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On the Road to Understanding Type Ia Progenitors: Precision Simulations of Double Degenerate Mergers

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Abstract. We review the current state of the art in double degenerate merger simulations to better understand the role this phenomenon plays in type Ia progenitors. Because the fate of a merged system may well depend on the exact evolution of the matter temperature (as well as mixing of the merged system), precision simulations are required to determine the true fate of these systems. Unfortunately, if we compare the results of current simulations, we find many-order of magnitude differences in quantities like mass-transfer rates in the merger process. We discuss these differences and outline an approach using verification and validation that should allow us to achieve a level of precision sufficient to determine the true fate (thermonuclear vs. collapse) of double degenerate mergers. Understanding the fate of lower-mass systems (e.g. R Coronae Borealis stars) may be key in our final testing phase.

1. Introduction

Supernovae (SNe) and their Gamma-Ray Bursts cousins mark the most powerful explosions in the universe. The quest for an understanding of the driving mechanism behind these explosions has been nearly as turbulent and heated as the events themselves. It is now nearly universally agreed that these explosions are produced in one of two engines: the collapse of a massive stellar core (the gravitational potential energy released in the collapse being the energy source of the supernova explosion), the thermonuclear explosion of a white dwarf (where the fusion of carbon and oxygen into heavier elements releases the energy to power the explosion). Nearly as fractious has been the discussion of the progenitors of these mechanisms. Understanding the progenitors of supernovae requires an understanding of stellar evolution, something we are far from doing. For “core-collapse” supernovae, SN 1987A provided our first direct glimpse of a supernova progenitor because the progenitor was observed prior to the explosion (Kirshner et al. 1987). It also profices an example of how wrong stellar evolution can be (past models insisted that only red supergiant stars would collapse to form supernovae). But for thermonuclear explosions (type Ia Supernovae), we have not been so lucky. All current type Ia SNe progenitors require mass transfer from a binary companion. To understand these progenitors, we must not only understand stellar evolution, but also binary interactions, introducing an entirely new set of uncertainties into our understanding of the type Ia progenitor.

One of the persistent progenitor scenarios of type Ia supernovae is the merger of two white dwarfs (double degenerate scenario), producing a single
white dwarf with mass above the Chandrasekhar limit that will contract and explode. The primary drawback of this scenario is that current theory argues that such a merger will not produce a type Ia supernovae. But its advantages, mostly that theory predicts that the rate of such mergers is consistent with the supernova rate (for many other proposed scenarios, this is not the case), has kept this proposed progenitor alive. To understand why this scenario is not believed to work, we must understand what happens when a white dwarf accretes matter. Nomoto and Kondo (1991) summarized this understanding in a single plot (Fig. 4 of that paper). They determined the fate of a Carbon/Oxygen (CO) white dwarf as a function of its birth mass and the rate at which we accrete material on this white dwarf. For the double degenerate scenario, the region of concern is at the topmost accretion rates. When the accretion rate is above a few times $10^{-6} \text{M}_\odot \text{s}^{-1}$, the carbon in the white dwarf ignites at its edge and burns inward, transforming the CO white dwarf into an Oxygen/Magnesium/Neon white dwarf. When such a white dwarf approaches the Chandrasekhar limit and contracts, neutrino emission cools the white dwarf sufficiently to prevent a thermonuclear runaway until the matter has contracted too much for the explosion to escape its own potential well. This material will continue to collapse until nuclear forces and neutron degeneracy halt the collapse at the formation of a neutron star. The fate of such an object is interesting (it is termed Accretion Induced Collapse “AIC” and has been used to explain a number of neutron star populations), but definitely not a type Ia supernova (Fryer et al. 1999). As we shall discuss below, all current results showing the merger of 2 white dwarfs suggest that this process is rapid and that the ultimate accretion rate onto the white dwarf will be very close to the Eddington rate. Hence, theory currently predicts that the double degenerate scenario produces AICs and not type Ia supernovae.

