Predictions for Fermion Pair Production at $e^+e^-$ Colliders

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We discuss the status and some ongoing upgrades of the ZFITTER program for applications at $e^+e^-$ colliders LEP 1/SLC, LEP 2, GigaZ, and TESLA. The inclusion of top quark pair production is under work.

1. Introduction

One of the great achievements in theoretical and experimental physics over the last three decades is the unique calculation of quantum corrections in the Standard Model (SM) of fundamental particle interactions, following the pioneering work of G. ’t Hooft and M. Veltman [1, 2, 3], and their observational evidence at the 10 $\sigma$ confidence level [4]. Especially the electroweak sector of the SM is being investigated at the high energy $e^+e^-$ colliding facilities LEP 1 and SLC, and now at LEP 2, with unprecedented precision. This, though, could well be surpassed in the future e.g. by the GigaZ mode of the TESLA Linear Collider (LC) project [5, 6]. One main focus, besides the search for direct signals of New Physics, was and still is on studying the properties of the neutral and charged weak gauge bosons.

Here we concentrate on the measurement of different cross sections and asymmetries in fermion pair production. Fermion pair production is an essential tool to determine parameters like the mass and total and partial decay widths of the weak neutral gauge boson $Z$ and the neutral current fermionic couplings. The theoretical break-through for this was the first complete evaluation of electroweak quantum corrections to $e^+e^- \rightarrow \mu^+\mu^-$ in the Weinberg model in [7], with a collection of the scalar one-loop integrals first given in [8].

For today’s predictions at the few per mil level, this calculation can only be seen as the first essential step. It did not include such important ingredients as a refined treatment of the $Z$ boson resonance, of QCD and higher order corrections, or realistic QED hard bremsstrahlung effects.

During the last twenty years many dedicated efforts were invested to perform the necessary improvements, to implement them into computer codes which may be applied in the analysis of experimental data and in regularly repeated comparisons of the codes. A representative collection of the underlying expressions, their implementations and numerical comparisons may be found in [1, 2, 3, 4]; see also [5, 6, 7, 8] for individual comparisons. The most recent workshop on LEP 2 physics is yet under way [9]. Prominent examples of such codes are:

- ALIBABA,
- BHM,
- KORALZ, KK,
- TOPAZO,
- ZFITTER.

On the latter, the semi-analytical Fortran program ZFITTER [10], we will now focus. Earlier program descriptions are [11, 12]. The core of ZFITTER relies on a complete electroweak one-loop calculation [21, 22, 23, 24, 25]. It is also based on many additional formulae; from [13] one may extract a list of papers that ZFITTER uses in addition.
2. Status of two-fermion codes at LEP and higher energies

ZFITTER [18] calculates radiative corrections to the muon decay constant [22, 26, 34, 11], to the $Z$ decay width [26, 54, 11], and to the $W$ decay width [83]. Cross sections and asymmetries are treated in a semi-analytical approach. Improved Born observables $\sigma_{T,FB}^0(s')$ containing the complete virtual weak and QCD corrections are convoluted with analytical flux functions $\rho(s'/s)$ (radiators) for the QED corrections [57, 23, 24, 25]. This is done in a one-dimensional numerical integration over the final-state invariant mass squared $s' = M_f^2$. The special Bhabha scattering case is treated in an effective Born approximation only following [1, 53]. As an example, the initial-state corrections to total cross sections for $s$-channel processes are reproduced in short ($v = 1 - s'/s$):

$$\sigma_T^{\text{ini}}(s) = \int d\left(\frac{s'}{s}\right) \sigma_T^0(s') \rho_T^{\text{ini}}\left(\frac{s'}{s}\right),$$  \hspace{1cm} (1)

$$\rho_T^{\text{ini}}\left(\frac{s'}{s}\right) = (1 + S^{\text{ini}}) \beta_\varepsilon v^{\beta_\varepsilon - 1} + H_T^{\text{ini}}\left(\frac{s'}{s}\right),$$  \hspace{1cm} (2)

$$\beta_\varepsilon = \frac{2\alpha}{\pi} Q_e^2 \left(\ln \frac{s}{m_e^2} - 1\right).$$  \hspace{1cm} (3)

