Characterization of surface crack width in plates using Rayleigh wave electromagnetic acoustic transducers

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Abstract. Surface crack in the plate like structures is one of the initial indication for its degradation. This indication could be very critical and requires some serious maintenance because this continuous exposure will lead to a severe damage to the safety and even to environment. To evaluate the structural performance and safety, a regular monitoring of such cracks is needed. A quantitative characterization method is utilized for the estimation of surface groove width instead of depth or length, which is previously done by many researchers, using Rayleigh waves in pitch-catch mode using electromagnetic acoustic transducers (EMATs). The method employs the experimentally determined reflection coefficient and transmission coefficient of Rayleigh waves scattered at a surface groove by a number of varying widths, which are compared to the reference curves, obtained from two-dimensional finite element method (2D-FEM) simulations for the same groove width as used in experiments. A good relation between FEM results and experimental results is found. Two different EMAT couples of different centre frequencies were employed to explore the measurement, and to compare and analyse the quantitative characterization of different groove widths. Both the FEM results and experimental results are compared and verified for the different groove widths. This comparison shows some percentage of error for the groove width ranging from 0.3 mm to 1.8 mm.

1. Introduction
Surface cracks generated by surface strains in materials, such as rails, steel plates, thick walled pipes, etc., can greatly affect the structural integrity, either during manufacturing process or in the service period. Non-destructive testing (NDT) is therefore becoming an increasingly essential tool for ensuring the structural integrity and safety[1]. Although to confirm the presence and location of cracks by NDT is of great importance, an accurate estimation of crack depth is much more crucial for structural health evaluation[2]. Rayleigh waves are extensively applied in NDT for the detection of surface defects, and are particularly appropriate for crack sizing[3]. Rayleigh wave propagates near the surface with its velocity independent of frequency, and attenuates exponentially when travelling from the surface. Rayleigh waves can also propagate along curved surfaces, such as pipes with fairly large diameters, in which variation of velocity and phase is negligible[4, 5]. The energy of Rayleigh waves is concentrated near the surface, making them more sensitive to surface cracks compared with bulk waves[6]. Furthermore, Rayleigh waves can be employed for the characterization of angled surface cracks[1-7]. Crack depth can be estimated by the scattering phenomena when Rayleigh waves incident on a crack. The travelling time and amplitude information of the incident, reflected, and transmitted Rayleigh waves carry the detail characteristics of the crack[3, 8]. In time domain, the propagating time of a surface wave along the crack’s cliffs can be measured to estimate the extent of crack penetration[9, 10]. It is an effective approach for depth characterization when the depth is greater than 0.8λ, in which the λ is the
corresponding wavelength. Date et al.\cite{8} employed the surface wave transmission method to measure the crack depth to a scale as small as 0.82 mm with a mean error of 0.01 mm using a 5 MHz transducer. Similarly, Baby et al.\cite{11} employed time-of-flight diffraction to estimate crack depth in the range of 1.68-19.04 mm with excellent accuracy by using 4 MHz angle beam probes. However, time-domain methods are not appropriate for the evaluation of a surface crack with a depth less than 0.8\l, due to the irregular time delay effect. Hence, the relative amplitudes of the incident, reflected, and transmitted Rayleigh waves at a crack, in form of reflection and transmission coefficients, become prospective for crack depth evaluation. Kino\cite{12} and Ault\cite{13} adopted the reciprocity theory to associate the scattering coefficients with the characteristics of the flaw, and Tien et al.\cite{14} extended this formulation for Rayleigh waves to observe the surface cracks in ceramics. Reflection and transmission coefficient curves can be obtained by numerical calculation and simulation\cite{15-17}. In contrast with time-domain methods, the methods based on reflection and transmission coefficients are more efficient for estimation of infinitesimal surface cracks. Resch et al.\cite{18} utilized the reflection coefficient of Rayleigh waves to estimate the crack depth less than 0.05 mm with an average error of 9.6\% using surface acoustic wave wedge transducers centred around 3.5 MHz. Hevin et al.\cite{19} detected cracks in concrete structures experimentally, and determined the crack depth within an error of 15\% of the actual size. However, when the crack depth is greater than 0.3\l, both the reflection and transmission coefficients oscillate severely as crack depth increases, resulting in an ambiguous relationship between the scattering coefficients and the crack depth\cite{20}. It is therefore difficult to accurately estimate crack depth by reflection and transmission coefficients of Rayleigh waves with a single centre frequency\cite{21}. In previous work, piezoelectric transducers and lasers have been widely used to generate and detect Rayleigh waves \cite{22-25}. Piezoelectric transducers are efficient for energy transformation, but the quality of received signal is critically affected by coupling conditions. Generally, laser ultrasonic equipment is much more expensive and complicated for practical applications. Edwards et al.\cite{2} and Jian et al.\cite{21} employed EMATs to generate low frequency wide band Rayleigh waves for estimation of surface crack depth. EMATs are non-contact ultrasonic transducers that do not require a couplant, and appear to be much more robust despite the coupling conditions than piezoelectric transducers\cite{20}. Meanwhile, the implementation of EMATs show great simplicity and repeatability, compared with laser ultrasonic\cite{27}.

