Departure from the Standard Model of Meson Decays and Cartan’s Supersymmetry

Sadataka Furui
Graduate School of Science and Engineering, Teikyo University
2-17-12 Toyosatodai, Utsunomiya, 320-0003 Japan *

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Abstract

The experimental decay branching ratios of mesons like $B_s \to \ell \bar{\ell}$ and $B_d \to \ell \bar{\ell}$ ($\ell = e$ or $\mu$) do not agree completely with the standard model (SM).

Cartan’s supersymmetry predicts relation of the coupling of vector particles $x_\mu, x'_\mu$, ($\mu = 1, 2, 3, 4$) to Dirac spinors of large components $\psi, \phi$ and small components $C\psi, C\phi$. In the decay of $B_d = \bar{b}d$, the Cabibbo-Kobayashi-Maskawa(CKM) model suggests that the contribution of $t$ quark dominates, while in the decay of $B_s = \bar{b}s$, contribution of $c$ quark in the intermediate state is expected to be large, since $s$ and $c$ quark belong to the same CKM sector. The relative strength of the $t$ quark and $c$ quark contribution in Cartan’s supersymmetric model has more freedom than that of the SM. Together with the problem of enhancement of $B_d \to J/\Psi K_0$ in high $\Delta t$ region, we can understand the problem of branching ratios of $B$ decay into $e\bar{e}$ and $\mu\bar{\mu}$, if the Nature follows Cartan’s supersymmetry.

*E-mail address: furui@umb.teikyo-u.ac.jp
1 Introduction

Recently universality of lepton flavor in the decay of $B$ mesons was questioned by the LHCb group\[1\] and conjectures based on the QCD with additional currents were given in \[2, 3, 4, 5\]. In \[6\], violation of the flavor symmetry due to admixture of $c$ quark contribution to the $t$ quark contribution was studied, and we want to extend the theory to the decay of $B$ mesons.

Decay branching ratios of mesons to lepton pairs or a pion and lepton pairs are good testing ground of the standard model (SM). Comparison of experimental decay branching ratios of $B_{s,d} \to \ell \bar{\ell}$, $K \to \pi \ell \bar{\ell}$, $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ with that of SM were discussed in \[7, 8, 9, 10, 11, 13, 14, 15, 16, 18, 19\].

In the SM the strength of transitions of a quark to different flavor states is defined by the Cabibbo-Kobayashi-Maskawa (CKM) matrices. For example, the branching ratio of $B_s \to \ell \bar{\ell}$ depends on the probability of the coupling of an $s$ quark + emitted vector particle to a $\bar{b}$ quark + absorbing vector particle which depends on $|V_{cb}|^2 \times |V_{tb}^* V_{ts}/V_{cb}|^2$, and the branching ratio of $B_d$ depends on the probability of the coupling of a $d$ quark + emitted vector particle to a $\bar{b}$ quark + absorbing vector particle which depends on $|V_{tb}^* V_{td}|^2$.

Comparisons of theoretical meson decay matrix elements and experiments were done in various processes, and recently a summary of lattice simulation results was given by the FLAG group \[20\]. But the lattice cannot give consistent branching ratios of $B_d$ and $B_s$ mesons together.

Violation of $CP$ symmetry or time reversal symmetry in the decay of $B$ mesons was studied in the difference of

\[
B^0 \to \ell^- \nu_\ell + X, J/\psi K^0_L, \quad \text{v.s.} \quad \bar{B}^0 \to \ell^+ \nu_\ell + X, J/\psi K^0_L
\]

\[
B_s \to \ell^- \nu_\ell + X, J/\psi K^0_L, \quad \text{v.s.} \quad \bar{B}_s \to \ell^+ \nu_\ell + X, J/\psi K^0_L
\]

and $CP$ violation was observed in $B^0$ decay but not in $B_s$ decay. Qualitative differences of $B^0 \to \ell \bar{\ell}$ and $B_s \to \ell \bar{\ell}$ were also observed.

Since complete non leading order corrections in $b \to s \gamma$ and $b \to s g$ transitions are not
available, we apply Cartan’s octonion theory to solve the problem of branching ratios of $B$ mesons.

In [21] we studied Cartan’s supersymmetry[22] and studied coupling of fermions and vector particles which transform under groups $G_{23}, G_{12}, G_{13}, G_{123}$ and $G_{132}$.

