Probing $\alpha$-cluster distribution via $\alpha$-transfer reaction

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Abstract. The $\alpha$-transfer reaction $^{16}\text{O}(^6\text{Li},d)^{20}\text{Ne}$ is investigated in order to extract the $\alpha$-clustering probability of $^{20}\text{Ne}$. The relative wave function for the $\alpha$-$^{16}\text{O}$ system is calculated by a microscopic cluster model. We show the angular distribution of the transfer cross section is a good probe to see the radial dependence of the $\alpha$-clustering probability in the surface region.

1. Introduction

The clustering phenomena have been predicted by theoretical studies that several states of unstable nuclei or of sd-shell nuclei have a cluster structure [1]. However, there is no direct measurement of the cluster structure except for resonance states decaying into constituent clusters. Therefore, it is desirable to establish how to extract the quantitative information on the clustering from observables.

In theoretical studies it is known that the $\alpha$-cluster state develops in the surface region of nuclei. We take here $^{20}\text{Ne}$ as a typical nucleus having an $\alpha$-$^{16}\text{O}$ cluster structure. The purpose of the present study is to extract the probability of the $\alpha$-clustering in the surface region from an $\alpha$-transfer reaction, $^{16}\text{O}(^6\text{Li},d)^{20}\text{Ne}$. The transfer reaction is analyzed by means of the conventional distorted wave Born approximation (DWBA) with the overlap between $^{20}\text{Ne}$ and $\alpha$-$^{16}\text{O}$, which is calculated by the microscopic cluster model.

2. Theoretical framework

2.1. Microscopic description of cluster wave function

As for the relative wave function between $\alpha$ and $^{16}\text{O}$, we adopt a microscopic cluster model. The total wave function of $^{20}\text{Ne}$ with the resonating group method (RGM) [2, 3, 4, 5] for the $\alpha$-$^{16}\text{O}$ configuration is given by

$$|\Psi_{\text{Ne}}\rangle = \frac{1}{\sqrt{20!}} A [\chi_l(r)Y_{l0}(\hat{r})\phi(\alpha)\phi(\text{Ne})],$$

(1)

where $r$ is the relative coordinate between $\alpha$ and $^{16}\text{O}$, $A$ stands for the antisymmetrization operator, and $\phi(C)$ is the intrinsic wave function of the nucleus $C$. $\chi_l$ can be expanded by the
orthonormal set $R_{nl}$ of the radial wave function of the harmonic oscillator (HO) as

$$\chi_i(r) = \sum_n a_n R_{nl}(r),$$

$$a_n = \int r^2 dr R_{nl}(r) \chi_i(r).$$

Here, $n$ and $l$ correspond to the principal quantum number and the orbital angular momentum of the HO, respectively. The relative wave function is defined by

$$u_l(r) = \sum_n a_n \sqrt{\mu_{nl}} R_{nl}(r)$$

with the eigenvalue $\mu_{nl}$ of the RGM norm kernel [6]. For a normalized cluster wave function satisfying $\langle \Psi | \Psi \rangle = 1$, the relative wave function $u_l$ is normalized to unity. Details of the formulation of the microscopic cluster model are given in Ref. [7].

2.2. Distorted wave Born Approximation (DWBA) formalism
In this paper the $\alpha$-transfer reaction $^{16}\text{O}(^{6}\text{Li},d)^{12}\text{C}$ is described with the post form distorted wave Born Approximation (DWBA) approach. The coordinates for the reaction system are illustrated in Fig. 1. The transition matrix for the $\alpha$-transfer reaction is given by

$$T_{\text{DWBA}}^{\text{(post)}} = \langle \Psi_{f}(-) | V_{\alpha\text{d}} | \Psi_{i}^{(+)} \rangle,$$

