Vibrational effects on the formation of quantum W states

H. G. Mendonça and F. M. Souza

Instituto de Física, Universidade Federal de Uberlândia - 38400-902 Uberlândia, MG, Brazil

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Abstract – We theoretically investigate the formation of W states in a tripartite system composed of three charge qubits coupled to vibrational modes. The electromechanical coupling is responsible for second-order virtual processes that result in an effective electron-electron interaction between neighbor qubits, which leads to the formation of W states. Based on the Lang-Firsov transformation and perturbation theory, we analytically solve the quantum dynamics, providing a mathematical expression for the maximally entangled W state. Dephasing is also taken into account, paying particular attention to the robustness of bipartite entanglement against local dephasing processes.

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Introduction. – Entanglement is one of the main features of quantum mechanics that make quantum computers so advantageous compared to classical computers [1]. Entangled states appeared early in the quantum mechanics development in the context of the Einstein, Podolsky and Rosen (EPR) paradox [2]. Since then quantum entanglement became a prominent resource for quantum communication and quantum information processing [3], with potential applications in problems such as prime factoring [4] and quantum simulations [5].

In the context of a solid-state system, superconducting-based quantum devices have received great attention in the last decade, as they constitute one of the leading systems for implementation of quantum computation [6]. For instance, transmon qubits in superconducting chips have been used to generate the Greenberg-Horner-Zeilinger state [7]. The GHZ state [8] was also found in a three-qubit superconducting circuit [9], and both GHZ and W [10] states were reported in superconducting phase qubits [11]. Superconducting qubits have also been applied for quantum networks [12]. Additionally, a recent experiment with 53 qubits is an outstanding example of the recent progress of the superconducting-based quantum computation [13].

Even though superconducting qubits have received a great deal of attention and significant progresses have been achieved, there are many other potential solid-state devices that can be applied to manipulate qubits. For instance, coherent oscillations of single electron spin in GaAs quantum dots were demonstrated [14], and coherent control of coupled electron spins was reported [15]. Additionally, initialization, control and readout of three-electron spin qubits were demonstrated [16]. More recently, silicon-based systems have received growing attention as long electron spin coherence times were found [17–20]. For instance, in the context of silicon-based systems the implementation of CNOT gates and single-qubit operations was recently reported [21]. An efficient resonantly driven CNOT gate for electron spins in a silicon double quantum dot structure was also demonstrated [22]. In addition, the SWAP two-qubit exchange gate between phosphorus donor electron spin qubits in silicon was recently reported [23]. Also, semiconductor quantum dot systems have been proposed as solid-state devices to construct a GHZ state [24]. Recently, it was shown that highly entangled two-qubit states can be achieved due to electron-vibrational mode coupling in molecular systems [25]. Here we extend this previous work by showing that electromechanical coupling can also be a useful tool to generate entangled three-qubit W states in electronic solid-state devices.

The W states consist of a special class of entangled states, being of the form [10]

\[
|W\rangle = a|010\rangle + b|001\rangle + c|100\rangle,
\]

(1)

\(|a|^2 + |b|^2 + |c|^2 = 1\) which when one of the qubits is traced out leaves a partially entangled pair of qubits. This is an example of a two-way entangled state [26], that is robust against losses in one of the qubits [10]. Being of great importance for quantum information processing, such as two-party quantum teleportation [27], and superdense coding, the W state has been both theoretically [28,29] and experimentally investigated in an array of superconducting microwave resonators [30], in spin systems [31,32], in...
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{0}, {1}, {2}, \cdots \}

\hbar \omega_1 \quad \cdots \quad \hbar \omega_2

\begin{align*}
\begin{array}{c}
g_1 \quad g_2 \quad g_3 \\
\ket{0} \quad \ket{1} \quad \ket{2} \quad \cdots \\
A \quad B \quad C \\
\end{array}
\end{align*}

Fig. 1: Illustration of the considered system. Three charge qubits couple to two bosonic environments. No direct coupling between the qubits is considered, so the qubits indirectly couple to each other via vibrational modes of the bosonic subsystems. As a result, the electrons in the qubits experience an attractive interaction, that gives rise to the formation of a highly entangled \(|W\rangle\) state.

superconducting quantum interference devices [33]. Recently, three-photon \(W\) states were demonstrated in optical fibers [34].

