Measurements of angular distance and momentum ratio distributions in three-jet and $Z +$ two-jet final states in pp collisions

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Abstract Collinear (small-angle) and large-angle, as well as soft and hard radiations are investigated in three-jet and $Z +$ two-jet events collected in proton-proton collisions at the LHC. The normalized production cross sections are measured as a function of the ratio of transverse momenta of two jets and their angular separation. The measurements in the three-jet and $Z +$ two-jet events are based on data collected at a center-of-mass energy of 8 TeV, corresponding to an integrated luminosity of 19.8 fb$^{-1}$. The $Z +$ two-jet events are reconstructed in the dimuon decay channel of the $Z$ boson. The three-jet measurement is extended to include events in the dimuon decay channel of the $Z$ boson. The three-jet measurement is extended to include events in the dimuon decay channel of the $Z$ boson. The three-jet measurement is extended to include events in the dimuon decay channel of the $Z$ boson. The three-jet measurement is extended to include events in the dimuon decay channel of the $Z$ boson.

1 Introduction

Collimated streams of particles, produced in interactions of quarks and gluons and reconstructed as jets, are described by the theory of strong interactions, quantum chromodynamics (QCD). Multijet events provide exemplary signatures in high-energy collider experiments, and modeling their characteristics plays an important role in precision measurements, as well as in searches for new physics. The understanding of the structure of multijet final states is therefore crucial for analyses of those events.

Theoretical predictions for multijet events are based on a matrix element (ME) expansion to a fixed perturbative order, supplemented by the parton shower (PS) approach to approximate higher-order perturbative contributions. The ME expansion incorporates color correlations between quarks and gluons, including interference terms, as well as kinematic correlations between the partons, without any approximation at fixed perturbative order. Its application is, however, currently limited to final states with less than $O(10)$ partons. The PS can simulate final states containing many partons, but with probabilities calculated using the approximations of soft and collinear kinematics and partial or averaged color structures. The best descriptions of multijet final states are based on a combination of both approaches [1–4]. Other features implemented in simulations, such as multiple parton interactions (MPI) and hadronization, also play an important role, e.g., in describing angular correlations between jets [5–7].

In this paper, we investigate collinear (small-angle) and large-angle radiation in different regions of jet transverse momentum ($p_T$) by concentrating on two different topologies, one using three-jet events and another with $Z +$ two-jet events. We label the hardest jet, or $Z$ boson as $j_1$, the next hardest as $j_2$, and the softest as $j_3$. We introduce two observables that are sensitive to the dynamic properties of multijet final states. One observable is the $p_T$ ratio of $j_3$ to $j_2$, $p_{T3}/p_{T2}$. The other observable is the angular distance between the jet centers of $j_2$ and $j_3$ in the rapidity-azimuth ($y$-$\phi$) phase space, $\Delta R_{23} = \sqrt{(y_3 - y_2)^2 + (\phi_3 - \phi_2)^2}$. The definition of rapidity is $y = \ln\left(\frac{E + p_z}{E - p_z}\right)$, and the definitions of other kinematic variables are given in Ref. [8]. As indicated in Fig. 1, we classify three-jet and $Z +$ two-jet events into different categories using these two observables:

(i) soft ($p_{T3}/p_{T2} < 0.3$) or hard ($p_{T3}/p_{T2} > 0.6$) radiation, depending on the ratio $p_{T3}/p_{T2}$;

(ii) small-angle ($\Delta R_{23} < 1.0$) or large-angle ($\Delta R_{23} > 1.0$) radiation, depending on the angular separation $\Delta R_{23}$.

According to these classifications, events in the soft and small-angle radiation region, as shown in Fig. 1a, can only be described if soft gluon resummation, e.g., in form of a parton shower, is included, whereas events in the hard and
large-angle radiation region, as shown in Fig. 1d, would be better described when including the ME calculations. The events in Fig. 1b and c are also of interest, since they should include effects from both the PS and ME.

