OFDM versus Single Carrier: A Realistic Multi-Antenna Comparison

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There is an ongoing discussion in the broadband wireless world about the respective benefits of orthogonal frequency division multiplexing (OFDM) and single carrier with frequency domain equalization (SC-FD). SC-FD allows for more relaxed front-end requirements, of which the power amplifier efficiency is very important for battery-driven terminals. OFDM, on the other hand, can yield improved BER performance at low complexity. Both schemes have extensions to multiple antennas to enhance the spectral efficiency and/or the link reliability. Moreover, both schemes have nonlinear versions using decision feedback equalization (DFE) to further improve performance of the linear equalizers. In this paper, we compare these high-performance OFDM and SC-FD schemes using multiple antennas and DFE, while also accounting for the power amplifier efficiency. To make a realistic comparison, we also consider most important digital imperfections, such as channel and noise estimation, transmit and receive filtering, clipping and quantization, as well as link layer impact. Our analysis shows that for frequency-selective channels the relative performance impact of the power amplifier is negligible compared to the frequency diversity impact. The higher frequency diversity exploitation of SC-FD allows it to outperform OFDM in most cases. Therefore, SC-FD is a suitable candidate for broadband wireless communication.

Keywords and phrases: single carrier, OFDM, multi-antenna, power amplifier, decision feedback equalization.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular, standardized technique for broadband wireless systems: it is used for wireless LAN [1, 2], fixed broadband wireless access [3], digital video & audio broadcasting [4, 5] and so forth. OFDM can reach high spectral efficiency at low equalization complexity [6]. In recent years, single carrier with frequency domain equalization (SC-FD) has received a lot of attention as an alternative technique for broadband wireless communications [7]. Studies [8, 9] show that SC-FD can allow for a more power efficient transmitter, which is a very important aspect for battery-operated mobile terminals.

Multiple antennas allow to increase the spectral efficiency and/or to improve the link reliability. Therefore, more and more systems, both in theory and in practice, make use of multiple antennas. Space division multiple access (SDMA) implements multiuser access spectrally efficiently...
Moreover, [10] shows that decision feedback equalization (DFE) improves the performance of OFDM-SDMA, namely by applying the so-called per-carrier successive interference cancellation (OFDM-pcSIC).

Recently, a single-user SC-FD scheme with frequency domain processing and DFE (SC-FD-DFE) has been introduced [11, 12]. The time domain DFE allows to eliminate intersymbol interference (ISI) based on previous decisions and thus to improve performance over linear equalization. Moreover, it explicitly assumes the feedforward equalization in the frequency domain, which enables a low complexity solution.

These properties make SC-FD-DFE a suitable scheme for a broadband wireless uplink: it can achieve high spectral efficiency, while at the same time it implies a simple and efficient transmitter, which is a real plus for mobile terminals with limited battery and processing power. Moreover, it puts all the processing complexity at the receiver, that is, a base station, where typically more processing power is available.

The authors of [11, 12] derived the SC-FD-DFE for a single-user single-antenna (SISO) case. To compare OFDM-pcSIC fairly to SC-FD, we need to compare it to a multiuser SC-FD-DFE. Therefore, we extend this scheme to incorporate multiple users and multiple antennas (MIMO).

We compare the performance between the SC-FD-DFE and OFDM-pcSIC. Since the introduction of OFDM and SC-FD schemes, the advantages and disadvantages between them have been compared frequently [7, 8, 9, 11, 12, 13]. Recently, this comparison has gained more attention, since both schemes have been included in the IEEE 802.16a standard for fixed broadband wireless access [3].

Two very important points of comparison are the performance in a multipath environment and the impact of the power amplifier (PA). OFDM and SC-FD are designed for transmission over a frequency-selective channel and their cyclic extension is useful only over such a channel. On the other hand, the back-off of the power amplifier determines the power efficiency of the transmitter, a very important aspect for a wireless uplink as the terminal is likely to be battery powered.

Previous comparisons have not investigated the impact of the power amplifier on the bit error rate (BER) performance over a multipath channel. For example, [7, 11, 12, 13] only compare the ideal multipath performance and do not consider the impact of the power amplifier. Struhaker and Griffin [9] only consider the spectral regrowth due to the power amplifier, but not the impact on the performance, while [8] considers the effect on the BER performance only for AWGN channels.

In this paper, we compare SC-FD and OFDM in a realistic multiuser scenario with DFE. In other words, we compare SC-FD-DFE and OFDM-pcSIC, taking into account the impact of the power amplifier and most important digital imperfections, such as channel and noise estimation, clipping and quantization, transmit and receive filtering. Moreover, we account for coding and retransmission enabling us to compare the useful throughput.

The paper is structured as follows. In Section 2, we briefly introduce OFDM and SC-FD as well as the multi-antenna DFE concept for both schemes. In Section 3, we derive the multi-antenna SC-FD-DFE scheme and assess its performance. Section 4 evaluates the performance of both SC-FD-DFE and OFDM-pcSIC under realistic multi-antenna conditions. Finally, the conclusions obtained in this paper are presented in Section 5.

2. OFDM VERSUS SC-FD

In this section, we briefly compare the basic properties of OFDM and SC-FD and indicate the main differences and similarities. We introduce the decision feedback concept for both schemes.

