How Plasma Composition affects the relativistic flows and the emergent spectra

Abstract. It has been recently shown that transonic electron-positron fluid is the least relativistic, compared to the fluid containing finite proportion of baryons. We compute spectra from these flows in general relativity (GR) including the effect of light bending. We consider the bremsstrahlung process to supply the seed photons. We choose accretion in the advective domain, and for simplicity the radial accretion or Bondi type accretion. We show that electron-positron accreting flow produces the softest spectra and the lowest luminosity.

1 Introduction

Relativistic flows are likely to be encountered in accretion discs around compact objects like neutron stars and black holes, astrophysical jets, Gamma Ray Bursts (GRBs) etc. A fluid is said to be relativistic if the bulk speed of the plasma is comparable to the speed of light ($c$), or if its thermal energy is comparable or greater than its rest energy — a fancy way of saying, that the random or jittery speed of the constituent particles of the fluid become comparable to the speed of light. This brings the issue of equation of state (EoS) of the fluid. The issue of relativistic equation of state was raised long ago (Chandrasekhar 1938, hereafter C38; Taub 1948; Synge 1957, hereafter S57), and was used for theoretical calculations too (Blumenthal & Mathews 1976, hereafter BM76; Fukue 1987), although did not become very popular in the community. Later Falls & Komissarov (1996) showed that the relativistic EoS of the form presented by C38 and S57, is computationally expensive which led Mignone et al. (2005) to use the EoS of the form presented by BM76. Ryu et al. (2006, hereafter RCC06), proposed a new EoS, which is very close to the EoS by C38 and S57, and better than the EoS proposed by BM76, and which can be efficiently implemented in a simulation code. However, these EoS states are for single species fluid. Chattopadhyay (2008; hereafter C08), Chattopadhyay & Ryu (2009, hereafter CR09) then modified the single species EoS for multi-species EoS, and analytically showed that the accreting and solutions around black holes are indeed dependent on the composition of the flow. Most interestingly, CR09 showed that electron-positron jets are the least relativistic when compared with flows containing protons. The most relativistic flow is the one with composition parameter (defined as the ratio proton to electron number density) $\xi \sim 0.24$. Later Chattopadhyay & Chakrabarti (2011; hereafter CC11) showed that fluids containing protons can produce accretion shocks while an entirely leptonic fluid ($\xi = 0$) will not be able to form accretion shocks.

Although it is difficult to envisage an accreting flow entirely composed of leptons i.e., electrons-positrons from close to the horizon to infinity, however, our main intention is to show that — (1) fluid behaviour do depend on composition even in absence of composition, and (2) it is erroneous to assume the most relativistic matter fluid is the lightest one that is pair plasma. In this paper therefore we want to compute the radiation produced by flow accreting onto a black hole, and show that indeed the spectra also depends on the composition of the fluid even if the outer boundary condition is the same. For that purpose we have developed a general relativistic Monte-Carlo code.

2 Governing Equations

We assume a non-rotating black hole described by the Schwarzschild radii

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

where, $G$, $M$, and $c$ are the universal gravitational constant, the mass of the central black hole and the speed of light, respectively, and $t$, $r$, $\theta$, $\phi$ are the usual four coordinates.

$$T^\mu_\nu - T^\nu_\mu = 0,$$
where, the energy momentum tensor of the fluid is \( T^{\mu \nu} = (\varepsilon + p) u^\mu u^\nu + p g^{\mu \nu} \), the radiation stress tensor being \( T_{\text{rad}}^{\mu \nu} \), where \( N \) is the total number density of the fluid. The fluid equation is solved theoretically assuming there is no radiation. And then it is fed to the general relativistic Monte Carlo code. The Monte Carlo code estimates bremsstrahlung emission from each grid point, and then the emitted photons travels and impinges on the electron at some other point. If the energy of the photon is less than electrons kinetic or thermal energy then inverse-Comptonization, i.e., the photon will take energy away. If the photon energy is more, then the opposite happens. The energy density of the fluid is given by (see, CR08, CR09).

\[
\varepsilon = n_e \epsilon_0 (\varepsilon + p) f
\]

\[(3)\]

\[
f = \left( 2 - \xi \right) \left[ 1 + \Theta \left( \frac{\theta_0 + \xi \theta}{\theta_0 + 2 \xi \theta} \right) \right] + \frac{\xi}{\eta} + \Theta \left( \frac{\theta_0 + \xi \theta}{\theta_0 + 2 \xi \theta} \right)\]

\[(4)\]

where, \( \xi = n_e / n_{-}\) is the composition parameter, where electron number density \( n_e \), is 0.5, the mass density \( \rho = n_e m_e (2 - \xi)(1 - \xi) \), \( \eta = n_m m_p \), \( \Theta = kT/m_r^2 \) and the pressure \( p = 2n_e \xi \Delta F \). For radial and adiabatic flow in steady state, the equation of motion simplifies to the form given in CR09.

