A Proposed New Test of General Relativity and a Possible Solution to the Cosmological Constant Problem

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Following a conjecture of Feynman, we explore the possibility that only those energy forms that are associated with (massive or massless) particles couple to the gravitational field, but not others. We propose an experiment to deflect electrons by a small charged sphere to determine if the standard general relativity or this modified one corresponds to reality. The outcome of this experiment may also solve the cosmological constant problem.

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The equivalence of mass and energy, expressed in his celebrated formula $E = mc^2$, led Einstein to postulate that the energy-momentum tensor $T^{\mu \nu}$ in the field equation of general relativity $^{[1]}$

$$R^{\mu \nu} - \frac{1}{2} g^{\mu \nu} R = \frac{8 \pi G}{c^4} T^{\mu \nu},$$

contains all kinds of energies, such as matter, radiation, electromagnetic, vacuum, etc. Thus assuming that the vacuum energy is negligible, the field equation $^{[1]}$ outside an object of total mass $M$ and static electric charge $Q$ containing no neutral and charged masses or other fields around it reduces to

$$R^{\mu \nu} = \frac{8 \pi G}{c^4} T^{\mu \nu}_{EM},$$

where $T^{\mu \nu}_{EM}$ is the traceless energy-momentum tensor of the electric field due to the charge $Q$ of the object. For the purpose of this letter we shall classify different energy types into two. The first class is the set of energy types with which massive or massless particles are associated. Thus the energy of an already existing mass distribution is obviously of this class $^{[1]}$. Since the energy in an electromagnetic wave (electromagnetic radiation) is carried in packages that behave like massless particles (photons) the electromagnetic radiation energy is also of this class $^{[2]}$. Energies associated with other massless particles like neutrinos and gravitons are further examples. Each energy type in this class may rightly be called 'mass energy' or 'particle energy'. The second class is the set of energy types with which no particles are associated $^{[3]}$. The energies in the electric fields of a static charge distribution and between the plates of a capacitor as well as the vacuum energy are of the second class.

There is plenty of empirical proof, such as the successes of the big-bang cosmology and the deflection of light by the sun, that the first class energies couple to the gravitational field. But there does not exist any empirical proof at present for the coupling of the second class energies to the gravitational field. Therefore, we do not know with certainty if Eq. $^{[2]}$ corresponds to a fact of nature. It lacks experimental support. There is the intriguing possibility that we shall consider in this letter, as first hinted by Feynman $^{[2]}$ when he said ‘...Now gravity is supposed to interact with every form of energy and should interact then with this vacuum energy. And therefore, so to speak, a vacuum would have a weight-an equivalent mass energy-and would produce a gravitational field. Well, it doesn’t! The gravitational field produced by the energy in the electromagnetic field in a vacuum-where there’s no light, just quiet, nothing-should be

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1The mass of the distribution may be constantly changing due to its mechanical energy, its absorption or loss of heat energy, etc. Furthermore, while we can estimate how much electromagnetic binding energy of an atom contributes to its rest mass we do not know, for example, how much weak and gravitational energies contribute to it. We also know from the Eötvös experiment that electromagnetic binding energy contributes equally to inertial and gravitational massess. We assume this is the case for the other energy types.

2Recall that though massless, an ‘effective mass’ can be assigned to photons.

3Of course, there is an ‘equivalent mass’ through $E = mc^2$ for such energies too. But the crucial point is that there is no already existing massive or massless particles associated with them.
enormous, so enormous, it would be obvious. The fact is, it’s zero! Or so small that it’s completely in disagreement with what we’d expect from the field theory. This problem is sometimes called the cosmological constant problem. It suggests that we’re missing something in our formulation of the theory of gravity. It’s even possible that the cause of the trouble—the infinities—arises from the gravity interacting with its own energy in a vacuum. And we started off wrong because we already know there’s something wrong with the idea that gravity should interact with the energy of a vacuum. So I think the first thing we should understand is how to formulate gravity so that it doesn’t interact with the energy in a vacuum…” According to this conjecture of Feynman there is the possibility that the right side of Eq. (2) may be zero:

\[ R^\mu{}\nu = 0. \] (3)

The importance of confronting with experiment the predictions of equations (2) and (3) is not merely academic. If it turns out that Eq. (3) is the one favored by nature, we then have a very simple solution (2) to the cosmological constant problem (4). It would mean that being of the second class, the vacuum energy does not couple to the gravitational field. The present value of the vacuum energy density is as large as it had been in the early universe. The cosmological constant, however, is simply zero, as it has always been.

