Training Convolutional Neural Networks With Hebbian Principal Component Analysis

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Abstract. Recent work has shown that biologically plausible Hebbian learning can be integrated with backpropagation learning (backprop), when training deep convolutional neural networks. In particular, it has been shown that Hebbian learning can be used for training the lower or the higher layers of a neural network. For instance, Hebbian learning is effective for re-training the higher layers of a pre-trained deep neural network, achieving comparable accuracy w.r.t. SGD, while requiring fewer training epochs, suggesting potential applications for transfer learning. In this paper we build on these results and we further improve Hebbian learning in these settings, by using a nonlinear Hebbian Principal Component Analysis (HPCA) learning rule, in place of the Hebbian Winner Takes All (HWTA) strategy used in previous work. We test this approach in the context of computer vision. In particular, the HPCA rule is used to train Convolutional Neural Networks in order to extract relevant features from the CIFAR-10 image dataset. The HPCA variant that we explore further improves the previous results, motivating further interest towards biologically plausible learning algorithms.

Keywords: Convolutional Neural Networks · Computer Vision · Unsupervised Learning · Principal Component Analysis · Hebbian · Biologically Inspired.

1 Introduction

The error backpropagation algorithm (backprop) has been used with great success for training neural networks (e.g. [10, 25]) on a variety of learning tasks, including computer vision. However, Neuroscientists doubt that it is biologically plausible and that it models the real learning processes of the brain [19]. A possible biologically plausible learning mechanisms could be based on the so-called Hebbian principle: “Neurons that fire together wire together”. Starting from this simple principle, it is possible to formulate different variants of the Hebbian learning rule which offer interesting opportunities in computer science and artificial intelligence. For example, Hebbian learning with Winner-Takes-All (HWTA) competition [9] allows a group of neurons to learn to perform clustering
on a set of data. Another interesting variant is Sanger’s rule \[23\], which allows to perform Principal Component Analysis (PCA) on the data in an online fashion. In essence, Hebbian algorithms can be employed to extract features of interest from data and provide a biologically plausible, efficient and online solution for unsupervised learning tasks.

In the context of Convolutional Neural Networks (CNNs) applied to computer vision, the various layers of the network act as feature extractors, with lower layers extracting low-level features and next layers extracting progressively higher-level features. Hebbian learning algorithms could represent a possible option for training such networks. Previous works \[2,26,27\] already showed that Hebbian learning variants are suitable for training relatively shallow networks (with two or three layers), which are appealing for applications on constrained devices. In \[1,16\], it was also shown that HWTA competition was effective to re-train higher layers of a pre-trained network, achieving results comparable with backprop, but requiring fewer training epochs, thus suggesting potential applications in the context of transfer learning.

In this work, we leverage on these results and we apply a nonlinear Hebbian Principal Component Analysis (HPCA) learning rule \[12,23\] to train CNNs for computer vision tasks. Specifically, a six layer network is trained using HPCA on the CIFAR-10 \[14\] dataset. We evaluate the quality of the features extracted from each layer on the image classification task by feeding these features to linear classifiers and evaluating the resulting accuracy. The results obtained by the HPCA variant are compared to those obtained by the corresponding network trained with backprop and to HWTA. In order to explore the possibility of integrating HPCA with backprop, we also consider hybrid network architectures, in which some layers are trained with backprop, and others with the Hebbian approach.

Results in this paper confirm previous findings showing that Hebbian learning can be integrated with backprop, providing comparable accuracy when used to train lower or higher network layers, while requiring fewer training epochs. Furthermore, we show that the HPCA variant provides further improvements over the results previously obtained with HWTA. On the other hand, the current limitation of the approach is that it is not suitable to train intermediate network layers, for which a significant accuracy drop is observed w.r.t. backprop. However, our results suggest that Hebbian approaches are worth being further explored, and we hope that our work can motivate further interest in this direction.

Biologically inspired learning approaches are attractive, in that they could give insights on peculiar features of human intelligence, such as generalization capabilities, learning speed, energy efficiency, robustness to adversarial examples. Moreover, Hebbian learning algorithms can be easily adapted to Spiking Neural Networks (SNNs) \[8\]. These are neural network models which mimic biological networks even more closely. Communication among neurons is achieved not by means of continuous signals, but by means of short pulses. This communication paradigm allows the networks to implement complex cognitive functions with very limited energy. This makes SNNs suitable for energy efficient imple-
mentations on neuromorphic hardware [7], thus enabling intelligent applications
also on constrained devices. Finally, the Hebbian learning rule is local, which
means that every layer of neurons performs updates independently of the suc-
cessive layers, encouraging the possibility of modular deep learning and parallel
training.

