Melting of the vortex lattice in layered superconductors

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Abstract

The structure function of the vortex lattice of layered superconductor is calculated to one-loop order. Based on a phenomenological melting criterion concerning the Debye-Waller factor, we calculate the melting line of the vortex lattice, and compare our results to Monte Carlo simulation and experiment. We find that our results are quantitatively in good agreement with the Monte Carlo results. Moreover, our analytic calculation of the melting line of BSCCO fits the experiment reasonably well in a temperature range not far from $T_c$.

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I. INTRODUCTION

Magnetic fields can penetrate into the layered high-$T_c$ superconductors (LHTS) and generate the well-known vortex matter. Thermal fluctuations and the effects of disorder are able to drive the vortex matter to undergo very complicated phase transitions among glass, solid and liquid phases. This led to a burst of investigations both experimental and theoretical, to understand the physical properties of such vortex matter. One interesting aspect concerning vortex matter is to determine the phase transition line of the melting between the vortex solid state and the vortex liquid state. Several significant experiments have observed the phase transition of the melting between the vortex solid state and the vortex liquid state. In addition, magnetization jumps and specific heat spikes were observed, which indicate that the vortex lattice melting is a first-order phase transition. An often used theoretical description of the vortex lattice is the elastic theory, which is based on the lowest energy excitations on a perfect vortex lattice and can retain in fact most of the interesting physics. In the framework of elastic theory, one uses a phenomenological criterion, Lindemann criterion, to calculate the phase transition line.

Near $T_c$ vortices overlap and the elastic theory is questionable. Furthermore for vortex liquid, the elastic theory is not applicable. Another theoretical approach to study the phase transition in vortex matter is studying the thermal fluctuations of the more fundamental model, the Ginzburg-Landau (GL) model. The model can describe very well both vortex lattice and vortex liquid in the region near $T_c$.

However, the full Ginzburg-Landau (GL) theory is too complicated, and one needs to use some sort of approximations to advance the theoretical investigation. Usually, the interested phase transition is located not far away from $T_c$. Near $T_c$, it is well known that one can use the lowest Landau level (LLL) approximation. A number of researchers had studied the vortex liquid phase. For solid phase, however, due to supersoft phonon modes, the perturbation theory was questioned. The problem was resolved by Rosenstein and it was found that all infrared divergencies are canceled. Non-perturbative Gaussian variational calculation had been carried out. Spinodal line was determined and recently was confirmed by experiments. By comparing the free energy of vortex liquid with vortex solid, the melting line had been obtained. The result was used to explain the melting transition of YBCO type (not too high anisotropy). Recently it was also used to explain the melting...
transition in low $T_c$ materials.\textsuperscript{29}

While for highly anisotropic ones such as the BSCCO, the case is far more complicated. The relevant model for highly anisotropic superconductors is the Lawrence-Doniach-GL model (LDGL).\textsuperscript{30} The LDGL model has been proposed to describe LHTS with weak Josephson interlayer coupling.\textsuperscript{1,31,32,33} However the full studying of this model is not tractable. We assume that we can start from an effective LLL LDGL by integrating the higher Landau level modes and focus our studies on this model. The phase diagram of the LLL LDGL model had been investigated numerically in Ref. 34, and the melting transition line was obtained. It is highly interesting if one can obtain the melting transition of the LLL LDGL model analytically. However even for the LLL LDGL, there is not a nonperturbative calculation of vortex liquid energy, therefore we have not yet determined the melting transition directly by comparing the liquid and solid free energy.

Therefore we come back to use more phenomenological criterions to determine the melting transition. We might use the Lindemmann criterion to study the melting transition, however unfortunately we do not know how to obtain the elastic modulii of the Lawrence-Doniach model and used them to obtain the melting transition.

In this paper, instead we will use another criterion to determine the transition, the Debye-Waller factor criterion (DW criterion).\textsuperscript{35,36} Based on this criterion, we will study analytically the model and the result will be compared quantitatively with the Monte Carlo (MC) one in Ref. 34, and the model will also be used to calculate the melting transition line in BSCCO.

