The lateral displacements analysis of space lattice beam with variable section based on the flexibility coefficients method

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Abstract. The space lattice beam is a complex statically indeterminate rod structure which, composed with several beams, is an important process for calculating the internal force and lateral stiffness in the design analysis. First, taking the single-chip truss structure with variable section as research object, each member’s force, lateral displacements and flexibility coefficients of the tapered truss with different arrangement in the form of webs are given. Finally, based on the flexibility coefficient of monolithic truss structure, considering it comprehensively with the effects of lateral stiffness caused by the chords and webs, the lateral displacement expressions are deduced for rectangular space truss structure. Result of analysis of examples show that the application of the method mentioned in this paper to calculate the lateral displacement of space lattice beam with variable section is correct and effective which can be used for practical engineering.

1. Introduction

Due to the reasonable stress, material saving and light weight, the lattice structure has become one of the main structural forms of crane metal structures and steel structures. Tower cranes’ body and booms, outrigger of gantry crane, power transmission towers, and other steel structures are all complex statically indeterminate structures composed of numerous members, its internal force and stiffness calculation are important for designing and analysis in engineering. The small flexural rigidity of the lattice members will result in large lateral displacement when the structure is working, and its overall stability will also become weaker. Therefore, the lateral deformation is clearly required to be controlled in the design specification of steel structure [1]. The space lattice structure with variable cross-section is a typical statically indeterminate rigid frame structure, and the connecting form of the members is a rigid connection, which makes the internal force of each member become difficult to be obtained. However, through theoretical studies, scholars found that, when the ratio of the length of the rod to its span is large enough, the bending stress caused by the joint stiffness is less than 5\% of the axial stress, and the effect of the stiffness of the joint can be ignored [2], the analysis of spatial lattice structure can be solved as spatial truss structure.

There are two basic methods to solve the internal force of the lattice member: One is to decompose the space truss into plane truss to calculate it, and then the internal force of the same rod should be superimposed to find the final answer [3]. Another method is to establish the finite element model, which is used for solution analysis [4, 5]. Although it is convenient to use finite element software to solve the internal forces of the structure, it has to establish different finite element models for various...
structures and various sizes; the modelling is complex and complicated and therefore cannot be applied to the subsequent theoretical research.

The lateral displacement of the lattice structure which is composed with several members is an important parameter of stiffness control in structural design. How to calculate the lateral displacement of such lattice structures quickly and accurately is an important issue which is worthy of attention. The lateral displacement of the truss is related to the arrangement form of the webs, so different arrangements will occur due to different answers. In the previous algorithms, due to the incomplete consideration of the connection between the chords, webs, and the relationship of internodes and internodes, a considerable error was always caused [6-8], and, moreover, the mentioned problem of space lattice beam with variable cross-section is researched rarely.

Based on the above reasons, the internal forces and lateral deformation of space lattice beam with variable cross-section are studied in this paper, the single-chip truss structure with variable section is used as the research object, according to the balance relationship between the forces of the internodes, the expressions of internal force and corresponding structural flexibility coefficients of plane truss are deduced. Then, the expressions of the lateral displacement of the space variable cross-section lattice member are obtained, which provides a new, simple and effective calculation method for the design calculation of the space lattice beam with variable cross-section.

2. Internal force analysis of lattice beam with variable cross-section

2.1. Web arrangements of planar lattice structure

The lattice structure with variable cross-section is usually composed by four monolithic planar structures, according to the arrangement of webs, the most used types of planar truss are shown in figure 1.

![Figure 1. Sample of truss with different web arrangements.](image)

The single-chip truss can form different types of space truss structures with different combinations; the expressions of the internal force of each type are different, but they have the same analytical method. Therefore, taking type I as an example, the derivation of the internal force expression of the chords and webs under the horizontal concentrated force are given. By similar use, the internal force expressions of other types are gained.

2.2. Internal force calculation of planar lattice structure

In order to save space, the truss structure of type I in figure 1 is rotated for 90° counterclockwise. The mechanical model is shown in figure 2.
Where, $n$—number of internodes; $\theta$—oblique deflection angle of chord, namely the angle between the chord and the horizontal direction; $l$—spacing of web, distance between each internodes is equal; $\beta$—the angle between oblique web and vertical direction; $b_0, b_2\ldots$—small end width and big end width of truss; $b_k$—width of truss cross section corresponding to the $k_{th}$ oblique web; $i, j, k$—the number of left chord, right chord and oblique web, it is numbered from left to right; $N_i, N_j, N_k$—the internal forces of the $i_{th}, j_{th}$ and $k_{th}$ member under lateral concentrated force $Q$.

