Implications of the LHCb Evidence for Charm CP Violation

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The LHCb collaboration recently announced preliminary evidence for CP violation in D meson decays. We discuss this result in the context of the standard model (SM), as well as its extensions. In the absence of reliable methods to evaluate the hadronic matrix elements involved, we can only estimate qualitatively the magnitude of the non-SM tree level operators required to generate the observed central value. In the context of an effective theory, we list the operators that can give rise to the measured CP violation and investigate constraints on them from other processes.

I. INTRODUCTION

Recently the LHCb collaboration reported a 3.5σ evidence for a non-zero value of the difference between the time-integrated CP asymmetries in the decays \(D^0 \rightarrow K^+K^-\) and \(D^0 \rightarrow \pi^+\pi^-\) [1],

\[
\Delta a_{CP} \equiv a_{K+K^-} - a_{\pi^+\pi^-} = -(0.82 \pm 0.21 \pm 0.11)\% \, .
\]  

(1)

The time-integrated CP asymmetry for a final CP eigenstate, \(f\), is defined as

\[
a_f = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(D^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(D^0 \rightarrow f)} \, .
\]  

(2)

Combined with previous measurements of these CP asymmetries [2–5], the world average is

\[
\Delta a_{CP} = -(0.65 \pm 0.18)\% \, .
\]  

(3)

Following [6] we write the singly-Cabibbo-suppressed \(D^0 (\bar{D}^0)\) decay amplitudes \(A_f (\bar{A}_f)\) to CP eigenstates, \(f\), as

\[
A_f = A^f T e^{i\phi_f} [1 + r_f e^{i(\delta_f + \phi_f)}] \, ,
\]  

(4a)

\[
\bar{A}_f = \eta_{CP} A^f T e^{-i\phi_f} [1 + r_f e^{i(\delta_f - \phi_f)}] \, ,
\]  

(4b)

where \(\eta_{CP} = \pm 1\) is the CP eigenvalue of \(f\), the dominant singly-Cabibbo-suppressed “tree” amplitude is denoted \(A^T f e^{\pm i\phi_f}\), and \(r_f\) parameterizes the relative magnitude of all the subleading amplitudes (often called “penguin” amplitudes), which have different strong (\(\delta_f\)) and weak (\(\phi_f\)) phases.

In the following we focus on the \(\pi^+\pi^-\) and \(K^+K^-\) final states. In general, \(a_f\) can be written as a sum of CP asymmetries in decay, mixing, and interference between decay with and without mixing. Mixing effects are suppressed by the \(D^0 - \bar{D}^0\) mixing parameters, and, being universal, tend to cancel in the difference between \(K^+K^-\) and \(\pi^+\pi^-\) final states [6]. Taking into account the different time-dependence of the acceptances in the two modes, LHCb quotes [1] for the interpretation of Eq. (1),

\[
a_{K+K^-} - a_{\pi^+\pi^-} \approx a^\text{dir}_K - a^\text{dir}_\pi + (0.10 \pm 0.01) a^\text{ind} \, .
\]  

(5)

Thus, because of the experimental constraints on the mixing parameters [see Eq. (18)], a large \(\Delta a_{CP}\) can be generated only by the direct CP violating terms,

\[
a_f^\text{dir} = -\frac{2r_f \sin \delta_f \sin \phi_f}{1 + 2r_f \cos \delta_f \cos \phi_f + r_f^2} \, ,
\]  

(6)

and we use the \(f = K, \pi\) shorthand for \(K^+K^-\) and \(\pi^+\pi^-\).

II. GENERAL CONSIDERATIONS AND SM PREDICTION

Independent of the underlying physics, a necessary condition for non-vanishing \(a_f^\text{dir}\) is to have at least two amplitudes with different strong and weak phases contribute to the final state \(f\). In the isospin symmetry limit, the condition on the strong phases implies that different isospin amplitudes have to contribute. Since the leading (singly-Cabibbo-suppressed) terms in the standard model (SM) effective Hamiltonian, defined below, have both \(\Delta I = 1/2\) and \(\Delta I = 3/2\) components, the subleading operators with a different weak phase may have a single isospin component. As far as amplitudes with a different weak phase are concerned, in the SM, as well as within its MFV expansions [7, 8], they are suppressed by \(\xi \equiv |V_{ub}/V_{cb}|/|V_{us}/V_{ub}| \approx 0.0007\).

