One-loop Radiative Corrections to the $\rho$ Parameter in the Left Right Twin Higgs Model

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We implement a one-loop analysis of the $\rho$ parameter in the Left Right Twin Higgs model, including the logarithmically enhanced contributions from both heavy fermion and scalar loops. Numerical analysis indicates that the one-loop corrections are dominant over the tree-level contributions in most regions of parameter space. The experimentally allowed values of the $\rho$-parameter divide the allowed parameter space into two regions; less than 670 GeV and larger than 1100 GeV roughly, for the symmetry breaking scale $f$. Therefore, our result significantly reduces the parameter space that is favorably accessible to the LHC.

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I. INTRODUCTION

The Standard Model (SM) has excellently described high energy physics up to energies of $\mathcal{O}(100)$ GeV. The only undetected constituent of the SM to date is the Higgs boson, which is essential to generating fermion and gauge boson masses. Theoretically, the Higgs boson’s mass squared is quadratically sensitive to any new physics scale beyond the Standard Model (BSM) and stabilization of the Higgs mass squared prefers an energy scale below $\mathcal{O}(1)$ TeV. However, electroweak precision measurements with naive naturalness assumption raise the energy scale of the BSM up to 100 TeV or even higher. Hence, a tension associated with the stabilization of the SM Higgs mass, remains between theory and experiment. With the latest data from the LHC the tension has gotten stronger.

The basic idea of little Higgs is that the SM Higgs is a pseudo-Nambu-Goldstone boson (pNGB) [1–7]. Stabilization of the Higgs mass in the little Higgs theories is achieved by the “collective symmetry breaking”, which naturally renders the SM Higgs mass much smaller than the global symmetry breaking scale. The distinct elements of little Higgs models are a vector-like heavy top quark, and various scalar and vector bosons. The former is universal while the latter are model-dependent.

Both of them contribute significantly to one-loop processes and, hence, set strict constraints on the parameter space of little Higgs models. At worst, electroweak precision tests push up the symmetry breaking scale to 5 TeV or higher, and even revive the fine-tuning problem of the Higgs potential.

The idea of twin Higgs shares the same origin with that of little Higgs. However, there is a difference between the two as for the stabilization of the Higgs mass squared. The twin Higgs mechanism introduces additional discrete symmetry to render no quadratic divergence in the Higgs mass squared. For instance, the mirror twin Higgs model containing a complete copy of the SM identifies the discrete symmetry with mirror parity. The SM world and its mirror world communicate only through the Higgs so that the mirror particles are very elusive in the SM world and yield poor phenomenology at the LHC.

The twin Higgs mechanism can also be realized by identifying the discrete symmetry with left-right symmetry in the left-right model [9]. The left-right twin Higgs (LRTH) model contains $U(4)_1 \times U(4)_2$ global symmetry as well as $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. The left-right symmetry acts on only the two $SU(2)$’s gauge symmetry. A pair of vector-like heavy top quarks play a key role in triggering electroweak symmetry breaking just as that of the little Higgs theories. On top of that, the non-SM Higgs particles acquire large masses not only at the quantum level but also at the tree level, causing the model to deliver much richer phe-
nomenclature at the LHC [10] compared with the mirror twin Higgs model. Moreover, they lead to large radiative corrections to one-loop processes, so the allowed parameter space can be significantly reduced. In this paper, we perform a one-loop analysis of the $\rho$-parameter in the LRTH model to reduce the parameter space.

The paper is organized as follows: The LRTH model is briefly reviewed in Section II. The renormalization procedure for the $\rho$-parameter is explained in Section III. The numerical analysis on the $\rho$-parameter is performed in Section IV. We present our conclusions in Section V. The technical details on the computation of the $\rho$-parameter are reckoned in the Appendices.

II. LEFT RIGHT TWIN HIGGS MODEL IN A NUTSHELL

We review the LRTH model in Ref. [10]. The LRTH model is based on the global $U(4)_1 \times U(4)_2$ symmetry, with a locally gauged subgroup $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. A pair of Higgs fields, $H$ and $\tilde{H}$, are introduced, and each transforms as $(4, 1)$ and $(1, 4)$, respectively, under the global symmetry. They are written as

$$H = \begin{pmatrix} H_L \\ \tilde{H}_L \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} \tilde{H}_L \\ H_R \end{pmatrix},$$

where $H_{L,R}$ and $\tilde{H}_{L,R}$ are two component objects that are charged under the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as

$$H_L : (2, 1, 1), \quad H_R \text{ and } \tilde{H}_R : (1, 2, 1).$$

The global $U(4)_1 (U(4)_2)$ symmetry is spontaneously broken down to its subgroup $U(3)_1 (U(3)_2)$ with VEVs:

$$(H)^T = (0, 0, 0, f), \quad (\tilde{H})^T = (0, 0, 0, \tilde{f}).$$

The spontaneous symmetry breaking results in seven Nambu-Goldstone bosons (NGB), which are parameterized as

$$H = f e^{i\pi/3} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \pi = \begin{pmatrix} -\frac{N}{2\sqrt{3}} & 0 & 0 & h_1 \\ 0 & -\frac{N}{2\sqrt{3}} & 0 & h_2 \\ 0 & 0 & C^* & \sqrt{N} \\ h_1^* & h_2^* & C^* & \sqrt{N} \end{pmatrix},$$

where $\pi$ is the corresponding Goldstone field matrix. $N$ is a neutral real field, $C$ and $C^*$ are a pair of charged complex scalar fields, and $h_{SM} = (h_1, h_2)$ is the SM $SU(2)_L$ Higgs doublet. Note that the normalization of $N$ is naturally altered when applying the unitary gauge. $H$ is parameterized in the identical way by its own Goldstone boson matrix, $\tilde{\pi}$, which contains $\tilde{N}$, $\tilde{C}$, and $\tilde{h} = (\tilde{h}_1^*, \tilde{h}_2^*)$. The two $U(4)/U(3)$’s symmetry breakings yield fourteen NGBs in all.

The linear combination of $C$ and $\tilde{C}$, and the linear combination of $N$ and $\tilde{N}$ are eaten by the gauge bosons of $SU(2)_R \times U(1)_{B-L}$, which is broken down to the $U(1)_{Y}$. The orthogonal linear combinations, a charged complex scalar $\phi^\pm$ and a neutral real pseudoscalar $\phi^0$, remain as NGBs. On top of that, the SM Higgs acquires a VEV, $\langle h_{SM} \rangle = (0, v/\sqrt{2})$, so electroweak symmetry $SU(2)_L \times U(1)_{Y}$ is broken down to $U(1)_{EM}$, but $h$’s do not get a VEV and remain as NGBs. At the end of the day, the two Higgs VEVs are given by

$$\langle H \rangle = \begin{pmatrix} 0 \\ f \cos x \\ f \sin x \end{pmatrix}, \quad \langle \tilde{H} \rangle = \begin{pmatrix} 0 \\ 0 \\ \tilde{f} \end{pmatrix},$$

where $x = \frac{\sqrt{2}f}{v}$. The values of $f$ and $\tilde{f}$ will be bounded by electroweak precision measurements. In addition, $f$ and $\tilde{f}$ are interconnected once we set $v = 246$ GeV.

