THE PATH FROM CHEMICAL TO THERMAL FREEZE-OUT

A. S. Kapoyannis
University of Athens, Division of Nuclear and Particle Physics,
GR-15771 Athens, Greece

Abstract

The evolution of a hadronic system after its chemical decomposition is described through a model that conserves the hadronic multiplicities to their values at chemical freeze-out. The state of the system is found as function of temperature and the corresponding baryon density is evaluated. The baryon density at thermal decoupling is also computed.

PACS numbers: 25.75.-q, 12.40.Ee, 05.70.Ce, 12.38.Mh
Keywords: chemical, thermal, freeze-out, hadron gas, baryon density
1. Introduction

Thermal approaches have extensively been used to describe the particle multiplicities which emerge from high energy collisions [1-16]. The results of such approaches are satisfactory since they are able to predict a large number of different experimentally measured hadronic multiplicities as functions of a few thermodynamic variables, such as temperature, volume and chemical potentials.

The extracted parameters from such approaches are associated with “chemical freeze-out”, i.e. the point where the chemical composition of the fireball is fixed. After this stage the particles continue to interact until their momentum distribution is fixed, as well. This second point is called “thermal freeze-out”.

Since a set of experimentally measured abundances define the chemical freeze-out point these abundances have to remain fixed through the whole cooling process until thermal freeze-out. After all, the particles are measured once freeze-out has been completed.

Various authors have used models of thermally equilibrated relativistic hadronic abundances to determine the chemical freeze-out parameters. In some of these models the hadrons are non-interacting particles [2-7] and in others a kind of interaction among them has been included [1,8-16]. In this paper the main focus will be on a model of a non-interacting hadronic gas formulated in the grand canonical ensemble, called “Ideal Hadron Gas” (IHG) [2-5]. It will be argued that in this model, as in the rest of thermal models, it is not possible to fulfil the requirement that all the particle multiplicities will remain fixed for the whole way from chemical decoupling to thermal freeze-out. Considering for example IHG, the partition function is expressed as function of \((V, T, \{\lambda\})\) where \(\{\lambda\}\) are fugacities associated with quantum numbers, such as Baryon Number, \(B\), Charge, \(Q\) and Strangeness, \(S\), as well as the departure from absolute chemical equilibrium. In this particular case the total number of these fugacities is limited to at most five. Not all of them are independent since quantities such as \(<B>\), \(<Q>\), \(<S>\) have to remain fixed, limiting the total number of independent parameters to four. The temperature at thermal freeze-out point is generally different from the chemical freeze-out one. So it will be impossible for someone who will use the same model at thermal freeze-out point to have all the multiplicities fixed to their chemical freeze-out values. The existing free parameters will not be enough.
In this work the necessity to have fixed particle numbers will be used to construct a thermal model which will determine the evolution of the hadronic system after its chemical freeze-out. With the use of this model the construction of the “path” followed by the system in the diagram of temperature as function of baryon density will be possible.

2. The model and its application

In the context of IHG the grand canonical partition function, formulated in the Boltzmann approximation, has the form

\[
\ln Z(V, T, \{\lambda\})_{IHG} = V \sum_i \lambda_{QN_i} \sum_j Z_{H_{ij}}(T) \equiv V \sum_i \lambda_{QN_i} \sum_j \frac{T}{2\pi^2} g_{ij} m_{ij}^2 K_2\left(\frac{m_{ij}}{T}\right),
\]

where \(i\) runs over all hadronic families such as mesons, \(N\) Baryons, \(\Lambda\) Baryons, etc. and \(j\) represents the specific member of the family with degeneracy factor \(g_{ij}\) and mass \(m_{ij}\). \(\lambda_{QN_i}\) stands for the product of all the fugacities associated with the particular family. These fugacities can either be quantum numbers fugacities related to Baryon number, Strangeness, etc. or to quark flavour\(^1\).

Using the above relation one can evaluate particle abundances if one extends the partition function by introducing a fugacity \(\lambda_{ij}\) for every particle. After calculating the particle number one has to set \(\lambda_{ij} = 1\) \([17]\), so again the particle number is only expressed as function of the quantum numbers fugacities.

