WAVELENGTH MEASUREMENTS OF K TRANSITIONS OF OXYGEN, NEON, AND MAGNESIUM WITH X-RAY ABSORPTION LINES

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ABSTRACT

Accurate atomic transition data are important in many astronomical research areas, especially for studies of line spectroscopy. Whereas transition data of He-like and H-like ions (i.e., ions in high-charge states) have been accurately calculated, the corresponding data of K transitions of neutral or low-ionized metal elements are still very uncertain. Spectroscopy of absorption lines produced in the interstellar medium (ISM) has been proven to be an effective way to measure the central wavelengths of these atomic transitions. In this work, we analyze 36 Chandra High Energy Transmission Grating observations to search for and measure the ISM absorption lines along sight lines to 11 low-mass X-ray binaries. We correct the Galactic rotation velocity to the rest frame for every observation and then use two different methods to merge all the corrected spectra to a co-added spectrum. However, the co-added spectra obtained by this method exhibit biases, toward to either observations with high counts or lines with high signal-to-noise ratios. We do a Bayesian analysis of several significantly detected lines to obtain the systematic uncertainty and the bias correction for other lines. Compared to previous studies, our results improve the wavelength accuracy by a factor of two to five and significantly reduce the systematic uncertainties and biases. Several weak transitions (e.g., 1s–2p of Mg iv and Mg v; 1s–3p of Mg iii and Mg v) are also detected for the first time, albeit with low significance; future observations with improved accuracy are required to confirm these detections.

Key words: ISM: atoms – methods: data analysis – X-rays: binaries – X-rays: ISM

Online-only material: color figures

1. INTRODUCTION

Since the launch of modern X-ray space telescopes like the Chandra and XMM-Newton X-ray observatories, X-ray astronomy has entered the epoch of grating observations that can produce spectra with much improved energy resolution. In these spectra, narrow absorption and emission lines, which have never before been observed, are now commonly detected. These X-ray absorption/emission lines can be generated in a variety of astronomical environments, e.g., stellar coronae, supernova (SN) remnants, X-ray binaries (XRBs), galaxies, active galactic nuclei (AGNs), and interstellar media (ISM) and intergalactic media (e.g., Raassen et al. 2003; Dewey et al. 2008; Schulz et al. 2002; Miller et al. 2006; Steenbrugge et al. 2003; Lee et al. 2001; Yao & Wang 2005; Fang et al. 2003; Nicastro et al. 2005). These lines carry valuable information about the absorbing/emitting material and thus are powerful diagnostic tools for stellar evolution, explosion mechanism of SNe, mass exchange in accretion systems, interplays of different galactic components, feedback of AGN, galaxy formation and evolution, evolution of our universe, and so on. Clearly, proper identification of these lines and subsequent scientific derivation and interpretation strongly rely on the accuracy of atomic databases.

There are several available databases of atomic transitions, among which the four most commonly referenced in the X-ray community are Verner et al. (1996, hereafter V96), National Institute of Standards and Technology (NIST), XSTAR (Kallman et al. 2004), and APED (Smith et al. 2001). While atomic data of ions in high-charge states (e.g., He- and H-like ions) are very accurate in these databases and consistent with observations (Juett et al. 2004, 2006, J0406 hereafter; Yao et al. 2009, Y09 hereafter), data of the K transitions of neutral and mildly ionized metal elements (e.g., O i–O iii, Ne i–Ne iii, Mg iii–Mg v, etc.) have not been included in any databases. There also exist other serious problems. First, although the statistical errors of the wavelengths of these highly ionized lines can be less than 10 mÅ in astronomical observations, they are still not precise enough for the study of low-velocity gases. Y09 measured these highly ionized lines; however, some lines have large uncertainties, such as O viii Kβ that has a statistical error 6.7 mÅ, equivalent to 125 km s\(^{-1}\). In addition, some lines (e.g., Ne x Kα) are too weak to be given statistical errors. Second, the methods commonly used (e.g., Y09) to obtain the co-added spectrum introduce biases, which must be corrected for. If the biases are not corrected for, serious estimation errors of the gas velocities will result, as well as errors in other parameters associated with the gas velocities. Finally, some of the line wavelengths are not consistent with each other between the commonly referenced databases. For example, the wavelength of O viii Kβ in NIST is given as 18.6270 Å but is given as 18.6288 Å in V96. The difference between the two values is ~30 km s\(^{-1}\), which is a serious problem for the study of low-velocity gases. Therefore, it is essential to make observational identifications of the K-shell transitions of the neutral, low-ionized, and highly ionized metals. Since most of the K transitions of these low-ionization ions are in the wavelength range of 9.5–24.0 Å, within which copious lines of highly ionized metal elements have also been observed, these missing atomic data are important not only in their own right but also for
the proper identification of other lines. Recently, several groups have calculated and updated these databases (e.g., Gorczyca 2000; Behar & Netzer 2002; Gorczyca & McLaughlin 2005; García et al. 2005), but even the centroid wavelengths, the most basic parameters of these lines, are yet to be standardized (see Table 1). In addition, the laboratory measurements are also very uncertain; e.g., the wavelengths of O i 1s–2p given by Stolte et al. (1997) range from 23.489 to 23.536 Å.

Some Galactic X-ray sources are very bright in soft X-ray bands. When the X-ray continuum radiation passes through the ISM in different phases, X-ray absorption lines of ions in various charge states are produced and thus are excellent calibration references for these atomic data. For instance, Schattenburg & Canizares (1986) used the Einstein observations of the Crab to obtain the wavelength of neutral O (1s–2p of O i), although the result is uncertain (23.46 ± 0.22 Å) according to modern standards. J0406 analyzed Chandra High Energy Transmission Grating (HETG) observations of several Galactic XRBs and obtained the wavelengths of some strong lines (i.e., 1s–2p of O i, O ii, O iii, Ne ii, and Ne iii and 1s–3p of O i). They found that the theoretically calculated line centroids of the K transitions of low-ionized neon and oxygen need to be shifted by >20 mÅ to match the observed values; these shifts correspond to 300–400 km s⁻¹. However, the ISM is rotating around the Galactic center and thus the observed X-ray lines are not in the restframe. Unfortunately, the above three studies did not apply a correction for Galactic rotation. It is also necessary to exclude X-ray sources with significant intrinsic absorption, such as accreting XRBs with winds from either their companions or accretion disks along the line of sight (LOS). This methodology can be easily accomplished by comparing the absorption line and/or the total absorption properties of a target with multiple observations, since all XRBs exhibit significant variability.

Although X-ray absorption lines produced in the ISM are indeed ideal sources for calibrating theoretical predictions of atomic transitions, high-quality spectra are crucial for accomplishing this important task. The measurements of J0406 are based on relatively poor quality spectra (with signal-to-noise ratio S/N ≤ 15 per 10 mA spectral bin) obtained with short observations of several sources; thus, only strong transitions (e.g., 1s–2p of O i, Ne ii, and Ne iii) were relatively well constrained and other transitions still have large uncertainties (Δλ/λ ~ 6–20 mÅ) or were not observed. Recently, Y09 presented an extensive study of ISM X-ray absorption lines in the spectrum of Cyg X–2 observed with the Chandra-HETG spectrograph. The high spectral quality not only allowed these authors to measure most of the transitions listed in J0406 as accurate (Δλ/λ ~ 1–4 mÅ, 1σ uncertainty), but also enabled them to constrain other faint transitions not included and/or misidentified in their list. For instance, Y09 detected and measured the 1s–3p transition of Ne iii that could not be revealed in previous poor quality spectra, and they did not confirm the reported line at 23.140 Å that was misidentified as the 1s–3p transition of O iii.

In this work, our aim is to obtain more accurate K transition data of neutral, low-, and highly ionized metals, as well as to find some weak absorption lines (e.g., 1s–2p of Mg iv–Mg v; 1s–3p of Mg iii–Mg v). We jointly analyze 36 Chandra-HETG observations of 11 Galactic XRBs and provide the most accurate wavelengths for the K-shell transitions of neutral, low-, and highly ionized atoms. In Section 2, we describe our methods of extracting the X-ray spectra and correcting for Galactic rotation. In Section 3, we present the results of a joint analysis of all the observational data. In Section 4, we estimate systematic uncertainties and make necessary corrections to the detected lines. A discussion and summary appear in Sections 5 and 6, respectively.

