Anytime Stochastic Task and Motion Policies

Naman Shah¹ and Siddharth Srivastava¹

Abstract

In order to solve complex, long-horizon tasks, intelligent robots need to carry out high level, abstract planning and reasoning in conjunction with motion planning. However, abstract models are typically lossy and plans or policies computed using them can be inexecutable. These problems are exacerbated in stochastic situations where the robot needs to reason about, and plan for multiple contingencies. We present a new approach for integrated task and motion planning in stochastic settings. In contrast to prior work in this direction, we show that our approach can effectively compute integrated task and motion policies whose branching structures encode agent behaviors that handle multiple execution-time contingencies. We prove that our algorithm is probabilistically complete and can compute feasible solution policies in an anytime fashion so that the probability of encountering an unresolved contingency decreases over time. Empirical results on a set of challenging problems show the utility and scope of our method.

Keywords

Stochastic task and motion planning, State and entity abstraction, Mobile manipulation, Hierarchical planning

1 Introduction

A long-standing goal in robotics is to develop robots that can operate autonomously in real-world environments and solve complex tasks such as cleaning or organizing a room. Although developments in sampling-based motion planning algorithms (Kavraki et al. 1996; Lavalle 1998; Sucan et al. 2012) have enabled robots to efficiently plan in configuration spaces that have infinite states and a large branching factor, solving real-world problems using such algorithms is infeasible given the requirement of reasoning over a long horizon to achieve complex goals. The problem becomes even more challenging when the robot’s actions and/or its environment are stochastic, as the agent has to not only deal with a long horizon, but also needs to have a contingent solution that deals with all possible situations that might arise while executing actions. Consider a household robot that is cleaning the floor with a vacuum cleaner. What should the robot do if some part of the floor is wet? What should the robot do if there are items on the floor?

A possible solution to this problem would be to compute a plan for the most likely scenario, and execute it until the agent reaches a state for which it has not yet planned an action. In such a case, the agent can replan from that state and compute a new plan that reaches the goal. While this approach may achieve the goal, it may not be safe in all the situations as on-the-fly replanning is prone to errors and may result in unwanted situations or dead ends. For e.g., a vacuum cleaner robot may end up in a water puddle and break itself if such a determinization (Yoon et al. 2007) based approach is employed.

A naïve approach to overcome such problems would be to first compute a symbolic high-level policy using an abstracted model of the domain defined using a symbolic language such as PPDDL (Younes and Littman 2004) or RDDL (Sanner 2010) and then refine each possible scenario in the policy by computing low-level motion plans for every action in the policy. This approach is naïve in the sense that it neither prioritizes more likely outcomes ahead of outcomes that are highly unlikely (but still possible) nor it is guaranteed that every action would admit a low-level motion plan as abstracted models are lossy and may lose important geometric information about the problem. Policies resulting from such approaches might not have any feasible motion planning refinements for some of their actions (Cambon et al. 2009; Kaelbling and Lozano-Pérez 2011a; Srivastava et al. 2014).

In this paper, we present a novel anytime approach for computing integrated task and motion policies. Our approach interleaves the computation of refinements with updating of abstractions to compute truly feasible task and motion policies. It continually improves the quality of the solution by ensuring that more likely situations are resolved earlier by the approach. It also provides a running estimate of

¹ Arizona State University, Tempe, AZ

Corresponding author:
Naman Shah, School of Computing, Informatics, and Decision Systems Engineering, Arizona State University, Tempe, AZ, 85281, USA.
the probability mass of likely executions covered in the current policy. This estimate can be used to start execution based on the level of risk acceptable in a given application, allowing one to trade-off precomputation time for on-the-fly invocation of our planner if an unhandled situation is encountered.

Our approach generalizes the methods of computing solutions for most likely outcomes during execution (Hadfield-Menell et al. 2015; Yoon et al. 2007) to the problem of integrated task and motion planning by using the anytime approaches in AI planning (Dean and Boddy 1988; Zilberstein and Russell 1993; Dean et al. 1995). The presented approach is the first probabilistically complete approach that uses sound abstractions to solve combined task and motion planning problems in stochastic environments. The framework works with arbitrary off-the-shelf symbolic solvers and motion planners. This allows it to scale automatically with improvements in either of these active areas of planning research and can also be used in deterministic settings as a special case.

The rest of the paper is structured as follows: Section 2 provides the background knowledge and discusses the related work, section 3 provides the formal framework and defines the stochastic task and motion planning problem, section 4 discusses our overall algorithm, and section 5 provides the empirical evaluation for our framework.

2 Background and Related Work

We now discuss some of the required concepts and related work. We start with the definition of classical planning and discuss a few of the many approaches that perform classical planning. Sections 2.2 and 2.3 discuss the stochastic variant of the planning problem and motion planning briefly. Lastly, section 2.4 discusses the recent work done related to the work presented in the paper.

2.1 Classical Planning

A classical planning problem is defined as,

Definition 1. A classical planning problem is defined as a 6-tuple \( P = (\mathcal{P}, S, \mathcal{A}, C, s_0, S_g) \) where,

- \( \mathcal{P} \) is a set of predicates.
- \( S \) is a set states of defined using the predicates from \( \mathcal{P} \).
- \( \mathcal{A} \) is a set of actions available to the agent.
- \( C : S \times \mathcal{A} \rightarrow \mathbb{R} \) is a cost function.
- \( s_0 \in S \) is an initial state.
- \( S_g \subseteq S \) is a set of goal states.

A solution to a task planning or a symbolic planning problem is a valid sequence of actions \( \langle a_0, \ldots, a_n \rangle \), such that when applied to the initial state, the resultant state \( s_n \in S_g \). Each operator \( a \in \mathcal{A} \) is defined using a tuple \( \langle \text{pre}_a, \text{eff}_a, \text{ff} \rangle \). An action \( a \) is executable in state \( s \) iff \( s \vdash \text{pre}_a \). \( \text{add}_a \) and \( \text{del}_a \) are sets of grounded predicates that are added and removed from the state when an action \( a \) is executed in any state respectively.

Many approaches have been proposed to perform classical planning efficiently. Bonet and Geffner (2001) introduce a way to synthesize domain-independent heuristics by relaxing the problem through ignoring predicates in the delete lists of all actions. The relaxed problem is easier to solve and the solution can be used to estimate heuristics for the states encountered in it. Planning graphs (Blum and Furst 1997) is another approach to relax the planning problem and automatically synthesize domain-independent heuristics such as \( h_{\text{add}} \) and \( h_{\text{max}} \). Hoffmann (2001) uses planning graphs to generate more tighter heuristic than \( h_{\text{add}} \) and \( h_{\text{max}} \) known as \( h_{\text{ff}} \) by avoiding re-counting actions that achieve similar predicates. Most classical planners use a relational language known as Planning Domain Definition Language (PDDL) (McDermott et al. 1998).

