METHODOLOGY FOR THE CHARACTERIZATION OF THE ELECTRICAL POWER DEMAND CURVE, BY MEANS OF FRACTAL ORBIT DIAGRAMS ON THE COMPLEX PLANE OF MANDELBROT SET

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ABSTRACT. The present article proposes a new geometric space in the complex plane of the Mandelbrot set, framed in the diagram of orbits and attractors, to characterize the dynamics of the curves of the demand of daily electrical power, with the purpose of discovering other observations enabling the elevation of new theoretical approaches. The result shows a different method to evaluate the dynamics of the electric power demand curve, using fractal orbital diagrams. This method is a new contribution that extends universal knowledge about the dynamics of complex systems and fractal geometry. Finally, the reader is informed that the data series used in this article was used in a previous publication, but using a different fractal technique to describe its dynamics.

1. Introduction. The purpose of this article is to present a methodology for characterizing the dynamics of electric power demand curves using fractal orbit diagrams defined in the complex plane of the Mandelbrot set. On this subject, the present authors have published two (2) articles. The fractal geometrical pattern of the Julia set [16] was determined with respect to the signs in the complex plane of the active and reactive electrical power; with new data based on [17] the conclusions were extended with respect to the fractal geometric pattern that produces the daily hourly demand of electric power. The main contribution to the theory of fractal geometry consisted in the identification and qualitative interpretation of the rules of formation of the topologies of Julia sets that produce the demand of hourly daily electrical power. The article [21] focuses on presenting a new methodology to analyze the dynamics of the series in reference, using the curve formed by attractors that move

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in the complex plane over the Mandelbrot set according to the law dictated by the load curve.

Articles [16, 17, 21] are the basis for the new contribution, so in this work substantive portions must be repeated regarding the state of the art and model of the Mandelbrot fractal set. However, in this article the authors extend the previously described results evaluating the dynamics of the demand curves, depending on the orbital fractal diagrams.

Like novelty, this also allows confirmation that:

(a) A new, discrete, dynamic system is used to analysis the demand of active and reactive daily power, through its orbits in the complex plane of the Mandelbrot set.
(b) The diagrams of the orbits in the complex plane of the Mandelbrot set establish the behavior of the iterated sequence for the given values of \((P_0, Q_0)\). With respect to the values of the demand of real and reactive electrical power, the obtained orbital geometry also allows for the characterization of the behavior of the demand curve.

For this reason, this work proposes the following hypothesis: The daily electric power demand curve may be characterized with a clear fractal pattern of orbits. In this new case, the working hypothesis, methods and results lead to new analyzes and conclusions.

1.1. Fractal geometry. In 1975, the mathematician Benoit Mandelbrot defined the concept of fractals as a semi-geometric element with a repetitive structure at different scales [1], with characteristics of self-similarity as seen in some natural formations such as snowflakes, ferns, peacock feathers, and Romanesque broccoli. Fractal theory has been applied to various fields such as biology [15, 8], health sciences [3, 13, 12, 5, 11], stock markets [10], network communications [7, 9, 18], and others. Fractal theory is one of the methods used to analyze data and obtain relevant information in highly complex problems. Thus, it has been used to study the price of highly variable markets, which are not always explainable from classical economic analysis.

For example, in [10], that paper demonstrates that current techniques have some issues to explain the real market operation and a better understanding is achieved by using techniques such as chaos theory and fractals. In that publication, the authors show how to apply fractal behavior to stock markets and refer to multifractal analysis and multifractal topology. The first describes the invariability of scaling properties of time series and the second is a function of the Hölder exponents that characterizes the degree of irregularity of the signal, and their most significant parameters.

In [4], the authors discuss the basic principle of fractal theory and how to use it to forecast the short-term electricity price. In the first instance, the authors analyze the fractal characteristic of the electricity price, confirming that price data have this property. In the second instance, a fractal model is used to build a forecasting model, which offers a wide application in determining the price of electricity in the markets.

Similarly, the authors of [20] demonstrate that the price of thermal coal has multifractal features by using the concepts introduced by Mandelbrot-Bouchaud. Hence, a quarterly fluctuation index (QFI) for thermal power coal price is proposed to forecast the price caused by market fluctuation. This study also provides a useful reference to understand the multifractal fluctuation characteristics in other energy prices.
Fractal geometry analysis has been also applied to study the morphology and population growth of cities, and electricity demand related to the demography of cities. In [14], a multifractal analysis is used to forecast electricity demand, explaining that two fractals are found that reflect the behavior pattern of power demand. Two concepts linked to fractal geometry are fractal interpolation and extrapolation, which are related to the resolution of a fractal-encoded image. In [19], an algorithm is used to forecast the electric load in which fractal interpolation and extrapolation are also involved; for the forecast dataset, the average relative errors are only 2.303% and 2.296%, respectively, indicating that the algorithm has advantages in improving forecast accuracy.

