Curved Membrane Solutions in D=11 Supergravity

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Abstract

We present a class of static membrane solutions, with non-flat worldvolume geometry, in the eleven dimensional supergravity with source terms. This class of solutions contains supersymmetric as well as a large class of non-supersymmetric configurations. We comment about near horizon limit and stability of these solutions and point out an interesting relation with certain two dimensional dilaton gravity system.
1 Introduction

Branes with curved geometries are objects of great interest in finding nonperturbative dynamics in general backgrounds. Spherical and other geometries in this context have been a subject of interest for long. In particular, D-branes in curved backgrounds and their dynamics have been studied from the point of view of conformal field theory, as well as using the Born-Infeld action in curved geometries such as WZW models. Inspired by these developments, we study in this paper the eleven dimensional supergravity, understood to be the low energy limit of M-theory, and analyze the possibility of obtaining a large class of non-flat membrane solutions.

The existence of a supergravity in D=11 is known for a long time. The massless spectrum contains a graviton, a gravitino and a three form gauge field. Since a string couples to a two form, it is only natural to expect a membrane to exist in the 11 dimensional theory which will couple to the three form. Indeed, such a membrane was eventually constructed that couples to this background. Subsequently, it emerged as an exact solution of the supergravity field equations. This solution has several interesting features. Firstly, the membrane has an electric charge that is conserved due to the equations of motion. Also, it has a $\delta$-function singularity so that it cannot be called a soliton in the usual parlance. Nevertheless, as is common with the soliton solutions, they break just one half of the spacetime supersymmetry and saturate a Bogomol’nyi bound between the mass per unit area and the conserved charge. As a result, one can stack an arbitrary number of membranes together without affecting their stability. After the connection between D=11 supergravity and the strong coupling limit of type IIA string theory was pointed out, it was observed that this membrane becomes the D2-brane of type IIA theory. Also, under double dimensional reduction, the membrane goes over to the well known macroscopic string solution of Dabholkar et al.

The flat membrane solution of was constructed by assuming dependence of the classical background on coordinates transverse to the membrane. As a result, one has translational isometries for these solutions along two spatial directions in addition to a time translation, leading to Poincaré invariance. To find out a non-flat membrane solution, we use an ansatz which is a variation of the one in by introducing dependence on spatial longitudinal coordinates. In particular, we now introduce metric
components in the two spatial directions along the membrane, parametrized by a single conformal degree of freedom. In addition, we modify the radial dependence of the membrane solution by a factor dependent on the world volume spatial coordinates. Finally, the 3-form ansatz is also modified by a similar factor.

The plan of the paper is as follows. In section 2, we give a review of the membrane solution as obtained in [8]. In section 3, we present our solution. Unlike the flat membrane, our ansatz has nonvanishing curvature on the worldvolume. As a result, generically it breaks all supersymmetry. We then solve the supergravity equations of motion with a membrane source term. While solving the equations of motion, we discern an interesting connection of our solutions with the solutions of two dimensional dilaton gravity [11]. For a class of solutions of dilaton gravity, there exists a corresponding class of curved membrane solutions. We also find, somewhat to our surprise, that these solutions have the same $AdS_4 \times S^7$ limit as that of a flat membrane. In section 4, we conclude by discussing the stability of these solutions against small perturbations and point out some open problems. We give various explicit mathematical expressions in an appendix.

## 2 Flat membrane solution

For later convenience we shall review the flat membrane solution in this section. Following [8], one starts with an ansatz for the $D = 11$ fields $g_{MN}$ and $A_{MNP}$ ($M, N = 0, 1, \ldots, 10$) corresponding to the most general three-eight split invariant under $P_3 \times SO(8)$, where $P_3$ is the three dimensional Poincaré group and $SO(8)$ is the eight dimensional rotation group. The $D = 11$ coordinates $x^M = (x^\mu, y^m)$, where $\mu = 0, 1, 2$ and $m = 3, \ldots, 10$. The metric is:

$$ds^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B} \delta_{mn} dy^m dy^n,$$  \hspace{1cm} (2.1)

and the three-form gauge field is:

$$A_{\mu\nu\rho} = \pm \frac{1}{3g} \varepsilon_{\mu\nu\rho} e^C,$$  \hspace{1cm} (2.2)

where $^3g$ is the determinant of $g_{\mu\nu}$ and $\varepsilon_{\mu\nu\rho}$ is the three dimensional Levi-civita tensor. All other components of $A_{MNP}$ and all components of the gravitino $\psi_M$ are set to


zero. Invariance under both $P_3$ and $SO(8)$ forces the arbitrary functions $\tilde{A}, \tilde{B}, \tilde{C}$ to depend on $r \equiv \sqrt{(y^m)^2}$. The requirement of some unbroken supersymmetry relates $\tilde{A}$ and $\tilde{B}$ to $\tilde{C}$ so that we are left with only one undetermined function $\tilde{C}$. To preserve some supersymmetry, there must exist Killing spinors $\varepsilon$ satisfying

$$\tilde{D}_M \varepsilon = 0,$$  \hspace{1cm} (2.3)

where

$$\tilde{D}_M = \partial_M + \frac{1}{4} \omega_M^{AB} \Gamma_{AB} - \frac{1}{288} (\Gamma^{PQRS}_M + 8 \Gamma^{PQR} \delta^S_M) F_{PQRS}$$  \hspace{1cm} (2.4)