2. SAMPLE SELECTION, DATA PROCESSING, AND GALACTIC ROTATION CORRECTION

2.1. Sample Selection

In order to obtain high S/N continuum spectra and absorption lines, we adopt the following criteria to select the sample.
1. Only Galactic low-mass X-ray binaries (LMXBs) are used as background light sources. Compared to extragalactic sources (e.g., AGNs), Galactic XRBs are usually located at low Galactic latitudes and thus the column densities of the ISM along the LOS are usually large if these objects are not too close to Earth. Thus, strong absorption lines are expected in their spectra. High-mass XRBs are excluded to avoid any possible contamination caused by their stellar winds that may not be stationary (e.g., Cyg X–1; Schulz et al. 2002).
2. Sources with intrinsic absorption or emission (e.g., GX 339–4, Miller et al. 2004; 4U 1916–05, Juett & Chakrabarty 2006) are excluded.
3. Only sources with Galactic latitudes above 2° are chosen to avoid too much Galactic absorption. Most lines we are concerned with have wavelengths longer than 10 Å; the photons in this regime suffer heavy absorption in the Galactic plane, which can reduce the significance of the line fitting (Appendix A).
4. Only HETG observations are used to ensure a high spectral resolution.
5. In order to ensure that there are significant absorption lines, we only include the sources for which the strongest O line (1s–2p of O i) and the strongest Ne line (1s–2p of Ne ii; Figures 1 and 2) are detected in the pipe line-produced spectra. We choose Ne ii Kα as an indicator, although it is always intrinsically weaker than O i Kα lines; the latter are not detected from several sources due to the rapidly

### Table 1

| Ion     | Transition | Wavelength (Å) |
|---------|------------|----------------|
|         | G05BN02    | G00GM05        | J0406 | Y09 |
| O i     | 1s–2p      | 23.4475        | 23.532 | 23.508 | 23.508 |
| O i     | 1s–3p      | ...            | 22.907 | 22.884 | ... |
| O ii    | 1s–2p      | 23.3100        | 22.781 | 23.330 | 23.384 |
| O ii    | 1s–3p      | ...            | 22.576 | ... | ... |
| O iii   | 1s–2p      | 23.0800        | ... | 23.140 | ... |
| O iii   | 1s–3p      | ...            | ... | ... | ... |
| Ne i    | 1s–3p      | ...            | 14.295 | 14.295 | 14.294 |
| Ne ii   | 1s–2p      | 14.6310        | 14.608 | 14.608 | 14.605 |
| Ne ii   | 1s–3p      | ...            | ... | 14.001 | ... |
| Ne iii  | 1s–2p      | 14.5260        | 14.508 | 14.508 | 14.507 |
| Ne iii  | 1s–3p      | ...            | ... | ... | 13.690 |
| Mg iii  | 1s–3p      | ...            | ... | ... | ... |
| Mg iv   | 1s–2p      | ...            | ... | ... | ... |
| Mg iv   | 1s–3p      | ...            | ... | ... | ... |

References. G05BN02: García et al. (2005) and Behar & Netzer (2002); G00GM05: Gorczyca (2000) and Gorczyca & McLaughlin (2005); J0406: Juett et al. (2004, 2006); Y09: Yao et al. (2009).
decreasing effective area of the HETG at wavelengths longer than 20 Å.

Finally, the 36 Chandra-HETG observations of the 11 LMXBs are listed in Table 2.

2.2. Data Reduction

We analyze all observations using CIAO 4.4 and CALDB 4.4.7. We use the standard tool tmerge to extract the spectra, i.e., PHA files. The energy redistribution matrix file (RMF) and the ancillary response file (ARF) are made by the standard tools mkgrmf and fullgarf, respectively. All the steps follow standard procedures except for the determination of the position of the zeroth-order image of each source, which is the key to fixing the wavelength scale. Since all the targets in our sample are bright sources, the zeroth-order source images are expected to be either severely piled-up in observations in the timed exposure (TE) mode or compressed into several pixels in observations in the continuous clocking (CC) mode. Rather than following the standard script to find the source positions, we use the mean position of the crosses between the CCD read-out streaks and each of the two arms (HEG and MEG) as the source positions for the TE-mode observations, and use two Gaussian profiles to fit the compressed image to determine the source positions for the CC-mode observations.

2.3. Velocity Correction Due to Galactic Rotation

Our aim is to accurately obtain the wavelengths of the absorption lines; we must therefore correct for the fact that the line-absorbing gas is not at rest with respect to the local standard of rest (LSR) frame. The gas in the Galactic plane
is rotating around the Galactic center and the rotation velocity increases as the radius decreases. The absorption lines in the X-ray band are produced by the multi-phase ISM along the LOS, and the motions are also different for the ISM in different phases. Therefore, we must correct for the Galactic rotation of neutral, low-ionized gas, and highly ionized gas separately. We adopt the method of Y09 that assumes that the gas is rotating around the Galactic center approximately in a circular orbit and that gas closer to the Galactic center rotates faster. For gas at a radius \( R \) and a rotational speed \( V \), the velocity relative to the LSR is

\[
V_r = V_{r, \text{gas}} - V_{r, \text{sun}} = R_0 \cos b \sin l \left( \frac{V}{R} - \frac{V_0}{R_0} \right),
\]

(1)

where

\[
R = \sqrt{R_0^2 + D^2 \cos^2 b + 2DR_0 \cos b \cos l}.
\]

(2)

Here, \( D \) is the distance between the gas and the observer and \( R_0 \) and \( V_0 \) are the radius of the LSR and its rotation velocity, respectively (Sparke & Gallagher 2000). The average velocity can be obtained by integrating all the gas with different \( R \) and \( V \) values along the path to the source.

For low-ionized gas, we use neutral H\textsc{i} 21 cm emission to trace the gas velocity. Data are derived from the Leiden–Argentina–Bonn Galactic H\textsc{i} Survey (Kalberla et al. 2005) that has a \( \sim 30' \) spatial grid and 1.3 km s\(^{-1}\) velocity resolution. To obtain the average profile \( \phi_m \) of the H\textsc{i} 21 cm emission toward the source, we average the emission profiles from four adjacent H\textsc{i} observations with respect to their angular separations, i.e.,

\[
\phi_m = \frac{\sum_{i=1}^{4} \phi_i/d_i^2}{\sum_{i=1}^{4} 1/d_i^2},
\]

(3)

where \( \phi_i \) is the profile of the H\textsc{i} 21 cm emission along each individual LOS and \( d_i \) is the angular separation between the LOS and the source. Then, we can obtain the average H\textsc{i} velocity from \( \phi_m \) and use this value as the projected velocity of the neutral or low-ionized gas along the LOS.
Table 2
Targets in Our Sample

| Source Name | (l, b)       | ObsID | EXPT (ks) | ACR (counts s\(^{-1}\)) | Distance (kpc) | \(V_1\) (km s\(^{-1}\)) | \(V_2\) (km s\(^{-1}\)) |
|-------------|--------------|-------|-----------|-------------------------|----------------|------------------------|------------------------|
| 4U 1705–44  | (343.32, –2.36) | 1923  | 25        | 117                     | 5.8\(^a\)      | –17                    | –37                    |
|             |              | 1924  | 6         | 171                     | ...            | ...                    | ...                    |
|             |              | 5500  | 27        | 37                      | ...            | ...                    | ...                    |
| GX 349+2    | (349.10, 2.75)  | 6715  | 11        | 267                     | 5.0\(^b\)      | –7                     | –10                    |
|             |              | 3354  | 35        | 281                     | ...            | ...                    | ...                    |
|             |              | 6628  | 13        | 301                     | ...            | ...                    | ...                    |
|             |              | 7336  | 12        | 264                     | ...            | ...                    | ...                    |
| 4U 0614+091 | (200.88, –3.36) | 10759 | 60        | 42                      | 3.2\(^b\)      | 19                     | 36                     |
|             |              | 10760 | 45        | 45                      | ...            | ...                    | ...                    |
|             |              | 10857 | 59        | 62                      | ...            | ...                    | ...                    |
|             |              | 10858 | 35        | 39                      | ...            | ...                    | ...                    |
| 4U 1735–44  | (346.05, –6.70) | 704   | 25        | 92                      | 5.9\(^a\)      | –14                    | –45                    |
|             |              | 6635  | 23        | 41                      | ...            | ...                    | ...                    |
|             |              | 6636  | 25        | 102                     | ...            | ...                    | ...                    |
| Ser X–1     | (36.12, 4.84)   | 700   | 78        | 110                     | 7.7\(^a\)      | 14                     | 57                     |
| 4U 1254–690 | (303.48, –6.42) | 3823  | 53        | 27                      | 15.5\(^a\)     | –2                     | 15                     |
| 4U 1636–536 | (332.92, –4.82) | 107    | 30        | 105                     | 5.95\(^a\)     | –14                    | –45                    |
|             |              | 1939  | 27        | 93                      | ...            | ...                    | ...                    |
|             |              | 6635  | 23        | 41                      | ...            | ...                    | ...                    |
|             |              | 6636  | 25        | 102                     | ...            | ...                    | ...                    |
| 4U 1820–303 | (2.79, –7.91)  | 1021  | 10        | 124                     | 4.94\(^a\)     | 6                      | –1                     |
|             |              | 1022  | 11        | 142                     | ...            | ...                    | ...                    |
|             |              | 6633  | 25        | 215                     | ...            | ...                    | ...                    |
|             |              | 6634  | 25        | 272                     | ...            | ...                    | ...                    |
|             |              | 7032  | 46        | 242                     | ...            | ...                    | ...                    |
| XTE J1817–330 | (359.82, –8.00) | 6615  | 50        | 998                     | 5.0\(^a\)      | 7                      | 0                      |
|             |              | 6616  | 50        | 560                     | ...            | ...                    | ...                    |
|             |              | 6617  | 47        | 293                     | ...            | ...                    | ...                    |
|             |              | 6618  | 51        | 109                     | ...            | ...                    | ...                    |
| 4U 1728–16  | (8.51, 9.04)   | 703   | 21        | 129                     | 5.0\(^a\)      | 7                      | –2                     |
|             |              | 11072 | 98        | 118                     | ...            | ...                    | ...                    |
| Cyg X–2     | (87.33, –11.32) | 1016  | 15        | 280                     | 11.0\(^a\)     | –8                     | –54                    |
|             |              | 1102  | 29        | 130                     | ...            | ...                    | ...                    |
|             |              | 8170  | 77        | 384                     | ...            | ...                    | ...                    |
|             |              | 8599  | 71        | 382                     | ...            | ...                    | ...                    |
|             |              | 10881 | 67        | 267                     | ...            | ...                    | ...                    |

Notes. Column 1: source name of our sample; Column 2: Galactic longitude and latitude; Column 3: observation ID; Columns 4–5: exposure time and average count rates of every ObsID; Column 6: distance; Columns 7–8: velocities relative to the LSR of the low-ionized and highly ionized gases. The references from which we obtained the distances of the targets are as follows:

\(^a\) Galloway et al. (2008).

