Configuration Generation of Aircraft Swarm Based on Communication Distance Constraint

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Abstract. Aiming at the requirements of the aircraft swarm combat task and information consensus, the control method of swarm configuration generation of aircraft swarm based on communication distance constraint is studied. We build the swarm communication relation model based on graph theory combined with the self-organizing obstacle avoidance control strategy and design the control protocol of aircraft swarm space configuration generation. We prove the stability and convergence of the system based on the Lyapunov stability theory and LaSalle invariable principle. Simulation results show that the proposed control protocol can ensure that the aircrafts can stabilize to the desired configuration under the premise of successfully avoiding environmental obstacles.

1. Introduction

After the aircraft swarm is assembled, different configurations have to be formed according to the different tasks. The formation of the configuration of the aircraft swarm is that the individual uses local information to enable all aircraft to implement the assigned task configuration. In the formation of configuration, communication between aircraft has an important impact on the implementation of the corresponding configuration. In the research of aircraft swarm configuration control, the existing literatures usually assume that individuals meet certain communication connectivity conditions without considering the limited communication distance in the actual system[5-7].

Considering aircraft swarm configuration generation communication distance constraint, this paper first constructs the artificial potential field function to achieve network connectivity to maintain communication, the basic idea is: when the adjacent aircraft communication distance tends to radius, the gradient of potential function tends to infinity so as to produce a sufficiently large attraction, the initial time of aircraft in the communication range capable of communicating, always maintain communication network. When carrying out combat tasks, aircraft swarm usually consist of different types and functions of aircraft. The mission is usually initiated by leaders. When other aircraft in the communication range receive instructions, they will cooperate with leaders to generate the specified task configuration. The control protocol which is designed with a leader considering the communication distance aircraft swarm configuration constraint generation control protocol can guarantee the initial communication to meet the constraints of distance of aircraft to avoid obstacles in the environment under the premise of success, and always maintain a spatial configuration within the communication range and stability to the desired.
2. Graph theory

Graph theory is an important theoretical analysis tool for the study of aircraft swarm. In the process of research, each individual in the swarm is usually regarded as a node in the graph. The communication connections between nodes are depicted with both directed and undirected edges. Given N state variables, $x_i \in \mathbb{R}^n, i = 1, 2, \ldots, N$, we can draw the following conclusions:

**Conclusion 1:** for the undirected graph of the adjacency matrix, there is an undirected graph of the adjacency matrix.

$$\sum_i \sum_j a_{ij}(x_i - x_j) = \frac{1}{2} \sum_i \sum_j a_{ij}\|x_i - x_j\|^2$$  \hspace{1cm} (1)

If the graph is connected, then:

$$\sum_i \sum_j a_{ij}\|x_i - x_j\|^2 = 0 \iff x_i = x_j, \forall i, j \in v$$  \hspace{1cm} (2)

**Proof:** since $a_{ij} = a_{ji}, \forall i, j$, then

$$\sum_i \sum_j a_{ij}(x_i - x_j) = \sum_i \sum_j a_{ij}(x_j - x_i) = \sum_i \sum_j a_{ij}(x_i - x_j)$$

Then

$$\sum_i \sum_j a_{ij}(x_i - x_j) = \frac{1}{2} \sum_i \sum_j a_{ij}(x_i - x_j) = \frac{1}{2} \sum_i \sum_j a_{ij}(x_i^T + x_j^T)x_i - 2x_i^Tx_j = \frac{1}{2} \sum_i \sum_j a_{ij}\|x_i - x_j\|^2$$

Where $\sum_i \sum_j a_{ij}\|x_i - x_j\|^2 = 0$, for all $a_{ij} > 0$, then $x_i = x_j$. And when it's a connected graph, you can get $x_i = x_j, \forall i, j$.

