The Cosmological Constant

U. Ellwanger

Laboratoire de Physique Théorique
Université Paris XI, Bâtiment 210, 91405 Orsay Cedex, France
E-mail: Ulrich.Ellwanger@th.u-psud.fr

Abstract

Various contributions to the cosmological constant are discussed and confronted with its recent measurement. We briefly review different scenarios – and their difficulties – for a solution of the cosmological constant problem.

Contents

1) Friedmann-Robertson-Walker Cosmology
2) Measurement of the Cosmological Constant
3) The Cosmological Constant in Classical and Quantum Field Theory
4) Towards Solutions of the Problem:
   a) Supersymmetry
   b) Dilaton
   c) 3-Form Field
   d) Quintessence
   e) Brane Universes
5) Conclusions

LPT Orsay 02-18
March 2002

*Lecture given at the XIV Workshop “Beyond the Standard Model”, Bad Honnef, 11-14 March 2002
†Unité Mixte de Recherche CNRS - UMR N° 8627
1 Friedmann-Robertson-Walker Cosmology

The most common physical interpretation of General Relativity \[1–3\] is that space-time has to be considered as a generally non-trivial four-dimensional Riemannian manifold: Let \( x^\mu \equiv \{ x^i, t \} \) be 3 + 1 coordinates on this manifold, then there exists a metric \( g_{\mu \nu}(x) \) which defines a line element

\[
d s^2 = g_{\mu \nu}(x) \, dx^\mu \, dx^\nu . \tag{1.1}
\]

Flat Minkowski space corresponds to

\[
g_{\mu \nu}(x) = \eta_{\mu \nu} = \text{diag}(1, -1, -1, -1) . \tag{1.2}
\]

An essential feature of General Relativity is that \( g_{\mu \nu}(x) \) is considered as a dynamical field which is determined by its equations of motion, the so-called Einstein equations. They involve the Ricci tensor \( R_{\mu \nu} \) and the Ricci scalar \( R \) constructed from the metric (1.2):

\[
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} \, R = - \kappa \, T_{\mu \nu} \tag{1.3}
\]

with

\[
\kappa = 8\pi G/c^2 \approx 1.865 \cdot 10^{-29} \, \text{m}/\text{g} ,
\]

\[
G = \text{Newton’s constant} \approx 6.668 \cdot 10^{-14} \, \text{m}^3/(\text{g} \, \text{sec}^2) . \tag{1.4}
\]

\( T_{\mu \nu} \) in (1.3) is the energy-momentum tensor of matter which acts as source for the gravitational field, the metric \( g_{\mu \nu}(x) \). Its role is analogous to the one of electro-magnetic currents \( J_\mu \), which act as sources for the electro-magnetic field \( A_\mu \) in Maxwell’s equations. Note, however, that the left-hand side of (1.3) is highly nonlinear in \( g_{\mu \nu} \) (in some analogy to non-abelian Yang-Mills field equations), and that also \( T_{\mu \nu} \) depends in general on \( g_{\mu \nu} \). This becomes clear if we derive (1.3) from an action principle. To this end we have to assume that the equations of motion for matter can be derived from a Lagrangian \( \mathcal{L}_M \), which can correspond to point particles or classical fields. For consistency (the Bianchi identities for the Riemann tensor

\[\text{The following section can not replace an introduction to General Relativity; to this end see the corresponding literature.}\]
which imply, via (1.3), that $T_{\mu\nu}$ is covariantly conserved (it is necessary that the space-time integrated matter Lagrangian is invariant under general coordinate transformations. A corresponding Lagrangian for a real scalar field $\varphi(x)$, e.g., is given by

$$\int d^4x \ L_M(\varphi, g_{\mu\nu}) = \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_\mu \varphi \ g^{\mu\nu} \partial_\nu \varphi - V(\varphi) \right)$$

(1.5)

where

$$g = \text{det} (g_{\mu\nu}) \quad (= -1 \text{ for } g_{\mu\nu} = \eta_{\mu\nu}) .$$

(1.6)

In general the matter energy momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g(x)}} \frac{\delta}{\delta g^{\mu\nu}(x)} \int d^4x' \ L_M(x') .$$

(1.7)

(The factor $\sqrt{-g}$ on the right-hand side of eq. (1.5) renders the volume integration invariant and is sometimes written explicitly under the $d^4x'$ integral in eq. (1.7)). The general dependence of $T_{\mu\nu}$ on $g_{\mu\nu}$ is now apparent.

The left-hand side of eq. (1.3) can also be derived from an action, the integrated Einstein Lagrangian

$$\int d^4x \ L_E = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \ R .$$

(1.8)

The variation of eq. (1.8) with respect to $g^{\mu\nu}$ gives, up to an integral of a total derivative (which can be important in the context of brane universes, but which is dropped here),

$$\frac{\delta}{\delta g^{\mu\nu}(x)} \int d^4x' \ L_E(x') = \sqrt{-g} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) ,$$

(1.9)

hence eq. (1.3) follows from $2\kappa/\sqrt{-g}$ times the vanishing variation of

$$\int d^4x \ L_{T\text{ot}} = \int d^4x \left( L_E + L_M \right) .$$

(1.10)

The following configurations of the metric $g_{\mu\nu}$ are of particular physical importance:

a) The Schwarzschild solution around a point-like source (and the Kerr solution around rotating point-like sources as pulsars): It determines the dynamics of astronomical objects (stars, planets, galaxies etc.) and reproduces, in the non-relativistic and weak field limit, the Newton potential $MG/r$. This solution of Einstein equations is tested down to scales of $O(1 \text{ mm})$.  


b) Gravitational waves, which are solutions of the Einstein equations in the vacuum ($T_{\mu\nu} = 0$) and await their discovery.

c) Cosmological configurations, which determine the global geometry of the universe and its temporal evolution. They are the essential subject of this chapter.

The search for such configurations of the metric $g_{\mu\nu}(x)$ is greatly simplified by the assumptions of isotropy of the universe (it looks the same in all directions) and homogeneity of the universe (no point is singled out, and physically relevant quantities as the Riemann scalar $R$ and the left-hand side of eq. (1.3), the Einstein tensor, do not depend on $x^i$). Experimental evidence for these assumptions is not overwhelming: Seemingly matter is lumped in our universe, from stars to galaxies to clusters of galaxies up to large scale structures of the size of the observed part of the universe.

