A non-data-aided SNR estimator based on maximum likelihood method for communication between orbiters

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Abstract

Signal-to-noise ratio (SNR) is an important metric for performance assessment in numerous scenarios. In order to ensure the reliability and effectiveness of the system performance, plenty of situations require the information of SNR estimate. At the same time, Mars exploration has been a hot topic in recent years, which leads to the research attention of scholars extending to deep space. In this paper, a new SNR estimator related to deep space scene is proposed. On the one hand, the time of essential data transmission is limited in Mars exploration system. On the other hand, the relative position and condition between orbiters vary quickly all the time, which makes it difficult to obtain the accurate and significant information for Mars exploration. Therefore, it is obvious that the information of SNR can promote the system to adjust the signal transmission rate automatically. Subsequently, the estimation of SNR becomes a fundamental research in automatic digital communications. In this paper, an SNR estimation method based on non-data-aided (NDA) with maximum likelihood (ML) estimation is proposed to enhance the accuracy and reliability of Mars exploration process. Additionally, the Cramer-Rao lower bound (CRLB) related to the designed ML algorithm is derived. Finally, the Monte Carlo simulation results demonstrate that the proposed ML estimator algorithm obtains a superior performance when compared to the existing SNR estimators.

Keywords: Signal-to-noise ratio estimate, Maximum likelihood, Cramer-Rao lower bound

1 Introduction

Since the Earth and Mars are far away from each other, there is an attenuation of signal and exists a delay of time for communication between them, both of which destroy the signal quality in receivers [1, 2]. On the one hand, the time of essential data transmission is limited in Mars exploration system, and on the other hand, the relative position and condition of orbiters vary quickly all the time, which makes it difficult to obtain the accurate and significant information for Mars exploration [3, 4]. Therefore, it is obvious that the information of signal-to-noise ratio (SNR) promotes the system to adjust the signal transmission rate automatically [5]. The existing SNR estimation methods can be
classified into two groups, namely data-aided (DA) and non-data-aided (NDA) [6, 7] estimators, respectively, which apply to the cases that the transmitted signals are known to the receivers for DA estimators and the receivers of the NDA estimators have no information about the transmitted symbols, respectively [8]. Although DA estimation algorithms outperform NDA-based methods in accuracy and effectiveness, it is at the expense of lower transmission efficiency, which is extremely worthwhile for deep space exploration [9]. Therefore, NDA-based estimators are considered for the deep space scene. Recently, many new researches are also investigated [10–16].

Generally, NDA-based SNR estimators include signal-to-variation ratio (SVR) [17], second- and fourth-order moments ($M_2 M_4$) [18], squared signal-to-noise variance (SNV), and subspace-based method [1]. SVR estimator, which can be applied to not only fading channels but also additive white Gaussian noise (AWGN) channels, is developed to monitor and evaluate the channel quality based on moment operations. It is noticeable that SVR estimator is not applicable to other forms of digital modulated signals except $M$-ary phase-shift keying (PSK) modulated ones. $M_2 M_4$ estimator was first proposed to estimate the strength of carrier and noise in real channels, and we further extended it to complex domain. Due to its independence of the transmitted symbols, namely, it only requires the estimation of the second and fourth order related to the received symbols, $M_2 M_4$ is also one of the NDA estimators. SNV estimator utilizes the first and second moment of the received signals, which is the output of the matched filter (MF). Finally, the subspace method is based on the singular value decomposition (SVD) theory.

This paper aims to develop a NDA SNR estimation method based on maximum likelihood (ML) method to improve the system performance of deep space model; at the same time, it can achieve a higher accuracy than other estimators, such as SVR, SNV, $M_2 M_4$, and subspace-based method. The Cramer-Rao lower bound (CRLB) of this proposed NDA estimation method is also derived to verify the effectiveness of the proposed ML-based estimator.

The rest of this paper is organized by the following structure. In Section 2, we discuss the system model. The maximum likelihood method based on data-aided is explained in Section 3. Section 4 demonstrates the NDA SNR estimation algorithm through maximum likelihood method. Section 5 shows the experimental figures. Section 6 concludes the contributions made by this paper.