\(^b\) Kuulkers et al. (2010).

\(^c\) We use the values from Kong et al. (2006), Iaria et al. (2004), and Sala et al. (2007) for the sources without confirmed distances.

For highly ionized gas, as its scale height is much larger than that of the low-ionized gas, the halo-lagging effect (Rand 1997, 2000) must be considered. Here, we follow the three assumptions in Y09: (1) the density of the hot gas is \(n = n_0 e^{-z/z_0}\), where \(n_0\) is the gas density in the Galactic plane and \(z\) is the height from the Galactic plane and \(z_0 = 3\) kpc (e.g., Bowen et al. 2008; Yao et al. 2008); (2) the rotation velocity linearly decreases from \(V_0\) to zero at a height \(z_0 = 8\) kpc (Rand 1997) above the Galactic disk; (3) the velocity at radii between \(R_0\) and \(R\) can be linearly interpolated from \(V_0\) and \(V\). For the LSR, we take \(R_0 = 8\) kpc and \(V_0 = 220\) km s\(^{-1}\). From Sparke & Gallagher (2000), the rotation velocity near the target can be inferred from the smallest velocity (for the targets with \(90 < |\ell| < 180\) and the close targets with \(270 < |\ell| < 360\)) and largest velocity (for the targets with \(180 < |\ell| < 270\) and close targets with \(0 < |\ell| < 90\)) of the H\(^1\) emission and then the average velocity of the hot gas can be obtained by integrating all the gas with different \(R\) and \(V\) values along the LOS. The average velocities of the neutral or low-ionized gas and highly ionized hot gas are listed in Table 2.

We transform all the spectra to the LSR and then use the IDL scripts writepha and wrt_ogip_rmf to produce the spectrum file (PHA) and the response file (RSP = RMF × ARF), respectively.

3. MERGING SPECTRA AND DETERMINING OF THE LINE CENTRAL WAVELENGTHS

In this section, we use two different methods to jointly analyze these 36 observations to obtain the wavelengths of the
absorption lines of neutral, low-ionized, moderately ionized and highly ionized gas (Table 4). The results and discussion will be presented in Sections 5 and 6, respectively.

3.1. Method 1: Direct Merging of All Full Spectra

All 36 spectra were co-added according to the following two steps (the method described in Y09) to increase the S/N: (1) add the counts of each spectrum channel by channel to make a new PHA; (2) merge all the response files to produce the RSP, where we used the total counts as the weights of each observation.

We fit the co-added spectrum with XSPEC (version 12.7.0) to analyze the co-added spectrum. The continuum was fit using a power law plus several broad Gaussian functions and the absorption lines were fit with narrow Gaussian functions:

$$\phi(\lambda_i) = a \frac{1}{\sqrt{2\pi}b} e^{-\frac{(\lambda - \lambda_i)^2}{2b^2}},$$  

where $\lambda$ is the wavelength of the line centroid, $b$ is the width of the line, and $a$ is the normalization of the line. From Figures 3 and 4, it can be seen that the co-added spectrum has a very high S/N. As a result, many weak lines (e.g., O vii Kγ) are visible. The S/Ns around the O i Kα, Ne ii Kα, and Mg xi Kα lines are about 20, 80, and 110, respectively.

Merging the spectra by this method can generally increase the S/N significantly. However, the spectra with weak lines may also decrease the S/N. We take two observations (ObsIDs 6618 and 8599) as examples. The wavelength of O vi Kα is $21.592_{-7}^{+14}$ Å obtained from ObsID 6618, whereas it is $21.597_{-10}^{+15}$ Å from the co-added spectrum of ObsID 6618 and ObsID 8599. The error actually increases as more observations are combined, since the significance of the absorption line in ObsID 8599 is too low to increase the significance of the line in the co-added spectrum. As a result, a spectrum with weak absorption lines can only increase the total counts of the continuum, which increases the background noise of the line (Figure 5). To avoid this problem, another method of merging the spectra is investigated next.

3.2. Method 2: Weighted Merging of Net Lines

Unlike the method in Section 3.1, we fit the spectrum to obtain the continuum and line parameters ($\lambda$, $b$, and $a$) of each observation first. All the data for each line flux were then merged to produce a co-added line flux after the removal of the continuum of each individual spectrum,

$$f_{\text{add}} = \frac{\sum_{i=1}^{36} f_i w_i}{\sum_{i=1}^{36} w_i},$$  

where $f_{\text{add}}$ is the flux of each co-added line flux and $f_i$ and $w_i$ are the flux and weight for each line in each observation, respectively. In order to avoid the problem caused by the spectrum with weak ISM absorption, we must evaluate $w_i$ carefully. Here, the weight of each absorption line is given as...
Figure 4. Same as Figure 1, but the spectrum is normalized to the best-fit continuum.

(A color version of this figure is available in the online journal.)

(Appendix B):

\[ w_i = \frac{a_i}{\sigma^2 b_i}, \quad (6) \]

where \( \sigma_i \) is the error of the continuum around the line and, \( a_i \) and \( b_i \) are the normalization and the width of the \( i \) line, respectively.

For the strong lines (e.g., O\textsc{i} K\alpha and Ne\textsc{ii} K\alpha), the significance is high enough to determine the line parameters for most observations, which can be used as the weights for merging the spectra. However, the significance of the weak lines (e.g., O\textsc{iii} K\alpha and Ne\textsc{iii} K\beta) is too low and the line parameters can be obtained for only a few observations. Among all 36 observations, \( N(\text{Ne}\textsc{ii} K\alpha; S/N > 1.645) = 26 \); but \( N(\text{Ne}\textsc{iii} K\beta; S/N > 1.645) = 3 \). To solve this problem, we make a simple assumption that the clouds in every LOS have the same ion fraction (i.e., all the lines in each spectrum have the same line-strength ratios). The ratios O\textsc{i} K\alpha/O\textsc{ii} K\alpha, O\textsc{vii} K\alpha/O\textsc{vii} K\beta, and Ne\textsc{ii} K\alpha/Ne\textsc{iii} K\alpha are shown in Figure 6, which proves the validity of the assumption. As shown in Table 3, all the lines are divided into five groups and the strongest line of each group is also found. Then, we can use \( w_i \) of the five strongest lines of each group to merge the spectra to obtain the wavelength of each line in each group. For the low-ionized O lines, \( w_i (\text{O}\textsc{i} K\alpha) \) is used as the weight. \( w_i (\text{Mg xi} K\alpha) \) is used as the weight to merge the spectra both for the low-ionized and the highly ionized Mg absorption lines, since the Mg xi K\alpha line is the only Mg absorption line detected significantly.

The co-added spectrum also exhibits very high S/N and we do the same Gaussian fit to every line in neutral, low-ionized, and highly ionized states. The results of neutral or low-ionized, moderately ionized and highly ionized lines are shown in Table 4.

4. CORRECTION OF THE CENTRAL WAVELENGTHS WITH BAYESIAN ANALYSIS

4.1. Bayesian Analysis

For a sample consisting of \( N \) observations, both Methods 1 and 2 merge all \( N \) spectra into a co-added spectrum. These methods can obtain the wavelength by spectral line fitting, but cannot calculate the systematic error. However, we can obtain the line wavelengths (\( \lambda_k; k = 1, 2, \ldots, N \)) and errors (\( \sigma_k; k = 1, 2, \ldots, N \)) of the \( N \) spectra, and then use Bayesian analysis to obtain the two-dimensional (2D) probability distribution of the wavelength (\( \lambda_B \); hereafter the subscript B denotes the parameter obtained from Bayesian analysis) and the systematic dispersion (\( \mathcal{D}_{\text{sys}} \); note \( \mathcal{D}_{\text{sys}} \) does not include the scatter caused by statistical uncertainties). Here, we use a 2D uniform distribution \( P(\lambda_B, \mathcal{D}_{\text{sys}}) \) as the a priori distribution of \( \lambda_B \) and \( \mathcal{D}_{\text{sys}} \). According to Bayes’ theorem, the posterior distribution \( P(\lambda_B, \mathcal{D}_{\text{sys}}|\Lambda) \) is given by

\[ P(\lambda_B, \mathcal{D}_{\text{sys}}|\Lambda) = \frac{P(\Lambda|\lambda_B, \mathcal{D}_{\text{sys}})P(\lambda_B, \mathcal{D}_{\text{sys}})}{P(\Lambda)}, \quad (7) \]
Figure 5. O\textsc{vii} K\textalpha line in ObsIDs 6618 and 8599 and in the co-added spectrum. Data in the right column have been normalized to the best-fit continuum. (A color version of this figure is available in the online journal.)