with $F_{MNPQ} = 4 \delta_M^{[M} A_{NPQ]}$. Here $\omega_M^{AB}$ are the spin connections and $\Gamma_A$ are the $D = 11$ Dirac matrices satisfying $\{ \Gamma_A, \Gamma_B \} = 2 \eta_{AB}$. $A, B$ refer to $D = 11$ tangent space, $\eta_{AB} = \text{diag}(-, +, \ldots, +)$ and $\Gamma_{AB \ldots C} = [A \Gamma_B \ldots C]$. We then make a three-eight split: $\Gamma_A = \left( \gamma^{\alpha} \otimes \Gamma_9, I \otimes \Sigma^a \right)$, where $\gamma^{\alpha}$ and $\Sigma^a$ are the $D = 3$ and $D = 8$ Dirac matrices respectively and $\Gamma_9 = \Sigma_3 \ldots \Sigma_{10}$.

The most general spinor consistent with $P_3 \times SO(8)$ is of the form $\varepsilon(x, y) = \epsilon \otimes \eta(r)$ where $\epsilon$ is a constant spinor of $SO(1, 2)$ and $\eta$ is an $SO(8)$ spinor. After some calculation, we find that (2.3) admits two non-trivial solutions $(1 \pm \Gamma_9) \eta = 0$ with $\eta = e^{-\hat{C}} \eta_0$ and $\tilde{A} = \frac{1}{3} \hat{C}$, $\tilde{B} = -\frac{1}{6} \hat{C} + \text{const}$. In each case, one half of the maximal possible rigid supersymmetry survives. At this stage, the three unknown functions $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ have been reduced to one by choosing the case where half supersymmetry survives. To determine this unknown function, we substitute the ansatz into the field equations which follow from the action

$$S_G = \int d^{11}x \mathcal{L}_G,$$  \hspace{1cm} (2.5)

where $\mathcal{L}_G$ is the supergravity lagrangian whose bosonic sector is given by

$$\kappa^2 \mathcal{L}_G = \frac{1}{2} \sqrt{-g} R - \frac{1}{96} \sqrt{-g} F_{MNPQ} F^{MNPQ} +$$

$$\frac{1}{2(12)^4} \varepsilon^{MNOPQRSU VW} F_{MNOP} F_{QRST} A_{UVW}.$$  \hspace{1cm} (2.6)

The three form field equation is given by

$$\partial_M (\sqrt{-g} F^{MUVW}) + \frac{1}{1152} \varepsilon^{UVW MNPQRST} F_{MNOP} F_{QRST} = 0.$$  \hspace{1cm} (2.7)

The equation (2.7), with the above ansatz, leads to $\delta^{mn} \partial_m \partial_n e^{-\hat{C}} = 0$. Imposing the boundary condition that the metric be asymptotically Minkowskian, we find $e^{-\hat{C}} =$
1 + \frac{K}{r^6}, \ r > 0 \text{ where } K \text{ is a constant. Hence we get,}

\begin{equation}
 ds^2 = (1 + \frac{K}{r^6})^\frac{4}{3} \eta_{\mu\nu} dx^\mu dx^\nu + (1 + \frac{K}{r^6})^\frac{1}{3} \delta_{mn} dy^m dy^n, A_{\mu\nu} = \pm \frac{1}{3} \varepsilon_{\mu\nu\rho} (1 + \frac{K}{r^6})^{-1}. \tag{2.8}
\end{equation}

These expressions solve the field equations everywhere except at \( r = 0 \). Hence, to obtain a solution that is valid everywhere, we have to modify the pure supergravity action by adding a membrane source at \( r = 0 \). Let us consider the combined supergravity and membrane equations which follow from the action

\begin{equation}
 S = S_G + S_M, \tag{2.9}
\end{equation}

where

\begin{equation}
 S_M = T \int d^3 \xi (-\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N g_{MN} + \frac{1}{2} \sqrt{-\gamma} + \frac{1}{3!} \varepsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P A_{MNP}), \tag{2.10}
\end{equation}

where \( T \) is the membrane tension. The Einstein equations are now

\begin{equation}
 R_{MN} - \frac{1}{2} g_{MN} R = \kappa^2 T_{MN}, \tag{2.11}
\end{equation}

where,

\begin{equation}
 \kappa^2 T_{MN} = \frac{1}{12} (F_{MPQR} F^P_{NQR} - \frac{1}{8} g_{MN} F_{PQRS} F^{PQRS}) - \kappa^2 T \int d^3 \xi \sqrt{-\gamma} \gamma^{ij} \partial_i X_M \partial_j X_N \frac{\delta^{11}(x - X)}{\sqrt{-g}}, \tag{2.12}
\end{equation}

while the three form equation is:

\begin{equation}
 \partial_M (\sqrt{-g} F^{MUVW}) + \frac{1}{1152} \varepsilon^{UVWMNOPQRST} F_{MNOP} F_{QRST} = \pm 2 \kappa^2 T \int d^3 \xi \varepsilon^{ijk} \partial_i X^U \partial_j X^V \partial_k X^W \delta^{11}(x - X). \tag{2.13}
\end{equation}