The SM effective weak Hamiltonian relevant for hadronic singly-Cabibbo-suppressed \(D\) decays, renormalized at a scale \(m_c < \mu < m_b\) can be decomposed as

\[
\mathcal{H}^\text{eff}_{\Delta c=1} = \lambda_q \mathcal{H}^d_{\Delta c=1} + \lambda_s \mathcal{H}^s_{\Delta c=1} + \lambda_b \mathcal{H}^b_{\Delta c=1} \, ,
\]  

(7)

where \(\lambda_q = V_{cq}^\ast V_{ub}\), and

\[
\mathcal{H}^d_{\Delta c=1} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i^d Q_i^d + \text{H.c.} \, , \quad q = s, d,
\]  

\[
Q_1^d = (\bar{u}_q v_{-A} (\bar{q} c) v_{-A} \, , \quad Q_2^d = (\bar{u}_q q_\beta v_{-A} (\bar{q} c_\alpha) v_{-A} \, ,
\]  

(8)

and \(a, \beta\) are color indices. The first two terms in Eq. (7) have \(O(1)\) Wilson coefficients in the SM. On the contrary,
the so-called penguin operators in \( \mathcal{H}_{\Delta c = 1}^{\text{penguin}} \) have tiny Wilson coefficients at scales \( m_c < \mu < m_b \) (see Refs. [9, 6] for the list of relevant operators and Wilson coefficients).

Let us first consider the \( D \to K^+ K^- \) amplitude. In the SM, it is convenient to use CKM unitarity, \( \lambda_d + \lambda_s + \lambda_b = 0 \), to eliminate the \( \lambda_d \) term, and obtain \( A_K = \lambda_s(A_K^d - A_K^s) + \lambda_b(A_K^b - A_K^s) \). For \( D \to \pi^+ \pi^- \), it is convenient to eliminate \( \lambda_s \) to obtain \( A_\pi = \lambda_d(A_\pi^d - A_\pi^s) + \lambda_b(A_\pi^b - A_\pi^s) \). This way, the first terms are singly-Cabibbo-suppressed, while the second terms are both CKM suppressed and have either vanishing tree-level matrix elements or tiny Wilson coefficients. The magnitudes of these subleading amplitudes are controlled by the CKM ratio \( \xi = |\lambda_b/\lambda_s| \approx |\lambda_b/\lambda_d| \approx 0.0007 \) and the ratio of hadronic amplitudes. We define

\[
R_K^{\text{SM}} = \frac{A_K^d - A_K^s}{A_K^d - A_K^s}, \quad R_\pi^{\text{SM}} = \frac{A_\pi^b - A_\pi^s}{A_\pi^b - A_\pi^s}.
\]

Since \( \arg(\lambda_b/\lambda_s) \approx -\arg(\lambda_b/\lambda_d) \approx 70^\circ \), we can set \( |\sin(\phi_{\pi}^{\text{SM}})| \approx 1 \) in both channels, and neglect the interference term in the denominator of Eq. (6).

In the \( m_c \gg \Lambda_{\text{QCD}} \) limit, one could analyze these decay amplitudes model independently. Given the valence-quark structure of the \( K^+ K^- \) final state, a penguin contraction is required for operators of the type \( c \to u d d \) or \( u b b \) to yield a non-vanishing \( D \to K^+ K^- \) matrix element. This is why \( R_K^{\text{SM}} \) is expected to be substantially smaller than one. A naïve estimate in perturbation theory yields \( |A_K^d/A_K^s| \sim \alpha_s(m_c)/\pi \approx 0.1 \) (and \( |A_\pi^b/A_\pi^s| \approx 0.01 \)). However, since the charm scale is not far from \( \Lambda_{\text{QCD}} \), non-perturbative enhancements leading to substantially larger values cannot be excluded [9]. The same holds for the ratio \( R_\pi^{\text{SM}} \) defined in Eq. (9).

To provide a semi-quantitative estimate of \( R_K^{\text{SM}, \pi} \) beyond perturbation theory, we note that penguin-type contractions are absent in the Cabibbo-allowed \( c \to u d d \) Hamiltonian, contributing to \( D \to K^+ \pi^- \). In the absence of penguin contractions, \( D \to K^+ K^- \) and \( D \to \pi^+ \pi^- \) amplitudes have identical topologies to \( D \to K^+ \pi^- \), but for appropriate \( s \leftrightarrow d \) exchanges of the valence quarks. The data imply \( |A_{KK}| \approx 1.3 |\lambda_s A_{K\pi}| \) and \( A_{\pi\pi} \approx 0.7 |\lambda_s A_{K\pi}| \). These results are compatible with the amount of \( SU(3) \) breaking expected in the tree-level amplitudes and show no evidence for anomalously large penguin-type contractions competing with the tree-level amplitudes. Further evidence that tree-level topologies dominate the decay rates is obtained from the smallness of \( \Gamma(D \to K^0 K^0)/\Gamma(D \to K^+ K^-) \), which is consistent with the vanishing \( D \to K^0 K^0 \) tree-level matrix element of \( \mathcal{H}_{(s-d)} \) in the \( SU(3) \) limit. However, it must be stressed that data on the decay rates do not allow us to exclude a substantial enhancement of the CKM suppressed amplitudes. The latter do not have an \( s-d \) structure as the leading Hamiltonian, and, if enhanced over naïve estimates as in the case of the \( \Delta I = 1/2 \) rule in \( K \to \pi \pi \) amplitudes, may account for \( |R_{K, \pi}^{\text{SM}}| > 1 \) [9].

