Zero Sound from Holography

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Abstract

Quantum liquids are characterized by the distinctive properties such as the low temperature behavior of heat capacity and the spectrum of low-energy quasiparticle excitations. In particular, at low temperature, Fermi liquids exhibit the zero sound, predicted by L. D. Landau in 1957 and subsequently observed in liquid He-3. In this paper, we ask a question whether such a characteristic behavior is present in theories with holographically dual description. We consider a class of gauge theories with fundamental matter fields whose holographic dual in the appropriate limit is given in terms of the Dirac-Born-Infeld action in $\text{AdS}_{p+1}$ space. An example of such a system is the $\mathcal{N} = 4$ $SU(N_c)$ supersymmetric Yang-Mills theory with $N_f$ massless $\mathcal{N} = 2$ hypermultiplets at strong coupling, finite baryon number density, and low temperature. We find that these systems exhibit a zero sound mode despite having a non-Fermi liquid type behavior of the specific heat. These properties suggest that holography identifies a new type of quantum liquids.

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**Introduction.**—Gauge/gravity duality \([1, 2, 3]\) has become a useful tool for investigating strongly coupled field theories. In the class of models where this tool can be applied, the strong coupling limit of the field theory is mapped into the weak-coupling, classical limit of a gravity theory, which can be studied either analytically or with minimal computer power. For example, a cousin of QCD—the \(N = 4\) supersymmetric Yang-Mills (SYM) theory—has been studied using this method. Such studies have pointed to a universal value of the viscosity/entropy density ratio in a wide class of strongly coupled theories (for a review, see \([4]\)). Somewhat surprisingly, the viscosity/entropy density ratio of the quark-gluon plasma created at the Relativistic Heavy Ion Collider seems to be close to this value, indicating that gauge/gravity duality may be useful for studies of QCD.

Here we would like to see what the gauge/gravity duality has to say about strongly coupled quantum liquids. By quantum liquids we mean translationally invariant systems at zero (or low) temperature and at finite density. Given the important role that quantum liquids play in physics, it is natural to ask whether the newly developed technique of gauge/gravity duality can give us any insights into their behavior.

The cornerstones of our understanding of quantum liquids are two phenomenological theories. These are Landau’s Fermi liquid theory \([5, 6, 7, 8, 9]\) and the theory of quantum Bose liquids \([7, 8]\). These two theories describe two distinct behaviors of a quantum liquid at low momenta and temperatures. In a Bose liquid, the only low-energy elementary excitation is the superfluid phonon with a linear dispersion. This leads to a \(T^3\) behavior of the specific heat at low temperatures. The Fermi liquid has a richer spectrum of elementary excitations, consisting of fermionic quasiparticles and a bosonic branch, which contains, in particular, the zero sound. The fermions dominate the specific heat, which scales as \(T^2\) at low \(T\).

In this paper, we found, through the gauge/gravity duality, a new type of quantum liquid. The quantum liquid we consider has a \(T^6\) behavior (\(\sim T^{2p}\) in \(p\) spatial dimensions) of the specific heat at low temperature. Despite the non-Fermi liquid behavior of the specific heat, the system supports a sound mode at zero temperature, which we will call “zero sound.” The mode is almost identical to the zero sound in Fermi liquids: not only the real part of its dispersion curve is linear in momentum (\(\omega = vq\)), but the imaginary part has the same \(q^2\) dependence predicted by Landau a long time ago for quantum attenuation of the zero sound. The difference is that in our case the zero sound velocity coincides with the first sound velocity, while in the case of a Fermi liquid the two velocities are not equal to each other.

In Fermi liquids, zero sound is a collective excitation involving fermions near the Fermi surface. It was predicted by Landau \([7, 8]\) and experimentally observed in liquid Helium-3. In a weakly interacting Fermi gas, the zero sound velocity is close to the Fermi velocity \(v_F\). The first sound, which is a hydrodynamic mode that exists at finite temperatures and wavelengths larger than the mean free path, has velocity \(v_F/\sqrt{3}\) at weak coupling. The quantum attenuation (i.e. damping at zero temperature) of zero sound was first considered by Landau, who showed that the imaginary part of the zero sound energy scales as \(q^2\) where \(q\) is the momentum. The experimental situation with the measurement of the zero sound
quantum attenuation is summarized in [10].