3. Problem formulation and some theories

The text of your paper should be formatted as follows:

**Theorem 1.** Global asymptotic stability theorem[10]. Suppose the system has an equilibrium point $x=0$, if there is a scalar function $V(x): \mathbb{R}^n \to \mathbb{R}^*$ satisfies the following three condition:

1. function $V(x)$ is positive in $\mathbb{R}^n_B$, that is $V(x) \geq 0$, and if and only $x=0$, $V(x) = 0$ exists;
2. The derivative $\dot{V}(x)$ in $\mathbb{R}^n_B$ of the function $V(x)$ is negative semidefinite.
3. When $x \to \infty$, $V(x) \to \infty$.

Then the equilibrium point $x=0$ is global stable. If the function $\dot{V}(x)$ satisfies in condition (2) is negative, then the equilibrium point of the system is globally asymptotically stable.

**Definition 1.** The system equation $\dot{x} = f(x)$, if $x(0) \in M \Rightarrow x(t) \in M$ is established for $\forall t \in \mathbb{R}$, then the set $M$ is the invariant set of the system, and if $x(0) \in M \Rightarrow x(t) \in M$ is established for $\forall t \geq 0$, the set $M$ is a positive invariant set of the system.

**Lemma 1.** LaSalle invariance theorem[9]. Let $\dot{x} = f(x)$ be defined on $D \subset \mathbb{R}^n$, and $\Omega \subset D$ is a positive invariant set of the system. Let $V: D \to \mathbb{R}$: it is a continuous and differentiable function, and there is a $\dot{V}(x) \leq 0$ in $\Omega$. Let $E$ be a set of all points where $\dot{V}(x) = 0$ in medium $\Omega$, and $M$ is the largest invariant set in $E$, then every solution that starts from $\Omega$ tends to a collection $M$ when $t \to \infty$.

**Lemma 2.** Barbalat lemma. Let $x: \text{the first continuous derivable on the } [0, \infty) \to \mathbb{R}$, and it has the limit when $t \to \infty$, if $\dot{x} = f(x)$ is consistent, then $\lim_{t \to \infty} \dot{x} = 0$.

**Suppose 1.** The diagram of the undirected communication is an initial connection, and

$$\forall(i, j) \in E, \|p_i(0) - p_j(0)\| < \rho$$  \hspace{1cm} (3)

The aim of this paper is to design a distributed control strategy under the suppose 1. The communication connectivity:

$$\forall t \geq 0, \forall(i, j) \in E, \|p_i(t) - p_j(t)\| < \rho$$  \hspace{1cm} (4)
2. speed consistence:

\[ q_i = q^L \]  

(5)

3. configuration generation:

\[ \forall (i, j) \in E, \lim_{t \to \infty} \| p_i(t) - p_j(t) - \Delta_{\gamma} \| = 0 \]  

(6)

4. successful obstacle avoidance:

\[ \| p_i - q_i \| > r \]  

(7)

Model 1. Individual dynamics model of aircraft swarm.

\[ \begin{cases} 
    \dot{p}_i = q_i, \\
    \dot{q}_i = u_i, 
\end{cases} \quad i=1,2,\ldots,N \]  

(8)

Where \( p_i \in R^3 \), \( q_i \in R^3 \) represents the position vector and velocity vector of \( i \), and \( u_i \in R^2 \) is the control input of \( i \). The expected track information can be expressed in the following form:

\[ \begin{cases} 
    \dot{p}^L = q^L \\
    \dot{q}^L = u^L 
\end{cases} \]  

(9)

\( p^L \in R^3, q^L \in R^3 \) and \( u^L \in R^2 \) are the position vector, velocity vector and control input of the leader respectively.

4. Design and analysis of control protocol

Considering the communication distance constraints, the generation control problem of leader swarm formation is considered. In this paper, each aircraft is regarded as a point in the potential field, and a smooth potential field function is defined for every two aircraft combination \((i, j) \in E\).

\[ \varphi_{ij} = \varphi_{ij}(\gamma_{ij}, \beta_{ij}) \]  

(10)

Where \( \gamma_{ij} \) is the control target that the aircraft combination \((i, j) \in E\) is minimization when the desired relative position is reached.

\[ \gamma_{ij} = \| p_i(t) - p_j(t) - \Delta_{ij} \| \]  

(11)

