3D Finite Element Modeling of Stresses in Filament Wound Structures

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Abstracts. A method for calculating stresses in the structure of a pressure cylinder made by winding a composite fibrous material has been developed. A special feature of the proposed method is that the problem is solved within the framework of the 3D formulation of the problem of the theory of elasticity, considering the curvilinear anisotropy of the structure, which changes from point to point. This technique allows one to consider the detailed microstructure of a composite material wound with overlapping tapes. To solve the problem, the 3D finite element method is used, as well as a special algorithm for constructing an additional mesh of elements, which makes it possible to relate the curvilinear anisotropy with the geometry of the thin-walled shell under study.

Keywords: composite winding, pressure vessel, modeling, stresses, composite materials, finite element method

1. Introduction

The winding method is one of the main methods for creating large-sized thin-walled structures from fibrous composite materials. This method is widely used, for example, to create pressure cylinders (vessels) [1] out of composite materials, which are applied in the automotive and aerospace industries, as well as a part of gas cylinder equipment for rescuers, firefighters, climbers, etc.

The method of manufacturing composite structures [2-3] by winding is a technological process in which a continuous reinforcing filler (filaments, ropes), impregnated with a polymer binder [4-5], is fed onto a rotating mandrel and placed on its surface in a given direction. Upon reaching the required reinforcement pattern and the specified thickness, the composite material is cured, as a rule, in a thermal furnace, and then the mandrel is removed from the product.

One of the known winding methods is spiral or geodesic winding. After the first turn, the filament wound around the mandrel forms a spiral line or a line close to it. The second turn has a certain offset in relation to the first. The displacement of the turn depends on the angle between the plane of the turn and the axis of the mandrel, the width of the tape and the overall dimensions of the structure. With the help of a dividing device, the gaps between the turns are filled with following turns of the tape [6-7].

The winding method is a universal, continuous, and high-performance process that makes it possible to manufacture a large class of convex shells from various composite materials.

To select the optimal winding schemes, as well as to select the optimal thicknesses of pressure cylinders, it is necessary to calculate the stresses in the composite structure [8-9]. Pressure cylinders are operated under high pressure, and belong to high-risk systems, especially when they are operated by people (firefighters, rescuers, climbers, motorists, etc.). With the improper design, or with poor-quality manufacturing, improper operation, rapid destruction of pressure cylinders is possible, which can result in an explosion. One of the most important advantages of composite pressure cylinders is their shatterproof destruction. When destroyed by internal pressure, the composite balloon "softens": first, the matrix that binds the composite filaments is destroyed, the balloon at the site of destruction loses its solidity and turns into a bundle of filaments, some of which break. Already at the stage of destruction of the composite matrix, gas escapes from the cylinder, the pressure drops sharply, and no fragments, which are most dangerous in case of the catastrophic destruction of the cylinder are formed. However, in the design of pressure cylinders, there are almost always metal embedded elements connecting them to hermetic shut-off valves. In the event of catastrophic destruction, these elements can become flying fragments, therefore, it is extremely important to accurately calculate the stresses even in a pressure cylinder, both in the composite structural elements themselves and in the zone of connection with embedded elements.

The need to carry out stress calculations in pressure cylinders to determine the safety factor is prescribed by regulatory documents on the design of composite pressure cylinders. For example, the standard GOST
ISO 11439-2014 [10] prescribes the need for stress calculations in the cylinder by the finite element method, while the mechanical properties of the composite must be calculated in advance. At present, analytical formulas [11], as well as numerical methods, including finite element methods [12], based on the Kirchhoff-Love or Timoshenko theory of thin-walled shells, are widely used to calculate stresses in composite pressure vessels. However, the theory of shells can lead to noticeable errors in the zone of connection of the composite structure with embedded elements and in the zone of the pole holes. The reason for this is essentially three-dimensional shape of the joint zone of the shell with the embedded elements, the change in the winding angles and thicknesses, as well as the properties of the composite in these zones, which is due to the peculiarities of the winding technology.

In this article, the modeling of the stress state of the structure of composite pressure cylinders manufactured by winding was considered, based on 3D finite element calculation, taking into account the detailed microstructure of the composite and the curvilinear anisotropy of the elastic properties of the structure.

2. Statement of a three-dimensional problem of elasticity for thin-walled composite structures, taking into account curvilinear anisotropy

A fragment of a typical pressure cylinder design is shown in Figure 1. Let us introduce a base surface for this structure, from which the entire 3D geometry of the pressure cylinder is constructed. To build such a 3D surface with a given parameterization (curvilinear coordinate system), a special tool is required, which can be used based on the formats of modern software for 3D geometric modeling, for example, SolidWorks, Compass-3D, etc. Assuming the presence of such a tool, given the base surface using the following conditions

\[ x = \rho(X^1, X^2), \]  
where \( x = x^i e_i \) – the radius-vector of points in Euclidean space \( \mathbb{R}^3 \), \( e_i \) – the vectors of the Cartesian (orthonormal basis), \( x^i \) – the Cartesian coordinates [13].