2.1. OFDM

The basic idea of OFDM [6] transmission is to divide the available bandwidth into \( N \) subcarriers. If the number of subcarriers is large enough, the bandwidth per subcarrier is narrow compared to the coherence bandwidth of the channel. Therefore, each subcarrier experiences approximately flat fading. A spectrally and computationally efficient method to modulate the data on the frequency domain subcarriers is by means of an IFFT.

The addition of a cyclic prefix ensures that the channel always appears cyclic and thus the linear convolution with the channel can be considered a circular convolution. This guarantees that the received signal can be equalized by means of a single-tap equalizer per subcarrier. This operation is performed in the frequency domain, thus after the received signal is passed through an FFT.

Uncoded OFDM loses all frequency diversity inherent in the channel: a dip in the channel erases the information data on the subcarriers affected by the dip and this information cannot be recovered from the other carriers. This mechanism results in a poor BER performance. Adding sufficiently strong coding spreads the information over multiple subcarriers. This recovers frequency diversity and improves the BER performance.

2.2. SC-FD

SC-FD [7] transmits the data in the time domain. The cyclic extension is added before transmission which ensures the channel appears cyclic at the receiver. This again allows to have the same simple one-tap equalizer in the frequency domain. The decisions have to be taken in the time domain, so after the equalization an IFFT is needed.

Compared to OFDM, SC-FD uses the same building blocks, but with the IFFT moved from the transmitter to the receiver. This also means single carrier transmits the data in the time domain, whereas OFDM puts the data in the frequency domain. This has a very important consequence: the information of each SC-FD symbol is spread out over the complete frequency band. This means that dips in the channel do not wipe out complete symbols, because the information of each symbol can be recovered from the other carriers. Therefore, SC-FD exploits the frequency diversity in the channel and thus has a better uncoded performance than OFDM [7].
To obtain a scalar (and thus low complexity) feedforward equalization section in the frequency domain, we need to insert a cyclic extension at the transmitter. We can obtain this by either inserting a cyclic prefix (as in OFDM) or a training sequence (TS). However, only the TS approach yields the same spectral efficiency as OFDM (as indicated in Figure 1) which is essential for a fair comparison: both schemes transmit 48 data symbols in a 4 microsecond period, compliant to the IEEE 802.11a standard. Note that the two schemes require a slightly different sampling rate ($F_s$) to use the same bandwidth. Moreover, the edges of the spectrum are filled with zero carriers for OFDM, while single carrier needs pulse shaping (see Section 4.2).

The TS offers some additional advantages: because it is a known sequence, it can be used for synchronization, tracking, and training [14] and to start up the DFE [12].

Coding improves the SC-FD performance, whereas for OFDM coding is needed to exploit the frequency diversity and improve the performance.

### 2.3. Decision feedback equalization

The knowledge of (part of the) data helps to reconstruct and thus to eliminate (part of) the interference on other data. In turn, this (partial) elimination of interference improves performance.

In this section, we aim to exploit DFE to eliminate interference caused by multipath or by other users.

#### 2.3.1. For OFDM

In single-user (uncoded) OFDM, the data on one subcarrier is not related to the data on any other subcarrier. Since the multipath channel effect is also separated per carrier, DFE reduces to the linear MMSE solution.

In a multiuser scenario, each subcarrier contains a superposition of data from different users. In this case, the knowledge of (part of the) data helps to reconstruct and subtract from the received signal. This eliminates the interference caused by this user on the given carrier and thus improves performance when detecting the other users. This process is repeated for the second strongest user and so on until all users on all carriers are detected.

pcSIC can considerably improve the MMSE performance with a reasonable complexity increase, for example, [10] presents a case study with $U = A = 4$ with a 5 dB improvement for a 20% increase in complexity. This clearly illustrates the attractive features of OFDM-pcSIC for multiuser wireless communication.

#### 2.3.2. For SC-FD

For SC-FD, data symbols are transmitted in the time domain. The multipath channel spreads a data symbol across a number of subsequent symbols, equal to the order of the channel. Consequently, knowledge of previous symbols (and channel information) allows to eliminate some interference on subsequent data symbols and thus to improve performance. This idea is exploited in SC-FD-DFE in [11, 12], which describe a DFE method with similar complexity as OFDM pcSIC.

To make a fair comparison with OFDM pcSIC, we need to extend this single-user SC-FD-DFE to the multiuser case: each user data symbol does not only experience interference from its own preceding symbols from that user, but from other users as well. Therefore, previous decisions from all users can be used to cancel interference and to improve performance. We derive the necessary equations for such a scheme in the following section.

### 3. Multi-antenna single carrier with DFE

#### 3.1. Scheme

In [11, 12] SC-FD-DFE is presented as in Figure 2, with a feedforward part in the frequency domain and a feedback section with $L_{\text{df}}$ taps in the time domain.

Throughout the rest of the paper, matrices are denoted by bold capital letters, vectors have a bold normal font and scalars have a normal font. Frequency domain signals are indicated by a tilde. $a$ represents the modulated data, $H$ is the channel matrix (it reduces to a vector for a SISO channel), $n$ is the noise, $r$ is the received time domain signal, while $\hat{r}$ is
the received signal in the frequency domain. $z$ is the equalized signal just before the slicer, $\hat{a}$ are the decisions and $F$ the feedback coefficients matrix (it reduces to a vector in the single-user case).

Since the TS is a known sequence, it can be used to start up the feedback process. Therefore, SC-TS suits the DFE structure better than the cyclic prefix approach.

We now derive the SC-FD-DFE equations for a scenario with $U$ users and $A$ receive antennas. The variables are indicated in Figure 2.

