Plasmas beyond MHD: two-fluids and symmetry breaking

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Abstract. Extended MHD effects are important in the nonlinear behavior of magnetically confined plasmas, even at the simplest level represented by the self-consistent diamagnetic ($\mathbf{B} \times \nabla p$) drifts. Allowing the electrons and ions to move independently, even as fluids, breaks certain geometrical symmetries preserved by the MHD equations that can be important for toroidal fusion burning plasmas. These symmetries are also broken by certain experimental designs and high temperature plasma conditions. Results are shown from the two-fluid and hybrid particle/fluid models in the M3D MPP code, part of the SciDAC CEMM (Center for Extended Magnetohydrodynamic Modeling) project.

1. Introduction
The magnetohydrodynamic (MHD) model has long set the standard for nonlinear, global plasma simulations because of its approachability and its agreement with the general properties of laboratory plasmas. Nevertheless, its limitations were clear even in the early decades of controlled fusion research. A two-fluid description, electrons and ions as separate fluids, is the simplest model that breaks the MHD constraint that the electrons and ions, and thus magnetic field, move together. Two-fluid theory was developed extensively for the study of plasma instabilities in the 1960’s, but mainly to derive dispersion relations for perturbed mode growth rates. MHD continued to be used for nonlinear plasma simulations.

Less well recognized is the fact that the MHD equations contain sets of geometrical constraints that apply to confined plasma configurations (cf [1] Appendix). MHD modellers realized this in the late 1970’s and early 1980’s, when 2D and reduced MHD nonlinear models first became feasible on existing computers, developing into fully compressible 3D MHD. Given the state of experimental measurements and the popularity of the toroidally axisymmetric tokamak, the MHD constraints had few obvious effects. The fastest growing MHD modes gave good agreement with experiment.

Ongoing development in experimental techniques and confinement design and the wealth of accumulated experimental data now seriously challenge the MHD model. Helical magnetic configurations (eg, stellarators) and strongly heated axisymmetric plasmas (eg, tokamaks) mix the two possible MHD toroidal symmetries. More importantly, the mixing implies that models beyond MHD, where the two states interact at lowest order, should be important.
Two-fluid nonlinear plasma models are being developed and extended under the SciDAC CEMM (Center for Extended Magnetohydrodynamic Modeling). High performance computation is crucial to understand the complex interactions that result. This paper presents selected results from the CEMM M3D code [2]. Two-fluids predicts very different properties and performance limits for helical fusion plasmas, such as the NCSX stellarator. The central ‘sawtooth’ pressure crash, common in high temperature plasmas, illustrates cases of axisymmetric plasmas with strong asymmetries (toroidal rotation, large perturbations, etc). Two-fluids better explain the \( n = 1, q = 1 \) sawtooth in the NSTX spherical torus with rotation and the axisymmetric \( n = 0 \) sawtooth that produces a central zero-current hole in JET and other experiments. Individual charged particle motions, more detailed than fluid models, are illustrated by fast fusion alpha-particle effects on the \( n = 1 \) sawteeth in ITER-FEAT, using the gyrokinetic particle simulation model in M3D[3].

2. Symmetries
A two-fluid nonlinear simulation model for lower frequency regimes can be defined [1] by self-consistently keeping the effects of the diamagnetic drifts \( \mathbf{v}_{\ast j} = \mathbf{B} \times \nabla p_j/q_j nB^2 \), for the electrons and ions, \( j = e, i \). (Note Eq. (2.24) in [1] should have a term \(+\mathbf{v}_{\ast e} \cdot \nabla n\) on the LHS). The standard MHD equations form a subset of these equations.

For toroidal plasmas, the MHD equations possess symmetries with respect to the toroidal \( (\phi) \) and poloidal \( (\theta) \) angles, the long and short circumferences of the torus [1]. Important parts of toroidal plasma dynamics lie in the space formed by linear combinations of functions that satisfy one of the two MHD parities \( f(\theta, \phi) = \pm f(-\theta, -\phi) \). Axisymmetric plasmas are further constrained. Ideal MHD equilibria can be described by nested magnetic surfaces, labeled by poloidal magnetic flux \( \psi \) (the magnetic field \( \mathbf{B} = \nabla \psi \times \nabla \phi + \nabla_\perp F + I \nabla \phi \), \( F \) and \( I \) being functions of space). Pressure, density, and temperature \( p = k n T \) are functions of flux. As long as the equilibrium is static (no rotation) and the plasma density does not evolve, fixing the parity (sign) of one quantity determines the parity of every other quantity. Furthermore, numerical simulation shows that most important MHD instabilities in axisymmetric plasmas, such tearing and ballooning modes, tend to grow most rapidly in the equilibrium subspace, even when the plasma density evolves (cf Fig. 3).

