Control System Based Modeling and Simulation of Cardiac Muscle With Optimization Using Performance Index

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Abstract

Because of the prolong use of the system, the performance (Output parameters of the system) can change and output of the system may start deteriorating from the desired value. If the performance of a system, based on control theory is not up to the expectations as per the desired specification, then some changes in the system are required to obtain the desired performance. The control system can be represented with a set of mathematical equations called system model which are used to answer questions via analysis and simulation. A model is a precise representation of a system dynamics which are the arrangement of physical elements and that physical elements are analyzed to make governing equations. Cardiovascular muscle senses the force generated due to the contraction and expansion of muscle wall. This can be well understood by the analytical approach of the transfer function generated by using a mechanical model of force displacement analogy. The efficiency of the work also lies in the measure of the movement of cardiovascular factors in the system. The mass of heart muscle varies with different age groups both for male and female. This work is based on the glimpses of changing transfer function with different age groups due to the variation of mass of heart muscle. Viscous drag has also been calculated considering different values of damping coefficient for a particular value of mass. For attending the optimality in the performance of the system one designed controller is used along with the derived transfer function in cascade arrangement. To get more stability of the system, damping coefficient is chosen for the system model considering less settling time and steady state error. The open loop transfer function in the forward path is simply the product of derived transfer function and designed transfer function of controller. The design emphasizes on the optimality in operation of the control process which has been determined by the performance index (PI) of the total process using integral square method.

Keywords: Transfer Function, Steady State Error, Performance Index, Integral Square Error.
I. Introduction

Human cardiovascular system and its abnormalities nowadays play an important role for research. To analyze cardiovascular system and its effects, control system based modeling of cardiovascular muscle is very much needed for researchers. Consideration is focused on the fact that, the cardiovascular system is a 2nd order system [XIX] having suitable parametric values of the damping coefficient & the natural frequency. The equation for cardiovascular system having damping coefficient and natural frequency can be written as

\[ \text{fd} = f_n \sqrt{1 - \xi^2} \]

Here \( \text{fd} \) is the damping frequency, \( f_n \) be the natural frequency and \( \xi \) is the damping coefficient. The dynamic behavior of 2nd order system can also be described by the above parameters. A cardiac muscle can be modeled with a mass \( (M) \) of it, viscous damping \( (B) \) which is proportional to the wall movement of the specimen and a torsion drag which is also proportional to the displacement of the specimen \( (K) \). \( f(t) \) is the force exerted by the cardiac muscle specimen generated by electrical and electromechanical activity effects on the cardiovascular system. Comparing the model equation with standard 2nd order system equation it is found that viscous damping \( (B) \) is a function of cardiac muscle mass ‘\( M \)’ is taken based on different age to calculate the value of ‘B’ for modeling of cardiac muscle.

This can be represented by the mathematical equation

\[ \frac{B}{M} = 2\xi \omega_n \]

Damping coefficient \( (\xi) \) has also been varied for individual ‘\( M \)’ to get different transfer function of the model stated above. Steady state error is calculated for different transfer function to get an optimized value of ‘B’. A quantitative measure of the performance of a system is necessary for the operation of the modern adaptive control systems for automatic parameter optimization of a control system & for the design of optimum systems. A performance index is a quantitative measure of performance of the system and is chosen so that emphasis is given to the important system specification. A system is considered an optimum control system when the system parameters are adjusted so that the index reaches an extreme value, commonly a minimum value. A performance index to be useful must be a number that is always positive or zero. Then the best system is defined as the system that minimizes this index.

A system design problem generally reaches the point at which one or more parameters are to be selected to give the best performance. If a measure or index of performance can be expressed mathematically the problem can be solved for the best choice of the adjustable parameter. The resulting system is termed optimal with respect to the selection criterion. The Z transform helps in the analysis of sample data control as Laplace transform does in the analysis and design of continuous data control system. The controllability and the observability of the system are tested and the system is found to be controllable and observable. MATLAB 7.1 is used in the concern analysis and design. The system designed in the present work is found to be stable with appropriate gain margin and phase margin. The controllability and observability of the total control system is tested. The system is found to be controllable and observable. The sample data analysis of the system is also done in the present study and system is found to be stable in the sample data system.
II. Control System Based Modeling of Human Heart Muscle:

The control system based modeling [X] is used to analyze the various parameters of cardiac muscle. To analyze the different parameters, a basic mechanical model of a dynamic system is taken [IV,XI]. In this same way the human cardiac muscle can be modeled with the following diagram, as cardiac muscle is a dynamic system.

