Asteroseismic analysis of 15 solar-like oscillating evolved stars

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ABSTRACT
Asteroseismology using space-based telescopes is vital to our understanding of stellar structure and evolution. CoRoT, Kepler, and TESS space telescopes have detected large numbers of solar-like oscillating evolved stars. Solar-like oscillation frequencies have an important role in the determination of fundamental stellar parameters; in the literature, the relations between the two is established by the so-called scaling relations. In this study, we analyse data obtained from the observation of 15 evolved solar-like oscillating stars using the Kepler and ground-based telescopes. The main purpose of the study is to determine very precisely the fundamental parameters of evolved stars by constructing interior models using asteroseismic parameters. We also fit the reference frequencies of models to the observational reference frequencies caused by the He II ionization zone. The 15 evolved stars are found to have masses and radii within ranges of 0.79-1.47 M⊙ and 1.60-3.15 R⊙, respectively. Their model ages range from 2.19 to 12.75 Gyr. It is revealed that fitting reference frequencies typically increase the accuracy of asteroseismic radius, mass, and age. The typical uncertainties of mass and radius are ∼ 3-6 and ∼ 1-2 per cent, respectively. Accordingly, the differences between the model and literature ages are generally only a few Gyr.

Key words: stars: evolution- stars: evolved -stars: fundamental parameters - stars: interiors - stars: oscillations.

1 INTRODUCTION
The determination of solar-like oscillation frequencies using information gathered by space telescopes is crucial to our understanding of stellar evolution and structure. Such oscillations are excited in stars with convective envelopes and have been detected in low-mass main-sequence (MS), sub-giant (SG), and red giant (RG) stars. Connecting interior stellar models to measured oscillation frequencies as well as other measurements, such as those obtained from spectroscopy, yields precise determinations of stellar parameters such as age (t), mass (M), and radius (R). It was also confirmed that fitting model reference frequencies to the observed reference frequencies decreases uncertainty in asteroseismic M, R, and t in comparison with uncertainties of the same parameters obtained by fitting large separations between oscillation frequencies (∆ν) and the frequency of maximum amplitude (νmax). Model stellar M and R are approximately several times more accurate than the M and R obtained from the scaling relations. Precisely determined fundamental parameters are broadly useful for a variety of efforts in astrophysics to test and develop theories of stellar and galactic evolution (e.g. Deheuvels et al. 2016; Nissen et al. 2017; Bellinger et al. 2017), and to understand the formation and evolution of exo-planetary systems (Chaplin and Miglio 2013, Campante et al. 2015, Campante et al. 2016, Kayhan, Yıldız & Çelik Orhan 2019, Jiang et al. 2020).

In this study, we constructed interior models of 15 evolved targets using the MESA code (Paxton et al. 2011, 2013) and used them to determine the stars’ fundamental properties under asteroseismic and non-asteroseismic constraints. The asteroseismic constraints used for calibration of interior models, which are derived from Kepler light curves for 14 targets (see Table 1 for the references) and from ground-based observations of one target (HD 2151, β Hyi, Bedding et al. 2007), included the reference frequencies discovered by Yıldız et al. (2014a) and customary asteroseismic parameters such as the ∆ν and νmax.

In the literature, there are many studies on the determination of the fundamental parameters of solar-like oscillating stars based on fitting of model adiabatic oscillation frequencies to observed frequencies (Li et al. 2020, Kayhan, Yıldız & Çelik Orhan 2019, Metcalfe et al. 2014, Mathur et al. 2012) and on the relating of asteroseismic parameters to non-asteroseismic properties (Yıldız, Çelik Orhan & Kayhan 2019; hereafter Paper IV, Mathur et al. 2012, White et al.
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2011. Tassoul (1980), Ulrich (1986) and Brown et al. (1991) have developed the general scaling relations between global asteroseismic observable parameters and fundamental stellar properties; subsequently Kjeldsen & Bedding (1995) scaled stellar properties to values observed in the Sun. We call these relations as the conventional scaling relations. \( M_{\text{scale}} \) and \( R_{\text{scale}} \) can be given as functions of mean \( \Delta \nu \) (\( \langle \Delta \nu \rangle \)), \( \nu_{\text{max}} \) and effective temperature \( (T_{\text{eff}}) \) as follows:

\[
\frac{M_{\text{scale}}}{M_{\odot}} = \left( \frac{\nu_{\text{max}}}{\nu_{\text{max,\odot}}} \right)^{3} \left( \frac{\langle \Delta \nu \rangle}{\langle \Delta \nu_{\odot} \rangle} \right)^{-4} \left( \frac{T_{\text{eff}}}{T_{\text{eff,\odot}}} \right)^{1.5}
\]

(1)

and

\[
\frac{R_{\text{scale}}}{R_{\odot}} = \left( \frac{\nu_{\text{max}}}{\nu_{\text{max,\odot}}} \right) \left( \frac{\langle \Delta \nu \rangle}{\langle \Delta \nu_{\odot} \rangle} \right)^{-5} \left( \frac{T_{\text{eff}}}{T_{\text{eff,\odot}}} \right)^{0.5},
\]

(2)

where \( \nu_{\text{max,\odot}} \) and \( \langle \Delta \nu_{\odot} \rangle \) are the solar values of \( \nu_{\text{max}} \) and \( \langle \Delta \nu \rangle \), respectively, and are taken as \( \nu_{\text{max,\odot}} = 3050 \) \( \mu \text{Hz} \) (Kjeldsen & Bedding 1995) and \( \langle \Delta \nu_{\odot} \rangle = 135.15 \) \( \mu \text{Hz} \) (Chaplin et al. 2014).