Cartan considered only electromagnetic interactions, but extension to weak interaction is possible[6]. We applied the Cartan’s supersymmetric model to $B^0 \rightarrow K_L J/\Psi$ and found modification of coupling occurs in a certain channel as compared to the SM[24, 25].

In the section 2, we summarize the status of meson decay branching ratios, and in the section 3 we explain the method of analyzing the meson decay using Cartan’s supersymmetry. Discussion and conclusion are given in section 4.

2 Deviation of $B$ meson decay branching ratios from the Standard Model

When one uses results of $K$ meson decay to $\pi \ell \bar{\ell}$ or $\ell \bar{\ell}$ states to analyses of $B_d$ meson or $B_s$ decay to $K\ell \bar{\ell}$ or $\ell \bar{\ell}$ states, one encounters problems.

2.1 $B_d \rightarrow K\ell \bar{\ell}$ and $B_s \rightarrow K\ell \bar{\ell}$

From the universality of leptons, the strength of $B^+ \rightarrow K^+ \mu \bar{\mu}$ and that of $B^+ \rightarrow K^+ e \bar{e}$ are expected to be the same, but experimentally

$$\frac{Br(B^+ \rightarrow K^+ \mu \bar{\mu})}{Br(B^+ \rightarrow K^+ e \bar{e})} = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst}).$$

is observed by the LHCb group[1].

The inclusive $b \rightarrow s\ell \bar{\ell}$ decay modes were studied by several authors and an extension of Randall and Sundrum model[26, 27] to $B \rightarrow K\ell \bar{\ell}$ transition was tried in [28, 29].

In the custodially protected Randall Sundrum model (RS$_c$), additional $U(1)$ current in dark sector was proposed to solve $b \rightarrow s$ anomalies [4, 5]. We do not assume this current,
but adopt the vertex with $1 - \gamma_5$ factor for the weak interaction, and consider two triangle diagrams in the coupling.

![Figure 1: The vertex of $B^+ \to K^+ \ell \bar{\ell}$ (left) and $B^0 \to K^0 \ell \bar{\ell}$ (right).](image1)

The $b \to s \ell \bar{\ell}$ vertex requires renormalization of penguin diagrams.

![Figure 2: The penguin diagram of $B^+ \to K^0 \ell \bar{\ell}$ (left) and $B^0 \to K^0 \ell \bar{\ell}$ (right).](image2)

The lepton pairs can be $e\bar{e}$ which transform mainly to $\gamma$ or $\mu\bar{\mu}$ which transform mainly to $Z$. The CKM model implies higher order effect of coupling of $\gamma$ to $b$ quark is dominated by the loop of $t$ quarks, but difficulty of the CKM model to explain the decay of $B \to K + X$ transitions implies contamination of the loop of $b - c - s$ quarks in addition to $b - t - s$ quarks, since $s$ and $c$ quark belong to the same CKM sector.

![Figure 3: The Feynman diagrams including two loops of $B^+ \to K^+ e\bar{e}$ (left), and $B^+ \to K^+ \mu\bar{\mu}$ (right).](image3)
2.2  $B_s \to \ell\bar{\ell}$ and $B_d \to \ell\bar{\ell}$

The decay of $B_{s,d} \to \ell^+\ell^-$ ($\ell = e, \mu$ and $\tau$) in the standard model contains two $\gamma_5$ vertices and the intermediate state is $Z$ or $H$, as shown in Fig. 4.

Figure 4: The electro-weak interaction quark diagrams of $B_s \to \ell\bar{\ell}$ (left), and $B_d \to \ell\bar{\ell}$ (right).

They defined the Lagrangian for $B_q \to \ell\bar{\ell}$ as

$$\mathcal{L}_{\text{weak}} = N C_A(\mu_b) (\bar{b}\gamma_\alpha q)(\bar{\ell}\gamma^\alpha \gamma_5 \ell) + O(\alpha_{em})$$

and averaged time-integrated branching ratios proportional to

$$\bar{B}_{s\ell} \propto |N|^2 |C_A(\mu_b)|^2 \tau_s^s$$

$$\bar{B}_{d\ell} \propto f_{B_d}^2 |V_{cb}|^2 \left(|V_{tb}^* V_{ts}| / |V_{cb}| \right)^2 \tau_d^s,$$

where $\tau_s^s = 1 / \Gamma_s^s$ is the lifetime of the heaviest mass eigenstate of $B_s$, $V_{tb}, V_{ts}$ and $V_{cb}$ are CKM matrix elements, was calculated for $B_s \to \mu^+\mu^-$, and