The main contributions of this paper can be summarized as follows:

– A nonlinear Hebbian PCA (HPCA) learning rule variant is used to train
neural network layers to extract features from an image dataset, which are
then used for the task of image classification;
– The learning rule is properly integrated with convolutional layers (Convolu-
tional Hebbian PCA);
– The results are compared with those obtained by an equivalent network
trained with backprop and Hebbian with Winner Takes All (HWTA) and
the potentials and limitations of the approach are highlighted;

The remainder of this paper is structured as follows: Section 2 provides a
background on the related literature; Section 3 presents the Hebbian PCA rule
used in our experiments; Section 4 delves into the details of our experimental
setup; In Section 5 the results of our simulations are illustrated; Finally, Sec-
tion 6 presents our conclusions and outlines possible future developments.

2 Background and related work

Consider a single neuron with weight vector \( w \) and input \( x \). Call \( y = w^T x \)
the neuron output. The Hebbian learning rule, in its most basic form, can be
expressed mathematically as:

\[
\Delta w = \eta y x
\]

where \( \eta \) is the learning rate. Basically, this rule states that the weight on a given
synapse is reinforced when the input on that synapse and the output of the
neuron are simultaneously high. Therefore, connections between neurons whose
activations are correlated are reinforced.

To prevent weights from growing unbounded, a weight decay term is generally
added. In the context of competitive learning [9, 13, 22], this is obtained as follows:

\[
\Delta w = \eta y x - \eta y w = \eta y (x - w)
\]

This rule has an intuitive interpretation: when an input vector is presented to
the neuron, its vector of weights is updated in order to move it closer to the
input, so that the neuron will respond more strongly when a similar input is
presented. When several similar inputs are presented to the neuron, the weight
vector converges to the center of the cluster formed by these inputs (Fig. 1).

When multiple neurons are involved in a complex network, the Winner-Takes-
All (WTA) [9, 22] strategy can be adopted to force different neurons to learn
different patterns, corresponding to different clusters of inputs. When an input
is presented to a WTA layer, the neuron whose weight vector is closest to the
Fig. 1: Hebbian updates with weight decay.

Fig. 2: Hebbian updates with Winner-Takes All competition.
current input is elected as winner. Only the winner is allowed to perform a weight update, thus moving its weight vector closer to the current input (Fig. 2). If a similar input will be presented again in the future, the same neuron will be more likely to win again. This strategy allows a group of neurons to perform clustering on a set of data points (Fig. 2).

In recent works [26, 27], WTA and the variant k-WTA (in which the k neurons with highest activations are elected as winners) were applied in the context of computer vision to train a three layer CNN to extract features from images, in order to perform classification. Similar paradigms were also studied in the context of SNNs [4, 5]. These works showed that the approach is suitable to train relatively shallow networks (e.g. with two or three layers), achieving accuracy around 65-70% on CIFAR-10 [14] and from 95% up to 98-99% on MNIST [17], which is comparable to backpropagation-based approaches on networks of the same depth.

In [1, 16], the authors went further by applying Hebbian-WTA learning to CNNs with up to six layers, comparing the results with those obtained by training the same network with backprop. The WTA approach, as it is, is unsupervised, but a supervised Hebbian learning variant was also proposed in order to train the final classification layer. The results confirmed that the approach was effective for training shallow networks. It was also found that the approach was effective for re-training the higher layers (including the final classifier) of a pre-trained network. In addition, the algorithm required much fewer epochs than backprop to converge.

3 Hebbian PCA

According to the definition given in Section 2, WTA enforces a kind of quantized information encoding in layers of neural network. Only one neuron activates to encode the presence of a given pattern in the input. On the other hand, neural networks trained with back propagation exhibit a distributed representation, where multiple neurons activate combinatorially to encode different properties of the input, resulting in an improved coding power. The importance of distributed representations was also highlighted in [6, 18].

