Numerical modelling of physical processes in a ballistic laboratory setup with a tapered adapter and plastic piston used for obtaining high muzzle velocities

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Abstract. Numerical modelling of a ballistic setup with a tapered adapter and plastic piston is considered. The processes in the firing chamber are described within the framework of quasi-one-dimensional gas dynamics and a geometrical law of propellant burn by means of Lagrangian mass coordinates. The deformable piston is considered to be an ideal liquid with specific equations of state. The numerical solution is obtained by means of a modified explicit von Neumann scheme. The calculation results given show that the ballistic setup with a tapered adapter and plastic piston produces increased shell muzzle velocities by a factor of more than 1.5–2.

1. Introduction

Laboratory installations designed to obtain high velocities for the purpose of studying high-velocity impact processes have broad applications in aeroballistics and space research [1]. Such installations especially have to be economic and easy to use. For this purpose, light-gas guns are now the most widespread, but they do not yet meet the listed requirements. Therefore, only a very small number of laboratories have such devices.

A ballistic setup with a tapered adapter and plastic piston can provide one possible alternative to light-gas guns. The barrel of such a setup consists of two cylindrical sections of diameters \(d_1\) and \(d_2\), joined by a tapered (or profiled) adapter (Figure 1). The accelerated body (assembly) at the initial time consists of a shell (4 in Figure 1) (as a rule, it is a compact element, such as a ball), the plastic piston (3) (for which polyethylene is used) and the inertial pallet (2). In a firing chamber (1) there is a propellant. At the first stage, the assembly accelerates in the first cylindrical section as a unit according to the classical artillery scheme. At this stage, the limit speed for classical ballistic installations can be reached. Then the assembly moves into the tapered channel (5), where the piston experiences plastic deformation. As a result, the forward part experiences additional acceleration (this process is known in light-gas guns as the hydrodynamic effect [1]). After leaving a barrel, the piston and the shell are separated, and the inertial pallet stops part of the propellant gases. Thus, the shell receives an additional increment of speed, which can exceed 50–100% of the shell speed at the entrance to the tapered section.

The design of the laboratory installation allows replaceable tapered sections to be used. Therefore, by varying the length of the piston and its material, as well as the length of the cone and diameter of the output cylindrical section, it is possible to produce various values of muzzle velocities.
In the present paper, the physical principles of the ballistic setup operation on the basis of a numerical solution to the one-dimensional gas dynamics equations are considered.

![Diagram](image.png)

**Figure 1.** The scheme of ballistic setup; 1 – propellant, 2 – inertial pallet, 3 – plastic piston, 4 – shell, 5 – tapered section.

### 2. Theory

#### 2.1. Mathematical model

Two-phase numerical modelling of the operation of guns has been performed in many studies [2], but in our case the detailed description of the combustion process in a firing chamber has no crucial importance. Therefore, we will limit our consideration of combustion processes to simple models. The motion of gas in the chamber is described by means of quasi-one-dimensional gas dynamics equations [3], to which the equations considering propellant burn within the framework of a geometrical law are attached [4]

\[
\begin{align*}
\frac{\partial (qS)}{\partial t} + \frac{\partial (fS)}{\partial z} &= h, \\
q &= \{\rho, \rho u, \rho E, \rho w\}, \\
f &= \{\rho u, p + \rho u^2, \rho u(E + p/\rho), \rho w\}, \\
h &= \{0, p \partial S/\partial z, 0, (v_1/\epsilon_1)\rho Sp\}.
\end{align*}
\]

Here, \( t \) is the time, \( z \) is the dimensional coordinate, \( u, \rho, p \) and \( E \) are the speed, density, pressure and a specific energy of gas, respectively, \( S \) is the variable cross sectional area of the barrel bore, \( f=1 \text{ MJ/kg} \) is the propellant’s impetus, \( v_1 \) and \( 2\epsilon_1 \) are the propellant’s burning rate constant and web thickness, respectively and \( w \) is the propellant’s relative burnt thickness.

