Representative scales of LASPEX wind data

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(Received 9 April 2003, Modified 15 July 2004)

1. Introduction

A major field experiment (LASPEX) was conducted in the semi-arid Sabarmati river basin in Gujarat, during the period January 1996 to March 1999. The details of this experiment are given in Verneker et al. (2003). Its main objective was to collect a complete surface and sub-surface atmospheric hydrological data base, against which parameterized models for land-surface process i.e., energy exchange, radiative, sensible and latent heat fluxes can be tested for improvement and further development. However, the general circulation and climate models can respond only to larger scales of motion due to their coarse resolutions. Since wind data is involved in the computation of most of the land-surface parameters, it is important to compute the wind data of the appropriate scale, from the raw data, so that it matches the model resolution.

The latitudes and longitudes of the stations covered by LASPEX, are given in Fig. 1. We define the wind components of the wind in a trigonometric form, as functions of wavelength and distance from the central point. The coordinates of each station is now redefined in terms of its $X$ and $Y$ distances from the central point, which is chosen as the origin. The $X$ and $Y$ ordinates are represented by the east and north directions respectively from the origin. The next step is the computation of the divergences analytically at each station. The mean value of the divergence, considering all the stations is computed for different sets of wavelengths. We now use the method of Yanai et al. (1973) to compute the large scale divergences over the area covered by LASPEX, using the same values of the wind and wavelengths. The aim is to identify the wavelengths for which the ratio of the two divergences are greater than 0.9. These wavelengths can be taken to be representative of the large scale wind field over the area.

2. Computation of divergences

The latitudinal and longitudinal wind fields are expressed as :

$$U_i = A \sin (2\pi/L_i X_i)$$  (1)
\[ V_i = A \sin \left(2 \frac{\pi}{L}, Y \right) \quad (2) \]

\( A \) is the arbitrary amplitude of the wave, \( L \), \( L \) are the wavelengths in the \( X \) and \( Y \) direction, respectively, and \( X_i, Y_i \) are the coordinates of the particular station. The divergences \( D\lambda_i \), are given by :

\[ D\lambda_i = \frac{\partial U_i}{\partial X_j} + \frac{\partial V_i}{\partial Y_j} \quad (3) \]

Thus, the divergences at each of the five points can be expressed as :

\[ D\lambda_i = 2\pi A \left( \cos \left( 2\frac{\pi}{L}, X \right) / L \right) + \cos \left( 2\frac{\pi}{L}, Y \right) / L \quad (4) \]

By the method of Yanai et al. (1973), the wind components are expressed as :

\[ U_i = A_1 X_i^2 + B_1 X_i Y_i + C_1 Y_i^2 + D_1 X_i + E_1 Y_i + F_1 \quad (5) \]

\[ V_i = A_2 X_i^2 + B_2 X_i Y_i + C_2 Y_i^2 + D_2 X_i + E_2 Y_i + F_2 \quad (6) \]

\( A_1, A_2, B_1, B_2 \ldots \ldots \), are the six coefficients of the regression equations which are to be determined from the five values of \( U_i, V_i \). Since there are only five stations, where values of \( U_i, V_i \) are defined, one constant has to be eliminated. A constraint is imposed that the curvature of the quadratic surface should be minimized, as in the case of skilled hand analysis. Thus, we minimize the quantity :

\[ \left[ \frac{\partial^2 U}{\partial X \partial Y} \right]^2 + \left[ \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right]^2 = \left[ B^2 + (A - C)^2 \right] \quad (7) \]

where the subscripts for the constants and variables, respectively, have been removed for convenience. Equation (5) can be rewritten as :

\[ A_1 X_i^2 + B_1 X_i Y_i + C_1 Y_i^2 + D_1 X_i + E_1 Y_i = U_i - F_1 \quad (8) \]

Differentiating the above equation w.r.t. the coefficient \( F_1 \), we get :

\[ (\partial A / \partial F) X^2 + (\partial B / \partial F) XY + (\partial C / \partial F) Y^2 + (\partial D / \partial F) X + (\partial E / \partial F) Y = -1 \quad (9) \]

where, as usual, the subscripts have been removed for convenience. We now minimize the R.H.S. of Equation (6), w.r.t. \( F \), to obtain :

\[ B \frac{\partial B}{\partial F} + (A - C) \left( \frac{\partial A}{\partial F} - \frac{\partial C}{\partial F} \right) = 0 \quad (10) \]

