Is the NANOGrav signal a hint of dS decay during inflation?

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Abstract

As suggested by the swampland conjectures, de Sitter (dS) space might be highly unstable if it exists at all. During inflation, the short-lived dS states will decay through a cascade of the first-order phase transition (PT). We find that the gravitational waves (GWs) yielded by such a PT will be “reddened” by subsequent dS expansion, which may result in a slightly red-tilt stochastic GWs background at low-frequency band, compatible with the NANOGrav 12.5-yr result.

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The primordial GW background (GWB) \[1, 2\] spans a broad frequency-band, \(10^{-18} - 10^{10}\)Hz, e.g.[3–5]. It is usually thought that the discovery of primordial GWs will solidify our confidence on inflation. The primordial GWs at ultra-low frequency \(10^{-18} - 10^{-16}\)Hz may induce the B-mode polarization in the cosmic microwave background (CMB) \[6, 7\]. The search for the primordial GWs with CMB has been still on the way.

The Pulsar Timing Array (PTA) experiments, sensitive to GWs with frequencies \(f \sim 1/yr\), have also been searching for such a background. Recently, the NANOGrav collaboration \[8\], based on the analysis of 12.5-yr data, reported evidence for a stochastic *common-spectrum* process with frequencies \(f \sim 1/yr\) \((\sim 10^{-8}\)Hz), which might be interpreted as a stochastic GWB with a spectrum \(\Omega_{GW} \sim f^{-1.5\sim0.5}\) at 1\(\sigma\) level. It is difficult for inflation to yield such a stochastic GWB, see recent \[9\], which seems to require a high blue-tilt GW spectrum e.g.[10, 11], see also [12, 13].

It is well-known that evolution of the Universe must be described in a UV-complete effective field theory (EFT). Recently, the Trans-Planckian Censorship Conjecture (TCC) has been proposed in Ref.[14], which states that the sub-Planckian fluctuations will never have its length scale larger than the Hubble scale, otherwise the EFTs belong to the swampland (without UV-completion). The swampland conjectures \[15, 16\] actually suggest that dS space is highly unstable if it existed at all and changed slowly, while AdS phase is ubiquitous, see e.g.[17, 18] for its implications to the observable Universe. According to TCC, inflation in the early Universe can only lasts for a limited efolding number \(\int H dt < \ln \frac{M_P}{H}\) \[19\], see also Refs.[20–23] for multi-stage inflation. It has been proposed in Ref.[24] that, during inflation, the short-lived dS space \((\Delta t < \ln \frac{M_P}{H})\) will decay to a dS state with lower energy through the non-perturbative nucleation of bubbles \[25\], so a cascade of dS decay will present, see also earlier \[26–28\]. It is significant to ask if such short-lived dS vacua have any observable imprint.

We will present this possibility. A novelty of our result is that the NANOGrav signal, if being a stochastic GWB, might be a hint of dS decay during inflation. The first-order PT that dS bubbles nucleate and collide yields a sub-horizon GWB with a peculiar spectrum \(P_{PT}^{T}\) \[29–31\], it will alter the initial state of GW mode if \(P_{PT}^{T} \gg P_{BD}^{T}\) for \(k \gg aH\), where \(P_{BD}^{T}\) is the Bunch-Davis spectrum. Usually, the lower-frequency GWB (at PTA band) requires the lower-energy PT. However, the inflation (with lower dS energy) after PT will stretch the corresponding sub-horizon GW mode outside the horizon, which not only redshifts their
frequency but also reddens their spectra [32, 33]. We will show how such a scenario works.

