Drag force on two disks moving in a granular bed

A Kuwabara¹, N Yoshioka², T Shimada¹, and N Ito¹,²
¹ Department of applied physics, graduate school of engineering, the university of tokyo, 7-3-1 hongo, bunkyo-ku, tokyo 113-8656, japan
² Riken aics, 7-1-26 minatojima-minami-machi, chuo-ku, hyogo 650-0047, japan
E-mail: kuwabara@serow.t.u-tokyo.ac.jp
E-mail: naoki.yoshioka@riken.jp
E-mail: shimada@ap.t.u-tokyo.ac.jp
E-mail: ito@ap.t.u-tokyo.ac.jp

Abstract. The drag forces acting on a disk and two disks dragged at constant speeds in a two-dimensional granular bed are measured by an event-driven molecular dynamics simulation. The normal drag force on a disk in case of two disks are dragged in parallel, \( F_{2x} \), is almost the same with the drag force on a disk in case of a single disk is dragged, \( F_{1x} \), when they are apart enough. As the distance between the disks \( D \) decreases, \( F_{2x} \) increases until it has the maximum at a certain point, \( D* \), and after that it decreases. When \( D \) is small enough, whether \( F_{2x} \) is larger than \( F_{1x} \) or not depends on the ratio of the average radius of the granular particles to the one of the dragged objects.

1. Introduction
To know the drag force on objects in a granular bed is not only interesting in terms of rheology but also important in terms of industrial application. There have been many researches about the drag force on a object in a granular bed [1–4], but there are few researches about the drag force on multiple objects in a granular bed. The research related to the drag force when multiple objects move in a granular bed is conducted by J. M. Solano-Altamirano et al [5]. They examined the interaction force in the horizontal direction when two intruders are falling simultaneously into a granular bed, but didn’t examined the drag force in the moving direction in detail. So in our research, we examined the drag force on a disk in the moving direction when two disks move in parallel and compared it with the one when a single disk moves in a two-dimensional granular bed by simulation.

2. Model
The simulation snapshots are shown in Fig. 1. At first the dragged objects is fixed as the midpoint of them (its center if there is only a single dragged object) is at the midpoint of two walls and their interval is \( D \) in \( y \) direction, and the granular particles which have random velocities are placed at random positions. Then two walls are approached to each other at constant speeds until the packing fraction of granular particles become \( \phi_0 \), and they are stopped. After that granular particles are given random velocities and the systems are made uniform by the elastic collisions between the granular particles. Finally the granular particles are stopped and the objects are dragged at constant speeds.

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2.1. granular bed
The granular particles are hard disks whose radius is $r(1.0 \pm 0.07)$ (uniform distribution). The normal coefficient of restitution is 0.9 and there is no tangential friction. In general it is known that there may be infinite collisions within a finite time in the system where the energies are lost when hard disks collide. So in this simulation, we avoid this by making collisions elastic if the relative speed in normal direction is less than $v_c = 10^{-4}$. $v_c$, which is the parameter that is arbitrarily introduced, should be small enough not to affect the drag force. The drag force are measured for various $v_c$ and it is made sure that the drag force doesn’t change so much if $v_c$ is small enough.

2.2. dragged object
The dragged objects are hard disks whose radii are 1.0. They collide with granular particles elastically. They are dragged at constant speeds $V_x = 1.0$ in $x$ direction.

2.3. boundary condition
There are periodic boundary in $x$ direction and fixed walls in $y$ direction. The system size is $2000r$ in $x$ direction and $500r$ in $y$ direction. The system size is large enough that there is no effect of the boundary condition.

![Fig. 1: The schematic picture of the simulation in case of dragging two object.](image)

3. Results and Discussions
In this paper packing fraction $\phi_0$ is fixed to 0.4. Drag force are averaged in the steady region and furthermore averaged over 10 samples. Drag force on a dragged object in case of only a disk is dragged is expressed as $F_1 = -F_{1x} e_x - F_{1y} e_y$. In case of two disks are dragged, the drag force on each object in $x$ direction is considered to be the same, so drag force on one object is measured. Drag force on a dragged object A in case of there are two dragged objects is expressed as follows, $F_2 = -F_{2x} e_x - F_{2y} e_y$. 

