Soft Rough q-Rung Orthopair m-Polar Fuzzy Sets and q-Rung Orthopair m-Polar Fuzzy Soft Rough Sets and Their Applications

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ABSTRACT The notion of a q-rung orthopair fuzzy soft rough set (\(q\)-ROFSRS) appeared as an extension of q-rung orthopair fuzzy set (\(q\)-ROFS) and q-rung orthopair fuzzy soft set (\(q\)-ROFSS) with the aid of rough set (RS) definition. Thus, \(q\)-ROFSRS and m-polar fuzzy set (\(m\)-PFS) are convenient to deal with uncertain knowledge which helps us to solve many problems in decision making. In this paper, we define the soft rough q-rung orthopair m-polar fuzzy sets (\(q\)-RO\(_m\)PFS) and q-rung orthopair m-polar fuzzy soft rough sets (\(q\)-RO\(_m\)PFSRS) through crisp soft and q-rung orthopair (q-RO) m-polar fuzzy soft approximation space. The related characteristics of these models are also studied. Then, we construct two new algorithms for these models to solve MADM issues. The successful application and corresponding comparative analyses proves that our proposed models are rational and effective.

INDEX TERMS q-rung orthopair fuzzy soft rough set, m-polar fuzzy set, soft rough q-rung orthopair m-polar fuzzy sets, q-rung orthopair m-polar fuzzy soft rough sets, multi-attribute decision making.

I. INTRODUCTION

The rapid of research articles become very huge, especially in mathematics. Numerous suggestions were made to solve real-world problems using mathematical techniques by way of appropriate equations or formulas in helping decision makers to make their best decisions. To solve problems involving uncertainty, fuzzy sets (FS) was introduced by Zadeh [1] in 1965.

Later in 1982, Pawlak introduced the notion called Rough Sets (RS) [2], [3]. The beauty of RS is it is able to divide the area into three parts (Lower, Upper, and Boundary region). This idea comes from the meaning of the topology concept. Eight years later, Dubois and Prade [4] combine the notion of RS and FS, to form rough fuzzy sets and fuzzy rough sets. Since then, many researchers studied further on RS and FS as in the following published articles [5]–[15].

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To reduce the uncertainty and vagueness of knowledge, Molodtsov [16] developed soft sets (SS). Feng et al. [17] established the soft rough sets (SRS) by merging SS and RS in 2011. Also, in 2017, Yager [18] defined a new concept called q-rung orthopair fuzzy sets (\(q\)-ROFS) as a refinement to the notion of Pythagorean fuzzy sets (PFS) [19], [20] and intuitionistic fuzzy sets (IFS) [21]. IFS and PFS are considered as special cases of \(q\)-ROFS, when \(q = 1\) and \(q = 2\), respectively. There are numerous research on IFS [22]–[27], PFS [28]–[36] and \(q\)-ROFS [37]–[45].

In 1994, as an extension of FS whose membership grade range is \([-1, 1]\), bipolar fuzzy sets (BFS) was proposed by Zhang [46]. In a BFS, the membership grade 0 of a variable means that the variable is irrelevant to the corresponding property, the membership grade (0, 1] of a variable points out that the variable somewhat fulfills the property, while the membership grade \([-1, 0)\) of a variable point out that the variable somewhat satisfies the implicit counter-property. The idea which lies behind such description is connected with...
the existence of “bipolar information” (e.g., plus information and minus information) about the given set. Plus information represents what is granted to be possible, while minus information represents what is considered to be impossible. Then to generalize the BFS to help experts to deal with uncertainty, the meaning of m-polar fuzzy sets ($m$PS) was mooted by Chen et al. [47]. They proved that bipolar fuzzy sets and 2-polar fuzzy sets are cryptomorphic mathematical tools. In many real-life complicated problems, data sometimes comes from an employee ($n \geq 2$), that is, multipolar information (not just bipolar information, which corresponds to two-valued logic) exists. There are many applications of m-polar fuzzy sets to decision-making problems when it is compulsory to make assessments with a group of agreements. Akram et al. [48], [49] proposed the soft rough m-polar fuzzy and m-polar fuzzy soft rough sets. By merging the concepts of SRS, PFS, and $m$PS, Riaz and Hashmi [50] investigated the Pythagorean m-polar fuzzy sets ($P_m$PFS), soft rough Pythagorean m-polar fuzzy sets ($SR_{m}$PFS) and Pythagorean m-polar fuzzy soft rough sets ($P_{m}$PFSRS). The concept of q-rung orthopair m-polar fuzzy sets ($q^m$ROFmPFS) was then defined by Riaz et al. [51].

Using the notions of SS, SRS, and $q$ROFS, Hussain et al. [52] proposed the q-rung orthopair fuzzy soft sets and their application. Wang et al. [53] explained the $q$ROF soft rough sets($q$ROFSRS) with a few applications. Riaz et al. [54] introduced the notion of soft rough q-rung orthopair fuzzy sets and some of their properties were discussed. Thereafter, many researchers studied SRS, SS, and their applications such as [55]–[60], [64].

From these interesting studies, we intend to develop a hybrid of SRS and $q$ROFmPFS and put forward a new model called q-rung orthopair m-polar fuzzy soft rough sets ($q^m$ROF$m$PFSRS) and soft rough q-rung orthopair fuzzy sets ($SR^q$mROF$m$PFS). These combinations provide us with the property of $q^m$ROF$m$PFS and soft rough sets together which maximize the handling of uncertain data. Thus our proposed methods are generalized extensions of Akram et al. [48], Riaz and Hashmi [50] and Riaz et al. [54]. When $q = 1$, the presented formula reduces to those methods in [48] and [49] and if $q = 2$, it reduces to those methods in [50]. Our proposed method will cater for $m$ sets which make our studies are reliable, compared to [54] which catered for only a single set. Their relevant properties will be investigated, a few definitions and theorems will be promulgated along with illustrative examples. We will then proceed to construct two algorithms along with their applications. Finally, we will run comparative analyses on the outcomes of those two algorithms.

The structure of this paper is as follows. The preliminary of basic notions will be introduced in Section 2. Section 3 will discuss the novel concept of $SR^q$mROF$m$PFS and the related characteristics. The hybrid concept of $SR^q$mROF$m$PFSRS will be proposed and its associated properties are discussed in Section 4. In Section 5, we will give an illustrative example to show the applicability of the proposed constructed algorithms along with the comparative analyses, followed by the conclusion in Section 6.

II. PRELIMINARIES

Now, we give some basic notions on IFS, PFS and $q$ROF before defining soft rough q-rung orthopair m-polar fuzzy sets $SR^q$mROF$m$PFS in the next section.

Definition 1 ([21], [22]): If $\mathcal{E}$ is the origin set. For every $\mathcal{H} \in \mathbb{E}$, we have a membership grade $\theta_{\mathcal{E}} : \mathbb{E} \rightarrow [0, 1]$ and a non-membership grade $\nu_{\mathcal{E}} : \mathbb{E} \rightarrow [0, 1]$. Define the IFS $\mathcal{E}$ as indicated below.

$$\mathcal{E} = ((\mathcal{H}, \theta_{\mathcal{E}}(\mathcal{H}), \nu_{\mathcal{E}}(\mathcal{H})))$$

where $0 \leq \theta_{\mathcal{E}}(\mathcal{H}) + \nu_{\mathcal{E}}(\mathcal{H}) \leq 1$.

Also, $\mathcal{H} = (\theta_{\mathcal{H}}, \nu_{\mathcal{H}})$ is said to be an intuitionistic fuzzy number (IFN), if

$$0 \leq \theta_{\mathcal{H}}, \nu_{\mathcal{H}} \leq 1, \theta_{\mathcal{H}} + \nu_{\mathcal{H}} = 1 - \theta_{\mathcal{H}} - \nu_{\mathcal{H}}.$$

To treat some problem in IFS which appeared in real issues, Yager in 2014 defined Pythagorean fuzzy sets (PFS) as indicated below.

$$\mathcal{E} = ((\mathcal{H}, \theta_{\mathcal{E}}(\mathcal{H}), \nu_{\mathcal{E}}(\mathcal{H})))$$

where $0 \leq \theta_{\mathcal{E}}(\mathcal{H}) + \sqrt{\nu_{\mathcal{E}}(\mathcal{H})} \leq 1$.

