Brain PET Synthesis from MRI Using Joint Probability Distribution of Diffusion Model at Ultrahigh Fields

Taofeng Xie*  
Inner Mongolia University  
Inner Mongolia Medical University  
tf.xie@mail.imu.edu.cn

Chentao Cao*  
SIAT, Chinese Academy of Sciences  
ct.cao@siat.ac.cn

Zhuoxu Cui  
SIAT, Chinese Academy of Sciences  
zx.cui@siat.ac.cn

Fanshi Li  
SIAT, Chinese Academy of Sciences  
fs.li@siat.ac.cn

Zidong Wei  
SIAT, Chinese Academy of Sciences  
zd.wei@siat.ac.cn

Yanjie Zhu  
SIAT, Chinese Academy of Sciences  
yj.zhu@siat.ac.cn

Ye Li  
SIAT, Chinese Academy of Sciences  
liye1@siat.ac.cn

Dong Liang  
SIAT, Chinese Academy of Sciences  
dong.liang@siat.ac.cn

Qiyu Jin  
Inner Mongolia University  
qyjin2015@aliyun.com

Guoqing Chen  
Inner Mongolia University  
egg@imu.edu.cn

Haifeng Wang†  
SIAT, Chinese Academy of Sciences  
hf.wang@siat.ac.cn

Abstract

MRI and PET are important modalities and can provide complementary information for the diagnosis of brain diseases because MRI can provide structural information of brain and PET can obtain functional information of brain. However, due to the expensive expense of PET scanning or radioactive exposure, some patients do not accept it, resulting in a lack of PET scans. Especially, simultaneous PET and MRI imaging at ultrahigh field is not achievable in the current. Thus, synthetic PET using MRI at ultrahigh field is essential. In this paper, we synthetic PET using MRI as a guide by joint probability distribution of diffusion model (JPDDM). On the public the Alzheimer’s Disease Neuroimaging Initiative (ADNI) dataset, we contrasted CycleGAN and score-based of SDE, and our model achieved a great result. Meanwhile, We utilized our model in 5T MRI and 7T MRI.

1 Introduction

Diagnosing the disease of brain disorder, (e.g. Alzheimer’s disease (AD)) jointed Positron emission tomography (PET) and magnetic resonance imaging (MRI) become a popular and useful method.

*Taofeng Xie and Chentao Cao contributed equally to this work.
†Corresponding author

Preprint. Under review.
because they can offer various information Johnson et al. [2012], Zhang et al. [2017], Cheng and Liu [2017]. MRI and PET include complementing information for enhancing the accuracy of AD diagnosis Calhoun and Sui [2016], Liu et al. [2017]. Positron emission tomography (PET) uses radiolabeled molecules like 18F-fluorodeoxyglucose (FDG) to offer metabolic information. MRI scan can provide structural information. However, the expensive expense of PET scanning or radioactive exposure, many patients will to receive MRI scans and to reject PET scans. Ultrahigh field MRI (e.g. 5T MRI and 7T MRI) can offers images which is higher resolution and high signal-to-noise ratio. But the PET corresponding to ultrahigh field cannot be obtained. Missing PET scans be synthesized urgently in order to make effective use of the multi-modality medical data, especially for PET at ultrahigh field. Generative models have generative adversarial networks (GAN) Goodfellow et al. [2020], likelihood-based method Graves [2013] and diffusion model Sohl-Dickstein et al. [2015] and so on. In recent years, diffusion model have remarkable advances comparable to GANs Goodfellow et al. [2020].

