Neutrino-Driven Mass Loading of GRMHD Outflows

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Abstract. A GRMHD model of disk outflows with neutrino-driven mass ejection is presented. The model is used to calculate the structure of the outflow in the sub-slow magnetosonic region and the mass loading of the outflow, under conditions anticipated in the central engines of gamma-ray bursts. It is concluded that magnetic launching of ultra-relativistic polar outflows is in principle possible along low inclination field lines (with respect to the symmetry axis), provided the neutrino luminosity is sufficiently low, $L_{\nu} \lesssim 10^{52}$ erg s$^{-1}$.

INTRODUCTION

Neutrino-driven mass loss plays an important role in the late phases of proto-neutron star evolution \cite{1}-\cite{7}, and in the outflows expelled from the hot, dense disks in hyper-accreting black hole systems (e.g., Refs. \cite{8, 9}). In weakly magnetized systems those neutrino-driven outflows are subrelativistic, because the entropy per baryon generated by neutrino absorption at the base of the wind ($s/k_B \lesssim 100$) does not reach the values required to accelerate the outflow to high Lorentz factors. However, in highly magnetized systems the rotational energy of the system can be extracted magnetically and the outflow can in principle accelerate to high Lorentz factors provided the mass loading of magnetic field lines is sufficiently low \cite{2, 10, 11, 12, 13}.

Magnetic extraction of the rotational energy of accreted matter in collapsars or, alternatively, a highly magnetized proto-neutron star is a plausible production mechanism for the $\gamma$-ray emitting jets inferred in long GRBs. To account for the characteristic luminosities and Lorentz factors observed in those systems two conditions must be fulfilled: (i) an ordered magnetic field with strength of the order of $10^{14} - 10^{15}$ G must be present at the flow injection point and (ii) the ratio of the neutrino-driven mass flux to magnetic flux must be sufficiently low. The possible existence of such strong magnetic fields in systems like those mentioned above appears to be indicated by observations, and is also supported by recent numerical simulations. However, the dependence of the neutrino-driven mass loading on the conditions in the disk and on the geometry of the magnetic field is yet an open issue.

A common approach in studies of magnetized disk outflows is to seek self-similar solutions of the trans-field equation (e.g., Refs. \cite{10, 11, 12}). While this approach allows the magnetic field geometry to be calculated self-consistently, incorporation of gravity (in the relativistic case) as well as neutrino heating in the flow equations is precluded. Furthermore, it does not allow simple matching of the self-similar outflow solution to a Keplerian disk (but c.f., Ref. \cite{12}). The immediate implication is that the mass-
to-magnetic flux ratio, which is determined by the regularity condition at the slow magnetosonic point, remains a free parameter. One must therefore adopt a different approach. One method is to seek solutions of the flow equations that pass smoothly through the slow magnetosonic point, assuming the magnetic field geometry is given. Such an analysis has been carried out recently for winds from a disk around a hyper-accreting black hole [9]. A brief account of the model and results is given below.

THE MODEL

We consider a stationary, axisymmetric MHD wind expelled from the surface of a hot, magnetized disk surrounding a non-rotating black hole. The range of conditions in the disk is envisioned to be similar to that computed by Popham et. al. [14] for hyperaccreting black holes with mass accretion rates $10^{-1} - 10^{-10} M_\odot$ s$^{-1}$ and viscosity parameters $\alpha_{vis} = 0.01 - 0.1$. Under such conditions the dominant cooling mechanism in the disk is neutrino emission. The major fraction of the neutrino luminosity is generated in the inner disk regions, within 10 Schwarzschild radii or so, and lies in the range $L_\nu = 10^{51} - 10^{54}$ erg s$^{-1}$ for the above range of accretion rates and viscosity parameter.

The model calculates the structure of the GRMHD outflow below the slow magnetosonic point for a given magnetic field geometry, treating the neutrinos emitted from the disk as an external energy and momentum source. To simplify the analysis the neutrino source is taken to be spherical with radius $R_\nu$. The model is characterized by three parameters: the black hole mass $M_{BH}$, the neutrino luminosity $L_\nu$, and the neutrinospheric radius $R_\nu$. It solves a set of coupled ODEs that are derived from the general relativistic energy-momentum equations, describing the change along a given streamline $\Psi(r, \theta) = \text{const}$ of the specific energy $E$, specific entropy $s$, and poloidal flow velocity $u_p$:

1. $(\rho/m_N)k_BT' = -u_\alpha q^\alpha$,  
2. $\rho c^2 s' = -q_t$,  
3. $(\ln u_p)' = \frac{F}{D}$.

