Limiting symmetry energy elements from empirical evidence

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Abstract

In the framework of an equation of state (EoS) constructed from a momentum and density-dependent finite-range two-body effective interaction, the quantitative magnitudes of the different symmetry elements of infinite nuclear matter are explored. The parameters of this interaction are determined from well-accepted characteristic constants associated with homogeneous nuclear matter. The symmetry energy coefficient $a_2$, its density slope $L_0$, the symmetry incompressibility $K_\delta$ as well as the density dependent incompressibility $K(\rho)$ evaluated with this EoS are seen to be in good harmony with those obtained from other diverse perspectives. The higher order symmetry energy coefficients $a_4$, $a_6$ etc are seen to be not very significant in the domain of densities relevant to finite nuclei, but gradually build up at supra-normal densities. The analysis carried with a Skyrme-inspired energy density functional obtained with the same input values for the empirical bulk data associated with nuclear matter yields nearly the same results.

Keywords: effective interaction, nuclear matter, equation of state, symmetry energy

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I. INTRODUCTION

Much attention has recently been drawn to a precise understanding of the different aspects of nuclear symmetry energy. For nuclei with extreme isospins they are the predominant factors in determining their stability and the nucleon distributions therein \([1–3]\). In astrophysics, they have seminal influence on the size, critical composition and maximum mass of neutron stars \([4, 5]\). The dynamical evolution of the core collapse of a massive star and the associated explosive nucleosynthesis \([6, 7]\) also depend sensitively on them.

Nuclear symmetry energy is the energy cost in converting asymmetric nuclear matter to a symmetric one. It is defined as

\[
e_{\text{sym}}(\rho, \delta) = e(\rho, \delta) - e(\rho, \delta = 0),
\]

where \(e\) is the energy per nucleon of nuclear matter, \(\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)\) is the nuclear asymmetry and \(\rho_n\) and \(\rho_p\) are the neutron and proton densities with \(\rho_n + \rho_p = \rho\). Expanding \(e(\rho, \delta)\) in powers of \(\delta\) around \(\delta = 0\) and keeping only the even powers of \(\delta\) (because of charge symmetry), one has

\[
e_{\text{sym}}(\rho, \delta) = a_2\delta^2 + a_4\delta^4 + a_6\delta^6 + \cdots,
\]

where

\[
a_2 = \left[\frac{1}{2} \frac{\partial^2 e_{\text{sym}}(\rho, \delta)}{\partial \delta^2}\right]_{\delta = 0},
\]

\[
a_4 = \left[\frac{1}{4!} \frac{\partial^4 e_{\text{sym}}(\rho, \delta)}{\partial \delta^4}\right]_{\delta = 0},
\]

\[
a_6 = \left[\frac{1}{6!} \frac{\partial^6 e_{\text{sym}}(\rho, \delta)}{\partial \delta^6}\right]_{\delta = 0},
\]

and so on.

Traditionally since the days of Bethe and Weizäcker \([8, 9]\), only the first term in the expansion (2) has been considered for symmetry energy. If so, the coefficient of symmetry energy as obtained from the double derivative of \(e_{\text{sym}}(\rho, \delta)\) is true for any value of \(\delta\) and the symmetry energy can then be taken as

\[
e_{\text{sym}}(\rho) = e(\rho, \delta = 1) - e(\rho, \delta = 0),
\]

(6)
which has been resorted to by some in its definition \[10\]. At low density when matter becomes clusterized, the two definitions given by Eqs. \[11\] and \[12\] show different behavior \[11\]. For homogeneous nuclear matter, however, up to around $\rho_0$, the symmetry energy $e(\rho, \delta)$ shows nearly a perfect linearity in $\delta^2$ \[11, 12\] in microscopic calculations with different energy density functionals (EDF) used to explain nuclear properties corroborating the Bethe-Weizäcker ansatz.

Even if terms beyond $\delta^2$ in Eq.\( (2) \) are unimportant for accounting the symmetry energy at normal density, at supra-normal densities, they can not be ignored as has recently been shown in calculations with Skyrme EDFs \[13\]. Mean-field calculations in a nonlinear relativistic framework \[14\] suggest also such an outcome. These higher order terms are important to reasonably describe the proton fraction of $\beta$-stable nuclear matter at high densities and the core-crust transition density in neutron stars \[15\].

In contrast to the generally accepted idea that terms beyond $\delta^2$ are relatively unimportant in symmetry energy at normal density, a recent analysis of the double differences of ‘experimental’ symmetry energies of neighboring nuclei \[16, 17\] indicates that the higher order terms in symmetry energy for finite nuclei may be sizeable even at saturation density. However, no firm conclusions could be drawn because of the model dependence in evaluating the nuclear masses. With the standard Skyrme energy density functionals the fourth order term (with $\delta^4$) comes out to be negative from the binding energy formula \[18\], whereas the latest Weizäcker-Skyrme formula and the extracted value \[16\] from the experimental data suggest positive values for this coefficient. In this context, a reexamination of the importance of the higher order terms in symmetry energy for infinite nuclear matter is called for.

The present communication is aimed towards that purpose.

Employing variants of the Bethe-Weizäcker mass formula, attempts were made to extract the value of the symmetry energy of nuclear matter from the known experimental nuclear masses \[13, 20\]. The symmetry energy of a finite nucleus has two components, the volume and the surface one. The volume term relates to the symmetry energy coefficient of infinite nuclear matter at the saturation density $\rho_0$, the surface term comes from finite-size effects. Extraction of the volume part of the nuclear symmetry energy from nuclear masses suffers some ambiguity because of the interference of the surface term. The nuclear binding energies may be well represented, but the volume and surface symmetry terms may vary over a considerable range \[16, 21\], a large volume term is compensated by a large surface term and
Microscopic theories built out of effective two-nucleon interactions \[22\] structured to explain selective experimental data have not yet been able to completely address the problem of properly delineating the symmetry elements of nuclear matter from finite nuclear properties. For example, both the relativistic NL3 interaction \[23\] and the non-relativistic BSk24 \[24\] give very good fit to the nuclear masses, but the symmetry element \(a_2\) at \(\rho_0\) in the former case is 37.4 MeV, in the latter case it is 30 MeV. The density slope of symmetry energy at \(\rho_0\), namely \(L(\rho_0)\) (= \(L_0\) defined as \(3\rho_0 \frac{\partial a_2(\rho)}{\partial \rho}\) ) varies even more significantly, \(L_0 = 46.4\) MeV for the BSk force, but is 118.5 MeV for the NL3 interaction. There is thus no clear consensus on the values of the different symmetry elements pertaining to nuclear matter from microscopic theories \[12\], though they are largely successful in fitting diverse experimental data.