![Basic mechanical model of cardiac muscle](image)

So from the above figure it is possible to derive the dynamical equation [IV,X] for the movement of cardiac muscle with the help of control modeling as given below.

\[
M \frac{d^2x}{dt^2} + B \frac{dx(t)}{dt} + k x(t) = f(t)
\]  

(1)

Here \(M\) is the mass of cardiac muscle, \(B\) is the constant of viscous drag of myocardial cell. \(x(t)\) is the movement of cardiac wall which is generated due to exerted force \(f(t)\) by electrical and electrochemical activity effects on the cardiovascular system. The viscous damping is proportional to the muscle wall movement of the specimen, so that the contribution to this viscous damping may be represented by the expression \(B \frac{dx(t)}{dt}\). Tensional drag is proportion to the displacement of the specimen, so that its contribution is given by the expression \(kx(t)\), \(k\) being the constant of proportionality[XVII]. Taking the Laplace transform of equation (1) the following equation can be written

\[
Ms^2X(s) + BsX(s) + kX(s) = F(s)
\]  

(2)

Here we have taken the Laplace Transform of the equation, where \(X(s) = L[x(t)]\) and \(F(s) = L[f(t)]\)

So from equation (2) it is possible to write

\[
\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}
\]  

(3)

\[
\frac{X(s)}{F(s)} = \frac{K_1}{s^2 + K_2s + K_3} = T(s)
\]  

(4)

Where \(K_1 = 1/M\), \(K_2 = B/M\), \(K_3 = k/M\).
The transfer function of the cardiac muscle movement due to the force can also be written with the help of three constants $K_1$, $K_2$, $K_3$.

The cardiovascular wall makes its movement incurring displacement despite hindrance due to frictional and torsional effects of the cardiovascular wall cells. The input force causing the displacement of cardiac muscle is generated by the electrical and electrochemical activity in the cells. The expression of the above transfer function thus gives the measure of the cardiovascular muscle cell wall due to the inherent force generated in the cardiovascular system. The values of the constants $K_1$, $K_2$, $K_3$ in the expression (iii) of the transfer function can be determined by experimentation on a cardiovascular system, and the expression (iii) thus may be standardized [VII]. Thus for known forcing function, the nature of the displacement (movement) of cardiovascular muscle, as function of time can be well predicted taking the Laplace inverse transformation on that equation.

III. Variation of Natural Frequency with the Variation of Damping Coefficient in a Under Damped System

As human heart is a second order under damped system [XV,XX] from the equation (4), the transfer function of the cardiovascular muscle dynamics can be written as

$$\frac{X(s)}{F(s)} = \frac{K_1}{s^2 + K_2 s + K_3} = T(s)$$

The above stated equation can also be compared with the transfer function of standard second order system. The transfer function of a 2nd order system can be represented by the following transfer function:

$$T(s) = \frac{X(s)}{F(s)} = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} \quad (5)$$

Where $\xi$ is the damping ratio, $\omega_n$ is the natural frequency. So the characteristic equation can be represented by the following equation

$$s^2 + 2\xi \omega_n s + \omega_n^2 = 0 \quad (6)$$

The dynamic behavior of the standard second-order system can then be described in terms of two parameters: the damping ratio and the natural frequency. If the damping ratio is between 0 and 1, the system poles are complex conjugates and lie in the left-half $s$ plane. The system is then called under damped, and the transient response is oscillatory. If the damping ratio is equal to 1 the system is called critically damped, and when the damping ratio is greater than 1 it behaves like an over damped system. The transient response of critically damped and over damped systems does not oscillate. The human heart is beating at an average rate of 72 beats per minute [XIII, XIV]. The frequency of heart beat is $f = 72/60 = 1.2$ Hz which is considered to be the damping frequency.

