NLO corrections to the polarized Drell-Yan cross section in proton-proton collisions

W.L. van Neerven

aInstituut-Lorentz, University of Leiden P.O. Box 9506, 2300 RA Leiden, The Netherlands

We present the full next-to-leading order (NLO) corrected inclusive cross section for massive lepton pair production in longitudinally polarized proton-proton collisions. All QCD partonic subprocesses have been included provided the lepton pair is created by a virtual photon, which is a valid approximation for $Q < 50$ GeV. Like in unpolarized proton-proton scattering the dominant subprocess is given by the $q(\bar{q})g$-channel so that massive lepton pair production provides us with an excellent method to measure the spin density of the gluon. Using our calculations we give predictions for the longitudinal spin asymmetry measurements at the RHIC.

1. Introduction

At this moment the NLO calculations of unpolarized quantities are almost finished so that one now is concentrating on the computations of the next-to-next-to-leading (NNLO) corrections. In the case of polarized processes this stage is not reached yet and in this contribution we will report on a recent calculation of the complete NLO contribution to massive lepton pair production (Drell-Yan process). From the study of the Regge pole model in the sixties we have learnt that the predictions are in better agreement with experiment when the reaction only involves unpolarized particles. In the case the particles become polarized, predictions and data are very often at variance with each other. The same also seems to happen for perturbative QCD and therefore it will be very interesting to study polarized reactions, as will be measured in the future at the RHIC (BNL, USA), because they can provide us with a deeper insight in QCD. At this moment there are only data available from polarized deep inelastic lepton hadron scattering. However like in unpolarized scattering they provide little information about the gluon and sea-quark densities which are important quantities under study. In our contribution we will mainly focus on the former density which can be much better extracted if the hadronic reaction is dominated by partonic subprocesses with a gluon in the initial state. Processes which are suitable to extract the polarized gluon density are jet production, charm quark production in photon-hadron collisions or deep inelastic lepton-hadron scattering and direct photon production. In the subsequent part of this paper we will concentrate under which conditions the polarized gluon density can be extracted from semi-inclusive massive lepton pair production in proton-proton collisions. The latter process is given by

$$ p + p \rightarrow l^+ + l^- + X', $$

where $X$ denotes an inclusive hadronic final state. The reaction above is described by the polarized cross section

$$ \frac{d^3\Delta\sigma_{pp}}{dQdp_Tdy} = $$

$$ \sum_{a,b=q,g} \Delta f_a^p(\mu^2) \otimes \Delta f_b^p(\mu^2) \otimes \frac{d^3\Delta\sigma_{ab}(\mu^2)}{dQdp_Tdy}. $$

(2)

Here $Q$ denotes the invariant mass of the lepton pair which has the transverse momentum $p_T$ and rapidity $y$. Further $\Delta\sigma_{ab}$ and $\Delta f_a(\mu^2)$ represent the polarized partonic cross section and polarized parton density respectively which both depend on the factorization scale $\mu^2$. Notice when the value of $Q$ is sufficiently small, e.g. $Q \ll M_Z$, the reaction in Eq. (1) is dominated by a virtual photon.
\( \gamma^* \) in the intermediate state so that one can neglect the \( Z \) contribution.

In lowest order (LO) of the strong coupling constant \( \alpha_s \), the following partonic subprocesses contribute to the cross section in Eq. 2

\[
q + \bar{q} \rightarrow g + \gamma^*,
\]
(3)

\[
g + q(\bar{q}) \rightarrow q(q) + \gamma^*.
\]
(4)

In next-to-leading order (NLO) one has to compute the one-loop contributions to the Born reactions appearing in the equation above and the two to three parton subprocesses

\[
q + \bar{q} \rightarrow g + g + \gamma^*,
\]
(5)

\[
g + q(\bar{q}) \rightarrow g + q(q) + \gamma^*,
\]
(6)

\[
q_1 + q_2 \rightarrow q_1 + q_2 + \gamma^*,
\]
(7)

\[
q_1 + \bar{q}_2 \rightarrow q_1 + \bar{q}_2 + \gamma^*,
\]
(8)

\[
g + g \rightarrow g + g + \gamma^*,
\]
(9)

where the (anti-)quarks in Eqs. (7), (8) can be identical \( q_1 = q_2 \) or non-identical \( q_1 \neq q_2 \). Notice that the one-loop corrections to the Born reaction in Eq. (3), the subprocess in Eq. (5) and the interference term appearing in the \( q\bar{q} \)-channel in Eq. (7) were calculated in [2] which are in agreement with our results in [1]. In [1] we have included the remaining contributions so that at this moment the complete NLO correction to the cross section in Eq. (2) is known. The outline of this calculation and the results predicted for the RHIC experiments will be presented in the next section.

