Radiative Decays of Hyperons in the Skyrme Model: E2/M1 Transitions Ratios†

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ABSTRACT

We study the radiative decays of $J^{\pi} = \frac{3}{2}^{+}$ baryons in the framework of the SU(3) collective approach to the Skyrme model. We present the predictions for the decay widths and the corresponding $E2/M1$ ratios. We find that all considered ratios are negative and of the order of a few percent only. We discuss the effects of flavor symmetry breaking and compare our results to those obtained in related models.

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1. Introduction

At present, only few data are available concerning the electromagnetic decays of hyperons. The most prominent one certainly is the reaction $\Delta \to N\gamma$. Recently, the ratio of the electric quadrupole ($E2$) to the magnetic dipole ($M1$) amplitude has been reanalyzed to be $E2/M1 = (-2.5 \pm 0.2)\%$ from a $\pi^0(\pm)$-photoproduction experiment performed at MAMI [1]. For the $J = \frac{3}{2}$ to $J = \frac{1}{2}$ transitions, which involve strange baryons, the empirical values for the $E2/M1$ ratios are still unknown. Upcoming experiments at CEBAF [2] and Fermilab [3] are expected to soon provide some data on these radiative decays. However, these transitions have already been studied within several models, which include the non–relativistic quark model [4, 5], the MIT bag model [6], heavy baryon chiral perturbation theory [7] as well as a quenched lattice calculation [8]. More recently, Schat et al. [8] presented a detailed analysis of the hyperon radiative decays within the bound state approach [9] to the Skyrme model [10]. In that treatment hyperons are considered as kaons bound in the background of the static soliton field. In the Skyrme model hyperons may alternatively be described within the SU(3) collective treatment. In the latter approach strange degrees of freedom are incorporated as SU(3) collective excitations of the non–strange soliton. Canonical quantization of the collective coordinates yields a Hamiltonian, which may be diagonalized exactly [11, 12] although it contains flavor symmetry breaking pieces. This procedure provides the baryon energies and wave–functions in the space of the collective coordinates. In the present work we employ this collective approach to the SU(3) Skyrme model for investigating the transitions $B(J = \frac{3}{2}) \to \gamma B'(J = \frac{1}{2})$. Further, we will compare these results with those obtained in the studies mentioned above.

2. The collective approach to the SU(3) Skyrme model

Our starting point is the non–linear realization, $U = \exp(i\Phi)$, of the pseudoscalar nonet, $\Phi$. Chirally invariant objects are conveniently constructed by introducing the derivative of $U$ via $\alpha_\mu = U^\dagger \partial_\mu U$. The Skyrme model contains the non–linear $\sigma$ model and the forth–order stabilizing term

$$L_S = \text{Tr} \left( -\frac{f^2}{4} \alpha_\mu \alpha^\mu + \frac{1}{32e^2} [\alpha_\mu, \alpha_\nu][\alpha^\mu, \alpha^\nu] \right) \quad (2.1)$$

\[\text{This preliminary result contains both, resonant and non–resonant contributions. These may add incoherently leading to an even larger (in magnitude) ratio for the resonant piece.}\]
which are flavor symmetric. In flavor SU(3) a minimal set of symmetry breaking terms is included \[13\]

\[
\mathcal{L}_{SB} = \text{Tr}(T + xS) \left[ \beta' (U \alpha_\mu \alpha^\mu + \alpha_\mu \alpha^\mu U^\dagger) + \delta' (U + U^\dagger - 2) \right].
\] (2.2)

Here \( T = \text{diag}(1,1,0) \) and \( S = \text{diag}(0,0,1) \) are the projectors onto the non–strange and strange degrees of freedom, respectively. The parameters are determined from the masses and decay constants of the pion and the kaon. Note, that the physical pion decay constant \( f_\pi = 93 \text{MeV} \) is given by \( f_\pi^2 = \tilde{f}_\pi^2 - 8 \beta' \). To be explicit \( \beta' = -26.4 \text{MeV}^2, \delta' = 4.15 \times 10^{-5} \text{GeV}^4 \) and \( x - 1 \approx 36 \) measures the flavor symmetry breaking \[13\]. To take proper account of the axial anomaly, the Wess–Zumino term

