A heuristic algorithm for medical staff’s scheduling problems with multiskills and vacation control

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Abstract

Introduction The main issue related to the duty schedule is to allocate medical staff to each medical department by considering personnel skills and personal vacation preferences. However, how to effectively use staff’s multiskill characteristics and how to execute vacation control have not been well investigated.

Objectives This article aims to develop duty scheduling and vacation permission decisions to minimize the sum of customers’ waiting costs, the overtime cost of medical staff, the cost of failing to meet medical staff’s vacation requirements, and the cost of mutual support between departments.

Methods This study formulated the problem as a multiperiod mixed integer nonlinear programming model and developed a hybrid heuristic based on evolutionary mechanism of genetic algorithm and linear programming to efficiently solve the proposed model.

Results Five types of problems were solved through Lingo optimization and the proposed approach. For small-scale problems, both methods can find the optimal solutions. For a slightly larger problem, the solutions found by the proposed approach are superior those of Lingo.

Conclusion This research discusses the complex decision-making problem of on-duty arrangement and vacation control of medical staff in a multidepartmental medical center. This research formulates the medical staff’s scheduling and vacation control problems as constrained mixed integer quadratic programming problems. Computational results indicate that the proposed approach can efficiently produce compromise solutions that outperform the solutions of the Lingo optimization software.

Keywords
Medical staff scheduling, multiskills, vacation control, heuristic

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Introduction

Operating a medical department usually needs to meet some requirements, which include (1) the combination of medical staff assigned to each department should meet each department’s manpower needs, and (2) the duty days of each staff should meet the requirements of the maximum and minimum working days. In general, the internal staff of each medical department is sufficient to meet these requirements. However, some gaps may exist between the manpower from internal members due to uncertainty demand. The uncertain factors include emergencies and uncertain operation times, uncertain surgery durations and so forth. When the medical manpower of a department is insufficient, many negative consequences may occur. First, patients must spend additional waiting time. The workload of the medical staff in the department is expected to increase. A department may take the risk of shortage of basic care services and poor patient prognosis due to insufficient nurses. In addition, employees who apply for a vacation may be requested to postpone their leave application due to insufficient night shift staff and insufficient staff on holidays.

Basically, the increase of manpower can enhance the quality of medical care service. However, from a cost–benefit point of view, merely increasing manpower is not economical because excessive increase in staff will only ineffectively increase the cost of medical services. Thus, properly arranging manpower under the established personnel and determining the correct number of manpower deployed on any given shift are important. When arranging medical manpower, planners can further notice two phenomena. First, staff have more than two technical skills. This feature enables staff in different departments to support one another across departments in the same branch or other branches because they have the skills required by the supported department. Second, medical staff may have their personal preferences to apply for their statutory holidays or to make up for certain overtime days. To avoid the cancellation of approved leave applications due to insufficient manpower, it is better to simultaneously arrange medical manpower and perform vacation permission control at the same time, because the staff’s vacation preference will affect the number of people that can be arranged.

However, no work in the literature simultaneously investigates how to effectively use staff’s multiskill characteristics and how to execute vacation control. In order to fill the research gap, this study establishes a mathematical model and an intelligent heuristic approach to construct the decisions of medical staff’s duty scheduling and vacation control to minimize the sum of customers’ waiting costs, the overtime cost of medical staff, the cost of failing to meet medical staff’s vacation requirements, and the cost of mutual support between departments.

Literature review

Baker categorized the personnel scheduling problems into three types, namely, the shift scheduling problem, days off scheduling problem, and tour scheduling problem. The shift scheduling problem type is of a daily time scheduling problem and is scheduled in daily scheduling. The days off scheduling problem means the length of the work week in the facility does not necessarily match the length of the work week of
employees. The third category is a combination of the first two problem types. In personnel scheduling, organization staff work seven days a week and work shift every day. This type of planned schedule not only considers the rules of the number of days that should be applied for work and vacation within a period but also includes the work and rest periods on the working day. The scope and type of the third category are complex, and problem solving is less easy. The problems discussed in this article are of this type.

The characteristic of large-scale personnel scheduling problem is that it is large in scale, with complicated constraints, and is difficult to solve. In terms of solution methods, Bechtold et al.\(^\text{10}\) classified the solution method of personnel scheduling problem into two types: linear programming or construction-based approaches. Erhard et al.\(^\text{11}\) provided a good overview of quantitative methods for physician scheduling problems. They addressed the characteristics of various physician scheduling problems, categorized the existing research methods, and explained the research gaps. Mathematical programming techniques, such as linear, integer, or mixed integer programming, are usually applied to formulate most personnel scheduling problems. Problem solving techniques, such as heuristic algorithms based on decomposition techniques and meta-heuristics, are used to solve them. However, it is difficult to obtain a satisfactory solution within a reasonable time using exactly methods since most personnel scheduling problems are computationally complex. Thus, many heuristics or meta-heuristics such as neural networks, particle swarm optimization, ant colony optimization, scatter search iterated local search and variable neighborhood search\(^\text{12-16}\) have been applied to solve various types of personnel scheduling problems because they can effectively generate reasonable feasible solutions within a limited computational time.

The components in the objective function and the constraints on decision variables in various scheduling problems are quite diverse. Personnel costs are usually considered. Some authors also consider customer waiting costs. Johnson et al.\(^\text{17}\) showed that nearly 77% of emergency department patients leave the hospital without receiving treatment due to the long waiting time. Bhandari et al.\(^\text{18}\) proposed an algorithm to solve a dynamic operator staffing problem in a call center. The objective of their model is to minimize cost associated with delay, hiring servers, waiting and using temporary servers. Nah and Kim\(^\text{19}\) established a mathematical model for the purpose of minimizing the costs related to waiting, labor, and abandonment to discuss the labor planning and deployment of hospital appointment call centers. Othman et al.\(^\text{20}\) balanced patient needs and human resources from the viewpoint of the workload of all medical staff, patient waiting time, and patient response time to optimize the functions of the pediatric emergency department.

The above literature does not consider personnel scheduling issues with cross-department support. Mabert and Raedels\(^\text{21}\) established a model with multisectors in which the holiday schedule is used to deal with changes in daily customer demand in the bank office. Later, Bechtold\(^\text{22}\) developed a transfer model to discuss the same problem. However, neither of these two seminal works consider the scheduling issues of mutual assistance among personnel across departments. Considering employees’ abilities and skills, Dahmen et al.\(^\text{23}\) dealt with a personalized multidepartment multiday shift planning problem in which employees can transfer between departments subject to certain restrictions. The goal of the problem is to assign workday shifts to employees.
within the scope of the multiday plan by considering some common restrictions related to the multiday shift plan and some restrictions that apply to the interdepartmental transfer flexibility option. They established a scheduling plan to minimize the costs related to under-coverage, over-coverage, transfer, and labor. However, in this research, the workdays of each employee are known. Schoenfelder et al.\textsuperscript{24} established a model to deal with a nurse scheduling problem through the determinations of resource sizes and allocations, cross-training levels, patient process strategies, and nurse arrangements by using flexible labor and patient transfer between inpatient units.

