Accelerated Sampling Research for Task Optimized Strategy for Multi-state Systems

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ABSTRACT The complex multi-state systems joint operation simulation and two task strategies are explored. For Monte Carlo sampling, based on the indirect sampling method, the principle of accelerated sampling methods, including forced transition FT and fault biasing FB, is studied. It is found that the direct adoption of the current FT and FB will result in systematic bias for the calculation of task success metrics. For the bias, state-transition chain truncation is studied, and its definition and truncation rules are proposed. In case study, after the simulation of various parameter combinations, it is found that when the sailing time is relatively smaller than the equipment reliability, which means a small probability event for the sampling of equipment faults, the ASM based on indirect sampling is applicable. Otherwise, the conventional sampling method should be used. The proposed state-transition chain truncation rule is also verified in case study.

INDEX TERMS Multi-state systems (MSS); sailing strategies; Monte Carlo simulation; accelerated sampling methods (ASM); simulation truncation rule; application rules

I INTRODUCTION
In the simulation study of several (multi-state) systems cooperative operation, the arrival of each system (i.e. a surface ship) at a specified destination at a specified time (neither later nor earlier) is an important issue, and various strategies can be used in this process, including the traditional sailing strategy [1] and an optimized strategy [2] developed on the basis of the traditional sailing strategy. The traditional sailing strategy holds an optimistic expectation of the subsequent task and does not consider the possible ship stop and speed reduction events in the subsequent sailing, and the sailing speed is calculated by dividing the remaining distance by the remaining time. The optimized strategy uses a digital simulation model to build a virtual ship, which predicts the possible stop and speed reduction events in the subsequent task in real-time, and the possible ship stop and speed reduction time need to be deducted from the remaining time when calculating the sailing speed, which is an improvement to the traditional sailing strategy.

Currently, simulation methods for several systems joint operation mission include Monte Carlo (MC) simulation [3-6], Petri nets [7], etc. MC simulation can be divided into direct and indirect sampling methods, both of which are conventional simulation methods that sample in the original state transfer space and simulate according to the true probability of event occurrence. MC simulation is less influenced by the system scale and structure, and is more capable of dealing with task reliability and task success ratio problems of complex system. However, with the increasing reliability of the equipment and the optimized design of the system structure, the sampling efficiency under the conventional method is getting lower and lower. In [8], accelerated sampling methods (ASM) were used to improve the simulation efficiency of the traditional sailing strategy. Because the optimized strategy requires real-time prediction of ship stop and speed reduction time for remaining sailing, it needs to nest a virtual ship simulation in each execution [2], and the number of simulations increases by a square multiple compared with the traditional strategy, thus the sampling efficiency problem is more prominent, and the research on accelerated sampling for the optimized strategy is more important.

The existing ASM can be classified into importance sampling, multilevel decomposition [9, 10], stratified sampling [11], and antithetic variates method [12, 13]. In this paper, two ASM based on the importance sampling, forced transition and fault biasing method [14, 15], are used to improve sampling efficiency under cooperative operation optimized strategy. The above two sampling methods increase equipment faults and improve the simulation efficiency by modifying the original system state transfer probability function. In our study, we found that when the above two ASM are used for actual simulation, some adjustments to current ASM must be made in different task scenarios. For example, in the simulation of joint operation optimized strategy, it is necessary to solve the related indexes of task success, during simulation, different indexes (such as system average downtime and task success ratio, etc.) should be set different truncation moments of state transition, otherwise it will easily lead to systematic bias. Therefore, this paper proposes the state-transition chain truncation rule to solve the systematic bias caused by accelerated sampling, which helps us to be further developed as a part of the digital twin model of complex MSS. Due to the wide applicability of MC-ASM, the rule can be easily extended to other fields, such as manned/unmanned system cooperative operation on air, replenishment at sea and on air with limited suppliers, intelligent railroad scheduling, and so on.
The rest of this paper is organized as follows, Section II is an analysis of the MSS joint operation and two sailing strategies. Section III analyzes two ASM, forced transition and fault biasing. Section IV proposes a state-transition chain truncation rule for calculating task success related metrics under ASM. Section V is a case study that discusses the application rules of conventional sampling methods and ASM in the background of this paper. Section VI concludes the full paper.

II. MULTI-STATE SYSTEMS JOINT OPERATION ANALYSIS
In this section, we analyze complex multi-state system transfer process and give the mission success criterion of several systems cooperative operation. The two sailing strategies are compared, and then the necessity of using accelerated sampling methods is discussed.

A. ANALYSIS OF COMPLEX SYSTEMS WITH MULTIPLE OPERATING STATES
In the following study, naval ship is used as an object for analysis. As complex multi-state systems (MSS), naval vessel is composed of many subsystems and their respective equipment. The traditional Binary-state theory classifies the working state of equipment into "fully working" and "completely failed", which is no longer applicable to modeling complex systems. A large number of shipboard electromechanical equipment, such as diesel engines and gas turbines of propulsion subsystem, are often in an intermediate working state between "fully working" and "complete failed". For example, if diesel engine has partial overheating or insufficient air intake, the diesel engine can still operate, but its power output cannot reach 100%, and the performance level at this time is a degraded level between 0 and 100%, as shown in Fig. 1.

![Multi-state systems state transition diagram](image)

**FIGURE 1** Multi-state systems state transition diagram
Other shipboard subsystems, such as electric power and communication, also have this multi-state characteristics extensively. Due to the space limitation, only the multi-state characteristics of propulsion subsystem is studied in this paper.