Also, we have the membrane field equations:

\begin{equation}
 \partial_i (\sqrt{-\gamma} \gamma^{ij} \partial_j X^N g_{MN}) + \frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^N \partial_j X^P \partial_M g_{NP} \pm \frac{1}{3!} \varepsilon^{ijk} \partial_i X^N \partial_j X^P \partial_k X^Q F_{MNPQ} = 0, \tag{2.14}
\end{equation}

\begin{equation}
 \gamma_{ij} = \partial_i X^M \partial_j X^N g_{MN}. \tag{2.15}
\end{equation}

One can easily verify that the correct source term is obtained by the the static gauge choice \( X^\mu = \xi^\mu \) and \( Y^m = \text{const} \), provided \( K = \frac{\kappa^2 T}{\Omega_7} \) where \( \Omega_7 \) is the volume of \( S^7 \).
3 A class of curved membrane solutions

We now proceed to present our solution. Our ansatz is as follows:

\[ ds^2 = e^{2A}[-(dx^0)^2 + \sqrt{f}(dx^a)^2] + e^{2B}(dy^m)^2, \]

and \[ A_{\mu\nu\rho} = \pm \frac{1}{3g} \varepsilon_{\mu\nu\rho} e^C, \]

with \[ e^{2A} = e^{2\tilde{A}} F^e, e^{2B} = e^{2\tilde{B}} g^b, e^C = e^{\tilde{C}} \chi^c, r^6 = h^d \tilde{r}^6, \]

\[ \tilde{r} = \sqrt{(y^m)^2}. \quad (3.1) \]

\( \tilde{A}, \tilde{B}, \tilde{C} \) are functions of \( r \) and \( F, g, \chi, f, h \) are functions of \( x^a \). In the above ansatz, \( \tilde{A}, \tilde{B}, \tilde{C} \) are related to each other as in section 2, viz., \( \tilde{A} = \frac{1}{3} \tilde{C}, \tilde{B} = -\frac{1}{6} \tilde{C} + \text{const} \). As is noticed, the worldvolume directions of the membrane for our ansatz is now curved due to the introduction of the conformal factor \( f \) above. In our ansatz, we have chosen this factor along purely spatial directions \( x^1, x^2 \). Alternatively, it is possible to have a conformal factor along \( x^0 \) and \( x^1 \). We mainly restrict our discussion to the former possibility.

3.1 Supersymmetry

We would like to see whether the ansatz we propose preserves some amount of space-time supersymmetry. For that we once again resort to the Killing spinor equation. We give below only the necessary equations and refer the reader to the appendix for more details.

We break the \( SO(1,10) \) spinor \( \varepsilon \) into a \( SO(1,2) \) spinor \( \epsilon \) and a \( SO(8) \) spinor \( \eta \). But now \( \epsilon \) depends on \( x^{1,2} \) and \( \eta \) depends on \( r \). In our notation, the three dimensional Dirac matrices are \( \gamma_0 = i\sigma_2, \gamma_1 = \sigma_1 \) and \( \gamma_2 = \sigma_3 \). The eight dimensional Dirac matrices are denoted by \( \Sigma_m \) as usual. Also, the hatted indices denote tangent space indices and the unhatted indices denote space-time indices. We now obtain the Killing spinor equations for our ansatz using equation \( (2.3) \).

\[ \bar{D}_0 \varepsilon = \frac{d}{6} f^{-\frac{1}{4}} (1 + \frac{K}{r^6})^{-\frac{1}{2}} \frac{h_a}{h} (\gamma_0 \gamma_a \epsilon) \otimes \eta - \]

\[ \frac{1}{6} (1 + \frac{K}{r^6})^{-\frac{3}{2}} h^{-\frac{1}{2}} (\frac{K}{r^6}) m (\gamma_0 \epsilon) \otimes (\Sigma_m \Gamma_9 \eta) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_a \epsilon) \otimes (\Sigma_a \Gamma_9 \eta) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \eta) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\nu \epsilon) \otimes (\Sigma_\nu \Gamma_9 \eta) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\nu \epsilon) \otimes (\Sigma_\nu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\rho \epsilon) \otimes (\Sigma_\rho \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\sigma \epsilon) \otimes (\Sigma_\sigma \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\tau \epsilon) \otimes (\Sigma_\tau \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\lambda \epsilon) \otimes (\Sigma_\lambda \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\nu \epsilon) \otimes (\Sigma_\nu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\rho \epsilon) \otimes (\Sigma_\rho \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\sigma \epsilon) \otimes (\Sigma_\sigma \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\tau \epsilon) \otimes (\Sigma_\tau \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\lambda \epsilon) \otimes (\Sigma_\lambda \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\nu \epsilon) \otimes (\Sigma_\nu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\rho \epsilon) \otimes (\Sigma_\rho \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\sigma \epsilon) \otimes (\Sigma_\sigma \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\tau \epsilon) \otimes (\Sigma_\tau \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\lambda \epsilon) \otimes (\Sigma_\lambda \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\nu \epsilon) \otimes (\Sigma_\nu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\rho \epsilon) \otimes (\Sigma_\rho \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\sigma \epsilon) \otimes (\Sigma_\sigma \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\tau \epsilon) \otimes (\Sigma_\tau \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\lambda \epsilon) \otimes (\Sigma_\lambda \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\nu \epsilon) \otimes (\Sigma_\nu \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\rho \epsilon) \otimes (\Sigma_\rho \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\sigma \epsilon) \otimes (\Sigma_\sigma \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\tau \epsilon) \otimes (\Sigma_\tau \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\lambda \epsilon) \otimes (\Sigma_\lambda \Gamma_9 \chi) - \]

\[ \frac{1}{2} \frac{h}{r^6} (\gamma_0 \gamma_\mu \epsilon) \otimes (\Sigma_\mu \Gamma_9 \chi) - \]
We will now discuss various cases.

