Possible tests of curvature effects in weak gravitational fields

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Abstract

We analyze the possibility of detecting, with optical methods, particle and photon trajectories predicted by general relativity for a weak spherically-symmetric gravitational field. We discuss the required sensitivities and the possibility of performing specific experiments on the Earth.

Key words: Experimental tests. Tidal effect. Local gravity effects.

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1 Introduction

Since the formulation of general relativity (GR), several experiments have been proposed and/or performed in order to test the predictions of this theory \[1\]. The most famous GR tests are concerned with the deflection of light by the Sun’s gravitational field, with the perihelion shift of Mercury and, more recently, with the radar echo delays \[2, 3\].

On-Earth experimental tests have also provided a richness of information, although they are extremely difficult because of the small size of the effects to be observed. The precision achievable in different experiments has been largely increased by the use of maser and laser sources. This accuracy was high enough to perform, for example, stringent tests of the isotropy of the light velocity \[4\]. Similar techniques have been used to improve the gravitational red shift experiment, first performed by Pound and Rebka \[5\], by comparing the frequency of a hydrogen maser in a ground laboratory with that of an identical maser on board of a rocket \[6\]. Moreover, several experiments have been recently performed to search for a "fifth-force" \[7\]. Such experiments provide an upper limit to a violation of the Einstein equivalence principle. Matter wave interferometers have also been used to study the influence of the homogeneous Earth gravitational field on quantum systems and the measurability of the influence of the space-time curvature was recently discussed \[8\].

In this paper we analyze the possibility of performing, on or near the Earth, experiments of the type based on the space curvature induced by the Sun. We discuss in particular three possible "local" experiments based on optical techniques to search for space-time curvature effects on the Earth or in its proximity, taking advantage of the extremely high sensitivity achievable with available optical techniques. On-Earth experiments offer the advantage of a better control of the experimental parameters. The first experiment we discuss is aimed to the detection of tidal acceleration using a freely-falling Michelson interferometer. The basic idea of the second experiment is instead to put into evidence, using laser ranging techniques, the relativistic correction, at the second order in the Schwarzschild radius, to the classical gravity acceleration. We investigate finally the possibility of detecting light deflection induced by space curvature by following its propagation in an optical cavity made of two high-reflectivity mirrors.

Although the purpose of this paper is not to design real experiments, we discuss the possibility of performing them and the sensitivities required in order to detect the relevant effects.
The paper is organized as follows: in Sect. 2, we give, with a relatively simple procedure, the expression for the trajectories of photons and of massive bodies in a weak Schwarzschild field [9]. This allows us to calculate the size of the relevant effects in the following discussion of possible experimental tests. In Sect. 3, we present the idea of using a falling Michelson interferometer to measure the tidal acceleration. In Sect. 4, we consider a possible scheme to measure the correction to \( g \) due to the space curvature. In Sect. 5, we discuss the possibility of a local detection of the light deflection due to the Earth gravitational field. In Sect. 6 we draw final conclusions about the feasibility or not of each of the experiments.

## 2 Light and particle motion in a Schwarzschild field

In this section we give the main results of a calculation of the trajectory of light and of massive particles in a weak spherically-symmetric gravitational field. Only the results which are important for the experiments discussed in the following are reported. A more detailed account of this calculation will be given elsewhere [10].

First, we consider photons. The equations for the null geodesics in a Schwarzschild field, described by the metric

\[
\text{ds}^2 = -\left(1 - \frac{2K}{r}\right)c^2\text{d}t^2 + \frac{1}{1 - \frac{2K}{r}}\text{d}r^2 + r^2(\text{sin}^2\theta\text{d}\phi^2 + \text{d}\theta^2) \tag{1}
\]

where \( K = GM/c^2 \) is the Schwarzschild radius, when expressed in terms of a suitable parameter \( \tau \), are the following: [11, 12]

