Cosmological Constant and Soft Terms in Supergravity

Kiwoon Choi\textsuperscript{(a)}, Jihn E. Kim\textsuperscript{(b)} and Hans Peter Nilles\textsuperscript{(c,d)}

\textsuperscript{(a)}Department of Physics, Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-Gu, Taejon 305-701, Korea

\textsuperscript{(b)}Department of Physics and Center for Theoretical Physics, Seoul National University
Seoul 151-742, Korea

\textsuperscript{(c)}Physik Department, Technische Universität München

D-85747 Garching, Germany

\textsuperscript{(d)Max-Planck-Institut für Physik, Werner-Heisenberg-Institut
D-80805 München, Germany

Abstract

Some of the soft SUSY breaking parameters in hidden sector supergravity model depend on the expectation value of the hidden sector scalar potential, \( \langle V_h \rangle \), whose tree level value is equal to the tree level cosmological constant. The current practice of calculating soft parameters assumes that \( \langle V_h \rangle = 0 \). Quantum correction to the cosmological constant can differ from the correction to \( \langle V_h \rangle \) by an amount of order \( m_{3/2}^2 M_{Pl}^2/8\pi \). This implies that, for the vanishing cosmological constant, the \( \langle V_h \rangle \)-dependent parts of soft terms can be sizable, and hence the supergravity phenomenology should be accordingly modified.
Presently supersymmetry (SUSY) is widely believed to be a leading candidate for physics beyond the standard model. This is largely due to the fact that SUSY provides the only known perturbative solution to the problem of quadratic divergence in the Higgs boson mass. Phenomenologically viable supersymmetric models contain soft supersymmetry breaking terms which are presumed to be induced by supergravity interactions in underlying $N = 1$ supergravity theories \cite{1}. Most of supergravity phenomenology then depend upon the nature of soft terms in the resulting globally supersymmetric effective theory.

An interesting feature of supergravity models is that many of the coefficients of soft terms are calculable at tree approximation. Furthermore, in some cases the model predicts certain tree level relations among the soft parameters renormalized at the Planck scale $M_{Pl}$. For instance, in the dilaton-dominated SUSY breaking scenario in string theory \cite{2,3}, one finds $A_{ijk} = -\sqrt{3}m_0\lambda_{ijk}$ where $A_{ijk}$ denotes a generic trilinear scalar coefficient, $\lambda_{ijk}$ is the associated Yukawa coupling, and $m_0$ is the soft scalar mass.

For physical applications of any tree level result, one needs to take into account quantum corrections. Ordinary renormalizable gauge and matter interactions would lead to one loop corrections proportional to $\frac{g^2}{8\pi^2}\ln(\Lambda^2/\mu^2)$ where $g$ denotes a gauge or Yukawa coupling constant, $\mu$ is a scale around the weak scale $m_W$ where the loop graph is being evaluated, and $\Lambda$ is the momentum cutoff above which the validity of four-dimensional $N = 1$ supergravity theory breaks down. For $g^2$ of order unity and $\Lambda$ around $M_{Pl}$, these corrections become important due to a large logarithm. A nice feature of this type of corrections is that they do not depend strongly on the cutoff $\Lambda$ and thus are calculable within the supergravity model by the aid of renormalization group analysis.

Besides the above type of logarithmic corrections, there are other types of corrections which depend strongly on $\Lambda$ and thus whose precise magnitudes are not calculable within the supergravity model \cite{4}. For instance, one loop graphs induced by nonrenormalizable gravitational interactions would give power-law divergent corrections which are proportional to $(\kappa_0\Lambda)^2/8\pi^2$ where $\kappa_0 = \sqrt{8\pi}/(M_{Pl})_{\text{bare}}$ denotes the dimensionful bare gravitational coupling constant. To estimate the size of such corrections, one needs to determine the dimension-
less coupling \( \kappa_0 \Lambda \), which requires an information of the theory underlying the supergravity model at energy scales above \( \Lambda \). For instance, for a supergravity model which corresponds to the low energy limit of heterotic string theory, it is most natural to set \( \Lambda \) to the string scale \( M_{\text{string}} \). We then have \( \kappa_0 \Lambda = \kappa_0 M_{\text{string}} = g_{\text{GUT}} \) where \( g_{\text{GUT}} \) is the unified gauge coupling constant. Inspired by this observation, throughout this paper, we will assume \( \kappa_0 \Lambda = g_{\text{GUT}} \) whenever we consider a power-law divergent quantum effect.

