TESLA: Potentials of $\gamma\gamma$ and $e^+e^-$ Options in Stoponium Searches

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Abstract

In some supersymmetric extensions of the Standard Model fairly light superpartner of t-quark is predicted, which may form bound states (stoponiums) under certain conditions. We estimate potentials of TESLA linear collider in search for stoponium, considering the basic electron-positron option and the $\gamma\gamma$ option (Photon Linear Collider - PLC).

It is found that PLC could be the best machine for discovery and study of these new narrow strong resonances. It can produce thousands stoponiums per 100 fb$^{-1}$ integrated $\gamma\gamma$ luminosity in the high energy peak. In the case of scenarios when stoponium decays mainly into two gluons the signal/background ratio is about 1/4. In addition the decay channel $S \rightarrow hh$ into two lightest Higgs bosons could be seen with high significance. Thus, several weeks run is sufficient for the discovery of stoponium, if its mass is approximately known (for example from observation of direct stops production at LHC). Then, in MSSM scenarios with dominant $S \rightarrow hh$ decay PLC shows excellent possibilities to discover bound state of stops, practically immediately after beginning of operating. Thus, PLC has good possibilities to study strong interaction of top quark superpartners in nonperturbative regime.

The $e^+e^-$ option also has some prospects to observe stoponium but only in the case of scenarios with dominant decay into two lightest Higgs bosons, with tens of events per 100 fb$^{-1}$. Interesting possibility appears in the case when the resonance is seated on 0.1% width luminosity peak – one could resolve the stoponium exited states.
1 Introduction

The broken supersymmetry is favorite among the different extensions of the Standard Model. It can happen that superpartners of top-quarks (stops, $\tilde{t}$) are long-living enough to compose (colorless) bound states, stoponiums, denoted as $S$ in what follows. In this scenario experimental study of the corresponding resonances could provide precise value of stop mass and stoponium partial widths, consequently yielding precise values of various parameters of SUSY Lagrangian. Then, if the difference between stop and LSP masses is very small, the search for stop evidence in collisions at high energy could be problematic. Observation of stoponium bound states will be the signature of such models confirming the existence of stop.

There are theoretical motivations for stop to be fairly light. First one appeals to the renormalization group behavior of soft mass terms. Indeed, gauge couplings raise while Yukawa couplings reduce these terms when energy scale evolves down, with Yukawa contributions being very large for stop. The next motivation concerns left-right mixing in squark sector, which is proportional to Yukawa coupling and decreases the mass of the lightest stop. Therefore, light stop may appear in different SUSY models (see, e.g., Refs. [1] for examples in the frameworks of supergravity and gauge mediation). Experimental bound on stoponium mass comes from searches for stop at LEP2 and TEVATRON, it depends on the MSSM spectrum [2]: lower bound is about 90 GeV for sneutrino masses larger than 45-50 GeV or for neutralino masses larger than 50 GeV (ALEPH), while CDF excludes stop mass up to 130 GeV for smaller sneutrino masses. The limitation is weaker if stop and neutralino masses are degenerate, it is about 60 GeV (ALEPH).

Stoponium was studied in Refs. [3,4] in detail, in particular its effective couplings and partial widths were calculated, and prospects to be discovered at LHC were estimated. In Ref. [4] it was briefly mentioned also the possibility to observe stoponiums in photon collisions, however, without analyzing this phenomenology. The first look at the PLC prospects on the stoponium search was done in [5].

Now, when main parameters of TESLA project are under technical discussion\footnote{see \url{http://www.desy.de/~njwalker/ecfa-desy-wg4/parameter_list.html} for current TESLA reference parameter set}, one should understand clearly signatures of stop bound states in $e^+e^-$ and $\gamma\gamma$ collisions, and compare potentials of these two options of future linear collider. In present analysis we will use year integrated luminosity for the $e^+e^-$ option equal to 300 fb$^{-1}$ for $\sqrt{s_{e^+e^-}} = 500$ GeV, and one should rescale this figure for lower collision energies approximately as $L \sim \sqrt{s_{e^+e^-}}$ [6]. Beamstrahlung and ISR will affect considerably the stoponium production rate due
to a condition to tune the collision energy at the resonance point. According to the current TESLA reference parameter set the average energy loss due to beamstrahlung is about 3%. The average energy loss due to ISR is about 5% [7]. Correspondingly, the luminosity distribution has a characteristic width about 5%.