1. 1/2 SUSY solution: If we set \( h, \bar{a} = f, \bar{a} = 0 \), we end up with a single condition on \( \eta \), viz. \( \eta = (1 + \Gamma_9) \eta_0 = 0 \) from (3.2) and (3.3). Finally, (3.4) implies that \( \eta_0 = (1 + \frac{K}{r^6})^{-\frac{1}{2}} \eta_0 \), where \( \eta_0 \) is a constant spinor. As a result, the membrane breaks 1/2 SUSY. This is the well known Duff-Stelle solution[8].

2. 1/4 SUSY solution: We can have 1/4 SUSY solution for the following conditions:

\[
\begin{align*}
\bar{a} \epsilon & = \partial_{a} \epsilon \otimes \eta + \frac{d}{6} (1 + \frac{K}{r^6})^{-1} \epsilon_{ab} \frac{h_{\bar{a}}}{h} \gamma_0 \eta \otimes \eta - \\
& + \frac{1}{4} \epsilon_{ab} f_{\bar{a} \bar{b}} \gamma_0 \eta \otimes \eta + \\
& + \frac{1}{6} (1 + \frac{K}{r^6})^{-\frac{3}{2}} \frac{f^4}{f^4} (\frac{K}{r^6}) \eta \otimes (\Sigma_m \Gamma_9 \eta)
\end{align*}
\]

\[
\begin{align*}
\bar{m} \epsilon & = \epsilon \otimes \partial_{m} \eta + \frac{d}{12} (1 + \frac{K}{r^6})^{-\frac{1}{2}} \frac{h_{\bar{a}}}{h} \gamma_0 \eta \otimes (\Sigma_m \Gamma_9 \eta)
\end{align*}
\]

\[
\begin{align*}
& + \frac{1}{12} (1 + \frac{K}{r^6})^{-1} \epsilon \otimes (\frac{K}{r^6}) \eta \otimes (\Sigma_m \Gamma_9 \eta)
\end{align*}
\]

\[
\begin{align*}
& + \frac{1}{6} (1 + \frac{K}{r^6})^{-\frac{1}{2}} (\frac{K}{r^6}) \eta \otimes (\Sigma_m \Gamma_9 \eta) = 0.
\end{align*}
\]

We will now discuss various cases.

1. 1/2 SUSY solution:

If we set \( h, \bar{a} = f, \bar{a} = 0 \), we end up with a single condition on \( \eta \), viz. \( (1 + \Gamma_9) \eta = 0 \) from (3.2) and (3.3). Finally, (3.4) implies that \( \eta = (1 + \frac{K}{r^6})^{\frac{1}{2}} \eta_0 \), where \( \eta_0 \) is a constant spinor. As a result, the membrane breaks 1/2 SUSY. This is the well known Duff-Stelle solution[8].

2. 1/4 SUSY solution:

We can have 1/4 SUSY solution for the following conditions:

\[
\begin{align*}
h_{z} = f_{z} = 0, \gamma_{z} \epsilon = 0,
\end{align*}
\]

in addition to the condition for the 1/2 SUSY case. Here the subscripts \( z(\bar{z}) \) refer to the complex coordinates \( x^1 + ix^2 \) and \( x^1 - ix^2 \) respectively.

In fact, one can obtain other curved membrane solutions by interchanging the role of one of the time coordinate with one of the worldvolume directions of the membrane. One then has a solution of the traveling wave type. These statements apply to the nonsupersymmetric cases discussed below as well.
3. Non-supersymmetric solutions:
For generic choice of $h$ and $f$, supersymmetry is completely broken. In the following, we discuss these solutions in detail.

### 3.2 Field Equations

The three-form equation (2.13) once again leads to
\[ e^{-\tilde{C}} = 1 + \frac{K}{r^6}, \tag{3.6} \]
provided we impose the condition
\[ F^{-\frac{3e}{2}} g^{3b} \chi^c f^{-\frac{1}{2}} h^{-d} = 1. \tag{3.7} \]

Similarly, the membrane field equation (2.14) gives the condition
\[ F^{-\frac{3e}{2}} f^{-\frac{1}{2}} \chi^c = 1. \tag{3.8} \]

Using (2.11) and (2.12), we can write the Einstein equations in the form
\[ R_{MN} = \kappa^2 (T_{MN} - \frac{1}{9} g_{MN} T), \tag{3.9} \]

The equations $R_{00} = 0$ and $R_{0m} = 0$ are identically satisfied by our ansatz. The equation $R_{am} = 0$ is satisfied provided $g = h$ and $d = 3b$. For the rest of the equations, we give below the details.

