1. INTRODUCTION

The study of the structure of compact stars requires the understanding of the equation of state describing the stellar matter under extreme conditions. It has been pointed out that most of the current equations of state describing quark matter are too soft and thus unable to explain the existence of massive neutron stars (Cottam et al. 2002; Özel 2006). Only stiff equations of state describing normal nuclear matter at high densities would be capable of explaining the stability of highly-compact star masses ($M \sim 2 M_\odot$). Thus, apparently these observational data tend to favor the existence of matter without deconfined quarks in the interior of neutron stars. Examples of massive compact stars are the isolated neutron star RX J1856.5−3754 (Trumper et al. 2008), some low-mass X-ray binaries, e.g., 4U 1636−536 (Barret et al. 2005), and the neutron star J0751+1807 (2.1 ± 0.2 $M_\odot$; Nice et al. 2005).

More recently, two possible pairs of mass and radius values have been attributed to neutron star EXO 1745−248, one of them centered around $M = 1.4 M_\odot$ with radius $R = 11$ km, and the other one centered around $M = 1.7 M_\odot$ and with a smaller radius $R = 9$ km (Özel et al. 2008). The neutron star in the low-mass X-ray binary 4U 1608−52 has mass determined to be $M \geq (1.84 \pm 0.009) M_\odot$ and radius $R \geq (9.83 \pm 1.24)$ km (Güver et al. 2010). Regarding the neutron star EXO 0748−676, the obtained lower limits on the mass and radius are: $M \geq (2.10 \pm 0.28) M_\odot$ and $R \geq (13.8 \pm 1.8)$ km (Özel 2006), even though these results are still unclear (Cottam et al. 2002).

It is believed that at high densities the strange quark matter is a more stable configuration than the ordinary nuclear matter (Bodmer 1971; Witten 1984), and hence it is claimed that neutron stars would be stellar objects entirely or partially composed of strange quark matter (Itoh 1970; Weber 2005), that is, a quark star or a hybrid star. However, some equations of state used in describing quark matter are too soft to support quark stars with large masses and thus, in this scenario, quark stars or hybrid stars seem to be incompatible with the observed massive neutron stars mentioned above. In fact, calculations using soft equations of state for quark matter provide values of maximum masses for hybrid neutron stars around 1.6 $M_\odot$, and lower values of maximum masses for strange quark stars.

Stiff equations of state of quark matter can be obtained when effects of strong interaction are taken into account (Alford et al. 2007). Calculations of compact star models using quark matter equations of state, generated by the modified MIT bag model (including perturbative corrections to QCD) or by the Nambu–Jona–Lasinio (NJL) model, are in general capable of reproducing a maximum star mass around 2 $M_\odot$, which is compatible with the observational data (Alford et al. 2005; Rodrigues et al. 2010).

A recent calculation for cold and dense QCD strange quark matter including corrections to order $\alpha_s^2$ indicates that massive compact stars with mass $\gtrsim 2 M_\odot$ up to maximal masses $\sim 2.75 M_\odot$ would be interpreted as possible candidates for strange quark stars, i.e., compact stars composed entirely of deconfined $u$, $d$, and $s$ quarks, and that compact stars with observed masses up to $\sim 2 M_\odot$ would be identified with hybrid stars, neutron stars with a central core made up of deconfined quark matter (Kurkela 2010). Motivated by these recent investigations, in this work we study the color superconducting quark matter by using a phenomenological model, namely, the density-dependent quark mass model (Chakrabarty et al. 1989; Chakrabarty 1991), in order to describe the strange quark star structure.

Assuming the color–flavor locked (CFL) phase is the ground state of strange quark matter, we consider the effects of the CFL gap energy on the global strange quark star structure.
The density-dependent quark mass model provides stiff equations of state, and hence large quark star masses can be obtained with this type of equations of state, which are compatible with some of those observational data, if reasonable values for the equation of state parameters are used. Note that these results contrast with the mass-radius relationships predicted by equations of state constructed with descriptions of quark matter based on a simple version of the MIT bag model, as discussed in Oliveira et al. (2008).

The study of the structure of the nucleonic crusts is also presented, and our results for quark star mass versus radius are compared with the data presented recently by Özel in Özel (2006), Özel et al. (2008), and Güver et al. (2010).