Some researchers deal with the duty scheduling problem in which the personnel have multiple skills. Bechtold et al.\textsuperscript{10} classified personnel scheduling solutions into three groups according to the flexibility of skills. The first group is defined by planners. In this case, the scheduler can freely define the skills of each person. The second group is related to the issue of hierarchical workforce. In this case, higher-level employees can perform the tasks of lower-level employees, and vice versa. The characteristic of the last group is that skills cannot be substituted. Tasks or jobs that require specific skills can only be performed by workers with that skill. Nevertheless, workers may still receive cross-training. Van den Bergh et al.\textsuperscript{25} divided personnel scheduling problems into three categories according to the flexibility of skills. The first category is user-defined skill types. In this type, everyone’s skills can be freely defined. This type is suitable for the situation where employees with certain specific skills can easily be hired or trained. The second group handles the case with hierarchical workforce. In this case, high-level skilled personnel can do jobs lower than their own skill level. The third group is where jobs with specific skills can only be performed by people with that skill and cannot be replaced by people with other skills. However, workers may still receive cross-training.

When personnel can be divided into different skill categories, a hard constraint is usually added to ensure that the number of workers required for each skill is sufficient during a specific period. In the case of soft constraints, when employees with the right skills are lacking, people with other skills can take over, but the disadvantage is that it damages the objective value. Harper et al.\textsuperscript{26} used simulation optimization methods to solve the problem of nursing team size and skills. Campbell\textsuperscript{27} proposed a two-stage stochastic procedure to schedule and assign cross-trained workers in a multidepartment service environment with random demands.

Among the mentioned literature with mutual support between departments, no research considers the multiskilled characteristics of personnel and the waiting time of patients. Long waiting time is an important reason for patient dissatisfaction. Therefore, in this study, in addition to considering the multiskilled characteristics of the personnel, the human support among multiple departments, and the vacation preferences of medical staff, waiting cost is considered in the objective function.

**Model description and assumption**

A medical institution provides medical services by its $S$ employees and $M$ departments. The number of skills needed in this medical institution is divided into $K$ types. The service provided by each department requires a different combination of skill types.
We use parameters $D_m^k = 1$ and $D_m^k = 1$ to indicate that skill $k$ is needed and not needed in department $m$, respectively. In department $m$, $P_m^k$ internal employees are allocated to serve jobs that require skill $k$. The total number of internal employees in department $m$ is $P_m = \sum_{k=1}^{K} P_m^k$ with $S = \sum_{m=1}^{M} P_m$. We respectively use binary parameters $e_s^m = 1$ and $e_s^m = 0$ to indicate whether or not employee $s$ is an internal employee of department $m$. Every employee in the department has a primary skill. In addition to primary skill, some staff have secondary skills. Let binary parameters $a_{sk} = 1$ and $a_{sk} = 0$ to indicate that staff $s$ has and does not have skill $k$, respectively.

The institution is planning a $T$ days’ staff schedule and divides a day into $H$ periods. During the planning days, the minimum and maximum working days of each staff are $T_1$ and $T_2$ days, respectively. The working time of each employee is $E$ consecutive periods, and each employee must have an allowable starting working period. We use binary parameters $a_s^h = 1$ and $a_s^h = 0$ respectively to indicate whether or not employee $s$ can start to work from the start of period $h$. In addition, employees apply for personal leaves on working days. We use symbol $r_{st}$ to represent whether employee $s$ applies for a leave on day $t$.

For department $m$ on day $t$, let $B_{tm}^m$ and $E_{tm}^m$ be the starting and ending operating periods, respectively. For convenience, we set $B_{tm}^m = 0$ and $E_{tm}^m = 0$ if no operation on day $t$. During the operation period of department $m$, the maximum service station that can be set up in department $m$ is $n_m^m = J_m$ due to capacity limitation; the minimum service station that needs to be setup is $n_m^m = 1$ due to the maintenance of basic operations. If $n_{tm}^m$ severs are setup in period $h$ of day $t$ in department $m$, the number of employees with skill $k$ needed is required to have at least $n_{tm}^m N_m^k$ for all $k$. The hourly service rate per sever of department $m$ is assumed to follow the exponential servicing time with parameter $\mu_m$. During the operation period of department $m$, the average customer arrival rate during period $h$ is assumed to follow the Poisson distribution with average arrival rate per hour, $\lambda_{mh}$. Each department has different workloads at different periods. To reach the purpose of mutual support between departments, the hospital performs staff transfer to adjust the number of service stations in each department.

In addition, the period waiting cost of customers in department $m$ is $c_m^m$, the daily overtime cost of staff $s$ is represented by $c_s^o$, and the cost of failing to allow staff $s$’ leave application is represented by $c_s^c$. In addition, we let $g_s^m$ represent the cost of dispatching employee $s$ to work in department $m$. If department $m$ is employee $s$’ own department, then the cost is $g_s^m = 0$.

The medical institution expects to minimize the sum of customers’ waiting cost, the cost of failing to allow vacation applications, the overtime cost and the cost of dispatching employees to support other departments. To reach this goal, the medical institution must seriously make the following decisions: (1) the decision of each employee’s starting working period, (2) the decision of each employee’s working days, (3) the decision of whether employees work in their own department, (4) the decision related to which departments’ employees are dispatched to support departments with short manpower, and (5) the decision to approve leave application. The relevant symbols and decisions are summarized as follows, and the mathematical model is formulated thereafter.
The total cost (TC) includes the cost of unapproving employees’ leave applications (CR), the overtime cost and the dispatch cost for cross-departmental personnel support (CO), and patients’ waiting cost (CW), and is given by (1).

\[ TC = CW + CR + CO + CD. \]  

where

\[ CR = \sum_{t=1}^{T} \sum_{s=1}^{S} (r_{st} - y_{st})c_{s}. \]  

\[ CO = \sum_{s=1}^{S} \left( \sum_{t=1}^{T} \sum_{h=1}^{H} x_{st}^{h} - T_{1} \right) c_{s}. \]  

\[ CD = \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{s=1}^{S} \sum_{h=1}^{H} z_{sm}^{hm} (1 - e_{s}^{m}) g_{s}^{m}. \]  

\[ CW = \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{h=1}^{H} W_{qmj}^{mh} c_{m}. \]  

