Research Article

Further Results on Stability Analysis for Uncertain Delayed Neural Networks with Reliable Memory Feedback Control

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1.Introduction

As we all know, because the structure of the NN model is similar to the synapse structure of the human brain, it can be described by a variety of differential equations [1–7]. The wide application of NNs in various fields has received widespread attention, such as signal processing [8], fault diagnosis [9], optimization problem solving [10], pattern recognition [11], image processing [12], and other fields [13–16]. However, artificial NNs always need to be maintained in practical engineering applications, so the stability of NNs has also been extensively studied by scholars at home and abroad [15, 17–19]. In the engineering application of NNs, the signal transmission between synapses has a time delay, and this delay may lead to instability of the NNs, increased oscillation, or performance degradation [20–22]. Therefore, the stability research of time-varying neural networks (DNNs) has also received extensive attention [23–26]. Compared with NNs, the latter requires more technical means and engineering requirements to maintain stability in technical analysis [27–29]. Therefore, the research on DNNs is obviously more important.

Thus, in order to effectively solve this problem, Lyapunov–Krasovskii functional (LKF) method is proposed [18, 30–33]. So far, many researchers have made a lot of contributions of how to establish a suitable LKF in order to better study the delayed systems [13, 34–37]. In [30], the authors proposed a novel LKF, which contains a common double-integral term, an augmented double-integral term, and two delay-product-type terms, was constructed to analyze the exponential stability. In [13], the authors proposed novel Hindawi Mathematical Problems in Engineering Volume 2021, Article ID 6698249, 16 pages https://doi.org/10.1155/2021/6698249
weighting-delay-based stability criteria for system research. In [36, 37], the authors mainly studied the sampled-data control and gave the analysis and proof of related stability.

However, there are many methods to construct a reasonable LKF, but only increasing the cross-sectional area will hardly improve, and it will cause a heavy calculation burden [23, 38, 39]. Therefore, the method of constructing LKF from a new perspective has become a hot issue in current research [34, 40]. Through in-depth study of existing work, this paper proposes an improved DTPF strategy to construct a new LKF, which fully considers information concerning time delays and the derivative information of both states and time delays, and the conservativeness of the guidelines can be further reduced. The issues discussed above have inspired the purpose of this study.

Based on the above discussions, we establish some new stability criteria for UDNNs, and a reliable memory feedback controller is designed to ensure that the considered system is asymptotically stable. Compared with the existing results [22–25], the main contributions of this paper are as follows:

1. A novel quadratic function \( V_1(x(t)) \) is constructed via developing an improved TDPF approach, which can fully excavate some intrinsic relationships between the delay derivative information and the time delay

2. Based on this construction method of LKF, the information storage performance of the function is strengthened, an appropriate integral inequality and linear convex combination method are adopted, and a more conservative stability criterion is obtained

3. Different from the earlier work, this paper designs a new RMPC, which fully considers the effective transmission of the three state signals of the controller while enhancing the performance of the controller

2. Preliminaries

Consider the following UDNN system:

\[
\dot{z}(t) = -\mathcal{A}(t)z(t) + \mathcal{W}_0g(z(t)) + \mathcal{W}_1g(z(t-h(t))) + u(t) + a,
\]

where \( z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \mathbb{R}^n \) denotes the neuron state vector and \( u(t) \) is the control input signal. \( g(.) = [g_1(.), g_2(.), \ldots, g_n(.)]^T \in \mathbb{R}^n \) represents the neuron activation function; \( \mathcal{A}(t) = \mathcal{A} + \Delta \mathcal{A}(t) \), and \( \Delta \mathcal{A}(t) \) is the revalued unknown matrix representing time-varying parameter uncertainties of (1) and satisfies \( \mathcal{A} + \Delta \mathcal{A}(t) = \mathcal{H} \mathcal{F}(t)E \). Here, \( \mathcal{A} > 0 \) is a symmetric matrix; \( H, E \) are known real constant matrices of appropriate dimensions. \( \mathcal{F}(t) \in \mathbb{R}^{k \times l} \) is an unknown time-varying matrix function satisfying \( \mathcal{F}(t)F(t) \leq I \). \( \mathcal{W}_0 \) indicates the connection weight matrix, and \( \mathcal{W}_1 \) expresses the delayed connection weight matrix: \( a = [a_1, \ldots, a_n]^T \in \mathbb{R}^n \) is a vector. \( h(t) \) is time varying and satisfies

\[
0 \leq h(t) \leq h, \\
0 \leq h(t) \leq \mu.
\]

Based on Assumption 1 in [25], suppose that \( z^* \) is the balance point of UDNN (1), which can be transferred to the origin by conversion, \( x(\cdot) = z(\cdot) - z^* \). Then, system (1) can be expressed as

\[
\dot{x}(t) = -\mathcal{A}x(t) + \mathcal{W}_0f(x(t)) + \mathcal{W}_1f(x(t-h(t))) + u(t),
\]

where \( x(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^T \in \mathbb{R}^n \) is the state vector of the transformed system and \( f(\cdot) = [f_1(\cdot), \ldots, f_n(\cdot)]^T \in \mathbb{R}^n \) with \( f_j(x_j(\cdot)) = g_j(x_j(\cdot) + z_j^* - g_j(z_j^*) \) being the activation function of the converted system. Based on (3), it holds that

\[
\begin{align*}
\mathcal{L}_i & \leq \frac{g_i(u_i + y_i^*) - g_i(y_i^*)}{u_i} = \frac{f_i(u_i)}{u_i} \\
& \leq I_i, \quad f_i(0) = 0, \quad \forall u \neq 0, \quad i = 1, \ldots, n.
\end{align*}
\]

In order to express a reliable control problem, the following actuator failure models are used in this technical description: for \( m = 1, \ldots, 0, n = 1, \ldots, n \):

\[
u_i^m(t) = (1 - \pi_i^m(t))u_i(m)(t), \quad 0 \leq \pi_i^m \leq \pi_i^m \leq \pi_i^m,
\]

where \( \pi_i^m \) is an unknown constant. Here, \( n \) is the nth case of failure, and \( j \) is the total number of failure cases. \( u_i^m(t) \) indicates the control signal from the signal of the nth actuator in the nth fault situation. \( \pi_i^m \) and \( \pi_i^m \) represent the lower and upper bounds of \( \pi_i^m \). First, when \( \pi_i^m = \pi_i^m = 0 \), in the nth failure mode, the nth actuator has no failure. When \( \pi_i^m = \pi_i^m = 1 \), the nth actuator \( u_i^m \) stops working in the nth fault situation. When \( 0 \leq \pi_i^m \leq \pi_i^m < 1 \), in the nth fault condition, the type of actuator failure is a gradual descent condition.

