

Given accurate relative motions and global motions, the
following equation can be established:

\[ I = M_iM_{ij}(M_j)^{-1}, \]

where \( I \) represents the identity matrix. However, the
relative motions are obtained by the pair-wise registration,
which inevitably involves error. Therefore, the following
increment can be defined as follows:

\[ \Delta M_{ij} = M_iM_{ij}(M_j)^{-1}. \]

According to [28], the motion averaging algorithm can be
summarized as Algorithm 1:

Algorithm 1 Motion Averaging Algorithm

Input: \( \{M_{i1}, M_{i2}, \cdots, M_{ijr}\} \)
Output: \( M_{\text{global}} = \{I, M_2, ..., M_N\} \)
Set \( M_{\text{global}} \) to an initial guess
Do
\[ \Delta M_{ij} = M_iM_{ij}(M_j)^{-1} \]
\[ \Delta m_{ij} = \log(\Delta M_{ij}) \]
\[ \Delta v_{ij} = \text{vec}(\Delta m_{ij}) \]
\[ \Delta \mathbb{S} = D^T\Delta V_{ij} \]
\[ \forall k \in [2, N], M_k = e^{\Delta m_k}M_k \]
Repeat till \( \|\Delta \mathbb{S}\| < \varepsilon \)

where \( \log(\cdot) \) are matrix operations and \( m_{ij} \) is the
corresponding Lie algebra of \( M_{ij} \). \( \Delta m_{ij} \) indicates the
corresponding Lie algebra element of \( \Delta M_{ij} \), and since
\( \Delta m_{ij} \) is a skew-symmetric matrix, it can be ex-
pressed by a 3-element vector given by \( \Delta v_{ij} \) which
is obtained by the operation \( \text{vec}(\cdot) \). \( \Delta \mathbb{S} \) contains all
the variables \( v_i \) stacked together into a single vector.
Assign a \( 6*(6n) \) vector \( D_{ij} \) to each relative motion:
\[ D_{ij} = [\ldots -I_{ij} \ldots I_{ij} \ldots ], \]
in which the \( 6*6 \) matrices \( -I_{ij} \) and \( I_{ij} \) are put into the position of \( i \)th block and \( j \)th block respectively. Stacking all these weighted
vectors into one single matrix and the formulation is as
below:
\[ D = [D_{i1j}, D_{i2j}, \ldots, D_{ijr}]^T, \]
where \( ijr \) is the number of participating scan pairs, and \( D^T \) is the pseudo-inverse of \( D \). Similarly, all \( \Delta v_{ij} \) vectors are stacked into one matrix,
which has the form:
\[ \Delta V_{ij} = [\Delta v_{ij1}, \Delta v_{ij2}, \ldots, \Delta v_{ijr}]^T. \]

In the multi-view registration, the pair-wise registration
approach should be utilized to obtain relative motions,
which are taken as the input of the motion averaging
algorithm. As there is no perfect registration approach, the
pair-wise registration result contains error, which can varied
from small to larage due to the outliers and noise of scan
pair. Therefore, the accuracy and reliability of each relative
motion are totally different. However, the original motion
averaging algorithm treats each relative motion equally,
which can lead to inaccurate multi-view registration result.
To obtain more accurate result, more effective motion aver-
aging algorithm should be designed.

3 THE PROPOSED MULTI-VIEW REGISTRATION AP-
PROACH

In this section, an effective approach is proposed for
multi-view registration of initially posed range scans and
its overview can be shown in Fig. 1. As shown in Fig. 1,
the proposed approach consists of the following three major
steps. 1) Estimate the overlapping percentage for each scan
pair; 2) Compute the relative motion and its corresponding
weight for each pair of range scans with high overlapping
percentage; 3) Calculate the multi-view registration by the
weighted motion averaging algorithm.
Subsequently, these three steps will be presented with more details.

### 3.1 Estimation of the overlapping percentage

To obtain reliable and accurate relative motions, the pair-wise registration algorithm can be only applied to these scan pairs which contain a certain degree of overlapping percentages. Here, we employ our previous method [27] to estimate the overlapping percentage $\xi_{ij}$ for each scan pair. It does not directly estimate the overlapping percentage for each individual scan pair. Instead, it firstly determines the distance threshold by considering all the scans and then utilize this threshold to estimate overlapping percentages. Since it uses all scans to estimate the overlapping percentage for each scan pair, the estimation result is reliable, even though the actual overlapping percentage of one scan pair may be low.

