The Strategic Exercise of Options Using Government Subsidies: An Analysis of Production Subsidies for the Ground Source Heat Pump

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Abstract. This study utilizes consolidation investment theory to incorporate with business strategies and government subsidy to develop a strategic exercise of options model. This empirical investigation examines the ground source heat pump (GSHP) government subsidy program, which is part of China’s 12th Five Year Plan. The developed model is applied to explain the behaviours of business investment with regard to strategic investment timing, option values, and the influence of government subsidies in duopolistic real-world investment decisions. The results indicate that subsidy policy can reduce the differences of investment timing among GSHP investors and has clearly evidenced the positive benefit–cost ratio of government subsidy, which facilitates China’s GSHP industry development.

1. Introduction
While the traditional real option approach (ROA) has been applied to analysing the optimal timing of execution, the lack of considerations to the interaction between two firms has been a remarkable deficiency. ROA ignores the potential impacts of other firms’ exercise of strategies, which assumes that there are no impacts on the other firm’s decision making, thus violating the reality of normal business strategic exercises. Therefore, this study develops an alternative real option games (ROG) model, which includes government subsidy within the framework of the strategic exercise of options to analyze the timing of competitors’ strategic exercises. The ROG analysis is an advanced investment decision technique that enables analysts to incorporate uncertain investment risks and strategic exercise of options by market participants. This study develops an ROG model by incorporating government subsidies to explain the behavior of strategic investment timings, option values, and interaction of market competitors with regard to the government subsidy policy in duopolistic real-world investment decisions. Dixit and Pindyck [1]
reveal that a competitive environment may have a considerable impact on the valuation of real options, because when opponents exercise strategies that influence the value of the underlying asset, optimal policies cannot be determined in isolation as in classical models of real options. In particular, this environment, incorporating government subsidies, it possibly intervene the investment survivor, market participants’ strategies, and their performance.

A subsidy is a form of financial or in-kind support extended to an economic sector generally with the aim of promoting economic and social policies. A government subsidy is an economic intervention that is commonly implemented to promote general welfare. Subsidies can be for various purposes, including subsidies for production, consumption, exports, employment, and taxes. This study focuses on real investment with production subsidies to ensure that producers are better off through either direct support or payments to factors of production. Fudenberg and Tirole [2] and Grenadier [3-6] highlight the importance of including strategic considerations in real option models as an advanced discipline of investment decisions. The ROG model was initially formulated based on a real option analysis (i.e., ROA) framework dependent on one or more stochastic underlying variables and from which the uncertain investment value can be obtained. The equilibrium of the investment game is employed to analyze the different market participants’ strategic exercises of options to analyze their investment timings and option value. Aguerrevere [7] and Goto et al. [8] use ROG to analyze competition in the manufacturing sector. Angelou and Economides [9] use concept of ROG to analyze information and communication technology (ICT) business decision analysis model under the threat of competition. Hsu and Lambrecht [10] and Leung and Kwok [11] study the patent race within the framework of ROG. Huisman and Kort [12] and Aye and Fujiwara [13] employ ROG to analyze the timing of new technology adoption by all market participants, and Wu et al. [14] use a ROG evaluation method to explain competition in the thin film transistor liquid crystal display (TFT-LCD) industry.

ROG theory has been broadly employed to analyze real investment issues; however, to the best of our knowledge, there are no published studies that incorporate government subsidies in the analysis of market participants’ investment timings and the option values. Central aspects unaddressed in the existing literature include under which conditions to conduct an investment, how much option values can be gained, and the influences of government subsidies on the investments. This study addresses the aforementioned questions within the framework of a continuous-time real options game approach. This empirical investigation reviews the ground source heat pump (GSHP) government subsidy program, which is the main program of the 12th Five Year Plan (hereafter 12th FYP) with regards to accelerating the development of renewable technologies and encouraging their utilizations. The government of China plans to invest approximately US$12–US$18 billion to support the GSHP industry. The GSHP is a renewable energy technology that has increasingly attracted worldwide attention due to its advantages in energy efficiency and environmental friendliness [15-18]. The development of GHSP can help address the problem of air pollution and smog in China. The principle of GSHP means that renewable technology that uses energy stored in the ground to heat buildings and provide the hot water and element of cooling [19]. GSHP investment is an irreversible investment in an oligopolistic market due to its uneconomical cost-performance ratio compared with the traditional central heating, ventilation and air conditioning (HVAC) [20], and it generally receives government subsidies. Thus, it is an appropriate sample for this investigation.

The study is organized as follows. Section 2 provides an overview of the model including the assumptions and model formulations and determines the equilibrium set of exercise strategies for both market participants. Section 3 describes the empirical investigation: a case study of subsidies received in relation with GSHP in China. Section 4 presents the conclusion and policy implications.

2. Method: assumptions and model formulations
The model development of ROG includes the following assumptions: there are two identical investors in the market, each investor has an option to develop the same product, and the investment option can last forever. Hence, investors can choose to wait for optimal entry. The option values are affected by the strategies of other market participants. To address the relationship between the strategic exercise of options and government subsidies, we assume that the investor can receive a unit of government
subsidy as long as a product can be sold. Upon the market reaching the Nash equilibrium, the investors’ investment strategies are optimized and cross-interactive with each other. At the beginning of the investment game, each investor contemplates two choices: to be the pioneer to exercise the option (become the leader) or to postpone exercising the option (become the follower), with each strategies having an optimal window for “time to act.”