Nevertheless the assumption is that matter can be described by an approximately $x^i$-independent (but $t$-dependent) energy-momentum tensor $T_{\mu\nu}(t)$. From isotropy it follows that the only non-vanishing components of $T_{\mu\nu}(t)$ are

$$ T_{00}(t) , T_{11}(t) = T_{22}(t) = T_{33}(t) \equiv T_s(t). \quad (1.11) $$

A convenient way to write a cosmological configuration of the metric $g_{\mu\nu}$ is in terms of the Robertson-Walker line element in spherical coordinates

$$ ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - k r^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right). \quad (1.12) $$

Here the constant $k$ can be chosen, after an appropriate rescaling of $r$ and $a(t)$, as $k = 0, \pm 1$. It determines the global geometry of three-dimensional space, which is flat for $k = 0$, a three-dimensional hypersphere (and hence closed) for $k = 1$, and hyperbolic (open) for $k = -1$. $a(t)$ in (1.12) is a scale factor of the three-dimensional space, and its time dependence is determined by the 00 or $ii$ components of Einstein equations (1.3):

$$ 3 \frac{\dot{a}^2 + k}{a^2} = \kappa \, T_{00}(t), \quad (1.13a) $$

$$ - 2a\ddot{a} + \dot{a}^2 + k \frac{\dot{a}^2}{a^2} = \kappa \, T_s(t) \quad (1.13b) $$

where $\dot{a} = da/dt$. Evidently eqs. (1.13) imply some relation between $T_{00}$ and $T_s$, which can be written as
\[ \dot{T}_{00} + 3 \frac{\dot{a}}{a} (T_{00} + T_s) = 0 \] (1.14)

and corresponds to the covariant conservation of the energy-momentum tensor. More properties of \( T_{00}, T_s \) have to be derived from ansätze for the properties of matter. Traditionally matter is modelled by a perfect fluid with density \( \rho(t) \) and pressure \( p(t) \). Even if stars (or galaxies) would be homogeneously distributed in our universe this model has its subtle problems: Evidently \( \rho(t) \) and \( p(t) \) play the role of effective spatial averages over “point-like” stars or galaxies, and \( a(t) \) corresponds to a spatial average of spatial diagonal components of \( g_{\mu\nu}(x) \) (the superposition of a large number of Schwartzschild solutions in the weak field limit). However, instead of writing eqs. (1.13) for these spatial averages, one should take the spatial average of Einstein’s equations (1.3). Due to the non-linear nature of these equations this is not the same. In addition, the spatial average of a metric on which spatial volumina depend is very difficult to define properly. For more discussions and literature on this subject see [4].

Within the perfect fluid model of matter the components of the energy-momentum tensor take the form

\[ T_{00} = \rho(t) + \Lambda \quad , \quad T_s = p(t) - \Lambda \] (1.15)

where \( \Lambda \) is the so-called cosmological constant. Its physical origin will be discussed in detail below.

The equation of state of a perfect fluid determines a relation \( p = p(\rho) \). Usually one assumes

\[ p = w\rho \] (1.16)

where the constant \( w \) depends on the microscopic properties of the fluid. Now eq. (1.14) (energy-momentum conservation) assumes the simple form

\[ \dot{\rho} + 3 \frac{\dot{a}}{a} (1 + \omega) \rho = 0 . \] (1.17)

It is instructive to derive expressions for the parameters \( w \) and \( \Lambda \) in the case where matter is modelled by plane waves of a scalar field. This model is possibly more realistic in an early hot and dense phase of the universe, but it also helps to interpret the physical significance of the above quantities at later epochs.
Let us consider the scalar field Lagrangian (1.5), with $V(\varphi)$ developed up to quadratic order in $\varphi$ around its minimum $V_0$:

$$\int d^4x \ L_M^{(2)}(\varphi, g_{\mu\nu}) = \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_\mu \varphi \ g^{\mu\nu} \partial_\nu \varphi - V_0 - \frac{1}{2} m^2 \varphi^2 \right). \quad (1.18)$$

If one constructs the components of the energy momentum tensor according to eq. (1.7) the following relation is useful:

$$\frac{\delta}{\delta g^{\mu\nu}} \sqrt{-g} = -\frac{1}{2} \sqrt{-g} \ g^{\mu\nu}. \quad (1.19)$$

Then one obtains

$$T_{00} = \partial_0 \varphi \partial_0 \varphi - g_{00} \left( \frac{1}{2} \partial_\mu \varphi g^{\mu\nu} \partial_\nu \varphi - \frac{m^2}{2} \varphi^2 - V_0 \right) \quad (1.20a)$$

$$T_{ij} = \partial_i \varphi \partial_j \varphi - g_{ij} \left( \frac{1}{2} \partial_\mu \varphi g^{\mu\nu} \partial_\nu \varphi - \frac{m^2}{2} \varphi^2 - V_0 \right). \quad (1.20b)$$

Subsequently we will replace, for simplicity, $g_{\mu\nu}(x)$ by $\eta_{\mu\nu}$ in $T_{\mu\nu}$; this avoids the need to discuss plane waves in curved space time. Hence we take $T_{\mu\nu}$ to be static; this still allows us to obtain expressions for $w$ and $\Lambda$ from eqs. (1.15) and (1.16). Then eqs. (1.20) (cf. our conventions (1.2)) simplify to

$$T_{00} = \frac{1}{2} \left( (\partial_0 \varphi)^2 + (\partial_i \varphi)^2 + m^2 \varphi^2 \right) + V_0 \quad (1.21a)$$

$$T_{ij} = \partial_i \varphi \partial_j \varphi + \frac{\delta_{ij}}{2} \left( (\partial_0 \varphi)^2 + (\partial_i \varphi)^2 + m^2 \varphi^2 \right) - \delta_{ij} V_0 \quad (1.21b)$$

Next we replace the fields $\varphi(x^i, t)$ by plane waves with constant amplitude $\varphi_0$:

$$\varphi(x^i, t) = \varphi_0 \sin(\omega t - \vec{k} \cdot \vec{x}) \quad (1.22)$$

where the equations of motion enforce the dispersion relation

$$\omega^2 = \vec{k}^2 + m^2. \quad (1.23)$$
Equations (1.21) become

\[ T_{00} = \frac{1}{2} \varphi_0^2 \left( (\omega^2 + \vec{k}^2) \cos^2(\omega t - \vec{k} x) + m^2 \sin^2(\omega t - \vec{k} x) \right) + V_0 \quad (1.24a) \]

\[ T_{ij} = \varphi_0^2 \left( \left( k_i k_j - \frac{\delta_{ij}}{2} \vec{k}^2 + \frac{\delta_{ij}}{2} \omega^2 \right) \cos^2(\omega t - \vec{k} x) - \frac{\delta_{ij}}{2} m^2 \sin^2(\omega t - \vec{k} x) - \delta_{ij} V_0 \right) . \quad (1.24b) \]