The actuator fault matrix \( \Pi \) is designed as the following:

\[
0 \leq \Pi = \text{diag}(\pi_1^m, \ldots, \pi_n^m) \leq \Pi \leq \Pi = \text{diag}(\pi_1^m, \ldots, \pi_n^m) \leq I.
\]

Next, we denote the following notation:

\[
\begin{align*}
\pi_{am}^m & = \frac{\pi_{am}^m + \pi_{an}^m}{2}, \quad \Pi_a = \text{diag}(\pi_{a1}^m, \ldots, \pi_{an}^m), \\
\pi_{bm}^m & = \frac{\pi_{bm}^m - \pi_{bn}^m}{2}, \quad \Pi_b = \text{diag}(\pi_{b1}^m, \ldots, \pi_{bn}^m).
\end{align*}
\]

From (7) and (8), \( \Pi \) can be expressed as follows:

\[
\Pi = \Pi_a + \Pi_b \Delta \Pi_0,
\]

where \( \| \Delta \Pi_0 \| \leq I_m \).

Thus, we get the reliable controller design as follows:

\[
u(t) = (\mathcal{F} - \Pi)u(t).
\]
Remark 1. Compared with the current design methods of the reliability controller [26–29], this paper introduces the RMFC design with effective lowering of the brake. The reliability control design considered in (10) is more comprehensive than the general reliability control design, which has a wider range of applications. Then, the RMFC is as follows:

\[ u(t) = \mathcal{H}_1 x(t) + \mathcal{H}_2 x(t - h(t)) + \mathcal{H}_3 x(t - h). \]  

(11)

From the above discussion, consider combining (10) and (11) to get the reliability controller design as follows:

\[
\dot{x}(t) = (-\mathcal{F} + (\mathcal{F} - \Pi)\mathcal{H}_1 x(t) + \mathcal{H}_0 f(x(t)) + \mathcal{H}_1 f(x(t - h(t))) + (\mathcal{F} - \Pi)\mathcal{H}_2 x(t - h(t)) + (\mathcal{F} - \Pi)\mathcal{H}_3 x(t - h).
\]  

(12)

\[
\text{Lemma 1 (see [38]). Given a symmetric positive definite matrix } \mathcal{R}, \text{ scalar } \alpha, \text{ scalar } \beta, \text{ and } \alpha < \beta, \text{ and } \epsilon \text{ in } [\alpha, \beta] \rightarrow \mathbb{R}^n. \text{ The following are the inequalities under the given conditions:}
\]

\[
\begin{align*}
(\beta - \alpha) \int_0^\beta \mathcal{R}(s) \mathcal{E}(s) ds & \geq \chi_1^T \mathcal{R} \chi_1, \\
(\beta - \alpha) \int_0^\beta \mathcal{R}(s) \mathcal{E}(s) ds & \geq \chi_2^T \mathcal{R} \chi_2, \\
(\beta - \alpha) \int_0^\beta \mathcal{R}(s) \mathcal{E}(s) ds & \geq \chi_3^T \mathcal{R} \chi_3,
\end{align*}
\]

(14)

where \( \chi_1 = e(\beta) - e(\alpha), \chi_2 = e(\beta) + e(\alpha) - (2/\beta - \alpha) \int_0^\beta e(s) ds, \chi_3 = \int_0^\beta e(s) ds, \chi_3 = \int_0^\beta e(s) ds - (2/\beta - \alpha) \int_0^\beta e(s) ds - (12/\beta - \alpha)^2 \int_0^\beta e(s) ds d\theta. \)

3. Main Results

In this section, we will provide a novel RMFC design scheme for (12). In the following theorem, the asymptotic condition for system (13) is provided under the designed gain matrices \( \mathcal{H}_i \) (i = 1, 2, 3). For simplicity, some relevant notations are defined as in Appendix A.

\[ V(x(t)) = \sum_{i=1}^3 V_i(x(t)), \]  

(20)

where

**Theorem 1.** Given positive scalars \( h, \mu, \) and \( \epsilon. \) System (13) is asymptotically stable if there exist symmetric positive definite matrices \( P_1, P_2, P_3, \) \( \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \) \( \mathcal{F}_4, \mathcal{F}_5, \) \( \mathcal{F}_6, \) and \( \mathcal{F}_7 \) (i = 1, 2, . . . , 5), any symmetric matrices \( \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \) \( \mathcal{H}_4, \) \( \mathcal{H}_5, \) \( \mathcal{H}_6, \) and \( \mathcal{H}_7 \) with appropriate dimensions such that the following LMIs hold:

\[
\begin{align*}
\mathcal{P}_{1,h(t)} & > 0, \\
\mathcal{P}_{2,h(t)} & > 0, \\
\mathcal{P}_{3,h(t)} & > 0, \\
\mathcal{P}_{4,h(t)} & > 0, \\
\mathcal{P}_{5,h(t)} & > 0,
\end{align*}
\]

(15)

\[
\begin{align*}
\mathcal{P}_{1,h(t)} & > 0, \\
\mathcal{P}_{2,h(t)} & > 0, \\
\mathcal{P}_{3,h(t)} & > 0, \\
\mathcal{P}_{4,h(t)} & > 0, \\
\mathcal{P}_{5,h(t)} & > 0,
\end{align*}
\]

(16)

\[
\begin{align*}
\mathcal{P}_{1,h(t)} & > 0, \\
\mathcal{P}_{2,h(t)} & > 0, \\
\mathcal{P}_{3,h(t)} & > 0, \\
\mathcal{P}_{4,h(t)} & > 0, \\
\mathcal{P}_{5,h(t)} & > 0,
\end{align*}
\]

(17)

\[
\begin{align*}
\mathcal{P}_{1,h(t)} & > 0, \\
\mathcal{P}_{2,h(t)} & > 0, \\
\mathcal{P}_{3,h(t)} & > 0, \\
\mathcal{P}_{4,h(t)} & > 0, \\
\mathcal{P}_{5,h(t)} & > 0,
\end{align*}
\]

(18)

\[
\begin{align*}
\mathcal{P}_{1,h(t)} & > 0, \\
\mathcal{P}_{2,h(t)} & > 0, \\
\mathcal{P}_{3,h(t)} & > 0, \\
\mathcal{P}_{4,h(t)} & > 0, \\
\mathcal{P}_{5,h(t)} & > 0,
\end{align*}
\]

(19)

where other symbols and related equations are listed in Appendix B.

