Original Article

Nonlinear Dynamic Response and Vibration of 2D Penta-graphene Composite Plates Resting on Elastic Foundation in Thermal Environments

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Abstract: This research discusses an analytical method for investigating vibration and dynamic response of plates structure made of 2-Dimensional (2D) penta-graphene. The density functional theory is used to figure out the elastic modulus of single layer penta-graphene. The classical plate theory is applied to determine basic equations of 2D penta-graphene composite plates. The numerical results are obtained by using the Bubnov-Galerkin method and Runge-Kutta method. The research results show high agreement in comparison with other studies. The results demonstrate the effect of shape parameters, material properties, foundation parameters and the mechanical load on the nonlinear dynamic response of 2D penta-graphene plates. The study also investigates the effect of the thermal environment on the behavior of 2D penta-graphene plates.

Keywords: Dynamic load, 2D penta-graphene composite plate, thermal environment, classical plate theory, stress function.

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1. Introduction

Graphene like one 2D allotrope of carbon was have been known in 2004 [1]. It has shown that 2D single-layer structure has many outstanding advantages. Hence, scientists have been attracted considerable attention to these materials in the last decade. Computation or experimental methods have been used to investigate various types of 2D monolayer [2–7]. In 2015, Zhang et al. [8] proposed a new 2D carbon allotrope that is penta-graphene (PG). PG unit cell consists entirely of carbon pentagons.

As a highly stable 2D allotrope of carbon, scientists have been attracted to this material in recent years. In [9], Sun et al. investigated the thermal transport property of penta-graphene which is affected by grain boundaries. A highlight point in [10] shown that the thermal conductivity of graphene is higher than that of penta-graphene. Tien et al. [11] studied the transport and electronic properties of sawtooth PG nanoribbons. Tien et al. figure out that the electronic and the transport properties of sawtooth PG nanoribbons can effectively modulate when doping by N and H. Alborznia et al. [12] examined the electronic and optical properties of 2D penta-graphene when this material is affected by vertical compressive strain using density functional theory. In [13], the effect of temperature on mechanical properties investigated using simulation method. The mechanical properties of penta-graphene were compared with pentaheptite, graphane, and graphene in [14]. The mechanical properties of penta-graphene when this material is rolled into penta-graphene nanotubes was examined by Chen et al. [15]. Previous studies mainly focus on the material properties of penta-graphene. We can see that the number of studies on 2D penta-graphene application in the field of structure is still limited. Thus, this research decided to investigate the composite plate structure which reinforced by 2D penta-graphene.

The structures, in reality, face various types of dynamic loads such as wind, wave, earthquake, vehicle, blast, etc. Therefore, it is necessary to study the behavior of structures subjected to dynamic loads. Zhang et al. [16] use analytical and numerical methods to analyse the effect of blast loading on the behavior of plate structure which has three layers with faces made from fiber-metal and core made from metal foam. Blast loading effect on the dynamic response of plate structures with two layers sandwich was investigated [17]. Song et al. [18] studied the effect of moving to load on the dynamic response of sandwich plates base on the first-order shear deformation theory. In [19], Duc et al. presented nonlinear dynamic response and vibration of plate. The plate made from functionally material with piezoelectric layer, and outside stiffeners. In [20], dynamic response and vibration of double-curved shells which made from functionally graded nanocomposite have been studied base on higher-order shear deformation theory. Li et al. [21] investigated the nonlinear dynamic response sheet with triple-layer. The behavior of the composite plate reinforced by CNT under impact loading was studied using analytical method [22]. There are many studies on the behavior of structure when subjected to dynamic loads. But the number of research on structures made from 2D penta-graphene has not been paid attention to far. So this study decided to carry out the investigation Nonlinear dynamic response and vibration of 2D penta-graphene composite plates resting on elastic foundation in thermal environments.

2. Analytical solution

2.1. Basic formulas

Figure 1 shown the 2D penta-graphene composite plate model in the Cartesian coordinate system $x, y, z$ on Pasternak foundation.
With \( xy \) is the mid-plane of the plates, \( z \) is the axis along with the thickness of the plate, \((-h/2 \leq z \leq h/2)\). \( h \) is the thickness of the plate. \( a, b \) are the length and width of the plate, respectively.

Figure 1. Geometry and coordinate system of the plate resting on Winkler and Pasternak foundations.

The reaction–deflection relation of the elastic foundation is expressed in Eq. (1):

\[
q_e = k_1 w - k_2 \nabla^2 w
\]

with \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \). \( w \) is the deflection of the plate. \( k_2 \) and \( k_1 \) are stiffness of Pasternak foundation and Winkler foundation, respectively.

The classical plate theory is used to build the compatibility, motions equations and examine the nonlinear dynamic response of the 2D penta-graphene composite plates in this research.