$$\bar{B}_{d\ell} \propto f_{B_d}^2 |V_{td}|^2 \tau_d^{av},$$

where $\tau_d^{av} = 2 / (\Gamma_d^d + \Gamma_d^L)$ is the average of the lifetime of the lighter mass and heaviest mass eigenstate of $B$, $V_{tb}$ and $V_{td}$ are CKM matrix elements, was calculated for $B_d \to \mu^+\mu^-$. In the SM, the decay branching ratio of $B_d$ and $B_s$ yields direct information on $V_{bt}, V_{st}$ and $V_{bc}$, but uncertainty in the CKM matrix element $V_{bc}$ was not seriously taken into account.

Blanke et al. [18] presented the review of $K_L \to \ell\bar{\ell}$, $B \to K\ell\bar{\ell}$ and $B \to \ell\bar{\ell}$. According to their work experimental world average branching ratio

$$B(B_s \to \mu^-\mu^+) = (2.9 \pm 0.7) \times 10^{-9}$$
is slightly smaller than the theoretical value \((3.65 \pm 0.23) \times 10^{-9}\)\[14\].

In the case of \(B_d \rightarrow \mu^- \mu^+\), there are larger experimental uncertainty\[15\], however the experimental world average branching ratio

\[
\bar{B}(B_d \rightarrow \mu^- \mu^+) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}
\]

is much larger than the theoretical value \((1.06 \pm 0.09) \times 10^{-10}\)\[14\].

A two loop calculation of branching ratio including the diagram of the penguin diagram was done by \[10\] for the \(K_L \rightarrow \mu^+ \mu^-\). If a neutrino is a Majorana neutrino and \(\nu\) and \(\bar{\nu}\) are indistinguishable, one can consider the decay through the box diagram\[11, 10\]. But there is no clear experimental results that suggests \(\nu|_{\text{Majorana}} = \bar{\nu}|_{\text{Majorana}}\). If neutrino is massive, it is difficult to identify a neutrino and an antineutrino.

We expect the decay of \(K_L \rightarrow \mu^+ \mu^-\) and \(B \rightarrow \mu^+ \mu^-\) are not dominated by Fig. we study \(c\) quark contribution is important. In the case of \(K_L \rightarrow \mu^+ \mu^-\) we consider the \(q-\bar{q}-W\) triangle diagram with a penguin loop studied as \[10\] and the \(W^+ - W^- - q\) triangle diagram. We do not assume that the penguin loop is given by a gluon but by a \(Z\) boson. A gluon has unphysical components and it can be replaced by a gauge potential and ghost fields\[23\].

Since the \(s\) quark and the \(c\) quark are in the same CKM sector, we expect in the \(q-\bar{q}-W\) triangle, some contribution of \(c\bar{c}\) quark to the usual \(t\bar{t}\) quark contribution. Since a \(c\) quark is heavier than a \(s\) quark, the contribution is less important than \(B_d \rightarrow \ell \bar{\ell}\) or \(B_s \rightarrow \ell \bar{\ell}\).

In \[24\] we analyzed \(B_d \rightarrow K_L J/\Psi\) decay process using Cartan’s supersymmetry and compared the results with SM using CKM matrix elements. We perform the similar analysis for \(K\) meson and \(B\) meson decays into lepton pairs.

In the previous analysis of \(B_d \rightarrow K_L J/\Psi\), a vector particle was emitted from a quark or an anti-quark in a meson and transformed to \(J/\Psi\), but in the present case we consider processes in which a vector particle transform to \(\ell \bar{\ell}\).

The coupling of the vector particle which was the source of \(J/\Psi\) was pseudo scalar type

\[
t\bar{\psi} \gamma_\mu \gamma_5 \psi.
\]
The agreement of branching ratios of meson decays in theoretical models and the SM is not sufficient. One could imagine new physics, in which heavy mesons $X^+X^0$ instead of $W^+Z$ contribute\[^{15}\]. However, it may be appropriate to study supersymmetry of quarks or leptons and vector particles more in detail.

