Electromagnetic characteristics of bilayer quantum Hall systems 
in the presence of interlayer coherence and tunneling

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I. INTRODUCTION

The electromagnetic characteristics of bilayer quantum Hall systems in the presence of interlayer coherence and tunneling are studied by means of a pseudospin-texture effective theory and an algebraic framework of the single-mode approximation, with emphasis on clarifying the nature of the low-lying neutral collective mode responsible for interlayer tunneling phenomena. A long-wavelength effective theory, consisting of the collective mode as well as the cyclotron modes, is constructed. It is seen explicitly from the electromagnetic response that gauge invariance is kept exact, this implying, in particular, the absence of the Meissner effect in bilayer systems. Special emphasis is placed on exploring the advantage of looking into quantum Hall systems through their response; in particular, subtleties inherent to the standard Chern-Simons theories are critically examined.

II. BILAYER SYSTEMS

Consider a bilayer system with average electron densities \( \rho^{(\alpha)}_{av} = \rho^{(1)}_{av} + \rho^{(2)}_{av} \) in the upper (\( \alpha = 1 \)) and lower (\( \alpha = 2 \)) layers. The two layers, each extending in the \( \mathbf{x} = (x_1, x_2) \) plane, are taken to be situated at position \( z^{(1)} = z_c + \frac{d}{2} \) and \( z^{(2)} = z_c - \frac{d}{2} \) with separation \( d \) in the vertical (z) direction. The system is placed in a common strong perpendicular magnetic field \( B_z = B > 0 \). We suppose that the electron fields \( \psi^{(\alpha)} \) in each layer are fully spin polarized and assemble them into a pseudospin \( \Psi = \psi^{(1)}\psi^{(2)} \). Our task in this paper is to study how the system responds to weak electromagnetic potentials \( A_\mu(x; z) \) and \( A_z(x; z) \) in three space. [We suppose \( \mu \) runs over \( (0, 1, 2) \) or \( (t, x_1, x_2) \) and denote \( A_k = (A_1, A_2) = A \) and \( x = (t, \mathbf{x}) \) for short.] We thus write the one-body Lagrangian in the form

\[
L_1 = \int d^2x \Psi^\dagger (i\partial_t - \mathcal{H}) \Psi, \quad (2.1)
\]

\[
\mathcal{H} = \frac{1}{2M}(p + A^B + A^+ - A^- \sigma_3)^2 + A_0^+ + A_0 \sigma_3, \quad (2.2)
\]

where \( A^\pm_\mu(x) = A_\mu(x; z^{(1)}) \pm A_\mu(x; z^{(2)}) \) in terms of the potentials acting on each layer, or explicitly,

\[
A^+_\mu(x) = A_\mu(x; z_c) + \cdots, \quad (2.3)
\]

\[
A^-_\mu(x) = (d/2) \partial_z A_\mu(x; z_c) + \cdots;
\]

\( A^B = eB(x_2, 0) \) supplies a uniform magnetic field \( B \); the electric charge \( e > 0 \) has been suppressed by

The tunneling and coherence regimes \( 3^,4^,2^2 \) and Josephson-like effects \( 1^,5^,16^,25^,26^ \) attract attention. We study the electromagnetic characteristics of bilayer systems by means of (i) a pseudospin-texture effective theory and (ii) an algebraic framework of the single-mode approximation, with essentially the same results. Our analysis shows that proper account of Landau-level projection is indispensable for deriving a low-energy effective theory of gauge-invariant form. The presence of interlayer coherence modifies even the leading long-wavelength features of the bilayer systems and we critically examine the CS approach to clarify its validity and limitations.

In Sec. II we consider the projection of a bilayer system into the Landau levels. We study the electromagnetic characteristics of the bilayer system in Secs. III and IV. In Sec. V we comment on the CS approach. Section VI is devoted to a summary and discussion.
rescaling $eA_\mu \to A_\mu$: [For conciseness, we shall write $\psi^{(a)}(x) = \psi^{(a)}(x, z^{(a)}(t))$, etc., and suppress reference to the $z$ coordinate or $z_c$, unless necessary.] Let us denote the number density and pseudospin densities as

$$\{\rho(x), S^a(x)\} = \Psi^\dagger(x) \{1, \frac{i}{2} \sigma_a\} \Psi(x) \quad (2.4)$$

with the Pauli matrices $\sigma_a$ ($a = 1, 2, 3$). The $A_0^\pm$ coupled to $\rho = \rho^{(1)} + \rho^{(2)}$ probes in-phase density fluctuations of the two layers while $A_0^\pm$ coupled to $S^3 = \frac{i}{2} (\rho^{(1)} - \rho^{(2)})$ probes the out-of-phase density fluctuations.

The electrons in the two layers are coupled through the intralayer and interlayer Coulomb potentials $V_{11}^\pm = V_{22}^\pm$ and $V_{12}^\pm = V_{21}^\pm$, respectively; $V_{11}^\pm = \frac{e^2}{2|e| |p|}$ and $V_{12}^\pm = e^2 d|p| V_{11}^\pm$ with $\epsilon$ being the dielectric constant of the substrate. The pseudospin structure of the Coulomb interaction is manifest by rewriting it as

$$H^C = \frac{1}{2} \sum_p \left( V_{p+}^\pm \rho \rho - 4V_{p-}^\pm S_{-p}^3 S_{3p}^3 \right), \quad (2.5)$$

$$V_{p+}^\pm = \frac{1}{2} \left(1 \pm e^{-d|p|}\right) V_{11}^\pm, \quad (2.6)$$

where $\rho = \rho^\dagger \rho$ and $S_{-p}^3$ stand for the Fourier transforms of $\rho(x)$ and $S^a(x)$ with obvious time dependence suppressed.

Note here that the electromagnetic gauge transformations in three space induce two sets of intralayer gauge transformations $A_{1\mu}(x, z^{(a)}) \otimes A_{2\mu}(x, z^{(a)}) + \partial_\mu \theta^{(a)}(x)$ and $\psi^{(a)}(x) \otimes e^{-i\theta^{(a)}(x)}\psi^{(a)}(x)$ with $\theta^{(a)}(x) \equiv \theta(x; z^{(a)})$, which can be regarded as totally independent (for $\mu \neq 0$) since $\theta(x; z)$ may have arbitrary dependence on $z$. The transformation laws read $\delta A_{1\mu}^\pm(x) = \partial_\mu \theta_\pm(x)$ in terms of $\theta_\pm = \frac{1}{2} \{\theta^{(1)} \pm \theta^{(2)}\}$. Thus, for bilayer systems electromagnetic gauge invariance turns into two separate U(1) gauge symmetries, $U(1)^{\text{em}} \supset U(1)^+ \times U(1)^-$. [We refer to this $U(1)^-$ as “interlayer” gauge invariance below. Note that it disappears in the $d \to 0$ limit.]

The tunneling phenomena must respect electromagnetic gauge invariance. A naive choice of interlayer coupling $S^3 + is^2 = \psi^{(1)}(1) \psi^{(2)}(2)$ should be promoted to a gauge-invariant form $^{27}$

$$H^{\text{tun}} = -\Delta_{\text{SAS}} \int d^2 x \frac{1}{2} \left\{ \psi^{(1)} \psi^{(2)} + \text{h.c.} \right\}, \quad (2.7)$$

with the line integral

$$\Gamma_z(x) = \int_{z(x)}^{\hat{z}(x)} \text{d}z \frac{1}{2} \left\{ A_x(x; z) + \frac{2}{e} B^\perp z \right\}, \quad (2.8)$$

connecting the two layers for each $x$. Here $\Gamma_z$ has the transformation law $\delta \Gamma_z = 20^{-}(x)$. The coupling strength $\Delta_{\text{SAS}}$ characterizes the energy gap between the symmetric and antisymmetric states.

