A Continuous Teleoperation Subspace with Empirical and Algorithmic Mapping Algorithms for Non-Anthropomorphic Hands

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Abstract — Teleoperation is a valuable tool for robotic manipulators in highly unstructured environments. However, finding an intuitive mapping between a human hand and a non-anthropomorphic robot hand can be difficult, due to the hands’ dissimilar kinematics. In this paper, we seek to create a mapping between the human hand and a fully actuated, non-anthropomorphic robot hand that is intuitive enough to enable effective real-time teleoperation, even for novice users. To accomplish this, we propose a low-dimensional teleoperation subspace which can be used as an intermediary for mapping between hand pose spaces. We present two different methods to define the teleoperation subspace: an empirical definition, which requires a person to define hand motions in an intuitive, hand-specific way, and an algorithmic definition, which is kinematically independent, and uses objects to define the subspace. We use each of these definitions to create a teleoperation mapping for different hands. We validate both the empirical and algorithmic mappings with teleoperation experiments controlled by novices and performed on two kinematically distinct hands. The experiments show that the proposed subspace is relevant to teleoperation, intuitive enough to enable control by novices, and can generalize to non-anthropomorphic hands with different kinematic configurations.

Index Terms — Telerobotics and Teleoperation, Grasping, Human Factors and Human-in-the-Loop

I. INTRODUCTION

Teleoperation is a valuable tool for robotic manipulators in highly unstructured environments, where a wide array of scenarios and objects can be encountered. In such conditions, the robot can rely on human cognition to deal with corner cases faster and more easily than fully autonomous manipulation planners. An important research direction for robot teleoperation aims to make the controls available to the operator as intuitive as possible: intuitive controls minimize the training time required for human teleoperators and can make teleoperation more accessible to novices. They also ensure a safe and effective workflow.

For manipulation, teleoperation controls which harvest the user’s hand motions, rather than using a joystick or a point-and-click interface, can provide an intuitive and user-friendly interface [1], because they harness motions which are already natural to the teleoperator. An example of this workflow is shown in Figure 1a.

Teleoperating a robot hand using a human hand as input requires a teleoperation mapping, which tells the robot hand how to move in response to movements of the human hand. Robot hand designs that are fully-actuated and highly anthropomorphic allow for a direct joint mapping to the human hand and thus are intuitive for a human to teleoperate; however, the hardware tends to be fragile and expensive. In contrast, non-anthropomorphic hands have proven to be robust and versatile in unstructured environments. However, finding an easy or intuitive mapping between the human hand and a non-anthropomorphic robot hand can be difficult, due to the different joint configurations, different axes, different numbers of fingers, or any number of dissimilarities between the hands.

In this paper, we seek to create a mapping between the human hand and a fully actuated but non-anthropomorphic robot hand that is intuitive enough to enable effective real-time teleoperation, even for novice users.

The method we propose uses a subspace relevant to teleoperation as an intermediary between the pose spaces of two different hands. Our method enables teleoperation by...
projecting the pose of the master hand into the defined teleoperation subspace, which it shares with the slave hand, and then projecting from the teleoperation subspace into the pose space of the slave hand.

At a conceptual level, each of the basis vectors that define the subspace corresponds to a hand motion: hand opening, finger curl, and finger spread (Figure 1b). While these concepts are natural for the human hand, we need to also define them in the context of non-anthropomorphic robotic hands. We show that this process can be done empirically: in this formulation, the person creating the teleoperation mapping defines what the motions of ‘open’, ‘curl’, and ‘spread’ mean for a specific robot hand. In this way, the mapping is tied to hand kinematics, since the hand motions mean different things for different hands. We show that this empirical mapping can be created following a series of simple steps (Section IV) and leads to effective teleoperation for novices.

One shortcoming of using an empirical mapping is its reliance on human intuition: effective teleoperation could be attributed to either the structure of the subspace, or simply to the hand-specific intuition provided by the person creating the mapping. We strive to show that the teleoperation subspace which we propose can also be defined without such hand-specific intuition.

We therefore propose a second, algorithmic, method where we formalize the notion of the hand motions used to define the subspace. Rather than considering, for example ‘hand opening’ as an intuitive concept to be defined by the person creating the mapping, this paradigm considers hand opening as the hand grasping a series of incrementally larger objects. In this way, we can use a set of objects to provide the same understanding of hand motions as the user provided in the empirical method.

This definition of the subspace is done exclusively through an object set, and is hand-independent. However, it lends itself to the algorithmic creation of a teleoperation mapping for any hand. We introduce a method which uses this algorithmic definition to generate subspace mappings for hands in a fully automated fashion (Section V). We aim to show that this algorithmic mapping also enables effective teleoperation for novices, implying that the value of the teleoperation subspace does not derive exclusively from hand-specific human intuition used to create it.

We create teleoperation mappings for two non-anthropomorphic robotic hands using both the empirical mapping and the algorithmic mapping. In teleoperation experiments with novices, we show that the mappings created with both these paradigms can enable teleoperation as fast as or faster than state-of-the-art teleoperation methods. Overall, the main contributions of this paper are:

- We introduce a continuous, low-dimensional teleoperation subspace as an intuitive way to map human to robot hand poses for teleoperation. We posit that this method allows for intuitive teleoperation as long as both the master and the slave hand poses can be projected into this subspace.
- We provide an empirical method to define the subspace and to create a projection into the subspace. This method requires the user to define the hand motions for the subspace using their intuition and based on the hand’s kinematics.
- We provide an algorithmic method for defining the subspace as well. This definition is independent of hand kinematics and lends itself to an automated algorithm which can create a mapping for hands of various kinematics. We are the first to show that an automated method for generating a teleoperation mapping can enable online teleoperation which is intuitive for novices.
- We show experimentally that the subspace is relevant to teleoperation for two different non-anthropomorphic hands and for two different manipulation tasks. These experiments demonstrate that our mappings created empirically and algorithmically allow novice teleoperators to pick and place objects and perform in-hand manipulation as fast as or faster than state-of-the-art teleoperation mapping methods using robot hands with non-anthropomorphic kinematics. As we discuss in the next section, in most literature on hand teleoperation, mappings are usually only validated on one or two expert users and on one robotic system. We validate the teleoperation subspace mappings on nine novice users and two different robotic hands.

