Understanding and responding the students in learning mathematics through the differentiated instruction

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Abstract. This research discusses the differentiated instruction, a mathematic learning which is as expected by the students in connection with the differentiated instruction itself, its implementation, and the students’ responses. This research employs a survey method which involves 62 students as the research respondents. The mathematics learning types required by the students and their responses to the differentiated instruction are examined through questionnaire and interview. The mathematics learning types in orderly required by the students, from the highest frequency cover the easily understood instructions, slowly/not rushing teaching, fun, not complicated, interspersed with humour, various question practices, not too serious, and conducive class atmosphere for the instructions. Implementing the differentiated instruction is not easy. The teacher should be able to constantly assess the students, s/he should have good knowledge of relevant materials and instructions, and properly prepare the instructions, although it is time-consuming. The differentiated instruction is implemented on the instructions of numerical pattern materials. The strategies implemented are flexible grouping, tiered assignment, and compacting. The students positively respond the differentiated learning instruction that they become more motivated and involved in the instruction.

1. Introduction
Thorndike [1] defines “instruction” as a method used to help students achieve the purpose of the instruction. Instruction in classroom is a complex activity. The teacher does not merely transfer knowledge, but should also attempt to help students achieve the competence planned, including the students’ mastery on materials and optimizing their potentials.

Students in classroom are greatly various, both in terms of capability and characteristics. Regarding to the mathematics instruction, these various students also have various needs. The teacher’s persistence is necessary in embracing all students, so that the instruction becomes effective and succeeds in optimizing the capability of the students. Department of National Education [2] teachers are required to understand the physical, moral, spiritual, social, cultural, emotional, and intellectual characteristics of the students. Teachers need to understand the thinking of students on mathematics, so that teachers may select or create tasks to help students build more valid concepts on mathematics. Developing various perspectives on students as learners of mathematics allows the teacher to build an environment that enables the students to learn mathematics with proper support and acceptance.
“Effective instruction” is an instruction that make the students become motivated and involved. McAdams [3] records that teachers show that their students become more motivated and enthusiastic in learning when the differentiation instruction approach is used. Christensen [4] reiterates that the implementation of differentiated instruction succeeds in improving the motivation of students in classroom activities. Heacox [5] finds that differentiation instruction overcomes the issue of academic variety in the classroom by focusing on use and development learning materials based on skill and processes.

Tomlinson [6] explains that differentiated instruction is a method to adjust the learning to the needs of students with the purpose of maximizing the potential of each student in the stipulated scope. Teachers consider individual students before determining the learning plan that the learning readiness, interest, and learning profile of the students are not neglected. Tomlinson [7] states that students may learn better if the task is appropriate to the students’ capability and understanding (readiness), the task stimulates the students’ curiosity or passion (interest), and the task encourages students to work in their preferred ways (instruction profile). The teacher differentiates the learning content, process, and product by considering the differences of the students' readiness, interest, and learning profile. Tomlinson [8] shows that differentiation may be done in various ways, and if the teacher is willing to use this philosophy in the classroom, the teacher selects a more effective instruction methods that respond to the diverse needs of students. Tomlinson [9] states that differentiation is not just a learning strategy. It is not a recipe for teaching either, but it is an innovative way of thinking about learning and teaching.

Hall [10] differentiated instruction is an effort to recognize the students' various backgrounds, readiness level, language, interest, and learning profile. Differentiated learning sees learning experience as social and collaborative. The persons in charge of what happens in the classroom are the teacher and students. Tomlinson [11] Building on this definition, Mulroy & Eddinger [12] also adds that differentiated instruction occurs in the context of increasingly diverse population of students. Each student is appreciated for his/her unique power. They are offered the opportunity to show their skill through various assessment techniques [7,12].

Tomlinson shows that the teacher is a professional in the classroom, a person trained to become mentor and leader, who use the correct techniques that to help each student to achieve their potential in the context of learning. Teachers are legally and ethically bound to become experts in developing students to full development [11]. Students are expected to respond to the teacher’s encouragement with independence and awareness so that they acknowledge the responsibility for their lives and learning. The relationship between teachers and students are clearly mutual: the responsibility of development becomes a joint effort.

Performing the differentiated instruction is not easy. Munro [13] teachers can differentiate their instruction effectively when they understand how their students learn and think, understand the various options to learning differentiation, may implement learning differentiation to the topics in their classrooms, have the correct motivational orientation, and may interpret the culture and climate of the school and the classroom in terms of this differentiation. For effective learning of mathematics, The Education Alliance [14] recommends learning strategy differentiated instruction through flexible grouping, individual learning, use of tiered tasks, and variable difficulty levels for questions.

