Semi-rigid operation of connection in timber engineering

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Abstract. Consideration the semi-rigid behaviour of connections is an essential part of the modern way of the design of timber load-bearing structures. The computational models using the finite element method allow us to easily replace the use hinge operation with the appropriate stiffnesses of connections in the design. Taking the stiffness of the joint into consideration can have a significant influence on the load distribution and on the stiffness of the object as a whole. The paper contains the calculation process of the stiffness of selected connections. The mentioned stiffnesses could be used in modelling of timber load-bearing structures.

1. Introduction

One of the important indicators in the building industry is the economic aspect. The sound design of a load-bearing structure and structural connections play an important role in wooden constructions. A high-quality load-bearing structure of a timber building consists predominantly well-designed and well-manufactured construction connections. In the engineering practice connections with the hinge effect are designed and utilized most frequently. The joint connection allows for free rotation. However, almost all connections have a certain degree of rotational stiffness. The real behaviour is somewhere in between the hinge behaviour and the rigid behaviour. The modern connections allow for semi-rigid behaviour. In the case of atypical connection where the structural elements are not connected under the right angle it is possible to use welded T-pieces made of steel plates. This paper deals with the welded T-connector that is used for connecting the adjoining beam to the main beam. The semi-rigid behaviour of connections is demonstrated based on this connection.

2. Input data

Figure 1. Structure.
Figure 2. Geometry of connection: (a) top view, (b) side view A-A’.

Table 1. Structural elements.

| Beam | Measure [mm] | Class of wood |
|------|--------------|---------------|
| A    | 200/600      | GL 24h        |
| B    | 200/360      | GL 24h        |

Secondary beam „B“ is connected to T-connector through dowels with (self perorating dowel) SFS WS 7/133 (part „1“). T-connector is connected to primary beam „A“ using screws VGS 6/100 (part „2“). Axial force is attached using notch. The aim of the calculation is the design and verification of the behaviour of the dowels and screws on shear force $V_d=25\text{kN}$. 

Table 2. Connection material.

| Type  | Diameter [mm] | Length [mm] | $F_{v,Rd}$ [kN] | $F_{v,Rd,30}$ [kN] | $F_{ax,Rd}$ [kN] |
|-------|---------------|-------------|-----------------|-------------------|------------------|
| WS    | 7             | 173         | 2.92            | 6.80              | -                |
| VGS   | 6             | 100         | 1.64            | -                 | 2.74             |

$F_{v,Rd}$ Design load-carrying capacity per shear plane per fastener.
$F_{v,Rd,30}$ Design load-carrying capacity per shear plane per fastener at angle of 30 degrees.
$F_{ax,Rd}$ Design value of axial withdrawal capacity of the fastener.

3. Rotation stiffness calculation according to [2]
To begin with, it is important to divide the connection into two parts – part 1 and part 2. Two different values of the rotational stiffness for the two parts will be obtained from the calculation. At the end, the FEM model will contain the stiffness values.
3.1 Part „1“

**Figure 3.** Distance of dowels.

Calculation of polar moment of inertia according to rotation point “X”:

\[ I_p = \sum r_i^2 \]

\[ I_p = 2\left(15^2 + 45^2 + 75^2 + 105^2 + 135^2\right) = 74,25 \times 10^3 \text{ mm}^2 \]

Calculation of middle value of slip modulus \( K_{ser} \) according to [1]:

\[ K_{ser} = \rho_{m}^{1,5} \cdot d \]

\[ K_{ser,1} = \frac{380^{1,5} \cdot 7}{23} \times 2 = 4509 \text{ Nmm}^{-1} \]

The \( K_{ser} \) is multiplied by two because this connection is of the wood + steel type. The variable \( \rho_{m} \) is the density for the GL 24h class. The variable \( d \) is the diameter of the dowel WS.

Calculation of stiffness for serviceability limit state according to [2]:

\[ C_{\phi, MSP} = K_{ser,1} \times n_{sr} \times I_p \]

\[ C_{\phi, MSP} = 4509 \times 2 \times 74,25 \times 10^3 \text{ mm}^2 = 0,67 \frac{MNm}{rad} \]

\( n_{sr} \) Number of shear planes.

Calculation of stiffness for ultimate limit state:

\[ C_{\phi, MSU} = \frac{2}{3} \times C_{\phi, MSP} \]

\[ C_{\phi, MSU} = \frac{0,67 \times 2}{3} = 0,45 \frac{MNm}{rad} \]

The verification of the calculation of stiffness through deformation of dowel:

\[ F_M = \frac{M \times r_{max}}{\sum r^2} \]

If \( M = 1\text{Nmm} \) then \( F_M = 1,818 \times 10^3 N \)
Deformation of dowel:
\[
\delta = \frac{F_M}{K_{ser,1}^*n_{sr}} = \frac{1.818 * 10^{-3}}{4509^*2} = 2.016 * 10^{-7} \text{mm}
\]

The deformation of the dowel is expressed using the angle of rotation, \( \varphi = 1,49257 * 10^{-9} \text{rad} \). 1 Nmm caused angle \( \varphi \). Using a simple calculation, we will obtain the stiffness of the connection, which is equal to \( 0,669 \frac{MNm}{rad} \). We have verified the calculation of stiffness of part „1“.  

