Modeling of characteristics of heat exchangers of heat supply systems in variable operating modes

T A Rafalskaya\(^1\) and V Ya Rudyak\(^1,2\)

\(^1\)Novosibirsk State University of Architecture and Civil Engineering (Sibstrin), Russia, 630008 Novosibirsk, Leningradskaya St., 113
\(^2\)Novosibirsk State University, Russia, 630090 Novosibirsk, Pirogov St., 2

rafalskaya.ta@yandex.ru

Abstract. Heat exchangers used in heating systems usually operate under conditions of variable flow rates and temperatures of heat carriers. However, the theory for calculating the variable modes of operation of such devices is based on the use of constant parameters of heat exchangers. At the same time it is clear that the parameter of the heat exchanger can be affected by all quantities that determine the change of the heat transfer coefficient. In addition, the groups of heat exchangers with a tied supply of heat, i.e. with redistribution of the heat flow in a variable mode of operation, are used. A new method for calculating the variable modes of operation of heat points with a tied heat supply has been developed. The method of mathematical modeling defines the dependencies that determine the change in the heat exchanger parameters in variable operating modes. The obtained expressions can be used to configure programmable regulators which will allow the correction of inefficient modes of operation of the heating system.

1. Introduction

Surface heat exchangers are widely used in thermal power plants and heating systems. Depending on the mode of operation of the heat exchanger, its heat capacity, flows and temperatures of the heat transfer media change:

\[ Q = G_p c \rho \left( t_{p1} - t_{p2} \right) = G_h c \rho \left( t_{h2} - t_{h1} \right) = kF\Delta t^m. \]  

(1)

where \( G_p \) and \( G_h \) are the flows of the heating and heated coolants, \( m^3/s \); \( c \) and \( \rho \) are specific heat capacity, \( J/(kg{°}C) \) and density, \( kg/m^3 \), of heat carriers; \( t_{p1}, t_{p2}, t_{h1}, t_{h2} \) are initial and final temperatures of heat carriers, \( °C \); \( k, W/(m^2{°}K) \) and \( F, m^2 \), are heat transfer coefficient and heat exchanger area, \( \Delta t^m \) is log average temperature difference of heat carriers, \( °C \).

In variable operation mode, the temperatures of the primary \( t_{p2} \) and secondary \( t_{h2} \) coolants at the outlet of the heat exchanger or the temperature at the outlet of the heat exchanger and the flow rate of one of the coolants or the temperature of one of the coolants and the flow rate of the other coolant may not be known, which does not allow the balance equations (1) to be used to calculate the variable modes. Therefore, the existing theories of calculating variable modes of heat exchangers are based on the use of constant dimensionless parameters, which reduce the number of unknown variables.
So, to calculate the efficiency of regenerative air heat exchangers [1] and heat exchangers of the recuperative type [2-4], heat exchangers with intermediate coolant [5-7], nozzle chambers [8, 9], aircraft engines [10], chemical industry apparatuses [11], as well as microchannel heat exchangers [12, 13], the method of numbers of transfer units NTU is used.

For heat carriers with the same and constant mass flow rate $G$, the number of transfer units NTU is found by the formula [12, 14]

$$NTU = \frac{(kF)_c}{W_{\text{min}}},$$

(2)

where $(kF)_c$ is the product of the heat transfer coefficient of the heat exchanger on its area in the installation (design) mode, W/K; $W_{\text{min}}$ is the minimum equivalent of the coolant flow rate, W/K; $W = cpG$.

For water-to-water heat exchangers, in which the coolant flow rates vary greatly, the number of transfer units (heat exchanger parameter) is defined as [15]

$$NTU = \frac{(kF)_c}{\left(W_{pc}W_{he}\right)^{0.5}},$$

(3)

where $W_{pc}$ and $W_{he}$ are installation equivalents of the flows of heating and heated fluids, W/K.

It is believed that the heat exchanger parameter is a constant value regardless of the operation mode. In our paper [16] it was shown that NTU can be affected by all the quantities that determine the change in the heat transfer coefficient. In addition, groups of heat exchangers with the redistribution of heat fluxes in a variable mode of operation, i.e. with a tied supply of heat are used (e.g., see figure 1).

![Figure 1. The scheme of Heating Point with a tied supply of heat](image)

2. The method of calculating the variable modes of heat exchangers with the associated supply of heat

2.1. The first approximation

The calculation starts with the heat exchanger of the hot water supply system (HWS) of the I stage, where the flow of network water in any mode is not less than the flow of network water required for the heating system. Therefore, the flow rate of the network water in the I stage of the HWS heat exchanger is assumed to be equal to the required heating flow rate of the network water. The flow rate of the network water supplied by the heating system to provide hot water is neglected due to its uncertainty.

