New CP observables in $B^0(t) \rightarrow \text{hyperon + antihyperon}$ from parity violation in the sequential decay

J. Charles, A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal
Laboratoire de Physique Théorique et Hautes Energies
Université de Paris XI, Bâtiment 210, F-91405 Orsay Cedex, France

Abstract
We consider the decay $B^0(t) \rightarrow \text{hyperon + antihyperon}$, followed by hyperon weak decay. We show that parity violation in the latter allows to reach new CP observables: not only $\text{Im} \lambda_f$ but also $\text{Re} \lambda_f$ can be measured. In the decay $B^0_d(t) \rightarrow \Lambda \bar{\Lambda} \left( BR \sim 10^{-6} \right)$, $\Lambda \rightarrow p\pi^-$ these observables reduce to $\sin 2\alpha$ and $\cos 2\alpha$ in the small Penguin limit, the latter solving the discrete ambiguity $\alpha \rightarrow \frac{\pi}{2} - \alpha$. For $\beta$ one could consider the Cabibbo suppressed mode $B^0_d(t) \rightarrow \Lambda^+ \bar{\Lambda}^- \left( BR \sim 10^{-4} \right)$, $\Lambda^+ \rightarrow \Lambda\pi^+$, $p\bar{K}^0$, ... (with $BR \sim 10^{-2}$). The pure Penguin modes $B^0_s(t) \rightarrow \Sigma^- \bar{\Sigma}^-, \Xi^- \bar{\Xi}^-, \Omega^- \bar{\Omega}^- \left( BR \sim 10^{-7} \right)$ can be useful in the search of CP violation beyond the Standard Model. Because of the small total rates, the study of these modes could only be done in future high statistics experiments. Also, in the most interesting case $\Lambda \bar{\Lambda}$ the time dependence of the asymmetry can be difficult to reconstruct. On the other hand, we show that $B_d$ mesons, being a coherent source of $\Lambda \bar{\Lambda}$, is useful to look for CP violation in $\Lambda$ decay. We also discuss $B^0_d(t) \rightarrow J/\Psi K^*0 \rightarrow \ell^+\ell^- K_S\pi^0$ where the secondary decays conserve parity, and angular correlations allow to determine terms of the form $\cos\delta \cos 2\beta$, $\delta$ being a strong phase. This phase has been measured by CLEO, but we point out that a discrete ambiguity prevents to determine sign($\cos 2\beta$). However, if one assumes small strong phases, like in factorization and as supported by CLEO data, one could have information on sign($\cos 2\beta$). Similar remarks can be done for $\cos 2\alpha$ in the decay $B^0_d(t) \rightarrow \rho\rho \rightarrow 4\pi$. 

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1 Introduction

The measurement of the CP angles of the Unitarity Triangle (UT) through time-dependent CP asymmetries is a major purpose of particle physics in the next years.

Concerning the angle $\alpha$, the present determination of the sides of the UT gives a range $40^\circ < \alpha < 140^\circ$ \[1\]. The CLEO upper bound on the favorite decay $B_d \rightarrow \pi^+\pi^-$ \[2\], although not inconsistent with naive expectations, points to a determination of $\alpha$ that will not be easy. The presence of Penguins complicates the picture, mostly if the expected color suppression of the $\pi^0\pi^0$ mode does not allow to perform an isospin analysis \[3\], \[4\]. Therefore, it may happen that one will only get an effective angle $\alpha_{eff}$, which is related to $\alpha$ via the Penguin contribution \[5\]. Also, even if $\sin 2\alpha_{eff}$ is some day measured in $B_d^0(t) \rightarrow \pi^+\pi^-$, the discrete ambiguity $\alpha_{eff} \rightarrow \frac{\pi}{2} - \alpha_{eff}$ (and also $\pi + \alpha_{eff}$) would be left \[6\]. The ambiguity $\alpha_{eff} \rightarrow \frac{\pi}{2} - \alpha_{eff}$ could be solved by the measurement of $\text{sign}(\cos 2\alpha_{eff})$. The measurement of $\sin 2\alpha$, $\cos 2\alpha$, and of the Penguin amplitudes could be made in principle by the study of the time-dependent Dalitz plot $B_d^0(t) \rightarrow \pi^+\pi^-\pi^0$ via $\rho$ decay \[7\]. However, the expected branching ratios for the different $\rho\pi$ decay modes make this study difficult, at least in the first generation of CP violation experiments in B mesons. It is therefore suitable to study all possible decay modes that can help to give hints on the different aspects of the measurement of $\alpha$. In this paper we propose to consider the sequential decay $B_d^0(t) \rightarrow \Lambda\bar{\Lambda}$, $\Lambda \rightarrow p\pi^-$ that
allows in principle to measure both \( \sin 2\alpha \) and also \( \cos 2\alpha \) (up to Penguins) thanks to parity violation in \( \Lambda \) decay. The new CP information is on the sign of \( \cos 2\alpha_{eff} \), where \( \alpha_{eff} \) is related to \( \alpha \) via the Penguin contributions [5]. The relevant features of this decay, with an amplitude \( A(B^0_d \rightarrow \Lambda\bar{\Lambda}) \sim |V_{ub}| \) and hence an expected branching ratio of the order of \( 10^{-5}-10^{-6} \), the excellent detection efficiency of \( \Lambda \rightarrow p\pi^- \) with a large \( BR(\Lambda \rightarrow p\pi^-) = 64\% \), and a sizeable parity violation parameter \( \alpha(p\pi^-) = 0.64 \), necessary to get information on \( \cos 2\alpha_{eff} \) as we will show below, make this mode very interesting. Let us emphasize however again that this mode gets contributions not only from current-current operators but also from local Penguin operators (Fig. 1), and long distance Penguin contributions as well [8].

The decay mode \( B^0_d(t) \rightarrow J/\Psi K_S \) will hopefully allow to measure \( \sin 2\beta \) with a high precision. Considering the decay modes studied in this paper, the same type of arguments can be applied to the angle \( \beta \) through the Cabibbo-suppressed decay \( B^0_d(t) \rightarrow \Lambda^+_c \bar{\Lambda}^+_c \) (Fig. 2) since \( \Lambda^+_c \) decays weakly, like in \( \Lambda^+_c \rightarrow \Lambda \pi^+, p\bar{K}^0, ... \) These sequential decays could in principle allow to measure \( \sin 2\beta \) and also \( \cos 2\beta \) up to possible Penguin pollution.

For \( \gamma \) one could naively consider \( B^0_s(t) \rightarrow \Lambda\bar{\Lambda}, \Xi^0\Xi^0 \), but branching ratios are expected to be very small in this case and Penguins are large (Fig. 3), as we will see below.

We also point out that the pure Penguin modes \( B^0_s(t) \rightarrow \Sigma^-\Sigma^-, \Xi^-\Xi^-, \Omega^-\Omega^- \) (Fig. 4), for which the Standard Model predicts very small asymme-
tries, could be useful in the search of CP violation beyond the Standard Model. Moreover, these pure Penguin modes can give a hint on the strength of the possible Penguin pollution in the decays relevant for $\alpha$, $\beta$ or $\gamma$.

Let us first estimate the order of magnitude of the branching ratios of the modes that we discuss in this paper. For $B^0_d \to \Lambda^+_c \bar{\Lambda}^+$ we can make a very crude estimate from the measured inclusive ratio $BR(B^\pm/B^0 \to \bar{\Sigma}_c + \text{anything}) \approx 5 \times 10^{-3}$ [9] :

$$BR(B^0_d \to \Lambda^+_c \bar{\Lambda}^+) \sim \frac{BR(\bar{\Sigma}_c + \text{anything})}{BR(D + \text{anything})} \times BR(D\bar{D}_s) \sim 2 \times 10^{-4} \quad (1)$$

and for $B^0_d \to \Lambda \bar{\Lambda}$ we rescale from the ratio $|V_{ub}/V_{cb}| \approx 0.08$ :

$$BR(B^0_d \to \Lambda \bar{\Lambda}) \sim 10^{-6} \quad (2)$$

For $B_s \to \Lambda \bar{\Lambda}$, $\Xi^0 \Xi^0$ there is a further Cabibbo suppression, yielding very small $BR \sim 10^{-7} - 10^{-8}$. The pure Penguin modes $B^0_s \to \Sigma^- \bar{\Sigma}^-$, $\Xi^- \bar{\Xi}^-$, $\Omega^- \bar{\Omega}^-$ can be rescaled from $BR(B^0_d \to K\pi) \sim 10^{-5}$ yielding $BR \sim 10^{-7}$, while the branching ratios of modes of $B^0_d$ to the same final states, that are also of Penguin type, are predicted to be very tiny, $BR \sim 10^{-8} - 10^{-9}$ [10].