### 3.2. Derivation of the coefficients

The equalizer output just before the slicer at time $n$ is

$$z_n = \frac{1}{N} \sum_{l=0}^{N-1} W_l \cdot \check{r}_l \cdot \exp \left( j \frac{2\pi l n}{N} \right) - F^* \cdot \hat{a}_{n-k}.$$  
(1)

For each frequency tap $l$, $\check{r}_l$ is an $A \times 1$ vector with the received signal in the frequency domain and $W_l$ is a $U \times A$ equalization matrix representing the feedforward part; $F$ is a $U \times (U \cdot L_{\text{dfe}})$ matrix containing the feedback coefficients; $\hat{a}_{n-k}$ is a $(U \cdot L_{\text{dfe}}) \times 1$ vector with the data decisions selected for feedback, for all users. $N$ is the number of time domain symbols per SC-FD block.

The derivation of the feedforward coefficients $W_l$ and the feedback coefficients $F$ extends the analysis by [11] to multiple antennas and multiple users and can be found in the appendix. The solution for the optimal feedforward coefficients $W_l$ is for every $l$

$$W_l = \tilde{\check{F}}_l \tilde{H}_l^* (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_A)^{-1}$$  
(2)

with $\tilde{H}_l$ an $A \times U$ matrix containing the coefficients for the channels from the $U$ users to the $A$ receive antennas at carrier $l$, $I_A$ the $A \times A$ identity matrix and $\sigma^2$ the noise power and

$$\tilde{F}_l = I_U + F^* \cdot X_l^*,$$

$$X_l = \begin{bmatrix}
\exp \left( j \frac{2\pi l k_1}{N} \right) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \exp \left( j \frac{2\pi l k_U}{N} \right)
\end{bmatrix},$$  
(3)

where $k_i$ is a $1 \times L_{\text{dfe}}$ vector containing the selected indices of the feedback taps for user $i$ and $I_U$ the $U \times U$ identity matrix.

The optimal feedback coefficients $F$ are

$$F = -T^{-1} \cdot t$$  
(4)

with

$$t = \frac{1}{N} \sum_{l=0}^{N-1} X_l^* (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1},$$

$$T = \frac{1}{N} \sum_{l=0}^{N-1} X_l^* (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1} X_l,$$

$W_l$ is a $U \times A$ matrix transforming the $A$ antenna streams into $U$ user streams. $F$ is of size $U \times (U \cdot L_{\text{dfe}})$ and contains for each user the optimal feedback coefficients to feed back the $U \cdot L_{\text{dfe}}$ selected decisions, $L_{\text{dfe}}$ from each user.

### Tap selection

By selecting the number of feedback taps, complexity can be exchanged for performance. With $L_{\text{dfe}} = 0$, the SC-FD-DFE scheme reduces to the SC-FD MMSE solution: (2) reduces to the linear equalization coefficients and there is no feedback part. A physically meaningful maximum of $L_{\text{dfe}} = L - 1$ feedback taps (with $L$ the order of the channel) can be selected, yielding the largest performance improvement. In this case, $X_l$ and $F$ are of maximum size and thus this operating point also introduces most processing power.

In the SISO case, the indices of the feedback taps are selected according to the strength of the channel response: the $L_{\text{dfe}}$ taps that contain most power are used. As we assume a channel of $L$ equal (average) power taps (see the following section), all channel taps can yield a significant improvement in the feedback process. Therefore, we set $L_{\text{dfe}} = L - 1$, which we assume to be known. The tap selection for the multiuser case is done for each user separately. In all multi-antenna simulations, we set the number of feedback taps $L_{\text{dfe}}$ equal to $L - 1$ for each user to provide the maximum performance gain. This means the feedback taps to be used are simply the indices from 1 to $L_{\text{dfe}}$.

### 3.3. Performance

We apply the multi-antenna SC-FD-DFE to two cases: first, we simulate an uplink SDMA scenario. Secondly, we investigate uplink receive diversity. The simulations presented in this section are performed for uncoded BPSK transmission with perfect channel knowledge, to illustrate basic DFE properties. More real-life results are presented in Section 4.

For all simulations in this paper, we apply a multipath channel with $L = 4$ Rayleigh fading taps; the taps are assumed to be independent and of equal average power. The channel is considered fixed for the duration of the data burst following the preamble. For the multiple-antenna simulations, we assume uncorrelated channel realizations.

#### 3.3.1. SDMA

We present the results for which each of the $U$ mobile terminals has one transmit antenna and the base station has a number of receive antennas $A$ equal to the number of users $U$. At the base station, the user streams are received at the $A$ antennas. This multiuser-multi-antenna scenario has a double effect on performance. On one hand, the $A$ receive antennas increase the receive diversity, which improves the performance. On the other hand, the $U$ user streams cause interuser...
Interference (IUI) which degrades the performance. Vandenameele et al. [10] show that overall the performance slightly improves as the number of users (equal to the number of antennas) increases for the linear MMSE receiver.

By applying DFE, $L_{\text{dfe}}$ decisions on the user’s preceding symbols are used to eliminate the ISI, while $(U - 1) \cdot L_{\text{dfe}}$ decisions on preceding symbols of the other users are used to eliminate the IUI. This nonlinear approach improves the performance over the linear SDMA. Figure 3 shows the performance curves comparing linear SDMA to DFE for 1 up to 4 users.

