Universal quantum gates on microwave photons assisted by circuit quantum electrodynamics

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Based on a microwave-photon quantum processor with multiple superconducting resonators coupled to one three-level superconducting qutrit, we construct the controlled-phase (c-phase) and controlled-controlled-phase (cc-phase) gates on microwave-photon-resonator qudits, by combination of the photon-number-dependent frequency-shift effect and the resonant operation between the qutrit and a resonator. This distinct feature provides us a useful way for achieving higher fidelity quantum logic gates on resonator qudits in a shorter operation time, compared with others. The fidelity of our c-phase gate can reach 99.51% within 92 ns. The fidelity of our cc-phase gate on three resonator qudits constructed here in the first time, can reach 92.92% within 124.64 ns.

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I. INTRODUCTION

Quantum communication and quantum computation [1, 2] attracted much attention in recent years. Many important schemes were proposed for quantum information processing and quantum computing by using different quantum systems, such as photonic systems [3–8], nuclear magnetic resonance [9–11], quantum dots [12–15], diamond nitrogen-vacancy (NV) centers [16–20], circuit quantum electrodynamics (QED) [26–31], and so on. Universal quantum gates are very important in quantum computing and quantum communication. They are the key elements in constructing a universal quantum computer. Moreover, they can be used to produce the entanglement of multipartite quantum systems. The controlled phase (c-phase) gate is one of the important universal two-qubit gates. It has the same role as the controlled-not gate in quantum computing. The controlled-controlled-phase (cc-phase) gate is an important three-qubit gate which can play the same role as the three-qubit Toffoli gate which can be used to construct a universal quantum computing with Hadamard operations.

Circuit QED, which combines superconducting circuits and cavity QED, provides a good platform for quantum information processing [26–31]. A superconducting Josephson junction can act as a perfect qubit and it has some good features, such as the large scale integration [32], a relatively long coherence time about 0.1 ms [33], the versatility in its energy-level structure with ξ, Λ, V, and even Λ types [34] which cannot be found in atom systems, the tunable coupling superconducting qubit [35–37], and so on. All these characters attract much attention focused on quantum information processing on superconducting qubits in circuit QED. Some interesting proposals for quantum information processing on qubits are presented, such as the fast reset of a superconducting qubit [38], universal quantum gates and entanglement generation [39], single-shot individual qubit measurement and the joint qubit readout [40], fast quantum entangling operation on superconducting qubits [31], and so on.

A superconducting coplanar resonator whose quality factors Q can be increased to be 10⁶ and even 10¹² [41], can be used as a qudit as it contains some microwave photons whose life time is much longer than that of a superconducting qubit [42]. The coupling strength between a resonator and a transmission line is tunable [43]. Moreover, the strong and even ultra-strong coupling [44] in circuit QED affords a strong nonlinear interaction between a superconducting qubit and a microwave-photon qudit. These good features make resonators act as a powerful platform for quantum information processing as well. There are many studies on resonator qudits, such as, resolving photon number states in a superconducting circuit [27], preparing individual Fock states and their superpositions [45, 46], realizing quantum non-demolition detection of single microwave photons [47], entangling resonator qudits [48–53], constructing coupled gates [54] on two resonator qudits, and so on. Aiming at constructing high-fidelity multi-qubit entangling gates on resonators for quantum information processing, we focus on the number-state-dependent interaction between a superconducting qubit and resonator qudits proposed by Schuster et al. [27], which provides an interesting way for achieving the state-selective qubit rotation. This rotation has been used for generating the quantum entanglement [55] and constructing the c-phase gate on two resonator qudits [56].

In this paper, we propose a microwave-photon quantum processor with several tunable resonators [57] coupled to a tunable superconducting qutrit, by combination of the number-state-dependent interaction between a superconducting qutrit and a resonator-qudit subsystem and the resonant operation between the qutrit and another resonator-qudit subsystem, different from those in Refs. [54, 56]. We construct the c-phase gate on two resonator qudits, in which the transition frequency of the superconducting qutrit depends on the photon number in one resonator and it resonates with the other resonator, different from the gate by Wu et al. [56] in which the transition frequency of the superconducting qubit relies on the photon numbers of the two resonators. This distinct feature provides us a useful way for achieving a higher fidelity c-phase gate on two resonator qudits within a shorter operation time, compared with others. The fidelity of our c-phase gate can reach 99.51% within the operation time 92 ns. Moreover, we construct the cc-phase gate on three-resonator qudits in the first time, by using a resonator to complete the resonant

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operation and the other two resonators to achieve the number-state-dependent interaction on the superconducting qutrit. Its fidelity can reach 92.92% within the operation time 124.64 ns.