Now we perform two averages: First, due to isotropy, we average over the angular dependence of the wave vectors \( \vec{k} \), using

\[ < k_i k_j >_{\varphi,\theta} = \frac{\delta_{ij}}{3} \vec{k}^2 . \quad (1.25) \]

Second, due to homogeneity, we average over space using

\[ < \cos^2(\omega t - \vec{k} x) >_x = < \sin^2(\omega t - \vec{k} x) >_x = \frac{1}{2} . \quad (1.26) \]

The final expressions for the components of the energy-momentum tensor are (with the notation \( T_s \) for its spacial components, cf. (1.11), and actually we should have written \( < T_{00} > \) and \( < T_s > \))

\[ T_{00} = \frac{1}{4} \varphi_0^2 \left( \omega^2 + \vec{k}^2 + m^2 \right) + V_0 \quad (1.27a) \]

\[ T_s = \frac{1}{4} \varphi_0^2 \left( \omega^2 - \frac{1}{3} \vec{k}^2 - m^2 \right) - V_0 . \quad (1.27b) \]

Comparing with eqs. (1.15) and (1.16) we immediately find for the cosmological constant

\[ \Lambda = V_0 \quad (1.28) \]

and for \( w \), the ratio of the pressure to density,

\[ w = \frac{\omega^2 - \frac{1}{3} \vec{k}^2 - m^2}{\omega^2 + \vec{k}^2 + m^2} = \frac{\vec{k}^2}{3(\vec{k}^2 + m^2)} \quad (1.29) \]

where we have used the dispersion relation (1.23) in the second step in (1.29). From (1.28) one finds for radiation, \( m^2 = 0 \),
\[ w_{rad} = \frac{1}{3}, \quad (1.30) \]

and for non-relativistic matter, \( \vec{k}^2 \ll m^2 \),

\[ w_{nr} \approx 0. \quad (1.31) \]

Sometimes \( w \) is not defined by the ratio of pressure and density as in (1.16), but directly by the ratio of \( T_s/T_{00} \). Then, if the cosmological constant \( \Lambda \) dominates over \( \rho \) over \( p \), one obtains an effective \( w_\Lambda \) with

\[ w_\Lambda = \frac{T_s}{T_{00}} = -1. \quad (1.32) \]

After this intermezzo on the physical meaning of \( w \) in eq. (1.16) we plug the first of eqs. (1.15) into eq. (1.13a), which becomes

\[ 3 \frac{\dot{a}^2}{a^2} = -\frac{3k}{a^2} + \kappa \rho(t) + \kappa \Lambda \quad (1.33) \]

and we recall eq. (1.17),

\[ \dot{\rho}(t) + 3 \frac{\dot{a}}{a}(1 + w)\rho(t) = 0. \quad (1.34) \]

If the term \( \kappa \rho(t) \) dominates the right-hand side of eq. (1.33) (and \( w \neq -1 \)), the solution of eqs. (1.33) and (1.34) is

\[ a(t) = a_0 \ t^{2(1+w)}/3 \quad (1.35a) \]

\[ \kappa \rho(t) = \frac{4}{3(1+w)^2} t^{-2}. \quad (1.35b) \]

Hence the universe expands, and \( \rho(t) \), the matter density, decreases.

If the term \( \kappa \Lambda \) dominates the right-hand side of eq. (1.33) (or, alternatively, \( w = -1 \) and \( \rho(t) = \text{const.} = \Lambda \)) the universe expands exponentially, which corresponds to an inflationary epoch.

The ratio \( \dot{a}/a \text{ today} \) is called the Hubble constant \( H_0 \):

\[ H_0 = \frac{\dot{a}}{a} \text{ today}. \quad (1.36) \]
Apparently our universe expands today, i.e. distant galaxies (with distances measured in $M_{pc} \approx 3, 1 \cdot 10^{22}$ m) move away with apparent speeds (of the order of km/sec) which are roughly proportional to $H_0$ times their distance. Hence one measures $H_0$ in these units,

$$H_0 = h_0 \cdot 100 \frac{\text{km}}{\text{sec} (M_{pc})^{-1}} ,$$  

with

$$h_0 \sim 0.65 .$$  

The motion of distant galaxies reveals itself in a redshift $z$, i.e. the measured wavelengths $\lambda_m$ and frequencies $\nu_m$ differ from the emitted wavelengths $\lambda$ and $\nu$:

$$\frac{\lambda_m}{\lambda} = \frac{\nu}{\nu_m} = 1 + z .$$  

For a time dependent scale factor $a(t)$ one obtains

$$1 + z = \frac{a(0)}{a(-t)}$$

where $-t$ is the time when the signal was emitted. For the light-like distance $D$ (or luminosity distance) of these galaxies one finds with the metric (1.12)

$$D = a(0) \int_0^r \frac{dr}{\sqrt{1 - kr^2}} = a(0) \int_{-t}^0 \frac{dt'}{a(t')} .$$

Here we have used (for light-like distances)

$$0 = ds^2 = dt^2 - a^2(t) \frac{dr^2}{1 - kr^2} .$$

These results will be used in the next chapter. Let us reconsider eq. (1.33) today, using (1.36)

$$3H_0^2 = -\frac{3k}{a(0)^2} + \kappa \rho(0) + \kappa \Lambda .$$

It is convenient to define

$$\Omega_M = \frac{\kappa \rho(0)}{3H_0^2} , \quad \Omega_\Lambda = \frac{\kappa \Lambda}{3H_0^2}$$

which turns eq. (1.43) into
\[ \Omega_M + \Omega_\Lambda - \frac{k}{H_0^2 a^2(0)} = 1. \] (1.45)

Let us recall that inflation predicts \( k = 0 \) (a spacially flat universe), hence in this case

\[ \Omega_M + \Omega_\Lambda \equiv 1. \] (1.46)

This value for \( k \) is also compatible with recent measurements of fluctuations of the Cosmic Microwave Background \([1]\). Note that \( \rho(0) \) in (1.44), and hence \( \Omega_M \), includes both baryonic (visible or invisible) and dark matter. The corresponding split will not concern us here; subsequently we will assume, however, that the equation of state for matter is such that \( w = 0 \) corresponding to nonrelativistic (baryonic and/or dark) matter.