We observe a 3 to 4 dB gain at a BER of $10^{-3}$ for the SC-FD-DFE over the conventional linear MMSE. The DFE advantage over the linear SDMA slightly increases as the number of users increases: more users mean more IUI for both schemes, but also mean more decisions available for the DFE to eliminate part of this interference. Therefore, as the number of users increases, so does the interference the DFE can eliminate relative to the linear SDMA and the DFE performance advantage.

### 3.3.2. Receive diversity

In this case, one mobile terminal in the system is active and the base station has $A$ receive antennas. This means the receive diversity increases with the number of antennas, but there is no IUI, only ISI caused by the frequency selective channel. The DFE can eliminate part of this ISI and thus creates a frequency diversity advantage. However, this frequency diversity advantage becomes relatively less important as the receive diversity increases.

Figure 4 shows that the DFE gain relative to the linear MMSE solution decreases if the number of receive antennas is increased.

#### 3.3.3. Summary

The results from the previous sections show that DFE improves the performance over the linear equalizer. Therefore, DFE can offer a targeted performance at lower signal-to-noise ratio (SNR) and thus at lower transmit power. This transmit power reduction comes at the cost of added receiver complexity. For uplink communication, the receiver is a base station and the transmitter is a user terminal. This means DFE allows to reduce terminal power consumption (which is crucial for battery-driven devices) by adding complexity at the base station (where the extra complexity is more easily accommodated). This clearly shows that DFE is a very useful technique for SC-FD uplink communication.

### 4. MULTI-ANTENNA SC-FD-DFE VERSUS OFDM-PCSIC

Section 3 illustrates the performance of SC-FD-DFE under ideal multipath conditions: an ideal analog front end is assumed, with a linear power amplifier; the channel and noise characteristics are assumed to be perfectly known; digital imperfections, such as clipping and quantization as well as transmit and receive filtering, are not taken into account. OFDM and SC-FD react differently to the deviations from these ideal conditions, so they need to be considered in order to make a fair comparison between both schemes.

We discuss the modeling and impact of the power amplifier, channel and noise estimation, clipping and quantization, and transmit and receive filtering. When presenting the performance results, we take the link layer efficiency into account as well.
4.1. Power amplifier

For nonconstant envelope signals, a linear power amplifier is needed. We assume a class A power amplifier with back-off because of its linearity. The back-off determines the power consumption of the power amplifier and also its linear dynamic range. Since the linear dynamic range directly relates to the distortion, the back-off also determines the BER.

The linearity of the power amplifier is quantified by the 1-dB-compression point $P_{1\text{dB}}$, defined as the input power at which the nonlinearity lowers the output power by 1 dB compared to the ideal amplifier (Figure 5).

Since we want to perform baseband simulations because of simulation speed, we use the baseband representation of the power amplifier. The transfer function of a class A power amplifier with linear amplification $G$ and a cubic nonlinearity is

$$y = x \cdot G \cdot \left(1 - \alpha \frac{3}{4} |x|^2\right), \quad (6)$$

with $x$ the input baseband representation of the signal and $y$ the output.

The coefficient $\alpha$ can be expressed as a function of $P_{1\text{dB}}$ as

$$\alpha = \frac{4}{3(1 - 10^{-1/20})P_{1\text{dB}}^2}. \quad (7)$$

In our setup, we set the average input power $P_{\text{in}} = 6$ dBm; the linear gain of the power amplifier is 23 dB, such that we operate at 29 dBm average output power, which is a specified maximum average output power for the 5 GHz band [1]. The higher the $P_{1\text{dB}}$ compression point, the further the signal is separated from the distortion area of the power amplifier transfer characteristic. The smaller the distortion added by the power amplifier, the smaller the BER performance degradation is. However, the larger the back-off between $P_{\text{in}}$ and $P_{1\text{dB}}$, the smaller the power amplifier efficiency, as can be seen in Figure 6. A class A power amplifier has a theoretical maximum efficiency of 50%. This efficiency drops rapidly with increasing back-off.

OFDM has a large dynamic range compared to SC-FD [9]. Therefore, the $P_{1\text{dB}}-P_{\text{in}}$ back-off needs to be larger for OFDM to accommodate the signal in the linear range of the power amplifier transfer function. Theory and simulations indeed show that in an AWGN channel the performance advantage of SC-FD over OFDM increases as the PA back-off decreases.

However, the comparison in a multipath environment yields different conclusions. Since uncoded OFDM only reaches frequency diversity 1, the additional degradation caused by the power amplifier nonlinearity is much smaller than over an AWGN channel. This is shown in Figure 7 for

\[\text{Figure 5: Transfer function of a class A power amplifier.}\]
\[\text{Figure 6: Power efficiency of a class A power amplifier as a function of the back-off $P_{1\text{dB}}-P_{\text{in}}$.}\]
\[\text{Figure 7: The impact of PA back-off $P_{1\text{dB}} = [\approx 6.4 4.4 2.4]$ dBm on SC-FD and OFDM in multipath with perfect channel knowledge and coding ($R = 3/4$).}\]
uncoded BPSK transmission over a multipath channel: the OFDM performance curves for decreasing back-off are situated quite close together as are the SC-FD curves. The dominating degradation for OFDM compared to SC-FD is caused by the lack of frequency diversity: with infinite back-off, SC-FD has an 8 dB performance advantage over OFDM at a BER of 10^-4. The additional gain for SC-FD because of the power amplifier back-off of [6.4 4.4 2.4] dBm is only [0.6 1 1.6] dB. This clearly illustrates that the exploitation of the frequency diversity is dominant over the power amplifier back-off impact.

