Clusters of galaxies are the most massive collapsed objects in the universe and sit at the top of the hierarchy of nonlinear structures. They were first identified as overdense regions in the projected number counts of galaxies (e.g., Abell 1958; Zwicky et al. 1968). However, nowadays clusters can be identified over the whole electromagnetic range, including as X-ray sources (e.g., Gladders & Yee 2005; Koester et al. 2007; Hao et al. 2010; Staniszewski et al. 2009; Vanderlinde et al. 2010; Marriage et al. 2011; Planck Collaboration et al. 2011; Song et al. 2012b).

Galaxy clusters have been unveiled in the last decade. These clusters are virialized and spherically symmetric, have very complex dynamics—are expected to be less affected by the complex baryonic physics affecting the intracluster medium. However, a number of possible systematics can affect dynamical mass estimations and must be carefully taken into account.

Biviano et al. (2006) for example studied a sample of 62 clusters at redshift \( z = 0 \) from a \( \Lambda \) CDM cosmological hydrodynamical simulation. They estimated virial masses from both dark matter (DM) particles and simulated galaxies in two independent ways: (1) a virial mass estimator that corrects for the surface pressure term and (2) a mass estimator based entirely on the velocity dispersion \( \sigma_v \). They also modeled interlopers by selecting galaxies within cylinders of different radius and length 192 \( h^{-1} \) Mpc and applying interloper rejection techniques. They found that the mass estimator based entirely on velocity dispersions is less sensitive on the radially dependent incompleteness. Furthermore, the effect of interlopers is smaller if early-type galaxies, defined in the simulations using their mean redshift of formation, are selected. However, the velocity dispersion–mass relations place a systematic floor on velocity dispersion mass calibration at the 5% level in dispersion.
dispersion of early type galaxies is biased low with respect to DM particles.

Evrard et al. (2008) analyzed a set of different simulations with different cosmologies, physics, and resolutions and found that the three-dimensional (3D) velocity dispersion of DM particles within the virial radius can be expressed as a tight function of the halo virial mass, regardless of the simulation details. They also found that the scatter about the mean relation is nearly log-normal with a low standard deviation \( \sigma_{\text{log}} \approx 0.04 \).

More recently, White et al. (2010) used high-resolution N-body simulations to study how the influence of large-scale structure could affect different physical probes, including the velocity dispersion based upon subhalo dynamics. They found that the highly anisotropic nature of infall material into clusters of galaxies and their intrinsic triaxiality is responsible for the large variance of the one-dimensional (1D) velocity dispersion under different lines of sight. They also studied how different interloper removal techniques affect the velocity dispersion and the stability of velocity dispersion as a function of the number of subhalos used to estimate it. They found that only when using small numbers of subhalos (<30) is the line-of-sight velocity dispersion biased low, and the scatter significantly increases with respect to the DM velocity dispersion. Furthermore, the effect of interlopers is different for different interloper rejection techniques and can significantly increase the scatter and bias low-velocity dispersion estimates.

Currently, IR, Sunyaev–Zel’dovich effect (SZE), and X-ray cluster surveys are delivering significant numbers of clusters at redshifts \( z > 1 \) (e.g., Stanford et al. 2005; Fassbender et al. 2011; Song et al. 2012b). Mass calibration of these cluster samples is challenging using weak lensing, making velocity dispersion mass estimates particularly valuable. At these redshifts, it is also prohibitively expensive to obtain spectroscopy of large samples of cluster galaxies, and therefore dispersion measurements must typically rely on small samples of 20–30 cluster members. This makes it critically important to understand how one can best use the dynamical information of a few dozen of the most luminous cluster galaxies to constrain the cluster mass. It is clear that with such a small sample one cannot obtain precise mass estimates of individual clusters. However, for mass calibration of a cluster SZE or X-ray survey, for example, an unbiased mass estimator with a large statistical uncertainty is still valuable.

In this work, we focus on the characterization of dynamical mass estimators of clusters with particular emphasis on high-\( z \) clusters with a small number of measured galaxy velocities. The plan of the paper is as follows. In Section 2, we briefly introduce the simulation and describe the adopted semi-analytic model (SAM). We present the results of our analysis based on the general galaxy population and in Section 3 and of the red-sequence-selected cluster galaxy population in Section 4. In Section 5, we investigate the effect of interlopers in red-sequence-selected galaxies. In Section 6, we summarize our findings and give our conclusions.

2. INPUT SIMULATION

This analysis is based on the study of clusters with \( M_{\text{vir}} \geq 10^{14} M_\odot \) up to \( z \approx 1.2 \) (Table 1) extracted from the publicly available galaxy catalog produced using the SAM by De Lucia & Blaizot (2007) on the Millennium Simulation (Springel et al. 2005). The Millennium Simulation adopts the following values for the parameters of a flat Λ cold dark matter model: \( \Omega_{\text{DM}} = 0.205 \) and \( \Omega_b = 0.045 \) for the densities in cold dark matter and baryons at redshift \( z = 0 \), \( \sigma_8 = 0.9 \) for the rms linear mass fluctuation in a sphere of radius \( 8 \, h^{-1} \) Mpc, \( h = 0.73 \) for the present dimensionless value of the Hubble constant, and \( n = 1 \) for the spectral index of the primordial fluctuation. The simulation follows the evolution of \( 2.16 \times 10^9 \) DM particles from \( z = 127 \) to the present day within a cubic box of \( 500 \, h^{-1} \) Mpc on a side. The individual DM particle mass is \( 8.6 \times 10^8 \, h^{-1} M_\odot \). The simulation was carried out with the massively parallel GADGET-2 code (Springel 2005). Gravitational forces were computed with the TreePM method, where long-range forces are calculated with a classical particle-mesh method while short-range forces are determined with a hierarchical tree approach (Barnes & Hut 1986). The gravitational force has a Plummer-equivalent comoving softening of \( 5 \, h^{-1} \) kpc, which can be taken as the spatial resolution of the simulation. Full data are stored 64 times, spaced approximately equally in the logarithm of the expansion factor. DM halos and subhalos were identified with the friends-of-friends (FOF; Davis et al. 1985) and SUBFIND (Springel et al. 2001a) algorithms, respectively. Based on the halos and subhalos within all the simulation outputs, detailed merger history trees were constructed, which form the basic input required by subsequently applied SAMs of galaxy formation.

We recall that the SAM we employ builds upon the methodology originally introduced by Kauffmann et al. (1999), Springel et al. (2001b), and De Lucia et al. (2004b). We refer to the original papers for details.

The SAM adopted in this study includes explicitly DM substructures. This means that the halos within which galaxies form are still followed even when accreted onto larger systems. As explained in Springel et al. (2001b) and De Lucia et al. (2004b), the adoption of this particular scheme leads to the definition of different galaxy types. Each FOF group hosts a Central galaxy. This galaxy is located at the position of the most bound particle of the main halo, and it is the only galaxy fed by radiative cooling from the surrounding hot halo medium. Besides central galaxies, all galaxies attached to DM substructures are considered satellite galaxies. These galaxies were previously central galaxies of a halo that merged

| \( z \) | \( N_{\text{clus}} \) |
|-----|-------|
| 0.00 | 3133  |
| 0.09 | 2953  |
| 0.21 | 2678  |
| 0.32 | 2408  |
| 0.41 | 2180  |
| 0.51 | 1912  |
| 0.62 | 1635  |
| 0.75 | 1292  |
| 0.83 | 1152  |
| 0.91 | 1020  |
| 0.99 | 867   |
| 1.08 | 702   |
| 1.17 | 552   |

Note. Column 1: redshift \( z \); Column 2: number of clusters \( N_{\text{clus}} \).
to form the larger system in which they currently reside. The positions and velocities of these galaxies are followed by tracing the surviving core of the parent halo. The hot reservoir originally associated with the galaxy is assumed to be kinematically stripped at the time of accretion and is added to the hot component of the new main halo. Tidal truncation and stripping rapidly reduce the mass of DM substructures (but not the associated stellar mass) below the resolution limit of the simulation (De Lucia et al. 2004a; Gao et al. 2004). When this happens, we estimate a residual surviving time for the satellite galaxies using the classical dynamical friction formula, and we follow the positions and velocities of the galaxies by tracing the most bound particles of the destroyed substructures.