The market price of a unit of product at time $t$, denoted by $P(t)$, equals the theoretical price $x(t)$, which is an exogenous shock process at time $t$, multiplied by the inverse demand function $D'[Q(t)]$, fluctuates stochastically over time, plus the production government subsidy $S$, based on Equation (1):

$$P(t)=X(t) \times D'[Q(t)]+S$$

where $D'[Q(t)]$ is the inverse demand function and $Q(t)$ represents the states of industry supplies (market conditions). There are two market conditions, $Q(t)=1$ and $Q(t)=2$, which represent the market as a monopoly and duopoly, respectively. When the market is a duopoly, investors’ strategies in the investment game are cross-interactive. Moreover, $D'[Q(t)]$ also represents the deterministic contribution to the investment value function. We assume that the inverse demand function is downward sloping, $D'[Q(t)] < 0$, to ensure “first-mover advantage” (i.e., FMA). Let $X(t)$ evolve following a geometric Brownian motion process:

$$dX(t) = \mu X(t) dt + \sigma X(t) dz$$

where $\mu$ is the instantaneous conditional expected percentage change in $X(t)$ per time; $\sigma$ is the instantaneous conditional standard deviation per unit time; and $dz$ is the increment of a standard Wiener process for $X(t)$. Consequently, parameters $\mu$ and $\sigma$ can be interpreted as the expected drift rate and volatility of log return, respectively. Upon the principle of risk neutrality, equation (2) can be rewritten as $dX(t) = (r-\delta)X(t) dt + \sigma X(t) dz$. $\delta$ is equal to risk premium, and $r$ is risk-free return (i.e., the risk-free rate).

In the solution procedure, we first solve for the follower’s optimal investment trigger and the option value function; then, we propose the analysis of the Nash equilibrium game.

### 2.1 Follower’s trigger and option value

First, we model the follower’s investment decision within the scenario of the leader’s investment option, which has already been exercised; thus, the follower can construct an exercise strategy without considering the strategic exercise by market competitor(s). The follower’s strategy is to maximize the option value as $F(X)$, which is equal to the follower’s investment value. We found $F(X)$ by solving an equilibrium differential equation using a standard real option analysis technique and obtained an expression for the expected drift rate on the follower’s option using Itô’s Lemma. The following expression gives us the expected drift rate on the follower’s option value $F(X)$ that solves differential Equation (3):

$$\frac{1}{2} \sigma^2 X(t)^2 F_{xx} + (r-\delta)X(t) F_x - rF = 0$$

when the consumer purchases a unit of a product, the market participants/investors receive a unit of $S$ from the government. $K$ is an exercise price which equal to the cost of investment. The differential Equation (3) must be solved subject to appropriate boundary conditions ensuring that the follower chooses the optimal trigger time (i.e., the “option to invest” is maximized):

$$F(0) = 0$$

$$F(X^*_f) = X(t)D(2)+S-K$$

(4-1)
\[ F(X_F^*) = D(2) \]  \hspace{1cm} (4-2)

The first boundary condition \( F(0) = 0 \) means that zero is an absorbing barrier of exercise price. The second boundary condition is the “value-matching” condition, where the option value is \( XD(2) + S - K \). The third boundary condition is the “smooth-pasting” or “high-contact” condition, which ensures that \( X_F^* \) triggers an entry strategy that maximizes the follower’s option value; hence, \( X_F^* \) is regarded as the follower’s optimal trigger timing (hereafter the follower’s trigger). Through this procedure, we obtain \( X_F^* \) closed-form solutions for the value of \( X(t) \), which triggers entry by standard dynamic programming methodology, and the follower’s option value \( F(X) \), in different steps:

\[ X^*_t = \frac{\beta}{\beta - 1} \left[ \frac{K-S}{D(2)} \right], \]  \hspace{1cm} (5)

\[ F(X(t)) = \begin{cases} 
\frac{K-S}{\beta - 1} \left( \frac{X(t)}{X^*_F} \right)^\beta & \text{for } X(t) < X^*_F \\
X(t)D(2) + S - K & \text{for } X(t) \geq X^*_F 
\end{cases} \]  \hspace{1cm} (6)

The first row of (6) is the present option value when follower does not invest immediately. Its first term is the value of the option to invest. The latter term equals the NPV of the follower’s investment discounted back from the (random) time of reaching the follower’s triggers entry \( X_F^* \). Consequently, \( (X(t)/X^*_F)^\beta \) is interpreted as a stochastic discount factor, which is equal to the present value of $1 received when starting from level \( X(t) < X^*_F \). The second row of (6) is the NPV of the profit flow when immediate investment is optimal [24].

where \( \beta \) is given by

\[ \beta = \frac{1}{2} \frac{(r-\delta)}{\sigma^2} + \sqrt{\left( \frac{(r-\delta)}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} > 1. \]  \hspace{1cm} (7)

Given the above results, we express both the leader’s and the follower’s triggers and investment value functions in the pre-emption games, as illustrated in figure 1, by referring to Fudenberg and Tirole [2] and Azevedo and Paxson [25].

