Simulation study on ride comfort of three-axle heavy vehicle spatial model based on rigid-elastic model and pseudo-excitation method

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Abstract
This research aims to develop a new ride comfort simulation technology for a three-axle heavy vehicle. To consider the elasticity of the frame, the finite element method (FEM) is used to analyze the free mode of the frame, and the elastic information of the frame is obtained. Based on the theory of rigid-elastic synthesis, a 17-degree-of-freedom (DOF) spatial rigid-elastic model of a three-axle heavy vehicle is established. The pseudo-excitation method (PEM) is adopted to improve the efficiency of the solution, thereby solving this problem. The pseudo road excitation is constructed, the pseudo responses of the vibration system and the response variables are derived, and the power spectral densities (PSDs) and the root mean square (RMS) values of the responses are deduced. Twenty response variables are used, including accelerations, suspension dynamic deflections, and relative dynamic loads of wheels, whose PSDs and RMS variables are used as evaluation indexes. Finally, a comparative study of ride comfort simulation is conducted. A comparison of the simulation results of the rigid-elastic and rigid models indicates that the elasticity of the frame considerably influences the ride comfort of the heavy vehicle and hence cannot be ignored in the study of this issue. Meanwhile, a comparison of the results of PEM and the Fourier method for the spatial rigid-elastic model of the three-axle heavy vehicle shows that PEM is accurate yet simpler and more efficient than the Fourier method. Therefore, the innovative simulation technology proposed in this work is practical and efficient and can reflect the essence of the problem.

Keywords: Ride comfort, Heavy vehicle, Rigid-elastic model, Pseudo-excitation method, Frame

1. Introduction

The working conditions of heavy vehicles are usually severe; the vibration of seats is 9–16 times that of passenger cars (Stephens, 1977). Moreover, heavy vehicle drivers usually work long hours continuously, which easily causes fatigue and is not conducive to safe driving (Thompson et al., 2014; Haworth et al., 1998). Three-axle heavy vehicles are the basic type of heavy vehicles. In a multi-axle semi-trailer train, the three-axle traction vehicle accounts for a large proportion. Therefore, studying the ride comfort of three-axle heavy vehicles offers important theoretical value and practical significance. Some scholars established three-axle heavy vehicle or three-axle tractor spatial model, and studied the vehicle ride comfort (Schade et al., 2000; Cassara et al., 2000; Chandrasekaran et al., 2002; Jain, 2007).

Heavy vehicles usually have long bodies, large wheelbases, and low frame stiffness, whose first natural frequencies are generally lower than 10 Hz (Gillespie, 1985; Qi, 2011). Therefore, the elastic performance of the frame is apparent. Moreover, with the development trend of vehicle lightweight and low energy consumption, the problem of frame elasticity will become increasingly prominent. To improve the accuracy of simulation models of heavy vehicles, frame
elastcity must be considered (Yang et al., 2003; Ibrahim et al., 1996). To date, rigid-elastic models of heavy vehicles are constructed through three techniques. The first method bases on the multi-body dynamics and the finite element method (FEM) to build a rigid-elastic coupled model (Forsén, 1999; Cosme et al., 1999; Jain, 2007; Ekberg et al., 2015; Schade et al., 2000; Anderson et al., 2001). The advantage is that it can build a complex model, but the disadvantage is that many parameters are needed and the mathematical model cannot be studied because the multi-body dynamics software belongs to the black box. The second method bases on elastic beam theory and the lumped parameter method to build a rigid-elastic vibration model (Hac, 1986; Elmadany, 1988; Sunder et al., 1993; Trangsrud et al., 2004; Spivey, 2007; Li et al., 2015). Its advantage is that it needs few parameters and easily understands the dynamics of heavy vehicles, but it is suitable only for plane models and not for spatial models. The third method bases on FEM and the lumped parameter method to establish a rigid-elastic model (Ibrahim, 2002, 2004). This method can overcome the shortcoming of the two previous methods.

The Fourier method is a traditional method of solving the ride comfort of heavy vehicles in the frequency domain. However, this technique becomes inefficient and inconvenient in the simulation of complex models. Compared with the structure of passenger cars, that of heavy vehicles is more complex, thereby increasing the complexity of their spatial models after frame elasticity is considered. Therefore, finding new solutions for the spatial rigid-elastic models of heavy vehicles is important. As an innovative theory of random vibration research, PEM has advantages of simplicity, high efficiency, and accuracy and has been widely used in earthquake engineering (Guo et al., 2011; Zhang et al., 2013; Jia et al., 2013), wind vibration engineering (Zhao et al., 2016; Hu et al., 2013; Rosa et al., 2015), and marine engineering (Zhou et al., 2014; Li et al., 2008; Liu et al., 2009). However, applications in vehicle engineering are scarce. Li et al. (2009) established a 7-DOF plane rigid model of a two-axle truck and simulated it in the frequency domain by PEM. Guo et al. (2010) established a 5-DOF plane rigid model of a two-axle vehicle and simulated it under nonstationary conditions by using PEM. Li et al. (2010) established a 9-DOF spatial rigid body model of a two-axle vehicle, simulated it in the frequency domain by the Fourier method and PEM, and compared the results. The above ride comfort simulation models are all simple rigid body models, therefore, adopting PEM to solve complex spatial rigid-elastic models of heavy vehicles offers considerable research value.

