Modeling and Analysis of Wireless Channels via the Mixture of Gaussian Distribution

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Abstract

Considerable efforts have been devoted to statistical modeling and the characterization of channels in a range of statistical models for fading channels. In this paper, we consider a unified approach to model wireless channels by the mixture of Gaussian (MoG) distribution. Simulations provided have shown the new probability density function to accurately characterize multipath fading as well as composite fading channels. We utilize the well known expectation-maximization algorithm to estimate the parameters of the MoG model and further utilize the Kullback-Leibler divergence and the mean square error criteria to demonstrate that our model provides both high accuracy and low computational complexity, in comparison with existing results. Additionally, we provide closed form expressions for several performance metrics used in wireless communication systems, including the moment generating function, the raw moments, the amount of fading, the outage probability, the average channel capacity, and the probability of energy detection for cognitive radio. Numerical Analysis and Monte-Carlo simulations are presented to corroborate the analytical results and to provide detailed performance comparisons with the other models in the literature.

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I. INTRODUCTION

Modeling the terrestrial wireless propagation is of very high importance for the design and performance analysis of wireless systems. In a typical mobile radio propagation situation, the received signal presents small scale power fluctuations, due to the multipath propagation, superimposed on large scale signal power fluctuations, also known as shadowing, which is due to the presence of large obstacles between the transmitter and receiver. The small scale fading results in very rapid fluctuations around the mean signal level, while shadowing gives rise to relatively slow variations of the mean signal level [1]. A popular example of such a composite fading channel is the Nakagami-Lognormal (NL) channel. In this case, the density function is obtained by averaging the instantaneous Nakagami-\(m\) fading average power over the conditional probability density function (pdf) of the log-normal shadowing, resulting in a complicated pdf that has no closed form expression [2].

The \(K\) [3] and generalized-\(K\) (\(K_G\)) distributions [4], [5] have been introduced as relatively simpler models to characterize composite fading channels, in which the lognormal distribution is replaced by the Gamma distribution in the Rayleigh-Lognormal (RL) and NL distributions, respectively. These models contain the modified Bessel function of the second kind, which complicates further analytical performance measures. In [6], the lognormal distribution was replaced by the Inverse-Gaussian distribution, resulting in the Rayleigh/Inverse Gaussian (RIGD) distribution, followed by its generalized versions, the \(\mathcal{G}\)-distribution [7], the \(\kappa - \mu\)/Inverse Gaussian distribution [8] and the \(\eta - \mu\)/Inverse Gaussian distribution [9]. The drawback of these distributions is their increased complexity due to a complicated algebraic form that also includes the modified Bessel function of the second kind. Recently, an interesting work has been proposed
by Atapattu et al. [10], where several channel models are expressed as a mixture Gamma ($MG$) distribution via Gauss-Quadrature approximation. The $MG$ model is more accurate than the aforementioned alternatives, and it has the advantage of simplicity as well.

Finite mixtures of distributions provide a mathematical-based approach to the statistical modeling of a wide variety of random phenomena [11]. In this paper, an alternative model, that represents both composite and non-composite fading channels by the Mixture of Gaussian (MoG) distribution is presented. The approximation method is based on the expectation-maximization (EM) algorithm, which was coined by Dempster et al. in their seminal paper [12]. The EM algorithm is essentially a set of algorithms exceptionally useful for finding the maximum likelihood estimator (MLE) of any distribution in the exponential family [13], and widely used for the missing data problem (i.e., modeling a mixture distribution). The main contribution of this paper can be summarized as:

- We propose a MoG distribution to model both the envelope and the signal-to-noise ratio (SNR) of wireless channels. The proposed distribution is proven to accurately model both composite and non-composite channels in a very simple expression, where the corresponding parameters for the mixture are evaluated using the EM algorithm.
- We show that, when considering the mean square error (MSE) and the Kullback-Leibler (KL) divergence, our model is either equally or more accurate than the other alternatives.
- We demonstrate the importance and tractability of our model by deriving several tools for the performance analysis of single-user communications in the MoG Channel such as the outage probability and raw moments.
- We derive the moment generating function (MGF) of the MoG channel, of which the symbol error rate (SER) of $L$-branch maximal ratio combining (MRC) diversity system is presented in closed form for various signaling schemes.
- We derive a generalized expression for the average detection probability for energy detection based spectrum sensing in cognitive radio networks.
• Numerical analysis and Monte Carlo simulation results are presented to corroborate the derived analytical results.

The rest of this paper is organized as follows, Section II gives a brief description of some wireless channel models. In Section III, the MoG distribution is introduced together with a brief description of the EM algorithm. Section IV presents a detailed comparison of the MoG model to the channel models it can approximate. In Section V, important performance analysis are derived using the MoG distribution. Simulation results and numerical analysis are presented in section VI. Finally Section VII concludes this work.

II. FADING CHANNELS

Radio-wave propagation through wireless channels is a complicated phenomenon characterized by numerous effects notably fading, multipath and shadowing. A precise mathematical description of this phenomenon is either unknown or too complex for tractable communications systems analysis. Having said that, considerable efforts have been devoted to the statistical modeling and characterization of these different effects resulting in a range of statistical models for fading channels [2]. In this section, we give a brief description of some well known models that have complicated statistical model making the performance analysis of wireless communication systems in such system intractable.