\[
\Gamma_{WZ} = -\frac{i N_c}{240 \pi^2} \int d^5 x \epsilon^{\mu \rho \sigma \kappa} \text{Tr} \left( \alpha_\mu \alpha_\nu \alpha_\rho \alpha_\sigma \alpha_\kappa \right)
\] (2.3)

is added. For the study of electromagnetic properties of baryons at finite momentum transfer a direct derivative coupling to the photon field, \( A_\mu \), has proven relevant

\[
\mathcal{L}_9 = i L_9 \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right) \text{Tr} \left( \xi^{\dagger} \left[ \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right] \xi \left[ \xi^{\dagger} \alpha^\mu \alpha^\nu \xi + \xi \alpha^\mu \alpha^\nu \xi^{\dagger} \right] \right),
\] (2.4)

where the square root of the chiral field has been introduced, i.e. \( U = \xi^2 \). In forth order chiral perturbation this term is needed to correctly reproduce the electromagnetic pion radius determining the dimensionless coefficient \( L_9 = (6.9 \pm 0.7) \times 10^{-3} \) \[14\]. The total action is the sum

\[
\Gamma = \int d^4 x \left\{ \mathcal{L}_S + \mathcal{L}_{SB} + \mathcal{L}_9 \right\} + \Gamma_{WZ}.
\] (2.5)

The associated electromagnetic current, \( J_m^{\mu} \), is obtained in two steps. First, the photon field is incorporated such that the action is invariant under the local \( U_{\text{e.m.}}(1) \) gauge transformation (\( \mathcal{L}_9 \) already has this property). Secondly, \( J_m^{\mu} \) is identified as the object which couples to the photon field linearly. The resulting covariant expression may e.g. be found in ref \[15\].

The SU(3) collective rotational approach for the description of the hyperons as chiral solitons employs the time dependent meson configuration \[13\]

\[
\xi(r, t) = A(t) \xi_k(r) \xi_H(r) \xi_k(r) A^{\dagger}(t)
\] (2.6)

Here \( \xi_H(r) \) refers to the hedgehog ansatz

\[
\xi_H(r) = \exp \left( i \mathbf{r} \cdot \mathbf{\tau} F(r) / 2 \right),
\]
while the SU(3) matrix $A$ contains the collective coordinates. The time dependence of $A$ is most conveniently parametrized in terms of the eight angular velocities $\Omega_a = -i \text{Tr} \lambda_a A^\dagger \dot{A}$. It is convenient to also introduce the adjoint representation of the collective rotation, $D_{ab} = (1/2) \text{Tr}(\lambda_a A\lambda_b A^\dagger)$. The kaon fields, which are induced by the collective rotation are contained in $\xi_k = \exp(iZ)$

$$Z = W(r)d_{ia\beta}\dot{r}_i\Omega_\alpha\lambda_\beta.$$

As usual, the convention $i = 1, 2, 3$ and $\alpha, \beta = 4, ..., 7$ is adopted. In this ansatz $\lambda_a$ and $d_{abc}$ denote the Gell–Mann matrices and symmetric structure functions of SU(3), respectively. Although the inclusion of these induced fields is mandatory to satisfy the PCAC–type relation for the kaon fields they introduce double counting effects because the overlap with the rotation of the pion fields into the strange flavor direction, $Z_0 \sim i[\lambda_\alpha, \xi_H(r)]$,

$$\langle Z_0|Z \rangle \propto \int_0^\infty r^2 dr W(r) \sin \frac{F(r)}{2}$$

vanishes for infinitely large symmetry breaking only [13]. Later we will alternatively consider the model (2.5) augmented by a Lagrange multiplier enforcing (2.8) to vanish.

The configuration (2.6) is then substituted into the action yielding the Lagrangian of the collective coordinates $L(A, \Omega_a)$. Canonical quantization of the collective coordinates provides a linear relation between the angular velocities and the right generators of SU(3), $R_a = -\partial L(A, \Omega_a)/\partial \Omega_a$. For $i = 1, 2, 3$ this relation defines the operator for the total angular momentum $J_i = -R_i$. Diagonalization of the associated Hamiltonian, $H(A, R_a) = -R_a\Omega_a - L$, generates the states corresponding to physical baryons. Although $H(A, R_a)$ contains flavor symmetry breaking terms it can be diagonalized exactly [11, 12] yielding the baryon wave–functions in the space of the collective coordinates.

