Errata for *Zeta integrals, Schwartz spaces and local functional equations*

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The numbering below follows the published version in the Lecture Notes in Mathematics, No. 2228 published in 2018 by Springer (ISBN: 978-3-030-01287-8). These corrections are incorporated into the arXiv version ≥ 5 (arXiv:1508.05594).

**Page 47, line -8** In the description of the topology on $C_\Omega(X^+)$ appeared an undefined norm $\|\cdot\|$ on the fibers of $E$ over $\Omega$. It suffices to take any continuous family over $\Omega$ of such norms, and the choice is irrelevant for the topology, as $\Omega$ is compact.

**Page 99, (7.4) $G \hookrightarrow X^+_\mathfrak{p}$.**

**Page 108, Lemma 7.4.5** The proof of the first assertion is incorrect, and the second assertion is unnecessary. This Lemma is only used in the proof of Theorem 7.4.7 (page 111). Specifically, I used it to argue that for any fundamental weight $\varpi$ of $G$ with respect to a Borel pair $(B,T)$, the defining equation $f_\varpi \in F[X^+]$ of the color $D_\varpi$ in $X^+$ satisfies

$$v_{\partial X}(tf_\varpi) = 0,$$

so that $tf_\varpi \in F[X]$ (recall that $X$ is normal and $\partial X$ is a prime divisor) and it cuts out $D_\varpi \subset X$. Below is a corrected argument for it.

Since $t \in F[X^+] \subset F(X)$ is a uniformizer for $\partial X$ (Lemma 7.2.5, 7.2.6), our goal is to prove $v_{\partial X}(f_\varpi) = -1$ for each fundamental weight $\varpi$.

Write $\mathbb{G}_m = \text{Spec } F[s,s^{-1}]$. By the proof of Lemma 7.2.6, for $g \in M_\text{ab} \times G \times G$ in general position we have

$$v_{\partial X}(f_\varpi) = \text{ord}_{s=0}(f_\varpi(c(s)g)) \cdot i(c(0), \partial X \cdot c; X)$$

$$= \text{ord}_{s=0}(f_\varpi(x_0(1,\mu(s))g)) \cdot i(c(0), \partial X \cdot c; X),$$

where the base point $x_0 \in X^+(F)$, the morphism $c : \mathbb{G}_m \to X^+$ and $\mu : \mathbb{G}_m \to G$ are defined in the cited proof, namely $c(s) = sx_0(1,\mu(s))$. Here we used the fact that $f_\varpi$ is invariant under $M_\text{ab}$-action. For the same reason, it suffices to take $g = (1,g_1,g_2)$ with generic $g_1,g_2 \in G$. 

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In the proof of Lemma 7.2.6, the local intersection number \( i(c(0), \partial X \cdot c; X) \) has been shown to be 1.

By Lemma 7.2.3, we identify \( X^+ \) with \( \mathbb{G}_m \times G \) so that \( x_0 = (1, 1) \); the \( \mathbb{G}_m \times G \times G \)-action is also specified there. Let \( \rho \) be the (right) \( G \)-representation of highest weight \( \varpi \); take vectors \( v_\varpi \) and \( \tilde{v}_{-\varpi} \) in \( \rho \) and \( \tilde{\rho} \), with weights \( \varpi \) and \( -\varpi \) respectively such that \( \langle \tilde{v}_{-\varpi}, v_\varpi \rangle = 1 \). Then

\[
\delta(s,g_1,g_2) := \langle \tilde{\rho}(g_2)\tilde{v}_{-\varpi}, \rho(\mu(s)^{-1})\rho(g_1)v_\varpi \rangle \in s^{-1}F[s],
\]

and \( \text{ord}_{s=0}(\delta(s,g_1,g_2)) \) for generic \((g_1,g_2)\) equals actually

\[
\inf_{(g_1,g_2) \in G^2} \text{ord}_{s=0}(\delta(s,g_1,g_2)), \quad \text{which is } \geq -1.
\]

Taking \( g_1 = 1 = g_2 \), the right hand side does attain \(-1\). This completes the proof of \( v_{\partial X}(f_\varpi) = -1 \).

Alternatively, if the right hand side is \( \geq 0 \) then \( f_\varpi \in F[X] \) and is invariant under the \( \mathbb{G}_m \)-dilation on \( X \). This would imply the constancy of \( f_\varpi \) by Lemma 7.4.5. Contradiction.

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1See N. Bourbaki, *Lie groups and Lie algebras*, Chapters 7—9, pp.206–207.