As shown in Fig. 1, the proposed approach should be implemented in iterative manner to achieve good multi-view registration. Given good registration parameters, the overlapping percentage can be estimated accurately by our previous method, which is helpful to multi-view registration. Intuitively, it seems that the overlapping percentage estimation can also be included in the iterative loop, so as to compute more accurate the overlapping percentage. However, the percentage estimation is time-consuming and the estimation results are only adopted to judge whether one scan pair contains high or low overlapping percentage. Therefore, the accuracy should yield to efficiency and estimation is only implemented once.

### 3.2 Pair-wise registration

To guarantee good pair-wise registration, the pair-wise registration can only by applied to these scan pairs with overlapping percentage $\xi_{ij} > \xi_{th}$. As shown in Fig. 1, the pair-wise registration approach should provide the motion and its corresponding weight for each pair of range scans with high overlapping percentage. Because it is easy to obtain relative motion by any pair-wise registration, so its corresponding weight will be designed as follows.

#### 3.2.1 Design of the weight

Given a set of relative motions, the original motion averaging algorithm views and treats each relative motion equally. However, some relative motions are accurate and other maybe not very accurate. Therefore, the weight of each motion should be introduced so as to deal with them differently.

Intuitively, more attention should be paid to these relative motions, which are more accurate and reliable than others. Therefore, it should find a function, which can indicates the accuracy and reliability of the relative motions. As the relative motions are estimated by the pair-wise registration algorithm, we utilized the TrICP algorithm to estimate relative motion between scan pairs with different overlapping percentages and recorded the corresponding registration error with respect to overlapping percentage.

As shown in Fig. 2, the pair-wise registration error is dramatically decreased with the increase of overlapping percentage. This is because a scan pair with low overlapping percentage contains more outliers, which can reduce the reliability and accuracy of pair-wise registration results. Subsequently, it is straightforward to design the weight of relative motions as follows:

$$w_{ij} = \xi_{ij}^2,$$

where $\xi_{ij} = \frac{|P_i \cap P_j|}{|P_i|}$, $P_i$ and $P_j$ denotes two range scans, $P_i \cap P_j$ denotes the overlapping regions of these two range scans. That means the overlapping percentage is utilized to indicate the accuracy and reliability of the corresponding relative motions.

#### 3.2.2 Computation of the relative motion

Obviously, it is expected that the pair-wise registration can estimate the relative motion and the corresponding overlapping percentage for a given pair of range scans simultaneously. Therefore, the TrICP algorithm can be adopted.

Suppose there are two partially overlapping range scans, a data shape $P \triangleq \{\tilde{p}_i\}_{i=1}^{N_p}$ and a model shape $Q \triangleq \{q_j\}_{j=1}^{N_q}$.

Denote $\xi, R \in \mathbb{R}^{3 \times 3}$, $t \in \mathbb{R}^3$ as the overlapping percentage, 3D rotation matrix and translation vector, respectively. The goal of partially overlapping registration is to find the optimal transformation $(R, t)$ with which $P$ is registered to be in the best alignment with $Q$. This problem can be formulated as follows:

$$\min_{R, t, \xi} \left\{ \frac{1}{|P_i|} \sum_{\tilde{p}_i \in P_i} \left\| R\tilde{p}_i + t - \tilde{q}_{c(i)} \right\|^2 \right\}$$

s.t. $R^T R = I_3, \det(R) = 1, \xi \in [\xi_{th}, 1], |P_i| \subseteq P, |P_j| = \xi |P|$

where $\tilde{q}_{c(i)}$ denotes the correspondence of the $\tilde{p}_i$ in model shape, $\lambda$ is a preset parameter, $|\cdot|$ denotes the cardinality of a scan and $P_j$ represents the overlapping part of data shape to model shape.

