A Reconstruction of Total Solar Irradiance Based on Wavelet Analysis

Haijun Xu¹, Bin Lei¹, and Zhen Li¹²

¹College of Surveying and Geo-informatics, North China University of Water Resources and Electric Power, Zhengzhou, China, ²State Key Laboratory of Geodesy and Earth's Dynamics, Wuhan, China

Abstract As an important indicator of solar activity, the long-term total solar irradiance (TSI) observations are needed to uncover the impact of solar activity on Earth's climate. In this paper, the periodic variation of TSI and its relationships with sunspot number and Ca II K index are analyzed by using the satellite observation data from 1979 to 2015 and the wavelet method. The results of continuous wavelet analysis show that TSI, sunspot number, and Ca II K index all have significant and stable oscillation periods of 9–13 years and intermittent oscillation periods of 2–6 months only during the time of intense solar activity. Moreover, the results of cross wavelet analysis indicate that the effect of sunspot number and Ca II K index on TSI is mainly reflected in the period of 9–13 years with one month phase lag, and sunspot number and Ca II K index cannot explain the TSI variation in the period of 3–6 months. Under the condition of considering the phase relationship and the model order, the TSI reconstruction model is established and the monthly TSI time series from 1907 to 1978 is restored.

1. Introduction

The total solar irradiance (TSI) refers to the sum of the solar electromagnetic radiation energy of all wavebands reaching the top of the earth's atmosphere per unit area in unit time at the average distance between the sun and the earth (Biktash & Lilia, 2017). Although the solar radiation energy received by the earth is only one in two billion of the total radiation energy from the sun to the space, but it is the main energy source of the earth's atmospheric movement, it is the most important energy source of the earth's atmospheric movement and an important external driving factor of global climate change (Haigh, 2007; Kren, 2015; Solanki et al., 2013). Before Hickey-Frieden cavity radiator (HF) was used to observe the solar radiation in October 1978, TSI was regarded as a constant due to the low accuracy of ground observation equipment, so it was called "solar constant." Since HF was launched, TSI has been continuously observed by several radiometers. Due to the high accuracy of these space radiometers, it is recognized that the solar radiation varies from several minutes to several decades (Kopp, 2016). TSI produces the earth's radiation environment and affects the earth's temperature and atmosphere, even a small change in TSI will have a profound impact on the earth's climate (Ermolli et al., 2013; Gray et al., 2010; Ineson et al., 2011; Lean & Rind, 2008). However, TSI space observation has only accumulated data for about 40 years, which is obviously not enough for the study of the long-term effects of climate change. Therefore, it is crucial to reconstruct the TSI with a longer time scale, and many studies have been carried out in TSI reconstruction. For example, Ambelu et al. (2011) and Zhao and Han (2012) utilized sunspot number (SN) to reconstruct TSI. Preminger and Walton (2005) and Vaquero et al. (2006) reconstructed TSI based on sunspot area. Krivova et al. (2007) reconstructed TSI from the surface magnetic flux. However, the phase relationship between TSI and its proxies (especially in the significant resonance period) is not considered in the above reconstruction methods. Moreover, Li et al. (2016) found that TSI decreases when dark sunspots are present on the solar disk, and increases due to bright structures. Considering the Ca II K index is usually used to represent the intensification by the bright structures (Bertello et al., 2016), in this paper, we focus on the TSI reconstruction based on sunspot number and Ca II K index. First, we analyze the variation characteristics of sunspot number, Ca II K index and TSI. Second, we discuss the phase relationship between sunspot number, Ca II K index and TSI in different scales. Finally, a linear regression model is established to reconstruct TSI based on the phase adjustment.
Figure 1. Monthly sunspot number, Ca II K index and total solar irradiance from 1979–2016.

Figure 2. Wavelet power spectrum for the sunspot number time series. The levels are represented by a color and the hotter the color the higher the power. The bold contour line denotes the 95% confidence level against red noise. The cone of influence, an area where the wavelet power suffers from the edge effect separates the reliable (full colors) and unreliable (white) regions with the thin solid line.
2. Data and Methods

2.1. Data

The daily TSI data are provided by Physikalisch-Meteorologisches Observatorium Davos/world Radiometric Center (PMOD/WRC) from November 1978 to September 2017. The detailed descriptions of the procedures used to construct the composite TSI data from the original data are given in [http://www.pmodwrc.ch](http://www.pmodwrc.ch). It should be noted that aiming at the problem of missing data (the percentage of missing data is 6.7%), we fill the data gaps using linear interpolation and calculate the average value.