where the $\alpha$-$d$ interaction $V_{\alpha\text{d}}$ in the final channel is adopted as the transition interaction, which causes the transition from the initial channel $i$ to the final channel $f$. The total wave functions $\Psi_{i}^{(+)}$ and $\Psi_{f}^{(-)}$ for the initial and final channels, respectively, are written as

$$\Psi_{i}^{(+)}(r_{\alpha\text{d}}, r_{i}) = \psi_{\alpha\text{d}}(r_{\alpha\text{d}}) \chi_{i}^{(+)}(r_{i}),$$

$$\Psi_{f}^{(-)}(r_{\alpha\text{O}}, r_{f}) = \psi_{\alpha\text{O}}(r_{\alpha\text{O}}) \chi_{f}^{(-)}(r_{f}),$$

where $\psi_{\alpha\text{d}} (\psi_{\alpha\text{O}})$ is the relative wave function of the $\alpha$-$d$ ($\alpha$-$^{16}\text{O}$) system and the distorted wave between $^{6}\text{Li}$ and $^{16}\text{O}$ ($d$ and $^{20}\text{Ne}$) is represented by $\chi_{i}^{(+)}$ ($\chi_{f}^{(-)}$). The superscript (+) and (−) represents the outgoing and incoming boundary conditions, respectively, on the scattering wave function. We adopt the cluster wave function defined by Eq. (4) for the radial part of $\psi_{\alpha\text{O}}$.

![Figure 1. Illustration of the three-body system.](image-url)
We adopt the Volkov No. 2 effective interaction with the Majorana parameter \( m = 0.62 \) [8] to calculate the \( \alpha^{16}\text{O} \) relative wave function \( u_\alpha \). The width parameter \( \nu = 0.16 \text{ fm}^{-2} \) is used for both \( \alpha \) and \( ^{16}\text{O} \). \( \psi_{ad} \) is calculated with a two-range Gaussian interaction \( V_{ad} \) [9].

We consider the \( ^{16}\text{O}(^6\text{Li},d)^{20}\text{Ne} \) reaction at four incident energies: 20, 38, 42, and 75 MeV. At 20 and 38 MeV, we adopt phenomenological distorting potentials of a Woods-Saxon form given in Ref. [10]. At 42 (75) MeV, potential parameters are taken from Ref. [11] (Ref. [12]) and Ref. [13] (set 2 of Ref. [14]) for the initial and final channels, respectively.

To investigate the role of the \( \alpha \)-cluster distribution in the transfer reaction \( ^{16}\text{O}(^6\text{Li},d)^{20}\text{Ne} \), the cross sections are calculated with the \( ^{20}\text{Ne} \) wave functions of the cluster model (CM), Eq. (4), and of the potential model (PM). In PM the \( \alpha^{16}\text{O} \) relative wave function is simply calculated with the Woods-Saxon potential \( V_{\alpha\text{O}} \) between \( \alpha \) and \( ^{16}\text{O} \): \( V_{\alpha\text{O}} = -V_0/[1 + \exp((r_{\alpha\text{O}} - r_0)/a)] \). The parameters of \( V_{\alpha\text{O}} \) are listed in Table 1. Figures 2(a) and 2(b) show the \( \alpha^{16}\text{O} \) relative wave functions of the 0\(^+\) (ground state) and the 1\(^-\) state (5.79 MeV), respectively. For the 1\(^-\) state we use a bound state approximation to calculate the relative wave function, taking the binding energy to be 0.2 MeV. By changing the parameter \( a \), the PM wave function (PM2) can reproduce the behavior of the CM wave function in the surface region \( r_{\alpha\text{O}} \gtrsim 5 \text{ fm} \).

