N = 2 Super-Conformal Filed Theory on the Basis of osp(2|2)

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Abstract

Using a unified and systematic scheme, the free field realization of irreducible representations of osp(2|2) is constructed. By using these realization, the correlation functions of N = 2 super-conformal model based on osp(2|2) symmetry and free field representation of osp(2|2) generators are calculated. Free field representation of currents are used to determine the stress-energy tensor and the central charge of the model.
1 Introduction

There are many evidences that 2-dimensional $N = 2$ super-conformal field theories are very important in string theory [1], disordered systems at criticality [2, 3], topological field theories [4], integrable models [5], mirror symmetry [6] and quantum $W$-gravity [7]. Such interests are enough to motivate one to study different aspects of $N = 2$ in 2-dimension.

A problem of much physical interest is the calculation of correlation functions, using super-conformal Ward identities.

From the other side free field representations of Lie (super)algebras [8, 9, 10] are of interest because, they have many applications in (super)conformal field theories [11], inverse problems and integrable systems [12, 13], and statistical mechanics [14, 2, 3]. In any conformal or super-conformal field theory all the properties of the theory can be encoded in its OPE’s. The main point in any OPE calculation is to use as much symmetry as possible for simplification. If somehow one can get free field representation of currents [8] the OPE calculation will be straight forward.

In the case of super-symmetric conformal field theories, it is often useful to work in a super-space formalism. Using super-space we will be able to find coherent states and the differential realization for generators of conformal symmetry. Such a realization can be used to find free field representations of current algebra in level zero, then using Feigin and Frankel method [15] to extend it to any arbitrary level. On the other hand we will be able to use differential realization in Ward identities to calculate correlation functions of the conformal field theory based on $osp(2|2)$ symmetries.

In this paper we give a very simple way of calculation for their free field representation, which is extendable to all other symmetric groups.

Our aim in this paper is to apply the unified and systematic scheme given in ref. [9, 10], for $osp(2|2)$ and use these realization to calculate correlation functions of $N = 2$ super-conformal models based on $osp(2|2)$ symmetry group.

The structure of this paper is as follows: In section 2, we construct the differential realization of $osp(2|2)$ for all its irreducible representations. In section 3, we will use the results of section 2 to calculate two and three point functions of super-conformal field theory based on $osp(2|2)$ symmetry group. In section 4, we will use the differential realization of $osp(2|2)$ to calculate free field representation of $osp(2|2)$ algebra, and stress-energy tensor of the theory.
2 Differential realization of $osp(2|2)$

The non-compact super-group $OSP(2|2)$ is generated by eight elements [16, 17, 18]. The four even generators, $X_{\pm}$, $H$ and $B$, close under commutation and generate the subgroup of $sl(2) \times U(1)$ and with four others odd generators $V_{\pm}$ and $W_{\pm}$, satisfy the following (graded)commutation relations

\[
[H, X_{\pm}] = \pm X_{\pm} \quad [X_{+}, X_{-}] = -2H \quad [B, X_{\pm}] = 0 \\
[B, H] = 0 \quad [H, V_{\pm}] = \pm \frac{1}{2} V_{\pm} \quad [H, W_{\pm}] = \pm \frac{1}{2} W_{\pm} \\
[X_{\pm}, V_{\pm}] = 0 \quad [X_{\pm}, W_{\pm}] = 0 \quad [X_{\pm}, V_{\mp}] = \mp V_{\pm} \\
[X_{\pm}, W_{\mp}] = \mp W_{\pm} \quad [B, V_{\pm}] = \frac{1}{2} V_{\pm} \quad [B, W_{\pm}] = -\frac{1}{2} W_{\pm} \\
\{V_{\pm}, V_{\pm}\} = 0 \quad \{V_{\pm}, V_{\mp}\} = 0 \quad \{W_{\pm}, W_{\mp}\} = 0 \\
\{W_{\pm}, W_{\mp}\} = 0 \quad \{V_{\pm}, W_{\pm}\} = X_{\pm} \quad \{V_{\pm}, W_{\mp}\} = H \mp B
\]  

(1)

