Self-Tuning and de Sitter Brane Intersections in the
6-Dimensional Brane Models

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Abstract

We study the self-tuning of general brane junctions and brane networks on
the 6-dimensional space-time. For the general brane junctions, there may
exist one fine-tuning among the brane tensions. For the brane networks,
similar to the 5-dimensional self-tuning brane models, the brane tensions can
be set arbitrarily and there exists the singularity for the metric and bulk
scalar. And if we want to regularize the singularity, we will introduce the
fine-tuning among the brane tensions. In addition, because the 4-dimensional
cosmological constant we observe may be positive and very small, we discuss
the brane network with de Sitter brane intersections by introducing a bulk
scalar.

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I. INTRODUCTION

There are some unattractive features in the Standard Model (SM) which may imply the new physics, although the SM is very successful from the experiments at LEP and Tevatron. One of these problems is that the gauge interactions and gravitational interaction are not unified. Another is the gauge hierarchy problem. As we know, several solutions to the gauge hierarchy problem have been proposed: the technicolor and compositeness which lacks calculability; the weak-scale supersymmetry which is the leading candidate for the extension of the Standard Model several years ago; and the conformality whose spirit is similar to that of supersymmetry: one just replaces one symmetry (supersymmetry) with another (conformal symmetry) above the TeV scale, and both approaches predict new physics at TeV scale [1].

About three years ago, it was suggested that the large compactified extra dimensions may also be a solution to the gauge hierarchy problem [2], because a low \((4+n)\)-dimensional Planck scale \((M_X)\) may result in the large 4-dimensional Planck scale \((M_{pl})\) due to the large physical volume \((V^n)\) of extra dimensions: \(M_{pl}^2 = M_X^{2+n} V^n\). In addition, Randall and Sundrum [3] proposed another scenario that the extra dimension is an orbifold, and the size of extra dimension is not large but the 4-dimensional mass scale in the Standard Model is suppressed by an exponential factor from the 5-dimensional mass scale due to the exponential warp factor. Furthermore, they suggested that the fifth dimension might be coordinate non-compact [4], and there may exist only one brane with positive tension at origin, however, there exists the gauge hierarchy problem. The remarkable aspect of the second scenario is that it gives rise to a localized graviton field. After that, a lot of 5-dimensional models with 3-branes were built [5-6], and the models with co-dimension one brane(s) were constructed on the 6-dimensional and higher dimensional space-time [7-11].

In above model buildings, all the models with warp factor in the metric have negative bulk cosmological constant. However, in string theory, it is natural to take the bulk cosmological constant to be zero since the tree-level vacuum energy in the generic critical closed string compactifications (supersymmetric or not) vanishes. And the zero bulk cosmological constant is natural in the scenario in which the bulk is supersymmetric (though the brane need not be), or the quantum corrections to the bulk are small enough to be neglect in a controlled expansion. So, how to construct the models with zero bulk cosmological constant is an interesting question in the model buildings, because such kinds of models are still interesting if the bulk corrections to bulk cosmological constant \(\Lambda\) were very small, which can be happened for instance if the supersymmetry breaking is localized in a small neighborhood of the branes, or if the supersymmetry breaking scale in the bulk is small enough. Moreover, if all the gauge fields and matter fields were confined to the branes, the quantum corrections of these fields to the brane tensions might not affect the models with \(\Lambda = 0\).

One scenario is that we introduce not only the space-like extra dimension, but also the time-like extra dimension [11]. The good aspect of this approach is that there is no singularity, however, there exists fine-tuning and might have the problems arising from the time-like extra dimension: unitarity and causality. The other scenario was proposed where a scalar \(\phi\), which does not have bulk potential, is introduced [12]. In the second scenario, \(\phi\) becomes singular at a finite distance along the extra dimension and the warp factor in the metric vanishes at singularity. The good aspect of this approach is that, the
brane tension can be set arbitrary. However, the $Z_2$ symmetric and 4-dimensional Poincare
invariant solution is unstable under the bulk quantum corrections, and any procedure which
regularizes the singularity will introduce the fine-tuning which the self-tuning is supposed to
avoid [13]. Furthermore, a simple no-go theorem [14] relating to the self-tuning
solutions to the cosmological constant for observers on the brane, which rely on a singularity
in an extra dimension, shows that it is impossible to shield the singularity from the brane by
a horizon [13], unless the positive energy condition is violated in the bulk or on the brane,
or the 3-brane has spatial curvature.