## 2 System model

We aim to find the best SNR estimator while consuming the least energy. Table 1 is the notation of variables for this paper. Generally, the statistical properties are averaged to generate the SNR estimate through a large amount of symbols. The discrete and complex binary phase-shift keying (BPSK) [19] signal constellations are adopted to generate the baseband-equivalent and band-limited transmit symbols in real AWGN channels as illustrated in Fig. 1. The length of the transmitted symbols, which is upsampling to $L = 16$ samples in each symbol, is denoted as $N$. The root raised-cosine (RRC) filter, whose rolloff $R$ equals to 0.5, and number of tap coefficients $Q$ equals to 65, is assumed [20]. The binary source symbols can be calculated by [21, 22]:

$$s_n = a e^{jx_n},$$  (1)
Fig. 1 The system model

Table 1 Notations of variables

| Notations | Description |
|-----------|-------------|
| $L$       | Sampling points |
| $N$       | The number of simulation symbols |
| $R$       | Rolloff coefficient |
| $Q$       | Tap coefficients |
| $\alpha$  | Signal amplitude |
| $\alpha_n$| The phase of signal |
| $s_n$     | The binary source symbols |
| $c_k$     | The upsampled information sequence |
| $\delta_f$| The Kronecker delta |
| $h_k$     | The tap coefficients of the RRC filter |
| $d_k$     | The signal after being sampled and pulse-shaped |
| $w_k$     | The additive white Gaussian noise |
| $\sigma^2$| The variance of AWGN |
| $r_k$     | The received signal |
| $y_k$     | The output of the MF |
| $y_n$     | The downsampled signal of $y_k$ |
| $f_k$     | The samples of the full raised-cosine |
| $f_0$     | The peak of $f_k$ |
| $z_n$     | The downsampled and filtered samples of noise |
| $\rho$    | Signal-to-noise ratio |
| $P(\cdot)$| The probability density function |
| $L_{\hat{\theta}}$ | The log-likelihood function |
| $\hat{\theta}$ | The estimated parameter vector |
| $g(\theta)$ | The decibel scale of $\theta$ |
| CRLB      | Cramer-Rao lower bound |
| $K(\theta)$| Fisher information matrix |
| $\hat{\rho}$ | The estimate value of SNR |
where $a$ represents the signal amplitude whose probability is equal by obtaining values from $[-A, A]$ and $\alpha_n$ represents one of the two evenly spaced phases on a unit circle with $n \in \{1, 2, \ldots, N\}$. The upsampled information sequence can be shown as:

$$c_k = \sum_n s_n \delta_{k,nN},$$

(2)

where $\delta_{ij}$ denotes the Kronecker delta. The signal after being sampled and pulse-shaped can be given as:

$$d_k = \sum_n s_n h_{k-nN},$$

(3)

where $h_k$ is the tap coefficients of the RRC filter with $k \in \{- (R - 1)/2, \ldots, -1, 0, 1, \ldots, (R - 1)/2\}$ and $h_k$ equals to zero for $|k| > (R - 1)/2$. The received signal can be presented as:

$$r_k = d_k + w_k,$$

(4)

where $w_k$ denotes the complex and sampled AWGN with zero mean and variance of $\sigma^2$. The output of the MF is expressed as:

$$y_k = r_k \otimes h_k^* = \sum_l h_l d_{k-l} + \sum_l h_l w_{k-l},$$

(5)

where $\otimes$ and $*$ denote the discrete convolution and complex conjugation, respectively, and $h_k = h_k^*$ can be attributed to the assumption that the impulse response of RRC is real and even symmetric. Finally, the downsampled symbols at the receiver can be expressed as:

$$y_n = y_k |_{k=nN} = s_n f_0 + z_n,$$

(6)

where $f_0$ denotes the maximum impulse response of the full raised-cosine, the samples of which can be written as:

$$f_k = h_k^* \otimes h_k = \sum_l h_{k-l} h_l,$$

(7)

and the downsampled and filtered samples of noise $z_n$ can be expressed as:

$$z_n = z_k |_{k=nN} = \sum_l h_l w_{k-l} |_{k=nN},$$

(8)

Subsequently, the SNR is deduced as:

$$\rho = \frac{E \left( |s_n f_0|^2 \right)}{\text{var} \{z_n\}},$$

(9)

where $E \{\cdot\}$ represents the expectation and $\text{var} \{\cdot\}$ represents the variance. The SNR can be independent of the channel by normalizing the corresponding square of tap coefficients of the RRC; subsequently, the SNR can be solely related to $A$ and $\sigma^2$, namely:

$$\rho = \frac{A^2}{2\sigma^2},$$

(10)

### 3 Maximum likelihood estimation method based on NDA

In this section, we propose an NDA estimation method [23, 24] based on ML algorithm. The probability density function (PDF) of $y_n$ can be expressed as:

$$P \left( y_n \right) = \frac{1}{2} \left[ P_+ \left( y_n \right) + P_- \left( y_n \right) \right] = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{y_n^2 + A^2}{2\sigma^2}} \cosh \left( \frac{y_n A}{\sigma^2} \right),$$

(11)
where $P_+ (y_n) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y_n-A)^2}{2\sigma^2}}$, $P_- (y_n) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y_n+A)^2}{2\sigma^2}}$, and $\cosh (x) = \frac{1}{2} (e^x + e^{-x})$.

