Spectrum Sharing Between Radar and Communication Systems Based on Overlapped Virtual Subarrays

YUEZHOU WU1, BING LI2,3, SHIQI LIU4, AND XINGYI WANG1

1School of Computer Science, Civil Aviation Flight University of China, Guanghan 618307, China
2School of Electrical Engineering, Southwest Jiaotong University, Chengdu 611756, China
3State Key Laboratory of Millimeter Waves, Southeast University, Nanjing 210096, China
4College of Electronic and Information Engineering, Shenzhen University, Shenzhen 518060, China

Corresponding authors: Bing Li (blljess@outlook.com) and Shiqi Liu (liu.shiqi@outlook.com)

This work was supported in part by the Natural Science Foundation of Sichuan under Grant 2022NSFSC1801, in part by the National Natural Science Foundation of China under Grant U2033213, in part by the Sichuan Science and Technology Program under Grant 2021YFS0319, in part by the National Key Research and Development Program of China under Grant 2021YFF0603904, in part by the Fundamental Research Funds for the Central Universities under Grant ZJ2022-004, and in part by the Foundation of State Key Laboratory of Millimeter Waves under Grant K202235.

ABSTRACT In this paper, we propose a new technique for spectrum sharing between radar and communication systems. The new technique enjoys the advantages of null-space projecting which can be used to mitigate the interference between radar and communication systems. In order to reduce the number of radar antennas and reduce the requirement of manufacture, deployment, and implementation, we also introduce virtual subarray architecture to reap a more effective transmit array aperture. The essence is to partition the multiple-input multiple-output (MIMO) antenna array into multiple overlapped subarrays to reap a compromise between coherent gain and diversity gain. We show that the configuration with overlapped subarrays can be used to form narrower beams with lower sidelobes, which is good for reaping higher performance in target detection. In order to evaluate the impact on the performance of radar systems with the implementation of spectrum sharing, the generalized likelihood ratio test for target detection is derived. It is shown that, although the implementation of the proposed spectrum sharing method would lose the radar system’s target detection performance, we can reap the benefits of spectrum sharing. That is to say, we achieve a good tradeoff between spectrum sharing and target detecting.

INDEX TERMS Spectrum sharing, MIMO radar, communication system, generalized likelihood ratio test, target detection.

I. INTRODUCTION

Over the last decade, the rapid development of wireless communication technologies increases the demand for more spectrum resources. However, most of the available radio spectrum has already been allocated to existing wireless systems, few spectrum resources are remaining. Nonetheless, the study by the Spectrum Policy Task Force (SPTF) of the Federal Communications Commission (FCC) has shown that some frequency bands are heavily used by licensed systems in particular locations and at particular times, but that there are also many frequency bands which are only partly occupied or largely unoccupied [1]. So, it is an emergency to design new paradigms to enhance the utilization of the radio frequency spectrum.

In a conventional spectrum licensing scheme, spectrum allocation is based on the command-and-control model, where the radio spectrum allocated to license users is not used, it cannot be utilized by unlicensed users and applications [2]. It is this static and inflexible allocation that leads to the problem of spectrum scarce problem. To solve the problem, cognitive radio is introduced to reuse the underutilized
licensed spectrum for unlicensed users [3], [4]. Whereas, it is an opportunistic access mechanism. Unlicensed users can only use the spectrum when it is unoccupied based on the implementation of spectrum sensing [5], [6], [7].

Recently, the efforts of exploring co-channel sharing approaches among secondary network entities have attracted a lot of interest [8]. In the United States, according to a Fast Track Report from the National Telecommunications and Information Administration (NTIA) [9], Federal agencies are now willing to share their spectrum with commercial users due to the high demand for spectrum by commercial operators. The 3550-3650 MHz band, currently used for military radar operations, is identified for spectrum sharing between military radars and communication systems [10].

However, it has not been possible for commercial communication systems, such as WiMAX or LTE, to directly operate in the vicinity of radar systems, on the same or adjacent frequency bands as the radar. This is because of the interference from radar signals. That’s why spectrum sharing between radars and communication systems requires a large separation distance in conventional implementation. So, in order to share radar bands for commercial operation, we have to address interference mitigation techniques. In this work, we focus on interference caused by radar systems to communication systems and propose methods to mitigate this interference.

A series of efforts have been made to exploit the potential for spectrum sharing between radar and telecommunication systems. Cooperative sensing-based spectrum sharing approaches can be utilized where the radar’s allocated bandwidth is shared with communications systems [11], [12]. In [13], the authors consider an uncooperative spectrum sharing scenario, wherein a radar system is to be overlaid with a pre-existing wireless communication system.

