Optical absorption and activated transport in polaronic systems

G. Schubert,1 G. Wellein,2 A. Weiße,3 A. Alvermann,1 and H. Fehske1

1Institut für Physik, Ernst Moritz Arndt Universität Greifswald, 17487 Greifswald, Germany
2Regionales Rechenzentrum Erlangen, Universität Erlangen, 91058 Erlangen, Germany
3School of Physics, The University of New South Wales, Sydney, NSW 2052, Australia
(Dated: November 6, 2018)

We present exact results for the optical response in the one-dimensional Holstein model. In particular, by means of a refined kernel polynomial method, we calculate the ac and dc electrical conductivities at finite temperatures for a wide parameter range of electron phonon interaction. We analyze the deviations from the results of standard small polaron theory in the intermediate coupling regime and discuss non-adiabaticity effects in detail.

PACS numbers: 71.38.-k, 71.38.Ht, 74.25.Fy

I. INTRODUCTION

The investigation of transport properties has been playing a central role in condensed matter physics for a long time. In recent years optical spectroscopy, for example, has contributed a lot to unravel the complex physics of highly correlated many-body systems, such as the one-dimensional (1d) MX chains, the quasi-2d high-temperature superconducting cuprates, or the 3d colossal magneto-resistive manganites. That way, optical measurements proved the importance of electron-phonon (EP) interactions in all these materials and, in particular, corroborated polaronic scenarios for modeling their electronic transport properties at least at high temperatures.

Polarons are quasi-particles composed of an electron and the surrounding ions which in a polar solid, provided the electron lattice interaction is sufficiently strong, are displaced from their equilibrium positions due to the presence of the electron. This bootstrap relation between electron and lattice displacement makes the particle heavy, because it has to drag with it the potential well due to the EP coupling. These quasi-particles are called large polarons or Fröhlich polarons. In the second case it is assumed that the bandwidth is small, whereas the dynamical mean-field approach inherently does not account for vertex corrections to the conductivity and for longer ranged hopping processes induced by the EP interaction. On the other hand, exact numerical investigations of the Holstein model provided quite a number of reliable results for the zero-temperature optical conductivity in the 1d and 2d (extended) Holstein models.

Motivated by this situation, in the present work we use a recently developed extended of the Kernel Polynomial Method (KPM) a refined numerical Chebyshev expansion technique, to compute both the dc and ac hopping conductivities at finite temperatures without any serious approximation.

II. MODEL AND METHOD

Our starting point is the 1d tight-binding Holstein Hamiltonian,

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j - \sqrt{\varepsilon_p \omega_0} \sum_i (b_i^\dagger + b_i) n_i + \hbar \omega_0 \sum_i b_i^\dagger b_i,$$

(1)

describing a single electron coupled locally to a dispersion-less optical phonon mode, where $c_i$ ($b_i^\dagger$) denotes the corresponding fermionic (bosonic) creation operator, and $n_i = c_i^\dagger c_i$. Setting $\hbar = 1$ and measuring all energies in units of the nearest-neighbor hopping integral $t$, the physics of the model is determined by the dimensionless EP coupling constants

$$\lambda = \varepsilon_p/2t \quad \text{and} \quad g^2 = \varepsilon_p/\omega_0$$

(2)
in the adiabatic ($\omega_0/t \ll 1$) and anti-adiabatic ($\omega_0/t \gg 1$) regimes, respectively. In 1d the crossover from large to small polaron behavior takes place at $\lambda \simeq 1$ ($g^2 \simeq 1$) in the former (latter) case. Whether large polarons form in the Holstein model for $d > 1$ is still under debate.
Addressing the linear response of our system to an external (longitudinal) electric field we consider the Kubo formula for the electrical conductivity at finite temperatures, which is

$$\text{Re} \sigma(\omega) = \pi \sum_{m,n} \frac{e^{-\beta E_n} - e^{-\beta E_m}}{ZL\omega} |\langle n|j|m \rangle|^2 \delta(\omega - \omega_{mn}).$$