For \( \kappa_0 \Lambda = g_{\text{GUT}} \), generic power-law divergent corrections would be expected to be of order \( \alpha_{\text{GUT}}/2\pi \) and thus not so significant. However, there is one important exception for this naive expectation. At one loop order, the cosmological constant (the vacuum energy density) receives a quadratically-divergent zero point energy contribution \([5]\). If the model contains \( N \) chiral multiplets with the boson mass \( m_B \) much larger than the fermion mass \( m_F \), the cosmological constant receives a contribution:

\[
N \int \frac{d^3k}{(2\pi)^3} \left( \sqrt{k^2 + m_B^2} - \sqrt{k^2 + m_F^2} \right) \simeq N \frac{m_B^2 \Lambda^2}{8\pi^2}.
\] (1)

As we will see later, due to this quantum correction to the cosmological constant, some soft parameters are changed by an amount of order \( N\alpha_{\text{GUT}}/2\pi \) times the tree level values. Then a simple but important point is that although \( \alpha_{\text{GUT}}/2\pi \) is small, \( N\alpha_{\text{GUT}}/2\pi \) can be significantly larger in realistic models. In this paper, we wish to discuss this point together with its implications in the context of a simple supergravity model.

To proceed, let us consider a simple supergravity model with the following Kähler potential and superpotential \([1]\)

\[
K = h_\alpha^* h_\alpha + \phi_i^* \phi_i + (\xi H_1 H_2 + \text{h.c.}) ,
\]

\[
W = W_h(h_\alpha) + \frac{1}{6} \lambda_{ijk} \phi_i \phi_j \phi_k ,
\] (2)

where \( h_\alpha \) denote hidden sector fields triggering SUSY breaking, and \( \phi_i \) are generic observable matter fields including quarks, leptons, and the two Higgs doublets \( H_1 \) and \( H_2 \). The corresponding supergravity lagrangian can be expanded in terms of \( \phi_i \) as

\[
\mathcal{L} = \mathcal{L}_h - (m_{3/2}^2 + \kappa_0^2 V_h) \phi_i^* \phi_i - [\xi (\kappa_0^2 V_h + 2m_{3/2}^2) H_1 H_2 + \text{h.c.}] + ..., \]

(3)
where $\mathcal{L}_h \equiv \mathcal{L}(\phi_i = 0)$, and the ellipsis stands for other observable-field-dependent terms. Here the local composite operators $m_{3/2}^2$ (the gravitino mass operator) and $V_h$ (the hidden sector scalar potential operator) are given by

$$m_{3/2}^2 = \kappa_0^4 |W_h|^2 \exp(\kappa_0^2 h_\alpha^* h_\alpha),$$

$$V_h = (|D_\alpha W_h|^2 - 3\kappa_0^2 |W_h|^2) \exp(\kappa_0^2 h_\alpha^* h_\alpha), \quad (4)$$

where $D_\alpha W_h = (\partial_{h_\alpha} + \kappa_0^2 h_\alpha^*) W_h$.

For a given supergravity lagrangian $\mathcal{L}$, one may integrate out the hidden fields to obtain the effective lagrangian $\mathcal{L}_\phi$ of the observable fields $\phi_i$:

$$\exp(i \int d^4x \mathcal{L}_\phi) = \int [\mathcal{D} h] \exp(i \int d^4x \sqrt{g} \mathcal{L}). \quad (5)$$

Here $[\mathcal{D} h]$ denotes the integration of the hidden fields including the gravity multiplet $(g_{\mu\nu}, \psi_\mu)$. If the vacuum values of $W_h$ and $D_\alpha W_h$ are nonzero and thus SUSY is broken, $\mathcal{L}_\phi$ would contain the soft SUSY breaking terms:

$$\mathcal{L}_\phi \ni - m_0^2 \phi_i^* \phi_i - (\frac{1}{6} A_{ijk} \phi_i \phi_j \phi_k + B H_1 H_2 + m_a \lambda^a \lambda^a + \text{h.c.}), \quad (6)$$

where $\lambda^a$ are the gauginos.

Since none of the $\phi_i$-modes are integrated out, the effective lagrangian $\mathcal{L}_\phi$ is essentially renormalized at the cutoff scale $\Lambda$, viz all the operators and the parameters in $\mathcal{L}_\phi$ are renormalized at $\Lambda$ in the sense of Wilson. Among the coefficients of soft terms, $m_0^2$ and $B$ depend on the expectation value of the hidden sector scalar potential, $\langle V_h \rangle$, while the others are independent of $\langle V_h \rangle$. From Eqs. (3) and (5), one easily finds

$$m_0^2(\Lambda) = \langle m_{3/2}^2 + \kappa_0^2 V_h \rangle, \quad B(\Lambda) = \langle \xi (\kappa_0^2 V_h + 2m_{3/2}^2) \rangle, \quad (7)$$

where the bracket means the average over the hidden fields, e.g.

$$\langle V_h \rangle = \int [\mathcal{D} h] V_h(h_\alpha) \exp(i S_h) / \int [\mathcal{D} h] \exp(i S_h), \quad (8)$$

where $S_h = \int d^4x \sqrt{g} \mathcal{L}_h$.  