Let us first discuss the angle $\beta$ and the well known angular correlations $B^0_d(t) \to J/\Psi K^{*0} \to \ell^+\ell^- K_S\pi^0$ [11] [12], where the secondary decays conserve parity and Penguin pollution is expected to be very small. This will make a natural introduction to the new angular correlations that appear if the secondary decay violates parity.

4
In the Standard Model, by the measurement of the sides of the unitarity triangle, $\beta$ is constrained to be in the range $10^\circ \leq \beta \leq 35^\circ$ [1]. Measurement of $\sin 2\beta$ in this range in e.g. $B_d^0(t) \to J/\Psi K_S$ would leave the ambiguities $\beta \to \pi/2 - \beta, \pi + \beta$, that would correspond, owing to the range predicted by the Standard Model, to possible physics beyond the Standard Model. Recently, some theoretical effort has been devoted to this question of the discrete ambiguities [3]. Concerning the ambiguity $\beta \to \pi/2 - \beta$, that needs the measurement of $\text{sign} (\cos 2\beta)$, we have put forward some ideas, like Dalitz plot analyses like $B_d^0(t) \to D^+ D^- K_S$ via $D_s^{*\pm}$ and other possible channels [13]. Another nice proposal involves interference between $B_d^0 - \bar{B}_d^0$ mixing and $K^0 - \bar{K}^0$ mixing in the cascade decay $B_d^0(\bar{B}_d^0) \to J/\Psi K \to J/\Psi \pi \ell \nu$ [14].

As we will see in detail below, in the time-dependent decay $B_d^0(t) \to J/\Psi K^{*0} \to \ell^+ \ell^- K_S \pi^0$ the observables obtained from angular analysis are [12] $|G_0(t)|^2, |G_\pm(t)|^2, |G_\mp(t)|^2, \text{Re}[G_0(t)G_+^*(t)], \text{Im}[G_0(t)G_+^*(t)]$ and $\text{Im}[G_0(t)G_-^*(t)]$, where $G_0(t), G_\pm(t), G_\mp(t)$ are transversity amplitudes to final states of definite CP (respectively $+, +, -$) [13]. The observables of the form $\text{Im}[G_{CP=+}(t) G_{CP=-}^*(t)]$ contain the term $\text{Re}[G_{CP=+}(0)G_{CP=-}^*(0)] \cos 2\beta \sin \Delta M t$, and the determination of $\text{sign} (\cos 2\beta)$, necessary to lift the ambiguity $\beta \to \pi/2 - \beta$, is polluted by a coefficient $\cos \delta$, $\delta$ being the strong phase $\delta = \arg[G_{CP=+}(0)G_{CP=-}^*(0)]$ [14]. This phase has been recently measured by CLEO [17], but, as we will show below there is a discrete ambiguity on $\delta$ that does not allow to measure $\text{sign} (\cos 2\beta)$ in a time-dependent analysis, unless
some reasonable model-dependent assumption is made. We will show that if one assumes small strong phase shifts in the decay \( B^0_d \to J/\Psi K^{*0} \) one could determine \( \text{sign}(\cos 2\beta) \) modulo this hypothesis. This is encouraging because CLEO finds amplitudes that are close to be relatively real, as predicted by factorization.

The difficulty of the pollution by strong phases in the determination of \( \cos 2\beta \) is lifted in principle if one considers the Cabibbo suppressed decay \( B^0_d(t) \to \Lambda_c^+ \Lambda_c^- \). The baryon \( \Lambda_c^+ \) decaying weakly, e.g. \( \Lambda_c^+ \to \Lambda \pi^+, pK^0, ... \), we will show below that other observables are accessible because of parity violation. Calling \( G_+(t), G_-(t) \) the amplitudes to \( \Lambda_c^+ \Lambda_c^- \) in a definite CP state, the observables are now \(|G_+(t)|^2, |G_-(t)|^2, \text{Re}[G_+(t)G^*_-(t)], \text{Im}[G_+(t)G^*_-(t)]|\), allowing to measure \( \sin 2\beta \) and also \( \cos 2\beta \) (in the small Penguin limit) without strong phase pollution. The same argument applies to the more interesting case of \( \alpha \), namely the sequential decay \( B^0_d(t) \to \Lambda \bar{\Lambda}, \Lambda \to p\pi^- \), the main result of this paper. Moreover, we emphasize that this decay could be useful to look for CP violation in \( \Lambda \) decay.

2 Remarks on angular analysis in \( B^0_d \to J/\Psi K^{*0} \)

To introduce the subject, let us first discuss the well-known case \( B^0_d(t) \to J/\Psi K^{*0} \to \ell^+\ell^- K_S\pi^0 \). As explained in detail in ref. [12], the angular dependent rate takes the form
\[ |M|^2 = \left( \frac{3}{4\pi} \right)^2 \sum_{\alpha = \pm 1} \sum_{\lambda = 0, \pm 1} A_\lambda D_{\lambda, \alpha}^1 (R_\Psi)^* D_{\lambda, 0}^1 (R_{K^*})^* \]  

(3)

where \( A_\lambda \) are helicity amplitudes, in general time-dependent: the angular dependence and the time dependence factorize. For the Jackson convention, \( R = (\theta, \varphi, 0) \) where \( \theta, \varphi \) are the decay angles in the vector meson rest frame. \(|M|^2\) can be written in the form

\[
|M|^2 = \left( \frac{3}{4\pi} \right)^2 \sum_{\alpha = \pm 1} \sum_{\lambda = 0, \pm 1} \sum_{\lambda' = 0, \pm 1} A_\lambda A_{\lambda'}^* (-1)^{2\lambda - \alpha} \sum_{J_L = 0, 1, 2} \sum_{J_R = 0, 1, 2} <1\alpha, 1 - \alpha|J_L 0 > <1\lambda', 1 - \lambda|J_L' \lambda - \lambda > D_{\lambda' - \lambda, 0}^{J_L} (R_\Psi) \\
<1\alpha, 1 - \alpha|J_L 0 > <1\lambda', 1 - \lambda|J_L' \lambda - \lambda > D_{\lambda' - \lambda, 0}^{J_R} (R_{K^*}) \\
= \left( \frac{3}{4\pi} \right)^2 \sum_{\alpha = \pm 1} \sum_{\lambda = 0, \pm 1} \sum_{\lambda' = 0, \pm 1} A_\lambda A_{\lambda'}^* (-1)^{2\lambda - \alpha} \sum_{J_L = 0, 1, 2} \sum_{J_R = 0, 1, 2} <1\alpha, 1 - \alpha|J_L 0 > <1\lambda', 1 - \lambda|J_L' \lambda - \lambda > \\
\sqrt{\frac{4\pi}{2J_L + 1}} \sqrt{\frac{4\pi}{2J_R + 1}} Y_{J_L, \lambda - \lambda}(\Omega_\Psi) Y_{J_R, \lambda' - \lambda}(\Omega_{K^*}).
\]  

(4)

Taking moments

\[ T_{J_L J_R M} = \int |M|^2 Y_{J_L M}(\Omega_\Psi) Y_{J_R M}(\Omega_{K^*}) d\Omega_\Psi d\Omega_{K^*} \]  

(5)

one finds
$$T_{J_L J_R} = \left( \frac{3}{4\pi} \right)^2 \sqrt{\frac{4\pi}{2J_L + 1}} \sqrt{\frac{4\pi}{2J_R + 1}} \sum_{\alpha = \pm 1} \sum_{\lambda = 0, \pm 1} \sum_{\lambda' = 0, \pm 1} A_\lambda A_{\lambda'}^* (-1)^{2\lambda - \alpha}$$

