Vibration Response Analysis of a Structural Metal Mild Steel under the Effect of Crack

N Ahmed¹, M Mamun² and A Ahmed³

¹ National Nanotechnology Research Centre, Bilkent University, Ankara, Turkey
² Department of Mechanical Engineering, Khulna University of Engineering & Technology, Khulna- 9203, Bangladesh
³ Department of Mechanical Engineering, Bangladesh University of Engineering & Technology, Dhaka, Bangladesh

E-mail: mamun1605099@stud.kuet.ac.bd

Abstract. When a crack is present in a structure then the probability of failure of that structure is higher. Failure occurs when the natural frequency of the periodic force and the natural frequency of the structure are in a state of superposition. To avoid this phenomenon, it is essential to calculate the natural frequency. The aim of this study is to analyze the behavior of natural frequency for different parameters such as crack position, crack depth, crack opening and mesh sensitivity. In this study, the first five modes of vibration of both cracked and un-cracked I beam has been extracted. A further analysis of resonance for a fixed vertical load has also been done. An amalgamation of hexahedral and wedge elements has been used to study the behavior of the cracked structure. As this is a problem of discontinuity, thus it is very troublesome to make a solution analytically. Therefore, ‘ABAQUS CAE’, one of the most popular finite element analysis software, was used for the analysis of cracked structure. The discoveries of this examination are that the presence of crack decreases the natural frequency of the beam, this decrease is identified with the position of the crack, its opening and its depth. Another finding of this examination is that the impact of vibration is huge for I beam when the break arrives at its web segment. It was also discovered that resonance happens before if the crack profundity is bigger or the existence of the crack is nearer to the fixed end.

Keywords: Natural Frequency, Resonance, Finite Element Method, Combined Meshing.

1. Introduction

Dynamics is considered as one of the major and important sectors of designed based mechanical engineering studies. Vibration possesses a large portion of this study. In order to make sure that a structure is safe, it is important to make a vibration analysis of that structure. In doing so, Natural frequency and resonance are the two most significant parameters which controls the safety of a structure. When a periodic load is applied, it is very common to show the tendency to vibrate. This vibration depends on the amplitude, frequency and other factors of the load. It is significant to notice that the existence of a crack in the beam changes its response in comparison with an un-cracked beam. When the amplitude of natural frequency of a beam and the periodic load applied on the beam are same, resonance takes place. This resonance increases the possibility of a beam failure. Therefore, to prevent the collapse of any structure because of resonance, it is necessary to make a vibration analysis of that structure. In any mechanical structure, the usage of a I beam is very common. This I beam can be continuous or cantilever in nature. In any structure, the usage of I beam is very banal, thus it is important to know about different natural frequencies of a I beam. This knowledge will allow to avoid such frequencies while applying periodic loads on it. A number of researches has been done on both numerical and experimental study of vibration analysis. For the change in various boundaries like crack location, crack opening, crack profundity and mesh sensitivity, the adjustment in natural frequency was
introduced by Sumon, Shahidul, Ghosh [1]. A parametric investigation of break profundity proportion and break area was also introduced. Regarding mesh sensitivity, there was not any verification done. The number of elements used was also not mentioned. Analysis on the resonance frequency was also absent. For this study, they considered the first three natural frequencies. Nitesh and Vaibhab [2] additionally introduced the adjustment in natural frequency for various crack depth, location and opening utilizing ANSYS Workbench programming. They also presented the parametric study for different crack location. A proposal was made for finding the crack location based on frequency contours. But there was no parting used in meshing and no study about resonance frequency for a load was present in the paper. Ostachowicz and Krawczuk [3] used a strategy to decide the impact of two breaks on natural frequency for transverse vibration of cantilever beam. But no analysis was present regarding various parameters in the paper. A method of determining the crack location using natural frequencies was presented by Rane, Barijbhe, patil [4]. They used Euler equation and solved it numerically for both cracked and un-cracked beam and the first three natural frequencies were calculated. Quila, Mondal and Sarkar [5] concentrated on the theoretical study of transverse vibration of a beam whose movement was restricted and then simulated the results through ANSYS. They also performed a parametric study. But there was no mentioning of mesh element type or size. Behzad, Meghdari and Ebrahimi[6] developed equations of motions and boundary conditions for a bent beam with open edged crack. By utilizing the recently evolved model related to the galerkin projection strategy, the normal frequencies of beam had been determined. It was seen from the outcome that the natural frequency lessens as the crack depth increments. In any case, no investigation was finished with respect to the recurrence proportion for different crack depth. In this study, the first five natural frequency of a I beam have been calculated. The mode shapes associated with transverse, lateral and torsional vibration was analysed. The change in the value of frequency with response to various crack depth, meshing element, crack opening and crack location has been studied. A further analysis of resonance for different crack depth and crack location was also done. In short the impact of crack on natural frequency and resonance condition has been thoroughly studied in this paper.