B. TASK SUCCESS CRITERION OF SEVERAL SYSTEMS COOPERATIVE OPERATION
This section provides an analysis of joint naval multi-force operation. When a ship (other forces may be submarines, manned and/or unmanned aircraft, etc.) is in such a joint mission, it cannot arrive early at its destination in the sailing phase, or it will expose itself early and be attacked one by one by the enemy; nor can it arrive late, or it will lose its cooperative combat opportunity. Since it will enter the combat phase immediately after the end of sailing phase, and the ship must maintain good maneuverability during the combat phase, so the ship's system must be in an available state at the moment the ship reaches its destination so that it can readily adjust its position for attack and defense. The results of the sailing phase can be classified into four types.

1) Task success (TS) status
At the end of the sailing task, \( t = T_{\text{max}} \), the ship's cumulative distance is \( S = S_{\text{max}} \) and the ship's system is in an available state, the sailing task is judged successful.

2) Not arriving (NA) status
During sailing, the ship may randomly experience equipment faults that lead to ship stop. As each stop causes a loss of time, when the ship resumes sailing (the fault is repaired), the ship needs to increase its speed to ensure that it can reach its destination at the end of the task. However, a long stop time may cause the ship's plan speed to exceed the maximum sustained speed, i.e. \( v_p > v_{\text{max}} \). This means that the ship cannot reach its destination on time in subsequent sailing, even if it sails at maximum speed. In this case, the sailing task is judged to be failed and the simulation process ends early. In MSS, a degradation of the propulsion subsystem may lead to \( v_p > v_{\text{max}} \cdot \theta \) situation and the task may not be judged as a failure for the time being. The task result will be determined after the degradation state has been removed.

3) Unfinished repair status
It means that ship's systems are still under repair at the end of the sailing task. As the ship will enter the combat phase immediately after the end of the sailing, the status of the system at this point does not meet the requirements of the combat phase and the sailing task is judged as a failure.

4) Fatal Fault (FF) status
During the sailing, some equipment may have a fatal fault (absorption state), which is generally considered to be irreparable during the sailing. And if this equipment is in a critical position, it will cause the ship to stop and cannot resume sailing, the task is judged to be a failure and the simulation process should end early.

In summary, the criteria for determining whether a sailing task is successful are summarized in Table 1.

| Event type         | Task Status | Ship's position at the end of the sailing | Ship system state at the end of the Sailing |
|--------------------|-------------|------------------------------------------|--------------------------------------------|
| Task Success(TS)   | success     | Arrive at destination                    | Ship system available, subsystems, and equipment in an normal state |
| Not Arriving(NA)   | Failure     | Not arrive at the destination            | Ship system is available                    |
| Unfinished Repair(UR) | Failure | Uncertainty of arrival at the destination | Ship system is unavailable, the ship stops sailing |
| Fatal Fault(FF)    | Failure     | Not arrive at the destination            | Ship system is unavailable, the ship stops sailing |

*TABLE 1* Criterion for sailing task results

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C. ANALYSIS TO THE SHIP'S SAILING STRATEGIES

1) THE TRADITIONAL SAILING STRATEGY

Here, the traditional sailing strategy [1] is first analyzed in the background of the above sailing task requirements. Define $T_{\text{max}}$ as the task time, $S_{\text{max}}$ as the task distance, $S_i$ as the cumulative distance at the current moment, and $v_{\text{max}}$ as the ship’s maximum speed. $v_{p\text{min}}$ is the minimum planned speed, i.e. the minimum speed required to complete the remaining distance, and the formula is shown in equation (1).

$$v_{p\text{min}} = (S_{\text{max}} - S_i)/(T_{\text{max}} - t)$$  \hspace{1cm} (1)

Define $\vartheta$ as the redundancy coefficient, i.e. the ratio of redundancy time to remaining task time [16]. At the initial moment $t=0$, let the ship be at the position $S=0$, and when there is a maximum value of the redundancy coefficient, the formula is shown in equation (2).

$$\max(\vartheta) = T_{\text{max}} - (S_{\text{max}} - S_i)/v_{\text{max}}$$  \hspace{1cm} (2)

That is the traditional strategy, we call it strategy A. Under the traditional strategy, the plan speed $v_p = v_{p\text{min}}$. The advantage of strategy A is that it ensures that the ship does not expose itself by arriving early (which would lead to task failure). If the ship suffers a stop or speed reduction event, the speed will be increased to compensate for the time lost due to the ship stop once the equipment fault is repaired and the ship resumes sailing.

The analysis suggests that strategy A is overly optimistic about the subsequent task, believing that there will be no system downtime or degradation during the subsequent sailing process, and therefore sails at the lowest plan speed $v_{p\text{min}}$, so as to avoid the ship arriving early and being defeat by the enemy individually. However, if the ship stops or speed reduction is too long during sailing, the ship may not arrive on time and thus miss the opportunity of cooperative combat. Therefore, an optimized strategy [2] is proposed based on the traditional strategy (named Strategy B in this paper).

2) OPTIMIZED SAILING STRATEGY

The principle of strategy B is to construct a virtual ship through digital simulation. And during the real ship sailing, the virtual ship performs real-time simulation and continuously predicts the possible downtime and degradation of the real ship’s system during the subsequent missions. The real ship adjusts its speed in advance according to the prediction, and the possible downtime and degradation time, as well as the delay time caused by the virtual ship simulation, are deducted in advance when calculating the speed under strategy B to ensure that the ship can reach the destination on time. Theoretically, it needs to restart the virtual ship simulation when the state of the real ship’s equipment is transferred or the task time and cumulative distance are changed, but the real ship needs to make the output of the propulsion subsystem stable and cannot adjust the speed too frequently, so the virtual ship cannot be started indefinitely. According to the above analysis, the following rules can be made for the start of the virtual ship.

