Study on damage location of shaft using model acoustic emission

Lu Wenxiu\(^1\), Xiao Yanyan, Chu Fulei
Department of Precision Instruments, Tsinghua University, Beijing, PR China 100084
E-mail: luwenxiu@tsinghua.edu.cn, chufl@tsinghua.edu.cn

Abstract. Acoustic Emission signal, which occurs in the structure materials, is a transient stress wave. The propagation of the stress wave in the structure is very complicated because of the various cross section, defect and damage. In this paper, a propagation model of AE in the shaft is established to analyze the propagation characteristics. The crack depth and location are considered to investigate the influence on the propagation characteristics of AE signal. A series of experiments are implemented and the results show that the AE signal is very effective to determine the damage location of shaft.

Introduction

Acoustic Emission (AE) is the class of phenomena whereby transient elastic waves are generated by rapid release of strain energy caused by a deformation or damage within or on the surface of a material [1]. Under the action of external conditions, the defects or potential defects of material will change their behaviour and release transient elastic waves. Therefore, the AE testing is a powerful method for examining the behaviour of material defects. The AE technology has great advantages comparing with the vibration-based measuring method:

- AE signal is generated at micro-level, which means that it is very sensitive to the small change of the structure. At early stage of fault, the vibration signal is very weak while the AE signal is obvious for many common faults such as crack, rubbing, looseness and so on.

- AE signal is high-frequency elastic waves, and the frequency of AE signal generated by machine fault is mainly above 20 kHz, which is far different from the natural frequency and the background noise frequency of most machines. This helps to restrain the disturbance of noise, and enhance the fault information.

- The AE sensor is non-directional sensor, which means that it can obtain the signals of different directions at the same time. Comparing with the vibration sensors, one AE sensor can perform the works that need multi-vibration-sensors, and it can not only simplify the device and program, but also reduce the negligence of early fault information.

A comprehensive research has been performed on the AE application in the industry. Recent years, application of Acoustic emission technique has been growing in fault diagnostics of rotating

\(^1\) Corresponding author. Tel: 86-10-62788308. E-mail: luwenxiu@tsinghua.edu.cn.
machinery. Rogers [2] used acoustic emission technology to monitor the condition of low-speed rolling element bearings of offshore production platform slewing cranes, and he found that when the speed is less than 5r/min, the bearing fault can be identified by the AE signal while the vibration-based method cannot. D.Mba et al. [3-6] applied the AE technology in the fault diagnosis of bearing fault, and they concluded that crack initiation and subsequent fracture can be detected using a range of data analysis techniques on AEs generated from natural degrading shafts, and the AE technology is very effective in fault diagnosis of bearing. Wang and Chu [7] used the AE technology and wavelet transform to determine the location of rubbing, their experimental results showed that the rubbing location could be identified rightly by cross-correlation method. AE signal is often used to detect whether these faults occur and to determine where the faults occur, especially to determine the crack location [8]. AE wave is an elastic wave, which is experimentally proved by Breckenridge in 1975; he concluded that the elastic wave generated by the damage of thin glass tube caused by impulse was a Lamb wave [9]. Dalton [10] studied the propagation of AE signals through metallic aircraft fuselage structure and found that only the fundamental modes A0 and S0 in a low frequency band, centred at about 70 kHz, exhibit long-range propagation. However, it is still difficult to analyse the propagation and attenuation characteristics of AE wave theoretically, the research about propagation and attenuation mainly focuses on experimental research.

In this paper, a propagation model of AE in the shaft is established to analyze the propagation characteristics. The crack depth and location are considered to investigate the influence on the propagation characteristics of AE signal. A series of experiments are implemented and the results show that the AE signal is very effective to determine the damage location of shaft.

One Dimension AE Propagation Equation

1.1. Wave propagation equation of a slender rod

As shown in figure 1, the shaft is considered as a slender rod. The cross-section tensile stiffness is $EA(x)$, $E$ is the elastic modulus, $A(x)$ is the area of cross section, $\rho$ is the density. Assuming any cross-section perpendicular to the shaft axis always remains to one plane, and ignore the lateral deformations, then the points of the cross-section keep the same displacement in $x$ direction. Then the internal force of the shaft can be obtained:

![Figure 1. Propagation equation deduction figure.](image-url)
\[ N = EA(x) \frac{\partial u}{\partial x} \]  

Thus, the equation of the differential element \( dx \) can be deducted:

\[ (N + \frac{\partial N}{\partial x} dx) - \rho A(x) dx \frac{\partial^2 u}{\partial t^2} - N = 0 \]  

From equations (1) and (2), the following equation can be obtained:

\[ \rho A(x) \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} [EA(x) \frac{\partial u}{\partial x}] \]  

For the rod with the same area of cross-section, that is, \( A(x) \) is a constant \( A \), and \( E \) is also a constant. Then equation (3) can be written as:

\[ \rho A \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} [EA \frac{\partial u}{\partial x}] = EA \frac{\partial^2 u}{\partial x^2} \]  

Then

\[ \frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial x^2} \]  

Where \( c = \sqrt{\frac{E}{\rho}} \), is the propagation velocity of AE wave.

