ABSTRACT. This paper reviews the current status of measurements of galaxy clustering at high redshifts \((z \gtrsim 0.3)\). The focus is on the inherent limitations in the observation and interpretation of the "evolution of clustering". It is likely that results from the first attempts to characterize galaxy clustering beyond the "local" universe have been significantly limited by sample variance, as the difficulty in assembling large samples over large volumes is exacerbated as the observations become more challenging. It is also argued that, because of the complicated relationship between galaxies and mass (i.e., bias), and the surprising degeneracies among different popular cosmological models, it is likely that studies of galaxy clustering as a function of cosmic epoch will never be useful for strong discrimination between different cosmological models. On the other hand, observations of galaxy clustering are capable of testing basic ideas about how (and where) galaxies form. Galaxy formation, as opposed to cosmography, will probably remain a fundamental question even beyond the MAP and Planck era.

1 Introduction

We are clearly living in an era in which the relatively nearby universe will be mapped out with exquisite precision with new dedicated telescopes and instruments. The aims of such surveys seem clear: to use galaxies as a means to map out the large-scale structure of the universe, in the hope that in so doing one can understand the details of the relationship between observable galaxies and the overall matter distribution, and ultimately, to test theories of structure formation and to measure the details of the power spectrum of mass fluctuations on scales that are capable of testing theoretical ideas about the origins of the fluctuations. Even at "zero" redshift, as we have heard as a major theme at this conference, there is considerable argument about the degree to which galaxies should be trusted as tracers of mass; in particular, we now know that the clustering properties of galaxies are not universal, but depend on galaxy color, luminosity, and other messy astrophysical properties that are correlated with, but not direct proxies for, mass. The days of treating all galaxies in a redshift survey as identical test particles are almost certainly over; the degree to which one must worry about things like population mixes and luminosity and color segregation depend on the nature and scale of the measurement being made. The point here is that to understand large-scale structure as traced by galaxies, one needs also to understand something about the galaxies themselves. This is either annoying, or incredibly interesting, depending on one's perspective.

For redshifts where the evolving properties of galaxies begin to be important, and where the quantity of available information drops off precipitously, we are still in very much an exploratory phase. At \(z \gtrsim 0.3\), there has been a great deal of progress, but we are happy enough to have any measurements at all—of the kind of careful scrutiny to which the current and future major "local" redshift surveys have been subjected has not yet descended. This relatively information-starved regime is the subject of this short review.

Now that observations of large samples of high redshift galaxies are feasible from a purely technical standpoint, it is worth revisiting the question of just what one learns by studying the evolution of clustering with redshift. It is also worth considering in some detail how the selection of galaxies, and the over-
all design of a survey, may significantly influence results. The organization of this review is to first discuss the status of observations of galaxy clustering at \(z \gtrsim 1\); the observations here consist of traditional apparent–magnitude selected redshift surveys designed primarily for studying galaxy evolution, to new wide–angle imaging surveys and applications of photometric redshift techniques. This will be followed by a discussion of results and prospects using large photometric and spectroscopic surveys at \(z \gtrsim 3\).

2 “Evolution” of the Correlation Function

Because of the relatively small numbers of galaxies in most of the high redshift samples, simple statistics are generally used to describe the overall level of clustering. Many have described the clustering in terms of the Groth & Peebles (1977) parameterization of the two–point correlation function,

\[
\xi(r, z) = \left( \frac{r}{r_o(z)} \right)^{\gamma} (1+z)^{-3} (1+z)^{\gamma + \epsilon}(1)
\]

where \(r\) is the co–moving coordinate distance, \(r_o\) is the co–moving correlation length, \(\gamma\) is the slope of the correlation function and \(\epsilon\) is the “evolutionary parameter”. In the context of this parameterization, with \(\gamma = -1.8\), one obtains the usual limiting cases of \(\epsilon = -1.2\) for clustering fixed in co–moving coordinates (i.e., \(r_o(z) = \text{constant}\)), \(\epsilon = 0\) for clustering fixed in physical (proper) coordinates \(r_o \propto (1+z)^{-2/3}\), and \(\epsilon = 0.8\) for linear growth of clustering in an Einstein–de Sitter universe \(r_o(z) \propto (1+z)^{-1}\), approximately. This would be a very good way of thinking about the evolution of clustering if it were the case that 1) one were seeing the same galaxies at all redshifts \(z\), and 2) if galaxy clustering were a monotonic function of scale factor. In practice, 1) is almost certainly not the case, since selection effects and galaxy evolution conspire to bring different types (masses?) of galaxies into and out of samples as a function of redshift, and quite probably 2) is also not satisfied.