2. Methodology

2.1. Geometry

There can be various types of cantilever beam depending on the geometry of the cross section. Here in this study a I shaped cross section has been used for designing the cantilever beam. For I shaped beam the dimensions were the same as the dimensions of a HE 1000×584 steel beam. The boundary condition used in ABAQUS analysis is encastre (U1=U2=UR1=UR2=UR3=0) meaning that it cannot move and rotate in x, y, z direction at the fixed end.

| Property         | Value    |
|------------------|----------|
| Flange thickness | 0.064 m  |
| Web thickness    | 0.0356 m |
| Depth            | 1.056 m  |
| Width            | 0.314 m  |
| Fillet radius    | 0 m      |
| Length           | 3.2 m    |
2.2. Crack Design
A wedge shaped crack was used to analyse the behaviour of vibration under cracks. The cracks were drawn in SolidWorks 2017 and then the files were imported and used into ABAQUS CAE. Three different openings of 0.002m, 0.004m, and 0.010m were used. Cracks were placed at 0.7 m distance interval from the free end. Here crack opening of 0.002, 0.004 m, 0.006 m and 0.010 m were used and crack depth of 0.025m, 0.0375m , 0.050m, 0.0625m and 0.075m were used.

2.3. Material Selection
For the examination, mild steel was used as a material because they are very commonplace in terms of usability in structural elements.

| Property                  | Value     |
|---------------------------|-----------|
| Modulus of elasticity     | 210 GPa   |
| Mass density              | 7860 kg/m³ |
| Poisson ratio             | 0.3       |

2.4. Governing Equation
Characteristic recurrence is the recurrence at which a framework or structure vibrates when exposed to an underlying excitation without any driving or damping power. To decide characteristic recurrence, in this manner free un-damped vibration is considered. For any split structure examination, the investigation of reverberation is imperative as it influences the structure in number of ways. At the point when the recurrence of connected burden ends up equivalent to related regular recurrence, the structure vibrates hypothetically at limitless adequacy prompting disappointment. To be alert about auxiliary disappointment because of intermittent burden, in this manner it is vital to decide full recurrence. The free bending vibration of an Euler-Bernoulli beam of a constant rectangular cross section is given by the following differential equation as given in[2]:

\[ EI \frac{d^4x}{dy^4} - m \omega^2 y = 0 \]

Where “m” is the mass of the beam per unit length (kg/m) “\( \omega_i \)” is the natural frequency of the i’th mode (rad/s), E is the modulus of elasticity (N/m²) and I is the moment of inertia (m⁴). By defining \( \theta^4 = \frac{m\omega^4}{EI} \) equation is rearranged as a fourth order differential equation as follows[2]:
\[
\frac{d^4 y}{dx^4} - \theta^4 y = 0 \quad \cdots \quad (2)
\]

The general solution to the equation is:
\[
y = A \cos \theta_i x + B \sin \theta_i x + C \cosh \theta_i x + D \sinh \theta_i x \quad \cdots \quad (3)
\]

Where A, B, C, D are constants and "\( \theta_i \)" is a recurrence parameter. Receiving Hermitian shape works, the firmness network of the two noded shaft component without a break is gotten utilizing the standard incorporation dependent on the variety in flexural inflexibility.