3. Radiative decays of hyperons

In order to extract information about the radiative decays of the $\frac{3}{2}^+$ baryons we need the quadrupole and monopole pieces of the electric and magnetic form factors, respectively. The former is extracted from the orbital angular momentum $l = 2$ component of the time component of the electromagnetic current, $J_{0}^{e.m.}$, while the latter is obtained from the spatial components, $J_{j}^{e.m.}$. It is therefore suitable to define the associated Fourier transforms

$$\hat{E}(q) = \int d^3r \, j_2(qr) \left( \frac{r^2}{r^2} - \frac{1}{3} \right) J_{0}^{e.m.}.$$
\[ \dot{M}(q) = \frac{1}{2} \int d^3r \, j_1(qr) \epsilon_{3ij} \dot{r}_i J_{j}^{e.m.}, \]  

where the \( j_i(qr) \) denote spherical Bessel functions. Substituting the ansatz \([2, 6]\) into \( J_{j}^{e.m.} \) \([13]\) yields the electric quadrupole operator

\[
\hat{E}(q) = -\frac{8\pi}{15\alpha^2} D_{e.m.,3} R_3 \int_0^\infty dr r^2 j_2(qr) V_0(r),
\]

\[
V_0(r) = s^2 \left( f_2^2 \frac{1}{e^2} \left( F'^2 + \frac{s^2}{r^2} \right) - 8\beta' \right) - 4L_9 \left( s^2 + \frac{s}{r} (r s)' - 3\frac{s^2}{r^2} \right), \tag{3.2}
\]

where contributions carrying total angular momentum zero have been omitted. Derivatives of radial functions with respect to the radial coordinate are denoted by a prime. Furthermore the abbreviations \( s = \sin F \) and \( c = \cos F \) have been introduced and the electromagnetic direction \( D_{e.m,i} = D_{3i} + D_{8i}/\sqrt{3} \) has been indicated. The moment of inertia \( \alpha^2 \) for rotation in coordinate space appears because the angular velocity \( \Omega_i \) has been substituted by the corresponding right generators, \( R_i = -\alpha^2 \Omega_i \). Similarly, the magnetic monopole operator becomes

\[
\dot{M}(q) = -\frac{4\pi}{3} \int_0^\infty dr r^2 j_1(qr) \left\{ V_1(r) D_{e.m.,3} - \frac{1}{\beta^2} V_2(r) d_{3\alpha\beta} D_{e.m.,\alpha} R_\beta + V_3(r) D_{88} D_{e.m.,3}
\right.
\]

\[
\left. + V_4(r) d_{3\alpha\beta} D_{e.m.,\alpha} D_{8\beta} + \frac{\sqrt{3}}{2\alpha^2} B(r) D_{e.m.,8} R_3 \right\}. \tag{3.3}
\]

The moment of inertia \( \beta^2 \) for rotation into the strange flavor direction stems from the replacement \( R_\alpha = -\beta^2 \Omega_\alpha \). Except for the contributions of the pion–radius term \([2, 4]\) to \( V_1 \) and \( V_2 \)

\[
V_1^{(L_3)}(r) = -L_9 \frac{1}{r^2} \left( \cos 2F \right)' + 4\frac{s^2}{r^2},
\]

\[
V_2^{(L_3)}(r) = \frac{4L_9}{r^2} \left\{ [c_2(1 + c_2)(W's + wc_2F') - WsF'(s + s_2)'] - \frac{2}{r^2} W s c_2 (1 + c_2) \right\}, \tag{3.4}
\]

the explicit expressions for the radial functions \( V_1(r), \ldots, B(r) \) in eq \((3.3)\) may be traced from refs \([13, 16]\). In addition to the abbreviations defined after eq \((3.2)\) we have introduced \( s_2 = \sin(F/2) \) and \( c_2 = \cos(F/2) \). The current \( J_{\mu}^{e.m} \) is formally identical in the two approaches I and II, i.e. in II we omit the explicit contribution stemming from the constraint \( \langle Z_0|Z\rangle = 0 \).