As has been discussed by several others at this conference (e.g., Peacock in these proceedings), a generic result of simulations and/or analytic models is that the correlation function for dark matter halos actually evolves very differently from the correlation function of the mass, and actually passes through a minimum at intermediate redshifts after having begun in a highly biased state when fluctuations of the particular mass threshold were very rare (see, e.g., Brainerd & Villumsen 1994; Bagla 1998b). In this picture, the evolution of the bias of the dark matter halos out–paces the growth of matter fluctuations (so that the galaxy clustering becomes weaker in co–moving units with decreasing redshift) until a characteristic redshift at which the mass threshold is a \(\sim 1\sigma\) fluctuation, after which the clustering strength increases again. For halos typical of bright galaxies today, \(\sim 10^{12} \, M_{\odot}\), this clustering minimum is expected to occur near \(z \sim 1\). Thus, under this picture, the observed evolution of clustering would depend upon the characteristic halo mass (or mix of mass scales) traced by galaxies satisfying the particular sample selection criteria at each redshift. Even if one could somehow isolate galaxies of fixed mass, the evolution of the clustering would not fit in well with the \(\epsilon\) parameterization shown above. In the real world, one is likely to be observing a complicated mix of galaxies/mass scales that is likely to be changing as a function of redshift, so that what one observes in a sample is a superposition of such complicated evolutionary sequences. The prediction would then be that describing the evolution of clustering with a single value of \(\epsilon\) that results from a “best–fit” to a heterogeneous sample (whether it be heterogeneous with respect to color, luminosity, etc.) is not likely to provide information that is particularly useful in understanding what is going on. Many have now suggested dispensing with the \(\epsilon\) parameterization altogether; this is an excellent suggestion.
3 Clustering at $z \lesssim 1$

Until very recently, the only information on the clustering of distant galaxies that was available was based on studies of the angular correlation function of faint galaxies (e.g., Efstathiou et al. 1991, Efstathiou 1995, Brainerd et al. 1995, Postman et al. 1998). The general result was much weaker clustering amplitude for faint galaxies than seen in local galaxy surveys, but because of the unknown extent of projection effects due to a lack of detailed knowledge of the redshift distribution $N(z)$ [in some cases the clustering results were used to place constraints on the behavior of $N(z)$] and the uncertainty that the galaxies seen at faint magnitudes are the same objects counted in local galaxy surveys, the implications were somewhat ambiguous. A tendency was seen in several of the above surveys for a flattening in the angular correlation function amplitude at the faintest magnitudes; however, the degeneracy between the issue of the redshift distribution and the evolution of the clustering does not permit the solution for one without knowledge of the other. This degeneracy/projection problem in the imaging surveys can be overcome, to some extent, by using multiple colors to assign photometric redshifts, as discussed by A. Connolly at this conference. While there are some limitations to the photometric redshift technique, it appears to work very well for $z < 1$ (e.g., Hogg et al. 1998 and references therein).

A first attempt at using a deep spectroscopic survey for measuring the spatial correlation function of faint galaxies (in this case at a median redshift of $z = 0.16$) was made by Côle et al. (1994), who obtained a correlation function that was indistinguishable, in co-moving units, from the local correlation function. There have been a number of subsequent spectroscopic surveys which have addressed galaxy clustering at $z \gtrsim 0.3$, many of which are summarized in Table 1. This summary includes minimal details describing the results on the co-moving correlation lengths, in particular, from the various surveys. There are other large spectroscopic surveys which reach similar redshift depths whose results were in preparation at the time of this meeting (e.g., Cohen et al. 1999, Small, Sargent, & Ma 1999) and which have not yet reported actual measured correlation functions in the literature. However, the Caltech Deep Redshift survey (a K-selected sample) of Cohen et al. (1999) has reported a strong tendency for objects that are red in their optical/IR colors and have absorption-dominated spectra to preferentially inhabit the most prominent structures in redshift space; the kinematics of these structures suggest that they are groups or poor clusters. There is clear evidence for both luminosity and color segregation in the clustering properties, but the quantitative comparison with the surveys presented in Table 1 is not yet possible.