① Activate the virtual ship for prediction when state transfer of real ship equipment occurs.

② When a real ship sails at a constant speed for a fixed time threshold $T_{\text{TH}}$.

Let the average system downtime and degradation time after the virtual ship $N_t$ simulations are $M_{T_F}$ and $M_{T_D}$, respectively. And the delay time of the virtual ship simulation be $\Delta t_{\text{ime}}$. The maximum speed of the ship decreases when the output coefficient $\vartheta_k$ of the propulsion subsystem is reduced. In summary, the calculation method of the ship’s plan speed under strategy B is shown in Equation (3).

$$v_B = \begin{cases} 
\frac{S_{\text{max}} - S_i}{(T_{\text{max}} - t) + \sum_{k=1}^{N_t} M_{T_D,k} (1 - \vartheta_k')} \left( T_{\text{max}} - t + \Delta t_{\text{ime}} \right) \quad & S_i < S_{\text{TH}} \\
\frac{(S_{\text{max}} - S_i) / (T_{\text{max}} - t)}{S_i \geq S_{\text{TH}}} \quad & \end{cases}$$  \hspace{1cm} (3)

$S_{\text{TH}}$ is the threshold for changing the plan speed, which is to prevent the ship reaching its destination early and causing task failure [17]. The real ship will not start the virtual ship for prediction after it sails to this point. In addition, the upper limit of the actual speed cannot exceed the maximum speed $v_{\text{max}}$ during the sailing.

The above is the calculation of the sailing speed under strategy B. It can be seen that each adjustment of the speed requires $N_t$ virtual ship simulations to get the average system unavailable time. And when calculating the final task success ratio, the real ship needs to perform $N$ simulations itself, resulting in high computational cost. Without taking into account the rule ① that equipment state transfer triggers the virtual ship, the total simulation times required to output the success rate of the sailing task will be at least.

$$\min(N_{\text{TH}}) = N_t \times N \times (T_{\text{max}} / T_{\text{TH}})$$  \hspace{1cm} (4)

For example, the virtual ship's sailing is simulated $N_t = 10^4$ times, and the virtual ship is activated at an interval $T_{\text{TH}} = 10$ h. The real ship should be also simulated $N=10^6$ times. In this way, the total simulation times reaches at least $N_t \times N \times (200/10) = 2 \times 10^9$ times in a $T_{\text{max}}=200$ h sailing task, without considering the state transfer of the equipment. If the conventional sampling method is still used to simulate in the original state transfer space, this optimized strategy to guide the ship's sailing will require so much simulations, consume respectively simulation time. And it may fail to achieve the near real-time simulation effect. In this case, it is necessary to use some accelerated sampling methods to improve the simulation sampling efficiency.

III. ACCELERATED SAMPLING ANALYSIS UNDER OPTIMIZED STRATEGY

As mentioned above, with the improvement of equipment reliability and optimized design of system structure, the efficiency of using conventional sampling methods in sailing task simulation is getting lower and lower. Because strategy B requires real-time or near-real-time prediction of stop and speed reduction time, and a virtual ship simulation needs to be nested in each simulation, and the simulation times increases by a higher multiple compared with the strategy A, making the sampling efficiency lower. In this section, we analyze the
accelerated sampling method under the optimized sailing strategy.

**A. DIRECT MONTE-CARLO SAMPLING**

The principle of Direct Monte-Carlo (DMC) sampling diagram is shown in Fig. 2. It is sampling in the original equipment state transition space [18], generates state transfer through the underlying component state change, and then takes the minimum value of the state transfer time in all directions for all components as the next state transfer time of the system, and the corresponding component and the entered state are the triggering events, and other sampling results are considered invalid. The advantage of DMC is that it is simple to model and suitable for various state transfer distributions and system structures. The disadvantage is that each state transfer generates a lot of random numbers, which affects the computational efficiency.

![Diagram](Image)

**FIGURE 2 The principal diagram of Direct Monte-Carlo**

The probability density function of equipment state transfer time is shown in equation (5), and the probability distribution function is shown in equation (6).

\[
f_{\phi}(t'|t) = \gamma_\phi(t) \cdot \exp \left[ -\int_{t}^{t'} \eta_\phi(t) \, dt \right], \quad t > t'
\]

\[
F_{\phi}(t'|t) = 1 - \exp \left[ -\int_{t}^{t'} \eta_\phi(t) \, dt \right], \quad t > t'
\]

In the equation, \( \eta_\phi \) denotes the \( i' \rightarrow i \) state transfer rate of equipment \( j \) at moment \( t' \), and \( t > t', \ i \in [1, 2, ..., m_j] \). For a multi-state equipment, the state transfer in each direction needs to be sampled independently. The state transfer time can be expressed as follows.

\[
\int_{t}^{t'} \eta_\phi(t) \, dt = -\ln(1 - \xi_\phi)
\]

\( \xi_\phi \) is a random number obeying uniform distribution between 0~1, then the final system state transfer moment is.

\[
t = \min(t_\phi), \quad j \in [1, 2, ..., n], \quad i \in [1, 2, ..., m_j]
\]

**B. INDIRECT MONTE-CARLO SAMPLING**

Indirect Monte-Carlo sampling (IMC) [19], performs state transfer from the system level. IMC needs to construct the state transfer probability density function of the system.

\[
K((s(t)|s'(t'))) = f(\tau|s',t') \cdot q(s|s',t) \tag{9}
\]

It consists of a transfer time function \( f(\tau|s',t') \) that determines the time of the system state transfer and a state transfer kernel \( q(s|s',t) \) that determines the direction of the system state transfer, where.