\[
\left(\frac{\text{d}r}{\text{d}\tau}\right)^2 + \frac{L^2}{r^2}\left(1 - \frac{2K}{r}\right) = E^2, \quad c\frac{\text{d}t}{\text{d}\tau} = \frac{E}{1 - \frac{2K}{r}}, \quad \frac{\text{d}\theta}{\text{d}\tau} = \frac{L}{r^2}, \tag{2}
\]

where \( r \) and \( t \) are the Schwarzschild variables and \( L \) is \( c \) times the angular momentum. The trajectories seen by a stationary observer in the planet reference frame are expressed in terms of the local variables \([13, 14, 15]\) \( \hat{r}, \hat{t} \) and \( \hat{\theta} \), which are obtained from the expressions of \( r, \theta \) and \( t \) along the trajectories from:

\[
\text{d}\hat{r} = \frac{\text{d}r}{\sqrt{1 - \frac{2K}{r}}}, \quad \hat{r}\text{d}\hat{\theta} = \text{r}\text{d}\theta, \quad \text{d}\hat{t} = \text{d}t\sqrt{1 - \frac{2K}{r}}. \tag{3}
\]
The symbol \( \tilde{=} \) is used to remark that its right hand side must be expressed in terms of the Schwarzschild solution. Our initial conditions are such that, at \( \tau = t = \theta = \hat{t} = \hat{\theta} = 0 \), the photon moves through a point at a distance \( \hat{r} = R \) from the center of the planet along a direction which makes an angle \( \alpha \) with the radial one; the local initial motion is then:

\[
\hat{r} = R \frac{\sin \alpha}{\sin(\alpha - \theta)}; \quad L = ER \sin \alpha
\]  

The procedure to derive the local motion is the following: first of all, we derive, from eqs. (2), the expressions for \( r, \theta, t \) at the fourth order in \( \tau \) and at the second order in \( K \); then, from eqs. (3), we derive the analogous polynomials for \( \hat{r}, \hat{\theta} \) and \( \hat{t} \), which satisfy the given initial conditions. These solutions, when expressed in terms of local Cartesian coordinates \( \hat{z} \equiv \hat{r} \cos \hat{\theta} - R \) and \( \hat{x} \equiv \hat{r} \sin \hat{\theta} \), assume the following form:

\[
\hat{z} = c \hat{t} \cos \alpha - \frac{1}{2} \frac{K}{R} \sin^2 \alpha \left[ \left( 1 + 3 \frac{K}{R} \right) \frac{c^2 \hat{t}^2}{R} - \left( \frac{4}{3} + \frac{11K}{3R} \right) \cos \alpha \frac{c^3 \hat{t}^3}{R^2} \right] + \frac{1}{3} \frac{K}{R} \sin^2 \alpha \left[ \frac{1}{2} - \frac{23}{8} \cos^2 \alpha + \left( 1 - 6 \cos^2 \alpha \right) \frac{K}{R} \frac{c^4 \hat{t}^4}{R^3} \right]
\]

\[
\hat{x} = c \hat{t} \sin \alpha + \frac{1}{2} \frac{K}{R} \sin \alpha \cos \alpha \left( 1 + 3 \frac{K}{R} \right) \frac{c^2 \hat{t}^2}{R} + \frac{1}{3} \frac{K}{R} \sin \alpha \left[ 2 \cos^2 \alpha + \left( \frac{1}{2} + \frac{11K}{2R} \cos^2 \alpha \right) \frac{K}{R} \frac{c^3 \hat{t}^3}{R^2} \right] + \frac{1}{6} \frac{K}{R} \sin^2 \alpha \cos \alpha \left[ -1 + \frac{23}{4} \cos^2 \alpha + \left( 1 + 12 \cos^2 \alpha \right) \frac{K}{R} \frac{c^4 \hat{t}^4}{R^3} \right]
\]

It is interesting to notice that, for \( \alpha = \pi/2 \), the trajectory in the Schwarzschild variables, which corresponds to the previous solution and solves eqs. (2), is such that:

\[
r^{-1} \simeq \frac{\cos(\theta)}{R} + \frac{K}{R^2} (2 - \cos^2 \theta)
\]  

As a consequence, \( r^{-1} \to 0 \) when \( \theta \to \pm \frac{\pi}{2} \pm \frac{2K}{R} \). The well-known expression \( 4K/R \) for light deviation follows.

