Manipulating quantum states with aspheric lenses

Zhi-Wei Wang, Xi-Feng Ren, Yun-Feng Huang, Yong-Sheng Zhang, and Guang-Can Guo

Key Laboratory of Quantum Information, Department of Physics,
University of Science and Technology of China,
Hefei 230026, People’s Republic of China

Abstract

We present an experimental demonstration to manipulate the width and position of the down-converted beam waist. Our results can be used to engineer the two-photon orbital angular momentum (OAM) entangled states (such as concentrating OAM entangled states) and generate Hermite-Gaussian (HG) modes entangled states.

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I. INTRODUCTION

Spontaneous parametric down-conversion (SPDC) generated in nonlinear crystals is one of the most effective methods to obtain entangled photon pairs, which forms the foundation of many applications in quantum communication, quantum cryptography and quantum computation etc. Up to now the theoretical discussions and experiments have mostly focused on two-dimensional spaces, namely qubits. However, entanglement is also embedded into the spatial modes of the two-photon states generated in SPDC. Such entanglement can exist in infinite-dimensional Hilbert space so that it has a promise to realize new-type protocols in quantum communication and quantum cryptography compared with two-dimensional spaces (qubits). It has been proven theoretically and experimentally that the generated two-photon state from SPDC is entangled in orbital angular momentum (OAM) which is related to spatial modes of the two-photon states.

Normally the two-photon state at the output of the nonlinear crystal can be written as a coherent superposition of eigenstates of the OAM operator, while the amplitudes of each eigenstate are related to the beam width of pump beam and signal (idler) beam. So we can manipulate the final state by a proper selection of the beam width. Recently Hermite-Gaussian (HG) modes entangled states have also been discussed. It is underlined that if the condition $\frac{\omega_0}{\omega_p} \ll 1$ ($\omega_0$ is the width of the down-converted beam and $\omega_p$ is that of the pump beam) is satisfied the HG modes of the signal and idler beam can be considered as quasi-conserved. In this paper, we find that we can manipulate the output state by properly choosing aspheric lenses for proper parameters (the width and position of the down-converted beam waist) of the down-converted photons can be selected with the help of lenses. So we can realize the concentration of OAM entangled states. Similarly the new-type HG modes entangled states can also be generated.

II. THE THEORY ANALYSIS

In the process of SPDC, the pump light is incident on the nonlinear crystal then the signal and idler photons can be generated with a small probability. As mentioned above the spontaneous parametric down-conversion can generate photon pairs entangled in OAM. It has also been shown that paraxial Laguerre-Gaussian (LG) laser beams carry a well-defined OAM. The beam $LG^l_p$ carries an OAM of $l \hbar$, where $l$ is referred to as the winding number and $(p + 1)$ is the number of radial nodes. So the generated OAM entangled state can be
The two-photon state can be simplified as

$$|\psi\rangle = \sum_{l_1} \sum_{l_2} C^{l_1, l_2}_{0, 0} |LG_{l_1}^{l_1}; LG_{l_2}^{l_2}\rangle,$$  

where $LG_{l_1}^{l_1}$ corresponds to the mode of the signal beam and $LG_{l_2}^{l_2}$ the mode of the idler beam.

The weights of the quantum superposition are given by $P^{l_1, l_2} = |C^{l_1, l_2}_{0, 0}|^2$, which denoting the joint detection probability for finding one photon in the signal mode $LG_{l_1}^{l_1}$ and the other in the idler mode $LG_{l_2}^{l_2}$.

In most applications, we only consider $p_1 = p_2 = 0$ for the sake of simplicity. In this way, the two-photon state can be simplified as

$$|\psi\rangle = \sum_{l_1 = -\infty}^{\infty} \sum_{l_2 = -\infty}^{\infty} C^{l_1, l_2}_{0, 0} |l_1, l_2\rangle.$$  

J. P. Torres has shown that $C^{l_1, l_2}_{0, 0}$ is related to the pump beam width $\omega_p$ and the chosen beam width of the LG base $\omega_0$. Generally speaking, $C^{l_1, l_2}_{0, 0}$ increases with $\omega_p$, but there is an optimal value of $\omega_0$ for which the contribution of $C^{0, 0}_{0, 0}$ is maximum.