This work is organized as follows. In Section 2, we describe the equation of state of the cold color superconducting strange quark matter. In Section 3, we study the equation of state of the unpaired quark matter (UQM) and discuss the color superconducting to UQM phases. In Section 4, we present and discuss our results obtained for bare quark stars structure and compare them to some values of mass and radii derived from observational data. In Section 5, we present some results for quark stars with nucleonic crusts. Conclusions and final remarks are presented in Section 6.

2. DESCRIPTION OF THE COLD COLOR SUPERCONDUCTING QUARK MATTER

Over the last few years, the study of the color superconducting phase in quark-gluon plasma (Alford et al. 1998, 1999a, 1999b; Alford 2004) has been attracting a great interest in discussing the possible states of quark matter. At the QCD perturbative regime the attractive quark interaction instabilities in the Fermi surface, producing a gap in the quasiparticle energy spectrum. The color and flavor symmetries of the three-flavor QCD are hence broken down, leading to the formation of pairs of quarks. (This mechanism is analogous to the electron pairing in the ordinary electric superconducting phenomenon.) The quark matter becomes a color superconductor, with an equal number of quark flavors scale inversely with the baryon number density, namely

\[ m_u = m_d = \frac{C}{3\rho_B} \quad \text{and} \quad m_s = m_{s0} + \frac{C}{3\rho_B}, \]

where \( C \) is interpreted as the constant energy density in the zero quark density limit, and is related to the QCD vacuum energy density present in the MIT bag model, and

\[ \rho_B = \frac{n_u + n_d + n_s}{3}, \]

is the baryon number density expressed in terms of the quark number densities \( n_u, n_d, \) and \( n_s \). The current mass of the strange quark, \( m_{s0} \), enters as a parameter of the model. The dependence of the quark masses on the baryon density mimics the quark interaction at different values of density, incorporating the effect of quark interactions in the model. Through this artifact, the density-dependent quark mass model recovers the asymptotic behavior of quark matter predicted by QCD at high densities, namely, the quark asymptotic freedom and the dynamical confinement of quarks for low-density regime as natural limit situations. Consequently, in this model quarks are dynamically massive, and thus chiral symmetry of the QCD Lagrangian is dynamically broken. Besides, as mentioned previously, it would be unrealistic adopting vanishing dynamical quark masses to describe quark matter for the range of densities currently accepted for the interior of compact stars (Schmitt 2010).

The pressure derived from the thermodynamic potential in Equation (1) thus reads

\[ P = -\Omega + B^*. \]
quark gas pressure and the thermodynamic potential, due to the dependence of the quark mass on the baryon density. The additional term \( B^* \) is similar to the QCD vacuum pressure present in the MIT bag model. The presence of this term enables a smooth model transition from the quark confinement regime to asymptotic freedom since its explicit form,

\[
B^* = \rho_B \left. \frac{\partial \Omega}{\partial \rho_B} \right|_{T=0, \mu},
\]

is a decreasing function of the baryon density.

We thus found the following explicit expression for the pressure:

\[
P = \sum_f \frac{3}{8\pi^2} m_f^4 \left[ f(x) + \frac{4}{m_f^2} \frac{C}{3\rho_B} g(x) \right] + \frac{3}{\pi^2} \left( v^3 \mu + \Delta^{2}_{\text{CFL}} \mu^2 \right),
\]

where the functions \( f(x) \) and \( g(x) \) are given by

\[
f(x) = \ln \left[ x + (x^2 + 1)^{1/2} \right] - x(x^2 + 1)^{1/2}(2x^2 + 1),
\]

and

\[
g(x) = \ln \left[ x + (x^2 + 1)^{1/2} \right] - x(x^2 + 1)^{1/2},
\]

with \( x \) defined by

\[
x = \frac{v}{m_f}.
\]

Finally, the energy density can be obtained from the thermodynamic relation

\[
\varepsilon = -P + \sum_f n_f \mu_f,
\]

from which we find

\[
\varepsilon_{\text{CFL}} = -P + 3\rho_B \mu.
\]

In order to complete the description of the CFL quark matter phase, we write the quark number density,

\[
n_f = \left. \frac{\partial \Omega}{\partial \mu} \right|_{\rho_B},
\]

which gives the baryon number density

\[
\rho_B = \frac{1}{\pi^2} \left( v^3 + 2\Delta^2_{\text{CFL}} \mu \right).
\]

