Topological defects: A problem for cyclic universes?

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We study the behaviour of cosmic string networks in contracting universes, and discuss some of their possible consequences. We note that there is a fundamental time asymmetry between defect network evolution for an expanding universe and a contracting universe. A string network with negligible loop production and small-scale structure will asymptotically behave during the collapse phase as a radiation fluid. In realistic networks these two effects are important, making this solution only approximate. We derive new scaling solutions describing this effect, and test them against high-resolution numerical simulations. A string network in a contracting universe, together with the gravitational radiation background it has generated, can significantly affect the dynamics of the universe both locally and globally. The network can be an important source of radiation, entropy and inhomogeneity. We discuss the possible implications of these findings for bouncing and cyclic cosmological models.

I. INTRODUCTION

Cosmological scenarios involving oscillating or cyclic universes have been known for a long time, with interest in them varying according to the latest theoretical prejudices or observational constraints. Recent interest has been associated with a cyclic extension of the ekpyrotic scenario. A related result was the realization that the presence of a scalar field seems to be necessary to make cosmological scenarios with a bounce observationally realistic. And if scalar fields are present, then one should contemplate the possibility of topological defects being formed.

It is thought that the early universe underwent a series of phase transitions, each one spontaneously breaking some symmetry in particle physics and giving rise to topological defects of some kind, which in many cases can persist throughout the subsequent evolution of the universe.

In the present work we study cosmic string evolution in a collapsing universe, following up on and generalizing the results of, and discuss in much greater detail some implications of the presence of cosmic strings (and cosmic defects in general) for bouncing universes. In a bouncing universe scenario the properties of the universe in the expanding phase depend on physics happening in a previous collapsing phase (before the bounce). For this reason, if defects do exist in these models, it is crucial to understand their evolution and consequences in both the expanding and collapsing phases. Up to now all these studies, be they analytic or numerical, have only been undertaken for the expanding case, and it is clear that while some results may be expected to carry over to the contracting phase, some others clearly won’t. This will have consequences not only for the standard string seeded (or hybrid) structure formation scenario, but also for other ‘non-standard’ scenarios involving defects, such as the production of adiabatic and nearly Gaussian density fluctuations, or those involving anisotropic or inhomogeneous universes.

In particular, we expect that cosmic strings will become ultra-relativistic, behaving approximately like a radiation fluid. This means that a cosmic string network, both directly and through the gravitational radiation emitted by the small loops it produces, will soon become a significant source of entropy (and also of inhomogeneity). Hence a cosmic string network is a further problem for cyclic universes if a suitable and efficient mechanism for diluting the entropy is not available.

We should point out at the outset that if/when during the collapse phase one reaches the Hagedorn temperature, one expects the string network to quickly dissolve. However this is largely irrelevant for the points being made in this paper: the radiation, entropy and anisotropy produced by the network will obviously still be left behind if the network does dissipate. On the other hand, it need not be the case that the Hagedorn temperature is reached or, more specifically, that the collapse has to continue all the way to the ‘Big Crunch’ (where there is no known sensible description of the physics involved). Cosmological models do exist where the bounce takes place at finite size. Indeed, such models seem to be relatively common in scenarios with extra dimensions and scalar fields, although this issue is somewhat debatable.

The outline of this paper is as follows. In Sect. we
present a very brief overview of previous work on cyclic universes, and discuss further motivation for our work. Then in Sect. V we introduce the basic dynamical properties of cosmic string networks, and after a ‘warm-up’ example of a circular string loop we successively discuss analytic scaling solutions in various regimes in a contracting universe. In Sect. VI we describe high-resolution numerical simulations of string networks in contracting universes using the Allen-Shellard string code [15], and then compare and contrast our analytic and numerical results. Possible cosmological consequences of these results are discussed in Sect. VII and finally we present some conclusions in Sect. VIII. Throughout the paper we will use units in which $c = \hbar = 1$.

II. AN OVERVIEW OF BOUNCING COSMOLOGICAL MODELS

Oscillating universes arise naturally as classical exact solutions of the Einstein equations, and have been explored in several contexts in an attempt to solve some of the cosmological enigmas. For example, they could conceivably solve the flatness and horizon problems. For oscillating universes whose size at maximum expansion increases in each cycle the flatness problem may be solved because it is ever less likely for an observer to find himself in a non-flat region. Provided there is a causal correlation for the microphysics at the bounce, the age for cyclic universes may also be large enough to solve the horizon problem.

However, oscillating models are not without their caveats. The first of these was pointed out a long time ago by Tolman [1]. By assuming a closed universe with zero cosmological constant in the context of the General Relativity theory, he noted that entropy is generated at each cycle and so the total entropy in the universe grows from cycle to cycle. Hence the period of each cycle is larger that the previous one, and extrapolating back in time one finds that the sum is finite. In other words, the universe must still have had a beginning. These points were also discussed by Rees [22], and formalized by Zel’dovich and Novikov [23], and subsequently by others [24, 25].

A thorough analysis of oscillating universe solutions was first carried out by Barrow and Dabrowski [26]. They showed that in closed universes filled with a perfect fluid of matter and/or radiation, with total entropy increasing from cycle to cycle, a positive cosmological constant or any non-zero stress violating the strong energy condition will eventually halt the oscillations after a finite number of cycles. In particular when the cosmological constant dominates the universe approaches the de Sitter solution. Dabrowski [27] subsequently generalized the analysis to include negative pressure matter which violated the strong energy condition, and also provided a somewhat simplistic description of the scalar field case.

Looking for an alternative model to inflation that could explain the scale invariance of the observed power spectrum of fluctuations, Durrer and Luekermann [28] presented a model of a closed universe dominated by radiation or with an intermediate matter-dominated period where small black holes could have been formed. They postulated that density perturbations produced gravitational entropy which would have to be transformed into radiation entropy at the bounce. This could explain most of the present day radiation entropy without having over-production, provided one also assumed that perturbations never became strongly non-linear in the previous cycle (as the mass fluctuation would in fact generally diverge at the big crunch).

More recently, a detailed analysis by Peter and Pinto-Neto [4] has shown quite generically that no detectable bounce is observationally allowed in any universe which around the epoch of the bounce is described by Einstein gravity together with hydrodynamical fluids, since under those conditions scalar density perturbations would become non-linear well before nucleosynthesis. However, this ‘no-bounce conjecture’ can in some circumstances be evaded by adding a free scalar field which can dominate the universe during the bounce [5].

