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Higgs inflation in minimal supersymmetric SU(5) GUT

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The Standard Model Higgs boson with large non-minimal coupling to the gravitational curvature can drive cosmological inflation. We study this type of inflationary scenario in the context of supersymmetric grand unification and point out that it is naturally implemented in the minimal supersymmetric SU(5) model, and hence virtually in any GUT models. It is shown that with an appropriate Kähler potential the inflaton trajectory settles down to the Standard Model vacuum at the end of the slow roll. The predicted cosmological parameters are also consistent with the 7-year WMAP data.

Introduction.— Recently the idea that the Standard Model (SM) Higgs field may be identified with an inflaton field, has attracted much attention [1–9]. The major rôle is played by the non-minimal coupling to gravity, which renders the Higgs mass to be within the range of $126 – 194$ GeV [1–4], while keeping the amplitude of the primordial curvature perturbation at the scale of $\sim 10^{-5}$. The idea of inflation by non-minimally coupled inflaton field itself is certainly not new [10]. Nevertheless, the striking agreement with the present-day cosmological data, combined with the minimalistic nature of the model, makes this type of scenario very attractive. The predicted mass range of the Higgs particle is also interesting for the physics of the Large Hadron Collider.

The Higgs potential in the SM is unstable against quantum corrections (the hierarchy problem) and it therefore is reasonable to reconsider Higgs inflation in supersymmetric theory [11, 12]. It is shown in [11] that Higgs inflation cannot be implemented within the minimal supersymmetric Standard Model (MSSM), as the field content of the latter is too restrictive. Instead, with an extra field (i.e. in the next-to-minimal supersymmetric Standard Model, NMSSM) a sensible scenario of Higgs inflation is found to be possible. The NMSSM model has tachyonic instability in the direction of the extra field, but this can be cured by considering a non-canonical Kähler potential [12].

In this Letter we discuss possibility of Higgs inflation in supersymmetric grand unified theory (GUT). There are several reasons to motivate this study. One obvious reason is that the energy scale of inflation is typically above the grand unification scale, and it is unnatural to suppose that the SM Lagrangian is valid all the way up to the scale of inflation; as the GUT scale destabilises the electroweak scale without supersymmetry, it seems that supersymmetric GUT is an appropriate theory to start with. Another reason is the puzzling necessity of the extra field besides the MSSM fields for successful Higgs inflation, as alluded to above; going beyond the MSSM is somewhat against the minimalistic guiding principle of the original Higgs inflation, and as the NMSSM is structurally similar to the SU(5) GUT model, it seems natural to conjecture that the SU(5) GUT, rather than the NMSSM, may be a more appropriate minimal supersymmetric theory that accommodates Higgs inflation. Obvious questions are then whether it is possible to obtain enough inflation (e-folding) somewhere between the Planck scale and the GUT scale, and if so whether the prediction of the cosmological parameters is consistent with the present observation. We shall address these issues below, and find that a viable Higgs inflationary scenario nicely fits into the minimal SU(5) model. We shall employ supergravity embedding of GUT [13], since the non-minimal coupling of the Higgs field to gravity naturally arises in that framework.

Supersymmetric SU(5) GUT.— The minimal supersymmetric SU(5) model consists of a vector supermultiplet transforming as an adjoint 24 of the SU(5), as well as 5 types of chiral supermultiplets, namely $N_1$ (the number of flavours) multiplets in 5 (that include $d$ and $L$ of the MSSM), $N_2$ multiplets in 10 (include $Q, \bar{u},$ and $\bar{e}$), one each in 24 (denoted $\Sigma$), 5 ($H$) and 5 ($\overline{H}$). $\Sigma$ is the Higgs multiplet responsible for breaking the GUT symmetry, while $H$ and $\overline{H}$ respectively play rôles in the dynamics of inflation. We shall hence disregard the other fields. The superpotential of our model is,

$$W = \overline{H} (\mu + \rho \Sigma) H + \frac{m}{2} \text{Tr}(\Sigma^2) + \lambda \frac{3}{3} \text{Tr}(\Sigma^3),$$

and the Kähler potential is $K = -3\Phi$, with

$$\Phi = 1 - \frac{1}{3} \left( \text{Tr} \Sigma^\dagger \Sigma + |H|^2 + |\overline{H}|^2 \right) - \frac{\gamma}{2} \left( \overline{H} H + H^\dagger \overline{H} \right)$$

$$+ \frac{\overline{\omega}}{3} \left( \text{Tr} \Sigma^\dagger \Sigma^2 + \text{Tr} \Sigma^2 \Sigma \right) + \frac{\zeta}{3} \left( \text{Tr} \Sigma^\dagger \Sigma \right)^2,$$

where $\mu, \rho, m, \lambda, \gamma, \zeta, \overline{\omega}$ are constant parameters (for simplicity we assume them to be real). The cubic and the quartic terms have been included in the Kähler po-

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tential, for reasons to be discussed shortly. We shall set the reduced Planck scale $M_P = 2.44 \times 10^{18}$ GeV to unity.