The structure of this paper is organized as follows. In Section 2, a 17-DOF spatial rigid-elastic model of a three-axle heavy vehicle is built with application of the principle of rigid-elastic synthesis. In Section 3, PEM for the ride comfort of the three-axle heavy vehicle is described. The ride comfort simulation of the three-axle heavy vehicle spatial model is studied in Section 4. The rigid-elastic and rigid-body models are compared, and PEM and the Fourier method are compared. Finally, conclusions are drawn in Section 5.

2. Spatial rigid-elastic model for three-axle heavy vehicle

2.1 Rigid-elastic synthesis principle

Rigid-elastic synthesis refers to a component’s movement that comprises rigid motion and elastic vibration. In this research, frame elasticity is considered, and the rigid-elastic synthesis motion of the frame is composed of the rigid motion and elastic vibration of the frame, as shown in Fig. 1.

The vertical displacement at any point of motion on the frame can be expressed as

\[ z_{rb}(x,y) = z_b + (x_b - x)\theta + (y - y_b)\psi + z_b(x,y) \]  

(1)
Where \( z_{v}(x, y) \) is the vertical displacement after rigid-elastic synthesis, \( z_{b} \) is the vertical rigid displacement of the centroid, \( \theta_{b} \) and \( \varphi_{b} \) are pitch and roll angular rigid displacements of the centroid, \( x_{b} \) and \( y_{b} \) are the centroid coordinates of the frame, and \( z_{e}(x, y) \) is the elastic vibration displacement.

According to the modal superposition principle (Craig, 1981), the elastic displacement at a point on the frame can be expressed as

\[
z_{e} = \sum_{i=1}^{n} \phi_{i}(x, y)z_{ei} \quad (2)
\]

Where \( \phi_{i}(x, y) \) is the \( i^{th} \)-order mode shape at position \( (x, y) \), \( z_{ei} \) is the \( i^{th} \)-order generalized coordinate, and \( n \) is the number of retained modes of the frame.

### 2.2 Finite element analysis of frame

To obtain the modal parameters of the frame, FEM is used for modal analysis of the frame. The frame is preprocessed using HyperMesh and then meshed by a SHELL63 element. The loads on the frame are loaded on the frame in the form of a mass point, which includes engine and payloads. A total of 370438 shell elements and 32226 mass elements are divided, and the finite element model of the frame is shown in Fig. 2.

The finite element model of the frame is imported into ANSYS for modal analysis. In the solution setting, the modal shape of the frame is set to orthogonalize the modal mass, so the modal mass matrix is a unit matrix.

The frame modal mass, stiffness, and damping matrix can be written as

\[
m_{e} = \begin{bmatrix}
m_{e1} \\
m_{e2} \\
\vdots \\
m_{en}
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix} \quad (3)
\]

Where \( m_{ei} \) is the \( i^{th} \)-order modal mass.

\[
k_{e} = \begin{bmatrix}
k_{e1} \\
k_{e2} \\
\vdots \\
k_{en}
\end{bmatrix} = \begin{bmatrix}
\omega_{e1}^{2} m_{e1} \\
\omega_{e2}^{2} m_{e2} \\
\vdots \\
\omega_{en}^{2} m_{en}
\end{bmatrix} = \begin{bmatrix}
\omega_{e1}^{2} \\
\omega_{e2}^{2} \\
\vdots \\
\omega_{en}^{2}
\end{bmatrix} \quad (4)
\]

Where \( k_{ei} \) is the \( i^{th} \)-order modal stiffness.

\[
c_{e} = \begin{bmatrix}
c_{e1} \\
c_{e2} \\
\vdots \\
c_{en}
\end{bmatrix} = \begin{bmatrix}
2\omega_{e1} m_{e1} \xi_{1} \\
2\omega_{e2} m_{e2} \xi_{2} \\
\vdots \\
2\omega_{en} m_{en} \xi_{n}
\end{bmatrix} = \begin{bmatrix}
2\omega_{e1} \xi_{1} \\
2\omega_{e2} \xi_{2} \\
\vdots \\
2\omega_{en} \xi_{n}
\end{bmatrix} \quad (5)
\]

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Where $c_i$ is the $i^{th}$-order modal damping, $\xi_i$ is the $i^{th}$-order damping ratio of the frame structure, $\omega_i$ is the $i^{th}$-order circular frequency.

The natural frequencies and modes of the frame are obtained by modal analysis. The first four modes are retained, as shown in Table 1. According to Eq. (3), Eq. (4) and Eq. (5), the modal mass, stiffness, and damping of each mode can be obtained. The elastic displacements of the key points, which include the frame centroid, and the connect points between the frame and suspensions and between the frame and the cab mounts, are shown in Table 2.