One of the earliest discussions of bouncing universes with scalar fields was by Hawking in the context of the no-boundary proposal [29]; of course, many of the discussions noted earlier are derivative of Hawking’s cosmological singularity theorems. Early in the development of quantum cosmology, he studied a simple chaotic inflation model and suggested that his boundary conditions selected only a special set of time-symmetric and eternally bouncing trajectories. Subsequent work, however, showed that these special bouncing states were not generic [30, 31], with typical universes diverging from what are unstable trajectories [32] and re-collapsing to singularities.

Kanekar et al. [33] have also considered a massive scalar field in a closed universe for the Einstein theory. This is a possible alternative mechanism which can still lead to an increase in the volume of an oscillating universe without requiring entropy production. They suggest that the oscillations of the universe force the scalar field to oscillate about the minimum of its potential. The work done by or on the scalar field creates an asymmetry between the expansion and collapse epochs which can result in the increasing volume of the universe. It was also argued that the presence of other matter fields will not change this picture, provided that the interactions between the matter and scalar fields are sufficiently weak.

Finally, we should mention the cyclic extension [2] of the ekpyrotic model [33, 34]. Here the contraction and expansion phases correspond to the epochs before and after the collision of two three-branes along a fifth dimension. During the expansion there are successive periods of radiation, matter and vacuum domination (the latter of which can dilute the entropy, black holes and other debris produced during each cycle), and the inter-brane potential can be carefully chosen to ensure the cyclic be-
haviour. Hence the model can potentially solve many of the cosmological issues raised above. However, it appears questionable whether density perturbations in this model, matched across the bounce, can reproduce the Harrison-Zel’dovich spectrum, as has been pointed out by a number of authors. [35, 36, 37, 38].

In the present work, motivated by the Kanekar et al. [8] results, we replace the scalar field by a cosmic string network, which is also expected to display an asymmetric behaviour between the contraction and expansion epochs. In particular, while during expansion a cosmic string network will quickly evolve towards a linear scalarnetwork, which is also expected to display an asymmetric behaviour between the contraction and expansion epochs. Hence the model can potentially solve many of the cosmological issues raised above. However, it appears questionable whether density perturbations in this model, matched across the bounce, can reproduce the Harrison-Zel’dovich spectrum, as has been pointed out by a number of authors. [35, 36, 37, 38].

In the following sections we will demonstrate this behaviour. Hence the model can potentially solve many of the cosmological issues raised above. However, it appears questionable whether density perturbations in this model, matched across the bounce, can reproduce the Harrison-Zel’dovich spectrum, as has been pointed out by a number of authors. [35, 36, 37, 38].

In the limit where the curvature radius of a cosmic string is much larger than its thickness, we can describe it as a one-dimensional object so that its world history can be represented by a two-dimensional surface in space-time (the string world-sheet)

$$x^\nu = x^\nu(\sigma^a); \quad a = 0, 1; \quad \nu = 0, 1, 2, 3$$  \hspace{1cm} (1)
microscopic speed of the loop. The microscopic string equations of motion\[^4\] then become
\[
\ddot{r} = (1 - \dot{r}^2) \left( -\frac{1}{r} - 2H\dot{r} \right), \tag{8}
\]
where \( H = \mathcal{H}/a \), with the velocity equation being obviously obtainable form this, or alternatively in terms of the invariant loop radius
\[
\frac{dR}{dt} = (1 - 2\nu^2)HR. \tag{9}
\]
This latter equation coincides with the averaged evolution equation for \( R \), obtained in the context of the velocity-dependent one-scale model\[^2\] [12, 13, 39,], while for the averaged velocity equation this model yields
\[
\frac{d\nu}{dt} = (1 - \nu^2) \left( \frac{\bar{k}(\nu)}{R} - 2H\bar{\nu} \right), \tag{10}
\]
where \( \bar{k}(\nu) \) is the momentum parameter which is thoroughly discussed in\[^13\].

Simple analytic arguments show that a loop whose initial radius is much smaller than the Hubble radius will oscillate freely with a constant invariant loop radius and an average velocity \( \bar{v} = 1/\sqrt{2} \) (Note that we are assuming units in which \( c = 1 \).) On the other hand, once the collapse phase begins, we will eventually get to a stage in which the physical loop radius becomes comparable to the Hubble radius \( ar \sim H^{-1} \) and then gets above it. In this regime the loop velocity is typically driven towards unity \( v \rightarrow 1 \) and it is straightforward to show that the invariant loop length grows as \( R \propto a^{-1} \) and the Lorentz factor as \( \gamma \propto a^{-2} \). Despite its growing energy \( R \), the actual physical loop radius \( ar = R/\gamma \propto a \), so the loop shrinks with the scale factor and inexorably follows the collapse into final big crunch singularity.

Importantly, note that this relativistic final state for a loop in a collapsing universe is generic and quite different to the initial condition usually assumed for superhorizon loops in the expanding phase (created, say, at a phase transition). In the latter case, the loops begin with a vanishing velocity which only becomes significant when each of them falls below the Hubble radius. Such evolution cannot be reproduced during the collapsing phase without fine-tuning the velocity as the loop crosses outside the Hubble radius. This simple fact introduces a fundamental time asymmetry for string evolution in a cyclic universe. In fact, something analogous happens for all other topological defects.

The analytic expectations for our circular loop solution have been confirmed by a numerical study, see Fig. 11. Here we plot both the microscopic and averaged loop sizes and velocities, for four loops of different sizes in a universe filled with matter (the radiation case being analogous, except for a slight difference in the timescale required to reach this asymptotic regime). Obviously a loop which has a considerable size relative to the horizon to begin with will not oscillate. We can see that the averaged quantities provide a very good description of the dynamics.

The main caveat with this solution is that it can not account for the presence of small-scale structures, whose build up we in fact expect to be enhanced in the collapsing phase. One expects that the presence of small-scale wiggles can significantly delay the onset of this asymptotic regime—an expectation that we will confirm in what follows.