It is possible to gauge away $\Gamma_z$ by setting $\Gamma_z = \Gamma_z + 2\theta^\perp = 0$ so that the transformed fields

$$\psi^{(1)}(x) = e^{i\frac{28}{3} \Gamma_{1\mu}} \psi^{(1)}(x), \quad \psi^{(2)}(x) = e^{-i\frac{28}{3} \Gamma_{2\mu}} \psi^{(2)}(x), \quad A_{1\mu}^\pm(x) = A_{2\mu}^\pm(x) - \frac{2\delta_{z, \mu}}{e} \partial_\mu \Gamma_{1\mu}, \quad \cdots, \quad (2.9)$$

and $A_{1\mu}^\pm(x) = A_{2\mu}^\pm(x)$ are inert under U(1)$^-$ gauge transformations. The $\psi^{(a)}(x)$ for the electron fields “projected” to the common plane $z = z_c$ and undergo only the U(1)$^+$ gauge transformations. Note that $A_{2\mu}^\pm(x)$ is gauge invariant and actually denotes a vertical electric field $A_{2\mu}^\pm \approx (d/2) E_z$ in-plane magnetic fields $A_{2\mu}^{-} \approx (d/2) (B_2, -B_1)$.

In view of this structure it is advantageous to restart with the Lagrangian written in terms of these $\psi^{(a)}(x)$ and $A_{1\mu}^\pm$, and recover the effect of $\Gamma_z$ at the very end. The interlayer gauge invariance is thereby kept exact. Accordingly we shall from now on regard $\psi^{(a)}$ and $A_{1\mu}^\pm$ as denoting the transformed fields $\psi^{(a)}(x)$ and $A_{1\mu}^\pm$.

In addition, it is rather natural and convenient to combine the one-body Lagrangian $L_1$ and $H^{\text{tun}}$ into a formally $U(1) \times SU(2)$ gauge symmetric form by setting

$$A_{1\mu}^\pm \sigma_3 \to A_{1\mu}^\pm \frac{i}{2} \sigma_3 \quad \sigma_3 \to A_{1\mu}^\pm (2.10)$$

in $H$ of Eq. (2.2) and by identifying the $SU(2)$ gauge field $A_{1\mu}^\pm = (A_{11}^\pm, A_{12}^\pm, A_{13}^\pm)$ with

$$A_{1\mu}^\pm = -\Delta_{\text{SAS}}^\pm \delta_{\mu 0}, \quad A_{2\mu}^\pm = 0, \quad A_{3\mu}^\pm = 2A_{1\mu}^\perp. \quad (2.11)$$

This SU(2) gauge symmetry, of course, is only superficial. The system has a global pseudospin SU(2) symmetry in the ideal limit $\Delta_{\text{SAS}} = 0$ and $V_{1\mu}^\pm \to 0$ with $A_{1\mu}^\pm = 0$; it gets broken to $U(1)$ either for $\Delta_{\text{SAS}} = 0$ or for $V_{1\mu}^\pm = 0$.

The “interlayer” gauge invariance has to do with interlayer out-of-phase U(1) rotations induced by the $z$ variation of $\theta(x; z)$, i.e., $\theta^\perp \propto \partial_\mu \theta(x; z)$. They are thus distinct from global U(1) rotations (with constant $\theta^\perp$) about the $S^3$ axis, which have to do with charge conservation.

As a result, the tunneling interaction $H^{\text{tun}} \propto S^1(x)$ defined in terms of $\psi^{(a)}(x)$ is gauge-invariant but transforms covariantly (i.e., breaks invariance) under global U(1)$^-$ rotations. (This in turn implies that there is no loss of generality in choosing $H^{\text{tun}} \propto S^1$.)

Let us now project our system onto the Landau levels. Let $|N\rangle = |n, y_0\rangle$ denote the Landau levels of a freely orbiting electron of energy $\omega_c(n + \frac{1}{2})$ with $n = 0, 1, 2, \cdots$, and $y_0 = \ell^2 p_z$, where $\omega_c = eB/M$ and $\ell = 1/\sqrt{eB}$: we frequently set $\ell \to 1$ below. We first pass into $N = (n, y_0)$ space via a unitary transformation $\Psi(x, t) = \sum_N \langle x | N \rangle \Phi_{n}(y_0, t)$ and, by a subsequent unitary transformation $\Phi_{n}(y_0, t) \to \Psi_{m}(y_0, t)$, make the one-body Hamiltonian diagonal in level indices; the relevant transformation is constructed in powers of $A_{1\mu}^\pm$ and $A_{2\mu}^\perp$. The resulting projected Hamiltonian is an operator in $r \equiv (r_1, r_2) = (i\ell^2 \partial / \partial y_0, y_0)$ with uncertainty $[r_1, r_2] \approx i\ell^2$. Such a systematic procedure of projection, developed earlier, is readily adapted to the present
SU(2) case. As a matter of fact, for $\Delta_{SAS} \ll \omega_c$ the result is essentially the same as in the single-layer case.

Let us focus on the lowest Landau level $n = 0$ in a strong magnetic field. The projected one-body Hamiltonian to $O(A^2)$ reads $\hat{H}^{\text{cyc}} + \hat{H}^{\text{em}} + \hat{H}^{\text{tun}}$ with

$$\hat{H}^{\text{cyc}} = \sum_{\alpha=1}^{2} \{ \hat{H}_{\alpha}^{(1)} + \hat{H}_{\alpha}^{(2)} \} \hat{F}_{\alpha}^{(p)}$$

$$\hat{H}^{\text{em}} = \sum_{p} \{ \chi^{+}_p \hat{p} - p + 2 \chi^{-}_p \hat{S}_p^3 \}$$

$$\hat{H}^{\text{tun}} = -\Delta_{SAS} \hat{S}_p^3 = 0,$$  \hspace{1cm} (2.12)

with $\chi^{\pm}_p = (A_{\alpha}^{\pm} + (1/2M) (A_{\alpha}^{\pm})_p$ and $A_{12}^{\pm} = \partial_1 A_1^{\pm} \mp \partial_2 A_2^{\pm}$; $(A_{\alpha}^{\pm})_p$ stands for the Fourier transform of $A_{\alpha}^{\pm}(x)$. Here the projected charges $\hat{P}_p = \hat{P}_{\alpha}^{(1)} \mp \hat{P}_{\alpha}^{(2)}$, etc., are defined by

$$\hat{P}_{\alpha} = \int dy \Psi_{\alpha}^\dagger(y, t) e^{-i/2 \alpha \vec{p} \cdot \vec{r}} \Psi_0(y, t),$$

$$\hat{S}_{\alpha} = \int dy \Psi_{\alpha}^\dagger(y, t) e^{-i/2 \alpha \vec{p} \cdot \vec{r}} \sigma_{\alpha}^{z} \Psi_0(y, t),$$

where the two-spinor $\Psi_0$, defining the true lowest Landau level, obeys the canonical commutation relation $\{ \Psi_0(y, t), \Psi_{\alpha}^\dagger(y, t) \} = \delta(y - y_0)$. The $U_{\alpha}^{(p)}$ denote the contributions quadratic in $A_{\mu}^{(p)}$, and are given (for $p = 0$) by $U_{\alpha}^{(p = 0)} = \int d^2x U_{\alpha}^{(x)}(x)$ with

$$U_{\alpha}^{(x)} = \frac{1}{2} \sum_{\mu} A_{\mu}^{(x)} D_{\alpha \mu \nu} \partial_\nu A_{\alpha}^{(x)} - \frac{1}{2 \omega_c} A_{\alpha}^{(x)} D_{k0} A_{\alpha}^{(x)} + \cdots,$$  \hspace{1cm} (2.15)

where $D = \omega_c^2 / (\omega_c^2 + \partial^2)$; $D_{\alpha \mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $\epsilon_{\mu \nu}^{\alpha}$ is a totally-antisymmetric tensor with $\epsilon_{12}^{012} = 1$. Here we have retained terms to $O(Q^2 / \omega_c)$; see Ref. 23 for an expression exact to all powers of $\partial_\nu$.

