Equivalence between a bosonic theory and a massive non-local Thirring model at Finite Temperature

M.V.Manías and M.L.Trobo
Departamento de Física, UNLP, C.C. 67, (1900) La Plata Argentina
Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina.

Using the path-integral bosonization procedure at Finite Temperature we study the equivalence between a massive Thirring model with non-local interaction between currents (NLMT) and a non-local extension of the sine-Gordon theory (NLSG). To this end we make a perturbative expansion in the mass parameter of the NLMT model and in the cosine term of the NLSG theory in order to obtain explicit expressions for the corresponding partition functions. We conclude that for certain relationship between NLMT and NLSG potentials both the fermionic and bosonic expansions are equal term by term. This result constitutes a generalization of Coleman’s equivalence at $T = 0$, when considering a Thirring model with bilocal potentials in the interaction term at Finite Temperature.

The study of this model is relevant in connection with the physics of strongly correlated systems in one spatial dimension. Indeed, in the language of many-body non-relativistic systems, the relativistic mass term can be shown to represent the introduction of backward-scattering effects.

PACS number(s): 11.10.Lm - 11.10.Wx - 05.30.Fk
Keywords: Quantum Field Theory - Finite Temperature - Strongly Correlated Systems.

I. INTRODUCTION

In the context of two dimensional Quantum Field Theories (2D QFT) the bosonization procedure [1], originally developed in the operator language [2], has been also fruitfully implemented in the path-integral framework [3], [4]. It has been shown to be very useful to study a great variety of problems in (1+1) dimensions. In particular the bosonization technique, based in a decoupling change of path-integral variables, has become a powerful tool for treating strongly correlated electron systems in 1D. The use of this technique enables to study some problems, which appear intractable when formulated in terms of fermions, in an easier way when formulated in terms of bosonic fields. The Tomonaga-Luttinger liquid theory (related with a quantum wire of interacting 1D electrons) [5], [6], quantum Hall edge states [7] and quantum impurity problems such as the Kondo effect [8] are some examples in which the application of the bosonization technique is more convenient.

The actual fabrication of the so called quantum wires [9] have renewed the interest on low-dimensional field theories, in particular, in the study of 1D fermionic gas. This kind of systems may present a deviation from the usual Fermi liquid behavior for which the Fermi surface disappear and the spectrum contains only collective modes. This situation is known as a Luttinger-liquid behavior [10].

In a recent series of works [11], the bosonization procedure in its path integral version was extended to non-local QFT; in particular, it was applied to a Thirring-like model with massless fermions and a non-local (and non-covariant) interaction between fermionic currents (NLT). For one particular choice of bilocal potentials this model displays the same forward scattering processes that are present in the zero temperature limit of the Tomonaga-Luttinger model [12], [13], which describes a one dimensional gas of highly correlated spinless fermions interacting through their density fluctuations.

On the other hand, following the functional treatment at finite temperature [14], it has been studied the thermodynamics of the NLT model [15], which constitutes a possible starting point to examine the thermodynamical properties of a LNT model through an alternative field-theoretical formulation.

In this paper we extend this path-integral approach to bosonization in presence of temperature to the case of a NLT model with massive fermions. From the point of view of the many-body systems, this relativistic mass term denotes the introduction of backward-scattering effects [16].

The equivalence between the massive Thirring and the sine-Gordon models was first derived, using the operational scheme, by Coleman [17], who showed that the perturbative series in the mass parameters of the massive Thirring and the sine-Gordon theories are equal term by term provided that certain relations hold. Coleman’s proof of the equivalence between both models was also extended to the finite temperature case [18].

Recently, following the path-integral procedures applied in Ref. [19] for the local case, it has been shown that the equivalence still holds for a certain non-local generalization of these two models at zero temperature [20]. In the present work we show that the non-local massive Thirring model is equivalent to a certain non-local extension of the sine-Gordon theory when both models are considered in their finite temperature version.

This paper is organized as follows: in Section 2 we present the non-local massive Thirring model at finite temperature. We make a perturbative expansion in the mass parameter and arrive to a completely bosonized expression for the generating functional. We also consider a non-
local version of the sine-Gordon theory and through an expansion in the cosine term we obtain the corresponding thermodynamical partition function. Comparing term by term the perturbative series for both models we show that they are equal if a certain relationship between the NLMT and the NLSG functional of the potentials is satisfied. Finally, in the last Section, we summarize the most interesting aspects of our investigations.

