Inference-based semantics in Data Exchange

Adrian Onet
Morgan Stanley, Montreal, Canada
Adrian.Onet@morganstanley.com

Abstract. Data Exchange is an old problem that was firstly studied from a theoretical point of view only in 2003. Since then many approaches were considered when it came to the language describing the relationship between the source and the target schema. These approaches focus on what it makes a target instance a “good” solution for data-exchange. In this paper we propose the inference-based semantics that solves many certain-answer anomalies existing in current data-exchange semantics. To this we introduce a new mapping language between the source and the target schema based on annotated bidirectional dependencies (abd) and, consequently define the semantics for this new language. It is shown that the ABD-semantics can properly represent the inference-based semantics, for any source-to-target mappings. We discovered three dichotomy results under the new semantics for solution-existence, solution-check and UCQ evaluation problems. These results rely on two factors describing the annotation used in the mappings (density and cardinality). Finally we also investigate the certain-answers evaluation problem under ABD-semantics and discover many tractable classes for non-UCQ queries even for a subclass of CQ

1 Introduction

The data-exchange problem is that of transforming a database existing under a source schema into another database under a different target schema. This database transformation is based on mappings that describe the relationship between the source and the target database. A mapping $M$ can be viewed as a, possibly infinite, set of pairs $(I, J)$, where $I$ is a source instance and $J$ a target instance. In this case, $J$ is called a data-exchange solution for $I$ and $M$. The mapping between the source and the target database is usually specified in some logic formalism. The most widely accepted mapping language is the one based on sets of tuple-generating dependencies (tgds) and equality-generating dependencies (egds). A tuple-generating dependency is a FO sentence of the form: $\forall \bar{x}(\forall \bar{y} \alpha(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} \beta(\bar{x}, \bar{z}))$, where $\alpha$ and $\beta$ represent conjunctions of atoms. In case all atoms from $\alpha$ are over the source schema and all the atoms from $\beta$ are over the target schema, then we name the dependency a source-to-target tgd ($s\rightarrow t$ tgd). An equality-generating dependency ($egd$) is a FO sentence
of the form: \( \forall \bar{x}(\alpha(\bar{x}) \rightarrow x = y) \), where \( \alpha(\bar{x}) \) is a conjunction of atoms over the target schema and variables \( x \) and \( y \) occur in \( \bar{x} \).

There may be several solutions for a given source instance and a mapping, thus the set of solutions can be viewed as an incomplete database \([22, 13]\). Most commonly, the target solutions need to be queried. This takes us to the problem of querying incomplete databases \([2]\). The **exact answers semantics** defines the answer to a query over an incomplete database as the set of all answers to the query for each instance from its semantics. This approach is rarely feasible in practice, therefore two approximations are commonly accepted: **certain answer** (answers occurring for all solutions) and **maybe answer semantics** (answers occurring for some solutions).

Based on the interpretation of the mapping language there may be several semantics in data exchange. The first semantics was introduced in \([8]\) and it is here referred to as the OWA-semantics (open world assumption). In order to improve the certain answer behavior for non-UCQ queries under OWA-semantics, Libkin \([23]\) introduced the first closed-world semantics in data exchange, known as CWA-semantics. This semantics was later on extended by Hernich and Schweikardt \([21]\) to include target \( \text{tgds} \). In CWA-semantics a solution incorporates only tuples that are “justified” by the source and the dependencies.

Hernich \([17]\) enumerated three rules that a “good” data-exchange semantics should follow: 1) Implicit information in schema mapping and source instances are taken into account; 2) Logical equivalence of schema mapping is respected; and 3) The standard semantics of \( \text{FO} \) quantifier is reflected. Based on these rules Hernich introduced the GCWA'-semantics for data exchange. Unfortunately most of these semantics have a rather strange behavior when looking for certain answers over the set of solutions. More recently Arenas et al. \([4]\) introduced a new semantics for data-exchange based on bidirectional constraints. This new semantics solves most of the query anomalies present in the other semantics but it comes with price of non-tractable data complexity even for the simplest data-exchange problems. Also, as we will see, there are simple mappings that can be specified by a set of \( \text{tgds} \), under a closed-world semantics, but can not be specified by only using bidirectional constraints.

To better understand the type of anomalies we may encounter under these semantics, consider the next simple example. A company has in the source schema a binary relation \( P \) representing the relationship between projects and employees. After some reorganization it was realized that each projects financing should be provided by one or more cost-centers, each employee belonging to one of these cost-centers. With this the company creates the target schema with two binary relations \( PC \) and \( CE \) representing the project to cost-center and cost-center to employee relationship respectively. In the case of the unidirectional semantics the process can be specified by the following \( \text{tgds} \):

\[
\xi_1 : \forall p \forall e \ P(p, e) \rightarrow \exists cc \ PC(p, cc) \land CE(cc, e).
\]

Let source \( I \) be \( P^I = \{(p_1, e_1), (p_1, e_2), (p_2, e_3)\} \), stating that there are two employees \( e_1 \) and \( e_2 \) working on project \( p_1 \) and only employee \( e_3 \) working on
Consider target boolean query: $Q := \forall p, cc \; PC(p, cc) \land CE(cc, "e_3") \rightarrow p = "p_2"$ (Is $e_3$ involved only in project $p_2$?). Because target instance $J$, with $CE^J = \{(cc_1, e_1), (cc_1, e_2), (cc_1, e_3)\}$ and with $PC^J = \{(p_1, cc_1), (p_2, cc_1)\}$, is part of all unidirectional semantics, the certain answer for the given query will be un-intuitively false under these semantics. One may expect this answer to be true based input data, thus instance $J$ should not be part of the semantics. For the same problem, consider the mapping represented by the next bidirectional constraint:

$$\xi_2 : \forall p \forall e \; (P(p, e) \leftrightarrow \exists cc \; PC(p, cc) \land CE(cc, e)).$$

(2)

This mapping together with the bidirectional-constraint semantics solves the previous anomaly but introduces others. Consider query: “Does each cost-center have an employee?” Under bidirectional constraint semantics the certain answer to this query will be counter-intuitively false. This happens because bidirectional semantics does not require target tuples to be “justified”, thus instance $J'$, with $CE^{J'} = \{(cc_1, e_1), (cc_1, e_2), (cc_2, e_3)\}$ and $PC^{J'} = \{(p_1, cc_1), (p_2, cc_2), (p_2, cc_3)\}$ is part of the bidirectional constraint semantics. Note that this anomaly can be fixed with a rather complicated bidirectional constraint mapping where the right-hand side is a first-order expression.

Contributions Motivated by certain answer anomalies and expressivity issues in the current data-exchange semantics, in this paper we propose a new data-exchange semantics based on logical inference for mappings represented by sets of s-t tgds and target egds. This semantics eliminates most anomalies related to certain-answers and keeps the same certain answers with the other semantics for union of conjunctive queries. To this, we show that for any set of s-t tgds and safe egds the inference-based semantic can easily be represented in a much richer language of annotated bidirectional dependencies (abd) and safe annotated target egds (safe aegd). The restriction to safe aegds instead of regular egds exists because by allowing non-safe aegds, it opens the door to certain answer anomalies.

We discovered two important characteristics for each set $\Sigma^*$ of abds, namely the annotation density ($\#D(\Sigma^*)$) and annotation cardinality ($\#C(\Sigma^*)$). Intuitively, the annotation density measures the number of occurrences a relational symbol is annotated with the same label in $\Sigma^*$. The annotation cardinality measures the number of labels used for a relational symbol in $\Sigma^*$. We found that data-exchange solution-existence and solution-check problems have a dichotomic behavior based on annotation density and cardinality, respectively.

On the expressibility side, we show that for any set $\Sigma$ of s-t tgds and egds one can simply compute $\Sigma^*$, a set of abds and egds of density 1 such that for any instance $I$ the inference-based semantics for $\Sigma$ and $I$ coincide with the ABD-semantics for $\Sigma^*$ and $I$. We also show that there exists sets of abds, even with density 1, that can’t be expressed under inference-based semantics. Next,

$^1$ Intuitively safe egds do not allow joining with attributes that may contain null values.
we found that UCQ query-evaluation problem is tractable when \( \#_D(\Sigma^{++}) = 1 \) and it is coNP-complete for \( \#_D(\Sigma^{++}) > 1 \). In case \( \#_D(\Sigma^{++}) = 1 \) we prove that there exists an exact table representation for the ABD-semantics (following that this representation can be used for the inference-based semantics too). This representation is computable with a new chase-based process. We call this table universal representative and we believe it is a good candidate to be materialized on the target schema as the result of the data-exchange process. We show that for \( \#_D(\Sigma) = 1 \) the evaluation of universal queries is tractable and introduce a large subclass of CQ\(^-\) for which the evaluation problem is tractable. To the best of our knowledge, none of the data-exchange semantics has tractable results for any subclasses of CQ\(^-\) containing at least one negated atom.

**Organization.** We start with preliminary notions, followed by Section 3 in which we overview existing semantics and introduce the new inference-based semantics. Next section is allocated to present the new mapping language based on abds. Section 4 introduces a new type of naive table called semi-naive, able to exactly represent the ABD-semantics specified by mappings with density 1. Section 5 is devoted to the problem of certain answers evaluation.

## 2 Preliminaries

This section reviews the basic technical preliminaries and definitions. More information on relational database theory can be obtained from [1]. We will consider the complexity classes \( P \), \( NP \) and \( coNP \). For the definition of these classes we refer to [28].

A finite mapping \( f \), where \( f(a_i) = b_i \), for \( i \in \{1, \ldots, n\} \), will be represented as \( \{a_1/b_1, a_2/b_2, \ldots, a_n/b_n\} \). When it is clear from the context \( f \), it will be also viewed as the following formula \( a_1 = b_1 \wedge a_2 = b_2 \wedge \ldots \wedge a_n = b_n \). For a mapping \( f \) and set \( A \), with \( f|_A \) will denote the mapping \( f \) restricted to values from \( A \). By abusing the notation, a vector \( \bar{x} = (x_1, x_2, \ldots, x_n) \) will be often viewed as the set \( \{x_1, x_2, \ldots, x_n\} \), thus we may have set operations like \( x_1 \in \bar{x} \) or \( \bar{x} \cap \bar{y} \).

**Databases.** A schema \( S \) is a finite set \( \{S_1, S_2, \ldots, S_n\} \) of relational symbols, each symbol \( S_i \) having a fixed arity \( \text{arity}(S_i) \). Let \( Cons \), \( Nulls \) and \( Vars \) be three countably infinite sets of constants, nulls and variables such that there are no common elements between any two of these sets. Elements from \( Cons \) are symbolized by lower case (possibly subscripted) characters from the beginning of the alphabet (e.g. \( a, b \)). Elements from \( Vars \) are represented by lower case (possibly subscripted) characters from the end of the alphabet (e.g. \( z, x \)). Each element from the countable set \( Nulls \) is represented by subscripted symbol \( \bot \) (e.g. \( \bot_i \)). A naive table \( T \) of \( S \) is an interpretation that assigns to each relational symbol \( S_i \) a finite set \( S^T_i \subset (Cons \cup Nulls)^{\text{arity}(S_i)} \), sometimes we also view \( S_i \) as a relation between elements of \( Cons \cup Nulls \). The set \( \text{dom}(T) \) means all elements that occur in \( T \), clearly \( \text{dom}(T) \subseteq Cons \cup Nulls \). A naive table \( T \) is called an instance if \( \text{dom}(T) \subseteq Cons \). In contrast to general naive tables, which are identified by capitalized characters from the end of the alphabet (e.g. \( T, V \)), instances are repre-
sent by capitalized characters from the middle of the alphabet (e.g., $I$, $J$). The set of all instances over schema $S$ is denoted $\text{Inst}(S)$. A valuation $v$ is a mapping over the set $\text{Cons} \cup \text{Nulls}$ such that $v(a) = a$, for all $a \in \text{Cons}$, and $v(\bot) \in \text{Cons}$, for all $\bot \in \text{Nulls}$. Valuations are extended to tuples and naïve tables as follows. For each tuple $t = (t_1, t_2, \ldots, t_n)$, let $v(t) := (v(t_1), v(t_2), \ldots, v(t_n))$; and for a naïve table $T$ over schema $S$, define $v(T)$ as $R^n(T) := \{v(t) : t \in R^n\}$, for all $R \in S$. The interpretation of a naïve table $T$ is given by $\text{Rep}(T) := \{v(T) : v$ valuation$\}$. 

**Schema mappings.** A data-exchange schema mapping is a triple $M = (S, T, \Sigma)$, where $S$ and $T$ are two disjoint schemas named the source and target schema respectively; $\Sigma$ is a set of formulae expressing the relationship between the source and the target database. Most commonly, $\Sigma$ is represented by a set of source-to-target tuple-generating dependencies (s-t tgds) and target equality-generating dependencies (egds). Where a source-to-target tuple-generating dependency is a FO sentence $\xi$ of the form: $\forall \bar{x} \ (\forall \bar{y} \ \alpha(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} \ \beta(\bar{x}, \bar{z}))$, where $\bar{x}$, $\bar{y}$ and $\bar{z}$ are vectors of variables from $\text{Vars}$; $\alpha(\bar{x}, \bar{y})$ (often referred to as the body of the tgd) is a conjunction of atoms over the source schema; and $\beta(\bar{x}, \bar{z})$ (often referred to as the head of the tgd) is a conjunction of atoms over the target schema. In case $\beta$ contains a single atom, the tgd is referred to as GAV (global as view) tgd. In case $\bar{z} = \epsilon$ (empty), the tgd is called a full tgd. We will often view a conjunction of atoms as a naïve table where each atom from the conjunction is a tuple and each variable from the conjunction corresponds to a null in the table. An equality-generating dependency is a FO sentence $\xi$ of the form: $\forall \bar{x} \ \alpha(\bar{x}) \rightarrow \bar{x} = \bar{y}$, where $\alpha(\bar{x})$ is a conjunction of atoms over the target schema and $x, y$ are variables from the vector $\bar{x}$. A source instance $I \in \text{Inst}(S)$ and a target instance $J \in \text{Inst}(T)$ are said to satisfy a s-t tgd $\xi$, denoted $(I, J) \models \xi$; if $I \cup J$ is a model of $\xi$ in the model-theoretic sense. Similarly, a target instance $J$ satisfies an egd $\xi$, denoted $J \models \xi$, if $J$ is a model for $\xi$ in the model-theoretic sense. This is extended to a set of s-t tgds and egds $\Sigma$ by stipulating that $(I, J) \models \xi$, for all s-t tgd $\xi \in \Sigma$ and $J \models \xi$, for all egd $\xi \in \Sigma$. When the schemas are known or not relevant in the context, we usually interchange the notion of schema mapping and the set of dependencies that defines it.

A data-exchange semantics $\mathcal{O}$ associates for a schema mapping $M = (S, T, \Sigma)$ and a source instance $I$ a possible infinite set of target instances $\llbracket I, M \rrbracket_\mathcal{O}$. We refer to each element of $\llbracket I, M \rrbracket_\mathcal{O}$ as a solution for $I$ and $M$ under semantics $\mathcal{O}$.