Let us define $< b, h; -2b, -2h, -2h > := < -2b, -2h >$ as the highest weight of an irreducible representation of $osp(2|2)$, where in left hand side the first two quantum numbers label the $osp(2|2)$ representation, while the next three labels are $U(1)$ quantum number, the $sl(2)$ quantum number and the corresponding third component, respectively. Such a highest weight is annihilated by all raising generators:

\[
< -2b, -2h | X_{\pm} = 0, \quad < -2b, -2h | V_{\mp} = 0, \\
< -2b, -2h | H = < -2b, -2h |(-2h), \quad < -2b, -2h | W_{\pm} = 0, \\
< -2b, -2h | B = < -2b, -2h |(-2b)
\]  

(2)

The relevant coherent state is as follows:

\[
< -2b, -2h | e^{(z-\frac{1}{2} \bar{\theta})X_{-}} e^{\bar{\theta}W_{-}} e^{\theta V_{-}} := < z, \theta, \bar{\theta} |
\]  

(3)

which $z$ is complex variable and $\theta$ and $\bar{\theta}$ are Grassmanian variables [18].

By the similar method as developed in [9, 10], by acting the group generators on the above coherent states and using the above (graded)commutation relations, the differential
realization of \(osp(2|2)\) for left action will be as follows:

\[
\begin{align*}
V_- &= \partial_\theta + \frac{1}{2} \theta \partial_z \\
W_- &= \partial_\theta + \frac{1}{2} \bar{\theta} \partial_z \\
V_+ &= z \partial_\theta + \frac{1}{2} z \theta \partial_z + \frac{1}{2} \theta \partial_\bar{\theta}_\theta - 2 \theta (h - b) \\
W_+ &= z \partial_\theta + \frac{1}{2} z \bar{\theta} \partial_z - \frac{1}{2} \theta \partial_\theta - 2 \bar{\theta} (h + b) \\
H &= \frac{1}{2} \theta \partial_\theta + \frac{1}{2} \bar{\theta} \partial_\theta + z \partial_z - 2h \\
B &= \frac{1}{2} \theta \partial_\theta - \frac{1}{2} \bar{\theta} \partial_\theta - 2b \\
X_- &= \partial_z \\
X_+ &= z^2 \partial_z + z \theta \partial_\theta + z \bar{\theta} \partial_\bar{\theta} - 2b \theta \bar{\theta} - 4hz.
\end{align*}
\]

The basis vectors of this super-vector space are

\[
\{< -2b, -2h |X_-, < -2b, -2h |(X_-)^2, \ldots, < -2b, -2h |(X_-)^4h, \\
< -2b, -2h |V_-, < -2b, -2h |X_+V_-, \ldots, < -2b, -2h |(X_-)^4h-1V_-, \\
< -2b, -2h |W_-, < -2b, -2h |X_-W_-, \ldots, < -2b, -2h |(X_-)^4h-1W_-, \\
< -2b, -2h |(V_-W_-), < -2b, -2h |(V_-W_-)^2, \ldots, < -2b, -2h |(V_-W_-)^{4h-2}\}.
\]

with dimension of \(16h\).

Geometrically, this realization describes the left action of the group on the sections of a holomorphic line bundle over the flag manifold \(osp(2|2)/T\), where \(T\) is maximal isotropic sub-algebra corresponding to the state \(< -2b, -2h | \) and \(| 2b, 2h >, \{H, B\} \). \(z\) and \(\theta\) are coordinates on the \(osp(2|2)/T\) and \(h\) and \(\beta\) are the coordinates on \(T\).

For right action we choose the highest weight such that

\[
\begin{align*}
X_+|2b, 2h >= 0, & \quad V_+|2b, 2h >= 0, \\
H|2b, 2h >= (2h)|2b, 2h >, & \quad W_+|2b, 2h >= 0, \\
B|2b, 2h >= (2b)|2b, 2h >.
\end{align*}
\]