Score matching with Langevin dynamics (SMLD) Song and Ermon [2019] calculates the score namely the gradient of the log probability density with respect to the data at each noise scale and sample from a series of diminishing noise scales in the process of generation by Langevin dynamics. Denoising diffusion probabilistic modeling (DDPM) Ho et al. [2020] uses functional knowledge of the reverse distributions to make training tractable. It trains a series of probabilistic models to reverse each stage of the noise corruption. The model of score-based generative solving the stochastic differential equation (SDE) Song et al. [2020] unified framework of SMLD Song and Ermon [2019] and DDPM Ho et al. [2020]. SDE diffuses a data point into random noise continuously and then reverse the process for molding random noise into raw data. This paper’s core task is synthesizing PET scans from MRI scans. Therefore, we need to estimate the probability distribution of PET \( x_{PET} \) conditioned on MRI \( x_{MRI} \), i.e., \( p(x_{PET} | x_{MRI}) \). However, it is not easy to estimate. In the framework of diffusion models described above, we only need to estimate \( \nabla x_{PET} p(x_{PET} | x_{MRI}) \). Given \( \nabla x_{PET} p(x_{PET} | x_{MRI}) = \nabla x_{PET} p(x_{PET}, x_{MRI}) \), we will construct an MRI and PET joint diffusion model and learn its joint distribution to achieve conditional generation.

2 Method

Diffusion modeling is a crucial method in the generating process. The method forecasts the score, namely the gradient of the log probability density with respect to the data at each noise scale and sample from a series of diminishing noise scales in the process of generation by Langevin dynamics. The diffusion process is forward SDE. The diffusion process is as follows

\[
X_{i+1} = X_i + \sigma_{\min} \left( \frac{\sigma_{\max}}{\sigma_{\min}} \right)^t, \quad i = 1, 2, ..., N - 1,
\]

\( X_i \) is the \( i-th \) joint perturbed data (e.g. \( X_i(x_{PET}, x_{MRI}, t) \)). \( X_0 \) is joint distribution of \( x_{PET} \) and \( x_{MRI} \). \( x_T \) obeys joint distribution of Standard Gaussian distribution and \( x_{MRI} \). \( \{\sigma_i\}_{i=1}^N \) is the
noise scales which \( \sigma_{\text{min}} \) is the minimum of the noise scales and \( \sigma_{\text{max}} \) is the maximum of the noise scales. Sample process is predictor-corrector sample namely PC sample. Predictor and corrector are executed alternately. Predictor is reverse diffusion (from joint distribution of noise and MRI to the distribution of PET) for the sample that can be described as

\[
X_i = X_{i+1} - f_i(X_{i+1}) + g_i(X_{i+1})^T s_{\theta^*}(X_{i+1}, i + 1) + g_i(X_{i+1}) z_{i+1}
\]

where \( f_i \) denotes the drift coefficient of \( X_i \). \( g_i \) denotes the diffusion coefficient of \( X_i \). \( s_{\theta^*}(X_{i+1}, i + 1) \) is to estimate \( \nabla_{X_i} \log p_t(X_i) \). \( p_t(X_i) \) is the distribution of \( X_i \). \( s_{\theta^*}(X_{i+1}, i + 1) \) is obtained by deep learning of UNet and objection function is

\[
L(\theta; \sigma) = \frac{1}{2} \mathbb{E}_{p_t(X)} \left[ \left\| \sigma_{\text{min}} \left( \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}} \right)^t s_{\theta}(X_{i+1}, \sigma) + z \right\|^2_2 \right]
\]

In this study, \( f_i = 0 \), \( g_i = \sqrt{\sigma_i^2 - \sigma_{i-1}^2} \). Corrector is Langevin dynamics. Langevin method can computes the sample by

\[
X_i = X_{i+1} + \varepsilon s_{\theta}(X, i + 1) + \sqrt{2\varepsilon} z
\]

where \( \varepsilon = 2\alpha_i (r\|z\|/\|s_{\theta}\|) \) denotes step size. The study utilized the Alzheimer's Disease Neuroimaging Initiative (ADNI) dataset [Jack Jr et al., 2008]. 14440 pairs image of MRI and PET had registered. All image are reshaped to 128 * 128.

3 Results

In this study, Peak Signal to Noise Ratio (PSNR) were used to assess image quality. Our model, CycleGAN [Zhu et al., 2017] and score-based of SDE model [Song et al., 2020] are contrasted for synthesis PET using MRI in our study. The experimental results are shown in Fig. 2.

