We investigate the consistency conditions for matter fields coupled to the four-dimensional ($N = 1$ supersymmetric) $\mathbb{C}P(1)$ nonlinear sigma model (the coset space $SU(2)_G/U(1)_H$). We find that consistency requires that the $U(1)_H$ charge of the matter be quantized, in units of half of the $U(1)_H$ charge of the Nambu-Goldstone (NG) boson, if the matter has a nonsingular kinetic term and the dynamics respect the full group $SU(2)_G$. We can then take the linearly realized group $U(1)_H$ to comprise the weak hypercharge group $U(1)_Y$ of the Standard Model. Thus we have charge quantization without a Grand Unified Theory (GUT), completely avoiding problems like proton decay, doublet-triplet splitting, and magnetic monopoles. We briefly investigate the phenomenological implications of this model-building framework. The NG boson is fractionally charged and completely stable. It can be naturally light, avoiding constraints while being a component of dark matter or having applications in nuclear physics. We also comment on the extension to other NLSMs on coset spaces, which will be explored more fully in a followup paper.

I. INTRODUCTION

The Standard Model (SM) is an extremely successful theory of particle physics, which has just seen a strong confirmation in the discovery of the Higgs boson. However, the SM has many seemingly arbitrary parameters and unexplained aspects. It is clear that some extension(s) to the SM is necessary to explain such phenomena as neutrino masses, dark matter, dark energy, quantum gravity and so on. That being said, the success of the SM means that it must be a low energy limit of any more complete theory which is valid at higher energies.

One of the most obvious (and earliest) frameworks for a completion of the SM is a grand unified theory (GUT), with or without supersymmetry (SUSY). Grand unified theories, if correct, would explain many otherwise mysterious aspects in the matter representations chosen by nature. In particular, they would explain the quantization of weak hypercharge $U(1)_Y$, which translates into a quantization of the electromagnetic charge $U(1)_{em}$, once the electroweak group $SU(2) \times U(1)_Y$ is broken down to $U(1)_{em}$. This point was made in the original proposal of GUTs by Georgi and Glashow [1]. This is an elegant answer to the longstanding question of charge quantization, which comes naturally on the road to a unified theory of all fundamental particles and forces.

However, the GUT framework has many seemingly intrinsic problems. GUT models tend to predict phenomena, such as cosmic monopole production and baryon decay, which are observationally excluded at the order of magnitude predicted in the simplest constructions. There is persistent difficulty in keeping Higgs doublets light against radiative corrections, while giving large masses to their partners under the GUT group, which carry $SU(3)_{color}$ charges. These scalar triplets thus interact with the quark and lepton sector via the Yukawa couplings, causing unacceptably large proton decay and other thus unobserved phenomena. This issue, the doublet-triplet splitting problem, is a major challenge for GUT model building.

In this Letter we explore an alternative framework that reproduces a major success of GUTs, namely $U(1)_{em}$ charge quantization, while at the same time eliminating the unwanted matter and massive gauge fields that plague GUT models.

Our proposal begins by treating the SM group $G_{SM} = SU(3)_{color} \times SU(2) \times U(1)_Y$ as a local symmetry, some part of which $H \subseteq G_{SM}$ is embedded in a larger global group $G$.

$$U(1)_Y \subseteq H \subseteq G_{SM} , \quad H \subset G.$$  

We work in a theory where $G$ is a nonlinearly realized symmetry. We never consider it to be linearly realized and spontaneously broken, nor gauged. In this work we will take $G = SU(2)_G$ and $H = U(1)_H$, and comment only briefly on generalizations which will be explored in a followup work.

Our first step is to consider the dynamics of a nonlinear $\sigma$-model (NLSM) on the coset space $C = G/H$, which has a natural set of global symmetries $G$ realized as isometries of the coset $C$ derived from its natural left $G$-action. With $G = SU(2)_G$ and $H = U(1)_H$, we have $C = \mathbb{C}P(1)$. Our second step is to add matter fields coupling to the NLSM that are linear representations of $H$, yet have dynamics that are invariant under $G$. Since $G$ is partially nonlinearly realized, the construction of such actions is nontrivial, and constitutes a major portion of
the detail of this Letter. For the third step, we gauge the linearly realized symmetry $H$, so that the matter fields $\chi$ are incorporated into the Standard Model, and the Nambu-Goldstone (NG) bosons described by the fields of $C$ are coupled to it through gauge interactions as well.