**Figure 2.** Mechanical model of monolithic tapered truss structure.

Assuming the pulled force of the member as positive, and the pressed force as negative, the section method is used to solve the internal force of each member.

1. **Right chord force**
   Take the section $f-f$ for analysis, taking moments on $O_1, O_3, O_{2j+1}$ points respectively, then, the internal force of the right chord of the tapered truss can be expressed as:
   \[
   N_j = -\frac{Q \cdot l \cdot (2j-1)}{b_{2j+1} \cos \theta} \quad (j = 1, 2, 3 \ldots n)
   \]  
   (1)

2. **Left chord force**
   Take the section $g-g$ for analysis, taking moments on $O_2, O_4, O_{2j-2}$ points respectively, then, the internal force of the right chord of the tapered truss is deduced.
   \[
   N_i = \frac{Q \cdot l \cdot (2i-2)}{b_{2i-2} \cos \theta} \quad (i = 1, 2, 3 \ldots n+1)
   \]  
   (2)

3. **Oblique web member force**
   a) As shown in figure 3(a), when $k=1, 3, 5\ldots 2n-1$, in the corresponding cross section $i=j=(k+1)/2$, according to the equilibrium condition of force
   \[
   N_i \sin \theta + N_k \cos \beta_k = Q + N_j \sin \theta
   \]  
   (3)

   **Figure 3.** Component’s cross-sectional view of oblique webs numbered odd-even.

Substitute the equation (1) and (2) into equation (3), we have:
   \[
   N_k = \frac{Q}{\cos \beta_k} \left[ 1 - \left( \frac{k}{b_2} + \frac{k-1}{b_{2j+1}} \right) l \tan \theta \right] \quad (k = 1, 3, 5\ldots 2n-1)
   \]  
   (4)
b) As shown in figure 3(b), when \( k = 2, 4, 6 \ldots 2n \), in the corresponding cross section \( i = (k+2)/2, j = k/2 \), based on the equilibrium condition of force, we find:

\[
N_i \sin \theta - N_k \cos \beta_k = Q + N_j \sin \theta
\]

Taking the formula (1) and (2) into formula (5), and we can get:

\[
N_k = \frac{-Q}{\cos \beta_k} \left[ 1 - \left( \frac{k}{b_k} + \frac{k-1}{b_{k-1}} \right) \tan \theta \right] \quad (k = 2, 4, 6 \ldots 2n)
\]

The synthesis of formula (4) and (6) are:

\[
N_k = (-1)^{k+1} \frac{Q}{\cos \beta_k} \left[ 1 - \left( \frac{k}{b_k} + \frac{k-1}{b_{k-1}} \right) \tan \theta \right] \quad (k = 1, 2, 3 \ldots 2n)
\]

Also known from figure 2,

\[
\tan \theta = \frac{b_k - b_0}{2kl} = \frac{b_{k-1} - b_0}{2(k-1)l}
\]

Substituting equations (8) and (9) into equation (7), the internal force of the oblique webs of the variable cross section truss can be expressed as:

\[
N_k = (-1)^{k+1} \frac{b_0}{l} \frac{Q \sin \beta_k}{1 - \sec^2 \theta \sin^2 \beta_k} \quad (k = 1, 2, 3 \ldots 2n)
\]

In the same way, the tapered truss structures of II, III and IV types in figure 1 are studied, the internal forces of the members under the horizontal force \( Q \) are calculated, and the expressions are listed in Table 1.

