Coherent transport and Andreev reflection in ferromagnetic semiconductor/superconductor/ferromagnetic semiconductor double tunelling junctions

Y. C. Tao
Department of Physics, Nanjing Normal University, Nanjing 210097, China
E-mail: taoyy@nju.edu.cn

Abstract. An extended Blonder-Tinkham-Klapwijk (BTK) approach is applied to study the coherent spin polarized transport in an ferromagnetic semiconductor (FS)/superconductor (SC)/FS double tunelling junction, in which the Andreev reflection, mismatches in the effective mass and Fermi velocity between the FS and SC, as well as strengths of potential scattering at the interfaces are included. It is demonstrated that they have different effects on the oscillations of differential conductances with energy, which is quite different from that in an ferromanet (FM)/SC/FM double tunneling junction. A direct robust measure of the spin polarization of the FS using conductance spectroscopy, being very weakly dependent on the strengths of potential scattering and energy, is also revealed.

1. Introduction
A fundamental understanding of new ferromagnetic semiconductor (FS) $\text{Ga}_{1-x}\text{Mn}_x\text{As}$ is very relevant in this context since this is a canonical FS that remains the most thoroughly studied of all such materials [1]. The large tunneling magnetoresistance observed in magnetic tunnel junctions derived from this material implies that the spin polarization ($P$) may be large even for small Mn concentrations [2, 3, 4]. However, there have been few direct measurements of $P$ for this important material except that J. G. Barden et al [5] carried out a direct measurement of $P$ in the FS $\text{Ga}_{1-x}\text{Mn}_x$, using Andreev reflection (AR) spectroscopy.

In this work, extending the Blonder-Tinkham-Klapwijk (BTK) [6] approach, we study the coherent spin-polarized tunneling in the FS/SC/FS double tunnel junction with the SC interlayer thin enough. Our motivation to study it in this work is not only to look for another direct measurement of $P$ but also reveal the related properties of coherent transport in the junction resulting from the mismatches in effective mass and Fermi velocity between the FS and SC. For a spin-$\sigma$ hole incident on the left FS/SC interface from the left FS, there are two sets of reflected quasiparticle waves in the left FS: normal reflection as a hole of spin-$\sigma$ with probability $B_\sigma$ and AR as an electron of the opposite spin $\overline{\sigma}$ with $A_\overline{\sigma}$, and two sets of transmitted waves: holelike quasiparticle with probability $F_\overline{\sigma}(E)$ and electronlike quasiparticle with probability $G_\overline{\sigma}(E)$ as shown in Fig. 1 (a). The conservation of probability requires that $A_\overline{\sigma}(E) + B_\sigma(E) + F_\overline{\sigma}(E) + G_\overline{\sigma}(E) = 1$. From this conservation condition, one can find that the current calculated at the left FS/SC interface by use of $1 + A_\overline{\sigma}(E) - B_\sigma(E)$ is unequal to that
calculated at the right interface by use of $F_\sigma(E) - G_\sigma(E)$. By the same approach similar to that in Refs. [7, 8] to solve this difficulty, it is proposed that in the presence of a voltage drop between the two FS electrodes, there are not only spin-polarized holes incident on the left FS/SC interface from the left FS, but also spin-polarized electrons incident on the right SC/FS interface from the right FS, as shown in Fig. 1(b). It is found that the quasiparticle interference in the central layer and resonant tunneling of tunneling conductance in the FS/SC/FS double junctions, are quite different from those in the FM/SC/FM double junction[7] due to the mismatches in effective mass and Fermi velocity between the FS and SC.

**Figure 1.** Schematic illustration of reflections and transmissions of quasiparticles in an FS/SC/FS double junction.

### 2. Theoretical model

Considering an FS/SC/FS double tunnel junction, in which the left and right electrodes are made up of the same FS GaMnAs: they are separated from the central SC electrode of thickness $a$ by two thin insulating barriers. The layers are assumed to be the x-y plane and to be stacked along the z-direction. The FS/SC interfaces are respectively described by two $\delta$-type barrier potentials $V(x) = U\delta(z \pm a/2)$, where $U$ depends on the product of the barrier height and width.

A two-band model of parabolic energy dispersion with $\Gamma$ is applied for the FS, where $\Gamma$ is the difference between the energies $E_M$ and $E_m$ of the tops of the majority (M) and minority (m) valence subbands with the spins parallel and antiparallel to the local magnetization, respectively. Here, the valence subbands comprise heavy and light holes with $m^*$ the effective mass and $h(r)$ the exchange energy. The two FS’ have the same exchange energy, which is described by $h(r) = h_0[\Theta(-z - a/2) \pm \Theta(z - a/2)]$, where the plus and minus signs respectively correspond to the parallel (P) alignment of the two FS electrodes with parallel magnetization directions and the antiparallel (AP) alignment with different magnetization directions, and $h_0$ is equal to $\Gamma/2$.

