Towards Quantum Mechanism of Sensory Transduction

Oleg A.KHRUSTALEV
N.N.Bogoliubov’s Institute of Theoretical Microphysics, Moscow State University,
Moscow, 119899, Russia
e-mail: khrust@bog.msu.ru

and

Olga D.TIMOFEEVSKAYA
Physics Department, Moscow State University,
Moscow, 119899, Russia
e-mail: olga@goa.bog.msu.ru

ABSTRACT

In this communication we consider wave equations describing the classical excitations in microtubules. We found that there exists double-periodic solution for these equations. Classical solutions form background of quantum excitations. Quantum excitations are considered by Bogoliubov’s group variables method. As a result these contain no zero-modes.

Keywords: Microtubules, Quantum computation, Bogoliubov’s transformation, Conditional density matrix, Polaron.

1. INTRODUCTION

Among the various structures of cytoskeleton microtubules appear to be prominent ones (and almost unknown for nonbiologists). Microtubules represent hollow cylinders formed by protofilaments aligned along their axes. The cylindrical walls of microtubules usually are assemblies of 13 longitudinal protofilaments, each of which is the series of subunit proteins — tubulin dimers. Each dimer has an electric dipole. Thus from the physical point of view microtubules may be described as oriented assemblies of dipoles.

The hypothesis that microtubules are actively involved in reception and transduction of sensory information was been proposed in the twenties of the past century [1].

Frohlich [2] developed the general theory of biological structures which allows nonlinear dipole excitations among substructures. Frohlich theory asserts that the coherent oscillations and the long-range order arise in biological structures.

The dimers form the hexagonal structures on the outer well of microtubules. Hameroff [3] connected their properties with possibility of the cellular automat. The states of this automat are changed as a result of Frohlich dipole oscillations. This array was the smallest classical computer.

Recently it was assumed [4] that the kink-like excitations may propagate along the microtubules (with velocity about 1 m/s) and that creation and detection of these excitations develop into the communication system of living organisms.

However there was serious objection against this model. It was connected with charge transport along microtubules. In particular Tegmark [5] has showed that this mechanism could not lead to the spread of excitations.

In this communication we study nonlinear wave equations describing microtubule excitations. We found that there exists double-periodic solutions for these nonlinear wave equa-
These solutions are connected with charge transport along the microtubules and define the structure of quantum excitations arising on classical background. There is no problem with coherence in this case.

The problem of quantization of nonlinear wave equations in the neighborhood of classical field, here classical double-periodic solution, is considered in this paper by Bogoliubov’s group variables method [6]. One of very important achievement of Bogoliubov’s method is absence of so named zero modes arising in straight methods of quantization on classical background. Thus this approach gives us possibility to avoid false conclusion about long-range interactions in biological systems. The symmetry of the system connected with oscillating motion of dimers is taking into account precisely. The introduction of the new dynamical variables which play the role of symmetry group parameters gives us possibilities to calculate the quantum corrections as perturbation to classical solution and to take account the conservation laws.

We propose the scheme of quantization and receive the quantum excitations in the neighborhood classical solutions. These results leads to the assumption that quantum communication can play the important role in sensory and cognition process. In particular it is possible to separate the structure inside the microtubules such that their function is similarly to quantum automats. The operating speed for quantum automats as usual is higher than the operating speed for corresponding classical automats. In addition the acceleration of operation is connected with higher organization of microtubule states but not with new physical mechanisms.

The analyze of that possibilities demands the higher than usual precision in definition of notions for divisions quantum systems into subsystems and reunification of subsystems into a joint system. This problem always arises in investigation of quantum scheme communications. The consideration of these processes is carried out with the help of conception of conditional density matrix [7]. This approach could solve all problems well known as “quantum paradoxes”. Consideration in the terms of conditional density matrix gives adequate to physical process description.

2. CLASSICAL FIELD IN MICROTUBULES

To considerable extend the physical foundation of Penrose’s and Hammeroff’s cognition [8],[9] theory is based on the hypothesis that there exist solitons in microtubules such that "quantum computation" processes are connected with them. It is very essential to the understanding of similar processes that although the generation of solitons is absolutely quantum effect the superposition of an abundance of quantum modes hides most of pure quantum soliton properties. As a result in the first place the soliton should be considered as classical object connected with the process of a quantum cognition.

In this paper we want to show that there is a possibility of generation a different state in microtubules on the classical level. This state could forms a background for an important "quantum cognition" processes.