C. String network evolution: analytic expectations

Two different but complementary approaches are available to study the evolution of a cosmic string network: one can resort to large numerical simulations\[^12\] [14, 15, 16] (which are intrinsically difficult and time consuming, as one is dealing with highly non-linear objects), or one can develop analytic tools\[^3\] [11, 12, 13, 39, 43] which provide an averaged (or ‘thermodynamical’) description of the basic properties of the network. In what follows we shall briefly describe the best motivated of these analytic models, the velocity-dependent one-scale (VOS) model\[^11\] [12, 13, 39,], and try to use it to deduce the basic properties of a cosmic string network during a phase of contraction. In the following subsection we will test these against numerical simulations.
1. The VOS model

The VOS model describes the string dynamics in terms of two ‘thermodynamical’ parameters: the string RMS velocity, $v_\infty$, defined by

$$v_\infty^2 \equiv \langle \dot{x}^2 \rangle = \frac{\int \dot{x}^2 e^{-\sigma} d\sigma}{\int e^{-\sigma} d\sigma}$$

and a single length scale, $L$, which can be variously interpreted as the long string correlation length, its curvature radius, or simply a measure of the energy density (see below). It is important to note that in the context of this model, all these three length scales are assumed to be identical [12]. Of course this assumption is not always realistic and models exist where it is relaxed [10, 12, 43], but it must be assumed if one wants to use the model in this formulation. The string network is thus assumed to be a Brownian random walk on large enough scales, characterized by a correlation length $L$. Hence one can simply relate it with the energy density in long strings as

$$\rho_\infty = \frac{\mu}{L^2},$$

where $\mu = \mu_0$ is the string mass per unit length. Note that given that we will be considering relativistic velocities, the commonly used ‘correlation length’ $L$ is really a measure of the invariant string length or energy, rather than the typical curvature radius of the strings. By including the appropriate Lorentz factor $\gamma_\infty = (1-v_\infty^2)^{-1/2}$ for the long strings, we can denote the physical correlation length by

$$L_{\text{phys}} = L\gamma_\infty^{1/2}.$$  \hspace{1cm} (13)

Note also that in defining (12) we are taking the string mass per unit length to be a constant. More generally, if one wanted to explicitly account for the presence of small-scale ‘wiggles’ one would need to define a varying ‘renormalized’ mass per unit length $\tilde{\mu}$, which would then be related to the ‘bare’ mass by

$$\mu = \tilde{\mu}\mu_0.$$  \hspace{1cm} (14)

With this assumption the VOS model has one phenomenological parameter $\tilde{c}$, commonly called the loop chopping efficiency, which describes the rate of energy transfer from the long-string network to loops. The evolution equations then take the following form [11, 13]

$$\frac{2dL}{dt} = 2(1 + v_\infty^2)HL + \tilde{c}v_\infty + 8\pi G\mu v_\infty^6,$$  \hspace{1cm} (15)

$$\frac{dv_\infty}{dt} = (1 - v_\infty^2) \left( \frac{k(v_\infty)}{L} - 2Hv_\infty \right).$$  \hspace{1cm} (16)

The final term in the evolution equation for the correlation length describes the effect of gravitational backreaction. Notice that we are not including in either equation additional terms arising from friction due to particle scattering [13], which could conceivably be important during the final stages of collapse. We shall return to this point below. Here $k(v_\infty)$ is the momentum parameter, which is thoroughly discussed in [12], and whose dependence on the string velocity,

$$k(v) = \frac{2\sqrt{2}}{\pi}(1 - v^2)(1 + 2\sqrt{2}v^3)\frac{1 - 8v^6}{1 + 8v^6}$$  \hspace{1cm} (17)

is shown in Fig. 2. Notice that this is positive for $0 < v^2 < 1/2$ and negative for $1/2 < v^2 < 1$; this turns out to be crucial for some of what follows.

As a final remark, we point out that these evolution equations are approximate, and are neglecting higher-order terms [12]. It is in principle possible that in this particular case ultra-relativistic corrections may become significant, for example there could be powers of $\gamma$ in various terms above. On the other hand, one physically expects that the velocity increase will also be accompanied by an enhanced build-up of small-scale ‘wiggles’, for which one might need further corrections [12, 43]. Thus in the present work we chose not to further complicate the model with these hypothetical corrections. We shall discuss these issues further below.

2. The ultra-relativistic regime

The first point to notice about eqns. (15-16) is that when the contraction phase starts and the Hubble parameter becomes negative the velocity will tend to increase—the Hubble term becomes an acceleration term, rather
than a damping one. This will be compounded be the fact that once one is beyond $v^2 = 1/2$ the momentum term in the velocity equation also changes sign (cf. Fig. 2), but it is clear that given the constant increase in the magnitude of $H$ this term will gradually prevail (more on this in the following sub-section).

Thus one can expect that, just as in the simple case of the circular loop, the string velocity will gradually tend towards unity. In this approximation, and neglecting for the moment the loop production and gravitational back-reaction terms, the evolution equation for the correlation length easily yields

$$L \propto a^2. \quad (18)$$

Note that this is the same overall scaling law for the string network as that in a radiation-dominated expanding universe; the string network effectively behaves like a radiation component. In terms of the physical correlation length $L_{phys}$ we have $L_{phys} \propto a$, which is as is if the strings were being conformally contracted (except for their rapidly growing velocities).

However, there are several factors that must be considered which complicate this simple state of affairs. First, there is the issue of loop production. Under the above assumptions on velocity, but putting the loop production term back in the correlation length equation, one finds the following approximate solution in the radiation-dominated case,

$$L_{rad} = \left(L_{max} - \frac{\dot{c}}{2} \ln a\right) a^2 \quad (19)$$

and, in the matter era,

$$L_{mat} = \left[L_{max} + \frac{\dot{c}}{2} \left(a^{-1/2} - 1\right)\right] a^2 \quad (20)$$

where $L_{max}$ is the string correlation length at the time of maximal size of the universe, and the scale factor at that time was chosen to be unity (so the logarithmic correction in the first case is positive). Hence we see that, if $\dot{c}$ remains constant (or is slowly varying), asymptotically the scale factor dependent terms will dominate, so that we expect $L \propto a^2 \ln a$ in the radiation era, and $L \propto a^{3/2}$ in the matter era. Again we note that the latter is also the scaling law for the correlation length in the matter-dominated, expanding universe (but again the behaviour of the velocities is very different in the two cases). This highlights the different roles played by loop production in the scaling behaviour of a cosmic string network in the radiation and matter eras, a point which has been noticed long ago [13, 20] in the usual expanding case.

A strong argument can be made, however, for an important relativistic correction to the loop production term in the evolution equation for the correlation length. In the simplest form of the VOS model there is a simple identification between the correlation length, $L$, and the physical distance $L_{phys}$ which a string segment is expected to travel before encountering another segment of the same size forming a loop in the process. However, taking into account the Lorentz factor in [13], this means that the probability $dP$ that a string segment will encounter another segment in a time interval $dt$ should be given approximately by

$$dP = \frac{d\rho_{\infty}}{\rho_{\infty}} = \frac{2}{L} \frac{dL}{L} \sim \frac{v_{\infty} dt}{L_{phys}} \sim \frac{\gamma_{\infty}^2 dt}{L}. \quad (21)$$

One would expect $\dot{c} \propto \gamma_{\infty}^{-1/2} \propto a$ thus driving $\dot{c}$ rapidly towards zero and asymptotically yielding our simple solution $L \propto a^2$ both in the radiation and matter eras.