Similar with Tomlinson and Department of National Education, NRC also emphasizes the importance of understanding students and of varied instruction that considers the characteristics and needs of students who are learning. NRC [15] states that there are 3 very important types of knowledge in teaching mathematics in school: knowledge of mathematics, knowledge of students, and knowledge of learning practice. Knowledge of students and how students learn mathematics includes general knowledge on how various mathematical ideas develop on children from time to time, and special knowledge on how to determining the direction of the students’ development.

Most teachers simply give one single instruction to all students. Single instruction is not sufficient for heterogeneous students. Hapsari, Putri, & Raharjo’s study [16] shows that students of lower and higher learning feel desperate when they come across questions that are too difficult, and they feel
motivated when come across questions that they can solve. Another characteristic of a good teacher is the understanding of students, including the students’ readiness level and interest that teachers may provide tasks and instructions appropriate with the students’ capability and interest level. Differentiated instruction is a cycle of learning that needs the understanding of the relevant students’ readiness level, interest, and learning profile, and the responding to these aspects with differentiated content, processes, and products. This was why the Researcher is interested in making a study entitled “Understanding and Responding to the Students in Learning Mathematics through the Differentiated Instruction”.

2. Methods
This research employs a survey method starting with literature studies on the differentiated instruction, followed by collecting the students’ responses on mathematic learning which is as expected by the students and figuring out their readiness, interest, and learning profiles. The next step is implementing the differentiated instruction in the class with the materials of number patterns and then examining the students’ responses. 62 Junior High School students of the 8th graders, participate in this study. The types of learning mathematics required by the students and their responses to the differentiated learning instruction are examined through questionnaire and interview.

3. Result and Discussion
3.1. Differentiated Instruction
Tomlinson [7,17] explains that differentiated instruction is instruction which differentiates content, process, and product according to the students' readiness level, interest, and learning profile. Readiness is entry point for students in obtaining certain knowledge or skills. Students with less readiness might need: a person to help them identify and bridge the gaps in learning, so that the students can move forward; more opportunities to direct learning or practice; activities or product that more structured or more concrete, with fewer steps, closer to the students’ experience, and emphasizing simple reading skill; a lot of practice with skill. In order to successfully differentiate instruction according to the students' readiness, Tomlinson [7] uses a certain device called the “equalizer”. It adjusts the correct “button” to the needs of various students – adjusting the proper challenge by using the content, process, and product in classroom as follows: basic to transformational, concrete to abstract, simple to complex, one step to a lot of steps, slow to fast, dependent to independent.

Tomlinson [7] “interest” is something that stimulates the students' curiosity and understanding upon the topic. A wise teacher knows that a key feature of the art of teaching is to have a plan that involves or stimulates students on familiar topics. “Involvement” is non-negotiable in learning and teaching. There are 2 (two) strong and relevant motivators to involvement: the interest of students and the options of students [18, 19]. If a student has a spark (or better, a flame) of curiosity on a topic, learning becomes more possible to him/her. Similarly, what options we have on what we instruct or how we instruct, also empowers, and therefore improves, the instruction. The problem is, of course, that not all students in the classroom have the same interest. Thus the need for differentiation.

Tomlinson [7] “learning profile” refers to the best ways a student learns something, on how we best learn as individuals. Each of us knows ways of learning that are effective for us, and others that obstruct learning or make learning feel awkward. Experience and study show to us that when teachers can make use ways that promote efficient and effective learning for students, the results get better. The purpose of differentiated learning profile is to help students understand ways of learning that work best for them, and to offer options to students so that each student finds the learning method that suits them in the classroom.

Tomlinson [7] content is input of learning and teaching, what the teacher teaches or what that teacher wants students to learn. Differentiated content can be thought of in two ways. Content can be differentiated as a response to the readiness level, interest, or learning profile. It can also be differentiated in response to each combination of readiness, interest, and learning profile.
Differentiation content strategies can be done with: concept-based teaching, compacting the curriculum, learning contract, small-group learning, and mentoring.

Tomlinson [7] “process” means opportunity for students to process content or ideas and skill. In scholastic language, process is frequently called “activities”. Process differentiation according to the readiness of the students means adjusting the complexity if the task to understanding and skill level of the students. Process differentiation according to the interest of the students involves providing options for students on aspects of the topic that help the students to connect to their personal interest, for the purpose of understanding the material. Process differentiation according to the learning profile of students generally means encouraging students to understanding ideas with preferred ways from learning samples, exploring or expressing what they learn kinaesthetically, spatially, verbally, or other means creatively; or deciding to work alone instead of a partner; or sitting on the floor to do their work instead of sitting on a chair. Differentiation process strategies can be done with: multiple intelligence, interest group, complex learning, independent learning, concept achievement.