\[
q(s|s',t) = \frac{\eta_{s'}(t)}{\eta_i(t)} \tag{10}
\]

\[
\eta_i(t) = \sum_{s \in \Lambda_i} \eta_{s'}(t) \tag{11}
\]

In the equation, \( \eta_i(t) \) denotes the rate at which the system transfers out of the current state \( s' \) at moment \( t \); \( \eta_{s'}(t) \) denotes the transfer rate of \( s' \rightarrow s \) at moment \( t \). Where \( O_r \) denotes the reachable states set of \( s' \). The system behaviors such as faults, degradation and repair are uniformly regarded as state transfer, and the transfer rate is set to \( \eta \). \( \lambda \) is the fault or degradation rate; \( \mu \) is the repair rate; \( \Lambda_i \) denotes the reachable faults and degradation states set of \( s' \), and \( \Psi_i \) denotes the reachable repair states set.

\[
\eta_{s'}(t) = \begin{cases} 
\lambda_{s'}(t), & s \in \Lambda_i \\
\mu_{s'}(t), & s \in \Psi_i 
\end{cases} \tag{12}
\]

\[
O_r = \Lambda_i \cup \Psi_i \tag{13}
\]

In the simulation using IMC, the system state transfer time \( \Delta t \) is first sampled by the equation (14), probability distribution function of the system state transfer time.

\[
F(t|t') = 1 - \exp \left[ -\int_{t'}^{t} \eta_i(t) \, dt \right], \quad t > t' \tag{14}
\]

In equation (14), replacing \( F(t|t') \) on the left side of the equation with \( \xi \), when the state transfer is exponentially distributed, the state transfer time of the system can be expressed as follows.

\[
\int_{t'}^{t} \eta_i(t) \, dt = -\ln(1 - \xi)
\]

In equation (15), \( \xi \) is a uniformly distributed random number between 0 and 1. After determining the state transfer time, the states are sampled using the state transfer kernel \( q(s|s',t) \). The specific state that the system enters is determined in a roulette wheel fashion according to equation (16).

\[
\sum_{n=1}^{s} \eta_{s'}(t) \leq \xi \leq \sum_{n=1}^{s+1} \eta_{s'}(t)
\]

In the equation, \( \zeta \) is also a random number uniformly distributed between 0 and 1. When \( \zeta \) falls within the interval of equation (16), the state is transferred from \( s \) to \( s' \), and
the state transferred equipment and its state are determined. The principal diagram is shown in Fig. 3.

![FIGURE 3 The principal diagram of Indirect Monte-Carlo](image)

IMC does not reduce the variance of the simulation results and has no difference on the simulation results compared with DMC. The difference between the two methods is that IMC requires only two random numbers to generate a system state transfer and does not require operations such as comparison, etc., which improves computational efficiency and is usually the basis for various accelerated sampling methods.

C. ANALYSIS TO ACCELERATED SAMPLING PRINCIPLE

The accelerated sampling method under IMC is based on importance sampling. Its principle is adjusting the system state transfer probability function \( \tilde{K} \). It is shown in the following equation.

\[
\tilde{K}(s,t) = \tilde{f}(t'|s,t') \cdot \tilde{q}(s'|s,t) 
\]  

(17)

It consists of a new system state transfer time probability density function \( \tilde{f}(t'|s') \) and a transfer kernel function \( \tilde{q}(s'|s,t) \). And in order to ensure the unbiased sampling results after each sampling, it is necessary to recalculate the current state transfer weights \( w_i \), so as to correct the current simulation sampling counts to maintain the unbiased results. In this section, two accelerated sampling methods, forced transition (FT) and fault biasing (FB), are analyzed.

1) ANALYSIS TO FORCED TRANSITION

The principle of FT is to adjust the sampling space of the state transfer time based on IMC, and it should ensure the next equipment fault must occur before the end moment of the task \( T_{\text{max}} \), as shown in Fig. 4. It forces the fault to occur before \( T_{\text{max}} \) and increases the fault frequency. FT samples the next state transition time of the system using the newly constructed system state transfer time probability density function \( f(t'|t') \) and probability distribution function \( \tilde{F}(t'|t') \). It is shown in the following equations.

\[
\tilde{f}(t'|t') = \frac{f(t'|t')}{1 - \exp\left[-\int_{t'}^{T_{\text{max}}} \eta_r(t) dt\right]}, \quad t' < t < T_{\text{max}}
\]  

(18)

\[
\tilde{F}(t'|t') = \frac{F(t'|t')}{1 - \exp\left[-\int_{t'}^{T_{\text{max}}} \eta_r(t) dt\right]}, \quad t' < t < T_{\text{max}}
\]  

(19)

Then the next state transition moment \( t \) of the system is shown in (20).

\[
\int_{t'}^{T_{\text{max}}} \eta_r(t) dt = -\ln\left[1 - \tilde{\xi}\left[1 - \exp\left(-\int_{t'}^{T_{\text{max}}} \eta_r(t) dt\right)\right]\right]
\]  

(20)

Since FT makes the probability of the fault event enlarged by a factor of \( \left[1 - \exp\left(-\int_{t'}^{T_{\text{max}}} \eta_r(t) dt\right)\right]^{-1} \), this sampling weight should be corrected according to equation (21) in order to ensure the unbiased simulation results.

\[
w_i = w_i \cdot \left[1 - \exp\left(-\int_{t'}^{T_{\text{max}}} \eta_r(t) dt\right)\right]
\]  

(21)

Since FT always makes the next state transfer sampling to occur within the task end moment, its use must be restricted (detailed analysis is below), otherwise FT will be executed infinitely before the task end moment, which will result in the inability to end the task.