We report on proton-proton (pp) collision data collected at the CMS experiment containing three-jet events at center-of-mass energies of 8 and 13 TeV, and Z + two-jet events at a center-of-mass energy of 8 TeV. The measurements are compared to calculations based on a leading-order (LO) or next-to-leading-order (NLO) ME supplemented with effects from PS, MPI, and hadronization. The NLO ME descriptions apply to the lowest parton multiplicities relevant to the selected events: 2 jets for the three-jet analysis and Z+1j for the Z + two-jet analysis. The measurements using three-jet final states are complementary to those with Z + two-jet events in a sense that different kinematic regions and initial-state flavor compositions are being probed. The jets are also fully color connected, while the Z boson is color neutral, so color coherence effects should not appear so strongly in Z + two-jet events.

The goal of the measurements is: (i) to untangle the different features of the radiation in the collinear and large-angle events; (ii) to investigate how well the PS approach describes the hard and large-angle radiation patterns; and (iii) to illustrate how ME calculations can attempt to describe the soft and collinear regions.

2 The CMS detector

The central feature of the CMS detector is a superconducting solenoid of 6 m internal diameter, providing a magnetic field of 3.8 T. A silicon pixel and strip tracker, a lead tungstate crystal electromagnetic calorimeter (ECAL), and a brass and scintillator hadron calorimeter (HCAL), each composed of a barrel and two endcap sections, reside within the volume of the solenoid. Charged-particle trajectories are measured in the tracker with full azimuthal acceptance within pseudorapidities $|\eta| < 2.5$. The ECAL, which is equipped with a preshower detector in the endcaps, and the HCAL cover the region $|\eta| < 3.0$. Forward calorimeters extend the pseudorapidity coverage provided by the barrel and endcap detectors to the region $3.0 < |\eta| < 5.2$. Finally, muons are measured up to $|\eta| < 2.4$ in gas-ionization detectors embedded in the steel flux-return yoke outside the solenoid. Events of interest are selected using a two-tiered trigger system [9]. The first level, composed of custom hardware processors, uses information from the calorimeters and muon detectors to select events at a rate of around 100 kHz within a fixed latency of about 4 $\mu$s. The second level, known as the high-level trigger (HLT), consists of a farm of processors running a version of the full event reconstruction software optimized for fast processing, and reduces the event rate to around 1 kHz before data storage.

A more detailed description of the CMS detector, together with a definition of the coordinate system and the kinematic variables, is given in Ref. [8].

3 Event samples and selection

The data in this study were collected with the CMS detector at the LHC using pp collisions at center-of-mass energies of 8 and 13 TeV. The $\sqrt{s} = 8$ TeV data, taken in 2012 during LHC Run 1, correspond to an integrated luminosity of 19.8 fb$^{-1}$, and the $\sqrt{s} = 13$ TeV data, taken in 2015 during LHC Run 2, correspond to an integrated luminosity of 2.3 fb$^{-1}$.

Particles are reconstructed and identified using a particle-flow (PF) algorithm [10], that utilizes an optimized combination of information from the various elements of the CMS detector. Jets are reconstructed by clustering the four-vectors of the PF candidates with the infrared and collinear-safe anti-$k_T$ clustering algorithm [11] using a distance parameter $R_{\text{jet}} = 0.5$ (0.4) at $\sqrt{s} = 8$ (13) TeV. The clustering is performed with the FASTJET software package [12]. The jets are ordered in $p_T$ and all events with additional jets are analyzed. In addition, three-jet events use the charged-hadron subtraction (CHS) technique [10] to mitigate the effect of extraneous pp collisions in the same bunch crossing (pileup, PU). The CHS technique reduces the contribution to the reconstructed jets from PU by removing tracks identified as originating from PU vertices.