**Proof.** Construct an augmented LKF as follows:

\[ V(x(t)) = \sum_{i=1}^3 V_i(x(t)), \]  

(20)
\[ V_1(x(t)) = \alpha^T(t)P_{h(t)}\alpha(t), \]
\[ V_2(x(t)) = \eta^T(t)Q\eta(t), \]
\[ V_3(x(t)) = \int_{t-h(t)}^{t} \hat{x}(s)M_{1,h(t)}\hat{x}(s)ds + \int_{t-h(t)}^{t} \hat{x}(s)M_{2,h(t)}\hat{x}(s)ds, \]
\[ V_4(x(t)) = \int_{0}^{\pi} \int_{t-h(t)}^{t} \hat{x}(s)\gamma(s)d\theta + \int_{t-h(t)}^{t} \hat{x}(s)\gamma_2(s)d\theta, \]
\[ V_5(x(t)) = 2\sum_{i=1}^{n} d_{ii} \int_{0}^{x(t-h(t))} (f_i(s) - l_i^2s)sds + 2\sum_{i=1}^{n} d_{ii} \int_{0}^{x(t-h(t))} (l_i^2s - f_i(s))ds, \]

\[ V_i(x(t)) = \alpha^T(t)P_{h(t)}\alpha(t), \]
\[ V_2(x(t)) = \eta^T(t)Q\eta(t), \]
\[ V_3(x(t)) = \int_{t-h(t)}^{t} \hat{x}(s)M_{1,h(t)}\hat{x}(s)ds + \int_{t-h(t)}^{t} \hat{x}(s)M_{2,h(t)}\hat{x}(s)ds, \]
\[ V_4(x(t)) = \int_{0}^{\pi} \int_{t-h(t)}^{t} \hat{x}(s)\gamma(s)d\theta + \int_{t-h(t)}^{t} \hat{x}(s)\gamma_2(s)d\theta, \]
\[ V_5(x(t)) = 2\sum_{i=1}^{n} d_{ii} \int_{0}^{x(t-h(t))} (f_i(s) - l_i^2s)sds + 2\sum_{i=1}^{n} d_{ii} \int_{0}^{x(t-h(t))} (l_i^2s - f_i(s))ds, \]

The time derivative of \( V(x(t)) \) along the trajectory of system (12) is given. Then, the derivative of \( V_i(x(t)) \) is derived:

\[
V_i'(x(t)) = 2\alpha^T(t)P_{h(t)}\dot{\alpha}(t) + \dot{h}(t)\alpha^T(t)P_{h(t)}\alpha(t)
\]
\[
= \dot{h}(t)\alpha^T(t)P_{h(t)}\alpha(t) + 2\alpha^T(t)P_{h(t)}\dot{\alpha}(t)
\]
\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{x}(t-h) \\
x(t) - (1 - \dot{h}(t))x(t-h(t)) \\
(1 - \dot{h}(t))x(t-h(t)) - x(t-h)
\end{bmatrix}
\]
\[
= \xi^T(t)\left[ \dot{h}(t)\Phi_1^T(h(t))P_2\Phi_1(h(t)) + \text{Sym}\left\{ \Phi_1^T(h(t))P_{h(t)}\Phi_2(h(t)) \right\} \right] \xi(t),
\]

\[
V_2'(x(t)) = 2\eta^T(\dot{\eta} + \dot{\xi}(t))
\]
\[
= 2\eta^T(t)\text{Sym}\left\{ \Phi_3^T\Omega_4(\dot{h}(t)) \right\} \xi(t),
\]

\[
V_3'(x(t)) = \hat{x}(t)M_{1,h(t)}\hat{x}(t) - (1 - \dot{h}(t))\hat{x}(t-h(t))M_{1,h(t)}\hat{x}(t-h(t)) + (1 - \dot{h}(t))
\]
\[
\times \hat{x}^T(t-h(t))M_{2,h(t)}\hat{x}(t-h(t)) - \hat{x}(t-h)M_{2,h(t)}\hat{x}(t-h)
\]
\[
+ \dot{h}(t)\int_{t-h(t)}^{t} \hat{x}^T(s)M_{1,h(t)}\hat{x}(s)ds - \dot{h}(t)\int_{t-h(t)}^{t} \hat{x}^T(s)M_{1,h(t)}\hat{x}(s)ds,
\]
\[
V_4(x(t)) = h(t)x^T(t)\mathcal{G}_1 \dot{x}(t) + (h - h(t))x^T(t)\mathcal{G}_2 \dot{x}(t) - (1 - \dot{h}(t)) \int_{t-h(t)}^{t} \dot{x}^T(s)\mathcal{G}_1 \dot{x}(s)ds \\
- \dot{h}(t) \int_{t-h(t)}^{t} \dot{x}^T(s)\mathcal{G}_2 \dot{x}(s)ds - \int_{t-h(t)}^{t-h(t)} \dot{x}^T(s)\mathcal{G}_2 \dot{x}(s)ds.
\] (25)

Combining (24) and (25), we can get the derivative of \( V_T(x(t)) \) as follows:

\[
V_T(x(t)) = x^T(t)\left[\mathcal{G}_{h(t)} + \mathcal{M}_{1,h(t)}\right]\dot{x}(t) + (1 - \dot{h}(t))x^T(t)\left[\mathcal{M}_{2,h(t)} - \mathcal{M}_{1,h(t)}\right]\dot{x}(t-h(t)) \\
- x^T(t-h)\mathcal{M}_{2,h(t)}\dot{x}(t-h) - \int_{t-h(t)}^{t} \dot{x}^T(s)\Pi_{1,h(t)}\dot{x}(s)ds - \int_{t-h}^{t-h(t)} \dot{x}^T(s)\Pi_{2,h(t)}\dot{x}(s)ds.
\] (26)