II. NON-LOCAL PARTITION FUNCTIONS AT FINITE TEMPERATURE

In this section we study two non-local 2D models, the massive Thirring model with a non-local interaction between fermionic currents and a non-local extension of the Sine-Gordon theory, both at finite temperature. We use the imaginary time formalism developed by Bernard [20] and Matsubara [21] and the path-integral approach.

A. Non-local massive Thirring model

We start with the Euclidean action given by

\[ S = \int d^{2}x \bar{\Psi} (i\partial_{\tau} - m) \Psi - \frac{g^{2}}{2} \int d^{2}x d^{2}y [J_{\mu}(x)V_{\mu}(x-y)J_{\mu}(y)] \]  \hspace{1cm} (2.1)

where \( \int d^{2}y \) means \( \int_{0}^{\beta} dy \int dy^{1} \) and \( \beta = \frac{1}{k_{B}T} \) with \( k_{B} \) the Boltzman’s constant and \( T \) the temperature. The fermionic currents \( J_{\mu}(x) = \bar{\Psi}(x)\gamma_{\mu}\Psi(x) \) are calculated using \( \gamma_{\mu} \) matrices defined as

\[ \gamma_{0} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]  \hspace{1cm} (2.2)

\[ \gamma_{1} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \]  \hspace{1cm} (2.3)

and Dirac fermions with \( \Psi_{1} \) and \( \Psi_{2} \) components

\[ \Psi = \begin{pmatrix} \Psi_{1} \\ \Psi_{2} \end{pmatrix} \]  \hspace{1cm} (2.4)

The arbitrary functions \( V_{\mu}(x,y) = V_{\mu}(|x-y|) \) represents the bilocal potentials which describe electron-electron forward-scattering interactions. Note that no sum over repeated indices will be implied when a subindex \((\mu)\) is involved.

The partition function of the model is given by

\[ Z = N N_{F}(\beta) \int_{\text{antiper}} D\bar{\Psi} D\Psi e^{-S} \]  \hspace{1cm} (2.5)

with \( N \) an infinite temperature-independent normalization constant and \( N_{F}(\beta) \) a \( \beta \)-dependent infinite factor to be determined.

In the case of the fermionic fields the functional integral must be extended over the paths with antiperiodicity conditions in the Euclidean time variable \( x^{0} \)

\[ \Psi(x^{0} + \beta, x^{1}) = -\Psi(x^{0}, x^{1}) \]

\[ \bar{\Psi}(x^{0} + \beta, x^{1}) = -\bar{\Psi}(x^{0}, x^{1}) \]  \hspace{1cm} (2.6)

As it is shown in [19] one can extend the path-integral approach to non-local bosonization developed in [14] at finite temperature. Indeed, as in the usual, local and covariant, massless Thirring model introducing auxiliary vector fields one can remove the fermionic quartic interaction and express the partition function in terms of a fermionic determinant. Let us start splitting the action in the form

\[ S = S_{0} + S_{\text{int}} \]  \hspace{1cm} (2.7)

where \( S_{0} \) contains massive free fermions

\[ S_{0} = \int_{\beta} d^{2}x \bar{\Psi} (i\partial_{\tau} - m) \Psi \]  \hspace{1cm} (2.8)

and \( S_{\text{int}} \) includes the interaction terms in the form

\[ S_{\text{int}} = \frac{g^{2}}{2} \int_{\beta} d^{2}x J_{\mu}(x)K_{\mu}(x) \]  \hspace{1cm} (2.9)

where we have defined a new current \( K_{\mu}(x) \) as

\[ K_{\mu}(x) = \int d^{2}y V_{\mu}(x,y)J_{\mu}(y). \]  \hspace{1cm} (2.10)

