HIGGS SECTOR CP-VIOLATION AT THE ELECTROWEAK PHASE TRANSITION

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Abstract

We consider explicit and spontaneous CP-violation related to the profile of the two Higgs doublets at the MSSM electroweak phase transition. We find, in accordance with previous results, that in principle spontaneous CP-violation could exist in the MSSM at finite temperatures, but when the constraints from experiment and the strength of the transition are taken into account, the relevant region of the parameter space is rather restricted. Nevertheless, we show that in this small region, perturbative estimates need not be reliable, and non-perturbatively the region might be slightly larger (or smaller). To allow for more precise perturbative studies and for a lattice study of the non-perturbative corrections, we construct an effective 3d theory for the light stops and the two Higgses in the regime of large $m_Q$. The 3d theory involves CP-violating parameters and allows to determine CP-odd observables.
1 Introduction

The physics problem motivating the studies of the cosmological electroweak phase transition is that it could contribute to the matter-antimatter (baryon) asymmetry of the Universe [1]. Several different ingredients are required for baryon asymmetry generation: one needs anomalous baryon number violating processes in the symmetric high temperature phase, microscopic C- and CP-violation, and thermal non-equilibrium (for a review, see [2]).

Recently, much progress has been made in understanding the order and strength of the electroweak phase transition (which determine whether there is non-equilibrium and what the rate of baryon number violation is in the broken phase), and also in understanding the rate of baryon number violation in the symmetric phase. It has been found that in the Standard Model, there is no electroweak phase transition at all for the experimentally allowed Higgs masses (for a review, see [3]). In the MSSM, in contrast, there can be a first order phase transition, the strength of which has been studied perturbatively up to 2-loop level [4]–[8] and, because of the potentially bad convergence of the perturbative series, non-perturbatively employing dimensional reduction and three-dimensional (3d) lattice simulations [9]–[12]. The conclusion of these studies was that, from the point of view of the strength of the transition, Higgs masses up to 105...110 GeV are allowed for electroweak baryogenesis, provided that the right-handed stop is light, \( m_{\tilde{t}_R} < m_{\text{top}} \).

As to the baryon number violation rate, its parametric form has been determined analytically [13, 14], and numerical estimates exist both in the symmetric phase [15] and in the broken phase [16] of the theory. The latter case serves to determine the magnitude of higher order corrections to the perturbative 1-loop saddle point computation; the corrections were found to be as small as expected.

However, it seems that the third requirement for baryon number generation, namely CP-violation, remains less well understood. If there were no CP-violation, one would produce the same amounts of baryons and anti-baryons, and no net asymmetry would arise. In the Standard Model, there is CP-violation as is experimentally observed in the \( K^0 \)-system, and this phenomenon is explained by the single complex parameter in the Kobayashi-Maskawa matrix. The Standard Model CP-violation is probably not large enough for the purpose of baryon number generation, though (for a review, see [2]). Hence one would need new sources of CP-violation which, at the same time, do not produce any observable effects violating the existing experimental constraints.

An interesting prospect for such a mechanism is provided by the MSSM. Indeed, there are new sources of CP-violation there. Fixing some regularization scale, one can redefine the fields so that the new CP-violating parameters appear in the trilinear couplings of the two Higgs doublets \( H_1, H_2 \) and the squarks:

\[
\mathcal{L}_{\text{CP}} = h_t (A_t^* \tilde{H}_2 - \mu H_1)^\dagger Q_\alpha U_\alpha + \text{H.c.} ,
\]  

(1.1)
where $\tilde{H}_2 = i\sigma_2 H_2^*$, $h_t$ is the top Yukawa coupling, $A_t, \mu$ are complex parameters with the dimension of mass ($A_t$ is induced by soft supersymmetry breaking while $\mu$ is assumed to be the supersymmetric mass parameter appearing in the Higgs sector), $Q_\alpha$ is the left-handed third generation squark, and $U_\alpha$ is the right-handed third generation squark. However, if the CP-violating effects are proportional to the complex phases of $A_t, \mu$, one is in the regime of explicit CP-violation, and then one again has to be careful in order not to violate the experimental constraints (for a review, see [17]).

There is also another possibility for CP-violation in the MSSM. Indeed, since one has two Higgs doublets in the theory, there is the possibility of spontaneous CP-violation [19]. This means that even though all the parameters were real, there could be a dynamically generated phase angle between $H_1, H_2$ (of course, without any explicit CP-violation, an angle and minus the angle are equally likely). The possibility of spontaneous CP-violation is in principle there already at $T = 0$, but in practice the parameter values needed (in particular, a small CP-odd Higgs mass $m_A$) are experimentally excluded [20]. On the other hand, it may be that spontaneous CP-violation is more easily realized at finite temperatures [21], or even that it takes place only in the phase boundary between the symmetric and broken phases [22, 23]. A non-trivial CP-violating profile in the phase boundary could conceivably be quite useful for electroweak baryogenesis [24]–[31], and at the same time, one would not need to worry about the constraints that have to be satisfied in the broken phase at $T = 0$.

The parameter region found in [21, 22, 23] for finite $T$ spontaneous CP-violation consists of relatively small values of $m_A$ and relatively large values of $\tan\beta$. Such values are not particularly ideal from the point of view of the strength of the transition, but could still be allowed in the small stop regime [8].

In this paper, we analyze the general prospects for finite $T$ spontaneous CP-violation in some detail. The emphasis is on understanding what kind of perturbative and non-perturbative effects there are, and whether the non-perturbative effects could be studied with lattice simulations. As a tool for these discussions, we construct an effective 3d theory which involves the CP-violating dynamics. Finally, we explain how this theory can be used for more precise perturbative and non-perturbative numerical studies than carried out here.

It should be noted that all these finite $T$ studies concern the thermodynamical equilibrium situation at $T = T_c$. During the actual history of the electroweak phase transition, the static equilibrium situation with $T = T_c$ is only reached if reheating to $T_c$ takes place after the transition, which is not generically true [32, 8]. However, understanding the equilibrium situation is clearly a good starting point.

The plan of the paper is the following. In Sec. 2 we briefly describe the functional form of the relevant dimensionally reduced effective 3d theory, and the parametric magnitude of its couplings. In Sec. 3 we study, parametrically, what kinds of “instabilities”
could in principle occur. Such possible instabilities are the deviation of an effective \( \tan \beta \)-parameter from a constant value within the phase boundary, spontaneous CP-violation, and the breaking of the charge neutrality of the vacuum. In Sec. 4 we discuss the profile of \( \tan \beta \) in some more detail, in particular with respect to whether it could be studied with lattice simulation. In Sec. 5 we do the same for spontaneous CP-violation. We conclude in Sec. 6. The details related to the derivation of the effective 3d theory described in Sec. 2 are presented in the Appendix.

2 The effective 3d theory for \( H_1, H_2, U \)

To discuss the physics of finite temperature CP-violation, we shall first construct the corresponding effective 3d theory, using the method of dimensional reduction applied to systems with phase transitions \[33\]. The motivation for this approach is that it implements automatically the resummations needed at finite temperatures and is thus the simplest way of seeing which infrared (IR) problems can be cured and which remain non-perturbative. The 3d theory may also allow for lattice simulations, as discussed below\[4\].

We work in the light stop regime, so that the dynamical degrees of freedom appearing in the effective theory are the SU(2) and SU(3) gauge fields, the two Higgs doublets \( H_1, H_2 \), and the right-handed stop \( U \) (in principle, the U(1) gauge field should also be kept there, but it is expected to induce only small corrections \[35\]). Moreover, we assume that the left-handed stop mass parameter is relatively large, \( m_Q \gtrsim 0.6 \) TeV, whereas the mixing parameters and the other mass parameters are not excessively so: \( |A_t| \lesssim m_Q \), and \( |\mu| \) and the other mass parameters are small compared with \( \pi T \). The details of the derivation of the effective theory are in the Appendix. We need here the main features only.

The effective 3d Lagrangian is the most general gauge-invariant Lagrangian involving the given degrees of freedom:

\[
\mathcal{L}_{3d} = \frac{1}{4} F_{ij} F_{ij} + \frac{1}{4} G_{ij}^A G_{ij}^A + (D_i^a H_1)\dagger (D_i^a H_1) + (D_i^a H_2)\dagger (D_i^a H_2) + (D_i^s U)\dagger (D_i^s U)
\]
\[
+ m_1^2(T) H_1\dagger H_1 + m_2^2(T) H_2\dagger H_2 + [m_{12}^2(T) H_1\dagger \tilde{H}_2 + \text{H.c.}] + m_1^2(T) U\dagger U + \gamma_1 U\dagger U H_1\dagger H_1 + \gamma_2 U\dagger U H_2\dagger H_2 + [\gamma_{12} U\dagger U H_1\dagger \tilde{H}_2 + \text{H.c.}] + \lambda_U(U\dagger U)^2
\]
\[
+ \lambda_1 (H_1\dagger H_1)^2 + \lambda_2 (H_2\dagger H_2)^2 + \lambda_3 H_1\dagger H_1 H_2\dagger H_2 + \lambda_4 H_1\dagger \tilde{H}_2 H_2\dagger H_1 + [\lambda_5 (H_1\dagger \tilde{H}_2)^2 + \lambda_6 H_1\dagger H_1 \tilde{H}_2 + \lambda_7 H_2\dagger H_2 \tilde{H}_1 + \text{H.c.}],
\]

\[(2.1)\]

\[\text{(2.1)}\]

It is interesting to note that the very good accuracy of dimensional reduction (in the Standard Model) has recently been confirmed also fully non-perturbatively with 4d finite temperature lattice simulations \[34\].
where \( D^\mu_i = \partial_i - igt^a A_i^a \), \( D^\mu_i = \partial_i - igsT^A C_i^A \) are the SU(2) and SU(3) covariant derivatives, \( t^a = \sigma^a / 2 \) where \( \sigma^a \) are the Pauli matrices, and \( U \) denotes the complex conjugate of the original right-handed stop field appearing in Eqs. (1.1), (A.94). Of the parameters in Eq. (2.1), \( m_{12}^2, \gamma_1, \lambda_5, \lambda_6, \lambda_7 \) can in principle be complex. The complex phases are due to the couplings \( A_t, \mu \) in Eq. (1.1). On the other hand, in the case of spontaneous CP-violation, all the parameters can be real.

It should be noted that after dimensional reduction, the effective theory also contains the zero components of the gauge fields \( A_0^a, C_0^A \), with mass terms \( (1/2)m_{10}^2 A_0^a A_0^a, (1/2)m_{10}^2 C_0^A C_0^A \), and, in principle, all the possible gauge-invariant quartic interactions involving \( A_0^a, C_0^A \) and the degrees of freedom in Eq. (2.1) (most of these can be constructed out of the gauge-invariant bilinears \( A_0^a A_0^a, C_0^A C_0^A, H_1^i H_1, H_2^i H_2, H_1^i H_2, U^i U \)). However, as argued in Appendix A.7, these terms are not essential for the present discussion, and thus \( A_0^a, C_0^A \) can be integrated out.

It is also important to note that all other CP-violating operators, such as those related to \( F \bar{F} \) (see, e.g., [24]), result in higher-order operators whose contributions to Higgs sector CP-violation are suppressed relative to the effects arising within the theory in Eq. (2.1) [36].

The expressions for the couplings in Eq. (2.1) arising from dimensional reduction are summarized in Appendix A.7 (see also [3, 10, 11, 21, 22, 23]). Here it is sufficient to know the leading parametric behaviour of the couplings: omitting multiplicative numerical factors and replacing logarithms by terms of order unity in the 1-loop terms proportional to \( 1/(16\pi^2) \), we get

\[
\begin{align*}
    m_1^2(T) & \sim m^2_1 + \left( \frac{3}{8} g^2 - \frac{1}{4} h_t^2 |\tilde{\mu}|^2 \right) T^2, \\
    m_2^2(T) & \sim m^2_2 + \left( \frac{3}{8} g^2 + \frac{1}{2} h_t^2 - \frac{1}{4} h_t^2 |\tilde{A}_t|^2 \right) T^2, \\
    m_{12}^2(T) & \sim m_{12}^2 + \frac{1}{4} h_t^2 \tilde{A}_t^\ast \tilde{\mu}^* T^2 - \frac{g^2}{16\pi^2} M_2 \mu^* + \frac{h_t^2}{16\pi^2} A_t^\ast \mu^*, \\
    m_{U^\ast}(T) & \sim m_U^2 + \left( \frac{2}{3} g^2 + \frac{1}{3} h_t^2 - \frac{1}{6} h_t^2 (|\tilde{A}_t|^2 + |\tilde{\mu}|^2) \right) T^2, \\
    \lambda_U & \sim \frac{1}{6} g^2, \quad \gamma_1 \sim -h_t^2 |\tilde{\mu}|^2, \quad \gamma_2 \sim h_t^2 (1 - |\tilde{A}_t|^2), \\
    \gamma_{12} & \sim h_t^2 \tilde{A}_t^\ast \tilde{\mu}^* - \frac{g^2 h_t^2}{16\pi^2} \left[ \frac{m_{12}^2}{(2\pi T)^2} + \frac{M_2 \mu^*}{(\pi T)^2} \right], \\
    \lambda_1 & \sim \frac{1}{8} (g^2 + g^2), \quad \lambda_2 \sim \frac{1}{8} (g^2 + g^2) + \frac{h_t^4}{16\pi^2}, \\
    \lambda_3 & \sim \frac{1}{4} (g^2 - g^2), \quad \lambda_4 \sim -\frac{1}{2} g^2, \\
    \lambda_5 & \sim \frac{1}{16\pi^2} \left[ h_t^4 (\tilde{A}_t^\ast \tilde{\mu}^*)^2 - g^4 \frac{m_{12}^4}{(2\pi T)^4} + g^4 \frac{(M_2 \mu^*)^2}{(\pi T)^4} \right],
\end{align*}
\]

(2.2) (2.3) (2.4) (2.5) (2.6) (2.7) (2.8) (2.9) (2.10)
\[ \lambda_6 \sim \frac{1}{16\pi^2} \left[ -g^2 h_t^2 \hat{A}_t^{\mu*} - g^4 \frac{m_{12}^2}{(2\pi T)^2} - g^4 \frac{M_2}{(\pi T)^2} \right], \quad (2.11) \]

\[ \lambda_7 \sim \frac{1}{16\pi^2} \left[ -h_t^4 \hat{A}_t^{\mu*} - g^4 \frac{m_{12}^2}{(2\pi T)^2} - g^4 \frac{M_2}{(\pi T)^2} \right], \quad (2.12) \]

where \( m_{1}^2, m_{2}^2, m_{12}^2, m_U^2 \) denote the zero-temperature mass parameters, \( M_2 \) is the SU(2) gaugino mass parameter, and

\[ \hat{A}_t \equiv \frac{A_t}{m_Q}, \quad \hat{\mu} \equiv \frac{\mu}{m_Q}. \quad (2.13) \]

The signs (which do have some significance) have been obtained by assuming that \( m_Q \sim 1 \text{ TeV}, T \sim 100 \text{ GeV}, |A_t| \sim |\mu| \lesssim 100 \text{ GeV}; \) for the general case, see Eqs. (A.137)–(A.151). It should be noted that loop effects within the effective theory may generate contributions which are larger than those in Eqs. (2.10)–(2.12); see Sec. 5.

We have written in Eqs. (2.1)–(2.12) the couplings in 4d units, so that there is an overall factor \( \int \beta_0 d\tau = 1/T \) in the action. This factor can, as usual, be defined away by a scaling of the fields, and the result is that the quartic couplings get simply multiplied by \( T \).

Note that the form of Eq. (2.1) imposes several constraints on the parameters. There are various kinds of constraints (see, e.g., [57]). First, the quartic part of the potential should grow (or remain the same) in every direction of the field space, for the field values that the effective theory is applicable. Otherwise the potential is “unbounded from below”. Second, the standard electroweak minimum should be at least a metastable one at low temperatures.

Denoting by \( v_1, v_2 \) the expectation values of the neutral components of \( H_1, H_2, \tan\beta = v_2/v_1 \), and

\[ \gamma_{\text{eff}} = h_t^2 \left[ -|\hat{\mu}|^2 \cos^2 \beta + (1 - |\hat{A}_t|^2) \sin^2 \beta + (\hat{A}_t^{\mu*} + \hat{A}_t \hat{\mu}) \cos \beta \sin \beta \right], \quad (2.14) \]

the first constraint says that for all values of \( \beta \) that lead to \( \gamma_{\text{eff}} < 0 \), one must require

\[ \gamma_{\text{eff}}^2 < \frac{1}{12} \tilde{g}^2 g^2 \cos^2 2\beta, \quad (2.15) \]

where \( \tilde{g}^2 = g^2 + g'^2 \). For instance, taking \( \tan\beta = 0 \), this means that there is an instability in the plane \( v_2 = 0 \) unless

\[ h_t^4 |\hat{\mu}|^4 < \frac{\tilde{g}^2 g^2}{12}, \quad (2.16) \]

Thus, we have to require at least that \( |\hat{\mu}| \ll 0.4 \). There is a similar but weaker constraint for \( \hat{A}_t, \hat{A}_t \lesssim 1 \).
The second constraint applies in the broken electroweak phase, for the physical value of \( \tan \beta \). Then the simplest requirement is just that the right-handed stop mass squared be positive, which means that for the physical value of \( \tan \beta \), \( \gamma_{\text{eff}} \) must be positive (for the values of \( m_U^2 \) we are interested in), and moreover, that

\[
1 - |\hat{A}_t - \hat{\mu}^* \cot \beta|^2 > -\frac{m_U^2}{m_{\text{top}}^2}.
\]

This constraint is typically weaker than the one in Eq. (2.13). Another version of Eq. (2.17) applies at finite temperatures when one requires that the history of the Early Universe leads us to the standard electroweak minimum [6, 7, 8].