Muons are reconstructed using a simultaneous global fit performed with the hits in the silicon tracker and the muon system. They are required to pass standard identification criteria [13,14] based on the minimum number of hits in each detector, quality of the fit, and the consistency with the primary vertex by requiring the longitudinal (transverse) impact parameters to be less than 0.5 (0.2) cm. The efficiency to reconstruct and identify muons is greater than 95% over the entire region of pseudorapidity covered by the CMS muon system ($|\eta| > 2.4$). The overall momentum scale is mea-
sured to a precision of 0.2% with muons from Z decays. The transverse momentum resolution varies from 1 to 6% depending on pseudorapidity for muons with \( p_T \) for a few GeV to 100 GeV and reaches 10% for 1 TeV muons [15]. Observed distributions for muons are well reproduced by Monte Carlo (MC) simulation. Corresponding scale factors for the difference between data and MC simulations are measured with good accuracy [16]. Muons must be isolated from other activity in the tracker by requiring the \( p_T \) sum of other tracks within a cone of radius \( \Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.3 \) centered on the muon candidate, is less than 10% of the muon \( p_T \). If the two muons with the highest \( p_T \) in an event are within the isolation cone of one another, the other muon candidate is removed from the isolation sum of each muon.

Three-jet events are collected using single jet HLT requirements that are not pre-scaled. The \( \sqrt{s} = 8 \) (13) TeV data use a 320 (450) GeV trigger \( p_T \) threshold. In the offline analyses, the \( p_T \) threshold starts at 510 GeV for both sets of data. The Z + two-jet events with the Z boson decaying into a pair of muons are collected at \( \sqrt{s} = 8 \) TeV with a single-muon HLT that requires a muon \( p_T > 24 \) GeV and \( |\eta| < 2.1 \).

In the three-jet systems, the leading jet is required to have a \( p_T > 510 \) GeV, because of a decreasing efficiency for single jet triggers below this value [9,17,18]. Events with at least three jets of \( p_T > 30 \) GeV are selected for further consideration. The leading and subleading jets must be within a rapidity range of \( |y| < 2.5 \), and the third jet is therefore implicitly restricted to \( |y| < 4 \) by requiring \( \Delta R_{23} < 1.5 \). A dijet topology with an extra jet is selected by requiring the difference in azimuthal angle between the first and second jet to be \( \pi - 1 < \Delta \phi_{12} < \pi \). The missing transverse momentum vector \( p_T^{\text{miss}} \) is defined as the projection onto the plane perpendicular to the beam axis of the negative vector sum of the momentum of all reconstructed PF objects in an event. Its magnitude is referred to as \( p_T^{\text{miss}} \). Events with a \( p_T^{\text{miss}} \) divided by the scalar sum of all transverse momenta > 0.3 are rejected to remove the contamination from Wor Z boson decays [19–21]. To avoid an overlap between \( j_2 \) and \( j_3 \), \( \Delta R_{23} \) is required to be larger than the distance parameter \( R_{\text{jet}} \). We thus require \( \Delta R_{23} \) to be larger than 0.6 (0.5) for \( \sqrt{s} = 8 \) (13) TeV data. The maximum \( \Delta R_{23} \) is set to 1.5 to ensure that \( j_3 \) is closer to \( j_2 \) than to \( j_1 \). We further require that 0.1 < \( p_T^{j_3}/p_T^{j_2} < 0.9 \) to avoid \( p_T^{j_3} \) threshold effects and to ensure \( p_T^{j_2} \) ordering for hard radiation.

In Z + two-jet events, the Z boson is reconstructed from a pair of oppositely charged, isolated muons with \( p_T > 25 \) (5) GeV and \( |y| < 2.1 \) (2.4) for the leading (subleading) muon. Muons are required to be from primary vertex with distance \( dr < 0.2 \) cm and \( dz < 0.5 \) cm. The dimuon invariant mass is required to be \( 70 < m_{\mu^+\mu^-} < 110 \) GeV with the dimuon momentum satisfying \( p_T^{j_1} > 80 \) GeV and \( |y_1| < 2 \). At least two jets are required in the final state with the leading jet (labeled \( j_2 \)) satisfying \( p_T^{j_2} > 80 \) GeV and \( |y_2| < 1 \) and the subleading jet (labeled \( j_3 \)) required to have \( p_T^{j_3} > 20 \) GeV with \( |y_3| < 2.4 \). The distance between muons from Z bosons and jets are requested to be more then 0.5. The Z + two-jet topology is further restricted by requiring a difference in the azimuthal angle between the Z boson and \( j_2 \) of \( \Delta \phi_{12} > 2 \).