A specific example considered in this paper is the $\mathcal{N} = 4$ SU($N_c$) supersymmetric Yang-Mills (SYM) theory with $N_f$ massless $\mathcal{N} = 2$ hypermultiplet fields. This theory has been suggested as a model which approximates QCD better than the theory without fundamental quarks. A string-theoretic description of this system is given by a low-energy limit of the D3/D7 brane configuration. The theory has been studied at finite temperature and density using the gauge/gravity duality [11, 12, 13, 14, 15, 16, 17, 18, 19]. Nevertheless, two striking aspects of this and similar systems (characterized by the Dirac-Born-Infeld action in Anti-de-Sitter space)—the unusual behavior of the low-temperature specific heat and the existence of the zero sound—have so far eluded attention. Their description is the main purpose and the main result of the paper.

Preliminaries.—The $\mathcal{N} = 4$ SYM theory contains fields in the adjoint representation of the gauge group only. Fields in the fundamental representation can be introduced by using the following construction [20]. In type IIB string theory, one considers a system of $N_c$ D3-branes and $N_f$ D7-branes aligned in flat ten-dimensional space as

$$
x_0 \times x_1 \times x_2 \times x_3 \times x_4 \times x_5 \times x_6 \times x_7 \times x_8 \times x_9
\begin{array}{c}
\text{D3} \\
\text{D7}
\end{array}
\quad \text{(1)}$$

In the limit of large number of colors ($N_c \gg 1$) and large ’t Hooft coupling ($g_{YM}^2 N_c \gg 1$), the D3-branes are replaced by the near-horizon AdS$_5 \times S^5$ geometry [1], while the $N_f$ D7-branes can be treated as probes embedded into this geometry as long as $N_f/N_c \ll 1$, i.e. as long as their backreaction on the geometry can be neglected [20].

The standard form of the near-horizon D3 brane metric is

$$ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} \left( dr^2 + r^2 d\Omega_5^2 \right), \quad \text{(2)}$$

where $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$, $R$ is the curvature radius of the AdS$_5$ (we shall set $R = 1$ in the following). The horizon is located at $r = 0$.

In this paper, we focus on adding $N_f$ massless $\mathcal{N} = 2$ hypermultiplets to $\mathcal{N} = 4$ SYM, keeping the theory at zero temperature. In the dual gravity this is described by a zero temperature “horizon-crossing” D7-brane embedding in which the distance between the D7 branes and the horizon in the $x_8 - x_9$ direction vanishes [13].

The action for the D7-branes is the Dirac-Born-Infeld (DBI) action

$$S_{\text{DBI}} = -N_f T_{D7} \int d^8 \xi \sqrt{-\det(g_{ab} + 2\pi \alpha' F_{ab})}, \quad \text{(3)}$$

where $T_{D7}$ is the D7-brane tension, $\xi_a$ are worldvolume coordinates, $g_{ab}$ is the induced worldvolume metric and $F_{ab}$ is the worldvolume $U(1)$ gauge field. At the boundary, the gauge field couples to the $U(1)_B$ “flavor” current $J^\mu$, where $U(1)_B$ is the “baryon number” subgroup of the global symmetry group $U(N_f)$ possessed by the $\mathcal{N} = 2$ hypermultiplet fields.
(The exact form of the $U(1)_B$ current operator is given in [11].) Considering finite “baryon” density $\langle j^0 \rangle \neq 0$ in the boundary gauge theory corresponds to turning on a non-trivial background worldvolume gauge field $A_0(r)$ in the bulk [11]. The DBI action becomes

$$S_{\text{DBI}} = -N V_3 \int dr r^3 \sqrt{1 - A_0^2},$$

where the factor $2\pi \alpha'$ is absorbed into $A_0$, $V_3$ is the spatial volume of the boundary gauge theory, and the prefactor $N = \lambda N_f N_c/(2\pi)^4$ is determined by the gauge/gravity duality dictionary [11].

