Temperature Width and Spin Structure of Superfluid $^3$He-$A_1$ in Aerogel

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Abstract

The influence of spin-exchange scattering centers on the triplet Cooper pairing is considered to explore the behavior of superfluid $^3$He in high porosity aerogel containing $^3$He atoms localized at the surface of silica strands. The homogeneously located and isotropically scattering system of spin-polarized “impurity” centers is adopted as a simple model to investigate the contribution of spin-exchange scattering channel for quasiparticles to the formation of non-unitary superfluid $A_1$-phase in aerogel environment. It is demonstrated that an interference between the potential and exchange parts of quasiparticle scattering against spin-polarized “impurity” centers can change considerably the temperature width and the spin structure of $A_1$-phase in aerogel.

Among recent achievements in physics of superfluid $^3$He the studies of the properties of this ordered Fermi liquid in presence of quasiparticle scattering medium is of a great importance. This situation is realized for liquid $^3$He confined to a high porosity aerogel.

The quasiparticle scattering against silica strands forming skeleton of aerogel has a profound influence on the properties of such superfluid as liquid $^3$He in the millikelvin temperature region. The gross effect of a finite mean free path $l$ of quasiparticles is manifested in a sizable suppression of the transition temperature of an ordered (superfluid) state, as is expected for a phase with an unconventional structure of the order parameter in the momentum space. This behavior of superfluid $^3$He in aerogel has been observed in a number of experiments using various techniques [1-5]

A more delicate question is about possible rearrangement of the phase diagram of superfluid $^3$He in presence of quasiparticle scattering medium [1-10]. In bulk superfluid $^3$He the isotropic $B$-phase is a favorable one in the sense that in major part of the $(P, T)$ phase diagram (in zero magnetic field) it appears as an equilibrium superfluid state. Only at sufficiently high pressures (above the polycritical value $P_{c0} \simeq 21$ bars) and at not too low temperatures an anisotropic
A-phase is a preferable equilibrium state due to so called strong-coupling effects [11] which take into account the inverse action of the ordering on the Cooper pairing interaction between quasiparticles.

In terms of $\beta_i \ (i = 1, 2, \ldots 5)$ coefficients which appear in the expansion of the free energy of superfluid $^3He$ near the transition to a normal state (at $T_{c0}(p)$) in power series of the order parameter components, the condition of thermodynamical stability of $^3He - A$ is (in what follows $\beta_{ij} = \beta_i + \beta_j + \ldots$)

$$\frac{2\beta_{345}}{3\beta_{13}} < 1.$$ (1)

This inequality is not satisfied in a weak-coupling approximation where

$$-2\beta_{1w} = \beta_{2w} = \beta_{3w} = \beta_{4w} = -\beta_{5w} = \frac{7\zeta(3)}{120} \frac{N_F}{\pi T_{c0}},$$ (2)

and $N_F$ stands for the quasiparticle density of states (DOS) at the Fermi level.

On introducing pressure-dependent strong-coupling corrections $\delta'_{sc}$ ($\beta_i = \beta_{iw} + \delta'_{sci}$) it can be shown that criterion of the priority of the $A$-phase over the $B$-phase reduces to

$$\delta'_{sc}(P) > 1/4,$$ (3)

where a dimensionless strong-coupling parameter $\delta'_{sc}(P)$ is defined according to an equation

$$\delta'_{sc} = 1 - \frac{\beta_{345}}{2\beta_{13}}.$$ (4)

It should be noted that along with $\delta'_{sc}$, which is useful to describe an interplay between $A$- and $B$-phases, some other strong-coupling parameters can be introduced in an appropriate way. On imposing an external magnetic field $^3He-A_1$, characterized by Cooper pairing in a single equal-spin-projection state, is stabilized in the vicinity of $T_{c0}$. $A_1$-phase appears as an equilibrium state below $T_{c1} > T_{c0}$ and extends down to $T_{c2} < T_{c0}$. As is well known, the $A_1 - A_2$ splitting asymmetry ratio

$$r = \frac{T_{c1} - T_{c0}}{T_{c0} - T_{c2}} = -\frac{\beta_5}{\beta_{245}}.$$ (5)