Table 1 shows a summary of the event selection requirements for both samples.

Generator jets are reconstructed from stable particles by clustering the four-vectors with an anti-\( k_T \) clustering algorithm excluding neutrinos. The kinematical requirements for muons and jets are the same as applied for reconstructed objects. For Z + two-jet events, the distance between muons from Z boson and jets must have \( \Delta R > 0.5 \). The \( p_T^{\text{miss}} \) selection is not applied at the generator level for QCD multijet events.

### 4 Theoretical models

Reconstructed data are compared to predictions from MC event generators, where the generated events are passed through a full detector simulation based on GEANT4 [22] and the simulated events are reconstructed using standard CMS software. Reconstruction-level predictions are obtained for three-jet events at \( \sqrt{s} = 8 \) TeV with the MADGRAPH [23] software package matched to PYTHIA 6 [24] with the CTEQ6L1 [25] parton distribution function (PDF) set and the Z2Star tune [26], as well as with standalone PYTHIA 8.1 [27] with the CTEQ6L1 PDF set and the 4C [28] tune. At 13 TeV, MADGRAPH interfaced to PYTHIA 8.2 [29] and standalone PYTHIA 8.2 are used with the NNPDF2.3LO [30] PDF set and the CUETP8M1 [31] tune. The SHERPA [32] event generator interfaced to CSSHOWER++ [33] with the CT10 [34] PDF set and the AMISIC++ [35] tune and MADGRAPH interfaced to PYTHIA 6 with the CTEQ6L1 PDF set and the Z2Star tune provide Z + two-jet events at 8 TeV. Table 2 summarizes the event generator versions, PDF sets and tunes.

Results corrected to stable-particle level are compared to predictions obtained with the models presented below. An overview of these models is given in Table 3.

The PYTHIA 8 [29] event generator provides hard-scattering events using a ME calculated at LO supplemented with PS. These event samples are labeled as “PYTHIA LO 2j+PS” for the three-jet and as “PYTHIA LO Z+1j+PS” for Z + two-jet events. The PDF set NNPDF2.3LO and the CUETP8M1 parameter set for the simulation of the underlying event (UE) are used with free parameters adjusted to measurements in pp collisions at the LHC and proton-antiproton collisions at the Fermilab Tevatron. The Lund string model [36] is applied for the hadronization process.

The MADGRAPH5_aMC@NLO event generator, labeled as “MADGRAPH” in the following, is used to simulate hard processes with up to 4 final-state partons at LO accuracy. It is
Table 1 Phase space selection for the three-jet and Z + two-jet analyses

| Event generator | PDF set | Tune |
|-----------------|---------|------|
| MADGRAPH 5.1.3.30 + PYTHIA 6.425 | CTEQ6L1 | Z2Star |
| PYTHIA 8.153 | CTEQ6L1 | 4C |
| MADGRAPH 5.2.3.3 + PYTHIA 8.219 | NNPDF2.3LO | CUETP8M1 |
| PYTHIA 8.219 | NNPDF2.3LO | CUETP8M1 |
| Z + two-jet events | | |
| SHERPA 1.4.0 + CSShower++ | CT10 | AMISIC++ |
| MADGRAPH 5.1.3.30 + PYTHIA 6.425 | CTEQ6L1 | Z2Star |

Table 2 Event generator versions, PDF sets, and tunes used to produce MC samples at reconstruction level

interfaced to PYTHIA 8 with the CUETP8M1 tune and the NNPDF2.3LO PDF set for the simulation of PS, hadronization, and MPI, for three-jet, and to PYTHIA 6 with the Z2Star tune and the CTEQ6L1 PDF set for Z + two-jet events. The three-jet sample is labeled as “MADGRAPH LO 4j+PS” and the Z + two-jet sample is labeled as “MADGRAPH LO Z+4j+PS”. The kT-MLM procedure [37] is used to match jets from the ME and PS with a matching scale of 10 GeV.

