Fluctuations in Hadronic and Nuclear Collisions*

Yogiro Hama\textsuperscript{(1)}, Takeshi Kodama\textsuperscript{(2)} and Samya Paiva\textsuperscript{(1)}\textsuperscript{†}

\textsuperscript{(1)} Instituto de Física, Universidade de São Paulo C.P.66318, 05389-970 São Paulo-SP, Brazil

\textsuperscript{(2)} Instituto de Física, Universidade do Rio de Janeiro C.P.68528, 21945-970 Rio de Janeiro-RJ, Brazil

Abstract

We investigate several fluctuation effects in high-energy hadronic and nuclear collisions through the analysis of different observables. To introduce fluctuations in the initial stage of collisions, we use the Interacting Gluon Model (IGM) modified by the inclusion of the impact parameter. The inelasticity and leading-particle distributions follow directly from this model. The fluctuation effects on rapidity distributions are then studied by using Landau’s Hydrodynamic Model in one dimension. To investigate further the effects of the multiplicity fluctuation, we use the Longitudinal Phase-Space Model, with the multiplicity distribution calculated within the hydrodynamic model, and the initial conditions given by the IGM. Forward-backward correlation is obtained in this way.

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I. INTRODUCTION

One of the main characteristics of the high-energy hadronic or nuclear collisions is the existence of large event-by-event fluctuations: fluctuation of multiplicity, of particle species, of inelasticity, of momentum distribution, of impact parameter, and so on. Usually, one tries to describe an average trend of such a phenomenon, of multiparticle production, by using for example hydrodynamic models \(^1\) and with success. However, if one analyzes the data more closely, one observes that in a given experimental setup, even under the same initial conditions of colliding objects, events with different final state configurations take place. This fluctuation has either a quantum mechanical or a statistical origin or even simply associated with the impact parameter. The so-called inclusive data for the final-particle distributions are the averages over such event-by-event fluctuations for a given set of experimental initial conditions. In the usual application of hydrodynamic models, to describing the inclusive data we presumably expect, by means of a sort of ergodic assumption \(^2\), that the average over event-by-event fluctuations is replaced by a statistical ensemble average. However, not all the average of physical fluctuations can be expressed in terms of the above average over statistical ensemble of the constituent configurations. For example, the impact-parameter and quantum-mechanical fluctuations that occur in the initial condition of each event can never be averaged out with the use of the ergodic hypothesis. Besides, a description in terms of the average quantities is clearly not satisfactory in treating such quantities as multiplicity, inelasticity, semi-inclusive rapidity distributions, correlations among particles, etc. The main aim of this work is to discuss the effects of such fluctuations on the observed quantities and to present our attempt to include them in the description of the data.

In the following, we shall give in the next two sections a brief account of a modified version \(^3,4\) of the Interacting Gluon Model (IGM) \(^5\), which will be used to generate the event-

\(^1\) The field-theoretical foundation of hydrodynamic model has first been given by M. Namiki and C. Iso \(^6\).
dependent fireballs. In Section II, we discuss how the impact-parameter fluctuation can be treated and in Section III, how IGM incorporates the energy and momentum fluctuations of the partons to obtain the event-dependent energy and momentum of the central fireball. Comparisons with some data, showing the effects of these are also performed there. These fluctuations are nevertheless not enough, for there are many data which require consideration of other kinds of fluctuations. In Section IV, we describe how the multiplicity fluctuation may be treated, concentrating especially in pp collisions and their multiplicity distributions. To treating the energy and momentum fluctuation of the observed particles, once the mass of the fireball and its multiplicity are defined, we use the longitudinal phase-space model. This is discussed in Section V. It is shown that these two kinds of fluctuations are essential in describing the forward-backward correlations. Conclusions are summarized in Section VI.

II. IMPACT-PARAMETER FLUCTUATION

In any collision process between particles or nuclei, the impact parameter cannot be fixed a priori. This is in part due to the quantum mechanical uncertainty, but, even if we could theoretically define the trajectories of the incident particles like in heavy-ion collisions where the incident objects are nearly classical, it would equally not be possible in practice due to the actual experimental conditions. We may recall that there exist some experimental techniques to discriminate, a posteriori, the central from the peripheral collisions in such reactions. But they do not eliminate ponderable uncertainties. So in any realistic description of hadronic or nuclear collisions the impact-parameter fluctuation must be included. In previous works [4,5], we have studied this fluctuation in connection with IGM and hydrodynamic model and shown that it affects the observables such as the inelasticity, leading-particle spectra, rapidity distributions of produced particles in a significant amount.