1. Gauge Sector

The whole gauge symmetry of the model is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. However, $SU(3)_C$ gauge symmetry is not taken into account in this paper because our interest lies in the electroweak symmetry breaking. The generators of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are given, respectively, as

$$\left( \begin{array}{ccc} \frac{1}{2}\sigma_1 & 0 & 0 \\ 0 & \frac{1}{2}\sigma_1 & 0 \\ 0 & 0 & 1 \end{array} \right), \quad \left( \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right),$$

and the corresponding gauge fields are $W_{+0}^\pm, W_{-0}^\pm$ and $B$. The covariant derivative is then given as

$$D_\mu = \partial_\mu - igW_\mu - ig'q_{B-L}B_\mu.$$
with \( q_{B-L} = 1 \). The Higgs mechanism for both \( H \) and \( \dot{H} \) makes the six gauge bosons massive, but one gauge boson, the photon, massless. For the charged gauge bosons, there is no mixing between the \( W_L \) and \( W_R \): \( W_L \) is identified with the SM weak gauge boson \( W^\pm \) while \( W_R \) is much heavier than \( W^\pm \) and is denoted as \( W_{R}^\pm \). Their masses are

\[
M_{W_L}^2 = \frac{1}{2} g^2 f^2 \sin^2 x, \quad M_{W_R}^2 = \frac{1}{2} g^2 (f^2 + f^2 \cos^2 x),
\]

For later use, we define the Weinberg angle of the LRTH model:

\[
s_w = \sin \theta_w = \frac{g'}{\sqrt{g'^2 + 2 g'^2}}, \quad c_w = \cos \theta_w = \frac{g}{\sqrt{g^2 + 2 g'^2}}, \quad c_2w = \cos 2 \theta_w = \frac{g}{\sqrt{g^2 + 2 g'^2}}.
\]

The unit of the electric charge is then given by

\[
e = g s_w = \frac{g g'}{\sqrt{g^2 + 2 g'^2}}.
\]

### 2. Fermion Sector

To cancel the quadratic sensitivity of the Higgs mass to the top quark loops, we incorporate a pair of vector-like, charge 2/3 fermion \((\bar{Q}_L, Q_R)\) into the top Yukawa sector:

\[
\mathcal{L}_{Yuk} = y_L \bar{Q}_L \tau_2 H_L^{+} Q_R + y_R \bar{Q}_R \tau_2 H_R^{+} Q_L - M \bar{Q}_L Q_R^{h.c.},
\]

where \( \tau_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \), \( Q_{L,3} = -i (u_{L,3}, d_{L,3}) \) and \( Q_{R,3} = (u_{R,3}, d_{R,3}) \) are the third generation up- and down-type quarks, respectively. The left-right parity indicates \( y_L = y_R(\equiv y) \). The mass parameter \( M \) is essential to the top mixing. The value of \( M \) is constrained by the \( Z \to b\bar{b} \) branching ratio. It can also be constrained by the oblique parameters, which we will do in the paper. Furthermore, it yields a large log divergence of the SM Higgs mass. To compensate for that, the non-SM gauge bosons also get large masses by increasing the value of \( f \). Therefore, it is natural for us to take \( M \lesssim g f \).

Expanding the \( H_{L,R} \) field in terms of the NGB fields, we acquire the mass matrix of the fermions. By diagonalizing it, we obtain not only the mass eigenstates for the SM-like and heavy top quarks, but also the mixing angles for the left-handed and right-handed fermions:

\[
m_t^2 = \frac{1}{2} (M^2 + g^2 f^2 - N_t),
\]

\[
m_T^2 = \frac{1}{2} (M^2 + g^2 f^2 + N_t),
\]

\[
\sin \alpha_L = \frac{1}{\sqrt{2}} \sqrt{1 - (g^2 f^2 \cos 2 \tau - M^2)/N_t},
\]

\[
\sin \alpha_R = \frac{1}{\sqrt{2}} \sqrt{1 - (g^2 f^2 \cos 2 \tau - M^2)/N_t},
\]

where \( N_t = \sqrt{(g^2 f^2 + M^2)^2 - g^4 f^4 \sin^2 2 \tau} \).

### 3. Higgs Sector

Among the fourteen NGBs in both \( \pi \) and \( \pi \), six NGBs are eaten by the gauge bosons. The remaining eight NGBs get masses through quantum effects and/or soft symmetry breaking terms, so called \( \mu \)-term. The Coleman-Weinberg potential, obtained by integrating out the heavy gauge bosons and top quarks, yields the SM Higgs potential, which determines the SM Higgs VEV and its mass, as well as the masses for the other
Higgs, $\phi^\pm, \phi^0, \tilde{h}_1^\pm$ and $\tilde{h}_2^0$. Moreover, the $\mu$-term contributes to the Higgs masses at the tree level:

$$V_\mu = -\mu^2 (H_R^\dagger H_R + c.c.) + \mu^2 \tilde{H}_L^\dagger \tilde{H}_L.$$  \hfill (22)

One may include $-\mu^2 (H_R^\dagger H_R + c.c.)$ to the Higgs potential, but here we choose $\mu_1 = 0$ so as to keep the original motivation of the model and to preserve the stability of $\tilde{h}_2$ dark matter [10]. We write the masses for the Higgs as

$$M_{\phi^0}^2 = \frac{\mu^2 f f}{f^2 + f^2 \cos^2 x} \left[ f^2 (\cos x + \frac{\sin x}{2} (4 + x^2)) + \frac{2 \cos x (\cos x + 4 \sin x)}{3 (\cos x + 2 \sin x)^2} + \frac{f^2 \cos^2 x (4 + \cos x)}{9 f^2} \right],$$

$$M_{\phi^\pm}^2 = \frac{3}{16 \pi^2} g^2 M_{W_R}^2 \left[ \left( \frac{M_{W_R}^2}{M_Z^2} - 1 \right) \mathcal{Z}(M_{Z_R}) - \left( \frac{M_{W_R}^2}{M_Z^2} - 1 \right) \mathcal{Z}(M_Z) \right] + \frac{\mu^2 f f}{f^2 + f^2 \cos^2 x} \left[ f^2 x^2 + 2 \cos x + \frac{f^2 \cos^3 x}{f^2} \right],$$

