Tensor-polarized structure functions:
Tensor structure of deuteron in 2020’s

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Abstract. We explain spin structure for a spin-one hadron, in which there are new structure functions, in addition to the ones \( F_1, F_2, g_1, g_2 \) which exist for the spin-1/2 nucleon, associated with its tensor structure. The new structure functions are \( b_1, b_2, b_3, \) and \( b_4 \) in deep inelastic scattering of a charged-lepton from a spin-one hadron such as the deuteron. Among them, twist-two functions are related by the Callan-Gross type relation \( b_2 = 2x b_1 \) in the Bjorken scaling limit. First, these new structure functions are introduced, and useful formulae are derived for projection operators of \( b_1 \) from a hadron tensor \( W_{\mu\nu} \). Second, a sum rule is explained for \( b_1 \), and possible tensor-polarized distributions are discussed by using HERMES data in order to propose future experimental measurements and to compare them with theoretical models. A proposal was approved to measure \( b_1 \) at the Thomas Jefferson National Accelerator Facility (JLab), so that much progress is expected for \( b_1 \) in the near future. Third, formalisms of polarized proton-deuteron Drell-Yan processes are explained for probing especially tensor-polarized antiquark distributions, which were suggested by the HERMES data. The studies of the tensor-polarized structure functions will open a new era in 2020’s for tensor-structure studies in terms of quark and gluon degrees of freedom, which are very different from ordinary descriptions in terms of nucleons and mesons.

1. Introduction
Spin structure of the nucleon has been extensively investigated for finding its origin. In a naive quark model, the spin-1/2 nucleon should consist of two quarks with parallel spins to the nucleon spin and a quark with antiparallel spin. In other words, quarks carry 100% of the nucleon spin. However, polarized lepton-nucleon scattering measurements clarified that this simple description does not work. The quarks carry only a small fraction of the nucleon spin. The remaining spin may be possibly carried by antiquark and gluon spins. However, antiquark spin contributions are small and gluon spin contribution seems to be also small according to recent measurements and their global analyses. Therefore, only remaining possibilities are orbital angular momentum effects of quarks and gluons. They are currently investigated by generalized parton distributions (GPDs) and transverse-momentum-dependent parton distributions (TMDs). Therefore, understanding of orbital motion is crucial in hadron spin physics.
For a spin-one hadron such as the deuteron, there exist new polarized structure functions, $b_1$, $b_2$, $b_3$, and $b_4$, which could probe dynamical aspects including orbital motion within the hadron. The new structure is associated with tensor structure, so that the tensor-polarized structure functions vanish if constituents are in the $S$ state. It is also important to investigate completely different spin quantities from the nucleon for establishing high-energy spin physics.

The tensor structure of deuteron and other nuclei has been investigated for a long time by nucleon and meson degrees of freedom, as illustrated in Fig. 1. However, the tensor structure has not been explored yet in terms of quark and gluon degrees of freedom. There are some studies. First, the new structure functions $b_{1-4}$ are introduced for a spin-one hadron [1, 2]. For the leading-twist structure function $b_1$, a useful sum rule was proposed by using a parton model [3]. Leptoproduction of spin-one hadrons is investigated in Ref. [4]. There are theoretical studies on the $x$ distribution of $b_1$. If the constituents are in the $S$ wave, it vanishes ($b_1 = 0$) [2]. A finite distribution with a node is expected in a convolution model with D-state admixture [2, 5], effects of pions and six-quark configuration [6], and shadowing effects in a nucleus [7, 8]. If we find a significant difference from these theoretical expectations, the studies could create a new field of high-energy spin physics. There are related theoretical studies such as new fragmentation functions [9], generalized parton distributions [10], target mass corrections [11], positivity constraints [12], lattice QCD estimate [13], projection operators of $b_{1-4}$ [14] for spin-one hadrons, and angular momenta for spin-1 hadron [15]. An alternative reaction for investigating the tensor structure is to use Drell-Yan processes. A theoretical formalism was developed in Ref. [16] to study the tensor-polarized distributions at hadron facilities by Drell-Yan processes with polarized deuteron.

The HERMES collaboration reported the measurement of the structure function $b_1$ in 2005 [17]. The data indicated a finite distribution at $x < 0.1$, which roughly agrees with a double scattering contribution estimated in Ref. [8]. The data are also consistent with the quark-parton model sum rule for $b_1$ [3] although experimental errors are still large. As a future experiment, a proposal was approved for measuring $b_1$ at Thomas Jefferson National Accelerator Facility (JLab) [18]. It could be also investigated at the Electron-Ion Collider (EIC) [19]. Tensor polarization $A_{zz}$ at large-$x$ ($x > 1$) can be also investigated [20]. In addition, the tensor-polarized quark and antiquark distributions could be studied by Drell-Yan processes with tensor-polarized deuteron at Fermilab [21], J-PARC (Japan Proton Accelerator Research Complex) [22], and GSI-FAIR (Gesellschaft für Schwerionenforschung - Facility for Antiproton and Ion Research) [23].

In this article, we explain the definition of tensor-polarized structure functions in Sec. 2. Then, operators to project out structure functions of a spin-1 hadron are shown in Sec. 3. A sum rule of $b_1(x)$ is explained in Sec. 4. A useful parametrization is proposed for tensor-polarized distributions in Sec. 5 for explaining the HERMES data. As shown in Sec. 6, the tensor-polarized distributions could be also investigated by Drell-Yan processes with tensor-polarized deuteron [16]. The results are summarized in Sec. 7.
2. Tensor polarized structure functions

We consider charged-lepton deep inelastic scattering (DIS) from a hadron in Fig. 2. Its cross section is described by a hadron tensor $W_{\mu\nu}$, multiplied by a lepton tensor $L^{\mu\nu}$. The hadron tensor for the spin-1/2 nucleon is described by four structure functions $F_1, F_2, g_1$, and $g_2$:

$$W^{\lambda\lambda_i}_{\mu\nu} = -F_1 \hat{g}_{\mu\nu} + \frac{F_2}{M^2} \hat{p}_{\mu} \hat{p}_{\nu} + \frac{i g_1}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + \frac{i g_2}{M^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma),$$

where $\epsilon_{\mu\nu\lambda\sigma}$ is a tensor with the convention $\epsilon_{0123} = +1$, $\nu$ is defined by $\nu = p \cdot q / M$ with the hadron mass $M$, hadron momentum $p$, and momentum transfer $q$, $Q^2$ is given by $Q^2 = -q^2 > 0$, and $s^\mu$ is the spin vector with the constraint $s \cdot p = 0$. The notations $\hat{g}_{\mu\nu}$ and $\hat{p}_{\mu}$ are introduced so as to ensure the current conservation $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$. The initial and final spin states are denoted by $\lambda_i$ and $\lambda_f$, respectively. In general, off-diagonal terms with $\lambda_f \neq \lambda_i$ are needed to consider higher-twist contributions [2].