A benchmark piece of work in terms of measuring the actual evolution of the correlation function as a function of redshift, within the same survey, was that of Le Fèvre et al. (1996) from the Canada-France Redshift Survey (CFRS). As can be seen in Table 1, the CFRS team had a large enough sample, over a large enough redshift range, that the data could be binned into redshift subsets. The measurements of the correlation lengths versus redshift, and of the sample as a whole, seemed to show strong evolution of the galaxy correlation function in the sense that the correlation strength was significantly weaker in the past; it was argued that is was very unlikely that they were seeing different galaxies as a function of redshift within the sample, so that for the first time one could see the growth of clustering of galaxies directly. Interestingly, the CFRS saw no evidence for color segregation (based on optical colors) of the clustering of galaxies for redshifts beyond $z \sim 0.3$.

However, a glance at Table 1 makes one worry slightly. Compare, for example, the preliminary CNOC-2 results at $z \sim 0.35$ with the lowest redshift bin of the CFRS survey, or with the results of the Hawaii Deep Survey K-band sample. The CNOC-2 sample finds significantly stronger clustering even for the “faint” subsample, and much stronger clustering for the “bright” subsample; the Hawaii Deep Survey, albeit a rela-
Table 1. Non-exhaustive summary of recent galaxy clustering results to $z \sim 1$. Correlation lengths are in co-moving units.

| Survey                | $\langle z \rangle$ | $N_{\text{gal}}$ | $r_0/h$ Mpc | Comments |
|-----------------------|---------------------|------------------|-------------|----------|
| CNOC-1                | 0.35                | 140              | 2.7 ± 0.6   | 1        |
| CFRS                  | 0.34                | 186              | 1.8 ± 0.2   | 2        |
|                       | 0.62                | 196              | 1.8 ± 0.2   | 2        |
|                       | 0.86                | 130              | 1.9 ± 0.2   | 2        |
| (Full CFRS Sample)    | 0.53                | 591              | 2.2 ± 0.1   | 2        |
| Hawaii Deep           | 0.34                |                  | 3.9 ± 0.2   | 3        |
|                       | 0.62                |                  | 3.2 ± 1.1   | 3        |
|                       | 0.97                |                  | 2.8 ± 1.2   | 3        |
|                       | 1.39                |                  | 2.4 ± 1.2   | 3        |
| Red Sub-sample        | 0.6                 | ~ 100            | 3.8         | 3        |
| Blue Sub-Sample       | 0.6                 | ~ 150            | 1.4         | 3        |
| KPNO I-band           | (0.5)               | 4e5              | 4.5 ± 0.6   | 4        |
| CNOC-2 (bright)       | 0.35                | ~ 1500           | 5.0 ± 0.2   | 5        |
| CNOC-2 (faint)        | 0.35                | ~ 1500           | 3.6 ± 0.2   | 5        |

1) Shepherd et al. (1996); found best-fit $\epsilon \sim 1 ± 1$
2) Le Fèvre et al. (1996); 5 10′ fields, to $I_{AB} = 22.5$
   Found $\gamma = -1.6, \epsilon \sim 0 - 2$, and no color segregation for $z > 0.3$.
3) Carlberg et al. (1997); K-selected sample, total $\sim 250 z$’s.
   Assumes fixed $\gamma, q_0 = 0.1$. Clear color segregation.
4) Postman et al. (1998); $w(\theta)$ de–projection using CFRS redshift distribution and magnitude cuts.
5) Carlberg et al. (1998); sample split into “bright and faint” at $M_R = -20$; find best fit evolutionary model is $r_0(z) \propto (1+z)^{-0.3±0.2}$. 

