TGAS Error Renormalization from the RR Lyrae Period-Luminosity Relation

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Abstract: The Gaia team has applied a renormalization to their internally-derived parallax errors $\sigma_{\text{int}}(\pi)$ based on comparison to Hipparcos astrometry. We use a completely independent method based on the RR Lyrae $K$-band period-luminosity relation to derive a substantially different result, with smaller ultimate errors $(A, \sigma_0) = (1.1, 0.12 \, \text{mas})$ (this paper).

We argue that our estimate is likely to be more accurate and therefore that the reported TGAS parallax errors should be reduced according to the prescription:

$$\sigma_{\text{true}}(\pi) = \sqrt{(0.79 \sigma_{\text{tgas}}(\pi))^2 - (0.10 \, \text{mas})^2}.$$ 

Key words: astrometry

1. INTRODUCTION

The Tycho-Gaia Astrometric Survey (TGAS) has just been released with approximately 2 million parallaxes, having typical reported precisions of $\sigma(\pi) \sim 300 \, \text{mas}$. Thus, while constituting only a tiny fraction of the ultimate Gaia product, TGAS is by far the largest and (with the exception of a tiny handful of Hubble Space Telescope parallaxes, e.g., [Benedict et al. 2011]), the most accurate optical astrometric catalog now available [Brown et al. 2016; Lindegren et al. 2016].

In order to validate the TGAS catalog, it is natural to compare with the best previously existing astrometric catalog, Hipparcos. This is not ideal as TGAS has, overall, significantly better parallax measurement compared to Hipparcos. However, it is feasible, in principle, because of the large number ($\sim 10^5$) of overlapping entries and because such validation requires only the measurement of two error-renormalization parameters $(A, \sigma_0)$

$$\sigma_{\text{tgas}}(\pi) = \sqrt{(A \sigma_{\text{int}}(\pi))^2 + \sigma_0^2}; \quad (A, \sigma_0) = (1.4, 0.20 \, \text{mas})$$

where $\sigma_{\text{int}}$ and $\sigma_{\text{tgas}}$ are respectively the internal and reported (renormalized) errors, $\sigma_0$ is the systematic error floor and $A$ is the renormalization factor.

However, it is notoriously difficult to calibrate superior data from inferior data and, in particular, requires superb knowledge of the error-structure of the inferior data set.

We therefore present an alternative method for calibrating the TGAS parallaxes, which does not require any external astrometric data.

2. NEW METHOD TO CALIBRATE TGAS

As we have previously discussed, the RR Lyrae (RRL) period-luminosity (PL) relation is expected to provide a powerful means to calibrate astrometric data — in particular Gaia data [Gould & Kollmeier 2016a]. In that paper, we discussed measurement of the parallax zero point $\pi_0$, which will not be feasible until the full Gaia release. However, as we show here, the RRL PL relation can also be used as a tool to determine the TGAS error renormalization. The RRL PL relation (in, for example, $K$ band) has the form

$$M_K = M_{K,0} + B \log(P/P_0)$$

where $M_{K,0}$ and $B$ are parameters, and $P_0$ is an arbitrarily chosen reference point. In this work we choose $\log(P_0/\text{day}) = -0.29$ so that $(M_{K,0}, B)$ are roughly uncorrelated in the TGAS data set. Thus, if the period

\textsuperscript{1}In principle, this relation also depends on metallicity. Because metallicity does not play a role in the current exercise as we show below, we ignore it. In principle, however, for a larger number of objects, we expect that metallicity information can further improve the errors reported here.

\textsuperscript{2}Theoretical models predict the period-luminosity relation to be of the form

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Given an ensemble of RRL parallax measurements and well understood errors \((\pi, \sigma)_i\), one can derive \((M_{K,0}, B)\) by minimizing \(\chi^2\),

\[
\chi^2(M_{K,0}, B) = \sum_i \frac{[\pi_i - \pi_{pred,i}(M_{K,0}, B)]^2}{\sigma_i^2 + (\epsilon \pi_{pred,i})^2}.
\]  

In the present case, however, we will extract \(K\) measurements from the 2MASS catalog, which are at a single epoch. Because RRL have full amplitudes of about 0.5 mag in \(K\) band, this introduces an rms flux error of roughly 16% into the relation, which corresponds to a fractional error in the \(\pi_{pred}\) of 8%. Therefore, for our particular case, Equation (4) must be rewritten as

\[
\chi^2(M_{K,0}, B) = \sum_i \frac{[\pi_i - \pi_{pred,i}(M_{K,0}, B)]^2}{\sigma_i^2} + \epsilon^2.
\]  

In principle one could augment \(\epsilon\) (in quadrature) by the photometric error, but this is negligible. Likewise, if the scatter in the relation were a major factor, then \(\epsilon\) could be augmented by this as well. We believe this is small (Madore et al., 2013; Beaton et al., 2016), and in any case it is likely smaller than the uncertainty in our estimate of \(\epsilon\) itself, so we ignore it.

