I discuss the question whether it is possible that the LHC will find no signal for the Higgs particle. It is argued that in this case singlet scalars should be present that could move in extra, even fractional, dimensions. A critical view at the existing electroweak data shows that this possibility might be favored over the simplest standard model. In this case one needs the ILC in order to study the Higgs sector.

1 Introduction

The standard model gives a good description of the bulk of the electroweak data. Only a sign of the Higgs particle is missing at the moment. The Higgs field is necessary in order to make the theory renormalizable, so that predictions are possible and one can really speak of a theory. A complete absence of the Higgs field would make the theory non-renormalizable, implying the existence of new strong interactions at the TeV scale. Therefore one is naively led to the so-called no-lose theorem. This theorem says that when one builds a large energy hadron collider, formerly the SSC now the LHC, one will find new physics, either the Higgs particle or otherwise new strong interactions. Since historically no-theorems have a bad record in physics one is naturally tempted to try to evade this theorem. So in the following I will try to find ways by which the LHC can avoid seeing any sign of new physics.

At the time of the introduction of the no-lose theorem very little was known about the Higgs particle. Since then there have been experiments at LEP, SLAC and the Tevatron, that give information on the Higgs mass. Through precise measurements of the W-boson mass and various asymmetries one can get constraints on the Higgs mass. The Higgs mass enters into the prediction of these quantities via radiative corrections containing a virtual Higgs exchange. Moreover at LEP-200 the direct search gives a lower limit of 114.4 GeV. The situation regarding the precision tests is not fully satisfactory. The reason is that the Higgs mass implied by
the forward-backward asymmetry $A_{FB}(b)$ from the bottom quarks is far away from the mass implied by the other measurements, that agree very well with each other. No model of new physics appears to be able to explain the difference. From $A_{FB}(b)$ one finds $m_H = 488^{+426}_{-219}$ GeV with a 95% lower bound of $m_H = 181$ GeV. Combining the other experiments one finds $m_H = 51^{+37}_{-22}$ GeV with a 95% upper bound of $m_H = 109$ GeV. The $\chi^2$ of the latter fit is essentially zero. Combining all measurements gives a bad fit. One therefore has a dilemma. Keeping all data one has a bad fit. Ignoring the $b$-data the standard model is ruled out. In the latter case one is largely forced towards the extended models that appear in the following. Accepting a bad fit one has somewhat more leeway, but the extended models are still a distinct possibility.

2 Hiding the Higgs boson

2.1 Invisible decay

When singlet scalar fields are added to the Higgs sector two effects appear, invisible decay and mixing. The general class of models is described in ref.\textsuperscript{2}. In the case that there is an unbroken symmetry in the singlet sector the singlets are stable and weakly interacting. They are a perfectly simple candidate for the dark matter of the universe. If they are light enough the Higgs boson will decay into these invisible particles, thereby leaving no obvious signal at the LHC.

However, is this Higgs boson completely undetectable at the LHC? Its production mechanisms are exactly the same as the standard model ones, only its decay is in undetectable particles. One therefore has to study associated production with an extra Z-boson or one must consider the vector-boson fusion channel with jet-tagging. Assuming the invisible branching ratio to be large and assuming the Higgs boson not to be heavy, as indicated by the precision tests, one still finds a significant excess of events. Of course one cannot study this Higgs boson in great detail at the LHC. For this the ILC would be needed, where precise measurements are possible in the channel $e^+e^- \rightarrow ZH$.

2.2 Mixing: fractional Higgses

Somewhat surprisingly it is possible to have a model that has basically only singlet-doublet mixing. If one starts with an interaction of the form $H\Phi^\dagger\Phi$, where $H$ is the new singlet Higgs field and $\Phi$ the standard model Higgs field, no interaction of the form $H^3$, $H^4$ or $H^2\Phi^\dagger\Phi$ is generated with an infinite coefficient. At the same time the scalar potential stays bounded from below. This means that one can indeed leave these dimension four interactions out of the Lagrangian without violating renormalizability. This is similar to the non-renormalization theorem in supersymmetry that says that the superpotential does not get renormalized. However in general it only works with singlet extensions. As far as the counting of parameters is concerned this is the most minimal extension of the standard model, having only two extra parameters.

The simplest model is the Hill model:

$$L = -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi) - \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda_0}{8}(\Phi^\dagger\Phi - f_0^2)^2 - \frac{\lambda_1}{8}(2f_1H - \Phi^\dagger\Phi)^2$$

(1)

Working in the unitary gauge one writes $\Phi^\dagger = (\sigma, 0)$, where the $\sigma$-field is the physical standard model Higgs field. Both the standard model Higgs field $\sigma$ and the Hill field $H$ receive vacuum expectation values and one ends up with two particles having the quantum numbers of the Higgs particle, but reduced couplings to standard model particles. One can call them fractional Higgs particles. A practical way to describe the situation is to replace in all experimental cross section calculations the standard model Higgs propagator by:

$$D_{\sigma\sigma}(k^2) = \frac{\cos^2(\alpha)}{(k^2 + m_\sigma^2) + \sin^2(\alpha)/(k^2 + m_\sigma^2)} + \frac{\sin^2(\alpha)}{(k^2 + m_\sigma^2)}$$

(2)
The generalization to an arbitrary set of fields $H_i$ is straightforward. One finds a number of (fractional) Higgs bosons $H_i$ with reduced couplings $g_i$ to the standard model particles.