The relation strain-displacement base on classical plate theory is:

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix} = \begin{pmatrix}
\varepsilon_{0\,x} \\
\varepsilon_{0\,y} \\
\gamma_{0\,xy}
\end{pmatrix} + \begin{pmatrix}
k_x \\
k_y \\
k_{xy}
\end{pmatrix} \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

with

\[
\begin{pmatrix}
\varepsilon_{0\,x} \\
\varepsilon_{0\,y} \\
\gamma_{0\,xy}
\end{pmatrix} = \begin{pmatrix}
u_{,x} + w_{,x}^2 / 2 \\
u_{,y} + w_{,y}^2 / 2 \\
u_{,xx} + v_{,x} + w_{,x} w_{,y}
\end{pmatrix},
\]

\[
\begin{pmatrix}
k_x \\
k_y \\
k_{xy}
\end{pmatrix} = \begin{pmatrix}
-w_{,xx} \\
w_{,yy} \\
2w_{,xy}
\end{pmatrix}
\]
in which the normal strains in the middle plane of the plate are $\varepsilon_x^0$ and $\varepsilon_y^0$. The shear strain of the plate is $\gamma_{xy}^0$ in the middle plane of the plate. The displacement long $x, y$ and $z$ axes are $u, v, w$, respectively. $k_x, k_y, k_{xy}$ are the curvature and twisted of the plates.

The relationship between stress and strain in the thermal environment is expressed through Hooke’s law in Eq. (4)

$$
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{pmatrix} = \begin{pmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{pmatrix} \begin{pmatrix}
\varepsilon_x - \alpha_1 \Delta T \\
\varepsilon_y - \alpha_2 \Delta T \\
\gamma_{xy}
\end{pmatrix},
$$

(4)

With $Q_{11} = \frac{E_{11}}{1 - v_{12}v_{21}}, Q_{12} = \frac{E_{12}}{1 - v_{12}v_{21}}, Q_{22} = \frac{E_{22}}{E_{11}}, Q_{12} = v_{12}Q_{22}, Q_{66} = G_{12}$.

The elastic modulus of 2D penta-graphene obtained by fit the strain energy equation and the density functional theory energies.

The relationship between forces, moments and stress of the penta-graphene plates are given by Eq. (5)

$$
(N_{i}, M_{i}) = \int_{-h/2}^{h/2} \sigma_i (1, z) dz, i = x, y, xy. \quad (5)
$$

Substituting Eqs. (2) into Eqs. (4) and the result into Eqs. (5) give the constitutive relations as

$$
\begin{align*}
N_x &= A_{11}\varepsilon_x^0 + B_{11}k_x + A_{12}\varepsilon_y^0 + B_{12}k_y + A_{16}\gamma_{xy}^0 + B_{16}k_{xy} - \Delta T(\alpha_1A_{11} + \alpha_2A_{12}), \\
N_y &= A_{12}\varepsilon_x^0 + B_{12}k_x + A_{22}\varepsilon_y^0 + B_{22}k_y + A_{26}\gamma_{xy}^0 + B_{26}k_{xy} - \Delta T(\alpha_1A_{12} + \alpha_2A_{22}), \\
N_{xy} &= A_{16}\varepsilon_x^0 + B_{16}k_x + A_{26}\varepsilon_y^0 + B_{26}k_y + A_{66}\gamma_{xy}^0 + B_{66}k_{xy} - \Delta T(\alpha_1A_{16} + \alpha_2A_{26}), \\
M_x &= B_{11}\varepsilon_x^0 + D_{11}k_x + B_{12}\varepsilon_y^0 + D_{12}k_y + B_{16}\gamma_{xy}^0 + D_{16}k_{xy} - \Delta T(\alpha_1B_{11} + \alpha_2B_{12}), \\
M_y &= B_{12}\varepsilon_x^0 + D_{12}k_x + B_{22}\varepsilon_y^0 + D_{22}k_y + B_{26}\gamma_{xy}^0 + D_{26}k_{xy} - \Delta T(\alpha_1B_{12} + \alpha_2B_{22}), \\
M_{xy} &= B_{16}\varepsilon_x^0 + D_{16}k_x + B_{26}\varepsilon_y^0 + D_{26}k_y + B_{66}\gamma_{xy}^0 + D_{66}k_{xy} - \Delta T(\alpha_1B_{16} + \alpha_2B_{26}),
\end{align*}
$$

(6)

where

$$
\begin{align*}
A_{ij} &= \int_{-h/2}^{h/2} Q_{ij} (z) dz, \quad ij = 11, 12, 16, 22, 26, 66. \\
B_{ij} &= \int_{-h/2}^{h/2} Q_{ij} (z) zdz, \quad ij = 11, 12, 16, 22, 26, 66. \\
D_{ij} &= \int_{-h/2}^{h/2} Q_{ij} (z) z^2 dz, \quad ij = 11, 22, 12, 66.
\end{align*}
$$