3 Cartan’s supersymmetry and meson decays

In the SM, the simplest amplitudes of $B \rightarrow \ell \bar{\ell}$ contains two $\gamma_5$ vertices, since electromagnetic decay is impossible. In the Cartan’s supersymmetric model, $W^+, W^-$ and $Z$ bosons are replaced by vector particles $x_1, x_2, x_3, x_4, x'_1, x'_2, x'_3$ and $x'_4$, and quarks and leptons are replaced by Dirac spinors $(\psi, \mathcal{C}\psi)$ and $(\phi, \mathcal{C}\phi)$. Incorporation of $\phi$ quarks allows decay amplitudes containing only one $\gamma_5$ vertex from a part of $i\bar{\phi}\gamma_\mu(1 - \gamma_5)\psi$.

3.1 $K_L \rightarrow \ell \bar{\ell}$ and $B_{s,d} \rightarrow \ell \bar{\ell}$

Simple diagrams of $K_L \rightarrow \ell \bar{\ell}$ containing the vertex corresponding to that of $W^+W^-Z$ are shown in Fig.5.

In our diagram of $K_L \rightarrow \ell \bar{\ell}$ shown in the left side of Fig.5, the symbol $x'_3$ corresponds to $Z$ boson with the momentum direction $x'_3$, the symbol $x_4$ corresponds to $W^+$ with the momentum direction $x_4$, and the symbol $x'_4$ corresponds to $W^-$ with the momentum direction $x'_4$. The vector particle $x_4$ couple with $\xi_{12}$ of $C\phi$ and create $\xi_{34}$ of anti-quark $\phi$ and the vector particle $x'_4$ is emitted from $\xi_3$ of $\psi$ and create the $\xi_{12}$.

In order to make coupling of a quark to $W^+$ and $W^-$ bosons, it is necessary to include a $\gamma_5$ vertex which couples $\phi$ and $\mathcal{C}\psi$ or $\psi$ and $\mathcal{C}\phi$. A quark emitted as $\mathcal{C}\phi = \xi_{12}$ can be absorbed as $\mathcal{C}\psi = \xi_{124}$ at the $\gamma_5$ vertex.

We extend the Cartan’s supersymmetric model to allow a spinor

$$\bar{\psi} = \xi_1i + \xi_2j + \xi_3k$$
and an antispinor

\[ \tilde{\phi} = \xi_{14}i + \xi_{24}j + \xi_{34}k \]

annihilate to a scalar particle \( Z = x'_4 \). Including the vertex \( \xi_4Z \) vertices, we can consider penguin diagrams of \([10]\), which are shown in the right hand side of Fig.5. In the diagrams there are triangle diagrams of \( q(t/c) - \bar{q}(\bar{t}/\bar{c}) - W \) boson \( (x'_4) \) and a loop of \( Z \) boson \( (x'_4/x_4) \). Due to this loop, coupling of a quark to the \( Z \) boson \( (x'_4) \) becomes possible, without including a \( \gamma_5 \) vertex. They appear as a correction from weak interactions.

In our model, we allow transformation of \( x_i \) to \( x'_i \) during propagations, and since \( K_L \) consists of \( \bar{s}d \), and \( \bar{s} \) quark and \( \bar{c} \) quark are in the same sector of CKM model, the \( q(\bar{q}) \) on the triangle is assumed to be \( t/c(\bar{t}/\bar{c}) \). In other words, we take into account the correction to the \( t \) quark loop dominance in the SM.

In the calculation of the right hand side of Fig.5 we fix the \( d \) quark and the \( \bar{s} \) quark, both large components and exchange a vector particle between \( d \) and \( \bar{s} \). In the first diagram, the \( d \) quark \( \xi_3 \) becomes after emitting a vector particle \( x_1' \), becomes a \( \bar{t} \) quark or a \( \bar{c} \) quark \( \xi_{24} \), and after emitting a particle \( x_4' \) corresponding to a \( Z \) boson becomes a quark \( \xi_2 \) and after emitting \( x'_4 \), becomes an anti-quark \( \xi_{31} \). The \( \bar{s} \) quark \( \xi_{34} \) emits a \( x_4 \) and becomes a quark \( \xi_{124} \). The \( C\psi\bar{C}\phi \) state of \( \xi_{124} \xi_{31} \) has an overlap with the vector particle \( x_1 \) and absorbs the previously emitted \( x'_1 \). The \( Z \)boson \( x'_4 \) becomes an \( \ell\bar{\ell} \) pair. By admitting an overlap of \( x_1x'_4 \) and \( x'_1x_4 \) state, \( d\bar{s} \rightarrow \ell\bar{\ell} \) transition becomes possible. The contribution of \( c \) or \( \bar{c} \) quark is expected to be not so important since \( m_c > m_s \) and virtual transition to \( t \) whose mass \( m_t > m_H \) is not expected to be affected.