Here $Z = \sum_0^{\infty} e^{-\beta E_n}$ is the partition function and $\beta = T^{-1}$ denotes the inverse temperature. Since the Holstein Hamiltonian involves bosonic degrees of freedom, the Hilbert space even of a finite $L$-site system has infinite dimension. In practice, however, the contribution of highly excited phonon states is negligible at the relevant temperatures, and the system is well approximated by a truncated phonon space with at most $M(\lambda, g, \omega_0; T)$ phonons. Then $|n\rangle$ and $|m\rangle$ are the eigenstates of $H$ within our truncated $D$-dimensional Hilbert space, $E_n$ and $E_m$ are the corresponding eigenvalues, and $\omega_{mn} = E_m - E_n$. In Eq. (3) the current operator has the standard hopping form, $j = i e \sum_i (c_i^\dagger c_{i+1} - c_{i+1}^\dagger c_i)$, and connects states with different parity. Thus, assuming that the ground state is non-degenerate, the expectation value $\langle 0|j|0 \rangle$ vanishes in the absence of an external electric field. The limit $\omega \rightarrow 0$ of Eq. (3) yields the dc conductivity, whereas the optical absorption is given by the finite frequency data. For small polarons both results are interesting and have been evaluated analytically at an early stage. At $T = 0$, the regular part of the optical conductivity, $\sigma^{\text{reg}}(\omega) = 2 \sum_{n>0} \omega_n^2 |\langle n|0 \rangle|^2 \delta(\omega - \omega_n)$, was calculated for finite 1d and 2d lattices with periodic boundary conditions (PBC) in a wide parameter range of the Holstein model, using a combination of the Lanczos algorithm and the KPM.

At finite temperatures, a similar straight-forward expansion of the conductivity is spoiled by the presence of the Boltzmann factors and the contribution of all matrix elements between eigenstates of the system. Instead, it turns out that a new generalised KPM scheme can be based upon a current operator density

$$j(x, y) = \sum_{m,n} |\langle n|j|m \rangle|^2 \delta(x - E_n) \delta(y - E_m).$$

Being a function of two variables, $j(x, y)$ can be expanded by a two-dimensional KPM,

$$j(x, y) = \sum_{k,l=0}^{N-1} \frac{\mu_{kl} w_{kl} y_l g_k^T T_k(x) T_l(y)}{\pi^2 \sqrt{(1 - x^2)(1 - y^2)}},$$

where the tilde refers to a rescaling of energy ($H \rightarrow \tilde{H}$) that maps the spectrum of $H$ into the domain $[-1, 1]$ of the Chebyshev polynomials of first kind $T_k(x)$. The finite order $N$ of the expansion leads to Gibbs oscillations which can be damped by introducing appropriate damping factors. Here we use $g_k$ derived from the Jackson kernels and $N = 512$ throughout the paper.

The core of the numerical work is the iterative calculation of the moments $\mu_{kl} = \text{Tr}(T_k(\tilde{H})j_l(\tilde{H}))$, where the trace can be replaced by an average over a relatively small number of random vectors $|r\rangle$. Finally, the factors $1/w_{kl} = (2 - \delta_{kl})(2 - \delta_{kl})$ account for the correct normalisation. Given the operator density $j(x, y)$ we find the optical conductivity by integration

$$\text{Re} \sigma(\omega) = \frac{\pi}{ZL\omega} \int_{-\infty}^{\infty} j(y + \omega, y) [e^{-\omega y} - e^{-\beta(y + \omega)}] dy.$$

The partition function $Z = \int_\infty^{\infty} \rho(E) \exp(-\beta E)$ is easily obtained by integrating over the density of states $\rho(E) = \sum_{n=0}^{D-1} \delta(E - E_n)$, which can be expanded in parallel to $j(x, y)$. Note the main advantage of this approach: The current operator density that enters the conductivity is the same for all temperatures, i.e., it needs to be expanded only once. Figure 1 exemplifies the different form of $j(x, y)$ in the weak and strong EP coupling regimes.

