Quantum Dynamical $\tilde{R}$-Matrix with Spectral Parameter from Fusion

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Abstract

A quantum dynamical $\tilde{R}$-matrix with spectral parameter is constructed by fusion procedure. This spin-1 $\tilde{R}$-matrix is connected with Lie algebra $so(3)$ and does not satisfy the condition of translation invariance.

1 Introduction

Since the classical dynamical $r$-matrix \cite{[1]} first appeared on the scene of integrable many body system, many dynamical $r$-matrices have been found in integrable models such as Calogero-Moser model \cite{[2]}, Sine-Gorden soliton case \cite{[3]} and the general case for the Ruijsenaars systems \cite{[4]}. These dynamical $r$-matrices do not satisfy the ordinary classical Yang-Baxter equation, so its quantization is rather nontrivial. The quantum dynamical Yang-Baxter (QDYB) equation, which appeared first in the quantization of Toda field theory \cite{[5]} and later in the quantization of KZB equation \cite{[6]}, had been studied widely for various integrable models and its algebraic structure was explored \cite{[7],[8],[9]}

In contrast to the non-dynamical one \cite{[10]}, only a few dynamical $R$ matrices are constructed explicitly and most of them can be obtained from Felder’s solution \cite{[6]} by taking a gauge transformations \cite{[9]}. So how to construct new $R$ matrix is still an interesting and challenging problem. As an efficient method to obtain higher-spin $R$ matrix, fusion procedure \cite{[11]} has been applied to dynamical $R$ matrix \cite{[12]}. In this paper, we construct a spin-1 quantum dynamical $R$-matrix with spectral parameter by “fusing” together the spin-$\frac{1}{2}$ $R$-matrices which satisfy the QDYB equation \cite{[7]}:

\[
R_{12}(\lambda_{12}, x + \gamma h^{(3)})R_{13}(\lambda_{13}, x - \gamma h^{(2)})R_{23}(\lambda_{23}, x + \gamma h^{(1)}) = R_{23}(\lambda_{23}, x - \gamma h^{(1)})R_{13}(\lambda_{13}, x + \gamma h^{(2)})R_{12}(\lambda_{12}, x - \gamma h^{(3)}).
\] (1)

Where the spectral parameters $\lambda_{ij}$ are defined as $\lambda_{ij} = \lambda_i - \lambda_j$, $x = \sum \nu x_{\nu} h_{\nu}$ is the dynamical variable and $h$ is the Cartan subalgebra of the underlying simple Lie algebra. Taking values in $\text{End}(V_1 \otimes V_2 \otimes V_3)$, $R$ matrix appears as $R_{12}(x + \gamma h^{(3)})(V_1 \otimes V_2 \otimes V_3) = (R_{12}(x + \gamma \mu)(V_1 \otimes V_2)) \otimes V_3$ if $h^{(3)}$ has weight $\mu$ in space $V_3$. Other symbols have a similar meaning.

In braid form, the QDYB equation (1) reads as

\[
\tilde{R}_{23}(\lambda_{12}, x + \gamma h^{(1)})\tilde{R}_{12}(\lambda_{13}, x - \gamma h^{(2)})\tilde{R}_{23}(\lambda_{23}, x + \gamma h^{(1)}) = \tilde{R}_{12}(\lambda_{23}, x - \gamma h^{(1)})\tilde{R}_{23}(\lambda_{13}, x + \gamma h^{(2)})\tilde{R}_{12}(\lambda_{12}, x - \gamma h^{(3)}),
\] (2)

where $\tilde{R}_{ij} = P_{ij} R_{ij}$ and $P_{ij}$ is the permutation operator acting on spaces $V_i \otimes V_j$. If $\tilde{R}$
matrices satisfy the condition of translation invariance:

\[
[D^{(i)} + D^{(j)}, \tilde{R}_{ij}(\lambda, x)] = 0; \quad D^{(i)} = \sum_\nu h^{(i)}_\nu \partial_{x^\nu},
\]

(3)

we can rewrite equation (2) as

\[
\tilde{R}_{23}(\lambda_{12}, x + 2\gamma h^{(1)})\tilde{R}_{12}(\lambda_{13}, x)\tilde{R}_{23}(\lambda_{23}, x + 2\gamma h^{(1)})
\]

\[
= \tilde{R}_{12}(\lambda_{23}, x)\tilde{R}_{23}(\lambda_{13}, x + 2\gamma h^{(1)})\tilde{R}_{12}(\lambda_{12}, x).
\]

(4)

This paper is organized as follows. In section 2, we obtain some useful properties of \(\tilde{R}^{(1/2, 1/2)}\) matrix. In section 3, using the \(\tilde{R}^{(1, 1)}\) matrix, we construct the \(\tilde{R}^{(1, 1)}\) matrix by fusion procedure and prove that the new matrix satisfies the QDYB equation too. Finally, we discuss our results and compare it with paper [12] in section 4.

