We study the mesons matter $D - \bar{D}$ in the framework of $\sigma$ and $\omega$ meson exchange model using Walecka’s mean field theory. We choose the equal number of $D$ and anti-$D$ meson then we get $\langle \omega^0 \rangle = 0$ and the field $\langle \sigma \rangle$ exhibits a critical temperature around 1.2 GeV. We investigate effective mass, pressure, energy density and energy per pair. We conclude that this matter is a gas and these results are not favorable for the existence of $D - \bar{D}$ bound state.

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I. INTRODUCTION

The possibility to found exotic mesons has been motivated many theoretical and experimental effort to understand the structure of the new mesons. The new mesons are called exotic, because they cannot be explained in terms of quark-antiquark picture. Since 2003, there are many candidates for exotic mesons called $X$, $Y$ and $Z$, they have been discovered in $B$ mesons decays \cite{1-4}. The nature of the new mesons are completely open and there are already many theoretical interpretations about their structure. One of these interpretations is that their structure is a bound state of two other mesons. It is analogous to a proton and a neutron binding together to form a deuteron. This idea is not new and it was first studied by Törnqvist in 1991 \cite{5} and he called these states a deuson.

In a second paper in 1994, Törnqvist \cite{6} used a meson potential model with one-pion-exchange interactions for to study charm meson molecules with isospin $I = 0$: $D^*\bar{D}/D\bar{D}^*$ and $D^*\bar{D}^*$, but he didn’t found any $D\bar{D}$ signal, on the other hand, Zhang and Huang \cite{7} calculated in QCD sum rules many molecular states, among them, $D\bar{D}$, but it is not clear its isospin and G-parity $I^G$ in them sum rules. The $D\bar{D}$ state has already predicted by Gamermann et al. \cite{8} in 2007. They used the unitarization, in couple channels, of the chiral perturbation amplitudes and this state was called $X(3700)$ and it has $I=0$. In recent paper, Gamermann et al. \cite{9} have suggested that it is possible to observe $X(3700)$ in radiative decay, $\psi(3770)$ into $X(3700)+\gamma$.

In 1974, Walecka \cite{11} constructed an effective Lagrangian composed of a baryon field and two mediated mesons $\sigma$ and $\omega$ to describe nucleon dynamics within highly condensed objects such a nucleus or neutron stars. The central idea of this model is the mean field theory (MFT), whereby we replace the meson field operators with their expectation values. The resulting equation of state for the system where the number of protons is equal the number of neutrons, called nuclear matter, exhibits nuclear saturation, where the two coupling constants in this theory are matched with binding energy and density of nuclear matter. A prediction of this theory at zero temperature $T=0$ is the neutron matter is unbound. Although this theory is trustful for large number mass, these results can be interpreted as consequence of there is a only possible configuration for two body N-N bound state can be organized, the neutron-proton system or deuteron. Another prediction of this theory at $T=0$ is that the nuclear matter is a liquid and the increasing of temperature it has a liquid-gas phase transition \cite{11}. The critical point has been measured by several experiments \cite{12} and this value is $T=(7 \pm 1)$ MeV.

For the pion matter, this situation is opposite case that of nuclear matter. Shuryak \cite{13} predicted that this system at finite temperature is a liquid. Kostyuk et al. \cite{14} have been proposed that the equation of state of pion matter gives a phase transition gas-liquid at $T<136$ MeV. More recently, Anchishkin and Nazarenko \cite{15} used mean field theory and predicted that the temperature of the phase transition gas-liquid is $T=43$ MeV and the coupling constant was extracted for using the data of $\pi^+\pi^-$ invariant
mass spectrum.
In this work, we apply the Walecka’s mean field theory to study the $D - \bar{D}$ matter. We focus our attention on the existence of $D - \bar{D}$ bound state or X(3700). We consider that if this bound state exists, then this matter should have a phase transition gas-liquid.

II. THEORY

Recently, Ding [10] studied the meson-meson system, $Y(4260)$ and $Z_2^+(4250)$, by using a procedure for converting a $T$-matrix into an effective potential, where the strong interactions are generated by $\sigma$, $\omega$, $\rho$, $\pi$ meson exchange in the framework of the SU(4) chiral invariant effective Lagrangian. In this work, we use the Lagrangians derived by Ding [10] for the interactions $\sigma - D\bar{D}$ and $\omega - D\bar{D}$:

$$
\mathcal{L} = (\partial_\mu D)(\partial^\mu D) - m_D^2 D D^\dagger \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu + \frac{1}{2} (\partial_\mu \sigma)(\partial^\mu \sigma) - \frac{1}{2} m_\sigma^2 \sigma^2
$$

$$
+ g_{D_D\sigma} D D^\dagger \sigma + ig_{D_D\omega} \omega^\mu [D \partial_\mu D^\dagger - (\partial_\mu D) D^\dagger],
$$