On day \( t \), the number of customer arrivals per hour in department \( m \) follows the Poisson distribution with an average of \( \lambda_{mh}^{t} \). According to the M/M/S queueing model, the average waiting time that a customer waits at the queuing system of department \( m \) having \( n_{mh}^{t} = j \) severs in period \( h \) of day \( t \) can be estimated by (6).

\[ W_{qmj}^{mh} = \frac{\lambda_{mh}^{t} \mu_{m}^{j} (\lambda_{mh}^{t} / \mu_{m}^{j})^{j}}{(j - 1)! (j \mu_{m}^{j} - \lambda_{mh}^{t})^{2}} P_{qmj}^{mh}, \quad \forall \ m, \ t, \ h, \]  

where \( P_{qmj}^{mh} \) is the probability that no one appears at department \( m \) having \( j \) service stations and is given by (7).

\[ P_{qmj}^{mh} = \frac{1}{\sum_{n=0}^{j-1} \frac{1}{n!} \left( \frac{\lambda_{mh}^{t} \mu_{m}^{j}}{\mu_{m}^{j}} \right)^{n} + 1 / j! \left( \frac{\lambda_{mh}^{t} \mu_{m}^{j}}{\mu_{m}^{j} \mu_{m}^{j} - \lambda_{mh}^{t}} \right), \quad \forall \ m, \ t, \ h. \]  

In addition, the decision variables are subject to the following constraints. The number of service stations in department \( m \) cannot exceed \( J_{m} \). Thus, we have:

\[ n_{mh}^{t} \leq J_{m}, \quad \forall \ m, \ t, \ h. \]  

The number of staff with skill \( k \) in department \( m \) in period \( t \) is the sum of staff dispatched from all departments to department \( m \). Thus, we have:

\[ q_{kjt}^{mh} = \sum_{s=1}^{S} z_{st}^{mh} a_{sk}^{y_{skjt}} \quad \forall \ k, \ m, \ k, \ t, \ h. \]
The number of employees with skill \( k \) allocated must be no less than \( N^m_k \) when \( n^m_{th} \) serves are setup in department \( m \) in period \( h \) of day \( t \). Thus, we have:

\[
q^m_{kt} \geq n^m_{th} N^m_k , \, \forall \, s, h.
\] (10)

Every employee has a shift starting period. Thus, we have:

\[
\sum_{h=1}^{H} z^h_s = 1, \, \forall \, s.
\] (11)

The start time of each employee must be selected from the set of periods that can start work. Thus, we have:

\[
z^h_s \leq a^h_s, \, \forall \, s, h.
\] (12)

For any day, if the first period of an employee’s shift time is not from time period \( h \), then the employee’s initial working time on any day is not from period \( h \). Thus, we have:

\[
x^h_{st} \leq z^h_s, \, \forall \, s, t, h.
\] (13)

The sum of the number of duty periods of employee \( s \) in all departments on any day cannot exceed \( E \) working periods on any day. Thus, we have:

\[
\sum_{m=1}^{M} \sum_{h=1}^{H} z^m_{sh} \leq E, \, \forall \, s, t.
\] (14)

Employees will not be on duty in more than one department at the same time. Thus,

\[
\sum_{m=1}^{M} z^m_{st} \leq 1, \, \forall \, s, t, h.
\] (15)

Employees will not be on duty at any time of any day with two different skills at the same time. Therefore, we have (16).

\[
\sum_{k=1}^{K} y^h_{skt} \leq 1, \, \forall \, s, t, h.
\] (16)

When employee \( s \) is on duty with his expertise \( k \) in period \( h \) on day \( t \), he must be dispatched to a department that needs such an expertise on duty. Thus, we have (17).

\[
y^h_{skt} \leq \sum_{m=1}^{M} z^m_{st} D^m_k, \, \forall \, s, t, h.
\] (17)

Suppose employee \( s \) is on duty in a certain department during the \( h \)-th period on any day. Then the shift start time corresponding to this duty period must satisfy (18–19).

\[
\sum_{m=1}^{M} z^m_{sh} = \sum_{h' = 1}^{h} x^h_{st} + \sum_{h' = H + h - E + 1}^{H} x^{h'}_{st}, \, \forall \, s, t, h \leq E - 1.
\] (18)
\[
\sum_{m=1}^{M} x_{st}^{mh} = \sum_{h'=h-E+1}^{h} x_{st}^{h'}, \quad \forall \ s, \ t, \ h \geq E.
\] (19)

If day \( t \) is an employee’s working day, then only one starting working period is recorded on that day. Thus, we have (20).

\[
\sum_{h=1}^{H} x_{st}^{h} \leq 1, \quad \forall \ s,
\] (20)

Each employee must work at least \( T_1 \) days. Thus, we have (21).

\[
\sum_{t=1}^{T} \sum_{h=1}^{H} x_{st}^{h} \geq T_1, \quad \forall \ s,
\] (21)

Each employee’s maximum working days is \( T_2 \). Thus, we have (22).

\[
\sum_{t=1}^{T} \sum_{h=1}^{H} x_{st}^{h} \leq T_2, \quad \forall \ s,
\] (22)

If no leave application exists, a decision whether to approve a leave is no longer needed. Thus, we have (23).

\[
y_{st} \leq r_{st}, \quad \forall \ s, \ t.
\] (23)

If employee \( s \) asks for a leave on day \( t \) and is approved, the employee will not be scheduled to work on that day. Thus, we have (24).

\[
x_{st}^{h} \leq 1 - y_{st}, \quad \forall \ s, \ t, \ h.
\] (24)

Finally, \( n_{m}^{nh} \) must be an integer, and \( x_{st}^{h}, y_{skt}^{h}, x_{skt}^{h}, \forall \ s, t, h, m, n, j, s, t \in \{0, 1\} \forall m, j, n, s, t.\)

**Solution method**

The problem is a mixed integer nonlinear programming problem. We develop a hybrid heuristic to solve the problem. The evolutionary mechanism of genetic algorithm is applied to determine the number of service stations, \( n_{m}^{nh} \), and skills used by employees on duty, \( y_{skt}^{h} \). The concept of the heuristic is addressed as follows.

**Encoding scheme of a chromosome for the number of service stations**

**Gene expression.** Let \( O_{i}^{m} \) be the smallest integer, such that \( O_{i}^{m} E \geq E_{i}^{m} - B_{i}^{m} \), and be used to determine the number of shifts on the operating day. A chromosome is composed of \( \sum_{m=1}^{M} \sum_{t=1}^{T} O_{i}^{m} \) integer genes.