The Debye-Waller factor is the number of the original height of the second Bragg peak (thermal fluctuation not considered) divided by the height of the second Bragg peak with thermal fluctuation considered. Due to thermal fluctuations, the Debye-Waller factor will be reduced. If the peak height is lowered to some extent, for example, 60\%, the lattice will be melted. The applications of the DW criterion in both Yukawa system\textsuperscript{36} and three dimensional (3D) anisotropic case of the high-$T_c$ superconductors\textsuperscript{26} indicate that the criterion is quite accurate in determining the melting transition line. By using the DW criterion, we find that our calculations fit the result very well obtained by the MC simulation for studying the effective LLL LD model. The analytic calculation of the melting transition line is compared reasonable well with experiment.\textsuperscript{37} We also stress that the calculations for the structure function and the Debye-Waller factor are fairly simple and straightforward.

The paper is organized as follows. The model is described and a perturbative mean-field
solution is developed in section II. Then in section III the structure function of the vortex lattice is calculated to one-loop. A melting criterion is discussed in section IV, comparison with MC simulations and experiment was also discussed in this section.

II. MODEL, MEAN FIELD SOLUTION, AND THE PERTURBATION THEORY

A. Model

We start from the following Lawrence-Doniach free energy:

\[
F_{GL} = d_0 \sum_n \int d^2 \vec{r} \left[ \frac{\hbar^2}{2m_{ab}} (|D\psi_n|^2) + \frac{\hbar^2}{2m_c d^2} \times |\psi_{n+1} - \psi_n|^2 + a(T)|\psi_n|^2 + \frac{b'}{2} |\psi_n|^4 \right],
\]

where \(\psi_n\) is the order parameter defined on discretely labelled continuum layers, \(d_0\) is the layer thickness, \(d\) is the interlayer spacing, the covariant derivative is defined by \(D \equiv \nabla - i(2\pi/\Phi_0)\), and \(\Phi_0 \equiv (hc/2e)\). In the limit that \(d_0 = d\) and \(d\) goes to 0, the LD model reduces to the 3D anisotropic Ginzburg-Landau model. For layered superconductors far from \(H_{c1}\) (this is the range of interest in this paper), the magnetic field is homogeneous due to the overlap of the vortices. We choose the Landau gauge \(A = (By, 0, 0)\), which describes a nonfluctuating constant magnetic field directed perpendicular to the layers. For simplicity, we assume \(a(T) = -\alpha (1 - t)T_c, t \equiv T/T_c\), and other parameters are temperature independent.

For convenience, within the LLL approximation, we use the following units to rescale the model: the units of length of the “ab” plane and the “z” direction are \(\xi_{ab} = \sqrt{\hbar^2/(2m_{ab}\alpha T_c)}\) and \(\xi_c = \sqrt{\hbar^2/(2m_c\alpha T_c)}\), respectively; the unit of magnetic field is \(H_{c2}\), and the order parameter field is rescaled as \(\psi_n^2 \rightarrow (2\alpha T_c/b')\psi_n^2\). The dimensionless free energy in these units is

\[
\frac{F_{GL}}{T} = \frac{d_0}{\omega} \sum_n \int d^2 \vec{r} \left[ \frac{1}{2} |D\psi_n|^2 + \frac{1}{2d^2} |\psi_{n+1} - \psi_n|^2 \right]
- \frac{1 - t}{2} |\psi_n|^2 + \frac{1}{2} |\psi_n|^4,
\]

The dimensionless coefficient is \(\omega = \sqrt{2Gi\pi^2 t}\), where the Ginzburg number is defined by \(Gi \equiv 1/2(32\pi e^2\kappa^2T_{c}\gamma/c^2\hbar^2)^2\) and \(\gamma \equiv \sqrt{m_c/m_{ab}}\) is the anisotropy parameter.
B. Mean field solution

By minimizing $F_{GL}$ with respect to $\psi_n$, this standard variation problem leads to the well-known GL equation

$$
\mathcal{H}\psi_n + \frac{1}{2d^2}(2\psi_n - \psi_{n-1} - \psi_{n+1}) - a_h\psi_n + |\psi_n|^2\psi_n = 0,
$$

where $a_h \equiv (1 - t - b)/2$, $b \equiv B/H_c^2$, and $\mathcal{H} \equiv -(D^2 + b)/2$. If $a_h$ is sufficiently small, the GL equation can be solved perturbatively. Within the LLL approximation, one gets the mean field solution of the GL equation

$$
\psi_n = \Phi = \sqrt{\frac{a_h}{\beta_A}} \varphi(x).
$$

Where $\varphi(x)$ is the Abrikosov’s lattice solution, its definition is

$$
\varphi(x) = \sqrt{\frac{2\pi}{\sqrt{\pi}a}} \sum_{l=-\infty}^{\infty} \exp \left\{ i \left[ \frac{\pi l (l-1)}{2} + \frac{2\pi \sqrt{b}}{a} l x \right] - \frac{1}{2} \left( y \sqrt{b} - \frac{2\pi}{a} l \right)^2 \right\},
$$

and where $a = \sqrt{4\pi/\sqrt{3}}$, $\beta_A \equiv \langle |\varphi|^4 \rangle = \int_{cell} d^2x |\varphi|^4(b/2\pi) \approx 1.1596$ is the Abrikosov’s constant, the “cell” here is a primitive cell of the vortex lattice. Obviously, the mean field solution is independent of the layer index $n$.

