An Optical Cavity Based on Valley Photonic Topological Insulator

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We demonstrate optical energy localization of a topologically protected unidirectional mode inside a cavity. The structure is reciprocal and requires no external magnetic field. The energy is localized in a subwavelength zero-dimensional mode. We use the coupled mode theory to analyze the interaction between the topological waveguide mode and the cavity mode. The analytic model matches perfectly with the results from first-principles simulations. Our design offers a new way of building resonant cavities by exploiting the unidirectionality of topological nontrivial photonic modes.

Photonic topological insulators (PTIs) have been an increasingly promising field for the past decade, because they share the fundamentally interesting properties with their condensed matter counterparts and have potential value in optical communication applications. One of the key features of PTIs is that they support robust unidirectional light propagations which are immune to the defects that preserve the corresponding topological invariant; whereas, in conventional waveguides, back-reflection is usually responsible for undesired loss. This advantage of PTIs naturally motivates the design of a topologically nontrivial waveguide-cavity system — the resonance time of light inside cavity can be largely increased, because the topological waveguide suppresses back-propagation from the cavity. Approaching such long time optical storage is theoretically and technologically interesting.

Recent proposal demonstrates that a heterostructure of a gyroelectric semiconductor and a dielectric slab with metallic covering can significantly localize energy. The two components of the heterostructure can be viewed as two media with different Chern numbers and the interface between two valley photonic crystals (VPCs) associated with opposite valley-Chern numbers. (Fig. 1 (a)) Because the structure has mirror symmetry about the \( z = \frac{h}{2} \) plane, the electric field profiles are either even or odd with respect to reflections about that mirror symmetric plane. We refer to the even and odd modes as TE- and TM- polarized, respectively.

Because the TE and TM modes are independent and for clearance, we focus on TE-polarization. The waveguide is the interface between two valley photonic crystals (VPCs) associated with opposite valley-Chern numbers. (Fig. 1 (a)) Because the structure has mirror symmetry about the \( z = \frac{h}{2} \) plane, the electric field profiles are either even or odd with respect to reflections about that mirror symmetric plane. We refer to the even and odd modes as TE- and TM- polarized, respectively. Because the TE and TM modes are independent and for clearance, we focus on TE-polarization infra. For TM-polarization, the same phenomenon of interest also exists. (see Supplemental Material)

We demonstrate optical energy localization at a cavity based on a photonic crystal emulating the quantum valley Hall effect. The cavity is at the junction between a waveguide that supports topologically protected chiral kink states (CKSs) and a zigzag termination formed by perfect electric conductor (PEC). The waveguide is the interface between two valley photonic crystals (VPCs) associated with opposite valley-Chern numbers. (Fig. 1 (a)) Because the structure has mirror symmetry about the \( z = \frac{h}{2} \) plane, the electric field profiles are either even or odd with respect to reflections about that mirror symmetric plane. We refer to the even and odd modes as TE- and TM- polarized, respectively. Because the TE and TM modes are independent and for clearance, we focus on TE-polarization infra. For TM-polarization, the same phenomenon of interest also exists. (see Supplemental Material)

As a valley-protected CKS propagates along the topologically nontrivial waveguide and encounters the PEC, it may leak out of the structure through two types of surface modes, related to downwards and upwards propagation. (Fig. 2) However, the dispersion relations of the surface states along the PEC do not span the entire spectrum of the valley CKS — there exists a no-surface-mode (NSM)
The dynamics of a one-port-two-cavity system (Fig. 4) can be described by,

\[ \frac{dA}{dt} = -\left(i\omega_0 + \gamma\right)A + K_a s_a^+ + K_b s_b^+ \]
\[ \frac{dB}{dt} = -\left(i\omega_0 + \gamma\right)B + K_b s_b^+ \]

