Picard-Fuchs Equations and Prepotential in $N = 2$ Supersymmetric $G_2$ Yang-Mills Theory

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Abstract

We study the low-energy effective theory of $N = 2$ supersymmetric Yang-Mills theory with the exceptional gauge group $G_2$. We obtain the Picard-Fuchs equations for the $G_2$ spectral curve and compute multi-instanton contribution to the prepotential. We find that the spectral curve is consistent with the microscopic supersymmetric instanton calculus. It is also shown that $G_2$ hyperelliptic curve does not reproduce the microscopic result.
Seiberg and Witten found the exact solution for the low energy effective theory of $N = 2$ supersymmetric $SU(2)$ Yang-Mills theory. Their $SU(2)$ solution has been generalized to other gauge groups. The low-energy effective theory in the Coulomb branch of $N = 2$ supersymmetric Yang-Mills theory with gauge group $G$ of rank $r$ is described by $r N = 1 U(1)$ vector multiplets $W_i = (\lambda_i^\alpha, v_m^i)$ and $r N = 1$ hypermultiplets $A^i = (a^i, \psi^n_a) \ (i = 1, \ldots, r)$. The low energy effective action is determined by a single holomorphic function $F(a)$, called the prepotential. In the semi-classical region, the prepotential $F(a)$ is expressed as

$$F(a) = \frac{i}{4\pi} \sum_{\alpha \in \Delta_+} (\alpha, a)^2 \log \frac{(\alpha, a)^2}{\Lambda^2} + \frac{1}{2} \tau_0 \sum_{i=1}^{r} a^i a^i + \sum_{n=1}^{\infty} F_n(a) \Lambda^2 \nu_n.$$

where $\Delta_+$ denotes the set of positive roots of the Lie algebra of $G$, $\nu$ the dual Coxeter number and $\Lambda$ the dynamically generated mass scale. The coefficient $F_n(a)$ comes from the $n$-instanton contribution.

In the exact solution, the Higgs fields $a^i$ and their duals $a_{Di} = \partial F(a)/\partial a^i$ are represented by the contour integral of the meromorphic one-form (the Seiberg-Witten differential) $\lambda_{SW}$ on certain algebraic curve:

$$a^i = \int_{A_i} \lambda_{SW}, \quad a_{Di} = \int_{B_i} \lambda_{SW}, \quad (2)$$

where $A_i$ and $B_i$ are 1-cycle on the algebraic curve. For classical gauge groups, it is known that the hyperelliptic curve provides the exact solution which satisfies several consistency conditions. This type of curves has been generalized to exceptional type gauge groups by embedding them to certain classical gauge groups.

Generalizing the work by Gorskii et. al., Martinec and Warner constructed the algebraic curves for any simple gauge group from the spectral curve of the periodic Toda lattice associated with the dual affine Lie algebra. For classical gauge groups, the spectral curve is shown to agree with the hyperelliptic type. For exceptional type gauge groups, however, the spectral curve has different from the hyperelliptic curves and shows different strong coupling physics. Since the singularity structure in the strong coupling region determines instanton terms in the prepotential by analytic continuation, the calculation of $n$-instanton contributions provides a non-trivial test to the exact solutions.
Recently, Sasakura and the present author [8] calculated the one-instanton effect $F_1(a)$ for any simple Lie group by using the microscopic supersymmetric instanton calculus [9]. These effects have been shown to agree with the exact solutions in the case of classical gauge groups [10, 11].

The purpose of the present paper is to study the exact solution for exceptional type gauge groups. We will consider the $G_2$ type Lie group as the simplest example. We study the exact solution by investigating the Picard-Fuchs equations that the period $\oint \lambda_{SW}$ obeys. The Picard-Fuchs equation has been extensively studied for classical gauge groups [14]-[17]. By solving these differential equations in the semi-classical region, we obtain instanton correction to the prepotential explicitly.

Another non-trivial consistency check for $G_2$ gauge group has been proposed by Landsteiner et al. [12]. They apply the method of confining phase superpotential [13] to $G_2$ and find that the discriminant of the spectral curve is consistent with that from the superpotential. The present approach provides another non-trivial and quantitative check to the exact solutions.

