Terahertz time-domain polarimetry (THz-TDP) based on the spinning E-O sampling technique: determination of precision and calibration

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Abstract: We have developed a terahertz time-domain polarimetry (THz-TDP) system by applying frequency modulation to electro-optic sampling detection in a nonlinear crystal. We characterized the precision of this system in determining the polarization angles to be 1.3° for fixed time delay, and 0.5° for complete time-domain waveform. Furthermore, we calculated the Jones matrix of the optical components used for beam propagation to calibrate the induced systematic error. The advantages of employing this calibration approach are demonstrated on a sapphire crystal investigated at different sample test positions in transmission configuration, and using high resistivity Si, AlN and quartz in reflection geometry. The new THz-TDP technique has the advantage of not using any external polarizers, and therefore is not constrained by their optical performance limitations, such as restricted bandwidths and frequency-dependent extinction ratio. Finally, the THz-TDP technique can be easily implemented on existing time-domain spectroscopy (TDS) systems.

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1. Introduction

Polarimetry refers to the measurement of the polarization state of electromagnetic waves transmitted through or reflected by samples. The polarization state is affected by dichroism [1], birefringence [2–4], optical activity [5–7] and the magneto-optic effect [8–10] of materials. It is also related to the complex dielectric functions and thickness of samples in ellipsometry measurements [11–13]. Polarization spectroscopy and ellipsometry in the terahertz frequency range have received significant research interest [14]. Their utility has been demonstrated in various fields of material characterization, including metamaterials [5–7], semiconductors [8–10], biomolecule crystals [15,16], polymer composites [17,18] and liquid crystals [19,20]. They are also critical to the study of microscopic physical processes, such as the mechanism of terahertz generation from laser filaments [21–24], the interaction between terahertz waves and chiral structures [25–27], and the transformation from phonon angular momentum into oriented photocurrent [28–30]. Polarization contrasts are also incorporated into terahertz imaging systems to map carrier mobility and doping density [8,31], to draw the distribution of internal strain [32] and to observe the sharp edges on samples [33,34].

One strategy to convert polarization angle into intensity measurements is to use a crossed-polarizers configuration. This approach has been applied to measure Faraday or Kerr rotation at sub-mrad levels [10,35,36]. Alternatively, the polarization state can be directly constructed by the field strength of two orthogonal components measured by terahertz time-domain spectroscopy (THz-TDS). The two components can also be separated by terahertz polarizers, which are used either as a transmissive analyzer [11,37] or beam splitter [8]. The accuracy of these systems is thus
restricted by the performance of polarizers. However, the extinction ratio of wire grid polarizers used in the terahertz regime is of the order of $10^{-3}$ [38], which is considerably worse than that of polarizers used in the visible range. Moreover, commercialized terahertz polarizers are usually designed for sources with bandwidth limited to around 1 THz, therefore their performance are not sufficient for ultrabroad bandwidth sources such as gas plasmas. In order to achieve high accuracy, the impact of the wire grid polarizers need to be calibrated by taking reference measurements [39] or using algorithms to model the leakage response of the polarizer [13,40].

There are several techniques available to extract the orthogonal components of terahertz waves without using wire grid polarizers. For instance, the efficiency of electro-optic (EO) detection in zinc-blende crystals depends on the orientation of terahertz polarization and crystal axis with respect to the polarization direction of the probe beam [41,42]. Similar angular dependence exists in uniaxial crystals used in the EO-sampling technique [43]. Polarization measurements are thus achieved by manually rotating the direction of the probe beam [44,45] or the detector crystal [34]. Detection with photoconductive antennas are also polarization-dependent, since the antenna gap has a certain orientation. Multi-contact antennas with more than one gap are thus fabricated to measure the orthogonal components simultaneously [46–48].

