DYNAMIC ANALYSIS OF MULTI DISC ROTATING STEPPED SHAFT THROUGH SIGNAL PROCESSING

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Abstract. This study identifies a method for detection of irregularities like open cracks or grooves on a rotating stepped shaft with multiple discs, based on the Wavelet Transforms. Cracks are represented as reduction in diameter of shaft (groove) with small width. Single as well as multiple grooves are considered on stepped shaft at locations of stress concentration. Translational or rotational response curves/mode shapes are extracted from finite element analysis of rotor with and without grooves. Discrete and continuous 1D wavelet transforms applied on resultant response curve or mode shapes. The results show that rotational response curves or mode shapes are more sensitive to shaft cracks and key contributors to identify the location of cracks than translation response curves or mode shapes. Discrete Wavelet Transforms are accurate enough to locate the groove of smaller size. Effectiveness of detection by Wavelets Transforms is analyzed for Single as well as Multiple grooves with increase in groove depth. Increase in groove depth can be quantified by increase in wavelet coefficient and it can be an indicator.

1. INTRODUCTION

Identification of damage in the machine elements has been the special interest among the researchers in last few decades. Increase in demands towards more efficient and high speed system demands more robust, reliable design and condition monitoring systems. Best and reliable defect identification techniques are available during and after manufacturing of components. This quality checks cannot stop the defects to generate during operation of machine due to fatigue, operating conditions, lack of maintenance and environment effects. Rotors are no more exemption to it. In fact this type of defects leads to catastrophic failure.

Studying the response of the system can reveal the changes in dynamic behaviour of rotor dynamic system. Many methods are available to identify the changes in the running behaviour of rotor dynamics system such as modal analysis, acoustic analysis, wear debris analysis, mode shapes, transfer functions and response of the system at different locations. The first model parameter used to diagnose the health of the rotor system is natural frequency. Signal process techniques like Fourier transforms along with natural frequency change can only capable of detecting damages and will not give any information about the location and severity of damage. The Sensitivity of this method is very low, possibility of damage identification is not guaranteed because there is very small change in frequency content of signal.

Wavelet Transforms (WT) is showed a better tool for extracting the salient feature from vibration response [8] to identify and locate cracks. WT carry the location information along with frequency and amplitude. Spatial vibration data such as operation deflection shapes (ODS) or mode shapes is reliable tools to detect the location of the cracks or damages [8]. The ODS at a particular frequency with different loading conditions is gaining importance for detection of crack in recent years [9]. Luc
Chouinard et al [1] estimated the reliability of WT in detection of cracks by analysing mode shapes. They defined a criteria based on probability and statics of wavelet coefficients to judge the severity of cracks in beams. S Swamy et al [4] proposed a method based on fundamental mode shapes with continuous wavelet transforms (CWT) to identify and quantify single as well as multiple damages in beam structure. S. Suresha and D. M. Reddy [5] used CWT to identify the sensitivity of mode shapes in detecting the damage location and quantification. They concluded that CWT is promising technology to identify the damages in plate structures [5, 8].

Rims Janeliukstis et al [7] defined a damage estimate reliability parameter based on wavelet scales and isotropic Pet Hat wavelet to validate scales of WT coefficients. Rouhollah Pour et al [2] applied WT on higher order mode shapes to detect the cracks in cantilever beam. Roger Serra and Lautaro Lopez et al [6] used stationary wavelet transforms to de-noise the signal of mode shapes with and without damage in cantilever beam and used CWT on difference between this signals to identify the damage.

Factor dimension of the detailed signals of discrete wavelet transforms (DWT) can be used to locate the cracks accurately [10]. Time synchronised wavelet analysis method was proposed and its effectiveness to detect faults in rotor system [12]. Maosen Cao and Pizhong Qiao [13] presented a method integrated WT to ODS for damage diagnosis. Curvature of mode shape is one the characteristic that can be used to detect the crack [2][15]. Mode shape curvatures and spatial wavelet transform can be used to identify the location of damage in rotor [11]. Many researchers have sued ODS to identify the crack or damage location in rotor systems using WT. C Bhargav Sai and D Mallikarjuna Reddy [14] proposed CWT on ODS can detect the cracks in the rotor. S. Rathna Prasad and A. S. Sekhar [9] used displacement and strain energy to locate and quantify the damage in noisy signal from ODS of rotor. Banks and colleagues [8], published a papers on crack detection using the wavelet Transforms. In this study we identified the sensitive parameter of ODS to detect, locate and quantify the dame or crack in rotor dynamics systems.

2. WAVELET TRANSFORMS AND ANALYSIS

Wavelet transform provides three outputs, scale (inverse of frequency), time or space and “wavelet coefficient”. This wavelet coefficient resembles signal behaviour [15]. Below subsection describe the basic theories of CWT and DWT.

