EDMs — signs of $CP$ violation
from physics beyond the Standard Model

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The limits placed by the non-measurement of atomic and neutron electric dipole moments on $CP$ violating phases beyond the SM are found to be not fully justified since the calculations of the expected EDMs lack the full understanding of the connection between perturbative and nonperturbative regimes of QCD for the measured boundstates. As a consequence rather old subroutines for the evaluation of EDMs are still usable.

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1. Introduction

Electric dipole moments (EDMs) are usually phenomena of extended objects, where the charge distribution can be described by a spherical multipole expansion. Fundamental pointlike particles can be expected to have no dipole moment since they have no internal structure. However, this is only true when one ignores the possibility of quantum corrections, i.e. in the classical limit. Looking closely enough at a particle will show the multitude of possible quantum corrections, which can and usually will induce an EDM to any particle.

At the level of the Lagrangian or Hamiltonian, an EDM violates the invariance of the Lagrangian (Hamiltonian) under parity $P$ and under time-reversal $T$. Assuming $CPT$ invariance this means, that the EDM is a $CP$ violating ($CPV$) quantity. In order to generate $CP$ violation, the Lagrangian has to have complex parameters.

Ignoring the neutrino sector, the perturbative Standard Model (SM) has as the single source of $CPV$ the complex phase of the CKM matrix $V$. The Jarlskog invariant $J = \Im m[V_{ij}V_{k\ell}^*V_{ij}^*V_{k\ell}^*] = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin \delta_{CP}$

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parametrizes the size of CPV from the CKM matrix. As seen in the kaon system, the CPV of the SM is tiny \[ \epsilon \sim 10^{-3} \text{ and } \epsilon' / \epsilon \sim 10^{-3} \].

But the SM also includes QCD which allows for an effective CPV low energy vacuum angle \( \theta \), that is generated as a non perturbative effect:

\[
L_{\theta} := L_{\text{dim=4}} = \frac{g_s^2}{32\pi^2} \theta \varepsilon^{\mu\nu\kappa\lambda} G^a_{\mu\nu} G^a_{\kappa\lambda} .
\]  

Estimating the contribution of this Lagrangian to the EDM of the neutron gives in the most simple model (cf. \[2\])

\[
d_n(\theta) \sim \frac{e_{\theta m_\pi}}{\Lambda_{\text{had}}^2} \sim \theta \cdot (6 \times 10^{-17}) \text{e cm} = 6 \times 10^9 \cdot \theta \cdot d_n^{\text{exp}},
\]  

which implies \( |\theta| < 10^{-9} \). This is the strong CP problem, which has no generally accepted unique solution yet.

\section{Measurements and calculations of EDMs}

EDMs are measured today with higher and higher precision: the EDM of Thallium \[3\], the EDM of the neutron \[4\], and most recently the EDM of mercury \[5\]. All of these measurements are done at very low energies, where QCD is non perturbative. The CPV parameters, on the other hand, are defined at high energies, where QCD is perturbative. In order to use the measurements to the fullest, one has to create a map between the low energy measurements and the high energy parameters. A map of this kind will necessarily involve the discussion of the different energy scales, as started by Khriplovich et al. 25 years ago. The recent treatment of Pospelov and Ritz (PR) \[2\] discusses in particular, how the effective parameters at the low scale should be calculated in terms of the CPV parameters at the high scale: the calculational method, connecting the parameters at the different scales as depicted in Fig \[4\] should

- possess chiral invariance, including relevant anomalous Ward identities,
- account for possible solutions of the strong CP problem,
- use the same method to calculate all low energy parameters,
- estimate the precision of the relevant QCD matrix elements,
- depend only on scale invariant combinations of parameters,
- give quantitative predictions for the dependence on the different high energy parameters.
Unfortunately, such a method does not yet exist. Lattice QCD seems to be a good candidate, but does not give definite results yet. In lack of such results, PR discuss four approaches to the calculations: the non-relativistic $SU(6)$ quark model, naive dimensional analysis, chiral techniques, and QCD sum-rules techniques.

2.1. Developments after PR

1. As predicted by PR, the experimental measurements of the EDMs of neutron [4] and of mercury [5] improved significantly but without measuring an EDM. So the constraints on the parameters are still limited by the understanding of the calculated EDMs.

2. In 2007 Liu et al. [6] analysed atomic EDMs and reformulated Schiff’s theorem [7] at the quantum mechanical operator level. Liu et al. note, that their obtained operator reduces to the usual Schiff moment, that is used in the literature to obtain the estimates of the atomic EDMs, only under certain additional assumptions. The authors comment also, that these assumptions are not generally justified.

That the issue is not settled yet can be seen from the ongoing discussion between [8, 9, 10] and [6, 11].

3. Already in 2005 Abel and Lebedev [12] studied correlations between
the neutron and electron EDMs in the context of supersymmetry and remark, that the EDM of Thallium, which is usually used to restrict the electron EDM, is dominated by the $\theta$-background of the SM, if $\theta > 10^{-15}$. But the experimental restriction is only $\theta < 10^{-10}$.

The reported change of $d_{\text{Schiff}}^{(199\text{Hg})}$ turns out to be an unresolved typo between the available version of [13] in the arXiv and the published version, which was not available at the time of the talk.

4. [14, 15] calculate the CPV of the effective action of the SM and find, that it is much bigger than the Jarlskog invariant. This indicates, that CPV should be treated as a non-perturbative effect, which in some way questions the perturbative relations between the parameters at the high and the low scale.