2.1.1 Hierarchical planning

Hierarchical planning uses abstractions to generate different hierarchies of relaxed planning problems to compute a solution for a complex planning problem. State abstraction generates hierarchies by removing certain predicates (in relational domains) or variables (in factored domains) from the domain description. ABSTRIPS (Sacerdoti 1974) is one of the earliest hierarchical planning approaches that assigns a rank to each literal using a predefined order and complexity of achieving that literal in the STRIPS planning process. Abstraction hierarchy is generated by dropping literals from the precondition of actions in the domain in the order specified by the rank of literals. The planning hierarchy generated using ABSTRIPS is common for all problems in the given domain and not catered to independent problems.

ALPINE (Knoblock 1990) uses ordered monotonicity property to overcome this issue by generating abstraction hierarchies tailored to each problem for the given domain. While this approach makes strong assumptions of ordered monotonicity and downward refinement, our approach does not require such critical assumptions. Seipp and Helmert (2013, 2018) use counter-example guided abstraction refinement to solve a complex planning problem hierarchically using cartesian abstraction - a variant of predicate abstraction. The proposed CEGAR-based approach starts with a naïve abstraction for the problem and computes an optimal plan for the abstract model. The approach tries to execute this plan in the original model. If it fails to execute the plan successfully, it computes a flaw and uses it to refine the abstract model. This approach requires a pre-image of each grounded operator and bounded branching factor for the search tree. Such approaches are not conducive to task and motion planning setups given that they rarely fulfill these requirements posed by such approaches.

Temporal abstractions generate high-level actions that are compositions of multiple low-level actions. Some hierarchical planning approaches employ temporal abstraction to create relaxed problems. Multiple approaches such as (Kambhampati et al. 1998; Bacchus and Kabanza 2000; Bercher et al. 2014) have used hierarchical task networks (HTNs) to compute plans efficiently for complex tasks. HTNs (Erol et al. 1995) use temporal abstractions to define tasks over primitive actions. The goal is to compute a final plan which is a composition of the high-level tasks that are achieved through the partial order planning of the
primitive actions. Marthi et al. (2007) compute hierarchical domain descriptions based on angelic semantics using temporal abstractions. They use a top-down forward search algorithm to refine the high-level actions into a sequence of primitive actions. While this approach and HTN-based approaches efficiently perform top-down planning using temporal abstraction, they fail to compute accurate plans in the models that do not fulfill downward refinement property. On the other hand, our approach handles such domains using an interleaved approach that refines actions while continually improving the abstract model.

2.2 Stochastic shortest path problems

We now discuss stochastic version of the planning problem named stochastic shortest path problem. (Bertsekas and Tsitsiklis 1991) defines the stochastic shortest path problems as follows:

**Definition 2.** A stochastic shortest path (SSP) problem is defined as 7-tuple \( P_s = (\mathcal{P}, \mathcal{S}, A, T, C, \gamma, H) \) where,

- \( \mathcal{P} \) is a set of predicates.
- \( \mathcal{S} \) is a set of states such that \( \forall s \in \mathcal{S}, s \subset \mathcal{P} \).
- \( A \) is a set of actions.
- \( T : S \times A \times S \to [0, 1] \) is a transition function that represents the probability of the agent being in state \( s' \) after executing an action \( a \) in state \( s \).
- \( C : S \times A \to \mathbb{R} \) is a cost function.
- \( \gamma = 1 \) is the discount factor.
- \( H \) is the horizon.

A solution to an SSP is a policy \( \pi \) of the form \( \pi : S \times \{1, \ldots, H\} \to A \) that maps all the states and time steps at which they are encountered to an action. The optimal policy \( \pi^* \) is a policy that reaches the goal state with the least expected cumulative cost. Due to finite horizon, SSP policies need not be stationary.

Solving SSPs

Dynamic programming algorithms such as value iteration and policy iteration can be used to compute these policies. They require computing optimal cost action for each state in the state space to converge. Real-time dynamic programming (RTDP) (Barto et al. 1993) generalizes Korf’s Learning-Real-Time-A* algorithm to a trial based dynamic programming method that ignores a large part of the state space by expanding states encountered in trials to solve SSPs faster. LAO* (Hansen and Zilberstein 2001) uses heuristics to expand the partial policy tree along with local value iteration to compute solutions. Labeled RTDP (Bonet and Geffner 2003) extends RTDP by labeling states that have converged greedy policy. Muise et al. (2012) use state relevance to guide the search to reduce the time to compute the policy. Abdelhadi and Cherki (2019) provide a method that decomposes an SSP into multiple smaller SSPs and combines the solution to handle dead ends.

While task planning efficiently computes solutions for complex goals, it can not handle manipulation problems with continuous domains that has an infinite branching factor. Though PDDL 2.1 (Fox and Long 2003) allows using continuous variables, it still struggles to handle infinite branching factor.

2.3 Motion Planning

**Definition 3.** A motion planning problem is a 4-tuple \( \langle C, f, p_0, p_t \rangle \) where,

- \( C \) is the space of all possible configurations or poses of a robot (a.k.a configuration space or C-space).
- \( p_0 \) is the initial configuration of the robot.
- \( p_t \) is the target configuration of the robot.
- \( f : C \to \{0, 1\} \) determines whether a pose \( p \in C \) is in a collision or not.

A trajectory is a sequence of poses. The solution to a motion planning problem is a trajectory in \( C \) from \( p_0 \) to \( p_t \) such that \( f \) is false for every pose in the trajectory.

Recent years have shown significant and groundbreaking improvements in sampling-based motion planners. Probabilistic roadmaps (PRM) (Kavraki et al. 1996) randomly samples from the C-space to generate a roadmap that can be lazily used to generate motion plans. Rapidly-exploring random trees (RRT) (LaValle 1998) computes a collision-free path from an initial robot configuration to the target configuration by connecting randomly sampled robot configurations from the C-space. Bi-directional RRT (BiRRT) (Kuffner and LaValle 2000) updates existing RRT to initiate search trees from the initial and goal configurations to boost the speed of motion planning. Constrained BiRRT (CBiRRT) (Berenson et al. 2009) extends the BiRRT technique constraining the search space by using projection techniques to explore configurations spaces and finds bridges between them.

While motion planning efficiently computes plans in high-dimensional continuous spaces, it lacks the capabilities to reason over a long horizon and to compute plans for complex tasks such as arranging a dining table. Our approach does not alter existing motion planning techniques and works with any off-the-shelf motion planner.