A. The Nakagami-Lognormal Channel

The NL fading model is a mixture of Nakagami-\(m\) distribution and lognormal distribution obtained by averaging the instantaneous Nakagami-\(m\) fading average power over the conditional pdf of the log-normal shadowing as follows

\[
f_{\alpha}(\alpha) = \int_{0}^{\infty} f_{\alpha}(\alpha|\sigma)f_{\sigma}(\sigma) \, d\sigma,
\]

where \(f_{\alpha}(\alpha|\sigma)\) is the Nakagami-\(m\) distribution given by

\[
f_{\alpha}(\alpha|\sigma) = \frac{2m^{m}}{\sigma^{m}\Gamma(m)}\alpha^{2m-1}e^{-\frac{\alpha^{2}}{\sigma^{2}}},
\]
where, \( \Gamma(\cdot) \) is the gamma function [14] and \( m \) is the fading parameter, which is inversely proportional to multipath fading severity i.e., as \( m \to \infty \), multipath severity diminishes. The parameter \( \sigma \) follows a Lognormal distribution, contributing to shadowing at longer routes, expressed as

\[
f_\sigma(\sigma) = \frac{e^{-(10 \log(\sigma) - M)^2}}{\sqrt{2\pi\sigma\lambda\zeta}}, \tag{3}\]

where \( \lambda = \frac{\ln 10}{10} \), while \( M \) and \( \zeta^2 \), measured in dB, are the mean and variance of the Gaussian RV \( V = 10 \log_{10}(\sigma) \), respectively. In order to compare (2) with that of the Gaussian RV \( X = \ln(\sigma) \), the following relations apply [15]

\[
X = \lambda V, \tag{4}
\]

\[
M_X = \lambda M, \]

\[
\zeta_X = \lambda \zeta.
\]

The relations in (4) are needed when comparing our model with other models, such as the \( K \), \( G \) and \( MG \) distributions.

An important remark regarding the Lognormal distribution is that, while \( \zeta \) essentially defines different Lognormal distributions, \( M \) is effectively a scaling factor [15]. Denote \( M_n = 10^M \) and \( x = \frac{\sigma}{M_n} \), then it is straightforward to show that

\[
f_\alpha(\alpha M_n) = \frac{1}{M_n} f_\alpha(\alpha | M = 0). \tag{5}\]

Therefore, it is only sufficient to perform an approximation for \( M = 0 \) dB, and generalize the results for other scaling factors. Let \( E_s \) denotes the energy per symbol, and \( N_0 \) be the single sided power spectral density of the complex additive white Gaussian noise (AWGN). Assuming \( \mathbb{E}[|\alpha^2|] = 1 \), where \( \mathbb{E}[.] \) denotes the expectation operator. By applying the following transformation to (1)

\[
\gamma = \alpha^2 \tilde{\gamma}, \tag{6}\]
where $\bar{\gamma} = E[\gamma] = \frac{E}{N_0}$ is the average SNR, we obtain the Gamma/Lognormal (GL) distribution as

$$f_\gamma(x) = \frac{(8.686)m^n}{\Gamma(m)\sqrt{2\pi}\zeta} \int_0^\infty \frac{x^{m-1}}{\gamma^{m\sigma^m+1}} e^{-\frac{m\gamma \sigma}{2}\gamma e^{-\frac{(20\log\gamma)^2}{2\zeta^2}}} d\sigma. \quad (7)$$

The SNR density function is not a closed form making the performance analysis of wireless communications under this particular channel very complicated or intractable. Note that the RL distribution is a special case of NL distribution where $m = 1$.

**B. The Weibull Channel**

Experimental data have shown that the Weibull fading channel model exhibits an excellent fit both for indoor and outdoor environments associated with mobile radio systems operating in the 800/900 MHz [16] [17]. Its instantaneous SNR pdf is expressed as

$$f_\gamma(x) = \frac{\beta^2}{2} \left( \frac{\Gamma(1 + \frac{2}{\beta})}{\bar{\gamma}} \right)^\frac{1}{2} x^{-\frac{1}{\beta} - 1} e^{-\left[\frac{\gamma}{\bar{\gamma}} \Gamma(1 + \frac{2}{\beta})\right]^\frac{1}{2}} \quad (8)$$

where $\beta$ is the Weibull fading parameter and $\Gamma(\cdot; \cdot)$ is the upper incomplete Gamma function [14]. The distributions described above provide examples of complicated pdfs that lack a closed form expression and/or make the performance analysis of wireless communication complicated. Next, we utilize the MoG distribution to model wireless channels statistics in a simple and generalized manner.

**III. The MoG Model**

We consider the problem of estimating the wireless channels’ density function. Mixtures of distributions have been used as models in such problems where the function is composed from a convex linear combination of two or more populations. Gaussian mixtures are often used because of their simplicity and their efficient representation in terms of the first two moments [18]. The MoG distribution is attributed to have the Universal-approximation property, as it has been proven by Weiners approximation theorem [19] that the MoG distribution can approximate
any arbitrarily shaped non-Gaussian density. The objective of this section is to provide a unified MoG distribution that can accurately represent the different wireless channels.