The joint PDF of the signal vector at the receiver $[y_1, y_2, \ldots, y_N]$ is hence given by (due to independence):

$$P_N (y_1, y_2, \ldots, y_N) = \prod_{n=1}^{N} P (y_n) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_n^2 + A^2}{2\sigma^2}} \cosh \left( \frac{y_nA}{\sigma^2} \right).$$

(12)

The log-likelihood function is described as:

$$L_N (y_1, y_2, \ldots, y_N | A, \sigma^2) = -N \log \left( \sqrt{2\pi}\sigma \right) - \frac{1}{2\sigma^2} \sum_{n=1}^{N} (y_n^2 + A^2) + \sum_{n=1}^{N} \ln \cosh \left( \frac{y_nA}{\sigma^2} \right).$$

(13)

Setting $\frac{\partial L_N (y_1, y_2, \ldots, y_N)}{\partial A} = 0$, we implicitly obtain the ML estimate of $A$ as the solution to the equation:

$$A = \frac{1}{N} \sum_{n=1}^{N} y_n \tanh \left( \frac{y_nA}{\sigma^2} \right).$$

(14)

We consider an iterative algorithm to explore the most suitable value of amplitude that satisfies (13). According to the signal vector at the receiver with $N$ samples of $y_n$, it is obvious to define the function:

$$F (x) = x - \frac{1}{N} \sum_{n=1}^{N} y_n \tanh \left( \frac{y_nx}{\sigma^2} \right).$$

(15)

By solving the equation above, we can obtain the amplitude estimation of the maximum likelihood method, namely, $F (x) = 0$ at $x = \hat{A}$, where $\hat{A}$ represents the estimate of $A$. We can determine the value of $\hat{A}$ by the following steps:

Step 1: The received vector is normalized as $\frac{1}{N} \sum_{n=1}^{N} y_n^2 = 1$. Find the maximum and minimum values of $A_{min}$ and $A_{max}$. At the same time, determine the total number of iteration $I$. Let $A_1 = A_{min}$ and $A_2 = A_{max}$. Initialize $i = 0$.

Step 2: Compute $A_m = \frac{A_1 + A_2}{2}$, and therefore, $\sigma_m^2 = 1 - A_m^2$.

Step 3: Compute $F (A_m) = A_m - \frac{1}{N} \sum_{n=1}^{N} y_n \tanh \left( \frac{y_nA_m}{\sigma_m^2} \right)$.

Step 4: If $F (A_m) > 0$, then let $A_2 = A_m$; otherwise, let $A_1 = A_m$.

Step 5: Let $i = i + 1$, if $i = I$, output $A_m = \frac{A_1 + A_2}{2}$ as to the ML estimate of the amplitude and $\frac{\sigma_m^2}{1-A_m^2}$ as the estimated SNR. Otherwise, go back to step 2.

4 Performance comparisons

4.1 Cramer-Rao lower bound (CRLB)

We consider CRLB [25, 26] to evaluate whether the estimators work effectively or not. The definition of the SNR has been given in (10). We intend to estimate $\rho$ by $N$ observed samples of $y_n$. The estimation task involves two parameters. The estimated parameter vector is denoted as:

$$\theta = [A \quad \sigma^2]^T.$$
Since the estimation of the SNR is generally expressed in decibel scale, the following form of CRLB is adopted:

\[ g(\theta) = 10 \log_{10} \left( \frac{A^2}{\sigma^2} \right), \]  

(17)

\[ \text{CRLB}(\rho) = \partial g(\theta) \partial \theta^{-1} \partial g(\theta)^T, \]  