With the rapid development of semiconductor technologies, the wideband transceiver is in the trend to be integrated on-chip with compact size and low power consumption. The authors in [14] investigate the integrated wideband chip-scale RF transceivers for radar applications including imaging, multimodal sensing, and recognition on gaits, where frequency modulated continuous wave (FMCW) radar sensors are investigated in detail. To seize future development opportunities, [15] introduces the space-air-ground-sea integrated network (SAGSINe) and related technologies, including the shape-adaptive antenna and radar-communication integration. Reference [16] identifies full-duplex operation as the key enabler for joint communication and sensing (JCAS) systems. Reference [17] presents a system architecture that unifies three types of sensing, investigates the required modifications to existing mobile networks, and exemplifies the signals applicable to sensing. Reference [18] reviews the entire trends that drive the development of radar sensing and wireless communication using joint radar and communication (JRC). Reference [19] proposes a centralized framework and the corresponding algorithm, which relies on the vehicle-to-everything (V2X) communication systems to allocate the spectrum resources for automotive radars to minimize interference. Reference [20] provides a signal processing perspective of mm-wave JRC systems with an emphasis on waveform design.

Besides the above studies, another line of work resorts to beamforming [21] and waveform-shaping [22] based schemes to mitigate radar interference in the communication system. In [22], the authors study the target detection performance of spectrum sharing multiple-input multiple-output (MIMO) radars. However, the number of transmit antennas in the radar system grows linearly with the number of users in the communication system. The requirement of manufacture and deployment for radar systems will become more rigorous when the communication system has a large scale.

In this paper, we consider a coexistence scenario where the MIMO radar system and communication system work at the same time with the same frequency. We propose a null-space-based method to mitigate the interference caused by radar systems to the communication system. Besides, with the help of using virtual subarrays of overlapped-MIMO architecture, we partition the MIMO antenna array into multiple overlapped subarrays to increase diversity gain and reduce the requirement of the number of antennas. In this work, we analyze the problem of target detection when sharing radar spectrum with cellular systems and show the advantages of using overlapped MIMO to alleviate interference. We study the performance of the considered method and show different measurements to be taken when the number of the radar antennas is different.

The main contributions of this paper can be summarized as follows:

- We analyze the advantage of overlapped virtual subarrays in restraining sidelobe and increasing diversity gain. Based on that, we construct a spectrum-sharing architecture between MIMO radar and the communication system.
We project the radar signal to the null-space of the interference channel to mitigate the interference between the two systems.

Virtual subarrays are used to reap a more effective transmit array aperture and increase the opportunity of spectrum sharing with a small number of antennas in the radar system. In other words, the requirement for manufacture, deployment, and utilization is reduced.

We analyze the relationship between the number of radar antennas and the number of communication users. Based on that, we propose different spectrum-sharing strategies to be implemented.

The remainder of this paper is organized as follows. Section II describes the considered MIMO radar, orthogonal waveforms, and communication system model. Section III discusses the spectrum sharing architecture between MIMO radar and the communication system, including the overlapped MIMO radar and the spectrum sharing algorithm we use. Section IV focuses on the target detection performance of lapped MIMO radar and the spectrum sharing algorithm we use. Section V presents numerical results. Section VI concludes the paper.

Notations: \( \otimes \) is the Kronecker product operator, \( \| \cdot \| \) is the \( \ell_2 \) norm.

II. SYSTEM DESCRIPTION

In this paper, we consider a scenario where a collocated MIMO radar shares spectrum with an LTE communication system, which is shown in Fig. 1. The MIMO radar has \( M_T \) transmit antennas and \( M_R \) receive antennas. The LTE communication system has \( N_{BS} \) base stations (BSs), each BS is equipped with \( N_{T}^{BS} \) transmit antennas and \( N_{R}^{BS} \) receive antennas, with the \( i \text{-th} \) BS supporting \( K_{i}^{UE} \) user equipments (UEs). Each UE is equipped with \( N_{T}^{UE} \) transmit antennas and \( N_{R}^{UE} \) receive antennas.

Let \( \phi(t) \) be the waveform emitted from the MIMO radar, which is defined as

\[
\phi(t) = [\phi_1(t) \; \phi_2(t) \ldots \phi_{M_T}(t)]^T,
\]

where \( t \) is the time index within the radar pulse and \( (\cdot)^T \) denotes the transpose operation. Note that the \( m \text{-th} \) transmit antenna emits the \( m \text{-th} \) element of the vector \( \phi(t) \), which is \( \phi_m(t) \). We assume that each element of the transmitted waveform satisfies the orthogonality principle

\[
\int T_0 \phi(t) \phi^H(t)dt = I_{M_T},
\]

where \( T_0 \) is the radar pulse width, \( (\cdot)^H \) stands for the Hermitian transpose, and \( I_{M_T} \) is the \( M_T \times M_T \) identity matrix.

Under point target assumption, the target signal can be written as

\[
x_{i}(t) = \beta_i \langle a(\theta_i) \phi(t) \rangle b(\theta_i),
\]

where \( \theta_i \) is the target direction, \( \beta_i \) is the complex-valued reflection coefficient of the focal point \( \theta_i \), \( a(\theta) \) and \( b(\theta) \)

are the actual transmit and actual receive steering vectors associated with the direction \( \theta \).