3. PROPERTIES OF THE FULL GALAXY POPULATION

3.1. Intrinsic Galaxy Velocity Dispersion

Evrard et al. (2008) showed that massive DM halos adhere to a virial scaling relation when one expresses the velocity dispersion of the DM particles as a function of the virial mass of the halo in the form

$$\sigma_{\text{DM}}(M_{\text{vir}}, z) = \sigma_{\text{DM,15}} \left( \frac{h(z)M_{\text{vir}}}{10^{15} M_\odot} \right)^{\alpha} ,$$

(1)

where \(\sigma_{\text{DM,15}} = 1082.9 \pm 4.0 \text{ km s}^{-1}\) is the typical 3D velocity dispersion of the DM particles within \(R_{\text{vir}}\) for a \(10^{15} h^{-1} M_\odot\) cluster at \(z = 0\) and \(\alpha = 0.3361 \pm 0.0026\). Similarly, we first compute for each cluster the 3D velocity dispersion \(\sigma_{3D}\) (divided by \(\sqrt{3}\)) of all the galaxies within \(R_{\text{vir}}\) and stellar mass predicted by the adopted SAM larger than \(5 \times 10^9 M_\odot\). In order to measure the velocity dispersion we use the robust estimator of Beers et al. (1990). We also consider the measurement uncertainties associated with each galaxy velocity to be negligible. We then fit the relation between \(M_{\text{vir}}\) and \(\sigma_{3D}\) in a more general form, allowing for a differential evolution of the normalization as a function of redshift. In more detail we assume log(\(\sigma_{3D}\)) \(\propto C \log(h(z)) + \log(M_{\text{vir}})/B\). As a result we can express the dynamical mass \(M_{\text{dyn}}\) as

$$M_{\text{dyn}} = \left( \frac{\sigma_v}{A \times h(z)C} \right)^B 10^{15} M_\odot ,$$

(2)

where \(\sigma_v\) is the observed velocity dispersion, \(A\) is a normalization that is a function of the Hubble constant, \(B\) is a characteristic exponent, and \(C\) is a calibration parameter. The best-fitting parameters and associated statistical uncertainties are highlighted in Table 2.

Table 2

| Variable | Best Value | Statistical Uncertainty |
|----------|------------|-------------------------|
| A        | 939        | 0.55                    |
| B        | 2.91       | 0.0021                  |
| C        | 0.33       | 0.0019                  |

After accounting for the differences in the Hubble parameter, our measured normalization of the galaxy velocity dispersion—mass relation is within \(\lesssim 3\%\) of Evrard et al. (2008). This reflects the differences between the subhalo and DM particle dynamics. As has been previously pointed out (e.g., Guo et al. 2004; Goto 2005; Faltenbacher & Diemand 2006; Evrard et al. 2008; White et al. 2010), the velocity bias between galaxies and DM is expected to be small, \(b_v \lesssim 5\%\). But to be absolutely clear, we adopt our measured galaxy velocity dispersion—mass calibration in the analyses that follow.

For the full sample of clusters analyzed (see Table 1), we then compute the dynamical masses by applying Equation (2) to (1) the 3D galaxy velocity dispersion (divided by \(\sqrt{3}\)) and Equation (2) to each orthogonal projected 1D velocity dispersion along the simulated box axes. Figure 1 shows the comparison between the virial masses \(M_{\text{vir}}\) and the resulting dynamical mass \(M_{\text{dyn}}\).
masses $M_{3D}$ (left panel) and $M_{1D}$ (right panel) for the full sample of clusters. The best fit of the relation (dashed black and white lines) is virtually indistinguishable from the one-to-one relation (dotted-dashed purple line) in the case of the 3D velocity dispersion. On the other hand, in the case of the 1D velocity dispersion there is a small but detectable difference between the one-to-one relation and the best fit. The best fit of the dynamical mass for the 1D velocity dispersion is about $\lesssim 1\%$ lower than the one-to-one relation. We will show in Section 3.2 that this difference can be explained in terms of triaxial properties of halos (Hopkins et al. 2005). Typical logarithmic scatters of $\sigma_{M_{1D}/M_{\text{vir}}} \simeq 0.145$ and $\sigma_{M_{1D}/M_{\text{vir}}} \simeq 0.334$ are highlighted with dotted black and white lines in $\log_{10}$ scale. We find that, similar to results by White et al. (2010), using the 1D velocity dispersion rather than the 3D velocity dispersion increases the intrinsic log scatter around the mean relation by a factor of $\sim 2.3$.

We further investigate the intrinsic scatter in the relation between the true virial masses and the dynamical mass estimates in Figure 2. Taking $\sigma$ to be the standard deviation of the logarithm of the ratio between the dynamical mass estimate and the virial masses, we show that in the case of the 3D velocity dispersion (dashed red line) and the 1D velocity dispersion (dotted black line) the scatter increases with redshift. The solid black line shows a linear fit to the evolution of the intrinsic $M_{\text{dyn,1D}}$ scatter and can be expressed as

$$\sigma_{\ln(M_{1D}/M_{\text{vir}})} \simeq 0.3 + 0.075 \times z.$$  

Velocity dispersions are $\sim 25\%$ less accurate for estimating single cluster masses at $z = 1$ than at low redshift.

The logarithmic scatter of the 1D velocity dispersion mass estimator $\sigma_{M_{1D}}$ around the true mass arises from two sources of scatter: (1) the logarithmic scatter between the 3D velocity dispersion mass estimator and the true mass $\sigma[M_{3D}/M_{\text{vir}}]$ (red dashed line in Figure 2), and (2) the logarithmic scatter between the 1D and 3D velocity dispersions $\sigma[\sigma_{1D}/\sigma_{3D}]$ (solid green line). The expected 1D dispersion mass scatter is then the quadrature addition of these two sources:

$$\sigma_{M_{1D}}^2 \sim \sigma^2[M_{3D}/M_{\text{vir}}] + [B \times \sigma[\sigma_{1D}/\sigma_{3D}]]^2,$$

where $B$ is the best-fitting slope parameter from Equation (2).

The expected $\sigma_{M_{1D}}$ estimate from Equation (4) appears as a dotted-dashed purple line in Figure 2; note that this estimate is in excellent agreement with the directly measured scatter (dotted black line). Therefore, we show—as pointed out by White et al. (2010)—that the dominant contributor to the scatter is the intrinsic triaxial structure of halos. Furthermore, its evolution with redshift is also the dominant source of the increasing scatter of the 1D dynamical mass estimates with redshift. By comparison, the scatter between the 3D velocity dispersion mass estimator and the true mass $\sigma[\ln(M_{3D}/M_{\text{vir}})]$, which is reflecting departures from dynamical equilibrium due to ongoing merging in the cluster population, is relatively minor. Ultimately, it is the lack of observational access to the full 3D dynamics and distribution of the galaxies that limits us from precise single cluster dynamical mass estimates. We stress however that the exact amount of this effect would significantly change according to different cluster selections, which can reflect different dynamical histories.

We finally investigate the shape of the observed distribution of the scatter in the mass–velocity dispersion relation. In particular, Evrard et al. (2008) found a roughly log-normal distribution with a significant tail to high-velocity dispersions due to merger transients. The histogram on the left panel of Figure 3 shows indeed that the probability distribution function (PDF) of $\ln(M_{3D}/M_{\text{vir}})$ is well fit by a Gaussian distribution in logarithmic space (solid red line), although the distribution has a significant skewness of 0.16. However, as shown on the right panel of Figure 3, when the dynamical mass is computed from the 1D line-of-sight velocities, the recovered distribution is both broader and more Gaussian in log space, with a skewness of only 0.08. The merger induced skewness is apparently largely masked by the variation in dispersion along different lines of sight.