In order to satisfy these constraints, we assume that \(|\hat{A}_t|^2, |\hat{\mu}|^2 \ll 1\).

### 3 Parametric estimates for instabilities, and when is further reduction possible?

Let us now inspect the Higgs doublet part of the action in Eq. (2.1). Taking into account the tree-level expressions for the parameters \( m_1^2, m_2^2 \) (see Eqs. (A.137), (A.138)), we observe that the sum of the eigenvalues of the Higgs mass matrix is

\[
m_1^2(T) + m_2^2(T) = m_A^2 + \left(\frac{3}{4}g^2 + \frac{1}{2}h_l^2 - \frac{1}{4}h_l^2(|\hat{A}_t|^2 + |\hat{\mu}|^2) + \mathcal{O}(g^3)\right)T^2 \sim (gT)^2.
\]

Here \( m_A \) is the CP-odd Higgs mass, we use the parametric convention \( g \sim h_t \sim g_S \), and we assume that \(|\hat{A}_t|^2 + |\hat{\mu}|^2 \ll 1\), see Sec. 3. On the other hand, at the phase transition point, there is a direction along which one of the Higgs doublets is light, \( m_{\text{light}}^2 \sim (g^2T)^2 \) (this is because the phase transition takes at the tree-level place when the second derivative of the effective potential vanishes in some direction). Then the other Higgs doublet must be heavy, \( m_{\text{heavy}}^2 \sim (gT)^2 \), since the sum of the eigenvalues is constrained by Eq. (3.1). Thus the question arises, how could there be instabilities such as spontaneous CP-violation, since one of the Higgs doublets is heavy and could possibly be integrated out, as done in [9, 10, 11]? If there is just one dynamical Higgs doublet, there cannot be any spontaneous CP-violation.

To establish rules for power-counting, recall that the first order electroweak phase transition is generated typically by gauge field (or stop) 1-loop terms of the form \( -Tm_3^3 \) (or \( -Tm_3^3 \)). Moreover, let us assume that the quartic scalar self-coupling \( \lambda \) is of the parametric order \( g^2 \). Then, according to the standard argument, the phase transition takes place when the tree-level quadratic and quartic terms are of the same order of magnitude as the 1-loop contribution:

\[
g^4T^2v^2 \sim g^3Tv^3 \sim g^2v^4,
\]

\( (3.2) \)
where \( v \) denotes the light Higgs field expectation value. It follows that \( v \sim gT \). Note that numerically, one would like to have in the broken phase that \( v \sim gT \) due to the sphaleron erasure bound \([1, 2]\), but since the bosonic temperature scale means really \( \sim 2\pi T \), even these values can parametrically be thought of as \( \sim gT \). The actual convergence of the perturbative expansion of course gets better with an increasing numerical value of \( v \).

Thus, we discuss values of order \( v \sim gT \) below. Let us stress that these estimates are assumed to hold also within the phase boundary, even though strictly speaking a fixed value of \( v \) is only obtained in one of the homogeneous (meta)stable phases.

### 3.1 Estimates in the diagonalized theory

Let us now diagonalize the two Higgs doublet model mass matrix (consisting of \( m_1^2(T), m_2^2(T), m_12^2(T) \) in Eqs. \((2.2)-(2.4)) at the phase transition point. We denote \( m^2 = m_{\text{light}}^2, M^2 = m_{\text{heavy}}^2 \), and the corresponding fields by \( h, H \), with vacuum expectation values \( v_h, v_H \) (note that \( h, H \) are linear combinations of \( H_1, \tilde{H}_2 \)). Then, the diagonalized Higgs potential is of the form

\[
V(h, H) = m^2 h^\dagger h + M^2 H^\dagger H + \lambda_1 (h^\dagger h)^2 + \lambda_2 (H^\dagger H)^2 + \lambda_3 h^\dagger H H^\dagger h + \lambda_4 h^\dagger H H^\dagger h \\
+ \left[ \lambda_5 (h^\dagger H)^2 + \lambda_6 h^\dagger h h^\dagger H + \lambda_7 H^\dagger H h^\dagger H + \text{H.c.} \right].
\]

The couplings \( \lambda_1, ..., \lambda_7 \) are some linear combinations of those in Eq. \((2.1)) (see, e.g., Eq. \((6.21)) in \([9]\)), but for simplicity we do not introduce a new notation.

Since one would expect that the dynamics is dominated by the light field \( h \), consider the set of Green’s functions with only \( h \) in the outer legs. Conceivably one can, order by order in perturbation theory, organize the contributions of the heavy fields \( H \), with vacuum expectation values \( v_h, v_H \) (note that \( h, H \) are linear combinations of \( H_1, \tilde{H}_2 \)). Then, the diagonalized Higgs potential is of the form

\[
V(h, H) = m^2 h^\dagger h + M^2 H^\dagger H + \lambda_1 (h^\dagger h)^2 + \lambda_2 (H^\dagger H)^2 + \lambda_3 h^\dagger H H^\dagger h + \lambda_4 h^\dagger H H^\dagger h \\
+ \left[ \lambda_5 (h^\dagger H)^2 + \lambda_6 h^\dagger h h^\dagger H + \lambda_7 H^\dagger H h^\dagger H + \text{H.c.} \right].
\]

The expressions for the effective parameters \( m_{\text{eff}}^2, \lambda_{\text{eff}} \) have been discussed in \([1]\): e.g.,

\[
\lambda_{\text{eff}} = \lambda_1 - \frac{T}{8\pi M} \left( \lambda_3^2 + \lambda_3 \lambda_4 + \frac{1}{2} \lambda_4^2 + 2|\lambda_5|^2 + 12|\lambda_6|^2 - 6\lambda_6^2 \lambda_7 - 6\lambda_6 \lambda_7^2 \right).
\]

The complication in the derivation of these parameters is that due to the interactions proportional to \( \lambda_6, \lambda_7 \), which break the \( h \to -h, H \to -H \) symmetries, there are reducible diagrams contributing to the effective parameters, related to the mixing between \( h, H \) generated at 1-loop level (see \([2]\)).

However, clearly the construction of the theory in Eq. \((3.4)) cannot be valid for arbitrary quartic couplings: one must require that \( \lambda_i T/M \ll 1 \) for the loop expansion parameters related to corrections such as those in Eq. \((3.3)) to be small. Parametrically,
this means that $\lambda_i \lesssim \mathcal{O}(g^2)$. Now, it is couplings at most of this order that arise from dimensional reduction, see Eqs. (2.6)–(2.12). Let us thus inspect whether this regime is consistent with instabilities such as spontaneous CP-violation.

To analyse the kind of instabilities that can appear in the theory in Eq. (2.1), it is illuminating to write $V(h, H)$ in terms of other sets of variables. Note first that $V(h, H)$ can be written as a 2nd order polynomial in the explicitly real and gauge invariant operators

$$O_1 = h^\dagger h, \quad O_2 = H^\dagger H, \quad O_3 = \text{Re} h^\dagger H, \quad O_4 = \text{Im} h^\dagger H.$$  \hspace{1cm} (3.6)

These operators are constrained by the inequality $O_3^2 + O_4^2 \leq O_1 O_2$, but the existence of four operators nevertheless shows that there are, in general, four independent physical directions in the field space. However, the representation of the potential in terms of the $O_i$'s is not exceedingly useful for studying these different directions, due to the constraint.

For perturbative studies, a more useful representation is obtained by using global SU(2) and U(1) transformations to rotate $h$ to unitary gauge and to remove one phase angle from $H$. Moreover, let us introduce an angular variable $\beta$ by defining $\tan \beta = v_H/v_h$, $v^2 = v_h^2 + v_H^2$ (note that this $\tan \beta$ is not the same as the usual zero-temperature parameter $\tan \beta = v_2/v_1$ of the MSSM, since we have already made a rotation to a basis where $m^2_{12}(T) = 0$, to arrive at Eq. (3.3)). Then, the two doublets can be written as

$$h = \frac{v}{\sqrt{2}} \begin{pmatrix} \cos \beta \\ 0 \end{pmatrix}, \quad H = \frac{v}{\sqrt{2}} \begin{pmatrix} \sin \beta \cos \theta e^{i\phi} \\ \sin \beta \sin \theta \end{pmatrix}.$$  \hspace{1cm} (3.7)

In terms of these variables, the tree-level potential in Eq. (3.3) becomes

$$V(h, H) = \frac{1}{2} v^2 \left( m^2 \cos^2 \beta + M^2 \sin^2 \beta \right)$$

$$+ \frac{1}{4} v^4 \left[ \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{1}{4} \sin^2 2\beta \left[ \lambda_3 + \cos^2 \theta (\lambda_4 + 2\lambda_5 \cos 2\phi) \right] \right.$$ 

$$+ \sin 2\beta \cos \theta \cos \phi (\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta) \right].$$  \hspace{1cm} (3.8)

There are now three possible types of deviations from the naive case that the heavy field $H$ is zero and only the light field $h$ is dynamical (i.e., $\beta = 0$). First, $H$ can get a non-vanishing expectation value, corresponding to $\beta \neq 0$. As we will see, this deviation generically takes place and does not introduce any qualitative changes in the thermodynamical properties of the system. It turns out that $\beta$ typically depends on the value of $v$.

The other two changes are more drastic in the sense that they correspond to genuinely new “phases”. Both of these deviations require that $\beta \neq 0$, see Eq. (3.7). The first possibility is that $\theta$ can be non-zero. This means that the doublet $H$ in Eq. (3.7) has both upper and lower components non-vanishing. When gauge fields are taken into
account, this would mean that the photon becomes massive, and acts thus as a (non-local) order parameter for this phase (another way to say it is that the vacuum is not neutral). Clearly, the parameters are expected to be such that this possibility is not realized.

The second possibility is that $\phi \neq 0$. This case corresponds to spontaneous CP-violation, if all the parameters appearing in Eq. (3.8) are real. Indeed, for the scalar fields, we can define the C-transformation as complex conjugation, while the P-transformation reverses the signs of the spatial coordinates but leaves the scalar fields intact (for a more general analysis, see [38]). Then

$$h \overset{\text{CP}}{\to} h^*, \quad H \overset{\text{CP}}{\to} H^*. \quad (3.9)$$

The action corresponding to Eq. (2.1) is invariant under this transformation, for real parameters. However, the angle $\phi$ and the gauge-invariant operator $O_4$ in Eq. (3.6) are not invariant. Let us denote $O_{\text{CP}} = O_4$: $O_{\text{CP}}$ acts as an order parameter for this phase.

**The implications of $\beta \neq 0$**

Due to the fact that $M^2 \sim (gT)^2$ is large, we can assume all the three kinds of deviations to be small. For $\beta$ this will be justified presently; for $\theta, \phi$ this assumption is no restriction, since, without a loss of generality, we are choosing to inspect when the origin $\theta = 0, \phi = 0$ becomes unstable (one may need to invert the signs of $\lambda_6, \lambda_7$ for this). Expanding to second order in the angles in Eq. (3.8) and noting that $\lambda_i v^4 \sim g^2(gT)^4 \ll M^2 v^2 \sim (gT)^4$, it is easy to see that

$$\beta \approx \sin \beta \approx -\frac{1}{2} \lambda_6 v^2 \sim O(g^2). \quad (3.10)$$

Here we assumed that $\lambda_6 \sim O(g^2), v \sim M \sim gT$. Thus, indeed, $\beta$ is in general non-vanishing but small, and depends on $v$.

Let us now inspect what a value $\beta \neq 0$ means for the thermodynamics of the system and, in particular, for the construction of the effective theory in Eq. (3.4), which is supposed to account for the thermodynamics. As seen from Eq. (3.8), a non-zero value of $\beta$ causes a shift in the free energy of the order

$$\frac{1}{2} M^2 v^2 \beta^2 + \frac{1}{2} \lambda_6 v^4 \beta \approx -\frac{1}{8} \lambda_6^2 v^6 \sim O(g^8 T^4). \quad (3.11)$$

Is this contribution contained in the theory in Eq. (3.4)? The answer is, no, since the term in Eq. (3.11) corresponds to a higher-order operator $\sim (h^\dagger h)^3$. However, this contribution is suppressed by the relative amount $O(g^2)$ with respect to contributions within the effective theory in Eq. (3.4), since they are of the order

$$m_\text{eff}^2 v^2 \sim \lambda_\text{eff} v^4 \sim O(g^6 T^4). \quad (3.12)$$

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(The same estimate arises by comparing Eq. (3.11) directly with the tree-level 6-point function generated within the effective theory, $\sim (\lambda_{\text{eff}}^2/m_{\text{eff}}^2)v^6$.) Due to the relative suppression $O(g^2)$, the negative sign of Eq. (3.11) is also not a problem in the range of $v$ considered. Thus, the effective theory is still useful and accurate in the phase transition region (for weak coupling), and $\beta \neq 0$ does not change the properties of the system qualitatively.

As a side remark, let us note that, for $\lambda_6 \sim O(g^2)$, the accuracy estimate $O(g^2)$ above differs from the one, $O(g^3)$, in [33, 36]. The reason is that with the present type of interactions, there can be reducible (left) diagrams with a heavy internal line, in additional to irreducible ones (right):

In the Standard Model case considered in [33, 36], in contrast, there are only irreducible diagrams, whose contribution is suppressed by one power of $\lambda_3 T/M \sim O(g)$ with respect to the reducible ones. In the MSSM, $\lambda_6$ is in fact typically parametrically smaller than $O(g^2)$, so that even there one may get errors smaller than $O(g^2)$.

**The implications of $\phi, \theta \neq 0$**

Let us then look at the directions $\phi, \theta$. To quadratic order, the part of the effective potential depending on them is

$$\frac{1}{4} v^4 \left\{ \beta^2 \left[ \lambda_4 (1 - \theta^2) + 2 \lambda_5 (1 - \theta^2 - 2 \phi^2) \right] + 2 \beta \lambda_6 \left[ 1 - \frac{1}{2} (\theta^2 + \phi^2) \right] \right\}. \quad (3.13)$$

Thus, $\phi$ can be non-zero if

$$\beta^2 (4 \lambda_5) + \beta \lambda_6 > 0, \quad (3.14)$$

while $\theta$ can be non-zero if

$$\beta^2 (\lambda_4 + 2 \lambda_5) + \beta \lambda_6 > 0. \quad (3.15)$$

Suppose that all the couplings $\lambda_i$ are of the same order of magnitude ($\sim g^2$). Then, the $\beta^2$- terms in Eqs. (3.14), (3.15) are of order $O(g^6)$ since $\beta \sim O(g^2)$ and can be neglected, and the L.H.S. of Eq. (3.14) is, according to Eq. (3.10),

$$\beta \lambda_6 \approx -\frac{1}{2} \frac{\lambda_6^2 v^2}{M^2} < 0. \quad (3.16)$$

In other words, for couplings of this order of magnitude, the inequality in Eq. (3.14) is never satisfied, $\phi$ is zero, and CP is not spontaneously violated!
When can \( \phi \) then be non-zero? It is seen from Eq. (3.14) that one must have \( \lambda_5 > 0 \) and

\[
\lambda_5 > \frac{\lambda_6}{4\beta} \approx \frac{1}{2} \frac{M^2}{v^2} \sim \mathcal{O}(1).
\]

(Recall that we assume \( v \sim M \sim gT \).) Hence, CP can be spontaneously violated only if \( \lambda_5 \) is of the parametric order \( \mathcal{O}(1) \)! Clearly, in that case the construction of the effective theory in Eq. (3.4) is not valid.

Another possibility is that \( \lambda_7 \), which did not play any role in the previous estimates, is large. But to be effective, it has to be at least of the order

\[
\lambda_7 \sim \frac{\lambda_6}{\beta^2} \sim \lambda_6 \mathcal{O}(g^{-4}).
\]

If \( \lambda_6 \sim \mathcal{O}(g^2) \), \( \lambda_7 \) needs to be even larger than \( \lambda_5 \), and the reduction in Eq. (3.3) again does not work.

It is seen from Eq. (3.15) that the conditions for the breaking of \( \theta \) are similar as those for the breaking of \( \phi \), except that \( \lambda_4 < 0 \) can have a further stabilizing effect.

In conclusion, CP could in principle be spontaneously violated, and the photon could even become massive, if the quartic couplings of the two Higgs doublet model are suitable. In such a case, further reduction into a single Higgs doublet model does not work, because some of the expansion parameters \( \mathcal{O}(\lambda_i T/M) \) would be large. However, this case is not a natural consequence of dimensional reduction in the MSSM, since the couplings produced this way are of too small a parametric magnitude, see Eqs. (2.6)–(2.12) (this statement is true even after radiative effects within the effective theory are taken into account, see Sec. 5). To get spontaneous CP-violation in practice, one thus needs somewhat large numerical factors and suitable parameter values, which overcome the parametric suppression. We will study these issues in Sec. 4.

### 3.2 Estimates in the original theory

In the estimates so far we used for simplicity the diagonalized effective theory in Eq. (3.3). For completeness, it is perhaps good to see how the same estimates arise in terms of the original variables, before a redefinition which allowed us to get rid of \( m_{12}^2 \).