(7)
The motion equations of plates supported by elastic foundations base on classical plate theory are

\[ N_{xx} + N_{yy} = 0 \]  
\[ N_{x,y} + N_{y,x} = 0 \]  
\[ M_{xx} + 2M_{xy} + M_{yy} + N_{x}w_{x}x + 2N_{x}w_{xy} + N_{y}w_{yy} + q - k_{1}w + k_{2}V^{2}w = \rho \frac{\partial^{2}w}{\partial t^{2}} \]  

where \( q \) is an external pressure uniformly distributed on the surface of the plate and \( \rho \) is the density. The stress function \( f(x, y, t) \) is established as

\[ N_{x} = \frac{\partial^{2}f}{\partial y^{2}}, N_{y} = \frac{\partial^{2}f}{\partial x^{2}}, N_{xy} = -\frac{\partial^{2}f}{\partial x \partial y}. \]  

From Eqs. (6), we get \( \varepsilon_{0}, \gamma_{0}^{y} \) as follows

\[ \varepsilon_{0}^{x} = A_{1}^{1}N_{x} + A_{1}^{2}N_{y} + A_{1}^{6}N_{xy} - B_{1}^{1}k_{x} - B_{1}^{2}k_{y} - B_{1}^{6}k_{xy} + \Delta(T(\alpha_{1}D_{1} + \alpha_{2}D_{2})), \]
\[ \varepsilon_{0}^{y} = A_{1}^{3}N_{x} + A_{1}^{2}N_{y} + A_{1}^{6}N_{xy} - B_{2}^{1}k_{x} - B_{2}^{2}k_{y} - B_{2}^{6}k_{xy} + \Delta(T(\alpha_{1}D_{2} + \alpha_{2}D_{2})), \]
\[ \varepsilon_{0}^{xy} = A_{1}^{6}N_{x} + A_{1}^{6}N_{y} + A_{1}^{6}N_{xy} - B_{6}^{1}k_{x} - B_{6}^{2}k_{y} - B_{6}^{6}k_{xy} + \Delta(T(\alpha_{1}D_{6} + \alpha_{2}D_{6})), \]  

In which the coefficients \( A_{ij}^{1}, B_{ij}^{1}, D_{ij}^{1} \) are explained in the Appendix.

The deformation compatibility equation of the perfect plates and imperfect plates are Eq. (14) and Eq. (15), respectively.

\[ \varepsilon_{x,xy}^{0} + \varepsilon_{y,xy}^{0} - \gamma_{y,xy}^{0} = w_{,xy}^{2} - w_{,xx}w_{,yy} \]
\[ \varepsilon_{x,yy}^0 + \varepsilon_{y,xx}^0 - \varepsilon_{x,y,y}^0 = w_{,xy}^2 - w_{,xx} w_{,yy} + 2 w_{,xy} w_{,xy}^* - w_{,xx} w_{,yy}^* - w_{,yy} w_{,xx}^* \]  

(15)

Substitution Eq. (10) and Eq. (11) into the deformation compatibility Eq. (15) leads to

\[ A_{22}^2 f_{xxx} + E_1 f_{xxyy} + A_{11}^2 f_{yyyy} - 2 A_{26}^2 f_{xxyy} - 2 A_{66}^2 f_{yyyy} + B_{21}^2 w_{,xxx} + B_{12}^2 w_{,yyyy} \]

\[ + E_2 w_{,xxyy} + E_3 w_{,xxyy} + E_4 w_{,yyyy} - \left( w_{,xy}^2 - w_{,xx} w_{,yy} + 2 w_{,xy} w_{,xy}^* - w_{,xx} w_{,yy}^* - w_{,yy} w_{,xx}^* \right) = 0, \]

(16)

In which \( E_i \) (\( i = 1-4 \)) are given in the Appendix.

The Eq. (13) and Eq. (16) accompany with initial conditions and boundary conditions are used to investigate the nonlinear dynamic response of 2D penta-graphene plates.

2.2. Boundary conditions

In this study, the 2D penta-graphene composite plate is assumed to be simply supported. Two boundary conditions, labeled as Case I and Case II are considered.

Case I. Four edges of the plate are simply supported and freely movable (FM)

\[ w = N_{xy} = M_x = 0, N_x = N_{x0} \text{ at } x = 0, a, \]

\[ w = N_{xy} = M_y = 0, N_y = N_{y0} \text{ at } y = 0, b. \]

(17)

Case II. Four edges of the plate are simply supported and immovable (IM)

\[ w = u = M_x = 0, N_x = N_{x0} \text{ at } x = 0, a, \]

\[ w = v = M_y = 0, N_y = N_{y0} \text{ at } y = 0, b. \]

(18)

in which \( N_{x0}, N_{y0} \) are compressive force along the direction \( x, y \), respectively.