These conventional scaling relations inherently assume that the Sun and the other solar-like oscillating stars have similar internal structures, an assumption that has still been seriously tested for the stars in various evolutionary phases. Accordingly, using these assumptions to investigate stars with varying stellar structures and in different evolutionary phases can lead to systematic errors in the values of \( M \) and \( R \) obtained from the scaling relations (see e.g. Yıldız et al. 2015; hereafter Paper II, Epstein et al. 2014, Chaplin & Miglio 2013, Corsaro et al. 2013, Huber et al. 2013, Miglio et al. 2012). In the literature, many approaches have been proposed to reduce such systematic errors (Zinn et al. 2019, Sahliholdt et al. 2018, Viani et al. 2017, Huber et al. 2017, Sharma et al. 2016, Yıldız, Çelik Orhan & Kayhan 2016; hereafter Paper III, Guggenberger et al. 2016, Yıldız et al. 2014a; hereafter Paper I, Mosser et al. 2013, White et al. 2011).

Because they have outer and inner structures that differ significantly from those in the Sun, it is particularly important to test the scaling relations for evolved stars. By calibrating these models, we are able to determine the \( M \), \( R \), luminosities (\( L \)), and \( t \) of the targets with high precision. To this end, we select and analyse fourteen solar-like evolved stars detected by Kepler and one star detected through ground-based observation (HD 2151) and used the obtained data to calculate their values of \( M \) and \( R \) by applying equations (1) and (2), respectively. We then constructed interior models of the stars using the MESA evolution code, to reobtain \( M \), \( R \), and the other stellar parameters.

Oscillation frequencies of the MESA models are computed using the ADIPLS pulsation package (Christensen-Dalsgaard, 2008). We determine reference frequencies (\( \nu_{\text{min1}}, \nu_{\text{min2}} \)) using the methods given in Paper I and Paper II, and \( \langle \Delta \nu \rangle \) from observed and model oscillation frequencies. The typical uncertainties for \( M \) and \( R \) are found to be approximately 3 and 1 per cent, respectively.

This paper is organized as follows. The observational properties of solar-like oscillating evolved stars are presented in Section 2. We explain the basic properties of the MESA code and the models used in the study in Section 3. In Section 4, we present and discuss the model results, and we compare our fundamental stellar model parameters with those found in the literature. Finally, we present conclusions in Section 5.

2 PROPERTIES OF SOLAR-LIKE OSCILLATING EVOLVED STARS

We analyse the solar-like oscillations of the 15 selected evolved stars. In Fig. 1, the stars are plotted on the Hertzsprung-Russell (HR) diagram. The luminosity values \( (L_{\text{scale}}) \) on the diagram are obtained from asteroseismic values of \( R_{\text{scale}} \) derived from equation (2) and from the respective spectral effective temperatures \( (T_{\text{eff}}) \): \( L_{\text{scale}} = 4\pi R_{\text{scale}}^{2} T_{\text{eff}}^{4} \), where \( \sigma \) is Stefan-Boltzmann constant. The thin and thick lines represent, respectively, the zero age main-sequence (ZAMS) and terminal age main-sequence (TAMS) obtained from APKI models with solar composition (Yıldız 2015), while the filled circles indicate the 15 evolved stars. The observed oscillation frequencies of the targets, which included 13 SGs and 2 RG stars (KIC 7341231 and KIC 8561221), are obtained with a high degree of precision. Asteroseismic and non-asteroseismic observational data for the stars are listed in Table 1.

The spectroscopic observational data of the Kepler target stars, i.e., the gravity (\( \log g \)), metallicity ([Fe/H]) and \( T_{\text{eff}} \), are adapted from Bruntt et al. (2012) and Molenda-Żakowicz et al. (2013), while their \( \langle \Delta \nu \rangle \) and \( \nu_{\text{max}} \) values are taken from Chaplin et al. (2014). In this study, observed \( T_{\text{eff}} \) and \( \log g \) values selected from different spectral studies in Table 1 are most compatible with \( T_{\text{eff}} \) and \( \log g \) values obtained by different methods in Paper IV. Thus, when these values obtained from different methods are compared, they are very close to each other (see Fig. 7). The observed oscillation frequencies of the stars, with the exception of HD 2151 (Bedding et al. 2007), are taken from Appourchaux et al. (2012).

For most stars, oscillation frequencies (\( \nu_{\text{ini}} \) of many
modes with low degrees (l) and high order (n) are available, allowing the computation of $\Delta \nu$ and the small separation between oscillation frequencies ($\delta \nu_{22} = \nu_{n0} - \nu_{n-1,2}$). $\langle \delta \nu_{22} \rangle$ is the mean value of $\delta \nu_{22}$, and listed in Table 1.

Determination of the reference frequencies is an efficient method for yielding the mass, radius, and effective temperature of solar-like oscillating stars (Papers I-IV). In this study, we obtained observed reference frequencies from a graph of $\Delta \nu$ as a function of $\nu$. Although values of $\nu_{\min 1}$ are available for 14 of the stars, value of $\nu_{\min 0}$ is obtained for 12 stars (see Table 1), $\nu_{\min 2}$ values are not obtained for any of the stars.

The reference frequencies are due to glitches caused by the He II ionization zone. To determine these frequencies, we follow the method described in Paper I: after determining the frequency interval corresponding to each minimum on the $\Delta \nu$ versus $\nu$ graph, we draw two lines from the neighboring intervals and set their intersection as the reference frequency $\nu_{\min}$ for that minimum. The maximum uncertainties in $\nu_{\min 0}$ and $\nu_{\min 1}$ are about half of the uncertainties in large separation (Paper III). The uncertainties in $\nu_{\min 0}$ and $\nu_{\min 1}$ are given in Table 1.