Figure 6. Ratio of line strengths between O\textsc{i} K\textalpha/O\textsc{ii} K\textalpha, O\textsc{viii} K\textalpha/O\textsc{vii} K\beta, and Ne\textsc{ii} K\textalpha/Ne\textsc{iii} K\alpha. The numbers of observations where both lines are well fit are: \(N(\text{O\textsc{i} K\alpha/O\textsc{ii} K\alpha}) = 8\), \(N(\text{O\textsc{viii} K\alpha/O\textsc{vii} K\beta}) = 11\), and \(N(\text{Ne\textsc{ii} K\alpha/Ne\textsc{iii} K\alpha}) = 16\). (A color version of this figure is available in the online journal.)

where

\[
P(\Lambda|\lambda_B, \mathcal{D}_{\text{sys}}) = \frac{1}{\sqrt{(2\pi)^N}} \prod_{k=1}^{N} \frac{1}{\sqrt{\mathcal{D}_{\text{sys}}^2 + \sigma_k^2}} e^{-\frac{(\lambda_B - \lambda_k)^2}{2(\mathcal{D}_{\text{sys}}^2 + \sigma_k^2)}},
\]

\[
P(\Lambda) = \int \int P(\Lambda|\lambda_B, \mathcal{D}_{\text{sys}}) P(\lambda_B, \mathcal{D}_{\text{sys}}) d\lambda_B d\mathcal{D}_{\text{sys}},
\]

where \(\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)\) and \(\sigma_k(k = 1, 2, \ldots, N)\) are the wavelength and its error, respectively, of each line of these \(N\) spectra. Note that when the a priori distribution is a 2D uniform distribution (with unspecified ranges), the Bayesian solution is the same as that obtained from the maximum likelihood estimation. From the 2D probability distribution, we can obtain the maximum a posteriori (MAP) estimates and errors of \(\lambda_B\) and \(\mathcal{D}_{\text{sys}}\). \(\sigma_{\text{sys}}\) can be calculated by:

\[
\sigma_{\text{sys}}^2 = \frac{\mathcal{D}_{\text{sys}}^2}{N}.
\]

As shown in Figure 7, in order to illustrate how to use Bayesian analysis to obtain \(\lambda_B\) and \(\mathcal{D}_{\text{sys}}\), we make a simulation as follows.

1. Make 36 absorption-line models (a normalized continuum plus a Gaussian absorption line) with the line parameters from Table 5. All the continua have the same spectral wavelength range (23–24 Å) and resolution (0.005 Å). One of the line models is shown in panel (a) of Figure 7.

2. Use the 36 models (step 1) to make 36 simulated spectra. Panel (b) of Figure 7 shows the simulated spectrum with the model from panel (a).

3. Fit the 36 simulated spectra with a Gaussian function to obtain 36 simulated line wavelengths (panel (c) in Figure 7).

4. Use Equations (7)–(9) to obtain the 2D probability distribution of \(\lambda_B\) and \(\mathcal{D}_{\text{sys}}\) (panel (d) in Figure 7).

From the 2D probability distribution, we obtain the MAP estimates of \(\lambda_B\) and \(\mathcal{D}_{\text{sys}}\) (red cross in panel (d)). The maximum width of the 1σ contour along the X-axis is \(\sigma_{\lambda_B}\), which is the 1σ error of \(\lambda_B\). The expectation of the wavelength is
Table 3

| Strongest | O I Kα | O (M & H) | Ne (L) | Ne (M & H) | Mg (A) |
|-----------|-------|-----------|--------|------------|--------|
| 1s–2p     | O I, O II, O III | O II, O VI, O VIII, O VII | Ne II, Ne III | Ne IV, Ne V, Ne VII, Ne IX, Ne X | Mg IV, Mg V, Mg XI |
| 1s–3p     | O I    | O II, O VI, O VIII | Ne II, Ne III | Ne IX | Mg III, Mg IV, Mg V |
| 1s–4p     |        | O VII     |        | Ne II |        |

Notes. The first row are the group names, where “L,” “M,” and “H” refer to the low-ionized, moderately ionized, and highly ionized states, respectively. “A” means all ionized states. The second row lists the strongest lines in each group.

Table 4

The Wavelengths of Low-ionized, Moderately ionized, and Highly ionized Elements

| Ion | Transition | λ₁ (Å) | S/N₁ | λ₂ (Å) | S/N₂ |
|-----|------------|--------|------|--------|------|
| O I | 1s–2p      | 23.5087±0.6 | 49.9 | 23.5091±0.7 | 44.1 |
| O I | 1s–3p      | 22.8834±2.3 | 5.5  | 22.8872±2.4 | 3.1  |
| O II| 1s–2p      | 23.3508±1.3 | 14.2 | 23.3507±1.1 | 13.4 |
| O III| 1s–2p     | 23.0392±5.4 | 4.2  | 23.0565±7.1 | 2.4  |
| O IV| 1s–2p      | 22.6969±37.3| 4.1  | 22.6987±33  | 2.9  |
| O V | 1s–3p      | 22.2849±47 | 4.4  | 22.2872±42  | 3.1  |
| O VI| 1s–2p      | 21.5915±1.6 | 7.1  | 21.5948±1.5 | 10.6 |
| O VII| 1s–3p     | 18.6259±1.1 | 13.0 | 18.6255±1.1 | 10.9 |
| O VIII| 1s–4p    | 17.7657±0.9 | 9.5  | 17.7673±0.8 | 6.1  |
| O IX| 1s–2p      | 18.9667±0.9 | 18.1 | 18.9664±0.8 | 16.3 |
| O VIII| 1s–3p     | 16.0046±1.3 | 8.3  | 16.0044±1.3 | 8.2  |
| Ne II| 1s–3p     | 14.2937±2.9 | 7.8  | 14.2942±2.7 | 10.7 |
| Ne III| 1s–2p    | 14.6068±0.4 | 29.4 | 14.6071±0.4 | 28.0 |
| Ne IV| 1s–3p     | 14.0029±1.0 | 11.6 | 14.0031±0.9 | 11.3 |
| Ne V | 1s–4p     | 13.9373±1.5 | 4.5  | 13.9339±2.9 | 4.5  |
| Ne VI| 1s–2p     | 14.5068±0.7 | 15.0 | 14.5073±0.7 | 18.1 |
| Ne VII| 1s–3p    | 13.6951±2.4 | 3.8  | 13.6953±2.5 | 5.1  |
| Ne VIII| 1s–2p   | 14.3471±64.3| 1.8  | ...     | ...  |
| Ne IX| 1s–2p     | 14.2127±38.6| 3.2  | 14.2090±13.7| 2.2  |
| Ne X | 1s–3p     | 13.8272±4.2 | 5.3  | 13.8269±3.3 | 3.2  |
| Ne XI| 1s–2p     | 13.4455±0.5 | 27.6 | 13.4453±0.4 | 26.1 |
| Ne XII| 1s–3p    | 11.5459±1.4 | 11.0 | 11.5456±1.6| 9.9  |
| Ne XIII| 1s–4p   | 11.0010±2.5 | 4.1  | 10.9987±1.9| 2.6  |
| Mg II| 1s–2p     | 12.1250±2.1 | 3.4  | 12.1253±2.2| 3.8  |
| Mg III| 1s–3p    | 9.4330±22.8| 1.5  | 9.4766±4.1 | 3.2  |
| Mg IV| 1s–2p     | 9.8539±29.1| 1.5  | ...     | ...  |
| Mg V | 1s–2p     | 9.8191±30.1| 3.3  | 9.8195±1.4 | 1.0  |
| Mg VI| 1s–3p     | 9.0569±28.0| 3.0  | 9.0553±2.5 | 2.3  |
| Mg VII| 1s–2p    | 9.1679±0.6 | 9.7  | 9.1670±0.6| 9.5  |
| Fe XVII| 2p–3d   | 15.0124±1.6 | 8.6  | 15.0123±0.9| 7.5  |
| Fe XVIII| 2p–3d  | 15.2596±4.2 | 4.7  | 15.2612±2.6| 3.7  |

Notes. λ₁ and S/N₁ (i = 1, 2) are the wavelength and signal-to-noise ratio of the absorption lines from Methods 1 and 2. S/N₂ = ∑EW/σEW and S/N₂ = a/σa, where EW is the equivalent width and a is the normalization of Gaussian-fit lines. All the errors are at the 1σ level and have units of mÅ.

\[
\lambda_e = \bar{\lambda} \pm \hat{\Delta}/\sqrt{N} = 23.5 \pm 0.00083 \text{ Å.}
\]

The weighted average of the wavelengths (\(\bar{\lambda}\)) can be calculated by:

\[
\bar{\lambda} = \frac{\sum_{i=1}^{36} \lambda_i / \sigma_i^2}{\sum_{i=1}^{36} 1 / \sigma_i^2}.
\] (11)

We have \(\bar{\lambda} = 23.49952 \pm 0.00025 \text{ Å}\) and the results from Bayesian analysis are \(\lambda_B = 23.49948 \pm 0.00088 \text{ Å}\) and \(\Delta \sigma_Y = 0.00494 \pm 0.00066 \text{ Å}\). In this simple simulation, \(\bar{\lambda}\), \(\lambda_B\), and \(\Delta \sigma_Y\) are consistent with expectations.

4.2. Comparison Among Method 1, Method 2, and the Bayesian Analysis

In our work, three methods (Method 1, Method 2, and the Bayesian analysis) can be applied to estimate the real
Figure 7. Simulation illustrating how to use Bayesian analysis to obtain $\lambda_B$ and $D_{\text{sys}}$. Panel (a) shows the absorption line model. Panel (b) shows the simulated spectrum with the model from panel (a). The absorption line is fit with a Gaussian function. The distribution of the 36 line wavelengths obtained from the 36 simulated spectra is shown in panel (c). The blue and red horizontal lines are the $\bar{\lambda}$ and the expected value of the mean value of the wavelengths $\hat{\lambda}_s$ of $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_{36}\}$ of the 36 simulated spectra. Panel (d) presents the 2D distribution of $\hat{\lambda}_s$ and $D_{\text{sys}}$ obtained from the Bayesian analysis of the data in panel (c); the red cross marks the MAP estimates of $\hat{\lambda}_s$ and $D_{\text{sys}}$; the three contours from the inside out were the 1$\sigma$, 2$\sigma$, and 3$\sigma$ credible intervals, respectively, which were obtained by calculations over a complete parameter space.