A more distributed coding scheme could be obtained by having neurons extract principal components from data, which can be achieved with Hebbian-type learning rules [3, 23]. In order to perform Hebbian PCA, a set of weight vectors has to be determined, for the various neurons, that minimize the representation error, defined as:

$$L(w_i) = E[(x - \sum_{j=1}^{i} y_j w_j)^2]$$  \hspace{1cm} (3)

where the subscript $i$ refers to the $i^{th}$ neuron in a given layer and $E[\cdot]$ is the mean value operator. It can be pointed out that, in the case of linear neurons and zero centered data, this reduces to the classical PCA objective of maximizing the output variance, with the weight vectors subject to orthonormality constraints.
From now on, we assume that the input data are centered around zero. If this is not true, we just need to subtract the average $E[x]$ from the inputs beforehand.

It can be shown that the following learning rule minimizes the objective in eq. 3:

$$\Delta w_i = \eta y_i (x - \sum_{j=1}^{i} y_j w_j)$$ (4)

In case of nonlinear neurons, a solution to the problem can still be found. Calling $f()$ the neuron activation function, the representation error

$$L(w_i) = E[(x - \sum_{j=1}^{i} f(y_j) w_j)^2]$$ (5)

can be minimized with the following nonlinear version of the Hebbian PCA rule:

$$\Delta w_i = \eta f(y_i) (x - \sum_{j=1}^{i} f(y_j) w_j)$$ (6)

Several variants of the Hebbian PCA approach were explored in literature for the linear case and applied in the context of computer vision, but only for shallow networks. In our experiments, we applied the nonlinear version of the Hebbian PCA rule also on deeper networks, as explained in the following sections.

## 4 Experimental Setup

In the following, we describe the details of our experiments and comparisons, discussing the network architectures and the training procedure.

### 4.1 Network architecture and learning

The core part of our experiments consisted in training the deep layers of a neural network on the CIFAR-10 dataset. We used a neural network consisting of six layers: five deep layers plus a final linear classifier. The various layers were interleaved with other processing stages (such as ReLU nonlinearities, max pooling, etc.), as shown in Fig. 3. The architecture was the same used in [16], it was inspired by AlexNet [15], but one of the fully connected layers

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3 The code to reproduce the experiments described in this paper is available at [https://github.com/GabrieleLagani/HebbianPCA](https://github.com/GabrieleLagani/HebbianPCA).

4 The network achieves 84.95% accuracy on CIFAR-10 when trained using backprop without data augmentation, and 91.5% with data augmentation. However, for simplicity, no data augmentation was used in the rest of the experiments described in this paper.
was removed and, in general, the number of neurons was decreased, in order to reduce the computational cost of the experiments. However, the architectural hyperparameters described above resulted from a parameter search to maximize the accuracy on CIFAR-10 [14] obtained from training the model with backprop.

The network was trained on CIFAR-10 using Stochastic Gradient Descent (SGD) with error backpropagation, and with the HPCA rule in eq. 6 (in which the nonlinearity was set to the ReLU function), in order to compare the results.

In order to evaluate the quality of the features extracted from the various layers of the trained models for the image classification task, we placed a linear classifier on top of each already trained layer, and we evaluated the accuracy achieved by classifying the corresponding features. This was done both for the SGD trained network and for the HPCA trained network. The linear classifier was trained with SGD in all cases. Notice that this does not raise biological plausibility issues, because backpropagation is not required when SGD is used to train a single layer. Although the Hebbian approach is unsupervised, it is also possible to apply a supervised variant [1, 16] for training the linear classifier, but we preferred to use SGD in all cases, in order to make comparisons on equal footings. Indeed, the SGD weight update can be considered as a form of supervised Hebbian update, modulated by a teacher signal.

4.2 Hybrid network models

We also implemented hybrid network models, i.e. networks in which some layers were trained with backprop and other layers were trained with HPCA, in order to assess up to which extent backprop layers in our model could be replaced with Hebbian equivalent without excessive impact on the accuracy. The models were constructed by replacing the upper layers of a pre-trained network with new ones, and training from scratch using different learning algorithms. Meanwhile, the lower layers remained frozen, in order to avoid adaptation to the new upper layers. Various configurations of layers were considered.
4.3 Convolutional HPCA

In order to be able to use the HPCA rule with CNNs, we had to define a proper way to integrate the HPCA rule with convolutional layers. In particular, neurons at different horizontal and vertical offset of the convolutional layer are constrained to have shared weights.