The purpose of this letter is to propose a deflection of electrons by a positively charged sphere experiment to distinguish between equations (2) and (3). To this end, we shall need the solutions of these equations. The solution of Eq. (2) for a static and spherical distribution of mass \( M \) and electric charge \( Q \) located at \( r = 0 \) is known as the Reissner-Nordstrøm solution (5). It is given by

\[ ds^2 = \left(1 - 2\frac{GM}{c^2r} + \frac{Gk_eQ^2}{c^4r^2}\right) c^2 dt^2 - \left(1 - 2\frac{GM}{c^2r} + \frac{Gk_eQ^2}{c^4r^2}\right)^{-1} dr^2 - \frac{r^2d\theta^2 - r^2\sin^2\theta d\phi^2}{1 - 2\frac{GM}{c^2r} + \frac{Gk_eQ^2}{c^4r^2}}, \] (4)

where \( k_e \) is the electric (Coulomb) constant. It should be noted that according to Eq. (5), the electric field of the sphere does not contribute to its gravitational field and hence must assert itself separately and independently. Therefore, for weak fields Eq. (3) must reduce to Laplace’s equation

\[ \nabla^2(\Phi_G + \Phi_E) = 0, \] (5)

where \( \Phi_G \) and \( \Phi_E \) are the gravitational and electric potentials of the sphere. Finding the solution of Eq. (3) proceeds along the lines of the Schwarzschild solution (5). We find

\[ ds^2 = \left(1 - 2\frac{GM}{c^2r} - 2\frac{e}{m}\frac{k_eQ}{c^2r}\right) c^2 dt^2 - \left(1 - 2\frac{GM}{c^2r} - 2\frac{e}{m}\frac{k_eQ}{c^2r}\right)^{-1} dr^2 - \frac{r^2d\theta^2 - r^2\sin^2\theta d\phi^2}{1 - 2\frac{GM}{c^2r} - 2\frac{e}{m}\frac{k_eQ}{c^2r}}, \] (6)

where \( -e \) and \( m \) are the charge and the mass of an electron—a test particle—in the vicinity of the spherical object (5). To find the trajectory of an electron deflected by a positively charged sphere we also need the equations describing the trajectory according to equations (2) and (3). They are

\[ \frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = -\frac{e}{mc^2} F^\mu_\alpha \frac{dx^\alpha}{ds}, \] (7)

according to Eq. (3), and

\[ \frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0, \] (8)

\(^4\)Eq. (1) can also be obtained intuitively by classical energy considerations. Consider an electron moving radially away from a sphere of mass \( M \) and charge \( Q \). For the electron to escape from this object at a distance \( r \) from its center and reach infinity with zero speed, the escape velocity \( v_{esc} \) satisfies \( mc^2v_{esc}^2/2 - GmM/r - ek_eQ/r = 0 \). Replacing \( v_{esc} \) with \( c \), the speed of light, (so that the electron cannot escape from the surface of radius \( r \)) and dividing it by \( mc^2/2 \) the left side of this equation becomes \( (1 - 2GM/c^2r - 2ek_eQ/mc^2r) \), which is the \( g_{00} \) in Eq. (1).
according to Eq. \( \Gamma^\nu_{\alpha\beta} \) and \( F_{\mu\nu} = \partial A_\nu / \partial x^\mu - \partial A_\mu / \partial x^\nu \) are the connection coefficients and the electromagnetic field strength tensor, with \( A_\mu = (k_\circ Q/r, 0) \) being the electromagnetic four-potential of the sphere. Before we indulge in obtaining the orbit equations in experimentally relevant form, we propose the following experiment. Consider a rectangular vacuum chamber. Let a small metallic sphere of radius \( R \approx 2 - 5 \text{ cm} \) positively charged to a voltage \( V(R) = k_\circ Q/R \) be hanged freely from an insulating thread. Let an electron gun be located at a distance \( d \) away from the equator (the \( \theta = \pi/2 \) plane) of the sphere with an impact parameter \( b \) which is the horizontal distance between the initial path of the ejected electron beam and the center of the sphere. Thus the initial position of the beam is \( (x_i, y_i) = (-b, d) \) at an angle \( \phi_i = \pi/2 + \arctan(b/d) \). Put a calibrated fluorescent screen on the side of the box facing the screen (or monitor the position of the electron beam on the screen electronically). Compare the reading of the position of the beam with the predictions of the equations that we obtain now.

Using spherical coordinates, we write the line element in the form

\[
ds^2 = e^\nu c^2 dt^2 - e^{-\eta} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2.\tag{9}\]

Inserting \( d\theta / ds = 0 \) in Eq. \((7)\) and integrating the equations obtained for the coordinates \( x^0 = ct \) and \( x^3 = \phi \) we get

\[
\frac{dt}{ds} = \frac{e^{-\eta}}{c} \left( -\frac{q k_\circ Q}{mc^2} \frac{1}{r} + a \right),
\]

\[
r^2 \frac{d\phi}{ds} = h,
\]

where \( a \) and \( h \) are integration constants. Using equations \((10)\) and \((11)\) in the equation obtained from the condition of timelike geodesics \( g_{\mu\nu}(dx^\mu / ds)(dx^\nu / ds) = 1 \), putting \( e^\eta \approx 1 \), and then differentiating with respect to \( du / d\phi \) we get

\[
\frac{d^2 u}{d\phi^2} + u = \frac{m_E h^2}{h^2 + 3 m_E u^2},
\]