### Table 1. Truss’s force under different arrangements of webs.

| Type          | I             | II            | III           | IV            |
|---------------|---------------|---------------|---------------|---------------|
| Left chord    | \( \frac{Ql(2i-2)}{b_{2i-2} \cos \theta} \) \((i = 1, 2 \ldots n + 1)\) | \( \frac{Ql(i-1)}{b_{i+1} \cos \theta} \) \((i = 1, 3 \ldots 2n-1)\) | \( \frac{Ql(i-1)}{b_{i+1} \cos \theta} \) \((i = 1, 2 \ldots n)\) | \( \frac{Ql(i-1)}{b_{i+1} \cos \theta} \) \((i = 1, 2 \ldots n)\) |
| Right chord   | \( \frac{-Ql(j-1)}{b_{2j-1} \cos \theta} \) \((j = 1, 2 \ldots n)\) | \( \frac{-Ql \cdot j}{b_{j+1} \cos \theta} \) \((j = 1, 3 \ldots 2n-1)\) | \( \frac{-Ql \cdot j}{b_{j+1} \cos \theta} \) \((j = 1, 2 \ldots n)\) | \( \frac{-Ql \cdot j}{b_{j+1} \cos \theta} \) \((j = 1, 2 \ldots n)\) |
| Left oblique web | \( \frac{b_0}{l} \frac{Q \sin \beta_k}{1 - \sec^2 \theta \sin^2 \beta_k} \) \((k = 1, 3 \ldots 2n-1)\) | \( \frac{b_0}{l} \frac{Q \sin \beta_k}{1 - \sec^2 \theta \sin^2 \beta_k} \) \((k = 1, 3 \ldots 2n-1)\) | \( \frac{b_0}{l} \frac{Q \sin \beta_k}{1 - \sec^2 \theta \sin^2 \beta_k} \) \((k = 1, 2 \ldots n)\) | \( \frac{Q \sin \beta_k}{4l(\cos \beta_k - \tan \theta \sin \beta_k)} \) \((k = 1, 2 \ldots n)\) |
unlabeled members default to zero

Note

monolithic truss structure in the oblique web and the tapered truss is gained.

member unit force acts in the web flexibility coefficient, the unit force in the $T_{ij}$ solution calculation is performed. Unit force in the $T_{ij}$

3. Analysis of lateral displacement of lattice structure with variable cross-section

3.1. Lateral displacement of plane structure

Taking the truss structure of the type I web arrangement shown in Fig. 1 as an example, a specific solution calculation is performed. Unit force in the $j$ direction $F_j=1$ produces a displacement $\delta_j$ along the $i$ direction, then $\delta_j$ is called the flexibility coefficient. Therefore, according to the definition of the flexibility coefficient, the unit load method can be used to obtain:

$$\delta_j = \sum_{k=1}^{\text{num}} \frac{\overline{N}_{ik} \cdot \overline{N}_{ij} \cdot l_k}{EA_k}$$

(11)

where $\text{num}$ is the total number of members; $\overline{N}_{ik}$ is the internal force of the $k_{th}$ member caused by the unit force acts in the $i$ direction $(N)$; $\overline{N}_{ij}$ is the internal force of the $k_{th}$ member caused by the unit force acts in the $j$ direction $(N)$; $l_k$ is the length of the $k_{th}$ member $(m)$; $A_k$ is the section area of the $k_{th}$ member $(m^2)$.

Then, the lateral displacement $\Delta_i$ under the corresponding force $F$ can be expressed as:

$$\Delta_i = F \delta_i$$

(12)

That is, the lateral displacement of the top of the truss depends on the flexibility coefficient of the structure.

In sum, combining the equation (11) with the equation (12), the top lateral displacement of planar tapered truss is gained:

$$\Delta = \sum_{i=1}^{m} \frac{\overline{N}_i N_i l_i}{EA_i} + \sum_{k=1}^{n} \frac{\overline{N}_k N_k l_k}{EA_k} + \sum_{k=0}^{n} \frac{\overline{N}_h N_h l_h}{EA_h}$$

(13)

where $A_i, A_k, A_h$ is the cross section area of truss chord, oblique webs and transverse webs respectively $(m^2)$; $n_i, n_k, n_h$ is the total number of truss chord, oblique webs and transverse webs respectively; $\overline{N}_i$, $\overline{N}_k, \overline{N}_h$ is the internal force of the $i_{th}$ chord, the $k_{th}$ oblique web and the $h_{th}$ transverse web under the influence of the unit horizontal force respectively $(N)$; $l_i, l_k, l_h$ is the length of the $i_{th}$ chord, the $k_{th}$ oblique web and the $h_{th}$ transverse web $(m)$.