The TrICP algorithm achieve pair-wise registration by iterations. Given the initial transformation $(R_0, t_0)$, three steps are included in each iteration:

1. Based on $(R_{k-1}, t_{k-1})$, assign the correspondence between two scans:

$$c_k(i) = \arg\min_{j \in \{1, 2, \ldots, N_q\}} \left\| R_{k-1}\tilde{p}_i + t_{k-1} - \tilde{q}_j \right\|_2$$

2. Update the overlapping percentage $\xi_k$ and subset $P_{c_k}$:

$$\left(\xi_k, P_{c_k}\right) = \arg\min_{\xi \in [\xi_{th}, 1]} \left\{ \frac{\sum_{\tilde{p}_i \in P_k} \left\| R_{k-1}\tilde{p}_i + t_{k-1} - \tilde{q}_{c_k(i)} \right\|^2}{|P_k| \xi^{1+\lambda}} \right\}$$

3. Calculate the $k$th transformation:

$$(R_k, t_k) = \arg\min_{R, t} \left\{ \frac{1}{|P_{c_k}|} \sum_{\tilde{p}_i \in P_{c_k}} \left\| R\tilde{p}_i + t - \tilde{q}_{c_k(i)} \right\|^2 \right\}$$

For the scan pair with a certain degree of overlapping percentage, its optimal rigid transformation can be computed by iterating these three steps until some convergence criteria are satisfied. Then the rigid transformation can be assembled into a relative motion.
3.3 Weighted motion averaging algorithm

Since the pair-wise registration can provide the relative motion and its corresponding weight, more effective motion averaging algorithm can be proposed as follows.

3.3.1 Principle of the weighted motion averaging

Suppose there are a set of motion \( \{ M_1^j, M_2^j, \ldots, M_N^j \} \) obtained through different methods for one same range scan, in which \( M_i^j \) refers to \( i \)th scan’s motion using method \( j \). And the corresponding weights for each motion \( \{ w_1, w_2, \ldots, w_N \} \). While the weighted average method \( 1/N \sum_{i=1}^{N} w_i M_i \) cannot enforce the orthonormality constraints for \( M_i \), we then turn to Riemannian distance for a more convenient solution, which minimizes \( \sum_{i=1}^{N} d^2(M_i, \overline{M}) \) and obeys the orthonormality constraints, in which \( \overline{M} \) is the result of weighted average and \( d(\cdots, \cdots) \) is the Riemannian metric. To acquire the above objective function, these motions are projected onto the tangent space and approximated Riemannian distance iteratively. The above weighted motion averaging in tangent space can be summarized as Algorithm 2:

**Algorithm 2: Averaging weighted motions**

| Input: \( \{ M_1, M_2, \ldots, M_N \} \) and \( \{ w_1, w_2, \ldots, w_N \} \) |
|--------------------------|
| Output: \( \overline{M} \) |

- Set \( \overline{M} = M_1 \)
- Do
  - \( \Delta M_i = \overline{M}^{-1} M_i \)
  - \( \Delta m_i = w_i \log m(\Delta M_i) \)
  - \( \Delta \overline{m} = \frac{1}{N} \sum_{i=1}^{N} \Delta m_i \)
  - \( \Delta \overline{M} = \exp m(\Delta \overline{m}) \)
  - \( \overline{M} = \overline{M} \Delta \overline{M} \)
- Until \( || \Delta \overline{m} || \leq \varepsilon \)

where \( \exp m \) and \( \log m \) are matrix operations and \( m \) is the corresponding Lie algebra of \( M \). Algorithm 2 can be extended to deal with the registration of multi-view range scans.

3.3.2 The weighted motion averaging for multi-view registration

A salient problem existing in motion average TrICP algorithm \([29]\) is that it will lead to inaccuracy to evenly distribute redundant information among related motions. Just like Fig. 3 demonstrates, there are as many as \( n(n-1) \) relative motions available in a connected graph. we add a weighted factor to each relative motion as a contributor to the related two global motions. Meanwhile, the conversion between Lie group and Lie algebra is still used to solve
this question, i.e. Transform relative motion $M_{ij}$ to its corresponding Lie algebra $m_{ij}$ to obtain a satisfactory result.