There are a number of caveats to this result. The Nomoto & Kondo result assumed constant accretion rates and did not account for the fact that the white dwarf could have a very complex rotation profile (Saio & Nomoto 2004; Yoon et al. 2007). This has led to small windows of opportunity for the double degenerate scenario to still produce type Ia supernovae. But to determine what the true fate of these objects is, we must simulate the merger. And we must be able to believe the results of our merger in detail.

This brings us to an important concept in the scientific method. In astronomy, we have two types of theoretical investigations: predictive and, for symmetry sake, “post-dictive”. Predictive science is what we strive for - to be able from first principles to determine how something should behave. Post-dictive is what happens with a lot of science. We know the answer we must get (e.g. explaining the solar abundance pattern) and we fit in some free parameters in our model to make sure to get this answer. Of course, we can not predict errors in the observations, and when the solar abundance pattern changes, we must then simply accept this change. Post-dictive science has many virtues: first, if we can match data with a reasonable set of parameters, we show that our basic model may also be reasonable; second, the parameters required for such a model to fit the data might teach us something about the underlying physics requirements. Unfortunately, scientists often forget that they have made parameter assumptions and start to believe they have predicted the answer. In such scenarios, post-dictive models can do more harm than good (scientists may
argue they have solved a problem prematurely, preventing continued work on a subject). Stellar evolution is rife with examples of such misuse of post-dictive models.

But truly predictive models are very difficult to do in astrophysics. Generally the problems we are interested in astrophysics are too complex to be solved with a simple analytic derivation. Once we resort to numerical models, we must be wary about numerical artifacts in our simulations. The national laboratories have focused their testing procedure on a process of Verification and Validation (V&V). Verification is the process by which scientists test to make sure their numerical models are solving correctly the physics in their code. Validation is the process by which scientists confirm that the physics in their code is the correct physics for the problem they are solving. Most of our time is spent on Verification, which can take many forms: comparison to analytic solutions, convergence studies, code comparison, and even comparison to laboratory experiments. Validation is almost entirely focused on comparison to some observation or experiment.

In this paper, we review the current status of simulations of WD mergers. Recent simulations suggest that understanding the details will require extremely accurate (∼10% in temperature) simulation results. Such accuracies will require a focused V&V effort and I will outline a basic approach for this problem. Validation requires an observational constraint similar to the problem we are solving and we will discuss the potential of hydrogen deficient stars (R Coronae Borealis stars) as a validation test for type Ia progenitors.

2. Status of Current Simulations

A great deal of work has already been done studying the merger of white dwarfs (e.g. Mochkovitch & Livio 1989, 1990; Benz et al. 1990; Segretain et al. 1997, Guerrero et al. 2004, Yoon et al. 2007). Let's focus on the work of the last two papers. The Guerrero et al. (2004) work studied a series of binary systems, with range of masses for the binary components: (0.4,0.4 M⊙), (0.4,0.6 M⊙), (0.4,1.2 M⊙), (0.6,0.8 M⊙), (0.6,1.0 M⊙), (1.0,1.2 M⊙). In all cases, the systems merged after a few orbital periods, or a few hundred seconds, corresponding to mass transfer rates of nearly 10^{-2} M⊙ s^{-1}. The white dwarf can’t incorporate this material on so such a timescale (it is limited to the Eddington accretion rate: ∼10^{-5} M⊙ yr^{-1}), so most of this material initial builds an atmosphere around the white dwarf. This material then accretes at the Eddington rate. Such high accretion rates would, using the Nomoto & Kondo analysis, ultimately collapse to form AICs.

But Saio & Nomoto (2004) and Yoon et al. (2007) have found that not all such systems need necessarily produce AICs. If the white dwarf is differentially rotating, we can expect very different results. Yoon et al. (2007) found that if the white dwarf is spinning fast enough, core contraction can be delayed until the core heats up sufficiently to ignite at low densities, driving a thermonuclear explosion and a type Ia supernova. Unfortunately, their results were not sufficiently accurate to determine exactly which fate each system would follow.