Through the maze of different experimental facts and their theoretical analyses, some empirical constants related to nuclear matter, however, have emerged that seem to lie in nearly tight limits. They are the saturation density \(\rho_0\) of symmetric nuclear matter (SNM) and its energy per nucleon \(e_0\) at that density \[12, 20, 25–28\]. The nuclear incompressibility \(K_0\) of SNM at \(\rho_0\) have been progressively refined and is now relatively well constrained \[29–31\]. We choose these empirical data as benchmarks to fix the isoscalar part of the effective interaction that would be used to explore nuclear matter properties. For the proper feel of the isovector component, we exploit an empirically observed characteristic of pure neutron matter (PNM). From a large number of ’best-fit’ EDFs \[32\] built in the Skyrme framework, it has been seen that the value of energy per particle for PNM at density \(\rho = 0.1\) fm\(^{-3}\) is practically the same, \(e_n \sim 10.9\) MeV. This is another benchmark we take recourse to. Incidentally, this value of \(e_n\) is in extremely good consonance with that obtained for PNM from the most realistic microscopic potential model calculations of Akmal and Pandharipande \[26\] and Akmal, Pandharipande and Ravenhall \[33\]. The agreement of this value for \(e_n\) is also excellent with that obtained from the ab initio advanced microscopic calculation by Baldo et al \[27\] within the Kohn-Sham density functional framework. The neutron-matter data is chosen so that extrapolation to highly asymmetric matter becomes reliable. In addition to the above benchmark empirical data, the value of the effective mass of the nucleon \(m^*(\rho_0)/m\) for SNM at saturation density is taken as a given input. The parameters describing the effective interaction can then be calculated from the given...
conditions. The value of $m_0^*/m$ (from now on, we write $m_0^*$ for $m^*(\rho_0)$) is constrained such that the observed maximum mass $M_{NS}^{max}$ of the neutron star \cite{34, 35} is in consonance with the calculated result.

To build the EDF, we confine ourselves in the non-relativistic framework. We start with a density and momentum dependent finite-range effective two-body interaction in the modified Seyler-Blanchard (SBM) prescription. This simple interaction with few parameters has been applied earlier to evaluate successfully many a nuclear properties \cite{36, 37}. A variant of this interaction has also been used by Myers and Swiatecki \cite{38} to calculate nuclear masses, nuclear deformations, charge distributions etc. and is seen to reproduce these properties very well. Calculations of EDF with empirical nuclear constants as base have been attempted earlier \cite{39, 40}. In Ref. \cite{39}, the SBM prescription for the form of the effective interaction was taken, in Ref. \cite{40}, the interaction was of the zero-range Skyrme class. The present calculations have been done in the same spirit, however, the chief difference with the earlier ones is that previously the parameters of the interactions were calculated with the symmetry energy element $a_2(\rho_0)$ being kept fixed at a predetermined value and that it was further equated with $e_{sym}(\rho_0)$. This masked the higher order effects in the asymmetry parameter $\delta$. Moreover, in the cases so mentioned, attempts were not made to find the maximum mass of the neutron star in relation to the interaction parameters.

The value of $e_0$, the energy per nucleon for SNM at $\rho_0$ is taken as $e_0 = -16.0 \pm 0.2$ MeV with $\rho_0 = 0.155 \pm 0.008$ fm$^{-3}$. There is a still no clear consensus on the strict bounds on $e_0$ or $\rho_0$. For example, some models lead to somewhat lower values for $e_0$ \cite{41-43}, we adhere to the value obtained from the recent version of the finite range droplet model (FRDM) \cite{20} that agrees better with the new mass database. The incompressibility $K_0$ is taken to be $215 \pm 25$ MeV \cite{31}. This is somewhat lower than the value of $K_0 = 230 \pm 40$ MeV as inferred in Ref. \cite{30}, but is consistent with the incompressibility of symmetric nuclear matter and its density slope at the sub-saturation crossing density $\rho_c$ as explained in Ref. \cite{31}. The value of the per particle energy of PNM at density $\rho = 0.1$ fm$^{-3}$ is taken as $e_n = 10.9 \pm 0.5$ MeV \cite{32}.

The paper is organized as follows. In Sec. II, we review the elements of theory. In Sec. III, the results and discussions are presented. The concluding remarks are drawn in Sec. IV.
II. THEORETICAL DETAILS

In the following, we describe the form of the effective two-body interaction and briefly outline the procedure for determining the parameters of this interaction from given empirical nuclear data. From the EDF constructed with this interaction, the different isovector elements pertaining to nuclear matter are then calculated, the question of the lower limit of the maximum mass of the neutron star $M_{\text{NS}}^{\text{max}}$ is further addressed.

A. Effective interaction and the nuclear EoS

The Seyler-Blanchard effective interaction $[44]$ in the modified version $[45]$ is taken to be of the form

$$v_{\text{eff}}(r, p, \rho) = C_{l,u} [v_1(r, p) + v_2(r, \rho)],$$

$$v_1 = -(1 - \frac{p^2}{b^2}) f(r_1, r_2),$$

$$v_2 = d^2 [\rho(r_1) + \rho(r_2)]^\alpha f(r_1, r_2),$$

with

$$f(r_1, r_2) = \frac{e^{-|r_1 - r_2|/a}}{|r_1 - r_2|/a}.$$  

Here the subscripts $l$ and $u$ to the interaction strength $C$ refer to like pair ($nn$, or $pp$) or unlike pair ($np$) interaction, $a$ is its spatial range and $b$ the strength of repulsion in its momentum dependence. The relative coordinate is $r = |r_1 - r_2|$, the relative momentum is $p = |p_1 - p_2|$, with 1 and 2 referring to the two interacting nucleons, $\rho(r_1)$ and $\rho(r_2)$ being the densities at their sites. The parameters $d$ and $\alpha$ are the measures of the strength of the density dependence in the interaction.

