Darcy-Benard surface tension driven convection in a composite layer with temperature dependent heat source

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Abstract
The effect of temperature dependent heat source on single component Benard Surface tension driven convection in a composite layer system comprising of an incompressible fluid saturated porous layer underlying a layer of same fluid, is studied. The lower surface of the porous layer is rigid and the upper surface is free with surface tension depending on temperature. The governing partial differential equations are non-dimensionalized using suitable transformation variables. The eigen value problem obtained after normal mode analysis is solved analytically using Exact Method. An expression for the eigenvalue, the Thermal Marangoni number is obtained for two sets of thermal boundary conditions on the boundaries of the composite layer, set (i) Adiabatic-Adiabatic and set (ii) Isothermal-Adiabatic. The effects of different physical parameters on the same are discussed in detail.

Keywords
Composite layer, Surface tension driven convection, Temperature dependent heat source.

AMS Subject Classification
78Axx.

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1. Introduction
Convection is the physical phenomenon that involves heat exchange through two systems. The interaction between a saturated porous layer overlying a fluid layer and convection in such a configuration is encountered in many industrial and environmental applications such as extraction of oil from underground reservoirs, packed-bed thermal storage systems, solidification of alloys, the manufacturing of composite materials used in aircraft and etc. Surface tension driven convection or Marangoni convection is the tendency for heat and mass to travel to areas of higher surface tension within a fluid and has many industrial applications such as semiconductor processing, welding, heat exchangers crystal growth, effect around vapour bubbles during nucleation and also in the field of space technology. There are few works available on Marangoni convection in two layer systems. Nield [5] has investigated the linear stability problem of superposed fluid and porous layers with buoyancy and surface tension effects at the deformable free upper surface by using Beavers-Joseph slip condition at the interface. While Nield [6] also argued about the modelling of Marangoni convection in a fluid saturated porous medium and has suggested the consideration of composite system in analyzing the problem of fluid-porous layer. The
A closed form solution is obtained for the thermal Marangoni phenomenon of non-uniform heat source in an inverted parabolic temperature profile. Also, it is observed that the effect of heat source/sink is dominant in the fluid layer. This research paper aims at understanding the effect of temperature dependent heat sources on surface tension driven convection in a composite system comprising of an incompressible fluid saturated porous layer over which lies a layer of the same fluid under microgravity condition. This composite layer is bounded below and above by rigid and free boundaries with surface tension effects depending on temperature at free boundary. The eigen value, the thermal Marangoni number is obtained for two sets of temperature boundary combinations set (i) Adiabatic-Adiabatic and set (ii) Adiabatic-Isothermal.

### 2. Formulation of the Problem

Consider an infinite horizontal layer of a Newtonian fluid of depth ‘d’ overlying a fluid saturated isotropic densely packed porous layer of depth ‘dm’ with heat sources Q and Qm respectively. The lower surface of the porous layer is rigid and the upper surface of the fluid layer is free with the surface tension depending on temperature. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z-axis, vertically upwards. The basic equations governing such a system are, For the Fluid Layer:

\[
\nabla \cdot \vec{q} = 0
\]  

\[
\rho_o \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla P + \mu \nabla^2 \vec{q}
\]  

\[
\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = \kappa \nabla^2 T + Q(T - T_o)
\]

For the porous layer:

\[
\nabla_m \cdot \vec{q}_m = 0
\]  

\[
\frac{\rho_o}{\phi} \frac{\partial \vec{q}_m}{\partial t_m} = -\nabla_m P_m - \frac{\mu}{K} \nabla_m \phi
\]  

\[
\kappa_m \nabla_m^2 T_m + Q_m(T_m - T_o)
\]

where \( \vec{q} \) is the velocity vector, \( \rho_o \) is the fluid density, \( t \) is the time, \( \mu \) is the fluid viscosity, \( P \) is the total pressure, \( \phi \) is the porosity, \( \kappa \) is the thermal diffusivity of the fluid, \( K \) is the permeability of the porous medium, \( A = \frac{\rho_o C_p}{\phi K} \) is the ratio of heat capacities where \( C_p \) is the specific heat, \( \kappa_m \) is the thermal diffusivity of the porous medium, \( T \) and \( T_m \) denotes temperatures in the fluid and porous medium respectively. Here subscripts \( m \) refers to the porous layer and \( f \) refers to the fluid layer.