An example: Suppose a medical center is planning a \( T=2 \) -day scheduling for its \( M = 2 \) departments. The number of periods per shift for each employee is \( E = 4 \); the start period
Formula (25).

The values of gene, $g$, $Em$, respectively. (3, 1, 2, 2, 1, 1, 22, 1) on the fourth row of Table 1 is an example of a chromosome. The number of service stations setup at periods 1, 5, and 9 are $max\{3, 1\}$, respectively. According to Equation (25), we know that for department 1, the number of service stations for department 1 for days 1 and 2, respectively. $(3, 1, 2, 2, 1, 1, 22, 1)$ on the fourth row of Table 1 is an example of a chromosome.

**Determination of the number of service stations.** The $i = \sum_{m=1}^{M-1} \sum_{t=1}^{T} O_{m}^{n} + \sum_{t=1}^{T-1} O_{m}^{n} + n$-th gene, $g^1$, is used to determine the values of $n_{t}^{mh}$ for $h = B_{t}^{m} + (n - 1) E$ through Formula (25).

$$n_{t}^{mh} = \max\{g_{t}^{1}, f_{t}^{mh}\} \forall m, t, h. \tag{25}$$

where $f_{t}^{mh}$ is given by (26).

$$f_{t}^{mh} = \min\{j | j\mu^{m} \geq \max(\lambda_{t}^{mh}, B_{t}^{m} + (n - 1) E) \leq h \leq B_{t}^{m} + n E - 1)\} \forall m, t, h. \tag{26}$$

For example, the periods corresponding to the first three data of the chromosomes in Table 1 are periods $h = 1, 5,$ and $9$. For department $m = 1$, suppose the minimum integers that satisfy $j\mu^{1} \geq \max(\lambda_{1}^{h}, 1 \leq h \leq 4)$, $j\mu^{1} \geq \max(\lambda_{1}^{h}, 5 \leq h \leq 8)$, and $j\mu^{1} \geq \max(\lambda_{1}^{h}, 9 \leq h \leq 12)$ are $j_{1}^{1} = 2, j_{2}^{1} = 2$, and $j_{3}^{1} = 2$, respectively. Then, consider that the values of $g_{t}$ in the chromosome that correspond to periods $h = 1, 5$, and $9$ are $3, 1, 1$, and $2$, respectively. According to Equation (25), we know that for department 1, the number of service stations setup at periods $1, 5$, and $9$ are $\max\{3, j_{1}^{1}\} = 3$, $\max\{3, j_{2}^{1}\} = 2$, and $\max\{2, j_{3}^{1}\} = 2$ service stations, respectively. That is, for department $m = 1$ in day $t = 1$, the values of $n_{j}^{mh}$ are $n_{1}^{1h} = 0$ for all $h$, except that $n_{1}^{1,1} = 3, n_{1}^{1,5} = 2$ and $n_{1}^{1,9} = 2$.

**Encoding scheme of a chromosome for skills used by employees on duty**

**Gene expression.** A chromosome is composed of SMTH binaries. Employee $s$ can only be on duty in departments with $e_{s}^{m} = 1$, and department $m$’s operating periods on day $t$ are from $B_{t}^{m}$ to $E_{t}^{m}$. On an operation day of department $m$, we let $T_{Bt}^{Em} = E_{t}^{m} - B_{t}^{m} + 1$ for $E_{t}^{m} > B_{t}^{m}$ and $T_{Bt}^{Em} = 0$ for $E_{t}^{m} = B_{t}^{m} = 0$. For employee $s$, we use $\sum_{m=1}^{M} e_{s}^{m} \sum_{t=1}^{T} T_{Bt}^{Em}$ binaries

**Table 1.** An example of a chromosome.

| M | 1 | 2 |
|---|---|---|
| T | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| No. of genes | 1 | 2 | 3 | 1 | 2 | 1 | 2 | 1 |
| Chromosome | 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 |
| Period | 1 | 5 | 9 | 3 | 7 | 3 | 7 | 3 | 7 |
to represent whether employee \( s \) is on duty in department \( m \) with skill \( k \) in time period \( h \) on day \( t \). A chromosome is composed of \( \sum_{s=1}^{S} \sum_{m=1}^{M} e_{s}^{m} \sum_{t=1}^{T} T^{Em_{t}} B_{t} \) binary genes.

**Determination of skills used by employees on duty.** For employee \( s \), \( y_{skt}^{h} \) is 0 in the following two situations: (a) department \( m \) is not a specialized department (i.e. \( e_{m}^{s} = 0 \)) or (b) non-operating time of possible duty department \( m \) (i.e. \( B_{m}^{h} = E_{m}^{h} = 0 \)). In addition to the above situation, the values of \( y_{skt}^{h} \) are determined by its gene value. However, for fixed \( s \), \( t \), and \( h \), given that only one of \( y_{skt}^{h} \) s can be 1, when the number of elements in \( \{ k | y_{skt}^{h} = 1, a_{sk} = 1 \} \) is larger than 1, we use the following criteria to determine which \( k \) will be one. We give different priorities on the basis of whether skill \( k \) with \( y_{skt}^{h} = 1 \) is the original department of employee \( s \). For all \( k \in \{ k | y_{skt}^{h} = 1, a_{sk} = 1 \} \), if \( D_{m}^{k} a_{sk} = e_{s}^{m} \), then the duty location of employee \( s \), who uses skill \( k \), is his/her original department. That is, we let \( y_{skt}^{h} = 0 \) for all \( k \), except \( y_{skt}^{h} = 1 \) for the \( k \) with \( D_{m}^{k} a_{sk} = e_{s}^{m} \).

**Fitness function**

After the values of \( n_{t}^{mh} \) and \( y_{skt}^{h} \) are determined, the value of \( CW \) is also known by Equations (5)–(7). We can compute the value of \( CW \). However, the number of employees may be insufficient to arrange feasible scheduling on the basis of the number of service stations generated by the previous step. Under a given policy of \( n_{t}^{mh} \), the required number of employees with skill \( k \) for period \( h \) of day \( t \) for department \( m \) is \( n_{t}^{mh} N_{k}^{m} \). If insufficient, then the policy of \( n_{t}^{mh} \) and \( y_{skt}^{h} \) becomes an infeasible solution. To avoid unfeasible solutions, we define \( z_{kt}^{m} \) as the shortage of employees with skill \( k \) in period \( h \) of day \( t \) in department \( m \), and a penalty value \( z_{kt}^{m} \) is given to a shortage of employee with skill \( k \) in department \( m \). We add a penalty cost function by (27).

\[
\tilde{CD} = \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{h=1}^{H} z_{kt}^{m} z_{st}^{m} .
\]

We also add Constraint (28).

\[
\sum_{s=1}^{S} z_{st}^{mh} a_{sk} + z_{kt}^{m} \geq n_{t}^{mh} N_{k}^{m}, \ \forall \ m, k, t, h.
\]

Subsequently, the fitness value and the decisions of \( z_{kt}^{h}, y_{skt}^{h}, z_{st}^{mh}, x_{ht}^{k}, z_{st}^{mh} \) under the policy of \( n_{t}^{mh} \) and \( y_{skt}^{h} \) are obtained by solving the following model.

\[
\text{Min } CW + CR + CO + CD + \tilde{CD}
\]

Subject to equations (2) to (4), (11) to (23), and (27) to (28).

