Lightweight LCP-Array Construction in Linear Time

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Abstract. The suffix tree is a very important data structure in string processing, but it suffers from a huge space consumption. In large-scale applications, compressed suffix trees (CSTs) are therefore used instead. A CST consists of three (compressed) components: the suffix array, the LCP-array, and data structures for simulating navigational operations on the suffix tree. The LCP-array stores the lengths of the longest common prefixes of lexicographically adjacent suffixes, and it can be computed in linear time. In this paper, we present new LCP-array construction algorithms that are fast and very space efficient. In practice, our algorithms outperform the currently best algorithms.

1 Introduction

A suffix tree for a string $S$ of length $n$ is a compact trie storing all the suffixes of $S$. It is an extremely important data structure with applications in string matching, bioinformatics, and document retrieval, to mention only a few examples; see e.g. [2]. The drawback of suffix trees is their huge space consumption of about 20 times the text size, even in carefully engineered implementations. To reduce their size, several authors provided compressed suffix trees (CSTs); see e.g. [12] and [8] for a survey. A CST of $S$ can be divided into three components: (1) the suffix array $SA$, specifying the lexicographic order of $S$'s suffixes, (2) the LCP-array, storing the lengths of the longest common prefixes of lexicographically adjacent suffixes, and (3) additional data structures for simulating navigational operations on the suffix tree.

Particular emphasis has been put on efficient construction algorithms for all three components of CSTs. (Here, “efficiency” encompasses both construction time and space, as the latter can cause a significant memory bottleneck.) This is especially true for the first component. In the last decade, much effort has gone into the development of efficient suffix array construction algorithms (SACAs); see [10] for a survey. Although linear-time direct SACAs were known since 2003, experiments showed [10] that these were outperformed in practice by SACAs having a worst-case time complexity of $O(n^2 \log n)$. To date, however, the fastest SACA is a linear time algorithm [9]. Interestingly, for ASCII alphabet its speed can compete with the fastest LCP-array construction algorithms (LACA) which uses equal or less space. This is somewhat surprising because sorting all suffixes seems to be more difficult than computing lcp-values.

As discussed in Section [2] today’s best LACAs [35] are linear time algorithms, but they suffer from a poor locality behavior. In this paper, we present two very space efficient (using...
or $2n$ bytes only) and fast LACAs. Based on the observation that one cache miss takes approximately the time of 20 character comparisons, we try to trade character comparisons for cache misses. The algorithms use the text (string) $S$, the suffix array, and the Burrows-Wheeler Transform (BWT). Since most CSAs are based on the BWT anyway, we basically get it for free. Our experiments show the significance of the algorithms. More precisely, experimental results show that our algorithms outperform state-of-the-art algorithms [3,5]. For large texts they are always faster than the previously best algorithms. The superiority of our new LACAs varies with the text size (the larger the better), the alphabet size (the smaller the better), the number of “large” values in the LCP-array (the less the better), and the runs in the BWT (the more the better). The algorithms work particularly well on two types of data that are of utmost importance in practice: long DNA sequences (small alphabet size) and large collections of XML documents (long runs in the BWT).