Now, if someone wishes to keep the particle numbers fixed, e.g. at their chemical freeze-out values, it is only natural to use the fugacities \(\lambda_{ij}\), but with the difference that they are allowed to be \(\lambda_{ij} \neq 1\). This model is called Fixed Particle Numbers (FPN) model and accordingly the partition function depends on \(\lambda_{ij}\)’s

\[
\ln Z(V, T, \{\lambda\})_{FPN} = V \sum_{ij} \lambda_{H_{ij}} Z_{H_{ij}}(T) \equiv V \sum_{ij} \lambda_{H_{ij}} \frac{T}{2\pi^2} g_{ij} m_{ij}^2 K_2\left(\frac{m_{ij}}{T}\right),
\]

where \(\lambda_{H_{ij}}\) is product of quantum numbers as well as particle number fugacities\(^2\). The mean particle number can be evaluated through the relation

\[
<N_{ij}> = \lambda_{ij} \frac{\partial \ln Z(V, T, \{\lambda\})_{FPN}}{\partial \lambda_{ij}} \bigg|_{\{\lambda\} \neq \lambda_{ij}},
\]

\(^1\)For example, for \(\Xi^-\) Baryons, \(\lambda_{QN}\) would read \(\lambda_B \lambda_Q^{-1} \lambda_{S^{-2}} \gamma_s^2\) or \(\lambda_d \lambda_{S^{-2}} \gamma_s^2\). One can look for example in \([16]\), eq. 14, to find out how the two sets of fugacities are related.

\(^2\)For example, for \(\Xi(1530)^-\) Baryon, \(\lambda_H\) would read \(\lambda_B \lambda_Q^{-1} \lambda_{S^{-2}} \gamma_s^2 \lambda_{\Xi(1530)^-}\).
where \( \{\lambda\} \neq \lambda_{ij} \) means that all fugacities except \( \lambda_{ij} \) are considered as constants.

A thermal model, like IHG, can then be used to extract the set of chemical freeze-out parameters \( \{V, T, \lambda_B, \lambda_S, \ldots\} \) which best fit a set of experimentally measured multiplicities. With these given parameters the numbers \( < N_{ij} > \) of all particles that compose the hadronic gas can be calculated. All these numbers have to stay fixed during the evolution of the system after chemical freeze-out, so \( \lambda_{ij} \) will be allowed to become different than one. With the use of (3) this requirement takes the form

\[
<N_{ij} > =< N'_{ij} > \Leftrightarrow V \lambda_{ij} Z_{H_{ij}}(T) = V' \lambda_{ij} Z_{H_{ij}}(T') \Leftrightarrow \lambda_{ij} = \frac{V \lambda_{ij} Z_{H_{ij}}(T)}{V' Z_{H_{ij}}(T')} . \tag{4}
\]

In the last equation \( \lambda_{ij} \) may contain quantum numbers fugacities as well the particle number fugacity. As it is shown in the Appendix it is not possible to evaluate each quantum number fugacity, but this is irrelevant since the full product of fugacities can be calculated. Let me point out that all quantum numbers are automatically conserved because they are linear combination of the particle numbers.

The volume of the system at chemical freeze-out, \( V \), on the other hand, need not necessarily stay fixed. If \( V' \) at temperature \( T' \) is different from \( V \), then all fugacities given by (4) depend on a multiplicand factor \( \frac{V'}{V} \), which cannot be determined from the constraints imposed by the conservation of particle numbers. An additional constraint has to be applied. For example conservation of entropy can be assumed.

The entropy of the system can be calculated from

\[
\tilde{S} = - \left( \frac{\partial\left[-T \ln Z(V, T; \{\mu\})\right]}{\partial T} \right)_{V;\{\mu\}} , \tag{5}
\]

where \( \mu \) represents the chemical potential associated with fugacity \( \lambda = \exp(\mu/T) \). Applying

\[\text{The primed variables in this paper will generally be related to subsequent points of the chemical freeze-out point.}\]

\[\text{A lot of authors assume isentropic evolution of the system, e.g. see [19].}\]

\[\text{The symbol of entropy is tilded in order not to be confused with the symbol of Strangeness. } K \text{ can be set equal to one.}\]
(5) to the partition function (2) the constraint of fixed entropy will read

\[ \tilde{S} = \tilde{S}' \iff \]

\[ \iff \ln Z(V, T, \{\mu\}) + VT \sum_{ij} \lambda_{QN_i} \frac{\partial Z_{H_{ij}}(T)}{\partial T} - VT \sum_{ij} \lambda_{QN_i} \frac{\mu_{QN_i}}{T^2} Z_{H_{ij}}(T) = \]

\[ = \ln Z(V', T', \{\mu'\}) + V'T' \sum_{ij} \lambda_{H_{ij}} \frac{\partial Z_{H_{ij}}(T')}{\partial T'} - V'T' \sum_{ij} \lambda_{H_{ij}} \frac{\mu_{H_{ij}}}{T'^2} Z_{H_{ij}}(T') . \]