In this work, Rayleigh wave EMATs are adopted to estimate the width of surface grooves based on reflection and transmission coefficient curves. The reflection coefficient and transmission coefficient are obtained through 2D-FEM simulations using COMSOL multi physics and through experiments. Two pairs of EMAT transducer having different central frequencies are selected to verify results more accurately. Multi-frequency method is used to estimate the crack width, by which the measurement accuracy is improved. The experimental curves obtained are matched with the reference curves obtained through 2D FEM simulations. The reference curves are calculated at a variable groove width values, and the measurement procedure is employed to compensate the amplitude decay of the received signals at a propagating distance, both of them are verified with a good accuracy. Two EMAT couples with centre frequencies of 1.0 MHz and 1.5 MHz are fabricated to estimate the grooves width in order to extend the measuring range. Each Rayleigh wave EMAT is composed of a permanent magnet and meander-line coil. When they are placed on aluminium plate and excited by alternating current, Rayleigh wave will be generated based on Lorentz force, magnetostriction force, and magnetization force mechanisms. An approach of multi frequency is used to estimate each groove’s width, to improve the measurement accuracy. So finally, for a specific groove, the reflection and transmission coefficient curves are detected by the Rayleigh waves at different frequencies and its width is characterized by the reference curves and measured curves.

2. Calculation of scattering coefficients using 2D-FEM simulation

2.1. FEM modeling

A two-dimensional finite element model (2D-FEM) is used to calculate the reflection coefficient and transmission coefficient using Rayleigh waves for a number of grooves of various width, using COMSOL multi-physics (COMSOL Inc., Sweden). An aluminium plate of thickness \( t \) and length \( w \) is used having a rectangular groove of width \( b \) and a constant depth \( a \) of value 0.6 mm perpendicular to the plate surface.

The origin of the coordinate system is the lower left corner of the aluminium plate. To ensure the propagation of waves at the surface of the plate the thickness of the plate is kept more than the 6\( \lambda \). The
value of groove width $b$ changes from 0.3 mm to 1.8 mm. The groove and upper surface of the aluminium plate are set to free boundaries, described by a bold line in Figure 1, whereas to simplify the received signal the other sides are set to low-reflecting boundaries. Rayleigh waves were generated at two different excitation frequencies of 1 MHz and 1.5 MHz, with its corresponding wavelengths $\lambda$ of 3 mm, 2 mm, and Rayleigh wave velocity of 3000 m/s using FEM simulations. The mesh grid in the model consists of a free triangular element mesh of size 0.1 $\lambda$ is used in the model. A five-cycle Hanning window excitation signal is used at the surface of the plate at a distance $l$ from the receiving point. Four loading points having adjacent spacing of one wavelength are used. To avoid the wave overlapping on the receiving points the distance $l$ should be properly decided.