2.4 Integrated Task and Motion Planning

Most of the prior work in the field of integrated task and motion planning has focused on solving deterministic task and motion planning problems. Most of these approaches can be classified into following three categories: 1) approaches that use symbols to guide the motion planning, 2) approaches that extend high-level representations to simultaneously search high-level plans along with continuous parameters, and 3) approaches that use interleaved search for valid high-level plans with low-level refinements for its actions. Garrett et al. (2021) present an exhaustive survey of these approaches; we discuss here only those that are most closely related with our approach. The approach presented in this paper falls under the last category.
Approaches that use symbols to guide the motion planning

Cambon et al. (2009) introduce one of the earliest approaches named aSyMov that uses symbolic knowledge to guide planning in geometric space using location references. Plaku and Hager (2010) use a similar approach to allow combined task and motion planning for robots with constrained manipulators. Such approaches employ task planning as a heuristic for planning in the C-space which may not always be efficient due to lack of knowledge of geometric constraints at the task-planning level. Our approach interleaves the process of computing motion plans and updating the high-level specification to incorporate geometrical knowledge to enable the task planner to compute sound plans.

Approaches that extend high-level representation

Another class of approaches (Hertle et al. 2012; Garrett et al. 2015, 2020) extends the high-level representation to allow the high-level planner to validate preconditions of the high-level actions in the geometric space while computing the high-level plan. Hertle et al. (2012) do so by developing semantic attachments for PDDL representation that check the validity of each high-level action using a motion planner in the low level. FF Rob (Garrett et al. 2015) uses a special representation that uses pre-sampled robot configurations to discretize the problem and build a roadmap to evaluate the preconditions of the high-level action. PDDLStream (Garrett et al. 2020) implements the notion of streams in PDDL that are used to sample continuous parameters for the high-level actions. While these approaches require a carefully crafted high-level domain description including an explicit specification of the geometric constraints for all high-level actions, our approach does not require such explicit constraint specification for each action. These approaches are computationally expensive compared to our approach as the motion planner is invoked much more frequently as part of high-level planning.

Approaches that perform an interleaved search

The last group of approaches perform an interleaved search to find a high-level solution that also has valid motion planning refinements in the low-level. These approaches incrementally update the high-level models using the feedback from the low-level while searching for the refinements. Srivastava et al. (2014) implement a modular approach that uses a planner-independent interface layer to allow communication between a task planner and a motion planner. The interface layer is used to compute refinements for high-level actions as well as update high-level models with the feedback received from the low-level environment while computing these refinements. Dantam et al. (2018) develop a constraint-based approach that incrementally adds constraints to the high-level specification of the problem discovered while trying to refine a high-level plan generated using an SMT-based planner. Because these approaches commit to a single high-level model, they may run into dead ends and fail to compute solution even if one exists. Our approach maintains multiple abstract models and a defined strategy to explore them. This provides more thorough guarantees of probabilistic completeness.

The only approaches known to be designed to handle stochastic task and motion planning problems are presented by Kaelbling and Lozano-Pérez (2011b) and Hadfield-Menell et al. (2015). These approaches consider a partially observable formulation of the problem. Kaelbling and Lozano-Pérez (2011b) utilize regression modules on belief fluents to develop a regression-based solution algorithm.

3 Formal Framework

3.1 Preliminaries

We use the concepts of abstraction to model the robot manipulation problem as a symbolic planning problem. The goal is to compute a solution for the obtained high-level symbolic problem with “refinements” that select a specific motion planning problem and its solution in the concrete space for each action in the high-level solution. In this view, each high-level action corresponds to infinite low-level problems in the concrete space, each defined by a specific initial and target configuration of the robot. For example, a high-level action of placing a cup on a table corresponds to infinite motion planning problems, each defined by a different location of the cup on the table. The refinement process would require selecting one of these problems and computes a valid motion planning solution for it.

Before defining the abstraction used in this work, we define some required terminology. Let $\tau_R$ be the type of variables that represent vectors of continuous real values and $\tau_0$ be the type of variables that represent names of the objects in the environment and symbolic references for the variables of type $\tau_R$. Let $U$ be the universe consisting object names of type $\tau_O$, continuous vectors of type $\tau_R$, and symbolic references for these vectors, $P$ be a set of predicates, $C$ be a set of constants. $V = P \cup C$ defines the vocabulary. In this
work, we consider two kinds of predicates: symbolic and hybrid. We define each of them as follows:

**Definition 4.** A predicate \( p_{sym}(y_1, \ldots, y_k) \) is a **symbolic predicate** iff its arguments \( y_1, \ldots, y_k \) are of type \( \tau_o \). \( P_{sym} \) is the set of all such symbolic predicates.

**Definition 5.** A predicate \( p_h(y_1, \ldots, y_k, \theta_1, \ldots, \theta_m) \) is a **hybrid predicate** iff its arguments \( y_1, \ldots, y_k \) are of type \( \tau_o \) and \( \theta_1, \ldots, \theta_m \) are of type \( \tau_r \). \( P_h \) is the set of all such hybrid predicates.

States are logical structures or models defined over predicates. A *structure* \( S \), of vocabulary \( V \) where \( P = P_{sym} \cup P_h \subset V \), consists of a universe \( U \), a predicate \( p^S \) over \( U \) for every predicate \( p \in V \), and an element \( \epsilon^S \) over \( U \) for every constant symbol \( c \in V \). An interpretation of a predicate \( p \in P \) provides a relation between objects in the universe \( U \). E.g., an interpretation, for a hybrid predicate \( at(o_1, loc) \), if true, specifies a relation between \( o_1 \) and \( loc \) representing that the object \( o_1 \) is at the location \( loc \), where \( o_1 \in U \) is the name of an object in the environment and \( loc \in \mathbb{R}^8 \) represents the pose of the object in the environment. Similarly, an interpretation, for a symbolic predicate \( on(o_1, o_2, loc) \), if true, specifies a relation between objects \( o_1 \) and \( o_2 \) representing that the object \( o_1 \) is on the object \( o_2 \), where \( o_1, o_2 \in U \) are the names of objects in the environment. Moving ahead, we use \( [p]^S \) and \( [\psi]^S \) to denote interpretations of the predicate \( p \) and a formula \( \psi \) in \( S \) respectively.

We use notion of actions in PPDDL ( McDermott et al. 1998 ) to represent the actions available to the robot. We classify actions available to the robot as symbolic and hybrid actions depending on the types of predicates that appear in the actions’ descriptions and their arguments. While both symbolic and hybrid actions use predicates from \( P_{sym} \) and \( P_h \) to specify their preconditions, symbolic actions use predicates only from \( P_{sym} \) to specify their effects, and hybrid actions may use predicates from \( P_{sym} \) and \( P_h \) to do so. E.g., a symbolic action \( \text{TurnOn(light)} \) is executed by the robot by using a wireless transmitter to turn on a light where the argument \( light \) is the name of an object in the environment and \( \text{IsTurnedOn(light)} \) is a predicate from \( P_{sym} \) that appears in the effect of the action. A concrete hybrid action \( \text{Place(obj, loc, traj)} \) is executed by the robot by placing an object \( obj \) at a certain location \( loc \) using the trajectory \( traj \), where \( obj \) is a name of the object in the environment while \( pose \) and \( traj \) are vectors or real values representing target pose of the object and trajectory to be used by the robot to execute the action. \( \text{holding(obj)} \) and \( at(obj, pose) \) are symbolic and concrete predicates respectively which appear in the effect of the action.