The decay widths (\( \Gamma \)) for the radiative decays of the \( \frac{3^+}{2} \) baryons to \( \frac{1^+}{2} \) baryons are obtained as the appropriate matrix elements of \( \hat{E} \) and \( \dot{M} \)

\[
\Gamma_{E2} = \frac{675}{8} \alpha_{lf} q \left| \langle B(\frac{1^+}{2})|\hat{E}(q)|B'(\frac{3^+}{2})\rangle \right|^2, \tag{3.5}
\]

\[
\Gamma_{M1} = 18\alpha_{lf} q \left| \langle B(\frac{1^+}{2})|\dot{M}(q)|B'(\frac{3^+}{2})\rangle \right|^2, \tag{3.6}
\]
where \( q \) refers to the momentum of the photon in the rest frame of the \( \frac{3}{2}^+ \) baryon and \( \alpha_{\text{hf}} = 1/137 \) denotes the electromagnetic structure constant. The matrix elements indicated in eqs (3.5) and (3.6) are computed in the space of the collective coordinates, employing the baryon wave–functions, which exactly diagonalize the collective Hamiltonian, \( H(A, R_a) \). This especially implies that the baryon states are not pure octet (decouplet) states for the \( \frac{1}{2}^+ (\frac{3}{2}^+) \) baryons but rather contain admixtures of higher dimensional SU(3) representations. For the details of the calculational procedure using an “Euler angle” decomposition for the rotation matrix \( A \) we refer the interested reader to appendix A of ref [15]. These analyses also allow us to compute the desired \( E2/M1 \)–ratio from

\[
\frac{E2}{M1} = \frac{5}{4} \frac{\langle B(1^+_{\frac{3}{2}})|\hat{E}(q)|B'(\frac{3}{2}^+)\rangle}{\langle B(1^+_{\frac{3}{2}})|\hat{M}(q)|B'(\frac{3}{2}^+)\rangle}.
\]

4. Numerical results

The Skyrme parameter \( e \) is fixed by optimizing the model predictions for the baryon mass differences. For this purpose we determine the minimum of \( \chi = (1/7)\sum (\Delta M_{\text{pred}} - \Delta M_{\text{expt}})^2 \) as a function of \( e \). The sum goes over the seven mass differences \( \Delta M = M_\Lambda - M_N, M_\Sigma - M_N, ..., M_\Omega - M_N \). This results in \( e = 4.0 \) with \( \chi = 13.0 \text{MeV} \), while a computation which includes the constraint, \( \langle Z_0|Z\rangle = 0 \), requires \( e = 3.9 \) yielding \( \chi = 12.4 \text{MeV} \). In the following we will refer to these two approaches by I and II, respectively. For later reference we also quote the model predictions for the magnetic moments [15] of the nucleon, \( \mu_p = 2.01 \) (1.98) and \( \mu_n = -1.53 \) (1.57) for the treatment I (II). The changes caused by the constraint are obviously minor and can easily be compensated by a slight variation of the Skyrme parameter. In both cases the experimental values (\( \mu_p = 2.79, \mu_n = -1.91 \)) are underestimated. It should also be mentioned that the pion–radius term (2.4) does not contribute to the magnetic moments because the corresponding current is a total derivative.

We now turn to the primary issue of this paper, the predictions for the radiative decays of the \( \frac{3}{2}^+ \) baryons. Our results are summarized in table 4.1. All considered \( E2/M1 \)–ratios are found to be negative and of the order of a few percent only. Furthermore the different treatments (I, II) do not cause significant changes. The inclusion of the pion radius term (2.4) tends to lower the \( M1 \) partial width leading to larger \( E2/M1 \)–ratios. We have already noted that the proton magnetic moment is predicted too low. This motivates the scaling \footnote{This treatment may be considered as an approximation to possible \( 1/N_C \) corrections [15]. However,
Table 4.1: The predictions for the total decay widths $\Gamma_{\text{tot}} = \Gamma_{M1} + \Gamma_{E2}$ (in keV) and ratios $E2/M1$ (in percent) in the collective approach to the Skyrme model. The treatments I and II are explained in the text. The data in parentheses refer to the ratios $E2/M1$ being rescaled by the proton magnetic moment. Also given are the non-relativistic quark model (QM, [4, 5]) and quenched lattice (Lat., [5]) predictions for the total widths. Note that the latter data are normalized to reproduce the magnetic moment of the proton.