### Table 1: First four natural frequencies and modes of the frame

| Order | Mode     | Frequency |
|-------|----------|-----------|
| First | Torsion  | 1.04      |
| Second| Bending  | 4.80      |
| Third | Torsion  | 5.43      |
| Fourth| Bending  | 9.94      |

### Table 2: Elastic displacements of the key points

| Key points | First order displacement (m) | Second order displacement (m) | Third order displacement (m) | Fourth order displacement (m) |
|------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| b          | 0                             | 0.00756                       | 0.00003                       | -0.00278                      |
| 1          | 0.01204                       | -0.01517                      | 0.01297                       | -0.00922                      |
| 2          | -0.00094                      | 0.00730                       | -0.00674                      | -0.00443                      |
| 3          | -0.00486                      | 0.00289                       | -0.00230                      | -0.00602                      |
| 4          | -0.01202                      | -0.01507                      | -0.01304                      | -0.00969                      |
| 5          | 0.00094                       | 0.00721                       | 0.00679                       | -0.00460                      |
| 6          | 0.00487                       | 0.00286                       | 0.00231                       | -0.00607                      |
| 7          | 0.01343                       | -0.01919                      | 0.01648                       | -0.01339                      |
| 8          | 0.00757                       | -0.00234                      | 0.00199                       | 0.00344                       |
| 9          | -0.01343                      | -0.01904                      | -0.01660                      | -0.01385                      |
| 10         | -0.00759                      | -0.00234                      | -0.00203                      | 0.00353                       |

### 2.3 Mechanical model

Fig. 3: Spatial mechanical model of three-axle heavy vehicle
The mechanical model of three-axis heavy vehicle spatial model is built in Fig. 3. The vehicle parameters are given in Table 3, and they come from the actual vehicle. In Fig. 3 and Table 3, some subscripts are explained as follows: "b" represents the body, "p" represents driver and seat, "c" represents the cab, "u" represents unsprung mass, "f" represents front, "m" represents middle, "r" represents rear in the mass parameters and dimension parameters, "l" represents left, the first "r" represents right and the second "r" represents rear in other parameters.

In this paper, the first four modes of the frame are used. Some considered the first mode of the frame (Elmadany, 1988; Sunder, et al., 1993, Trangsrud et al., 2004, Spivey, 2007). Others considered the first two modes of the frame (Li et al., 1988; Sunder, et al., 1993, Trangsrud et al., 2004, Spivey, 2007). Some took into account the first three modes of the frame (Ibrahim et al., 1996; Hać, 1986; Yang et al., 2003). The spatial rigid-elastic model of the three-axle heavy vehicle has 17 DOFs, including the vertical rigid displacement, the pitch and roll angular rigid displacement, the vertical rigid displacement of driver and seat (\(z_p\)), the vertical rigid displacement of the frame (Ibrahim et al., 1996; Hać, 1986; Yang et al., 2003). Others considered the first two modes of the frame (Li et al., 2015). Therefore, from the former researchers, it is sufficient to establish the rigid-elastic model considering the first four modes.

The spatial rigid-elastic model of the three-axle heavy vehicle has 17 DOFs, including the vertical rigid displacement, pitch and roll angular rigid displacement of the body mass center (\(z, \theta, \psi_x\)), vertical rigid displacements (\(z_p, z_m, z_r\)) and roll angular rigid displacement (\(\psi_p, \psi_m, \psi_r\)) of unsprung mass for the front, the middle and the rear axles, the vertical rigid displacement of driver and seat (\(z_p\)), the vertical rigid displacement, the pitch and roll angular rigid displacement of the cab mass center (\(z, \theta, \psi_x\)), and the first fourth-order elastic vibration displacement (\(z_p, z_m, z_r, z_k\)).