The function \( \beta_{ij} \) is used to ensure that the aircraft is always in the communication range. It makes the aircraft \( j \in N_i \), if it is in the communication scope of the aircraft \( i \) at the initial time, it will always be in the communication scope of the aircraft \( i \). The function \( \beta_{ij} \) is defined as:

\[ \beta_{ij} = \begin{cases} 
    1, & p_{ij} \leq \bar{\rho}^2 \\
    \frac{(\rho^2 - p_{ij})(\rho^2 - 2\rho + p_{ij})}{(\rho^2 - \bar{\rho}^2)^2}, & p_{ij} > \bar{\rho}^2 
\end{cases} \]  

(12)

where \( p_{ij} = \| p_i - p_j \| \), \( \bar{\rho} < \rho \) is constant, which satisfies the \( \bar{\rho} \geq \max_{i,j \in N} \| \Delta_{ij} \| \), and the function is shown in Figure 1.

![Figure 1. The function of the variable \( \beta_{ij} \).](image-url)
On the basis of the potential field function $\phi_{ij}$, and considering the obstacle avoidance in the external environment, the control protocol of the aircraft $i$ is designed.

$$u_i = -\sum_{j \in \mathcal{N}_i} c_{ij}(q_i - q_j) - \sum_{j \in \mathcal{N}_i} \frac{\partial \phi_{ij}}{\partial p_j} - c_i(q_i - q^*_i) + u^*_i - \nabla \phi_i U_{obs}$$  \hspace{1cm} (13)

The $c_{ij} = c_i > 0$ is the connection weight value of the connection to the aircraft $i, j, c_i \geq 0$. Bring the (13) into the (9):

$$\begin{cases}
\dot{p}_i = q_i \\
\dot{q}_i = -\sum_{j \in \mathcal{N}_i} c_{ij}(q_i - q_j) - \sum_{j \in \mathcal{N}_i} \frac{\partial \phi_{ij}}{\partial p_j} - c_i(q_i - q^*_i) + u^*_i - \nabla \phi_i U_{obs}, \quad i = 1, 2, \ldots, N
\end{cases}$$  \hspace{1cm} (14)

**Suppose 2.** The function $\phi_{ij}$ has the following properties:

1. $\phi_{ij} = \varphi_{ij} \geq 0$;  \hspace{1cm} 2. $\lim_{\beta_{ij} \to 0} \varphi_{ij} \to \infty$;  \hspace{1cm} 3. $\frac{\partial \varphi_{ij}}{\partial \beta_{ij}} > 0, \frac{\partial \varphi_{ij}}{\partial \rho_j} < 0$.

In order to avoid the obstacle of the external environment, the obstacle avoidance method used in this paper is shown in Figure 2.

![Figure 2. Schematic diagram of obstacle avoidance.](image)

**Theorem 2.** The system contains a swarm of aviation aircraft components, individual swarm motion model and the leader expected track information respectively (8) and (9) described in suppose 1 and control law (13) under the action, if there is at least one $0_{ic} > \alpha$, so the aircraft can converge to the desired configuration, and is expected to the speed of $q^c$ movement, and can effectively avoid the obstacles and reach the stable swarm.

**Lemma 3:** in suppose 1, the set $S(p) = \{ p \parallel p_i - p_j \parallel < \rho, \forall (i, j) \in E \}$ is an invariant set of the system (14), so the communication distance constraint condition (4) is always established.

**Proof** For the system (19), select the following Lyapunov function:

$$V = \frac{1}{2} \sum_{i} \dot{q}_i^T \dot{q}_i + \frac{1}{2} \sum_{j \in \mathcal{N}_i} \varphi_{ij}(\dot{p}_i, \dot{p}_j) + \sum_{t} U_{obs}$$  \hspace{1cm} (15)