A. Fading Channels Model

Let the \(i^{th}\) entry of a random data vector \(Y = (y_1, \ldots, y_n)\), which represents the envelopes of the composite models, be regarded as incomplete data and modeled as a finite mixture of Gaussians as follows

\[
p(y_i|\theta) = \sum_{j=1}^{C} \omega_j \phi(y_i, \theta_j), \quad i = 1, \ldots, n,
\]

where \(C\) represents the number of components. Each \(j^{th}\) component is expressed as

\[
\phi(y_i, \theta_j) = \frac{1}{\sqrt{2\pi\eta_j}} \exp \left(-\frac{(y_i - \mu_j)^2}{2\eta_j^2}\right),
\]

where the weight of the \(j^{th}\) component is \(\omega_j > 0\), with \(\sum_j \omega_j = 1\). Parameter \(\theta_j = (\mu_j, \eta_j^2)\) correspond to the mean and variance of the \(j^{th}\) component respectively.

Let the complete data \(X\) be the joint probability between \(Y\) and \(Z\), where \(Z \in \{1, \ldots, C\}\) is a hidden (latent) discrete RV that defines which Gaussian component the data vector \(Y\) come from, namely

\[
p(Z = j) = \omega_j, \quad j = 1, \ldots, C.
\]

Ideally, one would look for the MLE of \(\theta\) such that the log-likelihood value is maximized as follows

\[
\theta_{MLE} = \arg \max_{\theta \in \Theta} L(\theta) = \arg \max_{\theta \in \Theta} \log p(y|\theta).
\]

However, maximizing \(L(\theta)\) is not tractable and difficult to optimize [20]. Instead, the EM algorithm solves the MLE problem by maximizing the so-called \(Q\)-function as follows [21]

\[
\theta^{(m+1)} = \arg \max_{\theta \in \Theta} Q(\theta|\theta^{(m)}) = \arg \max_{\theta \in \Theta} \mathbb{E}_{X|y,\theta^{(m)}} [\log p_X(X|\theta)],
\]
where \( m \) is the iteration index. Practically, the EM algorithm is performed by two iterative steps, namely the expectation step (\( E \)-step), and the maximization step (\( M \)-step). We set initial guesses of the MoG coefficients, i.e. \( \omega^{(0)}, \mu^{(0)}, \eta^{(0)} \), whereby in the \( E \)-step, we compute the posterior probability (membership probability)

\[
\rho^{(m)}_{ij} = \frac{\omega^{(m)}_j \phi(y_i | \mu^{(m)}_j, \eta^{(m)}_j)}{\sum_{l=1}^C \omega^{(m)}_l \phi(y_i | \mu^{(m)}_l, \eta^{(m)}_l)}.
\] (14)

In the \( M \)-step, the coefficients are updated by differentiating the \( Q \)-function with respect to \( \omega \), \( \mu \), and \( \eta \), resulting in the following analytical \((m+1)\)th estimates

\[
\omega^{(m+1)}_j = \frac{1}{n} \sum_{i=1}^n \rho^{(m)}_{ij}, \ j = 1, \ldots, C,
\] (15)

\[
\mu^{(m+1)}_j = \frac{1}{n_j^{(m)}} \sum_{i=1}^n \rho^{(m)}_{ij} y_i, \ j = 1, \ldots, C,
\] (16)

\[
\eta^{(m+1)}_j = \frac{1}{n_j^{(m)}} \sum_{i=1}^n \rho^{(m)}_{ij} (y_i - \mu^{(m+1)}_j)^2, \ j = 1, \ldots, C.
\] (17)

This iterative procedure is terminated upon convergence, that is when \( |L^{(m+1)} - L^{(m)}| < \delta \), where

\[
L^{(m)} = \frac{1}{n} \sum_{i=1}^n \log \left( \sum_{j=1}^C \omega^{(m)}_j \phi(y_i | \mu^{(m)}_j, \eta^{(m)}_j) \right), \ i = 1, \ldots, n,
\] (18)

is the log-likelihood, and \( \delta \) is a preset threshold.

The EM algorithm is guaranteed not to get worse as iterates by, i.e. \( L^{(m+1)} \leq L^{(m)} \) [12]. Hence, the lower \( \delta \) is set, the more accurate the approximation would be. In addition, one can always increase the accuracy by increasing the number of components. Though, this technique might be stuck in a local maxima, since the likelihood is a marginal distribution. However, one could mitigate this problem by heuristics and multiple initial guesses. In this regard, Do et al. [20] suggest to initialize parameters in a way that breaks symmetry in mixture models. Finally, it is noteworthy to point out that the EM algorithm has an advantage of being a completely unsupervised learning algorithm, which makes it very convenient for our density estimation application. For more details, one can refer to [13], [21], and references therein.
B. The pdf of the Instantaneous SNR of the MoG Model

By the aid of the EM algorithm, all envelopes of composite fading models can be represented as

\[
f_\alpha(x) = \sum_{j=1}^{C} \frac{\omega_j}{\sqrt{2\pi\eta_j}} \exp\left(-\frac{(x - \mu_j)^2}{2\eta_j^2}\right). \tag{19}\]

By the change of variables \(\gamma = \frac{x}{2\eta_j}\), the pdf of the instantaneous SNR of the MoG model can be written as

\[
f_\gamma(\gamma) = \sum_{j=1}^{C} \frac{\omega_j}{\sqrt{8\pi\gamma\eta_j}} \frac{1}{\sqrt{\gamma}} \exp\left(-\frac{1}{4\eta_j^2} \gamma - \mu_j\right). \tag{20}\]

IV. MoG Model Analysis and Comparisons

In this section, several scenarios of the GL composite channels are approximated using the MoG model, as in (20). In order to validate the accuracy of the approximations, we use two criteria of error, namely the MSE and the KL, defined as

\[
\text{MSE} = \mathbb{E} \left[ f_\gamma(x) - \hat{f}_\gamma(x) \right], \tag{21}\]

\[
\text{KL}(f_\gamma(x), \hat{f}_\gamma(x)) = \int_{0}^{\infty} f_\gamma(x) \log \frac{f_\gamma(x)}{\hat{f}_\gamma(x)}, \tag{22}\]

respectively. Here, \(f_\gamma(x)\) is the exact pdf, and \(\hat{f}_\gamma(x)\) is the approximated pdf (MoG). Note that the MSE and KL measures are used in several related works, see e.g., [6], [7], [10]. The KL divergence, also known as relative entropy, is an information theoretic measure that quantifies the information lost when \(\hat{f}_\gamma(x)\) is used to approximate \(f_\gamma(x)\) [22].