### 3.2. Triaxiality and Dispersion Biases

The presence of pronounced departures from sphericity in DM halos (Thomas & Couchman 1992; Warren et al. 1992; Jing & Suto 2002), if not approximately balanced between prolate and oblate systems, could in principle not only increase the scatter in dynamical mass estimates, but also lead to a bias. If, for example, clusters were mainly prolate systems, with one major axis associated to a higher velocity dispersion and two minor axes with a lower velocity dispersion, there should be two lines of sight over three associated with a lower velocity dispersion. This could potentially lead to a bias in the 1D velocity dispersion with respect to the 3D velocity dispersion. To quantify this possible bias, we first compute the moment of inertia for each cluster in the sample. We then calculate the velocity dispersions along each of the three principal axes, in contrast with the velocity dispersions computed along the three orthogonal simulated box axes used in Section 3.1. As has been pointed out before (Tormen 1997; Kasun & Evrard 2005; White et al. 2010), the inertia and velocity tensor are quite well aligned, with a typical misalignment angle of less than $30^\circ$. In Figure 4, we show (at different redshifts) the lowest velocity dispersion $\sigma_0$ with black crosses, the highest $\sigma_2$ with green stars, and the intermediate one $\sigma_1$ with red diamonds normalized to the 3D velocity dispersion $\sigma_{3D}$ (divided by $\sqrt{3}$). Dashed blue lines
Figure 3. Left panel: black histograms show the probability distribution function of the logarithmic ratio between the dynamical mass computed from the 3D velocity dispersions and the virial mass. The red continuous line is the best Gaussian fit. Skewness of the distribution is highlighted at the top of the plot. Right panel: same as for the right panel, but for the 1D line-of-sight velocities follow a log-normal distribution with less skewness.

Figure 4. We show for each cluster the velocity dispersion along the three major axes of the inertia momentum (black crosses for the smaller, red diamonds for the intermediate, and green stars for the larger) normalized to the 3D velocity dispersion divided by $\sqrt{3}$ as a function of the cluster mass in different redshift bins. The black solid line is the best fit of the intermediate axis velocity dispersion, and the dashed blue lines are the median and the 16 and 84 percentiles of the full distribution. DEV is the associated standard deviation.

are the 16, 50, and 84 percentiles of the full distribution and DEV is the associated standard deviation which, as expected from Figure 2 is increasing with redshift. A perfectly spherical cluster in this plot would therefore appear with the three points lying all at the value 1, whereas prolate and oblate systems will have the intermediate velocity dispersions $\sigma_1$ closer to the lower one $\sigma_0$ and to the higher one $\sigma_2$, respectively. The black solid line is the best fit of the distribution of the intermediate $\sigma_1$ velocity dispersions and it is very close to unity, showing that dynamically, clusters do not have a very strong preference among prolate and oblate systems. This result holds for the range of redshifts and masses we examine here.

This can be better seen in Figure 5, where we show that we measure only a mild excess (at $\lesssim 5\%$ level) of prolate systems. In addition, for each cluster in the sample, we compute a “prolateness” quantity $Prol$ as

$$Prol = \frac{(\sigma_2 - \sigma_1) - (\sigma_1 - \sigma_0)}{\sigma_{3D}}.$$  

A prolate system will have a positive $Prol$ value whereas an oblate one will have a negative $Prol$. Figure 5 shows a map representing the distribution of the $Prol$ variable as a function of the cluster mass (left panel) and redshift (right panel). To
compute the former we stack clusters from all the redshifts, and for the latter we stack clusters from all masses. As it is shown in Figure 5, there are no clear dependencies of the $\text{Prol}$ variable on the cluster mass or redshift. The slight excess of prolate over oblate systems at all masses and redshifts would translate into 1D dynamical masses slightly biased toward smaller masses. Indeed, this is seen as a $\sim 1\%$ effect in Figure 1.

4. PROPERTIES OF RED-SEQUENCE-SELECTED CLUSTER GALAXIES

Results in the previous section relied on the full galaxy sample within each cluster virial region. We now study the possible systematics affecting the cluster velocity dispersion and associated dynamical mass estimates when more realistic selection for the member galaxies are taken into account. We do this in two steps: (1) we introduce red-sequence color selection here, focusing on the galaxy population that is physically associated with the cluster; and (2) we combine red-sequence color selection and spectroscopic selection on all galaxies projected near the cluster (i.e., including both cluster members and interlopers) in the following Section 5.

We model the selection carried out in real world circumstances by following the procedure we have developed for the South Pole Telescope (SPT) dynamical mass calibration program (S. Bocquet, in preparation). Namely, we preferentially choose the most luminous red-sequence galaxies that lie projected near the center of the cluster for our spectroscopic sample. To model this we select galaxies according to their colors, which are a direct prediction of the adopted SAM. In particular, we compute the following color–magnitude diagrams for different redshift ranges: $g-r$ as a function of $r$ for redshift $z \leq 0.35$, $r-i$ as a function of $i$ for redshifts $0.35 < z \leq 0.75$, and $i-z$ as a function of $z$ for redshifts larger than 0.75 (e.g., Song et al. 2012a). We report in Figure 6 the color–magnitude diagram at different redshifts for all the galaxies within the virial radius of each cluster. The model given by Song et al. (2012a), which has proven to be a good fit to the observational data, is highlighted with a dashed black-red line. The simulated cluster galaxy population has a red-sequence flatter than the observational results. Because the purpose of this work is not to study the evolution of the cluster galaxy population, but rather to see the effect of the selection of galaxies on the estimated dynamical mass, we adopt the following procedure: first we fit the red sequence at each analyzed redshift. Then, we symmetrically increase the area in color–magnitude space in order to encompass 68% of the galaxies and iterate the fit. The resulting best fit and corresponding areas are highlighted as green continuous and dashed lines in Figure 6. Table 3 describes the width in color space used to select red-sequence galaxies at each analyzed redshift.

In addition to color selection, we explore the impact of imposing a maximum projected separation $R_{\perp}$ from the cluster center, and of varying the spectroscopic sample size. In all cases we use the $N_{\text{gal}}$ most luminous galaxies in our selected sample. Table 4 shows the range of $N_{\text{gal}}$ and $a = R_{\perp}/r_{\text{vir}}$ that we explore as well as the sample binning in redshift.

![Figure 5](image.png)

**Figure 5.** Distribution of the prolateness variable $\text{Prol}$ (see Equation (5)) as a function of the cluster mass for all the clusters at different redshift stacked together (left panel) and as a function of redshift (right panel). To guide the eye the dashed black and white line is highlighting the value of $\text{Prol} = 0$, while the dotted red-white lines are respectively the 16, 50, and 84 percentiles for the two different distributions. The cluster ensemble exhibits a slight preference for prolateness at all masses and redshifts.

---

**Table 3**

| $z$   | $\text{mag}$ |
|-------|--------------|
| 0.00  | 0.05         |
| 0.09  | 0.06         |
| 0.21  | 0.08         |
| 0.32  | 0.10         |
| 0.41  | 0.06         |
| 0.51  | 0.08         |
| 0.62  | 0.08         |
| 0.75  | 0.08         |
| 0.83  | 0.09         |
| 0.91  | 0.09         |
| 0.99  | 0.07         |
| 1.08  | 0.06         |
| 1.17  | 0.05         |

---

**Table 4**

| $N_{\text{gal}}$ | $a = R_{\perp}/r_{\text{vir}}$ |
|-------------------|---------------------------------|
| 500               | 0.10                            |
| 1000              | 0.20                            |
| 2000              | 0.30                            |

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Figure 6. Color–magnitude relation for all the galaxies within $R_{\text{vir}}$ at six different redshifts. Color–magnitude relations are expressed as $g-r$ as a function of $g$ for redshift $z \leq 0.35$, $r-i$ as a function of $i$ for redshifts $0.35 < z \leq 0.75$, and as $i-z$ as a function of $z$ for redshifts larger than 0.75 (see the text for further details). Symbols with different colors refer to different galaxy clusters in each separate redshift bin. Dashed black-red line is the model given by Song et al. (2012a). The solid green lines are the best fit to the simulated red-sequence relation used in this work and dashed green lines enclose 68% of the galaxies. The area between them represents the color space used for the selection of galaxies described in Section 4.