The scenario we consider is sketched in Fig-1. The efolding number that the $j$-th stage of inflation lasts is bounded by $N_j \simeq H_j \Delta t_j < \ln \frac{M_P}{H_j}$, where $H_j^2 = \frac{\Lambda_j}{3M_P^2}$ with $\Lambda_j$ being the vacuum energy of the $j$-th stage. During $\Delta t_j$, a first-order PT must occur. Until $\Gamma_j/H_j^4 \gtrsim 1$ [34], the PT completed, where $\Gamma_j \sim e^{\beta(t-t^*)}$ is the nucleating rate of bubbles with vacuum energy $\Lambda_{j+1}$. The relevant physics is encoded in $\beta$, which we neglected here. When the bubbles collide, the energy of bubble walls is efficiently released, e.g.[35–37], and rapidly diluted with the expansion of the Universe. Hereafter, the $(j+1)$-th stage inflation with $\rho = \Lambda_{j+1}$ will start, and last $N_{j+1} < \ln \frac{M_P}{H_{j+1}}$ until $\Gamma_{j+1}/H_{j+1}^4 \gtrsim 1$.

![Fig. 1: A sketch of the dS cascade. Initially, the inflation with $\rho = \Lambda_j$ occurred. After the first-order PT, inflation continues but with lower $\rho = \Lambda_{j+1} < \Lambda_j$, and so on.

The tensor perturbation is $\gamma_{ij}(\tau, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{-i k \cdot x} \sum_{\lambda=\pm} \hat{\gamma}_\lambda(\tau, k)\epsilon_{ij}^{(\lambda)}(k)$, where $\hat{\gamma}_\lambda(\tau, k) = \gamma_k(\tau)\hat{a}_\lambda(k) + c.c.$ Its equation of motion is

$$\frac{d^2 u_k}{d\tau^2} + \left( k^2 - \frac{a''}{a} \right) u_k = 0,$$

where $u_k = \frac{a M_P}{2} \gamma_k$. Initially, the GW modes should be deep inside the horizon, i.e., $k^2 \gg \frac{a''}{a}$, so the initial state is $u_k = C(k)e^{-ik\tau}$ with $C = \frac{1}{\sqrt{2k}}$. However, the collision of bubbles will yield a sub-horizon stochastic GWB with a peculiar energy spectrum $\Omega_{GW}^j(k)$ [31],

$$\Omega_{GW}^j(k) = \Omega_{GW,c} \frac{(A + B)k^B_k}{B^2_k(A+B)} + A_k^{(A+B)},$$

where $A_k^{(A+B)}$ and $B^2_k(A+B)$ are given by

$$A_k^{(A+B)} = \frac{1}{2\pi^2} \int \frac{d^3 k}{(2\pi)^3} \frac{k^B_k}{(A+B)^2} \left( 1 + \frac{A_k^{(A+B)}}{B^2_k(A+B)} \right),$$

and

$$B^2_k(A+B) = \frac{1}{2\pi^2} \int \frac{d^3 k}{(2\pi)^3} \frac{k^B_k}{(A+B)^2}.$$
\[ \Omega_{GW,c} = \kappa^2 \left( \frac{\Delta \Lambda_j}{\Lambda_j} \right)^2 \left( \frac{H_j}{\beta} \right)^2 \times \frac{0.11v_b^3}{0.42 + v_b^2}, \]  

where \( \kappa \simeq 1 \) for \( \Lambda_{j+1} \ll \Lambda_j \), the bubble being sub-horizon requires \( H_j/\beta < 1 \), \( v_b \) is the bubble wall velocity and \( \Delta \Lambda_j = \Lambda_j - \Lambda_{j+1} \). Thus at the beginning of \((j+1)\)-th stage inflation, the initial state \( u_k \) of GWs modes will be inevitably modified as (2).

