On cosmic quantum tunneling from “nothing”

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Abstract. We extend to a general Λ-Friedmann-Lemaître-Robertson-Walker (ΛFLRW) a previous result by Vilenkin and others according to which a closed de Sitter universe could be created from “nothing”. More specifically, our main result is that only the closed ΛFLRW universe (but not the open and flat ones) could be created from a corresponding instanton, that is, from the corresponding solution with signature +4 of the Einstein field equations. Before getting this result the suitable corresponding instantons are calculated. The result is in accordance with previous results by another authors obtained by different methods.

1. Introduction
In four seminal papers [1, 2, 3, 4] Vilenkin has estimated the non zero probability of quantically creating from “nothing” a closed de Sitter universe, here “nothing” meaning a state without time. More specifically, our main result is that only the closed ΛFLRW universe (but not the open and flat ones) could be created from a corresponding instanton, that is, from the corresponding solution with signature +4 of the Einstein field equations. Before getting this result the suitable corresponding instantons are calculated. The result is in accordance with previous results by another authors obtained by different methods.

2. The general procedure for the quantum creation of a universe: its two prescriptions
Let $T_{\alpha\beta}$ be the energy-momentum tensor in General Relativity, and $g_{\alpha\beta}$ a solution of the corresponding Einstein field equations ($\alpha, \beta, \ldots = 0, 1, 2, 3$, signature +2). In order to provide a general proposal for the creation of a given universe, we advance the following two prescriptions.

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First prescription:
We define the instanton “energy-momentum” tensor $T^E_{\alpha\beta}$ as the one obtained putting in $T_{\alpha\beta}$ an instanton metric $g^E_{\alpha\beta}$ associated to the corresponding Lorentzian one. This associated instanton metric, $g^E_{\alpha\beta}$, is a suitable Euclidean (signature +4) solution of the Einstein field equations according to the following

Second prescription:
Given $g_{\alpha\beta}$, its quantum creation probability, $P$, can be estimated from $P \propto \exp (-|S_E|)$, by associating to $g_{\alpha\beta}$ some $g^E_{\alpha\beta}$ satisfying the Darmois continuity conditions. For a singular Lorentzian metric to be created these Darmois conditions are defined in the following way:

First, let it be a space-like 3-surface, $\Sigma_3$, where $g_{\alpha\beta}$ is singular, and let us consider the first, $3g_{ij}$, and the second, $K_{ij}$, fundamental forms, whose meaning respectively is $3g_{ij}$, the restriction of $g_{\alpha\beta}$ on $\Sigma_3$, and $K_{ij} = (\nabla_\mu n_\nu)\gamma^\mu_\alpha \gamma^\nu_\beta$. Here, $n_\mu$ is the unit normal 4-vector to $\Sigma_3$ and $\gamma^\alpha_\beta$ the projector on $\Sigma_3$.

Then, for the associated instanton metric $g^E_{\alpha\beta}$, we similarly define its first and second fundamental instanton forms: $3g^E_{ij}$, $K^E_{ij}$. Then, the imposed Darmois conditions are the vanishing of $3g_{ij} - 3g^E_{ij}$, and $K_{ij} - K^E_{ij}$, through $\Sigma_3$, that we will write in a concise way:

$$\begin{align*}
[3g_{ij}] &= 0, \\
[K_{ij}] &= 0.
\end{align*}$$

3. A particular canonical Einstein equations for ΛFLRW metrics, including instantons
The Einstein equations for a ΛFLRW metric and its instantons

$$ds^2 = \epsilon du^2 + a^2\left(\frac{d\rho^2}{1 + \epsilon k \rho^2} + \rho^2 d\sigma^2\right), \quad d\sigma^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

($a$, the cosmic expansion factor, $\epsilon = -1$ for the Lorentzian solutions, and $\epsilon = +1$ for the instanton solutions) can be written

$$\begin{align*}
\dot{a}^2 &= \frac{\kappa}{3} \mu - \frac{k}{a^2} - \frac{\epsilon}{3} \Lambda, \\
\ddot{a} &= -\frac{\kappa}{6} (\mu - 3 \epsilon p) - \frac{\epsilon}{3} \Lambda, \quad (k = +1, 0, -1)
\end{align*}$$

all in accordance with Ellis [6].

