Spontaneously Generated Gauge Invariance

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Abstract

We argue that the non-observability of the spontaneous breakdown of Lorentz symmetry (SBLS) caused by the vacuum expectation values of vector fields could provide the origin of all internal symmetries observed. Remarkably, the application of this principle to the most general relativistically invariant Lagrangian, with arbitrary couplings for all the fields involved, leads by itself to the appearance of a symmetry and, what is more, to the massless vector field(s) gauging this symmetry. A simple model for the SBLS based on massive vector and real scalar field interactions is considered; it is found that spontaneously broken gauge symmetries could also appear, when SBLS happens and is required to be physically unobservable.
1 Introduction

There are several indications in the literature (see [1] and extended references therein) that a local symmetry of all fundamental interactions of the matter and the corresponding massless gauge fields could be dynamically generated. They include observations stemming from Kaluza-Klein type theories, string theories, non-linear $\sigma$-models and non-standard interpretations of gauge symmetry.

In particular there has been considerable interest [2] in the interpretation of gauge fields as composite Nambu-Jona-Lasinio (NJL) bosons [3], possibly associated with the spontaneous breakdown of Lorentz symmetry (SBLS). A typical model realising this mechanism is based on the four fermion (current $\times$ current) interaction, where the gauge field appears as a fermion-antifermion pair composite state, in complete analogy with the massless composite scalar field in the original NJL model [3]. While this model is non-renormalisable and one must fix the momentum cut-off at some large (but finite) scale $\Lambda$, it is well-known to be largely equivalent to a gauge theory under the compositeness condition $Z_3 = 0$, where $Z_3$ is the wave function renormalisation constant of the gauge boson [4]. Under this condition, the gauge field becomes an auxiliary field without propagating degrees of freedom. The quantum fluctuations, then, give rise to its kinetic and self-interaction (for the non-Abelian case) terms, so that a dynamical gauge boson is finally induced as an independent composite field. However, in contrast to the belief advocated in the pioneering works [2], there appears a generic problem in turning the composite vector particles into genuine massless gauge bosons [5]. Actually, one must make a precise tuning of parameters, including a cancellation between terms of different orders in the $1/N$ expansion (where $N$ is the number of fermion species involved), in order to achieve the massless case needed.

In this note we would like to turn back to the role of Lorentz symmetry in a dynamical generation of gauge invariance. We argue that, generally, Lorentz invariance can only be imposed in the sense that all Lorentz non-invariant effects caused by its spontaneous breakdown are physically unobservable. We show here that the physical non-observability of the spontaneous breakdown of Lorentz symmetry (SBLS), taken as a basic principle, leads to genuine gauge invariant theories in both Abelian (Section 2) and non-Abelian (Section 3) cases, even though one starts from an arbitrary relativistically invariant
Lagrangian. In the original Lagrangian, the vector fields are taken as massive and all possible kinetic and interaction terms are included. However, when SBLS occurs and its non-observability is imposed, the vector bosons become massless and the only surviving interaction terms are those allowed by the corresponding gauge symmetry. Thus, the Lorentz symmetry breaking does not manifest itself in any physical way, due to the generated gauge symmetry converting the SBLS into gauge degrees of freedom of the massless vector bosons. Remarkably, even global symmetries are not required in the original Lagrangian - the SBLS induces them automatically.

A simple heuristic model for the SBLS, based on massive vector and real scalar field interactions, is considered (Section 4) and it is found that spontaneously broken gauge symmetries could also appear when SBLS occurs and is required to be physically unobservable.

Finally we summarise our conclusions in Section 5.

2 Abelian gauge invariance

The would-be Abelian model in the simplest case corresponds to massive electrodynamics including one charged fermion species. The Lagrangian density for the model is given by:

\[ L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} M^2 A_\mu^2 + \bar{\psi} \gamma_\mu \partial_\mu \psi - m \bar{\psi} \psi + e A_\mu \bar{\psi} \gamma_\mu \psi \]  

where \( e \) is the appropriate (would-be gauge) coupling constant of the vector field \( A_\mu \) with the charged fermion field. In this paper we shall assume that all the vector fields describe pure spin-1 fields satisfying the Lorentz condition:

\[ \partial_\mu A_\mu = 0 \]  

We impose this condition as an off-shell constraint, singling out a genuine spin-1 component in the four-vector \( A_\mu \) independent of the equations of motion. One can readily see that the Lagrangian density possesses a \( U(1)^\psi \) global symmetry, with the corresponding conserved Noether current \( j_\mu = \bar{\psi} \gamma_\mu \psi \) which, in this simple case, is closely connected to the current \( j^A_\mu \) associated with \( A_\mu \),

\[ j^A_\mu = \frac{\partial L}{\partial A_\mu} = e j_\mu + M^2 A_\mu \]
It is then clear that the $j^A_\mu$ current is also conserved, provided that the vector field satisfies the Lorentz condition (2) and preserves its transversality.

Let us consider now the SBLS in some detail. We propose that the vector field $A_\mu$ takes the form

$$A_\mu = a_\mu(x) + n_\mu$$

when the SBLS occurs. Here the constant Lorentz four-vector $n_\mu$ is a classical background field appearing when the vector field $A_\mu$ develops a VEV. We do not yet specify the mechanism which could induce the SBLS of type (4)—rather we study its general consequences for the possible dynamics of the matter and vector (gauge) fields and require it to be physically unobservable.