The energy density of GWs is [38]

\[ \rho_{GW} = \sum_{\lambda=+,-} \rho_{GW,\lambda} = \frac{M_p^2}{4} \int \frac{k^3}{2\pi^2} \left( \frac{|\gamma_k|^2 + k^2|\gamma_k|^2}{a^2} \right) d\ln k. \]  

According to \( \Omega_{GW} = \frac{d\rho_{GW}}{\rho_j(d\ln k)} \), we have

\[ |C(k)|^2 = 3\pi^2 M_p^2 H_j^2 a_*^4 \Omega_{GW}^j, \]  

where \( a_* \) is the scale factor at PT and \( \rho_j = \Lambda_j \). The inflation with \( \Lambda_{j+1} < \Lambda_j \) will start after the PT completed. Thus the sub-horizon GWs (with wavelength \( \lambda < H^{-1}_j \ll H^{-1}_{j+1} \)) will be stretched outside the horizon \( 1/H_{j+1} \). By requiring that the solution of Eq.(1) in the sub-horizon limit must be \( u_k = C(k)e^{-ik\tau} \), we obtain \( u_k(\tau) = -C(k)\sqrt{\frac{-\pi k^3}{2}} H^{(1)}_{3/2}(-k\tau) \) with \( C(k) \) being (5). On super-horizon scale, \( H^{(1)}_{3/2}(-k\tau) \approx -i\sqrt{2/(\pi k^3\tau^3)} \). Thus the primordial GW spectrum is

\[ P_T = \frac{4k^3}{\pi^2 M_p^2 a_*^2} |u_k|^2 = \frac{12H_j^2 H_{j+1}^2}{(k/a_*)^4} \Omega_{GW}^j. \]  

The physics of short-lived dS is encoded in \( \Omega_{GW}^j \), so \( P_T \). We see that the sub-horizon state (2) is reddened by the \((j+1)\)-stage inflation, \( P_T \sim \Omega_{GW}^j / k^4 \).

We have \( v_b = 1 \) for \( \Lambda_{j+1} \ll \Lambda_j \), noting that the swampland conjecture [24] requires \( \Delta \Lambda_j \approx \Lambda_j \). We set \( A \approx 3 \) and \( B \approx 1 \) [29–31] (see also [39, 40]) in (2). In addition, the peak momentum of \( \Omega_{GW}^j \) is \( \frac{k_c}{2\pi^3 a_*} = 0.62/(1.8 - 0.1v_b + v_b^2) \) [31], which suggests \( k_c \approx 1.4\beta a_* \) for \( v_b \approx 1 \). Thus Eq.(6) becomes

\[ P_T \simeq (\beta^{-1} H_j)^6 \left( \frac{\Lambda_{j+1}}{\Lambda_j} \right) \cdot \frac{\left( \frac{k}{k_c} \right)^{-1}}{1 + 3 \left( \frac{k}{k_c} \right)^4}, \]  

where \( V_{j/j+1} = 3M_p^2 H_{j/j+1}^2 \). The wavelength of GW mode is initially sub-horizon suggests a low-frequency cutoff \( k_{\text{cutoff}} = a_* H_j \) for \( P_T \). We have \( k_c/k_{\text{cutoff}} \approx 1.4\beta / H_j \), which is consistent with the requirement that the bubble is sub-horizon, \( H_j/\beta < 1 \). According to
\( P_T \sim k^{-5} \) for \( k \gg k_c \) is strongly red, while \( \sim k^{-1} \) for \( k \ll k_c \). The maximal value of \( P_T \), i.e., \( P_{T,\text{max}} \), is at \( k = k_{\text{cutoff}} \). We have

\[
P_{T,\text{max}} \simeq (\beta^{-1} H_j)^5 \left( \frac{\Lambda_{j+1}}{\Lambda_j} \right).
\] (8)

Thus if \( \Lambda_{j+1}/\Lambda_j = 0.2 \) and \( H_j/\beta \sim 0.4 \), we will have \( P_{T,\text{max}} \sim 10^{-3} \), far larger than that in slow-roll inflation scenario.

