BPS Dynamics of Triple \((p, q)\) String Junction

Soo-Jong Rey and Jung-Tay Yee

Physics Department
Seoul National University, Seoul 151-742 KOREA

jungtay@phya.snu.ac.kr, sjrey@gravity.snu.ac.kr

abstract

We study dynamics of triple junction of \((p, q)\) strings in Type IIB string theory. We probe tension and mass density of \((p, q)\) strings by studying harmonic fluctuations of the triple junction. We show that they agree perfectly with BPS formula provided suitable geometric interpretation of the junction is given. We provide a precise statement of BPS limit and force-balance property. At weak coupling and sufficiently dense limit, we argue that \((p, q)\)-string embedded in string network is a ‘wiggly string’, whose low-energy dynamics can be described via renormalization group evolved, smooth effective non-relativistic string. We also suggest the possibility that, upon Type IIB strings are promoted to M-theory membrane, there can exist ‘evanescent’ bound-states at triple junction in the continuum.

---

1 Work supported in part by the NSF-KOSEF Bilateral Grant, KOSEF SRC-Program, Ministry of Education Grant BSRI 97-2418, and the Korea Foundation for Advanced Studies Faculty Fellowship.
Triple String Junction: Among Dirichlet branes, the quantum solitons in string theory, Type IIB \((p, q)\) string is of particular interest in that it is the simplest non-threshold bound-state consisting of \(p\) fundamental strings (F-string) and \(q\) Dirichlet strings (D-string). A \((p, q)\)-string is BPS saturated, hence, string tension equals to mass density and is given by

\[
T_{(p,q)} = T\sqrt{p^2 + \frac{q^2}{g_{\text{IIB}}^2}},
\]

where \(T = T_{(1,0)} \equiv 1/2\pi\alpha\) is the F-string tension and \(g_{\text{IIB}}\) denotes Type IIB string coupling parameter. Indeed, for relatively prime integers \(p\) and \(q\), the entire string bound-states form an orbit of \(SL(2, \mathbb{Z})\) S-duality in Type IIB string theory.

Because of non-threshold nature of the bound-state, formation of \((p, q)\)-string entails intriguing interplay between gauge field on the D-string worldsheet and F-string charge density. To release latent binding energy into local recoil, the F-string charge had better spread over the D-string worldsheet. This is made possible via Cremmer-Scherk coupling that allows transmutation of the fused F-string charge to electric flux of D-string worldsheet gauge field.

Consider the fusion process of a F-string onto a D-string. If only part of the F-string is bound to the D-string and is transmuted to worldsheet gauge field, the resulting geometry is nothing but a triple junction of \((p, 0)\) F-string, \((0, q)\) D-string and \((p, q)\)-string. The configuration is depicted schematically in Fig. 1(a). At each junction F- and D-string charges are conserved separately:

\[
3 \sum_{a=1}^{3} p_a = 0 : \quad 3 \sum_{a=1}^{3} q_a = 0.
\]

Generically the F-string will not just stop at the configuration of Fig.1(a) but continue fusion process until they form a bound-state of \((p, q)\)-string. However, if configuration is such that the tension is balanced

\[
3 \sum_{a=1}^{3} T_{(p_a,q_a)} = 0,
\]

(where the string tension is treated as a complex quantity) the triple junction can be stabilized and become a BPS saturated configuration. It was previously conjectured that a triple string junction satisfying Eqs. (2) is BPS saturated. Recently, in the linearized approximation, this conjecture has been proven both in worldsheet and spacetime approaches. The former approach was inspired by recent new understanding of deformed branes.

