An improved superconducting neural circuit and its application for a neural network solving a combinatorial optimization problem

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Abstract. We have proposed a superconducting Hopfield-type neural network for solving the N-Queens problem which is one of combinatorial optimization problems. The sigmoid-shape function of a neuron output is represented by the output of coupled SQUIDs gate consisting of a single-junction and a double-junction SQUIDs. One of the important factors for an improvement of the network performance is an improvement of a threshold characteristic of a neuron circuit. In this paper, we report an improved design of coupled SQUID gates for a superconducting neural network. A step-like function with a steep threshold at a rising edge is desirable for a neuron circuit to solve a combinatorial optimization problem. A neuron circuit is composed of two coupled SQUIDs gates with a cascade connection in order to obtain such characteristics. The designed neuron circuit is fabricated by a 2.5 kA/cm² Nb/AlOx/Nb process. The operation of a fabricated neuron circuit is experimentally demonstrated. Moreover, we discuss about the performance of the neural network using the improved neuron circuits and delayed negative self-connections.

1. Introduction

We have proposed a superconducting Hopfield-type[1] neural network for solving the N-Queens problem which is one of combinatorial optimization problems[2]. The N-Queens problem is a famous combinatorial optimization problem. Its solution involves finding a placement of $N$ queens on an $N \times N$ chess board such that none of the queens are able to capture any other using standard chess moves for a queen. Figure 1 shows the circuit diagram of a neural network for solving the 4-Queens problem; it consists of 16 neurons representing a 4 x 4 chess board. The neuron circuits are composed of coupled SQUIDs(c-SQUIDs) gates[3] with sigmoid-shape functions. The output of a neuron, representing a queen, is connected to the inputs of other neurons restricted by the queen. These synaptic connections are represented by electrical resistances. In Fig. 1, the output of neuron(1,1) is connected to inputs of neurons placed to vertical, horizontal, and diagonal directions by cascade connections of inductances and resistances. Figure 2 shows an example of a dynamic simulation for solving the 4-Queens problem. The neuron outputs have negative polarity, which represents the constraint signals. We can confirm that the state shown in Fig. 2 fall into a correct pattern.

Unfortunately, not all network states can fall into correct patterns because of the local minima of the energy function of a neural network. We are interested in some characteristics of networks using analog Josephson devices; For example, network fluctuations caused by intrinsic Josephson voltage oscillations, network dynamics when a threshold of a neuron device has a hysteretic function, and so
on. These influences can be changed by changing device parameters, and the network can be effectively operated to avoid local minima of an energy function. One of the important factors for an improvement of the network performance is an improvement of a threshold characteristic of a neuron circuit. In this paper, we report an improved design of c-SQUIDs gates for a superconducting neural network. Moreover, we discuss about the performance of the neural network using the improved neuron circuits and delayed negative self-connections.

Figure 1. Circuit diagram of a neural network for solving the 4-Queens problem[1]. Synaptic connections from neuron(1,1) only are shown in this figure.

Figure 2. Dynamic simulation of a neural network for solving the 4-Queens problem. The polarity of the output voltages is negative to represent the constraining signals of neurons.

2. Improvement of the threshold characteristic of a neuron circuit

2.1. Design of an improved neuron circuit

The input-output characteristic of a conventional c-SQUIDs gate has a round shape at a vicinity of the rising edge of the threshold. This round shape causes a decrease of an output voltage, namely, a constraint signal to other neurons. A step-like function with a steep threshold at a rising edge is desirable for our neural network, because neurons generating intermediate states except 0 or 1 cause the network to tend to be trapped in local minima on the energy function. We propose two-stage c-SQUIDs with a cascade connection for obtaining such an activation function. Figure 3 shows the two-stage c-SQUIDs gate which is the improved neuron circuit. The first stage c-SQUIDs gate has the same parameters of the original one[2] therefore has the round characteristic(Output1) at the rising edge of the threshold as shown in Fig. 3(b). To obtain a flat output characteristic, $LI_0$ product of rf-SQUID(J4,L4-6) in the second stage is larger than that of the first stage. Although a hysteretic characteristic comes to appear on the single input-output characteristic because of the increase of $LI_0$, product, the output of the second stage(Output2) is not hysteretic because a discrete input current from Output1 passes through the hysteretic region on the second stage.