The description of quantum cognition processes is based on the conception the nature of biological order after Frohlich [2]. Frohlich argued that an ensemble of strongly interacting electrical dipoles that are capable of high-frequency oscillations under the influences of the external electric field may form the a metastable state characterized by long-range correlations.

Let the field $\phi(x)$ be a vector projection on the microtubule axis for microtubule dimer deflection from equilibrium. Here $x$ is a coordinate along the microtubule axis. Moreover we suppose that the time evolution of the field is described by nonlinear Klein-Gordon equation.

$$\frac{\partial^2 \phi(x,t)}{\partial t^2} - \frac{\partial^2 \phi(x,t)}{\partial x^2} + m^2 \phi(x,t) + \epsilon \phi^3(x,t) = 0,$$

$$0 \leq x \leq 2\pi. \quad (1)$$

Here the electrical dimers field [4] influence on the orientation of the field $\phi$ is not taken into account. Nevertheless these effects arise during successive description for micortubules processes on the basis of nonstationary polaron theory [11].

It is known two classes of double periodic solutions for this equation. The first one is traveling wave:

$$\phi(x,t) = \phi(x - vt). \quad (2)$$
In this case the wave equation is Duffing equation. The periodic solutions are elliptic Jacobi functions.

The second class of doubly periodic solutions consists of function in the standing wave form:

\[ \phi(x, t) = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} C_{kl} \sin(k(x - x_0)) \sin(l(\omega t - t_0)), \]

where \( x_0 \) and \( t_0 \) are constants determined by boundary and initial conditions. The wave equation is a translation-invariant, so we can restrict our consideration to zero \( x_0 \) and \( t_0 \).

In this case the solutions were under study in the paper [11] provided that \( \epsilon \) is small.

The possibility to obtain a uniform expansion, using standard asymptotic methods, depends on value of the frequencies in the zero approximation.

If \( \epsilon = 0 \), then wave equation is linear and has periodic solutions with frequencies in time approximation.

The resonance case, when there exist two frequencies \( \Omega_j \), whose relation is rational, is more difficult. The Krylov-Bogoliubov method can be used to find periodic solutions only to a few leading orders in \( \epsilon \).

Without loss of generality it can be assumed that under resonance conditions \( m = 0 \). We introduce the new time \( \tilde{t} = \omega t \) and look for a doubly-periodic solution in the form:

\[ \phi(x, \tilde{t}, \epsilon) = \sum_{n=0}^{\infty} \epsilon^n \phi_n(x, \tilde{t}), \]

\[ \omega = \omega(\epsilon) = 1 + \sum_{n=1}^{\infty} \epsilon^n \omega_n. \]

Expanding the wave equation in power series in \( \epsilon \) gives a sequence of equations for \( \phi_n \). Two leading equations are:

\[ \frac{\partial^2 \phi_0(x, \tilde{t})}{\partial \tilde{t}^2} - \frac{\partial^2 \phi_0(x, \tilde{t})}{\partial x^2} = 0, \]

\[ \frac{\partial^2 \phi_1(x, \tilde{t})}{\partial \tilde{t}^2} - \frac{\partial^2 \phi_1(x, \tilde{t})}{\partial x^2} = 2\omega^2 \frac{\partial^2 \phi_0(x, \tilde{t})}{\partial \tilde{t}^2} + \phi_0^3(x, \tilde{t}). \]

The general solution of the zero approximation is

\[ \phi_0(x, \tilde{t}) = \sum_{n=1}^{\infty} a_n \sin(nx) \sin(n\tilde{t}) \]

with arbitrary \( a_n \). We have to find coefficients \( a_n \) so that the function \( \phi_1(x, \tilde{t}) \) is a doubly-periodic:

\[ \phi_1(x, \tilde{t}) = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} b_{kl} \sin(kx) \sin(l\tilde{t}). \]

Whenever quantum phenomena for system with essentially nonlinear interaction are investigated it must be borne in mind that quantization develops on the classical field background. In other words the ground state of the system is no longer vacuum, i.e. the state without any quantum excitation, but another state. New "dressed vacuum" contains so many quantum excitations that it is easier to describe its properties on classical language. Namely this property of the ground state is principal for Frolich approach. The fundamentals for general theory of similar phenomena were found by Bogoliubov in 1950 [6].