The basic state of fluid and porous layers are assumed to be quiescent, pressure and temperatures are functions of \( z \) only. The temperature distributions are found to be

\[
T_o(z) = \frac{T_o - T_f}{\sin \left( \sqrt{\frac{Q}{\kappa}} \right)} \sin \left( \sqrt{\frac{Q}{\kappa}} z \right) + T_o, \quad 0 \leq z \leq d
\]
\[ T_{mb}(z_m) = \frac{T_o - T_i}{\sin \left( \sqrt{\frac{Q_m}{\kappa_m} z_m} \right)} - d_m \leq z_m \leq 0 \]  

where,

\[ T_o = \frac{T_o \sqrt{Q \sin \left( \sqrt{\frac{Q_m}{\kappa_m} d_m} \right)} + T_l \sqrt{Q \sin \left( \sqrt{\frac{Q_m}{\kappa_m} d} \right)} + \sqrt{Q \sin \left( \sqrt{\frac{Q_m}{\kappa_m} d_m} \right)} + \sqrt{Q \sin \left( \sqrt{\frac{Q_m}{\kappa_m} d} \right)}}{\sqrt{Q \sin \left( \sqrt{\frac{Q_m}{\kappa_m} d_m} \right)}} \]

is the interface temperature. 

Now an infinitesimal perturbations are superimposed in the form

\[ \vec{q} = \vec{q}_o + \vec{q}', \quad T = T_o(z) + \theta, \quad P = P_o(z) + P' \]

\[ \vec{q}_m = \vec{q}_m, \quad T_m = T_m(z_m) + \theta_m, \quad P_m = P_m(z_m) + P'_m \]

where the prime indicates a perturbed quantity and the subscript 'b' denotes the basic state. Equations (2.9) and (2.10) are substituted in equations (2.1) to (2.6) and linearized in the usual manner. The pressure term is eliminated from the equations (2.2) and (2.5) by taking curl twice on these two equations and then the resulting equations are non-dimensionalized using appropriate scale factors according to Vanishree et al. [14] and Sumithra et al. [11]

The dimensionless equations are subjected to normal mode analysis in the form

\[ \begin{bmatrix} W \\ \theta \end{bmatrix} = \begin{bmatrix} W(z) \\ \theta(z) \end{bmatrix} f(x,y)e^{a't} \] (2.11)

\[ \begin{bmatrix} W_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \theta_m(z_m) \end{bmatrix} f_m(x_m,y_m)e^{a'mt} \] (2.12)

with \( \nabla^2 f + a^2 f = 0 \) and \( \nabla^2 f_m + a^2 f_m = 0 \), where \( a \) and \( a_m \) are nondimensional horizontal wave numbers, \( n \) and \( n_m \) are frequencies, \( W(z) \) and \( W_m(z_m) \) are dimensionless vertical velocities in fluid and porous layers, the following equations are obtained: In \( 0 \leq z \leq 1 \),

\[ (D^2 - a^2 - \frac{n^2}{Pr}) (D^2 - a^2) W(z) = 0 \] (2.13)

\[ (D^2 - a^2 + R_f + n) \theta(z) + W(z) \sqrt{R_f} \frac{\cos(\sqrt{R_f} z)}{\sin \sqrt{R_f}} = 0 \] (2.14)