With the use of (4) the last equation becomes

\[ VT \sum_{ij} \lambda_{QN_i} \frac{\partial Z_{H_{ij}}(T)}{\partial T} - \sum_{ij} \lambda_{QN_i} \ln(\lambda_{QN_i}) Z_{H_{ij}}(T) = \]

\[ = V'T' \sum_{ij} \frac{V\lambda_{QN_i} Z_{H_{ij}}(T)}{V'Z_{H_{ij}}(T')} \frac{\partial Z_{H_{ij}}(T')}{\partial T'} - V' \sum_{ij} Z_{H_{ij}}(T') \frac{V\lambda_{QN_i} Z_{H_{ij}}(T)}{V'Z_{H_{ij}}(T')} \ln(\frac{V\lambda_{QN_i} Z_{H_{ij}}(T)}{V'Z_{H_{ij}}(T')}) \iff \]

\[ \iff T \sum_{ij} \lambda_{QN_i} \frac{\partial Z_{H_{ij}}(T)}{\partial T} - \sum_{ij} \lambda_{QN_i} \ln(\lambda_{QN_i}) Z_{H_{ij}}(T) = \]

\[ = T' \sum_{ij} \frac{Z_{H_{ij}}(T)}{Z_{H_{ij}}(T')} \frac{\partial Z_{H_{ij}}(T')}{\partial T'} - \sum_{ij} \lambda_{QN_i} \frac{Z_{H_{ij}}(T)}{Z_{H_{ij}}(T')} \ln(\frac{V\lambda_{QN_i} Z_{H_{ij}}(T)}{V'Z_{H_{ij}}(T')}) . \]

Setting \( x \equiv \frac{V'}{V} \), (6) can be solved for \( x \) to give

\[ x = \exp \left[ \frac{\sum_{ij} \lambda_{QN_i} Z_{H_{ij}}(T) \ln(Z_{H_{ij}}(T)/Z_{H_{ij}}(T')) + T \sum_{ij} \lambda_{QN_i} \frac{\partial Z_{H_{ij}}(T)}{\partial T} - T' \sum_{ij} \lambda_{QN_i} \frac{Z_{H_{ij}}(T)}{Z_{H_{ij}}(T')} \frac{\partial Z_{H_{ij}}(T')}{\partial T}}{\sum_{ij} \lambda_{QN_i} Z_{H_{ij}}(T)} \right] . \]

(7)

Equation (7) can be used to evaluate the volume expansion ratio as the system has cooled to a temperature \( T' \) less than the chemical freeze-out temperature \( T \). With the use of the same equation, quantities like the baryon density of the system can be calculated at \( T' \). One has to remember that baryon number is also fixed with the imposition of the constraints (4).

So

\[ n_B = \frac{<B'>}{V'} = \frac{<B>^{ch}}{V'} = \frac{V}{V'} \cdot \frac{<B>^{ch}}{V} = \frac{n_B^{ch}}{x} . \]

(8)

\[ ^6 \text{The IHG partition function (1), where } \lambda_{ij} = 1, \text{ can be used for the evaluation of the entropy at the chemical freeze-out point.} \]

\[ ^7 \text{All hadrons with masses up to 2400 MeV are included in the calculations corresponding to FPN and to IHG models.} \]
Other constraints which have to be applied to the system and are connected to quantum numbers, like
\[ < S > = 0 , \quad \frac{< B >}{< Q >} = \left( \frac{< B >}{< Q >} \right)^{ch} , \quad < |S| > = < |S| >^{ch}, \] (9)
are also satisfied.

Thus, the contour which is followed by the system after the fixation of its chemical composition until its thermal freeze-out can be evaluated. This contour can be defined on a \((T, n_B)\) plane with the use of eqs. (7) and (8) but not on a \((T, \mu_B)\) plane for the reasons explained in the Appendix.

As an example the model is used to depict the path followed by hadronic systems which have been formed at different interactions at SPS after their chemical freeze-out. The chemical freeze-out parameters used, along with the corresponding references are listed in Table 1. From a variety of thermal analyses performed by different authors the particular ones have been chosen because they allow for partial strangeness equilibrium \((\gamma_s \neq 1)\) and they use most recent available values for the experimentally measured hadronic multiplicities. The values of Table 1 are then used, for each interaction separately, as input to the equations
\[ < S > = 0 \quad \text{and} \quad \frac{< B >}{< Q >} = \beta \] evaluated through IHG, to determine the rest of the fugacities. Thus the whole set of chemical freeze-out parameters \((T, \mu_B, \mu_Q, \mu_S, \gamma_s)\) are calculated and also the products of fugacities \(\lambda_{QN_i}\) in (1) are also set.