In a spin-one hadron, there are four additional structure functions $b_1, b_2, b_3$ and $b_4$ in the hadron tensor [2, 14]:

$$W^{\lambda\lambda_i}_{\mu\nu} = -F_1 \hat{g}_{\mu\nu} + \frac{F_2}{M^2} \hat{p}_{\mu} \hat{p}_{\nu} + \frac{i g_1}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + \frac{i g_2}{M^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma) - b_1 r_{\mu\nu} + \frac{1}{2} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu}),$$

where $r_{\mu\nu}, s_{\mu\nu}, t_{\mu\nu},$ and $u_{\mu\nu}$ are defined by

$$r_{\mu\nu} = \frac{1}{\nu^2} \left[ q \cdot E^*(\lambda_f) q \cdot E(\lambda_i) - \frac{1}{3} \nu^2 \kappa \right] \hat{g}_{\mu\nu}, \quad s_{\mu\nu} = \frac{2}{\nu^2} \left[ q \cdot E^*(\lambda_f) q \cdot E(\lambda_i) - \frac{1}{3} \nu^2 \kappa \right] \hat{p}_{\mu} \hat{p}_{\nu},$$

$$t_{\mu\nu} = \frac{1}{2 \nu^2} \left[ q \cdot E^*(\lambda_f) \left( \hat{p}_{\mu} \hat{E}_\nu(\lambda_i) + \hat{p}_{\nu} \hat{E}_\mu(\lambda_i) \right) + \left( \hat{p}_{\mu} \hat{E}_\nu^*(\lambda_f) + \hat{p}_{\nu} \hat{E}_\mu^*(\lambda_f) \right) q \cdot E(\lambda_i) - \frac{4 \nu^2}{3 M} \hat{p}_{\mu} \hat{p}_{\nu} \right],$$

$$u_{\mu\nu} = \frac{M}{\nu} \left[ \hat{E}_{\mu}^*(\lambda_f) \hat{E}_\nu(\lambda_i) + \hat{E}_{\nu}^*(\lambda_f) \hat{E}_\mu(\lambda_i) + \frac{2}{3} \hat{g}_{\mu\nu} - \frac{2}{3 M^2} \hat{p}_{\mu} \hat{p}_{\nu} \right].$$

Here, $\kappa$ is given by $\kappa = 1 + Q^2 / \nu^2$, and $s^\mu$ is the spin vector of the spin-one hadron. The $E^\mu$ is the polarization vector of the spin-one hadron and it satisfies the conditions, $p \cdot E = 0$ and $E^* \cdot E = -1$. It is taken as the spherical unit vectors:

$$E^\mu(\lambda = \pm 1) = \frac{1}{\sqrt{2}}(0, \mp 1, \mp i, 0), \quad E^\mu(\lambda = 0) = (0, 0, 0, 1).$$
Then, the spin vector $s$ is given by the polarization vector as

$$
(s_{i_{\lambda_i}})^{\mu} = -\frac{i}{M} \epsilon^{\mu\alpha\beta\gamma} E_{\nu}^{\alpha}(\lambda_f) E_{\alpha}(\lambda_i) p_{\beta},
$$

(6)

where $M$ is the mass of the spin-1 hadron. The initial and final polarization vectors are denoted by $E_{\mu}^{\lambda_i}$ and $E_{\mu}^{\lambda_f}$, respectively, with the spin states $\lambda_i$ and $\lambda_f$. Explicit expressions for the spin vector $s^{\mu}$ are given for $s_{11}^{\mu}$, $s_{10}^{\mu}$, and $s_{01}^{\mu}$ as

$$
\begin{align*}
 s_{11}^{\mu} &= (0, 0, 0, 1), \\
 s_{10}^{\mu} &= \frac{1}{\sqrt{2}}(0, 1, -i, 0), \\
 s_{01}^{\mu} &= (s_{10}^{\mu})^*,
\end{align*}
$$

(7)

In Eq. (3), the coefficients of $b_1$, $b_2$, $b_3$ and $b_4$ are symmetric under $\mu \leftrightarrow \nu$, and they vanish under the spin average. The coefficients are defined so that $b_1$ and $b_2$ are twist-two functions and they satisfy the Callan-Gross type relation. In addition to the functions $F_1$, $F_2$, $g_1$, and $g_2$, which exist in a spin-1/2 hadron, there are four new structure functions, $b_1$, $b_2$, $b_3$ and $b_4$. They are related to the tensor structure of the spin-1/2 hadron.

3. Projection operators for tensor-polarized structure functions

In a convolution description of nuclear structure functions, the hadron tensor $W_{\mu\nu}$ is calculated by the convolution integral of nucleonic $W_{\mu\nu}$ with a lightcone momentum distribution of a nucleon in a nucleus. Then, the nuclear structure function $F_2^A$ is, for example, extracted from the nuclear tensor $W_{\mu\nu}^A$, by applying a projection operator to $W_{\mu\nu}^A$. Because there are eight structure functions for a spin-one hadron, eight independent combinations are needed for tensors with indices $\mu$ and $\nu$. Only $\lambda = 1$ and $\lambda = 0$ terms are used because $\lambda = -1$ terms make the same contributions as $\lambda = 1$ ones. We choose the following terms

$$
\begin{align*}
 g^{\mu\nu} \delta_{\lambda_f 1} \delta_{\lambda_i 1}, & \quad g^{\mu\nu} \delta_{\lambda_f 0} \delta_{\lambda_i 0}, \\
 g^{\mu\nu} \delta_{\lambda_f 1} \delta_{\lambda_i 0}, & \quad \frac{p^{\mu} p^{\nu}}{M^2} \delta_{\lambda_f 1} \delta_{\lambda_i 1}, \\
 \frac{1}{M} [p^\mu E^\nu(\lambda = 1) + p^\nu E^\mu(\lambda = 1)] \delta_{\lambda_f 1} \delta_{\lambda_i 0}, & \quad \frac{i}{M} \epsilon^{\mu\nu\alpha\beta} q_{\alpha} s_{\beta}^{11} \delta_{\lambda_f 1} \delta_{\lambda_i 1}, \\
 \frac{i}{M} \epsilon^{\mu\nu\alpha\beta} q_{\alpha} s_{\beta}^{10} \delta_{\lambda_f 0} \delta_{\lambda_i 1}.
\end{align*}
$$