Substitution of the form (4) into the Lagrangian (1) immediately shows that the kinetic term for the vector field $A_\mu$ translates into a kinetic term for $a_\mu$ ($F^{(A)}_{\mu\nu} = F^{(a)}_{\mu\nu}$), while its mass and interaction terms are correspondingly changed:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2(a_\mu + n_\mu)^2 + i\bar{\psi}\gamma_\mu\partial_\mu\psi - m\bar{\psi}\psi + e(a_\mu + n_\mu)\cdot \bar{\psi}\gamma_\mu\psi \quad (5)$$

As to the interaction term, one can always go by a unitary transformation\footnote{In momentum representation this transformation corresponds to displacing the momentum of each fermion by an amount $en_\mu$. In the presence of terms breaking the $U(1)^\psi$ global symmetry, this transformation would induce a breaking of momentum conservation in processes where the $U(1)^\psi$ charge is not conserved. This non-conservation of momentum would also break Lorentz symmetry, whose breaking we require to be unobservable, and so it is necessary here to have $U(1)^\psi$ charge conservation.} to a new fermion field $\Psi$

$$\psi = \exp[i\omega(x)] \Psi, \quad \omega(x) = n \cdot x \quad (6)$$

so as to exactly cancel the Lorentz symmetry-breaking term $n_\mu \cdot \bar{\psi}\gamma_\mu\psi$ in the Lagrangian (3). This cancellation occurs due to the appearance of a compensating term from the fermion kinetic term, provided that the phase function $\omega(x)$ is chosen to be linear in the coordinate four-vector $x_\mu$ (as indicated in Eq. (4)). Thus, the only trace left of SBLS is in the mass term

\footnote{A gauge function of this type was first considered in the framework of quantum electrodynamics \cite{6}. We use this form for $\omega$ here just for simplicity and convenience. In the general case, when the shift in the vector field depends on space-time in the form $A_\mu = a_\mu(x) + n_\mu f(n \cdot x)$ where $f$ is any integrable function of $y = n \cdot x = n_\mu x_\mu$, the gauge function $\omega$ is given by the integral $\omega = \int f(y)dy$. In the non-Abelian case (see Section}
for the vector field which contains the scalar product \((n \cdot a)\) and the physical non-observability of SBLS requires it to be set equal to zero:

\[
M^2(n \cdot a) = 0
\]

(7)

An extra gauge condition \(n \cdot a \equiv n_\mu a_\mu = 0\) would be incompatible with the Lorentz gauge (2), which has already been imposed for the massive vector field \(a_\mu\). Therefore, the only way to satisfy Eq. (7) is to take \(M^2 = 0\). Otherwise Lorentz symmetry is explicitly broken. In such a way all other terms but the gauge invariant ones (for which “compensating” local transformations of the type (6) are always available) can be excluded by our non-observability assumption.

Let, for example, the original Lagrangian include a term of the type

\[
\Delta L_1 = \frac{f}{4} A^2_\mu \cdot A^2_\nu
\]

(8)

which would cause non-conservation of the current

\[
j_\mu^A = c j_\mu + M^2 A_\mu + f A_\mu \cdot A^2_\nu
\]

(9)

related with the vector field \(A_\mu\). The condition (7) for the non-observability of SBLS now becomes

\[
[M^2 + f(a^2 + (n \cdot a) + n^2)](n \cdot a) = 0
\]

(10)

Again, this condition must be fulfilled identically, i.e. \(M^2 = 0\) and \(f = 0\). Otherwise it would either represent another supplementary condition on \(a_\mu\), in addition to the Lorentz gauge already taken (4), or it would impose another dynamical equation in addition to the usual Euler equation for the vector field \(a_\mu\). Hence physical Lorentz invariance requires the survival of \(j_\mu^A\) current conservation.

So far we have considered (a) the case of massive electrodynamics where the current \(j_\mu^A\) related to the vector field is conserved, and (b) the case where \(j_\mu^A\) would not be conserved due to the vector field self-interaction term (8). 

3) such a shift could have the general factorised form \(A_\mu^i = a_\mu^i(x) + n_\mu f^i(n \cdot x)\), with the gauge function \(\omega^i\) given by the integral \(\omega^i = \int f^i(y)dy\). The corresponding vacuum states would of course formally break translational invariance, as well as Lorentz invariance.
while the Noether current $j_\mu = \overline{\psi} \gamma_\mu \psi$ is still conserved. Let us now include in the Lagrangian (11) terms which break even global $U(1)^\psi$ invariance:

$$\Delta L_2 = \frac{G}{2} A_\mu \overline{\psi} C \gamma_\mu \gamma_5 \psi + \frac{G^*}{2} A_\mu \overline{\psi} \gamma_\mu \gamma_5 \psi_C$$

where $\psi_C$ is a charge-conjugated spinor, $\psi_C = C \overline{\psi}$, while $G$ and its complex conjugate $G^*$ stand for coupling constants (the term $A_\mu \overline{\psi} C \gamma_\mu \psi$ and its Hermitian conjugate are absent since they are identically equal to zero due to Fermi statistics). So, in writing down the Lagrangian (1) with the addition of $\Delta L_2$, we do not impose beforehand any restrictions connected with conservation of fermion number. Now, in distinct contrast to the ordinary vector field-current interaction in Lagrangian (1), no transformation of the type (6) is available for the fermion field $\psi$ which could eliminate the trace of SBLS in $\Delta L_2$ (11). The Lorentz invariance condition (10), extended now so as to include the SBLS terms from $\Delta L_2$, again requires the identical vanishing of the vector boson mass $M^2 = 0$ and coupling constants, $f = 0$ and $G = 0$. Otherwise there would be fewer degrees of freedom for the vector and fermion fields, $a_\mu$ and $\psi$, than is needed for describing the spins 1 and 1/2, respectively, which is inadmissible.