In the following we assume that \( \tau_f \ll 1 \) even in the presence of new physics (NP), and we can expand Eq. (6) to first order in this parameter. We can thus write

\[
a_K^{\text{dir}} \approx 2 \left[ \xi \text{Im}(R_K^{\text{SM}}) + \frac{1}{\sin \theta_C} \sum_i \text{Im}(C_i^{\text{NP}}) \text{Im}(R_{K,i}^{\text{NP}}) \right],
\]

and similarly in the \( \pi^+ \pi^- \) mode. Here \( R_{K,i}^{\text{NP}} \) denote the ratio of the subleading amplitudes generated by the operators \( Q_i \) in the NP Hamiltonian defined below in Eq. (14), normalized to the dominant SM amplitude, after factoring out the leading CKM dependence, \( \sin \theta_C \approx |\lambda_{s,d}| \approx 0.225 \), and the NP Wilson coefficients,\(^1\) \( C_i^{\text{NP}} \). This implies

\[
\Delta a_{CP} \approx (0.13%) \text{Im}(\Delta R_{K}^{\text{SM}}) + 8.9 \sum_i \text{Im}(C_i^{\text{NP}}) \text{Im}(R_{K,i}^{\text{NP}}),
\]

where we defined

\[
\Delta R_{K, \pi}^{\text{NP}} = R_{K, \pi}^{\text{SM}, \pi} + R_{\pi}^{\text{SM}, \pi}.
\]

In the \( SU(3) \) limit, \( R_K^{\text{SM}} = R_\pi^{\text{SM}} \), and therefore \( a_K^{\text{dir}} \approx -a_\pi^{\text{dir}} \), which add constructively in \( \Delta a_{CP} \) [9, 10].

Assuming the SM, the central value of the experimental result is recovered if \( \text{Im}(\Delta R_{K}^{\text{SM}}) \approx 5 \), as illustrated in Fig. 1. Such an enhancement of the CKM-suppressed amplitude cannot be excluded from first principles, but it is certainly beyond its naïve expectation [6].

Note that the applicability of \( SU(3) \) flavor symmetry should be questioned, because the \( D \to K^+ K^- \) and \( D \to \pi^+ \pi^- \) decay rates imply a large breaking of the symmetry. Without \( SU(3) \) as a guidance, one can no longer expect \( a_K^{\text{dir}} \approx -a_\pi^{\text{dir}} \); in particular, the strong phases relevant for direct CP violation in these two channels are no longer related. One might then expect \( |a_K^{\text{dir}}| < |a_\pi^{\text{dir}}| \), if the deviation from factorization is

\(^1\) Contrary to the SM case, where the CKM factors are explicitly factorized, in the NP case we include flavor mixing terms in the \( C_i^{\text{NP}} \) — see Eq. (14).
smaller in the $\pi^+\pi^-$ than in the $K^+K^-$ mode. Therefore, it will be very interesting for the interpretation of the results when the CP asymmetries are measured separately with increased precision. Recent measurements by CDF [2], Belle [3] and BaBar [4] yield for the average of the individual CP asymmetries (without LHCb, and dominated by CDF [2]) in the $\pi^+\pi^-$ and $K^+K^-$ modes $(2.0\pm2.2) \times 10^{-3}$ and $(-2.3\pm1.7) \times 10^{-3}$ [5], respectively, which does not yet allow us to draw definite conclusions [and are included in Eq. (3)]. Another important experimental handle to decide whether the observed signal can or cannot be accommodated in the SM would be observing or constraining CP violation in other decay modes, corresponding to the same quark-level transitions. These include pseudoscalar-vector or vector-vector final states, three-body decays, $D_s$ and $\Lambda_c$ decays. More precise measurements in such decays will help to decide whether the measured CP asymmetry in Eq. (1) is due to new short distance physics, or to a large enhancement of a hadronic matrix element in one particular channel.