Tomlinson [7] products must help students individually or in groups to rethink, use, and expand on what they have learnt for a specific period, sub-part, semester, or even a year. Product is an element of the curriculum that students can possess directly. Therefore, designing good product tasks can motivate because it would generate good work. Quality task product is a very good way to assess the knowledge, understanding, and of skill students. A lot of students can show what they know much better on products than written test. Therefore, in differentiated classrooms, teachers can replace some tests with rich task products, or combiines tests and options product so that the students’ scope becomes wide. They must have the maximum opportunity to think, implement, and show what they have learnt. Differentiated products can be: tests, posters, papers, task, articles.

3.2. Understanding and Responding the Students in Learning Mathematics

One of the teacher’s duties is trying to understand and respond to the students. One of the ways in understanding the students is by knowing what is required by the students. The students’ opinions on what instruction types they require are elicited using an open-ended questionnaire with this question: “What mathematics instruction types do you really require or like?” Based on the questionnaires completed by the students, there are 10 (ten) characteristics of mathematics instructions that are most frequently mentioned. They are in orderly presented in Table 1.

| Mathematics Learning Types Required by the Students | Frequency (%) |
|---------------------------------------------------|---------------|
| 1. Easy to understand                             | 21 (33.87)    |
| 2. Slowly /not rushing Teaching                   | 18 (29.03)    |
| 3. Fun                                            | 14 (22.58)    |
| 4. Not boring                                      | 13 (20.97)    |
| 5. Not complicated                                 | 10 (16.13)    |
| 6. Interspersed with humour                        | 10 (16.13)    |
| 7. Various question practices                      | 8 (12.90)     |
| 8. Not too serious                                 | 7 (11.29)     |
| 9. Conductive class atmosphere for the instruction | 7 (11.29)     |
| 10. Provided with rapid solution methods           | 7 (11.29)     |

The mathematics learning types required by the students is related to understanding of the students' readiness level, interest, and learning profiles. “slowly/not rushing Teaching” is related to the students' readiness. To differentiate learning based on the students’ readiness, Tomlinson [7] successfully utilizes an instrument called equalizer, that is, by pressing the proper button to meet the students’ needs adjusting the right challenge by having a faster learning more than before. “Fun instruction” is related to the students’ interest level, such as instruction related to a topic that the students are
interested in. Tomlinson [7] says that if a student has a spark (or better yet, flame) of curiosity on topic, instruction is more possible for students. “Interspersed with humour”, “not too serious”, conducive class atmosphere for instruction is related to their learning profile. The research shows that students at primary and secondary school levels have better achievements when learning is conducted based on their preference (Sternberg, [21]; Sternberg, Torff, & Grigorenko [22]). The students' readiness level, interest, and learning profile are various. In order to respond to this variation, teachers need to differentiate the content, process, and product so that the students require and need are not ignored and the learning goes properly.

Before starting differentiated learning, the teacher does three things:

1. Use diagnostic assessment to determine the readiness of students. This assessment can be formal or informal.
2. Determine the interest of students. This can be done using interest supply and/or involving students in the planning process. Teacher can ask students to tell about any specific interests that students may have in the topic, and then teacher can attempt to combine the interest into the lesson.
3. Identify learning style students and environmental preference. Learning style can be measured using the parameters in the list of learning styles.

The teacher varies learning strategy according to the needs of students. During the lesson unit, the teacher must assess students continuously. This assessment can be formal, but is frequently informal and can include recording the progress of students, checking the students’ work, and asking students questions on their understanding of the topic. The assessment result can then be used to direct the subsequent lessons. Strategies that can be implemented in differentiated learning are tiered tasks, compacting, interest centre and interest group, flexible grouping, learning contract, and options board [7,17].

Differentiated instruction in this study are flexible grouping, tiered assignment, and compacting. “Flexible grouping” is students work as part of various groups depending on task and/or content. Sometimes students placed in groups according to readiness, but other on times students are placed according to interest and/or learning profile. Groups can be determined by teacher or selected by students. This strategy creates teams without labelling whether students are “advanced” or “having difficulties”, so it allows them to work with various types of team mates. Radenchich and McKay [23] state that non-static classification may lead the students to frequently made various changes, such as working in pairs, small groups, or even with the whole class members.