We present the model of quantum information transport inside microtubules in terms of representation about nonstationary polaron [11]. Let \( \Psi(x) \) be the field connected with dimer distribution for microtubules; then it is possible to present the action of the system in the form:

\[ S = \frac{1}{2} \int d\tau (\psi_1^* \partial_\tau \psi_1 - \psi_2^* \partial_\tau \psi_2 - m^2 \psi_1^2) + \]

\[ g^2 \int d\tau (\Psi^*_t \Psi_t - \Psi^*_x \Psi_x - M^2 \Psi^* \Psi) - \]

\[ g \int d\tau \Psi^* \Psi \phi_1. \]

The translation invariance of the action results in two integrals of motion, the energy and the momentum. We obtain the expressions for the integrals of motion:

\[ H = \frac{1}{2} \int d\lambda (\rho^2 + \dot{\xi}^2 + m^2 \dot{\eta}^2) + \]

\[ g^2 \int d\lambda (\Psi^*_t \Psi_t - \Psi^*_x \Psi_x - M^2 \Psi^* \Psi) - \]
g \int d\lambda \Psi^* \Psi \hat{q}^2, \quad (11)

P = \int d\lambda \hat{p} \hat{q}_\lambda + g^2 \int d\lambda (\Psi_n^* \Psi_n + \Psi_n \Psi_n^*). \quad (12)

Inequality \( g \gg 1 \) clearly shows the energy relation between different subsystems: if the interaction is neglected; then monomers energy is rather more than the energy associated with the field \( \phi \).

To define quantized field \( \phi \) it is necessary to define the operators \( \hat{q}, \hat{p} \) such that the canonical commutation relation satisfies

\[ [\hat{q}(x, t), \hat{p}(x', t)] = i \delta(x - x'). \quad (13) \]

For our purposes the next representation is best suitable

\[ \hat{q}(x) = \frac{1}{\sqrt{2}} (\phi(x) + i \frac{\delta}{\delta \phi(x)}), \quad (14) \]

\[ \hat{p}(x) = \frac{1}{\sqrt{2}} (\phi_n(x) - i \frac{\delta}{\delta \phi(x)}), \quad (15) \]

It is supposed that the functions \( \phi(x), \phi_n(x) \) are independent here.

### 3. PERTURBATION THEORY

After separating large components from these functions

\[ \phi(x) = g v(x') + u(x'), \quad (16) \]

\[ \phi_n(x) = g v_n(x') + u_n(x'), \quad (17) \]

one gets that the operators \( \hat{q}, \hat{p} \) are transformed analogously. The terms have different powers of \( g \):

\[ \hat{q} = g F(x') + \hat{Q}(x') + \frac{1}{g} \hat{A}(x'), \quad (18) \]

\[ \hat{p} = g F_n(x') + \hat{P}(x') + \frac{1}{g} \hat{A}_n(x'). \quad (19) \]

Hamiltonian has a similar structure:

\[ \hat{H} = g^2 \hat{H}_{-2} + g \hat{H}_{-1} + \hat{H}_0 + \frac{1}{g} \hat{H}_1. \quad (20) \]

The first term is \( c - \) number. The operator \( \hat{H}_{-1} \) is linear form of \( \hat{Q}, \hat{P} \) and the operator \( \hat{H}_0 \) is bilinear form of \( \hat{q}, \hat{p}, \Psi, \Psi_n, \Psi^*, \Psi_n^* \). The linear powers of \( \hat{Q} \) and \( \hat{P} \) arise before quadratic ones. This circumstance inhibits the realization of regular perturbation theory. For elimination of these difficulties we put \( \hat{H}_{-1} \) equal zero. It is possible only in the case when function \( v(x, t) \) satisfies the equation

\[ v_{tt}(x, t) - v_{xx}(x, t) + \]

\[ m^2 v(x, t) + W(x, t) = 0, \quad (21) \]

where \( W(x, t) \) is functional of the third order according to \( v(x, t) \).

Degree of nonlocality is defined by effective dimension of microtubule dimer. To a first approximation when dimer is considered as point particle the equation for \( v \) becomes nonlinear Klein-Gordon equation which was analyzed earlier.

### 4. CONDITIONAL DENSITY MATRIX

After transformations made above we are close to construction of realistic model of quantum excitations in microtubules. Nevertheless the description how these excitations work needs the additional efforts. The famous Penrose and Hammeroff theory tries to overcome the arising difficulties. The proposed theory is very ingenious. However we want to note that in our opinion the authors are not completely right.

