Constraining Galaxy Mass Profiles with Strong Gravitational Lensing

David Rusin
Department of Physics and Astronomy, University of Pennsylvania, 209 S. 33rd St., Philadelphia, PA, 19104-6396

ABSTRACT

Measured time delays between the images of a gravitationally lensed source can lead to a determination of the Hubble constant ($H_0$), but only if the lensing mass distribution is well understood. The inability to sufficiently constrain galaxy mass models results in large uncertainties on the derived $H_0$, and severely hampers the cosmological application of this otherwise elegant method. At the same time, lensing must compete with new techniques that have the potential to measure the Hubble constant to within a few percent by the middle of the decade. In this letter we re-evaluate the role of strong gravitational lensing in the age of precision cosmology, and present a series of Monte Carlo simulations that demonstrate the effect of the galaxy mass distribution on Hubble constant determination. Though most gravitational lens systems are unlikely to contribute significantly to precision studies of the cosmological distance scale, the strong dependence of predicted time delays on the galaxy mass profile suggests that the most useful and interesting results may be obtained by running the traditional lensing problem in reverse – namely, combining measured time delays with a well-constrained value of $H_0$ to study galaxy mass distributions.

Subject headings: gravitational lensing

1. Introduction

Gravitational lensing offers a potent tool for determining the cosmological distance scale (Refsdal 1964), but only if the mass distribution of the deflector is well constrained. Since the mass model predicts time delays between the lensed images up to a factor of $h^{-1}$, where $H_0 = 100h\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, a simple comparison with measured time delays can yield the Hubble constant. This method is particularly tempting as it would determine $H_0$ directly out to cosmological distances, forgoing the calibration errors associated with traditional distance-ladder techniques (e.g. Mould et al. 2000). Much of the recent interest in finding arcsecond-scale gravitational lens systems through large, dedicated programs such as the Cosmic Lens All-Sky Survey (CLASS; Myers et al. 2000) has been motivated by this goal.

Measurements of $H_0$ from gravitational lenses are only as good as our understanding of the galaxies doing the lensing, as the model-predicted time delays contribute an equal fractional...
uncertainty to the derived Hubble parameter. Unfortunately, experience has shown that our knowledge of lensing mass distributions is not very good at all. This problem has been greatly exacerbated by the observation that a significant fraction of lenses, including several of those with measured time delays, are quite complex in structure (e.g. CLASS B1359+154, Rusin et al. 2000; QSO 0957+561, Barkana et al. 1999; PG 1115+080, Schechter et al. 1997; CLASS B1608+656, Koopmans and Fassnacht 1999). Mass models are difficult to construct for such systems due to limited constraints and parameter degeneracies (e.g. the mass-sheet degeneracy, Gorenstein et al. 1988).

Significant uncertainties in the mass model can remain even in systems lensed by a single isolated galaxy. This is because the positions and flux density ratios of the lensed images constrain derivatives of the potential, rather than the potential itself. Often a range of models can provide sufficient fits to the lens system while placing different potentials at the image positions. The predicted time delays will be greatly affected by model degeneracies, as they depend on the potential directly (Schneider, Ehlers and Falco 1992). This is particularly discouraging for many two-image lens systems, which do not offer a sufficient number of constraints to even attempt fits to the profile shape. For such systems the profile must be assumed, and is commonly taken to be isothermal. The uncertainty in the slope of the mass profile and its effect on Hubble constant determination are rarely included in most preliminary modeling analyses.

Attempts to constrain galaxy mass profiles through strong lensing generally focus on image positions, flux density ratios and, if available, the ratios of measured time delays. Studies of this type have thus far been performed in only a small handful of lens systems, and have had limited success in pinning down the mass profile. Assuming a power-law surface mass density ($\Sigma \sim r^{-\gamma}$), it has been found that the merging lens galaxies of CLASS B1608+656 are best fit by $0.8 \leq \gamma \leq 1.2$ (Koopmans and Fassnacht 1999), while the large elliptical galaxy in QSO 0957+561 is best fit by $0.82 \leq \gamma \leq 0.93$ (Grogin and Narayan 1996). However, each of these systems is affected by complicated deflectors. A profile analysis has also been done for MG 1654+134 (Kochanek 1995), which is a radio ring system lensed by an isolated galaxy. Because the lensed extended emission probes a wide range of radii (as opposed to the images of a compact source), ring systems can produce tighter constraints on the mass profile ($0.9 \leq \gamma \leq 1.1$ for MG 1654+134).