In the present work we investigated a tripartite system composed of three charge qubits that interact with vibrational degrees of freedom of a nearby molecular structure, that can be, for instance, carbon nanotubes, as pointed out in ref. [25]. Based on this previous work, here we derive a model Hamiltonian based on the Lang-Firsov transformation, that explicitly shows a charge-vibrational attractive-like interaction (a typical superconductivity-related phenomenon), that is behind the formation of the \(|W\rangle\) state being generated. Based on perturbation theory a simple three-dimensional model can be derived, expressed in the reduced computational base \({\{|000\rangle,|100\rangle,|010\rangle\}}\), that recovers the main physical ingredients behind our full numerical results. Our paper is organized as follows: in the next section we present our theoretical model and analytical results, in the third section we show the quantum dynamics of our system, paying particular attention to the formation of the \(|W\rangle\) state. In the fourth section we account for dephasing processes and finally in the last section we summarize our conclusions.

Theoretical model. – Consider a multipartite system composed of five subspaces, \(\mathcal{H} = \mathcal{H}_{\text{qb1}} \otimes \mathcal{H}_{\text{qb2}} \otimes \mathcal{H}_{\text{qb3}} \otimes \mathcal{H}_v \otimes \mathcal{H}_e\), where the first three subspaces correspond to the qubits A, B and C, while the last two subspaces are associated to the vibrational modes, as illustrated in fig. 1. The qubits Hamiltonian is given by

\begin{equation}
H_{\text{qubits}} = \bigoplus_{n=1}^{3} \left[ \frac{\delta_n}{2} \sigma_z^{(n)} + t_n \sigma_x^{(n)} \right],
\end{equation}

where the Kronecker sum means \(\bigoplus_{n=1}^{3} a_n A^{(n)} = a_1 A \otimes I \otimes I + a_2 I \otimes A \otimes I + a_3 I \otimes I \otimes A\). Also, \(\delta_n\) and \(t_n\) are the detuning between the electronic levels and the intra-qubit charge tunneling, respectively. The vibrational modes degrees of freedom are described according to

\begin{equation}
H_v = \omega_1 B^\dagger B \otimes I_v + \omega_2 I_v \otimes B^\dagger B,
\end{equation}

where \(\omega_i\) is the energy of the \(i\)-th vibrational mode. The operator \(B\) \((B^\dagger)\) annihilates (creates) vibrational excitations, and \(I_v\) is the identity matrix in the vibrational subspace. The electromechanical coupling is given in terms of projection operators as

\begin{equation}
V = \bigoplus_{n=1}^{3} \left[ g_n P_0^{(n)} \right] \otimes [(B^\dagger + B) \otimes I_v]
\end{equation}

\begin{equation}
+ \bigoplus_{n=1}^{3} \left[ g_n P_1^{(n)} \right] \otimes [I_v \otimes (B^\dagger + B)],
\end{equation}

where \(P_i = \ket{i}\bra{i}\) and \(g_n\) is the coupling parameter between the electronic and vibrational degrees of freedom. This means that state \(|0\rangle\) couples to the vibrational mode characterized by \(\omega_1\), while state \(|1\rangle\) couples to the vibrational model with \(\omega_2\) (see footnote 1). This feature is illustrated in fig. 1. As recently shown in ref. [25], to deal with this particular model (which considers the electron-vibrational mode coupling) it is more convenient to use the Lang-Firsov canonical transformation [35]. Defining the operator

\begin{equation}
S = \bigoplus_{n=1}^{3} \left[ \lambda_n P_0^{(n)} \right] \otimes [(B^\dagger - B) \otimes I_v]
\end{equation}

\begin{equation}
+ \bigoplus_{n=1}^{3} \left[ \lambda_n P_1^{(n)} \right] \otimes [I_v \otimes (B^\dagger - B)],
\end{equation}

where \(\lambda_n = g_n/\omega_n\), we can perform the transformation \(\tilde{H} = e^{i\delta} \bar{H} e^{-i\delta}\). After a straightforward calculation we find the following set of equations:

\begin{equation}
\tilde{H}_{\text{qubits}} = \bigoplus_{n=1}^{3} \left[ \frac{\delta_n}{2} \sigma_z^{(n)} - \frac{1}{\omega} (g_1^2 + g_2^2 + g_3^2) \right]
\begin{align*}
&- \frac{g_1 g_2}{\omega} (\sigma_z \otimes I \otimes \sigma_z \otimes I \otimes I) + I^\otimes 3) \\
&- \frac{g_1 g_3}{\omega} (\sigma_z \otimes I \otimes \sigma_z \otimes I \otimes I) + I^\otimes 3) \\
&- \frac{g_2 g_3}{\omega} (I \otimes \sigma_z \otimes \sigma_z \otimes I \otimes I) + I^\otimes 3).
\end{align*}
\end{equation}