The human cardiovascular system is considered to be an under damped system [X,XIX]. So the relation between $f_d, f_n, \xi$ is given by the below equation.
\[ f_d = f_n(1-\xi^2)^{1/2} \]

where \( f_d \) damping frequency , \( f_n \) natural frequency & \( \xi \) is the damping coefficient

\[ 1.2 = f_n(1-\xi^2)^{1/2} \]  

(7)

Here the frequency of heart and the damping frequency is considered to be same. \( \xi \) is the damping factor and \( f_n \) is the natural frequency, \( f_d \) is the damping frequency. It must be noticed that the various condition of ECG signals causes the variation in the values of damping frequency. As the system is under damped, it can be assumed that the (damping ratio) \( \xi = 0.7 \) and will vary the damping coefficient are for our next study. The value of \( \omega_n \) for different value of \( \xi \) has been calculated using equation (7) and given in table shown below.

**Table 1:** Value of \( \omega_n \) for different value of \( \xi \)

| \( \xi \) | \( \omega_n \) | \( \xi \) | \( \omega_n \) |
|-------|---------|-------|---------|
| 1     | 7.57    | 4     | 8.24    |
| 2     | 7.73    | 5     | 8.66    |
| 3     | 7.83    | 6     | 9.37    |

IV. Modeling Equations for Cardiac Muscle considering the variation of Age and Damping Coefficient

Mass (M) is one of the important parameter of muscle dynamics. It has been observed that mass of human cardiac muscle varies with age and given below in table 2.

**Table 2:** Variation of mass of heart muscle with age

| Age Group (Years) | Heart Muscle Mass (Male) (gm) | Heart Muscle Mass (Female) (gm) |
|-------------------|------------------------------|-------------------------------|
| 15-20             | 264                          | 196                           |
| 20-30             | 277                          | 251                           |
| 30-50             | 297                          | 267                           |
| 50-60             | 317                          | 256                           |
So the model is based on the glimpses of changing transfer function with different age groups due to the variation of mass of heart muscle. The variation of mass with different age group is given above [VIII]. Equating equation no (4) and (5) from the above stated, a mathematical relation has been established between the modeling parameters

\[ \frac{1/M}{s^2 + \frac{B}{M}s + \frac{K}{M}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]  

(8)

Again from the previous equation (3) numerically it has calculated that

\[ K_1 = 1/M \]  

(9)

\[ K_3 = k/M \]  

(10)

Since numerically, K1 and K3 are equal from equation (8), it can be concluded that

\[ 1/M = K/M \]  

(11)

From the above equation, it can be observed that, \( K = 1 \).

Again from equation (8) it is observed that,

\[ B = M*2\xi\omega_n \]  

(12)

So from the above stated equation it can be observed that \( B \) is function of \( M \) and \( \xi \).

V. Variation in Transfer Function Due to Different Value of Damping Coefficient and Mass

The various transfer functions has been derived using different damping frequency and the value of viscous drag has also been calculated for both male and female considering different mass. The settling time and the steady state error has also been simulated [IX] with the help of Matlab 13.0 and represented as follows.

Table 3.(a) Transfer Functions with the variation of mass and damping coefficient(Male)