In this Letter, we investigate BPS dynamics of the triple string junction. We do so by studying dynamics of harmonic fluctuation and extract information relevant to BPS condition of the triple junction. For a single string, in physical gauge, harmonic fluctuation is governed
by an action
\[ S = \int dt d\sigma \left[ \frac{1}{2} \rho (\partial_t X)^2 - \frac{1}{2} T (\partial_\sigma X)^2 \right]. \] (4)

Here, \( \rho \) and \( T \) denote \textit{inertial} mass density and tension of the string, that are in general functions of \((t, \sigma)\). Dynamics of the string is completely characterized only when the action Eq.(4) is supplemented by an ‘equation of state’ \( T = T(\rho) \). For example, propagation velocities of transverse and longitudinal oscillations are given by \( v_T^2 \equiv T(\rho)/\rho \) ; \( v_L^2 \equiv -dT(\rho)/d\rho \). For a fundamental BPS string, longitudinal oscillation is gauge redundant and \( T(\rho) = c^2 \rho \) is \((t, \sigma)\)-independent constant. However, this is no longer true for coarse-grained effective description of ‘wiggly \((p, q)\)-string’, on which we will return momentarily. Is it possible to probe the string equation of state \( T = T(\rho) \), dynamics for each string prong, and for the triple junction as a whole? We now answer these questions affirmatively and gain new understanding of \textit{BPS dynamics} of triple string junction that are visible only in full-fledged Dirac-Born-Infeld (DBI) analysis.

\textbf{Dirac-Born-Infeld Analysis:} Consider a \((0,q)\) D-string aligned initially along \( x_1 \) direction and a \((p,0)\) F-string along \( x_9 \) direction impinging on D-string at \( x_1 = x_9 = 0 \). See Fig. 1(b). In the static gauge \( X^0 = t, X^1 = x_1 \), worldsheet dynamics of the \((0,q)\) D-string is described by the following abelian part \([1]\) of the DBI Lagrangian:

\[ L_{\text{DBI}} = -\frac{q}{g_{\text{IIB}}} T \int dx_1 \sqrt{\det(\eta_{ab} + F_{ab} + \nabla_a X \cdot \nabla_b X)}. \] (5)

We have chosen a static gauge \( X^0 = t, X^1 = x_1, A_1 = 0 \), and have denoted the transverse collective coordinates as \( X \) and the worldsheet gauge field as \( F_{ab} \). Assuming that classical configuration excites \( X \) along the plane spanned by injected F-string and D-string (9-direction)
and is static, the total energy functional of D-string is given by

\[ H_{\text{DBI}} = \frac{q}{g_{\text{IIB}}} T \int dx_1 \frac{1 + (\nabla_1 X^9)^2}{\sqrt{1 + (\nabla_1 X^9)^2 - (\nabla_1 A_0)^2}}. \]  

(6)

Extremum of the energy functional is when \( A_0 \) and \( X^9 \) satisfy

\[ \nabla_1 \cdot \left( \frac{\nabla_1 A_0}{\sqrt{1 + (\nabla_1 X^9)^2 - (\nabla_1 A)^2}} \right) = g_{\text{IIB}} \frac{p}{q} \delta(x_1) \]

(7)

where \( a \) is a constant parameter \((0 < a < \infty)\). In the last equation, we have used the fact that \( p \) units of F-string charges are distributed among \( q \)-multiples of D-string. Comparison with previous results \([9, 10, 12]\) shows that \( a = 1 \) corresponds to the BPS limit. The equations are solved straightforwardly to yield an exact solution:

\[ X^9(x_1) = \sqrt{a} A_0(x_1) = - (\tan \theta) \ x_1 \quad (x > 0) \]

\[ = 0 \quad (x < 0) \]  

(8)

where

\[ \tan \theta \equiv \frac{X^9}{x_1} = \sqrt{\frac{a}{(1 - a) + \frac{1}{g_{\text{IIB}}^2} (q/p)^2}}. \]  

(9)