$< 1\alpha, 1 - \alpha | J_L 0 > < 1\lambda', 1 - \lambda | J_L M >$

$< 10, 10 | J_R 0 > < 1\lambda', 1 - \lambda | J_R M >$ \hspace{1cm} (6)

with the relation

$$T_{J_L, J_R}^* = T_{J_L, J_R} - M \hspace{1cm} (7)$$

In terms of transversity amplitudes

$$G_0 = A_0 \hspace{1cm} G_+ = \frac{A_{+1} + A_{-1}}{\sqrt{2}} \hspace{1cm} G_- = \frac{A_{+1} - A_{-1}}{\sqrt{2}}$$ \hspace{1cm} (8)

one finds that the non-vanishing moments are the following \[12\] :

$$T_{000} = \frac{2}{4\pi} (|G_0|^2 + |G_+|^2 + |G_-|^2)$$

$$T_{020} = \frac{2}{4\sqrt{5\pi}} \left( 2|G_0|^2 - |G_+|^2 - |G_-|^2 \right)$$

$$T_{200} = \frac{1}{4\sqrt{5\pi}} \left( -2|G_0|^2 + |G_+|^2 + |G_-|^2 \right)$$

$$T_{220} = -\frac{1}{5} \left( 4|G_0|^2 + |G_+|^2 + |G_-|^2 \right)$$

$$T_{22-1} = -\frac{3\sqrt{2}}{5} (\text{Re} G_+ G_0^* + i\text{Im} G_- G_0^*)$$

$$T_{22-2} = -\frac{3}{5} \left( |G_+|^2 - |G_-|^2 + 2i\text{Im} G_- G_+^* \right)$$ \hspace{1cm} (9)
We have used the notation $G_+$ and $G_-$ instead of $G_{1+}$ and $G_{1-}$ of ref. [12] for the transverse CP even and transverse CP odd amplitudes to make explicit the differences with the $\Lambda_c^+\Lambda_c^+$ case, where we will use the same notation $G_+$, $G_-$ for the CP even and CP odd amplitudes. We see that the observables are

\[
|G_0|^2 \quad |G_+|^2 \quad |G_-|^2 \quad \text{Re}G_+G_0^* \quad \text{Im}G_-G_0^* \quad \text{Im}G_-G_+^* \quad (10)
\]

that are in general time dependent. One can measure the real parts of the interferences between amplitudes of same CP and the imaginary parts of the interferences of amplitudes of opposite CP.

The time dependence of these amplitudes follows from the expression for the time evolution ($\Delta \Gamma \approx 0$ and $\frac{q}{p} = 1$ is assumed, as given by the Standard Model):

\[
|B_0^d(t) > = e^{-iM_t} e^{-\Gamma t/2} \left( \cos \frac{\Delta M_t}{2} |B_0^d > + i\frac{q}{p} \sin \frac{\Delta M_t}{2} |\bar{B}_0^d > \right)
\]

\[
|\bar{B}_0^d(t) > = e^{-iM_t} e^{-\Gamma t/2} \left( \cos \frac{\Delta M_t}{2} |\bar{B}_0^d > + i\frac{p}{q} \sin \frac{\Delta M_t}{2} |B_0^d > \right) \quad (11)
\]

that gives, for $B_0^d(t)$ decay amplitudes

\[
G_0(t) = G_0(0) e^{-iM_t} e^{-\Gamma t/2} \left( \cos \frac{\Delta M_t}{2} + \eta_f e^{-2i\beta} \sin \frac{\Delta M_t}{2} \right)
\]

\[
G_+(t) = G_+(0) e^{-iM_t} e^{-\Gamma t/2} \left( \cos \frac{\Delta M_t}{2} + \eta_f e^{-2i\beta} \sin \frac{\Delta M_t}{2} \right)
\]
\[ G_-(t) = G_-(0) e^{-i\Delta Mt} e^{-\Gamma t/2} \left( \cos \frac{\Delta Mt}{2} - i\eta_f e^{-2i\beta} \sin \frac{\Delta Mt}{2} \right) \] (12)

and analogously for \( B^0_d(t) \) decays. In these expressions, the sign \( \eta_f \) depends on the CP eigenstate in which \( K^{*0} \) decays, for example \( \eta(K_S \pi^0) = + \). Notice that in these expressions we have neglected possible Penguin contributions, that are expected to be very small in these \( J/\Psi K^* \) modes.

Then, the time dependent observables write,

\[
\begin{align*}
|G_0(t)|^2 &= |G_0(0)|^2 e^{-\Gamma t} (1 + \eta_f \sin 2\beta \sin \Delta Mt) \\
|G_+(t)|^2 &= |G_+(0)|^2 e^{-\Gamma t} (1 + \eta_f \sin 2\beta \sin \Delta Mt) \\
|G_-(t)|^2 &= |G_-(0)|^2 e^{-\Gamma t} (1 - \eta_f \sin 2\beta \sin \Delta Mt) \\
\text{Re} \ [G_+(t)G_0^*(t)] &= \text{Re} \ [G_+(0)G_0^*(0)] e^{-\Gamma t} (1 + \eta_f \sin 2\beta \sin \Delta Mt) \\
\text{Im} \ [G_-(t)G_0^*(t)] &= e^{-\Gamma t} \left\{ - \text{Re} \ [G_-(0)G_0^*(0)] \eta_f \cos 2\beta \sin \Delta Mt + \text{Im} \ [G_-(0)G_0^*(0)] \cos \Delta Mt \right\} \\
\text{Im} \ [G_-(t)G_+^*(t)] &= e^{-\Gamma t} \left\{ - \text{Re} \ [G_-(0)G_+^*(0)] \eta_f \cos 2\beta \sin \Delta Mt + \text{Im} \ [G_-(0)G_+^*(0)] \cos \Delta Mt \right\} \\
\end{align*}
\] (13)

CLEO has measured these observables in a time-integrated experiment [17], that amounts to determine these quantities at \( t = 0 \).

From these expressions we see that in a time-dependent angular analysis experiment where we could hopefully separate the time dependence \( e^{-\Gamma t} \sin \Delta Mt \), one could measure the products
\[ \text{Re} \left[ G_-(0)G_0^*(0) \right] \cos 2\beta \quad \text{Re} \left[ G_-(0)G_+^*(0) \right] \cos 2\beta \quad . \quad (14) \]

Therefore, one can only measure products of the form \( \cos \delta \cos 2\beta \) where \( \delta \) is the strong phase \( \text{Arg}[G_-(0)G_0^*(0)] \) or \( \text{Arg}[G_-(0)G_+^*(0)] \). Then, this measurement could not solve the ambiguity \( \beta \rightarrow \frac{\pi}{2} - \beta \) by itself because of the strong phase \( \delta \) [10]. However, as we will see below, there is some experimental knowledge on these phases that could allow to have information on \( \cos 2\beta \) if some additional theoretical input is assumed [18].

Let us go back to the decay \( J/\Psi(K^0_0)K_S\pi^0 \) and to the CLEO data [17]. CLEO has reported the phases, within the convention \( \varphi(G_0) = 0 \):

\[ \begin{align*}
\varphi(G_+) + \pi &= 3.00 \pm 0.37 \pm 0.04 \\
\varphi(G_-) &= -0.11 \pm 0.46 \pm 0.03
\end{align*} \quad (15) \]

(the \( \pi \) comes from the particular CLEO convention). These results are consistent with the amplitudes \( G_0, G_+, G_- \) being real relatively to each other, as expected within the hypothesis of factorization.

The CLEO results seem to solve the problem of determining \( \text{sign}(\cos 2\beta) \), since \( \varphi(G_+)-\varphi(G_0) \) and \( \varphi(G_-)-\varphi(G_0) \) have been measured. There is, however, a discrete ambiguity in the determinations of these phases, and therefore a second solution. From the angular distribution [17], i.e. simply the observables quoted above at \( t = 0 \), one sees that one can measure (within the CLEO convention \( \varphi(G_0) = 0 \)):

11
\[ \cos \varphi(G_+) \sin[\varphi(G_-) - \varphi(G_+)] \sin \varphi(G_-) \quad . \quad (16) \]

These quantities remain invariant under

\[ \varphi(G_+) \to -\varphi(G_+) \quad \varphi(G_-) \to \pi - \varphi(G_-) \quad (17) \]

and there is a second solution for these phases.

