Application of Inverse Filtering Technique in Power System Studies

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Abstract: This paper demonstrates the application of inverse filtering technique for power systems. In order to implement this method, the control objective should be based on a system variable that needs to be set on a specific value for each sampling time. A control input is calculated to generate the desired output of the system and the relationship between the two is used to design an auto-regressive model. The auto-regressive model is converted to a moving average model to calculate the control input based on the future values of the desired output. Therefore, required future values to construct the output are predicted to generate the appropriate control input for the next sampling time.

Keywords: Inverse filtering technique, deconvolution, auto-regressive, moving average, control objective, supplementary control input, wide-area control, inter-area oscillations.

1. INTRODUCTION

The concept of inverse filtering is widely used in control systems, signal processing and communications. It is applied for several purposes including damage detection in nonlinear imaging methods and image restoration (Ciampa et al., 2015, Dong et al., 2015); audio and telecommunications for creating virtual source; equalizing loudspeaker and room deconvolution (Norcross et al., 2004); and estimation of the power spectrum in a wide-sense stationary process (Chong-Yung and Wang, 1992). Inverse filtering helps to obtain the input of the system using the prior knowledge of the desired output. Generally, inverse filtering or deconvolution is categorized into two groups: the first category is for the cases where the dynamics of the plant is known and is modelled using time-invariant system; and the second category is where the plant dynamics is unknown (blind deconvolution) or partially known (Saberi et al., 2001).

For the first category of inverse filtering, the known plant can be modelled as (1)-(2).
\[
\begin{align*}
\dot{x} &= Ax + Bu + E\nu \\
y &= Cx + Du + F\nu
\end{align*}
\]

where \(x\) represents states of the plant, \(u\) is its unknown input, \(\nu\) is noise vector and \(y\) is output of system. Inverse filtering for this plant estimates the unknown input by having the output of system. Fig. 1 summarised the concept of the inverse filtering. The inverse filtering problem can be an ‘exact’ or ‘almost’ by choosing to achieve certain level of error \((e_u)\) between the estimated input and actual input of the system. For the exact inverse filter, the error \(e_u\to0\) as \(t\to\infty\); whereas, for almost inverse filter, the error is limited to a small desirable value. The design of the exact filter satisfying \(e_u\to0\) as \(t\to\infty\) makes the inverse filtering problem sometimes unsolvable.

![Fig. 1. Block diagram for inverse filtering](image1)

![Fig. 2. Block diagram in presence of uncertainty](image2)

For the second category of inverse filtering, where the plant is unknown or partially known as shown in Fig. 2, the system with uncertainty can be modelled as (3)-(6). Uncertainty for the systems, which is known partially, is modelled as a feedback around a nominally known plant model as block \(\Delta\) in Fig. 2 (Saberi et al., 2001).

\[
\begin{align*}
\dot{x} &= Ax + Bu + E\omega \\
y &= C_1x + D_1u \\
z &= C_2x + D_2u
\end{align*}
\]

The nominal transfer function of the system, \(G\), can be formulated as (6).
\[
\begin{bmatrix}
y \\
z
\end{bmatrix} =
\begin{bmatrix}
G_{yu} & G_{yw} \\
G_{zu} & G_{zw}
\end{bmatrix}
\begin{bmatrix}
u \\
w
\end{bmatrix}
\]

(6)
This technique has also been widely used in audio and telecommunications, where, magnitude and phase of the system can be recovered using the impulse response (IR). However, the outcome depends on the method used for inverse filtering and the type of IR. For instance, if the IR is non-minimum phase, obtaining the inverse filter requires appropriate methods to achieve satisfactory results (Widrow and Walach, 1984).

An adaptive inverse modelling process is capable of providing stable controller for both minimum and non-minimum phase plants. The schematic of an adaptive filter showing the input $u_k$; its output $y_k$ and the desired response, $o_k$, is illustrated in Fig. 3. An adaptive algorithm is used to determine the weighting coefficients of the filter using the error between actual output, $y_k$, and desired response, $o_k$. According to the adaptive inverse modelling method, an unknown plant can be constructed by calculating weights that can produce the best least squares fit for its output as the actual plant. The input for this adaptive plant model is the same as the original plant (Fig. 4). At this step by using the adaptive plant model, the inverse of the plant can be obtained. The input of the inverse of adaptive plant is the output of the real plant and the output of the adaptive plant inverse should be equal with the input of the system, using the error between actual inputs and obtained input, the adaptive plant model can be improved (Fig. 5). Applying the achieved inverse plant, as the controller for the plant, the desired output can be generated. Therefore, the input of the controller, adaptive plant inverse, is the desired output of the plant and the output of the controller is the input of the plant, the adaptive model of the plant can be updated during the simulation to adjust with the changes of the operating conditions (Fig. 6).