The radiators $\rho_T^{\text{ini}}$, $\rho_T^{\text{ini}+\text{fin}}$, and $\rho_T^{\text{fin}}$ with $S$ and $H_T$ are determined in [57], and including all relevant higher order terms e.g. in [18]. Eq. (4) can be straightforwardly generalized to different asymmetries $A_{FB}$, $A_{\text{pol}}$, $A_{LR}$ etc. or to scattering angle distributions like $d\sigma/d\cos\theta$, then with different effective Born terms and radiators. Kinematic cuts to the final-state phase space may also be applied [57, 23, 24, 25]. Also a more model-independent description of cross section observables, e.g. in form of an $S$-matrix approach, can be thought of [73, 94, 85, 86, 87, 88, 89]. Concerning the status of present codes for LEP 1 and SLC applications, one can summarize

3 A Fortran bug was corrected in ZFITTER v.6.30 [18] resulting in a 0.3% shift of the $W$ partial widths.

that the level of precision for cross section predictions in $s$-channel fermion pair production is now better than $10^{-4}$ on the $Z$ boson resonance and better than $0.3 \times 10^{-3}$ for center-of-mass energies $\sqrt{s} = M_Z \pm 3$ GeV (single contributions). For the recently updated branch with cuts on final-state maximum acollinearity and minimum energies in ZFITTER [16, 25], this is demonstrated in Table 1 comparing with program TOPAZ [66] when the QED initial-final state interference is switched on. For earlier comparisons with acollinearity cuts also consult [16, 57, 58, 59].

At LEP 2 center-of-mass energies up to roughly 200 GeV an agreement of the codes at the order of few per mil is being achieved [17]. This was studied earlier for the two-fermion programs ZFITTER [20, 18], BHM [104, 1], TOPAZ [66], and KORALZ/KK [101, 103, 1] or on maximum acollinearity and minimum energies of the final-state fermions [17, 87, 98, 99]. Similar studies of Bhabha scattering include the code ALIBABA [107] in [13, 106]. The conclusion was that precisions are of the order of few per mil to 1% for LEP 1 or LEP 2 energies respectively, excluding a radiative return to the $Z$ by sufficiently strong cuts. Compared with the final experimental precisions at LEP 1/SLC and LEP 2 these theoretical accuracies are satisfactory [107]. It is worth mentioning that for the envisaged accuracy several recent improvements had to be undertaken, e.g. a better treatment of initial- and final-state pair production corrections, the exponentiation of initial-final state corrections, convolution of the $ZZ$ and $WW$ box corrections, etc. As a result, ZFITTER v.6.30 [18] became more complicated and also slower, and the same might be true also for the other codes.

Thinking of applications of the ZFITTER code and of other programs at a future $e^+e^-$ Linear Collider (LC) with much higher luminosities and energies, the above observations at LEP/SLC energies might only prove to be preliminary and a further upgrade could become necessary. Two different scenarios may be envisaged for the use of the ZFITTER code at a LC:

First, precision physics could again be performed on the $Z$ boson resonance, but then with
Table 1

A comparison of predictions from ZFITTER v.6.30 \[18\] (Jun 2000) and TOPAZ0 v.4.4 \[96\] for muonic cross sections and forward-backward asymmetries around the Z peak. First row is without initial-final state interference, second row with, third row the relative/absolute effect of that interference in per mil \((M_Z = 91.1871 \pm 0.0021 \text{ GeV}, M_H = 125 \text{ GeV}, m_t = 173.8 \text{ GeV}, \alpha_S = 0.119, \Delta \alpha_b(5)(M_Z) = 0.0280398089)\).

| \(\theta_{\text{acc}} = 0^\circ\) | \(M_Z - 3\) | \(M_Z - 1.8\) | \(M_Z\) | \(M_Z + 1.8\) | \(M_Z + 3\) |
|------------------|------------|------------|--------|------------|--------|
| TOPAZ0 | 0.21928 | 0.46282 | 1.44814 | 0.67722 | 0.39362 |
|       | 0.21772 | 0.46077 | 1.44805 | 0.67891 | 0.39486 |
|       | -7.17   | -4.45     | -0.06  | +2.49     | +3.14  |
| ZFITTER | 0.21923 | 0.46278 | 1.44794 | 0.67716 | 0.39356 |
|       | 0.21768 | 0.46075 | 1.44790 | 0.67893 | 0.39485 |
|       | -7.16   | -4.41     | -0.03  | +2.61     | +3.27  |