The solution has the following simple geometric interpretation. At the impinging point of the F-string, the initially straight D-string is bent rigidly by angle \( \theta \) to the negative \( x_9 \) direction. We emphasize that the angle \( \theta \) is determined solely by the \((p, q)\) charges (at fixed \( a, g_{\text{IIB}} \) values), not by a requirement of tension balance. Note also that the bending angle \( \theta \) increases monotonically with increasing the parameter \( a \). Away from the triple junction location, the D- and F-string prongs at \( x_1 < 0 \) and \( x_9 > 0 \) are nothing but BPS saturated single string states with \( T(\rho) = \rho \) equals to \((q/g_{\text{IIB}})T\) and \( pT \) respectively. What about the \((p, q)\) string prong, now bent to the second quadrangle? We now show that, for all values of \( a \), the \((p, q)\)-string prong itself is also BPS saturated. To show this, we evaluate static mass density of the \((p, q)\) string from Eq.(8):

\[ E_{(p,q)\text{string}} = \int_0^\infty dx_1 \frac{q}{g_{\text{IIB}}} T \frac{1 + \tan^2 \theta}{\sqrt{1 + \frac{a}{1 - a} \tan^2 \theta}}. \]  

(10)

One expects the integrand to represent mass density of the \((p, q)\)-string prong. However, it looks nothing like the BPS formula for any value of \( a \), even including the expected BPS limit \([9, 10, 12]\) \( a = 1 \)! This puzzle is resolved neatly by noting that the \((p, q)\)-string prong has now been bent rigidly by angle \( \theta \) relative to the \( x_1 \) axis. Therefore, along \((p, q)\)-string prong, we introduce a
proper worldsheet coordinate $\sigma$ and measure the string mass density per unit $\sigma$-length. From elementary geometry,

$$\sigma = \frac{x_1}{\cos \theta} \quad (-\infty < \sigma < +\infty); \quad \frac{1}{\cos \theta} = \sqrt{1 + \frac{a}{1 - a + \frac{1}{g_{\text{IB}}}(q/p)^2}}. \quad (11)$$

After taking this simple geometric consideration into account to Eq.(10) we identify proper static mass density $\rho_{(p,q)}$ of the $(p, q)$-string prong:

$$E_{(p,q) \text{ string}} = \int_0^\infty ds \rho_{(p,q)} : \quad \rho_{(p,q)} \equiv T \sqrt{p^2 + \frac{q^2}{g_{\text{IB}}^2}}. \quad (12)$$

As claimed, for all values of parameter $a$, we have shown that the static mass density of the $(p, q)$-string prong of the triple junction is BPS saturated to the tension Eq.(1): $\rho_{(p,q)} = T_{(p,q)}$.

What is then special to the proclaimed BPS limit $a = 1$? We now show that, even though each string prong is always BPS saturated, it is only when $a = 1$ the string tensions sum to zero so that the triple string junction stays in equilibrium. For an arbitrary $a$, ratios of vector components of string tensions along $x_1, x_9$ directions are

$$x_1 : \quad T_{(p,q)} \cos \theta / T_{(0,q)} = -\sqrt{1 + (1 - a)g_{\text{IB}}^2(p/q)^2}$$

$$x_9 : \quad T_{(p,q)} \sin \theta / T_{(p,0)} = -\sqrt{a}. \quad (13)$$

It is clear that only when $a = 1$ vector sum of three string tensions vanish identically. If $a > 1$, net force is nonvanishing and acts on the triple string junction to the direction of third quadrant. As the triple string junction responds adiabatically to the force, the angle $\theta$ decreases monotonically until it reaches the BPS value $\theta_{\text{BPS}} = \tan^{-1}(p/q)g_{\text{IB}}$. Likewise, if $a < 1$, net force acts to the first quadrant so that the triple string junction moves in the direction of increasing $\theta$ to the BPS value. We thus conclude that any triple string junction with $a \neq 1$ relaxes always to $a = 1$ configuration. Strictly speaking, the assumption that triple string junction is a static configuration is not valid for $a \neq 1$.

