Introduction to Sterile Neutrinos

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Abstract

Model-building issues raised by the prospect of light sterile neutrinos are discussed in a pedagogical way. I first review the naïve proposal that sterile neutrinos be identified with “right handed neutrinos”. A critical discussion of the simple expedient of adding three gauge singlet fermions to the usual minimal standard model matter content is followed by an examination of right handed neutrinos in extended theories. I introduce the terminology of “fully sterile” and “weakly sterile” to classify varieties usually conflated under the sterile neutrino banner. After introducing the concepts of “technical naturalness” and plain “naturalness”, the unbearable lightness of being a sterile neutrino is confronted. This problem is used to motivate mirror neutrinos, whose connection with pairwise maximal mixing is emphasised. Some brief remarks about phenomenology are made throughout. The impossibility of identifying the sole sterile neutrino of the currently favoured 2 + 2 and 3 + 1 phenomenological constructs as a lone gauge singlet fermion added to the minimal standard model is explained. Finally, I remark on the beauty and subtlety of light sterile neutrino cosmology.

1 Introduction

The discovery of sterile neutrinos would rank in importance no lower than the discoveries of charm, bottom and tau. The role of charm in the theory of elementary particle interactions was presaged by Glashow, Iliopoulos and Maiani (GIM): it was needed to remove tree-level flavour changing neutral currents [1]. Similarly, Kobayashi and Maskawa [2] had anticipated the need for a third generation of quarks to introduce CP violation into the standard model and hence to explain the results of Christensen, Cronin, Fitch and Turlay [3]. What of sterile neutrinos? How might these new degrees of freedom, as yet hypothetical, be fitted into particle theory? Would their existence actually explain anything?

These are some of the questions I will explore in this lecture. I am going to use a model-builder’s perspective: starting with the standard model and the gauge theory rule book, how might sterile neutrinos enter the game? Quite deliberately, I will hardly address the phenomenological evidence for sterile neutrinos, because I do not think the time is yet ripe for drawing definite conclusions. Following every phenomenological twist and turn can be more a test of nimbleness than resolve! Nevertheless, I do wish to make an observation on the currently favoured 2 + 2 and 3 + 1 scenarios [4]: the simplest standard model extension featuring just one sterile neutrino cannot accommodade the parameter space required.\footnote{This extends the material delivered in the actual lecture.}

In the next section I review the naïve proposal that sterile neutrinos be identified with “right handed neutrinos”. A critical discussion of the simple expedient of adding three gauge singlet fermions to the usual minimal standard model matter content is followed in Sec. 3 by an examination of right handed
neutrinos in extended theories. Section 4 confronts the unbearable lightness of being sterile problem: why should these apparently alien degrees of freedom inhabit the same very low mass range as the active neutrinos? Cosmology is briefly discussed in Sec. 5, and a conclusion is then presented.

2 Sterile neutrinos and the standard model

Under the standard model gauge group,
\[ G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y, \] one generation or family of quarks and leptons forms the reducible representation,
\[ Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3, 2)(1/3), \quad d_R \sim (3, 1)(-2/3), \quad u_R \sim (3, 1)(4/3); \]
\[ \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2)(-1), \quad e_R \sim (1, 1)(-2), \quad ? \text{ MISSING ENTRY } ? \]
in the minimal model. A mismatch between quark and lepton degrees of freedom is immediately evident: while the up and down quarks have both left and right chiral components, the neutrino is purely left handed. Bearing in mind that electric charge \( Q \) is given by
\[ Q = I_L + \frac{Y}{2}, \] where \( I_L = \sigma_3/2 \) in weak isospin \( SU(2)_L \) space, we see that the “missing” right handed neutrino state should be
\[ \nu_R \sim (1, 1)(0). \] It has the quantum numbers of the vacuum, and is thus sterile with respect to the standard model gauge interactions. The putative right handed neutrinos (perhaps one per family) are the most obvious sterile neutrino candidates. Strictly speaking, the term “sterile fermions” should be preferred to “right handed neutrinos”, because the latter signifies the Dirac neutrino special case. On the other hand, “right handed neutrinos” does better emphasise the intrafamilial relationship between the sterile fermions and the other quarks and leptons.

It is interesting that the chiral structure of a quark-lepton family provides strong motivation for one sterile fermion per family. The presence of \( \nu_R \)'s would enhance two aesthetic qualities: left-right similarity (for each left handed fermion there is a right handed partner) and quark-lepton similarity (for each quark of a given chirality there is an associated chiral lepton). Historically, quark-lepton similarity was used by Bjorken and Glashow and others to predict the existence of the charm quark \[ \text{[5]} \]. Notice that this aesthetic motivation for charm preceded the technical motivation supplied by GIM. Aesthetics matter! One can even upgrade these similarities into symmetries, as per left-right symmetric models \[ \text{[6]}, \] SO(10) grand unification \[ \text{[7]}, \] and discrete quark-lepton symmetric theories \[ \text{[8]} \]. There is an exception to the rule that “tidier families imply right handed neutrinos”: SU(5) grand unification using the \( 5^* \oplus 10 \) representation has none \[ \text{[9]} \].