From the above equation we get an expression for the coefficient ‘\( B \)’ in terms of other coefficients and their derivatives, w.r.t. \( F \). We now have five unknown coefficients which can be calculated from the five known
Fig. 3. Total wavelength $L$ and corresponding ratios of divergences

values of ‘$U$’, at the five comers of the pentagon. Thus, the equation for the $U$ component is as follows:

$$U_i = A(X_i^2 + b X_i Y_i) + C(Y_i^2 - b X_i Y_i) + D X_i + E Y_i + F_i$$ (11)

where $b = B/(A-C)$. The value of ‘$b$’ is known from Eqn. 10. Thus, there are five unknown coefficients which are to be evaluated from five station data. A similar, expression can be obtained for the component $V_i$. The values of the coefficients, $A_1, A_2, b, C_1, C_2, D_1, D_2, ...$, can be computed by solving the matrix, as shown in Appendix 1. The large scale divergence was given by Eqn. 3, computed at the origin. Thus,

$$\frac{\partial U_i}{\partial X_i} + \frac{\partial V_i}{\partial Y_i} = D_1 + E_2$$ (12)

3. Results

Fig. 2 shows a plot between various combinations of the wavelengths $L_x$ and $L_y$ (meters$/10^5$) and the divergence ratio. In the legends, $L_x$ is shown as series 1, $L_y$ is shown as series 2 and the ratio of the divergences for a particular combination of wavelengths, as series 3. The wavelengths are to be multiplied by a factor $10^5$ to give the actual wavelengths in meters. Adjacent columns touching each other represent the components of the wavelengths and the point where it cuts the ratio curve (after being extended if necessary) the value of the corresponding ratio is obtained. It may be seen that even though the $L_y$ component of the wavelength is large in some cases, the corresponding ratio is less than 0.9, suggesting that it is not representative of the large scale field. It is therefore, the combination of the two components of the wavelength that decides large-scale representativeness of the wave. This point is made clearer in Fig. 3, which plots the total wavelength $L$ and the corresponding divergence ratio. It seen that the first value of $L$ for which the ratio is above 0.9, is $L = 2830$ km. There are several values of $L$ which are greater than this value, but the corresponding ratios are below 0.9.

4. Conclusion

This study indicates that the effective scales of the wind, used in derived parameters, such as divergence, depend upon both the components of the scales. The effective scale is representative of the large-scale field if the component scales combine in the proper proportion to generate the large-scale field. This aspect should be considered while examining field data for verification purposes by large-scale modelers.

Acknowledgements

The author is grateful to the Director of the Institute for providing the necessary facilities and to his colleagues for their sustained encouragement and help.

References

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Appendix 1

The derivatives of the coefficients $w.r.t.$ $F$, can be expressed as a product of an inverse matrix with $(-1)$, as shown:

$$\begin{bmatrix} X_1^2 & X_1 Y_1 & Y_1^2 & X_1 & Y_1 \\ X_2^2 & X_2 Y_2 & Y_2^2 & X_2 & Y_2 \\ X_3^2 & X_3 Y_3 & Y_3^2 & X_3 & Y_3 \\ X_4^2 & X_4 Y_4 & Y_4^2 & X_4 & Y_4 \\ X_5^2 & X_5 Y_5 & Y_5^2 & X_5 & Y_5 \end{bmatrix} \begin{bmatrix} \partial A/\partial F \\ \partial B/\partial F \\ \partial C/\partial F \\ \partial D/\partial F \\ \partial E/\partial F \end{bmatrix} = -1$$ (A1)

The first matrix represents the $X, Y$, distances of the respective stations from the origin. The second matrix represents the unknown derivative of the coefficients in
Eqns. 5 & 6, *w.r.t.* ‘F’. Thus, it can be evaluated as the product of inverse of matrix 1 with (-1), after which the value of ‘b’ in Eqn. (11), can be evaluated.

Now the coefficients in Eqn. 8 can be evaluated as follows:

\[
[A] = [X]^{-1} [U] \tag{A2}
\]

where [A] is the matrix of coefficients as in Eqn. 11, [X] represents the elements of distances expressed by the right hand side of Eqn. 11 and [U] represents the \(u\) - components of the velocities at each station. In a similar way the coefficients for the \(v\) - components of the velocity field can be evaluated and the large scale divergence field is computed from Eqn. 3.