(18)

where \( K(\theta) \) is the Fisher information matrix, which can be shown as:

\[
K(\theta) = \begin{bmatrix}
-\mathbb{E}\left[ \frac{\partial^2 \ln P(x|\theta)}{\partial A^2} \right] - \mathbb{E}\left[ \frac{\partial^2 \ln P(x|\theta)}{\partial A \partial \sigma^2} \right] & -\mathbb{E}\left[ \frac{\partial^2 \ln P(x|\theta)}{\partial \sigma^2 \partial A} \right] \\
- \mathbb{E}\left[ \frac{\partial^2 \ln P(x|\theta)}{\partial \sigma^2} \right] - \mathbb{E}\left[ \frac{\partial^2 \ln P(x|\theta)}{\partial \sigma^2 A^2} \right]
\end{bmatrix}. \]  

(19)

From (17), \( \partial g(\theta) \partial \theta \) is determined as:

\[
\frac{\partial g(\theta)}{\partial \theta} = \left[ \frac{20}{\ln(10)} \frac{-10}{\ln(10)\sigma^2} \right]. \]  

(20)

According to the \( N \) observed symbols, the Fisher information matrix is described as:

\[
K_{\text{BPSK-NDA}}(\theta) = \frac{N}{\sigma^4} \begin{bmatrix}
\sigma^2 - \sigma^2 J(\rho) & AJ(\rho) \\
AJ(\rho) & 1 - \frac{A^2}{\sigma^2} J(\rho)
\end{bmatrix}, \]  

(21)

where \( J(\rho) = \frac{e^{-\rho}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} u^2 e^{-u^2/2} \cosh(u\sqrt{2\rho}) du \). By substituting (21) to (17), the CRLB of the BPSK signals can be expressed as:

\[
\text{CRLB}_{\text{BPSK-NDA}}(\rho) = \frac{100 \left( \frac{2}{\rho} - J(\rho) + 1 \right)}{N(\ln(10))^2 \left( 1 - J(\rho) - 2\rho J(\rho) \right)} (dB)^2. \]  

(22)

### 4.2 Normalized mean square error (NMSE)

For the above iterative SNR estimator, we provide the required experimental results. To appropriately benchmark our proposed method, the results of SVR, M2M4, SNV, and subspace-based methods are also included. According to the system model given in Section 2, we obtain the NMSE of the aforementioned SNR estimators. The NMSE represents the deviation between estimated and true values and can be calculated as follows:

\[
\text{NMSE} = \frac{E \left[ (\rho - \hat{\rho})^2 \right]}{\rho^2}, \]  

(23)

where \( \hat{\rho} \) describes the estimated SNR while \( \rho \) represents the true SNR value. The NMSE of each estimator is calculated by using the Monte Carlo method as follows:

\[
\text{NMSE} = \frac{1}{N} \sum_{n=1}^{N} \frac{(\rho - \hat{\rho})^2}{\rho^2}. \]  

(24)

The number of symbols \( N \) should be large enough to promise the accuracy and objectivity of the experimental results. In terms of the ML estimator, the NMSE under different simulation numbers is also shown to further compare the performance of the proposed estimator.

### 5 Simulation results

In this section, the Monte Carlo simulation results of the NMSE performances of the aforementioned estimators are presented. The CRLB performance, the ML estimator
under several different simulation numbers, and the differences among perfect SNR, SNR under subspace-based method, and SNR under ML-based method are also displayed.

Figure 2 shows the NMSE performance among SNV, SVR, M2M4, subspace method, and ML-based estimator with the length of symbols $N = 1000$. From the simulation result, we can make a conclusion that SNV-based method performs worse than all the other estimators mentioned above. Subspace-based method outperforms SNV, SVR, and $M_2M_4$ in a reasonable SNR region. However, when compared to the proposed ML-based estimator, it suffers an SNR deficit under the same NMSE, which verifies the efficiency of the proposed ML-based method.

Figure 3 describes the CRLB of the proposed ML-based estimator under two cases, namely, $N = 300$ and $N = 600$, respectively. From the simulation result, we can find that the CRLB with $N = 300$ is higher than that of $N = 600$. The reason of this phenomenon can be easily understood by the essence that the larger simulation symbols lead to a higher estimation accuracy.

Figure 4 demonstrates the NMSE performance under different lengths of simulation symbols, which are $N = 100$, $N = 200$, and $N = 500$, respectively. It is obvious that with the increase of simulation symbols $N$, the NMSE performance becomes better, which is consistent with the theoretical analysis. This result is useful for engineering to choose the appropriate length of symbols.

