Statistical Performances of Resistive Active Power Splitter

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Abstract. In this paper, the synthesis and sensitivity analysis of an active power splitter (PWS) is proposed. It is based on the active cell composed of a Field Effect Transistor in cascade with shunted resistor at the input and the output (resistive amplifier topology). The PWS uncertainty versus resistance tolerances is suggested by using stochastic method. Furthermore, with the proposed topology, we can control easily the device gain while varying a resistance. This provides useful tool to analyse the statistical sensitivity of the system in uncertain environment.

1. Introduction
The power splitter (PWS) is one of key microwave devices that can be found in different stage radio-frequency communication system. The PWS is usually implemented in three ports: the two output ports are in same magnitude, 0° phase between any two output signals. Until now, many topologies of PWS have been already proposed in the literature and the majority is in passive circuit [1]. So, significant losses can appear, thus this proposition of active PWS topology which enables to compensate these losses and even to amplify the signal. Recently in [2], a new structure of the broadband PWS based on the topology of the active cell has been introduced [3]. Signal distortion occurs due to the gain variation, and limits usable frequency band. Consequently, a cell constituted by FET in cascade with a shunt resistor is suggested in this paper which is organized as follows; section 2 is devoted to the theoretical study of the proposed PWS. The simulation results are explored in section 3 jointly with a sensitivity analysis over resistive input parameters, before conclusion and prospects.

2. Theory of the proposed PWS
First and foremost, the PWS architecture is composed of the black box of circuits $C_1$ and $C_2$, and the input matching shunts resistor $R_m$. To synthesize this balanced active PWS, the circuits $C_1$ and $C_2$ should be identical (Fig. 1). Similarly to the approach introduced in [4], the elementary active cells should be comprised of a FET loaded by a shunt resistance named FET-R network. This cell is particularly advantageous for the PWS interbranch isolation. It is noteworthy that, for the sake of simplification along the paper, the considered FET is comprised of voltage controlled current source with a transconductance $g_m$, in cascade with the drain-source resistor, $R_{ds}$. The synthesis formulas for the case of single- and two-cell(s) in cascade in function of the desired gain and matching levels are expressed in the following.
By taking into account the matching resistor \( R_m \), we can establish the single cell two-ports S-parameters. The resistance values can be determined with the following synthesis relations:

\[
R_m = Z_0 (1 + S_{11}) / (1 - S_{11}),
\]

\[
R = Z_0 R_{ds} S_{21} / [g_m Z_0 R_{ds} (1 + S_{11}) - S_{21} (Z_0 + R_{ds})].
\]

**Figure 1.** Architecture of the proposed PWS.

We can synthesize the resistor value \( R \) knowing \( S_{22} \):

\[
R = Z_0 R_{ds} (1 - S_{22}) / [S_{22} (Z_0 + R_{ds}) - Z_0 + R_{ds}],
\]

and in this case, the transmission loss \( S_{21} \) must be equal to: \( S_{21} = -g_m Z_0 (1 + S_{11}) / (1 - S_{22}) \).

In the case of two-stages cascaded elements, the two cells can be different or identical. For the different FET-R network, it can be established that the transmission gain is expressed as:

\[
S_{21} = \frac{2g_m^2 R_{ds}^2 Z_0 R_m R_1 R_2}{(R_m + Z_0) (R_1 + R_{ds}) (Z_0 R_2 + Z_0 R_{ds} + R_2 R_{ds})}.
\]

Therefore, by using (1) and (3) to determine \( R_m \) and \( R_2 \), the intermediate resistance can be synthesized from the desired input/output matching levels and gain: \( R = 2R_{ds} S_{21} / [2S_{21} + g_m^2 Z_0 R_{ds} (1 + S_{11}) (1 - S_{22})] \). In such a case, the maximum gain can be expressed in function of the FET parameters \( g_m \) and \( R_{ds} \): \( S_{21max} = g_m^2 Z_0 R_{ds} (1 + S_{11}) (1 - S_{22}) / 2 \).

Under the assumption \( R_1 = R_2 = R \), the synthesis relation can be expressed as follows knowing \( S_{11} \) and \( S_{21} \):

\[
R = \frac{2R_{ds} [S_{21} (R_{ds} + 2Z_0) + R_{ds} \sqrt{S_{21} (4g_m^2 Z_0^2 (1 + S_{11}))}]}{2 [Z_0 g_m^2 R_{ds}^2 (1 + S_{11}) - S_{21} (R_{ds} + Z_0)]}.
\]

3. Numerical investigations

3.1. Ideal proof-of-concept

As a proof-of-concept (POC), the detailed circuit schematic depicted in Fig. 2a was designed and simulated using SPICE® environment.