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Obviously both $\langle m_{3/2}^2 \rangle$ and $\langle V_h \rangle$ are determined entirely by the hidden sector parameters. To be phenomenologically acceptable, those hidden sector parameters must be adjusted to the values leading to the vanishing cosmological constant. In most of the previous studies of soft terms, motivated by the vanishing cosmological constant, $\langle V_h \rangle$ has been taken to be zero or $\kappa_0^2 \langle V_h \rangle \ll m_{3/2}^2$.

Does the vanishing cosmological constant imply $\kappa_0^2 \langle V_h \rangle \ll m_{3/2}^2$? The cosmological constant $V_{\text{eff}}$ can be obtained by integrating out all fields in the theory [3]:

$$\exp(i \int d^4 x V_{\text{eff}}) = \int [D \phi D h] \exp(i \int d^4 x \sqrt{g} \mathcal{L}), \quad (9)$$

where $[D \phi]$ stands for the integration of all observable gauge and matter multiplets. Clearly at tree approximation where the path integral representations of $\langle V_h \rangle$ and $V_{\text{eff}}$ (see Eqs. (8) and (9)) are saturated by the field configurations satisfying the classical equations of motion, we have $\langle (V_h)_{\text{tree}} \rangle = (V_{\text{eff}})_{\text{tree}}$, and thus the hidden sector parameter values yielding a vanishing tree level cosmological constant automatically gives $\langle (V_h)_{\text{tree}} \rangle = 0$. However, as can be noticed by their definitions, $V_{\text{eff}}$ and $\langle V_h \rangle$ can receive completely different quantum corrections.

Even at one loop order, quantum fluctuations of both the observable fields and the hidden fields give a quadratically divergent correction to the cosmological constant [3]:

$$\delta V_{\text{eff}} \equiv V_{\text{eff}} - (V_{\text{eff}})_{\text{tree}} \simeq \frac{1}{8 \pi^2} \text{Str} (\mathcal{M}^2) \Lambda^2, \quad (10)$$

where $\mathcal{M}^2$ denotes the mass matrix of the supergravity action. For our simple model, we have

$$\text{Str} (\mathcal{M}^2) \simeq (N m_0^2 - \tilde{N} \tilde{m}^2) + ... \equiv N_{\text{eff}} m_0^2(\Lambda), \quad (11)$$

where $N$ is the number of observable chiral multiplets, $\tilde{N}$ is the number of gauginos which are assumed to have a common mass $\tilde{m}$, and the ellipsis denotes the contribution from the hidden sector. Here we have neglected the masses of matter fermions and gauge bosons. For the hidden sector contribution, the gravity multiplet gives a negative contribution (=...
−4m_{3/2}^2) while the hidden chiral multiplets give a positive contribution proportional to the number of multiplets.

Unlike the cosmological constant, \langle V_h \rangle receives a correction only from the hidden field fluctuations. For \kappa_0\Lambda = g_{\text{GUT}}, if (i) interactions among hidden sector fields are weak enough, and (ii) the number of hidden multiplets which contribute to SUSY breaking by having nonvanishing auxiliary components is of order unity, we have

\[ \delta \langle V_h \rangle = O\left(\frac{\alpha_{\text{GUT}}}{\pi} |F_\alpha|^2 \right) = O\left(\frac{\alpha_{\text{GUT}}}{\pi} \kappa_0^{-2} m_{3/2}^2 \right), \]

where \delta \langle V_h \rangle = \langle V_h \rangle - \langle \langle V_h \rangle \rangle_{\text{tree}}, and the F-term of \( h_\alpha \) is given by \( F_\alpha = D_\alpha W_h \exp(\kappa_0^2 h_\beta h_\beta^*/2) = O(\kappa_0^{-2} m_{3/2}^2) \). In fact, the conditions (i) and (ii) are met by many simple hidden sector models. One might think that these conditions are not fulfilled by the popular gaugino condensation model [4] for SUSY breaking in string theory. However, in the gaugino condensation model, strongly interacting gauge nonsinglet hidden multiplets can be integrated out without breaking SUSY. Then the effects of integrating out gauge nonsinglet fields can be summarized by an effective superpotential of a relatively small number of weakly interacting gauge singlet multiplets, e.g. the dilaton and moduli multiplets, for which the conditions (i) and (ii) are fulfilled.