For each pair of range scans, we firstly estimate their overlapping percentage to decrease computational complexity. Then using these selected range scans pairs equipped with weighted coefficient to reconstruct the whole model. In consideration of the weighted coefficient, Eq.(10) can be extended as:

$$w_{ij}M_{ij} = w_{ij}(M_{ij})^{-1}M_{j},$$ (11)

where the weighted coefficient $w_{ij}$ denotes the contribution of motion $M_{ij}$ in the weighted averaging process. Accordingly, the Lie algebra of Eq.(11) can be derived as:

$$w_{ij}m_{ij} \approx w_{ij}(m_{j} - m_{i}).$$ (12)

Every two range scan that conform to the requirement of overlapping percentage has a formula like Eq.(12), thus an equation system can be obtained composed of participants of relative motions. Taking advantage of block matrix, we assign a 6*(6n) vector $D_{ij}$ to each relative motion:

$$D_{ij} = \begin{bmatrix} \ldots & -I_{ij} & \ldots & I_{ij} & \ldots \end{bmatrix},$$ (13)

in which the 6*6 identity matrices $-I_{ij}$ and $I_{ij}$ are put into the position of $i$th block and $j$th block respectively. All these vectors are stacked into one matrix, which has the form:

$$D = [D_{ij1}, D_{ij2}, \ldots, D_{ijr}]^T,$$ (14)

where $ijr$ is the number of participating range scan pairs. Meanwhile, similarly with statements in section 2, noticing that $m$ relies on $u$ and the skew symmetric matrix in $\Omega$, the operation $v = mat2vec(m)$ is defined to extract parameters from $m$ to form a column wise vector $v = [\Omega_{21} \Omega_{31} \Omega_{32} u_1 u_2 u_3]^T$.

Similarly, stacking all these weighted vectors into one single matrix and the formulation is as below:

$$V = [w_{ij1}v_{ij1}, w_{ij2}v_{ij2}, \ldots, w_{ijr}v_{ijr}]^T.$$ (15)

Based on these derivations, the following equation can be acquired:

$$V = D \Xi,$$ (16)

in which $\Xi = [v_1, v_2 \ldots v_n]^T$ and $v_i$ is a $6^*1$ vector composed of effective elements in $m_i$. The above formula can be transformed as:

$$\Xi = D^\dagger V,$$ (17)

in which $D^\dagger$ is the pseudo-inverse of $D$.

Given enough relative motions and their corresponding weights, each columnwise vector $V$ can be restored according to Eq.(17). Accordingly, all of these Lie algebra $m_k$ can be determined, which then be used to recover the corresponding increment and then refine global motion $M_k$. The following algorithm summarizes the process of motion averaging for weighted relative motions:

Algorithm 3: The weighted motion averaging registration

Input: Initial $M_{global}^0 \triangleq \{I,M_2^0,\ldots ,M_n^0\}$ and relative motions and weights $\{w_{ijr}, M_{ijr}\}_{r=1}^R$

Output: Precise $M_{global} \triangleq \{I,M_2,\ldots ,M_N\}$

Do

1. $\Delta M_{ijm} = M_i^0M_{ijm}(M_j^0)^{-1}$
2. $\Delta m_{ijm} = logm(\Delta M_{ijm})$
3. $\Delta v_{ijm} = mat2vec(\Delta m_{ijm})$
4. $\Delta V = [w_{ij1}\Delta v_{ij1}, w_{ij2}\Delta v_{ij2}, \ldots, w_{ijr}\Delta v_{ijr}]^T$
5. $D = [D_{ij1}, D_{ij2}, \ldots, D_{ijr}]^T$
6. $\Delta \Xi = D^\dagger \Delta V$
7. $\Delta m_k = vec2mat(\Delta \Xi)$
8. $\forall k \in \{2, N\}$, $M_k = e^{\Delta m_k}M_k$

Until $\|\Delta \Xi\| < \varepsilon$

where $mat2vec(.)$ is defined to extract parameters from $m$ to form a column wise vector $v$ and $vec2mat(.)$ is defined to restore $m$ using $v$.

3.4 Algorithm implementation

Given a set of initial global motions $\{I,M_2^0,\ldots ,M_N^0\}$, the following 4 steps are contained in each iteration:

1. Compute the overlapping percentages for each range scan pair according to the method presented in [29].

2. According to Eq.(3), get initial parameters for pairwise registration presented in Section 3.2 and obtain relative motions and weights $\{w_{ijr}, M_{ijr}\}_{r=1}^R$ for scan pairs, whose overlapping percentage satisfies $\xi_{ijr} \geq \xi_{thr}$.