The subscript represents cavity a or b. A and B are the resonance amplitudes inside the two cavities. The two cavities are both single-mode cavities within the narrow NSM band gap, so A and B are both scalar quantities. \( K_{a,b} \) are the coupling coefficients from the incoming waves, to cavity a or b. \( s_a^+ \) and \( s_b^+ \) are the incoming waves to cavity a and b, respectively. Since the system has only one port, \( s_a^+ \) and \( s_b^+ \) are both scalars. Then \( K_{a,b} \) are both scalars too. In addition, the two cavities share the same eigen-frequency \( \omega_0 \) and the same decay rate \( \gamma \), because they are mirror images of each other. \( s_a^- \) and \( s_b^- \) are the outgoing waves from cavity a and b. The outgoing wave from cavity b is the incoming wave to cavity a, plus a phase delay due to the length of the waveguide: \( s_a \) and \( s_b \) are linked by \( s_a^+ = e^{-i\alpha} s_b^- \) and \( s_b^+ = e^{i\alpha} s_a^- \), where

\[ \alpha \]
\[ \alpha = -2\pi \frac{L}{\lambda} \text{mod} \lambda \] The negative sign is due to negative group velocity. \( D_{a,b} \) are coupling coefficients and \( C_{a,b} \) are the scattering coefficients. \( D \) and \( C \) are both scalars. Due to the lossless and time-reversal symmetric nature of this one-port-two-cavity system, the coupling coefficients and scattering coefficient satisfy \( D^* D = 2\gamma \), \( K = D \), and \( CD^* = -D \). \[ |\omega_1 - \omega_2|, |\omega_1 + \omega_2| \] are plotted, where \( \frac{\omega_0 + \omega_2}{2} = 0.7507 \) at the center of the NSM band gap and \( \frac{\omega_0 - \omega_2}{2} = 0.0132 \) spanning the NSM band gap width.

We solve for the eigen-solution to this dynamical system. Bounded by the fact that the two cavities are mirror images of each other, the eigen-modes can only be either symmetric or anti-symmetric (see Supplemental Material), with frequencies \( \omega_{sy} \) and \( \omega_{an} \), respectively. \( \omega_{sy} \) and \( \omega_{an} \) satisfy,

\[ \begin{align*}
\omega_{sy} &= \omega_0 - \gamma \tan \frac{\alpha}{2} \\
\omega_{an} &= \omega_0 + \gamma \cot \frac{\alpha}{2}
\end{align*} \]

We are only interested in the solutions that satisfy \( \omega_{sy,an} \approx \omega_0 \). Specifically, both \( \omega_{sy} \) and \( \omega_{an} \) must be within the NSM band gap where the SMs at the QVH/PEC surface is completely forbidden. When the solution of \( \omega_{sy,an} \) is deviated out of the NSM band gap, the model is no longer valid because, first, if a SM exists, the CKS would couple to that SM and leave the structure along the PEC surface. Then the system is no longer lossless. The loss through SM can be considered as an intrinsic loss that drains energy from the cavity and out of the system. Furthermore, if the solution of \( \omega_{sy,an} \) is at a frequency where no CKS exists (\( \frac{\omega_{sy,an}}{2\pi c} \leq 0.7185 \) or \( \omega_{sy,an} \geq 0.7714 \)), then the interface between two VPCs no longer functions as a topologically nontrivial waveguide. Overall, since the valley-dependent cavities operate only in the NSM band gap, the frequencies in the following discussions are set to be in that band gap.

The key feature of the symmetric and anti-symmetric modes is that the energy distribution between the cavity and the CKS would couple to that SM and leave the structure. Then the system is no longer functions as a topologically nontrivial waveguide. Overall, since the valley-dependent cavities operate only in the NSM band gap, the frequencies in the following discussions are set to be in that band gap.

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izontal direction from 6 to 21, by 1 at each time. For each configuration, we record the frequency of the symmetric and anti-symmetric eigen-modes, $\omega_{sy,an}$, and calculate the delay $\gamma$ by fitting data into Eqn. 3 (Fig. 7).

With zigzag termination, $\gamma_{0}/c = 5.634 \times 10^{-3}$ while, with armchair termination, $\gamma_{0}/c = 5.257 \times 10^{-2}$. The numerical results further confirms our previous qualitative understanding that stronger energy localization at the two cavities results from a smaller decay rate $\gamma$.

In conclusion, we have designed an optical cavity that utilizes the topologically protected unidirectional transmission property of the valley Hall PTI to achieve energy localization. The device is reciprocal and requires no external magnetic field. We present a concise coupled mode theory model to unfold the coupling of the topological CKS into the cavity, and the model fits precisely with numerical simulations.

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