Let us start by reviewing the general tree-level condition for spontaneous CP-violation within the theory in Eq. (2.1) (this discussion corresponds to those in [21, 22]). Putting now \( \theta \to 0 \), we choose, corresponding to Eq. (3.4),

\[
H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{-i\phi} \end{pmatrix}, \quad H_1^\dagger \tilde{H}_2 = \frac{1}{2} v_1 v_2 e^{i\phi}.
\]

Then, the part of the potential depending on \( \phi \) is

\[
V(\cos \phi) = m_{12}^2(T) v_1 v_2 \cos \phi + \frac{1}{2} \left[ \lambda_5 v_1^2 v_2 (2 \cos^2 \phi - 1) + \lambda_6 v_1^2 v_2 \cos \phi + \lambda_7 v_2^2 v_1 \cos \phi \right].
\]

(3.20)
Denoting again $\tan \beta = v_2/v_1$, we note that the second order polynomial $V(\cos \phi)$ has a non-trivial minimum, if $\lambda_5 > 0$ and

$$|f(\beta, v)| = \left| \frac{m_{12}^2(T) + (1/2)(\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta)v^2}{2\lambda_5 \sin \beta \cos \beta v^2} \right| < 1,$$

(3.21)

in which case $\cos \phi = -f(\beta, v)$. These are the standard (tree-level) conditions for the existence of spontaneous CP-violation [19].

Suppose that $\lambda_6, \lambda_7 \lesssim O(g^2)$, as one would like to have. Then, $m_{12}^2(T)$ must be of the order $O(g^4T^2)$ to compensate for the terms proportional to $\lambda_6 v^2, \lambda_7 v^2$ in Eq. (3.21). Since $m_{12}^2(T)$ is small, one of the masses $m_1^2(T), m_2^2(T)$ has to be large, $\sim (gT)^2$, in order for the eigenvalues to sum up to Eq. (3.1). Let us assume that this role is played by $m_2^2$: $m_2^2(T) = M^2 + O(g^2T^2)$. Then, it can be seen from the Higgs part in Eq. (2.1) that

$$\sin \beta \approx -\frac{m_{12}^2 + (1/2)\lambda_6 v^2}{M^2} \lesssim O(g^2).$$

(3.22)

But this means that the term proportional to $\lambda_7$ in Eq. (3.21) can again be neglected, unless $\lambda_7$ is very large, and that Eq. (3.21) becomes

$$|f(\beta, v)| \approx \frac{M^2 \sin \beta}{2\lambda_5 \sin \beta v^2} \approx \frac{M^2}{2\lambda_5 v^2}.$$  

(3.23)

Thus, we are again lead to the parametric estimate $\lambda_5 \gtrsim O(1)$ in order to have $|f(\beta, v)| < 1$. Large numerical values of $v$ (a strong transition) allow for smaller numerical values of $\lambda_5$.

To summarize, we have found that according to tree-level considerations, it is not enough to have $\lambda_5 > 0$ to get spontaneous CP-violation, but one should have $\lambda_5 \gtrsim O(1)$. The reason is that with the type of mass parameters and couplings that appear in the MSSM, the numerator and the denominator in Eq. (3.21) tend to vanish at the same point, when the equations of motion for $\sin \beta$ are taken into account.

### 4 The wall profile

In the previous section, we saw that even the heavy Higgs doublet $H$ can have a non-zero (even though small) expectation value in the broken phase, and a corresponding non-trivial profile at the phase boundary (since $\beta$ typically depends on $v$, see Eq. (3.10)). In the usual perturbative language in terms of the doublets $H_1, H_2$, the possible non-trivial profile means that the ratio of field values, $\tan \beta = v_2/v_1$, can change within the phase boundary. Since the typical values of $H$ are very small,

5For completeness, let us note that in the MSSM it is actually $m_2^2(T)$ which is large, but here it is convenient to exchange the fields, to keep the discussion as close to Sec. 3.1 as possible.
\[ \langle H \rangle \sim \langle h \rangle \tan \beta \sim g^3 T \] (see Eqs. (3.2), (3.10)), \( \tan \beta \) is expected to change only by a small amount. Nevertheless, in [27, 29] it is argued that the deviations of \( \tan \beta \) from a constant value are important for baryon number generation in the MSSM (this may not be a necessary requirement, though [30, 8]). The purpose of this section is to discuss briefly the prospects for perturbative and non-perturbative determinations of \( \tan \beta \).

Note first that, parametrically, the profile of \( \tan \beta \), or the expectation value of \( H \), is not computable in perturbation theory. This is simply because, as can be seen from Eq. (3.10), the value of \( H \) is determined by that of \( h \), which in turn is non-perturbative for \( h \sim gT \) (i.e., at least within the phase boundary). Numerically, the convergence might of course happen to be reasonably good, especially if the transition is strong as is required for baryogenesis. The wall profile obtained from the 2-loop Landau-gauge effective potential by solving the classical equations of motion has been computed in [39] (see also [8, 40]).

Let us then discuss whether the profile of \( \tan \beta \) could be determined with lattice simulations. We first give a gauge-invariant generalization for what one may mean with the profile of \( \tan \beta \). A static profile can only exist at the critical temperature. Then one may define

\[
\frac{1}{2} v_i^2 \equiv \left\langle H_i^I H_i \right\rangle - \left\langle H_i^I H_i \right\rangle_{\text{symm.}}, \quad \tan^2 \beta = \frac{v_2^2}{v_1^2},
\]

(4.1)

where \( i = 1, 2 \) and the latter expectation value is taken in the homogeneous “symmetric” high-temperature phase. This definition is clearly gauge and scale independent.

Suppose now that we have a lattice geometry such that planar interfaces are favoured. There is a zero mode related to the location of a planar interface, but this can in principle be removed. Averaging over the set of configurations and over the \( (x_1, x_2) \)-plane, we get a value for \( \tan \beta (x_3) \). This definition should not be sensitive to the lattice spacing. However, this definition is sensitive to the cross-sectional area of the lattice.

Indeed, denote by \( \Delta(x) \), \( x = (x_1, x_2) \), the deviation of the interface location from the average. Then a configuration with \( \Delta(x) \neq 0 \) has the action

\[
S_{\text{int}} \approx \sigma \int d^2 x \frac{1}{2} |\partial_3 \Delta(x)|^2,
\]

(4.2)

where \( \sigma \) is the surface tension. The fluctuations of the measured projection of the interface location \( \Delta \) are distributed as \( p(\Delta) = \int d^2 x \langle \delta(\Delta(x) - \Delta) \rangle \). As a consequence, the effective width \( l \) of the interface seen after averaging over the area \( A \), is expected to diverge as

\[
l^2 \sim \frac{\int d\Delta \Delta^2 p(\Delta)}{\int d\Delta p(\Delta)} \sim \frac{1}{A} \int d^2 x \langle \Delta(x) \rangle^2 \sim \int d^2 k \frac{1}{\sigma k^2} \sim \frac{1}{\sigma} \ln \frac{A}{\xi^2},
\]

(4.3)

where \( \xi \) is of the order of the longest Higgs correlation length.
In conclusion, the profile of the interface can only be studied by averaging over a finite area. This can be accomplished either by restricting the total cross-sectional area, or by averaging the surface profile only over a finite subarea of the large total area (‘coarse-graining’). However, we do not regard this as a fundamental problem. From the practical point of view, a more serious concern is that since the deviations from a constant value of \(\tan \beta\) are expected to be very small, they are most probably not visible from below the statistical noise.

5 Spontaneous CP-violation

In Sec. 3 we saw that, parametrically, spontaneous CP-violation is not naturally realized in the MSSM. However, parametric estimates need not always be reliable due to the actual numerical factors. In this section, we thus look at the issue from a more practical point of view. We will first make some perturbative estimates and then comment on the possibility of lattice simulations.

Let us start by formulating the problem as follows. The question concerning the existence of spontaneous CP-violation can be factorized into two parts:

(1) The perturbative computation of the parameters of the action in Eq. (2.1). This computation is not sensitive to the infrared problems of finite temperature field theory and is thus in principle well convergent.

(2) The determination of the constraint that the parameters of the 3d theory have to satisfy in order for there to be spontaneous CP-violation. As we will discuss, this constraint is sensitive to the infrared problems of finite temperature field theory and is ultimately to be determined non-perturbatively. As a concrete example of the non-perturbative nature of the problem, note that the parameter \(m_{12}^2(T)\) has a logarithmic scale dependence on the 3d renormalization scale parameter \(\Lambda\), \(m_{12}^2(T) \sim (\gamma_1 \gamma_{12} + \ldots) T^2/(16\pi^2) \ln[\Lambda/(g^2T)]\), and the relevant scale for, say, the statement \(m_{12}^2(T) = 0\), cannot be fixed perturbatively.

Concerning item (1), we will assume the expressions in Eqs. (A.137)–(A.151) to be precise enough for our purposes. If needed, the accuracy can be improved upon on two aspects, in particular: the zero-temperature \(\overline{\text{MS}}\)-scheme parameter \(m_{12}^2(\bar{\mu})\) could be fixed even more precisely in terms of the physical observables of the theory by a more accurate vacuum renormalization, and the finite temperature corrections to the parameter \(m_{12}^2(T)\) of the effective 3d theory could be derived up to 2-loop level, to fix its scale dependence (see, e.g., [33] and the last Ref. in [113]).

Concerning item (2), we will here work basically at tree-level, estimating the effects of loop corrections only in the regime where they can be reduced to the construction of a further simplified effective theory. We argue that this approximation contains the
largest uncertainties in the present computation, and we explain in Sec. 3 how it can be improved upon.

5.1 Tree-level constraints inside the 3d theory

The (first) tree-level condition for spontaneous CP-violation is given in Eq. (3.21). To see when it could be satisfied in practice, note first that for very large fields \( v \),

\[
|f(\beta, v)| = \left| \frac{\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta}{2\lambda_5 \sin 2\beta} \right| \gg 1, \tag{5.1}
\]

since typically \(|\lambda_5| \ll |\lambda_6|, |\lambda_7|\), see Eqs. (2.10)–(2.12). For very small fields \( v \to 0 \), on the other hand,

\[
|f(\beta, v)| = \left| \frac{m_{12}^2}{\lambda_5 \sin 2\beta v^2} \right| \gg 1. \tag{5.2}
\]

Moreover, the function \((a + bv^2)/(cv^2)\) is a monotonous function of \( v^2 \). Thus there can only be a region with \(|f(\beta, v)| < 1\) provided that the numerator of Eq. (3.21) is zero at some field values. This requires that

\[
|m_{12}^2(T)| \sim \{|\lambda_6|, |\lambda_7|\} v^2, \tag{5.3}
\]

and that the sign of \( m_{12}^2(T) \) is opposite to that of \( \lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta \).

The second tree-level constraint concerns \( \lambda_5 \):

\[
\lambda_5 > 0. \tag{5.4}
\]

The larger \( \lambda_5 \) is, the larger is the range of values of \( v \) where spontaneous CP-violation can take place. In Sec. 3.2 we got a stronger constraint, \( \lambda_5 \gtrsim O(g^2) \) (assuming now that \( v \sim T \) in Eq. (3.23), instead of \( v \sim gT \) as in Sec. 3.2). However, since we will see that the uncertainties in the constraint related to \( \lambda_5 \) are large (larger than those in the constraint related to \( m_{12}^2(T) \) in Eq. (3.3)), we consider here only the most relaxed version of the \( \lambda_5 \)-constraint, the one given in Eq. (5.4). The true requirement for \( \lambda_5 \) is eventually to be determined numerically.

Note now that, according to the expressions for the parameters related to CP-violation \( (m_{12}^2, \lambda_5, \lambda_6, \lambda_7 \text{ in Eqs. (2.4), (2.10)–(2.12)}) \), the inequality in Eq. (5.4) can be satisfied. However, assuming that \( v \lesssim T \), we see from Eqs. (2.10), (2.12) that Eq. (5.3) goes over into

\[
|m_{12}^2(T)| \lesssim \left\{ \frac{h_4^4}{16\pi^2} |\hat{A}_t^* \hat{\mu}^*|^2 T^2, \frac{g_4^4}{16\pi^2} \frac{|M_2 \mu^*|}{\pi^2} \right\}. \tag{5.5}
\]

Since we are in this section considering spontaneous CP-violation, we assume that \( \hat{A}_t, \hat{\mu} \) are real, but we nevertheless keep the same notation for them which arises in the complex case.
If the expression for $m_{12}^2(T)$ in Eq. (2.4) is dominated by contributions from some single particle species, then Eq. (5.5) clearly cannot be satisfied, as all the different types of terms in Eq. (2.4) are much larger than needed for Eq. (5.5). This holds independent of whether the dominant effects come from squarks ($\hat{A}_t, \hat{\mu} \neq 0$), gauginos and Higgsinos ($M_2, \mu \neq 0$), or Higgses ($A_t = \mu = 0$).

Thus, the only way to get spontaneous CP-violation is to have a cancellation between the contributions of the different particle species. Indeed, one can get $m_{12}^2(T) \sim 0$ at relatively large values of $m_A$, if $\text{sign}(\hat{A}_t^* \hat{\mu}^*) > 0$: according to Eqs. (2.4), (A.139),

$$m_{12}^2(T) \sim -\frac{1}{2} m_A \sin 2\beta + \frac{1}{4} h_t^2 \hat{A}_t^* \hat{\mu}^* T^2.$$ (5.6)

This behaviour is obtained for $m_U^2 \ll (2\pi T)^2 \ll m_Q^2$. Numerically, Eq. (5.6) could be close to zero if, for instance, $T \sim T_c \sim 85$ GeV, $\tan \beta \sim 15$, $\hat{A}_t \sim 0.6$, $\hat{\mu} \sim 0.3$, and $m_A \sim 70$ GeV.

5.2 Stop loops inside the 3d theory

In the discussion so far, we have completely ignored loop effects within the effective theory in Eq. (2.1), which might change the estimates noticeably (indeed, it is only loop effects which induce a first order transition and a stable phase boundary in the first place). We now proceed to estimate the loop effects related to the stop field $U$ in the case that it is heavy enough to allow for a perturbative treatment. By computing its effects on the parameters of the Higgs sector and applying then the tree-level estimates in Eqs. (5.3), (5.4), we show that the loop effects can be large in the regime where $m_{12}^2(T)$ given in Eq. (5.6) is small. On the other hand, if one is clearly outside this regime, then loop effects cannot trigger spontaneous CP-violation.

To be in the perturbative regime, we assume that the stop mass $m_U^2(T)$ is of the parametric order $(gsT)^2$. In other words, these estimates do not apply when one is very close to the “triple” point where the direction $U$ can get broken (see [6, 12]).

Inside the 3d theory, 1-loop contributions to the masses are of the form

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + m^2} = -\frac{m}{4\pi},$$ (5.7)

while 1-loop contributions to quartic couplings are of the form

$$-\int \frac{d^3p}{(2\pi)^3} \frac{1}{|p^2 + m_a^2|} = -\frac{1}{4\pi(m_a + m_b)},$$ (5.8)

It is interesting to note that the negative tree-level term in $m_{12}^2(T)$ can also be compensated for by a large stop vev $\sim h_t^2 \hat{A}_t^* \hat{\mu}^* (U^* U)$ in the stop breaking phase [6, 12]. However, there the Higgses are in the symmetric phase.
The stop contributions are

\[ \delta m_{12}^2(T) = -\frac{3}{4\pi} \gamma_{12} T m_U(T) \sim -h_t^2 \hat{A}_t^* \mu^* g s T^2, \]  

(5.9)

\[ \delta \lambda_5 = -\frac{3}{16\pi} \frac{\gamma_{12}^2 T}{m_U(T)} \sim -\frac{h_t^4 (\hat{A}_t^* \mu^*)^2}{g s T}, \]  

(5.10)

\[ \delta \lambda_6 = -\frac{3}{8\pi} \frac{\gamma_{12} \gamma_1 T}{m_U(T)}, \quad \delta \lambda_7 = -\frac{3}{8\pi} \frac{\gamma_{12} \gamma_2 T}{m_U(T)}. \]  

(5.11)

We observe that the contribution to the mass parameter \( m_{12}^2(T) \) is of order \( O(g^3)T^2 \) and is parametrically small compared with the leading \( T^2 \)-term. The contributions to the quartic couplings are, in contrast, of order \( O(g^3) \) and thus larger than the corresponding tree-level terms in Eqs. (2.10) - (2.12). However, even these corrections are not large enough to make Eq. (5.3) naturally satisfied for \( v \sim T \), unless the zero temperature and \( g^2 T^2 \)-terms in \( m_{12}^2(T) \) cancel, which was the requirement in Eq. (5.6).

The negative stop contribution to \( \lambda_5 \), in fact, tends to make spontaneous CP-violation less likely, since Eq. (5.4) is more difficult to satisfy. This effect should be compensated for by the gaugino-Higgsino term in Eq. (2.10).

Finally, consider what happens in the limit that the stop contributions become non-perturbative, \( m_U(T) \sim g_s^2 T \). In this case, one cannot look at contributions to effective parameters, but one may try to estimate the effects numerically by summing the contributions to the effective potential. For real parameters (\( \hat{A}_t^* \rightarrow \hat{A}_t, \mu^* \rightarrow \mu \)), the 1-loop stop contribution to the effective potential is of the form

\[ V \sim -\frac{1}{2\pi} \left[ m_{12}^2(T) - \frac{1}{2} h_t^2 v_1^2 |\mu|^2 + \frac{1}{2} h_t^2 v_2^2 (1 - |\hat{A}_t|^2) + h_t^2 v_1 v_2 \hat{A}_t \mu \cos \phi \right]^{3/2}. \]  

(5.12)

For large \( m_{12}^2(T) \) and sign(\( \hat{A}_t^* \mu^* \)) > 0, this term is minimized by \( \cos \phi = 1 \), which again corresponds to no CP-violation. However, for small \( m_{12}^2(T) \) the term becomes non-analytic and may induce non-trivial behaviour. At the same time, higher order corrections are of the same magnitude as the 1-loop term.