In the weak-coupling approximation $r = 1$ so that the critical temperatures $T_{c1}$ and $T_{c2}$ should be positioned symmetrically with respect to $T_{c0}$. In reality a sizable asymmetry of the $A_1 - A_2$ splitting has been observed experimentally [12,13], which is due to strong-coupling effects. In this case it is convenient to introduce another strong-coupling parameter $\delta''_{sc}$ defined by an equation

$$\frac{\beta_5}{\beta_{245}} = 1 + \delta''_{sc}/2 = 1 - \delta''_{sc}/2,$$ (6)

or alternatively as $\delta''_{sc} = -2(1 + 2\beta_5/\beta_{245})$
The phenomenological parameters $\delta_\text{sc}'$ and $\delta_\text{sc}''$ describe the role of the strong-coupling effects relative to a weak-coupling contribution $|\beta_1^{wc}|$. Generally spiking $\delta_\text{sc}' \neq \delta_\text{sc}''$. In a simple (static) paramagnon model [14] $\delta\beta_1^{sc} = \delta\beta_3^{sc} = 0$, $\delta\beta_2^{sc} = -\delta\beta_4^{sc} = -\delta\beta_5^{sc} = \delta\beta_{sc}$ and within this crude approximation $\delta_\text{sc}' = \delta_\text{sc}''$ (here $\delta\beta_{sc}$ describes the contribution to the strong-coupling effects stemming from an attractive interaction between quasiparticles via the exchange of magnetic excitations, the retardation being discarded).

The full temperature width $\Delta T = T_{c1} - T_{c2}$ of the $A_1$-phase in bulk $^3\text{He}$ is linear in the magnetic field strength (at list up to 10 T [12]) and is proportional to the Ambegaokar-Mermin coefficient $\eta$ [15]. In bulk $^3\text{He}$ $\eta \neq 0$ due to a small particle-hole asymmetry of DOS near the Fermi level. For $\Delta T$ we have

$$\frac{\Delta T}{T_{c0}} = \frac{2\eta h}{1 + \delta_\text{sc}''/2},$$

(7)

where $h = \gamma H/2T_{c0}$.

Now, on addressing the question of how the phase diagram of superfluid $^3\text{He}$ could be modified in aerogel environment, one has to understand in which way the key parameters $\delta_\text{sc}'$, $\delta_\text{sc}''$ and $\eta$, introduced above, react to quasiparticle scattering events.

The strong-coupling parameter $\delta_\text{sc}'$ defines, according to Eq.(3), the region of thermodynamical preference of the $A$-phase which in bulk superfluid $^3\text{He}$ is attained at $P > P_{c0} \simeq 21$ bars. The recent acoustic studies [4,5] of superfluid $^3\text{He}$ confined to 98% silica aerogel established that in zero magnetic field the $B$-phase-like superfluid state near $T_s(P)$ is stabilized at $P > P_{c0}$ up to the melting pressure $P_m$. This observation indicates that scattering of quasiparticles against spatial irregularities of a porous medium promotes the stability of the $B$-phase at the pressures where in bulk superfluid $^3\text{He}$ the $A$-phase is an equilibrium ordered state. In terms of the strong-coupling parameter $\delta_\text{sc}'$ this means that the equality $\delta_\text{sc}' = 1/4$ is not reached at $P < P_m$ in 98% porosity aerogel and the polycritical pressure $P_s$ in such quasiparticle momentum non-conserving environment is pushed to an unobservable region ($P > P_m$).

This conclusion is supported by theoretical investigations based on so-called homogeneous scattering model (HSM) treating the weak-coupling effect [16] and on a simple (static) paramagnon model estimating the strong-coupling contribution [17]. According to Ref.17 in the quasiparticle scattering medium

$$\delta_\text{sc}' = R(w_c)\delta_\text{sc}'0,$$

(8)

where the subscript “0” refers to the corresponding value in bulk superfluid $^3\text{He}$ and the “impurity” renormalization factor

$$R(w_c) = a(w_c) \frac{T_c}{T_{c0}},$$

(9)

with

$$a(w_c) = \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)} - \frac{\psi^{(2)}(1/2)}{\psi^{(2)}(1/2 + w_c)}.$$  