In the case of \( B_d \) we replace the \( \bar{s} \) quark to \( \bar{b} \) quark. In the case of \( B_s \) we replace the \( d \) quark in \( B_d \) to an \( s \) quark and \( t/\bar{t} \) quark to \( c/\bar{c} \) quark, since a \( c \) quark is in the same CKM sector as an \( s \) quark, and \( m_c < m_b \), we need to incorporate the effect of a \( c \) quark.

The quark diagrams of \( B_d \rightarrow \ell\bar{\ell} \) processes are given in Fig.6 and that of \( B_s \rightarrow \ell\bar{\ell} \) processes are given in Fig.7.

The qualitative difference of experimental and theoretical branching ratios of \( \bar{B}_{s\bar{u}} \) and
may originate from the difference of quarks propagating on the triangle of two loop diagrams between \( s \) and \( b \) and \( d \) and \( b \). In the case of \( B_s \), since the \( s \) quark belongs to the same sector as the \( c \) quark, after emitting a vector particle \( x_1' \), it can be transformed to an antiquark \( \bar{c} \) and emits a vector particle \( x_4' \) which changes to \( \ell \bar{\ell} \), returns to a quark \( c \) which emits a vector particle \( x_4' \). The \( c \) quark absorbs the emitted vector particles \( x_1' \) and \( x_4' \) and transforms to a \( \bar{b} \) quark.

In the case of \( B_d \), the \( d \) quark is transformed to \( \bar{t} \) quark after emitting a vector particle \( x_i' \) as shown in the right-hand part of diagrams. The product \( V_{ud}V_{ub}^* \) of CKM matrices gives the strength, and the \( \bar{t} \) quark emits an \( x_4' \) and transforms to a \( \bar{b} \) quark after absorbing the emitted \( x_4' \) on the \( \gamma_5 \) vertex.

The relative intensity of the left-hand side diagrams and the right-hand side diagrams of Figs.5, 6, and Fig.7 are not evident. We expect the left-hand side dominates, and reflects the CKM matrix elements, however in \( B_s \to \ell \bar{\ell} \) the right-hand side diagram containing a \( c \) quark propagation gives a slight suppression to the left-hand side dominant term, and in \( B_d \to \ell \bar{\ell} \) the right-hand side diagram containing a \( t \) quark propagation is expected to give an enhancement to the dominant term.

4 Discussion and conclusion

Description of Dirac fermions is not unique. The sub algebra \( R_{3,1}^+ \) with a basis

\[
\{1, e_1e_2, e_1e_3, e_2e_3, e_1e_4, e_2e_4, e_3e_4, e_1e_2e_3e_4\}
\]

where \( e_i \) satisfy

\[
e_1^2 = e_2^2 = e_3^2 = 1, e_4^2 = -1
\]

is equivalent to Pauli algebra, defined by

\[
e_1e_4 = \sigma_1, e_2e_4 = \sigma_2, e_3e_4 = \sigma_3 \quad \text{and}
\]

\[
e_1e_2e_3e_4 = \sigma_1\sigma_2\sigma_3 = i.
\]

CKM matrices were derived based on this algebra[31].
In the $n$ dimensional linear space $V$ over a field $F$ and exterior algebra $\wedge V$, octonion appears by defining $e_1 e_2 e_3 = \ell$, and choosing 

$$\{1, e_1, e_2, e_3, -e_3 \ell, -e_2 \ell, -e_1 \ell, \ell\}$$

as the basis of the field. The commutation relations of $e_i$ are not same as those of Cartan’s $\xi_i$.

The rules of multiplication of $e_i$ follow Clifford algebra. Lounesto pointed out that a product of Clifford numbers $x$ and $y$ in $n$ dimensional linear space $V$ over a field $F$ defined by Chevalley is

$$xy = x \wedge y + x \cdot y = x \wedge y + B(x, y)$$

where $x \wedge y$ is the antisymmetric product, and $x \cdot y$ is the contraction which depends on $F$.

When $F = \{0, 1\}$ and an exterior algebra $\wedge V$ has the basis $\{1, e_1, e_2, e_1 \wedge e_2\}$, there are two bilinear forms $B_1(x, y) = x_1 y_2$ and $B_2(x, y) = x_2 y_1$ which do not have a canonical multiplication table.

