An Estimate of Charge Symmetry Breaking in Nuclear Deep Inelastic Scattering

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Abstract

We estimate the magnitude of charge symmetry breaking effects in deep inelastic scattering from nuclei. The resulting contribution to systematic uncertainties in hadronic determinations of $\sin^2 \theta_W$ are found to be less than the current experimental accuracy, but may be important in the analyses of more precise future experiments. We expect the largest nuclear charge symmetry breaking effects in the Paschos-Wolfenstein ratio $R^-$, where the resulting uncertainty in the determination of $\sin^2 \theta_W$ reaches $10^{-3}$.

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I. INTRODUCTION

With the systematic improvement in hadronic determinations of $x_w \equiv \sin^2 \theta_W$ from deep inelastic scattering (DIS) experiments [1], it becomes increasingly important to get also the theoretical systematic uncertainties under control. While many theoretical and experimental uncertainties cancel when one considers the Paschos-Wolfenstein ratios [2],

$$ R_{\pm}^{\nu} = \frac{d\sigma_{\nu NC}/dy \pm d\sigma_{\nu CC}/dy}{d\sigma_{\nu CC}/dy \pm d\sigma_{\nu CC}/dy} \quad (1.1) $$

and

$$ R_{\nu}^{\nu} = \frac{d\sigma_{\nu NC}/dy}{d\sigma_{\nu CC}/dy}, \quad R_{\nu}^{\bar{\nu}} = \frac{d\sigma_{\bar{\nu} NC}/dy}{d\sigma_{\bar{\nu} CC}/dy} \quad (1.2) $$

there are still several important corrections to these ratios that arise, for example, from the strange and charm sea, radiative corrections [4] and charge symmetry breaking (CSB) effects in the nucleon [5]. While CSB effects for parton distributions in the nucleon have been considered [5,6], possible CSB effects arising from the Coulomb contribution to the nuclear binding energy have so far been neglected. In this Letter, we make a simple estimate of the size of such nuclear charge symmetry breaking effects on the Paschos-Wolfenstein ratios and concomitantly their effect on the determination of $x_w$ from these ratios.

II. THE MODEL

Let us first recall some formulae relevant for the determination of $x_w$ from neutrino-nucleus deep inelastic scattering. Given the momentum fractions $\langle q \rangle$ carried by the various flavors of quarks in the target, one finds for the inclusive cross sections [3]

$$ \frac{d\sigma_{\nu CC}}{dy} = \frac{G_F^2 s}{\pi} \frac{M_W^4}{(M_W^2 + Q^2)^2} \left\{ \left( 1 - y + \frac{y^2}{2} \right) \langle d + s + \bar{u} + \bar{c} \rangle + y \left( 1 - \frac{y}{2} \right) \langle d + s - \bar{u} - \bar{c} \rangle \right\} $$

$$ \frac{d\sigma_{\nu CC}}{dy} = \frac{G_F^2 s}{\pi} \frac{M_W^4}{(M_W^2 + Q^2)^2} \left\{ \left( 1 - y + \frac{y^2}{2} \right) \langle u + c + \bar{d} + \bar{s} \rangle - y \left( 1 - \frac{y}{2} \right) \langle u + c - \bar{d} - \bar{s} \rangle \right\} $$
\[
\frac{d\sigma^{\nu,\bar{\nu}}_{NC}}{dy} = \frac{G_F^2 s}{\pi} \left( \frac{M_Z^2}{M_Z^2 + Q^2} \right)^2 \left\{ \left( 1 - y + \frac{y^2}{2} \right) \left[ \alpha_{1+} \langle u + c + \bar{u} + \bar{c} \rangle + \alpha_{1-} \langle d + s + \bar{d} + \bar{s} \rangle \right] \right. \\
\left. \pm y \left( 1 - \frac{y}{2} \right) \left[ \alpha_{2+} \langle u + c - \bar{u} - \bar{c} \rangle + \alpha_{2-} \langle d + s - \bar{d} - \bar{s} \rangle \right] \right\}, \quad (2.1)
\]

where \( y \equiv [(E_{\text{initial}} - E_{\text{final}})/E_{\text{initial}}]_{\text{lab}} \) and the subscripts ‘CC’ and ‘NC’ refer to the charged and neutral currents, respectively. In the last of these equations, the ‘+’ refers to \( \nu \) and the ‘-’ to \( \bar{\nu} \), and

\[
\alpha_{1+} = \frac{1}{4} - \frac{2}{3} x_w + \frac{8}{9} x_w^2, \quad \alpha_{2+} = \frac{1}{4} - \frac{2}{3} x_w \\
\alpha_{1-} = \frac{1}{4} - \frac{1}{3} x_w + \frac{2}{9} x_w^2, \quad \alpha_{2-} = \frac{1}{4} - \frac{1}{3} x_w. \quad (2.2)
\]