1) The 15 target stars- HD 2151, KIC 5955122, KIC 7341231, KIC 7747079, KIC 7976303, KIC 8561221, KIC 8702606, KIC 10920273, KIC 11026764, KIC 11395018 and KIC 11414712 show mixed modes in their oscillation frequencies. Although the remaining stars are evolved, they featured no mixed mode behavior.

The effective temperature range of the stars is 5233 K (KIC 7341231) to 6145 K (KIC 10018963). The [Fe/H] values range from -1.64 (KIC 7341231) to 0.35 dex (KIC 11244118).

3 PROPERTIES OF THE MESA CODE AND MODELS

3.1 Properties of the MESA code

Stellar interior models are constructed using the MESA evolution code. Its version is 7184 (Paxton et al. 2011, 2013). Convection is treated using standard mixing-length theory (Bohm - Vitevce 1958). Convective overshooting (for an asteroseismic investigation on this subject, see Angelou et al. 2020) is not included. MESA incorporates the opacity tables developed by Iglesias & Rogers (1993, 1996) and includes their OPAL opacity tables in the high-temperature region supplemented by the low-temperature tables produced by Ferguson et al. (2005), with fixed metallicity set as the default option. The pre-main sequence is included in the construction of the stellar interior models. As the oscillation frequencies of the models are significantly influenced by atmospheric conditions, we applied the simple_photosphere option of the MESA code to construct the interior stellar models. The models do not include microscopic diffusion; nuclear reaction rates are adopted from Angulo et al. (1999) and Caughlan & Fowler (1988). The oscillation frequencies of the interior models are computed using the ADIPLS pulsation package (Christensen-Dalsgaard 2008).

3.2 Properties of the MESA models

The observed properties of the targets are used as either input parameters or constraints for the interior models. The input parameters of the MESA evolution code are $M$, helium abundance ($Y$), heavy element abundance ($Z$), and the convective parameter ($\alpha$). Stellar mass- the most important input parameter- is calculated from the scaling relations if no model value is available from the literature for the star in question. For the initial models, the solar values of $\alpha$ and $Y$ are used. To calibrate the interior model, we adjusted the values of $M$, $Y$, and $\alpha$ until a fit for the observed asteroseismic and non-asteroseismic constraints is obtained. Based on calibration of the solar model, the values of $Y$, $Z$, and $\alpha$ for the Sun are obtained as 0.2792, 0.0172, and 2.175, respectively.

In general, the $Z$ of a star is computed from its observed [Fe/H] value under the assumption that the heavy elements all have the same relative abundance as iron in the solar composition. However, iron is generally not the most abundant stellar element and is not a good indicator for all heavy elements. Thus, in the absence of detailed information on chemical composition, this issue can be partially overcome by using oxygen- generally the most abundant heavy element in a star- as superior indicator of total metallicity. For this purpose, Yıldız et al. (2014b) analysed the oxygen $O/H$ and [Fe/H] abundances produced by Edvardsson et al. (1993, see their fig. 15a-1) to derive a relationship between the two using data within the [Fe/H]=-0.5-0.5 dex. Following Yıldız et al. (2014b), we first calculate $O/H$ abundances from $[\text{Fe/H}]$ using this relationship and then used the results to obtain the total metallicities ($Z_0$). The model metallicities ($Z_{\text{mod}}$) of the 15 stars, taken as $Z_{\text{mod}} = Z_0$, are listed in Table 2. KIC 7341231, the [Fe/H] is found very low (-1.64) and, therefore, its total metallicity is set to 0.001.

The spectroscopic effective temperature and metallicity, ($\Delta \nu$) and the reference frequencies (and partly small separation between oscillation frequencies, $\langle \delta \nu_{22} \rangle$, see below) are all used as constraints to the interior models. This means that a calibrated model of any star should have the same value of these observables within the range of uncertainties. The input parameters $M$, $Y$, and $\alpha$ are modified in an amount found from comparison of model values with observational constraints during the calibration procedure:

i) At the beginning of calibration, we compute $M_{\text{calc}}$ and $R_{\text{calc}}$, using equations (1) and (2). We take initial value of $M$ as either $M_{\text{fit}}$ or $M_{\text{calc}}$. For $Y$ and $\alpha$, the initial values are the solar values. During the calibration, $R$ and $L$ are taken as $R_{\text{calc}}$ and $L_{\text{calc}}$, respectively.

ii) We calibrate model in the HR diagram by changing $\alpha$, and then compute $\langle \Delta \nu \rangle_{\text{mod}}$, $\nu_{\min 0, \text{mod}}$, and $\nu_{\min 1, \text{mod}}$.

iii) From comparison of model and observed (\$\langle \Delta \nu \rangle\$), we estimate new value of $M$. It is computed from the (\$\langle \Delta \nu \rangle$)-\$\rho$ relation: $\langle \Delta \nu \rangle_{\text{mod}} = \langle \Delta \nu \rangle_{\text{obs}} + \langle \Delta \nu \rangle_{\text{mod}} = \langle \Delta M / M_{\text{mod}} \rangle / 2$, because $R_{\text{mod}}$ is already fitted to $R_{\text{calc}}$ in the second step. If $\langle \Delta \nu \rangle_{\text{mod}}$ is fitted to $\langle \Delta \nu \rangle_{\text{obs}}$, then $M$ is fixed and we compare model and observed reference frequencies.

