First simulations of axion minicluster halos

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We study the gravitational collapse of axion dark matter fluctuations in the post-inflationary scenario, so-called axion miniclusters, with N-body simulations. Largely confirming theoretical expectations, overdensities begin to collapse in the radiation-dominated epoch and form an early distribution of miniclusters with masses up to \(10^{-12} M_\odot\). After matter-radiation equality, ongoing mergers give rise to a steep power-law distribution of minicluster halo masses. The density profiles of well-resolved halos are NFW-like to good approximation. The fraction of axion DM in these bound structures is \(\sim 0.75\) at redshift \(z = 100\).

The QCD axion is a hypothetical particle predicted in the Peccei-Quinn (PQ) mechanism for solving the strong CP problem, and is considered one of the best motivated dark matter candidates \([1,8]\). In the so-called post-inflation scenario, the axion field takes initial conditions after a phase transition happening after cosmic inflation, and its resulting DM density distribution has large fluctuations on sub-parsec comoving scales. Their gravitational collapse results in the formation of so-called axion miniclusters (MCs) with characteristic masses and radii of order \(M_{\text{MC}} \sim 10^{-12} M_\odot\) and \(R_{\text{MC}} \sim 10^{12} \text{cm}\) \([9-12]\), a range\(^1\) in which they could be detected in femto-, pico-\(^12\) and microlensing surveys \([13]\). Moreover, the clumping of dark matter axions in bound objects has a direct implication in the direct detection at terrestrial experiments \([15,16]\) and could have an impact in indirect detection \([19,20]\), see also \([21]\). Thus, quantitative predictions for the distribution of axions and the properties of MCs in this scenario are important.

The evolution of axion dark matter can roughly be split into three separate stages. The first encompasses the evolution of the axion field from PQ symmetry breaking until after the QCD phase transition when the axion mass has reached its low-temperature value, but well before the onset of gravitational instability. It is governed by the formation and decay of topological defects and nonlinear field dynamics. This early-universe epoch has recently been investigated with special focus on MC formation by means of large lattice simulations \([22,23]\). During the second stage, gravity takes over as the dominant force while scalar field gradients can be neglected on the scales of density perturbations, allowing their description with N-body methods for collisionless fluids \([24]\). Semi-analytic tools for structure formation can be employed to predict the properties of minicluster halos (MCHs) such as the minicluster halo mass function (MC-HMF) \([15]\). Finally, MCHs evolve into large-scale dark matter halos and become the sites of galaxy formation in the third epoch. Tidal disruption of MCHs and the formation of axion streams are of particular importance during this final stage in order to predict the statistics of axion clumping at the present time \([16,25]\).

This Letter reports the first results from large N-body simulations addressing the second stage of this process, the formation of axion MCHs by gravitational collapse of primordial axion density perturbations. In particular, we discuss the evolution of the MC-HMF, the fraction of axions bound into MCHs and the MCH density profile. More detailed statistics will be presented in a follow-up publication.

Simulations of axion density perturbations. – We start from initial conditions produced by early-universe simulations using the methods described in Ref. \([22]\). The frozen density distribution resulting from the evolution of the axion field at redshift \(z \simeq 10^6\) was converted to 1024\(^3\) particles in a box with comoving side length \(L = 0.864\text{ pc}\) and periodic boundary conditions. The length corresponds to \(24L_1\) where \(L_1 = 2(1 + z(t_1))t_1\) is the comoving coherence length of the axion field at the time \(t_1\) when its mass starts to dominate its dynamics \((m_A(t_1)t_1 = 1/2\), see Appendix A\). For simplicity, we assume that axions account for the total amount of DM.

We follow the gravitational evolution of the system with the GADGET-3 code to a final redshift determined by the time when perturbations on the scale of the computational volume become nonlinear (see Appendices A\(^3\) and B\(^3\) for details). A visualization of the full simulation box at the final redshift, \(z_f = 99\), is shown in Fig. 1. A zoom-in of the largest halo reveals its rich substructure.

