An Observation on $F_2$ at Low $x$

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Abstract

A simple parametrisation of H1 and ZEUS data at HERA is given for the ranges in $x$ and $Q^2$ of $10^{-4} - 5.10^{-2}$ and $5 - 250$ GeV$^2$, respectively. This empirical expression is based on a strikingly similar dependence of the average charged particle multiplicity $\langle n \rangle$ on the centre of mass system energy $\sqrt{s}$ in $e^+e^-$ collisions on the one hand, and the $x$ dependence of the proton structure function $F_2$ as measured at small $x$ on the other hand. To the best of our knowledge, this similarity has not been noted before.
One of the most successful tests of perturbative QCD is the quantitative explanation of scaling violations, i.e. the $Q^2$ dependence of the nucleon structure functions at fixed $x$-values. Here $Q^2$ and $x$ denote the usual deep-inelastic variables, the four-momentum squared and Bjorken-$x$. Previous fixed target experiments measured the structure function $F_2(x, Q^2)$ for the region $x > 0.01$ [3]. These data were, therefore, sensitive to the valence content of the nucleon. The DGLAP evolution equations [3] describe the $Q^2$ evolution of the structure functions in this region very well. However, the data from the electron-proton collider HERA explore a new kinematic region. Values of Bjorken-$x$ in the range $10^{-5} < x < 10^{-2}$ for $Q^2$ larger than $1 \text{ GeV}^2$, are reached. In this region the valence contribution is expected to be negligible and $F_2$ to be driven by the gluon in the proton.

Recently new data on $F_2$ from the H1 [4] and ZEUS [5] experiments at HERA, based on the 1994 data taking period, have been published. The data have reached a level of precision of 3-5% in a large region of the kinematical plane. They show very clearly that $F_2$ rises strongly for decreasing $x$ for all $Q^2$ values, and strong scaling violations are observed in the new deep-inelastic region at low-$x$. Originally it was thought that in the HERA region $\ln 1/x$ terms, not included in the DGLAP resummation, could become important. However, it has turned out that these evolution equations are still successful in describing the $Q^2$ dependence of the data [4].

The rise of $F_2$ at small $x$ was predicted more than twenty years ago [6] from the leading order renormalization group equations of perturbative QCD. Ball and Forte recently pursued these ideas [7] and proposed a way to demonstrate that the low-$x$ data at HERA exhibit double asymptotic scaling (DAS) dominantly generated by QCD radiation. They obtained an expression for $F_2(x, Q^2)$ in the double asymptotic limit of low-$x$ and large $Q^2$. The recent $F_2(x, Q^2)$ measurements of H1 for $Q^2 > 5 \text{ GeV}^2$ are broadly in agreement with such a scaling behaviour. Hence, in this region these data are expected to be sensitive to the fundamental QCD evolution dynamics, and not to depend on unknown (non-perturbative) starting distributions at sufficiently large $Q^2$ and small $x$. This idea has also been exploited in the dynamically generated parton distributions [8] which predicted, for the same reason, the rise of $F_2$ at small $x$ prior to data. Qualitatively these results can be understood by viewing the deep inelastic collision at low-$x$ as the interaction of a virtual photon with partons in a space-like parton cascade which stretches from $x$ of order one to $x << 1$, and thus covers a rapidity range $\propto \ln(1/x)$. For very small $x$, the rapidity range is large and a well-developed cascade can be formed. In the leading-log approximation this leads to an expression [3] for $F_2$

$$F_2 \sim \exp \sqrt{16N_c \ln(1/x) \ln \ln Q^2}. \quad (1)$$

Here $b$ is the leading coefficient in the $\beta$-function for the expansion of $\alpha_s$, namely $b = 11 - 2n_f/3$ with $n_f$ the number of flavours; $N_c$ is the number of colours.

Another cornerstone of the success of perturbative QCD are the calculations and predictions for particle production in time-like parton cascades in $e^+e^-$ collisions, based on the Modified Leading Log Approximation (MLLA) evolution equations and the assumption of Local Parton Hadron Duality (LPHD) [9, 10]. In this approximation, the average parton multiplicity of $e^+e^-$ collisions as function of the center of mass system energy $\sqrt{s}$ (CMS) is given by:

$$< n_p > \sim \Gamma(B)(\frac{z}{2})^{1-B}I_{B+1}(z), \quad (2)$$

with $z = \sqrt{16N_c/b} \cdot \ln(\sqrt{s}/2Q_1)$. The function $I_{B+1}(z)$ is a Bessel function of order $B + 1$, with, for four flavours, $B = (11 + 2n_f/27)/b = 1.355$. Here $\Gamma$ is the Gamma function. The parameter $Q_1$ is the $p_t$ cutoff of the partons in the shower and found to be in the range of 250-290 MeV [10] from fits to the data. Eqn. [2] gives a very good description of the averaged charged hadron multiplicity in $e^+e^-$ collisions for CMS energies in the range $\sqrt{s} = 3 - 130 \text{ GeV}$ [10].