Let \( V_l \) be a low-level vocabulary and \( V_h \) be a high-level vocabulary such that \( V_h \subset V_l \); the predicates in \( V_h \) are defined as identical to their counterparts in \( V_l \). We define relational abstractions as first-order queries that map structures over one vocabulary to structures over another vocabulary. A first-order query \( \alpha \) from \( V_l \) defines functions in \( V_h \)(also identified as \( \alpha(S_l) \)) using the \( V_l \)-formulas in \( S_l \): \( [\alpha]^S_l \) \( (a_1, o_2, \ldots, o_n) \) = \( True \) iff \( [\psi^S_1(a_1, o_2, \ldots, o_n)]^S_l \) = \( True \), where \( \psi^S_1 \) is a formula over \( V_l \). Such abstractions reduce the number of properties being modeled keeping number of objects the same.

3.2 Entity Abstraction

Let \( U_h (U_h) \) be the universe of \( V_l (V_h) \) such that \( |U_h| \leq |U_l| \). Let \( \rho : U_h \rightarrow 2^{U_h} \) be a collection function that maps elements in \( U_h \) to the collection of \( U_h \) elements that they represent, e.g., \( \rho(Kitchen) = \{loc : \lambda loc \cdot \text{BoundaryVector} < 0\} \) when kitchen has a polygonal boundary.

We define an entity abstraction \( \alpha_{p} \) using the collection function \( \rho \) as \( [\rho]^S_h \) \( (a_1, \ldots, a_n) \) = \( True \) iff \( \exists \alpha_1, \ldots, \alpha_n \) such that \( \alpha_i \in \rho(a_i) \) and \( [\psi^S_1(a_1, \ldots, a_n)]^S_h \) = \( True \). We omit the subscript \( \rho \) when it is clear from the context.

Entity abstractions define the truth values of predicates over abstracted entities as disjunction of the corresponding concrete predicate instantiations. E.g., an object is in the abstract region “kitchen” if it is at one of the any locations in that region and an object is on “table” if it is at any location on the table top. Such abstractions have been used for efficient generalized planning Srivastava et al. (2008) as well as answer set programming Saribatur et al. (2019). These type of abstractions instantiate terms that may not be identifiable at high level. E.g., the exact location of the table, the trajectory used to reach a configuration from current configuration.

We define an abstract hybrid predicate for each hybrid predicate in our vocabulary by replacing each continuous argument in the hybrid predicate with its symbolic reference. E.g., \( [at(o_1, loc)]^S_h \) is an abstract hybrid predicate corresponding to a hybrid predicate \( at(o_1, loc) \) where, \( loc \in U \) is a symbolic reference of type \( \tau_o \) for the continuous vector \( loc \).

To formally define an abstract hybrid predicate, let \( \alpha \) be a composition of entity abstraction and function abstraction. The abstract version of a concrete predicate \( p_h \) is denoted as \( [p_h]_{\alpha} \). We omit the subscript \( \alpha \) when it is clear from the context. We define \( [p_h]_{\alpha} \) as follows:

**Definition 6.** A predicate \( [p_h]_{\alpha} \) \( (y_1, \ldots, y_k, \theta_1, \ldots, \theta_m) \) is an **abstract hybrid predicate** corresponding to a concrete hybrid predicate \( p_h(y_1, \ldots, y_k, \theta_1, \ldots, \theta_m) \) iff all of its arguments \( y_1, \ldots, y_k, \theta_1, \ldots, \theta_m \) are variables of type \( \tau_o \) and \( \forall \theta_i \in \arg([p_h]_{\alpha}) \theta_i \in \rho(\overline{\theta}) \). \( [P_h]_{\alpha} \) is a set of all abstract hybrid predicates.

Let \( S \) be the set of abstract states generated when an abstraction function \( \alpha \) is applied on a set of concrete states \( X \). For any \( s \in S \), the concretization function \( \Gamma_{\alpha}(s) = \{x \in X : \alpha(x) = s\} \) denotes the set of concrete states represented by the abstract state \( s \). For a set \( C \subseteq X \), \( [C]_{\alpha} \) denotes the smallest set of abstract states representing \( C \). Generating the complete concretization of an abstract state can be computationally intractable, especially in cases where the concrete state space is continuous. In such situations, the concretization operation can be implemented as a generator that incrementally samples elements from an abstract argument’s concrete domain.

We define an abstract hybrid action for each hybrid action in the vocabulary using the abstraction function \( \alpha \). The abstraction function \( \alpha \) replaces each argument of type \( \tau_R \) with its symbolic reference of type \( \tau_O \) in the action’s arguments, and each concrete hybrid predicate in its description with its abstract counterpart.
Place(obj₁, config₁, config₂, target_pose, traj₁)  
precon RobotAt(config₁), holding(obj₁),  
IsValidMP(traj₁, config₁, config₂),  
IsCollisionFree(traj₁),  
IsPlacementConfig(obj₁, config₂, target_pose)

Concrete effect  
¬holding(obj₁), ∀ traj intersects(vol(obj, target_pose)),  
sweptVol(robot, traj) → Collision(obj₁, traj),  
RobotAt(config₁), at(obj₁, target_pose)

Abstract effect  
¬holding(obj₁), ∀ traj ⊇ Collision(obj₁, traj₁),  
¬RobotAt(config₁), RobotAt(config₂), at(obj₁, target_pose)

Figure 2. Specification of concrete (above) and abstract (below) effects of a one-handed robot’s action for placing an object

Example Consider the specification of a robot’s action of placing an item as a part of an SSP. In practice, low-level accurate models of such actions may be expressed as generative models or simulators. Figure 2 helps identify the nature of abstract representations needed for expressing such actions. For readability, we use a convention where preconditions are comma-separated conjunctive lists and universal quantifiers represent conjunctions over the quantified variables.

Figure 2 shows the specification of an action that places an object at the specified pose. Concrete description of the action requires action arguments representing object to be placed (obj₁), the initial and final configuration of the robot (config₁, config₂), target pose for the object (target_pose), and the motion trajectory that takes the robot from its initial configuration to final configuration (traj₁). The abstract counterpart is computed by replacing the continuous arguments in the concrete version with symbolic arguments representing regions as mentioned earlier. E.g., target_pose is a symbolic placeholder for all valid target poses for the object, traj₁ is a placeholder for all plans that take the robot from config₁ to config₂. Values of these arguments can not be determined precisely in the abstracted space; their values are assigned by the planning algorithm. E.g., it is not possible to determine, in the abstract model, what trajectories will be in a collision when an object is placed at a certain pose. Such predicates are annotated in the set of effects with the symbol ⊇. This results in a sound abstract model (Srivastava et al. 2014, 2016).