For the following discussion, it is also important to calculate the motion of massive particles. The equations governing the radial geodesics, in units \( c = 1 \), are [11]:

\[
\left( \frac{dr}{d\tau} \right)^2 + (1 - \frac{2K}{r}) = E^2, \quad \frac{dt}{d\tau} = \frac{E}{1 - 2K/r}.
\]
where \( r \) and \( t \) are the Schwarzschild variables. The same procedure used for the photon case can be followed. First of all, one looks for the generic polynomials at the fourth order in \( \tau \) and at the second order in \( K \) which solve equations (8). Then one finds the similar polynomials in \( \tau \) which give \( \hat{r} \) and \( \hat{t} \). Finally one fixes the arbitrary parameters in such a way that the initial conditions \( (\hat{r})_{i=0} = R, (d\hat{r}/dt)_{i=0} = v_0 \) are satisfied. Re-introducing \( c \), the final result is the following:

\[
\hat{r} = R + v_0 \hat{t} - \frac{K}{2R} \left( 1 - \frac{v_0^2}{c^2} \right) \left[ \left( 1 + \frac{3K}{R} \right) \frac{c^2 \hat{t}^2}{R} - \frac{v_0}{c} \left( \frac{2}{3} + \frac{5K}{3R} \right) \frac{c^3 \hat{t}^3}{R^2} \right] + \\
- \frac{1}{4} \frac{K}{R} \left[ 1 - \frac{v_0^2}{c^2} \right] \left( \frac{v_0^2}{c^2 - v_0^2} + \frac{1}{3} \left( 1 + 5 \frac{v_0^2}{c^2 - v_0^2} \right) \frac{K}{R} \right) \frac{c^4 \hat{t}^4}{R^3}.
\]

(9)

Notice that, in the limit \( v_0 \to c \), \( \hat{r} \to R + ct \). If the particle is initially at rest, the previous expression reduces to

\[
\hat{r} \simeq R - \frac{1}{2} g \hat{t}^2 - \frac{1}{12} \frac{g^2}{R} \hat{t}^4,
\]

(10)

where

\[
g \equiv \frac{GM}{R^2} \left( 1 + 3 \frac{GM}{Rc^2} \right).
\]

(11)

We verified that expression (11) for the gravity acceleration remains unchanged at the same level of approximation also if the Kerr metric is used to take into account possible effects induced by the rotation of the gravitational field generator.

The trajectory of a photon, initially moving through \( O \) perpendicularly to the radius \( (\alpha = \pi/2) \), is given by (see eqs (5) and (6)):

\[
\hat{x} \simeq c\hat{t} - \frac{1}{6} \frac{g \hat{t}^3}{c},
\]

(12)

\[
\hat{z} \simeq - \frac{1}{2} g \hat{t}^2 + \frac{1}{6} g \hat{t}^2 \frac{c^2 \hat{t}^2}{R^2} + \frac{1}{3} \frac{g^2}{R} \hat{t}^4.
\]

(13)

Notice that the light velocity along its trajectory is constant. It is obvious that the above expansion is valid only if the distance travelled by the light, which differs from the Pythagoric distance only at the second order in \( g \), is much shorter than the distance from the planet center. The last term in eq. (12), which coincides with the classical correction due to the non uniformity
of the gravitational field, and the last term in eq. (13) will be neglected in the next as they are too small. The second term at the r. h. s. of eq. (13) is apparently due to a repulsive force; the reason for this is that, when the light beam moves in the horizontal direction, it goes towards regions with decreasing gravity. In the following we will omit the sign $\hat{s}$ so that $x, z$ and $t$ will indicate the local variables.