Also, $\mathcal{H} = (\theta_{\mathcal{H}}, \nu_{\mathcal{H}})$ is said to be a Pythagorean fuzzy number (PFN), if

$$0 \leq \theta_{\mathcal{H}} + \sqrt{\nu_{\mathcal{H}}} \leq 1,$$

Generalizing further, Yager presented the notion of q-rung orthopair fuzzy sets in 2017 (q-ROFN), if $q \geq 2$, the following relation.

$$\mathcal{E} = ((\mathcal{H}, \theta_{\mathcal{E}}(\mathcal{H}), \nu_{\mathcal{E}}(\mathcal{H})))$$

where $0 \leq \theta_{\mathcal{E}}(\mathcal{H}) + \sqrt{\nu_{\mathcal{E}}(\mathcal{H})} \leq 1$, where $\nabla \geq 1$.

Also, $\mathcal{H} = (\theta_{\mathcal{H}}, \nu_{\mathcal{H}})$ is said to be a q-ROF number (q-ROFN), if

$$\theta_{\mathcal{H}} + \sqrt{\nu_{\mathcal{H}}} \leq \nu_{\mathcal{H}} \leq 1.$$

Definition 2 ([19], [20]): If $\mathcal{E}$ is the origin set. For every $\mathcal{H} \in \mathbb{E}$, if we have a membership grade $\theta_{\mathcal{E}} : \mathbb{E} \rightarrow [0, 1]$ and a non-membership grade $\nu_{\mathcal{E}} : \mathbb{E} \rightarrow [0, 1]$. Define the PFS $\mathcal{E}$ as indicated below.

$$\mathcal{E} = ((\mathcal{H}, \theta_{\mathcal{E}}(\mathcal{H}), \nu_{\mathcal{E}}(\mathcal{H})))$$

where $0 \leq \theta_{\mathcal{E}}(\mathcal{H}) + \nu_{\mathcal{E}}(\mathcal{H}) \leq 1$, where $\nabla \geq 1$.

Also, $\mathcal{H} = (\theta_{\mathcal{H}}, \nu_{\mathcal{H}})$ is said to be a q-ROFN number (q-ROFN), if

$$\theta_{\mathcal{H}} + \sqrt{\nu_{\mathcal{H}}} \leq \nu_{\mathcal{H}} \leq 1.$$

Definition 3 [18]: If $\mathcal{E}$ is the origin set. For every $\mathcal{H} \in \mathbb{E}$, if we have a membership grade $\theta_{\mathcal{E}} : \mathbb{E} \rightarrow [0, 1]$ and a non-membership grade $\nu_{\mathcal{E}} : \mathbb{E} \rightarrow [0, 1]$. Define the q-ROFs $\mathcal{E}$ as indicated below.

$$\mathcal{E} = ((\mathcal{H}, \theta_{\mathcal{E}}(\mathcal{H}), \nu_{\mathcal{E}}(\mathcal{H})))$$

where $0 \leq \theta_{\mathcal{E}}(\mathcal{H}) + \sqrt{\nu_{\mathcal{E}}(\mathcal{H})} \leq 1$, where $\nabla \geq 1$.

Also, $\mathcal{H} = (\theta_{\mathcal{H}}, \nu_{\mathcal{H}})$ is said to be a q-ROFN (q-ROFN), if

$$\theta_{\mathcal{H}} + \sqrt{\nu_{\mathcal{H}}} \leq \nu_{\mathcal{H}} \leq 1.$$
Next, Chen et al. [47] defined m-polynomial fuzzy sets as follows.

Definition 6 [47]: If $\Xi$ is the origin set, where $\phi: \Xi \to [0, 1]^m$ is the set of all m-polynomial fuzzy sets on $\Xi$.

Riaz and Hashmi [50] extended it to a Pythagorean form below.

Definition 7 [50]: If $\hat{E} = (\hat{\theta}, \hat{x})$ is a q-ROFNC, then we have the following.

$$\nabla \hat{E} = (\hat{\theta}, (1 - (\hat{x})^2)^{\frac{1}{2}})$$

$$\hat{E} = (\hat{x}, (1 - (\hat{x})^2)^{\frac{1}{2}})$$

where $0 \leq (\hat{x})^2 + (\hat{x})^2 \leq 1$, where $r = 1, 2, ..., m$.

Riaz et al. [51] further extended m-polynomial fuzzy sets of Chen et al. [47] to q-rung orthopair fuzzy form below.

Definition 8 [51]: If $\Xi$ is the origin set. For every $\hat{H} \in \Xi$, if we have a membership grade $\hat{\theta}_c : \Xi \to [0, 1]$ and a non-membership grade $\hat{x_c} : \Xi \to [0, 1]$. Define the Pythagorean m-polynomial fuzzy sets (P$_m$PFS) $E$ as indicated below.

$$E = \{(\hat{H}, \hat{\theta}_c(\hat{H}), \hat{x_c}(\hat{H}))\}$$

where $0 \leq (\hat{\theta}_c(\hat{H}))^2 + (\hat{x_c}(\hat{H}))^2 \leq 1$, where $r = 1, 2, ..., m$ and $\nabla \geq 1$.

Definition 9 [51]: If $\hat{E}_1 = (\hat{\theta}_1, \hat{x}_1)$ and $\hat{E}_2 = (\hat{\theta}_2, \hat{x}_2)$, for $\hat{E}_1, \hat{E}_2$ is qROFNC. Then $\forall \hat{H} \in \Xi$, we have the following relation.

(1) $\hat{E}_1 = \hat{E}_2$ $\iff$ $(\hat{\theta}_1, \hat{x}_1) = (\hat{\theta}_2, \hat{x}_2)$. Where $\hat{\theta}_1$ and $\hat{\theta}_2$ are $\forall \hat{H} \in \Xi$.

(2) $\hat{E}_1 = \hat{E}_2$ $\iff$ $\hat{\theta}_1 = \hat{\theta}_2$ and $\hat{x}_1 = \hat{x}_2$.

(3) $\hat{E}_1 \leq \hat{E}_2$ $\iff$ $\hat{\theta}_1 \leq \hat{\theta}_2$ and $\hat{x}_1 \leq \hat{x}_2$.

(4) $\hat{E}_1 \cap \hat{E}_2 = (\hat{\theta}_1, \hat{x}_1)$, $\hat{\theta}_1 \wedge \hat{x}_1$.

(5) $\hat{E}_1 \cup (\hat{\theta}_1, \hat{x}_1)$, $\hat{\theta}_1 \vee \hat{x}_1$.

(6) $\hat{E}_1 - \hat{E}_2 = \hat{E}_1 \cap \hat{E}_2$.

(7) $\hat{E}_1 \oplus \hat{E}_2 = (\hat{\theta}_1 \oplus \hat{\theta}_2, \hat{x}_1 \oplus \hat{x}_2)$.

(8) $\hat{E}_1 \otimes \hat{E}_2 = (\hat{\theta}_1 \otimes \hat{\theta}_2(\hat{H}), \sqrt{(\hat{\theta}_1(\hat{H}))^2 + (\hat{\theta}_2(\hat{H}))^2 - (\hat{\theta}_1(\hat{H})\hat{\theta}_2(\hat{H}))^2}, \hat{x}_1, \hat{x}_2)$. Where $\hat{\theta}_1, \hat{\theta}_2, \hat{x}_1, \hat{x}_2$ is a q-ROFNC, then we have the following.

where $0 \leq \hat{\theta}_1(\hat{H})^2 + \hat{\theta}_2(\hat{H})^2 \leq 1$, where $\hat{\theta} \geq 1$.