**Queries.** CQ, UCQ, UCQ and UCQ denote the classes of conjunctive queries, union of conjunctive queries, union of conjunctive queries with negation and union of conjunctive queries with inequalities, respectively. For complete definitions of these classes, please refer to [1]. For a given data-exchange semantics $\mathcal{O}$, schema mapping $M$ and source instance $I$, the certain answers for a given query $Q$ is defined as:

$$\text{cert}^\mathcal{O}(Q, (I, M)) := \bigcap_{J \in \llbracket I, M \rrbracket_\mathcal{O}} Q(J).$$ (3)
3 On data-exchange semantics

As mentioned in the introduction, there are many semantics considered in data-exchange. In this section we briefly review the most prominent of these semantics and present some certain-answers anomalies associated with these. In Section 3.4 we introduce a new semantics for mappings specified by sets of \( s-t \) tgds and \( egds \) that addresses all certain-answers anomalies presented in this paper. In the final part of this section we will review the semantics based on bidirectional constraints.

3.1 OWA Data Exchange

The OWA-Semantics is the first semantics considered in data exchange [8]. This is, by far, the most studied [8,3,10,7,5,16,19]. Under this semantics, given a data-exchange mapping specified by \( \Sigma \) a set of \( tgds \) and \( egds \) and given a source instance \( I \), the OWA-semantics for \( I \) under \( \Sigma \) is defined as

\[
[(I, \Sigma)]_{owa} := \{ J \in \text{Inst}(T) : I \cup J \models \Sigma \}.
\]

In their seminal paper Fagin et al. [8] show that, when dealing with conjunctive queries, the set of certain answers \( \text{cert}_{owa}(Q, (I, \Sigma)) \) can be computed by naively evaluating [11] the query \( Q \) on a special instance, called universal solution, obtainable in polynomial time, if exists, through the chase process.

Example 1. Consider source schema consisting of two relations: \( FT \) Employees and Consultants. The first relation maintains a list of full-time employees and the second relation maintains a list of all consultants from a company. Consider target schema consisting of relations \( AllEmp \) and \( Cons \) for all employees and consultants. The mapping between the source and target represented by the following \( s-t \) \( tgds \):

\[
\Sigma = \{ FT \text{Employees}(eid) \rightarrow AllEmp(eid); \ \\
\text{Consultants}(eid) \rightarrow Cons(eid), AllEmp(eid) \}.
\]

Let source instance \( I \) be \( FT \text{Employees}(\{dan\}) \) and \( \text{Consultants}(\{john\}) \). Under this settings, instance \( J \in [(I, \Sigma)]_{owa} \), where \( AllEmp(J) = \{john, dan\} \) and \( Cons(J) = \{john, dan\} \). Thus, the certain answer to the query: “Are there any employees that are not consultants?”, expressible by a simple \( CQ^- \) query, will return \text{false}. This is an unexpected result as clearly, based on the source instance, “\( dan \)” is an full-time employee and not a consultant.

In the previous example we note that the given \( CQ^- \) query will return \text{false} for any source instance \( I \). This uniformity in the certain query answering was first noted by Arenas et al. in [3], who have also shown that even for the “copy” dependencies there are source \( FO \) queries that can’t be rewritten as target \( FO \) queries that return the same results under OWA-semantics for all input sources.
To avoid some of the anomalies, like the one from the previous example, in [9] Fagin et al. proposed a more restricted semantics:

$$\llbracket (I, \Sigma) \rrbracket_{\text{cwa}} := \{ J \in \text{Rep}(T) : T \text{ universal solution for } I \text{ and } \Sigma \}.$$  \hspace{1cm} (5)

As shown in [18], even with this restriction the certain answers still preserve some of OWA anomalies with respect to certain answers.

3.2 CWA Data Exchange

To overcome the counter-intuitive behavior under the open-world semantics Libkin [23] introduced the CWA-semantics for mappings specified by a set of s-t tgds. Hernich and Schweikardt [21] extended the semantics by adding target tgds. Given a set \( \Sigma \) of s-t tgds and a source instance \( I \) a CWA-solution for \( I \) and \( \Sigma \) is defined as any naïve table \( T \) over the target schema that satisfies the following three requirements:

1. each null from \( \text{dom}(T) \) is justified by some tuples from \( I \) and a s-t tgd from \( \Sigma \); and
2. each justification for nulls is used only once; and
3. each fact in \( T \) is justified by \( I \) and \( \Sigma \).

**Example 2.** Consider mapping specified by the s-t tgd stating that each student that has a library card reads at least one book:

$$\text{LibraryCard}(\text{sid}, \text{cid}) \rightarrow \exists \text{bid} \ \text{Read}(\text{sid}, \text{bid}).$$  \hspace{1cm} (6)

Let source instance \( I \) be \( \text{LibraryCard}^I = \{ (\text{john}, \text{stateLib1}), (\text{john}, \text{univLib2}) \} \), reviling that student “john” has a library card at the municipal library “stateLib1” and one card for the university library “univLib1”. A CWA-solution for this settings is \( \text{Read}^I = \{ (\text{john}, \bot_1), (\text{john}, \bot_2) \} \).

Libkin [23] considered 4 query-answering semantics: certain, potential certain, persistent maybe and maybe. In this paper we will focus on certain-answer semantics. More detailed information on the other semantics can be obtained from [23,20]. The CWA-semantics for certain-answers is defined as:

$$\llbracket (I, \Sigma) \rrbracket_{\text{cwa}} := \{ J \in \text{Rep}(T) : T \text{ is a CWA-solution for } I \text{ under } \Sigma \}.$$  \hspace{1cm} (7)

Returning to the settings from Example 2 clearly, under this semantics the number of books read by a student depends on the number of library cards the student owns. Thus in our case any target instance from \( \text{Rep}(T) \), where \( T \) is a CWA-solution, will mention that student “john” read maximum two books. Following that the certain answer to the query: ”Did 'john' read exactly one or two book?” will be true. Clearly this is counter-intuitive as the dependencies do not exclude that “john” read more than 1 book from each library.
Later on, this semantics was improved by designating attributes from the mapping as either open or closed [24]. For example, consider the following annotated tgd that maps each graduated student with a course and with a grade for that course:

\[
\text{Graduate}(s) \rightarrow \exists c \exists g \text{Attend}(s, c, g), \text{Grade}(s, c, g).
\]

Without the open/closed annotation using the CWA-semantics each student is considered to have attended exactly one course for which the student received exactly one grade. To be able to represent that a student may have attended more than one course, the course attribute for both \text{Attend} and \text{Grade} relation are marked as open, all the rest being marked as closed. In this case, even if the annotation takes care of the previous issue, it introduces another problem as the certain-answer semantics for source \{\text{Graduate}(john)\} includes instance \(J\), where \text{Grade}^J = \{(john, c01, A), (john, c02, A)\} and \text{Attend}^J = \{(john, c01)\}. Consequently, the certain answer to the query "Does "john" have grades only for the courses he attended?" will be \text{false} even if from the mapping we expect the certain answer to be \text{true} for any student.

Without further details, it needs to be mentioned here that Grahne and O. [15] introduced another semantics for data exchange called the constructible-solution. This semantics, when restricted only on source-to-target tgds, coincides with the CWA-semantics.

### 3.3 GCWA* Data Exchange

The GCWA* semantics was inspired from Minker’s [26] GCWA- semantics and nicely adapted by Hernich [18] for data exchange. For a source instance \(I\) and \(\Sigma\) a set of s-t tgds and egds let the set \(\text{Sol}_{\text{min}}(I, \Sigma)\) denote all the subset-minimal target instances \(J\) with \(I \cup J \models \Sigma\). With this, the CGWA*-semantics is defined as:

\[
[(I, \Sigma)]_{\text{gcwa}^*} := \{J : J = \bigcup_{i=1}^{n} J_i \text{ for some } n, J_i \in \text{Sol}_{\text{min}}(I, \Sigma) \text{ and } I \cup J \models \Sigma\}
\]

Even if the GCWA* semantics solves all aforementioned anomalies, it introduces new ones exemplified hereafter.

**Example 3.** Consider source instance with binary relation \text{DeptC} for departments and names of consultant employees working in that department and ternary relation \text{DeptFTE} for departments and full-time employee (name and id) from the given department. Suppose the company hires all the consultants as full-time employee, thus the target schema will be the ternary relation \text{DeptEmp} with the same structure as \text{DeptFTE}. The exchange mapping is represented as:

\[
\text{DeptC}(\text{did}, \text{name}) \rightarrow \exists \text{eid} \text{DeptEmp}(\text{did}, \text{name}, \text{eid});
\]

\[
\text{DeptFTE}(\text{did}, \text{name}, \text{eid}) \rightarrow \text{DeptEmp}(\text{did}, \text{name}, \text{eid}).
\]
Consider source instance with consultants “john” and “adam” part of the “hr” department and full-time employee “adam” with employee id 1 part of “hr”. Let target query be: Is there exactly one employee named adam in hr department? Under the CGWA*-semantics the query will return the counter-intuitive answer true, even if based on the source instance and given mapping one would expect the answer to be false, because beside full-time employee “adam” there maybe a consultant named “adam” in the same “hr” department too.

3.4 Inference-based semantics

In order to avoid the certain-answer anomalies presented in this section and in the introduction, we present a new closed-world semantics for mappings specified by a set of s-t tgd and egds. For this let us first introduce a few definitions.

Definition 1. Let \( \xi : \alpha(\bar{x}, \bar{y}) \rightarrow \exists \beta(\bar{x}, \bar{z}) \) be a s-t tgd and \( I \) a source instance. A set of facts \( J' \) is said to be inferred from \( I \) and \( \xi \) with function \( f \) denoted with \( I \xrightarrow{f} J' \) if \( f(\alpha(\bar{x}, \bar{y})) \subseteq I \) and \( f(\beta(\bar{x}, \bar{z})) = J' \). Tuple \( t \in J' \) is said to be strongly inferred from \( I \) and \( \xi \) with function \( f \) if \( t \in f(\alpha(\bar{x}, \bar{y})) \), otherwise we say that \( t \) is weakly inferred.

Intuitively a tuple \( t \) is strongly inferred from \( I \) and \( \xi \) with \( f \) if \( t \) does not depend on the assignment given by \( f \) to existential variables from \( \xi \).

Definition 2. Let \( \Sigma \) be a set of s-t tgds, \( I \) a source instance and \( J \) a target instance. A function \( \kappa \) that assigns for each \( \xi \in \Sigma \) a set of functions \( \{ f_1, f_2, \ldots, f_n \} \) such that \( I \xrightarrow{f_i} J_i \subseteq J \) is called an inference strategy for \( I \) and \( J \) with \( \Sigma \). Note that the function that allocates the empty set for each \( \xi \in \Sigma \) is an inference strategy for any \( I \) and \( \Sigma \). Given an inference strategy \( \kappa \) and \( \xi \in \Sigma \), a tuple \( t \in J \) is said to be:

- strongly inferred with strategy \( \kappa \) for \( \xi \) if there exist \( f \in \kappa(\xi) \) such that \( t \) is strongly inferred from \( I \) and \( \sigma \) with \( f \);
- not inferred with strategy \( \kappa \) for \( \xi \) if there is no \( f \in \kappa(\xi) \) with \( I \xrightarrow{f} J' \ni t \) and
- weakly inferred with strategy \( \kappa \) for \( \xi \) otherwise.

A tuple \( t \) is said to be inferred with strategy \( \kappa \) for \( I \), \( \Sigma \) and \( J \) if there exists \( \xi \in \Sigma \) such that \( t \) is strongly or weakly inferred with \( \kappa \) for \( \xi \).

Example 4. Consider the following abstract set of s-t tgds \( \Sigma = \{ \xi \} \), where:

\[ \xi = R(x, y) \rightarrow \exists z S(x, z), T(z, y), T(x, y). \]

Consider also source instance \( I \) and target instance \( J \) with \( R^I = \{(a, b), (c, d)\} \), \( S^I = \{(a, a), (a, c), (c, a)\} \) and \( T^I = \{(a, a), (a, b), (c, b), (a, d), (c, d)\} \). With this we have \( I \xrightarrow{f_1} J_1 \), where function \( f_1 = \{x/a, y/b, z/a\} \), \( S^{J_1} = \{(a, a)\} \) and
$T^J = \{(a,b)\}$. In this case tuple $S(a,a)$ is weakly inferred from $I$ and $\xi$ because it is obtained by assigning constant $a$ to the existentially quantified variable $z$. On the other hand, tuple $T(a,b)$ is strongly inferred from $I$ and $\xi$, even if it can be obtained by assigning constant $a$ to the existentially quantified variable $z$ in the second atom in the head of $\xi$, because the tuple can also be obtained by valuating only universal variables $x$ and $y$ to $a$ and $b$ respectively in the third atom. Consider also functions: $f_2 = \{x/a, y/b, z/c\}$, $f_3 = \{x/c, y/d, z/a\}$ and $f_4 = \{x/a, y/b, z/b\}$. With this we have that for any set $A \in \mathcal{P}\{f_1, f_2, f_3\}$ function $\kappa(\xi) = A$ is an inference strategy for $I$ and $J$ with $\Sigma$. On the other hand, for any function $\kappa'$ such that $f_4 \in \kappa'(\xi)$ we have $\kappa'$ is not a valid inference strategy because tuple $T(b,b) \notin J$. Consider $\kappa(\xi) = \{f_1, f_2\}$, in this case tuples $S(c,a)$ and $T(a,d)$ are not inferred with strategy $\kappa$ for $\xi$; tuples $T(a,b)$ and $T(c,d)$ are strongly inferred with strategy $\kappa$ for $\xi$ and finally tuples $S(a,a), S(a,c), T(a,a)$ and $T(c,b)$ are weakly inferred with strategy $\kappa$ for $\xi$.

With this we are now ready to introduce the inferred-based semantics.

**Definition 3.** Given a source instance $I$ and $\Sigma$ a set of s-t tgds and egds the inference-based semantics for $I$ and $\Sigma$, denoted with $[\{I, \Sigma\}]_{inf}$, is the set of all target instances $J$ for which there exists an inference strategy $\kappa$ such that:

1. Every tuple $t \in J$ is inferred with $\kappa$ for $I$, $\Sigma$ and $J$;
2. For every $\xi : \alpha \rightarrow \beta$ $\in \Sigma$ and every function $f$ with $f(\alpha) \subseteq I$ there exists $f' \in \kappa(\xi)$ such that $f'$ is an extension of $f$;
3. For every $\xi : \alpha \rightarrow \beta$ $\in \Sigma$ and $J_{\kappa, \xi} = \bigcup_{\hat{f} \in \kappa(\xi)} J_{\hat{f}}$, there is no function $f$ with $f(\beta) \in J_{\kappa, \xi}$ and $f(\alpha) \subseteq I$, where $f(\beta)$ contains at least one weakly inferred tuple with strategy $\kappa$ for $\xi$.
4. $(I, J) \models \Sigma$ in the model-theoretic sense.