The charges $\hat{p}_p$, $\hat{S}_{\alpha}^{z}$ obey an SU(2) × $W_\infty$ algebra$^{20}$

$$[\hat{p}_p, \hat{p}_k] = -2is(p, k) \hat{p}_{p+k}; \quad [\hat{p}_p, \hat{S}_{\alpha}^{z}] = -2is(p, k) \hat{S}_{\alpha}^{z},$$

$$[\hat{S}_{\alpha}^{x}, \hat{S}_{\beta}^{y}] = c(p, k) \epsilon^{abc} \hat{S}_{\alpha}^{c} \hat{S}_{\beta}^{x} \delta^{ab} \frac{2}{s(p, k)} \hat{p}_{p+k},$$

where

$$s(p, k) = \sin \frac{(\vec{p} \times \vec{k})}{2} \epsilon^{\alpha \beta \kappa}.$$  \hspace{1cm} (2.17)

c(p, k) is given by $s(p, k)$ with $\sin \to \cos$. It is important to note here that the projected charges themselves, $\{ \rho_{\alpha}^{(1)} / \rho_{\alpha}^{(2)} + \Delta \rho_{\alpha}^{(2)} \}$, differ slightly from $\rho_{\alpha}^{(p)}$ by field-dependent corrections $\Delta \rho_{\alpha}^{(2)}$, which derive from the field-dependent projection employed. (See the Appendix.) As a result, the projected Coulomb interaction

$$\hat{H}^C = \frac{1}{2} \sum_{\alpha} \left( V^{+}_\alpha \hat{p} - \hat{p} \hat{p} + 4V^{+}_\alpha \hat{S}_{\alpha}^{3} \right) \hat{p} + \Delta \hat{H}^C$$  \hspace{1cm} (2.18)

acquires a field-dependent piece $\Delta \hat{H}^C$, which plays a crucial role, as we shall see.

The dynamics within the lowest Landau level is now governed by the Hamiltonian $\hat{H} = \hat{H}^C + \hat{H}^{\text{cyc}} + \hat{H}^{\text{em}} + \hat{H}^{\text{tun}}$. Suppose now that an incompressible many-body state $\langle G \rangle$ of uniform density $\rho_{av}(1) = \rho_{av}(2)$ is formed. Then, setting $\langle G | \rho_{av}^{(p)} | G \rangle = \rho_{av}^{(p)} (2n)^2 \delta^2(p)$ in $\hat{H}^{\text{em}}$ one obtains the effective action to $O(A^2)$:

$$S^{\text{cyc}} = -\int dt d^2x \sum_{\alpha} \rho_{av}^{(x)} U_{\alpha}^{(a)}(x),$$  \hspace{1cm} (2.19)

which summarizes the response due to electromagnetic inter-layer Landau-level mixing, i.e., due to the cyclotron modes (one for each layer).

The electromagnetic interaction in $\hat{H}$ also gives rise to intra-Landau-level transitions. For single-layer systems the intra-Landau-level excitations are only dipole inactive$^{11}$ (i.e., the response vanishes faster than $k^2$ for $k \to 0$) as a result of Kohn's theorem$^{29}$, and the incompressible quantum Hall states show universal $O(k)$ and $O(k^2)$ long-wavelength electromagnetic characteristics determined by the cyclotron mode alone$^{23}$.

The situation changes drastically for bilayer systems, where both in-phase and out-of-phase collective excitations arise. In-phase excitations generally remain dipole inactive, as a consequence of invariance under translations of both layers. Out-of-phase collective excitations, in contrast, become dipole active$^{10,17}$ (in the absence of interlayer coherence) and modify the electromagnetic characteristics of the bilayer systems substantially$^{23}$. Incompressible quantum Hall states well described by the Halperin $(m, n, m)$ wave functions$^{12}$, in particular, belong to this class of states. The presence of interlayer coherence is expected to cause further substantial changes in the systems, which we study in the next section.

### III. INTERLAYER COHERENCE AND ELECTROMAGNETIC RESPONSE

In this section we study how the presence of interlayer coherence affects the electromagnetic properties of bilayer systems. The particular set of states of our concern are the ground states at filling $\nu = 1/m$ for odd integers $m$, believed to have total pseudospin $S = N_c/2$, with their orbital wave functions well approximated by the Laughlin wave functions$^{9}$ or Halperin $(m, m, m)$ wave functions. For definiteness we shall concentrate on the $\nu = 1$ ground state, but our analysis will apply to other cases as well.

Suppose first that the SU(2) breaking Coulomb interaction $V^+_{\alpha} = (c^2/4e) d + O(D^2)$ is negligibly weak (i.e., $d \to 0$ and $\Delta_{SAS} \neq 0$). Then the $\nu = 1$ ground state is given by the total-pseudospin $S = N_c/2$ eigenstate $| G_0 \rangle$, fully polarized in the $\hat{S}_3$ direction via the tunneling interaction so that $\langle G_0 | \hat{S}_3^{\alpha} = 0 | G_0 \rangle = \frac{1}{2} N_c$, or
\[ \langle G_0 | S^a_p | G_0 \rangle = \delta^{a1} \frac{1}{2} \rho_0 \delta_{p,0}, \quad \langle G_0 | \bar{p}_p | G_0 \rangle = \rho_0 \delta_{p,0}, \quad (3.1) \]

where \( \rho_0 = \rho^{(1)}_\omega + \rho^{(2)}_\omega \) and \( \delta_{p,0} \equiv (2\pi)^2 \delta^2(p) \). We suppose that this \( S^a_{p=0} = N_c/2 \) eigenstate \( |G_0\rangle \) continues to be a good approximation to the \( \nu = 1 \) ground state as \( V_p^- \) kept weak, is turned on. It has been argued\(^{22}\) and supported experimentally\(^{22}\) that such \( |G_0\rangle \) well approximates the ground state for \( \Delta_{SAS} \to 0 \) with \( V_p^- \neq 0 \), where interlayer coherence \( \langle S^1 \rangle \neq 0 \) is realized spontaneously with \( \langle S^3 \rangle = 0 \) maintained so as to reduce the interlayer charging energy.

Further characterization of this \( S^1_{p=0} = N_c/2 \) state is given by the static structure factors

\[ \langle G_0 | S^a_{p \cdot k} | G_0 \rangle = \delta^{a1} \frac{1}{4} \rho_0 e^{-\frac{2}{2} p^2}; \]
\[ \langle G_0 | \bar{p}_p k | G_0 \rangle = \rho_0^2 \delta_{p,0} \delta_{k,0} \equiv R_{p,k}; \]
\[ \langle G_0 | \bar{p}_p S^a_{p \cdot k} | G_0 \rangle = 2 \langle G_0 | S^a_{p \cdot k} S^1 | G_0 \rangle = \delta^{a1} \frac{1}{2} R_{p,k}, \quad (3.2) \]

where \( b' \) runs over (2, 3). These relations are readily derived by rewriting \( \bar{p}_p \) and \( S^a_{p \cdot k} \) in terms of the eigenspinors \( (\psi_S, \bar{\psi}_A) \) of \( S^1 \), and by noting that \( |G_0\rangle \) involves no \( \psi_A \) component (of \( S^1 = -\frac{1}{2} \)). For the partially-filled \( \bar{\psi} \) Landau level of \( \nu = 1/3, 1/5, \ldots \) one has to retain in \( R_{p,k} \) a term\(^{11}\) \( \rho_0 \delta_{p,k,0} s^+(p) \) with \( s^+(p) \sim O(p^2) \), which vanishes in the present \( \nu = 1 \) case.