Now we introduce a vector field \( \tilde{A}_{\mu} \) as follows

\[ \exp \left\{ \frac{g^{2}}{2} \int_{\beta} d^{2}x J_{\mu}(x)K_{\mu}(x) \right\} = \int_{\text{per}} D\tilde{A}_{\mu} \delta(\tilde{A}_{\mu} - K_{\mu}) \exp \left\{ \frac{g^{2}}{2} \int_{\beta} d^{2}y \tilde{J}_{\mu}\tilde{A}_{\mu} \right\} \]  \hspace{1cm} (2.11)

and represent the delta functional through a \( \tilde{B}_{\mu} \)-field in the form

\[ \delta(\tilde{A}_{\mu} - K_{\mu}) = \int_{\text{per}} D\tilde{B}_{\mu} e^{-\int_{\beta} d^{2}x \tilde{B}_{\mu}(\tilde{A}_{\mu} - K_{\mu})} \]  \hspace{1cm} (2.12)

We have to impose periodicity conditions for the bosonic \( \tilde{A}_{\mu} \) and \( \tilde{B}_{\mu} \) fields over the range \([0, \beta]\). If we define

\[ \tilde{B}_{\mu}(x) = \frac{2}{g^{2}} \int_{\beta} d^{2}y V_{\mu}(y,x)\tilde{B}_{\mu}(y) \]  \hspace{1cm} (2.13)

the fermionic piece of the action results

\[ S = S_{0} - \frac{g^{2}}{2} \int_{\beta} d^{2}x J_{\mu}(x)[\tilde{A}_{\mu}(x) + \tilde{B}_{\mu}(x)] \]  \hspace{1cm} (2.14)

We can invert eq.(2.13) to obtain
we can deal with the massive fermionic determinant following the same procedure as in the local temperature-dependent case \[15\]; we perform a chiral expansion with m as perturbative parameter.

Then, we make a chiral change of variables in the fermionic fields.

\[ \Psi(x) = e^{g[\gamma_5 \phi(x) + im(x)]} \chi(x) \]  

\[ \bar{\Psi}(x) = \bar{\chi}(x) e^{g[\gamma_5 \phi(x) - im(x)]} \]  

which \( \chi(x) \) satisfying the boundary condition \( \chi(x_0, x_1) = -\chi(x_0 + \beta, x_1) \) and \( \phi \) and \( \eta \) are bosonic fields in terms of which we shall express the \( A_{\mu} \) field.

With this change of variables the measure transforms as

\[ D\bar{\Psi}D\Psi = J_F(\phi, \eta)D\bar{\chi}D\chi \]  

The Jacobian associated to this change in the fermionic path-integral measure has been first computed for the \( T \neq 0 \) case, by Reuter and Dittrich \[23\] and gives

\[ J_F = e^{\frac{g^2}{4\pi} \int_\beta d^2 x \phi \partial_\mu \phi} \]  

In 1+1 dimensions it is also possible to split the gauge field in a longitudinal plus a transversal component in the following way

\[ A_{\mu} = \epsilon_{\mu\nu} \partial_\nu \phi - \partial_\mu \eta \]  

This change of variables introduces another Jacobian in the path-integral measure give by

\[ DA_{\mu} = det_\beta(-\square) D\phi D\eta \]  

where \( \square = \partial_\mu \partial_\mu \). Note that this bosonic determinant is temperature-dependent and hence its contribution is relevant to the partition function in contrast to the zero-temperature case where it plays no role and can simply be absorbed in the normalization constant.

Putting all this together we obtain the following partition function

\[ Z = NN_F(\beta)det_\beta(-\square) \int D\bar{\chi}D\chi D\phi D\eta e^{-S_{\text{eff}}} \]  

where

\[ S_{\text{eff}} = S_{\text{fer}} + S_{\text{0NLB}} \]  

with

\[ S_{\text{fer}} = \int_\beta d^2 x [\bar{\chi}(i\partial - m e^{2g\gamma_5 \phi}) \chi] \]  

and
which describes a system of two bosonic fields coupled by distance-dependent coefficients. In order to study the generating functional for the massive non-local Thirring model we make the following expansion in the mass parameter \( m \).

\[
\exp \left\{ m \int d^2 x \, \bar{\chi}_j(x_j) e^{2g \gamma_5 \phi(x_j)} \chi(x_j) \right\} = \sum_{n=0}^{\infty} \frac{(m)^n}{n!} \prod_{j=1}^{n} \int d^2 x_j \, \bar{\chi}_j(x_j) e^{2g \gamma_5 \phi(x_j)} \chi(x_j)
\]

(2.31)