Let \( s^{UE}_j(t) \) be the signal transmitted from the \( j \text{-th} \) UE in the \( i \text{-th} \) cell. If there is no interference from the radar system, the received signal at the \( i \text{-th} \) BS receiver can be written as

\[
r_i(t) = \sum_{j=1}^{K^{UE}} H_j N_{T}^{BS} \times N_{T}^{UE} s_j^{UE}(t) + n(t),
\]

where \( H_j N_{T}^{BS} \times N_{T}^{UE} \) is the channel between the \( j \text{-th} \) UE transmitter and the \( i \text{-th} \) BS receiver. \( n(t) \) is the additive white Gaussian noise with zero mean and variance \( \sigma_n^2 \).

III. SPECTRUM SHARING ARCHITECTURE

In this section, we describe the architecture of the spectrum sharing problem, including the overlapped MIMO radar and the spectrum sharing algorithm we used.

A. OVERLAPPED MIMO RADAR

Considering that the overlapped MIMO architecture can increase the effective number of transmitting arrays, we implement it to reap more coherent processing gain and overall suppressed sidelobes [23]. The key idea behind this approach is to partition the transmit arrays into \( K \) subarrays where \( 1 \leq K \leq M_T \), which are allowed to overlap. The overlapped MIMO radar formulation is shown in Fig. 2.

With the overlapped MIMO architecture, the number of antenna elements in each subarray is

\[
M_n = M_T - K + 1.
\]

Then the effective transmit array aperture can be expressed as

\[
M_e = (M_T - K + 1)K.
\]

Obviously, \( M_e \geq M_T \), which illustrates that the overlapped MIMO architecture increases the effective number of transmitting arrays.
The complex envelop of the signal at the output of the $k^{th}$ subarray can be expressed as

$$s_k(t) = \frac{1}{\sqrt{M_T}} \frac{\phi_k(t)}{K} w_k, k = 1, \ldots, K,$$  

(7)

where $w_k$ is the $M_m \times 1$ unit-norm complex vector which consists of $M_m$ beamforming weights corresponding to the active antennas of the $k^{th}$ subarray.

The signal reflected by a target located at the direction $\theta$ in the far field can be expressed as

$$r(t, \theta) = \frac{1}{\sqrt{M_T}} \frac{\beta(\theta)}{K} \sum_{k=1}^{K} w_k^H a_k(\theta) e^{-j2\pi f \tau_k(\theta)} \phi_k(t),$$  

(8)

where $a_k(\theta)$ is the steering vector of the $k^{th}$ subarray, and $\tau_k(\theta)$ is the time required for the wave to travel from the first element to the $k^{th}$ element.

The received complex vector of the array observation can be written as

$$\mathbf{x}(t) = r(t, \theta_b) \mathbf{b}(\theta_b) + \sum_{i=1}^{D} r(t, \theta_i) \mathbf{b}(\theta_i) + \mathbf{n}(t),$$  

(9)

where $D$ is the number of interfering signals.

By matched-filtering $\mathbf{x}(t)$ to each of the waveforms, we can obtain the virtual data vector as [23]

$$\mathbf{y}_v = \sqrt{\frac{M_T}{K}} \beta(\theta_b) \mathbf{u}(\theta_b) + \sum_{i=1}^{D} \sqrt{\frac{M_T}{K}} \beta(\theta_i) \mathbf{u}(\theta_i) + \mathbf{n},$$  

(10)

where $\mathbf{u}(\theta) = (\mathbf{c}(\theta) \otimes \mathbf{d}(\theta)) \otimes \mathbf{b}(\theta)$ is the virtual steering vector with the coherent processing vector

$$\mathbf{c}(\theta) = [w_1^H a_1(\theta) \cdots w_K^H a_K(\theta)]^T,$$  

(11)

and the diversity vector

$$\mathbf{d}(\theta) \triangleq [e^{-j \tau_1(\theta)} \cdots e^{-j \tau_K(\theta)}]^T.$$  

(12)

In the case of non-adaptive beamforming, the corresponding beamformer weight vectors are given as

$$w_k = \frac{a_k(\theta_b)}{\|a_k(\theta_b)\|}, k = 1, \ldots, K.$$  

(13)

At the receive array, the receive beamformer weight vector is given by

$$\mathbf{w}_d = (\mathbf{c}(\theta_b) \otimes \mathbf{d}(\theta_b)) \otimes \mathbf{b}(\theta_b).$$  

(14)

Let $G(\theta)$ be the normalized overall beam pattern, that is

$$G(\theta) = \frac{|\mathbf{w}_d^H \mathbf{u}(\theta)|^2}{|\mathbf{w}_d^H \mathbf{u}(\theta_b)|^2} = \frac{|\mathbf{u}^H(\theta_b) \mathbf{u}(\theta)|^2}{\|\mathbf{u}(\theta_b)\|^4}. $$  

(15)

For the case of a uniform linear array (ULA), we have

$$a_i^H(\theta_b) a_i(\theta) = \cdots = a_k^H(\theta_b) a_k(\theta).$$  

(16)

Then, the beam pattern of the overlapped MIMO radar for a ULA with overlapped partitioning of transmitting subarrays can be expressed as

$$G_K(\theta) = \frac{|a_k^H(\theta_b) a_k(\theta)|^2 |d^H(\theta_b) d(\theta)|^2 |b(\theta)|^2}{\|a_k^H(\theta_b)\|^4 \|d(\theta)\|^4 \|b(\theta)\|^4}. $$  

