Supersymmetric quantum mechanics to study the 16.8 MeV resonance state of $^9$B

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The study of a high-lying resonant state of $^9$B is carried out using supersymmetric quantum mechanics (SQM). The resulting isospectral potentials are very deep and narrow and the generated wave functions identify the resonance at 16.84 MeV with a width of 69 keV. The present work shows that SQM can be successfully applied for detection of high-lying resonances in unstable nuclei.

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I. INTRODUCTION

With the development of new and upgraded rare isotope beam facilities worldwide, one can now carry out sophisticated experiments involving resonances in unstable nuclei $^\ddagger$ $^\ddagger$. This opens up immense opportunities to pursue pressing problems in nuclear astrophysics. The cosmological lithium problem $^\ddagger$ $^\ddagger$ is one such decades old and yet unresolved problem, where there is a pronounced abundance anomaly for $^7$Li between the observation and prediction of the Big Bang Nucleosynthesis theory. It has been argued that resonance enhancement through a high-lying state in the $^9$B nucleus might give a clue to the solution to this long standing problem $^\ddagger$ $^\ddagger$. We have developed a robust theoretical framework using supersymmetric quantum mechanics (SQM) to generate the resonant states and their wave functions for unstable and unbound nuclei with excellent results $^\ddagger$ $^\ddagger$. In the present work we study the high-lying resonance of $^9$B in this framework.

II. THEORY

Earlier, using SQM we have studied low-lying resonances in unstable and unbound nuclei $^\ddagger$ $^\ddagger$. However, we anticipated that detection of high-lying resonances in these nuclei would require very deep isospectral potentials resulting in serious challenges as described below.

In our present work, using SQM, a high-lying resonant state of $^9$B is investigated. In essence, this tests the effectiveness of our theoretical procedure for high-lying resonant states of unstable nuclei. The challenge in detecting such resonant states results from the shallowness of the two-body potential well $(d + ^7$Be for $^9$B), followed by a very low and wide barrier. Thus, for a finite barrier height, a system may temporarily be trapped inside the shallow well, when its energy is close to the resonance energy. In principle one can find quasi-bound states in such a shallow potential. However there is a large probability to tunnel out through the barrier which gives rise to broad resonance width. For high-lying resonant states, the probability of tunnelling becomes so high that it makes the accurate detection of such states practically impossible. In order to circumvent this problem, we resort to the very successful procedure adopted by us earlier in the detection of low-lying resonant states.

Since $^9$B is formed by the fusion of $^7$Be with a deuteron in the context of the lithium problem $^\ddagger$ $^\ddagger$, we study $^9$B in a two-body model consisting of a $^7$Be core and a deuteron. The two-body potential $v(r)$ is generated microscopically in a double folding model using densities of the deuteron and $^7$Be along with the density dependent M3Y (DDM3Y) effective interaction. The details of the framework are explained in $^\ddagger$ $^\ddagger$. The densities of deuteron and $^7$Be used in the present work are obtained from variational Monte Carlo calculations using the Argonne v18 two-nucleon and Urbana X three-nucleon potentials (AV18+UX) $^\ddagger$ $^\ddagger$. Earlier, the DDM3Y effective interaction was succesfully used to describe nuclear matter $^\ddagger$ $^\ddagger$, radioactivity $^\ddagger$ $^\ddagger$, scattering $^\ddagger$ $^\ddagger$ as well as resonances in unstable $^\ddagger$ $^\ddagger$ and unbound nuclei $^\ddagger$ $^\ddagger$. In the present work, it is used to study high-lying resonant states in unstable nuclei in the context of astrophysical problems.

From the above microscopic potential $v(r)$, inclusive of the centrifugal barrier, we use SQM to generate a family of isospectral potentials (IP) involving an arbitrary parameter $\lambda$. These potentials may appear quite different but they have exactly the same energy spectrum as the original one. The idea of isospectral potential has been extended by Pappademos et al. $^\ddagger$ $^\ddagger$ to scattering states with positive energy in the continuum. While the wave functions in the continuum are non-normalizable, following Pappademos et al., one can construct a normalizable wave function at a selected energy. This represents a bound state in the continuum (BIC). The BIC is a solution of the Schrödinger equation with an isospectral potential $\hat{v}(r;\lambda)$. The theory predicts that resonance energy does not depend on the choice of $\lambda$ and it is found in practice as well. So $\lambda$ is suitably chosen to optimize the stability of the

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resonant state. It preserves the spectrum of the original potential, only adding a discrete BIC at a selected energy.

From the constructed family of strictly isospectral potentials \( \hat{\psi}(r; \lambda) \), we extract normalizable wave functions \( \hat{\psi}_E(r; \lambda) \) following the procedures of BIC and could effectively calculate the trapping probability of the system within the enlarged well-barrier combination as

\[
C(E) = \int_{r_a}^{r_b} (\hat{\psi}_E(r'))^2 dr',
\]

where \( r_a \) and \( r_b \) are radial distances at the classical turning points \( a, b \) within the potential well. Resonance state which was not apparent in our microscopically constructed potential \( v(r) \) now gets prominence in our enlarged well-barrier combinations displayed by the probability plots of \( C(E) \). Judicious choice of parameter \( \lambda \) only helps in locating the high-lying resonant state while the result remains independent of the choice of \( \lambda \) as predicted by the theory.