### 2 Measurement of the Cosmological Constant

It turns out that we could measure \( \Omega_M \) and \( \Omega_\Lambda \) if we could determine the precise dependence of the distance of astronomical objects on the redshift \( z \), \( D(z) \): If we put \( w = 0 \) in (1.34) (correspond to nonrelativistic matter) one can solve eqs. (1.33) and (1.34) for \( a(t) \), \( \rho(t) \), with \( \dot{a}(0)/a(0) = H_0 \) and \( \rho(0) \) (or \( \Omega_M \)) and \( \Omega_\Lambda \) as boundary conditions. The solution for \( a(t) \) can be plugged into eq. (1.41) for the luminosity distance \( D \), and the integral \( dt' \) can be written as an integral over \( dz' \) using (1.40). The resulting expression for \( D(z) \) reads (re-installing the speed of light \( c \))

\[
D(z) = \frac{c(1+z)}{H_0 \cdot \sqrt{|\lambda|}} \sin \left( \sqrt{|\lambda|} \int_0^z \frac{dz'}{\sqrt{(1+z')^2(1 + \Omega_M z') - (2+z')z'\Omega_\Lambda}} \right) \tag{2.1}
\]

with

\[ \lambda = 1 - \Omega_M - \Omega_\Lambda \tag{2.2} \]

and

\[
\hat{\sin}(x) = \sin(x) \quad \text{for } \lambda < 0 \\
= x \quad \text{for } \lambda = 0 \\
= \sinh(x) \quad \text{for } \lambda > 0. \tag{2.3}
\]
For small $z$ one obtains approximately

$$D(z) \simeq \frac{cz}{H_0} \left( 1 + \frac{z}{2} \left( 1 + \Omega_\Lambda - \frac{1}{2} \Omega_M \right) \right) + \mathcal{O}(z^3) .$$

(2.4)

Hence, apart from $H_0$, $D(z)$ depends on the two parameters $\Omega_M$ and $\Omega_\Lambda$. Their separate determination requires, however, the measurement of the terms of $\mathcal{O}(z^3)$ in (2.1) or (2.4).

In order to determine the distance $D(z)$ of distant objects we need objects with very well known absolute magnitude $M$. These are supernovae of type Ia, which light up within a few weeks and fade away within a few months. From the light curve the absolute magnitude can be determined to high precision.

The luminosity distance $D$ then follows from a measurement of the apparent magnitude $m$, which is related to $M$ and $D$ by

$$m = M + 5 \log(D) + K ,$$

(2.5)

where $K$ includes a constant and a correction for the variation of the apparent magnitude with the redshift. (The factor 5 and the logarithm of $D$ in (2.5) have its origin in the logarithmic scale for apparent and absolute magnitudes.)

The search for high redshift ($z \sim 0.4 - 0.9$) supernovae starts with the observation of patches of sky with tens of thousands of galaxies. Some weeks later the same patches are observed again, and by comparison one finds a few dozens of supernovae, which have not been there before. Generally these have not yet reached peak brightness, and subsequently their light curves are tracked more or less continuously, partly with the help of the Hubble Space Telescope.

Two experimental groups lead by S. Perlmutter [5] and B. Schmidt [6] have pursued this program, and results from best fits to $D(z)$ up to $z \sim 1$ (one with $z \sim 1.7$ in [6]) can directly be plotted in the plane $\Omega_\Lambda$ versus $\Omega_M$, see fig. 1. The results of both groups agree well, and systematic errors as absorption of light by dust, supernovae evolution and selection bias are believed to be under control.

The figure shows that

- $\Omega_\Lambda = 0$ is ruled out at the 99 percent level, assuming $\Omega_M > 0$ (but no other assumptions on $\Omega_\Lambda + \Omega_M$)
- a flat universe ($k = 0$), i.e. $\Omega_M + \Omega_\Lambda = 1$, is consistent with the $1\sigma$ contour, and leads to $\Omega_\Lambda \sim 0.7, \Omega_M \sim 0.3$. 
The essential and amazing features of this result are

a) $\Omega_\Lambda$ is tiny, but non-vanishing: For the density $\Lambda$ one obtains (with $c = 1$)

$$\Lambda \approx 6 \cdot 10^{-24} \frac{g}{m^3}.$$  \hfill (2.6)

b) $\Omega_\Lambda$ is of the same order as $\Omega_M$, which is a priori difficult to understand, since both quantities depend very differently on the time: Whereas $\rho(t)$ decays like $t^{-2}$ (cf. (1.35b)), $\Lambda$ remains constant, hence the ratio $\Omega_M/\Omega_\Lambda$ also decays like $t^{-2}$.

Hence we happen to live in an epoch where $\Omega_M$ and $\Omega_\Lambda$ are of comparable order of magnitude; this is the so-called “coincidence problem”.

3 The Cosmological Constant in Classical and Quantum Field Theory

As long as we consider classical general relativity coupled to matter in the form of point-like sources (or a fluid made out of point particles) the cosmological constant appears as an arbitrary free parameter. Note that this framework is consistent with all present tests of general relativity.

Problems appear once we extrapolate from length scales of $\mathcal{O}(1 \text{ mm})$ (where gravity is tested) down to length scales of $1 \text{ Fermi}$, $1 \text{ TeV}^{-1}$ or even smaller, where we describe matter by classical or quantum field theory. Usually it is assumed that the consistent coupling of macroscopic matter to gravity has its origin in microscopic Lagrangians (for fundamental fields describing elementary particles) which are invariant under general coordinate transformations, cf. the Lagrangian for a scalar field in chapter 1.

The Higgs sector of the standard model is traditionally described by such a Lagrangian for the Higgs field $H$, including a potential

$$V^H(H^2) \cong V_0^H + \mathcal{O}(H^2)$$ \hfill (3.1)

where we have developed $V^H$ around its non-trivial minimum. The correct dimension of the constant $V_0^H$ is $g/(m \text{ sec}^2)$, which is obtained from the condition that the action

$$S = \frac{1}{\hbar} \int dt \, d^3x \, V_0^H$$ \hfill (3.2)
is dimensionless. (Recall that $\hbar \sim 10^{-31} \text{gm}^2/\text{sec.}$) This dimension coincides with the dimension of $\Lambda$ as obtained from eq. (1.44) with $\Omega_\Lambda$ dimensionless, and the dimensions of $H_0$ and $\kappa$ given in eqs. (1.37) and (1.4). Traditionally, however, particle physicists put $c = \hbar = 1$. In these units the natural scale of $V_0^H$ is simply given by the 4th power of the Higgs vev, i.e.

$$V_0^H \approx (100 \text{ GeV})^4 = 10^8 \text{ GeV}^4.$$  \hfill (3.3)

In order to bring eq. (3.3) into the form of eq. (2.6) (which has the dimension of a matter density) we have to use

$$1 \text{ GeV}^4 \approx 2.3 \times 10^{23} \frac{g}{m^3},$$  \hfill (3.4)

hence (3.3) corresponds to a natural value of the cosmological constant $\Lambda^H$ of the order

$$\Lambda^H \sim 2 \times 10^{31} \frac{g}{m^3}.$$  \hfill (3.5)

This is 55 orders of magnitude off the observed value (2.6).