**Harmonic Fluctuation:** Another quantities of physical interest are inertial mass density and tension of the triple string as defined via Eq. (4). We now probe these quantities by studying harmonic fluctuations of the D-string, part of which has now bent into $(p, q)$-string prong. In this approach, F-string serves only as a static background source of electric charge on the D-string worldsheet. Let us decompose fluctuation of $X^i$ around the background Eq.(8) into a fluctuation $\Xi$ parallel to 9-direction and perpendicular to 1-direction and a fluctuation $\Psi$ orthogonal to 1- and 9-directions (in- and out-of-plane fluctuations in Fig. 1(b)). The harmonic dynamics of D-string is governed by quadratic expansion of the DBI Lagrangian:

$$L^{(2)}_{\text{DBI}} = \int_{-\infty}^{+\infty} dx_1 \frac{T}{2g_{\text{IB}}} \frac{1}{\sqrt{1 + B^2 - E^2}} \left[ 1 + \frac{B^2}{1 + B^2 - E^2} (F_{01})^2 - 2 \frac{EB}{1 + B^2 - E^2} F_{01} \cdot \nabla_1 \Xi \right]$$
by spherically symmetric background fields. Fluctuation of the worldsheet gauge field can be integrated out exactly. This yields:

\[ L_{\text{DBI}}^{(2)} = \int_{-\infty}^{+\infty} dx_1 \frac{T}{2 g_{\text{IB}}} \frac{q}{\sqrt{1 + B^2 - E^2}} \left[ (\nabla_0 \Xi)^2 - \frac{1}{1 + B^2} (\nabla_1 \Xi)^2 + (1 + B^2) (\nabla_0 \Psi)^2 - (\nabla_1 \Psi)^2 \right]. \]  

Comparing this with Eq. (14) one might be tempted to conclude that the inertial mass density and tension differ from the static ones Eq. (12) and are also sensitive to the polarization directions, \( \Xi \) or \( \Psi \). We now show that this is not the case. Key observation is again associated with proper geometric interpretation. As for the identification of static mass density, we should measure fluctuation with respect to the \( \sigma \) coordinate introduced in Eq. (11). Furthermore, for in-plane fluctuation \( \Xi \) along \( x_9 \) direction, component parallel to the \((p, q)\)-string prong should be interpreted as a gauge redundant longitudinal mode, hence, only component perpendicular to the string is physical. Geometrically, these considerations amount to change of variable \( x \to \sigma \cos \theta \) and projection \( \Xi \to \Xi_\perp / \cos \theta \). Thus, we identify proper DBI Lagrangian describing harmonic fluctuation as:

\[ L_{\text{DBI}}^{(2)} = \int_{-\infty}^{+\infty} d\sigma \frac{T}{2 g_{\text{IB}}} \frac{q}{\sqrt{1 + aE^2}} \left[ (\nabla_\sigma \Psi)^2 - (\nabla_\perp \Psi)^2 + (\nabla_\perp \Xi)^2 - (\nabla_\perp \Xi)^2 \right], \]  

where \( 1/E^2 = a \cot^2 \theta = (1 - a) + (q^2/p^2)/g_{\text{IB}}^2 \). Thus, for all values of \( a \), the inertial mass density and the tension of the fluctuation per unit length of the \((p, q)\)-string prong are equal to \( \frac{q}{g_{\text{IB}}} T \sqrt{\frac{1 + aE^2}{1 + (a - 1)E^2}} \) or \( T \sqrt{\frac{q^2}{g_{\text{IB}}^2} + p^2} \). They are exactly the same as the static mass density and tension Eqs. (12, 14).

It is instructive to repeat the above analysis for a \((0, q)\) Dirichlet \( n \)-brane attached by a \((p, 0)\) F-string [3, 10, 11, 12]. On the world-volume of D-brane, the configuration is described by spherically symmetric background \( E = \hat{r} \nabla_r A_0 \) and \( B = \hat{r} \nabla_r X^9 \) satisfying \( B = \sqrt{\alpha} E \) and

\[ \frac{1}{E^2} = (1 - a) + \frac{(q/p)^{n-1}^2}{(n - 2)c_n} : \quad c_n = \frac{(2\pi/T)^{(n-1)/2}}{(n - 2)\Omega_{n-1}} g_{\text{IB}}. \]  