The $G_2$ type Lie algebra ($h^\vee = 4$) contains six positive roots. Let $\alpha_1 = (\sqrt{2}, 0)$ and $\alpha_2 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}})$ be simple roots. Among the the positive roots, $\alpha_1$, $\alpha_1 + 3\alpha_2$ and $2\alpha_1 + 3\alpha_3$ are long roots with squared length 2. Remaining roots $\alpha_2$, $\alpha_2 + \alpha_3$ and $\alpha_1 + 2\alpha_2$ are short roots with squared length 2/3. The fundamental weights $\lambda_1$ and $\lambda_2$ are defined by $\lambda_1 = 2\alpha_1 + 3\alpha_2$ and $\lambda_2 = \alpha_1 + 2\alpha_2$.

The representation with the highest weight $\lambda_2$ is seven-dimensional and may be embedded into that of the vector representation of the Lie algebra $so(7)$. One may use this embedding to construct the hyperelliptic curve for the $G_2$ gauge group from the gauge group $SO(7)$ with $N_f = 1$ flavor [3, 4]:

\[ y^2 = ((x^2 - \tilde{b}_1^2)(x^2 - \tilde{b}_2^2)(x^2 - \tilde{b}_3^2))^2 - \Lambda^8 x^4, \tag{3} \]

where

\[
\begin{align*}
\tilde{b}_1 &= b_2, \\
\tilde{b}_2 &= b_1 - b_2, \\
\tilde{b}_3 &= -b_1 + 2b_2.
\end{align*}
\tag{4}
\]
and $b_i = (\lambda_i, a)$ ($i = 1, 2$). This curve are shown to have correct monodromy in the weak coupling region. One can calculate one-instanton contribution to the prepotential by the restriction of $SO(7) \ N_f = 1$ theory \cite{10} to $G_2$. The result reads

$$F_1(b) \Lambda^8 = \frac{i \Lambda^8 (\bar{b}_1^2 - \bar{b}_3^2)^2 + \bar{b}_2^2 (\bar{b}_1^2 - \bar{b}_3^2)^2 + \bar{b}_3^2 (\bar{b}_1^2 - \bar{b}_2^2)^2}{(b_1^2 - b_3^2)^2 (b_2^2 - b_3^2)^2 (b_1 - b_2)^2}. \tag{5}$$

This is shown to be

$$F_1(b) \Lambda^8 = -\frac{\Lambda^8 i}{2^{5/2} \pi} \left( \frac{9}{2} \frac{9}{(2b_1 - 3b_2)^2 (b_1 - 3b_2)^2 b_1^2} + \frac{1}{2} \frac{1}{(b_1 - 2b_2)^2 (b_1 - b_2)^2 b_2^2} \right). \tag{6}$$

The first term is made of the contribution from the long roots. The second term contains the short roots only. One cannot expect the latter type of singularity from the microscopic instanton calculation. In fact, the microscopic instanton calculus \cite{8} shows that the one-instanton contribution to the prepotential is given by

$$F_1^{\text{inst.}} \Lambda^8 = -\frac{i \Lambda^8_{PV}}{2^{5/2} \pi} \frac{9}{(2b_1 - 3b_2)^2 (b_1 - 3b_2)^2 b_1^2}, \tag{7}$$

where $\Lambda_{PV}$ is the scale parameter defined in the Pauli-Villars regularization scheme. Therefore the hyperelliptic curve \cite{3} predicts additional singularities arising from the zero vacuum expectation value of a short root. The one-instanton term does not coincide with the result from the microscopic instanton calculation.