The correlations characteristic of interlayer order \( S^1_{p=0} = N_c/2 \) are involved in the structure factor

\[ s^-(p) = \frac{2}{N_c} \langle G_0 | S^3_{-p} S^3_{p} | G_0 \rangle = \frac{1}{2} e^{-\bar{p}^2}; \quad (3.3) \]

which is nonvanishing for \( p \to 0 \), in contrast to the case of the Halperin \((m, n, n) \) states where\(^{17,10}\) \( s^-(p) \sim p^2 \).

Let us now study low-energy excitations over this ground state. With polarization \( \langle G_0 | S^3_e_{p=0} = N_c/2 \), the Coulomb interaction \( H_{\text{Coul}} \) has an (approximate) U(1) symmetry about the \( S^1 \) axis, yielding two Nambu-Goldstone (NG) modes \( \{ \Omega^2_p (t), \Omega^3_p (t) \} \). These NG modes constitute the low-energy collective excitations in the system, and one can employ the technique\(^{20}\) of nonlinear realizations of the pseudospin symmetry for their description. To this end let \( \Psi_0^1(y_0, t) \) denote a classical configuration or the ground-state configuration, characterized by the expectation values in Eqs. (3.1) and (3.2). Let us set \( \Omega[r, t] = \sum_q (\sigma^{12}) \sum_{p} \Omega^p_q (t) e^{ipr} \) (with \( a = 2, 3 \)) and write the electron field \( \Psi_0 \) in the form of a small rotation in pseudospin from \( \Psi_0^1 \).

\[ \Psi_0(y_0, t) = e^{-i\Omega[r, t]} \Psi_0^1(y_0, t). \quad (3.4) \]

Here the NG modes serve as pseudospin textures in which the local pseudospin alignment varies slowly with position. Rewriting the Lagrangian in favor of \( \Psi_0^1 \) and \( \Omega^p_q \), and replacing the products of \( \Psi_0^1 \) and \( \Psi_0^1 \) by the expectation values (3.1) and (3.2) then yields a low-energy effective Lagrangian for the NG modes \( (\Omega^2, \Omega^3) \).

To facilitate such transcription it is convenient to express (3.4) in operator form

\[ \Psi_0(y_0, t) = \mathcal{P} \Psi_0^1(y_0, t) \mathcal{P}^{-1}, \quad \mathcal{P} = e^{i\Omega \cdot \mathcal{S}^1}, \quad \Omega \cdot \mathcal{S}^1 \equiv \sum_p e^{ip^2} \Omega^p \mathcal{S}_{p^1}^a. \quad (3.5) \]

Here \( \mathcal{S}_{p^1}^a \) stand for \( \mathcal{S}_{p}^a \) with \( \Psi_0 \) replaced by \( \Psi_0^1 \), and obey the same algebra (2.16) as \( \mathcal{S}_{p}^a \). Repeated use of the algebra then enables one to express \( S^3 = \mathcal{P} \mathcal{S}_{p^1}^3 \mathcal{P}^{-1} \) and \( p^2 = \mathcal{P} \mathcal{p}^2 \mathcal{P}^{-1} \) in powers of \( \mathcal{S}_{p^1}^3 \) and \( \mathcal{S}_{p^1}^a \). [Remember that the characterization (3.1) and (3.2) from now on applies to \( \mathcal{S}_{p^1}^a \) and \( \mathcal{S}_{p^1}^3 \).] In particular, for their expectation values one obtains to \( O(\Omega^2) \):

\[ \langle \mathcal{S}_{p}^3 \rangle = \frac{\rho_0}{2} \gamma_p \Omega^3_{p^1}; \quad \langle \mathcal{S}_{p}^3 \rangle = -\frac{\rho_0}{2} \gamma_p \Omega^2_{p^1}, \quad (3.6) \]

where \( \langle \cdots \rangle = \langle G_0 | \cdots | G_0 \rangle \) for short and \( \gamma_p = e^{-\bar{p}^2}; \) \( a' \) and \( b' \) run over (2, 3). These expressions suggest us to rename, following Moon et al.\(^{20}\), \( m_2^p = \Omega^3_{p} \) and \( m_3^p = -\Omega^2_{p} \) so that their \( x \)-space representatives \( m_2(x) = \langle m_2(x) \rangle, m_3(x) \) stand for the pseudospin density [with normalization \( \sum_{a=1}^3 (m_a)^2 \approx 1 \) classically]. Actually it is possible to generalize Eq. (3.6) to all powers of \( m_a \), if one ignores their derivatives \( \partial_k m_a \).

\[ \langle \mathcal{S}_{p}^3 \rangle \approx \frac{\rho_0}{2} \gamma_p \Omega^3_{p^1}; \quad \langle \mathcal{S}_{p}^3 \rangle \approx \frac{\rho_0}{2} \gamma_p \Omega^3_{p^1}. \quad (3.7) \]

where \( \mathcal{m} = (m_2(x), m_3(x)); \) \( \langle \cdots \rangle \approx \rho_0, \) etc.,

Moon et al.\(^{20}\) earlier made such a pseudospin-texture calculation and showed that the Coulomb interaction leads to the following low-energy effective Hamiltonian to \( O(\Omega^2) \) and \( O(p^2) \):

\[ \mathcal{H}^C = \sum_p \left\{ \beta[p] \left( (m_2(p))^2 + \frac{1}{2} \rho^E_p \mathcal{p}^2 (m_2(p))^2 \right) \right\}. \quad (3.8) \]

with

\[ \rho_s = \frac{1}{8} \rho_0 \sum_p V_{11}^p \mathcal{p}^2 e^{-\frac{2}{2} p^2} \]
\[ \rho^E_p = \left( 1 - \frac{1}{2} \sqrt{8/\pi} \partial^2 + (3/2) \bar{d}^2 + \cdots \right), \]
\[ \beta[p] = \rho_s \left\{ \left( \rho_0 / \rho_s \right) \left( c_0 + c_1 / \rho_s \right) \rho^E_p + \frac{1}{2} \left( 1 + c_2 \right) \mathcal{p}^2 \right\}, \]
\[ c_0 \approx \bar{d}^2, \quad c_1 \approx -\frac{1}{2} \sqrt{8/\pi} \partial^2, \quad c_2 \approx -\frac{1}{2} (1 - \bar{d}^2) / 3, \quad (3.9) \]

where we have used the pseudospin stiffness \( \rho_s \) as a common factor and recorded some corrections in powers of \( \bar{d} \equiv d / t \); \( \rho^E_p \) is given by the same expression as \( \rho_s \), with \( V_{11}^p \rightarrow V_{12}^p \).
Substituting $\Psi_0 = e^{-i\Omega_0 t + i \mathbf{q}_0 \cdot \mathbf{r}}$ into the electronic kinetic term $\langle \Psi_0 | i \partial_t \Psi_0 \rangle$ yields Berry’s phase,\(^\text{30}\) which turns into the kinetic term of the NG modes

$$L^{\text{kin}} = -\frac{1}{2} \rho_0 \sum_k e^{ab} \Omega_{\alpha k}^a \partial_t \Omega_{\alpha k}^b$$

(3.10)
to $O(\Omega^2)$. This shows that $\Omega^2 = -m_3$ is canonically conjugate to $\Omega^2 = m_2$.