1From the electronic point of view, state \(|0\rangle\) means that one electron occupies the up side of the qubit (see fig. 1), thus resulting in the coupling with phononic bath 1. Analogously, state \(|1\rangle\) corresponds to one electron in the lower part of the qubit, so interacting with phononic bath 2. The projection operator \(P_i\) gives the occupation of the upper \((i = 0)\) or the lower \((i = 1)\) part of the qubit. The presented form for \(V\) emphasizes the subspaces and the qubits structure of our model. However, one can describe the electron-vibrational mode interaction in a more standard way using second quantization. This is presented in the Supplemental Material Supplementarymaterial.pdf (SM).
Notice that the last three terms of eq. (6) can be associated to an attractive Coulomb-like interaction. The vibrational modes remain as $\tilde{H}_v = \omega_1 B_1^\dagger B_1 \otimes I_r + \omega_2 B_2 \otimes B_1^\dagger B_2$. Finally, the interaction term takes the form

$$
\tilde{V} = t_1 (\sigma_+ \otimes I \otimes I \otimes D(\lambda_1) \otimes D(-\lambda_1) + h.c.) + t_2 (I \otimes \sigma_+ \otimes I \otimes D(\lambda_2) \otimes D(-\lambda_2) + h.c.) + t_3 (I \otimes I \otimes \sigma_+ \otimes D(\lambda_3) \otimes D(-\lambda_3) + h.c.),
$$

where $\sigma_+ = |1\rangle \langle 1|$ and $D(\pm \lambda_i) = e^{\pm \lambda_i (B_1^\dagger B_2)}$ is the displacement operator [36]. In what follows we assume $t_1 = t$, $\omega_1 = \omega$ and $g_1 = g$, so that $\lambda_1 = \lambda = g/\omega$.

From the transformed model, we can easily find the system eigenenergies in the absence of tunneling, i.e., $\tilde{V} = 0$. In what follows, this coupling term will be considered as a perturbation. The eigenenergies of $\tilde{H}_0 = \hat{H} + \hat{H}_v$ can be divided into four energy groups. The higher and lower energetic ones,

$$
\varepsilon_{000,ml} = \frac{1}{2} (\delta_1 + \delta_2 + \delta_3) - \frac{3g^2}{\omega} + (m + l)\omega,
$$

and

$$
\varepsilon_{111,ml} = -\frac{1}{2} (\delta_1 + \delta_2 + \delta_3) - \frac{3g^2}{\omega} + (m + l)\omega,
$$

respectively. The upper intermediate ones,

$$
\varepsilon_{001,ml} = \frac{1}{2} (\delta_1 + \delta_2 - \delta_3) + \frac{g^2}{\omega} + (m + l)\omega,
$$

$$
\varepsilon_{010,ml} = \frac{1}{2} (\delta_1 - \delta_2 + \delta_3) + \frac{g^2}{\omega} + (m + l)\omega,
$$

$$
\varepsilon_{100,ml} = \frac{1}{2} (-\delta_1 + \delta_2 + \delta_3) + \frac{g^2}{\omega} + (m + l)\omega,
$$

and the lower intermediate ones,

$$
\varepsilon_{011,ml} = \frac{1}{2} (\delta_1 - \delta_2 - \delta_3) + \frac{g^2}{\omega} + (m + l)\omega,
$$

$$
\varepsilon_{101,ml} = \frac{1}{2} (-\delta_1 + \delta_2 - \delta_3) + \frac{g^2}{\omega} + (m + l)\omega,
$$

$$
\varepsilon_{110,ml} = \frac{1}{2} (-\delta_1 - \delta_2 + \delta_3) + \frac{g^2}{\omega} + (m + l)\omega.
$$

A constant energy shift of $-6g^2/\omega$ was omitted in all the energies above. If we set the detunings at $\delta_1 = \delta_2 = \delta_3 = \delta$ we find $\varepsilon_{000,ml} = \frac{3}{2} \delta - \frac{3g^2}{\omega} + (m + l)\omega$,

$$
\varepsilon_{111,ml} = -\frac{3}{2} \delta - \frac{3g^2}{\omega} + (m + l)\omega,
$$

and the degenerate levels $\varepsilon_{001,ml} = \varepsilon_{010,ml} = \varepsilon_{100,ml} = \frac{1}{2} \delta + \frac{g^2}{\omega} + (m + l)\omega$, and $\varepsilon_{011,ml} = \varepsilon_{110,ml} = \varepsilon_{111,ml} = -\frac{1}{2} \delta + \frac{g^2}{\omega} + (m + l)\omega$. In order to have a graphical view of these energy levels we calculate the spectral function of the system. Consider the retarded Green function [37],

$$
G_n^{\tau}(t - t') = -i\eta(t - t') \langle \phi_n | e^{-i\tilde{H}_0(t - t')} | \phi_n \rangle.
$$