### 3 Measurement of the tidal effect

In this section we analyze the possibility of detecting tidal acceleration effects, due to the non-uniformity of the gravitational field.

We consider a freely-falling Michelson interferometer characterized by the following scheme (see Fig. 1): inside an elevator a beam splitter $O$ and two mirrors $A$ and $C$ are placed in the horizontal plane; $A$ is freely falling while $C$, $O$, the laser source $S$ and a photodetector $B$ are rigidly connected and freely falling. We suppose that the distances $OC$ and $OA$ are initially equal. In the horizontal plane $(x, y)$ the initial positions of the components are the following: $O \equiv (0, 0), A \equiv (D, 0), C \equiv (0, D), S \equiv (-d, 0), B \equiv (0, -d); \ (d \ll D)$. Neglecting, for the moment, higher order terms in the frame in which $O$ and the light beam move like in the uniform case, the mirror $A$ moves along the geodesics:

$$x = L - \frac{1}{2}gt^2 \sin \theta; \quad z = -\frac{1}{2}gt^2 \cos \theta,$$

with $\sin \theta = D/R$. From the point of view of the frame in which $O$ is stationary, neglecting terms of the order $D^2/R^2$, $A$ is moving toward $O$ according to the law:

$$x' \approx D - \frac{1}{2}gt^2 \sin \theta \approx D - \frac{1}{2}g \frac{D}{R}t^2; \quad z' = 0; \quad (15)$$

if the light beam to $A$ leaves $O$ at the time $t'$, it will return in $O$ at the time $t + 2\frac{D}{c} - \frac{D}{R} \frac{d^2}{c^2}$. Neglecting the frequency variation due to the mirror motion, the phase difference between the beam from $A$ and the one from $C$ is given by $2\pi \nu' \frac{D}{c} gt^2$. The phase variation rate is then given by $2\pi \frac{D}{R} \frac{d^2}{c^2}$; in these expressions $\nu'(\lambda')$ is the laser source frequency (wavelength), which is seen by $\Sigma'$ to be constant. The phase change that would be observed for an
interferometer falling in proximity of the Earth can be evaluated considering visible light ($\lambda' = 0.6\mu m$), $D = 1$ m, $R = 6.4\times10^6$ m. This gives a phase difference numerically given by $15t'^2$, where $t'$ is expressed in seconds. A large effect would then be observed in experiments with a free-fall time of only a few seconds.

Obviously, in a real experiment several experimental details should be accurately considered. First of all the experiment must be performed in a region in which gravity acceleration is not affected by local non-uniform mass distributions. A critical point is the possibility of releasing the apparatus while keeping the interferometer well aligned. The optical components can be kept in place by means of small electro-magnets. The whole apparatus is then released and the magnets are switched off. The available time should be long enough to allow a slow release of the components in order to avoid spurious inductive effects. In fact, similar problems have been already considered in the development of gravitometers based on an interferometer with a falling mirror [7]. The use of retroreflectors (corner cubes), for example, reduces the sensitivity to small misalignments. A vacuum chamber is required in order to reduce the effect of air resistance to a negligible level. As in the modern version of the Michelson-Morley experiment [4], a large improvement in the sensitivity could be obtained by using two orthogonal optical cavities instead of a Michelson interferometer.

4 Corrections to g

This section is devoted to analyze the possibility of detecting GR effects in the vicinity of the Earth, which is supposed, for the moment, to be spherical (the eventual corrections will be discussed at the end of this section).