| \(\theta_{\text{acc}} = 0^\circ\) | \(M_Z - 3\) | \(M_Z - 1.8\) | \(M_Z\) | \(M_Z + 1.8\) | \(M_Z + 3\) |
|------------------|------------|------------|--------|------------|--------|
| TOPAZ0 | -0.28473 | -0.16935 | 0.00014 | 0.11494 | 0.16089 |
|       | -0.28181 | -0.16686 | 0.00068 | 0.11367 | 0.15919 |
|       | +2.92    | +2.49     | +0.54  | -1.27     | -1.70  |
| ZFITTER | -0.28519 | -0.16958 | 0.00005 | 0.11479 | 0.16068 |
|       | -0.28244 | -0.16731 | 0.00005 | 0.11375 | 0.15909 |
|       | +2.75    | +2.27     | +0.60  | -1.04     | -1.59  |

1000 times the luminosity of LEP 1 [\[3\],[4]] in order to search e.g. for virtual effects from a SM or MSSM (minimal supersymmetric) Higgs boson or from supersymmetric particles [\[108\],[5]]. With such a GigaZ option, experimental accuracies could increase by a factor of 100 or more [\[3\],[4]]. Updates of the codes could include higher order QED radiative effects, effects by beamstrahlung, or an update of the still critical Bhabha scattering case when demanding high precisions. What is more, complete electroweak two loop calculations [\[109\]] might become necessary; see also the many talks on this topic at this conference. At energies up to roughly 800 GeV like for the Tesla project [\[110\]], issues like experimental and theoretical precisions are still quite vague, but it is clear that the demands on codes for two-fermion production will be quite challenging due to higher beam energies, higher luminosities, and improved analysis techniques. On the one hand, electroweak and QED corrections become equally important which may demand a critical look at the numerical validity of the usually applied improved Born approximation at higher energies. On the other hand, higher order electroweak corrections will also grow in importance with increasing energies [\[111\]]. First studies of codes ZFITTER, TOPAZ0, and KK show some evidence that an agreement of 5 per mil to 1 per cent can be reached for the complete energy range [\[112\],[113\]] (see Fig. 1).

3. Top pair production at LC energies

At a future LC, top pair production will be studied in detail. One of the main topics of investigation will be the analysis of the elementary properties of the top quark, including the measurement of its mass \(m_t\), total and partial decay widths, and couplings to the weak gauge bosons. The recent progress in calculations at the \(t \bar{t}\) threshold itself is summarized e.g. in [\[114\]] and is continuously being updated in [\[115\]]. A determination of the weak neutral vector and axial-vector couplings of the top quark to the Z boson will be performed in the perturbative region sufficiently above \(\sqrt{s} \approx 350 \text{ GeV}\) by measuring cross sections, forward-backward and polarization asymmetries. Here ZFITTER may natu-
Figure 1. Cross section ratios for muon-pair production with different $s'$ cuts for codes ZFITTER v.6.22 (Oct 1999) [18], TOPAZ0 v.4.4 [96], KK v.4.12 [102] from 60 to 800 GeV c.m. energy; without initial-final state interference (INI PP: initial-state pair production; LL: leading logarithmic terms) [112].

rally step in to do the job. The one key point now to be considered is the inclusion of the mass of the final-state heavy fermions. Up to now final-state masses could be neglected for applications at the LEP or SLC center-of-mass energies.

For the massive case, the complete one-loop electroweak corrections in the SM with the running of the QED coupling, fermionic self energies, vertex corrections, and electroweak box contributions have been determined in [116]. A recent update of the QCD effects was given in [117]. Work is right now in progress treating the electroweak effects in form of effective couplings together with form factors in an improved Born approach [see term $\sigma^0_T(s')$ in (1)], suitable for a quick calculation of interesting observables with the ZFITTER code [118]. For results within the MSSM please refer to [119, 220].