To summarize the effects of the stop loops: \( m_{12}^2(T) \) cannot be made small unless it already is so, and thus spontaneous CP-violation cannot be triggered far away from where it was expected to take place according to Eq. (5.6). Moreover, even in this regime, stop loops in fact seem to make spontaneous CP-violation less likely at least as long as they allow for a perturbative treatment, since they give a large negative contribution to \( \lambda_5 \).

### 5.3 Higgs loops inside the 3d theory

Although the Higgs loops involve effects from the non-perturbative mass scale \( \sim g^2 T \), it is nevertheless illuminating to inspect what kind of contributions they would give to
the effective parameters according to the loop expansion. Note that the gauge fields (and $A_b^0, C_b^A$), in contrast, do not contribute to the CP-violating couplings at leading order.

It turns out that the Higgs field contributions to the CP-violating parameters are typically smaller than the stop contributions. For instance, in the limit $m^2_{12}(T) \approx 0$, there must be at least one appearance of the couplings $\lambda_5, \ldots, \lambda_7$, which are of order $h^4_t/(16\pi^2)$ instead of order $h^2_t$ as the couplings $\gamma_1, \gamma_2, \gamma_{12}$:

\[
\frac{\delta \lambda_5}{T} = \frac{(5/2)\lambda_6^2 + 2\lambda_1\lambda_5}{8\pi m_1(T)} - \frac{(5/2)\lambda_7^2 + 2\lambda_2\lambda_5}{8\pi m_2(T)} - \frac{6\lambda_4\lambda_5 + 4\lambda_3\lambda_6 + \lambda_6\lambda_7}{4\pi[m_1(T) + m_2(T)]},
\]

\[
\frac{\delta \lambda_6}{T} = \frac{12\lambda_1\lambda_6}{8\pi m_1(T)} - \frac{3\lambda_3\lambda_7 + 2\lambda_4\lambda_7 + 2\lambda_5\lambda_7^*}{8\pi m_2(T)} - \frac{3\lambda_3\lambda_6 + 4\lambda_4\lambda_6 + 10\lambda_5\lambda_6^*}{4\pi[m_1(T) + m_2(T)]},
\]

\[
\frac{\delta \lambda_7}{T} = \frac{3\lambda_3\lambda_6 + 2\lambda_4\lambda_6 + 2\lambda_5\lambda_6^*}{8\pi m_1(T)} - \frac{12\lambda_2\lambda_7}{8\pi m_2(T)} - \frac{3\lambda_3\lambda_7 + 4\lambda_4\lambda_7 + 10\lambda_5\lambda_7^*}{4\pi[m_1(T) + m_2(T)]}.
\]

The terms where there appears only the light Higgs with mass $\sim g^2T$ are seen to contain non-perturbative contributions which are parametrically as large as the original couplings. There are also other types of contributions of similar magnitude: expanding formally in $m^2_{12}(T) \neq 0$, one gets for example terms of the type

\[
\frac{\delta \lambda_i}{T} \sim \frac{\lambda^2 m^2_{12}(T)}{4\pi m_i(T)[m_1(T) + m_2(T)]^2},
\]

where $\lambda \sim \{\lambda_1, \ldots, \lambda_4\}, i = 1, 2$. All these contributions involve expansion parameters $\sim \{\lambda_1, \ldots, \lambda_4\}T/(g^2T) \sim 1$, and are thus not perturbatively computable.

### 5.4 Numerical values

Let us finally look at the numerical requirements for the parameter values after taking into account the dimensionally reduced effective couplings in Sec. A.7, together with the stop contributions in the regime where they are still assumed to be perturbative, Sec. 5.2. To compensate for the negative stop contribution to $\lambda_5$ in Eq. (5.10) by a positive gaugino-Higgsino contribution in Eq. (A.149), one needs that

- $m_Q$ is large, because then the gaugino-Higgsino term suppressed only by temperature is relatively more significant;

- $M_2$ is large in order to increase the gaugino-Higgsino contribution, but $\hat{A}_t$ is not very large, to decrease the stop contribution.

On the other hand, to allow for relatively large values of $m_A$ at the point where $m^2_{12}(T)$ in Eq. (A.133) is small (for fixed $T \sim T_c$), it is preferable to have sign($M_2\mu^*$) < 0 and
$|\mu|$ large (this may help also in making $\lambda_6, \lambda_7$ positive, so that $m_{12}^2(T)$ may remain slightly negative).

Using the expressions in Sec. A.7 and those in Eqs. (5.9), (5.10), we estimate that numerically the best region for these requirements is

$$m_Q \gtrsim 800 \text{ GeV}, \quad M_2 \gtrsim 70 \text{ GeV}, \quad |\mu| \gtrsim 50 \text{ GeV}, \quad |\hat{A}| \lesssim 0.2,$$

(5.17)

where $\hat{A}$ is as defined in Eq. (2.13). In these estimates, we have assumed that $T \sim T_c \sim 85$ GeV, as can be observed by equating Eq. (A.138) with zero in the limit that $\tan\beta$ is large (for numerical estimates of $T_c$ based on the 2-loop potential and lattice see, e.g., [6, 11, 12]). Nevertheless, even in the regime of Eq. (5.17) there remains the constraint

$$m_A^2 \sin^2 2\beta \lesssim 400 \text{ GeV}^2.$$

(5.18)

Hence one needs, say, $m_A \lesssim 65$ GeV and $\tan\beta \gtrsim 20$. Clearly this parameter region is at most barely consistent with what is required for the strength of the transition in order to preserve the baryon asymmetry generated, and with experimental constraints, which both provide a lower bound for $m_A$.

5.5 Prospects for lattice simulations

In the case of explicit CP-violation, some of the parameters appearing in the action have complex phases and the expectation values of CP-violating operators such as $O_4 = O_{\text{CP}}$ in Eq. (3.6) are proportional to these. Due to the experimental constraints on the complex phases which are thought to be relatively strong, the magnitude of explicit CP-violation should be relatively small for realistic parameter values. In this case, a lattice study of the CP-violating effects appears rather difficult from the practical point of view, for the same reasons as discussed in Sec. 4 for $\tan\beta$. Perhaps the best that could be done is to solve the classical equations of motion for the CP-violating wall profile using the 2-loop effective potential. Even though the effects may be small, this case is of interest since it has been argued that explicit CP-violation might suffice for baryogenesis [29, 30]. Note also that large explicit effects may not be excluded [13], in which case also a lattice study is feasible.

In the case of spontaneous CP-violation, on the other hand, the parameters can be real and yet the effects can be large and well detectable: the distribution of $O_{\text{CP}}$ should have two peaks at opposite values of $O_{\text{CP}}$ (perhaps only within the phase boundary), which gives an unambiguous signal. The role of explicit CP-violation is merely to choose one of the peaks as the physically preferred one.
6 Conclusions

In this paper, we have studied the general prospects for spontaneous CP-violation around the electroweak phase transition in the MSSM. Parametrically, spontaneous CP-violation seems not to be easily realized. Nevertheless, we have shown that numerically there is a small regime, given in Eqs. (5.17), (5.18), which could be favourable for spontaneous CP-violation. We have shown that in this regime, the IR-sensitive loop contributions related to the stops are large and even non-perturbative in the light stop regime. The Higgs field contributions are non-perturbative, as well, but are numerically smaller than the stop contributions. However, it does not seem obvious that the large effects would naturally increase the likelihood of spontaneous CP-violation. It is rather that one needs delicate cooperation from different particle species to get an effect. Moreover, there is the concern that the regime found for $m_A$ within the present approximations, is almost certainly excluded due to experimental constraints and the requirement for a strong 1st order phase transition [8].

It is conceivable that in many extensions of the MSSM, such as the NMSSM [11] and various two Higgs doublet models, the parameter region allowing for spontaneous CP-violation would be larger than in the MSSM. Moreover, even if there is no spontaneous CP-violation, it could be that in the case of explicit CP-violation, the non-perturbative dynamics of the 3d theory affects strongly the explicit effects (especially if the explicit effects are large, as proposed in [18]). It has also been argued that there could be CP-violating solitons in the broken phase of the theory, even though there is no spontaneous CP-violation [42]. Thus, in spite of the fact that the parameter space for spontaneous CP-violation in the MSSM seems to be quite small, we believe that a more precise study of these questions is motivated. We expect relatively large uncertainties in the present results.

The present results were based on deriving an effective action for the IR degrees of freedom of the theory, and studying then this effective action at the tree-level. The first part thereof, the derivation of the effective action, is a purely perturbative computation, but even its accuracy can in principle be improved upon. In particular, the $\overline{\text{MS}}$ parameter $m_{12}^2(\bar{\mu})$ could be related more precisely to physical observables such as $m_A$, and the parameter $m_{12}^2(T)$ of the effective 3d theory could be derived with 2-loop accuracy. Nevertheless, we expect that it is the non-perturbative IR-properties of the theory that are responsible for the largest uncertainties.

The first step of a more precise IR-study would be to compute the full 1-loop and 2-loop effective potentials within the effective theory, and then to solve the complete classical equations of motion for the wall profile consisting of $v_1, v_2, \phi$, to see when $\phi$ can deviate from zero. Solving the classical equations of motion is in principle straightforward but can lead, in practice, to somewhat difficult instabilities and slow convergence; a working algorithm has been recently proposed in [40].

At the second step, the theory in Eq. (2.1) could be studied with lattice simulations.
Concerning the lattice study, it should be realized that the wall profile can only be defined with a finite cross-sectional area, see Sec. 4. However, we do not consider this to be a fundamental problem (the area dependence is only logarithmic). It should be noted that a lattice study of spontaneous CP-violation is, from a practical point of view, more promising than for instance a study of the non-trivial profile of $\tan \beta$, since the effect can be observed by finding a non-trivial (two-peak) distribution of a single operator, such as $O_{CP}$ in Eq. (3.6). The breaking of global discrete symmetries in 3d gauge theories has previously been studied numerically in [33], for the SU(3)+adjoint Higgs model.

Finally, it is interesting to note that the leading parity violating effects are very difficult to study with lattice simulations, since they are induced by higher-dimensional operators in the 3d theory which are complex in Euclidian space [36]. The C-violating (and thus CP-violating) effects discussed in this paper are, in contrast, contained in a real super-renormalizable Euclidian action, and there should be no fundamental problems in the simulations.

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Appendix A

In this Appendix, we derive the 1-loop expressions for the parameters in Eq. (2.1). CP-violation is assumed to appear in the original theory (at some renormalization scale $\mu_0$) only in the trilinear couplings $A_t, \mu$, defined in terms of the Euclidian 4d Lagrangian as in Eq. (1.1). Thus, in particular, the Higgs sector mass parameter $m_{12}^2$ and the gaugino mass parameter $M_2$ are assumed to have been tuned real at $\mu_0$, employing field redefinitions. Of the parameters in Eq. (1.1), $\mu$ is taken to be equal to the supersymmetric mass parameter which appears also in the Higgsino mass matrix, although in principle there could be an arbitrary soft component in the trilinear term to which $\mu$ contributes [44]. The soft parameter $A_t$, in contrast, only appears in the interaction in Eq. (1.1).

We shall work in the limit $m_i^2 \ll (2\pi T)^2 \ll m_Q^2$, $A_t^2 \ll m_Q^2$, where $m_i^2$ signals all the mass parameters of the theory other than $m_Q^2$: $m_i^2 = \{m_1^2, m_2^2, m_{12}^2, m_{13}^2, |\mu|^2, M_2^2, M_3^2\}$. Note that the trilinear coupling $A_t$ need not, in principle, be smaller than $2\pi T$.

In this limit, the construction of the effective theory can be carried out in two steps. First, in Sec. A.1, we integrate out the left-handed squark field $Q$ at zero temperature. Second, we go to finite temperature and integrate out the non-zero Matsubara modes
of the remaining bosons (Sec. A.2), as well as all the fermions (Sec. A.3). For the sake of brevity, we discuss the field redefinitions related to all these steps at once in Sec. A.4. In Sec. A.5 we discuss issues which are somewhat less central to the present problem, namely the values of the gauge couplings and the removal of the zero Matsubara modes of the gauge fields $A_0, C_0$. Finally, for completeness, we discuss the most important effects related to vacuum renormalization in Sec. A.6. All the results are collected together and simplified final expressions for the parameters appearing in Eq. (2.1) are given in Sec. A.7.

It should be mentioned that many of the expressions given here have also been derived in other places. In particular, the dimensional reduction steps (but for $m_Q^2 \ll (2\pi T)^2$ and mostly only for the Higgs sector) have been carried out in [9]–[11] (see also [45]). In addition, expressions for the quadratic and quartic Higgs couplings involving CP-violation have been derived, in various limits, in [21, 22, 23].

The notation for the parameters of the final effective action, to be used below, is given in Eq. (2.1). In addition to the effective parameters, the individual reduction steps also give contributions to the wave functions, which we denote by

$$\delta \mathcal{L} = (D^w_i H_1)\dagger (D^w_i H_1) \delta Z_1 + (D^w_i H_2)\dagger (D^w_i H_2) \delta Z_2 + (D^s_i U)^\dagger (D^s_i U) \delta Z_U$$

$$+ \left[ (D^w_i H_1)\dagger (D^w_i \tilde{H}_2) \delta Z_{12} + \text{H.c.} \right]. \quad (A.1)$$

We do not write down the \( \overline{\text{MS}} \) scheme divergences \( 1/\epsilon \) in any of the expressions; these can be restored by \( \ln \mu^2 \to 1/\epsilon + \ln \mu^2 \). In the loop corrections, we always neglect terms proportional to the U(1) gauge coupling \( g'^2 \).

### A.1 Integrating out \( Q \) at \( T = 0 \)

Let us consider the case that the mass parameter related to the left-handed stops is much larger than the temperature, \( m_Q \gg 2\pi T \). Then, the field \( Q \) can be integrated out from the zero temperature theory. Note that this case is different from the consideration in [46], where all the squarks were integrated out with degenerate mass parameters. Evaluating some typical finite temperature integrals numerically, we expect the results to be well reliable when, say, \( m_Q \gtrsim 0.6 \text{ TeV} \) (\( T_c \lesssim 100 \text{ GeV} \)).

The left-handed stop field \( Q \) contributes to the effective action of the two Higgses \( H_1, H_2 \) and the right-handed stop \( U \) already at tree-level, due to graphs of the type

\[
\begin{array}{c}
\text{\( Q \)} \quad \to \quad \text{\( \bullet \)}
\end{array}
\]

where the dashed line denotes the light degrees of freedom, the solid line the heavy field \( Q \), and the filled circle an effective vertex.
Using the notation $\hat{A}_t, \hat{\mu}$ defined in Eq. (2.13), the tree-level couplings of the action in Eq. (2.1), after integrating out $Q$, are

$$
\lambda_U = \frac{g_S^2}{6}, \quad \gamma_1 = -h_t^2 \hat{\mu}^2, \quad \gamma_2 = h_t^2 (1 - \hat{A}_t^2), \quad \gamma_{12} = h_t^2 \hat{A}_t^2 \hat{\mu}^2, \\
\lambda_1 = \lambda_2 = \frac{g^2 + g'^2}{8}, \quad \lambda_3 = \frac{g^2 - g'^2}{4}, \quad \lambda_4 = -\frac{g^2}{2},
$$

where $h_t$ is the top Yukawa coupling, and $g_S, g, g'$ are the strong, weak and U(1) gauge couplings. There are no contributions to the wave functions in Eq. (A.1) at tree-level.

At 1-loop level, the following types of contributions arise (we expand in $m^2/m_Q^2, q^2/m_Q^2$):

\begin{align*}
\int \frac{dp}{p^2 + m_Q^2} &= -\frac{m_Q^2}{16\pi^2} \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 1 \right), \\
\frac{1}{(p^2 + m_Q^2)^2} &= \frac{1}{16\pi^2} \ln \frac{\bar{\mu}^2}{m_Q^2},
\end{align*}

where $m$ is the mass of the quark. The integrals are given by:

\begin{align*}
\frac{1}{(p^2 + m_Q^2)(p^2 + m^2)^2} - \frac{1}{m_Q^2} \int \frac{dp}{p^2 + m^2} &= -\frac{1}{m_Q^2} \frac{1}{16\pi^2} \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 1 \right), \\
\int \frac{dp}{(p^2 + m_Q^2)^2(p^2 + m^2)} - \frac{1}{m_Q^2} \int \frac{dp}{p^2 + m^2} &= \frac{1}{m_Q^2} \frac{1}{16\pi^2} \\
&= \frac{1}{m_Q^2} \frac{1}{16\pi^2}.
\end{align*}
\[ dp \equiv \frac{d^4 - 2^r}{(2\pi)^{4 - 2r}}. \]

Here the subtracted parts are contributions within the effective theory: after the subtractions the results are IR-safe as they should be. The effective six-point vertex in Eq. (A.7) is shown just for illustration; it gives a contribution suppressed by the relative amount \( m^2/m_Q^2 \) and is not kept in the actual theory in Eq. (7.1).

