A Model of Universality Violation Revisited

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Abstract

The possibility that the interactions of the third generation of quarks and leptons may violate universality by a small amount remains an open experimental question. The model of Li and Ma, which naturally accommodates such violations, is found to be highly constrained by newly obtained, high precision electroweak and $\tau$-lepton data once full Standard Model radiative corrections are incorporated into the analysis. A comparison of the predictions of this model with existing data and the expectations for future colliders is presented.

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One of the hallmarks of the Standard Model (SM) is the universality of the strong and electroweak interactions. Although well established experimentally for the first two generations, the possibility of a small universality violation (UV) for the third generation remains open\[1, 2]. In fact, there is potentially some evidence which may be interpreted as a signal for UV: the $\tau$-lifetime problem. Although the experimental situation has evolved significantly over the past year\[3], the possibility that $\tau$ decays may violate universality is still viable. If such a UV were indeed confirmed it would be an unmistakable signal for new physics beyond the SM.

The origin of any UV by the third generation could arise from a number of sources. One intriguing possibility, proposed more than a decade ago, is the model of Li and Ma\[4], in which each generation couples to its own left-handed weak isospin group, $SU(2)_\sigma$, ($\sigma = 1 - 3$) with its own gauge coupling, $g_\sigma$. The $U(1)$ hypercharge group remains universal, with a coupling $g_0$, and a global symmetry enforces the universality between the first two generations. Clearly, in such a model, the size of the UV in $\tau$ decays is not a prediction but rather an input parameter with which we can further analyze the implications of the model. For our purposes, we will depart from the nomenclature of Li and Ma and define

$$\tau^{\text{exp}}/\tau_{SM} = (1 + \epsilon)^2. \quad (1)$$

Experimentally, $\epsilon$ cannot be very large and two recent global analyses give $\epsilon = (7.6 \pm 6.6) \times 10^{-3}$\[3] and $\epsilon = (13.2 \pm 6.1) \times 10^{-3}$\[1].

The purpose of this paper is to further explore the ramifications of this model using these recently obtained values of $\epsilon$ as input. In particular, we are interested in its implications for physics at LEP as well as other $e^+e^-$ and hadron colliders. One possible approach is to treat $\epsilon$ as a small parameter and expand all expressions for observables only to leading order in $\epsilon$. The danger of this approach is that we can never be sure that the coefficient of the
next term in this expansion does not conspire to invalidate our conclusions. To this end, we will examine the predictions of the model numerically presenting lowest order expansions where appropriate. It is important to stress that our analysis of the model relies solely on its predictions of observable quantities. For the first two fermion generations, and for the gauge boson sector, the deviations from the predictions of the SM can be displayed by using the parameters introduced by Peskin and Takeuchi [5]: $\Delta S$, $\Delta T$, and $\Delta U$, and which are already constrained by existing data once SM radiative corrections are accounted for. The UV by the third generation may be observed at LEP, e.g., by comparing the $Z \rightarrow \tau^+\tau^-$ or $b\bar{b}$ widths with those expected on the basis of universality. As we will see, for a given value of $\epsilon$, this data already constrains the other parameters of the model. Let us first begin by reviewing the basics aspects of the model that we need in our analysis.

At first sight this model would appear to suffer from a proliferation of parameters but as we will now see there are actually only two new ones in the limit that $e - \mu$ universality is preserved. The first, $\epsilon$, we have already met above while a second can be defined in terms of the low-energy effective neutral current Hamiltonian

$$\mathcal{H}_{NC} = \frac{4G_F}{\sqrt{2}} \left[ (J_{3L} - x_w J_{em})^2 + C(J_{em})^2 \right].$$

