Pulsation-nature stresses on flat convergent walls of slot-type hopper under granular medium flow

AP Bobryakov, SV Klishin*, VP Kosykh** and AF Revuzhenko***
Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
E-mail: *sv.klishin@gmail.com, **v-kosykh@yandex.ru, ***revuzhenko@yandex.ru

Abstract. The authors investigate numerically the flow of granular materials in the converging radial channels by solving 3D problem using the finite element method. The pulsation nature of normal and shear stresses on the channel walls is demonstrated. The calculations are compared with the lab-scale experimental data.

1. Introduction
Flow of granular materials in converging channels is involved in many processing in mining: discharge from hoppers and bunkers, in ore passes etc. Direct investigation of this process in full-scale conditions faces some technical difficulties. So, numerical and experimental modeling on a laboratory scale takes on the key role. In this regard, the method of discrete elements acquires increasingly higher popularity in recent years. Laboratory-scale modeling consists of a few steps. The first step is the analysis of similarity criteria, the second step is the modeling and experimentation. The resultant outcome is the kinematics of deformation and failure and the stress distribution. There is a wide range of methods to investigate kinematics, starting from cross-hatching of material surface and finishing with X-ray imaging and various alternatives of PIV (Particle Image Velocimetry) [1, 2].

The issues connected with the stress measurements appear to be more difficult to solve. The experts in this area of research know well that it is a very intricate problem to measure stresses on a model (a fortiori in situ). Neither universal and common procedure nor equipment has been developed. This paper puts forward a variant of the specified problem solution.

2. Discrete element modeling
First, let us discuss issues of theoretical analysis. It is now popular to investigate kinematics using different variants of simulation models of flow [3, 4]. In this case, a cell is assigned, from which a material flows out. Then, certain probabilities of filling the vacant cell by the material from the neighbor cells are set. As a result, based on the very simple rules, it is possible to obtain rather complex patterns of flow, which, in some measure, simulate a real process (a sort of Conway’s Game of Life). However, this approach is unsuitable in terms of stresses. It is required to use closed models that account for stresses. An adequate and simulation-like model is the discrete element model [5–7]. This method is an alternative in principle to classical methods based on conventional continuum mechanics and is widely used this time to study various modes of flow of granular materials.

Figure 1 shows an intermediate result of the 3D calculation by DEM. The figure demonstrates the kinematic pattern of deformation of a flowing medium and distribution of horizontal velocities of discrete elements at fixed timing. The analysis of the number of discrete elements required for the
construction of continuum model equivalent to the assumed discrete element model was performed by
the authors in [7]. It was shown that the particles to be included in the numerical experiment varied
from $3 \times 10^4$ to $3 \times 10^9$. The continuum model is beyond the scope of this paper but the total number of
particles in the laboratory experiment is estimated as $57 \times 10^6$. The numerical model includes 350 000
particles. Apparently, it is impossible to parallel a laboratory and a numerical experiment at the
modern stage of computational technologies. The advantage of a numerical model is the possibility to
calculate coordinates, trajectories and displacements of particles, as well as forces and stresses at each
time step, which is problematic in the in situ measurement. Figure 2 shows epures of the normal forces
$F_n$ and tangential forces $F_t$ applied by granular material on a side wall of a hopper at a fixed moment
of time. It is seem that the stress distribution is nonstationary as in the experiments described below in
this paper.

![Figure 1](image1.png)

**Figure 1.** Kinematic pattern of deformation and distribution of horizontal PPV $v_x$ in
granular material at fixed time obtained from numerical calculation using discrete element
method.

![Figure 2](image2.png)

**Figure 2.** Distribution of the (a) normal and (b) tangential forces applied by granular
material on hopper wall at fixed time moment. The coordinate origin fist with the discharge
opening.

3. Stress measurement

Granular media possess elastoplastic properties and internal friction. The latter property is reflected by
the stiffness of sensors. Correct measurement is only possible with the sensors possessing maximum
stiffness.