If we now consider that the nominal errors \(\sigma_{int}\) must be modified as per Equation (1), then Equation (5) must be modified as well:

\[
\chi^2(M_{K,0}, B, A, \sigma_0) = \sum_i \frac{[\pi_i - \pi_{pred,i}(M_{K,0}, B)]^2}{A\sigma_{int}^2 + \sigma_0^2 + (\epsilon \pi_{pred,i})^2}.
\]

Using this equation, one can simultaneously determine the four parameters \((M_{K,0}, B, A, \sigma_0)\). For each set of trial error-renormalization parameters \((A, \sigma_0)\), one minimizes \(\chi^2\) over \((M_{K,0}, B)\). Then, following the method of Ye et al. (2012, 2013), we consider solutions to be acceptable if the cumulative \(\chi^2\) distribution (ordered by nominal errors) forms a straight line with a slope of unity. We note the following facts. If the real errors are subject to a systematic floor \(\sigma_0\) but this is not reflected in the formal errors, the cumulative distribution will initially rise more quickly than the “unity line”, before falling back to the line (if parameters have been chosen so that \(\chi^2/dof = 1\)). By contrast, if \(\sigma_0\) is overestimated, then the cumulative \(\chi^2\) distribution will initially climb more slowly than the line. Thus the morphology of this comparison gives us critical information on the error floor within the data.

3. DATA

We begin with the sample of 125 Hipparcos RRab RRL used in the PG98 statistical parallax study

\[
P, \text{the mean magnitude } K, \text{ and the extinction } A_K, \text{ are measured, the parallax can be predicted for a given assumed } (M_{K,0}, B)
\]

\[
\pi_{\text{pred}} = 10^{K - A_K - (M_{K,0} + B \log(P/\Pi_0))}/5+2 \text{ mas (3)}
\]

\[
\text{Given an ensemble of RRL parallax measurements and well understood errors } (\pi, \sigma)_i, \text{ one can derive } (M_{K,0}, B) \text{ by minimizing } \chi^2,
\]

\[
\chi^2(M_{K,0}, B) = \sum_i \frac{[\pi_i - \pi_{\text{pred},i}(M_{K,0}, B)]^2}{\sigma_i^2 + (\epsilon \pi_{\text{pred},i})^2} \quad \epsilon = 0.08.
\]

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Using this equation, one can simultaneously determine the four parameters \((M_{K,0}, B, A, \sigma_0)\). For each set of trial error-renormalization parameters \((A, \sigma_0)\), one minimizes \(\chi^2\) over \((M_{K,0}, B)\). Then, following the method of Ye et al. (2012, 2013), we consider solutions to be acceptable if the cumulative \(\chi^2\) distribution (ordered by nominal errors) forms a straight line with a slope of unity. We note the following facts. If the real errors are subject to a systematic floor \(\sigma_0\) but this is not reflected in the formal errors, the cumulative distribution will initially rise more quickly than the “unity line”, before falling back to the line (if parameters have been chosen so that \(\chi^2/dof = 1\)). By contrast, if \(\sigma_0\) is overestimated, then the cumulative \(\chi^2\) distribution will initially climb more slowly than the line. Thus the morphology of this comparison gives us critical information on the error floor within the data.

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Second, one expects that at least a few stars will be “corrupted” by real physical effects unrelated to the parallax measurement. Retaining these objects obviously contaminates our measurement. For example, as noted above, BX Dra is known to have additional light in the aperture from an eclipsing companion, so we know it should not be included in our analysis. Other stars undoubtedly have at least some such light due to non-eclipsing companions. Since RRL are giants, this usually will not matter much but it may in a few cases. Thus, removing these objects is important not only for making the the most accurate estimate of $(M_{k,0}, B)$ (which is not the main objective here), but also for making the most accurate estimate of $(A, \sigma_0)$ (which is).

4. Results

Figure 1B shows the RRL $K$-band PL relation and Figure 2 shows cumulative distributions for various choices of $(A, \sigma_0)$. The red curve shows the result of assuming $\sigma_0 = 0$. The cumulative distribution definitely rises too quickly. The cyan curve shows the best case if we assume that only a systematic term is needed (no rescaling factor: $A = 1$). In this case $\sigma_0 = 0.135$ mas. This would be acceptable. However, $(A, \sigma_0) = (1.1, 0.12$ mas) is clearly better. The TGAS adopted choice clearly falls below the “unity line”. Even if we accept the TGAS $\sigma_0 = 0.2$ mas and choose the minimum physically reasonable value of $A = 1$, the cumulative distribution still falls below the line. For completeness, if we eliminate only the 0 or 2 largest outliers (as opposed to the three largest, which is our preference) we obtain $A = 1.35$ and $A = 1.15$, respectively. We caution, however, that such procedures (particularly the former) would likely only be injecting corrupted measurements into the final result rather than respecting pure statistical protocol.