While isothermal ($\gamma = 1$) mass profiles are largely compatible with lensing, a range of near-isothermal profiles ($0.8 \leq \gamma \leq 1.2$) are equally feasible. In this Letter I outline the systematic effects of this uncertainty on the predicted time delays, and evaluate the possibility of measuring $H_o$ with the set of lens systems currently known. I then consider the largely-ignored reverse problem in lensing – namely, using measured time delays along with a well-determined value of $H_o$ to study galaxy mass profiles. The results offered by this technique have the potential to provide greatly improved constraints on the inner mass distribution for galaxies at intermediate redshift.
2. Power-law mass distributions

In this Letter I approximate the lensing mass of galaxies by a singular power-law ellipsoid (SPLE), with surface mass density

$$\Sigma(x_1, x_2) = q[x_1^2 + f^2 x_2^2]^{-\gamma/2}$$

where $\gamma$ is radial profile, $f$ is the axial ratio and $q$ is the normalization. Only recently have power-law mass profiles for arbitrary $\gamma$ become a feasible alternative for studying and modeling gravitational lens systems. All calculations presented in this paper make use of the rapidly converging series solutions for the deflection angles and magnification matrices derived by Barkana (1998) and implemented in the “FASTELL” software package.

While even small core radii greatly affect lensing cross-sections, they do not significantly alter the properties of the images in a given system and can therefore be ignored in this analysis. The SPLE probably oversimplifies the overall mass distribution in galaxies, but may be a feasible approximation when limited to describing the inner few $h^{-1}$ kpc that is probed by strong lensing. Moreover, it has been shown that a large number of gravitational lens systems can be successfully modeled using power-law mass profiles, isothermal or otherwise.

The lensing properties of the singular power-law ellipsoid depend strongly on the value of the profile slope $\gamma$. Here we summarize the possible image configurations produced by the SPLE for the range of profiles ($0.7 < \gamma < 1.3$) and axial ratios ($f > 0.5$) that seem to be favored by most gravitational lens systems:

- For the familiar isothermal case ($\gamma = 1$; Kormann, Schneider and Bartelmann 1994a), the SPLE will produce one, two or four images, as is typically observed in gravitational lens systems. The third image (for doubles) and fifth image (for quads) does not exist, as the radial critical curve that would enclose this image degenerates to a point at the lens center.

- For profiles steeper than isothermal ($\gamma > 1$), the deflection angle diverges as $r \to 0$, so the SPLE will multiply-image all source positions, producing either two or four images. The lensing cross-section for $\gamma > 1$ is thus nominally infinite. However, the second image is highly demagnified if the source is sufficiently misaligned with the lens, which leads to a large flux ratio between the images. This results in a finite “observational cross-section” that depends on the dynamic range limits of a given survey. The CLASS survey, for example, can only identify lens systems in which the flux density ratio between the brightest images is $\leq 10:1$.

- For profiles shallower than isothermal ($\gamma < 1$), the radial critical curve exists, allowing for the creation of one, three or five images. The third image and fifth image will be sufficiently demagnified to escape detection if $\gamma$ is close to 1. For shallower profiles, however, the SPLE commonly produces two distinct classes of three-image systems that would be readily observable – (1) fold-imaged configurations in which the positive parity image enclosed
by the radial critical curve is sufficiently bright (3F-type; Fig. 1a), and (2) naked-cusp configurations in which a source enclosed by the diamond caustic only is lensed into three bright images on the same side of the lens, with no counter images (3C-type; Fig. 1b). The paucity of observational evidence for three-image systems suggests that the typical mass profile of lens galaxies is unlikely to be much shallower than $\gamma = 0.75$ (Rusin and Ma 2000).

3. Mass profiles and time delays

How well can we hope to constrain galaxy mass profiles using gravitational lensing? How does the remaining uncertainty affect the predicted time delays, and therefore the extracted Hubble constant? Analyses of this type have thus far been been performed for only a few specific lenses, and in many instances have been significantly complicated by the presence a complex deflector system. Here I consider the case of lens systems produced by a single isolated galaxy, and address the above questions for both two and four-image morphologies.