In the dispersive strong regime ($\frac{\omega^d}{\Delta} \ll 1$), the photon-number-dependent transition frequency of the qubit is too small to distinguish the different transition frequencies of the qubit due to the different photon numbers in the resonator. In the quasi-dispersive regime ($\frac{\omega^d}{\Delta} < \frac{\omega^p}{\Delta} < 1$), the transition frequency of the qubit depends on the photon number largely, shown in Eq. (3). That is, if we apply a drive field with the frequency equivalent to the transition frequency of the qubit when $n = 1$, and take a proper amplitude $|\Omega| \ll \frac{\omega^p}{\Delta}$ to suppress the error generated by off-resonant transitions sufficiently, the field will flip the qubit only if there is one microwave photon in the resonator. On the other hand, if we apply a drive field with the frequency equivalent to the transition frequency of the qubit when $n = 0$, and take a proper amplitude, the field will flip the qubit only if there is no microwave photon in the resonator. To describe this effect, we consider a system with the resonator $r_1$ coupled to a $\Xi$-type three-level superconducting qutrit, whose Hamiltonian is (in the rotating-wave approximation)

$$H_2 = \sum_{l=g,e,a} E_l |l\rangle_\xi \langle l| + \omega^f a_1^+ a_1 + g^{e}_r(a_1^+ \sigma^-_{g,e} + a_1 \sigma^+_{g,e}) + g^{e}_r(a_1^+ \sigma^-_{e,a} + a_1 \sigma^+_{e,a}),$$

where $|g\rangle_q$, $|e\rangle_q$ and $|a\rangle_q$ are the first three lower energy levels of the qutrit. $\sigma^{e,g}_{e,a}$ and $\sigma^{e,g}_{g,e}$ are the creation operators for the transitions $|g\rangle_q \rightarrow |e\rangle_q$ and $|e\rangle_q \rightarrow |a\rangle_q$ of the qutrit $q$, respectively. $a_1^\dagger$ is the creation operator of the resonator $r_1$. The energy for the level $l$ of $q$ is $E_l$, and $\omega^f$ is the transition frequency of $r_1$. $g^{e}_r$ and $g^{a}_r$ are the coupling strengths between these two transitions of $q$ and $r_1$.

A microwave drive field $H_d = \Omega (|e\rangle_q \langle e| e^{-i\omega_d t} + |g\rangle_q \langle e| e^{i\omega_d t})$ with a proper amplitude $\Omega$ is applied to interact with the qutrit, and here the frequency $\omega_d$ is chosen to be equivalent to the transition frequency $(|e\rangle_q \leftrightarrow |a\rangle_q)$ of the qutrit $q$ when there is no microwave photon in the resonator. Due to the realistic quantum Rabi oscillation (ROT) occurring between the dress states of the system [59], we simulate the expectation value of $\text{ROT}_{1}^{e,a}$, $(|0\rangle_{r_1} |e\rangle_q)_{dress} \leftrightarrow (|0\rangle_{r_1} |a\rangle_q)_{dress}$ and $\text{ROT}_{1}^{g,e}$, $(|1\rangle_{r_1} |g\rangle_q)_{dress} \leftrightarrow (|1\rangle_{r_1} |e\rangle_q)_{dress}$, shown in Fig. 2. The transition frequencies of $|g\rangle_q \leftrightarrow |e\rangle_q$ and $|e\rangle_q \leftrightarrow |a\rangle_q$ of the qutrit are chosen to be $\omega^{g,e}/(2\pi) = E_e - E_g = 8.7GHz$ and $\omega^{e,a}/(2\pi) = E_a - E_e = 8.0GHz$, respectively. $\omega^f/(2\pi) = 7.5GHz$. The coupling strengths between two transitions of the qutrit and $r_1$ are taken in convenience with $g^{e}_r/(2\pi) = g^{a}_r/(2\pi) = 0.2GHz$. The frequency and amplitude of the drive field are $\omega^f/(2\pi) = 8.043GHz$ and $\Omega = 0.0115GHz$, respectively.