4. Matching ΛFRW universes with their instantons
The Darmois continuity conditions $[3g_{ij}] = 0, [K_{ij}] = 0$, through the singular space-like 3-surface $u = t = 0$, now give

$$[a] = 0, \quad [\dot{a}] = 0, \quad [\epsilon k] = 0.$$

These conditions are satisfied in the particular case considered by Vilenkin [1, 2]: the quantum creation from “nothing” of a closed de Sitter universe. Thus, our procedure to create any ΛFLRW model extends this previous result.

In particular, the continuity condition $[\epsilon k] = 0$ associates the closed instanton:

$$ds^2_E = du^2 + a^2_E(u)\left(\frac{d\rho^2}{1 - \rho^2} + \rho^2 d\sigma^2\right), \quad E \text{ from “Euclidean”},$$

to the closed ΛFRW universe

$$ds^2 = -dt^2 + a^2(t)\left(\frac{d\rho^2}{1 - \rho^2} + \rho^2 d\sigma^2\right).$$
where \( a_E \) and \( a \) satisfy Eqs. (1), for \( k = -1, \epsilon = +1 \) in the \( a_E \) case, and for \( k = +1, \epsilon = -1 \), in the \( a \) case, and with \( a_E|_{u \to 0} = a|_{t \to 0} \), \( \dot{a}_E|_{u \to 0} = \dot{a}|_{t \to 0} \) from the other two Darmois conditions \([a] = [\dot{a}] = 0\).

Since these two limiting conditions, near \( u = t = 0 \), because of the radiation dominance for the \( \Lambda \text{FRW} \) universe, we obtain for the approximate instanton behavior:

\[
\mu_E(u) \simeq \mu(u), \quad a_E \simeq \frac{\mu}{a_E e^{\sqrt{\mu u_e}}},
\]

\( a_E \) standing for the value of \( a_E \) in some “instant”, \( u_e \), of the “energy” instanton phase.

5. Quantum creation of a closed \( \Lambda \text{AFRW} \) universe

We try a particular instanton model by imposing in a natural way that \( \mu_E(u) = \mu(u), \forall u \). On the other hand the instanton action \( S_E \) in evident notation is

\[
S_E = \frac{1}{2\kappa} \int R_E \sqrt{\text{g}_E} \, d^4x + \int L_E \sqrt{\text{g}_E} \, d^4x.
\]

Further, extending to instanton metrics a procedure by Hawking and Ellis [7], we obtain \( L_E = \mu \), and then

\[
\frac{1}{2\kappa} R_E + L_E = \frac{1}{2}(\mu - 3\rho_E) + 2 \frac{\Lambda}{\kappa}.
\]

For a closed FLRW model, the calculation of this integral over \( \rho \in [0, 1] \) gives a finite value. Then, in this model the creation probability becomes finite. Thus the closed \( \Lambda \text{FRW} \) universe becomes quantically creatable.

Differently, it can be seen that the open non flat and flat models are not quantically creatable, and so, as quoted above, we recover similar results by Atkatz and Pagels [5] obtained by a different method.

These results remain true for an arbitrary (out of the above unavoidable condition \( \lim_{u \to 0} \mu_E(u) = \lim_{u \to 0} \mu(u) \)) \( \mu_E \) function, provided that for \( a_E \to a_{E_m} \) we have

\[
\frac{\dot{a}_E}{a_E} \propto (a_{E_m} - a_E)^n, \quad 0 \leq n < 1,
\]

where \( a_{E_m} \) is the minimal positive value of \( a_E \) such that \( \dot{a}_E = 0 \).

6. Final considerations

The closed and flat \( \Lambda \text{FLRW} \) models have vanishing intrinsic 4-momenta [8] (in particular, have vanishing intrinsic energy), while the open non flat one has a minus infinite intrinsic energy. But, the flat \( \Lambda \text{FLRW} \) model, perturbed in the standard inflationary scenario, has an infinite intrinsic 4-momenta [9]. Thus the suggestion by Fomin [10] and Tryon [11], according to which a quantically creatable universe should have vanishing energy, becomes reinforced by our present result on the quantum creatability of the closed \( \Lambda \text{FLRW} \) model but not of the open and flat ones.

An enlarged and detailed treatment of the topic covered by the present article will be published elsewhere.

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