In an earlier version of this study [2], we have introduced the concept of a teleoperation subspace defined exclusively via empirical mapping, and validated it with teleoperation experiments on a single robotic hand for a single task. Here we show that the subspace can be defined in a hand-independent fashion by considering variations in the grasped object shape, and introduce a fully automated process for creating mappings into this subspace. We also validate both mapping methods with multiple manipulation tasks on two kinematically distinct robotic hands.

II. RELATED WORK

The most common ways to create teleoperation mappings are: joint mapping [3], fingertip mapping [4], and pose mapping [5].

Joint mapping (also called joint-to-joint mapping) is used when the slave hand has similar kinematics to the human [6]. If the human and robot joints have a clear correspondence, the human joint angles can be imposed directly onto the robot joints with little or no transformation [3]. This mapping is most useful for power grasps [7], and is limited if the robot hand is non-anthropomorphic.

Fingertip mapping (also called point-to-point mapping) is the most common teleoperation mapping method. Forward kinematics transform human joint angles into Cartesian fingertip positions. These undergo scaling to find the desired robot Cartesian fingertip positions and then inverse kinematics determine robot joint angles. This mapping is useful for precision grasps [7].

For fingertip and joint mapping, how to reconcile kinematic differences between the human and robotic workspaces is an open question. Humans use only 3.6% of the potential workspace for the thumb during grasping [8]. So, if a robot finger maps to the human thumb, that finger will not move...
significantly during grasping tasks, unless the teleoperator adapts their grasping in unintuitive ways. To solve this problem, researchers combine different types of mappings [9], use virtual object mapping with special considerations for the workspace differences [10], optimize distances in task space [6], use error compensation [4], and alter the robotic hand frame for individual grasp postures to minimize the workspace differences [11]. All of this is additional work and computation for the human to create and test.

Pose mapping attempts to replicate the pose of the human hand with a robot hand, which is appealing because, unlike fingertip and joint mapping, it attempts to interpret the function of the human grasp rather than replicate hand position. Pao and Speeter define transformation matrices relating human and robot poses, using least squared error compensation when this transformation is not exact [5]. Others use neural networks to identify the human pose and map the pose to a robot either through another neural network [12] or pre-programmed joint-to-joint mapping [13]. Recently, an end-to-end solution was proposed, trained on a traditional pose mapping dataset but which tries to directly predict joint angles of the robot hand without first classifying the pose of the human hand [14].

Outside of a discrete set of known poses, pose mapping can lead to unpredictable hand motions. Furthermore, the mappings that use neural networks require classification of the human hand pose before it is mapped to the robotic hand. If this classifier misidentifies the human pose, the robot hand will move in undesirable ways. Our method also attempts to replicate hand shape, rather than fingertip or joint positions, making it most similar to pose mapping, but we do not require discrete classification of human pose before mapping.

This paper introduces a low-dimensional mapping. Other methods that define grasping in a low dimensional space include postural synergies, which are low dimensional and continuous [15]. Just as synergies move the description of human hand position from discrete, static poses [16] into a continuous space, we seek to allow pose mapping between the human and robotic hand to be continuous instead of interpolating between discrete poses.

Some works find synergies of robot hands by finding robot poses that resemble grasping poses for human hands, and then performing PCA on those poses. The poses are either found through joint mapping [17], pose mapping [18], or human intuition [19] [20]. Others use postural synergies to underactuate anthropomorphic hands, like the Pisa/IIT Softhand 2 [21], but user intuition is required to build synergies into the robotic hand design.

Other works use low dimensional latent variables which are not based on synergies to approximate human poses in non-anthropomorphic models. These latent variables have enabled both the animation of non-anthropomorphic creatures [22] and teleoperation. Gaussian process latent variable models (GP-LVM) can enable teleoperation of humanoid robots. In some formulations, the latent space changes with every different master-slave pairing [23]. In other formulations, multiple robots and a human share the same latent space [24].

Training data driven mappings, like some pose mappings [19], or GP-LVMs, requires the user to create many corresponding poses between the human and robotic hands. Creating these corresponding poses is often tedious and time consuming. We are inspired by a similar desire to find shared subspaces between robotic and human hands, but our algorithmic and empirical methods for generating teleoperation mappings reduce the burden placed on the user by eliminating the requirement to create tens or even hundreds of corresponding poses between the human and robot.

There are works which, like our algorithmic mapping, try to create teleoperation mappings without requiring that the user provide intuition about hand kinematics.

Kheddar et al. proposed high level abstraction teleoperation, where the operator manipulates a virtual environment and a bilateral transform translates changes in the virtual environment into commands for the robot, such that the slave mimics the change in the real environment [25]. The gripper control is object based, i.e. the robot must manipulate and transport an object in the real world in the same way it is being manipulated in the virtual environment [26]. Although they describe several possible ways to transform between the human and robotic hands, their ultimate solution is autonomous, and does not include a teleoperation mapping.

Kang et al. also introduced an object based approach to identifying human grasps using the contact web [27]. Once the human grasp has been identified, the robot hand is shaped based on virtual fingers and the human grasp. However, the user still provides an understanding about how the robot functions - for each new robot, they assign the fingers as being a primary finger, a secondary finger, or a palm.

Finally, Gioioso et al. defined an object based approach for mapping between hands with dissimilar kinematics using virtual objects [28], [29], [30], [31]. This work replicates the deformation of the virtual object in the human hand with the virtual object in the robot hand. This is the first time a (virtual) object set was used to define a teleoperation mapping. However, the authors have reported varying performance for the same hand with different number of virtual points and different numbers of synergies, meaning that creating the mapping for each hand requires the user to tune control parameters, including how many contact points to use, where to place these contacts, and which synergies to use.

All of the above work requires the user to give some input which provides an understanding about the hand’s kinematics or function. Some groups have moved towards teleoperation mappings which reduce the burden on the user, but our algorithmic mapping is the first to create a teleoperation mapping completely automatically, using a subspace definition which is independent of hand kinematics.

In the literature on hand teleoperation, most works validate their proposed teleoperation methods on one or two expert users or perform their experiments in simulation (e.g. [31], [4], [5], [7], [9], [10], [12], [13], [28], [29], [30]). We were only able to find two works that validate a proposed teleoperation method with novice users on a physical robot [14], [31]. Both of these works validated their method with five novices users teleoperating a single robotic hand. In this paper, we validate our work over two different tasks with a total of nine novice users, using two robotic hands with different kinematic
III. Teleoperation Subspace

As a general concept, we posit that, for many hands, there exists a three dimensional space $T$ isomorphic to $\mathbb{R}^3$ that can encapsulate the range of movement needed for teleoperation. The three dimensions of $T$ correlate to certain hand motions: opening and closing the hand, spreading the fingers, and curling the fingers. We will refer to these as the size $\sigma$, spread $\epsilon$, and curl $\psi$ basis vectors, respectively.