We next study the exact solution associated with the spectral curve which comes from the $(G_2^{(1)})^\vee$ Toda lattice\cite{7}. The spectral curve for $G_2$ reads

$$3 \left( z - \frac{\mu}{z} \right)^2 - x^8 + 2ux^6 - \left[ u^2 + \left( z + \frac{\mu}{z} \right) \right] x^4 + \left[ v + 2u \left( z + \frac{\mu}{z} \right) \right] x^2 = 0. \tag{8}$$

The Seiberg-Witten differential is given by

$$\lambda_{SW} = x \frac{dz}{z}. \tag{9}$$

It is convenient to introduce a new variable $y = z + \frac{\mu}{z}$. Then the Seiberg-Witten one-form take the form

$$\lambda_{SW} = x \frac{dy}{\sqrt{y^2 - 4\mu}}. \tag{10}$$

Here $y$ satisfies the quadratic equation

$$3y^2 - c_1 y - c_2 = 0, \tag{11}$$
where

\[ c_1 = 6x^4 - 2ux^2, \quad (12) \]
\[ c_2 = x^8 - 2ux^6 + u^2x^4 - vx^2 + 12\mu. \quad (13) \]

The equation (11) have two solutions:

\[ y = \frac{1}{6}(c_1 \pm \sqrt{c_1^2 + 12c_2}). \quad (14) \]

In the following analysis we take plus sign without loss of generality. The canonical holomorphic one-forms on the spectral curve are given by taking derivative of \( \lambda_{SW} \) with respect to \( u \) and \( v \):

\[ \frac{\partial \lambda_{SW}}{\partial t} = - \frac{\partial y}{\partial t \sqrt{y^2 - 4\mu}} dx - \frac{\partial}{\partial x} \left( \frac{x \frac{\partial y}{\partial t}}{\sqrt{y^2 - 4\mu}} \right) dx, \quad (15) \]

where \( t = u \) or \( v \).

We look for the Picard-Fuchs equations of the form

\[ a_{tt} \frac{\partial^2 \lambda_{SW}}{\partial t^2} + a_{uv} \frac{\partial^2 \lambda_{SW}}{\partial u \partial v} + a_u \frac{\partial \lambda_{SW}}{\partial u} + a_v \frac{\partial \lambda_{SW}}{\partial v} - \lambda_{SW} = d \left( \frac{f + g \sqrt{c_1^2 + 12c_2}}{\sqrt{c_1^2 + 12c_2 \sqrt{y^2 - 4\mu}}} \right) \quad (16) \]

where \( t = u \) or \( v \). \( f \) and \( g \) are polynomials of fourth order in \( x \). After some computations we find that the differential equations for the periods \( \Pi = \oint \lambda_{SW} \) are given by \( \mathcal{L}_i \Pi = 0 \) \( (i = 1, 2) \) where

\[ \mathcal{L}_1 = (\frac{8}{3}u^3v - 36v^2 + 960u^2\mu) \frac{\partial^2}{\partial v^2} + (\frac{8}{3}u^4 - 24uv + 2304\mu) \frac{\partial}{\partial u} \frac{\partial}{\partial v} + (4u^3 - 24v) \frac{\partial}{\partial v} - 1, \]
\[ \mathcal{L}_2 = \frac{2(720u^2\mu + 2u^3v - 27v^2)}{-vu + 24\mu} \frac{\partial^2}{\partial v^2} + \frac{4(256u^4\mu - 3u^2v^2 - 72uv\mu + 13824\mu^2)}{-vu + 24\mu} \frac{\partial}{\partial u} \frac{\partial}{\partial v} \]
\[ - \frac{6(-256u^3\mu + 96\mu v + 5v^2u)}{-vu + 24\mu} \frac{\partial}{\partial v} - 1. \quad (17) \]

Let us define differential operators \( \tilde{\mathcal{L}}_i \) by

\[ \tilde{\mathcal{L}}_1 = (1 - 2u^2) \mathcal{L}_1 - 2u^2 \mathcal{L}_2, \]
\[ \tilde{\mathcal{L}}_2 = \frac{v}{u} (\mathcal{L}_1 - \mathcal{L}_2). \quad (18) \]
These operators are convenient for studying solutions in the semi-classical region. In fact, these differential equations are written in the form of hypergeometric system by introducing new variables $x = \frac{u}{v^3}$ and $y = \frac{\mu u^2}{v^2}$:

$$\tilde{L}_1 = 1024y(\partial_x - 2\partial_y)(\partial_x - 2\partial_y - 1) + 2304xy(-3\partial_x + 2\partial_y)(\partial_x - 2\partial_y) - (8\partial_y + 1)^2,$$

$$\tilde{L}_2 = -32xy(\partial_x - 2\partial_y)(\partial_x - 2\partial_y - 1) + 2x(-3\partial_x + 2\partial_y)(-3\partial_x + 2\partial_y - 1)
+ \frac{2}{3}(\partial_x - 2\partial_y)(-4\partial_x + 1),$$

where $\partial_x = x\partial_x$ and $\partial_y = y\partial_y$ are the Euler derivatives.