In order to improve the precision and the speed of polarization measurements, modulation techniques have been considered. Mechanical modulations were made by rotating an optical component at a fixed frequency. For example, a rotating terahertz waveplate [49] or polarizer [6,50,51] was placed on the terahertz beam path. Electronic modulations are reported in systems using air-biased-coherent detection [52], or in setups using polarization [53] or amplitude [54] modulators of the probe beam in the EO sampling technique. A different modulation method in EO sampling is to rotate the detector crystal at angular frequency $\omega$ [55]. Subsequently, the polarization angle of terahertz pulses is precisely determined from the phase of $\omega$ and $3\omega$ frequency components of the measured signal in one scan. In this article, we demonstrate the feasibility of this technique by: (i) calibrating the intrinsic polarization changes induced by optical components and (ii) investigating the birefringence of a sapphire crystal probed in several sample position in both transmission (2f and 4f) and reflection (6f) geometries. The terms 2f, 4f and 6f describe the number and arrangement of off-axis parabolic mirrors with a focal distance $f$, which are used to collimate and focus the THz radiation in our setup. This method has the potential to fully exploit the entire bandwidth of the EO-sampling detection technique. Importantly, the THz-TDP technique can enable polarization-sensitive studies of new materials without the need for any external polarizers, and therefore is not restricted by their limited bandwidth or extinction coefficient.

2. Experiment

The schematic of our measurement system is illustrated in Fig. 1. The light source is the Spectra Physics Spitfire Ace system. The repetition rate, time width and central wavelength of the amplified pulses are 1 kHz, 35 fs and 800 nm, respectively. The beam is divided into two paths by a beam splitter. One of the two beams is used to generate THz waves through optical rectification in a 1 mm thick (110)-cut ZnTe crystal, and the other is used for electro-optic sampling. Here, we use another ZnTe crystal with the same parameters as the detector. The beam outside the amplifier is linearly polarized along the horizontal direction of the setup. A Glan-Thompson polarizer is placed on the path of the probe beam, defining the polarization direction of the transmitted light as the x-axis. We define $\gamma$ is the polarization angle of THz waves and $\beta$ is the angle of the crystal axis [-110] both with respect to the x-axis. The balanced signal obtained using the EO sampling technique is described by Eq. (1) [41,55],

$$\Delta I \propto E_{\text{THz}} \left\{ \frac{1}{2} \cos(\beta + \gamma) + \frac{3}{2} \cos(3\beta - \gamma) \right\}. \quad (1)$$
We mount the ZnTe detector on a hollow shaft motor and rotate it at a constant frequency \( \omega \) set to exactly \( 1/31 \) of the repetition rate of the laser \( (\omega/2\pi = 1 \text{ kHz}/31 = 32.25 \text{ Hz}) \), using a PLL circuit. The resulting time-dependent balanced signal \( \Delta I \) is described by Eq. (2) \[56\],

\[
\Delta I(t) \propto E_{\text{THz}} \left\{ \frac{1}{2} \cos(\omega t + \beta_0 + \gamma) + \frac{3}{2} \cos(3\omega t + 3\beta_0 - \gamma) \right\}.
\] (2)

where \( \beta_0 \) is the initial value of \( \beta \) when \( t = 0 \). The Fourier transform of the modulated \( \Delta I(t) \) gives \( \omega \) and \( 3\omega \) components, whose amplitudes and phases have the following relationships:

\[
E^\omega = \frac{C}{2} E_{\text{THz}} \quad \varphi^\omega = \beta_0 + \gamma + 2n_1 \pi
\]

\[
E^{3\omega} = \frac{3C}{2} E_{\text{THz}} \quad \varphi^{3\omega} = 3\beta_0 - \gamma + 2n_2 \pi,
\] (3)

where \( n_1 \) and \( n_2 \) are arbitrary integers and \( C \) is a constant. \( E_{\text{THz}} \) and \( \gamma \) can be calculated from these parameters and thus the polarization of THz waves is constructed.

Fig. 1. Schematic of the THz Time-Domain Polarimetry (THz-TDP) system. The ZnTe detector is rotating at a constant frequency, \( \omega \). BS1-BS2: beam splitters; ND: a variable neutral density filter wheel; GT1-GT2: Glan-Thompson polarizers; QWP1-QWP2: quarter-wave plates; HWP: a half-wave plate; S: position of sample holder; PM1-PM2: off-axis parabolic mirrors; HR-Si: a high resistivity silicon wafer used to block the NIR pulses; L: a plano-convex lens; WP: a Wollaston prism. Inset: configuration of the electro-optic sampling. The polarization direction of the probe beam is set parallel to the x-axis in the laboratory frame. \( \beta \) is the orientation of the [-110] axis of the (110)-cut ZnTe crystal and \( \gamma \) is the polarization angle of the terahertz electric field vector with respect to the probe beam.

