An adaptive intelligent collaborative optimization method based on inconsistent information

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Abstract—Multidisciplinary Design Optimization (MDO) is a design optimization method for dealing with large-scale and multi coupling complex engineering systems. The collaborative optimization (CO) has the characteristics of high degree of discipline autonomy, multi-level optimization and distributed computing. It can effectively solve the design optimization problems of large-scale complex engineering systems, and has been widely used in aerospace, ship, automobile, machinery and other fields. Because of its own optimization model and principle, the CO method has the defects of low computational efficiency and difficult convergence. In this paper, in order to overcome the convergence difficulty caused by the internal definition defects of the CO method, combined with the adaptive mechanism, the position relationship between the system level optimization points and the constraint conditions is analyzed, and the adaptive penalty function is constructed based on the inconsistent information of the system. The system level optimization model of the CO method is reconstructed by transforming the system level constraints. Finally, an example is given to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION
The typical aircraft design process is usually divided into three stages: conceptual design, preliminary design and detailed design. Designers choose different key disciplines to design and optimize the aircraft in different design stages. This kind of design essentially separates the disciplines of propulsion, aerodynamics, control and structure that affect the performance of aircraft artificially, and does not make full use of the synergy effect that may be produced by the mutual coupling of various disciplines. The consequence of this design is that it is very likely to lose the overall optimal solution of the system, thus reducing the overall performance of the aircraft [1].

Among many MDO methods, collaborative optimization (CO) is a new multi-level optimization method for coupled systems, which first appeared in Kroo’s aircraft preliminary optimization design in 1994 [2]. CO method is a distributed and multi-level optimization method for complex product design problems, which divides the problem into a system level optimization and multiple parallel subsystem level optimization; each subsystem does not interfere with other subsystems and optimizes independently and parallelly, so that the optimization results of the subsystem are close to the expected value of optimization variables provided by the system level; the coordinated optimization results at the system level can be achieved through multiple iterations between the system level and the subsystem, and the overall optimal solution is finally obtained. Starting from the study of optimization model, the
problem of convergence difficulty of CO method is solved. However, due to its own optimization model and principle, the CO method has the defects of low computational efficiency and difficult convergence. The former is caused by the conventional numerical algorithm of CO method, and the latter is caused by the imperfect system level definition of CO method.

In view of these problems, some researchers start with the optimization model to solve the problem of convergence difficulty of CO method. For example, the relaxation constraint method is used to transform the system level equality constraint into the inequality constraint [3], the penalty function method for adding the conversion system level constraint to the objective function [4], and the approximation technique [5]. In the above methods, the penalty function and relaxation constraint method will not increase the data transmission between the system level and the subsystem level, the calculation amount is less than the response surface surrogate model technology which needs function fitting approximation, and does not need additional calculation process, so it has reliable convergence. Therefore, this part focuses on the discussion of penalty function and relaxation constraint method.

Aiming at the problems of the CO method, in order to overcome the convergence difficulty caused by the internal definition defects of CO method, this paper analyzes the position relationship between the system level optimization points and the constraint conditions, constructs the adaptive penalty function based on the inconsistent information of the system, transforms the system level constraint conditions, and reconstructs the system level optimization model of the CO method. Finally, an example is given to demonstrate the effectiveness of the proposed method.

2. ADAPTIVE COLLABORATIVE OPTIMIZATION METHOD

The penalty function method is used to redefine the system level of CO method, that is, the equality constraints in the system level of CO method are added to the objective function by the penalty factor to solve the problem of convergence difficulty of CO method caused by uniform equality constraint. At the same time, the transformed unconstrained objective function can be easily handled by intelligent optimization algorithm. The penalty function method includes internal point and external point penalty function method. At present, the external point penalty function method is widely used, as shown in (1).

\[
\min F(z, \gamma) = F(z) + \gamma \times \sum_{i=1}^{m} |J_i(z)|
\]

s.t. \(Z_L \leq Z \leq Z_U\)