\begin{align*}
R_{00} &= \frac{1}{3} F^e g^{-b}(1 + \frac{K}{r^6})^{-3} \left[ \frac{K}{r^6} \right]_m^2 - \frac{1}{3} F^e g^{-b}(1 + \frac{K}{r^6})^{-2} \left[ \frac{K}{r^6} \right]_{mm} + \\
&\quad \frac{e}{2 \sqrt{f}} \left[ \frac{F_{aa}}{F} + (e - 1) \left( \frac{F_{a}}{F} \right)^2 + 4b \frac{g_{a} F_{a}}{g} \right] + \\
&\quad \frac{d}{3} \left[ \frac{K}{r^6} \right] (1 + \frac{K}{r^6})^{-1} \frac{1}{\sqrt{f}} \left[ \frac{h_{aa}}{h} \right] h^{-d} (1 + \frac{h_a}{h} - e F_{a} g_{a} - 4b \frac{g_{a} h_{a}}{g} h),
\end{align*}

\begin{align*}
R_{ab} &= d \left[ \frac{K}{r^6} \right] (1 + \frac{K}{r^6})^{-1} \left[ \frac{h_{ab}}{h} \right] + (d - 1) \frac{h_a h_b}{h^2} - \frac{1}{4h^2} (f_a h_{b} + f_b h_a - f_c h_c \delta_{ab}) - \\
&\quad \frac{d}{3} \left[ \frac{K}{r^6} \right] (1 + \frac{K}{r^6})^{-1} \left[ \frac{h_{cc}}{h} \right] h^{-d} \delta_{ab} - \frac{1}{3} \left( g^{-b} F^e \sqrt{f} \right) (1 + \frac{K}{r^6})^{-3} \left[ \frac{K}{r^6} \right]_{mm}^2 \delta_{ab} + \\
&\quad \frac{1}{3} \left( g^{-b} F^e \sqrt{f} \right)(1 + \frac{K}{r^6})^{-2} \left( \frac{K}{r^6} \right)_{mm} \delta_{ab},
\end{align*}

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\[ R_{mn} = -\frac{b}{2}(F^{-e}g^{b-1}f^{-\frac{1}{2}})\delta_{mn}[g_{,aa} + (4b - 1)\frac{g_{,a}^2}{g} + \frac{e}{2}\frac{g_{,a}F_{,a}}{F}] - \]
\[
\frac{1}{6}(1 + \frac{K}{r^6})^{-1}\frac{K}{r^6},pp\delta_{mn} - \frac{1}{2}(1 + \frac{K}{r^6})^{-2}(\frac{K}{r^6}),m(\frac{K}{r^6},n + \]
\[
\frac{1}{6}(1 + \frac{K}{r^6})^{-2}(\frac{K}{r^6}),p]^2\delta_{mn}. \tag{3.10}
\]

In writing \( R_{ab} \) components in equation (3.10) we have dropped terms independent of \( r \). This can be done self-consistently for our ansatz, as we discuss below after equation (3.14), using equation of motion (3.9). The source terms in the three cases are as follows.

\[
\kappa^2(T_{00} - \frac{1}{9}g_{00}T) = \frac{1}{3}F^e g^{-b}(1 + \frac{K}{r^6})^{-3}[(\frac{K}{r^6}),m]^2 + \]
\[
\frac{2}{3}\kappa^2 T \int d^3 \xi F^e g^{-db}(1 + \frac{K}{r^6})^{-2}\delta^{11}(x - X), \]
\[
\kappa^2(T_{ab} - \frac{1}{9}g_{ab}T) = -\frac{1}{3}(g^{-b}F^e\sqrt{f})(1 + \frac{K}{r^6})^{-3}[(\frac{K}{r^6}),m]^2\delta_{ab} - \]
\[
\frac{2}{3}\kappa^2 T \int d^3 \xi (g^{-b}F^e\sqrt{f})(1 + \frac{K}{r^6})^{-2}g^{-d}\delta^{11}(x - X), \]
\[
\kappa^2(T_{mn} - \frac{1}{9}g_{mn}T) = -\frac{1}{2}(1 + \frac{K}{r^6})^{-2}(\frac{K}{r^6},m(\frac{K}{r^6}),n + \]
\[
\frac{1}{6}(1 + \frac{K}{r^6})^{-2}(\frac{K}{r^6}),p]^2\delta_{mn} + \]
\[
\frac{1}{3}\kappa^2 T \int d^3 \xi g^{-d}(1 + \frac{K}{r^6})^{-1}\delta^{11}(x - X). \tag{3.11}
\]

The ansatz solves the equations of motion provided we set

\[ F = g = h, \ 2d = -3e = 6b. \tag{3.12} \]

We also obtain \( K = \frac{\kappa^2 T}{6} \). But what actually makes the solutions interesting is the following pair of equations:

\[ R_{ab}(\sqrt{f}F^e) + \frac{3e}{2}\nabla_a\nabla_b \ln F - \frac{3e^2}{4}\nabla_a \ln F\nabla_b \ln F = 0, \]
\[ \nabla^2(e^{-\frac{3e}{2}\ln F}) = 0. \tag{3.13} \]

which arise from (3.3) using (3.10)-(3.12). In (3.13), \( R_{ab}(\sqrt{f}F^e) \) denotes the Ricci tensor components with conformal metric, \( g_{ab} = \sqrt{f}F^e\delta_{ab} \). More precisely, from the (00) components in (3.10) and (3.11) we get the conditions, using (3.12):

\[ \frac{F_{,ab}}{F} + (d - 1)\frac{F_{,a}F_{,b}}{F^2} - \frac{1}{4} \frac{1}{Ff} (f_{,a}F_{,b} + f_{,b}F_{,a} - f_{,c}F_{,c}\delta_{ab}) = 0, \]
\begin{align}
F_{aa} + \left(4b + \frac{e}{2} - 1\right) \frac{F_{a}^2}{F} &= 0.
\end{align}

(3.14)

Identical conditions are obtained from other components in (3.10 and (3.11) as well. It can now be seen that (3.13) and (3.14) are identical. Indeed the LHS in equations (3.13) or (3.14) provide the explicit expressions for the terms that were dropped in writing \( R_{ab} \) explicitly. We have therefore obtained a necessary condition that our ansatz satisfies the supergravity equations of motion in eleven dimensions.