Based on Lemma 1, we can get

\[
- \int_{t-h(t)}^{t} \dot{x}^T(s)\Pi_{1,h(t)}\dot{x}(s)ds - \int_{t-h}^{t-h(t)} \dot{x}^T(s)\Pi_{2,h(t)}\dot{x}(s)ds \\
\leq \xi^T(t)\left(\frac{h - h(t)}{h^2}\mathcal{G}_1^T\mathcal{U}_1^{-1}\mathcal{G}_1 + \frac{h(t)}{h^2}\mathcal{G}_2^T\mathcal{U}_2^{-1}\mathcal{G}_2 \right) \\
\times \mathcal{G}_2^T - \frac{1}{h}\left[\begin{array}{c} 2h - h(t) \\ h \end{array}\right] \Pi_{1,h(t)} \left[ \begin{array}{c} \frac{h - h(t)}{h} \mathcal{U}_1 + \frac{h(t)}{h} \mathcal{U}_2 \\ \frac{1}{h - h(t)} \mathcal{U}_2 \end{array}\right] \xi(t)
\]
\[
= \xi^T(t)\left(\Psi(h(t)) - \frac{1}{h}\left[\begin{array}{c} 2h - h(t) \\ h \end{array}\right] \Pi_{1,h(t)} \mathcal{E}_1 + \frac{h - h(t)}{h} \mathcal{G}_1^T \mathcal{U}_1 \mathcal{G}_1 + \frac{h(t)}{h} \mathcal{G}_2^T \mathcal{U}_2 \mathcal{G}_2 \\
\times \mathcal{G}_2^T \mathcal{G}_1 + \frac{h - h(t)}{h} \mathcal{G}_1^T \mathcal{G}_2 \mathcal{G}_2 + \frac{h(t)}{h} \mathcal{G}_2^T \mathcal{G}_1 \Pi_{2,h(t)} \mathcal{G}_2 \right) \xi(t)
\]
\[
= \xi^T(t)(\Psi(h(t)) + \Omega(h(t)))\xi(t).
\]

Then, \( \dot{V}_T(x(t)) \) can be expressed as follows:

\[
\dot{V}_T(x(t)) \leq \xi^T(t)\left(e_4^T\left[\mathcal{G}_{h(t)} + \mathcal{M}_{1,h(t)}\right]e_4 + (1 - \dot{h}(t))e_5^T\left[\mathcal{M}_{2,h(t)} - \mathcal{M}_{1,h(t)}\right]e_5 \\
- e_6^T\mathcal{M}_{2,h(t)}e_6 + \Psi(h(t)) + \Omega(h(t)))\xi(t),
\] (28)

where \( \mathcal{G}_{h(t)}, \Pi_{1,h(t)}, \Pi_{2,h(t)}, \Psi(h(t)), \text{ and } \Omega(h(t)) \) are given in Appendix B.
Based on system (13), the following zero formula holds:

\[
0 = 2\left[ x^T(t)N_1x(t) + \dot{x}(t)N_2\right] \begin{bmatrix} -\Delta f(t) + \left( \mathcal{A} - \Pi \right) \mathcal{K}_1x(t) \mathcal{W}_0f(x(t)) \\ + \mathcal{W}_1f(x(t-h(t))) + \left( \mathcal{A} - \Pi \right) \mathcal{K}_2x(t-h(t)) + \left( \mathcal{A} - \Pi \right) \mathcal{K}_3x(t-h(t)) - \dot{x}(t) \end{bmatrix} \]

(30)

Then, based on Lemma 3 in [34], we can get

\[
2\Delta f(t)e_1 \leq \varepsilon (|\mathcal{F}|)^2 + \varepsilon^{-1}(B_1)(B_1)^T = d\Gamma^T + \varepsilon^{-1}\Theta^T. \tag{31}
\]

Based on the convex combination technique, \( \sum_{i=1}^{5} \dot{V}(x(t)) < 0 \) holds for all, if only if. By utilizing Schur complement, we can derive that \( \sum_{i=1}^{5} \dot{V}(x(t)) < 0 \) is equal to

\[
\begin{bmatrix}
\begin{array}{ccc}
\Sigma_{h(t);h(t)} & \sqrt{h-h(t)}U_1^T & \sqrt{h-h(t)}U_2^T \\
-\Pi_{2,h(t)} & 0 & 0 \\
0 & -\Pi_{1,h(t)} & 0 \\
0 & 0 & -\varepsilon^{-1}\mathcal{F} \\
0 & 0 & 0 \\
-\varepsilon^{-1}\mathcal{F}
\end{array}
\end{bmatrix} < 0.
\]

(32)

Based on (15)–(19), it is easy to come to the conclusion that system (13) is asymptotically stable. This concludes the proof.

Remark 2. In this paper, we consider \( \mathcal{D}_{h(t)} = \mathcal{P}_{h(t)} - (h-h(t))\mathcal{P}_{h(t)} \). (I) When \( h-h(t) = 0 \), \( \mathcal{P}_{h(t)} \) will be degenerated to the constant matrix \( \mathcal{P} \). (II) Compared with the existing methods [25], this paper only needs to consider that \( \mathcal{P}_{2} \) and \( \mathcal{P}_{3} \) are arbitrary symmetric matrices. Furthermore, as long as (25) and (26) are guaranteed, this constraint helps reduce the strength of positive definite conditions. (III) In addition, this construction makes full use of the delay information and the delay derivative information, thereby increasing the amount of LKF information storage, which helps to construct a more general LKF to further reduce the conservativeness of the criteria. At present, the method used in this paper is more general in constructing the LKF and includes a wider range of usage background and research significance.

Remark 3. Compared with existing research [24], this paper fully considers the relaxation of the requirements for matrix positive definiteness. In \( V_3(x(t)) \), by using \( \mathcal{A}_{h(t)} \) and \( \mathcal{A}_{h(t)}^T \), the LCCM is used to make constraints. Therefore, this method can obtain less conservative criteria through more relaxed positive definite conditions and increase the time-delay information contained in the LKF.

Remark 4. In order to better solve integral terms (24) and (25), \( -\int_{t-h(t)}^{t-h(t)} x^T(s) \mathcal{P}_2 \dot{x}(s) ds - \int_{t-h(t)}^{t-h(t)} x^T(s) \mathcal{P}_2 \dot{x}(s) ds \), in this paper, considers Lemma 1. Compared with the Wirtinger-based integral inequality, Jensen’s inequality, and other existing inequalities, Lemma 1 has a tighter bound in order to obtain a less conservative criterion.