Giving different values to temperature \(T\), equation (8) can be used to calculate the corresponding baryon density. The resulting paths for \(S+S\), \(S+Ag\) and \(Pb+Pb\) interactions are shown in Figure 1. For the \(Pb+Pb\) interaction the thermal freeze-out temperature is calculated in Refs. [21] and [22]. For these values baryon density at thermal freeze-out \(n_B^{ther}\) can be evaluated. The results are listed in Table 2. The path for \(Pb+Pb\) is followed until the lower temperature (of the two given in Refs. [21,22]) is reached.

In order to compare FPN with an IHG model which presents the closest characteristics with it, points that are subjected to the constraint that the baryon number and the entropy is fixed \((< B > = < B >^{ch} \quad \text{and} \quad \tilde{S} = \tilde{S}^{ch})\) are also depicted on Figure 1. Let me emphasise

\[ ^{8}\beta \quad \text{is fixed from the baryon number and charge of the participant nucleons, e.g. see [16].} \]

\[ ^{9}\text{The rest of the constraints on quantum numbers, like } < S >= 0, \text{ etc., are applied as well.} \]
that at these points the particle numbers are not conserved. But the IHG points have no problem to be depicted on a \((T, \mu_B)\) plane. This is done in Figure 2. On this Figure there is also depicted an “equivalent” value of baryon chemical potential \(\mu_{Beq}\) for FPN model as function of temperature. This chemical potential is calculated through IHG and the only connection it has with FPN is that it gives the same baryon density

\[ n_B(T, \mu_{Beq})_{IHG} = n_B(T)_{FPN}. \] (10)

For comparison with FPN, points that correspond to calculations through IHG for the given thermal freeze-out temperatures of Table 2 have also been depicted on Figs 1 and 2.

Finally in Figure 3 the ratio \(x = V'/V\), where \(V\) is the chemical freeze-out volume, is plotted as function of temperature for FPN and for IHG (with fixed entropy and baryon number).

3. Conclusion

After chemical freeze-out the collisions among hadrons that compose the hadronic gas can no longer change its chemical composition. Following this requirement an ideal hadron gas model (FPN) has been presented that keeps the multiplicity of every particle fixed to the value dictated by the chemical freeze-out conditions. In the context of FPN the constraints of conservation of quantum numbers are broken up to a larger number of constraints, these of conservation of particle numbers. The chemical potentials of quantum numbers are no longer “good” variables to describe the evolution of the system. Of course the fugacities of particle numbers used as variables in FPN are not “free” parameters. Their values are fixed from the given set of the quantum numbers fugacities at chemical freeze-out. So the evolution of a hadronic system is described as function of temperature and baryon density (after imposing conservation of entropy). This is done for three SPS interactions.

Following this evolution and using values of thermal freeze-out temperature extracted for the \(Pb+Pb\) interaction the baryon density at freeze-out is evaluated. As the temperature at thermal decoupling for various interactions can be calculated using transverse mass spectra or HBT analysis [23] the same procedure can be applied to evaluate the corresponding baryon density before free streaming for these interactions. Finally let me point out that the necessity to keep the particle multiplicities fixed after chemical freeze-out can be fulfilled to
any thermal model, apart from IHG, through the use of the particle fugacities.

**Acknowledgement** I would like to thank Professor N. G. Antoniou and Professor C. N. Ktorides for reviewing the manuscript and for useful remarks.

**Appendix**

It will be argued that an ambiguity presents itself when someone tries to evaluate the quantum numbers fugacities. Let us suppose that the system is initially in a state which is determined by volume $V$, temperature $T$, quantum fugacities $\lambda_B, \lambda_S, \cdots$ and particle number fugacities $\lambda_{ij}$. The question that arises is whether it is possible to determine the thermodynamic variables connected to a subsequent temperature $T'$. Normally the new fugacities could be calculated from a set of $n$ equations of the form\[< B(V, T, \{\lambda\}) > = < B'(V', T', \{\lambda\}') > ,\]

\[< S(V, T, \{\lambda\}) > = < S'(V', T', \{\lambda\}') > , \cdots \tag{11}\]

which insure for the conservation of the $n$ quantum numbers and a set of $m$ equations of the form