Proceeding in such a way with all possible interactions (including those with other fermion and scalar fields), one finally arrives at the gauge invariant Abelian theory as the only version of the theory which is compatible with physical Lorentz invariance when SBLS occurs. One may, of course, wonder whether it is at all possible for the lowest energy (vacuum) state to have SBLS. It is indeed possible, by introducing an $\frac{f}{2} A_\mu A_\mu$ term (8), to arrange that the $A_\mu$ field potential energy density term is minimised for $n_\mu = -M^2 f$. However we have seen that the non-observability of SBLS requires $M^2 = f = ...$

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*footnote*:

If we extend the SBLS non-observability assumption to the much stronger form required in the next section to derive a non-Abelian gauge symmetry, it is not necessary to impose the the Lorentz condition (2) as an off-shell constraint on the four-vector field $A_\mu$. The stronger form of the SBLS non-observability assumption requires the condition (8) or (11) to be satisfied for any vector $n_\mu$. In this case, the condition $n \cdot a = n_\mu \cdot a_\mu = 0$ would require the vector field to vanish identically, $a_\mu = 0$, which is also inadmissible. When the Lorentz condition (2) is dropped, there is an extra allowed contribution $\beta \cdot (\partial_\mu A_\mu)^2$ to the kinetic term for the vector field; however the SBLS non-observability assumption would require it to vanish ($\beta = 0$), if the vacuum vector field took the general space-time dependent form considered in footnote 2.
0, in which case the (vanishing) vector field potential obviously has many flat directions and the energy density for the SBLS and Lorentz invariant vacua happen to be the same. This means that the SBLS vacuum is quite possible.

We will extend this discussion to include scalar fields and the spontaneous breakdown of the gauge symmetry in section 4.

3 Non-Abelian gauge invariance

Let us come now to the many-vector field case which results in the non-Abelian gauge symmetry. We suppose there are a number of pure spin-1 vector fields of the type $A_{\mu}(x)$, satisfying the Lorentz gauge condition (2), considered in the previous section. Proposing not even a global symmetry at the start, one can simply collect them in some set $A_{\mu}^{i}(x)$ with $i = 1,...N$. The matter fields, say fermions again as in the Abelian case, are collected in another set $\psi = (\psi^{(1)},...,\psi^{(r)})$. The general Lagrangian describing all their interactions with dimensionless coupling constants (some exotic terms violating fermion number and parity are omitted for simplicity) is given by:

$$L = -\frac{1}{4}F_{\mu\nu}^{i}F_{\mu\nu}^{i} + \frac{1}{2}(M^2)_{ij}A_{\mu}^{i}A_{\mu}^{j} + \alpha^{ijk}\partial_{\nu}A_{\mu}^{i}A_{\mu}^{j}A_{\mu}^{k} + \beta^{ijkl}A_{\mu}^{i}A_{\nu}^{j}A_{\mu}^{k}A_{\nu}^{l} +$$
$$+ i\bar{\psi}\gamma^{\mu}\psi - \bar{\psi}m\psi + \bar{A}_{\mu}^{i}\psi^{\gamma}_{\mu}T^{i}\psi$$

Here $F_{\mu\nu}^{i} = \partial_{\mu}A_{\nu}^{i} - \partial_{\nu}A_{\mu}^{i}$, while $(M^2)_{ij}$ is a general $N \times N$ mass-matrix for the vector fields and $\alpha^{ijk}$ and $\beta^{ijkl}$ are their coupling constants—all as yet unknown numbers. The $r \times r$ matrices $m$ and $T^{i}$ contain, in a compact form, the still arbitrary fermion masses and coupling constants describing the interaction between the fermions and the vector fields (all the numbers mentioned are real and the matrices Hermitian, as follows in this case from the Hermiticity of the Lagrangian).

We assume now that the vector fields $A_{\mu}^{i}$ each take the form:

$$A_{\mu}^{i}(x) = a_{\mu}^{i}(x) + n_{\mu}^{i}$$

when SBLS occurs; here the independent constant Lorentz four-vectors $n_{\mu}^{i}$ ($i = 1,...N$) are the VEVs of the vector fields, analogous to the $n_{\mu}$ of the Abelian case. However, in contrast to the Abelian case where the constant
vector \( n_\mu \) in the form (4) may have any length, we consider here at first just infinitesimally small \( n_\mu^i \) four vectors (for the generalization to finite vectors \( n_\mu^i \) see below). Furthermore we require the non-observability of the SBLS for any set \( \{n_\mu^i\} \) of infinitesimal vectors \( n_\mu^i \).

Substitution of the form (13) into the Lagrangian (12) shows, again as in the Abelian case, that the kinetic term for the vector fields \( A_\mu \) translates into a kinetic term for the vector fields \( a_\mu \) \( (F(A)_\mu^i = F(a)_\mu^i) \), while their mass and interaction terms are correspondingly changed:

\[
L = -\frac{1}{4}F^\mu_\nu F^\nu_\mu + \frac{1}{2}(M^2)_{ij}(a^i_\mu + n^i_\mu)(a^j_\mu + n^j_\mu) \\
+ \alpha^{ijk}\partial_\nu a^i_\mu \cdot (a^j_\mu + n^j_\mu)(a^k_\nu + n^k_\nu) + \\
+ \beta^{ijkl}(a^i_\mu + n^i_\mu)(a^j_\nu + n^j_\nu)(a^k_\mu + n^k_\mu)(a^l_\nu + n^l_\nu) + \\
i\bar{\psi}\gamma^\nu\partial_\nu\psi - \bar{\psi}m\psi + (a^i_\mu + n^i_\mu)\bar{\psi}\gamma_\mu T^i\psi
\]