Theorem 2. Given positive scalars \( h, \mu, \varepsilon, \) and \( \kappa \). System (13) is asymptotically stable if there exist symmetric positive definite matrices \( \mathcal{P}_{1}, \mathcal{P}_{2}, \mathcal{M}_{2}, \mathcal{B}_{1}, \mathcal{B}_{2}, \mathcal{D}_{1}, \mathcal{D}_{2} \). Any symmetric matrices \( \mathcal{P}_{2}, \mathcal{M}_{2}, \mathcal{B}_{1}, \mathcal{B}_{2}, \) and \( \mathcal{Q}_{i} (i = 1, 2, \ldots, 5) \), any matrices \( \mathcal{U}_{1}, \mathcal{U}_{2}, \mathcal{Y}_{1}, \mathcal{Y}_{2}, \mathcal{Y}_{3} \) and \( \mathcal{V} \) with appropriate dimensions such that the following LMIs hold:

\[
\begin{align*}
\mathcal{P}_{h(t)} & > 0, \\
\mathcal{M}_{1,h(t)} & > 0, \\
\mathcal{M}_{2,h(t)} & > 0,
\end{align*}
\]

(33)

\[
\begin{bmatrix}
\Sigma_{1} & \sqrt{h} \mathcal{G}_{1}^T \mathcal{Y}_{2} & \Gamma & \Theta \\
\ast & -\Pi_{2,h(t)} & 0 & 0 \\
\ast & \ast & -\varepsilon^{-1}\mathcal{F} & 0 \\
\ast & \ast & \ast & -\varepsilon\mathcal{F}
\end{bmatrix} < 0,
\]

(34)

\[
\begin{bmatrix}
\Sigma_{2} & \sqrt{h} \mathcal{G}_{2}^T \mathcal{Y}_{1} & \mathcal{G} & \mathcal{O} \\
\ast & -\Pi_{1,h(t)} & 0 & 0 \\
\ast & \ast & -\varepsilon^{-1}\mathcal{F} & 0 \\
\ast & \ast & \ast & -\varepsilon\mathcal{F}
\end{bmatrix} < 0,
\]

(35)

\[
\begin{bmatrix}
\Sigma_{3} & \sqrt{h} \mathcal{G}_{2}^T \mathcal{Y}_{1} & \mathcal{G} & \mathcal{O} \\
\ast & -\Pi_{1,h(t)} & 0 & 0 \\
\ast & \ast & -\varepsilon^{-1}\mathcal{F} & 0 \\
\ast & \ast & \ast & -\varepsilon\mathcal{F}
\end{bmatrix} < 0,
\]

(36)
where other symbols and related equations are listed in Appendix C.

We define \( \mathcal{H}_1 = \mathcal{Y}_1 \mathcal{V}^{-1} \), \( \mathcal{H}_2 = \mathcal{Y}_2 \mathcal{V}^{-1} \), and \( \mathcal{H}_3 = \mathcal{Y}_3 \mathcal{V}^{-1} \) to solve the corresponding controller gain.

Proof. Define

\[
\begin{align*}
N'_1 &= \mathcal{V}^{-1}, \\
N'_2 &= \kappa \mathcal{V}^{-1}, \\
\bar{\mathcal{P}}_{h(t)} &= \text{diag} \{ \mathcal{V}^T, \mathcal{V}^T, \mathcal{V}^T, \mathcal{V}^T \} \mathcal{P}_{h(t)} \text{diag} \{ \mathcal{V}' \mathcal{V} \mathcal{V}' \mathcal{V}' \}, \\
\bar{\mathcal{G}}_1 &= \mathcal{V}^T \mathcal{G}_1 \mathcal{V}, \\
\bar{\mathcal{G}}_2 &= \mathcal{V}^T \mathcal{G}_2 \mathcal{V}, \\
\bar{\mathcal{G}}_3 &= \mathcal{V}^T \mathcal{G}_3 \mathcal{V}, \\
\bar{\mathcal{M}}_1 &= \mathcal{V}^T \mathcal{M}_1 \mathcal{V}, \\
\bar{\mathcal{M}}_2 &= \mathcal{V}^T \mathcal{M}_2 \mathcal{V}, \\
\bar{\mathcal{M}}_3 &= \mathcal{V}^T \mathcal{M}_3 \mathcal{V}, \\
\bar{\mathcal{M}}_4 &= \mathcal{V}^T \mathcal{M}_4 \mathcal{V}, \\
\bar{\mathcal{U}}_1 &= \text{diag} \{ \mathcal{V}^T, \mathcal{V}^T, \mathcal{V}^T \} \mathcal{U}_1 \text{diag} \{ \mathcal{V}' \mathcal{V} \mathcal{V}' \mathcal{V}' \}, \\
\bar{\mathcal{D}}_1 &= \mathcal{V}^T \mathcal{D}_1 \mathcal{V}, \\
\bar{\mathcal{D}}_2 &= \mathcal{V}^T \mathcal{D}_2 \mathcal{V}.
\end{align*}
\]

Premultiplying and postmultiplying (15)–(19) by \( \text{diag} \{ \mathcal{V}^T, \mathcal{V}^T, \mathcal{V}^T, \mathcal{V}^T \} \) and \( \text{diag} \{ \mathcal{V}' \mathcal{V} \mathcal{V}' \mathcal{V}' \} \), in view of the Schur complement, we can obtain LMIs (33)–(37). Thus, the proof is completed.

4. Illustrative Example

In this section, two simulation examples are exhibited to express the effectiveness of the established results.

\begin{align*}
\mathcal{H}_1 &= \begin{bmatrix}
-0.1488 & -0.4091 \\
0.2762 & -0.2079
\end{bmatrix},
\mathcal{H}_2 &= \begin{bmatrix}
-0.2986 & 0.1950 \\
-0.7407 & -0.1002
\end{bmatrix},
\mathcal{H}_3 &= \begin{bmatrix}
-0.5131 & -0.3584 \\
0.2236 & -0.1432
\end{bmatrix}.
\end{align*}

Here, the control input trajectory of DNNs is presented in Figure 2.
Table 1: Notations and descriptions.

| Notation | \( \mathbb{R}^m \) | \( \mathbb{R}^{l \times k} \) | \( \mathcal{I} \) |
|----------|-----------------|-----------------|-----------------|
| Description | \( m \)-dimensional Euclidean space | \( l \times k \) matrices | Identity matrix |
| Notation | \( \mathcal{P} > 0 \) | \( \mathcal{A}^{-1} \) | \( \mathcal{A}^T \) |
| Description | Positive definite matrix | Inverse of matrix \( \mathcal{A} \) | Inverse of matrix \( \mathcal{A} \) |
| Notation | \( \| \cdot \| \) | \( \text{sym}[X] \) | \( \text{diag}[\cdot] \) |
| Description | Inverse of matrix \( \mathcal{A} \) | \( X^T + X \) | Stands for a block diagonal matrix |

Table 2: The achieved MAUBs for different values of \( \mu \).

| Methods | \( \mu = 0.1 \) | \( \mu = 0.5 \) | \( \mu = 0.9 \) |
|---------|----------------|----------------|----------------|
| [19] | 3.3068 | 2.5802 | 2.2736 |
| [22] | 3.8025 | 2.7427 | 2.3811 |
| [23] | 3.8026 | 2.7440 | 2.3811 |
| [25] | 4.6754 | 3.0587 | 2.7368 |
| Theorem 1 | 5.5721 | 4.0916 | 3.9594 |
| Improvement | 16.0927\% | 25.2444\% | 30.8784\% |

Figure 1: State trajectories for \( \mu = 1 \) and \( h = 6.0917 \).