System (1) is supplemented with the equation of state

\[
p = \frac{(k-1)\varepsilon - (1-\psi)f}{1/\rho - (1-\psi)/\delta - \alpha \psi}.
\]

Here \( \varepsilon = E - u^2/2 \) is an internal energy per unit mass, \( \psi = \kappa \omega (1 + \lambda \omega) \) is the propellant’s burnt fraction, \( k = 1.24 \) is an adiabatic index, \( \alpha = 0.001 \text{ m}^3 \) is the covolume, \( \delta = 1600 \text{ kg/m}^3 \) is the mass density of the propellant and \( \kappa \) and \( \lambda \) are the propellant’s characteristic parameters.

With the high pressures accompanying the process of a shot, it is possible to approximately consider the plastic piston as a compressible ideal liquid. The equation of state of the plastic piston is taken in the following form [5]

\[
p = BR(R-1)/(C-R)^2,
\]

where \( R = \rho/\rho_0 \) is the compression rate, \( \rho_0 \) is the density of the material at zero pressure and \( B \) and \( C \) are the empirical constants characterizing a concrete material. For polyethylene, \( B = 1.19 \text{ GPa} \) and \( C = 1.73 \).

In one-dimensional gas dynamics problems, it is more convenient to use Lagrange mass coordinates, so we rewrite (1) in the following form
\[
\frac{\partial}{\partial t} \left( \frac{1}{\rho S} \frac{\partial u}{\partial \xi} \right) = \frac{\partial u}{\partial \xi}, \tag{4}
\]
\[
\frac{\partial u}{\partial t} + S \frac{\partial p}{\partial \xi} = 0, \tag{5}
\]
\[
\frac{\partial \varepsilon}{\partial t} + p \frac{\partial}{\partial \xi} (uS) = 0, \tag{6}
\]
\[
\frac{\partial \varepsilon}{\partial t} = \frac{p}{\rho \lambda}. \tag{7}
\]

Here \(\xi\) is a Lagrangian mass coordinate, \(d\xi/dz = \rho S\).

2.2. Initial and boundary conditions

At the initial time, the following propellant ignition conditions are used: \(p = p_0, u = 0, \rho = \Delta\) and \(w = 0\), where \(p_0 = 5\) MPa is the pressure of propellant ignition and \(\Delta\) is a charge density.

The nonpenetration condition is used at the bottom of the firing chamber \((z = 0)\). The nonpenetration boundary conditions are also applied at the boundaries with the piston and the shell. The velocity and coordinate of the shell are determined by the equation of motion

\[
\phi m_s (du_s / dt) = p_s \varepsilon, \tag{8}
\]

where \(m_s\) and \(u_s\) are the mass and velocity of the shell, respectively, \(p_s\) is the pressure on the shell and \(\phi\) is a coefficient accounting for secondary energy losses. The additional condition of assembly immobility is used until the chamber pressure reaches 60 MPa.

2.3. A numerical scheme

For the numerical solution of system (4) - (8), the explicit Neumann’s scheme (known as "cross") [6] was used. Non-conservatism of the scheme was compensated for by reduction of a spatial grid step. The propellant section was divided into 100 spatial cells, and the plastic piston section was divided into 80 cells. The time step was calculated from the Courant–Friedrichs–Lewy stability condition.

The preliminary calculations for other numerical schemes (Godunov and implicit Neumann scheme) have shown that they do not differ fundamentally from the explicit scheme, but are much more labour intensive.

3. Results and discussion

Calculations of a series of shots from a ballistic setup with a tapered section were carried out. The calculation parameters are specified in Table 1. In all calculations, the pallet mass is taken to be 0.005 kg, the mass of a shell, \(m_s = 0.01\) kg and the diameter of the first cylindrical section is \(d_1 = 30\) mm. The diameter of the second cylindrical section is defined by means of the coefficient of the barrel narrowing \(\gamma = d_2/d_1\). The shell muzzle velocity \(u_{sm}\) and the maximum pressure \(p_{max}\) are given as output data in Table 1. As the numerical analysis shows, the maximum pressure is usually not reached at the bottom of the chamber (as for a classical artillery system) or at the bottom of the shell, but falls on the internal section of the plastic piston when it passes the tapered section.