Then

$$\dot{V} = \sum_{i} \dot{q}_i^T \left( -\sum_{j \in \mathcal{N}_i} c_{ij}(\dot{q}_i - \dot{q}_j) - c_i \dot{q}_i + \sum_{j \in \mathcal{N}_i} \frac{\partial \varphi_{ij}}{\partial p_j} - \nabla \phi_i U_{obs} \right) + \frac{1}{2} \left( \sum_{i} \sum_{j \in \mathcal{N}_i} \left( \frac{\partial \varphi_{ij}}{\partial p_j} \right)^T \dot{q}_i + \sum_{i} \sum_{j \in \mathcal{N}_i} \left( \frac{\partial \varphi_{ij}}{\partial p_j} \right)^T \dot{q}_j \right) + \sum_{i} \dot{q}_i^T \nabla \phi_i U_{obs}$$

$$= \sum_{i} \dot{q}_i^T \left( -\sum_{j \in \mathcal{N}_i} c_{ij}(\dot{q}_i - \dot{q}_j) - c_i \dot{q}_i \right) - \sum_{i} \sum_{j \in \mathcal{N}_i} \frac{\partial \varphi_{ij}}{\partial p_j} \dot{q}_i + \frac{1}{2} \sum_{i} \sum_{j \in \mathcal{N}_i} \left( \frac{\partial \varphi_{ij}}{\partial p_j} \right)^T \dot{q}_j + \sum_{i} \dot{q}_i^T \nabla \phi_i U_{obs}$$

$$= -\sum_{i} c_i \dot{q}_i \dot{q}_i^T - \sum_{i} \sum_{j \in \mathcal{N}_i} \frac{\partial \varphi_{ij}}{\partial p_j} \dot{q}_i + \frac{1}{2} \sum_{i} \sum_{j \in \mathcal{N}_i} \left( \frac{\partial \varphi_{ij}}{\partial p_j} \right)^T \dot{q}_j + \sum_{i} \dot{q}_i^T \nabla \phi_i U_{obs}$$

$$= \frac{1}{2} \sum_{i} \sum_{j \in \mathcal{N}_i} c_{ij} \dot{q}_i \dot{q}_j + \frac{1}{2} \sum_{i} \sum_{j \in \mathcal{N}_i} \left( \frac{\partial \varphi_{ij}}{\partial p_j} \right)^T \dot{q}_j + \sum_{i} \dot{q}_i^T \nabla \phi_i U_{obs}$$

It is available from the inverse method: if there is at least one aircraft combination $(i, j) \in E$, so that $\beta_{ij} \to 0$ or $p_{ij} \to p^*$, then there is $\varphi_{ij} \to \infty$, so that $V \to \infty$ is not consistent with the known conditions. So $p(t) \in S(p), \quad \forall t \geq 0$ can be obtained. Therefore, the lemma 1 is proved that the system can always
be kept in the communication range.

According to the LaSalle invariance principle, formula (16) implies that when \( t \) tends to infinity,
\[
\dot{V} = 0, \quad \sum_{i,j \in N_i} c_{ij} \|q_i - \dot{q}_i\|^2 = 0 \quad \text{and} \quad \sum_{i,j \in N_i} c_{ij} \|q_i - \dot{q}_i\|^2 = 0
\]
indicate that all \( i, j, \dot{q}_i = \dot{q}_j \)
\[
\sum_{i} c_i \|\dot{q}_i\|^2 = 0
\]
and at least one \( c_i > 0 \) can be at least one \( \dot{q}_i = 0 \). This can be achieved for all \( i, \dot{q}_i = 0 \) or \( q_i = q^i \), that is, the follower will agree with the leader to expect speed.

Because every \( q_i \) has boundedness limit and \( \dot{q}_i \) is uniformly continuous, Barbalat lemma is applied to formula (14). When \( t \) tends to infinity, it has \( \dot{q}_i = 0 \) for all \( i \), and \( L_i = q_i \) means \( 1, 2, \ldots, n \) for all.

\[
\lim_{t \to \infty} \sum_{j \in N_i} \frac{\partial \phi_j}{\partial p_i}(t) = 0 \quad (17)
\]

In the global coordinate system, the \( p_i = \Delta_i \), \( q_i = \Delta_i \), \( \Delta_i \in R^2 \) are the position of the aircraft \( i \) in the relative global coordinate system when the desired configuration is realized.