The coherent state is

\[
e^{(z - \frac{1}{2} \theta \bar{\theta})X_-} e^{\bar{\theta}W_-} e^{\theta V_-} |2b, 2h > := |z, \theta, \bar{\theta} >.
\]
The relevant differential realization will be as follows:

\[
\begin{align*}
V_- &= \partial_{\bar{\theta}} + \frac{1}{2} \theta \partial_z \\
W_- &= \partial_{\theta} + \frac{1}{2} \bar{\theta} \partial_{\bar{z}} \\
V_+ &= -z \partial_{\bar{\theta}} - \frac{1}{2} z \theta \partial_z - \frac{1}{2} \theta \bar{\theta} \partial_{\bar{\theta}} + 2\theta (h - b) \\
W_+ &= -z \partial_{\theta} - \frac{1}{2} z \bar{\theta} \partial_{\bar{z}} + \frac{1}{2} \theta \bar{\theta} \partial_{\theta} + 2\theta (h + b) \\
H &= -\frac{1}{2} \theta \partial_{\theta} - \frac{1}{2} \bar{\theta} \partial_{\bar{\theta}} - z \partial_{\bar{z}} + 2h \\
B &= -\frac{1}{2} \theta \partial_{\theta} + \frac{1}{2} \bar{\theta} \partial_{\bar{\theta}} + 2b \\
X_- &= \partial_z \\
X_+ &= -z^2 \partial_z - z \theta \partial_{\theta} - z \bar{\theta} \partial_{\bar{\theta}} + 2b \theta \bar{\theta} + 4h z.
\end{align*}
\]

where \( \partial_z = \frac{\partial}{\partial z}, \partial_{\theta} = \frac{\partial}{\partial \theta}, \partial_{\bar{\theta}} = \frac{\partial}{\partial \bar{\theta}}, \) \([\partial_z, z] = 1, \{\partial_{\theta}, \theta\} = 1, \{\partial_{\bar{\theta}}, \bar{\theta}\} = 1, \{\theta, \bar{\theta}\} = 0 \) and \( \{\partial_{\theta}, \partial_{\bar{\theta}}\} = 0. \)

One can consider this, as the representation of \( \text{osp}(2|2) \) on the super-sub-space of analytic functions spanned by the monomial,

\[
\{1, z, z^2, \ldots, z^{4h}, \theta, \theta z, \ldots, \theta z^{4h-2}, \bar{\theta}, \bar{\theta} z, \ldots, \bar{\theta} \bar{\theta} z^{4h-4}\}
\]

whose dimension is \( 16h. \) We will find that this new operator realization, which satisfies the algebra of (1), is a finite-dimensional irreducible representation of \( \text{osp}(2|2). \)

Up to now, everywhere the covariant derivative in super-space was being defined \( [19] \) by

\[
\begin{align*}
D_{\theta} &:= \partial_{\theta} + \frac{1}{2} \bar{\theta} \partial_z \\
D_{\bar{\theta}} &:= \partial_{\bar{\theta}} + \frac{1}{2} \theta \partial_{\bar{z}}
\end{align*}
\]

As we can see this is not a definition but it represents the differential realization of \( V_- \) and \( W_- \) respectively on super-space of analytical functions of \( f(z, \bar{z}, \theta, \bar{\theta}). \)

### 3 Correlation functions of \( N=2 \) super-conformal field theory in \( d=2 \)

#### 3.1 Two point correlation functions

One of the simplest representations involves a complex scalar field \( \phi \) and a two-component spinor \( f_\alpha (\alpha = 1, 2) \). They form the so-called chiral multiplet. One also should include a field \( K \) that will turn out to be an auxiliary field. Such a chiral super-field in terms of \( z \) and \( \bar{z} \) as complex variables and \( \theta \) and \( \bar{\theta} \) as anti-commuting variables is as follows:

\[
\Phi(z, \theta) = \phi(z, \bar{z}) + \theta f(z, \bar{z}) + \bar{\theta} \bar{f}(z, \bar{z}) + \theta \bar{\theta} K(z, \bar{z}).
\]

\[\text{(10)}\]
For the ease of notation we have abbreviated \((z, \bar{z}, \theta, \bar{\theta})\) to \((z, \theta)\) on the left hand side.