To construct the EoS from the effective interaction, one needs to know the occupation probability $n_{\tau}(p, T)$ where $T$ is the temperature and $\tau$ referring to the isospin index (neutrons
or protons). The self-consistent occupation probability in asymmetric nuclear matter at $T$ is obtained by minimizing the thermodynamic potential $G$

$$G = E - TS - \sum \mu_\tau N_\tau,$$

(11)

where $E$ and $S$ are the total energy and entropy of the system, and $\mu_\tau$ and $N_\tau$ are the respective chemical potentials and total numbers of the isospin species. Following ref. [39], the minimization of the thermodynamic potential with this interaction leads to the expression for the occupation probability as

$$n_\tau(p, T) = \left[1 + e^{\left\{\frac{\left(\frac{p^2}{2m_\tau^*} + V_0^\tau + V_1^\tau - \mu_\tau\right)}{T}\right\}}\right]^{-1}.$$  

(12)

Here $m_\tau^*$ is the nucleon effective mass. The momentum-dependent part of the single-particle potential $V_0^\tau + p^2V_1^\tau$ defines the effective mass as

$$m_\tau^* = \left[\frac{1}{m_\tau} + 2V_1^\tau\right]^{-1},$$

(13)

where $m_\tau$ is the bare nucleon mass. The quantity $V_2^\tau$ is the rearrangement energy that vanishes for density-independent effective interactions.

Recently, symmetry energy and associated properties of finite nuclei have been studied at finite temperature [46]. In this paper we are dealing with the properties of nuclear matter in the ground state ($T = 0$). In the limit $T \to 0$, the occupation function $n_\tau(p)$ becomes the Heaviside theta function,

$$n_\tau(p) = \Theta[P_{F,\tau} - p],$$

(14)

where the Fermi momentum $P_{F,\tau}$ given by

$$\frac{P_{F,\tau}^2}{2m_\tau^*} = \mu_\tau - V_0^\tau - V_2^\tau,$$

(15)

is related to density as $P_{F,\tau} = (3\pi^2\rho_\tau)^{1/3}\hbar$. The expressions for different parts of the single-particle potential and the rearrangement term, at zero temperature are given as [39]

$$V_0^\tau = -4\pi a^3\left\{1 - d(2\rho)^a\right\}(C_l\rho_\tau + C_u\rho_{-\tau})$$

$$+ \frac{4}{5\pi}(3\pi^2)^{5/3}\frac{a^3\hbar^2}{b^2}(C_l\rho_\tau^{5/3} + C_u\rho_{-\tau}^{5/3}),$$

(16)
\[ V_\tau^1 = \frac{4\pi a^3}{b^2}(C_l\rho_\tau + C_u\rho_{-\tau}), \] (17)

\[ V_\tau^2 = 4\pi a^3 d^2(2\rho)^{\alpha-1}\alpha[(C_l\rho_\tau + C_u\rho_{-\tau})\rho_\tau \\
+ (C_l\rho_{-\tau} + C_u\rho_\tau)\rho_{-\tau}]. \] (18)

In Eqs. (16) - (18), if \( \tau \) refers to proton, \( -\tau \) refers to neutron and vice versa. The density is given by

\[ \rho_\tau = \frac{2}{\hbar^3} \int_0^{P_F} n_\tau(p)dp \]
\[ = \frac{2\sqrt{2}}{3\pi^2\hbar^3}(m^*_\tau)^{3/2}(\mu_\tau - V_\tau^0 - V_\tau^2)^{3/2}. \] (19)

The total energy of nuclear matter per nucleon is then written as

\[ e(\rho, \delta) = \frac{1}{\rho} \sum_\tau \rho_\tau \left[ \frac{3P_F^2}{20m_\tau}(1 + \frac{m_\tau}{m^*_\tau}) + \frac{3}{2}V_\tau^0 \right], \] (20)

and the total pressure is

\[ P = \sum_\tau \rho_\tau \left[ \frac{P_F^2}{2m_\tau}(\frac{7m_\tau}{10m^*_\tau} - \frac{3}{10}) + \frac{1}{2}V_\tau^0 + V_\tau^2 \right]. \] (21)

The expressions (20) and (21), for SNM reduce to

\[ e(\rho, \delta = 0) = \frac{3}{10}\left[ \frac{P_F^2}{2m}(1 + \frac{m}{m^*(\rho)}) \right] + \frac{1}{2}V_0, \] (22)

\[ P(\rho, \delta = 0) = \left[ \frac{P_F^2}{2m}(\frac{7m}{10m^*(\rho)} - \frac{3}{10}) + \frac{1}{2}V_0 + V_2 \right] \rho, \] (23)

where \( P_F = (\frac{3\pi^2}{2})^{1/3}\hbar\rho^{1/3} \) is the Fermi momentum, \( V_0, V_1, V_2 \) are the single-particle potentials and \( m^*(\rho) \) the effective mass, all for SNM. In our calculations, we have taken the bare neutron and proton masses to be equal \( (m_\tau = m) \).