The non-observability of the SBLS for any set of infinitesimal vectors \( n_\mu^i \) requires, as in the Abelian case, exact cancellations between non-Lorentz invariant terms of the same structure in the Lagrangian (14). One can first introduce a new set of vector fields \( a_\mu \) defined by the infinitesimal transformation

\[
a^i_\mu = a^i_\mu - \alpha^{ijk}\omega^j(x)a^k_\mu, \quad \omega^i(x) = n^i_\mu \cdot x_\mu
\]

which includes the above coupling constants \( \alpha^{ijk} \) and the linear “gauge” functions \( \omega^i(x) \). The condition that the Lorentz symmetry-breaking terms in the trilinear and quadrilinear self-interaction couplings of the vector fields \( a_\mu \), including those which arise from the kinetic term for the vector fields, should cancel for any infinitesimal vector \( n_\mu^i \) is then satisfied, if and only if the sets of real numbers \( \alpha^{ijk} \) and \( \beta^{ijkl} \) in these couplings satisfy the following conditions (a) and (b):

(a) \( \alpha^{ijk} \) is totally antisymmetric (in the indices \( i, j \) and \( k \)) and obeys the structure relations:

\[\text{As will be motivated in the conclusion, we are here using the very strong assumption that the SBLS shall be physically unobservable whatever the infinitesimal VEVs \( n_\mu^i \) would be. This means we are assuming that Nature hides the SBLS not only for the actually realised vacuum but also for the a priori possible vacua.}\]
\[
\alpha^{ijk} \equiv \alpha[^{ijk} \equiv \alpha[^{ijk} \equiv \alpha^{i[jk]} \ , \quad [\alpha^i, \alpha^j] = -\alpha^{ijk} \alpha^k
\]  
\]

where the \( \alpha^i \) are defined as matrices with elements \( (\alpha^i)^{jk} = \alpha^{ijk} \). This means that the matrices \( \alpha^k \) form the adjoint representation of a Lie algebra, under which the vector fields transform infinitesimally as given in Eq. (15). In the case when the matrices \( \alpha^i \) can be decomposed into a block diagonal form, there appears a product of symmetry groups rather than a single simple group. \(^5\)

(b) \( \beta^{ijkl} \) takes the factorised form

\[
\beta^{ijkl} = -\frac{1}{4} \alpha^{ijm} \cdot \alpha^{klm}
\]  

The above requirements (a) and (b) would of course also be fulfilled if the coupling terms \( \alpha^{ijk} \) were zero rather than antisymmetric structure constants, as is required to make the theory a true Yang-Mills theory. We would however like to argue that it is most natural and most likely that the gauge theory derived will be truly non-Abelian. Really it should be sufficient to just remark that there are many more possibilities for interacting Yang-Mills theories than for vector fields without self-interactions.

Let us turn now to the mass term for the vector fields in the Lagrangian (14). When expressed in terms of the transformed vector fields \( a^i_\mu \) (15), it contains should-be vanishing SBLS remnants of the type

\[
(M^2)_{ij}(\alpha^{ikl} \omega^k a^i_\mu a^j_\nu + a^i_\mu n^j_\nu) = 0
\]  

Here we have used the symmetry feature \( (M^2)_{ij} = (M^2)_{ji} \) for a real Hermitian matrix \( M^2 \) and have retained only the first-order terms in \( n^i_\mu \). These two types of remnant have different structures and hence must vanish independently. One can readily see that, in view of the antisymmetry of the structure constants, the first term in Eq. (18) may be written in the following form containing the commutator of the matrices \( M^2 \) and \( \alpha^k \)

\[
[M^2, \alpha^k]_\mu \omega^k a^i_\mu a^j_\mu = 0
\]  

It follows that the mass matrix \( M^2 \) should commute with all the matrices \( \alpha^k \), in order to satisfy Eq. (19) for all sets of “gauge” functions \( \omega^i = n^i_\mu \cdot x_\mu \). That

\(^5\)For simplicity we shall consider the case of a single simple group in the following.
means according to Schur’s lemma that, since the matrices $\alpha^k$ have been shown to form an irreducible representation of a (simple) Lie algebra, the matrix $M^2$ is a multiple of the identity matrix, $(M^2)_{ij} = M^2\delta_{ij}$, thus giving the same mass for all the vector fields. It then follows that the vanishing of the second term in Eq. (18) leads to the simple condition analogous to that (7) in the Abelian case:

$$M^2(n^i \cdot a^i) = 0$$

(20)

for any infinitesimal $n^i\mu$. Since the Lorentz gauge condition has already been imposed on $a^i\mu$ ($\partial_{\mu}a^i_{\mu} = 0$), we cannot impose extra gauge conditions of the type $n^i \cdot a^i = n^i\mu \cdot a^i_{\mu} = 0$. Thus, we are necessarily led to:

(c) masslessness of the vector fields, $(M^2)_{ij} = M^2\delta_{ij} = 0$.

