Charge Accretion Rate and Injection Radius of Ionized-Induced Injections in Laser Wakefield Accelerators

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Abstract. Ionization-induced injection has recently been proved to be a stable injection method with several advantages in laser wakefield accelerators. However, the controlling of this injection process aiming at producing high quality electron beams is still challenging. In this paper, we examine the ionization injection processes and estimate the injection rate with two-dimensional particle-in-cell simulations. The injection rate is shown to increase linearly with the high-Z gas density as long as its ratio is smaller than some threshold in the mix gases. It is also shown that by changing the transverse mode of the driving lasers one can control the injection rate.

1. Introductions
In laser wakefield accelerators (LWFA), one of the most important issues to obtain high beam qualities is optimizing the injection mechanism. Injection means placing the electrons into the accelerating and focusing phase with proper velocities that they can gain kinetic energy from the wake continuously. The injection schemes can be either pre-accelerating the electrons to the phase velocity of the wake such as the colliding pulse injection scheme [1, 2, 3], or slowing down the wake so that the electrons have enough time to catch up with the accelerating phase before deceleration such as the density ramping or transition injection scheme [4, 5, 6, 7]. A recent proposed injection scheme, called ionization-induced injections, generates free electrons inside a wake at appropriate phases [8, 9]. This idea is relatively easily operated in experiments due to its simple experimental configuration. Usually the ionization-induced injection is to utilize the higher ionization threshold of the inner shell of some high-Z gas (such as nitrogen, oxygen or argon) mixed with a low-Z gas (usually hydrogen or helium) to control the starting phase of the ionized electrons. Although the ionization-induced injection has been experimentally demonstrated by several groups [10, 11, 12], the detailed studies on the effect of the mixing ratio and injection length have not been conducted yet.

In this paper, we study the injection dynamics by two-dimensional (2D) particle-in-cell (PIC) simulations and find the relation between the gas mixing ratio and the injection rate. The transverse radius of the injection section is proved to be a few microns, and can be changed by using different transverse laser modes. At the same time, the betatron radiation can be enhanced when an appropriate laser mode is used.
In this section, we study the relation between the mixing ratio and the injected charge by PIC simulations and find out a way to estimate the growth rate of the injected beam charge.

In the following 2D simulations, the injection provider is chosen to be nitrogen and the background is helium. The simulation box size is 50 $\mu$m per simulation unit time $t_0 = 3.33$ fs.

The wake driver is a laser beam with wavelength of 800 nm, pulse duration of 33 fs in FWHM, waist of $W_0 = 7.48$ $\mu$m, and normalized vector potential of $a_0 = 2.8$, which corresponds to

$$ P \ [\text{GW}] = 21.5 \times \left( \frac{a_0 W_0}{\lambda_0} \right)^2 = 14.7 \ [\text{TW}] $$

The laser pulse satisfies the so called transverse matching condition\[13\]($k_p = \sqrt{0.2} \ \mu m^{-1}$ in our cases) for self-guided propagation $k_p W_0 = 2 \sqrt{a_0}$, which means that the laser can stably propagate for a distance much longer than the Rayleigh length. The mixed gas has a profile of a half infinite flat-top with a small up-ramp at the beginning, and the laser is focused at the up-ramp.

We measure the injected beam charge vs. time in each simulations. The charge variations with different nitrogen density cases are shown in Fig. 1 (a). One may see that the charge linearly increased with laser propagation distance in the first few microns. We fit the data in this linear region and obtain the beam charge increasing rate. After obtaining enough slops, one can get the charge increasing rate vs. nitrogen atom density as shown in Fig. 1 (b).

### Figure 1. (Color) (a) Injected charge vs. laser propagating distance from 2D PIC simulations. Only electrons in the first bucket are considered. The different symbols represent cases for different nitrogen atom densities as shown in the legend. The solid lines are linear fittings for data with propagation distance from 100 to 300 $\mu$m. $n_0 = 2.829 \times 10^{19} \ cm^{-3}$ is the density normalization unit in the simulations. Because the simulations are in 2D slab geometry, the charge unit is pico-Coulomb per $\mu$m. (b) Beam charge increasing rate vs. nitrogen atom density. Charge increasing rate is in pico-Coulomb per $\mu$m per simulation unit time $t_0 = 3.33$ fs.

#### 2. Charge accretion rate in ionization injection process

One of the most important issues in ionization-induced injection is to choose the mixing ratio of the injection provider and the background gas. The injection provider is a gas that has the electrons with higher ionization threshold, such as nitrogen, oxygen or argon. The background is another gas that has low Z number and provides the majority of the background plasma electrons which cannot be trapped, such as hydrogen or helium. Some reported experiments chose the mixing ratio smaller than 1:9 to reduce the defocusing effect owning to the preionization \[12\]. In this section, we study the relation between the mixing ratio and the injected charge by PIC simulations and find out a way to estimate the growth rate of the injected beam charge.

In the following 2D simulations, the injection provider is chosen to be nitrogen and the background is helium. The simulation box size is 50 $\times$ 100 $\mu$m$^2$, the cell size is 0.015625 $\times$ 0.25 $\mu$m$^2$ and the time step interval is 0.033 fs. The helium density is kept to be 0.1$n_0$, where $n_0 = 2.829 \times 10^{19} \ cm^{-3}$ is the density unit in the simulations. The nitrogen atom density varies from 0.0001 to 0.005$n_0$. The ADK ionization model is used in the simulation code.