1.2. Mandelbrot set. The Mandelbrot set, denoted as $M = c \in C/J_c$, represents sets of complex numbers $C$ obtained after iterating from the initial point $Z_n$ and the selected constant $C$ as shown in Equation 1. The results form a diagram with connected points remaining bounded in an absolute value. One property of $M$ set is that the points are connected, although in some zones of the diagram it seems that the set is fragmented. The iteration of the function generates a set of numbers called “orbits.” The results of the iteration of those points outside the boundary set tend to infinity:

$$Z_{t+1} = Z_t^2 + C$$

From the term $C$, a successive recursion is performed with $Z_0 = 0$ as the initial term. If this successive recursion is dimensioned, then the term $C$ belongs to the Mandelbrot set; if not, then they are excluded. Therefore, Fig. 1 shows the Mandelbrot set with points in the black zone called the “prisoners” while the points in other colors are the “escapists” and represent the escape velocity to infinity.

Figure 1. Graphical representation of the Mandelbrot set

From this figure, the value -1 is inside of the set while the number 1 is outside. In the Mandelbrot set, the fractal is the border and the dimension of Hausdorff is unknown. If the image is enlarged near the edge of the set, then many areas the
Mandelbrot set are represented in the same form. Besides, different types of Julia sets are distributed in different regions of the Mandelbrot set. If a complex number appears with a greater value than 2 in the 0 orbit, then the orbit tends to infinity.

The orbits that are generated are a sequence of complex numbers and their characteristics depend fundamentally on the values of the initial point $Z_n$ from which it is split and on the selected $C$ constant.

1.3. **Discrete dynamic system.** A dynamic system is a system that varies with time and is described by the state-space method together with the rule that determines the dynamics of the system. The state of the system is described by a point that travels in the phase-space and the path traced by the point represents the evolution of the state of the system from a particular initial condition. Thus, the phase diagram gives a qualitative idea of what happens to any initial condition. The state-space is the set $E$ where the system moves, which may have topological, metric, or differential space structure.

In general, a dynamic system is a triple $(E, G, f)$, where $E$ is a phase-space or state-space, $G$ is a semi-group of scalars or set of times, and $f$ is the system flow, which is an application of $E \ast G$ in $E$ with the following properties that describe a discrete dynamic system:

1. $f$ is a continuous application.
2. $f(0, x) = x$ for all $x \in E$.
3. $f(t, f(s, x)) = f(t + s, x)$ for all $t, s \in G$ and all $x \in E$.

One way to visualize the state of a dynamic system is by plotting the orbits in the phase-space. A detailed study on the matter is beyond the scope of this report. In [6], the Russian mathematician Katok Hasselblatt offers complete information. A recount of the matter is presented as follows. An orbit is a set of related points by the evolution function of a dynamic system. For discrete dynamic systems, the orbits are a succession, whereas for real dynamic systems the orbits are curves. The three basic classifications of orbits are: (a) constant orbits or fixed points: a fixed point verifies that $z = f(z)$; (b) periodic orbits: a periodic point verifies that $z = f^n(z)$; and (c) non-constant and non-periodic orbits.

The focus of this paper is on periodic orbits $x(t, x_0)$ where $x_0$ is not a fixed point and there exists a time $T > 0$ such as in Equation 2. As a result, $x(t, x_0)$ is a periodic orbit or limit cycle that forms simple closed curves. Additionally, the shortest time $T$ that meets this condition is the period of the cycle:

$$x(t + T, x_0) = x(t, x_0).$$

The way to visualize the behavior of the state variables can be in the form of a time series (a graph of the state variable against time) or in the form of a state-phase (phase diagram). In the phase-space of a dynamic system, singularities (points, cycles) that attract the trajectories that pass near them, and others that repel them, can be seen. It is said that a singularity of the phase-space is stable if every trajectory that begins near it approaches it as time passes. Finally, a singularity is unstable, a repulsor, or a source when it is not an attractor or Lyapunov-stable; that is, the trajectories that start close to it diverge as time passes.

The importance of the stability of the singularities lies in the fact that it determines the stability of the system in which the singularity occurs. In non-linear systems, singularities can present fixed points, limit cycles, and regions called “strange attractors.”
1.3.1. *Orbits and attractors*. One way to visualize the state of a system is through the orbit diagram. An orbit is a set of points related with the evaluation function of a dynamic system. For discrete dynamic systems the orbits are successions. The basic classifications can be defined as: (a) fixed points (b) periodic orbits, and (c) non-constant orbits [6]. With respect to the attractor, the main properties are a) compression, b) expansion, and c) folding [2]. If there is an attractor in the complex plane, the orbit associated with a complex number from the form \( z = a + bj \) is an orbit of complex numbers, with the same dynamics.