## 2 Related work

In their seminal paper [6], Manber and Myers did not only introduce the suffix array but also the longest-common-prefix (LCP) array. They showed that both the suffix array and the LCP-array can be constructed in $O(n \log n)$ time for a string of length $n$. Kasai et al. [5] gave the first linear time algorithm for the computation of the LCP-array. Their algorithm uses the string $S$, the suffix array, the inverse suffix array, and of course the LCP-array. Each of the arrays requires $4n$ bytes (under the assumption that $n < 2^{32}$), thus the algorithms needs $13n$ bytes in total (for an ASCII alphabet). The main advantage of their algorithm is that it is simple and uses at most $2n$ character comparisons. But its poor locality behavior results in many cache misses, which is a severe disadvantage on current computer architectures. Manzini [7] reduced the space occupancy of Kasai et al.’s algorithm to $9n$ bytes with a slow down of about $5\%$ – $10\%$. He also proposed an even more space-efficient (but slower) algorithm that overwrites the suffix array. Recently, Kärkkäinen et al. [8] proposed another variant of Kasai et al.’s algorithm, which computes a permuted LCP-array (PLCP-array). In the PLCP-array, the lcp-values are in text order (position order) rather than in suffix array order (lexicographic order). This algorithm takes only $5n$ bytes and is much faster than Kasai et al.’s algorithm because it has a much better locality behavior. However, in virtually all applications lcp-values are required to be in suffix array order, so that in a final step the PLCP-array must be converted into the LCP-array. Although this final step suffers (again) from a poor locality behavior, the overall algorithm is still faster than Kasai et al.’s. In a different approach, Puglisi and Turpin [11] tried to avoid cache misses by using the difference cover method of Kärkkäinen and Sanders [4]. The worst case time complexity of their algorithm is $O(nv)$ and the space requirement is $n + O(n/\sqrt{v} + v)$ bytes, where $v$ is the size of the difference cover. Experiments showed that the best run-time is achieved for $v = 64$, but the algorithm is still slower than Kasai et al.’s. This is because it uses constant time range minimum queries, which take considerable time in practice. To sum up, the currently best LACA is that of Kärkkäinen et al. [3].
3 Preliminaries

Let $\Sigma$ be an ordered alphabet whose smallest element is the so-called sentinel character $\$$. If $\Sigma$ consists of $\sigma$ characters and is fixed, then we may view $\Sigma$ as an array of size $\sigma$ such that the characters appear in ascending order in the array $\Sigma[0..\sigma - 1]$, i.e., $\Sigma[0] = \$ < $\Sigma[1] < \ldots < $\Sigma[\sigma - 1]$. In the following, $S$ is a string of length $n$ over $\Sigma$ having the sentinel character at the end (and nowhere else). For $0 \leq i \leq n - 1$, $S[i]$ denotes the character at position $i$ in $S$. For $i \leq j$, $S[i..j]$ denotes the substring of $S$ starting with the character at position $i$ and ending with the character at position $j$. Furthermore, $S_i$ denotes the suffix $S[i..n - 1]$ of $S$. The suffix array $SA$ of the string $S$ is an array of integers in the range $0$ to $n - 1$ specifying the lexicographic ordering of the $n$ suffixes of the string $S$, that is, it satisfies $SA[0] < SA[1] < \cdots < SA[n-1]$; see Fig. 1 for an example. In the following, ISA denotes the inverse of the permutation $SA$.

The LCP-array is an array containing the lengths of the longest common prefix between every pair of consecutive suffixes in $SA$. We use $\text{lcp}(u, v)$ to denote the length of the longest common prefix between strings $u$ and $v$. Thus, the lcp-array is an array of integers in the range $0$ to $n$ such that $\text{LCP}[0] = -1$, $\text{LCP}[n] = -1$, and $\text{LCP}[i] = \text{lcp}(SA[i-1], SA[i])$ for $1 \leq i \leq n - 1$; see Fig. 1. For $i < j$, a range minimum query $\text{RMQ}(i, j)$ on the interval $[i..j]$ in the LCP-array returns an index $k$ such that $\text{LCP}[k] = \min\{\text{LCP}[l] \mid i \leq l \leq j\}$. It is not difficult to show that $\text{lcp}(SA[i], SA[j]) = \text{LCP}[\text{RMQ}(i + 1, j)]$.

The Burrows and Wheeler transform [1] converts a string $S$ into the string $\text{BWT}[0..n - 1]$ defined by $\text{BWT}[i] = S[SA[i] - 1]$ for all $i$ with $SA[i] \neq 0$ and $\text{BWT}[i] = \$$ otherwise; see Fig. 1. The LF-mapping is defined by $\text{LF}[i] = \text{ISA}[SA[i] - 1]$ for all $i$ with $SA[i] \neq 0$ and $\text{LF}[i] = 0$ otherwise; see Fig. 1. Its long name last-to-first column mapping stems from the fact that it maps the last column $L = \text{BWT}$ to the first column $F$, where $F$ contains the first character of the suffixes in the suffix array, i.e., $F[i] = S[SA[i]]$. More precisely, if $\text{BWT}[i] = c$ is the $k$-th occurrence of character $c$ in $\text{BWT}$, then $j = \text{LF}[i]$ is the index such that $F[j]$ is the $k$-th occurrence of $c$ in $F$. The LF-mapping can be implemented by $\text{LF}[i] = C[c] + \text{occ}(c, i)$, where $c = \text{BWT}[i]$, $C[c]$ is the overall number (of occurrences) of characters in $S$ which are strictly smaller than $c$, and $\text{occ}(c, i)$ is the number of occurrences of the character $c$ in $\text{BWT}[1..i]$.