C. Fluctuation spectrum

In order to get the excitation spectrum one expands the free energy functional around the mean field solution. The fluctuating order parameter $\psi_n$ can be written as the sum of the mean field part and a small fluctuating part

$$
\psi_n(x) = \Phi(x) + \chi_n(x).
$$

We emphasize here that the argument “$x$” in (5) stands for a 3D vector (i.e., $x = (x, x_3)$), and the bold-face font (e.g., $x$) stands for the 2D vector in the “ab” plane in this paper. The field $\chi_n$ can be expanded in a basis of quasimomentum eigenfunctions $\varphi_k$ within the
LLL approximation):

\[ \varphi_k = \sqrt{\frac{2\pi}{\sqrt{\pi a}}} \sum_{l=-\infty}^{\infty} \exp \left\{ i \left[ \frac{\pi l(l-1)}{2} + \frac{2\pi}{\sqrt{b}} \left( \frac{k_y}{\sqrt{b}} - l \right) x_k \right] - \frac{1}{2} \left( \frac{y\sqrt{b} + k_x}{\sqrt{b}} - \frac{2\pi}{a} l \right)^2 \right\}. \]

(6)

In order to do the perturbation calculation more conveniently (this can be seen later), we define \( \beta_k \) and \( \gamma_k \)

\[ \beta_k = \langle |\varphi|^2 |\varphi_k|^2 \rangle, \]
\[ \gamma_k = \langle (\varphi^*)^2 \varphi - k \varphi_k \rangle, \]  

(7)

while \( k = 0, \beta_0, \gamma_0 \) are shorted as \( \beta, \gamma \), respectively. We get

\[ \chi_n(x) = \frac{1}{\sqrt{2}} \int d^3 k e^{-ik \cdot x} \frac{d_k \varphi_k(x)}{\sqrt{2\pi}} \left( O_k + iA_k \right), \]

(8)

where \( k_1, k_2 \in [-\infty, +\infty], k_3 \in [-\pi/d, \pi/d] \), \( d_k = \exp [-i\theta_k/2] \) and \( \gamma_k = |\gamma_k| \exp [i\theta_k] \).

For simplicity, we have used in (8) the “real” field \( O_k \) and \( A_k \), which satisfy the relations: \( O_k^* = O_{-k}, A_k^* = A_{-k} \). Within the LLL approximation, at order \( a_h \), the eigenstates are \( O_k, A_k \). We find that it is convenient for us to get the eigenvalues by using \( d_k \) in the expansion

\[ \epsilon_O = \tilde{\epsilon}_O + \frac{1}{d^2} (1 - \cos k_3 d) \]

\[ \epsilon_A = \tilde{\epsilon}_A + \frac{1}{d^2} (1 - \cos k_3 d) \]

of \( \chi_n(x) \). The eigenvalues are

\[ \epsilon_O = \tilde{\epsilon}_O + \frac{1}{d^2} (1 - \cos k_3 d) \]

(9)

In particular,

\[ \epsilon_A = \tilde{\epsilon}_A + \frac{1}{d^2} (1 - \cos k_3 d) \]

\[ \epsilon_A = a_h \left( -1 + \frac{2}{\beta} \beta_k + \frac{1}{\beta} |\gamma_k| \right) + \frac{1}{d^2} (1 - \cos k_3 d), \]

\[ \epsilon_A = a_h \left( -1 + \frac{2}{\beta} \beta_k - \frac{1}{\beta} |\gamma_k| \right) + \frac{1}{d^2} (1 - \cos k_3 d). \]

when \( k \to 0, \tilde{\epsilon}_A \approx 0.1 a_h |k|^4 \), while \( \tilde{\epsilon}_O \) has a finite gap.