2 Properties of spin- \(1/2\) \(\tilde{R}\)-matrix

According to spin- \(1/2\) chain, \(h^{(i)}(\otimes V_i) = \text{diag}\{\frac{1}{2}, -\frac{1}{2}\}(\otimes V_i)\), there is the simplest \(\tilde{R}\) matrix solution with spectral parameter [1]:

\[
\tilde{R}^{(1/2, 1/2)}(\lambda, x) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{\sinh \gamma \sinh (x + \lambda)}{\sinh x \sinh (\lambda - \gamma)} & \frac{\sinh \lambda \sinh (x + \gamma)}{\sinh x \sinh (\lambda - \gamma)} & 0 \\
0 & \frac{\sinh \lambda \sinh (x - \gamma)}{\sinh x \sinh (\lambda - \gamma)} & \frac{\sinh \gamma \sinh (x - \gamma)}{\sinh x \sinh (\lambda - \gamma)} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

(5)

This \(\tilde{R}^{(1/2, 1/2)}\) matrix satisfies the "weight zero" condition

\[
[h^{(i)} + h^{(j)}, \tilde{R}_{ij}(\lambda, x)] = 0,
\]

(6)

and it has one triple eigenvalue 1 and one single eigenvalue \(-\frac{\sinh(\lambda + \gamma)}{\sinh(\lambda - \gamma)}\).

To the triple eigenvalue, its right-acting eigenvectors are

\[
u^{(1)}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad u^{(0)}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}; \quad u^{(-1)}(x) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},
\]

(7)

and its left-acting eigenvectors are

\[
\overline{\nu}^{(1)}(x) = (1, 0, 0, 0) \\
\overline{\nu}^{(0)}(x) = \frac{1}{\sqrt{2}}(0, \sinh(x - \gamma), \sinh(x + \gamma), \sinh x \cosh \gamma) \\
\overline{\nu}^{(-1)}(x) = (0, 0, 0, 1).
\]

(8)

While the eigenvalue is \(-\frac{\sinh(\lambda + \gamma)}{\sinh(\lambda - \gamma)}\), the right-acting and left-acting eigenvectors are

\[
v^{(0)}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \frac{\sinh(x + \gamma)}{\sinh x \cosh \gamma} \\ -\frac{\sinh(x - \gamma)}{\sinh x \cosh \gamma} \\ 0 \end{pmatrix}; \quad \overline{v}^{(0)}(x) = \frac{1}{\sqrt{2}}(0, 1, -1, 0)
\]

(9)
respectively.
These eigenvectors satisfy
\[
\begin{align*}
\mathbf{\pi}^{(a)}(x)v_{(a)}(x) &= \mathbf{\pi}^{(0)}(x)u_{(a)}(x) = 0, & a = 1, 0, -1 \\
\mathbf{\pi}^{(0)}(x)v_{(0)}(x) &= 1; & \mathbf{\pi}^{(a)}(x)u_{(b)}(x) = \delta_{ab}, & a, b = 1, 0, -1
\end{align*}
\]
so we can construct two projection operators for the triplet and singlet
\[
\begin{align*}
P(x) &= \sum_{a} u_{(a)}(x)\mathbf{\pi}^{(a)}(x); & Q(x) &= v_{(0)}(x)\mathbf{\pi}^{(0)}(x) \\
id_{4(4)} &= P(x) + Q(x),
\end{align*}
\]
in which \(id_{4(4)} = \text{diag}\{1, 1, 1, 1\}\), \(P(x)\) and \(Q(x)\) have the properties:
\[
\begin{align*}
P^2(x) &= P(x); & Q^2(x) &= Q(x); & P(x)Q(x) = Q(x)P(x) = 0 \\
P(x)u_{(a)}(x) &= u_{(a)}(x), & \mathbf{\pi}^{(a)}(x)P(x) &= \mathbf{\pi}^{(a)}(x); & a = 1, 0, -1.
\end{align*}
\]
Now, we can rewrite \(\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x)\) as
\[
\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x) = P(x) - \frac{\sinh(\lambda + \gamma)}{\sinh(\lambda - \gamma)}Q(x).
\]
It is obvious that
\[
\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda = -\gamma, x) = P(x). \tag{12}
\]
Applying this property to equation (2), we obtain
\[
\begin{align*}
P_{23}(x + \gamma h^{(1)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x - \gamma h^{(3)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x + \gamma h^{(1)})
\end{align*}
\]
\[
\left.\begin{align*}
&= \tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x - \gamma h^{(3)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x + \gamma h^{(1)})P_{12}(x - \gamma h^{(3)}) \\
&= P_{12}(x - \gamma h^{(3)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x + \gamma h^{(1)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}(\lambda, x - \gamma h^{(3)}) \tag{13}
\end{align*}\right.
\]
\section{Construct spin- 1 \(\tilde{R}\)-matrix}