To manipulate or utilize this OAM entangled state, we need to detect different LG modes. The single-mode optical fibers and computer-generated holograms are frequently used to solve this problem. In this paper, we just consider the detecting of $LG_0^0$ mode (the Gaussian mode), while the other LG modes can be done similarly with the help of computer-generated holograms.

The detection efficiency for the single-mode fiber detecting photons in $LG_0^0$ mode at the beam waist $\omega_0$ is given as

$$Q_0 = \frac{\left( \int \int (LG_0^0)^* E(\rho) \rho d\rho d\varphi \right)^2}{\int \int (LG_0^0)^* LG_0^0 \rho d\rho d\varphi \int \int E(\rho)^* E(\rho) \rho d\rho d\varphi},$$  

where

$$E(\rho) = E(0) \exp \left(-\frac{\rho^2}{\omega_f^2}\right),$$  

$E(0)$ denoting the amplitude of the field at the fiber centre and $d = 2\omega_f$ is the mode field diameter of the fiber.

For the sake of simplicity, we assume that the maximal detection efficiency corresponds to the case that the beam waist of the down-converted light is incident on the input surface of the coupling lens of single-mode fiber. At this time, $\omega_0'$ is the beam width and $z'$ is the
distance from the lens to the beam waist after the lens. By virtue of transformation of the lens as shown in Figure 1:

\[
\omega_0^2 = \frac{\omega_0^2}{(1 - \frac{z}{f})^2 + \frac{\pi^2 \omega_0^4}{\lambda^2 f^2}} \quad (5)
\]

\[
z' = \left[1 - \frac{(1 - \frac{z}{f})}{(1 - \frac{z}{f})^2 + \frac{\pi^2 \omega_0^4}{\lambda^2 f^2}}\right] f. \quad (6)
\]

We can calculate the parameters \((\omega_0, z)\) of the down-converted beam before the lens if we know the values of \(\omega'_0\) and \(z'\).

In the experiment, we change the position of the coupling lens, while the positions of the BBO crystal and the single-mode fiber are stable. The detection efficiency \(Q_0\), substituted by the count rate \(R\), is changing when we change the position of the lens. From the experiment data, the optimal values of \(\omega'_0\) and \(z'\) can be obtained by means of “Curve Fitting”, which is the least squares fit program in OriginPro 7.0. Then from Eqs. (5) and (6), we can obtain the values of \(\omega_0\) and \(z\). While the value of the width of the pump beam \(\omega_p\) can be easily manipulated by adding another lens in the pump beam. Thus we realize the manipulation of the values of \(\omega_p\) and \(\omega_0\).

III. THE EXPERIMENT CONFIGURATION AND ANALYSIS

The experiment configuration is shown in Figure 2. A BBO (beta barium borate) crystal (thickness 1 mm) is illuminated by a quasimonochromatic argon-ion laser pump beam propagating in the \(z\) direction at the wavelength \(\lambda_p = 351.1\) nm. In the pump light path, an aspheric lens \(A (f_A = 500\) mm\) is placed to change the pump beam width \(\omega_p\) on the input surface of the crystal. After an appropriate choice of the phase matching angle, the down-converted photons are produced in type I SPDC at degenerated wavelength of 702.2 nm at an angle of 6° off the pump light direction. In the path of the signal photons, another aspheric lens \(B\) is placed to focus the photons into the detector. Before the detector are an interference filter (bandwidth 4 nm) and a fixed coupling lens (it is not shown in Figure 2) used to couple the down-converted photons into the single-mode fiber.

In Figure 3 the horizontal axis is the distance from the detector to lens \(B\) and the vertical axis is the count rate for the signal path. In the experiment, we scan the position of lens \(B\) and write down the corresponding count rate with the position of the crystal and the detector fixed (the distance from the detector to the BBO crystal is 852 mm). The dots are
the experiment data of single counts in the signal path, while the curves are based on the “Curve Fitting” with the dots. It can be seen that the experiment data fit the simulated curves very well near the peak. But far away from the peak, there is some deviation. Such deviation is probably due to the fact that when lens B is far away from the position of the peak, the light incident on the input surface of the lens group has a larger scattering angular. At this time, the Eq. (3) may be not appropriate. The values of \((\omega'_0, z')\) come from the “Curve Fitting” and the values of \((\omega_0, z)\) are calculated from the ones of \((\omega'_0, z')\) according to Eqs. (5) and (6).