**B. Determination of the interaction parameters and symmetry elements**

The effective interaction as given by Eqs. (7)-(10) contains six unknown parameters, \( C_l, C_u, a, b, d, \) and \( \alpha \). Out of these, as we find later, for infinite nuclear matter, the parameters \( C_l, C_u \) and \( a \) appear in combination as \( C_l a^3 \) and \( C_u a^3 \). It is then effectively five unknown
The parameters we need to determine. The given empirical data are the energy per particle $e_0$ at the saturation density $\rho_0$ for SNM when pressure is zero, its incompressibility coefficient $K_0$, and $e_n$, the energy per particle of neutron matter at $\rho = 0.1$ fm$^{-3}$. In addition, we take the value of $m^*_0/m$ for SNM as a free input such that a close contact of the calculated value of $M_{\text{NS}}^{\text{max}}$ from the EDF can be established with the current observed value of $M_{\text{NS}}^{\text{max}} = 2.01 \pm 0.04 M_\odot$. The quantities $e_0$ and $P(\rho_0)(= 0)$ are obtained from Eqs. (22) and (23) by setting $\rho = \rho_0$. The incompressibility is obtained from Eq. (24) as

$$K_0 = 9 \left. \frac{dP}{d\rho} \right|_{\rho=\rho_0},$$

which, after some algebraic manipulation, reduces to

$$K_0 = -3V_0 + (9\alpha + 3)V_2 + V_1 [10.8P_{F,0}^2 + 4.5b^2 \\
\quad \times \{(\alpha + 1)d^2(2\rho_0)^\alpha - 1\}].$$

In Eq. (25), $P_{F,0}$ is the Fermi momentum at $\rho_0$. The neutron matter energy at density $\rho_n$ can be obtained from Eq. (21) setting $\delta = 1$ as

$$e_n = \frac{3}{10m} (3\pi^2)^{2/3} \hbar^2 \rho_n^{2/3} + C_{l,a}^3 \left[ \frac{12\pi}{5} (3\pi^2)^{2/3} \hbar^2 \rho_n^{5/3} \\
-2\pi \rho_n \{ 1 - d^2(2\rho_n)^\alpha \} \right].$$

From the four given empirical data and a chosen value of $m^*_0/m$, the five unknown parameters of the interaction $C_{l,a}^3, C_{u,a}^3, b, d$ and $\alpha$ can be determined (see Appendix A). Since we are interested in properties of homogeneous nuclear matter, we do not need to determine $C_l, C_u$ and $a$ separately. That can be done if we take into consideration semi-infinite matter and put another constraint, say, a given value of its surface energy. The values of the interaction parameters are given in Tab. I for two values of $m^*_0/m$, namely, 0.65 and 0.75. This choice of the effective mass is consistent with the empirical values obtained from many recent optical-model analyses [47, 48]. Covariance analysis of symmetry observables from

| $m^*_0/m$ | $C_{l,a}^3$ | $C_{u,a}^3$ | $b$ | $d$ | $\alpha$ |
|-----------|-------------|-------------|-----|-----|--------|
| 0.65      | 471.9       | 1269.3      | 2430.6 | 0.982 | 0.0193 |
| 0.75      | 103.2       | 295.0       | 1477.4 | 0.942 | 0.1235 |
heavy ion flow data \cite{49, 50} would put the value of $m_0^*/m$ at $\sim 0.7-0.8$, the situation is, however, not unambiguous.

From Eqs. (2), (16) and (20), the symmetry coefficients at a density $\rho$, in terms of the potential parameters read as,

$$a_2 = \frac{P_F^2}{6m} + \frac{4\pi a^3 \rho P_F^2}{3b^2} (2C_l - C_u)$$

$$- \pi a^3 \rho \{1 - d^2(2\rho)^\alpha\} (C_l - C_u),$$

(27)

$$a_4 = \frac{P_F^2}{162m} + \frac{4\pi a^3 \rho P_F^2}{81b^2} (2C_u - C_l),$$

(28)

$$a_6 = \frac{7P_F^2}{4474m} + \frac{28\pi a^3 P_F^2}{10935b^2} (7C_u - 2C_l).$$

(29)

The total density slope of symmetry energy $L_t(\rho) = 3\rho(\partial e_{sym}/\partial \rho)$ is obtained using Eq. (3) as,

$$L_t(\rho) = 2^{2/3} P_F^2 \left( \frac{3}{5m} + \frac{12\pi}{b^2} C_l a^3 \rho \right)$$

$$- 6\pi C_l a^3 \rho \{1 - d^2(2\rho)^\alpha(1 + \alpha)\}.$$  

(30)

In the literature, the symmetry slope $L(\rho)$ has, however, been usually taken as

$$L(\rho) = 3\rho \frac{\partial a_2(\rho)}{\partial \rho},$$

(31)

which from Eq. (27) is evaluated as

$$L(\rho) = \frac{P_F^2}{3m} + \frac{20\pi a^3}{3b^2} (2C_l - C_u) \rho P_F^2$$

$$- 3\pi a^3 (C_l - C_u) \rho \left[ \{1 - d^2(2\rho)^\alpha\}$$

$$- \alpha d^2(2\rho)^\alpha \right].$$

(32)

III. RESULTS AND DISCUSSIONS

From the wealth of diverse theoretical enterprises like the liquid drop type models \cite{20, 28, 51}, the microscopic ab-initio or variational calculations \cite{26, 27} or different Skyrme or Relativistic mean field models (RMF) - all initiated to explain different experimental data, we choose saturation density as $\rho_0=0.155\pm0.008$ fm$^{-3}$ and the energy per nucleon for SNM
as \( e_0 = -16.0 \pm 0.2 \) MeV, respectively. The value of the nuclear incompressibility \( K_0 \), obtained from the microscopic analysis of isoscalar giant monopole resonances (ISGMR) in nuclei has gone through several revisions \([52, 54]\) from its early value of \( K_0 \approx 210 \pm 30 \) MeV \([55, 56]\). Now, with the understanding that the ISGMR centroid energy reflects better the density dependence of the incompressibility \([30, 57]\), its value has been reassessed \([31]\) to \( K_0 \approx 215 \pm 25 \) MeV. For \( K_0 \), we choose this input value. This is not much different from the early value quoted.