Refer to fusion procedures in papers [1], [2], we "fuse" dynamical \(\tilde{R}^{(1,1)}\) matrix with spectral parameter as follows
\[
\left[\tilde{R}^{(1,1)}_{12,34}(\lambda, x)\right]_{cd}^{ab} = \\
\mathbf{\pi}_{12}^{(a)}(x - \gamma h^{(3,4)})\mathbf{\pi}_{34}^{(b)}(x + \gamma h^{(1,2)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}_{23}(\lambda, x + \gamma h^{(1)} - \gamma h^{(4)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}_{12}(\lambda, x - \gamma h^{(3,4)})
\times\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}_{34}(\lambda, x + \gamma h^{(1,2)})\tilde{R}^{(\frac{1}{2}, \frac{1}{2})}_{23}(\lambda, x + \gamma h^{(1)} - \gamma h^{(4)})u_{12(c)}(x - \gamma h^{(3,4)})u_{34(d)}(x + \gamma h^{(1,2)}) \tag{14}
\]
in which \(a, b, c, d\) take values among 1, 0, -1 and \(h^{(i,j)}\) means \(h^{(i)} + h^{(j)}\), so this \(\tilde{R}^{(1,1)}\) matrix is a 9 \(\times\) 9 matrix.

In order to prove that equation (14) satisfies QDYB equation too, we define two \(4 \times 4\) matrices as follows:
\[
u = (u_{(1)}, u_{(0)}, 0, u_{(-1)}); \quad \mathbf{\pi} = \begin{pmatrix}
\mathbf{\pi}^{(1)} \\
\mathbf{\pi}^{(0)} \\
0 \\
\mathbf{\pi}^{(-1)}
\end{pmatrix}.
\]
then, we replace \( \varpi^{(a)} \) and \( \varpi^{(b)} \) by \( \varpi \) as well as replacing \( u(c) \) and \( u(d) \) by \( u \) in equation (14), such that \( \tilde{R}^{(1,1)} \) is changed into a 16 \( \times \) 16 matrix, where the added seven rows and seven columns are nothing but zero in fact. Such \( u \) and \( \varpi \) matrices not only keep \( u(x)\varpi(x) = P(x) \), \( P(x)u(x) = u(x) \) and \( \varpi(x)P(x) = \varpi(x) \), but also satisfy the weight zero condition too. Now the QDYB equation becomes

\[
\tilde{R}^{(1,1)}_{34,56}(\lambda_{12}, x + \gamma h^{(1,2)}) \tilde{R}^{(1,1)}_{12,34}(\lambda_{13}, x - \gamma h^{(5,6)}) \tilde{R}^{(1,1)}_{34,56}(\lambda_{23}, x + \gamma h^{(1,2)}) \\
= \tilde{R}^{(1,1)}_{12,34}(\lambda_{23}, x - \gamma h^{(5,6)}) \tilde{R}^{(1,1)}_{34,56}(\lambda_{13}, x + \gamma h^{(1,2)}) \tilde{R}^{(1,1)}_{12,34}(\lambda_{12}, x - \gamma h^{(5,6)}).
\]

(15)