4 First algorithm

In this section, we present our first LACA. A pseudo-code description can be found in Algorithm 1 and an application of it is illustrated in Fig. 1. Furthermore, Theorem 1 does not only prove its correctness but also explains it. The algorithm is based on Lemma 1, which in turn requires the following definition.

Define a function $\text{prev}$ by

$$\text{prev}(i) = \max\{j \mid 0 \leq j < i \text{ and } \text{BWT}[j] = \text{BWT}[i]\}$$
Fig. 1. Left: Suffix array, LCP-array, LF-mapping, and BWT of the string $S = \text{el}_\text{anele}_\text{lepanelen}$. Right: The LCP-array after the $j$th iteration of Algorithm 1 (omitted entries are not computed yet).

where $\text{prev}(i) = -1$ if the maximum is taken over an empty set. Intuitively, if we start at index $i$ and scan the BWT upward, then $\text{prev}(i)$ is the first index at which the same character $\text{BWT}[i]$ occurs.

Lemma 1.

$$\text{LCP}[\text{LF}[i]] = \begin{cases} 0, & \text{if } \text{prev}(i) = -1 \\ 1 + \text{LCP}[\text{RMQ}(\text{prev}(i) + 1, i)], & \text{otherwise} \end{cases}$$

Proof. If $\text{prev}(i) = -1$, then $S_{\text{LF}[i]}$ is the lexicographically smallest suffix among all suffix having $\text{BWT}[i]$ as first character. Hence $\text{LCP}[\text{LF}[i]] = 0$. Otherwise, $\text{LF}[\text{prev}(i)] = \text{LF}[i] - 1$. In this case, it follows that

$$\text{LCP}[\text{LF}[i]] = \text{lcp}(S_{\text{SA}[\text{LF}[i]-1]}, S_{\text{SA}[\text{LF}[i]]}) = \text{lcp}(S_{\text{SA}[\text{LF}[\text{prev}(i)]]}, S_{\text{SA}[\text{LF}[i]]}) = 1 + \text{lcp}(S_{\text{SA}[\text{prev}(i)]}, S_{\text{SA}[i]}) = 1 + \text{LCP}[\text{RMQ}(\text{prev}(i) + 1, i)]$$

Theorem 1. Algorithm 1 correctly computes the LCP-array.

Proof. Under the assumption that all entries in the LCP-array in the first $i-1$ iterations of the for-loop have been computed correctly, we consider the $i$-th iteration and prove:
**Algorithm 1** Construction of the LCP-array.

```
01 last_occ[0, σ − 1] ← [−1, −1, . . . , −1]
02 LCP[0] ← −1; LCP[n] ← −1; LCP[LF[0]] ← 0
03 for i ← 1 to n − 1 do
04     if LCP[i] = ⊥ then /* LCP[i] is undefined */
05         ℓ ← 0
06     if LF[i] < i then
07         ℓ ← max{LCP[LF[i]] − 1, 0}
08     if BWT[i] = BWT[i − 1] then
09         continue at line 12
10     while S[SA[i] + ℓ] = S[SA[i − 1] + ℓ] do
11         ℓ ← ℓ + 1
12     LCP[i] ← ℓ
13     if LF[i] > i then
14         LCP[LF[i]] ← LCP[RMQ(last_occ[BWT[i]] + 1, i)] + 1
15     last_occ[BWT[i]] ← i
```

1. If LCP[i] = ⊥, then the entry LCP[i] will be computed correctly.
2. If LF[i] > i, then the entry LCP[LF[i]] will be computed correctly.