The impact parameter $\vec{b}$ defines, in the first place, the probability density of occurrence of a reaction (apart from the normalization) $F(\vec{b}) = 1 - |S(\vec{b})|^2$, where the eikonal function is written as
\[ |S(\vec{b})|^2 = \exp\{-C \int d\vec{b}' \int d\vec{b}'' D_A(\vec{b}') D_B(\vec{b}'') f(\vec{b} + \vec{b}' - \vec{b}'')\} = \exp\{-C h_{AB}(\vec{b})\}, \]

where \( D_{A,B}(\vec{b}) \) are the thickness functions of the incident particle (nucleus) \( A \) and \( B \) respectively. \( C \) is an energy-dependent parameter and, though not necessarily so, in studying \( pp, p \)-nucleus and nucleus-nucleus collisions, we assume it to be universal, so that it may be determined by the condition \( \int F_{pp}(\vec{b}) d\vec{b} = \sigma_{inel}^{pp}(\sqrt{s}) \) for \( pp \) collision. Notice that, because of this, once \( pp \) cross-section is fixed, the \( AB \) cross-section \( \sigma_{inel}^{AB}(\sqrt{s}) = \int F_{AB}(\vec{b}) d\vec{b} \) may be calculated by using (1). We take, as an input \[ \sigma_{inel}^{pp} = 56 (\sqrt{s})^{-1.12} + 18.16 (\sqrt{s})^{0.16}. \]

The function \( f(\vec{b}) \) in (1), subject to the constraint \( \int f(\vec{b}) d\vec{b} = 1 \), accounts for the finite effective parton interaction range (with the screening effect taken into account). The simplest choice of \( f(\vec{b}) \) would be the point interaction \( \delta(\vec{b}) \), but we prefer to parametrize it as a Gaussian with a range \( \approx 0.8 \) \( fm \), which is more consistent with the character of the strong interaction and also describes better the data. For proton, we parametrize \( D_p(\vec{b}) \) as a Gaussian distribution. So, we have eventually \( D_p(\vec{b}) = f(\vec{b}) = (a/\pi) \exp(-ab^2) \), with \( a = 3/(2 R_p^2) \), where \( R_p \approx 0.8 \) \( fm \) is the proton radius. For nuclei, we take

\[ D_A(\vec{b}) = \int_{-\infty}^{+\infty} \rho_A(\vec{b}, z) dz = \int_{-\infty}^{+\infty} \frac{\rho_0}{1 + \exp\left\{(r - R_0)/d\right\}} dz, \]

where \( R_0 = r_0 A^{1/3}, r_0 = 1.2 \) \( fm \), \( d = 0.54 \) \( fm \) and \( \rho_A(r) \) is normalized to \( A \). Thus, in the particular case of \( pA \) collisions, we get

\[ h_{pA}(\vec{b}) = a \int_0^\infty db' b' D_A(\vec{b}') I_0(ab') e^{-a(b'^2 + b'^2)/2}, \]

where \( I_0 \) is a modified Bessel function.

Besides, the impact parameter determines the size of the fireball, because as \( b \) increases the overlap of the hadronic matter becomes smaller and consequently so does the average mass of the fireball generated in the collision. We incorporate this effect by writing the parton momentum distribution functions as
\[ G_A(x, \vec{b}) = D_A(\vec{b}) / x, \quad G_B(y, \vec{b}) = D_B(\vec{b}) / y, \]  

(5)

where \( x \) and \( y \) are the Feynman variables of partons in \( A \) and \( B \), respectively, in the equal-velocity (e.v.) frame. With this, we are assuming that the parton momentum distribution is independent of the particular type of nucleus and also equal for whole the nucleus, the only difference being their density (where the thickness \( D(\vec{b}) \) is large, more partons of a given momentum are found). Here, it is presumed that the same physics describes \( pp \), \( pA \) and \( AB \) collisions and that no correlation exists among the nucleons inside each of the incident nuclei. Then, given an impact parameter \( \vec{b} \), the density of parton pairs with momenta \((x, -y)\sqrt{s}/2\) that fuse contributing to the fireball formation is written as

\[
w(x, y; \vec{b}) = \int d\vec{b}' \int d\vec{b}'' G_A(x, \vec{b}'') G_B(y, \vec{b}'') \sigma_{gg}(x, y) f(\vec{b} + \vec{b} - \vec{b}'') \theta \left( xy - \frac{M_{\text{min}}^2}{s} \right) = h_{AB}(\vec{b}) w(x, y),
\]

(6)

with

\[
w(x, y) = \left[ \sigma_{gg}(x, y) / xy \right] \theta \left( xy - \frac{M_{\text{min}}^2}{s} \right),
\]

(7)

where \( M_{\text{min}} = 2m_\pi \) and the parton-parton cross-section is parametrized as \( \sigma_{gg}(x, y) = \alpha / (xys) \). Observe that in (6), \( \vec{b} \) dependence is factorized out.