(8)
Then, we obtain the projection operators for the structure functions of a spin-one hadron as \[ [14] \]

\[
F_1 = -\frac{1}{2} \left( g^{\mu\nu} - \frac{\kappa - 1}{\kappa - 1 \, M^2} \right) \frac{1}{3} \delta_{\lambda_1 \lambda_2} W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
F_2 = -\frac{x}{\kappa} \left( g^{\mu\nu} - \frac{\kappa - 1}{\kappa - 1 \, M^2} \right) \frac{1}{3} \delta_{\lambda_1 \lambda_2} W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
g_1 = -\frac{i}{2\kappa \nu} \varepsilon^{\mu\nu\alpha\beta} q_\alpha \left( s_{\beta}^{11} \delta_{\lambda_1 \lambda_2} - s_{\beta}^{10} \delta_{\lambda_0 \lambda_1} \right) W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
g_2 = \frac{i}{2\kappa \nu} \varepsilon^{\mu\nu\alpha\beta} q_\alpha \left( s_{\beta}^{11} \delta_{\lambda_1 \lambda_2} + s_{\beta}^{10} \delta_{\lambda_0 \lambda_1} \right) W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
b_1 = \left[ -\frac{1}{2\kappa} g^{\mu\nu} (\delta_{\lambda_1 \lambda_2} - \delta_{\lambda_1 \lambda_0}) + \frac{\kappa - 1}{2\kappa^2 - M^2} \delta_{\lambda_1 \lambda_2} \right] W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
b_2 = \frac{3(\kappa - 1)}{\kappa^2} g^{\mu\nu} \left( -\delta_{\lambda_1 \lambda_2} + 2(\kappa - 1) \delta_{\lambda_1 \lambda_0} + (2\kappa - 1) \delta_{\lambda_1 \lambda_0} \right) W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
b_3 = \frac{3(\kappa - 1)}{\kappa^2} g^{\mu\nu} \left( -\delta_{\lambda_1 \lambda_2} + 2(\kappa - 1) \delta_{\lambda_1 \lambda_0} + (2\kappa - 1) \delta_{\lambda_1 \lambda_0} \right) W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
b_4 = \frac{3(\kappa - 1)}{\kappa^2} g^{\mu\nu} \left( -\delta_{\lambda_1 \lambda_2} + 2(\kappa - 1) \delta_{\lambda_1 \lambda_0} + (2\kappa - 1) \delta_{\lambda_1 \lambda_0} \right) W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

where summations are taken over \( \lambda_1 \) and \( \lambda_2 \).

Because leading-twist structure functions \( b_1 \) and \( b_2 \) were experimentally measured first, it is useful to consider the Bjorken scaling limit, \( \nu, Q^2 \to \infty \) with finite \( x = Q^2/(2p \cdot q) \). In this limit, we have the relations

\[
\lim_{\nu} g^{\mu\nu} W_{\mu\nu}^{10} = \lim_{\nu} g^{\mu\nu} W_{\mu\nu}^{11} = \lim_{\nu} (\kappa - 1) \frac{p^\mu p^\nu}{M^2} W_{\mu\nu}^{\lambda_1 \lambda_2} = 0,
\]

\[
\lim_{\nu} \frac{\kappa - 1}{M} \left[ p^\mu E^\nu(\lambda = 1) + p^\nu E^\mu(\lambda = 1) \right] W_{\mu\nu}^{10} = 0,
\]

by noting \( \nu \to 1 \), \( 2xF_1 \to F_2 \), and \( 2xb_1 \to b_2 \). Then we obtain the expressions from the projection operator in the Bjorken scaling limit as

\[
F_1 = \frac{1}{2x} F_2 = \frac{i}{2} g^{\mu\nu} \frac{1}{3} \delta_{\lambda_1 \lambda_2} W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
g_1 = \frac{i}{2\nu} \varepsilon^{\mu\nu\alpha\beta} q_\alpha \delta_{\lambda_1 \lambda_2} \delta_{\lambda_1 \lambda_2} W_{\mu\nu}^{\lambda_1 \lambda_2},
\]

\[
b_1 = \frac{1}{2x} b_2 = \frac{1}{2} g^{\mu\nu} (\delta_{\lambda_1 \lambda_2} - \delta_{\lambda_1 \lambda_0}) W_{\mu\nu}^{\lambda_1 \lambda_2}.
\]
We discuss a useful sum rule for the structure function \( b \)

where \( i \). Sum rule for \( |\vec{p}| \gg M \).

### 4. Sum rule for \( b_1 \)

We discuss a useful sum rule for the structure function \( b_1 \). Sum rules of structure functions could be derived in a parton model by considering the relation between the structure function integrated over \( x \) and elastic form factor in the infinite momentum frame [24]. In the parton model, the structure function \( b_1 \) is expressed by the tensor-polarized distributions \( \delta_\tau q(x) \) as [2, 3]

\[
b_1(x, Q^2) = \frac{1}{2} \sum_i \frac{e_i^2}{2} \left[ \delta_\tau q_i(x, Q^2) + \delta_\tau \bar{q}_i(x, Q^2) \right], \quad \delta_\tau q_i \equiv q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2},
\]

where \( i \) indicates the flavor of a quark and \( e_i \) is its charge, and \( q_i^0 \) indicates an unpolarized-quark distribution in the hadron spin state \( \lambda \). Instead of \( \delta_\tau \), \( \delta \) could be used for the tensor-polarized distributions. Because \( \delta \) and \( \Delta_\tau \) are often used for transversity distributions of spin-1/2 nucleon, the notation \( \delta_\tau \) is used throughout this article. The function \( \delta_\tau q \) indicates an unpolarized-quark distribution in a tensor-polarized spin-one hadron. The idea how to derive a sum rule for \( b_1 \) is illustrated in Fig. 3. The hadron tensor is equal to the imaginary part of the forward scattering amplitude for virtual photon scattering from the hadron by the optical theorem. An integral of a DIS structure function is related to the form factor in the infinite momentum frame by expressing the form factor in terms of parton momentum distributions as discussed in Ref. [24], where the Bjorken sum rule is shown by this method.

Before stepping into the details, we may estimate the sum rule of \( b_1(x) \) intuitively by using the dimensional counting. The \( b_1 \) is defined in Eq. (3) as a dimensionless quantity. If it is integrated over \( x \), it should become electromagnetic quantities of the deuteron probed by the charged lepton. Due to the parity conservation, the new quantity, which does not exist in the spin-1/2 nucleon, should be the electric quadrupole moment. The electric quadrupole moment has the dimension of length squared. Therefore, the \( b_1 \) sum is expected to be \( \int dx b_1(x) \sim (\text{mass dim.})^2 \cdot Q_h \), where \( Q_h \) is the electric quadrupole moment of the hadron \( h \). The quantity of the mass dimension may be the deuteron mass \( M \), but it is not obvious, so we proceed to a more detailed calculation.

For the deuteron, we define the structure function \( b_1 \) and parton distribution functions (PDFs) by the ones per nucleon \( (b_1/2 \rightarrow b_1, q(x)/2 \rightarrow q(x)) \). Taking the integral of Eq. (12) over \( x \) and then using the relations, \( (\delta_\tau u_v)_D \equiv (\delta_\tau u - \delta_\tau \bar{u})_D = (\delta_\tau u^0_v + \delta_\tau u^+ \bar{v})/2 = (\delta_\tau u_v + \delta_\tau d_v)/2 \), \( (\delta_\tau d_v)_D = (\delta_\tau d_v + \delta_\tau u_v)/2 \), for valence quark distributions, we obtain

\[
\int dx b_1(x) = \frac{5}{36} \int dx \left[ \delta_\tau u_v(x) + \delta_\tau d_v(x) \right] + \frac{1}{18} \int dx \left[ 8 \delta_\tau \bar{u}(x) + 2 \delta_\tau \bar{d}(x) + \delta_\tau s(x) + \delta_\tau \bar{s}(x) \right]_D,
\]

\[1\] The overall factor 1/2 is introduced in \( b_1 \) as usual in defining \( F_1 \) and \( g_1 \) in terms of PDFs.
where \( Q^2 \) dependence is abbreviated.