Assuming the AE wave is a harmonic wave, the solution of equation (5) is:

\[ u(x, t) = A \sin \omega (t - \frac{x}{c}), \text{ that is, } u(x, t) = A \exp(j \omega (t - \frac{x}{c})) \]  

For a visco-elastic solid, the absolute modulus \( E = E_1 + j E_2 \), in which \( E_1 \) is storage modulus and \( E_2 \) is loss modulus. Let

\[ \frac{1}{c} = \sqrt{\frac{\rho}{E}} = s_1 - js_2 \]  

Then

\[ u(x, t) = A \exp(j \omega (t - (s_1 - js_2)x)) = A \exp(-\omega s_2 x) \exp(j \omega (t - s_1 x)) \]  

1.2. Elastic wave reflection and transmission in section break

In practical, there are frequently section breaks in rod. The crack defect can be also regarded as a section break in rod. In section break, the wave reflection and transmission will greatly impress the
wave propagation. As shown in figure 2, the modulus, wave propagation velocity, section area and internal stress are $E_1$, $c_1$, $A_1$, $N_1$ and $E_2$, $c_2$, $A_2$, $N_2$, respectively. $u_i$, $u_r$ and $u_t$ are the wave input, reflection and transmission displacement, respectively.

The displacement, velocity and stress should be continuous at section break. Therefore, the following equations can be obtained:

$$u_i(x,t) = u_2(x,t) \Rightarrow u_i(x,t) + u_r(x,t) = u_i(x,t)$$

$$v_i(x,t) = v_2(x,t) \Rightarrow v_i(x,t) + v_r(x,t) = v_i(x,t)$$

$$N_i(x,t) = N_2(x,t) \Rightarrow N_i(x,t) + N_r(x,t) = N_i(x,t)$$

(9)

Another form of the solution of equation (5) can be written as:

$$u(x,t) = f(x-ct) + g(x+ct)$$

(10)

In which, $f(x-ct)$ is the wave propagating along the $x$ direction with the velocity $c$, and $g(x+ct)$ is the wave propagating along the reverse $x$ direction with the velocity $c$.

From equation (10) and figure 2, assuming only the wave along $x$ direction exists. Let $\xi = x - ct$, then

$$\frac{\partial u_i}{\partial x} = \frac{\partial f_i(x-c\xi,t)}{\partial x} = \frac{\partial f_i(\xi)}{\partial \xi} \frac{\partial \xi}{\partial x} = \frac{\partial f_i(\xi)}{\partial \xi}$$

$$\frac{\partial v_i}{\partial t} = \frac{\partial f_i(x-c\xi,t)}{\partial t} = \frac{\partial f_i(\xi)}{\partial \xi} \frac{\partial \xi}{\partial t} = -c_i \frac{\partial f_i(\xi)}{\partial \xi} = -c_i \frac{\partial u_i}{\partial \xi}$$

(11)

For the reflection wave and transmission wave, the following equation can be also obtained:

$$v_r = c_1 \frac{\partial u_i}{\partial \xi}, v_r = -c_2 \frac{\partial u_i}{\partial \xi}$$

(12)

From equations (9), (11) and (12), the following equation can be obtained:

$$-c_1 \frac{\partial u_i}{\partial \xi} + c_1 \frac{\partial u_r}{\partial \xi} = -c_2 \frac{\partial u_i}{\partial \xi}$$

(13)

From equations (1) and (13), we can obtain:

$$-c_1 \frac{N_i}{E_i A_i} + c_1 \frac{N_r}{E_i A_i} = -c_2 \frac{N_i}{E_2 A_2}$$

(14)

Then

$$N_i = \frac{c_1 E_i A_i}{c_2 E_i A_i} (N_r - N_i) = \frac{\rho_2 c_2 A_2}{\rho_1 c_1 A_1} (N_r - N_i)$$

(15)

Let $Z = \rho c A$, then the following equations can be obtained:

$$\frac{N_r}{N_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \frac{N_r}{N_i} = \frac{2Z_2}{Z_2 + Z_1}, \frac{u_r}{u_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \frac{u_i}{u_i} = \frac{2Z_1}{Z_2 + Z_1}$$