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| Driver and seat mass \(m_p\) (kg) | 104 | Cab left rear suspension stiffness \(k_w\) (Nm\(^{-1}\)) | 3100 |
| Cab mass \(m_c\) (kg) | 608.4 | Cab right front suspension stiffness \(k_p\) (Nm\(^{-1}\)) | 3100 |
| Cab pitch moment of inertia \(I_p\) (kgm\(^2\)) | 355.1 | Cab right front suspension stiffness \(k_w\) (Nm\(^{-1}\)) | 3100 |
| Cab roll moment of inertia \(I_r\) (kgm\(^2\)) | 350 | Left front suspension stiffness \(k_w\) (Nm\(^{-1}\)) | 305000 |
| Body mass \(m_b\) (kg) | 21218 | Left middle suspension stiffness \(k_m\) (Nm\(^{-1}\)) | 1300000 |
| Body pitch moment of inertia \(I_b\) (kgm\(^2\)) | 210000 | Left rear suspension stiffness \(k_m\) (Nm\(^{-1}\)) | 1300000 |
| Body roll moment of inertia \(I_r\) (kgm\(^2\)) | 32000 | Right front suspension stiffness \(k_a\) (Nm\(^{-1}\)) | 305000 |
| Front axle mass \(m_f\) (kg) | 888 | Right middle suspension stiffness \(k_a\) (Nm\(^{-1}\)) | 1300000 |
| Middle axle mass \(m_m\) (kg) | 1367 | Right rear suspension stiffness \(k_a\) (Nm\(^{-1}\)) | 1300000 |
| Rear axle mass \(m_r\) (kg) | 1367 | Left front tire stiffness \(k_1\) (Nm\(^{-1}\)) | 678500 |
| Roll moment inertia of front axle \(I_{p1}\) (kgm\(^2\)) | 340 | Left middle tire stiffness \(k_m\) (Nm\(^{-1}\)) | 2714000 |
| Roll moment inertia of middle axle \(I_{m1}\) (kgm\(^2\)) | 650 | Left rear tire stiffness \(k_r\) (Nm\(^{-1}\)) | 2714000 |
| Roll moment inertia of rear axle \(I_{r1}\) (kgm\(^2\)) | 650 | Right front tire stiffness \(k_a\) (Nm\(^{-1}\)) | 678500 |
| Seat damping \(c_s\) (Nm\(^{-1}\)) | 680 | Right middle tire stiffness \(k_a\) (Nm\(^{-1}\)) | 2714000 |
| Cab left front suspension damping \(c_{pf}\) (Nm\(^{-1}\)) | 2250 | Right rear tire stiffness \(k_m\) (Nm\(^{-1}\)) | 2714000 |
| Cab left rear suspension damping \(c_{pr}\) (Nm\(^{-1}\)) | 2250 | Distance from seat to cab centroid \(l_1\) (m) | 0.164 |
| Cab right front suspension damping \(c_{pf}\) (Nm\(^{-1}\)) | 2250 | Centroid distance from body to cab \(l_2\) (m) | 4.09 |
| Cab right rear suspension damping \(c_{pr}\) (Nm\(^{-1}\)) | 2250 | Distance from cab front suspension to cab centroid \(l_1\) (m) | 0.977 |
| Left front suspension damping \(c_{pl}\) (Nm\(^{-1}\)) | 7700 | Distance from cab rear suspension to cab centroid \(l_2\) (m) | 1.229 |
| Left middle suspension damping \(c_{pm}\) (Nm\(^{-1}\)) | 7700 | Distance from body front end to front axle \(l_2\) (m) | 1.4 |
| Left rear suspension damping \(c_{pr}\) (Nm\(^{-1}\)) | 7700 | Distance from front axle to body centroid \(l_2\) (m) | 4.44 |
| Right front suspension damping \(c_{rf}\) (Nm\(^{-1}\)) | 7700 | Distance from middle axle to body centroid \(l_6\) (m) | 0.36 |
| Right middle suspension damping \(c_{rm}\) (Nm\(^{-1}\)) | 7700 | Distance from rear axle to middle axle \(l_6\) (m) | 1.35 |
| Right rear suspension damping \(c_{rr}\) (Nm\(^{-1}\)) | 7700 | Half of front axle wheelbase \(B_1\) (m) | 0.975 |
| Left front tire damping \(c_{pf}\) (Nm\(^{-1}\)) | 0 | Half of middle axle wheelbase \(B_2\) (m) | 0.925 |
| Left middle tire damping \(c_{pm}\) (Nm\(^{-1}\)) | 0 | Half of rear axle wheelbase \(B_3\) (m) | 0.925 |
| Left rear tire damping \(c_{pr}\) (Nm\(^{-1}\)) | 0 | Half of front axle suspension distance \(b_p\) (m) | 0.398 |
| Right front tire damping \(c_{rf}\) (Nm\(^{-1}\)) | 0 | Half of middle axle suspension distance \(b_m\) (m) | 0.398 |
| Right middle tire damping \(c_{rm}\) (Nm\(^{-1}\)) | 0 | Half of rear axle suspension distance \(b_r\) (m) | 0.398 |
| Right rear tire damping \(c_{rr}\) (Nm\(^{-1}\)) | 0 | Half of cab front suspension distance \(b_a\) (m) | 0.398 |
| Seat stiffness \(k_s\) (Nm\(^{-1}\)) | 29900 | Half of cab rear suspension distance \(b_s\) (m) | 0.398 |
| Cab left front suspension stiffness \(k_{wf}\) (Nm\(^{-1}\)) | 5100 | Lateral distance from cab centroid to seat \(b_1\) (m) | 0.3 |

2.4 Vibration System Energy

The system kinetic energy is written as
The system potential energy is described by

\[ T = \frac{1}{2} (m_x \dot{z}_x^2 + m_m \dot{z}_m^2 + m_a \dot{z}_a^2 + I_{\theta_m} \dot{\theta}_m^2 + I_{\theta_a} \dot{\theta}_a^2 + I_{\psi_m} \dot{\psi}_m^2 + I_{\psi_a} \dot{\psi}_a^2) + \frac{1}{2} (m_x \dot{z}_x^2 + I_{\theta_m} \dot{\theta}_m^2 + I_{\psi_m} \dot{\psi}_m^2) + \frac{1}{2} m_p \dot{z}_p^2 + \frac{1}{2} (m_x \dot{z}_x^2 + m_2 \dot{z}_2^2 + m_3 \dot{z}_3^2 + m_4 \dot{z}_4^2) \]