Let us suppose that very accurate measurements of the gravity acceleration in several points on the same radial direction at distances $R$ and $R + z_i$ from the center of the planet give the values $g$ and $g_i$ respectively. It is then possible, using eq. (11), to obtain values for $GM$ and for the reference distance $R$ with the same accuracy and to test the correctness of the motion laws (10) and eventually (9); in particular one might control whether or not the classical relation between the gravity acceleration and the gravitational constant is affected by the small correction $3GM_{\text{ref}}^3/Rc^3$ (of the order of $2\cdot10^{-9}$). Notice that this correction is of the second order in $K/R$,
while the classical GR experiments are of the first order. In principle, one might obtain gravity measurements using the gravitometer as the one realized by Niebauer et al. [7] which has the required accuracy, but its use is forbidden by the requirement of performing these measurements at very large distances. For this reason, in the following, \( g_i \) will be deduced from laser ranging measurements of the distances between the falling body and an observation point. This observation point may be placed on the Earth, in which case, although one is interested in distances between positions which are outside the atmosphere, the refraction and turbulence effects need to be considered. In alternative, the observation points will be supposed on the Moon or on two satellites. We discuss in the next the latter case.

Let \( C \) be a reflecting body, which is freely falling from a height of \( \sim 8000 \text{Km} \) from the center of the Earth in the plane \((x, z)\). For the moment, we suppose that the orbital angular momentum of \( C \) is absolutely negligible and neglect the Moon attraction. Let \( A \) and \( B \) be two satellites which continuously control their relative separation. A convenient (of the order of 100) number of \( \sim 1 \) ps light pulses are sent each \( \sim 10^{-9} \text{s} \) from \( A \) towards \( C \) and times are taken when the reflected light reaches both \( A \) and \( B \); in this way the relatives distances are known with a precision better than \( 10^{-9} \), if the relative distances are of the order of \( 1,500 \text{Km} \). If \( t_1 \) is the mean value of the times in which these flashes reached \( C \), its positions could be approximated by eq. (9) with the substitutions \( \hat{t} \rightarrow t - t_1, R \rightarrow R_1, v_0 \rightarrow v_1, K/R \rightarrow k'_1, K^2/R^2 \rightarrow k''_1 \); during this data acquisition time, we suppose that the polar coordinates of \( A \) and \( B \), if not yet known, change according to the laws \((r_A + \dot{r}_A(t - t_1), \theta_A + \dot{\theta}_A(t - t_1), \varphi_A + \dot{\varphi}_A(t - t_1)), (r_B + \dot{r}_B(t - t_1), \theta_B + \dot{\theta}_B(t - t_1), \varphi_B + \dot{\varphi}_B(t - t_1))\), which must be used also for the time data relativistic corrections. A best fitting of the measured and the calculated distances will give the values of these parameters at \( t = t_1 \).

This procedure is repeated several times with a suitable rate. If not only the products \( k'_m R_m \) are equal to a constant \( K \), but also the products \( k''_m R^2_m \) are constant and equal to \( K^2 \), then we may conclude that effectively the relation between \( g \) and \( G \) has the form (11).

The non-sphericity of the Earth implies that the expansion of \( g \) contains also latitude-dependent powers of \( R^{-2} \) [10], which can be taken into account by generalizing eq. (11). Moreover account must be taken for the Moon presence which, near the Earth, gives a constant contribution to the acceleration.

For these reasons, it might be considered the possibility of using for \( A, B \) and \( C \) three objects which are orbiting around the Sun, as in this case we
do not need to consider corrections of the type described above. This offers the advantage of operating in regions in which $K/R > 2 \cdot 10^{-9}$ (for example, at the level of our distance from the Sun, $K_{\text{Sun}}/R \approx 10^{-8}$).

The generalization of the method to the case in which $C$ has an initial orbital angular momentum is obtained by multiplying the second term at the l.h.s. of the eq. (8) by $(1 + L^2/r^2)$ and by considering also the last of eqs. (2); then one finds the polynomials which solve these equations at the fourth order in $t$ and at the second order in $K/R$. Following the procedure described in section 2 one finds the final, rather complicated, expressions which give $\hat{r}$ and $\hat{\theta}$ in terms of $\hat{t}$; these expressions contain also the GR corrections at the first order in $K/R$, analogous to the one which enter in the light deflection from a Schwarzschild field (see eqs (7), (6) and (5)). For a possible experiment a computational solution will suffice.