Fig. 1 provides the approximation results for several scenarios of the GL distribution. The number of components considered is \(C = 10\). As shown from the MSE and KL measures, the approximation is very accurate when both increasing the shadowing and the multipath fading severity. Parameters of the approximations are tabulated in Appendix A.
In order to further validate the accuracy of the new model, we compare the MoG model with the $G$ and the $MG$ models for two different scenarios. The first scenario is characterized by $(M = 0\ \text{dB},\ m = 2,\ \zeta = 1/4\ \text{dB})$ and the second scenario by $(M = 0\ \text{dB},\ m = 3,\ \zeta = 1\ \text{dB})$.

As Fig. 2 illustrates, the accuracy of the three models is quite similar in the first scenario. As for the second scenario, the MoG and the $MG$ models slightly outperforms the $G$ model in approximating the mode. Further verification of the accuracy via numerical means is addressed in section VI. The purpose of this approach is not to increase the accuracy of the approximation, as it
has already been achieved in all aforementioned fading alternatives, but rather to provide another unifying and simplifying model that has the potential of approximating all contemporary fading, composite and non-composite, distributions via the EM algorithm. Further approximations of non-composite fading channels, such as the Nakagami-$m$, Lognormal, and Weibull distributions are tabulated in Appendix A.
V. Performance Analysis of Wireless Channels

The MoG model provides a simplifying and unifying analysis for wireless communication systems over, but not limited to, generalized composite fading channel models. In this section, we first derive several performance metrics that can be used for the evaluation of wireless communication systems in a generalized manner. In particular, we derive expressions for the raw moments of the MoG model, the amount of fading (AF), the outage probability, the channel capacity, and the MGF. We further derive a closed form expression for the SER performance of $L$-branch MRC diversity system and the average probability of detection for cognitive radio networks.

A. Moment Generating Function

By definition, the MGF $M_{\gamma}(s) = \mathbb{E}[e^{-s\gamma}]$ is given by

$$M_{\gamma}(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{8\pi} \eta_i} \int_{0}^{\infty} \frac{1}{\sqrt{\gamma}} \exp \left( -\frac{\left( \sqrt{\frac{\gamma}{\pi}} - \mu_i \right)^2}{2\eta_i^2} \right) e^{-\gamma s} d\gamma. \quad (23)$$

Applying the change of variables $x = \sqrt{\frac{\gamma}{\pi}}$, and after expanding the exponentials and considerable mathematical simplifications, we get

$$M_{\gamma}(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{2\pi} \eta_i} \int_{0}^{\infty} \exp \left( -\frac{(2 - \beta_i)(x^2 - 2\mu_i x + \mu_i^2)}{2\eta_i^2} \right) dx, \quad (24)$$

where $\beta_i = 1 + 2\eta_i^2 s$. Then, after some mathematical manipulations, we obtain

$$M_{\gamma}(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{2\pi} \eta_i^2} \int_{-\mu_i/\eta_i\sqrt{2s}}^{\infty} \exp(-z^2) dz, \quad (25)$$

which leads to the following simple closed form expression

$$M_{\gamma}(s) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{\beta_i \eta_i}} \exp \left( \frac{\mu_i^2 s}{\beta_i} \right) Q \left( \frac{-\mu_i}{\eta_i \sqrt{\beta_i}} \right), \quad (26)$$

where $Q(.)$ is the Gaussian Q-function defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-\frac{u^2}{2}) du$. 

B. Raw Moments

The $n^{th}$ raw moment of the MoG distribution by definition, is

$$
E[\gamma^n] = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{8\pi \eta_i}} \int_0^\infty \frac{\gamma^n}{\sqrt{\gamma}} \exp \left( -\frac{(\sqrt{\gamma} - \mu_i)^2}{2\eta_i^2} \right) d\gamma.
$$

(27)

By taking the change of variables $x = \sqrt{\gamma}$, and after some mathematical simplifications, we get

$$
E[\gamma^n] = \sum_{i=1}^{C} \omega_i \gamma^n \int_0^\infty \frac{x^{2n}}{\sqrt{2\pi \eta_i}} \exp \left( -\frac{(x - \mu_i)^2}{2\eta_i^2} \right) dx,
$$

(28)

Alternatively, we can write (27) as

$$
E[\gamma^n] = \sum_{i=1}^{C} \omega_i \gamma^n \mathbb{E}[X_i^{2n}],
$$

(29)

where $X_i \sim \mathcal{N}(\mu_i, \zeta_i)$ is the $i^{th}$ Gaussian RV. Using the MGF approach, (29) can be expressed as

$$
E[\gamma^n] = \sum_{i=1}^{C} \omega_i \gamma^n \frac{d(2n)M_X(s)}{ds}|_{s=0},
$$

(30)

where $M_X(s) = \mathbb{E}\{e^{-sx}\}$ is the MGF of $X_i$ given by $\exp(\mu_is + \frac{\eta_i^2s^2}{2})$. Equation (30) is mathematically convenient for solving for the first few moments, where it will be utilized to calculate the AF.

