High-dimensional states of light with full control of path and OAM degrees of freedom

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We present here a compact scheme for the generation of high-dimensional states of light encoded in the path variable of photons that carry orbital angular momentum. Our method uses a programmable spatial light modulator in phase configuration to create correlations between these two spatial degrees of freedom. This setup allows us to independently control the relative phases and amplitudes of the path superposition, in addition to the topological charge of each path. Experimental results from the characterization of different generated states up to dimension 45, are in excellent agreement with the numerical simulations, and fidelities with respect to the target state are all above 95%.

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It is well known that photons can encode information in several degrees of freedom (DoFs) as polarization [1], frequency [2], time-bin [3] or transverse spatial modes [4, 5]. Since these DoFs can be accurately manipulated at room temperature by using standard commercial technology, photons have become one of the main platforms for several applications in quantum information processing [6]. For instance, in a quantum communication scenario, photons are the best choice as flying information carriers, since they are suitable for long-distance data transmission, in addition to the fact that current photonics implementations are compatible with conventional optical networks [7].

Of particular interest is the use of the transverse spatial modes with orbital angular momentum (OAM). As it was shown by Allen et al. [8], light with a twisted phase wavefront $e^{i\ell \theta}$ ($\theta$ the azimuthal angle with respect to the propagation axis) carries a quantized amount $\ell \hbar$ of OAM per photon, where $\ell$ is an integer that can be, in principle, arbitrarily large, and $\hbar$ is the reduced Planck’s constant. Hence, photons carrying OAM have been used to encode $d$-dimensional quantum states (namely qudits) allowing to increase the quantum complexity of a system without increasing the number of involved particles [9]. In this context, OAM-based communications could achieve a higher data transmission rate through free space [10] or optical fibers [11], and improve robustness to noise and security against to a potential eavesdropping.

The experimental generation of light with an OAM spectrum can be done using phase or amplitude masks [12]. One of the most versatile techniques to accomplish this task consists of displaying a fork hologram in a programmable device like a liquid crystal display (LCD) as part of a spatial light modulator (SLM) system. Moreover, these SLMs have also proved to be useful for generating qudits codified in the transverse spatial DoF of light with $\ell = 0$, by means of the discretization of the transverse linear momentum of single photons [13]. In this case, the encoding process is achieved when photons are passed through an aperture with $d$ slits (or alternatively, through $d$ bi-dimensional spatial regions), and it is the amount $d$ of possible paths to be follow by a photon that defines the dimension of the state. Optimally, it can be implemented using a single phase-only SLM system to control the complex superposition that define the path qudit, that is, to control independently the phase and amplitude modulation of each slit [14, 15].

More recently, different DoFs of multipartite-photonic states have been combined to defined high-dimensional quantum systems [16, 17]. It is a good option as strategy to increase the encoded information per particle and exploit the entire capacity of the quantum channel. Additionally, the linear superposition of multiple-DoFs states of a single photon, or even the use of various correlated DoFs of classical light, enables the generation of non-separable states [18], sometimes called classical entangled states. Although such encoding has a local nature and the resulting non-separable state is not, strictly speaking, a quantum entangled state, it can also brings advantages over conventional techniques of optical metrology [19] and be useful in applications of quantum information [20].

With this motivation, we propose here a codification that correlates two spatial DoFs of light: the path and the OAM modes. In our method, a binary phase hologram is programmed in each of the $d$ regions of a LCD screen, generating $d$ orthogonal path states that carry, each, a given $\ell$ quanta of OAM. By using a SLM operating in a phase only mode we are able to control both the OAM contribution and the complex amplitude of each path, regardless of the OAM content. As a result, we can obtain an arbitrary superposition of the OAM-path states giving a non-separable state. Besides, we exploit the regions of the SLM that are not used to encode in-

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formation to display a blazed grating whose diffracted order can be used as a reference beam in the state reconstruction stage. In this way we have implemented a compact design where only one SLM generates the emulated bipartite quantum system while the reference is used for the state characterization carried out by means of a phase shifting interferometry (PSI) technique.