The element stiffness matrix of the un-cracked beam is given as [2]:
\[
\begin{bmatrix} K_e \end{bmatrix} = \int [B(x)]^T EI [B(x)] dx \quad \cdots \quad (4)
\]

Where the Hermitian shape functions defined as:
\[
\begin{align*}
H_1(x) &= 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad \cdots \quad (5) \\
H_2(x) &= x - \frac{2x^2}{l} + \frac{x^3}{l^2} \quad \cdots \quad (6) \\
H_3(x) &= \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \quad \cdots \quad (7) \\
H_4(x) &= -\frac{2x^2}{l} + \frac{x^3}{l^2} \quad \cdots \quad (8)
\end{align*}
\]

the consistent mass matrix of the beam element can be formulated directly as [2]:
\[
\begin{bmatrix} M_e \end{bmatrix} = \int_0^1 \rho A [H(x)]^T [H(x)] dx \quad \cdots \quad (9)
\]

The natural frequency then can be calculated from the relation [2]:
\[
[\omega^2 [M] + [K]] [q] = 0 \quad \cdots \quad (10)
\]

Where: q=displacement vector of the beam

The theoretical formula for natural frequency can be obtained by Euler’s Beam theory as [17],
\[
f_n = \frac{\beta_n^2}{2\pi l^2} \sqrt{\frac{E I}{\rho A}} \quad \cdots \quad (11)
\]

Where, E= modulus of rigidity
\[\rho=\text{mass density}\]
\[L=\text{length of the beam}\]
\[A=\text{cross section area of the beam}\]
\[\beta_n^2=1.875 \text{ for the first mode, } 4.69 \text{ for the second mode, } 7.85 \text{ for the third mode}\]

2.5. Validation

The natural frequency analysis of a cantilever beam is validated by the natural frequency solution of M.S. Mia, M.S. Islam and U. Ghosh [1].

| Crack depth (m) | Mode 1 Present data (cycle/sec) | Reference Paper (cycle/sec) | Error (%) | Mode 1 Present data (cycle/sec) | Reference Paper (cycle/sec) | Error (%) | Mode 1 Present data (cycle/sec) | Reference Paper (cycle/sec) | Error (%) |
|----------------|-------------------------------|-------------------------------|-----------|-------------------------------|-------------------------------|-----------|-------------------------------|-------------------------------|-----------|
| 0.05           | 18.142                        | 18.2                         | 0.31      | 112.48                        | 112.75                       | 0.23      | 301.33                        | 302.36                       | 0.34      |
| 0.075          | 17.566                        | 17.733                       | 0.976     | 110.58                        | 111.3                        | 0.64      | 290.92                        | 293.74                       | 0.96      |
| 0.010          | 16.884                        | 16.936                       | 0.30      | 108.4                         | 108.77                       | 0.34      | 279.92                        | 280.81                       | 0.31      |
| 0.125          | 15.473                        | 15.471                       | 0.012     | 104.39                        | 104.66                       | 0.25      | 261.05                        | 261.18                       | 0.049     |
| 0.15           | 13.145                        | 13.21                        | 0.49      | 99.470                        | 99.175                       | 0.29      | 240.84                        | 238.64                       | 0.91      |
From the analysis it can be seen that the values of natural frequencies almost coincide. For a crack depth of 0.050 m the 1st natural frequency of the current study is 18.142 cycle/sec and the natural frequency calculated by M.S. Mia, M.S. Islam and U. Ghosh is 18.2 cycle/sec. So the difference between the two values here is only 0.3%. The highest deviation between two results is 0.976%.

3. Results and Discussion

This section is dedicated to compare and discuss the result for various parameters.

3.1. Natural frequency analysis based on Mesh elements

Without partition, I beam could not be analysed using hexahedral and wedge elements. So only tetragonal mesh elements were used while analysing without partition. When partition was used then both hexahedral and wedge elements were used for analysis. The crack was located at 1.4m location from the fixed end with an opening of 0.002 m and a depth of 0.05 m. ABAQUS analysis results for different mesh elements are shown in table 4. It is observed that partitioned meshing cause greater fluctuation in natural frequency which can be subjected to the decreased natural frequency theorem for a discontinuous structure. So for further analysis combined meshing was used.