As shown in Fig. 2 the maximal probability of $\text{ROT}_{1}^{e,a}$ can reach 100%. After a period of $\text{ROT}_{1}^{e,a}$, a π phase shift can be generated on the state $(|0\rangle_{r_1} |e\rangle_q)_{dress}$, and $\text{ROT}_{1}^{g,e}$ and the other oscillations take place with a very small probability, which indicate that the final state of the system composed of $r_1$ and $q$ becomes

$$|\phi_f\rangle = \frac{1}{2}(\langle 0|0\rangle_1 |g\rangle_q)_{dress} - (\langle 0|1\rangle_1 |e\rangle_q)_{dress} + (\langle 1|1\rangle_1 |e\rangle_q)_{dress} + (\langle 1|0\rangle_1 |g\rangle_q)_{dress}$$

(5)

after the state-selective qubit rotation if the initial state of the
Suppose that the initial state of the system is
$$|\psi_0\rangle = \frac{1}{2}(|00\rangle_1 |01\rangle_2 + |01\rangle_1 |10\rangle_2 + |10\rangle_1 |01\rangle_2) \otimes |g\rangle_q.$$
(9)

Our c-phase gate on the two resonators can be accomplished with three steps as follows.

First, by resonating $r_2$ and $q$ with $g_2^{e\pi}t = \frac{\pi}{2}$, and turning off the interaction between $r_1$ and $q$, the system evolves from the initial state $|\psi_0\rangle$ to the state
$$|\psi_1\rangle = \frac{1}{2}(|00\rangle_1 |g\rangle_q - i|01\rangle_1 |e\rangle_q + |10\rangle_1 |g\rangle_q - i|11\rangle_1 |e\rangle_q) \otimes |0\rangle_2.$$
(10)

Second, by turning on the coupling between $r_1$ and $q$, and turning off the coupling between $r_2$ and $q$, the state of the system becomes
$$|\psi_2\rangle = \frac{1}{2}[(|00\rangle_1 |g\rangle_q |dress\rangle - i|01\rangle_1 |e\rangle_q |dress\rangle + (|11\rangle_1 |g\rangle_q |dress\rangle - i|11\rangle_1 |e\rangle_q |dress\rangle) \otimes |0\rangle_2].$$
(11)

By applying a drive field $\frac{1}{2}(|\Omega |g\rangle_q (g\rightarrow e) + \Omega^* |g\rangle_q (e\rightarrow g))$ with the frequency equivalent to the transition frequency ($|e\rangle_q \leftrightarrow |g\rangle_q$) of the qutrit when there is no microwave photon in the resonator $r_1$, after an operation time of $\Omega t = \frac{\pi}{2}$, the state of the system is changed to be
$$|\psi_3\rangle = \frac{1}{2}(|00\rangle_1 |g\rangle_q |dress\rangle - i|01\rangle_1 |e\rangle_q |dress\rangle + (|11\rangle_1 |g\rangle_q |dress\rangle - i|11\rangle_1 |e\rangle_q |dress\rangle) \otimes |0\rangle_2.$$
(12)

Third, by turning off the coupling between $r_1$ and $q$, the state of the system evolves from $|\psi_2\rangle$ into
$$|\psi_4\rangle = \frac{1}{2}(|00\rangle_1 |g\rangle_q + i|01\rangle_1 |e\rangle_q + |11\rangle_1 |g\rangle_q - i|11\rangle_1 |e\rangle_q) \otimes |0\rangle_2.$$
(13)

By resonating $r_2$ and $q$ with $g_2^{e\pi}t = \frac{\pi}{2}$ again, and turning off the interaction between $r_1$ and $q$, the state of the system becomes
$$|\psi_q\rangle = \frac{1}{2}(|00\rangle_1 |00\rangle_2 + |01\rangle_1 |10\rangle_2 + |10\rangle_1 |01\rangle_2) \otimes |g\rangle_q.$$
(14)

This is just the result of the c-phase gate on $r_1$ and $r_2$ by using $r_1$ as the control qubit and $r_2$ as the target qubit. The reduced density operators of the two-resonator system in the initial state Eq. (9) and the final state Eq. (14) are shown in Fig. 4. One can see that the fidelity of our c-phase gate on two microwave-photon qubits is about 99.51% within about 93 ns. Here the fidelity is defined as $F = Tr(\sqrt{\rho_f\sqrt{\rho_{ideal}}} \sqrt{\rho_f})$.