At very low temperatures, the numerical evaluation of expression requires some caution, since the Boltzmann factors heavily amplify small numerical errors in $j(y + \omega, y)$. We can avoid these problems, occurring mainly at the lower bound of the spectrum, by treat-
The optical absorption in the 1d Holstein model at $T = 0$ (in units of $\pi e^2 t^2$) compared to the analytical small polaron result Eq. (8) [dashed blue lines]. Exact diagonalization data (ED) are obtained for a system with $L = 6$ and $M = 45$; $\sigma_0$ is determined to give the same integrated spectral weight of the $\omega > 0$ (regular part) of Re $\sigma$.

For sufficiently strong coupling this formula predicts a weakly asymmetric Gaussian absorption peak centered at $\omega = 2\varepsilon_p$. A similar analytical formula can be derived for finite temperatures. \cite{18,26}

Starting at zero temperature, Fig. 3 shows Re $\sigma(\omega)$ for various EP-parameters. For $\lambda = 2$ and $\omega_\tau/t = 0.4$, i.e., at rather large EP coupling, but not in the extreme small polaron limit, we find also a pronounced maximum in the low-temperature optical response, which, however, is located, somewhat below $2\varepsilon_p = 2g^2\omega_0$, being the value for small polarons at $T = 0$. At the same time, the lineshape is more asymmetric than in standard polaron theory, with a weaker decay at the high-energy side, which fits even better the experimental behavior observed in polaronic materials such as TiO$_2$. \cite{27} Varying the parameters significant discrepancies to a Gaussian-like absorption are found. Then the polaron motion is not adequately described as hopping of a self-trapped carrier almost localized on a single lattice site.

At finite temperature two different transport mechanisms can be distinguished. Clearly, coherent transport, which for large EP couplings is related to diagonal (zero-phonon) transitions within the lowest extremely narrow polaron band, will be negligible at high temperatures. For instance, the amplitude of the current matrix elements between the degenerate states with momentum $K = \pm \pi/3$ ($K = 0, \pm \pi/3, \pm 2\pi/3$, and $\pi$ are the allowed wave numbers of a 6-site system with PBC) is of the order of $10^{-7}$ only. Whereas phase coherence is maintained during a diagonal transition, the particle loses its phase coherence if its motion is triggered by (multi-)phonon absorption and emission processes. These so-called non-
diagonal transitions which, of course, can take place also with zero energy transfer, become more and more important as the temperature increases. Accordingly the main transport mechanism is thermally activated hopping, where each hop becomes a statistically independent event. In the small polaron limit, where the polaronic sub-bands are roughly separated by the bare phonon frequency (cf. Fig. 4 right panel), this happens for \( T \gtrsim \omega_0 \).

Let us consider the activated regime in more detail (cf. Fig. 4). With increasing temperatures we observe a substantial spectral weight transfer to lower frequencies, and an increase of the zero-energy transition probability in accordance with previous results.