Expression (2) can be approximated at large $z$ by

$$< n_p > \sim \exp \sqrt{16N_c \ln \sqrt{s}/2Q_1}. \quad (3)$$
Comparing expressions (1) and (3), one notices an intriguing similarity. For fixed \( Q^2 \) they have a similar functional dependence on \( 1/x \) and \( s/4Q_0^2 \), respectively. The connection \( s \to 1/x \) emerges naturally in Regge-inspired phenomenology, see e.g.\[1\]. In Fig. 1 we compare the \( e^+e^- \) data on average charged particle multiplicities versus \( \sqrt{s} \) and the HERA low-\( x \) \( F_2 \) data versus \( 2Q_1/\sqrt{x} \), with \( Q_1 = 270 \text{ MeV} \), as determined in \[3\]. The \( e^+e^- \) multiplicity data are represented by curves resulting from a phenomenological fit through the data as derived by OPAL\[12\]. The curves are normalized to the \( F_2 \) data for each \( Q^2 \) bin separately. It shows that at small \( x \) the evolution of \( F_2 \) with \( 1/x \) and the dependence of average charged particle multiplicity in \( e^+e^- \) collisions on \( \sqrt{s} \) are indeed quite similar as suggested by the expressions above.

This simple observation led us to study fits of the full expression (2) to the new low-\( x \) measurements of \( F_2 \) at HERA, with the change \( s/4Q_1^2 \to 1/x \). The \( Q^2 \) dependence (absent in \( e^+e^- \) ) was assumed to be given by a slowly varying function of \( Q^2 \) of the form \( (\ln \alpha_s(Q_0^2)/\alpha_s(Q^2))\delta \), with \( \delta \) taken to be a constant. The final expression fitted to the data is

\[
F_2(x, Q^2) = C(Q^2)\Gamma(B)\left(\frac{z}{2}\right)^{1-B}I_{B+1}(z),
\]

and

\[
z = \frac{16N_c}{b} \ln \sqrt{1/x} \cdot (\ln \alpha_s(Q_0^2)/\alpha_s(Q^2))\delta;
\]

where \( Q_0 \) is taken to be 1 GeV and the two-loop expression for \( \alpha_s \) is used with \( \Lambda_{QCD}^{(4)} = 263 \text{ MeV} \[13\]. Note that the normalization factor \( C(Q^2) \) and power \( \delta \) are the only fit parameters at any fixed \( Q^2 \).

The result is shown in Fig. 2, where a fit is made in each bin of \( Q^2 \) on the H1 and ZEUS data, separately. The expression (4) describes the data well over the whole kinematic region, except at large \( y = Q^2/xs \) values, where the contribution of valence quarks is expected to become important. The difference in the results obtained using the H1 or ZEUS data can hardly be distinguished. We find that the data are best reproduced for \( \delta \sim 0.7 \) (see below), definitely below the value \( \delta = 1 \) derived from the asymptotic form in perturbation theory\[3\].\[4\]. The result for the normalization \( C \) is shown in Fig. 3 as function of \( Q^2 \). In the range \( 5 < Q^2 < 250 \text{ GeV}^2 \), \( C \) is essentially constant with a value of about 0.38. For lower \( Q^2 \), a clear breaking of this regularity is observed, and hints that additional contributions to \( F_2 \) become important.

Encouraged by the results shown in Figs. 2 and 3 we perform a combined fit of the H1 and ZEUS data to eqn. (4) with \( C(Q^2) = C_0 \) constant over the whole \( Q^2 \) range, in the region \( 5 < Q^2 < 250 \text{ GeV}^2 \), \( x < 0.05 \), \( y > 0.02 \). The latter two conditions are imposed to avoid the valence quark region. The result is shown in Fig. 4. The fit has \( \chi^2/NDF = 265/231 \), using the full errors. The relative normalization of the H1 and ZEUS data was left free. The normalization factors found are 0.99 and 1.025 for H1 and ZEUS respectively, well within the quoted normalization uncertainties\[3\].\[6\]. The statistical errors on the fit parameters are from a fit with the statistical errors of the data only. Using the full error matrix of H1 and/or ZEUS each of the measured quantities entering the \( F_2 \) analysis is varied in turn. For the two fit parameters we find \( C_0 = 0.389 \pm 0.005(stat) \pm 0.012(syst) \) and \( \delta = 0.708 \pm 0.007(stat) \pm 0.028(syst) \). From the fits to data of the individual experiments we find for H1: \( C_0 = 0.385 \pm 0.007(stat) \pm 0.020(syst) \) and \( \delta = 0.683 \pm 0.010(stat) \pm 0.055(syst) \) (\( \chi^2/NDF = 76/97 \)); for ZEUS: \( C_0 = 0.384 \pm 0.007(stat) \pm 0.009(syst) \) and \( \delta = 0.723 \pm 0.010(stat) \pm 0.025(syst) \) (\( \chi^2/NDF = 186/134 \)). A point by point analysis shows that the region \( y < 0.04 \) is responsible for a substantial contribution to the \( \chi^2 \) for the ZEUS data.