However, stoponium is very narrow resonance, even more narrow than the initial beam energy spread. Its production rate is proportional to the differential luminosity, $dL/dW$, at the $W = \sqrt{s_{ee}}$ peak, where $W$ is a collision energy for hard subprocess. The initial (beam) energy spread for energies 200–500 GeV is about 0.07–0.1% and it is determined by the bunch compression system and undulator for the positron production. The fraction of the luminosity in this ±0.1% peak is determined by the ISR and beamstrahlung. The ISR leaves in this peak about 50% of the luminosity [7]. The average number of beamstrahlung photons with the energy more than $0.001\sqrt{s_{ee}}$ is about $N_{\gamma} \sim 1.15$ per electron for TESLA conditions. Thus, the probability of the $e^+e^-$ collision without such beamstrahlung photons can be estimated as $(1 - e^{-N_{\gamma}})^2/N_{\gamma}^2 \sim 0.35$. The probability of events without any photons (ISR or beamstrahlung ones) with the energies greater 0.1% is $0.5 \times 0.35 \sim 0.17$. So, about 17% of the luminosity is concentrated in ±0.1% range. The differential luminosity in this peak is higher than in 5% interval by a factor of $(5/0.2) \cdot 0.17/0.8 \sim 5.5$ times (here factor 0.8 is due to ISR in the 5% region). This peak could be very important factor for increasing significance of the resonance (when its mass is known), for measurement of its mass and even for resolving close excited states of the stoponium. Note, however, that for a search for the stoponium this peak will not help, because in presence of large background the scanning time in some wide energy interval, required for the resonance observation, has the following dependence on the bin width $\Delta W$ and the differential luminosity, $t_{scan} \propto 1/(\Delta W (dL/dW)^2)$ (here bin $\Delta W$ corresponds to the effective width of the luminosity distribution in the peak). So, the ratio of scanning times for the 5% and the 0.1% width peaks is $(0.2/5) \times (5.5)^2 \sim 1.2$, almost the same.

Photon colliders based on Compton backscattering of laser photons on high energy electrons has been proposed a long time ago [8]. This option has been included in the TESLA Conceptual Design Report [9] and work on the Technical Design Report is under way. Since the CDR parameters of TESLA were changed and luminosities have grown both in $e^+e^-$ and $\gamma\gamma$ collisions.

At the present workshop it was reported [10] that PLC luminosity can be further increased by a factor of 2.5 due to possible decrease of the horizontal beam emittance at the TESLA damping ring (however this has not improved $e^+e^-$ luminosity because it is restricted by collision effects). At present the $\gamma\gamma$ luminosity within the 20% interval just below $\sqrt{s_{\gamma\gamma}} = 0.8E_{ee}$ could be about 40% of luminosity in the $e^+e^-$ collisions (where $L_{e^+e^-} = 3 \times 10^{34}$ at
\( \sqrt{s_{ee}} = 500 \text{ GeV} \). In the analysis we will consider 60 fb\(^{-1} \) for PLC year luminosity at \( \sqrt{s_{\gamma\gamma}} \leq 400 \text{ GeV} \).

The most bright evidence of the narrow resonance is its direct s-channel production in \( e^+e^- \) annihilation or \( \gamma\gamma \) fusion. One can note, that high powers of the coupling constants, \( \alpha^2 \alpha_s^5 \), emerge in the squared matrix elements in the \( e^+e^- \) annihilation into stoponium. Indeed, \( \alpha^2 \) arises from two electroweak vertices, and \( \alpha_s^5 \) comes from squared derivative of the stoponium wave function (scalar stoponium can be created there only in P-wave by propagation of neutral vector particle: photon or Z boson). At the same time two powers of \( \alpha_s \) are eliminated in the case of \( \gamma\gamma \) fusion mechanism because the stoponium production can be proceeded in S-wave there. This circumstance makes for the relative enhancement of the stoponium production rate at PLC in comparison with \( e^+e^- \) option. Two powers of \( \alpha_s \) are eliminated also in the case of associated production of stoponiums, for example in the Higgs-like reactions \( e^+e^- \rightarrow ZS \) and \( e^+e^- \rightarrow \nu\bar{\nu}S \). Hereafter we denote stoponium as \( S \).