The balanced detector responds to the laser pulses as they arrive on the photodiodes, therefore, the analog form of \( \Delta I(t) \) consists of transient pulses of \( \mu \)s time scale with the same repetition rate as that of the laser. We have designed a lock-in type instrument to record the digitalized value of \( \Delta I(t) \) throughout the experiments (Fig. 2). The clock signal provided by the regenerative amplifier \( (f = 1 \text{ kHz}) \) is processed by several phase-locked loop (PLL) circuits to four channels of pulse signals of \( 1f, f/2, f/31 \) and \( f/62 \), respectively. The \( 1f \) clock signal is used to determine
the timings of readout from the data acquisition (DAQ) card, which digitalizes the analog signal \( \Delta I(t) \). The phase offset of this clock is carefully modified in the range of a few microseconds to optimize the signal-to-noise ratio (SNR). The recorded \( \Delta I \) are a series of discrete data which are indexed as \( \Delta I(i) \ (i = 1, 2, \ldots) \). The \( f/31 \) signal controls the rotation rate of hollow shaft motor, which guarantees the crystal orientation \( \beta(i) \) is always the same as \( \beta(i + 31) \), where \( \beta(i) \) is the value of \( \beta \) then \( \Delta I(i) \) collected. Every sequential 31 data points would form a complete period. The \( f/2 \) pulses are used as the reference signal of a mechanical chopper placed on the path of the pump laser. Consequently, the \( \Delta I(i) \) at all even indices do not contain the influence of the terahertz pulses. We rearrange and subtract the sequential 62 data points to a new set of 31 points in the following manner:

\[
\Delta I'(i) = \begin{cases} 
\Delta I(i) - \Delta I(i + 31) & (i = 1, 3, \ldots, 31) \\
\Delta I(i + 31) - \Delta I(i) & (i = 2, 4, \ldots, 30)
\end{cases}
\]  

(4)

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**Fig. 2.** Principle of the lock-in type readout in THz-TDP system. The E-O crystal is rotating at a period of 31 ms, and two revolutions of crystal rotation are taken as a complete measurement event. 62 data points are collected within a time interval of 1 ms, here 14 of them are illustrated and labeled with the index number \( i \). The E-O crystal is at the same angle for indices \( i \) and \( i + 31 \); Examples of the pairs of the same angles are shown such as \( (1, 32), (6, 37), (11, 42), (16, 47), (21, 52), (26, 57), \) and \( (31, 62) \). For even values of these two indices, the THz pulses are always blocked by the chopper. Therefore, we subtract the data of even indices (hollow) from those of odd indices (solid) to extract the net contribution by the THz waves.
This subtraction procedure would instantly remove the background signal during a short duration (~62 ms) of measurements, suggesting the results would be less affected by the long-term drift of laser power. The signal \( f/62 \) is the trigger for a new time-domain polarization measurement point. It guarantees that the periodic signal \( \Delta I(t) \) collected at all the time delays share the same initial phase.

3. Results and discussion

3.1. Performance of time-domain polarimetry system

Figure 3(a) shows the sequential 62 data points of the balanced signal \( \Delta I(i) \) during two revolutions of the ZnTe detector. The data points of odd and even indices are separated and sorted based on the procedure described by Eq. (4). The \( \Delta I(i) \) the even indices represent the measurement background, when pump laser is blocked by the chopper. This background arises from the response of the balanced photodiodes detectors to the probe laser alone. The angular dependence of this response originates from the residual birefringence of the EO crystal [57]. Figure 3(b) demonstrates that the contribution of background is removed from \( \Delta I(i) \) after subtracting the two sets of data in (a). We took \( N = 3000 \) measurements (each of them includes 62 data points) and averaged the data. The error of \( \Delta I'(i) \) is usually comparable to the error of photodiode response signal at index \( i \). One approach to improve the SNR is to increase the energy of the THz source, which can be increased by the power of the pump laser used for the optical rectification process. Further enhancement of the signal level can be achieved if THz sources with higher conversion efficiency were used, such as a high-field LiNbO\(_3\) source or two-color air plasma source. The other approach is to reduce the oscillation of the background response, which is related to the performance of the balanced detector, the timing of readout from the DAQ card and the stability of the PLL circuits.