The condensation energy of the Cooper pairs in the CFL phase given by Equation (2) (Alford et al. 1999a) requires the specification of the color superconducting gap. In order to make the present phenomenological approach compatible with microscopic treatments based on some results of QCD, here we use solutions provided by the NJL model with a local four-fermion interaction. A recent study in this direction discussing the solution of the gap equation is presented in Rajagopal & Wilczek (2001b), where it is given the following approximate gap expression:

\[
\Delta_{\text{CFL}} \approx 2\sqrt{\Lambda^2 - \mu^2} \exp \left( \frac{\Lambda^2 - 3\mu^2}{2\mu^2} \right) \exp \left( -\frac{\mu^2}{8G_D\mu^2} \right),
\]

where \( G_D = \eta G_S \) is the strength of the diquark pairing, with \( G_S \) being the quark–antiquark coupling constant, and \( \eta \) being a dimensionless parameter between 3/4, for the intermediate coupling strength, and one corresponding to the strong coupling between quarks (Ruster et al. 2005). In the above equation, we have introduced a quark chemical potential shift, \( \bar{\mu} = \mu - \mu^* \), where \( \mu^* \) establishes the quark chemical potential value for which the gap begins to take significant values.

At this point one can realize that the present calculation involves two parameters from the dynamical quark mass expression (\( C \) and \( m_{d0} \)) and the parameter \( \mu^* \) to adjust the gap in the density region of interest. The values of the first pair of parameters are chosen according to the stability criterion applied to the strange quark matter energy per baryon, which reads \( \epsilon/\rho_B \lesssim 930 \text{ MeV} \) at normal nuclear density \( \rho_B \), where 930 MeV is the energy per baryon in iron nuclei (Oliveira et al. 2008). The possible values of the parameter \( C \) lie in the interval 69.05 MeV fm\(^{-3} \lesssim C \lesssim 111.6 \text{ MeV fm}^{-3} \) and the strange quark mass lies in the interval 40 MeV \( \lesssim m_{d0} \lesssim 180 \text{ MeV} \) (Lugones 1994). The current masses of up and down quarks are set \( m_{u0} = m_{d0} = 0 \). For the free parameter \( \mu^* \), we have used the values 100, 150, 200, and 250 MeV to adjust the gap behavior properly in the range of densities covered by compact objects in analysis. Different combinations of the parameter values are considered in this work. In Table 1, we display the different sets of the model parameter values applied in this study. For the entire calculation we have fixed \( \Lambda = 603 \text{ MeV} \) and \( G_S \Lambda^2 = 1.822 \). We will consider only the case \( G_D = G_S \) for the diquark coupling strength.

In Figures 1 and 2, we show the dependence of the constituent quark masses and the color superconducting gap as functions of the quark chemical potential, for the cases A and D displayed in Table 1, covering four values of the free gap parameter \( \mu^* \). We can see that for \( \mu^* = 100 \text{ MeV} \), the color superconducting phase dominates even for low values of the quark chemical potential. However, for higher values of the parameter \( \mu^* \), the unpaired phase dominates for low values of the quark chemical potential, and thus we need to deal with a phase transition between the two phases. Similar behavior for the gap and the constituent masses is observed for the other sets of parameters given in Table 1. Consequently, it is necessary to analyze phase transitions between the color superconducting and UQM, as well as how sensitive the phase transition is to the parameter changes. We remark that the quark pairing lowers the constituent quark masses when compared to the values in the unpaired quark phase.

3. COLOR SUPERCONDUCTING TO UNPAIRED QUARK MATTER PHASE TRANSITION

For the UQM, the thermodynamic potential density reads

\[
\Omega_{\text{UQM}} = \sum_f \frac{3}{\pi^2} \int_0^{v_f} k^2 \left( \sqrt{k^2 + \mu^2} - \mu_f \right) dk,
\]

with \( f = \{u, d, s\} \) and \( v_f \) is the Fermi momentum of each quark flavor, which is given by

\[
v_f = \sqrt{\mu^2 - m_f^2}.
\]

The number density of each quark flavor is defined by

\[
n_f = -\left. \frac{\partial \Omega_{\text{UQM}}}{\partial \mu_f} \right|_{\rho_B},
\]

which provides explicitly

\[
n_f = \frac{1}{\pi^2} (\mu_f^2 - m_f^2)^{3/2}.
\]
Figure 1. Dependence of the quark masses and the color superconducting gap on the quark chemical potential for the models A1, A2, A3, and A4 given in Table 1. The thin dotted curves near the constituent quark masses curves represent the corresponding constituent quark masses for the unpaired quark phase.