The time(space) component of $\psi = \xi_4(\xi_i)$ and that of $\phi = \xi_0(\xi_i)$ satisfy the commutation relation

$$\frac{1}{2}(\xi_0 \xi_4 - \xi_4 \xi_0) = 1, \quad \frac{1}{2}(\xi_i \xi_i - \xi_i \xi_i) = 1 \quad (i = 1, 2, 3)$$

Similarly, those of $C \psi = \xi_{123}(\xi_{ij})$ and $C \phi = \xi_{ij3}(\xi_{ij})$ satisfy the commutation relation

$$\frac{1}{2}(\xi_{1234} \xi_{123} - \xi_{123} \xi_{1234}) = 1, \quad \frac{1}{2}(\xi_{ij} \xi_{ij} - \xi_{ij} \xi_{ij}) = 1 \quad (ij = 12, 23, 31).$$

Due to this difference of commutation relation of the space components and time components of $\psi$ and $\phi$, we can define unique octonion in $\mathbb{R}^8$ space.

We applied Cartan’s supersymmetry to analyze the decay branching ratios of $K$ and $B$ mesons to $\pi \ell \bar{\ell}$ and $\ell \bar{\ell}$. In the $B_{s,d} \rightarrow \mu^+ \mu^-$ process, we assigned vector particles $x_i$ to $Z$ boson of polarization $x_i$ and by allowing transition of $C \phi$ and $C \psi$ during the propagation of $b \rightarrow t \rightarrow s$, we could make $B_{s,d}$ composed of large components $\psi$ or $\phi$. 
The suppression of the $B_s \to \mu^+\mu^-$ can be explained, if one takes into account that the $c$ quark and $s$ quark are in the same sector, and correction to the CKM mechanism is large in $B_s \to \mu^+\mu^-$ than in $B_d \to \mu^+\mu^-$. When $s$ or $c$ quarks are not present in the meson system, the vertex of vector particles $x_4 - x_4' - x_j'(j = 1, 2, 3)$ and the triangular loop of top quarks and one vector particle $t - \bar{t} - x_j'$ defines the decay branching ratios. When $s$ or $c$ quarks are in the meson, the loop of $c - \bar{c} - x_j'$ modifies the $t - \bar{t} - x_j'$ contribution.

In the case of $B_d \to \mu^+\mu^-$, since $d$ quark is not in the sector of $c$, only the loop of $t - \bar{t} - x_j'$ contribute, and if it adds the contribution of the one loop of $x_4 - x_4' - x_j'(j = 1, 2, 3)$, we can explain the enhancement of $B_d \to \mu^+\mu^-$. In the processes of $B_s \to \mu^+\mu^-$ via exchange of Majorana neutrino[15], one allows a transition from $\nu$ to $\bar{\nu}$. In our model, $|e^-, \nu_e\rangle_L, |\mu^-, \nu_\mu\rangle_L, |\tau^-, \nu_\tau\rangle_L$ and their charge conjugates make sectors, and a neutrino in the sector of $|\mu^-, \nu_\mu\rangle_L$ can transform to $|\mu^+, \bar{\nu}_\mu\rangle^* \text{ or } |\mu^+, \bar{\nu}_\mu\rangle^{**}$ in the notation of [6], which can be interpreted as $\bar{\nu}_\mu$ in different triality sectors, which would not be detected by vector particles or leptons in the sector of the initial $|\mu^-, \nu_\mu\rangle_L$, since for detection of a particle $a_{i}^{k(j)}$ in the sector $k$, corresponding necessary condition $N_j$ also need to be transferred [30] from different sectors, and we may be allowed to ignore the $\nu \to \bar{\nu}$ transition.

If our electromagnetic detector can detect electromagnetic field transformed by $G_{23}$ but cannot detect electromagnetic field transformed by other four transformations, and there are six lepton sectors

$$|e, \nu_e\rangle^*, |\mu, \nu_\mu\rangle^*, |\tau, \nu_\tau\rangle^*, |e, \nu_e\rangle^{**}, |\mu, \nu_\mu\rangle^{**}, |\tau, \nu_\tau\rangle^{**}$$

which cannot be detected by our detectors, the presence of dark matter can be explained.

We observed that starting from the meson wave function given by the large component of a quark and the large component of an anti-quark, qualitative features of the strength of the decay branching ratios can be obtained from Cartan’s supersymmetry.