We chose these bases on intuition, guided by Santello’s research of postural synergies [15]. Since Santello et al. used principle component analysis (PCA), a linear dimension reduction method, to find postural synergies, we also assume that mapping between pose space and teleoperation subspace is linear.

We assume that many hands will be able to project their pose spaces into $T$. If this projection is possible, $T$ is embedded as a subspace in the pose space of the hand. $T$ is thus a subspace “shared” by all hands that can project their pose space into $T$. If the user can construct a projection matrix which projects pose space to teleoperation subspace in a meaningful way, our method will enable teleoperation. Experimentally, we will show that this is the case for at least the human hand, the Schunk SDH, and a two-fingered gripper.

We theorize that $T$ is also relevant to teleoperation for other hands.

To teleoperate using $T$, there are two steps:

1) Given joint values of the master hand, find the equivalent pose $\psi$ in teleoperation subspace $T$.
2) Given $\psi$ computed above, find the joint values of the slave hand, and move the slave hand to these values.

These steps are illustrated in Figure 2. In order to enact the teleoperation steps, we must first define the mapping between $T$ and the relevant pose spaces.

A. Teleoperation Subspace Mapping

For a given hand with $N$ joints, projecting from joint space $q \in \mathbb{R}^N$ (we use pose space and joint space interchangeably) into teleoperation subspace $T$ requires an origin pose $o \in \mathbb{R}^N$, a projection matrix $A \in \mathbb{R}^{N \times 3}$, and a scaling factor $\delta \in \mathbb{R}^3$.

1) Origin $o$: To project between joint space and $T$, we require a hand-specific, “neutral” origin pose $o \in \mathbb{R}^N$.

$$o = [o_1, o_2, ..., o_N]$$

This represents a hand position which will standardize the data we project between joint space and $T$. The origin pose of the master is arbitrary; however, it is crucial that the origin pose of the slave corresponds to the master’s origin. The two hands should assume approximately the same shape while positioned at their respective origins.

2) Projection Matrix $A$: The projection matrix $A \in \mathbb{R}^{N \times 3}$ is hand specific and consists of three basis vectors $\alpha_H, \sigma_H, \epsilon_H \in \mathbb{R}^N$. Whereas $\alpha$, $\sigma$, and $\epsilon$ represent the general concept of a hand motion, $\alpha_H$, $\sigma_H$, and $\epsilon_H$ are the projection of that motion into the pose space of a specific hand $H$.

$$A = [\alpha_H, \sigma_H, \epsilon_H]$$

$$\alpha_H = [\alpha_{H1}, \alpha_{H2}, ..., \alpha_{HN}]^T$$

$$\sigma_H = [\sigma_{H1}, \sigma_{H2}, ..., \sigma_{HN}]^T$$

$$\epsilon_H = [\epsilon_{H1}, \epsilon_{H2}, ..., \epsilon_{HN}]^T$$

3) Scaling Factor $\delta$: We wish to normalize such that any configuration in pose space will project to a pose in $T$ whose value is less than or equal to 1 along each of the basis vectors. We therefore require a scaling factor $\delta \in \mathbb{R}^3$ to normalize the projection:

$$\delta = [\delta_\alpha, \delta_\sigma, \delta_\epsilon]$$

To calculate $\delta$, we evaluate poses which illustrate the extrema of the hand’s kinematic limits along the basis vectors. For example, the maximum and minimum values along $\sigma$ are illustrated by projecting poses where the hand is holding the largest object possible and the smallest object possible from pose space into $T$. Once we select the illustrative poses for the hand, we project these poses from pose space into $T$ using $\psi = (q - o) \cdot A$, where $\psi \in T$. From this set of poses in $T$, we find the minimum and maximum values along each axis. Along $\alpha$, the minimum and maximum are referred to as $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$, respectively. From these values, we calculate $\delta_\alpha$ as:

$$\alpha_{\text{range}} = \text{abs}(\alpha_{\text{max}}) + \text{abs}(\alpha_{\text{min}})$$

$$\delta_\alpha = \begin{cases} 0 & \text{if } \alpha_{\text{range}} = 0 \\ 1/\alpha_{\text{range}} & \text{otherwise} \end{cases}$$

Finding $\delta_\sigma$ and $\delta_\epsilon$ uses the same calculation.

$\delta$ normalizes the projection from pose space to $T$; however, to project from $T$ back to pose space, we require an inverse scaling factor $\delta^*$:

$$\delta^* = \left[ \delta^*_{\alpha}, \delta^*_{\sigma}, \delta^*_{\epsilon} \right]$$

$$\delta^*_{\alpha} = \begin{cases} 0 & \text{if } \delta_\alpha = 0 \\ 1/\delta_\alpha & \text{otherwise} \end{cases}$$

where we find $\delta^*_\sigma$ and $\delta^*_\epsilon$ with similar calculations.
A specific matrix \( T \) teleoperation subspace

Fig. 3. Origin poses of two example hands.

4) A Complete Projection Algorithm: To project between teleoperation subspace \( T \) and joint space \( q \), we use the hand-specific matrix \( A \), the origin \( o \), and the scaling factor \( \delta \):

\[
\psi = (q - o) \cdot A \odot \delta \tag{11}
\]

\[
q = ((\psi \odot s^*) \cdot A^\top) + o \tag{12}
\]

where \( \odot \) represents element-wise multiplication.

To use \( T \) for teleoperation, Eq. [11] projects from the master hand’s pose space into the shared teleoperation subspace and then Eq. [12] projects from the shared teleoperation subspace into the slave hand’s pose space (Figure 2).

So, given the joint angles of the master hand, we are able to calculate the joint angles of the slave hand using:

\[
q_s = (((q_m - o_m) \cdot A_m) \odot \delta_m \odot \sigma^*) \cdot A_s^\top + o_s \tag{13}
\]

Now that we have formalized the subspace and what is required to map between pose space and the subspace, we propose two different methods to define the mapping. The first, empirical method, is dependent on hand kinematics and relies on the intuition of the person creating the mapping. The second, algorithmic mapping, is created automatically, using a definition of the subspace which is independent of hand kinematics.