### III. ENERGY AND MOMENTUM FLUCTUATION OF THE CENTRAL

**FIREBALL: INTERACTING-GLUON MODEL**

Interacting Gluon Model (IGM) \(^2\) is a simple QCD motivated model, especially designed to create the initial conditions for hydrodynamic descriptions, by incorporating in an intuitive way the microscopic fluctuations in the initial stage of the collision. It is based on an idea \(^8\) that in high-energy collisions valence quarks weakly interact so that they...
almost pass through, whereas gluons interact strongly, producing an indefinite number of mini-fireballs, which eventually form a unique large central fireball (all possible $q\bar{q}$ sea quarks are, in this model, “converted” to equivalent gluons). One of the nice features of this model is its easy handling. However, the main drawback of the original version was the neglect of the impact-parameter. In [4,5], we have improved it, by including the impact-parameter fluctuation as explained in the previous section. By reinterpreting the partons in that section as gluons, since only these are assumed to interact, the probability density $\chi(E, P; \vec{b})$ of forming a fireball with energy $E$ and momentum $P$ at a fixed $\vec{b}$ is obtained as follows.

We assume that the colliding objects form a fireball, via gluon exchanges, depositing in it momenta $x(\vec{b})\sqrt{s}/2$ and $-y(\vec{b})\sqrt{s}/2$, respectively. Let $n_i$ be the number of gluon pairs that carry momenta $x_i\sqrt{s}/2$ and $-y_i\sqrt{s}/2$. Thus,

$$\sum_i n_i x_i = x(\vec{b}) \quad \text{and} \quad \sum_i n_i y_i = y(\vec{b}).$$

(8)

In what follows, we will omit the explicit $\vec{b}$ dependence of $x$ and $y$ in order not to overload the notation. The energy and momentum of the central fireball in the e.v. frame of the incident particles are given by

$$E = (x + y) \sqrt{s}/2, \quad P = (x - y) \sqrt{s}/2$$

(9)

and its invariant mass $M$ and rapidity $Y$ are respectively

$$M = \sqrt{sxy} \equiv \kappa \sqrt{s} \quad \text{and} \quad Y = (1/2) \ln (x/y) .$$

(10)

With these notations, we can follow the prescription given in [6] and write the relative probability of forming a fireball with a specific energy and momentum as

$$\Gamma(x, y; \vec{b}) \simeq \exp\{-X^T G^{-1} X\} / \left[\pi \sqrt{\det(G)}\right],$$

(11)

where

$$X = \begin{pmatrix} x - \langle x \rangle \\ y - \langle y \rangle \end{pmatrix}, \quad G = 2 \begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix}.$$
with the notation
\[ \langle x^m y^n \rangle = \int dx' \int dy' \, x'^m y'^n \, w(x', y'; \vec{b}), \quad (12) \]
or, in terms of $E$ and $P$,
\[ \Gamma(E, P; \vec{b}) \sim [2\sqrt{a_1 a_2}/\pi] \exp\{-a_1 (E - \langle E \rangle)^2 - a_2 P^2\}, \quad (13) \]
where $a_1 = [s(\langle x^2 \rangle + \langle xy \rangle)]^{-1}$, $a_2 = [s(\langle x^2 \rangle - \langle xy \rangle)]^{-1}$ and $\langle E \rangle = (\langle x \rangle + \langle y \rangle)\sqrt{s}/2$ (don’t confuse this notation with the average value; it is not because $w(x, y; \vec{b})$ is not normalized). Apparently, $\Gamma(E, P; \vec{b})$ in (13) is normalized. However, both $E$ and $P$ are bounded because of the energy-momentum conservation constraint. It is also constrained by $M > M_{\text{min}} = 2m_\pi$. So, we put some additional factor $\chi_0(\vec{b})$,
\[ \chi(E, P; \vec{b}) = \chi_0(\vec{b}) \Gamma(E, P; \vec{b}), \quad (14) \]
such that
\[ \int dP \int dE \, \chi(E, P; \vec{b}) \, \theta(\sqrt{E^2 - P^2} - M_{\text{min}}) = \frac{F_{AB}(\vec{b})}{\sigma_{\text{inel}}^{AB}}. \quad (15) \]

As implied by (13) and remarked there, the gluon momentum distribution is independent of the particular type of nucleus, the only difference being their density. So, in the integral (12), $x'$ and $y'$ vary from some lower limit, defined by $\sqrt{sx'y'} = M_{\text{min}}$, up to 1, corresponding to the complete neglect of any collective effect of the nucleons in a nucleus. On the other hand, the integration limits of (15) are chosen differently. $x$ and $y$ in (8) may be larger than 1, because gluons from different nucleons may contribute to give the fireball a momentum transfer that is larger than $\sqrt{s}/2$, which is just the incident momentum of a single nucleon in our e.v. frame. We take as the upper limit of $x$ and $y$ the overlap $h_{A,B}(\vec{b})$, whenever it is larger than 1, and limited to $A$ and $B$, respectively ($x$ or $y$ remains $\leq 1$, if it corresponds to a proton). When $h_{A,B}(\vec{b}) < 1$, we take it $= 1$, because in such a case just a single nucleon of each nucleus interacts.
A. Inelasticity distributions