Assuming $|\phi_n\rangle$ as the eigenstate of $\tilde{H}_0$, $\tilde{H}_0|\phi_n\rangle = \varepsilon_n |\phi_n\rangle$, we can write

$$
G_n^{\tau}(t - t') = -i\eta(t - t') \delta_{ntn'} e^{-i\varepsilon_n(t - t')}.
$$

Fourier transforming $G_n^{\tau}(t - t')$ we find

$$
G_n^{\tau}(\epsilon) = \frac{\delta_{m'n'}}{\epsilon - \varepsilon_n + i\eta},
$$

where $\eta \to 0^+$. The spectral function, defined as $A_n(\epsilon) = -2\text{Im} [G_n^{\tau}(\epsilon)]$, is then given by

$$
A_n(\epsilon) = \frac{2\eta}{(\epsilon - \varepsilon_n)^2 + \eta^2}.
$$

In fig. 2 we show the total spectral function $A(\epsilon) = \sum_n A_n(\epsilon)$ as a function of $\epsilon$ and $\delta$ for $t_n = 0$. We observe two spaced groups of four branches each. The high energetic ones are basically replicas of the lowest ones, corresponding to vibrational modes $n = 1$ and $m = 0$ or $n = 0$ and $m = 1$. Focusing on the low energetic levels ($n = 0, m = 0$) we have $\varepsilon_{000,00} = \frac{3}{2} \delta - \frac{3g^2}{\omega}$ and $\varepsilon_{111,00} = -\frac{3}{2} \delta - \frac{3g^2}{\omega}$, with high slope $3/2$ (in modulus) as $\delta$ increases. The other two branches are given by $\varepsilon_{001,00} = \varepsilon_{010,00} = \varepsilon_{100,00} = \frac{1}{2} \delta + \frac{g^2}{\omega}$ and $\varepsilon_{011,00} = \varepsilon_{110,00} = -\frac{1}{2} \delta + \frac{g^2}{\omega}$. If $\delta$ is properly tuned such that the levels $\varepsilon_{001,00}, \varepsilon_{010,00}$ and $\varepsilon_{110,00}$ are relatively far from the other branches, the system becomes suitable for the generation of $|W\rangle$ states within subspace $\varepsilon_n = \text{span}\{100, 101, 010, 001\}$. It is important to emphasize that to form $W$ states within this electronic subspace.
the physical parameters should be set along the branches corresponding to the energies $\varepsilon_{00,00} = \varepsilon_{10,00} = \varepsilon_{10,00}$. Without loss of generality, we set our parameters to the values as indicated by the green dot in fig. 2. Other points could also be chosen, as long as they satisfy the condition of being far apart from other branches. With this assumption, it is convenient to divide the system into two subspaces, the relevant one, given by the projector operator $P = (|100\rangle\langle100| + |010\rangle\langle010| + |001\rangle\langle001|) \otimes |00\rangle\langle00|$, and the irrelevant one $Q = I - P$, composed by all the other states in the computational basis. In order to estimate the effective coupling between the states in the $P$ subspace, we apply the second-order perturbation theory. For instance, we can calculate the coupling between states $|100\rangle$ and $|010\rangle$,

$$\Omega_{100,010} = -\sum_{i,j,k} \frac{|100\rangle\langle i|V|j\rangle\langle j|V|010\rangle}{\varepsilon_{ijk,00} - \varepsilon_{100,00}},$$

(20)

where we consider only $n = m = 0$ for the vibrational states, as high energetic levels contributions to the sum are neglected. More specifically we have

$$\Omega_{100,010} = -\frac{|100\rangle\langle000|\langle000|V|010\rangle}{\varepsilon_{000,00} - \varepsilon_{100,00}} - \frac{|100\rangle\langle110|\langle110|V|010\rangle}{\varepsilon_{110,00} - \varepsilon_{100,00}},$$

(21)

which results in

$$\Omega_{100,010} = -\frac{4\alpha^2 r^2 e^{-2\lambda^2}}{\omega \delta (\delta - \frac{4\alpha^2}{\omega^2})},$$

