Neutrino factories — Physics potential and present status

Osamu Yasuda
Department of Physics, Tokyo Metropolitan University
Minami-Osawa, Hachioji, Tokyo 192-0397, Japan
E-mail: yasuda@phys.metro-u.ac.jp

Abstract. I briefly review the recent status of research on physics potential of neutrino factories including the discussions on parameter degeneracy.

1. Introduction

The observation of atmospheric neutrinos (See, e.g., Ref.[1]) and solar neutrinos (See, e.g., Ref.[2, 3]) gives the information on the mass squared differences and the mixings, which can be written in the three flavor framework of neutrino oscillations as $(|\Delta m^2_{32}|, \theta_{23})$ and $(\Delta m^2_{21}, \theta_{12})$, where I have adopted the standard parametrization [4] for the $3 \times 3$ MNSP [5, 6] matrix.

On the other hand, the CHOOZ result [7] tells us that $|\theta_{13}|$ has to be small ($\sin^2 2\theta_{13} < \sim 0.1$). So the MNSP matrix looks like

$$U_{\text{MNSP}} \simeq \begin{pmatrix}
  c_\odot & s_\odot & \epsilon \\
 -s_\odot/\sqrt{2} & c_\odot/\sqrt{2} & 1/\sqrt{2} \\
 s_\odot/\sqrt{2} & -c_\odot/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix},$$

where I have used $\theta_{23} \simeq \pi/4$, $\sin^2 2\theta_{12} \equiv \sin^2 2\theta_\odot \simeq 0.8$ and $|\epsilon| \ll 1$. With the mass hierarchy $|\Delta m^2_{21}| < |\Delta m^2_{32}|$ there are two possible mass patterns which are depicted in Fig. 1, depending on whether $\Delta m^2_{32}$ is positive or negative.

The next thing one has to do in neutrino oscillation study is to determine $\theta_{13}$, the sign of $\Delta m^2_{32}$ and the CP phase $\delta$. Among others, measurements of CP violation [8, 9, 10, 11] is the final goal of the neutrino oscillation experiments. During the past few years a lot of research have been done on the possibilities of future long baseline experiments. One is a super-beam experiment ‡ and the other one is a neutrino factory [13]. § The former is super intense conventional $\nu_\mu$ (or $\bar{\nu}_\mu$) beam which is obtained from pion decays where $\nu_e$ created in oscillation $\nu_\mu \rightarrow \nu_e$ is measured. The latter is beam from muon decays in a storage ring and it consists of $\nu_\mu$ and $\bar{\nu}_e$ ($\bar{\nu}_\mu$ and $\nu_e$) in the case of $\mu^-$ ($\mu^+$) decays. To measure the oscillation probability $P(E_\nu) \equiv P(\nu_e \rightarrow \nu_\mu)$ or $P(E_{\bar{\nu}}) \equiv P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ at a neutrino factory, it is very important that “wrong sign muons” which are produced in oscillation $\bar{\nu}_e \rightarrow \bar{\nu}_\mu \rightarrow \mu^+$ (or $\nu_e \rightarrow \nu_\mu \rightarrow \mu^-$) can be distinguished from right sign muons which are produced from $\nu_\mu$ (or $\bar{\nu}_\mu$) without oscillation. The background fraction $f_B$ in the case of super-beams is of order $10^{-2}$ [12], while in the case of a neutrino factory $f_B$ is of order $10^{-5}$ [20]. The advantage of a neutrino factory is such low background fraction and neutrino factories are expected to enable us to determine $\theta_{13}$ and the sign of $\Delta m^2_{32}$ ($\delta$) for $\sin^2 2\theta_{13} \gtrsim 10^{-5}$ ($10^{-3}$), respectively.

‡ The most realistic super beam project for the moment is the JHF experiment [12].
§ For research on a neutrino factory in the past, see, e.g., [14, 15, 16, 17, 18, 19] and references therein.
In this talk I will discuss measurements of these quantities at neutrino factories. The numbers $N_{\text{wrong}}(\mu^\pm)$ of the wrong sign muons are given by [13]

\[ N_{\text{wrong}}(\mu^-) = n_T \frac{12E^2_\mu}{\pi L^2 m^2_\mu} \int d \frac{E_\nu}{E_\mu} \left( \frac{E_\nu}{E_\mu} \right)^2 \left( 1 - \frac{E_\nu}{E_\mu} \right) \sigma_{\nu N}(E_\nu) P(E_\nu) \]

\[ N_{\text{wrong}}(\mu^+) = n_T \frac{12E^2_\mu}{\pi L^2 m^2_\mu} \int d \frac{E_\nu}{E_\mu} \left( \frac{E_\bar{\nu}}{E_\mu} \right)^2 \left( 1 - \frac{E_\bar{\nu}}{E_\mu} \right) \sigma_{\bar{\nu} N}(E_{\bar{\nu}}) P(E_{\bar{\nu}}), \]

where $E_\mu$ is the muon energy, $L$ is the length of the neutrino path, $n_T$ is the number of the target nucleons, $\sigma_{\nu N}(E_\nu)$ and $\sigma_{\bar{\nu} N}(E_{\bar{\nu}})$ are the (anti-)neutrino nucleon cross sections given by