Also, $\hat{E} = (\hat{\theta}, \hat{x})$ is said to be a q-ROFNC number (q-ROFNSN), if

$$\hat{E} = (\check{\theta}, (1 - (\hat{x})^2)^{\frac{1}{2}})$$

where $0 \leq (\hat{x})^2 + (\hat{x})^2 \leq 1$, where $r = 1, 2, ..., m$.

Wang et al. [53] defined qROF from blow.

Definition 12 [53]: If $\Xi$ is the origin set. For every $\hat{H} \in \Xi$, let $(\check{\theta}, \check{x})$ be a qROFNC. Then for $\forall \hat{E} \subseteq \Xi \times J$, is qROFNN relation is defined as follows.

$$\hat{E} = (\check{\theta}, (\hat{x})^2)$$

where $0 \leq \hat{\theta}_1(\hat{H})^2 + \hat{\theta}_2(\hat{H})^2 \leq 1$, where $\hat{\theta} \geq 1$.

Also, $\hat{E}$ is said to be a q-ROFSR number (q-ROFSRNN), if

$$\hat{E} = (\check{\theta}, (\hat{x})^2)$$

where $0 \leq (\hat{x})^2 + (\hat{x})^2 \leq 1$, where $\hat{\theta} \geq 1$.

In this section, we will define and illustrate the notion of soft rough q-rung orthopair m-polynomial fuzzy sets $S_{RO}RO_{m}F_{PS}$ and also discuss their relevant properties.

Definition 13: If $\Xi$ is the origin set, $\check{f}$ is the provisory features, and $\sigma$ is the crisp soft relation, then $(\Xi, \check{f}, \sigma)$ is a CSAS. For any $\hat{E} \in \check{3}RO_{m}F_{PS}(\check{f})$, the soft rough $\check{3}RO_{m}F_{PS}$-lower and soft rough $\check{3}RO_{m}F_{PS}$-upper approximations (SR$\check{3}RO_{m}F_{PSL}$A, SR$\check{3}RO_{m}F_{PSU}$), which are denoted by $\check{H}$ and $\check{H}$, respectively, are as follows.

$$\check{H}(\hat{E}) = \{(\hat{\theta}, \check{\theta}(\hat{E})) \subseteq \check{f}, \exists \theta \in (\check{\theta}(\hat{E}))\}$$

$$\check{H}(\hat{E}) = \{(\hat{\theta}, \check{\theta}(\hat{E})) \subseteq \check{f}, \exists \theta \in (\check{\theta}(\hat{E}))\}$$

where $\hat{H} \in \Xi$ and $\nabla = 1, 2, ..., n$. If $\check{H}(\hat{E}) \neq \check{H}(\hat{E})$, then $\hat{E}$ is a soft rough q-rung orthopair m-polynomial fuzzy sets, otherwise, it is definable.

Example 1: If $\Xi = \{\hat{H}_1, \hat{H}_2, \hat{H}_3, \hat{H}_4, \hat{H}_5\}$ is the origin set and $\check{f} = \{f_1, f_2, f_3, f_4\}$ is the features set. Suppose that

$$\check{H}(\hat{E}) = \{(\hat{\theta}, \check{\theta}(\hat{E})) \subseteq \check{f}, \exists \theta \in (\check{\theta}(\hat{E}))\}$$

where $\hat{H} \in \Xi$ and $\nabla = 1, 2, ..., n$. If $\check{H}(\hat{E}) \neq \check{H}(\hat{E})$, then $\hat{E}$ is a soft rough q-rung orthopair m-polynomial fuzzy sets, otherwise, it is definable.
Thus the relation is as follows

\[ S(f_1) = \{ H_1, H_2, H_3 \}, S(f_2) = \{ H_2, H_4, H_3 \}, S(f_3) = \{ H_1, H_2, H_4 \}, S(f_4) = \{ H_1, H_4, H_3 \}. \]

(2) Since \( \hat{H} \subseteq \mathcal{H}_1 \), so from Definition 13, we have

\[ H(\hat{H}) = \{ (\hat{X}, \bigwedge_{\theta \in \sigma(\hat{X})} (\theta^p_{\hat{H}}(\hat{X})), \bigvee_{\theta \in \sigma(\hat{X})} (\theta^s_{\hat{H}}(\hat{X})) \} \]

(3) \( H(\mathcal{H} \cap \hat{H}) = \{ (\hat{X}, \bigwedge_{\theta \in \sigma(\hat{X})} (\theta^p_{\mathcal{H}}(\hat{X}) \cap \theta^p_{\hat{H}}(\hat{X})), \bigvee_{\theta \in \sigma(\hat{X})} (\theta^s_{\mathcal{H}}(\hat{X}) \cap \theta^s_{\hat{H}}(\hat{X})) \} \]

(4) \( H(\mathcal{H} \cup \hat{H}) = \{ (\hat{X}, \bigwedge_{\theta \in \sigma(\hat{X})} (\theta^p_{\mathcal{H}}(\hat{X}) \cup \theta^p_{\hat{H}}(\hat{X})), \bigvee_{\theta \in \sigma(\hat{X})} (\theta^s_{\mathcal{H}}(\hat{X}) \cup \theta^s_{\hat{H}}(\hat{X})) \} \]

The proofs of (1’) - (4’) can be similarly proven as those proofs of (1) - (4).

IV. q-RUNG ORTHOPAIR m-POLAR FUZZY SOFT ROUGH SETS

Below, we construct the concept of q-rung orthopair m-polar fuzzy soft rough sets \( qRO_mPFSRS \), and will discuss their properties. Henceforth, the notions of \( I, J \) and \( (I, J) \)-cut sets will be proposed and their characteristics will be put forward.

\[ \text{Definition 14: Suppose } \Xi \text{ is the origin set and } \mathcal{F} \text{ is the provisory features for some } \hat{\mathcal{E}} \subseteq \Xi. \text{ If we have a mapping } \mu : \hat{\mathcal{E}} \rightarrow qRO_mPFSRS, \text{ then } (\mu, \hat{\mathcal{E}}) \text{ is called q-rung orthopair m-polar fuzzy sets } (qRO_mPFS), \text{ where } qRO_mPFS(\Xi) \text{ is the set of all q-rung orthopair m-polar fuzzy subsets of the origin set } \Xi. \]

\[ \text{Definition 15: If } (\mu, \hat{\mathcal{E}}) \text{ is a } qRO_mPFFS, \text{ then a q-rung orthopair m-polar fuzzy subset } v \text{ of } \Xi \times \mathcal{F} \text{ is called a q-rung orthopair m-polar fuzzy soft relation as below.} \]

\[ v = \{ ((\rho, \tau), \theta^q_v(\rho, \tau), \chi^q_v(\rho, \tau)) : (\rho, \tau) \in \Xi \times \mathcal{F}, \forall V = 1, 2, \ldots, n \}, \]

where \( \theta^q_v(\rho, \tau), \chi^q_v(\rho, \tau) \in [0, 1] \) are the membership and non-membership scale, respectively, under the term of

\[ 0 \leq \theta^q_v(\rho, \tau) + \chi^q_v(\rho, \tau) \leq 1. \]

This relation can be viewed as the following, \( v \), as shown at the bottom of the next page.
Definition 16: If $\Xi$ is the origin set, $\hat{f}$ is the provisory features, and $v$ is the $9RO_mPSFRS$ relation, then $(\Xi, \hat{f}, v)$ is a $9RO_mPSFS$-approximation space. For any $\hat{\mathcal{E}} \in 9RO_mPSFS(\hat{f})$, the $9RO_mPFS$ soft rough-lower and $9RO_mPFS$ soft rough-upper approximations, which are denoted by $\mathcal{L}(\hat{\mathcal{E}})$ and $\mathcal{T}(\hat{\mathcal{E}})$, respectively, are as follows.