Substitution of Eqs. (3.6) and (3.7) into $\tilde{H}_{\text{em}} + \tilde{H}_{\text{run}}$ yields the coupling of the NG modes to external fields, $(G_0) H_{\text{em}} + \tilde{H}_{\text{run}}(G_0) = \int d^2 x H_A$ with

$$H_A = \rho_0 \left\{ A^0_0 + \frac{1}{2} \chi^\perp \varepsilon^{ij} \partial_j m_2 \partial_i m_3 + \chi^- \gamma m_3 - \frac{1}{2} \Delta_{\text{SAS}} \cos |m| \right\},$$

(3.11)
to $O(m^2)$ and $O(\Omega^2)$, where $m = (m_2, m_3)$, $\gamma = e^4 \nabla^2$ and $\chi^\perp = A^0_0 + (1/2M) A^0_0$.

Similarly, the field-dependent Coulomb interaction $\Delta H^C$ leads to the effective interaction

$$\langle \Delta H^C \rangle = 2 \rho_s \int d^2 x \left\{ m_2 \partial_j A^-_j + (A^-_j)^2 + \cdots \right\};$$

(3.12)
see the Appendix for details.

Collecting terms so far obtained yields the effective action $S^{\text{coll}}_{\text{eff}} = \int dt d^2 x L^{\text{coll}}$ with

$$L^{\text{coll}} = \frac{\rho_0}{2} m_3 \left( \dot{m}_2 - 2 A^0_0 \right) - m_3 \left[ \beta[p] + \frac{1}{4} \rho_0 \Delta_{\text{SAS}} \right] m_3 - \frac{1}{2} \rho_s \left( \partial_j m_2 - 2 A^-_j \right)^2 + \frac{1}{2} \rho_0 \Delta_{\text{SAS}} \cos m_2 - \rho_0 A^0_0 \left( 1 + \frac{1}{2} \varepsilon^{ij} \partial_j m_2 \partial_i m_3 \right),$$

(3.13)
where $p \rightarrow -i \nabla$ in $\beta[p]$. Here we have simplified the result slightly by retaining only terms that contribute to the $O(\nabla^2)$ electromagnetic response eventually. The $L^{\text{coll}}$ is essentially the Lagrangian of a nonlinear sigma model that supports classical topological excitations\(^\text{31,32}\) Skyrmions, which constitute the low-lying charged excitations of the system; see Eq. (5.3) in Sec. V. Note that Eq. (3.13) correctly involves the topological charge density\(^\text{32}\) $(\rho_0/2) \varepsilon^{ij} \partial_j m_2 \partial_i m_3$, which implies that the Skyrmions carry electric charge of a multiple of $qe$.

Let us here focus on the neutral collective excitations described by the field $m_2$ or $m_3$. Eliminating $m_3$ from $L^{\text{coll}}$ yields the Lagrangian of the neutral field $m_2$

$$L^{\text{coll}}_{m_2} = \frac{1}{2} \rho_s \left[ \varepsilon^{ij} \partial_j m_2 - 2 A^-_0 \right]^2 - \frac{1}{2} \rho_0 \Delta_{\text{SAS}} \cos m_2,$$

(3.14)
with

$$v^2 = 2 \left( \rho_s / \rho_0^2 \right) \left( 4 \beta[0] + \rho_0 \Delta_{\text{SAS}} \right),$$

$$2 A^-_\mu = 2 A^-_\mu - \partial_\mu \Gamma \approx d \left( \partial_\mu A^-_\mu - \partial_\mu A_z \right).$$

(3.15)

Here we have indicated explicitly that $A^-_\mu$ so far used actually stands for $A^-_\mu$; we have also isolated the $\rho_0 A^0_0$ term which detects the charge of the ground state (\(G_0\)).

This collective mode $m_2$ gives rise to an electromagnetic response of the form

$$L^{\text{coll}}_{\text{em}} = 2 \rho_s \left( A^0_0 D_0 A^-_0 - v^2 A^-_0 D_0 A^-_0 \right) + \rho_0 \Delta_{\text{SAS}} \left( A^-_0 D_0 A^-_0 - v^2 A^-_0 D_0 A^-_0 \right)$$

(3.16)
in compact notation, where $D = 1/\{\omega^2 - (i \partial_t)^2\}$ and

$$\omega^2_p = \left\{ \Delta_{\text{SAS}} + \frac{4}{\rho_0} \beta[p] \right\} \left\{ \Delta_{\text{SAS}} + \frac{2 \rho_s}{\rho_0} p^2 \right\},$$

(3.17)
with $p \rightarrow -i \nabla$. Here we have recovered $\beta[p]$ to obtain the dispersion more accurately. In terms of the field strengths in three space one can write $L^{\text{coll}}_{\text{em}}$ as

$$L^{\text{coll}}_{\text{em}} \approx \frac{1}{2} \rho_s \left[ \partial_j E_0 - \partial_i E_\perp \right] \left( v^2 \partial_z B_\perp - \partial_\perp B_\perp \right) + \frac{\rho_0}{4} \Delta_{\text{SAS}} \left( E_\perp D E_\perp - v^2 B_\perp D B_\perp \right),$$

(3.18)
in obvious notation.

The response due to the cyclotron modes in Eq. (2.19) is generally suppressed by powers of $1/\omega_c$ compared with the collective-mode contribution, except for the Hall-drift or Chern-Simons term

$$L^{\text{cyc}}_{A^-} = -\frac{\rho_0}{2} A^-_{\mu} \left( \frac{\omega^2}{\omega^2 - \omega^2} e^{i \mu \rho} \partial_{\nu} A^-_{\nu} + \cdots \right),$$

(3.19)
which thus combines with $L^{\text{coll}}_{\text{em}}$ to form the principal out-of-phase response of the system at long wavelengths. Note here that the collective mode gives rise to no such Hall-drift term, unlike for the $(m, m, n)$ states.\(^\text{23}\) This implies that no appreciable interlayer Hall drag is expected for the present $\nu = 1$ state, in contrast to the case\(^\text{33}\) of the (gapful) $(m, m, n)$ states.

Some comments are in order here. First, the effective Lagrangian (3.14) essentially agrees with the one derived earlier\(^\text{20,27}\) if one sets $m_2 = \tilde{m}_2 - \Gamma_z$, where $\tilde{m}_2$ is taken to undergo the gauge transformation $\delta \tilde{m}_2 = 2 \theta^-$. The earlier derivations focused on the spectrum of the low-lying mode and its coupling to weak external electromagnetism was only guessed on the ground of gauge invariance. A direct derivation of such electromagnetic coupling, as shown in our approach, is quite nontrivial since it requires proper account of Landau-level projection, especially the field-dependent Coulomb interaction.

Second, in our approach electromagnetic gauge invariance is kept exact at each step of discussion by use of the gauge-covariant fields $\psi^{(a)}(x)$ and $A^\mu_{(a)}(x)$ in Eq. (2.9). Recall that the pseudospin densities $S^a$ are gauge-invariant while $S^1$ and $S^2$, defined in terms of $\psi^{(a)}$, are gauge-variant so that $S^1 + i S^2 = e^{i \chi^z} (S^1 + i S^2)^2$. Our characterization of interlayer coherence $\langle G_0 | S^a_{(\mathbf{p} = 0)} | G_0 \rangle =$
\[ S_{p=0}^1 = \frac{1}{2} N_e \cos \Gamma_z, \quad S_{p=0}^2 = \frac{1}{2} N_e \sin \Gamma_z, \]  

(3.20)

rotate in the pseudospin 1-2 plane under electromagnetic gauge transformations \( \Gamma_z \to \Gamma_z + 2\theta^- \), or under the action of in-plane magnetic fields \( \partial_j \Gamma_z \). In other words, a naive choice \( \langle S_{p=0}^1 \rangle \propto \theta^1 \) is not physically acceptable unless layer spacing \( d \to 0 \). This is the real reason why we have restarted with \( \psi^{(a)}(x) \) and \( A_{\mu}^+(x) \) after Eq. (2.9).