Table 4. Comparison of results for different types of meshing.

| Mesh Type                              | 1st Natural Frequency | 2nd Natural Frequency | 3rd Natural Frequency | 4th Natural Frequency | 5th Natural Frequency |
|----------------------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| Un-cracked beam                        | 18.825                 | 30.535                 | 95.642                 | 97.885                 | 113.73                 |
| Tetragonal element                     | 18.639                 | 30.782                 | 94.967                 | 97.702                 | 115.89                 |
| Partitioned (Hexahedral and wedge elements mixed) | 18.553                 | 30.114                 | 94.749                 | 95.518                 | 112.30                 |

The table shows that using tetragonal elements results in the increase in 2nd and 5th natural frequency than the natural frequency of the un-cracked beam. But all 5 modes decrease when partition is used. So it can be said that using partition would provide better accuracy in result.

3.2. Natural frequency analysis based on crack opening

The effect of crack opening size on natural frequency is shown in table 5.

Table 5. Frequency for different crack opening size.

| Crack opening size | 1st mode | 2nd mode | 3rd mode | 4th mode | 5th mode |
|--------------------|----------|----------|----------|----------|----------|
| 0.002              | 18.568   | 30.309   | 94.782   | 95.831   | 112.75   |
| 0.004              | 18.554   | 30.306   | 94.744   | 95.743   | 112.74   |
| 0.006              | 18.554   | 30.305   | 94.747   | 95.743   | 112.74   |
| 0.010              | 18.537   | 30.15    | 94.741   | 94.786   | 110.95   |

It can be seen from the table that the change in natural frequency is negligible unless the crack opening is significant. In this study for further analysis crack opening size of 0.010m is considered.

3.3. Natural frequency analysis based on crack location

Effect of crack location for specified crack depth of 0.05m and crack opening of 0.010m is represented in table 6 and figure 2(a), 2(b), 2(c), 2(d) and 2(e).
Table 6. Natural frequency for various crack location.

| Location of crack from free end | 1\textsuperscript{st} natural frequency (cycle/sec) | 2\textsuperscript{nd} natural frequency (cycle/sec) | 3\textsuperscript{rd} natural frequency (cycle/sec) | 4\textsuperscript{th} natural frequency (cycle/sec) | 5\textsuperscript{th} natural frequency (cycle/sec) |
|---------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| 0.7                             | 18.112                                          | 29.932                                          | 93.228                                          | 97.737                                          | 113.34                                          |
| 1.4                             | 18.568                                          | 30.309                                          | 94.782                                          | 95.831                                          | 112.75                                          |
| 2.1                             | 18.777                                          | 30.449                                          | 95.488                                          | 96.279                                          | 112.76                                          |
| 2.8                             | 18.826                                          | 30.525                                          | 95.65                                           | 97.856                                          | 113.69                                          |

(a) 1\textsuperscript{st} Natural Frequency
(b) 2\textsuperscript{nd} Natural Frequency
(c) 3\textsuperscript{rd} Natural Frequency
(d) 4\textsuperscript{th} Natural Frequency
(e) 5\textsuperscript{th} natural frequency

Figure 2. Natural frequency for various crack position

The graphs illustrate that the change in natural frequency for different crack position is not similar for all modes of vibration. For example, the natural frequency for the first three modes of vibration rises with the movement of crack location away from fixed end. But for the fourth and fifth mode of vibration the frequency decreases as the crack moves toward the middle section of the beam. Then increases as it
moves towards the end of the beam. In this case natural frequency has been determined for five different crack locations and other points were determined using the interpolation function available in Igor Pro software.

![Figure 3. Mode 1 representation of I beam in ABAQUS](image)

3.4. **Natural frequency analysis based on crack depth**

Natural frequency for various crack depth at specified crack location (1.4 m) is represented in Table 7.

| Crack depth (m) | 1<sup>st</sup> natural frequency (cycle/sec) | 2<sup>nd</sup> natural frequency (cycle/sec) | 3<sup>rd</sup> natural frequency (cycle/sec) | 4<sup>th</sup> natural frequency (cycle/sec) | 5<sup>th</sup> natural frequency (cycle/sec) |
|----------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
| 0.025          | 18.785                                      | 30.476                                      | 95.51                                       | 97.472                                      | 113.35                                      |
| 0.0375         | 18.709                                      | 30.412                                      | 95.247                                      | 96.895                                      | 113.12                                      |
| 0.05           | 18.555                                      | 30.307                                      | 94.744                                      | 95.747                                      | 112.74                                      |
| 0.0625         | 18.148                                      | 30.07                                       | 92.738                                      | 93.738                                      | 111.95                                      |
| 0.075          | 13.589                                      | 27.504                                      | 71.567                                      | 91.214                                      | 106.62                                      |

The graph shows us that as the crack depth increases the natural frequency reduces for all modes of vibration which justifies the theory of decrease in natural frequency in presence of discontinuities. The graph also represents the maximum decrease when the crack reaches the web section of the beam.