Now, defining \( \phi = \frac{3}{2} \ln F \), we can rewrite (3.13) as:

\begin{align}
R_{ab}(\sqrt{f} F^e) + \nabla_a \nabla_b \phi - \frac{1}{3} \nabla_a \phi \nabla_b \phi &= 0, \\
\nabla^2 e^{-\phi} &= 0.
\end{align}

(3.15)

The above equations match with the equations obtained from the following two dimensional dilaton gravity action\[11\] provided \( k = -\frac{1}{2} \) and the cosmological term \( \lambda \) is set to zero.

\begin{align}
S = -\int_M \sqrt{g} e^{-2\phi} [R + \frac{8k}{k-1} (\nabla \phi)^2 + \lambda^2] - 2 \int_{\partial M} e^{-2\phi} K.
\end{align}

(3.16)

where \( K \) is the trace of the second fundamental form, \( \partial M \) is the boundary of \( M \) and \( k \) is a parameter taking values \( |k| \leq 1 \). We therefore notice that a general class of membrane solutions can be constructed with our ansatz, for any solution of 2-dimensional gravity defined by equation (3.16). Since a world-volume dependent conformal factor \( \sqrt{f} \) appears explicitly in our solution, the translational isometries in these directions, unlike the flat membrane case, are now lost. In general we therefore have curved geometry for these branes.

At this point, we make the remark that the relation between the membrane source and the supergravity background turns out to be identical to that of \[8\]. It will be useful to find the ADM mass and charge of the solutions.

4 Discussions and Conclusions

To recapitulate our results, we found a class of curved membrane solutions in the eleven dimensional supergravity. The worldvolume Poincaré invariance is broken
while the isotropy in the directions transverse to the membrane survives. The metric and the three form for the general solution are given by:

\[ ds^2 = (1 + K F_{-\frac{3}{2} \rho \rho})^{-\frac{2}{3}} F^e \left[ -(dx^0)^2 + \sqrt{f} (dx^a)^2 \right] + (1 + K F_{-\frac{3}{2} \rho \rho})^{-\frac{1}{3}} F^{-\frac{2}{3}} \left( dy^m \right)^2, \]

\[ A_{\mu
u\rho} = \pm \frac{1}{3} g \epsilon_{\mu
u\rho} (1 + K F_{-\frac{3}{2} \rho \rho})^{-1} F^e \sqrt{f}. \]  

(4.1)

This represents general curved membrane solution in a class of embedding space. The geometry of the embedding space can be obtained by taking the asymptotic limit \( r \to \infty \). We are motivated by the current upsurge of studies of nonsupersymmetric brane solutions in various contexts and are mainly interested in non-supersymmetric solutions. However, our solution space turns out to be large enough to contain several supersymmetric solutions as well.

The solution presented here can be used to obtain the curved membrane solution in a class of embedding space provided it satisfies some necessary condition for being a solution which is the constraint imposed by (3.14). We can obtain the solution for various special cases, including the curved membrane in the flat space which is a special case of our general solution. At this point it may be useful to compare our result to that of [1]. They essentially use the conformal sigma models and obtain curved transverse and longitudinal geometries for higher and lower dimensional brane respectively. Our approach is more straightforward and we consider only membrane with a curved longitudinal world volume only. This method can be generalized for other branes and can be used for explicit solutions of branes wrapping cycles in asymptotically non-flat geometry.

One useful application of the present solution is to consider the near horizon limit. For the flat brane by setting \( f \) to unity and using the relation among the various exponents, we are left with essentially only one equation:

\[ F_{,aa} - \left( \frac{3e}{2} + 1 \right) \frac{F^a}{F} = 0. \]  

(4.2)

which is the condition on the embedding space. Then we find that in the limit \( K \to \infty \), the above metric goes over to the familiar \( AdS_4 \times S^7 \). As the gravity dual depends only on the brane and not on the embedding geometry, in our case we

---

\[ ^1 \text{Since the above equation (4.2) is integrable, we can consistently set } f \text{ to unity.} \]
find that although the generic solution is non supersymmetric, we end up with the same supergravity background as that of a supersymmetric solution. This analysis can be extended to generically curved branes. The gravity duals in the generic case correspond to non conformal world volume theory and it is interesting to construct the dual of a confining theory.