$$\mathcal{L}(\hat{\mathcal{E}}) = \{ (\hat{\mathcal{X}}, \bigcap_{\sigma \in \mathcal{H}(\hat{\mathcal{X}})} (1 - \theta^l_{\nu}(\rho, \tau) \lor \theta^l_{\nu}(\tau, \rho)), \bigcup_{\sigma \in \mathcal{H}(\hat{\mathcal{X}})} (\theta^l_{\nu}(\rho, \tau) \land \chi^l_{\mathcal{E}}(\tau)) \},$$

$$\mathcal{T}(\hat{\mathcal{E}}) = \{ (\hat{\mathcal{X}}, \bigcup_{\sigma \in \mathcal{H}(\hat{\mathcal{X}})} (\theta^l_{\nu}(\rho, \tau) \land \theta^l_{\nu}(\tau, \rho)), \bigcap_{\sigma \in \mathcal{H}(\hat{\mathcal{X}})} (1 - \theta^l_{\nu}(\rho, \tau) \lor \chi^l_{\mathcal{E}}(\tau)) \},$$

where $\hat{\mathcal{X}} \in \Xi$ and $\mathcal{V} = 1, 2, \ldots, n$. If $\mathcal{L}(\hat{\mathcal{E}}) \neq \mathcal{T}(\hat{\mathcal{E}})$, then $\hat{\mathcal{E}}$ is a $9RO_mPFS$-approximation space. Otherwise, it is definable.

Example 2: If $\Xi = \{ \hat{\mathcal{F}}_1, \hat{\mathcal{F}}_2 \}$ is the origin set and $\hat{f} = \{ f_1, f_2, f_3 \}$ is the features set. Suppose that the q-rung orthopair m-polar fuzzy soft relation $v : \Xi \rightarrow \hat{f}$ as set on the bottom of the page.

Suppose we have $\hat{\mathcal{E}} \in 9RO_mPFS(\hat{f})$ such that

$$\hat{\mathcal{E}} = \{ (f_1, (0.718, 0.318), (0.618, 0.118), (0.513, 0.213)), (f_2, (0.813, 0.518), (0.313, 0.513), (0.418, 0.713)), (f_3, (0.413, 0.318), (0.618, 0.412), (0.713, 0.312)) \},$$

Hence, we count the lower and upper approximations as below.

$$\mathcal{L}(\hat{\mathcal{E}}) = \{ (\hat{\mathcal{X}}_1, (0.413, 0.518), (0.381, 0.513), (0.513, 0.451)), (\hat{\mathcal{X}}_2, (0.482, 0.518), (0.487, 0.513), (0.418, 0.618)) \},$$

and

$$\mathcal{T}(\hat{\mathcal{E}}) = \{ (\hat{\mathcal{X}}_1, (0.718, 0.382), (0.519, 0.481), (0.513, 0.282)), (\hat{\mathcal{X}}_2, (0.718, 0.318), (0.618, 0.181), (0.617, 0.282)) \}.$$

Theorem 2: Let $(\Xi, \hat{f}, v)$ is a $9RO_mPFS$-approximation space. For every $\hat{\mathcal{E}}, \hat{\mathcal{E}}_1 \in \Xi$, then the next conditions hold.

1. $\mathcal{L}(\hat{\mathcal{E}}) = (\mathcal{L}(\hat{\mathcal{E}}))^c$.
2. If $\hat{\mathcal{E}} \subseteq \hat{\mathcal{E}}_1$, then $\mathcal{L}(\hat{\mathcal{E}}) \subseteq \mathcal{L}(\hat{\mathcal{E}}_1)$.
3. $\mathcal{L}(\hat{\mathcal{E}} \cap \hat{\mathcal{E}}_1) = \mathcal{L}(\hat{\mathcal{E}}) \cap \mathcal{L}(\hat{\mathcal{E}}_1)$.
4. $\mathcal{L}(\hat{\mathcal{E}} \cup \hat{\mathcal{E}}_1) \supseteq \mathcal{L}(\hat{\mathcal{E}}) \cup \mathcal{L}(\hat{\mathcal{E}}_1)$.
5. $\mathcal{L}(\hat{\mathcal{E}}) \subseteq \mathcal{E} \subseteq \mathcal{T}(\hat{\mathcal{E}})$.
6. $\mathcal{T}(\hat{\mathcal{E}}) = (\mathcal{T}(\hat{\mathcal{E}}))^c$.
7. If $\hat{\mathcal{E}} \subseteq \hat{\mathcal{E}}_1$, then $\mathcal{T}(\hat{\mathcal{E}}) \subseteq \mathcal{T}(\hat{\mathcal{E}}_1)$.
8. $\mathcal{L}(\hat{\mathcal{E}} \cap \hat{\mathcal{E}}_1) = \mathcal{L}(\hat{\mathcal{E}}) \cap \mathcal{L}(\hat{\mathcal{E}}_1)$.
9. $\mathcal{T}(\hat{\mathcal{E}} \cup \hat{\mathcal{E}}_1) = \mathcal{T}(\hat{\mathcal{E}}) \cup \mathcal{T}(\hat{\mathcal{E}}_1)$.

Proof: (1) From Definition 16, we have the following formulas.

$$\mathcal{L}(\hat{\mathcal{E}})^c = \{ (\hat{\mathcal{X}}, \bigcap_{\sigma \in \mathcal{H}(\hat{\mathcal{X}})} (1 - \theta^l_{\nu}(\rho, \tau) \lor \theta^l_{\nu}(\tau, \rho)), \bigcup_{\sigma \in \mathcal{H}(\hat{\mathcal{X}})} (\theta^l_{\nu}(\rho, \tau) \land \chi^l_{\mathcal{E}}(\tau)) \}.$$

(2) Since $\hat{\mathcal{E}} \subseteq \hat{\mathcal{E}}_1$, we have $\mathcal{L}(\hat{\mathcal{E}}) = \mathcal{L}(\hat{\mathcal{E}}_1)$.

(3) $\mathcal{L}(\hat{\mathcal{E}} \cap \hat{\mathcal{E}}_1) = \mathcal{L}(\hat{\mathcal{E}}) \cap \mathcal{L}(\hat{\mathcal{E}}_1)$.

(4) $\mathcal{L}(\hat{\mathcal{E}} \cup \hat{\mathcal{E}}_1) = \mathcal{L}(\hat{\mathcal{E}}) \cup \mathcal{L}(\hat{\mathcal{E}}_1)$.
Proof: The proofs are trivial.

Proposition 2: If we have \( \hat{\mathcal{E}}_1, \hat{\mathcal{E}}_2 \) and \( \hat{\mathcal{E}}_3 \) is \( \mathcal{QRO}_m\mathcal{PFSRS} \), then the following characteristics hold.

(1) \( \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 = \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 \).

(2) \( \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 = \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 \).

(3) \( \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 = \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 \).

(4) \( \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 = \hat{\mathcal{E}}_1 \cup \hat{\mathcal{E}}_2 \).

Example 4: If we have \( \hat{\mathcal{E}} = \{(\mathcal{H}_1, (0.531, 0.222), (0.412, 0.204), (0.555, 0.301), (0.156, 0.870)), (\mathcal{H}_2, (0.831, 0.231), (0.732, 0.444), (0.830, 0.010), (0.812, 0.110), (\mathcal{H}_3, (0.766, 0.244), (0.456, 0.140), (0.571, 0.473), (0.611, 0.142)), (\mathcal{H}_4, (0.514, 0.345), (0.819, 0.009), (0.700, 0.227), (0.153, 0.625), (\mathcal{H}_5, (0.712, 0.106), (0.513, 0.300), (0.729, 0.115), (0.822, 0.200), (\mathcal{H}_6, (0.632, 0.301), (1, 0), (0.768, 0.072), (0, 1))\} \) of \( \mathcal{QRO}_4\mathcal{PFSRS} \) through \( \mathcal{Z} \), then the following outcomes hold.