Predictions are also included using the POWHEG BOX library [38–40], with the CT10 NLO [34] PDFs and with the PYTHIA 8 CUETP8M1 tune applied to simulate PS, MPI, and hadronization. The POWHEG generator is run in the dijet mode [41] providing an NLO 2 + 2 calculation, labeled as “POWHEG NLO 2j+PS”. The matching between the POWHEG ME calculations and the PYTHIA UE [31] simulation is performed using the shower-veto procedure (UserHook option 2 [29]).

The SHERPA software package is used to simulate Z + two-jet events. The hard process is calculated at LO for a ME with up to four final-state partons and the CT10 PDF set is used. This sample is labeled as “SHERPA LO Z+4j+PS”. The SHERPA generator has its own PS [33], hadronization, and MPI tune [35].

Finally, the MADGRAPH5_aMC@NLO generator is also used in the MC@NLO mode, providing a Z + one-jet ME at NLO accuracy. This event generator is interfaced to PYTHIA 8, using the CUETP8M1 tune and the NNPDF3.0NLO [42] PDF set, to produce Z + two-jet events. The sample is labeled as “aMC@NLO NLO Z+1j+PS”.

The background from W, Z, top quark, and diboson production for the three-jet analysis is negligible and not further considered. The main background for Z + two-jet events comes from t(t), single top, and diboson production. The t(t), ZZ, and WZ events are simulated with MADGRAPH 5.1.3.30 + PYTHIA 6.425 using the same tune and PDF set as for generating Z + two-jet samples. WW events are generated with PYTHIA 6.425 with CTEQ6L1 PDF set and Z2Star tune. Single top events are generated with POWHEG (CT10 PDF set, Z2Star tune).
5 Data correction and study of systematic uncertainties

To facilitate the comparison of data with theory, the data are unfolded from reconstruction to stable-particle level, defined by a mean decay length larger than 1 cm, so that measurement effects are removed and that the true distributions in the observables are determined. The unfolding is performed using the D’Agostini algorithm [43] as implemented in the RooUNFOLD software package [44] for three-jet events, while the singular value decomposition method [45] is used for Z + two-jet events. The response matrices are obtained from the full detector simulation using MadGraph for three-jet events and Sherpa for Z + two-jet events.

We estimate the influence of $t\bar{t}$, single top, and diboson backgrounds by adding generated events produced with event generator MadGraph LO $Z+4j+PS$ and comparing the predictions for the observables $p_{T3}/p_{T2}$ and $\Delta R_{23}$ using the same generator without the backgrounds. For $t\bar{t}$ production with fully leptonic decay and dibosons the probability of $j_3$ emission increases from 2% (soft radiation) to 10% (hard radiation) depending on the phase space. For semileptonic and hadronic decays and single top production the change is negligible. Since the background effect is comparable to the systematic uncertainties, it is not included in the theoretical estimations and it is not subtracted from the data.

The distributions are normalized to the integral of the spectra for three-jet events and to the number of inclusive Z + one-jet events in the Z + two-jet analysis. The Z + two-jet analysis normalization thus reflects the probability to have more than one jet in the event.

Systematic uncertainties associated to the jet energy scale (JES) calibration, the jet energy resolution (JER), PU modeling, model dependence, as well as the unfolding method, are estimated. Muon-related uncertainties (single muon trigger efficiency, muon isolation, muon scale and resolution) for the Z + two-jet channel are negligible with respect to other systematic sources. The treatment of the uncertainty depends on the uncertainty source and is estimated separately for each bin (see below). The overall uncertainty for each bin is estimated summing in quadrature uncertainties from the various sources.