MCHs are defined as clusters of gravitationally bound particles in close analogy with dark matter halos in simulations of structure formation. We identify and characterize them by their virial masses and radii using the Subfind halo finder \([26]\). At \(z_f = 99\), the masses and

\(^1\) Note that these estimates depend strongly on the cosmological assumptions before big bang nucleosynthesis \([13,14]\).
radii span the ranges $2.5 \times 10^{-16} \sim 3.0 \times 10^{-9} M_\odot$ and 0.4 $\sim 92.0$ AU, respectively.

**Minicluster halo mass function.**– The minicluster halo mass function (MC-HMF) is the comoving number density of gravitationally bound MCHs per logarithmic mass interval as a function of MCH mass. It provides a quantitative picture of the dynamics of MCH formation.

The MC-HMFs computed from our simulation for different redshifts are shown in Fig. 2. At early times ($z \gg z_{eq}$, left panel), the MC-HMF grows quickly. It is dominated at first by halos near the low-mass resolution cutoff $\sim 10^{-15} M_\odot$ and develops a pronounced peak at $M_{mc} \sim 10^{-13} M_\odot$ by $z \sim 4 \times 10^4$. This rapid growth can be understood as the collapse of the density fluctuations that are deeply non-linear at high-$z$. Thus, we can identify the peak as due to the largest non-linear fluctuations, which should be the “canonical” MCHs. The abundance of low-mass MCHs is the result of the small density seeds found in $[22]$ when simulating axions with initial overdensities $[22]$.

During the post-equality evolution ($z \ll z_{eq}$, right panel in Fig. 2), the high-mass cutoff continues to grow at the expense of the total amplitude, which smoothly declines in time. Fitting the MC-MHF to a power-law times a high-mass cutoff still prefers the same overall slope $\alpha \simeq -0.7$. However, the fluctuations that collapse after $z_{eq}$ are already small (linear) and the semi-analytic Press-Schechter method predicts a MC-MHF $dn/d\log M \propto M^{-1/2}$ $[13]$, which is also compatible with the high-mass data. Indeed, a double-power law fit with cutoff provides a better fit to the MC-MHF in this regime. More statistics is needed to quantify it, which we leave for future work.

The late evolution is dominated by mergers with slowly diminishing accretion of unbound axions onto existing MCHs. This is confirmed by the slow saturation of the total fraction of bound axions (upper panel of Fig. 3) reaching $f_b \sim 0.75$ at $z_f = 99$, and the evolution of the total number of MCHs (lower panel of Fig. 3). Considering MCHs with masses above the low-mass resolution cutoff at $\sim 10^{-15} M_\odot$, we see that after their formation at $z \simeq 7 \times 10^6$ their number grows until $z \simeq z_{eq}$. Afterwards, their number is reduced as a result of ongoing mergers. By distinguishing between $N_{sub,tot}$ above certain mass scales we observe at which redshift MCHs with increasing masses emerge. Evidently, MCHs with masses up to $10^{-11} M_\odot$ begin to form before matter-radiation equality while higher-mass MCHs arise only for $z < z_{eq}$.

In order to characterize the distribution of sub-MCs within the MCHs, we compare the substructure of ten high-mass with ten medium-mass MCHs (mass samples are defined in Table I) in Fig. 4. For this, we identified all sub-MCs within the virial radius of each MCH and normalized the sub-MC masses to the virial mass of the corresponding parent MCH. Figure 4 shows the relative number of sub-MCs, i.e. the number of sub-MCs divided by the total number $N_{sub,tot}$ of sub-MCs contained within the parent MCH. For both subsets, the slopes of the averaged sub-MC-HMFs are similar to that of the MC-HMF, $\alpha \simeq -0.7$. The independence of the slopes from the parent MCH mass agrees with previous results for subhalo mass functions in CDM simulations $[27, 28]$.