**Example 5.** Let us consider the same settings as in Example 2 and consider the following inference strategies $\kappa_1(\xi) = \{f_1, f_2, f_3\}$ and $\kappa_2(\xi) = \{f_3\}$. Because tuple $S(c,a)$ $\in J$ is inferred neither by $\kappa_1$ or $\kappa_2$ it follows that inference strategies $\kappa_1$ and $\kappa_2$ does not respect Condition 1 from the definition. It is easy to observe that $\kappa_1$ satisfies Condition 2. On the other hand, for function $f = \{x/a, y/b\}$ we have that $f(R(x,y)) \subseteq I$ and there is no extension in $\kappa_2$, thus it follows that $\kappa_2$ does not respect Condition 2. The instances under condition 3 are $J_{\kappa_1, \xi} = \{S(a,a), S(a,c), T(a,b), T(c,b), T(c,d), T(a,d)\}$ and $J_{\kappa_2, \xi} = \{S(a,c), T(c,d), T(a,d)\}$. For $J_{\kappa_1, \xi}$ we have function $f = \{x/a, y/d, z/a\}$ such that $f(\beta) \subseteq J_{\kappa_1, \xi}$ and both $S(a,a)$ and $T(a,d)$ are weakly inferred tuples with $\kappa_1$ and $\xi$ but $f(\alpha) = \{R(a,d)\} \not\subseteq I$. Thus $\kappa_1$ does not respect Condition 3. It is easy to see that $\kappa_1$ respects this condition. Finally, we have that $(I, J) \models \Sigma$, thus Condition 4 is satisfied for any inference strategy with $I$, $\Sigma$ and $J$. Intuitively the first rule from the definition states that all tuples in the target instance needs to be inferred from the source instance and the mapping, this is taking care of the tuples not inferred present under OWA-semantics. This also allows the same nulls to be matched to different constant as long as there exists
an inference for each of these. This takes care of the query anomalies present under CWA-semantics. The second condition makes sure that all possible source triggers are fired. With this all tuples that can be inferred will be present in at least in one instance in the semantics, this solves the anomaly presented in Example 3 for GCWA∗. The third condition ensures that the assignment of nulls does not contradict with the inference strategy used, thus taking care of the query anomaly from the introduction. The last condition is needed in order to guarantee that the instances from the semantics are models for the egds. Need to mention here that by renouncing to Condition 3 from the previous definition we obtain the semantics defined in [14] in the context of exchange recovery.

It can be observed that the inference based semantics follows all the rules mentioned by Hernich [17] except the semantics closure under logical equivalence. That is, under inference-based semantics, there exists a source instance I and two logical equivalent sets of s-t tgds Σ1 and Σ2 such that [(I, Σ1)]inf ≠ [(I, Σ2)]inf. With the next example we argue that the closure under logical-equivalence for closed-world data-exchange semantics is not always a desirable property.

Example 6. Consider source schema consisting of a unary relation SalesPerson for the list of sales persons in a company. A sales person S1 can book trades on behalf of the accounts assigned to him and on behalf of the accounts assigned to a sales person S2 if sales person S1 is set to cover sales person S2. Consider target schema represented by a binary relation Cover that specifies if a sales person covers another sales person. In this case the mapping between the two schemata is represented by the following s-t tgds:

$$\Sigma_1 = \{\text{SalesPerson}(x) \rightarrow \text{Cover}(x, x)\}
\text{SalesPerson}(x) \rightarrow \exists y \text{Cover}(x, y)\}.$$  \hspace{1cm} (8)

The first dependency specifies that each sales person covers his own accounts and the second that each sales person covers one or more sales persons. Clearly Σ1 is logically equivalent to Σ2, where:

$$\Sigma_2 = \{\text{SalesPerson}(x) \rightarrow \text{Cover}(x, x)\}.$$  \hspace{1cm} (9)

Let source instance I contain two sales persons ‘john’ and ‘adam’. Consider target boolean query: Does ‘adam’ covers only his accounts? It is easy to observe that under any closed world semantics Σ2 and I the query will return certain answer true. On the other hand, the same query under mapping Σ1 is expected to return false, as ‘adam’ may cover other sales persons as well.

As we will see in Section 6 this new data-exchange semantics has a few desirable properties when it comes to certain-answers queries. That is, for any source-to-target mapping Σ, for any instance I and for any UCQ query Q we have certinf(Q, (I, Σ)) = certcwa(Q, (I, Σ)). Similarly for any full source-to-target mapping Σ, any source instance I and for any FO query Q we have certinf(Q, (I, Σ)) = certcwa(Q, (I, Σ)). In Section 4 we will also show the language of annotated bidirectional dependencies is a good language to represent this semantics.
3.5 Bidirectional constraints

Arenas et al. in \[4\] considered another approach to the certain answer anomaly problem by changing the language used to express the schema mapping. For this, the authors proposed the language of bidirectional constraints. Where a bidirectional constraints is a FO sentence of the form:

$$\forall \bar{x} \, \alpha(\bar{x}) \leftrightarrow \beta(\bar{x})$$

where $\alpha$ and $\beta$ are FO formulae over atoms from the source and target schema respectively with free variables $\bar{x}$. If the language of $\alpha$ is $L_S$ and of $\beta$ is $L_T$, then we are talking about a $\langle L_S, L_T \rangle$-dependency. Thus if $\alpha$ is represented as a union of conjunctive queries and $\beta$ as a conjunction of atoms, the bidirectional constraint is a $\langle \text{UCQ}, \text{CQ} \rangle$-dependency. With this, given a source instance $I$ and $\Sigma^{\leftrightarrow}$ a set of $\langle L_S, L_T \rangle$-dependencies, the bidirectional semantics is defined as:

$$[[I, \Sigma^{\leftrightarrow}]]_{\leftrightarrow} := \{ J \in \text{Inst}(T) : I \cup J \models \Sigma^{\leftrightarrow} \}.$$ \hspace{1cm} (10)

This approach did indeed solve most of the anomalies related to the other semantics. Unfortunately, this is achieved at a high cost, as even the most common data-exchange problems became non-tractable. For example, testing if the semantics is empty is an $\text{NP}$-hard problem \[4\] even for a set of $\langle \text{CQ}, \text{CQ} \rangle$-dependencies. Beside this, the semantics lacks of properties desirable for any closed-world semantics. Consider $\Sigma^{\leftrightarrow} = \{ \forall x \,(R(x) \leftrightarrow \exists y \, S(x,y) \land T(x,y)) \}$, source instance $I = \{R(a)\}$ and target instance $J$ where $S^I = \{(a,b)\}$ and $T^J = \{(a,b), (a_1,b_1), (a_2,b_2), \ldots, (a_n,b_n)\}$ with $a_i \neq a$ or $b_i \neq b$, for all $i \leq n$. Clearly we have $J \in [[(I, \Sigma^{\leftrightarrow})]]_{\leftrightarrow}$ even if none of the tuples under relation $T$, except $T(a,b)$, are inferred from source instance $I$ and $\Sigma^{\leftrightarrow}$. In order to fix this problem one may have to use a $\langle \text{CQ}, \text{FO} \rangle$-dependency.

Another issue with bidirectional semantics is that there are simple unidirectional mappings for which neither of the presented closed-world semantics are not expressible using bidirectional constraints without changing the target schema, as shown in the example below.

Example 7. Consider source schema with binary relations $PFT_E$, for projects and the full-time employees assigned on the project, and $PT$, that contains the tasks associated with each project. Let target schema consist of a binary relation $PE$, for projects and employee assigned to that project, and binary relation $TM$, for employee and the task they manage. Consider source instance $I$, with $PFT^I_E = \{(hr, adam)\}$ and $PT^I = \{(hr, comp)\}$, stating that full-time employee ‘adam’ works on the ‘hr’ project and the ‘hr’ project consists of one task ‘comp’. Consider the following mapping $\Sigma$ stating that each full-time employee working on a project is also an employee working on the project ($\xi_1$) and that for each project task there exists an employee working on that project that manages that task ($\xi_2$).

$$\xi_1 : PFTE(pid, eid) \rightarrow PE(pid, eid)$$

$$\xi_2 : PT(pid, tid) \rightarrow \exists eid \, PE(pid, eid), TM(eid, tid).$$
It is easy to observe that for any set $\Sigma^{\leftrightarrow}$ of bidirectional constraints there is no possibility to differentiate between the tuples from relation $PE$ as being inferred from $PFTE$ or $PT$ source relation. Thus, for $J_1 = \{PE(hr, adam)\}$, $J_2 = J_1 \cup \{TM(adam, comp)\}$ and $J_3 = J_1 \cup \{PE(hr, sal), TM(sal, comp)\}$, we have that either $J_1 \in [(I, \Sigma^{\leftrightarrow})]_{e_2}$, or $J_2 \notin [(I, \Sigma^{\leftrightarrow})]_{e_2}$ or $J_3 \notin [(I, \Sigma^{\leftrightarrow})]_{e_2}$, where “sal” is a consultant not a full-time employee. On the other hand, we have that $J_1 \notin [(I, \Sigma)]_*$ and $J_2, J_3 \in [(I, \Sigma)]_*$, for any semantics $\ast \in \{cwa, gcwa^*, inf\}$.

4 ABD-semantics

In this section we propose a new language and its corresponding semantics for data-exchange. The new language is based on annotated bidirectional dependencies and not only can properly express the inference-based semantics for any set of $s$-$t$ $tgds$ and safe target $egds$, but also can specify mappings not expressible in any of the previous semantics. Beside this, the language has syntactical characteristics (density and cardinality) that clearly delimit the complexity classes for different problems (solution-existence, solution-check and query evaluation).

First, let us introduce a few notions and notations. Given an instance $I$, a tuple-labeling function $\ell$ is a mapping that assigns a non-empty set of integers to each tuple from $I$.

**Definition 4.** Given a source schema $S$ and a target schema $T$, an annotated bidirectional dependency (abd) is a FO sentence of the form:

$$\forall x \ (\exists y \ a(x, y)) \leftrightarrow (\exists z \ \beta(x, z)),$$

(11)

where $\bar{x}, \bar{y}$ and $\bar{z}$ are disjoint vectors of variables; $a(\bar{x}, \bar{y})$ is a conjunction of atoms over $S$; $\beta(\bar{x}, \bar{z})$ is a conjunction of atoms over $T$ and each atom from $\beta$ is annotated with an integer. An annotated equality-generating dependency ($aegd$) is a target egd where each atom is annotated with an integer.

For an $abd$ $\xi$ of the form (11) body($\xi$) represents the conjunction $a(\bar{x}, \bar{y})$. With $\xi^{\rightarrow}$ is denoted $tgds$: $\forall x \ (\exists y \ a(\bar{x}, \bar{y})) \rightarrow (\exists z \ \beta(\bar{x}, \bar{z}))$, where $\beta'$ removes the annotations used in $\beta$. For a set $\Sigma^{\leftrightarrow}$ of abds and aegds with $\Sigma^{\rightarrow}$ we denote the set $\{\xi^{\rightarrow} : \xi \in \Sigma^{\leftrightarrow}\}$.

For a set $\Sigma^{\leftrightarrow}$ of abds and aegds with $\Sigma^{\leftrightarrow}_{ab}$ we denote the subset of $\Sigma^{\leftrightarrow}$ that contains all the abds and with $\Sigma^{\leftrightarrow}_{aeg}$ the set that contains all the aegd. Thus we have $\Sigma^{\leftrightarrow} = \Sigma^{\leftrightarrow}_{ab} \cup \Sigma^{\leftrightarrow}_{aeg}$. For a target relation symbol $R$ and $\xi \in \Sigma^{\leftrightarrow}$, the set $annot(\xi, R)$ contains all annotations used for atoms over $R$ in $\xi$. This notation is extended to $\Sigma^{\leftrightarrow}$: $annot(\Sigma^{\leftrightarrow}, R) := \cup_{\xi \in \Sigma^{\leftrightarrow}} annot(\xi, R)$.

For simplicity the quantifiers will be omitted when representing an $abd$ or $aegd$. The conjunctions will be represented with commas.

**Example 8.** Consider the abstract example with $\Sigma^{\leftrightarrow} = \{\xi_1, \xi_2, \xi_3\}$, where

\[
\begin{align*}
\xi_1 : & \quad R(x, y) \leftrightarrow T^1(x, z), T^1(y, z), T^2(x, y); \\
\xi_2 : & \quad S(x, x), R(x, x) \leftrightarrow V^1(x); \\
\xi_3 : & \quad T^1(x, y), V^1(x) \rightarrow x = y;
\end{align*}
\]
Atoms in the head of the rules are superscripted with their annotation. For this set we have $\Sigma_{\text{abd}}^\ast = \{\xi_1, \xi_2\}$ and $\Sigma_{\text{aegd}}^\ast = \{\xi_3\}$. Furthermore we have $\text{annot}(\xi_1, T) = \{1, 2\}$, $\text{annot}(\xi_1, V) = \emptyset$, $\text{annot}(\Sigma^\ast, V) = \{1\}$.

**Definition 5.** (Annotation Density) Given $Sigma^\ast$ and target relation $R$ the annotation density for $R$ in $Sigma^\ast$ is the maximum number of occurrences of $R^i$ in $\Sigma_{\text{abd}}^\ast$ for some $i$ and is denoted with $\#D(\Sigma^\ast, R)$. The annotation density for $\Sigma^\ast$ is defined as:

$$\#D(\Sigma^\ast) := \max\{\#D(\Sigma^\ast, R) : R \in T\}.$$  

**Definition 6.** (Annotation Cardinality) Given $Sigma^\ast$ and a target relation $R$ the annotation cardinality for $R$ in $Sigma^\ast$ is defined as $\#C(\Sigma^\ast, R) := |\text{annot}(\Sigma^\ast, R)|$. The annotation cardinality for $Sigma^\ast$ is defined as:

$$\#C(\Sigma^\ast) := \max\{\#C(\Sigma^\ast, R) : R \in T\}.$$  

For $\Sigma^\ast$ from Example 8, $\#D(\Sigma^\ast, T) = 2$, because $T$ is annotated with integer 1 two times in $\Sigma_{\text{abd}}^\ast$. For $V$ we have $\#D(\Sigma^\ast, V) = 1$, thus for the entire set $\#D(\Sigma^\ast) = 2$. Similarly, $\#C(\Sigma^\ast, V) = 1$ because only $V^1$ occurs in $\Sigma_{\text{abd}}^\ast$. On the other hand, $\#C(\Sigma^\ast, T) = 2$ thus $\#C(\Sigma^\ast) = 2$.

Similarly to the notations in [6], an annotated position in $\Sigma^\ast$ is a pair $(R^i, k)$, where $1 \leq i \leq \text{arity}(R)$. Given an aegd $\xi : \forall \bar{x} (\exists y \alpha(\bar{x}, y) \leftrightarrow \exists z \beta(\bar{x}, z))$, the affected positions in $\xi$, denoted $\text{aff}(\xi)$, is the set of annotated positions where elements of vector $\bar{z}$ occurs in $\xi$. This is extended to a set of aegds $\text{aff}(\Sigma^\ast) = \cup_{\xi \in \Sigma^\ast} \text{aff}(\xi)$.