(6)

The system dissipated energy is given by

\[ V = \frac{1}{2} k_p (\dot{z}_p - B \dot{\psi}_p - \dot{q}_p)^2 \]

\[ + \frac{1}{2} k_m (\dot{z}_m - B \psi_m - \dot{q}_m)^2 \]

\[ + \frac{1}{2} k_{\theta_m} (\dot{\theta}_m - B \psi_m - \dot{q}_m)^2 \]

\[ + \frac{1}{2} k_{\theta_a} (\dot{\theta}_a - B \psi_a - \dot{q}_a)^2 \]

\[ + \frac{1}{2} k_{\psi_m} (\dot{\psi}_m - B \psi_m - \dot{q}_m)^2 \]

\[ + \frac{1}{2} k_{\psi_a} (\dot{\psi}_a - B \psi_a - \dot{q}_a)^2 \]

\[ + \frac{1}{2} k_{\psi_m} (\dot{\psi}_m - B \psi_m - \dot{q}_m)^2 \]

\[ + \frac{1}{2} k_{\psi_a} (\dot{\psi}_a - B \psi_a - \dot{q}_a)^2 \]

\[ + \frac{1}{2} k_{\psi_m} (\dot{\psi}_m - B \psi_m - \dot{q}_m)^2 \]

\[ + \frac{1}{2} k_{\psi_a} (\dot{\psi}_a - B \psi_a - \dot{q}_a)^2 \]

\[ + \frac{1}{2} k_{\psi_m} (\dot{\psi}_m - B \psi_m - \dot{q}_m)^2 \]

\[ + \frac{1}{2} k_{\psi_a} (\dot{\psi}_a - B \psi_a - \dot{q}_a)^2 \]

(7)

The mathematical model is based on Lagrange’s equation, the mathematical equation of the spatial rigid-elastic model with 17 DOFs is written as

\[ M \ddot{z} + C \dot{z} + K z = k_i q + c_i \dot{q} \]

(9)

\[ z = [z_x, \dot{z}_x, \theta_x, \dot{\theta}_x, z_m, \dot{z}_m, \psi_m, \dot{\psi}_m, z_a, \dot{z}_a, \psi_a, \dot{\psi}_a, z_p, \dot{z}_p, \psi_p, \dot{\psi}_p]^T \]

(10)

\[ q = [q_x, \dot{q}_x, q_m, \dot{q}_m, q_a, \dot{q}_a]^T \]

(11)

\[ M = \text{diag}(m_x, I_{\theta_m}, m_m, I_{\theta_a}, m_a, I_{\psi_m}, I_{\psi_a}, m_p, I_{\psi_p}) \]

(12)
3. PEM for ride comfort analysis of three-axle heavy vehicle
3.1 Basic formula of PEM with single-point excitation

For a single-DOF linear system, when the PSD $G_s(f)$ of a stationary random excitation $x$ is known, the pseudo excitation $\tilde{x}$ can be constructed as (Wang et al., 2000)

$$\tilde{x} = \sqrt{G_s(f)} e^{j2\pi f_0}$$  \hfill (17)

Where $j$ is the square root of $-1$.

The pseudo response $\tilde{y}$ of response $y$ can be given by
\[ \tilde{y} = H(f)\tilde{x} = H(f)\sqrt{G_s(f)}e^{j\phi} \]

Subsequently, the PSD \( G_s(f) \) of response \( y \) can be expressed as

\[ G_s(f) = \tilde{y}^*\tilde{y} \]  

### 3.2 Basic formula of PEM with multi-point excitation

The stationary multi-point pseudo excitation problem can be solved by the superposition principle. For a multi-DOF linear system, the PSD matrix is generally a non-negative Hermitian matrix. Assuming its rank is \( r \), the PSD matrix can be expressed as (Lin et al., 1994)

\[ G_s(f) = \sum_{i=1}^{r} \lambda_i \phi_i \phi_i^* \]  

Where \( \lambda_i \) is the non-zero eigenvalue and \( \phi_i \) is the eigenvector.

The \( i \)-th independent pseudo excitations can be constructed as

\[ \tilde{x}_i = \phi_i \tilde{x}_i \quad i = 1, \ldots, r \]  

\[ \tilde{x}_i = \sqrt{\lambda_i}e^{j\phi} \quad i = 1, \ldots, r \]  

The pseudo response \( \tilde{y} \) can be written as

\[ \tilde{y}_i = H(f)\tilde{x}_i \]  

Under stationary multi-point pseudo excitation, the PSD matrix of the system’s actual response is

\[ G_s(f) = \sum_{i=1}^{r} \tilde{y}_i^*\tilde{y}_i \]  

### 3.3 Pseudo road excitation construction

The PSD of the six-wheel road excitation can be written as

\[ G_s(f) = G_s(n_0^2 n_f^2) \]

Where, \( G_s(n_0^2) \) is road roughness coefficient, \( n_0 \) is spatial frequency, \( u \) is the vehicle speed, \( coh_{ik} \) is the correlation coefficient between the \( i \)-th and \( k \)-th wheels, \( \tau_i \) is the time delay between the second and first axes, and \( \tau_2 \) is the time delay between the third and first axes.