In addition, we find a strong resonance in the absorption spectra at about \( \omega \sim 2t \), which can be easily understood using a configurational coordinate picture. Placing a homogeneous lattice distortion \( u \) at \( L_u \) consecutive sites by applying the unitary transformation \( S^i(u) = \prod_{i=L_u}^{1} S^i(u) = \prod_{i=L_u}^{1} \exp[u(b_i - b_j)] \), the transformed Holstein Hamiltonian takes the form \( \tilde{H} = (|0\rangle S^1(u) H S(u)|0\rangle)_{ph} = -t \sum_{\langle i,j \rangle} \epsilon_j - 2\sqrt{\epsilon \omega_0 u} \sum_{i=L_u}^{1} n_i + \omega_0 u^2 L_u \). In the adiabatic strong-EP-coupling regime, the ground state can be approximated as an electron localized at a certain single site with the lattice being in a shifted oscillator state (\( L_u = 1 \)). That is, \( |\Psi_0\rangle = |1\rangle_{el} \otimes S^1_1(0)0\rangle_{ph} \) and \( E_0 = -\epsilon_p \), in accordance with the Lang-Firsov approximation. Now let us consider excitations from this ground-state, where the lattice distortion spreads over two neighboring sites (\( L_u = 2 \)) and the electron is in a symmetric or antisymmetric linear combination of \(|1\rangle_{el}\) and \(|2\rangle_{el}\), i.e., the particle is mainly located at sites 1 and 2 but delocalized between these sites. We then find \( |\Psi_{1,\pm}\rangle = (|1\rangle_{el} \pm |2\rangle_{el}) \otimes \left[ S^1_1(g/2) S^1_2(g/2)0\rangle_{ph} \right] \) and \( E_{1,\pm} = \mp t - \epsilon_p/2 \). Whereas the (potential) energy related to the displacement field is reduced to \( -\epsilon_p/L_u \), the kinetic energy comes into play since hopping processes between 1 and 2 are allowed. The current operator \( j \) connects these different-parity states with perfect overlap \( \langle \Psi_{1,\pm} | j | \Psi_{1,\pm} \rangle = (|t\rangle^2) \), giving rise to a strong signal in the optical absorption. Note that the excitation energy \( \omega_{1,-2} = 2t \) is independent of \( \epsilon_p \). In order to activate these transitions thermally, the electron has to overcome the “adiabatic” barrier \( \Delta = E_{1+} - E_0 = \epsilon_p/2 - t \). A finite phonon frequency will relax this condition. From Fig. 4 we find the signature to occur above \( T \gtrsim 0.5t \). Obviously this feature is absent in the standard small-polaron transport description which essentially treats the polaron as a quasiparticle without resolving its internal structure. Owing to the infinite number of neighboring sites it is also absent in the DMFT calculation. Of course, one could also extend this scenario to excitations where the electron is delocalized over more than two distorted lattice sites, but for the present parameters the signature of these weakly bound states would be rather small.

Entering the non-adiabatic regime of large phonon frequencies at fixed \( \lambda = 2 \), the pattern of sub-bands separated roughly by \( \omega_0 \) becomes more pronounced, but is, of course, washed out at higher temperatures (see Fig. 5) for \( \omega_0/t = 0.8 \). In Fig. 5 the average number of phonons contained in the ground state \( (\sim g^2) \) is smaller \((g^2 = 5)\) than in the previous case where \( g^2 = 10 \) \((\omega_0/t = 0.4)\). This also concerns the activated region \( \omega_0/t \lesssim T \lesssim \Delta \) but for these parameters the simple adiabatic picture anticipated above breaks down anyway.

Now let us decrease the EP coupling strength at small phonon frequencies \( \omega_0/t = 0.2 \) keeping \( g^2 = 10 \) fixed.

![FIG. 5: (Color online) Optical absorption by Holstein polarons at finite temperatures in the non-adiabatic regime (\( L = 6, M = 30 \)). Note that now the abscissa is scaled with respect to the phonon frequency.](image)

![FIG. 6: (Color online) Optical absorption in the adiabatic intermediate EP-coupling regime (the notation is the same as in Fig. 4 again we use \( L = 6, M = 40 \)). The inset illustrates the finite-size dependence of Re \( \sigma(\omega) \) for \( T \sim 0 \) (the vertical dotted line gives the phonon absorption threshold). It demonstrates that the gap observed at low frequencies and temperatures is clearly a finite-size effect, i.e., at weak-to-intermediate couplings the discrete electronic levels of our finite system show up in the conductivity spectra. These effects, of course, are of minor importance at larger EP couplings, where the polaronic bandwidth is strongly reduced, as well as for high temperatures.](image)
Results for the optical response in the vicinity of the large to small polaron crossover ($\lambda = 1$) are depicted in Fig. 6. Here the small polaron maximum has almost disappeared and the 2$t$-absorption feature can be activated at very low temperatures ($\Delta \to 0$ for the two-site model with $\lambda = 1$). The overall behavior of Re $\sigma(\omega)$ resembles that of polarons of intermediate size. At high temperatures these polarons will dissociate readily and the transport properties are equivalent to those of electrons scattered by thermal phonons. Let us emphasize that many-polaron effects become increasingly important in the large-to-small polaron transition region. As a result, polaron transport might be changed entirely compared to the one-particle picture discussed so far.