The SC is assumed $s$-wave pairing and described by a BCS-like Hamiltonian with $\Delta$ the bulk superconducting gap. In the following, $E$ is the quasiparticle energy relative to the Fermi energy $E_F$, $\sigma$ is the spin opposite to $\sigma$ with $\uparrow$ and $\downarrow$, $\eta_\sigma = 1$ for $\sigma = \uparrow$, and $\eta_\sigma = -1$ for $\sigma = \downarrow$. 

**Figure 1.** Schematic illustration of reflections and transmissions of quasiparticles in an FS/SC/FS double junction.
Figure 2. Differential conductance for the heavy holes as a function of $E/\Delta$ at different $Z$ and $P$ with the P (solid lines) and AP (dot lines) alignments.

For the injection of a spin-up hole from the left FS into the SC as shown in Fig. 1(a), there are four possible trajectories: normal reflection ($b_{\sigma}$), AR ($a_{\sigma}$), transmission to the right FS as a holelike quasiparticle ($f_{\sigma}$), and transmission as an electronlike quasiparticle ($g_{\sigma}$),

$$\psi(z) = \begin{cases} 
  e^{-i\eta'hz} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + a_{\sigma} e^{-i\eta'hz} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_{\sigma} e^{i\eta'hz} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & z \leq -a/2, \\
  [e^{-ik_{\sigma}'hz} + e^{i\xi_{\sigma}'hz}] \begin{pmatrix} v \\ u \end{pmatrix} + [d e^{ik_{\sigma}'hz} + d' e^{-ik_{\sigma}'hz}] \begin{pmatrix} u \\ v \end{pmatrix}, & -a/2 \leq z \leq a/2, \\
  f_{\sigma} e^{-i\eta'hz} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + g_{\sigma} e^{i\eta'hz} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & z \geq a/2, \end{cases} \quad (1)$$

where $\eta_{(h)} = \sqrt{2m^*(E_{FS}^F + (-)E + \eta_0 h_0)}$, $k_{(h)} = \sqrt{2m_e(E_{SC}^F + (-)\Omega)/h}$, $u = \sqrt{(1 + \Omega/E)/2}$, and $v = \sqrt{(1 - \Omega/E)/2}$ with $\Omega = \sqrt{E^2 - \Delta^2}$. In Eq. (1), all coefficients will be determined by the matching conditions of the wave functions. From the matching conditions, we obtain the transmission and reflection coefficients. From them, we can get $A_{\sigma} = |a_{\sigma}|^2 q_{\sigma}/q_{\sigma}$, $B_{\sigma} = |b_{\sigma}|^2$, $F_{\sigma} = |f_{\sigma}|^2$, and $G_{\sigma} = |g_{\sigma}|^2 q_{\sigma}/q_{\sigma}$, respectively, corresponding to the AR and normal reflection probabilities, the transmission probabilities of holelike and electronlike quasiparticles. For a spin-$\sigma$ electron incident from the right, as shown in Fig. 1(b), the corresponding $A_{\sigma}$, $B_{\sigma}$, $F_{\sigma}$, and $G_{\sigma}$ can be obtained by a similar calculation, and then all the relative probabilities in the AP alignment can be similarly given. Here, two dimensionless parameters $Z = U m_e/(h^2k_{SC}^F)$ with $k_{SC}^F$ the Fermi wave vector of the SC and $P \equiv \Gamma/E_F$ are respectively introduced to describe the barrier strength and relative spin-polarized degree.
3. Results and conclusions

After obtaining all the transmission and reflection probabilities, we can perform the calculation of the currents in the P and AP configurations, which are respectively given by [7]

\[
G_P(E) = G_0 \sum_{\sigma = \uparrow, \downarrow} P_\sigma [1 + A_\sigma(E) - B_\sigma(E) + \mathcal{T}_\sigma(E) - \mathcal{G}_\sigma(E)],
\]

\[
G_{AP}(E) = G_0 \sum_{\sigma = \uparrow, \downarrow} P_\sigma [1 + A_\sigma(E) - B_\sigma(E) + \mathcal{T}_\sigma(E) - \mathcal{G}_\sigma(E)],
\]

(2)

where \( G_0 = 2e^2/h \).