### III. NEW PHYSICS CONTRIBUTIONS

The size of NP effects allowed in $\Delta\alpha_{CP}$ depends on $\text{Im}(\Delta R_{\text{SM}})$. In order to understand the scale probed by the measurement, we parametrize the NP contributions in terms of an effective NP scale $\Lambda_{\text{NDA}}$, normalized to the Fermi scale: $\text{Im}(C_{\text{NP}}^{\text{SM}}) = \sqrt{2} \text{Im}(C_{\text{NDA}})/(\Lambda_{\text{NDA}}^2 G_F)$. The resulting sensitivity for $\Lambda_{\text{NDA}} (\Lambda_{\text{NDA}})$ can be written as

$$\text{Im}(C_{\text{NDA}}) \left(\frac{10 \text{ TeV}}{\Lambda_{\text{NDA}}}\right)^2 = \left(0.61 \pm 0.17 \right) - 0.12 \text{Im}(\Delta R_{\text{SM}}).$$

(13)

In other words, assuming $\text{Im}(\Delta R_{\text{NP}}) \sim 1$, $|\Delta R_{\text{SM}}| \ll 5$ and $C_{\text{NDA}} = 1$ implies that a NP scale of $O(13 \text{ TeV})$ will saturate the observed CP violation; alternatively, setting $\Lambda_{\text{NDA}} \to 2^{1/4}/\sqrt{G_F}$ implies that $C_{\text{NDA}} \sim 7 \times 10^{-4}$ is required. As we discuss below, despite the large scale involved, after taking into account the bounds from CP violation in $|\Delta c| = 2$ and $|\Delta s| = 1$ processes, only a few NP operators may saturate the value in Eq. (13) in the limit $|\Delta R_{\text{SM}}| \ll 5$.

To discuss possible NP effects, we consider the following effective Hamiltonian

$$\mathcal{H}_{\Delta c=1}^{\text{eff-NP}} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2,5,6} \sum_{q} (C_i^{\delta q} Q_i^{\delta} + C_i^{\delta q} Q_i^{\delta})$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{i=7,8} (C_i Q_i + C_i^\prime Q_i^\prime) + \text{H.c.},$$

(14)

where $q = \{d, s, b, u, c\}$, and the list of operators includes, in addition to $Q_{i,2}^{\delta}$ given in Eq. (8),

$$Q_3^{\delta} = \sum_{i=1,2,5,6} \sum_{q} (\bar{u}_{c} \gamma_5 c_{\beta} (\bar{q}_{q} q_{d}) \gamma_5 V_A + a),$$

$$Q_6^{\delta} = \sum_{i=1,2,5,6} \sum_{q} (\bar{u}_{c} \gamma_5 c_{\beta} (\bar{q}_{q} q_{d}) \gamma_5 V_A + a),$$

$$Q_7 = -\frac{\alpha}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) G_{\mu\nu} c,$$

$$Q_8 = -\frac{\gamma_5}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) T^a G_{\mu\nu} c,$$

(15)

and another set, $Q_i^{(q)}$, obtained from $Q_i^{(q)}$ via the replacements $A \leftrightarrow -A$ and $\gamma_5 \leftrightarrow -\gamma_5$. This is the most general dimension-six effective Hamiltonian relevant for $D \to K^+K^-, \pi^+\pi^-$ decays, after integrating out heavy degrees of freedom around or above the electroweak scale.

### A. Bounds on NP effects from $D^0 - \bar{D}^0$ mixing

Charm mixing arises from $|\Delta c| = 2$ interactions that generate off-diagonal terms in the mass matrix for $D^0$ and $\bar{D}^0$ mesons. The $D^0 - \bar{D}^0$ transition amplitudes are defined as

$$\langle D^0 | \mathcal{H} | \bar{D}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}.$$  

(16)

The three physical quantities related to the mixing can be defined as

$$y_{12} \equiv \frac{\Gamma_{12}}{\Gamma}, \quad x_{12} \equiv 2 \frac{|M_{12}|}{\Gamma}, \quad \phi_{12} \equiv \arg \left( \frac{M_{12}}{\Gamma_{12}} \right).$$

(17)

HFAG has performed a fit to these theoretical quantities, even allowing for CP violation in decays, and obtained the following 95% C.L. regions [5]

$$\begin{align*}
    x_{12} &\in [0.25, 0.99] \%, \\
y_{12} &\in [0.59, 0.99] \% , \\
\phi_{12} &\in [-7.1^5, 15.8^5].
\end{align*}$$

(18)

We cannot reliably estimate the SM contributions to these quantities from first principles, and thus simply require the NP contributions to at most saturate the above experimental bounds on $x_{12}$, $y_{12}$, and $\phi_{12}$.