$$M_{h_2}^2 = \frac{3}{16 \pi^2} \left[ (\mathcal{Z}(M_W) - \mathcal{Z}(M_{W_R})) + \frac{2 g^2 + g^2 M_{W_R}^2 - M_{W_R}^2 (\mathcal{Z}(M_Z) - \mathcal{Z}(M_{Z_R}))}{4} + \frac{\mu^2 f f}{f^2} \cos x + \frac{\mu^2}{f^2} \right],$$

$$M_{h_1}^2 = M_{h_2}^2 + \frac{3}{16 \pi^2} g^2 M_{W_R}^2 \left[ \left( \frac{M_{W_R}^2}{M_Z^2} - 1 \right) \mathcal{Z}(M_{Z_R}) - \left( \frac{M_{W_R}^2}{M_Z^2} - 1 \right) \mathcal{Z}(M_Z) \right].$$  \hfill (23)

where

$$\mathcal{Z}(x) = -x^2 (\ln \frac{A^2}{x^2} + 1),$$  \hfill (27)

with $A$ being a UV cutoff. The SM Higgs potential arises mainly from both the top and the gauge sectors. The contribution of the fermion loops to the SM Higgs mass squared is negative, and its dominance over the contribution of the gauge boson loops and the tree level mass parameter $\mu^2 f f$ triggers electroweak symmetry breaking. We fix the SM Higgs VEV as $v = 246$ GeV.

### III. Renormalization Procedure

We follow the renormalization procedure in Ref. [11] to calculate the $\rho$-parameter at one-loop order. The $Z$-pole, $W$-mass, and neutral current data can be used to fix $A$ for and set limits on the deviations from the SM. The $\rho$-parameter is defined as

$$\rho = \frac{M_W^2}{M_Z^2 + c^2_w}.$$  \hfill (28)

The electroweak mixing angle $s^2_\theta (\equiv 1 - c^2_\theta)$ in the effective leptonic (electronic) vertex of the $Z$ boson is defined as

$$s^2_\theta = \frac{1}{4} \left( 1 + \text{Re} \frac{g^L_V}{g^L_A} \right)$$  \hfill (29)

in terms of the effective vector and axial vector couplings $g^L_V, A$ of the $Z$ to electrons;

$$\mathcal{L} = i e g_L (g_V + g_A g_3) e Z^\mu.$$  \hfill (30)

The effective Lagrangian of the charged current interaction in the LRTH model is given by

$$\mathcal{L}_{ee} = \frac{g}{\sqrt{2}} \left( W_{\mu L}^+ J_{\mu e}^- + W_{\mu R}^- J_{\mu e}^+ \right) + (L \rightarrow R),$$  \hfill (31)

where $J_{\mu e}^\pm_L$ is the charged currents. For momenta quite small compared to $M_W$, this effective Lagrangian gives rise to the effective four-fermion interaction with the Fermi coupling constant $g_{\nu L} = \frac{g \sqrt{2}}{8 M_W}$, and the vector and the axial vector parts of the neutral current $Z ee$ coupling constants are given to the order $v^2/f^2$ as

$$g^L_V = \frac{g}{2 c_w} \left[ (-\frac{1}{2} + 2 s^2_w) + \frac{v^2}{4 (f^2 + f^2)} \frac{s^2_w (c^2_{2w} - 2)}{c^4_w} \right],$$  \hfill (32)

$$g^L_A = \frac{g}{2 c_w} \left[ \frac{1}{2} + \frac{v^2}{4 (f^2 + f^2)} \frac{s^2_w c^2_w}{c^4_w} \right].$$  \hfill (33)

The effective leptonic mixing angle $s^2_\theta$ in Eq. (29) is then related to the mixing angle $s^2_w$ as

$$s^2_\theta = s^2_w - \frac{v^2}{f^2 + f^2} \frac{s^4_w c^2_w}{c^4_w},$$  \hfill (34)

which can then be inverted to

$$s^2_w = s^2_\theta + \Delta s^2_\theta,$$  \hfill (35)
where
\[
\frac{\Delta s_\theta^2}{s_\theta^2} = -\zeta + \frac{1}{2} \frac{c_\theta^2}{s_\theta^2} - \sqrt{\zeta + \left( -\frac{1}{2} \frac{c_\theta^2}{s_\theta^2} \right)^2},
\]
with
\[
\zeta \equiv \frac{v^2}{f^2 + f_2^2}.
\]

The SM SU(2)_L gauge coupling constant, \( g \), is expressed by using both the effective leptonic mixing angle, \( s_\theta^2 \), and the fine-structure constant, \( \alpha \), as
\[
g^2 = \frac{e^2}{s_\theta^2} = \frac{4\pi\alpha}{s_\theta^2} \left( 1 - \frac{\Delta s_\theta^2}{s_\theta^2} \right).\]

The \( \rho \)-parameter at tree level is
\[
\rho^{\text{tree}} = \frac{\pi\alpha}{\sqrt{2} G_F c_\theta^2 s_\theta^2 M_Z^2} \left( 1 - \frac{\Delta s_\theta^2}{s_\theta^2} + \Delta r_Z \right),
\]
where \( \Delta r_Z \) includes radiative effects from various sources:
\[
\Delta r_Z = \frac{\delta\alpha}{\alpha} - \frac{G_F}{G_\text{F}} - \frac{M_Z^2}{M_Z^2} - \left( \frac{c_\theta^2}{c_\theta^2} - \frac{s_\theta^2}{s_\theta^2} \right) \frac{\delta s_\theta^2}{s_\theta^2}.
\]
We should mention that \( \Delta r_Z \) in Eq. (41) differs from the usual \( \Delta r_Z \) defined in the SM by extra corrections due to the renormalization of \( s_\theta^2 \). In general, the vertex and box contributions to the radiative effects are relatively small compared to the other corrections [12, 13]; hence, we consider only the “oblique corrections”, i.e. the vacuum polarization corrections to \( W, Z \) and the photon. The correction due to the vacuum polarization of the photon, \( \delta\alpha \), is given by
\[
\frac{\delta\alpha}{\alpha} = \Pi^{\gamma\gamma}(0) + 2 \left( g_\gamma^2 - g_\text{F}^2 \right) \Pi^{\gamma Z}(0) \frac{M_Z^2}{M_Z^2}.
\]
Because we ignore the vertex and the box corrections, the electroweak radiative correction to the Fermi constant, \( \delta G_F \), stems from the W-boson vacuum polarization:
\[
\frac{\delta G_F}{G_F} = - \frac{\Pi^{WW}(0)}{M_W^2}.
\]

The counterterms for the \( Z \)-boson mass, \( \delta M_Z^2 \), and the leptonic mixing angle, \( \delta s_\theta^2 \), are given, respectively, by [13]
\[
\delta M_Z^2 = Re[\Pi^{ZZ}(M_Z^2)],
\]
\[
\frac{\delta s_\theta^2}{s_\theta^2} = Re \left[ \frac{c_\theta^2}{s_\theta^2} \left( \frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} + \frac{v^2}{a_e} \right) + \frac{v^2}{2 s_\theta c_\theta} \left( \frac{\Lambda^{\gamma e e}(M_Z^2)}{v} - \frac{\Lambda^{\gamma e e}(M_Z^2)}{v} \right) \right].
\]
where \( \Lambda^{\gamma e e} \) are the vector and the axial vector form factors of the unnormalized one-loop \( Zee \) vertex corrections, and \( \Sigma_A \) is the axial part of the electron self-energy. Once again, we ignore the non-oblique corrections.