(1) \( \hat{\mathcal{E}} \oplus \hat{\mathcal{D}} = \{(\mathcal{H}_1, (0.687, 0.076), (0.852, 0.002), (0.804, 0.068), (0.217, 0.544)), (\mathcal{H}_2, (0.921, 0.025), (0.816, 0.133), (0.924, 0.001), (0.943, 0.022)), (\mathcal{H}_3, (0.867, 0.073), (1, 0), (0.850, 0.034), (0.611, 0.142)), (\mathcal{H}_4, (0.592, 0.238), (0.376, 0.482), (0.605, 0.115), (0.667, 0.210)), (\mathcal{H}_5, (0.484, 0.346), (0.456, 0.140), (0.439, 0.474), (0, 1))\} \).
Proof:

(1) \( \hat{E}_1 \oplus \hat{E}_2 \equiv (\hat{E}_1 \oplus \hat{E}_2) \cap (\hat{E}_1 \oplus \hat{E}_3) \).

(2) \( \hat{E}_1 \oplus (\hat{E}_2 \vee \hat{E}_3) = (\hat{E}_1 \oplus \hat{E}_2) \vee (\hat{E}_1 \oplus \hat{E}_3). \)

(3) \( \hat{E}_1 \oplus (\hat{E}_2 \vee \hat{E}_3) = (\hat{E}_1 \oplus \hat{E}_2) \vee (\hat{E}_1 \oplus \hat{E}_3). \)

(4) \( \hat{E}_1 \oplus (\hat{E}_2 \vee \hat{E}_3) = (\hat{E}_1 \oplus \hat{E}_2) \oplus (\hat{E}_1 \oplus \hat{E}_3). \)

(2) The proof is similar to the proof of (1).

Definition 18: If we have \( \hat{E} = \{ (\theta_1, x_1), (\theta_2, x_2), \ldots, (\theta_m, x_m) \} \) is \( R_m \text{PFN} \), then we define the assort (\( A \)) and accuracy (\( R \)) functions of \( \hat{E} \) as follows.

\[
A(\hat{E}) = \frac{1}{2m} \sum_{i=1}^{m} (\theta \vee x)
\]

\[
R(\hat{E}) = \frac{1}{m} \sum_{i=1}^{m} (\theta \vee x)
\]

Definition 19: If we have two \( R_m \text{PFN} \) \( \hat{E}_1 = \{ (\theta_1, x_1), (\theta_2, x_2), \ldots, (\theta_m, x_m) \} \), \( \hat{E}_2 = \{ (\theta_1', x_1'), (\theta_2', x_2'), \ldots, (\theta_m', x_m') \} \), then the following hold.

(1) If \( R(\hat{E}_1) > R(\hat{E}_2) \), then \( \hat{E}_1 > \hat{E}_2 \).

(2) If \( R(\hat{E}_1) = R(\hat{E}_2) \) and \( R(\hat{E}_1) > R(\hat{E}_2) \), then \( \hat{E}_1 > \hat{E}_2 \).

Definition 20: If \( \hat{E} = (\theta, x') \) is a \( R_m \text{PFNSRS} \), then we have the following.

\[
\square \hat{E} = (\theta \vee 1 - (\theta \vee x'))
\]

\[
\Diamond \hat{E} = (x' \vee 1 - (x' \vee \theta))
\]

Proposition 5: If we have \( \hat{E} \) is \( R_m \text{PFNSRS through} \) \( \Sigma \) and \( \hat{H} \in \Sigma \), then the following characteristics hold.

(1) \( \square \hat{E} = \square \hat{E} \).

(2) \( \diamond \hat{E} = \diamond \hat{E} \).

(3) \( \square \hat{E} = \square \hat{E} \).

(4) \( \square \hat{E} = \square \hat{E} \).

(5) \( \square \hat{E} = \square \hat{E} \).

(6) \( \diamond \hat{E} = \square \hat{E} \).

Proof:

(1) \( \hat{E}_1 \oplus \hat{E}_2 \equiv (\hat{E}_1 \oplus \hat{E}_2) \cap (\hat{E}_1 \oplus \hat{E}_3) \).

(2) \( \hat{E}_1 \oplus \hat{E}_2 \equiv (\hat{E}_1 \oplus \hat{E}_2) \vee (\hat{E}_1 \oplus \hat{E}_3). \)

(3) \( \hat{E}_1 \oplus (\hat{E}_2 \vee \hat{E}_3) = (\hat{E}_1 \oplus \hat{E}_2) \vee (\hat{E}_1 \oplus \hat{E}_3). \)

(4) \( \hat{E}_1 \oplus (\hat{E}_2 \vee \hat{E}_3) = (\hat{E}_1 \oplus \hat{E}_2) \oplus (\hat{E}_1 \oplus \hat{E}_3). \)
Proposition 7: If we have 

\[ \Box (e_1 \lor e_2) = \Box (e_1 \lor e_2) \]

Proof: The proofs follow from Propositions 5 and 6.

Proposition 8: If we have \( \dot{e}_1 \) and \( \dot{e}_2 \) are \( \text{RO}_m \)PFSRS through \( \mathbb{E} \) and \( \mathcal{H} \in \mathbb{E} \), then the following characteristics hold.

\[ \Box (\dot{e}_1 \lor \dot{e}_2) = \Box (\dot{e}_1 \lor \dot{e}_2) \]

Proof: The proofs follow from Proposition 5.

Proposition 9: If we have \( \dot{e}_1 \) and \( \dot{e}_2 \) are \( \text{RO}_m \)PFSRS through \( \mathbb{E} \) and \( \mathcal{H} \in \mathbb{E} \), then the following characteristics hold.

\[ \Box (\dot{e}_1 \lor \dot{e}_2) = \Box (\dot{e}_1 \lor \dot{e}_2) \]

Proof: The proofs follow from Proposition 5, 7 and 8.

B. \( (I, J) \)-CUT SETS

Definition 21: If \( \dot{e} \in \text{RO}_m \)PFSRS and \( I \in [0, 1] \), then the \( I \)-cut for \( \dot{e} \) is defined as,

\[ \dot{e}_I = \{ \mathcal{H} \in \dot{e} : \mathcal{H} \geq I \} \]

and is called a strong (robust) \( I \)-cut if

\[ \dot{e}_I^c = \{ \mathcal{H} \in \dot{e} : \mathcal{H} > I \} \]

Example 5: From Example 4, if \( I = 0.456 \), we get the next values. \( \dot{e}_{0.456} = \{ \dot{e}_2, \dot{e}_3 \} \) and \( \dot{e}_{0.456}^c = \{ \dot{e}_2 \} \)

Proposition 10: If we have \( \dot{e}, \dot{d} \) are \( \text{RO}_m \)PFSRS through \( \mathbb{E} \) and \( I \in [0, 1] \), then the following characteristics hold.

\[ \Box (\dot{e} \lor \dot{d}) = \Box (\dot{e} \lor \dot{d}) \]

Proof: The proofs follow from Definition 21.

(3) Since \( \dot{e} \lor \dot{d} = \{ \mathcal{H} : \mathcal{H} \geq \mathcal{H}, \mathcal{H} \geq \mathcal{H}, \mathcal{H} \lor \mathcal{H} \} \), and \( \dot{e}^c = \{ \mathcal{H} : \mathcal{H} \lor \mathcal{H} \} \). Hence, \( \dot{e}_I = \{ \mathcal{H} : \mathcal{H} \geq I \} \) and \( \dot{e}_I^c = \{ \mathcal{H} : \mathcal{H} > I \} \). Thus \( \dot{e}_I^c = \{ \mathcal{H} : \mathcal{H} > I \} \).