| $\Delta \to \gamma N$ | $\Sigma^*0 \to \gamma \Lambda$ | $\Sigma^*0 \to \gamma \Sigma^-$ | $\Sigma^+ \to \gamma \Sigma^+$ | $\Xi^- \to \gamma \Xi^-$ | $\Xi^0 \to \gamma \Xi^0$ |
|----------------------|-------------------------------|-----------------------------|-------------------------|-----------------|-----------------|
| $L_9 = 0$            | $L_9 = 6.9 \times 10^{-3}$    | $L_9 = 0$                   | $L_9 = 6.9 \times 10^{-3}$ | $L_9 = 0$     | $L_9 = 6.9 \times 10^{-3}$ |
| $E2/M1$              | $E2/M1$                        | $E2/M1$                     | $E2/M1$                  | $E2/M1$       | $E2/M1$         |
| $\Gamma_{\text{tot}}$ | $\Gamma_{\text{tot}}$          | $\Gamma_{\text{tot}}$      | $\Gamma_{\text{tot}}$    | $\Gamma_{\text{tot}}$ | $\Gamma_{\text{tot}}$ |
| $339$                | $-3.1(-2.3)$                   | $313$                       | $-3.7(-2.7)$             | $348$          | $-3.1(-2.2)$    |
| $313$                | $-3.7(-2.7)$                   | $322$                       | $-3.7(-2.6)$             | $330$          | $-3.3(-2.6)$    |
| $348$                | $209$                         | $194$                       | $232$                    | $18$           | $17$            |
| $322$                | $2$                           | $2$                         | $12$                     | $100$          | $100$           |
| $330$                | $430$                         | $232$                       | $3$                      | $3$            | $3$             |
| $348$                | $3$                           | $5$                         | $4$                      | $3$            | $129$           |

With this procedure the model reproduces the newest data ($-2.5 \pm 0.2$) for the $E2/M1$–ratio of the process $\Delta \to \gamma N$ [1]. For the widths this scaling, which also has been performed in the quenched lattice computation of ref [5], results in values approximately twice as large as those displayed in table 4.1, e.g. $\Gamma_{\Delta \to \gamma N} \approx 650\text{keV}$. This is similar to the particle data group estimate of $660\ldots730\text{keV}$ for the width of the decay $\Delta \to \gamma N$ [19].

For the proceeding discussion it is convenient to categorize the radiative decays according to the magnitude of their widths into large (l): $\Delta \to \gamma N$, $\Sigma^*0 \to \gamma \Lambda$, $\Xi^0 \to \gamma \Xi^0$, moderate (m): $\Sigma^+ \to \gamma \Sigma^+$, $\Sigma^0 \to \gamma \Sigma^0$ and tiny (t): $\Xi^- \to \gamma \Xi^-$, $\Sigma^- \to \gamma \Sigma^-$. We have considered the two limiting cases of flavor symmetric and strongly distorted baryon wave functions. In the former case the $E2$ transition matrix elements for the t–type reactions vanish identically. In this limit, together with the omission of the symmetry breakers $V_3$ and $V_4$ in eq (3.3), we also observe vanishing $M1$ transition matrix elements for these reactions. Of course, this just reflects the $U$–spin selection rule of the flavor symmetric formulation [20]. Although the use of SU(3) symmetric baryon wave–functions results in widths, which are up 30% smaller than those presented in table 4.1, a small deviation from flavor symmetric wave–functions yields already results similar to those shown in table 4.1. In the large symmetry breaking limit we find that those $E2$ transition matrix elements, which do in contrast to the simple scaling, these corrections can in principle vary with the momentum transfer $q$. Fortunately, only small momentum transfers are involved in the radiative hyperon decays considered.
not change the baryon isospin, tend to be proportional to the corresponding isospin projection. Actually this feature is similarly observed in the bound state computation \[8\]. However, in the collective treatment this proportionality is only slowly approached with increasing symmetry breaking in the baryon wave–functions, \textit{i.e.} the isoscalar component decreases only slowly with increasing symmetry breaking. Quantitatively the effects of symmetry breaking can be investigated by varying the symmetry breaking parameter \(x\) in eq (2.2). Even for values as large as \(x \approx 100\) a 30% deviation from these proportionalities is obtained. In the realistic case \((x \approx 37)\) the isoscalar contribution is still sizable leading to the intermediate situation where \(e.g.\) \(\langle \Xi^-|\hat{E}(q)|\Xi^*-\rangle \approx -(1/3)\langle \Xi^0|\hat{E}(q)|\Xi^{*0}\rangle\). In a wide range of the symmetry breaking parameter \(x = 20 \ldots 50\) the total widths of the \(l\)–type reactions exhibit almost no variation \((< 5\%)\). Also the absolute change of the \(m\)–type widths is only about 10keV. The \(t\)–type widths may increase by a factor two in this range, however, these are small in any event. We therefore conclude that the flavor symmetry breaking has no significant impact on the predictions for the radiative hyperon decays. This is in contrast to other quantities, especially those which are related to the strangeness content of the nucleon \[21\].