3.5. **Resonance frequency analysis based on crack location**

The conditions applied for resonance frequency analysis are shown in Table 8.

| Conditions         | Value            |
|--------------------|------------------|
| Load               | 1 kN             |
| Imposed Frequency  | 0-120 cycle/sec  |
| Structural Damping | 5%               |

In this section of the study a vertical force of 1 kN of various frequencies has been applied to I beam and for various imposed frequency the displacement of the beam was studied. To avoid infinite displacement, 5 percent structural damping was applied to the beam. The displacement of the beam for various crack location is presented in figure 3.
The graph and table shows that resonance frequency for 1 kN vertical load increases as the crack location moves away from the fixed end. Which is similar to the first three modes of natural frequency. There is a sudden decrease in displacement near the 90-100 range frequency. The 5 percent structural damping is responsible for that. In fact the frequency in which the displacement is zero is the real resonance frequency for this vertical load. As 5 percent structural damping has been applied hence resonance has shifted and the maximum displacement has also been reduced. Without damping the displacement will be very high resulting in the collapse of the structure. From the mode shape analysis, it can be said that the resonance frequency for the applied load is associated with the 3rd and 4th mode of vibration of the beam. It can also be said that resonance occurs earlier if the crack is located closer to the fixed end of the cantilever beam.

3.6. Resonance frequency analysis based on crack depth

It tends to be seen from the diagram that reverberation happens prior when the crack depth is bigger. Likewise it can be seen from the diagram that for crack depth of 0.075 m displacement is sensibly higher than others. This is due to the fact that a crack of 0.075m reaches to the web section of the I beam.
4. Conclusions

The effect of cracks is very much significant in the vibration analysis. From this paper the following results are summarized.

- When there is discontinuity in the structure in the form of crack then the natural frequency reduces. The quantity of reduction depends upon the crack opening, crack depth and crack location.
- For a certain location of crack, the natural frequency reduces as the crack depth increases.
- While designing cracked structure mesh refinement should be considered for more accurate result.
- As the number of elements increases the accuracy of the calculation of natural frequency increases.
- Effect of crack is different for different modes of vibrations.
- For I beam natural frequency reduces significantly if the crack reaches the web section of the beam.
- Resonance occurs at earlier imposed frequency if the crack is located closer to the fixed end.
- Resonance occurs at lower imposed frequency if the crack depth increases.
- The effect of vibration will be significantly higher if the crack reaches the web section of a I beam.

References

[1] M. S. Mia, M. S. Islam, and U. Ghosh Modal analysis of cracked cantilever beam by finite element simulation 2017 Procedia Eng. vol. 194 pp. 509–516
[2] Nitesh A. Meshram and Prof. Vaibhav S. Pawar Analysis of Crack Detection of A Cantilever Beam using Finite Element Analysis 2015 Int. J. Eng. Res. vol. V4 no. 04 pp. 713–718
[3] W. M. Ostachowicz and M. Krawczuk Analysis of the effect of cracks on the natural frequencies of a cantilever beam 1991 J. Sound Vib. vol. 150 no. 2 pp. 191–201
[4] P. M. Jagdale and M. A. Chakrabarti Free Vibration Analysis of Cracked Beam 2013 vol. 3 no. 6 pp. 1172–1176
[5] M. Quila, P. S. Ch. Mondal, and P. S. Sarkar Free Vibration Analysis of an Un-cracked & Cracked Fixed Beam 2014 IOSR J. Mech. Civ. Eng. vol. 11 no. 3 pp. 76–83
[6] M. Behzad, a Meghdari, and a Ebrahimi Archive of SID A NEW APPROACH FOR VIBRATION ANALYSIS OF A CRACKED BEAM Archive of SID 2005 Int. J. Eng. vol. 18 no. 4 pp. 319–330
[7] S. Orhan Analysis of free and forced vibration of a cracked cantilever beam 2007 NDT E Int. vol. 40 no. 6 pp. 443–450