(10)
Here $\psi^{(m)}(z)$ is the poly-gamma function of $m$-th order, $w_c = \Gamma/2\pi T_c$, where the “impurity” scattering rate $\Gamma = v_F/2l$, and the critical temperature $T_c$ of the “dirty” superfluid $^3He$ is found according to the Abrikosov-Gorkov equation

$$\ln \left( \frac{T_c}{T_{c0}} \right) + \psi(1/2 + w_c) - \psi(1/2) = 0. \quad (11)$$

The two co-factors in Eq. (10) have opposite behavior as concerns their dependence on the scattering parameter $w_c: a(w_c)$ is an increasing function of $w_c$ whereas the ratio $T_c/T_{c0}$ decreases with increasing $w_c$. This competition is in favor of $T_c/T_{c0}$ so that $R(w_c) < 1$ at $w_c \neq 0$ (for $w_c \approx 1$ $R(w_c) = 1 - 2.56w_c$).

As a result strong-coupling parameter $\delta_{sc}'$ is suppressed in quasiparticle scattering medium thus opening a way to the appearance of a $B$-like superfluid state in the pressure region $P > P_{c0}$.

In what follows we concentrate on the $A_1 - A_2$ splitting of superfluid transition in aerogel in presence of an external magnetic field. This effect is characterized by the temperature width of the $A_1$-phase

$$\Delta T = \frac{\eta}{1 + \delta_{sc}''/2} \gamma H, \quad (12)$$

and by the field-independent splitting asymmetry ratio

$$r = \frac{1 + \delta_{sc}''/2}{1 - \delta_{sc}''/2}. \quad (13)$$

According to an estimate of strong-coupling effects, mentioned above, it is expected that $\delta_{sc}''$ is suppressed in aerogel and $r < r_0$. On the other hand, the $A_1$-phase width $\Delta T$ needs a more careful examination. In bulk $^3He$ the splitting coefficient $\eta_0$ stems from a small particle-hole asymmetry of DOS at the Fermi level. In the weak-coupling approximation

$$\eta_0 = \frac{N_F'}{N_F} T_{c0} \ln \left( \frac{2\gamma_E}{\pi} \cdot \frac{\omega_c}{T_{c0}} \right), \quad (14)$$

where $N_F'$ is the derivative of DOS $N(\varepsilon)$ with respect to the quasiparticle excitation energy, $\gamma_E$ stands for the Euler constant and $\omega_c$ is a cut-off parameter.

In aerogel environment $\eta_0$ is suppressed because of suppression of the critical temperature, although this is not the only source of modification of the splitting parameter $\eta$. Below it will be shown that more generally

$$\eta = \eta_0 T_c/T_{c0} + \delta \eta, \quad (15)$$

where an extra contribution $\delta \eta$ is due to the interference part of the spin-exchange scattering of the quasiparticles against localized $^3He$ “impurity” atoms adsorbed at the surface of silica strands of aerogel and spin-polarized under the action of an externally imposed magnetic field. The presence of such “frozen” layers of $^3He$ atoms covering aerogel silica strands was demonstrated in Ref. 18.
The spin-triplet Cooper pair condensate is described by an order parameter $\vec{\Delta}(\hat{k})$ transforming as a vector on the rotation in spin space. The lowest ordered contribution in $\vec{\Delta}$ to the free energy is proportional to $\langle |\vec{\Delta}|^2 \rangle$ with brackets $\langle \ldots \rangle$ showing an average across the Fermi surface (over the direction of an unity vector $\hat{k}$ in the momentum space). In presence of a magnetic field $\vec{H} = H\hat{h}$ a new term $i\langle \vec{\Delta} \times \vec{\Delta}^* \rangle \vec{H}$ appears which contributes to the free energy of superfluid $^3$He, as long as the particle-hole asymmetry (proportional to $\eta_0$) is taken into account.