![FIGURE 4 Forced transition sampling principle](image)
2) ANALYSIS TO FAULT BAISING
When determining state transfer events under IMC, if there is maintenance of the equipment in the system, the fault events are often difficult to be sampled because the maintenance rate of the equipment is usually much larger than the fault rate, i.e., \( \mu(t) \gg \lambda(t) \). In order to quickly make the system into the unavailable state, FB adjusts the state kernel function \( q(s|s',t) \) to amplify the probability of fault-type events and reduce the probability of repair-type events to increase the proportion of fault events being selected, the principle is shown in Fig. 5.

![Fault baising sampling principle](image)

\[ q(s|s',t) = \begin{cases} \frac{q(s|s',t)}{\sum_{s' \in \Lambda} q(s|s',t)} \cdot x, & s \in \Lambda^f \\ \frac{q(s|s',t)}{\sum_{s' \in \Psi} q(s|s',t)} \cdot (1-x), & s \in \Psi^f \end{cases} \]  

(22)

In the equation, \( x \) is the proportion of fault-type events to all events in this sampling; \( (1-x) \) is the proportion of repair-type events to all events in this sampling, and the value of \( x \) usually ranges from 0.5 to 0.7 [14], taking a higher value may make the variance increase further. The biased system fault rate will be much greater than the unbiased rate. Which state should be transferred is determined according to equation (16). To keep the unbiased simulation results, we should correct the simulation sampling counts according to equation (23)

3) WEIGHT CORRECTION BASED ON STATE-TRANSITION CHAIN
In the above analysis process, if each state transfer process is represented as \( \{s,w,t\} \), it consists of the state \( s \), the transition weight \( w \) and the moment \( t \) when it occurs. Then the system forms a random walking sequence of states after \( L \) state transfers, which is called a state transition chain in this paper.

\[ \Gamma_L = (s_1, w_1, t_1), (s_2, w_2, t_2), \ldots, (s_L, w_L, t_L) \]  

(24)

Then the cumulative correction value of the state transition weight under FT and FB after the \( L \)th transfer is

\[ w_L = w_0 \prod_{l=1}^{L} \frac{f_w(t_l | t_{l'})}{f_w(t_{l'} | t_l)} \prod_{l=1}^{L} q(s | s', t_l) \]  

(25)

It is the standard cumulative state transition weight equation. The state transition chain truncation rule is based on FT, FB, and this equation. And the rule will be presented below in this paper.

IV. STATE TRANSITION CHAIN TRUNCATION AND THE RULE
In the simulation, as mentioned before, we found that in some cases when sampling directly using the above ASM (including FT and FB), some results always have errors compared to the conventional sampling methods.

For example, when the ASM is used to calculate the task success probability, the state transition chain should be truncated after the task is transferred to some specific status, such as FF, NA. If the simulation is continued, it will produce meaningless results and the simulation results will have certain systematic bias. It is called problem No.1.

Meanwhile, when using the virtual ship to calculate the average system unavailable time, it is necessary that each simulation should be run completely until the task end moment, without early termination. The truncation time of the state transition chain should be placed at the end of the task, and ASM can be used to improve the sampling efficiency until the truncation time. That is, the state transition chain must be simulated completely at this time, and cannot be truncated early, otherwise the accuracy of the calculation results will be affected. It is called problem No.2.

In this paper, we define them as the truncation problem of state-transition chain under ASM, and give rules and rules, which can determine the truncation moment of state transition chain in different scenarios.

Section A will discuss problem No.1 and Section B will discuss problem No.2.
A. TRUNCATION MOMENT FOR TASK SUCCESS RATIO

About problem No.1, we find that when calculating the task success ratios, we focus on the task status and are no longer limited to the system reliability. When the task status is determined and cannot be changed (e.g., when a NA event occurs during the sailing), the task event (i.e., this sailing process) is actually over, and the weight transition should end at this point and should not continue.

As shown in Fig. 6, when the task status $S_{TA}$ turns to NA, although the simulation clock has not yet reached the end moment of the task, i.e., $s_{T_{max}}$, the task should be judged as failure according to the task criteria, and the ship cannot reach the destination on time even if it sails at maximum speed. This is an irreversible failure situation (it can be considered that the task status has entered the absorbing state), and the task status cannot be changed from this moment on, and the corresponding weights should not be changed. Subsequent sampling for the task status using FT and FB is invalid, otherwise it is a forced change for the task event weights without changing the task status, resulting in weights deviating from the status. The systematic bias mentioned earlier originates from this, as shown in the state transition chain of NA in Fig.6.

![Task state-transition chain truncation rule under accelerated sampling methods](image)

FIGURE 6 Task state-transition chain truncation rule under accelerated sampling methods

The sailing task status $S_{TA}$ transition is shown in Fig. 7.

![Task status transition diagram](image)

FIGURE 7 Task status transition diagram

The correct calculation is as follows. If there are $n$ state transfers in one sailing task simulation, the state transfer moment is

$$t_0 < t_1 < \ldots < t_k < T_{TA} < t_{k+1} < \ldots < t_n < T_{max}$$

(26)

The final state $S_{TA}$ of the task is determined at time $T_{TA}$. Then the task status transition chain should be truncated at time $T_{TA}$, i.e., the final cumulative weight of the task status transition chain is

$$w_{TA} = w_0 \prod_{i=0}^{k} f_i(\tau_i) \prod_{j=k+1}^{n} q_j(s_j, t_j)$$

(27)

This equation takes into account the task state-transition chain truncation and eliminates the bias in equation (25).

