On decay width of heavy quarkonia in QGP

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Quarkonia are some of the most important probes of the medium created in relativistic heavy ion collision experiments, but it is still difficult to get quantitative results for its behavior in the plasma. Here I discuss the decay width of a heavy \(QQ\) system, and calculate the gluodissociation width of bottomonia. In the end I comment on study of quarkonia as open quantum systems.

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I. INTRODUCTION

Quarkonia, mesons of heavy quark and antiquark, constitute one of the most popular probes of quark-gluon plasma. In the plasma the reduced binding between quark and antiquark and the presence of energetic thermal gluons lead to a reduction in yield of a \(Q\bar{Q}\) meson [1]. Since the vector quarkonia like \(J/\psi\) and \(\Upsilon(1S)\) can be clearly detected through their decay into dileptons, it has become one of the most studied observables in relativistic heavy ion collisions, both experimentally and theoretically [2].

While the large mass of the quark provides various simplifications in the theory side, quantitative predictions have remained difficult, at least in the temperature range of interest for heavy ion collision experiments. Direct lattice studies are difficult due to the requirement of analytic continuation to connect to experimental observables. Early lattice results [3, 4] predicting very small temperature effects on \(J/\psi\) yield has been questioned [5], and a later study has suggested substantial thermal modification on crossing \(T_c\) [6]. For bottomonia, nonrelativistic effective field theory on lattice has been employed; however, there is a wide spread in the thermal width estimates [7, 8]. On the other hand, very interesting theoretical insights have been obtained, from use of various effective field theory techniques [9, 10], as well as from using concepts of open quantum systems [11, 12]. However, most of these works are based on perturbation theory, and therefore, it is difficult to extract quantitatively accurate predictions from them.

In this note I discuss use of lattice studies to complement the effective field theory works, and in particular, use it to calculate the decay width of \(\Upsilon(1S)\) in plasma. Then I make some general observations about constraining open quantum system studies.

II. HEAVY QUARKONIA IN QGP

Since we will work with heavy quarks whose mass is larger than all other scales in the theory including the temperature scale, it is natural to start with the nonrelativistic action,

\[
\mathcal{L}_{NR} = \mathcal{L}_Q + \bar{\psi} \left( i \frac{D}{m} - m \right) \psi - \frac{1}{2} \text{Tr} G_{\mu\nu} G_{\mu\nu}
\]

\[
\mathcal{L}_Q = \phi^\dagger \left( iD_0 + \frac{D^2}{2M} \right) \phi + \chi^\dagger \left( iD_0 - \frac{D^2}{2M} \right) \chi + ...
\]

where \(\phi, \chi\) are the two-component fields that annihilate heavy quark and antiquark, respectively, and \(\mathcal{L}_Q\) is the nonrelativistic heavy quark part of the action. The other parts in Eq. (1) relate to the light quark action and the gauge action, respectively, and \(\mathcal{L}_{NR}\) is an effective lagrangian for energy scales \(\ll M\). We will use the generic term "heavy quarks" to denote a configuration of heavy quarks and antiquarks.

If we further assume that temperature scale is much larger than all other scales in the theory, \(T \gg Mv, \Lambda_{QCD}\), then one can integrate out this scale and get a complex potential to describe the \(Q\bar{Q}\) pair [2]. The real part of the potential is the well-known debye-screened Yukawa potential, and the imaginary part describes decay of the \(Q\bar{Q}\) meson via

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In field theory, the corresponding quantity of interest is \( M_H \) where scale orderings was carried out in Ref. [10], where it was shown that for the physically more interesting case \( Mv \gtrsim T \) the leading mechanism of the decay of the \( \bar{Q}Q \) meson is gluodissociation.

The main purpose of this note is to make a nonperturbative estimate of the gluodissociation width. We will do that in the leading order of \( L_0 \) in Eq. (1), and assuming the scale hierarchy

\[
Mv \sim 1/r \gg T \gtrsim m_D \gg \epsilon_B,
\]

where \( \epsilon_B \) is the binding energy of the \( \bar{Q}Q \) meson.