However, with the optimization, the Hesse matrix of the function will gradually deteriorate, and the solution of the problem will become difficult. The external penalty function is usually non-differentiable on the boundary of feasible region, which depends heavily on the selection of initial points. The unconstrained optimization method will be limited, and the final optimization point may not meet the actual situation. Without the combination of variable range, the objective function cannot be guaranteed to be optimized in the feasible region. In order to avoid this problem, based on the study of the penalty function optimization model, this paper uses the mixed penalty function which combines the advantages of the external point method and the internal point method, redefines the system level optimization model of the CO method, and improves the convergence performance of the CO method, as shown in (2):

\[
\min F(z, \mu, \gamma) = F(z) - \mu \sum_{i=1}^{m} \ln\left(-g_i(z)\right) + \gamma \times \sum_{i=1}^{m} |J_i(z)|
\]

s.t. \(Z_L \leq Z \leq Z_U, g_i(z) \leq 0, J_i(z) = 0, \mu \times \gamma = 1\)

Where F(z) is the original objective function and F(z, \(\mu, \gamma\)) is the objective function transformed by the mixed penalty function. The constraint \(g_i\) of the design variables and the objective function \(J_i\) of the subsystem are transformed into penalty terms by the internal and external point methods respectively. The former ensures that each variable is always constrained by the boundary, and the latter enables the designer to select the optimal initial
point arbitrarily. In the process of optimization, the external penalty factor $\gamma$ increases and the internal penalty factor $\mu$ decreases gradually, which can be defined as the reciprocal relationship.

When the difference between the system level optimization point and the subsystem optimization point is large, the penalty must be increased to decrease the inconsistent information of the system; with the optimization going on, the objective function gradually approaches the optimization point, and the weight of the penalty term in the objective function should be reduced to make the CO method converge to the optimization point. The penalty factors $\mu$ and $\gamma$ ensure that the objective function approaches the optimization point step by step, which plays an important role in the optimization properties of the CO method. If the penalty factor is fixed, it is difficult to meet the above requirements. It must be defined that the value of penalty factor can be adjusted adaptively with the system level optimization process of CO method.

According to the relationship between system level optimization points and their constraints of CO method and subsystem level optimization points, two different situations are considered respectively. Taking two-dimensional space as an example, the principle can be extended to high-dimensional space, as shown in Fig. 1.

![Fig. 1. Schematic diagram of relative position of system level optimization points and constraints](image)

The part enclosed by the constraint curve and coordinate axis represents the variable space of the CO method. The system level optimization point $Z_{01}$ does not satisfy all the constraints, among which the distance from constraint $J_1$ is the farthest. In this case, the penalty factor should control $Z_{01}$ to move to the farthest constraint and move in the direction of the vector $\|Z_{01} - J_1\|$, which is the inconsistent information of the system. With the optimization, $Z_{01}$ has moved into the space position $Z_{02}$ which satisfies the local constraints. At this time, the penalty factor should also control $Z_{02}$ to move to the farthest constraint, that is, along the direction of vector $\|Z_{02} - J_2\|$. When the CO method is optimized to the later stage, the system level optimization point $Z_0$ enters the region that satisfies all constraints, and $Z_0$ should be prevented from jumping out in reverse. A threshold can be set. When the vector is less than the threshold, it means that $Z_0$ is located in the region that satisfies all the constraints, and any position in the region satisfies the constraint requirements. The optimal solution obtained has practical significance. In conclusion, $\gamma$ is defined by formula (3).

$$\gamma = \max \|J_t(z_t - x_t)\|, \quad \mu = \frac{1}{\nu}, \quad t = 1, 2, \ldots n$$ (3)

In the initial stage of optimization, the distance between subsystem level optimization point and system level optimization point is large. The maximum distance is selected to construct $\gamma$, and the most inconsistent subsystem optimization point is punished to make it gradually close to the system level optimization point. With the optimization going on, the inconsistency information, i.e. the radius $\gamma$, decreases, indicating that the subsystem optimization point has approached the system level optimization point, and the penalty term can be reduced to ensure that the CO method converges to the system level optimization point. $\gamma$ makes the objective function converge to the best point adaptively along the direction of maximum inconsistent information, avoiding repeated calculation; the mixed penalty function transforms all constraints to make the system level objective function meet the Karush-Kuhn-Tucker condition, which reduces the difficulty of convergence, meets the requirements of
diversified constraints of complex product design and optimization problems, avoids the risk of reducing the reliability of the optimization results that may be caused by relaxing all system-level constraints, increases the probability of convergence of objective function, and improves the optimization performance of CO method. This is of great significance for the application of CO method in engineering practice.