Of course, during re-collapse we expect that $\dot{c}$ will depend on a number of other properties of the string network such as the enhanced build-up of small scale structure due to the contraction. Eventually, however, the Hubble radius will fall below even the length scale of wiggles on the string after which our asymptotic solution $L \propto a^2$ should be valid. In what follows, we shall consider the two well-motivated cases, first $\dot{c} = \text{const.} \neq 0$, and secondly $\dot{c} = 0$, the probable asymptotic case.

3. Approaching the ultra-relativistic regime

Returning to our analytic solutions for the constant loop production case [19] and [20], we can use the velocity equation to find an approximate solution for the evolution of the velocity as it approaches unity. Taking the simplifying assumption that the momentum parameter is slowly varying, one can arrive at the following implicit solution

$$1 - v^2 \propto a^4 \exp \left[\frac{2k(v)}{\lambda} \int \frac{a^{1/\lambda} da}{L(a)}\right], \quad (22)$$

where $\lambda = 1$ in the radiation era and $\lambda = 2$ in the matter era. Substituting [19, 20] one respectively obtains

$$1 - v_{rad}^2 \propto a^4 (-\ln a)^{4k(v)/\dot{c}}, \quad (23)$$

in the radiation-dominated case, and

$$1 - v_{mat}^2 \propto a^{4+2k(v)/\dot{c}} \quad (24)$$

in the matter-dominated case. Recall that the momentum parameter $k(v)$ is negative in the ultra-relativistic regime. Hence in the limit where $v \to 1$ and therefore $k \to 0$ the asymptotic solution would have the form

$$\gamma^{-2} \propto (1 - v^2) \propto a^4. \quad (25)$$

The presence of the momentum corrections, which phenomenologically account for the existence of small scale structures on the strings, imply that convergence will be slower than this power.
So far we have been neglecting the contribution of friction due to particle scattering for the evolution of the cosmic string network. The reason is that previous work [11, 12, 33, 41] has shown that for heavy strings (say, those formed around the GUT scale) this will only be significant very close to the Hagedorn temperature, that is, for temperatures close to those of the string-forming phase transition. For the rest of the cosmic history of the network, this friction is always subdominant relative to the damping generated by the expansion itself. However, for light strings (say, those formed around the electroweak scale) friction is significant for a longer period, and hence it is important to discuss how our previous results might be changed in this case.

One can show [3, 12] that the effect of friction due to particle scattering may be characterized by a friction length scale

$$\ell_f = \frac{\mu}{\beta T^3} \approx \frac{\eta^2 a^3}{\beta T_0^3}, \quad (26)$$

where $T$ is the temperature ($T_0 \sim 1$ meV today), $\eta$ is the symmetry-breaking scale of the string and $\beta$ is a phenomenological parameter counting the number of particle species interacting with the strings ($\beta \sim O(1)$). Including this term in the string equations of motion is then fairly straightforward—see [12, 34] for a derivation. One finds

$$\frac{2dL}{dt} = 2(1+v_\infty^2)HL + \frac{L}{\ell_f} v_\infty^2 + \ddot{v}_\infty + 8\Gamma \mu v_\infty^6, \quad (27)$$

$$\frac{dv_\infty}{dt} = (1-v_\infty^2) \left[ k(v_\infty) - \frac{2H + 1/\ell_f}{L} v_\infty \right]. \quad (28)$$

The epoch at which the frictional force becomes subdominant is given by

$$\frac{t_*}{t_{hag}} = \left( \frac{a}{a_{hag}} \right)^2 \approx \beta^2 \left( \frac{45}{16\pi^3 N} \right)^{1/2} \frac{1}{\Gamma \mu}, \quad (29)$$

where $N$ counts the total number of effectively massless degrees of freedom in the model (hence $N = 106.75$ for a minimal GUT model, but it can be more than an order of magnitude higher for particular extensions of it).

One can then proceed as before and look for scaling solutions for the particular case when this extra frictional term dominates over the damping due to the Hubble flow. Again we start by considering the simplest case where both loop production and gravitational back-reaction are neglected. Assuming a relativistic network $v \approx 1$, the criterion for friction dominating the velocity evolution is $\ell_f^{-1} > 2H$ which in the collapsing radiation era occurs at when the scale factor again reaches [29] or alternatively in terms of the time $t_*$ before the big crunch

$$t_c - t_* \approx (\Gamma \mu)^{-2} m_{pl}^{-1}, \quad (30)$$

or, still equivalently, the temperature $T_* \approx \eta^2 / m_{pl}$ with $a_* \approx \eta^2 / (m_{pl} T_0)$. This expectation can be confirm by solving the above evolution equations numerically, as shown in Fig. 4. For $t > t_*$ and while the string remains relativistic ($v \approx 1$), it is easy to see that exponential brakes are applied to the momentum with

$$\gamma v = \gamma_* v_* \exp \left[ \frac{1}{2} \left( 1 - \frac{a_*}{a} \right) \right]. \quad (31)$$

Even if $\gamma_* v_*$ was extremely large at $t = t_*$, the strings would soon be driven non-relativistic when the scale factor had shrunk by only the logarithmic factor $a_{nr}/a_* \approx 1/\ln(\gamma_* v_*)$.

From this point onwards, the velocity evolution becomes analogous to friction domination in flat space, with Hubble anti-damping irrelevant and the small velocities simply reflecting a balance between the curvature scale and the friction length:

$$v \approx \frac{\ell_f}{L}. \quad (32)$$

When the velocity is driven away from unity $v << 1$ at $t_{nr}$, the previous scaling law $L \propto a^2$ breaks down. Substituting (22) into the evolution equation for the correlation length, then yields the simple asymptotic solution for $a << a_{nr}$,

$$L \propto a, \quad v \propto a^2. \quad (33)$$

The string network ends in friction domination by coming to a standstill in comoving coordinates and then being genuinely conformally contracted. The energy density in the string network now behaves as $\rho_{\infty} \propto a^{-2}$, so if it has not dominated the energy density before $t_{nr}$, then it never will before the next bounce or final big crunch. This final demise for the network is expected. As the background temperature and density approaches those of the original string-forming phase transition the strings should effectively dissolve back into the high density radiation background from whence they had come.

D. Further improvements

1. Dynamical friction

We now discuss some further contributions to the dynamics of the cosmic string network in some detail, as well as a few caveats to this approach. An effect related to friction from particle scattering is dynamical friction, first discussed in [10, 15]. Cosmic string dynamics will be damped by dynamical friction, the magnitude of the effect depending both on the nature of the background fluid, the string mass per unit length and the amount of small scale structure of the cosmic string network.