For the effective mass, as explained in Appendix B, the minimum value with the given central values of the empirical inputs for this effective interaction is \( (m_0^*/m)_{\text{min}} \approx 0.64 \). We keep \( (m_0^*/m) \) as a free parameter above this value. We find, as shown later, that a low value of \( m_0^*/m \) explains better the lower limit of \( M_{\text{NS}}^{\text{max}} \), it increases with decreasing effective mass. We therefore fix the central value of \( m_0^*/m \) at 0.65, close to the lower limit, with an uncertainty of \( \pm 0.1 \). As already mentioned, this value of the isoscalar effective mass is coincident with that obtained recently \([48]\) from a global analysis of nucleon-nucleus scattering data within an isospin-dependent optical model. In finite nuclei, the effective mass is typically closer to unity \([32, 58]\) because of its enhancement due to the coupling of the single-particle motion to the surface vibrations, but this has not been included in the optical model analysis \([48]\). The value of the energy per particle for neutron matter is taken to be \( e_n = 10.9 \pm 0.5 \) MeV at \( \rho = 0.1 \) fm\(^{-3}\) \([32]\). Out of several hundred Skyrme EDFs, sixteen of them nicely reproduced a selected set of experimental nuclear matter properties. They gave \( e_n = 11.4 \pm 1.0 \) MeV at \( \rho = 0.1 \) fm\(^{-3}\) for PNM, among them six 'best-fit' results gave a more restricted range (10.9\( \pm 0.5 \) MeV) which we have chosen for \( e_n \).

A. The isovector elements of nuclear matter

In Fig. 1, the symmetry coefficients \( a_2, a_4, a_6 \) as defined in Eqs.\((27)\),\((28)\) and \((29)\) are displayed as a function of density. The coefficient \( a_2 \) increases with density up to \( \sim 4\rho_0 \), then decreases slowly; \( a_4 \) and \( a_6 \), however, monotonically increase with density. At the saturation density \( \rho_0 \) of symmetric nuclear matter, the values of \( a_2, a_4 \) and \( a_6 \) come out to be \( 32.18 \pm 0.78, 1.02 \pm 0.23 \) and \( 0.23 \pm 0.04 \) MeV, respectively. The higher order coefficients are seen to be negligible at low densities, even around \( \rho_0 \) they are not appreciable validating the Bethe-Weizäcker conjecture. The value of symmetry energy is seen to agree very well
FIG. 1: (color online) The symmetry energy coefficients $a_2$, $a_4$ and $a_6$ displayed as a function of density. Their central values are shown by the red lines. The shaded regions are their uncertainties. With the estimate of $31\pm 2$ MeV extracted from a combination of various experiments \cite{59, 60}. At higher densities, the relative importance of the higher order coefficients starts to show up. The shades in the figure refer to the uncertainties in the coefficients which are quite significant as the density increases. The emergence of the relative importance of the higher order coefficients with increasing density is shown in Fig. 2. The growing difference of the total symmetry energy $e_{sym}$ (which is the sum of all orders of the symmetry coefficients) from $a_2 + a_4 + a_6$ with density shows that still higher order terms need to be taken into consideration.
at very high densities and asymmetries prevalent near the core of the neutron star. The relatively smaller values of the higher order symmetry coefficients in our calculation at low densities and their growing importance with increasing density are in fair agreement with those obtained from both non-relativistic \cite{13} and relativistic calculations \cite{14}. Even with reasonable variations of the empirical input data, no sizeable values for them are obtained near the normal density $\rho_0$. At the highest density considered, the coefficient $a_4$ and $a_6$ are larger by about a factor of two in the present calculation as compared to those presented in reference \cite{14} and \cite{13} reminding us of the associated uncertainty in the calculated results in all models as one moves further away from the normal density around which the interaction parameters are determined.

The total density slope of symmetry energy $L_t$, the nuclear incompressibility $K$ and its density derivative $M = 3\rho dK/d\rho$ are displayed in the three panels of Fig. 3 as a function of density. They grow with density, so also their variances as shown by the shaded areas. The total symmetry density slope $L_t(\rho)$ is more relevant for asymmetric nuclear matter than the conventional $L(\rho)$. The pressure of neutron matter $P(\rho, \delta = 1)$ is intimately related to $L_t(\rho)$ as $P(\rho, \delta = 1) = P(\rho, \delta = 0) + \frac{2}{3}\rho L_t(\rho)$. We have therefore chosen to display the density variation of $L_t(\rho)$ rather than that of $L(\rho)$ which is very similar. At saturation density $L_{t,0} = 63.8 \pm 8.6$ MeV, $L_0 = 58.5 \pm 6.5$ MeV and $K_0$ is the same as the input value as it ought to be. The value of $L_0$ is seen to be somewhat lower than those obtained from earlier studies using different methodologies \cite{3, 61–63}, but is in good agreement with those obtained from fitting of selective experimental data on nuclear masses across the periodic table \cite{64, 65} that includes highly neutron-rich nuclear systems. The value of incompressibility $K_c$ at a density $\rho_c(= 0.71 \pm 0.005\rho_0)$ is argued to be more relevant \cite{30, 66} as an indicator of the ISGMR centroid. The incompressibility $K(\rho)$ calculated with a multitude of EDFs of the Skyrme class, when plotted against density are seen to cross close to this single density point $\rho_c$. The reported value of $K_c \sim 35 \pm 4$ MeV \cite{66} compares extremely well with our calculated value of $34.1 \pm 1.2$ MeV. The computed value of $M_c (= 3\rho dK/d\rho|_{\rho_c}) = 1062 \pm 102$ MeV also compares very favorably with $M_c = 1050 \pm 100$ MeV \cite{66} as obtained from the analysis of known experimental ISGMR data. The value of $M_0 = M(\rho_0)$ at saturation density cannot be compared with any benchmark value, but since $M_0 = 12K_0 + Q_0$ where $Q_0 = 27\rho_0^3\partial^3 e(\rho, 0)/\partial \rho^3|_{\rho_0}$, $Q_0$ can be estimated (as $K_0$ is given). The value $Q_0 = -360$ MeV conforms well with the one $Q_0 = -350 \pm 30$ MeV \cite{12} obtained from examination of
a host of standard Skyrme interactions. The evaluated value of symmetry incompressibility

\[ K_\delta = -382 \pm 60 \text{ MeV} \]  

is also in good consonance with the reported value of \(-370 \pm 120 \text{ MeV}\) extracted from measurements of isospin diffusion in heavy ion collisions and with \(-350 \text{ MeV}\) obtained from analysis of ISGMR data in Sn-isotopes. The total uncertainties \(\Delta X\) in the various observables \(X\) are evaluated as \(\Delta X = \sqrt{\sum_i (\Delta X_i)^2}\),

where \(\Delta X_i = \frac{\partial X}{\partial Y_i} \Delta Y_i\), \(\Delta X_i\) is the partial uncertainty induced by the uncertainty \(\Delta Y_i\) in the input quantity \(Y_i\) (say, 25 MeV in \(K_0\)). The derivatives \(\frac{\partial X}{\partial Y}\) are calculated numerically.