To complete this brief review of possible production mechanisms one can note that in hadron collisions the stoponium resonance production is available in S-wave through the gluon fusion. Here the effective \( ggS \) vertex includes \( \alpha_s^{5/2} \) and one can anticipate large stoponium cross sections. However, main decay channel \( gg \) is too dirty due to huge QCD 2\( jets \) background. Then, the most promising decay channel at LHC is \( \gamma\gamma \) [4], but in order to discover stoponiums one year of LHC operating at high luminosity is needed, or even more depending on SUSY scenario.

We consider stoponium mass range \( M_S = 200 - 400 \text{ GeV} \), which could be surely probed by first TESLA run. It is worth to note that the same interval is not an exceptional case for SUSY models with stoponiums as a quasistationary state, as we discuss briefly in the next section.

Cross sections for various tree-level background processes were evaluated with the help of CompHEP package [11].

2 Stop bound states

It is clear that gluons try to bind two stops as well as ordinary quarks. The corresponding bound state can be described as a quasistationary system with energy levels \( E_n (< 0) \) and masses \( M_n = 2m_{\tilde{t}} + E_n \) similarly to quarkonium. For stoponium mass \( M_S = 200 - 600 \text{ GeV} \) the binding energies \( E_n \) are of order 1 GeV [12]. This treatment is valid if the formation process (time scale \( \sim |E_n|^{-1} \)) is faster than destroying one.
Among destroying mechanisms the obvious ones are the stop decays\(^2\): \(\tilde{t} \rightarrow t + \text{LSP}, \ b + \text{chargino} \) and \(c + \text{neutralino}\). At first, let us consider the third decay. It proceeds only through loop diagrams, if Universal boundary condition on soft terms is imposed (that is motivated by the absence of FCNC). So, partial width is highly reduced by a factor of \(\sim 10^{-7}\) in comparison with the first two tree-level decay processes [13]. The rates of latter decays depend on the parameters of the model. As an example, in the framework of gravity mediation, where LSP is neutralino, these decays proceed at the tree level and the corresponding partial widths are of order \(O(\alpha m_\tilde{t})\). In the framework of models with gauge mediated supersymmetry breaking [14], where LSP is gravitino, the first process is strongly suppressed by supersymmetry breaking scale, but remaining one has the same partial width as in gravity mediation. Hence, the possibility of existence of stoponium is a subject of special study in each concrete model. For instance, in models with the lightest chargino being mostly wino and the lightest stop being mostly right stop (i.e., \(m_{t_L} > m_{t_R}\)), decay into chargino is damped and stoponium could exist if \(m_\tilde{t} - m_{\text{LSP}} < m_t\), i.e., when the first decay channel is kinematically forbidden. One can state that SUSY scenario, where tree-level decays, \(\tilde{t} \rightarrow t + \text{LSP}\) and \(\tilde{t} \rightarrow b + \text{chargino}\), are somehow suppressed and, therefore, stop decay can not destroy the stoponium formation, is not an exceptional case.

Next destroying mechanism is related to the stop annihilation. Here two gluon channel is always open with partial width about 1 MeV. Generally the gluon channel is dominant. However, for the certain choice of model parameters, partial width into two lightest Higgs bosons, \(S \rightarrow hh\), can be larger, increasing the stoponium total width by a factor of \(\sim 5 - 10\). In Ref. [4] these figures were analyzed and found that quasistationary description is valid for \(M_S < 600\) GeV in models with forbidden stop tree-level decays and neutralino being mostly bino. The worst case is a model with chargino and neutralino states are both higgsino-like. Here the stoponium total width increases rapidly with \(M_S\) and quasistationary treatment fails for \(M_S > 300\) GeV.