So, let us add one right handed neutrino per family, and examine implications for neutrino mass. As everyone knows, quark and charged lepton mass generation is associated with spontaneous electroweak symmetry breakdown,
\[ SU(2)_L \otimes U(1)_Y \to U(1)_Q. \] In the standard model, this is induced by Higgs boson self-interactions that lead to a nonzero vacuum expectation value (VEV) for a Higgs doublet field \( \Phi \), where
\[ \Phi \sim (1, 2)(1) \] and
\[ \langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}. \]
Quark and charged lepton (Dirac) masses are produced through the Yukawa couplings

\[ \mathcal{L}_{\text{Yuk}} = h_d \bar{Q} d_R \Phi + h_u \bar{Q} u_R \Phi + h_e \bar{\ell} e_R \Phi + \text{H.c.} \]  

with \( m_f = h_f v \), and where \( \Phi \equiv i \sigma_2 \Phi^* \). (The \( h_f \)'s and the \( m_f \)'s are \( 3 \times 3 \) matrices in family space.)

If \( \nu_R \)'s are absent, and the Higgs sector remains minimal, then the neutrinos are massless. The individual family lepton numbers \( L_{e,\mu,\tau} \) emerge as accidental exact symmetries in this case.

But if \( \nu_R \)'s exist, then neutrinos are naturally massive. First, there are Dirac masses induced through \( \langle \Phi \rangle \):

\[ h_u \bar{\ell} L \nu_R \tilde{\Phi} \Rightarrow m_D = h_\nu v. \]  

But, the trivial gauge quantum numbers of the \( \nu_R \)'s also allow a bare Majorana mass matrix \( M \) through

\[ \mathcal{L}_{\text{Maj}} = M (\nu_R)^c \nu_R + \text{H.c.} \]  

Dirac mass mixing in general violates individual family lepton number conservation but preserves total lepton number \( L = L_e + L_\mu + L_\tau \). Majorana masses and mixings violate all of the leptonic global symmetries including \( L \), with their main phenomenological signature being neutrinoless double \( \beta \)-decay, another topic covered at this School.

So, sterile neutrinos are a natural addition to the fermionic zoo of the minimal standard model. They arguably fill a gap in the quantum number spectrum of a family, and they generally lead to nonzero neutrino masses and mixings. But, the appealing see-saw mechanism \[^{[10]}\] requires them to be very massive and hence irrelevant for present neutrino phenomenology.

What is the see-saw mechanism and why is it considered appealing? According to Eqs. 9 and 10, the full neutrino mass matrix is

\[ \left( \begin{array}{c} \nu_L \\ (\nu_R)^c \end{array} \right) \left( \begin{array}{cc} 0 & m_D \\ m_D^T & M \end{array} \right) \left( \begin{array}{c} (\nu_L)^c \\ \nu_R \end{array} \right) . \]  

Now, Dirac masses in the standard model, including those for neutrinos, are proportional to the electroweak symmetry breaking scale \( v \). So, while we do not understand the pattern of quark and charged lepton masses revealed experimentally, neutrino Dirac masses in a similar range are a natural expectation. However, \( M \) has a completely different origin in that it is not proportional to \( v \). Without further theoretical input, there can be no strong prejudice about its value.

Let us specialise to just one family for ease of exposition. The see-saw model supposes that

\[ M \gg m_D \]  

so that the eigenvalues become approximately

\[ \frac{m_D^2}{M} \text{ and } M, \]  

with eigenvectors

\[ \nu'_L \simeq \nu_L - \frac{m_D}{M} (\nu_R)^c, \quad \nu'_R \simeq \nu_R + \frac{m_D}{M} (\nu_L)^c, \]  

respectively. (Put another way, the mixing angle \( \theta \) is approximately equal to \( m_D/M \ll 1 \).) The predominantly sterile eigenstate \( \nu'_R \) is very massive.

The parameter space defined by Eq. 12 is considered appealing because then the small eigenvalue obeys

\[ \frac{m_D^2}{M} \ll m_D \sim m_{u,d,e}, \]  

\[^2\] Just as an example: A Higgs triplet coupling to the left handed lepton bilinear \( \bar{\ell}_L (\ell_L)^c \) would be required to induce tree-level neutrino masses in the absence of right handed neutrinos. The nonzero triplet VEV would also spontaneously break lepton number, and produce a Goldstone boson called the Majoron. To make the Majoron phenomenologically acceptable would then require an epicyclic construction.

\[^3\] Except indirectly by allowing the nonzero neutrino masses and mixings.
so we have a sketchy explanation for why neutrinos are much lighter than all other known fermions. Strictly, though, this argument just replaces the small-neutrino-mass mystery with the large-Majorana-\(\nu_R\)-mass mystery. We will see shortly that in many extended theories, \(M\) is proportional to a high symmetry breaking scale rather than being a bare mass. The additional theoretical assumption that there is a symmetry breaking scale much larger than the electroweak seems necessary to flesh out the see-saw paradigm. But one should acknowledge that this is an assumption, as yet empirically unsupported. One can only hope that direct experimental evidence for new very short-distance physics will eventually be produced.

The limiting case opposite to that of the see-saw is also amusing [11]. If

\[
M \ll m_D, 
\]

then the eigenvalues are approximately

\[
m_D \pm \frac{M}{2} 
\]

and the mixing angle is given by

\[
\tan 2\theta = -\frac{2m_D}{M} \Rightarrow |\theta| \simeq \frac{\pi}{4}.
\]

This is called “pseudo-Dirac structure” because the neutrino becomes fully Dirac as \(M \to 0\). The signatures for pseudo-Dirac neutrinos are:

- a nearly degenerate pair with a mass gap \(m_D\) above zero;
- nearly maximal active-sterile mixing.