The construction above is specific to the D3–D7 system, but we can be more general and consider D$q$ probe branes whose worldvolume include an $\text{AdS}_{p+2}$ factor. For probe branes corresponding to massless flavors the embedding is independent of the internal directions. One example with $p = 2$ would be the defect D5 on $\text{AdS}_4 \times S^2$ in $\text{AdS}_5 \times S^5$ [21]. The DBI action reads

$$S_{\text{DBI}} = -N_q V_p \int dr r^p \sqrt{1 - A_0^2},$$

where the normalization now includes the tension of the $N_f$ D$q$-branes [13]. The solution to the embedding problem is given by

$$A_0' = d \sqrt{r^2 + d^2},$$

where $d \equiv (2\pi \alpha' N_q)^{-1} \rho$ is proportional to the baryon number density $\rho$ [13]. In all subsequent formulas, the results for the D3–D7 case are trivially recovered by setting $p = 3$ and using the relation $\alpha'^{-1} = \sqrt{\lambda}$.

Low-temperature limit of the specific heat.— One interesting hint to the nature of the phase of matter described by the probe branes with finite chemical potential is the behavior of the specific heat at low temperature. To extract this information one first needs to generalize the setup to a background that contains a black hole, that is the $\text{AdS}_{p+2}$ part of the metric is modified to

$$ds^2 = \frac{r^2}{R^2} \left[ - \left( 1 - \frac{r_H^{p+1}}{r^{p+1}} \right) dt^2 + \frac{1}{R^p} \left( 1 - \frac{r_H^{p+1}}{r^{p+1}} \right)^{-1} \frac{R^2}{r^2} dr^2 \right] + \frac{1}{R^p} \frac{1}{r^2} d\vec{x}^2.$$

Here $r_H$ is the horizon radius of the black hole related to the temperature of the black hole by $r_H = 4\pi T/(p + 1)$ (with $R = 1$). The action describing the system at finite density and finite temperature is almost identical to the one in the zero temperature case

$$S_{\text{DBI}} = -N_q V_p \int_{r_H}^{\infty} dr r^p \sqrt{1 - A_0^2},$$

the only difference being that the integration starts at $r_H$ rather than at 0. All powers of the redshift factor cancel between $g^{tt}$ and $g^{rr}$. The solution for $A_0'$ is still given by Eq. (6).

1 Throughout this paper, we work in a gauge with $A_r = 0$. 

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The on-shell value of the total action \(^2\) directly gives us minus the thermodynamic potential \(\Omega\) in the grand canonical ensemble. Substituting the solution (3) for \(A'_0\) into the action, we can write \(\Omega = \Omega_{ad} + \Omega_{\text{fun}}\), where \(\Omega_{ad} \sim T^{p+1}\) is the contribution of the adjoint degrees of freedom (for the D3/D7 system \(\Omega_{ad} = -\pi^2 N_c^2 T^4/8\) is the free energy of \(\mathcal{N} = 4\) SYM at strong coupling), and

\[
\Omega_{\text{fun}} = N_q V_p \int_{r_H}^{\Lambda} dr \frac{r^{2p}}{\sqrt{r^{2p} + d^2}} - \frac{N_q}{p+1} \int d^{p+1} x \sqrt{-h(\Lambda)},
\]

where \(\Lambda\) is the ultraviolet cutoff. In Eq. (9), the local counterterm action built from the metric \(h_{\mu\nu}\) induced on the slice \(r = \Lambda\) by the ambient metric (7) has been added in the spirit of the holographic renormalization [22]. In the grand canonical ensemble, the potential \(\Omega\) is a function of \(T\) and \(\mu\), where \(\mu\) is the baryon number chemical potential related to the density and temperature via the condition

\[
\mu = \int_{r_H}^{\infty} dr A'_0.
\]