Dimensionless characteristics of the I stage of the HWS heater are as follows:
\[
\varepsilon^{(l)}(t_{\text{out}}) = \sqrt{\frac{a(W_h^l/W_p^h) + b + NTU^l(W_h^l/W_p^h)^{0.5}}{1}} \leq 1. \tag{4}
\]

Thermal power of the I stage of the HWS heat exchanger:
\[
Q_h^{(l)}(t_{\text{out}}) = \varepsilon^{(l)}(t_{\text{out}}) W_h^l D_t^{(l)}, \tag{5}
\]

where \(D_t^{(l)}\) is the maximum temperature difference between the heat transfer carriers in the heater; for the I stage of HWS heat exchanger it is the temperature difference between the heating water at the inlet of the heater and the water being heated at the inlet equal to the temperature of water in the cold water supply pipe (figure 1). The temperature of the supply water at the inlet to the heater of the first stage of HWS is assumed to be equal to the temperature of the return water after the heating system. The maximum temperature difference at the inlet to the heater is:
\[
D_t^{(l)}(t_{\text{out}}) = t_{p2}^h - t_{p1}^h. \tag{6}
\]

The temperature of the heated water at the outlet of the I stage of HWS heat exchanger is:
\[
t_{h2}^{(l)}(t_{\text{out}}) = t_{h1}^l + Q_h^{(l)}/W_h^l. \tag{7}
\]

The thermal power of the II stage of HWS heater is determined by the total thermal capacity of the I and II stages of HWS heat exchangers:
\[
Q_h^{(II)}(t_{\text{out}}) = Q_h - Q_h^{(I)}. \tag{8}
\]

In the first approximation, subject to the un-tied supply of heat, the equivalent flow of network water through the II stage of the HWS heater can be defined as:
\[
W_{p2}^{(II)}(t_{\text{out}}) = Q_h^{(II)}/(t_{p1} - t_{p2}^{(II)}). \tag{9}
\]

where \(t_{p1} = t_{p1}\) (see figure 1); \(t_{p2}^{(II)}\) is the temperature of the heating water at the outlet of the II stage of the HWS heater. In view of the fact that \(t_{p2}^{(II)}\) in the first approximation is unknown, it is accepted that \(t_{p2}^{(II)} = t_{h2}^{(II)}\).

The total equivalent of the flow rate of network water for heating and HWS is:
\[
W_p^{(l)}(t_{\text{out}}) = W_{p1}^{\text{hs,req}} + W_{p2}^{(II)}. \tag{10}
\]

The temperature of the network water at the inlet to the I stage of the heater from the balance equation is:
\[
t_{p1}^{(l)}(t_{\text{out}}) = \left(W_{p1}^{\text{hs,req}}/W_p^{(l)}\right)_p^{h} + \left(W_{p2}^{(II)}/W_p^{(l)}\right)_p^{h}. \tag{11}
\]

2.2. The second approximation

The calculations performed in the first approximation did not take into account the dependence of the thermal power of the heating system on the change in the thermal power of the heat exchanger of the second stage of HWS.

Specifies thermal capacity of the I stage of the HWS heat exchanger is as follows
\[
Q_h^{(I)}(t_{\text{out}}) = \varepsilon^{(I)}(t_{\text{out}}) W_h^l D_t^{(I)}. \tag{12}
\]
The temperature of the heated water after the I stage of the HWS heater is:

\[ T_{h1} = \frac{Q_{h}^{I(2)}}{W_{h}} + t_{h1}^{I(1)} \quad (13) \]

Thermal power of the II stage of the HWS heater is:

\[ Q_{h}^{II(2)} (t_{out}) = Q_{h} - Q_{h}^{II(2)} \quad (14) \]

The equivalent of the flows of network water in the II stage of the HWS heater can be determined by the formula

\[ W_{p2}^{II(2)} (t_{out}) = c (NTU^{II})^{2} W_{h}^{II} \left\{ -1 + \left[ 1 + d (NTU^{II})^{2} \left( D_{t}^{II(2)} W_{h}^{II} / Q_{h}^{II(2)} - e \right) \right]^{0.5} \right\}^{2} \geq \frac{Q_{h}^{II(2)}}{D_{t}^{II(2)}}, \quad (15) \]

where \( c \) and \( d \) are coefficients. If \( W_{h}^{II} < W_{p2}^{II(2)} \) then \( c = 1.7; d = 2.6; e = a = 0.35 \). If \( W_{h}^{II} = W_{p2}^{II(2)} \) then \( c = 0.5; d = 1.4; e = b = 0.65 \).