1.3.2. *Orbits in the complex plane of Mandelbrot set*. In the Mandelbrot fractal set, there are three types of orbital diagrams: Fixed point diagram if \( C \) is inside the set \( M \); Periodic orbit diagram if \( C \) is the limit of the set \( M \); and the orbital diagram is chaotic if \( C \) escapes to infinity and, as such, does not belong to the set \( M \). In this case, the orbital diagram does not exhibit a discernible pattern.

In summary, the set \( M \) is then like an infinite catalog of orbital diagrams.

Figure 2. Relationship between Mandelbrot set and orbital diagrams

Figure 2 shows the characteristic of the orbital diagrams: a) it is a point \( C(0.3 + j0.3) \) inside the main bulb of \( M \) and its orbital diagram corresponds to a fixed point. b) \( C(0.25 + j0.51) \) is over the limit of the Mandelbrot set, and its orbital diagram is of the periodic type. Finally c) the orbital diagram is chaotic because \( C(0.25 + j0.58) \) it is outside of the \( M \) set.

The pseudocode used to generate orbit diagrams and find the attractor of a complex number inside the \( M \) set is described as follows.

Start
Read \( C_i \)
Fix \( Z_0=C_i \)
For \( t = 1 \) to \( t_{\text{maxNumOrb}} \) Do
    Calculate \( Z_t = Z_t^2 + C \)
    If \( |Z_{t+1}| > 2 \) then
        Break
End if
Paint orbit $Z_t$

$Z_t = Z_{t+1}$

End For

End

2. Methodology. In order to obtain universally valid results with respect to the patterns of orbit formations in the complex plane of the Mandelbrot set that represent the dynamics of the real and reactive demand curves found in electrical power, the procedure described below was followed.

2.1. Discrete dynamic system. Load demand curve. The following are the typical records of the demand for real and reactive electrical power in the city of Medellin-Colombia, in a period of 24 hours. Although in the original template, the records are taken every 15 minutes, for practical purposes the table only contains the time records.

The demand for electric power has a strong daily seasonal pattern that can be seen on all working days because they have a very similar demand profile. Thus, the time series is seasonal because it has a regular repetition pattern during the same period of time. Its periodic behavior is reflected in parameters such as mean, standard deviation, asymmetry, and auto-correlation asymmetry.

For the purpose of studying the orbital diagram of the set of related points by the evolution function $Z_{t+1} = Z_t^2 + C$, which moves on the set of Mandelbrot, according to the law dictated by the curve of electrical power, the registers of active and reactive electrical power became per unit as:

$$pu = \frac{(Actual\,\,Power)}{Base\,\,Power} \quad (3)$$

In this case, the base power is 4000 MVA, the experimental value with which all the electrical power records are scaled within the $M$ set. The above procedure differs from the conventional power base used for power systems analysis, which considers apparent power to obtain the per unit values.

The number of orbits of each real and reactive electrical power listed in Table 1.

The third part was to graph for each record the respective orbits-attractor diagram.

In order to validate the working hypothesis, the fourth part consisted of evaluating the topological properties of the orbital diagram, in relation to the dynamics of the demand for real and reactive electrical power.

In the fifth part the orbits are classified, depending on whether they are compressed or expanded, and selected according to which appear most interesting by their degree of dispersion.

Finally, the fractal formation pattern of the orbit diagram was concluded in relation to the discrete dynamics of the data under study.

Figure 3 presents a summary of the previously described process:

3. Results and discussion. Table 1 presents the curve of daily load electric demand. $P$ is real power, $Q$ reactive power. $P_{pu}$ and $Q_{pu}$ are $P$ and $Q$ in per unit.

Figure 4 presents the typical demand curves plotted with the data of Table 1 and Figure 5 presents the phase diagram of power demand plotted in the first quadrant of the complex plane, which moves on the Mandelbrot set, according to
Figure 3. Algorithm with the steps used to obtain the fractal of the power demand