The initial configuration of the fluid is semi-analytical calculation of an adiabatic radial inflow for Bernoulli parameter \( E = 0.001 \) in units of \( c^2 \). The electron number density \( n_e \) is plotted assuming \( M_e = 10 M_e \) and particle flux rate to be \( N = 1.724 \times 10^{20} \text{ cm}^{-3} \text{ s}^{-1} \). This value \( N \) is the particle flux corresponding to accretion rate \( 0.5 M_{\odot}/\text{yr} \) for an electron-proton \( (e^- - p) \) fluid or a fluid with \( \xi = 1 \). All types of fluid starts with these values of \( N \) and \( \xi \). In Fig. 1a, b, we plot total lepton density \( (n_e - n_{-}) \) (long dashed), temperature \( T \) (dashed) and the radial three velocity \( v \) (solid) are plotted with \( r \) in log-log scale, for two kinds of fluid the \( \xi = 0 \) or \( e^- - e^+ \)(a) and \( \xi = 1 \) or \( e^- - p \)(b). Both the fluids starts with \( N = 1.724 \times 10^{20} \text{ cm}^{-3} \text{ s}^{-1} \) and \( \xi = 0.001 \) in units of \( c^2 \).

\[
\begin{align*}
\Delta E &= 16 \frac{\varepsilon^3}{\pi} - 2 \xi \varepsilon_0^2 \frac{\Theta^3}{2} \\
\Theta &= 1 + \frac{\varepsilon_0}{\varepsilon} \\
\frac{\Delta E}{\varepsilon} &= 128 \frac{kT}{\varepsilon} + (1 - \xi^2)^2 \\
\frac{\Delta E}{\varepsilon} &= 128 \frac{kT}{\varepsilon} (1 - \xi)^2 \\
\frac{\Delta E}{\varepsilon} &= 128 \frac{kT}{\varepsilon} (1 - \xi)^3 \\
\frac{\Delta E}{\varepsilon} &= 128 \frac{kT}{\varepsilon} (1 - \xi)^2 \\
\frac{\Delta E}{\varepsilon} &= 128 \frac{kT}{\varepsilon} (1 - \xi)^3
\end{align*}
\]

\[(5)\]

\[(6)\]

\[(7)\]

where, \( a = v/2kT \), \( r = \lambda \), \( n = e^2/16 \pi \), \( \lambda = h \alpha /mc, \) and \( f(\varepsilon) = 1/43(\varepsilon + K_0(a)/K_0(\varepsilon) + a(1 - K_0(a)/K_0(\varepsilon)))\), \( \varphi(\varepsilon) = (\varepsilon + K_0(a)/K_0(\varepsilon) + a(1 - K_0(a)/K_0(\varepsilon))) \). The \( K_0 \)s are modified Bessel’s function of various kind. Now there are at least three reference frames involved, the local comoving frame, the local rest frame and the Schwarzschild frame. The three velocities are measured in the local fixed frame and the cooling rates are calculated in the comoving frame. The photons, however will be moving in a curved geometry, and would follow geodesic equations. The coordinate transformations between the locally fixed frame \( (x^a) \) and the coordinate frame \( (x^a) \) are given by (Park 2006).

\[
\frac{\partial}{\partial \theta} = \frac{1}{(1 - 2 \theta^2)^{1/2}} \frac{\partial}{\partial \tilde{\theta}}
\]

\[
\frac{\partial}{\partial \phi} = \frac{1}{1 - \theta^2} \frac{\partial}{\partial \tilde{\phi}}
\]

\[
\frac{\partial}{\partial \phi} = \frac{1}{3} \frac{\partial}{\partial \tilde{\phi}}
\]