Table 1

| $\mu^*$ | $C = 100, m_{\Delta 0} = 50$ | $C = 90, m_{\Delta 0} = 80$ | $C = 80, m_{\Delta 0} = 100$ | $C = 70, m_{\Delta 0} = 150$ |
|---------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 100     | A1                         | B1                          | C1                          | D1                          |
| 150     | A2                         | B2                          | C2                          | D2                          |
| 200     | A3                         | B3                          | C3                          | D3                          |
| 250     | A4                         | B4                          | C4                          | D4                          |

Notes. The density-dependent mass model parameters $C$ and $m_{\Delta 0}$ are in units of MeV fm$^{-3}$ and MeV, respectively. The free characteristic gap parameter $\mu^*$ is in units of MeV.

For degenerate and massless free electrons, the free energy density is given by

$$\Omega_e = -\frac{1}{12\pi^2} \mu_e^4,$$  \hspace{1cm} (23)

and thus the electron number density reads

$$n_e = \frac{1}{3\pi^2} \mu_e^3.$$  \hspace{1cm} (24)

In beta equilibrium, the chemical potentials of quarks and electrons must satisfy the following relationships:

$$\mu_d = m_u + \mu_e, \quad \mu_s = \mu_d.$$  \hspace{1cm} (25)
Figure 2. Dependence of the quark masses and the color superconducting gap on the quark chemical potential for the models D1, D2, D3, and D4 given in Table 1. The thin dotted curves near the constituent quark mass curves represent the corresponding constituent quark masses for the unpaired quark phase.

The condition of electric charge neutrality provides the equation

\[ \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0. \]  

(26)

Note that the existence of different quark masses makes it impossible for the UQM to satisfy Equations (25) and (26) simultaneously, without the presence of electrons in bulk.

In order to analyze the occurrence of the phase transition, the behavior of the quark matter pressure as a function of the quark chemical potential is depicted in Figure 3 for all sets of parameter combinations given in Table 1. For the class A models, we can identify subtle intersections of the pressure curves for cases A3 and A4 with the curve of the unpaired phase, which indicate the phase transitions between the two phases, occurring at \( \mu \approx 335 \text{ MeV} \) (case A3) and at \( \mu \approx 390 \text{ MeV} \) (case A4). For the set of parameters D2, D3, and D4, the phase transition between the two phases occurs at \( \mu \approx 333 \text{ MeV} \) (case D2), \( \mu \approx 370 \text{ MeV} \) (case D3), and at \( \mu \approx 413 \text{ MeV} \) (case D4).

For the last one, the phase transition occurs at a baryon density \( \rho \approx 5.5 \rho_0 \), with \( \rho_0 = 0.15 \text{ fm}^{-3} \) being the normal nuclear matter density. We remark that small values of the parameter \( \mu^* \) combined with the high values of the current strange quark mass provide stiffer equations of state, leading to more massive quark stars, as we will discuss latter.

4. QUARK STAR STRUCTURE

The calculation of the structure of the quark star models for given values of the central density is given by solving numerically the Tollman–Oppenheimer–Volkoff (TOV) equations (Tolman 1939; Oppenheimer & Volkoff 1939)

\[
\frac{dP}{dr} = -G \frac{m(r) \epsilon(P)}{(rc)^2} \left( 1 + \frac{P}{\epsilon} \right) \left( 1 + \frac{4 \pi r^3 P}{m(r)c^2} \right) \times \left( 1 - 2G \frac{m(r)}{rc^2} \right)^{-1},
\]  

(27)
and

\[ \frac{dm(r)}{dr} = 4\pi r^2 \varepsilon. \] 

(28)

The obtained results show that larger values of $\Delta_{CFL}$ and the current strange quark mass tend to give more massive and larger quark stars. With an appropriate choice of the gap parameter $\mu^*$, it is possible to obtain masses and radii compatible with the neutron stars calculated with equation of states widely used to nuclear and hadronic matter. Consequently, suitable selection of the quark matter equation of state may lead to similar observational results obtained in different constitutive theoretical frameworks to the dense stellar matter composition.