A 1-loop computation of all the graphs of the types in Eqs. (A.3)–(A.8) results in the following contributions to the effective parameters in Eqs. (2.2)–(2.12) and to the wave functions in Eq. (A.1):

\[
\begin{align*}
\delta Z_1 & = \frac{3}{2} \frac{h_t^2}{16\pi^2} |\mu|^2, \quad \delta Z_2 = \frac{3}{2} \frac{h_t^2}{16\pi^2} |\tilde{\mu}|^2, \\
\delta Z_{12} & = -3 \frac{h_t^2}{16\pi^2} \tilde{A}_t^* \mu^*, \quad \delta Z_U = \frac{h_t^2}{16\pi^2} (|\tilde{A}_t|^2 + |\tilde{\mu}|^2), \\
\delta m_1^2(T) & = -3 \frac{h_t^2}{16\pi^2} \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 1 \right) (|\mu|^2 + |\tilde{\mu}|^2 m_U^2), \\
\delta m_2^2(T) & = -3 \frac{h_t^2}{16\pi^2} \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 1 \right) (m_Q^2 + |A_t|^2 + |\tilde{A}_t|^2 m_U^2), \\
\delta m_{12}^2(T) & = 3 \frac{h_t^2}{16\pi^2} \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 1 \right) (A_t^* \mu^* + \tilde{A}_t^* \mu^* m_U^2), \\
\delta m_{12}^2(T) & = -2 \frac{h_t^2}{16\pi^2} \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 1 \right) (m_Q^2 + |A_t|^2 + |\mu|^2 \\
& \quad + |\tilde{A}_t|^2 m_2^2 + |\tilde{\mu}|^2 m_1^2 - (\tilde{A}_t^* \mu^* + \tilde{A}_t \mu) m_{12}^2), \\
\delta \lambda_U & = \frac{1}{16\pi^2} \left[ \left(-h_t^4 + \frac{2}{3} h_t^2 g_s^2 - \frac{1}{6} g_s^4 \right) \ln \frac{\bar{m}_2^2}{m_Q^2} - \frac{1}{12} g_s^4 \ln \frac{\bar{m}_2^2}{m_D^2} + 2 h_t^4 \left( |\tilde{\mu}|^2 \\
& \quad - |\tilde{A}_t|^2 \ln \frac{\bar{m}_2^2}{m_Q^2} \right) - \frac{2}{3} h_t^2 g_s^2 (|\tilde{A}_t|^2 + |\tilde{\mu}|^2) + h_t^4 (|\tilde{A}_t|^2 + |\tilde{\mu}|^2)^2 \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 2 \right) \right], \quad (A.15) \\
\delta \gamma_1 & = \frac{h_t^2}{16\pi^2} |\mu|^2 \left[ h_t^2 - \left( \frac{4}{3} g_s^2 + \frac{3}{4} g^2 \right) \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 1 \right) \\
& \quad + h_t^2 (|\tilde{A}_t|^2 + |\tilde{\mu}|^2) \left( \ln \frac{\bar{m}_2^2}{m_Q^2} + 2 \right) \right], \quad (A.16)
\end{align*}
\]
\[ \delta \gamma_2 = \frac{h_t^2}{16\pi^2} \left[ -h_t^2 \ln \frac{\mu^2}{m_Q^2} + h_t^2 (2|\hat{A}_t|^2 + |\hat{\mu}|^2) - |\hat{A}_t|^2 \left( \frac{4}{3} g_5^2 + \frac{3}{4} g^2 \right) \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 1 \right) \right. \\
+ h_t^2 |\hat{A}_t|^2 (|\hat{A}_t|^2 + |\hat{\mu}|^2) \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right], \tag{A.17} \]

\[ \delta \gamma_{12} = \frac{h_t^2}{16\pi^2} \hat{A}_t^* \hat{\mu}^* \left[ -h_t^2 \left( \frac{4}{3} g_5^2 - \frac{3}{4} g^2 \right) \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 1 \right) \right. \\
- h_t^2 (|\hat{A}_t|^2 + |\hat{\mu}|^2) \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right], \tag{A.18} \]

\[ \delta \lambda_1 = \frac{1}{16\pi^2} \left\{ h_t^2 |\hat{\mu}|^2 \left[ \frac{3}{4} g^2 + \frac{3}{2} h_t^2 |\hat{\mu}|^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right] - \frac{3}{16} g^4 \ln \frac{\bar{\mu}^2}{m_Q^2} \right\}, \tag{A.19} \]

\[ \delta \lambda_2 = \frac{1}{16\pi^2} \left\{ h_t^2 \left[ \left( -\frac{3}{2} h_t^2 - 3h_t^2 |\hat{A}_t|^2 + \frac{3}{4} g^2 \right) \ln \frac{\bar{\mu}^2}{m_Q^2} - \frac{3}{4} g^2 |\hat{A}_t|^2 \right. \\
+ \frac{3}{2} h_t^2 |\hat{A}_t|^4 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right] - \frac{3}{16} g^4 \ln \frac{\bar{\mu}^2}{m_Q^2} \right\}, \tag{A.20} \]

\[ \delta \lambda_3 = \frac{1}{16\pi^2} \left\{ h_t^2 \left[ \frac{3}{4} g^2 \ln \frac{\bar{\mu}^2}{m_Q^2} - 3h_t^2 |\hat{\mu}|^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 1 \right) + \frac{3}{4} g^2 (|\hat{\mu}|^2 - |\hat{A}_t|^2) \right. \\
+ 3h_t^2 |\hat{A}_t|^2 |\hat{\mu}|^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right] - \frac{3}{8} g^4 \ln \frac{\bar{\mu}^2}{m_Q^2} \right\}, \tag{A.21} \]

\[ \delta \lambda_4 = \frac{1}{16\pi^2} \left\{ h_t^2 \left[ -\frac{3}{2} g^2 \ln \frac{\bar{\mu}^2}{m_Q^2} + 3h_t^2 |\hat{\mu}|^2 + \frac{3}{2} g^2 (|\hat{\mu}|^2 - |\hat{A}_t|^2) \right. \\
+ 3h_t^2 |\hat{A}_t|^2 |\hat{\mu}|^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right] + \frac{3}{4} g^4 \ln \frac{\bar{\mu}^2}{m_Q^2} \right\}, \tag{A.22} \]

\[ \delta \lambda_5 = \frac{3}{2} \frac{h_t^4}{16\pi^2} \left( \hat{A}_t^* \hat{\mu}^* \right)^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right), \tag{A.23} \]

\[ \delta \lambda_6 = - \frac{h_t^2}{16\pi^2} \hat{A}_t^* \hat{\mu}^* \left[ \frac{3}{4} g^2 + 3h_t^2 |\hat{\mu}|^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right], \tag{A.24} \]

\[ \delta \lambda_7 = \frac{h_t^2}{16\pi^2} \hat{A}_t^* \hat{\mu}^* \left[ 3h_t^2 \ln \frac{\bar{\mu}^2}{m_Q^2} + \frac{3}{4} g^2 - 3h_t^2 |\hat{A}_t|^2 \left( \ln \frac{\bar{\mu}^2}{m_Q^2} + 2 \right) \right]. \tag{A.25} \]

### A.2 Integrating out bosonic non-zero Matsubara modes

The bosonic non-zero Matsubara modes are integrated out in the theory defined by the tree-level couplings in Eq. (A.2). Thus, there are loop contributions involving Higgses, right-handed stops and the gauge fields.

The types of graphs and integrals appearing have been discussed in [12] (see also [9]–[11]). For completeness, let us briefly review the results here, as well. We use the
following notation:

\[
\bar{\mu}_T = 4\pi e^{-\gamma}T \approx 7.0555T,
\]

(A.26)

\[
\oint \frac{1}{J_{\mu}(p^2)}^3 = \frac{\zeta(3)}{128\pi^4 T^2} = \frac{B_6}{16\pi^2}, \quad B_6 = \frac{\zeta(3)}{2} \frac{1}{(2T)^2},
\]

(A.27)

\[
\oint \frac{1}{J_{\mu}(p^2)}^4 = \frac{\zeta(5)}{1024\pi^6 T^4} = \frac{B_8}{16\pi^2}, \quad B_8 = \frac{\zeta(5)}{4} \frac{1}{(2T)^4},
\]

(A.28)

where \(p_b\) denotes the bosonic Matsubara momenta and a prime means that the zero Matsubara mode is omitted. Then the basic contributions are of the types

\[
\oint \frac{1}{p^2 + m^2} = \frac{T^2}{12} - \frac{m^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2},
\]

(A.29)

\[
\oint \frac{(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{p^2} = \frac{T^2}{4},
\]

(A.30)

\[
\oint \frac{(2q_\mu + p_\mu)(2q_\nu + p_\nu)}{p^2[(p + q)^2 + m^2]} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) = \frac{3q^2}{16\pi^2} \left( \ln \frac{\mu^2}{\mu_T^2} - \frac{10}{9} m^2 B_6 \right).
\]

(A.31)

\[
\oint \frac{1}{(p^2 + m_1^2)(p^2 + m_2^2)} = \frac{1}{16\pi^2} \left[ \ln \frac{\mu^2}{\mu_T^2} - (m_1^2 + m_2^2) B_6 + (m_1^4 + m_1^2 m_2^2 + m_2^4) B_8 \right],
\]

(A.32)

\[
\oint \frac{(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{(p^2)^2} = \frac{1}{16\pi^2} \left[ 3 \ln \frac{\mu^2}{\mu_T^2} - 2 \right].
\]

(A.33)

Here the external momentum is \(q = (0, q)\). Note that in contrast to Sec. A.1, there are no subtractions to be made at 1-loop level: due to momentum conservation, all the internal lines have a non-zero Matsubara mode and are thus heavy.

In most places, we keep only the dominant terms of the different qualitative types in the high temperature expansion in \(m_1^2/(2\pi T)^2, m_2^2/(2\pi T)^2, m_3^2/(2\pi T)^2, m_4^2/(2\pi T)^2, m_5^2/(2\pi T)^2\). Note that to obtain these expressions, it is convenient to write the Higgs field propa-
gators as
\[
\langle H_m^{ix}H_n^{ij} \rangle = \delta_{mn} \delta^{ij} \left( \frac{1}{p^2} - \frac{m_n^2}{(p^2)^2} \right), \quad \langle H_m^{ix}H_n^{js} \rangle = \epsilon_{mn} \epsilon^{ij} \frac{m_{12}^2}{(p^2)^2}.
\] (A.34)

We then obtain that
\[
\delta Z_1 = -\frac{9}{4} \frac{g_s^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2}, \quad \delta Z_2 = -\frac{9}{4} \frac{g_s^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2}, \quad \delta Z_{12} = \frac{5}{2} \frac{g_s^2}{16\pi^2} m_{12}^2 B_6, \quad \delta Z_U = -\frac{4}{3} \frac{g_s^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2}, \quad \delta Z_{12} = \frac{5}{2} \frac{g_s^2}{16\pi^2} m_{12}^2 B_6, \quad \delta Z_{UV} = \frac{4}{3} \frac{g_s^2}{16\pi^2} m_{12}^2 B_6, \quad \delta Z_{UU} = -\frac{4}{3} \frac{g_s^2}{16\pi^2} m_{12}^2 B_6, \quad \delta Z_{UUV} = \frac{4}{3} \frac{g_s^2}{16\pi^2} m_{12}^2 B_6.
\] (A.35)

\[
\delta m_1^2 = \frac{3}{16} g_s^2 T^2 - 3 h_t^2 |\hat{\mu}|^2 \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right) + \frac{3}{4} g_s^2 \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right),
\] (A.37)

\[
\delta m_2^2 = \frac{3}{16} g_s^2 T^2 + 3 h_t^2 (1 - |\hat{A}|^2) \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right)
+ \frac{3}{4} g_s^2 \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right), \quad \delta m_{12}^2 = 3 h_t^2 \hat{A}_t^* \hat{\mu}^* \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right) + \frac{3}{4} g_s^2 \frac{m_{12}^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2},
\] (A.38)

\[
\delta m_U^2 = \frac{1}{3} g_s^2 \frac{T^2}{12} + \frac{4}{3} g_s^2 \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right) + \frac{1}{6} h_t^2 (1 - |\hat{A}|^2 - |\hat{\mu}|^2) T^2
+ \frac{2}{3} \ln \frac{\mu^2}{\mu_T^2} \left[ |\hat{\mu}|^2 m_1^2 - (1 - |\hat{A}|^2) m_2^2 - (\hat{A}_t^* \hat{\mu}^* + \hat{A}_t \hat{\mu}) m_{12}^2 \right] \ln \frac{\mu^2}{\mu_T^2}, \quad \delta m_{12}^2 = 3 h_t^2 \hat{A}_t^* \hat{\mu}^* \left( \frac{T^2}{12} - \frac{m_U^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2} \right) + \frac{3}{4} g_s^2 \frac{m_{12}^2}{16\pi^2} \ln \frac{\mu^2}{\mu_T^2},
\] (A.39)

\[
\delta \lambda_U = \frac{1}{16\pi^2} \left\{ -h_t^4 \left[ (|\hat{\mu}|^4 + (1 - |\hat{A}|^2)^2 + 2 |\hat{A}|^2 |\hat{\mu}|^2) \ln \frac{\mu^2}{\mu_T^2} - 2 m_{12}^2 B_6 (1 - |\hat{A}|^2 - |\hat{\mu}|^2) (\hat{A}_t^* \hat{\mu}^* + \hat{A}_t \hat{\mu}) \right] - g_s^4 \left[ \frac{13}{12} \left( \ln \frac{\mu^2}{\mu_T^2} - \frac{2}{3} \right) + \frac{7}{18} \ln \frac{\mu^2}{\mu_T^2} \right] \right\}, \quad \delta \lambda_{12} = \frac{1}{16\pi^2} \left\{ -h_t^4 \left[ (|\hat{\mu}|^4 + (1 - |\hat{A}|^2)^2 - |\hat{A}|^2 |\hat{\mu}|^2 - 2 |\hat{A}|^2 |\hat{\mu}|^2) \ln \frac{\mu^2}{\mu_T^2} + \frac{1}{4} m_{12}^2 B_6 (\hat{A}_t^* \hat{\mu}^* + \hat{A}_t \hat{\mu}) (3 g_s^2 - 8 h_t^2 |\hat{\mu}|^2) \right] \right\},
\] (A.41)

\[
\delta \gamma_1 = \frac{h_t^2}{16\pi^2} \left[ |\hat{\mu}|^2 \left( -2 h_t^2 (1 - |\hat{A}|^2)^2 - 2 h^2 |\hat{A}|^2 |\hat{\mu}|^2 - 4 g_s^2 + \frac{3}{4} g^2 \right) \ln \frac{\mu^2}{\mu_T^2} + \frac{1}{4} m_{12}^2 B_6 (\hat{A}_t^* \hat{\mu}^* + \hat{A}_t \hat{\mu}) (3 g_s^2 - 8 h_t^2 |\hat{\mu}|^2) \right],
\] (A.42)

\[
\delta \gamma_2 = \frac{h_t^2}{16\pi^2} \left[ \left( -2 h_t^2 (1 - |\hat{A}|^2)^2 - 2 h^2 |\hat{A}|^2 |\hat{\mu}|^2 - 4 g_s^2 + \frac{3}{4} g^2 \right) \ln \frac{\mu^2}{\mu_T^2} + \frac{1}{4} m_{12}^2 B_6 (\hat{A}_t^* \hat{\mu}^* + \hat{A}_t \hat{\mu}) (3 g_s^2 + 8 h_t^2 (1 - |\hat{A}|^2^2)) \right],
\] (A.43)

\[
\delta \gamma_{12} = \frac{h_t^2}{16\pi^2} \left[ \hat{A}_t^* \hat{\mu}^* \left( -2 h_t^2 (1 - |\hat{A}|^2^2 - |\hat{\mu}|^2^2 - 4 g_s^2 + \frac{3}{4} g^2 \right) \ln \frac{\mu^2}{\mu_T^2} + \frac{1}{4} m_{12}^2 B_6 [-3 g^2 (1 - |\hat{A}|^2^2 - |\hat{\mu}|^2^2) + 8 h_t^2 (\hat{A}_t^* \hat{\mu}^*)^2 - 8 h_t^2 |\hat{\mu}|^2 (1 - |\hat{A}|^2^2)] \right],
\] (A.44)
\[ \delta \lambda_1 = -\frac{3}{2} \frac{h_t^4}{16 \pi^2} |\bar{\mu}|^2 \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - \frac{g^4}{16 \pi^2} \left[ \frac{9}{16} \left( \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - \frac{2}{3} \right) + \frac{1}{4} \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} \right], \] (A.45)

\[ \delta \lambda_2 = -\frac{3}{2} \frac{h_t^4}{16 \pi^2} (1 - |\hat{A}_t|^2) |\bar{\mu}|^2 \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - \frac{g^4}{16 \pi^2} \left[ \frac{9}{16} \left( \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - \frac{2}{3} \right) + \frac{1}{4} \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} \right], \] (A.46)

\[ \delta \lambda_3 = 3 \frac{h_t^4}{16 \pi^2} |\bar{\mu}|^2 (1 - |\hat{A}_t|^2) \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - \frac{g^4}{16 \pi^2} \left[ \frac{9}{8} \left( \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - \frac{2}{3} \right) + \frac{1}{2} \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} \right], \] (A.47)

\[ \delta \lambda_4 = -3 \frac{h_t^4}{16 \pi^2} |\hat{A}_t|^2 |\bar{\mu}|^2 \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} + \frac{g^4}{16 \pi^2} \frac{1}{4} \ln \frac{\bar{\mu}^2}{\bar{p}_T^2}, \] (A.48)

\[ \delta \lambda_5 = -3 \frac{1}{16 \pi^2} \left[ 8 h_t^4 (\hat{A}_t^* \hat{\mu})^2 \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} + g^4 m_{12}^2 B_3 \right]. \] (A.49)

\[ \delta \lambda_6 = \frac{3}{8} \frac{1}{16 \pi^2} \left[ 8 h_t^4 |\bar{\mu}|^2 \hat{A}_t \hat{\mu}^* \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} - g^4 m_{12}^2 B_6 \right], \] (A.50)

\[ \delta \lambda_7 = -3 \frac{1}{8} \frac{1}{16 \pi^2} \left[ 8 h_t^4 (1 - |\hat{A}_t|^2) \hat{A}_t \hat{\mu}^* \ln \frac{\bar{\mu}^2}{\bar{p}_T^2} + g^4 m_{12}^2 B_6 \right]. \] (A.51)

### A.3 Integrating out fermions

The fermions to be integrated out, interacting with the Higgs and stop degrees of freedom, include the third generation quarks, the Higgsinos, and the SU(2) and SU(3) gauginos. The mass parameters of the latter are denoted by $M_2, M_3$.