The work that has been completed recently in the $z \lesssim 1$ regime, it is that most of the samples have probably not been large enough to yield universal results on the galaxy correlation function there is a trend of increasing measured $r_0$ at a given redshift with increasing sample size, a telltale sign that sample variance has been a problem. The results of the very large photometric survey by Postman et al. (1998) perhaps illustrates the problem best, in that one can isolate many independent sub–samples that are as large as (for example) the CFRS, and it is clear that in a sample the size of the CFRS there is a quite large probability of measuring clustering strength that is significantly smaller than that observed over very large volumes. Using the CFRS photometric selection criteria for their angular correlation function, and the CFRS–observed $N(z)$ for de–projecting to form the real–space correlation function, Postman et al. obtain a value for the correlation length that is more than two times larger, at the
median redshift, than that obtained by Le Fèvre et al., a difference that is significant at about the 4σ level if one takes the error bars at face value. On the other hand, the same large imaging survey of Postman et al., while making a measurement of unprecedented precision in terms of shear reduction in sample variance and counting statistics, is unable to distinguish between a number of widely different models for the growth of clustering; this indicates the inadequacy of the ε parameterization in general, and the need for cutting down on projection effects that hamper deep imaging surveys intended for clustering analysis. One very sensible means of overcoming this limitation is to use multi-bandpass imaging and the photometric redshift method to isolate cosmic epochs, galaxy luminosity classes, and color cuts, exploring the behavior of the clustering as a function of galaxy properties in a multi-dimensional manner. In doing this, one clearly increases the noise in any clustering estimate, but it is almost certainly worth sacrificing precision in favor of information content. Most useful of all, although rather painstaking in terms of the resources necessary to obtain the observations, are wide angle, deep spectroscopic surveys such as CNOC–2 (and future surveys such as that planned by the DEEP collaboration—see Davis & Faber 1998) where in addition to simple correlation statistics, precision galaxy redshifts might yield more detailed dynamical information that might help relate the galaxy clustering to the clustering of the dominant dark matter component on small scales. Both large volumes and accurate redshifts, with a good sampling of redshifts using sensible selection criteria, will be invaluable.

4 Clustering at \( z \gg 1 \)

Given that we do not really understand the details of what is going on with galaxy clustering at intermediate redshifts, why would one want to bother exploring the higher redshift universe, where the problem of making clear evolutionary connections to galaxies at the present is even more difficult? The simplest answer is that the pure novelty of compiling a large sample of high redshift galaxies would be bound to yield something interesting; naively, one might have thought that one of the cleanest possible tests of the idea that matter fluctuations grow by gravitational instability would be to obtain a “snapshot” of galaxy clustering at very high redshifts, where one might expect the clustering to be much weaker if gravity really were the dominant factor in producing the structure observed in the local universe. Also naively, one might have expected that the amount by which the clustering should be different would be rather sensitive to \( \Omega_m \), with smaller differences expected for lower \( \Omega_m \). All of this would be true if galaxies formed at infinite redshift and then evolved quiescently to the present epoch, acting like conserved test particles. As it turns out, the clustering is a sensitive test of our collective wisdom about how galaxies form, but probably a relatively poor cosmological discriminant.

The method used for compiling samples of very high redshift galaxies to date has been almost exclusively the “Lyman–break” technique, where one makes use of the essentially guaranteed “break” in the spectrum of high redshift star forming objects at 912 Å in the rest frame due to photo-electric absorption both in the galaxy itself and in the intergalactic medium. The feature is so strong that the coarse spectrophotometry allowed by broadband imaging can be used to isolate particular ranges of redshift that can be controlled based on the adopted filter system. The technique as successfully implemented has been described recently in many places (e.g., Steidel, Pettini, & Hamilton 1995, Steidel et al. 1996, 1998b, Madau et al. 1996), so I will not go into any details here. Basically, the method is a highly efficient means of selecting a nearly volume–limited sample of objects on the basis of their rest–frame far–UV luminosity. As with any other galaxy sample, it is important to understand what selection effects are implicit in the detection technique: here, one is quite insensitive to stellar mass, but sensitive almost exclusively to unobscured
high-mass star formation. Extremely dusty galaxies, or galaxies which have ceased forming stars prior to the epoch at which they are observed (or those going through a quiescent phase between star formation episodes) are unlikely to be included.

The particular implementation of the Lyman break technique for which the most data have been obtained to date selects galaxies in the redshift range $2.7 < z < 3.4$, as shown in Figure 1. The primary goal of the survey is to accumulate a large enough sample of high redshift galaxies that proper statistics on the luminosity distribution, spectral properties, reddening, and, most relevant here, their large-scale distribution, are possible. The survey is in many ways similar to the CFRS in design: there are 5 primary survey regions, typically $9' \times 18'$ by 23h$^{-1}$ Mpc for the LBG fields at $z \sim 3$, for $\Omega_m = 0.3$, $\Omega_L = 0.7$. The CFRS depth along the line of sight, on the other hand, is between 2 and 4 times larger than in the LBG fields, depending on cosmology. Practical matters, mainly having to do with the faint apparent magnitudes of even the more luminous LBGs, limit the sampling density for galaxies at $z \sim 3$ in these larger volumes, so that scales much smaller than the field size are not probed well. This bears significantly on what types of clustering statistics can be measured well using the data. Each LBG survey field samples an effective co-moving volume of $\sim 2.2 \times 10^4$ h$^{-3}$ Mpc$^3$ for an Einstein-De Sitter model ($\sim 8.3 \times 10^4$ h$^{-3}$ Mpc$^3$ for $\Omega_m = 0.3$ and $\Omega_L = 0.7$), so that the total volume surveyed is somewhere between $10^{5}$ and $10^{6}$ h$^{-3}$ Mpc$^3$.