The task success ratio $D$ can be derived from the task failure frequency $N_d$, which is calculated as in (28). Since $N_d$ contains multiple task failure types, it can be extended to vector $N_d = [N_{NA}, N_{UR}, N_{FF}]$, which denotes NA ratio, UR ratio, and FF ratio, respectively. The final status transition weights at the end of each simulation is

$$w_{TA} = \left[ w_{TA}^{(0)}, w_{TA}^{(2)}, \ldots, w_{TA}^{(N)} \right]$$

(28)

$$D = 1 - \frac{1}{N} w_{TA} \cdot N_d$$

$$= 1 - \frac{1}{N} \sum_{i=0}^{N} w_{TA} \left( I_{\left( i \right)}^{(i)} \right) \cdot I\left( I_{\left( i \right)}^{(i)} \right)$$

(29)

With such an adjustment, the systematic bias between conventional and ASM can be eliminated and the simulation results of both can be made consistent.

B. TRUNCATION MOMENT FOR AVERAGE SYSTEM UNAVAILABLE TIME

About problem No.2, under ASM, the virtual ship finishes $N_c$ complete simulations, and after the results are stable, the performance matrix of the overall system is generated as shown in equation (30). The duration of each segment performance level $g_{\left( i \right)}$ is $T_{\left( i \right)}$.

$$G = \left[ g_{\left( i \right)}, g_{\left( i \right)}, \ldots, g_{\left( i \right)} \right]$$

(30)

The adjusted value of each task transition weight is shown in equation (31).

$$w_{\left( i \right)} = \left[ w_{\left( i \right)}, w_{\left( i \right)}, \ldots, w_{\left( i \right)} \right]$$

(31)

The average system downtime $MT_F$ during the remaining task time is calculated as shown in equation (32).

$$MT_F = \frac{1}{N_c} \sum_{i=1}^{N_c} \sum_{i=0}^{N} w_{\left( i \right)} \cdot I\left( g_{\left( i \right)} \right)$$

(32)

And

$$I\left( g_{\left( i \right)} \right) = \begin{cases} 1 & g_{\left( i \right)} - g_0 = 0 \\ 0 & \text{others} \end{cases}$$

(33)
For MSS, there exists a system average degradation time $MT_D$, calculated as shown in (34), where $0% < g_D < 100%$.

$$\begin{align*}
MT_D &= \frac{1}{N_i} \sum_{i=1}^{N_i} \sum_{k=1}^{k_i} w_k^{(i)} \cdot T_k^{(i)} \cdot I(g_k^{(i)}, g_D), \\
i &= 1, 2, \ldots, N_i
\end{align*}$$

(34)

The calculation of $I(g_k^{(i)}, g_D)$ is the same as equation (33). The formal definition of the state transition chain truncation rule and its rules are given below.

C. DEFINITION OF STATE-TRANSITION CHAIN TRUNCATION RULE AND ITS RULES

Based on the above study, this section defines the state-transition chain truncation rule (SCTR) and gives the corresponding rules.

Through the analysis, the state-transition chain truncation rule can be defined as the truncation moment of the state transfer chain when performing accelerated sampling.

Truncation rule No.1: When calculating task success ratios, the status-transition chain truncation moment is the moment when the task status is determined, and in some specific task status the truncation moment will be earlier than the task end moment. In the sailing task, since TS and UR are determined at the end of the task and no accelerated sampling is required after FF (see TS, UR, and FF in Fig. 6 for details), the NA task failure type needs to be focused on. NA's determination is always accompanied by system state transfer, and NA's determination is required after each system state transfer, and the task state-transition chain is truncated at this moment when entering NA status.

Truncation rule No.2: When calculating for the average system unavailability time, the state transition chain truncation moment must be placed at the end of the task, i.e., the accelerated sampling method can be used throughout the task period. The calculation of $MT_F$ and $MT_D$ requires the simulation process to run completely from the current moment $t$ to the end of the task $T_{max}$, simulation process should not be terminated earlier to the end of the task, so as to ensure the unbiased results of the unavailable time.

This is because when calculating the average system unavailability time, the event we concerned about is in the entire remaining task time, i.e., the possible downtime and degradation time from the current moment $t$ until the end of the task, so the state transition chain truncation moment should be placed at the end of the task.

Generally speaking, the reason for using the above two rules is that task success ratios are the cumulative count of task types (including different types of task success and failure event) at a given moment based on a specific task profile. When calculating the task success ratios, what we are paying attention to is an entire task profile, task success ratio cannot be changed after the task status is determined, meaning that the task we are focusing on is actually over (even if the simulation clock has not yet put to the task end moment $T_{max}$) and the transfer of task weights should be ended. Continuing to accelerate sampling would force a change in the task event weights, which will cause the weights to diverge from the task status when the task status has already been determined, which is the source of systematic bias when calculating the task success ratios. And the average system unavailable time is the cumulative count of some system states at each moment, so its simulation process should not be terminated earlier.

It can be expected that the IMC explored in this section, and IMC-based FT, FB and the corresponding weight correction methods, are in general only applicable to small probability events in complex multistate system tasks. That is, the equipment reliability is very large compared to the task time, or the task time is short compared to the equipment reliability. If it does not belong to the above two cases, then the efficiency of sampling in the normal state space will not be reduced, and thus there is no need for the above accelerated sampling, as verified by the case study in this paper (see Section 5 of this paper).

D. SIMULATION PROCESS UNDER ACCELERATED SAMPLING METHODS

Here gives the complete simulation flow of a sailing task using the optimized sailing strategy, ASM, and SCTR. Since FT and FB always make the system status shift toward failure, if FT and FB are not restricted, then the system will rapidly enter the fatal fault state and eventually fail completely without obtaining the complete full picture of system unreliability (see the analysis in A and B for details). Therefore, in the task scenario of this paper, the rules for the use of FT and FB should be determined as follows.

