Neutrino emissivities in 2SC color-superconducting quark matter

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Abstract

The phase structure and equation of state for two-flavor quark matter under compact star constraints is studied within a nonlocal chiral quark model. Chiral symmetry breaking leads to rather large, density dependent quark masses at the phase transition to quark matter. The influence of diquark pairing gaps and quark masses on density dependent emissivities for the direct URCA is discussed. Since $m_u > m_d$, the direct URCA process due to quark masses cannot occur. We present cooling curves for model quark stars and discuss their relation to observational data.

1 Introduction

Emissivities and mean free paths of photons and neutrinos are essential for most astrophysical phenomena (supernovae, neutron star cooling, gamma ray bursts(GRB), pulsar kicks [8] etc.). Their correct treatment is one of the challenging tasks in astrophysics. Theoretical predictions of quark matter properties inside compact stellar objects including the possibility of different color-superconducting phases, resulted in a number of emissivity calculations in quark matter until now (e.g. [1, 2, 3, 4]). The first calculation of neutrino emissivities in quark matter has been done by Iwamoto [1]. He found that the matrix element of the direct URCA process would vanish and this important cooling process in quark stars could not occur if one neglects quark-quark interactions and quark masses. Most calculations consider the effect of quark-quark interactions to obtain a finite matrix element since the current up and down quark masses are small and their influence on the emissivity is negligible. However, the up and down quark masses can be up to almost two orders of magnitude larger than the current quark mass in the vicinity of the phase transition to quark matter. This can be shown within NJL-type chiral quark models [5]. The influence of both quark masses and diquark pairing on the direct URCA emissivities is discussed in the following sections.

2 Relativistic chiral quark model

The grand canonical potential for quark matter in a 2SC superconductor is

$$\Omega(\mu_B, \mu_Q, \mu_s, T) = \left(1 - \alpha\right)\frac{\phi_u^2 + \phi_d^2}{8 G_s} + \alpha \frac{\phi_u \phi_d}{4 G_s} + \frac{\Delta^2}{4 G_D}$$

$$-2 \int \frac{d^3p}{(2\pi)^3} \sum_{a=1}^{12} \left[\frac{\lambda_a}{2} + T \ln \left(1 + e^{-\lambda_a/T}\right)\right] + \Omega_e - \Omega_0,$$  

with $\Omega_e = -\mu_e^4/12\pi^2 - \mu_Q^2 T^2/6 - 7\pi^2 T^4/180$ being the thermodynamic potential of ultra-relativistic electrons, where $\mu_Q = -\mu_e$, and $\Omega_0$ is the divergent
vacuum contribution which has to be subtracted to assure vanishing energy and pressure of the vacuum. Out of the twelve eigenvalues \( \lambda_a \), four belong to the ungapped blue quarks and can be determined easily by using textbook methods \[6\] as \( \lambda_{1,4} = E_f(p) \pm \mu_f \), with the dispersion relation \( E_f(p) = \sqrt{p^2 + M_f^2(p)} \) containing the dynamical mass function \( M_f(p) = m_f + g_f(p) \phi_f \) for the two flavors \( f = u, d \). We have introduced the chemical potentials for the quarks of unpaired color \( \mu_{ub} = \mu_B/3 + 2\mu_Q/3 - 2\mu_8/2 \) and \( \mu_{db} = \mu_{ub} - \mu_Q \). The other eight eigenvalues belong to the red and green quarks which are paired in the 2SC state and have therefore identical eigenvalue spectra. It is thus sufficient to determine the four eigenvalues for the red quarks by solving a quartic equation similar to the one discussed in \[7\] for the CFL phase. The parameter \( \alpha \) describes flavor mixing due to instanton induced interactions, see \[9,5\]. For the form factor we used a simple NJL cut off \( g(p) = \theta(1 - p/\Lambda) \). The numerical solutions of this section are obtained with the parameters from Table 5.2 of \[5\]; current quark mass \( m_{u,d} = 5.5 \) MeV, coupling constant \( G_s \Lambda^2 = 2.319 \), cut off \( \Lambda = 602.3 \) MeV and diquark coupling \( G_D = \eta_D G_s \) with \( \eta_D = 1 \) (strong coupling). Fig. 1 shows

![Figure 1](image)

Figure 1: Quark masses for asymmetric matter under charge conservation without diquark gap and flavor mixing (left panel), with diquark gap and without flavor mixing (center), and with diquark gap and flavor mixing (right panel).

that after the chiral phase transition is first order, signalled by a sudden drop of the constituent quark masses at the critical chemical potential \( \mu_{q}^{\text{crit}} = 340 \) MeV. The region just above the transition, \( \mu_{q}^{\text{crit}} < \mu < 400 \) MeV, relevant for quark stars (or quark cores of neutron stars), shows still nonperturbatively large quark masses strongly decreasing towards their current values with increasing \( \mu \). In asymmetric quark matter under charge conservation the up quark mass is always larger than the down quark mass. The magnitude of this mass difference decreases with the flavor mixing factor \( \alpha \), being maximal for \( \alpha = 0 \). Note that
the diquark gap $\Delta$ has a similar but smaller mixing effect on the quark flavors.