We refer to the above model as Model 1. The procedure is addressed as follows.

**Solution procedure:**

(A) Parameter input.
(B) Generate an initial population of a group of individuals, let the best cost be \( TC^* \) and be a large number, and start the iteration index from \( I = 1 \).

(C) Perform the following procedures while \( \text{Iter} < \text{Iter}_{\max} \) or the stop criterion has not been met.

1. Perform the following steps for each individual.
   (a) For each department, determine the values of \( n_{t}^{m_h} \) and \( y_{st}^{h} \) by using the decoding value generated by the proposed algorithm and Equations (24) and (25).
   (b) Substitute \( n_{t}^{m_h} \) and \( y_{st}^{h} \) as the parameters into Model 1 and solve Model 1 to obtain the decisions of \( z_{s}^{h}, x_{st}^{h}, g_{st}^{m_h}, y_{t k}, z_{st}^{m_h} \) and the fitness, \( TC \), of an individual.
   (c) If no shortage occurs, that is \( \bar{CD} = 0 \), and the current fitness, \( TC \), is better than the best solution so far, \( TC^* \), then let \( TC^* = TC \) and replace the decisions of \( z_{s}^{h}, x_{st}^{h}, g_{st}^{m_h}, y_{t k}, z_{st}^{m_h} \), and \( n_{t}^{m_h} \) with \( z_{s}^{h}, x_{st}^{h}, g_{st}^{m_h}, y_{t k}, z_{st}^{m_h} \), respectively.
2. If \( \text{iter} = \text{iter}_{\max} \) or the stop criterion has been met, then report the result; otherwise, let \( \text{iter} = \text{iter} + 1 \); perform three operators, including election, crossover, and mutation; back to Step (a).

Numerical example

In this section, the duty scheduling problem of \( M = 8 \) departments is used to illustrate the duty scheduling planning. A total of 191 employees are recorded in these eight departments. Each staff’s belonging department, \( e_{s}^{m} \), and possessing skills, \( a_{sk} \), are shown in Table 2. The values of \( P_{k}^{m} \) and \( P_{m}^{m} \) can also be obtained from Table 2. The minimum number of medical staff with skill \( k \) requires to set up a service station in department \( m \), \( N_{k}^{m} \) is as shown in Table 3. To shorten this paragraph, the remaining parameters are assumed to be in formula form to save space. Each department’s starting and ending operating periods are assumed to be \( B_{t}^{m} = 5 \) and \( E_{t}^{m} = 8 \) for \( 1 \leq t \leq 5 \) for departments \( 1 \leq m \leq 4 \), \( B_{t}^{m} = 5 \) and \( E_{t}^{m} = 10 \) for \( 2 \leq t \leq 6 \) for departments \( 5 \leq m \leq 6 \), and \( B_{t}^{m} = 1 \) and \( E_{t}^{m} = 12 \) for \( 1 \leq t \leq 7 \) for departments \( 7 \leq m \leq 8 \). The value of \( a_{s}^{h} \) is assumed to be \( a_{s}^{h} = 1 \) for \( \max\{B_{t}^{m}, \forall m\} \leq h \leq \{E_{t}^{m}, \forall m\} \). The value of \( r_{st} \) is assumed to be \( r_{st} = 1 \), where \( t' = \text{mod}(s, 7) + 1 \). The value of \( \mu^{m} \) is assumed to be \( 5 + \text{mod}(m, 4) \). The value of \( \lambda_{s}^{m_h} \) is assumed to be \( 0.7\mu^{m} + 0.2\text{mod}(t + m + h, 4) \). Costs of \( c_{s}^{m}, c_{s}^{r}, c_{s}^{o} \) and \( g_{s}^{m} \) are assumed to be \( c_{s}^{m} = 60 + 20\text{mod}(m, 5) \), \( c_{s}^{r} = 100 + 100\text{mod}(m, 5) \), \( c_{s}^{o} = 1500 + 200\text{mod}(s, 3) \), and \( g_{s}^{m} = 20 \), respectively. Suppose that the belonging department of staff \( s \) is department \( n \) and the department on duty is department \( m \). For \( m = n \), suppose \( g_{s}^{m} = 0 \). For \( m \neq n \), suppose \( g_{s}^{m} = 20 \) for \( m \leq 6 \), \( g_{s}^{m} = 90 \) for \( m = 7 \), and \( g_{s}^{m} = 100 \) for \( m = 8 \).

Design of testing problems and computational environment

In this section, five problem types were designed to investigate the performance of the proposed approach and impacts of parameters on the computational results. For each
Table 2. The values of $a_{jk}$ and each employee’s possessing skills.