3 Stoponium in \(\gamma\gamma\) collisions

Let us begin with study of stoponium events in photon-photon scattering. The main effect associated with stoponium would be a direct resonance production, where stoponium is produced in spin 0 state. The corresponding cross section is described by the Breit-Wigner formula (similarly to the light Higgs production

\(^2\) We suppose R-parity to be conserved, as favored by the absence of rapid proton decay and lepton flavor violating processes.
discussed in Ref. [15])

\[
\sigma_{\gamma\gamma \to S \to f}(\hat{s}) = 8\pi \frac{\Gamma_{\gamma\gamma} \Gamma_f}{(\hat{s} - M_S^2)^2 + \Gamma_{tot}^2 M_S^2} (1 + \lambda_1 \lambda_2),
\]

(1)

where \( \hat{s} = W^2 \) is squared collision energy for hard subprocess, \( \Gamma_f \) is the stoponium partial width for the decay into state \( f \), \( \Gamma_{tot} \) is stoponium total width, \( \lambda_{1,2} \) are helicities of initial photons.

At photon colliders the width of the luminosity distribution is much wider than that of the stoponium. After integration over the luminosity distribution we obtain the effective cross section

\[
\sigma_f = \frac{1}{L_{\gamma\gamma}} \frac{dL_{\gamma\gamma}}{dW} \frac{4\pi^2 \Gamma_{\gamma\gamma} Br_f}{M_S^2} (1 + \lambda_1 \lambda_2),
\]

(2)

The differential luminosity, \( dL_{\gamma\gamma}/dW \), at \( W = M_S \) can be estimated as \( L_{\gamma\gamma}/0.15M_S \) according to 15\% width of the high energy luminosity peak (note, we define the \( \gamma\gamma \) luminosity as the luminosity in the interval \( \Delta W/W_{\max} = 20\% \)). So the effective cross section is

\[
\sigma_f = \frac{(4/0.15)\pi^2 \Gamma_{\gamma\gamma} Br_f}{M_S^2} (1 + \lambda_1 \lambda_2).
\]

(3)

Photon beams are planned to be highly polarized. Hence, as stoponium is a scalar the production cross section will be enhanced by factor two if initial photons have total helicity equal to zero. Hereafter we assume initial total helicity \( 0 \) (\( \lambda_1 \lambda_2 = 1 \)) in numerical estimates. Finally, we parameterize the stoponium cross sections as follows

\[
\sigma_f \approx 50\text{fb} \cdot (1 + \lambda_1 \lambda_2) \cdot \left( \frac{Br_{\gamma\gamma}}{4 \cdot 10^{-3}} \right) \cdot Br_f \cdot \left( \frac{\Gamma_{tot}}{1\text{MeV}} \right) \cdot \left( \frac{200\text{GeV}}{M_S} \right)^3.
\]

(4)

One should take into account that squared stoponium wave function at the origin, attending in \( \Gamma_{tot} \), scales as a square root of its mass [12].

As it has been stressed above one can discuss two main variants of the SUSY models, one with dominant \( gg \) decay mode and another with stoponium total width being saturated by \( hh \) mode. Let us make qualitative signal/background analysis for different decay channels within these two variants. The signal significance can be evaluated by ratio \( N_S/\sqrt{N_B} \) because one deals with resonance and background rate in the signal bin can be fixed as average cross section in neighboring bins (here \( N_{S,B} \) are numbers of signal and background events).
We used the results for stoponium width and branching ratios calculated in Ref. [4] with some corrections [5].

1. In the first scenario stoponium total width $\approx 1.3$ MeV and photon-photon branching is $\text{Br}^{\gamma\gamma} \approx 3.4 \cdot 10^{-3}$. By making use of Eq.(4) one obtains the signal rate at the level of 110 fb for $M_S = 200$ GeV. So, more than six thousand stoponiums will be produced per year if $L_{\text{year}}^{\text{PLC}} = 60$ fb$^{-1}$. Background is two jet production, where subprocess $\gamma\gamma \rightarrow q\bar{q}$ gives main contribution with very large unpolarized cross section, $\sim 6$ pb, if optimal cut on the jet angle of 45$^\circ$ is applied. However, production of fermions is suppressed in collisions of photons with the total helicity 0 [16]:

$$d\sigma(\gamma\gamma \rightarrow q\bar{q})/d\cos\theta \propto \frac{1 + \cos^2\theta}{1 - \cos^2\theta} (1 - \lambda_1\lambda_2) \quad \text{for } \beta \rightarrow 1 ,$$

