Modeling of non-contact atomic force microscope with two-term excitations

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Abstract. The goal of this article is to study the dynamical behavior of atomic force microscope cantilever in its non-contact mode of operation. The lumped parameter model is used to construct the mathematical model of the cantilever. The tip of the cantilever is excited by two harmonic terms and is in interaction with the sample surface. The Van der Waals force, tip-sample interaction force, makes the system nonlinear. Using multiple scales method, the frequency response equation is found. The effects on the amplitude of excitations, the damping coefficient, and initial sample – tip distance is studied and presented as the results.

1. Introduction
The atomic force microscope (AFM) which is a high-resolution scanning the probe microscope was invented in 1986 [1]. It is necessary to determine the surface topography with a resolution up 1000KX. Primarily, the atomic force microscope was a profilometer, only the radius of the tip (probe) was about tens of angstroms. The desire to improve lateral resolution has led to the development of dynamic methods.

The superior advantage of the AFM is scanning the sample surface and constructing the topography in three dimensions in comparison with other microscope types. The other feature of AFM is the capability of scanning any samples type and in different fields of science and industry AFM may use [2-5].

As presented in figure 1(a), a conventional AFM system includes a microcantilever with a sharp tip end, laser unit, photodetector (photodiode), controller, and PZT (mechanical stage). The cantilever excited by harmonic force faces deflection as the result of resultant interaction force (tip-sample interaction force). The deflection can be detected utilizing the reflection of the laser beam gathering by photodiode unit which provides feedback through the controller to the mechanical stage to keep the tip-sample distance constant. Thus, recording the magnitude of the deflection, we can construct the topography of the surface. Therefore, the study of the dynamic behavior of cantilever plays an important role in studying AFM.

Works [6-12] presented different methods to study the dynamic behavior of the cantilever in the different modes of operation.

This article, which is the extended work of [8, 12], deals with the construction of mathematical models for a situation in which the AFM cantilever tip excited by two harmonic terms under the influence of the Van der Waals force, and solving using multiple scales method. It should be noted that to create the mathematical model of the AFM cantilever lumped parameter model is utilized.
2. Modeling of non-contact AFM cantilever

In this paper, to construct the mathematical model of the cantilever, we use a lumped parameter model (figure 1(b)).

The equation of motion of the system is:

\[ m \frac{d^2 \zeta}{d \tau^2} + b \frac{d \zeta}{d \tau} + k \zeta = f_{\text{tip}} + f_{\text{ext}}, \] (1)

where \( m \) is the effective mass of the cantilever attached to the end of a massless spring with the stiffness of \( k \) and corresponding damping coefficient \( b \).

Two harmonic forces, \( f_{\text{ext}} \), excite the cantilever tip which is acted upon the resulting force from the tip-sample interaction, \( f_{\text{tip}} \). Excitations are:

\[ f_{\text{ext}} = F_1 \cos(\omega_1 \tau + \theta_1) + F_2 \cos(\omega_2 \tau + \theta_2), \] (2)

and Van der Waals sphere–plane that is considered as the interaction force presented as follows:

\[ f_{\text{tip}} = -\frac{A_H R}{6(\zeta_0 - \zeta)^2}. \] (3)

Introducing the dimensionless variables,

\[ x = \frac{\zeta}{\zeta_0}, \quad t = \omega_0 \tau, \quad P_n = \frac{F_n}{m \zeta_0}, \quad \eta = \frac{\zeta_0}{\zeta}, \]

\[ c = \frac{b}{m \omega_0}, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \Omega_n = \frac{\omega_n}{\omega_0}, \quad d = \frac{4}{27}, \] (4)

and finding the relax point of the system setting time and external force terms in (1) to zero,

\[ \zeta_\text{x} = 1.5(2H)^{1/3}, \quad H = \frac{A_H R}{6k}, \] (5)

one can obtain the dimensionless equation of motion as
\[ \ddot{x} + c\dot{x} + x = \frac{d}{(\eta - x)^2} + P_1 \cos(\Omega_1 t + \theta_1) + P_2 \cos(\Omega_2 t + \theta_2) \quad (6) \]

In equation (6) dot denotes the derivative with respect to \( t \).