In the experiment, first the role of \(\omega_p\) played in the detection of the spatial mode is investigated. We change the value of \(\omega_p\) by varying the position of lens A. In Figure 3(a), (b), (c), \((f_B = 100 \text{ mm})\), the relations \(\omega_{p,a} < \omega_{p,b} < \omega_{p,c}\) can be derived by proper position of lens A. From the three charts, we find the count rate gets larger as \(\omega_p\) becomes smaller. For further justification of this result, in Figure 3(d),(e), lens B with the focus of 200 mm is used. The same result is derived as above. The result is in good accords with the theory put forward by Torres et al. that the count rate of \(LG_0^0\) mode can be made larger by decreasing the pump beam width\([11]\).

Comparing the case of \(f_B = 200 \text{ mm}\) with that of \(f_B = 100 \text{ mm}\), one can find that the similar values of \((\omega_0, z)\) are got for the same lens B. For example, in the case of \(f_B = 100 \text{ mm}\), we get the values of \((\omega_0, z)\) \((0.024, 115.1), (0.026, 114.7)\) and \((0.029, 114.5)\) respectively, and when \(f_B = 200 \text{ mm}\), the values of \((\omega_0, z)\) are \((0.083, 281.6), (0.078, 282.6)\) respectively. We can also find that the parameters with regard to different focus of lens B are obviously distinct from each other, which means that lens B is able to select the values of \((\omega_0, z)\). Since different values of \((\omega_0, z)\) represent different spatial distribution of down-converted light field, we can draw the conclusion that different lens B can select different spatial distribution of down-converted light field.

From the analysis above, we can vary the values of \(\omega_p\) and \(\omega_0\) by means of proper operation on lenses. As the amplitude \(C_{l_1,l_2}^{p_1,p_2}\) in Eq. (1) is related to \(\omega_p\) and \(\omega_0\), we can engineer the state of Eq. (1) by selecting the focus and position of lens. So we can obtain the maximal OAM entangled states using this method with the technology available. In the same way, partial entangled states can also be derived for they both have many important applications\([19, 20]\). Similarly, the condition \(\omega_0/\omega_p \ll 1\) can be satisfied easily, allowing to produce the HG modes entangled states\([12]\).
IV. CONCLUSIONS

In conclusion, we find that using detection efficiency of the down-converted photons and “Curve Fitting” function, we can acquire the parameters of the down-converted light field of SPDC. We can manipulate the state in Eq. (1) by selecting the focus and position of lenses. The condition to generate HG modes entangled states can also be satisfied using this method. Additionally our conclusion is useful to increase detection efficiency of the down-converted photons.

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Figure 1 The transformation of the Gaussian beam through the lens. $\omega_0 (\omega'_0)$ and $z (z')$ are the beam width and the distance from the beam waist to the lens before (after) the lens respectively. The relations between them are shown in Eqs. (5)-(6).

Figure 2 Our experiment configuration. The pump beam ($\lambda_p = 351.1 \text{ nm}$) from the argon-ion laser is incident on the BBO crystal focused by lens A ($f_A = 500 \text{ mm}$). In the experiment, we move lens B while fixing the positions of the crystal and the detector (the distance from the crystal to the detector is 852 mm). $F$ is the interference filter (bandwidth 4 nm).

Figure 3 The single count rate $R$ varies when the position of lens B is changed. In (a), (b), (c), lens B with the focus of 100 mm is used. The dots are the experiment data and the curves are derived from “Curve Fitting” with the dots. The value of $(\omega_0, z) ((\omega'_0, z'))$ corresponds to the parameters of the down-converted light before (after) lens B. The relation between the pump beam widths on the input surface of the crystal is $\omega_{p,a} < \omega_{p,b} < \omega_{p,c}$. In (d),(e), lens B ($f_B = 200 \text{ mm}$) is used and the relation between $\omega_{p,d}$ and $\omega_{p,e}$ is $\omega_{p,d} > \omega_{p,e}$. 

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Figure 1
Figure 2
Figure 3