On the other hand, the increase in the stiffness of elastic elements of sensors lowers their
sensitivity down to a level comparable with the noise of measurement equipment. Thus, a problem of
stress measurement in granular medium is design of sensors of sufficient stiffness.

Some designs of normal stress sensors possessing sufficient stiffness and sensitivity are known
from literature. For example, hydraulic and mechanical pressure transducers are described in [8, 9].
The other designs are presented in [10–13]. General deficiency of the known sensor designs is the size
and complexity. Such sensors are not always suitable for the analysis of properties of granular
materials and soil under low stresses, for instance, on a laboratory scale. In this study, the authors used
the original sensors possessing admissible sensitivity for the investigation of stress state of granular
material in the laboratory conditions.
The sensitive element of a normal stress sensor was a spring steel beam rigidly attached to a steel frame on two sides. As a strain transducer, a strain gauge rosette was glued on the inner side of the beam. The size of the rosette was selected so that strain gauges on the opposite sides extended and constricted pairwise to ensure the maximum sensitivity of the sensor.

The temperature error of the sensor was decreased by making the strain gauges of a material having the temperature compensation factor conformable with the heat expansion of the beam.

The shear stress sensor [14] determines pulling forces under shear of two plates pressed to each other and placed in the granular medium. The sensor (Figure 3) has a body composed of cover 1 and bottom 2 which are simultaneously the force-transmitting elements. For pre-tensed strain gauges 3 are arranged between the cover and bottom so that one their ends are glued in the center of the cover 1 and their other ends are glued at the opposite edges of the bottom 2. Friction between the bottom and cover is reduced by means of steel balls 4 with a diameter of 0.2 mm placed between the cover and bottom. Sealing of the sensor is ensured by the thin elastic shell 5 through which contacts 6 are let out to be connected to the measurement equipment.

![Figure 3](image1.png)

**Figure 3.** Shear stress sensor design 1—cover; 2—bottom; 3—strain gauges; 4—steel balls; 5—elastic shell; 6—contacts of the strain gauges.

This design totally eliminates the influence of normal stresses on the measurement of shear stresses, and heat compensation is reached thanks to the close-spaced location of all strain gauges.

The sensors are connected to LTR-212-M1 modules of crate data acquisition system L-Card. The crate data acquisition system allows amplification, digitalization and synchronous recording of signals.

### 4. Flow of granular materials in radial channels

Figure 4 offers a general view of the experimental slot-type hopper with the flat convergent walls 1 and the flap gate 2 of the outlet slot. On the inner surface of the transparent wall 3, a thin tube 4 filled with a colored granular material is attached. When involved in motion, the colored particles generate a marker stream to visualize the trajectory of the flow.

The stress measurement is carried out by four shear stress sensors and four normal stress sensors. On the visible wall of the hopper, sensors $\tau_1\sigma_1$ and $\tau_2\sigma_2$ are arranged at the top and the bottom, respectively; sensors $\tau_3\sigma_3$ and $\tau_4\sigma_4$ are placed at the top and bottom of the opposite wall. Thus, $\tau_1\sigma_1$ and $\tau_3\sigma_3$, as well as sensors $\tau_2\sigma_2$ and $\tau_4\sigma_4$ are arranged on the opposite walls of the hopper at the height of 235 and 140 mm, respectively.

![Figure 4](image2.png)

**Figure 4.** Slot-type hopper, general view: 1—convergent walls; 2—discharge hole gate; 3—transparent front wall; 4—source nozzle of marker stream. Hopper height $H = 310$ mm, hopper width $D = 52$ mm, upper boundary length $L = 182$ mm.
The tests involved quartz sand with particles 0.3 and 1.5 mm in size. The outlet hole size was 1.5×52 mm for particles of 0.3 mm and 3×52 mm for particles of 1.5 mm.