We find that in order to write down kinetic terms for the matter that are nonsingular and invariant under the full group $G$ everywhere on $C$, the $U(1)_H$ charges of the matter fields $\chi$ must be quantized in units of half the charge of the Nambu-Goldstone boson. Thus the structure of the NLSM itself and its interactions with matter eliminates the need for a GUT group, with all its phenomenologically troublesome baggage, in order to explain the quantization of electric charge.

The organization of the Letter is as follows. In Section II we introduce the $\mathbb{CP}(1)$ model. We derive the couplings of matter fields $\chi$ to $C$ under the assumption of exact $SU(2)_G$ symmetry, and in particular show that the $U(1)_H$ charge of $\chi$ must be an integer multiple of $\frac{1}{2}$ the charge of a NG boson. In Section III we then gauge the $U(1)_H$ and examine the phenomenological consequences of embedding this into the Standard Model gauge group as $U(1)_Y$. We discuss some further aspects of the model, including briefly commenting on generalizations, and conclude in Section IV.

II. THE $\mathbb{CP}(1)$ MODEL $SU(2)_G/U(1)_H$

We start by introducing our conventions for $\mathbb{CP}(1)$. With a straightforward analysis, we can show charge quantization outside of the usual setting (e.g. a GUT or monopole). We derive a charge quantization condition by considering a complex charged matter field. This field will transform linearly under the unbroken $U(1)_H$, but nonlinearly under the $SU(2)_G$. By explicitly determining the transformation properties for the field, and requiring that all transformations be smooth over the entire manifold, we find that the charge of the field is quantized: the charge is a half-integer multiple of the Nambu-Goldstone boson's charge.

If we identify the $U(1)_Y$ of the SM as $\mathbb{CP}(1)$, we can match the known hypercharges of the SM by fixing the NG boson charge. This can explain charge quantization in the SM. Additionally, the NG boson is a color-neutral, fractionally charged particle which can be a stable component of dark matter or have applications in nuclear physics. We discuss this briefly in the following section.

A. Symmetries and dynamics of the NLSM

The complex projective space $\mathbb{CP}(1)$ has two (complex) homogeneous coordinates, $\phi_{1,2}$. These satisfy $(\lambda \phi_1, \lambda \phi_2) = (\phi_1, \phi_2)$. We can then define affine coordinates as their ratio, namely $z_+ \equiv \nu \phi_1 / \phi_2$, where the vev, $\nu$, is used to give $z_+$ mass dimension one.

We label the infinitesimal generators of $SU(2)_G$ as $T_\pm$ and $T_0$. On the field $z_+$ the generators act as

$$\delta T_+ \circ z_+ = -\frac{1}{\nu} z_+^2, \quad (1a)$$
$$\delta T_- \circ z_+ = \nu, \quad (1b)$$
$$\delta T_0 \circ z_+ = z_+, \quad (1c)$$

so we can write the action of $SU(2)_G$ on holomorphic functions of $z_+$ as

$$\delta T_+ \equiv \delta^{(\text{hol.})}_{T_+} = -\frac{1}{\nu} z_+^2 \partial z_+, \quad (2a)$$
$$\delta T_- \equiv \delta^{(\text{hol.})}_{T_-} = \nu \partial z_+, \quad (2b)$$
$$\delta T_0 \equiv \delta^{(\text{hol.})}_{T_0} = z_+ \partial z_+, \quad (2c)$$

which obey the commutators

$$[\delta T_+, \delta T_-] = \pm 2 \delta T_0, \quad (3a)$$
$$[\delta T_+, \delta T_+] = 2 \delta T_0. \quad (3b)$$