Now, it is convenient to write \( Z_{\text{NLMT}} \) in terms of the thermal averages \(< >_\beta \) corresponding to a theory of free massive fermions and non-local bosons in the form

\[
Z_{\text{NLMT}} = N N_F(\beta) \det_\beta(-\Box) \det_\beta(i\partial)
\times \int D\phi D\eta e^{-S_{\text{NLMB}}} \sum_{n=0}^{\infty} \frac{(m)^n}{n!} n \int d^2 x_j \, \bar{\chi}_j(x_j) e^{2g \gamma_5 \phi(x_j)} \chi(x_j) > 0
\]

(2.32)

We can use the identity

\[
\bar{\chi}_j(x_j) e^{-2g \gamma_5 \phi(x_j)} \chi(x_j) = e^{-2g \phi(x_j)} \bar{\chi}_j(x_j) \frac{1 + \gamma_5}{2} \chi(x_j) + e^{2g \phi(x_j)} \bar{\chi}_j(x_j) \frac{1 - \gamma_5}{2} \chi(x_j)
\]

(2.33)

and since

\[
\bar{\chi}_j(x_j) \frac{1 + \gamma_5}{2} \chi(x_j) = \bar{\chi}_1 \chi_1(x_j)
\]
\[
\bar{\chi}_j(x_j) \frac{1 - \gamma_5}{2} \chi(x_j) = \bar{\chi}_2 \chi_2(x_j)
\]

(2.34)

where

\[
\chi = \left( \begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right)
\]

\[
\bar{\chi} = \left( \begin{array}{c} \bar{\chi}_1 \\ \bar{\chi}_2 \end{array} \right)
\]

(2.35)

(2.36)

applying the Wick’s theorem the eq. (2.32) can be written as

\[
Z_{\text{NLMT}} = N N_F(\beta) \det_\beta(-\Box) \det_\beta(i\partial)
\times \int D\phi D\eta e^{-S_{\text{NLMB}}} \sum_{n=0}^{\infty} \frac{(m)^n}{n!} \prod_{j=1}^{n} \chi_1 \chi_1(x_j) \chi_2 \chi_2(y_j) > 0
\]

(2.37)

in which \(< >_\beta \) means v.e.v in a theory with free fermions and \(< >_{\text{NLMB}} \) corresponds to the bosonic action given by eq. (2.30).

It is simpler to evaluate the generating functional in the momentum space. To do this, we Fourier transform the non-local bosonic action. We expand the bosonic fields \( \phi(x^0, x^1) \) and \( \eta(x^0, x^1) \) which are periodic in the interval \( 0 \leq x \leq \beta \) in a Fourier series

\[
\phi(x^0, x^1) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{dk}{2\pi} e^{ikx^1} e^{i\omega_n x^0} \tilde{\phi}_n(k)
\]

(2.38)

where

\[
\tilde{\phi}_n(k) = \int dx^1 \int_0^\beta dx^0 e^{-ikx^1} e^{-i\omega_n x^0} \phi(x^0, x^1)
\]

(2.39)

with \( \omega_n = \frac{2\pi n}{\beta} \) the Matsubara frequencies. A similar expansion corresponds to \( \eta(x^0, x^1) \). For the bilocal potentials we have:

\[
b_{\mu(n)}(x^1, y^1, x^0, y^0) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{dk}{2\pi} e^{ik(x^1-y^1)} e^{i\omega_n(x^0-y^0)} \tilde{b}_{\mu(n)}(k).
\]

(2.40)

As it is habitual working at finite temperature, we use discrete and continuous delta functions given by

\[
\int_0^\beta dx^0 e^{i(\omega_n - \omega_n') x^0} = \beta \delta_{n,n'}
\]

(2.41)

\[
\int \frac{dx^1}{2\pi} e^{i(k-k') x^1} = \delta(k-k').
\]

(2.42)

and then the Fourier transformed bosonic action results

\[
S_{\text{NLMB}} = \frac{1}{2\beta} \sum_{n=-\infty}^{\infty} \int \frac{dk}{2\pi} \tilde{\phi}_n(k) \tilde{\phi}_{-n}(-k) \left[ \tilde{B}_n(k) \tilde{B}_{-n}(-k)
+ \tilde{C}_n(k) \tilde{C}_{-n}(-k) \right]
\]