The approximate solutions satisfying the boundary conditions are

\[ \left( w, w^* \right) = (W, \mu h) \sin \lambda_m x \sin \delta_n y, \]

(19)

\[ f = A_1 \cos 2 \lambda_m x + A_2 \cos 2 \delta_n y + A_3 \sin \lambda_m x \sin \delta_n y + A_4 \cos \lambda_m x \cos \delta_n y + \frac{1}{2} N_{x0} y^2 + \frac{1}{2} N_{y0} x^2 \]

(20)

with \( \lambda_m = \frac{m \pi}{a}, \beta_n = \frac{n \pi}{b} \), \( W \) - the amplitudes of the deflection of the plate. \( \mu \) - imperfect parameter. The coefficients \( A_i \) (\( i = 1 \div 4 \)) found are as follow

\[ A_1 = \frac{1}{32 A_{22}} \frac{\lambda_m^2}{\lambda_m} W(W + 2 \mu h), \]

\[ A_2 = \frac{1}{32 A_{11}} \frac{\delta_n^2}{\delta_n} W(W + 2 \mu h), \]

\[ A_3 = \frac{(F_2 F_3 - F_1^2 F_4)}{F_2^2 - F_1^2} W, \]

\[ A_4 = \frac{(F_2 F_3 - F_1^2 F_4)}{F_2^2 - F_1^2} W, \]

(21)

With \( F_j (i = 1 \div 4) \) are given in the Appendix.
Replacing Eq. (21), Eq. (20) and Eq. (19) into the Eq. (13) and then using Galerkin method we obtain Eq. (22)

\[
\frac{ab}{4}(P_1 \left(\frac{F_2 - F_1}{F_1}\right) \xi^4 + P_2 \left(\frac{F_2 - F_1}{F_1}\right) \xi^2 \delta^2 + P_3 \left(\frac{F_2 - F_1}{F_1}\right) \xi^2 \delta_n^2 \\
- P_4 \left(\frac{F_2 - F_1}{F_1}\right) - P_5 \left(\frac{F_2 - F_1}{F_1}\right) + P_6 \xi^4 + P_8 \xi^2 \delta_n^2 - k_1 - k_2 \left(\xi^2 + \delta_n^2\right))W \\
+ \left[ \frac{1}{3} \xi^2 \delta_n^2 \left(\frac{P_1}{A_{22}} + \frac{P_2}{A_{11}}\right)\right] W(W + 2\mu h) + \frac{8}{3} \left(\frac{F_2 - F_1}{F_1}\right) \xi^2 \delta_n^2 W(W + \mu h) \\
- \frac{ab}{64} \left(\frac{1}{A_{22}} \delta_n^4 + \frac{1}{A_{11}} \xi^4\right) W(W + \mu h)(W + 2\mu h) \\
+ \frac{ab}{4} \left(N_{00} \xi^2 + N_{00} \delta_n^2\right)(W + \mu h) + \frac{4}{\xi^2 \delta_n^2} q = \rho_i \frac{ab \xi^2 W}{\partial t^2}.
\]

2.3. Plates subjected to mechanical load

Consider the composite plates with Case I of boundary condition. The composite plates are assumed that subjected uniform compressive forces \( P_x \) and \( P_y \) (Pascal) on the edges \( x=0, a \), and \( y=0, b \)

\[ N_{x0} = -P_x h, \quad N_{y0} = -P_y h. \]  

2.4. Plates with effect of temperature

Consider the composite plates with Case II of boundary conditions in the thermal environment. The condition expressing the immovability on the edges, \( u = 0 \) (at \( x = 0, a \)) and \( v = 0 \) (at \( y = 0, b \)), is satisfied in an average sense as

\[ \int_0^a \frac{\partial u}{\partial x} dx dy = 0, \quad \int_0^b \frac{\partial v}{\partial y} dx dy = 0. \]  

From Eq. (3) and Eq. (14), we can obtain Eq. (25)

\[
\frac{\partial u}{\partial x} = A_{11} f_{yy} + A_{12} f_{xy} - A_{16} f_{xx} + B_{11} w_{xx} + B_{12} w_{xy} + 2B_{16} w_{xy} \\
+ \Delta T(D_1 \xi + D_2 \xi) - \frac{1}{2} w_{,x}^2 - w_{,y}^2 \\
\frac{\partial v}{\partial y} = A_{21} f_{yy} + A_{22} f_{xy} - A_{26} f_{xx} + B_{21} w_{xx} + B_{22} w_{xy} + 2B_{26} w_{xy} \\
+ \Delta T(D_1 \xi + D_2 \xi) - \frac{1}{2} w_{,x}^2 - w_{,y}^2 \]  

Substitution Eq. (19-20) into Eq. (25) and then results into Eq. (24), We can obtain the equation of \( N_{x0}, N_{y0} \) as below
\[ N_{x_0} = J_1 W + J_2 W(W + 2\mu h) + J_3 \Delta T \]
\[ N_{y_0} = J_4 W + J_3 W(W + 2\mu h) + J_6 \Delta T \]

(26)

where \( J_i (i = 1/6) \) are shown in the Appendix.