In the time-dependent analysis, the terms proportional to \( \cos 2\beta \) change sign under this transformation

\[ \cos [\varphi(G_-) - \varphi(G_+)] \cos 2\beta \to -\cos [\varphi(G_-) - \varphi(G_+)] \cos 2\beta \]
\[ \cos \varphi(G_-) \cos 2\beta \to -\cos \varphi(G_-) \cos 2\beta \quad . \quad (18) \]

There is a sign ambiguity on \( \cos[\varphi(G_-) - \varphi(G_+)] \) and on \( \cos \varphi(G_-) \) and therefore a sign ambiguity on \( \cos 2\beta \) remains \cite{18}. One of the solutions for \( \text{sign}[\cos[\varphi(G_-) - \varphi(G_+)] \) and on \( \text{sign}[\cos \varphi(G_-)] \) will correspond to the relative sign between CP even and CP odd amplitudes as given by factorization \cite{13}, \cite{20}. The other solution will correspond to the (awkward ?) solution in which the relative sign has been exactly reversed by a very large FSI. Then, the measurement of the relative phases plus the hypothesis of small strong phase shifts can give a hint on the \( \beta \to \frac{\pi}{2} - \beta \) ambiguity. Let us emphasize that there are two observables of this kind, and then if \( \text{sign}[\cos \delta \cos 2\beta] \) is observed to be the same for both, consistent with \( \beta \) as constrained by the Standard Model and with small strong phase \( \delta \), we could confidently conclude
about the Standard Model solution for $\beta$. It is important to note that the same remarks can be done to have information on $\cos 2\alpha$, since observables of the same form are in principle measurable in $B_d^0(t) \to \rho \rho \to 4\pi$, although Penguin pollution is expected to be sizeable in this case.

3 Angular analysis in $B^0 \to \text{hyperon} + \text{anti-hyperon}$

Let us now turn back to the decays $B^0(t) \to \text{hyperon} + \text{antihyperon}$, taking first as an example $B_d^0(t) \to \Lambda^+_c \Lambda_c^-$. One can write the angular distribution

$$|M|^2 = \frac{1}{4\pi^2} \sum_{\alpha=\pm 1/2} \sum_{\beta=\pm 1/2} \sum_{\lambda=\pm 1/2} A_\lambda B_\alpha B_\beta D^{1/2}_{\lambda,\alpha} (R_{\Lambda_c})^* D^{1/2}_{\lambda,\beta} (R_{\Lambda_c})^*$$

where $A_\lambda$ are the helicity amplitudes of the decay $B_d^0 \to \Lambda^+_c \Lambda_c^-$ in the $B$ center-of-mass and $B_\alpha (B_\beta)$ are the helicity amplitudes of the decay $\Lambda^+_c \to \Lambda \pi^+ (\Lambda_c^- \to \bar{\Lambda} \pi^-)$ in the $\Lambda_c (\bar{\Lambda}_c)$ rest frames. After some angular momentum calculations, one finds, analogously to (4) :

13
\[ |M|^2 = \frac{1}{4\pi^2} \sum_{\alpha=\pm1/2} \sum_{\beta=\pm1/2} \sum_{\lambda=\pm1/2} \sum_{\lambda'=\pm1/2} A_{\lambda'} A_{\lambda'}^* B_\alpha B_\alpha^* \bar{B}_\beta \bar{B}_\beta^* \]

\[ (-1)^{2\lambda-\alpha-\beta} \sum_{J_L=0,1} \sum_{J_R=0,1} \]

\[ < \frac{1}{2}\alpha, \frac{1}{2} -\alpha |J_L0 > < \frac{1}{2}\lambda', \frac{1}{2} -\lambda |J_L\lambda' -\lambda > \]

\[ < \frac{1}{2}\beta, \frac{1}{2} -\beta |J_R0 > < \frac{1}{2}\lambda', \frac{1}{2} -\lambda |J_R\lambda' -\lambda > \]

\[ \sqrt{\frac{4\pi}{2J_L+1}} \sqrt{\frac{4\pi}{2J_R+1}} Y_{J_L\lambda'-\lambda}^* (\Omega_{\Lambda_c}) Y_{J_R\lambda'-\lambda}^* (\Omega_{\bar{\Lambda}_c}) \] (20)

and defining moments:

\[ T_{J_LJ_RM} = \int |M|^2 Y_{J_LM} (\Omega_{\Lambda_c}) Y_{J_RM} (\Omega_{\Lambda_c}) d\Omega_{\Lambda_c} d\Omega_{\bar{\Lambda}_c} \] (21)

one obtains

\[ T_{J_LJ_RM} = \frac{1}{4\pi^2} \sqrt{\frac{4\pi}{2J_L+1}} \sqrt{\frac{4\pi}{2J_R+1}} \]

\[ \sum_{\alpha=\pm1/2} \sum_{\beta=\pm1/2} \sum_{\lambda=\pm1/2} \sum_{\lambda'=\pm1/2} A_{\lambda'} A_{\lambda'}^* B_\alpha B_\alpha^* \bar{B}_\beta \bar{B}_\beta^* (-1)^{2\lambda-\alpha-\beta} \]

\[ < \frac{1}{2}\alpha, \frac{1}{2} -\alpha |J_L0 > < \frac{1}{2}\lambda', \frac{1}{2} -\lambda |J_LM > \]

\[ < \frac{1}{2}\beta, \frac{1}{2} -\beta |J_R0 > < \frac{1}{2}\lambda', \frac{1}{2} -\lambda |J_RM > \] (22)

and the same relation \( [\cdot] \), \( T_{J_LJ_RM}^* = T_{J_LJ_R-M} \).

One finds the independent moments
\[
T_{000} = \frac{1}{4\pi} \left[ |A_{+1/2}|^2 + |A_{-1/2}|^2 \right] \left[ |B_{+1/2}|^2 + |B_{-1/2}|^2 \right] \left[ |\bar{B}_{+1/2}|^2 + |\bar{B}_{-1/2}|^2 \right]
\]
\[
T_{100} = \frac{1}{4\pi \sqrt{3}} \left[ |A_{+1/2}|^2 - |A_{-1/2}|^2 \right] \left[ |B_{+1/2}|^2 - |B_{-1/2}|^2 \right] \left[ |\bar{B}_{+1/2}|^2 + |\bar{B}_{-1/2}|^2 \right]
\]
\[
T_{010} = \frac{1}{4\pi \sqrt{3}} \left[ |A_{+1/2}|^2 - |A_{-1/2}|^2 \right] \left[ |B_{+1/2}|^2 + |B_{-1/2}|^2 \right] \left[ |\bar{B}_{+1/2}|^2 - |\bar{B}_{-1/2}|^2 \right]
\]
\[
T_{110} = \frac{1}{12\pi} \left[ |A_{+1/2}|^2 + |A_{-1/2}|^2 \right] \left[ |B_{+1/2}|^2 - |B_{-1/2}|^2 \right] \left[ |\bar{B}_{+1/2}|^2 - |\bar{B}_{-1/2}|^2 \right]
\]
\[
T_{11-1} = \frac{1}{6\pi} A_{+1/2} A_{1/2}^* \left[ |B_{+1/2}|^2 - |B_{-1/2}|^2 \right] \left[ |\bar{B}_{+1/2}|^2 - |\bar{B}_{-1/2}|^2 \right] . \quad (23)
\]