The systematic uncertainty from the JES is 0.15 (0.24)% at $\sqrt{s} = 8$ (13) TeV for the three-jet case and 5–10% for the Z + two-jet events. The JER observed in data differs from that obtained from simulation and simulated jets are therefore smeared to obtain the same resolution as in the data [46]. The systematic uncertainty from JER is estimated by varying the simulated JER uncertainty up and down by one standard deviation, which results in a systematic uncertainty of 0.16 (0.12)% at $\sqrt{s} = 8$ (13) TeV for three-jet and 2–3% for Z + two-jet events when the distributions of $Z + \text{two-jet}$ events are normalized to the integrals of the histograms, instead of the number of $Z + \text{one-jet}$ events, the systematic uncertainties due to the JES and JER decrease to 0.3–0.5%, except for the $p_{T3}/p_{T2}$ shape, which is still sensitive to the JES with changes of up to 3%.

The distribution in the number of primary vertices is sensitive to the PU difference between data and simulation. To estimate the uncertainty due to the PU modeling, the number of PU events in simulation is changed by shifting the total inelastic cross section by ±5% [47]. The resulting PU uncertainties are 0.10 (0.17)% at $\sqrt{s} = 8$ (13) TeV for the three-jet and 1% for the Z + two-jet events.

The dependence on the event generator used for the unfolding is estimated with MC event samples from MadGraph and Pythia for three-jet, and Sherpa and MadGraph for the Z + two-jet events. The means of both sets vary the simulated JER uncertainty up and down by one standard deviation, which results in a systematic uncertainty of 0.16 (0.12)% at $\sqrt{s} = 8$ (13) TeV for three-jet and 2–3% for Z + two-jet events. When the distributions of $Z + \text{two-jet}$ events are normalized to the integrals of the histograms, instead of the number of $Z + \text{one-jet}$ events, the systematic uncertainties due to the JES and JER decrease to 0.3–0.5%, except for the $p_{T3}/p_{T2}$ shape, which is still sensitive to the JES with changes of up to 3%.

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Table 4 summarizes the systematic uncertainties in the measurements.
The systematic uncertainties from various sources are similar for the three-jet samples at $\sqrt{s} = 8$ and 13 TeV, except for unfolding and model dependence at $\sqrt{s} = 8$ TeV. The systematic uncertainties between the three-jet and Z + two-jet analysis cannot be compared directly because each analysis uses a different normalization and also differs in statistical significance. The JES uncertainty is especially sensitive to the jet $p_T$ range, and the Z + two-jet phase space has a lower $p_T$ threshold than the one used in the three-jet events.

The figures of Sect. 6 show the total systematic uncertainty as a band in the panels displaying the ratio of predictions over data.

### 6 Results

We compare the distributions in the ratio $p_{T3}/p_{T2}$ in data to predictions for events with small-angle ($\Delta R_{23} < 1.0$) and large-angle radiation ($\Delta R_{23} > 1.0$). We also compare the $\Delta R_{23}$ distributions in data to predictions with soft ($p_{T3}/p_{T2} < 0.3$) and hard radiation ($p_{T3}/p_{T2} > 0.6$). The events with $0.3 < p_{T3}/p_{T2} < 0.6$ are not used in the comparisons for the $\Delta R_{23}$ observable because we focus on the limits in soft and hard radiation. This classification is summarized in Fig. 1, within the phase space defined in Table 1.

The data measurements are provided at the Durham High Energy Physics Database (HEPData) [48].

The uncertainties in the PDF and in the renormalization and factorization scales are investigated for the POWHEG and amC@NLO models. Other theoretical predictions are expected to have comparable uncertainties. The PDF uncertainties are calculated as recommended in PDF4LHC [49] following the description of the PDF sets: for CT10 using the Hessian approach; and for NNPDF using MC replicas. The renormalization and factorization scales are varied by a factor 2 up and down, excluding the (2,1/2) and (1/2,2) cases. Finally, the theoretical uncertainties are obtained as the quadratic sum of the PDF variance and the envelope of the scale variations, and displayed as a band around the theoretical predictions in the Figs. 2, 3, 4, 5, 6 and 7.