\[ \sigma_{\nu N}(E_\nu) = \left( \frac{E_\nu}{\text{GeV}} \right) \times 0.67 \times 10^{-38} \text{cm}^2 \]

\[ \sigma_{\bar{\nu} N}(E_{\bar{\nu}}) = \left( \frac{E_{\bar{\nu}}}{\text{GeV}} \right) \times 0.33 \times 10^{-38} \text{cm}^2, \]

and $P(E_\nu)$ and $P(E_{\bar{\nu}})$ are the oscillation probabilities $P(E_\nu) \equiv P(\nu_e \rightarrow \nu_\mu)$ and $P(E_{\bar{\nu}}) \equiv P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$.

\section*{2. The sign of $\Delta m^2_{32}$}

As was mentioned in the Introduction, the mass pattern corresponds to either Fig. 1(a) or 1(b), depending on whether $\Delta m^2_{32}$ is positive or negative. Determination of this mass pattern is important, since Figs. 1(a) and 1(b) correspond to one and two mass states, assuming that the lowest mass is almost zero. \[ \| \]

\| The mixed dark scenario in which neutrinos have masses of order 1 eV seems to be disfavored by cosmology [21, 22]. It has been reported that signals of neutrinoless double $\beta$ decay was found [23]. If the claim is correct, then there must be overall shift to the mass, since the mass range 0.05 eV – 0.84 eV is larger than $\sqrt{\Delta m^2_{\text{atm}}}$.\]
In the limit $\Delta m^2_{21} \to 0$ the oscillation probability essentially becomes that of two flavors and is given by (See, e.g., [24])

$$
\begin{align*}
\{ P(\nu_e \to \nu_\mu) \} &= \sin^2 \theta_{23} \sin^2 2\tilde{\theta}^{(\mp)}_{13} \sin^2 \left( \frac{\Delta E_{32}^{(\mp)} L}{2} \right),
\end{align*}
$$

where $A \equiv \sqrt{2} G_F N_e$ stands for the matter effect [25, 26] of the Earth, $\tilde{\theta}^{(\pm)}_{13}$ is the effective mixing angle in matter given by

$$
\tan 2\tilde{\theta}^{(\mp)}_{13} \equiv \frac{\Delta E_{32} \sin 2\theta_{13}}{\Delta E_{32} \cos 2\theta_{13} \pm A},
$$

and $\Delta E_{32} \equiv \Delta m^2_{32} / 2E \equiv (m_3^2 - m_2^2) / 2E$.

As can be seen from (1), if $\Delta m^2_{32} > 0$ then the effective mixing angle $\tilde{\theta}^{(\pm)}_{13}$ is enhanced and $P(\nu_e \to \nu_\mu)$ increases. On the other hand, if $\Delta m^2_{32} < 0$ then $\tilde{\theta}^{(\mp)}_{13}$ is enhanced and $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ increases. So, at neutrino factories where baseline is relatively large and therefore the matter effect plays an important role, the sign of $\Delta m^2_{32}$ may be determined by looking at the difference between neutrino and anti-neutrino events which should reflect the difference between $P(\nu_e \to \nu_\mu)$ and $P(\bar{\nu}_e \to \bar{\nu}_\mu)$.

Since the cross section $\sigma_{\nu N}$ and $\sigma_{\bar{\nu} N}$ are different (the ratio is 2 to 1), it is useful to look at the quantity

$$
\frac{N_\nu - 2N_\bar{\nu}}{\delta(N_\nu - 2N_\bar{\nu})} = \frac{N_\nu - 2N_\bar{\nu}}{\sqrt{N_\nu + 4N_\bar{\nu}}}
$$

whose absolute value should be much larger than one to demonstrate $\Delta m^2_{32} > 0$ or $\Delta m^2_{32} < 0$. Now let me introduce the quantity

$$
R \equiv \frac{[N_{\text{wrong}}(\mu^-) - 2N_{\text{wrong}}(\mu^+)]^2}{N_{\text{wrong}}(\mu^-) + 4N_{\text{wrong}}(\mu^+)},
$$

Figure 2. The contour plot of the ratio $R$ in (2) to distinguish $\text{sgn}(\Delta m^2_{32})$ for $\theta_{13} = 1^\circ$, $\theta_{13} = 9^\circ$, respectively [27].
If $R \gg 1$ then one can deduce the sign of $\Delta m^2_{32}$. The contour plot of $R=\text{const.}$ is given in Fig. 2(a) and 2(b) for typical values of $\theta_{13}$ with $\Delta m^2_{32} = 3.5 \times 10^{-3}\text{eV}^2 > 0$, $\sin^2 2\theta_{23} = 1.0$ [27]. If $\sin^2 2\theta_{13}$ is not smaller than $10^{-3}$, it is possible to determine the sign of $\Delta m^2_{32}$. Irrespective of the value of $\theta_{13}$, $L \sim 5000\text{km}$, $E_\mu = 50\text{ GeV}$ seem to optimize the signal, as far as the quantity $R$ is concerned. Similar results were obtained in [28, 29] by considering the ratio $N(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)/N(\nu_e \rightarrow \nu_\mu)$.