**Definition 7.** (Safe aegd) An aegd $\xi \in \Sigma^\ast$ is said to be safe if every variable $y$ that occurs at least two times in the body($\xi$) occurs only in positions other than $\text{aff}(\Sigma^\ast)$. The set $\Sigma^\ast$ is said to be safe if all aegds from $\Sigma^\ast$ are safe.

Returning to Example 8 $\text{aff}(\Sigma^\ast) = \{(T^1, 2)\}$. Clearly $\xi_3$ is a safe aegd because variable $x$ occurs in positions $(T^1, 1)$ and $(V^1, 1)$ neither being part of $\text{aff}(\Sigma^\ast)$.

In our semantics we will restrict the set of aegds from the mappings to be safe. This restriction is necessary in order to avoid unwanted anomalies in the certain answers. To be more specific, consider the following example.

**Example 9.** Let $\Sigma = \{\xi_1, \xi_2\}$. The mapping copies employees from the source table Emp$_0$ into the target table Emp by replacing the current ssn key with a new employee id (eid) key. Where $\xi_1$ copies the employee data from source to target and $\xi_2$ enforces the primary-key constraint. Note that $\xi_2$ is not safe.

$$\xi_1 : \text{Emp}_{0}(\text{ssn}, \text{name}) \rightarrow \exists \text{eid} \text{Emp}(\text{eid}, \text{name});$$  

$$\xi_2 : \text{Emp}(\text{eid}, \text{name}_1), \text{Emp}(\text{eid}, \text{name}_2) \rightarrow \text{name}_1 = \text{name}_2.$$  

Let $I$ be specified by $\text{Emp}' = \{(s1, john), (s2, adam), (s3, adam)\}$ and let $J$ be specified by $\text{Emp}'' = \{(a, john), (b, adam)\}$. It can be easily verified that instance $J \in [(I, \Sigma)_{\text{owa}} \cap [(I, \Sigma)_{\text{owa}} \cap [(I, \Sigma)]_{\text{owa}}]$. Thus, the following query
expressible in FO: \(Q := \text{Are there exactly two employees named "adam"?}\) will return the counter-intuitive answer false. To avoid such behavior we decided to not allow unsafe \textsf{aegds} at the cost of reducing the expressivity of the mapping language.

In order to define the data-exchange semantics for this new language, let us first see what it means for a target instance over schema \(T\) to satisfy a set \(\Sigma^{**}\) of \textsf{abds} and \textsf{aegds}. For this, let \(T_\circ\) be the schema \(T_\circ = \{R_i : i \in \text{annot}(\xi, R) \mid \xi \in \Sigma^{**}, R \in T\}\). Intuitively \(T_\circ\) contains a distinct relation name for each pair of relation name and annotation used in \(\Sigma^{**}\). Similarly, dependency set \(\Sigma^{**}\) over \(T_\circ\) is computed from \(\Sigma^{**}\) by replacing each annotated atom \(R^i(\vec{x})\) in \(\Sigma^{**}\) with atom \(R_i(\vec{x})\). Given \(J \in \text{Inst}(T)\) and \(\ell\) a tuple-labeling function, with \(J_\ell\) is denoted the instance over schema \(T_\circ\) such that \(R_i(\vec{a}) \in J_\ell\) iff \(R(\vec{a}) \in J\) and \(i \in \ell(R(\vec{a}))\). Instance \(J\) is said to be inferred from \(I\) and \(\Sigma^{**}\) with tuple-labeling function \(\ell\), denoted with \((I, J) \models_\ell \Sigma^{**}\), iff for any tuple \(R(\vec{a}) \in J\) and for any \(i \in \ell(R(\vec{a}))\) there exists \(J_i \subseteq J\) such that \(R_i(\vec{a}) \in J_i\), \((I', J'_i) \models_\ell \Sigma^{*}\) in the model-theoretic sense and for no instance \(J_i' \subseteq J'_i\) it is that \((I', J'_i) \models_\ell \Sigma^{**}\).

**Definition 8.** The ABD-semantics for a source instance \(I\) and a set \(\Sigma^{**}\) of \textsf{abds} and \textsf{aegds} is defined as: \([[(I, \Sigma^{**})]]_{\text{abd}} := \{J : (I, J) \models_\ell \Sigma^{**} \text{ for some } \ell\}\).

Intuitively \([[(I, \Sigma^{**})]]_{\text{abd}}\) contains all target instances which tuples can be labeled in such a way that it satisfies all the rules from \(\Sigma^{**}\) wrt. their annotation and each tuple is inferred from the source and the dependencies.

**Example 10.** Consider source schema containing 3 relations: one for full-time employees (\textsf{Emp}) one for consultant employees (\textsf{Cons}) and binary relation (\textsf{Proj}) that maps projects to their location. The target schema consists of unary relation (\textsf{AllEmp}) for all employees and ternary relation (\textsf{EmpP}) that pairs employees with projects and location. The following mapping specifies: that each full-time employee (\(\xi_1\)) and consultant employee (\(\xi_2\)) works on at least one project in the location specified by \textsf{Proj} ("\(-\to\)" from \(\xi_3\) and \(\xi_4\)) for each project there exists at least one full-time and at least one consultant employee that works on that project ("\(-\to\)" from \(\xi_3\) and \(\xi_4\)) and all full-time employees are involved only in projects from the same location (\(\xi_5\)).

\[
\begin{align*}
\xi_1 : & \hspace{1em} \textsf{Emp}(eid) \leftrightarrow \textsf{EmpP}^1(eid, pid, lid), \textsf{AllEmp}^1(eid) \\
\xi_2 : & \hspace{1em} \textsf{Cons}(cid) \leftrightarrow \textsf{EmpP}^2(cid, pid, lid), \textsf{AllEmp}^2(cid) \\
\xi_3 : & \hspace{1em} \textsf{Proj}(pid, lid) \leftrightarrow \textsf{EmpP}^1(eid, pid, lid) \\
\xi_4 : & \hspace{1em} \textsf{Proj}(pid, lid) \leftrightarrow \textsf{EmpP}^2(cid, pid, lid) \\
\xi_5 : & \hspace{1em} \textsf{EmpP}^1(eid, pid_1, lid_1), \textsf{EmpP}^1(eid, pid_2, lid_2) \\
& \quad \rightarrow \text{lid}_1 = \text{lid}_2
\end{align*}
\]

Let instance \(I\) be \(\textsf{Emp}^I = \{(e_1), (e_2)\}\), \(\textsf{Cons}^I = \{(c_1)\}\) and \(\textsf{Proj}^I = \{(p_1, ny), (p_2, hk)\}\). Consider target instances \(J_1, J_2\) and \(J_3\) together with their tuple-labeling functions \(\ell_1, \ell_2\) and \(\ell_3\):
Clearly \( J_1 \notin \llbracket (I, \Sigma^*) \rrbracket_{\text{abd}} \) because it does not satisfy \( \text{abd} \) \( \xi_1 \), offending tuple \( \text{Emp}(e_2) \). On the other hand, even if \( (I, J_2, \ell_2) \models \Sigma^* \), we have that \( (I, J_2) \not\models \ell_2 \Sigma^* \) because \( 3 \in \ell_2(\text{Emp}(e_1, p_1, ny)) \) and no \( \text{abd} \) infers annotation 3. Finally, for \( J_3 \) and \( \ell_3 \) we have \( (I, J_3) \models_3 \Sigma^* \).

The following theorem relates the inference-based semantics to ABD-semantics showing that the language of annotated bidirectional dependencies is a good language to express the inference-based semantics. This, together with the results from Section 5 and 6 will help to develop a framework to compute “universal” target tables used to evaluate general queries over the target schema under this semantics.

**Theorem 1.** Let \( \Sigma \) be a set of s-t tgds and safe egds, then there exists a set \( \Sigma^* \) of abds and aegds, with \( \#_D(\Sigma^*) = 1 \), such that for any instance \( I \) we have \( \llbracket (I, \Sigma) \rrbracket_{\text{inf}} = \llbracket (I, \Sigma^*) \rrbracket_{\text{abd}} \).

One of the important result of this theorem is that the annotation density needed to represent the inference-based semantics is 1 and, as it will be shown next, this ensures a sufficient condition for a “universal” target table to exists. Clearly the converse of this theorem does not hold, for this let \( \Sigma^* = \{ R(x, y) \leftrightarrow T^s(x), S^t(y) \} \) and consider source instance \( R^I = \{(a, b), (c, d)\} \). We have \( \llbracket (I, \Sigma^*) \rrbracket_{\text{abd}} = \emptyset \), on the other hand for any \( \Sigma \) \( \llbracket (I, \Sigma) \rrbracket_{\text{inf}} \neq \emptyset \). The following example gives us a hint on how the set \( \Sigma \) from the theorem is transformed into \( \Sigma^* \).

**Example 11.** Consider schema for projects (\( P \)) project tasks (PT) and task employee (TE). To this we add target unary relation PR for the list of projects on the target schema. Consider the set \( \Sigma \) containing a single source-to-target
dependency: \( P(p, e) \rightarrow \exists t\ PT(p, t), TE(t, e), PR(p) \). For the set \( \Sigma^{++} = \{ \xi_1, \xi_2 \} \), where
\[
\begin{align*}
\xi_1 : & \ P(p, e) \leftrightarrow PT^1(p, t), TE^1(t, e); \\
\xi_1 : & \ P(p, e) \leftrightarrow PR^1(p);
\end{align*}
\]
we have \([I, \Sigma]_{\text{inf}} = [I, \Sigma^{++}]_{\text{abd}}\) for any \( I \).

### 4.1 ABD Solutions

We will now investigate the membership and the existence problems under ABD-semantics. The complexity of these problems will give us an intuition on under what circumstances one will be able to compute a finite representation of the semantics on the target database. We found that the complexity for the existence problem is directly related to the annotation density and the membership problem is connected to the annotation cardinality. The ABD-solution existence problem asks if there exists at least one instance part of the semantics for the given input instance and mapping:

| Problem      | SOL-EXISTENCE_{ABD}(\Sigma^{++}) |
|--------------|-----------------------------------|
| Input        | \( I \in \text{Inst}(S) \).       |
| Question     | Is \( [I, \Sigma^{++}]_{\text{abd}} \neq \emptyset \)? |

Similarly, the ABD-solution checking problem verifies if a target instance is part of the semantics under the given mapping and input source instance:

| Problem      | SOL-CHECK_{ABD}(\Sigma^{++}) |
|--------------|-------------------------------|
| Input        | \( I \in \text{Inst}(S) \) and \( J \in \text{Inst}(T) \). |
| Question     | Is \( J \in [I, \Sigma^{++}]_{\text{abd}} \)? |

The following dichotomy result directly relates the solution-existence problem to annotation density.

**Theorem 2.** Let \( \Sigma^{++} \) be a set of abds and safe aegds, then SOL-EXISTENCE_{ABD}(\Sigma^{++}) problem:

- can be solved in polynomial time if \( \#D(\Sigma^{++}) = 1 \),
- is in NP if \( \#D(\Sigma^{++}) > 1 \) and there exists \( \Sigma^{++} \) with \( \#D(\Sigma^{++}) = 2 \) such that the problem is NP-hard.

Note that the complexity of solution existence problem is not influenced by the annotation cardinality. That is the tractable result for annotation density equal to 1 is maintained for any annotation cardinality. Also, as shown in the reduction, the problem is NP-hard for annotation density 2 even with annotation cardinality 1.

Similarly to the existence problem, we have a dichotomy result for the solution check problem. In this case the complexity delimiting factor is the cardinality.
Theorem 3. Let $\Sigma^{**}$ be a set of abds and safe egds, then the $\text{SOL-CHECK}_{\text{ABD}}(\Sigma^{**})$ problem:

- can be solved in polynomial time if $\#_C(\Sigma^{**}) = 1$,
- is in NP if $\#_C(\Sigma^{**}) > 1$ and there exists a $\Sigma^{**}$ with $\#_C(\Sigma^{**}) = 2$ such that the problem is NP-hard.

The proof attached in the Appendix B clarifies that the complexity of the solution-existence problem depends only on the annotation cardinality. The NP-hardness result is obtained even with annotation density 1.

5 Universal Representatives

As mentioned in the introduction, data exchange transforms a database existing under a source schema into another database under the target schema. This means that for a given semantics it would be preferable to be able to materialize one or more table representations on a target. The materialized table(s) could later be used to obtain answers for different queries over the target database. In [8] it was shown that for the OWA-semantics expressed as a set of s-t tgds and target egds, there exists a universal solution that can be represented as a naïve table under the target database. This universal solution can be used to obtain the certain answers to any UCQ query. Need to mention that Deutsch et al. in [7] materialize a set of naïve tables and Grähne and O. in [15] use conditional tables to be able to retrieve certain answers for a larger range of queries under their semantics. In this section we will introduce a new type of table capable of representing, under some restrictions, all solutions for ABD-semantics. Thus this new table can be used to obtain the certain answers for any FO query.

Definition 9. Let $\text{Nulls}$ be partitioned in two countable infinite sets $\text{Nulls}^o$ and $\text{Nulls}^c$. A semi-naïve table is a naïve table $T$ for which each null is identified as being either from $\text{Nulls}^o$ or $\text{Nulls}^c$. The semi-naïve table $T$ has the following interpretation:

$$\text{Rep}(T) := \{ J = v(\bigcup_{i=1}^{n} v_i(T)) : n \text{ an integer}, \\ v \text{ valuation over } \text{Nulls}^c, v_i \text{ valuation over } \text{Nulls}^o \}$$

The nulls from $\text{Nulls}^o$ are called open and denoted $\bot^o$ (possibly subscripted). The ones from $\text{Nulls}^c$ are called closed and denoted $\bot^c$ (possibly subscripted).

Example 12. Let $T$ be the semi-naïve table with $R^T = \{(a, \bot^o_1, \bot^c_1, \bot^o_2)\}$. We have $I_1, I_2, I_3 \in \text{Rep}(T)$, where $R^{I_1} = \{(a, a, b, c)\}$, $R^{I_2} = \{(a, a, b, c), (a, b, b, c)\}$ and $R^{I_3} = \{(a, a, b, a), (a, b, b, a), (a, c, b, a)\}$. On the other hand, for $R^{I_4} = \{(a, a, b, c), (a, a, b, d)\}$, we have that $I_4 \notin \text{Rep}(T)$, because closed null $\bot^c_2$ was valued to both $c$ and $d$. 
To a semi-na"ive table \( T \) we may add a global condition \( \varphi^* \), denoted \( (T, \varphi^*) \),
as a conjunction of the form \( \delta_1 \wedge \delta_2 \wedge \ldots \wedge \delta_n \) and each conjunct \( \delta_i \), \( 1 \leq i \leq n \), is a
disjunction of unequations over the elements from \( \text{dom}(T) \). Given \( v \) a valuation
over Nulls\(^c\) and \( v_1, v_2, \ldots, v_n \) valuations over Nulls\(^o\), for some integer \( n \),
we say that \( (v, \{v_1, v_2, \ldots, v_n\}) \) satisfies \( x \neq y \), denoted \( (v, \{v_1, v_2, \ldots, v_n\}) \models (x \neq y) \),
iff:

- \( v(x) \neq a \), when \( x \in \text{Nulls}\(^c\) \) and \( y = a \in \text{Cons} \);
- \( v_i(x) \neq a \), for all \( i \leq n \), when \( x \in \text{Nulls}\(^o\) \) and \( y = a \in \text{Cons} \);
- \( v(x) \neq v_i(y) \), for all \( i \leq n \), when \( x \in \text{Nulls}\(^c\) \) and \( y \in \text{Nulls}\(^o\) \);
- \( v(x) \neq v(y) \), when \( x, y \in \text{Nulls}\(^c\) \); and
- \( v_i(x) \neq v_j(y) \), for all \( i, j \leq n \), when \( x, y \in \text{Nulls}\(^o\) \).