For the simple case of a wiggly string oriented along the $z$ axis moving in a matter background this effect can be easily computed from the fact that in the rest frame of the
order in mass particle should be the same before and after the δv due to particles passing above and below the string. This means that the total momentum transferred to the string in the direction perpendicular to the string experienced by a matter particle is given by [8]:

\[ \delta v_y = -\frac{2\pi G(\tilde{\mu} - \tilde{T})}{v_s \gamma^2} - 4\pi G\tilde{\mu}v_s, \]  

(34)

where \( v_s \) is the string velocity with respect to the matter particles and \( \tilde{\mu} \) and \( \tilde{T} \) are the effective string mass and tension per unit length, respectively. It should be obvious that the total momentum transferred to the particles in the \( y \) direction vanishes because there is a cancellation due to particles passing above and below the string. This is not the case in the \( x \) direction where we may calculate \( \delta v_x \) from the fact that the value of the velocity of the mass particle should be the same before and after the interaction has taken place. This means that to second order in \( \delta v_y \) we have \( \delta v_x = -\frac{1}{2}(\delta v_y)^2/v_s \) so that

\[ \delta v_x = -\frac{2\pi G^2(\tilde{\mu} - \tilde{T})^2}{v_s^2 \gamma^4} - 8\pi^2 G^2 \tilde{\mu}^2 v_s - \frac{8\pi^2 G^2(\tilde{\mu} - \tilde{T})\tilde{\mu}}{v_s \gamma^2}. \]  

(35)

The total momentum transferred to a particle in the \( x \) direction in the rest frame of the string is then \( m\gamma \delta v_x \). This means that the total momentum transferred to the string in the interval of time \( dt \) in this frame is given approximately by

\[ -\delta v_x \rho v_s dt \int_{-\xi}^{\xi} dz \int_{-\xi}^{\xi} dy - \frac{3\rho \delta v_x dt}{32\pi G}, \]  

(36)

where \( \xi \sim H^{-1} \) is the string length and we have assumed the universe to be flat with background density \( \rho = 3H^2/(8\pi G) \). Making a Lorentz transformation to the rest frame of the fluid we finally obtain that the fraction, \( f \), of the string momentum, \( \rho = \tilde{\mu} \xi v_s \), lost in one Hubble time is given by

\[ f \equiv \frac{dp H^{-1}}{dt} \rho \sim \frac{3\pi G}{4} \left( \frac{(\tilde{\mu} - \tilde{T})^2}{4\tilde{\mu}^2 \gamma^4} + \tilde{\mu}v_s + \frac{(\tilde{\mu} - \tilde{T})}{v_s \gamma^2} \right). \]  

(37)

We note that this simple analysis is linear on \( \delta v_y \) which effectively means that we are assuming \( G\tilde{\mu} \ll 1 \) and \( v_s \gg G(\tilde{\mu} - \tilde{T}) \).

In the context of a collapsing universe the ultra-relativistic \( (\gamma \gg 1) \) regime is the most relevant. In this regime we have \( f \sim O(G\tilde{\mu}) \) during the matter dominated era. The effect of dynamical friction in a radiation background is less dependent on the small scale structure on the string and is simply given by [46] [47] :

\[ f \equiv -\frac{dp H^{-1}}{dt} \rho \sim \frac{3\pi G\tilde{\mu} \gamma(1 + v_s^2/3)}{32\pi}. \]  

(38)

In the ultra-relativistic regime we have \( f \sim O(G\tilde{\mu} \gamma) \) during the radiation dominated era. We note that these results correct equation typos (by a factor of \( \gamma \)) in refs. [8] [47].

Hence, we conclude that in a collapsing universe and during the matter era dynamical friction is never able to slow the strings down significantly so long as \( G\tilde{\mu} \ll 1 \). On the other hand, during the radiation era dynamical friction will have a bigger impact on string dynamics and could in principle halt the evolution of \( \gamma \) at \( \gamma \sim (G\tilde{\mu})^{-1} \). Having said this, it is important to keep in mind that this analysis assumes a homogeneous and isotropic background and so may not strictly apply in our case. Moreover, the fact that a significant amount of the momentum will be transferred from the strings to the background (specially in the radiation era regime with \( \gamma \sim (G\tilde{\mu})^{-1} \)) will in itself add to the anisotropies which naturally occur in our model.

2. A string-dominated collapsing universe

If the energy density in the cosmic string network becomes a non-negligible contribution to the overall energy density of the universe before \( t_{\text{cr}} \), then the Friedmann eqn. (46) must be modified to include the string density contribution on the right-hand side. Furthermore, it may even happen that the string network becomes the dominant contribution to the dynamics of the universe, and in that case, the scaling laws can be significantly...
modified—this type of scenario was first studied, for the case of expanding universes, in [48–9].

In our case, one finds that to zeroth order the radiation-like behaviour of the network, given by Eqns. [13, 25], is maintained. However, if one calculates (using the same method as sketched above) the first order corrections to this behaviour, one finds that they are slightly different, namely

\[ L = (L_{\text{max}} - \tilde{c} \ln a) a^2 \quad (39) \]

and

\[ 1 - v^2 \propto a^{4 + 2k(v)/\Delta}, \quad (40) \]

where we have defined \( \Delta^2 = 8\pi G \mu/3 \), that is a measure of the string energy scale.

Notice that even though the loop chopping efficiency \( \tilde{c} \) still affects the behaviour of the correlation length, it no longer affects the behaviour of the velocities (at least in an explicit way). In this case the corrections to the \( \gamma \propto a^{-2} \) scaling depend only on the momentum parameter \( k(v) \) and on \( \Delta \).

Also notice that \( \Delta \) is a very small number, e.g. \( \Delta_{\text{GUT}} \sim 3 \times 10^{-3} \) for a GUT-scale network and \( \Delta_{\text{EW}} \sim 3 \times 10^{-17} \) for an electroweak-scale network. The correction term to the Lorentz factor scaling law is still becoming less and less important as the velocity increases, since the momentum parameter is approaching zero, but the fact that \( \Delta \) is so small implies that the convergence towards the asymptotic solution is slower in this case—the more so the lighter the strings are.

3. Other effects and caveats

In the analysis in section [13–15] we have not explicitly included the effect of gravitational back-reaction. However, these solutions will still hold when this is incorporated. Indeed, in the context of the VOS model one can rigorously show [15] that although this term will clearly affect the quantitative values of the parameters in a given scaling solution, as well as the timescale needed for such solution to be reached within a given accuracy, it cannot affect the existence of such solutions.

One relevant issue is that of loop reclassification. In the expanding phase a small loop chopped off by the string network will slowly decay into gravitational radiation. However, in the collapsing phase it is possible that an initially small loop becomes a large loop (that is with a size in the center of mass (CM) frame comparable to \( H^{-1} \)) before it can lose all its energy. The size of the loop in the CM frame is given approximately by:

\[ L(t) = L_0 - \Gamma G \mu(t - t_0) \]

where \( t_0 \) is the physical time when the loop was produced, \( L_0 \) is the initial loop size, and the numerical coefficient \( \Gamma \) is independently of the loop size, but does depend on its shape. Numerical simulations have shown that \( (\Gamma) \sim 65 \).