(A color version of this figure is available in the online journal.)

### Table 5

| Wavelength ($\bar{\lambda} = 23.5$, $\hat{\Delta} = 0.005$) | FWHM (mÅ) | EW (mÅ) | S/N |
|-----------------|-----------|---------|-----|
| $\bar{\lambda}$ | 60        | 50      | 10  |

**Note.** The wavelengths obey a Gaussian distribution ($\bar{\lambda}$, $\hat{\Delta}$) and the other parameters are constant.

wavelengths of the lines from a sample. However, only the strong lines have enough observations with enough S/N to do the Bayesian analysis. It is very important to know which method will give unbiased results. We answer this question by a simulation, as follows.

1. Step 1 is the same as in Section 4.1, except that both the EWs and the S/Ns of the 36 continua obey a uniform distribution.
   All the model parameters are sampled once and then fixed, as shown in Table 6.

2. Repeat steps 2–4 from Section 4.1 10,000 times to obtain the distributions of the results from Method 1, Method 2, and the Bayesian analysis (Figure 8).

In this simulation, the wavelengths of the absorption lines ($\lambda_s$; $s = 1, 2, \ldots, N$) in these 36 simulated spectra are set to obey a Gaussian distribution ($\bar{\lambda} = 23.5$, $\hat{\Delta} = 0.005$Å), thus the expected value of the mean value of the wavelengths ($\lambda_s$) should also obey a Gaussian distribution ($\bar{\lambda}$, $\hat{\Delta}/\sqrt{N}$), i.e., $23.5 \pm 0.00083$ Å (the lined region in the left panel of Figure 8 is the 1σ confidence region of $\lambda_s$). We have $\lambda_1 = 23.50284 \pm 0.00024$ Å (Method 1; red histogram) and $\lambda_2 = 23.50319 \pm 0.00022$ Å (Method 2; blue histogram); these values are more than 3σ deviant from $\lambda_s$. This result occurs because the co-added spectra depend on the spectra with high count rates (Method 1) or high line significances (Method 2), which significantly deviate from the expected value of 23.5 Å. However, the Bayesian analysis gives $\lambda_B = 23.50054 \pm 0.00041$ Å (green histogram), which is consistent with $\lambda_s$. $D_{\text{sys}} (0.00508 \pm 0.00034)$ Å is also

### Table 6

| $\lambda_s$ (Å) | EW (mÅ) | S/N | $\lambda_s$ (Å) | EW (mÅ) | S/N |
|-----------------|---------|-----|-----------------|---------|-----|
| 23.5053         | 20.9    | 28.9| 23.4995         | 14.9    | 11.2|
| 23.5091         | 49.9    | 29.7| 23.4944         | 38.0    | 22.3|
| 23.4916         | 33.5    | 9.1 | 23.4970         | 26.7    | 15.3|
| 23.4967         | 34.7    | 12.7| 23.4926         | 16.6    | 8.7 |
| 23.5092         | 14.2    | 16.1| 23.5016         | 49.3    | 25.3|
| 23.5002         | 23.2    | 14.7| 23.5008         | 40.5    | 22.4|
| 23.4950         | 20.0    | 9.4 | 23.5023         | 47.9    | 22.9|
| 23.4999         | 31.7    | 9.8 | 23.4959         | 26.9    | 17.4|
| 23.4934         | 49.6    | 9.2 | 23.5090         | 44.7    | 28.5|
| 23.4987         | 22.3    | 10.2| 23.5002         | 28.3    | 8.9 |
| 23.5001         | 35.2    | 12.4| 23.4950         | 29.4    | 18.7|
| 23.5037         | 31.4    | 23.0| 23.5076         | 18.2    | 23.7|
| 23.5020         | 17.8    | 20.5| 23.4957         | 21.8    | 17.1|
| 23.5003         | 22.0    | 26.3| 23.4990         | 44.9    | 5.8 |
| 23.5116         | 33.4    | 25.8| 23.5022         | 31.3    | 24.3|
| 23.5033         | 45.3    | 17.2| 23.5002         | 27.6    | 20.3|
| 23.5044         | 33.4    | 24.3| 23.4998         | 20.6    | 7.2 |
| 23.4961         | 28.3    | 7.7 | 23.4971         | 40.8    | 18.7|

**Notes.** $\lambda_s$ (Gaussian distribution; $\bar{\lambda} = 23.5$, $\hat{\Delta} = 0.005$ Å), EWs (uniform distribution; 10–50 mÅ), and S/Ns (uniform distribution; 5–30).
consistent with the expected value (Δ), which obeys a Gaussian distribution (Δ, Δ/√2N), i.e., 0.005 ± 0.00059 Å (the lined region in the right panel of Figure 8 is the 1σ confidence region of Δc). Both the errors on λ1 and λ2 are smaller than that on λB; we will explain this finding in Section 4.3. Therefore, co-added spectral fitting can produce biased results, but Bayesian analysis can give unbiased results for both λB and Δc.

### 4.3. Correction to the Wavelength Obtained from the Co-added Spectra

Since the Bayesian analysis can give unbiased results for both λB and Δc, we can use the results of the Bayesian analysis (λB, Δc, and σB) to correct the results obtained from the co-added spectra (λm and σm; m = 1, 2 for Methods 1 and 2).

In our statistical model, the unbiased λB can be expressed as

\[
λ_B = λ_m + Δλ_m \pm σ_{sys}, \quad m = 1, 2,
\]

where λm is the biased result, Δλm is the correction quantity, and σsys is the systematic error. Thus, σB can be written as:

\[
σ_{Bm}^2 = σ_{xm}^2 + σ_{Δxm}^2 + σ_{sys}^2, \quad m = 1, 2,
\]

where σxm and σΔxm are the statistical errors of λm and Δxm, respectively. For convenience, σxm is the mean value of the asymmetric errors of λm in Table 4. As described in Section 4.1, σB can be obtained from the Bayesian analysis and σsys can be calculated by Equation (10) from Δc. From Equations (12) and (13), we can obtain

\[
Δλ_m = λ_B - λ_m, \text{ and } σ_{Δxm}^2 = σ_{xm}^2 - σ_{sys}^2, \quad m = 1, 2.
\]

From the Bayesian analysis, we can extract the statistical component (σstat) from σB by

\[
σ_{stat}^2 = σ_{Bm}^2 - σ_{sys}^2,
\]

where σstat is the total statistical error that consists of both σxm and σΔxm in our model. Therefore, we have σstat > σxm; this result is why σxm < σB in Section 4.3.

As discussed above, λm needs to be corrected as

\[
λ_{m,c} = λ_m + Δλ_m \pm σ_{sys}, \quad m = 1, 2.
\]

\[
σ_{λm,c}^2 = σ_{xm}^2 + σ_{Δxm}^2 + σ_{sys}^2, \quad m = 1, 2.
\]

The final errors consist of three parts: the statistical error (σxm) from the co-added spectra, the statistical error of the correction (σΔxm), and the systematic error (σsys) obtained from multiple observations. Because the systematic dispersion of the 36 spectra is partly converted into the broadening of lines in the co-added spectrum, σΔm also partially includes σsys. Therefore, σλm,c is a conservative estimate of the final total error.

### 4.4. Application of Bayesian Analysis to Our Sample

Although the co-added spectrum has an extremely high S/N that is useful for finding some weak lines, it can be biased toward observations with high count rates (Method 1) or high line significances (Method 2). Bayesian analysis can obtain an unbiased result; however, only the strong lines have enough observations with enough S/N to do the Bayesian analysis.

As shown in Table 3, all the lines have been divided into five groups and the strongest line of each group was found. Since the absorption clouds within each group are assumed to have the same velocities, we can obtain an unbiased result for all the lines as follows: (1) use Methods 1 and 2 to obtain the results of high S/N co-added spectra; (2) use the results of the Bayesian analysis of the strongest line of each group to correct all the other lines in the same group, e.g., Δλm (O I Kα) is used to correct the low-ionized O lines; see Tables 3 and 7 and Figure 9 for details. The final results are shown in Tables 8–10.

The Mg xix Kα line is very special. We find that for the Mg xix Kα line σsys > σstat. This result means that, unlike O viii Kα and Ne xix Kα, σst,2 of the Mg xix Kα lines is not dominated by statistical errors but rather by the components caused by the line broadening when a co-added spectrum is produced, i.e., σsys increases σλm significantly. Δλm of the Mg xix Kα line in Table 7 is very uncertain and we cannot obtain σΔm with Equation (14). In order to correct the wavelength of the Mg lines, we must make the assumption that the motion of both the low-ionized and the highly ionized Mg are the same as that of Ne. Thus, we can use Δλm (Ne ii Kα) to correct the low-ionized Mg lines and use Δλm (Ne xix Kα) to correct the moderately ionized and the highly ionized Mg lines, respectively.

For the Fe 2d–3p double lines, we also use Δλm (Ne xix Kα) to correct the value obtained from the co-added spectra (Methods 1 and 2; Table 4).