The interaction of static \( \bar{Q}Q \) singlet with a gluonic field has already bin worked out by Peskin [16], who showed that the interaction of the gluonic field with the \( \bar{Q}Q \) pair is like a color electric dipole term: summing over the time evolution, one gets

\[
\frac{g^2}{2N_c} \int dt \int_0^\infty d\tau \langle \bar{r}E_{a}(t)e^{-(H_a-H_s)r}rE_{a}(t-\tau) \rangle
\]

where \( H_a, H_s \) are the hamiltonians for the octet and singlet, respectively, and \( E_a \) is the color electric field.

To find the contribution of this term to the decay width of \( \Upsilon(1S) \), following Peskin we write the \( \Upsilon(1S) \) state formally as \( |\Upsilon(R)\rangle \equiv |R\rangle |L_c\rangle |\psi(r)\rangle \), where \( R \) denotes the c.m. and \( \psi(r) \) corresponds to the wavefunction in relative coordinates. Also (following Peskin) we can set the energy of the adjoint state to 0 (compared to free particle state) and so the energy difference in the exponential \( \sim \epsilon_B \).

In the vacuum cross-section calculation, the energy exponentia plays a crucial role in the total matrix element. On the other hand, here \( EE \) thermal correlator is expected to have a range \( \sim 1/m_D \), and \( m_D \gg \epsilon_B \) (Eq. (2)). So we can ignore the effect of this term (it is of the same order as subleading terms) [17]. Then we get

\[
\Gamma_g = 2\frac{g^2}{2N_c} \langle \phi|r_{r}r_{j}\phi \rangle \int d\tau (E_{a}^{(\alpha}(\tau)E_{j}^{(\beta}(\tau))_{T})
\]

\[
= \frac{g^2}{6N_c} \int d\tau (E_{a}^{(\alpha}(\tau)E_{j}^{(\beta}(\tau))_{T}) \int d^3r \phi(\tau r)^2 r^2
\]

where \( \phi(r) \) is the spatial wavefunction of the \( \Upsilon(1S) \).

The thermal matrix element in Eq. (4) has already been calculated on lattice in Refs. [18, 19] in the context of study of momentum diffusion coefficient of heavy quarks in plasma. The momentum diffusion coefficient, \( \kappa \), is defined through a Langevin equation for a heavy quark in plasma [20, 21].

\[
\frac{d\vec{p}}{dt} = -\frac{1}{2MT} \gamma \vec{p} + \xi(t)
\]

\[
\langle \xi(t) \xi_m(t') \rangle = \kappa \delta_{mn} \delta(t-t').
\]

In field theory, the corresponding quantity of interest is \( M\vec{J} \), where \( \vec{J} \) is the number density current. Using Eq. (1) one gets

\[
M\vec{J} = \phi^\dagger g\bar{E} \phi - \chi^\dagger g\bar{E} \chi.
\]

Using fluctuation-dissipation theorem and some manipulation one can then show that [22]

\[
\kappa \propto \lim_{\omega \to 0} \int dt e^{i\omega t} \int dx \langle \{ M\vec{J}(t, \vec{x}), M\vec{J}(0, 0) \} \rangle.
\]

On the lattice, one can calculate the Matsubara correlator of the electric field, and extract \( \kappa \) [18, 19, 22]:

\[
\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega).
\]

Here \( \rho \) is the spectral function for the electric field operator.

To calculate the spatial matrix element, we solve for the ground state singlet wave function \( \phi(r) \) using the real part of the \( \bar{Q}Q \) potential. As mentioned before, this potential has been calculated on the lattice [13]. We use the
parametrization of the potential given in [23, 28] and take as vacuum potential the Cornell form $V(r) = -\frac{\alpha}{r} + \sigma r$, with $\alpha = 0.3872$, $\sigma = 0.2025 \text{GeV}^2$, and $m_b = 4.68 \text{GeV}$.