For simplicity, we introduce \( \tilde{R}_{ij}(\lambda) \) := \( \tilde{R}^{(1,1)}_{ij}(\lambda, x + \gamma \sum_{k=1}^{i-1} h^{(k)} - \gamma \sum_{l=j+1}^{6} h^{(l)}) \), and replace \( u_{ij}(x + \gamma \sum_{k=1}^{i-1} h^{(k)} - \gamma \sum_{l=j+1}^{6} h^{(l)}) \) and \( \varpi_{ij}(x + \gamma \sum_{k=1}^{i-1} h^{(k)} - \gamma \sum_{l=j+1}^{6} h^{(l)}) \) by \( \cap_{ij} \) and \( \cap_{ij} \) respectively. After these notations, the weight zero condition means

\[
[A_{i}, i+1(\lambda), B_{j}, j+1(\lambda')] = 0; \quad if \quad i + 1 < j \ or \ j + 1 < i
\]

in which \( A, B \in \{ \tilde{R}, \cap, \bar{\cap} \} \). By the relation (13) and its analogue, we can reduce equation (15) to

\[
\begin{align*}
\text{l.h.s.} &= \cap_{12} \cap_{34} \cap_{56} \cap_{12}(\lambda_{12}) \cap_{34}(\lambda_{13}) \cap_{56}(\lambda_{23}) \cap_{12} \cap_{34} \cap_{56} \\
\text{r.h.s.} &= \cap_{12} \cap_{34} \cap_{56} \cap_{12}(\lambda_{12}) \cap_{34}(\lambda_{13}) \cap_{56}(\lambda_{23}) \cap_{12} \cap_{34} \cap_{56} \\
S_{i+1}(\lambda) &= (\tilde{R}_{i+1,i+2}(\lambda, x) \tilde{R}_{i+1,i+2}(\lambda) \tilde{R}_{i+1,i+2}(\lambda) \tilde{R}_{i+1,i+3}(\lambda)) \tilde{R}_{i+1,i+2}(\lambda + \gamma)).
\end{align*}
\]

Using QDYB equation (2) and its analogue, we have proved \( S_{34}(\lambda_{12})S_{12}(\lambda_{13})S_{34}(\lambda_{23}) = S_{12}(\lambda_{23})S_{34}(\lambda_{13})S_{12}(\lambda_{12}) \), or l.h.s. = r.h.s. in above equation. In other words, the fusion procedure is practicable.

If we rewrite equation (15) in the standard 9 \( \times \) 9 matrix form \( \tilde{R}^{(1,1)}(\lambda, x) \), it becomes

\[
\begin{align*}
\tilde{R}^{(1,1)}_{JK}(\lambda_{12}, x + \gamma h^{(I)}) \tilde{R}^{(1,1)}_{IJ}(\lambda_{13}, x - \gamma h^{(K)}) \tilde{R}^{(1,1)}_{JK}(\lambda_{23}, x + \gamma h^{(I)}) \\
= \tilde{R}^{(1,1)}_{IJ}(\lambda_{23}, x - \gamma h^{(K)}) \tilde{R}^{(1,1)}_{JK}(\lambda_{13}, x + \gamma h^{(I)}) \tilde{R}^{(1,1)}_{IJ}(\lambda_{12}, x - \gamma h^{(K)}).
\end{align*}
\]

(17)

It is just the original QDYB equation (2). Notice that this \( \tilde{R}^{(1,1)}(\lambda_{12}, x + \gamma h^{(I)}) \) matrix is of spin- 1 since \( h^{(I)}(\otimes V_{t}) \) (in which \( t \in \{ I, J, K \} \)) becomes \( \text{diag}\{ 1, 0, -1 \} (\otimes V_{t}) \) by taking the singlet of spin-0 away.

With the \( \tilde{R}^{(1,1)}_{ij} \) matrix (5) and the fusion method (14), we obtain

\[
\tilde{R}^{(1,1)}(\lambda, x) =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a(\lambda, x) & 0 & b(\lambda, -x) & 0 & 0 & e(\lambda, x) & 0 & 0 \\
0 & c(\lambda, x) & 0 & d(\lambda, x) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b(\lambda, x) & 0 & 0 & 0 & 0 & f(\lambda, -x) & 0 & 0 \\
0 & 0 & f(\lambda, x) & 0 & 0 & 0 & 0 & a(\lambda, x) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b(\lambda, x) \\
0 & 0 & e(\lambda, -x) & 0 & 0 & 0 & 0 & 0 & c(\lambda, -x) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
\]