3.2 Part „2“  
Part 2 consists of two rows of screws VGS 6/100. The T-connector is attached to the main beam using the screws.  

![Distance of screws](image)

**Figure 4.** Distance of screws.  

Calculation of polar moment of inertia according to rotation point “Y”:
\[
I_p = 2(15^2 + 35^2 + 55^2 + ... + 275^2 + 295^2) = 944.75 * 10^3 \text{mm}^2
\]

Calculation of middle value of slip modulus \( K_{ser} \) according to [1]:
\[
K_{ser} = 100 * l_{ef}  \tag{6}
\]
\[
K_{ser,2} = 100 * 100 = 10 \, 000 \frac{N}{mm}
\]

Calculation of stiffness for serviceability limit state according to [2]:
\[
C_{\varphi,\text{MS}} = K_{ser,2} * n_{sr} * I_p
\]
\[
C_{\varphi,\text{MS}} = 10000 * 1 * 944.75 * 10^3 \text{mm}^2 = 9,45 \frac{MNm}{rad}
\]
Calculation of stiffness for ultimate limit state according to (4):

\[
C_{\phi, MSU} = \frac{9.45 \times 2}{3} = 6.3 \text{ MNm rad}
\]

The verification of the calculation of stiffness through deformation of screw according to (5):

If \( M = 1 \text{Nmm} \) then \( F_M = 3.1217 \times 10^{-4} \text{ N} \)

Deformation of screw:

\[
\delta = \frac{F_M}{K_{ser,1} \times n_{sr}} = \frac{1.818 \times 10^{-3}}{10000} = 3.12 \times 10^{-8} \text{ mm}
\]

The deformation of the screw is expressed using the angle of rotation, \( \varphi = 1.0571 \times 10^{-10} \text{ rad} \). 1 Nmm caused angle \( \varphi \). Using a simple calculation, we will obtain the stiffness of the connection, which is equal to 9.45 MNm rad. We have verified the calculation of stiffness of part „2“.

**Table 3.** Rotational stiffness of connection.

| Part | \( C_{\phi,MSU} \) \([\text{MNm rad}]\) | \( C_{\phi,MSP} \) \([\text{MNm rad}]\) |
|------|---------------------------------|---------------------------------|
| „1“  | 0.45                            | 0.67                            |
| „2“  | 6.3                             | 9.45                            |

4. FEM model
To examine the connection, we will model the construction. The model will contain the calculated stiffness values of the connection from parts „1“ and „2“.

![Figure 5. FEM model.](image)

When designing a similar connection, the bending moment at the point of the connection is usually not taken into consideration. After a detailed analysis and using the semi-rigid impact we have obtained the bending moment which is equal to \( M_{max} = 3 \text{kNm} \).
5. Report

Table 4. Final forces for the most stressed dowel or screw.

| Part | $F_M$ [kN] | $F_V$ [kN] |
|------|------------|------------|
| „1“  | 5,50       | 2,50       |
| „2“  | 0,94       | 0,83       |

5.1 Examination of part „1“

\[ F_{\text{fin}} = \sqrt{F_M^2 + F_V^2} \]  
\[ F_{\text{fin}} = \sqrt{5,5^2 + 2,5^2} = 6,04 \text{kN} \]
\[ \frac{F_{\text{fin}}}{F_{V,Rd,30}} \leq 1,0 \]
\[ 0,88 < 1,0 \]

5.2 Examination of part „2“

\[ \left( \frac{F_M}{F_{ax,Rd}} \right)^2 + \left( \frac{F_V}{F_{V,Rd}} \right)^2 \leq 1,0 \]
\[ \left( \frac{0,94}{2,74} \right)^2 + \left( \frac{0,83}{1,64} \right)^2 \leq 1,0 \]
\[ 0,38 < 1,0 \]

6. Conclusion

When modelling the structure, we have utilized the real stiffness values of the connection in question. The detailed calculation better explains the behaviour of the structure connection and the overall allocation of the forces in the load-bearing structure. The results of considering the connection as a semi-rigid will be become evident not only in the area of safety but using more connection will also be more economical. Hinge behaviour is a less safe because this way doesn’t consider additional bending moment. By effect of bending moment the dowels are more stresses, and they should be designed properly.

7. References

[1] STN EN 1995-1-1 + A1: Eurocode 5 – Design of timber structures – Part 1-1: General – Common rules and rules for buildings, SUTN, 2008
[2] Schickhofer G 2006 Holzbau Nachweisführen für Konstruktionen aus Holz (Graz: Institut für Holzbau & Holztechnologie Technische Universität Graz)
[3] Porteous J and Kermani A 2007 Structural timber design to Eurocode 5 (UK: Blackwell Publishing Ltd)
Acknowledgements
This paper has been supported by the Scientific Grant Agency of the Ministry of Education, science, research and sport of the Slovak Republic and the Slovak Academy of Sciences – grant VEGA 1/0773/18.

[4] Rothoblaas catalogue, available 21.2.2019