The temperature of the network water at the outlet of the II stage of the HWS heater is:

\[ t_{h2}^{II(2)} (t_{out}) = t_{h1}^{II(2)} \quad (16) \]

The flow of network water to provide heat power of HWS is:

\[ W_{p2}^{II(2)} (t_{out}) = 0.5 (NTU^{II})^{2} W_{h}^{II} \left\{ -1 + \left[ 1 + 1.4 (NTU^{II})^{2} \left( D_{t}^{II(2)} W_{h}^{II} / Q_{h}^{II(2)} - 0.65 \right) \right]^{0.5} \right\}, \quad (17) \]

The temperature of the network water at the outlet of the heater stage I is:

\[ t_{p2}^{II(2)} (t_{out}) = t_{p1}^{II(2)} \quad (18) \]

The flow of network water from the heat supply system to the heating system depends on the flow of water to the II stage of the HWS heat exchanger. The total equivalent of the flow of network water can be determined through the I stage of the HWS heat exchanger at [12]

\[ W_{p2}^{II(2)} (t_{out}) = \frac{Q_{h}}{D_{t}^{II(2)}} \quad (19) \]

Equivalent of the flow of network water to provide heat power of HWS is:

\[ W_{p2}^{II(2)} (t_{out}) = W_{p}^{II} - W_{p2}^{hs,eq} \quad (20) \]

The actual equivalent of the flow of network water to the heating system is:

\[ W_{p2}^{II(2)} (t_{out}) = W_{p2}^{II} - W_{p2}^{II(2)} \quad (21) \]

The equation of heat balance of heating and hot water systems to determine the temperature of the network water at the inlet of the I stage of the heater will be:

\[ t_{p1}^{II(2)} (t_{out}) = \left( W_{p}^{II} - W_{p2}^{II(2)} \right) / W_{p2}^{II(2)} t_{p2}^{II(2)} + \left( W_{p2}^{II(2)} / W_{p2}^{II(2)} \right) t_{p2}^{II(2)}. \quad (22) \]

The temperature of the network water at the outlet of the heater stage I is:
The temperature of the heated water after the first stage of the HWS heater is:

$$t_{h_1}^{(2)}(t_{out}) = t_{p_1}^{(2)} + \frac{Q_h^{(2)}}{W_h^{(1)}}.$$  

(25)

The temperature of the heated water entering the HWS after the II stage of the heat exchanger is:

$$t_{h_2}^{(2)}(t_{out}) = t_{h_1}^{(2)} + \left(t_{p_2}^{(2)} - t_{p_1}^{(2)}\right)\frac{W_p^{(2)}}{W_h^{(1)}}.$$  

(26)

2.3. The third approximation

The product of the heat transfer coefficient on the area for heaters of the I and II stages:

$$(k^I F^I)^{(3)}(t_{out}) = \frac{Q_h^{(1)}}{\Delta t^{(2)}}.$$  

Variable parameters of the I and II stages of the heaters are:

$$NTU^{(3)}(t_{out}) = \frac{(k^I F^I)^{(3)}}{(W_h^{(1)} W_p^{(2)})^{0.5}} \quad NTU^{H(3)}(t_{out}) = \frac{(k^II F^II)^{(3)}}{(W_h^{II} W_p^{(2)})^{0.5}}.$$  

The found variable parameters of the Heating Point can be used to calculate the modes of operation of heating and hot water supply using exponential expressions [15]:

$$\varepsilon^{(3)}(t_{out}) = \left\{1 - \exp\left[\frac{-(k^I F^I)^{(3)}}{W_h^{(1)}} \left(1 - \frac{W_h^{(1)}}{W_p^{(2)}}\right)\right]\right\} \left\{1 - \frac{W_h^{II}}{W_p^{(2)}} \exp\left[\frac{-\left(k^II F^II\right)^{(3)}}{W_h^{II}} \left(1 - \frac{W_h^{II}}{W_p^{(2)}}\right)\right]\right\} \leq 1.$$  

(29)

The thermal power of the I stage of the HWS heater $Q_h^{H(3)}(t_{out})$ is specified according to formula (12) with $D_{r}^{H(3)}(t_{out})$ according to the formula (14). The temperature of the heated water after the I stage of the HWS heater $t_{h_1}^{H(3)}(t_{out})$ is specified according to formula (25) and the log average temperature difference in the heater of the second stage $\Delta t^{H(3)}(t_{out})$.