3. Multiple scales method

In this section, we use the method of multiple scales [13], to construct a uniformly valid approximate solution. The equation (14) is scaled as:

\[ \ddot{x} + x = -\varepsilon c\dot{x} + \varepsilon \frac{d}{(\eta - x)^2} + P_1 \cos(\Omega_1 t + \theta_1) + P_2 \cos(\Omega_2 t + \theta_2) \quad (7) \]

Defining time scales as \( T_n = \varepsilon^n t \) (for \( n = 0, 1, 2 \ldots \)) the derivatives with respect to \( t \) will be:

\[
\begin{align*}
\frac{d}{dt} &= D_0 + \varepsilon D_1 + 2\varepsilon D_2 + \\
\frac{d^2}{dt^2} &= D_0^2 + \varepsilon D_1 D_0 + \varepsilon^2 (D_1^2 + 2D_0 D_1) + \ldots 
\end{align*}
\quad (8)
\]

where \( D_n = \frac{\partial}{\partial T_n} \).

An approximate solution of equation (7) is

\[ x(t; \varepsilon) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + O(\varepsilon^2). \quad (9) \]

Substituting equation (9) in equation (7) using the system of (8) and equating the coefficients of \( \varepsilon^0 \) and \( \varepsilon^1 \) on both sides, we obtain:

\[
\begin{align*}
D_0 \dot{x}_0 + x_0 &= +P_1 \cos(\Omega_1 T_0 + \theta_1) + P_2 \cos(\Omega_2 T_0 + \theta_2), \\
D_0 \dot{x}_1 + x_1 &= -2D_0 D_1 x_0 - cD_0 x_0 + \rho x_0^3, \quad \rho = \frac{4d}{\eta^2}.
\end{align*}
\quad (10, 11)
\]

The solution of equation (10) can be constructed as:

\[
\begin{align*}
x_0 &= A(T_1) e^{iT_0} + W_1 e^{\Omega_1 T_0} + W_2 e^{\Omega_2 T_0} + \bar{A}(T_1) e^{-iT_0} + \bar{W}_1 e^{-\Omega_1 T_0} + \bar{W}_2 e^{-\Omega_2 T_0}, \\
W_n &= \frac{P_n}{2(1 - \Omega_n^2)} e^{i\alpha_n}.
\end{align*}
\quad (12)
\]

Inserting equation (12) into equation (11) leads to
\[
D_0^2 x_1 + x_1 = - (iA' + icA + \rho A (3A\bar{A} + 6W_1\bar{W}_1 + 6W_2\bar{W}_2) + 3\rho W_1 W_2 e^{i\sigma T_0}) e^{i\Omega T_0} + A^2 e^{2i\Omega T_0} \\
+ W_1^2 e^{2i\Omega T_0} + W_2^2 e^{2i\Omega T_0} + 2(A\bar{A} + W_1\bar{W}_1 + W_2\bar{W}_2) + 2AW_1 e^{i(1+\Omega)T_0} \\
+ 2AW_2 e^{i(1+2\Omega)T_0} + 2\bar{A}\bar{W}_1 e^{i(1-\Omega)T_0} + 2\bar{A}\bar{W}_2 e^{i(1-2\Omega)T_0} + 2W_1 W_2 e^{i(\Omega + 2\Omega)T_0} \\
-W_1 (ic\Omega + 3\rho (2A\bar{A} + W_1\bar{W}_1 + 2W_2\bar{W}_2)) e^{i\Omega T_0} - \rho A^3 e^{3i\Omega T_0} - \rho W_1^3 e^{3i\Omega T_0} \\
-W_2 (ic\Omega + 3\rho (2A\bar{A} + 2W_1\bar{W}_1 + 2W_2\bar{W}_2)) e^{i\Omega T_0} - \rho W_2^3 e^{3i\Omega T_0} \quad (13)
\]

where \(cc\) is the conjugate parts.