The previous notion is naturally extended to a disjunction of unequations and
to the conjunctive formula \( \varphi^* \) where each conjunct represents a disjunction
of unequations. This is denoted \( (v, \{v_1, v_2, \ldots, v_n\}) \models \varphi^* \). With this we can define
the interpretation of \( (T, \varphi^*) \) as:

\[
\text{Rep}(T) := \{ J = \{ v(\bigcup_{i=1}^{n} v_i(T)) : n \text{ an integer}, \
\quad \text{valuation over Nulls}\(^c\), v_i \text{ valuation over Nulls}\(^o\), \\
\quad \text{and } (v, \{v_1, \ldots, v_n\}) \models \varphi^* \} \}
\]

Example 13. Consider \( (T, \varphi^*) \), where \( T \) and the instances are the same as in
Example 12 and global condition \( \varphi^* := (\bot \neq a \vee \bot \neq a) \). It can be verified that
we have \( I_1, I_2 \in \text{Rep}(T, \varphi^*) \) and \( I_3 \notin \text{Rep}(T, \varphi^*) \) because the tuple \( R(a, a, b, a) \)
in \( I_3 \) was obtained from valuations \( v = \{ \bot, a, a \} \) and \( v_1 = \{ \bot, a \} \) and
\( (v, \{v_1\}) \models \varphi^* \).

The following result shows that for a fixed \( \Sigma^* \) with \( \#_D(\Sigma^*) = 1 \) and an
input source instance, we may compute in polynomial time a representative for its
ABD-semantics.

**Theorem 4.** Let \( \Sigma^* \) be a set of abds and safe aegds with \( \#_D(\Sigma^*) = 1 \). Then
either there exists \( (T, \varphi^*) \), computable in polynomial time in the size of \( I \), such
that \( \{I, \Sigma^*\}_{\text{abd}} = \text{Rep}(T, \varphi^*) \), or \( \{I, \Sigma^*\}_{\text{abd}} = \emptyset \).

The pair \( (T, \varphi^*) \) from the previous theorem, if it exists, is called *universal
representative* for \( I \) and \( \Sigma^* \). In Theorem 4 the universal representative is computed
using a 3-step chase process: a) \( \rightarrow \)”, similar with the chase from 3 with
the difference that the abds will create new nulls from Nulls\(^o\); b) \( = \)” when an
aegd equates two open nulls or an open null with a closed null in the result,
they both will be replaced with a new closed null; c) \( \leftarrow \)”, the chase in the
other direction will be used to construct the global condition \( \varphi^* \) or it fails in
case there exists a constant in two positions supposed to be distinct. For the
complete annotated-chase algorithm please refer to Appendix A.
Example 14. Consider \( \Sigma^{**} = \{ \xi_1, \xi_2, \xi_3 \} \), where

\[
\begin{align*}
\xi_1 & : \quad S(x, y) \leftrightarrow K^1(x, z), V^1(z, y); \\
\xi_2 & : \quad R(x) \leftrightarrow U^1(x, y); \\
\xi_3 & : \quad U^1(x, y), K^1(x, z) \rightarrow y = z.
\end{align*}
\]

Let instance \( I \) be \( S^I = \{(a, b), (c, d)\} \) and \( R^I = \{(a)\} \). In this case a universal representative for \( I \) and \( \Sigma^{**} \) is the pair \((T, \varphi^*)\), where \( K^T = \{(a, \perp^1), (c, \perp^0)\} \), \( V^T = \{(\perp_1^1, b), (\perp_0^0, d)\} \), \( U^T = \{(a, \perp^1_1)\} \), \( \varphi^* = (\perp_1^1 \neq \perp_0^0) \). The open null \( \perp_1^1 \) was obtained by simply chasing \( \xi_1 \) in the right direction with source tuple \( S(c, d) \); the closed null \( \perp_1^1 \) was obtained by equating the two open nulls obtained by the chase process in the right direction from \( \xi_1 \) and \( \xi_2 \) with the source tuples \( S(a, b) \) and \( R(a) \) respectively. Finally, the condition \( \perp_1^1 \neq \perp_0^0 \) is obtained by chasing to the left with tuples \( \{K(a, \perp_1^1), V(\perp_0^0, d)\} \). Note that when chasing to the left we consider that null values may equate to any other value.

In order to give a syntactic restriction for an \( \Sigma^{**} \) that will ensure that the global condition is tautological, we need to introduce first few notations.

For a set \( \Sigma^{**} \), with \( \Sigma^{**} \) is denoted the set of \( s-t \) tgds and egds obtained from \( \Sigma^{**} \) by removing all annotations and each \( abd \) specified as in [11] is replaced with the following \( s-t \) tgd: \( \forall \bar{x} (\forall y \alpha(x, y) \rightarrow \exists z \beta(x, z)) \). A set \( \Sigma \) of \( s-t \) tgds is said to be GAV-reducible if there exists a set \( \Sigma' \) of GAV \( s-t \) tgds such that \( \Sigma \) is logically equivalent to \( \Sigma' \). A set \( \Sigma \) is said to be non-redundant if none of the \( s-t \) tgds from \( \Sigma \) contains repeated atoms in the head.

The following proposition gives a necessary and sufficient condition for a set of \( s-t \) tgds to be GAV-reducible.

**Proposition 1.** A non-redundant set \( \Sigma \) of \( s-t \) tgds is GAV-reducible iff for each tgd in \( \Sigma \) every existentially quantified variable occurs only in one atom in the head of the tgd.

With this, we can presents a syntactical condition for a set of dependencies \( \Sigma^{**} \) to ensure that the universal representative does not have a global condition for input instance \( I \) and set \( \Sigma^{**} \).

**Proposition 2.** Let \( \Sigma^{**} \) be a set of abds and safe aegds with \( \#D(\Sigma^{**}) = 1 \) such that \( \Sigma^{**} \) is GAV-reducible and let \( I \) be a source instance. Then either there exists a semi-na"ive table \( T \), computable in polynomial time in the size of \( I \) such that \( \lbrack (I, \Sigma^{**}) \rbrack_{\text{abd}} = \text{Rep}(T) \) or \( \lbrack (I, \Sigma^{**}) \rbrack_{\text{abd}} = \emptyset \).

6 Query Answering

If in the previous section we showed how we can compute universal representatives for ABD-semantics, we will focus here on when and how these representatives can be used to compute certain answers for different query classes and check the complexities of such evaluations. Based on Theorem [1] it follows that
all tractable result presented here are applicable for mappings specified by s-t tgds and safe egds under the inference-based semantics.

Let us first start by defining the certain answer evaluation problem for a schema mapping defined under the ABD-semantics. For this, let $\Sigma^{**}$ be a set of abds and safe aegds. Let $Q$ be a query over the target schema $T$. The query evaluation problem for mapping $\Sigma^{**}$ and query $Q$ is the following decision problem:

\[
\text{Problem Eval}_{\text{ABD}}(\Sigma^{**}, Q)
\]

**Input:** $I \in \text{Inst}(S)$ and $i \in \text{dom}(I)^{\text{arity}(f)}$.

**Question:** Is $i \in \text{cert}_{\text{abd}}(Q, (I, \Sigma^{**}))$?

The next dichotomy result tells us that one may search for tractable query evaluation only for mappings with annotation density equal to 1.

**Theorem 5.** Let $\Sigma^{**}$ be a set of abds and safe aegds. If $Q \in \text{UCQ}$, then $\text{Eval}_{\text{ABD}}(\Sigma^{**}, Q)$ problem:

- is polynomial if $\#D(\Sigma^{**}) = 1$ and one may use a universal representative $(T, \varphi^*)$, if it exists, to answer the problem and
- is in coNP if $\#D(\Sigma^{**}) > 1$ and there exists a mapping $\Sigma^{**}$ with $\#D(\Sigma^{**}) = 2$ and $Q \in \text{CQ}$ such that the problem is coNP-hard.

Note that if the universal representative $(T, \varphi^*)$ exists, then certain answers for UCQ queries can be computed using the naïve query evaluation on $T$ (see [8]). The following theorem ensures that for any mapping $\Sigma$ of s-t tgds and egds we can find a corresponding $\Sigma^{**}$ that agrees with $\Sigma$ on UCQ certain answers for any source instance.

**Theorem 6.** Let $\Sigma$ be a set of s-t tgds and safe egds. Then there exists a set $\Sigma^{**}$ of abds and safe aegds such that for any source instance $I$ and $q \in \text{UCQ}$ we have $\text{cert}_{\text{owa}}(Q, (I, \Sigma)) = \text{cert}_{\text{abd}}(Q, (I, \Sigma^{**}))$.

From this and Theorem 1 we have the following corollary that states that the OWA and inference-based semantics agree on UCQ certain answers.

**Corollary 1.** Let $\Sigma$ be a set of s-t tgds and target egds, then $\text{cert}_{\text{owa}}(Q, (I, \Sigma)) = \text{cert}_{\text{inf}}(Q, (I, \Sigma))$ for any source instance $I$ and $Q \in \text{UCQ}$.

From Theorem 5 we know that if the annotation density is 1, then we can compute in tractable time certain answers for UCQ. The following negative result shows that not all queries are tractable for mappings with annotation density 1.

**Theorem 7.** There exists a set $\Sigma^{**}$ of abds with $\#D(\Sigma^{**}) = 1$ and there exists a query $Q \in \text{CQ}$ such that the problem $\text{Eval}_{\text{ABD}}(\Sigma, Q)$ is coNP-complete.

From this it follows that for tractable query evaluation under ABD-semantics we need to restrict either the set $\Sigma^{**}$ or the query class used or both. In the last part of this section we will present such restrictions that ensure tractability for certain answers evaluation.
Proposition 3. Let $\Sigma^{*\ast}$ be a set of abds and safe aegds such that $\Sigma^{\rightarrow}$ is a collection of full s-t tgds and safe egds. Then for any FO query $Q$ the $\text{EVAL}_{\text{ABD}}(\Sigma^{*\ast},Q)$ is tractable.

Note that in the previous proposition there is no need for annotation density restriction. The following corollary can be easily verified.

Corollary 2. Let $\Sigma$ be a set of full s-t tgds, then there exists $\Sigma^{*\ast}$ such that $\Sigma^{\rightarrow}$ is a collection of full s-t tgds and $\llbracket (I, \Sigma^{*\ast}) \rrbracket_{\text{ab}} = \llbracket (I, \Sigma) \rrbracket_{\text{inf}} = \llbracket (I, \Sigma) \rrbracket_{\text{owa}}$.

With UCQ$^{\neq,1}$ is denoted the set of UCQ$^\neq$ queries with at most one inequality per disjunct.

Theorem 8. Let $\Sigma^{*\ast}$ be a set of abds and safe aegds with $\#D(\Sigma^{*\ast}) = 1$. Then for any UCQ$^{\neq,1}$ query $Q$ the $\text{EVAL}_{\text{ABD}}(\Sigma^{*\ast},Q)$ problem is tractable and one may use only the universal representative, if it exists, to evaluate the query.

Similar result was also shown for OWA [8] and CWA-semantics [20]. Because the aegds in the mapping are safe, we can use the universal representative, if it exists, to compute the certain answers for any UCQ$^{\neq,1}$. This can be extended to OWA-semantics too. Thus, if the schema mapping is defined as a set of tgds and safe egds, we may use any universal solution to evaluate certain answers for any UCQ$^{\neq,1}$ under OWA-semantics eliminating the need to re-chase the source instance for each query. The same as for OWA-semantics in case the UCQ$^\neq$ contains at most 2 inequalities per disjunct [25], the certain answers evaluation becomes intractable.

Theorem 9. There exists a set $\Sigma^{*\ast}$ of abds with $\#D(\Sigma^{*\ast}) = 1$ and there is a conjunctive query with two inequalities such that $\text{EVAL}_{\text{ABD}}(\Sigma^{*\ast},Q)$ problem is coNP-complete.

The following example shows that the certain answers under inference-based semantics (and implicit under ABD-semantics) may differ from the certain answers under OWA-semantics even for UCQ$^{\neq,1}$ queries (based on Corollary 1 the semantics agree on UCQ queries).

Example 15. Let $\Sigma = \{ R(x, y) \rightarrow \exists z S(x, z), V(z, y) \}$ and source let instance $I$ be $R^I = \{(a, b), (c, d)\}$. The universal representative for the corresponding abd set $\Sigma^{*\ast} = \{ R(x, y) \leftrightarrow S^1(x, z), V^1(z, y) \}$ and for the given source instance $I$ is $(T, \varphi^*)$, with $S^T = \{(a, \bot_1), (c, \bot_2)\}$, $V^T = \{ (\bot_1, b), (\bot_2, d) \}$, and $\varphi^*:=(\bot_1 \neq \bot_2)$. Consider $Q := \exists x \exists y \exists z_1 \exists z_2 S(x, z_1), V(z_2, y), z_1 \neq z_2$. It can be verified that $\text{cert}_{\text{ow}}(Q, (I, \Sigma)) = \text{false}$ and $\text{cert}_{\text{inf}}(Q, (I, \Sigma)) = \text{cert}_{\text{ab}}(Q, (I, \Sigma)) = \text{true}$.

In [15] Hernich showed that if the mapping is given by a restricted set of s-t tgds (packed s-t tgds), then the certain answers evaluation problem may be answered in polynomial time for universal queries under the GCWA*-semantics. Where a universal query is one of the form $Q(\vec{x}) := \forall \vec{y} \beta(\vec{z}, \vec{y})$, with $\beta$ a quantifier-free FO formula over the target schema. In our next result we show that similar polynomial time can be achieved under the ABD-semantics even without any restriction on the s-t tgds and also by adding safe target aegds.
Theorem 10. Let \((T, \varphi^*)\) be a universal representative for some source instance \(I\) and a set \(\Sigma^{**}\), of abds and safe aegds with \(\#_D(\Sigma^{**}) = 1\). Then there exists a polynomial time algorithm, with input \((T, \varphi^*)\) and \(t \subset \text{Cons}\), such that for any universal query \(Q\) decides if \(t \in \bigcap_{J \in \text{Rep}(T, \varphi^*)} Q(J)\).

From this theorem and Theorem 4 it directly follows that the \(\text{Eval}_{\text{ABD}}\) problem is polynomial for universal queries.