The NP operators present in $\mathcal{H}_{|\Delta c|=1}^{\text{eff-NP}}$ may affect $D^0 - \bar{D}^0$ mixing parameters at the second order in the NP coupling, $T \left[ \mathcal{H}_{|\Delta c|=1}^{\text{eff-NP}}(x) \mathcal{H}_{|\Delta c|=1}^{\text{eff-NP}}(0) \right]$. Such a contribution, which formally corresponds to a quadratically divergent one loop diagram, is highly UV sensitive. If we assume a fully general structure for our effective theory, where operators are of NDA strength, then the scaling in Eq. (13) would imply much too large contributions to $D - \bar{D}$ mixing and CP violation (see, e.g., [11]). This could be a major constraint for many SM extensions. However, being a genuine UV effect, it is also highly model dependent. On the other hand, assuming that $\mathcal{H}_{|\Delta c|=1}^{\text{eff-NP}}$ is generated above the electroweak scale and the UV completion of the theory cures the above mentioned problem, we can derive (model-independent) bounds on
the coefficients of $\mathcal{H}^{\text{eff-NP}}_{|\Delta c|=1}$ from the effective $|\Delta c| = 2$ operators generated at low energies, by considering its time ordered product with the SM charged-current interactions, $T\{\mathcal{H}^{\text{eff-NP}}_{|\Delta c|=1}(x) \mathcal{H}^{\text{SM}}_{|\Delta c|=0}(0)\}$ (schematically depicted Fig. 2).

The effective Hamiltonian thus obtained integrating out all the heavy fields is

$$\mathcal{H}^{\text{eff}}_{|\Delta c|=2} = \frac{G_F}{\sqrt{2}} \left( \sum_{i=1}^{5} C_{i}^{cu} Q_{i}^{cu} + \sum_{i=1}^{\Delta} C_{i}^{cu} Q_{i}^{cut} \right),$$

(19)

where

$$Q_{1}^{cu} = (\bar{u}c)_{V-A} (\bar{u}c)_{V-A},
Q_{2}^{cu} = (\bar{u}c)_{S-P} (\bar{u}c)_{S-P},
Q_{3}^{cu} = (\bar{u}c)_{S-P} (\bar{u}c)_{S-P},
Q_{4}^{cu} = (\bar{u}c)_{S-P} (\bar{u}c)_{S-P},
Q_{5}^{cu} = (\bar{u}c)_{S-P} (\bar{u}c)_{S-P},$$

(20)

and, as before, the $Q_{1,2,3}^{cu}$ operators are obtained from $Q_{1,2,3}^{cut}$ by the replacements $A \leftrightarrow -A$ and $P \leftrightarrow -P$.

We perform the matching at one-loop at the matching scale $\mu \gtrsim m_W$. Some of the contributions generate logarithmic divergences, which can be canceled by the appropriate counterterms, genuine short-distance contributions to the $|\Delta c| = 2$ Hamiltonian in Eq. (19). We denote the corresponding contributions to the $|\Delta c| = 2$ Wilson coefficients $\delta C_{i}^{cu(r)}$. Using dimensional regularizarion with the MS prescription we obtain for the renormalized $|\Delta c| = 2$ Wilson coefficients

$$C_{1}^{cu} = \delta C_{1}^{cu} + \frac{g^2}{32\pi^2} \sum_{q} \lambda_q \left( C_{2}^{u} - C_{1}^{u} \right) \ln \frac{\mu^2}{m_W^2},
C_{4}^{cu} = \delta C_{4}^{cu} - \frac{g^2}{16\pi^2} \sum_{q} \lambda_q C_{6}^{q} \ln \frac{\mu^2}{m_W^2},
C_{5}^{cu} = \delta C_{5}^{cu} - \frac{g^2}{16\pi^2} \sum_{q} \lambda_q C_{5}^{q} \ln \frac{\mu^2}{m_W^2},$$

(21)

where here and below we neglect contributions proportional to $r_q = m_q^2/m_W^2$. In particular, the leading order contributions to $C_{1,2}$ and $C_{5,6}$ which are proportional to $r_q \ln r_q$ were set to zero. Similarly, contributions of the gluonic and electromagnetic dipole operators, $Q_{7,8}$, both at tree-level via two insertions, as well as at one loop, are parametrically suppressed by $r_q \alpha/\sin^2 \theta_W$. Numerically this leads to bounds of order unity on the corresponding Wilson coefficients, well above the values obtained in Eq. (13), and thus no useful constraint is obtained from $D - \bar{D}$ mixing.