Here $'$ denotes derivative along streamlines $u^\alpha \partial_\alpha$, $q^\beta$ denotes the source terms associated with energy and momentum exchange with the external neutrino source, $u^\alpha$ is the outflow 4-velocity, $\rho$ is the baryon rest mass density and $T$ is the temperature. The denominator on the R.H.S. of Eq. (3) is given explicitly as $D = -(\alpha^2 - R^2 \Omega^2 - M^2)(u_p^2 - u_{SM}^2)/(u_\alpha^2)$, where $u_\alpha$, $u_{SM}$ and $u_{FM}$ are the Alfvén, slow and fast magnetosonic wave speeds, respectively, $\alpha$ is the lapse function, $\Omega$ is the angular velocity defined below, $R$ is the cylindrical radius and $M = u_p/u_\alpha$ is the Alfvén Mach number. The term $F$ can be expressed as $F = \zeta_1(\ln B_p)' + \zeta_2(\ln \alpha)' + \zeta_3(\ln R)' + \zeta_4(\ln s)' + \zeta_5(\ln s)'$, where the coefficients $\zeta_k$ are functions of the flow parameters, given explicitly in Ref. [9], and $B_p$ is the poloidal field component. Since the derivatives $(\ln B_p)'$ and $(\ln R)'$ depend on the magnetic field geometry which is unknown a priori, additional equation is needed. Our approach is to invoke a given field geometry. To examine the dependence of mass flux on the latter, we obtained solutions for different magnetic field configurations, focusing particularly on split monopole and $r$ self-similar geometries. Equations
are augmented by an equation of state for the mixed fluid of baryons, photons and electron-positron pairs. In addition there are three integrals of motion of the MHD system: the mass-to-magnetic flux ratio $\eta(\Psi)$, the angular velocity of magnetic field lines $\Omega(\Psi)$, and the specific angular momentum $L(\Psi)$. The two invariants $\Omega(\Psi)$ and $L(\Psi)$ are fixed by a choice of boundary conditions. The mass flux $\eta(\Psi)$ is an eigenvalue of the problem. The integration starts sufficiently deep in the disk atmosphere where the density is high, the flow velocity is very small, and the ratio of baryonic pressure and the light fluid pressure is about unity so that the specific entropy is dominated by the baryons. The value of $\eta$ is adjusted iteratively by repeating the integration, changing the boundary value of the poloidal velocity $u_{p0}$ each time, until a smooth transition across the slow magnetosonic point is obtained.

**RESULTS**

Typical solutions are shown in Figs 1 and 2. Those were obtained using a split monopole geometry (see Ref. [9] for more details). In Fig. 1 the change along a field line of the flow quantities indicated is plotted against the normalized height above the disk midplane, $z/R_0$, where $R_0$ is the radius at which the field line meets the disk. The inclination angle of the field line in this example is about $12^\circ$ with respect to the rotation axis, so it lies in the so-called stable regime. The mass flux is thermally driven in this regime rather than centrifugally driven. As seen, the slow magnetosonic point is located at $z \simeq R_0$, which we find to be typical for injection along low inclination field lines. The integration started well beneath the slow magnetosonic point where the specific entropy is dominated by the baryons. In this region adiabatic cooling is negligible and kinetic equilibrium is established. The net energy deposition rate nearly vanishes, viz., $q(T_0) \simeq 0$, and the temperature is $T_0 \simeq T_\nu$, where $T_\nu$ is the neutrino equilibrium temperature. As the flow accelerates its temperature starts falling, and since the neutrino cooling rate is very sensitive to the temperature $q'$ increases rapidly leading to the steep rise of the entropy per baryon seen in Fig 1. The terminal value of $s$ is however modest, $s/k_B < 100$ in all cases examined, so the net outflow energy is practically not affected. The main effect of neutrino heating is not to change the net outflow energy along streamlines, but rather to enhance the baryon load by increasing the light fluid pressure in layers of higher baryon density. Thus, the initial value of $E$ presents, essentially, an upper limit for the asymptotic Lorentz factor of the wind. The values of the mass flux, which we somewhat arbitrarily define as $M = \rho_0 u_{p0} 2\pi R_0^2$ (for a two-sided outflow), that corresponds to the cases depicted in Fig. 1 are $10^{28}$ and $7 \times 10^{30}$ g s$^{-1}$ for $T_\nu = 2$ and 4 MeV, respectively, and the corresponding values of the total energy per baryon are $E = 5 \times 10^2 m_pc^2$ and $2m_pc^2$ for a surface poloidal field strength of $10^{15}$ Gauss.