| $a$ | $N_{\text{gal}}$ | $z$ | $M_{\text{vir}}$ ($10^{14} M_\odot$) |
|-----|----------------|-----|----------------------------------|
| 0.2 | 10             | 0.00| 1.0                             |
| 0.4 | 15             | 0.09| 2.0                             |
| 0.6 | 20             | 0.21| 4.0                             |
| 0.8 | 25             | 0.32| 6.0                             |
| 1.0 | 30             | 0.41| 8.0                             |
| 1.2 | 40             | 0.51| 10.0                            |
| 1.4 | 50             | 0.62| 20.0                            |
| 1.6 | 60             | 0.75|                                 |
| 1.8 | 75             | 0.83|                                 |
| 2.0 | 100            | 0.91|                                 |
| 2.2 | 99             | 0.99|                                 |
| 2.4 | 108            | 1.08|                                 |
|     |                | 1.17|                                 |

Table 4
Parameter Space Explored for the Mock Observations

and mass. Note that for SZE-selected clusters from SPT or equivalently X-ray-selected samples of clusters, once one has the cluster photometric redshift, one also has an estimate of the cluster virial mass and virial radius from the SZE signature or X-ray luminosity (e.g., Reiprich & Böhringer 2002; Andersson et al. 2011); we routinely use this information when building spectroscopic masks to limit our spectroscopic sample to lie within the projected virial region of the cluster.

We also note that observationally $M_{\text{vir}}$ and therefore $R_{\text{vir}}$ are not known precisely. However, as the radius scales as the cubic root of the cluster mass, uncertainties on the mass result in much smaller uncertainties on the virial radius. Therefore, apart from catastrophic cases, typical uncertainties on the radius would imply errors well below the systematics we see here.

4.1. Impact of Selecting Luminous Red Galaxies

In Section 3.1, we showed the presence of a tight relation between the 3D dynamical mass and the virial mass $M_{\text{vir}}$ for galaxy clusters (see also Evrard et al. 2008). We also showed that when dynamical masses are computed from the 1D velocity dispersion instead of the 3D one, we significantly increase the scatter of this relation (see also White et al. 2010) and introduce a negligible bias ($\lesssim 1\%$) due to the triaxial properties of DM halos. We now study the effect of the selection of a finite number of galaxies and its effect on the estimated dynamical masses. To do this, for each cluster we select a number of red-sequence galaxies within the virial radius $R_{\text{vir}}$ that ranges from 10 to 100 galaxies as described in Table 4. We sort galaxies according to their luminosity (different bands were used at different redshift as described in Section 4). This results in a “cumulative” selection.
Therefore, for example, for each cluster the 10 most luminous red-sequence galaxies are present in all the other samples with larger numbers of galaxies. However, when a cluster field is spectroscopically observed, completeness to a given limiting magnitude is not always achieved. In fact, the number of slits per mask in multi-slit spectrographs is fixed, hence the central, high-density regions of galaxy clusters can often be sampled only to brighter magnitudes than the external regions. As a consequence, the spatial distribution of the galaxies selected for spectroscopy can be more extended than the parent spatial distribution of cluster galaxies.

For example, Biviano et al. (2006) simulated an observational setup in which clusters were observed with a multi-slit spectrograph with four quadrants, each 13 arcmin on a side, with a typical slit separation of 10 arcsec. They found that the fraction of interlopers increased from 18% to 26%. They also argued that if it is not possible to increase the density of spectroscopic targets in the central regions of the clusters by applying multiple masks, it is better to rely on the dynamical mass estimator rather than the virial mass estimator, due to its smaller sensitivity to problems of incompleteness (Girardi et al. 1996).

In the analyses presented here, we do not model this observational limitation. Indeed, as described in the companion paper S. Bocquet (in preparation), such difficulty could be easily overcome by applying multiple masks to the same field, which would allow one to achieve high completeness in the central portion of the cluster. For each cluster and for all the three orthogonal projections, we compute the robust estimation of the velocity dispersion with different numbers of galaxies and compare it with the intrinsic 1D velocity dispersion. Figure 7 shows the PDF of the ratio between the velocity dispersion computed with different numbers of bright red-sequence cluster galaxies ($\sigma_{N_{\text{gal}}}$) and the intrinsic 1D velocity dispersion ($\sigma_{1D}$) obtained by stacking results from all lines of sight of the cluster sample. Different colors refer to different numbers of galaxies and the mean of each distribution is highlighted at the top of the plot with a vertical line segment. We note that when large numbers of galaxies are used to estimate the velocity dispersion, the PDF is well represented by a log-normal distribution centered at zero. As a result dynamical masses obtained from large numbers of bright red-sequence cluster galaxies are unbiased with respect to the intrinsic 1D dynamical mass. However, when the number of red-sequence galaxies used to estimate the velocity dispersion is lower than ~30, the corresponding PDF starts to deviate from a symmetric distribution and its mean is biased toward lower values. This effect is evidence of a dynamically cold population of luminous red galaxies. The existence of this cold population is compatible with the expected effects of dynamical friction (Chandrasekhar 1943). Indeed, dynamical friction is more efficient for more massive galaxies, hence the velocity bias is expected to be more important for the bright end of the galaxy population (e.g., Merritt 1985).

To verify that the bias is associated with the most massive galaxies, we compute $\sigma_{N_{\text{gal}}}$ in the same way described above, but starting from galaxies that are randomly selected with respect to luminosity. Note that in this case we only randomly select galaxies, but we do not change the “cumulative nature” of our selection and the subsequent estimated velocity dispersion when using larger numbers of galaxies. We then calculate the corresponding dynamical masses in the case of galaxies selected according to the procedure described in Section 4 and in the case of random selection using the different number of galaxies listed in Table 4. The resulting stacked dynamical masses for the full sample of clusters and for the three orthogonal projections are shown in Figure 8 as a function of the intrinsic virial mass $M_{\text{vir}}$. The dashed purple-black line is the one-to-one relation and the solid green lines are the median and 16 and 84 percentiles of the distributions. The left panel of Figure 8 represents the original distribution (luminosity-selected galaxies), while the right panel represents the randomly selected distribution. As expected from Figure 7, if velocity dispersions are computed from red-sequence galaxies selected according to their luminosity, a clear bias is introduced in the estimated dynamical mass. This is because the more massive galaxies are a dynamically cooler population relative to the full galaxy population within the cluster.

Furthermore, it appears that the bias present in the estimated dynamical mass does not depend on the cluster mass. On the other hand, if we randomly select galaxies (right panel), the bias is reduced, and we obtain a more symmetric distribution. We also check for a possible redshift dependence of the velocity bias. For this purpose, we split our sample of clusters into two different redshift bins and examine the relationship between the true cluster virial mass and the estimated dynamical masses computed with the different number of bright red-sequence galaxies selected according to their luminosity. There is no evidence that the velocity bias is changing as a function of redshift.

Using the results of these mock observations we express the velocity dispersion bias, represented by the position of the vertical segment at the top of Figure 7, with the following parameterization:

$$\langle \ln(\sigma_{N_{\text{gal}}}/\sigma_{1D}) \rangle = 0.05 - 0.51/\sqrt{N_{\text{gal}}}$$  \hfill (6)$$

This parameterization is valid only in the limit of the number of galaxies used in this study (between 10 and 100). One can use this relation to apply corrections dependent on the number of galaxies used for the dispersion and thereby recover unbiased dynamical mass estimates.
4.2. Impact of Poisson Noise

Using the results of these mock observations we express the characteristic width of each distribution shown in Figure 7 with the following parameterization:

\[
\sigma_{\ln(\sigma_{\text{gal}}/\sigma_{\text{1D}})} = -0.037 + 1.047/\sqrt{N_{\text{gal}}}. \tag{7}
\]

Remember that this is the scatter of the predicted 1D velocity distribution from the red-sequence subsample as compared to the underlying 1D velocity distribution of the cluster. So we are really only measuring the ability to recover the true underlying 1D distribution with a limited, red-sequence-selected sample of galaxies. This parameterization is valid only in the limit of the number of galaxies used in this study (between 10 and 100). As described below, this expression describes a width that cannot be explained by Poisson noise alone.