III. CONTROLLED-CONTROLLED-PHASE GATE ON THREE RESONATORS

The principle of our cc-phase gate on a three-resonator system is shown in Fig. 5. Here the resonator $r_1$ has the same role.
FIG. 3: (Color online) (a) The density operator ($\rho_0$) of the initial state $|\psi_0\rangle$ of the system composed of the two resonators and the qutrit in our c-phase gate. (b) and (c) are the real part (Real$[\rho_1]$) and the imaginary part (Imag$[\rho_1]$) of the final state $|\psi_1\rangle$ of the system, respectively.

as $r_2$ and they both are used to provide the effect of the photon-number-dependent transition frequency of the $\Xi$-type three-level qutrit to accomplish the state-selective qubit rotation, different to the resonator $r_1$. The photon-number-dependent transition frequency between $|e\rangle_q$ $\leftrightarrow$ $|a\rangle_q$ of the qutrit can be written as $\omega_{e,a}^{r_1} = \omega_e^{r_1} + \frac{(g_e^{r_1})^2}{\omega_e^{r_1} - \omega_r^{r_1}}(2n_1 + 1) + \frac{(g_a^{r_1})^2}{\omega_a^{r_1} - \omega_r^{r_1}}(2n_2 + 1)$.

Here $n_1$ and $n_2$ are the photon numbers in the resonators $r_1$ and $r_2$, respectively. The photon-number-dependent transition frequency of $q$ depends on the relationship of the photon numbers in two resonators. That is, one can afford a drive field with the frequency equivalent to the changed transition frequency of the qutrit to achieve the state-selective qubit rotation with different relations between $n_1$ and $n_2$.

Suppose that $\omega_{e,a}^{r_1} = \omega_e^{r_1} + \frac{(g_e^{r_1})^2}{\omega_e^{r_1} - \omega_r^{r_1}}(N+4)$, where $N = 2n_1 + 6n_2$. The transition frequency of $q$ can be divided into four groups, according to the photon-number relations between $r_1$ and $r_2$. That is, $|0_10_2, 1_10_2, 0_11_2, 1_11_2\rangle$ with $N = 0, 2, 6,$ and 8, respectively. Considering $N = 8$, a drive field with the frequency $\omega^d = \omega_e^{r_1} + \frac{(g_e^{r_1})^2}{\omega_e^{r_1} - \omega_r^{r_1}}$ can flip the qutrit between $|e\rangle_q$ and $|a\rangle_q$ only if there is one microwave photon in each of the two resonators $r_1$ and $r_2$.

If we take the initial state of the hybrid system composed of $r_1$, $r_2$, and $q$ as $|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|0_10_2|g\rangle_q^{dress} + |0_11_2|e\rangle_q^{dress} + |1_11_2|g\rangle_q^{dress} + |1_10_2|e\rangle_q^{dress} + |0_10_2|q\rangle_q^{dress} + |1_10_2|q\rangle_q^{dress} + |1_11_2|q\rangle_q^{dress} - |0_11_2|q\rangle_q^{dress})$, applying a drive field to complete a state-selective qubit rotation operation on the transition between $|e\rangle_q$ $\leftrightarrow$ $|a\rangle_q$ when there is one microwave photon in each of the two resonators $r_1$ and $r_2$, the state of the hybrid system becomes $|\Phi_f\rangle = \frac{1}{\sqrt{2}}(|0_10_2|g\rangle_q^{dress} + |0_11_2|e\rangle_q^{dress} + |1_10_2|q\rangle_q^{dress} + |1_11_2|q\rangle_q^{dress} - |0_11_2|q\rangle_q^{dress} - |1_10_2|q\rangle_q^{dress})$.