## 5 Light deflection in a Schwarzschild field

In analogy with the GR test on the light deflection by the Sun, we discuss in this section the possibility of locally detecting the light deflection due to the Earth gravitational field. The deflection is in this case extremely small; for a path of 5000 Km it is only $\approx 2 \cdot 10^{-9}\text{rad}$. For this purpose we consider the light propagation in an optical cavity made of two high-reflectivity mirrors. Optical cavities with a length of the order of a meter and with finesse in the range $10^5 - 10^6$ can already be realized with present technology \[17, 18\]. We consider here cavities with a finesse of the order of the ones presently achievable, the distance between the mirrors being of the order of 100 m.

We first consider a cavity made of two plane mirrors; such a configuration is not appropriate in a real experiment, because of the sensitivity to misalignment and because of the unavoidable divergence of the laser beams which would mask any effect due to gravity. However, its simplicity allows to get a first insight into the relevant effects. We then discuss the more realistic case of stable cavities including non-planar mirrors.

Let us follow the motion of a photon which enters the cavity, at $t = 0$, going in the horizontal direction. For a mirror reflectivity $r_e$ such that $1 - r_e = 10^{-5}$ and for a distance between the mirrors $2d \simeq 100$ m, the cavity optical decay time is $30\text{ ms}$. If we consider only the first term in eq.(13),
that is if $z = z_0 - \frac{1}{2}gt^2$, the corresponding vertical displacement of the beam is 5 mm. If we include the second non-uniform term of eq.(13), that is, if $z = z_0 - \frac{1}{2}gt^2 \left(1 - \frac{1}{3} \frac{c^2 t^2}{R^2}\right)$, the photon fall is slightly retarded. For time intervals of the order of 30 ms this expansion is not appropriate and further terms should be included, but it suffices for the transit time between the two mirrors; the photon motion is then correctly described by eqs. (5) and (6) if, after any reflection, the new initial conditions are introduced.

As already mentioned, the configuration to be considered for a real experiment would be a cavity with mirrors of spherical type. This allows, with a proper choice of the curvature of the mirrors and of the distance between them, to control the light beam divergence. In order to analyze if, in a real experiment, it is possible to see this beam displacement, one may calculate, using eqs. (5) and (6), the trajectory followed by the photons while traveling between two mirrors of appropriate form. For brevity, we do not give here details on these calculations.

The result indicates that, if the two mirrors have spherical form any gravity effect is cancelled. This result, which is in contrast with what one might have expected considering the simple flat-mirrors model discussed above, excludes the possibility of detecting gravity effects in typical very-high-Q optical cavities. The qualitative explanation for this is that any photon which tends to go downwards because of the beam divergence (or the gravity deflection) is reflected by the cavity mirrors in the opposite direction. Incidentally, this conclusion allows to exclude the effect of light deflection in those experiments, such as gravitational wave detection, in which very-high-Q optical cavities are used.

A possibility of recovering the effect suggested by the plane case is to take mirrors with a more complex shape. We have analyzed the behaviour of a light beam, with a Gaussian distribution characterized by a spread in $\varphi (\simeq 10^{-4} - 10^{-5})$ much smaller than the spread in $\zeta$, in the case the two mirrors are essentially made of a central cylindrical part (of the order of 1 cm) with spherical confocal concavities at the upper and lower borders. The conclusion of this analysis is that the beam motion appears to be partially chaotic and only a qualitative evidence of the light deflection might be reached when the flat parts of the mirrors were parallel. The problem is that the experimental optimization of the cavity would be realized not by this configuration, but when the mirrors form an angle equal to the angular deviation of the photon in going from the center of a mirror to the center of the other one; this angle is equal to $2gd/c^2$. As a consequence, the photon
at the center of the beam moves from one of the mirrors in the direction perpendicular to it and hits the other one perpendicularly; it remains then, after any reflection, on the same geodesics \cite{19}. If this is the case, the cavity is optimized and no beam fall could be detected.