One last set of results has thrown yet another wrench into our current understanding of double degenerate mergers. Motl et al. (2002) and D’Souza
et al. (2006) found a very different result for the fate of a binary system with mass ratios similar to those used by Guerrero et al. (2004). Their work found that, instead of the catastrophic destruction of the mass-losing star, the mass transfer is stable. If the mass transfer is stable, the accretion rate would then be determined by the coalescence time from gravitational wave radiation:

$$T_{\text{merger}} = 5 \times 10^5 \left( \frac{A}{10^{10} \text{ cm}} \right)^4 \left( \frac{M_1}{M_\odot} \right) \left( \frac{M_2}{M_\odot} \right) \left( \frac{M_{\text{tot}}}{M_\odot} \right) \text{yr},$$

where $A$ is the orbital separation, $M_1$, $M_2$, $M_{\text{tot}}$ are, respectively, the masses of the primary, secondary and total binary mass of the system. For typical separations of a few times $10^9 \text{ cm}$, the mass transfer timescale is on par with the Eddington rate. If we need to solve the temperature evolution of the accreting matter to better than 10%, such differences (10 orders of magnitude - although the corresponding error in the temperature is probably less than a factor of 2) in the mass transfer rate will make a large difference.

3. What do we expect?

If we require 10% accuracies in the temperature, we need to take numerical testing to a new level. We must understand the deficiencies and strengths of each code and we must track down the differences in the simulations. D’Souza et al. (2006) use a very different code than that used in all other studies. Their code is grid, not particle, based. The disadvantage of such a technique is that grid-based codes have trouble conserving angular momentum, but this team has worked very hard to remove this issue from their simulations. An advantage of this scheme is that it can easily model low mass-transfer rates, but our preliminary SPH calculations have shown that with the SPH particle counts we can afford today (1-10 million particles), modeling low mass transfer rates becomes more tractable. Also, grid codes model shocks differently than SPH codes (performing better on shock-tube problems where the shock is along the grid). Currently, the D’Souza et al. (2006) result does not include shocks.

What do we expect the result to be from analytic (or semi-analytic) estimates? When gravitational radiation brings together the two white dwarfs in a double degenerate merger, the lower-mass white dwarf overfills its Roche lobe and accretes onto the higher-mass white dwarf. The subsequent accretion is affected by two processes. First, because the lower-mass white dwarf is supported by degeneracy pressure, it will expand as it loses mass. If the orbital separation were kept constant, the accretion process would quickly run away and the white dwarf would be completely disrupted in a few orbits. The Second effect is the fact that, if orbital angular momentum is conserved, the orbital separation will increase as the lower-mass star accretes onto its higher-mass companion. This effect would try to push the system into stable accretion. In reality, orbital angular momentum is not strictly conserved, but it is still likely that the orbit will expand during the accretion process. It is then the competition between the expansion of the white dwarf pushing toward runaway accretion and the expansion of the orbit pushing toward stable accretion that drives the evolution of the accretion.

Clayton et al. (2007) provide a more quantitative analysis of this process. To estimate the expansion of the white dwarf, they used the following formula
Figure 1. Semi-analytic estimate of the evolution of the orbital separation (solid line) and separation where the lower-mass white dwarf overfills its Roche lobe (dashed line). As the white dwarf loses mass (left to right), it expands and the orbital separation where it will overfill its Roche lobe also increases. Depending upon how much orbital angular momentum is lost in the mass transfer phase (and depending upon the relative masses of the two stars), the actual orbital separation may decrease or increase. To understand this graph, let’s study one or two possible tracks. If \( j_{\text{spin+disk}} = 0.2 \) for the 0.4,1.2 \( M_\odot \) star merger (left panel), the orbital separation will increase faster than the white dwarf expands: fate - steady mass transfer. If \( j_{\text{spin+disk}} = 0.4 \) for the 0.4,1.2 \( M_\odot \) star merger (left panel), the white dwarf will expand faster than the orbit until the mass of the white dwarf falls below 0.14 \( M_\odot \): fate - runaway accretion.

for the white dwarf radius (Nauenberg 1972):

\[
R_{\text{WD}} = 10^4(M_{\text{WD}}/0.7M_\odot)^{-1/3}(1 - M_{\text{WD}}/M_{\text{CH}})^{1/2}(\mu_e/2)^{-5/3}\text{km},
\]