(2.43)

where

\[
\tilde{A}_n(k) = \frac{g^2}{\pi} \left[ \omega_n^2 + k^2 \right] + k^2 \tilde{b}_{(0),n}(k) + \tilde{b}_{(1),n}(k) \omega_n^2
\]

(2.44)
\[ B_n(k) = \tilde{b}_{(1),n}(k) \omega_n^2 + \tilde{b}_{(0),n}(k) k^2 \] (2.45)

\[ C_n(k) = 2[\tilde{b}_{(1),n}(k) - \tilde{b}_{(0),n}(k)] \omega_n k. \] (2.46)

The bosonic v.e.v appearing in eq.(2.37) can be written in the form

\[ <e^{2g \sum_n [\phi(x_j) - \phi(y_j)]}_0^{NLB} = \frac{1}{Z_{0^{NLB}}} \int D\tilde{\phi}(p) \tilde{D}\tilde{\eta}(p) \times \exp \left\{ -\frac{1}{2} \int_{-\infty}^{\infty} \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_n(k) A_n \tilde{\phi}_n(-k) + \tilde{\eta}_n(k) B_n \tilde{\eta}_n(-k) + \tilde{\phi}_n(k) C_n \tilde{\eta}_n(-k) - 4g \sum_{j=1}^{l} (D(\omega_n, k; x, y) \tilde{\phi}_n(k)) \right\} \] (2.47)

with

\[ Z_{0^{NLB}} = \int D\tilde{\phi}(p) \tilde{D}\tilde{\eta}(p) e^{-S_{0^{NLB}}} \] (2.48)

and

\[ D_n(\omega_n, k; x, y) = e^{i\omega_n x_i k x_i} - e^{i\omega_n y_i k y_i} \] (2.49)

We have introduced the vector \( p \) which stands for \( p = (\omega_n, k) \). In order to solve this path-integral we diagonalize the action in eq.(2.43) through change of variables

\[ \tilde{\phi}_n(k) = \phi_n(k) + E_n(k) \]

\[ \tilde{\eta}_n(k) = \eta_n(k) + F_n(k) \] (2.50)

with

\[ E_n(k) = 8g \sum_{n=-\infty}^{\infty} \frac{B_n(k) D_n(k)}{\Delta_n(k)} \]

\[ F_n(k) = -4g \sum_{n=-\infty}^{\infty} \frac{C_n(k) D_n(k)}{\Delta_n(k)} \] (2.51)

and the eq.(2.48) gets

\[ Z_{0^{NLB}} = [det A(k)]^{-1/2} [det [B(k) - C(k)^2/4A(k)]]^{-1/2} \] (2.52)

we finally obtain for the bosonic factor

\[ <e^{2g \sum_j [\phi(x_j) - \phi(y_j)]}_0^{NLB} = [det A(k)]^{-1/2} \times \{ det [B(k) - C(k)^2/4A(k)] \}^{-1/2} \times \exp \left\{ -\frac{8g^2}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4 k}{(2\pi)^4} \frac{2\pi}{N L B} \Delta_n(k) \sum_{j,j'} D(\omega_n, k; x, y) D(\omega_n, -k; x', y') \right\} \] (2.53)

where

\[ \Delta_n(k) = C_n^2(k) - 4A_n(k) B_n(k) \] (2.54)

The fermionic thermal average is also evaluated giving

\[ <\prod_{j=1}^{m} \chi_1(x_j) \chi_2(y_j)>_0^{NLB} = det_{\beta}(i \phi) \times e^{-\sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} D(\omega_n, k; x, y) D(\omega_n, -k; x, y)} \] (2.55)

Putting all this together in eq.(2.37) we obtain for the complete partition function

\[ Z_{NLMT} = N \cdot \frac{N_L}{F} \cdot \frac{det_{\beta}(i \phi)}{\det \frac{1}{2} \left[ \frac{g^2}{\pi} \frac{\omega_n^2}{b_1^2} + \frac{k^2}{b_0^2} + p^2 \right]} \times \sum_{i=0}^{m} \frac{m_i^2}{(i!)^2} \sum_{j=1}^{i} \int d^2 x_j d^2 y_j \times \exp \left\{ \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^4 k}{(2\pi)^4} \frac{2\pi}{N L B} \Delta_n(k) \sum_{j,j'} D(p, x_j, y_j) D(-p, x_j, y_j) \right\} \] (2.56)

where for simplicity we have omitted the \( p \)-dependence of the potentials.