where $\beta$ is velocity of the quarks. So, $q\bar{q}$ pairs are produced only in collisions of photons with the total helicity 2. This fact is used for suppression of similar background in the analysis of the Higgs production in $\gamma\gamma$ collisions [17,15,18]. The corresponding luminosity spectra for total helicity 0 ($L_0$) and 2 ($L_2$) can be found elsewhere [10]. In the region $\sim 10\%$ near the maximum energy (detector resolution or intrinsic resolution of the PLC for two collinear jets) one can obtain the ratio of the luminosities $L_2/L_0 < 0.1$, that assumes at least one order suppression of the $q\bar{q}$ background. Note that the cross section 6 pb for background corresponds to the case of unpolarized beams. It is zero for collisions of photons with the total helicity 0 and is equal to 12 pb for the total helicity 2, therefore the remaining cross section is 1.2 pb. Note that due to gluon emission $q\bar{q}$ pairs can be produced even in collisions of photons with the total helicity 0. Detailed studies have shown that with proper cuts this process is not important [19,20] and contribution from resolved photons is also not significant [20].

Furthermore, the cross section is proportional to the fourth power of the electric charge of quarks, so the main contribution is given by $u$ and $c$ quarks. The later can be easily suppressed by the vertex detector. This gives additional factor of 2 in the background suppression. Additional improvement can give the detector energy resolution which is at least factor of 2 smaller than the width of the $\gamma\gamma$ luminosity peak. All three methods give a suppression factor of 40. The remaining background is about 0.3 pb, while the isotropic signal is smaller by factor 1.4 only if the cut on the jet angle of 45$^\circ$ is applied. Hence in the scenario with dominant $gg$ channel the signal/background ratio is $\sim 1/4$. The signal significance is about 35 for 60 fb$^{-1}$ and $M_S = 200$ GeV.

Note, that in [5] this background was overestimated by taking the unpolarized cross section.
These figures can be related to the first year of PLC operation if the stoponium mass is known approximately, for example from the observation of the direct stops production at LHC. In opposite case, for a search of a stoponium one should make scanning with the energy bin $\Delta W/W \sim 10\%$. It is clear that stoponium can be found in this scenario during several month work in the whole energy region under discussion, 200-400 GeV.

For two photon channel the background process, $\gamma \gamma \rightarrow \gamma \gamma$, proceeds through one-loop diagrams, so the corresponding cross section is small, about 10 fb [21]. One should note that the photon-photon invariant mass bin can be taken equal to $2 \text{GeV} \cdot \sqrt{M_S/100\text{GeV}}$ for CMS-like crystal electromagnetic calorimeter [22]. Thus, for $M_{\gamma\gamma} = 200 \pm 1.4$ GeV window the background rate can be estimated at the level of 1 fb. The signal rate is $0.4 \div 0.14$ fb for $M_s = 200 \div 300$ GeV, providing the signal significance about $3 \div 1.1$ for statistics 60 fb$^{-1}$.

Some other decay channels in the framework of the first scenario should be discussed. First note, that WW final state has no chance for the detection of stoponiums due to huge SM background, $\sigma^{tot}_{\gamma\gamma \rightarrow WW} \sim 60$ pb at $\sqrt{s_{\gamma\gamma}} = 200$ GeV. More promising are decay channels with background processes emerging due to the higher order corrections from perturbation theory. For instance, SM background to $\gamma Z$ and $ZZ$ final states comes from 1) one-loop $\alpha^4$ processes $\gamma \gamma \rightarrow \gamma Z$ ($10 - 15$ fb [23]) and $\gamma \gamma \rightarrow ZZ$ ($\sim 50$ fb [24]), and 2) from tree-level $\alpha^3$ processes (e.g. $\gamma \gamma \rightarrow \gamma q\bar{q}$ for $S \rightarrow \gamma Z \rightarrow \gamma + 2\text{jets}$), with total cross section smaller than 1 fb within cuts on final $\gamma$ and jets reasonably motivated by 2-body ($\gamma + Z$) kinematics of the signal events.

As to signal $\gamma Z$ rate one can get from Ref. [4] the branching $\text{Br}_{\gamma Z} \sim 2 \cdot 10^{-3}$, so $\sigma_{\gamma Z} \sim 0.22$ fb already for $M_S = 200$ GeV. It means very low level of the signal significance, lower than 0.5 for statistics 60 fb$^{-1}$.