This is a noteworthy result, especially since BPSK is the modulation scheme which yields the largest power amplifier advantage for SC-FD over OFDM.

Coding helps to overcome the lack of frequency diversity, bringing the OFDM and SC-FD curves closer together. However, coding does not change the impact of the power amplifier back-off. This means that even on coded OFDM, the impact of the power amplifier back-off is comparable to that of coded SC-FD. The difference in performance between OFDM and SC-FD remains dominated by the difference in frequency diversity; this, in turn, is determined by the code rate. For high code rates (and uncoded transmission) SC-FD outperforms OFDM; for lower code rates, the performance becomes comparable.

These results only indicate the impact of the power amplifier back-off in a single-user multipath scenario with perfect channel knowledge. In the following sections, we introduce other imperfections which need to be taken into account, together with the multi-antenna aspect.

### 4.2. Digital imperfections

To make a realistic comparison, we include transmit and receive filtering, channel and noise estimation, and clipping and quantization in our simulation model as shown in Figure 8.

#### Transmit and receive filtering

The OFDM and SC-FD symbols are not transmitted as such, but they need to be filtered to limit the out-of-band radiation. Therefore, we apply a square root raised cosine (SRRC) filter as a transmit pulse shaping filter \( g_T \) and a receive filter \( g_R \) with a rolloff \( \alpha = 0.2 \), a delay of 5 taps, and an oversampling by 4. Moreover, at the receiver, an optimal channel matched filter is added in the frequency domain before the equalization.

#### Channel and noise estimation

Until now, we assumed perfect channel knowledge. In practice, the channel needs to be estimated and quite often a noise estimate is required as well. We use the estimation methods as described in [16].

The channel estimation is based on the C sequence (BPSK symbol \( \mathbf{1} \)) of the OFDM-based IEEE 802.11a standard. The following frequency domain estimation can also be directly applied to SC-FD, ensuring a fair comparison. In practice, a time domain sequence will be used for SC-FD channel estimation, which is designed to have a frequency response which is as flat as possible, but that is beyond the scope of this paper.

For a channel with frequency response \( \mathbf{c} \) and noise \( \mathbf{n} \), we obtain the channel estimate \( \hat{\mathbf{h}} \) as

\[
\hat{\mathbf{h}} = \mathbf{\tilde{t}} \star (g_R \star \mathbf{\tilde{c}} \star (g_T \star \mathbf{\tilde{t}} + \text{FFT}\{\mathbf{n}\}))
\]

\[
\hat{\mathbf{h}} = g_R \star \mathbf{\tilde{c}} \star \mathbf{\tilde{g}_T} + \mathbf{\tilde{n}},
\]

with \( \star \) the elementwise vector multiplication, \( \mathbf{\tilde{n}} = g_R \star \mathbf{\tilde{t}} \star \text{FFT}\{\mathbf{n}\} \) and \( \mathbf{\tilde{g}_T}, \mathbf{\tilde{g}_R} \) the frequency responses of the transmit pulse shaping filter \( g_T \) and the receive filter \( g_R \) respectively.

Since the channel as well as the transmit, and receive filters have a limited number of taps in the time domain (assuming the total number of taps to be \( L \)), we know that all the power in the taps \( \geq L + 1 \) can be attributed to noise. This allows us to remove part of the noise on the time domain channel estimate. This has two consequences: first, it allows to have an improved channel estimate, since we remove part of the noise. Second, the power of the noise which is removed is used to obtain a noise estimate. The analysis of these estimates can be found in [16].

In case of multiple transmit and/or receive antennas, we repeat the above process for each antenna pair separately. Vandenameele et al. [10] show that more performant multi-antenna channel estimation schemes exist. However, we do not need the most performant scheme; we just want to make a realistic and fair comparison between the OFDM and SC-FD schemes.

#### Clipping and quantization

OFDM is clipped and quantized to limit the signal’s dynamic range or the so-called peak-to-average power ratio (PAPR). This is beneficial since the limited dynamic range enables a smaller back-off. Côme et al. [15] show clipping at \( 4\sigma \) and a quantization of 8 bits to be a good solution for a realistic OFDM system. SC-FD needs to be clipped and quantized as well, but because of its limited dynamic range the impact is smaller.
4.3. Performance

We present the simulation results of the OFDM versus SC-FD comparison for a WLAN case study: we use the system parameters for OFDM as described in the IEEE standard [1] and choose the equivalent SC-FD such that the comparison is fair. For both schemes, the results for coded BPSK transmission \((R = 3/4)\) are presented.

We first show the SISO results and extend them to the multiple-antenna multiple-user case: we compare OFDM to SC-FD for SDMA and receive diversity, both schemes in the linear MMSE and the nonlinear DFE version.

4.3.1. Single user

The single-user single-antenna performance results also apply to a multiuser scenario if the multiple access to a single-antenna base station is provided by TDMA or FDMA.

As shown in Figure 9, the impact of the PA for \(P_{1\text{dB}} = [6.4 4.4 2.4] \text{dBm}\) is a \([0.75 1.1 1.9] \text{dB}\) degradation compared to the reference case (with infinite back-off) for SC-FD and \([0.75 1.2 2.2] \text{dB}\) for OFDM at a BER of \(10^{-4}\). This means that the additional advantage for SC-FD to a decreasing back-off \(P_{1\text{dB}} = [6.4 4.4 2.4] \text{dBm}\) is only \([0.1 0.3] \text{dB}\), while the difference in frequency diversity is still 8 dB.

