Design of dewar supports through topology optimization

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Abstract. Dewars are used to store and transport cryogens like LNG, LN2, LOX, LHe etc. These comprise two vessels, one placed inside the other and held together either at the “neck” (input/output port) or by support systems, depending on the capacity, the mechanical loads on the vessel and the boil-off characteristic of the stored cryogen. Support system based dewars are more common for real-life and industrial applications. Design of the support system are based on the principles that are used for high temperature pressure vessels. On the other hand, support system to be used for cryogenic fluid storage should also address the heat inleak through the supports along with the imposed mechanical load and thermal contraction-expansion effects. Some safety factors are prescribed in the literature to address these concerns; however, the scientific basis of design strategies available in the open literature so as to give a more scientific basis of design is absent. This would result in reduction in use of excessive dimensions or material thereby reducing the payload and the capital cost. Considerations of mechanical load and thermal heat inleak often lead to diametric conclusions in terms of the diameter/thickness of the support system, leading to pareto-optimal solution. Topology optimization (TO) is often used to design structures like bridges, vehicles, robotic arms etc. by a systematic and sequential removal of the mass of the material being used to fabricate the given structure while meeting the constraints in terms of load bearing capacity of the structure and heat inleak. This methodology may be followed to arrive at an optimized geometry for the support system when the designer is unsure of the initial shape to start working. In this work, TO has been tested with various thermal and mechanical boundary conditions to arrive at optimized support geometry.

1. Introduction

Cryogens such as oxygen and hydrogen (used as oxidizer and fuel for propulsion), helium (medical and scientific research), LNG (fuel applications) etc require efficient storage and transportation in liquefied form to their site of application. Owing to their low boiling points and low latent heat of vaporization, cryogens tend to boil off easily. Hence, the
A major challenge in designing a storage system for cryogens is to prevent or minimize boil off due to heat inleak from the ambient. Dewar vessels, developed in 1892, is the most popular means of storing cryogenic liquids. These are double walled containers with their annular space suitably insulated. The inner tank, also called the product or cargo containment, is essentially a pressure vessel that holds the cryogen. The outer tank holds the inner tank and the insulation in place. For smaller capacities, the two vessels are held together by holding at the “neck”, even though this arrangement leads to heat leak from the surroundings. For helium and hydrogen, whose normal boiling points are very small (4K and 20K respectively), neck-holding method is therefore not used even for small capacities. For these cryogens and also for high capacity storage, separate support system are embedded in the annular space between the two vessels. Support systems are designed to provide requisite mechanical strength and allow minimal heat inleak. Most of the reported studies on the design of support systems for cryogenic vessels are concerned with space application[1]. These design strategies are not directly applicable to any land-based or sea-based support system design due to significant differences in the magnitude and nature of the stresses acting on the supports, as well as the duration of storage. Patents are available for land-based support system design [2, 3]. The support system design is generally based on consideration of high temperature pressure vessels. Some arbitrary safety factors are incorporated without citing any scientific basis. This leads to overdesign, without necessarily achieving any benefit in terms of strength and heat inleak. In view of the above, in this work an attempt has been made to obtain an optimized design of a dewar support system using Topology Optimization (TO), which is often used for shape optimization in the design of bridges [4], aircraft [5], automobile parts [6], bicycles [7], robotic arms [8] etc. Thus a more scientific design approach of support systems for cryogenic vessels, is achieved to reduce payload and cost.

2. Modelling
Optimized geometry is obtained on the basis of stress distribution in the support system. A typical support system in a dewar is shown in Fig. 1. The supports are welded to the inner and outer vessels. One end of the support is considered to be at ambient temperature($T_{amb}$), while the other is at the cryogenic temperature($T_{cryo}$) when the inner vessel is filled with the cryogen.

![Figure 1: Schematic of a dewar being supported and the enlarged view of the support](image-url)
The combined weight of the inner vessel and the cargo contained within, tends to bend the support\cite{9}. The supports are pre-tensioned while being welded to the walls of the vessels. This imposes an axial force on them. Furthermore, the temperature gradient in the support generates an additional axial thermal stress in the support. Bending stress is absent if the support is under pure pre-tension and/or contraction. Mechanical loading on the support is the resultant of these three stresses.

The heat inleak through the support is dictated by the geometry, temperature gradient and the material of construction. A model is developed to obtain the temperature and stress distribution in the support system, based on the following assumptions:

(i) Steady state heat transfer from outer to inner tank of the dewar.

(ii) Material of construction of the support is isotropic and homogeneous.

(iii) Only axial heat transfer, that is, heat transfer is one dimensional.