To compute the contributions of $\mathcal{H}^{\text{eff}}_{|\Delta c|=2}$ to $M_{12}$, we take into account the running and mixing of the operators between the matching scale $\mu$ and the scale $m_D$. This is performed using the formula [12]

$$\langle \bar{D}^0 \rangle \mathcal{H}^{\text{eff}}_{|\Delta c|=2} | D^0 \rangle_i = \frac{G_F}{\sqrt{2}} \sum_{j=1}^{5} \sum_{r=1}^{5} \left( b_j^{(r,i)} + \eta c_j^{(r,i)} \right) \eta^i, \times C_{i}^{cu} (\mu) \langle \bar{D}^0 \rangle Q_{r}^{cu} | D^0 \rangle_i,$$

(22)

where all the relevant parameters are defined in Ref. [12], including the relevant hadronic operator matrix elements. Requiring that such contributions do not exceed the bounds on $x_{12}$ or $x_{12} \sin \phi_{12}$ in Eq. (18), we obtain the bounds on $C_{i}^{cu}$ at the matching scale $\mu \sim 1$ TeV

$$|C_{1}^{cu}| \lesssim 5.7 \times 10^{-8}, \quad \text{Im} (C_{1}^{cu}) \lesssim 1.6 \times 10^{-8},
|C_{2}^{cu}| \lesssim 1.6 \times 10^{-8}, \quad \text{Im} (C_{2}^{cu}) \lesssim 4.3 \times 10^{-9},
|C_{3}^{cu}| \lesssim 5.8 \times 10^{-8}, \quad \text{Im} (C_{3}^{cu}) \lesssim 1.6 \times 10^{-8},
|C_{4}^{cu}| \lesssim 5.6 \times 10^{-9}, \quad \text{Im} (C_{4}^{cu}) \lesssim 1.6 \times 10^{-9},
|C_{5}^{cu}| \lesssim 1.6 \times 10^{-8}, \quad \text{Im} (C_{5}^{cu}) \lesssim 4.5 \times 10^{-9}.$$

(23)

Inserting expressions (21) into the above constraints we can obtain bounds on the combinations of $\delta C_{i}^{cu}$ and $C_{i}^{cu}$ at the high scale. In the following we put all counter term contributions to zero and consider only a single chirality operator structure at a time.

In order to control the QCD induced RGE evolution of the $|\Delta c| = 1$ operators between the matching scale and the hadronic charm scale $\mu_D \sim 2$ GeV, it is convenient to change flavor basis and consider the following set of operators, both for $|\Delta c| = 1$ (and $|\Delta s| = 1$, see below) NP Hamiltonians ($i = 1, 2, 5, 6$):

$$Q_{i}^{(s-d)} = Q_{i}^s - Q_{i}^d,
Q_{i}^{(c-u)} = Q_{i}^c - Q_{i}^u,$n
Q_{i}^{(sd)} = Q_{i}^s + Q_{i}^d - 2Q_{i}^b,$n
Q_{i}^{(b)} = Q_{i}^s + Q_{i}^d + Q_{i}^t + Q_{i}^r + Q_{i}^l,$n
Q_{i}^{(o)} = Q_{i}^s + Q_{i}^d + Q_{i}^t + Q_{i}^r + Q_{i}^u,$

(24)

and similarly for the primed operators. With this choice, the $Q_{i}^{(o)(i)}$ are the standard QCD penguin operators, whose RGE evolution can be found, for instance, in [9].
Moreover, penguin contractions are completely absent in the RGE evolution at $\mu \gtrsim m_c$ of the first two sets of terms in (24) and, to a good approximation (i.e., for $\mu \gtrsim m_b$), are safely negligible also in the case of $Q_i^{(b,8d)}(t)$. For these operators we can thus consider, to lowest order, a simplified RGE evolution in terms of $2 \times 2$ blocks of same flavor and chirality:

$$
\frac{dC_i^{(f)}}{d\ln \mu} = \gamma_{LL}^{(f)} C_i^{(f)}, \quad \gamma_{LL}^{(f)} = \left( \begin{array}{cc}
\frac{5}{6} & \frac{6}{6} \\
\frac{6}{6} & \frac{-6}{6}
\end{array} \right), \quad i = 1, 2,
$$

$$
\frac{dC_i^{(f)}}{d\ln \mu} = \gamma_{LR}^{(f)} C_i^{(f)}, \quad \gamma_{LR}^{(f)} = \left( \begin{array}{cc}
\frac{6}{6} & \frac{-6}{6} \\
\frac{6}{6} & \frac{6N_c^2 - 1}{6}
\end{array} \right), \quad i = 5, 6,
$$

where $f = \{s - d, c - u, 8d, b\}$, $N_c = 3$ is the number of colors, and the same equations hold for primed operators.