3. According to algorithm 3, utilize $\{w_{ijr}, M_{ijr}\}_{r=1}^R$ to refine global motions $M_{global} \triangleq \{I,M_2,\ldots ,M_N\}$.

4. Repeat steps (1)~(3) until $\sum_{i=1}^n \|\Delta M_i\| < \delta$, where $\delta$ is a preset threshold.

By introducing the weighted motion averaging, the proposed approach can effectively achieve good registration of multi-view range scans.

4 Experimental results

To verify its superior performance, the proposed approach was compared to the low-rank and sparse matrix decomposition [22], the motion average TrICP algorithm [29] and the coarse-to-fine TrICP approach [21], which are abbreviated as LRS-L1alm, MATrICP and CFTrICP. Experiments were tested on seven data sets from the Stanford repository [30], where the Bunny, Dragon, Happy Buddha, Chicken, Parasaurolophus, Chef and Trex include 8, 15, 15, 16, 22 and 21 range scans, respectively. As each of these seven data sets contains a huge number of scans, the multi-view registration is time-consuming. In order to save time, the testing data sets are sampled from the raw
data sets with the sampling frequency set to be 8. During experiments, parameters are set as follows: registration control parameters $\lambda = 2$, minimum overlapping percentage $\xi_{\text{min}} = 0.3$, overlapping percentage threshold $\xi_{\text{thr}} = 0.4$, the minimum value of the iterative termination condition of the motion average algorithm $\varepsilon = 10^{-4}$, the minimum value of the iterative termination condition for the multi-view registration $\sigma = 4.5(N-1) \times 10^{-4}$, maximum number of iterations $K = 30$. All the competed approaches adopted nearest-neighbor search method based on k-d tree to assign the correspondences and were implemented in Matlab. Experiments were performed on an eight-Core 3.60GHZ computer with 8GB of memory.

4.1 Efficiency and accuracy

Since all these four approaches require the initial global motions, it only needs to compare the runtime in multi-view registration step. For comparison of different approaches, the objective function presented in [29] is adopted as the error criterion for accuracy evaluation of multi-view registration results. During experiment, the same noise is added to the initial registration parameters obtained by Eq.(3). Accordingly, four approaches can be applied to register multi-view range scans. As shown in Table 1, it records the runtime and objective function value of the final registration result for all these competed approaches.

It can be seen from Table 1 of the four kinds of multi-view range scan registration algorithm, the stability of the WMAA is optimal, and the number of minimum value of the objective function and the shortest running time is the most. Since the objective function value of LRS-L1alm is too large from Chicken and Chef data sets, it is the most unstable among all the competed approaches. The output image has been can’t see the initial appearance, it unable to correctly match. The objective function value is lower and the running time is short by using the CFTrICP algorithm when the data sets is smaller. But with the increase of the datasets, the results of Happy data demonstrate that its running time and objective function value are the largest. Similarly, in the results of Chef data sets, the running time of the CFTrICP algorithm is more than two times of the MATrICP and the WMAA algorithm, so the CFTrICP algorithm is very unstable when the data set is too large. Compared to all the experimental results of the MATrICP and the WMAA, it is obvious that the objective function value of WMAA algorithm is less than that of the MATrICP algorithm, and its running time is smaller.

In order to evaluate the results in a more intuitive way, Fig. 4 shows the Bunny, Dragon, Happy, Buddha and Parasaurolophus range scan data sets from cross-section of multi-view registration results for four kinds of algorithms, three-dimensional models are more accurate if cross-section outline is more detailed.

It can be seen from the whole image, the WMAA can obtain the most efficient and accurate multi-view registration result among these competed approaches. In the Parasaurolophus group image section, the initial image due to the large error shows a tail part. Cross-section of the LRS-L1alm effect is the worst, it almost can’t see the initial effect. The other three algorithms matching effect is better. As shown in Fig. 5 compare amplified cross-section of multi-view registration results for four competed approaches, the LRS-L1alm algorithm and the effect of CFTrICP algorithm is relatively poor. The curve obtained by MATrICP algorithm is not smooth enough, it has a large number of outliers. Algorithm of WMAA achieves the best result, in cross-section subfigure, it has almost no outliers.