In this model, the parameter $C$ is highly constrained and we can in fact define a new quantity, $p$, which is forced to lie on the unit interval:

$$0 \leq p \equiv \frac{1}{x_w} \sqrt{\frac{C}{\epsilon}} \leq 1.$$  

One sees from this Hamiltonian that $x_w$ is the effective mixing angle probed by low energy $\nu$ data. At LEP or higher collider energies, we must decompose this ‘composite’ interaction
into its ‘components’ which arises from the exchange of the individual \(Z\) and \(Z'\) gauge bosons:

\[
\mathcal{L}_{Z,Z'} = -\frac{e}{\sqrt{x(1-x)}} (J_{3L} - x J_{em}) Z \\
+ \frac{e}{\sqrt{x}} \left[ \frac{p(1+\epsilon)}{1-p} \right]^{1/2} \left( J_{3L} - \frac{1+\epsilon p}{p(1+\epsilon)} J_{3L}\delta_{3\sigma} \right) Z',
\]

with \(\sigma\) being a generation label and \(e\) being the running electric charge at, e.g., the \(Z\) mass scale. Here, an apparently new parameter has appeared, \(x\), but we find that it is directly related to \(x_w\), \(p\), and \(\epsilon\):

\[
x \equiv x_w(1 + \epsilon p).
\]

We can, of course, freely choose either \(x\) or \(x_w\) as the independent parameter and write all of the various combinations of gauge couplings in terms of this choice together with \(p\) and \(\epsilon\):

\[
\begin{align*}
e^2 g_0^{-2} &= x, & e^2 g_3^{-2} &= x p(1+\epsilon)/(1+\epsilon p), \\
e^2 g_{123}^{-2} &= 1 - x, & e^2 g_{12}^{-2} &= x(1-p)/(1+\epsilon p).
\end{align*}
\]

where we follow Li and Ma and employ the short-hand notation: \(g_{123}^{-2} = g_1^{-2} + g_2^{-2} + g_3^{-2}\) and similarly \(g_{12}^{-2} = g_1^{-2} + g_2^{-2}\). We find the parameter \(x\) to be the more convenient choice for our analysis. \(Z - Z'\) mixing arises naturally from the \(Z - Z'\) mass matrix

\[
\mathcal{M}_{ZZ'}^2 = \frac{1}{2} \begin{pmatrix} a_z v_3^2 & b_z v_3^2 \\ b_z v_3^2 & (c_z + d_z/\epsilon)v_3^2 \end{pmatrix},
\]

with \(v_3\) being a vacuum expectation value and where the coefficients are given by

\[
\begin{align*}
a_z &= g_0^2 + g_{123}^2, & c_z &= g_3^2 - g_{123}^2, \\
b_z &= e^{-1} g_{12} g_0 g_3 g_{123}^2, & d_z &= g_{12}^2 + g_3^2.
\end{align*}
\]
The mass eigenstates are defined via the rotation

\[
\begin{pmatrix}
Z \\
Z'
\end{pmatrix} = \begin{pmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix}.
\]
(9)

The angle, $\phi$, is found through the simple relation

\[\tan 2\phi = \frac{-2b_z}{c_z - a_z + d_z/\epsilon},\]
(10)

while $v_3$ is directly related to the Fermi constant

\[G_F \sqrt{2} = \frac{1 + \epsilon}{4v_3^2}.
\]
(11)

We note, for our analysis below, that the corresponding $W - W'$ mass matrix is given by

\[
M_{WW'}^2 = \frac{1}{2} \begin{pmatrix}
a_w v_3^2 & b_w v_3^2 \\
b_w v_3^2 & (c_w + d_w/\epsilon) v_3^2
\end{pmatrix},
\]
(12)

with

\[
\begin{align*}
a_w &= g_{123}^2, \\
b_w &= -g_{123}(g_3^2 - g_{123}^2)^{1/2}, \\
c_w &= c_z, \\
d_w &= d_z.
\end{align*}
\]
(13)