3. The magnitude of $\theta_{13}$

To establish non-zero value of $\theta_{13}$, the following quantity has to be large:

$$\Delta \chi^2 \equiv \sum_j \left[ \frac{N_j(\nu_e \rightarrow \nu_\mu; \sin^2 2\theta_{13}) - N_j(\nu_e \rightarrow \nu_\mu; \sin^2 2\theta_{13} = 0)}{N_j(\nu_e \rightarrow \nu_\mu; \sin^2 2\theta_{13}) + f_B N_j(\nu_e \rightarrow \nu_\mu; \sin^2 2\theta_{13})} \right]^2,$$

where $j$ stands for the label of energy bins and $N_j$ stands for the numbers of events. Taking realistic systematic errors and detection efficiencies into consideration, sensitivity of a neutrino factory to $\sin^2 2\theta_{13}$ was obtained in [30] for baselines $L=732\text{km}$, $3500\text{km}$, $7332\text{km}$ with the muon energy $E_\mu=50\text{GeV}$, the detector volume $40\text{kt}$, useful muon decays $2 \times 10^{20}\mu$’s, and is depicted in Fig. 3. Fig. 3 shows that neutrino factories have sensitivity to $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{13} \gtrsim 10^{-5}$. Such high sensitivity is possible because background fraction at neutrino factories is small ($\lesssim 10^{-5}$ [20]). The discussions in this section are devoted to establishment of non-zero value of $\theta_{13}$, and when it comes to determination of the precise value of $\theta_{13}$, one has to take into account the problem of parameter degeneracy, which will be discussed later.
4. Measurements of the CP phase $\delta$

4.1. Oscillation probabilities in vacuum and in matter

To discuss the dependence of oscillation probability on $\delta$, let me start with the oscillation probability in vacuum which is given by

$$\begin{align*}
\left\{ \begin{array}{c}
P(\nu_\alpha \to \nu_\beta) \\
P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)
\end{array} \right\} &= \delta_{\alpha\beta} - 4 \sum_{j<k} \text{Re} \left( U_{\alpha j} U_{\alpha \beta j}^* U_{\alpha k}^* U_{\beta k} \right) \sin^2 \left( \frac{\Delta E_{jk} L}{2} \right) \\
&\pm 2 \sum_{j<k} \text{Im} \left( U_{\alpha j} U_{\alpha \beta j}^* U_{\alpha k}^* U_{\beta k} \right) \sin (\Delta E_{jk} L).
\end{align*}$$

(3)

The CP violation in vacuum is given by

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 4 \sum_{j<k} \text{Im} \left( U_{\alpha j} U_{\alpha \beta j}^* U_{\alpha k}^* U_{\beta k} \right) \sin (\Delta E_{jk} L)$$

$$= 4 J \left[ \sin (\Delta E_{12} L) + \sin (\Delta E_{23} L) + \sin (\Delta E_{31} L) \right]$$

$$= -16 J \sin \left( \frac{\Delta E_{31} L}{2} \right) \sin \left( \frac{\Delta E_{32} L}{2} \right) \sin \left( \frac{\Delta E_{21} L}{2} \right),$$

where

$$J \equiv \text{Im} \left( U_{\alpha 1} U_{\alpha 1}^* U_{\alpha 2}^* U_{\beta 2} \right)$$

is the Jarlskog factor, and

$$\text{Im} \left( U_{\alpha 1} U_{\alpha 1}^* U_{\alpha 2}^* U_{\beta 2} \right) = \text{Im} \left( U_{\alpha 2} U_{\beta 2}^* U_{\beta 3}^* U_{\alpha 3} \right) = \text{Im} \left( U_{\alpha 3} U_{\beta 3}^* U_{\alpha 1}^* U_{\beta 1} \right),$$

which follows from the unitarity relations, has been used. This Jarlskog factor, which is written as

$$J = c_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \delta$$

in the standard parametrization [4], contains a small factor $\sin 2\theta_{13}$ which is constrained by the CHOOZ data ($\lesssim \sqrt{0.1}$). So the Jarlskog factor is expected to be small. In vacuum the asymmetry factor

$$A \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}$$

(4)

is a useful quantity to measure the CP phase $\delta$. In vacuum the CP violation happens to be the same as the $T$ violation $P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$.