For the model to be phenomenologically consistent, the $SU(5)$ symmetry needs to be broken down to the SM gauge group $SU(3) \times SU(2) \times U(1)$. This is accomplished as usual by setting,

$$\Sigma = \sqrt{\frac{2}{15}} S \text{ diag}(1, 1, 1, \frac{3}{2}, \frac{3}{2}),$$

(3)

with $S$ a chiral superfield. The MSSM Higgs doublets $H_u$, $H_d$ and the Higgs colour triplets $H_c$, $\overline{H}_c$ are embedded in $H$ and $\overline{H}$ as

$$H = \begin{pmatrix} H_c \\ H_u \end{pmatrix}, \quad \overline{H} = \begin{pmatrix} \overline{H}_c \\ H_d \end{pmatrix}.$$  

(4)

The superpotential now reads

$$W = \left( \mu + \sqrt{\frac{2}{15}} \rho S \right) \overline{H}_c H_c + \left( \mu - \sqrt{\frac{3}{10}} \rho S \right) H_u H_d + \frac{m}{2} S^2 - \frac{\lambda}{3\sqrt{30}} S^3.$$  

(5)

The masses of $H_u$ and $H_d$ are in the electroweak scale, which is negligible smaller than the typical scale $M_P$ of the inflationary dynamics. Thus the expectation value of the second term in (5) must vanish, $\mu = \sqrt{\frac{3}{10}} \rho \langle S \rangle$, where $\langle S \rangle = v \equiv 2 \times 10^{16}$ GeV is the GUT scale. The first term of (5) indicates that $H_c$ and $\overline{H}_c$ have GUT scale masses. For the colour symmetry to be unbroken we require that they are already stabilised at $\langle H_c \rangle = \langle \overline{H}_c \rangle = 0$, from the onset of the inflation. During inflation the dominant rôle is played by the MSSM Higgs fields $H_u$ and $H_d$, which settle down to the present values after the inflation. When $H_u$, $H_d \ll 1$ (i.e. close to the end of inflation) the stationarity condition $\delta W/\delta S = 0$ with $H_c = \overline{H}_c = 0$ yields $S(m - \lambda S/\sqrt{30}) = 0$. Since the GUT symmetry must be broken, $\langle S \rangle = v \neq 0$ and we must have $m = \frac{\lambda}{\sqrt{30}} v$. The charged Higgs can be consistently set to be zero,

$$H_u = \begin{pmatrix} 0 \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ 0 \end{pmatrix},$$

(6)

and parametrizing $S = se^{i\alpha}$, $H_u^0 = \frac{1}{\sqrt{2}} h_1 e^{i\alpha_1}$, $H_d^0 = \frac{1}{\sqrt{2}} h_2 e^{i\alpha_2}$, with $s, h_1, h_2, \alpha, \alpha_1, \alpha_2 \in \mathbb{R}$, and further setting $h_1 = h \sin \beta$ and $h_2 = h \cos \beta$, the model depends on five parameters $\rho, \lambda, \gamma, \omega, \zeta$, and six real scalar fields $s, h, \alpha, \beta, \alpha_1, \alpha_2$. Note that $\rho$ and $\lambda$ are parameters appearing in the GUT superpotential and are typically of order one, while there is no such restriction for $\gamma, \omega, \zeta$. Analysing the scalar potential, we find stability at $\alpha = \alpha_1 = \alpha_2 = 0$. Furthermore, the D-flat condition sets the value of $\beta$ to be $\pi/4$. Thus the model reduces to a system of two real scalars $h$ and $s$, with the scalar-gravity part of the Jordan frame Lagrangian (cf. [12]),

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{1}{2} \Phi R_J - \frac{1}{2} g_{\mu \nu} \partial_\mu h \partial_\nu h - \kappa g_{\mu \nu} \partial_\mu s \partial_\nu s - V_J \right].$$