By substituting Eq. (26) into Eq. (22), leads to the basic equations used to investigate the nonlinear dynamic response of the 2D penta-graphene composite plates in the Case II of boundary condition

\[
\begin{align*}
\frac{ab}{4} & \left[ P_1 \frac{(F_2 F_4 - F_1 F_3)}{F_2^2 - F_1^2} \lambda_m^4 + P_2 \frac{(F_2 F_4 - F_1 F_3)}{F_2^2 - F_1^2} \delta_n^4 + P_3 \frac{(F_2 F_4 - F_1 F_3)}{F_2^2 - F_1^2} \lambda_m^2 \delta_n^2 \\
- & P_4 \frac{(F_2 F_3 - F_1 F_4)}{F_2^2 - F_1^2} + \frac{P_5 (F_2 F_3 - F_1 F_4)}{F_2^2 - F_1^2} + P_6 \lambda_m^4 + P_7 \delta_n^4 + P_8 \lambda_m^2 \delta_n^2 - k_1 - k_2 (\lambda_m^2 + \delta_n^2) \right] W \\
+ & \frac{ab}{4} (J_3 \Delta T \lambda_m^2 + J_6 \Delta T \delta_n^2) (W + \mu h) + \left[ \frac{1}{3} \lambda_m^2 \delta_n \left( P_{11} \frac{1}{A_{11}} + P_{22} \frac{1}{A_{22}} \right) \right] W (W + 2\mu h) \\
+ & \left[ \frac{8}{3} \frac{(F_2 F_4 - F_1 F_3)}{F_2^2 - F_1^2} \lambda_m \delta_n + \frac{ab}{4} (J_3 \lambda_m^2 + J_6 \delta_n^2) \right] W (W + \mu h) \\
+ & \left[ \frac{ab}{4} (J_3 \lambda_m^2 + J_6 \delta_n^2) - \frac{ab}{64} \left( \frac{1}{A_{22}} \delta_n^4 + \frac{1}{A_{11}} \lambda_m^4 \right) \right] W (W + \mu h) (W + 2\mu h) \\
+ & \frac{4}{\lambda_m \delta_n} \frac{\partial^2 W}{\partial t^2}.
\end{align*}
\]

(27)

Eq. (27) is used to study behavior 2D penta-graphene composite plates subject dynamic load in the thermal environment on elastic foundation.

3. Numerical results and discussion

This research studied composite plates under the present of an exciting force \( q = Q \sin \Omega t \). \( Q \) is the amplitude of exciting force and \( \Omega \) is the frequency of the force. Numerical results for dynamic response and vibration of the composite plates are obtained by Runge–Kutta method.

We performed density functional theory calculations to estimate the elastic modulus of the single-layer penta-graphene. The structure of a penta-graphene sheet was derived from T12-carbon. Using fitting coefficients, we have estimated \( Q_{11} \) of 201.4 GPa nm and \( Q_{22} \) of 208.4 GPa nm, \( Q_{32} \) of 18.6 GPa nm and the \( Q_{66} \) elastic constant was approximately 149.8 GPa, and thermal expansion coefficients of penta-graphene \( (\alpha_1, \alpha_2) \) are (6.128, 6.128) \( 10^6 /K \), respectively.

| \( Q_{11} \) | \( Q_{22} \) | \( Q_{32} \) | \( Q_{66} \) | \( \alpha_1 \) | \( \alpha_2 \) |
|---|---|---|---|---|---|
| 201.4 | 208.4 | -18.6 | 149.8 | 6.128 | 6.128 |
| GPa nm | GPa nm | GPa nm | GPa | \( 10^6 /K \) | \( 10^6 /K \) |

Table 1. The elastic of the 2D penta-graphene by our calculations
3.1. Validation

Park and Choi [23] studied the vibration of isotropic plates base on first-order shear deformation theory. In order to evaluate the accuracy of the method used in this research, we compared the value of the fundamental frequency parameter $\bar{\omega} = \omega_c a^2 \sqrt{\frac{\rho h}{D}}$ of plates in case of homogeneous plates without elastic foundations with results of Park and Choi [23].

Table 2 presents the influence of ratio length to thickness and ratio length to width on the fundamental frequency of the isotropic plates. From Table 2, the values of the fundamental frequencies obtained in this study very close to the results of Park and Choi [23]. The biggest difference is only 1.456%. This confirms that the method used in this study is completely reliable.