3.1. Two-image systems

Gravitational lenses consisting of two unresolved images comprise the majority of known lens systems. Such systems provide only five constraints to the lens model (four image coordinates and the flux density ratio). Even if the mass distribution can be fixed to a well known galaxy position, one must still solve for the axial ratio, position angle and normalization of the model, as well as the two coordinates of the unlensed source. Because there are no degrees of freedom, any mass profile can fit the data perfectly ($\chi^2 = 0$). Such lens systems are thus not capable of constraining the galaxy mass profile, if only image positions and flux density ratios are used.

To study the effect of the mass profile on the predicted time delays, a series of fake two-image lens systems was generated using a range of SPLE mass models with $0.7 \leq \gamma \leq 1.3$ and $0.5 \leq f \leq 0.9$. To limit the analysis to the types of lens systems that are actually observed, configurations with a flux density ratio of $\geq 10:1$ between the two brightest images or with an observable third image (flux density ratio between the brightest and third brightest image of $\leq 50:1$) were discarded. Next, the fake lens systems were remodeled using an isothermal ($\gamma = 1$) mass profile. The mass distribution was fixed at the same position as the SPLE to constrain the model. Finally, the time delays of the isothermal and power-law distributions were compared.

The predicted time delay relative to the isothermal case ($\Delta t_{\text{pred}}(\gamma)/\Delta t_{\text{pred}}(\gamma = 1)$) is displayed in Fig. 2. The error bars indicate the range containing 95% of the Monte Carlo trials. The

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1The results described here were originally presented at the 195th AAS Meeting in January. Since then, an interesting paper has been submitted by Witt, Mao and Keeton (2000), which references a similar set of simulations describing the effect of the galaxy mass profile on time delays. These results agree well with those shown in Fig. 2.
scatter in the results is remarkably small, given that the lens systems were created by placing
sources at random locations behind the galaxy. The plot is shown for SPLEs with \( f = 0.7 \), but the
results are only dependent on axial ratio at the few percent level. The results are also independent
of the angular separation of the gravitational lens system. (A simple calculation shows that the
size of the caustics and critical curves scale as \( q^{1/\gamma} \), and the absolute time delays scale as \( q^{2/\gamma} \).)

For a two-image lens system, the predicted time delays vary almost linearly with \( \gamma \), as found
independently by Witt, Mao and Keeton (2000). Steeper mass profiles lead to longer predicted
delays, since the ray paths traverse deeper potentials. Similar trends have been noted in several
previous investigations (Barkana et al. 1999, Koopmans and Fassnacht 1999, Williams and Saha
2000). A longer predicted delay translates into a larger value of the Hubble constant. In almost all
trials, the position angle of the SIE was recovered to within a few degrees of that of the original
SPLE. The axial ratio required to fit the lens system decreases (becomes flatter) as the profile
is made steeper. However, since there is no reason to assume that the axial ratio of the mass
distribution must match that of the observed surface brightness distribution of the lensing galaxy,
this does not provide a useful constraint on the model.

The range of profiles consistent with lensing \((0.8 \leq \gamma \leq 1.2)\) translates into a spread of \(\sim 20\%\)
in the predicted time delays. This uncertainty should be folded into preliminary modeling analyses
of all two-image lenses. It is therefore doubtful whether any basic two-image gravitational lens
system will produce a precision measurement of the Hubble constant.

In a fraction of radio-loud gravitational lens systems, VLBI imaging is able resolve
milliarcsecond-scale substructure in the lensed components. Correlated substructure, such as a
core and jet emission feature in each image, can provide additional constraints to the model.
Images resolved into two subcomponents offer a total of ten constraints (eight image coordinates
and two flux density ratios), but the number of free parameters also increases by two to account for
a second set of unlensed source coordinates. For such systems a sufficient number of constraints are
available to study different mass profiles. However, Monte Carlo results using sources consisting
of two closely-spaced subcomponents show that in order to determine the profile to within 10\%,
the galaxy position must be fixed by observations and the subcomponents must be separated by
several milliarcseconds (mas) in the lensed images.