The action on antiholomorphic functions is obtained by the replacements

holomorphic $\to$ antiholomorphic, $T_0 \to -T_0$, $T_\pm \to -T_\mp$, (4)

that is,

$$\delta^{(\text{ant.})}_{T_+} = -\nu \partial z_-, \quad (5a)$$
$$\delta^{(\text{ant.})}_{T_-} = -\frac{1}{\nu} z_-^2 \partial z_-, \quad (5b)$$
$$\delta^{(\text{ant.})}_{T_0} = -z_- \partial z_-, \quad (5c)$$

when acting on antiholomorphic coordinates. The full generators are simply a sum of holomorphic and antiholomorphic pieces,

$$\delta T_+ \equiv \delta^{(\text{hol.})}_{T_+} + \delta^{(\text{ant.})}_{T_+}, \quad \delta T_- \equiv \delta^{(\text{hol.})}_{T_-} + \delta^{(\text{ant.})}_{T_-}, \quad (6)$$

We will omit the labels (hol.) and (ant.) when clear.

The same $SU(2)_G$-action on holomorphic functions, eq. (2a)-(2c), can be written, using the chain rule, in terms of the variable $z_- \equiv v^2 / z_+$:

$$\delta T_+ = +\nu \partial z_-, \quad (7a)$$
$$\delta T_- = -\frac{1}{\nu} z_-^2 \partial z_-, \quad (7b)$$
$$\delta T_0 = -z_- \partial z_-, \quad (7c)$$

which obeys the same commutators, as it must.

The nonlinearly realized $SU(2)_G$ symmetry fixes the form of the kinetic term for the Goldstone bosons uniquely, up to an overall coefficient. The metric is Kähler with respect to the same complex coordinate, $z_+$, with which the symmetry is holomorphic, and the Kähler potential is fixed to be proportional to $v^2 \ln(v^2 + |z_+|^2)$.

We are working so far purely in the target space $\mathbb{CP}(1)$, which is a complex Kähler manifold, and the action of
SU(2)G on the holomorphic fields $z_+$ is holomorphic. The Kähler property of the metric and the holomorphy of the isometries are automatic consequences of the geometry of the $\mathcal{Q}_+$ with Fubini-Study metric. The model therefore admits a natural $\mathcal{N} = 1$ supersymmetrization, and we analyze principally the supersymmetric version in this Letter. However, our conclusion that charge is quantized for matter coupled to the sigma model holds even if supersymmetry is broken explicitly. We will comment further on this point in Section III. The phenomenology of the model, with supersymmetry, is discussed in Section III.

B. Consistency conditions for matter

Now let us introduce a complex charged field $\chi$, that transforms linearly under $T_0$,

$$\delta T_0 = \alpha \chi \partial \chi,$$

with $\alpha$ the $U(1)_H$ charge, and in some not yet determined nonlinear way under $T_{\pm}$. We have:

$$\delta T_+ = F_+(\chi, z_+) \partial \chi,$$

$$\delta T_- = F_-(\chi, z_+) \partial \chi.$$

For now we only define the action of the generators in the southern hemisphere ($z_+$$ \neq 0$). We will examine the extension to the northern hemisphere ($z_+ = 0$) later; we shall see that the condition that the action extends smoothly will impose precisely the condition of charge quantization.

Now let us examine the conditions for the closure of the commutation relations. The full generators are:

$$\delta T_+ = F_+(\chi, z_+) \partial \chi - \frac{1}{v} z_+^2 \partial z_+, \tag{10a}$$

$$\delta T_- = F_-(\chi, z_+) \partial \chi + v \partial z_+, \tag{10b}$$

$$\delta T_0 = \alpha \chi \partial \chi + z_+ \partial z_+ . \tag{10c}$$

The commutation relations become first-order differential equations on $F_+$:

$$[\delta T_+, \delta T_+] = (-\alpha F_+ + \alpha \partial F_+ \chi + z_+ F_+, z_+) \partial \chi - \frac{1}{v} z_+^2 \partial z_+ . \tag{11a}$$

$$[\delta T_0, \delta T_-] = (-\alpha F_- + \alpha \partial F_- \chi + z_+ F_-, z_+) \partial \chi - v \partial z_+, \tag{11b}$$