(7)

The subscript J denotes quantities in the Jordan frame, $\kappa \equiv K_{SSJ} = 1 - 4\omega s - 4\zeta s^2$ is the non-trivial component of the Kähler metric, $\omega \equiv -\bar{\omega}/\sqrt{30}$, and

$$\Phi = 1 - \frac{1}{3} s^2 + \frac{2\omega}{3} s^3 + \frac{\zeta}{3} s^4 + \left( \frac{\gamma}{4} - \frac{1}{6} \right) h^2.$$  

(8)

$V_J$ is the F-term scalar potential in the Jordan frame, computed in the standard way [14], as

$$V_J = \frac{3}{10} \left\{ \frac{\rho^2}{2} (s - v)^2 h^2 + \frac{1}{2} \left[ \frac{\rho}{4} h^2 - \frac{\lambda}{3} s (s - v) \right]^2 \right\}$$

$$- \left\{ \frac{2\zeta + \omega}{6 \kappa} \left[ \frac{\rho h^2}{4} - \frac{\lambda s (s - v)}{3} \right] s^2 + \frac{2\rho h^2}{6} - \frac{\lambda s^2}{6} - \frac{3\zeta \rho h^2 (s - v)}{4} \right\}$$

$$10 \left[ 1 + \frac{2}{3} (\zeta - 1) h^2 + \frac{\omega s^2}{3 \kappa} s^4 \right].$$  

(9)
The inflation dynamics.— The dynamics of inflation is encoded in the scalar potential $V_E = \Phi^{-2} V_J$ in the Einstein frame. If we take the canonical form of the Kähler potential (i.e. $\omega = \zeta = 0$), the potential exhibits tachyonic instability in the direction of the $s$-field. Just as in the case of the NMSSM Higgs inflation [11, 12] the instability is controlled by introducing a quartic term ($\zeta \neq 0$) in the Kähler potential. In the GUT model, however, this is not the whole story, as the quartic term has a serious side effect: the SM vacuum becomes disfavoured and the $SU(5)$ symmetry tends to be restored at the end of inflation. This problem is resolved by allowing a cubic term\(^1\), $\omega \neq 0$. Note that these terms are perfectly consistent with the supergravity embedding. The bottom line is that for a wide range of the parameter space with up to quartic order terms in the Kähler potential, there exist reasonable trajectories of the inflaton field. In Fig.1 we show the shape of the scalar potential $V_E$ (the left panel), the inflaton trajectory (centre), and the values of $V_E$ at local minima (bottom of the valley) for given $h$ (right).

In this example we have taken $\rho = \lambda = 0.5$, $\omega = -100$, $\zeta = 10000$, and $\gamma = 1.86 \times 10^4$ (this value of $\gamma$ is determined for the $s$-field control breaking of the GUT symmetry, the trajectory shows that $SU(5)$ is broken from the onset, indicating that problematic topological defects are not produced during inflation. For this parameter set the dynamics of the slow roll inflation is dominated by the $s$-field, as the displacement of $s$ is negligibly small ($\Delta s / \Delta h \lesssim 2\%$, with suitable normalisation $d\tilde{s} = \sqrt{2\kappa} ds$). Assuming that $s$ is nearly constant\(^2\), the model simplifies to single field inflation. The Lagrangian (7) can then be written in a form similar to the SM Higgs inflation [1–8],

\[
\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{M^2 + \xi h^2}{2} R_J - \frac{1}{2} g_J \partial_i \phi \partial^i \phi - V_J \right],
\]

with $M^2 = 1 - \frac{1}{2} \frac{dV_E}{dh} \frac{dV_E}{dh}$ and $\xi = \frac{1}{2} \frac{d^2 V_E}{dh^2}$.