The monthly sunspot numbers from 1749 to present are provided by the Solar Influences Data analysis Center (SIDC), Royal Observatory of Belgium ([http://www.sidc.be/silso/datafiles](http://www.sidc.be/silso/datafiles)). On July 1st, 2015, the sunspot data has been replaced by a new improved version (version 2.0) that includes several corrections of past inhomogeneities in the sunspot number series. The detailed information about the corrections can be found in Clette and Lefèvre (2016).

The monthly Ca II K index from 1907 to 2016 are provided by Harvard Dataverse which are available at [https://doi.org/10.7910/DVN/VF5BMO](https://doi.org/10.7910/DVN/VF5BMO). Three different Ca II K observations from Kodaikanal Observatory, the National Solar Observatory Sacramento Peak and the National Solar Observatory in Arizona are combined into a single disk-integrated Ca II K index time series, which is an indicator of solar chromospheric activity. The details of the composite disk-integrated Ca II K index are given in Bertello et al. (2016).

2.2. Methodology of Analysis

The mathematical methods adopted in this study include the continuous wavelet transform (CWT) and the cross wavelet transform (XWT). The CWT of a time series \( x_n \) \( (n = 1, 2, ..., N) \) is defined as (Grinsted et al., 2004):

![Figure 3. Wavelet power spectrum for the Ca II K time series. The levels are represented by a color and the hotter the color the higher the power. The bold contour line denotes the 95% confidence level against red noise. The cone of influence, an area where the wavelet power suffers from the edge effect separates the reliable (full colors) and unreliable (white) regions with the thin solid line.](image-url)
\[ W_n^s(S) = \sqrt{\delta t/s} \sum_{n'=1}^{N} x_n \psi_{0}^n \left( \frac{n' - n}{s} \right) \]  

where \( W_n^s(S) \) is the wavelet coefficient, \( \delta t \) is the uniform time step, \( s \) is the wavelet scale, \( n' \) is the reversed time, \( \psi_0 \) is the mother function and we specially choose the Morlet wavelet (\( w_0 = 6 \)) as the month wavelet since it offers an optimal balance between frequency and time localization (Luo et al., 2019; Xu et al., 2020). The Morlet wavelet is defined as (Grinsted et al., 2004):

\[ \psi_{0}(\eta) = \pi^{-1/4} e^{i\pi\eta^2} e^{-\eta^2/2} \]  

where \( \omega_0 \) and \( \eta \) are dimensionless frequency and time, respectively. Based on the CWT and cross spectrum, the XWT of two time series \( X_n \) and \( Y_n \) (\( n = 1, 2, ..., N \)) is defined as (Li et al., 2018):

\[ XWT_{XY} = \sum_{n=1}^{N} \Re \{ W_{XY}^n \}^* \]  

where the symbol * means complex conjugation, the absolute value of \( XWT_{XY} \) denotes the cross wavelet power and the argument of the complex number \( W_{XY} \) (or \( \arg(W_{XY}) \)) is the local relative phase between \( X_n \) and \( Y_n \). The XWT can find the high common power regions and further reveal the phase difference between two time series in time frequency space. If two series are physically related a consistent or slowly varying phase lag would be detected and the circular mean of the phase angles is calculated to quantify the phase relationship. The circular mean of a set of angles \( \alpha_i \) (\( i = 1, 2, ..., N \)) is defined as (Li et al., 2018):

\[ \alpha_m = \arctan \left( \frac{1}{N} \sum_{i=1}^{N} \cos(\alpha_i), \frac{1}{N} \sum_{i=1}^{N} \sin(\alpha_i) \right) \]  

where \( \alpha_m \) is the circular mean of the phase angles and \( \arctan \) is the inverse tangent.

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**Figure 4.** Wavelet power spectrum for the TSI time series. The levels are represented by a color and the hotter the color the higher the power. The bold contour line denotes the 95% confidence level against red noise. The cone of influence, an area where the wavelet power suffers from the edge effect separates the reliable (full colors) and unreliable (white) regions with the thin solid line.
3. Results and Discussions

3.1. Continuous Wavelet Transform Analysis

Figure 1 shows the time series of SN, Ca II K index and TSI from 1979 to 2016, respectively. It is clear from Figure 1 that the time series of SN, Ca II K index and TSI all show significant periodic variations, and there is some trend when SN and Ca II K index decrease or increase then this trend also can be found in TSI. The whole trend of SN and Ca II K index is decreased while the whole trend of TSI is also decreased.