To construct Ward identity according to the method which has been given in [10], we take following ”co-product” and differential realization of [4], to calculate the two point function,

\[
\Delta(g) = g \otimes I + I \otimes g; \quad g \in osp(2|2)
\]

where \(I\) means the identity operator. Then the Ward identity is given by

\[
\begin{align*}
\Delta H &< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = \left( \sum_{i=1}^{2} \left( \frac{1}{2} \theta_i \partial_{\theta_i} + \frac{1}{2} \bar{\theta}_i \partial_{\bar{\theta}_i} + z_i \partial_{z_i} - 2 h_i \right) \right) < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = 0 \\
\Delta B &< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = \left( \sum_{i=1}^{2} \left( \frac{1}{2} \theta_i \partial_{\theta_i} - \frac{1}{2} \bar{\theta}_i \partial_{\bar{\theta}_i} - 2 h_i \right) \right) < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > \\
\Delta X_+ &< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = \left( \sum_{i=1}^{2} \theta_i \partial_{\theta_i} \right) < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > \\
\Delta W_+ &< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = \left( \sum_{i=1}^{2} \theta_i \partial_{\theta_i} + 2 h_i \right) < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > \\
\Delta X_- &< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = \left( \sum_{i=1}^{2} \bar{\theta}_i \partial_{\bar{\theta}_i} \right) < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > \\
\Delta W_- &< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = \left( \sum_{i=1}^{2} \bar{\theta}_i \partial_{\bar{\theta}_i} - 2 h_i \right) < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > \\
\end{align*}
\]

where \(h_1\) and \(h_2\) are conformal weight of \(\Phi(z_1, \theta_1)\) and \(\Phi(z_2, \theta_2)\) respectively.

By solving the above Ward identities for super-fields, \(< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) >\), and using the realization given by [4], we have the following expression for the two point function which is well-known [21]

\[
< \Phi(z_1, \theta_1)\Phi(z_2, \theta_2) > = (z_{12} - \frac{1}{2} (\theta_1 \bar{\theta}_2 + \bar{\theta}_1 \theta_2))^\Lambda.
\]

Our second attempt is to solve the above set of equations for two point functions of bosonic and spinor fields. The first equation in the above set will give the answer up to a constant. The second equation will fix the representation of the symmetric group \(osp(2|2)\), which here for chiral super-field, [11], the representation is \((2h, 2\bar{b} = 0)\). The rest of equations will give the relations between different two point functions.

\[
\begin{align*}
< \phi(z_1)\phi(z_2) > &\sim z^\Lambda \\
< f(z_1)\bar{f}(z_2) > &\sim f(z_1)f(z_2) >\sim \frac{1}{2} \Lambda z^{(\Lambda - 1)} \\
< K(z_1)K(z_2) > &\sim \frac{1}{2} \Lambda (\Lambda - 1) z^{(\Lambda - 2)}
\end{align*}
\]

\[\text{(14)}\]
where
\[ z = (z_1 - z_2), \quad \Lambda = 2(h_1 + h_2) \]

Some remarks are in order:
1- Correlation functions are scaling.
2- A specific representation of the algebra has been chosen.

These two point are the indications that such a model is a non-unitary model \[2\].

### 3.2 Three point functions

A method similar to that in last section can be used to calculate three point correlation functions. For three point function the result is as follows:

\[ < \Phi(z_1, \theta_1)\Phi(z_2, \theta_2)\Phi(z_3, \theta_3) > \sim z_{12}^{(h_1+h_2-h_3)} z_{13}^{(h_1-h_2+h_3)} z_{23}^{(-h_1+h_2+h_3)} (1 + \beta \hat{\eta}), \]

where \( \beta \) is constant,

\[ \hat{\eta} = \frac{\theta_1 z_{23} + \theta_2 z_{13} + \theta_3 z_{12}}{\sqrt{z_{12} z_{13} z_{23}}}, \quad z_{ij} = (z_i - z_j - \theta_i \theta_j). \]