Finally we consider the interaction term between the vector and fermion fields in the Lagrangian (14). In terms of the transformed vector fields $a^i_{\mu}$ (15), it takes the form

$$\left(a_{\mu}^i - \alpha^{ijk} \omega^j a^k_{\mu} + n_{\mu}^i \cdot \Psi \gamma^\mu T^i \Psi \right)$$

(21)

One now readily confirms that the Lorentz symmetry-breaking terms (the second and third ones) can be eliminated, when one goes to a new set of fermion fields $\Psi$ using a unitary transformation of the type:

$$\psi = \exp[iT^i\omega^i(x)]\Psi, \quad \omega^i(x) = n^i \cdot x$$

(22)

A compensating term appears from the fermion kinetic term and the compensation occurs for any set of “gauge” functions $\omega^i(x)$ if and only if:

(d) the matrices $T^i$ form a representation of the Lie algebra with structure constants $\alpha^{ijk}$

$$[T^i, T^j] = i\alpha^{ijk}T^k$$

(23)

In general this will be a reducible representation but, for simplicity, we shall take it to be irreducible here. This means that the matter fermions $\Psi$ are all assigned to an irreducible multiplet determined by the matrices $T^i$. At the same time, the unitary transformation (22) changes the mass term for the fermions to

$$\Psi(m + i\omega^k[m, T^k])\Psi$$

(24)
The vanishing of the Lorentz non-invariant term (the second one) in Eq. (24) for any set of “gauge” functions $\omega^i(x)$ requires that the matrix $m$ should commute with all the matrices $T^k$. According to Schur’s lemma, this again means that the matrix $m$ is proportional to the identity, $m_{rs} = m \delta_{rs}$, thus giving:

(e) the same mass for all the matter fermion fields within the irreducible multiplet determined by the matrices $T^i$; in the case when the fermions are decomposed into several irreducible multiplets, their masses are necessarily equal only within each multiplet.

The above argument for gauge symmetry as a consequence of the non-observability of SBLS also works in the absence of matter fields. However, if fermions are present, there is a technically shorter derivation of all the above conclusions. Firstly, one can transform to the new sets of vector and fermion fields according to Eqs. (15) and (22) and, as in the Abelian case, consider the vector field and fermion-current interaction terms (21). After compensation of all the linear (under $n^{i\mu}_i$) SBLS terms, one is led to the commutator (23) for the fermion current matrices $T^i$. Then the standard Jacobi identity for the $T$ matrices immediately leads to the basic structure relation (16) for the $\alpha^{ijk}$, whose antisymmetry with respect to indices $i$ and $j$ follows from the commutator (23) by itself. Its antisymmetry under indices $j$ and $k$ stems from the required cancellation of the linear SBLS terms (containing a vector-field derivative) in the trilinear self-interaction couplings in the Lagrangian (14) with the terms that appear from the kinetic term for the vector fields. The cancellation of the linear SBLS terms (containing no vector-field derivative) in the trilinear and quadrilinear vector field couplings in the Lagrangian results in the relation (17) for their coupling constants. Finally, the conclusions (c) and (e) for the masses of the vector and fermion fields are derived.

Now, collecting together all the outcomes (a)-(e) derived from the non-observability of the SBLS for any set of infinitesimal vectors $n^{i\mu}_i$ applied to the general Lagrangian (12), we arrive at a truly gauge invariant Yang-Mills theory for the new fields $a^{i\mu}_i$ and $\Psi$:

$$L_{YM} = -\frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu} + i \overline{\Psi} \gamma \partial \Psi - m \overline{\Psi} \Psi + g a^{i\mu}_i \overline{\Psi} \gamma_{\mu} T^i \Psi$$ (25)

Here $F_{\mu\nu}^i = \partial_\mu a^i_\nu - \partial_\nu a^i_\mu + ga^{ijk}_a a^{j\mu}_a a^{k\nu}_a$ and $g$ is a universal gauge coupling constant extracted from the corresponding matrices $\alpha^{ijk} = g a^{ijk}$ and $T^i = g T^i$. 10
Let us now consider the generalisation of the vector field VEVs from infinitesimal to finite background classical fields $n^i_\mu$. Unfortunately one cannot directly generalise the SBLS form (13) to all finite $n^i_\mu$ vectors. Due to the non-cancellation of the high-order $n^i_\mu$ terms, one would inevitably come to an Abelian rather than a non-Abelian symmetry, when applying the principle of the SBLS non-observability to the Lagrangian (14). Otherwise, one has a non-vanishing field strength $F^a_{\mu\nu}$ in the vacuum, implying a real physical breakdown of Lorentz symmetry. This problem can be automatically avoided if the finite SBLS shift vector $n^i_\mu$ in the basic equation (13) takes the factorised form

$$A^i_\mu(x) = a^i_\mu(x) + n_\mu \cdot f^i \tag{26}$$

where $n_\mu$ is a constant Lorentz vector as in the Abelian case, while $f^i$ ($i = 1, 2, ..., N$) is a vector in the internal charge space. So we now formulate the strong form of our SBLS non-observability assumption, needed to derive non-Abelian gauge invariance, as follows: the SBLS must remain hidden for any set of VEVs for the vector fields $A^i_\mu$ of the factorised form $n^i_\mu = n_\mu \cdot f^i \tag{26}$. Using the Lagrangian (24) derived for infinitesimal VEVs, it is now straightforward to show that there will be no observable effects of SBLS for any set of finite factorised VEVs $n^i_\mu = n_\mu \cdot f^i \tag{26}$. For this purpose, it is sufficient to generalise Eq. (13) to the finite transformation:

$$a_\mu \cdot \alpha = \exp[(\omega \cdot \alpha)]a_\mu \cdot \alpha \exp[-(\omega \cdot \alpha)] \tag{27}$$

Then Eqs. (13, 22, 27) combine to form a genuine finite gauge transformation for the Yang-Mills Lagrangian (24) and the SBLS is simply transformed away as a gauge degree of freedom.

If other fermion and scalar fields are included, the Lorentz invariance condition applied to their SBLS remnants will assign them to appropriate symmetry multiplets. Thus the gauge symmetry is readily extended to the Yukawa and Higgs sectors as well.