The integrals in Eqs. (9) and (10) can be expressed in terms of the Gauss hypergeometric function:

\[
\Omega_{\text{fun}} = \Omega_0 - \frac{N_q V_p r_H^{2p+1}}{(2p+1)d} {}_2F_1\left(\frac{1}{2}, 1 + \frac{1}{2p}; 2 + \frac{1}{2p}; -r_H^{2p}/d^2\right) + \frac{N_q V_p r_H^{p+1}}{2(p+1)} 
\]

\[
\mu = \mu_0 - r_H {}_2F_1\left(\frac{1}{2}, 1 + \frac{1}{2p}; 1 + \frac{1}{2p}; -r_H^{2p}/d^2\right),
\]

where \(\Omega_0\) and \(\mu_0\) are the zero-temperature values,

\[
\mu_0 = \alpha d^p, \quad \Omega_0 = -\frac{N_q V_p}{(p+1)\alpha^p} \mu_0^{p+1},
\]

and \(\alpha(p) = \Gamma\left(\frac{1}{2} - \frac{1}{2p}\right) \Gamma\left(1 + \frac{1}{2p}\right)/\Gamma\left(\frac{1}{2}\right)\). We note that the last term in Eq. (11), \(N_q V_p r_H^{p+1}/2(p+1) \equiv \Omega_{\text{ct.}}\), is independent of the matter density and has the same temperature dependence as the free energy of adjoint fields \(\Omega_{ad}\). We shall focus on the density-dependent part of the thermodynamic potential \(\Delta \Omega \equiv \Omega_{\text{fun}} - \Omega_{\text{ct.}}\). Equations (11) and (12) determine \(\Delta \Omega\) as a function of temperature and chemical potential. At low temperature, both equations can be treated as series expansions in \(T/\mu_0 \ll 1\). The baryon number density is proportional to \(d\),

\[
\rho = -\frac{1}{V_p} \frac{\partial \Omega_{\text{fun}}}{\partial \mu} = N_q d.
\]

One then computes the entropy density \(s(\mu, T)\) in the grand canonical ensemble

\[
s(T, \mu) = \frac{1}{V_p} \left(-\frac{\partial \Delta \Omega(T, \mu)}{\partial T}\right)_{\mu, V_p}.
\]

\(^2\) The total action is the sum of the DBI action (8) describing fundamental degrees of freedom and the bulk gravitational action dual to the adjoint sector of the system.
Using Eq. (12), we find the entropy density as a function of temperature and charge density

\[ s(T, d) = s_0 + N_q \left( \frac{4\pi}{p+1} \right)^{2p+1} \frac{T^{2p}}{2d} \left[ 1 + O \left( T d^{\frac{1}{p}} \right) \right], \quad (16) \]

where \( s_0 = 4\pi \rho / [(p + 1)(2\pi \alpha')] \) is the entropy at zero temperature. This entropy is related to the quark thermal mass (free energy), which is negative and proportional to \( T \). Finally, the specific heat (heat capacity per unit volume) \( c_V \) at constant volume and density is determined by

\[ c_V = T \left( \frac{\partial s(T, d)}{\partial T} \right)_\rho. \quad (17) \]

At low temperature (\( T \ll \mu_0 \)) the density-dependent part of the specific heat\(^3\) is proportional to \( T^{2p} \):

\[ c_V = N_q p \left( \frac{4\pi}{p+1} \right)^{2p+1} \frac{T^{2p}}{d} \left[ 1 + O \left( T d^{\frac{1}{p}} \right) \right]. \quad (18) \]

This has to be contrasted with a gas of free bosons whose low temperature specific heat is proportional to \( T^p \) (a sphere of volume \( T^p \) of occupied states in momentum space, each with energy \( T \)) or a gas of fermions, whose low temperature specific heat scales as \( T \) for any \( p \) (a shell of thickness \( T \) of occupied states above the Fermi surface contributing an energy \( T \) each). The behavior of the specific heat in Eq. (18) is suggestive of a new type of quantum liquid.