However, if the experiment were performed with a shorter and mechanically rigid cavity, a rotation of the cavity around the longitudinal axis, should show the resulting misalignment. Another possibility is to align the cavity and then let it fall freely. In this case, the effect of the space curvature would produce a deformation of the eigen-modes of the cavity. A quantitative calculation of this effect would then require an analysis of the propagation of a light wave in a curved space-time, but this is beyond the scope of this work.

6 Conclusions

We described effects of a weak spherically-symmetric gravitational field which might be detected exploiting the high sensitivity achievable with laser sources and modern optical methods.

Three possible experiments near the Earth were discussed. The result of our analysis seems to exclude the possibility of detecting the light deflection in an optical cavity. It seems instead possible, using Michelson interferometers, to detect the tidal acceleration. Finally, we proposed an experiment to measure GR corrections for $g$ on the Earth or in its proximity; this correction can be put into evidence by comparing the motion of a falling body with the Schwarzschild local solutions.

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References

[1] A comprehensive survey of the classical and more recent experiments to search for the effects predicted by general relativity is given in: C.M. Will, Theory and experiment in gravitational physics, Cambridge University Press, Cambridge (1993).
[2] I.I. Shapiro et al. Phys. Rev. Lett. 26, 1132 (1971)
[3] R.D. Reasenberg et al. Astr. J. 234, L219 (1979)
[4] A. Brillet and J. L. Hall, Phys. Rev. Lett. 42, 549 (1979).
[5] R.V. Pound and G.A. Rebka, Phys. Rev. Lett. 4, 337 (1960).
[6] R.F.C. Vessot et al., Phys. Rev. Lett. 45, 2081 (1980).
[7] T.M. Niebauer, M.P. McHugh, and J.E. Faller, Phys. Rev. Lett. 59, 609 (1987).
[8] J. Audretsch and K.-P. Marzlin, Phys. Rev. A 50, 2080 (1994) and references therein.
[9] K. Schwarzschild, Sitzber. Deut. Akad. Wiss. Berlin, Kl. Math.-Phys. Tech. p.424 (1916).
[10] B. Preziosi, Local trajectories in weak gravitational fields, work in progress.
[11] S. Chandrasekhar, The mathematical Theory of Black Holes, pages 123 ff. and 98 ff. Clarendon Press, Oxford 1983
[12] A. Papapetrou, Lectures on general relativity, Reidel (1974)
[13] Misner, C.W., Thorne, K.S. and Wheeler, J. A., Gravitation, Freeman, S. Francisco (1973)
[14] A.P. Lightman, W.P. Press, R.H. Price, S.A. Teukolsky, Problem book in relativity and gravitation, Princeton University Press (1975).
[15] I.R. Kenyon, General relativity, Oxford University Press (1990).
[16] W.A. Heiskanen and F.A. Vening Meinesz, The Earth and its Gravity Field, Mc Graw Hill, New York (1958), Chapt. 3.
[17] J.L. Hall in Frontiers in laser spectroscopy, Proceedings of the International School of Physics E. Fermi, Course CXX, edited by T. W. Haensch and M. Inguscio, North Holland, Amsterdam (1994)
[18] A.M. De Riva et al., INFN report DFPD 95/EP/60; A.M. De Riva et al., Rev. Sci. Instrum. (submitted).
[19] We thank Giuseppe La Rocca for drawing our attention on this point.
Figure captions

Fig. 1. Schematic diagram of the falling Michelson interferometer. The mirror A is falling freely. The source S, the beam splitter O, the mirror C and the detector B are connected together and freely-falling.