1) When there is no faulty equipment in the system, FT should be used for sampling.
2) FB should be used when there is maintenance equipment in the system and the system is not in a down state.
3) In other cases, such as when equipment generates a repairable fault and causes system down, conventional sampling methods should be used.

As mentioned before, since there is no difference between DMC and IMC on final simulation results, DMC should be used during the simulation when ASM is not used, and IMC is used when ASM is used. The simulation flow is shown in Fig. 8.

Firstly, the real ship simulation is started and the simulation parameters are initialized, then the system state transition is generated continuously with the ship sailing, and whether ASM is used will be judged before each state transition sampling. After the sampling is completed, the real ship equipment state is updated, and then a virtual ship simulation is triggered, and a virtual ship's triggering may be between state transfers. After the virtual ship completes the simulation, $MT_F$ and $MT_D$ are output to update the speed of the real ship, and then the next state transition sampling is performed until the end of the task.
V. CASE STUDY AND APPLICATION

A. CASE BACKGROUND

1) ASSUMPTIONS FOR MODELLING

Assumption No.1: The state transition time of the equipment obeys exponential distribution, and the equipment returns to original normal state (we repair it as a newer) after the repair is completed, and the fatal fault is not repairable during the sailing task.

Assumption No.2: State transition of equipment is independent and do not affect each other.

Assumption No.3: The multi-state performance output of the ship's system is only affected by the propulsion subsystem. When the propulsion subsystem is degraded, the ship's maximum speed is also reduced, and the electrical power subsystem only affects whether the ship's system is down.

Assumption No.4: Maintenance resources are sufficient and there are no restrictions.

2) DATA SETTINGS

Shipboard equipment is generally divided into two categories: platform and load. The platform includes subsystems such as propulsion, electric power, communication and navigation, and the load includes subsystems such as surveillance detection, artillery, missile and anti-submarine. The platform is mainly used during the sailing task, so the propulsion and electrical power systems are modeled as an example for the simulation of the sailing task. The propulsion subsystem is driven by diesel engines, gas turbines, and is usually operated by combined diesel and gas turbine (CODAG) or combined diesel or gas turbine (CODOG). To ensure the ship moves, it also requires reduction speed gearboxes, shaft, propellers, and supporting monitoring and auxiliary equipment.

The electric power consists of diesel generator sets, which generate, distribute and transmit electrical energy for the whole ship to ensure the normal operation of the shipboard equipment. The propulsion subsystem and the electrical subsystem are in series. Assuming that the propulsion subsystem is operated in CODOG mode, the reliability diagram of the main equipment is drawn according to the requirements of the sailing task as shown in Fig. 9.
Diesel engine
Diesel engine
Monitoring equipment
Auxiliary equipment
Propeller & Shaft & Gearbox
Diesel generator
Diesel generator
Diesel generator
Propulsion subsystem
Electrical power subsystem
Gas turbine

Diesel engine 1
Diesel engine 2
Gas turbine
Propeller & Shaft & Gearbox
Monitoring equipment
Auxiliary equipment

Diesel generator 1
Diesel generator 2
Diesel generator 3
Diesel generator 4

In the sailing, the equipment can be divided into 3 types according to the working condition. (1) diesel and gas turbines are Type I equipment with four states, normal, degradation, repairable fault and fatal fault; (2) propellers, shaft, reduction speed gearbox and diesel generator sets are Type II equipment with three states, normal, repairable fault and fatal fault; (3) monitoring and auxiliary equipment are Type III equipment, which are assumed not to have fatal fault during the sailing task and have two states, normal and repairable fault. The equipment state transition is shown in Fig. 10.

![FIGURE 9 Propulsion and electrical power subsystem reliability block diagram](image)

The maximum speed of the ship is related to the performance output of the propulsion subsystem, propulsion performance level decreasing makes the maximum speed decreasing, and the performance output of the propulsion subsystem is expressed by $\theta$, it is

$$v'_{\text{max}} = v_{\text{max}} \cdot \theta$$  \hspace{1cm} (35)

The output of the propulsion subsystem is determined by the working states of the diesel and gas turbines, as shown in Table 2.

Table 2 The performance output of the propulsion subsystem table

| Diesel engine ① state | Diesel engine ② state | Gas turbine state | $\theta$ (%) | Diesel engine ① state | Diesel engine ② state | Gas turbine state | $\theta$ (%) |
|----------------------|----------------------|------------------|-------------|----------------------|----------------------|------------------|-------------|
| Normal               | Normal & Degradation | Normal           | 100.00      | Normal               | Degradation          | Fault            | 60.00       |
| Normal & Degradation | Normal               | Normal           | 100.00      | Normal               | Normal               | Fault            | 60.00       |
| Normal               | Normal & Degradation | Degradation      | 60.00       | Fault                | Normal               | Normal           | 60.00       |
| Normal               | Degradation          | Degradation      | 60.00       | Non-normal           | Non-normal           | Normal           | 60.00       |
| Degradation          | Normal               | Fault            | 60.00       | All other state combinations | 0.00 |

Let the maximum ship speed $v_{\text{max}} = 35\text{kn}$, the planned speed change threshold $S_{TH} = 0.95 \cdot S_{\text{max}}$ under strategy B, and the virtual ship start interval $T_{TH} = T_{\text{max}} / 20$. The reliability and maintainability of the equipment are shown in Table 3.
### TABLE 3 Equipment reliability & maintainability parameters

| Equipment                      | Degradation (h) | Reliability (h) | Maintainability (h) |
|-------------------------------|-----------------|-----------------|---------------------|
| Diesel engine                 | 1 000           | 1 000           | 8                   |
| Gas turbine                   | 850             | 850             | 32                  |
| Reduction speed gearbox       | --              | 2 500           | 10                  |
| Propeller & Shaft monitoring & Auxiliary equipment | --              | 5 000           | 12                  |
| Diesel generator              | --              | 2 000           | 2                   |
|                               | --              | 1 000           | 40                  |

#### B. SIMULATION CALCULATION AND TRUNCATION RULES APPLICATION

**1) ANALYSIS TO AVERAGE SYSTEM UNAVAILABLE TIME**

The following figures show the standard deviation (S.D.) of the average system unavailable time under ASM and conventional sampling method. Where the horizontal axis is the average system unavailable time, the vertical axis is the task time. Fig. 11 and 12 show the S.D. for 5 to 50h of task time, and Fig. 13 and 14 show the S.D. for 100 to 1000h.