On the other hand, in the presence of matter, the expression (3) for the probability is modified. The eigen matrix in matter can be formally diagonalized by a unitary matrix $\tilde{U}^{(\pm)}$:

$$U^{(\pm)} \text{ diag } (E_1, E_2, E_3) U^{(\pm)-1} \pm \text{ diag } (A, 0, 0) = \tilde{U}^{(\pm)} \text{ diag } (\tilde{E}_1^{(\pm)}, \tilde{E}_2^{(\pm)}, \tilde{E}_3^{(\pm)}) \tilde{U}^{(\pm)-1},$$

where the sign for the matter term $A \equiv \sqrt{2} G_F N_e$ is reversed and the complex conjugate $U^*$ of the unitary matrix is used instead of $U$ for antineutrinos $\bar{\nu}_\alpha$, so the unitary matrix $\tilde{U}^{(\pm)}$ and the eigenvalues $\tilde{E}_j^{(\pm)}$ for $\bar{\nu}_\alpha$ are different from $\tilde{U}^{(\pm)}$ and $\tilde{E}_j^{(\pm)}$ for neutrinos $\nu_\alpha$. Assuming constant density, the oscillation probability can be written as

$$\begin{align*}
\left\{ \begin{array}{c}
P(\nu_\alpha \to \nu_\beta) \\
P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)
\end{array} \right\} &= \delta_{\alpha\beta} - 4 \sum_{j<k} \text{Re} \left( \tilde{U}_{\alpha j}^{(\pm)} \tilde{U}_{\beta j}^{(\pm)} \tilde{U}_{\alpha k}^{(\pm)} \tilde{U}_{\beta k}^{(\pm)} \right) \sin^2 \left( \frac{\Delta \tilde{E}_{jk}^{(\pm)} L}{2} \right) \\
&\pm 2 \sum_{j<k} \text{Im} \left( \tilde{U}_{\alpha j}^{(\pm)} \tilde{U}_{\beta j}^{(\pm)} \tilde{U}_{\alpha k}^{(\pm)} \tilde{U}_{\beta k}^{(\pm)} \right) \sin \left( \Delta \tilde{E}_{jk}^{(\pm)} L \right),
\end{align*}$$

(5)
as in the case of the probability in vacuum, and $\Delta \tilde{E}_{jk}^{(+)} \equiv \tilde{E}_{j}^{(+)} - \tilde{E}_{k}^{(+)}$. Therefore the asymmetry factor $A$ in (4) is not illuminating to see the CP violation in matter, since it contains both terms which vanish in the limit $\delta \to 0$ and those which do not:

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$= -4 \sum_{j<k} \text{Re} \left[ \tilde{U}_{\alpha j}^{(-)} \tilde{U}_{\beta j}^{(-)*} \tilde{U}_{\alpha k}^{(-)} \tilde{U}_{\beta k}^{(-)*} \sin^2 \left( \frac{\Delta \tilde{E}_{jk}^{(-)} L}{2} \right) \right]$$

$$\quad + 2 \sum_{j<k} \text{Im} \left[ \tilde{U}_{\alpha j}^{(-)} \tilde{U}_{\beta j}^{(-)*} \tilde{U}_{\alpha k}^{(-)} \tilde{U}_{\beta k}^{(-)*} \sin \left( \Delta \tilde{E}_{jk}^{(-)} L \right) \right].$$

It has been pointed out [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52] that T violation is useful to probe the CP violating phase in the presence of matter. In fact from (5) I have T violation in matter:

$$\Delta P \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

$$= 4 \sum_{j<k} \text{Im} \left( \tilde{U}_{\alpha j}^{(-)} \tilde{U}_{\beta j}^{(-)*} \tilde{U}_{\alpha k}^{(-)} \tilde{U}_{\beta k}^{(-)*} \sin \left( \Delta \tilde{E}_{jk}^{(-)} L \right) \right)$$

$$= 4 \tilde{J}^{(-)} \left[ \sin \left( \Delta \tilde{E}_{12}^{(-)} L \right) + \sin \left( \Delta \tilde{E}_{23}^{(-)} L \right) + \sin \left( \Delta \tilde{E}_{31}^{(-)} L \right) \right]$$

$$= -16 \tilde{J}^{(-)} \sin \left( \frac{\Delta \tilde{E}_{31}^{(-)} L}{2} \right) \sin \left( \frac{\Delta \tilde{E}_{32}^{(-)} L}{2} \right) \sin \left( \frac{\Delta \tilde{E}_{21}^{(-)} L}{2} \right),$$

where

$$\tilde{J}^{(-)} \equiv \text{Im} \left( \tilde{U}_{\alpha 1}^{(-)} \tilde{U}_{\beta 1}^{(-)*} \tilde{U}_{\alpha 2}^{(-)} \tilde{U}_{\beta 2}^{(-)*} \right)$$

is the modified Jarlskog factor in matter. It is known [53, 54, 43] that this modified Jarlskog factor can be written in terms of $J$ as

$$\tilde{J}^{(-)} = \frac{\Delta E_{31} \Delta E_{32} \Delta E_{21}}{\Delta E_{31}^{(-)} \Delta E_{32}^{(-)} \Delta E_{21}^{(-)}} J.$$

Unfortunately it is known that measurements of T violation is experimentally difficult, and most of the discussions in the past have been focused on a kind of indirect measurements of the CP phase, i.e., one deduces the values of $\delta$ etc. by comparing the energy spectra of the data and of the theoretical prediction with neutrino oscillations assuming the three flavor mixing.