The S-parameters of the commercial FET CF004 − 01 are used during the numerical computations. The active PWS simulation results show a rather constant 0° output phase (0° ± 5°), insertion losses above −1.5dB and an excellent isolation below −30dB for all three ports, for a bandwidth from 1GHz to 3GHz (see Fig. 2b). The system acts as an active PWS using in each arm a two-stages active FET-R cell. It can be pointed out that the outputs of each branch are in phase to the output in the ideal case. Thanks to synthesis relation (4), the insertion loss can change inversely with either \( R_m \), \( R_1 \) or \( R_2 \). Since insertion loss \( S_{12} \) of the FET CF004 − 01 is relatively low, \( R_1 \) can be varied to fit \( S_{21} \) without changing the return losses. The parametric analysis based on the swept variation of the matching resistance \( R_1 \) from 40Ω to 55Ω was addressed (not shown here). It can be emphasized that the insertion
losses $\Delta S_{21} = \Delta S_{31} = f(R_1)$ presented about 2.5dB maximal variation. Then, the return losses $\Delta S_{11} = \Delta S_{22} = \Delta S_{33} = g(R_1)$ are rather less sensitive with 0.5dB maximal variation. Although parametric study is useful, the assessment of active PWS performances may be improved throughout statistical investigations in the following.

3.2. Statistical and sensitivity analyses

A sensitivity analysis is proposed following previous numerical model and considering 4 different cases including parameters considered as random one at a time (i.e. $R_1$, $R_2$ and $R_m$ respectively), and entirely randomly chosen (Gaussian distribution with mean 51Ω and standard deviation 5.1Ω). Among the diversity of stochastic techniques available [5, 6], crude Monte Carlo (MC) was used. Fig. 3a depicts the statistical results obtained under Gaussian assumption and well-known Student’s t-distribution as follows:

$$CI_\mu^\alpha = \left[ \bar{x} - t_{\bar{x},n-1}^{n-1, \alpha} \sqrt{\bar{var}} , \bar{x} + t_{\bar{x},n-1}^{n-1, \alpha} \sqrt{\bar{var}} \right],$$

where $CI_\mu^\alpha$ stands for confidence interval (CI) of mean $\mu$ from population (here $S_{31}$ parameter), $t_{\bar{x},n-1}^{n-1, \alpha}$ represents $(1-\alpha)$-quantile of Student’s distribution with $n-1$ degrees of freedom, $\bar{x}$ and $var$ are mean and variance computed from $n$ samples. Fig. 3 is obtained from 25 MC simulations regarding 98% testing confidence interval with Student’s distribution (i.e. $\alpha = 0.01$.
and $1 - \alpha = 0.99$ in relation (6)). Transmission parameter $S_{31}$ is mainly driven by $R_1$ parameter over the whole frequency bandwidth as depicted in Fig. 3b with variances. In accordance with previous remark, errors around mean trends (Fig. 3a) allow ranking inputs effects.

![Figure 4.](image)

**Figure 4.** Reflection coefficients $S_{11}$ (a) and $S_{22}$ (b) from random models including: $R_1$ (red), $R_2$ (green), $R_m$ (blue) parameters solely, and together (black). Error bars are given to assess dispersion around mean trend following Student’s distribution.

Based upon relation (6), Fig. 4 provides CI around mean S-parameters trends ($x/y$-linear drawing). Reflection coefficients are not influenced by $R_1$ parameter whereas $R_m$ and $R_2$ plays a key role respectively considering $S_{11}$ and $S_{22}$ (which may be physically expected). This overview of system’s sensitivity clearly demonstrates the importance of modeling random variations of inputs (e.g by ranking effects of parameters) since this may involve different drifts: from $-0.2$ to $1.2$dB $S_{21}$-mean, $S_{32}$ isolation is lower than $-41$dB (not shown here), and mean trends from reflection parameters $S_{11}/S_{22}$ are lower than $-15$dB.

4. Conclusion
An active resistive PWS is proposed. The analytical relation indicating how to synthesize the PWS resistive parameters are established. The POC based on two-stages circuit is analyzed. The sensitivity of PWS S-parameters in function of the resistive data are analyzed. Further works relying on stochastic methods [5] should efficiently provide accurate results to ensure security levels when computing S-parameters, leading to enforce the reliability of systems [6].

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