(1) If the if-condition in line 6 is not true, then S_{SA[i−1]} and S_{SA[i]} are compared character by character (lines 10-11) and LCP[i] is assigned the correct value in line 12. Otherwise, if the if-condition in line 6 is true, then ℓ is set to max{LCP[LF[i]] − 1, 0}. We claim that ℓ ≤ LCP[i]. This is certainly true if ℓ = 0, so suppose that ℓ = LCP[LF[i]] − 1 > 0. According to (the proof of) Lemma 1, LCP[LF[i]] − 1 = lcp(S_{SA[prev(i)]}, S_{SA[i]}). Obviously, lcp(S_{SA[prev(i)]}, S_{SA[i]}) ≤ lcp(S_{SA[i−1]}, S_{SA[i]}), so the claim follows.

Now, if BWT[i] ≠ BWT[i − 1], then S_{SA[i−1]} and S_{SA[i]} are compared character by character (lines 10-11), but the first ℓ characters are skipped because they are identical. Again, LCP[i] is assigned the correct value in line 12. Finally, if BWT[i] = BWT[i − 1], then prev(i) = i − 1. This, in conjunction with Lemma 1 yields LCP[LF[i]] − 1 = lcp(S_{SA[prev(i)]}, S_{SA[i]}) = lcp(S_{SA[i−1]}, S_{SA[i]}) = LCP[i]. Thus, ℓ = LCP[LF[i]] − 1 is already the correct value of LCP[i]. So lines 10-11 can be skipped and the assignment in line 12 is correct.

(2) In the linear scan of the LCP-array, we always have last_occ[BWT[i]] = prev(i). Therefore, it is a direct consequence of Lemma 1 that the assignment in line 14 is correct.

We still have to explain how the index \( j = \text{RMQ}(\text{last}_\text{occ}[\text{BWT}[i]] + 1, i) \) and the lcp-value LCP[j] in line 14 can be computed efficiently. To this end, we use a stack \( K \) of size \( O(\sigma) \). Each element on the stack is a pair consisting of an index and an lcp-value. We first push \((0, −1)\) onto the initially empty stack \( K \). It is an invariant of the for-loop that the stack elements are strictly increasing in both components (from bottom to top). In the \( i \)-th iteration of the for-loop, before line 13, we update the stack \( K \) by removing all elements whose lcp-value is greater than or equal to LCP[i]. Then, we push the pair \((i, \text{LCP}[i])\) onto
\(K\). Clearly, this maintains the invariant. Let \(x = \text{last\_occ}[\text{BWT}[i]] + 1\). The answer to RMQ\((x, i)\) is the pair \((j, \ell)\) where \(j\) is the minimum of all indices that are greater than or equal to \(x\). This pair can be found by an inspection of the stack. Moreover, the lcp-value \(\text{LCP}[x] + 1\) we are looking for is \(\ell + 1\). To meet the \(O(\sigma)\) space condition of the stack, we check after each \(\sigma\)th update if the size \(s\) of \(K\) is greater than \(\sigma\). If so, we can remove \(s - \sigma\) elements from \(K\) because there are at most \(\sigma\) possible queries. With this strategy, the stack size never exceeds \(2\sigma\) and the amortized time for the updates is \(O(n)\). Furthermore, an inspection of the stack takes \(O(\sigma)\) time. In practice, this works particularly well when there is a run in the BWT because then the element we are searching for is on top of the stack.

Algorithm \([\text{I}1]\) has a quadratic run time in the worst case, consider e.g. the string \(S = \text{ababab...ab}\).

At first glance, Algorithm \([\text{I}1]\) does not have any advantage over Kasai et al.’s algorithm because it holds \(S, \text{SA}, \text{LF}, \text{BWT},\) and \(\text{LCP}\) in main memory. A closer look, however, reveals that the arrays \(\text{SA},\) \(\text{LF},\) and \(\text{BWT}\) are accessed sequentially in the for-loop. So they can be streamed from disk. We cannot avoid the random access to \(S\), but that to \(\text{LCP}\) as we shall show next.