4.2 Robustness

To verify the robustness of the proposed approach, all competed approaches were tested on Bunny data sets with varied initial parameters, which can be obtained by adding the uniform noises to the initial rotation obtained by initial rotation matrix $\{R_{10}, R_{20}, \ldots, R_{N0}\}$. In order to eliminate the effect of random error in the experiment results, 50 MC trials were carried out with respect to three noise levels for all competed approaches.

To compare the robustness of the proposed approach. As shown in Table 2 it depicts the mean value, standard deviation of objective function and the mean runtime for these approaches. The random error of applying different size on the initial global motion, the optimal objective function is the minimum standard deviation, the running time is the shortest, and with the increase of random error, the effect of robustness is more obvious. It explains that the WMAA achieves the best robustness.

As shown in Fig. 6 it displays the objective function value of the registration results for all competed approaches in each MC trial. when the longitudinal coordinate ranges from 0 to 6, some objective function values in subgraph
Figure 4: Cross-section of multi-view registration results for four competing approaches. From the first to fourth rows are Stanford Bunny, Dragon, Happy Buddha, and Parasaurolophus. (a) The 3D model obtained by the proposed approach; (b) Cross-section of the initial model; (c) Cross-section of LRS-L1alm; (d) Cross-section of CFTrICP; (e) Cross-section of MATrICP; (f) Cross-section of WMAA.

Figure 5: Cross-section of partially amplified the dragon and Parasaurolophus model results for four competing approaches. (a) Cross-section of the initial model; (b) Amplified cross-section of the initial model; (c) Amplified cross-section of LRS-L1alm; (d) Amplified cross-section of CFTrICP; (e) Amplified cross-section of MATrICP; (f) Amplified cross-section of WMAA.

Table 2: Performance comparison of three approaches under varied noise levels

|                  | [-0.02,0.02]rad |                  | [-0.04,0.04]rad |                  | [-0.06,0.06]rad |                  |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                  | Obj T(min)      | Obj T(min)      | Obj T(min)      | Obj T(min)      | Obj T(min)      | Obj T(min)      |
|                  | Mean Std        | Mean Std        | Mean Std        | Mean Std        | Mean Std        | Mean Std        |
| LRS-L1alm        | 0.6500 0.0047   | 1.5772          | 0.6513 0.0040   | 1.7477          | 1.0424 1.1452   | 3.4957          |
| CFTrICP          | 0.6333 0.0161   | 1.0182          | 0.7201 0.0395   | 1.2277          | 0.7942 0.3358   | 1.3293          |
| MATrICP          | 0.7163 0.0026   | 1.2829          | 0.6361 0.0046   | 1.0653          | 0.6426 0.0085   | 1.0268          |
| WMAA             | 0.6298 0.0002   | 0.8802          | 0.6299 0.0009   | 0.9674          | 0.6317 0.0032   | 0.9538          |

(a) and (b) are greater than 1, which show that in addition to random error, LRS-L1alm and CFTrICP results of registration error is too large, the robust stability is poor; There is no difference between the subgraph (c) and (d); The longitudinal coordinate is 0.62 to 0.66 when amplify the subgraph (c) and (d). Contrast (e) with (f) we can see that
and accuracy of the pair-wise registration results can be raised with the increase of the overlapping percentage of scan pair, it then adopts the TrICP algorithm to estimate the overlapping percentage and obtain the relative motion of each scan pair with a certain degree of overlapping percentage. By viewing the estimated overlapping percentages as weights, it can apply the weighted motion averaging algorithm to achieve good registration of multi-view range scans. Experimental results tested on several public datasets demonstrate that the proposed approach has superior performances in accuracy, robustness and efficiency over the state-of-art approaches.

As this approach requires initial parameters for multi-view registration, our future work will be committed to estimating the initial registration parameters.

5 Conclusion

This paper presents a novel approach for registration of multi-view range scans. The main contribution is proposing the weighted motion averaging algorithm, which can take into account the reliability and accuracy of each motion obtained from the pair-wise registration. Since the reliability of the objective function value of WMAA algorithm is the basic trend in the 0.63, and the objective function value of MA-TrICP algorithm in the range of 0.63-0.66 amplitude changes obviously, which illustrates that the WMAA algorithm has a big error in the initial global motion situation, it can still get precise results for multi-view range scan’s registration and has strong robustness.

Figure 6: The objective function value of the registration results for the competed approaches in each MC trial, the subgraph (e) and (f) are amplified results of (c) and (d)
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