In this work, we restrict our analyses to all the galaxies with stellar masses predicted by the adopted SAM larger than \(5 \times 10^8 M_\odot\). The total number of galaxies within the virial radius \(R_{\text{vir}}\) is therefore quite large and even for the poorer clusters with \(M_{\text{vir}} \sim 10^{14} M_\odot\), the number of galaxies used to compute the 1D velocity dispersion is \(N_{\text{1D}} \sim 200\). As a result, in the absence of any further selection bias, the associated characteristic scatter to the ratio \(\sigma_{\text{gal}}/\sigma_{\text{1D}}\) is well represented by the Poissonian factor \(\sqrt{2N_{\text{gal}}}\). To demonstrate this, we show in Figure 9 the evolution of the scatter in the relation between the true virial masses and the dynamical masses as a function of redshift. For each cluster, dynamical mass is estimated starting from the velocity dispersion of the 100 most luminous red-sequence galaxies through Equation (2). The resulting scatter is highlighted as a cyan solid line. We also show the evolution of associated scatter when dynamical mass is computed from the intrinsic 3D (dashed red line) and 1D (dotted black line) velocity dispersions. Moreover, similarly to Figure 2, we separately show the predicted scatter obtained by adding in quadrature the scatter associated with the 1D velocity dispersion with the Poisson term \(\sqrt{2N_{\text{gal}}}\) (dashed-triple dotted green line) or with the factor given by Equation (7) (dashed-dotted purple line). We note that, as expected, both predictions agree very well with the measured evolution of the scatter.

However, if a lower number of galaxies is used to calculate the dynamical mass, a difference in the two predictions emerges. For example, in Figure 9 we show the same computation highlighted but with the number of galaxies equal to 15. We note in particular that the observed evolution of the scatter of the relation among the virial mass and the dynamical mass is well described by adding in quadrature to scatter associated to the intrinsic 1D dynamical mass the term given by the fitting formula of Equation (7). On the contrary, if only the Poisson term \(\sqrt{2N_{\text{gal}}}\) is taken into account, the predicted scatter is underestimated with respect to the measured one. This effect is quite apparent for larger samples of 50 galaxies, too.

Thus, there is clear evidence for an additional source of scatter beyond Poisson when one uses a red-sequence-selected subsample of galaxies to estimate the dynamical mass. We attribute this additional scatter to the presence of multiple kinematic subpopulations within these simulated clusters of galaxies. We see this source of scatter even when we randomly select small numbers of galaxies from the cluster population. But we note that this additional scatter is larger when small numbers of the most luminous galaxies are used to estimate the dynamical mass. Given that clusters are undergoing merging quite often, it is not surprising that there would be multiple kinematic subpopulations reflecting the remnants of the last few remaining sub-clusters that merged. The enhanced scatter from the luminous galaxy population suggests something more; we suspect that the additional process of dynamical friction that drives the velocity dispersions of the most massive galaxies lower over time provides a natural mechanism to explain this enhanced scatter. We note however that an accurate prediction
in this sense should also take into account the cluster selection, which might bias the selected cluster population toward systems with different dynamical histories.

5. PROPERTIES OF RED-SEQUENCE AND SPECTROSCOPICALLY SELECTED GALAXIES TOWARD CLUSTERS

Finally, to add more realism to our approach, we investigate the effect of interlopers as a possible source of systematics in the computation of cluster dynamical masses. For this purpose, for each snapshot and projection, we construct a number of cylinders centered at each cluster with length equal to the full simulated box-length and different radii spanning the interval 0.2 to 2.4 $R_{\text{vir}}$ (see Table 4). We then apply an initial cut of 4000 km s$^{-1}$ to select galaxies within the cylinders. For each cylinder realization, we then initially select a different number of members in the color–magnitude space described in Section 4 ranging from 10 to 100 galaxies as shown in Table 4. Several techniques have been developed to identify and reject interlopers. Such methods have been studied before typically using randomly selected DM particles (e.g., Perea et al. 1990; Diaferio & Geller 1997; Łokas et al. 2006; Wojtak et al. 2007, 2009) and more recently using subhalos (Biviano et al. 2006; White et al. 2010). In this work, we simply apply an iterative 3$\sigma$ clipping procedure using the robust estimation of the velocity dispersion Beers et al. (1990) to reject interlopers. This leads to a final spectroscopic sample of galaxies for each cluster, at each redshift, for each projection, within each different aperture and for each different initially selected number of red-sequence galaxies.

5.1. Fraction of Interlopers

Because red-sequence galaxies exist also outside clusters and because the galaxies tracing the large-scale structure surrounding clusters have similar redshifts to those of cluster galaxies, the spectroscopic samples selected in this manner are contaminated by interlopers. From the final spectroscopic sample of galaxies described above, we compute the fraction of interlopers $f_{\text{interlopers}}$ (arbitrarily defined here as galaxies lying at a cluster centric distance larger than 3 times the virial radius) as a function of the aperture by stacking together the sample in different bins according to their redshift, to the number of galaxies used to evaluate their velocity dispersion and to the cluster masses. Results in Figure 10 are in good agreement with previous works (e.g., Mamon et al. 2010). Panels (A)–(C) show the fraction of interlopers as a function of aperture, color coded according to the number of galaxies (panel (A)), the redshift (panel (B)), and the cluster mass (panel (C)). As expected, the fraction of interlopers increases with the aperture within which the simulated red-sequence galaxies were initially chosen. This indicates that even red sequence, spectroscopically selected samples are significantly contaminated by galaxies lying at distances more than three times the virial radius from the cluster.

Figure 10(A) shows that whether one selects small or large numbers of galaxies the fraction of interlopers remains almost the same. Panels (B) and (C) indicate that the fraction of interlopers is larger at larger redshifts consistent with expectations, given that the universe is denser (B) and is a steeper function of aperture for lower mass clusters (C). Because more massive halos form at later times than lower mass halos, the cluster mass range and redshift are clearly correlated within a mass- and volume-limited sample like ours. Thus, we also show in panel (D) how the fraction of interlopers varies as a function of redshift by stacking together the sample in different mass bins. Most massive clusters are not formed yet at high redshifts, therefore above certain redshifts the high-mass (redder) lines go to zero. Although noisy, our measurement shows a tendency for increasing interloper fraction at larger redshift, whereas at fixed redshift $z$ there is no clear dependency of the interloper fraction on cluster mass.

5.2. Impact on Velocity Bias

In a similar way, we compute the mean velocity bias (defined as the ratio between the measured velocity dispersion and the
intrinsic line-of-sight velocity dispersion: \( \sigma(N_{\text{gal}}, R_\perp, M_{\text{vir}}, z) / \sigma_{1D} \) as a function of the aperture by stacking together the sample in different bins according to their redshift, the number of spectroscopic galaxies, and the cluster mass. This can be seen in Figure 11. Panels (A)–(C) show the velocity bias as a function of aperture, color coded according to the number of galaxies (panel (A)), the redshift (panel (B)), and the cluster mass (panel (C)). Interestingly, the velocity bias has a minimum when velocity dispersions are evaluated within \( R_\perp \sim R_{\text{vir}} \), and rises at both smaller and larger radii. In particular, for projected radii \( \lesssim R_{\text{vir}} \), where the effect of interlopers is smaller, we recover the expected decrease of the average velocity dispersion profile (e.g., Biviano et al. 2006) as a function of aperture. On the other hand, for \( R_\perp \gtrsim R_{\text{vir}} \), the larger contamination from interlopers is significantly affecting and boosting the velocity bias. Furthermore, as expected, the velocity bias described in Section 4.1 is also affecting the estimated velocity dispersion, when the latter is computed with a small number of selected red-sequence galaxies (A). Indeed, by applying Equation (6) to the estimated velocity dispersion we are able to successfully remove the dependence of the velocity bias on the number of galaxies within projected aperture \( R_\perp \lesssim R_{\text{vir}} \).

