Throat destabilization
(for profit and for fun)

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Abstract

Two recent results indicate that the addition of supersymmetry-breaking ingredients can destabilize the Klebanov-Strassler warped deformed conifold throat. The first comes from an analytic treatment of the interaction between anti-D3 branes and the complex-structure modulus corresponding to the deformation of the conifold \cite{1}. The second comes from the numeric construction of Klebanov-Strassler black holes \cite{2}, which stop existing above a certain value of the non-extremality. We show that in both calculations the destabilization energies have the same parametric dependence on $g_s$ and the conifold flux, and only differ by a small numerical factor. This remarkable match confirms that anti-D3 brane uplift can only work in warped throats that have an $\mathcal{O}(1000)$ contribution to the D3 tadpole.

\textit{Dedicated to the memory of Steve Gubser}
Adding antibranes to warped throats of flux compactifications is the preferred method to up-
lift the negative cosmological constant of supersymmetric flux compactifications and obtain
de Sitter space in string theory [3]. In previous work [1], two of the authors together with
Graña and Dudaš have shown that antibranes added to the Klebanov-Strassler warped de-
fomed conifold (KS) solution [4] can destabilize the complex structure modulus correspond-
ning to the size of the three-cycle of the deformed conifold, thus collapsing the Klebanov-
Strassler warped deformed conifold to the Klebanov-Tseytlin (KT) singular warped conifold
solution [5].

The destabilization mechanism found in [1] is triggered when the number of antibranes
is larger than a certain bound that depends on the RR-flux $M$ on the three-cycle of the
deformed conifold. Furthermore, this happens both when the KS solution is infinite, and
when the KS solution is glued to a compact space. This result has led the authors of [1] to
conjecture the existence of a KS black hole. The intuition behind this conjecture was based
on the fact that both antibranes and a black hole horizon break supersymmetry by making
the mass of the solution larger than the charge. Hence, if a small number of antibranes does
not destabilize the three-cycle of the deformed conifold when $M$ is large enough, neither one
would expect a small mass above extremality brought about by a finite-temperature event
horizon to do it.

In parallel to this work, one of the authors had succeeded in constructing numerically the
Klebanov-Strassler black hole [2], despite prevailing expectations based on previous work [6]
that this black hole should not exist and any attempt to add an event horizon to the KS
solution would shrink the size of the three-cycle (or alternatively reduce the vev of the
operator parameterizing chiral symmetry breaking to zero) and result in the Klebanov-
Tseytlin black hole [7]. Interestingly enough, the KS black hole found in [2] also exists only
when the energy above extremality is below a certain value. When the energy is above this
value chiral symmetry is restored and the only possible solution is the KT black hole.

The purpose of this note is to compare the (numerically found) energy above extremality
at which the numerically constructed KS black hole stops existing with the (analytically
found) energy above extremality at which a configuration of antibranes in a KS throat stops
existing. At first glance one could argue that these two energies should have nothing to do
with each other, as one energy is calculated using a fully-backreacted black-hole solution
in which the nonlinear effect of the mass above extremality has been taken into account,
while the other is calculated in a probe approximation which ignores the backreaction of
the antibranes. Furthermore, it is well known that the KS black hole suffers from Gregory-
Laflamme-like instabilities corresponding to clumping along the field theory directions [2],
while antibranes also have tachyons on their worldvolume which make them unstable to
moving away from each other on the Coulomb branch [8,9].

One can however argue that the instabilities of the KS black hole and antibranes, while
important for holography and for flux compactifications, are not of direct relevance here.
What we are comparing are the energies at which the two non-extremal configurations stop
existing as solutions to the equations of motion, and this comparison is meaningful because the underlying phenomenon is the same: the size of the $S^3$ of the deformed conifold, or alternatively the vev of the field dual to chiral symmetry breaking is shrunk by the addition of mass above extremality.