Within these survey regions, to the adopted apparent magnitude cutoff of $\mathcal{H}_{AB} = 25.5$, there are $\sim 1500$ photometrically selected candidates, and the aim is for a total spectroscopic sample of $\sim 750$. Although the survey is not yet completed, several interim results have already been published. Many of the results have been discussed elsewhere, e.g. Steidel et al. 1998a,b, Adelberger et al. 1998, Giavalisco et al. 1998, so the discussion here will be brief.

First, as can be seen in Figure 2, the Lyman break galaxies are strongly clustered, and large overdensities, or “spikes” in the redshift distribution are evident in each survey field. The galaxies within these over-densities are not obviously concentrated on the plane of the sky, and angular correlations of the photometric samples of Lyman break galaxies have rather poor S/N because of the aforementioned low surface density and therefore poor sampling of the small scales where most of the angular correlation signal would lie. At present, the most robust statistic for evaluating the level of clustering for the LBGs is a “counts-in-cells” analysis. Here one simply counts the number of objects in cubical cells of roughly $10h^{-1}$ Mpc on a side (the scale being defined by the transverse size of the survey field, which of course varies as a function of cosmology) in the spectroscopic sample, corrects for the shot-noise contribution to the variance and modest redshift-space distortions, and evaluates $\sigma_{cell}$ (see Adelberger et al. 1998). The variance in galaxy counts (relative to the overall expectation value from the selection function) in cells of this size is very closely related to the commonly-used $\sigma_g$ statistic for normalizing mass fluctuations using cluster abundances (see, e.g., White et al. 1993, Eke, Cole, & Frenk 1996). Since it is straightforward to compute the expected mass fluctuations on the same scales at $z \sim 3$ (using present-day cluster normalization) for a given cosmology, one can essentially “read off” the required galaxy effective bias on the scale of a “cell” from a plot similar to the ones shown in Figure 2. The most recent numbers resulting from such an analysis of the current LBG survey sample, assuming cluster normalization of Eke, Cole, & Frenk (1996) are

$$b_{eff} = \begin{cases} 5.2 \pm 0.9 & \Omega_m = 1 \\ 3.6 \pm 0.6 & \Omega_m = 0.3, \Omega_L = 0.7 \\ 1.7 \pm 0.4 & \Omega_m = 0.2 \end{cases}$$

(2)
Clustering at High Redshift

Figure 1. Redshift histogram of Lyman break galaxies in the $z \sim 3$ sample, selected using color criteria in the $U_nG_R$ color–color plane from wide–field ground–based images. All of the confirming spectroscopic redshifts were obtained with the Keck telescopes and the Low Resolution Imaging Spectrograph (Oke et al. 1995).

where $b_{eff}$ is the effective linear bias on scales of $\sim 10h^{-1}$ Mpc.

Thus, it can be seen that bright LBGs must be strongly biased tracers of mass fluctuations in order to be accommodated easily by standard hierarchical models. It is somewhat more difficult, given the nature of the data, to turn this into a correlation function (although, of course, $b_{eff}$ is equivalent to an integral over the correlation function over the cell volume)–while we have attempted to de-project the angular correlation function of LBGs to obtain a real–space correlation function (Giavalisco et al. 1998), the current sample is not well–suited to measuring $w(\theta)$ nor $w_p(\theta)$ accurately. The central problem is that the depth of one of our survey fields (in co–moving units) far exceeds its width, and as a result the vast majority of angular pairs are simply chance projections of galaxies at very different redshifts. For example, even at separations as small as $20''$ approximately 90% of galaxy pairs in our sample are chance projections (Giavalisco et al. 1998). Including these chance pairs in a clustering analysis—as is required for $w(\theta)$ and some forms of $w_p$—results in a disastrous reduction in signal–to–noise ratio. This seems an unnecessarily high price for avoiding peculiar velocity distortions, which are after all relatively minor on large scales at these redshifts. Measuring $w(\theta)$ is a reasonable approach when the sample is large enough that random fluctuations in the number of chance pairs are small compared to the number of true pairs, and is often the only approach when only a small fraction of the galaxies in the sample have redshifts; however, neither condition is true for the $z \sim 3$ LBG sample.