Most problematic are the “jumps” upwards (line 7 when \(\text{LF}[i] < i\)) and downwards (line 14 when \(\text{LF}[i] > i\)). The key idea is to buffer lcp-values in queues (FIFO data structures) and to retrieve them when needed.

First, one can show that the condition \(\text{LCP}[i] = \bot\) in line 4 is equivalent to \(i \geq C[F[i]] + \text{occ}(\text{BWT}[i], i)\). The last condition can be evaluated in constant time and space (increment a counter \(\text{cnt}(\text{BWT}[i])\) in iteration \(i\), so it can replace \(\text{LCP}[i] = \bot\) in line 4. This is important because in case \(j = \text{LF}[i] > i\), the value \(\text{LCP}[j]\) is still in one of the queues and has not yet been written to the LCP-array. In other words, when we reach index \(j\), we still have \(\text{LCP}[j] = \bot\) although \(\text{LCP}[j]\) has already been computed. Thus, by the test \(i \geq C[F[j]] + \text{occ}(\text{BWT}[j], i)\) we can decide whether \(\text{LCP}[j]\) has already been computed or not.

Second, \(\text{LF}[i]\) lies in between \(C[\text{BWT}[i]]\) and \(C[\text{BWT}[i]] + \text{occ}(\text{BWT}[i], n - 1)\), the interval of all suffixes that start with character \(\text{BWT}[i]\). Note that there are at most \(\sigma\) different such intervals. We exploit this fact in the following way. For each character \(c \in \Sigma\) we use a queue \(Q_c\). During the for-loop we add (enqueue) the values \(\text{LCP}[[c]], \text{LCP}[[c]+1], \ldots, \text{LCP}[[c] + \text{occ}(c, n - 1)]\) in exactly this order to \(Q_c\). In iteration \(i\), an operation \(\text{enqueue}(Q_c, x)\) is done for \(c = \text{BWT}[i]\) and \(x = \text{LCP}[\text{RMQ}(\text{last\_occ}[\text{BWT}[i]] + 1, i) + 1\) in line 14 provided that \(\text{LF}[i] > i\), and in line 12 for \(c = F[i]\) and \(x = \ell\). Also in iteration \(i\), an operation \(\text{deque}(Q_c)\) is done for \(c = \text{BWT}[i]\) in line 7 provided that \(\text{LF}[i] < i\). This dequeue operation returns the value \(\text{LCP}[\text{LF}[i]]\) which is needed in line 7. Moreover, if \(i < C[F[i]] + \text{occ}(\text{BWT}[i], i)\), then we know that \(\text{LCP}[i]\) has been computed previously but is still in one of the queues. Thus, an operation \(\text{deque}(Q_c)\) is done for \(c = F[i]\) immediately before line 13, and it returns the value \(\text{LCP}[i]\).

The space used by the algorithm now only depends on the size of the queues. We use constant size buffers for the queues and read/write the elements to/from disk if the buffers
are full/empty (this even allows to answer an RMQ by binary search in $O(\log(\sigma))$ time). Therefore, only the text $S$ remains in main memory and we obtain an $n$ bytes semi-external algorithm.

5 Improved algorithm

Our experiments showed that even a careful engineered version of Algorithm 1 does not always beat the currently fastest LACA \[3\]. For this reason, we will now present another algorithm that uses a modification of Algorithm 1 in its first phase. This modified version computes each LCP-entry whose value is smaller than or equal to $m$, where $m$ is a user-defined value. (All we know about the other entries is that they are greater than $m$.) It can be obtained from Algorithm 1 by modifying lines 8, 10, and 14 as follows:

08 if $\text{BWT}[i] = \text{BWT}[i - 1]$ and $\ell < m$ then
10 while $S[\text{SA}[i] + \ell] = S[\text{SA}[i - 1] + \ell]$ and $\ell < m + 1$ do
14 $\text{LCP}[\text{LF}[i]] \leftarrow \min\{\text{LCP}[\text{RMQ(last_occ[\text{BWT}[i]] + 1, i]) + 1, m + 1]\}$

In practice, $m = 254$ is a good choice because LCP-values greater than $m$ can be marked by the value 255 and each LCP-entry occupies only one byte. Because the string $S$ must also be kept in main memory, this results in a total space consumption of $2n$ bytes.