3. EXAMPLE VERIFICATION

An example of design optimization of aviation gear transmission system. Aeronautical gear transmission is one of the classic test cases proposed by NASA to evaluate the performance of MDO method. As a common mechanism installed between propeller and piston engine of small aircraft, aviation gear transmission system transmits the rotation between them and outputs appropriate rotational speed to obtain maximum output power. The objective of multidisciplinary design optimization of aviation gear transmission system is to obtain the minimum volume of aviation gear transmission system (the weight is the lightest when the manufacturing material density is constant) while meeting the constraints of gears and shafts in the transmission mechanism. The optimization problem includes seven design variables, i.e.: x₁ is the width of gear surface, x₂ is the module of gear, x₃ is the teeth number of small gear, x₄ and x₅ are the bearing spacing, x₆ and x₇ are the spacing of large and small gear shafts.

The constraints to be considered in function optimization are shown in Table 1. In Table 1, the maximum bending stress g₁ of the gear shall not exceed the specified value; the maximum contact stress g₂ of the gear shall meet the design value; g₃ and g₄ are the maximum transverse deflection of the large and small gear shaft, which shall not exceed the specified value; g₅ and g₆ are the maximum internal stress of the large and small gear shaft, which shall meet the strength requirements; the gear size shall meet the dimensional and spatial constraints such as g₇, g₈ and g₉; g₁₀ and g₁₁ are empirical formulas for calculating gear shaft dimensions. In addition, each variable is constrained by upper and lower bounds.

| Constraint | Constraint Description | Expression                        |
|------------|------------------------|-----------------------------------|
| g₁         | Gear bending stress constraint | \((x_1x_2x_3)/27.0 - 1.0 \geq 0\) |
| g₂         | Gear contact stress constraint | \((x_1x_2^2)/397.5 - 1.0 \geq 0\) |
| g₃         | Lateral displacement constraint of large axis | \((x_1x_3^2)/1.925x_1 - 1.0 \geq 0\) |
| g₄         | Lateral displacement constraint of small axis | \((x_1x_4^2)/1.925x_1 - 1.0 \geq 0\) |
| g₅         | Large axis stress constraint | \[110x_1^2 \sqrt{\frac{745x_1^2 + 1.691 \times 10^8}{x_2x_3}} - 1.0 \geq 0\] |
| g₆         | Small axis stress constraint | \[85x_1^2 \sqrt{\frac{745x_1^2 + 1.575 \times 10^8}{x_2x_3}} - 1.0 \geq 0\] |
| g₇         | Dimensional constraints based on space and experience | \(x_1/(5.0x_2) - 1.0 \geq 0\) |
| g₈         | Dimensional constraints based on space and experience | \(12.0x_1/x_1 - 1.0 \geq 0\) |
| g₉         | Dimensional constraints based on space and experience | \(40.0/(x_2x_3) - 1.0 \geq 0\) |
| g₁₀        | Dimensional constraints based on space and experience | \(x_2/(0.5x_1 + 1.9) - 1.0 \geq 0\) |
| g₁₁        | Dimensional constraints based on space and experience | \(x_2/(1.1x_1 + 1.9) - 1.0 \geq 0\) |
ACO method divides the optimization problem of aviation gear transmission system into three subsystems and one system level. The system level optimization adopts intelligent optimization algorithm, and the optimization model of ACO aviation gear transmission system is shown in Fig. 2.

![System level optimization model of aviation gear transmission system](image)

Fig. 2. Design optimization model of aviation gear transmission system

Among them, \( z_1 \sim z_7 \) are system level design variables, \( x_{11} \sim x_{13} \), \( x_{15} \) and \( x_{17} \) are subsystem 1 optimization variables, \( x_{21} \sim x_{23} \), \( x_{24} \) and \( x_{26} \) are subsystem 2 optimization variables, \( x_{31} \sim x_{33} \) are subsystem 3 optimization variables, and \( \gamma \) are penalty factors constructed by using difference information.