It is interesting to connect (6) with experimental results closely. The analysis result of NANOGrav 12.5-yr data is modeled as a signal with the characteristic strain amplitude [8] \( h_c(f) = A (f/f_{yr})^{(3-\gamma)/2} \). Thus the corresponding energy spectrum \( \Omega_{GW} = \frac{2\pi^2 f^2 h_c^2(f)}{3H_0^2 f_{yr}^2 A^2} \) is

\[
\Omega_{GW} = 2\pi^2 \frac{f^2}{3H_0^2 f_{yr}^2 A^2} \left( \frac{f}{f_{yr}} \right)^{5-\gamma},
\] (9)

where \( f_{yr} = 1/yr \). Present energy spectrum \( \Omega_{GW}(\tau_0) \) of GWs (6) is [41]

\[
\Omega_{GW}(\tau_0) = \frac{k^2}{12a_0^2 H_0^2} P_T(k) \left[ \frac{3\Omega_m j_1(k\tau_0)}{k\tau_0} \sqrt{1.0 + 1.36 \frac{k}{k_{\text{eq}}} + 2.50 \left( \frac{k}{k_{\text{eq}}} \right)^2} \right]^2.
\] (10)

see also [38, 42, 43]. Here, \( \Omega_m = \rho_m/\rho_c \) and \( \rho_c = 3H_0^2/(8\pi G) \) is the critical energy density, \( 1/k_{\text{eq}} \) is the Hubble scale at matter-radiation equality. According to (9), if the NANOGrav signal is regarded as the stochastic GWB (10), we have

\[
A = \left( \frac{3H_0^2 \Omega_{GW}(f_0)}{2\pi^2 f_{yr}^2} \right)^{1/2} \left( \frac{f_{yr}}{f_0} \right)^{(5-\gamma)/2} \quad \text{for} \quad f_0 = 5 \times 10^{-9} \text{Hz}
\] (11)

and \( \gamma \simeq 6 \). We plot \( \Omega_{GW}(\tau_0) \) in the left panel of Fig-2 for \( \Lambda_{j+1}/\Lambda_j = 0.2 \). Thus if \( H_j/\beta \sim 0.4 \), we have \( \Omega_{GW} \sim 10^{-9} \) with the frequency at \( (k_{\text{cutoff}}, 1.4k_{\text{cutoff}}\beta/H_j) \), which might explain the NANOGrav result for \( k_{\text{cutoff}} \simeq 2.3 \times 10^{-9} \).

We plot the 1\( \sigma \) and 2\( \sigma \) NANOGrav contours of \( \{A-\gamma\} \) result in the right panel of Fig-2. As expected, if \( H_j/\beta \sim 0.4 \), the short-lived dS decay predicts \( A \sim 10^{-15} \) and \( \gamma \simeq 6 \), which fits the NANOGrav data at 1\( \sigma \) level. The subsequent PT will occur after the \( j \)-th inflation with lower dS energy \( \Lambda_{j+1} \) lasting for efolds \( N_{j+1} < \ln \left( \frac{M_P}{H_{j+1}} \right) \), see Fig-1, so a “redden” stochastic GWB will be also possibly imprinted in high frequency, which might be detectable by GW detectors LISA, TAIJI, TianQin. It is noted that the stochastic GWB in the cosmic string scenario corresponds to \( \gamma \lesssim 5 \) [44, 45], see also [46–55] for other GW sources, while ours is \( \gamma \simeq 6 \).
In summary, we showed that the NANOGrav signal might be telling us the existence of short-lived dS vacua in the primordial Universe. As suggested by the swampland conjectures [15, 16, 24], the dS space is highly unstable. During inflation, dS cascade seems to be inevitably present. We found that the GWs yielded by the corresponding PT will be reddened by subsequent dS expansion, which results in a stochastic GWB at low frequency, compatible with the recent NANOGrav result at 1σ level. It should be mentioned that the NANOGrav collaboration did not claim a detection of GWs, since the signal seems not exhibiting quadrupole correlations. However, our spectrum (except the amplitude) of the stochastic GWB is universal for the dS decay during inflation. Though the model we consider is quite simplified, it highlights an unexpected point that the short-lived dS vacua, emerging in a consistent UV-complete theory, might imprint unique signal in our observable Universe.

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\(^1\) https://github.com/nanograv

\(^2\) http://vallis.github.io/libstempo/
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