Thus far we have been quoting a number for the correlation lengths that is based on assuming a value for the slope $\gamma$ of the correlation function (Adelberger et al. 1998), and these number range from 4–6$h^{-1}$ Mpc, with the lower end of the range applying to Einstein-de Sitter and the upper range to low $\Omega_m$ models. We plan to do a more careful job with this when the spectroscopic sample has been completed, which we anticipate will be quite soon. Regardless of the precise values for $r_0$, the clustering of the $z \sim 3$ LBGs is as strong, or stronger, than most local galaxy samples, and significantly stronger than most of the
intermediate redshift numbers. It is still unclear whether we have sampled enough volume to asymptote to the “truth”, but these correlation lengths are likely to be firm lower limits.

Strong clustering/bias is expected theoretically for rare peaks in the density field (Kaiser 1984), and many theoretical papers, both predating and interpreting the clustering observations, can easily explain the strong clustering of the LBGs through this “high peaks” biasing (e.g., Baugh et al. 1998, Coles et al. 1998, Wechsler et al. 1998, Bagla 1998a, Mo and Fukugita 1996, Jing & Suto 1998, Governato et al. 1998, Mo, Mao, & White 1998, Katz, Hernquist, & Weinberg 1998). One can get quite good agreement with both the abundance and clustering properties of dark matter halos (using either N-body or analytic techniques) and the real galaxies provided that the typical LBG in the sample is associated with a dark matter halo mass scale of \( \sim 10^{12} \, M_\odot \) (Steidel et al. 1998a,b; Adelberger et al. 1998; Mo, Mao, & White 1998). This good agreement suggests that there should be a monotonic relationship between dark matter halo mass and UV luminosity, and that most, if not all, dark matter halos of a given mass contain a LBG exhibiting a star formation rate with relatively small scatter (Adelberger et al. 1998). This provides empirical evidence that the use of star formation prescriptions that are based on the dark matter halo properties (as in most semi-analytic models of galaxy formation) may be on the right track.

A power spectrum shape parameter of \( \Gamma \sim 0.2 \) is most consistent in matching the inferred bias and the abundance of galaxies and dark matter halos, but otherwise surprisingly little difference is expected for the clustering of objects of a given abundance among the currently popular dark matter models which have this kind of shape parameter (i.e., \( \tau \)CDM, open CDM, \( \Lambda \) CDM). As discussed by Steidel et al. 1998b, Adel-
berger et al. 1998, and Giavalisco et al. 1999, a more stringent test of such a simple association of LBGs with dark matter halos in a hierarchical model would come from examining the clustering of much fainter LBGs, which would presumably trace much smaller mass dark matter halos which should be significantly less clustered at high redshift. Preliminary indications show that most of the models remain consistent with the data when faint LBG samples from the HDF are compared with the ground based results, with significantly smaller correlation lengths for objects with abundances $\sim 20$ times larger than the bright galaxies in the ground–based survey. Larger samples, particularly of the faint objects, will be required in order to be able to exert much pressure on any of the currently popular dark matter models.

On the other hand, there is a very substantial difference among the various models for the masses of objects of a given abundance and clustering level. This difference is large enough (a difference of a factor of $\sim 3$ in circular velocity) between low $\Omega_m$ models and $\Lambda$-CDM that observations of line widths (even with all of the inherent uncertainties in using them for dynamical mass estimates) may be able to resolve the degeneracy. The main problem is that line widths are essentially always providing lower limits on $v_c$, and some theoretical predictions suggest (e.g., Mo, Mao, & White 1998) that the observed line widths may not be radically different despite the very different $v_c$ because of the fact that the highest star formation efficiency would occupy regions that are still on the rising part of the rotation curve. Some observations along these lines have already been attempted using the familiar nebular lines in the rest–frame optical (observed near-IR) (Pettini et al. 1998), but the advent of IR spectrographs on 8–10m telescopes should result a huge amount of progress in this area.

The UV spectra of LBGs, on the other hand, represent possibly the most frustrating limitation of the spectroscopic samples. While in principle the redshift accuracy achievable at $z \sim 3$ is the same as one could achieve at intermediate redshift, the problem is that essentially none of the commonly–observed far–UV lines is trustworthy as an indication of the systemic redshift of the galaxy. We have estimated the intrinsic uncertainty (independent of measurement errors) to be on the order of $\sim 300$ km s$^{-1}$. This means that it may be difficult to explore any statistics based on small-scale dynamics (e.g., pairwise velocity dispersions) without wholesale IR spectroscopy.