The comparison of the numerical results of [2] with the analytical results of [1] is easiest to illustrate if one converts the energy above extremality of the black hole in “antibrane” units, corresponding to the energy above extremality brought about by a single antibrane placed in the warped deformed conifold. The KS black hole solution exists when

$$g_s M^2 \geq \gamma_{BH}^2 N_{D3},$$

while the antibranes do not have a runaway when

$$g_s M^2 \geq \gamma_{D3}^2 N_{D3}.$$  

In this paper we evaluate $\gamma_{BH}$ and $\gamma_{D3}$ and find them to be

$$\gamma_{BH} \approx 4.16 \quad \text{and} \quad \gamma_{D3} \approx 6.8.$$ 

This match is quite remarkable, both because the functional expression of the critical energies as functions of the parameters of the solution are the same, and because the coefficients differ by so little despite the fact that we are comparing two very different sources of non-extremality. Furthermore, one should not forget that the value of $\gamma_{D3}$ found in [1] is based on the analysis of [10], in which the values of certain numerical coefficients were only estimated and not rigorously computed. Hence, this value also has error bars.

This remarkable match strongly reinforces the conclusion of [1], that the only warped KS throats in flux compactifications that are not destabilized by the addition of a single antibrane must have a RR three-form flux, $M$, larger than the value given in equation (2). If one is to build a de Sitter flux compactification with stable moduli, the length of this throat also needs to be quite large, which puts a lower limit on the amount of the NS-NS three-form flux on the B-cycle. This in turn implies that the contribution of the fluxes of this KS throat to the D3 tadpole of the compactification is larger than about 1000 [1].

Hence our analysis greatly reduces the space of Calabi-Yau manifolds where one may hope to build a de Sitter solution in String Theory, by eliminating from the landscape all the manifolds whose geometrical structure does not allow a negative contribution to the tadpole whose absolute value is smaller than about 1000. In an upcoming paper [11] two of the authors and collaborators will show that most of these manifolds have unfixed moduli, and reduce the question of the existence of de Sitter flux compactifications to a rigorous problem that can be proved or disproved mathematically.
The Calculation

The (Einstein-frame) ten-dimensional metric and five-form flux of the Klebanov-Strassler solution \[4\] take the form
\[
ds^2 = H^{-1/2} ds_4^2 + H^{1/2} ds_6^2, \\
F_5 = (1 + \ast) \text{vol}_4 \wedge dH^{-1},
\]
where \(ds_6^2\) is the metric of the deformed conifold. For \(M\) units of RR flux on the compact three-cycle of the deformed conifold,
\[
\frac{1}{4\pi^2 \alpha'} \int_{S^3} F_3 = M,
\]
the warp factor is given by
\[
H(\tau) = 2^{2/3} g_s(\alpha' M)^2 |S|^{1/3} I(\tau),
\]
with \(\tau\) a radial coordinate on the deformed conifold and \(S\) its deformation parameter (\(|S| \equiv \epsilon^2\) in \[4\]), corresponding to the size of the three-sphere at the tip of the cone. In Appendix A we outline this solution in more detail, and explain some of the subtleties related to its expressions in String frame and Einstein frame. The function \(I(\tau)\) is given by an integral expression, here we will only need its numerical value at the tip of the throat,
\[
I(0) \approx 0.718.
\]

The black hole numerically constructed in \[2\] is stable as long as its energy density satisfies
\[
\mathcal{E} \leq \mathcal{E}_{\chi SB},
\]
for larger values the chiral symmetry is restored \[6\]. After introducing
\[
\hat{\mathcal{E}} \equiv 216\pi^4 (\alpha')^4 \mathcal{E},
\]
the value of \(\hat{\mathcal{E}}_{\chi SB}\) was numerically determined in \[2\],
\[
\hat{\mathcal{E}}_{\chi SB} = 1.270093(1)\Lambda^4.
\]
Here, \(\Lambda\) denotes the strong coupling scale, given by
\[
\Lambda = \frac{3^{1/2} \epsilon^{1/3} |S|^{1/3}}{2^{7/12}}.
\]

Following \[4\] the energy density of an anti-D3-brane placed in the (Einstein-frame) Klebanov-Strassler background can be computed from its DBI and Chern-Simons action,
\[
S_{D3} = S_{DBI} + S_{CS} = -T_3 \int d^4x \sqrt{-g_4} - T_3 \int C_4,
\]
where its tension is given by
\[ T_3 = \frac{1}{(2\pi)^3\alpha'^2}. \] (13)