III. STRUCTURE FUNCTION OF THE VORTEX LATTICE

In this section we calculate the structure function to order \( \omega \) within the LLL approximation, i.e., neglecting higher \( a_h \) correlations. Firstly, we calculate the density-density
correlation function defined by

\[ \tilde{S}(z, z_3) = \langle \rho(x, x_3) \rho(x + z, x_3 + z_3) \rangle_x, \]  

(10)

where \( \rho(x) \equiv |\psi(x)|^2 \), and the subscript \( x \) here indicates the average over the unit cell. The correlation function is calculated using the well-known Wick expansion:

\[ \tilde{S}(z, z_3) = \tilde{S}_{mf} + \omega \tilde{S}_{fluct}, \]  

(11)

where the first term is the mean field part, while the second term is the correction due to thermal fluctuations.

A. Mean field contribution

The mean field part is

\[ \tilde{S}_{mf} = \langle |\Phi(x)|^2 |\Phi(x + z)|^2 \rangle_x. \]  

(12)

The structure function is the fourier transform \( S(q, 0) = \int dz \exp[iq \cdot z] \tilde{S}(z, z_3 = 0) \), hence, the mean field part of the structure function is

\[ S_{mf}(q, 0) = \int dz \exp[iq \cdot z] \langle |\Phi(x)|^2 |\Phi(x + z)|^2 \rangle_x \]

\[ = \left( \frac{ah}{\beta_A} \right)^2 \frac{b}{2\pi} \int dy e^{iqy} |\varphi(y)|^2 \]

\[ \times \int_{cell} dxe^{-iqx} |\varphi(x)|^2 \]

\[ = \left( \frac{ah}{\beta_A} \right)^2 4\pi^2 \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right]. \]

(13)

In order to derive Eq.(13), we have used the following relation:

\[ \int_A d\mathbf{x} \varphi(\mathbf{x}) \varphi^*_k(\mathbf{x}) \exp[-i\mathbf{x} \cdot \mathbf{q}] \]

\[ = 4\pi^2 \delta_n(\mathbf{q} - \mathbf{k}) \exp \left[ \frac{\pi i}{2} (n_1^2 - n_1) \right] \]

\[ \times \exp \left[ -\frac{\mathbf{q}^2}{4b} - \frac{id_1 q_y}{2b} + \frac{ik_2 q_y}{b} \right], \]

(14)

where \( A \) is the sample area, and where we have used the notation: \( \delta_n(\mathbf{q}) \equiv \sum_{n_1, n_2} \delta(\mathbf{q} - n_1 \mathbf{d}_1 - n_2 \mathbf{d}_2) \), \( n_1 = (1/2\pi)\mathbf{d}_1 \cdot \mathbf{q} \), \( n_2 = (1/2\pi)\mathbf{d}_2 \cdot \mathbf{q} \), \( \mathbf{d}_1 \), \( \mathbf{d}_2 \) are the reciprocal lattice basis
vectors
\[ \tilde{d}_1 = \frac{2\pi \sqrt{b}}{a} \left(1, -\frac{1}{\sqrt{3}}\right); \quad \tilde{d}_2 = \left(0, \frac{4\pi \sqrt{b}}{a \sqrt{3}}\right), \]
which are dual to the lattice basis vectors
\[ d_1 = \left(a/\sqrt{b}, 0\right); \quad d_2 = \left(a/2\sqrt{b}, a\sqrt{3}/2\sqrt{b}\right). \]