4.2. Establishing non-zero value of $\delta$

To establish non-zero value of $\delta$, what one has to do is to show that the deviation of the numbers of events with $\delta \neq 0$ from those with $\delta = 0$ has to be larger than the errors (cf. Fig. 4(a)). Also, since there are other oscillation parameters as well as the density (cf. [55]) of the Earth whose values are not exactly known, one has to take into account correlations of errors of these parameters.\footnote{A different form of the quantity $\tilde{J}^{(-)} / J$ was given in [32].} To explain the situation, let me consider an example of the correlation of the CP phase $\delta$ and the matter effect $A \equiv \sqrt{2}G_F N_e$. First of all, to be able to show that $\delta \neq 0$, the theoretical estimation for the numbers of events with $\delta \neq 0$ and $\delta = 0$ under an experimental setup has to look like Fig. 4(a), where the deviation of the numbers of events with $\delta = \pm \pi/2$ is larger than the error of that with $\delta = 0$. However, if the number of events with $\delta = 0$ with $A = A_0 + \epsilon$ (dotted lines) mimics the case of $\delta = \pi/2$ with $A = A_0$ as depicted in Fig. 4(b), then there is no way to distinguish the case of $\delta = \pi/2$ with $A = A_0$.

\footnote{Correlations of errors at neutrino factories were studied in [30, 56, 57, 58, 59].}
from that of $\delta = 0$ with $A = A_0 + \epsilon$. Therefore, one has to make sure that the case with $\delta = \pi/2$ is distinct from that with $\delta = 0$, no matter which value the oscillation parameters and the density of the Earth may take within the allowed region of the parameters. In other words, one has to choose the baseline and the muon energy of a neutrino factory such that the correlations of errors of the parameters are small.

Thus I introduce the following quantity to see the significance of the case with nonvanishing $\delta$:

$$
\Delta \chi^2 \equiv \min_{\delta, \theta_{\ell\ell}, \Delta m^2_{\ell\ell}} \sum_j \left\{ \frac{[N_j(\nu_e \to \nu_\mu) - \bar{N}_j(\nu_e \to \nu_\mu)]^2}{\sigma_j^2} + \frac{[N_j(\bar{\nu}_e \to \bar{\nu}_\mu) - \bar{N}_j(\bar{\nu}_e \to \bar{\nu}_\mu)]^2}{\sigma_j^2} \right\},
$$

(7)

where $N_j(\nu_\alpha \to \nu_\beta) \equiv N_j(\nu_\alpha \to \nu_\beta; \theta_{\ell\ell}, \Delta m^2_{\ell\ell}, \delta, A)$, $\bar{N}_j(\nu_\alpha \to \nu_\beta) \equiv \bar{N}_j(\nu_\alpha \to \nu_\beta; \theta_{\ell\ell}, \Delta m^2_{\ell\ell}, \delta = 0, \bar{A})$ stand for the numbers of events of the data and of the theoretical prediction with a vanishing CP phase, respectively, and $\sigma_j^2$ stands for the error which is given by the sum of the statistical and systematic errors. At neutrino factories appearance and disappearance channels for $\nu$ and $\bar{\nu}$ are observed, and I have included the numbers of events of all the channels in (7) to gain statistics. In the present analysis, $\bar{N}_j(\nu_\alpha \to \nu_\beta)$ is substituted by theoretical prediction with a CP phase $\delta$ and $\Delta \chi^2$ obviously vanishes if $\delta = 0$.* The quantity $\Delta \chi^2$ reflects the strength of the correlation of the parameters, i.e., if $\Delta \chi^2$ turns out to be very small for a certain value of $\bar{A}$ then the correlation between $\delta$ and $\bar{A}$ would be very strong and in that case there would be no way to show $\delta \neq 0$. To reject a hypothesis “$\delta = 0$”

* This is the reason why the quantity in (7) is denoted as $\Delta \chi^2$ instead of absolute $\chi^2$. $\Delta \chi^2$ represents deviation from the best fit point rather than the goodness of fit.
Ref. | correlations of $\theta_{ij}, m^2_{ij}$ | $|\Delta A/A|$ | $f_B$ | $\Delta m^2_{21}/10^{-3}$eV$^2$ | $E_{\mu}$/GeV | optimized $E_{\mu}$/GeV | optimized $L$/km
--- | --- | --- | --- | --- | --- | --- | ---
KOS [57] | included | 10% | 0 | 5 | 1 | $\lesssim 6$ | 600 – 800
 |  |  |  | 10 | 1 | $\lesssim 50$ | 2000
FHL [58] | included | 0 | 0 | 10 | 4 | 30 – 50 | 2800 – 4500
H [64] | included | 10% | 0 | 10 | 0.1 | $\lesssim 20$ | 2000
 |  |  |  | 10 | 3.2 | $\sim 10$ | 800
 |  |  |  | 10 | 3.2 | $\sim 10$ | 800
 |  |  |  | 10 | 3.2 | $\sim 8$ | 500
 |  |  |  | 10 | 3.2 | $\sim 6$ | 500
 |  |  |  | 10 | 3.2 | $\sim 25$ | 1500

Table 1. Comparison of different works The reference value is $\sin^2 2\theta_{13} = 0.1, 10^{21} \mu$s.

at the $3\sigma$ confidence level, I demand
\[
\Delta \chi^2 \geq \Delta \chi^2(3\sigma CL) \tag{8}
\]
where the right hand side stands for the value of $\chi^2$ which gives the probability 99.7% in the $\chi^2$ distribution with a certain degrees freedom, and $\Delta \chi^2(3\sigma CL)=20.1$ for 6 degrees freedom. From (8) I get the condition for the detector size to reject a hypothesis “$\delta = 0$” at $3\sigma$CL.