Using (4) the energy density of \( N_{D3} \) anti-D3-branes, placed at the tip of the KS-throat, is therefore given by
\[ \mathcal{E}_{D3} = 2N_{D3}T_3C_4 = \frac{2N_{D3}}{(2\pi)^3\alpha'^2} H(0)^{-1}. \] (14)

Using the warp factor (6) we obtain
\[ \hat{\mathcal{E}}_{D3} = N_{D3}3^32^{1/3}\pi |S|^{4/3} \frac{I(0)}{g_sM^2}, \] (15)
and thus
\[ \frac{\hat{\mathcal{E}}_{D3}}{\Lambda^4} = \beta \frac{N_{D3}}{g_sM^2}, \] (16)
where we abbreviated the numerical prefactor,
\[ \beta \equiv \frac{2^{8/3}3\pi}{e^{4/3}I(0)} \approx 22.0. \] (17)

If we now naively replace in (9) the energy density \( \mathcal{E} \) of the black hole with the energy density of the anti-D3-branes, we find a bound of the same functional form as the one derived in [1],
\[ g_sM^2 \geq \gamma_{BH}^2 N_{D3}, \] (18)
where combining (10) and (17) gives
\[ \gamma_{BH} \approx 4.16. \] (19)

This result is of the same order of magnitude as the value \( \gamma_{D3} \approx 6.8 \) determined in [1] by completely different methods.

Appendix A explains in detail the KS geometry and how to do holography therein. Appendix B explains how to do holography in the black hole solution of [2] and how to obtain equations (9) and (11) which are crucial for relating the two calculations.

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\[ ^1 \text{The factor of } g_s \text{ was corrected in } [12]. \]
A KS geometry

We review here the relevant details of the extremal supersymmetric Klebanov-Strassler geometry \[4\] following \[13,14\]. This would allow to precisely define the strong coupling scale \(\Lambda\) \[11\] of the dual cascading gauge theory used in the thermodynamic analysis of the model in \[2\,6\,7\]. Additionally, we introduce the order parameter \(\xi\) (see Appendix \[3\]) for the chiral symmetry breaking in the cascading gauge theory and its geometrical counterpart representing the relative sizes of the 2- and 3-cycle of the conifold.

The \(\mathcal{N} = 1\) supersymmetric Klebanov-Strassler solution (in Einstein frame) takes the form:
\[
\begin{align*}
\left(ds_{10}ight)^2 &= ds_5^2 + dY_5^2, \\
&= H_{K_S}^{-1/2}(−dt^2 + dx^2) + H_{K_S}^{1/2} d\tau^2, \\
dY_5^2 &= \Omega_1^2 g_5^2 + \Omega_2^2 [g_5^2 + g_4^2] + \Omega_3^2 [g_1^2 + g_2^2], \\
\Omega_i &= \omega_i, K_S H_{K_S}^{1/4}, \\
h_i &= h_i, K_S, \\
e^\Phi &= g,
\end{align*}
\]
with
\[
\begin{align*}
h_{1,K_S} &= Pg_s \cosh \tau - 1 \frac{\tau \cosh \tau}{18 \sinh \tau} - 1, \\
h_{2,K_S} &= P \frac{1 - \tau}{18 \sinh \tau}, \\
h_{3,K_S} &= Pg_s \cosh \tau + 1 \frac{\tau \cosh \tau}{18 \sinh \tau} - 1, \\
g &= g_s, \\
\omega_{1,K_S} &= \frac{\epsilon^{2/3}}{\sqrt{6}K_S}, \\
\omega_{2,K_S} &= \frac{\epsilon^{2/3} K_S^{1/2}}{\sqrt{2}} \cosh \frac{\tau}{2}, \\
\omega_{3,K_S} &= \frac{\epsilon^{2/3} K_S^{1/2}}{\sqrt{2}} \sinh \frac{\tau}{2}, \\
\hat{K}_K &= (\sinh (2\tau) - 2\tau)^{1/3} \frac{2^{1/3} \sinh \tau}{2}, \\
H_{K_S} &= 16(9h_{2,K_S} - P)h_{1,K_S} - 9h_{3,K_S} h_{2,K_S}, \frac{9\epsilon^{8/3} K_S^{2} \sinh^{2} \tau}{9\epsilon^{8/3} K_S^{2} \sinh^{2} \tau},
\end{align*}
\]
where the radial coordinate \(\tau \in [0, +\infty]\) and the parameter \(P\) is related to the quantized RR flux \([5]\) as
\[
P = \frac{9}{2} M\alpha'.
\]