Maximal mixing is certainly an interesting feature, both theoretically and phenomenologically. There is an obvious drawback, though, because in this case phenomenology requires \(m_D\) to be tiny compared to all other Dirac masses: the fermion mass hierarchy puzzle becomes even more profound. Nevertheless, since we do not understand the origin of this hierarchy, tiny neutrino Dirac masses remain a possibility. We will return to this issue in the next section.

The issue of neutrino mixing angles is just as interesting as the origin and magnitudes of neutrino masses. So far, we have uncovered a connection between light sterile neutrinos and large mixing angles in the pseudo-Dirac limit. Later we will see that the mirror matter model supplies a rationale for both light effectively sterile neutrinos and large active-sterile mixing angles. But neither the see-saw limit nor the pseudo-Dirac scenario nor the mirror matter hypothesis implies constraints on the pattern of interfamily mixing. Prior to the confirmation of the atmospheric neutrino anomaly, there was a strong theoretical prejudice in favour of small active-active mixing angles, simply because that was the observed situation in the quark sector. In recent years, this prejudice has been “revised”. Many have made the observation that the quark and lepton sectors need not be qualitatively similar, because of the presence of the Majorana mass matrix \(M\) for neutrinos. Since this is a lecture on sterile neutrinos, I will refrain from developing the active-active mixing angle story further, except to note that a direct neutrino oscillation resolution to the LSND anomaly [12] requires at least one active-active mixing angle to be small.

The see-saw and pseudo-Dirac cases are two interesting limits. But what can one say in general about the mass matrix of Eq. [11]? It is interesting that it is not an arbitrary \(6 \times 6\) symmetric matrix: the \(3 \times 3\) zero matrix in the top left block ensures that. Physical possibilities are consequently constrained.

To illustrate this, consider again just a single family. The mass matrix

\[
\begin{pmatrix}
0 & m \\
m & M
\end{pmatrix}
\]

has eigenvalues

\[
m_{\pm} = \frac{M \pm \sqrt{M^2 + 4m^2}}{2}.
\]
and the mixing angle is given by \( \tan 2\theta = -2m/M \). (The sign of the negative eigenvalue can be absorbed into the corresponding Majorana eigenfield.) The zero in Eq. \([19]\) has as an important consequence in that the three quantities “overall mass scale”, “mass difference” and “mixing angle” are not arbitrary, but satisfy a relation. Defining

\[
\begin{align*}
\Delta m^2 &\equiv m_+^2 - m_-^2 = M\sqrt{M^2 + 4m^2}, \\
\Sigma m^2 &\equiv m_+^2 + m_-^2 = M^2 + 2m^2
\end{align*}
\]  

\(\text{(21)}\)

we can write this relation as

\[
\Sigma m^2 = \frac{1}{2} \Delta m^2 (\cos 2\theta + \sec 2\theta).
\]

\(\text{(22)}\)

Within this (unrealistic one family) model, a measurement of the oscillation parameters \(\Delta m^2\) and \(\theta\) would immediately specify the absolute mass scale \(\Sigma m^2\).

A more realistic scenario is to add one gauge singlet fermion to the three standard families. This is the simplest “three active plus one sterile neutrino model” imaginable. In the absence of Higgs triplets, the most general mass matrix is

\[
\begin{pmatrix}
0 & 0 & 0 & m_1 \\
0 & 0 & 0 & m_2 \\
0 & 0 & 0 & m_3 \\
m_1 & m_2 & m_3 & M
\end{pmatrix}
\]

\(\text{(23)}\)

in the \([\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, (\nu_R)^c]\) basis. The \(m_i\) are Dirac masses while \(M\) is the \(\nu_R\) Majorana mass. It is easy to see that this matrix has two zero eigenvalues, so there are only two different \(\Delta m^2\) parameters. This is amusing, because it means that the currently favoured phenomenological fits, the 2 + 2 and 3 + 1 so-called models, \textit{cannot} be accomodated within this minimal framework. Recall that \textit{three} unrelated \(\Delta m^2\) values are required to simultaneously resolve the solar \([13, 14]\), atmospheric \([15]\) and LSND \([12]\) anomalies through oscillations. So, if you want a gauge theoretic underpinning for the aforementioned phenomenological fits, you need to increase the number of sterile flavours or introduce Higgs boson triplets or both \([16]\).

### 3 “Sterile” neutrinos beyond the standard model

The naïve minimal standard model extension discussed in the previous section sees sterile neutrinos identified with right handed neutrinos and having the gauge quantum numbers of the vacuum. In fact we can still sensibly call the resulting theory the “standard model”, though it should no longer be called the “\textit{minimal} standard model”. In this section, we will consider extensions of the standard model obtained by enlarging \(G_{SM}\). It turns out that, in most such theories, the right handed neutrinos are \textit{not} sterile with respect to the new gauge interactions.

It may be helpful to introduce some terminology. Let us define

- **“fully sterile”** to mean \textit{feels no gauge interactions of any sort, including hypothetical forces beyond those of the standard model}, and
- **“weakly sterile”** to mean \textit{does not feel standard model gauge interactions (strong, electromagnetic or left handed weak)}.

The following points should be noted:

- Right handed neutrinos in the standard model are fully sterile.
- Full sterility is defined with respect to gauge interactions only. Such species may well interact through Higgs boson exchange and of course they may partake of mass mixing. They necessarily couple via gravity.
As far as current neutrino phenomenology is concerned, fully and weakly sterile neutrinos are indistinguishable. However, there is an important theoretical difference between the two, and phenomenological differentiation will be evident in other contexts (such as the early universe).