Figure 2: Control input of the system.
Example 2. Consider DNN (13) with the following matrix parameters, and these matrices are based on [19–25]:

\[
\mathcal{A} = \text{diag}[1.2769, 0.6231, 0.9230, 0.4480],
\]
\[
\mathcal{L}_1 = \text{diag}[0.1137, 0.1297, 0.7994, 0.2368],
\]
\[
\mathcal{W}_0 = \begin{bmatrix}
-0.0373 & 0.4852 & -0.3351 & 0.2336 \\
-1.6033 & 0.5988 & 0.3224 & 1.2352 \\
0.3394 & -0.0860 & -0.3824 & -0.5785 \\
-0.1311 & 0.3253 & -0.9534 & -0.5015
\end{bmatrix},
\]
\[
\mathcal{W}_1 = \begin{bmatrix}
0.8674 & -1.2405 & -0.5325 & 0.0220 \\
0.0474 & -0.9164 & 0.0360 & 0.9816 \\
1.8495 & 2.6117 & -0.3788 & 0.8428 \\
-2.0413 & 0.5179 & 1.1734 & -0.2775
\end{bmatrix},
\]
\[
\mathcal{L}_2 = 0,
\]
\[
\Pi = \text{diag}[0.5, 0.5, 0.5, 0.5].
\]

By employing Theorem 1, for different \(\mu\), Table 3 lists the MAUBs based on Theorem 1. From Table 3, we can see that when \(\mu\) takes different values, the MAUBs obtained in this paper are always the largest. For example, when \(\mu = 0.5\), the existing result shows \(h = 3.0587\), but the result obtained by applying Theorem 1 in this paper is \(h = 4.0916\). Compared with results in [25], the criterion proposed in this paper improves MAUBs by up to 25.2444%.

When \(\kappa = 1\), by solving LMIs in Theorem 2, the controller gains are obtained as follows:

\[
\mathcal{K}_1 = \begin{bmatrix}
-2.8807 & -1.3407 & -0.4210 & -0.1567 \\
-0.1579 & -0.1483 & -0.1396 & 1.5440 \\
-1.9637 & -1.0886 & -0.1666 & -0.1519 \\
-0.1573 & -0.1539 & 0.9762 & 1.9872
\end{bmatrix},
\]
\[
\mathcal{K}_2 = \begin{bmatrix}
-1.6769 & -0.1564 & -0.1578 & -0.1525 \\
-0.1651 & 0.3745 & 1.3234 & 3.1923 \\
-0.1484 & -0.1550 & -0.1652 & -0.1642 \\
-0.1488 & -0.4091 & 0.2762 & -0.2079
\end{bmatrix},
\]
\[
\mathcal{K}_3 = \begin{bmatrix}
-0.1979 & -0.1569 & -0.1676 & -0.2986 \\
0.1950 & -0.7407 & -0.1002 & -0.2256 \\
-0.1683 & -0.1543 & -0.5131 & -0.3584 \\
-0.1432 & 0.3253 & -0.1054 & -0.2237
\end{bmatrix}.
\]

Letting \(\mu = 0.1\), \(h(t) \leq 4.2290\), \(f(x(t)) = \text{diag}[0.1137, 0.1297, 0.7994, 0.2368] \text{tanh}(x(t))\), and \(h(t) = (42290/10000) \sin((1000/42290)t) + (42290/10000)\). We verify the stability of the system by choosing different initial values. As shown in Figures 3–8, we give a dynamic response graph through node combination. Through the figures, we can directly see that the system eventually tends to a stable state under the controller involved. In addition, the control input trajectory of DNNs is presented in Figure 9.

### Table 3: The achieved MAUBs for different values of \(\mu\).

| Methods         | \(\mu = 0.8\) | \(\mu = 0.9\) |
|-----------------|---------------|---------------|
| [13]            | 2.5406        | 1.7273        |
| [14]            | 2.2495        | 1.5966        |
| [15]            | 2.1105        | 1.4268        |
| [16]            | 2.8794        | 1.9562        |
| [17]            | 2.8980        | 1.9562        |
| [18]            | 3.1409        | 1.6375        |
| [19]            | 4.8167        | 3.4245        |
| [20]            | 5.2756        | 4.6281        |
| Theorem 1       | 6.7491        | 5.9291        |
| Improvement     | 21.8325%      | 21.9426%      |
Figure 4: State trajectories $x_3(t)$ and $x_4(t)$ for $\mu = 0.1$ and $h = 4.2290$.

Figure 5: State trajectories $x_1(t)$ and $x_3(t)$ for $\mu = 0.1$ and $h = 4.2290$.

Figure 6: State trajectories $x_2(t)$ and $x_4(t)$ for $\mu = 0.1$ and $h = 4.2290$. 
5. Conclusion

This study has proposed further results for the stability analysis issue of UDNNs based on the RMFC scheme. First, an improved quadratic function method has been introduced for constructing a novel $V_1(x(t))$, which can fully excavate some intrinsic relationships between the delay derivative information and the time delay. Based on the TDPF and LCCM, the information storage has been further improved for obtaining new theoretical results. Second, by using resultful integral inequalities and correlation analysis approaches, several relaxed criteria have been established with respect to the asymptotical stability of the considered UDNNs. Third, a new RMFC has been designed, which can ensure the system stability of UDNNs. Lastly, two numerical experiments have been given to illustrate the significance of the theoretical results. In the future research, we need to further study the UDNNs based on quantized measurements.
in order to improve the entire system [41]. In addition, the results obtained in this paper will be extended to the chaotic Lurie system [42, 43], quaternion-valued or memristor-based neural networks [44–46], T-S fuzzy NNs [47, 48], Markov jump systems [49, 50], and complex dynamical networks [51–53]. These will occur in the near future.