B. Fluctuation contribution

We calculate the fluctuation contribution of the structure function \( S_{\text{corr.}} \) to one loop. For convenience, the results are divided into four parts:
\[ S_{\text{corr.}}(q, 0) = S_1(q, 0) + S_2(q, 0) + S_3(q, 0) + S_4(q, 0), \]
where \( S_1(q, 0) \) is the fourier transform of \( \langle \Phi(x)\Phi(x + z)\chi_n^*(x)\chi_n(x + z) + c.c. \rangle_x \), \( S_2(q, 0) \) is the fourier transform of \( \langle \Phi(x)\Phi^*(x + z)\chi_n^*(x)\chi_n(x + z) + c.c. \rangle_x \), \( S_3(q, 0) \) is the fourier transform of \( \langle \Phi(x)^2\chi_n(x + z)^2 + |\Phi(x + z)|^2\chi_n(x)^2 \rangle_x \), and \( S_4(q, 0) \) is the fourier transform of \( \frac{2a_B}{n_B} \langle |\varphi(x)|^2|\varphi(y)|^2 \rangle_x (\nu_1^2) \). We emphasize that the final term is due to the vacuum renormalization, which cause the shift \( \nu \) in \( \psi_n(x) = \nu \varphi_n(x) + \chi_n(x) \) be renormalized. To one-loop order, let \( \nu^2 = \nu_0^2 + \omega \nu_1^2 \), here \( \nu_0^2 = a_h/\beta_A \), and \( \nu_1^2 \) is given by minimizing the effective one-loop free energy
\[-\ln Z = \frac{L_x L_y L_z}{\omega} \left(-a_h \nu^2 + \frac{1}{2} \nu^4 \beta_A \right) + \frac{1}{2} \text{Tr} \ln \left[ \frac{1 - \cos k_3 d}{d^2} \right] + \frac{1}{2} \text{Tr} \ln \left[ \frac{1 - \cos k_3 d}{d^2} \right] \]
where \( L_x, L_y, L_z \) are the scales of the sample. From Eq.(18), we get
\[ \nu_1^2 = \frac{-\sqrt{2}}{16\pi^2 \beta_A} \int_k \left[ \frac{2\beta_k + |\gamma_k|}{\sqrt{\tilde{\epsilon}_O + \frac{d^2 \tilde{\epsilon}_O^2}{2}} + \frac{2\beta_k - |\gamma_k|}{\sqrt{\tilde{\epsilon}_A + \frac{d^2 \tilde{\epsilon}_A^2}{2}}} \right] \]
Each term of the r.h.s of (17) is given as follows:
\[ S_1(q, 0) = \frac{\omega a_B}{2\beta_A} \cos \left[ k_x q_y + \frac{(k \times Q)_z}{b} + \theta_k \right] \exp \left[ -\frac{q^2}{2b} \right] \left[ \sqrt{\frac{2}{\tilde{\epsilon}_O + \frac{d^2 \tilde{\epsilon}_O^2}{2}}} - \sqrt{\frac{2}{\tilde{\epsilon}_A + \frac{d^2 \tilde{\epsilon}_A^2}{2}}} \right] \]
\[ S_2(q, 0) = \frac{\omega a_B}{2\beta_A} \exp \left[ -\frac{q^2}{2b} \right] \left[ \sqrt{\frac{2}{\tilde{\epsilon}_O + \frac{d^2 \tilde{\epsilon}_O^2}{2}}} + \sqrt{\frac{2}{\tilde{\epsilon}_A + \frac{d^2 \tilde{\epsilon}_A^2}{2}}} \right] \]
where \( q = k + Q \), \( k \) is the fractional part of \( q \), while \( Q \) is the integer part of \( q \). After combining the mean field part and the one loop correction part of the structure function, we get

\[
S(q, 0) = S_{mf} + S_{corr.} = \left( \frac{a_h}{\beta_A} \right)^2 4\pi^2 \delta_n(q) \exp \left[ -\frac{q^2}{2b} \right] + \omega a_h \exp \left[ -\frac{q^2}{2b} \right] \left[ f_1(q) + \delta_n(q)(f_2(Q) + f_3) \right]
\]

where \( f_1(q) \), \( f_2(Q) \), \( f_3 \) are given as follows:

\[
f_1(q) = \left[ 1 + \cos \left( \frac{k_x k_y + (k \times Q)_z + \theta_k}{b} \right) \right] \sqrt{\frac{2}{\epsilon_O(k) + \frac{d^2 \epsilon}{2} \epsilon_O(k)^2}}
\]

\[
+ \left[ 1 - \cos \left( \frac{k_x k_y + (k \times Q)_z + \theta_k}{b} \right) \right] \sqrt{\frac{2}{\epsilon_A(k) + \frac{d^2 \epsilon}{2} \epsilon_A(k)^2}}
\]

\[
f_2(Q) = \int_k \left[ -1 + \cos \left( \frac{(k \times Q)_z}{b} \right) \right] \left[ \sqrt{\frac{2}{\epsilon_O(k) + \frac{d^2 \epsilon}{2} \epsilon_O(k)^2}} + \sqrt{\frac{2}{\epsilon_A(k) + \frac{d^2 \epsilon}{2} \epsilon_A(k)^2}} \right]
\]

\[
f_3 = -\frac{1}{a_h} \int_k \left[ \sqrt{\frac{2 \epsilon_O(k)}{1 + \frac{d^2 \epsilon}{2} \epsilon_O(k)}} + \sqrt{\frac{2 \epsilon_A(k)}{1 + \frac{d^2 \epsilon}{2} \epsilon_A(k)}} \right].
\]