Concerning the integrals appearing, let us define

\[ \sum_{\mathbf{p}_f} \frac{1}{(p^2)^3} = \frac{7 \zeta(3)}{128 \pi^4 T^2} \equiv F_6, \quad F_6 = \frac{7 \zeta(3)}{8} \frac{1}{(\pi T)^2}, \] (A.52)

\[ \sum_{\mathbf{p}_f} \frac{1}{(p^2)^4} = \frac{31 \zeta(5)}{1024 \pi^6 T^4} \equiv F_8, \quad F_8 = \frac{31 \zeta(5)}{64} \frac{1}{(\pi T)^4}, \] (A.53)

where $\mathbf{p}_f$ denotes the fermionic Matsubara momenta. Then the basic graphs give, in the high temperature expansion, contributions of the form

\[ \sum_{\mathbf{p}_f} \frac{\text{Tr} [i \hat{\mu} (p + q)]}{[p^2 + M_2^2] [(p + q)^2 + |\mu|^2]} \]

\[ = \frac{T^2}{6} + \frac{1}{16 \pi^2} (4 M_2^2 + 4 |\mu|^2 + 2q^2) \ln \frac{(4 \mu)^2}{\bar{p}_T^2}, \] (A.54)

\[ \sum_{\mathbf{p}_f} \frac{1}{[p^2 + M_2^2] [(p + q)^2 + |\mu|^2]} \]

\[ = \frac{1}{16 \pi^2} \left( \ln \frac{(4 \mu)^2}{\bar{p}_T^2} - \left( M_2^2 + |\mu|^2 + \frac{1}{3} q^2 \right) F_6 \right). \] (A.55)
\[ \mathcal{L}_{\beta}\left[ \frac{1}{p^2 + M_\alpha^2} \right] = \frac{1}{16\pi^2} \left[ \ln \left( \frac{44\mu^2}{\mu_T^2} \right) - (M_\alpha^2 + |\mu|^2)F_6 + (M_\alpha^2 + M_\beta^2)|\mu|^2 + |\mu|^4)F_8 \right]. \] (A.56)

As the fermions appearing are Majorana particles, the Lorentz-structure of the contractions is

\[
\langle \tilde{W}_\alpha(p)\tilde{W}_\beta(q) \rangle = \delta(p - q) \left[ \frac{-i\bar{\gamma} + M}{p^2 + M^2} \right], \\
\langle \tilde{W}_\alpha(p)\tilde{W}_\beta(q) \rangle = \delta(p + q) \left[ \frac{[-i\bar{\gamma} + M][C^{-1}]_{\alpha\beta}}{p^2 + M^2} \right], \\
\langle \tilde{W}_\alpha(p)\tilde{W}_\beta(q) \rangle = \delta(p + q) \left[ \frac{[C^{-1}(i\bar{\gamma} + M)]_{\alpha\beta}}{p^2 + M^2} \right],
\] (A.57)

where the charge conjugation matrix \( C \) satisfies

\[ C = -C^\dagger = -C^T = -C^{-1}, \quad C^{-1}\gamma\mu C = (\gamma^\mu)^T, \quad C^{-1}\gamma^5C = (\gamma^5)^T, \] (A.58)

and \( \tilde{W}_\alpha(x) = \tilde{W}_\beta(x)C_{\beta\alpha} \).

The final results for the couplings are (terms proportional to the gauge couplings come from loops involving gauginos, terms proportional to the Yukawa coupling from loops involving left-handed quarks):

\[
\delta Z_1 = \frac{3}{2} \frac{g^2}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \quad \delta Z_2 = \left( \frac{3}{2} g^2 + 3h_\mu^2 \right) \frac{1}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \quad \delta Z_{12} = -\frac{g^2}{16\pi^2} M_2\mu^* F_6, \quad \delta Z_U = \left( \frac{8}{3} g_\mu^2 + \nu h_\nu^2 \right) \frac{1}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \] (A.59)

\[
\delta m_1^2(T) = \frac{3}{4} g^2 \left( \frac{T^2}{6} + 4(M_2^2 + |\mu|^2) \frac{1}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right) \right), \quad \delta m_2^2(T) = \frac{3}{4} g^2 \left( \frac{T^2}{6} + 4(M_2^2 + |\mu|^2) \frac{1}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right) \right) + \frac{1}{4} h_\mu^2 T^2, \] (A.60)

\[
\delta m_{12}^2(T) = 3 \frac{g^2}{16\pi^2} M_2\mu^* \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \quad \delta m_U^2(T) = \frac{4}{3} g_\mu^2 \left( \frac{T^2}{6} + 4M_2^2 \frac{1}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right) \right) + h_\nu^2 \left( \frac{T^2}{6} + 4|\mu|^2 \frac{1}{16\pi^2} \ln \left( \frac{4\mu^2}{\mu_T^2} \right) \right), \] (A.61)

\[
\delta \lambda_U = \frac{1}{16\pi^2} \left( 2h_\mu^4 + \frac{22}{9} g_\mu^4 \right) \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \quad \delta \gamma_1 = -3 \frac{g^2 h_\mu^2}{16\pi^2} |\mu|^2 F_6, \quad \delta \gamma_2 = \frac{h_\mu^2}{16\pi^2} \left( 2h_\mu^4 + \frac{16}{3} g_\mu^4 + 3g_\mu^2 \right) \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \] (A.62)

\[
\delta \gamma_3 = \frac{1}{16\pi^2} \left( 2h_\mu^4 + \frac{22}{9} g_\mu^4 \right) \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \quad \delta \gamma_4 = \frac{h_\mu^2}{16\pi^2} \left( 2h_\mu^4 + \frac{16}{3} g_\mu^4 + 3g_\mu^2 \right) \ln \left( \frac{4\mu^2}{\mu_T^2} \right), \] (A.63)
\[\delta \gamma_{12} = -3 \frac{g^2 h_i^2}{16\pi^2} M_2 \mu^* F_6, \quad (A.67)\]

\[\delta \lambda_1 = \frac{5}{4} \frac{g^4}{16\pi^2} \ln \left(\frac{4\bar{\mu}^2}{\pi^2 T}\right), \quad \delta \lambda_2 = \frac{1}{16\pi^2} \left(\frac{5}{4} g^4 + 3 h_i^4\right) \ln \left(\frac{4\bar{\mu}^2}{\pi^2 T}\right), \quad (A.68)\]

\[\delta \lambda_3 = \frac{5}{2} \frac{g^4}{16\pi^2} \ln \left(\frac{4\bar{\mu}^2}{\pi^2 T}\right), \quad \delta \lambda_4 = -2 \frac{g^4}{16\pi^2} \ln \left(\frac{4\bar{\mu}^2}{\pi^2 T}\right), \quad (A.69)\]

\[\delta \lambda_5 = \frac{3}{2} \frac{g^4}{16\pi^2} M_2^2 (\mu^*)^2 F_8, \quad (A.70)\]

\[\delta \lambda_6 = -3 \frac{g^4}{16\pi^2} M_2 \mu^* F_6, \quad \delta \lambda_7 = -3 \frac{g^4}{16\pi^2} M_2 \mu^* F_6. \quad (A.71)\]

For completeness, let us also write down the results for the CP-violating parameters in the limit that \(M_2, |\mu|\) are large compared with \(\pi T\) (this limit corresponds to the zero temperature contributions considered in [20]). This limit is of interest since it was seen in Eq. (5.17) that non-vanishing values of \(M_2, |\mu|\) are preferred from the point of view of spontaneous CP-violation, and it is thus important to know when the high-temperature expansion used is reliable. We obtain

\[\delta Z_{12} = 3 g^2 M_2 \mu^* \int dp \frac{1}{(p^2 + |\mu|^2)(p^2 + M_2^2)^2} \left[\frac{4}{3} \frac{D^2}{p^2 + M_2^2} - 1\right]\]

\[\approx -3 \frac{g^2}{2} M_2 \mu^* \frac{|\mu|^4 - M_2^4 + 4|\mu|^2 M_2^2 \ln \frac{M_2}{|\mu|}}{2} \quad (A.72)\]

\[\delta m_{12}^2(T) = 3 g^2 M_2 \mu^* \int dp \frac{1}{(p^2 + M_2^2)(p^2 + |\mu|^2)^2}\]

\[= 3 \frac{g^2}{16\pi^2} M_2 \mu^* \left[\ln \frac{M_2}{|\mu|} + 1 - \frac{M_2^4 + |\mu|^2}{M_2^2 - |\mu|^2} \ln \frac{M_2}{|\mu|}\right]\]

\[\approx 3 \frac{g^2}{16\pi^2} M_2 \mu^* \left[\ln \frac{M_2}{|\mu|} + O\left(\frac{M_2^2 - |\mu|^2}{M_2^2}\right)^2\right], \quad (A.73)\]

\[\delta \gamma_{12} = -3 g^2 h_i^2 M_2 \mu^* \int dp \frac{1}{(p^2 + M_2^2)(p^2 + |\mu|^2)^2}\]

\[= -3 g^2 h_i^2 \frac{M_2 \mu^*}{16\pi^2 M_2^2 - |\mu|^2} \left[2 \frac{M_2^2}{M_2^2 - |\mu|^2} \ln \frac{M_2}{|\mu|} - 1\right]\]

\[\approx -3 g^2 h_i^2 \frac{M_2 \mu^*}{2} \left[\ln \frac{M_2}{|\mu|} + O\left(\frac{M_2^2 - |\mu|^2}{M_2^2}\right)^2\right], \quad (A.74)\]

\[\delta \lambda_5 = \frac{3 g^4}{2} M_2^2 (\mu^*)^2 \int dp \frac{1}{(p^2 + M_2^2)(p^2 + |\mu|^2)^2}\]

\[= 3 \frac{g^4}{16\pi^2} M_2^2 (\mu^*)^2 \left[\frac{M_2^4 + |\mu|^2}{M_2^2 - |\mu|^2} \ln \frac{M_2}{|\mu|} - 1\right]\]
\[ \approx \frac{1}{4} g^4 \left( \frac{\mu^*}{|\mu|} \right)^2 \left[ 1 + \mathcal{O} \left( \frac{M_2^2}{M_2^2} - \frac{|\mu|^2}{M_2^2} \right)^2 \right], \tag{A.75} \]

\[ \delta \lambda_6 = \delta \lambda_7 = -3g^4 M_2 \mu^* \int \frac{d^2 p}{(p^2 + M_2^2)(p^2 + |\mu|^2)} = -3 \frac{g^4 M_2 \mu^*}{16 \pi^2 |\mu|^2} \left[ M_2^4 - |\mu|^4 - 4 M_2^2 |\mu|^2 \ln \frac{M_2}{|\mu|} \right], \tag{A.76} \]

where \( dp = d^{4-2\epsilon} p / (2\pi)^{4-2\epsilon} \). Comparing with the expressions in Eqs. (A.59)-(A.71), we observe that the high-temperature expansion breaks down when \( \{M_2, |\mu|\} \gtrsim 2T \). In the range \( 2T \lesssim \{M_2, |\mu|\} \lesssim 4T \), a numerical evaluation of the integrals appearing is needed, while at \( \{M_2, |\mu|\} \gtrsim 4T \), the values of the couplings saturate and the zero-temperature results in Eqs. (A.72)-(A.76) can be used. In Eqs. (A.72)-(A.76) we have also shown the direct integral representations which can easily be evaluated at any temperature (see, e.g., [23]).

### A.4 Field redefinitions

In Secs. A.1-A.3, the removal of various ultraviolet degrees of freedom from the theory resulted in contributions to the effective parameters characterizing the interactions of the infrared degrees of freedom. In addition, there were contributions to the field normalizations, \( \delta Z_1, \delta Z_2, \delta Z_{12}, \delta Z_U \). It is convenient to make a redefinition of the fields so that these extra wave function terms disappear. This results in new contributions to the effective parameters, rendering them gauge and scale independent at 1-loop level.

Summing the tree-level terms with those in Eq. (A.1), the kinetic terms are of the form

\[ \mathcal{L}_{\text{kin}} = (D_i^w H_1) \dagger (D_i^w H_1)(1 + \delta Z_1) + (D_i^w H_2) \dagger (D_i^w H_2)(1 + \delta Z_2) \]

\[ + \left[ (D_i^w H_1) \dagger (D_i^w \tilde{H}_2) \delta Z_{12} + \text{H.c.} \right] + (D_i^w U) \dagger (D_i^w U)(1 + \delta Z_U). \tag{A.77} \]

One can now get rid of \( \delta Z_1, ..., \delta Z_U \) by writing

\[ H_1 = \left( 1 - \frac{1}{2} \delta Z_1 \right) H_1^{(\text{new})} - \frac{1}{2} \delta Z_{12} \tilde{H}_2^{(\text{new})}, \tag{A.78} \]

\[ \tilde{H}_2 = -\frac{1}{2} \delta Z_{12} H_1^{(\text{new})} + \left( 1 - \frac{1}{2} \delta Z_2 \right) \tilde{H}_2^{(\text{new})}, \tag{A.79} \]

\[ U = \left( 1 - \frac{1}{2} \delta Z_U \right) U^{(\text{new})}. \tag{A.80} \]

When the field redefinitions combine with the tree-level couplings in Eq. (A.3), there are new contributions to the effective parameters as follows:

\[ \delta m_1^2(T) = -m_1^2 \delta Z_1 - \frac{1}{2} m_{12}^2 (\delta Z_{12} + \delta Z_{12}^*), \tag{A.81} \]
\[
\delta m_2^2(T) = -m_2^2 \delta Z_2 - \frac{1}{2} m_{12}^2 (\delta Z_{12} + \delta Z_{12}^*),
\]
(A.82)

\[
\delta m_{12}^2(T) = -\frac{1}{2} m_{12}^2 (\delta Z_1 + \delta Z_2) - \frac{1}{2} (m_1^2 + m_2^2) \delta Z_{12},
\]
(A.83)

\[
\delta m_U^2(T) = -m_U^2 \delta Z_U, \quad \delta \lambda_U = -\frac{1}{3} g_5^2 \delta Z_U,
\]
(A.84)

\[
\delta \gamma_1 = h_t^2 |\tilde{\mu}|^2 (\delta Z_1 + \delta Z_U) - \frac{1}{2} h_t^2 (\delta Z_{12}^* \hat{A}_t^* \mu^* + \delta Z_{12} \hat{A}_t \mu),
\]
(A.85)

\[
\delta \gamma_2 = -h_t^2 (1 - |\hat{A}_t|^2) (\delta Z_2 + \delta Z_U) - \frac{1}{2} h_t^2 (\delta Z_{12}^* \hat{A}_t^* \mu^* + \delta Z_{12} \hat{A}_t \mu),
\]
(A.86)

\[
\delta \gamma_{12} = -\frac{1}{2} h_t^2 (1 - |\hat{A}_t|^2 - |\tilde{\mu}|^2) \delta Z_{12} - \frac{1}{2} h_t^2 \hat{A}_t^* \mu^* (2 \delta Z_U + \delta Z_1 + \delta Z_2),
\]
(A.87)

\[
\delta \lambda_1 = -\frac{1}{4} g^2 \delta Z_1, \quad \delta \lambda_2 = -\frac{1}{4} g^2 \delta Z_2,
\]
(A.88)

\[
\delta \lambda_3 = -\frac{1}{4} g^2 (\delta Z_1 + \delta Z_2), \quad \delta \lambda_4 = \frac{1}{2} g^2 (\delta Z_1 + \delta Z_2).
\]
(A.89)

There are no new contributions to \(\lambda_5 \ldots \lambda_7\).

### A.5 Gauge couplings and \(A_0, C_0\)-integrations

So far we have been considering couplings related to the Higgs fields and to the stop fields. Let us now review what happens with the gauge fields.

We start by inspecting the spatial sector, where the plasma does not screen the gauge fields. What temperature does is that it changes the effective gauge coupling related to the static fields [33, 47]:

\[
g_T^2 = g^2(\tilde{\mu}) \left[ 1 + \frac{g^2}{48 \pi^2} \left\{ \beta_{\text{gauge}} \left( \ln \frac{\tilde{\mu}}{T} + \frac{1}{22} \right) + \beta_{\text{scalar}} \ln \frac{\tilde{\mu}}{T} + \beta_{\text{fermion}} \ln \frac{4 \tilde{\mu}}{T} \right\} \right]
\]
\[= g_T^2 + \frac{g_{T_0}^2}{48 \pi^2} (\beta_{\text{gauge}} + \beta_{\text{scalar}} + \beta_{\text{fermion}}) \ln \frac{T_0}{T}, \quad (A.90)\]

where \(T_0\) is any reference temperature, and the standard contributions to \(\beta\) are, for the weak gauge coupling (\(N = 2\)),

\[
\begin{align*}
\beta_{\text{gauge}} &= 22N, \quad (A.91) \\
\beta_{\text{scalar}} &= -N_H (\text{Higgses}) - 3N_g (\text{squarks}) - N_g (\text{sleptons}), \quad (A.92) \\
\beta_{\text{fermion}} &= -4N (\text{gauginos}) - 2N_H (\text{Higgsinos}) - 6N_g (\text{quarks}) - 2N_g (\text{leptons}). \quad (A.93)
\end{align*}
\]

Here \(N_H = 2\) is the number of Higgs doublets and \(N_g = 3\) the number of generations. For the superpartners, often only one generation (squarks) or none at all (sleptons)
is taken to be effective. The expressions for the strong gauge coupling are completely analogous (see also [6]).