The concept of inelasticity, understood as the fraction of the incident energy $E_0$ which is lost while a particle interact with another one, is crucial in cosmic-ray data analysis where the primary mass composition, hence an important information about the Universe, is deduced by using models of cascade development with appropriate inelasticity distributions and cross sections as the inputs. In this case, the usual definition is $k = (E_0 - E')/E_0$, where $E'$ is the leading (or surviving) particle energy. At high energy, a nucleus hardly supports any collision without suffering a breakup, so the term inelasticity in this sense is commonly reserved to a hadron projectile and, in this paper, we shall restrict the consideration only to proton incident on nuclear targets. However, in a wider sense, as the fraction of the incident energy $E_0$ which is materialized into produced particles, it is also important in connection with the production of a quark-gluon plasma in heavy-ion collisions \cite{10–14,6}. In Ref. \cite{6}, $\kappa$ appearing in (10) is called inelasticity and this is the quantity of interest in heavy-ion collisions. In the present paper, we shall adopt the usual definition $k$ given above and, as for $\kappa$, call it simply $\kappa$. In any case, it is clear that we can talk about inelasticity distribution only when there exists some event-by-event fluctuation.

Having obtained $\chi(E, P; \vec{b})$, we can readily compute such distributions. The $\kappa$-distribution has been obtained in \cite{4} and reads

$$\chi(\kappa) = \int d\vec{b} \int dE \int dP \chi(E, P; \vec{b}) \delta(\sqrt{(E^2 - P^2)/s - \kappa}) \theta(\sqrt{E^2 - P^2} - M_{\text{min}}).$$

(16)

Then, by fitting the only existing $\chi(\kappa)$ data \cite{15} at $\sqrt{s} = 16.5$ GeV, we fix the parameter $\alpha$ of the model as $\alpha = 21.35$. A comparison with the data is shown in Fig.1, where we have also put the result of \cite{6}. It is seen that the original version of IGM already gives a reasonable description of the data, but the inclusion of the impact-parameter fluctuation drastically improves the agreement. The latter enhances the small-$\kappa$ events and makes the

\footnote{For a recent comparative study of the main existing models on inelasticity, see for example \cite{9}.}
overall shape flatter. The enhancement of large-\( \kappa \) events is simply due to the larger value of \( \alpha \) which is necessary for an overall fitting now.

The inelasticity distribution \( \chi(k) \) has been obtained in \([5]\) and reads

\[
\chi(k) = \int d\vec{b} \int dE \int dP \chi(E, P; \vec{b}) \delta((E + P)/\sqrt{s} - k) \theta(\sqrt{E^2 - P^2 - M_{\text{min}}}). \tag{17}
\]

Often, the inelasticity is defined in the lab. frame but, except when \( k \to 1 \), the difference between \( k \) defined in this frame and the one given in the e.v. frame is quite negligible. So, we will not make any distinction here and compute everything in the latter. We show, in Fig.2, the results for several \( pA \) collisions at \( \sqrt{s} = 550 \) GeV. No accelerator data at such a high energy exists, but it is seen that \( \chi(k) \) is nearly \( k \)-independent for \( pp \), in agreement with ISR data \([16,17]\). In a recent cosmic-ray experiment \([18]\), hadron-\( Pb \) inelasticity distribution at an average energy of \( \langle \sqrt{s} \rangle = 550 \) GeV has been estimated. The result\(^4\) is \( \chi(k) \simeq (-0.25 \pm 0.50)(1 - k)^{1.85 \pm 1.77} + (3.1 \pm 1.7)k^{1.85 \pm 1.77} \). We find that the qualitative features of our result agree with this estimate, except in the low-\( k \) region. Some of the origins of the discrepancy may be the difference between \( \pi - Pb \) and \( p - Pb \) collisions and the absence of the leading-particle fragmentation in our model. However, the uncertainty in their estimate is quite large.

We show in Fig.3 the average inelasticity \( \langle k \rangle \) as function of \( \sqrt{s} \), for several target nuclei. In Ref. \([6]\), several models for inelasticity have been compared with their estimates obtained with cosmic-ray data, which show slowly decreasing behavior with \( E_0 \). According to their comparison, IGM is the model which predicts the most quick fall of the average inelasticity with \( E_0 \) in clear conflict with their estimates. Our curves in Fig.3 still decrease as \( \sqrt{s} \) increases but, compared with the results of \([6]\), the energy-dependence is quite small now and compatible with the estimates obtained in \([6]\). The main origin of this contrast is the factor \( \sigma_{pp}^{\text{inel}} \) which has been dropped out in \([6]\), because it is not necessary in our version.