The tree-level and radiative corrections, except for the \( W \)-boson, are summed and expressed as
\[
\Delta \hat{r} = - \frac{\Delta s_\theta^2}{s_\theta^2} \frac{Re[\Pi^{ZZ}(M_Z^2)]}{M_Z^2} + \Pi^{\gamma\gamma}(0)
\]
\[
+ 2 \left( g_\gamma^2 - g_\text{F}^2 \right) \Pi^{\gamma Z}(0) \frac{M_Z^2}{M_Z^2} + \frac{c_\theta^2}{s_\theta^2} Re[\Pi^{\gamma Z}(M_Z^2)],
\]
and then, Eq. (40) is rewritten as,
\[
\frac{\delta s_\theta^2}{s_\theta^2} = \frac{\pi\alpha(M_Z)}{\sqrt{2} G_F M_Z^2 \rho} \left[ 1 + \frac{\Pi^{WW}(0)}{M_W^2} + \Delta \hat{r} \right].
\]
Solving Eqs. (28) and (47) for the \( W \)-boson mass, we get
\[
M_W^2 = \frac{1}{2} \left[ a(1 + \Delta \hat{r}) + \sqrt{a^2(1 + \Delta \hat{r})^2 + 4a\Pi^{WW}(0)} \right],
\]
where \( a \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2} G_F M_Z^2} \). Finally, the \( \rho \) parameter is calculated using Eq. (28).

IV. NUMERICAL ANALYSIS

In order to take the precision measurements, we need the standard experimental values as input parameters. These are the input parameters we take [14]:
\[
G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2},
\]
\[
M_Z = 91.1876(21) \text{ GeV},
\]
\[
\alpha(M_Z)^{-1} = 127.918(18),
\]
\[
s_\theta^2 = 0.23153(16).
\]
We also take the top and the bottom quark masses as [14–16]
\[
m_t = 172.3 \text{ GeV},
\]
\[
m_b = 3 \text{ GeV},
\]
where \( m_t \) is the central value of the electroweak fit and \( m_b \) is the running mass at the \( M_Z \) scale with the \( \tilde{M}_S \)
scheme. The $\rho$-parameter itself is measured very accurately [14]:

$$\rho \equiv \frac{\rho_0 \tilde{\rho}}{M^2_{\rho_0} \lambda^2_{\rho_0}}$$

$$\rho_0 = 1.0009^{+0.0007}_{-0.0007},$$  \hspace{0.5cm} (54)

$$\tilde{\rho} = 1.01043 \pm 0.00034.$$  \hspace{0.5cm} (55)

Including all the SM corrections (top quark loop, bosonic loops), we take the allowed range of the $\rho$ parameter as

$$1.00989 \leq \rho_{\text{exp}} \leq 1.01026.$$  \hspace{0.5cm} (56)

The input parameters of the LRTH model are $f, M, \mu_r,$ and $\hat{\mu},$ where $f$ is the Higgs VEV in Eq.(3), $M$ is the heavy top quark mass scale, and both $\mu_r$ and $\hat{\mu}$ are soft symmetry breaking terms. The masses of the top and the heavy top quarks are determined mainly by $f$ and $M$ while those of the scalar particles $h_1, h_2, \phi^\pm$ and $\phi^0$ largely depend on $\mu_r, \mu_r$ and $f.$ Another scale $\hat{f}$ associated with the masses of the heavy gauge bosons, can be determined by the electroweak symmetry breaking condition: there is a generic relation between $f$ and $\hat{f}$ because the Coleman-Weinberg potential of the Higgs boson mostly depends on $M, f$ and $\hat{f}.$ For the scalar potential, there is a tree-level mass term proportional to $\mu_r^2,$ so we may not acquire the negative mass squared term that is necessary for the electroweak symmetry breaking, which gives an upper bound for the value of $\mu_r.$

Figure 1 shows $f$ versus $\hat{f}$ for various values of the heavy top mass scale, $M.$ For a given $f,$ $\hat{f}$ becomes larger as $M$ increases. The heavy top loop through $M$ contributes positively to the Higgs mass while the heavy gauge boson loop through $f$ contributes negatively to the Higgs mass. The two contributions cancel out in order to retain $v = 246$ GeV. There is also a contribution from the tree-level mass term $\mu_r^2,$ but in most cases, it makes little difference to the relation as long as $\mu_r$ is much smaller than the Higgs mass scale. This insensitivity can be figured out with a simple evaluation. First, from the electroweak symmetry breaking condition, the mass-squared contribution from the soft symmetry-breaking term $\mu_r^2 f$ should be smaller than that from the fermion loop. This can be written approximately as

$$\mu_r^2 f \lesssim \frac{3}{8\pi^2} (M^2 + g^2 f^2).$$  \hspace{0.5cm} (57)

In the above inequality, we ignore the gauge boson loop contributions because they are small compared to the fermion loop contributions. In general, $\hat{f}$ is larger than $f$ by about 5 times or more and $\frac{3}{8\pi^2}$ is very small, so we can see that $\mu_r$ should be very much smaller than $f.$ To get the $\hat{f}$ that reproduces the electroweak symmetry-breaking scale $v = 246$ GeV, we should solve the equation

$$\frac{3g^4}{64\pi^2} f^2 + \mu_r^2 \hat{f} + 2\lambda v f - \frac{3}{4\pi^2} f (M^2 + f^2) + \frac{3g^4}{64\pi^2} = 0$$  \hspace{0.5cm} (58)

for given $f, M$ and $\mu_r.$ $\lambda$ in the above equation is the coefficient of the quartic term and is less than 1 in general. Note that we derive the above equation with some degree of approximation. For example, we ignore the logarithmic terms, but the crude behavior will be similar. In this equation, the coefficients of $\hat{f}^2$ and $\hat{f}$ are much smaller than constant term, so the solution $\hat{f}$ is almost insensitive to the value of $\mu_r.$

Plots in Fig. 2 illustrate the behavior of one-loop $\rho$-parameter for various values of $M.$ At $M = 0,$ there is no mixing between the top and the heavy top quarks, $\Delta \rho$ increases monotonically with $f.$ For nonzero $M$ where the mixing is turned on, the mass of the heavy top quark becomes large as $M$ increases for a given $f,$ and the fermionic loop contributions tend to become large, too. The effects of mixing angles on the fermionic loops become significant as either $f$ or $M$ increases while the condition of electroweak symmetry breaking is retained. In other words, because the mixing angles are determined...
by $f$ and $M$, the one-loop corrected $\rho$ parameter begins to waver as $f$ increases even with the fixed scalar mass parameters. Because of this, fine tuning in the $\rho$-parameter is inevitable for large $f$, as will be shown later.