Figures 11(B) and (C) show that, consistent with the fraction of interlopers, the velocity bias computed within \( R_\perp \gtrsim R_{\text{vir}} \) is larger at larger redshifts (B) and is a steeper function of aperture for lower mass clusters (C). Finally, a mild dependence of redshift for fixed mass is highlighted in panel (D).

### 5.3. Impact on Scatter in Dynamical Masses

Finally, we compute the interloper contributions to the scatter in the dynamical mass estimates as a function of the aperture by stacking together the sample in different bins according to their redshift, to the number of spectroscopic galaxies, and to the cluster mass. In particular, we measure the contribution of the scatter due to interlopers by subtracting in quadrature the measurement uncertainty computed through Equation (7):

\[
\sigma_{\text{interlopers}}^2 = \sigma_{\text{total}}^2 - \sigma_{\text{statistical}}^2,
\]

where \( \sigma_{\text{total}} \) is the total scatter measured in the distribution of the logarithmic ratio between the measured velocity
Figure 11. (A)–(C) The stacked mean velocity bias as a function of maximum projected separation from the cluster $R_\perp$ normalized to $R_{\text{vir}}$ color coded by numbers of galaxies used to estimate the velocity dispersion (A), redshift (B), and mass of the cluster (C). (D) The stacked mean velocity bias as a function of redshift color coded according to the cluster mass.

dispersion and the intrinsic line-of-sight velocity dispersion
$\ln(\sigma_{\text{vel},R_\perp M_{\text{gal}}}/\sigma_{\text{1D}})$, where $M_{\text{vir}}$ and $N_{\text{gal}}$ are averaged over
the sample in each bin. We also note that $\sigma_{\text{interlopers}}$, estimated in
this way is properly meaningful only under the assumption of
normal distributions. As we will show in Figure 14, while inter-
lopers can significantly skew this distribution, we still choose to
use this relatively simple definition as a first approximation of
the scatter induced by interlopers.

Results are shown in Figure 12. Panels (A)–(C) show typical
scatter due to the presence of interlopers as a function of aperture
respectively color coded according to the number of galaxies
(panel (A)), the redshift (panel (B)), and the cluster mass (panel
(C)). We note that, consistent with Figure 10, the contribution of
interlopers in the total scatter is a strong function of the projected
radii $R_\perp$ within which the spectroscopic sample has been
selected. Furthermore, as expected from Figure 10, panel (A)
shows that, once the statistical uncertainty is properly subtracted
through Equation (7), we remove most of the dependence of $\sigma_{\text{interlopers}}$ from the number of galaxies used to estimate
the velocity dispersion. Moreover, panel (B) shows a weak
dependence of the scatter due to interlopers as a function of redshift, with higher redshift clusters exhibiting a larger scatter,
while, as highlighted in panel (C), the dependence of $\sigma_{\text{interlopers}}$
from the cluster mass is much stronger. In particular, the effect
of interlopers as a function of projected radii $R_\perp$ is more critical
for low-mass systems than for massive clusters. Since, as explained
above, more massive clusters are formed only at lower redshift,
to disentangle the two effects we are also showing in panel (D) the mean stacked scatter due to interlopers as a function of redshift in different mass bins. We note that indeed the dependence of $\sigma_{\text{interlopers}}$ from the cluster mass is dominant with respect to the dependence from the cluster redshift.

5.4. Focus on Massive Clusters

To better understand how interlopers affect the inferred
velocity dispersion we select as an example all clusters with
$M_{\text{vir}}$ larger than $5 \times 10^{14} M_{\odot}$. For each of the three orthogonal
projections we then initially select the 25 most luminous red-
sequence galaxies as described in Section 4 within a projected
distance of 1.5 $R_{\text{vir}}$. We then apply the same procedure described
above to reject interlopers and obtain a final list of galaxies. From
this list of galaxies we then identify the “true” cluster members
and the interlopers. We show in the left panel of Figure 13 a
map representing the stacked distribution of the velocity of this
remaining sample of galaxies as a function of the projected separation from the cluster center \( R_{\perp} / R_{\text{vir}} \). Note the typical trumpet shape of the expected caustic distribution (Regos & Geller 1989; Diaferio & Geller 1997; Serra et al. 2011; Zhang et al. 2011). On the top of this map, we overplot as contours the stacked distribution of the interloper population that the 3\( \sigma \) clipping procedure was not able to properly reject from the remaining sample of galaxies. A large fraction of high-velocity interlopers are still present after foreground and background removal and thus they will bias high the estimated velocity dispersion.

We also show in the right panel of Figure 13 as solid black and dashed red histograms respectively the distribution of velocities for both the cluster galaxies and the interloper population. The expected Gaussian velocity distribution is overplotted as a solid black Gaussian with a standard deviation given by Equation (6) and \( N_{\text{gal}} = 25 \). The absolute normalization of the histograms are arbitrary, but the relative ratio of the two histograms is representative of the ratio between the number of cluster galaxies and interlopers. Note also that a large fraction of low-velocity interlopers is present. These interlopers are mostly red-sequence galaxies which lie at about the turnaround radius of the cluster overdensity and therefore have associated redshifts which are consistent with the cluster redshift. As discussed above, a simple 3\( \sigma \) clipping technique is not able to effectively remove high-velocity interlopers, and therefore is biasing high the inferred velocity dispersion. On the contrary, caustic-based methods are able to better remove the high-velocity interlopers population, but they require a larger number of spectroscopic members. However, they are not effective at rejecting these low-velocity galaxies at around the turnaround radius. As a net result, velocity dispersions computed after rejection of interlopers based upon caustic techniques will be biased low (Wojtak et al. 2007; Zhang et al. 2011).

As mentioned above, for each cluster along all the projections we end up with different samples of red-sequence galaxies that the 3\( \sigma \) clipping procedure recognizes as “spectroscopic members.” Therefore, for each different initially selected number of red-sequence galaxies, we measure the robust estimation of the velocity dispersion. We then apply Equation (2) to estimate the dynamical mass. The left panel of Figure 15 shows the corresponding relation between the resulting dynamical mass and the true virial mass for all the sample stacked together. The dashed black-purple line is the one-to-one re-
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Figure 13. Left panel: the color map represents the distribution of the line-of-sight velocity of cluster galaxies (within $3 R_{\text{vir}}$) normalized to the intrinsic 3D velocity dispersion of clusters as a function of the projected distance from the cluster center in units of $R_{\text{vir}}$ for the sample described in the text. The contour lines represent the same distribution for the interloper galaxies. Right panel: the distribution of velocities in units of the intrinsic 3D velocity dispersion for the cluster galaxy population (solid black histogram) and for the interloper population (dashed red histogram). The normalization is arbitrary, while the relative ratio of the two histograms reflects the sample described in the text. The solid black Gaussian is the expected distribution with width given by the Equation (6) and $N_{\text{gal}} = 25$.

In particular, the median of the distribution shows that a systematical overestimation of the dynamical mass is present at all cluster masses, as expected from the interloper contribution previously discussed. Furthermore, especially at the low-mass end of the cluster galaxy distribution, the presence of a significant population of high-velocity outliers is making the relation among virial mass and dynamical mass very asymmetric and causing a severe boosting of the dynamical mass.

Figure 14 shows how the presence of interlopers can affect the distribution of the estimated dynamical mass for a sample of clusters with $M_{\text{vir}} > 3 \times 10^{14} M_\odot$, where a number of luminous red galaxies (between 15 and 60) has been spectroscopically observed within $R_{\perp} < R_{\text{vir}}$. Clearly the distribution is not lognormal as it was for the intrinsic line-of-sight velocities (see Figure 3). We note that a tail of outliers, for which the dynamical mass is largely overestimated, makes the observed distribution much more skew. Indeed we measure a value for the skewness of 0.39.