**Zero sound.**—The zero sound mode would manifest itself as a pole of the zero-temperature retarded flavor current density correlator \([7, 8, 9]\). In the dual gravity language, the pole arises as the quasinormal frequency of the background geometry \([23, 24, 25]\). Generically, the quasinormal spectrum is determined by fluctuations of all background fields including the metric. However, in the particular case we are dealing with, it is sufficient to consider fluctuations of the DBI U(1) field in the gravitational background \((2)\) with the non-trivial background component \( A_0 \). Moreover, since the dual quantum field theory is isotropic, we can choose the fluctuations to depend on time, radial coordinate and one of the spatial coordinates (e.g. \( x_p \)) only

\[ A_\mu(r) \rightarrow A_\mu(r) + a_\mu(r, x_0, x_p). \quad (19) \]

Substituting (19) into the DBI action \((5)\) and expanding to second order in fluctuations, we find that the quadratic part of the resulting action is given by the sum of the actions describing longitudinal \( (a_0, a_p) \) and transverse fluctuations. The action for the longitudinal fluctuations is (we use the \( a_r = 0 \) gauge)

\[ S^{(2)} = \frac{N_q}{2} \int d^{p+1}x \, d^p r \, r^p \left\{ \frac{a_0^2}{(1 - A_0^2)^{3/2}} + \frac{(\partial_0 a_p - \partial_p a_0)^2}{r^4 \sqrt{1 - A_0^2}} - \frac{a_p^2}{\sqrt{1 - A_0^2}} \right\}. \quad (20) \]

\(^3\) The density-independent part of the specific heat is proportional to \( T^p \).
Introducing the Fourier components

\[ a_\mu(r, x_0, x_p) = \int \frac{d\omega dq}{(2\pi)^2} e^{-i\omega x_0 + iq x_p} a_\mu(r, \omega, q), \] (21)

we find the equations of motion for the fluctuations

\[ \frac{d}{dr} \left[ \frac{r^p a'_0}{(1 - A_0'^2)^{3/2}} \right] - \frac{r^{p-4}}{\sqrt{1 - A_0'^2}} (\omega q a_p + q^2 a_0) = 0, \] (22)

\[ \frac{d}{dr} \left[ \frac{r^p a'_p}{\sqrt{1 - A_0'^2}} \right] + \frac{r^{p-4}}{\sqrt{1 - A_0'^2}} (\omega q a_0 + \omega^2 a_p) = 0. \] (23)

There is also a constraint arising as a consequence of the residual gauge invariance of the components \( a_0 \) and \( a_p \)

\[ \omega a'_0 + (1 - A_0'^2) q a'_p = 0. \] (24)

Introducing a new radial coordinate \( z = 1/r \), Eqs. (22, 23, 24) can be written as

\[ \partial_z \left( f^2 z^{-2-p} a'_0 \right) - f z^{-2-p} (\omega q a_p + q^2 a_0) = 0, \] (25a)

\[ \partial_z \left( f z^{-2-p} a'_p \right) + f z^{-2-p} (\omega q a_0 + \omega^2 a_p) = 0, \] (25b)

\[ f^2 \omega a'_0 + q a'_p = 0, \] (25c)

where \( f(z) = \sqrt{1 + d^2 z^{2p}} \). Following the approach of [25], we use Eq. (25) to derive the equation for the gauge-invariant variable \( E = \omega a_p + q a_0 \):

\[ E'' + \left[ \frac{f'(3q^2 - \omega^2 f^2)}{f (q^2 - \omega^2 f^2)} - \frac{p - 2}{z} \right] E' + \frac{\omega^2 f^2 - q^2}{f^2} E = 0. \] (26)

The horizon, \( z = \infty \), is an irregular singular point of the differential equation (26). The solution in the vicinity of \( z = \infty \) is given by \( E(z) \sim e^{\pm i\omega z}/z \). The incoming wave boundary condition at the horizon [24] singles out one of the exponents

\[ E(z) \sim C e^{i\omega z}/z \] (27)

where \( C \) is a constant. For \( \omega z \ll 1 \) we have

\[ E(z) = \frac{C}{z} + i\omega C. \] (28)