An alternative approach that yields a closed form expression can be attained by following the same method in [23], where the $v^{th}$ raw moments of $X_i$ are derived as

$$
E[x_v^n] = \eta_i^v 2^\frac{v}{2} \Gamma\left(\frac{v}{2} + \frac{1}{2}\right) \sqrt{\pi} \frac{1}{\sqrt{2\pi \eta_i}} 1F_1\left[-\frac{v}{2}, \frac{1}{2}, -\frac{-\mu_i^2}{2\eta_i^2}\right],
$$

(31)

where $v$ is an even integer (note that there is no loss in generality), and the function $1F_1$ is the confluent hypergeometric function [14, eq. 9.210.1]. By substituting (31) into (29), the $n^{th}$ raw moment of the MoG model is derived as

$$
E[\gamma^n] = \sum_{i=1}^{C} \omega_i \gamma^n \eta_i^{2n} 2^n \frac{\Gamma(n + \frac{1}{2})}{\sqrt{\pi}} \frac{1}{\sqrt{2\pi \eta_i}} 1F_1\left[-n, \frac{1}{2}, -\frac{-\mu_i^2}{2\eta_i^2}\right],
$$

(32)
C. Amount of Fading

The AF measure was firstly introduced by Charash [24], as a measure of the severity of the fading channel. The AF requires the knowledge of only the first two moments in the corresponding fading channel, where it is defined by

\[ AF = \frac{\mathbb{E}[\gamma^2] - (\mathbb{E}[\gamma])^2}{(\mathbb{E}[\gamma])^2}. \]  

(33)

By solving (32) for the first two moments, we obtain

\[ AF = \frac{\sum_{i=1}^{C} \omega_i (\mu_i^2 + 6\mu_i^2\eta_i^2 + 3\eta_i^4)}{[\sum_{i=1}^{C} \omega_i (\mu_i^2 + \eta_i^2)]^2} - 1. \]  

(34)

D. Outage Probability

The outage probability is a standard performance criterion used over fading channels. It is defined as \( F(\gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(x) \, dx \). By performing the following change of variables applied to (20)

\[ y = \frac{x - \mu_i}{\sqrt{2}\eta_i}, \]  

(35)

and after some mathematical manipulations, the CDF of (20) can be written as

\[ F(\gamma_{th}) = \sum_{i=1}^{C} \frac{\omega_i}{\sqrt{\pi} \, \sqrt{2\eta_i}} \int_{-\mu_i/\sqrt{2\eta_i}}^{\sqrt{2\eta_i} - \mu_i} \exp(-y^2) \, dy. \]  

(36)

Further simplifications yield

\[ F(\gamma_{th}) = \sum_{i=1}^{C} \omega_i \left[ Q\left(\frac{-\mu_i}{\eta_i}\right) - Q\left(\frac{\sqrt{2\eta_i} - \mu_i}{\eta_i}\right)\right], \]  

(37)

\[ F(\gamma_{th}) = \sum_{i=1}^{C} \omega_i Q\left(\frac{\sqrt{2\eta_i} - \mu_i}{-\eta_i}\right). \]  

(38)
E. Average Ergodic Channel Capacity

When only the receiver has knowledge about the channel state information (CSI), the ergodic capacity \( C_{\text{erg}} \) is expressed as

\[
C_{\text{erg}} = \frac{B}{\ln 2} \int_{0}^{\infty} \ln(1 + \gamma) f_\gamma(\gamma) \, d\gamma.
\]  

(39)

where \( B \) is the channel bandwidth measured in Hertz. Unfortunately, the exact solution of (39) is intractable. Instead, a computationally simple and very accurate form can be obtained by following [25], where \( \ln(1 + \gamma) \) is expanded about the mean value of the instantaneous SNR, \( \mathbb{E}[\gamma] \), using Taylor’s series, yielding

\[
\ln(1 + \gamma) = \ln(1 + \mathbb{E}[\gamma]) + \sum_{w=1}^{\infty} \frac{(-1)^{w-1}}{w} \frac{(x - \mathbb{E}[\gamma])^w}{(1 + \mathbb{E}[\gamma])^w}.
\]  

(40)

Substituting (40) into (39), i.e. taking the expectation of \( \ln(1 + \gamma) \), the ergodic capacity can be re-written as

\[
C_{\text{erg}} \approx \frac{B}{\ln 2} \left[ \ln(1 + \mathbb{E}[\gamma]) - \frac{\mathbb{E}[\gamma^2] - \mathbb{E}^2[\gamma]}{2(1 + \mathbb{E}[\gamma])^2} \right],
\]  

(41)

where

\[
\mathbb{E}[\gamma] = \sum_{i=1}^{C} \omega_i (\mu_i^2 + \eta_i^2),
\]  

(42)

\[
\mathbb{E}[\gamma^2] = \sum_{i=1}^{C} \omega_i (\mu_i^4 + 6\mu_i^2\eta_i^2 + 3\eta_i^4).
\]  

(43)