B. Sum rules

Before we consider the temperature dependence of the dc conductivity, it is useful to test the sum rules for the real part of the optical response. First we have the so-called $f$-sum rule,

$$S_{\text{tot}} := \int_{-\infty}^{\infty} \text{Re} \sigma(\omega) d\omega = -\frac{\pi e^2}{L} E_{\text{kin}},$$

which relates the $\omega$-integrated Re $\sigma(\omega)$ to the kinetic energy $E_{\text{kin}} = -t \sum_{(i,j)} \langle c_i^\dagger c_j \rangle_T$. Note that the $\omega = 0$ (Drude) contribution is included in (9). The second sum rule for Re $\sigma(\omega)$ is

$$\int_{0}^{\infty} \omega \text{Re} \sigma(\omega) d\omega = \frac{\pi}{L} \langle j^2 \rangle_T.$$  

Throughout our calculations Eq. (10) was fulfilled within numerical accuracy, where the thermal average $\langle j^2 \rangle_T$ was determined again using a 1d KPM. Testing sum rule (9) we gain important information about finite-size effects.

Figure 7 compares $E_{\text{kin}}$ and $S_{\text{tot}} L/\pi e^2$ obtained from our finite-cluster calculation. At weak EP couplings the kinetic energy is a strictly monotonic increasing function of temperature and becomes strongly suppressed at high temperatures due to scattering of the electron by thermal phonons. Whereas the $f$-sum rule is almost perfectly fulfilled for smaller values of $\beta$, we found pronounced deviations at low temperatures which, without doubt, can be assigned to the finite size of our Holstein ring (cf. the $L$ dependence of $S_{\text{tot}}$ shown in the upper panel of Fig. 7 and also the discussion of the dc conductivity below). Clearly finite-size effects become important when the temperature is comparable to the energy gaps in the spectrum of $H$ (see Fig. 2). At strong EP couplings the transport is hopping-dominated and the kinetic energy exhibits a maximum at a finite temperature that can be related to the thermal activation energy of polarons. Now, in view of the narrow small polaron band (cf. Fig. 2), finite size gaps are small and the $f$-sum rule is fulfilled down to very low temperatures.

Small deviations appear to vanish rapidly with increasing system size (see inset, lower panel of Fig. 7). In order to analyze the contributions of different transport processes to $E_{\text{kin}}$ in some more detail, we have decomposed $S_{\text{tot}} = S_{\text{tot}}^{\text{ED}} + \sum_{s=0}^{S-1} S_{\text{tot}}^{1\text{d}(s)} + S_{\text{tot}}^{2\text{d}}$ in analogy to Eq. (7). Figure 7 shows that the coherent (intra-band) contribution ($\propto S_{\text{tot}}^{\text{ED}}$) is almost negligible at the temperatures considered. Inter-band transitions connecting eigenstates of the lowest polaron band to higher excited states (1d KPM) are the determining factor at low $T$. If the renormalised polaron bandwidth is small enough, all states in the band are equally populated, leading to pretty much the same values of $S_{\text{tot}}^{1\text{d}(s)}$. At high temperatures, of course, the transitions covered by the 2d KPM are predominant.