This again clearly illustrates that in a realistic system the advantage of SC-FD over OFDM comes almost completely due to its effective use of the frequency diversity present in the channel. The impact of the PA power efficiency is negligible in the comparison of OFDM and SC-FD in a multipath scenario.

Another way of quantifying the difference in performance is not based on (SNR, BER) curves, but on \((P_{TR}, \text{Goodput})\) curves.

Goodput indicates the actual throughput at MAC level. It starts from the raw physical throughput (based on the constelllation size and the sampling time) and takes all overhead up to MAC level into account; it accounts for coding, the number of actual data symbols, the cyclic extension, and protocol overhead. Since we have chosen our system parameters to make a fair comparison between OFDM and SC-FD, both schemes can achieve the same maximum effective throughput or goodput \(R_{\text{max}}\). Therefore, we can normalize \(R_{\text{max}} = 1\) without loss of generality. Finally, erroneously received packets need to be retransmitted: (1-\text{PER}) accounts for the loss of actual throughput because of retransmissions. This leads to

\[
\text{Goodput} = R_{\text{max}} \cdot (1 - \text{PER}) = 1 - \text{PER}. \tag{9}
\]

The same relative measure can be derived for the total power consumed at the transmitter. If we assume the path loss, noise power, noise figure, and implementation loss are identical for OFDM and SC-FD (which is necessary for a fair comparison), we define the relative consumed power at the transmitter as

\[
P_{TR} = \frac{\text{SNR}}{\mu(P_{1\text{dB}})}. \tag{10}
\]

This means the total consumed power at the transmitter is proportional to the SNR (since higher SNR means more transmitted power) and inversely proportional to the PA efficiency \(\mu(P_{1\text{dB}})\), which is determined by the back-off from the 1 dB-compression point \(P_{1\text{dB}}\) (Figure 6). We assume the power consumption at the transmitter is largely determined by the PA power consumption, thus by the transmitted power and the PA efficiency.

The (BER / PER, SNR) curves as in Figure 9 can be transformed into (Goodput, \(P_{TR}\)) curves as in Figure 10 through
Table 1: SC-FD advantage (dB) over OFDM for MMSE SDMA as a function of the number of antennas (A).

| Number of antennas | 1   | 2   | 3   | 4   |
|--------------------|-----|-----|-----|-----|
| Goodput = 0.90     | 6.8 | 4.8 | 3.7 | 2.9 |
| Goodput = 0.95     | 7.2 | 5.5 | 4.5 | 3.6 |

Table 2: SC-FD advantage (dB) over OFDM for DFE SDMA as a function of the number of antennas (A).

| Number of antennas | 1   | 2   | 3   | 4   |
|--------------------|-----|-----|-----|-----|
| Goodput = 0.90     | 6.8 | 2.6 | 1.8 | 1.5 |
| Goodput = 0.95     | 7.2 | 3.0 | 2.1 | 1.7 |

As the number of antennas goes up, the relative diversity impact also increases, so does the receive diversity. As the number of users increase, so does the receive diversity, and the IUI for both schemes. The receive diversity helps OFDM to overcome the lack of frequency diversity. Therefore, as the number of antennas goes up, the relative diversity difference between OFDM and SC-FD becomes smaller and their performances converge. Table 1 shows that the SC-FD advantage gradually decreases as the number of users increases. The number in the following tables are given for $P_{1\text{dB}} = 4.4$ dBm; as shown in Figure 10, the impact of $P_{1\text{dB}}$ on the performance comparison is negligible.

Also in a multiuser scenario, the SC-FD advantage increases if a larger goodput is targeted. DFE allows to eliminate interference. In case of OFDM, pcSIC allows to eliminate precursor and postcursor ISI, whereas in case of SC-FD, SC-FD-DFE only eliminates postcursor ISI [12, 17]. This means OFDM possibly can eliminate the IUI completely, while SC-FD can only partly. Therefore, the SC-FD advantage over OFDM is smaller for DFE (Table 2) than for MMSE (Table 1).

4.3.3. Multi-antenna receive diversity

In this scenario, we consider one active user with one antenna while the base station has A receive antennas. As the number of receive antennas increases, the receive diversity rapidly increases the performance of both schemes, since there is no extra IUI to counter the diversity benefit. Since the receive diversity also helps to compensate the lack of frequency diversity, both OFDM and SC-FD converge to the same performance. Simulations show that the SC-FD advantage over OFDM rapidly decreases with increasing number of receive antennas.

In the single-user case, SC-FD performs the DFE through the SC-FD-DFE as described in [11, 12], while for single-user OFDM, the pcSIC algorithm reduces to linear equalization. Therefore, the single-user SC-FD advantage over OFDM increases for the DFE (Table 4) compared to MMSE (Table 1).

Table 3: SC-FD advantage (dB) over OFDM for MMSE receive diversity as a function of the number of antennas (A).

| Number of antennas | 1   | 2   | 3   | 4   |
|--------------------|-----|-----|-----|-----|
| Goodput = 0.90     | 6.8 | 2.6 | 1.8 | 1.5 |
| Goodput = 0.95     | 7.2 | 3.0 | 2.1 | 1.7 |

4.4. Summary of the results

The comparison between OFDM and SC-FD can be summarized as follows. The comparison is dominated by the frequency diversity rather than the power amplifier impact. SISO SC-FD outperforms OFDM by 4–5 dB in our case study, because of its higher frequency diversity exploitation. Adding multiple antennas increases the receive diversity for both schemes and thus decreases the relative diversity gap.