\(^4\)In \([5]\), we cited the preliminary result communicated by E. Shibuya, a member of the collaboration, which is slightly different.
B. Leading-particle spectra

A related quantity is the leading-particle spectrum, as shown in Fig.4 at $\sqrt{s} = 14$ GeV [19]. Since data on $p_T$ dependence are scarce, we have assumed an approximate factorization of $x_l(= 2p_l/\sqrt{s})$ and $p_T$ dependences,

$$E_l(d^3\sigma/dp^3) \approx f(x_l)h(p_T),$$

where

$$f(p_l) = \int d\vec{b} \int dP \int dE \chi(E, P; \vec{b}) \theta(\sqrt{E^2 - P^2 - M_{\text{min}}}) \delta(\sqrt{\sqrt{s} - (E + P)} / 2 - p_l),$$

and parametrized $h(p_T)$ as

$$h(p_T) = (\beta^2 / 2\pi) e^{-\beta p_T},$$

determining the average $\beta$ by using the $p_T$ dependence of the data [19]. The curves obtained with these $\beta$ values (with an interpolation for Al and Ag) are shown in Fig.4. The result of [6] for $pp$ is also shown for comparison. Again, it is seen that a reasonable agreement with the data is obtained with IGM only, but the inclusion of the impact-parameter fluctuation improves it remarkably. We did not put their curves for the other targets, but the behavior is similar, namely they are more bent showing a definite deviation from the data in the largest-$x_l$ region. This is a consequence of the neglect of the peripheral events there. Some authors [11, 13, 14] have obtained good fits to $pA$ data, but in those works it is not clear which is the connection to other relevant quantities such as momentum distributions, correlations,... of the secondary particles. Also, $pp$ is usually treated as a separate case.

C. Rapidity distributions

Let us now study the effects of fluctuations we have just introduced on the final particle spectra. As mentioned in the Introduction, the so-called inclusive rapidity (or pseudo-rapidity) distributions are the averages over such event-by-event fluctuations for a given set
of experimental initial conditions. However, in usual computations of these distributions, say, by use of hydrodynamic models, instead of taking such averages

\[
\langle \frac{dN}{dy} \rangle = \int d\vec{b} \int dP \int dE \frac{dN}{dy}(E, P; \vec{b}) \chi(E, P; \vec{b}) \theta(\sqrt{E^2 - P^2}/s - \kappa_{\text{min}}),
\]

one considers some average initial conditions and calculates the distributions, starting from them. Namely,

\[
\frac{dN}{dy} \left( \langle E \rangle, \langle P \rangle = 0, \langle \vec{b} \rangle \right).
\]

It is evident that only under very special conditions that these two quantities can coincide.

In [4], we studied the deviation of the latter from the more realistic former distribution \( \langle dN/dy \rangle \), by adopting the one-dimensional Landau’s hydrodynamic model for an ideal gas in order to compute the rapidity distribution \( dN(E, P, \vec{b})/dy \) for a definite initial conditions. Despite all the simplifications, this model is known to reproduce the main features of the measured momentum (or rapidity) distributions and has advantage of having an analytical solution over the whole rapidity range [20]. The only inputs of the model are the total energy, momentum and the geometrical size of the initial fireballs. Remark that we are talking about the central fireballs and not about the rest of the system. In the case the incident particles are nucleons, the latter appears most frequently as leading particles. In order to avoid additional complexities, let us consider only this case, namely pp collisions.

The invariant momentum distribution of produced particles in a hydrodynamic model is usually given by Cooper-Frye formula [21]

\[
E \frac{dN}{d\vec{p}} = \int_{\sigma(T_d)} f(p^\mu u_\mu) p^\mu \, d\sigma_\mu,
\]

where \( \sigma(T_d) \) is a constant-temperature freeze-out hypersurface, \( f(p^\mu u_\mu) \) is Bose-Einstein (or Fermi-Dirac) distribution, \( p^\mu \) is the 4-momentum of the emitted particle and \( u^\mu \) is the 4-velocity of the fluid. Although it is possible to use more realistic freeze-out criteria [22–25], here we limit ourselves to the simplest choice [23] without sophistication. This is enough for our present purpose of studying how the initial condition fluctuations affect the final particle spectra.
\( \sigma(T_d) \) in (23) is determined by solving the hydrodynamic equation

\[
\partial_\mu T^{\mu\nu} = 0 , \quad \text{with} \quad T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu},
\]

with an appropriate equation of state \( p = \varepsilon/3 \) in our case) once the dissociation temperature \( T_d \) is given. As for the “initial volume” for a fireball of mass \( M \), in the lack of a better justification founded on a physical basis, we adopt in the present work

\[
V_0 = \frac{2m_p^2}{M} V ,
\]