(17)

### B. SPECTRUM SHARING ALGORITHM

In the spectrum sharing scenario, the radar system is sharing $N_{BS}$ interference channels with the communication system. The received signal at the $i^{th}$ BS receiver can be written as

$$\mathbf{r}_i(t) = \mathbf{H}_{i}^{N_{BS} \times M} \mathbf{s}(t) + \sum_{j=1}^{K_{BS}} \mathbf{H}_{j}^{N_{BS} \times N_{BS}} \mathbf{s}_{j}^{N_{BS}}(t) + \mathbf{n}_i(t),$$  

(18)

where the first part is the interference from the radar system with interference channels $\mathbf{H}_{i}^{N_{BS} \times M}$, and $M$ is the number of the transmit antennas, which is equal to $M_T$. $M_k = (M_T - K + 1)K$ for the traditional MIMO and the overlapped MIMO architecture, respectively. $\mathbf{s}(t)$ is the transmitted complex vector of the array. The second part is the received communication signal. The third part is the noise.

In order to avoid interference to the $i^{th}$ BS, we map $\mathbf{s}(t)$ onto the null-space of $\mathbf{H}_{i}^{N_{BS} \times M}$, then the first part of (18) will equal $\mathbf{0}$, and we will have (4) instead of (18). In this circumstance, the two systems will coexist in the same spectrum. And the aim of spectrum sharing will be achieved.

In the traditional MIMO architecture, when $M_T$ is less than $N_{BS}$, it is impossible to map the radar signal to the null-space of the interference channel because there is no sufficient degree of freedom (DoF). Whereas, with the advantage of using overlapped architecture, we can reap more DoF to mitigate interference to the communication system.

Because the DoF available at the radar is associated with the effective transmit array aperture. We consider two spectrum sharing scenarios which are discussed as follows.

Case 1 ($M_T - K + 1)K > N_{BS}N_{BS}$):

Consider a scenario in which the effective transmit array aperture is very large as compared to the combined antenna array of $N_{BS}$ BSs. In this scenario, the available DoF is sufficient enough for the overlapped MIMO radar to simultaneously mitigate interference to all the $N_{BS}$ BSs.

The combined interference channel that the radar shares with the communication system can be written as

$$\mathbf{H} = [\mathbf{H}_{1}^{N_{BS} \times M} \mathbf{H}_{2}^{N_{BS} \times M} \cdots \mathbf{H}_{N_{BS}}^{N_{BS} \times M}] .$$  

(19)

In order to implement spectrum sharing, we need to find the projection matrix to project the radar signal onto the null space of the combined interference channel $\mathbf{H}$.

At first, based on the singular value decomposition (SVD) theorem, we find SVD of $\mathbf{H}$ as

$$\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H.$$  

(20)
where $U$ and $H$ are complex unitary matrices. $\Sigma$ is the matrix of singular values, which can be written as
\[
\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_l),
\tag{21}
\]
where $l = \min(N_{BS}^{R}, N_{BS}^{C})$, in this case $l = N_{BS}^{R}$ and $\sigma_1 > \sigma_2 > \cdots > \sigma_l > \sigma_{l+2} = \cdots = \sigma_l = 0$ are the singular values.

Now, we define
\[
\Sigma' = \text{diag}(\sigma_1', \sigma_2', \ldots, \sigma_{M_c}'),
\tag{22}
\]
with
\[
\sigma_u' = \begin{cases} 
0, & u \leq n \\
1, & u > n
\end{cases}.
\tag{23}
\]

Based on that, we can define the projection matrix as
\[
P = \Sigma'\Sigma' \Sigma' H^H.
\tag{24}
\]

The radar signal projected onto the null space of the interference channel will become
\[
s'(t) = Ps(t).
\tag{25}
\]

And the first part of (18) can be expressed as
\[
HS'(t) = HPs(t)
= U\Sigma V^H \times V\Sigma' V^H \times s(t)
= 0
\tag{26}
\]

It is obvious that the interference from the radar system is eliminated, and the two systems coexist in the same spectrum.

Case 2 ($N_{BS}^{R} \geq (M_T - K + 1)K > N_{BS}^{C}$):

In this case, we consider a scenario in which the effective transmit array aperture is not very large. In this scenario, the available DoF is not sufficient enough to simultaneously mitigate interference to all the $N_{BS}^{R}$ BSs. However, the amount of the transmit array aperture is large enough to implement interference mitigation to one of the $N_{BS}^{C}$ BSs. So, in this case, it is important to find a tradeoff between spectrum sharing and interference mitigation. Considering that, we select the optimal BS to minimize the degradation of the radar waveform. The other $N_{BS}^{C} - 1$ BSs in the communication system can be moved to non-radar frequency bands to avoid interference from the radar system through spectrum sensing and access in cognitive radio techniques [25], [26], [27].

If spectrum sharing is implemented with the $i$th BS, we need to find the projection matrix to project the radar signal onto the null space of the interference channel.