We note in passing that indirect measurements of CP violation are also considered in the $B^0 - \bar{B}^0$ system. To measure the phase $\phi_1$, direct CP violating process $B(\bar{B}) \to J/\psi K_s$ is used [60, 61], while those to measure $\phi_2$ and $\phi_3$ are $B \to 2\pi$ [62] and $B \to DK$ [63], which are not necessarily CP–odd processes. Their strategy is to start with the three flavor framework, to use the most effective process, which may or may not be CP–odd, to determine the CP phases and to check unitarity or consistency of the three flavor hypothesis.

The optimized muon energy $E_{\mu}$ and baseline $L$ for the measurement of CP the phase at a neutrino factory have been investigated by several groups by taking into consideration the correlations of $\delta$ and all other parameters and the results are summarized in Table 1. The results in Ref.[59] are given in Fig. 5, which shows that the more uncertainty one has in the density, the shorter baseline one has to choose because the correlation between $\delta$ and $A$ becomes stronger for larger baseline and muon energy. The works [58, 59, 64], which basically used $\Delta \chi^2$ in (7), agree with each other to certain extent (there are some differences on the reference values for the oscillation parameters) while the result by Koike et al.[57] is quite different from others. This discrepancy is due to the fact that they adopted different quantity:
\[
\Delta \chi^2 \equiv \min_{\nu_{e,\mu}, \Delta m^2_{\nu,\mu}} \sum_j \frac{1}{\sigma_j^2} \left[ \frac{N_j(\nu_e \rightarrow \nu_\mu)}{\bar{N}_j(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)} - \frac{\bar{N}_j(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)}{N_j(\nu_e \rightarrow \nu_\mu)} \right]^2, \tag{9}
\]

\[\] The term “indirect” in the B system is different from that in neutrino oscillations.
Figure 5. The contour plot of equi-number of data size required to reject a hypothesis $\delta = 0$ at $3\sigma$ with the background fraction $f_B = 10^{-5}$ or $10^{-3}$ and the uncertainty of the matter effect $\Delta A = 5\%$, 10\% or 20\%. [59]

which turns out to be a combination in which the correlation between $\delta$ and $A$ improves for low energy and worsens at high energy.

There are slight differences between the results by the groups [58, 64] and those by the other [59] and this appears to come from different statistical treatments. In the future it should be studied what makes a difference to get the optimum set $(E_\mu, L)$. The detector size required to reject a hypothesis ”$\delta=0$” as a function of $\theta_{13}$ is given in Fig. 6 (taken from Ref.[65]). Fig. 6 shows the sensitivity of neutrino factories to the CP phase.

4.3. High energy behaviors of $\Delta \chi^2$

Some people have questioned whether the sensitivity to the CP phase at a neutrino factory increases infinitely as the muon energy increases, and Lipari[66] concluded that the sensitivity is lost at high energy. In the work [59] the behaviors of $\Delta \chi^2$ in (7) was studied analytically for
high muon energy and it was shown after the correlations between $\delta$ and any other oscillation parameter or $A$ is taken into account that

$$\Delta \chi^2 \propto \left( \frac{J}{\sin \delta} \right)^2 \frac{1}{E_\mu} \left( \sin \delta + \text{const} \frac{\Delta m^2_{32} L}{E_\mu} \cos \delta \right)^2$$

(10)

for large $E_\mu$, where $J \equiv (c_{13}/8) \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \delta$ stands for the Jarlskog parameter. The behavior (10) is the same as that for $\Delta \chi^2$ in (9) and it is qualitatively consistent with the claim by Lipari [66]. It is remarkable that (10) is different from a naively expected behavior

$$\Delta \chi^2_{\text{naive}} \propto E_\mu (\cos \delta - 1)^2.$$

This is because the correlation between $\delta$ and other parameters is taken into account.
5. Parameter degeneracy

The discussions in the previous sections have been focused on rejection of a hypothesis \("\theta_{13} = 0\)" or \("\delta = 0\)". Once \(\theta_{13}\) or \(\delta\) is found to be nonvanishing, it becomes important to determine the precise value of \(\theta_{13}\) or \(\delta\). Since the work [67] it has been known that various kinds of parameter degeneracy exist. Burguet-Castell et al.[67] found degeneracy in \((\delta, \theta_{13})\), Minakata and Nunokawa [68] found the one in the sign of \(\Delta m_{23}^2\), and Barger et al.[69] found the one in the sign of \(\pi/4 - \theta_{23}\). In general, therefore, I have eight-fold degeneracy. To understand the parameter degeneracy, it is instructive to consider the appearance probabilities \(P(\nu_\mu \rightarrow \nu_e)\) and \(\bar{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)\) analytically. Up to second order in \(\alpha\) and \(\theta_{13}\) and assuming constant density of matter, the oscillation probabilities for \(\Delta m_{31}^2 > 0\) and \(\Delta m_{21}^2 > 0\) are given by [30]