| Table 1. Grooves dimensions |
|----------------------------|
| Width (mm) | Depth (mm) | Length (mm) |
| 0.3        | 0.6        | 30          |
| 0.6        | 0.6        | 30          |
| 0.9        | 0.6        | 30          |
| 1.2        | 0.6        | 30          |
| 1.5        | 0.6        | 30          |
| 1.8        | 0.6        | 30          |

2.2. FEM results

Considering two receiver points $P$ and $Q$, the displacements using FEM simulations at both the central frequencies of Rayleigh waves for the different values of groove width as given in Table 1 are obtained. Figs. 2(a) and 2(b) show the out-of-plane displacements at receiving points $P$ and $Q$ for a centre frequency of 1.5 MHz, at a constant depth $a$ of 0.6 mm and width $b$ of 0.3 mm of the groove. The received Rayleigh wave signal at point $P$ generated from the load point is shown in Figure 2(a), where the first wave packet is the direct received signal and the second wave packet is reflected signal from the groove of width 0.3 mm and depth 0.6 mm. Figure 2(b) shows the received signal at a receiving point $Q$, which is transmitted through the groove at the same width value of 0.3 mm.

Similarly Figure 2(c) shows the received signal at point $P$ at the same depth value and width of the groove is 1.2 mm, containing direct signal and reflected signal, at a central frequency of 1.0 MHz. The received transmitted signal at the same central frequency and for the same groove parameters is shown in Figure 2(d). Using the received Rayleigh wave signal, the reflection and transmission coefficients are calculated. The reflection coefficient is the ratio of the amplitude of reflected signal to the direct signal at the receiving point $P$ and transmission coefficient is the ratio of the amplitude of transmitted signal at point $Q$ to direct signal amplitude at point $Q$ without a groove. The reflection and transmission
coefficients calculated from the FEM simulations at both the central frequencies for the given range of width $b$ values are shown in Figure 3.

(a) Point $P$ at a center frequency of 1.5 MHz, and width $b = 0.8$ mm (reflected signal)

(b) Point $Q$ at a center frequency of 1.5 MHz, and width $b = 0.8$ mm (transmitted signal)

(c) Point $P$ with a center frequency of 1.0 MHz, and width $b = 1.2$ mm (reflected signal)
Figure 3(a) shows the reflection coefficient curve calculated from FEM simulation results for a varying width ranging from 0.3 mm to 1.8 mm at a constant depth of 0.6 mm at central frequency of 1.5 MHz, whereas the transmission coefficient curve calculated from the FEM simulation results for the same parameters of groove is shown in Figure 3(b). Similarly, the reflection coefficient curve and transmission coefficient curve are calculated at a central frequency of 1.0 MHz for the same parameters of groove, which are shown in Figure 3(c) and Figure 3(d) respectively.

3. Experimental arrangement

3.1. Specimen
An aluminium plate having the dimension of length 600 mm, width 450 mm and thickness 25 mm is used as an inspected sample. Six grooves of constant depth value 0.6 mm and varying width values of 0.3 mm to 1.8 mm with a step size of 0.3 mm are machined on the specimen. The groove width is denoted by \( b \). The length of all the grooves is 30 mm. Three grooves are designed on each side of the plate at a different end, the adjacent groove has a vertical separation distance of 90 mm.

3.2. Rayleigh wave EMATs
The outstanding reliability and repeatability presented by EMATs are very important for the accurate estimation of groove width. Two Rayleigh wave EMAT pair of transducers were designed and
(b) Calculated transmission coefficient curve at a surface groove of varying width $b$ and a constant depth $d = 0.6$ mm with a central frequency of 1.5 MHz

(c) Calculated reflection coefficient curve at a surface groove of varying width $b$ and given depth $d = 0.6$ mm with a central frequency of 1.0 MHz

(d) Calculated transmission coefficient curve at a surface groove of varying width $b$ and given depth $d = 0.6$ mm with a central frequency of 1.0 MHz