In case of spin-triplet Cooper pairing in presence of spin-polarized scattering centers one more contribution to the free energy emerges proportional to $i\langle \vec{\Delta} \times \vec{\Delta}^* \rangle \vec{S}_T$, where $\vec{S}_T$ is the thermal average of the localized “impurity” spins [19]. In order to establish explicitly the quasiparticle spin-exchange scattering contribution $\delta \eta$ (as defined by Eq. (15)) we adopt the Abrikosov-Gorkov HSM which mimics the effects of incoherent scattering of quasiparticles against a system of localized $^3$He atoms adsorbed at the surface of aerogel silica strands. The details about HSM of aerogel could be found in Ref. 20. In the AG HSM the “impurity” scattering interaction is described by $2 \times 2$ matrix

$$\vec{U} = u_0\vec{I} + u_\text{ex}\vec{\alpha}\vec{S}. \quad (16)$$

The rate of potential (spin-independent) part of scattering is characterized by

$$\Gamma = n_\text{imp} \sin^2 \frac{\delta_0}{\pi N_F} = \frac{v_F}{2l}, \quad \tan \delta_0 = -\pi N_F u_0, \quad (17)$$

where $n_\text{imp}$ stands for an effective concentration of paramagnetic centers and $\delta_0$ is an $s$-wave phase shift. In presence of a magnetic field interference part of the scattering becomes operative as long as the polarization of the impurity spins $\vec{S}_T \neq 0$. As a result the interference scattering rate

$$\Gamma_{\text{int}} = 2\pi N_F n_\text{imp} u_\text{ex} u_0 \quad (18)$$

appears in the field-dependent contribution to the free energy:

$$\delta F_{\text{SH}} = -N_F \left[ \left( \frac{N_F'}{N_F} \right) \left( \frac{\gamma H}{2} \right) a_1(T_c) i\langle \vec{\Delta} \times \vec{\Delta}^* \rangle \vec{h} - \Gamma_{\text{int}} \cos^4 \delta_0 a_2(T_c) i\langle \vec{\Delta} \times \vec{\Delta}^* \rangle \vec{S}_T \right], \quad (19)$$

where

$$a_1 = 2\pi T \sum_{\omega > 0} \frac{1}{\omega + \Gamma} = \ln \left( \frac{2\gamma_F \omega_c}{\pi T} \right) + \psi(1/2) - \psi(1/2 + \Gamma/2\pi T), \quad (20)$$

$$a_2 = 2\pi T \sum_{\omega > 0} \frac{1}{(\omega + \Gamma)^2} = \frac{1}{2\pi T} \psi^{(1)}(1/2 + \Gamma/2\pi T) \quad (21)$$

Noticing that $\Gamma_{\text{int}} \cos^2 \delta_0 = (v_F/l)(u_\text{ex}/u_0)$ and adopting a free impurity spin model with $S_T = \frac{1}{2} \tanh (\gamma H/2T)$, it is concluded that quadratic-in-$\vec{\Delta}$ contribution to the free energy of superfluid $^3$He in aerogel reads as
\[ F_S^{(2)} = N_F \left( t(\bar{\Delta})^2 - \eta h i(\bar{\Delta} \times \bar{\Delta}^\ast) \bar{h} \right), \]  
(22)

where \( t = (T - T_c)/T_c \), \( h = \gamma H/2T_c \) and the \( A_1 - A_2 \) splitting parameter \( \eta \) is given by Eq. (15) with the spin-exchange scattering contribution

\[ \delta \eta(h) = -\frac{\pi^2 \xi_{c0} T_{c0} u_{ex} \tanh(h)}{8 \frac{l}{T_c} u_0 h}. \]  
(23)

Here the coherence length \( \xi_{c0} = v_F/(2\pi T_{c0}) \) and it is assumed that \( \cos^2 \delta_0 \to 1/2 \) and \( \Gamma \ll 2\pi T_c \).

According to Eq. (12) the temperature width of the \( A_1 \)-phase in aerogel (relative to the bulk value) reads as

\[ \frac{\Delta T}{(\Delta T)_0} = \frac{\eta 1 + \delta_{sc0}/2}{\eta_0 1 + \delta_{sc}/2}, \]  
(24)

As is evident from Eq. 15, in case of \( \delta \eta < 0 \) (realized at \( u_{ex}/u_0 > 0 \)) the \( A_1 - A_2 \) splitting parameter \( \eta \) may attain negative values (see below). The measurement of temperature width \( \Delta T \) do not contain information about the sign of \( \eta \) which can be fixed only in the experiments where the spin structure (\( \uparrow \uparrow \) or \( \downarrow \downarrow \)) of the \( A_1 \)-phase Cooper condensate is established (see Ref.21 and citations therein). That is why \( |\eta| \) stands in Eq. 24.