| s | $S_m$ | AS | s | $S_m$ | AS | s | $S_m$ | AS | s | $S_m$ | AS | s | $S_m$ | AS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6 | 29 | 3 | 9.10 | 57 | 5 | 10.11 | 85 | 6 | 9 | 113 | 7 | 2.7 | 141 | 7 | 12 | 169 |
| 2 | 1.4 | 30 | 3 | 10.12 | 58 | 5 | 10.11 | 86 | 6 | 9 | 114 | 7 | 7.8 | 142 | 7 | 12 | 170 |
| 3 | 1.5 | 31 | 3 | 10.11 | 59 | 5 | 11.12 | 87 | 6 | 10 | 115 | 7 | 7.7 | 143 | 7 | 9.12 | 171 |
| 4 | 1.3 | 32 | 3 | 10.11 | 60 | 5 | 10.11 | 88 | 6 | 9 | 116 | 7 | 3.7 | 144 | 7 | 12 | 172 |
| 5 | 1.5 | 33 | 3 | 10.12 | 61 | 5 | 11.12 | 89 | 6 | 10.12 | 117 | 7 | 7.8 | 145 | 7 | 12 | 173 |
| 6 | 1.8 | 34 | 3 | 10.11 | 62 | 5 | 10.11 | 90 | 6 | 9.12 | 118 | 7 | 2.7 | 146 | 7 | 9.12 | 174 |
| 7 | 1.0 | 35 | 3 | 10.12 | 63 | 5 | 11.1 | 91 | 6 | 6.1 | 119 | 7 | 1.7 | 147 | 7 | 9.12 | 175 |
| 8 | 1.0 | 36 | 3 | 11.1 | 64 | 5 | 11.12 | 92 | 6 | 6.12 | 120 | 7 | 9.10 | 148 | 8 | 1.8 | 176 |
| 9 | 1.0 | 37 | 4 | 4.6 | 65 | 5 | 10.11 | 93 | 7 | 1.7 | 121 | 7 | 9.10 | 149 | 8 | 3.8 | 177 |
| 10 | 1.0 | 38 | 4 | 4.8 | 66 | 5 | 11 | 94 | 7 | 5.7 | 122 | 7 | 9.10 | 150 | 8 | 5.8 | 178 |
| 11 | 1.0 | 39 | 4 | 10 | 67 | 5 | 10.11 | 95 | 7 | 7 | 123 | 7 | 10 | 151 | 8 | 8 | 179 |
| 12 | 1.0 | 40 | 4 | 9.10 | 68 | 5 | 11 | 96 | 7 | 7 | 124 | 7 | 10 | 152 | 8 | 2.8 | 180 |
| 13 | 2.7 | 41 | 4 | 9.10 | 69 | 5 | 9.12 | 97 | 7 | 7.8 | 125 | 7 | 9.10 | 153 | 8 | 5.8 | 181 |
| 14 | 2.6 | 42 | 4 | 10 | 70 | 5 | 11.12 | 98 | 7 | 5.7 | 126 | 7 | 10 | 154 | 8 | 5.8 | 182 |
| 15 | 2.9 | 43 | 4 | 9.10 | 71 | 5 | 9.12 | 99 | 7 | 3.7 | 127 | 7 | 10.12 | 155 | 8 | 7.8 | 183 |
| 16 | 2.0 | 44 | 4 | 12 | 72 | 5 | 10.12 | 100 | 7 | 4.7 | 128 | 7 | 10.11 | 156 | 8 | 11 | 184 |
| 17 | 2.0 | 45 | 5 | 5.7 | 73 | 6 | 2.6 | 101 | 7 | 7.8 | 129 | 7 | 9.10 | 157 | 8 | 9.11 | 185 |
| 18 | 2.9 | 46 | 5 | 1.5 | 74 | 6 | 6.8 | 102 | 7 | 5.7 | 130 | 7 | 9.10 | 158 | 8 | 11.8 | 186 |
| 19 | 2.10 | 47 | 5 | 4.5 | 75 | 6 | 2.6 | 103 | 7 | 7 | 131 | 7 | 10.11 | 159 | 8 | 9.11 | 187 |
| 20 | 2.11 | 48 | 5 | 4.5 | 76 | 6 | 6.7 | 104 | 7 | 5.7 | 132 | 7 | 9.10 | 160 | 8 | 10.11 | 188 |
| 21 | 2.1 | 49 | 5 | 3.5 | 77 | 6 | 10.12 | 105 | 7 | 6.7 | 133 | 7 | 10 | 161 | 8 | 11.8 | 189 |
| 22 | 2.9 | 50 | 5 | 1.5 | 78 | 6 | 10 | 106 | 7 | 4.7 | 134 | 7 | 10 | 162 | 8 | 11 | 190 |
| 23 | 2.13 | 51 | 5 | 1.5 | 79 | 6 | 9.12 | 107 | 7 | 4.7 | 135 | 7 | 10 | 163 | 8 | 10.11 | 191 |
| 24 | 3.4 | 52 | 5 | 5.8 | 80 | 6 | 9.10 | 108 | 7 | 7.8 | 136 | 7 | 9.10 | 164 | 8 | 10.11 | |
| 25 | 3.7 | 53 | 5 | 1.5 | 81 | 6 | 9 | 109 | 7 | 5.7 | 137 | 7 | 10.11 | 165 | 8 | 11 | |
| 26 | 3.5 | 54 | 5 | 4.5 | 82 | 6 | 9.11 | 110 | 7 | 2.7 | 138 | 7 | 9.12 | 166 | 8 | 9.11 | |
| 27 | 3.4 | 55 | 5 | 1.5 | 83 | 6 | 9.10 | 111 | 7 | 2.7 | 139 | 7 | 10.12 | 167 | 8 | 11 | |
| 28 | 3.8 | 56 | 5 | 2.5 | 84 | 6 | 9 | 112 | 7 | 7 | 140 | 7 | 11.12 | 168 | 8 | 10.11 | |

Symbol AS represents available skills.
Table 3. The number of technicians $k$ required to set up a service station, $N_k^m$.

| $m$ | Required skills | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|-----------------|---|---|---|---|---|---|---|---|
|     | 1               | 10| 12| 3 | 9 | 12| 5 | 11| 12|
|     | 2               | 11| 12| 4 | 10| 12| 6 | 9 | 12|
|     | 3               | 9 | 12| 5 | 11| 12| 7 | 10| 12|
| $N_k^m$ | 3 | 2 | 1 | 1 | 3 | 1 | 3 | 2 | 1 | 1 | 3 | 1 |
problem type, five test cases were examined in which all parameters are the same as the basic dataset, except one parameter was changed. The number of departments for the five problem types was set from 4 to 8. In addition, for problem types 1 and 2, only the waiting cost was changed, and the cost was set at \((1 + 0.1(n - 3))c_{m}^{n}\) for the \(n\)-th case. For problem type 3, only the cost of unapproved leave was changed and became \((1 + 0.1(n - 3))c_{s}^{n}\) for the \(n\)-th case. For problem type 4, only the cost of unapproved leave was changed and became \((1 + 0.1(n - 3))g_{s}^{m}\) for the \(n\)-th case. For problem type 5, only the customer arrival rate was changed and became \((1 + 0.1(n - 3))\lambda_{mh}^{m}\) for the \(n\)-th case.

We used symbol HGA to stand for the proposed approach. The Visual C++ programming language was used to code the solution procedure of HGA. All of the test problems were solved by HGA and well-known Lingo optimization solver. The computing platform was implemented on an Intel® Core™ i5-7200 U CPU 2.7 GHz notebook computer with 16.0 GB RAM. The calculation time was set to a maximum of four hours. In HGA, 20 individuals comprised each generation, among which four individuals with the first four best fitness values were retained as the elite group and were directly copied to the next generation without evolution operators. The mating and mutation rates were set to 0.95 and 0.06, respectively. The criteria for the algorithm to stop include the conditions for convergence and when the number of iterations reaches 20,000. The condition of convergence is when the best solution appears continuously for 20 generations and remains unchanged. The comparison result of the HGA and Lingo was used to examine the performance of the proposed method.