The effectiveness and efficiency of the adaptive collaborative optimization strategy (ACO) proposed in this paper is verified by the above optimization example. The conventional CO(COO), GA-ACO and SA-ACO are used to optimize the aviation gear transmission system. The system level adopts the following algorithms: SQP, GA, SA algorithm; the subsystems level all adopt SQP algorithm with the same initial point \( x = (3.5, 0.7, 17.0, 7.30, 7.715, 3.35, 5.287) \). The calculation results based on the above three optimization strategies are shown in Table 2. Fig. 3 shows the iterative process of inter-disciplinary difference information of the MDO problem under the two adaptive collaborative optimization strategies. Among them, Fig. 3 (a) is the iterative process of inter-disciplinary difference information under GA-ACO strategy, and Fig. 3(b) is the iterative process of inter-disciplinary difference information under SA-ACO strategy.

**TABLE 2 RESULTS OF DETERMINISTIC MULTIDISCIPLINARY DESIGN OPTIMIZATION FOR AVIATION GEAR TRANSMISSION SYSTEM**

| Optimization Method | \( x_1/cm \) | \( x_2/mm \) | \( x_3/n \) | \( x_4/cm \) | \( x_5/cm \) | \( x_6/cm \) | \( x_7/cm \) | \( f/cm^3 \) | Number of function calls |
|---------------------|--------------|--------------|------------|--------------|--------------|--------------|--------------|--------------|--------------------------|
| CCO                 | 3.512        | 0.700        | 17.000     | 7.300        | 7.715        | 3.351        | 5.287        | 2993.5       | 288                      |
| GA-ACO              | 3.564        | 0.746        | 17.033     | 7.997        | 7.850        | 2.971        | 5.024        | 2992.9       | 216                      |
| SA-ACO              | 3.513        | 0.707        | 17.050     | 7.430        | 7.953        | 3.202        | 5.267        | 2997.8       | 180                      |

It can be seen from the optimization results in Table 2: firstly, GA algorithm and SA algorithm are used to replace the traditional numerical optimization algorithm at the CO system level, the optimization results converge to the theoretical optimal solution, and the ideal optimal solution is obtained. Secondly, in terms of the number of function calls (including system level iterations and subsystem level analysis and optimization iterations), the traditional CCO with numerical optimization algorithm has the most function calls, up to 288 times; while the two optimization strategies based on intelligent algorithm, GA-ACO and SA-ACO, have 216 and 180 iterations respectively, and their computational efficiency increases in turn.
According to the principle of ACO method described in the second section, in the initial stage of optimization, the inconsistent information among disciplines makes the punishment factor $\gamma$ greatly increase so as to accelerate the convergence speed. With the optimization, the disciplines gradually tend to be consistent, and the increasing speed of penalty factor $\gamma$ slows down, so the convergence speed also slows down obviously. It can also be seen from Fig. 3 that the increase of inconsistent information among disciplines in the fifth optimization iteration of GA-ACO strategy and in the third optimization iteration of SA-ACO strategy both significantly accelerate the convergence speed of the objective function.

The optimization results also show that the ACO strategy has high numerical stability and computational efficiency. It not only converges to the vicinity of the optimal solution, but also reduces the inconsistent information among subjects in the optimization process. When the objective function finally converges, all disciplines basically become consistent. It can be seen that the proposed adaptive intelligent collaborative optimization (ACO) method has good optimization effect and can meet the actual needs of MDO problems.

4. **CONCLUSION**

Starting from the defects of the standard CO method, this paper discusses several common ways to improve the CO method at present. Through analysis and comparison, it is determined that the mixed penalty function method can improve the convergence performance of the CO method, which has the best comprehensive effect and has better realizability and operability. On the basis of the existing research, this paper proposes to redefine the system level optimization model with the mixed penalty function, so that the CO method has mathematically stable solution conditions, and meets the operational requirements of intelligent optimization algorithm; the relationship between the system level optimization points and the constraint conditions in the optimization process is analyzed, and the inconsistent information of the system is extracted to construct the adaptive penalty factor, and the system level optimization model of CO method is reconstructed. Finally, an example is given to show the effectiveness of the method.

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