Figure 5 shows the differences among the perfect SNR, SNR under subspace-based method, and SNR under ML-based method under $N = 600$. We can obtain from the curves that compared to the subspace-based method, the ML-based method achieves a higher estimation accuracy and the gap across the perfect SNR is smaller. Therefore, the
Fig. 3 The CRLB of the proposed algorithm. ML under $N = 300$, ML under $N = 600$, CRLB under $N = 300$, and CRLB under $N = 600$

Fig. 4 The NMSE under different length of simulation symbols. $N = 100$, $N = 200$, and $N = 500$, respectively
The difference comparison among the subspace method-based SNR, ML-based SNR, and perfect SNR under $N = 600$. Perfect SNR ($N = 600$), subspace-based method ($N = 600$), and ML-based method ($N = 600$) proposed ML-based method performs better than all the other estimators mentioned above.

6 Conclusions
We proposed a novel NDA-based ML estimator, which is based on an iterative algorithm and achieves a higher SNR transmission accuracy compared to several traditional SNR estimators. The simulation results showed that the proposed new ML estimation method achieves a superior NMSE performance compared to the existing ones, which is significant for the researches of Mars exploration. The CRLB of the proposed ML-based method was also derived. Furthermore, the NMSE of the ML-based method under different simulation symbols was compared to further verify the effectiveness of the proposed algorithm.

Abbreviations
SNR: Signal-to-noise ratio; NDA: Non-data-aided; ML: Maximum likelihood; CRLB: Cramer-Rao lower bound; DA: Data-aided; SVR: Signal-to-variation ratio; $M_2/M_4$: Second- and fourth-order moments; SNV: Squared signal-to-noise variance; AWGN: Additive white Gaussian noise; PSK: Phase-shift keying; MF: Matched filter; RRC: Root raised-cosine; PDF: Probability density function; NMSE: Normalized mean square error

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Authors’ contributions
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The simulation was performed using MATLAB in Intel Core IS (64bit).

Competing interests

The authors declare that they have no competing interests.