IV. Empirically Defining the Subspace Mapping

The teleoperation subspace mapping can be created empirically through a relatively simple process. For each hand, the user must:

- Select an origin pose. Figure 3 shows the pose we chose for the human hand and the Schunk SDH robot in our experiments.
- Determine poses which illustrate the extrema of the hand’s kinematic limits along the basis vectors. It is up to the user to determine poses which illustrate the full range of values for each basis vector. Figure 4 shows the poses which demonstrate these ranges for the human hand.
- Define what the hand motions (finger spread, finger curl, and hand opening) mean in the context of the hand’s kinematics, then identify which joints contribute to that motion. This is a winner-take-all approach, so a joint may only contribute to a single motion. We set joints which adduct the fingers to 1 in \( \sigma^H \), joints which open the hand to 1 in \( \epsilon_H \), and joints which curl the fingers to 1 in \( \epsilon_H \).

We then normalize the vectors to create \( A \). Table I shows this process for the Schunk SDH hand.

Creating the empirical mapping is a simple, winner-take-all, three step approach. Despite this simplicity, we show experimentally that these calculations are sufficient to meaningfully project pose space into \( T \) in a way that enables teleoperation for novice users.

V. Algorithmically Defining the Subspace Mapping

In the previous section, we rely on a human to look at the hand’s kinematics, define each of the motions associated with the different subspace basis vectors, and then determine which joints contribute to that motion.

We would like to demonstrate that we can define the subspace in a way which is independent of hand kinematics. We also hypothesize that this subspace definition allows us to create a teleoperation subspace mapping for a hand automatically (i.e. an algorithmic mapping). If the algorithmic mapping can enable teleoperation for novices, this would demonstrate that the value of the teleoperation subspace does not derive exclusively from the human intuition used to create it.

To create a subspace mapping algorithmically, we must formalize the notion of a hand motion in a way that does not depend on the hand’s kinematics. We do this using objects. Hand opening can be thought of as the hand grasping a series of objects that grow incrementally larger. Spreading the fingers results from the hand grasping a series of objects whose curvature increases incrementally. Finger curl is binary, and can be defined as the difference between a precision grasp and a power grasp for the same object.

Based on this formalized notion of hand movements, we can use object characteristics to predict the location of a grasp in \( T \). We posit that when a hand, regardless of kinematics, is holding an object, we can predict where the resulting grasp will lie in the teleoperation subspace, based on the object’s size, shape, and the type of grasp used. This is illustrated in Figure 5.

If we can use the object’s characteristics to predict where a grasp will lie on the subspace, we can create a set of objects which we predict will result in grasps along the basis vectors of \( T \). Regardless of a hand’s kinematics, when the hand holds any of the objects in this set, the resulting grasp will lie along one of the basis vectors of \( T \). The object set we design consists of 8 objects and is described in more detail in Section V.A.
TABLE I

| Hand Motion | Motion Defined by Hand-Specific Kinematics | Joints which affect the motion | Basis vector |
|-------------|-----------------------------------------|-----------------------------|--------------|
| Finger Spread | | | \( \alpha_{\text{schunk}} = [1, 0, 0, 0, 0, 0, 0] \) |
| Hand Opening | | | \( \sigma_{\text{schunk}} = [0, 0.577, 0, 0.577, 0, 0.577, 0] \) |
| Finger Curl | | | \( \epsilon_{\text{schunk}} = [0, 0, 0.577, 0, 0.577, 0, 0.577] \) |

If a hand of any kinematic configuration grasps all the objects in our set, the result will be a set of grasps in the pose space of that hand, but that we predict can be used to find the basis vectors of \( T \). So, given a hand with a specific kinematic configuration, for each of the objects in our object set, we can generate a set of grasps \( G_{\text{object}} \). Each of the grasps \( g \) in \( G_{\text{object}} \) shows one possible way for a hand to grasp that object in a stable configuration. Each grasp \( g \) is an \( N \) dimensional vector, where \( N \) is the number of degrees of freedom for the hand. Once we have generated grasps for each of the objects, we can combine these individual sets into one grasp set \( \mathcal{G} \), which encompasses all the objects:

\[
\mathcal{G} = \bigcup_{i=1}^{8} G_{\text{object}i}
\]

Since \( \mathcal{G} \) is a set of grasps in pose space which spans \( T \), we can find a model of \( T \) by fitting a subspace to \( \mathcal{G} \). The model for \( T \) provides us with the subspace mapping needed to teleoperate the hand. The model of the subspace includes the origin and the directions of the basis vectors, which translate to \( o \) and \( A \) in the teleoperation mapping. We can then find \( \delta \) with a simple iterative method.

Once the object set has been designed, algorithmically creating a subspace teleoperation mapping requires three steps. For both the master and the slave hand, we need to:

- Generate a set of grasps \( \mathcal{G} \) where the hand is grasping each of the objects in the object set.
- Fit a subspace to the grasps. The subspace model provides us with the projection between \( T \) and joint space.
- Use an iterative approach to find \( \delta \).

Once the mapping has been generated for a given hand, it does not have to be generated again for a new master slave pairing. For example, once we generate the human mapping,
it will work with slave hand mappings that we generate in the same way.

We discuss the design of the object set, and the steps needed to implement teleoperation in the sections below.

A. Object Set

We hypothesize that we can design a set of objects to elicit grasps which lie along the basis vectors of \( T \). Table II and Figure 6a show the objects in our set, and where in the subspace we predict hands grasping those objects will lie.

The object set consists of eight objects. We use disks and boxes as our shape primitives. We specify the type of grasp (power or precision) which must be used with each object, in order to guarantee the grasp’s location along the curl basis vector of \( T \). In the set, there are objects that have the same dimensions, but are grasped with a different grasp type.

The approach direction of the hand is along the z axis, and we orient the objects in the same way relative to the hand.

B. Grasp Generation

Once we define our object set, we generate a set of grasps \( G \), which demonstrate how a hand of a specific kinematic configuration can hold the objects in our set in stable configurations. For robotic hands, we generate \( G \) using a grasp planner, and for human hands, we use human subjects.

We acknowledge that there are many ways to grasp an object. To compensate, we generate multiple grasps for each object, and use a subspace fitting method which is robust to outliers. In this way, we assume that we have sufficiently sampled grasps for the object set which would fall along the basis vectors of the subspace.