The eigenvalues and eigenvectors of Eq. (25) are obtained, and the pseudo excitation is then constructed according to Eq. (21) and Eq. (22).

### 3.4 Pseudo response of vibration system and vibration response variables

The pseudo response vector of the vibration system is expressed as
\[ Z = (\tilde{z}_x, \tilde{\theta}_x, \tilde{\psi}_x, \tilde{\zeta}_x, \tilde{z}_m, \tilde{\theta}_m, \tilde{\psi}_m, \tilde{\zeta}_m, \tilde{z}_b, \tilde{\theta}_b, \tilde{\psi}_b, \tilde{\zeta}_b, \tilde{z}_c, \tilde{\theta}_c, \tilde{\psi}_c, \tilde{\zeta}_c)^T \]  

(26)

Accelerations, dynamic deflections, and relative dynamic loads are regarded as vibration response variables. The pseudo response of the vertical acceleration of the body centroid is described by

\[ \tilde{z}_{rb} = \tilde{z}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(27)

The pseudo responses of the vertical accelerations of the connecting points between the body and the suspensions are given as

\[ \tilde{z}_{rl} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(28)

\[ \tilde{z}_{rv} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(29)

\[ \tilde{z}_{rv} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(30)

\[ \tilde{z}_{re} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(31)

\[ \tilde{z}_{rv} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(32)

\[ \tilde{z}_{re} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} \]  

(33)

The pseudo responses of suspension dynamic deflections are written as

\[ \tilde{f}_{ld} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} - \tilde{z}_m + b_{vl} \tilde{\psi}_m \]  

(34)

\[ \tilde{f}_{ld} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} - \tilde{z}_m - b_{vl} \tilde{\psi}_m \]  

(35)

\[ \tilde{f}_{ld} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} - \tilde{z}_m + b_{vl} \tilde{\psi}_m \]  

(36)

\[ \tilde{f}_{ld} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} - \tilde{z}_m - b_{vl} \tilde{\psi}_m \]  

(37)

\[ \tilde{f}_{ld} = \tilde{z}_b + \Delta x \tilde{\theta}_b + \Delta y \tilde{\psi}_b + \phi_1 \tilde{z}_{b1} + \phi_2 \tilde{z}_{b2} + \phi_3 \tilde{z}_{b3} + \phi_4 \tilde{z}_{b4} - \tilde{z}_m - b_{vl} \tilde{\psi}_m \]  

(38)

The pseudo responses of wheel relative dynamic loads are denoted by

\[ \tilde{F}_{ld} = B_m m_{w} \tilde{z}_m - m_{w} \tilde{\psi}_m - (b_{w} + B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) + (b_{w} - B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) \]  

\[ \frac{G_1}{2B_{j} G_1} \]  

(40)

\[ \tilde{F}_{ld} = B_m m_{w} \tilde{z}_m - m_{w} \tilde{\psi}_m - (b_{w} + B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) + (b_{w} - B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) \]  

\[ \frac{G_2}{2B_{j} G_2} \]  

(41)

\[ \tilde{F}_{ld} = B_m m_{w} \tilde{z}_m - m_{w} \tilde{\psi}_m - (b_{w} + B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) + (b_{w} - B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) \]  

\[ \frac{G_3}{2B_{j} G_3} \]  

(42)

\[ \tilde{F}_{ld} = B_m m_{w} \tilde{z}_m - m_{w} \tilde{\psi}_m - (b_{w} + B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) + (b_{w} - B) (k_{w} \tilde{f}_{ld} + c_{w} \tilde{f}_{ld}) \]  

\[ \frac{G_4}{2B_{j} G_4} \]  

(43)
\[ \frac{\vec{F}_{\text{red}}}{G_s} = \frac{B_m m_{s0} \vec{z}_{s0} + m_{s0} \vec{\psi}_{s0} + (b_{s0} - B_s)(k_{s0} \vec{f}_{s0} + c_{s0} \vec{f}_{s0}) - (b_s + B_s)(k_{s0} \vec{f}_{s0} + c_{s0} \vec{f}_{s0})}{2B_s G_s} \]  
\[ \frac{\vec{F}_{\text{red}}}{G_s} = \frac{B_m m_{s0} \vec{z}_{s0} + m_{s0} \vec{\psi}_{s0} + (b_{s0} - B_s)(k_{s0} \vec{f}_{s0} + c_{s0} \vec{f}_{s0}) - (b_s + B_s)(k_{s0} \vec{f}_{s0} + c_{s0} \vec{f}_{s0})}{2B_s G_s} \]

Where \( G_s \) is the static load of the \( i^\text{th} \) wheel.