**Continuous Wavelet Transform (CWT)**

Let \( \psi(t) \) is a real or complex valued function in limited time domain. That is \( \psi(t) \) has values in a certain time range and zeros elsewhere. The CWT coefficient of mother wavelet \( \psi_{u,s}(t) \) which is given as

\[
Wf(s,u) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} \psi^* \left( \frac{t-u}{s} \right) dt
\]

(1)

With this transformation, one can map a one dimensional signal \( f(t) \) to a two dimensional coefficients \( Wf(s,u) \).

![Wavelet Decomposition Tree](image)

**Discrete Wavelet Transform (DWT)**

DWT process the signals in two channel sub bands approximation and detail. Low frequency content in a signal is referred as Approximation and High frequency content in a signal is referred as Detail. Discrete wavelet approximation and detail coefficients can be calculated as follows. The
decomposition can be iterative, the filtering process for multilevel decomposition is shown figure (1).

\[ W(j_0, k) = \frac{1}{\sqrt{s}} \sum f(n) \psi_{j_0,k}(n) \]  

(2)

\[ W(j,k) = \frac{1}{\sqrt{s}} \sum f(n) \psi_{j,k}(n) \]  

(3)

3. METHODOLOGY

The following block diagram as shown in the figure (2) indicates steps flowed in this study. This diagram provides details about the process and techniques used for the current analysis.

4. NUMERICAL MODEL AND EQUATION OF MOTION

In this study finite element analysis of stepped shaft with 3 Disc rotor model with four degree of freedom per node is considered. Rotor shaft is modelled using Euler and Timoshenko beam theory. Shear, rotary inertia and gyroscopic effects are considered in this analysis. The equation of motion of stepped rotor systems in fixed coordinates is given as

\[ [M]\ddot{q} + [C + G]q + [K]q = \dot{[Q]} \]  

(4)

Where [M] is mass matrix, [C+G] gyroscopic and damping matrix and [K] stiffness matrix of shaft, bearings and groove or crack elements. Two translational displacements u and v are along OX and OY axis respectively. Two Rotational Displacements \( \theta \) and \( \phi \) are along OX and OY respectively. The rotor rotates clockwise with angular velocity \( \Omega \) about OZ as shown in figure (1). It is most common to term translation and rotation displacement as bounce and tilt motion, respectively. Bearing are isotropic, stiffness in x and y direction are \( K_{xx}=K_{yy}=1e9 \text{ N/m}^2 \).

DESCRIPTION OF STEPPED SHAFT ROTOR MODEL

Uniform diameter shaft are rare in the any physical applications. Steps are necessary for some functionality and cannot be avoided. Shaft with three diametric steps (\( \varnothing 40.0 \text{ mm} \), \( \varnothing 50.0 \text{ mm} \) and \( \varnothing 60.0 \text{ mm} \) ) are considered in this study. The total length of the shaft is 1.0m. Figure (3) describes the positional, dimensional and nodal details of stepped rotor, bearings, discs and grove or cracks.
Discretization with more number of elements can increase the accuracy but need more computational time and data to store. A convergence test based on mode-4 natural frequency reveals that a minimum 80 number of elements required for discretization, however we considered 105 elements for this study, Refer figure 4 for details.

Table 1. Natural frequency of different cases

|   | WG | C1 | C2 | C3 | C4 | C5 |
|---|----|----|----|----|----|----|
| NF1 | 22.88 | 22.88 | 22.82 | 22.82 | 22.75 | 22.75 |
| NF2 | 54.39 | 54.38 | 54.27 | 54.29 | 53.92 | 54.19 |
| NF3 | 213.2 | 213.1 | 211.6 | 211.8 | 209.4 | 210.4 |
| NF4 | 436.5 | 435.0 | 414.2 | 414.7 | 396.0 | 397.9 |
| NF5 | 629.0 | 628.7 | 604.2 | 604.6 | 574.2 | 575.2 |

Groove or open cracks are modelled by reduction in the diameter of the shaft element. Groove is provided at highly stress concentrations locations or areas. At these locations the section changes occur which have an effect on modal displacement. It is highly important to clearly distinguish the causes of these singularities. Studying the Effectiveness of the crack or groove detection method at this location is vital.

NUMERICAL SIMULATION

A finite element code is generated in MATLAB for stepped rotor model with different groove location and combinations as follows

- A single groove or an open crack between node 6 & 7 (Near bearing LH).
- A single groove or an open crack between node 29 & 30 (Near disc-1)
- Two groove or an open crack between node 6 & 7, 29 & 30 (Near bearing LH & disc-1)
- Three groove or an open crack between node 6 & 7, 29 & 30, 76 & 77 (Near bearing LH, disc-1 & disc-2)
- Four groove or an open crack between node 6 & 7, 29 & 30, 76 & 77, 99 & 100 (Near bearing LH, disc-1, disc-2 & bearing RH)

Model analysis is performed on healthy rotor and rotor with groove or cracks. Table 1 shows the details of first five natural frequencies against healthy rotor (WG) and cracked...
rotor Case C1 to C5 with 40% reduction in diameter of groove elements. These natural
frequencies are taken at a rotor speed of 3100 RPM which is close to second natural
frequency. Reductions in natural frequency are evident for presence of groove. Response of
all four degrees of freedom at every node is recorded and Operational Deflection Shapes are
formed separately for translational and rotational degrees of freedom. Bounce mode or
translation mode shape curve is formed by resultant of X-displacement and Y-displacement.
Tilt mode or rotational mode shape is formed by resultant of rotational Displacements \( \theta \) and \( \varphi \).