![Figure 1. 3D geometry of a composite pressure cylinder (cross-section shown) with embedded elements](image)

Let us construct a 3D area \( \Omega \), which corresponds to the entire structure of the pressure cylinder together with the embedded elements. Then the coordinates of an arbitrary point \( M \in V \) of the area can be represented in the following form (Fig. 2):

\[ x = \rho(X^1, X^2) + X^3 n(X^1, X^2), \]  
where \( n \) – the normal vector to the surface \( \Sigma \), \( X^3 \) – the third coordinate, measured along the normal to the surface and changing in a some area: \( X^3 \in V_X(X^1, X^2) \), which depends on the coordinates \( (X^1, X^2) \). For a structure of constant thickness (shell), this area is given by the inequalities: \( -\frac{h}{2} < X^3 < \frac{h}{2} \), where \( h \) – the thickness.

Consider the three-dimensional elasticity problem [11] in a bounded domain \( \Omega \) with Lipschitz boundary \( \partial V = \Sigma_u \cup \Sigma_p \):

\[ \nabla \cdot \sigma = 0; \]
\[ \sigma = \mathcal{C}(x) \cdot \varepsilon, \]
\[\varepsilon = \text{def}(\mathbf{u}) = \frac{1}{2}(\nabla \otimes \mathbf{u} + \nabla \otimes \mathbf{u}^T)\]
\[\mathbf{n} \cdot \mathbf{\sigma}|_{\Sigma_d} = S_{\varepsilon}, \quad \mathbf{u}|_{\Sigma_d} = \mathbf{u}_e, \quad (3)\]

where \(\mathbf{\sigma}\) – stress tensor; \(\varepsilon\) – small strain tensor; \(\mathbf{u}\) – vector of displacements; \(4 \mathbf{C}(\mathbf{x})\) – variable tensor of elasticity modulus (tensor of the 4th rank) [11], depending on the radius-vector of the coordinates of the points of the area; \(\nabla\) – nabla operator; \(\mathbf{n}\) – the normal vector to the area; \(S_{\varepsilon}, \mathbf{u}_e\) – vectors of stresses and specified external displacements [11].

Let us introduce another orthonormal coordinate system: \(\xi^1\) with unit vectors \(\mathbf{c}_i(\mathbf{x})\), which corresponds to the principal axes of anisotropy of the composite
\[\mathbf{c}_i(\mathbf{x}) = Q^j(\mathbf{x})\mathbf{e}_j, \quad (4)\]

where \(Q^j(\mathbf{x})\) – the transformation matrix between the coordinate systems introduced above, depending on the coordinates and calculated on the basis of the vector of the function (2) [13].

In the basis \(\mathbf{c}_i\), the components of the elastic modulus tensor have the minimum reduced form corresponding to the type of composite anisotropy [2]. Composites formed by winding can be considered transversally isotropic with principal axes \(MC_i(\mathbf{x})\). The structure components \(C_{pqrs}\) of the tensor \(4 \mathbf{C}(\mathbf{x})\) in the basis \(\mathbf{c}_i(\mathbf{x})\) is given in [13]. In the basis \(\mathbf{e}_i\), the components of this tensor have the form
\[C_{ijkl}(x) = C_{pqrs}Q^p_i(x)Q^q_j(x)Q^r_k(x)Q^s_l(x). \quad (5)\]

3. Numerical algorithms for solving a three-dimensional problem of elasticity for composite structures taking into account curvilinear anisotropy

The Voronoi diagram [14] was used as one of the ways to take into account the curvilinear anisotropy. With its help, it was possible to decouple the method of accounting for curvilinear anisotropy from direct reference to the geometric model of the structure.

The finite element method was applied [12] for solving the three-dimensional elasticity problem (3) - (5). Consider a curved shell \(\Omega\) formed by winding by anisotropic filaments. Each winding thread was considered as a cylindrical body, the center line of which is geodesic \(\Gamma\) on the base surface \(\Sigma\) of the shell \(\Omega\). This geodesic line \(\Gamma\) satisfies Clairaut’s theorem [15-18]. A numerical algorithm was proposed for approximating the indicated geodesics using broken lines. The centers of the links of such broken lines were assumed to be the barycenter’s of the cells of the Voronoi diagram. Inside each cell, the orientation of the anisotropy axes \(MC_i(\mathbf{x})\) – the local curvilinear orthogonal coordinate system \(\xi^1\), and the properties of materials were assumed to be unchanged.