4.1 General Implications

The nature and clustering of the $z \sim 3$ LBGs are very consistent with the overall “paradigm” that galaxies would form at the highest, “biased” peaks in the dark matter distribution at early epochs, and that these objects should be strongly clustered at high redshift. These clustering properties, together with the observed space densities, imply the individual galaxies are associated with dark matter halos of the order of $\sim 10^{12} M_\odot$. Within the context of these models, a large fraction of the LBGs seen in the bright ground–based samples would end up in richest environments in the present–day universe, and the prominent “spikes” at $z \sim 3$ are likely to be the progenitors of present–day rich galaxy clusters (e.g., Steidel et al. 1998a, Governato et al. 1998). The incidence of these prominent over–densities is indeed broadly consistent with this hypothesis. Observations of the clustering properties as a function of space density for LBGs over a wide range of luminosities have the potential to measure the shape of the power spectrum on scales of $\sim 1 – 10h^{-1}$ Mpc (i.e., between galaxy and cluster scales) if the case can be made that the UV luminosity really is a good proxy for dark matter halo mass. Here again the problem is somewhat circular, in the sense that the dark matter models can be tested rigorously only if some observable property of the galaxies can be closely tied to the mass, but if one knew the underlying structure of the dark matter, it would be possible to close in on how star formation is related to the dark matter distribution. At present, the assumption that UV luminosity and dark matter halo mass are very closely related, with a power
spectrum shape very close to that which works best to explain the local large scale structure (see e.g., Peacock & Dodds 1996), works very well indeed, but this solution cannot be said to be unique at this point in time.

5  $z \sim 4$ and Beyond

At the time of this writing, a handful of galaxies have been identified beyond $z \sim 5$. Should one be thinking about large surveys beyond $z \sim 3$? As always, the answer depends on what it is one wants to learn. It is conceptually straightforward to locate higher redshift galaxy candidates using variations on the Lyman break technique, particularly with good sensitivity in the near-IR for redshifts beyond $z \sim 5$ or so. However, practical matters will probably prevent large and successful surveys useful for examining large scale structure. We have recently completed a pilot spectroscopic survey for LBGs at redshifts $z \lesssim 4$ (Steidel et al. 1999), and find that the surface density of candidates objects to $I_{AB} = 25.0$ is just barely high enough to take advantage of multiplexing using imaging spectrographs on 8–10m class telescopes. Fainter than this, because of the much brighter background one must fight to get spectra of the higher redshift objects (the features that secure the redshifts tend to be in the range $1200 - 1700$ Å in the rest–frame), anything close to spectroscopic completeness would be extremely painful. On the other hand, if one can get around the more significant contamination by interlopers in the $z \sim 4$ samples ($\sim 20\%$) it might be possible to use de–projection of the angular correlation function of photometrically–selected candidates to quite faint magnitudes using the new generation of wide–field imagers. The question is whether the clustering of similarly–selected objects at $z \sim 4$ is providing much additional information over the much more easily–easured statistics at $z \sim 3$.

On the other hand, it has been possible to compare the properties of the galaxies even in modest–sized samples at $z \sim 4$ and $z \sim 3$ using the ground–based surveys. This is a bit of a diversion from the topic of large–scale structure in general, but perhaps serves as an interesting example of how sample variance can lead to somewhat misleading results, and in any case allows me to present a new result that had just been obtained at the time of the meeting in August 1998. Most readers would be familiar with the very exciting results from the Hubble Deep Field regarding the star formation history of the universe as revealed by various galaxy surveys, with the highest redshift points being obtained using Lyman break galaxies within the $\sim 5$ square arc minute HDF. The implication from the work of Madau et al. (1996) and follow–up papers is that the UV luminosity density of the universe, a proxy for the total SFR as a function of cosmic epoch, reached a peak somewhere in the neighborhood of $z \sim 2$ and declined steadily beyond that redshift. Fearing that perhaps the HDF might not be a representative region of the universe, we have compiled a sample covering about 830 square arc minutes ($\sim 160$ times larger area than the HDF) to the relatively bright magnitude of $I_{AB} = 25.0$, using photometric selection designed to be as analogous as possible to the one implemented at $z \sim 3$, and spectroscopic redshifts for about 50 $z \sim 4$ galaxies to secure the redshift–selection function. A comparison of the luminosity density represented in the bright ends of the $z \sim 3$ and $z \sim 4$ luminosity functions indicates that the luminosity density is essentially constant in the two redshift ranges (see Figure 3), and indications are that the HDF is under–dense in $z \sim 4$ galaxies relative to a survey covering a much larger volume. For amusement, I have reproduced a figure showing the latest incarnation of the “star formation history diagram” in Figure 4, showing previous results as well as the new points that come purely from the large ground–based surveys. The moral of the story is that one can never survey too much volume, and one should always be concerned about the lumpiness of the universe when convincing oneself that one is seeing something “universal”.
Clustering at High Redshift