Further, Figures 2–4 display the CWT analysis results for time series of SN, Ca II K index and TSI, respectively. The statistical significance of wavelet power can be evaluated relative to the null hypothesis that the signal is generated by a stationary process with a given background power spectrum. Most of geophysical time series have obvious red noise characteristics, which can be well simulated by first-order autoregressive (AR1) process (Xu et al., 2019). The Fourier power spectrum of an AR1 process is given by Allen and Smith (1996). From Figures 2–4 we can see that (a) the energy distribution of the power spectrum of SN, Ca II K index is decreased while the whole trend of TSI is also decreased.

Figure 5. Cross wavelet spectrum for sunspot number (SN) and total solar irradiance (TSI). The hot colors imply high common power and cold colors imply low common power. The bold contour line indicates the 5% significance level against red noise. The reliable (full colors) and unreliable (white) regions are separated by COI. The phase difference is shown as arrows (with in-phase pointing right, anti-phase pointing left, SN leading TSI by 90° pointing straight up and TSI leading SN by 90° pointing straight down).

Further, Figures 2–4 displays the CWT analysis results for time series of SN, Ca II K index and TSI, respectively. The statistical significance of wavelet power can be evaluated relative to the null hypothesis that the signal is generated by a stationary process with a given background power spectrum. Most of geophysical time series have obvious red noise characteristics, which can be well simulated by first-order autoregressive (AR1) process (Xu et al., 2019). The Fourier power spectrum of an AR1 process is given by Allen and Smith (1996). From Figures 2–4 we can see that (a) the energy distribution of the power spectrum of SN, Ca II K index and TSI are generally consistent; (b) the high power is mainly concentrated in the period between 108 and 156 months (9~13 years); (c) the low frequency band of 9~13 years passes the significance test of 0.05 in the whole period, and the high frequency band of 2~6 months also passes the significance test of 0.05, but only appeared in the period of intense solar activity (solar cycle 21, 22 and 23). The results of Figures 2–4 indicate that SN, Ca II K index and TSI all have significant 9~13 years oscillation periods and intermittent 2~6 months quasi oscillation periods.

3.2. Cross Wavelet Transform Analysis

In order to quantify the phase relationship between SN and TSI and between Ca II K index and TSI, we adopt the XWT to reveal their time-varying phase differences.
Figure 5 displays the XWT analysis results for SN and TSI. Since the low-frequency component around 128 months (approximately 11 years) is more dominant than the other components, we focus on the phase differences in the period band of 108–156 months. From Figure 5 we can see that the arrows in this belt entirely point to right, suggesting that the phase relationships between SN and TSI is basically synchronous. Although fluctuation of arrow directions are not obvious, the phase asynchrony are still detected. The average phase differences between SN and TSI are −2.6°, indicating that SN variation leads TSI variation by one month. Additionally, the phase angles in the period band of 2–6 months fluctuate violently, demonstrating that the phase relationship between SN and TSI in this belt is not stable.

Figure 6 displays the XWT analysis results for Ca II K index and TSI. It can be seen that the arrows in the period band of 108–156 months generally point to right, implying that Ca II K index and TSI is approximately in-phase. The average phase differences between Ca II K index and TSI are −2.5°, indicating that Ca II K index variation leads TSI variation by one month. In addition, the sharp change of phase angle indicates that the phase relationship between Ca II K index and TSI in the period band of 2–6 months is not stable.

It is known that TSI variations are mainly caused by the combination of the sunspots' blocking and the intensification due to bright faculae and network elements which are represented by Ca II K index. The phase relationship between SN, Ca II K index and TSI can further reveal the physical mechanism of SN, Ca II K and TSI. The first occurrence of a new sunspot active region is accompanied by some faculae, then the sunspots typically die away after one solar rotation while the faculae persist for several rotation periods. Considering the typical 7-month long lifetime

Table 1
| Model     | SSE   | RMSE  | R     |
|-----------|-------|-------|-------|
| Model_A   | 32.78 | 0.2687| 0.7575|
| Model_B   | 30.42 | 0.2602| 0.7727|
| Model_C   | 28.21 | 0.2496| 0.7957|
| Model_D   | 20.89 | 0.2150| 0.8534|
Figure 7. Comparison between the original total solar irradiance (TSI) and the reconstructed TSI with Model_A and Model_B respectively.

Figure 8. The goodness-of-fit between observed total solar irradiance (TSI) and reconstructed models with different orders. The top panel shows TSI versus sunspot number while the bottom panel shows TSI versus Ca II K index.
for active regions (Preminger & Walton, 2005) whereby the first month has a dark (negative) TSI impact followed by several months of bright (positive) TSI variation, this is inferred to be the reason why both sunspot number and Ca II K lead TSI by one month.