Now we construct the solution of the Picard-Fuchs equations $\tilde{L}_i\Pi = 0$ ($i = 1, 2$) in the semi-classical region where $\mu$ is small. Consider a formal power series solution around $(x, y) = (0, 0)$ of the form

$$\omega_{\alpha,\beta}(x, y) = \sum_{m,n \geq 0} c_{\alpha,\beta}(m, n)x^{m+\alpha}y^{n+\beta},$$

where $c_{\alpha,\beta}(0, 0) = 1$. The indicial equations become

$$(8\beta + 1)^2 = 0,$$

$$(\alpha - 2\beta)(-4\alpha + 1) = 0.$$  \hspace{1cm} (21)

The equations (21) have two degenerate solutions $(\alpha, \beta) = (-1/4, -1/8)$ and $(1/4, -1/8)$. Applying the Frobenius method, we find two other solutions of logarithmic type. Finally four solutions of the Picard-Fuchs equations (19) are given by

$$\Omega_1(x, y) = \omega_{-1/4, -1/8}(x, y),$$

$$\Omega_2(x, y) = \omega_{1/4, -1/8}(x, y),$$

$$\Omega_{D1}(x, y) = \left(\frac{\partial}{\partial\alpha} + \frac{1}{2}\frac{\partial}{\partial\beta}\right)\omega_{\alpha,\beta}(x, y) \mid_{(\alpha,\beta)=(-1/4, -1/8)}$$

$$= \Omega_1(x, y) \log(xy^{1/2}) + \sum_{m,n \geq 0} \left(\frac{\partial}{\partial\alpha} + \frac{1}{2}\frac{\partial}{\partial\beta}\right)c_{\alpha,\beta}(m, n) \mid_{(\alpha,\beta)=(-1/4, -1/8)} x^{m+\alpha}y^{n+\beta},$$

$$\Omega_{D2}(x, y) = \left.\frac{\partial}{\partial\beta}\omega_{\alpha,\beta}(x, y)\right\mid_{(\alpha,\beta)=(1/4, -1/8)}$$

$$= \Omega_2(x, y) \log(y) + \sum_{m,n \geq 0} \left.\frac{\partial}{\partial\beta}\right)c_{\alpha,\beta}(m, n) \mid_{(\alpha,\beta)=(1/4, -1/8)} x^{m+\alpha}y^{n+\beta},$$

\hspace{1cm} (22)
Here the coefficients $c_{\alpha,\beta}(m, n)$ obey the recursion relations

$$
c_{\alpha,\beta}(m, n) = A_{\alpha,\beta}(m, n)c_{\alpha,\beta}(m, n-1) + B_{\alpha,\beta}(m, n)c_{\alpha,\beta}(m-1, n-1),$$
$$
c_{\alpha,\beta}(m, n) = C_{\alpha,\beta}(m, n)c_{\alpha,\beta}(m-1, n) + D_{\alpha,\beta}(m, n)c_{\alpha,\beta}(m-1, n-1),
$$
(23)

where

$$
A_{\alpha,\beta}(m, n) = \frac{1024(m - 2n + 2 + \alpha - 2\beta)(m - 2n + 1 + \alpha - 2\beta)}{(8n + 1 + \beta)^2},$$
$$
B_{\alpha,\beta}(m, n) = \frac{2304(-3m + 2n + 1 - 3\alpha + 2\beta)(m - 2n + 1 + \alpha - 2\beta)}{(8n + 1 + \beta)^2},$$
$$
C_{\alpha,\beta}(m, n) = \frac{-3(-3m + 2n + 3 - 3\alpha + 2\beta)(-3m + 2n + 2 - 3\alpha + 2\beta)}{(m - 2n + \alpha - 2\beta)(-4n + 1 - 4\alpha)},$$
$$
D_{\alpha,\beta}(m, n) = \frac{48(m - 2n + 1 + \alpha - 2\beta)(m - 2n + \alpha - 2\beta)}{(m - 2n + \alpha - 2\beta)(-4n + 1 - 4\alpha)}.
$$
(24)