It is important to note the relations $c_w = c_z$ and $d_w = d_z$ result from a residual $SU(2)$ symmetry in the model. Because of these relationships and the rather small size of $\epsilon$ the $W'$ and $Z'$ in this model are highly degenerate. For fermions in the first two generations, the couplings of the $Z_1$, the state probed at LEP and SLC, can be written as

\[\mathcal{L} = \frac{\sqrt{-e}}{\sqrt{x(1-x)}} N(J_{3L} - x_{eff} J_{em}) Z_1,
\]
(14)
with $x_{\text{eff}}$ being an ‘effective’ mixing angle parameter and $N$ a normalization correction both of which can be determined experimentally and are given by

$$x_{\text{eff}} = \frac{x \cos \phi}{N}, \quad (15)$$

$$N = \cos \phi - \sqrt{1 - x \sin \phi} \left[ \frac{p(1 + \epsilon)}{1 - p} \right]^{1/2}.$$  

with $e = e(M_{Z_1})$ being understood. The $Z_1$ mass can be obtained directly from the $Z - Z'$ mass matrix and can written in the form

$$M_{Z_1}^2 = \frac{\pi\alpha(M_{Z_1})}{\sqrt{2}G_F x(1-x)} \left[f(\epsilon) - \epsilon^2 g(\epsilon, x)\right], \quad (16)$$

where $f, g$ can be calculated to any desired order in $\epsilon$. Unfortunately, unlike all other parts of our analysis, we cannot analytically determine $x$ to all orders in $\epsilon$ from this equation. However, a series solution is possible and can be generated with relative ease to any desired order. Since $\epsilon$ is relatively small, we find that this series expansion rapidly converges numerically especially since we find that it can be partially resummed. For example, to order $\epsilon^2$, and including this resummation, we can invert the $Z_1$ mass relation above for $x$ and obtain

$$x = x_r \left[ 1 - \frac{2\epsilon^2 g_0 A x_r}{(1 - x_r)(1 - 2x_r)} \right], \quad (17)$$

where to this order we have defined

$$f = \frac{1 + \epsilon}{1 + \epsilon \left( \frac{1-p}{1+ep} \right)^2},$$

$$g_0 = \frac{p(1-p)^3(1+\epsilon)^2}{(1+ep)^4}, \quad (18)$$

$$x_r = \frac{1}{2} \left[ 1 - \sqrt{1 - 4fA} \right].$$
with \( A = \pi \alpha(M_{Z_1})/\sqrt{2}G_F M_{Z_1}^2 \). In our analysis presented below, we will analytically include all terms of order \( \epsilon^3 \), together with partial resummation, with all the higher order terms explicitly verified as being numerically insignificant as can be shown from a complete iterative numerical solution.

In order to proceed with the calculation of the contributions to \( \Delta T \), \( \Delta S \), and \( \Delta U \) in this model, we follow the analysis of Peskin and Takeuchi (PT). The first thing we do is define the usual auxiliary quantity, \( x_0 \), which is given solely in terms of the observed \( Z_1 \) mass, \( G_F \), and \( \alpha(M_{Z_1}) \):

\[
x_0(1-x_0) \equiv A \tag{19}
\]

Note that this is a model independent quantity and is used simply as an input from experiment into our analysis. It does not, e.g., depend on the shift in the \( Z \) mass due to \( Z - Z' \) mixing as claimed by an earlier analysis. (The fact that it is independent of new physics is reason it was introduced by PT.) Using the latest data from LEP as input,[4] we obtain \( x_0 = 0.23136 \pm 0.00022 \). In terms of \( x_0 \), we can write \( \Delta S \), \( \Delta T \), and \( \Delta U \) as linear functions of three observables as given by PT:

\[
\frac{M_{W_1}^2}{M_{Z_1}^2} - (1-x_0) = \frac{\alpha(1-x_0)}{1-2x_0} \left[ \frac{1}{2} \Delta S + (1-x_0) \Delta T + \frac{(1-2x_0)}{4x_0} \Delta U \right],
\]