where the field tensor is defined by $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $D$ field is a doublet ($D^0, D^+$), $m_D, m_\sigma$ and $m_\omega$ are respectively the masses of the $D$ meson, $\sigma$ meson and $\omega$ meson. The coupling constants of this theory are: $g_{D_D\sigma}$ and $g_{D_D\omega}$. Applying the Lorentz gauge $\partial_\mu \omega^\lambda = 0$, the equations of motion were obtained from Eq. (1) are:

$$
\partial_\mu \partial^\mu \sigma + m_\sigma^2 \sigma = g_{D_D\sigma} D D^\dagger,
$$

$$
\partial_\mu \partial^\mu \omega^\rho + m_\omega^2 \omega^\rho = -ig_{D_D\omega} [D \partial^\rho D^\dagger - (\partial^\rho D) D^\dagger],
$$

$$
\partial_\mu \partial^\mu D + m_{\text{eff}}^2 D = 0,
$$

where $\partial_\mu^\prime = \partial_\mu + ig_{D_D\omega} \omega^\mu$ and $m_{\text{eff}}^2 = m_D^2 - g_{D_D\sigma} \sigma + g_{D_D\omega}^2 \omega^2$.

The equations Eq. (2) and Eq. (3) are field equations with massive quanta and $D$ meson currents as source. The equation Eq. (4) is a Klein-Gordon equation for the $D$ field with the meson fields $\omega$ and $\sigma$ included in a minimal substitution.

This situation is analogous to Walecka’s study of nuclear matter [11], where Walecka’s baryon field is now represented by $D$ meson field.

Considering Walecka’s mean field theory [11] for a uniform and static system of $D$ mesons where the $\sigma$ and $\omega$ meson fields can be replaced by classical fields. Thus, the equations Eq. (2), Eq. (3) and Eq. (4) reduce to:

$$
\langle \sigma \rangle = \frac{g_{D_D\sigma}}{m_\sigma^2} \rho_\sigma,
$$

$$
\langle \omega^0 \rangle = \frac{g_{D_D\omega}}{m_\omega^2} \rho_\omega,
$$

$$
\partial_\mu \partial^\mu D + (m_D^2 - g_{D_D\sigma} \langle \sigma \rangle) D + 2ig_{D_D\omega} \langle \omega^0 \rangle \partial_\mu D = 0,
$$

where the scalar density is $\rho_\sigma = \langle D D^\dagger \rangle$ and the vector density is $\rho_\omega = \langle [\partial^\mu D D^\dagger - D (\partial^\mu D^\dagger)] \rangle$. These mean value is done in thermal state and integrate these densities in a box with volume $V$ and divide this result by $V$.

The solution for the $D$ field is given by:

$$
D(\vec{x}, t) = e^{-ig_{D_D\omega} \langle \omega^0 \rangle t} \int \frac{d^3 k}{2q^0(k)} \left[ a(q) e^{-i\vec{q} \cdot \vec{x}} (2\pi)^{3/2} + b^\dagger(q) e^{i\vec{q} \cdot \vec{x}} (2\pi)^{3/2} \right],
$$
where \( q_x = q^0_x - \vec{k} \vec{x} \) and \( q^0(\vec{k}) = \sqrt{\vec{k}^2 + m_{\text{eff}}^2} \). After the second quantization, the commutation relations between operators are given by:

\[
\begin{align*}
\left[ a(\vec{k}), a(\vec{k}') \right] &= \left[ b(\vec{k}), b(\vec{k}') \right] = \left[ a(\vec{k}), b^\dagger (\vec{k}') \right] = \left[ b(\vec{k}), a(\vec{k}') \right] = 0, \\
\left[ a(\vec{k}'), a^\dagger (\vec{k}) \right] &= \left[ b(\vec{k}'), b^\dagger (\vec{k}) \right] = 2q^0 (\vec{k}) \delta^3 (\vec{k} - \vec{k}').
\end{align*}
\]

The thermodynamic quantities are obtained by grand potential \( \Phi(T, V, \mu) \) at specified chemical potential, volume and temperature, is defined by:

\[
\Phi(T, V, \mu) = -T \ln \Xi,
\]

where,

\[
\Xi(T, V, \mu) = \text{Tr} \left[ e^{-(\hat{H} - \mu \hat{N})/T} \right].
\]

The \( \hat{H} \) is the Hamiltonian operator and \( \hat{N} \) is the number operator, are given by:

\[
\hat{N} = \int \frac{d^3k}{2q^0(\vec{k})} \left\{ a^\dagger (\vec{k}) a(\vec{k}) - b^\dagger (\vec{k}) b(\vec{k}) \right\},
\]

\[
\hat{H} = \frac{1}{2} \int d^3k \left[ a^\dagger (k) a(k) + b^\dagger (k) b(k) \right] + g_{D \bar{D} \omega} \langle \omega^0 \rangle \hat{N} + \left( \frac{m_\sigma^2 \langle \sigma \rangle^2}{2} - \frac{m_\omega^2 \langle \omega^0 \rangle^2}{2} \right) V.
\]