(2)

where \( M_{\text{WD}} \) is the white dwarf mass, \( M_{\text{CH}} \) is the Chandrasekhar mass and \( \mu_e \) is the mean molecular weight per electron of the white dwarf. For the evolution of the orbital separation, Clayton et al. (2007) used (Podsiadlowski et al. 1992; Fryer et al. 1999):

\[
A/A_0 = (M_{\text{low-mass}}/M_{\text{low-mass}}^0)^{C_1}(M_{\text{high-mass}}/M_{\text{high-mass}}^0)^{C_2}
\]

(3)

where \( A_0, M_{\text{low-mass}}^0, M_{\text{high-mass}}^0 \) are the initial values for the orbital separation and masses of the lower and higher mass white dwarfs. The angular momentum conservation or lack thereof is including in two coefficients: \( C_1 \equiv -2 + 2j_{\text{disk+spin}} \) and \( C_2 \equiv -2 - 2j_{\text{disk+spin}} \) where \( j_{\text{disk+spin}} \) is the term for the specific angular momentum of the accreted material that is lost to either spinning up the companion or to an accretion disk (see Fryer et al. 1999 for details).

Figure 1 shows the competition between these two effects for two different binary systems: 0.4,1.2 \( M_\odot \) components and 0.6,0.9 \( M_\odot \) components. The orbital separation where the lower-mass white dwarf overfills its Roche lobe expands as it loses mass. The evolution of the orbital separation depends upon our value of
Figure 2. Orbital separation as a function of time for our two runs: co-rotating with shocks (solid), no co-rotation without shocks (dashed). The oscillations occur because the system is not in perfect circular orbits. The decrease in the orbital separation in the case of our non-corirotating case is primarily due to the fact that orbital angular momentum is converted into spin angular momentum in the stars.

\[ j_{\text{disk+spin}} \]. Of course, the actual fate of a merging system will depend upon a more exact representation of the white dwarf radius, but the primary uncertainty in the fate depends upon the determination of the value of \( j_{\text{disk+spin}} \). The differing results between D’Souza et al. (2006) and the SPH calculations all probably reside in different values for this quantity.

4. On the Path to Predictive Simulations

Code comparison has long been used effectively in astrophysics to estimate theoretical errors in a numerical solution. If we can understand the differences in the simulations, we can not only estimate the numerical errors, but we can also find ways to improve the codes and minimize these errors. Here is a first attempt at understanding these differences.

To help better explain these differences, we produce some of our own calculations using the SNSPH code (Fryer et al. 2006). The SPH algorithm in this code copies that used by Fryer et al. (1999) and has already been used on a number of binary calculations (e.g. Fryer & Heger 2005). We present 2 preliminary calculations both using the initial setup from D’Souza et al. (2006) with a mass ratio of 0.4. The first simulation uses the same polytropic equation of state from D’Souza et al. (2006). With this equation of state, shocks do not occur. In this calculation, we did not initially put the binary components in co-rotation. In the second simulation we use an ideal gas equation of state to include the
effects of shocks. In this simulation, we placed the stars in co-rotation prior to starting the simulation.

Our initial orbital separation places the binaries close enough that the lower-mass star overfills its Roche radius and is accreting onto the more massive star. Figure 2 shows the time evolution of this orbital separation. In the first simulation, the fact that the stars were not co-rotating meant that after about 10 orbits, the orbital separation decreased by nearly 2%. This star will ultimately be disrupted in less than a 20-30 orbits. This timescale is longer than most past calculations of WD mergers, but much shorter than that predicted by D’Souza et al. (2006).