Right handed neutrinos in extensions of the standard model are often just weakly sterile, and one can have both fully and weakly sterile states in the same theory. Some examples are:

- The “usual” left-right symmetric model (LRSM): weakly sterile.
- One can construct an “unusual” LRSM which has both weakly and fully sterile fermions.
- The mirror matter or exact parity model \[17\]: weakly sterile or both.
- The Pati-Salam model \[18\]: usually weakly sterile.
- SU(5) grand unification: fully sterile.
- SO(10) grand unification: usually weakly sterile.

I now expand on a couple of these examples.

### 3.1 Left-right symmetric models: usual incarnation

Left-right symmetric models are defined by the gauge group

\[
G_{LR} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L},
\]

with a fermionic family assigned as per

- Quarks : \( Q_L \sim (3, 2, 1)(1/3), \quad Q_R \sim (3, 1, 2)(1/3) \),
- Leptons : \( \ell_L \sim (1, 2, 1)(-1), \quad \ell_R \sim (1, 1, 2)(-1) \).

The basic motivation is to treat left and right handed fermions more symmetrically than does the standard model. Parity violation is usually induced spontaneously rather than engineered explicitly, and phenomenology of course requires the breaking scale for right handed weak isospin \( SU(2)_R \) to be suitably high (greater than a few TeV).

Different incarnations of LRSMs are defined by their Higgs sectors and additional multiplets of fermions (if any). In the standard incarnation, three copies of the multiplets in Eq. \[25\] specify the complete fermion spectrum. The left handed and right handed neutrinos reside within \( \ell_L \) and \( \ell_R \), respectively:

\[
\ell_{L,R} = \left( \begin{array}{c} \nu_{L,R} \\ e_{L,R} \end{array} \right).
\]

Rather than being fully sterile, the \( \nu_R \)'s now participate in right handed weak interactions, mediated by exotic \( W \)-like bosons and an additional neutral gauge boson \( Z' \). The right handed neutrinos are weakly sterile in the usual LRSM.

The usual LRSM is completed by specifying the Higgs sector, which is constructed to yield a see-saw structure for neutrinos while at the same time spontaneously breaking \( G_{LR} \) in two stages:

\[
G_{LR} \to G_{SM} \to SU(3)_c \otimes U(1)_{Q}.
\]

The Higgs multiplets are

\[
\Phi \sim (1, 2, 2)(0), \quad \Delta_L \sim (1, 3, 1)(2), \quad \Delta_R \sim (1, 1, 3)(2),
\]

which participate in the Yukawa couplings

\[
\mathcal{L}_{\text{Yuk}} = h_Q Q_L Q_R \Phi + h'_{Q} \bar{Q}_L Q_R \Phi^c + h_{\ell} \ell_L \ell_R \Phi + h'_{\ell} \bar{\ell}_L \ell_R \Phi^c + \lambda_{\nu}(\ell_L)^c \ell_L \Delta_L + \lambda_{\nu}(\ell_R)^c \ell_R \Delta_R + \mathcal{H} c.
\]
(A left-right or parity discrete symmetry is also usually imposed.) The required VEV hierarchy

\[ \langle \Delta_R \rangle \sim v_R \gg \langle \Phi \rangle \sim v_{\text{ew}} \gg \langle \Delta_L \rangle \sim v_L \]  

is arranged by a suitable choice of Higgs potential parameters. At the scale \( v_R \), right handed weak isospin is spontaneously broken and the \( W_R \) and the \( Z' \) bosons acquire large masses. The most interesting point for our discussion is that the same VEV generates large Majorana masses, \( \lambda R v_R \), which immediately connects the see-saw limit with an \( a \ priori \) separate physical phenomenon: the spontaneous breakdown of an enlarged gauge group. There are in general two electroweak scale VEVs within \( \langle \Phi \rangle \), through which all Dirac masses are induced via the \( h \)- and \( h' \)-terms in the Yukawa Lagrangian. The left handed triplet VEV must be very small for the achievement of the see-saw limit, since the Majorana mass matrix for left handed neutrinos is proportional to it (there is also a phenomenological constraint from the electroweak \( \rho \)-parameter).

The LRSM sketched above is an example of a standard model extension that can incorporate the see-saw mechanism. The generic lesson is that the large right handed neutrino Majorana masses required for this mechanism can often be correlated with a high symmetry breaking scale for a non-standard gauge interaction that couples to the now just weakly sterile \( \nu_R \)'s. This is why one often hears the claim that neutrino oscillation phenomenology is a “window” into high-energy-scale physics. It may be or it may not be. The see-saw idea is attractive, but it can hardly be considered as established. It is interesting to ponder how enough experimental information could ever be gathered to establish such a scenario beyond reasonable doubt. The difficulty of this is just an example of the general problem of testing theories that postulate new physics at very high energy scales. In any case, we will obviously have to explore other paths in our search for a decent theory of light sterile neutrinos.