Construction auxiliary function:
\[
F = \sum_{i} p_i \sum_{j \in N_i} \frac{\partial \phi_j}{\partial p_i} \quad (18)
\]

Then
\[
F = \sum_{i} \sum_{j \in N_i} 2\eta_i (p_i - p_j) + \sum_{i} \sum_{j \in N_i} 2 \xi_i (p_i - p_j + \Delta_i) = \sum_{i} \sum_{j \in N_i} \eta_i \|p_i - p_j\|^2 + \sum_{i} \sum_{j \in N_i} \xi_i (p_i - p_j)^T (p_i - p_j + \Delta_i)
\]
\[
= \sum_{i} \sum_{j \in N_i} \eta_i \|p_i - p_j\|^2 + \sum_{i} \sum_{j \in N_i} \xi_i (p_i - p_j - \Delta_i)^T (p_i - p_j) = \sum_{i} \sum_{j \in N_i} \eta_i \|p_i - p_j\|^2 + \sum_{i} \sum_{j \in N_i} \xi_i \left(\|p_i - p_j\|^2 - \Delta_i^T (p_i - p_j)\right)
\]
\[
\xi_i = \begin{cases} \frac{\partial \phi_j}{\partial p_i}(p_i - p_j)^2, & p_i \leq p_j^2 \\ \frac{\partial \phi_j}{\partial p_i}(p_i - p_j)^T (p_i - p_j), & p_i > p_j^2 \end{cases}
\]

It is clear that \( \sum_{i} \sum_{j \in N_i} \eta_i \|p_i - p_j\|^2 \geq 0 \) is also able to get \( \xi_i \geq 0 \) because of
\[
\sum_{i} \sum_{j \in N_i} \xi_i \left(\|p_i - p_j\|^2 - \Delta_i^T (p_i - p_j)\right) = \sum_{i} \sum_{j \in N_i} \xi_i \left(\|p_i - p_j\|^2 - \|\Delta_i\|^2\right) > 0
\]

Then
\[
F \geq \sum_{i} \sum_{j \in N_i} \eta_i \|p_i - p_j\|^2 + \sum_{i} \sum_{j \in N_i} \xi_i \left(\|p_i - p_j\|^2 - \|\Delta_i\|^2\right) > 0 \quad (21)
\]

The formula (17) is brought into the type (18) to get \( \lim_{t \to \infty} F(t) = 0 \), that is,
\[
\sum_{i} \sum_{j \in N_i} \xi_i \left(\|p_i - p_j\|^2 - \Delta_i^T (p_i - p_j)\) = 0 \quad \text{and} \quad \sum_{i} \sum_{j \in N_i} \eta_i \|p_i - p_j\|^2 = 0 \quad (21)
\]

When the communication graph \( G \) is connected, for all \( i, j, \ p_i = p_j \) is also \( p_i - p_j - \Delta_i = 0 \), that is, the follower and the leader will maintain the desired configuration movement.

Because of \( V \leq 0 \), it can be concluded that \( V \) is bounded, and may as well assume that \( V \leq h \). The former formula shows that when \( \|p_i - o_i\| \to r, U_{\text{dual}} \to \infty \), and \( V \to \infty \) can be derived from the formula (15), it is inconsistent with the hypothesis \( V \leq h \), so the aircraft swarm can realize the obstacle avoidance under the effect of the control protocol (13), and theorem 2 can be proved.

5. Simulation results and analysis

In this paper, we use the proposed control algorithm[11] to simulate the aircraft swarm cooperative anti stealth configuration in the paper “three rounds” and verify the effectiveness of the proposed method. In
the simulation, we take the anti-stealth detection configuration of $R_t = r_t = 52km$. At this time, the three receivers (Rec1, Rec2, Rec3) are distributed in the front of the transmitter in the $[3\pi/20, 17\pi/20]$ range, and are distributed counter-clockwise according to the equal angular distance. The 10000 times the distance of the space configuration is $R_s = r_s = 5.2m$, and the rate of 200 times the contraction ratio is $1m/s$. The target matrix of the control configuration is obtained.