The dependence of the mass flux on the inclination angle of the field line is shown in Fig. 2. As seen, the mass flux is quite sensitive to the inclination of the field line. For field lines in the unstable regime (inclination angles larger than $30^\circ$ to the vertical) the wind is centrifugally driven and, for the parameter range explored, is found to be always subrelativistic, owing to the large neutrino-driven mass flux expelled along the field lines. The slow magnetosonic point in this case is located very near the surface,
FIGURE 1. Profiles of various quantities in the sub-slow magnetosonic region, computed using a split monopole magnetic field with $\tan \theta = 0.2$, where $\theta$ is the inclination angle of the field line with respect to the symmetry axis, Keplerian rotation $\Omega = \Omega_k$, and surface magnetic field $B_{\phi 0} = -10 B_{\theta 0} = 10^{15}$ G. Each panel corresponds to a run with a different neutrino luminosity (indicated in terms of the effective black-body temperature $T_\nu$). The quantities plotted in each panel are: the slow magnetosonic Mach number $M_{SM}$ (solid line), the dimensionless entropy per baryon $s$ (dashed line), the temperature $T$ in units of the initial temperature $T_0$ (dotted-dashed line), and the energy deposition per baryon per unit length along the streamline measured in units of $m_p c^2 / R_0$, $dE/ dl$ (dotted line). All quantities are given as functions of the normalized height above the disk midplane $z/R_0$.

FIGURE 2. Dependence of mass-to-magnetic flux ratio $\eta$ (left axis) on the inclination angle $\theta$ of the split-monopole field line. The corresponding values of the mass flux $\dot{M}$ are indicated on the right axis.

at $z/R_0 << 1$. A systematic study of the dependence of $\eta$ on the various parameters is presented in Ref. [9].
CONCLUSION

Our main conclusion is that ejection of relativistic outflows from the innermost disk radii, within several $r_s$ or so, is possible in principle for certain magnetic field configurations even in non-rotating black holes, provided the neutrino luminosity emitted by the disk does not exceed $10^{52}$ ergs s$^{-1}$ or so, and the magnetic field is sufficiently strong ($B_p \sim 10^{15}$ G). The neutrino-driven mass flux depends rather sensitively on the neutrino luminosity, on magnetic field geometry in the sub-slow magnetosonic region, and on the angular velocity of magnetic flux surfaces, but is highly insensitive to the strength of poloidal and toroidal magnetic field components near the surface provided the Alfvén Mach number is sufficiently small there. The picture often envisioned, of an ultra-relativistic core surrounded by a slower, baryon rich wind [2, 15] seems a natural consequence of a neutrino-assisted MHD disk outflow.

The hyperaccretion process is likely to be intermittent, leading to temporal changes in the neutrino luminosity. This should result in large variations of the Lorentz factors of consecutive fluid shells expelled from the disk in the polar region, owing to the sensitive dependence of mass loading on $L_\nu$. In this situation we expect strong shocks to form in the outflow. If the polar disk outflow is associated with the GRB-producing jet, then the observed gamma-ray emission can be quite efficiently produced behind those shocks.

Finally, the conditions we find to be optimal for launching an ultra-relativistic jet in the polar region, are also the conditions favorable for large neutron-to-proton ratio in the disk. However, for the steady flow considered above we estimate the ratio of the neutronization timescale and flow time to be of order unity, and so the electron fraction is expected to evolve as the flow accelerates. Detailed analysis of the composition profile in the outflow is left for future work.

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