$\text{Goodput} = \frac{\text{Number of antennas}}{\mu}$
This decreases the SC-FD advantage over OFDM until the linear MMSE performances converge for a large number of antennas. In single-user scenarios (such as SISO and the receive diversity scenario), DFE further increases the SC-FD advantage, while for multi-user SDMA with DFE, the SC-FD advantage decreases with increasing number of users, because of OFDM-pcSIC’s better IUI elimination. Targeting a larger goodput always increases the SC-FD advantage over OFDM.

4.5. Remarks

Power amplifier
Apart from impact on the BER/PER performance or goodput, we should note that other considerations have to be made. The power amplifier nonlinearity also determines the amount of out-of-band radiation for which specifications exist. This has been studied in [9]. The authors indicate OFDM is indeed more sensitive to PA impact as far as out-of-band radiation is concerned. Therefore, a large back-off might still be required for OFDM, if not to assure link performance, then at least to limit out-of-band radiation.

Code rate
The difference in performance between OFDM and SC-FD depends also on the code rate $R$ and the constellation size. In this paper, we have taken $R = 3/4$ as specified in both the IEEE802.11a and HIPERLAN-II standards for BPSK transmission. The code $R = 3/4$ is a frequently used code rate, making a tradeoff between code performance and code overhead. For code rates larger than $R = 3/4$, the frequency diversity advantage of SC-FD increases, while for smaller code rates it decreases. For example, Table 5 indicates the performance advantage of SC-FD for BPSK $R = 1/2$, which is decreased by about 3 dB compared to the $R = 3/4$ case.

While the absolute numbers of the respective degradations change, the general conclusions as formulated in Section 4.4 remain valid.

Modulation
The BPSK transmission is the modulation scheme with the largest range and applicability area and will be used quite often. Moreover, for BPSK the power amplifier back-off creates the largest impact difference between SC-FD and OFDM. Since we have seen that this impact is small in a multipath environment, BPSK simulations provide the strongest support for this assertion. Therefore, we believe the ($R = 3/4$, BPSK) setting is a relevant case study.

Multipath channel
As the SC-FD advantage over OFDM is based on frequency diversity exploitation, the channel characteristics are of key importance. The results presented here are obtained for a channel length $L = 4$. For longer channel lengths, the inherent channel frequency diversity is larger and thus the SC-FD advantage will be larger as well, while for shorter channel lengths, the frequency diversity and the corresponding SC-FD advantage is smaller.

Table 5: SC-FD advantage (dB) over OFDM for SISO BPSK $R = 1/2$.

| Goodput | MMSE | DFE |
|---------|------|-----|
| 0.90    | 4.5  | 4.7 |
| 0.95    | 4.6  | 4.8 |

5. CONCLUSIONS

In this paper, we compared the performance of OFDM and SC-FD as modulation schemes for broadband wireless communication. In high-performant systems, multiple antennas are used to increase spectral efficiency and/or link reliability and DFE is used to improve the linear performance. For OFDM, this results in the so-called OFDM-pcSIC scheme, while for SC-FD we first extended the single-antenna SC-FD-DFE scheme to multiple-antenna scenarios. We have taken into account most important nonlinearities to obtain a realistic comparison: the power amplifier of the transmitter is very important for battery-driven terminals; we included most digital imperfections, such as channel and noise estimation, clipping and quantization, transmit and receive filtering, as well as link layer efficiency.

Our analysis shows that for frequency-selective channels the relative impact of the power amplifier on the multipath performance is negligible compared to the frequency diversity impact. Because of its higher frequency diversity exploitation, SISO SC-FD outperforms OFDM by about 7 dB in our case study. Adding multiple antennas decreases the relative diversity gap between both schemes and thus decreases the SC-FD advantage over OFDM until the linear equalizer performances converge for a large number of antennas. DFE increases the SC-FD advantage in single-user scenarios, while it decreases the advantage in a multiuser context.

In summary, the higher frequency diversity exploitation of SC-FD allows it to outperform OFDM in frequency-selective channels. Therefore, SC-FD is a suitable scheme for broadband wireless communication.

APPENDIX

DERIVATION OF THE FEEDFORWARD AND FEEDBACK COEFFICIENTS

The equalizer output just before the slicer at time $n$ is as in (1). As mentioned, for each frequency tap $l$, $\tilde{f}$ is an $A \times 1$ vector with the received signal in the frequency domain and $\tilde{W}_l$ is a $U \times A$ equalization matrix representing the feedforward part; $F^*$ is a $U \times (U \cdot L_{dfe})$ matrix containing the feedback coefficients; $\tilde{a}_{n-k}$ is a $(U \cdot L_{dfe}) \times 1$ vector with the data decisions selected for feedback, for all users. $N$ is the number of time domain symbols in an SC-FD block.

For the mathematical tractability of the analysis, we assume perfect feedback (i.e. no decision errors, $\tilde{a}_{n-k} = a_{n-k}$); for the simulated performance assessment, we do take decision errors into account.