So, (\( \dot{e} \lor \dot{d} \)) \( = \{ \mathcal{H} : \mathcal{H} \geq \mathcal{H}, \mathcal{H} \geq \mathcal{H}, \mathcal{H} \lor \mathcal{H} \} \) and \( \dot{e}_I = \{ \mathcal{H} : \mathcal{H} \geq I \} \).

(4) Since \( \dot{e} \lor \dot{d} = \{ \mathcal{H} : \mathcal{H} \geq \mathcal{H}, \mathcal{H} \lor \mathcal{H} \} \), and \( \dot{e}_I = \{ \mathcal{H} : \mathcal{H} \geq I \} \).

(6) Since \( \dot{e} \lor \dot{d} = \{ \mathcal{H} : \mathcal{H} \geq \mathcal{H}, \mathcal{H} \lor \mathcal{H} \} \), and \( \dot{e}_I = \{ \mathcal{H} : \mathcal{H} \geq I \} \).

(7) \( \Box (\dot{e}_1 \lor \dot{e}_2) = \Box (\dot{e}_1 \lor \dot{e}_2) \).

(8) \( \Box (\dot{e}_1 \land \dot{e}_2) = \Box (\dot{e}_1 \land \dot{e}_2) \).
So, \((\hat{E} \lor D)_I = \{\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}) \geq I\} = \{\hat{H}, \vartheta_E^I(\hat{H}) \geq I\} \lor \{\hat{H}, \vartheta_D^I(\hat{H}) \geq I\}\}

(5) Since \(\hat{E} \land D = \{(\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}), x_E^I(\hat{H}) \lor x_D^I(\hat{H})\}\).

So, \((\hat{E} \land D)_I^V = \{\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}) > I\} = \{\hat{H}, \vartheta_E^I(\hat{H}) > I\} \land \{\hat{H}, \vartheta_D^I(\hat{H}) > I\}\}

(6) Since \(\hat{E} \lor D = \{(\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}), x_E^I(\hat{H}) \lor x_D^I(\hat{H})\}\).

So, \((\hat{E} \lor D)_I = \{\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}) \geq I\} = \{\hat{H}, \vartheta_E^I(\hat{H}) \geq I\} \lor \{\hat{H}, \vartheta_D^I(\hat{H}) \geq I\}\}

Definition 22: If \(\hat{E} \in \mathcal{RO}_mPFSRS\) and \(J \in [0, 1]\), then the \(J\)-cut for \(\hat{E}\) is defined as,

\(\hat{E}_J = \{\hat{H} \in \xi : x_E^I(\hat{H}) \leq J\}\)

and is called a strong (robust) \(J\)-cut if \(\hat{E}_J = \{\hat{H} \in \xi : x_E^I(\hat{H}) < J\}\).

Example 6: From Example 4, if \(J = 0.140\), we get the next values. \(\hat{E}_{0.145} = \{\hat{H}_1, \hat{H}_3\}\) and \(\hat{E}_{0.145}^J = \{\hat{H}_1\}\). We consider \(\hat{E}_J \leq \hat{E}_J^P\). Then \(\hat{E}_J = \{\hat{H} \in \xi, \vartheta_E^I(\hat{H}) \leq J\} \lor \{\hat{H} \in \xi, \vartheta_E^I(\hat{H}) < J\}\). Thus \(\hat{E}_J^P = \{\hat{H} : \vartheta_E^I(\hat{H}) \leq J\}\).

(2) Follows from Definition 22.

(3) As \(\hat{E} \land D = \{(\hat{H}, \vartheta_E^I(\hat{H}) \land \vartheta_D^I(\hat{H}), x_E^I(\hat{H}) \land x_D^I(\hat{H})\}\).

So, \((\hat{E} \land D)_J = \{\hat{H}, x_E^I(\hat{H}) \land x_D^I(\hat{H}) \leq J\} = \{\hat{H}, \vartheta_E^I(\hat{H}) \leq J\} \land \{\hat{H}, \vartheta_D^I(\hat{H}) \leq J\}\)

(4) As \(\hat{E} \lor D = \{(\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}), x_E^I(\hat{H}) \lor x_D^I(\hat{H})\}\).

So, \((\hat{E} \lor D)_J = \{\hat{H}, x_E^I(\hat{H}) \lor x_D^I(\hat{H}) \leq J\} = \{\hat{H}, \vartheta_E^I(\hat{H}) \leq J\} \lor \{\hat{H}, \vartheta_D^I(\hat{H}) \leq J\}\)

(5) As \(\hat{E} \land D = \{(\hat{H}, \vartheta_E^I(\hat{H}) \land \vartheta_D^I(\hat{H}), x_E^I(\hat{H}) \lor x_D^I(\hat{H})\}\).

So, \((\hat{E} \land D)_J = \{\hat{H}, x_E^I(\hat{H}) \land x_D^I(\hat{H}) < J\} = \{\hat{H}, \vartheta_E^I(\hat{H}) < J\} \lor \{\hat{H}, \vartheta_D^I(\hat{H}) < J\}\)

(6) As \(\hat{E} \lor D = \{(\hat{H}, \vartheta_E^I(\hat{H}) \lor \vartheta_D^I(\hat{H}), x_E^I(\hat{H}) \lor x_D^I(\hat{H})\}\).

So, \((\hat{E} \lor D)_J = \{\hat{H}, x_E^I(\hat{H}) \lor x_D^I(\hat{H}) < J\} = \{\hat{H}, \vartheta_E^I(\hat{H}) < J\} \lor \{\hat{H}, \vartheta_D^I(\hat{H}) < J\}\)

Proposition 12: If we have \(\hat{E}, \hat{D} \in \mathcal{RO}_mPFSRS\) through \(\Xi\) and \(J \in [0, 1]\), then the following characteristics hold.

(1) \(\hat{E}_J = \hat{E}_I \land \hat{E}_J^P = \hat{E}_I \land \hat{E}_J^P\)

(2) \(\hat{E} \leq \hat{D} \iff \hat{E}_J \leq \hat{D}_J\)

(3) \(\hat{E} \lor \hat{D})_I(\hat{E}_J \lor \hat{D}_J\)

(4) \(\hat{E} \lor \hat{D}\) \lor \hat{D}_J(\hat{E}_J \lor \hat{D}_J\)

(5) If \(I_1 \geq I_2\) and \(J_1 \leq J_2\), then \(\hat{E}_{I_1} \leq \hat{E}_{I_2}, \hat{E}_{J_1} \leq \hat{E}_{J_2}\) and \(\hat{E}_{(I_1, J_1)} \leq \hat{E}_{(I_2, J_2)}\)

Proposition 21: If \(\hat{E} = \hat{E}_I \land \hat{E}_J\), then \(\hat{E} \lor \hat{D} \leq \hat{E}_I \lor \hat{D}\), \(\hat{E}_J \lor \hat{D}_J\)

(4) As \(\hat{E} \leq \hat{E} \lor \hat{D}\) and \(\hat{D} \leq \hat{E}_I \lor \hat{D}\), then from (2), we have \(\hat{E}_{I_1} \leq \hat{E}_I \lor \hat{D}\) and \(\hat{E}_{(I_1, J_1)} \leq \hat{E}_{I_1} \lor \hat{D}_{(I_1, J_1)}\), \(\hat{E}_J \lor \hat{D}_J\)

(5) Follows from Definitions 21, 22, and 23, and the property (1) of Proposition 4.28.

V. APPLICATIONS

Here, we construct two algorithms to solve MCDM issues via soft rough q-rungh orthopair m-polar fuzzy sets (SRmROqPFSRS) and q-RO m-polar fuzzy soft rough sets (SRmROqPFSR).
These algorithms will aid managers to make decisions using our proposed models via the lower and upper approximations.