Figure 3. Calculated coefficient curves.
fabricated to generate and detect Rayleigh waves using aluminium plate. Each EMAT is composed of a meander-line coil having six turns of wire, where the length of the wire is 20 mm. The spacing between adjacent lines of each EMAT couple are 1.5 mm and 1.0 mm, respectively, which has a Rayleigh wave wavelength of 3.0 mm and 2.0 mm with corresponding centre frequencies of 0.1 MHz and 1.5 MHz, respectively. The coils are 56 mm long and 30 mm wide. The permanent magnet used in the transducer is NdFeB35 with a dimension of 50 mm × 30 mm × 10 mm. The experimental frequency performance was assessed, and the results presented are in good agreement with the required specifications. The frequency characteristics are shown in Figure 4.

3.3. Experimental setup

The experimental setup consists of RPR-4000 High Power Pulser/Receiver (RITEC Inc., USA), impedance analyser (Agilent Technologies, USA), digital oscilloscope (Tektronix Inc., USA), impedance matching box for transmitter, impedance matching box for receiver, aluminium plate, transmitter EMAT and receiver EMAT as shown in Figure 5.

The EMATs were matched using an impedance analyser. The reflected wave, direct incident wave and transmitted wave are measured by using pitch-catch mode, these modes are denoted as R-wave, I-wave and T-wave, respectively. EMAT impedance matching networks and impedance analyser was used to find out the impedance for transmitter and receiver for the maximum transfer of energy. A 5-
cycle Hanning window sinusoidal tone burst signal was given to the transmitter as an excitation signal for the generation of Rayleigh waves into the specimen. Centre frequencies of 1 MHz and 1.5 MHz were used, for the two different EMAT couples used in this work. The transmitter and receiver EMATs were placed perpendicular to the grooves, and aligned along their centre lines. The distance between the transmitter EMAT and groove is denoted by \( (1+d) \) whereas the distance between receiver EMAT and groove is denoted by \( d \) and similarly the distance between a receiver EMAT, for the transmitted signal, and groove is denoted by \( d \). The transmitter and receiver EMATs are placed on the same side of the groove for the measurement of incident wave and reflected wave as shown in Figure 6(a) whereas to measure the transmitted wave the receiver is placed on the other side of the groove as shown in Figure 6(b). The received signal is then filtered by a band pass filter (400 kHz to 2500 kHz for 1.0 MHz transducer and 800 kHz to 2500 kHz for 1.5 MHz transducer), and is being averaged 256 times in oscilloscope.

3.4. Measurement procedure

The reflection and transmission coefficients were achieved using 2-D FEM simulations, which is the ratio of reflected and transmitted signal amplitudes to incident signal amplitude, and no attenuation was considered during wave propagation. However, using experimental data, the amplitudes of Rayleigh waves are affected because of attenuation through propagating distance because of diffusion of the acoustic signal and due to absorption in material. Clearly, the reflection and transmission coefficients calculated through experiments would be quite different as compared to the values obtained without attenuation. So it is noticeable, that attenuation should be taken into account during experiments, and the reflection and transmission coefficients should be compensated and calculated as: (i) the Rayleigh waves propagation with attenuation curves should be measured first using a pair of transducers, (ii) the incident, reflected, and transmitted signals are measured using the receiver EMAT as described before, (iii) the amplitudes of incident, reflected, and transmitted waves are calculated and compensated according to their propagation distances, (iv) the reflection and transmission coefficients denoted by \( C_{\text{ref}} \) and \( C_{\text{tran}} \) respectively, and are compared with the reflection and transmission coefficients obtained through FEM results. Due to some factors, such as, surface texture and roughness the attenuation curves of Rayleigh waves propagating in the specimen may be varied. Therefore, the attenuation curves should be calculated for the inspecting material through experiments. During the experiments, Rayleigh waves were generated by placing a transmitter transducer at a distance relatively far from the groove to exclude the disturbances included by the generated body waves. The receiver was located, for both reflected signal and transmitted signal on either side of the groove, at a suitable distance to improve their detection quality. The Rayleigh wave EMATs arrangement in the experiment is shown in Figure 6.
Where the distance between transmitter and groove is 300 mm, the distance between receiver and groove is 50 mm. The receiver was located on either side of the groove at a distance of 50 mm to detect reflected or transmitted waves.