Regarding steady-state conditions of discharge, it is known that in a granular material, owing to pores that are nonuniformly spread in the material volume, there is always a scatter in the mechanical properties, which results in bad reproducibility of test results. This, first of all, relates the material fill conditions since fill in the mode of ‘rain’ or ‘jet’ can offer both dense and loose packing of particles. It is possible to improve structural uniformity and to stabilize mechanical parameters of a medium by subjecting it to low-amplitude alternating shears \[15, 16\]. In the presented studies, aimed to stabilize the experimental result on granular material discharge from hopper, it was provided that the material occurred in stationary conditions during flowing. After filling the hopper and opening the discharge hole gate, the material was continuously fed in so that the hopper was permanently filled. The feed-in was carried out through a continuous dosing unit (not shown in Figure 4). The constant weight of the material ensured stable average pressure on side walls of the hopper and the continuous discharge velocity.

The tests show that the discharge velocity of quartz sand 1.5 mm in size with the feed-in from the dosing unit is constantly 14.7 g/s, while without the feed-in the material discharge velocity grows with the height of the material column in the hopper. In the end, the average velocity makes 11.43 g/s.

Adherence to the condition of continuity was checked by simultaneous shutting the gate and the dosing unit with the subsequent weighing of the material in the hopper at the fixed moments of time. The average weight of seven samples was 2059 g at the average deviation of not higher than 6%.

So, the introduction of a stabilizing continuous-action dosing device with the adjustable capacity has made it possible to reach almost steady-state condition of the granular medium in the hopper. The average bulk density of sand is 1.4 g/cm\(^3\) and porosity is 45%.

Figure 5 shows characteristic epures of normal and shear stresses (Figures 5a and 5b, respectively) measured by the bottom sensors \(\tau_2\sigma_2\) and \(\tau_4\sigma_4\) (see Figure 4) during discharge of quartz sand particles 0.3 mm in size.

![Figure 5](image)

**Figure 5.** Diagrams of (a) normal and (b) shear stresses measured under flow of quartz particles of 0.3 mm by the bottom sensors \(\tau_2\sigma_2\) and \(\tau_4\sigma_4\).

The curves display the pulsation nature of the stresses during the discharge. The maximum amplitude of the normal stresses is not more than 5% of the static pressure while the amplitude of the shear stresses is 70% of the average value.
It is seen that the external stationary conditions do not make the stresses stationary. Accordingly, the flow is not stationary, either. This is evident in the kinematic patterns of the flow, too.

(a)  
(b)  
(c)

**Figure 6.** Deformation of the colored particle flow at the fixed time moments.

The flow is associated with the complex processes of plane shearing, localization and clustering of strains, generation of new flows and the change in their direction [17]. All that induces redistribution of stresses, generation of new stresses and their re-orientation. As an illustration, Figure 6 shows the pictures of the marker stream deformed by the flow of quartz sand with particles 0.3 mm in size. The pictures are made at the lag of 10 s. It is seen that the flow toward the discharge hole is nontrivial. At a distance of 120 mm from the hopper top, a new structure is formed, and the marker stream begins bending and stretching on either side of the direction of gravity alternatively. The new flow trajectory is characterized by the change of direction in time.

The change in the length of the straight-line portion of the marker stream versus the flow time is depicted in Figure 7. It is seen that the flow is uniform and straight-line in this portion in a relatively long distance making 1/3 of the hopper height $H$. In Figure 6 the level beneath which the change in the stream direction starts is marked by the horizontal line.

![Figure 7](image)

**Figure 7.** Change in the length of the straight-line portion of the marker stream versus time.

Repeatability of the straight-line portion length and the related flow time are illustrated in Figure 8 by the data of seven identical tests with formation of a straight marker stream of maximum length $AB$.

![Figure 8](image)

**Figure 8.** Formation of straight-line portion of marker stream in seven tests.
The average velocity of the marker stream in the tests was 1.69 at the standard deviation of 0.05 mm/s and maximum deviation of 6.5%.

5. Conclusions
1. The three-dimensional variant of DEM is an adequate approach to calculating gravity flows of granular media.
2. The lab-scale experiments using original stress sensors display the pulsation nature of stresses.
3. The comparison of the experimental and calculated parameters of flow shows their good agreement.

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