It is worth mentioning at this juncture that the recent analyses of the nuclear masses suggest a rather high value for the fourth order coefficient (order with \(\delta^4\)) of symmetry energy for finite nuclei. This coefficient is, however, not to be equated with \(a_4\) of Eq. (2), but possibly is indicative of the term with \(\delta^4\) \((E_{\text{sat},4})\), in the notation of \([12]\) in the series expansion in \(\delta\) of the binding energy per nucleon at saturation density of nuclear matter of asymmetry \(\delta\). It is related to \(a_4(\rho_0)\) as

\[ E_{\text{sat},4} = a_4(\rho_0) - \frac{L_0^2}{2K_0}. \]

With values of \(L_0\) and \(K_0\) in our model, this fourth order coefficient \((E_{\text{sat},4})\) is then \(\sim -6.7\) MeV. The magnitude of this coefficient may be compared with those obtained for a multitude of Skyrme interactions in Ref. \([12]\) which is \(\sim -4.6\) MeV.

The isospin splitting of the nucleon effective mass is a useful reference mark for an easy comprehension of the strength of the momentum dependence of the nucleon isovector potential. This is still a poorly known quantity, even the signature of the mass difference \((m_n^* - m_p^*)|_{\rho_0}\) is seen to be rather uncertain within the Skyrme-Hartree-Fock approach. There has been some recent interest in understanding it from different perspectives. Analyzing comprehensive nucleon elastic scattering data over a wide energy domain for a large number of systems, Li et al. \([48]\) have reported a value for \((m_n^* - m_p^*)|_{\rho_0}/m = (0.41 \pm 0.15)\delta\) at saturation density. On the other hand, exploring the giant resonances and the electric dipole polarizability in \(^{208}\text{Pb}\), a somewhat lesser value \((0.33 \pm 0.16)\delta\) of the said isovector splitting is obtained \([70]\). From our calculation, it is easy to show, from Eq. \([13]\) and \([17]\) that at any density

\[ \frac{1}{m_n^*} - \frac{1}{m_p^*} = 2V_n^1 - 2V_p^1 \]
FIG. 2: (color online) The contributions of different orders of symmetry energy coefficients $a_2, a_4, a_6$ to the total symmetry energy coefficients $e_{\text{sym}}$ shown as a function of density.

\[
e_{\text{sym}} = \frac{8\pi}{b^2} (C_l - C_u) a^3 \rho \delta
\]  

(35)

A little algebra leads to

\[
\frac{(m_n^* - m_p^*)}{m} = 2 \frac{K_2}{(K_1)^2} \left( \delta + \left( \frac{K_2}{K_1} \right)^2 \delta^3 + \ldots \right),
\]  

(36)

where

\[
K_1 = 1 + \frac{4\pi a^3}{b^2} m \rho (C_l + C_u),
\]  

(37)
and

\[ K_2 = \frac{4\pi a^3}{b^2} \rho (C_u - C_l). \]  

(38)

At \( \rho_0 \), plugging in the values of the interaction parameters and noting that the higher order terms in \( \delta \) in Eq. (36) are negligible, we get,

\[ \left. \frac{(m_n^* - m_p^*)}{m} \right|_{\rho_0} \simeq (0.209 \pm 0.017) \delta. \]  

(39)

The results on the symmetry elements presented so far pertain to calculations with an energy density functional constructed with the momentum and density dependent SBM interaction, the parameters of which are fixed from empirical bulk nuclear data. To check the consistency of the results, the calculations have been repeated in the Skyrme framework.

FIG. 3: (color online) The total density slope of symmetry energy \( L_t \), the incompressibility \( K \) and its density derivative \( M \) are shown in panels (a), (b) and (c), respectively, at different values of density. The full black line refer to their central values with the shaded regions representing their uncertainties.
The energy per nucleon $e(\rho, \delta)$ in this framework is

$$e(\rho, \delta) = g_1 \left[ \left( \frac{1 + \delta}{2} \right)^{5/3} + \left( \frac{1 - \delta}{2} \right)^{5/3} \right] \rho^{2/3} + (b_1 + b_2 \delta^2) \rho + (c_1 + c_2 \delta^2) \rho^{\gamma + 1} + \left[ d_1 \left\{ \left( \frac{1 + \delta}{2} \right)^{5/3} + \left( \frac{1 - \delta}{2} \right)^{5/3} \right\} + d_2 \left\{ \left( \frac{1 + \delta}{2} \right)^{8/3} + \left( \frac{1 - \delta}{2} \right)^{8/3} \right\} \right] \rho^{5/3}. \quad (40)$$

The first term on the right-hand side is the free Fermi-gas energy, $g_1 = \hbar^2 \frac{2}{m^3} \left( \frac{3 \pi^2}{2} \right)^{2/3} = 119.14$ MeV fm$^2$. There are seven parameters in this EDF, namely, $b_1, b_2, c_1, c_2, d_1, d_2$ and $\gamma$. The parameter $\gamma$ is related to the isoscalar bulk data $m^*, e_0, K_0$ and $\rho_0$ as

$$\gamma = -e_0 - \frac{K_0}{9} + \left( \frac{4}{3} \frac{m}{m_0} - 1 \right) \frac{g_1 \rho_0^{2/3}}{3 \lambda^{2/3}}. \quad (41)$$