In Fig. 1 I show our estimates for the gluodissociation width thus obtained. In the left hand figure is shown the calculation of the momentum diffusion coefficient $\kappa$ in SU(3) gluon plasma. This is a slightly updated figure from that in Ref. [19]: for the renormalization constant the one-loop result of [24] is used. Also here we only show the statistical (including fitting) errorbar. Here we are using this calculation only to get the decay width, which has its own, large but different, set of systematic errors; at the moment we are not making an attempt to make a serious assessment of the systematic error. One such obvious issue is the fact that the lattice measurement of $\langle E - E \rangle$ correlator was done for a gluon plasma, with $T_c \sim 260 \text{MeV}$. We note, however, that plotted in units of $T_c$, $\kappa$ agrees well with experimental measurements of the quantity. Encouraged by this, for our estimate we assume that $\kappa/T_c^3$ has similar values for full QCD at similar value of $T/T_c$, and expect the estimate of $\gamma/T_c$ to be reasonably good also for QCD.

For estimate of $\Gamma$ in MeV, one can use $T_c = 172.5 \text{MeV}$.

Before we discuss our result, a few comments are in order. The connection of decay width of $\bar{Q}Q$ system to the electric field correlator, at a similar level of approximation, was discussed first in Ref. [17]. Using the pNRQCD effective field theory approach, they came directly to the electric field electric field correlator (see also [10]). In pNRQCD one writes an effective lagrangian for the $\bar{Q}Q$ system. The pNRQCD lagrangian is

$$\mathcal{L}_{pNR} = \int \mathcal{d}^3 r \text{ Tr } \left\{ S^\dagger (i\partial_0 - V_s) S + O^\dagger (iD_0 - V_o) O \right\} + Z_A(r) \text{ Tr } \left\{ O^\dagger \vec{r}.g\vec{E} S + S^\dagger \vec{r}.g\vec{E} O \right\} + ...$$

where $S, O$ refer to octet and singlet configurations of $\bar{Q}Q$, $r$ is the relative coordinate, and $Z_A$, the matching coefficient, is 1 in leading order (see [25] for a review). The dipole interaction vertices $\vec{r}.\vec{E}$ connect the singlet to the octet. Therefore one immediately gets Eq. (8). The difference between their work and ours is in our treatment of the spatial part $\langle \phi | r^2 | \phi \rangle$, which will lead to a different behavior with temperature since this factor changes quite substantially in the temperature range we have discussed.

Another recent estimation of gluodissociation width was made in [23]. While our treatment of the spatial wavefunction is similar to theirs, they used the perturbative estimate of [16] for the electric field correlator, leading to an order-of-magnitude smaller width at comparable temperatures. Their approach leads to the somewhat counterintuitive result that the decay width starts decreasing with temperature after $\sim 1.4 T_c$.

Now let us look at the results obtained in Fig. 1. The decay width of $\Upsilon(1S)$ is small at small temperatures, but rises quite fast with temperature, reaching $\sim 100 \text{MeV}$ by $1.5 T_c$. Note that this is quite a large width, since the plasma lasts for almost 10 fm. However, it is considerably smaller than the estimate made in [7] from direct lattice studies. Due to the large statistical error it is difficult to identify a trend at higher temperatures, but over this temperature range a linear rise with $T - T_c$ is consistent with data within error. Of course, this linearity is the result of combined effect of different factors; at very high temperatures one expects a $\sim T^3$ behavior with temperature [10]. It will be interesting to see if the approach to such behavior sets in at moderately high temperatures.
III. HEAVY QUARK SYSTEM IN PLASMA

In the previous section we connected the calculation of decay width of $\Upsilon(1S)$ to the motion of a meandering $b$ quark. This is a pointer to the idea that rather than thinking about the quarkonia separately, it may be more useful to think of the heavy quark system as a whole, and its interactions with the plasma. More generally, treating the problem of quarkonia in plasma as an open quantum system has become popular, see, e.g., [11, 12]. Setting of the pNRQCD effective theory in the open quantum system setup has also been considered in [17]. Phenomenological study of quarkonia within the open quantum system framework has also been considered [26, 27].