For describing the elastic scattering, we define the helicity amplitude with the charge operator \( J_0 \) by

\[
\Gamma_{H,H} = \langle p, H | J_0(0) | p, H \rangle.
\]

(14)

As shown in Fig. 3, the frame with large longitudinal momentum \( |\vec{p}| \gg M \) is considered. Then, the parton model can be used for estimating the amplitude by assuming that the quarks move along the longitudinal direction with the momentum fraction \( x \) and we obtain

\[
\Gamma_{H,H} = \sum_i e_i \int dx \left[ q_i^H(x) - \bar{q}_i^H(x) \right]_D \equiv \sum_i e_i \int dx \left[ q_{i,v}^H(x) \right]_D.
\]

(15)

The tensor spin combination of the amplitudes is written in terms of the valence-quark distributions in the deuteron by the proton and neutron contributions

\[
\Gamma_{0,0} - \frac{\Gamma_{1,1} + \Gamma_{-1,-1}}{2} = \frac{1}{3} \int dx \left[ \delta_r u_v(x) + \delta_r d_v(x) \right],
\]

where the relation \( \int dx \left[ \delta_r s(x) - \delta_r \bar{s}(x) \right] = 0 \) is used. Substituting this relation into Eq. (13), we obtain

\[
\int dx b_1(x) = \frac{5}{12} \left[ \Gamma_{0,0} - \frac{\Gamma_{1,1} + \Gamma_{-1,-1}}{2} \right] + \frac{1}{9} \int dx \left[ 4 \delta_r \bar{u}(x) + \delta_r \bar{d}(x) + \delta_r \bar{s}(x) \right]_D.
\]

(17)

The next step is to calculate the tensor helicity combination by macroscopic quantities for the deuteron. The elastic scattering amplitudes can be described in terms of electric charge and quadrupole form factors, \( F_C(t) \) and \( F_Q(t) \) where \( t \) is the momentum-transfer squared, of the deuteron as

\[
\Gamma_{0,0} = \lim_{t \to 0} \left[ F_C(t) - \frac{t}{3M^2} F_Q(t) \right], \quad \Gamma_{1,1} = \Gamma_{-1,-1} = \lim_{t \to 0} \left[ F_C(t) + \frac{t}{6M^2} F_Q(t) \right].
\]

(18)

Here, the \( t \to 0 \) limit is taken, and the units of \( F_C \) and \( F_Q \) are given by \( e \) and \( e/M^2 \), respectively, where \( e \) is the unit charge and \( M \) is the mass of the deuteron. Then, the tensor spin combination is expressed in terms of the quadrupole form factor as

\[
\Gamma_{0,0} - \frac{\Gamma_{1,1} + \Gamma_{-1,-1}}{2} = -\lim_{t \to 0} \frac{t}{4} F_Q(t) = 0.
\]

(19)

Substituting this relation into Eq. (17), we finally obtain

\[
\int dx b_1(x) = -\lim_{t \to 0} \frac{5}{24} t F_Q(t) + \frac{1}{9} \int dx \left[ 4 \delta_r \bar{u}(x) + \delta_r \bar{d}(x) + \delta_r \bar{s}(x) \right]_D.
\]

(20)

We should note that the derived sum rule is not a rigorous one, but it is based on the parton model. The situation is the same as the Gottfried sum rule. There is a similarity between these sum rules:

\[
\int dx b_1(x) = 0 + \frac{1}{9} \int dx \left[ 4 \delta_r \bar{u}(x) + \delta_r \bar{d}(x) + \delta_r \bar{s}(x) \right],
\]

\[
\int dx \left[ F^p_2(x) - F^n_2(x) \right] = \frac{1}{3} + \frac{2}{3} \int dx [\bar{u}(x) - \bar{d}(x)].
\]

(21)
The factor of 1/3 in the Gottfried sum rule comes from the flavor dependence of the valence quark distributions: \( \int [u_i(x) - d_i(x)] / 3 = 1/3 \). In the same way, the factor \( -\lim_{t \to 0}(5/24)tF_2(t) = 0 \) comes from the tensor-polarized valence quark distributions. The difference of zero and a finite number in both sum rules could be summarized as the valence-quark number depends on flavor, whereas the number does not depend on the tensor polarization. As the violation of the Gottfried sum rule led to fruitful studies of flavor asymmetric antiquark distributions \( \bar{u} \) and its physics origins [25], there is a good possibility that a finite value of the \( b_1 \) sum indicates the tensor-polarized antiquark distributions according to Eq. (21). There were HERMES measurements on \( b_1 \) and it indicated [17]

\[
\int_{0.02}^{0.85} dx b_1(x) = [1.05 \pm 0.34 \text{ (stat)} \pm 0.35 \text{ (sys)}] \times 10^{-2},
\]

\[
\int_{0.02}^{0.85} dx b_1(x) = [0.35 \pm 0.10 \text{ (stat)} \pm 0.18 \text{ (sys)}] \times 10^{-2} \text{ for the range } Q^2 > 1 \text{ GeV}^2,
\]

which suggests a finite tensor polarization for antiquarks. It is very interesting to find the physics origin of finite tensor polarized distributions. It will be tested by the approved JLab experiment on \( b_1 \) [18].

Both \( b_1 \) and Gottfried sum rules are obtained in the parton model, and they provide useful guidelines for the tensor-polarized antiquark distributions \( \delta_T \bar{q} \) and the light-quark flavor dependence of the unpolarized antiquark distributions \( \bar{u} - \bar{d} \). However, one should be careful that they are not rigorous sum rules. For example, the \( \bar{u}(x) \) and \( \bar{d}(x) \) distributions are assumed to be equal at very small \( x \) according to current parametrizations on the unpolarized PDFs. Therefore, the integral \( \int dx(\bar{u} - \bar{d}) \) seems to converge at this stage, but it could become infinite if a slight difference exists between \( \bar{u}(x) \) and \( \bar{d}(x) \) at small \( x \). In the same way, the integral \( \int \delta_T \bar{q}(x) \) may not be finite depending on the \( x \) dependence of \( \delta_T \bar{q}(x) \) or on its \( Q^2 \) dependence. As noted in recent studies of Ref. [6], the sum rule are not satisfied in some theoretical models. In a convolution model of deuteron structure function, the \( b_1 \) sum is given by the product of a moment of lightcone momentum distribution of the nucleon with a moment of unpolarized quark distribution. The first nucleon part identically vanishes but the second PDF part could diverge at large \( Q^2 \), although it converges in some PDF parametrization at small \( Q^2 \), so that the whole integral is not certain. We need more theoretical efforts on an appropriate description of the tensor-polarized distributions, especially on the antiquark part, together with experimental measurements.