From a Higgs sector of a grand unified theory, where

$$V_0^{\text{GUT}} \approx \left(10^{16} \text{ GeV}\right)^4,$$  \hfill (3.6)

we would obtain

$$\Lambda^{\text{GUT}} \sim 2 \times 10^{87} \frac{g}{m^3},$$  \hfill (3.7)

which is 111 orders of magnitude away from (2.6).

What about quantum field theory? We cannot consistently quantize gravity, but we can couple the metric $g_{\mu\nu}(x)$ as an external field to the bare action of a quantum field theory such that it becomes invariant under general coordinate transformations. For its partition function we write schematically

$$e^{iG(J, g_{\mu\nu})} = \int \mathcal{D}\varphi e^{iS_{\text{bare}}(\varphi, g_{\mu\nu}) + iJ \cdot \varphi}$$  \hfill (3.8)

where $\varphi$ are quantum fields and $J$ the corresponding sources. Then $g_{\mu\nu}$ plays actually the role of a source for the (composite) energy-momentum operator, i.e. functional derivatives of (3.8) with respect to $g_{\mu\nu}$ generate the corresponding matrix elements. Instead of the functional $G(J, g_{\mu\nu})$ in (3.8) it is more convenient to work with the effective action $\Gamma_{\text{eff}}(\varphi_{ct}, g_{\mu\nu})$, which
is obtained by a Legendre transform from $G(J, g_{\mu\nu})$, keeping $g_{\mu\nu}$ fixed. Now the components of the energy momentum tensor, which have to be inserted into the Einstein equations (1.3), are obtained in analogy to eq. (1.7):

$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g(x)}} \frac{\delta}{\delta g^{\mu\nu}(x)} \Gamma_{eff}(\varphi_{cl}, g_{\mu\nu}) \quad (3.9)$$

where, in the vacuum, the fields $\varphi_{cl}$ are extrema of $\Gamma_{eff}(\varphi_{cl}, g_{\mu\nu})$. The leading diagrams, which contribute to the diagonal components of $T_{\mu\nu}$ and hence to the cosmological constant, are quartically (!) divergent tadpole diagrams. Hence an UV cutoff $\Lambda_{UV}$ is required, and the quantum contribution $\Lambda^{Quant}$ to the cosmological constant is of the order

$$\Lambda^{Quant} \sim \frac{1}{16\pi^2} \left(\Lambda_{UV}\right)^4 . \quad (3.10)$$

If one uses the Planck scale $\sim 10^{19}$ GeV as an UV cutoff (under the assumption that local quantum field theory is valid up to this scale) eq. (3.10) gives

$$\Lambda^{Quant} \sim 10^{97} \frac{g}{m^3} \quad (3.11)$$

which is 120 orders of magnitude off the experimental result (2.6). This is the world record on disagreement between theory and experiment.

What size of the UV cutoff $\Lambda_{UV}$ could we tolerate in order not to be in conflict with the experimental result (2.6)? A rapid calculation show that we would need

$$\Lambda_{UV} \lesssim 8 \cdot 10^{-3} \text{ eV} \quad (3.12)$$

or

$$\left(\Lambda_{UV}\right)^{-1} \gtrsim 0.025 \text{ mm} . \quad (3.13)$$

Hence even very low energy quantum effects, at sizes much larger than the size of atoms (where quantum mechanics is well tested), lead to untolerable contributions to the cosmological constant.

The contradiction between the field theoretical results for $\Lambda^H$, $\Lambda^{GUT}$ and $\Lambda^{Quant}$ and the experimental result (2.6) is the famous "cosmological constant problem" [8].
4 Towards Solutions of the Problem

a) Supersymmetry

Unbroken global supersymmetry \[9, 10, 11\] would lead to a nice solution of the cosmological constant problem. This is because the energy operator \(P^0\) can be written as a sum over squares of supercharges \(Q_\alpha\):

\[
P^0 = \frac{1}{4} \sum_\alpha \left( Q_\alpha + \overline{Q}_\alpha \right) \left( Q_\alpha + \overline{Q}_\alpha \right),
\]

hence matrix elements of \(P^0\) can be written as

\[
< \psi | P^0 | \psi > = \frac{1}{4} \sum_\alpha < \psi_\alpha | \psi_\alpha >
\]

with

\[
\psi_\alpha = \left( Q_\alpha + \overline{Q}_\alpha \right) | \psi >.
\]

The right-hand side of eq. (4.2), and hence all expectation values of \(P^0\), is semi-positive in a semi-positive Hilbert space. Thus, if a single state \(\psi_0\) with \(< \psi_0 | P^0 | \psi_0 >= 0\) exists, it is the state of lowest (vanishing) energy and hence the vacuum state. This argument includes all quantum effects, and would nicely explain a vanishing energy in the vacuum. However, eqs. (4.2) and (4.3) show that in this vacuum state supersymmetry would be unbroken,

\[
\left( Q_\alpha + \overline{Q}_\alpha \right) | \psi_0 > = 0
\]

which would imply untolerable Fermion/Boson degeneracies in the physical spectrum.

In classical \(N = 1\) supergravity it is possible to break spontaneously global supersymmetry, with vanishing vacuum energy and without fine tuning, provided the kinetic Lagrangian of the \(n\) matter fields is invariant under a non-linearly realized global \(SU(n, 1)\) symmetry, i.e. if it parametrizes the coset space of a \(SU(n, 1)/SU(n) \times U(1)\) non-linear sigma model \[11, 12\]. Such \(N = 1\) supergravity theories in \(d = 4\) appear naturally by compactifying \(N = 1\) supergravity in \(d = 10\) down to \(d = 4\), hence in most realistic superstring theories to lowest order in \(\alpha'\) \[13\].