Central issues. Let us start with the generation of OAM-path states. Suppose a paraxial and monochromatic light field $U(\rho, z)$, where $\rho = \rho \left(\hat{x} \cos \theta + \hat{y} \sin \theta\right)$ denotes the transverse coordinates of the position and the axis $z$ is taken along the direction of propagation. When this field passes through an aperture at $z = 0$ described by the complex transmission function $T(\rho)$, its state is transformed from $U(\rho) \equiv U(\rho, z = 0)$ to $\tilde{U}(\rho) \equiv T(\rho) U(\rho)$.

We will consider, without loss of generality, a transmittance defining an array of $d$ circular spatial regions of radius $r$

$$T(\rho) = \sum_{\mu=0}^{d-1} \text{circuit} \left(\frac{|\rho - \rho_\mu|}{r}\right) f_\mu(\rho - \rho_\mu),$$

where $\text{circuit}(\rho) = 1$ if $\rho \leq 1$, zero otherwise. Here $f_\mu(\rho)$ is the complex modulation function introduced by the SLM in the $\mu$-region, corresponding, in our implementation, to a diffraction grating with fork dislocations centered on $\rho_\mu$. Under the condition $|\rho_\mu - \rho_{\mu'}| > 2r$, such an aperture divides a Gaussian wavefront in $d$ non-overlapping vortex beams with topological charge $\ell_\mu$

$$\tilde{U}(\rho) = \sum_{\mu=0}^{d-1} \beta_\mu U_{\ell_\mu}(\rho - \rho_\mu) e^{i\ell_\mu \theta_\mu},$$

where $e^{i\ell_\mu \theta_\mu}$ is an OAM eigenstate with $\ell_\mu \hbar$ of OAM per photon in the electromagnetic field, and $U_{\ell_\mu}(\rho) \equiv \text{circuit}(\rho/\tau) A_{\ell_\mu}(\rho)$ is the radial amplitude function of this mode which, in general, will depend on $\ell_\mu$. Note that, as the optical vortex in each path is defined with respect to the individual center $\rho_\mu$, the angular coordinate is $\theta_\mu = \text{arg}(\rho - \rho_\mu)$.

Hence, the complex amplitude $\tilde{U}(\rho)$ of the resulting $d$-fold beams at $z = 0$, is described by a non-separable function given by the correlations between two independent DoFs. It emulates the bipartite entangled state

$$|\psi\rangle = \sum_{\mu=0}^{d-1} \tilde{\beta}_\mu |\mu\rangle \otimes |\ell_\mu\rangle,$$

with complex coefficients $\tilde{\beta}_\mu = \beta_\mu/\sqrt{\sum_{\mu=0}^{d-1} |\beta_\mu|^2}$ and kets $|\mu\rangle$, $|\ell_\mu\rangle$ representing the path and the OAM state, respectively. These two DoFs are independent in the sense that it is possible to set the amount of OAM per photon in each of the $d$ available paths. In addition, we can control the complex coefficients $\beta_\mu$ that define the superposition, i.e., the intensity and relative phase of each path.

To prepare the state in Eq. (3), the SLM display is divided in $d$ circular regions showing a binary phase hologram each. These holograms are fork-shaped containing the OAM information given by the number of central dislocations. When such a hologram with $\ell$ dislocations is illuminated with a plane wave, two beams are generated with a topological charge $\pm \ell$ and a doughnut-shaped intensity in the positive and negative first diffraction order, respectively [21]. With the purpose of controlling the complex amplitude $\beta_\mu$ and to redirect the information in the first diffraction order for a spatial filtering, each hologram is combined with a blazed grating. This codification is achieved in a single phase-only SLM by following a similar procedure to that described in Refs. [14, 15]:

i. A binary $0-\pi$ phase hologram multiplied by a phase diffraction blazed grating is displayed in the spatial region defining a particular path state. Designing the fork holograms in this way enables the first diffraction order to be situated far away from the zeroth order, in both the $\hat{x}$ and $\hat{y}$ directions, on the Fourier plane. In this way it is easily filtered and the light distribution is not corrupted by unwanted noise. As an example, in Fig. 1(a) we can see the programmed phase mask corresponding to the state $|\psi_1\rangle = \frac{1}{\sqrt{2}} (|\mu = 0 \rangle \otimes |\ell = -1\rangle + |\mu = 1 \rangle \otimes |\ell = 1\rangle)$, isomorphic to the Bell state $|00\rangle + |11\rangle$. The typical single-fork diffraction corresponding to $\ell = 1$ is clearly seen in the zoomed image.