In the panel shown in Figure 4, we compare the limits of mass and radius given in Özel’s works with those of quark stars calculated in this work, considering only the regime of strong coupling with $G_D = G_S$. Our results have been obtained for all sets of values of the parameters given in Table 1. The corresponding quark star models can reach a mass of $2.25 M_\odot$ and radius 11.4 km, for $\mu^* = 250$ MeV, as shown in part-a of the figure, and up to the maximum value of $2.73 M_\odot$ with the radius 13.3 km, for $\mu^* = 100$ MeV, as shown in part-d of the figure. For the sake of comparison, we show in the same panel the lower limits on the mass and radius of the neutron star EXO 0748–676, with the values of mass and radius given by $M = 2.10 \pm 0.28 M_\odot$ and $R = 13.8 \pm 1.8$ km. The 1σ and 2σ error bars of the results obtained by Özel are shown (dashed bars). Özel has found two pairs of values for the mass and radius of the neutron star in EXO 1745–248, which are centered around $M = 1.4 M_\odot$ and $R = 11$ km or around $M = 1.7 M_\odot$ and $R = 9$ km. The 1σ and 2σ contours for its mass and radius are represented by the dark and soft gray regions, respectively. For the neutron star in 4U 1608–52, the obtained values are $M = 1.84 \pm 0.09 M_\odot$ and $R = 9.83 \pm 1.24$ km.

We can see that the obtained values of masses and radii for pure strange quark star models are in good agreement with those given by Özel in Özeli (2006) and Güver et al. (2010), and only in fair accordance with the present data obtained for the compact star EXO 1745–248, which correspond to the smallest radius of the set of compact stars shown in the figure. From the result obtained for this class of object we thus can infer that such types of small compact stars, with small radii, are not natural candidates for pure quark stars, as it happens for the other ones that are more massive and with larger radii. Nevertheless, small compact stars may be identified with hybrid stars, which are composed of a quark matter core surrounded by a hadronic envelope. The present quark matter equation of state could still be helpful in describing the quark matter in the core of these hybrid structures.

5. QUARK STAR WITH A NUCLEAR CRUST

The strange quark matter in the CFL phase is electrically neutral since the Cooper quark pairing minimizes the energy
if the quarks have equal Fermi momenta. In this situation, the
number of $u$, $d$, and $s$ quarks is equal and the bulk electric
charge is zero without the need for electrons in the medium.
However, even for strange quark matter in the CFL phase, when
surface effects are taken into account, the number of massive
quarks is reduced near the boundary of the star relative to the
number of massless quarks at fixed Fermi momenta values.
Consequently, there is a net quark charge at the very thin surface
of the system. The thickness of the charged surface layer of the
CFL quark star is $\sim 1$ fm, as it was discussed in Berger & Jaffe
(1987, 1991), Madsen (2001), and Usov (2004). The presence of
such strong electric fields near the surface of the CFL strange star
make possible the existence of nuclear crusts spatially separated
from the deconfined CFL quark core (Weber 2005).

Therefore, in order to investigate the change in the quark star
structure with the inclusion of a nuclear matter crust, we have
constructed an equation of state for the crust assuming that cold
stellar matter in this crust is composed of a nuclear lattice im-
merged in an electron gas. To describe this crust matter we have
used the equation of state given in Baym et al. (1971). Hence, we
can find equilibrium configurations compatible with the solution
of the TOV equations using these equations of state connected
to the CFL matter equation of state previously constructed.

In Figure 5, we depict the energy density as a function
of radius for two quark star models with the central density
$1.0 \times 10^{15}$ g cm$^{-3}$, for the set of parameters D1 and D4 in
Table 1. In the figure, the density is depicted as a function of
radius for two compact stars with dense and large cores. The one

corresponding to the set of parameters D4 ($\mu^* = 100$ MeV) is
composed entirely of deconfined quarks in the CFL phase, and

Figure 4. Mass–radius diagrams for the stable quark stars. The cases A1 to A4 given in Table 1 are shown in part-a; the cases B1 to B4, in part-b; the cases C1 to C4, in part-c; and the cases D1 to D4 in part-d. In all parts of the panel, the solid lines stand for the models A1, B1, C1, and D1 ($\mu^* = 100$ MeV), the long-dashed curves are for the models A2, B2, C2, and D2 ($\mu^* = 150$ MeV), the short-dashed curves represent the models A3, B3, C3, and D3 ($\mu^* = 200$ MeV), and the thin dotted curves represent the models A4, B4, C4, and D4 ($\mu^* = 250$ MeV). It is also shown the lower limits on the mass and radius of the neutron star EXO 0748–676, given by $M = 2.10 \pm 0.28 M_\odot$ and $R = 13.8 \pm 1.8$ km. The 1$\sigma$ and 2$\sigma$ error bars of the results obtained by Özel are also shown (dashed bars). Regarding the neutron star in EXO 1745–248, Özel has found two pairs of values for the mass and radius, which are centered around $M = 1.4 M_\odot$ and $R = 11$ km or around $M = 1.7 M_\odot$ and $R = 9$ km. The 1$\sigma$ and 2$\sigma$ contour lines for its mass and radius are represented by the two gray regions. For the neutron star in 4U 1608–x : 52, the obtained values are $M = 1.84 \pm 0.09 M_\odot$ and $R = 9.83 \pm 1.24$ km. The 1$\sigma$ and 2$\sigma$ error bars of the new results obtained by Özel are shown (solid bars). For all error bars in the