We have handled two NG modes \( (m_2, m_3) \) associated with \( \text{SU}(2) \to U(1) \) breaking. They, being gauge-invariant, are neutral physical fields. They, however, happen to form a pair of canonical conjugates and thus actually describe only one physical mode \( m_2 \). Note here that, since \( m_2 \sim \Omega^\dagger \), a shift \( m_2 \to m_2 + \text{const} \) induces a rotation about the \( S^0 \) axis so that

\[ \bar{q} [S_{p=0}^3, m_2] = 1 \neq 0. \]  

(3.21)

This shows that \( m_2 \) can also be interpreted as a NG mode associated with the spontaneous breaking of the global \( U(1) \) symmetry about the \( S^3 \) axis. Because this global \( U(1)^- \) is only approximate, \( m_2 \) is a pseudoscalar NG mode and acquires a finite energy gap \( \propto \Delta_{SAS} \). In the absence of tunneling \( (\Delta_{SAS} = 0) \), the response is exact but spontaneously broken; the energy gap closes and \( m_2 \) disperses linearly.

Unlike the global \( U(1)^- \), the gauged \( U(1)^- \) or \( U(1)^\text{em} \) is kept exact, as seen clearly from the gauge-invariant response (3.16). This implies, in particular, that there is no Anderson-Higgs mechanism or no Meissner effect working in the present bilayer system. Here we see a peculiar instance of spontaneous breaking of a global symmetry with the related gauge symmetry kept exact; this derives from the special character of the “interlayer” gauge invariance remarked in Sec. II.

Finally, one can use the effective theory to discuss the tunneling phenomena. The equation of motion of \( m_2 \) implies the conservation law for the three-current

\[ -\partial \mathcal{L}_{m_2}^\text{coll} / \partial A_{\mu}^- = j_{\mu}^{(1)} - j_{\mu}^{(2)}, \]  

from which one can read off the tunneling current \( j_{\mu}^{\text{tun}} \sim -\partial j_{\rho}^{(1)} \) as

\[ j_{z}^{\text{tun}} = \frac{1}{2} e \rho_0 \Delta_{SAS} \sin m_2. \]  

(3.22)

Adding a source term \( a_z j_{\text{tun}}^z \) to \( \mathcal{L}_{m_2}^\text{coll} \) and calculating the response yields the tunneling current

\[ j_{z}^{\text{tun}} = \frac{1}{2} e \rho_0 \Delta_{SAS} \frac{1}{\omega^2 - \omega_p^2} \partial_t V_z, \]  

(3.23)

in response to an alternating interlayer voltage \( V_z = -2 \Delta_0 \approx -d E_\perp \).

IV. RELATION TO THE SINGLE-MODE APPROXIMATION

In this section we present a derivation of the electromagnetic response (3.16) by an alternative means, the single-mode approximation (SMA). Let us first suppose that \( V_p^- = 0 \), in which case the ground state is exactly given by \( S_{p=0}^1 = N_e / 2 \) eigenstate \( |G_0 \rangle \) in Eq. (3.1). We consider the phonon-rotor mode coupled to \( A_0 \) and represent it as \( |\phi_k^- \rangle \sim S_{k}^3 |G_0 \rangle \). The basic quantity in the SMA is the static structure factor \( s^- (k) \sim \langle \phi_k^- | \phi_k^- \rangle \), which, in view of Eq. (3.3), is given by

\[ s^- (k) = (1/2) e^{-\frac{4}{2} k^2}. \]  

(4.1)

To determine the collective-excitation spectrum in the SMA one considers the projected oscillator strength

\[ f^- (k) = (2/N_e) \langle G_0 | S_{k}^3 | H, S_{k}^0 | G_0 \rangle, \]  

(4.2)

which is calculable by use of the algebra (2.16). With \( \bar{H}^\text{tun} = -\Delta_{SAS} S_{p=0}^1 \) included, it is given to \( O(k^2) \) by

\[ f^- (k) = \frac{1}{2} e^{-\frac{4}{2} k^2} \left[ \Delta_{SAS} + 2(\rho_e^E / \rho_0) k^2 + \cdots \right]. \]  

(4.3)

Here the coefficient of \( k^2 \) term derives from the general expression

\[ \frac{1}{2} \sum_p p^2 V_p^{12} \{ s^- (p) - s^+ (p) \} \]  

(4.4)

upon substitution of \( s^- (p) \) above: \( s^+ (p) = 0 \) for \( \nu = 1 \). Saturating \( f^- (k) \) with the single mode \( |\phi_k^- \rangle \) then yields the SMA excitation spectrum \( \epsilon_k^- = f^- (k) / \bar{s}^- (k) \) or

\[ \epsilon_k^- = \Delta_{SAS} + 2(\rho_e^E / \rho_0) k^2 + \cdots. \]  

(4.5)

This agrees with the spectrum derived by the pseudospin-structure calculation in Eq. (3.17) with \( V_p^- \to 0 \).

To calculate the electromagnetic response one may resort to the previous SMA analysis, which, though developed originally for the case of a dipole-active response \( s^- (k) = (e^2 / 2) k^2 + \cdots \), is adapted to the present case as well: One may simply replace \( 2s^- (k) \epsilon_k^- \) in Eq. (3.20) of Ref. 23 by \( 2f^- (k) = \Delta_{SAS} e^{-\frac{4}{2} k^2} + 2(\rho_e^E / \rho_0) k^2 \) and \( c^- \epsilon_0^- \) in Eq. (3.28) of Ref. 23 by Eq. (4.4) or \( 2p_e^E / \rho_0 \). Then our result (3.16) is correctly reproduced, apart from the \( v^2 A_{12} \cdots A_{17} \) and \( v^2 A_{12} \cdots A_{17} \) terms, which, being \( O(\nu^2) \sim O([H^C]^2) \) higher in the Coulomb interaction, were not covered in the previous SMA treatment.

It is possible to include the effect of \( V_p^- \) and make the agreement complete if one appeals to the low-energy effective theory in Eq. (3.13). With the identification \( S_{p=0}^3 = (\rho_0 / 2) \gamma_p (m_3) p \), as implied by Eq. (3.6), one can calculate \( \bar{s}^- (k) \) from the vacuum expectation value \( \langle \gamma_k \rangle^2 (0|m_3) - k (m_3) k |0 \rangle \). The task is thus reduced to
determining the uncertainty $\langle 0 | (m_3)^2 | 0 \rangle$ for a collection of harmonic oscillators described by the Hamiltonian

$$H_{\text{coll}} \approx \sum_k \frac{1}{4} \left[ g^{(12)} |(m_2)_{k}|^2 + g^{(11)} |(m_3)_{k}|^2 \right],$$

(4.6)

where $g^{(12)} = \rho_0 \Delta_{\text{SAS}} + 2 \rho_s E_k^2$ and $g^{(11)} = \rho_0 \Delta_{\text{SAS}} + 4 \beta |k|$. Via rescaling $(m_3)^2 (\rho_0 / 2) / \sqrt{g^{(11)} / g^{(12)}}$ is seen to attain the minimum uncertainty $(\hbar / 2) \int d^2 x$, yielding

$$\bar{s}^-(k) = \frac{1}{2} e^{-i k^2 / 2} \sqrt{g^{(12)} / g^{(11)}}. \quad (4.7)$$

The $\bar{s}^-(k)$ in Eq. (4.3), being already exact to $O(V_p^{-1})$, remains unmodified. The excitation spectrum and the response thereby agree with those in Eq. (3.16). Since $\bar{s}^-(k)$ shows the importance of Landau-level projection, the differences of pseudospin on the ground state due to two competing sources of SU(2) breaking, $V_p^-$ and $\Delta_{\text{SAS}}$. It is seen from $\langle (m_3)^2 \rangle / \langle (m_2)^2 \rangle = g^{(12)} / g^{(11)} \propto (\bar{s}^{-1})(k)$ that $\langle (m_3)^2 \rangle$ gets rapidly squeezed with decreasing $\Delta_{\text{SAS}}$, i.e., in passing from the tunneling regime to the correlation regime (where $\bar{s}^{-1}(k) \propto |k|$ for $\Delta_{\text{SAS}} = 0$). It is an advantage of the pseudospin-texture theory that it accommodates different types of correlations in a single framework.

