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ELECTROMAGNETIC PROPERTIES OF NON-DIRAC PARTICLES
WITH REST SPIN 1/2

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We resolve a number of questions related to an analytic description of electromagnetic form factors of non-Dirac particles with the rest spin 1/2. We find the general structure of a matrix antisymmetric tensor operator. We obtain two recurrence relations for matrix elements of finite transformations of the proper Lorentz group and explicit formulas for a certain set of such elements. Within the theory of fields with double symmetry, we discuss writing the components of wave vectors of particles in the form of infinite continued fractions. We show that for \( Q^2 \leq 0.5 \text{ (GeV/c)}^2 \), where \( Q^2 \) is the transferred momentum squared, electromagnetic form factors that decrease as \( Q^2 \) increases and are close to those experimentally observed in the proton can be obtained without explicitly introducing an internal particle structure.

Keywords: electromagnetic form factor, tensor operator, finite Lorentz transformation, infinite continued fraction

1. Introduction

The difference between the proton and the electron that shows up in their electromagnetic interaction was first evidenced in the 1933 measurement of the magnetic moment of the proton [1] and was finally established after the McAllister and Hofstadter experiment on elastic scattering of electrons on protons [2]. The essence of this difference is that one or more of the following three electron characteristics are inapplicable to the proton. First, the electron is assigned a Dirac representation of the proper Lorentz group \( L^\uparrow_+ \). Second, up to the experimentally attainable sizes of the order \( 10^{-16} \text{ cm} \), the electron behaves as a pointlike particle, which corresponds to the locality of the Lagrangian describing its interaction with the photon, where the fields of all particles are taken at the same point. Third, the Lagrangian of the electromagnetic interaction of the electron is minimal: expressed in terms of the vector potential \( A^\mu \) of the electromagnetic field, it is free of derivatives.

The theoretical concept of the proton and neutron that subsequently formed appeals to their nonminimal electromagnetic interaction and to a complicated internal structure but hardly challenges the assumption that the nucleon is a Dirac particle. This is reflected in the general form of the nucleon electromagnetic current in the momentum space [3]:

\[
    j^\mu(p, p_0) = ie\bar{u}(p) \left[ \gamma^\mu F_1(Q^2) + \frac{i\kappa}{2M} \sigma^{\mu\nu} q_\nu F_2(Q^2) \right] u(p_0),
\]

where \( \bar{u} \) and \( u \) are Dirac spinors, \( \kappa \) is the anomalous magnetic moment, \( M \) is the nucleon mass, \( q = p - p_0 \), and \( Q^2 = -q^2 \). Along with the Dirac \( (F_1) \) and Pauli \( (F_2) \) form factors in (1), the Sachs form factors \( G_e \) and \( G_m \) are introduced as [4]

\[
    G_e = F_1 - \kappa F_2, \quad G_m = F_1 + \kappa F_2,
\]

where \( \kappa \) is the anomalous magnetic moment.

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where $\tau = Q^2/4M^2$; these form factors are believed to describe the distribution of the electric charge and the magnetism of the nucleon. Using the quantities $G_e$ and $G_M$ brings the Rosenbluth formula [5] for the elastic scattering of nonpolarized electrons on nonpolarized nucleons in the laboratory frame to the simplest form

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E' \cos^2(\theta/2)}{4E^3\sin^2(\theta/2)} \left[ \frac{G_e^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right], \quad (3)$$

where $E$ and $E'$ are the respective electron energies in the initial and final states and $\theta$ is the electron scattering angle. A detailed review of the results of experimental measurements of electromagnetic form factors of the proton and the neutron and their various theoretical interpretations can be found in [6].

Rejecting the description of the proton by a Dirac spinor entails numerous indeterminacies. One type of indeterminacy arises because a particle with the rest spin 1/2 can be assigned an infinite number of irreducible representations of the proper Lorentz group $L^\uparrow_+$ of the form $|l_1\rangle$ of $(-1/2,l_1)$ and $(1/2,l_1)$, where $l_1$ is an arbitrary complex number, and infinitely many reducible representations constructed from these. Another type of indeterminacy arises from the arbitrariness in the field theory constants allowed by the relativistic invariance [7], [8], and this arbitrariness can be infinite, for example, if ISFIR-class fields are considered, which transform under representations decomposable into an infinite direct sum of finite-dimensional irreducible representations of the $L^\uparrow_+$ group.

Arguments in favor of the possible description of hadrons by ISFIR-class fields were adduced in [9], [10]. We note that these fields had not been investigated until the appearance of a symmetry approach for eliminating the infinite arbitrariness in the corresponding Lagrangians. The general definition of the double symmetry notion [11], including the Gell-Mann–Lévy $\sigma$-model symmetry [12] and supersymmetry as particular cases, allowed supplementing [9] the relativistic invariance (the primary symmetry) of ISFIR-class field theories with the requirement of their additional (secondary) symmetry generated by transformations of the form

$$\psi(x) \to \psi'(x) = \exp[-iD^\mu \theta_\mu] \psi(x), \quad (4)$$

where the $D^\mu$ are matrix operators and the parameters $\theta_\mu$ are components of a polar or an axial four-vector of the orthochronous Lorentz group $L^\uparrow$. To avoid infinite degeneracy with respect to spin in the mass spectrum of the resulting theory because the Lorentz group is extended, spontaneous secondary symmetry breaking is introduced [13]. As a result, as shown in [14], the free relativistic invariant equations for ISFIR-class fermionic fields thus constructed yield mass spectra that agree with the experimental picture of baryon resonances and with the parton model of hadrons supplemented by the confinement hypothesis.

We note that all the previously considered relativistically invariant free field theories with infinitely many degrees of freedom, namely, bilocal equations (see [15]) and Gelfand–Yaglom-type equations [7], [8] (with the corollaries refined in [16]) for FSIIR-class fields transforming under representations decomposable into a finite direct sum of infinite-dimensional irreducible representations of the $L^\uparrow_+$ group, have mass spectra that are unsuitable for particle physics because of an accumulation point at zero. The infinite number of states in the theory of ISFIR-class fields with double symmetry seems to reflect some internal structure of the corresponding particles, and the physically entirely satisfactory mass spectra of the theory are evidence that the proposed approach describing a hadron by an infinite-component monolocal field is essentially correct. It is now desirable to find out how the physical content of the existing structure characteristics of particles with rest spin 1/2 is affected by passing from the Dirac representation of the

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1Irreducible representations of the $L^\uparrow_+$ group are labeled [7], [8] by two indices $(l_0,l_1)$, where $2l_0$ is an integer and $l_1$ is an arbitrary complex number. The canonical basis of the $(l_0,l_1)$-representation space is related to the $SO(3)$ subgroup and is denoted by $\xi_{(l_0,l_1)l}^m$, where $l$ is the spin and $m$ is its projection on the third axis, with $m = -l,-l+1,\ldots,l$ and $l = |l_0|,|l_0| + 1,\ldots$. In general, the range of values of $l$ is infinite. The $(l_0,l_1)$ representation is finite dimensional, and the above sequence of spins terminates at the number $|l_1| - 1$ if $2l_0$ and $2l_1$ are integers of the same parity and $|l_1| > |l_0|$. The Dirac representation is $(-1/2,3/2) \oplus (1/2,3/2)$. 1276