5. This year An, Ji, and Xu [16] give the first systematic treatment of the contribution of the four quark operators to $d_n$ and compare chiral perturbation theory with the results of QCD sum-rules. When evaluated with these different descriptions some four quark operators give similar, other four quark operators give very different contributions to $d_n$. That result shows a limit of our understanding, how we should describe non-perturbative QCD.

6. Already in 2002 Asaga and Fujita [17] noted, that the first order contribution of the quarks to $d_n$ can be shuffled between the EDM and the chromo-EDM of the quarks and thus provide an additional suppression of $d_n$.

From the arguments [4] and [6] I draw the conclusion, that the perturbative calculations that connect the high scale parameters $C_{ff}$, $d_f$, and $\tilde{d}_q$ with the lower scale parameters $C_{S,P,T}^{(i)}$, $d_n$, and $\tilde{g}_{SNN}^{(i)}$ are not capturing the whole effect, as they cannot include the non-perturbative nature of QCD.

2.2. What we really know

Though expecting to see EDMs, all measurements up to now could not detect any permanent electric dipole moment. These null measurements of atomic EDMs can be explained by the Schiff’s screening [7]. But the null measurement of the neutron EDM still lacks a convincing explanation, which constitutes the strong $CP$ problem.

$CPV$ from the CKM matrix is strongly suppressed by the small mixing angles and the expected EDMs generated by the phase of the CKM matrix are several orders of magnitude smaller than the precision of the null measurements. This means that any detection of an EDM points at a source beyond the SM.
On the other hand, the null measurements of the EDMs do not restrict
the possible $CPV$ phases in the same way, as the coefficients with which
these phases contribute to the EDMs are not very well known. One obstacle
to the exact calculation of these coefficients is that QCD is non perturbative
in the energy range where the EDMs are measured.

The only EDMs that would not suffer from the QCD uncertainties are
the EDMs of the leptons. A direct measurement of a leptonic EDM comes
as a by-product from the measurement of the anomalous magnetic moment
of the muon [18] and gives a limit of $d_\mu = (3.7 \pm 3.4) \times 10^{-19}$ e.cm. There
are ongoing efforts to improve this measurement directly: [19, 20].

**2.3. What we should improve**

If we want to use the improving measurements of atomic EDMs we will
have to use the same low energy Lagrangians and the same corresponding
Hamiltonians, most probably with the stable particles, i.e. electron, proton,
and neutron, including their EDMs, as the only available degrees of freedom.
Since there is quite some freedom of choosing a suitable Hamiltonian this
step might take the community a while to figure out.

The next missing piece is the description of the atomic bound state, in-
cluding the tiny admixtures of the $P$ and $T$ violating interactions, that are
necessary to describe a measured EDM. For the electron-nucleus part the
consensus seems to be growing, but for the nucleon-nucleon part the full
ab-initio nuclear boundstate calculations are not yet at the point where the
needed heavy nuclei can be calculated reliably. This makes the proposed
measurement of deuterium [21, 22, 23] a very good alternative to the sole
reliance on leptonic EDMs.

One step further is the understanding of the nucleons as boundstates
of quarks and gluons, which are the degrees of freedom at the high scale.
There are several models that describe the binding of the quarks into the
nucleons [24, 25, 26], but there is no final agreement [27] about the calculated
properties of the nucleon: the different models and the measurements do not
agree about all properties. This basically means, that we do not yet fully
understand the connection between perturbative QCD, where the high scale
parameters are defined, and nonperturbative QCD, where the boundstates
are formed.

One idea might be explored while trying to estimate the effect of EDMs
of the constituents of the bound states: the nucleon as a bound state of
quarks and gluons and the nucleus as a boundstate of nucleons. The argu-
ment of the Schiff’s screening of EDMs for the atomic boundstates operates
with the groundstate energy and a displacement operator that summarises
the EDMs. When the constituents are seen as points, as the quarks in the
nucleons are assumed to be, then the charge and the dipole densities are the same and the only violation of the screening can come from the relativistic motion of the constituents. In this way one could formulate a screening argument similar to [7] for the nucleons and in the following maybe also for the nucleus itself.

A formulation of such a screening and its evaluation with the agreed upon model of the nucleons might also solve the strong CP problem, as it gives a symmetry argument for cancellations and hence suppression of the estimated EDM of the neutron: the different contributions to the neutron EDM have to cancel because of the symmetry that describes the screening.

3. Summary and Conclusion

I failed in the attempt to find significantly improved constraints on the CPV phases of the MSSM in the recent literature. Instead I found, that the bounds that were published as restrictions should be understood as the possible reach of experiments that measure atomic or neutron EDMs for the detection of CPV phases.

Null measurements of the EDMs do not restrict the underlying CPV phases in a strict sense, though some phases are more likely small, as they would give EDMs that are bigger than the excluded ones, assuming that there are no cancellations. But these cancellations can be explained by an argument similar to Schiff’s screening.

In absence of needed adjustments the subroutines written around 2004 are still usable to estimate $d_e$, $d_n$, and $d_{\text{Hg}}$ in the MSSM. The calculation includes the Weinberg dim=6 operator and the 2-loop Barr-Zee type Higgs contributions. $d_n$ is calculated with two different models for the neutron. These subroutines work as an addition to the FeynArts/FormCalc model and driver files for the MSSM. They can be downloaded from http://terra.ar.fi.lt/~garfield/EDMs/.

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