(22)

where the identity $\langle m|D(\lambda)|0\rangle = \lambda^m e^{-\lambda^2/2}/\sqrt{m!}$ was applied [36]. Similar results hold for $\Omega_{100,001}$ and $\Omega_{010,001}$. Therefore, we can write an effective model in the subspace $P$ as $H_{\text{eff}} = \Omega \hbar$, where

$$h = |100\rangle\langle010| + |100\rangle\langle001| + |010\rangle\langle001| + \text{h.c.}$$

(23)

The eigenvalues of $h$ are given by $a' = 2$, $b' = c' = -1$, with corresponding eigenvectors

$$|\bar{0}\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle),$$

$$|\bar{1}\rangle = \frac{1}{\sqrt{6}}(|0\rangle - |1\rangle - 2|2\rangle),$$

$$|\bar{2}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

(24-26)

From here on, we take the shorthand notations $|\bar{0}\rangle = |100\rangle$, $|\bar{1}\rangle = |010\rangle$ and $|\bar{2}\rangle = |001\rangle$. With this simple model we can proceed to the dynamics analysis.

**Physical parameters.** From the experimental point of view, our model can describe carbon nanotube (CNT) quantum dots, as recently proposed in ref. [25] for two qubits entanglement. In the context of CNT the vibrational mode frequency will be assumed around $\omega = 20 \text{meV}$, in agreement with typical values found in CNT [38]. The intra-qubit electron hopping will be considered $t = 0.1 \text{meV}$, in accordance with the coupling parameter found in experimental setup on parallel CNT quantum dots [39]. Also, the electron-phonon coupling parameter $g$ can be experimentally adjusted in CNT [40], here in particular we set $g = 0.1\omega$. Different $g$ values were explored in the context of quantum transport in CNT quantum dots [41,42]. Even though we assume experimentally feasible parameters in the context CNT quantum dots, the theoretical results presented below can in principle be found in different systems, as soon as the system set of parameters matches the conditions discussed in fig. 2.

**Quantum dynamics.** In this section we calculate the evolution of the quantum state, searching for the formation of quantum entanglement between the three qubits. Consider

$$|\psi(\phi)\rangle = e^{-i\phi} |\psi_{0}\rangle,$$

(27)

where $\phi = \Omega t$, with $|\psi_{0}\rangle = |0\rangle$ as initial state. Writing the relevant states $|0\rangle$, $|1\rangle$ and $|2\rangle$ in terms of the $h$ eigenstates, we have

$$|0\rangle = \frac{1}{\sqrt{3}} |\bar{0}\rangle + \frac{1}{\sqrt{6}} |\bar{1}\rangle + \frac{1}{\sqrt{2}} |\bar{2}\rangle,$$

$$|1\rangle = \frac{1}{\sqrt{3}} |\bar{0}\rangle + \frac{1}{\sqrt{6}} |\bar{1}\rangle - \frac{1}{\sqrt{2}} |\bar{2}\rangle,$$

$$|2\rangle = \frac{1}{\sqrt{3}} |\bar{0}\rangle - \frac{2}{\sqrt{6}} |\bar{1}\rangle.$$

(28-30)

Applying the evolution operator on $|\psi_{0}\rangle$ we find

$$|\psi(\phi)\rangle = e^{-i\phi} \frac{1}{\sqrt{3}} |\bar{0}\rangle + \frac{i}{3} e^{i\phi} |\bar{1}\rangle + \frac{e^{i\phi}}{\sqrt{2}} |\bar{2}\rangle,$$

(31)

which in the original computational basis reduces to

$$|\psi(\phi)\rangle = \left[ \cos(\alpha) + i \frac{\sin(\alpha)}{3} |0\rangle - \frac{2i}{3} \sin(\alpha) |1\rangle + |2\rangle \right],$$

(32)

with $\alpha = (3/2)|\phi|$. The maximum entangled state can be found at

$$\sin(\alpha_{\text{max}}) = \pm \sqrt{3}/2,$$

(33)

so that $\alpha_{\text{max}} = \pm \frac{\pi}{6} + \pi n$, or

$$\frac{\alpha_{\text{max}}}{2\pi} = \pm \frac{1}{6} + \frac{n}{2}.$$  

(34)

This provides the following sequence of integer numbers:

$$\beta_{\text{max}} = \frac{6\alpha_{\text{max}}}{2\pi} = 1, 2, 4, 5, 7, 8, 10, 11, 13, \ldots,$$