| Mass(Male) | Damping Frequency | Viscous Drag(B) | Transfer Function | Settling Time | Steady State Error |
|------------|-------------------|----------------|-------------------|---------------|-------------------|
| 0.1        | 399.696           | \( \frac{1}{264s^2 + 399.699s + 1} \) | 1.56*\( e^{-03} \) | 1.26*\( e^{-04} \) |
| 0.2        | 816.288           | \( \frac{1}{264s^2 + 816.288s + 1} \) | 3.192*\( e^{-03} \) | 1.26*\( e^{-04} \) |
| 0.3        | 1249.776          | \( \frac{1}{264s^2 + 1249.776s + 1} \) | 4.89*\( e^{-03} \) | 1.26*\( e^{-04} \) |
| 0.4        | 1740.288          | \( \frac{1}{264s^2 + 1740.288s + 1} \) | 6.80*\( e^{-03} \) | 1.589*\( e^{-04} \) |
| \(a\) | \(b\) | \(c\) | \(d\) |
|------|------|------|------|
| 0.5  | 2286.24 | \(\frac{1}{264s^2 + 2286.24s + 1}\) | \(8.94 \times 10^3\) | \(1.589 \times 10^{-4}\) |
| 0.6  | 2968.416 | \(\frac{1}{264s^2 + 2968.416s + 1}\) | \(1.16 \times 10^3\) | \(1.589 \times 10^{-4}\) |
| 0.7  | 3880.8 | \(\frac{1}{264s^2 + 3880.8s + 1}\) | \(1.52 \times 10^3\) | \(1.589 \times 10^{-4}\) |
| 0.8  | 5280 | \(\frac{1}{264s^2 + 5280s + 1}\) | \(2.07 \times 10^3\) | \(1.258 \times 10^{-4}\) |
| 0.9  | 8316 | \(\frac{1}{264s^2 + 8316s + 1}\) | \(3.25 \times 10^3\) | \(1.258 \times 10^{-4}\) |
| 1    | 419.378 | \(\frac{1}{277s^2 + 419.378s + 1}\) | \(1.64 \times 10^3\) | \(1.2609 \times 10^{-4}\) |
| 0.2  | 856.484 | \(\frac{1}{277s^2 + 856.484s + 1}\) | \(3.35 \times 10^3\) | \(1.259 \times 10^{-4}\) |
| 0.3  | 1311.318 | \(\frac{1}{277s^2 + 1311.318s + 1}\) | \(5.13 \times 10^3\) | \(1.259 \times 10^{-4}\) |
| 0.4  | 1825.984 | \(\frac{1}{277s^2 + 1825.984s + 1}\) | \(7.142 \times 10^3\) | \(1.259 \times 10^{-4}\) |
| 0.5  | 2398.82 | \(\frac{1}{277s^2 + 2398.82s + 1}\) | \(9.384 \times 10^3\) | \(1.259 \times 10^{-4}\) |
| 0.6  | 3114.588 | \(\frac{1}{277s^2 + 3114.588s + 1}\) | \(1.218 \times 10^4\) | \(1.259 \times 10^{-4}\) |
| 0.7  | 4071.9 | \(\frac{1}{277s^2 + 4071.9s + 1}\) | \(1.592 \times 10^3\) | \(1.258 \times 10^{-4}\) |
| 0.8  | 5540 | \(\frac{1}{277s^2 + 5540s + 1}\) | \(2.167 \times 10^3\) | \(1.258 \times 10^{-4}\) |
| 0.9  | 8725.5 | \(\frac{1}{277s^2 + 8725.5s + 1}\) | \(3.413 \times 10^3\) | \(1.258 \times 10^{-4}\) |
| 0.1  | 449.658 | \(\frac{1}{297s^2 + 449.658s + 1}\) | \(1.757 \times 10^3\) | \(1.26 \times 10^{-4}\) |
| 0.2  | 918.324 | \(\frac{1}{297s^2 + 918.324s + 1}\) | \(3.591 \times 10^3\) | \(1.259 \times 10^{-4}\) |
| 0.3  | 1405.998 | \(\frac{1}{297s^2 + 1405.998s + 1}\) | \(5.499 \times 10^3\) | \(1.259 \times 10^{-4}\) |
|   |   |   |   |   |
|---|---|---|---|---|
| 0.4 | 1957.824 | \( \frac{1}{297s^2 + 1957.824s + 1} \) | 7.658e+03 | 1.259e-04 |
| 0.5 | 2572.02  | \( \frac{1}{297s^2 + 2572.02s + 1} \) | 1.006e+03 | 1.259e-04 |
| 0.6 | 3339.468 | \( \frac{1}{297s^2 + 3339.468s + 1} \) | 1.306e+03 | 1.259e-04 |
| 0.7 | 4365.9   | \( \frac{1}{297s^2 + 4365.9s + 1} \) | 1.708e+03 | 1.258e-04 |
| 0.8 | 5940     | \( \frac{1}{297s^2 + 5940s + 1} \) | 2.323e+03 | 1.258e-04 |
| 0.9 | 9355.5   | \( \frac{1}{297s^2 + 9355.5s + 1} \) | 3.659e+03 | 1.258e-04 |
| 1.0 | 479.938  | \( \frac{1}{317s^2 + 479.938s + 1} \) | 1.875e+03 | 1.260e-04 |
| 0.2 | 980.164  | \( \frac{1}{317s^2 + 980.164s + 1} \) | 3.833e+03 | 1.259e-04 |
| 0.3 | 1500.678 | \( \frac{1}{317s^2 + 1500.678s + 1} \) | 5.870e+03 | 1.259e-04 |
| 0.4 | 2089.664 | \( \frac{1}{317s^2 + 2089.664s + 1} \) | 8.174e+03 | 1.259e-04 |
| 0.5 | 2745.22  | \( \frac{1}{317s^2 + 2745.22s + 1} \) | 1.703e+03 | 1.259e-04 |
| 0.6 | 3564.348 | \( \frac{1}{317s^2 + 3564.348s + 1} \) | 1.394e+03 | 1.259e-04 |
| 0.7 | 4659.9   | \( \frac{1}{317s^2 + 4659.9s + 1} \) | 1.823e+03 | 1.258e-04 |
| 0.8 | 6340     | \( \frac{1}{317s^2 + 6340s + 1} \) | 2.480e+03 | 1.258e-04 |
| 0.9 | 9985.5   | \( \frac{1}{317s^2 + 9985.5s + 1} \) | 3.906e+03 | 1.258e-04 |
### Table 3(b) Transfer Functions with the variation of mass and damping coefficient (Female)