Assuming \( \langle s \rangle = \langle \bar{s} \rangle \) and \( \langle c \rangle = \langle \bar{c} \rangle \), the Paschos-Wolfenstein ratios may be written as\[1\]

\[
R^- = \frac{f_2(y) \left[ (\alpha_{2+} + \alpha_{2-}) \langle Q_+ \rangle - (\alpha_{2+} - \alpha_{2-}) \langle Q_- \rangle \right]}{f_2(y) \langle Q_+ \rangle + f_1(y) \langle Q_- \rangle} \\
R^+ = \frac{f_1(y) \left[ (\alpha_{1+} + \alpha_{1-}) \langle Q_+ + 2Q_1 \rangle - (\alpha_{1+} - \alpha_{1-}) \langle Q_- + Q_2 \rangle \right]}{f_1(y) \langle Q_+ + 2Q_1 \rangle + f_2(y) \langle Q_- + 2Q_2 \rangle} \\
R^{\nu,\bar{\nu}} = \left\{ f_1(y) \left[ \alpha_{1+} \langle Q_+ - Q_- + 4Q_{uc} \rangle + \alpha_{1-} \langle Q_+ + Q_- + 4Q_{ds} \rangle \right] \pm f_2(y) \left[ \alpha_{2+} \langle Q_+ - Q_- \rangle + \alpha_{2-} \langle Q_+ + Q_- \rangle \right] \right\} \\
\times \left\{ f_1(y) \langle Q_+ \pm Q_- + 2Q_1 \rangle \pm \langle Q_+ \pm Q_- \pm 2Q_2 \rangle \right\}^{-1}, \quad (2.3)
\]

where \( \langle Q_\pm \rangle = \langle d_v \pm u_v \rangle, \langle Q_{uc} \rangle = \langle \bar{u} + \bar{c} \rangle, \langle Q_{ds} \rangle = \langle \bar{d} + \bar{s} \rangle, \langle Q_1 \rangle = \langle \bar{d} + \bar{u} + \bar{s} + \bar{c} \rangle \) and \( \langle Q_2 \rangle = \langle \bar{d} + \bar{s} - \bar{u} - \bar{c} \rangle \). Also, \( f_1(y) = 1 - y + y^2/2 \) and \( f_2(y) = y - y^2/2 \). In the last of the equations, ‘+’ refers to \( \nu \) and ‘-’ to \( \bar{\nu} \).

As the above ratios, Eq. (2.3), are dominated by the valence quark contributions, \( \langle Q_+ \rangle \), the scale of CSB effects in an isoscalar target is set by the ratio \( \langle Q_- \rangle / \langle Q_+ \rangle \). Evidently, for an isoscalar target \( \langle Q_- \rangle \) vanishes in the absence of CSB effects. In order to obtain a simple and transparent estimate for this ratio, and hence nuclear CSB effects in DIS, we employ the convolution approach of Dunne and Thomas [8], where the momentum fractions of a

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\[1\] We consider kinematics where \( Q^2 << M_W^2 \).
quark in the nucleus, \langle q \rangle, are

\begin{equation}
A\langle q \rangle = \sum_i \left\{ n_{ip} \frac{M_{ip}}{M_p} \langle q \rangle_p + n_{in} \frac{M_{in}}{M_n} \langle q \rangle_n \right\},
\end{equation}

where \( A \) is the nuclear mass number, \( M_{p(n)} \) is the proton (neutron) mass, \( n_{ip(n)} \) is the proton (neutron) occupation number for the i-th shell, \( M_{ip(n)} = M_{p(n)} - \varepsilon_{ip(n)} \) and \( \varepsilon_{ip(n)} \) is the proton (neutron) separation energy for the i-th shell. There are some difficulties with the overall momentum sum rule in the convolution approach, so one should not consider Eq. (2.4) an exact formula. However, for the purpose of this work, namely obtaining a rough estimate of the size of nuclear charge symmetry breaking in the Paschos-Wolfenstein ratios, such subtleties can be ignored.