In \(-1 \leq z_m \leq 0\),

\[ \left( \frac{n_m^2 \beta^2}{Pr_m} - 1 \right) (D_m^2 - a_m^2) W_m(z_m) = 0 \] (2.15)

\[ (D_m^2 - a_m^2 - R_{lm} + n_m A) \theta_m(z_m) + W_m(z_m) \sqrt{R_{lm}} \frac{\cos(\sqrt{R_{lm}} z_m)}{\sin \sqrt{R_{lm}}} = 0 \] (2.16)

where \( Pr = \frac{\mu}{\rho c_p} \) is the prandtl number in the fluid layer, \( Pr_m = \frac{\mu}{\rho c_p} \) is the Prandtl in the porous layer, \( \beta^2 = \frac{K}{\rho c_p} = Da \) is the darcy number, \( R_f = \frac{Q}{\kappa} d^2 \) is the internal Rayleigh number for the fluid layer and \( R_{lm} = \frac{Q_m}{\kappa_m} d_m^2 \) is the internal Rayleigh number for the porous layer. Obtaining the relevant neutral stability \( (n = n_m = 0) \), we get, in \( 0 \leq z \leq 0 \),

\[ (D^2 - a^2) (D^2 - a^2) W(z) = 0 \] (2.17)

\[ (D^2 - a^2 + R_f) \theta(z) + W(z) \sqrt{R_f} \frac{\cos(\sqrt{R_f} z)}{\sin \sqrt{R_f}} = 0 \] (2.18)

In \(-1 \leq z_m \leq 0\),

\[ (D_m^2 - a_m^2) W_m(z_m) = 0 \] (2.19)

\[ (D_m^2 - a_m^2 - R_{lm}) \theta_m(z_m) + W_m(z_m) \sqrt{R_{lm}} \frac{\cos(\sqrt{R_{lm}} z_m)}{\sin \sqrt{R_{lm}}} = 0 \] (2.20)

Equations (2.17) to (2.20) are decoupled ordinary differential equations. To solve these equations we need six velocity boundary conditions and four temperature boundary conditions.

### 3. Boundary Conditions

The suitable boundary conditions are non-dimensionalized and then subjected to normal mode analysis. They are:

\[ W(1) = 0 \] (3.1)

\[ W(0) = \frac{\xi}{\varepsilon_T} W_m(0) \] (3.2)

\[ (D^2 + a^2) W(0) = \frac{\varepsilon_T \mu}{\varepsilon_T \mu} (D_m^2 + a_m^2) W_m(0) \] (3.3)

\[ (D^3 - 3a^2 D) W(0) = -\frac{\varepsilon_T \mu}{\varepsilon_T \mu} D_m W_m(0) \] (3.4)

\[ W_m(1) = 0 \] (3.5)

\[ D^2 W(1) + M \theta(1) = 0 \] (3.6)

Here \( \mu = \frac{\mu_m}{\mu} \) is the viscosity ratio, where \( \mu_m \) is the effective viscosity of the fluid in the porous layer, and in this paper the value of \( \mu \) is considered as unity, that is viscosity of the fluid in the fluid layer is same as the viscosity of the fluid in the porous layer, \( \xi = \frac{\varepsilon_T \mu}{\varepsilon_T \mu} \) is the depth ratio, \( \varepsilon_T = \frac{\varepsilon_T}{\varepsilon_T} \) is the ratio of thermal diffusivity and \( M = -\frac{\sigma}{\sigma} \left( T_o - T_m \right) \frac{d}{\mu_k} \) is the thermal Marangoni number, where \( \sigma \) is the surface tension and \( T \) is the temperature.

### 4. Method of Solution

The velocity equations (2.17) and (2.19) are solved exactly for vertical velocity distributions \( W(z) \) and \( W_m(z_m) \) and are
suitably obtained as,

\[ W(z) = A_1 \cosh(az) + A_2 \sinh(az) + A_3 \cosh(az) + A_4 z \sinh(az) \]  
\[ W_m Z_m = A_1 \cosh(a m_{zm}) + A_{m2} \sinh(a m_{zm}) \]

where \( A_1, A_2, A_3, A_4, A_m, \) and \( A_m \) are constants which are determined by velocity boundary conditions given by equations (3.1), (3.2), (3.3), (3.4) and (3.5) and they are:

\[ A_2 = -\frac{\zeta m_{am}}{2 \alpha^2 D a} \cosh(a m), \quad A_3 = [1 + (A_2 + A_4) \tanh(a)] \]
\[ A_4 = \frac{1}{\alpha} [\zeta^2 m_{am}^2 - a^2], \quad A_{m1} = \frac{\epsilon_T}{\zeta}, \quad A_{m2} = -\frac{\epsilon_T}{\zeta} \cosh(a m) \]