Figure 3. TMR for an FS/SC/FS double junction as a function of \( P \).

Figure 2 shows differential conductances as a function of \( E \) at different \( Z \) and \( P \) for the heavy holes in Figure 2 shows differential conductances as a function of \( E \) at different \( Z \) and \( P \) for the heavy holes in the P and AP alignments. The parameters in the calculation are taken to be \( a = 200/k_F \), \( E_F = 100.0 mev \), \( \Delta = 1.4 mev \), \( m_e = 1.0 \), and \( m^* = 0.45 m_e \) for the heavy hole and 0.08\( m_e \) for the light hole [7]. It is found that the conductances exhibit amplitude-varying oscillatory behaviors at a fixed \( Z \), arising from the quantum interference effects of quasiparticle in SC, however, there is no \( \pi \) phase difference between \( G_P \) and \( G_{AP} \), which appears in the FM/SC/FM double tunnel junctions[7]. In addition, with increasing \( Z \), the conductance peaks are not obviously split into two peaks, which is also different from that for the FM/SC/FM double tunnel junctions [7]. Furthermore, as \( Z \) is increased, the difference between the peak values of conductances in the P and AP alignments decreases. On the other hand, we show that, with the enhancement of \( P \), the difference gradually increases. As a result, it is interesting that when \( P \) and \( Z \) are simultaneously enhanced to certain values such as \( P = 0.6 \) and \( Z = 1.0 \),
the values of conductance in the P and AP configurations are almost equal. In the light of our calculation, for the light holes, there are the same features as those for the heavy holes except that in the P and AP alignments, the peak values of conductance are approximately equal and the increase of $P$ at bigger values almost can not cause any changes of the conductances. Here, the absence of $\pi$ shift between the conductance oscillations of the P and AP configurations, which was found in [7] for $m^*/m_e = 1$, is really due to different effective masses between the FS and SC. According to our calculations, with the value of $m^*/m_e$ decreased, the $\pi$ shift more easily disappears especially for smaller $m^*/m_e$ and the critical value of $m^*/m_e$ is 0.8. Last and most importantly, at smaller $Z$, the values of the conductance peaks for the heavy and light holes in the P and AP alignments nearly have no variations with energy $E$, and depend only weakly on the presence of potential at the interface, which can be used to estimate $P$ through measuring TMR given by $(G_P - G_{AP})/G_{AP}$, where $G_P$ and $G_{AP}$ are the values of the conductance peaks. In Fig. 3 is illustrated the dependence of TMR on $P$ for $Z = 0.0, 0.05$, and $0.15$ [5]. We find that TMR nearly does not change with $Z$ and exhibits linear variations with increasing $P$. In experiments, we can obtain an FS/SC/FS double tunnel junction approaching to the metallic contact ($Z = 0$), since such a single FS/SC junction has been fabricated [5].

In summary, taking into account the AR, mismatches in the effective masses and Fermi velocity between the FS and SC, and strengths of potential at the interface, we extend the BTK approach to studying the coherent transport properties of an FS/SC/FS double tunnel junction. We show that they possess different influences on the oscillations of differential conductances with energy, which are quite different from those in the FM/SC/FM double junctions. Also we uncover a direct measure of the spin polarization in the FS using conductance spectroscopy.

Acknowledgments
This work was supported in part by the National Science Foundation of China under Grant No. 10347118. Tao’s work was also supported by Natural Science Foundation of Jiangsu Education Department of China under Grant No. 2004102TSJB141.

References
[1] H. Ohno, in Semiconductoe Spintronics and Quantum Computation, edited by D.D. Awschalom, N. Samarth, and D. Loss (Springer, New Yok, 2002), p. 1.
[2] Y.C. Tao, J.G. Hu, and H. Liu 2004 J. Appl. Phys. 96 498.
[3] M. Tanaka and Y. Higo 2001 Phys. Rev. Lett. 83 026602.
[4] S.H. Chun et al 2002 Phys. Rev. B 66 100408.
[5] J.G. Braden, J.S. Parker, P. Xiong, S.H. Chun, and N. Samarth 2003 Phys. Rev. Lett. 91 056602.
[6] G.E. Blonder, M. Tinkham, and T.M. Klapwijk 1982 Phys. Rev. B 25 4515.
[7] Z.C. Dong, R. Shen, Z.M. Zheng, D.Y. Xing, and Z.D. Wang 2003 Phys. Rev. B 67 134515.
[8] C.J. Lambert 1991 J. Phys: Condens. Matter. 3 6579.