Fig. 3. (a) Time-dependent balanced signal \( \Delta I(i) \) obtained during two revolutions of the ZnTe detector. The time interval between each index of data \( i \) is 1 ms. The THz waves are blocked at all even indices by an optical chopper rotating at \( f/2 = 500 \text{ Hz} \). \( \Delta I \) obtained at odd (solid) and even indices (open circle) are rearranged and plotted separately. (b) The net signal induced by the THz pulses is shown. We conducted \( N = 3000 \) measurements to calculate the mean value and standard deviation of data. (c) Distribution of deviations \( \Delta \gamma \) of the measured polarization angle \( \gamma \) from the mean value \( \bar{\gamma} \) using data presented in (b).

Figure 3(c) presents the distribution of the polarization angle \( \gamma \) at a fixed time delay position, which is extracted from the same \( N = 3000 \) measurement events summarized in Fig. 3(b). The distribution can be fitted with a Gaussian curve, resulting in the fitted standard deviation of \( \gamma \) to be approximately 1.3°. To test the reproducibility of these results over a more feasible measurement
where \( N = 100 \) and takes approximately 6 seconds to accomplish, we calculated the 95% confidence interval of the mean value to be \( (\tilde{\gamma} - 0.26^\circ, \tilde{\gamma} + 0.26^\circ) \).

Subsequently, we evaluated the precision of the THz-TDP technique by scanning the entire THz waveform as a function of time delay. The THz waves generated from ZnTe crystals are linearly polarized and their directions are described by Eq. (5) [55,58],

\[
E_{\text{THz}} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \propto \begin{pmatrix} \sin(\psi - \phi_0) \cos(\psi - \phi_0) \cos \psi + \frac{1}{2} \cos^2(\psi - \phi_0) \sin \psi \\ \sin(\psi - \phi_0) \cos(\psi - \phi_0) \sin \psi - \frac{1}{2} \cos^2(\psi - \phi_0) \cos \psi \end{pmatrix}, \tag{5}
\]

where \( \psi \) is the angle between the [-110] axis of the ZnTe emitter and the x-axis, and \( \phi_0 \) is the angle between the polarization direction of the pump laser and the x-axis. In the simplified scheme when \( \psi - \phi_0 \) is kept at a fixed value (e.g., the ZnTe crystal and a pump polarizer are affixed to a circular tube), the change in the polarization angle of the electric vector \( E_{\text{THz}} \) equals the increment of \( \phi_0 \) and \( \psi \). To rotate the polarization of pump laser, we used a quarter-waveplate to convert the linearly polarized IR pump into a circularly polarized beam, and then selected the polarized component in the desired direction defined by the axis of a Glan-Thompson polarizer. The THz pulse waveform obtained from this configuration is shown in Fig. 4(a). We describe the polarization state by the complex ratio \( \tilde{r} \) between the \( y \) and \( x \) components:

\[
\tilde{r} = \frac{\tilde{E}_y}{\tilde{E}_x} = \tan \Psi e^{i\Delta}, \tag{6}
\]

where \( \tilde{E}_x \) and \( \tilde{E}_y \) are the frequency domain signal obtained by the Fourier transform, \( \tan \Psi \) is the amplitude ratio and \( \Delta \) is the phase difference. For a linearly polarized waves, we expect the relation \( \Psi = |\gamma| \), \( \Delta = 0^\circ\text{or}180^\circ \). Figure 4(b) shows the \((\Psi, \Delta)\) coordination of the THz wave when \( \gamma \) is set to -20°. From 0.1 to 1.4 THz, the standard deviation (SD) of \( \Psi \) is less than 0.5°, while the SD of \( \Delta \) is less than 1.1°. The average value of \( \Delta \) is close to 0° with an offset of <2°, which indicates that the THz source is near linearly polarized. Beyond 1.4 THz, the error increases due to the lower SNR. The mean values of \( \Psi \), however, have nonnegligible differences from the expected value of 20°. Similar phenomena were observed when \( \gamma \) was set to other polarization angles, which is attributed to the effect of optical components in the THz propagation path, including one pair of off-axis parabolic mirrors and a silicon wafer used as an IR beam block (Fig. 1). To calibrate for the effect of the propagation path optics, we will next evaluate the Jones matrix representation of our experimental setup. The goal of our calibration methodology introduced in the next section is to quantify and suppress this systematic error.

