Theoretical Study on the Size Effect of the Rate Characteristics of Concrete Cubes Compressive Strength

Hongyu Zhou*, Yanan Liu, Yun Zhou and Qi Tang

College of Architectural and Civil Engineering, Faculty of Urban Construction, Beijing University of Technology, Architectural & Civil Engineering West Building, Chaoyang, Beijing, China
Email: zhouhy@emails.bjut.edu.cn

Abstract. In order to study the effect of size effect and rate correlation on the mechanical properties of concrete, a total of 228 concrete cube test blocks of three sizes (70, 100 and 150mm) were tested and studied under four strain rates (10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}/s). The test results show that under the same strain rate, the peak stress of concrete decreases with the increase of the size. Under the same size, the peak stress of concrete increases with the increase of strain rate. Experimental data were used to verify and modify Weibull size effect equation based on statistics and Bazant size effect equation based on fracture mechanics. The experimental data fit well with the equation, and the size effect theory of concrete strength with correlation of holdup is established.

Keywords. Concrete, compressive strength, size effect, strain rate.

1. Introduction
In recent years, many scholars at home and abroad have done a lot of research on the size effect of concrete, and some achievements have been made. Neville [1] conducted compressive tests on three sizes of concrete cube specimens (70, 125, 150 mm), and the results showed that small specimens had higher strength than large specimens. Sabnis [2] analyzed and summarized the experimental data obtained by 12 researchers under different conditions of concrete, different curing methods and different ages, and obtained the empirical equation of concrete compressive strength considering the size effect, indicating that the size effect of concrete material is affected by many factors. Yang Chengqiu and Wu Zheng [3] conducted uniaxial compressive strength tests on fully graded and corresponding wet sifted concrete cube specimens, and the results showed that both the strength of fully graded and wet sifted concrete had size effect, and its change law was consistent with Weibull statistical theory. In recent years, many scholars have also studied the size effect of concrete under different strain rates. Among them, Chen Wei [4], Hu Weihua [5], Wang Min [6] and Hu Haizhou [7] all conducted monotone compression tests for cube test blocks with three sizes (150, 300 and 450mm) under four dynamic loading rates (10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}/s), and analyzed the variation rules of concrete peak stress and strain under different strain rates and specimen sizes.

By introducing the influence of strain rate into Weibull and Bazant size effect models, the size effect model of concrete strength considering strain rate effect is established.
2. Experiment

2.1. Overview
Cube test blocks with side lengths of 70, 100 and 150 mm were used in the test. The concrete mix ratio is shown in Table 1. The steel film is formed and compacted on the vibrating table. After 24h, the mold is removed and cured for 28 days [8] under standard curing conditions, and then placed for 60 days naturally to reduce the influence of age on strength [9].

| Concrete grade | Material utilization amount (kg/m³) |
|----------------|-------------------------------------|
| C50            |                                    |
|                | Cement 491 | Water 158 | Sand 610 | Stone 1138 |

Four dynamic loading rates ($10^{-5}$, $10^{-4}$, $10^{-3}$, $10^{-2}$/s) of monotone compression tests were performed on test blocks of each size [10]. The number of test blocks is shown in Table 2 below. A total of 228 test blocks were tested in 12 different loading types.

| Size(mm) | Strain rate(/s) |
|----------|-----------------|
|          | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ |
| 70       | 10        | 10        | 20        | 36         |
| 100      | 10        | 10        | 20        | 36         |
| 150      | 10        | 10        | 20        | 36         |
| Total    | 228       |

2.2. Results
The experimental results obtained are shown in Table 3.

| Size (mm) | Item                   | Strain rate(/s) |
|-----------|------------------------|-----------------|
|           |                        | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ |
| 70        | Peak stress (MPa)      | 57.071    | 58.016    | 58.872    | 59.856    |
|           | Standard deviation (MPa)| 0.188    | 2.611    | 4.588    | 3.685    |
|           | Coefficient of variation| 3%       | 4.5%     | 7.8%     | 6.2%     |
| 100       | Peak stress (MPa)      | 54.218    | 56.558    | 57.853    | 59.358    |
|           | Standard deviation (MPa)| 3.194    | 3.594    | 3.331    | 3.534    |
|           | Coefficient of variation| 5.9%     | 6.4%     | 5.8%     | 6%       |
| 150       | Peak stress (MPa)      | 52.326    | 53.172    | 54.143    | 55.052    |
|           | Standard deviation (MPa)| 2.198    | 2.806    | 3.628    | 4.456    |
|           | Coefficient of variation| 4.2%     | 5.3%     | 6.7%     | 8.1%     |

3. Study on Size Effect Characteristics Based on Different Strain Rate

3.1. Research on Weibull Size Effect Theory Based on Statistics
For concrete test blocks (volume $V_1$ and $V_2$) with the same loading form, the size effect equation based on statistical theory is as follows [11]:

$$\frac{\sigma_1}{\sigma_2} = (V_2/V_1)^{1/m}$$

(1)

where, $V_1$ and $V_2$ represent the specimen volume, and $M$ is the material parameter; $\sigma_1$ and $\sigma_2$ are
the peak stress corresponding to the concrete test block of volume $V_1$ and $V_2$. Equation (1) is used to fit the peak stress of concrete and specimen volume under different strain rates to obtain material parameter $M$ and correlation coefficient $R^2$, as shown in the table and figure 1 below.