Several radio lens systems are currently known to have image substructure. For example,
QSO 0957+561 (Barkana et al. 1999) features unusually rich jet emission consisting of several
correlated knots, which has allowed for the mass profile of the lensing galaxy to be determined to
5–10\%. Each image of CLASS B0218+357 (Biggs et al. 1999) is resolved into two subcomponents
separated by only \(\sim 1\) mas, and the position of the lens galaxy is very poorly known. CLASS
B2319+051 (Marlow et al. 2000) has well-resolved substructure, but thus far the lensed images
have not been detected in the optical, so the relative position of the lensing galaxy must be left as
a free parameter in the modeling process. The model is further complicated by the presence of a
nearly shear contributor at a different redshift than that of the primary galaxy (Lubin et al. 2000).
3.2. Four-image systems

The situation with four-image lens systems is greatly improved, due to the fact that the images probe the mass distribution at four distinct locations. Image positions and flux density ratios provide a total of 11 constraints to the lens model, which allows the mass profile to be studied directly. Monte Carlo simulations using quads produced with power-law deflectors show that modeling can recover the true mass profile to within a percent in $\gamma$, while strongly excluding neighboring profiles. Therefore, quads lensed by an isolated galaxy offer the best possibility of precisely measuring the Hubble constant through gravitational lensing.

Unfortunately, a very large fraction of known four-image lens systems have lensing potentials that are significantly more complicated than that of a single galaxy, which again leads to large uncertainties on both $H_o$ and the profile. This is particularly true of the radio quads. Of the four-image systems in the CLASS sample, two have multiple galaxies inside the Einstein ring (B1359+154, Rusin et al. 2000; B1608+656, Koopmans and Fassnacht), and two more appear to be significantly perturbed by strong external shear contributors (B2045+265, Fassnacht et al. 1999; B1422+231, Kormann, Schneider and Bartelmann 1994b). Two additional systems are lensed by a relatively isolated galaxy (B0712+472, Jackson et al. 1998; B1933+507, Sykes et al. 1998) but have thus far not shown themselves to be variable, which would prevent the measurement of time delays. Finally, B1555+375 (Marlow et al. 1999) has an inverted radio spectrum and is likely to be variable, but the very small image separation ($\sim 420$ mas) makes a standard VLA monitoring program infeasible. It is therefore questionable whether any of the currently known set of four-image lens systems will provide tight constraints on $H_o$. A subset of the systems may still be useful in studying the mass profile, and advanced modeling of these lenses will be presented in a future paper (Rusin 2000).

4. Turning the problem around

Gravitational lensing now competes with new methods seeking to measure the Hubble constant. One technique in particular, combining the power spectrum of the cosmic microwave background (CMB) with data from large galaxy redshift surveys, has the potential to determine $H_o$ to within a few percent (Eisenstein, Hu and Tegmark 1999) by the middle of the decade. Gravitational lensing is unlikely to match this precision, due to aforementioned problems with constraining the galaxy mass profiles. Witt, Mao and Keeton (2000) point out that the uncertainties can be beaten down by measurements of $H_o$ in a number of different lens systems. However, if the typical uncertainty is 20% on any particular measurement (as is currently the case for most lenses), time delays for an unreasonably large number of systems ($\sim 50$) would be required to reduce the overall error down to a competitive level. This argument also assumes that the typical mass profile for lensing galaxies corresponds to some known value (such as isothermal) which is input to the lens models. If the wrong value is assumed, the derived Hubble constants...
will converge to a systematically incorrect result.

Should the Hubble constant be sufficiently well determined by some other means, however, the measured time delays become a direct and powerful constraint on the slope of the mass profile. This astrophysical application of the time delay method has been largely ignored, in favor of the cosmological one. However, this approach may provide the best tool for studying the inner part of galaxy mass profiles and directly testing the isothermal assumption. Two-image systems will be particularly important to this process, as the majority of doubles appear to be dominated by a single lensing galaxy, which will greatly ease the analysis.