In this approach, we thus obtain

\begin{equation}
\langle Q \pm \rangle = \sum_i n_{ip} \frac{M_{ip}}{M_p} \langle u_v \pm dv \rangle_p + n_{in} \frac{M_{in}}{M_n} \langle u_v \pm dv \rangle_n.
\end{equation}

As we are interested in nuclear CSB effects, we assume charge symmetry for the nucleon and obtain

\begin{equation}
\frac{\langle Q_- \rangle}{\langle Q_+ \rangle} = \frac{\langle u_v - dv \rangle_p \left\{ \sum_i n_{in} M_{in} - n_{ip} M_{ip} \right\}}{\langle u_v + dv \rangle_p \left\{ \sum_i n_{in} M_{in} + n_{ip} M_{ip} \right\}}.
\end{equation}

**III. NUMERICAL RESULTS**

Since the \( y \)-dependence in Eqs. (2.3) is not very large except for \( R^- \) at very small \( y \), we consider from now on the ratios of the \( y \)-integrated cross sections in order to get some overall quantitative understanding about the relevance of CSB in these ratios. Since the CSB contribution to the Paschos-Wolfenstein ratios, Eqs. (1.1), (1.2), are rather small, it is not necessary to include other corrections (charm, etc.) when calculating the CSB corrections. To estimate \( M_{ip} \) and \( M_{in} \), we take the separation energies given in Ref. [8] and estimate

\[2\text{As we are ultimately interested in ratios, the “rescaling” effects may be neglected.}\]
the Coulomb correction, $V_c$, by considering the difference in masses between the nucleus of interest the nucleus with one proton or neutron removed. For example, to estimate $V_c$ for $^4\text{He}$, we compare its mass with the masses of $^3\text{He}$ and $^3\text{H}$. For heavier nuclei such as $^{40}\text{Ca}$, this comparison actually gives the difference in separation energies for the outer most shell. However, detailed calculations of the separation energies including the Coulomb interaction show that the difference in separation energies is largely independent of the shell. Given $V_c$, we then take $M_{ip} = M_{iN} + V_c/2$ and $M_{in} = M_{iN} - V_c/2$, where the $M_{iN}$ are given in Ref. [3]. Our results for $V_c$ and $\langle Q^- \rangle / \langle Q^+ \rangle$ are given in Table I, where we have taken $\langle d_v \rangle / \langle u_v \rangle = 0.4434$ [9].

The scale of the results in Table I can easily be understood by noting the the sum in the denominator of Eq. (2.6) is nearly equal to $AM$ and the sum in the numerator is $ZV_c$. Taking, for simplicity, the SU(6) result of $\langle d_v \rangle / \langle u_v \rangle = 1/2$, one obtains

$$\frac{\langle Q^- \rangle}{\langle Q^+ \rangle} \approx \frac{V_c}{6M}.$$  

As an additional check on these results, the sum in the numerator in Eq. (2.6) is a sum over the Coulomb energies of the individual protons, and is thus equal to twice the Coulomb energy of the nucleus, $E_c$. Assuming a uniform charge density and a nuclear radius that scales like $R = 1.2\text{fm} A^{1/2}$, we find for isoscalar nuclei

$$\frac{\langle Q^- \rangle}{\langle Q^+ \rangle} \approx -\frac{\langle u_v - d_v \rangle_p}{\langle u_v + d_v \rangle_p} \frac{0.57\text{MeV}}{M} Z^{2/3}.$$  

The numbers obtained with this latter formula are in good agreement with those in Table I, with the exception of $^4\text{He}$.

Given the numbers in Table I, it is now easy to determine the nuclear CSB effects on the ratios. In particular, we are interested in how much the ratios change when the CSB effects are included. Thus, we show in Table II the results for $\delta R = R_c - R_0$ where $R_c$ is the ratio including CSB effects. We also show the results for $\delta w = x_{w'} - x_w$ [4], where $x_{w'}$ is determined by demanding that $R$ does not change when CSB effects are included, i.e. $R_0(x_{w'}) = R_c(x_{w'})$. For calculations, we use $x_w = 0.23$, and thus have $R_0^- = 0.2700$, $R_0^+ = 0.3288$, $R_0^\nu = 0.3092$, and $R_0^\bar{\nu} = 0.3876$. 

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Evidently, of the Paschos-Wolfenstein ratios it is $R^-$ that is most influenced by CSB effects. For the other ratios, the CSB effect is roughly 1/2 the size of the $\langle c \rangle$ contribution\footnote{Based on Owens’\cite{Owens} parameterization at $Q^2 = 10 \text{ GeV}^2$.} for $^{28}\text{Si}$ and $^{40}\text{Ca}$ and about 1/6 the size of the $\langle c \rangle$ contribution for $^4\text{He}$ and $^{12}\text{C}$. For $\delta_w$, it is $\bar{\delta}_w^c$ that is most affected by CSB effects, but this contribution is at most 1/3 of the $\langle c \rangle$ contribution. Compared to the $\langle c \rangle$ contribution, the CSB effect in $\delta_w^c$ is quite important, being about 2/3 the size of it for $^{40}\text{Ca}$.