**4.1 Thermal Marangoni number for the set (i) Adiabatic - Adiabatic (A-A) condition**

The Adiabatic-Adiabatic temperature boundary conditions on the boundaries of the composite layer are used to solve the temperature distributions \( \theta(z) \) and \( \theta_m(z_m) \), where both the boundaries of the composite layer are adiabatic. They are as follows:

\[ D \theta(1) = 0, \quad \theta(0) = \frac{\epsilon_T}{\zeta} \theta_m(0), \quad D \theta(0) = D_m \theta_m(0), \]
\[ D_m \theta_m(-1) = 0. \]

The temperature distributions obtained by solving equations (2.18) and (2.20) using velocity distributions (4.1) and (4.2) are,

\[ \theta(z) = A_1 [c_1 \cosh(bz) + c_2 \sinh(bz) - f(z)] \]
\[ \theta_m(z_m) = A_1 [c_3 \cosh(b m_{zm}) + c_4 \sinh(b m_{zm}) - f_m(z_m)] \]

where \( b = \sqrt{\alpha^2 - R_I} \) and \( b_m = \sqrt{\alpha^2 - R_{l_m}} \).

The coefficients \( c_1, c_2, c_3 \) and \( c_4 \) are obtained by using temperature boundary conditions (4.3) and as follows:

\[ c_1 = \delta_1 + \frac{\epsilon_T}{\zeta} c_3, \quad c_2 = \frac{\delta_3 - c_1 b \sinh(b)}{b \cosh(b)}, \quad c_4 = \frac{c_3 b m \sinh(b m)}{b m \cosh(b m)} \]

\[ c_3 = -b \delta_3 \sinh(b) \cosh(b_m) - \delta_5 \cosh(b) \cosh(b_m) + \delta_1 \cosh(b_m) + \delta_2 \cosh(b) \]

\[ \delta_1 = \frac{A_4}{2 \alpha \sqrt{R_I} \sin(\sqrt{R_I})}, \]
\[ \delta_2 = \left( \frac{\sqrt{R_I}}{2 \alpha \sin(\sqrt{R_I})} \right) \left( A_2 - \frac{A_3 + a A_3}{a R_I} - \frac{A_{m2} \sqrt{R_{l_m}}}{2 a m \sin(\sqrt{R_{l_m}})} \right) \]
\[ \delta_3 = \frac{1}{2 \alpha \sin(\sqrt{R_I})} (A_1 + A_2 A_2 + A_3 A_3 + A_4 A_4) \]

\[ A_1 = a \cosh(a) \sin(\sqrt{R_I}) + \sqrt{R_I} \sinh(a) \cos(\sqrt{R_I}) \]
\[ A_2 = a \sinh(a) \sin(\sqrt{R_I}) + \sqrt{R_I} \cosh(a) \cos(\sqrt{R_I}) \]
\[ A_3 = a \sin(\sqrt{R_I}) \cosh(a) + \sqrt{R_I} \cos(a) \sin(\sqrt{R_I}) \]
\[ -\frac{1}{a} \sqrt{R_I} \cos(\sqrt{R_I}) \cosh(a) \]
\[ -\sqrt{R_I} \sinh(a) + \frac{a}{\sqrt{R_I}} \cos(\sqrt{R_I}) \cosh(a) \]
\[ A_4 = A_{i1} + A_{i2} \]
\[ A_{i1} = a \sin(\sqrt{R_I}) \sinh(a) + \sqrt{R_I} \cos(\sqrt{R_I}) \cosh(a) \]
\[ -\sqrt{R_I} \sinh(a) \]
\[ A_{i2} = \frac{a}{\sqrt{R_I}} \sin(\sqrt{R_I}) \sinh(a) \]
\[ + \frac{1}{a} \sqrt{R_I} \cos(\sqrt{R_I}) \cosh(a) \]