4. Results and discussion

The measurement results obtained through experiments for width value of groove $b = 0.3$ mm is presented in Figure 8 with a centre frequency of 1.5 MHz. The results presented in Figure 7(a) and Figure 7(b) correspond to the measurement setups as shown in Figure 6(a) and Figure 6(b), respectively. Wave package $WP_1$ as shown in Figure 7(a) with an arrival time of $t = 83 \mu s$ is a direct received Rayleigh wave at point $P$ which is called incident wave and denoted by $I$-wave. This $I$-wave after propagation is reflected from the groove and received at the same receiver, denoted by $WP_2$. Similarly, as shown in Figure 7(b), the wave packet $WP_3$ is a transmitted signal received at point $Q$ denoted by $R$-wave. The $R$-wave and $T$-wave can be monitored by the receivers located at the left and right sides of the groove respectively.

![Diagram](image)

Figure 6. The arrangement of transmitter and receiver in experiments for the detection of $I$-wave, $R$-wave, and $T$-wave at a groove on the steel specimen
Figure 7. Average signals based on summations of over 256 individual five cycle excitations detected by the receiver with a center frequency of 1.0 MHz and a crack width $b = 0.3$ mm.
Similarly the direct incident wave, reflected wave and transmitted wave at the same receiving points are received at a frequency of 1 MHz as shown in Figure 7(c) and Figure 7(d). The peak-to-peak amplitudes of incident wave, reflected wave and transmitted wave were compensated for the calculation of \( C_{\text{ref}} \) and \( C_{\text{tran}} \). The other wave packages shown in the figures (as denoted by \( WP_3, WP_4, WP_6, \text{and} WP_7 \)) are the Rayleigh waves generated from the groove’s edges, which was previously scattered by the incident wave, and reflected wave from the bottom edge, as shown in Figure 8\[28,29\]. The wave packets \( WP_3 \) and \( WP_6 \) were generated because of the first reflected shear wave, and \( WP_4 \) and \( WP_7 \) were generated due to second reflected shear wave.

### 4.1. Experimental verification of reflection and transmission coefficients

Through experiments the reflection coefficient and transmission coefficient are calculated and compared with reflection and transmission curves obtained through FEM simulation to be verified. These curves are obtained for different width values \( b \) of groove. The measurements are done for two different central frequencies on the specimen. Figure 9(a) and Figure 9(b) show the comparison of reflection and transmission coefficient curves calculated from the FEM results and experimental results with a centre frequency of 1.5 MHz. These curves are plotted with respect to width of groove \( b \), to evaluate the effect on \( C_{\text{ref}} \) and \( C_{\text{tran}} \) with the excitation frequency. Similarly, Figure 9(c) and Figure 9(d) show the comparison of both the FEM simulation and experimental results at a central frequency of 1 MHz. Based on the multi-frequency measurements, as shown in Figure 9, for a given width \( b \), the calculated coefficients are positioned in different states of coefficient curves. For example, at a frequency 1.5 MHz, and \( b = 0.4 \text{ mm} \), \( C_{\text{ref}} \) and \( C_{\text{tran}} \) are more likely to be situated in the same region.

![Wave propagation path in experiments.](image)

**Figure 8.** Wave propagation path in experiments.

![Comparison of reflection coefficients versus groove width in the range \( b = 0.3 – 1.8 \text{ mm} \)](chart)

(a) Comparison of reflection coefficients versus groove width in the range \( b = 0.3 – 1.8 \text{ mm} \)
Comparison of transmission coefficients versus groove width in the range $b = 0.3 – 1.8$ mm

Comparison of reflection coefficients versus groove width in the range $b = 0.3 – 1.8$ mm