Let $I = \{i \mid 0 \leq i < n \text{ and } \text{LCP}[i] \geq m\}$ be an array containing the indices at which the values in the LCP-array are $\geq m$ after phase 1. In the second phase we have to calculate the remaining $n_I = |I|$ many LCP-entries, and we use Algorithm 2 for this task. In essence, this algorithm is a combination of two algorithms presented in \[3\] that compute the PLCP-array: (a) the linear time $\Phi$-algorithm and (b) the $O(n \log n)$ time algorithm based on the concept of irreducible lcp-values. Let us recapitulate necessary definitions from \[3\].

Definition 1. For all $i$ with $1 \leq i \leq n - 1$ let $\Phi[\text{SA}[i]] = \text{SA}[i - 1]$, and for all $j$ with $0 \leq j \leq n - 1$ let $\text{PLCP}[j] = \text{lcp}(S_j, S_{\Phi[j]})$. An entry $\text{PLCP}[j]$, where $j > 0$, is called reducible if $S[j - 1] = S[\Phi[j] - 1]$; otherwise it is irreducible.

Note that $\text{PLCP}[\text{SA}[i]]$ is reducible if and only if $\text{BWT}[i] = \text{BWT}[i - 1]$. This is because $\text{BWT}[i] = S[\text{SA}[i] - 1]$ and $\text{BWT}[i - 1] = S[\text{SA}[i - 1] - 1] = S[\Phi[\text{SA}[i]] - 1]$.

Lemma 2. For all $j$ with $0 \leq j \leq n - 1$, we have $\text{PLCP}[j] \geq \text{PLCP}[j - 1] - 1$. Moreover, if $\text{PLCP}[j]$ is reducible, then $\text{PLCP}[j] = \text{PLCP}[j - 1] - 1$.

Proof. See \[57,3\].

The preceding lemma has the following two consequences:

- If we compute an entry $\text{PLCP}[j]$ (where $j$ varies from 1 to $n - 1$), then $\text{PLCP}[j - 1]$ many character comparisons can be skipped. This is the reason for the linear run time of Algorithm 2; cf. \[3\].
– If we know that PLCP\([j]\) is reducible, then no further character comparison is needed to determine its value. At first glance this seems to be unimportant because the next character comparison will yield a mismatch anyway. At second glance, however, it turns out to be important because the character comparison may result in a cache miss! (Note that in contrast to the \(O(n \log n)\) time algorithm in \(3\), the \(\Phi\)-algorithm does not make use of this property.)

\begin{algorithm}
\caption{Phase 2 of the construction of the LCP-array. (In practice \(SA[n-1]\) can be used for the undefined value \(\perp\) because the entries in the \(\Phi\)-array are of the form \(SA[i-1]\), i.e., \(SA[n-1]\) does not occur in the \(\Phi\)-array.)}