For simplicity, we restrict fluctuations around the above configuration to S-wave partial wave modes only. After integrating out world-volume gauge field, the DBI Lagrangian of harmonic fluctuation is given by:

\[ L_{\text{DBI}}^{(n)} = \frac{T^n}{2} \frac{q}{g_{\text{IB}}} \int d^n r \frac{1}{\sqrt{1 + B^2 - E^2}} \left[ (1 + B^2) (\nabla_0 \Xi)^2 - (\nabla_\perp \Xi)^2 + (\nabla_0 \Psi)^2 - \frac{1}{(1 + B^2)} (\nabla_\perp \Psi)^2 \right]. \]
where \( T^{(n)} \equiv \Omega_{n-1}(2\pi)^{(1-n)/2}T^{(n+1)/2} \), and \( \Xi \) and \( \Psi \) denote fluctuations along \( x_9 \)-direction and perpendicular to \( x_9 \) and Dirichlet \( n \)-brane directions respectively. Following the same geometric reasoning as in D-string case, proper description of fluctuations is obtained once we make a change of variable \( r \rightarrow \sigma \cos \theta \) and orthogonal projection \( \Xi \rightarrow \Xi_{\perp} / \cos \theta \), where \( \cos \theta = \sqrt{1/(1 + B^2)} \). The DBI Lagrangian for proper fluctuation of Dirichlet \( n \)-brane is then given by

\[
L_{\text{DBI}} = \frac{T^{(n)}}{2} \frac{q}{g_{\text{IIB}}} \int d^n \sigma \cos^n \theta \sqrt{\frac{1 + aE^2}{1 + (a - 1)E^2}} \left[ (\nabla_0 \Psi)^2 - (\nabla_\sigma \Psi)^2 + (\nabla_0 \Xi_{\perp})^2 - (\nabla_\sigma \Xi_{\perp})^2 \right]. \tag{19}
\]

Therefore, for all possible polarization, we find that the triple Dirichlet \( n \)-brane junction is a BPS saturated configuration with equal inertial mass density and tension:

\[
\rho_{(p,q)}^{(n)} = T_{(p,q)}^{(n)} = T^{(n)} \left( \frac{(2\pi/T)^{(n-1)/2}}{\Omega_{n-1}r^{(n-1)}} \right)^2 p^2 + \frac{q^2}{g_{\text{IIB}}^2}. \tag{20}
\]

A novelty not encountered for triple \((p,q)\) string is that the tension varies continuously as one moves in from asymptotic infinity to the center, where the F-string is impinging on the Dirichlet \( n \)-brane. The bending angle \( \theta \) is position-dependent and increases monotonically from zero to maximum value, \( \theta^* = \cos^{-1} \sqrt{1/(1 - a)} \) (for \( a < 1 \)) or \( \pi/2 \) (for \( a > 1 \)). Far away from the center, the BPS mass density Eq. (20) approaches that of an isolated Dirichlet \( n \)-brane: \((q/g_{\text{IIB}})T^{(n)}\). Near the center, the BPS mass density per unit \( n \)-dimensional volume Eq.(20) diverges. However, mass density and tension measured per unit \( \sigma \)-length is finite. For example, for \( a < 1 \), \( r^{(n-1)}T_{(p,q)}^{(n)} \rightarrow pT \), the mass density of \((0, p)\) F-string. It is straightforward to recognize that tension is balanced at every point on the Dirichlet \( n \)-brane only when \( a = 1 \) but not for \( a \neq 1 \). Hence, we interpret this as an indication that deformed Dirichlet \( n \)-brane with \( a \neq 1 \) always relax to stable \( a = 1 \) configuration. Details will be reported elsewhere.