**Density profiles.**– We study the angular-averaged density profiles $\rho(r)$ of MCHs in the last snapshot of our simulation, $z_f = 99$, for which we separated them into three mass samples (cf. Table I). The stacked density profiles of 20 MCHs in each sample, truncated at the numerical
softening length, are plotted in Fig. 5 (upper panel) together with their best-fit NFW parameterizations given by \[ \rho_{\text{NFW}}(r) = \frac{\rho_0}{r/r_s(1 + r/r_s)^2}, \] where \( \rho_0 \) is the characteristic density of the halo and \( r_s \) the scale radius. As seen in the lower panel of Fig. 5, high-mass MCHs are in good agreement with NFW-profiles across the entire radial range and medium- and low-mass MCHs are slightly underdense only at large radii \( r \sim r_{\text{vir}}/2 \).

The resulting concentration parameter, \( c = r_{\text{vir}}/r_s \), is of the order of several \( 10^2 \) (cf. Table I) and increases for decreasing MCH masses, in agreement with CDM N-body simulations [30]. In order to examine the stability of the fits, their radial range was reduced by 5% which varies the concentration parameter of the high- and medium-mass sample by a few percent. The increased sensitivity for the low-mass sample is related to the fact that the scale radius is only well-resolved for the high- and the
medium-mass MCHs, as is also evident from Fig. 5. The MCHs from the low-mass sample together with MCHs of masses down to \(10^{-13} M_\odot\), which make up 9% of the total number of MCHs above the low-mass resolution cutoff, have a density profile consistent with the outer \(r^{-3}\)-slope of NFW profiles. A verification of its convergence to Eq. (1) would require higher mass resolution and will be addressed in a follow-up publication. Nevertheless, we conclude that the density profiles at \(z = 99\) do not match a \(\rho \sim r^{-9/4}\) power-law predicted for spherical accretion from a homogeneous background [31]. Instead, our results are consistent with high-resolution simulations of ultracompact minihalos producing NFW density profiles for even mild deviations from spherical symmetry [32].

Studying the MCH density profiles at earlier times we observe that they slowly converge to NFW profiles. The detailed evolution as well as questions concerning possible differences between MCHs and MCs formed from mergers or monolithic collapse, are left to future work.

Discussion. We have studied the formation of axion miniclusters (MCs) and their clustering into minicluster halos (MCHs) from post-inflationary symmetry breaking initial conditions. Our results are based on the highest resolution simulations performed to date, both for the initial conditions and their gravitational evolution. The main conclusions are a nearly scale-invariant MC-HMF with slope \(\alpha \approx -0.7\), density profiles that converge toward an NFW-shape for \(z \ll z_{eq}\) at least for sufficiently massive MCHs with concentration parameters of order several \(10^2\), and a final bound fraction of \(f_b \sim 0.75\).

Of these, the bound fraction is the least robust prediction for axion DM at \(z = 0\). Improving it will require a better understanding of tidal interactions with stars in the Milky Way. For example, current estimates for tidal disruption by stellar encounters scale with the mean MC density [10] [25] which is an ambiguous concept for MCHs with NFW-like density profiles. We hope that our results provide a starting point for better models, as the problem is probably intractable for full simulations.

More work is also needed to explore the morphology of MCHs, including their mass-dependent substructure and evolution of density profiles as a function of redshift. In particular, it is plausible that features of the “original” MCs that clustered into MCHs remain distinguishable even at late times.

Finally, let us consider the predicted population of axion stars in the context of MCHs. Recent studies have shown that the formation of axion stars in the cores of MCHs is a firm prediction [33] [34]. Following the relation between the mass of the axion star and the host MCH found in simulations [31] [50], \(M_* \sim M_{MCH}^{1/3}\), we can expect axion stars with masses ranging from \(10^{-17} - 10^{-15} M_\odot\). Although the mass ranges of the axion stars and the smallest identified MCHs overlap, we note that our simulations are not capable of resolving them. This is because the value of the de Broglie wavelength \(\lambda_{IB} = (mv)^{-1}\) which determines the scale of the axion star radius does not exceed the numerical softening length across the entire mass range of the MCHs.