Multiple-input/output inverse theorem (MINT) is another principle that is applied to obtain inverse filtering of gathered acoustic impulse responses in a room (Miyoshi and Kaneda, 1988). For this method it is assumed that the plant is a multiple input or multiple output linear finite impulse response (FIR) plant. This technique obviates the main problems of the conventional LSE method which caused because of using just one input for the plant. According to the conventional LSE method, it is not possible to obtain exact inverse filtering since the error energy of the plant does not converge to zero due to having non-minimum phase impulse response.

2. APPLICATION OF INVERSE FILTERING IN POWER SYSTEM

The concept of inverse filtering can be applied for the purpose of controlling excitation system of a power plant. It is assumed that the model of the network is unknown and the internal parameters of generators and exciters are known. Using several Phasor Measurement Units (PMUs) and a Kalman estimator, the required parameters of the system can be obtained (Vahidnia et al., 2014). These state values are used in the process of inverse filtering. In order to improve the damping of the system, a supplementary control input should be exerted on the excitation system. This supplementary control input ($u_c$) will be obtained by designing the appropriate inverse filtering controller. An Auto-regressive (AR) model for the system can be easily converted to a moving average (MA) model and is used for inverse filtering. The continuous transfer function, as in (7), between an input and output of a typical plant is discretized into an autoregressive Moving average (ARMA) model as in (8). For the purpose of this nonlinear inverse filtering
controller, (8) is modified to an AR model as in (9), which is turn is transformed to MA model as in (10).

\[ H(s) = \frac{Y(s)}{U(s)} = \frac{1}{1 + \sum_{i=1}^{n} a_i z^{-i}} \]  
(7)

\[ H(z) = \frac{1}{1 + \sum_{i=1}^{n} c_i z^{-i}} \]  
(8)

\[ H(z) = \frac{Y(z)}{U(z)} = \frac{1}{1 + \sum_{i=1}^{n} c_i z^{-i}} \]  
(9)

\[ u_i(k) = (1 + \sum_{j=1}^{l} c_{ij} z^{-j}) y(k) \]  
(10)

In order to design the proposed nonlinear inverse filtering controller the following steps required to be performed.

### 2.1 Objective of the Controller

The first step to design the inverse filtering controller is determining the objective of the controller, as the desired output of the system will be obtained by setting the objective of the control system. For the purpose of this research, the objective of the inverse filtering controller is to improve the damping of the inter-area oscillations. The minimization of the kinetic energy function of the whole system can be one of the ways to achieve the desired damping. As it is known, when a power system is subjected to a fault, the velocity of generators start oscillating. By using the center of inertia (COI) reference frame for the rotor speed of generators, the quantity of the total inertia of the system in steady-state condition is zero. However, the rotor speeds fluctuate and the total kinetic energy of the system is no longer zero for post-fault case. As a result, by decreasing the total kinetic energy of the system, the amplitude of the oscillations should also decrease which is the objective of the proposed controller. The total kinetic energy of the system is constructed based on the kinetic energy of the generators (Palmer and Ledwich, 1999). It is known that each power system consists of several coherent generators in each area that oscillate with the same phase. By considering an equivalent generator for each group which its inertia is the sum of generators’ inertias in that group and its rotor speed equal to (11), the total kinetic energy is expressed in (12) and its derivative in (13).

\[ \omega_{area,i} = \frac{\sum_{i=1}^{n} J_i \omega_i}{\sum_{i=1}^{n} J_i} \]  
(11)

\[ KE = \frac{1}{2} \sum_{i=1}^{n} J_{area,i} \omega_{area,i}^2 \]  
(12)

\[ \dot{KE} = \sum_{i=1}^{n} J_{area,i} \omega_{area,i} \dot{\omega}_{area,i} \]  
(13)

