Parameter Identification of Lithium-ion Battery Equivalent Circuit Model Based on Limited Memory Recursive Least Squares Algorithm with Variable Forgetting Factor

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Abstract: Equivalent circuit method is the most widely used methodology in dynamic modeling of lithium-ion battery. An equivalent circuit with second-order RC network is used to model lithium-ion battery, and a limited memory recursive least square with variable forgetting factor (VFF-LMRLS) is proposed to identify the model parameters in this paper. Firstly, based on the current and voltage data measured from the battery cyclic discharging experiment, the VFF-LMRLS algorithm is used to identify the time-varying parameters of equivalent circuit model. Then, the model verification system is constructed by taking the average value of the identification results in the stable stage as the component parameter value of the equivalent circuit. Finally, through the comparative experiment and analysis with the variable forgetting factor RLS (VFFRLS), it is verified that the terminal voltage error of the proposed method is smaller, indicating that the identified model parameters are closer to the actual parameters.

1. Introduction

Lithium-ion battery has been of extensive use for electric vehicles and energy storage system due to its advantages of high energy density, long cycle life, low self-discharging rate and environmental friendliness [1]. Among them, lithium batteries with ternary nickel cobalt manganese (NCM) and lithium ferric phosphate (LFP) as cathode materials are widely used in electric vehicles [2], becoming the main power batteries at present. Among many indexes of power battery, state of charge (SOC) is of great significance for energy optimization management and health life management of electric vehicles [3]. In the online estimation methods of the SOC, the filtering algorithm is the best choice to replace the coulomb accumulation algorithm because of its unique closed-loop correction structure, good robustness and fast convergence [4]-[5]. However, the SOC online estimation accuracy based on the filtering depends highly on the adopted battery model [6]-[7] and parameter identification method [8]-[9].

At present, the models of the battery mainly include electrochemical model, neural network model and equivalent circuit model (ECM). Compared with the previous two models, the ECM based on experimental data is a network composed of simple components such as ideal voltage source, resistance and capacitance in series and parallel to equivalent the external characteristics of actual battery. The common battery ECMS include Thevenin model, PNGV model and multi-order RC network model [10] - [13]. Among these ECMS, considering the accuracy and complexity of the model, the second-order RC
ECM is employed to simulate the battery characteristics in this paper. For a specific ECM, the reliability and accuracy of the parameters in the model are directly determined by the quality of the model parameter identification method. Recursive least squares (RLS) [14] is the most widely used method for parameter identification of lithium battery model at present. RLS can be used not only for static and dynamic systems, but also for linear and nonlinear systems. To improve the convergence speed and reduce the sensitivity to noise, forgetting factor is often introduced in RLS algorithm [15]. For the system with fast time-varying and irregular parameter changes, the RLS algorithm based on fixed forgetting factor [16] has the phenomenon of “data saturation”, which cannot effectively track the changes of parameters. RLS algorithm based on variable forgetting factor [2], [15], improves the performance and speed of tracking time-varying system parameters by timely adjusting the dependence on past data, but the identified data is always related to the old data. To overcome the “data saturation” and remove the influence of the old data on the algorithm, a variable forgetting factor is added to the limited memory RLS algorithm, and a VFF-LMRLS algorithm for parameter identification of lithium battery ECM is proposed. Through the limited measured current and voltage data collected by the cyclic discharge shelving experiment (CDSE), the lithium battery model parameters are identified by VFF-LMRLS and VFFRLS, and the Simulink verification model is built. The rest is arranged as follows:

Section 2 introduces the modeling process of battery. Section 3 gives the detailed derivation process of battery ECM parameter identification method, namely VFF-LMRLS algorithm. In section 4, the verification model is built to compare, analyze and verify the proposed methods. Section 5 is the summary of the full text.

2. Battery Modeling

2.1. Second-order RC ECM

The circuit topology of the battery second-order RC ECM is shown in Fig. 1. $U_{OC}$ represents the open-circuit voltage, $I$ and $U_L$ represent the load current and voltage, respectively. $R_0$ represents the ohmic internal resistance. $R_1$ and $R_2$ represent the electrochemical and concentration polarization resistance, respectively. $C_1$ and $C_2$ represent the electrochemical and concentration polarization capacitance, respectively. $U_1$ and $U_2$ represent the voltages of polarization resistors $R_1$ and $R_2$, respectively.