\footnote{Note that $\exp[(n \cdot x)(f \cdot \alpha)]\partial_\mu \exp[-(n \cdot x)(f \cdot \alpha)] = -n_\mu(f \cdot \alpha)$ and hence Eq. (26) correctly represents the derivative term in the gauge transformation for finite factorised VEVs $n^i_\mu = n_\mu \cdot f^i$.}
4 Spontaneously broken gauge invariance

So far our considerations have been quite general and model-independent, as they concerned the impact of the SBLS upon the possible dynamics of the matter and vector (gauge) fields. However, we have not yet explicitly discussed the role of scalar fields, particularly in the situation when these fields develop VEVs. An interesting question then arises: how does the SBLS interplay with the spontaneously broken internal symmetries? The presence of scalar fields may also provide a possible mechanism for inducing the appearance of non-zero classical fields \( n_{\mu} \) and \( n^i_{\mu} \), as the VEVs of the original vector fields \( A_{\mu} \) and \( A^i_{\mu} \). The simplest case to consider involves just vector and scalar field interactions.

In the Abelian case, with one vector and one real scalar field, the corresponding general Lagrangian density is given by:

\[
L = -\frac{1}{4} F_{\mu \nu} F_{\mu \nu} + \frac{h}{2} A^2_\mu \sigma^2 + \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{2} \mu^2 \sigma^2 - \frac{\lambda}{4} \sigma^4 + L'
\]  

(28)

Here we have divided the total Lagrangian into two parts: the explicitly written terms are, as we will see, compatible with Lorentz invariance, while the part \( L' \) includes all other terms having an independent structure, which break Lorentz invariance once the SBLS appears. These include a fundamental mass term for the vector field \( \frac{\mu^2}{2} A^2_\mu \), and a quadrilinear term \( A^2_\mu \cdot A^2_\mu \), both of which were considered in section 4. Also there are the interaction terms \( \partial_\mu A_\nu \cdot A_\mu A_\nu \), \( A_\mu \cdot \sigma \partial_\mu \sigma \), and all the odd-power couplings of the \( \sigma \) (which for simplicity we exclude, by requiring the reflection \( Z_2 \) symmetry \( \sigma \rightarrow -\sigma \)).

Let us consider first the possible vacuum configurations corresponding to the potential in \( L \) (28):

\[
U = -\frac{h}{2} A^2_\mu \sigma^2 + \frac{1}{2} \mu^2 \sigma^2 + \frac{\lambda}{4} \sigma^4 + U'
\]  

(29)

One can see that the explicitly written part of the potential \( U \) (29) drives the vector field \( A_\mu \) to have a VEV as large as possible—it is in fact unstable in the absence of a quadrilinear term \( A^2_\mu \cdot A^2_\mu \), or some other stabilizing term (see below). For the moment, let us ignore \( U' \) and this instability and take the VEV of \( A_\mu \) as some given parameter. Variation of the potential \( U \) with respect to \( \sigma \) yields one equation between the VEVs \( n_\mu \) and \( v \) of the fields (for \( v \neq 0 \):
\[ A_\mu = a_\mu + n_\mu, \quad \sigma = \rho + v, \quad h n_\mu^2 = \lambda \rho^2 + \mu^2 \] (30)

This leads to a non-trivial minimum of the potential \( U \)

\[ U_{\text{min}} = -\frac{1}{4\lambda} (h n_\mu^2 - \mu^2)^2 \] (31)

provided that \( h n_\mu^2 > \mu^2 \). The trivial minimum \( U_{\text{min}} = 0 \) with \( v = 0 \) corresponds to the case \( h n_\mu^2 \leq \mu^2 \). As mentioned above, the potential \( U \) is unbounded below and drives the VEV of \( A_\mu \) towards large values, \( h n_\mu^2 \to \infty \). So it is necessary to add a term to the potential \( U \) in order to cut-off this instability for some large (but finite) value of \( A_\mu^2 \). As a simple example, we could add a cut-off term \( U_{\text{cut-off}} = (A_\mu^2)^2 \theta \left( (A_\mu^2)^2 - \Lambda^4 \right) \) and consider the limit \( \Lambda^2 \to \infty \) at the end of the calculation, when Lorentz symmetry breaking effects from such a term become unobservable. Similar results can be obtained by including a large dimensional irrelevant term, such as \( U_{\text{cut-off}} = \frac{1}{6} (A_\mu^2)^3 \Lambda^2 \); however note that it is necessary to take \( \mu^2 \) (and \( v^2 \)) to be of order the cut-off scale \( \Lambda^2 \).

Now, apart from the standard symmetrical case \( (n_\mu = 0, v = 0) \), the following vacua are possible for the potential \( U \) (depending on the sign and value of \( n_\mu^2 \) and \( \mu^2 \)): an ordinary Goldstone-Higgs (GH) mode \( (n_\mu = 0, v \neq 0) \) which breaks the above reflection symmetry, the SBLS mode \( (n_\mu \neq 0, v = 0) \) and the combined SBLS-GH mode \( (n_\mu \neq 0, v \neq 0) \). When the cut-off term \( U_{\text{cut-off}} \) is added, one finds that the preferred vacuum configuration in the potential \( U \) corresponds to the case when the vector field \( A_\mu \) develops a VEV \( n_\mu \), having a large value \( |n_\mu^2| = \Lambda^2 \). The pure GH minimum is degenerate with the symmetrical one \( (U_{\text{min}} = 0) \) for \( \mu^2 > 0 \) or located at \( U_{\text{min}} = -\frac{\mu^4}{4\lambda} \) for \( \mu^2 < 0 \). This means that, provided \( h \Lambda^2 > |\mu^2| \), the combined SBLS-GH mode dominates over the other possible vacua.