(iv) Weight of a support is negligible compared to the loads acting on it. For example, a typical SS 304 support member of diameter 54 mm and length 300 mm weighs about 5.5 kg which is insignificant compared to the typical load acting on the support (1000 kg).

(v) Thermal resistance of the outer wall of the dewar is negligible.

A free body diagram of a single support in the support system is shown in Fig. 2. The fixed end (outer vessel) is represented by hatching, while the deformable end on the inner vessel is kept free. Bending of the support inherently leads to the generation of a two-dimensional stress field. To determine the stress distribution in the support, a force balance may be written over a control volume inside the support (shown by a dashed box in Fig. 2(a)) as given in Eqn. 1. A magnified view of the control volume is shown in Fig. 2(b).

\[ \sum \vec{F} = \vec{F}_B + \vec{F}_{PT} + \vec{F}_C - \vec{F}_R = 0 \]  \hspace{1cm} (1)

Figure 2: (a)Free body diagram of the support showing the various loads acting on it, dashed box denotes the control volume for force balance (b) Magnified view of the control volume showing the various forces acting on it
The stresses developed in and perpendicular to the axis of the support is its failure criteria. These stresses may be derived from Eqn. 1 and be written as,

$$\sigma_{\text{total}} = \sigma_B + \sigma_{PT} + \sigma_C$$  \hspace{1cm} (2)

2.1. Stresses in the support:

(i) The bending stress

$$\sigma_B = \frac{My}{I}$$  \hspace{1cm} (3)

where $M$ is the moment at any cross section due to the bending load, $y$ is the vertical distance from the neutral axis, and $I$ is the moment of inertia of the cross section. The moment $M$, acting at any cross-section of the support is the product of the vertical force acting on the support and the distance of the cross-section from the rigid fixed end.

(ii) The stress due to pre-tension

$$\sigma_{PT} = \frac{F_{PT}}{A}$$  \hspace{1cm} (4)

where $A$ is the area of cross section of the support.

(iii) The stress generated due to the thermal gradient is given as,

$$\sigma_C = E\alpha (\Delta T)$$  \hspace{1cm} (5)

where, $E$ and $\alpha$ are the Young’s modulus and coefficient of thermal expansion of the support material respectively, and $\Delta T$ is the temperature difference between the two ends of the support.

The total stress developed in the support is thus given as,

$$\sigma_{\text{total}} = \frac{My}{I} + \frac{F_{PT}}{A} + E\alpha (\Delta T)$$  \hspace{1cm} (6)

2.2. Thermal analysis:

The temperature distribution in the support has been determined by solving the one dimensional steady state energy balance equation given by,

$$\frac{d}{dx} (k(T) \frac{dT}{dx}) = 0$$  \hspace{1cm} (7)

where $k$ is the thermal conductivity. The heat flux through the support from the ambient to the cryogen has been obtained from Fouriers law of heat conduction

$$\dot{q} = +k \frac{dT}{dx}$$  \hspace{1cm} (8)

The positive sign on the RHS of Eq. 8 is due to the fact that temperature increases with distance from the origin ($x$).
2.3. Topology Optimization

Topology optimization (TO) is a tool used to optimize material layout under a given set of optimization goals and constraints. In the present case, TO has been used to remove excess material from a support system to provide the required strength and heat inleak. In implementing TO here, “inefficient” regions have been identified from stress distribution as those that are not critically stressed. Regions having stress value less than a prescribed value are termed as inefficient region.

The maximum stress ($\sigma_{\text{max}}$) is identified from the stress profile. The ratio of stress in each control volume to the maximum stress is the stress ratio (SR).

\[
SR = \frac{\sigma}{\sigma_{\text{max}}} \tag{9}
\]

The material removal from a particular location is based on the following criterion.

\[
\text{if } \frac{\sigma}{\sigma_{\text{max}}} = SR < RR \rightarrow \text{Material removal}
\]

\[
\text{if } \frac{\sigma}{\sigma_{\text{max}}} = SR > RR \rightarrow \text{No material removal} \tag{10}
\]

Rejection rate (RR) is initially a user specified value which acts as the threshold for the stress ratio. The more the RR, the more the material removal. The material removal is done in a stepwise manner. After material removal in a given step, stress profile is computed based on this new geometry taking new $M$ and $A$ values, and material is removed by the same former criteria. This continues until there is no more material that can be removed with the current RR. Under this condition, RR is updated as per Eqn. 11.

\[
RR^{(l)} = RR^{(l-1)} + ER ; \quad 0 \leq ER \leq 1 \tag{11}
\]

Where ($l$) denotes iteration level and ER is Evolution rate. ER signifies the rate of material removal at each iteration level. Continuing with this procedure, one of the two situations would arise: it could so happen that the stress generated in the support exceeds the allowable stress of the material of construction or the stress remains constant iteration after iteration. The former case is abandoned as it means design failure. In the second case, all the points are stressed to their maximum capability and further removal of material from any point will cause the support to fail. Thus procedure is stopped at this juncture.