3.5 PSDs and RMS values of response variables

According to Eq. (24), the power spectral density of the vibration response can be written as

\[ G_p(f) = \sum_{i=1}^{\infty} \vec{p}_i^* \vec{p}_i^f \]  

Where \( \vec{p} \) is the pseudo response of vibration response, \( \vec{p}^* \) is the conjugate of \( \vec{p} \).

According to the PSD of the vibration response, RMS can be expressed as (Wong, 2001)

\[ \sigma_p = \sqrt{\int_{f_l}^{f_u} G_p(f) df} \]

Where \( f_l, f_u \) are the lower and upper limits of frequency, respectively.

4. Simulation study on ride comfort of three-axle heavy vehicle spatial model

4.1 Comparison between rigid-elastic model and rigid model

| Response variables | Rigid-elastic model | Rigid model | (a-b)/b% |
|--------------------|---------------------|-------------|---------|
| \( \vec{z}_{\text{reb}} \) (ms\(^{-2}\)) | 1.643042 | 1.402069 | 17.19% |
| \( \vec{z}_{\text{rel}} \) (ms\(^{-2}\)) | 2.702573 | 1.217901 | 121.90% |
| \( \vec{z}_{\text{rse}} \) (ms\(^{-2}\)) | 1.885319 | 1.442338 | 30.71% |
| \( \vec{z}_{\text{rse}} \) (ms\(^{-2}\)) | 1.917960 | 1.577861 | 21.55% |
| \( \vec{z}_{\text{rse}} \) (ms\(^{-2}\)) | 2.863349 | 1.206669 | 137.29% |
| \( \vec{z}_{\text{rse}} \) (ms\(^{-2}\)) | 1.892786 | 1.429717 | 32.39% |
| \( \vec{z}_{\text{rse}} \) (ms\(^{-2}\)) | 1.917446 | 1.5652 | 22.48% |
| \( \vec{z}_{\text{rse}} \) (ms\(^{-2}\)) | 1.553569 | 1.470644 | 5.64% |
| \( f_{\text{y0}} \) (m) | 0.015702 | 0.009772 | 60.68% |
| \( f_{\text{y0}} \) (m) | 0.009351 | 0.006183 | 51.24% |
| \( f_{\text{y0}} \) (m) | 0.009416 | 0.006304 | 49.37% |
| \( f_{\text{y0}} \) (m) | 0.015856 | 0.009718 | 63.16% |
| \( f_{\text{y0}} \) (m) | 0.009305 | 0.006139 | 51.57% |
| \( f_{\text{y0}} \) (m) | 0.009361 | 0.006268 | 49.35% |
| \( f_{\text{y0}} / G_s \) | 0.154100 | 0.139546 | 10.43% |
| \( f_{\text{y0}} / G_s \) | 0.213311 | 0.214083 | -0.36% |
| \( f_{\text{y0}} / G_s \) | 0.263953 | 0.277874 | -5.01% |
| \( f_{\text{y0}} / G_s \) | 0.155221 | 0.139294 | 11.43% |
| \( f_{\text{y0}} / G_s \) | 0.212814 | 0.213768 | -0.45% |
| \( f_{\text{y0}} / G_s \) | 0.263284 | 0.277533 | -5.13% |
Fig. 4 Comparison of the rigid-elastic model and the rigid model (a) vertical acceleration of the body centroid (b) vertical acceleration of driver and seat mass (c) vertical acceleration of key 1 (d) vertical acceleration of key 2 (e) vertical acceleration of key 3 (f) vertical acceleration of key 4 (g) vertical acceleration of key 5 (h) vertical acceleration of key 6 (i)
dynamic deflection of left-front suspension (j) dynamic deflection of left-middle suspension (k) dynamic deflection of left-rear suspension (l) dynamic deflection of right-front suspension (m) dynamic deflection of right-middle suspension (n) dynamic deflection of right-rear suspension (o) relative dynamic load of left-front wheel (p) relative dynamic load of left-middle wheel (q) relative dynamic load of left-rear wheel (r) relative dynamic load of right-front wheel (s) relative dynamic load of right-middle wheel (t) relative dynamic load of right-rear wheel

The corresponding simulation program is compiled via MATLAB. The simulation condition is B-grade road, and the speed is 70 km/h. The simulation results are shown in Fig. 4 and Table 4.

As shown in Fig. 4, the vertical acceleration PSDs of the body centroid and the connecting points between the suspensions and the body are remarkably different. The curves of the rigid-elastic model are two peaks more than those of the rigid model, occurring at the frame natural frequencies of 4.80 Hz (bending) and 9.94 Hz (bending).