**PROPOSITION 1.** Conditional on the leader having begun investment in an investment game, the follower does not fear a threat of pre-emption and pre-gain from the government subsidy, which the leader already possesses. The follower’s optimal strategy is to begin at \( X(t) \). This is equal to or exceeds the trigger value \( X_F^* \), where the investors’ investment values are maximized. Figure 1 illustrates that the follower’s investment is lower than the leader’s investment below \( X_F^* \).

2.2 **Leader’s trigger and option value**

In the investment game, the leader retains pre-emption and is the first exerciser in the market. In a monopoly, the leader receives monopoly profits \( X(t)D(1) \) [see Equation (8)] till the follower is triggered for entry. Hence, let \( L(X(t)) \) denote the leader’s option value, which solves the equilibrium differential Equation (8):

\[ 0 = \frac{1}{2} \sigma^2 X(t)^2 L_{xx} + (r-\delta) X(t)L_x - rL + X(t)D(1). \]  \hspace{1cm} (8)

However, once the follower’s option is exercised, the leader faces competition in the market and only receives duopoly profits. Differential Equation (8) must be solved subject to the boundary condition.
This boundary condition reflects the fact that the follower exercises the option at $X_F^*$, and the leader’s option value equals that of the follower in the investment game:

$$L(X^*_L) = X^*_F D(2) + S - K$$

The solution for $L(X(t))$ can be expressed as

$$L(X(t)) = \begin{cases} X(t)D(1) + S - K + \left( \frac{\beta}{\beta - 1} \right) (K - S) \left( \frac{D(2) - D(1)}{D(2)} \right) \left( \frac{X(t)}{X_F^*} \right)^\beta & \text{for } X(t) < X_F^* \\ X(t)D(2) + S - K & \text{for } X(t) \geq X_F^* \end{cases}$$

There exists a unique point $X_L^*$ between 0 and $X_F^*$ at which the option value for the leader and follower are equal. $X_L^*$ is the point proposed by Fudenberg and Tirole [2] at which a duopoly market is in equilibrium. Further, Paxson and Pinto [26] indicate that till this point, either investor is better off being the follower; after this point, the leader is better off. Therefore, this point should be the leader’s trigger $X_L^*$; however, a closed-form solution for $X_L^*$ cannot be obtained because the resulting function is highly non-linear. Thus, the trigger can be obtained numerically by solving the following non-linear equation (where $X_L^*$ is unknown):

$$X_L^* D(1) + S - K + \left( \frac{\beta}{\beta - 1} \right) (K - S) \left( \frac{D(2) - D(1)}{D(2)} \right) \left( \frac{X_L^*}{X_F^*} \right)^\beta - \left( \frac{K - S}{\beta - 1} \right) \left( \frac{X_L^*}{X_F^*} \right)^\beta = 0$$

**PROPOSITION 2.** Conditional on the investors not yet beginning the investment game, when competition is added to the investment decision problem, other investors’ actions are important factors that contribute to the leader’s exercise strategy. Because of monopoly power, the leader tends to invest earlier to earn a government subsidy in advance. However, the leader may also be vulnerable to risk of loss in the early market stages. Figure 1 shows that the leader’s option value is lower than that of the follower below $X_L^*$ and higher than that of the follower between $X_L^*$ and $X_F^*$. The leader’s optimal strategy is to begin investment when $X(t)$ equals or exceeds the trigger $X_L^*$.

2.3 **Equilibrium exercise strategies**

Two types of equilibria can occur in the investors’ choices of strategy—sequential and simultaneous equilibria—depending on the initial state of the theoretical price $X(t)$.

![Figure 1. Investors’ triggers and value in the pre-emption games](Source: Fudenberg and Tirole [2]; Azevedo and Paxson [25].)
2.3.1 Equilibrium with sequential exercise: \( X(t) < X_L^* \). Based on the distribution of \( X(t) \) and the relative position of \( X_L^* \) and \( X_F^* \), the price horizon can be divided into two different regions on the \( X(t) \) line, \( 0 < X(t) < X_L^* \) and \( X_L^* \leq X(t) < X_F^* \), where sequential equilibrium occurs. These situations are described as follows:

**Region 1:** For \( 0 < X(t) < X_L^* \)

If the investment game begins at the initial level, \( X(t) \) is below \( X_L^* \). As the market is not yet booming, the follower’s option value is higher than the leader. (figure 1). Most investors prefer to be followers rather than leaders. If other investors have not yet entered, they initiate investment as soon as \( X(t) \) equals or exceeds \( X_L^* \). If their competitor has already entered, they wait till \( X(t) \) rises to \( X_L^* \) before initiating an investment. Given the above strategy, sequential equilibrium will occur.