Really quantum description of quantum brain function is a problem of quantum communication theory. Its solution needs creation of systematic methods for description of subsystem unification and system separation into subsystems. Although this problem was resolved in 1927 by von Neumann the present researchers as rule do not use the advantages of his approach and commonly methods used for system and subsystems description are not quite clear. The density matrix method could be developed by conditional density matrix introducing. We suppose that conditional density matrix utilization helps to resolve the problems of quantum problem description in microtubules.

The possibility to separate the system \( S_{12} \) into subsystems \( S_1 \) and \( S_2 \) naturally leads to following definition.

Let an operator \( \hat{F} = \hat{f}(x) \hat{P}_2(y) \), where \( \hat{P}_2 \) is a projector on the \( \Psi_2 \), correspond to the physical
variable $F(x, y)$ then

\[<\hat{F}>_{\rho} = Tr_{1,2}(\hat{f}\hat{P}_2) = Tr_1(\hat{f}Tr_2(\hat{\rho}\hat{P}_2)).\]

The operator $P_2$ may be connected with the state of subsystem II such that some observable $F$ has an exact value $f_2$. The operator $Tr_2(\hat{\rho}\hat{P}_2)$ is proportional to some density matrix of the first system $\hat{\rho}_1$. The $\hat{\rho}_1$ defines state of system I under condition that $F_2$ has the exact value $f_2$. After normalization we get an operator

\[\hat{\rho}_{1/2} = \frac{Tr_2(\hat{P}_2\hat{\rho})}{Tr(\hat{P}_2\hat{\rho})}\]

It is Conditional Density Matrix for the subsystem 1 when the subsystem 2 is selected in the pure state $\hat{P}_2 = \hat{\rho}_2$. It is the most important case for quantum communications.

5. CONCLUSION

In present communication for the first time the successive description of interaction of the field $\phi(x)$ with tubulin dimers is proposed. The field $\phi(x)$ is deflection vector projection of dimers on the axis of microtubule with finite length.

In the proposed formalism the new type of space-time microtubule organization is found out. This organization is connected with double-periodic solution $\phi(x)$

Quantum description for microtubules processes is made with utilization of Bogoliubov's method. This method gives us possibility to take into account the existence of classical component and translation invariant quantum excitations simultaneously.

The processes of unification and separation composite system into subsystems are basic for information theory. It is proposed to use conditional density matrix method for construction information quantum transport mechanism in microtubules.

REFERENCES

[1] A.E.Hopkins, J. comp. Neurol., Vol. 41, 1926, pp. 253.

[2] H.Frohlich, "Evidence of Bose condensation-like excitation of coherent modes in biological systems", Physics Letters, Vol.51A, No.1,1975, pp.21-22.

[3] S.P.Hameroff, R.C.Watt, "Information processing in microtubules", J.Theor.Biol., Vol.98, 1982, pp.549-561.

[4] M.V.Satarich, J.A.Tuszyński, R.B.Zakula, "Kinklike excitations as an energy-transfer mechanism in microtubules", Physical Review E, Vol.48, No.4, 1993, pp.589-597.

[5] M.Tegmark, "Importance of quantum decoherence in brain processes", Phys.Rev., Vol.E 61, No.4, 2000, pp.4194-4206.

[6] N.N.Bogoliubov, Ukrainian Math. Journal, Vol.2, 1950, pp.2-34.

[7] V.V.Belokurov, O.A.Khrustalev, V.A.Sadovnichy, O.D.Timofeevskaya, "Systems and subsystems in quantum communication", Preprint, Moscow State University, Moscow, 2001; [arXiv:quant-ph/0111164].

[8] S.P.Hameroff, "Quantum computation in brain microtubules? The Penrose-Hamerof "Orh Or" model of consciousness", Philosophical Transactions Royal Society London (A), Vol.356, 1998, pp.1869-1896.

[9] R.Penrose, The Emperer’s New Mind, Oxford Press, Oxford, U.K., 1989.

[10] O.Khrustalev, S.Vernov, Construction of doubly periodic solutions via the Poincare-Lindstedt method in the case of massless $\phi^4$ theory, Mathematics and Computers in Simulation, Vol. 57, 2000, pp. 239-252.

[11] E.Yu.Spirina, O.A.Khrustalev, M.V.Tchichikina, "Nonstationary polaron", Theor.and Math.Physycs, Vol.122, No.3, 2000, pp.347-354.

[12] S.R. Hameroff, R. Penrose, Orchestrated reduction of quantum coherence in brain microtubules: A model for consciousness, Neural Network World", Vol. 5, No. 5, 1995, pp.793-804.