Table 2. Comparison of the fundamental natural frequencies $\bar{\omega} = \omega_c a^2 \sqrt{\frac{\rho h}{D}}$.

| $b/a$ | $h/a$ | $b/a$ | $h/a$ |
|-------|-------|-------|-------|
| Park and Choi [23] | 0.01 | 0.1 | 0.2 | 0.1 | 0.2 |
| Present | 49.5845 | 39.3847 | 19.7322 | 19.0840 | 17.5055 |
| Difference (%) | 0.296 | 0.807 | 1.093 | 0.606 | 0.995 | 1.456 |

3.2. The natural frequency and dynamic response of 2D penta-graphene plates

Table 3. Effect of foundation and ratio $b/a$ on natural frequencies $\left(s^{-1}\right)$ of 2D penta-graphene plates with $\mu = 0, b/h = 100, (m, n) = (1, 1)$.

| $(k_1, k_2)$ | $b/a$ | $b/a$ | $b/a$ |
|--------------|-------|-------|-------|
| (0.1, 0.02) | 1629.0 | 1711.0 | 2401.6 |
| (0.1, 0.04) | 1971.3 | 2063.6 | 2486.9 |
| (0.1, 0.06) | 2152.0 | 2599.6 |
| (0.3, 0.02) | 2275.2 | 2627.1 |
| (0.5, 0.02) | 2543.0 | 2862.2 |

Table 4. Effect of thickness and foundation on natural frequencies $\left(s^{-1}\right)$ of 2D penta-graphene plates with $\mu = 0, b/a = 2$.

| $b/h$ | $(k_1, k_2)$ | $(m, n)$ |
|-------|--------------|----------|
| (1.1) | (1.3) | (1.5) | (3.5) |
| 80 | (0.1, 0.02) | 1227.8 | 1711.0 | 2401.6 | 3353.4 |
| (0.3, 0.04) | 1702.1 | 2115.8 | 2763.1 | 3708.0 |
| (0.5, 0.06) | 2070.5 | 2454.7 | 3082.6 | 4031.6 |
| 90 | (0.1, 0.02) | 1151.7 | 1564.8 | 2166.2 | 3003.4 |
| (0.3, 0.04) | 1641.8 | 1985.7 | 2537.5 | 3357.1 |
| (0.5, 0.06) | 2016.0 | 2331.9 | 2861.0 | 3676.9 |
| 100 | (0.1, 0.02) | 1094.1 | 1451.1 | 1980.8 | 2725.7 |
| (0.3, 0.04) | 1597.2 | 1887.2 | 2362.9 | 3081.6 |
| (0.5, 0.06) | 1976.1 | 2240.0 | 2691.2 | 3400.5 |
Table 3 and Figure 2 show the effect of ratio length to width $a/b$ on the behavior of 2D penta-graphene composite plate. Table 2 shows the natural frequencies of penta-graphene with three case ratio $b/a = (1, 1.5, 2)$. The natural frequencies of the plate increase significantly when the ratio of $b/a$ increases. In case $(k_1, k_2) = (0.1, 0.02)$, respectively, the natural frequencies increase 45.4% when ratio $b/a$ increase from 1 to 2. Figure 3 shows the dynamic response of penta-graphene plate with three cases of $b/a = (2, 2.5, 3)$. From Figure 3, it is noticeable that the amplitude of the fluctuation of the structure is larger when the ratio of $b/a$ decreases.

Table 4 and Figure 3 present the effect of length to thickness $b/h$ on the behavior of 2D penta-graphene composite plate. A glance at Table 4 provided reveals the effect of ratio $b/h$ on natural frequencies of the penta-graphene plate. We considered three values of ratio $b/h = (80, 90, 100)$. It can be seen that natural oscillation frequency decreases when $b/h$ increase. In case $(k_1, k_2) = (0.1, 0.02)$, respectively, the natural frequency decrease from 1227.8 ($s^{-1}$) to 1094.1 ($s^{-1}$) when $b/h$ increase from 80 to 100. Figure 3 provided show the ratio length to thickness $b/h$ affect the dynamic response of penta-graphene plate. Three case of ratio $b/h = (60, 80, 100)$. The value of the amplitude of the penta-graphene plates increases when the value of ratio $b/h$ increase. When the plate is thick, the structure is stronger.

**Figure 2.** The ratio length to width $a/b$ affect the behavior of the plate.

**Figure 3.** The ratio length to thickness $b/h$ affect the behavior of the plate.
We can see the effect of the elastic foundation on the behavior of penta-graphene plate from Table 3, Table 4, Figure 4 and Figure 5. Table 3 and Table 4 present the influence of the elastic foundation on the natural frequency of the plate. From those two tables, the frequency of plate increases significantly when the stiffness of the elastic foundation increases. Figures 4 and 5 show the influence of the foundation on the dynamic response of penta-graphene plates. Figure 4 considered three values of $k_1 = (0.1, 0.5, 0.9)$ (GPa/m). Figure 5 considered three values of $k_2 = (0.02, 0.06, 0.1)$ (GPa.m). Elastic foundation especially Pasternak foundation has a positive effect on the dynamic response of penta-graphene plates. When the stiffness of the elastic foundation increases, the amplitude of the plate will decrease.