With the hybrid cc-phase gate above, we can construct the cc-phase gate on three resonator qudits, shown in Fig 4. The Hamiltonian of the hybrid system composed of the three resonators $r_1$, $r_2$, and $r_3$ and the superconducting qubit $q$ is $H_3 = \sum_{i=1,2,3} E_i |q\rangle\langle i| + \sum_{i=1,2,3} [\omega_i^e a_i^+ a_i + g_i^e (a_i^+ \sigma_{e,e}^- + a_i \sigma_{e,e}^+) + g_i^a (a_i^+ \sigma_{e,a}^- + a_i \sigma_{e,a}^+)]$.

Suppose that the initial state of the system is $|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|0_10_20_3 + |0_11_20_3|)$.\]
The cc-phase gate can be achieved with the three steps as follows.

First, we turn off the interaction between the two resonators \(r_1 \text{ and } q\), and then resonate \(r_3\) and the qutrit \(q\) in the transition between \(|g\rangle_q\) and \(|e\rangle_q\). After the operation time \(t = \frac{\pi}{2\omega}\), the state of the system evolves from \(|\Psi_0\rangle\) into

\[
|\Psi_1\rangle = \frac{1}{2\sqrt{2}} [ |(0) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(0) \langle 1| (1) \rangle_2 |g\rangle_q + |i(1) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 1| (1) \rangle_2 |g\rangle_q + |i(1) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \otimes |0\rangle_3].
\]

Second, we turn off the interaction between \(r_3\) and \(q\), and turn on the interactions between \(r_1\) and \(q\), and \(r_2\) and \(q\). The state \(|\Psi_1\rangle\) is changed to be

\[
|\Psi_1\rangle = \frac{1}{2\sqrt{2}} [ |(0) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(0) \langle 1| (1) \rangle_2 |g\rangle_q + |i(0) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 1| (1) \rangle_2 |g\rangle_q + |i(1) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \otimes |0\rangle_3].
\]

By taking the hybrid cc-phase gate on \(r_1, r_2, \text{ and } q\), we can get

\[
|\Psi_2\rangle = \frac{1}{2\sqrt{2}} [ |(0) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(0) \langle 1| (1) \rangle_2 |g\rangle_q + |i(0) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 1| (1) \rangle_2 |g\rangle_q + |i(1) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \otimes |0\rangle_3].
\]

Third, we turn off the coupling between \(r_1, r_2, \text{ and } q\), the state of the system becomes

\[
|\Psi_3\rangle = \frac{1}{2\sqrt{2}} [ |(0) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(0) \langle 1| (1) \rangle_2 |g\rangle_q + |i(0) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 1| (1) \rangle_2 |g\rangle_q + |i(1) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \otimes |0\rangle_3].
\]

By resonating \(r_3\) and \(q\), we can get the final state of the system as

\[
|\Psi_f\rangle = \frac{1}{2\sqrt{2}} [ |(0) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(0) \langle 1| (1) \rangle_2 |g\rangle_q + |i(0) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 0| (0) \rangle_2 |g\rangle_q + |i(0) \langle 0| (0) \rangle_2 |e\rangle_q \rangle_{dress} \\
+ |(1) \langle 1| (1) \rangle_2 |g\rangle_q + |i(1) \langle 1| (1) \rangle_2 |e\rangle_q \rangle_{dress} \otimes |g\rangle_q. 
\]

This is just the outcome of the cc-phase gate operation on \(r_1, r_2, \text{ and } r_3\), by using \(r_1\) and \(r_2\) as the control qutrits and \(r_3\) as the target qutrit.

**IV. DISCUSSION AND SUMMARY**

The deterministic approaches to realize the nonlinear interaction between two photons for quantum information processing are usually based on the Kerr effect. Here we constructed the local c-phase and cc-phase gates on the resonator qubits in a microwave-photon quantum processor by using the number-state-dependent interactions between the superconducting qubit and the resonator qubits. The effect of number-state-dependent interaction suggests a photon-to-qubit conditional logic gate was proposed by Schuster et al. [27] in 2007. In 2010, Strauch, Jacobs, and Simmonds [55] used this
effect to achieve an arbitrary control of entanglement between two superconducting resonators. In 2012, Wu et al. [56] constructed the c-phase gate on two resonators for the one-way quantum computer using on-chip resonator qubits. The feasibility of the microwave photon quantum processor has been discussed in the applications for generating entanglement and constructing quantum logic gates [49, 56, 62]. The processor needs a tunable coupling superconducting qubit and some tunable resonators. The experiments showed that a tunable coupling strength between a superconducting qubit and a superconducting resonator is feasible [35–37]. Johansson et al. [63] and Ong et al. [64] gave the way for tuning the frequency of a resonator. In order to avoid to shorten the relaxation time of the qutrit, the processor needs high-Q resonators. That is, the present c-phase and cc-phase gates are feasible, similar to those in Refs. [49, 55, 56, 62].