Inserting the operators \( \hat{H} \) and \( \hat{N} \) in equation Eq.(11), the grand potential is:

\[
\Phi(T, V, \mu) = T \frac{V}{2\pi^2} \int_0^\infty k^2 dk \left\{ \ln \left[ 1 - e^{-q^0(k) - \mu + g_{D \bar{D} \omega} \langle \omega^0 \rangle / T} \right] + \left[ 1 - e^{-q^0(k) + \mu - g_{D \bar{D} \omega} \langle \omega^0 \rangle / T} \right] \right\}
\]

\[
+ \left( \frac{m_\sigma^2 \langle \sigma \rangle^2}{2} - \frac{m_\omega^2 \langle \omega^0 \rangle^2}{2} \right) V.
\]

Finally, we get the mean value of the number operator

\[
\langle N \rangle = -\frac{\partial \Phi}{\partial \mu},
\]

match the equations Eq.(12) with the mean value of Eq.(12), we obtain

\[
\langle a^\dagger (\vec{k}) a(\vec{k}) \rangle = \frac{V}{4\pi^3} \frac{q^0(k)}{e^{q^0(k) - \mu + g_{D \bar{D} \omega} \langle \omega^0 \rangle / T} - 1},
\]

\[
\langle b^\dagger (\vec{k}) b(\vec{k}) \rangle = \frac{V}{4\pi^3} \frac{q^0(k)}{e^{q^0(k) + \mu - g_{D \bar{D} \omega} \langle \omega^0 \rangle / T} - 1}.
\]

In \( \langle N \rangle = 0 \) case, we get \( \mu = g_{D \bar{D} \omega} \langle \omega^0 \rangle \). Inserting the mean values given above in Eq.(11), we thus obtain the self consistent equation for sigma field:

\[
\langle \sigma \rangle = \frac{g_{D \bar{D} \omega}}{m_\sigma^2} \frac{1}{2\pi^2} \int_0^\infty \frac{k^2 dk}{q^0(k) [e^{q^0(k) / T} - 1]}.
\]

Inserting the equations Eq.(16) and Eq.(17) in Eq.(13), we get the equation for omega field:

\[
\langle \omega^0 \rangle = \frac{2g_{D \bar{D} \omega}^2}{m_\omega^2} \langle \omega^0 \rangle \rho_s,
\]

where the solution is given by

\[
\langle \omega^0 \rangle = 0.
\]
III. RESULTS

The parameters used are: $g_D\bar{D}\sigma = 2.85$ GeV \cite{10}, $m_D = 1.87$ GeV and $m_\sigma = 0.5$ GeV \cite{12}. We use a procedure \cite{15} to solve the self consistent equation Eq.(18) by a method to find the roots of the function $F(\langle \sigma \rangle, T)$,

$$F(\langle \sigma \rangle, T) = \langle \sigma \rangle - LHS(Eq.(18)).$$

Numerically, we get the behavior of sigma field with the temperature. Inserting the sigma field in an effective mass equation,

$$m_{eff} = \sqrt{m_D^2 - g_D\bar{D}\sigma \langle \sigma \rangle},$$

we get the mass of the D meson in hadronic medium Fig.(1). The effective mass reduces with increasing temperature and has a sudden drop at temperature $T \approx 1.2$ GeV. The minimum value of the ratio $m_{eff}/m$ is 0.08. This value is much smaller than the value 0.40 for the pion matter \cite{15}. Our result is similar in nucleon-antinucleon matter \cite{17}, but in our result, there is a maximum temperature around 1.4 GeV, where there isn’t any real sigma field. In recently paper, Kummar and Mishra \cite{18} predicted a different behavior of the D meson mass in nuclear medium. They get the mass grows with increasing temperature.

![FIG. 1: Effective mass as a function of temperature.](image)

We study the thermodynamics functions in three situations: $\sigma \neq 0$, $\sigma = 0$ and $m_D = \sigma = 0$. The Fig.(2) shows the pressure as function of temperature for the $\sigma \neq 0$ case. It exhibits a phase transition around $T \approx 1.2$ GeV. The Fig.(3) shows that the other two systems have a single phase, corresponding to free gas. The Fig.(4) shows the behavior of the energy density with temperature.

As if our system has the same number of D and anti-D mesons, we create the quantity energy per pair where we divide the total energy per the number of D mesons, $N_p$. The Fig.(5) shows the behavior for the energy per pair with temperature. For the $\sigma \neq 0$ case, the energy per pair has more lower values than the case $\sigma = 0$. This result is in agreement on the fact that the sigma field play an important role to create bound states.

These results are analogous to nucleon-antinucleon matter case \cite{17} and support the existence of phase transition for the $\sigma \neq 0$ case are an indication that this new phase is almost free zero mass D mesons Fig.(1) and not a liquid phase.
FIG. 2: Pressure as a function of temperature.

IV. CONCLUSIONS

These results indicate that our system is a gas. With increasing temperature has a phase transition at temperature $T \approx 1.2$ GeV. These new state is not a liquid and looks like a almost free zero mass D mesons gas. These results could be interpreted as an indication that the interaction between $D - \bar{D}$ is not so strong to form a liquid phase and it becomes difficult to understand the existence of the $D - \bar{D}$ molecules or $X(3700)$ in this theory.
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