F. Symbol Error Analysis

In order to further demonstrate the significance of the MoG model, we study the performance of independent but not identically distributed (i.n.i.d.) \( L \)-branch MRC diversity receiver over various composite and non-composite fading scenarios. The MRC scheme is the optimal combining scheme at the expense of increased complexity, where the receiver requires knowledge of all channel fading parameters [2]. Here, the receiver sums up all received instantaneous SNR replicas \( \gamma_k \) as follows

\[
\gamma_{\text{MRC}} = \sum_{k=1}^{L} \gamma_k.
\]  

(44)
The corresponding MGF is thus

$$M_{\gamma_{MRC}}(s) = \mathbb{E}\{e^{-s\sum_{k=1}^{L}\gamma_k}\} = \prod_{k=1}^{L} M_{\gamma_k}(s), \quad (45)$$

where $M_{\gamma_k}(s)$ is derived in (26). The SER $P_s(E)$, for coherent binary signals, can be computed as follows [2]

$$P_s(E) = \mathbb{E}_{\gamma_{MRC}}[Q(\sqrt{2g\gamma_{MRC}})], \quad (46)$$

where $g$ is some constant resembling several coherent binary signals, such as the coherent binary phase shift keying (BPSK) and coherent orthogonal binary frequency shift keying (BFSK) corresponding to $g = 1$ and $g = \frac{1}{2}$, respectively. By substituting the Q-function by its closed form representation in [2, eq. 4.2], the SER is written as

$$P_s(E) = \int_{0}^{\frac{\pi}{2}} \int_{0}^{\infty} \exp\left(\frac{g\gamma_{MRC}}{\sin^2(\theta)}\right) f_{\gamma_{MRC}}(\gamma_{MRC}) d\gamma_{MRC} d\theta. \quad (47)$$

The inner infinite integral in (47) is the equivalent MGF derived in (45). Hence, the SER is expressed as

$$P_s(E) = \int_{0}^{\frac{\pi}{2}} \prod_{k=1}^{L} \sum_{i=1}^{C} \frac{w_i}{\sqrt{1 + \frac{2\eta^2_i\gamma_{MRC}}{\sin^2(\theta)}}} \exp\left(\frac{\mu_i^2 g}{\sin^2(\theta)} + \frac{2\eta^2_i\gamma_{MRC}^2}{\sin^2(\theta)}\right) d\theta. \quad (48)$$

Following a similar approach, and by utilizing [2, eq. 8.22] and [2, eq. 8.10], the SER expressions for $M$-PSK and square $M$-QAM signaling schemes are given, respectively, by

$$P_s(E) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{M}} \prod_{k=1}^{L} M_{\gamma_k}\left(\frac{-\sin^2\left(\frac{\pi}{M}\right)}{\sin^2(\theta)}\right) d\theta, \quad (49)$$

$$P_s(E) = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right) \left[ \int_{0}^{\frac{\pi}{2}} \prod_{k=1}^{L} M_{\gamma_k}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right) d\theta - \int_{0}^{\frac{\pi}{2}} \prod_{k=1}^{L} M_{\gamma_k}\left(\frac{g_{QAM}}{\sin^2(\theta)}\right) d\theta \right], \quad (50)$$

where $g_{QAM} = \frac{3}{4}(M - 1)$. 
G. Probability of detection

Cognitive radio (CR) is a promising technology that can enhance the performance of wireless communications [26]. The basic concept behind opportunistic CR, is that a secondary user is allowed to use the spectrum, which is assigned to a licensed primary user (PU), when the channel is idle [27]. The CR users perform spectrum sensing in order to identify idle spectrum, while energy detection is the most common sensing technique in CR networks, due to its low implementation complexity and no requirements for knowledge of the signal [28]. Several studies have been devoted to the analysis of the performance of energy detection-based spectrum sensing for different communication and fading scenarios [29]. The probability of detection, $P_d$ is an important performance metric for CR networks, since it measures the probability that the PU is detected. The generalized expression for the average detection probability is evaluated by averaging the the conditional $P_d$ in the AWGN ($P_d = Q_u(\sqrt{2\gamma}, \sqrt{\lambda})$) case over the SNR fading distribution as follows

$$P_d = \int_0^\infty Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) f_\gamma(\gamma) d\gamma,$$

(51)

where $Q_u(.,.)$ is the generalized Marcum-Q function [30], $\lambda$ is a predefined energy detection threshold, $u$ is the time bandwidth product which corresponds to the number of samples of either the in-phase ($I$) or the quadrature ($Q$) component, and $\gamma \triangleq \frac{\alpha^2 E_s}{N_0}$ is the received SNR, where $E_s$ is the signal energy, $N_0$ the one-sided noise power spectral density, and the channel gain $\mathbb{E}[|\alpha^2|] = 1$ for AWGN channel. The Marcum Q-function can be expressed as [31]

$$Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) = e^{-\gamma} \sum_{n=0}^{\infty} \frac{\gamma^n}{n!} \frac{\Gamma(u + n, \frac{1}{2})}{\Gamma(u + n)}.$$

(52)

By substituting (20) and (52) in (51), we obtain

$$P_d \approx \sum_{i=1}^{G} w_i \eta_i e^{-\frac{\mu_i^2}{2\eta_i^2}} \frac{1}{\sqrt{2\pi \eta_i^2}} \left( \sum_{n=0}^{\infty} \frac{\Gamma(u + n, \frac{1}{2})\Gamma(2n + 1)(2\gamma\eta_i^2 + 1) - \frac{2n+1}{2}}{n!\Gamma(u + n)(\frac{1}{\eta_i^2})^{\frac{2n+1}{2}}} \right) D_{-(2n+1)} \left( \frac{-\mu_i}{\eta_i \sqrt{2\gamma\eta_i^2 + 1}} \right).$$

(53)

In (53), $D_n(i)$ is the parabolic cylinder function [14].
VI. SIMULATION RESULTS

In this section, we present some analytical and simulation results for the outage probability, the average ergodic capacity and the SER of MRC scheme.