### 5. DISCUSSION

#### 5.1. The Two Methods of Merging the Spectra

In Section 3, two different methods were used to merge all 36 observations. Method 1 is very simple: we only need to add the counts of each spectrum channel by channel; i.e., every spectrum has the same weight. However, Method 1 implicitly assumes that all the spectra have the same absorption column densities, i.e., the same weight. When we merge two spectra where one has a strong absorption line (e.g., the O vi Kα in ObsID 6618) and the other has a weak absorption line (e.g., the O vi Kα in ObsID 8599), the spectrum with the weak absorption line
Figure 9. Distributions of the wavelengths of O i Kα, Ne ii Kα, O viii Kα, Ne ix Kα, and Mg xi Kα obtained from all the well-fit observations. The contours and the red crosses in the right panels are the same as in Figure 7. (A color version of this figure is available in the online journal.)

Table 7

| Ion  | Transition | ∆λB | σB | σstat | ∆λ1 | ∆λ2 | λi,c (i = 1, 2) |
|------|------------|------|----|--------|------|------|----------------|
| O i  | 1s–2p      | 23.5088 | 0.8 | 0.0    | 0.8  | 0.1(0.5) | −0.3(0.3)       |
| O viii | 1s–2p    | 18.9676 | 1.1 | 0.6    | 1.0  | 0.6(0.6) | +0.8(0.6)       |
| Ne ii | 1s–2p      | 14.6076 | 0.7 | 0.4    | 0.5  | 0.9(0.4) | +0.6(0.4)       |
| Ne ix | 1s–2p      | 13.4465 | 0.7 | 0.5    | 0.5  | 1.1(0.3) | +1.3(0.3)       |
| Mg xi | 1s–2p      | 9.1670  | 0.9 | 0.7    | 0.6  | −0.8(*)  | −0.9(*)         |

Notes. ∆λB, σB, and σstat are obtained from the Bayesian analysis in Figure 9. ∆λB is in units of Å and all the other data are in units of mÅ. All the errors in parentheses are at 1σ levels. σstat is calculated with Equation (15). ∆λ1 and ∆λ2, as well as their errors, are calculated with Equation (14). "*" means that we cannot obtain σΔλi because σstat is smaller than σi in Table 4. The last column lists the lines to be corrected and “L,” “M,” and “H” are the same as those defined in Table 3. Please see Section 4.3 for details.

can dilute or even wipe out the absorption line in the co-added spectrum, i.e., reduce the significance of the absorption line. To avoid this problem, we use the significance of the absorption line as the weight to merge the spectra instead (Method 2). The spectra that only contribute to the continuum are omitted in this method, since the continuum is actually the noise for the absorption lines. From Table 4, we can see that the results obtained by the two methods are consistent with each other for most of the absorption lines, except for some weak lines (e.g., O iii Kα) that are too weak to be fit with a Gaussian function. In addition, several suspicious lines may not be genuine, as they are only present in the co-added spectrum obtained with Method 1 (Mg iv Kα and Ne iv Kα). The significance levels of the lines in the co-added spectrum obtained with Method 2 are...
not remarkably higher than those obtained with Method 1. This result means that the assumptions of both Methods 1 and 2 are reasonable for our sample. More high-quality observations of additional targets are the key to determining the wavelengths of these weak transitions.

5.2. Systematic Errors of the Lines in Co-added Spectra

As described in Section 4.2, although both Methods 1 and 2 can produce spectra with high S/Ns, both \( \lambda_1 \) and \( \lambda_2 \) are biased, however. In addition, \( \sigma_{\lambda_1} \) and \( \sigma_{\lambda_2} \) are smaller than their true values, because \( \sigma_{\lambda_{sys}} \) is not considered. In this subsection, we emphasize another important source of \( \sigma_{\lambda_{sys}} \), i.e., \( \sigma_{\lambda_{sys}} \) caused by the uncertainty of the spectral fitting.

The S/Ns of some observations are so low that the fluctuations of the continua will seriously affect the fitting result of the weak lines. We can quantify this uncertainty with simulations. In this simulation, we take the S/N of the simulated spectrum as the only variable to test the dependence of the systematic uncertainty on the S/Ns of the spectra. The S/Ns range between 1 and 10, with a step of 0.5, as shown in Figure 10. For each S/N, we conduct the simulation as follows.

1. The same as step 1 in Section 4.1, except that the line central wavelength is fixed at 23.5 Å.
2. Repeat steps 2–4 in Section 4.1 10,000 times to obtain \( \lambda_{sys} \) and the error on \( \Delta \lambda_{sys} \).

Finally, for each S/N, we obtain \( \Delta \lambda_{sys} \) and its error, as shown in Figure 10. We find that \( \Delta \lambda_{sys} \) increases dramatically as S/N decreases. The relationship between the S/N and \( \Delta \lambda_{sys} \) is similar to the form of a power law. The reason for this anti-correlation is that the error propagation of the Gaussian fitting is nonlinear when the statistical error is large. Therefore, the joint analysis of all the observations can reduce the systematic uncertainty due to the improvement of the S/Ns of the co-added spectra.

### Table 8

Comparison of the Wavelengths of Low-ionized Elements: the Corrected Values in This Paper, Observations from Y09, and Theoretical Calculations

| Ion   | Transition | \( \lambda_{1,c} \) (Å) | \( \lambda_{2,c} \) (Å) | \( \lambda_{w} \) (Å) | Y09  | GSBN02 | G00GM05 |
|-------|------------|-------------------------|-------------------------|----------------------|------|--------|---------|
| O I   | 1s–2p      | 23.5088\(^{+0.8}_{-0.8}\) | 23.5088\(^{+0.8}_{-0.8}\) | 23.5088\(^{+0.8}_{-0.8}\) | 23.5088\(^{+1.6}_{-1.6}\) | 23.4475 | 23.532 |
| O I   | 1s–3p      | 22.8834\(^{+2.4}_{-2.4}\) | 22.8869\(^{+2.4}_{-2.4}\) | 22.8852\(^{+2.4}_{-2.4}\) | 22.886(4)  | ...    | 22.907 |
| Ne II  | 1s–2p     | 23.3504\(^{+0.8}_{-0.8}\) | 23.3503\(^{+0.8}_{-0.8}\) | 23.3505\(^{+1.3}_{-1.1}\) | 23.3484\(^{+2.2}_{-2.0}\) | 23.310 | ...    |
| Mg III | 1s–2p     | 23.0392\(^{+5}_{-5}\) | 23.0562\(^{+7.1}_{-7.0}\) | 23.0477\(^{+2.8}_{-2.0}\) | ...   | 23.0692\(^a\) | ...    |

Notes. \( \lambda_{1,c} \) (i = 1, 2) is the corrected value of the low-ionized elements in Table 4. \( \lambda_{2,c} \) is the average value of \( \lambda_{1,c} \). The O lines are corrected by \( \Delta \lambda_{o} \) of O I \( \lambda_{5577} \) in Table 7, and the Ne and Mg lines are both corrected by that of Ne \( \lambda_{3869} \). \( \sigma_{\lambda_{sys}} \) are asymmetric; the upper and lower 1σ errors are calculated by \( \sigma_{\lambda_{sys}} = \sigma_{\lambda_{o}} + \sigma_{\lambda_{sys}} + \sigma_{\lambda_{sys}} \) (J = u, l).

Values of oxygen in Columns 7 and 8 are from García et al. (2005) and Gorczyca (2000), respectively. Values of neon in Column 7 and 8 are from Behar & Netzer (2002) and Gorczyca & McLaughlin (2005), respectively. All the errors are in units of mA at the 1σ level and “f” in parentheses means “fixed.”

\(^a\) The value obtained from M. Gu (2010, private communication, hereafter Gu10).

### Table 9

Comparison of the Wavelengths of Moderately ionized Elements: Corrected Values in This Paper, Observations from Y09, and Theoretical Calculations

| Ion    | Transition | \( \lambda_{1,c} \) (Å) | \( \lambda_{2,c} \) (Å) | \( \lambda_{w} \) (Å) | Y09  | Gu10  |
|--------|------------|-------------------------|-------------------------|----------------------|------|-------|
| O IV   | 1s–2p      | 22.6975\(^{+3.1}_{-3.2}\) | 22.6964\(^{+3.4}_{-3.4}\) | 22.6969\(^{+3.3}_{-2.5}\) | ...  | 22.7515 |
| O V    | 1s–2p      | 22.2855\(^{+5.8}_{-5.8}\) | 22.2866\(^{+5.3}_{-4.9}\) | 22.2861\(^{+4.0}_{-3.9}\) | 22.2682 | 22.368 |
| O VI   | 1s–2p      | 22.0287\(^{+2.6}_{-2.6}\) | 22.0289\(^{+1.9}_{-2.0}\) | 22.0288\(^{+2.3}_{-2.1}\) | 22.0264\(^{+1.0}_{-1.0}\) | 22.0403 |
| Ne IV  | 1s–2p      | 14.3482\(^{+6.4}_{-6.4}\) | 14.3492\(^{+6.4}_{-6.4}\) | 14.3482\(^{+6.4}_{-4.3}\) | ...  | 14.3710 |
| Ne V   | 1s–2p      | 14.2137\(^{+3.0}_{-3.0}\) | 14.2096\(^{+5.6}_{-1.3}\) | 14.2117\(^{+6.1}_{-2.0}\) | 14.2126 | ...    |
| Ne VII | 1s–2p      | 13.8284\(^{+12.0}_{-12.0}\) | 13.8275\(^{+12.3}_{-12.3}\) | 13.8273\(^{+12.3}_{-12.3}\) | 13.8262 | ...    |
| Mg IV  | 1s–2p      | 9.8550\(^{+0.1}_{-0.1}\) | 9.8550\(^{+0.1}_{-0.1}\) | 9.8550\(^{+0.1}_{-0.1}\) | 9.8570\(^{+0.9}_{-0.9}\) | 9.8786 |
| Mg V   | 1s–2p      | 9.8202\(^{+0.1}_{-0.1}\) | 9.8200\(^{+0.1}_{-0.1}\) | 9.8201\(^{+0.1}_{-0.1}\) | 9.8206\(^{+0.1}_{-0.1}\) | 9.8304 |
| Mg V   | 1s–3p      | 9.0581\(^{+0.1}_{-0.1}\) | 9.0581\(^{+0.1}_{-0.1}\) | 9.0581\(^{+0.1}_{-0.1}\) | 9.0576\(^{+0.0}_{-0.0}\) | 9.0766 |
| Fe XVII| 2p–3d      | 15.0135\(^{+0.1}_{-0.1}\) | 15.0135\(^{+0.1}_{-0.1}\) | 15.0135\(^{+0.1}_{-0.1}\) | 15.0135\(^{+0.1}_{-0.1}\) | 15.0158 |
| Fe XVII| 3p–3d      | 15.2607\(^{+0.3}_{-0.3}\) | 15.2607\(^{+0.3}_{-0.3}\) | 15.2607\(^{+0.3}_{-0.3}\) | 15.2613\(^{+0.4}_{-0.4}\) | 15.2626|