Figure 2: comparison synthetic results using different generation methods(joint distribution, CycleGAN, score-based of SDE)

Efficiency of our model is better and PSNR of our model is big her than others. We applied the trained model to the 5T MRI images acquired by a 5T MRI scanner (uMR Jupiter, United Imaging, Shanghai, China) and the 7T MRI images acquired by a 7T MRI scanner (MAGNETOM Terra, Siemens Healthcare, Erlangen, Germany). All of the protocols were approved by our Institutional Reviews Board (IRB). The results of 5T MRI show in Fig. 3. The results of 7T MRI show in Fig. 4.
4 Conclusions and Discussion

This study synthetic PET from MRI using joint probability distribution of diffusion model. It not only improves the stability of the generation model but also enables more accurate recovery of PET from MRI. The method has high potential for cross-modal synthesis. However, the disadvantage of our method is the slow of imaging speed. In future research, accelerated imaging speed is one of the research directions.
References

V. D. Calhoun and J. Sui. Multimodal fusion of brain imaging data: a key to finding the missing link (s) in complex mental illness. *Biological psychiatry: cognitive neuroscience and neuroimaging*, 1(3):230–244, 2016.

D. Cheng and M. Liu. Cnns based multi-modality classification for ad diagnosis. In *2017 10th international congress on image and signal processing, biomedical engineering and informatics (CISP-BMEI)*, pages 1–5. IEEE, 2017.

I. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative adversarial networks. *Communications of the ACM*, 63(11):139–144, 2020.

A. Graves. Generating sequences with recurrent neural networks. *arXiv preprint arXiv:1308.0850*, 2013.

J. Ho, A. Jain, and P. Abbeel. Denoising diffusion probabilistic models. *Advances in Neural Information Processing Systems*, 33:6840–6851, 2020.

C. R. Jack Jr, M. A. Bernstein, N. C. Fox, P. Thompson, G. Alexander, D. Harvey, B. Borowski, P. J. Britson, J. L. Whitwell, C. Ward, et al. The alzheimer’s disease neuroimaging initiative (adni): Mri methods. *Journal of Magnetic Resonance Imaging: An Official Journal of the International Society for Magnetic Resonance in Medicine*, 27(4):685–691, 2008.

K. A. Johnson, N. C. Fox, R. A. Sperling, and W. E. Klunk. Brain imaging in alzheimer disease. *Cold Spring Harbor perspectives in medicine*, 2(4):a006213, 2012.

M. Liu, Y. Gao, P.-T. Yap, and D. Shen. Multi-hypergraph learning for incomplete multimodality data. *IEEE journal of biomedical and health informatics*, 22(4):1197–1208, 2017.

J. Sohl-Dickstein, E. Weiss, N. Maheswaranathan, and S. Ganguli. Deep unsupervised learning using nonequilibrium thermodynamics. In *International Conference on Machine Learning*, pages 2256–2265. PMLR, 2015.

Y. Song and S. Ermon. Generative modeling by estimating gradients of the data distribution. *Advances in Neural Information Processing Systems*, 32, 2019.

Y. Song, J. Sohl-Dickstein, D. P. Kingma, A. Kumar, S. Ermon, and B. Poole. Score-based generative modeling through stochastic differential equations. *arXiv preprint arXiv:2011.13456*, 2020.

X. Y. Zhang, Z. L. Yang, G. M. Lu, G. F. Yang, and L. J. Zhang. Pet/mr imaging: new frontier in alzheimer’s disease and other dementias. *Frontiers in molecular neuroscience*, 10:343, 2017.

J.-Y. Zhu, T. Park, P. Isola, and A. A. Efros. Unpaired image-to-image translation using cycle-consistent adversarial networks. In *Proceedings of the IEEE international conference on computer vision*, pages 2223–2232, 2017.