![Figure 4. Winkler foundation $k_1$ affect the behavior of the plate.](image)

![Figure 5. Pasternak foundation $k_2$ affect the behavior of the plate.](image)

Figure 6 illustrates the impact of the initial imperfection of plate on the dynamic response of penta-graphene plate. Three case $\mu = (0, 0.1, 0.2)$ was investigated in Figure 6. The plate is perfect when $\mu = 0$. When the initial imperfect coefficient increase, the fluctuation range of the plate decreases.

![Figure 6. Initial imperfection of plate affect the dynamic response of plate.](image)
Figure 7 demonstrates the impact of the thermal environment on the deflection amplitude – time curve of the plate. Three values $\Delta T = (900, 700, 500)$ were investigated in Figure 7. Obviously, the thermal environment has a negative effect on the nonlinear dynamic response of penta-graphene plate. The temperature of the environment causes the amplitude fluctuation of the plate to increase.

![Figure 6. Imperfect coefficient $\mu$ affects the behavior of the plate.](image)

Figure 6. Imperfect coefficient $\mu$ affects the behavior of the plate.

![Figure 7. Thermal environment affects the behavior of the plate.](image)

Figure 7. Thermal environment affects the behavior of the plate.

Figure 8 shows the effect of exciting force amplitude on the dynamic response of penta-graphene plates. In Figure 8, three cases of the excitation force amplitude $Q = \left(500 \text{ kN/m}^2, 400 \text{ kN/m}^2, 300 \text{ kN/m}^2\right)$ were considered. The amplitude of the excited force has a clear negative impact on the dynamic response of penta-graphene plate.

Figure 9 demonstrates the impact of pre-loaded axial compression $P_x$ on the behavior of penta-graphene plates. From Figure 9, the value of the amplitude fluctuation of the plate to increase when the value of pre-loaded axial compression increase from 0 GPa to 0.6 GPa.

Figure 10 introduces the effect of the amplitude $Q$ on the graph of the frequency and amplitude. Obviously, the frequency – amplitude curve in the middle of the frequency – amplitude curves of forced vibration. When the value of the amplitude $Q$ increase, the frequency – amplitude curves of force vibration and free vibration are farther apart.
Figure 8. The amplitude $Q$ affects the behavior of plate.

Figure 9. The pre-loaded axial $P_x$ affect the behavior of plate.

Figure 10. The amplitude $Q$ affects the frequency – amplitude curves of plate.
4. Conclusions

This research analysed the behavior of penta-graphene composite plates on elastic foundation based on the classical plate theory and Airy stress function. The elastic modulus of 2D penta-graphene obtained by fit the strain energy equation and the density functional theory energies.

One highlight in this study was the study of the effect of the thermal environment on the behavior of penta-graphene plate. From the results of the study, the conclusions obtained are:

- The value of the natural frequency of the penta-graphene plate affected by geometrical parameters and elastic foundations has been examined.

- The influences of geometrical parameters on the nonlinear dynamic response curves of the penta-graphene plate are examined.

- The elastic foundations have a positive effect while excitation force and mechanical loads have negative effect on the nonlinear dynamic response curves of the penta-graphene plates.

- The influence of temperature field on the nonlinear dynamic response curves of penta-graphene plates are investigated. Specifically, increasing temperature of the environment adversely affects the behavior of the penta-graphene plates.