Notes. The same as Table 8, but \( \lambda_{1,c} \) (i = 1, 2) is the corrected wavelength of the moderately ionized elements in Table 4. The O lines are corrected by \( \Delta \lambda_{o} \) of O IV \( \lambda_{5577} \) in Table 7, and the Ne, Mg, and Fe lines are corrected by that of Ne \( \lambda_{3869} \). The theoretical calculations of O, Ne, and Mg are obtained from Gu10.

\(^a\) The value obtained from NIST.
The Astrophysical Journal, 774:116 (18pp), 2013 September 10

LIAO, ZHANG, & YAO

Table 10
Comparison of the Wavelengths of Highly ionized Elements: Corrected Values in This Paper, Observations from Y09, and Theoretical Calculations

| Ion   | Transition | \( \lambda_{1,2} \) (Å) | \( \lambda_{2,3} \) (Å) | \( \lambda_{\text{sys}} \) (Å) | Y09 (Å) | NIST (Å) | V96 (Å) |
|-------|------------|-------------------------|-------------------------|---------------------------|---------|---------|---------|
| O iv  | 1s–2p      | 21.5921^{+0.006}_{-0.005} | 21.5957^{+0.006}_{-0.007} | 21.5959^{+0.006}_{-0.007} | 21.6020 | 21.6020 | 21.6019 |
| O iv  | 1s–3p      | 18.6265^{+0.004}_{-0.003} | 18.6263^{+0.004}_{-0.003} | 18.6264^{+0.004}_{-0.003} | 18.6257^{+0.004}_{-0.003} | 18.6270 | 18.6288 |
| O iv  | 1s–4p      | 17.7663^{+0.001}_{-0.001} | 17.7682^{+0.001}_{-0.001} | 17.7673^{+0.001}_{-0.001} | 17.7658 | ...     | 17.7686 |
| O iv  | 1s–2p      | 18.9673^{+0.003}_{-0.003} | 18.9673^{+0.003}_{-0.003} | 18.9673^{+0.003}_{-0.003} | ...     | 18.9689 | 18.9689 |
| O iv  | 1s–3p      | 16.0052^{+0.003}_{-0.003} | 16.0053^{+0.003}_{-0.003} | 16.0053^{+0.003}_{-0.003} | 16.0063^{+0.003}_{-0.003} | 16.0059 | 16.0059 |
| Ne ix| 1s–2p      | 13.4466^{+0.007}_{-0.007} | 13.4466^{+0.007}_{-0.007} | 13.4455^{+0.007}_{-0.007} | 13.4470 | 13.4470 | 13.4471 |
| Ne ix| 1s–3p      | 11.5470^{+0.009}_{-0.009} | 11.5469^{+0.009}_{-0.009} | 11.5469^{+0.009}_{-0.009} | 11.5494^{+0.009}_{-0.009} | 11.5470 | 11.5466 |
| Ne ix| 1s–4p      | 11.0021^{+0.026}_{-0.026} | 10.0000^{+0.026}_{-0.026} | 11.0010^{+0.026}_{-0.026} | ...     | 11.0128 | ...     |
| Ne x | 1s–2p      | 12.1621^{+0.041}_{-0.041} | 12.1266^{+0.030}_{-0.025} | 12.1264^{+0.026}_{-0.025} | 12.134{f} | ...     | 12.1339 |
| Mg x | 1s–2p      | 9.1690^{+0.010}_{-0.010}  | 9.1685^{+0.010}_{-0.010}  | 9.1687^{+0.010}_{-0.010}  | 9.170^{+0.016}_{-0.016}  | 9.1689 | 9.1688 |

Notes. The same as Table 8, but \( \lambda_{i,j} \) (\( i = 1, 2 \)) is the corrected wavelength of the highly ionized elements in Table 4. The O lines are corrected by \( \Delta \lambda \) of O iv K\( \alpha \) in Table 7, and the Ne and Mg lines are both corrected by that of Ne ix K\( \alpha \).  

\( a \) The value obtained from Gu10.  

* The values of the wavelengths of these lines are the weighted centroids of the doublets in the databases listed in this table.

\[ \text{Figure 10. Dependence of } \Delta \lambda_{\text{sys}} \text{ on the S/Ns of the spectra.} \]

(A color version of this figure is available in the online journal.)

\( \sigma_{\text{sys}} \) can be caused by other uncertainties, such as the imprecise Galactic rotation correction that depends on several uncertain models (e.g., the distribution of the gas inside and above the Galactic plane). Because the parameters of the Galactic rotation model are still uncertain, we cannot provide an accurate value of the systematic uncertainty caused by this uncertainty. Nevertheless, the total systematic uncertainty can be obtained from the distribution of the lines of the 36 observations, as described in Section 4.1. Due to the limitation of the S/Ns, we cannot apply Bayesian analysis to all the lines. However, since the absorption clouds with a similar degree of ionization are assumed to have the same velocities as shown in Table 3, we can use \( \sigma_{\text{sys}} \) of the strongest line to represent the systematic uncertainty of all the lines in the same group, e.g., \( \sigma_{\text{sys}}(\text{Ne } \alpha \text{ K}\alpha) \) represents the systematic uncertainty of the low-ionized Ne lines.

5.3. Detection of the K Transitions in the Soft X-Ray Band

As described above, our results of the wavelengths of the K transitions of highly ionized O, Ne, and Mg are the most accurate so far, because more sources were used to reduce the systematic uncertainty induced by the Galactic rotation correction and more observations were used to reduce the statistical error. Our results are also unbiased, because we do the bias correction for each line. Among all the lines, Ne ix K\( \alpha \) is the most accurate, with an error of only 0.7 mÅ, which is equivalent to about 14 km s\(^{-1}\). This error is so small that it may be used to measure the low-velocity gas, e.g., the hot highly ionized gas in the Galactic halo. For the lines whose wavelengths are consistent between NIST and V96, our results are also consistent with those. For those lines whose wavelengths are inconsistent between NIST and V96, our results are closer to the values given in NIST. Compared to Y09, our results are two times more accurate (e.g., O viii K\( \alpha \)) and are also consistent with theoretical calculations except for the O vii K\( \alpha \) and Ne x K\( \alpha \) lines (Table 10). In our work, \( \lambda_{\text{sys}}(\text{Ne } \alpha \text{ K}\alpha) = 12.1264^{+0.026}_{-0.025} \text{ Å} \) is 7.5 mÅ lower than the value of 12.1339 Å in V96. We note that NIST does not include this line. The O vii K\( \alpha \) line is more complicated and it is also 10 mÅ lower than that in NIST and V96. We will discuss this result in the next subsection.

Unlike the highly ionized elements, there still exist large discrepancies between the astronomical observations and the theoretical calculations of the K transitions of the low-ionized and moderately ionized elements, in the soft X-ray band (Table 1). In fact, some of these lines (e.g., 1s–3p of Mg iii and 1s–2p of Mg iv) have never even been detected. The co-added spectra have excellent S/Ns that provides us an unique opportunity to detect these weak absorption lines. We are particularly concerned about these transitions and thus we searched for the signals of these absorption lines around their theoretical values. For the 11 transitions of the low-ionized elements shown in Table 8, \( S/N_{\text{S/N}} > 7 \) and \( N_{\text{S/N}} < 5 \). Only \( S/N_{\text{S/N}}(\text{Mg } \text{iii K}\alpha) < 3 \), but \( S/N_{\text{S/N}}(\text{Mg } \text{iv K}\alpha) > 3 \). Despite the low S/Ns, it is the first detection of the Mg iii K\( \alpha \) line so far. Several moderately ionized ions, i.e., the O, Ne, and Mg lines shown in Table 9 (except for O vii K\( \alpha \), which has been detected in Y09), also appear for the first time around the theoretical values in the co-added spectra. As shown in Table 9, only \( S/N_{\text{S/N}}(\text{O } \text{vi K}\alpha \text{ and Ne } \text{v K}\alpha) > 5 \). However, among the other lines, \( S/N_{\text{S/N}}(\text{Ne } \text{iv K}\alpha \text{ and Mg } \text{iv K}\alpha) \leq 3 \). Moreover, the Ne iv K\( \alpha \) and Mg iv K\( \alpha \) lines are only present in the co-added spectrum of Method 1, which hints that these three lines
may be artificial and need to be confirmed by higher quality observations in the future.