**Wiggly \((p,q)\) String:** Using the triple string junction as a building block, one can construct a network of \((p,q)\)-strings. Indeed, Sen [8] has suggested the string network as a novel mechanism of string compactification. Being so, *dynamical aspect* of the string network is also of interest. Consider, for definiteness, a dense string network in weakly coupled Type IIB string theory. In this limit, any prongs carrying D-string Ramond-Ramond charge \((q \neq 0)\) will become much heavier than those carrying F-string charge only. Because of the D-string charge conservation, these heavy prongs should either form a closed loop or extend to infinity. See Fig. 2(a). What are then distinguishing characteristics, if any, of a heavy loop in the network from a \((p,q)\)-string in isolation? We now argue that coarse-grained picture of the heavy-prong loop is a smooth, non-relativistic string by showing that ‘wiggles’ present on the loop renormalize the microscopic equation of state \( T(\rho) = \rho \) to a non-trivial renormalization-group fixed point \( T(\rho)\rho = T^2_{(p,q)} \) in the infrared.
Figure 2: (a) massive \((p,q)\) string embedded in string network, (b) noise generation via lowest-energy fluctuation.

We begin with an argument that the heavy-prong loop is constantly wiggled by the background F-string network. Consider low-energy excitations of the BPS string network. One possible excitation is to boost prongs in each triple string junction while maintaining balance of the tension forces. An example involving 4 adjacent junctions is illustrated in Fig. 2(b). At macroscopic scale on the heavy-prong loop, net effect of such gapless excitation is to put wiggles to the loop. Generically, wiggles of all possible sizes will be present. Hence, we will call the loops made out of heavy prongs as ‘wiggly strings’ – they are nothing but D-strings with small scale structures induced by dense F-string network background.

Suppose the heavy-prong loop has wiggles on it with characteristic size \(\lambda\) but is straight otherwise. To an observer with resolution \(\ell \gg \lambda\), the wiggly string appears to be a straight string with an effective mass density \(\rho_{\text{eff}} > \rho_{(p,q)}\) and tension \(T_{\text{eff}} < T_{(p,q)}\). Low-energy dynamics of the wiggly string is most conveniently described by a coarse-grained, effective smooth string in which the small-scale wiggles of sizes \(\lambda < \ell\) are integrated out. Intuitively, the wiggles increase string mass density but decrease string tension. Hence, we expect that the microscopic equation of state \(T(\rho) = \rho\) is unstable under coarse-graining and flows into a renormalization-group fixed point in the infrared. It is our aim to find out non-trivial infrared fixed point, if present. A similar question has been addressed previously [13] in the context of noisy cosmic and Nambu-Goto strings [14].

For small amount of wiggliness, the effective parameters have the following schematic form:

\[
\begin{align*}
\rho_{\text{eff}} &= \rho_{(p,q)} + \langle V^2 \rangle F(\rho_{(p,q)}, T_{(p,q)}, \cdots) \\
T_{\text{eff}} &= T_{(p,q)} - \langle V^2 \rangle G(\rho_{(p,q)}, T_{(p,q)}, \cdots).
\end{align*}
\]

(21)

Here, \(\langle V^2 \rangle\) denotes the average velocity-squared (both transverse and longitudinal), and can
be calculated from harmonic fluctuations of the wiggly string. The $F, G$ are positive-definite functions of string mass density and tension that are determined solely by the microscopic equation of state.

At first sight, it appears that wiggly $(p, q)$-string under consideration should be significantly different from Nambu-Goto string studied in Ref. [13] – for example, $(p, q)$-string has nontrivial worldsheet gauge field excitations. However, this is not the case. We have shown already that, in deriving the proper DBI Lagrangian Eq. (13), it was crucial not to discard the worldsheet gauge field fluctuations but integrate them out explicitly. Harmonic fluctuation described by the resulting DBI Lagrangian is exactly the same as that of Nambu-Goto string. Since all one needs for determining the structure of Eq. (21) are harmonic fluctuation $\langle V^2 \rangle$ and microscopic equation of state, the renormalization group equation can be derived straightforwardly by endowing scale dependence to string mass density and tension and utilizing the techniques of Ref. [13]. Demanding that energy-momentum conservation and equations of motion are satisfied for the wiggly string with worldsheet 2-velocity $u^a(\sigma) (u^a u_a = +1)$, we find the renormalization-group equation in Fourier mode $k$-space at the lowest non-trivial order:

$$\frac{d}{d \ln k} \rho(k) = -W_T(k) \rho(k) - W_L(k) \{ \rho(k) - T(k) \} + \cdots$$

$$\frac{d}{d \ln k} T(k) = -\frac{1}{2} W_T(k) \{ \rho(k) + T(k) \} + W_L(k) \{ \rho(k) - T(k) \} + \cdots,$$

where power-spectrum of effective transverse and longitudinal fluctuations is given by:

$$W_T(k) \equiv k \int_0^\ell \frac{d \sigma}{\ell} e^{ik \sigma} \left[ 7 \langle \nabla_0 \Psi(\sigma) \cdot \nabla_0 \Psi(0) \rangle + \langle \nabla_0 \Xi(\sigma) \cdot \nabla_0 \Xi(0) \rangle \right]$$

$$W_L(k) \equiv k \int_0^\ell \frac{d \sigma}{\ell} e^{ik \sigma} \left[ \langle (u^1(\sigma)/u^0(\sigma)) \cdot (u^1(0)/u^0(0)) \rangle \right].$$

We have ignored second- or higher-order corrections on the right-hand side of Eq. (22). It should then become clear that, using the microscopic equation of state $T(\rho) = \rho \equiv T_{(p,q)}$ as the ultra-violet boundary condition of the renormalization-group equations, there exists a non-trivial infrared fixed point, where

$$T(k) \to T_{IR} \ll T_{(p,q)}, \quad \rho(k) \to \rho_{IR} \gg T_{(p,q)} \quad \text{such that} \quad T_{IR} \cdot \rho_{IR} = T^2_{(p,q)}.$$ 

Note that, even though the microscopic $(p, q)$-string has no physical longitudinal excitation, effective smooth string acquires non-trivial longitudinal dynamics via coarse-graining and has propagation velocity $v^2_L = -dT(\rho)/d\rho$. The infrared fixed point is characterized by the fact that the longitudinal propagation velocity become equal to the transverse propagation velocity, $v^2_T, v^2_L \to (T_{(p,q)}/\rho_{IR})^2$. We note that this is much less than speed of light, hence, conclude that low-energy excitation of wiggly $(p, q)$-string is conveniently described by a non-relativistic dynamics of effective smooth string.
M-Theory Limit and Evanescent Bound-State: We finally discuss a possible existence of novel bound-state in the middle of continuum excitations once the triple string junction is promoted to M-theory configuration. It has been known [5, 6] that the triple string junction arises from M-theory by starting with a "pant" configuration of membrane and wrapping each of the membrane prongs on different cycles of compactified two-dimensional torus.

Consider a D-string impinged by several F-strings. In the M-theory limit, the configuration look like a ‘twisted tube’ as depicted in Fig. 3 (only the cycle associated with D-string charge is shown explicitly). Excitations of such a triple membrane junction is carried by waves propagating on the surface of ‘twisted tube’. Along the direction of common $S_1$ cycle, the waves satisfy periodic boundary condition. Hence, for each periodic normal modes, propagation of low-energy waves along the tube direction is described by a one-dimensional Helmholtz equation. Is there any new phenomena due to the fact that the tube is twisted instead of being straight? We now provide an argument that suggests a positive answer to this question.

For definiteness, we will take the limit that the common $S_1$ cycle is small but non-zero. In describing the propagation of effective one-dimensional waves along the direction of the twisted tube (See Fig. 3), it is more convenient to use the natural coordinate $\sigma$ introduced earlier in Eq.(11). Crucial point is that the projection angle $\theta$ varies every time one passes the triple membrane junction. Geometrically, it is clear then that the junction region (denoted as I in Fig. 3) has effectively larger surface of the tube than the region in between. This results in relative decrease of the normal-mode frequency in the junction region compared to the membrane-prong or asymptotic prong regions. More concretely, let us start from a small fluctuation equation of motion of the membrane in terms of physical gauge coordinates $x_0, x_1, x_2$ (the analogs of $x_0, x_1$ in Eq.(8)). If we make a change of variables to natural curved coordinates $\sigma$ along the tube.
direction, the effective one-dimensional wave equation takes the following form:

\[
\left[ \partial_t^2 - \partial_\sigma^2 - V(\sigma) + (k_{2n})^2 \right] X_n^i(\sigma, t) = 0,
\]