| Strain rate (/s) | $m$  | $R^2$ |
|------------------|------|------|
| $10^{-5}$        | 26.270 | 0.964 |
| $10^{-4}$        | 26.121 | 0.817 |
| $10^{-3}$        | 26.080 | 0.821 |
| $10^{-2}$        | 25.837 | 0.832 |

**Figure 1.** Relationship between peak stress and volume of concrete under different strain rates.

It can be seen from table 4 that the relationship between the ratio of peak stress and the ratio of volume of concrete under different strain rates is fitted with the statistical Weibull size effect equation, and the obtained parameter $M$ is not a fixed value, but decreases with the increase of strain rate. Through linear regression analysis, equation (2) can be used for fitting. The expression is:

$$m = a \log \varepsilon + b$$  \hspace{1cm} (2)

The fitting results are shown in figure 2 and table 5.
It can be seen from figure 2 that there is a linear relationship between the M value in the Weibull size effect equation and different strain rates. The function curve of Weibull modulus m changing with strain rate obtained by equation (2) is in good agreement with the experimental results obtained in this paper. Equation (2) can better reflect the law of M value changing with strain rates.

Table 5. Fitted value and correlation coefficient.

| Strain rate (1/s) | D0   | B    | R²   |
|-------------------|------|------|------|
| 10^{-3}           | 360.977 | 1.241 | 0.950 |
| 10^{-4}           | 357.418 | 1.276 | 0.986 |
| 10^{-5}           | 353.732 | 1.293 | 0.957 |
| 10^{-2}           | 351.232 | 1.319 | 0.901 |

3.2. Research on Bazant Size Effect Based on Fracture Mechanics

The Bazant size effect equation based on fracture mechanics [12-13] is:

$$\sigma_N = B f_{cu} / \sqrt{1 + \beta} = B f_{cu} / \sqrt{1 + D / D_0}$$  (4)

Where, $\beta = D / D_0$, $D$ is the size of the specimen, $f_{cu}$ is the compressive strength of the material cube, $B$ is the compendium constant, and $D_0$ is the geometric constant of the dependent structure.

Equation (4) is used to fit the relationship between the peak stress of concrete and specimen size under different strain rates. The geometric constants, compoundless constants and correlation coefficients of the structure-dependent structures under different strain rates are obtained. The specific values are shown in the table 6.
The curve of the relationship between peak stress and size of concrete tests under different strain rates obtained by fitting is shown in the figure 3 below.

![Graph showing the relationship between peak stress and size under different strain rates.](image)

**Figure 3.** Relationship between concrete strength and size under different strain rates.

It can be seen from table 6 that the Bazant size effect equation based on fracture mechanics is adopted to fit the relationship between concrete strength and size under different strain rates, and the value of parameter \( B \) in the equation is not a constant. However, with the increase of strain rate, the value of \( B \) also increases, while \( D_0 \), another parameter in Bazant size effect equation based on fracture mechanics, decreases with the increase of strain rate. Through regression analysis, equation (5) can be used to fit \( D_0 \) and \( B \) respectively. Their expressions are as follows:

\[
D_0 = a \lg \varepsilon + b, \quad B = a \lg \varepsilon + b
\]  

As shown in figures 4 and 5:

![Graph showing the relationship between \( D_0 \) and strain rate.](image)

**Figure 4.** Relationship between \( D_0 \) and strain rate.

![Graph showing the relationship between \( B \) and strain rate.](image)

**Figure 5.** Relationship between \( B \) and strain rate.
Table 7. Fitted value and correlation coefficient.

| Coefficient | Equation       | a           | b           | R²   |
|-------------|----------------|-------------|-------------|------|
| $D_0$       | $D_0 = a \log \varepsilon + b$ | -3.292      | 344.317     | 0.993|
| $B$         | $B = a \log \varepsilon + b$    | 0.025       | 1.370       | 0.982|

The curves of $B$ and $D_0$ obtained by using the equation in table 7 fit well with the test results. It shows that the fitting equation can well reflect the variation law of $B$ and $D_0$ values with strain rate.

Equation (6) is derived from equation (4) and equation (5). Its expression is:

$$\sigma_N = B f_{cu} / \sqrt{1 + D/D_0} = (0.025 \log \varepsilon + 1.370) f_{cu} / \sqrt{1 + D/(-3.292 \log \varepsilon + 344.317)}$$

(6)

4. Conclusion

(1) Under the same strain rate, the peak stress of concrete decreases with the increase of specimen size, which indicates that the size effect exists in concrete materials. Under the same specimen size, the peak stress of concrete increases with the increase of strain rate, indicating that the concrete material has a rate correlation.

(2) Combined with the experimental data in this paper, the Weibull size effect equation based on statistics and Bazant size effect equation based on fracture mechanics are verified. The theoretical model of size effect of concrete strength with strain rate effect was established and fitted well with the test data.

Acknowledgement
This work was financially supported by National Natural Science Foundation of China (51978012), Beijing Tianjin Hebei cooperation project of Beijing Natural Science Foundation (J160002).

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