5. Parametrization of tensor-polarized distributions

Theoretical models are used for calculating structure functions, and there are studies on low moments of the structure functions by lattice QCD. However, it is almost impossible to obtain \( x \)-dependent distributions at this stage, so that global analyses of world experimental data are used for determining reliable PDFs. In the same way, a useful parametrization of the tensor polarized distributions can be proposed by using the existing HERMES data, although the number of data is not sufficient for an accurate determination. It should be useful for proposing future experiments (e.g. [18]) and for testing theoretical calculations.

We consider that certain fractions of the unpolarized distributions are tensor polarized:

\[
\delta_T q^D_{u}(x) = \delta_T w(x) q^D_{u}(x), \quad \delta_T \bar{q}^D(x) = \alpha_q \delta_T w(x) \bar{q}^D(x),
\]

where \( \delta_T w(x) \) and \( \alpha_q \delta_T w(x) \) are such fractions for valence quarks and antiquarks, respectively. The \( x \) dependence of \( \delta_T w(x) \) could be different between valence quarks and antiquarks, and flavor dependence may exist. However, it is not the stage to investigate such details because
the HERMES data are the only ones. The tensor-polarized distributions cannot be determined accurately at this stage, so that simplifying assumptions are employed for the unpolarized PDFs. Nuclear modifications in the PDFs of the deuteron are considered as a few percent effects [26], and they are neglected in our studies. Namely, the PDFs of the deuteron are given by the contributions from proton and neutron: $q^{D}_i = (q^p_i + q^n_i)/2$ and $\bar{q}^{D}_i = (\bar{q}^p_i + \bar{q}^n_i)/2$. Next, isospin symmetry is used for relating the PDFs of the neutron to the ones of the proton: $u_n = d$, $d_n = u$, $\bar{u}_n = \bar{d}$, and $\bar{d}_n = \bar{u}$, and flavor symmetric tensor-polarized distributions are assumed. Then, we have the distributions

$$\delta_\tau q^D(x) \equiv \delta_\tau u^D_v(x) = \delta_\tau d^D_u(x) = \delta_\tau w(x) \frac{u_v(x) + d_v(x)}{2},$$

$$\delta_\tau \bar{q}^D(x) \equiv \delta_\tau \bar{u}^D_v(x) = \delta_\tau \bar{d}^D_u(x) = \delta_\tau s^D(x) = \delta_\tau \bar{s}^D(x) = \alpha_\bar{q} \frac{2\bar{u}(x) + 2\bar{d}(x) + s(x) + \bar{s}(x)}{6},$$

(24)

for an analysis of the HERMES data. Any available unpolarized PDFs could be used, but the LO version of the MSTW parametrization [27] is employed. From these tensor-polarized distributions, we finally obtain $b_1$ for the deuteron as

$$b^D_1(x) = \frac{1}{36} \delta_\tau w(x) \left[ 5\{u_v(x) + d_v(x)\} + 4\alpha_\bar{q}\{2\bar{u}(x) + 2\bar{d}(x) + s(x) + \bar{s}(x)\} \right].$$

(25)

The important point of the analysis is how to choose the $x$ dependence of $\delta_\tau w(x)$. From the derivation of the $b_1$ sum rule, the tensor-polarized valence-quark distributions should satisfy the sum $\int dx b_1(x)_{\text{valence}} = 0$. In order to satisfy this relation, the function $\delta_\tau w(x)$ should have a node at least, so we may take the parametrization

$$\delta_\tau w(x) = ax^b(1 - x)^c(x_0 - x),$$

(26)

where $a$, $b$, $c$, and $x_0$ are the parameters determined by the analysis. The existence of the node is also supported by the convolution model for $b_1$ with a D-state admixture. However, if the first moment of $\delta_\tau w(x)$ vanishes, $x_0$ can be expressed by other ones as

$$x_0 = \frac{\int_0^1 dx x^{b+1}(1 - x)^c\{u_v(x) + d_v(x)\}}{\int_0^1 dx x^b(1 - x)^c\{u_v(x) + d_v(x)\}}.$$  

(27)

There is no data to probe the scaling violation at this stage, so that the $Q^2$ dependence is neglected in this analysis. For calculating the unpolarized PDFs Eq. (25), the average $Q^2$ value ($Q^2 = 2.5$ $\text{GeV}^2$) of the HERMES experiment is used.

In order to find the impact of the tensor-polarized antiquark distributions, we made two types of analyses:

- Set 1: Tensor-polarized antiquark distributions are terminated ($\alpha_\bar{q} = 0$).
- Set 2: Finite tensor-polarized antiquark distributions are allowed ($\alpha_\bar{q}$ is a parameter).

| Analysis | $\chi^2$/d.o.f. | $a$ | $\alpha_\bar{q}$ | $b$ | $c$ | $x_0$ |
|----------|----------------|-----|-----------------|-----|-----|-------|
| Set 1    | 2.83           | 0.378 ± 0.212 | 0.0 (fixed) | 0.706 ± 0.324 | 1.0 (fixed) | 0.229 |
| Set 2    | 1.57           | 0.221 ± 0.174 | 3.20 ± 2.75 | 0.648 ± 0.342 | 1.0 (fixed) | 0.221 |
The constant $c$ is fixed at $c = 1$ in these analyses due to the lack of data at large $x$ to constrain it. The obtained parameters are listed in Table 1. The smaller $\chi^2$ value of the set-2 analysis indicates that there is a significant improvement in the fit by including the tensor-polarized antiquark distributions.

Obtained $b_1$ structure functions are compared with the HERMES data in Fig. 4. The dashed and solid curves are for the set-1 and the set-2, respectively. A reasonable fit was obtained by the set-2 analysis, whereas the small-$x$ ($< 0.1$) data cannot be explained by the set-1. Namely, the HERMES data at small $x$ suggest the existence of a finite tensor-polarized antiquark distributions according to this analysis. The medium and large $x$ regions are well explained by the tensor-polarized valence-quark distributions.

Determined tensor polarized distributions are shown in Fig. 5. The dashed and solid curves are tensor-polarized valence-quark distributions for the set-1 and set-2, respectively, and the dotted curve indicates the tensor-polarized antiquark distribution of the set-2. The valence-quark distribution is negative at medium $x$ and it turned to negative at $x < 0.2$. The antiquark distribution becomes large at $x < 0.2$. The functional form of the valence-quark distribution is expected in the convolution description of the deuteron structure function [5]. However, the physics origin of the antiquark distribution is not obvious at this stage. More theoretical efforts are needed to understand the origin. From the obtained distributions of the set-2, the $b_1$ sum is estimated as

$$\int dx b_1(x) = \frac{5}{24} \lim_{t \to 0} tF_Q(t) + \frac{1}{18} \int dx \left[ 8\delta_x \bar{u}(x) + 2\delta_x \bar{d}(x) + \delta_x s(x) + \delta_x \bar{s}(x) \right]$$

This finite value is due to the existence of antiquark tensor polarization. There will be measurements on $b_1$ at JLab [18], so that more details will become clear experimentally. On the other hand, theoretical studies are needed for the tensor-polarized quark and antiquark distributions.