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| TABLE I. Selected mass samples of MCHs, their respective concentration parameter from an NFW-fit and its sensitivity to the radial fit range (details in the text). |
|---------------------------------|-----------------|-----------------|-----------------|
| \(M_{MCH} [10^{-11} M_\odot]\)  | \(r_{vir} [\text{AU}]\) | \(c\) sensitivity |
| high-mass                      | 26 – 300        | 40.8 – 92.0     | 160             | 3%               |
| medium-mass                    | 3.4 – 4.6       | 20.7 – 22.8     | 400             | 6%               |
| low-mass                       | ~ 0.8           | ~ 12.7          | 450             | 11%              |

FIG. 5. Top: averaged radial density profiles of 20 miniclusters in each mass bin (dark solid lines) truncated at the softening length. The light solid lines represent NFW-fits, where vertical lines mark the corresponding scale radii. Bottom: deviations from the fit shown in the upper panel.
Appendix A: Early Universe simulation

Recent numerical simulations \cite{22, 23} have described the evolution of the axion field from the very early Universe through the period where it becomes non-relativistic and starts behaving as a coherent-state of very cold particles (temperatures $T_1 \sim \text{GeV}$ in a radiation dominated pre-BBN Universe). Simulations have included for the first time the effects of cosmic strings and domain walls. Albeit the string tensions available are very far from physical, the current direct results appear largely insensitive to these parameters. This can very well be a result of the small tensions themselves \cite{37}, but at least, an effective model reaching physically relevant high tensions \cite{38} shows very little dependence \cite{39}. Further work to understand the dynamics of high-tension strings and axion DM is required to clarify these issues.

Because of the fast increase of the axion mass with decreasing $T$ the DM distribution at the large scales of interest freezes rapidly below $T_1$. The resulting axion DM density distribution has density fluctuations with a standard deviation $\delta \rho_A/\rho_A \simeq 0.45$ at distance $\sigma = L_1$, where $L_1$ is set by the horizon size at $T_1$ \cite{22}.

$$L_1 = \frac{1}{a_1 H_1} = 0.0362 \left( \frac{50 \mu \text{eV}}{m_A} \right)^{0.167} \text{pc}.$$  \hspace{1cm} (A1)

As $\sigma$ grows encompassing more correlation lengths, the white noise fluctuations in the number of $\sim L_1^3$ make the fluctuations decrease as $\propto \sigma^{-3/2}$. At small scales, fluctuations grow until they saturate around $\delta \rho/\bar{\rho} \sim \sqrt{3}$ at the smallest scales, see Fig. 21 of \cite{22}. Note that when our field-simulations end, pseudo-breathers called axitons would continue to evolve to increasingly smaller objects but they are expected to diffuse away to a considerable extent after the axion mass saturates and are thus irrelevant for the $\sim L_1$ scales of interest here \cite{22}.

The size of our simulation will constrain the minimum redshift at which we can trust our gravitational evolution and the maximum “typical” mass of our MCHs. This is because periodic boundary conditions start to play a significant role when fluctuations of the order of the box size become nonlinear and respond to their “periodic” copies. In order to be able to reach small redshifts we require large boxes. Using a large box also has the advantage to increase the statistics of typical mass MCHs. In order to evolve to $z_f \simeq 99$, we produced early-universe axion field simulations with $L = 24 L_1$ in 8192\(^3\) grids using the techniques of \cite{22}. Simulating larger boxes compromises the resolution of the string cores or the requirements of sufficient tension to avoid nonphysical destruction of domain-walls by string creation.