The lifetime of the loop is then:

\[ \tau \sim \frac{L}{\Gamma G \mu}. \]

Hence, we see that for \( \tau > H^{-1} \) the loop will not have enough time to lose all its energy before the big crunch which means that loops produced with sizes greater than \( \Gamma G \mu H^{-1} \) will eventually re-join the long string network. This effectively contributes to a smaller value of \( \tilde{c} \).

A final important element not included in the above solution is the presence of small-scale ‘wiggles’ on the string network and the loops [14–15]. The momentum parameter \( k(v) \) can to some extent account for this, but only in a very simple and phenomenological way. As discussed elsewhere in this article, in the case of a contracting universe, we expect that the effect of the small-scale structures on the string dynamics should be proportionally larger than that in the case of an expanding universe.

Although one could attempt to use more elaborate analytic models to model this [10, 12, 43, 44], we shall leave this for further work, and for the moment restrict ourselves to the simple and more intuitive VOS model.

Having said that, and even considering that the model has been extensively tested and shown to be accurate, one should keep in mind that in this case it is not expected to do as well as in the previously studied cases. Apart from the issue of small-scale structures, it is also worth emphasizing that the VOS model assumes that the long string network has a Brownian distribution on large enough scales, which may not be a realistic approximation in a closed, collapsing universe. This point clearly deserves further investigation. Indeed, given the non-trivial (fractal) properties of a defect network even in the simplest linear scaling regime, it is quite interesting to ascertain what are the statistical properties of the network in a closed universe around the epoch of maximal volume. We leave this topic for future analysis.

E. The equation of state

An approximate equation of state for a cosmic string network in the relativistic limit is easy to obtain (see for example Kolb and Turner [50]):

\[ p_s = (2v_s^2 - 1) \frac{\rho_s}{3}. \quad (41) \]

This result can be obtained by taking an average over all possible directions of the energy-momentum tensor of a straight string. The generalization of this result for a domain wall network is also straightforward [50]:

\[ p_w = (v_w^2 - 2) \frac{\rho_w}{3}. \quad (42) \]

The same asymptotic limit is obtained when \( v \to 1 \) for both cosmic strings and domain walls: they behave essentially as radiation. The caveat here is that this derivation explicitly assumes a ‘perfect gas’ of strings or walls:
in other words, it assumes that there are no dissipative effects, either due to the defect motion or to defect interactions. Another minor caveat is that one should have the gas in a box much larger than the network correlation length, which will not be the case in a closed universe around maximum expansion, just like the Brownian assumption.

This result could also be obtained from energy-momentum considerations taking into account that the comoving momentum should be proportional to $a^{-1}$. For a point mass this means that $\gamma_p v_p \propto a^{-1}$, for a straight string $a \gamma_s v_s \propto a^{-1}$ and for a planar wall $a^2 \gamma_w v_w \propto a^{-1}$. This means that when $v \to 1$ we should have respectively

$$\gamma_p \propto a^{-1} \quad (43)$$

$$\gamma_s \propto a^{-2} \quad (44)$$

$$\gamma_w \propto a^{-3} \quad (45)$$

Taking into account the variation of the comoving volume in obtaining the density of each one of these objects during the collapsing phase it is straightforward to show that $\rho_{p,s,w} \propto a^{-4}$ which is just radiation-like behaviour.

IV. STRING NETWORK EVOLUTION: NUMERICAL SIMULATIONS

With the caveats discussed above in mind, we are now ready to study the numerical evolution of a cosmic string network.

A. The numerical code and basic checks

We have performed a number of very high resolution Goto-Nambu simulations on the COSMOS supercomputer, using a modified version of the Allen-Shellard string code [15].

We have simulated a simplified scenario, where a radiation or matter dominated universe is evolved in the expansion phase for a while, to allow the initial conditions of the string box to be erased away (for example, the initial string network is set up in a cubic lattice), and then reversed the sign of (the square root of) the Friedmann equation, thus forcing the universe to collapse. This setting is numerically simpler to simulate than a closed universe, or one whose collapse is induced by a negative cosmological constant, but is still good enough to allow us to test the validity of the above solutions.

In addition to the main set of simulations, we have performed a range of other control simulations to test various numerical issues and satisfy ourselves that our results had good enough numerical accuracy. Part of these control simulations will be briefly described below.

We emphasize that given the expected enhancement of small-scale structure on the strings, having very high resolution is a crucial factor, especially in what concerns the loop population. In particular, time-saving schemes like point joining are not accurate enough, even if one keeps a constant physical resolution. This is illustrated in Fig. 4 which shows the outcome of evolving the same simulation box with and without a point joining approximation. One sees that the number of loops produced is very different, and an even more dramatic discrepancy can be noticed by looking at images of the long strings in the box as the simulation evolves. We believe that the main simulations shown below have a sufficient resolution (in terms of points per correlation length) and dynamic range to provide statistically significant results.

The first point to be established is that there is a solution with the rough properties of the one described above, namely with the velocity evolving from approximately $v \sim 1/\sqrt{2}$ (corresponding to the usual scaling solution in the expanding case) to $v \sim 1$ as the universe collapses, and that such solution is stable. With this aim we have, among other tests, carried out some simulations which start collapsing after an expansion of a single time step, but which start out with $v \sim 1$. The outcome of these is illustrated in Fig. 5.

There is a very clear decrease of the velocity at early times, and an apparent tendency for an increase at late times, though it must be said that the numerical accuracy slightly deteriorates at the very end of the simulation. Notice that the decrease is much stronger for the radiation case. At the same time, the power law dependence

![FIG. 4: The same string network box is run without (solid) and with (dashed) physical resolution point joining. This crucially affects the loop production. The two panels show the fraction of the total string energy that is in the form of long strings (left), and the total number of loops in the simulation box (right), as a function of the conformal time (defined such that maximal expansion occurs at $\eta = 1$ and the big crunch at $\eta = 2$). This clearly shows that high resolution is crucial in this case.](image-url)
of the correlation length on the scale factor, $L \propto a^\beta$, is approximately constant and in the range $3/2 < \beta < 2$, except for very early in the simulation where the network hasn’t yet erased its initial conditions. Hence we can conclude that these results are consistent with the existence of an attractor solution of the type described above. Further evidence for this convergence has been observed by other means, such as starting simulations at maximal expansion with much lower velocity (say zero).