\[
x_{eff} - x_0 = \frac{\alpha}{1-2x_0} \left[ \frac{1}{4} \Delta S - x_0(1-x_0) \Delta T \right], \hspace{1cm} \tag{20}
\]

\[
Z_* - 1 = \frac{\alpha}{4x_0(1-x_0)} \Delta S,
\]

with \( \alpha = \alpha(M_{Z_1}) \) and where \( Z_* \) is defined via the leptonic decay width of the \( Z_1 \) as given by PT:

\[
\Gamma_\ell = Z_* \frac{\alpha(M_{Z_1}) M_{Z_1}}{48x_{eff}(1-x_{eff})^2} \left[ 1 + (1-4x_{eff})^2 \right] \left( 1 + \frac{3\alpha(M_{Z_1})}{4\pi} \right). \tag{21}
\]
The three observable quantities on the left-hand side of Eq.(20) can be calculated explicitly in terms of the model parameters, \( p \) and \( \epsilon \), and \( x_0 \). While the \( W_1 \) to \( Z_1 \) mass ratio can be obtained directly from the mass matrices above, \( x_{\text{eff}} \) is given by Eq.(15) while \( Z_* \), defined via Eq.(21), can be written simply as

\[
Z_* = \frac{N^2 x_{\text{eff}} (1 - x_{\text{eff}})}{x (1 - x)}. \tag{22}
\]

The PT parameters can thus be directly obtained and are shown as a function of \( p \) in Fig. 1 for the two central values of \( \epsilon \) obtained by the global analyses. Approximate, lowest order in \( \epsilon \) expressions are given by

\[
\begin{align*}
\Delta S &= 4\epsilon p (1 - p) x_0 / \alpha, \\
\Delta T &= -\epsilon p^2 / \alpha, \\
\Delta U &= -4x_0 \Delta T,
\end{align*} \tag{23}
\]

and provide a reasonably adequate description of the results shown in the figure. The important result to notice is that while \( \Delta T \) is negative, both \( \Delta S \) and \( \Delta U \) are positive. These results differ somewhat from those previously obtained in the literature as we rely solely on observables to define \( \Delta S \), \( \Delta T \), and \( \Delta U \). We remind the reader that the values we obtain are due only to the new physics contained in this model and are over and above those contributions arising from shifts in the SM radiative corrections reference point, i.e., other choices of the top-quark and Higgs boson masses \((m_t, m_H)\), respectively. Of course, for a specific choice of these quantities we can ask for the shift in various observables due to the new physics contained in this model. Fig. 2 shows the shifts in \( x_{\text{eff}}, x_w, \) the \( W_1 \) to \( Z_1 \) mass ratio

\[
\delta \rho_0 \equiv \frac{M_{Z_1}^2}{(1 - x_0) M_{W_1}^2} - 1, \tag{24}
\]
and the shift in the overall normalization of $Z_1$ partial width to lepton pairs in the ‘$G_F$’ scheme, $\rho_Z$, defined via

$$\Gamma_\ell \equiv \rho_Z \frac{G_F M_{Z_1}^3}{24\sqrt{2} \pi} \left[ 1 + (1 - 4x_{eff})^2 \right] \left( 1 + \frac{3\alpha(M_{Z_1})}{4\pi} \right),$$

(25)

as functions of $p$ for the same choice of $\epsilon$ values as in Fig. 1. Recent analyses which attempt to determine the allowed ranges for the PT parameters are generally found to favor negative values for both $\Delta S$ and $\Delta T$ while $\Delta U$ is hardly constrained by existing data \cite{7}. We note that for small values of $p$, all of the PT parameters are quite small while for large $p$, e.g., $p \geq 0.6$, $\Delta S$ is small and positive while $\Delta T$ can be large and negative. We will see that this large $p$ region is particularly interesting when the UV experienced by the third generation are considered and to this we now turn. By combining the constraints from limits on UV together with those from an analysis of the PT parameters we will find that the the model of Li and Ma is now highly constrained and that $\epsilon$ is forced to be small.