**Region 2:** For \( X_L^* \leq X(t) < X_F^* \)

If the investment game starts from \( X_L^* \), the benefits of a potentially temporary monopoly only equal the costs of early investment. The option value for both the leader and follower is equal at \( X_L^* \), where each investor has a 50% chance of being the leader. If the other investor earns monopoly profits in the region where \( X(t) \) is between \( X_L^* \) to \( X_F^* \), they become the leader. As soon as one investor earns the leadership in the investment game, the follower chooses the optimal time at point \( X_F^* \) to invest.

If the investment game begins with an initial level of \( X(t) \), which is between \( X_L^* \) and \( X_F^* \), the market is more developed; the leader’s option value is higher than that of the follower due to monopoly profits, and each investor would prefer to be the leader. If the leader’s option has been exercised, the follower waits till the trigger \( X_F^* \) is reached.

2.3.2 Equilibrium with simultaneous exercise: \( X_F^* \leq X(t) \). We indicate that the investment game begins with an initial level of \( X(t) \) and that each investor has the same investment value above \( X_F^* \). In this range, any equilibrium will be characterized by a simultaneous exercise. If either investor’s trigger \( X(t) \) is equal to or greater than \( X_F^* \), the other enters immediately thereafter.

3. Results: case study of the ground source heat pump subsidies in china

To demonstrate that this ROG model including government subsidy can be applied to real-world project evaluation under different scenarios, we investigate the GSHP subsidies specified in China’s 12 FYP. The GSHP is a central heating and/or cooling system that transfers heat to/from the ground. It takes advantage of the moderate ground temperatures to boost efficiency and reduce the system’s operational costs; moreover, it can be combined with solar heating to form a geo-solar system with even greater efficiency. The US Environmental Protection Agency (EPA) described the GSHP as the most energy-efficient, environmentally clean, and cost-effective space conditioning system available. Heat pumps offer significant emission reduction potential, particularly when used for both heating and cooling and when the electricity is produced from renewable resources. UK Environment Agency also indicated that ground pumps could help to achieve energy policy targets by 2020.

The data for the empirical investigation are sampled from sunshi100.com and describe a certain brand of GSHP central air conditioning. The observed sample unit services an indoor area up to 170 square meters. The parameters for empirical investigation are revealed in table 1.

We simulate a sensitivity analysis of investors’ triggers by simulating \( \mu, \sigma \), and \( X(t) \) and run simulations for the inverse demand function \( D(I) \), the purchase price that consumers are willing to pay \( K \), and government subsidies \( S \). Importantly, the investors’ cross-interactive strategic exercises and government subsidies \( S \) are also addressed in different scenarios and provide policy implications.
3.1 Sensitivity analysis of investors’ triggers by simulating \( \mu \)

Figure 2 depicts the changes in \( X^*_L \) and \( X^*_F \) and the difference between them, \( G_\delta \), by simulating \( \mu \) changes from 0%, 0.5%, 1.0%, 1.5%, 2.0%, 2.5%, and 3.0% to 3.5% under the scenario of \( r = 3.8\% \), \( \sigma = 5.0\% \), \( D(1) = 1.0, D(2) = 0.75 \), \( K = 36,364 \), and \( S = 10,909 \). The results of this simulation show that investors’ triggers rise with increasing \( \mu \), regardless of the leader’s or follower’s triggers. \( G_\mu \) also simultaneously increases (see figure 2), and with the increasing risk premium, the trigger rises sharply. When the risk premium change \( \mu \) is 0%, the leader’s and follower’s triggers are US$26,637 and US$40,679, respectively, and the \( G_\mu \) is US$14,042. When the risk premium change \( \mu \) is 0.5%, the leader’s trigger is US$28,270 and follower’s trigger increase to US$44,500 with a \( G_\mu \) of US$16,230. When \( \mu \) is equal to 1.0%, the leader’s and follower’s reach US$31,330 and US$50,653, and the \( G_\mu \) is US$19,323. The leader’s and follower’s triggers are US$36,516 and US$60,294, respectively, with a \( G_\mu \) of US$23,778 when \( \mu \) is 1.5%. When \( \mu \) is equal to 2.0%, the leader’s trigger increases to US$45,246 and the follower’s trigger rises to US$75,913 with a \( G_\mu \) of US$30,667. When \( \mu \) is equal to 2.5%, the leader’s trigger reach US$61,269 and follower’s trigger increase to US$104,052, respectively, and the \( G_\mu \) is US$42,783. When \( \mu \) is equal to 3.0%, the leader’s and follower’s trigger increase to US$97,916 and US$69,945, respectively, with a \( G_\mu \) of US$69,945. In addition, the leader’s and follower’s triggers reach US$257,777 and US$445,212, respectively, with a \( G_\mu \) of US$187,435 when \( \mu \) is 3.5%. The observed triggers and \( G_\mu \) rise sharply, as shown in figure 2.
These results demonstrate that $\mu$ has a positive relationship with both the leader’s and follower’s triggers in the GSHP investment game. In other words, when the risk premium is high, potential investors are more likely to postpone exercising their investment options. The findings in this study are in line with previous studies on energy investment stimulus programs, such as Moon [23].