With two free parameters only: the normalization \( C_0 \) and \( \delta \), we are able to account for the \( x \) and \( Q^2 \) dependence of \( F_2 \) starting from a parametrisation which successfully describes the energy dependence of the mean charged multiplicity in \( e^+e^- \) annihilation, provided \( s \) is identified with \( 1/x \). We also note that according to eqn. (4) \( F_2 \) grows slower than any power of \( 1/x \) but faster than any
power of $\ln 1/x$. In particular, for most of the regions in $Q^2$ shown in Fig. 4, the $F_2$ data indeed increase faster than $\ln 1/x$, contrary to the claims in [14].

Fig. 4 shows $\lambda = d\ln F_2(x, Q^2)/d\ln(1/x)$ calculated from (4) for a number of $x$ values. A rise of $\lambda$ with $Q^2$ is observed. Note that its value depends on the $x$-region: $\lambda$ increases with increasing $x$. The growth of $\lambda$ with $Q^2$ is often considered to be indicative of a transition from a region of “soft” pomeron exchange ($\lambda \sim 0.1$) at low $Q^2$ to a regime of “hard” pomeron exchange ($\lambda \sim 0.3 - 0.4$) at high $Q^2$. This argument is based on measurements of $d\ln F_2(x, Q^2)/d\ln(1/x)$ which cover, however, different ranges in $x$ as $Q^2$ changes. Fig. 4 demonstrates that the so-called “soft” to “hard” transition is much less spectacular when $x$ is kept fixed. In addition, we note that the slopes at a given $Q^2$ are larger at large $x$ than at small $x$. This runs contrary to the often expressed opinion that the small $x$ region in deep-inelastic scattering probes the “hard” pomeron.

In first instance we regard eqn. (4) as a compact parametrisation of the $F_2$ data at small $x$, where the dynamics of the $F_2$ evolution is expected to be dominated by gluons. Since it is based on a result of the MLLA evolution equations, which include coherence, it is well adapted to be used e.g. as an ansatz for starting distributions in QCD fits of proton structure data.

However, it is tempting to speculate that the similarity observed here is more than just a mathematical coincidence. It indeed suggests that, at least qualitatively, the evolution of the structure function at low-$x$ can be attributed to the development of an unhindered QCD parton shower in “free” phase space quite similar to that in $e^+e^-$. For $F_2$ this also follows essentially from the observation of DAS and the success of the dynamically generated GRV parton distributions. Whether a more profound explanation for the empirical regularity reported here exists, remains an interesting open question.

Summary

A striking similarity between the rise with energy ($\sqrt{s}$) of the charged particle multiplicity in $e^+e^-$ and the rise of $F_2$ at HERA is observed. To the best of our knowledge, this similarity has not been noted before. For $Q^2 \geq 5$ GeV$^2$ and $10^{-4} < x < 0.05$, the phenomenologically successful MLLA expression for the average multiplicity in $e^+e^-$ collisions, with the transformation $s \to 1/x$, and adding a QCD inspired $Q^2$ dependence, describes the HERA data on $F_2$ at small $x$ very well. The result suggests that both deep inelastic small-$x$ scattering and $e^+e^-$ annihilation can be adequately described by angular ordered QCD radiation in an essentially free phase space.

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Figure 1: Comparison of $e^+e^-$ data on average charged particle multiplicities versus $\sqrt{s}$ and the HERA low-$x$ $F_2$ data versus $2Q_1/\sqrt{x}$, with $Q_1 = 270$ MeV, for $Q^2 = 22$ GeV$^2$ (ZEUS) and 25 GeV$^2$ (H1). The $e^+e^-$ multiplicity data (solid lines) are represented by curves resulting from a phenomenological fit through the data [12]. The curves are normalized to the $F_2$ data for each $Q^2$ bin separately.
Figure 2: The proton structure function $F_2$ measured by the H1 and ZEUS experiments at HERA together with fits of eqn. (4) through both data sets separately. The normalization constant $C(Q^2)$ was fitted separately for each $Q^2$ value.
Figure 3: The normalization constant $C$ of eqn. (4) as function of $Q^2$. 
Figure 4: The proton structure function $F_2$ in the range $5 < Q^2 < 250 \text{GeV}^2$, $x < 0.05$ and $y > 0.02$ by the H1 and ZEUS experiments together with a fit of eqn. (4), for which the normalization constant $C$ is taken to be independent of $Q^2$. 
Figure 5: The slope parameter $\lambda = d \ln F_2(x, Q^2)/d \ln(1/x)$ as function of $Q^2$, derived from the final parametrisation for several $x$ values.