Where \( \dot{\omega} \) is the velocity of generators, \( J \) is inertia, \( m \) is the number of generators in each coherent group, \( KE \) is the kinetic energy of the power grid and \( n \) is the number of coherent groups in the system. The derivative of the \( KE \) in (13) can be expressed as a function of generator fluxes by using classic model of the synchronous generator as in (14)-(16) to obtain the electrical power.

\[ \frac{d\delta}{dt} = \omega_l - \omega_i \]  
(14)

\[ J \frac{d\omega}{dt} = (P_m - P_e) - D (\omega_l - \omega_i) \]  
(15)

\[ P_{ci} = \sum_{j=1}^{n} E_j E_j' \sin(\delta_j - \delta_i) \]  
(16)

where \( \delta \) is rotor angle of the generator, \( P_m \) is mechanical power of the generator, \( D \) is the damping ratio, \( E_q \) is the q component of internal voltage which is proportional to field flux linkage and \( X_q \) is the reactance between generator area \( i \) and \( j \). By substituting (15) and (16) in (13), the derivative of total kinetic energy is expressed as the function of generator fluxes (17).

\[ K\dot{E} = \sum_{i=1}^{n} (P_{mi} - \sum_{j=1}^{n} E_j E_j' \sin(\delta_{area,i} - \delta_{area,j})) \]  
(17)

Generator flux is the parameter that is controlled to maximize the reduction rate of the total kinetic energy. Therefore, by differentiating with respect to the generator flux, the value of the flux variations that fulfills the objective of this controller is obtained (18).

\[ \Delta E_{qi} = \frac{\partial K\dot{E}}{\partial E_{qi}} = \sum_{j=1}^{n} E_j X_j' \sin(\delta_{area,i} - \delta_{area,j})(\dot{\delta}_{area,i} - \dot{\delta}_{area,j}) \]  
(18)

where \( K_{ef} \) is a gain, determined to improve the performance of the controller by avoiding flux saturation after first or second swing which leads into undesirable transients after the fault. Having the desired output of the system an MA model can be obtained that calculates the supplementary control input. However to acquire the desired flux as given in (18), rotor angle and velocity of each coherent area and also the reactance between coherent areas should be available. The equivalent parameters of coherent areas are obtained by using a nonlinear Kalman estimator to estimate the angle and velocity of each area which are the states of the reduced order power system.

\[ \dot{\delta}_{area,i} = \dot{\alpha}_{area,i} - \omega_i + \sum_{j=1}^{l} L_{y,ij}(Y_{PMU} - \hat{Y}_j) \]  
(19)

\[ \dot{\alpha}_{area,i} = (P_{mi} - \sum_{j=1}^{n} E_j E_j' \sin(\delta_{area,i} - \delta_{area,j})) \]  
(20)

\[ \hat{Y}_j = C \dot{\delta}_{area,j} \]  
(21)

where \( Y_{PMU} \) is the measurements, \( \hat{Y}_j \) is the measurement estimation, \( l \) is the number of measurements, \( C \) is the relation between estimated measured data and the reduced states of
the system and \( L \) is the gain of Kalman estimator that will be updated for each sampling time to be accurate for different operating conditions and is calculated by using the co-variances for the Kalman filter noises as in (Vahidiab et al., 2014). It is stated that the presence of local modes in the obtained angles and velocities is reduced significantly and for the purpose of damping area-oscillations, estimated angles and velocities provide more appropriate values to be applied in the proposed nonlinear controller.

### 2.2 Inverse filtering approach

The desired output of the system, which is the flux of generator, is achieved using Kalman estimator. Now by performing inverse filtering, an appropriate supplementary control input can be obtained to force the excitation system generate the desired output. The generator is assumed to be modelled by using flux decay representation and a first order exciter is used for generator. The dynamics of generator and excitation system is given below. It is notable that flux saturation of the generator is also considered which expressed by the function \( S_E \) which is proposed by (Arrillaga and Arnold, 1990).

\[
\frac{d\delta}{dt} = \omega - \omega_s 

\]  

(22)

\[
J_I \frac{d\omega}{dt} = P_m - D_I (\omega - \omega_s) - E'_{qk} I_q + (X_{d'} - X_{d'}) I_d I_q 

\]  

(23)

\[
T_{d'} \frac{dE'_{d'}}{dt} = -S_E E'_{d'} - (X_{d'} - X_{d'}) I_d + E_{\rho d} 