After symmetry breaking (30), the potential \( U \) (29) takes the form

\[ U = -\frac{1}{2} (a_\mu + n_\mu)^2 (\rho + v)^2 + \frac{1}{2} \mu^2 (\rho + v)^2 + \frac{\lambda}{4} (\rho + v)^4 + U'(a_\mu + n_\mu, \rho + v) \] (32)

The pure SBLS mode \( (n_\mu \neq 0, v = 0) \) corresponds to the particular choice \( n_\mu^2 = \frac{\mu^2}{v} \) and is degenerate with the trivial symmetrical case with \( U_{\text{min}} = 0 \).
It can be seen from (32) that, due to the SBLS and the spontaneous breakdown of the reflection $Z_2$ symmetry\footnote{Another physical consequence is the formation of domain walls separating the vacua with $<\sigma> = +v$ and $<\sigma> = -v$.} $\sigma \rightarrow -\sigma$, the whole vector field $A_\mu = a_\mu + n_\mu$ has acquired a soft mass in addition to the fundamental mass term contained in $U'$. However, the situation drastically changes when $U'$, which simultaneously induces (as we already know from section 2) the physical breakdown of Lorentz symmetry, is required to vanish identically. Thereby, the potential (32) becomes nothing but the potential of spontaneously broken scalar electrodynamics. This can be converted into its well-known gauge invariant form by making an inverse Higgs transformation:

$$a_\mu \rightarrow a_\mu + \frac{1}{gv} \partial_\mu \phi, \quad \phi = \frac{\rho + v}{\sqrt{2}} e^{i\frac{\phi}{v}}, \quad \phi^* = \frac{\rho + v}{\sqrt{2}} e^{-i\frac{\phi}{v}} \quad (33)$$

So, one finally arrives at the standard QED Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + ig(a_\mu + n_\mu) (\phi^* \partial_\mu \phi - \partial_\mu \phi^* \phi) + g^2 (a_\mu + n_\mu)^2 \phi^* \phi + \partial_\mu \phi^* \partial_\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \quad (34)$$

of the massive charged scalars $\phi$ and $\phi^*$ with a gauge coupling constant $g = \sqrt{h}$. Going then by a unitary transformation to a new scalar field $\phi \rightarrow e^{i\alpha_{\rm H} x} \phi$, one can completely eliminate\footnote{The consequent spontaneous breakdown of momentum conservation, mentioned in footnote 1 and postulated to be unobservable since it also violates Lorentz symmetry, is in fact hidden by the global charge conservation manifest in the Lagrangian (34). This global charge conservation arose through the inverse Higgs transformation (33).} any dependence on the constant vector $n_\mu$ in the Lagrangian (34) thus excluding it from the VEV of the charged field $\phi$ (33) as well. This means that the predominant SBLS-GH mode, while being physical in the Lorentz non-invariant phase, turns into a false vacuum when Lorentz invariance is restored and gauge symmetry appears. Thus, any terms in the original Lagrangian (28) which have not evolved to a gauge invariant form (they are collected in $L'$) must definitely vanish—otherwise the SBLS-GH mode will dominate, inevitably inducing an explicit Lorentz non-invariance. The actual scenario for the physical vacuum depends only on the mass of the scalar field $\phi$. If $\mu^2$ is positive one has standard scalar electrodynamics with a massless photon, if it is negative
then a spontaneously broken gauge theory emerges. So, we come to the conclusion that spontaneously broken gauge symmetries could also appear when the SBLS happens, as a consequence of the non-observability of the Lorentz symmetry breakdown.

The emergence of the massive charged scalar $\phi$ (or neutral scalar $\rho$ in the case of the spontaneously broken gauge theory) is, as a matter of fact, the most direct prediction of the model considered. In the non-Abelian case, to which this model is straightforwardly extended, it follows from the generalisation of Eq. (33) that the scalars should belong (like the vector fields do) to the adjoint representation of the gauge symmetry group.

In the model of section 2 with a charged fermion field, the SBLS arose as a consequence of the existence of “flat directions”, meaning that from gauge symmetry the energy for a vacuum state with $A_\mu = 0$ is the same as for one with $A_\mu = n_\mu$. If the reader does not like this idea of getting SBLS as the result of a flat direction, the price to be paid is all the above scalars. Alternatively, instead of using fundamental vector fields $A_\mu$, one could consider them as being effective composite fields and then again ask the question from which we started in this section: what could induce the SBLS? Some possibilities may be related, for example, with a Lorentz-symmetry breaking fermion-antifermion pair condensate, as in the composite models mentioned earlier [2], rather than with the scalar condensate considered here. However, independent of its origin, if SBLS appears and is unobservable, it inevitably generates some gauge invariance, as was demonstrated in the previous sections.

5 Conclusion

We have shown that gauge invariant Abelian and non-Abelian theories can be obtained from the requirement of the physical non-observability of the SBLS rather than by using the Yang-Mills gauge principle.