Here \(g_i\) are the usual 1-forms on the deformed conifold defined as
\[
\begin{align*}
g_1 &= \frac{\alpha^1 - \alpha^3}{\sqrt{2}}, \quad g_2 = \frac{\alpha^2 - \alpha^4}{\sqrt{2}}, \quad g_3 = \frac{\alpha^1 + \alpha^3}{\sqrt{2}}, \quad g_4 = \frac{\alpha^2 + \alpha^4}{\sqrt{2}}, \quad g_5 = \alpha^5,
\end{align*}
\]
where
\[
\begin{align*}
\alpha^1 &= -\sin \theta_1 d\phi_1, \quad \alpha^2 = d\theta_1, \quad \alpha^3 = \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2, \\
\alpha^4 &= \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2, \quad \alpha^5 = d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2,
\end{align*}
\]
with \(0 \leq \psi \leq 4\pi, 0 \leq \theta_1 \leq \pi\) and \(0 \leq \phi_i \leq 2\pi\).

Topologically, the compact manifold \(Y_5\) is an \(S^2\) fibration over \(S^3\). In the deep infrared, as \(\tau \to 0\),
\[
\left.dY_5^2 \right|_{\tau \to 0} \overset{\tau \to 0}{\longrightarrow} \frac{1}{6} 4^{1/3} 2^{1/3} 3^{2/3} H_{K_S}^{1/2} \left[ \left( \frac{1}{2} g_5^2 + g_3^2 + g_4^2 \right) + \frac{\tau^2}{4} \left( g_1^2 + g_2^2 \right) \right].
\]
In (25) we can easily identify the 2− and 3−cycles:

\[ 2 \text{− cycle : } g_2 + g_3 \left| _{\psi=0, \theta_2=-\theta_1, \phi_2=-\phi_1} = 2 \left( (d\theta_1)^2 + \sin^2 \theta_1 \left( d\phi_1 \right)^2 \right), \]

\[ 3 \text{− cycle : } \frac{1}{2} g_2^2 + g_3^2 + g_4^2 \left| _{\phi_2=\theta_2=0, \psi=\xi_1+\xi_2, \phi_1=\xi_2-\xi_1, \theta_1=2\eta} = \frac{1}{2} \left( (d\eta)^2 + \sin^2 \eta \left( d\xi_1 \right)^2 + \cos^2 \eta \left( d\xi_2 \right)^2 \right). \]  

(26)

In the ultraviolet, as \( \tau \to \infty \),

\[ \frac{dY_5^2}{\Omega_2^2} \to \begin{cases} 6 \left[ \frac{1}{9} g_5^2 + \frac{1}{6} \left( g_3^2 + g_4^2 \right) + \frac{1}{6} \left( g_1^2 + g_2^2 \right) \right] \end{cases}, \]  

(27)

which is the standard metric on the coset \( T^{1,1} = \frac{SU(2) \times SU(2)}{U(1)} \) of radius \( R_{T^{1,1}} = \sqrt{6} \). As in (26) we can identify the 2− and 3−cycles

\[ \frac{dY_5^2}{\Omega_2^2} \to \begin{cases} \delta_{\infty} \left[ \frac{1}{2} g_5^2 + g_3^2 + g_4^2 \right] + \left[ g_1^2 + g_2^2 \right] \end{cases}, \quad \delta_{\infty} = \frac{4}{3}. \]  

(28)

Note that at the tip of the deformed conifold, \( Y_5 \), the 3-cycle is a round \( S^3 \), and the 2-cycle is a collapsed \( S^2 \). Because \( \delta_{\infty} \neq 1 \), the 3−cycle is a deformed \( S^3 \) as \( \tau \to \infty \). It is convenient to (formally) assign radii for the 2− and 3-cycles as:

\[ R_3 \equiv \sqrt{2} \Omega_2, \quad R_2 \equiv \sqrt{2} \Omega_3. \]  

(29)

We can now introduce parameters

\[ \xi \equiv 1 - \frac{R_2^2}{R_3^2} = 1 - \frac{\Omega_3^2}{\Omega_2^2}, \quad \delta \equiv \frac{2 \Omega_1^2}{\Omega_2^2}, \]  

(30)

where \( \xi \) is the order parameter for the chiral symmetry breaking \( U(1) \to Z_2 \) in the dual gauge theory and geometrically encodes the relative sizes of the compact cycles on the deformed conifold; \( \delta \) encodes the deformation of the 3−cycle relative to a round \( S^3 \).