ii. The efficiency of the first diffracted order for an ideal blazed grating can be calculated by the expression $\eta_1 = \text{sinc}^2 \left(1 - \frac{2\pi}{2\Gamma}\right)$, where $\phi_0$ is the phase modulation depth of the grating. In a real grating there is a discretization in the phase levels of the blazed profile given by the number of pixels $N$ in a grating period [22]. A schematic profile of a blazed grating with $N = 10$ is shown in Fig. 1(b). Therefore, it is possible to modulate the amount of light diffracted by each region by selecting the phase modulation $\phi_0$, and consequently, the weights in the superposition of Eq. (3). To the maximum grating efficiency reached with our SLM, we assign the maximum value of the real amplitude $|\beta_\mu| = 1$.

iii. A lateral displacement of the grating by $\Gamma$ pixels corresponds, in the Fourier plane, to an additional constant phase in the $n$-th diffraction order of $\delta = 2\pi \frac{\Gamma}{P} \mu$, where $P$ is the grating period measured in pixel units. Hence, introducing relative displacements between blazed gratings of different regions we set the value $\arg(\beta_\mu)$. In this way, we have a complete control of the complex coefficients $\beta_\mu$ in equation Eq. (3). Finally, after a spatial filtering, we obtain the desired OAM-path state in the first diffraction order.

In order to characterize the states we have implemented the four-step PSI method [23]. As it is well known PSI techniques are used to accurately measure the phase distribution of a wavefront for which successive phase shifts are introduced in the reference beam that is made to interfere with the object beam. In each step, the intensity distribution of the interferogram is given by
FIG. 1: (a) Phase mask representing the state $|\psi_1\rangle = |0\rangle - |1\rangle + \sqrt{2}|11\rangle$. The state is codified in the circular regions while the background generates a reference beam used in the state reconstruction stage. (b) Schematic profile of an ideal (red) and real (black) blazed grating.

$I(\rho; \Delta) = I_0 + \gamma \cos (\Phi(\rho) + \Delta)$, where $\Phi(\rho)$ represents the phase of interest, $\Delta$ is the constant phase value added to the reference in each step ($\Delta = 0, \pi/2, \pi, 3\pi/2$), $I_0$ is the arithmetic sum of the intensity in the reference ($I_1$) and the object ($I_2$) beams, and $\gamma = \frac{2\sqrt{I_1 I_2}}{I_0}$ is the modulation of the interference fringes. From here, the phase distribution is calculated by

$$\Phi(\rho) = \arctan \left( \frac{I(\rho; 3\pi/2) - I(\rho; \pi/2)}{I(\rho; 0) - I(\rho; \pi)} \right).$$

In our experimental implementation, the regions on the SLM external to the fork holograms do not contain any information about the OAM-path state. Hence, this background region can be used for generating the reference beam for a full characterization of the state. As it is schematically shown in Fig. 1, we display a tilted blazed grating in the region without information, sending the reference to a different position in the Fourier plane to which the state is sent. The required phase shifting $\Delta$ in each step of the PSI is achieve by a lateral displacement of this grating.

Figure 2 shows the experimental setup used for the generation and characterization of the states described by Eq. (3). In the first part, where the state is generated, a 810 nm laser diode is expanded, filtered and collimated. Linear polarizers (P) and quarter wave plates (QWP) are set to control the polarization of light both at the input and output of a reflective LCD (Holoeye LC-R 2500). We used elliptical polarization since it allows us to modulate mostly the phase [24] of the wavefront as required by our encoding. The modulated light is split in two arms by the beam splitter BS2. On each arm the first diffracted order, which carried the required information, is filtered in the focal plane of the transforming lens L3. In the second part of the setup, where the state characterization is performed, intensity measurements are carried out with charge-coupled device (CCD) cameras. The far-field distribution is registered by CCD1 in one of the arm, while in the other arm the image of the LCD is obtained by CCD2.