V. COMPARISON WITH THE CHERN-SIMONS APPROACH

In this section we examine the bilayer system within the Chern-Simons theory. For the $\nu = 1$ quantum Hall state, as naively described by the $(1,1,1)$ state, one introduces a single CS field\textsuperscript{16} to convert the electron fields $\psi^{(a)}(x)$ [of Eq. (2.9)] into the composite-boson fields $\psi_{\text{ch}}^{(a)}$.

Let us set $\psi_{\text{ch}}^{(a)}(x) = \sqrt{\rho^{(a)}(x)} e^{i\eta^{(a)}(x)}$, rewrite the Lagrangian in favor of $\rho^+ = \rho^{(1)}(1)^2$ and $\eta^+ = \eta^{(1)}(1)^2$, and expand it around the mean-field $\rho^+(x) \sim \rho_0$. Then the $(\rho^+, \eta^+)$ sector, coupled to $A_{\mu}^\pm$, is seen to be essentially the same as in the single-layer case. The $(\rho^-, \eta^-)$ sector, on the other hand, is sensitive to the SU(2) breaking interactions $\propto V_p^-$ or $\Delta_{\text{SAS}}$. Integration over $\rho^-$ leads to a low-energy Lagrangian, that takes essentially the same form as $\mathcal{L}_{\text{coll}}$ in Eq. (3.14) with $m_2 \rightarrow \eta^-$, apart from some differences in scale.

The difference is subtle for the $(\delta_0 m_2 - 2 A_\mu^0)^2$ term:

$$\psi^2 / \rho_s E \leftrightarrow 4 V_{p=0}^- + 2 \Delta_{\text{SAS}} / \rho_0. \quad (5.1)$$

These coincide if $V_{p=0}^-$ reads $V_{p=0}^- = (1 / \rho_0) \sum_{p} V_p^- e^{-i \cdot p^2}$; this shows the importance of the Landau-level projection, of which no explicit account is taken in the CS approach. For the $(\delta_0 m_2 - 2 A_\mu^0)^2$ term the discrepancy is

$$\rho_s E \leftrightarrow (\rho_0 / 4 M) = \omega_c / (8 \pi). \quad (5.2)$$

Here we see that the CS approach attributes the pseudospin stiffness improperly to inter-Landau-level processes. Another difficulty is that an important Hall-drift response (3.19) is missing from the CS theory.

All these subtleties derive from the fact that the CS approach, because of the lack of Landau-level projection, fails to distinguish between the cyclotron modes and the collective modes. The flux attachment in the CS approach properly introduces some crucial correlations among electrons, but unfortunately not all of them.

Finally, it will be instructive to refer to the full effective Lagrangian to make it clear what is missing. Let us employ the decomposition\textsuperscript{34} $\psi_{\text{ch}}^{(a)}(x) = \sqrt{\rho(x)} Z^a(x)$ in terms of a $CP^1$ field $Z = (Z^1, Z^2)^T$ with $Z^1 Z = 1$, particularly suited for studying the dynamics of Skyrmions and vortices. Making use of the dual transformation of Lee and Zhang\textsuperscript{2} then enables one to rewrite the Lagrangian in terms of $Z^a$ and a vector field $b_{\mu}$ (representing the cyclotron mode coupled to $A_{\mu}^\pm$):

$$\mathcal{L}_{\text{CS}} = -(A_{\mu}^b + A_{\mu}^b - i Z^1 D_\mu Z) (\delta_0 \delta_{0\mu} + e^{\mu
u\rho} \partial_{\nu} b_{\rho}) \quad + \frac{\pi}{\nu} \left\{ b_{\mu} e^{\mu
u\rho} \partial_{\nu} b_{\rho} + \frac{1}{\omega_c} (b_{\mu} b_{\mu})^2 \right\} \quad - \frac{1}{2} K \left\{ |D_{\mu} Z|^2 + (Z^1 D_{\mu} Z)^2 \right\} \quad + \frac{1}{2} \delta_0 \Delta_{\text{SAS}} Z^1 \sigma_1 Z + \cdots, \quad (5.3)$$

where only the principal terms are shown; $D_{\mu} = \partial_{\mu} + i A_{\mu}^\pm \sigma_3$ and $b_{\mu} = \partial_{\mu} b_{\mu} - \partial_{\mu} b_{\mu}$. The last two terms, constituting a $CP^1$ nonlinear sigma model with a breaking interaction, essentially coincide with our $\mathcal{L}_{\text{coll}}$ in Eq. (3.13) if one replaces the stiffness $K = \rho_0 / M$ in this CS theory by $K = 4 \rho_s$ [in accordance with Eq. (5.2)] and includes some SU(2) breaking terms coming from $V_p^-$. The full effective Lagrangian is obtained by supplying to this modified $\mathcal{L}_{\text{CS}}$ the missing cyclotron-mode contribution with another vector field $b_{\mu}^-$:

$$\mathcal{L}^- = -A_{\mu}^b - e^{\mu
u\rho} \partial_{\nu} b_{\rho} \quad + \frac{\pi}{\nu} \left\{ b_{\mu} e^{\mu
u\rho} \partial_{\nu} b_{\rho} + \frac{1}{\omega_c} (b_{\mu} b_{\mu})^2 \right\} + \cdots. \quad (5.4)$$

VI. SUMMARY AND DISCUSSION

In this paper we have studied the electromagnetic characteristics of bilayer quantum Hall systems in the presence of interlayer coherence and tunneling by means of a pseudospin-texture effective theory and the single-mode approximation (SMA). It will be clear from the analysis that a proper choice of the fields to start with, as well as proper account of Landau-level mixing, is crucial for deriving a long-wavelength effective theory in gauge-invariant form. We have seen from the response that electromagnetic gauge invariance is kept exact, this, in
particular, implying the absence of the Anderson-Higgs mechanism or the Meissner effect in bilayer systems. The response also shows that no appreciable Hall drag is expected for the $\nu = 1$ state, in contrast to the case of the gapful $(m, m, n)$ states. We have further seen that the identification of the low-lying neutral collective mode with a (pseudo) Nambu-Goldstone mode offers a peculiar instance of a spontaneously-broken (approximate) global symmetry with the related gauge symmetry kept intact. Our approach offers a critical look into the Chern-Simons theories, and we have observed that the lack of Landau-level projection is the principal source of subtleties inherent to them.