\]  

(24)

\[
T_{d'} \frac{dE_{\rho d}}{dt} = -E_{\rho d} + K_{d} (V_{d'} - u_s) 

\]  

(25)

\[
T_{d'} \frac{dV_{d'}}{dt} = -V_{d'} + (V_{ref} - V_{d'}) 

\]  

(26)

where \( n \) is the number of generators, \( E_{\text{ref}}, V_{\text{ref}} \) are the states of the system and represent field voltage and exciter input, respectively. \( X_{d'} \) and \( X_{d''} \) represent direct-axis synchronous and transient reactances, respectively, and \( X''_{d} \) is called quadrature-axis reactance. \( T_{\text{dref}} \) represents open-circuit \( d' \)-axis transient time constant, \( \omega_s \) is synchronous speed, \( J_I \) is inertia and \( D_I \) is damping coefficient. \( K_{d} \) and \( T_{d'} \) are the gain and time constant of the exciter and \( V_{\text{ref}} \) denotes reference voltage and \( u_s \) is the supplementary control input which is added as the output of the nonlinear inverse filtering controller. The supplementary control input can be obtained using the differential equations of the generator excitation system. This stage is carried out in three steps. Fig. 7 which is illustrated with respect to (24), is used to demonstrate how step 1 and 2 of the inverse filtering process is performed. Since the desired output of the system, \( E'_{qk} \) is the known parameter of the plant, the variations of \( E_s \) (an internal parameter) can be calculated by completing the first step of inverse filtering. To perform this step, the continuous transfer function is discretized into an AR model, which in turn is converted into MA model to obtain \( \Delta E_{\phi} \). The effect of flux saturation and \( l_i \) is considered as shown in Fig. 7.

At Step 2, the variation of the field voltage is obtained using (30) and (31). At Step 3, the inverse filtering is used to determine the required supplementary control for the excitation system. It is notable that the value of \( T_R \) should be large enough to have \( V_R \) as constant during a short time interval after the fault occurrence. Fig. 8 is obtained by using (25) and the last step for calculating \( u_s \) is as (32)-(34).

![Fig. 7. Step1 of inverse filtering](image)

\[
\frac{\Delta E'_{\phi}}{\Delta E_{\phi}} = \frac{1}{1 + s T_{d'}} 

\]  

(27)

\[
\Delta E'_{\phi}(k) = \frac{1}{1 + \sum_{i=1}^{p} c_i z^i} 

\]  

(28)

\[
\Delta E_{\phi}(k) = c_k \Delta E'_{\phi} + \ldots + c_{\phi} \Delta E_{\phi,y} 

\]  

(29)

Step 2 of inverse filtering:

\[
\Delta E_{\phi} = (X_{d'} - X_{d'}) I_d - S_E 

\]  

(30)

\[
\Delta E_{\phi,s} = \Delta E_{\phi} + S_{\phi} + (X_{d'} - X_{d'}) I_d 

\]  

(31)

\[
\Delta E_{\phi,s} = \frac{K_I}{1 + s T_d} \Delta V_g 

\]  

(32)

\[
\Delta U_s = \frac{1}{1 + s T_{d'}} \Delta V_g 

\]  

(33)

\[
\Delta u_s(k) = d_0 \Delta E_{\phi,s} + \ldots + d_\phi \Delta E_{\phi,y} 

\]  

(34)

In order to guarantee that the obtained supplementary control input can generate the desired output, the actual flux of the generator is compared with the desired one. The flux tracking graph verifies the accuracy of inverse filtering process that is performed.

Analysing the variations of generator velocity with and without inverse filtering controller proves that the appointed objective and the obtained supplementary control input lead into improvement in the damping of inter-area oscillations. The next section confirms the effectiveness of the proposed approach by using time-domain simulation for 16 generator-68 bus test system.
3. SIMULATION RESULTS