Material removal under TO:

While the grid is made on the support, an address table containing the grid number, its $x$ and $y$ coordinates, and a boolean is constructed. The purpose of this boolean is to identify the presence (1) or absence (0) of material at the grid point. Initially all the grid points are given the boolean equal to unity. As the optimization procedure proceeds and material removal takes place, the boolean may assume zero value. Area, moment of inertia and bending moment calculations are done on only those grid points whose boolean equals unity.
2.4. Constitutive relationships

The variations of \( k \), \( E \) and \( \alpha \) with temperature may be obtained by regressing data available in standard tables or charts. In this work, SS 304 has been chosen as the material of construction of the support for which following regression equations were found by taking the data from [10],

\[
k = AT + B \quad (12)
\]

\[
70 < T < 120 : A = 0.0529, \quad B = 3.8502
\]

\[
120 < T < 300 : A = 0.0284, \quad B = 6.8653
\]

\[
E = -0.0001T^2 - 0.0244T + 217.2 \quad (13)
\]

\[
\alpha = 1.2983T - 390.79 \quad (14)
\]

3. Solution Methodology

A rectangular mesh is superimposed on a member of the support system leading to the formation of a rectangular grid as shown in Fig. 3. The grid points are addressed in \((i, j)\) format, where \(i\) is the grid increment number in the horizontal direction and \(j\) in the vertical direction. Eqn. 7 was solved along with Eqn. 12 to obtain the temperature distribution in the support member, as

\[
\frac{A}{2}T^2 + BT = C_1x + C_2 \quad (15)
\]

where \(C_1\) and \(C_2\) are the constants of integration that were obtained from the boundary conditions given by,

at \( x = 0; \quad T = T_{\text{cryo}} \)

at \( x = L; \quad T = T_{\text{amb}} \quad (16)\)

The temperature is used to find the stress due to thermal expansion. Eqn. 6 is used to evaluate the stress distribution as given by Eqn. 17.

\[
[\sigma]_{i,j} = \frac{M_i y_j}{I_i} + \frac{F_{PT}}{A_i} + E\alpha(\Delta T)_i \quad (17)
\]
3.1. Uncertainty analysis
The uncertainty involved in this procedure owing to the grid size selection is determined by Richardson’s extrapolation method [11, 12]. The formula for the same is given as,

$$\varepsilon_n = \frac{\phi_n - \phi_{an}}{\beta - 1}$$  \hspace{1cm} (18)

4. Results and discussion
The model has been applied to analyse two axial supports employed in a cylindrical dewar. The operating conditions imposed in this work are given in Table 1. The total horizontal and vertical loads are the sum of their static and dynamic components[13]. Grid independence was performed using uncertainty analysis and the model equations were solved using 170 × 170 grid. The evolution of the stress distribution and geometry are shown in Fig. 4. The initial stress distribution shows higher stress at the cylinder surface than at the center. This indicates the presence of inefficient material at the center. On implementation of TO, it is observed that material from the central region gets removed, eventually leading to a pipe like geometry. As seen from Table 2, the final geometry of the support has lower volume (and hence mass) and allows lower heat transfer through the support. The uncertainty in the result is $3.906 \times 10^{-6}$ m$^3$ of volume removed.

| Parameter                  | Value |
|----------------------------|-------|
| Length of the chunk (mm)  | 300   |
| Diameter of the chunk (mm)| 54    |
| Temperature of the cryogen (K) | 70 |
| Ambient temperature (K)    | 300   |
| Total horizontal load (N)  | 10000 |
| Total vertical load (N)    | 10000 |
| Density of SS 304 (kg/m$^3$)| 8000 |

Table 2: Comparison of the volume, mass and heat transfer through the support

|                         | Volume ($\times 10^{-4}$m$^3$) | Mass ($\rho \times $Volume)(kg) | Heat transfer($\times 10^{-5}$W/mK) |
|-------------------------|---------------------------------|---------------------------------|-----------------------------------|
| Before TO               | 6.87                            | 5.49                            | 2.84                              |
| After TO                | 2.41                            | 1.92                            | 1.023                             |

5. Conclusion
Topology optimization has been used to arrive at an optimized geometry for a dewar support system. The effects of pre-tension, bending loads and thermal contraction/expansion have been considered while determining the total stress in the continuum. A reduction of 65% in mass and 64% in heat transfer has been obtained in the study. The established method may be extended for other loading conditions and for multi-objective optimization.
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