**FIGURE 11 Downtime and S.D.**

In Fig. 11, the longer the task time, the greater the average system downtime. Where the width of the blue bar indicates the $MT_F$ when without ASM, and light blue bar is the corresponding S.D. The width of the red bar indicates the $MT_F$ calculated by ASM, and the light red bar indicates the S.D. under ASM. It can be seen that $MT_F$ are basically the same between accelerated and unaccelerated sampling (conventional sampling), but the S.D. is significantly lower when ASM is used, indicating that ASM reduces the variance of the simulation results and can improve the simulation sampling efficiency.

**FIGURE 12 Degradation time and S.D.**

Fig. 12 shows the corresponding average system degradation time, and the conclusion is consistent with Fig. 11. It shows that the results have less fluctuation and require fewer simulation times under ASM, especially when the task time is short (much shorter than reliability of the equipment), ASM can really improve the simulation efficiency and reduce the variance.

**FIGURE 13 Downtime and S.D.**

In Fig. 13 and 14, the task time is expanded to 1000h, $MT_F$ and $MT_D$ have no significant difference between conventional sampling and ASM. However, when the task time exceeds 500h, the S.D. of $MT_F$ and $MT_D$ when using ASM are greater than conventional sampling.

Now it is clear that ASM is suitable for statistically small probability events, and the smaller the event probability, the higher the ASM simulation accuracy.

In other words, the method performs better in higher reliable systems (e.g., when the mission time is much smaller than the reliability of the equipment, especially in the propulsion
subsystem). When the mission time is longer (e.g., when the mission time is equivalent or near to equipment reliability) or equipment reliability in the system are poor, sampling from original space can get needed failure events, i.e., conventional simulation method already has better simulation performance. Meanwhile, ASM may increase the statistical variance and confidence interval range under this condition. In summary, ASM is suitable for high reliability systems, or the probability of an event sampling is small. When sampling probability is large, sampling from original space is suitable. In application, which method is used can also be determined by trial simulation.

2) ANALYSIS TO TASK SUCCESS RATIO

Three task simulations (1) $S_{\text{max}} = 650\text{nm}$, $T_{\text{max}} = 20\text{h}$; (2) $S_{\text{max}} = 15600\text{nm}$, $T_{\text{max}} = 480\text{h}$ and (3) $S_{\text{max}} = 31200\text{nm}$ were performed under strategy B using ASM and conventional sampling methods, respectively. The results are shown in Fig. 15-17.

- The convergence rate of UR and FF of accelerated sampling at $T_{\text{max}} = 20\text{h}$ is faster than that of the conventional simulation, and FF events are not even once drawn under the conventional sampling;
- while at $T_{\text{max}} = 480\text{h}$, the convergence rates of various task rates under the two methods are similar;
- when $T_{\text{max}} = 960\text{h}$, the fluctuation of each type of task ratio after accelerated sampling is greater than that of the conventional simulation method, i.e., the convergence rate after accelerated sampling is slower than that of the conventional simulation method under this condition.

The simulation results in this section show that ASM can be used to calculate the task success ratio for strategy B under shorter task time; and as the task time increases, the effect of ASM and conventional sampling simulation methods are close to each other, and both methods are available; ASM decreases the sampling efficiency when the task time is longer. Here, conventional simulation methods should be used for simulation. This conclusion is consistent with the previous section.

3) RESULTS AND ANALYSIS WITH SCTR

In order to verify the SCTR, a comparative analysis of the task success ratio simulation with and without SCTR is completed in this section. Still using the data in subsection V.B.2, but without SCTR to re-run the simulation, and the results are shown in grey line of Table 4.
From the above results, it is clear that the task ratio with SCTR is consistent with that under the conventional sampling method. In contrast, the task ratio without SCTR has a larger deviation compared with the conventional method. Taking the 480h task as an example, without SCTR, the deviation between TS is 11.5% and the deviation between NA is 11.3%. It means that the deviation between the two task ratios is too large and there is an obvious error when without SCTR, which verifies the previous analysis in this paper.

**VI. CONCLUSION**

In this paper, accelerated sampling method and the corresponding state-transition chain truncation rule SCTR, can effectively complete task strategy simulation of multi-state systems. The two accelerated sampling methods, forced transition FT and fault biasing FB, effectively solve the simulation times and sampling efficiency under the optimized strategy in the condition of little probability. The proposed SCTR ensures the unbiased simulation results under the accelerated sampling method and eliminates the systematic errors existing in the original ASM rule. In the case study, the application rules of the conventional and accelerated sampling methods are analyzed by adjusting the task time, which can provide guidance for the joint operation of multi-state systems.

Generally speaking, the research in this paper can make the simulation for task optimized strategy of multi-state systems with near real-time capability and make the simulation results unbiased compared with conventional sampling methods, which lays a solid foundation for the task planning model of complex multi-state systems digital twin.

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