C. dc conductivity

The dc conductivity is obtained by taking the limit $\omega \to 0$ of (8), which with $\lim_{\omega \to 0} (1 - e^{-\beta \omega})/\omega = \beta$ yields
polaron hopping transport manifests in the temperature dependence, leading to the well-known absorption maximum in $\text{Re } \sigma_{dc}(T)$ (cf. Fig. 8 lower panel). Since this signature is related to (rather local) polaron excitation processes the position of the maximum is almost independent of the system size. In comparison to the $d = \infty$ (DMFT) results we find the same qualitative behavior in the relevant temperature regime $\omega_0 \lesssim T \sim 2\varepsilon_p$ but a different location of the conductivity maximum. Generally in DMFT the activation energy for polaron hopping turns out to be lower than expected from commonly accepted arguments for finite-d systems. Increasing the lattice size, our 1d Holstein data indicates that this discrepancy does not necessarily imply the failure of standard theory of hopping conduction but may partly arise from dimensionality effects on polaron transport in infinite dimensions. Conversely, and in light of the deviations found for the ac conductivity (cf. Fig. 1), the standard (anti-adiabatic) strong-coupling description can only be supposed to provide estimates on relevant energy scales in the intermediate adiabatic EP-coupling regime.

![FIG. 8: (Color online)](image)

Re $\sigma_{dc}$ essentially counts the number of thermally accessible current carrying (degenerate) states. Since we have $\langle 0|j|0 \rangle = 0$ the conductivity almost vanishes for small $T$ below the finite size gap between the ground state and the first excited state. In the thermodynamic limit, $L \to \infty$, Re $\sigma_{dc}$ is related to the charge stiffness $D_c$, or the so-called Drude weight (at $T = 0$)\textsuperscript{20}

The temperature dependence of the dc conductivity is illustrated in Fig. 8. Again the weak coupling results appear to have a rather strong finite-size dependence (note that Re $\sigma_{dc}$ depicted in Fig. 8 is an intensive quantity). When $L$ increases a pronounced peak develops at low temperatures. This peak can be attributed to a normal “metallic-like” behavior. Independent of the coupling strength $\varepsilon_p$ the polaron is an itinerant quasiparticle thus, for $T \to 0$, always leading to band conduction. When $\varepsilon_p > 0$, we expect that Re $\sigma(\omega \to 0)$ is finite for $T > 0$ in contrast to an ideal conductor.

The shoulder observed for the 10 and 12 site systems at $T/t \gtrsim 1$, again is an artifact of our phonon truncation procedure as can be seen by comparing the data obtained for $L = 6$ and different numbers of the phonon cut-off $M$. In the strong-EP-coupling polaronic regime, band-like transport becomes extensively suppressed (the Drude weight is exponentially small). Nevertheless quantum zero-point phonon fluctuations cause polaron delocalization at $T = 0$. At higher temperatures incoherent

IV. SUMMARY

In this work, we have investigated the motion of a charge carrier in response to ac and dc external fields for strongly correlated 1d electron-phonon systems. The combination of Lanczos diagonalization and the Kernel Polynomial Method has enabled us to calculate for the first time quasi-exactly the temperature dependence of the optical absorption spectra and the dc conductivity in the framework of the one-dimensional Holstein model. Besides the well-known polaron maximum a pronounced absorption feature at about $2\pi$ is found in the optical conductivity. Finite-size effects were identified and assessed, e.g., on the basis of the $f$-sum rule. In the physically most interesting range of intermediate coupling strengths and phonon frequencies, we find that the conductivity deviates from the standard small polaron results.

Acknowledgments

We are grateful to M. Hohenadler and J. Loos for helpful discussions. This work was supported by the Deutsche Forschungsgemeinschaft through SPP1073, by KONWIHR and by the Australian Research Council. H. F. acknowledges the hospitality at the University of New South Wales sponsored by the Gordon Godfrey Bequest. Special thanks go to NIC Jülich and HLRN Berlin for granting access to their supercomputer facilities.

1 A. R. Bishop and B. I. Swanson, Los Alamos Science 21, 133 (1993); H. Fehske, M. Kinateder, G. Wellein, and A. R. Bishop, Phys. Rev. B 63, 245121 (2001).
E. Dagotto, Rev. Mod. Phys. 66, 763 (1994); S. Tajima, Y. Fudamoto, T. Kakeshita, B. Gorshunov, V. Železní, K. M. Kojima, M. Dressel, and S. Uchida, Phys. Rev. B 71, 094508 (2005).