In order to meet this constraint, the learning rule was adapted as follows: each set of neurons looking at the same portion of the image computed their updates by applying rule in eq. 6, the input $x$ being the patch extracted from the image at the specific horizontal and vertical position. We then averaged the updates over the horizontal and vertical dimensions. The resulting update was applied to the kernel shared by all the neurons at different horizontal and vertical locations. When mini-batches of inputs were used during training, the update averaging was performed also over the mini-batch dimension.

4.4 Details of training

We implemented our experiments using PyTorch. Training was performed in 20 epochs (although, for the Hebbian approach, convergence was typically achieved in much fewer epochs) using mini-batches of size 64.

For SGD training, the initial learning rate was set to $10^{-3}$ and kept constant for the first ten epochs, while it was halved every two epochs for the remaining ten epochs. We also used momentum coefficient 0.9, Nesterov correction, dropout rate 0.5 and L2 weight decay penalty coefficient $6 \cdot 10^{-2}$.

In the HPCA training, the learning rate was set to $10^{-3}$. No L2 regularization or dropout was used in this case, since the learning method did not present overfitting issues.

The linear classifiers placed on top of the various network layers were trained with supervision using SGD in the same way as we described above for training the whole network, but the L2 penalty term was reduced to $5 \cdot 10^{-4}$.

All the above mentioned hyperparameters resulted from a parameter search to maximize the accuracy on CIFAR-10 in the respective scenarios.

In all the experiments, we used 40000 CIFAR-10 samples for training, 10000 for validation and 10000 for testing, as this is the standard approach with this dataset. In order to obtain the best possible generalization, early stopping was used in each training session, i.e. we chose as final trained model the state of the network at the epoch when the highest validation accuracy was recorded.

5 Results

In Table 1, we report the CIFAR-10 test accuracy obtained by classifiers placed on top of the various convolutional layers of the network. We compare the results obtained on the network trained with backprop (BP) and HPCA. We also included the results of the Hebbian-WTA (HWTA) method from [16], in order to make comparisons. In all the cases, the Hebbian approach required fewer
Table 1: CIFAR-10 accuracy on features extracted from convolutional network layers (results within 2% accuracy from BP or higher, but achieved with fewer training epochs, are highlighted in bold).

| Layer | BP Acc.(%) | HPCA Acc.(%) | HWTA Acc.(%) |
|-------|------------|--------------|--------------|
| Conv1 | 60.71      | 63.40        | 63.92        |
| Conv2 | 66.30      | 65.42        | 63.81        |
| Conv3 | 72.39      | 65.40        | 58.28        |
| Conv4 | 82.69      | 63.60        | 52.99        |

The results show a general improvement of the HPCA approach w.r.t. the HWTA approach. In Table 1, we can observe that both Hebbian approaches reach comparable performance w.r.t. backprop for the features extracted from the first two layers (but in fewer epochs), suggesting possible applications of Hebbian learning for training relatively shallow networks.

The HWTA approach suffers from a decrease in performance when going further on with the number of layers. With the HPCA approach, this problem seems to alleviate, and the accuracy remains pretty much constant when we move to deeper layers. In particular, the HPCA approach exhibits an increase of almost 11% points w.r.t. HWTA on the features extracted from the fourth convolutional layer. Still, further research is needed in order to close the gap with backprop also when more layers are added, in order to make the Hebbian approach suitable as a biologically plausible alternative to backprop for training deep networks.

In Table 2, we report the results obtained on the CIFAR-10 test set with hybrid networks. In each row, we reported the results for a network with a different combination of Hebbian and backprop layers (the first row below the header represent the baseline fully trained with backprop). We used the letter "H" to denote layers trained using the Hebbian approach, and the letter "B" for layers trained using backprop. The letter "G" is used for the final classifier (corresponding to the sixth layer) trained with gradient descent. The final classifier (corresponding to the sixth layer) was trained with SGD in all the cases (except the last two rows, see later), in order to make comparisons on equal footings. Notice that, as we already said, this does not raise biological plausibility problems, because backprop is not required on the last layer when SGD training is used. The last two columns show the resulting accuracy obtained with the corresponding combination of layers.