**Illustrative example**

In this subsection, case 3 of problem type 2 was used to explain the on-duty period and working department of each staff in each department. In this case, 72 staff were recorded. We summarized the calculation results according to staff’s belonging departments \(S_{m}\), duty days \(D\), and working periods, including each employee’s working departments, as presented in Table 4. The table shows that most staff were on duty in their own departments. However, employee number 21 belonging to department 2 supported department 5 in periods 5–8 of the sixth day, and employee number 48 belonging to department 5 supported department 4 in periods 5–8 from day 2 to day 5. The interdepartmental staff support cost incurred was $400. Although employee numbers 14 and 28 applied for the first day of leave, their applications were not approved. The unapproved leave cost was $500. The total waiting cost was $8807. Summing the three costs, \(TC = $9707\).

**Performance of heuristic procedure**

All five types of problems were solved through Lingo optimization and HGA. The computational results of the five problem types are shown in Tables 5–9, in which the percentage gap of the TC between Lingo and HGA was given by the formula of \((LS-HS)/LS\), where symbols LS and HS stand for the TC of Lingo method and the TC of HGA method, respectively.
1. Ability to find optimal solutions

Lingo software is a software that can confirm the optimal solution. When the solution it finds is the best, the solution report will show the message of the global optimal solution found. If the optimal solution cannot be found, but a feasible solution can be found, then the solution report will show the message of the feasible solution found. Among the five problems, Lingo can find the optimal solution for all test cases in problem type 1 but can only find feasible

| Table 4. Computational result for case 3 of problem type 2. |
|-------------|-------------|-------------|-------------|-------------|-------------|
| S  | S_0 | D  | W_0 | W_0 | s   |
| 1–12 | 1  | 1–5 | 5–8 | 1  | 45,47,49,53,55,65,69 |
| 13–20,22 | 2  | 1–5 | 5–8 | 2  | 46,51,54,58,61–63,68 |
| 21  | 2  | 2–5 | 5–8 | 2  | 48,50,52,56,57,59,60,64,66,67,70–72 |
| 21  | 2  | 6   | 5–8 | 5  | 48  |
| 23–36 | 3  | 1–5 | 5–8 | 3  | 48  |
| 37–44 | 4  | 1–5 | 5–8 | 4  |       |

| Table 5. Computational result of problem type 1. |
|-------------|-------------|-------------|-------------|-------------|-------------|
| Lingo       | HEU         | LA vs. HA  |
| No.         | CW  | CR | CO | CD | TC  | CPU | CW  | CR | CO | CD | TC  | CPU | LA vs. HA  |
| 1           | 6360 | 600 | 0  | 0  | 6960 | 35  | 6360 | 600 | 0  | 0  | 6960 | 25  | 0.0%        |
| 2           | 7154 | 600 | 0  | 0  | 7754 | 44  | 7154 | 600 | 0  | 0  | 7754 | 31  | 0.0%        |
| 3           | 7950 | 600 | 0  | 0  | 8550 | 92  | 7950 | 600 | 0  | 0  | 8550 | 25  | 0.0%        |
| 4           | 8745 | 600 | 0  | 0  | 9345 | 56  | 8745 | 600 | 0  | 0  | 9345 | 26  | 0.0%        |
| 5           | 9540 | 600 | 0  | 0  | 10,140 | 36  | 9540 | 600 | 0  | 0  | 10,140 | 31  | 0.0%        |

| Table 6. Computational result of problem type 2. |
|-------------|-------------|-------------|-------------|-------------|-------------|
| Lingo       | HEU         | LA vs. HA  |
| No.         | CW  | CR | CO | CD | TC  | CPU | CW  | CR | CO | CD | TC  | CPU | LA vs. HA  |
| 1           | 7546 | 500 | 0  | 120 | 8166 | 7313 | 500  | 0  | 80 | 7893 | 155 | 3.3%        |
| 2           | 8449 | 500 | 0  | 160 | 9109 | 8227 | 500  | 0  | 80 | 8807 | 158 | 3.3%        |
| 3           | 9387 | 500 | 0  | 160 | 10,047 | 8807 | 500  | 0  | 400 | 9707 | 177 | 3.4%        |
| 4           | 10,285 | 500 | 0  | 200 | 10,985 | 9687 | 500  | 0  | 400 | 10,587 | 142 | 3.6%        |
| 5           | 11,220 | 500 | 0  | 200 | 11,920 | 10,568 | 500  | 0  | 400 | 11,468 | 195 | 3.8%        |
| 37–44       | 9378 | 500 | 0  | 168 | 10,046 | 8920 | 500  | 0  | 272 | 9692 | 3.49% |
solutions for the four other types of problems. Table 4 presents that Lingo and HGA can find the optimal solution for all cases of problem type 1. This feature shows that HGA can also find the optimal solution similar to the Lingo method.

2. Comparison of solution quality

The number of decision variables increases rapidly with a small increase in the number of departments. When the number of departments increases from five in problem type 2 to eight departments in problem type 5, the scale of the problem also increases rapidly. For problem type 2, the data are the same as those of problem type 1 but only the number of departments increases from four departments in problem type 1 to five departments. The increase in variables makes Lingo unable to find the optimal solution within the limited time. The last column of Table 6 shows that the HGA solution is slightly better than Lingo’s solution in all test cases of Problem 2. On average, the HGA solution is better than Lingo’s solution by 3.49%. From the last columns of Tables 5–9, we can find that as the problem size increases, the percentage gap of HGA over Lingo method tends to increase.
| No. | Lingo         | HEU       |        |      |       |       | CPU | LA vs. HA |
|-----|--------------|-----------|--------|------|-------|-------|-----|-----------|
|     | CW           | CR        | CO     | CD   | TC    | CW    |     |           |
| 1   | 36605        | 300       | 0      | 2200 | 58905 | 19404 | 0   | 42.9%     |
| 2   | 59310        | 300       | 13000  | 34580| 107190| 22159 | 0   | 66.1%     |
| 3   | 87759        | 300       | 2200   | 36920| 127179| 25388 | 0   | 68.9%     |
| 4   | 114099       | 0         | 0      | 33100| 147199| 38440 | 0   | 50.1%     |
| 5   | 327456       | 0         | 67000  | 38990| 433446| 51546 | 0   | 80.0%     |
|     | 125046       | 180       | 16440  | 33118| 174784| 39551 | 0   | 61.58%    |
Discussions

From the above numerical example results, we can find the following:

1. From the results of problem types 2 and 3 (last line of Tables 6 and 7), the average percentage gap of the TC between the two methods is only 3% and 6%, respectively. On average, the waiting cost of method Lingo is much higher than the waiting cost of HGA, and the dispatching cost of HGA is only slightly higher than that of Lingo. This result suggests that HGA can better use the manpower between departments to support one another for weighing various costs to achieve the goal of reducing the TC. Therefore, although the dispatching cost of HGA is higher, it can make the waiting cost much lower than that of Lingo. Therefore, TC is lower than that of Lingo.