Table 4 shows that the frame elasticity strongly influences the vertical acceleration of the key points and suspension dynamic deflections. The maximum difference of the vertical acceleration RMS value at the connection point between the left-front suspension and the body is 137.29% mainly due to the influence of the first and second bending modes of the frame. Frame elasticity also has a certain effect on wheel dynamic loads. The maximum relative percentage is 11.43% in the right-front wheel dynamic load, and the minimum is -5.13% in the left-rear wheel dynamic load because of the influence of the first torsion mode of the frame.

A comparison of the simulation results of the two models shows that the frame elasticity substantially affects the ride comfort of the heavy vehicle. Yang et al., (2003) built five-axle heavy vehicle spatial elastic model, and a real vehicle test is carried out to verify the simulation results of ride comfort. Their conclusion is that the test is similar to the elastic model, which is much larger than the results of the rigid model. Therefore, frame elasticity must be considered in the study of the ride comfort of heavy vehicles.

4.2 Comparison between PEM and Fourier method

The simulation results are shown in Fig. 5 and Table 5.

| Response variables | PEM       | Fourier   |
|--------------------|-----------|-----------|
| $\ddot{x}_{reb}$ (ms$^{-2}$) | 1.643042 | 1.643042 |
| $\ddot{x}_{rel}$ (ms$^{-2}$) | 2.702573 | 2.702573 |
| $\ddot{x}_{rem}$ (ms$^{-2}$) | 1.885319 | 1.885319 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 1.917960 | 1.917960 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 2.863349 | 2.863349 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 1.892786 | 1.892786 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 1.917446 | 1.917446 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 1.553569 | 1.553569 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.015702 | 0.015702 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.009351 | 0.009351 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.009416 | 0.009416 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.015856 | 0.015856 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.009305 | 0.009305 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.009361 | 0.009361 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.154100 | 0.154100 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.213311 | 0.213311 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.263953 | 0.263953 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.155221 | 0.155221 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.212814 | 0.212814 |
| $\ddot{x}_{re}$ (ms$^{-2}$) | 0.263284 | 0.263284 |
Fig. 5 Comparison of PEM and the Fourier method (a) vertical acceleration of the body centroid (b) vertical acceleration of driver and seat mass (c) vertical acceleration of key 1 (d) vertical acceleration of key 2 (e) vertical acceleration of key 3 (f) vertical acceleration of key 4 (g) vertical acceleration of key 5 (h) vertical acceleration of key 6 (i)
dynamic deflection of left-front suspension (j) dynamic deflection of left-middle suspension (k) dynamic deflection of
left-rear suspension (l) dynamic deflection of right-front suspension (m) dynamic deflection of right-middle suspension (n)
dynamic deflection of right-rear suspension (o) relative dynamic load of left-front wheel (p) relative dynamic load of
left-middle wheel (q) relative dynamic load of left-rear wheel (r) relative dynamic load of right-front wheel (s) relative
dynamic load of right-middle wheel (t) relative dynamic load of right-rear wheel

Table 5 and Fig.5 show that the simulation results of PEM and the Fourier method are completely the same, which
verifies the validity of PEM for the ride comfort of heavy vehicles. From the solution process, PEM does not require
the frequency response function of the response variables and is thus simpler than the Fourier method. The running
times of the Fourier method and PEM are 24.49 s and 12.06 s, respectively, and the running speed of PEM is increased
by 50.8%. Therefore, PEM is correct and is simpler and more efficient than the Fourier method in solving the ride
comfort of heavy vehicles.

5. Conclusion

In this research, an innovative simulation technology based on a rigid-elastic model and PEM is presented for the
study of the ride comfort of three-axle heavy vehicle. First, through modal analysis of the frame, the elastic information
of the frame is obtained accurately. On the basis of the theory of rigid-elastic synthesis and the lumped parameter
method, a 17-DOF spatial rigid-elastic model of a three-axle heavy vehicle is established. Subsequently, the formula of
PEM is deduced, which is used to solve the ride comfort of the three-axle heavy vehicle. Based on this study, some
conclusions can be obtained as follows:

(1) This paper uses FEM and the lumped parameter method to establish the rigid-elastic spatial model. This method
requires only a few parameters, is convenient for understanding the dynamics of heavy vehicles, and can be used in
three-dimensional model and two-dimensional model.

(2) By the comparison of the rigid-elastic and rigid models, the maximum difference of the responses RMS value is
137.29%, therefore, the frame elasticity substantially affects the ride comfort of the heavy vehicle. The frame elasticity
not only affects the acceleration of the key points on the frame, but also affects the suspension dynamic deflection and
the wheel dynamic load. The first two bending modes of the frame mainly affect the vertical acceleration of the key
points. The first torsion mode of the frame mainly affects the suspension dynamic deflection and the wheel dynamic
load. In order to establish an accurate simulation model for the ride comfort of heavy vehicle, the elasticity of the frame
must be considered.

(3) By the comparison of PEM and the Fourier method, the simulation results are completely the same and the
running speed of PEM is increased by 50.8%. Meanwhile, PEM does not require the frequency response function of the
response variables and is thus simpler than the Fourier method. Therefore, PEM is correct and is simpler and more
efficient than the Fourier method.

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