**B. Detailed scaling solutions**

With some confidence in the robustness of our methods, we can now proceed to study very high resolution simulations of the behaviour of a string network in a universe with a period of expansion followed by contraction. The result of two such simulations, for universes filled with radiation and matter, is shown in Fig. 5 and two snapshots (typical of the expansion and contraction phases) of a fraction of the simulation box in one of the runs is shown in Fig. 6.

Notice that for times up to $\eta \sim 0.5$ the network hasn’t yet erased its initial (lattice) conditions, so this period should be disregarded when considering the scaling analysis. We should also mention that for simplicity we are only plotting every twentieth time step in the simulation, and also we are not plotting the error bars associated with each data point. Depending on the quantity, these are of the order of 10%, and are typically larger at later than at earlier times.

During the expanding phase we confirm the usual linear scaling regime, namely

$$L_{\text{exp}} \propto t \propto a^2, \quad v_\infty = \text{const.}$$

in the radiation-dominated case, and

$$L_{\text{exp}} \propto t \propto a^{3/2}, \quad v_\infty = \text{const.}$$

in the matter era.

As soon as the contraction phase starts, these laws are modified. As expected, the velocity starts increasing, and the scaling of the correlation length with the scale factor also drops, being approximately constant to begin with, and then rising slowly. One can roughly identify a transient scaling phase, valid in the period $\eta \sim 1.0 - 1.4$, where one approximately has

$$L_{\text{trans}} \propto a$$

in the radiation-dominated case, and

$$L_{\text{trans}} \propto a^{5/4}$$

in the matter era. (These can not be easily recovered by analytic methods using the evolution equations for the VOS model discussed above.) Unfortunately, the extremely demanding requirements in terms of resolution of...
the simulation do not currently allow us to run simulations with longer dynamic range to establish beyond reasonable doubt whether this scaling law approaches $\beta = 2$, as predicted above. However, there are strong indications that the networks are evolving towards this asymptotic regime, as shown by the relatively rapid climb of the exponent in Fig. 6.

It is clearly noticeable that the velocity rises much faster in the matter era than in the radiation era. It is also interesting to point out that during the collapse phase the loop and long string velocities are noticeably different, and this difference (which is more significant in the radiation than in the matter case) increases with time. The plot also shows an apparent difference in this velocity ratio in the expanding phase, but this is not significant. The initial lattice conditions of our simulations do tend to give equal velocities to ‘long’ strings and small loops, but as they start evolving, the fact that the strings are on a lattice means that there can be no inter-commutings for the first few time steps, and this artificially makes the long string velocities fall behind those of the loops. Eventually, once the inter-commutings start and the network erases the ‘memory’ of its initial conditions, this velocity difference is also gradually erased.

Finally, we also notice that the network keeps chopping off loops throughout the simulation, and that there is a dramatic increase in the small scale structure of the network, particularly at later times. Visually, the string network develops large numbers of ‘knots’, highly convoluted strings regions where the wiggly long strings have collapsed inhomogeneously. These small scale features have proved to be difficult to evolve numerically, and this in fact turns out to be the main limiting factor which at present preventing us from running the simulations closer to the big crunch.

C. Contrasting the analytical and numerical approaches

We end this section by testing our solutions for the evolution of the string network in the contracting phase, obtained with the VOS model, against our numerical simulations. The outcome of these tests is summarized in Fig. 8.

We compare our solutions (19-20) for the long string correlation length and (23-24) for the long string RMS velocity with the numerical simulations described above, by plotting respectively $L/a^2$ and $(1 - v^2)/a^4$. We have assumed the usual ansatz for the momentum parameter $k(v)$—see Eqn. 17 and 18—while for the loop chopping efficiency we have assumed a constant value $\tilde{c} = 0.23$ which was obtained in previous studies of high resolution simulations in expanding universes 13 14.

We can see that, given the approximate nature of our solutions and the numerical errors in the simulations, the matching between the two is fairly remarkable. Naturally the quality of the fit will be crucially determined by our
assumptions about $\tilde{c}$ and $k(v)$, and as such one could consider this an independent test on the behaviour of these parameters. However, given the uncertainties discussed above, one cannot really meaningfully use these simulations to ‘measure’ $\tilde{c}$ directly from the simulations (with robust error bars) and test the $k(v)$ ansatz.

Among other reasons this is pertinent because, as mentioned above, there could be further corrections to the equations in the analytic model.

We note that tentative measurements of the loop chopping efficiency $\tilde{c}$ directly from the simulations (that is, without using any VOS model dependent assumptions) are consistent with no variation throughout the range probed by the simulations, though again error bars in this particular measurement are significant. This is because this measurement requires the calculation of second derivatives of quantities in the simulation, whereas all quantities plotted in this paper require, at most, the calculation of first derivatives, and are therefore much more stable numerically). However, the limited dynamical range of the simulation does not allow us to distinguish between the two discussed ansätze for the evolution of $\tilde{c}$ (namely $\tilde{c} = \text{constant}$ and $\tilde{c} \propto \frac{1}{\sigma}$).

If anything, one could use the observation that in Fig. 8 the numerical curves are steeper than the analytic ones to infer that our approximate solution is underestimating the loop production, though on the other hand this could be due to further corrections on $k(v)$ rather than to corrections on $\tilde{c}$. In any case, it is clear from direct inspection of snapshots of the simulation boxes that the amount of small-scale wiggles is gradually building up.

Be that as it may, the two extreme cases outlined above are likely to be applicable only in asymptotic regimes. The loop chopping efficiency is clearly important initially but it cannot continue to grow arbitrarily large, since the amount of energy transferred to loops (and small scale wiggles) at any time is limited by causality. On the other hand, a scenario in which loop production switches off completely may be attained ultimately during the final collapse. However, this is clearly delayed by the obvious build-up of small-scale structure on the strings. So at the moment, given the finite resolution of the simulations, the quantitative behaviour of the network is somewhat open to debate. Any more accurate modelling can only be meaningfully done in the context of a proper multi-scale string evolution model which, although possible is beyond the scope of the present work.

Finally it is also worth keeping in mind that any discussion of the evolution of a cosmic string network with the present formalism is only applicable while one is well below the Hagedorn temperature, at which the strings would ‘dissolve’ in a reverse phase transition. Discussions of asymptotic regimes should be taken with some caution, since a cosmic string network will only survive the bounce intact if this happens before the Hagedorn temperature is reached.