To extract meaningful information on the model parameters from the $\rho$-parameter, we scan the parameter space generally, i.e.,

$$500 \text{ GeV} \leq f \leq 2500 \text{ GeV}, \quad 0 \leq M, \mu_r, \hat{\mu} \leq f.$$  \hfill (60)

We take a rather large value of $f$, 2.5 TeV, as an upper limit for completeness of scanning. As a result of the $\rho$-parameter calculation, we can obtain the allowed regions of parameter space. As an example, Fig. 3 shows the allowed regions of parameter space (a) for $f$ versus $M$ and (b) for $f$ versus $\mu_r$. Note that the allowed parameter space for $f$ is divided into two regions: less than 670 GeV and larger than 1100 GeV roughly. This can be figured out as follows: The loop corrections tend to be larger as $f$ increases because the masses of the particles involved in the one-loop correction increase, in general, as $f$ increases. However, at the same time, the mixing angles of the top-heavy top quarks also vary. Because the mixing angles depend on not only $f$ but also $M$, these two effects compete during the increase of $f$. Because of this interplay of top mixing angles and masses, we have two distinct allowed parameter spaces. For small $f$, solution points prefer very small values of $M$. This means there is no large mixing between the top and the heavy top quarks. In general, $\Pi^{WW}(0)$ is large for small $f$ and decreases as $f$ increases. Thus, for fitting the observed W-boson mass in the small $f$ region, which is directly related to the $\rho$-parameter, we restrict the $\Delta \hat{\rho}$ to a rather small range. Because the $\Delta \hat{\rho}$ is mostly determined by $\Pi^{ZZ}(M_Z^2)$, it should also be small. For doing that, we should take a small value of $M$, which makes the masses and mixing angles of heavy top quark small. We find that in the small $f$ region, $M$ should be smaller than about 22 GeV. The soft symmetry-breaking parameter $\mu_r$ is restricted to the values less than around 60 GeV. This bound arises mainly from the electroweak symmetry breaking condition and is generically independent of the $\rho$-parameter. Another free parameters $\hat{\mu}$ is not restricted by the one-loop corrected $\rho$-parameter. The reason is that $\hat{\mu}$ only contributes to the masses of $\hat{h}_1$ and $\hat{h}_2$, and their contributions are effectively cancelled among the relevant loop diagrams. This is pointed out in Appendix C.

This region of parameter space can provide constraints on the masses of the many particles that appear in this model. First, let us consider the masses of the heavy top
we can give the lower bound for \( f \) as 1.1 TeV from our calculation of the \( \rho \)-parameter and for the many particles that appear in the model. Another constraint on the \( m_{W_R} \) from CDF and D0 is a lower bound of 650 \( \sim \) 786 GeV [19,20]. Our results remain safe based on these experimental bounds. The heavy \( Z \) boson has also been studied in detail by many experimentalists. The current experimental bound is about 500 \( \sim \) 800 GeV from precision measurements [14] and \( \sim \) 630 GeV from CDF [14]. In this case, also safe is the mass of the heavy \( Z \) boson.

With the parameters allowed by the \( \rho \)-parameter, the masses of new scalar bosons \( \hat{h}_{1,2}, \phi^0 \) and \( \phi^\pm \) can be constrained. \( \hat{h}_{1,2} \) have almost degenerate masses and depend on both \( \mu_r \) and \( \hat{\mu}_r \), unlike the \( \phi^{0\pm} \), which depend only on \( \mu_r \). Their masses are substantially constrained according to the value of \( f \). Unfortunately, we cannot give a lower bound on the mass of \( \phi^0 \). In fact, its mass, though it is quite small, arises from radiative corrections. For \( \phi^\pm \), the loop effects are rather large, so they are heavier than \( \phi^0 \) as shown in Fig. 5.

The distribution of the SM Higgs mass as a function of \( f \) is shown in Fig. 6. As for the lower bound of the SM Higgs mass we adopt the LEP bound for the Higgs mass, 114.4 GeV [21], because its structure is the same as that of the SM. Its upper bound is approximately given as 167 GeV.

We summarize the results of our analysis as follows: With the observed \( \rho \)-parameter, the allowed parameter space is divided into two separate regions: \( f \) smaller than about 670 GeV and \( f \) larger than about 1.1 TeV. We can give the mass bounds of the particles in the LRTH model for either region. However, the heavy gauge bosons remain safe from the experimental constraints. Unlike the other particles, we cannot set a lower bound for the neutral \( \phi^0 \) scalar. The loop corrections play an important role in the charged \( \phi^\pm \) scalars, yielding a mass difference between the charged and the neutral scalars. Further analysis is required to reduce the allowed region. If the small \( f \) region is excluded, for example by Ref. [10], we can provide exact lower bounds for the masses of \( T, Z_H, W_H, \hat{h}_{1,2} \), and \( \phi^\pm \) but even in that case, we cannot do so for \( \phi^0 \) and the SM Higgs boson.
The left-right twin Higgs model is a concrete realization of the twin Higgs mechanism. The model predicts a heavy top quark, heavy gauge bosons and various scalar bosons, along with a light SM Higgs boson, and will, in turn, yield rich phenomenology of the new particles at the LHC. We have performed an indirect search for the existence of the particles. The heavy top and new scalars contribute significantly to the isospin violating the $\rho$-parameter. One-loop radiative corrections to the $\rho$-parameter reduce the parameter space of the model and can set rough bounds for the masses of the heavy particles. In particular, we demonstrated that the symmetry breaking parameter $f$ can be either smaller than 660 GeV or larger than 1.1 TeV, which is a crucial region in parameter space. More analysis on other one-loop processes, as well as study of collider physics, is mandatory to further reduce the region of parameter space.

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V. CONCLUSION

We summarize the relevant coupling constants relevant to our calculation. The gauge-fermion interaction is given by

$$\mathcal{L} = i \bar{\psi} \gamma_\mu (g_V + g_A \gamma_5) \psi_2 X^\mu,$$

$$= i \bar{\psi} \gamma_\mu (c_L P_L + c_R P_R) \psi_2 X^\mu, \quad (A1)$$

where $P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)$ are the projection operators. We make a list of the gauge coupling constants of the fermions in Table 1.