2. Regularization in n dimensions with the \( \gamma_5^- \) matrix

The computation of the virtual contributions and the radiative corrections to the partonic cross sections reveals the presence of ultraviolet, infrared and collinear divergences in loop and phase space integrals. The usual method to regularize these singularities is given by \( n \) dimensional regularization. The advantage of this method is that before one has to carry out renormalization and mass factorization all Ward-identities are automatically preserved. However this is not longer true when the \( \gamma_5^- \)-matrix and the Levi-Civita tensor appear. The latter quantities show up in electro-weak interactions and in the computation of polarized processes. The terms which violate the Ward identities are called evanescent and they have to be removed before renormalization and mass factorization are carried out. One of the most chosen prescription for the \( \gamma_5^- \)-matrix is given by the HVBM approach [3]. This prescription violates the Ward identity for the non-singlet axial vector current and the Adler-Bardeen theorem [4] so that one needs evanescent counter terms. The HVBM method is rather complicated since it requires that the \( n \)-dimensional space has to split up in a 4 and an \( n - 4 \) dimensional subspace. Accordingly the gamma-matrices and the momenta have to be split up which complicates the gamma-matrix algebra and the phase space integrals. This will complicate the calculations in particular if one wants to compute NNLO corrections. To avoid this complication we have chosen the approach in [4] and replace the \( \gamma_5^- \)-matrix by

\[
\gamma_\mu \gamma_5^- = \frac{i}{6} \epsilon_{\mu\rho\sigma\tau} \gamma^\rho \gamma^\sigma \gamma^\tau, \quad \text{or}
\]

\[
\gamma_5^- = \frac{i}{24} \epsilon_{\rho\sigma\tau\kappa} \gamma^\rho \gamma^\sigma \gamma^\tau \gamma^\kappa.
\]
(10)

In this way one can apply the usual gamma-matrix algebra in \( n \) dimensions. Moreover the integration over the final state momenta is the same as in processes where the \( \gamma_5^- \)-matrix and the Levi-Civita tensor do not appear. Furthermore we contract the Levi-Civita tensors in four dimensions before the phase space integrals are carried out. We checked that in this procedure the matrix elements are independent of an arbitrary axial gauge vector \( l \) which appears in the polarization sum

\[
\sum_{\alpha = L, R} \epsilon^\alpha(p, \alpha) \epsilon'^\alpha(p, \alpha) = -g_{\mu\nu} + \frac{l^\mu p^\nu}{l \cdot p} + \frac{l^\nu p^\mu}{l \cdot p}, \quad \text{with} \quad l^2 = 0.
\]
(11)

This method leads to more evanescent counter terms than shown by the HVBM approach. They
can be extracted from a more simple cross section than the one given in Eq. (2). Notice that the ultraviolet divergences do not need evanescent counter terms because in process (4) only the virtual photon is attached to the loop graphs. Therefore the coupling constant renormalization can be performed in the usual \( \overline{\text{MS}} \)-scheme and no additional evanescent counter term is needed. Only the collinear divergences which are removed by mass factorization

\[
d d\hat{\sigma}_{ij} \left( \frac{1}{\varepsilon} \right) = \sum_{k,l=q,g} \Delta \Gamma_{ki} \left( \frac{1}{\varepsilon}, \mu^2 \right) \otimes \Delta \Gamma_{lj} \left( \frac{1}{\varepsilon}, \mu^2 \right) = \Delta \Gamma_{ij} \left( \frac{1}{\varepsilon}, \mu^2 \right) \sigma_{ij}(\mu^2),
\]

with

\[
\Delta \Gamma_{ij} = \delta_{ij} + \frac{\alpha_s}{2\pi} \left[ \left( \frac{2}{\varepsilon} + \gamma_E - \ln 4\pi \right) \Delta P_{ij} \right],
\]

need evanescent counter terms for all four splitting functions \( \Delta P_{ij} \). The evanescent counter terms for the splitting functions \( \Delta P_{qg} \) and \( \Delta P_{gg} \) are extracted from the Drell-Yan polarized cross section \( d\Delta \sigma/dQ \) of the processes in Eqs. (3) and (4) respectively. This is achieved by comparing the coefficient functions using our method above with the ones obtained from a four dimensional regularization scheme where there is no problem with the \( \gamma_5 \)-matrix and the Levi-Civita tensor. For instance one can regularize the collinear divergences by taking the external quark and gluon legs off-shell \( (p^2 < 0) \) or the quark gets a mass \( m \) and one puts the external legs on-shell. In this case the kernels are given by

\[
\Delta \Gamma_{ij} = A_{ij}(p^2, m^2, \mu^2) = \langle j(p)\mid O_i \mid j(p) \rangle,
\]

where \( A_{ij} \) \( (i,j=q,g) \) denote the renormalized operator matrix elements corresponding to the local operators appearing in the operator product expansion for the product of two electromagnetic currents. To obtain the evanescent counter terms for \( \Delta P_{qg} \) and \( \Delta P_{gg} \) we followed the same procedure for the total cross section for polarized Higgs production given by the subprocesses

\[
q + g \to q + H, \quad g + g \to g + H.
\]