IEEE 16-machine, 68-bus test system, as shown in Fig. 9, (Rogers, 2000), is used to evaluate the performance of the proposed inverse filtering excitation controller. This test system consists of 16 generators which four of them are large equivalent generators and to make the system more realistic, no excitation system is considered for them. This test system has unstable and under-damped modes when the system is not equipped with Power System Stabilizers (PSS). Therefore, after the system is subjected to a fault, the generators lose synchronism and the system collapses. High gains of the voltage regulators are also responsible for the instability of the power system. Five coherent areas and 4 inter-area modes are identified for this test system (Jing et al., 2013). Non-linear excitation controllers are installed on generators 1 to 12. The equivalent rotor angles and velocities of all 5 coherent areas are used to obtain the objective function of the controller. Nine PMUs are installed on selected locations to enable the Kalman filter estimate the area rotor angles and velocities. By using (18), flux variations for generators in area 1 and 2 are obtained and then by performing the discretization process and calculating MA models as stated in (29), (31) and (34), the supplementary control input for excitation systems are obtained.

The test system is subjected to a 10 cycles self-clearing three-phase short-circuit fault at bus 11. The system without controller and PSS is unstable. However, with proposed inverse filtering excitation controller, the system is stable and the inter-area modes are well damped. The area equivalent rotor speeds and the generator rotor speeds are plotted in Fig. 10. The test system is equipped with 12 conventional PSSs installed on generator 1 to 12 and area rotor speeds are illustrated in Fig. 10.

Fig. 9. 16-machine, 68-bus New York-New England Test system

![Diagram of the 16-machine, 68-bus New York-New England Test system]

(a) Generator Rotor Speeds (COI)

Time (Sec)

(b) Generator Rotor Speeds (COI)

Time (Sec)

(c) Rotor Area Speed (COI)

Time (Sec)

(d) Area Rotor Speed (COI)

Time (Sec)
for this test system are low pass filters that can be fitted in its AR model. If the transfer function has zeros, then AR model fitting can be performed by reducing it to the frequency range of interest.

The proposed nonlinear excitation controller is designed to damp inter-area oscillations by maximizing the reduction rate of the total kinetic energy of the system by controlling generator flux. In order to control the generator flux, the inverse filtering technique is used to calculate a control input that can impose the excitation system to generate the desired variations for the generator flux. The controller is evaluated by employing 16-machine, 68-bus test system and time-domain simulations verify that the approach is effective in damping inter-area oscillations.

5. REFERENCES

ARRILLAGA, J. & ARNOLD, C. 1990. Computer analysis of power systems.

CHONG-YUNG, C. & WANG, D. 1992. An improved inverse filtering method for parametric spectral estimation. Signal Processing, IEEE Transactions on, 40, 1807-1811.

CIampa, F., SCARELLI, G. & MEO, M. 2015. Nonlinear Imaging Method Using Second Order Phase Symmetry Analysis and Inverse Filtering. Journal of Nondestructive Evaluation, 34, 1-6.

DONG, Y., ZHANG, H. & LI, M. Analysis and Comparison of Image Restoration Methods. 2015 International Symposium on Computers & Informatics, 2015. Atlantis Press.

JING, M., TONG, W., ZENGPING, W. & THORP, J. S. 2013. Adaptive Damping Control of Inter-Area Oscillations Based on Federated Kalman Filter Using Wide Area Signals. Power Systems, IEEE Transactions on, 28, 1627-1635.

MIYOSHI, M. & KANEDA, Y. 1988. Inverse filtering of room acoustics. Acoustics, Speech and Signal Processing, IEEE Transactions on, 36, 145-152.

NORCROSS, S. G., SOULODRE, G. A. & LAVOIE, M. C. 2004. Subjective investigations of inverse filtering. Journal of the Audio Engineering Society, 52, 1003-1028.

PALMER, E. & LEDWICH, G. 1999. Switching control for power systems with line losses. Generation, Transmission and Distribution, IEE Proceedings-, 146, 435-440.

ROGERS, G. 2000. Power system oscillations, Kluwer Academic.

SABERI, A., STOORVOGEL, A. A. & SANNUTI, P. 2001. Inverse filtering and deconvolution. International Journal of Robust and Nonlinear Control, 11, 131-156.

VAHIDNIA, A., LEDWICH, G., PALMER, E. & GHOSH, A. 2014. Identification and estimation of equivalent area parameters using synchronised phasor measurements. Generation, Transmission & Distribution, IET, 8, 697-704.

WIDROW, B. & WALACH, E. Adaptive signal processing for adaptive control. Acoustics, Speech, and Signal Processing, IEEE International Conference on ICASSP’84., 1984. IEEE, 191-194.