The finite element approximation \(\Omega^h_i\) of the area \(\Omega_i\) of the shell \(\Omega\) was immersed in the Voronoi diagram \(\Omega^h_{ij}\):
\[\Omega^h_i \subset \bigcup_{j=1}^{M_i} \Omega^h_{ij}, \quad \Omega^h_{ij} = \left\{ \mathbf{x} \in \mathbb{R}^3; \rho(\mathbf{x}, \mathbf{w}_j) = \min_{k=1, M_i} \rho(\mathbf{x}, \mathbf{w}_k) \right\}, \quad (6)\]

where \(\mathbf{w}_j\) – barycenter of the cell \(\Sigma^h_j\) in Delaunay triangulation \(\Sigma^h\) of the middle surface of the shell \(\Omega\), while:
\[Q(x)|_{\mathbf{x} \in \Omega^h_{ij}} = \left( c_j \frac{n_j, \mathbf{e}_j}{|n_j, \mathbf{e}_j|} n_j \right) = \text{const}, \quad (7)\]

where \(n\) – normal to \(\Sigma^h\); \(\mathbf{e}_j\) – unit vector of the nearest to the \(\mathbf{w}_j\) ink of the network of broken lines on \(\Sigma^h\), defining the direction of the axis \(O \xi_1\).

During modeling of the winding, the filament overlap was also taken into account, which was carried out as follows. Consider two filaments of one tape: let the filament of the first turn be adjacent to the base surface, and the filament of the second turn crosses it at a certain angle, making an overlap. We introduce two points: the point of ascent or descent \(P_1(\mathbf{x}, y, z)\), when the filament of the first turn passes to the filament of the second turn, and the point of intersection of the filament of the second turn with the extreme filament of the first turn \(P_2(\mathbf{x}, y, z)\) [19]. It was proposed to change the radius-vector of the point \(P_2(\mathbf{x}, y, z)\), by increasing it along the normal to the surface of the pressure cylinder at this point by a certain height \(h\).
corresponding to the thickness of the tape filament. In addition, the midpoint \( P^*(x_*,y_*,z_*) \) belonging to the tape filament and located between points \( P_1(x,y,z) \) and \( P_2(x,y,z) \), in which the radius-vector was increased by the height \( h/2 \).

4. Results of calculating the stress state of a composite pressure vessel

For the finite element solution of problem (2) - (5) with the help of the proposed algorithm, the SMCM software complex was used, developed by in the Scientific and Educational Center "Supercomputer Engineering Modeling and Development of Software Systems" of the Bauman Moscow State Technical University. [20-22]. The design of a typical composite pressure cylinder (Fig. 1) was considered, which consisted of three subareas: a cylindrical shell, a shell constructed according to the optimal formulas described in [23-27] and a metal flange (embedded element). Glass/epoxy composite were considered. The effective elastic characteristics of the glass/epoxy composite in the local coordinate system are presented in Tabl.1.

| Elastic constants | Value |
|------------------|-------|
| \( E_1 \), GPa   | 216   |
| \( E_2 \), GPa   | 57    |
| \( E_3 \), GPa   | 57    |
| \( G_{12} \), GPa| 12,41 |
| \( G_{13} \), GPa| 12,41 |
| \( G_{23} \), GPa| 3,677 |
| \( \nu_{12} \)    | 0.21  |
| \( \nu_{13} \)    | 0.21  |
| \( \nu_{23} \)    | 0.3   |

The following boundary conditions were set: the upper base of the flange is fixed motionless by the symmetry condition; the internal pressure in the cylinder is 50 atmospheres, the rest of the surfaces are free from the load, the conditions of ideal contact were set at the section between the composite shell and the flange.

The results of calculating the stress state of the investigated pressure cylinder using the SMCM software package based on the finite element method in the Cartesian coordinate system are shown in Figures 2–4. Figures 5-8 show the stress fields in the coordinate system \( M_{ci}(x) \).

![Figure 2](image2.png)

**Figure 2.** Stress fields in a composite pressure vessel: a) \( \sigma_{11} \), GPa; b) \( \sigma_{22} \), GPa
Figure 3. Stress fields in a composite pressure vessel: \( \sigma_{33} \), GPa

Figure 4. Stress fields in a composite pressure vessel: a) \( \sigma_{12} \), GPa; b) \( \sigma_{13} \), GPa

Figure 5. Stress fields in a composite pressure cylinder: \( \sigma_{11}^{loc} \), GPa
Figure 6. Stress fields in a composite pressure cylinder: $\sigma_{22}^{loc}$, GPa; b) $\sigma_{33}^{loc}$, GPa

Figure 7. Stress fields in a composite pressure cylinder: $\sigma_{12}^{loc}$, GPa; b) $\sigma_{13}^{loc}$, GPa

Figure 8. Stress intensity, GPa
Analyzing the results obtained, it can be seen that the maximum stresses are achieved in the area of connection of the pressure cylinder with the flange and in the area of the pole holes. This is due to the peculiarities of the winding technology: with a change in winding angles and an increase in thickness.

5. Conclusion
A finite element method has been developed for modeling the stress state of a winding composite pressure cylinder, taking into account curvilinear anisotropy. An algorithm has been developed to take into account the overlap of composite tapes. Numerical modeling of the stress state of a typical pressure cylinder formed by winding with overlapping composite tapes is carried out. It is shown that the maximum stresses are achieved in the zone of connection of the pressure cylinder with the flange and in the zone of the pole holes, which does not contradict the theoretical justification.

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