At the end of the simulation the axion field is evolved with the linearised equations (using the WKB approximation) until the redshifts of interest $z \sim 10^6$, similarly to \cite{23}. This process accounts for the free-streaming of axions until the redshift of interest but is only relevant for the highest-momentum axions. Therefore only the smallest scales of our 8192\(^3\) grid are softened.

The linear growth of gravitational perturbations of a scalar field is generally hampered by the “quantum” or gradient pressure at length scales smaller than the comoving axion Jeans wavelength,

$$\lambda_J = \frac{2\pi}{(16\pi G \rho_a(t_0)/(1+z))^{1/4} m_a^{1/2}},$$

$$\sim \left( \frac{m_a}{10^{-5} \text{eV}} \right)^{-1/2} \left( \frac{1+z}{10^4 \Omega_a h^2} \right)^{1/4} \text{mpc},$$

which is smaller than our resolution at all times and only comparable at the initial time $z_i \simeq 10^6$ deep in the radiation domination epoch where gravity is still frozen for our moderate perturbations.

In order to sample our density field into particles, we first smooth the density into a 1024\(^3\) grid of $\sim$ mpc grid-spacing. We then compute the density normalised to the average $n_i = \rho_i/\bar{\rho}$. The sum of the normalised density is by definition $\sum_i n_i = 1024^3$ so we create a number of particles equal to $floor(n_i)$ and distribute them around the grid point $i$ with coordinates displaced by a Gaussian probability distribution with standard deviation equal to half the grid spacing $L/2048$. We decide whether to put a last particle or not by sampling a binomial distribution with probability $n_i - floor(n_i)$. This procedure does not produce exactly a set of 1024\(^3\) particles – in this case it felt short by 28783 – but the sampling is adequate for our purposes. We checked that the resulting dimensionless power spectrum $\Delta_2$ coincides with the original grid up to momenta $k \sim 1500$ pc\(^{-1}\), above which white-noise from discretisation kicks in. Note that at these scales $\Delta_2$ is already decreasing. The velocities of the particles are set to zero, which is compatible with the last smoothing procedure.

Appendix B: Power spectrum and Linear evolution

The initial dimensionless power spectrum of density fluctuations is a white-noise power law $\Delta_2 \propto k^3$ at large scales $k \ll k_1 = 1/L_1$ saturating around $1/L_1$ and slowly decreasing, cf. Fig. 5 (down). The large dynamical range of 1024\(^3\) particles allows our simulation to start probing the decrease of $\Delta_2$ at small scales. At large scales the variance of the fluctuations within a Gaussian window-function follows the power law $\langle \delta_2^2 \rangle = 0.019(5) (L_1/\sigma)^3$ as a function of the Gaussian width $\sigma$. Fourier modes of the density field corresponding to large scales are small and evolve linearly through our simulation, as

$$\delta_k \propto 1 + \frac{31 + z_{eq}}{2} \frac{z}{1+z},$$

where we have assumed $\partial_t \delta_k = 0$ at our initial time (which sets the well-known log a growth of matter perturbations to zero during radiation domination). The

\footnote{Here $\sigma$ is the width of the Gaussian window function.}
largest scales in our simulation \( \sigma \sim L/2 \) would then become non-linear at a redshift where \( \langle \delta^2(z_\sigma) \rangle \sim 0.1 \) which gives \( z_\sigma \sim 60 \) for \( \sigma = 12L_1 \). When these modes become non-linear, the box starts effectively reacting to the gravitational potential of the periodic “copies” of our box outside it and our simulations can no longer be trusted.

Power spectra of mass density fluctuations in physical and in dimensionless units are shown in Fig. 6 for different redshifts. Comparing the power spectra at the beginning of the simulations and at \( z = 9999 \), we observe enhanced growth of high-\( k \) modes. As can be seen from the dimensionless power spectrum, the scales of the length of the box start to become nonlinear at \( z = 99 \). Hence, our simulations are reliable until this point.