The eigenvalue thermal Marangoni number \( M \) obtained from the boundary condition (3.6) is:

\[ M = -\frac{D^2 W(1)}{\alpha^2 \theta(1)} \]

and using the same condition for the set (i) Adiabatic-Adiabatic condition, the thermal Marangoni number is:

\[ M_{AA} = -\frac{m_{11} \cosh(a) + m_{12} \sinh(a)}{a^2} \left( m_{21} - \frac{1}{2 a \sin(\sqrt{R_I})} (m_{22} + A_3 m_{23} + A_4 m_{24}) \right) \]

\[ m_{11} = a^2 (1 + A_3) + 2 a A_4, \quad m_{12} = a^2 (A_2 + A_3) + 2 a A_3 \]
\[ m_{21} = c_1 \cosh(b) + c_2 \sinh(b) \]
\[ m_{22} = \sinh(a) \sin(\sqrt{R_I}) + A_3 \cosh(a) \sin(\sqrt{R_I}) \]
\[ m_{23} = \sinh(a) \sin(\sqrt{R_I}) - \frac{1}{a} \sin(\sqrt{R_I}) \cosh(a) \]
\[ + \frac{1}{\sqrt{R_I}} \cos(\sqrt{R_I}) \sinh(a) \]
\[ m_{24} = \sin(\sqrt{R_I}) \cosh(a) - \frac{1}{a} \sin(\sqrt{R_I}) \sinh(a) \]
\[ + \frac{1}{\sqrt{R_I}} \cos(\sqrt{R_I}) \cosh(a) \]

**4.2 Thermal Marangoni number for the set (i) Adiabatic - Isothermal (A-I) condition**

The Adiabatic-Isothermal temperature boundary conditions on the boundaries of the composite layer are used to solve the temperature distributions \( \theta(z) \) and \( \theta_m(z_m) \), where the upper boundary of the fluid layer is adiabatic and the lower boundary of the porous layer is isothermal and they are as follows:

\[ D \theta(1) = 0, \quad \theta(0) = \frac{\epsilon_T}{\zeta} \theta_m(0), \quad D \theta(0) = D_m \theta_m(0), \]
\[ \theta_m(-1) = 0. \]
The temperature distributions are obtained by solving equations (2.18) and (2.20) using the velocity distributions (4.1) and (4.2) and are as follows:

\[
\begin{align*}
\theta(z) &= A_1 [c_{11} \cosh(bz) + c_{12} \sinh(bz) - f(z)] \\
\theta_m(z_m) &= A_1 [c_{23} \cosh(b_m z_m) + c_{24} \sinh(b_m z_m) - f_m(z_m)]
\end{align*}
\]

The coefficients \(c_{11}, c_{12}, c_{23}\) and \(c_{44}\) are obtained by using temperature boundary conditions (4.7) and are as follows:

\[
\begin{align*}
c_{11} &= \delta_1 + \frac{\varepsilon_T}{\zeta} c_{23}, \quad c_{12} = \frac{\delta_{12} + c_4 b_m}{b}, \quad c_{23} = \frac{\Delta_1 + \Delta_2}{\Delta_{12}}, \\
c_{24} &= \frac{c_{23} \cosh(b_m)}{\sinh(b_m)}, \quad \Delta_1 = -b \delta_1 \sinh(b_m) - b \cosh(b_m) \sinh(b_m) \\
\Delta_2 &= \delta_m b_m \cosh(b_m) + \delta_3 \sinh(b_m), \\
\Delta_{12} &= b_m \cosh(b_m) + \frac{\varepsilon_T}{\zeta} b \sinh(b_m) \sinh(b_m)
\end{align*}
\]