Defining now amplitudes of definite CP final state for the decay \( B_d^0 \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- \)

\[
G_{\pm} = \frac{1}{\sqrt{2}} \left( A_{+1/2} \pm A_{-1/2} \right) \quad (\text{CP} = \pm) \quad (24)
\]

and amplitudes of definite parity for the decay \( \Lambda_c^+ \rightarrow \Lambda \pi^+ \)

\[
B_{pc} = \frac{1}{\sqrt{2}} \left( B_{+1/2} + B_{-1/2} \right) \quad B_{pv} = \frac{1}{\sqrt{2}} \left( B_{+1/2} - B_{-1/2} \right) \quad (25)
\]

one can rewrite the moments in the form

\[
T_{000} = \frac{1}{4\pi} \left[ |G_+|^2 + |G_-|^2 \right] \left[ |B_{pc}|^2 + |B_{pv}|^2 \right] \left[ |\bar{B}_{pc}|^2 + |\bar{B}_{pv}|^2 \right]
\]
\[
T_{100} = \frac{1}{4\pi \sqrt{3}} \left[ 2 \text{Re}(G_+ G_-^*) \right] \left[ 2 \text{Re}(B_{pc} B_{pv}^*) \right] \left[ |\bar{B}_{pc}|^2 + |\bar{B}_{pv}|^2 \right]
\]
\[
T_{010} = \frac{1}{4\pi \sqrt{3}} \left[ 2 \text{Re}(G_+ G_-^*) \right] \left[ |B_{pc}|^2 + |B_{pv}|^2 \right] \left[ 2 \text{Re}(\bar{B}_{pc} \bar{B}_{pv}^*) \right]
\]
\[
T_{110} = \frac{1}{12\pi} \left[ |G_+|^2 + |G_-|^2 \right] \left[ 2 \text{Re}(B_{pc} B_{pv}^*) \right] \left[ 2 \text{Re}(\bar{B}_{pc} \bar{B}_{pv}^*) \right]
\]
\[
T_{11-1} = \frac{1}{12\pi} \left[ |G_+|^2 - |G_-|^2 \right] \left[ 2 i \text{Im}(G_+ G_-^*) \right] \left[ 2 \text{Re}(B_{pc} B_{pv}^*) \right] \left[ 2 \text{Re}(\bar{B}_{pc} \bar{B}_{pv}^*) \right] . \quad (26)
\]
If parity were conserved in the baryon decay, the only observable would be $T_{000}$ or $|G_+|^2 + |G_-|^2$ (as pointed out in ref. [12]), but since parity is in general violated, one has in general $\text{Re}(B_{pc}B_{pv}^*)$, $\text{Re}(\bar{B}_{pc}\bar{B}_{pv}^*) \neq 0$. In terms of the parity violating parameters in the secondary decays,

$$
\alpha_\Lambda = \frac{2\text{Re}(B_{pc}B_{pv}^*)}{|B_{pc}|^2 + |B_{pv}|^2}, \quad \bar{\alpha}_\Lambda = \frac{2\text{Re}(\bar{B}_{pc}\bar{B}_{pv}^*)}{|\bar{B}_{pc}|^2 + |\bar{B}_{pv}|^2} \quad (27)
$$

new moments appear due to parity violation $T_{100}$, $T_{010}$, $T_{110}$ and $T_{11-1}$ respectively proportional to $\alpha_\Lambda$, $\bar{\alpha}_\Lambda$, $\alpha_\Lambda \bar{\alpha}_\Lambda$ and $\alpha_\Lambda \bar{\alpha}_\Lambda$. Neglecting now CP violation in $\Lambda^+_c$ decay, a safe assumption in the Standard Model, then $\alpha_\Lambda + \bar{\alpha}_\Lambda \approx 0$ [21], and we see that the observables of the primary decay are now

$$
|G_+|^2, \quad |G_-|^2, \quad \text{Re}(G_+G_+^*), \quad \text{Im}(G_+G_-^*) \quad (28)
$$

in general time dependent. CP violation in $\Lambda^+_c$ decay will be discussed below, together with the present limits on CP violation in $\Lambda$ decay. Therefore we have more information than in the case $J/\Psi K^*$ because of the term $\text{Re}(G_+G_+^*)$ that at $t = 0$ will give $\cos[\text{Arg}(G_+G^+_*)]$. The time dependence of the amplitudes is given by (assuming $\Delta \Gamma = 0$):

$$
G_+(t) = G_+(0)e^{-iMt}e^{-\Gamma t/2}\left(\cos\frac{\Delta Mt}{2} + i\lambda_+ \sin\frac{\Delta Mt}{2}\right)
$$

$$
G_-(t) = G_-(0)e^{-iMt}e^{-\Gamma t/2}\left(\cos\frac{\Delta Mt}{2} - i\lambda_- \sin\frac{\Delta Mt}{2}\right) \quad (29)
$$

where
\[
\lambda_{\pm} = \frac{q}{p} \frac{\tilde{G}_{\pm}(0)}{G_{\pm}(0)} .
\]  

(30)

Notice that there is no general argument to claim that \( \lambda_+ = \lambda_- \) because of the presence of Penguins, that can differently affect the two CP amplitudes, i.e. their contribution with possible FSI phases can depend on the helicity [22]. In terms of \( \lambda_{\pm} \) the time-dependent observables (28) write:

\[
|G_{\pm}(t)|^2 = e^{-\Gamma t} \left[ X^{(1)}_{\pm} + X^{(2)}_{\pm} \cos \Delta M t + X^{(3)}_{\pm} \sin \Delta M t \right]
\]

\[
\text{Im} \left[ G_{-}(t)G^{*}_{\pm}(t) \right] = e^{-\Gamma t} \left[ Y^{(1)} + Y^{(2)} \cos \Delta M t + Y^{(3)} \sin \Delta M t \right]
\]

\[
\text{Re} \left[ G_{-}(t)G^{*}_{\pm}(t) \right] = e^{-\Gamma t} \left[ Z^{(1)} + Z^{(2)} \cos \Delta M t + Z^{(3)} \sin \Delta M t \right]
\]  

(31)

where

\[
X^{(1)}_{\pm} = |G_{\pm}(0)|^2 \frac{1}{2} \left( 1 + |\lambda_{\pm}|^2 \right)
\]

\[
X^{(2)}_{\pm} = |G_{\pm}(0)|^2 \frac{1}{2} \left( 1 - |\lambda_{\pm}|^2 \right)
\]

\[
X^{(3)}_{\pm} = \mp |G_{\pm}(0)|^2 \text{Im} \lambda_{\pm}
\]  

(32)

and

\[
Y^{(1)} = \text{Re} \left[ G_{-}(0)G^{*}_{\pm}(0) \right] \frac{1}{2} \left( \text{Re} \lambda_- \text{Im} \lambda_+ - \text{Re} \lambda_+ \text{Im} \lambda_- \right)
\]

\[
+ \text{Im} \left[ G_{-}(0)G^{*}_{\pm}(0) \right] \frac{1}{2} \left( 1 - \text{Im} \lambda_- \text{Im} \lambda_+ - \text{Re} \lambda_- \text{Re} \lambda_+ \right)
\]

17
\[ Y^{(2)} = -\text{Re} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Re}\lambda_{-}\text{Im}\lambda_{+} - \text{Re}\lambda_{+}\text{Im}\lambda_{-}) \]
\[ + \text{Im} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (1 + \text{Im}\lambda_{-}\text{Im}\lambda_{+} + \text{Re}\lambda_{-}\text{Re}\lambda_{+}) \]

\[ Y^{(3)} = -\text{Re} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Re}\lambda_{-} + \text{Re}\lambda_{+}) \]
\[ + \text{Im} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Im}\lambda_{-} - \text{Im}\lambda_{+}) \]

\[ Z^{(1)} = \text{Re} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (1 - \text{Re}\lambda_{+}\text{Re}\lambda_{-} - \text{Im}\lambda_{+}\text{Im}\lambda_{-}) \]
\[ - \text{Im} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Im}\lambda_{+}\text{Re}\lambda_{-} - \text{Re}\lambda_{+}\text{Im}\lambda_{-}) \]

\[ Z^{(2)} = \text{Re} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (1 + \text{Re}\lambda_{+}\text{Re}\lambda_{-} + \text{Im}\lambda_{+}\text{Im}\lambda_{-}) \]
\[ + \text{Im} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Im}\lambda_{+}\text{Re}\lambda_{-} - \text{Re}\lambda_{+}\text{Im}\lambda_{-}) \]

\[ Z^{(3)} = -\text{Re} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Im}\lambda_{+} - \text{Im}\lambda_{-}) \]
\[ + \text{Im} \left[ G_{-}(0)G_{+}^{*}(0) \right] \frac{1}{2} (\text{Re}\lambda_{+} + \text{Re}\lambda_{-}) \]  