(35)

that gives the times with maximum entangled states. At this point it becomes instructive to compare our analytical predictions with a numerical calculation derived from
We can also analytically find the exact quantum W state generated at $\beta_{\text{max}}$ values. Using eq. (33) in eq. (32) we obtain

$$|\psi(\alpha_{\text{max}})\rangle = \frac{1}{\sqrt{3}} \left[ |0\rangle + e^{\mp i \frac{\pi}{2}} (|1\rangle + |2\rangle) \right],$$

where global phases are omitted. Defining the target density matrix

$$\sigma_{\text{tar}} = |\psi(\alpha_{\text{max}})\rangle\langle\psi(\alpha_{\text{max}})| \otimes |00\rangle\langle00|,$$

we can calculate the fidelity

$$\mathcal{F} = \text{Tr} \{ \sigma_{\text{tar}} \rho(t) \}. $$

In fig. 3(c) we show $\mathcal{F}$ as a function of $\beta$, for both signs of the relative phase $e^{\mp i \frac{\pi}{2}}$. Notice that $\mathcal{F}$ becomes close to one at the $\beta$ values given by eq. (35), alternating the relative phase sign.

Finally, in fig. 3(d) we show the concurrences $C_{BC}$ (black line) and $C_{AB}$ (red line) against $\beta$. The peaks of these concurrences indicate the formation of bipartite entanglement. In particular, the peaks of $C_{BC}$ appear above 0.8, thus indicating a relative high entanglement degree. Notice that we can minimize the probability amplitude of state $|0\rangle$ in eq. (32), in order to find the corresponding state for the bipartite subsystem $BC$. This amplitude cannot reach zero, though its minimum value can result in partially entanglement between qubits $B$ and $C$. Writing this probability as

$$p(\alpha) = \cos^2(\alpha) + \frac{1}{9} \sin^2(\alpha),$$

and taking the derivative with respect to $\alpha$ we find the condition $\sin(\alpha) \cos(\alpha) = 0$. The minimum is reached for $\cos(\alpha) = 0$, thus $\alpha = \pi/2, 3\pi/2, 5\pi/2, \ldots$. Expressing these values in terms of $\beta$ we find the following sequence:

$$2\beta = 3, 9, 15, \ldots.$$  

**Dephasing.** – In this last section we discuss how dephasing mechanisms taken in one of the three qubits can affect the entanglement of the other two qubits. To do so, we assume that phase flip errors are present in qubit A. As pointed out in ref. [10] the entanglement of the $|W\rangle$ state is more robust against loss mechanisms in one of the qubits, when compared to other states, e.g., the $|\text{GHZ}\rangle$ state. Now we see this effect in action in our modeled device. The Lindblad equation is written in its standard form as [44]

$$\dot{\rho} = -i[H, \rho] + \sum_i \left( L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right),$$

where $H$ is the Hamiltonian of the system.
In contrast, can lead to the formation of highly entangled between charge degrees of freedom and vibrational modes in the presence of dephasing in qubit $\beta$. As a result of dephasing in qubit $A$, the concurrence $C_{AB}$ is suppressed. In contrast, $C_{BC}$ is preserved for longer times. Parameters: $g = 0.1\omega$, $\delta = 0.1\omega$ and $t = 0.005\omega$, $\Gamma = 1 \cdot 10^{-4}\omega$ (0.5 GHz).

where the Lindblad operator is given by $L_i = \sqrt{\Gamma} \sigma_i \otimes I \otimes I$ for $i = A$, with $\Gamma$ being a phase flip error rate. Here we take $\Gamma = 1 \cdot 10^{-4}\omega$, which corresponds to 0.5 GHz for $\omega = 20$ meV. We also assume $L_B = L_C = 0$, i.e., Bob and Claire’s qubits are assumed free of dephasing.

Similarly to fig. 3, in fig. 4 we show the quantities $E_\tau$ and $C_{\text{min}}^2$, the fidelity $F$ and concurrences for the bipartite subsystems $C_{AB}$ (red line) and $C_{BC}$ (black line). The peaks of both $E_\tau$ and $C_{\text{min}}^2$ are significantly suppressed, thus departing from the respective ideal values of 4/3 and 4/9. The fidelity now shows damped oscillations. Interestingly, though, the concurrences $C_{AB}$ and $C_{BC}$ present contrasting features, with $C_{BC}$ remaining finite much longer than $C_{AB}$. This shows that the subsystem composed of qubits $B$ and $C$ preserves for relatively large times some degree of entanglement even when dephasing is present in qubit $A$.