which has been suggested by a phenomenological analysis [26] of the \( M \) dependence of average multiplicity data [27] and also consistent with the \( M \) dependence of the momentum distribution data [28]. The initial temperature \( T_0 \) is then computed by putting \( M \) into this volume. As remarked in [4], nowadays we know that neither the hypothesis of instantaneous thermalization nor the appearance of extremely high values of the initial temperature are physically reasonable. However, in spite of these rather non-conventional initial conditions, many of the qualitative and the quantitative results (average multiplicity, particle ratios, momentum distributions, \ldots) are surprisingly good when compared with data. In our point of view, perhaps the equilibrium is attained at a later time when the system has already suffered some expansion, but then the temperature and the rapidity distributions at the onset of the hydrodynamic regime would be approximately those of Landau’s model whose initial conditions correspond to high temperature and energy density if extrapolated back in time. So, for any practical purpose, we can use Landau’s solution to describe the system. We emphasize, however, that the fluctuation effects which are the central object of the present study do not depend sensibly on such a choice.

We show in Fig.5 a comparison of the results obtained in this way for \( <dN/d\eta> \) and \( dN/d\eta \) at \( \sqrt{s} = 53 \) GeV. It is seen that the rapidity or equivalently pseudo-rapidity distributions are very sensitive to the fluctuations in the initial conditions. The peak, in the case of \( dN/d\eta \) computed with one fireball of mass \( <M> \), corresponds to the simple-wave solution. When the fluctuations are taken into account, such a peak is completely smoothed.
away. They also cause a widening and a lowering of the distributions. Although the main purpose of this work is just to show the influences of the fluctuations, we may also compare them with some data. We see that the behavior of the first one is more similar to the data than the other one and the presence of the simple-wave peaks in each event does not invalidate the overall agreement with data.

IV. MULTIPLICITY FLUCTUATION

We have shown in the last two sections how the impact-parameter and the energy-momentum fluctuations in the initial stage of the collision affect some of the observables. However, there are many more quantities whose description cannot be given only in terms of the fluctuations considered up to this point. Even after the mass of the fireball has been defined, quantities such as the multiplicity, particle species, their momentum distributions, ... vary from event to event and hydrodynamic model we have used in the last section only describes the average behavior. Under certain conditions (constant dissociation temperature \( T_d \)), it does give the moments of the multiplicity distribution, so in principle also the multiplicity distribution itself, but not the fluctuating events.

Let us discuss in this section how the multiplicity fluctuation may be implemented. One way of doing this is to conveniently parametrizing the multiplicity distribution for a fixed mass \( M \) and determining the parameters by imposing certain constraints. We choose a very simple parametrization for the multiplicity distribution

\[
\psi(M, z) = Az^{\nu}e^{-\alpha z}, \tag{26}
\]

where, as usual,

\[
\psi(M, z) = \langle N(M) \rangle \ P_N(M), \tag{27}
\]

\[
z = N/ \langle N(M) \rangle
\]

with \( P_N(M) \) indicating the probability of a fireball of mass \( M \) decaying into \( N \) charged particles, and impose the conditions
\[ \sum_N P_N(M) = 1 \quad \rightarrow \quad \frac{1}{2} \int_0^\infty \psi(z) \, dz = 1, \quad (28) \]
\[ \sum_N NP_N(M) = \langle N(M) \rangle \quad \rightarrow \quad \frac{1}{2} \int_0^\infty z \, \psi(z) \, dz = \langle z \rangle = 1, \]
\[ \sum_N N^2 P_N(M) = \langle N(M)^2 \rangle \quad \rightarrow \quad \frac{1}{2} \int_0^\infty z^2 \, \psi(z) \, dz = \frac{\langle N(M)^2 \rangle}{\langle N(M) \rangle^2}. \]

By substituting (26) into these equations, we obtain
\[ \alpha = \frac{1}{\langle z^2 \rangle - 1}, \quad (29) \]
\[ \nu = \alpha - 1, \quad (30) \]
\[ A = \frac{2\alpha^2}{\Gamma(\alpha)}. \quad (31) \]

The moments \( \langle z^n \rangle = \langle N(M)^n \rangle / \langle N(M) \rangle^n \) have been calculated in \[31\] and, in particular, it is found that \( \langle z^2 \rangle = 1 + a_2 / \langle N(M) \rangle \), with \( a_2 = 1.105 \). What we can do is, once \( M \) is fixed, to produce events following this distribution by the use of the Monte Carlo method. In doing so, we have indeed to consider also the charge fluctuation.

The overall multiplicity distributions calculated in this way is shown in Fig.6. As seen, the results reproduce quite well the qualitative features of the data in all the ISR energy region. They are slightly narrower than the data and, as the energy increases, the discrepancy becomes more pronounced, but probably the data begins to suffer the influence of the mini-jets there.

