System identification from ambient vibration measurements using output-only modal identification techniques

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Abstract. Frequency domain decomposition (FDD) method and Random decrement technique are proposed to analyze ambient vibration data. An ambient vibration test was conducted on an 18-story reinforced concrete building, and the proposed methods were applied to estimate modal parameters. The estimated natural frequencies by the FDD method and the RD technique are in good agreement, and comparisons for damping values estimated by the EFDD method and the RD technique show a 72-246 percent difference. The results imply that the proposed methods can accurately evaluate structural dynamic properties from ambient vibration data in a non-destructive manner.

1. Introduction
Output-only modal identification techniques have been popular due to some advantages. In output-only modal testing, only output response data is measured, and the input signals are ignored. This means the output-only modal testing can be carried out under natural conditions, and the test structure can be in normal operation during the test.

Based on the white noise assumption, some output-only modal identification methods have been developed. The output-only modal identification techniques can mainly be classified into two groups, including the frequency domain methods and the time domain methods. In the frequency domain, the Peak-picking method has been used for decades, and the Frequency domain decomposition (FDD) method improves Peak-picking method by taking singular value decomposition of the output spectral matrix\cite{1,2}, later the Enhanced frequency domain decomposition (EFDD) method was developed to estimate modal damping \cite{3}. In the time domain, the Ibrahim time domain method (ITD), Eigensystem realization algorithm (ERA), Stochastic Subspace Identification (SSI) and Random decrement (RD) technique have been widely used.

In this paper, the Frequency domain decomposition method, Enhanced frequency domain decomposition method, and Random decrement technique were adopted to estimate modal parameters from ambient vibration response of an 18-story reinforced concrete building. These two techniques represent two different classes of identification. The results clearly illustrate that the output-only modal identification techniques in both frequency and time domain perform excellently in dealing with ambient vibration response data.
2. Output-only modal identification techniques

2.1 Frequency domain decomposition method

The Frequency domain decomposition method removes limitations of the PP method. The response power spectral density matrix is decomposed into a set of auto spectral density functions, and each function corresponds to a single degree of freedom (SDOF) system, then modal parameters are obtained.

Considering an $n$ degree of freedom structure. The relationship between the measured output response $y(t)$ and the unknown input $x(t)$ can be expressed as

$$ G_y(j\omega) = \overline{H}(j\omega)G_x(j\omega)H(j\omega)^T $$

(1)

where $G_x(j\omega)$ is the power spectral density matrix of the input, $G_y(j\omega)$ is the power spectral density matrix of the output, $H(j\omega)$ is the frequency response function (FRF), the overbar and the superscript T denote the complex conjugate and transpose, respectively.

$$ H(j\omega) = \sum_{k=1}^{n} \frac{R_k}{j\omega - \lambda_k} + \frac{\bar{R}_k}{j\omega - \bar{\lambda}_k} $$

(2)

where $n$ is the DOF number and the mode number, $R_k$ is the residue, $\lambda_k$ is the pole, $\phi_k$ is the mode shape vector, $\gamma_k$ is the modal participation vector.

With the white noise assumption, the input PSD matrix $G_x(j\omega)$ is reduced to a constant matrix $C$. Assume that the structure is lightly damped and there is no closely spaced mode, the Eq.(1) can be simplified as follows

$$ G_y(j\omega) = \sum_{k=1}^{n} \left( \frac{d_k\phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k\bar{\phi}_k^T}{j\omega - \bar{\lambda}_k} \right) = \Phi \cdot \text{diag} \left( 2\text{Re} \left( \frac{d_k}{j\omega - \lambda_k} \right) \right) \cdot \Phi^T $$

(3)

The power spectral density matrix of the $k$th order can be represented as follows

$$ G_{y_k}(j\omega) = \phi_k \cdot \text{diag} \left( \frac{2d_k \zeta_k \omega_k}{(\zeta_k \omega_k)^2 + (\omega - \omega_k)^2} \right) \cdot \phi_k^T $$

(4)

From Eq.(4) it is noticed that only one mode can contribute to the maximum of power spectral energy, so that modal parameters can be estimated by the FDD method.