Why are we getting a result that lies in between these two extremes. If we look more carefully at the angular momentum (Fig. 3), we see that it increases as a function of time. The total angular momentum in our simulations has increased by 0.02% after 10 orbits. This numerical artifact should cause the orbital separation to increase. So why does it decrease in the case of the non co-rotating run? The answer lies in the amount of orbital angular momentum converted into the spin of the angular momentum. The lower panel shows the total amount of the angular momentum in stellar spin angular momentum. In the non co-rotating case, the spin angular momentum increases with time. This angular momentum is being taken from the orbital angular momentum. Although the total angular momentum is increasing at the 0.02% level over 10 orbits, over 3% of the orbital angular momentum is being converted into spin angular momentum at the same time, causing a net loss in the orbital angular momentum and forcing the binary to coalesce. Note that in our co-rotating initial conditions, very little orbital angular momentum is converted into spin angular momentum.

So it seems that when we are careful about our initial conditions, the merger time is very much longer than the timescale predicted by all of the previous SPH calculations. This is probably because most of the past work did not worry too much about the initial conditions. But does this mean that as we remove numerical artifacts we will ultimately reach the result of D’Souza et al. (2006)? Not quite. Recall that D’Souza et al. (2006) did not include shock heating. Figure 4 shows the evolution of matter for both our simulations. In the unshocked simulation, the material is immediately incorporated into the accreting star. Since the star is co-rotating, the material incorporated into the white dwarf must have given its angular momentum back to the orbit and it is unlikely that much angular momentum is lost at all form the binary system. But in the shocked simulation, the accreting material forms an atmosphere around the entire binary system. First, this material must take angular momentum from the orbit.

We now can understand the stable accretion in the D’Souza et al. (2006) results and why that may not be the right solution either. Without shocks, the D’Souza et al. (2006) essentially guaranteed that the value for $j_{\text{disk+spin}}$ was set to 0 because it allows the accreting object to incorporate all of the accreting material. With shocks, an atmosphere forms and $j_{\text{disk+spin}}$ is definitely more than 0. The D’Souza et al. (2006) result is also not the final answer. We simply have yet to converge on the exact value for this factor, and without it, we can
Figure 3.  top: Total angular momentum in the binary system as a function of time.  bottom: angular momentum in the spin of the stars (as a fraction of total angular momentum) as a function of time. In both our simulations, numerical artifacts which we have yet to determine allow the angular momentum to increase 0.02% over about 10 orbits. But the biggest effect on the angular momentum is the conversion of orbital angular momentum to spin angular momentum in the case of our stars that are initially non co-rotating. For the co-rotating stars, the stars do not extract much angular momentum.
Figure 4. Three snapshots in time for both our “no-shocks” simulation (left column) and “shocked” simulation (right column). In the no-shock case, all of the matter accretes directly onto the accretor. In the shocked case, an atmosphere builds up around the accreting star that ultimately envelopes both stars.
not know the mass transfer rate. And we definitely can not predict accurate temperature profiles for the accreting matter or the accreting white dwarf.

5. Summary

So what can we say at this point in time? First, no calculation has yet accurately calculated the mass transfer rate in double degenerate mergers. We know that it is probably not as fast as that shown in most SPH calculations of type Ia progenitors, but it is probably faster than those results using unshocked matter. If we only had to distinguish between a mass transfer rate above or below the Nomoto & Kondo (1991) line, these simulations would be sufficient: the mass transfer rate is almost certainly above this line, possible even above the Eddington accretion rate (that means it will form a thick disk). But if we want to get accurate temperature evolution profiles, we will have to do more accurate calculations.
Figure 5 shows the accretion rates for our current simulations. Although we have much more testing to do before such quantities like accretion rate can be trusted, we can still study a number of trends that may have implications in the observations to help constrain the simulations. For example, the accretion is periodic caused by slight errors in the initial orbit. How does this alter the nuclear burning? It will affect the temperature as well as the mixing. We may be able to use observations of R Coronae Borealis stars to place limits on the level of eccentricity in nature.

Such high-precision calculations can not be done without systematic verification and validation. Here we have shown some of the tests that can be used in V&V: comparing (and understanding) the differences in code results in concert with analytic comparisons. This requires a step-by-step process, adding physics one piece at a time to understand its effect. Convergence studies will also prove useful in this problem. This all fits under the “verification” of a code. It would be nice to also conduct some validation tests - comparison to a very similar problem where the data is more plentiful. It could well be that stars like R Coronae Borealis stars provide such a validation experiment.

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