where \( u = 1/r \) and the constant \( a \) has been set to 1 so that when \( h = l/mc \), with \( l = m r^2 \dot{\phi} \) being the ordinary angular momentum, the first term on the right side of Eq. \((12)\) agrees with the corresponding Newtonian expression. Here \( m_E = e k_\circ Q/mc^2 = e R V(R)/mc^2 \) has the dimension of length and corresponds to \( m_G = G M/c^2 \) in the Schwarzschild solution. On the other hand, we obtain from Eq. \((8)\)

\[
\frac{dt}{ds} = \frac{e^{-\eta}}{c}
\]

and Eq. \((11)\) remains intact. By putting \( e^\eta = (1 - 2 m_G/r - 2 m_E/r) \approx (1 - 2 m_E/r) \) and proceeding as above we get

\[
\frac{d^2 u}{d\phi^2} + u = \frac{m_E}{h^2} + 3 m_E u^2.
\]

\( mch \) is the conserved angular momentum of the electron in its rest frame. \( h \) can be expressed in terms of \( l \), the angular momentum in the laboratory frame, using equations \((10)\) and \((11)\) in the Reissner-Nordstrøm case and equations \((13)\) and \((11)\) in our case. They are, respectively, \( h = l(1 + m_E u)/mc \) and \( h = l(1 - 2 m_E u)^{-1}/mc \). Inserting these in equations \((12)\) and \((14)\) we finally obtain

\[
\frac{d^2 u}{d\phi^2} + u = \frac{m_E}{l^2} \frac{m_E}{(1 + m_E u)}.\tag{15}\]

\(^5\)Note that charged particles follow the geodesics, Eq. \((8)\), of the metric \( g_{\mu\nu} \). This is a consequence of Eq. \((3)\). Note also that there is a different metric for each particle with a different charge-to-mass ratio. The resulting theory, therefore, is a multi-metric theory.

\(^6\)For a sphere of \( M = 1kg \), \( R = 5cm \), \( V(R) = 10^4V \), we have for an electron just grazing the sphere \( g_{\phi\phi}^{RN} = (1 - 2 m_G/R + G V(R)^2/k_\circ c^4) = (1 - 1.48 \times 10^{-26} + 9.19 \times 10^{-49}) \approx 1 \).
which is the orbit equation for the Reissner-Nordstrøm solution, and

$$\frac{d^2 u}{d\phi^2} + u = \frac{m^2 c^2}{l^2} m_E (1 - 2m_E u)^2 + 3m_E u^2,$$

which is the orbit equation in our case which we call modified general relativity. By using the initial conditions stated above these equations can be solved numerically for \( r = 1/u \), the predicted position of the electron beam on the screen from the center of the sphere. By comparing the experimental value with these predictions, the correct theory can be determined. In Figures 1 and 2 we depict the trajectory of the electrons according to the two theories. It is seen that the difference between the two predictions is large enough. Hence the experimentally favored one can be picked up rather easily.

Before we conclude, we wish to clarify the implications of the Eötvös experiment in regard to the theory presented here. The electromagnetic energy of the atoms, or any other form of energy, in an object has already been converted to mass ( and thus belongs to the first type in our classification of the energy types above). What the Eötvös experiment tells us is that the electromagnetic energy contributes in equal amounts to gravitational and inertial masses. It does not tell us that this energy couples to the gravitational field independently as energy. Had the electromagnetic energy coupled to the gravitational field independently, a deflection in the balance of the Eötvös apparatus would have been seen when two equal massless having considerably different electromagnetic binding energies were used. Of course this does not happen.

In conclusion, we have explored the conjecture of Feynman on a reformulation of general relativity. In this new scheme only the ‘mass energy’ couples to the gravitational field, but not other energy forms. We have proposed a deflection of electrons by a charged sphere experiment. The significance of this experiment is that it not only provides a new test of general relativity but also may point out to the solution of the cosmological constant problem. Not being a wave, the energy of the vacuum is not associated with quantized packages and a formula like \( E \propto f \), with \( f \) being the frequency, cannot be written and an effective mass \( m_{eff} \propto f/c^2 \) cannot be defined. Thus according to the scheme presented here the field equation for vacuum is \( R^{\mu\nu} = 0 \), implying that the cosmological constant \( \lambda = (8\pi G/c^4)\rho_V \) of standard general relativity, with \( \rho_V \) being the vacuum energy density, is \( \lambda = 0 \times \rho_V = 0 \) here. \( \lambda \) has always been equal to zero! Keeping on mind that (i) standard general relativity remains one of the least tested of scientific theories, and (ii) the theory presented here offers a very simple solution to the cosmological constant problem, the immediate performance of the experiment suggested here cannot be overemphasized.

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FIG. 1. The trajectories of the electron beam according to the Reissner-Nordstrøm (the bottom curve) and the Modified General Relativity (the top curve) theories for an anode-cathode voltage of 30 kV for the electron gun located at a vertical distance of 20 cm with an impact parameter of 7 cm, for a sphere of $R = 2.5$ cm and $V(R) = 5$ kV.
FIG. 2. Same as Fig.1, but $R = 5\, \text{cm}$. 