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6 Concluding Remarks

Whereas the evolution of clustering of galaxies used to be seen as a sure-fire cosmological test, this somewhat naive view has at this point probably gone the way of all other cosmological tests that hoped to ignore the vagaries of galaxy evolution and treat galaxies like test particles. If there are any points that I would hope to get through in this highly qualitative talk (the data do not yet justify anything much more sophisticated!), it is that one needs to cover large volumes in order to hope to have reliable estimates of even the simplest statistics, one needs to worry a great deal about comparing apples to oranges in comparing galaxy samples at different cosmic epochs, and one must remain highly suspicious of what galaxy clustering is telling one about the development of structure. It is essential to be able to isolate cosmic epochs to avoid being overwhelmed by complicated projection effects—photometric redshifts seem like a very powerful tool for surveying both large volumes and being able to slice a survey in ways that will reveal what is really going on. Even better are large spectroscopic surveys, in combination with photometric redshifts.

In the end, because of complex epoch-dependent, luminosity-dependent, type-dependent bias, which is expected theoretically (and now seen observationally) to be worse at high redshift, one must accept the fact that galaxy clustering may never constitute a powerful cosmological test, even with all of the fantastic

Figure 3. Luminosity functions obtained by combining our ground-based, wide-area surveys, with data on faint objects in the Hubble Deep Field. The HDF points are based on the catalogs of LBGs presented by Dickinson (1998) and Madau et al. 1998, but we have reanalyzed the effective survey volumes with knowledge of the true color distributions of the LBGs based on our spectroscopic samples. The bright ends of the $z \sim 3$ and $z \sim 4$ luminosity functions are strikingly similar, in both shape and normalization (the $z \sim 4$ curve is simply the fit at $z \sim 3$ shifted by the distance modulus between $z = 3.04$ and $z = 4.13$, with the normalization multiplied by 0.8). The integrals of UV luminosity to the equivalent of $I_{AB} = 25$ in the higher $z$ sample is within about 20% of that over the same luminosity range at $z \sim 3$, independent of cosmology. Note the indications that the HDF is underdense in $z \sim 4$ galaxies.
Figure 4. A revised version of the “star formation history” diagram, with new points from the large ground–based surveys indicated with the crosses. The circles come from Lilly et al. (1996), the squares from Connolly et al. (1997), and the triangles from Madau et al. (1998). Note that with internally consistent corrections for extinction (see Steidel et al. 1999 for details), there is no indication for any significant change in the universal star formation density for any $z > 1$.

data that will continue to roll in over the next several years. On the bright side, it seems that our basic ideas about how galaxies form within halos of dark matter whose distribution is easily understood using relatively simple statistics or N-body simulations are holding up very well, and (from an observer’s point of view, at least) it is very encouraging that theorists and observers seem by and large to be talking about the same universe. There is enormous potential for progress in the area of understanding the interface between galaxy formation and structure formation, and it will involve a lot of interaction between theory and observations.

Studying large scale structure by using galaxies ultimately involves having to understand galaxies themselves, and the problems of structure formation and galaxy formation are intimately related, and largely inseparable, problems. The very obvious galaxy bias seen in the high redshift samples (at least, within the context of generic models in which structure grows by gravitational instability from initial Gaussian perturbations) have perhaps emphasized the problem that has long been implicit in the theory—that galaxies are not to be trusted as reliable tracers of mass, and you should trust the young ones even less than the older ones. On the other hand, as eloquently pointed out by Carlos Frenk in his closing remarks, it is human nature that one’s interest in anyone (or anything) that one understands or trusts quickly wanes, whereas the mysterious and untrustworthy seem all the more attractive, despite one’s better judgment. This probably means that many cosmologists will be trying to understand galaxy formation after all of the cosmological parameters are sorted out by MAP and Planck.
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