$$[\delta T_+, \delta T_-] = (F_+ \cdot F_-, \chi - \frac{1}{v} z_+^2 F_+ \cdot F_-, z_+) \partial \chi - v F_+ \cdot z_+ \partial \chi + 2 z_+ \partial z_+, \tag{11c}$$

and by matching with the algebra and the generators above we have

$$-\alpha F_+ + \alpha \partial F_+ \chi + z_+ F_+, z_+ = +F_+ \tag{12a}$$

$$-\alpha F_- + \alpha \partial F_- \chi + z_+ F_-, z_+ = -F_- \tag{12b}$$

$$F_+ \cdot F_-, \chi - \frac{1}{v} z_+^2 F_-, z_+ - F_+ \cdot F_+ \chi - v F_+ \cdot z_+ = +2 \alpha \chi. \tag{12c}$$

It’s easiest to begin by solving the first two equations, which are linear first-order PDEs. The general solutions to the first and second equations are

$$F_+ = v^{-\alpha} z_+^{\alpha+1} f_+ \left( \frac{v^{\alpha-1} \chi}{z_+^\alpha} \right), \tag{13}$$

$$F_- = v^{2-\alpha} z_+^{\alpha-1} f_- \left( \frac{v^{\alpha-1} \chi}{z_+^\alpha} \right), \tag{14}$$

where $f_{\pm}$ are two arbitrary functions of the dimensionless ratio $(v^{\alpha-1} \chi)/z_+^\alpha$.

Now restrict to transformations linear in $\chi$ (but involving a priori unknown nonlinear functions of $z_+$). We do not lose any generality by doing this. If $F_{\pm}$ contain $\chi^0$ terms, we can remove them by redefining $\chi$ additively by a function of $z_+$. Then $F_{\pm}$ can be assumed to be linear in $\chi$ plus terms of order $\chi^2$ and higher. By examining the action on $\chi$ at small $\chi$, we can see that the transformations must close among themselves at the order $\chi^1$ level. So we lose no generality by taking the transformations of $\chi$ to be strictly linear in $\chi$ (but involving unknown nonlinear functions of $z_+$). The constraints on the order $\chi^1$ terms in the transformation law for $\chi$ are independent of the order $\chi^2$ and higher-order terms.

We then take the transformations to be smooth at the south pole $z_+ = 0$, which fixes

$$F_- = 0,$$  \hspace{1cm} (15)

$$F_+ = (\text{const.}) \ z_+ \chi. \hspace{1cm} (16)$$

Imposing the third commutator equation fixes the constant, and we find

$$F_- = 0,$$  \hspace{1cm} (17)

$$F_+ = -\frac{2\alpha}{v} z_+ \chi. \hspace{1cm} (18)$$

So all in all, the holomorphic generators are

$$\delta (T_+) = \frac{2\alpha}{v} z_+ \chi \partial \chi - \frac{1}{v} z_+^2 \partial z_+, \tag{19a}$$

$$\delta (T_-) = v \partial z_+, \tag{19b}$$

$$\delta (T_0) = \alpha \chi \partial \chi + z_+ \partial z_+. \tag{19c}$$

Again, the action on the antiholomorphic $\chi^\dagger$ is obtained by extending the rules in eq. (11) to include $\chi^\dagger$:

$$\delta (T^+) = -v \partial \bar{z}_+, \tag{20a}$$

$$\delta (T^-) = + \frac{2\alpha}{v} \bar{z}_+ \chi^\dagger \partial \chi^\dagger + \frac{1}{v} \bar{z}_+^2 \partial \bar{z}_+, \tag{20b}$$

$$\delta (T_0) = -\alpha \chi \partial \chi^\dagger - \bar{z}_+ \partial \bar{z}_+. \tag{20c}$$

C. Proof of charge quantization

Now we would like this to be well-defined in the northern hemisphere, which is to say, manifestly smooth in
coordinates $z_\pm = v^2/z_\mp$. In order to make this so, we must also transform the field $\chi$ as well, to a new field $\chi'$. Since the $SU(2)_G$ transformations are linear in $\chi$, the transformation between $\chi$ and $\chi'$ should be linear in $\chi$. That is, $\chi$ should transform as a section of a vector bundle over $\mathbb{CP}(1)$. A general linear change of basis is:

$$\chi' = f(z_+) \chi, \quad \chi = \frac{1}{f(z_+)} \chi',$$

and we would like to choose $f(z_+)$ such that the $SU(2)_G$ transformations act smoothly in the northern hemisphere as well as in the southern hemisphere.