A. DESCRIPTION
Let $\Xi = \{\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_t\}$ be $t$ number of computer programmers and $f = \{f_1, f_2, \ldots, f_r\}$ be $r$ features required of these programmers by the institution which placed the advertisement. The institution establishes several criteria to best choose desirable candidates with the following features: Communication Skill $f_1$, Personality $f_2$, Experience $f_3$, Self-Dependability $f_4$. We will build a crisp soft relation for the first method $\sigma$ over $\Xi \times f$ and q-RO m-polar fuzzy soft relation for the second method $\nu : \Xi \rightarrow f$. Therefore, through the proposed methods $SR^4RO_mFPS$ and $qRO_mPFSRS$, we introduce the following two subsections to aid with the managerial decision.

B. $SR^4RO_mFPS$ APPROACH
The following steps in Algorithm 1 establishes our new approach using the q-ROF m-polar fuzzy sets and crisp soft approximation space.

Algorithm 1 Algorithm for $SR^4RO_mFPS$

**Input:** $\Xi$ is the origin set and $f$ is the provisory features.

**Output:** Decision Making.

1. Investigate the crisp soft relation $\sigma$ based on the data provided.
2. Establish $\hat{\mathcal{E}} \in 4RO_mFPS(f)$.
3. Compute $\mathcal{K}(\hat{\mathcal{E}})$ ($SR^4RO_mFPSLA$) and $\mathcal{K}(\hat{\mathcal{E}})$ ($SR^4RO_mFPSLA$).
4. Compute $\mathcal{K}(\hat{\mathcal{E}}) \oplus \mathcal{K}(\hat{\mathcal{E}})$ from Definition 17.
5. Compute the consequence of all features in $\mathcal{K}(\hat{\mathcal{E}}) \oplus \mathcal{K}(\hat{\mathcal{E}})$ from Definition 18.
6. Assort the features by Definition 19.
7. Obtain the decision.

Now, we give the following illustrated example of the proposed approach.

Suppose $\Xi = \{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4, \mathcal{H}_5\}$ is the origin set of candidates and $f = \{f_1, f_2, f_3, f_4\}$ is the features set. Thus the relation is as follows

$\sigma = (\mathcal{H}_1, f_2), (\mathcal{H}_2, f_3), (\mathcal{H}_3, f_1), (\mathcal{H}_4, f_2), (\mathcal{H}_5, f_4)$.

Hence, we have the following results.

$S(\mathcal{H}_1) = \{f_2, f_3\}$, $S(\mathcal{H}_2) = \{f_1, f_4\}$, $S(\mathcal{H}_3) = \{f_1, f_4\}$, $S(\mathcal{H}_4) = \{f_1, f_4\}$, $S(\mathcal{H}_5) = \{f_2, f_4\}$.

Then we set the q-RO 3-polar fuzzy subsets of $\Xi$ as follows.

$\hat{\mathcal{E}} = \{\{ f_1, (0.67, 0.21), (0.71, 0.28), (0.78, 0.31)\},$ 
$\{ f_2, (0.81, 0.21), (0.73, 0.31), (0.69, 0.18)\},$ 
$\{ f_3, (0.89, 0.12), (0.78, 0.31), (0.74, 0.44)\},$ 
$\{ f_4, (0.81, 0.38), (0.67, 0.17), (0.65, 0.44)\}\}.$

Through these data, we can now compute the lower and upper approximations of $\hat{\mathcal{E}}$ as follows.

$\mathcal{K}(\hat{\mathcal{E}}) = \{\mathcal{H}_1, (0.81, 0.21), (0.73, 0.31), (0.69, 0.44),$ 
$\mathcal{H}_2, (0.67, 0.38), (0.67, 0.31), (0.65, 0.31),$ 
$\mathcal{H}_3, (0.67, 0.38), (0.67, 0.28), (0.65, 0.31)\},$ 
$\mathcal{K}(\hat{\mathcal{E}}) = \{\mathcal{H}_1, (0.81, 0.38), (0.67, 0.17), (0.65, 0.16),$ 
$\mathcal{H}_2, (0.81, 0.38), (0.67, 0.31), (0.65, 0.18)\}.$
Finally, we rank the alternatives as follows.

If $\nabla = 1$.

$$\hat{\mathcal{H}}_1 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_2 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_4.$$ 

If $\nabla = 2$.

$$\hat{\mathcal{H}}_1 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_2 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_4.$$ 

If $\nabla = 3$.

$$\hat{\mathcal{H}}_1 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_2 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_4.$$ 

If $\nabla = 5$.

$$\hat{\mathcal{H}}_1 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_2 > \hat{\mathcal{H}}_3 > \hat{\mathcal{H}}_4.$$ 

### C. $9RO_m$PFSRS APPROACH

The following steps in Algorithm 2 establishes our new approach using the q-ROF m-polar fuzzy soft rough sets and crisp soft approximation space.

Now, we give the following illustrated example using the proposed approach.

Presume that $\mathcal{Z} = \{\hat{\mathcal{H}}_1, \hat{\mathcal{H}}_2, \hat{\mathcal{H}}_3, \hat{\mathcal{H}}_4, \hat{\mathcal{H}}_5\}$ is the origin set and $f = \{f_1, f_2, f_3, f_4\}$ is the features set.

Hence, we have the q-RO 3-polar fuzzy soft relation as in the following matrix.

$$\nu = \left( \begin{array}{cccc}
\hat{\mathcal{H}}_1 & \hat{\mathcal{H}}_2 & \hat{\mathcal{H}}_3 & \hat{\mathcal{H}}_4 \\
 f_1 & f_2 & f_3 & f_4 \\
\end{array} \right)$$

Then we set the q-RO 3-polar fuzzy subsets of $\mathcal{Z}$ as follows.

$$\hat{\mathcal{E}} = \{(f_1, (0.67, 0.38)), (0.73, 0.31), (0.81, 0.31))\}, (f_2, (0.86, 0.18), (0.75, 0.41), (0.73, 0.21))\),

$$\hat{\mathcal{H}}_2 = \{(f_1, (0.91, 0.15), (0.83, 0.41), (0.81, 0.51))\),

$$\hat{\mathcal{H}}_3 = \{(f_1, (0.85, 0.41), (0.73, 0.35), (0.69, 0.23))\),

$$\hat{\mathcal{H}}_4 = \{(f_1, (0.73, 0.41), (0.71, 0.37), (0.83, 0.41))\),

$$\hat{\mathcal{H}}_5 = \{(f_1, (0.82, 0.43), (0.78, 0.31), (0.68, 0.23))\),

$$\hat{\mathcal{H}}_6 = \{(f_1, (0.81, 0.31), (0.78, 0.41), (0.72, 0.18))\),

$$\hat{\mathcal{H}}_7 = \{(f_1, (0.79, 0.53), (0.68, 0.46), (0.67, 0.51))\),

$$\hat{\mathcal{H}}_8 = \{(f_1, (0.71, 0.51), (0.69, 0.41), (0.76, 0.51))\),

$$\hat{\mathcal{H}}_9 = \{(f_1, (0.82, 0.52), (0.69, 0.41), (0.63, 0.28))\),

$$\hat{\mathcal{H}}_{10} = \{(f_1, (0.85, 0.41), (0.71, 0.51), (0.73, 0.11))\),

$$\hat{\mathcal{H}}_{11} = \{(f_1, (0.75, 0.18), (0.67, 0.41), (0.63, 0.43))\),

$$\hat{\mathcal{H}}_{12} = \{(f_1, (0.73, 0.31), (0.75, 0.13), (0.78, 0.32))\),

$$\hat{\mathcal{H}}_{13} = \{(f_1, (0.85, 0.13), (0.71, 0.11), (0.68, 0.28))\),

$$\hat{\mathcal{H}}_{14} = \{(f_1, (0.86, 0.23), (0.68, 0.51), (0.69, 0.19))\),

$$\hat{\mathcal{H}}_{15} = \{(f_1, (0.78, 0.17), (0.63, 0.31), (0.61, 0.38))\),

$$\hat{\mathcal{H}}_{16} = \{(f_1, (0.73, 0.13), (0.81, 0.21), (0.85, 0.16))\),

$$\hat{\mathcal{H}}_{17} = \{(f_1, (0.89, 0.11), (0.81, 0.31), (0.78, 0.21))\),

$$\hat{\mathcal{H}}_{18} = \{(f_1, (0.96, 0.12), (0.86, 0.21), (0.83, 0.31))\),

$$\hat{\mathcal{H}}_{19} = \{(f_1, (0.87, 0.36), (0.76, 0.26), (0.74, 0.14))\),

Using these information, we can now compute the lower and upper approximations of $\hat{\mathcal{E}}$ as follows.