To facilitate the comparison of the extremal KS geometry (20) with that of the KS BH we introduce a new radial coordinate \( \rho \) as

\[ \frac{(d\rho)^2}{\rho^4} \equiv g_{xx} g_{rr} \left( d\tau \right)^2 = (w_{1,KS}(\tau))^2 (d\tau)^2. \]  

(31)

Note that the asymptotic boundary \( \tau \to \infty \) corresponds to \( \rho \to 0 \). With

\[ e^{-\tau} \equiv \frac{3\sqrt{6}\epsilon^2}{8} z^3, \]  

(32)
we find from (31)

$$z = \rho \left( 1 + \rho^6 \epsilon^4 \left( \frac{27}{800} + \frac{27}{80} \ln 3 - \frac{9}{16} \ln 2 + \frac{27}{40} \ln (\rho \epsilon^{2/3}) \right) + \mathcal{O}(\rho^{12} \epsilon^8 \ln^2 (\rho \epsilon^{2/3})) \right), \quad (33)$$

leading to

$$K_{1,KS} \equiv 12P \; h_{1,KS} = P^2 g_s \left( -\ln 3 + \frac{5}{3} \ln 2 - \frac{4}{3} \ln \epsilon - \frac{2}{3} - 2 \ln \rho + \mathcal{O}(\rho^3 \epsilon^2 \ln (\rho \epsilon^{2/3})) \right). \quad (34)$$

In Appendix B we will use (34) to relate $\Lambda$ and $\epsilon$.

We conclude this section relating the Einstein frame KS solution (21) with the String frame solution reviewed in [13]. Substituting explicit expressions in (21) for $H_{KS}$ we find

$$H_{KS} \equiv \frac{2^{2/3} g_s}{\epsilon^{8/3}} \left( \frac{2P}{9} \right)^2 \int_0^\infty dx \; x \coth x - \frac{1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} \cdot \quad (35)$$

Note that the integral $I(\tau)$ in (35) is exactly the same as in eq.(66) of [13]. In the limit $\tau \to 0$,

$$I(\tau \to 0) = I(0) \quad \implies \quad H_{KS}(\tau \to 0) = \frac{2^{2/3} g_s}{\epsilon^{8/3}} \left( \frac{2P}{9} \right)^2 I(0). \quad (36)$$

We already established (from the quantization of 3-form flux (5)) a relation between $P$ and $M$ (22), so

$$H_{KS}(\tau \to 0) = \frac{2^{2/3} M^2 (\alpha')^2 g_s I(0)}{\epsilon^{8/3}}. \quad (37)$$

The Einstein frame metric as $\tau \to 0$ is then

$$ds_{10}^2 \bigg|_{KS}^{Einstein} = \frac{\epsilon^{4/3}}{2^{1/3}(I(0))^{1/2} g_s^{1/2} M \alpha'} dx_n dx_n$$

$$+ (I(0))^{1/2} 6^{-1/3} g_s^{1/2} M \alpha' \left\{ \frac{1}{2} (d\tau)^2 + \frac{1}{2} g_5^2 + g_3^2 + g_4^2 + \frac{1}{4} \tau^2 [g_1^2 + g_2^2] \right\}. \quad (38)$$

Since

$$ds_{10}^2 \bigg|_{string} = g_s^{1/2} \; ds_{10}^2 \bigg|_{Einstein},$$

we have

$$ds_{10}^2 \bigg|_{string}^{KS} = \frac{\epsilon^{4/3}}{2^{1/3}(I(0))^{1/2} M \alpha'} dx_n dx_n$$

$$+ (I(0))^{1/2} 6^{-1/3} g_s M \alpha' \left\{ \frac{1}{2} (d\tau)^2 + \frac{1}{2} g_5^2 + g_3^2 + g_4^2 + \frac{1}{4} \tau^2 [g_1^2 + g_2^2] \right\}. \quad (39)$$
Note that
\[ H_{KS, string} = \frac{1}{g_s} H_{KS}. \] (40)

