Thermalization of gluons in spatially homogeneous systems

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in collaboration with

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After a heavy-ion collision, an out-of-equilibrium system of gluons is produced.

In the weak coupling limit, the thermalization follows a bottom-up fashion.

The only tool used for a quantitative study of these systems before is the Effective Kinetic Theory (EKT).

Our study uses the Boltzmann Equation in Diffusion Approximation (BEDA) as an alternative approach.
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*Phys. Rev. D* 55 (1997). Jalilian-Marian et al.  
*Nucl. Phys. B* 529 (1998). Kovchegov and Mueller

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*Phys. Lett. B* 502 (2001). Baier et al.

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*JHEP* 01 (2003). Arnold, Moore, and Yaffe

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The QCD Boltzmann equation at leading order:

\[
(\partial_t + \mathbf{v} \cdot \nabla_x) f = C^{2\leftrightarrow2}[f] + C^{1\leftrightarrow2}[f]
\]

The thermalization process is mainly determined by the Debye mass and the jet quenching parameter.

\[
m_D^2 = 8\pi\alpha_s N_c \int \frac{d^3 p}{(2\pi)^3} \frac{f}{|p|}, \quad \hat{q} = 8\pi\alpha_s^2 N_c^2 \ln \frac{\langle p_t^2 \rangle}{m_D^2} \int \frac{d^3 p}{(2\pi)^2} f(1 + f)
\]
In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be rewritten as a Fokker-Planck equation.

\[
C^{2\leftrightarrow2} = \frac{1}{4} \hat{q}(t) \nabla_p \cdot \left( \nabla_p f + \frac{v}{T^*_t(t)} f(1 + f) \right)
\]

\[
T^*_t(t) \equiv \frac{\hat{q}}{2\alpha_s N_c m_D^2 \ln \frac{\langle p_t^2 \rangle}{m_D^2}}
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The $1 \leftrightarrow 2$ kernel is computed considering the LPM splitting rate.

\[ C^{1 \leftrightarrow 2}[f] = \int \frac{d^3 p'}{(2\pi)^3} \int_0^1 dx \frac{d^2 I(p')}{dx dt} \times \left\{ \begin{array}{l} \text{Statistics} \\ \text{contribution} \end{array} \right\} \]

\[ \frac{d^2 I(p)}{dx dt} = \frac{\alpha_s N_c}{\pi} \frac{(1 - x + x^2)^{\frac{5}{2}}}{(x - x^2)^{\frac{3}{2}}} \sqrt{\frac{\hat{q}}{p}} \]

When we include the inelastic collisions, no BEC appears.
At very early times, the inelastic kernel dominates for small $p$.

$$f(p) \approx \frac{T_*}{p} \quad \text{for} \quad p \lesssim p_*$$

where the momentum scale $p_*$ has been introduced.

$$p_* \equiv \left( \hat{q} m_D^4 t^2 \right)^{\frac{1}{5}}$$

This means that the soft sector is always in a thermal distribution.
Under-populated scenario I

Three different stages for thermalization.

1. Soft gluon radiation and overheating.
   - $T_*$ is almost constant.

2. Cooling and overcooling of soft gluons.
   - Soft gluons start to contribute to $m_D^2$.
   - $T_*$ decreases.

3. Reheating of soft gluons and mini-jet quenching.
   - $\hat{q}$ receives dominant contribution from soft gluons.
   - $T_*$ increases until it reaches $T_{eq}$.

Parametric estimation for $f_0 \ll 1$
Under-populated scenario I

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Parametric estimation for $f_0 \ll 1$
Under-populated scenario II

Parametric estimation for $f_0 \ll 1$

Numerical results for $f_0 = 0.01$

$T_*/Q$

$m_D^2/Q^2$

$\dot{q}/Q^3$

$\alpha_s f_0^{1/2}$

$\alpha_s f_0^{2/3}$

$\alpha_s^2 f_0^{3/4}$

$\alpha_s^2 f_0$

$f_0^{1/4}$

$f_0^{1/3}$

$a_s^2 f_0^{1/3}$

$a_s^2 f_0^{3/8}$

$\alpha_s^2 f_0$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-2}$

$10^{-1}$

$10^0$

$10^1$

$10^2$
Time evolution of $T_*$, entropy density $s$ and gluon number density $n$. 

$f_0 = 0.01, \alpha_s = 0.1$
Over-populated scenario

Two-stage thermalization

1. Soft gluon radiation and overheating.
   - $T_*$ is almost constant

2. Momentum broadening and cooling (no overcooling)
   - $T_*$ starts to decrease until it reaches thermal equilibrium.
   - All the quantities evolve according the universal scaling solution (dashed lines).

See also:
**Phys. Rev. D 86** (2012). Kurkela and Moore
**Phys. Rev. D 89.7** (2014). Abraao York et al.

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Parametric estimation for $f_0 \ll 1$
The Boltzmann Equation in Diffusion Approximation (BEDA) provides a framework to study the thermalization of a system of gluons. The qualitative features of thermalization described by the BEDA agree with previous studies using EKT. The soft sector quickly achieves a thermal distribution due to inelastic processes. We identify the reheating of the gluons, which agrees with the increasing of the temperature identified in the bottom-up scenario for initially under-populated systems.
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Summary and conclusions

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Currently, we are including quarks and antiquarks in our calculations for BEDA.
Thank you for your attention