Therefore the coefficients are obtained recursively

$$
c_{\alpha,\beta}(m, 0) = \left(\frac{27}{4}\right)^m (\alpha - 2\beta) \frac{m(\alpha - 2\beta + \frac{1}{3})}{(\alpha - 2\beta + 1)m(\alpha + \frac{1}{3})},$$
$$
c_{\alpha,\beta}(m, 1) = A_{\alpha,\beta}(m, 1)c_{\alpha,\beta}(m, 0) + B_{\alpha,\beta}(m, 1)c_{\alpha,\beta}(m - 1, 0),$$
$$
c_{\alpha,\beta}(m, 2) = A_{\alpha,\beta}(m, 2)c_{\alpha,\beta}(m, 1) + B_{\alpha,\beta}(m, 2)c_{\alpha,\beta}(m - 1, 1),
$$
(25)

e tc. where $(a)_m = \Gamma(a + m)/\Gamma(a)$. The first few terms of the series expansions of the solutions are given by

$$
\tilde{\Omega}_1(x, y) = \sqrt{u} - \frac{3}{8} \frac{v}{u^{5/2}} - \frac{105}{128} \frac{v^2}{u^{11/2}} + \frac{15}{2} \frac{\mu}{u^{7/2}} + \frac{693}{16} \frac{\mu v}{u^{13/2}} + \cdots,
$$
$$
\tilde{\Omega}_2(x, y) = \frac{\sqrt{u}}{v} + \frac{3}{4} \frac{v^3/2}{u^4} + \frac{17}{16} \frac{v^4}{u^{11/2}} - \frac{14}{40} \frac{\mu v}{u^{13/2}} + \cdots,
$$
$$
\tilde{\Omega}_{1D}(x, y) = \tilde{\Omega}_1(x, y) \log \frac{\mu^{1/2}}{u^{2}} + \frac{3}{4} \frac{v}{u^{5/2}} + \frac{17}{16} \frac{v^2}{u^{11/2}} - \frac{14}{40} \frac{\mu v}{u^{13/2}} + \cdots,
$$
$$
\tilde{\Omega}_{2D}(x, y) = \tilde{\Omega}_2(x, y) \log \frac{\mu v}{u^2} - \frac{5}{3} \frac{v^{3/2}}{u^4} - \frac{53}{10} \frac{v^{5/2}}{u^{11/2}} + \frac{8}{u^2} \frac{\mu v}{u^{13/2}} + \cdots,
$$
(26)

where $\tilde{\Omega}_i = \mu^{1/8}\Omega_i$ and $\tilde{\Omega}_{Di} = \mu^{1/8}\Omega_{Di}$ (i = 1, 2). One may construct the classical solutions

$$
b_1 = \sqrt{3}\tilde{\Omega}_1 - \frac{\sqrt{3}}{2}\tilde{\Omega}_2,
$$
\[ b_2 = \frac{2}{\sqrt{3}} \tilde{\Omega}_1, \]
\[ b_{D1} = \frac{i}{2\pi} \sqrt{3\tilde{\Omega}}_{D2} + t_0(2b_1 - 3b_2), \]
\[ b_{D2} = \frac{i}{2\pi} \left(-2\sqrt{3\tilde{\Omega}}_{D1} - \frac{3\sqrt{3}}{2} \tilde{\Omega}_{D2}\right) + t_0(-3b_1 + 6b_2), \quad (27) \]

where \( t_0 \) is a constant which is obtained by evaluation of the contour integral. But the value of \( t_0 \) is not necessary for the determination of the instanton effects to the prepotential. From these solutions, we may obtain the identities

\[ \sum_{i=1}^{2} (\partial_t b_{Di} b_i - b_{Di} \partial_t b_i) = \frac{i}{4\pi} \delta_{t,u} \quad (28) \]

where \( t = u \) or \( v \). Due to the complicated structure of poles in \( \lambda_{SW} \), it is difficult to prove (28) directly in a similar way as [19]. We have explicitly checked (28) up to order \( \mu^5 \). Hence the present results are exact up to 5-instanton level. By integration the identities (28) over \( u \) or \( v \), we get the scaling equation [18, 19]

\[ \frac{iu}{4\pi} = \sum_{i=1}^{2} b_i \frac{\partial F(b)}{\partial b_i} - 2F(b). \quad (29) \]