$$\mathcal{L}(\hat{\mathcal{E}}) = \{(\hat{\mathcal{H}}_1, (0.81, 0.38)), (0.73, 0.31), (0.69, 0.44))\),$$

$$\mathcal{U}(\hat{\mathcal{E}}) = \{(\hat{\mathcal{H}}_2, (0.67, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_3, (0.67, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_4, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_5, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_6, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_7, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_8, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_9, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{10}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{11}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{12}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{13}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{14}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{15}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{16}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{17}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{18}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\), (\hat{\mathcal{H}}_{19}, (0.81, 0.38)), (0.67, 0.31), (0.65, 0.44))\),
TABLE 1. Table for scores using different $\nabla$ for $SR^4RO_mPFS$.

| Different approaches | Obtain a decision |
|----------------------|-------------------|
| $\mathcal{H}_1$      | $\mathcal{H}_2$  |
| $\mathcal{H}_3$      | $\mathcal{H}_4$  |
| $\mathcal{H}_5$      | $\mathcal{H}_6$  |
| $\mathcal{H}_7$      | $\mathcal{H}_8$  |
| $\mathcal{H}_9$      | $\mathcal{H}_10$ |

$V = 1$, $0.9317$, $0.933$,
$V = 2$, $0.923$, $0.932$,
$V = 3$, $0.933$, $0.932$,
$V = 4$, $0.932$, $0.931$,
$V = 5$, $0.931$, $0.932$,

Then, we compute the order of each variable as next.
If $V = 1$,
$\mathcal{R}(\mathcal{H}_1) = 0.9317$, $\mathcal{R}(\mathcal{H}_2) = 0.9122$, $\mathcal{R}(\mathcal{H}_3) = 0.9125$, $\mathcal{R}(\mathcal{H}_4) = 0.9095$, $\mathcal{R}(\mathcal{H}_5) = 0.925$.

If $V = 2$,
$\mathcal{R}(\mathcal{H}_1) = 0.9155$, $\mathcal{R}(\mathcal{H}_2) = 0.891$, $\mathcal{R}(\mathcal{H}_3) = 0.892$, $\mathcal{R}(\mathcal{H}_4) = 0.8855$, $\mathcal{R}(\mathcal{H}_5) = 0.9032$.

If $V = 3$,
$\mathcal{R}(\mathcal{H}_1) = 0.9043$, $\mathcal{R}(\mathcal{H}_2) = 0.8769$, $\mathcal{R}(\mathcal{H}_3) = 0.8778$, $\mathcal{R}(\mathcal{H}_4) = 0.8742$, $\mathcal{R}(\mathcal{H}_5) = 0.8893$.

If $V = 5$,
$\mathcal{R}(\mathcal{H}_1) = 0.89$, $\mathcal{R}(\mathcal{H}_2) = 0.8604$, $\mathcal{R}(\mathcal{H}_3) = 0.8599$, $\mathcal{R}(\mathcal{H}_4) = 0.8528$, $\mathcal{R}(\mathcal{H}_5) = 0.872$.

Finally, we rank the alternatives as follows.
If $V = 1$,
$\mathcal{H}_1 > \mathcal{H}_3 > \mathcal{H}_5 > \mathcal{H}_2 > \mathcal{H}_4$.

If $V = 2$,
$\mathcal{H}_1 > \mathcal{H}_3 > \mathcal{H}_5 > \mathcal{H}_2 > \mathcal{H}_4$.

If $V = 3$,
$\mathcal{H}_1 > \mathcal{H}_3 > \mathcal{H}_5 > \mathcal{H}_2 > \mathcal{H}_4$.

If $V = 5$,
$\mathcal{H}_1 > \mathcal{H}_3 > \mathcal{H}_5 > \mathcal{H}_2 > \mathcal{H}_4$.

D. COMPARATIVE ANALYSES

In this section, we will explain the merits of the proposed methods by comparisons between ours, that is, $SR^4RO_mPFS$ and $\sigma RO_mPFSRS$, and the previous methods, that is, soft rough m-polar fuzzy sets and m-polar fuzzy soft rough sets by Akram et al. [48], soft rough Pythagorean fuzzy set and Pythagorean fuzzy soft rough set by Riaz and Hashmi [50] and soft rough q-rung orthopair fuzzy sets and q-rung orthopair fuzzy soft rough sets by Riaz et al. [54]. The novel approaches to solve MADM issues can be seen as illustrated in Tables 1 and 2.

Table 1 shows the ordering outcomes for different $\nabla$ (i.e., Akram et al. [48], Riaz and Hashmi [50] and our proposed methods) for $SR^4RO_mPFS$. The best selection of the proposed different approaches is by hiring programmer $\mathcal{H}_1$. This means that our model is reliable and rational.

Table 2 shows the ordering outcomes for different $\nabla$ (i.e., Akram et al. [48], Riaz and Hashmi [50] and our proposed methods) for $\sigma RO_mPFSRS$. The best selection of the proposed different approaches is by hiring programmer $\mathcal{H}_1$. This means that our model is reasonable and effective.

We can also show the differences between different $\nabla$ (i.e., Akram et al. [48], Riaz and Hashmi [50] and our proposed methods) using the following two figures, Figure 1 and Figure 2.

Figure 1 illustrates the comparisons on the outcomes for $\nabla = 1, 2, 3, 5$ for $SR^4RO_mPFS$, which means that the $\mathcal{H}_1$ alternative is the best choice for this institution under the given requirements.

Figure 2 illustrates the comparisons on the outcomes for $\nabla = 1, 2, 3, 5$ for $\sigma RO_mPFSRS$, which means that the $\mathcal{H}_1$
alternative is the best choice for this institution under the given requirements.

Figure 2 illustrates the comparisons on the outcomes for $V = 1, 2, 3, 5$ (i.e., Akram et al. [48], Riaz and Hashmi [50] and our proposed methods) for $\mathcal{RO}_m\text{PFSRS}$, which means that the $\mathcal{RO}_m$ alternative is the best choice for this institution under the given requirements. Note that the data used here cannot be processed by the methods of Riaz et al. [54] which can only handle a single set. Hence, our proposed methods have overcome the hurdle of set limitations of the previous existing methods of Akram et al. [48], Riaz and Hashmi [50] and Riaz et al. [54].

VI. CONCLUSION

We have constructed new algorithms using soft rough q-RO m-polar fuzzy sets ($\mathcal{RO}_m\text{PFSRS}$) and q-RO m-polar fuzzy soft rough sets ($\mathcal{RO}_m\text{PFSRS}$) to provide us with novel approaches to help make a decision on managerial problems. These new models proved their effectiveness and reliability, as can be seen in Tables 1 and 2, and displayed on Figures 1 and 2. The characteristics related to these models have also been discussed. We have established two different groups of steps for these new models according to the crisp soft and q-RO m-polar fuzzy soft approximation space to solve MADM problems. The comparative analyses indicated that the proposed approaches yield consistent results. In future, we shall extend the proposed methods to a variety of other environments such as the T-spherical power Muirhead operators [62], multi-objective programming [64], neurogenetics [65] and polynomial zeros [66]–[68].

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