Natural level of $ZZ$ branching is about $4 \cdot 10^{-2}$ for stoponium masses far from the threshold, 250-400 GeV, although in some points it could fall down due to opening of new channels or degeneration of stoponium and Higgs masses. This provides signal rate $\sim 2.5$ fb for $M_S = 250$ GeV and significance at the level of 2.7 if one uses formula (4). However, the threshold effect is significant still for this value of the stoponium mass, and these figures should be improved to 1.8 fb for signal rate and to 2 for the significance.

The $hh$ decay channel, where $h$ is the lightest Higgs boson, is open if $M_S > 2m_h$. As current limit on $h$ mass is about 80-100 GeV this channel could exist for $M_S > 200$ GeV. If consider mass region not very close to the threshold (say $M_S > 300$ GeV for $m_h = 115$ GeV) the $hh$ branching is about $2 \cdot 10^{-2}$ or even higher. In this case the signal rate is about 1 fb or larger, if one uses formula (4), and if take into account the threshold factor one gets signal cross section at the level of 0.2 fb or larger. The background from direct
double $hh$ production through one-loop diagrams can be estimated by the cross section of this process in SM, $\sim 0.2$ fb [25]. There are no reasons for very large additional contributions to this process in supersymmetric models. Then, we found that direct electroweak production of four $b$ quarks ($\gamma\gamma \rightarrow bbbb$) together with contribution from $cccc$ final state (assuming 10% of $b/c$ misidentification) has the rate smaller than 0.1 fb. These cross sections for background processes correspond to 15% width of the $\gamma\gamma$ luminosity spectrum. The detector resolution is at least factor of two better (full width), therefore the total cross section of background processes can be estimated as $(0.2+0.1)/2 = 0.15$ fb. We see that $hh$ decay can also be studied in the considered scenario. For $M_S = 300$ GeV, $m_h = 115$ GeV and $60$ fb$^{-1}$ integrated luminosity about 20 stoponium events will be produced in the decay channel $S \rightarrow hh$ with $S/B$ ratio about 2.3 and statistical significance about 7.

![Fig. 1. Signal significance of stoponium events in various channels for the first scenario; mass of the lightest Higgs boson is taken equal to 115 GeV.](image)

In Fig. 1 the signal significances are represented for main stoponium decay channels in the case of the first scenario. As a resume for this scenario we conclude that stoponium can be found at PLC during several months scan of the 200–400 GeV region in $gg$ and $hh$ decay modes. If its mass is known approximately, it will be found during first weeks. More than a year is necessary in order to observe stoponium in $\gamma\gamma$ and $ZZ$ channels.

Note that LHC (at the high luminosity operating stage) has good prospects to observe light stoponium in $\gamma\gamma$ mode in this scenario [4]. Thus, these two colliders could be complementary in study of different effective stoponium couplings, $S\gamma\gamma$ and $Sgg$ at LHC and the photon collider, respectively and $Shh$ at PLC if this decay channel is open kinematically.
In the second scenario stoponium total width could be about 10 MeV or even larger. The photon-photon branching in this case is smaller, $\sim (2 - 4) \cdot 10^{-4}$. So, about thousand of stoponions will be produced per year and almost all of them will decay to pairs of lightest Higgs bosons. This result suggests, that stoponium will be discovered practically immediately after PLC start, since the background ($hh$, $bbbb$ ...) is very small. Again the scanning over the energy interval is necessary if the stoponium mass is not known. In Fig. 2 the year yields of stoponions are represented for different masses of lightest Higgs bosons.

Due to rather high statistics for the signal and absence (practically) of the background one gets PLC as a stoponium factory. The detailed study of the stoponium characteristics will be available in this case, in particular measurement of its mass and total width and effective couplings $S_{hh}$ and $S_{\gamma\gamma}$. One can stress the importance of study the $S_{hh}$ coupling, which relates directly to the $stop$-$Higgs$ interaction and, thus, to the mechanism of the superpartner mass generation.