### 3.2 A model with both weakly and fully sterile neutrinos

Just for amusement, let us construct a scenario featuring both weakly and fully sterile fermions. We will adopt the gauge group of the LRSM, but choose a different fermion and Higgs boson content. In addition to the quarks, we have

\[
\begin{align*}
\text{Leptons:} & \quad \ell_L \sim (1, 2, 1)(-1), \quad \ell_R \sim (1, 1, 2)(-1); \\
\text{Sterile fermion:} & \quad S_L \sim (1, 1, 1)(0); \\
\text{Higgs bosons:} & \quad \Phi \sim (1, 2, 2)(0), \quad \chi \sim (1, 1, 2)(1)
\end{align*}
\]

We have not imposed the \( L \leftrightarrow R \) discrete symmetry. The neutrino Yukawa and bare mass terms can be assembled into

\[
\begin{pmatrix}
\nu_L & (\nu_R)^c & S_L
\end{pmatrix}
\begin{pmatrix}
0 & \Phi & 0 \\
\Phi & 0 & \chi \\
0 & \chi & M_S
\end{pmatrix}
\begin{pmatrix}
(\nu_L)^c \\
\nu_R \\
(S_L)^c
\end{pmatrix},
\]

where I am being schematic rather than technically accurate. The \( \nu_R \) is weakly sterile, whereas \( S_L \) is fully sterile.

A VEV for Higgs multiplet \( \chi \) is required to break right handed weak isospin at a high scale, while \( \langle \Phi \rangle \) sets the electroweak scale. The fully sterile fermion has a bare Majorana mass \( M_S \). For \( M_S \ll \langle \Phi \rangle \ll \langle \chi \rangle \), the lightest eigenstate is a Majorana fermion of mass \( M_S \langle \Phi \rangle^2/\langle \chi \rangle^2 \) which is predominantly \( \nu_L \). The other two eigenstates (weakly and fully sterile) are of order \( \langle \chi \rangle \) and thus very massive.

This scenario illustrates that there can in principle be a “hierarchy of sterility” for neutrino-like particles, but in the above model neither of the sterile states is light.

### 3.3 The story so far

Let us pause to summarise what we have deduced so far:

- There are varieties of “sterile” neutrino, grouped into the broad categories of weakly and fully sterile.
Their existence can be very well motivated by quark-lepton and left-right similarity (or symmetry, if you want to go that far).

The pseudo-Dirac limit hints at a connection between large mixing angles and sterile neutrinos.

The main issue is whether they are expected to be heavy or light. The see-saw mechanism favours heavy sterile neutrinos. Such particles are usually called “heavy neutral leptons”, and while they may have an important role to play in cosmological baryogenesis (a topic beyond the scope of this lecture), they play no direct role in neutrino oscillation phenomenology.

The model obtained by adding a single sterile fermion to the minimal standard model cannot serve as a gauge theoretic underpinning for the presently favoured 2 + 2 and 3 + 1 phenomenological fits.

4 The unbearable lightness of being sterile

Let us now confront what has emerged as a core issue: can one theoretically justify light sterile neutrinos?

4.1 Naturalness and technical naturalness

Clearly, one can just add gauge singlet fermions with arbitrary masses to any model, so what is the problem? It is considered a question of naturalness rather than mere possibility. (And one has to worry about how gauge singlet fermions couple to the other degrees of freedom.)

“Naturalness” is an aesthetic concept, and physicists can have different opinions about what it means. “Technical naturalness” is a precise mathematical criterion, which is usually weaker than naturalness per se. A parameter choice is technically natural if its adoption increases the symmetry of the theory. (Parameter choice means either a special numerical value, or a special relationship between parameters.) Increased symmetry ensures that the special parameter choice is not altered by radiative corrections. Actually, a slightly weaker statement is more pertinent: Consider a parameter $\lambda$, and suppose that for $\lambda = \lambda_0$, the symmetry of the theory is increased. Radiative corrections cannot move $\lambda$ from having the value $\lambda_0$. But then one also deduces that values of $\lambda$ in the neighbourhood of $\lambda_0$ enjoy a kind of stability, because radiative corrections to the parameter must be proportional to the difference $\lambda - \lambda_0$, which by hypothesis has a small value. Since the radiative corrections are thus also small, points in the neighbourhood of $\lambda_0$ never move out of that regime. Such a parameter choice is also termed technically natural.

Well, taking the mass of a sterile neutrino to zero is a special parameter choice. If technical naturalness holds, then massless sterile neutrinos remain massless to all orders.

For definiteness, consider the standard model with right handed neutrinos added. Taking the $\nu_R$ Majorana mass to zero increases the symmetry of the theory, because total lepton number conservation then results. So, having Dirac rather than Majorana neutrinos is technically natural. But is it natural? What do you think? Many would say “No!” because it is nicer to include all renormalisable and gauge invariant terms in the Lagrangian. In response, one might add small Majorana masses. Then, technically, they will remain small to all orders in perturbation theory. This remark is obviously relevant for the pseudo-Dirac option.

Taking in addition the neutrino Dirac mass $m_D$ to zero increases the symmetry further, because the chiral transformation,

$$\nu_R \rightarrow e^{i\alpha} \nu_R,$$

(33)

with every other field just going into itself, is now an invariance. Indeed, this is just an instance of the well known fact that zero fermion masses go hand-in-hand with increased chiral symmetry.

Thus we conclude that having light sterile (and active!) neutrinos is technically natural. But is it natural, is it nice? Again, many would say “No!” typical opinions being:

4Actually, one can question if the concept of technical naturalness is truly meaningful. The point is that it presupposes a perturbative analysis: One chooses a special parameter choice at tree-level. One calculates 1-loop corrections and asks...
• Small Majorana masses are not nice, because in many extensions of the standard model they are proportional to a higher symmetry breaking scale (as we have seen). (Such an opinion contains the implicit assumption that additional symmetry breaking scales are desirable and/or are likely to exist.)

