An detailed study of the thermodynamical state of nuclear matter in transport calculations of heavy–ion reactions is presented. In particular we determine temperatures from an analysis of the local momentum space distribution on one hand, and from a fit to fragment energy spectra in terms of a blast model with radial flow and temperature on the other. We apply this to spectator and participant matter. In spectator we find regions of spinodal instability with temperatures and densities which are consistent with experiments. In the participant we find different temperatures for different fragment masses, indicating that the fragments are not emitted from a source in thermal and chemical equilibrium.

1 Introduction

One of the challenges in the study of heavy–ion collisions is the understanding of multifragmentation in relation to liquid–gas phase transitions. In spite of many experimental and theoretical efforts these processes have not been fully understood yet. This is largely due to the fact that a heavy ion collision is a dynamical process where the state of nuclear matter varies strongly in space and time, and which during much of the reaction is not in global or even local equilibrium. This is, e.g. seen by looking at local momentum space distributions in transport calculations which are found to be highly anisotropic even during the compression phase of the collision. Only at the later stages of the reaction the local momentum distributions become more and more thermalized without necessarily leading to global thermal equilibrium. Therefore, non–equilibrium effects are important for a reliable description of
heavy ion collisions [1, 3]. The influence of these non-equilibrium effects on the determination of the equation of state of nuclear matter has been discussed in Ref. [1]. In this contribution we concentrate on the question of the applicability of thermodynamical concepts in the non-equilibrium situation of heavy ion collisions [3] and, in particular, with respect to phase transitions and multifragmentation. We want discuss the question whether, starting from a transport description of a heavy ion collision, where in principle everything is known about the system, a thermodynamical picture of multifragmentation can be deduced.

2 Determination of temperature

In this work we make use of two different methods of determining temperature: In the first, we determine local temperatures by fitting the local momentum distributions (obtained in our case from relativistic transport calculations) to covariant Fermi–Dirac distributions at finite temperature in the local rest frame [3]. Non-equilibrium effects are taken into account by allowing a parametrization of the momentum space distribution in terms of two thermalized Fermi spheres [1] (or covariantly by ellipsoids). With this method we obtain a local microscopic temperature, $T_{\text{loc}}$.

In the second method we follow the experimental method of fitting fragment energy spectra. These are generated in our calculations by applying a phase space coalescence algorithm [1] to the final stages of the transport calculations. As in experimental analyses these spectra are interpreted in a Siemens–Rasmussen [4] or blast model [5], which assumes a thermalized freeze–out configuration of nucleons and fragments with a collective radial flow profile and a unique temperature [5, 6]. In this model the fragment spectra are given by

$$\frac{dN}{dE} \sim pE \int \exp(-\gamma E/T) \left[ \frac{\sinh \alpha \left( \gamma + \frac{T}{E} \right)}{\alpha} - \frac{T}{E} \cosh \alpha \right] \times n(\beta)\beta^2d\beta,$$

(1)

where $n(\beta)$ is the flow profile, $\gamma = \sqrt{1 - \beta^2}$ and where $T$ is the global temperature ($\alpha \equiv \gamma \beta p/T$). The flow profile is obtained from the simulation. Then the remaining parameter is the temperature $T$ which is fitted to experimental, resp. generated fragment spectra. In experimental analyses a global temperature is assumed, which characterizes the shapes of all fragment spectra. This is not obvious and should be clarified in the analyses of our transport calculations.
3 Analysis of spectator matter (semi-central collisions)

We first discuss the thermodynamical properties of spectator matter in semi-central Au on Au reactions at intermediate energies. This reactions has been studied extensively by the ALADIN collaboration [7]. We determine the spectator temperature from fits to local momentum distributions (for more details see Ref. [3]). Fig. 1 shows the time evolution of the temperature in the spectator (left side) for different beam energies. When the spectators are clearly developed in the transport calculations after about 40 fm/c, their temperatures approach a rather constant value of about $T \approx 5$ MeV which remains fairly stable up to about 80 fm/c, and furthermore is rather independent on the incident energy considered.

![Temperature Evolution](image1)

Fig. 1. Left: Temperature evolution in the spectator in semi-central Au on Au reactions at different beam energies indicated in the figure. Right: Density-pressure trajectories for the spectator matter for the same reaction at 600 AMeV. The solid and dashed curves represent longitudinal and transverse pressure, respectively. The squares and circles are the values at different times starting from $t = 35$ fm/c in steps of 5 fm/c. The dotted curves are the nuclear matter isothermal equation of state for $T = 5$ and 7 (lower and upper curve, respectively).