In northern-hemisphere coordinates $z_-, \chi'$, we have

$$\partial_{z_+} = \frac{\partial \chi'}{\partial \chi}, \quad f(z_+) \partial_{\chi} = \partial_{z_+} \chi', \quad (22a)$$

$$\partial_{z_+} = -v^{-2} z_+^2 \partial_{z_-} + f'(z_) \chi \partial_{\chi'}, \quad (22b)$$

So

$$\chi \partial_{z_+} = \chi' \partial_{\chi'}, \quad (23a)$$

$$z_+ \chi \partial_{z_+} = z_+ \chi \partial_{\chi'}, \quad (23b)$$

$$z_+ \partial_{z_+} = -z_- \partial_{z_-} + \frac{z_+ f'(z_+)}{f(z_+)} \chi \partial_{\chi'}, \quad (23c)$$

$$z_+ \partial_{z_+} = -v^2 \partial_{z_-} + \frac{z_+^2 f'(z_+)}{f(z_+)} \chi \partial_{\chi'}, \quad (23d)$$

The unbroken $U(1)_H$ at the north pole is the same as the unbroken $U(1)_H$ at the south pole, because any two antipodal points are fixed by the same rotation generator. Therefore the matter $\chi$ at the south pole and $\chi'$ at the north pole must each have definite eigenvalues under the same $U(1)_H$. Thus we can take the same generator $T_0$ to act simultaneously on $\chi$ and $\chi'$ as multiplication by constants. It follows that the transition function $f(z_+)$ is a pure power, $f(z_+) = K f z_+^p = v^2 K f z_+^{-p}$, where $K$ is an arbitrary constant that can be absorbed into the normalization of $\chi'$. Then we have:

$$\chi \partial_{z_+} = \chi' \partial_{\chi'}, \quad (24a)$$

$$z_+ \chi \partial_{z_+} = z_+ \chi' \partial_{\chi'}, \quad (24b)$$

$$z_+ \partial_{z_+} = -z_- \partial_{z_-} + \frac{z_+ f'(z_+)}{f(z_+)} \chi' \partial_{\chi'}, \quad (24c)$$

$$z_+ \partial_{z_+} = -v^2 \partial_{z_-} + z_+ \chi' \partial_{\chi'}, \quad (24d)$$

Then form of the transformations $\delta_{T_0}, \delta_{T_2}$ in the northern hemisphere is

$$\delta_{T_0} = -z_- \partial_{z_-} + (\alpha + p) \chi' \partial_{\chi'}, \quad (25a)$$

$$\delta_{T_2} = \frac{z_-}{v} (z_- \partial_{z_-} - p \chi' \partial_{\chi'}), \quad (25b)$$

$$\delta_{T_2} = v \partial_{z_2} - v (p + 2\alpha) z_2^{-1} \chi' \partial_{\chi'}, \quad (25c)$$

We want this transformation to be nonsingular everywhere in the northern hemisphere $z_- \in \mathbb{C}$, including the north pole, where $z_- = 0$. This imposes the condition

$$p = -2\alpha. \quad (26)$$

The change of variables between $\chi$ and $\chi'$ must be single-valued everywhere in the overlap region $z_\pm \in \mathbb{C} - \{0\}$, which fixes

$$p \in \mathbb{Z}. \quad (27)$$

This forces $\alpha$ to live in $\mathbb{Z}/2$, giving the charge-quantization condition.

In mathematical terms, the matter field $\chi$ can be thought of as the fiber coordinate of a line bundle over $\mathbb{CP}(1)$, and the charge of the matter field corresponds to half the degree of the line bundle. The charge quantization for matter coupled to $\mathbb{CP}(1)$ follows from the Birkhoff-Grothendieck theorem [2], which classifies holomorphic bundles over $\mathbb{CP}(1)$.