The other gauge-scalar interactions are also taken into account. We choose the unitary gauge in which all gauge-scalar mixing terms vanish. The various gauge coupling constants of the scalar fields are given in Table 2, 3, and 4.

APPENDIX A: COUPLING CONSTANTS OF THE LRTH MODEL

We list scalar integrals relevant for one-loop Feynman diagrams. The one-loop scalar integrals are decomposed in terms of Passarino-Veltman functions [23], which are defined in $d = 4 - 2\epsilon$ dimensions:

$$Q^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2 + i\epsilon} \equiv \frac{i}{16\pi^2} A_0 (m^2), \quad (B1)$$

$$Q^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)} \equiv \frac{i}{16\pi^2} B_0 (p^2, m_1^2, m_2^2), \quad (B2)$$

$$Q^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)} \equiv \frac{i}{16\pi^2} \mu_\nu B_1 (p^2, m_1^2, m_2^2), \quad (B3)$$

$$Q^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)} \equiv \frac{i}{16\pi^2} [g_\mu_\nu B_{22} (p^2, m_1^2, m_2^2) + p_\mu p_\nu B_{11} (p^2, m_1^2, m_2^2)], \quad (B4)$$

where $Q$ is the renormalization scale and $1/\epsilon = (4\pi)^2 \Gamma(1 + \epsilon)/\epsilon$. We also define the following integrals:

$$I_1 (a) \equiv \int_0^1 dx \ln [1 - ax (1 - x)], \quad (B5)$$

$$I_3 (a) \equiv \int_0^1 dx [x(1 - x) \ln [1 - ax (1 - x)]], \quad (B6)$$

$$I_4 (a, b) \equiv \int_0^1 dx \ln [1 - x + ax - bx (1 - x)], \quad (B7)$$

$$I_5 (a, b) \equiv \int_0^1 dx x \ln [1 - x + ax - bx (1 - x)], \quad (B8)$$

$$I_6 (a, b) \equiv \int_0^1 dx (1 - x) \ln [1 - x + ax - bx (1 - x)], \quad (B9)$$

$$I_7 (a) \equiv \int_0^1 dx x (1 - x) \ln [1 - x + ax - bx (1 - x)], \quad (B10)$$

$$I_7 (a) \equiv \int_0^1 dx x (1 - x) \ln [1 - x + ax - bx (1 - x)], \quad (B10)$$
Table 1. Relevant coupling constants $X^\psi\psi$. The mixing angles $C_L = \cos\alpha_L, C_R = \cos\alpha_R$, etc. are given in Eq. (19).

| Parameter | $eL$ | $eS_L$ | $eR$ |
|-----------|------|--------|------|
| $Wb$      | $g_L = eC_L/\sqrt{2}$ | $g_L = eS_L/\sqrt{2}$ | $g_R = 0$ |
| $Wtb$     | $g_L = eC_L/\sqrt{2}$ | $g_L = eS_L/\sqrt{2}$ | $g_R = 0$ |
| $Ztt$     | $g_L = eC_L/\sqrt{2}$ | $g_L = eS_L/\sqrt{2}$ | $g_R = 0$ |
| $Zbb$     | $g_L = eC_L/\sqrt{2}$ | $g_L = eS_L/\sqrt{2}$ | $g_R = 0$ |
| $ZTT$     | $g_L = eC_L/\sqrt{2}$ | $g_L = eS_L/\sqrt{2}$ | $g_R = 0$ |
| $ZTt$     | $g_L = eC_L/\sqrt{2}$ | $g_L = eS_L/\sqrt{2}$ | $g_R = 0$ |

APPENDIX C: GAUGE BOSON SELF-ENERGIES IN THE LRTH MODEL

We calculate the four gauge boson self-energies, $\Pi^\gamma\gamma(0)$, $\Pi^\gamma Z(M_Z^2)$, $\Pi^{WW}(0)$ and $\Pi^{ZZ}(M_Z^2)$. In general, the gauge independence in the bosonic sector can be retained by using the pinch technique [24, 25] or by using the background field method [26]. In our calculations, three one-loop diagrams are involved with an internal gauge boson propagator. In these diagrams, we take only gauge-invariant parts that are proportional to $\ln(M_Z^2)/16\pi^2$.

1. Contributions to $\Pi^\gamma\gamma(0)$

The one-loop corrections to the self-energy $\Pi^\gamma\gamma$ of the LRTH model are shown in Fig. 7. The total contribution to the self-energy is

$$\Pi^\gamma\gamma(0) = \frac{\alpha}{4\pi} \left[ \frac{16}{9} \ln \frac{Q^2}{m_t^2} + \frac{4}{9} \ln \frac{Q^2}{m_b^2} + \frac{16}{9} \ln \frac{Q^2}{m_T^2} + \frac{1}{3} \ln \frac{Q^2}{m_{h_1}^2} + \frac{14}{3\epsilon} \right].$$  \hspace{1cm} (C1)

2. Contributions to $\Pi^\gamma Z(M_Z^2)$

The one-loop corrections to the self-energy $\Pi^\gamma Z(M_Z^2)$ are shown in Fig. 8. These are (i) fermionic loops having $(tt), (TT)$ and $(bb)$, (ii) the scalar loops due to SSV coupling, $(\phi^+\phi^-)$, $(\hat{h}_1^+\hat{h}_1^-)$, and (iii) $\phi^+$ and $\hat{h}_1^+$ scalar loops due to SSV quartic couplings. The contributions to $\Pi^\gamma Z(M_Z^2)$ due to the fermion loops through the couplings in Table 1 are

$$\Pi^\gamma Z_t(M_Z^2) = \frac{N_c\alpha}{\pi} \left[ \frac{2}{3} \frac{g_L^2}{c_w^2} \left( \frac{1}{4} C_L^2 - \frac{2}{3} \frac{c_w^2}{s_w^2} \right) M_Z^2 \left[ \frac{1}{3} \left( \ln \frac{Q^2}{m_t^2} + \frac{1}{\epsilon} \right) - 2I_3 \left( \frac{M_Z^2}{m_t^2} \right) \right] \right];$$  \hspace{1cm} (C2)

$$\Pi^\gamma Z_T(M_Z^2) = \frac{N_c\alpha}{\pi} \left[ \frac{2}{3} \frac{g_L^2}{c_w^2} \left( \frac{1}{4} S_L^2 - \frac{2}{3} \frac{s_w^2}{c_w^2} \right) M_Z^2 \left[ \frac{1}{3} \left( \ln \frac{Q^2}{m_T^2} + \frac{1}{\epsilon} \right) - 2I_3 \left( \frac{M_Z^2}{m_T^2} \right) \right] \right];$$  \hspace{1cm} (C3)