V. ENERGY AND MOMENTUM FLUCTUATION OF THE OBSERVED PARTICLES: PHASE-SPACE MODEL

The multiplicity fluctuation, discussed in the preceding section, does not manifest itself only in the multiplicity distribution. With the inclusion of this fluctuation, we are considering that the momentum distribution of the secondary particles for a given \( M \) is a superposition of distributions with different multiplicity \( N \). Moreover, even with a fixed \( N \), the momentum distribution will vary from event to event. As will be shown below, there are observables such as the forward-backward multiplicity correlation, which depends on
this kind of fluctuation. But, then we face the following problem: “How to compute the rapidity distribution of a system having a definite mass $M$ and a definite multiplicity $N$ and in an event-dependent way?” This question did not arise when computing the inclusive distribution, because the hydrodynamic model does take such a fluctuation into account, as mentioned in the preceding section. What it does not do is to generate each fluctuating event.

We propose to use the one-dimensional phase-space model to generate these events. First, one-dimensional because we know from the data that the momentum distributions in high-energy is essentially longitudinal. In a previous work [33], we have shown that the rapidity distributions predicted by the one-dimensional phase-space model, given $M$ and $N$, are approximately Gaussian, as in hydrodynamic model. It was also shown that these are insensitive to a certain class of dynamical factors introduced in the model. Although it is indeed not guaranteed $a$ priori that the superposition of these distributions shall give the one obtained by the hydrodynamic model, the result of this model has the qualitative features of that one and an advantage of being a sum of event-dependent distributions with fixed $M$ and $N$.

Given a mass $M$ and a multiplicity $N$, the one-dimensional phase-space model tells us that the probability of finding an event with the particles in the longitudinal momentum intervals

$$[p_i, p_i + dp_i], \quad i = 1, \ldots, N,$$

is given by

$$d^N P = \frac{1}{R_N(M)} \frac{dp_1}{2E_1} \frac{dp_2}{2E_2} \cdots \frac{dp_N}{2E_N} \delta\left(\sum_{j=1}^{N} p_j\right) \delta\left(\sum_{j=1}^{N} E_j - M\right),$$

(33)

where

$$R_N(M) = \int \frac{dp_1}{2E_1} \int \frac{dp_2}{2E_2} \int \cdots \int \frac{dp_N}{2E_N} \delta\left(\sum_{j=1}^{N} p_j\right) \delta\left(\sum_{j=1}^{N} E_j - M\right).$$

(34)

The invariant one-particle distribution is then given by
\[
\frac{E}{dp} \frac{dN}{dy} = \left( \frac{N R_{N-1}(M')}{R_N(M)} \right),
\]

normalized to \(N\) and where \(M' = \sqrt{(M - E)^2 - p^2}\) represents the invariant mass of the system after subtracting the observed particle. The correct expression of the probability would be (33) with some dynamical factor. In [33], in order to simulate the hydrodynamic motion, we have included a factor \(f(y) = \alpha e^{-\beta m_T \cosh y}\) for each particle, where \(y\) is the rapidity, and shown that this is entirely irrelevant, the final result being the same.

In Fig.7, we show one-particle pseudo-rapidity distributions in \(pp\) collision at 53 GeV given by (33). Since we have completely neglected any dynamical factor, we could not expect to obtain a perfect agreement with the data. However, it is seen that the qualitative features of the data are reproduced.

A. Forward-backward correlation

One of the data, which cannot be understood without the multiplicity fluctuation discussed in Section IV and nicely reproduced with the longitudinal phase-space model proposed here, refers to the so called forward-backward charged multiplicity correlation. First of all, the data show that the charged multiplicities in the two hemispheres are very little correlated [16], presenting a large fluctuation. The correlation is usually presented as a graph of the average charged multiplicity in the backward hemisphere \(\langle N_b \rangle\) as a function of the effectively observed charged multiplicity \(N_f\) in the forward hemisphere as in Fig.8.

To begin with, remark that in the usual application of the hydrodynamic model, without any fluctuation taken into account, the graph would reduce to a single point. The consideration of the impact-parameter and the initial-state fluctuations, implemented through IGM as we are proposing, improves considerably the agreement with the data. Notice that the momentum fluctuation of the fireball is essential, otherwise the correlation would be too large. Now, without the inclusion of the multiplicity fluctuation and especially the momentum fluctuation of the final particles, the correlation begins to deviate from the experimental
trends for large values of $N_f$, because, in that case, large $N_f$ means large $M$ with the fireball sitting more or less in the center of mass, with a symmetrical distribution of particles.