Results and discussion. To test the feasibility of the proposed method for generating OAM-path states we start by using an aperture $T(\rho)$ with $d = 2$ like the one shown in the scheme of Fig.1. In each of the $d$ circular regions defining the dimension of the path sector we displayed a binary fork hologram of 4 pixels of period. It was multiplied by a blazed grating, of 10 pixels of period, in the vertical direction. Each region has a diameter of 56 pixels wit a separation between centers of 112 pixels. Outside the circular regions, a second blazed grating that generate the reference for the PSI was oriented at 190 degrees with respect to the previous ones.
It is enough for sending the backlight in the far field to a region distant from the light coming from the fork holograms. The states \(|\psi_0\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}\), \(|\psi_{1,2}\rangle = \frac{|01\rangle \pm |11\rangle}{\sqrt{2}}\), and \(|\psi_{3}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{10}}\) were implemented. These examples were chosen as representative examples that emulate different types of two-qubit states: a separable state \(|\psi_0\rangle\), two maximally entangled states \(|\psi_{1,2}\rangle\), and a non-maximally entangled state \(|\psi_{3}\rangle\). Figure 3(a) shows the experimental and simulated far-field distributions from which the one-dimensional interference pattern is obtained and shown in Fig. 3(b). One can clearly see the interference fringe structures corresponding to a coherent superposition of two beams with the given complex amplitudes. Figure 3(c) shows the intensity distributions obtained in the near-field after the spatial filtering and the corresponding phase distributions reconstructed by the PSI method. It can be seen the typical spiral phases of a vortex beam anticipating the topological charge of each path. Last, in Fig. 3(d) the density matrices obtained from the experimental data can be seen. In each case, the imaginary part is null in agreement with the expected one.

In Fig. 4 we show the results of the proposed encoding for five and nine circular regions represented onto the LCD in two different spatial arrays. As an example, two arbitrary states were represented: \(|\psi_4\rangle = \frac{1}{\sqrt{5}} \sum_{\mu=0}^{4} e^{i \pi (\mu+1)/3} |\mu\rangle\) and \(|\psi_{5}\rangle = \frac{1}{\sqrt{5}} \sum_{\mu=0}^{4} e^{i \pi (\mu+1)/6} |\mu\rangle\), with \(\mu = \{0, 1, -2, -1, 2\}\) and \(\mu = \{-2, 0, 1, 0, -2, 2, 1, -2, 0\}\), respectively. In these cases, each circular region onto the LCD has a 56-pixels diameter while the centers are separated by 44 pixels. The Fig. 4(a) shows the experimental and numerical results for the generated state \(a\) in the far field. The phase and amplitude distributions obtained from the measurement in the near field are shown in Fig. 4(b) and their corresponding density matrices in Fig. 4(c). As it can be appreciated even for this high-dimensional states the fidelities of reconstruction are still better than 96%. Finally, Fig. 5 shows the OAM power spectrum of the total generated states where an excellent performance of the used encoding method is appreciated. It should be noted that the performance of the proposal codification for a given state is limited by the resolution of the LCD. In general, the fidelities of the reconstructed states will decrease both by the number \(d\) of spatial regions and the maximum topological charge \(l\) that can be represented. This is mainly due to the limitation of generating fork holograms with a low number of pixels.

To summarize, we have implemented a method that exploits the versatility of programmable SLMs to control and correlate two spatial DoFs. It can be useful for encoding high-dimensional path-OAM states, increasing the ability of photonic systems to handle information. The method is based on the encoding of fork holograms using superimposed blazed gratings and a spatial filtering process. We were able to generate photonic states with a high fidelity (above 95%) in, for example, dimension \(2 \times 2\), \(5 \times 5\) and \(9 \times 9\). The proposed setup is compact and allows us to characterize the generated states by using the same SLM. It also exploits the benefits of using a binary phase grating to maximize light efficiency.

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