1) Robot Datasets: To generate the robot grasps, we use a grasp planner provided by the GraspIt! simulator. Given a hand and an object, the planner returns grasp configurations in which the hand stably grasps that object, ranked by the epsilon quality metric [32]. This quality metric is a geometric method that determines the total space of possible wrenches, within certain friction constraints, for a given grasp.

For grasp planning we apply a random search: we randomly sample an object pose (3 dimensions, we do not consider object rotation) that lies within the workspace of the hand. We also sample pre-grasp pose joint angles (\( N \) dimensions, where \( N \) is the number of degrees of freedom of the hand) that lie within the joint limits. We then close the fingers until they make contact with the object and evaluate the resulting grasp. In order to ensure robustness of the resulting grasps, particularly with respect to small deviations in object and pre-grasp pose, we also evaluate the grasps that arise when small perturbations are applied. Specifically we apply both positive and negative disturbances along each coordinate axis of the search space individually. Thus, for the \( 3+N \) dimensions from which candidate object and pre-grasp poses are sampled, we evaluate a total of \( 3(3+N) \) grasps. We choose the minimum quality encountered across these trials to represent the sampled grasp overall. This process is repeated until an iteration limit is reached and the sampled grasps are stored in a database.

| Identifier | Object | Grasp Type | Predicted location of grasp in \( T \) |
|------------|--------|------------|--------------------------------------|
| 1          | Disk   | x y z       | \( \psi = [0, 0, 0] \)               |
| 2          | Disk   | x y z       | \( \psi = [1, 1, 0] \)               |
| 3          | Box    | x y z       | \( \psi = [0, 0, 0] \)               |
| 4          | Box    | x y z       | \( \psi = [0, 0, 0] \)               |
| 5          | Disk   | x y z       | \( \psi = [0, 0, 0] \)               |
| 6          | Disk   | x y z       | \( \psi = [1, 0, 1] \)               |
| 7          | Box    | x y z       | \( \psi = [0, 0, 1] \)               |
| 8          | Box    | x y z       | \( \psi = [0, 0, 1] \)               |

Given a hand and an object, the planner returns up to 1,000 stable grasp configurations for that object. We parse the dataset by removing grasps which are closer than a parsing threshold \( \xi \) in Euclidean distance to a higher ranking grasp. \( \xi \) starts at 0.0 and is increased in intervals of 0.1. Each time \( \xi \) increases, the dataset is re-parsed. This is repeated until each object has fewer than 20 grasps remaining. We note that this means that the final parsed grasp set may have a different number of grasps for each object.

The object set we present is sized to the human hand. However, some robot hands are larger than the human hand. We therefore scale the objects based on hand size. For example, the Schunk SDH hand is approximately 1.5 times the size of the human hand. We therefore multiply the dimensions of the objects by 1.5 when we plan grasps for the Schunk SDH.

2) Human Dataset: Our grasp planner does not have a robust model of the human hand, so we instead generate a dataset for the human hand using grasps generated by test subjects.

Subjects were asked to don a instrumented dataglove (a Cyberglove III) and grasp objects in the object set. After the subjects grasp a given object stably, their joint angles are collected from the Cyberglove. We collected grasps from five subjects. The human dataset is not parsed because there are no metrics available which would tell us how well each of the subjects grasped the objects.

C. Fitting a Subspace to a Grasp Dataset

We hypothesized that grasps created by holding the objects in our set would exist in the pose space of the hand, but lie along the basis vectors of \( T \). If this is true, then we can find a model of \( T \) by fitting a subspace to the grasps in \( G \). We wish for this model of \( T \) to explain enough of \( G \) to enable teleoperation.

A model of \( T \) would provide us with the information necessary to create a teleoperation mapping for the hand. The model of the subspace consists of an origin and three \( N \)-dimensional orthogonal vectors, which describe the bases of the subspace. For a given hand, the origin pose of the subspace provides us with an origin pose \( o \) for \( T \), and the basis vectors provide us with a projection matrix \( A \).

To find the model of \( T \), we fit a subspace to the set of grasps \( G \) using RANSAC [33, 34]. RANSAC is a consensus based algorithm used to find the model underlying data with
a large number of outliers. The basic algorithm of RANSAC is as follows:

- Generate a model hypothesis using random samples from the dataset. The number of samples selected should be the minimum number needed to define your model.
- Looking at all the points in the dataset, determine how well the hypothesis model explains/supports the data. If it is better than the best hypothesis to date, update the best model to your current hypothesis.

This process is repeated $M$ times, where $M$ is a number high enough to ensure that the probability of finding a model that is better than the current best model is sufficiently low. For our algorithm, we use $M = 2,000,000$. When RANSAC is parallelized, its runtime is 187 minutes on a computer with 24 CPUs.

Since our subspace is three dimensional, our model hypothesis consists of an origin grasp and three basis vectors. We also keep track of which of the three basis vectors corresponds to size, spread, and curl.

To generate a model hypothesis, we select random samples from the dataset. We first select an origin grasp. We specify that the origin must come from the set of grasps where the hand is holding Object 8 ($G_8$). Preliminary tests with several hands showed that the performance for this origin was the highest. We hypothesize that this is because the constraints of the envelopes grasps are greater than the constraints of fingertip grasps. This gives the grasp planner (and the human) fewer options in how to grasp the objects, so the grasps have less variability.

Next, we select three additional grasps. We specify that each additional grasp must be selected from an object whose position in the subspace is identical to the origin object, except along a single basis vector. Since we have specified the origin, we randomly select one grasp from the set where the hand is holding Object 7 ($G_7$), another grasp from $G_4$, and the last grasp from $G_6$. These objects correspond to the size, curl and spread directions, respectively.

After we choose four random samples, we generate our model hypothesis. We subtract the three non-origin grasps from the origin and normalize the result to find the three basis vectors of the subspace. We randomize the order of the three basis vectors, then orthogonalize these three vectors using Gram-Schmidt orthogonalization [35].