**D. Kinetic terms for matter fields**

For a more concrete way of understanding the origin of charge quantization, we can think of the condition on the charge as a requirement on the kinetic terms for the fields of the coupled matter-NG system. The kinetic terms must be smooth in both hemispheres and consistent under coordinate changes from hemisphere to hemisphere. We take the Kähler potential to be quadratic in $\chi, \chi'$, and invariant under the separate flavor symmetry that acts as a phase rotation on $\chi$. Imposing invariance under the combined generators $\delta \equiv \delta^{(\text{hol.})} + \delta^{(\text{ant.})}$ forces the Kähler potential for the matter to be

$$K_{\text{matter}} = \left(1 + \frac{|z_+|^2}{v^2}\right)^{-2\alpha} |\chi|^2,$$

in the southern hemisphere, up to an overall coefficient of proportionality that can be absorbed into the normalization of $\chi$. In the northern hemisphere, if we demand that the kinetic term have the same form under the simultaneous replacement $(z_+, \chi) \rightarrow (z_-, \chi')$, then equality of the northern and southern hemisphere expressions uniquely fixes $\chi' = \frac{z_+^{2\alpha}}{z_-^{1+2\alpha}} \chi$. This transformation is single valued if and only if $p \equiv -2\alpha \in \mathbb{Z}$.

We have so far assumed that our theory has $N = 1$ supersymmetry, which restricts the combined metric defining the kinetic term for the NG bosons $z_+$ and matter fields $\chi$, to be Kähler. This assumption simplifies our derivations considerably. Unlike the case for $\mathbb{CP}(1)$ in isolation, the Kähler property of the combined kinetic term for $z_+$ and $\chi$ is not an automatic consequence of the (bosonic) global symmetries. In the absence of SUSY, the Kähler condition on the kinetic term is not preserved under renormalization group (RG) flow. If we were to add mass terms for the fermions, breaking supersymmetry, this would alter the quantum properties of the theory.
and the quantum-corrected kinetic term for NG bosons and matter fields need not in general be a Kähler metric. However, charge quantization is a RG-invariant property, and therefore our demonstration of charge quantization for matter coupled to the NLSM does not depend on supersymmetry or on the existence of a Kähler metric on field space at all.

III. PHENOMENOLOGY OF THE \( \mathbb{CP}(1) \) MODEL

To show that we can use this \( \mathbb{CP}(1) \) model as the \( U(1)_Y \) part of the SM, we must be able to reproduce all of the hypercharges. There is an inherent freedom in the choice of the charge of the NG boson, and all other charges are proportional to this one. We will fix this charge through a type of “minimality:” the charge of the NG boson is chosen such that the smallest possible hypercharge is 1/6, the smallest charge in the SM.

The generator of weak hypercharge is then defined as one-third the Cartan generator \( T_0 \), normalized as in eq. (19c):

\[
Q_Y = \frac{1}{3} T_0. \tag{29}
\]

We have \( T_0 = 1 \) for the Nambu-Goldstone boson, so it has hypercharge 1/3. All other matter fields then have hypercharge

\[
Q_Y = \frac{n}{6}. \tag{30}
\]

where \( n/2 \) is the charge in units of the charge of the NG boson. Thus we see that we can easily reproduce the SM hypercharges with matter coupled to \( \mathbb{CP}(1) \).

It is clear that at this stage we have a massless, fractionally charged, and stable Nambu-Goldstone boson. The stability is ensured by its fractional charge under \( U(1)_Y \): The NG boson is a color singlet, weak isosinglet, and has \( U(1)_Y \) electric charge equal to its weak hypercharge, 1/3. It is therefore absolutely stable as there are no fractionally charged color singlets to which it can decay. Also, since it carries electric charge, once we gauge the \( U(1)_H \) gauge interactions will give it a mass, of order \( \sqrt{\alpha} M_{G/H} \). We can take \( M_{G/H} \sim M_{\text{Planck}} \), and thus the NG boson is very heavy. However, with supersymmetry we expect the mass to be protected down to about \( \sqrt{\alpha} M_{\text{ SUSY}} \).