VI. CONCLUSIONS AND FURTHER OUTLOOKS

We have investigated, in this paper, effects of several kinds of fluctuations which appear in hadronic and nuclear collision, analyzing different kinds of observables. The Interacting Gluon Model, improved by the inclusion of the impact-parameter fluctuation and complemented by an appropriate hydrodynamic model seems to describe well the the bulk of the phenomenon, such as the cross-section, inelasticity, leading-particle spectrum, average multiplicity and the inclusive momentum distribution. Other quantities depend explicitly on the multiplicity and the final-particle momentum fluctuations. In this paper, we have treated the first one by considering the thermodynamics of the fireball and then parametrizing the multiplicity distribution for each mass $M$ in a convenient way. More microscopic description of this fluctuation would also be possible. We have treated the momentum fluctuation of the final particles by using the one-dimensional phase-space model. Although not completely satisfactory, this treatment has shown the importance of such a fluctuation in an event-dependent basis.

There are many other properties which clearly depend on fluctuations. One of these quantities is the so-called semi-inclusive rapidity distribution, namely, distribution with a fixed multiplicity interval. In principle, it would be possible to obtain this distribution with the ingredients we have considered here, but there is something which is missing. Especially in the low multiplicity intervals, the effects of the diffractive processes cannot be neglected. We are studying how to incorporate the diffractive processes in IGM, in a consistent way. Also, to compare with the data, a treatment which is somewhat more realistic than (33) is required. Another interesting quantity that could be studied, which certainly depends on event-by-event fluctuations we have discussed, is the Bose-Einstein correlations of produced particles, or the so-called Hanbury-Brown Twiss effect [35], frequently used in heavy-ion
collisions to infer about the space-time development of the hadronic matter which is formed in such collisions. Suggestions for such a study has been given by M. Namiki et al. in [3].

In this paper, we have completely neglected the fragmentation of the leading particles, which certainly give non-negligible contributions to the semi-inclusive rapidity distributions mentioned above, especially when the multiplicity is small. In the case of nucleus-nucleus collisions, how to treat the fragmentation of the leading nuclei is a completely open question.

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Figure Captions

Fig.1: $\kappa$-distribution for $pp$ at $\sqrt{s} = 16.5$ GeV. The data are from [15]. The solid line is our result, whereas the dashed one is from [6].

Fig.2: Inelasticity distribution for $pA$ collisions with several targets at $\sqrt{s} = 550$ GeV.

Fig.3: Energy dependence of the average inelasticity for $pA$ collisions.

Fig.4: Leading-particle spectra as function of $x_l$ at $p_T = 0.3$ GeV. The data are from [19] at $\sqrt{s} = 14$ GeV. The solid curves are our results, whereas the dashed one is from [6]. The slope parameter has been extracted from [19] as $\beta = 4.20, 3.22, 3.21, 3.26, 3.47$ and $3.78 \text{GeV}^{-1}$ for $p, C, Al, Cu, Ag$ and $Pb$ targets, respectively.

Fig.5: Pseudo-rapidity distributions calculated in the usual procedure (solid line) and with fluctuations (dashed line) at $\sqrt{s} = 53$ GeV. Experimental data [29] are shown for comparison.

Fig.6: Charged-particle multiplicity distribution in $pp$ collisions at $\sqrt{s} = 30$ and 62 GeV, compared with data [32].

Fig.7: One-particle inclusive pseudo-rapidity distributions for $pp$ collisions at $\sqrt{s} = 53$ GeV, computed by using the longitudinal phase-space model. The solid curve represents the distribution without any fluctuation, whereas the dashed one is the result with all the fluctuations included in the way described in the text. The data are from [29].

Fig.8: Forward-backward multiplicity correlation in $pp$ collisions at $\sqrt{s} = 24$ GeV, computed with the longitudinal phase-space model (□), compared with the result without the multiplicity and final-particle-momentum fluctuations (*). The data (●) are from [34].
Figure 1

\[ \chi(\kappa) \]

\[ \kappa \]

0.0 0.2 0.4 0.6 0.8 1.0

2.0

1.0

0.0

0.0 0.2 0.4 0.6 0.8 1.0
Figure 2

\[ \chi(k) \] vs. \[ k \]

- p
- C
- Al
- Cu
- Ag
- Pb
Figure 3

\[\langle k \rangle\]

\[s^{1/2} \text{ (GeV)}\]
Figure 4

\[ \frac{E}{dx} \left( \text{mb.GeV}^{-2} \right) \]

\[ x_l \]

A graph showing the energy density per unit length \( x_l \) for various materials: Pb, Ag, Cu, Al, C, and p. The data points and lines indicate the behavior of the energy density for different materials as a function of \( x_l \).
Figure 6

Data $^{1/2} = 30$ GeV
Data $^{1/2} = 44$ GeV
Data $^{1/2} = 53$ GeV
Data $^{1/2} = 62$ GeV
Theory $^{1/2} = 30$ GeV
Theory $^{1/2} = 62$ GeV

$z = N_{ch}/\langle N_{ch} \rangle$
Figure 7
Figure 8