Cartan’s octonion satisfies a triality automorphism that is supersymmetric, which octonions of [32, 33] dont satisfy. In our system of fermions and vector fields with interaction
\( t\phi C(1 - \gamma_5)X\psi \), the operator \( \gamma_5 \) yields coupling between a large component of a quark and a large component of an antiquark which enhances certain \( B \) meson decay processes. It makes the branching ratio of \( B \) meson decays different from the CKM model.

If in the universe there are world which are transformed by \( G_{ij} \) and \( G_{ijk} \), and our electromagnetic detector can detect the world transformed by \( G_{12}, G_{13}, G_{123} \) and \( G_{132} \), and the uncertainty principle applies not only in our world but also whole universe, we can understand the presence of dark matter.

Departure from the standard model of \( B_s \to \mu\bar{\mu} \) and \( B_d \to \mu\bar{\mu} \) is due to the large correction from the \( c \) quark in the decay of \( B_s \) meson. We ignored \( K \to \pi\nu\bar{\nu} \) and \( B \to D\ell\nu \) etc, whose information will serve for establishing the model of \( B \) mesons based on Cartan’s supersymmetry.

References

[1] R. Aaij et al. (LHCb Collaboration), Measurement of form-factor independent observables in the decay \( B^0 \to K^{*0}\mu^+\mu^- \), Phys. Rev. Lett.111, 19,191801 (2013). arXiv:1308.1707[hep-ex].

[2] C. Bobeth, G. Hiller and D. van Dyk, General analysis of \( \bar{B} \to \bar{K}(\ast)\ell^+\ell^- \) decays at low recoil, Phys. Rev. D87,034016 (2013).

[3] C.Bobeth et al., \( B_{s,d} \to \ell^+\ell^- \) in the Standard Model and Reduced Theoretical Uncertainty, Phys. Rev. Lett.112, 101801(2014).

[4] R. Alonso, B. Grinstein and J.M. Camalich, \( SU(2) \times U(1) \) Gauge Invariance and the Shape of New Physics in Rare \( B \) Decays, Phys. Rev. Lett.113,241802 (2014).

[5] D. A. Sierra, F. Staub and A. Vicente, Shedding light on the \( b \to s \) anomalies with a dark sector, arXiv: 1503.0607 v1(2015).
[6] S. Furui, Cartan’s Supersymmetry and Weak and Electromagnetic Interactions, Few Body Syst. 56, 703(2015), DOI 10.1007/s00601-015-1016-6, arXiv:1502.04524 [hep-ph].

[7] G. Buchalla and A.J. Buras, The Rare Decays of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \mu^+\mu^-$ Beyond Leading Logarithms, Nucl. Phys. B412, 106(1994).

[8] M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, Minimal Flavour Violation Waiting For Precise Measurements of $\Delta M_s, S_{\psi\phi}, A_{S_{L}}^s, |V_{ub}|, \gamma$ and $B_{s,d}^0 \rightarrow \mu^+\mu^-$, arXiv:hep-ph/0604057v5.

[9] J. Brod, M. Gorbahn and E. Stamou, Two-Loop Electroweak Corrections for the $K \rightarrow \pi\nu\bar{\nu}$ Decays, Phys. Rev. D83, 034030 (2011); arXiv:1009.0947v2 [hep-ph].

[10] M. Gorbahn and U. Haisch, Charm-Quark Contribution to $K_L \rightarrow \mu^+\mu^-$ at Next-to-Next-to-Leading Order, arXiv hep-ph/0605203 v3.

[11] T. Inami and C.S. Lim, Prog. Theor. Phys.(Kyoto)65, 297 (1981).

[12] C.M. Becchi and G. Ridolfi, An Introduction to Relativistic Processes and the Standard Model of Electroweak Interactions 2nd Ed. Springer (2014).

[13] A.J. Buras, J. Girrbach-Noe, C. Niehoff and D.M. Straub, $B \rightarrow K^{(*)}\nu\bar{\nu}$ decays in the Standard Model and beyond, arXiv:1409.4557.

[14] C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou and M. Steinhauser, $B_{s,d} \rightarrow \ell^+\ell^-$ in the Standard Model with Reduced Theoretical Uncertainty, Phys. Rev. Lett.112, 101801(2014).