Appendix

A. Some Relevant Notations

\(v_1(t) = \int_{t-h(t)}^{t} \frac{x^T(s)}{h(t)} ds,\)

\(v_2(t) = \int_{t-h(t)}^{t-h} \frac{x^T(s)}{h-h(t)} ds,\)

\(v_3(t) = \int_{t-h(t)}^{t} \int_{t}^{s} \frac{x^T(s)}{h^T(t)} dsd\theta,\)

\(v_4(t) = \int_{t-h}^{t} \int_{t}^{s} \frac{x^T(s)}{(h-h(t))^2} dsd\theta,\)

\(\alpha(t) = \text{col}[x(t), x(t-h), h\nu_1(t), (h-h(t))\nu_2(t)],\)

\(\eta(t) = \text{col}[x(t), x(t-h(t)), x(t-h)],\)

\(\xi(t) = \text{col}\left\{x(t), x(t-h(t)), x(t-h), \dot{x}(t), \dot{x}(t-h(t)), \dot{x}(t-h), f(x(t)), f(x(t-h(t))), v_1(t), v_2(t), v_3(t), v_4(t)\right\},\)

\(\mathcal{P}_{h(t)} = \mathcal{P}_1 - (h-h(t))\mathcal{P}_2,\)

\(\mathcal{Q} = \begin{bmatrix} \frac{\mathcal{Q}_1 + \mathcal{Q}_2 - \mathcal{Q}_2}{2} & \mathcal{Q}_2 & \mathcal{Q}_3 \\ \ast & -\mathcal{Q}_2 - \mathcal{Q}_2 - \mathcal{Q}_2 \frac{2}{2} & \mathcal{Q}_4 \\ \ast & \ast & \mathcal{Q}_5 + \mathcal{Q}_5 \end{bmatrix},\)

\(\mathcal{M}_{1,h(t)} = h(t)\mathcal{M}_1 + \mathcal{M}_2,\)

\(\mathcal{M}_{2,h(t)} = (h-h(t))\mathcal{M}_3 + \mathcal{M}_4.\)

B. Related Equations of Theorem 1

The appendix lists some notations and equations given in Theorem 1:
\[ \Sigma_{h(t),h(t)} = \sum_{j=1}^{4} \varepsilon_{j} + \text{Sym}\{\mathcal{L}\varphi\}, \]

\[ \Sigma_{1} = \Sigma_{h(t)=0,h(t)=0} \]
\[ \Sigma_{2} = \Sigma_{h(h)=h,h(t)=0} \]
\[ \Sigma_{3} = \Sigma_{h(h)=h,h(t)=\mu} \]
\[ \Sigma_{4} = \Sigma_{h(h)=0,h(t)=\mu} \quad (B.1) \]

\[ \Xi_{1} = \hat{h}(t)\Phi^{T}_{4}(h(t))\mathcal{P}_{2}\Phi_{1}(h(t)) + \text{Sym}\{\Phi^{T}_{4}(h(t))\mathcal{P}_{h(t)}\Phi_{2}(\hat{h}(t))\}, \]

\[ \Xi_{2} = \Phi^{T}_{4}\Phi_{4}\hat{h}(t), \]

\[ \Xi_{3} = \varepsilon_{4}^{T}\left[ \mathcal{G}(h(t)) + \mathcal{M}_{1,h(t)} \right]e_{4} + (1 - \hat{h}(t))\varepsilon_{5}^{T}\left[ \mathcal{M}_{2,h(t)} - \mathcal{M}_{1,h(t)} \right]e_{5} - \varepsilon_{6}^{T}\mathcal{M}_{2,h(t)}e_{6} + \Omega(h(t)), \]

\[ \Xi_{4} = 2(1 - \hat{h}(t))\varepsilon_{6}^{T}(\mathcal{D}_{1} - \mathcal{D}_{2})e_{5} + 2(1 - \hat{h}(t))\varepsilon_{2}(\mathcal{D}_{2}\mathcal{L}_{2} - \mathcal{D}_{1}\mathcal{L}_{1})e_{5}, \]

\[ \Gamma(h(t)) = \frac{h - h(t)}{h^{2}}e_{1}^{T}\mathcal{U}_{2}^{T}\mathcal{U}_{2}, \]

\[ \Omega(h(t)) = \frac{1}{h^{2}}\left( \frac{2h - h(t)}{h}e_{1}^{T}\mathcal{U}_{2}^{T}\mathcal{U}_{2} + \frac{h - h(t)}{h}e_{2}^{T}\mathcal{U}_{2}^{T}\mathcal{U}_{2} \right), \quad (B.2) \]

\[ \mathcal{G}(h(t)) = h(t)e_{1} + (h - h(t))e_{2}, \]

\[ \mathcal{L} = e_{1}^{T}\mathcal{N}_{1} + e_{4}^{T}\mathcal{N}_{2}, \]

\[ \varphi = (-\mathcal{A} + \mathcal{J} - \mathcal{I})e_{1} + \mathcal{W}_{0}e_{7} + \mathcal{W}_{1}e_{8} + \mathcal{J}(\mathcal{I} - \mathcal{P})e_{2} + \mathcal{J}(\mathcal{I} - \mathcal{P})e_{3} - e_{4}, \]

\[ \Gamma = \mathcal{L}\mathcal{H}, \]

\[ \Theta = \mathcal{G}e_{1}, \]

\[ \Phi_{1}(h(t)) = \text{col}\{e_{1}, e_{3}, h(t)e_{9}, (h - h(t))e_{10}\}, \]

\[ \Phi_{2}(\hat{h}(t)) = \text{col}\{e_{4}, e_{6}, e_{1} - (1 - \hat{h}(t))e_{2}, (1 - \hat{h}(t))e_{2} - e_{3}\}, \]

\[ \Phi_{3} = \text{col}\{e_{1}, e_{2}, e_{3}\}, \]

\[ \Phi_{4}(\hat{h}(t)) = \text{col}\{e_{4}, (1 - \hat{h}(t))e_{5} - e_{6}\}, \]

\[ \mathcal{E}_{1} = \text{col}\{e_{1} - e_{2}, e_{1} + e_{2} - 2e_{9}, e_{1} - e_{2} + 6e_{9} - 12e_{11}\}, \]

\[ \mathcal{E}_{2} = \text{col}\{e_{3} - e_{3}, e_{3} + e_{3} - 2e_{10}, e_{3} - e_{3} + 6e_{10} - 12e_{12}\}, \]

\[ \Pi_{1,h(t)} = (1 - \hat{h}(t))\mathcal{G}_{1} + \hat{h}(t)\mathcal{G}_{2} - \hat{h}(t)\mathcal{M}_{4}, \]

\[ \Pi_{2,h(t)} = \mathcal{G}_{2} + \hat{h}(t)\mathcal{M}_{3}, \]

\[ e_{i} = \begin{bmatrix} I_{\text{odd}(1)n} & 0_{\text{even}} & 0_{\text{odd}(9-n)} \end{bmatrix}, \quad i = 1, 2, \ldots, 12, \]

\[ \Pi_{1,h(t)} = \begin{bmatrix} \Pi_{1,h(t)} & 0 & 0 \\ 0 & 3\Pi_{1,h(t)} & 0 \\ 0 & 0 & 12\Pi_{1,h(t)} \end{bmatrix}, \]

\[ \Pi_{2,h(t)} = \begin{bmatrix} \Pi_{2,h(t)} & 0 & 0 \\ 0 & 3\Pi_{2,h(t)} & 0 \\ 0 & 0 & 12\Pi_{2,h(t)} \end{bmatrix}. \]
C. Related Equations of Theorem 2