Hopefully separating the different time dependences $e^{-\Gamma t}$, $e^{-\Gamma t} \cos \Delta M t$ and $e^{-\Gamma t} \sin \Delta M t$, all the quantities $X_{\pm}^{(i)}$, $Y^{(i)}$ and $Z^{(i)}$ ($i = 1, 2, 3$) are in principle observable. We have four unknowns related to CP violation, namely $\text{Im}\lambda_{+}$, $\text{Im}\lambda_{-}$ and $\text{Re}\lambda_{+}$, $\text{Re}\lambda_{-}$. As usual, the imaginary parts $\text{Im}\lambda_{+}$, $\text{Im}\lambda_{-}$ can be measured by the different terms contributing to $|G_{\pm}(t)|^2$ \text{(32)} since $|\lambda_{\pm}|^2$
can also be known. Moreover, we have two strong interaction unknowns, namely \( \text{Re}[G_-(0)G^*_+(0)] \) and \( \text{Im}[G_-(0)G^*_+(0)] \). As we now show, the other four unknowns \( \text{Re}\lambda_+ \), \( \text{Re}\lambda_- \) and \( \text{Re}[G_-(0)G^*_+(0)] \), \( \text{Im}[G_-(0)G^*_+(0)] \) can be determined without discrete ambiguities by the six observables (33). First, one finds:

\[
\text{Im} \left[ G_-(0)G^*_+(0) \right] = Y^{(1)} + Y^{(2)} \\
\text{Re} \left[ G_-(0)G^*_+(0) \right] = Z^{(1)} + Z^{(2)} \quad .
\]

(34)

Calling now the combinations of observables

\[
r = \frac{\text{Re} \left[ G_-(0)G^*_+(0) \right]}{\text{Im} \left[ G_-(0)G^*_+(0) \right]} \\
A = \frac{r}{1 + r^2} \left[ \frac{Y^{(1)} - Y^{(2)}}{Y^{(1)} + Y^{(2)}} - \frac{Z^{(1)} - Z^{(2)}}{Z^{(1)} + Z^{(2)}} \right] \\
B = \frac{2r}{1 + r^2} \left[ \frac{Z^{(3)}}{Z^{(1)} + Z^{(2)}} - \frac{Y^{(3)}}{Y^{(1)} + Y^{(2)}} \right]
\]

(35)

we find

\[
\text{Re}\lambda_+ = -\frac{A - B\text{Im}\lambda_+}{\text{Im}\lambda_+ + \text{Im}\lambda_-} \quad \text{Re}\lambda_- = \frac{A + B\text{Im}\lambda_-}{\text{Im}\lambda_+ + \text{Im}\lambda_-} \quad .
\]

(36)

The conclusion is that the six unknowns \( \text{Im}\lambda_+ \), \( \text{Im}\lambda_- \), \( \text{Re}\lambda_+ \), \( \text{Re}\lambda_- \), \( \text{Re}[G_-(0)G^*_+(0)] \) and \( \text{Im}[G_-(0)G^*_+(0)] \) can be determined in principle without discrete ambiguities, and the system is overconstrained since we have 12
observables.

4 Particular cases

Let us first examine the case \( B_d^0(t) \to \Lambda^+_c \Lambda^+_c \), related to the angle \( \beta \) (Fig. 2). The decay amplitude will write, in all generality

\[
G_\pm = G_{\pm}^{(u)} V_{ud}^* V_{ub} + G_{\pm}^{(c)} V_{cd}^* V_{cb} + G_{\pm}^{(t)} V_{td}^* V_{tb}
\]  
(37)

where \( G_{\pm}^{(c)} \) and \( G_{\pm}^{(t)} \) are respectively the current-current (tree) and short distance Penguin amplitudes, and \( G_{\pm}^{(u)} \) is a long distance \( u \)-Penguin, responsible for rescattering effects \[8\]. Using unitarity \( V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0 \), in all generality \( \lambda_\pm \) will be of the form

\[
\lambda_\pm = \pm \frac{e^{-i\beta} - |R_\pm| e^{i\delta_\pm}}{e^{i\beta} - |R_\pm| e^{i\delta_\pm}}
\]  
(38)

where

\[
R_\pm = z r_\pm, \quad z = \frac{V_{ud}^* V_{ub}}{V_{cd}^* V_{cb}}, \quad r_\pm = \frac{P_\pm}{T_\pm} = |r_\pm| e^{i\delta_\pm}
\]  
(39)

\( z \) being a ratio of CKM matrix elements, and \( r_\pm \) the ratio of strong amplitudes:

\[
P_\pm = G_{\pm}^{(t)} - G_{\pm}^{(u)}, \quad T_\pm = G_{\pm}^{(c)} - G_{\pm}^{(u)}.
\]  
(40)
The notation \( P_\pm, T_\pm \) means that these are respectively Penguin and dominantly tree amplitudes. One can measure in principle \( \text{Re} \lambda_\pm \) and \( \text{Im} \lambda_\pm \) and we have 5 unknowns, namely \( \beta, |R_\pm| \) and \( \delta_\pm \). Therefore, to get information on \( \beta \) we need theoretical input on a single parameter. This is the general situation in these decays if Penguins are sizeable. At this stage, one may use the formalism of [5]: one can get \( \beta \) as a function of this unknown parameter and of the observables. Then, measuring \( \text{Re} \left( \frac{\mathcal{G}}{\mathcal{G}} \right) \) means that a discrete ambiguity \( (\beta_{\text{eff}} \rightarrow \frac{\pi}{2} - \beta_{\text{eff}}) \) is solved in this procedure, and allows to get more information than in, e.g., \( B_d^0 \rightarrow D^+D^- \), even in the presence of sizeable Penguin uncertainties.

Notice that \( |z| \sim O(1) \) in powers of the Wolfenstein parameter \( \lambda \), although presumably Penguins are not large because of their small short distance coefficient, or loop suppression for long distance Penguins. If we assume that the Penguin contribution is small, i.e. \( R_\pm \) is small, the time dependence of the observables is given in terms of \( \beta \) as follows:

\[
|G_+(t)|^2 = |G_+(0)|^2 e^{-\Gamma t} (1 + 2\beta \sin \Delta Mt)
\]

\[
|G_-(t)|^2 = |G_-(0)|^2 e^{-\Gamma t} (1 - 2\beta \sin \Delta Mt)
\]

\[
\text{Im} \left[ G_-(t)G_+^*(t) \right] = e^{-\Gamma t} \left\{ \text{Im} \left[ G_-(0)G_+^*(0) \right] \cos \Delta Mt - \text{Re} \left[ G_-(0)G_+^*(0) \right] \cos 2\beta \sin \Delta Mt \right\}
\]
\[
Re \left[ G_-(t)G_+^*(t) \right] = e^{-\Gamma t} \left\{ Re \left[ G_-(0)G_+^*(0) \right] \cos \Delta Mt + \right.
\]
\[
\left. \quad \text{Im} \left[ G_-(0)G_+^*(0) \right] \sin \Delta Mt \right\}.
\]

One can in principle separate both \( \text{Im} \left[ G_-(t)G_+^*(t) \right] \) and \( \text{Re} \left[ G_-(t)G_+^*(t) \right] \) by the angular analysis. The term in \( e^{-\Gamma t} \sin \Delta Mt \) from \( \text{Im} \left[ G_-(t)G_+^*(t) \right] \) measures \( \text{sign}(\cos \delta \cos 2\beta) \) where \( \delta = \text{Arg} \left[ G_-(0)G_+^*(0) \right] \), but the \( \text{sign}(\cos \delta) \) can be known from the coefficient of \( e^{-\Gamma t} \cos \Delta Mt \) in the new observable \( \text{Re} \left[ G_-(t)G_+^*(t) \right] \). Conversely, one could measure \( \text{sign}(\sin \delta \cos 2\beta) \) from the term in \( e^{-\Gamma t} \sin \Delta Mt \) from \( \text{Re} \left[ G_-(t)G_+^*(t) \right] \), and \( \text{sign}(\sin \delta) \) can be known from the coefficient of \( e^{-\Gamma t} \cos \Delta Mt \) in the observable \( \text{Im} \left[ G_-(t)G_+^*(t) \right] \). Therefore, \( \cos 2\beta \) is overdetermined in this case in which parity is violated in the secondary decay.