S. Jin, T. H. Tiefel, M. McCormack, R. A. Fastnach, R. Ramesh, and L. H. Chen, Science 264, 413 (1994); A. Weiße and H. Fehske, New J. Phys. 6, 158 (2004).

D. Emin, Phys. Rev. B 48, 13691 (1993).

A. S. Alexandrov and N. F. Mott, Polaron and Bipolaron (World Scientific, Singapore, 1995).

D. C. Worledge, L. Miéville, and T. H. Geballe, Phys. Rev. B 57, 15267 (1998).

S. Fratini, F. de Pasquale and S. Ciuchi, Phys. Rev. B 63, 153101 (2001); S. Fratini and S. Ciuchi, Phys. Rev. Lett. 91, 256403 (2003).

H. Fehske, J. Loos, and G. Wellein, Z. Phys. B 104, 619 (1997); G. Wellein and H. Fehske, Phys. Rev. B 58, 6208 (1998); H. Fehske, J. Loos, and G. Wellein, Phys. Rev. B 61, 8016 (2000).

C. Zhang, E. Jeckelmann, and S. R. White, Phys. Rev. B 60, 14092 (1999).

R. N. Silver, H. Röder, A. F. Voter, and D. J. Kress, J. of Comp. Phys. 124, 115 (1996).

A. Weiße, G. Wellein, A. Alvermann, and H. Fehske (2005), URL http://arXiv.org/abs/cond-mat/0504627.

T. Holstein, Ann. Phys. (N.Y.) 8, 325 (1959); ibid 8, 343 (1959).

G. Wellein and H. Fehske, Phys. Rev. B 56, 4513 (1997).

M. Capone, W. Stephan, and M. Grilli, Phys. Rev. B 56, 4484 (1997).

D. Emin, Phys. Rev. B 48, 3973 (1986); V. V. Kabanov and O. Y. Mashtakov, Phys. Rev. B 47, 6060 (1993).

H. Fehske, H. Röder, G. Wellein, and A. Mistriotis, Phys. Rev. B 51, 16582 (1995); L.-C. Ku, S. A. Trugman, and J. Bonča, Phys. Rev. B 65, 174306 (2002).

V. Cataudella, G. D. Filippis, and G. Iadonisi, Phys. Rev. B 60, 153163 (1999); ibid 63, 052406 (2001).

G. D. Mahan, Many-Particle Physics (Kluwer Academic/Plenum Publishers, New York, 2000).

B. Bäuml, G. Wellein, and H. Fehske, Phys. Rev. B 58, 3663 (1998).

O. S. Barisić, Phys. Rev. B 65, 144301 (2002); H. Fehske, M. Sekania, A. P. Kampf, and G. Wellein, Eur. Phys. J. B 31, 11 (2003).

I. G. Lang and Y. A. Firsov, Zh. Eksp. Teor. Fiz. 43, 1843 (1962); Y. A. Firsov, Polaron (Izd. Nauka, Moscow, 1975); Y. A. Firsov, Semiconductors 29, 515 (1995).

A. Weiße, Eur. Phys. J. B 40, 125 (2004).

D. Jackson, Trans. Amer. Math. Soc. 13, 491 (1912).

D. A. Drabold and O. F. Sankey, Phys. Rev. Lett. 70, 3631 (1993).

H. G. Reik and D. Heese, J. Phys. Chem. Solids 28, 581 (1967).

H. Böttger and V. V. Bryksin, Hopping conduction in solids (Akademie Verlag, Berlin, 1985).

E. K. Kudinov, D. N. Mirlin, and Y. A. Firsov, Fiz. Tverd. Tela 11, 2789 (1969).

E. L. Nagaev, Sov. Phys. Solid State 4, 1611 (1963).

M. Hohenadler, D. Neuber, W. von der Linden, G. Wellein, J. Loos, and H. Fehske, Phys. Rev. B 71, 245111 (2005).

B. S. Shastry and B. Sutherland, Phys. Rev. Lett. 65, 243 (1990); B. N. Narozhny, A. J. Millis, and N. Andrei, Phys. Rev. B 58, R2921 (1998).