| Mass (Female) | Damping Frequency | Viscous Drag (B) | Transfer Function | Settling Time | Steady State Error |
|---------------|------------------|-----------------|------------------|--------------|-------------------|
| 196           | 0.1              | 296.744         | $196s^2 + 296.744s + 1$ | 1.158e+03    | 1.261e+04         |
|               | 0.2              | 606.032         | $196s^2 + 606.032s + 1$ | 2.369e+03    | 1.259e+04         |
|               | 0.3              | 927.864         | $196s^2 + 927.864s + 1$ | 3.629e+03    | 1.259e+04         |
|               | 0.4              | 1292.032        | $196s^2 + 1292.032s + 1$ | 5.054e+03    | 1.259e+04         |
| 251           | 0.1              | 3920            | $196s^2 + 3920s + 1$ | 1.533e+03    | 1.258e+04         |
|               | 0.2              | 776.092         | $196s^2 + 776.092s + 1$ | 2.415e+03    | 1.258e+04         |
|               | 0.3              | 1188.234        | $196s^2 + 1188.234s + 1$ | 4.647e+03    | 1.259e+04         |
|               | 0.4              | 1654.592        | $196s^2 + 1654.592s + 1$ | 6.472e+03    | 1.259e+04         |
| 267           | 0.1              | 3689.7          | $251s^2 + 3689.7s + 1$ | 1.443e+03    | 1.258e+04         |
|               | 0.2              | 5020            | $251s^2 + 5020s + 1$ | 1.963e+03    | 1.258e+04         |
|               | 0.3              | 7906.5          | $251s^2 + 7906.5s + 1$ | 3.093e+03    | 1.258e+04         |
|               | 0.4              | 1263.978        | $267s^2 + 1263.978s + 1$ | 4.944e+03    | 1.259e+04         |
|               |                  | 1760.064        | $267s^2 + 1760.064s + 1$ | 6.885e+03    | 1.259e+04         |

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VI. Response of different Transfer Functions & Simulation Output

It is observed from the step response of the modeled transfer function, the settling time and steady state error has been minimized for a particular value of damping coefficient and viscous drug. In the next study, these values will be used for modeling purpose to get the optimum design. The step response of the two modeled transfer function has been shown in Figure 2.

![Figure 2](image)

Figure 2. This is a figure, shows the step responses of two transfer function.
VII. Best Modeling value of $\xi$& $B$ for the Muscle dynamics

Finally the Transfer function is obtained for each case to design the model of human cardiac muscle. It has been observed that, for a particular age interval, there is certain value of damping coefficient for which the step response gives minimum settling time and minimum steady state error. The optimized value of damping coefficient and the viscous drag for different age range for both male and female with settling time and steady state error with the least value has been stated below in Table 4(a) & Table 4(b).

Optimized value of $\xi$& $B$ for the Muscle dynamics with least Settling time & Steady state error (for male)

Table 4(a). Optimum value of Damping Coefficient and Viscous Drag

| Age(Years) | Mass of Cardiac Muscle(M) | Damping Coefficient($\xi$) | Viscous Drag($B$) | Settling Time | Steady State Error |
|------------|---------------------------|-----------------------------|-------------------|---------------|-------------------|
| 15-20      | 264gm                     | 0.8                         | 5280              | $2.07*e^{03}$ | $1.258*e^{-04}$   |
| 20-30      | 277gm                     | 0.6                         | 3114.588          | $1.218*e^{03}$ | $1.259*e^{-04}$   |
| 30-50      | 297gm                     | 0.5                         | 2572.02           | $1.006*e^{03}$ | $1.259*e^{-04}$   |
| 50-60      | 317gm                     | 0.6                         | 3564.348          | $1.394*e^{03}$ | $1.259*e^{-04}$   |