At first, based on the singular value decomposition (SVD) theorem, we find SVD of $H_i$ as
\[
H_i = U_i\Sigma_i V_i^H,
\tag{27}
\]
where $U_i$ and $H_i$ are complex unitary matrices. $\Sigma_i$ is the matrix of singular values, which can be written as
\[
\Sigma_i = \text{diag}(\sigma_{i,1}, \sigma_{i,2}, \ldots, \sigma_{i,l}),
\tag{28}
\]
where $l = \min(N_{BS}^{C}, M_c)$, in this case $l = N_{BS}^{C}$ and $\sigma_{i,1} > \sigma_{i,2} > \cdots > \sigma_{i,n} > \sigma_{i,n+1} = \sigma_{i,n+2} = \cdots = \sigma_{i,l} = 0$ are the singular values.

Now, we define
\[
\Sigma'_i = \text{diag}(\sigma'_{i,1}, \sigma'_{i,2}, \ldots, \sigma'_{i,M_c}).
\tag{29}
\]

With
\[
\sigma'_{i,u} = \begin{cases} 
0, & u \leq n \\
1, & u > n
\end{cases}.
\tag{30}
\]

Based on that, we can define the projection matrix as
\[
P_i = \Sigma'_i\Sigma_i V_i^H.
\tag{31}
\]

The radar signal projected onto the null space of the interference channel will become
\[
s'(t) = P_i s(t).
\tag{32}
\]
And the first part of (18) can be expressed as
\[
H_s(t) = H_s(t) = U_i \Sigma_i V_i^H \times V_i \Sigma_i V_i^H \times s(t)
\]
\[
= 0
\]

(33)

It is obvious that the interference from the radar system is eliminated, and the radar system coexists with the degradation of radar waveform, the projection matrix can be set as
\[
\hat{P} = P_{\text{min}},
\]

(34)

where
\[
i_{\text{min}} = \arg \min_{1 \leq i \leq N^{\text{BS}}} \| P_i s(t) - s(t) \|_2.
\]

(35)

IV. PERFORMANCE ANALYSIS

In this paper, we project the radar signal onto the null space of the interference channel to avoid interference to the communication system. It is a tradeoff between spectrum sharing and the performance of the radar system. The implementation of spectrum sensing is acted at the cost of the degradation of radar waveform. So, we should pay attention to the radar system’s performance. In this section, we focus on the target detection performance.

For target detection and estimation, a radar system has to distinguish the following two hypotheses
\[
y(t) = \begin{cases} 
\hat{n}(t) & H_0 \\
\beta A(\theta)s(t) + \hat{n}(t) & H_1
\end{cases}
\]

(36)

where \(s(t)\) is the transmitted signal, \(y(t)\) is the received signal, \(\beta\) is the complex-valued reflection coefficient of the focal point \(\theta\), \(A(\theta)\) is the transmit-receive steering matrix, and \(\hat{n}(t)\) is the additive white Gaussian noise. Hypotheses \(H_0\) and \(H_1\) represent the absence and presence of the target, respectively.

Since \(\theta\) and \(\beta\) are unknown, but deterministic, we can use the generalized likelihood ratio test (GLRT). The advantage of using GLRT is that we can replace the unknown parameters with their maximum likelihood (ML) estimates. The ML estimator for target localization is given by [28]
\[
(\hat{\theta}, \hat{\beta})_{\text{ML}} = \arg \min_{\theta, \beta} \| y - \beta A(\theta)s(t) \|_2.
\]

(37)

After optimization with respect to \(\beta\), the ML estimator for \(\theta\) is given by
\[
\hat{\theta}_{\text{ML}} = \arg \max_{\theta} L(\theta),
\]

(38)

where
\[
L(\theta) = \frac{|u^H(\hat{\theta}_{\text{ML}})u^H(\hat{\theta}_{\text{ML}})|^2}{M u^H(\hat{\theta}_{\text{ML}}) R_s^T u(\theta)}.
\]

(39)

and
\[
E = \int_{T_0} y(t) s^H(t) dt,
\]

(41)

Then, the GLRT can be written as
\[
L_y = \max_{\theta, \beta} \left\{ \log f_y(y, \theta, \beta; H_1) - \log f_y(y; H_0) \right\}.
\]

(42)

where \(f_y(y, \theta, \beta; H_1)\) and \(f_y(y; H_0)\) are the probability density functions of the received signal under hypotheses \(H_1\) and \(H_0\), respectively. \(\delta\) is the threshold required for a decision between the two hypotheses. Hence, the GLRT can be written as
\[
L(\hat{\theta}_{\text{ML}}) = \frac{|u^H(\hat{\theta}_{\text{ML}})Eu^H(\hat{\theta}_{\text{ML}})|^2 H_1}{M u^H(\hat{\theta}_{\text{ML}}) R_s^T u(\hat{\theta}_{\text{ML}}) H_0} \geq \delta.
\]

(43)