Finally it is important to check the stability of these solutions. Since these solutions are not supersymmetric in general, their stability is not guaranteed. As a result, one would like to know if these solutions are stable against small perturbations of the metric. This is not so innocuous a question as it might appear. The gravitational stability of black holes was a long standing problem\cite{13}. In \cite{14}, it was settled in the affirmative for Schwarzschild black holes. Subsequently, the stability of Reissner-Nordström and Kerr black holes was also established. In low energy string theory and M-theory, there is a plethora of black holes and p-branes. In particular, extended objects having event horizons enclosing a curvature singularity emerge as classical solutions\cite{15} of supergravity. They can be thought of as higher dimensional cousins of the familiar black holes. It is thus of paramount importance to study the stability of these solutions. In a series of papers\cite{16}, it was argued that both uncharged as well as charged non-extremal black p-branes are unstable while the charged extremal black p-branes are stable. As a specific example, a black string solution was considered and argued to be unstable against decay into black holes. Recently, in \cite{17}, the authors objected to this argument and proposed that such unstable black string will finally settle down to some other black string. This proposal has the virtue that it is able to avoid the bifurcation of event horizon. But there are still many unresolved questions. A full fledged perturbation analysis of the solutions presented in our paper is therefore important.

We now end with some speculations. We presented the supergravity solutions which essentially provide the long distance bulk behavior. A complementary study of the short distance behavior using matrix model \cite{18}, may be illuminating. We also think that the Euclidean version of these branes might have some relevance to the brane world scenario. Finally, the similarity between the bosonic sector of the supersymmetric M-Theory and the Bosonic M-Theory in \cite{12} and the existence of nonsuper-

\footnote{A different type of instability still persists in the vicinity of the inner Cauchy horizon.}
symmetric 2-branes in the Bosonic M-Theory makes us strongly feel that it might be possible to relate the 2-brane solutions in these two apparently disparate theories. We hope to address some of these issues in future.

5 Acknowledgements

The work of S.M is supported by the National Science Foundation under grant no. 9801875.

6 Appendix

For convenience, we provide the Christoffel connections below for our ansatz given in sections 2 and 3.

\[
\begin{align*}
\Gamma^0_{0a} &= -\frac{1}{3}(e^{3\tilde{A}}\partial_a e^{-3\tilde{A}} - \frac{3e}{2} \partial_a \ln F), \\
\Gamma^0_{0m} &= -\frac{1}{3}e^{3\tilde{A}}\partial_m e^{-3\tilde{A}}, \\
\Gamma^a_{00} &= -\frac{1}{3}f^{-\frac{1}{2}}(e^{3\tilde{A}}\partial_a e^{-3\tilde{A}} - \frac{3e}{2} \partial_a \ln F), \\
\Gamma^a_{0c} &= -\frac{1}{3}e^{3\tilde{A}}F^{\frac{3\tilde{B}}{2}}[\partial_c(e^{-3\tilde{A}}F^{-\frac{3\tilde{B}}{2}})\delta^a_b + \partial_b(e^{-3\tilde{A}}F^{-\frac{3\tilde{B}}{2}})\delta^a_c - \\
&\partial_a(e^{-3\tilde{A}}F^{-\frac{3\tilde{B}}{2}})\delta_{bc}] + \\
&\frac{1}{4f}(\partial_c f \delta^a_b + \partial_b f \delta^a_c - \partial_a f \delta_{bc}), \\
\Gamma^a_{bm} &= -\frac{1}{3}\delta^a_b e^{3\tilde{A}}\partial_m e^{-3\tilde{A}}, \\
\Gamma^a_{mn} &= -\frac{1}{2}(F^{-e} f^{-\frac{1}{2}})e^{-2\tilde{A}}\partial_a(e^{2\tilde{B}} g^b)\delta_{mn}, \\
\Gamma^m_{00} &= \frac{1}{2}F^e g^{-b}e^{-2\tilde{B}}\partial_m e^{2\tilde{A}}, \\
\Gamma^m_{ab} &= -\frac{1}{2}(g^{-b} F^e f^{\frac{1}{2}})e^{-2\tilde{B}}\partial_m e^{2\tilde{A}}\delta_{ab}, \\
\Gamma^m_{an} &= \frac{1}{2}e^{-2\tilde{B}} g^{-b}\partial_a(e^{2\tilde{B}} g^b)\delta^m_n, \\
\Gamma^l_{mn} &= \frac{1}{6}e^{-6\tilde{B}}[\partial_n e^{6\tilde{B}} \delta^l_m + \partial_m e^{6\tilde{B}} \delta^l_n - \partial_l e^{6\tilde{B}} \delta_{mn}] \\
\end{align*}
\]

The simplified form of these Christoffel connections (in the notation explained in
the text) are also given below.

\[
\Gamma^0_{0a} = \frac{d}{3} \kappa^0 (1 + \frac{K}{r^6})^{-1} \frac{h_a}{h} + \frac{e}{2} F_a
\]

\[
\Gamma^0_{0m} = -\frac{1}{3} (1 + \frac{K}{r^6})^{-1} (\frac{K}{r^6})_m
\]

\[
\Gamma^a_{00} = \frac{d}{3} \kappa^0 (1 + \frac{K}{r^6})^{-1} \frac{h_a}{\sqrt{f}} \frac{h}{h} + \frac{e}{2\sqrt{f}} F_a
\]

\[
\Gamma^a_{bc} = \frac{d}{3} \kappa^0 (1 + \frac{K}{r^6})^{-1} (h_c \delta^b_a + h_b \delta^a_c - h_a \delta_{bc}) + \frac{e}{2F} (F_c \delta^a_b + F_b \delta^a_c - F_a \delta_{bc}) + \frac{1}{4f} (F_c \delta^a_b + F_b \delta^a_c - F_a \delta_{bc})
\]

\[
\Gamma^a_{bm} = -\frac{1}{3} (1 + \frac{K}{r^6})^{-1} (\frac{K}{r^6})_m \delta^a_b
\]