iv) $X_{\text{calc}}$ is used in order to examine the fit of the model parameters with the observed parameters. If $X_{\text{calc}}$ is small enough, $X_{\text{calc}} < 1$, model is calibrated. If $X_{\text{calc}}$ is not small, we try to fit model reference frequencies to observed reference frequencies by changing $Y$ and go to the second step.

v) If agreement between model and observations is not satisfactory, we slightly modify $T_{\text{eff}}$ and then start from
Table 1. Observed properties of evolved solar-like oscillating stars. The listed asteroseismic and non-asteroseismic properties are $T_{\text{eff}}$, logarithm of surface gravity (cgs), $[\text{Fe/H}]$, $(\Delta \nu)$, mean small separation $(\langle \delta \nu \rangle)$, $\nu_{\text{max}}, \nu_{\text{min0}}, \nu_{\text{min1}},$ and references for the observational data.

| Star           | $T_{\text{eff}}$ (K) | $\log g$ (cgs) | $[\text{Fe/H}]$ (dex) | $(\Delta \nu)$ (µHz) | $\langle \delta \nu \rangle$ (µHz) | $\nu_{\text{max}}$ (µHz) | $\nu_{\text{min0}}$ (µHz) | $\nu_{\text{min1}}$ (µHz) | ref         |
|----------------|----------------------|----------------|------------------------|-----------------------|---------------------------------|---------------------------|---------------------------|---------------------------|-------------|
| HD 2151        | 5873±45              | 3.98±0.02      | -0.11±0.08             | 57.5±0.6              | 5.11±1.21                      | 10000.6±30                | 1120.44±0.30             | 890.33±0.30               | 2.3,4       |
| KIC 5955122    | 5865±70              | 3.88±0.08      | 0.06±0.06              | 49.2±0.9              | 5.20±0.31                      | 861.0±24                  | 952.67±0.45              | 721.07±0.45               | 1.5,7,13,15 |
| KIC 7341231    | 5233±50              | 3.55±0.03      | -1.64±0.06             | 29.2±0.7              | 3.40±0.17                      | 408.0±8                   | —                         | 342.04±0.35               | 1.7,9       |
| KIC 7747078    | 5840±70              | 3.98±0.08      | -0.11±0.06             | 53.9±0.3              | 4.67±0.17                      | 936.0±32                  | 1037.89±0.15             | 793.81±0.15               | 1.5,7,14,15 |
| KIC 7976303    | 6053±70              | 3.87±0.08      | -0.41±0.06             | 51.7±0.6              | 4.46±0.39                      | 851.0±20                  | 980.4±0.30               | 777.44±0.30               | 5,7,13      |
| KIC 828742     | 6042±70              | 4.02±0.08      | 0.00±0.06              | 61.8±0.6              | 4.80±0.37                      | 1171.0±34                 | —                         | 1038.97±0.30              | 1.5,7,13,15 |
| KIC 8524425    | 5643±70              | 3.98±0.08      | 0.13±0.05              | 59.4±0.6              | 4.95±0.13                      | 1081.0±28                 | 1128.00±0.30             | 897.61±0.30               | 1.5,7,14,15 |
| KIC 8561221    | 5247±70              | 3.61±0.06      | -0.04±0.06             | 29.6±0.4              | 2.40±0.14                      | 467.0±23                  | 530.46±0.20              | 398.87±0.20               | 5,12,15     |
| KIC 870260     | 5540±70              | 3.98±0.08      | -0.06±0.05             | 39.9±0.5              | 3.59±0.16                      | 664.0±16                  | 688.67±0.25              | 530.5±0.25                | 1,5,10,15   |
| KIC 10018663   | 6145±112             | 3.95±0.21      | -0.16±0.05             | 55.5±0.5              | 5.06±0.23                      | 987.0±32                  | 997.0±0.25               | —                         | 1,7,14,15   |
| KIC 1090273    | 5710±75              | 4.15±0.08      | -0.02±0.07             | 57.3±0.1              | 4.84±0.23                      | 990.0±60                  | 1111.50±0.05             | 882.77±0.05               | 6,8,11      |
| KIC 11026764   | 5682±70              | 3.88±0.08      | 0.11±0.06              | 50.5±0.6              | 4.50±0.18                      | 895.0±29                  | 924.01±0.30              | 723.52±0.30               | 1,5,7,15    |
| KIC 11244118   | 5745±70              | 4.09±0.08      | 0.35±0.06              | 71.3±0.9              | 5.50±0.15                      | 1420.0±31                 | —                         | 1166.25±0.45              | 1,5,7,14,15 |
| KIC 11395018   | 5445±85              | 3.84±0.12      | 0.13±0.07              | 47.5±0.1              | 5.50±0.30                      | 834.0±50                  | 875.30±0.05              | 685.40±0.05               | 1,8,11      |
| KIC 11414712   | 5635±70              | 3.80±0.08      | -0.02±0.05             | 43.5±0.7              | 4.19±0.21                      | 707.0±20                  | 800.43±0.35              | 628.85±0.35               | 1,5,7,15    |

1: Appourchaux et al.(2012), 2: Bedding et al.(2007), 3: Brandão et al.(2011), 4: Bruntt et al.(2010), 5: Bruntt et al.(2012), 6: Campante et al.(2011), 7: Chaplin et al.(2014), 8: Creevey et al.(2012), 9: Deheuvels et al.(2012), 10: Deheuvels et al.(2014), 11: Doğan et al.(2013), 12: García et al.(2012), 13: Mathur et al.(2012), 14: Metcalfe et al.(2014), 15: Molenda-Zakowicz et al.(2013).
Thereof have no available reference frequencies, we limited MS models with various masses, as function of $\Gamma_{1s}$. The values of $\Delta\nu$ for the best-fitting models are plotted as function of $\Gamma_{1s}$ in Fig. 2, in which the fitting formula given in equation (6) is represented by the solid line. The evolved stars and the MS models from Paper III are represented by filled circles and circles, respectively. All of SGs and the RG (KIC 8561221) are located slightly above the MS models and clustered around a $\Gamma_{1s}/\Gamma_{1s}^{\odot} = 0.570$.