Imposing the condition that the Lorentz symmetry breaking be unobservable of course restricts the values of the coupling constants and mass parameters in the Lagrangian density. These restrictions may naturally also depend on the direction and strength of the Lorentz symmetry breaking vector field vacuum expectation values (VEVs), whose effects are to be hidden. This allows us a choice as to how strong an assumption we make about
the non-observability requirement. Actually, in the Abelian case, we just assumed this non-observability for the physical vacuum that really appears (5), but we needed a stronger assumption in the non-Abelian case. A stronger assumption is needed because we believe that, for a given translational invariant but Lorentz symmetry breaking vacuum state, it is always possible to hide SBLS by only imposing restrictions on the couplings which lead to an Abelian gauge theory. We could ensure the derivation of a non-Abelian gauge symmetry by choosing a stronger version of the non-observability assumption: the SBLS is unobservable for all conceivable vacuum field configurations, including even those vacua that could never be realised. This is clearly a very strong assumption. Then we note that we let the coupling constants and mass parameters be restricted in a way which a priori depends on the vacuum from which we have to hide the SBLS. So, as the vacuum state itself depends on the values of these coupling constants and mass parameters, there is a back reaction problem unless one simply imposes that all possible vacua must have their SBLS hidden. Nevertheless this assumption looks to be too strong, in the sense that even Yang-Mills theories would in general not be able to hide the SBLS unless all the \( n_i^\mu \) commute. The reason for this is that, if the \( n_i^\mu \) did not commute, the Yang-Mills field tensor \( F_{\mu\nu}^a \) would not vanish in the vacuum and would break Lorentz symmetry spontaneously. If, for some combination of the Lorentz indices \( \mu, \nu \) we have some components \( F_{\mu\nu}^a \neq 0 \), there is no way to gauge that property away. Thus once the field tensor \( F_{\mu\nu}^a \) is non-zero, it is virtually impossible to gauge the SBLS away. We have therefore invented a milder assumption, which can be used for deriving non-Abelian gauge symmetry: the SBLS is unobservable in any vacuum for which the vector fields have VEVs of the factorised form \( n_i^\mu = n_\mu \cdot f^i \) (26). This factorised form is a special case in which the \( n_i^\mu \) commute with each other.

A working assumption in the present work is that the vector fields are taken to be pure spin-1 fields, satisfying the Lorentz condition (4). Due to this constraint \( \partial_\mu A_\mu = 0 \), there is a difficulty, a priori, in prescribing the functional measure \( D A_\mu \) over the vector fields in the path integral for the theory. Therefore a priori we have only worked in the classical approximation (although most of our arguments are also true quantum mechanically). In the Abelian case it is essentially obvious how to define the measure \( D A_\mu \). However, in the non-Abelian case, it would be necessary to introduce ghost fields as usual, in order to keep the gauge invariance beyond tree-level. We do
not address here the interesting and important question of how these ghost fields arise in our argumentation, when calculating quantum mechanically. At the same time, we recall from footnote 3 that the Lorentz condition (3) is not actually required, when we use the strong form of the SBLS non-observability assumption and space-time dependent vacuum vector fields.

Gauge transformations having parameters $\omega^i(x) = n^i \cdot x$ varying linearly with $x^\mu$ are important in our calculations, in as far as they induce changes in the VEVs of the vector fields. Such linear gauge functions $\omega^i(x)$ do not obey the boundary conditions of vanishing at infinity—not even their gradients vanish at infinity. However, due to the required commutation of the different components $n^i_\mu$ discussed above, we effectively work with a Cartan algebra. Since we are only using a Cartan algebra, the immediately needed large fields at infinity do not imply the large gauge transformations required to allow non-Abelian instantons (it is essential that the instantons are genuinely non-Abelian even at infinity). So the danger of introducing non-trivial topological effects is avoided.

There could of course be other (non-perturbative) sources of spontaneous breaking of Lorentz invariance, causing the field strengths $F^a_{\mu\nu}$ to acquire non-zero VEVs in Abelian or non-Abelian models. In this case, however, there would be no way to rescue the Lorentz invariance, even by having a gauge symmetry in the theory. Such a theory would be totally excluded by our main assumption of no observable Lorentz invariance breaking. So logically our strong requirement of the non-observability of SBLS excludes this possibility, which otherwise could easily happen. It was precisely to avoid this possibility that we introduced our assumption of factorised VEVs (26) in the non-Abelian case.

Remarkably, the application of the here proposed non-observability principle to the most general relativistically invariant Lagrangian density, with arbitrary couplings for all the fields involved, leads by itself to the appearance of a continuous symmetry in terms of conserved Noether currents and, what is more, to the massless vector field(s) gauging this symmetry. As a consequence, matrices constructed out of the coupling constants constitute representations of the Lie algebra corresponding to the above symmetry and (in the non-Abelian case) the coupling constants for the vector field self-interaction terms prove to be the structure constants for this algebra. Thus the vector fields are then a source of the symmetries, rather than local sym-
mertics being a source of the vector fields as in the usual formulation\(^1\). We considered a simple model for SBLS based on the interaction between a massive vector and a real scalar field and found that the gauge, or spontaneously broken gauge, symmetry phase has to appear. So, to conclude, when Lorentz symmetry spontaneously breaks through the appearance of constant background vector fields then, simultaneously, a stable gauge (or spontaneously broken gauge) symmetry phase is created in order to avoid a physical breakdown of Lorentz invariance.

It is difficult to confirm our SBLS hypothesis phenomenologically since, by assumption, there can be no non-Lorentz invariant evidence for it. However, the very fact that there are many well-established gauge symmetries in the Standard Model might be taken as a weak confirmation of the viewpoint in this paper. There could be some indirect phenomenological manifestation of this picture, related to the underlying mechanism responsible for the genesis of the SBLS, such as the existence of scalar or composite fermion states at higher scales. We leave this and other open questions for further investigation.

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\(^1\)In a similar manner, it was shown a long time ago\(^2\) that the massive Abelian or non-Abelian theories can be obtained from the requirement that the vector fields preserve their transversality\(^3\) in all their interactions. However, this requirement cannot lead to the massless vector field case, and for good reason.
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