Our expression (39) differs from eq.(68) of [13] (HKO):
\[
\left. ds^2_{10} \right|_{K\text{O}}^{KS} = \frac{e^{4/3}}{2^{1/3} I(0)^{1/2} g_s M \alpha'} dx_n dx^n + I(0)^{1/2} 6^{-1/3} g_s M \alpha' \left\{ \frac{1}{2} (d\tau)^2 + \frac{1}{2} g_5^2 + g_3^2 + g_4^2 + \frac{1}{4} \tau^2 [g_1^2 + g_2^2] \right\};
\] (41)

while the warp factors in \( ds^2 \) agree, there is a typo in the \( dx_n dx^n \) warp factor.

B The parameters \( \Lambda, \xi \) and \( \delta \) of the KS black hole

We recall first relevant details of the KS BH constructed numerically in [2]. The background geometry is
\[
ds_{10}^2 = H^{-1/2} \left( -(1-x)^2 dt^2 + (dx)^2 \right) + g_{xx} dx^2 + \Omega_1^2 g_5^2 + \Omega_2^2 \left[ g_3^2 + g_4^2 \right] + \Omega_3^2 \left[ g_1^2 + g_2^2 \right], \] (42)

where the radial coordinate is \( x \in [0, 1] \), with
\[
\begin{align*}
h_1 &= \frac{1}{12P} K_1, & h_2 &= \frac{P}{18} K_2, & h_3 &= \frac{1}{12P} K_3, & H &= (2x - x^2) h, \\
\Omega_1 &= \frac{1}{3} f_c^{1/2} h^{1/4}, & \Omega_2 &= \frac{1}{\sqrt{6}} f_a^{1/2} h^{1/4}, & \Omega_3 &= \frac{1}{\sqrt{6}} f_b^{1/2} h^{1/4}.
\end{align*}
\] (43)

Asymptotically near the boundary \( (x \to 0) \),
\[
\begin{align*}
K_1 &= P^2 g_s \left[ k_a - \frac{1}{2} P^2 g_0 \ln x + \sum_{n=3}^{\infty} \sum_k k_{1nk} x^{n/4} \ln^k x \right], \\
K_2 &= 1 + \sum_{n=3}^{\infty} \sum_k k_{2nk} x^{n/4} \ln^k x, \\
K_3 &= P^2 g_s \left[ k_a - \frac{1}{2} P^2 g_0 \ln x + \sum_{n=3}^{\infty} \sum_k k_{3nk} x^{n/4} \ln^k x \right], \\
f_a &= a_0 \left[ 1 + \sum_{n=3}^{\infty} \sum_k f_{ank} x^{n/4} \ln^k x \right], \\
f_b &= a_0 \left[ 1 + \sum_{n=3}^{\infty} \sum_k f_{bnk} x^{n/4} \ln^k x \right],
\end{align*}
\] (44-48)
\[ f_c = a_0 \left[ 1 + \sum_{n=2}^{\infty} \sum_{k} f_{cnk} x^{n/4} \ln^k x \right], \]  
\[ h = \frac{P^2 g_s}{a_0^2} \left[ \left( \frac{1}{8} + \frac{k_s}{4} \right) - \frac{1}{8} \ln x + \sum_{n=2}^{\infty} \sum_{k} h_{nk} x^{n/4} \ln^k x \right], \]  
\[ g = g_s \left[ 1 + \sum_{n=2}^{\infty} \sum_{k} g_{nk} x^{n/2} \ln^k x \right], \]

and the solution is characterized by four microscopic parameters \( \{P^2, g_s, a_0, k_s\} \) and seven expectation values \( \{ f_{a_0}^{30} \equiv (f_{a_0} - f_{b_0})/2, dk_{10} \equiv (k_{130} - k_{330})/2, f_{a40}, g_{40}, f_{a60}, k_{a80}, f_{a80} \} \).