3.3 Problem Statement

The robot may require to change its pose as part of executing some actions which which indeed requires it to have an explicit motion plan. E.g., to execute the action Place(obj₁, pd_pose, traj₁), the robot must have a valid trajectory that changes robot’s pose to pd_pose in order to place the object obj₁. We define such actions as motion planning actions. Action arguments for such motion planning actions specify trajectories required to execute these actions and preconditions can be used to specify constraints on these motion planning trajectories. Values for these motion planning arguments can be “sampled” using a motion planner. E.g, the action place (Fig. 2) contains a motion planning argument traj₁ and its precondition specifies a constraint that it should be a valid collision-free trajectory (IsCollisionFree(traj₁)). We formally define motion planning actions as follows:

Definition 7. A motion planning action \( a_{mp}(a₁, ..., o_k, \theta₁, ..., \theta_j, t₁, ..., t_n) \) is a hybrid action where \( o₁, ..., o_k \) are of type \( \tau_o \), \( \theta₁, ..., \theta_j \) are of type \( \tau_R \) and \( t₁, ..., t_n \) are motion planning trajectories. pre\((a_{mp})\) contains constraints on \( t₁, ..., t_n \) and eff\((a_{mp})\) represents the effective pose of the robot after executing action \( a_{mp} \). \( A_{mp} \subset A_h \) is the set of all motion planning actions.

We use these components to define concrete and abstract planning problems as follows:

Definition 8. A concrete planning problem \( P \) is defined as a 6-tuple \( M = \langle O^M, P^M, X^M, A^M, T^M, C, x₀, X_g, γ, H \rangle \), where,

- \( O^M \) is a set of names for the objects in the environment,
- \( P^M = P_{sym} \cup P_h \) is a set of predicates,
- \( X^M \) is a set of states defined using predicates in \( P^M \),
- \( A^M = A_{sym} \cup A_h \) is a set of actions available to the robot, where \( A_{mp} \subset A_h \) is a set of motion planning actions,
- \( T : X × A × X \rightarrow [0, 1] \) is a transition function,
- \( C : X × A \rightarrow \mathbb{R} \) is a cost function,
- \( x₀ ∈ X^M \) is the initial state,
- \( X_g ⊂ X^M \) is the set of goal or terminal states,
- \( γ = 1 \) is the discount factor,
- \( H \) is the horizon.

For ease of reading, we omit the superscript when it is clear from the context. The solution to a concrete planning problem is a valid sequence of actions \( π = ⟨a₀, ..., a_n⟩ \) such that every action, when applied sequentially from the initial state \( x₀ \), the system reaches one of the goal states in \( X_g \).

Definition 9. Given a concrete planning problem \( M \), an abstract planning problem \( [M] = \langle O, [P], [X], [A], T, C [x₀], [X_g], γ, H \rangle \), where,

- \( O \) is a set of names for the objects in the environment and symbolic references for entities in the environment,
- \( [P] = P_{sym} \cup [P_h] \) is a set of abstract predicates,
- \( [X] \) is a set of abstract states,
- \( [A] = A_{sym} \cup [A_h] \) is a set of abstract actions available to the robot,
- \( T : [X] × [A] × [X] \rightarrow [0, 1] \) is a transition function,
- \( C : [X] × [A] \rightarrow \mathbb{R} \) is a cost function,
4 Computing Task and Motion Policies

4.1 HPlan Algorithm

We extend the idea of planning with abstraction briefly discussed by (Srivastava et al. 2016) to perform task and motion planning using abstraction hierarchies. The goal is to find a valid “high-level” policy that has valid “low-level” refinements for each of its actions. We propose HPlan algorithm (Alg. 1) that performs hierarchical planning with arbitrary abstraction and concretization function.

Our approach uses a plan refinement graph (PRG) to keep track of different abstract models and their corresponding policies. As shown in Figure 3, each node $u$ in the PRG contains an abstract model $[M]_u$, an abstract policy $[\pi]_u$, and the current state of refinement for each action $[\alpha]_u \in [\pi]_u$. An edge $(u, v)$ in the PRG from the node $u$ to the node $v$ consists of a partial refinement $\sigma_u$ and a failed precondition of the first action from $[\pi]_u$ that does not have a valid refinement.

Our approach combines two processes: 1) Concretizing the abstract policy, and 2) Refining the abstract model.

HPlan algorithm (Algorithm 1) performs the above-mentioned two steps in an interleaved manner. The algorithm starts with a single node in the PRG with the initial provided abstract model. Line 1 initializes the PRG with a node containing the initially provided abstract model $[M]_0$, and an abstract policy $[\pi]_0$ that achieves the goal $G$ computed using an off-the-shelf symbolic solver. Each iteration of the main loop (line 2) selects a node $u$ from the PRG using a defined strategy and extracts a root-to-leaf (RTL) path from

\[
\begin{align*}
\text{Algorithm 1: HPlan Algorithm} \\
\text{Input: model } M, \text{ abstraction function } \alpha, \\
\text{concretization function } \gamma, \text{ abstract model } \ [M]_0, \text{ symbolic planner } P \\
\text{Output: anytime, contingent policy that is executable in } M \\
1 \text{ Initialize PRG with a node with an abstract policy } [\pi]_0 \text{ for } G \text{ computed using } P; \\
2 \text{ while solution of desired quality not found do } \\
3 \quad u \leftarrow \text{GetPRNode}(); \\
4 \quad [M]_u \leftarrow \text{GetAbstractModel}(u); \\
5 \quad [\pi]_u \leftarrow \text{GetAbstractPolicy}([M]_u, G, P, u); \\
6 \quad \text{Choice} \leftarrow \text{NDChoice}\{\text{Concretization,} \\
7 \quad \text{RefineAbstraction}\}; \\
8 \quad \text{if Choice = Concretization then } \\
9 \quad \quad \text{while } [\pi]_u \text{ has an unrefined RTL path and resource limit is not reached do} \\
10 \quad \quad \quad \text{path} \leftarrow \text{GetUnrefinedRTLPath}([\pi]_u); \\
11 \quad \quad \quad \text{if explore // non-deterministic} \\
12 \quad \quad \quad \quad \text{then replace a suffix of refined partial path} \\
13 \quad \quad \quad \quad \quad \text{with a random action;} \\
14 \quad \quad \quad \quad \text{Search for a feasible concretization of path; } \\
15 \quad \quad \text{if Choice = RefineAbstraction then } \\
16 \quad \quad \quad \text{path} \leftarrow \text{GetUnrefinedRTLPath}([\pi]_u); \\
17 \quad \quad \quad \sigma \leftarrow \text{ConcretizeFirstUnrefinedAction(path)}; \\
18 \quad \quad \quad \text{failure_reason} \leftarrow \text{GetFailedPrecondition}(\sigma); \\
19 \quad \quad \quad [M'] \leftarrow \text{UpdateAbstractPolicy}(M, \sigma, \text{failure_reason}); \\
20 \quad \quad \quad [\pi'] \leftarrow \text{merge}([\pi], \text{GetAbstractPolicy}(M', G, solver)); \\
21 \quad \quad \quad \text{generate new_pr_node}([\pi'], [M']); \\
\end{align*}
\]
current PRG node’s policy $[\pi]_u$ such that the path has at least one action that has not been instantiated (line 3-5). Arbitrary strategies can be used to select a node from the PRG and the RTL path at each iteration. Each iteration of the main loop non-deterministically selects one of the two above-mentioned modes (line 7). The algorithm then carries out the interleaved search in the following manner:

a) Concretizing the abstract policy

Lines 8-13 search for valid concretization (refinement) of the partial path selected on line 6 by concretizing the abstracted actions with actions from the concrete domain $\mathcal{M}$ using the concretization function $\gamma$. Every abstracted entity in each action is concretized using a local backtracking search (line 13). A concretization $c_0, a_1, c_1, \ldots, a_k, c_k$ is a valid concretization of the path $[s_0], [a_1], [s_1], \ldots, [a_k], [s_k]$ is valid if $c_{i+1} \in a_{i+1}(c_i)$ and $c_i = precon(a_i+1)$ for $i = 0, \ldots, k-1$. Due to the lossy nature of the abstraction, it may be possible that no valid concretization exists for the policy $[\pi]_u$. For example, consider an abstraction which drops $\text{InCollision}$ predicate that checks whether a trajectory is in collision with some object or not from an action that places an object at a desired pose. Such high-level actions would not have any valid concretization if all the trajectories are being obstructed by some object in the low level.

b) Refining the abstract model

Lines 15-20 fix a concretization for the partially refined path selected on line 6 and identify the earliest abstract state in the selected path whose subsequent action’s concretization is infeasible. The abstract model is refined by adding the true form of the violated precondition. Continuing the same example, if all the trajectories from the current state to the state that has the object at the desired pose are in collision with some other object $obj_x$, the concrete precondition $\text{InCollision}(\text{traj}_x, obj_x)$ is violated at the concrete level and is added to the current abstract model. The rest of the policy after this abstract state is discarded. Lines 19-20 use the new model to compute a new policy. The symbolic planner is invoked to compute a new policy from the updated state; its solution policy is unrolled as a tree of bounded depth and appended to the partially refined path. This allows the time horizon of the policy to be increased dynamically.

**Theorem 1.** If there exists a proper policy that reaches the goal within horizon $h$, i.e. the probability of reaching the goal is 1.0, and has feasible low-level concretization, then Alg. 1 will find it with probability 1.0 in the limit of infinite samples.

**Proof.** (Sketch) Let $\pi_p$ be a proper policy. Consider a policy $\pi$ in the PRG; let $k$ denote the minimum depth up to which $\pi_p$ and $\pi$ match. $k$ will be used as a measure of correctness. When $\pi$’s PRG node is selected, suppose we try to refine one of the child nodes of depth $k + 1$ in the partial path that had the $k$-length prefix consistent with the solution.

The algorithm selects the correct child action with non-zero probability under the explore steps (line 11) and then generates a plan to reach the goal from the resultant state. The finite number of discrete actions and the fixed horizon ensures that at time bounded in expectation, $\text{HPlan}$ will generate a policy with the measure of correctness $k + 1$. Once the algorithm finds the policy with the measure of correctness $h$, it stores it in the PRG and is guaranteed to find feasible refinements with probability one if the measure of these refinements under the probability-density of the generators is non-zero.

4.2 HPlan with entity abstraction for STAMP

We enhance the basic Alg. 1 in two primary directions to facilitate $\text{STAMP}$ problems. The optimizations allow Alg. 1 to compute anytime policies and improve the search of concretization of abstract policies.

**Search for concretization**

Sample-based backtracking search performed by Alg. 1 (line 13) to concretize the abstract actions suffers from a few limitations in stochastic settings that are not present in the deterministic settings. Fig. 4 illustrates the problem. The grey nodes in the image show the actions which are concretized. White nodes are yet to be concretized. Sibling nodes represent the non-deterministic action outcomes. If $B$ does not accept any valid concretization, backtracking to $A$ and changing its concretization would invalidate concretizations for the entire subtree rooted at $A$. Algorithm 1 handles such scenarios by non-deterministically selecting whether to perform backtracking searching or not (line 7) and by maintaining different abstract models through PRG and employing a resource limit (line 8) to explore them simultaneously.

**Anytime computation for task and motion policies**

The main computational challenge for the Alg. 1 in stochastic settings is that the number of root-to-leaf (RTL) branches grow exponentially with the time horizon and the contingencies in the domain. Each RTL branch has a certain probability of being encountered; refining it incurs a computational cost. Waiting for a complete refinement of the policy tree results in wasting a lot of time as most of the situations have a very low probability of being encountered. The optimal selection of the paths to refine within a fixed computational budget can be reduced to the knapsack problem. Unfortunately, we do not know the precise computational costs required to refine a path. However, we can approximate this cost depending on the number of actions and the size of the domain of the arguments in those actions. Furthermore, the knapsack problem is NP-hard. However, we can compute provably good approximate solutions to this problem using a greedy approach: we prioritize the selection of a path to refine based on the probability of encountering that path $p$ and the estimated cost of refining that path $c$. We compute $p/c$ ratio.
for all the paths and select the unrefined path with the largest ratio for refinement (line 9 and 15). \( p/c \) ratio for each path is updated after each iteration of the main loop (line 23).

**Theorem 2.** Let \( t \) be the time since the start of the algorithm at which the refinement of any RTL path is completed. If path costs are accurate and constant then the total probability of unrefined paths at time \( t \) is at most \( \frac{1 - \text{opt}(t)/2}{\text{opt}(t)/2} \), where \( \text{opt}(t) \) is the best possible refinement (in terms of the probability of outcomes covered) that could have been achieved in time \( t \).

**Proof.** (Sketch) The proof follows from the fact that the greedy algorithm achieves a 2-approximation for the knapsack problem. In practice, we estimate the cost as \( c \hat{c} \), the product of measures of the true domains of each the symbolic argument in the given RTL. Since, \( \hat{c} \geq c \) modulo constant factors, the priority queue never can only underestimate the the relative value of refining a path, and the algorithm’s coverage of high-probability contingencies will be closer to optimal than the bound suggested in the theorem above. This optimization gives a user the option of starting execution when a desired value of the probability of covered contingencies has been reached.