Equations (1), (2) and (10) are substituted into equation (13), then the lateral displacement of the monolithic truss structure in the webs arrangement of type I is obtained:
According to the different arrangement of web members, the commonly used tapered spatial lattice structure is given in figure 4.

| Tip lateral displacements |  |
|---------------------------|--|
| I | \[ \Delta = \frac{2Ql^3}{EA \cos^3 \theta} \sum_{i=1}^{n} \left[ \left( \frac{2i-1}{b_{2i-1}} \right)^2 + \left( \frac{2i-2}{b_{2i-2}} \right)^2 \right] + \frac{4Ql^2 n^2}{EA b_{2n} \cos^3 \theta} + \frac{Qb_0^2}{EA} \sum_{k=1}^{2n} \left[ \sin \beta_k \left( 1 - \sec^2 \theta \sin^2 \beta_k \right) \right] + \frac{Qb_0}{EA} \] |
| II | \[ \Delta = \frac{2Ql^3}{EA \cos^3 \theta} \sum_{i=1}^{n} \left[ \left( \frac{i-1}{b_{i-1}} \right)^2 + \left( \frac{i}{b_i} \right)^2 \right] + \frac{4Ql^2 n^2}{EA b_{2n} \cos^3 \theta} + \frac{Qb_0^2}{EA} \sum_{k=1}^{2n} \left[ \sin \beta_k \left( 1 - \sec^2 \theta \sin^2 \beta_k \right) \right] + \frac{Qb_0}{EA} \] |
| III | \[ \Delta = \frac{Ql^3}{EA \cos^3 \theta} \sum_{i=1}^{n} \left[ \left( \frac{i-1}{b_{i-1}} \right)^2 + \left( \frac{i}{b_i} \right)^2 \right] + \frac{Qb_0^2}{EA} \sum_{k=1}^{2n} \left[ \sin \beta_k \left( 1 - \sec^2 \theta \sin^2 \beta_k \right) \right] + \frac{Qb_0}{EA} \sum_{k=1}^{b_n} \sum_{k=0}^{n} \frac{1}{b_k} \] |
| IV | \[ \Delta = \frac{2Ql^3}{EA \cos^3 \theta} \sum_{i=1}^{n} \left[ \left( \frac{i-1}{b_{i-1}} \right)^2 + \frac{Qb_0^2}{8EA l} \sum_{k=1}^{2n} \frac{\sin \beta_k}{\cos^4 \beta_k \left( 1 - \tan \theta \tan \beta_k \right)^2} \right] + \frac{Qb_0}{EA} \sum_{k=1}^{b_n} \sum_{k=0}^{n} \frac{1}{b_k} + \frac{Qb_0}{2EA} \] |

where \( b_k = b_0 + 2k \tan \theta \); \( \beta_k = \arctan \left( \frac{l}{b_k - l \tan \theta} \right) \).

The same method is used to obtain the tip lateral displacements of the tapered monolithic truss of various web arrangements as shown in figure 1, the corresponding expressions are shown in table 2.

3.2. Lateral displacement of space structure

According to the different arrangement of web members, the unfolded schematic diagram of the commonly used tapered spatial lattice structure is given in figure 4.
The mechanical model of space truss structure is shown in figure 5, when the spatial truss bears horizontal concentrated load $Q$ at the top, the force of the entire structure can be decomposed into two plane trusses on the $A$ surface, each bearing a horizontal load of $Q/2$, that is, the lateral displacement of the entire structure is equivalent to the lateral displacement caused by the horizontal force of the monolithic truss of $A$ which subjected to $Q/2$.

![Image](image1.png)

**Figure 5. Mathematical model of spatial truss.**

Taking the form of type $A$ shown in figure 4 as an example, the mentioned method is used to calculate the displacement of the variable-section space truss structure, and, as is shown in figure 5, the webs arrangement type of $A$-plane belongs to the monolithic I type structure, therefore, based on the lateral displacement equation (14) of the monolithic truss, the lateral displacement of the spatial truss can be obtained as follows:

$$
\Delta' = \frac{Ql^3}{EA_h \cos^3 \theta_a} \sum_{i=1}^{n} \left[ \left( \frac{2i-1}{a_{2i-1}} \right)^2 + \left( \frac{2i-2}{a_{2i-2}} \right)^2 \right] + \frac{2Ql^3n^2}{EA_h a_{n}^2 \cos^3 \theta_a} + \frac{Qa_0^2}{2EA_h} \sum_{i=1}^{2n} \frac{\sin \alpha_k}{1 - \sec^2 \theta_a \sin^2 \alpha_k} + \frac{Qa_0}{2EA_h}
$$