According to [29], [30], [31], the asymptotic statistic of \(L(\hat{\theta}_{\text{ML}})\) under the two hypotheses has the following distribution
\[
L(\hat{\theta}_{\text{ML}}) \sim \begin{cases} 
\chi^2_2 & H_0 \\
\chi^2_2(\rho) & H_1
\end{cases},
\]

(44)

where \(\chi^2_2\) and \(\chi^2_2(\rho)\) are central and non-central chi-square distributions respectively, each with two degrees of freedom and a non-centrality parameter of \(\rho\) for the latter one, which is equal to
\[
\rho = \frac{\beta^2}{\sigma_n^2} | u^H(\theta) R_s^T u(\theta) |^2.
\]

(45)

Based on the distribution, the probability of false alarm can be expressed as
\[
P_f = P[L(\hat{\theta}_{\text{ML}}) \geq \delta | H_0] = \frac{\Gamma(2, \delta/(2\sigma_n^2))}{\Gamma(2)}
\]

(46)

where \(\Gamma(\cdot)\) is the gamma function, \(\Gamma(\cdot, \cdot)\) is the incomplete gamma function.

Given the target false alarm probability \(P_f\), the threshold \(\delta\) can be determined as
\[
\delta = 2\sigma_n^2 \Gamma^{-1}(2, P_f \Gamma(2))
\]

(47)

where \(\Gamma^{-1}(\cdot, \cdot)\) denotes the inverse of the incomplete gamma function.

Then, the probability of detection can be given as
\[
P_d = P[L(\hat{\theta}_{\text{ML}}) \geq \delta | H_1] = Q_2(\sqrt{\frac{\rho}{\sigma_n^2}}, \sqrt{\frac{\delta}{\sigma_n^2}}),
\]

(48)

where \(Q_2(\cdot, \cdot)\) is the generalized Marcum Q-function.

When \(s(t)\) is an orthogonal signal,
\[
R_s = I_M.
\]

(49)

In this case, the GLRT can be expressed as
\[
L_{\text{Orthog}}(\hat{\theta}_{\text{ML}}) = \frac{|u^H(\hat{\theta}_{\text{ML}})Eu^H(\hat{\theta}_{\text{ML}})|^2 H_1}{M u^H(\hat{\theta}_{\text{ML}}) R_s^T u(\hat{\theta}_{\text{ML}}) H_0} \geq \delta_{\text{Orthog}}.
\]

(50)
and the asymptotic statistics of $L(\hat{\theta}_{ML})$ in this case is

$$L_{\text{Orthog}}(\hat{\theta}_{ML}) \sim \begin{cases} \chi^2_\nu^2(H_0) & \nu \in \{0, 1\}, \\ \chi^2_\nu^2(\rho_{\text{Orthog}}) & H_1, \end{cases}$$  \tag{51}$$

where

$$\rho_{\text{Orthog}} = \frac{M^2 |\beta|^2}{\sigma_n^2}. \tag{52}$$

For the spectrum sharing scenario considered in this paper, we have

$$\mathbf{R}_{\text{Share}} = \mathbf{R}_{\text{Share}}' = \int_{t_0}^{t_f} \mathbf{s}(t)\mathbf{s}^H(t)dt. \tag{53}$$

Therefore, the GLRT can be expressed as

$$L_{\text{Share}}(\hat{\theta}_{ML}) = \frac{|\mathbf{u}^H(\hat{\theta}_{ML})\mathbf{u}^H(\hat{\theta}_{ML})|^2}{M|\mathbf{u}^H(\hat{\theta}_{ML})\mathbf{R}_{\text{Share}}\mathbf{u}(\hat{\theta}_{ML})|^2} \frac{H_1}{H_0} \delta_{\text{Share}}, \tag{54}$$

and the asymptotic statistics of $L(\hat{\theta}_{ML})$ in this case is

$$L_{\text{Share}}(\hat{\theta}_{ML}) \sim \begin{cases} \chi^2_\nu^2(H_0) & \nu \in \{0, 1\}, \\ \chi^2_\nu^2(\rho_{\text{Share}}) & H_1, \end{cases}$$  \tag{55}$$

where

$$\rho_{\text{Share}} = \frac{|\beta|^2}{\sigma_n^2} |\mathbf{u}^H(\theta)\mathbf{R}_{\text{Share}}\mathbf{u}(\theta)|^2. \tag{56}$$

V. NUMERICAL RESULTS

In this section, in order to study the performance of the spectrum sharing architecture considered in this work, we demonstrate the simulation results. A ULA of a few omnidirectional antennas spaced with half a wavelength apart from each other is considered. The signal passes through a Rayleigh distributed channel and is subject to additive white Gaussian noise (AWGN).

In order to evaluate the advantage of using virtual subarrays, Fig. 3 and Fig. 4 demonstrate the overall beampatterns implementing conventional transmit-receive beamformer and the overlapped subarrays architecture, respectively. We assume two interfering targets located at directions $-30^\circ$ and $-10^\circ$. The target of interest is assumed to reflect a plane-wave that impinges on the array from the direction $\theta_s = 10^\circ$. Here the number of the antenna elements is set as 16.