### 3.2. Jones matrix calibration

Adopting the matrix representation for an electric field (E-field) vector, the impact of linear optical elements can be written in the following matrix form:

\[
E'(\omega) = T(\omega)E_0(\omega), \tag{7}
\]

\[
T(\omega) = \begin{bmatrix} \tilde{r}_{xx}(\omega) & \tilde{r}_{xy}(\omega) \\ \tilde{r}_{yx}(\omega) & \tilde{r}_{yy}(\omega) \end{bmatrix} \propto \begin{bmatrix} 1 & \tilde{r}_1(\omega) \\ \tilde{r}_2(\omega) & \tilde{r}_3(\omega) \end{bmatrix}, \tag{8}
\]

where \( E_0 \) is the initial THz E-field vector and \( E' \) is its final state after passing through the elements being investigated, \( T \) is the Jones matrix, while \( \tilde{r}_1, \tilde{r}_2 \) and \( \tilde{r}_3 \) are entries of \( T \) normalized \( \tilde{r}_{xx} \). In our system, \( E_0 \) is predicted by Eq. (5), \( E' \) is the actual polarization measured at the surface of the EO detector crystal. Equation (8) is simplified when we create linearly polarized waves in the
Fig. 4. (a) Polarization-resolved time-domain waveform of a THz pulse polarized at $\gamma = 20^\circ$. (b) The rotation angle $\Psi$ and (c) phase lag $\Delta$ obtained for the THz pulses polarized at $\gamma = 20^\circ$. The average value (black line) is calculated for 10 separate instances of polarimetric measurements (red dots).

The choice of $E_0 = [E_0^x, E_0^y]$ in Eq. (11) is arbitrary, therefore it offers an approach to verify the robustness of our Jones matrix calculations. The results of Jones matrix calibration are presented in Fig. 5. Here, we used $\gamma = -60^\circ$ ($E_0 = [1, -\sqrt{3}]$) or $\gamma = -45^\circ$ ($E_0 = [1, -1]$) in Eq. (11). In Figs. 5(a)–5(b), we have plotted the real and imaginary part of $\tilde{t}_3$ is a function of frequency, calculated from two sets of measurements when $\gamma = -60^\circ$ (solid blue) and $\gamma = -45^\circ$ (dashed blue). Figures 5(a)–5(b) shows that the term $\tilde{t}_3$ is robust to this choice of the polarization angle of $E_0$. The off-diagonal term $\tilde{t}_1$ and $\tilde{t}_2$ are nearly zero (not presented). Therefore, the changes in the THz polarization due to optical propagation are mostly caused by $\tilde{t}_3$. In Figs. 5(c)–5(f), we have used these two Jones matrices to calibrate the measured $\Psi$ angle for when $\gamma = -80^\circ$, $-20^\circ$, $+20^\circ$ and $+40^\circ$, respectively. Note that $\Psi$ is by definition positive. The calibrated angles $\Psi$ have much smaller offset from the expected values, suggesting that the error introduced by the optical components can be calibrated and suppressed effectively, and that our control of the polarization of the THz source is reliable.
significant phase changes. If we take the real parts of all entries at 0.45 THz as representative, the Jones matrix would be:

\[
T = \begin{bmatrix}
1 & -0.003 \\
0.011 & 0.80
\end{bmatrix}.
\] (12)

The off-diagonal term \( t_{xy} \) and \( t_{yx} \) implies a small amount of cross polarization, which can be attributed to the off-axis parabolic mirrors [59]. The diagonal term \( t_{yy} \) is smaller than \( t_{xx} \), suggesting that the optical elements may reflect or absorb more s-polarized THz waves (parallel to y-axis) than p-polarized THz waves (parallel to x-axis). Anisotropic absorption is not expected for parabolic mirrors or silicon wafers. However, from the Fresnel equations we anticipate that the reflectivity for s- and p- polarized components can be different for a medium with finite permittivity. To confirm that the reflection by the Si wafer is responsible for this effect, we directly measured the reflectance of the silicon wafer using a pyroelectric detector (THZ5l-BL-BNC, Gentec E-O). When the THz source was switched from p- to s-polarized, the detector response to transmitted THz wave decreased from 7.02 mV to 4.59 mV, or about 65.4% of the p-polarized transmission coefficient. This power ratio \( \eta \) agrees with the value estimated from the calculated Jones matrix \( \eta = 0.80^2 = 64\% \). Fresnel equations also predict that the reflectivity of s- and p- polarized components would become closer at smaller incident angles. We verified this by repeating the Jones matrix calibration for the same silicon wafer, with the incident angle reduced from \( \sim 45^\circ \) to \( \sim 20^\circ \). The term \( \tilde{t}_3 \) in this new matrix (as shown in Figs. 5(a)–5(b) with black solid lines) is essentially real and is close to 1, suggesting that the difference in reflectivity is negligible when the incident angle of the THz waves is sufficiently small. It should be noted that the reflection at silicon wafer is only one of the factors that contribute to the Jones matrix \( T \), therefore it is not reliable to calculate \( T \) based on Fresnel equations.