3 Direct URCA emissivities

The resulting neutrino emissivity from the direct URCA reactions

$$d \rightarrow u + e^- + \bar{\nu}_e, \quad u + e^- \rightarrow d + \nu_e$$

has been calculated in [1] and can be expressed as

$$\varepsilon_\nu \simeq \frac{914 \pi}{1680} G^2 \cos^2 \theta_e p_{F,u} p_{F,e} p_{F,d} T^6 \left( \frac{1}{3} + \frac{2}{3} \eta \right) \theta_{ue}^2.$$  

Here $\theta_c$ is the Cabibbo angle, $G$ the weak coupling constant and $\theta_{ue}$ is the angle between the up-quark and electron momenta obtained from the condition of momentum conservation in the matrix element, see Fig. 2. We take into account that in the 2SC phase $2/3$ of the quarks are paired and thus the corresponding direct URCA emissivity is suppressed by a factor $\eta = (C(\zeta) + \exp[(\Delta - 0.5\mu_e)/T])^{-1}$ taken from Ref. [2], where we choose $C(\zeta) = 1$. For the late cooling stage with temperatures $T \ll 1$ MeV, neutrinos are untrapped and their effects on momentum conservation and chemical potentials can be neglected. Trigonometric relations can be used to find an appropriate analytical expression for the momentum conservation from Fig. 2 which in lowest order of $\theta_{de}$ is given by

$$p_{F,d} - p_{F,u} - p_{F,e} \simeq -\frac{1}{2} p_{F,e} \theta_{de}^2.$$  

For small angles holds $\theta_{de} \simeq \theta_{ue}$, so that an expression for the matrix element of the direct URCA process can be derived. Following Iwamoto [1] we take into account either quark-quark interactions to lowest order in the strong coupling constant $\alpha_s$ (5) or the effect of finite masses (6)

$$\mu_i = p_{F,i} \left( 1 + \frac{2}{3\pi} \alpha_s \right), \quad i = u, d \quad \text{(5)}$$

$$\mu_i \simeq p_{F,i} \left[ 1 + \frac{1}{2} \left( \frac{m_i}{p_{F,i}} \right)^2 \right], \quad i = u, d, e \quad \text{(6)}$$

![Figure 2: Triangle of momentum conservation for the URCA process](image-url)
From Eqs. (4)-(6) with the $\beta$-equilibrium condition $\mu_d = \mu_u + \mu_e$ an expression for the angle $\theta_{de}$ determining the emissivity Eq. (3) can be found similar terms for Eq. (4)

$$\theta_{de}^2 \simeq \frac{m_s^2}{p_{F,d} p_{F,u}} \left[ 1 - \left( \frac{m_u}{m_d} \frac{p_{F,d}}{p_{F,u}} \right)^2 - \left( \frac{m_e}{m_d} \frac{p_{F,d}}{p_{F,u}} \right)^2 \right].$$

(7)

The mass effect for current quark masses is negligible in comparison to the perturbative quark-quark interaction effect, as stated in [1] (see left panel of Fig. 3). However, this statement is restricted to the region of the QCD phase diagram, where the perturbative treatment is possible at all and the quark masses are of the order of the current values $m_{u,d} \sim 5 - 9$ MeV. If $m_u > m_d$ the momentum conservation is violated and the URCA process via the mass effect does not work, despite the large quark masses. The influence of the quark masses on the Fermi momentum of up and down quarks in a relativistic chiral quark model together with the perturbative one-gluon exchange (OGE) interactions results in slightly higher emissivities compared to the purely perturbative ones.
Emissivities in superconducting quark matter are suppressed by the factor $\eta$, leading to lower emissivities, cf. Fig. 3.

## 4 Cooling

In order to qualitatively estimate the consequences of the emissivity effects discussed above on the cooling behavior of compact stars, we consider the simplified model of a homogeneous quark star. The temperature-age behavior is obtained by inverting the solution of the cooling equation

$$t - t_0 = - \int_{T(t_0)}^{T(t)} \sum_{i,j} \frac{C_i(T)}{L_j(T)} \, dT; \quad i = \text{quark, } e^-, \gamma, \text{gluon}; \quad j = \nu, \gamma. \quad (8)$$

The contributions of the different species to the specific heat and the photon contribution to the star luminosity can be found in [10]. The double logarithmic

![Cooling curves](image)

Figure 4: Cooling curves for the corresponding emissivities from Fig. 3 without flavor mixing (center) for a homogeneous quark star ($\mu_B = 1.1$ GeV) with a radius of 9 km and a mass $1.2 \, M_\odot$. The plot Fig. 4 shows the surface temperature $T_s$ of the star vs. its age. The relation between crust and surface temperatures is taken from the Tsuruta law $T_s = (10 \, T_c)^{2/3}$. The cooling curves are sensitive to the quark masses and the diquark gap suppression. Note that the configuration of a homogeneous quark star without hadronic shell is quite unrealistic and the comparison with observational data is only made to give an orientation about the relations in a T-t plot. A realistic treatment of quark star cooling including heat flux, as well as neutrino and photon transport with temperature and density profiles can alter these results strongly.
5 Conclusion

Within our study of the classical URCA process due to perturbative OGE interactions we find that constituent quark masses and chemical potentials obtained from a NJL type relativistic chiral quark model lead to emissivities comparable to the results from the ideal Fermi gas. Neutrino emissivities due to the quark masses can only occur if \( m_d > m_u \). Since in the chiral quark model under charge conservation the selfconsistent masses obey \( m_d < m_u \), momentum conservation is violated and the URCA process can not occur. In hybrid stars (neutron stars with a quark matter core) or model quark stars we can proceed from the assumption that the quark matter close to the phase transition is in a non-perturbative density region rather than in a perturbative one. It is questionable whether OGE interactions are applicable in the non-perturbative density regime close to the chiral phase transition. Therefore, non-perturbative effects like, e.g., \( \rho \) and \( \omega \) meson exchanges could be possible candidates to replace the perturbative OGE interaction in the non-perturbative density regime.

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