In Y09, only one of the 2d–3p double lines, i.e., the Fe\textsubscript{XVII} 15.01 Å line, was detected, but no statistical error was given. In our work, both the Fe\textsubscript{XVII} 15.01 Å and the 15.26 Å lines are detected significantly for the first time simultaneously (Figures 3 and 4; Table 9). All the lines with S/N\textsubscript{1} < 5 are shown in Figure 11.

5.4. Wavelength of the O\textsubscript{VII} K\textalpha Lines

By analyzing the co-added spectrum, the wavelengths of all the strong absorption lines can be determined accurately. In our work, several O\textsubscript{VII} lines, including even the very weak O\textsubscript{VII} K\gamma, are found. However, the wavelength of O\textsubscript{VII} K\alpha is still uncertain. The observations of Cyg X–2 and XTE J1817–330 are the most important components of the co-added spectrum, which have enough counts and thus can greatly determine the shape of the co-added spectrum. Y09 jointly analyzed four Chandra-HETG observations of Cyg X–2 and found the O\textsubscript{VII} K\beta line clearly but failed to detect the O\textsubscript{VII} K\alpha line. This problem can be interpreted by unknown emission that fills in the absorption (Cabot et al. 2013). The spectrum of XTE J1817–330 is similar to Cyg X–2 but more complex, which may be caused by not only two absorption lines but also an absorption line plus an emission line. Thus, the co-added spectrum can also be well fit by two different models (Figure 12). For the model of a strong absorption line plus a weak redshifted absorption line, the fitted value of the O\textsubscript{VII} K\alpha line is 21.5915 ± 0.0015 Å. However, for the model of an absorption line plus a weak emission line, the fitted value is 21.6074 ± 0.0015 Å. Both values are very different from the theoretical value of 21.6020 Å given by NIST or 21.6019 Å given by V96. We cannot distinguish which model is correct due to the limitation of the S/N of the spectrum.

6. SUMMARY

So far, the wavelengths of the K\alpha absorption lines of neutral, low-ionized, and moderately ionized O, Ne, and Mg have
not been determined precisely in theoretical calculations (e.g., V96; NIST), laboratory measurements (Stolte et al. 1997), or astronomical observations (J0406; Y09). In order to obtain the wavelengths of the K transitions of O, Ne, and Mg more accurately, we jointly analyzed 36 Chandra-HETG observations of 11 LMXBs at low Galactic latitudes in this work. We corrected the Galactic rotation velocity to the rest frame for every observation and then used two different methods to merge all the corrected spectra into a co-added spectrum. Finally, we fit and obtained the wavelengths of every absorption line in the co-added spectrum obtained by the above two methods (Section 3). Both methods give similar and consistent results for most lines, as shown in Figure 3.

We note that the co-added spectra obtained by the usual methods (Methods 1 and 2) exhibit biases, which are very important for the calibration of the lines. The co-added spectrum can be biased toward observations with high counts (Method 1) or lines with high S/Ns (Method 2). We applied a Bayesian analysis to O i Kα, Ne ii Kα, O viii Kα, and Ne ix Kα to obtain the systematic uncertainty and the bias correction of the O, Ne, and Mg lines in all phases. The final results after the bias correction are summarized as follows.

1. For the neutral, low-ionized, and the highly ionized lines, the accuracy of our result is five and two times higher than that of J0406 and Y09, respectively (Table 4). Several lines that were not detected (i.e., O iii Kα, Ne ii Kγ, Mg iii Kβ, and Ne ix Kγ) or too weak to give measurement errors (i.e., O ii Kβ, O vii Kγ, and Ne x Kα) in Y09 are detected clearly in our work. We also find the moderately ionized lines of these elements (O iv Kα, O v Kα, Ne iv Kα, Ne v Kα, Ne vii Kα, Mg iv Kα, Mg v Kα, and Mg v Kβ; Table 9) whose significances are so low that they need to be confirmed in the future. Besides the remarkable improvement in the accuracy, it is worth mentioning that all the biases of the measurements here have been corrected (Tables 8–10).

2. The systematic uncertainty in our measurement mainly comes from the Galactic rotation correction and the spectral fitting. The former depends on the Galactic model. For the latter, we make simulations to estimate the effect. The total systematic uncertainties are: σsys = 0 mÅ and 0.6 mÅ for low-ionized and the highly ionized O, respectively. σsys = 0.4 mÅ and 0.5 mÅ for low-ionized and the highly ionized Ne, respectively. σsys = 0.7 mÅ for the Mg lines (Table 7).

3. For highly ionized lines of O vii Kβ and O viii Kα, our results are consistent with those of NIST, but 2 mÅ lower than those of V96. For Ne x Kα, our result is about 8 mÅ lower than that given by V96; NIST does not provide the theoretical value. For the moderately ionized lines, the discrepancy between our measurements and the theoretical calculations are generally between 1 and 80 mÅ. Because the statistical errors of these lines are also similar to the discrepancy, the measurements are consistent with the theoretical calculations.

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APPENDIX A

THE RELATIONSHIP BETWEEN THE ABSORPTION COLUMN DENSITY (N_H) AND THE ERROR OF THE WAVELENGTH OF THE ABSORPTION LINE (σ_λ)

For the spectrum with a continuum plus a Gaussian absorption line, the wavelength of the line center (λ) can be determined by line fitting with a Gaussian function

\[ \phi(\lambda) = d e^{-\left(\frac{\lambda - \lambda_0}{\sigma_\lambda^2}\right)^2}, \]  

where \( d, b, \) and \( \lambda \) are the depth, broadening, and center wavelength of the line respectively. As described in Landman et al. (1982) and Lenz & Ayres (1992), \( \sigma_\lambda \) (the error on \( \lambda \)) can be expressed as

\[ \sigma_\lambda^2 \propto \frac{\sigma^2 b}{d^2}, \]  

where \( \sigma \) is the error of the continuum around the line. For an observed spectrum, if the observation time is fixed, \( \sigma^2 \) and \( d \) can be expressed as:

\[ \sigma^2 \propto f \propto e^{-\frac{\tau_c}{\tau}}, \quad \tau_c = \sigma_c N_H, \]  

and

\[ d \propto f \cdot (1 - e^{-\tau_1}) \propto e^{-\tau_1} (1 - e^{-\tau_1}), \quad \tau_1 = \sigma_1 f_1 N_H, \]  

where \( f \) is the flux of the continuum, \( \tau_c \) and \( \tau_1 \) are the optical depths of the continuum and the line center, respectively, \( \sigma_1 \) and \( \sigma_2 \) are the cross sections of the continuum and the line center, respectively, and \( f_1 \) is the abundance of the ion producing the absorption line. Thus, \( \sigma_\lambda^2 \) can be written as

\[ \sigma_\lambda^2 \propto \frac{1}{e^{-\sigma_c N_H} (1 - e^{-\sigma_1 f_1 N_H})^2}. \]  

We use observations of the O viii Kα line in Y09 to show the relationship between \( N_H \) and \( \sigma_\lambda \). Here, \( \sigma_c = 5.9 \times 10^{-22} \text{ cm}^2 \) is obtained from Morrison & McCammon (1983) and \( \sigma_1 f_1 = 1.1 \times 10^{-22} \text{ cm}^2 \) is calculated from the observations in Y09. As shown in Figure 13, the error on O viii Kα can increase significantly for either extremely low or high \( N_H \) values.
APPENDIX B
THE WEIGHTS OF THE ABSORPTION LINES FOR MERGING TO A CO-ADDED SPECTRUM

In Equation (A2), \( d \) can also be described by \( d \propto (a/b) \), where \( a \) is the normalization of the line. Thus, we have:

\[
\sigma_i^2 \propto \frac{\sigma^2 b^3}{a^2}.
\]  

(B1)

When merging two spectra with the same absorption lines with weights \( w_1 \) and \( w_2 \), we have:

\[
\sigma^2 = \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2
\]  

(B2)

\[
a' = a_1 w_1 + a_2 w_2
\]  

(B3)

\[
d' = d_1 w_1 + d_2 w_2 \propto \frac{a_1}{b_1} w_1 + \frac{a_2}{b_2} w_2
\]  

(B4)

\[
b' \propto \frac{a'}{d'} \propto \frac{a_1 w_1 + a_2 w_2}{b_1 w_1 + b_2 w_2},
\]  

(B5)

where the superscript “’” refers to the parameters of the co-added spectra. Thus, \( \sigma_i^2 \) can be expressed as:

\[
\sigma^2 \propto \frac{\sigma^2 b^3}{a^2} \propto \left( \sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 \right) \left( \frac{a_1 w_1 + a_2 w_2}{b_1 w_1 + b_2 w_2} \right)^2.
\]  

(B6)

Defining \( k = (w_1/w_2) \), \( m = (a_1/a_2) \), and \( n = (b_1/b_2) \), we then have:

\[
\sigma^2 \propto \left( \sigma_1^2 k^2 \sigma_1^2 + \sigma_2^2 \right) \left( \frac{m k + 1}{n k + 1} \right)^2.
\]  

(B7)

Usually, the absorption lines to be merged typically have a similar broadening, i.e., \( n \sim 1 \), as exemplified in Figure 14. Therefore, the above equation can be simplified as:

\[
\sigma^2 \propto \left( \sigma_1^2 k^2 + \sigma_2^2 \right) \frac{1}{(n k + 1)^2}.
\]  

(B8)

Therefore, we adopt Equation (B10) as the weight of each line for merging data into a co-added spectrum.

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