![Fig 1. Second-order RC ECM of lithium-ion battery](image)

According to KVL theorem, the mathematical expression of load voltage $U_L$ in Fig.1 is

$$U_L = U_{OC} - IR_0 - U_1 - U_2$$

(1)

Where $U_1$ and $U_2$ satisfy the relationship as follows

$$\begin{align*}
R_1 C_1 \frac{dU_1}{dt} + U_1 &= IR_1 \\
R_2 C_2 \frac{dU_2}{dt} + U_2 &= IR_2
\end{align*}$$

(2)

2.2. Least Square Form of Battery ECM

Laplace transform is performed on Equation (2) and substituted into Equation (1) to obtain the frequency domain expression of the battery model.
\[ U_{k}(s) = U_{OC}(s) - I(s) \left( R_{0} + \frac{R_{1}}{1 + R_{1}C_{1}s} + \frac{R_{2}}{1 + R_{2}C_{2}s} \right) \] (3)

Time constant are \( \tau_{1} = R_{1}C_{1}, \tau_{2} = R_{2}C_{2}, a = \tau_{1} + \tau_{2}, b = \tau_{1} + \tau_{2}, c = R_{0} + R_{1} + R_{2}, d = (R_{0} + R_{1})\tau_{2} + (R_{0} + R_{2})\tau_{1} \). Then equation (3) is equivalent to

\[
\frac{U_{k}(s) - U_{OC}(s)}{I(s)} = -\frac{aR_{0}s^{2} + ds + c}{as^{2} + bs + 1} \] (4)

Where \( U(s) = U_{k}(s) - U_{OC}(s) \), and \( T \) is the sampling time. Introducing \( s = \frac{2}{T} - \frac{1}{z^{-1}} \) for linear transformation, equation (4) is transformed into

\[
\frac{U(z^{-1})}{I(z^{-1})} = \frac{1}{1 - \theta_{1}z^{-1} - \theta_{2}z^{-2}} \] (5)

Where, the expressions of \( \theta_{1} \sim \theta_{5} \) are as follows

\[
\begin{align*}
\theta_{1} &= \frac{8a - 2T^{2}}{4a + 2bT + T^{2}} \\
\theta_{2} &= \frac{2bT - 4a - T^{2}}{4a + 2bT + T^{2}} \\
\theta_{3} &= \frac{-4aR_{0} + 2dT + cT^{2}}{4a + 2bT + T^{2}} \\
\theta_{4} &= \frac{8aR_{0} - 2cT^{2}}{4a + 2bT + T^{2}} \\
\theta_{5} &= \frac{-4aR_{0} - 2dT + cT^{2}}{4a + 2bT + T^{2}}
\end{align*}
\] (6)

The discrete recurrence formula of voltage \( U_{k} \) obtained from equation (5) is

\[
U_{k} = \theta_{1}U_{k-1} + \theta_{2}U_{k-2} + \theta_{3}I_{k-1} + \theta_{4}I_{k-2} + \theta_{5}I_{k-3}
\] (7)

Let \( h_{k} = [U_{k-1}, U_{k-2}, I_{k-1}, I_{k-2}]^{T}, \theta_{k} = [\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}, \theta_{5}]^{T}, \) and the recursive least squares standard form of equation (7) is

\[
U_{k} = h_{k}^{T}\theta_{k}
\] (8)

Where \( h_{k} \) is the information vector composed of input and output data, and \( \theta_{k} \) is the parameter vector to be identified of the battery system.

3. Parameter identification method

3.1. Recursive Least Squares

Assuming that the output of the battery ECM system is \( y_{k} \), the least squares recursive form of the parameters to be identified can be expressed as

\[
\hat{\theta}_{k} = \hat{\theta}_{k-1} + K_{k}e_{k}
\] (9)

where

\[
e_{k} = y_{k} - h_{k}^{T}\hat{\theta}_{k-1}
\] (10)

\[
K_{k} = \frac{P_{k-1}h_{k}}{1 + h_{k}^{T}P_{k-1}h_{k}}
\] (11)

\[
P_{k} = (I - K_{k}h_{k}^{T})P_{k-1}
\] (12)

In equation (9) - equation (11), \( \hat{\theta}_{k} \) and \( \hat{\theta}_{k-1} \) represent the parameter vectors of the \( k \)-th and \( (k-1) \)-th identification, \( e_{k} \) is the new information of the \( k \)-th innovation, \( K_{k} \) is the correction gain vector of the \( k \)-th innovation, \( h_{k} \) is the observation vector of the \( k \)-th input and output, \( I \) is the identity matrix, and \( P_{k} \) and \( P_{k-1} \) represent the covariance matrix of the \( k \)-th and \( (k-1) \)-th errors.
3.2. **VFF-LMRLS**

In most cases, the parameter identification method of the battery model based on RLS can estimate accurate results, but RLS is an algorithm with infinite memory length. In the identification process, the problem of "data saturation" will occur with the increase of observed data [2]. For lithium battery systems with time-varying parameter characteristics, RLS cannot track parameters well. To weaken the influence of the old data on the parameter identification process, the forgetting factor is introduced into RLS [16]. However, the introduction of forgetting factor cannot get rid of the interference of old data to the identification process, which will inevitably affect the identification accuracy of the algorithm. Therefore, based on the limited memory RLS algorithm, a VFF-LMRLS algorithm is proposed to identify the battery model parameters of the. The discrete recursive formula of the algorithm is divided into two stages:

**STAGE ONE:** receive new data

\[
\begin{align*}
\hat{e}_{k+L} &= y_{k+L} - h_{k+L}^T \hat{\theta}_{k+L} \\
\lambda_{k+L} &= 1 - \frac{e_{k+L}^2}{1 + h_{k+L}^T P_{k+L-1} h_{k+L}} \\
K_{k+L, k} &= \frac{P_{k+L} h_{k+L}}{\lambda_{k+L} + h_{k+L}^T P_{k+L-1} h_{k+L}} \\
\hat{\theta}_{k+L} &= \hat{\theta}_{k+1} + K_{k+L, k} \hat{e}_{k+L} \\
P_{k+L} &= \frac{I + K_{k+1} h_{k+L}^T}{\lambda_{k+L}} P_{k+L-1} 
\end{align*}
\]  

**STAGE TWO:** remove old data

\[
\begin{align*}
\hat{e}_{k+i+L} &= y_{k+i+L} - h_{k+i+L}^T \hat{\theta}_{k+i+L} \\
\lambda_{k+i+L} &= 1 - \frac{e_{k+i+L}^2}{1 + h_{k+i+L}^T P_{k+i+L-1} h_{k+i+L}} \\
K_{k+i+L, k+i} &= \frac{P_{k+i+L} h_{k+i+L}}{\lambda_{k+i+L} + h_{k+i+L}^T P_{k+i+L-1} h_{k+i+L}} \\
\hat{\theta}_{k+i+L} &= \hat{\theta}_{k+i+1} - K_{k+i+1} h_{k+i+L} \\
P_{k+i+L} &= \frac{I + K_{k+i+1} h_{k+i+L}^T}{\lambda_{k+i+L}} P_{k+i+L-1} 
\end{align*}
\]

where, \( \lambda_{k+i+L} \) or \( \lambda_{k+i+L} \) is the updating formula of variable forgetting factor, and \( L \) is the memory length.

According to the parameter vector identified by the \( k \)-th time \( \hat{\theta} = [\hat{\theta}_1(k), \hat{\theta}_2(k), \hat{\theta}_3(k), \hat{\theta}_4(k), \hat{\theta}_5(k)] \) and equation (6), the ohmic internal resistance \( R_0(k) \) and the intermediate parameters \( a_k, b_k, c_k \) and \( d_k \) of the battery are inversely solved as follows:

\[
R_0(k) = \frac{\hat{\theta}_1(k) - \hat{\theta}_2(k) + \hat{\theta}_3(k)}{1 + \hat{\theta}_4(k) - \hat{\theta}_5(k)}
\]  

\[(15)\]
Further, $R_0(k)$, $R_2(k)$, $C_1(k)$ and $C_2(k)$ are calculated as follows

\[
\begin{aligned}
    R_1(k) &= \frac{R_0(k)\tau_1(k) + c_1\tau_2(k) - d_k}{\tau_1(k) - \tau_2(k)} \\
    R_2(k) &= \frac{d_k - R_0(k)\tau_1(k) - c_1\tau_2(k)}{\tau_1(k) - \tau_2(k)} \\
    C_1(k) &= \frac{\tau_1(k)}{R_1(k)} \\
    C_2(k) &= \frac{\tau_2(k)}{R_2(k)}
\end{aligned}
\]  

In equations (17) and (18), $\tau_1(k)$, $\tau_2(k)$ is

\[
\begin{aligned}
    \tau_1(k) &= \frac{b_1 - \sqrt{b_1^2 - 4a_1}}{2} \\
    \tau_2(k) &= \frac{b_1 + \sqrt{b_1^2 - 4a_1}}{2}
\end{aligned}
\]

To sum up, the steps of identifying lithium battery ECM parameters based on VFF-LMRLS algorithm are as follows:

1) Given the initial conditions, set $\hat{\theta}(0,0)$ as a sufficiently small real column vector, $P(0,0)=\delta I$ ($\delta$ is a sufficiently large real number, $I$ is the identity matrix) and the memory length $L$.

2) The RLS is used to preliminarily estimate results as the initial parameters $\hat{\theta}(0, L-1)$ and $P(0, L-1)$ of VFF-LMRLS algorithm.

3) Equation (8) is iterated by VFF-LMRLS algorithm to obtain a single iteration result $\hat{\theta}_k$. Wherein, equation (13) is used to receive new data and equation (14) is used to remove old data.