In the remaining part of this contribution, we focus on the top mass dependent radiative corrections from QED bremsstrahlung which correspond to the radiators $\rho_A^a(s'/s)$, $a = ini, fin, int$, $A = T, FB$ [see (1)]. One may expect at most some additional several per cent photonic corrections from the finite top quark mass. Initial-state corrections to $\sigma_T$ and final-state effects to the angular distribution $d\sigma/d\cos \vartheta$ (i.e. also to $A_{FB}$) are known [121, 117].

For the massive cross sections $\sigma_T, A_{FB} = \sigma_T/\sigma_{FB}, d\sigma/d\cos \vartheta$, analytic expressions like (1) would be nice to have. Soft and virtual QED corrections with final-state masses have been known for a long time for pure QED [123] and also in the SM [116, 119]. For the hard massive radiators $\rho_A^a(s'/s; m_f^2/s)$ the situation is the following: With an $s'$ cut only, hard photonic corrections for total cross sections $\sigma_T(s)$ with $m_f \neq 0$ were shown in [121] for the initial- and final-state corrections respectively. Having in mind a cut on the cosine of the scattering angle $\cos \vartheta$ of one final-state fermion, the differential cross section $d\sigma/d\cos \theta$ first has to be determined. For this case, only formulae with final-state bremsstrahlung are given in [30]; without cuts, the total cross section and forward-backward asymmetry had been presented earlier in [122].

It should be mentioned that the QED corrections to cross section contributions $\sigma_L$ and $\sigma_R$ for a left- or right-handed polarized $e^-$ beam are not automatically given by the results $\rho_A^a$ for unpolarized cross sections; there are additional radiators that do depend on the initial helicities [124]. Of course, the effective Born results $\rho_T^a$ also have to be replaced by the corresponding massive terms $\sigma^0_{LR}$ with polarization. Nevertheless, the predictions for $A_{LR}$, as being measured at SLC at the $Z$ resonance using a polarized $e^-$ beam, may be to a very good precision described by the massless radiator functions (as is presently done in ZFITTER) since there hard bremsstrahlung is strongly suppressed, and, in addition, for polarization asymmetries the photonic corrections cancel to a large extent (see e.g. [83]).
3.1. The distribution \( \frac{d\sigma_{\mathrm{int}}}{ds'} \)

The hard-photon contribution from QED interference to total cross sections for heavy fermions,

\[
e^-(p_1) + e^+(p_2) \rightarrow f(p_3) + \bar{f}(p_4) + \gamma(k),
\]

will be calculated here by two methods; one is tensor integration. The final-state phase space is split into two two-particle phase spaces. The \((f \bar{f})\) rest frame is boosted with respect to the center-of-mass system (cms). The phase space integration is then conveniently carried out in two steps: First, the final-state tensor integration is then conveniently carried out in two steps: First, the final-state tensor \( T_{\mu\nu} \) is integrated over \( p_3 \) and \( p_4 \) in the \((f \bar{f})\) rest frame, leaving Lorentz covariant expressions in \( k^\mu \) and \( q^\mu \equiv p_3^\mu + p_4^\mu \). Contracting then with the initial-state tensor \( I_{\mu\nu} \), the remaining integration over the cosine of the photon polar angle with respect to the beam axis is carried out in the center-of-mass system. Evidently, the cms fermion production angle is not accessible in this approach and thus neither the angular distribution nor \( A_{FB} \).

The integrated distribution gets:

\[
\frac{d\sigma_{\mathrm{int}}}{ds'} = \sum_{V_1,2=\gamma,Z,Z'} \frac{d\sigma_{\mathrm{int}}}{ds'}(V_1V_2),
\]

where

\[
\frac{d\sigma_{\mathrm{int}}}{ds'}(Z'Z) = -4\alpha^3 Q_e Q_f \cdot \Re[Z'(s')(Z^2)](v_e a'_e + a_e v'_e) + \frac{1}{s^2} \frac{1 + s'/s}{1 - s'/s} \left[ \beta_f(v_f a'_f + a_f v'_f) - \frac{2m_f^2}{s} \frac{s}{s} L_f \left( \frac{s}{s} v_f a'_f + a_f v'_f \right) \right],
\]

and

\[
L_f = \log \left( \frac{1 + \beta_f}{1 - \beta_f} \right),
\]

\[
\beta_f = \sqrt{1 - \frac{4m_f^2}{s}}.
\]

\[
\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2} \pi \alpha s - M_Z^2 + iM_Z V_Z}.
\]

For Z-boson exchange we have: \( v_f = I_3^f - 2Q_f \sin^2 \theta_W \) and \( a_f = I_3^f \) with \( I_3^f = -1/2 \), \( Q_e = -1 \). For the contributions from \( \gamma \)-exchange, for example, we have \( v_f = Q_f \), \( a_f = 0 \), etc. and \( \chi_{\gamma} \rightarrow 1 \). Massive contributions are proportional to \( 2m_f^2/s \cdot L_f \) which drop out for \( \beta_f \rightarrow 1 \) in the massless case. The formula (8) is quite compact.