Thus we have been able to extend an explicit expansion for the partition function of a massive Thirring model with arbitrary (symmetric) bilocal potentials coupling the fermionic currents considered at zero temperature to the finite temperature case.

### B. Non-local extension of the Sine-Gordon model

Now, we consider a non-local version of sine-Gordon partition function of the well known sine-Gordon model. To this end we add in the Lagrangian density an arbitrary potential function \( d_\mu(x) \) in the form

\[ \mathcal{L}_{NLG} = \frac{1}{2} (\partial_\mu \Phi)^2 + \frac{1}{2} \int d^2 y \partial_\mu \Phi(x) d_\mu(x - y) \partial_\mu \Phi(y) - \frac{\alpha_0}{\lambda^2} \cos(\lambda \phi) \] (2.57)

where \( \alpha_0 \) plays the role of a squared mass and \( \lambda \) is a dimensionless coupling constant.

The Euclidean partition function reads

\[ Z_{NLG} = N_0 \cdot N'(\beta) \int D\Phi e^{-\int d^2 x \mathcal{L}_{NLG}} \] (2.58)

As in the fermionic case, the renormalization constants \( N_0 \) and \( N'(\beta) \) absorb the infinite \( \beta \)-independent and \( \beta \)-dependent term respectively.
The functional integration runs now over scalar fields periodic in the time direction
\[ \Phi(x^0, x^1) = \Phi(x^0 + \beta, x^1) \] (2.59)
In order to treat this theory we make a perturbative expansion in the \( \alpha_0 \) the sine-Gordon partition function and obtain
\[
Z_{\text{NLG}} = N \ N' (\beta) \int D \Phi \ \exp \left\{ \int d^2 x \int \frac{1}{2} (\partial_\mu \Phi(x))^2 - \frac{1}{2} \int d^2 x \int d^2 \omega \partial_\mu \Phi(x) d(\omega)(x - y) \Phi(y) \right\} \\
\times \sum_{l=0}^{\infty} \left( \frac{\alpha_0}{\lambda^2} \right)^{2l} \int d^2 x_j \int d^2 y_j e^{\lambda \sum_j [\Phi(x_j) - \Phi(y_j)]} \] (2.60)
It is simpler to evaluate the generating functional in the momentum space. After transforming Fourier the action reads
\[
S_{\text{NLG}} = \frac{1}{\beta} \sum_n \int \frac{dk}{2\pi} \left[ \frac{\omega_n + k^2}{2} \tilde{\Phi}_n(k) \tilde{\Phi}_n(-k) \right] + \frac{1}{2} \left[ \omega_n \tilde{d}_n(k) + k^2 \tilde{d}_n(k) \right] \tilde{\Phi}_n(-k) - i\lambda \sum_j D_n(\omega_n, k; x_j, y_j) \tilde{\Phi}_n(-k) \] (2.61)
with \( D_n(\omega_n, k; x_j, y_j) \) given by eq.(2.49) which for simplicity we will denote by \( D_n \).
Again we solve the path integral in eq.(2.60) translating the quantum fields \( \tilde{\phi}_n(k) \) in the form
\[ \tilde{\phi}_n(k) = \phi_n(k) + \eta_n(k) \] (2.62)
in order to diagonalize the action given by eq.(2.61). We arrive at
\[
Z_{\text{NLG}} = N_0 \ N' (\beta) \sum_{l=0}^{\infty} \left( \frac{\alpha_0}{\lambda^2} \right)^{2l} \frac{1}{(l!)^2} \int d^2 x_j \int d^2 y_j \\
\times \det_{j=1}^{\infty} \left[ \frac{\omega_n^2 (1 + \tilde{d}_0) + k^2 (1 + \tilde{d}_1)}{4\pi (1 + \tilde{d}_0)} \right] \\
\times \prod_{j=1}^{l} \int d^2 x_j d^2 y_j e^{\lambda \sum_j [\Phi(x_j) - \Phi(y_j)]} \] (2.63)
which constitutes the thermodynamical partition function of the actual non-local Sine-Gordon model written in terms of arbitrary potentials \( \tilde{d}_\mu \).