4) According to the $k$-th iteration result $\hat{\theta}_k$, the second-order RC equivalent circuit parameters $R_0(k)$, $R_1(k)$, $R_2(k)$, $C_1(k)$ and $C_2(k)$ of lithium battery are inversely solved.

5) Return to step 3) and step 4), and repeat the iteration to finally obtain the identification result.

4. Experimental verification and analysis

4.1. CDSE and Its Parameter Identification

In this section, the 3.7V/50Ah NCM lithium-ion battery of AVIC lithium battery is selected as the experimental object for CDSE, and then the parameters are identified according to the current and voltage curves collected in the experiment.

In fact, the problem of model parameter identification is to solve the five parameter values of $R_0$, $R_1$, $R_2$, $C_1$ and $C_2$ in the model according to the current and voltage data collected in the test. Firstly, select
the appropriate SOC sampling points, and select the SOC sampling points as 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1. At the same time, record the corresponding open circuit voltage (OCV), and use polynomial fitting to establish the mathematical relationship between SOC and OCV, the real data and fitting curve are as shown in Fig. 2.

![Fig.2 SOC-OCV relationship curve.](image)

Secondly, CDSE is performed on the tested battery and the experimental data are collected. The current and voltage data are shown in Fig. 3 and Fig. 4 respectively.

![Fig.3 Discharge current profile of CDSE.](image)

![Fig.4 Discharge voltage profile of CDSE.](image)

Import the data obtained by CDSE into VFF-LMRLS algorithm, set the memory length $L$ to 1500, and obtain the parameter identification results are as shown in Fig. 5.

![Fig.5 Parameter identification results.](image)
Take the average value of each identification parameter $R_0$, $R_1$, $R_2$, $C_1$ and $C_2$ as the parameters of ECM, which are $2.3841 \times 10^{-3}$ Ω, $9.8761 \times 10^{-4}$ Ω, $6.0566 \times 10^{-4}$ Ω, $3.9721 \times 10^{3}$ F and $1.05979 \times 10^{5}$ F, respectively.

4.2. Model Verification and Analysis
Compared with VFFRLS, VFFRLS adopts the same variable forgetting factor calculation formula as VFF-LMRLS, and takes the average value of $L$ data points after the identification result is stable as the parameters of ECM. $R_0$, $R_1$ and $C_1$ of Thevenin ECM are $2.4781 \times 10^{-3}$ Ω, $1.4274 \times 10^{-3}$ Ω and $7.8899 \times 10^{3}$ F, respectively; $R_0$, $R_1$, $R_2$, $C_1$ and $C_2$ of the second-order RC ECM are $2.4469 \times 10^{-3}$ Ω, $1.0129 \times 10^{-3}$ Ω, $5.9700 \times 10^{-4}$ Ω, $5.0734 \times 10^{3}$ F and $8.14379 \times 10^{4}$ F, respectively.

After obtaining the identified model parameters, build the corresponding simulation circuits for verification and analysis. The MATLAB/Simulink simulation system diagrams are shown in Fig. 6.
Fig. 6. Model simulation system

Fig. 7 shows the error between the predicted value and the actual measured value of model terminal voltage under two algorithms and two ECMs. It can be seen from the figure that the voltage errors are all within the range of 0.03V, indicating the rationality of using ECM and the effectiveness of the proposed method.

To compare the accuracy of different algorithms and different ECMs, the absolute average error and root mean square error are set to measure the error between the voltage predicted value and the measured value. Through calculation, the absolute average voltage errors of VFFRLS (Thevenin ECM), VFFRLS (Second-order RC ECM) and VFF-LMRLS (Second-order RC ECM) are 6.57 mV, 4.77 mV and 4.16 mV, respectively. The root mean square errors of voltage are 5.40 mV, 5.33 mV and 3.64 mV respectively. The calculation results show that the voltage deviation predicted by the proposed algorithm is the smallest, which shows that the parameters applied to model identification are closer to the actual parameters, have better dynamic tracking performance, and the performance is better than VFFRLS algorithm.
5. Conclusion
The second-order RC ECM can not only well reflect the dynamic and static characteristics of the battery, but also be easily implemented in the real-time system, which is very suitable for the development of BMS. Based on the second-order RC ECM, a parameter identification method based on VFF-LMRLS is proposed, this method can overcome the inherent influence of the old data in VFFRLS on the algorithm, and improve the accuracy of identifying model parameters by setting a reasonable memory length. The simulation experiment of lithium battery verifies the effectiveness of the proposed method.

Acknowledgments
This research work was funded by Defense Industrial Technology Development Program (JCKY2019130C002), The Institute of Electrical Engineering, CAS (E155610201), Youth Innovation Promotion Association of Chinese Academy of Sciences (2,020,144).

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