4.2 $\nu_{\text{min}}$ of evolved stars

$\Delta\nu$ of the model oscillation frequencies and observed frequencies of each star are plotted as a function of $\nu$ (see Fig.
9). Based on this comparison, we are able to ensure that the observed and model minimum frequencies are close to each other. In addition to fitting the observed $\Delta \nu$ values to the models, more precise models could be obtained by fitting them to the observed minima values.

The $\chi^2_{\nu_{\text{min}}}^2$ values obtained form equation (4) are used to assess the similarities between the model and the observed oscillation frequencies interms of $\nu_{\text{min}}$. The results obtained in this process are listed in Table 3.

In Fig. 3, the observed and model minimum frequencies, $\nu_{\text{min0}}^{\text{obs}}$ (filled circles) and $\nu_{\text{min0}}^{\text{mod}}$ (pluses) are compared. For KIC 7341231 and KIC 11244118, the values of $\nu_{\text{min0}}$ corresponding to the high frequencies could be obtained from neither the observations nor the models. There are good agreement between model and observational values of $\nu_{\text{min0}}$. The same is true for $\nu_{\text{min1}}$.

Fig. 4 compares the values of $\nu_{\text{min1,obs}}$ (filled circles) and $\nu_{\text{min1,mod}}$ (pluses). It is seen from the figure that the observed and model oscillation frequencies are in excellent agreement, with the model values generally falling within the error ranges of the observed $\nu_{\text{min1}}$ values.

### 4.3 Comparison of radii

The oscillation frequencies observed in a star depend on the its mean density and the mean density is inversely proportional to the radius. By fitting the observed oscillation frequencies with those obtained from the model, it is possible to determine the stellar radius very precisely from the interior models. Fig. 5, shows a comparison of the stellar radii produced by MESA model ($R_{\text{mod}}$), with those obtained from the literature ($R_{\text{lit}}$). For majority of stars, they are in good agreement, so that the difference is less than 4 per cent. Significant discrepancy appears for KIC 8702606 and KIC 11414712. For these two stars, the difference between $R_{\text{lit}}$ and $R_{\text{mod}}$ is about 6 per cent.

In Paper IV, fitting formulæ are derived for radius as a function of $\nu_{\text{min0}}$ ($R_{\text{dias0}}$) and $\nu_{\text{min1}}$ ($R_{\text{dias1}}$) from the MS models. In Fig. 6, $(R_{\text{dias0}} - R_{\text{mod}})/R_{\text{mod}}$ and $(R_{\text{dias1}} - R_{\text{mod}})/R_{\text{mod}}$ are plotted with respect to $R_{\text{mod}}$. Both of $R_{\text{dias0}}$ and $R_{\text{dias1}}$ (MS models) are in very good agreement with $R_{\text{mod}}$ (post-MS models) if $R_{\text{mod}} < 2.2R_\odot$. Despite the different evolutionary phases. For these stars $R_{\text{dias0}}$ and $R_{\text{dias1}}$ are slightly greater than $R_{\text{mod}}$, about 1.5 per cent ($\approx 0.029R_\odot$). This implies that the derived formulæ for $R_{\text{dias0}}$ and $R_{\text{dias1}}$ are also suitable for such stars, namely SGs. For the larger stars, the difference becomes larger. The three stars with $R_{\text{mod}} > 2.2R_\odot$ (KIC 7341231, KIC 8561221 and KIC 8561221) have $T_{\text{eff}} < 5450$ K and log $g < 3.77$.

The model results and the literature values for mass and radius are summarized in Table 4. For comparison, masses and radii found in Papers III and IV are also listed.
4.4 Comparison of $T_{\text{eff}}$

$T_{\text{eff}}$ is another important parameter in asteroseismic scaling relations for stellar mass and radius. $T_{\text{eff}}$, obtained from stellar spectrum, has an uncertainty of nearly 200 K (Bruntt et al. 2012), which has a critical role in the scaling relations (see equations 1 and 2). For example, if the effective temperature of a star is $T_{\text{eff}} = 5800$ K with an uncertainty of 100 K, the uncertainty in the mass and radius values obtained from the scaling relations will be 2.6 and 0.9 per cent, respectively.

In Paper IV, the effective temperatures of 90 solar-like oscillating stars are computed using five different methods: the obtained results are $T_{\text{VK}}$ (from V-K colour), $T_{\text{AV}}$ (from B-V colour), $T_{\text{av0}}$ (from $\nu_{\text{min0}}$), $T_{\text{av1}}$ (from $\nu_{\text{min1}}$) and $T_{\text{av2}}$ (from $\nu_{\text{min2}}$). Fig. 7 shows a comparison of $T_{\text{mod}}$ with $T_{\text{av}}$, $T_{\text{av0}}$, $T_{\text{av1}}$ and $T_{\text{mod}}$ (from $\nu_{\text{min2}}$). The reason behind this discrepancy should be investigated.