Signals of translational and rotational mode shapes are analysed separately and collectively
with Continuous and discrete wavelet transforms. Grove depth is varied from 10% to 50%
of base diameter for all cases as stated above. Signal of translational and rotational mode
shapes are generated by subtracting the nodal displacement without groove from without
groove. DWT coefficients are calculated for level 6 decompositions. It is observed that crack
or groove location can be identified by plot of detail-1 of the DWT and CWT transforms.

RESULTS AND DISCUSSION

1st Translational ODS and its DWT, CWT with db4 mother wavelet for healthy rotor are
shown in the figure (5). It is observed that ODS curve 5(a) is having discontinuities at
location where diameter of the shaft changes. Correspondingly we can see the spikes at
location of discontinuities in DWT plot 5(b) which is detail at Level-1. CWT plot 5(c)
doesn’t show these discontinuities. Generally CWT will show higher amplitudes at bearing
restrains.

Figure 5. Healthy rotor 1st translational response plots. (a) ODS, (b)DWT details-L1, (c)
CWT

Figure 6. Case-1:10% grove depth 1st translational response plots. (a) RODS, (b)DWT, (c)
CWT

ODS of rotors with groove or crack are formed by subtracting the deflection without groove
from with groove for all mode shapes. We termed this ODS as Resultant ODS (RODS). 1st
Translational RODS and its DWT, CWT with 10% grove-1 depth is shown in the figure (6).
It is observed that DWT 6(b) and CWT 6(c) plots are similar as healthy rotor in figure (5)
with change in magnitude of coefficients. Therefore 1st translation mode is not effective in
identifying the location of damage it can only give information about discontinuities in shaft.

1st Rotational RODS and its DWT, CWT with 10% grove-1 depth (Case-1) is shown in the
figure (7). It is observed that DWT 7(b) plot showing spikes with higher magnitude at groove-
1 location. In and CWT 7(c) plot the higher magnitude (bright colour) at low scales i.e. scale
-1 (High frequency) indicates the discontinuity. We observe a small discontinuity at middle
of the disc. Therefore rotational modes are more sensitive to the groove than translation
modes but other small spikes are also observed. This may be conflating the claim to identify
the groove or crack location clearly.
Figure 7. Case-1: 10% groove depth 1st rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

Figure 8. Case-1: 10% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

2nd Rotational RODS and its DWT, CWT with 10% groove-1 depth (Case-1) is shown in the figure (8). It is observed that DWT 8(b) plot showing spikes with higher magnitude only at groove-1 location. In and CWT 8(c) plot the higher magnitude (bright colour) at low scales i.e. scale -1 (High frequency) observed only at groove locations. Therefore 2nd rotational modes are more sensitive and reliable to identify the crack or groove location clearly.

Figure 9. Case-2: 10% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

Figure 10. Case-3: 10% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

Figure 11. Case-4: 10% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT
Figure 12. Case-5: 10% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

2nd Rotational RODS and its DWT, CWT with 10% groove depth for Case-2, Case-3, Case-4 and Case-5 are shown in the figure (9), figure (10), figure (11) and figure (12) respectively. It is observed that DWT and CWT plot showing spikes with higher magnitude only at groove location.

Figure 13 to 16 plots related to increase in groove depth from 20% to 50% for case-2 of this study. Observing figures 9, 13 to 16 it is clear that DWT coefficient magnitude increase as the crack or groove depth increases. 2D-CWT plots cannot revile the increase in magnitude of wavelets. A 3D CWT plots can provide this information. To proceed further, in future crack depth or groove depth can be quantified by analyse the wavelet coefficients for different depths.

Figure 13. Case-2: 20% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

Figure 14. Case-2: 30% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

Figure 15. Case-2: 40% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

Figure 16. Case-2: 50% groove depth 2nd rotational response plots. (a) RODS, (b) DWT details-L1, (c) CWT

5. CONCLUSIONS

This paper present a non-destructive damage detection and location identification technique based on operational mode shapes along with discrete and continuous wavelet transforms. Daubechies mother wavelet for continuous wavelet transforms and daubechies order 4 mother wavelet with level 6 decomposition is used for discrete wavelet transforms. Finite element method is used to analyse the stepped rotor with 3 discs. Grove or crack was created at stress concentration location by reducing the diameter of the groove element. Operational
mode shape data is processed through wavelet transforms to identify the location of damage. 1\textsuperscript{st} Translational mode shapes can reveal the information about discontinuity in the rotor while 2\textsuperscript{nd} rotational mode shapes can accurately locate the groove or crack location. Despite of groove or crack locations are at discontinuities the proposed method can effectively and accurately locate the groove or crack element. This research study proved that wavelet transforms of 2nd operational mode shapes is a promising method to identify and locate the crack in rotor machine condition monitoring.

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