$$\Pi^\gamma Z_b(M_Z^2) = \frac{N_c\alpha}{4\pi} \left[ \frac{1}{2} \frac{g_L^2}{c_w^2} \left( \frac{1}{2} - \frac{2}{3} \frac{c_w^2}{s_w^2} \right) M_Z^2 \left[ \frac{1}{3} \left( \ln \frac{Q^2}{m_b^2} + \frac{1}{\epsilon} \right) - 2I_3 \left( \frac{M_Z^2}{m_b^2} \right) \right] \right];$$  \hspace{1cm} (C4)
The terms proportional to $M_{h_1}^2$ and $M_{h_2}^2 \ln(Q^2/M_{h_1}^2)$ in Eqs. (C5) and (C7) cancel between themselves, as do the terms proportional to $M_{\phi^+}^2$ and $M_{\phi^-}^2 \ln(Q^2/M_{\phi^+}^2)$ in Eqs. (C6) and (C8). For $p^2 = 0$, it can be easily checked that the total fermionic and scalar contributions vanish individually. As expected in the unitary gauge, no mixing between the two gauge boson gauges at one-loop due to

$$\Pi^Z(0) = 0.$$  \hfill (C9)

3. Contributions to $\Pi^{WW}(0)$

The contributions to $\Pi^{WW}(0)$ from the fermion loops through the couplings in Table 1 are given as follows,

$$\Pi^{WW}_{b b}(0) = \frac{N_c G_F}{4\pi} \frac{C_2}{2\alpha_w^2} f_1(m_t^2, m_b^2),$$  \hfill (C10)

$$\Pi^{WW}_{t b}(0) = \frac{N_c G_F}{4\pi} \frac{S_4}{2\alpha_w^2} f_1(m_t^2, m_b^2),$$  \hfill (C11)

where $1/\epsilon$ terms are omitted, and $f_1(m_t^2, m_b^2)$ is defined as

$$f_1(m_t^2, m_b^2) = \frac{1}{2} \left( \frac{m_t^2 + m_b^2}{m_t^2 - m_b^2} \right)$$

$$\ln \frac{Q^2}{m_t^2} - \frac{m_b^2}{m_t^2 - m_b^2} \ln \frac{Q^2}{m_b^2}.$$  \hfill (C12)

The contributions to $\Pi^{WW}(0)$ from the scalar loops through the couplings in Table 3 are given as,

$$\Pi^{WW}_{h_1 h_1}(0) = \frac{\alpha}{4\pi} \frac{c_{w_s}^2}{c_w s_w} \left[ 1 + \ln \frac{Q^2}{M_{h_1}^2} + \frac{1}{\epsilon} \right] M_{h_1}^2,$$ \hfill (C13)

$$\Pi^{WW}_{h_2 h_2}(0) = \frac{\alpha}{4\pi} \frac{c_{w_s}^2}{c_w s_w} \left[ 1 + \ln \frac{Q^2}{M_{h_2}^2} + \frac{1}{\epsilon} \right] M_{h_2}^2.$$ \hfill (C14)

$$\Pi^{WW}_{h_2 h_2}(0) = \frac{\alpha}{4\pi} \frac{c_{w_s}^2}{c_w s_w} \left[ 1 + \ln \frac{Q^2}{M_{h_2}^2} + \frac{1}{\epsilon} \right] M_{h_2}^2.$$ \hfill (C15)

$$\Pi^{WW}_{\phi^+ \phi^-}(0) = -\frac{\alpha}{4\pi} \frac{c_{w_s}^2}{c_w s_w} \left[ 1 + \ln \frac{Q^2}{M_{\phi^+}^2} + \frac{1}{\epsilon} \right] M_{\phi^+}^2.$$ \hfill (C16)

$$\Pi^{WW}_{\phi^- \phi^-}(0) = -\frac{\alpha}{4\pi} \frac{c_{w_s}^2}{c_w s_w} \left[ 1 + \ln \frac{Q^2}{M_{\phi^-}^2} + \frac{1}{\epsilon} \right] M_{\phi^-}^2.$$ \hfill (C17)

Note that $\Pi^{WW}_{\phi^+}$ and $\Pi^{WW}_{\phi^-}$ are much smaller than the other contributions due to the suppression factor $x^2$.

The contribution to $\Pi^{WW}(0)$ from the scalar loops through the couplings in Table 2 has the following form

$$\Pi^{WW}_{h_1 h_2}(0) = \frac{\alpha}{4\pi} \frac{c_{w_s}^2}{c_w s_w} \left[ 1 + \ln \frac{Q^2}{M_{h_1}^2} + \frac{1}{\epsilon} \right] M_{h_1}^2.$$ \hfill (C18)
Table 3. Relevant gauge coupling constants of the scalar fields, $C_{XXXS}$ [22].

| $XXXS$ | $C_{XXXS}$ | $XXXS$ | $C_{XXXS}$ |
|--------|-------------|--------|-------------|
| $W^+W^-h\bar{h}$ | $e^2/(2c_w^2)$ | $ZZh\bar{h}$ | $e^2/(2c_w^2)$ |
| $W^+\phi\phi^0$ | $-e^2/2\xi(54c_w^2)$ | $ZZe\phi^0\phi^0$ | $-e^2/2\xi(54c_w^2)$ |
| $W^+W^-h\bar{h}_1$ | $e^2/(16s_w^2)$ | $Z\phi\phi^0\bar{h}_1$ | $2e^2s_w^2/16c_w^2$ |
| $W^+h\bar{h}_1h_2$ | $2e^2/(2s_w^2)$ | $ZZh\bar{h}_1h_2$ | $e^2/(2c_w^2)$ |

where $g_i(m_1^2, m_2^2)$ is defined as

$$g_i(m_1^2, m_2^2) = \frac{3}{8}(m_1^2 + m_2^2)$$

$$+ \frac{1}{4(m_1^2 - m_2^2)}[m_1^4\ln\frac{Q^2}{m_1^2} - m_2^4\ln\frac{Q^2}{m_2^2}].$$

The terms proportional to $M_h^2$ and $M_{h_1}^2 \ln(Q^2/M_h^2)$ in Eqs. (14) and (18) canceled partially between themselves and so do the terms proportional to $M_{h_1}^2$ and $M_{h_2}^2 \ln(Q^2/M_{h_1}^2)$ in Eqs. (15) and (18). Although the terms proportional to $M_{\phi^+\phi^-}^2$ and $M_{\phi^+\phi^-}^2 \ln(Q^2/M_{\phi^+\phi^-}^2)$ in Eqs. (16) and (17) do not cancel out, their coefficients are significantly small and so are their contributions to $\Pi^{WW}(0)$.