In this paper, we would like to discuss the self-tuning of general brane junctions [11],
a simple brane intersection, and a brane network [9] on the 6-dimensional space-time by
introducing a bulk scalar without bulk potential. For the general brane junction models,
there may exist one fine-tuning among the total brane tensions [1]. However, in the brane
intersection models or brane networks, where the branes are co-dimension one hypersurfaces,
the brane tensions can be set arbitrarily because the constraint in the brane junctions is
satisfied automatically. Similar to the 5-dimensional brane models, there exists the singu-
larity for the metric and bulk scalar in the self-tuning of brane networks. And if we want
to regularize the singularity, for example, we require the extra dimensions be compact and
introduce the cut-off branes, we will have the fine-tuning among the brane tensions.

In addition, as we know, our universe may have a very small positive cosmological con-
stant, so, we discuss the brane network with de Sitter brane intersections. Suppose we have
$n$ space-like extra dimensions whose coordinates are $y^i$ where $i=1, 2, ..., n$, and the branes
are co-dimension one hypersurfaces which are determined by the algebraic equations $y^i = r$
where $r$ is a real number. If one assumed the metric

$$ds^2 = \Omega^{-2} (-dt^2 + \sum_{i=1}^{3} e^{2Ht} dx^i dx^i + \sum_{i=1}^{n} dy^{i2}) \ ,$$

(1)

it is not difficult for one to show that there does not exist the solution for $n > \frac{1}{2}$ i. e.,
we do not have such kind of brane networks with de Sitter brane intersections. In order to
obtain the solutions, we introduce a bulk scalar which has bulk potential, in other words,
we add one degree of freedom to the system. We present a 6-dimensional brane network
with three (two) 4-branes whose extra dimension coordinates are $y$ and $z$, one along the y
direction and two (one) along the z direction. The solution has no singularity and all the
brane tensions have similar forms in terms of the scalar. Similarly, one can also discuss the
general brane networks with de Sitter or Anti-de Sitter brane intersections.

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1 Although it is not essentially self-tuning if there was one fine-tuning among the brane tensions,
we still call it “self-tuning”, which means the bulk potential for $\phi$ is zero.

2 For $n = 1$ solution, for example, see ref. [16].
II. SELF-TUNING OF THE BRANE JUNCTIONS AND NETWORKS

First, let us discuss the self-tuning of the general brane junctions. The set-up is given in Fig. 1, we use the metric with signature \((-, +, +, +, +, +)\). The 6-dimensional gravitational action describing the system is

\[
S = \frac{1}{2\kappa^2} \int d^4x dy dz \sqrt{-g} (R - \partial_A \phi \partial^A \phi) - \sum_{i=1}^{k} \int d^4x dy dz \sqrt{-g^{(i)}} V_i(\phi) \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}) ,
\]

where \(\kappa^2 = M_X^{-4}\) is the 6-dimensional coupling constant of gravity, \(M_X\) is the 6-dimensional Planck scale, and \(R\) is the curvature scalar. The vectors

\[
\vec{n}_i = (-\sin \alpha_i, \cos \alpha_i), \quad \vec{l}_i = (\cos \alpha_i, \sin \alpha_i) \quad \text{and} \quad \vec{w} = (y, z)
\]

are defined so that \(\delta(\vec{n}_i \cdot \vec{w})\) is the line in the \(y - z\) plane which contains the \(i^{th}\) brane and \(\theta(\vec{l}_i \cdot \vec{w})\) serves to cut the irrelevant half of the line. Thus, \(\delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w})\) defines the location of the \(i^{th}\) semi-infinite brane. The 5-dimensional metric on the \(i^{th}\) half-brane is

\[
g^{(i)}_{AB} \equiv g_{AB}(z = y \tan \alpha_i) .
\]