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Appendix

\[
A_{11}^* = \frac{A_{12} A_{66} - A_{16}^2}{\Delta},
A_{12}^* = \frac{A_{16} A_{66} - A_{12} A_{66}}{\Delta},
A_{16}^* = \frac{A_{12} A_{26} - A_{16} A_{66}}{\Delta},
\]
\[
A_{22}^* = \frac{A_{11} A_{66} - A_{16}^2}{\Delta},
A_{26}^* = \frac{A_{12} A_{16} - A_{16} A_{26}}{\Delta},
A_{66}^* = \frac{A_{11} A_{22} - A_{12}^2}{\Delta},
\]
\[
\Delta = A_{11} A_{22} A_{66} - A_{11} A_{36}^2 + 2 A_{12} A_{16} A_{26} - A_{16}^2 A_{66} - A_{16}^2 A_{22},
\]
\[
B_{11}^* = A_{11} B_{11} + A_{12} B_{12} + A_{16} B_{16},
B_{12}^* = A_{11} B_{12} + A_{12} B_{12} + A_{16} B_{16},
B_{16}^* = A_{11} B_{16} + A_{12} B_{16} + A_{16} B_{66},
B_{21}^* = A_{12} B_{11} + A_{12} B_{12} + A_{16} B_{16},
B_{22}^* = A_{12} B_{12} + A_{16} B_{26},
B_{26}^* = A_{12} B_{26},
B_{51}^* = A_{16} B_{11} + A_{16} B_{12} + A_{16} B_{66},
B_{52}^* = A_{16} B_{12} + A_{16} B_{22} + A_{16} B_{66},
B_{56}^* = A_{16} B_{26},
D_{11}^* = A_{11} B_{11} + A_{12} B_{12} + A_{16} B_{16},
D_{12}^* = A_{11} B_{12} + A_{12} B_{22} + A_{26} B_{26},
D_{16}^* = A_{11} B_{16} + A_{16} B_{66},
D_{22}^* = A_{12} B_{12} + A_{12} B_{22} + A_{16} B_{66},
D_{26}^* = A_{12} B_{26},
\]

**Appendix**
\( P_1 = B_{21}^* + B_{12}^* + B_{11}^* + B_{22}^* - 2B_{66}^*, P_4 = 2B_{26}^* - B_{61}^* \),
\( P_2 = 2B_{16}^* - B_{62}^*, P_6 = B_{11}B_{12} + B_{12}B_{11}^* + B_{26}B_{61}, P_7 = B_{12}B_{22} + B_{62}B_{26}^* \),
\( P_8 = B_{11}B_{21}^* + B_{12}B_{22}^* + B_{16}B_{62}^* + B_{12}B_{11}^* + B_{22}B_{21}^* + B_{26}B_{61}^* + 4B_{16}B_{16}^* + 4B_{26}B_{26}^* + 4B_{66}B_{66}^* \),
\( P_9 = 2(B_{11}B_{16} + B_{12}B_{26} + B_{16}B_{66}^* + B_{16}B_{11}^* + B_{26}B_{21}^* + B_{66}B_{61}^* ) \),
\( P_{10} = 2(B_{12}B_{16}^* + B_{23}B_{26}^* + B_{26}B_{66}^* + B_{66}B_{12}^* + B_{66}B_{22}^* + B_{66}B_{63}^* ) \).

\( E_1 = 2A_{12} + A_{66}^*, E_2 = B_{11}^* + B_{22}^* - 2B_{66}^*, \)
\( E_3 = 2B_{26}^* - B_{61}^*, E_4 = 2B_{16}^* + B_{62}^* \).

\( F_1 = A_{22}^* \lambda_m^4 + A_{11}^* \delta_n^4 + E_4 \lambda_m^2 \delta_n^2 \),
\( F_2 = 2A_{26}^* \lambda_m^3 \delta_n + 2A_{16}^* \lambda_m \delta_n^3 \),
\( F_3 = -B_{24}^* \lambda_m^4 + B_{12}^* \delta_n^4 - E_2 \lambda_m^2 \delta_n^2 \),
\( F_4 = E_3 \lambda_m^3 \delta_n + E_2 \lambda_m \delta_n^3 \).

\[ J_1 = \frac{A_{12}^* L_1 - A_{12}^* L_2}{A_{11}^* A_{22} - A_{12}^2}, J_2 = \frac{A_{12}^* L_1^* - A_{12}^* L_2}{A_{11}^* A_{22} - A_{12}^2}, J_3 = \frac{A_{12}^* H_1 - A_{12}^* H_2}{A_{11}^* A_{22} - A_{12}^2}, J_4 = \frac{A_{12}^* L_3 - A_{12}^* L_4}{A_{11}^* A_{22} - A_{12}^2} \]
\[ J_5 = A_{12}^* H_1^* - A_{12}^* H_2^*, J_6 = A_{11}^* H_1^* - A_{12}^* H_2^* \]

\[ L_1 = \frac{1}{ab} \left( A_m^* \delta_n^2 + A_m^* \lambda_m^2 \right) \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} + 4A_{16}^* \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} + 4B_{16}^* \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} + 4B_{66}^* \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} \]
\[ L_2 = \frac{1}{ab} \left( A_m^* \delta_n^2 + A_m^* \lambda_m^2 \right) \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} + 4A_{26}^* \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} + 4B_{26}^* \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} + 4B_{66}^* \frac{(Q_2Q_3 - Q_1Q_4)}{Q_1^2 - Q_2^2} \]
\[ L_2 = \frac{\delta_n^2}{8}, L_4 = \frac{\delta_n^2}{8} \]

\( H_1 = D_{11} \alpha_1 + D_{12} \alpha_2 \)
\( H_2 = D_{11} \alpha_1 + D_{22} \alpha_2 \)