Let us now take a look at \(\text{CQ}^\neg\) queries. Theorem 7 showed that the certain answers evaluation is \(\text{coNP}\)-hard for \(\text{CQ}^\neg\) queries even if the mapping has the annotation density 1. Next we will present a subclass of \(\text{CQ}^\neg\) that has tractable query evaluation properties for a restricted class of abds and safe aegds. Let \(\text{CQ}^\neg_{1}\) denote the subclass of \(\text{CQ}^\neg\) such that each query of this class has exactly one positive atom. With this we have the following positive result:

Theorem 11. Let \(\Sigma^{**}\) be a set of abds and safe aegds with \(\#_D(\Sigma^{**}) = 1\) and such that \(\Sigma^{\rightarrow}\) is \(\text{GAV}\)-reducible and each aegd does not equate two variables both occurring in affected positions. Then for any \(\text{CQ}^\neg_{1}\) query the \(\text{Eval}_{\text{ABD}}(\Sigma^{**}, Q)\) problem is polynomial and can be decided using a universal representative.

Intuitively, the restrictions on the mapping language from the previous theorem ensure that the universal representative does not have any global condition (see Proposition 2), it contains only open nulls and because the query contains only one positive atom it ensures that the Gaifman-blocks, from the universal representative that match this atom, are bounded in size by a constant value (depending only on \(\Sigma^{**}\)).

Table 1 summarize the tractable results presented in this paper together with the known tractable query evaluation results from the other semantics. Note that for all semantics we considered only \(s-t\ tgd\)s and target egds, even if some of these results also hold for restricted classes of target tgd\s.

| Query/ Semantics | UCQ | \(\text{UCQ}^\neg_{1}\) | universal queries | FO | \(\text{CQ}^\neg_{1}\) |
|------------------|-----|-----------------|------------------|----|-----------------|
| OWA              | \(s-t\ tgd\) + egds [8] | \(s-t\ tgd\) + egds [8] | - | - | - |
| CWA              | \(s-t\ tgd\) + egds [20] | \(s-t\ tgd\) + egds [20] | full \(s-t\ tgd\) + egds [20] | - | - |
| GCWA*            | \(s-t\ tgd\) + egds [18] | packed \(s-t\ tgd\) [18] | full \(s-t\ tgd\) + egds [18] | - | - |
| ABD \(\Sigma^{**}\) | ABD \(\Sigma^{**}\) | ABD \(\Sigma^{**}\) | ABD \(\Sigma^{**}\) | ABD \(\Sigma^{**}\) | ABD \(\Sigma^{**}\) | ABD \(\Sigma^{**}\) |
| \(\text{Theorem}\) | \(\text{Theorem}\) | \(\text{Theorem}\) | \(\text{Proposition}\) | \(\text{Theorem}\) | \(\text{Theorem}\) |
7 Conclusions

In this paper we introduced two new semantics. One of them (inference-based semantics) relying on s-t tgds and target egds language and the second one based on richer language of annotated bidirectional dependencies. We showed that the inference-based semantics solves most of the certain-answers anomalies existing in the existing semantics and using the language of abds one may compute a universal representative that exactly represents the semantics.

As shown, the language based in abds is much more expressive than the one based on tgds, as we could express the later one using only abds with density 1. Thus, the work presented here is only the first step for a full understanding of this new language and semantics. Even with this, one may be interested in considering target abds, to further increase its expressibility. For the certain answer semantics it remained an open problem if one can evaluate certain-answers for any \( CQ^{-1} \) queries in polynomial time for any \( \Sigma^{++} \) and not only for the restricted class of dependencies presented here.

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A ABD Chase

In this section we will describe the chase process for a given source instance \( I \) and \( \Sigma \leftrightarrow \) a set of abds and safe aegds. The annotated chase algorithm can be represented as a sequence of three steps that will result in computing a pair \((T, \varphi^*)\):

1. “→” chase;
2. “egd” chase; and
3. “←” chase.

1) The “→” chase constructs a table \( T_1 \) and a tuple-labeling function \( \ell \) by chasing the tgds from \( \Sigma \rightarrow \) with \( I \), similarly to the oblivious chase algorithm \([6]\) with the difference that the new nulls created are from Nulls\(^o\) and the labeling function maps the generated tuple with the annotation of the atom in the dependency that generated that tuple.

2) The “egd” chase step takes the table \( T_1 \) and tuple-labeling function \( \ell \) obtained from the previous step and either change it into table \( T_2 \) or will fail the chase algorithm. Let \( \xi_e \) be an aegd of the form: \( \alpha(\bar{x}) \rightarrow x = y \), where \( \alpha \) is a conjunction of annotated atoms from the target schema and both variables \( x \) and \( y \) occur in \( \bar{x} \). Given a target naïve table \( T \) and a labeling function \( \ell \), a trigger \( \tau \) for \( T \) and \( \xi_e \) is a pair \((h, \xi_e)\), where \( h \) is a homomorphism such that \( h(\alpha(\bar{x})) \subseteq T \) respecting the annotations given by \( \ell \). An egd chase step with trigger \((h, \xi_e)\) is said to fail, denoted \( T(h, \xi_e) \rightarrow^\ell \perp \), if \( h(x) \neq h(y) \) and both \( h(x), h(y) \in \text{Cons} \). A non-failing egd chase step transforms instance \( T \) into \( T' \), denoted \( T(h, \xi_e) \rightarrow^\ell T' \), where

- \( T' \) is equal with \( T \), if \( h(x) = h(y) \);
- \( T' \) is obtained from \( T \) by replacing each occurrence of \( h(x) \) and \( h(y) \) with constant \( a \), if either \( h(x) = a \) or \( h(y) = a \), this change is reflected as well to \( \ell \);
- \( T' \) is obtained from \( T \) by replacing each occurrence of \( h(x) \) and \( h(y) \) with new closed null from Nulls\(^c\), otherwise. This change is reflected as well to \( \ell \).

Starting with \( T_1 \) the previous steps are applied either until the table is not changed, or if one egd step fails. In case it fails we say that the annotated chase fails. Otherwise, let \( T_2 \) be the instance obtained in this step.

3) The “←” chase step takes semi-naïve table \( T_2 \) and labeling function \( \ell \) computed in previous steps and either fails or outputs the global condition \( \varphi^* \). For this given an abd \( \xi \) of the form: \( \alpha(\bar{x}, \bar{y}) \leftrightarrow \beta(\bar{x}, \bar{z}) \), let \( \xi^- \) denotes the following sentence: \( \beta(\bar{x}, \bar{z}) \rightarrow \exists \bar{y} \alpha(\bar{x}, \bar{y}) \). For a set \( \Sigma^* \), we denote \( \Sigma^- \) the set of annotated target-to-source tgds obtained by replacing each bidirectional dependency with the unidirectional dependency as mentioned before. A “←” trigger, for \( T_2 \) and a target-to-source annotated tgd \( \xi^- \) is a pair \((H, \xi^-)\), where \( H \) is a set of homomorphisms \( \{h_1, h_2, \ldots, h_k\} \), \( k \) represents the number of atoms in the body of \( \xi^- \), such that for each atom \( R_i(\bar{x}) \) from the body of \( \xi^- \) we have
$h_i(R_i(\bar{v})) \subseteq T_2$, the annotation of atom $R_i(\bar{v})$ in $\xi$ is in $\ell(h_i(R_i(\bar{v})))$ and for all variables $x \in \bar{x} \cup \bar{z}$ we have either:

- $h_i(x) = h_j(x)$ for all $i, j \in \{1, \ldots, k\}$, or
- if $h_i(x) \neq h_j(x)$, for some distinct $i, j \in \{1, \ldots, k\}$, then either $h_i(x)$ or $h_j(x)$ is a null.

For the set $H$ of homomorphisms the mapping $h_H$ is defined such that for each $x \in \bar{x} \cup \bar{z}$:

$$h_H(x) = \begin{cases} h_1(x), & \text{if } h_1(x) = h_2(x) = \ldots = h_k(x) \\ h_j(x), & \text{otherwise, where } h_j(x) \text{ is a constant} \\ \text{for some } j \in \{1, \ldots, k\}, \\ \text{or } \forall i \in \{1, \ldots, k\}, h_i(x) \notin \text{Cons} \end{cases}$$

We say that $(H, \xi^-)$ generates naïve table $T'$, denoted $T_2 \xrightarrow{(H, \xi^-)} T'$, if $T' = h'(\alpha(\bar{x}, \bar{y}))$, for some extension $h'$ of $h_H$ that assigns a new null value for each variable from $\bar{y}$. If there is no homomorphism from table $T'$ into $I$, then let $\varphi(H, \xi^-):=\neg(\bigwedge_{x \in \bar{x}} \bigwedge_{1 \leq i, j \leq k} h_i(x) = h_j(x))$. If $\varphi(H, \xi^-)$ for some trigger $(H, \xi^-)$ is a contradiction, we say that the annotated chase algorithm fails. Otherwise, let $\varphi^*$ be the conjunctions of all formulae $\varphi$ constructed in the previous process, that is:

$$\varphi^* := \bigwedge_{\xi' \in \Sigma^{**}} \bigwedge_{(H, \xi^-) \text{ trigger}} \varphi(H, \xi^-).$$

Finally, if the algorithm does not fail it will return pair $(T_2, \varphi^*)$.

**Example 16.** Consider the following set $\Sigma^{**}$:

$$\xi_1 : \quad R(x, y) \leftrightarrow S^1(x, z), S^2(y, z), V^1(x, z);$$

$$\xi_2 : \quad V^1(x, z_1), S^2(x, z_2) \rightarrow z_1 = z_2.$$  

Let source instance $I$ be $R^I = \{(a, b), (c, a)\}$. The “→” step will construct $T_1$ and tuple-labeling function $\ell$, with $S^T_1 = \{(a, \bot), (c, \bot), (1, b, \bot), (2, a, \bot) : 2, (a, \bot) : 2\}$, and $V^T_1 = \{(a, \bot) : 1, (c, \bot) : 1\}$. After applying the “ød” steps we obtain semi-naïve table $T_2$ and corresponding tuple-labeling function $\ell$, where $S^{T_2} = \{(a, \bot), (c, \bot), (1, b, \bot) : 2, (a, \bot) : 2\}$, $V^{T_2} = \{(a, \bot), (c, \bot) : 1\}$. Finally, for “ecd” step we have that $(H, \xi^-)$ is a trigger for $T_1$, where $H = \{h_1 = \{x/a, z/\bot\}, h_2 = \{y/a, z/\bot\}, h_3 = \{x/a, z/\bot\}\}$. With this we obtain $\varphi(H, \xi^-) = \neg\alpha = a \land \bot_1 = \bot_1$ which is a contradiction. From this it follows that the annotation chase will fail. Note that if we remove $\xi_2$ from $\Sigma^{**}$, $T_2$ would be equal to $T_1$ and the global condition $\varphi^*$ would be equivalent to $\varphi^* := (\bot_1 \neq \bot_2)$.
B Sketch Proofs

In this section we provide sketch proofs for the main results presented in the paper. The complete proofs will be provided in the full version of this paper.

In order to show the proof of Theorem 1 we need to introduce first a few notations.

Definition 10. The Gaifman graph $G^I$ for a table $T$ is an undirected graph with vertex set $\text{dom}(T) \cap \text{Nulls}$ and an edge between two vertices $x$ and $y$ if $x$ and $y$ occurs together in a tuple of $T$. A block is a connected set of nulls in $G^T$.

Definition 11. Let $T$ be a table. A set $\{T_1, T_2, \ldots, T_n\}$ is called a Gaifman partition of $T$ if the following holds:

- $\{T_1, T_2, \ldots, T_n\}$ is a partition of $T$; and
- for each $x \in \text{Nulls}(T)$ there exists a exactly one $i$ such that $x \in \text{Nulls}(T_i)$; and
- if nulls $x$ and $y$ are in the same block of $G^T$, then there exists exactly one $i$ such that $x, y \in \text{Nulls}(T_i)$; and
- if $t \in T$ and $t$ does not contain any nulls, then there exists $i$ such that $T_i = \{t\}$.

Clearly for each table $T$ there exists a unique Gaifman partition of $T$.

Theorem 1. Let $\Sigma$ be a set of s-t tgds and safe egds, then there exists a set $\Sigma^\leftrightarrow$ of abds and aegds, with $\#D(\Sigma^\leftrightarrow) = 1$, such that for any instance $I$ we have $\llbracket (I, \Sigma) \rrbracket_{\text{inf}} = \llbracket (I, \Sigma^\leftrightarrow) \rrbracket_{\text{abd}}$.

Proof. Construct $\Sigma^\leftrightarrow$ from $\Sigma$ as follows: for each $\text{tgd} \ \xi \in \Sigma$, where $\xi : \alpha(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} \beta(\bar{x}, \bar{z})$, add abds $\alpha(\bar{x}, \bar{y}) \leftrightarrow \beta'_i(\bar{x}, \bar{z})$, where $\beta'_i(\bar{x}, \bar{z})$ is a table from the Gaifman partition of $\beta(\bar{x}, \bar{z})$ (where each variable from $\bar{x}$ is treated as a constant and variables from $\bar{z}$ are treated as nulls) and $\beta'_i$ is obtained from $\beta$ by annotating each relation with a distinct integer. For each $\text{egd} \ \xi \in \Sigma$, where $\xi : \alpha(\bar{x}, \bar{y}) \rightarrow x = y$ add aegds $\alpha_i(\bar{x}, \bar{y}) \rightarrow x = y$ to $\Sigma^\leftrightarrow$, where $\alpha_i$ is obtained from $\alpha$ by annotating each relation with annotations used in abds for the same relation. Note that if an egd from $\Sigma$ contains a relation name not occurring in any tgds, then that egd will not be reflected in $\Sigma^\leftrightarrow$. Because each tgds is mapped to a set of abds in $\Sigma^\leftrightarrow$ based on the Gaifman partitioning it follows that there always exists a solution for any source instance $I$ and the constructed $\Sigma^\leftrightarrow$, of course if the equality dependencies are satisfied. From here it is a simple exercise to verify that all properties of the inference-based semantics are fulfilled by $\Sigma^\leftrightarrow$ under ABD-semantics. Thus, it follows that for any source instance $I$, $\llbracket (I, \Sigma^\leftrightarrow) \rrbracket_{\text{abd}} = \llbracket (I, \Sigma) \rrbracket_{\text{inf}}$

For the next theorem we need the following result.
Lemma 1. If a graph is 3-colorable but not 2-colorable, then for any 3 coloring of the graph there exists at least an edge between any two distinct colors.

With this we can now prove the first dichotomy theorem.

**Theorem 2.** Let $\Sigma^{**}$ be a set of abds and safe aegds, then $\text{Sol-Existence}_{\text{ABD}}(\Sigma^{**})$ problem:

- can be solved in polynomial time if $\#D(\Sigma^{**}) = 1$,
- is in $\textbf{NP}$ if $\#D(\Sigma^{**}) > 1$ and there exists $\Sigma^{**}$ with $\#D(\Sigma^{**}) = 2$ such that the problem is $\textbf{NP}$-hard.