Assuming the metric to be conformally flat, it can be written as

\[
ds^2 = \Omega^{-2} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + dz^2) ,
\]

where \(\Omega \equiv \Omega(y, z)\). Then, the Einstein equations are

\[
G_{AB} = \kappa^2 T_{AB} = \partial_A \phi \partial_B \phi - \frac{1}{2} g_{AB} (\partial \phi)^2 - \kappa^2 \Omega^{-1} \sum_{i=1}^{k} \Gamma^{(i)}_{AB} V_i(\phi) \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}) ,
\]

where

\[
\Gamma^{(i)}_{AB} = \begin{pmatrix}
-1 & 1 & 1 \\
1 & \cos^2 \alpha_i & \sin \alpha_i \cos \alpha_i \\
1 & \sin \alpha_i \cos \alpha_i & \sin^2 \alpha_i
\end{pmatrix} .
\]

These equations can be put in a form amenable to easy solution by transforming to the conformally related space-time,

\[
\tilde{g}_{AB} = \Omega^2 g_{AB} .
\]

In six dimensions the Einstein tensor in the two metrics are related by

\[
G_{AB} = \tilde{G}_{AB} + 4 \left[ \Omega^{-1} \tilde{\nabla}_A \tilde{\nabla}_B \Omega + \tilde{g}_{AB} \left( -\Omega^{-1} \tilde{\nabla}^2 \Omega + \frac{5}{2} \Omega^{-2} (\tilde{\nabla} \Omega)^2 \right) \right] ,
\]
where the covariant derivatives \( \tilde{\nabla} \) are evaluated with respect to the metric \( \tilde{g} \). Since the metric is conformally flat, the covariant derivatives are identical to ordinary derivatives and \( \tilde{G}_{AB} = 0 \). Using above form of the Einstein tensor, the Einstein equations are

\[
4 \frac{\partial^2 \Omega}{\partial y^2} = \Omega \left( \frac{\partial \phi}{\partial y} \right)^2 + \kappa^2 \sum_{i=1}^{k} V_i(\phi) \sin^2 \alpha_i \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}) ,
\]

\[
4 \frac{\partial^2 \Omega}{\partial z^2} = \Omega \left( \frac{\partial \phi}{\partial z} \right)^2 + \kappa^2 \sum_{i=1}^{k} V_i(\phi) \cos^2 \alpha_i \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}) ,
\]

\[
20 \left( \frac{\partial \Omega}{\partial y} \right)^2 + 20 \left( \frac{\partial \Omega}{\partial z} \right)^2 = \Omega^2 \left( \frac{\partial \phi}{\partial y} \right)^2 + \Omega^2 \left( \frac{\partial \phi}{\partial z} \right)^2
\]

\[
4 \frac{\partial^2 \Omega}{\partial y \partial z} = \Omega \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial z} - \kappa^2 \sum_{i=1}^{k} \sin \alpha_i \cos \alpha_i V_i(\phi) \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}) ,
\]

which must be supplemented with the equation of motion of the scalar field \( \phi \),

\[
\Omega \frac{\partial^2 \phi}{\partial y^2} + \Omega \frac{\partial^2 \phi}{\partial z^2} - 4 \frac{\partial \Omega}{\partial y} \frac{\partial \phi}{\partial y} - 4 \frac{\partial \Omega}{\partial z} \frac{\partial \phi}{\partial z} = \kappa^2 \sum_{i=1}^{k} \frac{\partial V_i}{\partial \phi} \delta(\vec{n}_i \cdot \vec{w}) \theta(\vec{l}_i \cdot \vec{w}) .
\]

The solution to Eq.s (10–14) is

\[
\Omega = \left\{ \sum_{i=1}^{k} (\vec{r}_i \cdot \vec{w}) \theta(\vec{n}_i \cdot \vec{w}) \theta(-\vec{n}_{i+1} \cdot \vec{w}) + C \right\}^{-\frac{3}{2}} ,
\]

\[
\phi = -2 \sqrt{5} \ln \Omega ,
\]

where \( \vec{n}_{k+1} \equiv \vec{n}_1 \) and \( \vec{r}_i = (p_i, q_i) \) are the integration constants which are not all independent of each other. The brane tensions are

\[
\kappa^2 V_i(\phi) = \frac{p_i - p_{i-1}}{\sin \alpha_i} e^{-\frac{2\kappa \phi}{5}} \text{ for } \sin \alpha_i \neq 0 ,
\]
or

$$\kappa^2 V_i(\phi) = \frac{q_i - q_i}{\cos \alpha_i} e^{-\frac{\sqrt{5}}{2} \phi} \quad \text{for } \cos \alpha_i \neq 0.$$  

(18)

Of course, $\sin \alpha_i = 0$ and $\cos \alpha_i = 0$ imply $p_i = p_{i-1}$ and $q_i = q_{i-1}$, respectively. We obtain that there may exist one fine-tuning among the total brane tensions from Eq.s (17) and (18), which will be automatically satisfied when we consider the self-tuning of brane intersections or brane networks.