On the other hand, for \( \omega z \ll 1 \) and \( qz \ll 1 \) with \( \omega/q \) fixed, Eq. (26) reduces to the equation

\[ E'' + \left[ \frac{f'(3q^2 - \omega^2 f^2)}{f (q^2 - \omega^2 f^2)} - \frac{p - 2}{z} \right] E' = 0, \] (29)

whose solution is given in terms of the Gauss hypergeometric function

\[ E(z) = C_1 + C_2 z^{p-1} \left[ \frac{q^2}{p f(z)} + \frac{(q^2 - p \omega^2)}{p(p-1)} {}_2F_1 \left( \frac{1}{2}, \frac{1}{2} - \frac{1}{2p}; 3 - \frac{1}{2p}; -\omega^2 z^{2p} \right) \right]. \] (30)
For $z \to \infty$ we find

$$E(z) \to C_1 + C_2 \left( \frac{a}{z} + b \right) + O(1/z^2),$$

where the coefficients $a$ and $b$ are given by

$$a = \frac{\omega^2}{d}, \quad b = \frac{(q^2 - p\omega^2) d^{p-1} \Gamma \left( \frac{1}{2} - \frac{1}{2p} \right) \Gamma \left( \frac{1}{2p} \right)}{2p^2 \Gamma \left( \frac{1}{2} \right)}.$$

Matching to the expansion (28), we find the coefficients $C_1$ and $C_2$:

$$C_1 = \left( i\omega - \frac{b}{a} \right) C, \quad C_2 = \frac{C}{a}.$$  

The lowest quasinormal frequency is found by imposing the Dirichlet condition at the boundary, $E(0) = 0$ [25]. This condition gives the equation $C_1 = 0$ or, equivalently,

$$i\omega = \left( \frac{q^2}{\omega^2} - p \right) \frac{d^{p-1} \Gamma \left( \frac{1}{2} - \frac{1}{2p} \right) \Gamma \left( \frac{1}{2p} \right)}{2p^2 \Gamma \left( \frac{1}{2} \right)}.$$

Solving Eq. (34) for small $\omega$ and $q$, we find the dispersion relation for the zero sound

$$\omega = \pm \frac{q}{\sqrt{p}} - \frac{i \Gamma \left( \frac{1}{2} \right) q^2}{d^{p} \Gamma \left( \frac{1}{2} - \frac{1}{2p} \right) \Gamma \left( \frac{1}{2p} \right)} + O(q^3).$$

Using the expression for the chemical potential at zero temperature from Eq. (13), the zero sound dispersion relation can be written as

$$\omega = \pm \frac{q}{\sqrt{p}} - \frac{i q^2}{2p \mu_0} + O(q^3).$$

What is the nature of this excitation? First, one can exclude the possibility that it is a superfluid phonon. Indeed, our background does not break the particle number symmetry, hence the ground state is not a superfluid. Furthermore, superfluid phonon width has a low-momentum behavior different from $q^2$, namely $q^3$ in 3 spatial dimensions [25] and $q^{p+2}$ in $p$ spatial dimensions (provided that phonon decay is kinematically allowed). The $q^2$ behavior of the imaginary part is characteristic of zero sound quantum attenuation, thus we call this mode the zero sound. Yet in other respects (such as the specific heat temperature dependence) the system does not show Fermi-liquid behavior. It is notable that the zero sound velocity in our system coincides with the velocity of the finite-temperature first sound, while in a weakly-coupled Fermi liquid it is $\sqrt{p}$ times larger than the first-sound velocity.

**Conclusion.**—In this paper we have considered a general theory described by a DBI action in AdS space. We found that by turning on a chemical potential one arrives to a new type of quantum liquid. The specific heat $c_V$ has an unusual non-Fermi liquid $T^{2p}$ behavior ($T^6$ in 3+1 dimensions and $T^4$ in 2+1 dimensions). The low energy spectrum contains a gapless mode with a dispersion relation similar to the zero sound in Fermi liquids. One can
speculate that the mode observed here is what the Fermi-liquid zero sound becomes when the interaction is infinitely strong. In this connection, we note that in a simple model of the Fermi liquid, the velocity of the zero and first sounds approach each other in the limit where the interaction strength (parameterized by the Fermi-liquid parameter $F_0$) is infinite \[9\].

The systems described here are strongly coupled, as they have gravity duals. It would be interesting to investigate the properties of the ground state and the zero sound in the weak-coupling regime of the $\mathcal{N} = 4$ SYM theory with $\mathcal{N} = 2$ matter hypermultiplets. We leave this problem for future work.

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