We determine the quality of our model hypothesis by how many objects from the object set can be grasped using the hypothesized subspace. To determine how well the hypothesis model explains the grasp data, we find the inliers in $G$ by calculating the distance from each grasp to the subspace defined by the hypothesis model. The distance $d$ from each grasp $g$ to the hypothesis subspace model is found by projecting the grasp onto the subspace $g_{proj}$ and then finding the distance between the true grasp and the projected grasp:

$$
P = \omega_1^\top \cdot \omega_1 + \omega_2^\top \cdot \omega_2 + \omega_3^\top \cdot \omega_3
$$

$$
g_{proj} = P \cdot (g - o) + o
$$

$$
d = \| g - g_{proj} \|
$$

where $\omega_1$, $\omega_2$, and $\omega_3$ are the basis vectors of the hypothesis model, and $o$ is the origin of the hypothesis model. Grasps which are closer than $\xi$ (the final threshold used when we parsed the datasets) in Euclidean distance to the subspace are considered inliers:

$$
g = \begin{cases} 
\text{inlier} & \text{if } |d| < \xi \\
\text{not an inlier} & \text{otherwise}
\end{cases}
$$

Since we did not parse the human grasps, we simply set $\xi$ for the human dataset as 0.1.

Many RANSAC algorithms use the total number of inliers to estimate how well the model explains the data; however, we wish all parts of our subspace to fit equally well. If the grasps for a few objects contain all the inliers and grasps for all other objects are far from the subspace, we do not consider this to be a sufficiently good model, even if it has the highest total number of inliers. We want our model to be able to grasp all the objects in our dataset. So, we use a tiered metric which considers the quality of fit in all parts of the subspace.

Our tiered metric has 4 components, ranked by importance:

1) Minimum number of inliers per object, over all the objects in our set. For example, if each object has at least one inlying grasp, then we consider that model to be better than a model where one or more of the objects have no inliers, because we can grasp all the objects in our object set.
2) Number of objects which have the minimum number of inliers. If the minimum number of inliers is one, if only one object has one inlier and all other objects have more
than one inlier, this is preferable to all of the objects only having one inlier.
3) Total number of inliers across all grasps. The higher the number of inliers, the better the model.
4) Sum of the distances (error) between all the grasps and the subspace. The model with the lower error is better.

When two models tie in one or more of the metrics, the subsequent tier is used as a tiebreaker to determine the best model between two hypotheses.

Once we have generated and tested a sufficient number of hypotheses, the hypothesis model which explained the data the best, as defined by our metric, is considered to be the model of our subspace.

We perform one more processing step to find our final model. The same preliminary testing which indicated the best origin for the subspace model was Object 8 also showed that this was not the best origin when we combined the mappings for two hands into a complete teleoperation pipeline. For the final processing step, we choose a grasp from a different object to serve as the origin; empirically, we have found Object 1 to serve best in this role. For the robot hand, we move the origin to the grasp from $\mathcal{G}_{ob1}$ that is closest to the original subspace. For the human hand, we ask the teloperator to grasp a model of Object 1, and use the resulting pose as the subspace origin. Performing this additional step for every teleoperator also calibrates the mapping to the dimensions of their hand.

D. A Complete Mapping

1) Projection Matrix: We use the three basis vectors of the subspace model found by RANSAC as the vectors which make up the projection matrix $A$. During RANSAC, we keep track of which of the three vectors corresponds to size, spread, and curl. We use this information to determine which vector is $\sigma_H$, $\alpha_H$, and $\epsilon_H$, respectively.

2) Origin Pose: For a robot hand, the origin of the subspace model found by RANSAC becomes $\alpha$, the origin of the teleoperation mapping. For a human hand, we find the origin by asking the user to perform a calibration pose at the beginning of teleoperation. A standardized pose will not work for humans because user hand size varies.

3) Scaling Factor: To determine the scaling factors for our mapping, we require poses for the hand at the extremes of the subspace. We could select grasps from the dataset to determine these ranges, but it is faster to use a simple assumption and an iterative solution to find them.

For a robot, we assume that the hand will achieve its minimum and maximum value along each basis vector when the joints relevant to that basis are at some combination of their maximum and minimum values. We are given the maximum and minimum values for each joint from our robot model and the projection matrix tells us which joints are relevant to each subspace basis (if they are non-zero, they are relevant). For each subspace basis, we iterate through all the combinations of the relevant joints at their maximum and minimum values. These become the poses which show the hand’s kinematic extrema.

For the human hand, we require the human to perform four calibration poses which will give us the ranges along each basis vector (see Figure 4). We require these poses because the differences in user hand size mean ranges which work for one person may not work for another.

For both human and robot hands, we project all the poses into the subspace, using the projection matrix and the origin of our subspace model. We use the largest and the smallest value for each of the dimensions to calculate the range of that basis, and use Eq. 8 and Eq. 10 to find $\delta$ and $\delta^*$. We can use Equation 13 to teleoperate the slave hand.

VI. EXPERIMENTS

To validate that both the algorithmic and empirical mappings project to a subspace which is relevant to teleoperation, we asked nine novice users to complete manipulation tasks using both our mappings, and two state-of-the-art mappings as baselines. Four of the novices performed pick and place experiments with a Schunk SDH hand, and five of them performed in-hand manipulation tasks with a two fingered gripper.

For both the pick and place and the in-hand manipulation experiments, subjects were presented with the objects in the same order, and completed objects with one control before moving on to another control. We randomized the order in which the subjects used the controls. We also did not tell subjects how each of the control methods worked, but gave them two minutes to play with the hand when they were introduced to a new control method. The subjects gave their informed consent and the study was approved by the Columbia University IRB.

Below, we describe the mappings used in both experiments and the experiments themselves.

A. Subspace Teleoperation Mappings

We generated teleoperation mappings for the human hand, the Schunk SDH and a two fingered gripper. For each hand, we created teleoperation mappings empirically and algorithmically, using the procedures outlined in Section IV and Section V respectively. Figure 7 shows the resulting mappings for all three hands.

B. State-of-the-Art Comparisons

We selected two state-of-the-art teleoperation mappings with which to compare our subspace mappings:

1) Fingertip Mapping: We use fingertip mapping as a state-of-the-art comparison because it is one of the most common mapping methods and it is applicable to precision grasps, particularly with smaller objects [4]. The fingertip mapping was designed as follows: first, we found the cartesian positions of the thumb, index, and ring fingers of the human hand using the joint values from the Cyberglove and forward kinematics. The kinematic model we used for the human hand is described elsewhere [36]. We multiplied these positions by a scaling factor of 1.5, the ratio between an average human finger and the robot fingers. This ratio is 1.5 for both the Schunk SDH
Fig. 7. Teleoperation mappings generated for the human hand, Schunk SDH, and two finger gripper, both empirically and algorithmically. Each of the spokes represents a degree of freedom for the hand, and the blue (spread), red (size) and green (curl) values along those spokes indicate the values in the $\alpha_H$, $\sigma_H$, and $\epsilon_H$, respectively, at that degree of freedom.

and the two fingered gripper. We assign each human finger a corresponding robot finger (for the two finger gripper, only the thumb and the index fingers are used). We translated the coordinates from the hand frame into the finger frame to find the desired robotic fingertip positions. Finally, inverse kinematics determined the joint angles which placed the robot fingertips at these positions. This process is documented elsewhere for the Schunk SDH [11], and we use the same approach for the two fingered gripper.