The coefficient \(\delta_{11}\) is obtained from boundary condition (3.6) as:

\[
\delta_{11} = \frac{1}{2a_m \sin(\sqrt{R_{lm}})} (A_{m1} \Delta_3 - A_{m2} \Delta_4)
\]

The eigen value, thermal Marangoni number obtained from boundary condition (3.6) is:

\[
M = -\frac{D^2 W(1)}{a^2 \theta(1)}
\]

and using the same condition for the set (ii) Adiabatic-Isothermal condition, the thermal Marangoni number is:

\[
M_{AI} = -\frac{m_{11} \cosh(a) + m_{12} \sinh(a)}{a^2 \left( m_{21} - \frac{1}{2a_m \sin(\sqrt{R_{lm}})} (m_{22} + A_3 m_{23} + A_4 m_{24}) \right)}
\]

The eigen value problem is solved exactly and analytical expression for the thermal Marangoni number is obtained for two types of temperature boundary conditions, viz., (i) both the boundaries of the composite layer are adiabatic(A-A), (ii) lower rigid boundary is isothermal and upper free surface is

5. Result and Discussion

The effect of surface tension driven convection in a composite system consisting of a fluid layer saturated by the same system is investigated theoretically.
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The effects of $R_I$, $R_{Im}$, the internal Rayleigh numbers for the fluid and porous layers, the horizontal wave number $a$, the thermal diffusivity ratio $\varepsilon_T$, and the Darcy number $Da$ on the Marangoni number are displayed in the following figures where the supplementary parameters are fixed.

The comparison of thermal Marangoni numbers as a function of depth ratio $\zeta$ for the Adiabatic-Adiabatic (A-A) and Adiabatic-Isothermal (A-I) temperature boundary conditions is shown in Figure 1. It is observed that in case of A-A boundaries, the thermal Marangoni number $M_{AA}$ increases initially with $\zeta$ and reaches maximum and then decreases with further increase in $\zeta$. In case of A-I boundaries, thermal marangoni number $M_{AI}$ increases with increase in $\zeta$ without any decreasing trend as noticed in case of A-A boundaries. For smaller values of depth ratio, the system is stable when A-A temperature boundary conditions are deployed, whereas for larger values of depth ratio the same system is stable for A-I temperature boundary combinations. But both $M_{AA}$ and $M_{AI}$ coincide when the value of the depth ratio is $\zeta = \frac{d_L}{d_m} = 0.75$.

Figures (2a) and (2b) are the plots of $M_{AA}$ and $M_{AI}$ versus the depth ratio $\zeta$ for the linear temperature distribution for different porous internal Rayleigh number $R_{Im}$ when the other parameters are fixed. In case of A-A boundaries, it is observed that $M_{AA}$ increases initially with $\zeta$ and reaches maximum and then decreases with further increase in $\zeta$. One important observation that can be made from fig (2a) is that when the strength of the heat source is small ($R_{Im} = 0.01$), $M_{AA}$ attains maximum with increasing $\zeta$ up to a certain depth ratio and then remains the same with further increase in $\zeta$, whereas for $R_{Im} = 0.35$ and $0.6$, $M_{AA}$ increase with $\zeta$, attains maximum and decreases with further increase in $\zeta$. This indicates that the choice of the strength of heat source plays a crucial role in the stability of the system. It is also observed that increase in $R_{Im}$ decreases $M_{AA}$ and thus destabilizes the system. This indicates that the increase in the strength of the heat source in the porous layer decreases the surface tension thereby decreasing $M_{AA}$.