2.3 A higher dimensional Higgs boson

The mechanism described above can be generalized to an infinite number of Higgses. The physical Higgs propagator is then given by an infinite number of very small Higgs peaks, that cannot be resolved by the detector. Ultimately one can take a continuum limit, so as to produce an arbitrary line shape for the Higgs boson, satisfying the Källén-Lehmann representation.

\[ D_{\sigma\sigma}(k^2) = \int ds \frac{\rho(s)}{(k^2 + \rho(s) - i\epsilon)} \]  

One has the sum rule \[ \int \rho(s) ds = 1, \] while otherwise the theory is not renormalizable and would lead to infinite effects for instance on the LEP precision variables. Moreover, combining mixing with invisible decay, one can vary the invisible decay branching ratio as a function of the invariant mass inside the Higgs propagator. There is then no Higgs peak to be found any more. The general Higgs propagator for the Higgs boson in the presence of singlet fields is therefore determined by two functions, the Källén-Lehmann spectral density and the s-dependent invisible branching ratio. Unchanged compared to the standard model are the relative branching ratio’s to standard model particles.

Since a sharp mass peak is absent, this is a promising way to hide the Higgs boson at the LHC. The general case is rather arbitrary, but it contains an elegant subclass. Because the $H\Phi^4\Phi$ interaction is superrenormalizable one can let the $H$ field move in more dimensions than four, without violating renormalizability. One can go up to six dimensions. The precise form of the propagator depends on the size and shape of the higher dimensions. The exact formulas can be quite complicated. However it is possible that these higher dimensions are simply open and flat. In this case one finds simple formulas. One has for the generic case a propagator of the form:

\[ D_{\sigma\sigma}(q^2) = \left[q^2 + M^2 - \mu_{\text{thd}}^{8-d}/q^2 + m^2 \frac{d-6}{2}ight]^{-1}. \]  

The parameter $M$ is a four-dimensional mass, $m$ a higher-dimensional mass and $\mu_{\text{thd}}$ a higher-to-lower dimensional mixing mass scale. For six dimensions a limiting procedure is needed. Calculating the corresponding Källén-Lehmann spectral densities one finds a low mass peak and a continuum that starts a bit higher in the mass.

If one does not introduce further fields no invisible decay is present. If the delta peak is small enough it is too insignificant for the LHC search. The continuum is practically invisible at the LHC, since the low mass Higgs search depends on the presence of a sharp peak.

3 Comparison with the LEP-200 data

We now confront the higher dimensional models with the results from the direct Higgs search at LEP-200. Within the pure standard model the absence of a clear signal has led to a lower limit on the Higgs boson mass of 114.4 GeV at the 95% confidence level. Although no clear signal was found the data have some intriguing features, that can be interpreted as evidence for Higgs bosons beyond the standard model. There is a 2.3 $\sigma$ effect seen by all experiments at around 98 GeV. A somewhat less significant 1.7 $\sigma$ excess is seen around 115 GeV. Finally over the whole range $s^{1/2} > 100$ GeV the confidence level is less than expected from background. We interpret these features as evidence for a spread-out Higgs-boson spectrum. The peak at 98 GeV is taken to correspond to the delta peak in the Källén-Lehmann density. The other excess data are interpreted as part of the continuum, that peaks around 115 GeV. We vary the data within
the uncertainties of the experiment. We start with the case $d = 5$. There is no problem fitting the data with these conditions. As allowed ranges we find:

\begin{align*}
95 \text{ GeV} &< m < 101 \text{ GeV} \\
111 \text{ GeV} &< M < 121 \text{ GeV} \\
26 \text{ GeV} &< \mu_{\text{thd}} < 49 \text{ GeV}
\end{align*}

(5)

We now repeat the analysis for the case $d = 6$. Here the data can only be fitted in a restricted range within the uncertainties of the experiment. The six-dimensional case therefore seems to be somewhat disfavoured compared to the five-dimensional case. We found the following limits:

\begin{align*}
95 \text{ GeV} &< m < 101 \text{ GeV} \\
106 \text{ GeV} &< M < 111 \text{ GeV} \\
22 \text{ GeV} &< \mu_{\text{thd}} < 27 \text{ GeV}
\end{align*}

(6)

4 Conclusion

Taking the analysis at face value we find a roughly $3\sigma$ effect with the following conclusions:

a) The Higgs field has been found at LEP-200.
b) Its properties are consistent with the electroweak precision data.
c) A dark matter candidate can be included.
d) The LHC will see no Higgs signal.

We note that also the Tevatron appears to have an excess at low masses in the Higgs search. However the experimental significance is hard to estimate, as the data were not analysed with this type of model in mind.

Acknowledgments

This work was supported by the BmBF Schwerpunktsprogramm "Struktur und Wechselwirkung fundamentaler Teilchen".

References

1. M.S. Chanowitz, in XXIII International Conference on High Energy Physics, Berkeley, California (1986).
2. J.J. van der Bij, Phys. Lett. B 636, 56 (2006).
3. A. Hill and J.J. van der Bij, Phys. Rev. D 36, 3463 (1987).
4. R. Akhoury, J.J. van der Bij and H. Wang, Eur. Phys. J. C 20, 497 (2001).
5. J.R. Espinosa and J.F. Gunion, Phys. Rev. Lett. 82, 1084 (1999).
6. The LEP working group for Higgs boson searches, Phys. Lett. B 565, 61 (2003).
7. J. J. van der Bij and S. Dilcher, Phys. Lett. B 638, 234 (2006).