The isoscalar equations for $e_0, K_0$ and $P(=0$ at $\rho_0$) yield the values of $b_1, c_1$ and $(d_1 + d_2)$. For the remaining parameters, in addition to the constraints $e_n$ for neutron matter at $\rho = 0.1$ fm$^{-3}$, we need two other isovector entities. We choose them to be $a_2(\rho_0) = 32.1 \pm 0.31$ MeV [72] and $a_2(\rho_1) = 24.1 \pm 0.8$ MeV [73]. The former have been obtained recently from a meticulous study of the double differences of "experimental" symmetry energies [72], the latter is obtained from giant dipole resonance analysis [73]. All the other empirical data are chosen to be the same as in the SBM framework. Equations for $a_2(\rho_0), a_2(\rho_1)$ and $e_n(\rho_1)$ yield the values of $b_2, c_2$ and $(d_1 + 2d_2)$. The values of all the parameters entering the Skyrme EDF are thus known. Details about finding out the parameters in the Skyrme framework are given in Ref. [40]. In Tab. II the central values of the symmetry elements we deal with at and around the saturation density obtained from the two frameworks are compared. They are compatible, the difference in the neutron-proton effective mass is seen to be larger in the Skyrme prescription. Both are positive.

### B. Supranormal density and neutron stars

Now that the EDF so constructed in the SBM framework produces results that are in reasonably good agreement with those obtained from different perspectives (both in experiment and theory) at normal and subnormal densities, it would be interesting to see how
TABLE II: Comparison of the values of the symmetry energy elements in the SBM and Skyrme framework at the densities indicated.

|        | $a_2(\rho_0)$ | $a_2(\rho_1)$ | $a_4(\rho_0)$ | $a_4(\rho_1)$ | $a_6(\rho_0)$ | $a_6(\rho_1)$ | $L(\rho_0)$ | $L_t(\rho_0)$ | $K_\delta$ | $\left(\frac{m^*_n-m^*_p}{m}\right)_{\rho_0}$ |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|--------------|------------|---------------------------------|
| SBM    | 32.2           | 24.4           | 1.01           | 0.61           | 0.23           | 0.14           | 58.3         | 63.8         | -382       | 0.21\delta                         |
| Skyrme | 32.1           | 24.1           | 1.47           | 0.83           | 0.25           | 0.14           | 57.3         | 65.6         | -412       | 0.49\delta                         |

The EDF behaves at high densities, how the pressure changes as a function of baryon density and asymmetry. The baryon pressure is an essential element in shaping properties of neutron star matter, in understanding the lower limit of the maximum mass of neutron star $M_{\text{NS}}^{\text{max}}$. Our calculations show that at low densities, the pressure for SNM is lower compared to that for PNM, however, it catches up at higher densities. This crossing density is found to be dependent on the value of $m^*_0/m$. For higher values of effective mass, the crossing density is lower ($\sim 3\rho_0$ for $m^*_0/m = 0.75$), but moves up as $m^*_0/m$ decreases ($\sim 5.5\rho_0$ for $m^*_0/m = 0.65$). In Fig. 4, the pressure-density relation is portrayed in the upper panel for SNM and in the lower panel for PNM with $m^*/m = 0.65 \pm 0.1$. The violet shades show the calculated uncertainties. The shaded red and orange regions in the upper panel display the ‘experimental’ EoS for SNM extracted from collective flow data [74] and from data for Kaon production [75, 76], respectively. The shaded green region in the EoS of PNM is a theoretically obtained result where the density dependence of symmetry energy is taken to be soft. The red shaded region is the one where the said density dependence is modeled as stiff [77]. These regions have very good overlap with the one obtained from our calculation.

Solving the general relativistic Tolman-Oppenheimer-Volkoff (TOV) equation [78], we have calculated $M_{\text{NS}}^{\text{max}}$ for neutron star with different values of $m^*/m$. The EoS for the crust was taken from the Baym, Pethick and Sutherland model [79]. The EoS for the core region was calculated under the assumption of a charge neutral uniform plasma of neutrons, protons, muons and electrons in $\beta-$ equilibrium. Possible phase transition to exotic phases such as hyperons, kaons etc. at high densities softens the EoS somewhat. This is not taken into consideration here. The maximum mass calculated within this framework is shown in Fig. 5 as a function of $m^*/m$. We note that $M_{\text{NS}}^{\text{max}}$ goes up with decreasing $m^*/m$. At $m^*/m = 0.65$, the calculated value for $M_{\text{NS}}^{\text{max}} = (1.95 \pm 0.14)M_\odot$ is consistent with the currently observed values of $(1.97 \pm 0.04)M_\odot$ for the pulsar PSR J1614-2230 [34] and also
FIG. 4: (color online) The EoS for symmetric nuclear matter (upper panel) and for pure neutron matter (lower panel). The calculated results and the experimental data are as indicated. See text for details.
FIG. 5: (color online) Dependence of the lower limit of the maximum mass of neutron star $M_{\text{NS}}^{\text{max}}$ (in units of solar mass $M_\odot$) on the effective nucleon mass for symmetric nuclear matter.

with the value of $(2.01 \pm 0.04)M_\odot$ \cite{35}.

IV. CONCLUDING REMARKS

From well-constrained empirical data relevant for nuclear matter at saturation and subsaturation densities, we have constructed an energy density functional based on a finite-range, momentum and density dependent interaction. The different elements related to symmetry energy and their density dependence are then analyzed with this EDF. The density slope of
symmetry energy \( L(\rho) \), the density dependence of nuclear incompressibility \( K(\rho) \), its density slope \( M(\rho) \), the symmetry incompressibility \( K_\delta \) at saturation for asymmetric matter — all these are seen to be in excellent agreement with their recently obtained values from different perspectives. Calculations done in a Skyrme-inspired framework for the EDF with the same input empirical data do not change the conclusions much. We modeled the EoS with the SBM EDF so that the calculated \( M_{\text{max}}^{\text{NS}} \) conforms well with the experimentally observed one; keeping this in mind, still it must be said that the agreement of our constructed EoS with the ‘experimental’ one over an extended density plane is very striking. From this overall consistency of our constructed EoS and the derivative results built from empirical data, we infer that the higher order symmetry coefficients \( a_4, a_6 \) etc. of infinite nuclear matter are not sizeable at and around saturation density, but grow with increasing density. This is in fair agreement with earlier investigations \([13–15]\) and confirms that the EoS of asymmetric nuclear matter, though conforms to the parabolic approximation at normal and sub-saturation density deviates significantly from it as the density rises.