| Case | \(l_{km}\) | \(l_{pst}\) | \(l_{dsp}\) | \(l_{cn}\) | \(\gamma\) | \(\Delta_s\) | \(I_f\) | \(\kappa\) | \(\lambda\) | \(p_{max}\) | \(u_{sm}\) |
|------|-----|-----|-----|-----|-----|-------|-----|-----|-----|------|------|
| 1    | 0.20 | 0.30 | 1.50 | 0.50 | 2/3 | 700   | 0.50 | 1.0 | 0.0 | 143  | 992  |
| 2    | 0.25 | 0.40 | 1.35 | 0.80 | 0.6 | 700   | 0.30 | 0.5 | 0.1 | 375  | 1560 |
| 3    | 0.25 | 0.25 | 1.55 | 0.9  | 0.5 | 600   | 0.25 | 0.6 | 0.2 | 396  | 1830 |
| 4    | 0.35 | 0.50 | 1.60 | 1.0  | 0.5 | 800   | 0.25 | 0.6 | 0.2 | 2802 | 3972 |
| 5    | 0.45 | 0.40 | 1.65 | 1.0  | 0.5 | 600   | 0.25 | 0.6 | 0.2 | 3420 | 2953 |
For a clear demonstration of the discussed effects, we will consider the results of one calculation in detail (case 1 in table 1). This case is chosen because the calculation results are more obvious with these parameters. Figure 2 gives calculated curves of the shell velocity versus the shell coordinate with and without a tapered section. These curves are correlated to the scheme of the ballistic setup. From Figure 2, it follows that the shell receives a considerable acceleration when passing a tapered section (curve I), which doesn't occur if the barrel is purely cylindrical (curve II).

**Figure 2.** Shell muzzle velocity versus shell coordinate; curve I refers to a calculation with a tapered section and curve II refers to a calculation without a tapered section.

Figure 3 shows curves representing the pressure on the shell and on the bottom of the firing chamber (dashed curve II) versus time.

**Figure 3.** Pressure on the shell (solid curve I) and pressure on the bottom of a firing chamber (dashed curve II) versus time.

In this case, the maximum pressure is reached at the bottom of the chamber. In other cases, the maximum pressure is attained in the tapered section, because the pressure in the piston is much higher due to more rigid conditions of the shot. Proceeding from this result, one can conclude that the main drawback of the ballistic setup with the tapered section is a high level of pressure in the tapered section. It can be avoided in several ways. First, it is possible to make replaceable tapered...
sections. Secondly, it is possible to reduce pressure, by making a profiled section instead of a tapered one, or by making several tapered sections that are separated by a certain distance.

The performed calculations show that the ballistic scheme presented produces muzzle velocities in the range from 1500 m/s to 2500 m/s and higher. Ballistic setups with tapered sections can now find applications as rather inexpensive means of obtaining high velocities for high velocity impact research.

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References
[1] Zlatin N A, Krasilchikov A P, Mishin G I, Popov N N 1974 Ballistic systems and their use in experimental studies [in Russian] eds. N A Zlatin and G I Mishin (Nauka, Moscow)
[2] Jaramaz S, Mickovic’ D, Elek P 2011 Two-phase flow in a gun barrel: theoretical and experimental studies Int J of multiphase flow 37 475-87
Kasimov V Z, Ushakova O V, Khomenko Yu P 2003 Numerical modeling of interior ballistics processes in light gas guns J. of App. Mech. and Tech. Phys. 44 612-9
[3] Landau L D and Lifshitz E M 1987 Fluid Mechanics (Butterworth-Heinemann)
[4] Carlucci D E and Jacobson S S 2008 Ballistics: theory and design of guns and ammunition (CRC Press/Taylor & Francis Group)
[5] Mills E J 1965 Hugoniot Equations of State for Plastics: A Comparison AIAA, 3, 742-3
[6] Kalitkin N N 1975 Numerical methods [in Russian] (Nauka, Moscow)