However, in the case of \( \beta \), this procedure could be quite difficult to put in practice, since, as we have seen above, the branching ratio of \( B_d^0 \to \Lambda_c^+\Lambda_c^- \) being Cabibbo-suppressed can be expected to be of the order \( 10^{-4} \) and the exclusive \( \Lambda_c^+ \) two-body decays like \( \Lambda_c^+ \to \Lambda\pi^+, p\bar{K}^0, \Sigma^0\pi^+, \ldots \) have already been measured with BR of the order of \( 10^{-2} \). Notice also that the parity violation \( \alpha \) parameter of some of these modes has already been measured: \( \alpha(\Lambda\pi^+) = -0.98 \pm 0.19, \alpha(\Sigma^0\pi^+) = -0.45 \pm 0.32 \). Since the combined branching ratios in the case of \( \beta \) are small, one should look for inclusive arguments along the same lines, in order to increase statistics. This might be possible to carry out, since the solution of the discrete ambiguity \( \beta \to \frac{\pi}{2} - \beta \)
only needs the determination of $\text{sign}(\cos 2\beta)$.

Let us now turn to one major purpose of this paper, the determination of $\sin 2\alpha$ and $\cos 2\alpha$ (up to Penguins) in the sequential decay $B_0^q(t) \rightarrow \Lambda\bar{\Lambda}$, $\Lambda \rightarrow p\pi^-$. The time dependence of the amplitudes is given by the same expressions (29) with $\lambda_\pm$ now given by

$$
\lambda_\pm = \pm \frac{e^{i\alpha} - |R_\pm|e^{i\delta_\pm}}{e^{-i\alpha} - |R_\pm|e^{i\delta_\pm}}
$$

(42)

where

$$
R_\pm = z r_\pm \quad z = \frac{V_{td}^* V_{tb}}{V_{ud}^* V_{ub}} \quad r_\pm = \frac{P_\pm}{T_\pm} = |r_\pm|e^{i\delta_\pm}
$$

(43)

$z$ being a ratio of CKM matrix elements, and $r_\pm$ is the Penguin to tree ratio of strong amplitudes, different than in the $\beta$ case (39), but defined along the same lines. Since $z \simeq \frac{1 - \rho + i\eta}{\rho - i\eta}$, there is no CKM suppression of Penguins. From the values of the short distance QCD coefficients, one naively expects $|R_\pm| \simeq 0.05 \times \left| \frac{1 - \rho + i\eta}{\rho - i\eta} \right| \sim 0.15$, not inconsistent with CLEO data [3].

Neglecting Penguins, the observables will read:

$$
|G_+(t)|^2 = |G_+(0)|^2 e^{-\Gamma t} (1 - \sin 2\alpha \sin \Delta M t)
$$

$$
|G_-(t)|^2 = |G_-(0)|^2 e^{-\Gamma t} (1 + \sin 2\alpha \sin \Delta M t)
$$

$$
\text{Im} \left[ G_-(t)G_+^*(t) \right] = e^{-\Gamma t} \left\{ \text{Im} \left[ G_-(0)G_+^*(0) \right] \cos \Delta M t - \text{Re} \left[ G_-(0)G_+^*(0) \right] \cos 2\alpha \sin \Delta M t \right\}
$$
\[ \text{Re} \left[ G_-(t)G_+^*(t) \right] = e^{-\Gamma t} \left\{ \text{Re} \left[ G_- (0) G_+^*(0) \right] \cos \Delta Mt + \right. \\
\left. \text{Im} \left[ G_- (0) G_+^*(0) \right] \cos 2\alpha \sin \Delta Mt \right\} . \tag{44} \]

As pointed out above, in view of the intrinsic difficulties in the determination of \( \alpha \) with any decay mode, we are here in a relatively favorable situation. One expects a \( BR(B_0^0 \to \Lambda \bar{\Lambda}) \sim 10^{-5} - 10^{-6} \), one has an excellent detection efficiency for \( \Lambda \to p\pi^- \) with a large \( BR(\Lambda \to p\pi^-) = 64 \% \), and a sizeable parity violation parameter \( \alpha_\Lambda (p\pi^-) = 0.64 \). Notice however that in order to extract the observables (44) one needs to neglect CP violation in \( \Lambda \) decay [21], which, in the Standard Model, would come from CKM suppressed Penguin diagrams. Anyhow, to extract (44) one needs to assume that the CP violation parameter \( \alpha + \bar{\alpha} \) is very small. Taking into account the expected size of CP violation in \( B \) decays, \( \alpha_\Lambda + \bar{\alpha}_\Lambda \simeq 0 \) is a safe assumption, since present data give for the CP violation parameter in \( \Lambda \) decay [3] :

\[ \frac{\alpha_\Lambda + \bar{\alpha}_\Lambda}{\alpha_\Lambda - \bar{\alpha}_\Lambda} = -0.03 \pm 0.06 . \tag{45} \]

The Standard Model predicts a tiny value of \( O(10^{-5}) \) [21] due to Penguins. Penguin diagrams being much smaller in charm decay, the Standard Model prediction of CP violation for \( \Lambda_c^+ \) decay is even smaller. Another difficulty of quite a different nature is that since the \( \Lambda \) decays far away from the primary vertex, it may be experimentally difficult to measure the time dependence of the observables [23].

Let us now turn to the possible determination of \( \sin 2\gamma \) and \( \cos 2\gamma \) through
the decays $B_0^0(t) \to \Lambda \bar{\Lambda}, \Xi^0 \Xi^0$. The time dependence of the amplitudes is given by the same expressions \(^{(29)}\) (making the rough approximation of neglecting $(\Delta \Gamma)_{B_s}$) with now $\lambda_{\pm}$ given now by

$$\lambda_{\pm} = \pm \frac{e^{-i\gamma} - |R_{\pm}| e^{i\delta_{\pm}}}{e^{i\gamma} - |R_{\pm}| e^{i\delta_{\pm}}} \quad (46)$$

where

$$R_{\pm} = z r_{\pm} \quad z = \frac{V_{ts}^* V_{tb}}{V_{us}^* V_{ub}} \quad r_{\pm} = \frac{P_{\pm}}{T_{\pm}} = |r_{\pm}| e^{i\delta_{\pm}} \quad (47)$$

$z$ being a ratio of CKM matrix elements, and $r_{\pm}$ the ratio of dominantly Penguin to tree strong amplitudes, different than in the $\beta$ and $\alpha$ cases \(^{(39)}\) and \(^{(47)}\). Since now we have $z \approx \frac{1}{\lambda^2 (\rho - i\eta)}$, there is CKM enhancement of the Penguin to tree ratio, that could compensate the ratio $|r_{\pm}| \sim 0.05$, giving $|R_{\pm}| \sim \frac{|r_{\pm}| \frac{1}{\lambda^2 (\rho - i\eta)}}{\lambda^2 (\rho - i\eta)} \sim 3$. To conclude, branching ratios are very small $\sim 10^{-7} - 10^{-8}$, and Penguins could be relatively quite important in this case due to the tree suppression. These modes do not seem suitable to get information on $\gamma$.

Let us now comment about the pure Penguin modes $B_d^0(t)$ or $B_s^0(t) \to \Sigma^- \Sigma^-, \Xi^- \Xi^-, \Omega^- \Omega^-$. If short distance Penguin dominates, the weak phase of $\frac{q}{p}$ cancels the weak phase of the ratio of decay amplitudes $\frac{\tilde{G}_f}{G_f}$ for both $B_d$ or $B_s$, so that the product $\lambda_f = \frac{q}{p} \frac{\tilde{G}_f}{G_f}$ is real. However, long distance Penguins induced by $u$ or $c$ quark loops could contribute also. These terms describe rescattering effects of the form $B_d^0(t) \to (\bar{u}q)(u\bar{q}) \to \Sigma^- \Sigma^-, \Xi^- \Xi^-, \Omega^- \Omega^-$. 