### 6.1 Three-jet selection

We show the $\sqrt{s} = 8$ TeV measurements of $p_{T3}/p_{T2}$ in Fig. 2 and of $\Delta R_{23}$ in Fig. 3, and compare them to theoretical expectations. In Figs. 4 and 5 the distributions are given for $\sqrt{s} = 13$ TeV. Figure 2 (upper) shows the $p_{T3}/p_{T2}$ distribution for the small $\Delta R_{23}$ region. All predictions show significant deviations from the measurements. Interestingly, the LO 4j+PS prediction shows different behavior compared with LO 2j+PS and NLO 2j+PS. We see that the number of partons in the ME calculation and the merging method with the PS in the present simulations lead to different predictions. In Fig. 2 (lower) the $p_{T3}/p_{T2}$ distribution is shown for large $\Delta R_{23}$. This region of phase space is well described by the LO 4j+PS calculations, while the LO 2j+PS and NLO 2j+PS predictions show large deviations from the measurements.

In Fig. 3, the $\Delta R_{23}$ distribution is shown for two regions of $p_{T3}/p_{T2}$. Figure 3 (upper) shows $p_{T3}/p_{T2} < 0.3$. The predictions from LO 2j+PS and NLO 2j+PS describe the measurement well, while the prediction from LO 4j+PS shows a larger deviation from the data. In Fig. 3 (lower) the $\Delta R_{23}$ distribution is shown for $p_{T3}/p_{T2} > 0.6$. In contrast to Fig. 3 (upper), the predictions for distributions from LO 2j+PS differ from the measurement, whereas the predictions from NLO 2j+PS and LO 4j+PS agree well with it. This indicates that in this region the contribution from higher-multiplicity ME calculations supplemented with PS should be included. The same comparisons are performed for the $\sqrt{s} = 13$ TeV measurements as shown in Figs. 4 and 5. A similar behavior is observed for $\sqrt{s} = 8$ TeV. In conclusion, none of the simulations simultaneously describes to simultaneously describe both the $p_{T3}/p_{T2}$ and the $\Delta R_{23}$ distributions in three-jet events.

### 6.2 Z + two-jet selection

The measurement of $p_{T3}/p_{T2}$ for Z + two-jet events is presented in Fig. 6 for data at $\sqrt{s} = 8$ TeV. All distributions are normalized to the selected number of Z + one-jet events. All predictions from PYTHIA, SHERPA, MADGRAPH, and amC@NLO agree with data within the uncertainties of the measurement except for the phase space region with hard radiation.

Figure 7 shows the measurement as a function of $\Delta R_{23}$. The amC@NLO prediction deviates from the data at high $\Delta R_{23}$ and small $p_{T3}/p_{T2}$, while PYTHIA, SHERPA, MADGRAPH, and amC@NLO describe the shape of the distribution in the high-$p_{T3}/p_{T2}$ range, but underestimate the data due to fluctuations.
to a smaller contribution from production of $j_3$. This feature is based on the normalization of Z + two-jet distributions by the number of inclusive Z + one-jet events selected.

Figures 8 and 9 compare the event distributions with predictions from PYTHIA 8 with the final-state PS and MPI switched off. The initial-state PS was kept, because one of the jets must originate from PS when Z + two-jet events are selected. Multiple parton interactions play a very minor role, while the final-state PS in PYTHIA 8 is very important. When the final-state PS is switched off, events where both jets come from the initial-state PS are kept with a tendency to be close to each other in $\Delta R_{23}$.