The transfer cross sections of \( ^{16}\text{O}(^6\text{Li},d)^{20}\text{Ne}(0\(^+\)+1\(^-\)) \) as a function of the neutron emitting angle \( \theta \) in the center-of-mass frame are compared with the experimental data [10, 12, 15] in Fig. 3. One sees the result with CM (solid line) agrees well with that with PM2 (dotted line) up to the third maximum at all energies. On the other hand, the result with PM1 (dashed line) deviates from the other two significantly. As shown in Fig. 2, CM and PM1 gives the same distribution.

| Table 1. Potential parameters for \( V_{\alpha\text{O}} \) in fm. The depth \( V_0 \) of \( V_{\alpha\text{O}} \) is determined so as to reproduce the binding energy 4.73 MeV and 0.20 MeV for 0\(^+\) and 1\(^-\) states, respectively. |
|-----------------|-------|----------|-----------------|
|                 | \( r_0 \) | \( a \)  | \( r_0 \) | \( a \)  |
| PM1             | \( 1.25 \times 16^{1/3} \) | 0.65 | \( 1.25 \times 16^{1/3} \) | 0.65 |
| PM2             | \( 1.25 \times 16^{1/3} \) | 0.76 | \( 1.25 \times 16^{1/3} \) | 0.83 |

\[ \text{Figure 2.} \ (a) \text{ The } \alpha^{16}\text{O} \text{ relative wave functions for the } 0^+ \text{ state calculated with CM (solid line) and two parameter sets of PM: PM1 (dashed line) and PM2 (dotted line). (b) Same as in (a) but for the } 1^- \text{ state.} \]
in the surface region but are different from each other in the inner region. Whereas the two sets of PM show a difference only in the surface region. Thus, the results of Fig. 3 suggest that the transfer cross section is not sensitive to the inner part of the structure of \(^{20}\text{Ne}\) but it probes the \(^{16}\text{O}\) radial wave function in the surface region.

Note that the radial wave functions used here are normalized to unity. Nevertheless, PM1 gives a significant difference from other two models on the cross sections. This indicates the difficulty of a precise determination of the spectroscopic factor (SF) from transfer reactions. It is also important that the surface region in this study means about 5–8 fm in the \(^{16}\text{O}\) relative distance, i.e., still within a range of the nuclear interaction between the two clusters. Thus, the transfer process considered here is not governed by the asymptotic normalization coefficient (ANC). The \(\alpha\)-clustering probability in the surface region will be a third alternative to the SF and the ANC for nuclear structural information to be extracted from reaction observables.

Unfortunately, however, agreement of the calculations with CM and PM with the experimental data is not satisfactorily well. This may be due to ambiguity of the distorting potentials. We will fix this possible problem by adopting an \(\alpha + d + ^{16}\text{O}\) three-body model in describing the transfer reaction. In this case we need the \(^{16}\text{O}\) and \(d + ^{16}\text{O}\) distorting potentials, for which some global parameterizations can be used.
Figure 4. Same as in Fig. 3 but for the transfer cross section to the $1_{1}^{-}$ state of $^{20}$Ne.

Figure 4 shows the transfer cross section populating the $1_{1}^{-}$ state of $^{20}$Ne. One may draw a similar conclusion on this result to that for the transfer to the $0_{1}^{-}$ state. It implies a possibility to probe a cluster structure also in a resonance state of a nucleus. This will be one of the advantages to use transfer reactions for the study of clustering phenomena.

4. Summary
We have analyzed the transfer reaction $^{16}$O($^{6}$Li,$d$)$^{20}$Ne to investigate the radial dependence of the $\alpha$-cluster probability. The $\alpha$-$^{16}$O relative wave function calculated microscopically are adopted in the DWBA analysis. We have found that the angular distribution of the transfer cross section is a good probe to see the radial dependence of the $\alpha$-clustering probability. The procedure proposed in the present study can be useful and applicable to probe the cluster structure via observables in general systems such as unstable nuclei and sd-shell nuclei. As future work, to take into account the breakup channels of $^{6}$Li with an $\alpha + d$+$^{16}$O three-body model by means of the continuum-discretized coupled-channels method (CDCC) [16, 17, 18] will be important. This will also minimize ambiguity of distorting potentials required in reaction calculations.

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