3.3. TSI Reconstruction

TSI is a very important index in the sun-earth environment study. However, it is unfortunate that the reliable satellite observation data of TSI are only about 40 years, which is obviously insufficient for determining the impact of solar irradiation variation on climate. Therefore, SN and Ca II K index with longer time observation records are expected to reconstruct the longer TSI time series. If using single-index to reconstruct TSI, the linear regression model based on SN is:

Model_A:

\[ TSI(t) = 1361 + 0.004284SN(t) \]  

The reconstructed model based on Ca II K index is:

Model_B:

\[ TSI(t) = 1353 + 91.19Ca(t) \]  

If using double-index to reconstruct TSI, the model based on SN and Ca II K index is:

Model_C

\[ TSI(t) = 1361 + 0.05849SN(t) + 0.2721Ca(t) \]  

Considering the influence of phase on the model performance, the model_C can be modified as:

![Figure 9. The monthly and 13-month smoothed total solar irradiance reconstruction time series from February 1907 to December 1978.](image-url)
Model_D

\[ \text{TSI}(t) = 1361 + 0.05849 \text{SN}(t - 1) + 0.2721 \text{Ca}(t - 1) \]  

(8)

Table 1 shows the statistical parameters of Model_A, Model_B, Model_C and Model_D respectively. It is clear from Table 1 that in case of choosing one index to reconstruct TSI, the model based on Ca II K is better than on SN; the double-index model based on SN and Ca II K is better than the single-index model based on SN or Ca II K; the model considering the phase relationship is superior to that without considering the phase relationship. Meanwhile, we find that the statistical parameters of Model_C are consistent with those of TSI reconstruction model in Zhao and Han (2012), and a considerable improvement in Model_D is observed when compared with the results in Zhao and Han (2012).

Figure 7 displays the comparison between the original TSI and the reconstructed TSI with two different models respectively. From Figure 7 we can see that the peaks and troughs of the reconstructed TSI with Model_B are more consistent that of the reconstructed TSI with Model_A with the original TSI.

In addition to taking the phase relationship into account, the effect of model order on TSI reconstruction is further investigated.

From Figure 8 we can see that the goodness-of-fit between observed TSI and reconstructed models is not significantly improved by increasing the order. The results of quantitative analysis in Table 2 are consistent with those of qualitative analysis in Figure 8. In addition, Table 2 further shows that when the order is increased, the accuracy of the model has not been significantly improved. The results of Figure 8 and Table 2 indicate that the relationship between sunspot number, Ca II K index and TSI is approximately linear. Therefore, the TSI reconstruction form February 1907 to December 1978 is done by using the linear regression model (Figure 9).

It is clear from Figure 9 that there exists a secular increase for the entire TSI. The secular slope with 95% confidence bounds is 0.00114 ± 0.00061. Further, a detailed insight into the TSI variability range (TSI maximum minus TSI minimum) for the different solar cycles (SC) is given in Table 3. Considering that NOAA/NASA standard for defining solar cycle minima and maxima is to use 13-month averages, the monthly TSI reconstruction time series are smoothed by 13 months.

From Table 3 we can see that the solar cycle with the largest TSI fluctuation range is in SC23, while the solar cycle with the smallest TSI fluctuation range of solar radiation is in SC16. The TSI fluctuation range shows an increasing trend from SC15 to SC23, it is because the TSI minimum shows a downward trend and the TSI maximum shows an upward trend (Figure 10).
4. Conclusion

This paper mainly focuses on the phase relationship between sunspot number, Ca II K index and TSI in the time-frequency domain, and the reconstruction of total solar irradiance for a longer time. The results of continuous wavelet transform show that sunspot number, Ca II K index and total solar irradiation all have a local cycle in the period band of 2~6 months and a globally dominant cycle in the period band of 108~156 months. Further, the results of cross wavelet transform indicate that the phase relationships between sunspot number, Ca II K index and TSI are unstable in the intermittent resonance period band of 2~6 months, whereas the phases of both sunspot number and Ca II K index lead that of TSI by one month in the continuous resonance period band of 108~156 months. Compared with the TSI time series reconstructed without considering the phase relationship, the reconstructed TSI time series based on the phase analysis results of the XWT has higher reliability and accuracy. Finally, the influence of model order on the TSI reconstruction is discussed and the result suggests that the relationship between sunspot number, Ca II K index and TSI is approximately linear. Therefore, the monthly TSI time series from February 1907 to December 1978 is reconstructed based on the linear regression model.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

The daily TSI data can be downloaded from http://www.pmodwrc.ch. The monthly SN data can be downloaded from http://www.sidc.be/silso/datafiles. The monthly Ca II K index data can be downloaded from https://doi.org/10.7910/DVN/VF5BMO. The processed data shown in the figures can be accessed at https://doi.org/10.5281/zenodo.5153355.
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