V. DISCUSSION: COSMOLOGICAL CONSEQUENCES

The evolution of the string energy density is dependent on the dynamics of the universe. In an expanding universe the long string energy density will evolve as

$$\rho_\infty \propto a^n$$

where $n = -4$ during the radiation-dominated era, $n = -3$ during the matter-dominated era and $n = -2$ during a curvature-dominated or accelerated expansion era. The overall density of strings remains constant relative to the background density $\bar{\rho}$ in both radiation and matter eras

$$\frac{\rho_\infty}{\bar{\rho}} = \sigma G \mu,$$  

with $\sigma_r \approx 400$ and $\sigma_m \approx 60$ respectively. During curvature domination or accelerated expansion, the string density grows relative to the other matter as $\rho_\infty/\rho_m \propto a$. For GUT-scale strings with $G \mu \sim 10^{-6}$ this gives the interesting conclusion that today strings have a comparable energy density to the cosmic microwave background radiation $\rho_{\text{cmb}}/\rho_0 \sim 10^{-4}$. However, a realistic cyclic model will continue to expand well beyond $t_0$, so the string density at maximum expansion will end up being much greater than the radiation density (even for strings considerably lighter than GUT scale). In addition, the gravitational (or other) radiation produced through the continuous decay of the string network evolves as

$$\rho_{\text{gr}} \propto a^{-4}.$$
It might appear that this contribution would become negligible during the matter era but in each Hubble time the strings lose about half their energy into gravitational radiation, so this background always remains comparable to the string density $\rho_{\text{str}} \sim \rho_{\infty}$.

A. A string-dominated universe

Now consider the collapsing phase in which the string network, like the gravitational waves they have produced, begins to behave like radiation. Globally, the density of both the strings and the gravitational waves will grow as $a^{-4}$ and, together with any other radiation components, they will eventually dominate over any nonrelativistic matter. In a realistic cyclic model reaching maximum expansion in the far distant future, sufficiently massive strings and their decay products will have a greater density than the cosmic microwave and neutrino backgrounds. As the universe contracts, then, it will eventually reach a state in which the relativistic string network and/or their gravitational waves dominate the global dynamics of the universe! This seems likely to lead to a dramatically different energy content for the universe after it emerges from the next bounce.

At this stage we should note, however, that if the gravitational radiation background becomes dominated by the longer wavelength modes then the radiation fluid approximation will eventually break down, and one then expects that this background will behave as ‘curvature’ rather than radiation—this is shown, in an the context of inflation, in 51. Obviously perturbations in such modes can’t be directly detected or have a direct impact on cosmological observables, but they can have an impact on the background in which they propagate. On the other hand, it is not yet clear if these results are directly applicable to the contracting case, and if and when that regime is reached, since in order for the long wavelength modes to become dominant one requires that loop production (and hence gravitational radiation) switches off fast enough on small (sub-horizon) scales.

Even for lighter strings which do not dominate the universe, they would end up with a much greater density in the collapsing phase than they had previously during expansion. If the universe went through a bounce, the energy density in the cosmic strings and gravitational radiation produced by the network would be much greater after the bounce than before it (though the exact amount is dependent on the model details, in particular on the duration of the matter and curvature and/or accelerated expansion era). For example, bounds on the string mass per unit length obtained in order not to overproduce a gravitational radiation background may be severely modified, in addition to the more general constraints on additional relativistic fluids 52, 53, 54.

B. An inhomogeneous universe

Furthermore, unlike the uniform CMB background, the energy density in both cosmic strings and gravitational radiation will be very inhomogeneous. In the collapsing regime, we expect that an increasingly small fraction of Hubble regions will have a string passing through them. Those that do will become string dominated since the string energy density in those regions will approximately evolve as

$$\frac{\rho_{\infty}}{\rho} \propto \gamma_{\infty} \propto a^{-2},$$

up to the corrections described in the previous section. This means that for these regions the assumption of a FRW background will cease to be valid at sufficiently late times, and the defects can make the universe very inhomogeneous 21 or even anisotropic 24. Even Hubble regions without strings can be expected to have large fluctuations in their gravitational radiation content. For sufficiently massive strings, both of these effects can be expected to survive the bounce to create large inhomogeneities in the next cycle.

We conclude that a cosmic string network will be a significant source of radiation, entropy and inhomogeneity which may be problematic in the cyclic context. An attempt to solve the problem of the overproduction of entropy and unwanted relics has been proposed in the ekpyrotic context 2. They suggest a cyclic model for the universe where an extended period of cosmic acceleration at low energies is used to remove the entropy, black holes, and other debris produced in the previous cycle. It is clear that when quantitative calculations are carried out to establish the amount of acceleration required to dilute unwanted debris, then the answer will depend on whether or not cosmic strings or other topological defects are present; a much longer period could be required if they are.

We note that something analogous happens for the case of black holes—see 24 for a discussion of this case. From cycle to cycle one expects that they will accumulate, since they will tend to have Hawking lifetimes longer than the duration of the cycle. The only ways to get rid of them are having a bouncing universe that is very close to flatness (so as to increase the duration of the cycle) or having then annihilated or torn apart at the bounce singularity.

Finally, some of the results described in this paper are also expected to be valid for other topological defects, in particular domain walls 6. We shall reserve a more detailed analysis of the evolution and implications of strings and other defects in the context of bouncing universes to a forthcoming publication.

VI. CONCLUSIONS

In this paper we have presented a first study of the basic evolutionary properties of cosmic string networks in
contracting universes, using both analytic methods and high-resolution numerical simulations. We have shown that the string network becomes ultra-relativistic, and at late times will approximately behave like a radiation fluid. We have derived new analytic scaling solutions describing this behaviour and shown, through high-resolution numerical simulations, that these analytic solutions are a good approximation to the actual string dynamics.

The main cosmological consequence of this asymmetric behaviour in the evolution of cosmic string networks in the collapsing and expanding phases is that it makes them a significant source of entropy and inhomogeneity, and therefore establishes the need for a suitable entropy dilution mechanism if they are present in a bouncing cosmological scenario. This mechanism will also operate, mutatis mutandis, for other stable topological defects. Conversely, if direct evidence is found for the presence of topological defects (with a given energy scale) in the early universe, their existence alone will impose constraints on the existence and characteristics of any previous phases of cosmological collapse.

Acknowledgments

We thank John Barrow, Kostas Dimopoulos, Ruth Durrer, Ruth Gregory and David Wands for useful discussions and comments at various stages of this work. The string evolution code used for the numerical simulations was developed by E.P.S. in collaboration with Bruce Allen. C.S. thanks the Mathematical Sciences Institute in Durham for hospitality during part of this work.

C.M. and C.S. are funded by FCT (Portugal), under grants nos. FMRH/BPD/1600/2000 and BPD/22092/99 respectively. Additional support for this project came from grant CERN/FIS/43737/2001.

This work was done in the context of the COSLAB network, and was performed on COSMOS, the Origin3800 owned by the UK Computational Cosmology Consortium, supported by SGI, HEFCE and PPARC.

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