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3.1. In case of only a single disk is dragged
Fig. 2(a) shows the snapshot of the simulation in case of only a disk is dragged. As can be seen from the figure, we can find that the granular particles are swept up by the dragged object and there are dense regions, sparse regions and regions in which there are almost no particles. To determine the boundary between the dense region and other regions, local packing fraction is calculated, and \( \phi_{\text{max}} \) and \( \phi_{\text{th}} \) are defined. \( \phi_{\text{max}} \) is defined as the maximum value of \( \phi_l \) in the region where \( x \geq 0 \) and the threshold of local packing fraction \( \phi_{\text{th}} \) is defined as follows,

\[
\phi_{\text{th}} = \frac{\phi_{\text{max}} + \phi_0}{2}.
\] (1)

We define the boundary as the coordinate which satisfies the equation \( \phi_l(x, y) = \phi_{\text{th}} \). As can be seen form Fig. 2(a), this definition seems to be reasonable. We can find a boundary inside and outside the dense region. We call the outer boundary as the interface in this paper. Fig. 2(b) shows the interface for various \( r \). The interface is less extended to the outside when \( r \) is small. It is considered as follows. If \( r \) is small the granular particles are more likely to collide with each other and lose energy. So granular particles are less likely to extend to the outside when they are smaller.

![Fig. 2: (a): The snapshot of the simulation when \( r = 1/10 \). \( \triangle \) denotes the coordinates of the interface where local packing fraction \( \phi_l \) equals \( \phi_{\text{th}} (= \frac{\phi_{\text{max}} + \phi_0}{2}) \). (b): The shape of the interface for various \( r \).](image)

3.2. In case of two disks are dragged
Fig. 3(a) shows \( F_{2x}/F_{1x} \) as a function of \( D \) for various \( r \). \( F_{2x} \) is found to increase as the distance between the disks \( D \) increases until it has the maximum at a certain point, \( D^* \), and then relaxes to \( F_{1x} \) when \( D \) is large enough. Fig. 3(b) shows the snapshot of the simulation. When \( D \) is large enough, we can find the two interfaces in front of the dragged objects and they don’t interact with each other so much, so \( F_{2x} \sim F_{1x} \). As can be seen from the figure, the distance \( D \) when two interfaces start to interact increases as \( r \) decreases. This is because the interface is more likely to be extended as \( r \) decreases (See Fig. 2(b)). As two dragged objects approaches two interfaces start to interact with each other and it becomes more difficult for granular particles to move through the objects, so drag force becomes larger. However if \( D \) is smaller than \( D^* \), it becomes easier for granular particles to move outside the dragged objects, so drag force becomes...
smaller. When two dragged objects are close enough, whether $F_{2x}$ is smaller than $F_{1x}$ or not depends on $r$. Basically, if the two dragged objects are close enough, drag force are assumed to be smaller when they are apart enough because the area where granular particles can collide with the dragged object become smaller than the one when they are apart enough. However if $r$ is smaller the granular particles are more likely to be stuck just in front of the two dragged objects and granular particles in the region are hardly to move outside the two dragged objects. So drag force are considered to become larger when they are apart enough because of the frequent collision.

Fig. 3: (a): $D$ dependence of $F_{2x}$ for various $r$. (b): The simulation snapshot where $r = 1/10$.

4. Summary and Conclusion
We examined the drag forces acting on a disk and two disks dragged at constant speeds in a two-dimensional granular bed altering the average radius of the granular particles $r$. In case of a single disk is dragged, the granular particles are swept up by the dragged object and there are dense regions, sparse regions and regions in which there are almost no particles. The range of dense regions in front of the dragged object is wider when $r$ is larger because they are less likely to collide than when $r$ is smaller. In case of dragging two objects, The normal drag force on a disk in case of two disks are dragged in parallel, $F_{2x}$, is almost the same with the drag force on a disk in case of a single disk is dragged, $F_{1x}$, when they are apart enough. As the distance between the disks $D$ decreases, $F_{2x}$ increases until it has the maximum at a certain point, $D*$, and after that it decreases. When $D$ is small enough, $F_{2x}$ is smaller than $F_{1x}$ if $r$ is large. However $F_{2x}$ is larger than $F_{1x}$ if $r$ is small. This reason is considered to be that if $r$ is smaller the granular particles are more likely to be stuck just in front of the two dragged objects and granular particles in the region are hardly to move outside the two dragged objects, so drag force become larger because of the frequent collision.

References
[1] Wassgren C R, Cordova J a, Zenit R and Karion a 2003 Phys. Fluids 15 3318–3330
[2] Chehata D, Zenit R and Wassgren C R 2003 Phys. Fluids 15 1622
[3] Albert R, Pfeifer M, a L Barabási and Schiffer P 1999 Phys. Rev. Lett. 82 205–208
[4] Takehara Y and Okumura K 2014 Phys. Rev. Lett. 112 1–5
[5] Solano-Altamirano J M, Caballero-Robledo G a, Pacheco-Vázquez F, Kamphorst V and Ruiz-Suárez J C 2013 Phys. Rev. E 88 032206