**Conclusion.** -- We investigate how the coupling between charge degrees of freedom and vibrational modes can lead to the formation of highly entangled $|W\rangle$ states. We numerically calculate quantities such as $E_\tau$ and $C_{\text{min}}^2$, that peak on the formation of $|W\rangle$ states. Based on perturbation theory and the Lang-Firsov transformation, we derive analytical expressions that recover our full numerical calculation, thus providing a simple physical picture of the complex dynamics driven by the many-body interaction. In particular, we analytically predict with great accuracy the times in which the $|W\rangle$ state is being formed. Additionally, via concurrence calculation we find partial entanglement between subsystems composed of only two electronic qubits, i.e., tracing out one of the qubits. We show that even in the presence of dephasing mechanisms taken in one of the qubits, a partial entanglement between the other two qubits is preserved for relatively large times, thus revealing the robustness of the present system against local dephasing processes.

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**REFERENCES**

[1] Barnett S. M., *Quantum Information* (Oxford University Press, New York) 2009.
[2] Einstein A., Podolsky B. and Rosen N., *Phys. Rev.*, **47** (1935) 777.
[3] Nielsen M. A. and Chuang I. L., *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge) 2004.
[4] Lucero E., Barends R., Chen Y., Kelly J., Mariantoni M., Megrant A., O’Malley P., Sank D., Vainsencher A., Wenner J., White T., Yin Y., Cleland A. N. and Martinis J. M., *Nat. Phys.*, **8** (2012) 719.
[5] Qi X., Zou J., Qi X. and Li X., *npj Quantum Inf.*, **6** (2020) 87.
[6] Devoret M. H. and Schoelkopf R. J., *Science*, **339** (2013) 1169.
[7] Barends R., Kelly J., Megrant A., Veitia A., Sank D., Jeffrey E., White T. C., Mutus J., Fowler A. G., Campbell B., Chen Y., Chen Z., Chiaro B., Dunsworth A., Neill C., O’Malley P., Roushan P., Vainsencher A., Wenner J., Korotkov A. N., Cleland A. N. and Martinis J. M., *Nature*, **508** (2014) 500.
[8] Greenberger D. M., Horne M. A. and Zeilinger A., in *Bell’s Theorem, Quantum Theory, and Conceptions of the Universe*, edited by Kafatos M. (Kluwer, Dordrecht) 1989, pp. 69-72 (arXiv:0712.0921 (2007)).
[9] DiCarlo L., Reed M. D., Sun L., Johnson B. R., Chow J. M., Gambetta J. M., Frunzio L., Girvin S. M., Devoret M. H. and Schoelkopf R. J., *Nature*, **467** (2010) 574.
[10] Dürr W., Vidal G. and Cirac J. I., *Phys. Rev. A*, **62** (2000) 062314.
[11] Neeley M., Bialczak R. C., Lenander M., Lucero E., Mariantoni M., O’Connell A. D., Sank D., Wang H., Weides M., Wenner J., Yin Y., Yamamoto T., Cleland A. N. and Martinis J. M., *Nature*, **467** (2010) 570.
[12] Yin Z. Q., Yang W. L., Sun L. and Duan L. M., *Phys. Rev. A*, **91** (2015) 012333.
[13] Arute F., Arya K., Babbush R. et al., *Nature*, **574** (2019) 505.
[14] Koppens F. H. L., Buizert C., Tielrooij K. J., Vink I. T., Nowack K. C., Meunier T., Kouwenhoven L. P. and Vandersypen L. M. K., *Nature*, **442** (2006) 766.
Vibrational effects on the formation of quantum $W$ states

[15] PETTA J. R., JOHNSON A. C., TAYLOR J. M., LAIRD E. A., YACOHAY A., LUKIN M. D., MARCUS C. M., HANSON M. P. and GOSSARD A. C., Science, 309 (2005) 2180.

[16] MEDFORD J., BEIL J., TAYLOR J. M., BARTLETT S. D., DOHERTY A. C., RASHBA E. I., DI VINCENZO D. P., LU H., GOSSARD A. C. and MARCUS C. M., Nat. Nanotechnol., 8 (2013) 654.

[17] TYRYSHIN A. M., TOJO S., MORTON J. J. L., RIEHMANN A., ABROSI V. N., BECKER P., POHL H.-J., SCHENKEL T., THEWALT M. L. W., ITOH K. M. and LYON S. A., Nat. Mater., 11 (2012) 143.