Using Eqs. (7), (10), and (12) together with \( (V_{\text{eff}})_{\text{tree}} = (\langle V_h \rangle)_{\text{tree}} \), we easily find that the phenomenological requirement of \( V_{\text{eff}} = 0 \) leads to

\[ \kappa_0^2 \langle V_h \rangle \simeq -\epsilon m_0^2(\Lambda) \simeq -\frac{\epsilon}{1 + \epsilon} \langle m_{3/2}^2 \rangle, \]

where

\[ \epsilon = \frac{N_{\text{eff}}}{8\pi^2} (\kappa_0\Lambda)^2. \]

In many cases, the gaugino mass contribution to \( N_{\text{eff}} \) is expected to be significantly smaller than the chiral matter contribution. It is then quite conceivable that \( N_{\text{eff}} \) is positive and of \( O(8\pi^2) \), implying that \( \epsilon \) can be essentially of order unity for \( \kappa_0\Lambda = g_{\text{GUT}} \). Note that \( N_{\text{eff}} \) receives a contribution from all chiral multiplets with masses far below \( M_{\text{Pl}} \), particularly from those in the minimal supersymmetric standard model containing 49 chiral multiplets.
So far, we have considered only the quadratically divergent one loop contribution to the cosmological constant, which is of order \( \kappa_0^{-2} m_{3/2}^2 \) due to a large value of \( N_{\text{eff}} \) compensating over the loop suppression factor \( 1/8\pi^2 \). Higher loops also give quadratically divergent contributions to the cosmological constant. However, contrary to the one loop effect higher loop effects do not contain any additional large factor which may compensate the additional loop suppression factor \( 1/8\pi^2 \). For example, two loop diagrams involving gauge interactions give a contribution smaller than the one loop effect by the small factor \( \alpha_{\text{GUT}}/2\pi \). For two loop diagrams involving trilinear and/or Yukawa interactions also, it is suppressed by \( (\text{coupling constant})^2 \). For the trilinear couplings, the two loop diagram is shown in Fig. 1 and its contribution is roughly \( \frac{1}{(4\pi)^2} \sum_{ijk} A_{ijk} A^*_{ijk} \Lambda^2 \). For Yukawa interactions, the contribution is roughly \( \frac{1}{(4\pi)^2} \sum_{ijk} \lambda_{ijk} \lambda^*_{ijk} m_{3/2}^2 \Lambda^2 \). Usually \( A_{ijk} \) is of order \( m_{3/2}^2 \lambda_{ijk} \), and then clearly these two loop effects are negligible compared to the one loop effect of Eq. (10). Then our result of Eq. (13) which was derived within the one-loop approximation would remain valid even after taking into account higher loop effects. One may also include the effects associated with the electroweak symmetry breaking and the nonperturbative QCD effects. Since \( \Lambda_{\text{QCD}}^4 \ll m_w^4 \ll \kappa_0^{-2} m_{3/2}^2 \), again this point does not affect at all our result of Eq. (13).

In the above discussion, we have pointed out that, if one implements a fine tuning of the hidden sector parameters to have the vanishing tree level cosmological constant, the corresponding hidden sector scalar potential \( V_h \) has an expectation value \( |\langle V_h \rangle| \ll \kappa_0^{-2} m_{3/2}^2 \), which is the relation frequently used in the previous analyses of soft parameters. However, if the hidden sector parameters are adjusted to yield the vanishing renormalized cosmological constant, \( \langle V_h \rangle \) is likely to have a negative value of \( O(\kappa_0^{-2} m_{3/2}^2) \). Note that this is not due to a quantum correction of \( \langle V_h \rangle \) in the path integral evaluation, which is of \( O((\alpha_{\text{GUT}}/\pi)\kappa_0^{-2} m_{3/2}^2) \) if the two plausible conditions specified above Eq. (12) are met, but due to a change in the fine tuning of the hidden sector parameters required for the vanishing renormalized cosmological constant.