However, here the vanishing of the vacuum energy is a purely classical phenomenon and spoiled by quantum corrections. If the spontaneous breakdown of supersymmetry manifests
itself in the form of gaugino masses, positive definite scalar masses or trilinear scalar couplings of order $M_{\text{susy}}$, the quantum corrections to the cosmological constant are still of the order

$$\frac{1}{16\pi^2} M_{\text{susy}}^2 \left(\Lambda^{\text{UV}}\right)^2,$$

which is much too large for $M_{\text{susy}} \sim 100 \text{ GeV}$ and $\Lambda^{\text{UV}} \sim M_{\text{Planck}} \sim 10^{19} \text{ GeV}$.

If the spontaneous breakdown of supersymmetry manifests itself only in the form of opposite mass shifts among scalars and pseudo-scalars of a massive chiral multiplet (so-called $F$-type splitting) \[11, 12, 14\] the quantum corrections to the cosmological constant are reduced to the order

$$\frac{1}{16\pi^2} M_{\text{susy}}^4$$

to all orders in perturbation theory. However, this value is still too large, and comparable to the contribution of the classical Higgs potential in the classical standard model. Hence, since a supersymmetry breaking scale $M_{\text{susy}} \lesssim 10^{-2} \text{ eV}$ (cf. eq. (3.12)) is out of question, supersymmetry alone is finally not able to explain the measured value of the cosmological constant.

**b) Dilaton**

It is tempting to obtain a vanishing vacuum energy as a consequence of the equation of motion of a scalar field. A natural candidate for such a field is the dilaton $\phi$, which helps to realize scale invariance non-linearly: In the presence of a dilaton all mass scales $M$ appear multiplied with an exponential of $\lambda \phi$. (The smallest possible value for the coupling $\lambda$ is $\lambda \sim \sqrt{G} = M_{\text{Planck}}^{-1}$..) Hence the “vacuum energy” $V_0(\phi)$, which is of the order $M^4$, is roughly of the form

$$V_0(\phi) \sim M^4 e^{4\lambda \phi}.$$

(4.7)

Scalars with this property appear in string theory and compactified supergravity theories; often the dilaton $\phi$ with the property (4.7) in $d = 4$ is a linear combination of several such scalars.

At first sight eq. (4.7) implies indeed

$$\frac{dV_0(\phi)}{d\phi} = 0 \leftrightarrow V_0(\phi) = 0.$$

(4.8)
The value \( \phi_0 \) which solves (4.8) is obviously \( \phi_0 = -\infty \). This is the "dilaton run away problem": In string theory and compactified supergravity theories couplings depend typically on \( \phi \), and these couplings vanish (or even tend to infinity in some cases) in this limit.

Moreover particle masses \( m \) depend necessarily on the dilaton in the form \( m(\phi) \sim M e^{\lambda \phi} \). Hence, whenever \( V_0(\phi) \) vanishes, all masses vanish as well (and scale invariance is restored). In addition, near the minimum of its potential the dilaton itself is nearly massless, and the coupling of the dilaton to particles of mass \( m \) is of the order of \( \sqrt{G} \cdot m = m/M_{\text{Planck}} \). Light scalars induce long-range interactions which are strongly constraint by limits on "fifth forces" and/or violations of the equivalence principle.

Note that these arguments apply also if mass scales are generated by dimensional transmutation (as in QCD or technicolour) once the \( \phi \) dependence of the UV cutoff is correctly taken into account. Hence, despite many efforts in this direction \( \mathbb{R} \), no working model could be constructed up to date.

c) 3-Form-Field

A 3-form-field \( A_{\mu \nu \rho} = A_{[\mu \nu \rho]} \) appears in \( N = 8 \) supergravity \( \mathbb{R} \). It has a field strength \( F_{\mu \nu \rho \sigma} = \partial_{[\mu} A_{\nu \rho \sigma]} \) and satisfies the equations of motion

\[
\partial^\mu F_{\mu \nu \rho \sigma} = 0 \quad (4.9)
\]

(suitably covariantized). The equation of motion (4.9) follows from a Lagrangian

\[
\mathcal{L}_A = \lambda \, F_{\mu \nu \rho \sigma} \, F^{\mu \nu \rho \sigma} . \quad (4.10)
\]

The only non-trivial solutions of (4.9) which respect Lorentz covariance are of the form

\[
F_{\mu \nu \rho \sigma} = \Sigma \, \varepsilon_{\mu \nu \rho \sigma} \quad (4.11)
\]

where \( \Sigma \) is an arbitrary constant. If one naively plugs (4.11) back into \( \mathcal{L}_A \), one obtains an effective contribution to the cosmological constant

\[
V_A = -24 \, \lambda \, \Sigma^2 . \quad (4.12)
\]

This does not solve the cosmological constant problem, but the arbitrary value of \( \Sigma \) could be used to cancel other contributions to it. Based on a path integral approach to Euclidean
quantum gravity Hawking has argued [16] that, if $S_{Eucl}$ contains an arbitrary parameter as in (4.12), the most probable value of $\exp(-S_{Eucl})$ is where $S_{Eucl}$ is minimal. Chosing the four sphere $S^4$ for Euclidean space time, the minimum of $S_{Eucl}$ (actually $-\infty$) is achieved for a vanishing cosmological constant, i.e. infinite volume. Subsequently Duff has pointed out [17], however, that one should not plug ansätze for solutions back into the action, but rather vary the unconstrained action, and that consequently the $A$-contribution to the Euclidean action is positive, invalidating Hawking’s argument.

Brown and Teitelboim [18] have coupled a $d-1$-form field $A$ (in $d$ space-time dimensions) to a $d-2$-brane. Based on a 2-dimensional toy model they proposed that then the value of $\Sigma$ could relax dynamically to the one corresponding to a vanishing cosmological constant.

But, all in all no convincing mechanism is known which naturally generates such a value of $\Sigma$ in $d = 4$.

\textbf{d) Quintessence}

Quintessence [19] does not aim at a solution of the problem of the smallness of the cosmological constant with respect to particle physics scales, but at a solution of the coincidence problem: Why is $\Omega_\Lambda$ of the order of $\Omega_M$?

In this approach the cosmological constant $\Lambda$ is replaced by a potential $V(\phi)$ of a scalar field $\phi$, which is not assumed to have reached its minimum value by now. One uses that for potentials with

$$\frac{V''V}{(V')^2} \geq 1 \quad (4.13)$$

the values of the scalar fields $\phi$ evolve with time such that, for a wide range of initial conditions, $V(\phi)$ is of the order of the density of background (standard) matter today. The solution of the coupled Einstein equations and equations of motion for $\phi$ is required for this result. As a by-product one finds that the effective parameter $w_\phi$ (cf. eq. (1.16) ff), associated to the equation of state for $\phi$, is often time-dependent.