Consider then the temporal components of the gauge fields. They do get screened. Neglecting higher order $O(g^4)$-terms related to $A_0, C_0$, the (tree-level) part of the Lagrangian involving them is

$$\mathcal{L}_{A_0, C_0} = \frac{1}{2} \left( D^w_i A_0 \right)^a \left( D^w_i A_0 \right)^a + \frac{1}{2} \left( D^s_i C_0 \right)^A \left( D^s_i C_0 \right)^A + \frac{1}{2} m^2_{A_0} A_0^a A_0^a + \frac{1}{2} m^2_{C_0} C_0^A C_0^A$$

$$+ \frac{1}{4} g^2 A_0^a A_0^a (H_1^I H_1 + H_2^I H_2) + g_5^2 C_0^A C_0^B U_\alpha (T^A T^B)_{\alpha\beta} U^*_{\beta}.$$  \hspace{1cm} (A.94)

Here the covariant derivatives are in the adjoint representation and the Debye masses are (see, e.g., [48])

$$m^2_{A_0} = \frac{5}{2} g^2 T^2, \quad m^2_{C_0} = \frac{8}{3} g^2 S T^2,$$  \hspace{1cm} (A.95)

where we took into account gauginos and Higgsinos ($\delta m^2_{A_0} = (1/2) g^2 T^2$), as well as gluinos ($\delta m^2_{C_0} = (1/2) g^2 S T^2$), but only the 3rd generation $U$-squarks.

Now, usually the fields $A_0, C_0$ can be integrated out, since they have the mass scale $\sim gT$, while the phase transition dynamics is related to the scale $\sim g^2 T$. However, we have argued in Sec. 3 that to study spontaneous CP-violation, one has to keep also the Higgs degrees of freedom with masses $\sim gT$ in the action. Thus it is, in terms of the original 4d power counting, not strictly speaking consistent to integrate out some degrees of freedom with masses $\sim gT$ and leave others in the action. However, we can introduce a different power counting within the 3d theory, in particular since numerically the Debye masses in Eq. (A.95) are relatively large. Moreover, they do not affect the CP-violating couplings at leading order. Thus it seems very reasonable to integrate out also $A_0, C_0$. The 1-loop results of that integration are [3]:

$$g^2(\text{new}) = g^2 \left(1 - \frac{g^2 T}{24 \pi m_{A_0}}\right), \quad g_5^2(\text{new}) = g_5^2 \left(1 - \frac{g_5^2 T}{16 \pi m_{C_0}}\right),$$  \hspace{1cm} (A.96)

$$\delta m_1^2(T) = \delta m_2^2(T) = \frac{3}{16 \pi} g^2 T m_{A_0}, \quad \delta m_U^2(T) = \frac{1}{3 \pi} g_5^2 T m_{C_0},$$  \hspace{1cm} (A.97)

$$\delta \lambda_U = -\frac{13}{36 \pi} g_5^2 T, \quad \delta \lambda_1 = \delta \lambda_2 = -\frac{3}{16 \pi} g^4 T, \quad \delta \lambda_3 = -\frac{3}{8 \pi} g^4 T.$$  \hspace{1cm} (A.98)

**A.6 Vacuum renormalization**

In the previous sections, we have derived an effective theory in terms of renormalized parameters in the $\overline{\text{MS}}$ scheme. In order to make connection to physics, the $\overline{\text{MS}}$ scheme parameters should, in turn, be expressed in terms of measurable quantities. For completeness, we will discuss here some of the most important effects. A more detailed
discussion can be found, e.g., in \cite{9–11}. In this section, we take, for simplicity, the couplings $A_t, \mu$ to be real:

\begin{equation}
A_t^* \rightarrow A_t, \quad \mu^* \rightarrow \mu.
\end{equation}

Of course, these quantities could still be negative. For the case of complex couplings see, e.g., \cite{19}.

Let us note first that the problem with the renormalization of the gauge couplings $g_2^S, g_2'$ and the top quark Yukawa coupling $h_t$ is that the same parameters appear in many different vertices due to supersymmetry, while in the softly broken theory, this equivalence is lost beyond tree-level. Thus, it is not sufficient to compute three different physical observables to fix these three parameters appearing in different places. However, as the renormalization effects related to these parameters are not too essential for the problem discussed in this paper, we shall not discuss the fixing of $g, h_t, g_2$ in great detail: it is sufficient to remark that the effective finite temperature gauge couplings in the gauge sector are related to the zero temperature \overline{\text{MS}} scheme couplings as discussed in Sec. A.4, and the top quark Yukawa coupling can be fixed at tree-level,

\begin{equation}
h_t^2 \sin^2 \beta = \frac{g^2 m_t^2}{2 m_W^2},
\end{equation}

where $\tan \beta = v_2/v_1$ is a parameter determined by the Higgs mass, see below.

Let us, instead, discuss the values of the mass parameters $m_1^2(\bar{\mu}), m_2^2(\bar{\mu}), m_{12}(\bar{\mu}), m_{11}(\bar{\mu})$. For them, there are relatively large renormalization effects proportional to $h_t^2$, and we would like to account for the dominant terms of such types, in particular those proportional to $m_Q^2$.

The four running mass parameters can be fixed by computing the 1-loop pole masses $m_Z^2, m_A^2, m_h^2, m_t^2$ of the Z boson, the CP-odd Higgs particle, the lightest CP-even Higgs particle, and the lightest stop, respectively. For the dominant terms we are interested in, this computation can be simplified since the momentum dependence of the 2-point functions can be neglected: indeed, the momentum dependence results essentially in multiplicative terms proportional to $p^2 \sim m^2$, where $m^2$ is the pole mass in question. These terms are suppressed with respect to the dominant additive ones, $\sim m_Q^2$. Since the momentum dependence can be neglected, one can derive some of the 2-point functions from the effective potential.

There is one further simplification. It turns out that the only large additive ($\sim m_Q^2$) contributions to the Z boson mass $m_Z^2$ come from the tadpole graphs which account for a shift in the location of the broken Higgs minimum $(v_1, v_2)$. In other words, in the one-particle-irreducible graphs, the terms proportional to $m_Q^2$ cancel due to gauge invariance (for explicit expressions see, e.g., Eqs. (A.23)–(A.25) in \cite{9}). This means that if we denote by $(v_1, v_2)$ the location of the radiatively corrected broken minimum, then we can write, within the present approximation, $m_Z^2 = \tilde{g}^2 (v_1^2 + v_2^2) / 4$, where $\tilde{g}^2 = g^2 + g'^2$. Then, all appearances of $v_1, v_2$ can be expressed in terms of $\tan \beta = v_2/v_1$ and $m_Z$, as $\tilde{g} v_1 = 2 m_Z \cos \beta$, $\tilde{g} v_2 = 2 m_Z \sin \beta$. 

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To solve for $m^2_1(\mu), m^2_2(\mu), m^2_{12}(\mu)$, we now proceed as follows. First, we solve from the conditions $\partial V(v_1, v_2)/\partial v_1 = 0, \partial V(v_1, v_2)/\partial v_2 = 0$ for $m^2_i(\bar{\mu}), m^2_2(\bar{\mu})$ in terms of $\tan \beta, m^2_2, m^2_{12}(\bar{\mu})$. We feed these results into the mass matrix of the CP-odd Higgs (and of a Goldstone boson). Solving for the physical eigenvalue $m^2_A$, we obtain $m^2_{12}(\mu)$ as a function of $\tan \beta, m^2_A, m^2_Z, \bar{\mu}$. With this expression, the results are known also for $m^2_1(\mu), m^2_2(\mu)$. It remains to eliminate $\tan \beta$: this is done by plugging the expressions found into the mass matrix of the CP-even Higgses, and thus finding the relation of $\tan \beta$ to the lightest CP-even Higgs mass $m_h$.

To proceed with this program, we write the Higgs doublets in component form as

$$ H_1 = \frac{1}{\sqrt{2}} \left( \begin{array} {c} h^0_1 + ih^3_1 \\ -h^2_1 + ih^1_1 \end{array} \right), \quad H_2 = \frac{1}{\sqrt{2}} \left( \begin{array} {c} h^0_2 + ih^1_2 \\ h^0_2 - ih^3_2 \end{array} \right). \quad (A.102) $$

The tree-level potential ($\langle h^0_1 \rangle = v_1, \langle h^0_2 \rangle = v_2$) is then

$$ V^{\text{tree}}(v_1, v_2) = \frac{1}{2} m^2_1 v^2_1 + \frac{1}{2} m^2_2 v^2_2 + m^2_{12} v_1 v_2 + \frac{1}{32} g^2 (v^2_1 - v^2_2). \quad (A.103) $$

The $h^2_i$-contributions of the 1-loop potential to $\partial V/\partial v_i$ can be read, e.g., from Eqs. (A.29), (A.30) in [9]:

$$ \frac{\partial V^{1\text{-loop}}(v_1, v_2)}{\partial v_1} = v_1 C_h(\bar{\mu}), \quad \frac{\partial V^{1\text{-loop}}(v_1, v_2)}{\partial v_2} = v_2 S_h(\bar{\mu}). \quad (A.104) $$

where

$$ C_h(\bar{\mu}) = 3 \frac{h^2_i}{16\pi^2} \left[ \mu (-\mu + A_t \tan \beta) \left( \ln \frac{\bar{\mu}^2}{m^2_T m^2_{T}} + 1 + \frac{m^2_T + m^2_{i}}{m^2_T - m^2_{i}} \ln \frac{m^2_T}{m^2_{T}} \right) \right], \quad (A.105) $$

$$ S_h(\bar{\mu}) = 3 \frac{h^2_i}{16\pi^2} \left[ 2 m^2_{\text{top}} \left( \ln \frac{\bar{\mu}^2}{m^2_{\text{top}}} + 1 \right) \right. 

$$

$$ - (m^2_{iL} + m^2_{iR}) \left( \ln \frac{\bar{\mu}^2}{m^2_{iL} m^2_{i}} + 1 \right) - (m^2_{T} - m^2_{i}) \ln \frac{m^2_{T}}{m^2_{i}} 

$$

$$ + A_t (-A_t + \mu \cot \beta) \left( \ln \frac{\bar{\mu}^2}{m^2_T m^2_{i}} + 1 + \frac{m^2_{T} + m^2_{i}}{m^2_T - m^2_{i}} \ln \frac{m^2_{T}}{m^2_{i}} \right). \quad (A.106) $$

Here

$$ m^2_{iL} = m^2_{Q} + m^2_{\text{top}} + \frac{1}{2} m^2_Z \cos 2\beta, \quad m^2_{iR} = m^2_{L} + m^2_{\text{top}}, \quad (A.107) $$

$$ m^2_{iL} = m_{\text{top}} (A_t - \mu \cot \beta), \quad (A.108) $$

$$ m^2_T = \frac{1}{2} \left[ m^2_{iL} + m^2_{iR} + \sqrt{(m^2_{iL} - m^2_{iR})^2 + 4m^4_{iL}}, \quad (A.109) $$

$$ m^2_{i} = \frac{1}{2} \left[ m^2_{iL} + m^2_{iR} - \sqrt{(m^2_{iL} - m^2_{iR})^2 + 4m^4_{iL}} \right]. \quad (A.110) $$

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Note that in the notation of [9], \( w_s \rightarrow -h_t \mu, u_s \rightarrow h_t A_t, e_s = d_s \rightarrow 0, m_{\ell_1}^2 \rightarrow m_{\ell_i}^2, m_{\ell_2}^2 \rightarrow m_{\ell_i}^2, m_{U_{12}}^2 \rightarrow m_{U_{iR}}^2, m_{U_{2i}}^2 \rightarrow m_{U_{i}}^2, m_{U_{1i}}^2 \rightarrow m_{U_{i}}^2 \). In addition, no Higgsino and gaugino contributions were considered in [9], but this does not affect the leading \( h_t^2 \) terms we are considering here (see, however, the discussion below).

It then follows from the conditions \( \partial (V^{\text{tree}} + V^{1\text{-loop}}) / \partial v_i = 0 \) that

\[
m_{12}(\bar{\mu}) = -m_{12}(\bar{\mu}) \tan \beta - \frac{1}{2} m_Z^2 \cos 2\beta - C_h(\bar{\mu}), \tag{A.111}
\]
\[
m_{12}(\bar{\mu}) = -m_{12}(\bar{\mu}) \cot \beta + \frac{1}{2} m_Z^2 \cos 2\beta - S_h(\bar{\mu}). \tag{A.112}
\]

The mass matrix for the CP-odd Higgs mass \( m_A \) and for a neutral Goldstone boson is given by the fields \( h^3, h^2 \), and is of the form

\[
M_{\text{CP-odd}} = \begin{pmatrix}
m_{12}^2(\bar{\mu}) + \frac{1}{8} g^2 (v_1^2 - v_2^2) + \Pi_{11}^A(\bar{\mu}) & m_{12}^2(\bar{\mu}) + \Pi_{12}^A(\bar{\mu}) \\
\Pi_{12}^A(\bar{\mu}) & m_{12}^2(\bar{\mu}) + \frac{1}{8} g^2 (v_2^2 - v_1^2) + \Pi_{22}^A(\bar{\mu})
\end{pmatrix}.
\tag{A.113}
\]

The expressions for \( \Pi_{ij}^A(\bar{\mu}) \) can be obtained, e.g., from Eqs. (A.19)–(A.22) in [9] (the functions \( F_H \) vanish within the zero-momentum approximation considered, and the overall sign should be reversed):

\[
\Pi_{11}^A(\bar{\mu}) = C_h(\bar{\mu}) - \tan \beta A_t \mu D(\bar{\mu}), \quad \Pi_{12}^A(\bar{\mu}) = A_t \mu D(\bar{\mu}), \tag{A.114}
\]
\[
\Pi_{22}^A(\bar{\mu}) = S_h(\bar{\mu}) - \cot \beta A_t \mu D(\bar{\mu}), \tag{A.115}
\]

where

\[
D(\bar{\mu}) = 3 \frac{h_t^2}{16 \pi^2} \left( \ln \frac{\bar{\mu}^2}{m_t m_\ell} + 1 + \frac{m_t^2 + m_\ell^2}{m_t^2 - m_\ell^2} \ln \frac{m_t}{m_\ell} \right). \tag{A.116}
\]

Plugging in the values of \( m_{12}^2(\bar{\mu}), m_{12}^2(\bar{\mu}) \) from Eqs. (A.111), (A.112), the CP-odd mass matrix becomes

\[
M_{\text{CP-odd}} = \begin{pmatrix}
-\left[ m_{12}^2(\bar{\mu}) + A_t \mu D(\bar{\mu}) \right] \tan \beta & m_{12}^2(\bar{\mu}) + A_t \mu D(\bar{\mu}) \\
m_{12}^2(\bar{\mu}) + A_t \mu D(\bar{\mu}) & -\left[ m_{12}^2(\bar{\mu}) + A_t \mu D(\bar{\mu}) \right] \cot \beta
\end{pmatrix}.
\tag{A.117}
\]

The (scale-independent) eigenvalues of \( M_{\text{CP-odd}} \) are zero and \( m_A^2 \), which finally gives the expression for \( m_{12}^2(\bar{\mu}) \), and after insertion into Eqs. (A.111), (A.112), for \( m_{12}^2(\bar{\mu}), m_{12}^2(\bar{\mu}) \):

\[
m_{12}^2(\bar{\mu}) = -\frac{1}{2} m_A^2 \sin 2\beta - A_t \mu D(\bar{\mu}), \tag{A.118}
\]
\[
m_{12}^2(\bar{\mu}) = \frac{1}{2} m_A^2 - \frac{1}{2} (m_A^2 + m_Z^2) \cos 2\beta + |\mu|^2 D(\bar{\mu}), \tag{A.119}
\]
\[
m_{12}^2(\bar{\mu}) = \frac{1}{2} m_A^2 + \frac{1}{2} (m_A^2 + m_Z^2) \cos 2\beta + |A_t|^2 D(\bar{\mu}) \tag{A.120}
\]
\[
+ 3 \frac{h_t^2}{16 \pi^2} \left( m_Q^2 + m_T^2 \right) \left( \ln \frac{\bar{\mu}^2}{m_T m_\ell} + 1 \right) + 2 m_{\text{top}}^2 \ln \frac{m_{\text{top}}^2}{m_t m_\ell} + (m_T^2 - m_\ell^2) \ln \frac{m_t}{m_\ell}.
\]

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There is the following useful observation to be made. If one expands Eqs. (A.118)–(A.120) in terms of $m_i^2/m_Q^2, m_{top}^2/m_Q^2$ in the limit of large $m_Q$, then it turns out that the dominant terms containing $m_Q$ cancel against the terms in Eqs. (A.11)–(A.14). For example, summing the dominant 1-loop contributions to $m_{12}^2(T)$ from Eqs. (A.14), (A.39) together with Eq. (A.118), the scale dependence cancels and we obtain, up to terms of relative order $m_i^4/m_Q^4, m_{top}^4/m_Q^4$:

$$
\delta m_{12}^2(T) = 3\frac{h_i^2}{16\pi^2} A_i^* \mu^* \left\{ m_U^2 \left( \ln \frac{m_T^2}{m_i^2} + 1 \right) + m_{top}^2 \left[ 1 + \frac{A_i^2}{m_Q^2} + 2 \left( 1 - \frac{A_i^2}{m_Q^2} \right) \ln \frac{m_Q}{m_i} \right] \right\},
$$

(A.121)

where we have denoted $m_i^2/LR = m_{top} A_i$. The largest term here is that induced by the symmetry breaking, $\sim A_i^* \mu^* (m_{top}/m_Q)^2$. For $m_1^2(T), m_2^2(T)$, in particular (where the result is similar), such terms are typically much smaller than the tree-level mass terms. Thus, after vacuum renormalization is taken into account, the terms proportional to $h_i^2/(16\pi^2)$ in $m_1^2(T), m_2^2(T)$ can be neglected as a first approximation, and it is sufficient to consider the tree-level mass terms and the thermal $T^2$-terms.