Asymptotically near the regular horizon (as \( y \equiv 1 - x \to 0 \)) the KS BH solution

\[ K_i = P^2 g_s \sum_{n=0}^{\infty} k_{inh} y^{2n}, \quad i = 1, 3, \quad K_2 = \sum_{n=0}^{\infty} k_{2hn} y^{2n}, \]

\[ f_\alpha = a_0 \sum_{n=0}^{\infty} f_{\alpha hn} y^{2n}, \quad \alpha = a, b, c, \]

\[ h = \frac{P^2 g_s}{a_0^2} \sum_{n=0}^{\infty} h_{hn} y^{2n}, \quad g = g_s \sum_{n=0}^{\infty} g_{hn} y^{2n}, \]

is characterized by the “nice” parameters \( \{ k_{1h_0}, k_{2h_0}, k_{3h_0}, f_{a_0h_0}, f_{b_0h_0}, f_{c_0h_0}, f_{ch_0}, h_{h_0}, g_{h_0} \} \).

The microscopic parameters \( k_s \) and \( a_0 \) determine the strong-coupling scale \( \Lambda \) of the theory via

\[ k_s = \frac{1}{2} \ln \left( \frac{a_0^2}{\Lambda^4} \right), \]  

and the entropy and the energy densities are given by

\[ 16\pi G_5 \frac{s}{\Lambda^3} = 4\pi P g_s^{1/2} e^{3k_s/2} h_{h_0}^{1/2} f_{ch_0}^{1/2} f_{a_0h_0} f_{b_0h_0}, \]

\[ 16\pi G_5 \frac{E}{\Lambda^4} = e^{2k_s} (3 - 12 f_{a40}), \]

where

\[ \frac{1}{16\pi G_5} = \frac{\text{vol}(T^{1,1})}{16\pi G_{10}} = \frac{16\pi^3}{27} \]

\[ \frac{1}{16\pi G_{10}} = \frac{1}{16\pi G_{10}} = \frac{1}{216\pi^4(\alpha')^4}. \]  

We now proceed to relate \( \Lambda \) to the conifold deformation parameter \( \epsilon \), as we did for the supersymmetric solution in Appendix A. We first identify the “holographic” radial coordinate, \( \rho \), exactly as in equation (31):

\[ \frac{(d\rho)^2}{\rho^4} \equiv g_{xx} g_{xx} (dx)^2. \]  

\[ ^2To \ avoid \ confusion, \ we \ should \ remember \ that \ g_{xx} \ is \ the \ metric \ along \ the \ 3 \ spatial \ directions, \ denoted \ generically \ by \ x, \ and \ g_{xx} \ is \ the \ metric \ along \ the \ radial \ direction, \ denoted \ by \ x. \]
Using the expansions in (44)-(50) we find
\begin{equation}
\frac{x^{1/4}}{\Lambda^4} = 2^{-1/4}\rho_{0}^{1/2}\left(1 + \rho_{0}^{4}\left(\frac{1}{4}f_{a40} - \frac{5}{48}\right) + \rho_{0}^{6}a_{0}^{3}\frac{3\sqrt{2}}{400}df_{0}^{2} + O\left(\rho_{0}^{8}a_{0}^{4}\ln(\rho_{0}^{1/2})\right)\right),
\end{equation}
and using equation (52) we can express $K_1$ in terms of the "holographic" radial variable
\begin{equation}
K_1 = P^2g_s\left(-2\ln\Lambda + \frac{1}{2}\ln 2 - 2\ln\rho + O\left(\rho_{0}^{3/2}\ln(\rho_{0}^{1/2})\right)\right).
\end{equation}
Comparing $K_1$ from (57) and $K_{1,KS}$ from (34) we identify
\begin{equation}
\Lambda = \frac{3^{1/2}e^{1/3}}{2^{7/12}}\epsilon^{2/3}.
\end{equation}
Given the numerical solution of the KS BH it is straightforward to evaluate the parameters $\xi$ and $\delta$ (30) at the horizon
\begin{equation}
\xi_{\text{horizon}} = 1 - \frac{f_{bh0}}{f_{ah0}}, \quad \delta_{\text{horizon}} = \frac{4f_{bh0}}{3f_{ah0}}.
\end{equation}
Numerical results for these parameters are presented in Fig. 1.

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