In general, the measurements with Z + two-jet events are well described by all theoretical predictions, except for the underestimation of the $j_3$ emission. The contribution of background from $t\bar{t}$ production and dibosons can partially compensate the lack of the $j_3$ emission. The contribution of the background ($t\bar{t}$ production with fully leptonic decay and dibosons) increases the probability of $j_3$ emission from 2% (soft radiation) to 10% (hard radiation) depending on the phase space region. The effect of the other processes ($t\bar{t}$ pro-

\[ \frac{dN}{d\Delta R_{23}} \]

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duction with semileptonic and hadronic decays, single top production) is negligible. In comparison with the three-jet measurements, we observe significant differences; only in the region of large $\Delta R_{23}$ and large $p_T^3/p_T^2$ (hard and large-angle radiation) do the theoretical predictions agree with the measurement. The accessible range in $p_T$ is rather small in $Z +$ two-jet events because of the limit in the $p_T$ of the $Z$ bosons ($p_T^1 > 80$ GeV), while the three-jet selection, on the contrary, can have a rather large range ($p_T^1 > 510$ GeV). This may explain why the region of small $p_T^3/p_T^2$ is better described by predictions that include PS in the latter case. In addition, the large-angle radiation is best described by fixed-order ME calculations.

In conclusion, the $Z +$ two-jet measurement has a different distribution in $p_T^3/p_T^2$, which originates from the different kinematic selection criteria relative to three-jet events, thus reducing the sensitivity in the soft and collinear region. Within the available phase space, the measurements are in reasonable agreement with both PS and ME calculations, apart from the emission of $j_3$ in the high-$p_T^3/p_T^2$ region.
Two kinematic variables are introduced to quantify the radiation pattern in multijet events: (i) the transverse momentum ratio \( p_{T3}/p_{T2} \) of two jets, and (ii) their angular separation \( \Delta R_{23} \). The variable \( p_{T3}/p_{T2} \) is used to distinguish between soft and hard radiation, while \( \Delta R_{23} \) classifies events into small- and large-angle radiation types. Events with three or more energetic jets as well as inclusive \( Z + \) two-jet events are selected for study using data collected at \( \sqrt{s} = 8 \) TeV corresponding to an integrated luminosity of 19.8 fb\(^{-1}\). Three-jet events at \( \sqrt{s} = 13 \) TeV corresponding to an integrated luminosity of 2.3 fb\(^{-1}\) are also analyzed. No significant dependence on the center-of-mass energy is observed in the differential distributions of \( p_{T3}/p_{T2} \) and \( \Delta R_{23} \).

Overall, large-angle radiation (large \( \Delta R_{23} \)) and hard radiation (large \( p_{T3}/p_{T2} \)) are well described by the matrix element (ME) calculations (using LO 4j+PS formulations), while the parton shower (PS) approach (LO 2j+PS and NLO 2j+PS) fail to describe the regions of large-angle and hard radia-
The collinear region (small $\Delta R_{23}$) is not well described; LO 2j+PS, NLO 2j+PS, and LO 4j+PS distributions show deviations from the measurements. In the soft region (small $p_{T3}/p_{T2}$), the PS approach describes the measurement also in the large-angle region (full range in $\Delta R_{23}$), while for large $p_{T3}/p_{T2}$ higher-order ME contributions are needed to describe the three-jet measurements. The distributions in $Z$ + two-jet events are reasonably described by all tested generators. Nevertheless, we find an underestimation of third-jet emission at large $p_{T3}/p_{T2}$ both in the collinear and large-angle regions, for all of the tested models. Contribution from $t\bar{t}$ and dibosons production may partially cover the difference. These results illustrate how well the collinear/soft, and large-angle/hard regions are described by different approaches. The different kinematic regions and initial-state flavor composition may be the reason why the three-jet measurements are less consistent with the theoretical predictions relative to the $Z$ + two-jet final states. These results clearly indicate
that the methods of merging ME with PS calculations are not yet optimal for describing the full region of phase space. The measurements presented here serve as benchmarks for future improved predictions coming from ME calculations combined with parton showers.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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