Second, we present a simple brane intersection model which is a special case of above solution. Suppose that we have two 4-branes, which are determined by the equations $y = 0$ and $y \sin \alpha = z \cos \alpha$, respectively. The setup is given at Fig. 2. The action and Einstein equation can be obtained from above general discussions, so, we will not repeat them here. We just give the solution where the conformal factor and $\phi$ are

$$\Omega = (a \abs{z} + b \abs{y \sin \alpha - z \cos \alpha} + c y + d z + e)^{-\frac{1}{4}},$$

(19)

$$\phi = -2 \sqrt{5} \ln (a \abs{z} + b \abs{y \sin \alpha - z \cos \alpha} + c y + d z + e)^{-\frac{1}{4}},$$

(20)

and the brane tensions are

$$\kappa^2 V_1(\phi) = -2 a e^{-\frac{\sqrt{5}}{2} \phi}, \quad \kappa^2 V_2(\phi) = -2 b e^{-\frac{\sqrt{5}}{2} \phi}.$$  

(21)

We also assume $e > 0$. If $a < 0$ and $b < 0$, then, two branes have positive tensions and the brane tensions can be set arbitrarily. However, $\phi$ will have singularity on some lines and the conformal factor $\Omega^{-1}$ will vanish there. The singular points form co-dimension one curves, and can be calculated easily. For example, assuming $c = d = 0$, there are 4 singular points along the two branes: $(y = -e/a, z = 0)$, $(y = e/a, z = 0)$, $(y = -e/b \cos \alpha, z = -e/b \sin \alpha)$, and $(y = e/b \cos \alpha, z = e/b \sin \alpha)$, we can draw 4 straight lines from $(y = -e/a, z = 0)$ and $(y = e/a, z = 0)$ to $(y = -e/b \cos \alpha, z = -e/b \sin \alpha)$ and $(y = e/b \cos \alpha, z = e/b \sin \alpha)$. On those 4 lines, $\phi$ is singular and $\Omega^{-1}$ is zero.