(25)

where \(k_n\) denotes the normal frequency around the common \(S_1\) direction and \(X_n^i\) is the normal mode of membrane fluctuation. The crucial term \(V(\sigma) = (\kappa(\sigma)/2)^2\) is a potential induced during the course of the change of variables from the ambient flat space coordinate \(x_1, x_2\) to the curved one \(\sigma, x_2\), and is expressible in terms of extrinsic curvature \(\kappa\) of the tube \([15, 16]\). It is transparent that the induced potential can be interpreted as a local decrease of the normal frequency \((k_{1m})^2\). If the induced potential is smooth enough, \(V(\sigma) \approx \langle V \rangle = \text{constant}\), effective dispersion relation locally in the neighborhood of the triple membrane junction is

\[
\omega^2 = (\tilde{k}_{1m})^2 + (k_{2n})^2 \quad \text{where} \quad (\tilde{k}_{1m})^2 \equiv (k_{1m})^2 - \langle V \rangle.
\]

(26)

As stated, the normal frequency along the tube direction is lowered effectively by twisting of the membrane. In particular, when \(k_{1m}^2 \to 0\), \(\tilde{k}_{1m}^2 < 0\) and \((k_{2n}/\omega) > 1\)\!. This is an indication that there may exist effectively one-dimensional evanescent bound-states \([17]\) in the middle of the scattering continuum, whose wave function is localized near the triple membrane junction region. Note that these excitations are non-BPS since both \(\tilde{k}_1\) and \(k_2\) should be nonvanishing.

We conjecture that this is not specific to the triple membrane junction but very generic to small fluctuations of any curved membrane. We will report further details elsewhere.

References

[1] J.H. Schwarz, Phys. Lett. 360B (1995) 13.
[2] E. Witten, Nucl. Phys. B460 (1996) 335.
[3] S. Lee and S.-J. Rey, Nucl. Phys. B508 (1997) 107, hep-th/9706115.
[4] E. Cremmer and J. Scherk, Nucl. Phys. B72 (1974) 117.
[5] O. Aharony, J. Sonnenschein and S. Yankielowicz, Nucl. Phys. B474 (1996) 309.
[6] J.H. Schwarz, Nucl. Phys. [Proc. Suppl.] 55B (1997) 1.
[7] K. Dasgupta and S. Mukhi, hep-th/9711094.
[8] A. Sen, hep-th/9711130.
[9] C.G. Callan and J. Maldacena, hep-th/9708147.
[10] G. Gibbons, hep-th/9709027.

[11] S. Lee, A. Peet and L. Thorlacius, hep-th/9710097.

[12] A. Hashimoto, hep-th/9711097.

[13] J. Hong, J. Kim and P. Sikivie, Phys. Rev. Lett. 69 (1992) 2611 (Erratum-ibid. 74 (1995) 4099);
    J. Kim and P. Sikivie, Phys. Rev. D50 (1994) 7410.

[14] A. Vilenkin, Phys. Rev. D41 (1990) 3038;
    B. Carter, Phys. Rev. D41 (1990) 3869.

[15] R.L. Schult, D.G. Ravenhall and H.W. Wyld, Phys. Rev. B39 (1989) 5476;
    Y. Avishai et.al, Phys. Rev. B44 (1991) 8028.

[16] P. Exner and P. Seba, J. Math. Phys. 30 (1989) 2574;
    J. Goldstone and R.L. Jaffe, Phys. Rev. B45 (1992) 14100.

[17] See, for example, L.D. Landau and E.M. Lifschytz, Quantum Mechanics, pp. 58, (Pergamon Press, 1977, New York); R. Blanckembecler, M.L. Goldberger and B. Simon, Ann. Phys. 108 (1977) 69.