The degree of agreement with the non-asteroseismic ob-
Table 4. Listing of masses and radii of all the evolved stars in this study. $M_{\text{mod}}$ and $R_{\text{mod}}$ are determined from conventional scaling relations. $M_{\text{HII}}$ and $R_{\text{HII}}$, and $M_{\text{IV}}$ and $R_{\text{IV}}$ are calculated using formulas from Paper III and IV, respectively. MESA masses and radii are given as $M_{\text{lit}}$ and $R_{\text{lit}}$, respectively. $M_{\text{HII}}$ and $R_{\text{HII}}$ are masses and radii, respectively, from the literature.

| Star          | $M_{\text{mod}}$ | $R_{\text{mod}}$ | $M_{\text{HII}}$ | $R_{\text{HII}}$ | $M_{\text{IV}}$ | $R_{\text{IV}}$ | $M_{\text{lit}}$ | $R_{\text{lit}}$ |
|---------------|------------------|------------------|------------------|------------------|-----------------|-----------------|-----------------|-----------------|
| HD 2151       | 1.01±0.01        | 1.75±0.02        | 1.01±0.15        | 1.75±0.10        | 1.10±0.15       | 1.83±0.03       | 1.09±0.03       | 1.83±0.02       |
| KIC 5955122   | 1.18±0.23        | 2.06±0.15        | 1.16±0.24        | 2.05±0.16        | 1.25±0.03       | 2.11±0.02       | 1.17±0.04       | 2.09±0.02       |
| KIC 7341231   | 0.7±0.159        | 2.58±0.20        | 0.79±0.10        | 2.58±0.21        | 0.81±0.04       | 2.69±0.20       | 0.79±0.02       | 2.59±0.03       |
| KIC 7747078   | 1.26±0.17        | 1.99±0.10        | 1.26±0.17        | 1.99±0.10        | 1.12±0.02       | 1.94±0.10       | 1.10±0.03       | 1.92±0.02       |
| KIC 7976303   | 1.45±0.15        | 2.17±0.10        | 1.42±0.15        | 2.14±0.10        | 1.15±0.02       | 2.11±0.01       | 1.06±0.03       | 1.97±0.02       |
| KIC 8228742   | 1.42±0.18        | 1.90±0.10        | 1.39±0.19        | 1.87±0.10        | 1.24±0.02       | 1.81±0.01       | 1.15±0.03       | 1.78±0.02       |
| KIC 8524425   | 1.11±0.15        | 1.78±0.09        | 1.12±0.16        | 1.79±0.09        | 1.10±0.02       | 1.79±0.01       | 1.07±0.03       | 1.79±0.02       |
| KIC 8561221   | 1.39±0.10        | 3.07±0.08        | 1.38±0.10        | 3.07±0.08        | 1.56±0.01       | 3.19±0.01       | 1.47±0.04       | 3.15±0.04       |
| KIC 8702606   | 1.31±0.19        | 2.47±0.14        | 1.32±0.19        | 2.48±0.14        | 1.13±0.02       | 2.37±0.01       | 1.27±0.04       | 2.49±0.03       |
| KIC 1001863   | 1.34±0.20        | 1.99±0.12        | 1.30±0.21        | 1.96±0.12        | 1.24±0.02       | 1.95±0.01       | 1.16±0.03       | 1.92±0.02       |
| KIC 10920273  | 1.07±0.20        | 1.81±0.17        | 1.08±0.30        | 1.82±0.17        | 1.11±0.02       | 2.03±0.01       | 1.06±0.03       | 1.82±0.02       |
| KIC 11026764  | 1.28±0.21        | 2.10±0.13        | 1.29±0.21        | 2.11±0.13        | 1.13±0.02       | 2.03±0.01       | 1.11±0.03       | 2.03±0.02       |
| KIC 11244118  | 1.07±0.18        | 1.57±0.09        | 1.08±0.18        | 1.57±0.09        | 1.21±0.02       | 1.64±0.01       | 1.09±0.03       | 1.60±0.02       |
| KIC 11395018  | 1.27±0.31        | 2.17±0.19        | 1.28±0.31        | 2.17±0.19        | 1.11±0.02       | 2.10±0.01       | 1.13±0.03       | 2.11±0.02       |
| KIC 11414712  | 1.15±0.18        | 2.23±0.14        | 1.16±0.18        | 2.24±0.14        | 1.08±0.03       | 2.18±0.02       | 1.06±0.03       | 2.19±0.03       |

Figure 8. Comparison of stellar ages obtained using the MESA models and the ages from the literature. Uncertainty of $t_{\text{lit}}$ is not available for KIC 8702606, KIC 7341231 and KIC 11414712. Filled circles are the ages from Li et al. (2020).

4.5 Comparison of ages

Stellar age is one of the essential parameters for the understanding of stellar structure. The age of star cannot be directly determined using observational data but must instead be estimated using an interior model. As many different models for reconstructing age from limited observational data are available, a given star will have conflicting age estimates. To delimit age of stars in our study, we used observed asteroseismic and non-asteroseismic data.

Fig. 8 shows the estimated ages of the target stars obtained from interior models ($t_{\text{mod}}$) versus ages in the literature ($t_{\text{lit}}$). $t_{\text{lit}}$ is taken from Deheuvels et al. (2014), Metcalfe et al. (2014), Garcia et al. (2014), Doğan et al. (2013) and Brandono et al. (2011). For majority of the stars, $t_{\text{lit}}$ and $t_{\text{mod}}$ are in good agreement, the difference between them is less than 1 Gyr for the nine targets. Also shown in Fig. 8 is the recent ages ($t_{\text{Li}}$) obtained by Li et al. (2020) for ten stars. The greatest difference between $t_{\text{Li}}$ and $t_{\text{mod}}$ is for KIC 10920273, about 5.19 Gyr. However, $t_{\text{mod}} - t_{\text{lit}}$ is just 0.15 Gyr for this star.