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**Figure 1. (Color) (a) Injected charge vs. laser propagating distance from 2D PIC simulations. Only electrons in the first bucket are considered. The different symbols represent cases for different nitrogen atom densities as shown in the legend. The solid lines are linear fittings for data with propagation distance from 100 to 300 $\mu$m. $n_0 = 2.829 \times 10^{19} \ cm^{-3}$ is the density normalization unit in the simulations. Because the simulations are in 2D slab geometry, the charge unit is pico-Coulomb per $\mu$m. (b) Beam charge increasing rate vs. nitrogen atom density. Charge increasing rate is in pico-Coulomb per $\mu$m per simulation unit time $t_0 = 3.33$ fs.**
Figure 2. (Color) (a) Nitrogen ionization level plot after laser propagates a few hundred microns. The nitrogen inner shell includes the 6th and 7th electrons. Applying the analysis in Sec. 2, we can define a “Accretion Tube” inside which the electrons ionized from the inner shell of nitrogen atoms are injected. The Accretion Tube encircles the region of ionization level of 6 or above as one can see in the figure. (b) Forward betatron radiation spectra for the cases of using Gaussian and super-Gaussian driver lasers. The laser parameters are the same: $a_0 = 2.8$, $W_0 = 7.48 \mu m$, and the measurements were taken after the injection saturation, which means that the injected charge is almost the same in these two cases.

The physical modelling is straightforward. Assume in the first few hundred microns in the gas target, the laser keeps its shape and amplitude so that the ionization-induced injection is relatively stable. And in low $n_N$ limit, only the inner shell electrons within few microns away from the central region of nitrogen are ionized and injected as shown in Fig. 2 (a). We call this few microns’ section the “Accretion Tube”, inside which the ionized 6th and 7th electrons of nitrogen atoms are trapped in the wake. Thus the increasing rate of the injected beam charge, or “Accretion Rate” $R_A$ can be written as

$$R_A = \pi r^2 \cdot v_g \cdot 2n_N \cdot e$$ \hspace{1cm} (2)

in 3D geometry, or

$$R_A = 2r_\sigma \cdot v_g \cdot 2n_N \cdot e$$ \hspace{1cm} (3)

in 2D slab geometry, where $r_\sigma$ is the radius of the accretion tube, $v_g$ is the laser group velocity and $e$ is the electron charge. Notice that in 3D, $R_A$ is defined as charge gain per unit time, while in 2D slab geometry $R_A$ is actually charge gain per unit time per unit length (in the 3rd direction). By taking $v_g \approx c$ and applying our simulation parameters, one obtain

$$R_A[pC \cdot \mu m^{-1} \cdot t_0^{-1}] = -18.1 \times r_\sigma[\mu m] \cdot \frac{n_N}{n_0}$$ \hspace{1cm} (4)

where $t_0 = 1 \mu m/c = 3.33$ fs is the time unit in the simulations. Finally, by comparing with the linear fitting in Fig. 1 (b) $R_A[pC \cdot \mu m^{-1} \cdot t_0^{-1}] = -56.3 \times \frac{n_N}{n_0}$, one obtains the accretion radius $r_\sigma = 3.1 \mu m$, which is consistent with Fig. 2 (a).

In high $n_N$ limit, the accretion rate is shown to be saturated. This may because of the beam loading effect [14] or ionization etching effect on the laser pulse.

3. Charge accretion rate and betatron radiation with super-Gaussian lasers

In this section, we use the same parameters as those in Sec. 2, but change the laser transverse mode to super Gaussian, i. e. $E_{\text{laser}} \propto \exp[-(r/W_0)^4]$. The total laser energy changes due to the mode change is small and neglected. We repeat the simulations shown in Sec. 2, i. e. a few simulations with $n_N$ varying from 0.0001 to 0.0005 $n_0$, calculate the injected charge increasing rate $R_A$, and find the linear relation between $n_N$ and $R_A$, $R_A[pC \cdot \mu m^{-1} \cdot t_0^{-1}] = -72.5 \times \frac{n_N}{n_0}$.
Compared with Eq. 4, the accretion radius is obtained as $r_a = 4.0 \, \mu\text{m}$ in the super-Gaussian mode driven case, which is larger than the Gaussian mode case. In a real experiment, the laser beam is commonly neither a pure Gaussian mode nor a pure super Gaussian mode. This makes the precise prediction of the accretion radius difficult. However, the approach described above can be applied in real experiments to measure the accretion radius.

Measuring the betatron radiation is another possible way to diagnose the injected beam radius. We use the “particle tracking” technique of the PIC simulations to obtain the trajectories of the injected particles, and calculate the far-field radiation spectra plotted in Fig. 2 (b). From simulation results we know that the total injected charge is almost the same in Gaussian or super-Gaussian driver case. This is because the measurements were taken after injection saturation. However, the radiation of the super-Gaussian laser driven case is about twice of the Gaussian driver case. There are two reasons. The first is that the super-Gaussian case has larger charge increasing rate, so that a larger number of electrons are injected earlier and thus gain higher energy compared with the Gaussian case. The second is that in the super-Gaussian laser driven case, the off-axial laser field is slightly higher, so that there are more inner-shell electrons ionized and injected off-axially compared with the Gaussian case, thus the averaged transverse displacement of the injected electrons is larger. As measured from the simulations, the emittance after injection saturation is 1.05 mm · mrad for the gaussian driver case, and 2.13 mm · mrad for the super-gaussian driver case. These explains why the betatron radiation of the later is about twice of the former.

4. Summary
We present a method to measure the injection rate of the ionization-induced injection in laser wakefield accelerators. The injection rate is proportional to the density of the injection provider (nitrogen in our case) when the density is small. It is also proportional to the injection area which can be controlled by using different laser modes. By using super-Gaussian transverse modes, one can obtain increased injection section areas and amplitudes of the betatron motions, thus enhanced betatron radiations.

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