Different from the c-phase gate on two resonator qudits proposed by Strauch [54], in which two resonator qudits are coupled to two capacitive-coupled-superconducting qubits, our microwave-photon quantum processor can be extended to the general case in which there are multiple resonators coupled to a superconducting qubit. Although the c-phase gate on two resonator qudits was constructed by Wu et al. [56] with the number-state-dependent interaction between a superconducting qubit and a two-resonator-qudit subsystem, our c-phase gate is based on the combination of the number-state-dependent interaction between a superconducting qutrit and a resonator-qudit subsystem and the resonant operation between the qutrit and another resonator-qudit subsystem. That is, in our scheme for the c-phase gate on two resonator qudits, the transition frequency of the qutrit depends on the photon number in one resonator, not those in two resonators [56], and the information of the qutrit comes from the resonant operation with another resonator. This difference makes us get a higher-fidelity c-phase gate on two resonator qudits with a shorter operation time, compared with previous proposals. The fidelity of our c-phase gate is about 99.51% within the operation time 93 ns.

There are, by far, no works about the construction of the cc-phase gate on three microwave-photon-resonator qudits. In our scheme for the cc-phase gate on three microwave-photon-resonator qudits, two resonators are used to complete the number-state-dependent interaction with the superconducting qutrit, which is similar to the c-phase gate in Ref. [56]. The other resonator is used to accomplish the resonant operation with the qutrit. Our scheme does not require the three resonators to complete the number-state-dependent interaction with the superconducting qutrit, which makes us get a higher-fidelity cc-phase gate. The fidelity of our cc-phase gate can reach 92.92% within about 124.64 ns.

In our calculation, the parameters of the qutrit are taken as the same as those of a transmon qutrit in Ref. [65]. Actually, the coupling strength for the two different transitions of a transmon qutrit and a microwave-photon resonator is asymptotically increased as $(E_J/E_C)^{1/3}$ (for a transmon qubit, $20 < E_J/E_C < 5 	imes 10^5$) [66]. That is, it is reasonable to use the same coupling strength for the two different transitions of a transmon qutrit and a resonator for convenience. The amplitudes of the drive fields for constructing the c-phase and cc-phase gates are too small (compared with the anharmonicity between the two transitions of the qutrit) to induce the influences coming from the higher excited energy level of the superconducting qutrit.

The coherence time of a transmon qutrit approaches to 0.1 ms [33] and the life time of microwave photons contained in resonators are always longer than that of a qutrit [42], which means our gates can be operated several hundreds of times within the life time of the processor. In order to evolve the systems from the dress states to the computational states in our schemes for constructing the gates, one needs to tune on or off the interaction between the resonators and the qubit, as the same as those in Refs. [55, 56]. The quantum error coming from this method is decided by the technique of the tunable transition frequency of a superconducting resonator and the tunable coupling strength between the qutrit and the resonators.

It is worth pointing out that the techniques for catching and releasing of microwave-photon states from a resonator to the transmission line [43] and the single-photon router in the microwave regime [68] have been realized in experiments. The microwave photon quantum processor can play an important platform for quantum communication as well.

In summary, we have constructed two universal quantum gates, i.e., the c-phase and cc-phase gates in a microwave-photon quantum processor which contains multiple superconducting microwave-photon- resonator qudits coupled to a Z-type superconducting qutrit. Our gates are based on the combination of the number-state-dependent interaction between a superconducting qutrit and a resonator-qudit subsystem and the resonant operation between the qutrit and another resonator-qudit subsystem, and they have a high fidelity in a short operation time. The algorithms of our gates are based on the Fock states of the resonators, and the microwave photon number in each resonator is limited to none or just one. As a universal quantum processor on microwave photons in resonators, these processors can deal with the quantum computation with microwave photons, and they can act as a platform for quantum communication as well.

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