This basis also has the benefit of clearly distinguishing between various contributions to $D - \bar{D}$ mixing observables suppressed by different CKM prefactors. Most severe constraints are expected for the flavor combination $Q_i^{(s-d)}(t)$ proportional to $\lambda_s - \lambda_d \approx 2\lambda_s$. On the other hand, $Q_i^{(8d)}(t)$ contributions are suppressed by $\lambda_s + \lambda_d - 2\lambda_b \approx -3\lambda_b$. An even stronger suppression of $r_b\lambda_b$ is expected for the flavor combinations $Q_i^{(b,0)}(t)$, while $Q_i^{(c-u)}(t)$ do not contribute to $|\Delta c| = 2$ observables at one electroweak loop order.

Considering thus only the cases $Q_i^{(s-d)}(t)$ and $Q_i^{(8d)}(t)$, we obtain the bounds on $C_i^{(s)}$ in Table I. We also verified that due to $r_q$ suppression, $C_{1,2}, C_{5,6}$, and $C_{7,8}$, as well as the contributions of $C_{12}^{(b,0)}$ and $C_{5,6}^{(b,0)}$ are presently allowed by $D - \bar{D}$ data to be $O(1)$. We observe that $Q_i^{(s-d)}$ and $Q_i^{(8d)}$ are also the only effective operators generated at the one-loop level from $T[H_{\Delta s = 1}^{\text{eff-NP}}(x) H_{\Delta c}^{\text{SM}}(0)]$ in the limit where we neglect light quark masses. It is also clear that these receive non-suppressed contributions only from the $Q_i^q$ in $H_{\Delta s = 1}^{\text{eff-NP}}$; the contributions of $Q_i^q$ and dipole operators are doubly Yukawa suppressed (in addition to the loop suppression), and thus can be safely neglected.

Proceeding as before, we get

$$
C_i^{(s)} = \delta C_i^{(s)} + C_i^q \frac{g^2}{32\pi^2} \ln \frac{\mu^2}{m_W^2},
$$

for all the relevant four-quark operators. To compute the contributions of $H_{\Delta s = 1}^{\text{eff}}$ to $K \rightarrow \pi\pi$ amplitudes we
need to take into account the running and mixing of the operators between the matching scale and a scale \( \mu \sim 1 \) GeV. Again it is done in the flavor basis (24), and using Eq. (25) analogous to the \( |\Delta c| = 1 \) Wilson coefficients at the scale \( \mu = 1 \) TeV, from the contributions to \( |\epsilon'|/\epsilon |.

\[
\left| \frac{\epsilon'}{\epsilon} \right| \approx \frac{\omega}{\sqrt{2}|\epsilon| \text{Re} A_0} \left| \text{Im} A_0 - \frac{1}{\omega} \text{Re} A_2 \right|, \tag{29}
\]

\[
\text{Im} A_I = \frac{G_F}{\sqrt{2}} \sum_{i,f} C_i^{(f)}(d) \langle (2\pi)f|Q_i^{(f)}(d)K\rangle,
\]

where \( \omega = \text{Re} A_2/\text{Re} A_0 \approx 0.045 \) (from now on we omit the superscript \( (sd) \) on the coefficients and operators of the \( |\Delta s| = 1 \) Hamiltonian). Evaluating the matrix elements of \( H^{\text{NP}}_{|\Delta s|=1} \) in the large \( N_c \) limit leads to

\[
\left| \frac{\epsilon'}{\epsilon} \right|_{\text{NP}} \approx 10^2 \left| \text{Im} \left[ 3.5 C_1^{(3/2)} + 3.4 C_2^{(3/2)} - 1.7 \rho^2 C_5^{(3/2)} - 5.2 \rho^2 C_6^{(3/2)} - 0.04 C_1^{(1/2)} - 0.12 C_2^{(1/2)} - 0.04 \rho^2 C_5^{(1/2)} + 0.11 \rho^2 C_6^{(1/2)} \right] \right|, \tag{30}
\]