We do not intend to go to the machinery of open quantum system here. We will, however, see how Section II sits within a more general framework introduced in [12]. Starting from a configuration of heavy quarks $Q_i$ at time $t_i$, the probablitiy of finding the heavy quark system in a configuration $Q_f$ at time $t_f$ can be written as [11, 12]

$$P[Q_f,t_f|Q_i,t_i] = \int DQ \int D[\bar{\psi},\psi] \exp(iS[Q,\psi]) \equiv \int D[\phi,\chi] e^{i\Phi[A]}$$ (10)

where the influence functional, $\Phi[Q]$ [11, 12]

$$e^{i\Phi} = \int DA_0 e^{-i\int \rhoA_0} e^{iS'}$$ (11)

includes all the terms with light quarks and gluons. One can write a path integral expression for $\Phi$ using the well-known Schwinder-Keldysh contour.

It can be shown [12] that the above path integral can be obtained from the generalized Langevin equation

$$M \frac{d^2R}{dt^2} = -M\gamma(R)\dot{R} - \nabla_R V(R) + \xi(R,t)$$ (12)

by averaging over $\xi$. $\xi$ is a white noise:

$$\langle \xi(R,t)\xi_m(R,t') \rangle = \kappa_{im}(R) \delta(t-t').$$ (13)

The 2n-component column vector $R$ has the position vectors of the $n$ quarks and antiquarks, $R^T = (\vec{r}_1, \vec{r}_1)$ and $V(R)$ is the potential due to other heavy quarks and antiquarks. $\nabla_R V(R)$ is a shorthand for interaction terms between all possible pairs: it is a column vector with terms like

$$\nabla_R V(R) = \left[\nabla V(r_i - r_j), \ldots, i=1, n, \nabla V(\vec{r}_i - \vec{r}_j), \ldots, i=1, n\right]$$

while the matrix $\gamma$ is $2n \times 2n$ matrix with elements like $H(r_i - r_j) = \nabla_{\vec{r}_i}W(r_i - r_j)$, with $W$ the imaginary part of the 2-body potential. The medium information is included in the noise term. Note that an individual collision with the medium particles with momenta $T$ imparts a momenta kick $\lesssim T$, which does not change the heavy particle momentum $P \sim \sqrt{2MT}$ substantially, therefore justifying the white noise assumption [21, 21].

If we take an isolated heavy quark, sufficiently far away from all the other quarks and antiquarks, then for it the $V(R)$ term does not contribute and we get back Eq. [5]. On the other hand for $Q\bar{Q}$ pair at separation $r \ll T$, from Eq. [12] one gets the equation of the pair under the influence of a mutual potential and scattering from the electric gluons.

This generic formalism generalizes the connection of Section II. Similarly, this also opens up the possibility of parametrizing the theory of the generic open quantum system using studies of isolated quarkonia on the lattice.

[1] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
[2] For a review, see, e.g.,
  A. Mocsy, P. Petreczky and M. Strickland, Int. J. Mod. Phys. A 28 (2013) 1340012.
  S. Datta, Pramana 84 (2015) 881.
  G. Aarts, et al., Eur. Phys. J. A 53 (2017) 93.
Ref. [28] uses the form of the 1-dimensional screened Cornell potential,

\[ V(r,T) = -\frac{\alpha}{r}e^{-m_D r} + \frac{\sigma}{m_D} (1 - \exp(-m_D r)) \]

where \( m_D \), the Debye mass, is fitted to the form

\[ \frac{m_D}{T_c} = a\left(\frac{T}{T_c} - b\right)^c + d. \]

Using the lattice results of [15], and setting \( T_c = 172.5 \) MeV, they obtain \( a=6.32, b=0.885, c=0.1035 \) and \( d=-4.058 \).