2) Joint Mapping: We chose joint mapping as the second state-of-the-art comparison because of its common use in the field, its applicability to power grasps, and because we predicted that explicit control over individual joints of the robotic fingers would be intuitive for novice users [3]. To implement joint mapping, we assigned each of the robot joints to a corresponding human hand joint. This mapping can be found in Table III for the Schunk SDH and Table IV for the two fingered gripper. Once we received joint angles from the Cyberglove, we set the corresponding joints of the robot hand to the same values. Preliminary tests showed teleoperation is difficult if the robot thumb’s proximal joint maps to the human thumb’s metacarpophalangeal (MCP) joint. We therefore mapped the Schunk thumb’s proximal joint and the left proximal joint of the two finger gripper to the human thumb’s adductor.

### Table III

| Cyberglove Sensor | Robotic Hand Joints |
|-------------------|---------------------|
| Joint Label Name  | Joint Label Name    |
| e Index/Middle adduction | 0 Finger 1 adduction |
| a Thumb adduction  | 1 Thumb proximal flexion |
| b Thumb distal flexion | 2 Thumb distal flexion |
| c Index proximal flexion | 3 Finger 1 proximal flexion |
| d Index medial flexion  | 4 Finger 1 distal flexion |
| f Middle proximal flexion | 5 Finger 2 proximal flexion |
| g Middle medial flexion  | 6 Finger 2 distal flexion |

C. Pick and Place Experiments

We asked four novice users to complete pick and place experiments with our mappings and with state-of-the-art mappings.

We asked our novice users to pick and place the ten objects shown in Figure 8 using a Schunk SDH mounted on a Sawyer arm. The Sawyer’s end effector position and orientation are controlled with a simple cartesian controller (completely separate from the hand control) using a magnetic tracker (Ascension 3D Guidance trakSTAR™) placed on the back of the user’s hand. Figure 8 shows the experimental setup. Subjects were asked to pick up one object at a time and move the object across a line based on visual feedback.
TABLE IV
JOINT MAPPING FROM THE CYBERGLOVE TO THE TWO FINGER GRIPPER

| Cyberglove Sensor Joint | Robotic Hand Joints Joint |
|-------------------------|--------------------------|
| a Thumb adduction      | 0 Finger 1 proximal flexion |
| b Index distal flexion | 1 Finger 1 distal flexion |
| c Middle proximal flexion | 2 Finger 2 proximal flexion |
| d Middle medial flexion | 3 Finger 2 distal flexion |

D. In-Hand Manipulation Experiments

We asked five novice users to perform in-hand manipulation tasks with a two fingered gripper. The gripper is stationary and placed on a table. An object was placed on the table between the distal links of the fingers in a precision grasp. We then asked the subjects to transition the object to a power grasp by moving the object closer to the palm and enveloping it with the robot fingers. For a transition to be counted as successful, the subject had to move an object so that it was in contact with both the proximal and distal links on one finger and at least one link on the other finger. Figure 8 shows the experimental setup and the objects used for the in-hand manipulation experiments.

VII. RESULTS

For both experiments, we use three metrics to determine performance: time to complete the task, how many tries the subject needed to complete the task and how many objects for which the task was completed. We describe these metrics below.

Our first metric was time to completion: we timed how long it took for the user to perform the task with each object. If the user did not complete the task in the given time limit (two minutes for pick and place experiments and one minute for in-hand manipulation experiments), they were considered to be unable to pick up the object and their final time was set to the respective time limit. We elected to shorten the time limit for the in-hand manipulation because there is no arm to move, and therefore the task should be completed faster.

Our second metric is the number of tries needed to complete a task. We define a try as a completed task, an attempt where the user drops the object, an attempt where the user knocks over the object, or an attempt where the user knocks an object out of the range of the robot hand. In the last two scenarios, the object is reset by the experimenter. If the subject was unable to pick the object, we report the number of tries the user took before the time elapsed.

Our final metric is how many objects for which the task was completed: for each mapping we counted the number of objects with which the user was able to successfully complete the assigned task.
A. Pick and Place Results

We report our results as the average across all subjects. We report averages for all objects, for large objects (the box, ball, wire, screwdriver, and water bottle), for small objects (the peg, valve, and marble), and for irregular objects (the drill, screwdriver, and lego stack). We classify the drill, screwdriver, and lego as irregular objects because their width to length ratios and their irregular shapes allow users to pick up the objects with a wide variety of grasps. Users tended to pick up the large objects and the small objects with consistent grasps.

We report the average time to pick and place across all subjects in Table VIa. Across all subjects and all objects, novices using the fingertip mapping took 3 times longer than when using the empirical subspace mapping, and 1.7 times longer than when using the algorithmic subspace mapping. Similarly, joint mapping took 2.5 times longer than the empirical subspace mapping and 1.4 times longer than the algorithmic subspace mapping.

For the four combinations of objects we look at (all objects, small, large, and irregular), the empirical subspace mapping took the least amount of time, with the algorithmic subspace mapping coming in second in every case. The algorithmic subspace mapping was at least 1.6 times slower than the empirical subspace mapping for all of these object combinations. However, in turn, the state-of-the-art mappings were at least 1.3 times slower than the algorithmic subspace mapping.

We report the average number of tries in Table VIb. In all object combinations, users were able to pick and place objects with the fewest amount of tries using the empirical subspace mapping. The algorithmic subspace mapping came in second in all cases except for with the small objects, where fingertip mapping came in second.

Finally, we report the average number of objects the users were able to pick up with each of the mappings in Table VIc. For all the objects, the maximum number of objects that can be picked is 10, for the small and irregular objects, the maximum is three, and for the large objects, the maximum is four.

The empirical subspace mapping allowed every novice to pick up every object. The algorithmic subspace mapping allowed the novices to pick up most objects, and the state-of-the-art mappings allowed novices to pick up the majority of objects, but still fewer than either of the subspace mapping methods.