$$
\begin{bmatrix}
0 & 0 & -4.2 & -3.1 & -5.2 & 0 & -4.2 & 3.1 \\
4.2 & 3.1 & 0 & 0 & -1 & 3.1 & 0 & 6.2 \\
5.2 & 0 & 1 & -3.1 & 0 & 0 & 1 & 3.1 \\
4.2 & -3.1 & 0 & -6.2 & -1 & -3.1 & 0 & 0 \\
\end{bmatrix}
$$

The aircraft swarm system contains four aircraft platforms. The initial location is $p_i(0) = [-12.5, 0]^T$, $p_s(0) = [-7.5, 12.5]^T$, $p_r(0) = [-12.5, -5]^T$, where $p_i$ is transmitter, namely leader position, and the remaining three are receiver, follower location.

The location of the obstacle is set to $[50, 0]^T$, the radius is $5m$, and the radius of the buffer zone is $30m$. The desired speed is set to $Lq = 10$, and the speed is $1m/s$. In this paper, the perception radius of aircraft is set to $\rho = 20m$, that is to say, when the distance between aircraft platforms is $d_{ij} = \|p_i - p_j\| < 20m$, that is, communication between aircraft can be carried out. In simulation, the parameter $\beta$ in function $\varphi$ is set to $\beta = 15m$, and the communication topology of swarm at the initial time is shown in Figure 3.

![Figure 3. Aircraft swarm initial location and communication topology.](image_url)

The communication relationship between $i$ and $j$ is $c_{ij} = 2$ when there is no communication relationship $c_{ij} = 0$. Since the leader is known to expect speed information, that is, in the control protocol (12), $c_1 = 4$. In this paper, the potential field function $\varphi$ is defined as $\varphi = \lambda_1 \gamma - \lambda_2 \ln \beta$, in which $\ln(\cdot)$ is a logarithmic function. When the parameters $\lambda_1 = 3, \lambda_2 = 0, \lambda_3 = 3, \lambda_4 = 2$ are taken, the simulation results are shown as shown in Figure 4 and Figure 5 respectively.
From the trajectories of the graph 4 (a) swarm, we can see that the aircraft swarm can not form the desired configuration when the communication maintenance item is not considered (i.e. $\lambda_2 = 0$). This is because the distance between number three follower and other aircraft is increasing gradually, beyond the critical value, making the system communication network no longer connected.

After considering the communication maintenance, the simulation results are shown in Figure 5. From Figure 5 (a), we can see that in the simulation process, the aircraft gradually moved away from the obstacle center in the obstacle buffer area, and successfully achieved the obstacle avoidance. With the passage of time, after the successful obstacle avoidance, the aircraft gradually converged to form a stable configuration. In Figure 5, Figure 5 (b) (c) to adjust the speed of the system oscillation can be seen from the beginning, gradually dynamic stability, in the face of obstacles when the aircraft speed appeared oscillation changes, this is because the obstacle by force, appeared on the direction of adjustment of speed change in obstacle avoidance after the success and gradually converge to the long machine speed. Figure 5 (d) shows that the three followers tend to be stable relative to the long machine, reaching the desired configuration of the 5.2m. aviation.
To sum up, it can be seen that the communication distance constraint has an important influence on the performance of the control algorithm, and it is a problem that must be considered in the design of the control protocol. The control protocol designed in this paper can realize that the aircraft which always meets the communication distance constraint at the initial time always stays in the communication range, and can successfully avoid obstacles in the environment, and ultimately achieve the gradual stable state and generate the desired configuration.

6. Conclusions
Aiming at the problem of communication distance constraint in the process of aircraft swarm configuration, a distributed control protocol is designed based on consensus theory and artificial potential field method. From the actual operation of the aircraft swarm, the protocol can also realize the speed following and obstacle avoidance for the leader. The use of Lyapunov tools for system stability and convergence are proved. The simulation results show that the initial time in communication within the scope of the aircraft, can always stay connected communication network and can realize the obstacle avoidance and the leader of the speed to follow in the control protocol design; by considering the communication distance constraint and without considering the simulation the communication distance constraint under two kinds of situations, This paper further demonstrated the important role of the communication distance to achieve aircraft swarm configuration.

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