Fig. 1 Total lepton number density \( (n_e - n_{-}) \) (long dashed), temperature \( T \) (dashed) and the radial three velocity \( v \) (solid) are plotted with \( r \) in log-log scale, for two kinds of fluid the \( \xi = 0 \) or \( e^- - e^+ \)(a) and \( \xi = 1 \) or \( e^- - p \)(b).
And the transformation between local fixed frame (x) and the comoving frame (x_c) are related by Lorentz transformation of the form
\[ \partial_{x} = A^\mu_{\nu}(\partial_{x^\mu}) \delta_{\nu}^{\mu}. \]

Where \( \alpha, \beta \) indicates the tetrad and \( A^\mu_{\nu} \) are the components of Lorentz transformation. The photons in strong gravity moves in curved trajectories and the geodesic equations given by Weinberg (1972), can be written as,

\[ \frac{d^2\rho}{dt^2} = 2 \frac{1}{2(1 - \theta)} \frac{d\xi}{dt} \left( - (r - 1) \frac{dr}{dt} \right)^2. \]

Moreover the definition of optical depth in curved space and moving media in radial direction, is modified to \( \dot{\tau} = \sigma_{\nu}(2 - (\zeta \nu_{\nu}) (1 - v_n/c_h) / \theta) \).

In Schwarzschild metric, the trajectories of the photons, which follows geodesic equations, are curved. It is to be noted that the seed photons generated will depend on \( \zeta \), as well as temperature and other relevant quantities. If the fluid is \( e^{\nu} - e^{\nu} \) then \( \xi = 1 \), then the contribution due to Eq. (7) is zero, while for \( e^{\nu} - e^{\nu} \) fluid or when \( \zeta = 0 \), contribution due to Eq. (5) is zero.

3 Result and Discussion

In Fig. 2 we show that, photon trajectories of two photons originating from same location at S. One photon escapes and suffers no scattering (blue), while the other scatters (magenta), changes direction and is captured by the black hole at the centre (location +). The arrow heads are the velocity field or \( v \) as shown in Fig. 1a.
close to the horizon the temperature rises, since the cooling time scale is much larger than the infall timescale at those distances. The temperature converges after few iterations. Final spectra is obtained from the converged temperature (black).

In Fig. 4, we present the combined spectra due to bremsstrahlung and Comptonization for fluids of different composition $\xi = 0$ (red), $\xi = 0.5$ (blue), and $\xi = 1$ (magenta), and $\xi = 1$ (magenta), and $\xi = 0$ (blue). Clearly, $e^- e^\pm$ fluid which was coldest and the least relativistic is the least luminous and the less energetic in terms of the spectra as well. This is to be expected since the temperature of $e^- e^\pm$ fluid is much lower than that by fluids containing protons. Therefore, inverse-Comptonization produces much less energetic photons. In this paper, we have compared with flows starting with the same outer boundary condition, namely, at $r_{out} = 500 \, r_g$, $\mathcal{P} = 0.001 \, s^3$ and $N = 1.724 \times 10^{22} \, s^{-1}$. We have not injected flows with the same accretion rate, because in that case the number density of particles at the outer boundary for $e^- e^\pm$ fluid will be much higher than fluids with $\xi = 0.5$ and $e^- e^\mp$ fluid.

4 Conclusion

It has been shown earlier (Ch08, CB09, CC11) that, the solution of relativistic, transonic, and adiabatic accretion depends on the composition of the plasma. More interestingly, we also showed that, contrary to the expectation, we found that $e^- e^\mp$ fluid is the least relativistic fluid. And we conjectured that a purely leptonic flow will be less luminous and of low energy. Since $e^- e^\mp$ flow is least relativistic and slowest, density should be higher. This would increase the optical depth, and therefore should be less luminous. Moreover, the temperature of $e^- e^\mp$ is very low too, as higher energies will not be available through inverse-Comptonization. Chattopadhyay et al. (2013) showed that this is indeed true for a toy model of radiation, where the seed photons were mono-energetic and artificially injected. In this paper, we present preliminary solutions of the total spectra from radial accretion onto black holes by considering realistic seed photons (bremsstrahlung) and the Comptonization of those photons, and we vindicate the conclusions of Chattopadhyay et al. (2013). It may be noted though, that $e^- e^\mp$ fluid cannot completely describe an accretion flow from infinity to the horizon. This is shown here as an extreme case, the realistic cases are $\xi = 0$ and $\xi = 0.5$ fluid. In view of the results of CC11, extension of these methods to accretion disc, in presence of all kind of cooling processes, is very interesting. We are working on it and will be reported elsewhere.

Acknowledgements We acknowledge Central Department of Physics, Tribhuvan University for providing various supports during the conference.

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