• Small Dirac masses are not nice, because they are proportional to the electroweak VEV, so you would need an extremely small Yukawa coupling constant.

In response to such criticisms, one can try to invent models that purport to explain why Majorana and Dirac masses should be so small. This is either a model-building challenge, or an epicyclic indulgence, depending on your point of view.

4.2 Mirror neutrinos

There is no known model for fully sterile light neutrinos that is generally accepted as being “nice”. I should comment that the right handed neutrino identification is not the only possible one, especially in supersymmetric theories which abound with states such as axinos and modulinos that are sterile “neutrino” candidates. However, I will concentrate on a completely different possibility: mirror neutrinos.

The mirror matter or exact parity model is essentially the standard model squared [17]. The gauge group is

\[ G = G_{\text{SM}} \otimes G'_{\text{SM}}, \]  

where both factors are isomorphic to SU(3)\otimes SU(2)\otimes U(1). Ordinary particles transform non-trivially under the first factor and are singlets under the second. This means that ordinary particles interact amongst themselves in the standard way. The extension comes from postulating a new sector called “mirror matter”. In addition to the gauge bosons of \( G'_{\text{SM}} \), new fermions and Higgs bosons are added which transform non-trivially under the second factor but trivially under the first. If a given ordinary fermion \( f_L \) transforms as \( (R, 1) \), then its mirror partner \( f'_R \) transforms as \( (1, R) \). Furthermore a discrete non-standard parity symmetry,

\[ f_L \leftrightarrow f'_R, \]  

is imposed on the Lagrangian. Mirror particles interact amongst themselves through \( G'_{\text{SM}} \) gauge forces that have the same form and strength as their ordinary counterparts. The only difference is that mirror weak interactions are right handed, to offset the left handed nature of the ordinary weak interactions. Our original motivation for this scheme was to demonstrate that nature could be invariant under improper Lorentz transformations despite the left handed nature of the weak force. (We found out later that Lee and Yang had sketched the mirror matter idea in their famous paper proposing parity violation [19]!)

The mirror matter model is interesting for neutrino physics because the mirror neutrinos are, first of all, sterile with respect to ordinary weak interactions and, secondly, guaranteed to be light. Mirror neutrinos are weakly sterile. Let us have a look at this in more detail. Under \( G_{\text{SM}} \otimes G'_{\text{SM}} \) the ordinary lepton doublets transform as per

\[ \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim [ (1, 2)(-1) ; (1, 1)(0) ], \]  

using a slightly cumbersome but obvious notation, whereas the mirror lepton doublet behaves according to

\[ \ell'_R = \begin{pmatrix} \nu'_R \\ e'_R \end{pmatrix} \sim [ (1, 1)(0) ; (1, 2)(-1) ]. \]  

if the special choice still holds. It will if the choice is associated with an increase in symmetry, be it exact or approximate. Otherwise it probably will not hold in general, and one may then have to fine-tune to maintain the special value after 1-loop renormalisation. One then calculates 2-loop corrections, and repeats the process. And so on. Technically natural choices display perturbative stability, but why should stability with respect to a certain approximation scheme be so fundamentally important? This is an interesting question in light of the usual motivation for supersymmetry as well. Even if one accepts this criticism, parameter space regions near points of enhanced symmetry are still mathematically special compared to generic regions.
The $\nu_R$ fields are the mirror neutrinos. Notice that they are completely different states from what we have been calling “right handed neutrinos”. Indeed, the minimal mirror matter model does not contain right handed neutrinos (nor their parity partners, the left handed mirror neutrinos); both ordinary and mirror neutrinos are then exactly massless. Why are mirror neutrinos massless? For exactly the same reasons that ordinary neutrinos are massless: absence of the “missing” singlet fermion per (ordinary or mirror) family, and absence of Higgs triplets. The discrete parity symmetry ensures that the physics of the ordinary sector is replicated by the mirror sector.

The fact that mirror neutrinos are weakly sterile and massless in the minimal mirror matter model provides a good starting point for developing our coveted theory for light, effectively sterile neutrinos. All we need to do is extend the standard sector so that ordinary neutrinos get tiny masses. Whatever mechanism we use to achieve this (e.g. see-saw) will operate analogously in the mirror sector! If ordinary neutrinos are light, then so also will be the mirror neutrinos. In fact, we do not have to understand exactly why ordinary neutrinos are light in order to conclude that mirror neutrinos must also be light! Whatever the reason, it will have its mirror analogue.