Two-fluid models can be rigorously ordered in a way that satisfies desired conservation properties [4]. The diamagnetic drifts constitute a major part of the differential motion of the electrons and ions in lower frequency regimes. They allow the plasma mass (ions) to move independently of the magnetic field, whose motion is mainly electron-based. Inspection shows that all the major two-fluid terms break the MHD symmetries. This is an expression of the general property that kinetic, ie particle-motion-based, plasma processes increase the independence of the electron and ion dynamics and therefore further break the MHD symmetries.

3. Helical plasmas
Helical confinement configurations, such as stellarators, are fundamentally composed of both MHD parities. Due to the difficulty of analysis, stellarator plasmas to date have been designed and analyzed using MHD, with piecemeal additions from other physics models. Although reasonably successful, their stability and confinement remain poorly understood. M3D simulations show[5, 6] that the basic two-fluid diamagnetic processes predict fundamentally different steady state forms and beta limits compared to MHD, and better conform to experiment. Theoretically, the most stringent limits on stellarator equilibria at high beta are imposed by ideal and resistive MHD ballooning and interchange modes with relatively high mode numbers, near the plasma boundary. Nonlinear simulation of the NCSX quasi-axisymmetric stellarator[7] shows that these modes are easily stabilized by ion diamagnetic effects at realistic two-fluid strength \( H \), due to smearing out of their short wavelengths by finite ion Larmor radius.
size. (Here $H = c/(\omega_R R)$, where $\omega_R = (4\pi n m_i/e^2)^{1/2}$ is the ion plasma frequency, $c$ the speed of light, and $R$ the plasma major radius.)

Electron effects, on the other hand, increase magnetic reconnection rates and saturated island sizes at interior magnetic surfaces with low order rational winding numbers $\lambda = m/n$, through the action of the the electron pressure gradient in Ohm’s law. Its parallel-to-$B$ component allows the parallel electric field, through which the magnetic field reconnects, to become large at modest perturbed currents $\tilde{J}_\parallel$. Magnetic reconnection rates may thus impose the intrinsic limit on stellarator beta, a “soft” limit on the electron $\beta_e$ ($\beta_e = p_e/(B^2/2)$ in mks units), due to steadily degrading confinement in the presence of ever larger magnetic islands. Figure 1 shows the increase of island size with $p_e/p$ at fixed total volume averaged $\beta = 7\%$, well above the MHD stability limit and official design value of $\beta = 4.2\%$.

Two-fluid physics may explain a number of long-standing puzzles regarding stellarator behavior[6]. Ideal MHD ballooning and interchange modes, although often theoretically unstable in MHD, are almost never observed nor is confinement reduced. The corresponding resistive MHD modes, also at high mode numbers, are almost always theoretically unstable and again, almost never seen. (The resistive modes were stable in the two-fluid simulations even at unrealistically high resistivities.) Experiments, such as W7-AS, have seen some beta limits that behave as if due to islands, very unlike turbulent transport, although measurements were not good enough to detect islands directly. Definitive experimental tests do not yet exist for most of these predictions, although work is underway.

4. Toroidal Rotation

Global toroidal rotation, as a direct or indirect result of applied plasma heating, is a common feature of axisymmetric plasmas. In bumpy plasmas, rotation is damped by dissipation, but strict axisymmetry allows toroidally rotating steady states with velocity of the form $v_\phi = R\Omega(\psi)$, for an arbitrary function $\Omega$ of the poloidal magnetic flux $\psi$. The rotational part of $v_\phi$ has opposite MHD parity to that of the axisymmetric equilibrium, + rather than -. Strong rotation drives an outward, centrifugal shift in the plasma density and pressure, matched by a corresponding outward shift of the magnetic surfaces. It has little effect on the structure of the MHD solution, except to drive a balancing radial electric field $E_\psi = -\nabla\Phi(\psi)$.

At higher temperatures, toroidal plasmas often experience periodic “crashes” of the central pressure. Such crashes would strongly reduce fusion reaction rates, as well as possibly triggering other instabilities, so their control is important in a burning plasma. In many axisymmetric plasmas, the central magnetic field safety factor $q$ (inverse of the magnetic field winding number) falls below unity as the central temperature and current rise. The crash is associated with predominantly $m = 1$ poloidal and $n = 1$ toroidal harmonics and affects the entire region inside the flux surface where the magnetic field safety factor $q = m/n = 1$. 