Optimized value of $\xi$& $B$ for the Muscle dynamics with least Settling time & Steady state error (for female)

Table 4(b). Optimum value of Damping Coefficient and Viscous Drag

| Age(Years) | Mass of Cardiac Muscle(M) | Damping Coefficient($\xi$) | Viscous Drag($B$) | Settling Time | Steady State Error |
|------------|---------------------------|-----------------------------|-------------------|---------------|-------------------|
| 15-20      | 196gm                     | 0.7                         | 2881.2            | $1.127*e^{03}$ | $1.259*e^{-04}$   |
| 20-30      | 251gm                     | 0.6                         | 2822.244          | $1.104*e^{03}$ | $1.259*e^{-04}$   |
| 30-50      | 267gm                     | 0.6                         | 3002.148          | $1.174*e^{03}$ | $1.259*e^{-04}$   |
| 50-60      | 256gm                     | 0.6                         | 2878.464          | $1.726*e^{03}$ | $1.259*e^{-04}$   |

VIII. Transfer Function of Human Heart Muscle Considering the optimum value of $\xi$& $B$

In Control theory, transfer functions are commonly used to characterize the input output relationships of systems that can be described by linear time invariant
differential equation. In this study, our proposed model is based on the transfer function approach. To get the optimal design, the transfer functions will be used for muscle dynamics in the modeling of human cardiovascular system based on age are given below in Table 5 (for male and female both).

Table 5(a). Modeled Transfer Functions with considering the optimum value of value
of $\xi$ & B

| Type  | Age    | Value of $\xi$ | Mass of Cardiac Muscle(M) | Value of B | T.F                                  |
|-------|--------|---------------|---------------------------|------------|--------------------------------------|
| Male  | 15-20  | 0.8           | 264                       | 5280       | $\frac{1}{264s^2 + 5280s + 1}$     |
| Female| 15-20  | 0.7           | 196                       | 2881       | $\frac{1}{196s^2 + 2881.2s + 1}$   |

Table 5(b). Modeled Transfer Functions with considering the optimum value of value
of $\xi$ & B

| Type  | Age    | Value of $\xi$ | Mass of Cardiac Muscle(M) | Value of B | T.F                                  |
|-------|--------|---------------|---------------------------|------------|--------------------------------------|
| Male  | 20-30  | 0.6           | 277                       | 3114       | $\frac{1}{277s^2 + 3114.588s + 1}$ |
| Female| 20-30  | 0.6           | 251                       | 2821       | $\frac{1}{251s^2 + 2822.244s + 1}$ |

Table 5(c). Modeled Transfer Functions with considering the optimum value of value
of $\xi$ & B

| Type  | Age    | Value of $\xi$ | Mass of Cardiac Muscle(M) | Value of B | T.F                                  |
|-------|--------|---------------|---------------------------|------------|--------------------------------------|
| Male  | 30-50  | 0.6           | 297                       | 2572       | $\frac{1}{297s^2 + 2572.02s + 1}$  |
Table 5(d). Modeled Transfer Functions with considering the optimum value of value of $\xi$ & B

| Type | Age   | Value of $\xi$ | Mass of Cardiac Muscle(M) | Value of B | TF                  |
|------|-------|----------------|---------------------------|------------|---------------------|
| Male | 50-60 | 0.6            | 317                       | 3564       | $\frac{1}{317s^2 + 3564.348s + 1}$ |
| Female | 50-60 | 0.6            | 256                       | 2878       | $\frac{1}{256s^2 + 2878.464s + 1}$ |

IX. Performance Index and Optimal System

It is generally used to evaluate the performance of the system. A quantitative measure of the performance or degree of performance of a system is necessary to design an optimum system. A system is considered an optimum system when the parameters of the system are adjusted so that the index reaches an extreme value commonly a minimum value. A performance index must be a number that is always positive or zero.