Fig. 3 shows the overall beam pattern for three different radar formulations with the number of subarrays being 1, 5, and 16, respectively. And the overlapped MIMO radar will become a phased-array radar or a MIMO radar when $K$ equals to 1 and 16, respectively. We can see that the phased-array and MIMO radars have the same overall transmit/receive beampatterns. This is because the effective number of transmitting arrays of them are the same. Besides, it is worth noting that the overall beampattern shape for the overlapped MIMO radar is significantly improved as compared to the beampatterns of the phased-array and MIMO radars. It is a good advantage for the use of target detection.

Fig. 4 shows the overall beam pattern for the three different radar formulations with the implementation of null-space projecting. We can observe that, although the implementation of projecting has reduced sidelobe suppression, the beam pattern of the overlapped MIMO radar still provides encouraging suppression compared to the other two radars. Meanwhile, we can reap the benefits of spectrum sharing because the implementation of null-space projecting minimizes interference from the radar system to the communications system.

In order to evaluate the detection performance of the radar system with the implementation of spectrum sharing, in Fig. 5, we consider the case when the effective transmit array aperture of the overlapped MIMO radar is very large as compared to the combined antenna array of all BSs.
in order to get a desired $P_d$ for a fixed $P_f$, we need more SNR for spectrum sharing. For example, according to Fig. 5 (a), with $P_f = 10^{-3}$, if we desire $P_d = 0.9$, then we need 2 to 10 dB more gain in SNR for spectrum sharing when $N_{RBS}$ is 4, 6, 8, 10, and 12, respectively, to get the same result produced by the orthogonal waveforms. According to Fig. 5 (b), with $P_f = 10^{-5}$, if we desire $P_d = 0.9$, then we need 3 to 11 dB more gain in SNR for spectrum sharing when $N_{RBS}$ is 4, 6, 8, 10, and 12, respectively, to get the same result produced by the orthogonal waveforms.

In order to illustrate the necessity of selecting the best sharing channel when the available DoF of the radar system is not very large, in Fig. 6, we consider the case when the effective transmit array aperture is not large enough to simultaneously share spectrum with all the communication BSs but large enough to share spectrum with one of them. In this scenario, we assume a ULA with 8 antenna elements at the transmitter. In overlapped radar, the number of the subarrays is $K = 3$, and each subarray has 6 antenna elements. In a communications system, there are $N_{BS} = 5$ BSs, each BS is equipped with $N_{RBS} = 8$ receive antennas. It can be noted that $N_{BS}N_{RBS} \geq (MT - K + 1)K > N_{RBS}$ is satisfied. In this case, the radar system tries to share the spectrum with one of the BSs. And it is important to find the optimal BS to minimize the degradation of radar waveform. In Fig. 6, detection performance for orthogonal waveform signal and five different spectrum sharing signals with the implementation of null space projecting onto five different BSs is illustrated, where the probability of. We can observe that, with $P_f = 10^{-3}$, in order to achieve a detection probability of 90%, we need 6 dB to 9 dB more gain in SNR as compared to the orthogonal waveform. With $P_f = 10^{-5}$, in order to achieve a detection probability of 90%, we need 5 dB to 7 dB more gain in SNR as compared to the orthogonal waveform. Using the method shown in (35), we “principle” can select the interference channel with the least degradation of radar waveform. In this scenario, the first BS and the fourth BS will be selected to coexist with the radar system when the probability of false alarm is set as $P_f = 10^{-3}$, and $P_f = 10^{-5}$, respectively.

**Fig. 6.** Detection performance when the radar system simultaneously share spectrum with one of the 5 BSs in the case that the effective transmit array aperture of the overlapped MIMO radar is not large enough to simultaneously share spectrum with all the communication BSs but large enough to share spectrum with one of them.

**VI. CONCLUSION**

In this paper, we considered a scenario where a MIMO radar shares a spectrum with an LTE communication system. A spectrum-sharing architecture was designed to realize the coexistence of the two systems and mitigate the interference between them. Based on null-space theory, we proposed to project the radar signal to the null-space of the interference channel, so as to avoid interference to the communication system. Additionally, the virtual subarray architecture was used to reap a more effective transmit array aperture. This configuration increased diversity gain and formed narrower beams. We enjoyed this advantage to ensure the performance in target detection and reduce the requirement of the number of antennas and naturally reduce the requirement of manufacture, deployment, and implementation.
In the case that the effective transmit array aperture of the overlapped MIMO radar is very large as compared to the combined antenna array of all BSs, the available DoF is sufficient enough to mitigate interference to all the BSs. We proposed to simultaneously share the spectrum with all the BSs. In the case that the effective transmit array aperture of the overlapped MIMO radar is not very large, we proposed to select the optimal BS to minimize the degradation of radar waveform.

Theoretical analysis and simulation results demonstrated that, although the implementation of the proposed spectrum sharing method would reduce the radar system’s target detection performance, we reaped the benefits of spectrum sharing. Taken together, the proposed technique achieved a good tradeoff between spectrum sharing and target detecting.

REFERENCES

[1] Federal Communications Commission Spectrum Policy Task Force: Report of the Spectrum Efficiency Working Group, Federal Communications Commission, Washington, DC, USA, Nov. 2002.