These results are in good agreement with experiments of the ALADIN collaboration, which from measurements with different "thermometers" determine the same value of $T \approx 5$ MeV, depending only moderately on the beam energy of the reaction. We also generate pressure–density trajectories for the spectator matter as a function of time (right side of Fig. 1). Dynamical instabilities should arise when the pressure increases with decreasing density indicating a negative effective compressibility, which occurs here at $t \geq 50$ fm/c. The system at this stage therefore enters an instability region and should break up into fragments. Comparing to the nuclear matter isothermal equation of state for temperatures of $T = 5$ and 7 MeV corresponding to the range of spec-
tator temperatures in Fig. 1 one sees that the thermodynamical conditions, as determined here, are close to but not identical to those of equilibrated nuclear matter. Only at the final stages the spectator closely follows the nuclear matter behavior at temperatures of about $T \approx 5$ MeV. The densities at the instability condition are about $1/3 - 1/2$ of saturation density. It thus appears that the spectator closely approaches a freeze–out configuration in thermal and chemical equilibrium.

4 Analysis of the fireball (central collisions)

![Graph](image)

Fig. 2. Temperatures (left) and radial flow (right) obtained from blast model fits to fragment energy spectra as function of the beam energy. The theoretical results are shown for two mean field models (see text). The data are taken from [5, 6].

In central collisions the situation is rather different. If very central events are selected experimentally using charged particle multiplicities [5] or theoretically at polar angles near mid rapidity [6], there is no spectator matter. Rather one observes a hot dense fireball which expands isotropically as found by our calculations and also by other groups [5]. Thus, assuming thermalization one can use Eq. (1) to extract the mean collective radial flow $\beta_f = <\beta>$ and a slope temperature $T_{slope}$ from fits to the fragment energy spectra. Fig. 2 shows the energy dependence of these quantities as determined from our calculations and from experiments for central Au on Au collisions. Two parametrizations of the mean field (non–linear Walecka model and configuration dependent Dirac–Brueckner mean fields) were used in the calculations to demonstrate the moderate dependence of $\beta_f$ and $T$ on the mean field. As seen in Fig. 2 the experimental data for the radial flow are reproduced very well. The comparison of the extracted slope temperatures $T_{slope}$ (left side of Fig. 2) is, however, only qualitative.
It is of interest, to discuss the relation of these slope temperatures to the local temperatures $T_{loc}$ determined from the momentum distribution of the calculation (we used a Maxwell–Boltzmann distribution here, in order to be consistent with eq. (1), but the difference is $\leq 5$ MeV in the final stages of the collision). It is also of interest to make the comparison separately for different fragment masses in order to determine whether a freeze-out scenario with a unique temperature is realistic.

![Graph showing slope parameter vs. fragment mass for temperatures and radial flow](image)

The results of this analysis are shown in Fig. 3. Here we show the slope temperatures and radial flow velocities from blast model fits to the spectra of different fragment masses $A_f$. The temperature increases and the radial flow decreases with increasing mass. In the coalescence picture such a behavior is reasonable, since a larger fragment has to be generated more inside the fireball, where the flow velocity is smaller and the temperature higher. In Fig. 3 we also show the result of a simultaneous blast model fit to all fragments and to fragments with mass $A_f \geq 2$. Since the fragment multiplicities are roughly exponential and thus dominated by the nucleons the results for all fragments are close to those for $A_f = 1$ alone. On the other hand the fit to the heavier fragments alone has lower radial flow and higher temperature and is the one compared in Fig. 2 to the corresponding experimental value.

Also shown in Fig. 3 is the local temperature from the momentum distributions determined at about 35 fm/c. At this time the fireball in the calculations approaches a freeze-out configuration (nucleon–nucleon collisions cease).
in equilibrium (pressure isotropic). It is then a consistent check that the local
temperature for this situation agrees approximately with the one determined
from a blast model fit to the $A_f = 1$ energy spectra.

5 Conclusions

We have studied the thermodynamical state of nuclear matter in heavy ion
collisions by analyzing local phase space configurations and by analyzing frag-
ment energy spectra. For the spectator a consistency with temperatures and
breakup conditions with results of the ALADIN collaboration was found. For
the participant matter we have applied in addition a blast model analysis to
fragment spectra generated in the coalescence model. We see that the slope
temperatures in such a description do not yield a unique value for all fragments.
This does not favour the picture of a freeze–out configuration in thermody-
namical equilibrium. Rather it appears that fragment emission is a dynamical
process which occurs during a longer stage of the heavy ion reaction.

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