The largest decay width for reactions involving strange baryons is obtained for \(\Gamma_{\Sigma^{*0} \rightarrow \gamma \Lambda} \approx 240\text{keV}\). This is similar to other model predictions like the bound state approach to the Skyrme model \[8\] or the non–relativistic quark model \[6\]. Our results for \(\Gamma_{\Xi^{*0} \rightarrow \gamma \Xi^0} \approx 110\text{keV}\) and \(\Gamma_{\Sigma^{*+} \rightarrow \gamma \Sigma^+} \approx 80\text{keV}\) are also of the same order as these model calculations. Although the absolute values for the decay widths predicted by the bound state method exhibit some parameter dependencies \[8\], the pattern

\[
\Gamma_{\Sigma^{*0} \rightarrow \gamma \Lambda} > \Gamma_{\Xi^{*0} \rightarrow \gamma \Xi^0} > \Gamma_{\Sigma^{*+} \rightarrow \gamma \Sigma^+} \gg \Gamma_{\Sigma^{*0} \rightarrow \gamma \Sigma^0} \gg \Gamma_{\Xi^{*0} \rightarrow \gamma \Xi^0} \approx \Gamma_{\Sigma^{*+} \rightarrow \gamma \Sigma^+} \approx 0 \quad (4.1)
\]

is recovered in the collective treatment. On the whole our predictions for the widths tend to be slightly smaller than those of the non–relativistic quark model or the quenched lattice calculation.

5. Conclusion

We have computed the decay widths for the radiative decays of the \(\frac{3}{2}^+\) baryons in the framework of the collective approach to the \(\text{SU}(3)\) Skyrme model by separately evaluating the magnetic dipole \((M1)\) and electric quadrupole \((E2)\) transition matrix elements. The total decay widths have been found to be strongly dominated by the \(M1\) contribution yielding
$E2/M1$ ratios which are of the order of a few percent only. Hence the quadrupole deformation of the baryons is predicted to be small in the collective approach to the SU(3) Skyrme model. All these ratios are predicted to be negative. The resulting decay widths agree reasonably not only with predictions of the bound state approach to the Skyrme model \cite{8} but also with those obtained within other models of the baryon \cite{4,5,8}.

We have observed that these transition matrix elements are not very sensitive to flavor symmetry breaking. This naturally explains why even in the realistic case, where the baryon wave–functions significantly deviate from pure octet and decouplet states, the $U$–spin selection rule \cite{20} is almost exactly reproduced. As for the magnetic moments of the $\frac{1}{2}^+$ baryons the collective approach approximately satisfies the relations, which reflect the $U$–spin symmetry \cite{22}. Since a treatment, which incorporates the flavor orientation in the stationary condition for the chiral angle, predicts the experimentally demanded deviations from the $U$–spin relations \cite{22}, one may speculate whether this treatment yields significantly different results for the transition matrix elements.

In contrast to the bound state model \cite{8} we observe a non–vanishing isoscalar contribution to the $E2$ transition matrix elements. Only in the unrealistic case of infinitely large flavor symmetry breaking these matrix elements are proportional to the isospin projection. Actually this is similar to the situation for the quadrupole moments of the $\frac{3}{2}^+$ baryons. In the bound state approach these moments are found to be proportional to the isospin projection \cite{23}. In the collective approach this proportionality only appears in the large symmetry breaking limit, while for small breaking the quadrupole moments happen to be linked to the baryon charge \cite{24}. 

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