Let us briefly discuss the see-saw route by way of example. In addition to the ordinary and mirror lepton doublets, we add a singlet fermion to each family. As mentioned above, the singlet added to an ordinary family is just a right handed neutrino $\nu_R$, whereas the mirror version is called a left handed mirror neutrino $\nu'_L$. We now write down all renormalisable Yukawa coupling and bare mass terms consistent with the gauge symmetry $G_{SM} \otimes G'_{SM}$, and we take the see-saw limit. Switch off inter-family mixing for simplicity. It is easy to see that the light eigenstate sector then consists of two states per family, which are maximal mixtures of ordinary and mirror neutrinos:

$$\nu_\pm = \frac{\nu \pm \nu'}{\sqrt{2}}. \tag{38}$$

The state $\nu$ is mostly the $\nu_L$, while the mirror state $\nu'$ is mostly the antiparticle of $\nu'_R$. (This is the left handed mirror antineutrino. In the literature, it is usually called a mirror neutrino for simplicity, even though strictly speaking it is an antiparticle.) The masses $m_{\pm}$ are arbitrary, except for the qualitative constraint that they are both small due to the see-saw. The appearance of pairwise ordinary-mirror maximal mixing is reminiscent of the pseudo-Dirac option discussed earlier. In this case, however, maximal mixing is enforced by the exact discrete parity symmetry (and the lightness of the effectively sterile state has an elegant theoretical explanation). Recall from basic quantum mechanics that symmetry eigenstates must also be Hamiltonian eigenstates. The maximal linear combinations $\nu_\pm$ are exactly the even and odd parity eigenstates, respectively. The mixing between ordinary and mirror neutrinos arises from some of the Lagrangian terms containing the singlet species, for example

$$\bar{\ell}_L(\nu'_L)^c \phi' + \bar{\nu}'_R(\nu_R)^c \phi', \tag{39}$$

plus other mixed terms involving the singlets only, where $\phi'$ is the mirror Higgs doublet. The result in Eq. 38 is model independent though: every neutrino mass model must produce that result, provided that any additional neutrino-like states are made sufficiently massive. Notice that the strength of the ordinary-mirror neutrino mixing is governed by the mass splitting $m_+ - m_-$, since the mixing angle is constrained to be maximal. In terms of neutrino phenomenology, this means that the wavelength is a free parameter but the amplitude is not.

Maximal mixing is interesting because of both the atmospheric and solar neutrino problems. Many non-trivial experimental results are consistent with maximal $\nu_\mu \leftrightarrow \nu'_\mu$ and maximal $\nu_e \leftrightarrow \nu'_e$ oscillations. But, the SuperKamiokande collaboration claim that $\nu_\mu \to \nu_\tau$ is preferred over the sterile channel by the atmospheric neutrino data, and the combined SuperKamiokande and SNO data [13, 14] suggest a $3\sigma$ preference for $\nu_e \to \nu_{\mu,\tau}$ over $\nu_e \to \nu_\tau$. This is disappointing from the mirror neutrino perspective, and it will be interesting to see if future data will support these initial findings.

In closing the mirror neutrino discussion, let us compare the effective $2 \times 2$ mass matrix for the light ordinary plus mirror one-family situation with the alternative $\nu_L$ plus $\nu_R$ scenario of Eq. 19. For the mirror case, the analogous matrix must be of the form

$$\begin{pmatrix}
m_1 & m_2 \\
m_2 & m_3
\end{pmatrix} \tag{40}$$
due to the discrete symmetry. As before, there are just two parameters to describe three quantities: mass splitting, overall mass scale and mixing angle. In this case, the mixing angle is uniquely singled out as the constrained parameter, since it must be $\pi/4$, whereas the previous case resulted in an algebraic relation, Eq. [22], involving all three of the quantities. Mirror neutrinos are in general distinguishable from right-handed neutrinos.

5 Cosmology

Light sterile neutrinos can have important cosmological implications. The most dramatic possible effect concerns big bang nucleosynthesis (BBN), the processes thought to be responsible for generating the light isotopes $^4\text{He}$, $^3\text{He}$, D and $^7\text{Li}$. This is a rather complicated topic, that I can only summarise here.

The BBN epoch occurs shortly after neutrinos thermally decouple from the $e^\pm/\gamma$ plasma at about $T \simeq 1 \text{ MeV}$, where $T$ is temperature. The plasma contains some nucleonic contamination, with neutrons and protons being interconverted through the processes

$$\nu_e n \leftrightarrow e^- p, \quad \bar{\nu}_e p \leftrightarrow e^+ n, \quad n \leftrightarrow p e^- \bar{\nu}_e.$$ 

(41)

These contaminants form the raw material for nucleosynthesis. The most abundantly produced isotopes are H (just unsynthesised protons) and $^4\text{He}$, the latter being a tightly bound nucleus. The relative abundance of neutrons to protons essentially determines the relative yield of $^4\text{He}$ to H, since almost all of the neutrons eventually get incorporated into $^4\text{He}$. This ratio is determined by the relative rates of the reactions in Eq. [41] as well as by the expansion rate of the universe during the relevant period. Light sterile neutrinos can alter the course of BBN via both of these avenues.

The expansion of the universe during BBN is determined by the relativistic component of the plasma. In standard BBN, the relativistic species are the three active neutrinos and antineutrinos, electrons and positrons, and photons. A significant light sterile neutrino component would increase the expansion rate relative to the standard value. This would bring forward “weak freeze out”, the time when the reactions of Eq. [41] cease maintaining the $n/p$ ratio at its equilibrium value, which for the zero chemical potential case is $\exp[(m_p - m_n)/T]$. If weak freeze-out occurs earlier, then $T$ is larger, and hence $n/p$ is also larger. This can increase the $^4\text{He}$ yield unacceptably.

An interesting set of unknown cosmological parameters are the neutrino-antineutrino number density asymmetries for each flavour. In standard BBN, these are put for simplicity to zero. However, the relic neutrino background has never been detected, so we have no direct empirical justification for this simplifying assumption. It turns out that active-sterile neutrino and antineutrino oscillations can induce large asymmetries in the active flavours [20]. Two important consequences flow from this. First, an asymmetry in the $e$-like neutrino flavour will directly affect the rates for Eq. [41], and thus also $n/p$ [21]. Second, neutrino asymmetries generate effective Wolfenstein potentials [22] for active-sterile neutrino oscillation modes, and hence act to suppress them. This is important, because oscillations into sterile species can populate the plasma with an extra relativistic component.