These outliers are likely related to cases where the simple $3\sigma$ clipping procedure is not sophisticated enough to effectively separate the foreground and background interloper galaxies from the proper cluster galaxies. To verify this hypothesis we show in the right panel of Figure 15 the same computation as for the left panel, but restricting our sample to only the cases in which the presence of interlopers is smaller than 5%. We note how this subsample qualitatively looks very similar to left panel of Figure 8 which by construction contains only cluster galaxies. Furthermore, once the contribution from interlopers is removed, the bias of dynamical mass over the true mass disappears. However, remember that Figure 15 shows that without interlopers dynamical masses are on average underestimated compared to the true virial mass, as expected from the velocity bias discussion presented in Section 4.1. Clearly, the interloper effect on the dynamical mass is more severe at the low-mass end of the cluster population.

5.5. Sensitivity to Color Cuts in Selection

Because the color selection of cluster members is a crucial point in this analysis, the results presented here obviously depend on the adopted galaxy formation model at some level. On the one hand it is true that the model is not perfectly reproducing the observed properties of the cluster galaxy population. On the other hand, we also do not take into account any observational uncertainty which will instead affect the real data, for example broadening the observed red-sequence at fainter magnitudes.
First, we explore the properties of the full galaxy sample within galaxy clusters and subhalos and the effect of multiwavelength cluster selection (i.e., SZE or X-ray) on the velocity dispersions. We show in the left panel of Figure 15 the resulting 16, 50, and 84 percentiles overplotted as red continuous lines. We note that a larger effect from the interlopers is present in comparison with the previous analyses, as expected from the broader color selection adopted. In particular, larger differences appear at the low-mass end of the cluster galaxy population, where a significant increase of catastrophic outliers in the overestimation of the dynamical mass is visible. On the other hand, the average population is not affected by much. As a net result, a change in the color selection of a factor of $\sim 2$ implies a change in the estimated velocity dispersion by less than $\sim 3\%$. In particular, this difference reduces to less than $\sim 1\%$ for clusters with $M_{\text{vir}}$ larger than $5 \times 10^{14} M_{\odot}$.

6. DISCUSSION AND CONCLUSIONS

We have examined the use of velocity dispersions for unbiased mass estimation in galaxy clusters using the publicly available galaxy catalog produced with the SAM by De Lucia & Blaizot (2007) coupled with the $N$-body cosmological Millennium Simulation (Springel et al. 2005). In particular, we selected all galaxies in the SAM with stellar mass larger than $5 \times 10^8 M_{\odot}$ and analyzed a sample consisting of more than $\sim 20,000$ galaxy clusters with $M_{\text{vir}} \geq 10^{14} M_{\odot}$ up to $z \sim 1.2$ (Table 1).

First, we explore the properties of the full galaxy sample and then we increase the level of complication to mimic the spectroscopic selection that is typically undertaken in real world studies of clusters. Then we work through a series of controlled studies in an attempt to disentangle the different effects leading to biases and enhanced scatter in velocity dispersion mass estimates. Ultimately our goal is to inform the dispersion-based mass calibration of the SPT cluster sample (S. Bocquet, in preparation), but we explore a broad range in selection in hopes that our results will be of general use to the community.

Our primary conclusions for the full subhalo population within galaxy clusters are as follows.

1. We measure the galaxy (i.e., subhalo) velocity dispersion–mass relation and show that it has low scatter ($\sim 0.14$ in $\ln(M)$) and that subhalo dispersions are $\lesssim 3\%$ lower than DM dispersion in Evrard et al. (2008). This difference corresponds to a $\lesssim 10\%$ bias in mass for our halos if the DM dispersion–mass relation is used, and is consistent with previous determination of subhalo velocity bias.

2. We explore line-of-sight velocity dispersions of the full galaxy populations within the cluster ensemble and confirm

Figure 15. Left panel: the distribution of the dynamical mass estimated through Equation (2) as a function of the true $M_{\text{vir}}$ for the whole sample described in Section 4 used in this work (green lines). Red lines represent the same distribution obtained from a different color selection of red-sequence galaxies as explained in Section 5. Right panel: same as for the left panel, but only for the cases where the fraction of interlopers is smaller than 5%. Dashed purple-black line is showing the one-to-one relation, while solid green and red lines are the 16, 50, and 84 percentiles.

(A color version of this figure is available in the online journal.)
that the triaxiality of the velocity dispersion ellipsoid is the dominant contributor to the characteristic \( \sim 35\% \) scatter in dispersion-based mass estimates (White et al. 2010). We show that this scatter increases with redshift as \( \sigma(z) \approx 0.3 + 0.075z \). We also show that the distribution of dynamical masses estimated through line-of-sight velocity dispersions is more log-normal than the same one computed from the full 3D velocity dispersions.

3. We measure the principal axes and axial ratios of the spatial galaxy distribution ellipsoid, showing that there is a slight (\( \sim 5\% \)) preference for prolate distributions; this property has no clear variation with mass or redshift. We examine the line of sight velocity dispersions along the principle axes, showing that the slight preference toward prolate geometries translates into a slight (\( \sim 1\% \)) bias in the dispersion mass estimates extracted from line-of-sight measures.

Our primary conclusions for the spectroscopic and red-sequence-selected subsamples of subhalos physically associated with clusters are as follows.

1. We characterize the bias (Equation (6)) and the scatter (Equation (7)) in the line-of-sight velocity dispersion introduced by selecting a subset \( N_{\text{gal}} \) of the most luminous red-sequence galaxies within a cluster. The bias is significant for samples with \( N_{\text{gal}} < 30 \) and is compatible with the expected effect due to dynamical friction of these most massive subhalos.

2. The scatter in the dynamical mass estimator cannot be fully explained through a combination of intrinsic scatter in the relation between the mass and the 3D dispersion of all galaxies (i.e., departures from equilibrium), scatter of the line-of-sight dispersion around the 3D dispersion (halo triaxiality), and Poisson noise associated with the number of subhalos \( N_{\text{gal}} \). A further component of scatter is needed to explain the full measured scatter, and we argue that this is evidence of multiple kinematic subpopulations that are poorly sampled with small numbers of spectroscopic redshifts. In addition, we show that the scatter is larger for small subsamples of the most luminous red galaxies.

Our primary conclusions for the spectroscopic and red-sequence-selected subsamples that include interlopers from the surrounding large-scale structure are as follows.

1. We explore the impact of interlopers by creating spectroscopic samples using (1) red-sequence color selection, (2) a maximum projected separation from the cluster center, and (3) \( N \)-sigma outlier rejection in line-of-sight velocity. In these samples the interloper fraction (contamination) can be significant, growing from \( \sim 10\% \) at the projected virial radius to \( \sim 35\% \) at twice the project virial radius. The contamination fraction has a much weaker dependency on the sample size \( N_{\text{gal}} \). We explore the dependence on mass and cluster redshift, showing that within a fixed aperture, contamination is a factor of \( \sim 2 \) worse at redshift \( z \approx 1 \) than at \( z = 0 \). Furthermore, we show that the fraction of interlopers is a steeper function of aperture for low-mass clusters, but that at fixed redshift contamination does not change significantly with mass.

2. We show that contamination is significant even if a more sophisticated caustic approach is used to reject interlopers, demonstrating that even clusters with large numbers of spectroscopic redshifts for red-sequence-selected galaxies suffer contamination from non-cluster galaxies.

3. We further study how interlopers are affecting the estimated velocity bias. We find that the velocity bias has a minimum if computed within \( R_{\perp} \sim R_{\text{vir}} \). This is due to the balancing effect of larger intrinsic velocity bias at smaller radii and larger contamination at larger radii. Furthermore, we show that if velocity dispersions are computed within projected aperture \( R_{\perp} \) larger than \( R_{\text{vir}} \), the velocity bias is a steeper function of \( R_{\perp} \) for higher redshifts and lower cluster mass, as expected from the contamination fraction.