C. Equivalence between the two non-local models at finite temperature

In this section we compare the expressions for the two expansions, one for the massive non-local Thirring model, eq.(2.60), and the other for the sine-Gordon like model given by eq.(2.63), both at finite temperature. Before doing this let us note that up to now we have considered a very general situation in which the potentials \( b(\mu) \) depend on both distances and temperature. However from a physical point of view it is reasonable to assume that the interactions are temperature-independent. In that case \( \det^{-1/2}(\tilde{b}_0 \tilde{b}_1) \) which appears in \( Z_{\text{NLMT}} \) turns to be a \( \beta \)-independent factor that can be absorbed in the normalization constant \( N \).
On the other hand the finite \( \beta \)-dependent contributions of \( \det^{1/2}_\beta (-\Box) \) and \( \det_\beta(\partial \partial) \) cancel each other \[20], \[24\] and the infinite \( \beta \)-dependent terms of both determinants fix the value of \( N_F(\beta) \) in eq.(2.56).
Having this comments in mind we can conclude that the two theories we have considered, the NLMT and the NLG, are equivalent provided the following relations hold:

\[ m = \frac{\alpha_0}{\lambda^2} \] (2.64)
and
\[ \frac{3\lambda^2}{4\pi (p^2 + d_0 \omega_n^2 + \tilde{d}_1 k^2)} = \frac{1}{2} \left( \frac{\omega_n^2}{\omega_0^2} + \frac{\tilde{d}_1}{\tilde{d}_0} + p^2 \right) \] (2.65)
This extension of the Coleman’s \[3\] equivalence constitutes the main result of this paper. It is a generalization to the non-local case of the recently proved equivalence between sine-Gordon and massive Thirring models at finite temperature \[15\]. To check this result we only have to take \( b_0 = b_1 = 1 \) in eq.(2.56) and \( d_0 = d_1 = 0 \) in eq.(2.63). These lead us to
\[
Z_{\text{TH}} = Z_{\text{FD}} \det^{-1/2}_\beta (1 + g^2/\pi) \sum_{l=0}^{\infty} \frac{m^{2l}}{(l!)^2} \times \\
\int \prod_{j=1}^{l} d^2 x_j d^2 y_j e^{\lambda \sum_j [\Phi(x_j) - \Phi(y_j)]} \] (2.66)
and
\[
Z_{\text{SG}} = Z_{\text{BE}} \sum_{l=0}^{\infty} \frac{1}{(l!)^2} \frac{\alpha_0}{\lambda^2} \frac{2l}{\pi^2} \prod_{j=1}^{\infty} \int d^2 x_j d^2 y_j e^{\lambda \sum_j [\Phi(x_j) - \Phi(y_j)]} \] (2.67)
These expressions are the momentum space version of the result presented in \[15\]. Indeed, both expansions are identical if the following identification is made
\[ m = \frac{\alpha_0}{\lambda^2} \] (2.68)
\[ \frac{4\pi}{\lambda^2} = 1 + \frac{g^2}{\pi} \] (2.69)
which coincide with the relations imposed in \cite{18} when studying the equivalence between the partition functions corresponding to the usual massive Thirring model and the ordinary sine-Gordon model in presence of temperature and is the well known Coleman's result \cite{3}. Moreover, in the limit $T \to 0$ we can also recover a recent result obtained in \cite{19}. In this reference the authors proved the equivalence of the two non-local models: the massive Thirring model and a sine-Gordon one at $T = 0$. To this end they imposed the same conditions as in the present work, eqs. (2.68) and eq. (2.69), but with the momentum continuous variables $p_0$ instead of the discrete one $\omega_n$. It is important to stress that eq. (2.68) representing the equivalence between both non-local theories is in terms of arbitrary potentials. By specifying the interactions it is possible to show that these two general theories contain certain models already discussed in the literature related to 1-D strong correlated systems.

### III. SUMMARY

Working in the path-integral formalism at finite temperature we have shown the equivalence between a non-local massive Thirring model and a non-local extension of the sine-Gordon theory in Coleman’s sense. To be precise, we have considered a massive Thirring model with bilocal fermionic current-current interaction and a sine-Gordon model with a non-local kinetic-like term in its action. Following the original procedure developed by Coleman \cite{3} but in the present case with the time variable compactified into a circle of radius $\beta = 1/T$ (that is at a fixed temperature), we made perturbative expansions in the mass parameter of the NLMT model and in the cosine term of the SNSG one. We have shown that both series are term by term identical provided the relations given by eqs. (2.68), (2.69) hold. This identities constitutes a generalization of the Coleman’s equivalence for the non-local and finite temperature case.