\[< N_{ij}(V, T, \{\lambda\}) > = < N'_{ij}(V', T', \{\lambda\}') > , \cdots \tag{12}\]

which insure for the conservation of the number of the $m$ particle species that are available in the hadronic gas. If one tries to solve the above set of the $n + m$ equations one will find out that it is impossible to determine all the fugacities. The reason for this is that the $n + m$ equations are not linearly independent. When the particle numbers are fixed, automatically the quantum numbers are fixed as well (the opposite, of course, is not true). For example the equation for the conservation of the baryon number can be expressed as a linear combination of equations for the conservation of the number of certain particle species

\[< B > - < B' > = (< N_n > - < N'_n >) + (< N_p > - < N'_p >) + \cdots \]

\[- (< N_\bar{n} > - < N'_\bar{n} >) - (< N_\bar{p} > - < N'_\bar{p} >) - \cdots . \tag{13}\]

One might think that a way out of the problem of the linear dependency of the equations (11) and (12) would be to reduce the number of particle number fugacities by $n$ in which\[\text{The primed quantities are connected to } T'.\]
case $n$ particle species would be described by only quantum numbers fugacities. Let us suppose for simplicity that the hadronic gas is composed only of particles 1 and 2 and the only relevant quantum number is $B$. If one decided to describe the hadronic gas with the fugacities $\lambda_B$ and $\lambda_2$, then two equations would have to be satisfied

$$\langle B \rangle = \langle B' \rangle \Leftrightarrow V(Z_{H_1}(T)\lambda_B + Z_{H_2}(T)\lambda_B\lambda_2) = V'(Z_{H_1}(T')\lambda_B' + Z_{H_2}(T')\lambda_B\lambda_2')$$  \hspace{1cm} (14)$$

$$\langle N_2 \rangle = \langle N_2' \rangle \Leftrightarrow V(Z_{H_2}(T)\lambda_B\lambda_2) = V'(Z_{H_2}(T')\lambda_B'\lambda_2') .$$  \hspace{1cm} (15)$$

Using (15) in (14) one can solve for the final $\lambda_B'$ to find

$$\lambda_B' = \frac{V}{V'}\lambda_B \frac{Z_{H_1}(T)}{Z_{H_1}(T')} = \frac{VT}{VT'}\lambda_B K_2(m_1/T) \frac{K_2(m_1/T)}{K_2(m_2/T')} .$$  \hspace{1cm} (16)$$

On the other hand if someone had decided to use the set of fugacities $\lambda_B$ and $\lambda_1$ he would arrive in a similar way to the relation

$$\lambda_B' = \frac{V}{V'}\lambda_B \frac{Z_{H_2}(T)}{Z_{H_2}(T')} = \frac{VT}{VT'}\lambda_B K_2(m_2/T) \frac{K_2(m_2/T)}{K_2(m_2/T')} .$$  \hspace{1cm} (17)$$

It is obvious from comparing (16) and (17)\cite{11} that the value of baryon number fugacity depends on the choice of which particle number fugacities are kept. This, of course, is undesirable.

Two alternative choices thereby present themselves. The first is to drop the quantum numbers fugacities after chemical freeze-out and describe the evolution of the system with only the particle number fugacities. The second is to keep the quantum numbers fugacities with the ambiguity that accompanies them. In either case the product of fugacities which accompany the part of the partition function associated with each particle species has no problem to be evaluated. Thus quantities like the baryon density can be calculated.

References

\begin{itemize}
  \item [1] J. Cleymans, H. Satz, Z. Phys. C 57 (1993) 135.
  \item [2] J. Cleymans, K. Redlich, H. Satz, E. Suhonen, Z. Phys. C 58 (1993) 347.
\end{itemize}

\textsuperscript{11}The hadron masses are in general different ($m_1 \neq m_2$).
[3] J. Sollfrank, M. Gaździcki, U. Heinz, J. Rafelski, Z. Phys. C 61 (1994) 659.

[4] J. Letessier, A. Tounsi, U. Heinz, J. Sollfrank, J. Rafelski, Phys. Rev. D 51 (1995) 3408.

[5] A. D. Panagiotou, G. Mavromanolakis, J. Tzoulis, Phys. Rev. C 53 (1996) 1353.

[6] F. Becattini, U. Heinz, Z. Phys. C 76 (1997) 269.

[7] F. Becattini, M. Gaździcki, J. Sollfrank, Eur. Phys. J. C 5 (1998) 143.