(16)

1.3. Recurrence formula of elastic wave
It is very difficult to invest the propagation characteristics in the rod with variable cross sections analytically. The method similar to finite element method is often used to establish the recurrence formula of variable units, just as shown in figure 3. The rod is divided by \( \text{lnum} \) units, and the length of each unit is just equal to the product of the propagation velocity \( c \) and the sampling interval \( \Delta t \).

\[
\begin{align*}
\text{Figure 3.} & \quad \text{Recurrence propagation of each unit.} \\
\end{align*}
\]

Assuming the impedance \( Z \) changes at exact interface of unit, the following equations can be obtained:

\[
\begin{align*}
f(n,i) &= \beta \frac{Z(n) - Z(n-1)}{Z(n) + Z(n-1)} g(n,i-1) + \beta \frac{2Z(n-1)}{Z(n) + Z(n-1)} f(n-1,i-1) \\
g(n,i) &= \beta \frac{Z(n) - Z(n+1)}{Z(n+1) + Z(n)} f(n,i-1) + \beta \frac{2Z(n+1)}{Z(n) + Z(n+1)} g(n+1,i-1) \\
f(1,i) &= h(i) + \theta g(1,i-1) \\
g(\text{lnum},i) &= \theta f(\text{lnum},i-1)
\end{align*}
\]

Where \( \beta = \exp(-\omega \Delta x) \) is the attenuation coefficient, \( \theta \) is the reflection attenuation coefficient, and \( h(i) \) is the input AE signal.

With equation (17), the AE wave propagation characteristics can be simply simulated by program.

AE propagation experiment and simulation

1.4. Experiment research of AE propagation

The overall layout of experimental rig is as shown in figure 4. A signal generator can produce voltage signal with variable frequency, and the voltage signal is used to excite the AE signal by AE sensor. Then the AE signal is transmitted to one end of the shaft. Another AE sensor is installed at the other end of the shaft to obtain the AE signal. The AE sensors used for this experiment were broadband type sensors with a relative flat response in the region between 50 kHz and 200 kHz (Physical Acoustic Corporation). All the signals from sensors were pre-amplified with 40 dB, filtered with 20-1200 kHz, and connected directly to a commercial data acquisition card, where a sampling rate of 1 or 10 MHz per channel was used during the test.

The impulse wave is a sine wave produced by the signal generator with amplitude 0.1 V, frequency 150 kHz, interval 1ms.
Two experiments were carried out to invest the propagating of AE waves. The first is the propagation characteristics of the normal shaft with the length 400 mm and diameter 16 mm. The other is the defect shaft with ring groove in the middle of the shaft, and the diameter in the middle shaft is 8 mm.

The normal shaft experiment result is shown in figure 5. $F_1(t)$ is the impulse AE wave obtained by sensor 1, and $F_2(t)$ is the AE wave transmitted by the normal shaft. It can be calculated that the time difference between $F_1(t)$ and $F_2(t)$ is 88 us, then the longitudinal wave propagating speed is about 4545 m/s. And it can be clearly observed that there are four echo waves in $F_2(t)$, in which the interval is equal and the attenuation is $10.70\text{dB}$, $8.21\text{dB}$, $5.10\text{dB}$, respectively. Furthermore, the impulse
wave width is becoming bigger and bigger, that is, the wave dispersion occurs during the AE wave propagation.

![Figure 6](image)

**Figure 6.** AE wave of defect shaft.

Figure 6 shows the AE wave propagation of defect shaft. It can be observed that the AE wave of defect shaft is more complex than the normal shaft, and the waveform is more concentrated, which is caused by the more reflections by ring groove. In fact, one concentrated region is a reflection wave. Comparing with figure 5, the amplitude of the first concentrated region is more less because of the defect. Similarly, it can be calculated that the time difference between $F_1(t)$ and $F_2(t)$ is 89 us, then the longitudinal wave propagating speed is about 4494 m/s. The time difference between the first concentrated region and the second concentrated region is 88 us, and the defect location can be determined at 197.75 mm with a error ratio of 1.1%.

1.5. Simulation research of AE propagation

From the layout of experimental rig as shown in figure 4, the sampled AE signals is a combined effect of signal generator, transducer response, coupling medium and transmission medium. Then the mathematical equation of the sampled AE signals can be expressed as the following:

$$F_1(t) = S(t) * R'(t) * C(t) * R(t)$$
$$F_2(t) = S(t) * R'(t) * C(t) * M(t) * C(t) * R(t)$$

(18)

In which, $S(t)$ is the produced signal by signal generator, $R'(t)$ is impulse frequency response function of piezoelectric actuator, $R(t)$ is impulse frequency response function of transducer, $C(t)$ and $M(t)$ are the impulse frequency response function of coupling medium and transmission medium (just the shaft), respectively.