[18] MAUNE B. M., BORSELI M. G., HUANG B., LADD T. D., DEELMAN P. W., HOLABIRD K. S., KISELEV A. A., ALVARADO-RODRIGUEZ L., ROSS R. S., SCHMIDT A. E., SOKOLICH M., WATSON C. A., GYURE M. F. and HUNTER A. T., Nature, 481 (2012) 344.

[19] VELDHOE R. M., HWANG J. C. C., YANG C. H., LEENSTRA A. W., DE RONDE B., DEHLA I. P., MUHONEN J. T., HUDSON F. E., ITOH K. M., MORELLO A. and DZURAK A. S., Nat. Nanotechnol., 9 (2014) 981.

[20] KAWAKAMI E., SCARLINO P., WARD D. R., BRAKMAN F. R., SAVAGE D. E., LAGALLY M. G., FRIESEN M., COPPERSMITH S. N., ERIKSSON M. A. and VANDERSYPEN L. M. K., Nat. Nanotechnol., 9 (2014) 666.

[21] VELDHOE R. M., YANG C. H., HWANG J. C. C., HUANG W., DEHLA I. P., MUHONEN J. T., SIMMONS S., LAUGT A., HUDSON F. E., ITOH K. M., MORELLO A. and DZURAK A. S., Nature, 526 (2015) 410.

[22] ZAJAC D. M., SIGILLITO A. J., RUSI M., BORJANS F., TAYLOR J. M., BURKARD G. and PETTA J. R., Science, 359 (2018) 439.

[23] HE Y., GORMAN S. K., KEITH D., KRANZ J. A., KEIZER J. G. and SIMMONS M. Y., Nature, 571 (2019) 371.

[24] NOGUERIA J., OLIVEIRA P. A., SOUZA F. M. and SANZ L., Phys. Rev. A, 103 (2021) 032438.

[25] SOUZA F. M., OLIVEIRA P. A. and SANZ L., Phys. Rev. A, 100 (2019) 042309.

[26] WONG A. and CHRISTENSEN N., Phys. Rev. A, 63 (2001) 044301.

[27] JUNG E., HWANG M.-R., JU Y. H., KIM M.-S., YOO S.-K., KIM H., PARK D. K., SON J.-W., TAMARYAN S. and CHA S.-K., Phys. Rev. A, 78 (2008) 012312.

[28] SHENG Y. B., ZHOU L. and ZHAO S. M., Phys. Rev. A, 85 (2012) 042302.

[29] SHENG Y. B., PAN J., GUO R. and WANG L., Sci. China Phys. Mech. Astron., 58 (2015) 060301.

[30] GANGAT A. A., MCCULLOCH I. P. and MILBURG G. J., Phys. Rev. X, 3 (2013) 031009.

[31] LI C. and SONG Z., Phys. Rev. A, 91 (2015) 062104.

[32] CHEN J., ZHOU H., DUAN C. and PENG X., Phys. Rev. A, 95 (2017) 032340.

[33] KANG Y.-H., CHEN Y.-H., SHI Z.-C., SONG J. and XIA Y., Phys. Rev. A, 94 (2016) 052311.

[34] FANG B., MENOTTI M., LISCIDINI M., SIPE J. E. and LORENZ V. O., Phys. Rev. Lett., 23 (2019) 070508.

[35] MAHAN G. D., Many-Particle Physics, 3rd edition (Plenum, New York) 2000.

[36] SCULLY M. O. and ZUBAIRY M. S., Quantum Optics (Cambridge University Press, Cambridge) 1997.

[37] HAUG H. and JAUH A.-P., Quantum Kinetics in Transport and Optics of Semiconductors, 2nd edition (Springer, Berlin, Heidelberg) 2008.

[38] LEROY B. J., LEMAY S. G., KONG J. and DEKKER C., Nature, 432 (2004) 371.

[39] GOß K., LELINSE M., SMERAT S., WEGEWILS M. R., SCHNEIDER C. M. and MEYER C., Phys. Rev. B, 87 (2013) 035424.

[40] BENIAMIN A., HAMO A., KUSMINSKII S. V., VON OPPEN P. and ILANI S., Nat. Phys., 10 (2014) 151.

[41] WALTER S., TRAUZETTEL B. and SCHMIDT T. L., Phys. Rev. B, 88 (2013) 195425.

[42] SOWA J. K., MOL J. A., BRIGGS G. A. D. and GAUGER E. M., Phys. Rev. B, 95 (2017) 085423.

[43] WOOTTERS W. K., Phys. Rev. Lett., 80 (1998) 2245.

[44] LINDBLAD G., Commun. Math. Phys., 48 (1976) 119.