### Appendix C: Comparison to adiabatic perturbations

The size of the axion isocurvature fluctuations \( \Delta^2(k) = 0.03(1)(kL_1)^3 \) becomes comparable to the scale-invariant adiabatic density fluctuations assumed from inflation, \( \Delta^2 \sim 2 \times 10^{-9} \), at a wavenumber \( k \sim 0.005 \text{ pc}^{-1} \). We would need to simulate boxes \( \sim 200 \) times larger to be sensitive to those scales. The isocurvature fluctuations that we simulate here correspond to sizes and densities of minicluster seeds. The adiabatic fluctuations originate from the temperature fluctuations, which shift “locally” the time \( t_1 \). At large scales, they correspond to a very small overall up or downwards shift in the axion content of each minicluster seed in our simulation.

### Appendix D: N-body simulations and Halo finder

We used the OpenMP/MPI optimized developer version of GADGET-3 which is a successor of GADGET-2 [10]. The simulations were performed with \( 1024^3 \) particles having a mass of \( 2.454 \times 10^{-17} M_\odot \). The numerical softening length, which sets the limit of the spatial resolution, was adjusted to be \( 1 \text{ AU}/h \), slightly below the Power criterion [11] and even the revisited lower value in [42] by a factor of \( \sim 4 \).

We chose a comoving box side length of \( L = 0.864 \) corresponding to \( 24L_1 \) with an axion mass of \( m_A = 50 \mu eV \), with a total of 3 massless neutrino species. Although masses within current cosmological bounds will not make a difference at these scales, we explicitly take into account the calculation of the Hubble parameter,

\[
H^2(z) = H_0^2 \left( \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0} \right).
\]

We used the standard \( \Lambda \text{CDM} \) parameters \( \Omega_{m,0} = 0.3, \Omega_{r,0} = 8.486 \times 10^{-5}, \Omega_{\Lambda,0} = 0.7 \) and \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \) with \( h = 0.7 \). We do not include baryons. At our very small scales they are tightly coupled to photons and their density fluctuations would be irrelevant.

MCHs and sub-MCs were identified by deploying the SUBFIND algorithm [26, 28]. SUBFIND starts with a halo list identified through the Friends-of-Friends algorithm,
applying a linking length of $l = 0.16$ and considering halos with at least 32 particles. In order to estimate the local density at each particle belonging to an identified halo, it is adopted an adaptive kernel estimation based on all particles using 50 neighbors. Starting from isolated density peaks, additional particles are added in sequence to disjoint the sub-halo candidates. All of them then undergo an iterative unbinding procedure with a tree-based calculation of the potential, where the Hubble flow is taken into account. Finally, only sub-halos with at least 20 bound particles are considered. To calculate the properties of the halos their center is set to the position of the lowest potential. Virial quantities of the halos are then computed as spherical averages using again all particles. Specifically, we used the virial parameter

$$\Delta_v = (18\pi^2 + 82x - 39x^2), \quad (D2)$$

where $x = \Omega_m(z) - 1$ and

$$\Omega_m(z) = \frac{\Omega_{m,0}(1+z)^3}{\Omega_{m,0}(1+z)^3 + \Omega_r,0(1+z)^4 + \Omega_{\Lambda,0}}. \quad (D3)$$

In our range of interest $z \in (10^6 - 10^2)$, we find $\Delta_v = \Delta_{v,0} \sim (50, 180)$.

The virial radius is defined as the radius for which the average density of the MCH matches the virial parameter times the critical density,

$$\frac{\int_0^{r_{vir}} 4\pi r^2 \rho(r) dr}{4\pi r_{vir}^3} = \Delta_{v,0} \rho_c. \quad (D4)$$

where $\rho_c = 3H^2/(8\pi G)$ with $H$ the Hubble expansion rate. The virial mass is the mass contained within that radius, i.e. $M_{vir} = 4\pi/3 \Delta_{v,0} \rho_c^2 r_{vir}^3$. 

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