The genuine \( \overline{\text{MS}} \)-scheme for mass factorization is now given by the following replacement in Eq. (13)

\[
\Delta P_{ij} \to \Delta P_{ij} + \text{evanescent counter term},
\]

where \( \Delta P_{ij} \) can e.g. be found in [1]. In order to check that the same evanescent counter terms also apply to the cross section in Eq. (2) we recalculated the latter using a four dimensional regularization method. It turned out that the evanescent counter terms are the same for both the reactions in Eqs. (5)-(9) and the processes mentioned above.

3. Results

The hadronic cross section in Eq. (2) has been plotted using the following input. For the C.M. energy of proton-proton collisions at the RHIC we have chosen \( \sqrt{s} = 200 \) GeV. Further we adopted the NLO approximation for the running coupling constant and the polarized parton densities given by the parametrizations in [7], [8]. For the factorization scale, which is set to be equal to the renormalization scale, we have taken \( \mu^2 = p_T^2 + Q^2 \). Both parametrizations are presented in two scenarios depending on the size of the gluon density. The parametrizations in [7] are represented by the valence scenario (VS) and the standard scenario (SS). Those in [8] are given by scenarios 1 (S1) and 2 (S2) respectively. Over the whole \( x \)-region the polarized gluon densities for all four scenarios approximately satisfy the following inequalities

\[
\Delta f_g^{VS}(x) < \Delta f_g^{SS}(x) < \Delta f_g^{S2}(x) < \Delta f_g^{S1}(x).
\]

From this behaviour it turns out that the \( qg \)-channel dominates the cross section \( d^2\Delta \sigma^{pp}/dQ/dp_T \) provided \( p_T > Q/2 \). An exception is the VS-scenario where the \( qg \) subprocess becomes of equal importance over the whole \( p_T \)-range. This feature was already discovered in LO in [8] so that the NLO corrections do not change this picture. However both scenarios (VS) and (SS) lead to the same transverse momentum distributions so that one cannot distinguish them. The same holds for the S2 scenario in [8] of which the cross section is slightly larger than the ones.
given by VS and SS. Only the S1 scenario leads to a larger cross section than the other ones. This is revealed by Figure 1 where we have plotted in NLO the longitudinal asymmetry defined by

$$A_{LL} = \frac{d^2\Delta\sigma^{pp}/dQ/dp_T}{d^2\sigma^{pp}/dQ/dp_T},$$

(18)

for $Q = 6$ GeV. For the computation of the unpolarized cross section in the denominator we have chosen the GRV98 set in [10] with the same factorization scale as given above. From this figure we infer that one cannot distinguish between the VS and SS scenario. At $p_T = 20$ GeV/c the difference between both scenarios and the S2 scenario can be observed when the polarized cross section is known up to 12.5% assuming 100% polarization for the proton beams which is very unlikely. However when we allow for a 25% uncertainty in the polarized cross section one can distinguish between scenarios S1 and S2 even if the protons are not fully polarized (e.g. 75%). In Figure 2 we have studied the effect of the NLO corrections to the longitudinal asymmetry when compared with the LO approximation. The effect is rather small for the SS and S1 scenarios but amounts to 30-40% at large $p_T$ for the VS and S2 scenarios. From this one can conclude that the K-factors for the polarized and unpolarized cross sections are about the same in particular for scenarios SS and S1.

Figure 1. Longitudinal asymmetry $A_{LL}$ in percentage.

Figure 2. Ratio $A_{LL}^{NLO}/A_{LL}^{LO}$.

REFERENCES

[1] V. Ravindran, J. Smith, W.L. van Neerven, hep-ph/0207076.
[2] S. Chang, C. Coriano, R.D. Field and L.E. Gordon, Nucl. Phys. B512 (1998) 393; S. Chang, C. Coriano, R.D. Field, Nucl. Phys. B528 (1998) 285.
[3] G.'t Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189; P. Breitenlohner and B. Maison, Commun. Math. 53 (1977) 11, 39, 55.
[4] S.L. Adler and W. Bardeen, Phys. Rev. 182 (1969) 1517.
[5] D. Akyeampong and R. Delbourgo, Nuov. Cim. 17A (1973) 578, 18A (1973) 94, 19A (1974) 219.
[6] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
[7] M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D63 (2001) 094005.
[8] M. Blumlein and H. Böttcher, Nucl. Phys. B636 (2002) 225.
[9] E.L. Berger, L.E. Gordon and M. Klasen, Phys. Rev. D58 (1998) 074012, D62 (2000) 014014.
[10] M. Glück, E. Reya and A. Vogt, Eur. Phys. J. C5 (1998) 461.