What would be the implications of our observation that \( \kappa_0^2 \langle V_h \rangle = -O(m_{3/2}^2) \)? First of all, it implies that the \( \langle V_h \rangle \)-dependent parts of soft parameters can be sizable, and thus must
be included when one computes soft parameters for a given supergravity model, as was done recently by Brignole, Ibanez and Munoz [3]. In some cases, using the input $\langle V_h \rangle = 0$, one obtains a very simple pattern of soft parameters renormalized at $\Lambda$. For instance, in the dilaton-dominated SUSY breaking scenario in string theory, if the $\mu$-term is induced entirely by the Kähler potential term $\xi H_1 H_2$ [8], one finds the relation $m_0 : B/\mu : A_{ijk}/\lambda_{ijk} = 1 : 2 : -\sqrt{3}$ [3]. We have already noted that $m_0^2(\Lambda)$ and $B(\Lambda)$ associated with $\xi H_1 H_2$ contain $\langle V_h \rangle$-dependent pieces, while the other soft parameters are independent of $\langle V_h \rangle$. Clearly then, for $\kappa_0^2\langle V_h \rangle = -O(m_0^2/\mu^2)$, the above relation must be modified and only the property of soft parameters which is independent of the input $\langle V_h \rangle = 0$ must be seriously taken into account [3].

There is another implication of our observation. It has been pointed out that in multi-gaugino condensation models [3] for SUSY breaking in string theory, the hidden sector potential, i.e. the dilaton potential, appears to have a negative value of $O(\kappa_0^{-2}m_0^2)$ for a reasonable range of the hidden sector parameters. This negative dilaton potential has been often considered as an undesirable feature of the model. However, our discussion above indicates that a negative value of the hidden sector potential is not a problem, but rather a desirable feature, for the fully renormalized cosmological constant to vanish.

A few comments should be made. To obtain the soft parameters renormalized at $m_W$ from those at $\Lambda$, one must integrate out the $\phi_i$-modes with frequencies greater than $m_W$. This would lead to a subsequent renormalization of soft parameters. Such subsequent renormalizations, e.g. $m_0^2(\Lambda)/m_0^2(m_W)$, are determined mainly by the observable sector couplings in $L_\phi$, e.g. the gauge and Yukawa couplings, and thus are independent of the one-loop renormalization of the cosmological constant which affects only the hidden sector parameters. Note that both $(V_{\text{eff}})_{\text{tree}}$ and the one-loop correction $\delta V_{\text{eff}}$ of Eq. (10) depend only on the hidden sector parameters.

Since the quantum corrections to the cosmological constant of order $\kappa_0^{-2}m_0^2 \gg m_W^4$ are added to the tree level value, one might wonder whether the vacuum structure of the observable sector fields $\phi_i$ is changed. However, due to SUSY, the effective potential of $\phi_i$
does not contain any field-dependent quadratic divergence. As a result, the corresponding vacuum structure is not touched at all by the quadratically divergent correction to the cosmological constant. Also our results of Eqs. (13) explicitly show that \( m_0^2(\Lambda) \) is still positive even after including the contribution from a negative \( \langle V_h \rangle \).

In conclusion, we have pointed out in this paper that the quantum correction to the cosmological constant in supergravity models is likely to be positive and can be of \( O(\kappa_0^{-2} m_{3/2}^2) \), while the quantum correction to \( \langle V_h \rangle \) is of \( O(\pi^{-1} \alpha_{\text{GUT}} \kappa_0^{-2} m_{3/2}^2) \). This is for a reasonable choice of the cutoff scale, \( \kappa_0 \Lambda = g_{\text{GUT}} \), and mainly due to a large number of chiral multiplets compensating over the loop suppression factor. Although they are equal at tree level, the renormalized cosmological constant and the expectation value of the hidden sector scalar potential \( V_h \) can differ from each other by an amount of \( O(\kappa_0^{-2} m_{3/2}^2) \). Then, the condition of the vanishing cosmological constant leads to a negative \( \kappa_0^2 \langle V_h \rangle \) of \( O(m_{3/2}^2) \). This means the \( \langle V_h \rangle \)-dependent parts of the soft parameters cannot be ignored, and thus the conventional studies of supergravity phenomenology based on the input \( \kappa_0 \langle V_h \rangle \ll m_{3/2}^2 \) should be accordingly modified. This also implies that it might not be a problem, but rather a desirable feature (for the fully renormalized cosmological constant to vanish) that the hidden sector scalar potential has a negative minimum whose magnitude is of \( O(\kappa_0^{-2} m_{3/2}^2) \).

**ACKNOWLEDGMENTS**

This work is supported in part by Korea Science and Engineering Foundation through Center for Theoretical Physics at Seoul National University (KC, JEEK), KOSEF–DFG Collaboration Program (JEK, HPN), the Ministry of Education of The Republic of Korea (KC, JEEK), Deutsche Forschungsgemeinschaft (HPN) and EC grants SC1-CT91-0729 and SC1-CT92-0789 (HPN).
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FIGURES

FIG. 1. A two loop contribution to the vacuum energy from the $A$ terms.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9404311v1