Cosmological parameters.— The slow roll parameters,

\[
\epsilon = \frac{1}{2} \left( \frac{1}{V_E} \frac{dV_E}{dh} \right)^2, \quad \eta = \frac{1}{V_E} \frac{d^2 V_E}{dh^2},
\]

are defined for the scalar potential $V_E$ and the canonically normalised inflaton field $h$ in the Einstein frame. The latter is related to $h$ by

\[
dh = \sqrt{\frac{M^2 + \xi h^2 + 6\xi h^2}{M^2 + \xi h^2}} dh.
\]

For given $(\lambda, \rho, \omega, \zeta)$, the non-minimal coupling $\xi$ is determined from the power spectrum of the curvature perturbation $\mathcal{P}_R = V_E / 24\pi^2 c^3$. The slow roll terminates when either of the slow roll parameters ($\epsilon$ in the present case) becomes $O(1)$. The values of the inflaton $h = h_s$ at the end of slow roll and $h_k$ at the horizon exit of the comoving CMB scale $k$, are related by the $s$-field control breaking of the GUT symmetry, the trajectory shows that $SU(5)$ is broken from the onset, indicating that problematic topological defects are not produced during inflation. For this parameter set the dynamics of the slow roll inflation is dominated by the $s$-field, as the displacement of $s$ is negligibly small ($\Delta s / \Delta h \lesssim 2\%$, with suitable normalisation $d\tilde{s} = \sqrt{2\kappa} ds$). Assuming that $s$ is nearly constant\(^2\), the model simplifies to single field inflation. The Lagrangian (7) can then be written in a form similar to the SM Higgs inflation [1–8],

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dh = \sqrt{\frac{M^2 + \xi h^2 + 6\xi h^2}{M^2 + \xi h^2}} dh.
\]

\(^1\) Higher (say sextic) terms in the Kähler potential can also solve this problem.

\(^2\) The value of $s = s(h)$ is taken at the local minimum of $V_E$ for a given $h$, and derivatives of $s$ are set to be zero.
fit with today’s observational constraints.

The non-minimal coupling is consistent with the symmetries of general relativity and the SM, and it naturally arises in quantum field theory in curved spacetime [17]. The value of the coupling \(\xi \sim 10^4\), however, is rather large. This is a generic feature of Higgs inflation, since successful slow-roll requires \(h^2 \lesssim M_P^2 \lesssim \xi h^2\) [1]. It has been argued that such large non-minimal coupling could violate the unitarity bound, since the cut-off scale evaluated as \(M_P/\xi\) is considerably lower than the Planck scale [5–8]. Others contend that such a criticism is not valid, arguing that at large field values \(\geq M_P/\xi\) the cut-off scale is actually field-dependent [4, 9, 12]. The large non-minimal coupling is, at any rate, a key feature of the Higgs inflation and it is certainly worthwhile understanding possible dangers arising from this. Another type of criticism concerns the quantum stability of the classical potential. This problem was studied using renormalisation group (RG) analysis [2–4], and the effects of renormalisation are found to be small except for some extreme values of parameters. We have also performed RG analysis in our GUT model and verified that the effects are small (less than 3% for \(r\), less than 2% for \(\xi\), and less than 0.1% for \(n_s\)). This is expected, since inflation takes place in a narrower energy range of \(10^{16} \to 10^{18}\) GeV and the RG effects should be smaller than the SM case.

A closer look at the potential \(V_5\) shows that its minimum is at a small negative value, \(\sim -2 \times 10^{-16} M_P^4\), for our parameter choices. This is offset by a contribution from the supersymmetry breaking sector and the scenario does not suffer from the cosmological constant problem. In our scenario the energy scale of inflation is in the GUT scale and the Higgs fields are directly coupled to the SM particles. This indicates that the reheating temperature is high, typically from the intermediate to the GUT scale. It would be interesting to discuss further phenomenological implications, such as the gravitino problem and baryogenesis.

In this Letter we considered a single-field Higgs inflation model appropriate for our parameter choice \(\xi = 10000\), \(\omega = -100\) of the Kähler potential. These values are not too exotic, as \(\langle \Phi \rangle\) is still very close to 1 and the Planck scale after inflation is nearly \(M_P\). For smaller values of \(\xi\) and \(|\omega|\), the displacement of \(s\) during inflation becomes large. This leads to two-field inflation, which is also of interest, in particular, due to possible generation of detectable large non-Gaussianity. Supersymmetric models of Higgs inflation necessarily involve multiple fields [11]. The engendered isocurvature mode can, in principle, distinguish various models of Higgs inflation.

Finally, the scenario can also be extended to other GUT models whose gauge group contains \(SU(5)\) as a subgroup. When the Higgs multiplets of the GUT model contain \(5, \bar{5}\) and \(24\) of the minimal \(SU(5)\) GUT, a superpotential like (1) can be introduced. Then a viable model of Higgs inflation is implemented, as described in this Letter. One such simple example is the \(SO(10)\) GUT with Higgs multiplets in \(10\) and \(54\) representations.

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