The stellar mass has the most significant influence on the determined age of a star. Although the masses of stars calculated in the literature differ from those calculated using MESA, the differences between the two ages are generally only a few Gyr. Based on this, the model oscillation frequencies could be fitted to the observed oscillation frequencies to obtain age estimates that are independent of the input parameters of the model.

5 CONCLUSIONS

In this study, we constructed models of 15 evolved stars using the MESA code and used the MESA models in conjunction with the stars' observed oscillation frequencies to determine the fundamental properties of the stars. As oscillation frequency data, we used observations from both a ground-based telescope and the *Kepler* space telescope. The observed asteroseismic ($\Delta\nu$, $\nu_{\text{max}}$, $\nu_{\text{min1}}$, $\nu_{\text{min2}}$) and non-asteroseismic ($T_{\text{eff}}$, $\log g$, [Fe/H]) parameters are compared with model properties obtained using the MESA code. The 15 evolved stars are estimated to have masses and radii within the ranges of 0.79 – 1.47 $M_{\odot}$ and 1.60 – 3.15 $R_{\odot}$, respectively, with typical uncertainties of ~ 3-6 per cent and

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~ 1-2 per cent in terms of mass and radius, respectively. By fitting the reference frequencies, typically the accuracy of the asteroseismic radius, mass, and age is much better than that determined from $\Delta \nu$ and $\nu_{\text{max}}$. Also, the stellar $R$ and $M$ determined from the interior models are more accurate than the $R$ and $M$ obtained from the different scaling relations.

Metallicity is crucial to the understanding of the evolution and structure of stars. In the literature, the metallicity of a star is generally calculated from its [Fe/H] value, a factor that might not in fact represent the exact metallicity of the star. In this study, metallicity is calculated from the relation between [O/H] and [Fe/H] obtained from Yıldız et al. (2014b). Based on this metallicity, the stellar interior models are constructed using the MESA evolution code.

It is quite difficult to determine the age of a star from the observations; instead, the age of individual stars must be calculated from interior models. The results of these models should be tested using different methods or observed data. The estimated ages for the 15 evolved stars in this study (Table 2) are all compared with ages available in the literature. In addition, the values of $M$, $Z$, mixing-length parameter ($\alpha$), and $Y$ determined using MESA are found to vary significantly from those produced by models in the literature. However, values of $t_{\text{mod}}$ (Fig. 8) are found to be in close agreement with $t_{\text{mod}}$. The difference between the ages obtained from different evolution codes provided correct ages within a range of several Gyr.

Determining the asteroseismic scaling relations for solar-like stars is very important because they can be used to derive the fundamental parameters of the single stars from observed properties ($\langle \Delta \nu \rangle$, $\nu_{\text{max}}$ and $T_{\text{eff}}$). In this study, we tested the scaling relations for the selected evolved stars. In doing so, we attempted to fit the reference frequencies, $\langle \Delta \nu \rangle$, and $\nu_{\text{max}}$ to ensure compatibility between the model and observed oscillation frequencies. In particular, the models that are constructed to fit the minima provided much more accurate results.

It is found that the asteroseismic effective temperature $T_{\text{eff}}$ are in good agreement with $T_{\text{mod}}$ and $T_{\text{mod}}$. However, there is significant temperature difference between $T_{\text{eff}}$ and $T_{\text{mod}}$. Despite the different evolutionary phases, new expressions obtained for $R_{\text{mod}}$ and $R_{\text{ast}}$ from MS models in Paper IV give very consistent results with $R_{\text{mod}}$, if $R_{\text{mod}} < 2.2R_\odot$. For these stars $R_{\text{ast}}$ and $R_{\text{mod}}$ are only 1.5 per cent greater than $R_{\text{mod}}$. Much more explicit formulae can be developed for $R_{\text{ast}}$ and $R_{\text{mod}}$ for SGs and RGs.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