3.2 Sensitivity analysis of investors’ triggers by simulating $\sigma$

Figure 3 show the changes in $X^*_L$ and $X^*_F$ and $G_\sigma$ by simulating $\sigma$ from 5%, 10%, 15%, 20%, 25%, and 30% to 35% under the scenario of $r=3.8\%$, $\delta=1.9\%$, $D(1)=1.0$, $D(2)=0.75$, $\kappa=36,364$, and $S=10,909$. The simulation results indicate that when $\sigma$ is 5%, the leader’s trigger is US$43,095 and the follower’s is US$72,098 with a $G_\sigma$ of US$29,003. When $\sigma$ is 10%, the leader’s trigger increase to US$49,256 and follower’s trigger increase to US$82,991. The $G_\sigma$ is US$33,735. When $\sigma$ is 15%, the leader’s and follower’s triggers increase to US$58,114 and US$98,534, respectively, with a $G_\sigma$ of US$40,421. The leader’s and follower’s trigger are increase to US$69,275 and US$118,026, respectively, with a $G_\sigma$ of US$48,751. When $\sigma$ is 25%, the leader’s trigger is US$82,663 and follower’s is US$141,342 with a $G_\sigma$ of US$58,678. When $\sigma$ is 30%, the leader’s and follower’s triggers reaches US$98,303 and US$168,533, respectively, with a $G_\sigma$ of US$70,230. When $\sigma$ is 35%, the leader’s and follower’s triggers reaches US$116,243 and US$199,693, respectively, with a $G_\sigma$ of US$83,450. This result demonstrates that uncertainty $\sigma$ has a positive relationship with both the leader’s and follower’s triggers in the GSHP investment game.

The above property is consistent with the observations by Lukas and Welling [22] and Moon [23], who indicated that the higher the uncertainty (volatility), the higher the propensity to postpone exercising their investment options.

3.3 Sensitivity analysis of investors’ triggers by simulating $X(t)$

Figure 4 shows the changes in $L(X)$ and $F(X)$ and the difference between them, $G_p = F(X) - L(X)$, by simulating the changes in $X(t)$ from 0 to 120,000, in step sizes of 10,000 and under the scenario of $r=3.8\%$, $\delta=1.9\%$, $\sigma=5\%$, $D(1)=1.0$, $D(2)=0.75$, $\kappa=36,364$, and $S=10,909$. The simulation results can be divided into three regimes. In regime 1, $X(t) < X^*_L = US$43,095, When the theoretical price is US$10,000, the leader’s and the follower’s option values are US$−15,886 and US$685, respectively, with a $G_p$ of is US$16,571. The leader’s and the follower’s option values are US$−7,053 and US$2,538, respectively, and $G_p$ is US$9,591 when the theoretical price is US$20,000. When the theoretical price is US$30,000, the leader’s and the follower’s option values are US$1,107 and US$5,459, respectively, with a $G_p$ of is US$4,352. As the market is not yet booming, the leader’s option value is lower than that of the follower due to weak initial market conditions. Hence, the leader faces an investment loss. Most investors prefer to be followers rather than leaders. However, with increasing theoretical price $X(t)$, when the theoretical price reaches $X^*_F$, all investors’ option values equal US$10,824, with a $G_p$ of is zero; then, investors in the GSHP market decide to invest immediately.

When the theoretical price is located between $X^*_L$ and $X^*_F$, firms follow regime 2: the leader’s and the follower’s option values are US$15,518 and US$14,332, respectively, and $G_p$ is US$−1,186 when the theoretical price is US$50,000. When theoretical price reaches US$60,000, the leader’s and the follower’s option values are US$21,806 and US$20,227, respectively, with a $G_p$ of US$−1,579. When the theoretical price is increased to US$70,000, the leader’s option value increases to US$27,499 and the follower’s option value increases to US$27,066. The $G_p$ reduces to US$−433. In this regime, the investment market is more developed, the leader enjoys the profits from pre-emption;
thus, the leader’s option value is higher than that of the follower, and each investor would prefer to be
the leader.
When $X(t)$ is higher than $X^*_L = \text{US$72,098}$, firms follow regime 3: option values of both investors
are equal, their option values increase to US$28,619, and $G_p$ is zero. When theoretical price reach
US$80,000, the investor’s option values increase to US$34,545, with a $G_p$ of US$0$. When theoretical
price increase to US$90,000, the leader’s option value increases to US$42,045, follower’s option value
also increase to US$42,045, with a $G_p$ of US$0$. The investors’ option values are equal, their
option values increase to US$49,545, and $G_p$ is US$0$ when the theoretical price is US$100,000.
When theoretical price is increased to US$110,000, the all investors’ option values are both
US$57,045, the difference is zero. The investors’ option values are US$64,545 when theoretical price
increase to US$120,000. In this range, each investor would prefer to enter immediately. If either
investor’s trigger is equal to or greater than $X^*_L$, the other also enters immediately thereafter.
The empirical investigation demonstrates that the developed model is consistent with those in existing
studies on standard real option games, such as Fudenberg and Tirole [2], Pawlina and Kort [24], and
Azevedo and Paxson [25]. Thus, this empirical investigation supports the use of the developed model
in the practical analysis of GSHP investments.