\begin{align*}
< \phi(z_1)\phi(z_2)\phi(z_3) > & \sim z_{12}^{2\alpha \gamma^{2\gamma}} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< \phi(z_1)f(z_2)\phi(z_3) > & \sim \frac{1}{2} \alpha z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< f(z_1)\phi(z_2)\phi(z_3) > & \sim \frac{1}{2} \gamma z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< f(z_1)f(z_2)\phi(z_3) > & \sim \frac{1}{2} \beta z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< \phi(z_1)K(z_2)K(z_3) > & \sim \frac{1}{2} \gamma (\gamma - 1) z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< K(z_1)\phi(z_2)K(z_3) > & \sim \frac{1}{2} \beta(\beta - 1) z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< K(z_1)K(z_2)\phi(z_3) > & \sim \frac{1}{2} \alpha(\alpha - 1) z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< f(z_1)f(z_2)K(z_3) > & \sim \frac{1}{2} \beta\gamma z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< K(z_1)f(z_2)f(z_3) > & \sim \frac{1}{2} \alpha\beta z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< K(z_1)K(z_2)f(z_3) > & \sim \frac{1}{2} \alpha\gamma z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma} \\
< K(z_1)K(z_2)K(z_3) > & \sim \alpha\beta\gamma z_{12}^{2\gamma} z_{13}^{2\gamma} z_{23}^{2\gamma}
\end{align*}

and the rest are zero. Here,

\[ z_{ij} = z_i - z_j, \quad \alpha = h_1 + h_2 - h_3, \quad \beta = h_1 - h_2 + h_3, \quad \gamma = -h_1 + h_2 + h_3. \]

In the case of three point functions also, we find out that the representation, \((2h, 0)\), is the only one valid for the super-field given in \((14)\) and all other correlations are zero.
4 Free Field Representation of $\hat{osp}(2|2)_k$

In parallel to Wakimoto’s method \[21\] of free field realization of given current algebra, $\hat{G}$, we use the simple observation that at the zero level these currents, $\hat{g} \in \hat{G}$, become differential operator realization of $G$ given by \((4)\). By using the Feigin and Frenkel \[15\] method we can extend level zero to any arbitrary level $k$.

To get the free field realization of currents in standard form, we make the following changes:

\[
\begin{align*}
  z & \rightarrow \chi, \quad \partial_z \rightarrow W, \quad 2h \rightarrow \sqrt{2(k-1)}\partial\phi := j_0, \quad \theta \rightarrow \psi_-,
  \partial_\theta \rightarrow \psi_+, \quad \bar{\theta} \rightarrow \bar{\psi}_-, \quad \partial_{\bar{\theta}} \rightarrow \bar{\psi}_+, \quad 2b \rightarrow 2\partial\phi' := j_0'.
\end{align*}
\]

According to section (2), the classical part of affine algebra $G$ (zero level affine algebra $\hat{G}$) in fact comes from the action of $G$ on homogenous space (flag manifold) $G/T$ as an algebra of vector fields. Then $\chi_i$’s and $\psi_i$’s are the corresponding complex and Grassmanian coordinates on $G/T$ respectively, and $W_i \sim \frac{\partial}{\partial \chi_i}$. The fields $\phi$ and $\phi'$ are the corresponding coordinates on $T$. The Killing vectors $J(W, \chi, \psi, \bar{\psi})$ of zero level affine algebra $\hat{G}$ do not depend on $\phi$ and $\phi'$, but such decoupling no longer takes place when $W, \chi, \psi, \bar{\psi}, \phi$ and $\phi'$ are considered $z$-dependent, and affine algebra $\hat{G}$ arises instead of classical finite-dimensional $G$. So, we considered only flag manifolds with $T$ being a product of $U(1)$ factors, and this provided us with the free field realization of the model.

As a result the free field realization of $\hat{G}$ generators will be

\[
\begin{align*}
  V_- &= \bar{\psi}_+ + \frac{1}{2}\psi_- W \\
  W_- &= \psi_+ + \frac{1}{2}\bar{\psi}_- W \\
  V_+ &= \chi\bar{\psi}_+ + \frac{1}{2}\chi\psi_- W + \frac{1}{2}\bar{\psi}_- \bar{\psi}_+ - \psi_-(\alpha j_0 - \alpha' j_0') + \beta \partial \psi_- \\
  W_+ &= \chi\psi_+ + \frac{1}{2}\chi\bar{\psi}_- W - \frac{1}{2}\psi_- \bar{\psi}_+ - \psi_-(\alpha j_0 + \alpha' j_0') + \beta' \partial \bar{\psi}_- \\
  H &= \frac{1}{2}\psi_- \psi_+ + \frac{1}{2}\bar{\psi}_- \bar{\psi}_+ + \chi W - \alpha j_0 \\
  B &= \frac{1}{2}\psi_- \psi_+ - \frac{1}{2}\bar{\psi}_- \bar{\psi}_+ - \alpha' j_0' \\
  X_- &= W \\
  X_+ &= \chi^2 W + \chi\psi_- \psi_+ + \chi\bar{\psi}_- \bar{\psi}_+ - \alpha' j_0' \psi_- \bar{\psi}_- - 2\alpha\chi j_0 + \gamma \partial \chi.
\end{align*}
\]