REFERENCES

Angelou G. C., Bellinger E. P., Hekker S., Mints A., Elsworth Y., Basu S., Weiss A., 2020, MNRAS, 493, 4987
Angulo C. et al., 1999, Nucl. Phys. A, 656, 3
Appourchaux T. et al., 2012, A&A, 543, A54
Bedding, T. R., Kjeldsen, H., Arensfoft, T., et al. 2007, ApJ, 663, 1315
Brown T. M., Gilliland R. L., Noyes R. W., Ramsey L. W., 1991, ApJ, 368, 599
Bellinger E. P., Basu S., Hekker S., Ball W. H., 2017, ApJ, 851, 80. doi:10.3847/1538-4357/aa9f48
Böhm- Vitense E., 1958, Z. Astrophys., 46, 108
Brandao I. M., Doğan G., Christensen-Dalsgaard, J., Cunha M. S., Bedding T. R., Metcalfe T. S., Kjeldsen H., Bruntt H., Arentoft T., 2011, A&A, 527, A37
Broomhall A.-M., Chaplin W. J., Davies G. R., Elsworth Y., Fletcher S. T., Hale S. J., Miller B., et al., 2009, MNRAS, 396, L100. doi:10.1111/j.1745-3933.2009.00672.x
Bruntt H. et al., 2010, MNRAS, 405, 1907
Bruntt H. et al., 2012, MNRAS, 231, 122
Campante T. L. et al., 2011, A&A, 534,A46
Campante T. L., Barclay T., Swift J. J., Huber D., Adibekyan V. Z., Cochran W., Burke C. J., et al., 2015, ApJ, 799, 170. doi:10.1088/0004-637X/799/2/170
Campante T. L., Schofield M., Küsselwicz J. S., Bouma L., Chaplin W. J., Huber D., Christensen-Dalsgaard J., et al., 2016, ApJ, 830, 138. doi:10.3847/0004-637X/830/2/138
Caughalten G. R., Fowler W. A., 1988, At. Data Nucl. Data Tables, 40, 283
Chaplin W. J., Miglio A., 2013, ARA&A, 51, 353
Chaplin W. J. et al., 2014, ApJS, 210, 1
Christensen-Dalsgaard J., 2008, Ap&SS, 316, 113
Creevey O. L. et al., 2012, A&A, 545, A17
Corsaro E., Frohlich H.-E., Bonanno A., Huber D., Bedding T. R., Benomar O., De Ridder J., Stello D., 2013, MNRAS, 430, 2313
Deheuvels S. et al., 2012, ApJ, 756, 19
Deheuvels S. et al., 2014, A&A, 564, A27
Deheuvels S. et al., 2016, A&A, 589, A93. doi:10.1051/0004-6361/201527967
Doğan G. et al., 2013, ApJ, 763, 49
Edvardsson, B., Andersen, J., Gustafsson, B., et al., 1993, A&A,275, 101E
Epstein C. R. et al., 2014, ApJ, 785, L28
Ferguson J. W., Alexander D. R., Allard F., Barman T., Bodnarik J. G., Hauschildt P. h., Heffner- Wong A., Tammanai A., 2005, ApJ, 623, 585
Garcia R. A., et al., 2014, A&A, 572, A34
Guggenberger E., Hekker S., Basu S., Bellinger E., 2016, MNRAS, 460, 4277
Huber D. et al., 2013, ApJ, 767 127
Huber D., Zinn J., Bojsen-Hansen M., Pinsonneault M., Saliholdt C., Serenelli A., Silva Aguirre V., et al., 2017, ApJ, 844, 102. doi:10.3847/1538-4357/aa75ca
Iglesias C. A., Rogers F. J., 1993, ApJ, 412, 752
Iglesias C. A., Rogers F. J., 1996, ApJ, 464, 943
Jiang C., Bedding T. R., Stassun K. G., Veras D., Corsaro E., Buzasi D. L., Mikolajczek P., et al., 2020, ApJ, 896, 65. doi:10.3847/1538-4357/ab8f29
Kayhan C., Yıldız M., Çelik Orhan Z., 2019, MNRAS, 490, 1509
Kjeldsen H., Bedding T. R., 1995, A&A, 293, 87
Z. Çelik Orhan, M. Yıldız and C. Kayhan

Li T., Bedding T. R., Christensen-Dalsgaard J., Stello D., Li Y., Keen M. A., 2020, MNRAS, 495, 3431
Mathur S. et al., 2012, ApJ, 749, 152
Metcalfe T. S. et al., 2014, ApJS, 214, 27
Metcalfe T. S. et al., 2010, ApJ, 723, 1583
Miglio A. et al., 2012, MNRAS, 419, 2077
Molenda-Zakowicz J. et al., 2013, MNRAS, 434, 1422
Mosser B. et al., 2013, A&A, 559, A137
Nissen P.E. et al., 2017, A&A, 608, A112
Paxton B., Bildsten L., Dotter A., Herwing F., Lesaffre P., Timmes F., 2011, ApJS, 2011, 192
Paxton B., Catteino M., Arras P., Bildsten L., Brown E. F., Dotter A., Mankovich C., Montgomery M. H. et al., 2013, ApJS, 208, 49
Sahlholdt C. L., Silva Aguirre V., Casagrande L., Mosumgaard J. R., Bojsen-Hansen M., 2018, MNRAS, 476, 1931. doi:10.1093/mnras/sty319
Sharma S., Stello D., Bland-Hawthorn J., Huber D., Bedding T. R., 2016, ApJ, 822, 15
Silva Aguirre V., Davies G. R., Basu S., Christensen-Dalsgaard J., Creevey O., Metcalfe T. S., Bedding T. R., et al., 2015, MNRAS, 452, 2127. doi:10.1093/mnras/stv1388
Tassoul M., 1980, ApJS, 43, 469
Ulrich R. K., 1986, ApJ, 306, L37
Viani L. S., Basu S., Chaplin W. J., Davies G. R., Elsworth Y., 2017, ApJ, 843, 11. doi:10.3847/1538-4357/aa729c
White T. R. et al., 2011, ApJ, 742, L3
Yıldız M., 2008, MNRAS, 388, 1143
Yıldız M., Çelik Orhan Z., Aksoy C., Ok S., 2014a, MNRAS, 441, 2148 (Paper I)
Yıldız M., Çelik Orhan Z., Kayhan C., Türkoglu G. E., 2014b, MNRAS, 445, 4395
Yıldız M., Çelik Orhan Z., Kayhan C., 2015, MNRAS, 448, 3689 (Paper II)
Yıldız M., 2015, RAA, 15, 2244
Yıldız M., Çelik Orhan Z., Kayhan C., 2016, MNRAS, 462, 1577 (Paper III)
Yıldız M., Çelik Orhan Z., Kayhan C., 2019, MNRAS, 489, 1753 (Paper IV)
Zinn J. C., Pinsonneault M. H., Huber D., Stello D., Stassun K., Serenelli A., 2019, ApJ, 885, 166. doi:10.3847/1538-4357/ab44a9

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