Figure 1. Two-fluid island size increases with electron pressure in the NCSX stellarator at high $\beta = 7\%$. Magnetic puncture plots for $p_e/p = 0.05$, 0.5, and 0.95 (islands still growing).
In the NSTX spherical torus at Princeton Plasma Physics Laboratory, the high temperature $m = 1$ mode behavior is anomalous compared to MHD. When the plasma is heated with neutral beams, the beam ions drive a strong toroidal rotation, with Mach number $M_A \simeq 0.2–0.3$. The beams are aimed tangentially in the same direction as the plasma current (“co”-direction). M3D simulations have shown[8] that MHD reproduces basic features of the growing instability and crash, including the observed final redistribution of $v_\phi$ similarly to pressure. MHD fails, however, to explain the long saturated phase that precedes and delays a crash.

Two-fluid effects, on the other hand, can be strongly stabilizing in the presence of toroidal rotation. The mechanism seen in the simulation is nonlinear and the effect depends on the direction of the plasma rotation. Co-rotation leads to strong stabilization at the NSTX values of $M_A$. Counter-rotation speeds up the instability. Under co-rotation, two-fluid effects pull the hot region away from the magnetic X-point where magnetic reconnection takes place. This is stabilizing, since for the crash to occur, the central plasma within the $q = 1$ surface must flow out through the X-point. In the usual MHD crash, the outward shift of pressure toward the X-point combines with the magnetic field perturbation to nonlinearly accelerate motion toward the X-point (plasma and field move together, except at the reconnection spot). In two-fluids, even in the small amplitude mode[1], the diamagnetic $\omega_{\ast i}$ drift allows poloidal displacement, ie, $\theta$-rotation, of temperature and pressure relative to the magnetic field. Toroidal rotation and nonlinear effects increase the poloidal displacement and, in NSTX, saturate the instability at an early stage, soon after a small magnetic island forms at $q = 1$.

5. Current hole at finite beta
Rotation of the perturbation itself can also facilitate reconnection. The “current hole” observed in JET and other large tokamaks is a spontaneously formed, self-sustained central region with essentially zero toroidal current. Such regions are interesting for fusion, since they support high pressures without the unfortunate instabilities associated with current fluctuations. While MHD can explain the formation of the current hole as the consequence of a series of $m = 0$, $n = 0$ sawtooth reconnections (JET at zero beta[9]), it does not explain the observed clamping of the current at zero when beta is finite. MHD predicts that the hole eventually stabilizes reconnection, and the current grows increasingly negative. At finite beta, the peak pressure region, preserved through the reconnection, moves to the outboard (low field, large major radius) side of the torus, since this is energetically favorable. The shift disconnects it from the magnetic X-point, which
remains on inboard side, and reconnection ceases. Lowering beta sufficiently allows the pressure to shift back inward so that reconnection occurs.

Two-fluid, ion diamagnetic $\omega_{\|i}$ effects allow reconnection to continue at finite beta, by making the mode rotate poloidally, in the $\omega_{\|i}$-direction at $p_e = p_i$. Figure 2 shows the magnetic flux for a two-fluid JET-like case at a peak $\beta = 1\%$. The pressure peak, located in the crescent shaped island, remains mostly outboard (right side), but the X-point rotates around to meet it. Successive crashes, at varying orientations, occur with complete reconnection and keep the current clamped near zero. The important driving effect is the mode rotation, since an artificial initial rotation applied to the strictly MHD mode, with the same direction and magnitude, causes a similar fast reconnection.

6. Fast particles

Particle dynamics also breaks the MHD symmetries. The M3D code can simulate a gyrokinetic hot particle population on top of an MHD background plasma[2]. The hybrid model has been carefully benchmarked[3]. Recent work on the $q = 1$ sawtooth in the proposed ITER-FEAT D-T fusion experiment shows[3] that the stabilizing effects of the fast fusion-produced alpha particles are significantly weaker than have been predicted by analytical estimates. The vertical elongation of the plasma reduces the stabilizing effect, by a factor of 2.5 for ITER-FEAT, measured by the hot particle contribution to the perturbed free energy, $\delta W_\alpha$. Previous studies suggested that elongation should improve stabilization, but simulation shows that it also changes the diamagnetic frequency $\omega_\alpha = k_\perp v_\alpha$, significantly, cancelling its stabilizing effect on the other main frequency, the $\alpha$ precessional drift. Figure 3 contrasts the MHD and hybrid modes.

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