One’s ability to constrain the galaxy mass profile using this method depends on the accuracy to which the image positions, flux density ratio and time delay, as well as the galaxy position, can be measured. Radio imaging can determine image positions to within a fraction of a milliarcsecond, so this contributes very little to the uncertainty. History has also shown that flux density ratios and time delays can measured from light curves to within a few percent, given some persistence and luck (Kundic et al. 1997).

Since two-image systems require that the deflector position be fixed to constrain the model, any uncertainty in the center of the galaxy surface brightness distribution will translate into a spread of predicted time delays, and therefore weaken constraints on the mass profile. Typically the brightness center of a lens galaxy can be determined to no better than 10 – 25 mas using the Hubble Space Telescope. For a lens system with an image separation of ∼ 1 – 1.5 arcseconds, this translates to an uncertainty of ∼ 10% in the predicted time delays when all the remaining galaxy parameters are allowed to float. The error can be reduced if the position angle of the mass model is constrained to be compatible with that of the observed surface brightness distribution of the lens galaxy. This position angle may be a good modeling constraint, as it is reasonable to believe that the light and mass distributions are similarly oriented.

If both the Hubble constant and time delay are known to within a few percent, the lens model is required to reproduce $\Delta t_{\text{pred}} = h \Delta t_{\text{meas}}$ (where $\Delta t_{\text{pred}}$ is the predicted delay for $h = 1$) to within ∼ 5%. Since Fig. 2 shows that $\Delta t_{\text{pred}} \propto \gamma$ for the parameter range of interest, the mass profile would be measured to this accuracy, assuming the position of the galaxy is perfectly known. Including the uncertainty in the galaxy position but using the orientation of the surface brightness distribution as a constraint will likely increase the error in the derived mass profile to $\Delta \gamma = 0.05 – 0.1$ for most two-image lens systems.

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2See the CASTLES webpage, [http://cfa-www.harvard.edu/castles/](http://cfa-www.harvard.edu/castles/) for imaging data on most of the currently known gravitational lenses.
5. Discussion

Gravitational lensing has already contributed to our knowledge of both galaxy mass distributions and the Hubble constant. The profiles of several lens galaxies have been constrained at the $10\%$ level, while the ensemble of lenses with measured time delays may have determined the Hubble constant to $\sim 10\%$ (Koopmans and Fassnacht 1999). In this age of precision cosmology and astrophysics, however, new techniques and observations promise very accurate measurements of a wide range of quantities. It is therefore necessary to reassess the goals and merits of more familiar methods.

The traditional application of gravitational lensing seeks to measure the Hubble constant, and requires that the mass distribution of the lensing galaxy be modeled independently of the absolute time delays. Uncertainties in the mass profile dominate the error budget in $H_0$ for every system in which time delays are currently measured. At the same time, lensing must compete with new techniques that have the potential to determine the Hubble constant to within a few percent.

The time delay method, however, can be elegantly inverted to provide powerful constraints on galaxy mass profiles, once the Hubble constant has been accurately measured by some other means. Whereas highly constrained lens systems, such as quads or doubles with well-separated substructure, would be needed to derive $H_0$ independent of any assumptions regarding the mass profile for a given galaxy, time delay determinations in almost any simple two-image system can make interesting statements about the mass distribution present. In time this approach may provide the best tool for probing the inner few $h^{-1}\text{kpc}$ in elliptical galaxies, and will nicely complement weak lensing techniques that are sensitive to the mass profile at large radius (Fischer et al. 2000). Because photometric studies by the CASTLES collaboration (Kochanek et al. 2000) have found that lensing galaxies appear to be consistent with an ordinary population of passively evolving ellipticals, the results derived from lensing can be applied to typical galaxies at intermediate redshift. In conclusion, the precision studies of galaxy mass profiles offered by lensing suggest that the most significant results from Refsdal’s method may be astrophysical in nature, rather than cosmological.

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Fig. 1.— Two classes of triple-imaging. (a) Left: Fold imaging. (b) Right: Naked-cusp imaging. The critical curves are shown as dark lines and the caustics are shown as light lines. The position of the unlensed source is marked with a filled circle. The positions of the lensed images are marked with open circles. The model shown is for $f = 0.6$, $\gamma = 0.6$. 
Fig. 2.— The variation of predicted time delays with mass profile for two-image gravitational lens systems. Error bars indicate the range including 95% of the Monte Carlo results.