So, what can sterile neutrino cosmology look like? The physics is quite interesting and some aspects of it are even subtle. Relevant issues are:

- **Sterile neutrino decoupling temperature.** At some high temperature, sterile neutrinos decouple from the rest of the plasma. The temperature is high, because by definition sterile neutrinos interact very weakly with all other particles. Exactly how high depends on the precise model, on what other interactions the “sterile” neutrinos feel, on whether they are fully or weakly sterile. (Note that the mirror matter model is a case unto itself, because of the self-interactions within the mirror sector [23].)

- **Dilution of sterile component.** As the universe cools, species mutually annihilate and reheat the main component of the plasma. Since the sterile neutrinos have decoupled, they do not get reheated and their number density becomes negligible compared to the reheated species.

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5 And in the sterile flavour(s).

6 In equilibrium, neutrino asymmetries are synonymous with nonzero neutrino chemical potentials.
• **Repopulation through oscillations?** However, the sterile species can make a “comeback” through active-sterile neutrino oscillations. For an interesting range of oscillation parameters, this becomes an issue in the epoch immediately prior to BBN. For some parameter ranges, the sterile neutrinos will be repopulated in the plasma, entailing a higher expansion rate during BBN and thus potentially leading to $^4\text{He}$ overproduction.

• **Large neutrino asymmetries?** However, if large enough neutrino asymmetries exist in the plasma, then the active-sterile oscillation modes will be suppressed [24]. Acceptable cosmology despite the existence of light sterile neutrinos can result.

• **Oscillation generated asymmetries.** Remarkably, large neutrino asymmetries will be generated by the active-sterile oscillations themselves, provided the oscillation parameters are in the correct regime [20]! Depending on the model and the parameters, different asymmetry values can be produced for the different active flavours, including the $e$-like flavour which affects the $n \leftrightarrow p$ reactions directly.

Cosmology with light sterile neutrinos requires careful analysis, with the outcome depending on the model and on the actual values of the parameters. Successful cosmologies can result.

### 6 Conclusion

“Sterile neutrino” is a class of fermions whose place in nature has yet to be finalised. While acknowledging that an oscillation resolution for the combined atmospheric, solar and LSND problems requires at least one light sterile neutrino, I have taken my cues from theory rather than phenomenology, describing some model building perspectives on how such states may fit into the standard model or extensions thereof. A particular concern was developing theoretical frameworks for how such particles could have tiny masses.

We have drawn the following conclusions:

• The “missing entry” in the minimal standard model family is the right handed neutrino. It is an obvious sterile neutrino candidate. Its existence would in a sense balance out each family, by enhancing both left-right and quark-lepton similarity. Three sterile neutrino flavours would be implied.

• However, there is no really compelling reason to give such particles very small masses. In fact, the see-saw mechanism for producing light active neutrinos proposes that right handed neutrinos are very massive Majorana fermions. Such species may play an important role in cosmology, but would not be directly relevant for neutrino oscillation phenomenology. The Devil’s Advocate points out, though, that assigning $\nu_R$’s small masses is a technically natural procedure.

• The right handed neutrino paradigm in the absence of Higgs triplets produces a restricted Majorana mass matrix. Not all phenomenological parameter regimes are possible, and in particular, the currently favoured $2+2$ and $3+1$ phenomenological fits cannot be accommodated by the minimal standard model augmented by a single sterile fermion.

• The opposite limit to the see-saw produces pseudo-Dirac neutrinos, featuring maximal active-sterile mixing. However, there is no good understanding for why such states should be light, despite having noted that technical naturalness is satisfied.

• One has to be careful about the meaning of “sterile”. The strict definition requires sterility with respect to the standard interactions, but leaves open the possibility of nonzero influence under hypothetical very weak forces such as right handed weak interactions. The terminology “fully sterile” and “weakly sterile” was introduced to deal with this.

• Mirror neutrinos are a radical (?) reinterpretation of the theoretical origin of sterile neutrinos. The lightness of being problem is solved, and pairwise ordinary-mirror mixing is predicted.
• Cosmology with sterile neutrinos is subtle, interesting and complicated, with acceptable cosmological outcomes quite possible.

So, let us return to the questions posed in the first paragraph. What of sterile neutrinos? How might these new degrees of freedom, as yet hypothetical, be fitted into particle theory? Would their existence actually explain anything? We have seen that they might signal a completion of a quark-lepton family. On the other hand, they might be the first indication for a mirror sector. Pairwise active-sterile maximal mixing can be explained by either the pseudo-Dirac configuration or the mirror matter hypothesis. Current phenomenological indications might lead one to be pessimistic about this elegant explanation for large mixing, but perhaps all hope is not yet lost.

In any case, the most urgent task is to perform new experiments so as to either establish the existence of light sterile neutrinos, or to make them clearly irrelevant. The discovery of light sterile neutrinos would be epochal, simply because they would be genuinely new degrees of freedom. If the mirror matter idea is correct, then they would also be the tip of the iceberg. In addition, light sterile neutrinos would make early universe cosmology very interesting indeed. Here’s hoping that in the future these particles enrich real physics and not just our imaginations.

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