While $R^-$ is certainly the best ratio to look for CSB effects, particularly at small $y$, we remind the reader that we have assumed $\langle s \rangle = \langle \bar{s} \rangle$. If this assumption is relaxed, there will be contributions to $R^-$ of the form $\langle s - \bar{s} \rangle$. \footnote{For a discussion of the $s - \bar{s}$ asymmetry in nucleons, see e.g. Ref.\cite{Ref} and references therein.} Although this is certainly small, its size needs to be known before one can get information about $\langle Q_- \rangle$ from $R^-$. $R^\nu$ receives a similarly large CSB correction, but $R^\nu$ is more strongly affected by the strange and charm corrections, relative to which nuclear CSB plays only a minor role.

\section*{IV. SUMMARY}

We have investigated the contribution of nuclear binding effects to charge symmetry breaking for parton momentum fractions in isoscalar targets. Due to the Coulomb repulsion, protons are less strongly bound in nuclei and thus carry a larger fraction of the nucleus’ momentum. Up to a factor of 3 suppression, which results from a partial cancelation between quarks of the same flavor in protons and neutrons, this translates directly into $u$ quarks carrying more momentum than $d$ quarks due to the Coulomb energy in nuclei.

Naively, one would expect nuclear charge symmetry breaking effects to be of the order of the Coulomb energy of the nucleus divided by its mass, i.e. an effect of the order of 1\% for larger nuclei. Actually, the average effect per nucleon is only 0.5\%, since only protons, but not neutrons, are affected. However, the 0.5\% charge symmetry breaking in the distribution
of nucleons in nuclei gets “diluted” roughly by abovementioned factor of three when one considers quarks. As a result, the net charge symmetry breaking at the quark level is only about 0.1-0.2%, i.e. almost an order of magnitude smaller than the most naive estimate.

Of course, our model is very naive and one should thus not view our result as a prediction, but rather as an estimate for the systematic uncertainties that arise due to nuclear charge symmetry breaking effects. Depending on the target and on which of the Paschos-Wolfenstein ratios one uses for the determination of $\sin^2 \theta_W$, the resulting systematic error due to charge symmetry breaking nuclear effects is thus of the order of $0.1 - 0.5\%$, i.e. slightly smaller than the expected effects due to charm quarks in the target. As a result, nuclear isospin symmetry breaking effects are still too small to be detected, but this may soon change as more precise experimental data become available.

We predict the largest nuclear CSB corrections in the ratio $R^-$. Incidentally, most other corrections to the naive Paschos-Wolfenstein result cancel in this particular ratio. Thus for $R^-$, nuclear CSB might be, besides CSB in the nucleon and a possible charge conjugation asymmetry in the strange sea [10], the largest nonperturbative correction to the naive Paschos-Wolfenstein ratio.

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TABLE I. Our estimate of the average difference between proton and neutron separation energies due to the Coulomb interaction, $V_c$, and the ratio $\langle Q_- \rangle / \langle Q_+ \rangle$ for various N=Z nuclei.

| Nucl. | $V_c$ (MeV) | $\langle Q_- \rangle / \langle Q_+ \rangle$ |
|-------|-------------|----------------------------------|
| $^4$He | 0.77        | -0.0007661                       |
| $^{12}$C | 2.77        | -0.0008523                       |
| $^{28}$Si | 5.60        | -0.0014512                       |
| $^{40}$Ca | 7.31        | -0.0018153                       |

TABLE II. The results for $\delta R$ and $\delta w$ (see text) due to nuclear CSB effects for various N=Z nuclei.

| Nucl. | $\delta^- R$ | $\delta^- w$ | $\delta^+ R$ | $\delta^+ w$ | $\delta^\nu R$ | $\delta^\nu w$ | $\delta^R \bar{w}$ | $\delta^\nu \bar{w}$ |
|-------|---------------|---------------|---------------|---------------|----------------|----------------|------------------|--------------------|
| $^4$He | 0.0002        | 0.0002        | 0.0001        | 0.0001        | 0.0000        | 0.0001        | -0.0001         | -0.0006           |
| $^{12}$C | 0.0004       | 0.0004       | 0.0001        | 0.0002        | 0.0001        | 0.0003        | -0.0001         | -0.0011           |
| $^{28}$Si | 0.0007       | 0.0007       | 0.0002        | 0.0003        | 0.0003        | 0.0005        | -0.0003         | -0.0018           |
| $^{40}$Ca | 0.0008       | 0.0008       | 0.0002        | 0.0003        | 0.0003        | 0.0006        | -0.0003         | -0.0023           |