25
Let us first consider \( B^0_d(t) \to \Sigma^-\Sigma^-, \Xi^-\Xi^-, \Omega^-\Omega^- \). In this case, in the Standard Model, the time dependence of the amplitudes is given by the same expressions (29) with now \( \lambda_\pm \) given now by

\[
\lambda_\pm = \pm \frac{1 - |R_\pm| e^{i\alpha} e^{i\delta_\pm}}{1 - |R_\pm| e^{-i\alpha} e^{i\delta_\pm}}
\]

(48)

where now

\[
R_\pm = z r_\pm, \quad z = \frac{V_{us}^* V_{ub}}{V_{ts} V_{tb}}, \quad r_\pm = \frac{P_{\pm}^{LD}}{P_\pm} = |r_\pm| e^{i\delta_\pm}
\]

(49)

where \( r_\pm \) is now the ratio of \( P_{\pm}^{LD} \) (the difference between long distance \( u \) and \( c \) Penguins) to \( P_\pm \) (difference between \( t \) and \( c \) Penguins) strong amplitudes. Since \( z \approx \frac{\rho - i\eta}{1 - \rho + i\eta} \), \( |z| \sim 0.36 \), there is no CKM suppression of \( LD \) Penguins. Neglecting however these rescattering effects, that might be small, \( \lambda_\pm \) will be real and these modes can be useful to get information on possible sources of CP violation beyond the Standard Model. However, branching ratios are quite unfavorable in this case, as pointed out above. The situation is better for \( B^0_s(t) \to \Sigma^-\Sigma^-, \Xi^-\Xi^-, \Omega^-\Omega^- \), for which \( \lambda_\pm \) is given by

\[
\lambda_\pm = \pm \frac{1 - |R_\pm| e^{-i\gamma} e^{i\delta_\pm}}{1 - |R_\pm| e^{i\gamma} e^{i\delta_\pm}}
\]

(50)

where

\[
R_\pm = z r_\pm, \quad z = \frac{V_{us}^* V_{ub}}{V_{ts} V_{tb}}, \quad r_\pm = \frac{P_{\pm}^{LD}}{P_\pm} = |r_\pm| e^{i\delta_\pm}
\]

(51)
Since now $z \approx -\lambda^2(\rho - i\eta)$ is CKM suppressed, we can safely neglect long distance Penguins and $\lambda_{\pm}$ is predicted to be real in the Standard Model. Analogously to $B_d^0 \to \varphi K_S$, these modes can then be useful to get information on possible sources of CP violation beyond the Standard Model, i.e., to obtain $\text{Im}\lambda_{\pm}$ and $\text{Re}\lambda_{\pm}$ whatever the origin of CP violation could be.

5 CP violation in $\Lambda$ decay from $B_d^0 \to \Lambda\bar{\Lambda}$

As a quite different application of the formalism developed here, let us consider CP violation in $\Lambda$ decay. A simple inspection of (26) shows that the CP violation parameter in $\Lambda$ decay is given by

$$\frac{\alpha_{\Lambda} + \bar{\alpha}_{\Lambda}}{\alpha_{\Lambda} - \bar{\alpha}_{\Lambda}} = \frac{T_{100} + T_{010}}{T_{100} - T_{010}}. \quad (52)$$

As we now discuss, this quantity is, in general, independent of CP violation in $B_d$ decay, and the formula holds also for time-integrated moments. This can be interesting since the $B_d^0$ is a coherent source of $\Lambda\bar{\Lambda}$ and provides automatically a calibration of the measure of $\alpha_{\Lambda}$ and $\bar{\alpha}_{\Lambda}$.

Formulas (26) show that $T_{100}$ and $T_{010}$ are proportional to $\text{Re}[G_+(t)G_-^*(t)]$. In these expressions, there are three kinds of time dependence:

$$e^{\Gamma t/2} \text{Re} \left[ G_+(0)G_-^*(0) - \bar{G}_+(0)\bar{G}_-^*(0) \right] \quad (53)$$
We have assumed in these expressions that \( \frac{q}{p} \) is a pure phase, as given in the Standard Model, and coherent with \( \Delta \Gamma = 0 \). Here the \( \bar{G}_\pm(0) \) amplitudes are defined from the CP transformation of \( G_\pm(0) \) by the relation \( \bar{G}_\pm(0) = \pm CP[G_\pm(0)] \). The first term (53) corresponds to direct CP violation, because it vanishes if Penguins are absent. This term survives for untagged events and obviously it survives also if one integrates over time. The second term (54) conserves CP. It survives integrating over time but does not contribute to untagged events. Note the interesting point that, just opposite to the usual case, e.g. \( B_0^0 \to \pi^+\pi^- \), the terms in \( e^{i\Gamma t} \) and \( e^{i\Gamma t} \cos \Delta M t \) in these angular correlations correspond to direct CP violation and to CP conservation. This is due to the fact that we have here the real part of the interference between two opposite CP amplitudes. Also unusual is the role of the third term (55) that in the present case conserves CP, and is sensitive to \( \cos 2\Phi \), \( \Phi \) being a CP angle. This term survives in general the time integration, except in \( e^+e^- \) at the \( \Upsilon(4S) \), although tagging is necessary.

To conclude, formula (52) holds in general, even if one integrates over time. There is one exception, namely the case of untagged events in the limit of vanishing Penguin. Indeed, in this last case both \( T_{100} \) and \( T_{010} \) vanish. Of course,
the possibility of integrating over time is very welcome although tagging is needed in practice. However, using $B_d^0 \rightarrow \Lambda \bar{\Lambda}$ would need large statistics, maybe available in hadronic machines. In view of the present accuracy (45), it would be suitable to reach at least a 1% upper limit on the $\Lambda$ CP asymmetry. To this aim, one would need of the order of $10^{10}$ $B_d^0$ mesons, since this number is inversely proportional to the $BR(B_d^0 \rightarrow \Lambda \bar{\Lambda}) \sim 10^{-6}$ and to the square of the CP asymmetry.

6 Conclusion

A matter of principle is the main point of this paper, namely that not only $\sin 2\beta$ ($\sin 2\alpha$) but also $\cos 2\beta$ ($\cos 2\alpha$) (up to Penguin pollution) could be reached in decays of the type $B^0 \rightarrow$ hyperon + antihyperon, because of parity violation in the hyperon decay, provided one neglects CP violation in the latter, a safe assumption within the Standard Model. On the other hand, we have also shown that this decay can be useful to look for CP violation in hyperon decay. However, one should keep in mind the smallness of the needed combined branching ratios and, also, in the interesting case $\Lambda \bar{\Lambda}$, that the time-dependence of the decay could be hard to measure because the $\Lambda$ decays far away from the primary vertex.
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We are indebted to Y. Grosman and H. Quinn for pointing out to us that in the $B_d^0(t) \to J/\Psi K^{*0} \to \ell^+\ell^- K_S\pi^0$ angular correlations one does not measure $\cos 2\beta$ but $\cos \delta \cos 2\beta$, $\delta$ being a strong phase. We also acknowledge useful remarks from B. D’Almagne, A. Gaidot and G. Vasseur.

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Note added

When this work was finished, we noticed two recent papers by A. S. Dighe et al. [24] that discuss the determination of $\text{sign}(\cos 2\beta)$ using the decays $B_{u,d} \to J/\psi K^*$ and $B_s \to J/\psi \varphi$. The $\text{sign}(\cos 2\beta)$ could be determined using $SU(3)$ and interference effects involving the sizeable $\Delta\Gamma$ of the $B_s$-$\bar{B}_s$ system, which is neglected in our paper.
Figure Captions

Fig. 1:

Decay $B^0_d \rightarrow \Lambda \bar{\Lambda}$. The four-fermion interaction represents local operators, current-current (related to the CP angle $\alpha$), or Penguins.

Fig. 2:

Decay $B^0_d \rightarrow \Lambda^+_c \bar{\Lambda}^+_c$. The four-fermion interaction represents local operators, current-current (related to the CP angle $\beta$), or Penguins.

Fig. 3:

Decay $B^0_s \rightarrow \Lambda \bar{\Lambda}$. The four-fermion interaction represents local operators, current-current (related to the CP angle $\gamma$), or Penguins.

Fig. 4:

Decay $B^0_s \rightarrow \Sigma^- \Sigma^-$. The four-fermion interaction represents Penguin local operators.
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