It is very interesting to notice that each of the four terms \( S_i(i = 1, \ldots, 4) \) is divergent, respectively, as \( k \to 0 \), however, the sums \( S_1, S_2 \) and \( S_3, S_4 \) are not. Here we just take the sum \( S_1 + S_2 \) as an example:

\[
S_1(q, 0) + S_2(q, 0) = \frac{\omega a_h}{2\beta_A} \exp \left[ -\frac{q^2}{2b} \right] f_1(q).
\]

In order to see it more clearly, we use \( \sqrt{b} \) to rescale the momentum. As it can be shown that \( k_x k_y + \theta_k = O(k^4) \) when \( k \to 0 \), the function \( (k_x k_y + (k \times Q)_z + \theta_k) \to (k \times Q)_z \), and \( 1 - \cos(k_x k_y + (k \times Q)_z + \theta_k) \to (k \times Q)_z^2 \), hence it will cancel the \( 1/k^2 \) singularity of \( \sqrt{2/(\epsilon_A + d^2 \epsilon_A/2)} \). It is also easy to show that \( S_3 + S_4 \) is the case. Consequently, we get a not divergent result. The fluctuation correction of the structure function (for non-peak region) is shown in Fig.1.
IV. MELTING OF THE VORTEX LATTICE

A. A melting criterion

The above calculations also indicate that thermal fluctuations will reduce the intensity of the Bragg peak of the structure function. In fact, the Debye-Waller factor has been used to describe the melting of the lattice system. If the intensity of the peak is lowered to some extent, for example, 60%, the lattice will be melted. In 3D case, we know the exact melting transition temperature via a different method. With this temperature, we find that the one loop calculation of the Debye-Waller factor is reduced to 50%. This does not mean that the criterion “60%” is wrong as the higher order correction to the one loop calculation usually will increase this value to some number above 50% (it is too complicated to calculate the structure function to two loop and we will leave it to future studies). In this paper, we also calculate the structure function to one loop order. Thus we will use the "one loop criterion" that the Debye-Waller factor calculated to one loop is reduced to 50% at the melting transition line.

According to the definition of the Debye-Waller factor, we denote the ratio of the one-loop value to the mean field value of the intensity of the second Bragg peak by $\rho$, we get

$$\rho = \frac{\left[f_2(Q_1) + f_3\right] \omega/2 + 4\pi^2a_h/\beta_A}{4\pi^2a_h/\beta_A},$$

(29)

where $Q_1$ denotes the shortest reciprocal vector of the triangular vortex lattice. We define the critical value of $\rho$ corresponding to melting by $\rho_c$. According to the above discussion, the one loop criterion corresponding to $\rho_c$ is about 50%.

B. Comparison with MC simulations

Now we compare our results with MC simulations of the LLL layered system in Ref. 34. In Ref. 34, the authors use two dimensionless parameters $g$ and $\eta$ to describe the melting transition of the system. The $g$ and $\eta$ measure the intralayer and interlayer coupling, respectively, and they are equivalent to the $t$ and $b$ in this paper (in fact, they are the functions of $t$ and $b$). In order to carry out the comparison, first, we make our notations consistent with the ones of Ref. 34.

$$\alpha_B \equiv a(T) + \frac{\hbar eB}{m_{ab}c} = -2\alpha_T a_h,$$

(30)
\[ g \equiv \alpha B \sqrt{\frac{\pi hcd_0 \xi_c}{2\beta k_B T e B}} = -a_h \sqrt{\frac{\pi d_0}{b\omega}}, \]  \hspace{1cm} (31)

\[ \eta \equiv \frac{\hbar}{2mc(d\xi_c)^2|\alpha_B|} = \frac{1}{2d^2 a_h}. \] \hspace{1cm} (32)

Furthermore, from Eq.(26), (27), it is easy to see that \( f_2(Q_1) + f_3 \) is only dependent on \( d^2a_h \), we define \( f(d^2a_h) = f_2(Q_1) + f_3 \), then, we have

\[ f(d^2a_h) = (\rho - 1) \frac{8\pi^2 a_h^2 d_0}{\beta_A b\omega d a_h^{1/2}}. \] \hspace{1cm} (33)

After combining Eq.(31),(32),(33) together, we get

\[ \frac{1}{\sqrt{2\eta}} f\left(\frac{1}{2\eta}\right) = (\rho - 1) \frac{8\pi g^2}{\beta_A}. \] \hspace{1cm} (34)

According to the knowledge from 3D anisotropic model, we choose \( \rho_c = 0.475 \), hence, Eq.(34) gives out the relationship between \( g \) and \( \eta \).