3.4 Sensitivity analysis of investors’ triggers by simulating $D(1)$
Figure 5 show the changes in $X^*_L$ and $X^*_F$, and the difference, $G_{D(1)}$, by simulating the changes in
$D(1)$ from 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, and 4.5 to 5.0 under the scenario of $r = 3.8\%$, $\delta = 1.9\%$,
$\sigma = 5\%$, $D(2) = 0.75$, $\kappa = 36,364$, and $s = 10,909$. First, we set the follower’s trigger $X^*_F$ at US$72,098.$
The simulation result shows that the leader’s trigger is US$43,095 when the monopoly inverse
demand function $D(1)$ is 1.0, and $G_{D(1)}$ is US$29,003$. When $D(1)$ is 1.5, the leader’s trigger
decrease to US$23,714. The $G_{D(1)}$ is US$48,384. When $D(1)$ is 2.0, the leader’s trigger decrease to
US$16,307, and the difference increases to US$55,791. The leader’s trigger fall to US$12,410, and
$G_{D(1)}$ is US$59,688$. When $D(1)$ is 3.0, the leader’s triggers decrease to US$10,010, and difference
increases to is US$62,088. When $D(1)$ is 3.5, the leader’s trigger decrease to US$8,385, and
$G_{D(1)}$ increases to is US$63,713. When $D(1)$ is 4.0, the leader’s trigger decrease to US$7,212, and
difference increases to is US$64,886. When $D(1)$ is 4.5, the leader’s trigger decrease to US$6,326, and
$G_{D(1)}$ increases to is US$65,772. When $D(1)$ is 5.0, the leader’s trigger decreases to US$5,633
and the $G_{D(1)}$ increases to US$66,465. This result means that a higher $D(1)$ reduces the leader’s
trigger due to strong demand and government subsidy in a monopoly market. In other words, the
stronger inverse demand function $D(1)$ promotes investors to be the leader. When considering the
traditional real option, investors select an entry trigger that maximizes the investment value because
the inverse demand function is always identical. In this study, we add the monopoly inverse demand
function to determine the impacts of investors’ option value, triggers, and strategies. We obtain more
investment information using a model of the real option game compared with traditional real options.

3.5 Sensitivity analysis of investors’ triggers by simulating $K$
Figure 6 show the changes in $X^*_L$ and $X^*_F$, and the difference between them, $G_k = X^*_F − X^*_L$, by simulating the changes in $K$ from 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, and 4.5 to 5.0 times purchase
price that consumers are willing to pay (exercise price) under the scenario of $r = 3.8\%$, $\delta = 1.9\%$,
$\sigma = 5\%$, $D(1) = 1.0$, $D(2) = 0.75$, and $s = 10,909$. The results of this simulation reveal that the leader’s
and follower’s triggers are US$21,548 and US$36,049, respectively, with a $G_k$ of US$14,052 when consumers are willing to pay 0.5 times the purchase price. When $K$ increase to 1.0 times the purchase
price, the leader’s and follower’s triggers are US$43,095 and US$72,098, respectively, with a $G_k$ of US$29,003. Investor’s triggers and $G_k$ are gradually increased along with increasing purchase price $K$. When $K$ increases to 1.5 times the purchase price, the leader’s and follower’s triggers are US$64,643 and US$108,148, respectively, and $G_k$ rises to US$43,505. When $K$ increase to 2.0 times the purchase price, the leader’s and follower’s triggers are US$86,191 and US$144,197, respectively, with a $G_k$ of US$58,006. When $K$ increase to 2.5 times the average purchase price, the leader’s and follower’s triggers are US$107,738 and US$180,246, respectively, with a $G_k$ of US$72,508. In addition, the leader’s and the follower’s trigger are US$129,286 and US$216,295, respectively, with a $G_k$ of US$87,009 when the purchase price that consumers are willing to pay is 3.0 times the purchase price. When purchase price is 3.5 times the purchase price, the leader’s and the follower’s trigger are US$150,833 and US$252,344, respectively, with a $G_k$ of US$101,511. The leader’s and the follower’s triggers decrease to US$215,476 and US$360,492, respectively, with a $G_k$ of US$130,514. When $K$ reaches to 4.5 times the purchase price, the leader’s and follower’s triggers are US$92,698 and US$137,848, respectively, with a $G_k$ of US$145,295. Depending on the purchase price that consumers are willing to pay, $k$, higher costs (exercise price) postpone investment decisions, which is consistent with the results in Moon [23] that indicated investors wait longer if the costs are increased.