Dimensionless characteristics of the II stage of the HWS heater are as follows:

$$\varepsilon^{H(3)}(t_{out}) = \left\{1 - \exp\left[\frac{-(k^II F^II)^{(3)}}{W_h^{II}} \left(1 - \frac{W_h^{II}}{W_p^{(2)}}\right)\right]\right\} \left\{1 - \frac{W_h^{II}}{W_p^{(2)}} \exp\left[\frac{-\left(k^II F^II\right)^{(3)}}{W_h^{II}} \left(1 - \frac{W_h^{II}}{W_p^{(2)}}\right)\right]\right\} \leq 1.$$  

(30)

Next, let us determine the maximum temperature difference at the inlet of the II stage of the HWS heater by formula (18) and by formula (16):

$$W_p^{II}(t_{out}) = \left(\frac{(k^II F^II)^{(3)}}{NTU^{II(3)}}\right)^2 1/W_h^{II} = \left(\frac{(k^II F^II)^{(3)}}{NTU^{H(3)}}\right)^2 1/W_h^{II}.$$  

(31)

The temperature of the networks water after the II stage of the heater is determined by (19).

Similarly, for the I stage of HWS heater:

$$W_p^{(3)}(t_{out}) = \left(\frac{(k^I F^I)^{(3)}}{NTU^{(3)}}\right)^2 \varepsilon^{I(3)} D_{r}^{I(3)} / Q_h^{I(3)}.$$  

(32)
Further, the equivalent of the flow rate of the supply water for the HWS $W_{p2}^{(3)}(t_{out})$ is determined according to formula (21), the temperature of the supply water at the inlet of the I stage of the HWS heater $t_{p1}^{(3)}(t_{out})$ is determined by formula (23), the network water at the outlet of the I stage of the heater $t_{p2}^{(3)}(t_{out})$ is determined by formula (24), the heated water after the I stage $t_{h2}^{(3)}(t_{out})$ is determined by formula (25) and after the II stage of the HWS heater $t_{h2}^{II}(t_{out})$ is determined according to formula (26).

To reduce the calculation error, the following approximations are performed similarly to the third stage of calculation until a given convergence is obtained.

3. Results and discussion
The calculation according to the developed method allows determining and predicting the flow rates and temperatures of network water in the heating system. Figure 2 shows the temperature of the water in the supply and return pipes of the heating network, and figure 3 shows the flow of water in the heating network and operating data. Calculations were carried out for a Heating Point in the city of Novosibirsk, with a calculated heat output: for heating and ventilation $Q_{omax}=5.22$ MW; for hot water supply $Q_h=3.30$ MW. The temperature schedule is 150/70°C with cuts at 114°C and 75°C.

![Figure 2](image)

**Figure 2.** Temperature in heat supply network:
- $t_{p1}^{gr}$ - in supply pipe of heat supply network on graph; $t_{p1}^{op}$ - the same operational; $t_{p2}^{h,gr}$ - in return pipe of heating system on the graph; $t_{p2}^{h,cal}$ - the same, calculation by the proposed method; $t_{p2}^{cal}$ - in return pipe of heat supply network, by method; $t_{p2}^{op}$ - the same operational.

![Figure 3](image)

**Figure 3.** The flow rates in heat supply network at the maximum water flow rate in the HWS:
- $G_{p1}^{cal}$ - in supply pipe of heat supply network, calculation by the proposed method; $G_{p1}^{op}$ - the same operational; $G_{p2}^{II}$ - in the second stage of HWS heater; $G_{p1}^{h}$ - in the heating system.

The calculation results showed that the dependences obtained fairly accurately describe the modes of operation of the heat supply system. At the same time, it is clear that to set up programmable regulators you need to have expressions that allow you to describe the desired parameters without conducting cumbersome calculations. Based on the results of the proposed method, new formulas that allow determination of the variable value of NTU depending on the mode of operation of heat exchangers by the method of mathematical modeling in the MathCad software were obtained:
• at unknown temperatures of heat carriers in a variable mode [17]

$$NTU(W) = \frac{Q}{\Delta t \left( \frac{W_p}{W_h} \right)^{0.5}} \left\{ \left( \frac{W_p}{W_h} \right)^{a_0} \frac{Q}{Q} - c_1 \right\} + \frac{Q}{Q} \left[ \left( \frac{W_p}{W_h} \right)^{b} \left( \frac{W_p}{W_h} \right)^{c} \right]^{0.5}, \quad (33)$$

for the I stage of the HWS heater: $a_1=0.2; b_1=0; c_1=0.4$; for the II stage: $a_1=0.02; b_1=0.03; c_1=1$.