From Fig. 2b we notice that the thermal Marangoni number $M_{AI}$ increases with increase in $\zeta$ and decreases with increase in $R_{Im}$ for the A-I boundaries and thus destabilizes the system. One can see from these graphs that the effect of $R_{Im}$ is more dominant in the case of A-A temperature boundary conditions. It is also observed that for different $R_{Im}$, both $M_{AA}$ and $M_{AI}$ remain same till $\zeta$ attains a certain value. This implies that the effect of $R_{Im}$ is significant in the porous dominant composite.
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Thermal Marangoni numbers $M_{AA}$ and $M_{AI}$ as a function of depth ratio $\zeta$ for linear temperature distribution are shown in Figs. 3a and 3b. The effect of fluid internal Rayleigh number, $R_I$ for a set of fixed physical parameters $a = 1, R_{lm} = \varepsilon_T = 0.5, Da = 10$ and different values of $R_I = 0.25, 0.4, 0.5$. In case of A-A boundary combination(Fig. 3a), it is observed that $M_{AA}$ increases initially with $\zeta$ and reaches maximum and then decreases with further increase in $\zeta$. In case of A-I boundary combination (Fig.3b), $M_{AI}$ increases with increase in $\zeta$ without any decreasing trend as noticed in case of A-A boundaries. Also, it is observed that $M_{AA}$ and $M_{AI}$ decreases with increase in $R_I$ for both the cases and thus destabilize system. All other observations are qualitatively similar to those observed in Figs. 2a and 2b.

The variations in $M_{AA}$ and $M_{AI}$ as a function of $\zeta$ in Figs. 4a and 4b, respectively for $Da = 5, 10, 100$ when $R_I = R_{lm} = \varepsilon_T = 0.5$ and $a = 1$. In case of A-A boundaries(Fig.4a), it is seen that $M_{AA}$ increase initially with $\zeta$ and reaches maximum and then decreases with further increase in $\zeta$. It is also observed that increase in $Da$ initially increase the thermal Marangoni number $M_{AA}$ and thus stabilize system. This result is in tune with the intuition, physically $Da$ increase implies the increase in the permeability there by allowing the flow of more fluid which delays the onset of convection. In case of A-I boundaries (Fig. 4b) also $M_{AI}$ increases with increase in $\zeta$ as well as $Da$ and thus stabilize system.

The effect of thermal diffusivity ratio $\varepsilon_T$ as a function of $\zeta$ for a set of fixed physical parameters $R_I = R_{lm} = \varepsilon_T = 0.5, a = 1, Da = 10$ and different values of $\varepsilon_T = 0.15, 0.2, 0.25$ in case of A-A and A-I boundaries are depicted in Figs. 5a and 5b. From Fig. 5a, it is seen that $M_{AA}$ increases initially with $\zeta$ reaches maximum and then decreases with further increase in $\zeta$, whereas in case of A-I boundaries, $M_{AI}$ increases with increase in $\zeta$ as shown in Fig. 5b. Also it is observed that increase in $Da$ and $M_{AA}$ and $M_{AI}$ in both the cases and thus having a destabilizing effect on the system. Physically, increase in thermal diffusivity increases the heat transfer rate which destabilizes the system.

Figures 6a and 6b show variations in $M_{AA}$ and $M_{AI}$ with $\zeta$ for different values of horizontal wave number $a = 1.2, 1.5, 1.8$ when $R_I = R_{lm} = \varepsilon_T = 0.5$ and $Da = 10$. In case of A-A and A-I boundaries, we observe that an increase in $\zeta$ increases $M_{AA}$ and $M_{AI}$ respectively. This is because increase in depth ratio decreases the temperature at the surface ther by increasing the surface tension due to the increase in cohesive force
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6. Conclusion

The effect of temperature dependent heat source on surface tension driven convection in a composite system comprising of an incompressible fluid saturated porous layer over which lies a layer of the same fluid is analyzed theoretically. The following conclusions are drawn from the above results.

1. The effect of internal Rayleigh numbers, both $R_{\text{m}}$ and $R_{\text{i}}$, and thermal diffusivity ratio $\varepsilon_T$ is to destabilize the system and that of porous parameter $Da$ and the wave number $a$ is to stabilize the system for both the boundaries viz., A-A and A-I.

2. The strength of heat source in both porous and fluid layer, $R_{\text{m}}$ and $R_{\text{i}}$, can be effectively chosen to control the convection.

3. $M_{\text{AA}}(\zeta) < M_{\text{AI}}(\zeta)$ for all parameters except for smaller depth ratios.

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