The contribution to $\Pi^{WW}(0)$ from the SM Higgs-W boson loops has the following form:

$$\Pi^{WW}(0) = \frac{\alpha}{4\pi} \frac{M_W^2}{s^4_w} \left[\frac{5}{8} \frac{M_h^2}{M_W^2} + \frac{3}{4} \frac{M_{h_1}^2}{M_{h_1}^2} \ln\frac{Q^2}{M_W^2} + \frac{M_{h_1}^2}{M_W^2} \ln\frac{Q^2}{M_{h_1}^2} \right].$$

We take only the contribution proportional to $\ln(M_h^2)/16\pi^2$, which is gauge invariant:

$$\Pi^{WW}(0) = \frac{\alpha}{4\pi} \frac{M_W^2}{s^4_w} \ln\frac{Q^2}{M_h^2} \left[\frac{M_h^2}{M_W^2} - \frac{1}{4} \frac{M_{h_1}^2}{M_{h_1}^2} \right].$$

4. Contributions to $\Pi^{ZZ}(M_{ZZ})$

The one-loop corrections to the self-energy function $\Pi^{ZZ}(p^2)$ are shown in Fig. 10. The complete list of fermionic contributions to the self-energy function are given below:
The contributions to $\Pi^{ZZ}(M_Z^2)$ from the scalar loops through the couplings in Table 3 have the following form,

$$
\Pi^{ZZ}_{(h)}(M_Z^2) = \frac{\alpha}{16\pi} \frac{1}{c_w^2 s_w^2} M_h^2 \left[ 1 + \ln \frac{Q^2}{M_h^2} + \frac{1}{\epsilon} \right],
$$

(C26)

$$
\Pi^{ZZ}_{(h_1)}(M_Z^2) = \frac{\alpha}{8\pi} \frac{c_{2u}^4}{s_w^2} M_{h_1}^2 \left[ 1 + \ln \frac{Q^2}{M_{h_1}^2} + \frac{1}{\epsilon} \right],
$$

(C27)

$$
\Pi^{ZZ}_{(h_2)}(M_Z^2) = \frac{\alpha}{8\pi} \frac{1}{c_w^2 s_w^2} M_{h_2}^2 \left[ 1 + \ln \frac{Q^2}{M_{h_2}^2} + \frac{1}{\epsilon} \right],
$$

(C28)

$$
\Pi^{ZZ}_{(\phi^+)}(M_Z^2) = \frac{\alpha}{2\pi} \frac{s_w^2}{c_w^2} M_{\phi^+}^2 \left[ 1 + \ln \frac{Q^2}{M_{\phi^+}^2} + \frac{1}{\epsilon} \right],
$$

(C29)

$$
\Pi^{ZZ}_{(\phi^0)}(M_Z^2) = -\frac{\alpha}{8\pi} \frac{x^2}{54 c_w^2 s_w^2} M_{\phi^0}^2 \left[ 1 + \ln \frac{Q^2}{M_{\phi^0}^2} + \frac{1}{\epsilon} \right].
$$

(C30)

The contributions to $\Pi^{ZZ}(M_Z^2)$ from the scalar loops through the couplings in Table 2 have the following forms:

$$
\Pi^{ZZ}_{(h_1 h_2)}(M_Z^2) = -\frac{\alpha}{8\pi} \frac{c_{2u}^4}{s_w^2} \left[ \left( M_{h_1}^2 - \frac{1}{6} M_{h_2}^2 \right) \left( \ln \frac{Q^2}{M_{h_1}^2} + \frac{1}{\epsilon} \right) + \left( \frac{1}{6} M_Z^2 - \frac{2}{3} M_{h_1}^2 \right) I_1 \left( \frac{M_Z^2}{M_{h_1}^2} \right) + M_{h_1}^2 - \frac{1}{9} M_Z^2 \right].
$$

(C31)

$$
\Pi^{ZZ}_{(h_1 h_1)}(M_Z^2) = -\frac{\alpha}{16\pi} \frac{c_{2u}^4 c_{2w}^2}{s_w^2} \left[ \left( M_{h_1}^2 - \frac{1}{6} M_Z^2 \right) \left( \ln \frac{Q^2}{M_{h_1}^2} + \frac{1}{\epsilon} \right) + \left( \frac{1}{6} M_Z^2 - \frac{2}{3} M_{h_1}^2 \right) I_1 \left( \frac{M_Z^2}{M_{h_1}^2} \right) + M_{h_1}^2 - \frac{1}{9} M_Z^2 \right].
$$

(C32)

$$
\Pi^{ZZ}_{(h_2 h_2)}(M_Z^2) = -\frac{\alpha}{16\pi} \frac{c_{2u}^4 c_{2w}^2}{s_w^2} \left[ \left( M_{h_2}^2 - \frac{1}{6} M_Z^2 \right) \left( \ln \frac{Q^2}{M_{h_2}^2} + \frac{1}{\epsilon} \right) + \left( \frac{1}{6} M_Z^2 - \frac{2}{3} M_{h_2}^2 \right) I_1 \left( \frac{M_Z^2}{M_{h_2}^2} \right) + M_{h_2}^2 - \frac{1}{9} M_Z^2 \right].
$$

(C33)

$$
\Pi^{ZZ}_{(h_1 \phi^0)}(M_Z^2) = 0.
$$

(C34)

The terms proportional to $M_{h_1}^2$ and $M_{h_2}^2 \ln(Q^2/M_{h_1}^2)$ in Eqs. (C27) and (C31) cancel between themselves, as do the terms proportional to $M_{h_1}^2$ and $M_{h_2}^2 \ln(Q^2/M_{h_2}^2)$ in Eqs. (C28) and (C32). The terms proportional to $M_{\phi^+}^2$ and $M_{\phi^0}^2 \ln(Q^2/M_{\phi^+}^2)$ in Eqs. (C29) and (C33) also cancel between themselves. There are contributions of scalar-gauge boson loops to $\Pi^{ZZ}(M_Z^2)$. We take only the contribution proportional to $\ln(M_Z^2)/16\pi^2$, which is gauge invariant:

$$
\Pi^{ZZ}_{(Z h)}(M_Z^2) = \frac{\alpha}{8\pi} \frac{M_W^2}{c_w s_w} \ln \frac{Q^2}{M_h^2} \left[ 1 - \frac{3 M_Z^2 + 2 M_h^2}{12 M_Z^2} \right].
$$

(C35)

$$
\Pi^{ZZ}_{(Z h h)}(M_Z^2) = \frac{\alpha}{16\pi} \frac{f^2 x^2}{c_w^2 c_{2w}^2} \ln \frac{Q^2}{M_h^2} \left[ 1 - \frac{3 M_Z^2 + 3 M_Z^2 s_w^2 - M_Z^2}{12 M_Z^2} \right].
$$

(C36)

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