[15] V. Khachatryan et al. (CMS and LHCb Collaborations), Observation of the rare $B_s \rightarrow \mu^+\mu^-$ decay from the combined analysis of CMS and LHCb data, arXiv:1411.4413.

[16] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ in the Standard Model: Status and Perspectives, arXiv:1503.02693.
[17] Y. Amhis et al. (Heavy Flavour Averaging Group), arXiv:1207.1158 updates at http://www.slac.stanford.edu/xorg/hfag.

[18] M. Blanke, A.J. Buras and S. Recksiegel, Quark flavour observables in the Littlest Higgs model with T-parity after LHC Run 1, arXiv:1507.06316 v1 [hep-ph].

[19] F. Beayjean, Ch. Bobeth and S. Jahn, Constraints on tensor and scalar couplings from $B \to K \bar{\mu}\mu$ and $B_s \to \bar{\mu}\mu$, arXiv.1508.01526 v1[hep-ph].

[20] S. Aoki et al. (Flavour Lattice Averaging Group), Review of lattice results concerning low energy particle physics, arXiv:1310.8555.

[21] S. Furui, Fermion Flavors in Quaternion Basis and Infrared QCD, Few Body Syst. 52, 171(2012);

[22] É. Cartan, The theory of Spinors, Dover Pub. (1966) p.118.

[23] C.M. Becchi and G. Ridolfi, An Introduction to Relativistic Processes and the Standard Model of Electroweak Interactions, 2nd Edition, Springer (2014).

[24] S. Furui, Cartan’s Supersymmetry and the violation of CP symmetry, arXiv:1505.05830 physics.gen-ph].

[25] S. Furui, Cartan’s Supersymmetry and the Decay of $H^0 (0+, 125 \text{ GeV})$ to $\gamma\gamma, WW$ and $ZZ$, PoS, contribution to Flavor Physics & CP violation, Nagoya (2015).

[26] L. Randall and R. Sundrum, A Large Mass Hierarchy from a Small Extra Dimension, Phys. Rev. Lett. 83, 3370(1999); arXiv:hep-ph/9905221.

[27] L. Randall and R. Sundrum, An Alternative to Compactification, Phys. Rev. Lett. 83, 4690(1999); arXiv:hep-th/9906094.

[28] P. Biancofiore, P. Colangelo and F. De Fazio, Rare semileptonic $B \to K^*\ell^+\ell^-$ decays in RS$_c$ model, arXiv:1403.2944v2 (2014).
[29] P. Biancofiore, P. Colangelo, F. De Fazio and E. Scrimieri, Exclusive $b \rightarrow s\bar{u}\bar{u}$ induced transitions in $RS_c$ model, arXiv:1408.5614v1 (2014).

[30] M. Bitbol, The Concept of Measurement and Time Symmetry in Quantum Mechanics, Philosophy of Science 55, 349 (1988).

[31] G. Aragón-Camarasa et al., Clifford Algebra with Mathematica, J. Comput. Appl. Math. (2008).

[32] T. Dray and C.A. Manogue, Finding Octonionic Eigenvectors Using Mathematica, Computer Physics Communications (1998)

[33] T. Dray and C.A. Manogue, The Octonionic Eigenvalue Problem, Advances in Applied Clifford Algebras 8 341 (1998).

[34] I. R. Porteous, Clifford Algebra and the Classical Groups., Cambridge University Press (1995).

[35] P. Lounesto, Clifford Algebras and Spinors, 2nd Ed., Cambridge University Press (2001)

[36] C. Chevalley, Theory of Lie Groups, Overseas Publications LTD. (1965).
Figure 5: $K_L \rightarrow \ell \ell$ quark diagrams containing $x_4 x_4' x_{k'} (k = 1, 2, 3)$ vertices with different $\gamma_5$ positions (left), and $q - \bar{q} - W$ triangle diagrams with different penguin loop positions (right).
Figure 6: $B_d \rightarrow \ell \bar{\ell}$ quark diagrams containing $x_4 x'_4 x_{k'}$ ($k = 1, 2, 3$) vertices with different $\gamma_5$ positions (left), and $q - \bar{q} - W$ triangle diagrams with different penguin loop positions (right).
Figure 7: $B_s \rightarrow \ell \ell$ quark diagrams containing $x_4x_4'x_k'$ ($k = 1, 2, 3$) vertices with different $\gamma_5$ positions (left), and $q - q - W$ triangle diagrams with different penguin loop positions (right).