B. In-Hand Manipulation Results

We report our results as the average across all subjects. We report averages for all objects, for circular objects (the bottle, peanut butter container, and goblet), and for irregularly shaped objects (the wheels, legos, lettuce, and mustard).

We report the average time to perform the in-hand manipulation task across all subjects in Table VIa. Across all subjects and all objects, novices using the fingertip mapping took 2 times longer than when using the empirical subspace mapping, and 1.3 times longer than when using the algorithmic subspace mapping. Joint mapping performed about the same as the empirical subspace mapping and was 1.5 times faster than the algorithmic subspace mapping.

For the three combinations of objects we look at (all objects, circular, and irregular), manipulation with the empirical subspace mapping took the least amount of time for all objects and the circular objects, with joint mapping taking the least amount of time for the irregular objects. In all cases, the algorithmic subspace mapping was third and fingertip mapping took the longest.

We report the average number of tries subjects took to manipulate the objects in Table VIb. In all object combinations, users were able to transition the objects with the fewest amount of tries using the empirical subspace mapping.

Finally, we report the average number of objects the users were able to manipulate with each of the mappings in Table VIc. For all objects, the maximum number of objects that can be manipulated is 7, for the circular objects, the maximum is three, and for the irregular objects, the maximum is four.

The joint and algorithmic subspace mappings allowed every novice to manipulate every object. For the empirical subspace mapping, one subject was not able to transition one object, and for the fingertip mapping, one subject was not able to manipulate two objects.

VIII. Discussion

We begin by discussing the teleoperation mappings generated algorithmically and empirically. In both cases, novice users were able to complete two different types of manipulation tasks using two different non-anthropomorphic robot hands. This shows that the mappings created with both methods rely on a subspace which is relevant to teleoperation and which can encompass the range of motion necessary to manipulate a variety of objects in different ways. Similarly, these experiments show that the subspace is relevant for more than one hand.

The algorithmic subspace mapping, in particular, not only shows that the subspace we propose is relevant to teleoperation, but that the benefit of using such a subspace does not
derive exclusively from the human intuition used to create the mapping. Since this mapping is created automatically, without hand-specific intuition from the mapping creator, and can still enable teleoperation, we conclude that $T$ is a concept that has value even when there is no human intelligence ‘built into’ the mapping. That being said, we do note that the empirical subspace mapping, which is defined with the benefit of human intuition, outperforms the algorithmic subspace mapping. The use of human intuition to define the basis vectors, while not exclusively defining the value of the subspace, can make it a more powerful and intuitive control.

Both experiments showed that the empirical and algorithmic subspace mappings were as intuitive as or more intuitive than the state-of-the-art mappings we selected as baseline comparisons. We measure intuitiveness as the combination of our three metrics: we hypothesize that controls which allow the user to manipulate more objects in less time, with fewer tries are more intuitive. We note that the measure of which method is preferable is a trade off between intuitiveness for the teleoperator and intuitiveness for the person who must generate the teleoperation mapping. For our real-time experiments, the three metrics we have selected only measure intuitiveness for the teleoperator.

For the pick and place experiments, the empirical subspace mapping was the most intuitive control for novices, and the algorithmic subspace mapping was the second most intuitive control. In all metrics, the empirical and algorithmic subspace mappings outperformed the state-of-the-art. The empirical subspace mapping provides the greatest advantage for small objects, but still has a significant edge for all other object combinations. It is worth noting that the standard error we report for all the metrics is also lowest for the empirical subspace mapping. We hypothesize that this means the novices were able to use the empirical subspace mapping more consistently than the other controls.

For the in-hand manipulation experiments, our empirical subspace mapping proved the most effective in terms of time to perform the experiments, followed by joint mapping and the algorithmic subspace mapping. A similar result was observed for the average number of tries required to succeed; for total objects manipulated, all three of these mappings showed similar performance, with joint mapping and algorithmic subspace mapping having a very slight advantage. This ranking is thus less definitive than for the pick and place experiments because different mappings performed better for different metrics.

We note that the in-hand manipulation experiments lend themselves particularly well to joint mapping, which allows users to individuate the robot digits, an advantage when performing in-hand manipulation, and something which our subspace mappings do not allow. This individuation provides a particular advantage for irregular objects and is likely why joint mapping outperformed the empirical subspace mapping in time to completion for that particular object category.

These experiments show that our two subspace mapping methods can generalize across different hands and different manipulation tasks.

IX. CONCLUSIONS AND FUTURE WORK

In this paper, we propose an intuitive, low dimensional mapping between the pose spaces of the human hand and non-anthropomorphic robot hands. We present an empirical algorithm to generate this mapping that leverages the user’s intuition to define hand motions for a specific kinematic configuration. We also propose an algorithmic method to generate the mapping, which is completely automatic. This automated process is made possible by defining the subspace independently of hand kinematics, using objects to define hand motions that span the desired subspace.

We validate both the empirical and algorithmic subspace mappings with real-time teleoperation experiments with novice users on two kinematically different robotic hands. We found that, for pick and place experiments, our empirical subspace mapping was most intuitive for users, with the algorithmic subspace mapping still performing better than state-of-the-art alternatives. For the in-hand manipulation experiments, we found that our empirical subspace mapping performed as well as joint mapping, one of the state-of-the-art methods, and better than fingertip mapping, the other baseline we employed. For the in-hand manipulation experiments, the algorithmic subspace mapping was generally less intuitive for novices than joint mapping, but more intuitive than fingertip mapping.

The real-time teleoperation experiments show that the subspace we propose is relevant to teleoperation for multiple hands with distinct kinematics and for different manipulation tasks. This is the first time, to our knowledge, that a teleoperation mapping generated without requiring a user’s understanding of hand-specific kinematics has been shown to be intuitive for novices for real-time teleoperation. The fact that the algorithmic mapping can enable teleoperation shows that the subspace encodes useful information for teleoperation that does not rely exclusively on human intuition.

The future of this work could take a number of directions. Our experiments in this paper show that the subspace is relevant for at least three different hands, and we would like to continue to show that is relevant for other hands with different kinematic configurations. We have already shown that the teleoperation subspace is suitable for lower dimensional controls, like electromyography (EMG) [37] and would like to validate this control with more kinematic configurations as well. Finally, we would like to show that the subspace is useful for more complex tasks, like assembling and disassembling machinery.

ACKNOWLEDGMENT

We would like to thank Rami J. Hamati for his help setting up the hardware for our experiments.

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