Fig. 3 depicts the outage probability, as in (38), versus the threshold SNR $\gamma_{th}$ for the two GL scenarios, where the multipath severity is reduced from $m = 1$ to $m = 3$. Here, one can notice how accurate the approximation is. Also, it is very noticeable how the multipath fading severity affects the outage probability performance.

![Graph showing outage probability versus threshold SNR](image)

**Fig. 3.** Analytical and simulated outage probability for two scenarios.

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Fig. 4 depicts the capacity, as in (41), for the two aforementioned scenarios. The term simulation refers to cross-validating the results via the recursive adaptive simpson quadrature method performed by the aid of a mathematical package. Here, we notice that the composite channel in the second scenario exhibits a constant improvement of the capacity of about 0.2 (bits/s/Hz) throughout the whole operating average SNR.

![Graph](image)

Fig. 4. Analytical and simulated ergodic capacity for two scenarios, $B=\frac{1}{2}$.

Fig. 5 illustrates the analytical SER of the BPSK signaling scheme for various NL scenarios, including mild and severe fading cases in both shadowing and multipath. The solid squared line represents the corresponding Monte Carlo simulation. It is quite noticeable how the multipath
severity plays greater role in determining the SER, where as observed, incrementing $m$ by only 1 yields an SER performance improvement of about an order of magnitude at observed mid-range average SNR values. On the other hand, increasing $\zeta$ from 2 to 6 dB, while fixing $m$, yields a very similar SER performance.

![Graph showing SER vs. Average SNR](image)

*Fig. 5. Analytical and simulation SER of 2-branch MRC diversity receiver for BPSK signaling scheme for RL and NL fading channels.*

Fig. 6 features the analytical SER of the 16-QAM signaling scheme for some non-composite fading models: The Nakagami-$m$ and Weibull-$m$ fading models, with $m = 3$. Here, $L$ corresponds to the number of antennas, and it is noticeable that our model is still very accurate for high
antenna diversity order.

Fig. 6. Analytical and simulation SER of $L$-branch MRC diversity receiver for 16-QAM signaling scheme for Nakagami-$m$ and Weibull-$m$ Fading Channels.

Fig. 7 depicts the receiver operating characteristic (ROC) (the probability of missed detection ($P_m$) versus the probability of false alarm ($P_f$)) where $P_d$ obtained from (53) is compared to the theoretical expression from (51) substituting $f_\gamma(\gamma)$ by the composite fading channel pdf.
Theoretical NL for $m = 3, \zeta = 2$

Proposed expression

Fig. 7. ROC curves for Nakagami-lognormal channel with $u = 3$ and $\gamma = 5$ dB

VII. Conclusion

In this paper, the MoG distribution has been considered to model the envelop and the SNR statistics for wireless propagation. The parameters of the mixtures are evaluated by the means of the EM algorithm and the MSE and KL have been evaluated to challenge the proposed models accuracy. The proposed model enjoys both simplicity and accuracy. We have shown that the proposed expression can accurately model a wide range of both composite and non-composite channels. It should be highlighted that the adopted approach provides a generalized model for wireless communication systems where all channels can be modeled with the same analytical
expressions. Several analytical tools essential for the evaluation of performance analysis of digital communications were presented. This new model can be applied to various scenarios including, diversity systems, cooperative communications, and cognitive radio networks.

**APPENDIX A**

**MoG Parameters for Selected Scenarios**

The following tables provide the approximation parameters for all scenarios presented in the paper.

**TABLE I**

MoG Parameters for RL Fading Channel with $\zeta = 1$ dB

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|-----|-------|-------------------|-------|-------------------|
| 1/6 | 0.0051164 | 0.060411, 0.025507 | 0.16838 | 0.96294, 0.15747 |
| 2/7 | 0.021518  | 0.14045, 0.046689  | 0.14568 | 1.2183, 0.20628  |
| 3/8 | 0.062099  | 0.26306, 0.076786  | 0.13552 | 1.4884, 0.31467  |
| 4/9 | 0.14837   | 0.43713, 0.11547   | 0.040294 | 1.8528, 0.43084  |
| 5/10| 0.26982   | 0.68518, 0.15515   | 0.0031988 | 2.274, 0.56255   |