In Ref. 34, the MC simulation was employed to determine the melting transition. On the melting transition the value of \( g \) is a function on \( \eta \). We denote the function \( g \) in Ref. 34 by \( g_{HM} \), and the \( g \) determined by Eq.(34) is denoted by \( g \). The result of the comparison is given in Table I. For typical LHTS such as BSCCO, \( \xi_{ab} \) is about 25Å, \( \gamma \) is about 200, \( H_{c2} \) is about 50T, \( T_c \) is about 90K, and the interlayer spacing \( d_0 \) is about 4Å. For temperature and magnetic field at 75K and 400G respectively, the value of \( \eta \) is about 0.01 (actually \( \eta \) on the all points on the theoretical curve on fig.2 is less than 0.1). when \( \eta \) increases, \( |g| \) is gradually larger than \( |g_{HM}| \). As discussed in Ref. 34 that the finite size effects of MC simulation become stronger as \( \eta \) increased, and the finite size effects lead the \( |g_{HM}| \) to be less than its actual value. In summary, we find the two results fit very well for not too big \( \eta \) (less than 0.1 \)). This also demonstrates that the DW criterion works well.

C. Comparison with experiment

In Ref. 37, the material parameters describe BSCCO: \( \kappa = 100, \gamma = 270, T_c = 86K, d = 15Å \). The interlayer spacing \( d_0 \) and \( H_{c2} \) have not given, we find \( d_0 = 4.1Å, H_{c2} = 50T \) give the best fit to the experimental data, the results is shown in Fig.2. The deviation become large as \( T \) reduces, this is expected as the effects of disorder will be enhanced as temperature lowed. The effects of disorder tend to lower the curve. However, the effects of thermal
fluctuations dominated in the region of our interest (near \( T_c \)). The comparison indicate that the effective LLL LD model is quite good to describe the melting phase transition of the LHTS near \( T_c \). In future work we will include the disorder effect, and we expect that the result can be extended to the region with lower temperature.

V. CONCLUSIONS

To conclude, we have calculated the structure function of layered superconductors and the melting line can be obtained quantitatively by "one-loop" DW criterion, i.e. the ratio of the one-loop value of the intensity of the second Bragg peak of the structure function to the mean-field value is about 50%, the solid melts. With this criterion, we calculate the melting line and compare the results with existing MC results and experiments. Our results fit the MC results very well. Moreover, our results fit the experimental data reasonably well in the range not far from \( T_c \) (for BSCCO, \( T_c = 86K \), the range we fitted is from 72K to 86K).

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1 G. Blatter, M. V. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994), and references therein.
2 T. Nattermann and S. Scheidl, Adv. Phys. 49, 607 (2000), and references therein.
3 T. Giamarchi and S. Bhattacharya, in High Magnetic Fields, edited by C. Berthier, L. P. Levy, and G. Martinez, (Springer-Verlag, Berlin, 2002), p.314 [cond-mat/0111052] and references therein.
4 D. Li, B. Rosenstein, and V. Vinokur, Journal of Superconductivity and Novel Magnetism 19, 369 (2006), and references therein.
5 D. R. Nelson, Phys. Rev. Lett. 60, 1973 (1988).
6 J. Kierfeld and V. Vinokur, Phys. Rev. B 61, R14928 (2000).
7 E. Zeldov, D. Majer, M. Konczykowski, V. B. Geshkenbein, V. M. Vinokur, and H. Shtrikman, Nature(London) 375, 373 (1995).
8 R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. Lett. 76, 835 (1996).
9 M. Willemin, A. Schilling, H. Keller, C. Rossel, J. Hofer, U. Welp, W. K. Kwok, R. J. Olsson, and G. W. Crabtree, Phys. Rev. Lett. 81, 4236 (1998).
10 N. Kobayashi, T. Nishizaki, K. Shibata, T. Sato, M. Maki, and T. Sasaki, Physica C 362, 121 (2001).
11 A. Schilling, R. A. Fisher, N. E. Phillips, U. Welp, D. Dasgupta, W. K. Kwok, and G. W. Crabtree, Nature (London) 382, 791 (1996).
12 M. Roulin, A. Junod, and E. Walker, Science 273, 1210 (1996).
13 F. Bouquet, C. Marcenat, E. Steep, R. Calemczuk, W. K. Kwok, U. Welp, G. W. Crabtree, R. A. Fisher, N. E. Phillips, and A. Schilling, Nature (London) 411, 448 (2001).
14 E. H. Brandt, Phys. Rev. Lett. 63, 1106 (1989).
15 A. Houghton, R. A. Pelcovits and A. Sudbo, Phys. Rev. B 40, 6763 (1989).
16 T. Giamarchi and P. Le Doussal, Phys. Rev. B 52, 1242 (1995).
17 F. Lindemann, Phys. Z 11, 69 (1910).
18 G. P. Mikitik and E. H. Brandt, Phys. Rev. B 64, 184514 (2001).
19 G. P. Mikitik and E. H. Brandt, Phys. Rev. B 68, 054509 (2003).
20 M. Tinkham, Introduction to Superconductivity (McGraw-Hill, New York, 1996).
21 A. A. Abrikosov, Zh. Eksp. Teor. Fiz. 32, 1442 (1957)[Sov. Phys. JETP 50, 1174 (1957)].
22 G. J. Ruggeri and D. J. Thouless, J. Phys. F 6, 2063 (1976).
23 Z. Tesanovic, L. Xing, L. Bulaevskii, Q. Li, and M. Suenaga, Phys. Rev. Lett. 69, 3563 (1992); Z. Tesanovic and A.V. Andreev, Phys. Rev. B 49, 4064 (1994).