6. Polarized proton-deuteron Drell-Yan processes

6.1. Structure functions and spin asymmetries

The HERMES data indicated an existence of finite tensor-polarized antiquark distributions. One of methods to probe antiquark distributions is to use Drell-Yan processes. For example,
the polarized proton-deuteron (pd) Drell-Yan processes could be used with tensor-polarized
deuterons. There is no experiment for the pd Drell-Yan, and there are only a few theoretical
formalisms [16].

We consider the Drell-Yan process of \( A (\text{spin } 1/2) + B (\text{spin } 1) \rightarrow \ell^+ \ell^- + X \). Our formalism
can be used for any spin-1/2 and spin-1 hadrons, but the most realistic reaction is the proton-deuteron
Drell-Yan. Its cross section is written in terms of the lepton tensor \( L_{\mu\nu} \) and the hadron
tensor \( W^{\mu\nu} \)
\[
\frac{d\sigma}{d^3Q d\Omega} = \frac{\alpha^2}{2 s Q^4} L_{\mu\nu} W^{\mu\nu},
\]
where \( \alpha = e^2/(4\pi) \) is the fine structure constant, \( s \) is the center-of-mass energy squared
\( s = (P_A + P_B)^2 \), \( Q \) is the total dilepton momentum, and \( \Omega \) is the solid angle of the momentum
\( \vec{k}_\ell^- - \vec{k}_\ell^+ \).

A general spin-density formalism, by expressing structure functions in terms of helicity amplitudes and Clebash-Gordan coefficients with the conditions of Hermiticity, parity
conservation, and time-reversal invariance, indicates that there exist 108 structure functions for the unpolarized and polarized pd Drell-Yan processes, whereas there are 48 structure functions
for the pp. There are 60 new structure functions, which should be associated with the deuteron
tensor structure. Of course, all of them are not important for the first investigation. If the cross
section is integrated over the lepton-pair transverse momentum \( Q_T \), there are only 22 ones which
include 11 new functions associated with the tensor structure of the deuteron. This spin-density
formalism is rather lengthy, so that we refer the paper [16] for the details.

In the general hadron tensor formalism, the hadron tensor of the Drell-Yan process
\[
W^{\mu\nu} = \int \frac{d^4\xi}{(2\pi)^4} e^{iQ\cdot\xi} \langle P_A S_A P_B S_B | J^{\mu}(0) J^{\nu}(\xi) | P_A S_A P_B S_B \rangle,
\]
is expanded in terms of possible Lorentz index combinations including the hadron momenta and
spins by considering the conditions

\[
\begin{align*}
\text{Hermiticity:} & \quad [W^{\mu\nu}(Q; P_A S_A; P_B S_B)]^* = W^{\mu\nu}(Q; P_A S_A; P_B S_B), \\
\text{Parity conservation:} & \quad W^{\mu\nu}(Q; P_A S_A; P_B S_B) = W^{\mu\nu}(Q; -P_A; -S_A; P_B - S_B), \\
\text{Time-reversal invariance:} & \quad [W^{\mu\nu}(Q; P_A S_A; P_B S_B)]^* = W^{\mu\nu}(Q; P_A S_A; P_B S_B),
\end{align*}
\]

where \( \bar{P} \) is defined by \( \bar{P}^{\mu} = (P^0, -\vec{P}) \). For the expansion, the Lorentz vectors \( X^\mu, Y^\mu, \) and \( Z^\mu \)
\[
\begin{align*}
X^\mu &= P_A^{\mu} Q^2 Z \cdot (P_B - P_B^{\mu} Q^2 Z) \cdot P_A + Q^\mu (Q \cdot P_B Z \cdot P_A - Q \cdot P_A Z \cdot P_B), \\
Y^\mu &= \epsilon^{\mu\alpha\beta\gamma} P_A^{\alpha} P_B^{\beta} Q_\gamma, \quad Z^\mu = P_B^{\mu} Q \cdot P_B - P_B^{\mu} Q \cdot P_A,
\end{align*}
\]
are used. We also define the vector \( T^\mu \) by
\[
T^\mu = \epsilon^{\mu\alpha\beta\gamma} S_B^{\alpha} Z_\beta Q_\gamma.
\]

In addition to these vectors, \( Q^\mu, S_A^{\mu}, S_B^{\mu}, T_A^{\mu}, \) and \( T_B^{\mu} \) can be used to expand \( W^{\mu\nu} \). However,
instead of \( S_A^{\mu} \) and \( S_B^{\mu} \), it is more convenient to use the transverse vectors \( S_A^{\mu T} \) and \( S_B^{\mu T} \):
\[
S_T^\mu = \left( g^{\mu\nu} \frac{Q^\nu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \frac{S_\nu}{Q^2}.
\]

In the general case, it is too lengthy to write them down here because there are 108 structure
functions. The transverse momentum \( Q_T \) is roughly restricted by the hadron size \( R \) by
$Q_T < 1/R$, so that the limit of $Q_T \to 0$ is considered in the following formalism. Then, $X^\mu$ and $Y^\mu$ do not have to be considered because $|\vec{X}|$ and $|\vec{Y}|$ are proportional to $Q_T$ and $X^0 = Y^0 = 0$ in the dilepton rest frame. Then, we obtain

\[
W^{\mu\nu} = - g^{\mu\nu} A - \frac{Z^\mu Z^\nu}{Z^2} B + Z^{(\mu T_A^\nu) \nu} C + Z^{(\mu T_B^\nu) \nu} D + Z^{(\mu S^\nu) \nu} E + Z^{(\mu S^\nu) \nu} F
\]

\[ - S^{(\mu S^\nu) \nu} G - S^{AT}_{BT} H + T^{(\mu\nu) \nu} I + S^{(\mu\nu) \nu} J + Q^{(\mu\nu) \nu} K + Q^{(\mu\nu) \nu} L
\]

\[ + Q^{(\mu\nu) \nu} M + Q^{(\mu\nu) \nu} N + Q^{(\mu\nu) \nu} O + Q^{(\mu\nu) \nu} P,
\]

where $Q^{(\mu\nu) \nu}$ is defined by $Q^{(\mu\nu) \nu} \equiv Q^{(\mu Z^\nu)} + Q^{(\nu Z^\nu)}$. Next, we impose the current conservation $Q_\mu W^{\mu\nu} = 0$, and then the coefficients $A$, $B$, $\cdots$, which still contain spin factors, are expanded by the scalar and pseudoscalar terms with the spins. We finally obtain