The structural damping can be estimated by the Enhanced frequency domain decomposition method. The piece of the SDOF density function obtained around the peak of the PSD is first isolated and then is taken back to the time domain using IFFT. The damping ratio can then be obtained from the decay of autocorrelation functions.

2.2 Random decrement technique

Random decrement technique was first put forward to estimate damping of airplane wings [4]. It assumes the output response data to be white noise. Base on the white noise assumption, the measured random response is divided into a series of short segments, and the free vibration response is extracted by averaging the short segments. The obtained free vibration signal can be written as

$$ z(\tau, c) = \frac{1}{2N} \sum_{i=1}^{2N} \text{sgn}[y(t_i + \tau, c)] \cdot y(t_i + \tau, c) $$

(5)

where $z(\tau, c)$ is the extracted free vibration signal, $y(t, c)$ is the displacement, velocity or acceleration response at time $t$ with respect to the given trigger value of $c$, $\text{sgn}[y(t, c)]$ is the signum function of $y(t, c)$, $t_i$ is the $i$-th time instant corresponding to the trigger value $c$, $2N$ is the number of time segments.
3. Applications

Frequency domain decomposition method, Enhanced frequency domain decomposition method, and Random decrement technique were applied to evaluate modal parameters from ambient vibration response of an 18-story reinforced concrete building, as shown in Figure 1. Layout of the test building is shown in Figure 2. Ambient vibration test was conducted. The response acceleration data along the width direction and the associated PSD function are shown in Figure 3.

![Test building](image1)
![Layout of the structural system](image2)

Figure 1.Test building  
Figure 2. Layout of the structural system

![Acceleration record along width direction](image3)
![Power spectral density](image4)

(a) Acceleration along width direction  
(b) Power spectral density

Figure 3. Acceleration record along width direction

Figure 3 shows singular values of the output response PSD matrix. It can be noticed that the 1st mode and the 2nd mode are coupled, and the 4th mode and the 5th mode are coupled. However, the closely spaced modes are clearly illustrated in the figure.

Figure 4 shows curve fitting process of the RD technique. The free vibration response is extracted from the measured random response by averaging short segments, then the curve fitting technique is applied to the free vibration signal to estimate modal parameters. It can be seen that the curve fitting result fits well with the free vibration data.

![Singular values of the output response PSD matrix](image5)

Figure 4. Singular values of the output response PSD matrix
The estimated natural frequencies and damping ratios by the FDD method, EFDD method, and RD technique are shown in Table 1. The first six modes are identified by the FDD method, but the RD technique fails in identifying the 4\textsuperscript{th} mode. The estimated natural frequencies by the FDD method and the RD technique are in good agreement. However, an obvious distinction between the EFDD method and the RD technique can be noticed. The damping ratios estimated by the EFDD method are generally larger than those estimated by the RD technique, except the 2\textsuperscript{nd} mode. Comparisons for damping values estimated by the two methods show a 72-246 percent difference.

Table 1. Natural frequencies and damping ratios of the building.

| Mode | FDD and EFDD | RD technique |
|------|--------------|---------------|
|      | Natural frequency (Hz) | Damping ratio (%) | Natural frequency (Hz) | Damping ratio (%) |
| 1\textsuperscript{st} | 1.07 | 1.11 | 1.07 | 0.53 |
| 2\textsuperscript{nd} | 1.27 | 0.81 | 1.27 | 2.54 |
| 3\textsuperscript{rd} | 2.44 | 1.36 | 2.44 | 0.52 |
| 4\textsuperscript{th} | 4.98 | 1.35 | - | - |
| 5\textsuperscript{th} | 5.47 | 1.31 | 5.47 | 0.76 |
| 6\textsuperscript{th} | 8.30 | 2.29 | 8.30 | 0.66 |

4. Conclusions

In this paper, the frequency domain and time domain output-only modal identification methods are introduced, including the FDD method, EFDD method, and the RD technique. The proposed methods are applied to ambient vibration response of a concrete building. The results show that natural frequencies estimated by the frequency and time domain methods are in good agreement, but damping ratios show significant difference. The results imply that both frequency and time domain methods can evaluate modal parameters from ambient vibration data in a non-destructive manner.

References

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