III. RESULTS

In finding a solution to the cosmological \( ^7\text{Li} \) problem as mentioned above, it was suggested that the 16.8 MeV state in \(^9\text{B} \) needs to be studied in greater detail \(^2\). It is a possibility that substantial amount of \(^7\text{Be} \) destruction through resonant reaction with deuteron may lead to this state in \(^9\text{B} \). Since in standard Big Bang Nucleosynthesis theory, most \(^7\text{Li} \) is produced in the form of \(^7\text{Be} \), its destruction leads to a reduction in \(^7\text{Li} \) abundance. Therefore, inclusion of the resonant contribution of the 16.8 MeV state in \(^9\text{B} \) might lead to a solution to the lithium problem.

The probability \( C(E) \) of the system for \( \frac{5}{2}^+ \) state of \(^9\text{B} \) for \( \lambda = 1 \times 10^{-6}, 5 \times 10^{-7} \) and \( 1 \times 10^{-7} \) is shown in Fig. 1 where each curve is normalized to a peak value of 100. The figure clearly indicates the presence of a resonant state at energy \( E = 16.84 \) MeV. This is in excellent agreement to the experimental finding of 16,800(10) MeV with a width of 81(5) keV \(^10\). We calculated the width (\( \Gamma \)) of the resonance as \( 69 \) keV using WKB method as described in \(^8\).

The plot of isospectral potentials \( \hat{\psi}(r; \lambda) \) are shown in Fig. 2 at resonance energy \( E_R \) for the same \( \lambda \) values as in Fig. 1 for the \( \frac{5}{2}^+ \) state of \(^9\text{B} \) along with the original DDM3Y potential and the centrifugal barrier. The figure shows the deep well-barrier combination that actually succeeds in trapping the high-lying resonant state. The narrow and deep potential wells result in numerical difficulties in computation that indirectly limits the choice of \( \lambda \) also. The deep and narrow isospectral potentials of depth around 200-300 MeV for the present high-lying resonance at 16.84 MeV can be compared to the isospectral potentials of depth around 20-50 MeV for the low-lying resonance of \( 1.8 \) MeV in \(^{10}\text{Be} \) \(^3\).

The wave functions \( \hat{\psi}_E(r; \lambda) \) at the resonance energy 16.84 MeV for the same \( \lambda \) values as in Fig. 1 are shown in Fig. 3. The wave function plots are confirmation of the presence of the high-lying resonant state as it has an appreciable amplitude within the well. In the asymptotic region, the sinusoidal nature represents a free system after it leaks out of the well-barrier combination. The inset of Fig. 3 shows the wave function plot for \( \lambda = 5 \times 10^{-7} \) in an expanded scale up to 50 fm. We carried out similar calculations for a \( \frac{5}{2}^+ \) resonance as shown by the dotted line in Fig. 3. The low amplitude of such a wave function within the well as well as similar oscillatory behaviour in the asymptotic region rules out \( \frac{5}{2}^+ \) resonance at this energy. The observed angular distribution \(^10\) of the 16.8 MeV state is also consistent with the \( \frac{5}{2}^+ \) assignment.

IV. CONCLUSION

We have generated the wave function of a high-lying resonant state in the \(^9\text{B} \) nucleus in the SQM framework with a DDM3Y microscopic potential. The resulting resonance energy and width agree very well with the experimental data. The single parameter \( \lambda \) in the present formalism, appears to enhance resonance effect but it has no role in locating the exact resonance energy. Wave functions for different values of this parameter are in a sense equivalent as they reproduce the same resonance energy and width of the state.

In a nutshell, the SQM is the only procedure by which resonant state wave functions are extracted and utilized to effectively reproduce an experimental observable like
resonance width. The present work successfully showed the effectiveness of SQM and its range of applicability in the detection of high-lying resonant state and computation of the wave functions. We could establish that the present theoretical procedure works extremely well in the study of high-lying resonant states in unstable nuclei. Future works may be directed towards the decay properties of such resonances which are very relevant in the context of nuclear astrophysics.

[1] J. Snyder et al., Phys. Rev. C 88, 031303(R) (2013).
[2] N. Rijal, et al, Phys. Rev. Lett. 122, 182701 (2019).
[3] R. H. Cyburt, B. D. Fields, K. A. Olive, Journal of Cosmology and Astroparticle Physics 2008(11), 012 (2008).
[4] R. Boyd, C. R. Brune, G. M. Fuller, C. J. Smith, Phys. Rev. D 82, 105005 (2010).
[5] C. Angulo et al., The Astrophysical Journal 630(2), L105 (2005).
[6] N. Chakraborty, B. D. Fields, K. A. Olive, Phys. Rev. D 83, 063006 (2011).
[7] S. K. Dutta, D. Gupta, D. Das, Swapan K Saha, Jour. Phys. G: Nucl. Part. Phys. 41, 095104 (2014); see references therein.
[8] S. K. Dutta, D. Gupta, Swapan K Saha, Phys. Lett. B 776, 464 (2018); see references therein.
[9] O. S. Kirsebom, B. Davids, Phys. Rev. C 84, 058801 (2011).
[10] D. N. Basu, Phys. Lett. B 566, 90 (2003).
[11] R. B. Wiringa, Phys. Rev. C 43, 1585 (1991); https://www.phy.anl.gov/theory/research/density/
[12] D. N. Basu, J. Phys. G: Nucl. Part. Phys. 30 B7 (2004)
[13] D. N. Basu, P. Roy Chowdhury and C. Samanta Phys. Rev. C 72 051601 (2005).
[14] D. Gupta, E. Khan and Y. Blumenfeld, Nucl. Phys. A 773, 230 (2006).
[15] J. Pappademos, U. Sukhatme and A. Pagnamenta, Phys. Rev. A 48, 3525 (1993).
[16] C. Scholl et al, Phys. Rev. C 84, 014308 (2011).