## 5 Empirical Evaluation

We use a total of six domains with varying configurations to evaluate our approach. Out of these six domains, only a single domain had only deterministic actions, while the rest of the domains had a mix of deterministic and stochastic actions. We use FF (Hoffmann 2001) as a high-level classical planner for deterministic settings and implementation of LAO* (Hansen and Zilberstein 2001) from the MDP-Lib (Pineda 2014) repository for stochastic settings. We use the OpenRAVE (Dankov 2010) robot simulation system with its collision checkers to represent 3D environments and CBiRT (Berenson et al. 2009) implementation from the PrPy (Koval 2015) suite for computing motion plans. In practice, fixing the horizon \( H \) for the SSP solver a priori is infeasible and renders some problems unsolvable. Instead, we implemented a variant that dynamically increases the horizon until the goal is reached with a probability \( p > 0 \). The source code of the framework along with the videos of our experiments can be found at [https://aair-lab.github.io/stamp.html](https://aair-lab.github.io/stamp.html)

Lagriouf et al. (2018) propose several framework-independent benchmark domains for task and motion planning systems. While these benchmarks are proposed for deterministic TAMP systems, characteristics of the domains can still be used to evaluate TAMP systems. Fig. 7 shows the criteria fulfilled by every domain used to evaluate our approach. We include the average number of branches in the policy tree as an additional criterion to depict the complexity of stochastic problems.

**Problem 1: Cluttered table**

In this problem, we have a table cluttered with cans, each having different probabilities of being crushed when grabbed by the robot. Some cans are delicate and are highly likely to be crushed when the robot grabs them, incurring a high cost (probability for crushing was set to 0.1, 0.5 & 0.9 in different experiments in Fig. 9(a)), while others are normal cans that cannot be crushed. Delicate cans are always crushed when grasped in the deterministic variant. The goal for the robot is to pick up a specified can. We used different numbers of cans (15, 20, 25) and different random configurations of cans to extensively evaluate the proposed framework. We also used this scenario to evaluate our approach in the real-world (Fig. 5) using the Fetch robot (Wis et al. 2016).

**Problem 2: Aircraft inspection**

In this problem, an unmanned aerial vehicle (UAV) is employed to inspect possibly faulty parts of an aircraft in an airplane hangar. The goal for the agent is to locate the fault and notify the human supervisor about it. Fig. 6 shows the simulated environment. The UAV’s sensors are inaccurate and may fail to locate the fault with some non-zero probability (failure probability was set to 0.05, 0.1, & 0.15 for experiments in Fig. 9(b)) while inspecting the location; it may also drift to another location while flying from one location to another or while inspecting the parts. The UAV has a limited amount of battery charge. A charging station is available for the UAV to dock and charge itself. All movements use some amount of battery charge depending on the length of the trajectory, but the high-level planner cannot determine whether the current level of charge is sufficient for the action or not as it lacks the details such as current battery level, length of previous and next trajectories, etc. This makes it necessary to have an interleaved approach that searches for a high-level policy that has valid low-level refinements.

**Problem 3: Building structures with keva planks**

In this problem, the YuMi robot (ABB 2015) is used to build different structures using Keva planks. Keva planks are laser-cut wooden planks with uniform geometry. Fig. 5 and Fig. 1 show the target structures. Planks are placed one at a time by a user after each pickup and placement by the YuMi. Each new plank may be placed at one of a few predefined locations, which adds uncertainty in the planks’ initial location. For our experiments, two predefined locations were used to place the planks with a probability of 0.8 for the first location and a probability of 0.2 for the second location. In this problem, hand-written goal conditions are used to specify the desired target structure. The YuMi needs a task and motion policy for successively picking up and placing planks to build the structure. There are infinitely many configurations in which one plank can be placed on another, but the abstract model blurs out different regions on the plank. The generator that samples put-down poses for planks on the table uses the target structure to concretize each plank’s target put-down pose. The number of branches in a solution tree grows exponentially with the number of planks in the structure and can quickly become huge. For example, a solution tree for a structure with just 10 planks would have a total of 1024 branches. Our observation shows that even the state-of-the-art SSP solvers fail to compute high-level solution policies for structures that have greater than 6 planks. However, these structure-building problems exhibit repeating substructure every 1-2 layers that reuse minor variants of the same abstract policy. We used this observation to develop an SSP solver (Vasudevan 2020) that computes generalized policies for such repeating structures. For our
Figure 5. Top: The Fetch mobile manipulator uses a STAMP policy to pickup a target bottle while avoiding those that are likely to be crushed. It replaces a bottle that wasn’t crushed (left), discards a bottle that was crushed (center) and picks up the target bottle (right). Bottom: ABB YuMi builds Keva structures using a STAMP policy: 12-level tower (left), twisted 12-level tower (center), and 3-towers (right).

Figure 6. Left: UAV inspects faulty parts of an aircraft in an airplane hangar and alerts the human about the location of the fault. UAV’s movements and sensors are noisy so it may drift from its location or fail to locate the fault. Right: Fetch searches for a can in drawers. The can can be placed in one of the drawers stochastically.

| Criteria                  | Cluttered Table | Aircraft Inspection | Building Keva Structures | Sort Clutter | Kitchen | Find the can |
|---------------------------|-----------------|---------------------|--------------------------|--------------|---------|--------------|
| Deterministic             | ✓               | ✓                   | ✓                        | ✓            | ✓       | ✓            |
| Stochastic                | ✓               | ✓                   | ✓                        | ✓            | ✓       | ✓            |
| Infeasible Tasks          | ✓               | ✓                   | ✓                        | ✓            | ✓       | ✓            |
| Large task spaces         | ✓               | ✓                   | ✓                        | ✓            | ✓       | ✓            |
| Motion/task trade-off     | ✓               | ✓                   | ✓                        | ✓            | ✓       | ✓            |
| Non-monotonicity          | ✓               | ✓                   | ✓                        | ✓            | ✓       | ✓            |
| #branches                 | $O(2d)$         | $O(3^n)$            | $O(2^n)$                 | $O(2d)$      | 1       | 2            |

Figure 7. Criteria defined by Lagriffoul et al. (2018) evaluated in each of the test domains.

experiments, we use a primitive implementation of this algorithm that incrementally calls LAO* to compute iterative policies. Approaches for generalized planning (Srivastava et al. 2008; Bonet et al. 2009; Hu and De Giacomo 2011; Srivastava et al. 2011) could be used to automatically extract and utilize such patterns in other problems with repeating structures.

**Problem 4: Sort cluttered table**

In this problem, as shown in Fig. 8a, the Fetch robot is used to sort objects placed on a table. The goal is to have all $N$ blue blocks on the left table and all $N$ green blocks on the right table while $2N$ red blocks act as obstacles. The cluttered configuration of objects on the table renders some actions infeasible that makes ordering of the actions critical. The interleaved framework proposed by HPlan algorithm searches for an abstraction that is sufficient to compute a valid order of the cans to be moved to solve the problem. For our experiments, we use $N = 3, 5, 8$ (total number of objects 12, 20, and 32 respectively).

**Problem 5: Setting up a dining table**

In this problem, the Fetch robot arranges a dining table with two plates and two glasses (Fig. 8b). A tray is available for the robot to use for carrying multiple items at once. If the robot tries to carry more than two objects on a tray at once, the objects can fall from the tray with a probability $0.2$ and that would break the object. While using the tray can reduce...
(a) Fetch sorts a cluttered table. All the blue cans have to be placed on the left table and all the green cans have to be placed on right table. Red cans act as obstacles. Left: The initial state for a problem. Right: The goal state.