in terms of the \( |\Delta s| = 1 \) Wilson coefficients at the low scale (\( \mu = 1.4 \) GeV), where \( C_1^{(3/2)} = [-C_1^{(s-d)} + C_1^{(c-u)} + C_1^{(8d)}]/2 + (5/4) C_2^{(b)} \), \( C_2^{(1/2)} = [C_2^{(s-d)} + C_2^{(c-u)} - C_2^{(8d)}]/2 + (1/4) C_6^{(b)} - C_5^{(0)} \), and \( \rho = m_K/m_s \). Imposing the conservative bound \( |\epsilon'/\epsilon|_{\text{NP}} < |\epsilon'/\epsilon|_{\text{exp}} \approx 1.7 \times 10^{-3} \), leads to severe constraints on all the coefficients. In terms of \( |\Delta s| = 1 \) Wilson coefficients at the high scale \( (\mu = 1 \) TeV) the constraints read

\[
\begin{align*}
\text{Im}(C_1^{(s-d)}) & \lesssim 1.4 \times 10^{-5}, & \text{Im}(C_2^{(s-d)}) & \lesssim 1.4 \times 10^{-5}, \\
\text{Im}(C_5^{(8d)}) & \lesssim 1.9 \times 10^{-7}, & \text{Im}(C_5^{(c-u)}) & \lesssim 6.1 \times 10^{-8}, \\
\text{Im}(C_6^{(c-u)}) & \lesssim 1.3 \times 10^{-5}, & \text{Im}(C_2^{(c-u)}) & \lesssim 1.6 \times 10^{-5}, \\
\text{Im}(C_6^{(8d)}) & \lesssim 1.9 \times 10^{-7}, & \text{Im}(C_6^{(c-u)}) & \lesssim 6.4 \times 10^{-8}, \\
\text{Im}(C_1^{(8d)}) & \lesssim 1.4 \times 10^{-5}, & \text{Im}(C_2^{(8d)}) & \lesssim 1.4 \times 10^{-5}, \\
\text{Im}(C_6^{(c-u)}) & \lesssim 1.9 \times 10^{-7}, & \text{Im}(C_6^{(8d)}) & \lesssim 6.1 \times 10^{-8}, \\
\text{Im}(C_1^{(b)}) & \lesssim 5.4 \times 10^{-6}, & \text{Im}(C_2^{(b)}) & \lesssim 5.9 \times 10^{-6}, \\
\text{Im}(C_6^{(b)}) & \lesssim 7.5 \times 10^{-8}, & \text{Im}(C_6^{(b)}) & \lesssim 2.5 \times 10^{-8}, \\
\text{Im}(C_1^{(0)}) & \lesssim 1.5 \times 10^{-5}, & \text{Im}(C_2^{(0)}) & \lesssim 4.5 \times 10^{-5}, \\
\text{Im}(C_6^{(0)}) & \lesssim 9.6 \times 10^{-6}, & \text{Im}(C_6^{(0)}) & \lesssim 1.9 \times 10^{-6}, \\
\end{align*}
\tag{31}
\]

Inserting the matching conditions (28), we obtain bounds on the \( |\Delta c| = 1 \) Wilson coefficients in Table II. We observe that all \( Q_i^{(f)} \) except \( Q_5^{(c-)} \) are excluded from contributing significantly to \( \Delta c_{CP} \). The remaining operators are only marginally constrained and can give observable effects in the charm sector provided \( |\Delta R^{\text{NP}}| \) have significant values as also shown in Fig. 3.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
f & s - d & c - u & 8d & b & 0 \\
\hline
\text{Im}(C_1^{(1/2)}) \times 10^4 & 2.0 & 2.0 & 2.0 & 0.79 & 2.2 \\
\text{Im}(C_2^{(1/2)}) \times 10^4 & 2.0 & 2.3 & 2.0 & 0.88 & 6.6 \\
\text{Im}(C_5^{(1/2)}) \times 10^5 & 2.7 & 2.8 & 2.7 & 1.1 & 142 \\
\text{Im}(C_6^{(1/2)}) \times 10^5 & 0.90 & 0.94 & 0.90 & 0.37 & 28 \\
\hline
\end{array}
\]
given the constrains from $D - \bar{D}$ mixing and $\epsilon'/\epsilon$, is shown in Table III. It is also worth noting that in cases where the new physics contributions are large, we generically expect sizable contributions to CP violation in $D - \bar{D}$ mixing (and in $\epsilon'/\epsilon$) to arise. This will be tested when the constraints on CP violation in $D - \bar{D}$ mixing will improve substantially with more LHCb and future super-B-factory data.

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