Figure 5. Energy density vs. radius of quark stars with central density $\rho_c = 1.0 \times 10^{15}$ g cm$^{-3}$ for the models D1 and D4 given in Table 1.
the other one, for the case A1 ($\mu^* = 250$ MeV), is composed of unpaired quarks. The obtained masses for these two compact stars are $2.69\, M_\odot$, for the first model, and $2.37\, M_\odot$, for the second one.

Figure 6 shows the detailed structure of the nuclear crusts enveloping the quark matter cores of the model compact stars shown in the previous figure. The thickness of these nuclear crusts is 196 m, for the model D1 ($\mu^* = 100$ MeV), and 231 m for the model D4 ($\mu^* = 250$ MeV).

In Figure 7, the energy density is displayed as a function of radius for two quark stars with the central density $\rho_c = 3.0 \times 10^{15}\, \text{g}\,\text{cm}^{-3}$, again using the set of parameters of the models D1 and D4. For the model D1, the obtained quark star has a large core composed entirely of deconfined quarks in the CFL phase, and for the model D4, the large and dense core is composed of the CFL quark phase enveloped by an unpaired quark shell. The obtained masses are $2.56\, M_\odot$, for the case D1, and $2.28\, M_\odot$, for the case D4. The detailed structures of the nuclear crusts enveloping the quark stars shown in the previous figure are displayed in Figure 8. The thickness of these nuclear crusts is 150 m, for the first one, and 169 m, for the second one.

The obtained results indicate that smaller values of $\mu^*$ correspond to more massive and less compact quark stars, almost entirely composed of quark matter. An interesting implication of the obtained results is that for increasing CFL gap energies one obtains smaller nuclear crusts. However, the CFL quark star will be naked only for a very unrealistic high value of the CFL gap energy.

Observe that the maximum density at the base of the nuclear crust which envelopes the quark star is determined by the threshold density for neutron drip ($\rho \sim 4.0 \times 10^{11}\, \text{g}\,\text{cm}^{-3}$). On the other hand, the equal values of pressure at the surface of the quark matter core and the base of the nuclear crust are necessary to assure the hydrostatic equilibrium of the quark star as a whole.

6. CONCLUSIONS

In this work, we have studied the properties of quark matter by applying the density-dependent quark mass model to the calculation of the equation of state of the color superconducting quark matter. Within this framework, we have investigated the quark star structure for suitable values of the model parameter C, the current mass of the quark $\bar{s}$, and the CFL gap energy, and so we have discussed the dependence of the observable properties of quark stars, such as maximum mass and radius, on the CFL gap energy.

In the present phenomenological analysis, we have used a color superconducting gap parameterization derived from microscopic approaches widely applied to describe color superconducting quark matter. The density-dependent quark mass model is also used in order to mimic the quark chiral symmetry behavior.

According to our results, the existence of the reported massive compact stars can be compatible with stable quark stars with a nuclear crust, or even bare quark stars.
The results presented here show that massive compact stars with mass $\gtrsim 2 M_\odot$ up to maximal masses $2.73 M_\odot$ would be interpreted as compact stars composed entirely of deconfined quarks, in the color superconducting phase or even in the unpaired phase. Moreover, less massive and smaller compact stars would be identified with hybrid stars composed of a deconfined quark phase in the inner core enveloped by a hadronic phase, spatially separated due to the gravitational action. This configuration is allowed since the burning of nuclear matter into quarks must be an exothermic process. This specific situation can occur for a range of densities, which are determined by the equations of state used to describe both phases, as shown in Lugones (1994, 1995).

As a final remark, we call attention to the fact that the observational data obtained for compact objects, which are possible candidates to be quark stars, could be used to constrain the parameters of phenomenological models involved in the quark matter equation construction.

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