From the results in Figs. 1 and 2 it would appear that in the $p \to 0$ limit one would recover the predictions of the SM. This is in fact true only for the first two generations. Eq.(4) tells us that the additional couplings of the $Z'$ for the third generation grow with decreasing $p$ and thus we expect to see the largest effect from UV in the $p \to 0$ limit. This is shown explicitly in Fig. 3 where we display the fractional change in both the $Z \to b\bar{b}$ and $Z \to \tau^+\tau^-$ partial widths from the expectations of universality as functions of $p$. Analytically, these deviations from universality for both these decay modes can be expressed as

$$\frac{N_2^2[1 + (1 - 4x_3)^2]}{N_2^2[1 + (1 - 4x_{eff})^2]} - 1$$

(26)

for the case of taus(with $x_{eff} \to x_{eff}/3$ and $x_3 \to x_3/3$ in the $b$ case), where we define, in a manner similar to what we did above for the first two generations, the third generation
effective coupling parameters, $N_3$ and $x_3$, that are given by

\[ x_3 = \frac{x \cos \phi}{N_3}, \]

\[ N_3 = \cos \phi + \sin \phi \sqrt{1 - x} \left[ \frac{1 - p}{p(1 + \epsilon)} \right]^{1/2}. \]  

(27)

While sufficiently precise data is not yet available on the $Z \rightarrow b \bar{b}$ partial width, such data does exist in the $\tau$ case\[1, 3\]. Correcting for the finite mass of the $\tau$, we find that the deviation from universality can be at most $-1.26\%$ at the 95% CL. This would rule out values of $p$ below $0.22(0.55)$ for the smaller(larger) choice of $\epsilon$ used in this analysis. Of course this implies larger deviations from the SM for the first two generations and, as discussed above, is just the region where $\Delta T$ can be large and negative while $\Delta S$ remains small and positive. Fig. 4 shows the region excluded in the $p - \epsilon$ plane using the constraints from LEP on violations of $\tau$ universality. To go further we must make some assumptions about the usual SM radiative corrections.

To be definitive, let us assume that the SM Higgs mass is 250 GeV with a top quark mass of 120(150, 180) GeV so that SM radiative corrections can be performed completely. If we then calculate $x_3$ in the SM (which is just the usual $\sin^2 \theta_{eff}$) and compare with the LEP data\[1\] for the $\tau$ polarization and forward-backward asymmetry as well as the corresponding $b$ quark asymmetry, we find that another sizeable region of the $p - \epsilon$ plane is now excluded at the 95% CL as shown in Fig. 4. Also shown in the figure are the corresponding regions which are excluded by a recent $\Delta T - \Delta S$ analysis\[7\] with $m_t$ now fixed at 150 GeV. As can be seen, a combination of all these constraints results in the requirement that $\epsilon$ must be reasonably small for all values of $p$ and thus favoring the smaller value obtained in the global fits of $\tau$ data.

In addition to improved $Z_1$ and $\tau$ data, this model can be probed at hadron colliders
by searches for the $Z'$ and $W'$. Given a set of fixed values for $p$ and $\epsilon$, the $Z - Z'$ and $W - W'$ mass matrices can be used to determine the masses of these new gauge bosons. To a excellent approximation, the $W'$ and $Z'$ are degenerate with masses given by