**TABLE II**

MoG Parameters for RL Fading Channel with $\zeta = 2$ dB

| $i$ | $w_i$ | $[\mu_i, \eta_i]$ | $w_i$ | $[\mu_i, \eta_i]$ |
|-----|-------|-------------------|-------|-------------------|
| 1/6 | 0.0045723 | 0.054824, 0.023214 | 0.20138 | 0.8864, 0.18134 |
| 2/7 | 0.018487  | 0.12622, 0.041603  | 0.20286 | 1.1829, 0.25922  |
| 3/8 | 0.053141  | 0.23603, 0.068681  | 0.13306 | 1.5772, 0.37511  |
| 4/9 | 0.11657   | 0.39073, 0.10468   | 0.034063 | 2.1289, 0.53072  |
| 5/10| 0.23304   | 0.60606, 0.15207   | 0.0028262 | 2.8129, 0.81739  |
### TABLE III
**MoG Parameters for RL Fading Channel with \( \zeta = 3 \text{ dB} \)**

| \( i \) | \( w_i \) | \([\mu_i, \eta_i]\) | \( w_i \) | \([\mu_i, \eta_i]\) |
|---|---|---|---|---|
| 1/6 | 0.013628 | 0.087034,0.037077 | 0.1804 | 1.1221,0.24894 |
| 2/7 | 0.050433 | 0.19744,0.065056 | 0.1419 | 1.4935,0.35483 |
| 3/8 | 0.11614 | 0.35142,0.10021 | 0.067797 | 2.0375,0.49984 |
| 4/9 | 0.20896 | 0.55843,0.14439 | 0.017268 | 2.8308,0.75869 |
| 5/10 | 0.20232 | 0.82448,0.183 | 0.0011544 | 4.0701,1.3529 |

### TABLE IV
**MoG Parameters for NL Fading Channel with \( m = 2, \zeta = 1 \text{ dB} \)**

| \( i \) | \( w_i \) | \([\mu_i, \eta_i]\) | \( w_i \) | \([\mu_i, \eta_i]\) |
|---|---|---|---|---|
| 1/6 | 0.018334 | 0.30455,0.08798 | 0.14674 | 1.2847,0.24108 |
| 2/7 | 0.11932 | 0.50407,0.12491 | 0.0439 | 1.4702,0.28394 |
| 3/8 | 0.31224 | 0.73853,0.15534 | 0.0062887 | 1.9316,0.36121 |
| 4/9 | 0.166 | 0.97151,0.15317 | 0.010059 | 1.6849,0.26647 |
| 5/10 | 0.1669 | 1.1294,0.20025 | 0.010214 | 1.7405,0.2531 |

### TABLE V
**MoG Parameters for NL Fading Channel with \( m = 3, \zeta = 1 \text{ dB} \)**

| \( i \) | \( w_i \) | \([\mu_i, \eta_i]\) | \( w_i \) | \([\mu_i, \eta_i]\) |
|---|---|---|---|---|
| 1/6 | 0.028243 | 0.45395,0.1075 | 0.060344 | 1.3236,0.21195 |
| 2/7 | 0.27669 | 0.68858,0.1457 | 0.027664 | 1.4728,0.26251 |
| 3/8 | 0.26612 | 0.91492,0.14596 | 0.020771 | 1.5543,0.25319 |
| 4/9 | 0.22083 | 1.1092,0.19932 | 0.004416 | 1.7682,0.33435 |
| 5/10 | 0.080896 | 1.1454,0.16278 | 0.014031 | 1.4453,0.15585 |
### TABLE VI

**MoG Parameters for NL Fading Channel with** \( m = 2, \zeta = \frac{1}{4} \text{ dB} \)

| \( i \) | \( w_i \) | \( [\mu_i, \eta_i] \) | \( w_i \) | \( [\mu_i, \eta_i] \) |
|---|---|---|---|---|
| 1/6 | 0.0092819 | 0.2603,0.076272 | 0.13024 | 1.1019,0.21364 |
| 2/7 | 0.094244 | 0.45325,0.11001 | 0.13245 | 1.2135,0.22628 |
| 3/8 | 0.17939 | 0.65027,0.11392 | 0.095181 | 1.3958,0.27931 |
| 4/9 | 0.17555 | 0.88055,0.14857 | 0.018117 | 1.5449,0.34947 |
| 5/10 | 0.13466 | 0.99209,0.17982 | 0.030879 | 0.81428,0.08503 |

### TABLE VII

**MoG Parameters for Nakagami-3 Fading Channel**

| \( i \) | \( w_i \) | \( [\mu_i, \eta_i] \) | \( w_i \) | \( [\mu_i, \eta_i] \) |
|---|---|---|---|---|
| 1/6 | 0.012256 | 0.40355,0.092473 | 0.10878 | 1.037,0.18181 |
| 2/7 | 0.1741 | 0.62361,0.12718 | 0.096799 | 1.1327,0.18953 |
| 3/8 | 0.12848 | 0.78859,0.11074 | 0.095857 | 1.1476,0.1952 |
| 4/9 | 0.1319 | 0.92625,0.13577 | 0.10312 | 1.3028,0.22945 |
| 5/10 | 0.11989 | 0.96646,0.15685 | 0.028822 | 1.3492,0.29308 |

### TABLE VIII

**MoG Parameters for Weibull-3 Fading Channel**

| \( i \) | \( w_i \) | \( [\mu_i, \eta_i] \) | \( w_i \) | \( [\mu_i, \eta_i] \) |
|---|---|---|---|---|
| 1/6 | 0.0052097 | 0.17592 0.058515 | 0.11426 | 1.0215,0.19211 |
| 2/7 | 0.029342 | 0.32431 0.089484 | 0.11301 | 1.0945,0.1984 |
| 3/8 | 0.17097 | 0.54879 0.1332 | 0.097193 | 1.2318,0.24213 |
| 4/9 | 0.1445 | 0.76212 0.12345 | 0.096228 | 1.2365,0.23923 |
| 5/10 | 0.13386 | 0.9294 0.15251 | 0.095419 | 1.3711,0.27779 |
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