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References

1. Z. Z. Sun, C. H. Wang, X. F. Zhang. Subspace method-based blind SNR estimation for communication between orbiters in Mars exploration. IFIP Web Conf. 11, 03006 (2017). https://doi.org/10.1051/itm2conf/20171103006
2. G. Cates, C. Stromgren, B. Mattfeld, W. Cirillo, K. Goodliff, in 2016 IEEE Aerospace Conference, The exploration of Mars launch assembly simulation, (2016), pp. 1–12. https://doi.org/10.1109/AERO.2016.7500766
3. D. S. Bass, R. C. Wales, V. L. Shalini, in 2005 IEEE Aerospace Conference, Choosing Mars time: analysis of the Mars exploration rover experience, (2005), pp. 4174–4185. https://doi.org/10.1109/AERO.2005.1599722
4. T. Iida, T. Arieoto, Y. Suzuki, T. Li, The future Mars human community: a story of high speed needs beyond explorations. IEEE Aerosp. Electron. Syst. Mag. 26(2), 19–25 (2011). https://doi.org/10.1109/MAES.2011.5739485
5. M. Simon, S. Dölinar, in IEEE Global Telecommunications Conference, 2004. GLOBECOM ’04, Signal-to-noise ratio estimation for autonomous receiver operation, vol. 1, (2004), pp. 282–287.1 https://doi.org/10.1109/GLOCOM.2004.1377955
6. F. Socheleau, A. Aissa-El-Bey, S. Houcke, Non-data-aided SNR estimation of OFDM signals. IEEE Commun. Lett. 12(11), 813–815 (2008). https://doi.org/10.1109/LCOMM.2008.081134
7. W. Gappmaier, R. Lopez-Valcarce, C. Mosquera, Joint NDA estimation of carrier frequency phase and SNR for linearly modulated signals. IEEE Signal Proc. Lett. 17(5), 517–520 (2010). https://doi.org/10.1109/LSP.2010.2045545
8. H. Xiao, D. Wu, Y. Li, W. Liu, X. Chu, Improved signal-to-noise ratio estimation algorithm for asymmetric pulse-shaped signals. IET Commun. 9(14), 1788–1792 (2015). https://doi.org/10.1049/iet-com.2014.1162
9. Y. Wang, L. Jin, in 2014 International Conference on Computational Intelligence and Communication Networks, A joint blind channel order estimation algorithm, (2014), pp. 1158–1162. https://doi.org/10.1109/CICN.2014.242
10. F. Adi Al-Turjman, Smart-city medium access for smart mobility applications in internet of things. Trans. Emerg. Telecommun. Techn. (2019). https://doi.org/10.100
11. F. Al-Turjman, I. Baali, Machine learning for wearable IoT-based applications: a survey. Trans. Emerg. Telecommun. Techn. n/a/v/val, 3635. https://doi.org/10.1006/tetc.2002.0031 R2
12. F. Al-Turjman, Inteligence and security in big 5g-oriented iot: an overview. Futur. Gener. Comput. Syst. 102, 357–368 (2020). https://doi.org/10.1016/future.2019.08
13. Z. Ullah, F. Al-Turjman, L. Mostarda, Cognition in UAV-aided 5G and beyond communications: a survey. IEEE Trans. Cogn. Commun. Netw. 1–1 (2020). https://doi.org/10.1109/TCCN.2020.2968
14. K. Z. Ghafoor, L. Kong, S. Zeadally, A. S. Sadiq, G. Epiphanio, M. Hammoud, A. K. Bashir, S. Mumtaz, Millimeter-Wave Communication for Internet of Vehicles: Status, Challenges and Perspectives. IEEE Internet Things J., 1–1 (2020). https://doi.org/10.1109/JIOT.2020.2992460
15. Q. Li, M. Wen, B. Clerckx, S. Mumtaz, A. Al-Dulaimi, R. Qingyang Hu, Subcarrier Index Modulation for Future Wireless Networks: Principles, Applications, and Challenges. IEEE Wirel. Commun., 1–8 (2020). https://doi.org/10.1109/MWC.2020.1900335
16. Q. Li, M. Wen, D. Di Renzo, H. V. Poor, S. Mumtaz, F. Chen, Dual-Hop Spatial Modulation With A Relay Transmitting Its Own Information. IEEE Trans. Wirel. Commun., 1–1 (2020). https://doi.org/10.1109/TWC.2020.2983696
17. T. Salman, A. Badawy, T. M. Eloufly, T. Khatbab, A. Mohamed, in 2014 IEEE 10th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob), Non-data-aided SNR estimation for QPSK modulation in AWGN channel, (2014), pp. 611–616 https://doi.org/10.1109/WiMob.2014.6962233
18. M. B. Ben Salah, A. Samet, NDA SNR estimation using fourth-order cross-moments in time-varying single-input multiple-output channels. IET Commun. 10(11), 1348–1354 (2016). https://doi.org/10.1049/iet-com.2015.0954
19. M. Bakkali, A. Stephenne, S. Affes, in IEEE Vehicular Technology Conference, Iterative SNR estimation for MPSK modulation over AWGN channels, (2006), 1–5. https://doi.org/10.1109/VTc.2006.350
20. E. Fishter, M. Grosman, H. Messer, Detection of signals by information theoretic criteria general asymptotic performance analysis. IEEE Trans. Signal Process. 50(5), 1027–1036 (2002). https://doi.org/10.1109/78.995060
21. D. R. Pauluzzi, N. C. Beaulieu, in IEEE Pacific Rim Conference on Communications, Computers, and Signal Processing. Proceedings, A comparison of SNR estimation techniques in the AWGN channel, (1995), pp. 36–39. https://doi.org/10.1109/PACRIM.1995.519404
22. N. C. Beaulieu, A. S. Torns, D. R. Pauluzzi, Comparison of four SNR estimators for QPSK modulations. IEEE Commun. Lett. 4(2), 43–45 (2000). https://doi.org/10.1109/4234.824751
23. F. Belili, R. Melfesi, S. Affes, A. Stephenne, Maximum likelihood SNR estimation of linearly-modulated signals over time-varying flat-fading simo channels. IEEE Trans. Signal Process. 63(2), 441–456 (2015). https://doi.org/10.1109/ TSP.2014.2364017
24. L. Bin, R. Di Fazio, A. Zeira, A low bias algorithm to estimate negative SNRs in an AWGN channel. IEEE Commun. Lett. 6(11), 469–471 (2002). https://doi.org/10.1109/LCOMM.2002.805546
25. W. Gappmair, Cramer-Rao lower bound for non-data-aided SNR estimation of linear modulation schemes. IEEE Trans. Commun. 56(5), 689–693 (2008). https://doi.org/10.1109/TCOMM.2008.060275
26. N. S. Alagha, Cramer-Rao bounds of SNR estimates for BPSK and QPSK modulated signals. IEEE Commun. Lett. 5(1), 10–12 (2001). https://doi.org/10.1109/4234.901810

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