(15)

where $l_a$, $l_b$ is the span between each internodes of plan-A and plane-B respectively (m), $l_a = \frac{H_a}{2n}$, $l_b = \frac{H_b}{2n}$; $H_a$, $H_b$ is the total height of plan-A and plane-B respectively (m), $H_a = \sqrt{H^2 + \left( (b_{2n} - b_0)/2 \right)^2}$, $H_b = \sqrt{H^2 + \left( (a_2 - a_0)/2 \right)^2}$; $a_k$, $b_k$ is the cross-sectional width of the truss corresponding to the $k_{ib}$ oblique web of plan-A and plane-B respectively (m), $a_k = a_0 + 2kl_a \tan \theta_a$, $b_k = b_0 + 2kl_b \tan \theta_b$; $\alpha_k$, $\beta_k$ is the angle between the $k_{ib}$ oblique web of plan-A and plane-B with the horizontal direction respectively (rad), $\alpha_k = \arctan \left( \frac{l_a}{a_k - l_a \tan \theta_a} \right)$, $\beta_k = \arctan \left( \frac{l_b}{b_k - l_b \tan \theta_b} \right)$.

**4. Examples**

The verification model is the $A$ type of web arrangement shown in figure 4, the main model parameters are as follows: the section of chords are $\phi108 \times 10\text{mm}$, the section of webs are $\phi70 \times 3.5\text{mm}$, the top spans of each plane are $a_0 = 1.0\text{m}$, $b_0 = 1.0\text{m}$ respectively, the bottom spans of
each plane are $a_{2x} = 2.5m$, $b_{2y} = 2.5m$ respectively, and the elastic modulus $E = 200GPa$. Assuming the spacing of each internodes is constant, $l = 1.0m$, the total height $H$ of the structure is changing, while other parameters keep unchanged. Taking transverse load as $Q = 1.0 \times 10^4 N$, the finite element results of truss-frame and rigid-frame are compared with the results of the proposed formula, which are listed in table 3.

### Table 3. Comparison results of spatial truss with height changing.

| Number of Internodes $n$ | Total Height $H$ (m) | Proposed formula value(1) (mm) | Finite Element Analysis value (mm) | Error |
|--------------------------|----------------------|-------------------------------|-----------------------------------|-------|
|                          |                      | Truss-frame(2)                | Rigid-frame(3)                    |       |
|                          |                      | $(2) - (1)$                   | $(3) - (1)$                       |       |
| 4                        | 8.0                  | 1.011                         | 1.003                            | 0.7976% |
| 8                        | 16.0                 | 5.938                         | 5.921                            | 0.2871% |
| 12                       | 24.0                 | 18.771                        | 18.734                           | 0.1975% |
| 16                       | 32.0                 | 43.464                        | 43.390                           | 0.1705% |
| 20                       | 40.0                 | 83.969                        | 83.834                           | 0.1610% |

As is shown in table 3, the results of the proposed method are exactly the same with the values of the truss-frame structure, which is analysed by finite element software. The results show that the proposed equations are correct, and the same time, the proposed method which divides the spatial structure into 2 plane truss for force calculation, then composing the plane truss to form space truss for calculating structural displacement is feasible. It can also be seen that, with the increasing of height, the error between the lateral displacement of space truss-frame structure and space rigid-frame structure is gradually reduced, the errors are all within 1%, which means that it is satisfactory for practical engineering when making the spatial lattice structure a special truss for lateral displacement calculation.

### 5. Conclusions

In this paper, the solving of lateral displacement of the complex spatial lattice structure is transformed into a simple planar truss force analysis problem, the influence of the chord and web on the stiffness of the space truss is fully considered, and a high-precision expression of the lateral displacements of the variable-section space truss is obtained. The results show that the mentioned method can be used for lateral displacement analysis of tapered spatial lattice structure, which has the advantages of being a clear concept, of simple calculation and high precision, and of being easily applied in engineering.

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