2. For small sizing problems, planners can apply Lingo optimization software or HGA to plan manpower arrangements because both approaches can find optimal solutions. However, as problem size increases, planners should choose HGA to efficiently find satisfactory solutions.

3. For large problems, the Lingo and HGA methods can find feasible solutions. However, from the large gap between the Lingo and the HGA solutions shown in Tables 8 and 9, the quality of the Lingo solution strictly becomes poor with the increase in scale. One of the possible reasons for the HGA method to find a solution superior to the Lingo method in a short time is that the number of service stations and the skills of the personnel on duty by HGA are generated through algorithms. In addition, HGA uses the relationship between variables to further determine the variable of staff’s starting working time. These features greatly reduce the uncertain variables and the solution searching space. Furthermore, through the mechanism of evolutionary algorithm, the solution scheme is continuously improved to produce improved solutions. Thus, HGA approach outperforms the well-known commercial software Lingo solver for large-scale problems.

4. In the case of changes in waiting costs, unsatisfied vacation costs, and cross-departmental support costs, the proposed algorithm initiates a low-cost scheme first, weighing various costs to avoid a rapid increase in TC. For example, for problem type 2, when the waiting cost rises, the proposed algorithm uses further cross-departmental support, so that the waiting cost does not rapidly increase, and the TC does not increase rapidly. For another example, in problem type 5, when the demand is not high (cases 1–3), only low-cost personnel scheduling is used to properly arrange the department’s manpower, but when the demand rises to more than 10% (cases 4–5), high-cost employee overtime programs begin to be launched.

Conclusion

Medical Staff’s duty scheduling and leave control are practical problems in many medical institutions. The result of scheduling that does not consider vacation issues may lead to
poor on-duty schedule, because the vacation preference of medical staff will affect the actual number of staff that can be arranged. To overcome this problem, this study establishes a mathematical model to simultaneously deal with these two issues and formulate the problem as a constrained mixed-integer programming problem to minimize the sum of customer waiting costs due to insufficient allocation of staff, the overtime cost of medical staff, the cost of failing to meet medical staff’s vacation requirements, and the cost of mutual support between departments. Since the formulated model is a highly complicated combinatorial optimization problem. We propose a hybrid intelligent approach based on a genetic algorithm to efficiently solve the problem. Computational results indicate that our proposed approach outperforms the Lingo optimization software. In addition, when waiting costs and demand rise, the proposed solution procedure can weigh the waiting costs and support costs, and adjust cross-departmental support decisions in a timely manner, so that the total cost will not increase rapidly. In practice, this effect can reduce the operating expenditure of medical institutions.

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Appendix

Notation

\[ a^h_s \]
A binary parameter, \( a^h_s = 1 \) if employee \( s \) can start working from period \( h \), and \( a^h_s = 0 \) otherwise.

\[ a_{sk} \]
A binary parameter, \( a_{sk} = 1 \) if employee \( s \) has skill \( k \), and \( a_{sk} = 0 \) otherwise.

\[ B^m_t \]
The starting operating period for department \( m \) on day \( t \).

\[ c^{jn} \]
Waiting cost per unit hour for customers waiting for treatment in department \( m \).

\[ c^o_s \]
Overtime cost of employee \( s \) per period.

\[ c^f_s \]
Cost of unapproving employee \( s \)'s leave application.

\[ D^m_k \]
A binary parameter, \( D^m_k = 1 \) if skill \( k \) is needed in department \( m \), and \( D^m_k = 0 \) otherwise.

\[ E \]
The number of periods per shift.

\[ E^m_t \]
The ending operating period for department \( m \) on day \( t \).

\[ e^m_s \]
A parameter, \( e^m_s = 1 \) if employee \( s \) belongs to department \( m \), and \( e^m_s = 0 \) otherwise.

\[ g^m_s \]
Cost of dispatching employee \( s \) to support department \( m \).

\[ H \]
The total number of periods in a day.

\[ J^m \]
The maximum number of service stations that can be set at department \( m \).

\[ K \]
The number of total skill types.

\[ M \]
The number of departments in medical institutions.

\[ N^m_k \]
The minimum number of employees with skill \( k \) needed to be allocated when a service station is set at department \( m \).

\[ P^m \]
The number of internal employees in department \( m \) of the medical institution.

\[ P^m_k \]
The number of internal employees with primary skill \( k \) allocated to department \( m \).

\[ r_{st} \]
A parameter, \( r_{st} = 1 \) if employee \( s \) applies for a leave on day \( t \), and \( r_{st} = 0 \) otherwise.

\[ S \]
The total number of employees in medical institutions.

\[ T \]
The total number of planning days.

\[ T_1 \]
The number of basic shifts during the planning period.

\[ T_2 \]
The maximum number of shifts during the planning period.

\[ \mu^m \]
The service rate per period per service station of department \( m \).

\[ \lambda^{mh}_t \]
The average number of customers arriving at department \( m \) in period \( h \) of day \( t \).

Decision variables:

\[ n^{mh}_t \]
The number of service stations set in department \( m \) in period \( h \) of day \( t \).

\[ q^{mh}_{sk} \]
The number of staff with skill \( k \) in department \( m \) in period \( h \) of day \( t \).

\[ x^{st}_h \]
A binary variable, \( x^{st}_h = 1 \) if the starting period of employee \( s \)'s shift on day \( t \) is period \( h \) and \( x^{st}_h = 0 \) otherwise.

\[ y^{jh}_{skt} \]
A binary variable, \( y^{jh}_{skt} = 1 \) if employee \( s \) uses skill \( k \) to work in period \( h \) period on day \( t \), and \( y^{jh}_{skt} = 0 \) otherwise.

\[ y_{st} \]
A binary variable, \( y_{st} = 1 \) if the leave application of employee \( s \) for day \( t \) is permitted and \( y_{st} = 0 \) otherwise.
$z_{st}^{mh}$ A binary variable, $z_{st}^{mh} = 1$ if employee $s$ works in department $m$ in period $h$ of day $t$, and $z_{st}^{mh} = 0$ otherwise.

$z_{s}^{h}$ A binary variable, $z_{s}^{h} = 1$ if the shift type of employee $s$ includes period $h$, and $z_{s}^{h} = 0$ otherwise.