Note that in this scenario several years of operating at high luminosity is needed in order to observe stoponium at LHC [4].
4 Stophonium in $e^+e^-$ collisions

1. First we discuss direct resonant production of stoponiums in $e^+e^-$ collisions where initial electron and positron would be in P-state (thus, stoponium is produced in spin 1 state). As in case of $\gamma\gamma$ collisions one gets for narrow resonance after the integration of Breit-Wigner distribution the following formula for production cross section

$$\sigma_S = \frac{1}{L_{e^+e^-}} \frac{dL_{e^+e^-}}{dW} \cdot \frac{6\pi^2 \Gamma_{e^+e^-}}{M_S^2} \cdot (1 - \lambda_- \lambda_+),$$ (6)

where $\lambda_{\mp}$ are helicities of initial electrons and positrons. The differential luminosity at $W = M_S$ can be estimated as $L_{e^+e^-}^{\Delta W}/\Delta W$, where $L_{e^+e^-}^{\Delta W}$ is a fraction of the $e^+e^-$ luminosity corresponding to some interval $\Delta W$ near the $W_{max} = \sqrt{s_{e^+e^-}}$ peak. Let us consider two methods of the stoponium detection:

1) $L_{\Delta W} = 80\% \ L_{e^+e^-}, \ \Delta W/W \sim 5\%$;
2) $L_{\Delta W} = 15\% \ L_{e^+e^-}, \ \Delta W/W \sim 0.2\%$.

Then, the stoponium partial width into $e^+e^-$ pair, $\Gamma_{e^+e^-}$, is equal to [26]

$$\Gamma(S \to e^+e^-) = \frac{32}{3} \cdot \alpha^2 \cdot \frac{|R_P(0)|^2}{M_S^4}. \quad (7)$$

with $R_P(\vec{r})$ being wave function of P-state. In correspondence with Ref. [12] one can approximate $\frac{|R_P(0)|^2}{M_S^4} = 5 \cdot 10^{-6}$ GeV.

We assume, also, 80% circular polarization for the electron beam and 60% for the positron beam, and neglect fairly weak enhancement by Z-boson pole for light stoponiums.

Finally, for the first method of the stoponium detection one gets for the production rate in $e^+e^-$ collisions the following estimate

$$\sigma_S \sim 0.2 \text{fb} \cdot (200 \text{ GeV}/M_S)^3, \quad (8)$$

while it is about 5 times larger in case of second method.

Let us now consider background for the direct resonance production of stoponiums in $e^+e^-$ collisions. One should note that stoponium, being produced in excited state with spin 1, will be transferred to the basic state with spin
0 by the emission of a photon. Therefore, one can consider decay modes of scalar stoponium in case of $e^+e^- \to 2jets$ process. It has a cross section about 10 pb for $\sqrt{s} = 200$ GeV. So, this channel is too dirty with negligible significance.

In the case of first MSSM scenario with $gg$ decay channel being dominant the main background is $e^+e^- \to 2jets$ process. It has a cross section about 10 pb for $\sqrt{s} = 200$ GeV. So, this channel is too dirty with negligible significance.

In the case of second MSSM scenario with $hh$ dominant decay mode almost all stoponiums will decay into pair of lightest Higgs bosons, and the number of events for $M_S = 300$ GeV and 100 $fb^{-1}$ is 5 (25) (the second number is for the case of seating on the 0.1% luminosity peak). As direct pair production of Higgs bosons, $e^+e^- \to hh$, is negligible since the corresponding $eeh$ coupling includes electron mass, this channel is should be practically free of background. Indeed, direct production of four b-quarks in electron-positron collisions has very small cross section if exclude Z peaks (less than 0.05fb if apply the cut $M_{bb} > 95$ GeV). So, stoponium can be observed at $e^+e^-$ machine if the collision energy is tuned at $M_S$.

Note, that in this scenario the yield of stoponiums at PLC with $L_{\gamma\gamma}/L_{e^+e^-} = 0.4$ is higher than that in $e^+e^-$ collisions by a factor of 250(50) (second number for the case of seating on the 0.1% peak of the $e^+e^-$ luminosity). Thus, the scanning time in $e^+e^-$ mode will be about factor of 500 longer than at PLC due to smaller cross section and smaller energy bin (see also discussion on the scanning time in the Introduction).