\[
P = P(\nu_e \rightarrow \nu_\mu) = x^2 f^2 + 2 x y f g (\cos \delta \cos \Delta - \sin \delta \sin \Delta) + y^2 g^2,
\]

\[
\bar{P} = P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = x^2 \bar{f}^2 + 2 x y \bar{f} g (\cos \delta \cos \Delta + \sin \delta \sin \Delta) + y^2 g^2,
\]

respectively, where I follow the notations of [69]:

\[
x \equiv \sin \theta_{23} \sin 2\theta_{13},
\]

\[
y \equiv \alpha \cos \theta_{23} \sin 2\theta_{12},
\]

\[
\begin{cases} f \\ \bar{f} \end{cases} \equiv \frac{\sin((1 \mp \hat{A})\Delta)}{(1 \mp \hat{A})},
\]

\[
g \equiv \frac{\sin(\hat{A}\Delta)}{\hat{A}},
\]

\[
\Delta \equiv \frac{|\Delta m_{31}^2| L}{4 E_\nu} = 1.27 \frac{\Delta m_{31}^2 / eV^2 (L/\text{km})}{(E_\nu / \text{GeV})},
\]

\[
\hat{A} \equiv \frac{2 A E_\nu}{|\Delta m_{21}^2|},
\]

\[
\alpha \equiv \frac{|\Delta m_{21}^2|}{|\Delta m_{31}^2|}.
\]

Since (11) and (12) are quadratic in \(x \equiv \sin \theta_{23} \sin 2\theta_{13}\), given \(P\) and \(\bar{P}\), there are two solutions of (11) with respect to \(x\). In fact it has been known [70, 71, 72, 73] that the solution of (11) constitutes an ellipse in the \((P, \bar{P})\) plane as \(\delta\) ranges from 0 to \(2\pi\).

5.1. \((\theta_{13}, \delta)\) ambiguity [67]

As the two ellipses indicate in Fig. 7(a), given \(P\) and \(\bar{P}\), there exist two sets of solution \((\theta_{13}, \delta)\) and \((\theta'_{13}, \delta')\), and they are given by [69]

\[
x'^2 - x^2 = \frac{4 y g \sin 2\Delta \left(y g \sin 2\Delta + x f \sin(\Delta - \delta) + x \bar{f} \sin(\Delta + \delta)\right)}{f^2 + f^2 - 2 f \bar{f} \cos 2\Delta},
\]

and \(\delta'\) is obtained by

\[
x' \cos \delta' = x \cos \delta + \frac{(f + \bar{f})(x^2 - x'^2)}{4 y g \cos \Delta},
\]

\[
x' \sin \delta' = x \sin \delta - \frac{(f - \bar{f})(x^2 - x'^2)}{4 y g \sin \Delta}.
\]
Figure 7. The CP trajectory in the \((P, \bar{P})\) plane. (a–c): Orbit ellipses showing \((\delta, \theta_{13})\) ambiguity for \(L = 1290\) km with (a) \(E_\nu = 2.09\) GeV \((\Delta = 3\pi/4)\), (b) \(E_\nu = 3.13\) GeV \((\Delta = \pi/2)\), and (c) \(E_\nu = 1.565\) GeV \((\Delta = \pi)\), for \(\sin^2 2\theta_{13} = 0.01\) and 0.00298. The other parameters are \(\Delta m_{21}^2 = 3 \times 10^{-3} \text{ eV}^2, \Delta m_{31}^2 = 5 \times 10^{-5} \text{ eV}^2, \sin^2 2\theta_{23} = 1\), and \(\sin^2 2\theta_{12} = 0.8\). The value of \(\delta\) varies around the ellipse. In (b) and (c) the ellipse collapses to a line and the ambiguity reduces to a \((\delta, \pi - \delta)\) or \((\delta, 2\pi - \delta)\) ambiguity, respectively, and different values of \(\theta_{13}\) do not overlap (for the same \(\text{sgn}(\Delta m_{13}^2)\)). (d–f): \(\text{sgn}(\Delta m_{13}^2)\) ambiguity for \(L = 730\) km with (d) \(E_\nu = 3.54\) GeV \((\Delta = \pi/4)\), (e) \(E_\nu = 1.77\) GeV \((\Delta = \pi/2)\), and (f) \(E_\nu = 0.885\) GeV \((\Delta = \pi)\). The other parameters are \(\Delta m_{21}^2 = 5 \times 10^{-5} \text{ eV}^2, \sin^2 2\theta_{23} = 1\), and \(\sin^2 2\theta_{12} = 0.8\). (g–i): \((\theta_{23}, \pi/2 - \theta_{23})\) ambiguity for \(L = 1290\) km with (g) \(E_\nu = 2.09\) GeV \((\Delta = 3\pi/4)\), (h) \(E_\nu = 3.13\) GeV \((\Delta = \pi/2)\), and (i) \(E_\nu = 1.565\) GeV \((\Delta = \pi)\). The other parameters are \(\Delta m_{21}^2 = 5 \times 10^{-5} \text{ eV}^2, \sin^2 2\theta_{12} = 0.8\) [69].
5.2. \((\Delta m^2_{31}, -\Delta m^2_{31})\) ambiguity [68]