\[
W^{\mu\nu} = - \left[ g^{\mu\nu} - \frac{Q^{\mu\nu} Z^\nu}{Z^2} \right] \left\{ W_{0,0,0} + \frac{M_{AMB}}{s Z^2} Z \cdot S_A Z \cdot S_B V_{0,0,0}^{LL} - S_{AT} \cdot S_{BT} V_{0,0}^{TT} \right. \\
\left. - 2 g^{\mu\nu} \left( \frac{M_B^2 (Z \cdot S_B)^2}{s^2 (Q \cdot P_B)^2} + \frac{4}{3} s_B^2 \right) V_{0,0,0}^{UQ_0} + \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B T_A \cdot S_{BT} V_{0,0}^{TQ_1} \right\}
\]

\[ - \left[ Z^\mu Z^\nu \frac{1}{Z^2} \right] \left\{ W_{0,0,0} + \frac{M_{AMB}}{s Z^2} Z \cdot S_A Z \cdot S_B V_{0,0,0}^{LL} - S_{AT} \cdot S_{BT} V_{0,0,0}^{TT} \right. \\
\left. - 2 g^{\mu\nu} \left( \frac{M_B^2 (Z \cdot S_B)^2}{s^2 (Q \cdot P_B)^2} + \frac{4}{3} s_B^2 \right) V_{0,0,0}^{UQ_0} + \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B T_A \cdot S_{BT} V_{0,0,0}^{TQ_1} \right\}
\]

\[ - Z^{(\mu T_A^\nu)} \frac{1}{\sqrt{Q^2 Z^2}} \left\{ U_{2,1}^{TU} - \left( \frac{8 M_B^2 (Z \cdot S_B)^2}{s^2 (Q \cdot P_B)^2} + \frac{4}{3} s_B^2 \right) U_{2,1}^{TQ_0} \right\}
\]

\[ - Z^{(\mu T_B^\nu)} \frac{1}{\sqrt{Q^2 Z^2}} \left\{ U_{2,1}^{TU} + \frac{M_{AMB}}{s Z^2} Z \cdot S_A Z \cdot S_B U_{2,1}^{UQ_1} + S_{AT} \cdot S_{BT} U_{2,1}^{TQ_1} \right\}
\]

\[ + Z^{(\mu S^\nu) \nu} \frac{\sqrt{Q^2 M_B}}{Z^2 Q \cdot P_B} Z \cdot S_B U_{2,1}^{TL}
\]

\[ + Z^{(\mu S^\nu) \nu} \frac{\sqrt{Q^2 M_B}}{Z^2 Q \cdot P_B} Z \cdot S_B U_{2,1}^{TL}
\]

\[ + Z^{(\mu S^\nu) \nu} \frac{\sqrt{Q^2 M_B}}{Z^2 Q \cdot P_A} Z \cdot S_A U_{2,1}^{UQ_1} - \frac{1}{\sqrt{Q^2 Z^2}} T_A \cdot S_{BT} U_{2,1}^{TQ_2}
\]

\[ - \left[ 2 S_{BT}^2 S_B^2 - S_{BT}^2 \left( g^{\mu\nu} - \frac{Q^{\mu\nu} Z^\nu}{Z^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] U_{2,2}^{UQ_2}
\]

\[ - \left[ S^{(\mu S^\nu) \nu} - S_{AT} \cdot S_{BT} \left( g^{\mu\nu} - \frac{Q^{\mu\nu} Z^\nu}{Z^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] U_{2,2}^{TQ_2}
\]

\[ - \left[ T^{(\mu S^\nu) \nu} - T_A \cdot S_{BT} \left( g^{\mu\nu} - \frac{Q^{\mu\nu} Z^\nu}{Z^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B U_{2,2}^{TQ_1}
\]

\[ + S_{BT}^2 T_B^2 \frac{M_A}{Z^2 Q \cdot P_A} Z \cdot S_A U_{2,2}^{UQ_2}. \]

Therefore, in the limit of $Q_T \to 0$, there are 22 structure functions:

\[
W_{0,0}, V_{0,0}^{LL}, V_{0,0}^{TT}, V_{0,0}^{UQ_0}, V_{0,0}^{TQ_1}, W_{2,0}, V_{2,0}^{LL}, V_{2,0}^{TT}, V_{2,0}^{UQ_0}, V_{2,0}^{TQ_1}, U_{2,1}^{TU}, U_{2,1}^{TQ_0}, U_{2,1}^{TQ_1}, U_{2,2}^{TQ_1}, U_{2,2}^{TQ_2}, U_{2,2}^{UQ_2}.
\]

\[ W_{LM} \]

where $W$, $V$, and $U$ are an unpolarized structure function, a polarized one without the spin factors in the hadron tensor, and a polarized one with the spin factor. The function $W_{LM}$
is obtained by the integral $\int d\Omega Y_{LM} d\sigma/(d^4Q d\Omega) \propto W_{L,M}$ of the unpolarized reaction. The superscripts $U, L,$ and $T$ show unpolarized, longitudinally polarized, and transversely polarized states. The quadrupole polarizations $Q_0, Q_1,$ and $Q_2$ are associated with the spherical harmonics $Y_{20}, Y_{21},$ and $Y_{22}$ as shown in Fig. 6. They are the polarizations in the $xz, yz,$ and $xy$ planes. The structure functions with $Q_0, Q_1,$ and $Q_2$ are specific for the spin-1 deuteron.

In the pp Drell-Yan processes, the unpolarized, longitudinal, and transverse combinations exist: $<\sigma>, A_{LL}, A_{LT}, A_{LT},$ and $A_T.$ In addition, the following fifteen quadrupole spin asymmetries could be investigated in the pd Drell-Yan:

$$<\sigma>, A_{LL}, A_{LT}, A_{LT}, A_{LT}, A_{LT},$$

$$A_{UQ_0}, A_{QTQ_0}, A_{UQ_1}, A_{LTQ_1}, A_{UQ_2}, A_{LTQ_2}.$$

These asymmetries are expressed in terms of the structure functions in Eq. (37). For example, the quadrupole spin asymmetry $A_{UQ_0}$ is measured with the unpolarized proton and the $Q_0$-type tensor polarized deuteron, and it is expressed in terms of the structure functions $V_{0,0}^{UQ_0}, V_{2,0}^{UQ_0}, W_{0,0},$ and $W_{2,0}:

$$A_{UQ_0} = \frac{1}{2} <\sigma> \left[ \sigma(\bullet, 0_L) - \sigma(\bullet, +1_L) + \sigma(\bullet, -1_L) \right] = \frac{2 V_{0,0}^{UQ_0} + (1/3 - \cos^2\theta) V_{2,0}^{UQ_0}}{2 W_{0,0} + (1/3 - \cos^2\theta) W_{2,0}},$$

where $\bullet$ indicates the unpolarized case.