[2] S. Haykin, “Cognitive radio: Brain-empowered wireless communications,” IEEE J. Sel. Areas Commun., vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] J. Mitola and G. Q. Maguire, Jr., “Cognitive radio: Making software radios more personal,” IEEE Pers. Commun., vol. 6, no. 4, pp. 13–18, Apr. 1999.

[4] M. T. Mason, M. Mzyece, and N. Nilatlapa, “Spectrum decision in cognitive radio networks: A survey,” IEEE Commun. Surveys Tuts., vol. 15, no. 3, pp. 1088–1107, 3rd Quart., 2013.

[5] S. Q. Liu, B. J. Hu, and X. Y. Wang, “Hierarchical cooperative spectrum sensing based on double thresholds energy detection,” IEEE Commun. Lett., vol. 16, no. 7, pp. 1096–1099, Jul. 2012.

[6] S.-Q. Liu and B.-J. Hu, “Analysis of sensing efficiency for cooperative spectrum sensing with malicious users in cognitive radio networks,” IEEE Commun. Lett., vol. 18, no. 9, pp. 1645–1648, Sep. 2014.

[7] O. Alttrad, S. Muhaidat, A. Al-Dweik, A. Shami, and P. D. Yoo, “Opportunistic spectrum access in cognitive radio networks under imperfect spectrum sensing,” IEEE Trans. Veh. Technol., vol. 63, no. 2, pp. 920–925, Feb. 2014.

[8] B. Gao, J.-M. J. Park, and Y. Yang, “Uplink soft frequency reuse for self-coexistence of cognitive radio networks,” IEEE Trans. Mobile Comput., vol. 13, no. 6, pp. 1366–1378, Jun. 2014.

[9] NTIA, An Assessment of the Near-Term Viability of Accommodating Wireless Broadband Systems in the 1675–1710 MHz, 1755–1780 MHz, 3500–3650 MHz, and 4200–4220 MHz, 4380–4400 MHz: Bands: National Telecommunications and Information Administration, Washington, DC, USA, Oct. 2010.

[10] FCC Proposes Innovative Small Cell Use in 3.5 GHz; Band, Federal Communications Commission, Washington, DC, USA, Dec. 2012.

[11] S. S. Bhat, R. M. Narayanan, and M. Rangaswamy, “Bandwidth sharing and scheduling for multimodal radar with communications and tracking,” in Proc. IEEE 7th Sensor Array Multichannel Signal Process. Workshop (SAM), Jun. 2012, pp. 233–236.

[12] L. S. Wang, J. P. Mcgeehan, C. Williams, and A. Doufexi, “Application of cooperative sensing in radar-communications coexistence,” JET Commun., vol. 2, no. 6, pp. 856–868, Jul. 2008.

[13] Y. Li, L. Zheng, M. Lops, and X. Wang, “Interference removal for radar/communication co-existence: The random scattering case,” IEEE Trans. Wireless Commun., vol. 18, no. 10, pp. 4831–4845, Oct. 2019.

[14] Z. Fang, W. Wang, J. Wang, B. Liu, K. Tang, L. Lou, C.-H. Heng, C. Wang, and Y. Zheng, “Integrated wideband chip-scale RF transceivers for radar sensing and UWB communications: A survey,” IEEE Circuits Syst. Mag., vol. 22, no. 1, pp. 40–76, 1st Quart., 2022, doi: 10.1109/MCAS.2022.3142689.

[15] T. Hong, M. Lv, S. Zheng, and H. Hong, “Key technologies in 6G SAGS IoT: Shape-adaptive antenna and radar-communication integration,” IEEE Netw., vol. 35, no. 5, pp. 150–157, Sep. 2021, doi: 10.1109/MNET.001.2100148.
BING LI received the B.S. degree in electronic information science and technology and the M.S. degree in radio physics from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2010 and 2013, respectively, and the Ph.D. degree in information and communication engineering from the South China University of Technology (SCUT), Guangzhou, China, in 2016.

She is currently a Lecturer with the School of Electrical Engineering, Southwest Jiaotong University (SWJTU), Chengdu. She is also a Postdoctoral Researcher with the Institut Langevin, Paris, France. Her research interests include the time reversal method, imaging method, non-destructive damage detection, wireless power transmission, antennas, and radio technology.

SHIQI LIU received the B.S. degree in electronic and information engineering from the Wuhan University of Science and Technology, in 2009, and the Ph.D. degree in information and communication engineering from the South China University of Technology, in 2014.

He has been working as a Senior Engineer with the Southwest China Research Institute of Electronic Equipment, since 2014. Since 2019, he has been a Research Associate Professor with Shenzhen University. His research interests include cognitive radio, spectrum sensing, radar signal processing, SAR jamming/anti-jamming, and integration of radar and communication.

XINGYI WANG received the B.S. degree in computer science and technology from Sichuan Normal University, Chengdu, Sichuan, China, in 2020. She is currently pursuing the M.S. degree in transportation with the Civil Aviation Flight University of China.

Her research interests include computer vision and machine learning.