The appendix lists some notations and equations given in Theorem 2:

\[
\bar{\Sigma}_{h(t), \dot{h}(t)} = \sum_{i=1}^{4} \bar{\Sigma}_i + \text{Sym}\{\mathcal{D}\varphi\},
\]

\[
\begin{align*}
\bar{\Sigma}_1 &= \bar{\Sigma}_{h(t)=0, \dot{h}(t)=0}, \\
\bar{\Sigma}_2 &= \bar{\Sigma}_{h(t)=h, \dot{h}(t)=0}, \\
\bar{\Sigma}_3 &= \bar{\Sigma}_{h(t)=-h, \dot{h}(t)=0}, \\
\bar{\Sigma}_4 &= \bar{\Sigma}_{h(t)=0, \dot{h}(t)\neq 0}, \\
\bar{\Sigma}_5 &= \dot{h}(t)e_4^T(h(t))\bar{\Phi}_4(h(t)) + \text{Sym}\{\Phi_1^T(h(t))\bar{\Phi}_2(h(t))\}, \\
\bar{\Sigma}_6 &= \Phi_1^T(h(t)), \\
\bar{\Sigma}_7 &= e_4^T[h\bar{\mathcal{H}}_h + \bar{\mathcal{M}}_{1,h(t)}]e_4 + (1 - \dot{h}(t))e_5^T[\bar{\mathcal{M}}_{2,h(t)} - \bar{\mathcal{M}}_{1,h(t)}]e_5 - e_6^T\bar{\mathcal{M}}_{2,h(t)}e_6 + \Omega(h(t)), \\
\bar{\Sigma}_8 &= 2(1 - \dot{h}(t))e_5^T(\bar{\mathcal{D}}_1 - \bar{\mathcal{D}}_2)e_5 + 2(1 - \dot{h}(t))e_4(\bar{\mathcal{D}}_2\mathcal{L}_2 - \bar{\mathcal{D}}_1\mathcal{L}_1)e_5,
\end{align*}
\]

\[
\bar{\Gamma}(h(t)) = \frac{h - h(t)}{h^2}[\bar{\mathcal{H}}_1^T\bar{\mathcal{U}}_2^T\bar{\Pi}_{1,h(t)}^{-1}\bar{\mathcal{U}}_2 \bar{\mathcal{F}}_1 + \frac{h(t)}{h^2}\bar{\mathcal{U}}_2^T\bar{\Pi}_{1,h(t)}^{-1}\bar{\mathcal{U}}_1\mathcal{E}_2],
\]

\[
\bar{\Omega}(h(t)) = \frac{1}{h} \left( \frac{2h - h(t)}{h} \bar{\mathcal{H}}_1^T\bar{\mathcal{U}}_2^T\bar{\Pi}_{1,h(t)}^{-1}\bar{\mathcal{U}}_2 \bar{\mathcal{F}}_1 + \frac{h - h(t)}{h^2}\bar{\mathcal{U}}_2^T\bar{\Pi}_{1,h(t)}^{-1}\bar{\mathcal{U}}_1\mathcal{E}_2 + \frac{h - h(t)}{h} \bar{\mathcal{U}}_2^T\bar{\Pi}_{2,h(t)}^{-1}\bar{\mathcal{U}}_2 \bar{\mathcal{F}}_1 \right),
\]

\[
\begin{align*}
\bar{\mathcal{H}}_{h(t)} &= h(t)\bar{\mathcal{H}}_1 + (h - h(t))\bar{\mathcal{H}}_2, \\
\bar{\mathcal{D}}_1 &= e_4^T + \kappa e_1, \\
\bar{\varphi} &= (\mathcal{L}\mathcal{H} + (\mathcal{I} - \Pi)\mathcal{Y}_4)e_1 + \mathcal{W}_0\mathcal{V}e_7 + \mathcal{W}_1\mathcal{V}e_8 + (\mathcal{I} - \Pi)\mathcal{Y}_2e_2 + (\mathcal{I} - \Pi)\mathcal{Y}_3e_3 + e_4\mathcal{V}, \\
\bar{\Gamma} &= \bar{\mathcal{D}}\bar{\mathcal{H}}, \\
\bar{\Theta} &= \mathcal{E}e_1, \\
\bar{\Pi}_{1,h(t)} &= (1 - \dot{h}(t))\bar{\mathcal{D}}_1 + \dot{h}(t)\bar{\mathcal{D}}_2 - \dot{h}(t)\bar{\mathcal{H}}_1, \\
\bar{\Pi}_{2,h(t)} &= \bar{\mathcal{D}}_2 + \dot{h}(t)\bar{\mathcal{H}}_2,
\end{align*}
\]

\[
\begin{align*}
\bar{\Pi}_{1,h(t)} &= \begin{bmatrix} \bar{\Pi}_{1,h(t)} & 0 & 0 \\ 0 & 3\bar{\Pi}_{1,h(t)} & 0 \\ 0 & 0 & 12\bar{\Pi}_{1,h(t)} \end{bmatrix}, \\
\bar{\Pi}_{2,h(t)} &= \begin{bmatrix} \bar{\Pi}_{2,h(t)} & 0 & 0 \\ 0 & 3\bar{\Pi}_{2,h(t)} & 0 \\ 0 & 0 & 12\bar{\Pi}_{2,h(t)} \end{bmatrix},
\end{align*}
\]
Data Availability
The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

Disclosure
The authors declare that the work described is original research that has not been published previously and not under consideration for publication elsewhere, in whole or in part.

Conflicts of Interest
The authors declare no conflicts of interest.

Authors’ Contributions
All the authors listed approved the manuscript that is enclosed.

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