It may of course also be used in the context of searches for extra heavy gauge bosons \( Z' \) e.g. through \( ZZ' \) mixing effects (see e.g. \[ 71, 74, 123 \] and program \text{ZEFIT} \[ 82 \]). The massless limit is given in Eq. (1.3.14) in \[ 13 \].

Numerical results are shown in Fig. 2. We obtained that figure also with another approach (see the next section), where we also include a short discussion.

3.2. The distribution \( \frac{d\sigma_{FB}^{int}}{ds'} \)

Another phase space parameterization makes the fermion production angle accessible. The approach goes back to \[ 51 \] and is explained in sections (1.2) and (1.5.1) of \[ 18 \]; see also \[ 16 \] for more details, where also the first analytical integration is described. After that, the integration is much simplified, and it is here where presently the Fortran program \text{topfit.f} \[ 124 \] starts the numerical treatment. We have some understanding of the resulting integrand, and it seems to us that the remaining analytical integrations may also be performed while retaining a finite final fermion mass \( m_f \) \[ 124 \]. This would open the way to treat the massive initial-final state interference on the same footing as the other corrections, namely with only one numerical integration (over \( s' \)).

The numerical effects from final mass corrections to the initial-final state interference QED bremsstrahlung are depicted in Fig. 2. There, both the invariant mass distributions \( \frac{d\sigma_{T,FB}}{d\sqrt{s}} \) are shown without and with final-state mass effects for \( m_f = m_t = 174 \text{ GeV} \) and \( \sqrt{s} = 500 \text{ GeV} \). The tree-level cross section, shown for comparison, is \( \sigma^0 = 0.51 \text{ pb} \).

For large invariant masses, \( \sqrt{s'} \rightarrow \sqrt{s} \), the infrared peak clearly dominates. It has to be regularized by soft and virtual corrections. The main difference between the massive and massless cases is a suppression of cross sections near the threshold \( \sqrt{s'} \approx 2m_t \). This suppression is stronger for \( \sigma_{FB}^{int} \) than for \( \sigma_{T}^{int} \). The total corrections from bremsstrahlung are roughly by a factor four larger.
Figure 2. Invariant mass cross section distributions $\frac{d\sigma_{F}}{d\sqrt{s}'}$ and $\frac{d\sigma_{T}}{d\sqrt{s}'}$.

for $d\sigma_{FB}$ than for $d\sigma_{T}$ in both cases. Including final-state masses $m_{t}$, however, reduces the radiative corrections for both $d\sigma_{FB}$ and $d\sigma_{T}$ again by roughly a factor of four.

4. Conclusions

Summarizing, the semi-analytical Fortran program ZFITTER for $e^{+}e^{-} \rightarrow ff$ with radiative corrections is applied in electroweak precision physics at LEP and SLC. With its quick cross section evaluations and precision of about $10^{-3}$ at the $Z$ peak and of order few per mil to 1% up to $\sqrt{s} \approx 800$ GeV, the code is well-equipped for the latest data analyses at present accelerators and to become an important tool for data-fitting in fermion pair production at a future LC.

In the massive case, the code has to be upgraded for the still missing branch of continuum $t\bar{t}$ physics, a determination of the weak couplings of the top quark may be studied. ZFITTER, or topfit respectively, would then describe all fermion pair final-states including radiative corrections and different realistic cuts for the complete range of energies e.g. at DAPHNE, LEP/SLC, and at a LC, i.e. for $\sqrt{s} \approx O(1 \text{ GeV})$ up to $O(1 \text{ TeV})$. However, at DAPHNE energies a specialized code might be better suited in view of the different hadronic final states there which are observed besides fermion pairs.

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