\[
\Gamma^a_{mn} = -\frac{1}{2} (F^e g^b f^{-\frac{1}{2}}) [\frac{d}{3} \kappa^0 (1 + \frac{K}{r^6})^{-1} \frac{h_a}{h} + b (1 + \frac{K}{r^6}) \frac{g_a}{g}] \delta_{mn}
\]

\[
\Gamma^m_{00} = -\frac{1}{3} (F^e g^b) (1 + \frac{K}{r^6})^{-2} (\frac{K}{r^6})_m \delta_{ab}
\]

\[
\Gamma^m_{ab} = \frac{1}{3} (F^e g^b \sqrt{f}) (1 + \frac{K}{r^6})^{-2} (\frac{K}{r^6})_m \delta_{mn}
\]

\[
\Gamma^m_{an} = -\frac{d}{6} \kappa^0 (1 + \frac{K}{r^6})^{-1} \frac{h_a}{h} \delta_{mn} + \frac{b g_a}{2 g} \delta_{mn}
\]

\[
\Gamma^m_{np} = \frac{1}{6} (1 + \frac{K}{r^6})^{-1} [(\frac{K}{r^6})_p \delta_{mn} + (\frac{K}{r^6})_n \delta_{mp} - (\frac{K}{r^6})_m \delta_{np}]
\]

To present the relevant spin connections, let us first fix our notation. We denote the spacetime indices by 0, a and m and the tangent space indices by 0, a and m. The nonvanishing spin connections are as follows.

\[
\omega_{\hat{0}a} = \frac{1}{3} f^{-\frac{1}{2}} (e^{3\hat{A}} \partial_a e^{-3\hat{A}} - \frac{3}{2} \partial_a \ln F),
\]

\[
\omega_{\hat{0}m} = -F_{\hat{A}m} g^{-\frac{1}{2}} e^{-\hat{A}} \partial_m e^{\hat{A}},
\]

\[
\omega_{\hat{A}c} = \frac{1}{3} e^{3\hat{A}} F^{\hat{A}} \partial_c (e^{-3\hat{A}} F^{-\hat{A}}) \delta_{\hat{A}c} - \partial_c (e^{-3\hat{A}} F^{-\hat{A}}) \delta_{\hat{A}c} - \frac{1}{4f} (\partial_b f \delta_{\hat{A}c} - \partial_c f \delta_{\hat{A}b})],
\]

\[
\omega_{\hat{A}m} = \frac{1}{2} (F_{\hat{A}m} f^{\hat{A}} g^{-\frac{1}{2}}) e^{-(\hat{A} + \hat{B})} \partial_m e^{2\hat{A}},
\]

\[
\omega_{\hat{m}m} = -\frac{1}{2} (F_{\hat{A}m} f^{\hat{A}} g^{-\frac{1}{2}}) e^{-(\hat{A} + \hat{B})} (e^{-2\hat{B}} \partial_m e^{2\hat{B}} + g^{-\hat{B}} \partial_m g^{\hat{B}}),
\]

\[
\omega_{\hat{m}m} = e^{-\hat{B}} (\partial_n g^{\hat{B}} \delta_{\hat{m}n} - \partial_n e^{\hat{B}} \delta_{\hat{m}n}).
\]

(6.3)
The above spin connections become simplified once we use the explicit form of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$.

\[
(\omega_0)^{\hat{b}}_{\hat{a}} = \frac{d}{3} f^{-\frac{7}{4}} \frac{h}{h}(1 + \frac{K}{r^6})^{-1},
\]
\[
(\omega_0)^{\hat{b}}_{\hat{m}} = \frac{1}{3} h^{-\frac{7}{2}} (1 + \frac{K}{r^6})^{-\frac{7}{4}} (\frac{K}{r^6})_{,m},
\]
\[
(\omega_a)^{\hat{b}c} = \left[-\frac{d}{3} (1 + \frac{K}{r^6})^{-1} \frac{1}{h} (h_{,\hat{e}} \delta_{\hat{a} \hat{b}} - h_{,\hat{b}} \delta_{\hat{a} \hat{e}}) + \frac{1}{4f} (f_{,\hat{c}} \delta_{\hat{a} \hat{b}} - f_{,\hat{b}} \delta_{\hat{a} \hat{c}})\right],
\]
\[
(\omega_a)^{\hat{b}m} = -\frac{1}{3} h^{-\frac{7}{2}} (1 + \frac{K}{r^6})^{-\frac{7}{4}} f^{-\frac{7}{4}} \delta^{\hat{b}}_{\hat{a}} (\frac{K}{r^6})_{,m},
\]
\[
(\omega_m)^{\hat{a}n} = -\frac{d}{6} h^{\frac{7}{2}} f^{-\frac{7}{4}} (1 + \frac{K}{r^6})^{-\frac{7}{4}} \hat{h}_{,\hat{a}} \delta_{\hat{m} \hat{n}},
\]
\[
(\omega_m)^{\hat{a}\hat{p}} = \frac{1}{6} (1 + \frac{K}{r^6})^{-1} \left[ (\frac{K}{r^6})_{,\hat{p}} \delta_{\hat{m} \hat{n}} - (\frac{K}{r^6})_{,\hat{m}} \delta_{\hat{p} \hat{n}} \right].
\]

(6.4)

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