2. The second effect related to stoponium at $e^+e^-$ collider would be a production in Higgs-like channels. In present analysis we neglect Higgs-stoponium mixing as well as Higgs influence on stoponium-involved processes. First, let us evaluate effective coupling constants between stoponium and weak bosons,

$$ L = \lambda_{SZZ} S Z_{\mu\nu} Z^{\mu\nu} + \lambda_{SWW} S W_{\mu\nu} W^{\mu\nu} ,$$

$$ \lambda_{SZZ} = \frac{0.152}{M_S^{5/4}} \cdot M_{SZZ} \cdot \left( 1 - \frac{4M_Z^2}{M_S^2} + \frac{6M_Z^4}{M_S^4} \right)^{-1/2} ,$$

$$ \lambda_{SWW} = \frac{0.152}{M_S^{5/4}} \cdot M_{SWW} \cdot \left( 1 - \frac{4M_W^2}{M_S^2} + \frac{6M_W^4}{M_S^4} \right)^{-1/2} ,$$

where we used stoponium wave function evaluated at the origin in Ref. [12]; $M_{SZZ}$ and $M_{SWW}$ are corresponding amplitudes of $\tilde{t}\tilde{t} \to ZZ(W^+W^-)$ processes evaluated in Ref. [4]. Certainly, these effective couplings are obtained
with all particles being on-shell, that is rather rough approximation. However, one can hope that it is acceptable for the estimate.

The calculation with the effective couplings above gives the cross section about 0.03 fb for $e^+e^- \rightarrow ZS$ (in case of no mixing in stop sector and $M_{\tilde{b}} = 1$ TeV). So, only a few signal events will be produced for 100 fb$^{-1}$ statistics. The $Zjj$ background is too heavy for this level of the signal, so only the second scenario could have some prospects. The main background will come from associated double Higgs bosons production, $e^+e^- \rightarrow Zhh$, the corresponding cross section is of order $0.2 \div 0.5$ fb for $\tan\beta = 3 \div 50$ \cite{27}. It gives the signal significance less than 1.

In the case of $W$-fusion the signal cross section is very small, less than $10^{-3}$ fb, that closes this channel.

5 Conclusions

In two considered scenarios of supersymmetric extension of the Standard Model (1st one with dominant $gg$ decay mode, and $hh$ being dominant in 2nd scenario) Photon Linear Collider will be the best machine to discover and study bound state of stops, if it exists.

In case of 1st scenario stoponium will be observed at PLC in the $gg$ and $hh$ (later if permitted kinematically) in the beginning of the operation – several months for scanning of whole energy region or several weeks if the stoponium mass is approximately known.

In case of 2nd scenario, about thousand of stoponiums will be produced free of background. It means that stoponium can be discovered at PLC practically immediately.

Study of the effective stoponium couplings with photons, gluons and lightest Higgs bosons will be available at PLC, latter two depending on the MSSM scenario. Measurement of the stoponium mass and total width (extracted from the measured signal rate) will be possible also.

Stoponium discovery mass range will be limited only by attainable values of the $\gamma\gamma$ collision energy, which is discussed up to $0.8 \cdot 500$ GeV. The tuning of PLC collision energy at the resonance point is necessary within the 15% window. These estimates were done for the case of PLC year luminosity being 60 fb$^{-1}$.

In $e^+e^-$ collisions stoponium could be observed only in the scenario with $hh$ decay channel being dominant, but with the rate lower than in the PLC case.
by a factor of 250 (50) (the second number for seating on the 0.1% luminosity peak). In this comparison it was assumed that $\gamma\gamma$ luminosity in the high energy peak is equal 40% of $e^+e^-$ luminosity. Search time here is about 500 times longer than in $\gamma\gamma$ collisions. However, there is one important advantage of $e^+e^-$ collisions: by use of very monochromatic part of the luminosity spectrum (0.1%) one can make precise measurement of the stoponium mass and resolve its excited states. Although this is possible only in the scenario when $hh$ decay dominates.

A few further comments can be made. The first one is related to the circumstance that at photon colliders ground state of stoponium could not be distinguished from excited states due to the detector resolutions. Therefore, the resonance peak will include contributions from ground state and all excitations, leading to enhancement factor of about 2 in all cross sections [4]. At the same time there is a big uncertainty because of poor understanding of the stoponium wave function, that results in 30-50% error when the stoponium rates are estimated [4].

Then, stoponium has the same quantum numbers as neutral Higgs bosons. Thus, interesting phenomena could appear due to the interference of stoponium with Higgs sector. This point was discussed briefly in Ref. [5].

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