(b) Fetch uses STAMP policy to set up a dining table. A tray is available to carry multiple items at a time but carrying more than two items on the tray may break the items. Left: The initial state. Right: The goal state.

the number of trips between tables, breaking the objects would render the problem unsolvable. As our approach considers all possible outcomes of stochastic actions, it successfully computes a policy that prevents any object from breaking compared to determinization-based approaches that only consider the most likely outcome for stochastic actions that may fail to solve such problems as most-likely scenarios might fail to capture dead ends in the domain.

Problem 6: Find the can
In this problem, the Fetch robot searches for a can that may be present in one of the drawers. Fig. 6 shows the simulated environment for the problem. The can is placed in one of the drawers with a given prior distribution. The robot does not have access to the can’s location apriori and has to open the drawer to check whether the can is present in the drawer or not. In our experiments, the can is placed in the upper drawer with a probability 0.6 and in the bottom drawer with a probability 0.4.

Analysis of the results
Nature of the solutions
The most distinct characteristic of the solutions generated through our framework is that they capture all possible contingencies that may arise while executing the policy. E.g., solutions generated for setting up the dinner table (problem 5) avoids placing more than two items on the tray to completely eliminate the possibility of incurring higher expected cost, and solutions for picking up a can from the cluttered table (problem 1) avoid picking up a delicate can for similar reasons.

Quality of the solution over time
While our approach computes refinements for every action in the policy, the anytime property allows the agent to start executing the actions before all the actions are refined. Our approach computes anytime policies with respect to the possible outcomes handled by a policy at any point in time. Fig. 9 shows the anytime property of our approach in stochastic test domains. The y-axis shows the probability with which the policy available at any point of time during the algorithm’s computation will be able to handle all possible outcomes, and the x-axis shows the time (in seconds) required to compute task and motion policies that handle these outcomes. The results show that with time, the likelihood with which the solution would be able to handle any scenario increases. The agent can use this observation to decide a threshold at which it can start executing the actions. For our experiments, we use a threshold of 60% of all possible outcomes to start the execution of the policy. Our experiments show that in most cases, the problem was
Figure 9. Anytime performance of ATM-MDP, showing the time in seconds (x-axis) vs. probability mass refined (y-axis).

| Problem             | % Solved | Avg. Time (s) |
|---------------------|----------|---------------|
| Cluttered-15        | 100      | 367.89 ± 854.52 |
| Cluttered-20        | 97       | 654.1541 ± 1641.98 |
| Cluttered-25        | 86       | 990.93 ± 1011.07 |
| Aircraft Inspection | 100      | 278.94 ± 30.54 |
| 3π                  | 100      | 227.04 ± 38.11 |
| Twisted-Tower-12    | 100      | 805.31 ± 102.10 |
| Three-Tower-12      | 100      | 1367.27 ± 144.29 |
| Sort Clutter (N = 3)| 100      | 687.65 ± 103.36 |
| Sort Clutter (N = 5)| 52       | 2384.91 ± 540.65 |
| Sort Clutter (N = 8)| 12       | 3687.65 ± 301.21 |

Figure 10. Summary of times taken to solve the TAMP problems. Timeout: 4000 seconds

| Problem             | % Solved | Avg. Time (s) |
|---------------------|----------|---------------|
| Cluttered-15        | 100      | 1120.21 ± 1014.54 |
| Cluttered-20        | 83       | 1244.32 ± 990.65 |
| Cluttered-25        | 75       | 1684.54 ± 890.78 |
| Aircraft Inspection | 100      | 2875.01 ± 103.65 |
| 3π                  | 100      | 1356.34 ± 75.8 |
| Tower-12            | 100      | 2232.36 ± 104.84 |
| Twisted-Tower-12    | 80       | 3249.92 ± 773.69 |
| Setting up a dining table | 100 | 1287.23 ± 321.32 |
| Find the can        | 100      | 36.74 ± 0.13 |

Figure 11. Summary of times taken to solve the STAMP problems. Timeout: 4000 seconds.
solved significantly faster compared to starting execution after refining the entire policy tree (Fig. 10 and 11).

Impact of prioritized RTL path selection

The results presented in Fig. 9 indicate that when RTL paths are selected using the p/c ration (blue line), the framework can quickly handle outcomes with most likely outcomes, compared to a randomized selection of RTL paths for refinements (red line). In most cases, 80% of probable executions are covered within about 30% of the total computation time. This characteristic is most evident in the aircraft inspection problem due to a large number of possible outcomes and differences in the probability of different outcomes. Such a prioritization does not make a significant impact if all the outcomes are equally probable. E.g., such impact is least evident in the cluttered table problem with the probability of crushing the objects set to 0.5 given each outcome becomes equally probable and the sequence in which they are handled does not make any difference.

Scalability of the framework

Figures 10 and 11 show the time taken by our approach to compute complete TAMP and STAMP solutions by concretizing every action in the entire policy for the given test problems respectively. We combine results for different variants of the test problem as variations in the probabilities of outcomes do not affect the time required to concretize all actions in the entire policy. Values in the figures 10 and 11 are averages of 50 runs with standard deviation. It is evident from the figures that solving a STAMP problem requires significantly more time than a TAMP problem for the same environment setup. E.g., the stochastic variant of the aircraft inspection problem takes nearly 15 times more time than the deterministic version as the stochastic variant had 780 branches in the solution tree compared to a single branch in the deterministic variant. Results for larger problems such as Sort Clutter(N = 5), Twisted-Tower-12, and Cluttered-25 show scalability of our system. While our approach requires more time to solve these larger problems, our approach was able to compute solutions successfully for the majority of the problems. Most problem instances for Sort Clutter(N = 8) faced timeout due to the magnitude of the problem as it requires the framework to compute at least 64 motion plans in the rare best-case scenario.

6 Conclusion

In this paper, we formalize the stochastic task and motion planning (STAMP) problem and introduce an anytime, sound, and probabilistically complete HPlan algorithm that uses entity abstractions to compute contingent task and motion solutions for the STAMP problems using off-the-shelf task and motion planners. The HPlan algorithm interleaves search for concretizations of the actions in the current model with computing refinements of the current abstract model. Policies generated through HPlan are complete in the sense that it provides an action for each scenario that may arise while executing the policy, unlike previous works that provide partial policies for most likely scenarios only. The approach uses greedy selection to prioritize more likely actions over less likely ones to allow the robot to start executing actions rather than waiting for a complete solution.

The approach assumes access to accurate descriptions for the robot’s action models and an abstraction function that can be used to formulate the STAMP problem. As part of the future work, we intend to relax these assumptions by learning the abstraction function. Shah et al. (2020) present a framework to learn critical regions for arbitrary motion planning problem. We plan to extend this work to learn the abstract representation for the STAMP problems. The current approach requires the robot to have full observability over its state. We also aim to relax this requirement and support partially observable setups as part of our future work.

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