In calculating the maximum mass \( M_{\text{max}}^{\text{NS}} \) for neutron star, we have confined ourselves to homogeneous nuclear matter in \( \beta \)-equilibrium. Exotic degrees of freedom near the interior of the star may change the calculated value of \( M_{\text{max}}^{\text{NS}} \) somewhat, this has been left out of our consideration in the present description.

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Appendix A

Here we show how the parameters of the interaction are determined. The single-particle potentials \( V_0, V_1, V_2 \) and the effective mass \( m_0^* \) refer to the entities for SNM at the saturation density \( \rho_0 \).

From Eq. (22), we know \( V_0 \) from the empirical inputs,

\[
V_0 = 2e_0 - \frac{3}{5} \left\{ \frac{P_{F,0}^2}{2m} \left( 1 + \frac{m}{m_0^*} \right) \right\},
\]

(A1)
where \( P_{F,0} \) is the Fermi momentum obtained from

\[
\rho_0 = \frac{2P_{F,0}^3}{3\pi^2\hbar^3}.
\]  \hspace{1cm} (A2)

The momentum dependent part \( V_1 \) is known from

\[
V_1 = \frac{1}{2m} \left( \frac{m}{m^*} - 1 \right),
\]  \hspace{1cm} (A3)

which for symmetric matter is \([\text{Eq. (17)}]\)

\[
V_1 = \frac{2\pi a^3}{b^2} (C_l + C_u) \rho_0.
\]  \hspace{1cm} (A4)

The rearrangement term \( V_2 \) and the potential \( V_0 \) for symmetric matter are

\[
V_2 = \pi a^3 d^2 (2\rho_0)^\alpha (C_l + C_u) \rho_0, \tag{A5}
\]

\[
V_0 = V_1 \left[ \frac{3}{5} P_{F,0}^2 - b^2 \left\{ 1 - d^2 (2\rho_0)^\alpha \right\} \right]. \tag{A6}
\]

Eqs. (A4 - A6) give a relation between the three potentials,

\[
V_0 = V_1 \left[ \frac{3}{5} P_{F,0}^2 - b^2 \right] + \frac{2V_2}{\alpha}. \tag{A7}
\]

From Eq. (23), \( V_2 \) at saturation (where pressure is zero) can also be calculated in terms of known quantities

\[
V_2 = -\epsilon_0 + \frac{P_{F,0}^2}{10m} (3 - \frac{m}{m^*}). \tag{A8}
\]

From Eqs. (A4) and (A5)

\[
d^2 (2\rho_0)^\alpha = \frac{2V_2}{\alpha b^2 V_1}. \tag{A9}
\]

Putting this in Eq. (25) gives

\[
K_0 = -3V_0 + V_2 (9\alpha + \frac{9}{\alpha} + 12) + V_1 (10.8 P_{F,0}^2 - 4.5b^2). \tag{A10}
\]

Eqs. (A7) and (A10) give \( b \) and \( \alpha \). Eq. (A9) then gives \( d^2 \). The values of \( \alpha, b^2 \) and \( d^2 \) are given as,

\[
\alpha = \frac{1}{V_2} \left[ \frac{K_0}{9} - \frac{V_0}{6} - \frac{9}{10} P_{F,0}^2 V_1 \right] - \frac{4}{3}, \tag{A11}
\]

\[
b^2 = \frac{3}{5} P_{F,0}^2 + \frac{1}{V_1} \left[ \frac{2V_2}{\alpha} - V_0 \right], \tag{A12}
\]

\[
d^2 = \frac{2V_2}{\alpha b^2 (2\rho_0)^\alpha V_1}. \tag{A13}
\]

The value of \( C_l a^3 \) is determined from Eq. (26). Eqs. (A3) and (A4) then gives \( C_u a^3 \).
Appendix B

In the framework of the effective interaction chosen, the effective mass $m^*_0/m$ is seen to have a lower bound. In Eq. (A10), putting the value of $b^2 V_1$ from Eq. (A7) we have,

$$\alpha = \left[ K_0 - 1.5V_0 - 12V_2 - 8.1P_{F,0}^2 V_1 \right] / 9V_2.$$  \hfill (B1)

With values of $V_0, V_1$ and $V_2$ from Eqs. (A1), (A3) and (A8), one gets an equation for $\alpha$ from Eq. (B1) in terms of empirical quantities,

$$\alpha = \left[ K_0 - 3e_0 + \frac{9P_{F,0}^2}{2m} (1 + \frac{m}{m_0}) - 8.1\frac{P_{F,0}^2}{2m} (\frac{m}{m_0} - 1) \right] / 9V_2 - \frac{4}{3}.$$  \hfill (B2)

With given values of $K_0, e_0, \rho_0$ etc., examination of Eq. (B2) shows that as $m^*_0/m$ starts decreasing from unity, the value of $\alpha$ starting from a positive value become lower and lower until at some value of $m^*_0/m$, it crosses zero and then becomes negative. The value of $b^2$ then makes a sudden transition from a large positive value to a large negative value. Since the density-dependent part of the interaction

$$d^2(2\rho_0)^\alpha = \frac{2V_2}{\alpha b^2 V_1}$$  \hfill (B3)

should be repulsive and should increase with density, $\alpha$ should be positive; the physically accepted minimum value of $m^*_0/m$ is then determined from the condition ($V_2$ is still finite from the empirical inputs)

$$K_0 - 1.5V_0 - 12V_2 - 8.1P_{F,0}^2 V_1 = 0,$$  \hfill (B4)

which yields

$$\left( \frac{m^*_0}{m} \right)_{\text{min}} = \frac{6P_{F,0}^2}{m} + 45e_0 + 5K_0 + 4.5\frac{P_{F,0}^2}{m}.$$  \hfill (B5)

With the values of the empirical quantities chosen, $\left( \frac{m^*_0}{m} \right)_{\text{min}}$ is $\sim 0.64.$

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