[8] D. H. Rischke, M. I. Gorenstein, H. Stöcker, W. Greiner, Z. Phys. C 51 (1991) 485.

[9] R. Hagedorn, Nuovo Cimento Suppl. III (1965) 147.

[10] R. Hagedorn, J. Ranft, Nuovo Cimento Suppl. VI (1968) 169; R. Hagedorn, Nuovo Cimento Suppl. VI (1968) 311.

[11] R. Hagedorn, Nuovo Cimento LVI A (1968) 1027.

[12] R. Hagedorn, J. Rafelski, Phys. Lett. B 97 (1980) 136.

[13] A. S. Kapoyannis, C. N. Ktorides, A. D. Panagiotou, J. Phys. G 23 (1997) 1921.

[14] A. S. Kapoyannis, C. N. Ktorides, A. D. Panagiotou, Phys. Rev. D 58 (1998) 034009.

[15] A. S. Kapoyannis, C. N. Ktorides, A. D. Panagiotou, Phys. Rev. C 58 (1998) 2879.

[16] A. S. Kapoyannis, C. N. Ktorides, A. D. Panagiotou, Eur. Phys. J. C 14 (2000) 299.

[17] R. Hagedorn, K. Redlich, Z. Phys. C 27 (1985) 541.

[18] F. Becattini, M. Gaździcki, J. Sollfrank, Eur. Phys. J. C 5 (1998) 143.

[19] J. Cleymans, K. Redlich, Phys. Rev. C 60 (1999) 054908.

[20] F. Becattini, J. Cleymans, A. Keränen, E. Suhonen, K. Redlich, Phys. Rev. C 64 (2001) 024901.

[21] H. Appelshäuser et al., NA49 Collaboration, Nucl. Phys. A 638 (1998) 91c; Eur. Phys. J. C 2 (1998) 661.
J. R. Nix et al., [nucl-th/9801043]

U. Heinz, J. Phys. G 25 (1999) 263; R. Stock, Nucl. Phys. A 661 (1999) 282c.

Figure Captions

Figure 1 Contours (thick lines) that follow hadronic systems after chemical freeze-out on \((T, n_B)\) plane for 3 interactions at SPS, calculated through FPN (model of Fixed Particle Numbers). On the same graph points (dotted lines) calculated through an IHG model that conserve entropy and baryon number are depicted.

Figure 2 The points of the IHG model of Figure 1 on the \((T, \mu_B)\) plane (dotted lines). The thick lines represent calculation through IHG of the baryon chemical potential that leads for a given temperature to the same baryon density as FPN.

Figure 3 The ratio of the volume \(V'\) of the hadronic system to its volume \(V^\text{ch}\) at chemical freeze-out as function of temperature calculated for the models FPN and IHG of Figure 1.

Table Captions

Table 1 Chemical freeze-out parameters calculated for different interactions at SPS and the corresponding references.

Table 2 Thermal freeze-out temperature calculated in two different references for the \(Pb + Pb\) interaction and the corresponding computation of baryon density through FPN. The upper errors of baryon density correspond to the upper errors of temperature. The same is true for the lower errors.

| Experiment  | \(T^\text{ch}(MeV)\) | \(\mu_B^\text{ch}(MeV)\) | \(\gamma_s^\text{ch}\) | Reference |
|-------------|----------------------|--------------------------|-----------------|----------|
| S+S 200 A·GeV | 180.5 ± 10.9 | 220.2 ± 18.0 | 0.747 ± 0.048 | [18,19] |
| S+Ag 200 A·GeV | 178.9 ± 8.1 | 241.5 ± 14.5 | 0.711 ± 0.063 | [18,19] |
| Pb+Pb 158 A·GeV | 174.7 ± 6.7 | 240 ± 14 | 0.900 ± 0.049 | [20] |
### Table 1.

| Experiment       | $T^\text{ther} (MeV)$ | Reference | $n_B^\text{ther} (fm^{-3})$ |
|------------------|------------------------|-----------|-----------------------------|
| Pb+Pb 158 $A \cdot GeV$ | 120 ± 12               | [21]      | 0.099$^{+0.022}_{-0.019}$   |
| Pb+Pb 158 $A \cdot GeV$ | 95.8 ± 3.5             | [22]      | 0.0627$^{+0.0047}_{-0.0045}$|

### Table 2.
\[ x = V' / V_{ch} \]

- **FPN**
- **IHG**
- **Chemical Freeze-Out**
- **Thermal Freeze-Out**

Temperature, \( T \) (MeV)