By similar arguments, we can also easily derive the dominant contributions of the parameters $M_2, |\mu|$ in the limit that $M_2, |\mu| \gg m_W$. Indeed, in this limit the expressions for vacuum renormalization must cancel the large direct contributions in Eq. (A.73), just as happened above for $m_Q$ in the limit $m_Q \gg m_{top}$. Thus, e.g.,

$$
m_{12}^2(\mu) \approx m_{12}^2(\mu) \bigg|_{(A.118)} - \delta m_{12}^2(T) \bigg|_{(A.73)}.
$$

(A.122)

Finally, let us connect $\tan \beta$ to the Higgs mass. The mass matrix for the CP-even Higgses $m_h, m_H$ is given by the fields $h_1^0, h_2^0$:

$$
M_{CP-even} = \begin{pmatrix}
m_1^2(\mu) + \frac{1}{8} g^2 (3v_1^2 - v_2^2) + \Pi_{11}^H(\mu) & m_{12}^2(\mu) - \frac{1}{4} g^2 v_1 v_2 + \Pi_{12}^H(\mu) \\
- \frac{1}{4} g^2 v_1 v_2 + \Pi_{12}^H(\mu) & m_2^2(\mu) + \frac{1}{8} g^2 (3v_2^2 - v_1^2) + \Pi_{22}^H(\mu)
\end{pmatrix},
$$

(A.123)

where the $\Pi_{ij}^H(\mu)$'s can be obtained from the second derivatives of the effective potential $V(v_1, v_2)$, or directly from a diagrammatic computation in, e.g., Eqs. (A.12)–(A.18) in [3]. Denoting

$$
\delta_1 = 12 \frac{h_i^2}{16\pi^2} \left( 1 + \frac{m_{top}^2 + m_i^2}{m_T^2 - m_i^2} \ln \frac{m_i}{m_T} \right) \left( \frac{A_i}{m_T^2 - m_i^2} \right)^2, \\
\delta_2 = 12 \frac{h_i^2}{16\pi^2} \frac{m_{top}^2}{m_T^2 - m_i^2} \ln \frac{m_T}{m_i}, \\
\delta_3 = 12 \frac{h_i^2}{16\pi^2} \frac{m_i m_T}{m_{top}^2},
$$

(A.124)–(A.126)
where again $\tilde{A}_t = A_t - \mu \cot \beta$, we obtain

$$\Pi_{11}^H(\bar{\mu}) = -|\mu|^2\left[D(\bar{\mu}) - \delta_1\right], \quad \Pi_{12}^H(\bar{\mu}) = A_t \mu \left[D(\bar{\mu}) - \delta_1 - \tilde{A}_t \mu \delta_2\right],$$

(A.127)

$$\Pi_{22}^H(\bar{\mu}) = -|A_t|^2\left[D(\bar{\mu}) - \delta_1\right] + 2\tilde{A}_t A_t \delta_2 + \frac{3}{2} m_{\text{top}}^2 \delta_3,$$

(A.128)

Plugging these expressions together with those in Eqs. (A.118)–(A.120) into Eq. (A.123), the explicit scale dependence cancels and the physical mass matrix remaining is (see also [50])

$$M_{\text{CP-even}} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix},$$

(A.129)

$$M_{11} = m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta + |\mu|^2 \delta_1,$$

(A.130)

$$M_{12} = -(m_Z^2 + m_A^2) \sin \beta \cos \beta - A_t \mu \delta_1 - \tilde{A}_t \mu \delta_2,$$

(A.131)

$$M_{22} = m_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta + |A_t|^2 \delta_1 + 2\tilde{A}_t A_t \delta_2 + m_{\text{top}}^2 \delta_3.$$  

(A.132)

The lightest CP-even Higgs mass is then

$$m_h^2 = \frac{1}{2} \left[M_{11} + M_{22} - \sqrt{(M_{11} - M_{22})^2 + 4|M_{12}|^2}\right].$$

(A.133)

The renormalization of $m_{\tilde{t}_1}^2(\bar{\mu})$ and its relation to the lightest stop mass $m_{\tilde{t}}$ (which, in the absence of mixing, is the right-handed stop mass $m_{\tilde{t}_R}$) can be handled with similar methods as the Higgs sector. However, we have already argued what the result will approximately be: there are large renormalization effects related to the integration out of $Q$, see Eq. (A.14), but when vacuum renormalization is taken into account, then these effectively cancel. Thus, it is enough for our purposes to ignore the effects in Eq. (A.14) and replace $m_{\tilde{t}_1}^2$ with the tree-level value, which is obtained from the expression for the lightest stop mass:

$$m_{\tilde{t}}^2 \approx m_{\tilde{t}_1}^2 + m_{\text{top}}^2 \left[1 - (\tilde{A}_t - \bar{\mu} \cot \beta)^2\right].$$

(A.134)

**A.7 Summary: the effective theory**

In this section we collect together the results of the previous sections. For the quartic couplings, we will work at the accuracy of the vacuum renormalization discussed in Sec. A.6. Thus, we ignore all the contributions of order $\mathcal{O}(g^4)$ to the quartic Higgs couplings, contributions of order $\mathcal{O}(g_3^4)$ to the quartic stop coupling, and contributions of order $\mathcal{O}(h_t^4)$ to the top Yukawa coupling squared appearing in the quartic scalar interactions between stops and Higgses. In other words, we keep only contributions for
which the scale dependence is cancelled within the accuracy of vacuum renormalization discussed in Sec. A.4. It should be noted, though, that for the quartic stop coupling, neglecting effects of order $O(g^4)$ and keeping effects of order $O(h^4)$ is numerically not a very useful approximation scheme.

We also make an expansion in $m_1^2/m_Q^2, m_1^2/\pi T^2$, where $m_1^2$ denotes the mass parameters other than $m_Q$. In general, we keep the dominant effects of the different qualitative types appearing. Note that $|A_t|$ can, in principle, be larger than $|\mu|$, since it is not subject to constraints from the high-temperature expansion. The Debye masses appearing are given in Eq. (A.95).

For the mass parameters $m_1^2(T), m_2^2(T), m_3^2(T)$, we only keep the leading terms below, which should already give a reasonable approximation as explained after Eq. (A.121). More precise results can be obtained by using the full expressions in Eqs. (A.119),(A.120) (see also [9]–[11]).

For $m_1^2(T)$, which affects directly spontaneous CP-violation, we display a more precise expression, with the dominant additive corrections of various qualitative types included. Multiplicative corrections to $m_A^2 \sin 2\beta$ have been neglected. It should also be noted that the high temperature expansion for the gaugino-Higgsino contribution is reliable only when the expression in the square brackets on the first row in Eq. (A.139) is negative: without a high temperature expansion, this term is

$$\delta m_{12}^2(T) = 3g^2 M_A \mu^* \left[I_f\left(\frac{|\mu|}{T}\right) - I_f\left(\frac{M_2}{T}\right)\right],$$

(A.135)

where

$$I_f(y) = -\frac{T^2}{2\pi^2} \int_0^\infty dx \frac{1}{x^2 + y^2} e^{\frac{1}{x^2 + y^2}} + 1.$$  

(A.136)

If $M_2, |\mu|$ are large ($\gtrsim \pi T$), then the term in Eq. (A.135) vanishes, but also the contributions of these degrees of freedom to the $T^2$-terms in the mass parameters, Eqs. (A.61)–(A.64), and to the Debye masses, Eq. (A.95), should be left out.

Note that the square bracket on the second row in Eq. (A.139) is also negative, so that the whole term is positive for sign($\hat{A}_t \hat{\mu}^*$) > 0; see Eq. (A.121) for an approximation.

Finally, let us point out that we write here the couplings in 4d units, so that there is an overall factor $g^2/\sqrt{4}$ in the action. This factor can, as usual, be defined away by a rescaling of the fields, and the result is that the quartic couplings of the 3d action get simply multiplied by $T$.

With these conventions and approximations, we obtain that\(^8\)

$$m_1^2(T) = \frac{1}{2} m_A^2 - \frac{1}{2} (m_A^2 + m_Z^2) \cos 2\beta$$

\(^8\)Note that as can be seen from the $h^4$-term in Eq. (A.140), there is an error in Eq. (3.7) of the latter of [12]: there should be no extra factor of $e^{3/4}$ inside the logarithm. (This does not affect any of the conclusions in [14] concerning the non-perturbative effects.)
\[ m_2^2(T) = \frac{1}{2} m_A^2 + \frac{1}{2} (m_A^2 + m_2^2) \cos 2\beta \]
\[ + \left( \frac{3}{8} g^2 + \frac{1}{2} h_i^2 - \frac{1}{4} h_i^2 |\hat{A}_i|^2 \right) T^2 - \frac{3}{16\pi} g^2 T m_A, \quad (A.137) \]

\[ m_{12}^2(T) = -\frac{1}{2} m_A^2 \sin 2\beta + \frac{3}{16\pi^2} m_2 \mu^* \left[ \ln \frac{M_2 |\mu|}{\mu_T^2} - 1 + \frac{M^2_2 + |\mu|^2}{m^2_2 - |\mu|^2} \ln \frac{M_2}{|\mu|} \right] \]
\[ - 3 \frac{h_i^2}{16\pi^2} \hat{A}_i^* \mu^* \left[ \ln \frac{m_Q^2}{m_T^2} - \frac{m_i^2}{m_T^2 - m_i^2} \ln \frac{m_T}{m_i} + 2 \frac{m^2_U}{m_Q^2} \ln \frac{m_Q}{m_T} \right] \]
\[ + m_A^2 \left( \frac{3}{4} h_i^2 \hat{A}_i^* \mu^* + \frac{7\zeta(3)}{16} g^2 M_2 \mu^* (\pi T)^2 \right) + \frac{1}{4} h_i^2 \hat{A}_i^* \mu^* T^2, \quad (A.139) \]

\[ m_U^2(T) = m_{t_{\text{top}}}^2 \left[ 1 - |\hat{A}_t - \hat{\mu}^* \cot \beta|^2 \right] \]
\[ + \left( \frac{2}{3} g^2 S + \frac{1}{3} h_i^2 - \frac{1}{6} h_i^2 |\hat{A}_i|^2 + |\hat{\mu}|^2 \right) T^2 - \frac{1}{3\pi} g^2 T m_{c_0}, \quad (A.140) \]

\[ \lambda_U = \frac{1}{6} g_S^2 - \frac{13}{36} \frac{g^4 S T}{\pi m_{c_0}} + \frac{1}{3\pi^2} \left[ - g^2 S \left( 4 \ln \frac{4m_Q}{\mu_T} + 3 |\hat{A}_i|^2 + 3 |\hat{\mu}|^2 \right) \right. \]
\[ + 6h_i^2 \left( 2 |\hat{A}_i|^2 + 2 \ln \frac{m_Q}{\mu_T} + 2 |\hat{\mu}|^2 - (|\hat{A}_i|^2 + |\hat{\mu}|^2) \ln \frac{m_Q}{\mu_T} \right), \quad (A.141) \]

\[ \gamma_1 = -h_i^2 |\hat{\mu}|^2 - \frac{\zeta(3)}{4} \frac{g^2 h_i^2}{16\pi^2} \left[ - \frac{1}{2} \frac{m_A^2 \sin 2\beta}{(2\pi T)^2} (\hat{A}_i^* \mu^* + \hat{A}_i \mu) \right] \]
\[ + 7 \frac{|\hat{\mu}|^2}{(2\pi T)^2} \left( 6 - \frac{M_2}{m_Q} (\hat{A}_i^* + \hat{A}_i) \right), \quad (A.142) \]

\[ \gamma_2 = h_i^2 (1 - |\hat{A}_i|^2), \quad (A.143) \]

\[ \gamma_12 = h_i^2 \hat{A}_i^* \mu^* - \frac{\zeta(3)}{4} \frac{h_i^2}{16\pi^2} \left[ - 2 \frac{m_A^2 \sin 2\beta}{(2\pi T)^2} \left( g^2 (1 - |\hat{A}_i|^2 - |\hat{\mu}|^2) \right) \right. \]
\[ + h_i^2 |\hat{\mu}|^2 (1 - |\hat{A}_i|^2) + 7 g^2 \frac{M_2 \mu^*}{(2\pi T)^2} (5 + |\hat{A}_i|^2 + |\hat{\mu}|^2) \], \quad (A.144) \]

\[ \lambda_1 = \frac{1}{8} (g^2 + g^2) - \frac{3}{16} \frac{g^4 T}{8\pi m_{A_0}} + 3 \frac{h_i^2}{8} \frac{|\hat{\mu}|^2}{16\pi^2} \left( g^2 - 8 h_i^2 |\hat{\mu}|^2 \ln \frac{m_Q}{\mu_T} \right), \quad (A.145) \]

\[ \lambda_2 = \frac{1}{8} (g^2 + g^2) - \frac{3}{16} \frac{g^4 T}{8\pi m_{A_0}} + 3 \frac{h_i^2}{8} \frac{|\hat{\mu}|^2}{16\pi^2} \left[ - g^2 \left( 4 \ln \frac{4m_Q}{\mu_T} + 3 |\hat{A}_i|^2 \right) \right. \]
\[ + 8h_i^2 \left( 2 |\hat{A}_i|^2 + 2 \ln \frac{m_Q}{\mu_T} + \ln \frac{m_Q}{\mu_T} \right), \quad (A.146) \]

\[ \lambda_3 = \frac{1}{8} (g^2 + g^2) - \frac{3}{8} \frac{g^4 T}{8\pi m_{A_0}} + 3 \frac{h_i^2}{8} \frac{|\hat{\mu}|^2}{16\pi^2} \left[ - g^2 \left( 4 \ln \frac{4m_Q}{\mu_T} + 3 |\hat{A}_i|^2 - |\hat{\mu}|^2 \right) \right] \]
\[ +16h_t^2 |\hat{\mu}|^2 \left( \ln \frac{m_Q}{e^{\gamma_e} \mu_T} - |\hat{A}_t|^2 \ln \frac{m_Q}{e^{\gamma_e} \mu_T} \right), \]  
(A.147)

\[ \lambda_4 = -\frac{1}{2} g^2 + \frac{3}{4} \frac{h_t^2}{16\pi^2} \left[ g^2 \left( 4 \ln \frac{4m_Q}{\mu_T} + 3|\hat{A}_t|^2 - |\hat{\mu}|^2 \right) 
+ 4h_t^2 |\hat{\mu}|^2 \left( 1 - 2|\hat{A}_t|^2 \ln \frac{m_Q}{e^{\gamma_e} \mu_T} \right) \right], \]  
(A.148)

\[ \lambda_5 = \frac{1}{16\pi^2} \left[ -3h_t^4 (\hat{A}_t^* \hat{\mu}^*)^2 \ln \frac{m_Q}{e^{\gamma_e} \mu_T} 
- \frac{3\zeta(5)}{64} g^4 \left( \frac{1}{4} m_A^2 \sin^2 2\beta \frac{2}{(2\pi T)^2} - 31 (M_2 \mu^*)^2 \right) \right], \]  
(A.149)

\[ \lambda_6 = \frac{1}{16\pi^2} \left[ -\frac{3}{4} h_t^2 \hat{A}_t^* \hat{\mu}^* \left( g^2 - 8h_t^2 |\hat{\mu}|^2 \ln \frac{m_Q}{e^{\gamma_e} \mu_T} \right) 
- \frac{3\zeta(3)}{16} g^4 \left( -\frac{1}{2} m_A^2 \sin 2\beta \frac{2}{(2\pi T)^2} + 14 \frac{M_2 \mu^*}{(\pi T)^2} \right) \right], \]  
(A.150)

\[ \lambda_7 = \frac{1}{16\pi^2} \left[ \frac{3}{4} h_t^2 \hat{A}_t^* \hat{\mu}^* \left( g^2 - 8h_t^2 \ln \frac{m_Q}{\mu_T} + 8h_t^2 |\hat{A}_t|^2 \ln \frac{m_Q}{e^{\gamma_e} \mu_T} \right) 
- \frac{3\zeta(3)}{16} g^4 \left( -\frac{1}{2} m_A^2 \sin 2\beta \frac{2}{(2\pi T)^2} + 14 \frac{M_2 \mu^*}{(\pi T)^2} \right) \right]. \]  
(A.151)

In these formulas, all the scale dependence has cancelled.

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