Tiered tasks are designed to train students on important skills. These tasks are given in various levels of complexity and abstraction, and they are open ended. The content and the curricular purpose are the same, but the process and/or product vary according to the readiness level of the students. Heacox [5] explains that in the differentiation classes, a teacher may use various assignment levels to make sure that the students are able to explore their ideas and use their skills based on their levels of understanding and encourage their progress. When students are learning with different difficulty levels regarding to their assignments, all students may explore their essential ideas and similar tasks with various thinking levels. The groups will eventually gather to share each other and learn together. The gradual assignments should be: different pieces of work; active; highly attractive; fair in terms of the tasks and time allotments; considering on the implementation of major concepts, skills, and ideas.

Compacting is the process of adjusting instruction by considering the mastery of students. Compacting provides talented students the opportunity to learn advanced concepts instead of just sitting around following the instruction that they do not need. Renzulli & Reis [20] compacting curriculum can be used in all classrooms to help students who have proven mastery of the material, while their peers have not mastered it. “What must I do now?” is a question that a lot of teachers and students ask in frustration. Compacting curriculum has been shown as an intervention that succeeds in maintaining the involvement of students with high potential. Notes on the implemented strategies can be seen in the following Table 3.2.
Table 3.2 Strategies Used on the Differentiated Instruction

| Strategy                                | Focus Differentiation          | Implementation                                                                                                                                 |
|-----------------------------------------|---------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| Flexible grouping                       | Readiness, interest, learning profile | - Students grouped according to readiness level, students with high readiness are grouped with students with low readiness. Students with high readiness explain to students with low readiness in the form of group discussions.  
- Students with the same interest are grouped into the same groups and given learning materials and tasks relating to the interest.  
- Students with the same learning style are grouped into one and given exercise sheets appropriate to the learning style. |
| Tiered assignment                        | Readiness                       | - Students are grouped into 3 levels (high, medium, low) according to diagnostic tests. Teacher creates 3 question levels. Questions level 1 is related with basic questions. They use formulas.  
Questions level 2 in relation with basic questions and partially questions applications. Level 3 in relation with questions questions applications that in relation with material arithmetical series.  
Students are given questions and solve them individually according to their levels. After all individual work is completed, the questions are discussed in the classroom.  |
| Compacting                               | Readiness                       | - The classroom learns about arithmetical series. Diagnostic tests show that 3 students have already mastered the concept of arithmetical series. These 3 students are released from reviewing the concept of arithmetical series. As replacement, they are given questions related to the concept of arithmetical number series. |

3.3. Students’ Response to the Differentiated Instruction

According to observation and interview of students, they respond positively to differentiated instruction. Students who were initially reluctant to get involved with mathematical learning become involved voluntarily after given tasks that are not difficult (appropriate to their individual capabilities).
Students who have mastered mathematics become more excited and challenged when given difficult tasks. Students love the tiered task options. This is in line with what [7] said: students learn better if the task is appropriate to their capability and understanding (readiness).

McAdamis [3]; Christensen [4] also states that learning differentiation improves the motivation and enthusiasm of students in learning. Students do not feel enthusiastic and feel bored when following the teacher’s explanation on materials that they have understood. Students are happy when released from such an instruction and when the instruction is replaced with advanced challenges. This is in line with Renzulli & Reis’ [20] finding: compacting curriculum is shown as an intervention that succeeds in maintaining the involvement of students with high potential.

Students respond positively to flexible grouping in differentiated instruction. Group learning allows students to discuss with their friends and ask questions when they have difficulties. Flexible grouping makes students not bored, because it avoids exclusive grouping. According to the questionnaire, 90% of students like learning groups. Students said this is because with group learning, when they do not understand something they can ask their friends and can be discuss with them so that they can understand better. The work becomes easier and they can socialize and cooperate at the same time.

Furthermore, according to the questionnaire, 70% of students prefer that group members are appointed by their teacher. Students feel that when the teacher determines the group members, the division becomes fairer. When groups select themselves, the students tend to keep on picking the same people, and smart students tend to group themselves with equally smart students. This is in line with what Butler & Luwe [6] say: differentiated learning is effective in group work. Through groupings, students will find it easy to interact with their friends. This is a strength that helps them understand and solve the instructions.

4. Conclusion
Students expect mathematical instruction to be easy to understand, fun, not rushed, and not too serious. Implementing the differentiated instruction to mathematics instruction is not easy. The teacher must master various knowledge, assess students continuously, and plan instruction properly although time consuming. Students respond positively to the differentiated instruction. They become more motivated and involved in learning.

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