$$M_{Z',W'}^2/M_Z^2 = \frac{(1-x)}{ep(1-p)}$$

and is shown in Fig. 5 without the use of any approximations. Taking the more conservative determination of $\epsilon$ from the global fits to $\tau$ data we see that the $Z'$ and $W'$ must have masses in excess of 1 TeV at the 95% CL. We can, of course, ask what limits can be set on the $Z'$ and $W'$ from Tevatron data which is presently available, or will be available before the turn-on of the SSC/LHC; this is shown in Fig. 6. As we can see from this figure, the Tevatron will be barely able to touch the region of interest even if an integrated luminosity as large as 1 $fb^{-1}$ is accumulated. The $Z'$ and $W'$ physics in this model must be left to the SSC/LHC or TeV $e^+e^-$ linear colliders to probe. Fig. 7, for example, shows the anticipated signal rate for the $Z'$ in this model at the SSC after cuts and efficiencies are accounted for assuming an integrated luminosity of 10 $fb^{-1}$, corresponding to 1 ‘SSC-year.’ Certainly a $Z'$ in the several TeV mass range will prove to be easily observable. In the near future, however, it is thus most likely that stricter constraints upon the parameter space of this model will only result from further refinements in $\tau$ and $Z_1$ data, provided universality violations are not in fact observed.

In this paper we have re-examined the predictions of the universality violating model of Li and Ma in light of recent high precision data from LEP on the properties of the $Z$ as well as new data on the properties of the $\tau$. We find that a combination of this data, taken together with a knowledge of the SM radiative corrections, places very strong constraints on the two parameters of the model: $p$ and $\epsilon$. For a top quark mass of 150 GeV and a Higgs scalar mass of 250 GeV, we find for example that values of $\epsilon > 0.009$ are excluded for all
values of $p$, while for $p$ less than 0.3, even small values of $\epsilon$ are excluded. Existing constraints on the model parameters were shown to forbid the direct discovery of the new $Z'$ and $W'$ gauge bosons at the Tevatron but they may be copiously produced at, *e.g.*, the SSC if their masses do not exceed a few TeV.

Searches for the possibility of universality violation by the particles in the third generation must continue.

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Figure Captions

Figure 1. (a) $\Delta T$, (b) $\Delta S$, and (c) $\Delta U$ as functions of the parameter $p$ with the smaller(larger) value of $\epsilon$ discussed in the text corresponding to the dotted(dashed) curve.

Figure 2. Shifts in the values of (a) $x_{eff}$, (b) $\delta\rho_0$, (c) $x_w$, and (d) $\delta\rho_Z$ as functions of $p$ as described in the text for the same two choices of $\epsilon$ as shown in Fig. 1.

Figure 3. Fractional change in the (a) $Z \rightarrow bb$ and (b) $Z \rightarrow \tau^+\tau^-$ partial widths as functions of $p$ for the two choices of $\epsilon$ shown in Fig. 1 relative to the predictions of universality.

Figure 4. Allowed region in the $p-\epsilon$ plane from LEP data on the $Z \rightarrow \tau^+\tau^-$ partial width(solid) and from a radiative corrections analysis determination of the bound on $x_3$ assuming a SM Higgs mass of 250 GeV and a top quark mass of 120(leftmost), 150(center), or 180(rightmost) GeV represented by the sequence of dashed curves. The allowed region is to the left of each curve. The corresponding bounds from a $\Delta S$(dotted) and $\Delta T$(dash-dots) analysis is also shown for the $m_t=150$ GeV case.

Figure 5. Lower bound on the $Z'$ or $W'$ masses in this model as functions of the parameter $p$ for the same choices of $\epsilon$ as in Fig. 1.

Figure 6. Tevatron search limits for the $Z'$ or $W'$ as functions of $p$: present limits(dotted), and future limits from a total integrated luminosity of 25(100, 400, 1000) $pb^{-1}$ shown as a dashed(dash dotted, solid, square-dotted) curve.

Figure 7. Number of $Z'$-induced dilepton events which would be observed at the SSC, as a function of $p$, assuming an integrated luminosity of 10 $fb^{-1}$, a lepton identification efficiency of 0.85, and demanding both leptons lie in the pseudorapidity interval $-2.5 \leq \eta \leq 2.5$. The $Z'$ mass is assumed to be 1(2, 3, 4, 5) TeV in the case of the dotted(dashed, dash-dotted, solid, square-dotted) curve.