The idea underlying our approach is to explore the quantum Hall systems via their electromagnetic response, which in some cases is calculable without the details of the microscopic dynamics. An immediate example is the case of single-layer systems where it is generally known that intra-Landau-level collective excitations are dipole-inactive; the leading long-wavelength response of the single-layer systems to $O(k^2)$ therefore is governed by the cyclotron mode alone. The second example is offered by bilayer systems (without interlayer coherence), for which one can construct from the response an effective gauge theory properly realizing the SMA spectrum of collective excitations. The third example is the analysis of the effects of interlayer coherence and tunneling given in the present paper. These would combine to enforce the identification of the low-lying neutral collective mode which in some cases is calculable without the details of the microscopic dynamics. An immediate example is the case of single-layer systems where it is generally known that intra-Landau-level collective excitations are dipole-inactive; the leading long-wavelength response of the single-layer systems to $O(k^2)$ therefore is governed by the cyclotron mode alone. The second example is offered by bilayer systems (without interlayer coherence), for which one can construct from the response an effective gauge theory properly realizing the SMA spectrum of collective excitations. The third example is the analysis of the effects of interlayer coherence and tunneling given in the present paper. These would combine to enforce the identification of the low-lying neutral collective mode which in some cases is calculable without the details of the microscopic dynamics.

ACKNOWLEDGMENTS

The author wishes to thank Z. F. Ezawa for useful discussions. This work is supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education of Japan, Science and Culture (No. 14540261).

APPENDIX A: FIELD-DEPENDENT COULOMB INTERACTION

In this appendix we display some expressions related to the field-dependent Coulomb interaction $\Delta H^C$. The charge densities $\rho_p^{(a)}$ projected to the lowest Landau level read $\rho_p^{(a)} + \Delta \rho_p^{(a)}$ with

$$\Delta \rho_p^{(a)} = \sum_k u_{p,k}^{(a)} \rho_{p-k}^{(a)} + \sum_{q,k} u_{p,q,k}^{(a)} \rho_{p-q-k}^{(a)} + \cdots,$$

$$u_{p,q,k}^{(a)} = i e A^{(a)}_k \langle A^{(a)}_k \rangle_k + \cdots,$$

where we have retained only terms with no derivatives acting on $A^{(a)}_\mu$, the portion relevant to our discussion. They give rise to the field-dependent piece $\Delta H^C$ in the Coulomb interaction. See Ref. 23 for the explicit form of the $O(A)$ contribution, which involves operator products of the form

\[ \tilde{I}^+_p = \{ \tilde{\rho}_p, \tilde{\rho}_{p-k} \}, \quad \tilde{I}^-_p = 2 \{ \tilde{\rho}_p, \tilde{S}^3_{p-k} \}, \]  

and those with $\tilde{\rho} \leftrightarrow 2\tilde{S}^3$ in the above.

In Sec. III we evaluate the expectation value $\langle \Delta H^C \rangle = \langle G_0 | \Delta H^C | G_0 \rangle$ to derive an effective electromagnetic coupling following from $\Delta H^C$. A direct calculation to $O(\Omega)$ shows that $\langle \tilde{I}^-_{k,p} \rangle = -\langle \tilde{I}^-_{k-p,k} \rangle \propto \Omega^3 \delta_{k,0}$ while $\langle \tilde{I}^+_p \rangle \propto \delta_{k,0}$ fail to contribute. As a result, the $O(A)$ coupling is written as

\[ \rho_0 \sum_{p,k} u_{p,k}^{(a)} V^2_{p} \gamma_p \gamma_{k-p} \sin \left( \frac{p \times k}{2} \right) \Omega^3_{k}. \]  

The calculation of the $O(A^2)$ term is somewhat tedious, though straightforward, eventually leading to Eq. (3.12).

1. S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. 62, 82 (1989).
2. D. H. Lee and S.-C. Zhang, Phys. Rev. Lett. 66, 1220 (1991).
3. S. C. Zhang, Int. J. Mod. Phys. B 6, 25 (1992).
4. B. Blok and X. G. Wen, Phys. Rev. B 42, 8133 (1990).
5. A. Lopez and E. Fradkin, Phys. Rev. B 44, 5246 (1991); ibid. 47, 7080 (1993).
6. B. I. Halperin, P. A. Lee and N. Read, Phys. Rev. B 47, 7312 (1993).
7. S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. 58, 1252 (1987); N. Read, Phys. Rev. Lett. 62, 86 (1989); J. K. Jain, Phys. Rev. Lett. 63, 199 (1989).
8. D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).
9. R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
10. A. H. MacDonald and S.-C. Zhang, Phys. Rev. B 49, 17 208 (1994).
11. S. M. Girvin, A. H. MacDonald, P. M. Platzman, Phys. Rev. B 33, 2481 (1986).
12. B. I. Halperin, Helv. Phys. Acta 56, 75 (1983).
13. T. Chakraborty and P. Pietilainen, Phys. Rev. Lett. 59, 2784 (1987); D. Yoshioka, A. H. MacDonald, and S. M. Girvin, Phys. Rev. B 39, 1932 (1989); H. A. Fertig, Phys. Rev. B 40, 1087 (1989).
14. A. H. MacDonald, P. M. Platzman, and G. S. Boebinger, Phys. Rev. Lett. 65, 775 (1990).
15 X. G. Wen and A. Zee, Phys. Rev. Lett. 69, 1811 (1992); Phys. Rev. B 47, 2265 (1993).
16 Z. F. Ezawa and A. Iwazaki, Int. J. Mod. Phys. B 6, 3205 (1992); Phys. Rev. B 47, 7295 (1993).
17 S. R. Renn and B. W. Roberts, Phys. Rev. B 48, 10 926 (1993).
18 K. Yang, K. Moon, L. Zheng, A. H. MacDonald, S. M. Girvin, D. Yoshioka, and S.-C. Zhang, Phys. Rev. Lett. 72, 732 (1994).
19 A. Lopez and E. Fradkin, Phys. Rev. B 51, 4347 (1995).
20 K. Moon, H. Mori, K. Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka, and S.-C. Zhang, Phys. Rev. B 51, 5138 (1995).
21 Y. W. Suen, L. W. Engel, M. B. Santos, M. Shayegan, and D. C. Tsui, Phys. Rev. Lett. 68, 1379 (1992); J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, K. W. West, and S. He, Phys. Rev. Lett. 68, 1383 (1992).
22 S. Q. Murphy, J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, K. W. West, Phys. Rev. Lett. 72, 728 (1994).
23 K. Shizuya, Phys. Rev. B 65, 205324 (2002).
24 E. Fradkin and F. A. Schaposnik, Phys. Lett. B 338, 253 (1995); F. A. Schaposnik, Phys. Lett. B 356, 39 (1995).
25 I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 84, 5808 (2000); ibid. 87, 036803 (2001).
26 L. Balents and L. Radzihovsky, Phys. Rev. Lett. 86, 1825 (2001); A. Stern, S. M. Girvin, A. H. MacDonald, N. Ma, Phys. Rev. Lett. 86, 1829 (2001); M. M. Fogler and F. Wilczek, Phys. Rev. Lett. 86, 1833 (2001).
27 Z. F. Ezawa, Phys. Rev. B 51, 11 152 (1995); Phys. Rev. B 55, 7771 (1996).
28 K. Shizuya, Phys. Rev. B 63, 245301 (2001); ibid. 45, 11143 (1992).
29 W. Kohn, Phys. Rev. 123, 1242 (1961).
30 M. V. Berry, Proc. R. Soc. Lond. A392, 45 (1984) .
31 D.-H. Lee and C. L. Kane, Phys. Rev. Lett. 64, 1313 (1990).
32 S. L. Sondhi, A. Karlhede, S. A. Kivelson and E. H. Rezayi, Phys. Rev. B 47, 16 419 (1993).
33 S. R. Renn, Phys. Rev. Lett. 68, 658 (1992).
34 T. Nakajima and H. Aoki, Phys. Rev. B 56, 15549 (1997).