In the rest of the article, we consider the case in which \(\tilde{\omega}_0 = 1 = 2\Omega_1 + \Omega_2\). Now we introducing detuning parameter \(\sigma\) as

\[
\tilde{\omega}_0 = 1 = 2\Omega_1 + \Omega_2 - \epsilon \sigma. \quad (14)
\]

To eliminate the singular parts of equation (13) we have:

\[
iA' + icA + \rho A (3A\bar{A} + 6W_1\bar{W}_1 + 6W_2\bar{W}_2) + 3\rho W_1 W_2 e^{i\sigma T_0} = 0. \quad (15)
\]

Introducing \(A = \frac{1}{2} a e^{i\theta}\), inserting in equation (15) and separating real and imaginary parts we have:

\[
a' = - \frac{1}{2} ca - \rho \nu_1 \sin(\sigma T_1 - \beta_1 + 2\theta_1 + \theta_2), \\
\beta' = \rho \nu_2 + \frac{3\rho}{8} a^2 + \frac{\rho}{a} \nu_1 \cos(\sigma T_1 - \beta_1 + 2\theta_1 + \theta_2), \\
\nu_1 = \frac{P_1^2 P_2}{8 (1-\Omega_1)^2 (1-\Omega_2)^2}, \\
\nu_2 = \frac{3}{4} \left( \frac{P_1^2}{(1-\Omega_1)^2} + \frac{P_2^2}{(1-\Omega_2)^2} \right). \quad (16)
\]

The setting \(\varphi = \sigma T_1 - \beta_1 + 2\theta_1 + \theta_2\), we can rewrite equation (16) as:

\[
a' = - \frac{1}{2} ca - \rho \nu_1 \sin(\varphi), \\
\varphi' = \sigma - \rho \nu_2 - \frac{3\rho}{8} a^2 - \frac{\rho}{a} \nu_1 \cos(\varphi). \quad (17)
\]

Setting \(a' = \varphi' = 0\) in equation (17) one can obtain frequency response equation:
\[
\left(\frac{c}{2}\right)^2 + \left(\sigma - \rho v_2 - \frac{3}{8} \rho a^2\right)^2 = \left(\frac{\rho v_1}{a}\right)^2.
\] (18)

Now, to investigate the influence of excitations, damping, and initial sample–tip distance on the behavior of the cantilever excited by two harmonic forces acted on the cantilever tip, we use equation (18) (figure 2).

The excitations’ effect on the frequency response is presented in figure 2(a) and figure 2(a). As shown in figure 2(a) by increasing the amplitude of the excitation of the first excitation term, \(v_1\), the amplitude increases, but by increasing the amplitude of the excitation of the second excitation term, \(v_2\), the graph shifts to the right without any change in the amplitude.

In figure 2(c) effect of nonlinearity, the initial tip-sample distance is shown.

Figure 2(d) illustrates the damping coefficient effect on the frequency response. As expected, by increasing the damping coefficient the peak response amplitude decreases respectively.

![Graphs showing the effect of excitations, damping, and initial sample-tip distance on the frequency response](image)

**Figure 2.** (a) Effect of \(v_1\), (b) effect of \(v_2\), (c) effect of \(\rho\), (d) effect of \(c\).

In nonlinear systems, multivaluedness phenomena occur. This means there exist two values for amplitude by having a given frequency. This phenomenon which is called the jump phenomenon is shown in figure 3.
Figure 3. Jump phenomenon.

As illustrated in figure 3 by increasing the detuning parameter, $\sigma$, the magnitude of the amplitude of the response increases ($-\infty$, point 1]. A further increase of $\sigma$ leads to a jump from point 1 to point 2 and from this point the magnitude of the amplitude decreases (point 2, $\infty$). Now, we consider the other situation in which the detuning parameter, $\sigma$, decreases till point 3 the magnitude of the amplitude of the response increases, and then a jump takes place to the higher amplitude, point 4, from which the amplitude decreases [13-16].

4. Conclusion
This article deals with mathematical modeling of the atomic force microscope cantilever under the influence of the Van der Waals sphere plane interaction force. The cantilever tip excited by two terms harmonic forces. To model, the cantilever lumped model has been used. The system has been solved using the multiple scales method and the frequency response equation has been found to investigate the nonlinearity, amplitude of excitations, and damping coefficient effects.

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