Table 2 allows us to understand what is the effect of switching a specific layer (or group of layers) in a network from backprop to Hebbian training. The first row represents our baseline for comparison, i.e. the network fully trained with backprop. In the next rows we can observe the results of a network in which a
Table 2: CIFAR-10 accuracy of hybrid network models (results within 2% accuracy from BP or higher, but achieved with fewer training epochs, are highlighted in bold).

| L1 | L2 | L3 | L4 | L5 | L6 | HPCA Acc.(%) | HWTA Acc.(%) |
|----|----|----|----|----|----|--------------|--------------|
| B  | B  | B  | B  | B  | G  | 84.95        | 84.95        |
| H  | B  | B  | B  | B  | G  | 83.84        | 84.93        |
| B  | H  | B  | B  | B  | G  | 81.12        | 80.36        |
| B  | B  | H  | B  | B  | G  | 79.50        | 80.68        |
| B  | B  | B  | H  | B  | G  | 81.53        | 80.92        |
| B  | B  | B  | B  | H  | G  | 83.90        | 83.75        |
| H  | H  | B  | B  | B  | G  | 78.60        | 78.61        |
| B  | H  | H  | B  | B  | G  | 75.42        | 72.12        |
| B  | B  | H  | H  | B  | G  | 77.00        | 74.98        |
| B  | B  | H  | H  | G  |    | 79.17        | 76.86        |
| H  | H  | H  | B  | B  | G  | 72.92        | 67.87        |
| B  | H  | H  | H  | B  | G  | 69.38        | 63.68        |
| B  | B  | H  | H  | G  |    | 68.44        | 62.43        |
| H  | H  | H  | H  | B  | G  | 66.04        | 57.56        |
| B  | H  | H  | H  | G  |    | 55.91        | 47.24        |
| H  | H  | H  | H  | G  |    | 54.71        | 41.78        |
| B  | B  | B  | B  | B  | H  | 84.88        | 84.88        |
| B  | B  | B  | B  | H  | H  | 83.47        | 83.16        |

single layer was switched. Both HPCA and HWTA exhibit competitive results with the baseline when they are used to train the first or the fifth network layer. A small, but more significant drop is observed when inner layers are switched from backprop to Hebbian, but the HPCA approach seems to perform generally better than HWTA. In the successive rows, more layers are switched from backprop to Hebbian training, and a higher performance drop is observed, but still, the HPCA approach exhibits a better behavior than HWTA. The most prominent difference appears when we finally replace all the network layers with Hebbian equivalent, in which case the HPCA approach shows an increase of 13% points over HWTA. The last two rows aim to show that it is possible to replace the last two layers (including the final classifier) with new ones, and re-train them with Hebbian approach (in this case, the supervised Hebbian algorithm [1,16] is used to train the final classifier), achieving accuracy comparable to backprop, but requiring fewer training epochs (2 vs 10, respectively). This suggests potential applications in the context of transfer learning [28].

6 Conclusions and future work

In summary, our results confirm previous findings, i.e. that the Hebbian approach is suitable for training relatively shallow models (e.g. with just one or
two layers) or to re-train the final layers of a pre-trained deep neural network, while requiring fewer training epochs. This suggests potential applications in the context of constrained devices, where shallow networks might be appealing (while deeper models could be prohibitive), or in the context of transfer learning, where an experimenter wants to re-train or fine-tune higher network layers of a pre-trained model on a new task. In addition, our results show a general accuracy improvement up to 13% of the HPCA rule w.r.t. the HWTA method. On the other hand, we also showed that HPCA is unable to replace SGD for training also intermediate layers of deep neural networks. Further research is still needed to tackle this limitation.

In future work, further improvements might come from exploring more complex feature extraction strategy, which can also be formulated as Hebbian learning variants, such as Kernel-PCA [24] and Independent Component Analysis (ICA) [11]. In addition, it would be interesting to move this work also to the context of Spiking Neural Networks (SNNs), where the Hebbian principle is implemented by the Spike Timing Dependent Plasticity (STDP) learning rule [8]. In this case, it is necessary to map the variants of the Hebbian rule to corresponding STDP variants and test their effectiveness for SNN training. Another interesting aspect would be to explore in more depth the convergence and generalization properties of Hebbian learning algorithms when few training samples are available. For example, we suggest to train the network on CIFAR-10 using only 5000 or 500 training samples, instead of the whole training set, and compare the the results with SGD. Further experiments should be performed also on other datasets and tasks, in order to obtain a more complete picture of the potentialities of this approach. Finally, an exploration on the behavior of such algorithms w.r.t. adversarial examples also deserves attention.

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