\begin{algorithmic}
\State \(b[0..n-1] \leftarrow [0,0,\ldots,0]\)
\For {\(i \leftarrow 0\) to \(n-1\)}
\If {\(\text{LCP}[i] > n\)}
\State \(b[\text{SA}[i]] \leftarrow 1\) /* the \(b\)-array can be computed in phase 1 already */
\EndIf
\EndFor
\State \(\Phi[0..n-1] \leftarrow[\perp,\perp,\ldots,\perp]\)
\State initialize a rank data structure for \(b\)
\For {\(i \leftarrow 0\) to \(n-1\)} /* stream \(SA\), \(LCP\), and \(BWT\) from disk */
\If {\(\text{LCP}[i] > n\) and \(BWT[i] \neq BWT[i-1]\) then} /* \(\text{LCP}[\text{SA}[i]]\) is irreducible */
\State \(\Phi[\text{rank}_1(\text{SA}[i])] \leftarrow \text{SA}[i-1]\)
\EndIf
\EndFor
\State \(j_i \leftarrow 0\)
\State \(\ell \leftarrow m+1\)
\State \(\text{PLCP}[0..n \ell - 1] \leftarrow [0,0,\ldots,0]\)
\For {\(j \leftarrow 0\) to \(n-1\)} /* left-to-right scan of \(b\) and \(S\), but random access to \(S\) */
\If {\(\text{b}[j] = 1\)}
\State \(\ell \leftarrow \ell - 1\) /* at least \(\ell - 1\) characters match by Lemma 2 */
\Else
\State \(\ell \leftarrow m+1\) /* at least \(m+1\) characters match by phase 1 */
\EndIf
\If {\(\Phi[j_i] \neq \perp\) then} /* \(\text{PLCP}[j_i]\) is irreducible */
\While {\(S[j + \ell] = S[\Phi[j_i] + \ell]\)}
\State \(\ell \leftarrow \ell + 1\)
\EndWhile
\State \(\text{PLCP}[j_i] \leftarrow \ell\) /* if \(\text{PLCP}[j_i]\) is reducible, no character comparison was needed */
\State \(j_i \leftarrow j_i + 1\)
\EndIf
\EndFor
\For {\(i \leftarrow 0\) to \(n-1\)} /* stream \(SA\) and \(LCP\) from disk */
\If {\(\text{LCP}[i] > m\)}
\State \(\text{LCP}[i] \leftarrow \text{PLCP}[\text{rank}_1(\text{SA}[i])]\)
\EndIf
\EndFor
\end{algorithmic}
\end{algorithm}

Algorithm 2 uses a bit array \(b\), where \(b[\text{SA}[i]] = 0\) if \(\text{LCP}[i]\) is known already (i.e., \(b[j] = 0\) if \(\text{PLCP}[j]\) is known) and \(b[\text{SA}[i]] = 1\) if \(\text{LCP}[i]\) still must be computed (i.e., \(b[j] = 1\) if \(\text{PLCP}[j]\) is unknown); see lines 1–4 of the algorithm. In contrast to the \(\Phi\)-algorithm \(3\),
our algorithm does not compute the whole $\Phi$-array (PLCP-array, respectively) but only the $n_I$ many entries for which the lcp-value is still unknown (line 5). So if we would delete the values $\Phi[j]$ (PLCP$[j]$, respectively) for which $b[j] = 0$ from the original $\Phi$-array (PLCP-array, respectively), we would obtain our array $\Phi[0..n_I - 1]$ (PLCP$[0..n_I - 1]$, respectively). We achieve a direct computation of $\Phi[0..n_I - 1]$ with the help of a rank data structure for the bit array $b$ such that rank queries $\text{rank}_1(j)$ can be answered in constant time, where $\text{rank}_1(j)$ returns the number of 1’s up to position $j$ in $b$. The for-loop in lines 7–9 fills our array $\Phi[0..n_I - 1]$ but again there is a difference to the original $\Phi$-array: reducible values are omitted! After initialization of the counter $j_I$, the number $\ell$ (of characters that can be skipped), and the PLCP array, the for-loop in lines 13–23 fills the array PLCP$[0..n_I - 1]$ by scanning the $b$-array and the string $S$ from left to right. In line 14, the algorithm tests whether the lcp-value is still unknown (this is the case if $b[j] = 1$). If so, it determines the number of characters that can be skipped in lines 15–18. If PLCP$[j_I]$ is irreducible (equivalently, $\Phi[j_I] \neq \perp$) then its correct values is computed by character comparisons in lines 20–21. Otherwise, PLCP$[j_I]$ is reducible and PLCP$[j_I] = \text{PLCP}[j_I - 1] - 1$ by Lemma[2]. In both cases PLCP$[j_I]$ is assigned the correct value in line 22. Finally, the missing entries in the LCP-array (lcp-values in suffix array order) are filled with the help of PLCP-array (lcp-values in text order) in lines 24–26.

Clearly, the first phase of our algorithm has a linear worst-case time complexity. The same is true of the second phase as explained above. Thus, the whole algorithm has a linear run-time.

References

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