4. We estimate the effect of interlopers as an additional source of scatter. We find that the effect of interlopers could be the dominant source of uncertainty (up to \( \sim 49\% \)) and is a steep function of the projected aperture \( R_{\perp} \) that is worse for low-mass systems. In particular, this effect has to be considered when developing a strategy for using dynamical masses to calibrate galaxy cluster masses. We summarize here in Figure 16 the expected total scatter in velocity dispersion at fixed mass for three different galaxy cluster mass classes as a function of the number of spectroscopic galaxies (with projected radius \( r < R_{200} \)) used to estimate the velocity dispersion. In particular, the amount of scatter increases dramatically once the number of spectroscopic members falls below \( \sim 30 \) galaxies. The results presented here for total scatter are found to be in good agreement with those reported by R. Ruel et al. (2013, in preparation) from the SPT collaboration. We note that a more sophisticated interloper rejection technique could well lead to reductions in scatter.

5. We study how changing the color selection affects the fraction of interlopers and the subsequent effect on the estimated velocity dispersion and dynamical masses. We find that doubling the width of the color selection window centered on the red sequence has only a modest impact on the interloper fraction. The primary effect of changing the color selection is on the filtering of catastrophic outliers. This results in changes to the estimated velocity dispersion–virial mass relation at the level of 1% in mass. We also show that uncertainties in the color selection are more important for low-mass clusters than for the high-mass end of...
the cluster population, which is because the dispersions of low-mass clusters are more sensitive to catastrophic outliers. The rather weak dependence of the dispersion-based mass estimates on the details of the color selection suggests also that uncertainties in the star formation histories (and therefore colors) of galaxy populations in and around clusters are not an insurmountable challenge for developing unbiased cluster mass estimates from velocity dispersions.

In a companion paper, S. Bocquet (in preparation) apply this model to the dispersion mass calibration of the SPT SZE-selected cluster sample. We identify the following key remaining challenges in using dispersions for precise and accurate mass calibration of cluster cosmology samples. Surprisingly, the larger systematic uncertainty has to be ascribed to our relative poor knowledge of the velocity bias between galaxies or subhalos and DM. A conservative estimate of this systematic is at the level of <5% and arises from the comparison of different simulations and different algorithms for subhalo identification (e.g., Evrard et al. 2008). The systematic uncertainty in the color selection of galaxies and its subsequent mapping between line-of-sight velocity dispersion and mass is at relatively smaller level. Indeed, we can estimate it at a <1% level for samples selected as the ones described in S. Bocquet (in preparation), despite the fact that galaxy formation models involve a range of complex physical processes. In other words, systematics in predicting galaxy properties (e.g., luminosity, colors, etc.) due to subgrid physics associated with magnetic fields, AGNs, and supernova feedback, radiative cooling, and the details of star formation, do not appear to significantly change the spectroscopic sample selection. On the other hand, simulations including different physical treatments of gravity are affecting the dynamics of the spectroscopically selected sample at a higher level than we expected. Given that the current dominant contributor to the systematics floor is an issue associated with the treatment of gravitational physics, there are reasons to be optimistic that future simulations will be able to reduce the current systematics floor on dispersion-based cluster mass estimates.

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REFERENCES

Abell, G. O. 1958, ApJS, 3, 211
Andersson, K., Benson, B. A., Ade, P. A. R., et al. 2011, ApJ, 738, 48
Barnes, I., & Hut, P. 1986, Natur, 324, 446
Beers, T. C., Flynn, K., & Gebhardt, K. 1990, AJ, 100, 32
Benson, A. J. 2010, PhR, 495, 33
Biviano, A., Murante, G., Borgani, S., et al. 2006, A&A, 456, 23
Börhinger, H., Voges, W., Huchra, J. P., et al. 2000, ApJS, 129, 435
Chandrasekhar, S. 1943, ApJ, 97, 255
Davis, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371
De Lucia, G., & Blaizot, J. 2007, MNRAS, 375, 2
De Lucia, G., Kauffmann, G., Springel, V., et al. 2004a, MNRAS, 348, 333
De Lucia, G., Kauffmann, G., & White, S. D. M. 2004b, MNRAS, 349, 1101
Diaferio, A., & Geller, M. J. 1997, ApJ, 481, 633
Dressler, A., & Shectman, S. A. 1988, AJ, 95, 985
Evrard, A. E., Bialek, J., Busha, M., et al. 2008, ApJ, 672, 122
Faltenbacher, A., & Diemand, J. 2006, MNRAS, 369, 1698
Fassbender, R., Börhinger, H., Nastasi, A., et al. 2011, NJPh, 13, 125014
Gao, L., De Lucia, G., White, S. D. M., & Jenkins, A. 2004, MNRAS, 352, L1
Geller, M. J., & Beers, T. C. 1982, PASP, 94, 421
Goto, T. 2005, MNRAS, 359, 1415
Hao, J., McKay, T. A., Koester, B. P., et al. 2010, ApJS, 191, 254
Hopkins, P. F., Bahcall, N. A., & Bode, P. 2005, ApJL, 618, 1
Jing, Y. P., & Suto, Y. 2002, ApJ, 574, 538
Kasun, S. F., & Evrard, A. E. 2005, ApJ, 629, 781
Kauffmann, G., Colberg, J. M., Diaferio, A., & White, S. D. M. 1999, MNRAS, 303, 188
Koester, B. P., McKay, T. A., Annis, J., et al. 2007, ApJ, 660, 221
Locatelli, L., Wojtak, R., Gottlöber, S., Mamont, G. A., & Prada, F. 2006, MNRAS, 367, 1463
Mamon, G. A., Biviano, A., & Murante, G. 2010, A&A, 520, A30
Marriage, T. A., Acquaviva, V., Ade, P. A. R., et al. 2011, ApJ, 737, 61
Mezzetti, D. 1985, ApJ, 299, 18
Mogh, H. J., Evrard, A. E., Fabricant, D. G., & Geller, M. J. 1995, ApJ, 447, 8
Pacaud, F., Pierre, M., Adami, C., et al. 2007, MNRAS, 382, 1289
Pereira, J., del Olmo, A., & Moles, M. 1999, A&A, 327, 319
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2011, A&A, 536, A8
Reiprich, T. H., & Börhinger, H. 2002, ApJ, 567, 716
Serra, A. L., Diaferio, A., Borgani, S., & Bongi, S. 2011, MNRAS, 412, 800
Song, J., Mohr, J. J., Barkhouse, W. A., et al. 2012a, ApJ, 747, 58
Song, J., Zenteno, A., Stalder, B., et al. 2012b, ApJ, 761, 22
Springel, V. 2005, MNRAS, 364, 1105
Springel, V., White, S. D. M., Jenkins, A., et al. 2005, Natur, 435, 629
Springel, V., White, S. D. M., Tormen, G., & Kauffmann, G. 2001a, MNRAS, 328, 726
Springel, V., Yoshida, N., & White, S. D. M. 2001b, Natur, 463, 629
Stanford, S. A., Eisenhardt, P. R., Brodin, M., et al. 2005, ApJL, 634, L129
Staniszewski, Z., Ade, P. A. R., Aird, K. A., et al. 2009, ApJ, 701, 32
Sunyaev, R. A., & Zel’dovich, Y. B. 1972, CoASP, 4, 173
Szabo, T., Pierpaoli, E., Dong, F., Pipino, A., & Gunn, J. 2011, ApJ, 736, 21
Thomas, P. A., & Couchman, H. M. P. 1992, MNRAS, 257, 11
Tormen, G. 1997, MNRAS, 290, 411
Vikhlinin, A., Burenin, R. A., Ebeling, H., et al. 2001, ApJ, 563, 22
Wojtak, R., Lokas, E. L., Gatto, S., & Gottlöber, S. 2009, MNRAS, 399, 405
White, M., Cohn, J. D., & Smit, R. 2010, MNRAS, 408, 1818
Wojtak, R., Lokas, E. L., Gatto, S., & Gottlöber, S. 2009, MNRAS, 399, 812
Zwicky, F., Herzog, E., & Wild, P. 1968, Catalogue of Galaxies and of Clusters of Galaxies (Pasadena, CA: California Institute of Technology)