3.6 Sensitivity analysis of investors’ triggers by simulating $S$

Figure 7 shows the changes in $X_L^r$ and $X_L^f$, and $G_s = X_L^r - X_L^f$, by simulating the changes in $S$ from 0%, 5%, 10%, 15%, 20%, 25%, 30%, 35%, 40%, and 45% to 50% times the purchase price that consumers are willing to pay, under the scenario of $r=3.8\%$, $\sigma = 1.9\%$, $\sigma = 5\%$, $D(1) = 1.0$, $D(2) = 0.75$, $K = 36,364$, and $s = 10,909$. The simulation result reveals that the leader’s trigger is US$61,565, the follower’s is US$102,998, and $G_s$ is US$41,433 within the scenario of zero government subsidies. When government subsidies rise to 5% times the purchase price that consumers are willing to pay, the leader’s and follower’s triggers decrease to US$58,486 and US$97,848, respectively, and $G_s$ decreases to US$39,361. The leader’s and follower’s triggers decrease to US$55,408 and US$92,698, respectively, with a $G_s$ of US$37,290 when the government subsidies reach to 10% times the purchase price that consumers are willing to pay. When government subsidies increase to 15% times the purchase price, the leader’s and follower’s triggers decrease to US$52,330 and US$87,548, respectively, and $G_s$ decreases to US$35,218. The leader’s and follower’s triggers decrease to US$49,252 and US$82,398, respectively, with a $G_s$ of US$33,146 when the government subsidies reach to 20% times the purchase price that consumers are willing to pay. When government subsidies reach to 25% times the purchase price, the leader’s and follower’s triggers decrease to US$46,174 and US$77,248, respectively, and $G_s$ decreases to US$31,075. In GSHP market, investor’s triggers and $G_s$ are gradually decreased along with increasing government subsidies. When government subsidies reach to 30% times the purchase price, the leader’s and follower’s triggers decrease to US$43,095 and US$72,098, respectively, with a $G_s$ of US$29,003. When government subsidies rise to 35% times the average sample price, the leader’s and follower’s triggers decrease to US$40,017 and US$66,948, respectively, and $G_s$ decreases to US$26,931. When government subsidies rise to 40% times the purchase price, the leader’s and follower’s triggers decrease to US$36,939 and US$61,799, respectively, and $G_s$ decreases to US$24,860. When government subsidies rise to 45% times the purchase price, the leader’s and follower’s triggers decrease to US$33,861 and US$56,649, respectively, and $G_s$ decreases to US$22,788. When the government subsidies rise to 50% times the purchase price, the leader’s and follower’s triggers fall to US$30,782 and US$51,499, respectively, and $G_s$ decreases to US$20,716. The above simulation demonstrates the negative relationship between investors’ triggers and government subsidies. In other words, higher government subsidies
attract GSHP investors to enter the market earlier. In other words, more investors are attracted to and more likely to invest because of government subsidies. Investors stimulate the market as they are more willing to join the GSHP market and produce more GSHP products, thus achieving the goal of government policy.

3.7 The effects of GSHP subsidy policy on investors’ option value and government’s benefit–cost analysis (benefit–cost analysis, BC ratio)

To examine China’s subsidy policy (30% times the average purchase price that consumers are willing to pay) and how to influence the investors’ option value and its benefit and cost for the goal of promoting the GSHP industry, we simulate the investors’ option values under the scenario of $S = 28\%–32\%$. Therein, we display changes in the leader’s and follower’s option values under the scenario of $S = 28\%–32\%$, $L(X)$, $F(X)$, as well as the difference in option values, $G_L$, and $G_F$, and the benefit ratio, $B$ from adding government subsidies of 1% by simulating the theoretical price changes in $X(i)$ from 0 to 90,000, in step sizes of 10,000 and under the scenario of $r = 3.8\%$, $\sigma = 1.9\%$, $\delta = 5\%$, $D(1) = 1.0$, $D(2) = 0.75$, and $K = 36,364$.

Under the scenario of $S = 28\%$, the leader’s and the follower’s option values are US$7,741 and US$2,475, respectively. When the theoretical price is US$20,000, and when government adds subsidies of 1% to $-29\%$, the investors’ option values increases to US$7,397 and US$2,506, at which point the differences in option values are US$344 and US$31, respectively. When the theoretical price reaches US$60,000, the differences in option values are US$208 and US$247, respectively. Evidently, the leader’s (follower’s) option value experiences a decrease (increase) when adding the theoretical price under the same scenario of subsidy policy. Hence, the results conclude with certainty that the policy of government subsidy can decrease the differences in option values between investors of GSHP.

As previously mentioned previously, we will display the changes in the benefit–cost ratio under the different scenarios in examining the effect of China’s subsidy policy on GSHP investors’ option values and its implication of benefit and cost for subsidy policy. When the theoretical price of GSHP is US$36,364 (i.e. the average purchase price that consumers are willing to pay), the leader’s and follower’s values are US$5,662 and US$7,754, respectively and differences of option values are US$303 and US$96, respectively, under the scenario of a 29% subsidy. The government can earn 1.098 times the benefits for promoting overall GSHP industry when it increases costs by 1% (i.e. subsidy of 1%). Under the current China subsidy policy (i.e. 30%) and the same level of the theoretical prices of GSHP, the benefit–cost ratio increases to 1.100, even though the difference in subsidy is also equal to 1%. The results indicate a positive and strong relationship between subsidy policy and promoting the GSHP industry. More specifically, China’s current GSHP production subsidies policy will earn 1.1 times the benefits for GSHP industry development.