This identity allows us to calculate the \( n \)-instanton effects explicitly. The first three terms are given as

\[ F_1(b)\Lambda^8 = -\frac{3^4 i \Lambda^8}{\pi} \frac{1}{(2b_1 - 3b_2)^2(b_1 - 3b_2)^2b_1^2}, \]
\[ F_2(b)\Lambda^{16} = -\frac{3^{10} 5 i \Lambda^{16}}{2\pi} \frac{(b_1^2 - 3b_1 b_2 + 3b_2^2)^2}{(2b_1 - 3b_2)^6(b_1 - 3b_2)^6b_1^6b_2^6}, \]
\[ F_3(b)\Lambda^{24} = -\frac{2^3 3^{17} i \Lambda^{24}}{\pi} \frac{(b_1^2 - 3b_1 b_2 + 3b_2^2)^4}{(2b_1 - 3b_2)^{10}(b_1 - 3b_2)^{10}b_1^{10}} + \frac{i116 \cdot 3^{12} \Lambda^{24}}{\pi} \frac{(b_1^2 - 3b_1 b_2 + 3b_2^2)}{(2b_1 - 3b_2)^8(b_1 - 3b_2)^8b_1^8b_2^8} - \frac{i2^6 3^6 \Lambda^{14}}{\pi} \frac{(b_1^2 - 3b_1 b_2 + 3b_2^2)}{(2b_1 - 3b_2)^6(b_1 - 3b_2)^6b_1^6b_2^6(b_1 - b_2)^2(-b_1 + 2b_2)^2}, \quad (30) \]

where we put \( \Lambda = \mu^{1/8} \). If the parameter \( \mu \) satisfies the relation

\[ \Lambda_{PV}^8 = 3^2 2^4 \mu, \quad (31) \]

we find the one-instanton term in eqs. (31) agrees with that from the microscopic calculation [6]. As another consistency check, let us consider the \( SU(2) \) limit \( b_2 \rightarrow \infty \) with
the matching condition
\[ \mu = \frac{b^4_2 \Lambda^4_{SU(2)}}{4}, \] (32)
and finite \( \Lambda_{SU(2)} \). In this limit, we can show that the above instanton series reduces to that of the prepotential for \( SU(2) \) gauge group [13, 15, 16, 18].

Note that in the classical limit \( \mu = 0 \), \( \pm \tilde{b}_i \) \((i = 1, 2, 3)\) obey the equation
\[ x^6 - 2ux^4 + u^2x^2 - (\pm v + \frac{4}{27}u^3) = 0, \] (33)
which is obtained from the classical characteristic polynomial \( x^6 - 2ux^4 + u^2x^2 - v \) by the transformation
\[
\begin{align*}
&\{ u \rightarrow u, \\
&v \rightarrow -v + \frac{4}{27}u^3.
\end{align*}
\] (34)
The necessity of this replacement of variables has been also noticed in ref. [12].

In this paper we have studied the exact solutions represented by the hyperelliptic and spectral curves for the exceptional gauge group \( G_2 \). We have shown that the spectral curve (8) gives the prepotential which is consistent with the microscopic instanton calculation. But the hyperelliptic curve (3) does not agree with the microscopic result. The present analysis suggests that the spectral curves provides a systematic approach to the exact solutions to the Seiberg-Witten theory. It is interesting to generalize the present analysis to other exceptional gauge groups. In particular, \( E_6 \) type gauge groups would be treated in a similar way. \( E_r \) type gauge groups are particularly interesting in viewpoint of string duality, since the ALE fibration [20] gives systematic construction of the spectral curve [21]. The microscopic one-instanton calculation would provide quantitative test to the exact solutions and string duality in these cases.

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