Gauging the \( U(1)_H \) explicitly breaks the \( SU(2)_C \). However, this is controlled by the gauge coupling; as we take the coupling to zero, we flow continuously to the exact global case. Charge quantization cannot be changed by this explicit breaking. In the followup paper we will consider more general explicit breaking and its relation to the charge quantization condition.

In a supersymmetric theory, the NG boson is accompanied by a fermion partner and hence we have gauge anomalies. To cancel the anomalies we can introduce a chiral matter multiplet whose \( U(1)_H \) charge is conjugate to that of the NG multiplet. This matter multiplet can also cancel the nonlinear sigma model anomalies \( [6] \). It may be natural that the fermions receive a Dirac mass of order the gravitino mass once supersymmetry and the R-symmetry are broken. These Dirac fermions can be lighter than the NG boson, depending on the supersymmetry breaking mechanism. Here, however, we will assume that the NG boson is the lightest among them. In this case it could be probed by the LHC when the supersymmetry scale is not too high, say of order \( \sim 10 \text{ TeV} \).

This NG boson is then a candidate for dark matter. Charged dark matter was first considered and constrained tightly some time ago \( [6, 7] \), and has more recently been revived \( [8] \). The immediate concern is for production of this charged particle in the early universe, which is stringently constrained both astrophysically and terrestrially. One can simply take the reheating temperature to be far enough below the mass of the Nambu-Goldstone so that production is negligible. Even for low-scale supersymmetry, this is plausible. With some small amount of production to avoid constraints, however, the NG boson could be a component of dark matter.

We see that it is possible then to have a light, fractionally-charged boson that is accessible in colliders while satisfying current experimental constraints. Besides the discovery of such a particle to be a rather unique prediction of this scenario (given the assumptions of the relevant scales), what are some other consequences of such an unusual fundamental particle?

This Nambu-Goldstone boson could have profound consequences in nuclear physics. One very enticing idea is to use such a particle to catalyze nuclear fusion \( [9] \). By forming a bound state between the NG boson and, for instance, the deuteron, the Coulomb potential is screened. This lowers the required energy (or temperature) for fusion to occur. While a distant thought, this could make fusion-based energy accessible. As a stable, charged, and heavy particle, the NG boson could also be used as a probe of the structure of heavy nuclei by analyzing the interactions as it penetrates into the nucleon.

While we see that it is possible to have the NG boson as a light state in the low energy theory, this is not an unavoidable prediction. It is entirely possible that there are no residual effects besides charge quantization. This differs greatly from the standard GUT scenario and we will not observe or predict gauge coupling unification. Even with no new phenomena to be seen (e.g. if the supersymmetry scale is very high) we still have charge quantization in the SM via the \( \mathbb{CP}(1) \) structure.

---

1 While the hypercharges are easy to reproduce in this model, in other models the charges will be related in a more complicated manner.

2 We note here that this model can produce cosmic strings (but not magnetic monopoles), but at a scale much above the inflation scale, so they are of no concern.
IV. DISCUSSION AND CONCLUSIONS

In this Letter we have only considered the CI P(1) model, but it is clear there should be a generalization to other cases. The CI P(1) model already encapsulates all of the important aspects of our approach and has clear phenomenological applications. Furthermore, by analyzing this model, we already learn about more complicated constructions.

Consider the more general model CI P(k). Rather than directly following the procedure of Section III, we can instead add mass terms to the additional Nambu-Goldstone modes and flow down to the CI P(1) model we have already studied. It is straightforward to do this and arrive at a generalization of the quantization formula we have derived in this Letter. In a followup paper we work this out in detail, along with other models, and consider their phenomenological consequences.

To summarize, we have found a new way of quantizing electromagnetic charge, without introducing the problems typically associated with standard GUTs. The basic idea is simple: consider a nonlinear sigma model, where there is an unbroken $U(1)$ factor. From the constraints of the NLSM, namely that matter fields and their transformations under $G$ are well-defined everywhere, the charges of any matter are restricted and related to the NG boson charge. We explicitly analyzed the CI P(1) model $SU(2)C/U(1)H$, where we see all of the matter charges are half-integer multiples of the NG boson charge. This $U(1)H$ is gauged and identified as the weak hypercharge group of the SM. Thus, the NG boson has electromagnetic charge 1/3.