Moreover, we also discover that the benefit–cost ratio will exhibit an increasing trend with “increasing theoretical price of GSHP with same subsidy policy” or “increasing subsidy policy with same theoretical price of GSHP”. For the first case, when the theoretical price rises to US$20,000 from US$10,000, the benefit–cost ratio increases to 1.032 from 1.009 (1.032–1.009 = 0.023). When the theoretical price rises to US$70,000 from US$60,000, the benefit–cost ratio increases to 1.336 from 1.251 (1.336–1.251 = 0.085). The results suggest that subsidy policy may have a stronger impact on the higher GSHP price. For the second case, when the theoretical price rises to US$70,000 from US$60,000, the benefit–cost ratio increases to 1.336 from 1.251 (1.336–1.251 = 0.085), and 1.365 from 1.273 (1.365–1.273 = 0.092) under the scenarios of subsidy ratios of 28% and 32%, respectively. These findings support the claim that higher subsidy policy will create more benefits for industry development. Thus, the government is able to expand the economic sector through subsidies to promote the GSHP and energy industries in China.
Figure 2. Sensitivity analysis of investors’ triggers by simulating expected drift rate $\mu$.

Figure 3. Sensitivity analysis of investors’ triggers by simulating volatility $\sigma$.

Figure 4. Sensitivity analysis of investors’ option value by simulating theoretical price $X(t)$.

Figure 5. Sensitivity analysis of investors’ triggers by simulating inverse demand function $D(1)$.

Figure 6. Sensitivity analysis of investors’ triggers by simulating purchase price that consumers are willing to pay $K$.

Figure 7. Sensitivity analysis of investors’ triggers by subsidies $S$.

Table 2. The effects of GSHP subsidy policy on investors’ option value and government’s benefit–cost analysis

| $X(t)$ | $L_{GSHP}(X) = \left( \frac{g_2 - g_1}{L_2 - L_1}(X) \right)$ (in US$) | $F_{GSHP}(X) = \left( \frac{f_2 - f_1}{F_2 - F_1}(X) \right)$ (in US$) | $B_{GSHP} = \frac{(G_2 + G_{GSHP})}{1%(K)}$ |
|--------|-------------------------------------------------|-------------------------------------------------|-------------------|
| 0      | -25,455 (364)                                   | 0 (0)                                           | 1.000             |
| 10,000 | -15,886 (358)                                   | 685 (9)                                         | 1.009             |
| 20,000 | -7,053 (344)                                    | 2,538 (32)                                      | 1.032             |
| 36,364 | 5,964 (302)                                     | 7,853 (98)                                      | **1.100**         |
| 40,000 | 8,624 (289)                                     | 9,402 (118)                                     | 1.120             |
| 50,000 | 15,518 (250)                                    | 14,332 (180)                                    | 1.183             |
Examining currently China’s GSHP production subsidies policy from viewpoint of Benefit Cost ratio, this study also discovers that government can earn 1.1 times benefits for GSHP industry development (see table 2).

5. Conclusion and policy implications
This proposed model is the first attempt to incorporate the real option game with subsidies into investors’ decision making for the development of new products from the investors’ viewpoint. Utilizing consolidation investment theory, this study considers investment-related business strategies and develops a relevant strategic option model. The model is applied to explain the behaviors of energy industries with regard to strategic investment timing, option values, and the influence of government subsidies in duopolistic real-world investment decisions. This empirical investigation examined the GSHP government subsidy program, which is part of China’s Twelfth Guideline (2011). The GSHP is part of the green energy industries, which often utilize government subsidies because.....
they are unable to maintain economic viability. The empirical investigation simulates the influences of interior parameters, $\mu$, $\sigma$, and $X(t)$, and of underlying and exterior parameters, $D(\ast)$, $K$, and $S$. The results show that investors’ triggers and the difference between these triggers rise with the increasing risk premium and volatility of market participants (leaders or followers). Concerning the simulative influence of $X(t)$, when $X(t) < X^\ast$, due to weak initial market conditions, the leader faces a loss due to weak initial market conditions. Further, when $X^\ast < X(t) < X^\ast$, the leader enjoys the profits arising from pre-emption and has an option value higher than that of the follower. In addition, when $X^\ast < X(t)$, both investors’ option values are equal. The investigation demonstrates the consistency of the developed model with those in relevant existing studies, such as Fudenberg and Tirole [2], Pawlina and Kort [24], and Azevedo and Paxson [25]. Moreover, we find that a higher inverse demand function also increases the leader’s triggers due to monopolistic power. Further, a higher purchase price postpones the investment timing. The government subsidy enhances investors’ desires to trigger earlier GSHP investment. The 12th FYP includes targets for developing energy efficient technologies; increasing the share of renewable forms of energy; and reducing emissions, carbon intensities, and pollution [27]. China’s government invests approximately US$12–US$18 billion in the GHSP government subsidy program, which is highly emphasized in the 12th FYP. This study demonstrates that the more valuable a government subsidy the more it diminishes the differences of triggers (investment timing) or option values between GSHP investors. This empirical study also clearly evidenced that China’s current GSHP production subsidies policy will earn 1.1 times the benefits for the GSHP industry, and hence can effectively enhance GSHP industry development, which also means that the government subsidy is effectively achieving its goal for increasing the share of renewable forms of energy and is able to expand the economic sector through subsidies to promote the GSHP industry. The aforementioned results are on the whole consistent with those of existing studies [20, 21].

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