• at unknown equivalents of heat carrier flow rates in a variable mode

$$NTU(t) = \frac{Q}{\Delta t \left( \frac{W_p}{W_h} \right)^{0.5}} \left\{ a_2 \left( \frac{Q}{Q} \right)^{b_2} - c_2 \right\} + d_2, \quad (34)$$

for the I stage: $a_2=2.3; b_2=-0.2; c_2=1; d_2=0$; for the II stage: $a_2=1.6; b_2=0.05; c_2=1.14; d_2=0.57$.

4. Conclusion
The proposed method of calculating the heating points of the heat supply systems with the tied heat supply can be used in predicting the variable operating modes of the heat supply systems. The obtained formulas (33) and (34) can be applied when setting up programmable regulators, according to control signals of temperatures and flow rates of network water. The use of the obtained relations will allow correcting the inefficient modes of operation of the heating system.

References
[1] Nellis G F and Klein S A 2006 Regenerative heat-exchangers with significant entrained heat capacity J. International Journal of Heat and Mass Transfer vol 49 no 1-2 pp 329-340
[2] Fernández-Torrijos M, Sobrino C and Santana D 2016 $e$-NTU relationships in parallel series arrangements: application to plate and tubular heat exchangers J. Applied Thermal Engineering vol 99 pp 1119-32
[3] Samarin O D 2017 Accounting for uneven water consumption in exhaust air heat recovery systems for the needs of hot water supply J. Vestnik MGSU Proc. of Moscow State University of Civil Engineering vol 3(102) iss 12 pp 341-345
[4] Koubasi E, Kurt H 2018 Simulation of heat exchangers and heat exchanger networks with an economic aspect Mag. Engineering Science and Technology pp 70-76
[5] Samarin O D 2009 Evaluation of the temperature efficiency of heat exchangers with an intermediate coolant by dimensionless parameters J. News of Higher Educational Institutions. Construction no 2 pp 54-58
[6] Navarro H A, Filho J R B Z and Ribatski G 2010 Effectiveness – NTU data and analysis for air-conditioning and refrigeration air coils J. Journal of the Brazilian Society of Mechanical Sciences and Engineering vol 32 no 3 pp 218-226
[7] Mirgolbabaei H 2018 Numerical investigation of vertical helically coiled tube heat exchangers thermal performance J. Applied Thermal Engineering vol 136 no 3 pp 252-259
[8] Ren C Q 2008 Effectiveness NTU-relation for pack bed-liquid desiccant-air contact systems with a double film model for heat and mass transfer J. International Journal of Heat and Mass Transfer vol 51 no 7-8 pp 1793-1803
[9] Eremitkin A I and Averkin A G 2015 Improving the method of calculating contact devices for heat and moisture treatment of air Mag. Construction and Reconstruction no 2(58) pp 105-114
[10] Arbekov A N, Surovtsev I G and Dermer P B 2014 Heat transfer efficiency in recuperative heat exchangers with high-speed gas flows at low Prandtl numbers J. High Temperature Thermal Physics vol 52 no 3 pp 463-468
[11] Havin G L 2011 Calculation of a plate heat exchanger with channels of different types in one unit Mag. Problems of mechanical engineering vol 14 no 4 pp 40-45
[12] Valueva E P, Garyaev A B and Klimenko A V 2016 Features of hydrodynamics and heat transfer during flow in microchannel technical devices (Moscow: Publishing house MEI) 140 p
[13] Popescu T 2012 *Microchannel heat exchanger - present and perspectives* J. U.P.B. Sci. Bul. Series D vol 74 iss 3 pp 55-70
[14] Belonogov N V and Pronin V A 2004 *Calculation of the efficiency of cross-precision plate heat exchangers* J. Bulletin of the International Academy of Cold no 4 pp 12-15
[15] Sokolov E Ya 2001 *Heating and heat networks* (Moscow: Publishing house MEI) 472 p
[16] Rafalskaya T A and Rudyak V Ya 2019 Research of variable characteristics of heat exchange equipment *Proc. Int. Conf. of Young Scientists “Energy Systems Research 2019”* (Irkutsk, Russia) J. E3S Web of Conferences 114, 07001 (2019) https://doi.org/10.1051/e3sconf/201911407001
[17] Rafalskaya T A 2019 *Investigating the Possibility of Using Low-Temperature Heat Supply with the Central Qualitative Regulation* J. Thermal Engineering vol 66 no 11 pp. 858-867 DOI: 10.1134/S0040601519110041