24 E. Brezin, D.R. Nelson, and A. Thiaville, Phys. Rev. B 31, 7124 (1985); E. Brezin, S. Hikami, and A. I. Larkin, Phys. Rev. B 60, 3589 (1999).

25 B. Rosenstein, Phys. Rev. B 60, 4268 (1999).

26 D. Li and B. Rosenstein, Phys. Rev. Lett. 86, 3618 (2001); Phys. Rev. Lett. 90, 167004 (2003); Phys. Rev. B 65, 220504(R) (2002); Phys. Rev. B 70, 144521 (2004).

27 Z. L. Xiao, O. Dogru, E. Y. Andrei, P. Shuk, and M. Greenblatt, Phys. Rev. Lett. 92, 227004 (2004).

28 A. D. Thakur, S. S. Banerjee, M. J. Higgins, S. Ramakrishnan, and A. K. Grover, Phys. Rev. B 72, 134524 (2005).

29 N. Kokubo, K. Kadowaki, and K. Takita, Phys. Rev. Lett. 95, 177005 (2005); N. Kokubo, T. Asada, K. Kadowaki, K. Takita, T. G. Sorop, and P. H. Kes, Phys. Rev. B 75, 184512 (2007).

30 W.E. Lawrence and S. Doniach, in Proceeding of the Twelfth International Conference on Low Temperature Physics, Kyoto, Japan, 1971, edited by E. Kanda (Keigaku, Tokyo, 1971), p.316.

31 D. Feinberg, S. Theodorakis, and A. M. Ettouhami, Phys. Rev. B 49, 6285 (1994).

32 L. I. Glazman, A. E. Koshelev, Phys. Rev. B 43, 2835 (1991).

33 A. Zamora, Phys. Rev. B 69, 054506 (2004).

34 J. Hu, A. H. MacDonald, Phys. Rev. B 56, 2788 (1997).

35 N. W. Ashcroft and N. D. Mermin, Solid State Physics (Saunders, Orlando, 1976)

36 M. Stevens and M. Robbins, J. Chem. Phys. 98, 2319 (1993).

37 H. Beidenkopf, T. Verdiene, Y. Myasoedov, H. Shtrikman, E. Zeldov, B. Rosenstein, D. Li, and T. Tamegai, Phys. Rev. Lett. 98, 167004 (2007).

38 D. Li and B. Rosenstein, Phys. Rev. B 60, 10460 (1999); Phys. Rev. B 60, 9704 (1999).

39 J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (Clarendon, Oxford, 2002)
Figure Captions

**Figure 1** Fluctuation correlation to the structure function of the Abrikosov vortex lattice. The peaks at reciprocal lattice points are removed, only the correction to the non-peak region is plotted (i.e. only $f_1(q)$ is plotted.

**Figure 2**
Comparison of the theoretical melting curve(line) of highly over doped BSCCO with experimental results.