Also for this ambiguity, there are two solutions, as can be expected from Fig. 7(d). The general equations for the \(\text{sgn}(\Delta m^2_{31})\) ambiguity are not easy to work out, so for simplicity let me consider the case \(\Delta = (n - \frac{1}{2})\pi\). The values of \((x', \delta')\) for \(\Delta m^2_{31} < 0\) that give the same \(P\) and \(\bar{P}\) as \((x, \delta)\) for \(\Delta m^2_{31} > 0\) are determined by [69]

\[
x'^2 = \frac{x^2(f^2 + \bar{f}^2 - f \bar{f}) - 2yg(f - \bar{f})x \sin \delta \sin \Delta}{ff},
\]

\[
x' \sin \delta' = x \sin \delta \frac{f^2 + \bar{f}^2 - f \bar{f}}{ff} - \frac{x^2}{\sin \Delta} \frac{f^2 + \bar{f}^2 - f \bar{f}}{2yg}.
\]

(13)

In particular, if \(\sin \delta = 0\) then Eq. (13) gives

\[
\sin \delta' = -x \frac{f^2 + \bar{f}^2 - f \bar{f}}{ff} \frac{ff}{2yg \sin \Delta} \sqrt{f^2 + \bar{f}^2 - f \bar{f}},
\]

which indicates that even if \(\delta = 0\), there is some contribution from matter effects which contributes to CP violation.

5.3. \((\theta_{23}, \pi/2 - \theta_{23})\) ambiguity [69]

From Fig. 7(g) two solutions are expected. As with the \(\text{sgn}(\Delta m^2_{31})\) ambiguity, the solutions for the \(\theta_{23}\) ambiguity are complicated in the general case. For the special case \(\Delta = (n - \frac{1}{2})\pi\), I have [69]

\[
\sin^2 2\theta'_{13} = \sin^2 2\theta_{13} \tan^2 \theta_{23} + \frac{\alpha^2 g^2 \sin^2 2\theta_{12}}{ff} (1 - \tan^2 \theta_{23}),
\]

\[
\sin 2\theta'_{13} \sin \delta' = \sin 2\theta_{13} \sin \delta + \frac{\alpha g(f - \bar{f}) \sin 2\theta_{12}}{ff} \cot \theta_{23},
\]

where \((\delta, \theta_{13})\) are the parameters that give a certain \((P, \bar{P})\) for \(0 < \theta_{23} < \pi/4\) and \((\delta', \theta'_{13})\) are the parameters that give the same \((P, \bar{P})\) for \(\pi/2 - \theta_{23}\). Again in this case \(\sin \delta = 0\) does not necessarily imply \(\sin \delta' = 0\).

5.4. Resolution of the degeneracies

There have been several works to resolve the degeneracies mainly in the context of conventional beams [74, 75, 76, 77, 78].

If one tunes the neutrino energy so that \(|\Delta m^2_{31}| L/4E_\nu = \pi/2\) then the ellipse collapses to a line as depicted in 7(b),(c) and the degeneracy in \((\theta_{13}, \delta)\) is lifted [74, 75], assuming that \(\theta_{23}\) is close to \(\pi/4\). If one has experiments at different neutrino energies then the four-fold degeneracy \((\theta_{13}, \delta) \times \text{sgn}(\Delta m^2_{13})\) can be removed [77]. These scenarios assume approximately monoenergetic beams in conventional neutrino beams. In the case of a neutrino factory, since the neutrino energy spectrum is continuous, these techniques may not be applicable unless the energy resolution is good enough to distinguish the neutrino energies of different bins.

As was discussed in Sect. 2, if one performs a very long baseline experiment with \(L \sim 5000\text{km}\) then the sign of \(\Delta m^2_{31}\) can be definitely determined, so eventually the degeneracy in \(\text{sgn}(\Delta m^2_{13})\) will be resolved.

As for the one in \((\theta_{23}, \pi/2 - \theta_{23})\), the most effective way to lift the degeneracy is to measure the appearance probability \(P(\nu_e \rightarrow \nu_\tau)\), which is proportional to \(\cos^2 \theta_{23}\) assuming
that the contribution from $\Delta m^2_{21}$ is negligible. A neutrino factory may be the only experiment which can resolve this degeneracy.

6. Summary

Neutrino factories can establish the existence of the non-zero CP phase $\delta$ with the detector size larger than $10^{21} \mu \cdot 100 \text{kt} \cdot \text{yr}$ for $\sin^2 2\theta_{13} \gtrsim 10^{-3}$ unless $|\delta|$ is small. To determine the precise value of $\theta_{13}$ and $\delta$, one needs to resolve the eight-fold degeneracy, and it will require more than one neutrino energy and more than one channel (and maybe more than one baseline) to lift the degeneracy completely. Comprehensive study on the degeneracy by taking into account statistical and systematic errors as well as the error correlations in the case of a neutrino factory still needs to be done.

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