As it was stressed \cite{1} there is a close relation between a non-local Thirring model and the physics of strongly correlated systems in one spatial dimension. For a particular choice of the bilocal potential corresponding to the case $\tilde b_1 \to \infty$ and $\tilde b_0$ associated to the density-density interaction, the NLT model coincides with the Tomonaga-Luttinger model, which constitutes one of the starting points to the study of the 1d many-body problems. In this kind of systems the presence of a relativistic fermion mass is related to a Luther-Emery model for which backward scattering processes are present \cite{1}. In that sense the non-local Sine-Gordon model proposed in this paper could be the bosonic arena to analyze the thermodynamics of collective modes in 1d systems when both forward and backward scattering effects are taken into account.

**Acknowledgements:** We would like to thank F.A.Schaposnik and C.M.Naón for a critical reading of the manuscript.

\begin{thebibliography}{99}
\bibitem{1} M. Stone, Bosonization, World Scientific Publishing Co.1994.
\bibitem{2} J.Lowenstein and J.Swieca Ann.Phys.(N.Y.) 68, 172 (1971).
\bibitem{3} S.Coleman, Phys. Rev.D 11, 2088, (1975).
\bibitem{4} S.Mandelstam, Phys. Rev.D 11, 3026, (1975).
\bibitem{5} R.Gamboa Sarví, F.Schaposnik and J.Solomin, Nucl. Phys. B 185, 239 (1981); R.Roskies, F.Schaposnik Phys. Rev. D 23, 558 (1981).
\bibitem{6} K.Furuya, R.Gamboa Saraví and F.Schaposnik Nucl.Phys. B 208, 159 (1982).
\bibitem{7} C.M. Naón, Phys. Rev. D 31, 2035 (1985).
\bibitem{8} Tomonaga, Prog.Theor.Phys.5, 544 (1950).
\bibitem{9} J.Luttinger, J. Math. Phys. 4, 1154 (1963).
\bibitem{10} B.I. Halperin, Phys. Rev. B 25, 2185 (1982).
\bibitem{11} E. Fradkin, F. Schaposnik and M. C. von Reichenbach, Nucl. Phys. B 316, 710 (1989).
\bibitem{12} J. Voit, Rep. Prog.Phys. 58, 977 (1995).
\bibitem{13} F. Haldane, J. Phys. C14, 2585 (1981).
\bibitem{14} C. M.Naón, M.C. von Reichenbach and M. L. Trobo, Nucl. Phys. [FS] B435, 567 (1995); Nucl. Phys. [FS] B485, 665,(1997).
\bibitem{15} M.V.Manías, C.M.Naón and M.L.Trobo, Phys.Lett.B 416, 157 (1998).
\bibitem{16} M.V.Manías, C.M.Naón and M.L.Trobo, to appear in Nucl. Phys. B.
\bibitem{17} A. Luther and V.A.Emery, Phys. Rev. Lett 33, 589 (1974); R. Heidenreich, B.Schoer, R.Seiler and D.A.Uhlenbrock, Phys. Lett 54, 119 (1975).
\bibitem{18} D.Delepine, R. Gonzalez Felipe and J.Weyers, Phys. Lett. B 419, 296 (1998).
\bibitem{19} Kang-li and C.M.Naón, to appear in Jour.of Phys. A, hep-th 9706156.
\bibitem{20} C.Bernard, Phys.Rev.D.09, 3312 (1974).
\bibitem{21} T.Matsubara, Prog.Theor.Phys.14, 351 (1955).
\bibitem{22} B.Klaiber, Lectures in Theoretical Physics, Boulder, Colorado, 1967, Vol.10A, eds. A.Barut and W.Brittin (Gordon and Breach, New York), p.141.
\bibitem{23} M.Reuter and W.Dittrich Phys. Rev. D 32, 513 (1985).
\bibitem{24} L.Dolan and R.Jackiw, Phys.Rev.D 9 3320 (1974).
\end{thebibliography}