### 6.2. Parton model expressions

Possible structure functions and spin asymmetries were introduced for the pd Drell-Yan processes. Here, we express them in terms of parton distribution functions of the proton and deuteron [16]. As shown in Fig. 7. The hadron tensor $W^{\mu\nu}$ of the Drell-Yan processes is
The correlation functions $\Phi_{a/A}(P_A S_A; k_a)$, $\Phi_{b/B}(P_B S_B; k_b)$ are defined by

$$W^{\mu\nu} = \frac{1}{3} \sum_{a,b} \delta_{ba} \epsilon_a^2 \int d^4k_a d^4k_b \delta^4(k_a + k_b - Q) Tr[\Phi_{a/A}(P_A S_A; k_a)\gamma^\mu \Phi_{b/B}(P_B S_B; k_b)\gamma^\nu].$$

(40)

The correlation functions $\Phi_{a/A}$ and $\Phi_{b/B}$ are defined by

$$\Phi_{a/A}(P_A S_A; k_a)_{ij} = \int \frac{d^4\xi}{(2\pi)^4} e^{ik_a \cdot \xi} \langle P_A S_A | \bar{\psi}_j^{(a)}(0) \psi_i^{(a)}(\xi) | P_A S_A \rangle,$$

$$\Phi_{b/B}(P_B S_B; k_b)_{ij} = \int \frac{d^4\xi}{(2\pi)^4} e^{ik_a \cdot \xi} \langle P_B S_B | \bar{\psi}_j^{(a)}(0) \psi_i^{(a)}(\xi) | P_B S_B \rangle,$$

(41)

where link operators for the gauge invariance are not explicitly written. Using a Fierz transformation, we write the hadron tensor in a factorized form:

$$W^{\mu\nu} = \frac{1}{3} \sum_{a,b} \delta_{ba} \epsilon_a^2 \int d^2k_{aT} d^2k_{bT} \delta^2(k_{aT} + k_{bT} - \bar{Q}_T) \left\{ -\Phi_{a/A}[\gamma^\alpha] \Phi_{b/B}[\gamma^\alpha] - \Phi_{a/A}[\gamma^\alpha\gamma_5] \Phi_{b/B}[\gamma_5] + \frac{1}{2} \Phi_{a/A}[i\sigma_{\alpha\beta}\gamma_5] \Phi_{b/B}[i\sigma^{\alpha\beta}\gamma_5] \right\} g^{\mu\nu} + \Phi_{a/A}[\gamma^\mu] \Phi_{b/B}[\gamma^\nu] + \Phi_{a/A}[\gamma^5] \Phi_{b/B}[\gamma^\alpha] + O(1/Q).$$

(42)

Then, these correlation functions are expressed by the unpolarized, longitudinally-polarized, and transversity distributions of the proton and deuteron, together with the tensor-polarized distributions of the deuteron, as illustrated in Fig. 8. Particularly, it is important that the correlation function $\Phi[\gamma^\mu]$ contains the tensor-polarized distributions $\delta_{TQ}(x)$. The details are found in Ref. [16].

In the naive parton model, we find 19 structure functions, which become four by the $\bar{Q}_T$ integration. We define $\bar{W} = \int d^2\bar{Q}_T W$, and $\overline{V}$ and $\overline{U}$ are defined in the same way. The pd Drell-Yan cross section is then given by

$$\frac{d\sigma}{dx_A dx_B d\Omega} = \frac{\alpha^2}{4Q^2} \left[ (1 + \cos^2\theta) \left\{ \bar{W}_T + \frac{1}{4} \lambda_A \lambda_B \overline{V}_T^{LL} + \frac{2}{3} \left( 2 |\bar{S}_{BT}|^2 - \lambda_B^2 \right) \overline{V}_T^{UQ_0} \right\} + \sin^2\theta |\bar{S}_{AT}| |\bar{S}_{BT}| \cos(2\phi - \phi_A - \phi_B) \overline{V}_T^{TT} \right].$$

(43)
The structure functions are expressed by the parton distributions in the process $q$(in p)+$\bar{q}$(in d)$\rightarrow \ell^+ + \ell^-$ as

$$W_T = \frac{1}{3} \sum_i e_i^2 q_i(x_A) \bar{q}_i(x_B),$$

$$V_T^{LL} = -\frac{4}{3} \sum_i e_i^2 \Delta q_i(x_A) \bar{\Delta q}_i(x_B),$$

$$U_{2,2}^{TT} = \frac{1}{3} \sum_i e_i^2 \Delta \bar{q}_i(x_A) \Delta \bar{q}_i(x_B),$$

$$V_T^{UQ_0} = \frac{1}{6} \sum_i e_i^2 q_i(x_A) \delta \bar{q}_i(x_B),$$

(44)

where $\Delta q_i$ and $\Delta \bar{q}_i$ are longitudinally-polarized and transversity distributions. The tensor-polarized distributions can be studied by the asymmetry $Q_0$:

$$A_{UQ_0} = \frac{V_T^{UQ_0}}{W_T} = \frac{\sum_a e_a^2 [q_a(x_A) \delta \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta \bar{q}_a(x_B)]}{2 \sum_a e_a^2 [q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B)]}.$$  

(45)

This asymmetry indicates that the tensor-polarized distributions $\delta \bar{q}$ and $\delta \bar{q}$ should be found in the Drell-Yan in addition to the charged-lepton scattering. It is especially important that the antiquark distribution $\delta \bar{q}$ is measured, because it was suggested that the finite $\delta \bar{q}$ was indicated by the HERMES experiment.

The possibility of polarized deuteron acceleration was once considered at RHIC [28], but it was not attained. However, there are future possibilities to investigate the Drell-Yan process with a fixed tensor-polarized deuteron target at hadron facilities such as Fermilab [21], J-PARC [22], GSI [23], and CERN-COMPASS. The JLab $b_1$ measurement will clarify the details of the tensor-polarized distributions in 2020’s. Together with their data, we expect that the Drell-Yan measurements will clarify the tensor-polarized antiquark distributions.

7. Summary

We explained our studies on the tensor-polarized structure function $b_1$ and tensor-polarized quark and antiquark distributions, $\delta q$ and $\delta \bar{q}$. First, the projection operators are shown for all the eight structure functions of the deuteron from the hadron tensor $W^{\mu\nu}$. The projection operators should be useful in a convolution description of the deuteron structure functions. Second, the sum rule was explained for $b_1$ by using the parton model. It is valuable for indicating the existence of antiquark tensor polarization, as the Gottfried sum rule violation indicated a $\bar{u}/d$ asymmetry in the nucleon. Third, the parametrization of the tensor-polarized quark and antiquark distributions was proposed by analyzing the HERMES data on $b_1$. The analysis indicated an existence of antiquark tensor polarization, and its origin should be investigated theoretically. The tensor-polarized antiquark distributions should be studied by the Drell-Yan processes with tensor-polarized deuteron. We showed the general formalisms for the structure functions in the proton-deuteron Drell-Yan processes and also their expressions in terms of the parton distribution functions in the proton and deuteron. With the tensor-polarized deuteron, the Drell-Yan process probes the tensor-polarized antiquark distributions directly. The JLab experiment on $b_1$ was approved and the actual measurement is expected to start in 2019. There are other possibilities to investigate the tensor structure at EIC, Fermilab, J-PARC, GSI, and CERN-COMPASS. The studies of tensor-polarized structure functions could open a new era of high-energy spin physics.

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