The spin-exchange scattering part \( \delta \eta \) can be contribute appreciably to \( \Delta T \) at

\[ \frac{\xi_{c0}}{l} \frac{|u_{ex}|}{u_0} \frac{\tanh(h)}{h} \sim \left( \frac{T_c}{T_{c0}} \right)^2 \eta_0. \]  
(25)

In the low magnetic field case \( (\gamma H \ll T_c) \) this condition is fulfilled at \( l = 100 \) nm, \( P = 15 \) bars and \( |u_{ex}/u_0| = 0.1 \).

According to existing experimental data (see Ref. 5) the \( A_1 \)-phase temperature width \( \Delta T \) is suppressed in aerogel environment. Adopting a view that this happens due to the presence of quasiparticle spin-exchange scattering contribution \( \delta \eta < 0 \), we concentrate on this possibility. Figs.1 and 2 show the dependence \( \eta = \eta(h) \) for the pressures \( P = 21 \) bars and \( P = 15 \) bars. It is seen that the spin-polarized scattering centers \( ^3He \) atoms adsorbed at the surface of aerogel silica strands) suppress considerably the \( A_1 - A_2 \) splitting parameter \( \eta \) in relatively low magnetic fields (Fig. 1). In the limit of high magnetic fields \( \eta \) tends to its asymptotic value \( (T_c/T_{c0})\eta_0 \).

The spin-exchange scattering contribution \( \delta \eta \) can even change sign in low fields.
Figure 1: $\eta$ as a function of $h$, for $P = 21$ bar

Figure 2: $\eta$ as a function of $h$, for $P = 15$ bar
In Figs. 3 and 4 the values of
\[ \frac{\Delta T}{H} = \frac{|\eta|}{1 + \delta_{sc}^\eta/2 k_B} = \frac{1.56|\eta|}{1 + \delta_{sc}^\eta/2} \frac{mK}{T} \] (26)
are plotted as a function of \( h \) for the pressures \( P = 21 \) bars and \( P = 15 \) bars.
A rather peculiar situation is expected for \( P = 15 \) bars at \( u_{ex}/u_0 = 0.2 \) and \( l = 200 \) nm (Fig. 4). At low magnetic fields (where \( \eta < 0 \)) the \( A_1 \)-phase with a reversed spin configuration \( \downarrow \downarrow \) of Cooper pairs is stabilized. On the increase of the magnetic field the temperature width of this superfluid state decreases and vanishes at a field strength for which \( \eta = 0 \) (see Fig. 2). On further increase of the magnetic field the \( A_1 \)-phase reappears, this time in a spin configuration \( \uparrow \uparrow \) (appropriate to bulk \( A_1 \)-phase with \( \eta > 0 \)). The reversing of the Cooper pairs spin configuration from \( \downarrow \downarrow \) to \( \uparrow \uparrow \) is shown in the inset of Fig. 4 at \( u_{ex}/u_0 = 0.2 \) and \( l = 150 \) nm.

In summary, it has been shown that the spin-exchange scattering of quasiparticles against magnetically polarized \( ^3 \)He atoms adsorbed at the surface of aerogel silica strands can cause substantial modification of \( (P, T, H) \) phase diagram of superfluid \( ^3 \)He in the region where non-unitary \( A_1 \)-phase is stabilized by an externally imposed magnetic field. This effect could be manipulated by the variation of the magnetic field strength or by preplating \( ^3 \)He in aerogel with some amount of \( ^4 \)He atoms which remove paramagnetic scattering centers from silica strands surface.

After having completed this article, we learned from Prof. W.P. Halperin about the Archive preprint (cond-mat/0306099) by J.A. Sauls and P. Sharma on the same subject.
Figure 4: $\Delta T/H$ as a function of $h$, for $P = 15$ bar

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