The derivation of the coefficients extends the analysis by [11] to multiple antennas and multiple users.
Because of the use of a TS the channel and the data symbols appear cyclic at the receiver and thus at each frequency tap l we can write

\[ \tilde{r}_l = \tilde{H}_l \tilde{a}_l + \tilde{v}_l, \]  
(A.1)

with

\[ \tilde{H}_l = \sum_{n=0}^{N-1} H_n \exp \left( -j2\pi \frac{ln}{N} \right), \]
\[ \tilde{a}_l = \sum_{n=0}^{N-1} a_n \exp \left( -j2\pi \frac{ln}{N} \right), \]  
(A.2)

\[ \tilde{v}_l = \sum_{n=0}^{N-1} n_n \exp \left( -j2\pi \frac{ln}{N} \right), \]

where \( \tilde{H}_l \) is an \( A \times U \) matrix containing the coefficients for the channels from the U users to the A receive antennas at carrier \( l \), \( a_n \) is the \( U \times 1 \) vector with the user data and \( n_n \) is the \( A \times 1 \) noise vector.

The autocorrelation functions associated with the data and the noise are

\[ E[\tilde{a}_l \tilde{a}_l^*] = N \delta(l_1 - l_2)_{\text{mod}N} I_U, \]
\[ E[\tilde{v}_l \tilde{v}_l^*] = N \sigma^2 \cdot \delta(l_1 - l_2)_{\text{mod}N} I_A, \]  
(A.3)

\( I_U \) is the identity matrix of size \( U \times U \); \( I_A \) of size \( A \times A \). The error just before the slicer can be written as a \( U \times 1 \) vector containing the error for each user at each timing instant \( n \):

\[ e_n = z_n - a_n. \]  
(A.4)

The average error is

\[ \text{MSE} = E[e_n \cdot e_n^*] = E[(z_n - a_n)(z_n^* - a_n^*)] \]
\[ = \frac{1}{N} \sum_{l=0}^{N-1} (\tilde{H}_l \tilde{H}_l^* - \tilde{H}_l^* \tilde{H}_l) + \sigma^2 \sum_{l=0}^{N-1} \tilde{W}_l^* \tilde{W}_l^* \]  
(A.5)

with (3), where \( k_i \) is a \( 1 \times L_{\text{diff}} \) vector containing the selected indices of the feedback taps for user \( i \). Minimizing the MSE from (A.5) with respect to the feedforward coefficients means solving the following equation:

\[ \frac{\delta\text{MSE}}{\delta \tilde{W}_l} = \sum_{l=0}^{N-1} (\tilde{H}_l \tilde{H}_l^* + \sigma^2 I_A) \tilde{W}_l^* - \tilde{H}_l^* \tilde{F}_l^* = 0. \]  
(A.6)

The solution for the optimal feedforward coefficients \( \tilde{W}_l \) is, for every \( l \),

\[ \tilde{W}_l = \tilde{F}_l \tilde{H}_l^* (\tilde{H}_l \tilde{H}_l^* + \sigma^2 I_A)^{-1}. \]  
(A.7)

The matrix inversion lemma states that

\[ (A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA. \]  
(A.8)

Applying this lemma to our case results in

\[ \tilde{H}_l^* (\tilde{H}_l \tilde{H}_l^* + \sigma^2 I_A)^{-1} \tilde{H}_l = I_U - \sigma^2 (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1}. \]  
(A.9)

Substituting (A.9) and (A.7) in (A.5) gives us

\[ \text{MSE} = \frac{N}{\sigma^2} \sum_{l=0}^{N-1} \tilde{F}_l (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1} \tilde{F}_l^*. \]  
(A.10)

Minimizing (A.10) with respect to the feedback coefficients \( \tilde{F} \) means solving

\[ \frac{\delta\text{MSE}}{\delta \tilde{F}} = \frac{N}{\sigma^2} \sum_{l=0}^{N-1} (I_U + F^* X_l^*) (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1} X_l = 0. \]  
(A.11)

This leads to

\[ -\frac{1}{N} \sum_{l=0}^{N-1} X_l^* (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1} t \]  
(A.12)

\[ = \frac{1}{N} \sum_{l=0}^{N-1} X_l^* (\tilde{H}_l^* \tilde{H}_l + \sigma^2 I_U)^{-1} X_l \cdot F \]  
\[ t \]

from which we get the final result

\[ \tilde{F} = -T^{-1} \cdot t. \]  
(A.13)

Using (A.7) and (A.13) realizes the SC-FD-DFE.

REFERENCES

[1] IEEE Standard 802.11a-1999, “Part 11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications—amendment 1: high-speed physical layer in the 5 GHz band,” September 1999.

[2] HIPERLAN/2 standard, “Functional specification data link control (DLC) layer,” October 1999.

[3] I. Koffman and V. Roman, “Broadband wireless access solutions based on OFDM access in IEEE 802.16,” IEEE Communications Magazine, vol. 40, no. 4, pp. 96–103, 2002.

[4] U. Reimers, “DVB-T: the COFDM-based system for terrestrial television,” Electronics & Communication Engineering Journal, vol. 9, no. 1, pp. 28–32, 1997.

[5] P. Shelswell, “The COFDM modulation system: the heart of digital audio broadcasting,” Electronics & Communication Engineering Journal, vol. 7, no. 3, pp. 127–136, 1995.

[6] J. Bingham, “Multicarrier modulation for data transmission: an idea whose time has come,” IEEE Communications Magazine, vol. 28, no. 5, pp. 5–14, 1990.

[7] A. Czylik, “Comparison between adaptive OFDM and single carrier modulation with frequency domain equalization,” in
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