(18)
in which
\[ a(\lambda, x) = \frac{\sinh(2\gamma) \sinh(\lambda + x)}{\sinh(2\gamma - \lambda) \sinh x} \quad b(\lambda, x) = \frac{\sinh(\lambda) \sinh(2\gamma - x)}{\sinh(2\gamma - \lambda) \sinh x} \]

\[ c(\lambda, x) = \frac{\sinh \gamma \sinh(2\gamma) \sinh(\lambda + x) \sinh(\gamma + \lambda + x)}{\sinh(\gamma - \lambda) \sinh(2\gamma - \lambda) \sinh x \sinh(\gamma + x)} \]
\[ d(\lambda, x) = \frac{\sinh(2\gamma) \sinh(\lambda) \sinh(2\gamma + x) \sinh(\lambda + x) \cosh \gamma}{\sinh(\gamma - \lambda) \sinh(2\gamma - \lambda) \sinh(\gamma - x) \sinh(\gamma + x)} \]
\[ e(\lambda, x) = -\frac{\sinh \lambda \sinh(\gamma + \lambda) \sinh(\gamma + x) \sinh(2\gamma + x)}{\sinh(\gamma - \lambda) \sinh(2\gamma - \lambda) \sinh(\gamma - x) \sinh(\gamma + x)} \]
\[ f(\lambda, x) = \frac{2 \sinh \gamma \sinh \lambda \sinh(\gamma - x) \sinh(\lambda + x)}{\sinh(\gamma - \lambda) \sinh(2\gamma - \lambda) \sinh x \sinh(\gamma + x)} \]
\[ g(\lambda, x) = \frac{\sinh(\gamma + \lambda) + \sinh \lambda (\cosh(2x) - \cosh(2\gamma) - \sinh^2(2\gamma))}{\sinh(\gamma - \lambda) \sinh(2\gamma - \lambda) \sinh(\gamma - x) \sinh(\gamma + x)}. \]

The obtained \( \hat{R}^{(1,1)} \) matrix has three distinct eigenvalues, say, 1, \(-\frac{\sinh(\lambda + 2\gamma)}{\sinh(\lambda - 2\gamma)}\) and \( \frac{\sinh(\lambda + \gamma) \sinh(\lambda + 2\gamma)}{\sinh(\lambda - \gamma) \sinh(\lambda - 2\gamma)} \) whose multiplicity are 5, 3 and 1 respectively. This \( \hat{R}^{(1,1)} \) is connected with Lie algebra \( so(3) \). By direct calculation, we can show that it does satisfy the QDYB equation (17) with \( h^{(l)}(\otimes \hat{V}_l) = \text{diag}\{1, 0, -1\}(\otimes \hat{V}_l) \).

4 Discussion

From the expression (18), we find the \( \hat{R}^{(1,1)} \) matrix does not satisfy the translation invariance condition (3). In other words, if we want to translate it to the form of equation (4), we will obtain a more complex \( \hat{R}^{(1,1)} \) matrix form. In fact, it is just the matrix of \( \hat{R}^{(1,1)}(\lambda, x + \gamma h^{(I,J)}) \) where \( h^{(I,J)} \) means \( h^{(I)} + h^{(J)} \), so we have to construct new commuting operators different from those in paper [7], in which the condition (3) was used in constructing commuting operators. For the simplicity of the expression of \( R \)-matrix, we had better use more symmetric form as equation (1) or (2), rather than the form as equation (4).

Now we compare our results with paper [12]. At first, QDYB equation (4) tends to be independent from the spectral parameter by requiring \( \lambda \rightarrow \pm \infty \). Secondly we need change dynamical variable \( x \rightarrow -\gamma x \) in our \( \hat{R}^{(\frac{1}{2}, \frac{1}{2})} \) and \( \hat{R}^{(1,1)} \) matrices because QDYB equation takes different forms in these two papers. At last, we need translate expression (18) to \( \hat{R}^{(1,1)}(\lambda, x + \gamma h^{(I,J)}) \) as discussed before. After these changes of \( \lambda \) and \( x \) in \( \hat{R}^{(1,1)}(\lambda, x + \gamma h^{(I,J)}) \), we indeed obtain the \( \hat{R}^{(1,1)} \) matrix gauge equivalent to the one in paper [12]. The single eigenvalue \( q \) of \( \hat{R}^{(\frac{1}{2}, \frac{1}{2})} \) matrix in paper [12] is connected to \( e^{\pm 2\gamma} \) when we take \( \lambda \rightarrow \pm \infty \) respectively.

For the six-vertex elliptic solution of QDYB equation, eigenvalues of \( \hat{R}^{(\frac{1}{2}, \frac{1}{2})} \) matrix are not made of one triplet and one singlet. It is still an open problem about how to construct its higher-spin \( \hat{R} \) matrix.
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