A simple potential satisfying (4.13) is given by [20, 21]

$$V(\phi) \sim e^{-\lambda\phi} \quad (4.14)$$

However, now the full cosmological evolution has to be reconsidered, and notably nucleosyn-
thesis constraints (which require the quintessence energy density not to be too large at that epoch) rule out the simple model (4.14). Among the many working models is [20]:

\[ V(\phi) \sim \frac{\lambda}{\phi^\alpha}, \quad 0 < \alpha \lesssim 2. \] (4.15)

Motivations for quintessence fields and potentials can again be found in dilaton (or moduli) sectors of superstring or supergravity models, where the “dilaton run away problem” is now turned into a goody. However, the coupling of \( \phi \) to matter has to be reduced ad hoc in order not to generate additional dangerous long-range forces. Moreover, of course, the absolute value of the cosmological constant (the minimum of all scalar potentials) is still fine-tuned to zero here; the finite observed value of \( \Omega_\Lambda \) is just explained by the fact that the quintessence field \( \phi \) has not yet reached the minimum of its potential.

It should be noted that once \( D(z) \) (cf. (2.1)) can be determined to even better accuracy by measuring even more supernovae, various quintessence scenarios, i.e. various dependences of the cosmological “constant” on time, can be distinguished experimentally.

e) Brane Universes

In brane worlds the dimension of space-time is extended beyond \( d = 4 \). In contrast to the standard Kaluza-Klein approach, however, matter fields are confined to live on \( 3 + 1 \) dimensional manifolds (3-branes) which are embedded into the higher dimensional space-time manifold. Only gravity (and so-called bulk fields) lives in this higher dimensional manifold.

The most studied examples consist in one extra fifth dimension (denoted by \( y \)) and one or two embedded 3-branes. The fifth dimension is compact, i.e. \( y \) varies between \( -\pi R_5 \leq y \leq +\pi R_5 \) where the points \( y = \pm \pi R_5 \) are identified. The branes are located at \( y = 0 \) (and \( y = \pm \pi R_5 \)), and only symmetric modes of the \( y \)-dependent metric \( g_{\mu\nu}(x,y) \) under \( y \leftrightarrow -y \) are allowed.

This example is motivated by the Hořava-Witten construction of the strong coupling limit of the heterotic superstring [22], which is formulated in 11 dimensions with two 9-branes. After compactification of 6 space dimensions the above picture in 5 dimension emerges. Many more general brane worlds have been constructed since then, motivated by the presence of Dirichlet branes in string theories [23].

In this approach one can put constant energy densities \( \Lambda_1, \Lambda_2 \) on either of the two branes,
and another constant energy density \( \Lambda_b \) in the bulk outside the branes. Which of these plays the role of the cosmological constant in the Friedmann-Robertson-Walker cosmology (1.33)?

In order to answer this question one has to start with the five dimensional Einstein equations for \( g_{\mu \nu}(x, y) \). Assuming translational invariance in the three spacial \( x^i \) dimensions and a constant diagonal energy momentum tensor \( T^{\mu \nu} \) in the bulk the \( y \) dependence of \( g_{\mu \nu} \) can be fixed completely. Actually the first derivatives of \( g_{\mu \nu} \) with respect to \( y \) are discontinuous across the branes. This solution for the \( y \) dependence of \( g_{\mu \nu} \) can be plugged back into the Einstein equations, and the \( t \) dependence of \( g_{\mu \nu} \) on the brane(s) can be parametrized by a Friedmann-Robertson-Walker ansatz (1.12).

Although the five-dimensional Einstein equations contain additional terms compared to the four dimensional Einstein equations (involving non-vanishing terms \( \sim \partial_y g_{\mu \nu} \)) the resulting equations for \( a(t) \) can be written in the form of eq. (1.33) plus corrections on its right-hand side [24], provided \( \Lambda_2 = -\Lambda_1 \) (as in the Hořava-Witten construction). The term \( \kappa \Lambda \) in eq. (1.33) is now replaced by

\[
\kappa \Lambda_{eff} = \frac{\kappa_5}{2} \Lambda_b + \frac{\kappa_5}{12} \Lambda_1^2
\]  

(4.16)

where \( \kappa_5 \) is the five dimensional gravitational coupling.

More generally, in the presence of bulk fields \( \varphi_i \) with potentials \( V^1(\varphi) = -V^2(\varphi) \) on the branes, a potential \( V_b(\varphi) \) in the bulk and a general sigma model metric \( G_{ij}(\varphi) \) for the kinetic terms in the bulk, \( \Lambda_{eff} \) corresponds to the minimum of the effective potential [25]

\[
V_{eff}(\varphi) = \frac{1}{2} V_b - \frac{1}{32} V^1 G^{ij} V^1_{,i} V^1_{,j} + \frac{\kappa_5}{12} (V^1)^2 .
\]  

(4.17)

(Curiously enough \( V_{eff} \) resembles the scalar potential in \( N = 1 \) supergravity, with \( V^1 \) playing the role of the superpotential).

Generically even two five tunings (\( \Lambda_2 = -\Lambda_1 \), and the vanishing of the right-hand side of eq. (4.16) through an appropriate choice of \( \Lambda_b \)) are required in order to reproduce both a small effective cosmological constant and a time independent effective four dimensional gravitational constant in brane worlds with two 3-branes. Scenarios with just one 3-brane and a non-compact fifth dimension also exist [26] where just one fine-tuning is required.

It has been claimed that even this fine-tuning can be avoided if one puts a scalar field in the bulk, with either no potential at all in the bulk and an exponential potential on the brane [27] or an exponential potential in the bulk [28]; Then the combined equations of motion for the metric and the scalar field generically possess a (static) solution which
corresponds to a vanishing effective cosmological constant. This scenario seems to violate the above theorem (4.17) on the effective cosmological constant. Indeed this “self-tuning”-scenario always involves a \( y \)-dependent metric with naked singularities in \( y \). As shown in \[29\], any attempt to regularize these singularities requires a new fine-tuning. Moreover this scenario would require fine-tuned initial conditions.

The previous scenario involving a 3-form-field has also been applied to brane universes \[30\]. However, again fine-tuned initial conditions are required if the static solution corresponding to a vanishing effective cosmological constant is to be realized \[31\].

5 Conclusions

None of the attempts listed above has led to a successful explanation for the observed smallness of the cosmological constant. We did not have time to discuss the approach of Verlinde et al. \[32\], which is based on an interpretation of the AdS/CFT correspondence as an AdS/Renormalization Group correspondence. This concept is reviewed in \[33\].