The constituents are free fields whose commutation relations are encoded by the singular OPEs

\[
\begin{align*}
  W(z)\chi(\omega) & \sim \frac{1}{z-\omega}, \quad \partial\phi(z)\partial\phi(\omega) \sim \frac{1}{(z-\omega)^2}, \\
  \psi^i_-(z)\psi^i_+(\omega) & \sim \frac{1}{z-\omega}, \quad \psi^1 = \psi, \quad \psi^2 = \bar{\psi}.
\end{align*}
\]

(19)
The non-trivial OPE of the above currents will be as follow:

\[
\begin{align*}
H(z)H(\omega) & \sim \frac{-(\frac{1}{2}+\alpha^2)}{(z-\omega)^2} + \cdots \\
H(z)V_+(\omega) & \sim \frac{\beta}{(z-\omega)} + \cdots \\
B(z)V_+(\omega) & \sim \frac{\gamma}{(z-\omega)} + \cdots \\
H(z)W_+(\omega) & \sim \frac{\delta}{(z-\omega)} + \cdots \\
B(z)W_+(\omega) & \sim \frac{\epsilon}{(z-\omega)} + \cdots \\
\end{align*}
\]

where \( \cdots \) are non-singular terms. In calculation of the above OPEs we found the system of equations for \( \alpha, \alpha', \beta \) and \( \gamma \), which result in

\[
\alpha' = \alpha = 0, \quad \beta = \frac{1}{2}, \quad \gamma = 1.
\]

Such a result shows that the representation given in (18) is free field representation of \( \text{osp}(2|2) \) in level one \( (k=1) \). The above expression can be put in the following compact form:

\[
J^a(z)J^b(\omega) \sim k \frac{\kappa^{ab}}{(z-\omega)^2} + f^{ab} \frac{J^c(\omega)}{z-\omega}.
\]  

(21)

where \( f^{ab} \) are the structure constants of \( \text{osp}(2|2) \), and \( \kappa^{ab} \) is proportional to its non-degenerate Killing form.

The stress-energy tensor of the above model can be obtained by means of the Sugawara construction,

\[
T(z) = \frac{1}{k} \kappa_{ab} : J^a(z)J^b(z) : 
\]

where \( \kappa_{ab} \kappa^{ab} = 1, +1 \) for compact and \( -1 \) for non-compact groups. The result is

\[
T(z) = 3W(z)\partial \chi(z) + \chi(z)W(z)\bar{\psi}_-(z)\bar{\psi}_+(z) + \chi(z)W(z)\psi_-(z)\psi_+(z) + \bar{\psi}_-(z)\partial \bar{\psi}_+(z) + \bar{\psi}_+(z)\partial \bar{\psi}_-(z) + \psi_-(z)\partial \psi_+(z) + \psi_+(z)\partial \psi_-(z) + \psi_-(z)\psi_+(z)\bar{\psi}_-(z)\bar{\psi}_+(z)
\]  

(22)

By using the OPE of stress-tensor

\[
T(z)T(\omega) \sim \frac{c/2}{(z-\omega)^4} + \frac{2T(\omega)}{(z-\omega)^2} + \frac{\partial T(\omega)}{z-\omega}
\]

one can get the central charge, \( c \), as zero. This result was evident from

\[
c = 2k \frac{\kappa_{ab} \kappa^{ab}}{\kappa}
\]
where $\kappa$ as a constant is such that $J^a(z)$ to be a primary field of conformal weight one:

$$T(z) J^a(\omega) \sim \frac{J^a(\omega)}{(z-\omega)^2} + \frac{\partial J^a(\omega)}{z-\omega}.$$ 

Hence, the conformal field theory with $osp(2|2)$ symmetry is a non-unitary $c = 0$ model.

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