Third, we discuss a brane network with four 4-branes on the extra space manifold $R^1/Z_2 \times R^1/Z_2$. Two branes along $y$ direction are located at $y = 0$ and $y = y_1$, respectively, and two
branes along the \( z \) direction are located at \( z = 0 \) and \( z = z_1 \), respectively. The set-up is given in Fig. 3. The action for this model is
\[
S = S_{\text{Bulk}} + S_{\text{Branes}},
\]
where
\[
S_{\text{Bulk}} = \frac{1}{2\kappa^2} \int d^4xdydz \sqrt{-g(R - \partial_A \phi \partial^A \phi)},
\]
\[
S_{\text{Branes}} = -\int d^4xdydz \left( \sqrt{-g^{(1)}} V_1(\phi) \delta(y) + \sqrt{-g^{(3)}} V_3(\phi) \delta(y - y_1) \right) - \int d^4xdydz \left( \sqrt{-g^{(2)}} V_2(\phi) \delta(z) + \sqrt{-g^{(4)}} V_4(\phi) \delta(z - z_1) \right),
\]
where \( g^{(i)} \) for \( i = 1, 2, 3, 4 \) is the metric on the \( i \)-th brane, which can be obtained by restriction. The detail calculation is similar, so, we just give the result. Assuming the conformal metric
\[
ds^2 = \Omega^{-2} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2 + dz^2),
\]
we obtain the solution for conformal factor and \( \phi \)
\[
\Omega = (a | y - y_1 | + b | z - z_1 | + c y + d z + e)^{-\frac{1}{4}},
\]
\[
\phi = -2 \sqrt{5} \ln (a | y - y_1 | + b | z - z_1 | + c y + d z + e)^{-\frac{1}{4}},
\]
and the brane tensions are
\[
\kappa^2 V_1(\phi) = -(c - a) e^{-\frac{\sqrt{5}}{2} \phi}, \quad \kappa^2 V_2(\phi) = -(d - b) e^{-\frac{\sqrt{5}}{2} \phi},
\]
\[
\kappa^2 V_3(\phi) = -2 a e^{-\frac{\sqrt{5}}{2} \phi}, \quad \kappa^2 V_4(\phi) = -2 b e^{-\frac{\sqrt{5}}{2} \phi}.
\]
So, if \( c > a, d > b \) and \( e > 0 \), the metric and \( \phi \) does not have singularity or vanish at finite distance from origin, but they are divergent at infinity. And the brane tensions can be set arbitrarily, for instance, if \( a < 0 \) and \( b < 0 \), the brane tensions \( V_3 \) and \( V_4 \) are positive, and the brane tensions \( V_1 \) and \( V_2 \) are negative. In order to avoid the divergence in the metric and \( \phi \), we can introduce two cut-off 4-branes: \( V_5 \) which is located at \( y = y_2 \), and \( V_6 \) which is located at \( z = z_2 \), where \( y_2 > y_1 \) and \( z_2 > z_1 \). So, the extra space manifold is \( S^1/Z_2 \times S^1/Z_2 \). Because the tensions for the cut-off 4-branes are
\[
\kappa^2 V_5(\phi) = (c + a) e^{-\frac{\sqrt{5}}{2} \phi}, \quad \kappa^2 V_6(\phi) = (d + b) e^{-\frac{\sqrt{5}}{2} \phi},
\]
we will have the fine-tuning among the brane tensions, which is similar to the 5-dimensional self-tuning models [13].
III. BRANE NETWORK WITH DE-SITTER BRANE INTERSECTION

Because the 4-dimensional cosmological constant we observe is positive although it is very small, we would like to discuss the brane network with de Sitter brane intersections. In order to have the solutions, we introduce one bulk scalar $\phi$ whose bulk potential does not vanish. Assume we have three 4-branes, one along the $y$ direction at $y = 0$, two along the $z$ direction at $z = 0$ and $z = z_1$. The set-up is given at Fig. 4 (b). The metrics on the branes can be obtained by restriction

$$ g^{(1)}_{AB} \equiv g_{AB}(y = 0), \text{ where } A, B \neq y, \quad (31) $$

$$ g^{(2)}_{AB} \equiv g_{AB}(z = 0), \text{ where } A, B \neq z, \quad (32) $$

$$ g^{(3)}_{AB} \equiv g_{AB}(z = z_1), \text{ where } A, B \neq z. \quad (33) $$

And the action for this system is

$$ S = S_{\text{Bulk}} + S_{\text{Branes}}, \quad (34) $$

where

$$ S_{\text{Bulk}} = \int d^4x dy dz \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{2} \partial_A \phi \partial^A \phi - \Lambda(\phi) \right), \quad (35) $$

$$ S_{\text{Brane}} = - \int d^4x dy dz \sqrt{-g^{(1)}V_1(\phi)} \delta(y) $$

$$ - \int d^4x dy dz (\sqrt{-g^{(2)}V_2(\phi)} \delta(z) + \sqrt{-g^{(3)}V_3(\phi)} \delta(z - z_1)). \quad (36) $$

With the following conformal metric

$$ ds^2 = \Omega^{-2} (-dt^2 + \sum_{i=1}^{3} e^{2Ht} dx^i dx^i + dy^2 + dz^2), \quad (37) $$

we obtain the Einstein equations.
FIG. 4. (a) Two 4-branes with de Sitter brane intersection; (b) Three 4-branes with two de Sitter brane intersections.