From equation (18), we can obtain:

$$F_2(t) = F_1(t) * M(t) * C(t)$$

(19)
Since the piezoelectric actuator and transducer are the same kind of piezoelectric film, the impulse frequency response functions are similar in shape, except the amplification factor; that is, \( R'(\omega) = cR(\omega) \), and \( c \) is constant, maximal value of \( R'(\omega) \times R(\omega) \) is about 1. The impulse frequency response function graph is shown in figure 7. The sine waves are produced by the signal generator with amplitude 0.02 V, frequency 130 and 150 kHz, the AE signals \( F_1(t) \) are as shown in figure 8. The amplitude of 150 kHz sine wave is 4.3 V, and the one of 130 kHz is 3.88 V. Then the constant \( c \) can be calculated as about 1e9. And the impulse frequency response function of coupling medium is calculated as shown in figure 7, from which it can be seen that there are peak values at 100, 150 and 200 kHz and the values at other frequency are very small.

![Figure 7. Impulse frequency response function of AE sensor and coupling medium](image)

![Figure 8. AE waves produced by different frequency of sine waves.](image)

The wavelet package decomposition and reconstruction technology is used to decompose signals into a series of time domain signals, and then the signals covering a specific octave frequency band are reconstructed. DB5 wavelet is adopted to decompose the AE signal into 3 levels, and 8 band signals are reconstructed. The 8 AE signals have different attenuation coefficient and propagating speed, as shown in figure 9.

![Figure 9. Attenuation coefficients of different band frequency](image)

![Figure 10. Propagation speeds of different band frequency](image)

For every band frequency signal, the equation (17) is used to obtain simulation signal of AE. Then the 8 signals are combined into the real AE signal. For the normal shaft, the simulation signal is shown in figure 11. Comparing with figure 5, it can be seen that the simulation signal wave is similar to the experimental signal, and the time difference between the echo waves are almost the same. However,
the wave shape are not some different. The experimental wave is high in middle and low on both sides, which is like a beat-wave; while the simulation signal is similar to the shot signal $F_1(t)$. It can be caused by the transverse wave, the wave mode conversion, the coupling medium and so on.

For the defect shaft, the simulation waves with different crack depth are shown in figure 12. Comparing with figure 6, it can be seen that the differences of result between the experiment and simulation are some bigger. However, the maximal value of the first and second regions is almost in the same time place, that is, the location of the defect can be exactly simulated. From figure 12, it can also be observed that the first region of AE wave is becoming smaller with the diameter of defect position becoming small. It can be easily explained that the transmission wave of AE signal is becoming small with the decreasing of area. Surprisingly, the second region of AE wave may become larger with the decreasing of with the diameter of defect position.

![Figure 11. AE simulation wave of normal shaft.](image)

![Figure 12. AE simulation wave of defect shaft with different crack depth.](image)
Conclusions

In this paper, a propagation model of AE in the shaft is established to analyze the propagation characteristics. The crack depth is changed to investigate the influence on the propagation characteristics of AE signal. An AE impulse signal is generated to actuate the shaft, and the AE impulse is propagating along the shaft. The reflection and attenuation of AE impulse are very obvious in the experiments. The results also show that the AE signal is very effective to determine the damage location of shaft.

Acknowledgments

This research is supported by National Natural Science Foundation of China (Grant No. 10702031, 10732060) and the Beijing Natural Science Foundation (Grant No. 3112013).

References

[1] Mathews J R 1983 *Acoustic Emission* (New York: Gordon and Breach Science Publishers Inc.)
[2] Rogers L M 1979 *Tribology International* 12 51
[3] Elforjani M and Mba D 2009 *Engineering Failure Analysis* 16 521
[4] Mba D, Bannister R H and Finadlay GE 1999 *Proc Instn Mech Engrs* 213 153
[5] Jamaludin N and Mba D 2002 *NDT&E International* 35 349
[6] Elforjani M and Mba D 2009 *Engineering Failure Analysis* 16 2121
[7] Wang Q and Chu F 2001 *Journal of Sound and Vibration* 248 91
[8] Sedlak P, Hirose Y, et al 2009 *Ultrasonics* 49 254
[9] Breckenridge F R and Tschiegg C E and Greenspan M 1975 *J. Acoust. Soc. Am* 57 626
[10] Dalton R P, Cawley P and Lowe M J 2001 *IEE Proceedings-Science Measurement and Technology* 148 169