Figure 9. Reflection and transmission coefficients curves: FEM result and experimental measurements.
Similarly, when the frequency is 1.0 MHz, and \( b = 0.9 \text{ mm} \), \( C_{\text{ref}} \) is more likely to be placed in the oscillating state, whereas the \( C_{\text{tran}} \) is in a state of monotonic. So, varying \( C_{\text{ref}} \) and \( C_{\text{tran}} \) can be calculated for a given groove using Rayleigh waves at various centre frequencies. Hence, the width \( b \) of the groove can be expected through the comparison of both the \( C_{\text{ref}} \) and \( C_{\text{tran}} \). The detected deviations between the experimental and simulation results is due to different causes. The Rayleigh wave attenuation curve used to compensate the amplitudes of \( I \)-wave, \( R \)-wave, and \( T \)-wave. The calculated results through the experiments will be effected by the measured attenuation curve. Moreover, the measured results are effected through electromagnetic interference from the measurement system.

4.2. Measurement of a groove

Each groove in the testing material is detected and calculated using two different central frequency EMATs, and \( C_{\text{ref}} \) and \( C_{\text{tran}} \) are obtained. Comparing experimentally obtained \( C_{\text{ref}} \) and \( C_{\text{tran}} \) with the reference coefficient curves obtained through FEM simulations a number of grooves with a varying width value \( b \) is obtained owing to the non-monotonicity of the curves. Based on this comparison, to eliminate the errors, the value of the groove was highly approximated to the actual width value \( b \). To estimate the width of all the six grooves were obtained by comprehensively analysing the reflection and transmission coefficients of Rayleigh waves, and the results are presented in Figure 9 for values of \( b = 0.3 \) to 1.8 mm. The calculated values of \( b \) correlate well with the actual width values \( b \) over the whole range, and offer good accuracy. As shown the above figures, the accuracy of the estimated and calculated grooves varies at different points in the given range. During the measurements, Rayleigh waves contribute different weights to the evaluation of width at different width ranges owing to their different sensitivities at different frequencies. In this study, the measurements of the reflection and transmission coefficients for \( b = 1.2 \text{ mm} \) are peculiar, in that a width of 2.5 mm is located near local maxima in the reflection coefficient curves for centre frequencies of 1.5 MHz and 1 MHz (Figs. 9(b), (c) and (d)). Though, the slopes are relatively flat near the extreme value, such that the reflection coefficient is not sensitive to variations in the crack width within this range. Similarly, the observed dispersion for \( b = 1.2 \text{ mm} \) is not presently explainable as shown in Figure 9(d). A groove depth of 3.0 mm is possibly the upper limit of the measurement range for the proposed approach with centre frequencies of 1.0 MHz and 1.5 MHz. For a groove depth more than 3 mm, the reflection coefficients of Rayleigh waves employing 1.0 MHz and 1.5 MHz centre frequencies are nearly constant, and are useless to estimate the crack depth. Similarly, the lower limit of groove is 0.2 mm.

5. Conclusion

A methodology using EMATs to generate and receive Rayleigh waves in a pitch-catch mode is developed for the width characterization of surface grooves, and the relative error of width characterization ranges from 14\% (with respect to crack width of 0.3 mm) to 2\% (with respect to crack width of 1.5 mm). The reflection and transmission coefficients are experimentally obtained and compensated by the measured attenuation curves. Then, the compensated coefficients are compared with the reference curves obtained through 2D-FEM using COMSOL multi physics, where the width of the groove varies from 0.3 mm to 1.8 mm. Two different types of central frequency EMATs (1 MHz and 1.5 MHz) are used to carry out the experiments for the width characterization, where the measurement range and reliability are improved significantly. This research on EMATs demonstrates a suitable approach to generate and detect Rayleigh waves. The measured reflection and transmission coefficients are compensated according to each of their propagation distance, and the experimental results with compensation exhibit good agreement with the corresponding simulation curves. For a given width, the measured coefficients using each excitation frequency are plotted and presented. This cross-referring operation plays a vital role in the characterization of crack width, by which the width can be finally estimated.

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