This model does not have any of the usual difficulties associated with GUTs: there are no extra gauge bosons to mediate proton decay, no monopoles, no colored Higgs partners, and so on. Charge quantization, a major success of GUTs, is a consequence of the geometry and dynamics of the nonlinear sigma model. A product of this construction is a Nambu-Goldstone boson that can appear in the low energy theory. The NG boson is not forced to be very light (for instance, even if protected by supersymmetry, the breaking scale could be very high), so it is possible to have no new low energy effects but still have successful charge quantization.

Given the right circumstances, though, this NG boson can play a very interesting phenomenological role. It is absolutely stable, has fraction electromagnetic charge, and can even be light enough to be seen in present colliders. Such a charged particle is not a typical prediction of supersymmetric or many other models. While avoiding cosmological constraints it can have a profound impact on nuclear physics, especially fusion. In an optimistic scenario then, we can see a smoking gun signature of charge quantization which can have very promising everyday applications.

ACKNOWLEDGMENTS

T.T.Y. would like to thank Yuji Tachikawa for discussions about CI P(1). J.K. would like to thank Brian Feldstein for conversations about charged dark matter. This work was supported by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. The work of S. H. was also supported in part by a Grant-in-Aid for Scientific Research (22740153) from the Japan Society for Promotion of Science (JSPS).

REFERENCES

[1] H. Georgi and S.L. Glashow, “Unity of All Elementary Particle Forces,” Phys.Rev.Lett. 32, 438–441 (1974)
[2] George D. Birkhoff, “Singular points of ordinary linear differential equations,” Transactions of the American Mathematical Society 10, pp. 436–470 (1909)
[3] Gregory W. Moore and Philip C. Nelson, “Anomalies in Nonlinear Sigma Models,” Phys.Rev.Lett. 53, 1519 (1984)
[4] A. Grothendieck, “Sur la classification des fibres holomorphes sur la sphere de riemann,” American Journal of Mathematics 79, pp. 121–138 (1957)
[5] Haim Goldberg and Lawrence J. Hall, “A New Candidate for Dark Matter,” Phys.Lett. B174, 151 (1986)
[6] Haag Martens, David Eichler, Rahim Esmailzadeh, and Glenn D. Starkman, “Getting a Charge Out of Dark Matter,” Phys.Rev. D41, 2388 (1990)
[7] Andrew Gould, Bruce T. Draine, Roger W. Romani, and Shmuel Nussinov, “Neutron Stars: Graveyard of Charged Dark Matter,” Phys.Lett. B238, 337 (1990)
[8] Leonid Chuzhoy and Edward W. Kolb, “Reopening the window on charged dark matter,” JCAP 0907, 014 (2009) [arXiv:0809.0436 [astro-ph]]
[9] Samuel D. McDermott, Hai-Bo Yu, and Kathryn M. Zurek, “Turning off the Lights: How Dark is Dark Matter?” Phys.Rev. D83, 063509 (2011) [arXiv:1011.2907 [hep-ph]]
[10] George Zweig, “Quark Catalysis of Exothermal Nuclear Reactions,” Science 201, 973–979 (1978); B.L. Ioffe, L.B. Okun, Mikhail A. Shifman, and M.B. Voloshin, “Heavy Stable Particles and Cold Catalysis of Nuclear Fusion,” Acta Phys.Polon. B12, 229 (1981); K. Hamaguchi, T. Hatsuda, and T.T. Yanagida, “Stau-catalyzed nuclear fusion,” (2006) [arXiv:hep-ph/0607256 [hep-ph]]
[11] T. Yanagida and Yukinori Yasui, “Supersymmetric Nonlinear Sigma Models Based on Exceptional Groups,” Nucl.Phys. B254, 235 (1985) [arXiv:hep-ph/0607256 [hep-ph]]