**Proof.** In case $\#D(\Sigma^{**}) = 1$, from Theorem 1 we know that if for a source instance $I$ the universal representative $(T, \phi^*)$ exists, then $\text{Rep}(T, \phi^*) = \llbracket (I, \Sigma^{**}) \rrbracket_{\text{abd}}$. From the construction of $(T, \phi^*)$ we know that $\phi^*$ is always satisfiable (just assign new distinct constant values for each null). Thus, the problem reduces itself to find if such universal representative exists. This can be done in polynomial time in the size of $I$ using the annotated chase algorithm from Section A.

For the second part it is clear that the solution existence problem is in $\textbf{NP}$ for any $\#D(\Sigma^{**}) > 1$ as one may guess a target instance $J$ and labeling function $\ell$ that labels each tuple with a set of integer, the values from the set being restricted from a constant set given by $\Sigma^{**}$. Then test in polynomial time that $(I, J) \models_{\ell} \Sigma^{**}$.

For the completeness part we will use a reduction from the graph 3-colorability problem known to be $\textbf{NP}$-complete. We could not use directly the reduction from 4 because that reduction has $\#D(\Sigma^{**}) = 4$. Our reduction not only has $\#D(\Sigma^{**}) = 2$ but $\#C(\Sigma^{**}) = 1$. The restriction that $\#C(\Sigma^{**}) = 1$ is important in order to show that the annotation cardinality does not influence the solution existence problem. For this let us consider the following set of abds $\Sigma^{**}$:

$$
\begin{align*}
V(x) &\leftrightarrow B^1(x, v), C^1(x, v); \\
E_0(x, y) &\leftrightarrow E^1(x, y); \\
D(z, v) &\leftrightarrow B^1(x, z), C^1(y, v), E^1(x, y).
\end{align*}
$$

To this consider source instance $I$ that represents under unary relation $V$ all the vertexes of a graph $G$, under binary relation $E_0$ all the edges in the graph. And finally, under binary relation $D$ we’ll have all the combinations of two distinct colors from the set $\{r, g, b\}$. Based on the previous lemma, we know that if the graph is 3-colorable but not 2-colorable, there will be an edge between any 2 distinct colors, thus the third abd holds if the graph is 3-colorable but not 2-colorable. This means that our reduction needs to verify first in polynomial time if graph $G$ is 2-colorable. If it is, then return $\text{true}$. If it is not 2-colorable we create source instance $I$ from $G$ as mentioned before. With this and Lemma 1 it is obvious that the graph $G$, not 2-colorable is 3-colorable iff the solution existence problem returns $\text{true}$. From this it follows that there exists $\Sigma^{**}$ with $\#D(\Sigma^{**}) = 2$ for which the problem is $\textbf{NP}$-complete.
Theorem 3. Let $\Sigma^{**}$ be a set of abds and safe aegds, then the SOL-CHECK\textsubscript{ABD}(M) problem:

- can be solved in polynomial time if $\#_C(\Sigma^{**}) = 1$,
- is in NP if $\#_C(\Sigma^{**}) > 1$ and there exists a $\Sigma^{**}$ with $\#_C(\Sigma^{**}) = 2$ such that the problem is NP-hard.

Proof. In case $\#_C(\Sigma^{**}) = 1$, the solution-check problem is the same as equivalent to check if the instance $J$ together with $I$ satisfies a set of FO formulae, problem known to be tractable for a fix set of formulae. For the second part, given target instance $J$, one may guess in polynomial time a labeling function $\ell$ for $J$ that takes values from a fixed set of integers determined by $\Sigma^{**}$. It can be verified in polynomial time, in the size of $I$, if $(I, J) \models \ell \Sigma^{**}$. For the completeness we will use a reduction from the graph 3-colorability problem as follows. Let $\Sigma^{**}$ be:

\[ D(z, v) \leftrightarrow B^1(x, z), C^1(y, v), E^1(x, y); \]
\[ V(x, v) \leftrightarrow B^2(x, v); \]
\[ V(x, v) \leftrightarrow C^2(x, v). \]

Note that $\#_C(\Sigma^{**}) = 2$ and $\#_D(\Sigma^{**}) = 1$. Consider source instance $I$ representing binary relation $D$ with a pair of each distinct color from the set \{r, g, b\}. For each vertex $x$ of the graph $G$ and each color $c$ from \{r, g, b\} binary relation $V$ will contain tuple $(x, c)$. Finally, target instance $J$ will contain under relations $B$ and $C$ the same tuples as $V^{I}$ and binary relation $E$ will contain the edges from the graph. It can be verified that if there exists a labeling function $\ell$ such that $(I, J) \models \ell \Sigma^{**}$, then the tuples in $B$ which labeling contains integer 1 represent the 3-coloring mapping for graph $G$. Clearly the converse is also true. From this it follows that graph $G$ is 3-colorable iff $J \in [(I, \Sigma^{**})]_{\text{abd}}$

Theorem 4. Let $\Sigma^{**}$ be a set of abds and safe aegds with $\#_D(\Sigma^{**}) = 1$. Then either there exists $(T, \varphi^*)$, computable in polynomial time in the size of $I$, such that $[(I, \Sigma^{**})]_{\text{abd}} = \text{Rep}(T, \varphi^*)$, or $[(I, \Sigma^{**})]_{\text{abd}} = \emptyset$.

Proof. The result follows from the construction of $(T, \varphi^*)$ in the annotated chase algorithm presented in Section \textsection. Thus, in case the algorithm fails, then there is no target instance part of the ABD-semantics. In case the annotated chase algorithm returns table $(T, \varphi^*)$, then $\text{Rep}(T, \varphi^*) = [(I, \Sigma^{**})]_{\text{abd}}$

Proposition 1. A non-redundant set $\Sigma$ of s-t tgds is GAV-reducible iff for each tgd in $\Sigma$ every existentially quantified variable occurs only in one atom in the head of the tgd.
Proof. For the “if” direction, consider \( \Sigma \) a set of s-t tgd s such that no existentially quantified variable occurs in two distinct atoms in the head. We will construct \( \Sigma' \) such that for each \( \xi \in \Sigma \), with \( \xi \) a sentence of the form:

\[
\alpha(\bar{x}, \bar{y}) \rightarrow \exists \bar{z} R_1(x, \bar{z}_1), R_2(x, \bar{z}_2), \ldots, R_k(x, \bar{z}_k);
\]

where \( \{\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_k\} \) is a partition of \( \bar{z} \), we will add the following \( k \) s-t tgd s to \( \Sigma' \):

\[
\xi'_i := \alpha(\bar{x}, \bar{y}) \rightarrow R_i(x, \bar{z}_i);
\]

for all \( 1 \leq i \leq k \). It is easy to verify that \( \Sigma' \) is a set of GAV dependencies and \( \Sigma' \) is logically equivalent to \( \Sigma \).

For the “only if” direction, consider that there exists \( \Sigma \) logically equivalent to \( \Sigma' \) a set of GAV dependencies such that \( \Sigma \) contains a s-t tgd \( \xi \) and existentially quantified variable \( z \), with two distinct atoms in the head of \( \xi \) that share \( z \). Let these two atoms be \( R(\bar{x}, z) \) and \( S(\bar{y}, z) \). Clearly, these relational symbols occur in the head of some s-t tgd in \( \Sigma' \). If \( z \) is existentially quantified in both dependencies in \( \Sigma' \), it follows that there exists a subset minimal target instance with those positions different that models a source instance \( I \) and triggers at least one of those two dependencies. But that instance \( J \) is not a model for \( \Sigma \), contradicting with our assumption that \( \Sigma \) and \( \Sigma' \) are logically equivalent.

Similarly, it can be proved if one of the positions for \( z \) is universally quantified.

In this case an adequate source instance needs to be selected.
closed null and open null respectively to a new distinct constant from Cons, then we have that $v \circ v_1(\varphi^*) \equiv \text{true}$ and $v \circ v_1(T) \in [[(I, \Sigma^*)]]_{\text{abd}}$. 

In case $\#_D(\Sigma^*) > 1$, one may guess $J \in [[(I, \Sigma^*)]]_{\text{abd}}$, such that $Q(J) = \text{false}$, thus making the problem in coNP. For the completeness part consider the set $\Sigma^*$ defined as:

$V(x) \leftrightarrow B^1(x, v), C^1(x, v)$;
$M(v) \leftrightarrow C^1(x, v)$;
$E_0(x, y) \leftrightarrow E^1(x, y)$.

Given a graph $G$ we construct in polynomial time source instance $I$ such that $V^I$ will contain all the vertexes from $G$, $M^I$ will contain three tuples, one tuple for each color, and $E^I_0$ representing the edges in the graph. To this, consider boolean query $Q: B(x, z), C(y, z), E(x, y)$. It is verifiable that $\text{cert}^{\text{abd}}(Q, (I, M)) = \text{true}$ iff the graph is not 3-colorable.

**Theorem 6.** Let $\Sigma$ be a set of s-t tgds and safe egds. Then there exists a set $\Sigma^*$ of abds and safe aegds such that for any source instance $I$ and $q \in \text{UCQ}$ we have $\text{cert}^{\text{smo}}(Q, (I, \Sigma)) = \text{cert}^{\text{abd}}(Q, (I, \Sigma^*))$.

**Proof.** Construct $\Sigma^*$ from $\Sigma$ the way it was shown in the proof of Theorem 1. From the ABD-chase algorithm and from the way the $\Sigma^*$ was constructed, it follows that there exists a universal representative $(T, \varphi^*)$ for $I$ and $\Sigma^*$ if and only if there exists a universal solution $U$ for $I$ and $\Sigma$. From the construction of $T$ we have that $T$ and $U$ are homomorphically equivalent and also $\varphi^*$ satisfiable (just assign distinct constant for each null). From this and from [8] follows that $\text{cert}^{\text{smo}}(Q, (I, \Sigma)) = \text{cert}^{\text{abd}}(Q, (I, \Sigma^*))$ for any $q \in \text{UCQ}$.

**Theorem 7.** There exists a set $\Sigma^*$ of abds with $\#_D(\Sigma^*) = 1$ and there exists a query $Q \in \text{CQ}^-$ such that the problem $\text{Eval}_{\text{ABD}}(\Sigma, Q)$ is coNP-complete.

**Proof.** Clearly the problem is in coNP. For the hardness result the reduction is adapted from Hernich’s [13] reduction from the CLIQUE problem, that is to decide, for an undirected graph without loops, if it contains a clique of size $k$. Let $\Sigma^*$ be defined by:

$E_0(x, y) \leftrightarrow E^1(x, y)$;
$C_0(x, y) \leftrightarrow C^1(x, y), A^1(x, z), B^1(y, v)$.

Note that $\#_D(\Sigma^*) = 1$. Let source instance $I$ being defined such that $E^I_0$ has all the edges from the graph and relation $C^I$ has all pairs of disjoint elements from $\{c_1, c_2, \ldots, c_k\} \subseteq \text{Cons}$. Consider boolean CQ$^-$ query:

$Q: \exists x \exists y \exists z_1 \exists z_2 C(x, y), A(x, z_1), B(y, z_2), \neg E(z_1, z_2)$.
It can be verified that \( \text{cert}^{\text{abd}}(Q, (I, \Sigma^{**})) = \text{true} \) if the graph does not contain a clique of size \( k \), because for each target instance \( J \in [(I, \Sigma^{**})]_{\text{abd}} \), for any complete graph of size \( k \) in \( J \) there exists an edge that is not in \( E \). The converse holds as well, thus proving that \( \text{cert}^{\text{abd}}(Q, (I, \Sigma^{**})) = \text{true} \) if the graph does not contain a clique of size \( k \).

**Proposition 3.** Let \( \Sigma^{**} \) be a set of abds and safe aegds such that \( \Sigma^{\rightarrow} \) is a collection of full s-t tgds and safe egds. Then for any FO query \( Q \) the \( \text{Eval}_{\text{ABD}}(\Sigma^{**}, Q) \) is tractable.

**Proof.** It follows directly from Theorem 4, the annotation chase algorithm defined in Section A and the observation that during the chase process no new nulls will be created, thus the result of the abd-chase process will be an instance, if exists.

**Corollary 2.** Let \( \Sigma \) be a set of full s-t tgds, then there exists \( \Sigma^{**} \) such that \( \Sigma^{\rightarrow} \) contains only full tgds.

**Theorem 8.** Let \( \Sigma^{**} \) be a set of abds and safe aegds with \( \#D(\Sigma^{**}) = 1 \). Then for any UCQ query \( Q \) the \( \text{Eval}_{\text{ABD}}(\Sigma^{**}, Q) \) problem is tractable and one may use only the universal representative, if it exists.

**Proof.** Next we describe the polynomial algorithm that decides the evaluation problem for mapping \( \Sigma^{**} \), source instance \( I \) and boolean query \( Q \in \text{UCQ}^{\phi 1} \).

Let \( (T, \varphi^*) \) be a universal representative for \( I \) and \( \Sigma \). Consider query \( Q \) of the form:

\[
Q := \exists \bar{x} \ q(\bar{x}) \bigvee_{i \leq k} (q_i(\bar{x}_i) \land x_i \neq y_i).
\]

Where \( q(\bar{x}) \) is a UCQ and each \( q_i(\bar{x}_i) \) is a conjunctive query. First, if \( q(T) \models \text{false} \) (naive evaluation of query \( q \) over \( T \)) returns \text{true}, then the algorithm will return \text{true} for \( Q \) too. Using similar methodology as in the proof of Theorem 5.12 in [8], in case \( q(T) \models \text{false} \) we construct the following set of egds:

\[
\begin{align*}
q_1(\bar{x}_1) &\rightarrow x_1 = y_1; \\
q_2(\bar{x}_1) &\rightarrow x_2 = y_2; \\
&\vdots \\
q_k(\bar{x}_1) &\rightarrow x_k = y_k.
\end{align*}
\]

Next we use the standard chase on \( T \) with the previous set of egds also with respect to \( \varphi^* \). Compared with the proof of Theorem 5.12 in [8], because the

\footnote{This condition makes the difference between the certain answer result in OWA compared with the ABD-semantics.}
ae
gds in $\Sigma^{**}$ are safe we can directly use the materialized target table $T$ and
don’t need to repeat the chase process for each query as done in [5]. In case the
chase process does fail either due to trying to equate two distinct constants or
due to contradicting with $\varphi^*$, then it returns false. Otherwise it returns true.

**Theorem 9.** There exists a set $\Sigma^{**}$ of abds with $\#D(\Sigma^{**}) = 1$ and there is a
conjunctive query with two inequalities such that $\text{Eval}_{A\mathcal{B}D}(\Sigma^{**}, Q)$ problem is
coNP-complete.