| Hour     | P   | Q   | P_{pu} | Q_{pu} | NumOrbs |
|----------|-----|-----|--------|--------|---------|
| 00:00:00 | 889 | 371 | 0.222  | 0.092  | 5       |
| 01:00:00 | 834 | 405 | 0.287  | 0.101  | 5       |
| 02:00:00 | 792 | 337 | 0.197  | 0.082  | 5       |
| 03:00:00 | 790 | 324 | 0.199  | 0.081  | 5       |
| 04:00:00 | 804 | 323 | 0.201  | 0.080  | 3       |
| 05:00:00 | 925 | 355 | 0.231  | 0.088  | 5       |
| 06:00:00 | 1041| 482 | 0.260  | 0.120  | 9       |
| 07:00:00 | 1105| 556 | 0.276  | 0.139  | 9       |
| 08:00:00 | 1191| 610 | 0.297  | 0.152  | 18      |
| 09:00:00 | 1256| 704 | 0.314  | 0.176  | 30      |
| 10:00:00 | 1309| 744 | 0.327  | 0.186  | 32      |
| 11:00:00 | 1366| 775 | 0.341  | 0.193  | 50      |
| 12:00:00 | 1385| 793 | 0.346  | 0.198  | 53      |
| 13:00:00 | 1356| 774 | 0.339  | 0.193  | 44      |
| 14:00:00 | 1337| 759 | 0.334  | 0.189  | 38      |
| 15:00:00 | 1350| 774 | 0.337  | 0.193  | 41      |
| 16:00:00 | 1336| 773 | 0.334  | 0.193  | 41      |
| 17:00:00 | 1312| 749 | 0.328  | 0.187  | 41      |
| 18:00:00 | 1287| 687 | 0.321  | 0.171  | 41      |
| 19:00:00 | 1420| 683 | 0.355  | 0.170  | 89      |
| 20:00:00 | 1389| 660 | 0.351  | 0.167  | 89      |
| 21:00:00 | 1311| 605 | 0.327  | 0.151  | 41      |
| 22:00:00 | 1175| 544 | 0.293  | 0.136  | 18      |
| 23:00:00 | 1030| 489 | 0.257  | 0.122  | 14      |

Table 1. Daily load demand represented by hour
the law dictated by the curve of electrical power. As real and reactive powers are positive, they represent a load demand related to inductive elements.

Figure 4. The typical demand curves of active and reactive electric power

Figure 5. Electric power demand curve plotted in the first quadrant of the complex plane of Mandelbrot set

Under these conditions, the three most interesting values of the power demand are selected such as the lowest demand at 3:00, the highest demand at 19:00, and the approximate average demand at 09:00. Orbits for each \((P_{pui}, Q_{pui})\) were calculated using the algorithm previously described.

Figure 6 shows the orbit diagram generated for each point of Figure 5. These fractals are created by performing iterations of the complex numbers obtained from the daily load demand.
The process of generation of the orbital diagram reveals folds with the following properties: from 0:00 to 11:00 the orbits expand from 5 to 32 orbits, causing stretching of their space, associated with the progressive increase in the demand for electrical power for hourly. From 12:00 to 17:00 the orbital space goes from 53 to 41 orbits, corresponding with a slight decrease in the demand of the hourly electrical power. The point of greatest interest corresponds to the orbital diagram generated at 19:00. It is a diagram that presents its maximum expansion and whose 89 orbits diverge exponentially from each other, related to the record of maximum demand for electrical power. From 20:00 and until 24:00 the demand for electric power decreases and the diagram is compressed with only 14 orbits, the folding approaches each other causing the remote orbits to approach each other. In summary, the orbital diagram in the complex plane of the Mandelbrot set obeys the daily periodic curve profile of the electrical power demand, which is continuously compressed from 20:00 to 5:00, from which the diagram presents continuous expansion.

4. Conclusions. In this article, the working hypothesis was validated. The daily electric power demand curve may be characterized with a clear fractal pattern of orbits, in the complex plane of Mandelbrot set, and from which other relevant observations emerge from its analysis, such as:

The compression or expansion of the folding of the orbits depends on the point evaluated within the Mandelbrot set. The folding expansion increases when the evaluated point is close to the Mandelbrot limit. Its possible physical interpretation is related to the electrical demand curve. In this article was proved that the number of orbits increases when the magnitude of the power increases and vice versa.
Fractal diagrams of orbits that are scaled within the Mandelbrot set may be related to the variations in the demand curve load, to identify changing behaviors of the electric daily load demand. The orbit diagrams were obtained using an algorithm that considers the mathematical model of Mandelbrot set.

The results identified a new space of analysis of the discrete dynamic system of the daily load demand of real and reactive electric powers. As the load demand constantly changes, the orbit diagrams and its folding density also change by expanding or decreasing accordingly. Stability of the fractal periodic orbits may be related to the low variation in the daily demand curve. The demand for electrical power changes constantly throughout the day, which corresponds to the permanent variation of the orbital diagram. Dense folding fractals may be related to the increased demands of electrical power.

This is a new way to look at fractal geometry and its complex dynamic systems as it relates to electrical load demand curve. For future works, a specific methodology must be developed to calculate within Mandelbrot set the base value for scaling the real and reactive power as “per unit”.

Conflicts of Interest. The authors declare that there is no conflict of interests regarding the publication of this paper.

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