5. Discussion

We provide an independent method to calibrate the TGAS catalog errors that does not rely in any way on previous generation astrometric data. This purely photometric method of the standard candles avoids the issues associated with relying on lower quality data and ultimately yields more precise values for the TGAS catalog.

The method presented here is likely to be more accurate than one based on comparison to Hipparcos parallaxes. This is particularly true for measuring the zero-point floor $\sigma_0$, for which we find $\sigma_0 = 0.12$ mas and the TGAS team found $\sigma_0 = 0.20$ mas.

The Hipparcos catalog very likely has systematic errors at the $\sim 0.1$ mas level. This is well below the typical statistical error for individual stars, so the only place that these systematics have surfaced is in the measurement of the distance to the Pleiades, which was

\footnote{With $\chi^2 = (10.5, 10.5, 8.2)$, these objects have probabilities of, respectively, about (0.1, 0.1, 0.6) of appearing in the Gaussian tails of 111 objects. Thus, excluding the first two is clearly indicated, while the third is basically indicated, but borderline. However, whether the third is included or excluded has almost no effect. See main text.}

![Figure 2. Cumulative distribution of $\chi^2$ ordered from smallest to largest formal error. According to prescription of Yee et al. (2012, 2013), the parameters $(A, \sigma_0)$ entering Equation 1 should be adjusted so that this distribution is a straight line with unit slope (black). If there is no systematic floor (red), then cumulative $\chi^2$ rises too quickly at small errors (left). If the rescaling factor $A$ is set to its minimum physical value $A = 1$ (cyan), the behavior is acceptable, but not optimal. The magenta curve $(A, \sigma_0) = (1.1, 0.12$ mas) is optimal. The values adopted by TGAS $(A, \sigma_0) = (1.4, 0.2$ mas) strongly overestimate the errors (green), and this remains true even if one sets the rescaling factor at its minimum physical value $A = 1$, but leaves $\sigma_0 = 0.20$ (blue).}
based on combining measurements of many stars. The original Hipparcos parallax was larger than the “traditional” value by almost 1 mas, and it was suggested at that time that this could be due to correlated errors (Pinsonneault et al., 1998; Soderblom et al., 1998). Narayan & Gould (1999) demonstrated strong evidence for correlated Hipparcos errors in the Hyades field which, by chance, they showed did not lead to an error in the distance estimate. However, after reanalyzing the Hipparcos data, van Leeuwen (2009) could find no internal evidence of these correlations and published a similar Pleiades distance measurement as originally with yet smaller error bars. This conflict was resolved by TGAS in favor of the “traditional”, longer Pleiades distance (Brown et al., 2016).

While the distance to the Pleiades is settled, the cause of the Hipparcos error in its estimate of this distance is not. It may be that the problem is entirely explained by correlations, but these may also be masking other problems. In particular, since the cause of these correlations (if they are in fact the root cause) have not been tracked down, it cannot be assumed that the Hipparcos error profile is understood at a level well below the precision of its measurement.

By contrast, the external inputs into RRL PL relation (P, K, A) are quite well understood. Therefore, the approach of the current paper appears more secure and in any case independent from the purely astrometric approach.

One important difference between the sample studied here (108 RRL) and the one studied by the Gaia team (10^5 Hipparcos stars) is that the RRL have intrinsically similar colors while the Hipparcos stars cover a full range of stellar colors. In principle, this difference could be important since the Gaia team did not attempt to correct TGAS for color-dependent astrometric deviations (as they will for subsequent releases).

This issue clearly deserves further investigation, a path to which we outline below. However, if there are color-dependent systematic errors, these are likely to be larger for RRL than average stars because RRL have a larger color offset relative to the mean reference frame set by other stars.

Nevertheless, this question can be further investigated by applying the same method that we have used here, but to Cepheids. Cepheids lie in the same instability strip and so, like RRL, they are systemically bluer than other stars. However, unlike RRL, they virtually all lie in the Galactic plane. Moreover, they are more luminous than RRL and so are typically seen at greater distances and so through more dust. In our sample of 108 RRL, there are only 19 stars with E(B - V) > 0.1 and only four of these have E(B - V) > 0.2. This is not enough to probe a broad range of observed colors. By contrast, Cepheids will probe a broad range. Note in particular that while distant Cepheids provide relatively little information about the PL relation, they can provide excellent information on TGAS error characterization. This is because, once the PL relation is determined from nearby stars, the parallaxes of distant stars can be determined photometrically to much higher precision than the parallax errors. This is the same principle used in the Gould & Kollmeier (2016a) method of measuring \( \pi_0 \).

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