*Proof.* This reduction from the 3CNF satisfiability problem is similar to the one
provided by Madry in [25] with the set of $\Sigma^{**}$ changed to (note the new relation
symbols $M$ and $V$):

$P(x, y) \leftrightarrow P^{\text{rl}}(x, y)$;
$L(x, y) \leftrightarrow P^{\text{rl}}(x, z), P^{\text{st}}(z, y)$;
$R(x, y) \leftrightarrow P^{\text{rl}}(x, u), P^{\text{st}}(u, v), P^{\text{st}}(v, t),$
$P^{\text{nl}}(t, y), V^{\text{nl}}(x), M^{\text{nl}}(y)$;
$N(x, y) \leftrightarrow N^{\text{rl}}(x, y)$;
$V(x) \leftrightarrow V^{\text{rl}}(x)$;
$M(x) \leftrightarrow M^{\text{rl}}(x)$.

Where instance $I$ is defined similarly as in [25] to which we add $M^I = \{(x_1^k) : i \in$
$\{1, \ldots, m\}, k \in \{1, 2, 3\}\}$ and $V^I = \{(T)\}$. The query is the same as the one in
[26]. The new relational symbols $M$ and $V$ are needed in order to ensure that
the third dependency creates only paths between the variables from the 3CNF
formula and vertex $T$.

Some new definitions are needed in order to sketch the next proofs. Some of
these definitions are slight modifications of definitions from [27]. These modifi-
cations were needed because of the different behavior between open and close
nulls.

**Definition 12.** Let $T$ be a semi-naïve table and $C$ be a finite set of constants.
A $C$-retraction for $T$ is a mapping $h$ from $\text{dom}(T)$ to $\text{dom}(T) \cup C$ that is identity on
Cons $\cup \text{dom}(h(T))$.

**Definition 13.** Let $V$ be a naïve table and $T$ a semi-naïve table. A unifier for $V$ and $T$, if it exists, is a pair $(\theta_1, (\theta_2^1, \theta_2^2, \ldots, \theta_2^k))$, where $\theta_1$ is a homomor-
phism from the set $\text{dom}(V)$ to $\text{dom}(T) \cup \text{cons}(V)$, $\theta_2$ is a cons($V$)-retraction for
$T$ identity on $\text{Nulls}^p$ and $\theta_2^i, 1 \leq i \leq k$, is a cons($V$)-retraction for table $T$ iden-
tity on $\text{Nulls}^c$ such that $\theta_1(V) = \theta_2(\bigcup_{1 \leq i \leq k} \theta_2^i(T))$. Where cons($V$) represents
the set of constants occurring in $V$. 
Note the asymmetrical role of instances $V$ and $T$. In our case $V$ will usually play the role of the naïve table associated with a conjunctive query $\alpha$, where each variable is replaced with a null value. The semi-naïve table $T$ will be a subset of the universal representative $(T, \varphi^*)$ returned by the annotated chase algorithm.

**Definition 14.** A unifier $(\theta_1, (\theta_2, \{\theta_1^2, \theta_2^2, \ldots, \theta_k^2\}))$ for tables $V$ and $T$ is more general than another unifier $(\mu_1, (\mu_2, \{\mu_1^2, \mu_2^2, \ldots, \mu_k^2\}))$, if there exist mappings $(f, \{f^1, f^2, \ldots, f^k\})$ on $\text{dom}(T)$, with at least one mapping not identity, such that $\mu_2 = f \circ \theta_2$ and $\mu_i = f^i \circ \theta_i$ for all $1 \leq i \leq k$.

**Definition 15.** A table $T$ is said to be minimally unifiable with table $V$ if for any unifier $\theta$ of $V$ and $T$, there is no $T' \subsetneq T$ such that $\theta$ is unifier of $V$ and $T'$.

From the previous definition we have the following important result that ensures tractability for our last theorems.

**Proposition 4.** If $T$ is minimally unifiable with table $V$, then $|T| \leq |V|$.

**Definition 16.** A unifier $(\theta_1, (\theta_2, \{\theta_1^2, \theta_2^2, \ldots, \theta_k^2\}))$ is a most general unifier (mgu) for tables $V$ and $T$, if all unifiers $(\mu_1, (\mu_2, \{\mu_1^2, \mu_2^2, \ldots, \mu_k^2\}))$ of $V$ and $T$ that are more general than unifier $(\theta_1, (\theta_2, \{\theta_1^2, \theta_2^2, \ldots, \theta_k^2\}))$ actually are isomorphic with it. We denote $\text{mgu}(V, T)$ the set of (representatives of the) equivalence classes of all mgu’s of $V$ and $T$.

By abusing the notations with $\text{mgu}(V, T)$ we will also denote the following set $\text{mgu}(V, T) := \bigcup_{T' \subseteq T} \text{mgu}(V, T')$. The set will contain only the representatives of the equivalence classes of mgu’s. The following lemma ensures that the set of most general unifiers can be computed in polynomial time.

**Lemma 2.** Let $V$ be a naïve table and $T$ a semi-naïve table with $c = |V|$ and $n = |T|$. Then one may compute the set $\text{mgu}(V, T)$ in $O((2c)^2c^2n^c)$.

**Proof.** The proof directly follows from Proposition 18 in [27]

**Theorem 10.** Let $(T, \varphi^*)$ be a universal representative for some source instance $I$ and a set $\Sigma^{**}$, of abds and safe aegds with $\#D(\Sigma^{**}) = 1$. Then there exists a polynomial time algorithm, with input $(T, \varphi^*)$ and $\bar{t} \in \text{Cons}$, such that for any universal query $Q$ decides if $\bar{t} \in \bigcap_{J \in \text{Rep}(T, \varphi^*)} Q(J)$. 
Proof. A universal query is a FO query of the form $Q(\bar{x}) := \forall \varphi(\bar{x}, \bar{y})$, where $\varphi$ is a quantifier-free FO formula. It is easy to verify that $\ell \notin \text{cert}^{abd}(Q, (I, \Sigma^{**}))$ if and only if there exists a nonempty ground instance $J$ in $[(I, \Sigma^{**})]^{abd}$ such that $J \models \neg Q$. Note that $\neg Q$ is logically equivalent with $\exists X_{\bar{y}}$. Let $\bar{y} := \{y_1, \ldots, y_n\}$, then

$$Q(\bar{x}) := \bigvee_{i=1}^{m} (\exists y_i \bigwedge_{j=1}^{n} (\delta_{ij})),$$

(12)

where $\delta_{ij}$ is an atomic formula or the negation of an atomic formula. In order to compute $\text{EVAL}^{abd}(Q, \Sigma^{**})$ it is enough to check that there exists an instance $J$ in $[(I, \Sigma^{**})]^{abd}$ such that $J \models \bar{Q}(\ell)$. If such instance exists, then $\text{EVAL}^{abd}(Q, \Sigma^{**})$ will return $\text{false}$. For this it is enough to find an integer $i \leq m$ and instance $J$ such that $J \models \varphi_i(\ell)$, where:

$$\varphi_i(\bar{x}) := \exists y_i \bigwedge_{j=1}^{n} (\delta_{ij}).$$

(13)

Figure 1 lists the algorithm which for a fix query of the form (13), a given semi-naïve table $T$ and a tuple $\ell$ decides if there exists a ground instance $J \in \text{Rep}(T)$ with $J \models \varphi_i(\ell)$. Thus the result of $\text{EVAL}^{abd}(Q, \Sigma^{**})$ will be the negation of the result returned by the algorithm below. In the following we will also view $f = \{x_1/a_1, x_2/a_2, \ldots, x_n/a_n\}$ as formula $(x_1 = a_1 \land x_2 = a_2 \land \ldots \land x_n = a_n)$. Thus $\neg f$ will represent formula $(x_1 \neq a_1 \lor x_2 \neq a_2 \lor \ldots \lor x_n \neq a_n)$. A conjunctive formula $\bar{Q}^+ := \bigwedge_{i=1}^{\ell} R_i(\nu(\bar{u}_i))$ we view it also as a naïve table where each variable from the formula is replaced by a new distinct null.

For our problem membership problem is $\ell \in \bigcap_{J \in \text{Rep}(T, \Sigma^{**})} Q(J)$? we will return $\text{true}$ if $\text{EXISTS}_\text{EVAL}(T, \Sigma^{**}, \ell)$ returns $\text{false}$, and it returns $\text{false}$ otherwise. Note that the problem from Step 23 of the algorithm is solvable in polynomial time as $\varphi$ is a conjunction of disjunctive formulae of the following form $(x_1 \neq y_1 \lor x_2 \neq y_2 \lor \ldots \lor x_n \neq y_n)$, $p \leq |\bar{Q}^+|$ and $\delta_1, \delta_2$ and each $\delta_i$, $1 \leq i \leq p$, are of fixed size bounded by the maximum arity of a relation in $\bar{Q}^+$.

**Theorem 11.** Let $\Sigma^{**}$ be a set of abds and safe aegds with $\#(\Sigma^{**}) = 1$ and such that $\Sigma^{**}$ is GAV-reducible and each aegd does not equate two variables both occurring in affected positions. Then for any $\text{CQ}^{-1}$ query the $\text{EVAL}_{\text{ABD}}(\Sigma^{**}, Q)$ problem is polynomial and can be decided using a universal representative.

**Proof.** Let $Q$ be a $\text{CQ}^{-1}$, that is $Q$ has exactly one positive atoms. The polynomial algorithm listed in Figure 2 decides if $\ell \in \text{cert}^{abd}(Q, (I, \Sigma^{**}))$, where $I$ is an input instance and $T$ is the universal representative for $I$ and $\Sigma^{**}$. Table $T$ may be computed in polynomial time by the annotated-chase algorithm. Because none of the aegds equate variable occurring in affected positions in the body of the aegds, the same annotated chase algorithm will return $T$ as a table containing only nulls from $\bot^{a}$.
Few clarifications are in order here. Let $k$ be the maximum number of atoms occurring in the head of any $tgd$ from $\Sigma^\rightarrow$. It can be easily verified that for each block $B$ from Step 11 we have $|B| \leq k$. From this it follows that the size of table $U_i$ from Step 13 is also bounded by the same $k$. At Step 14 for each $i$ the list of mgu’s can be listed in time $O((2k)^{2k^2k^2k!})$, where $n = |T|$. Note that in this case the mgu considers two semi-naïve tables that may share null values. At Step 17, the size of instance $J_i$ is not bounded by any constant, still we may verify if for an instance $J_i$ and table $T$ if there exists the set of homomorphisms, as per Step 13, with the following algorithm:

**STEP18.CHECK(T,J_i)**

1. Let $v : J_i \rightarrow \{\text{true}, \text{false}\}$;
2. Let $v(\bar{t}) := \text{false}$, for all $\bar{t} \in J_i$;
3. for all $\bar{s} \in T$ and all $h$ such that $h(\bar{s}) \in J_i$
4. do
5. Let $v(h(\bar{s})) := \text{true}$;
6. if $v(\bar{t}) = \text{true}$ for all $\bar{t} \in J_i$
7. then return true;
8. else return false;

It can be verified that $\text{CQ}^{-}\text{EVAL}(T,\bar{t})$ algorithm is sound and complete in deciding if $\bar{t} \in \text{cert}^{abd}(Q,(I,\Sigma^\rightarrow))$
EXISTS_EVAL((T, \varphi^*), \tilde{t})

1. Let \( \tilde{Q}(\tilde{x}) := \exists \tilde{y} \bigwedge_{i=1}^n R_i(\tilde{u}_i) \bigwedge_{j=1}^m \neg S_j(\tilde{w}_j) \bigwedge_{k=1}^n t_{k1} = t_{k2}; \)
2. where \( \tilde{x} \subseteq (\bigcup_{i=1}^n \tilde{u}_i \cup \bigcup_{j=1}^m \tilde{w}_j); \)
3. \( \tilde{y} = (\bigcup_{i=1}^n \tilde{u}_i \cup \bigcup_{j=1}^m \tilde{w}_j) \setminus \tilde{x}; \) and
4. \( t_{k1}, t_{k2} \in \tilde{x} \cup \tilde{y} \cup \text{Cons}, \) for \( 1 \leq k \leq n; \)
5. Let \( v : \tilde{x} \cup \tilde{y} \rightarrow \tilde{t} \cup \tilde{y}, \) such that \( v(\tilde{x}) = \tilde{t} \) and \( v(y) = y \) for all \( y \in \tilde{y}; \)
6. Let \( \tilde{Q}^+ := \bigwedge_{i=1}^n R_i(v(\tilde{u}_i)); \)
7. for all \( (\theta_1, (\theta_2, \{\theta_2^1, \theta_2^2, \ldots, \theta_2^p\}))) \) mgu for \( \tilde{Q}^+ \) and \( T \)
8. do
9. Let \( \varphi \leftarrow \varphi^*; \)
10. for \( j := 1 \) to \( m \)
11. do
12. for all \( (\xi^j_1, (\xi^j_2, \{\xi^j_2\}))) \) mgu for \( S_j(v(\tilde{w}_j)) \) and \( T \)
13. do
14. Let \( \varphi \leftarrow \varphi \land (\neg \xi^j_1 \lor \neg \xi^j_2 \lor \neg \xi^j_2); \)
15. for \( k := 1 \) to \( n \)
16. do
17. Let \( \varphi \leftarrow \varphi \land (v(t_{k1}) \neq v(t_{k2})); \)
18. if \( (\theta_1 \land \theta_2 \land \theta_2^1 \land \theta_2^2 \land \ldots \land \theta_2^p \land \varphi) \) is satisfiable
19. then return true
20. return false
Fig. 2. $CQ^-\text{EVAL}$ algorithm

$\text{CQ}^-\text{EVAL}(T,\bar{t})$

1. Let $Q(\bar{x}) := \exists \bar{y} \ R_i(\bar{u}) \land \bigwedge_{i=1}^{n} \neg S_i(\bar{w}_i)$;
2. where $\bar{x} \cup \bar{y} = \bar{u} \setminus \text{Cons}$;
3. $\bigcup_{i=1}^{n} \bar{w}_i \subseteq \bar{u} \setminus \text{Cons}$;
4. Let $v : \bar{x} \cup \bar{y} \rightarrow \bar{t} \cup \bar{y}$, such that $v(\bar{x}) = \bar{t}$ and $v(y) = y$ for all $y \in \bar{y}$;
5. Let $g_i$ be the function that maps vector $\bar{u}$ to $\bar{w}_i$, for $i \in \{1, \ldots, n\}$;
6. Let $V := \{R(h \circ v(u)) : R(h \circ v(u)) \in T \text{ for homomorphism } h\}$;
7. Let $J = V \cap \left(\text{Cons}\right)^{arr_{\text{type}}(B)}$;
8. Let $V' = V \setminus J$;
9. for each set $B$ from the Gaifman-partition of $V'$
10. 
11. do
12. 
13. Let $U_i := \{S_i(g_i(\bar{z})) : R(\bar{z}) \in B\}$, for $i \in \{1, \ldots, n\}$;
14. 
15. if $\neg \exists \theta_1 \land \exists \theta_2 \land \exists \theta_3 \land \ldots$ such that $U_i$ and $T$ have mgu
16. 
17. then return true;
18. 
19. for each set $B$ from the Gaifman-partition of $V'$
20. 
21. do
22. 
23. if $\neg \exists h \land \forall h_2, h_3, \ldots, c$ such that $h(U_i(T)) \subseteq J_i$
24. 
25. then return true;
26. 
27. return false;