2.2. Fabrication and measurement of an improved neuron circuit

A neuron circuit consisting of two-stage c-SQUIDs was fabricated using the AIST standard process(STP2) in order to confirm the characteristic. Figure 4 shows a measurement result of a fabricated circuit. Figures 4 (a) and (b) denote the first stage and the second stage output voltages, respectively. An offset current $I_{offset}$ was injected from the offset bias point shown in Fig. 3(a). The output characteristic of the first stage c-SQUIDs shown in Fig. 4(a) has the similar round shape to
conventional c-SQUIDs. On the other hand, that of the second stage has a step-like function with a steep threshold at a rising edge as shown in Fig. 3(b).

![Figure 3](image)

**Figure 3.** Improved design of a neuron circuit. (a) Circuit diagram. (b) Numerical simulation. The device parameters are designed as $I_{c1} = I_{c4} = 0.188 \text{ mA}$, $I_{c2} = I_{c3} = I_{c5} = I_{c6} = 1.50 \text{ mA}$, $L_1 = 1.5 \text{ pH}$, $L_2 = L_3 = 0.75 \text{ pH}$, $L_4 = L_5 = L_6 = 1.35 \text{ pH}$, $L_c = 10.0 \text{ pH}$, $R_c = 0.10 \Omega$, $I_{offset} = 0.32 \text{ mA}$, $I_{bias} = 2.85 \text{ mA}$.

![Figure 4](image)

**Figure 4.** Measurement result of a fabricated neuron circuit. (a) Output voltage $V_{output1}$ of 1st stage c-SQUIDs. (b) Output voltage $V_{output2}$ of 2nd stage c-SQUIDs.

### 3. Numerical estimation of the network performance

To improve the network performance, we introduce the improved neuron circuits described in the previous section and delayed negative self-connections. The self-connection consists of a feedback loop of an inductance ($L = 500 \text{ pH}$) and a synaptic weight ($R = 1.0 \Omega$). The self-connection is added to each of 16 neuron circuits. The delayed negative self-connections cause oscillations when the neuron output is close to the threshold value. Figure 5 shows an example of a dynamic simulation for solving the 4-Queens problem. We can confirm that an incorrect pattern changes into a correct one.

We investigate the network performance using original c-SQUIDs gates and that using the proposed method by numerical simulations. A performance of one network is evaluated using 1000 individual simulation results including unavoidable fluctuations, i.e. thermal noise and circuit parameter variations[2]. Figure 6 shows the dependence of the correct pattern ratio of the 4-Queens problem on the MaCumber parameter $\beta_c ( = 2\pi I_c CR^2/\Phi_0)$. $\beta_c$ is changed by adjusting the two shunt resistors in the double-junction SQUID. Changing $\beta_c$ causes changing the fluctuation of the network due to Josephson oscillation[2]. Open squares denote simulation results of the network using original c-SQUIDs gates. Black circles denote those using two-stage c-SQUIDs gates. We assume that the
networks start to operate from two initial conditions[2] when the offset biases were decreased from an over-biased state (I), and when those were increased from zero (II), respectively. An increase of the ratios of achieving a correct answer is confirmed from Fig. 6. Increasing a fluctuation of the network by increasing $\beta$ induces an increase in escape probability from local minima. So, these results suggest that flat output characteristics generated by two-stage c-SQUIDs gates contribute an increase in escape probability from local minima when a fluctuation arising from Josephson oscillation is relatively large. We confirm that the improved design of neuron circuits and delayed negative self-connections contribute better network performance.

![Figure 5. Dynamic simulation of a neural network two-stage c-SQUID gates and delayed negative self-connections for solving the 4-Queens problem.](image)

**Figure 5.** Dynamic simulation of a neural network two-stage c-SQUID gates and delayed negative self-connections for solving the 4-Queens problem.

![Figure 6. MaCumber parameter dependence of the correct pattern ratio of the 4-Queens problem. (I) and (II) denote the two cases where network offset biases were decreased from an over biased state and increased from zero, respectively.](image)

**Figure 6.** MaCumber parameter dependence of the correct pattern ratio of the 4-Queens problem. (I) and (II) denote the two cases where network offset biases were decreased from an over biased state and increased from zero, respectively.

4. Conclusion
We proposed an improved design of coupled SQUIDs gates for a superconducting neural network. A neuron circuit was composed of cascade connected two coupled SQUIDs gates to obtain the flat output characteristics. The designed neuron circuit was fabricated by AIST standard process(STP2). The operation of the neuron circuit was experimentally confirmed. The improved neuron circuits and delayed negative self-connections were introduced to the network for solving 4-Queens problem in order to improve the network performance. An increase of the ratios of achieving a correct answer was confirmed by numerical simulations of the improved network.

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