Optimal Control

Optimal control is one particular branch of modern control that sets out to provide analytical designs of especially appealing type. The system that is the end result of an optimal design [XIII] is not supposed merely to be stable, have a certain bandwidth, or satisfy any one of the desirable constraints associated with classical control, but it is supposed to be the best possible system of a particular type – hence, the word optimal. If it is both optimal and possesses a number of the properties that classical control suggest are desirable [III,XIV,XVII], so much the better.

Integral Square

Instead of the time domain calculation of $J$ (integral square error), the complex frequency domain [XXI] can be used. According to a theorem in mathematics by Parseval,

$$J = J_{SE} = \int_{0}^{\infty} e^2 (t) dt = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} E(s)E(-s)ds$$  \hspace{1cm} (13)
where $E(s)$ can be expressed as follows:

$$E(s) = \frac{a_n^{-1} N_{n-1} + \cdots + a_n s + N_0}{s^n + D_n + s^{n-1} D_{n-1} + \cdots + D_1 + D_0}$$  \hspace{1cm} (14)$$

Assuming type 1 behavior, $J$ follows from complex variable theory. To clarify the effect of system order, the subscript for $J$ will be the system order. For an $n$th order system, the performance index is given by the relation given below.

$$J = (-1)^{n-1} \frac{B_n}{2D_n H_n}$$  \hspace{1cm} (15)$$

where $H_n$ and $B_n$ are determinants. $H_n$ is the determinant of the $n \times n$ matrix. The first two rows of the Hurwitz matrix are formed from the coefficients of $D(s)$, while the remaining rows consist of right-shifted versions of the first two rows until the $n \times n$ matrix is formed. Thus we write,

$$H = \begin{bmatrix} D_{n-1} & D_{n-3} & \cdots & \cdots \\ D_n & D_{n-2} & \cdots & \cdots \\ 0 & D_{n-1} & D_{n-3} & \cdots \\ 0 & D_0 & D_{n-2} & \cdots \end{bmatrix}$$

$$N(s)N(-s) = b_{2n-2} s^{2n-2} + \cdots + b_2 s^2 + b_0$$  \hspace{1cm} (16)$$

then first row of the Hurwitz matrix is replaced by the coefficients of $N(s)N(-s)$, while the remaining rows are unchanged.

$$B = \begin{bmatrix} b_{2n-2} & \cdots & b_2 & b_0 \\ D_n & D_{n-2} & \cdots & \cdots \\ 0 & D_{n-1} & D_{n-3} & \cdots \\ 0 & D_0 & D_{n-2} & \cdots \end{bmatrix}$$

**System Design**

One simplified model of a control system for regulating the heart rate of a patient in an efficient way is considered as shown in Figure 3.

**Figure 3.** Controller with plant in series with a unity feedback

$$G_c(s) = K(s + 30)$$  \hspace{1cm} (17)$$

$H(s)=1$; $R(s) =$ Input ; $Y(s) =$ Desired output.

$G_M(s)$ = Transfer Function obtained for different values of mass $M$ of heart muscle and different values of viscous drag $B$ by changing the value of damping coefficient. The overall control system study consists of cascade connection of $G_c(s)$ & $G_M(s)$. 

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with negative unity feedback. Under present situation overall transfer function is given by the relation
\[
\frac{C(s)}{R(s)} = \frac{G_C(s)G_M(s)}{1+G_C(s)G_M(s)+1}
\]
(18)

**Calculation of Integral Square Error**

To find out the error by using integral square method, equation (15) is used. One of the modeled transfer function is taken for calculation. Here the transfer function 
\[
G_M(s) = \frac{1}{277s^2+3114.588s+1}
\]
is chosen to derive the integral square error which has been calculated for muscle having mass of 277gm.

\[
G_C(s) = k(s+30)
\]

Now \(G_M(s) \cdot G_C(s) = \frac{1}{277s^2+3114.588s+1} \cdot k(s+30)
\]
(19)

\[
T(s) = C(s) = \frac{ks+30k}{277s^2+(3114+k)s+(30k+1)}
\]
(20)

Characteristic Equation: 
\[
277s^2+(3114+k)s+(30k+1)=0
\]
(21)

Applying Routh Stability Criterion

\[
\begin{array}{ccc}
 s^2 & 277 & 30k + 1 \\
 s^1 & 3114 + k & 0 \\
 s^0 & 30k + 1 & . \\
\end{array}
\]

\[
T_E(s) = \frac{1-T(s)}{s} = \frac{277s^2+(3114+k)s+30k+1-ks-30k}{s[277s^2+(3114+k)s+(30k+1)]}
\]
(22)