\[
4 \frac{\partial^2 \Omega}{\partial y^2} = \Omega \left( \frac{\partial \phi}{\partial y} \right)^2 + V_1(\phi) \delta(y) + 3H^2 \Omega, \quad (38)
\]

\[
4 \frac{\partial^2 \Omega}{\partial z^2} = \Omega \left( \frac{\partial \phi}{\partial z} \right)^2 + V_2(\phi) \delta(z) + V_3(\phi) \delta(z - z_1) + 3H^2 \Omega, \quad (39)
\]

\[
20 \left( \frac{\partial \Omega}{\partial y} \right)^2 + 20 \left( \frac{\partial \Omega}{\partial z} \right)^2 = \Omega^2 \left( \frac{\partial \phi}{\partial y} \right)^2 + \Omega^2 \left( \frac{\partial \phi}{\partial z} \right)^2 - 2\Lambda(\phi) + 18H^2 \Omega^2, \quad (40)
\]

\[
4 \frac{\partial^2 \Omega}{\partial y \partial z} = \Omega \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial z}. \quad (41)
\]

and the equation of motion for \( \phi \)

\[
\Omega^2 \frac{\partial^2 \phi}{\partial z^2} + \Omega^2 \frac{\partial^2 \phi}{\partial z^2} = +4 \frac{\partial \Omega}{\partial y} \frac{\partial \phi}{\partial y} + 4 \frac{\partial \Omega}{\partial z} \frac{\partial \phi}{\partial z} + \Omega \frac{\partial V_1(\phi)}{\partial \phi} \delta(y) \\
+ \Omega \frac{\partial V_2(\phi)}{\partial \phi} \delta(z) + \Omega \frac{\partial V_3(\phi)}{\partial \phi} \delta(z - z_1) + \frac{\partial \Lambda(\phi)}{\partial \phi}. \quad (43)
\]

If \( V_3(\phi) = 0 \), i.e., there are two 4-branes and the set-up is given in Fig. 4 (a). The conformal factor and \( \phi \) are

\[
\Omega = \exp \left\{ \frac{3}{8} H^2 \left[ (|y| + c_1)^2 + (|z| + c_2)^2 + c_3 \right] \right\}, \quad (44)
\]

\[
\phi = \frac{3}{4} H^2 \left[ (|y| + c_1)^2 + (|z| + c_2)^2 + c_3 \right], \quad (45)
\]

the bulk potential for \( \phi \) is

\[
\Lambda(\phi) = 9(1 + \frac{1}{2}H^2 c_3)H^2 e^{\phi} - 6H^2 \phi e^{\phi}, \quad (46)
\]

and the brane tensions are

9
\[ V_1(\phi) = 6\ e^{\phi/2}, \ V_2(\phi) = 6\ e^{\phi/2}. \]  \tag{47}

So, both branes have positive tensions.

Now, we consider the case \( V_3(\phi) \neq 0 \), because we need at least three 4-branes to solve the gauge hierarchy problem. The conformal factor and \( \phi \) are

\[ \Omega = \exp\left\{ \frac{3}{8} H^2 \left[ (|y| + c_1)^2 + (|z| - |z - z_1| - z + c_2)^2 + c_3 \right] \right\}, \tag{48} \]

\[ \phi = \frac{3}{4} H^2 \left[ (|y| + c_1)^2 + (|z| - |z - z_1| - z + c_2)^2 + c_3 \right], \tag{49} \]

the bulk potential for \( \phi \) is

\[ \Lambda(\phi) = 9(1 + \frac{1}{2} H^2 c_3) H^2 e^\phi - 6H^2 \phi e^\phi, \tag{50} \]

and the brane tensions are

\[ V_1(\phi) = 6\ H^2\ e^{\phi/2}, \ V_2(\phi) = 6\ H^2\ e^{\phi/2}, \tag{51} \]

\[ V_3(\phi) = -6\ H^2\ e^{\phi/2}. \tag{52} \]

Thus, the third brane has negative tension. By the way, all the brane tensions have similar forms in terms of the scalar.

Similarly, we can discuss the general brane networks with de Sitter or Anti-de Sitter brane intersections.

**IV. CONCLUSION**

We study the self-tuning of general brane junctions and brane networks on the 6-dimensional space-time. For the general brane junctions, there may exist one fine-tuning among the brane tensions. For the brane networks, similar to the 5-dimensional self-tuning brane models, the brane tensions can be set arbitrarily and there exists the singularity for the metric and bulk scalar. And if we want to regularize the singularity, we will introduce the fine-tuning among the brane tensions. In addition, because the 4-dimensional cosmological constant we observe may be positive and very small, we discuss the brane network with de Sitter brane intersections by introducing a bulk scalar.

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