Let us recall that the origin of the problem is the coupling of local four dimensional quantum field theory (supposedly valid down to length scales of \( \mathcal{O}((100 \text{ GeV})^{-1}) \)) to gravity, whose Einstein action is tested down to length scales of \( \mathcal{O}(1 \text{ mm}) \). Any attempt to solve the cosmological constant problem – involving extra dimensions, branes, AdS/CFT or whatsoever – must either fit into these frameworks (possibly with new extra fields) or be precise on its modifications.

Modifications of the Einstein action – either through effects due to quantum gravity, “large” extra dimensions, curvature squared terms from string theory etc. – face the following obstacle: Gravitational interactions from millimeters to astronomical distances should not be (drastically) modified, but there should be an effect on the equation of motion of the “global” Robertson-Walker mode \( a(t) \) of the gravitational field – without spoiling the very successful part of the cosmological standard model. Notably no “short-distance” modifications (affecting only small wave length modes of the gravitational field) can do this job.

Recall also that extra dimensions – with or without branes – can always be represented in terms of effective four dimensional fields (including possibly infinite towers of massive states) with local interactions. Hence any cancellation of the vacuum energy within an
approach based on extra dimensions must be representable by an effective four-dimensional Lagrangian with peculiar properties.

Then one finds oneself automatically in one of the frameworks (dilaton etc.) considered and discarded before. It seems that any dynamics sensitive to a vacuum energy density of $\mathcal{O}(10^{-3} \text{ eV})^4$ must involve fields with masses of this order or lighter. On the one hand these fields have to couple to matter and/or gravity (in order to detect the vacuum energy), but they should neither imply new long-range interactions nor have disastrous cosmological effects like important relic densities. Altogether these constraints seem to be self-contradictory.

In this situation some scientists appeal to anthropic principles as, e.g., “we happen to live in one long-living among $\sim 10^{100}$ possible universes”. Otherwise we possibly have to touch at one of the “hidden assumptions” – the local coupling of fundamental fields to gravity. Work in this direction is in progress [34].

**Acknowledgement**

I would like to thank the participants of the Workshop for various critical and helpful comments on the subject.
References

[1] S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, 1972.

[2] R. Sexl, H. Urbantke, *Gravitation and Kosmologie*, B I Mannheim, 1983.

[3] R. Wald, *General Relativity*, The University of Chicago Press, Chicago, 1984.

[4] A. Krasiński, *Inhomogeneous Cosmological Models*, Cambridge University Press, Cambridge (UK), 1997.

[5] S. Perlmutter et al., Int. J. Mod. Phys. A15 S1 (2000) 715, eConf/990809, Astrophys. J. 517 (1999) 565.

[6] A. Riess et al., Astron. J. 516 (1998) 1009, Astrophys. J. 560 (2001) 49 (astro-ph/0104455).

[7] P. de Bernardis et al., Nature 404 (2000) 955.

[8] For reviews and references see S. Weinberg, Rev. Mod. Phys. 61 (1989) 1, and S. M. Carroll, astro-ph/0004075.

[9] J. Wess, J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, Princeton, 1983.

[10] H. P. Nilles, Phys. Rept. 110C (1984) 1.

[11] N. Dragon, U. Ellwanger, M. G. Schmidt, Progress in Part. and Nucl. Phys. 18 (1987) 1.

[12] N. Dragon, U. Ellwanger, M. G. Schmidt, Phys. Lett. B145 (1984) 192; Nucl. Phys. B255 (1985) 549; Phys. Lett. B154 (1985) 373.

[13] E. Witten, Phys. Lett. B155 (1985) 151; U. Ellwanger, M. G. Schmidt, Nucl. Phys. B294 (1987) 445.

[14] U. Ellwanger, Phys. Lett. B349 (1995) 57.

[15] A. Aurilia, H. Nicolai, P. Townsend, Nucl. Phys. B176 (1980) 509.

[16] S. Hawking, Phys. Lett. B134 (1984) 403.
[17] M. Duff, Phys. Lett. B226 (1989) 36.

[18] J. Brown, C. Teitelboim, Phys. Lett. B195 (1987) 177, Nucl. Phys. B297 (1988) 787.

[19] For reviews see P. Binétruy, Int. J. Theor. Phys. 39 (2000) 1859, hep-ph/0005037, and V. Sahni, astro-ph/0202076.

[20] B. Ratra, P. Peebles, Phys. Rev. D37 (1988) 3406.

[21] C. Wetterich, Nucl. Phys. B302 (1988) 668;
   P. Ferreira, M. Joyce, Phys. Rev. Lett. 79 (1997) 4740 Phys. Rev. D58 (1998) 023503.

[22] P. Hořava, E. Witten, Nucl. Phys. 460 (1996) 506, Nucl. Phys. B475 (1996) 94.

[23] J. Polchinski, Tasi Lectures on D-Branes, hep-th/9611050.

[24] P. Binétruy, C. Deffayet, U. Ellwanger, D. Langlois, Phys. Lett. B477 (2000) 285.

[25] O. De Wolfe, D. Freedmann, S. Gubser, A. Karch, Phys. Rev. D62 (2000) 046008;
   U. Ellwanger, Phys. Lett. B473 (2000) 233.

[26] M. Gogberashvili, Mod. Phys. Lett. A14 (1999) 2025;
   L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4670.

[27] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, R. Sundrum, Phys. Lett. B480 (2000) 193.

[28] S. Kachra, M. Schulz, E. Silverstein, Phys. Rev. D62 (2000) 045021.

[29] S. Förste, Z. Lalak, S. Lavignac, H. P. Nilles, Phys. Lett. B481 (2000) 360, JHEP 0009 (2000) 34.

[30] J. Kim, B. Kyae, H. Lee, Phys. Rev. Lett. 86 (2001) 4223, Nucl. Phys. B613 (2001) 306.

[31] A. Medved, hep-th/0109180.
[32] E. Verlinde, Class. Quant. Grav. 17 (2000) 1277;
   E. Verlinde, H. Verlinde, JHEP 0005 (2000) 34.

[33] U. Ellwanger, Lectures at the LPT Orsay, hep-th/0009006.

[34] U. Ellwanger, hep-th/0201163.
Figure 1: Confidence regions in the $\Omega_M - \Omega_\Lambda$ plane based on data from 42 type Ia supernovae at large redshift discovered by the Supernova Cosmology Project [5], and 18 supernovae at low redshift.