From the above we can derive the coefficients as follows:

\[
N_1 = 277 \quad N_2 = 3114 \\
D_2 = 277 \quad D_1 = (3114+k) \quad D_0 = (30k+1)
\]

So for a 2nd order system the performance index is given by formula

\[
J_2 = \frac{-B_2}{2D_2H_2}
\]
(23)

Now calculating the value of \(H_2\) matrix as follows

\[
H_2 = \begin{bmatrix} D_1 & 0 \\ D_2 & D_0 \end{bmatrix} = \begin{bmatrix} 3114 + k & 0 \\ 277 & 30k + 1 \end{bmatrix}
\]

\[
= 93420k+3114+30k^2+k
\]

\[
= 30k^2 + 93421k+3114
\]

\[
= 30(k^2 + 3114.03k+103.8)
\]
(24)

Now the value of \(N(s)\) can be written as \(N(s) = 277s^2+3114\quad \& N(-s) = -\)
Now \( N(s) \ast N(-s) = -76729s^2 + 9696996 \) (25)

So deriving the coefficient of \( b_2 \) & \( b_0 \) from the above equation as \( b_2 = -76729 \) \( b_0 = 9696996 \)

Now calculating the value of \( B_2 \) matrix as follows.

\[
B_2 = \begin{vmatrix} -76729 & 9696996 \\ 277 & 30k + 1 \end{vmatrix}
\]

\[
= -2301870k - 76729 - 2686067892
\]

\[
= -2301870(k+1166.94)
\]

So \( J_2 = \frac{2301870(k+1166.94)}{2.277.30(k^2+3114.03k+103.8)} = \frac{138.5(k+1166.94)}{2.277.30(k^2+3114.03k+103.8)} \) (26)

Similarly calculating the performance index \( (J_2) \) modeled transfer functions:

**Table 6(a). Performance Index for modeled Transfer Function**

| Muscle Mass(Male) | Transfer Function | \( J_2 \) (Performance Index) |
|-------------------|------------------|-------------------------------|
| 264gm             | \( \frac{1}{264s^2 + 5280s + 1} \) | 132(k + 3520.03) |
| 297gm             | \( \frac{1}{297s^2 + 2572.02s + 1} \) | 148.5(k + 742.47) |
| 317gm             | \( \frac{1}{317s^2 + 3564.348s + 1} \) | 158.5(k + 1335.7) |

**Table 6(b). Performance Index for modeled Transfer Function**

| Muscle Mass(Female) | Transfer Function | \( J_2 \) (Performance Index) |
|---------------------|-------------------|-------------------------------|
| 196gm               | \( \frac{1}{196s^2 + 2881.2s + 1} \) | 9.8(k + 1411.63) |
| 251gm               | \( \frac{1}{251s^2 + 2822.244s + 1} \) | 125.5(k + 1102.31) |
| 267gm               | \( \frac{1}{267s^2 + 3002.148s + 1} \) | 133.5(k + 1125.13) |

**X. Conclusion**

In this paper the cardio vascular control model has been developed representing the muscle M, viscous drag B which is proportional to the movement of the wall of the specimen. Here cardiac muscle specimen suffers the displacement \( x(t) \) due to input force represented by \( f(t) \). The progress and calculation has been done by
Considering the cardiovascular system to be the 2nd order under damped system. Comparing the modeled equation with standard 2nd order control system equation, the conclusion can be derived that viscous damping \((B)\) is a function of cardiac muscle mass \(\text{‘M’}\), damping coefficient \(\xi\) and the natural frequency \(\omega_n\). The different value of viscous drag \((B)\) has been calculated taking different muscle mass \(\text{‘M’}\) based on different age.

The damping coefficient \((\xi)\) is varied from 0.1 to 0.9 for individual \(\text{‘M’}\) to get different transfer function of the model stated above. Steady state error has also been calculated for different transfer function to get an optimized value of \(\text{‘B’}\). A performance index is a quantitative measure of performance of the system and is chosen so that emphasis is given to the important system specification. To get the optimal design of the total system performance index has also been calculated by using integral square method. The system offers an in depth sight for the biomedical engineering application of human heart muscle operation in optimal condition.

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