In summary, we have shown, in Fig. 3(c), that the THz-TDP technique has a precision of approximately 1.3 degrees in determining the polarization angles, and after calibration methodology described in Section 3.2, we estimate the accuracy of our measurement to be better than 0.5 degrees between 0.3 and 1.8 THz. Moreover, the THz-TDP has the advantage of simultaneously measuring both the s and p component of the electric field - i.e. the magnitude and angle of the THz electric field can be sampled directly as a function of time, as shown in Fig. 4(a).

### 3.3. Measurement of terahertz birefringence

To demonstrate the utility and accuracy of the THz-TDP technique in material characterization, we measured the birefringence properties of a sapphire crystal (Al\(_2\)O\(_3\)). The sample consisted of a 0.52 mm thick square-shape substrate, polished on both sides, oriented with the a-axis as the surface normal and the c-axis in the plane. We determined the orientation of the c-axis by rotating the sample. We set the linearly polarized THz source in the x direction and recorded the transmitted THz waveforms at different crystal orientations. Minimum changes in polarization would be observed when the ordinary or extraordinary axis is parallel to the x-axis. After identifying the c-axis, we rotated the sample so that the o- and e-axes were at \( 45^\circ \) with respect to the x-axis. The Jones matrix of a pure birefringent material is given by Eq. (13):

\[
S = R(-\alpha) \begin{bmatrix}
e^{i\phi_x} & 0 \\
0 & e^{i\phi_y}
\end{bmatrix} R(\alpha) = 
\begin{bmatrix}
e^{i\phi_x} \cos^2 \alpha + e^{i\phi_y} \sin^2 \alpha & (e^{i\phi_x} - e^{i\phi_y}) \cos \alpha \sin \alpha \\
(e^{i\phi_x} - e^{i\phi_y}) \cos \alpha \sin \alpha & e^{i\phi_y} \sin^2 \alpha + e^{i\phi_x} \cos^2 \alpha
\end{bmatrix},
\] (13)
Fig. 5. (a) The real and (b) imaginary part of the normalized Jones matrix entry $\tilde{t}_3$. Matrix #1 and #2 are calculated when the polarization angle $\gamma$ equals $-60^\circ$ or $-45^\circ$, respectively. Matrix #3 is obtained when the silicon wafer on the path of THz pulses is set at a smaller incident angle. (c-f) Comparison between the $\Psi$ before (red) and after calibration (blue) using Matrix #1 (Solid) and Matrix #2 (dashed), when $\gamma$ is set at (c) $-80^\circ$, (d) $-20^\circ$, (e) $20^\circ$ and (f) $40^\circ$. Note that $\Psi$ is by definition a positive number. The error in $\Psi$ after calibrating with Jones Matrix #1 and #2 are shown separately in (g) and (h), represented by the mean value and range of the data in (c-f).
where $\alpha$ is the angle of e-axis with respect to the x-axis (labeled in Fig. 6(a)), $\varphi_x$ and $\varphi_y$ are the phase changes when the electric vectors propagate through the e- and o-axis, respectively. Since incoming electric vector $E_0 = [1, 0]$, the transmitted $E'$ takes the form:

$$E' = SE_0 = \begin{bmatrix} e^{i\varphi_x} \cos^2 \alpha + e^{i\varphi_y} \sin^2 \alpha \\ (e^{i\varphi_x} - e^{i\varphi_y}) \cos \alpha \sin \alpha \end{bmatrix}. \quad (14)$$

A typical temporal waveform of $E'$ is shown in Fig. 6(b). The orientation ($\alpha$) of the e-axis and anisotropic phase retardation ($\delta$) can be extracted from the complex ratio of $E'[60]$:

$$\begin{align*}
\delta &= \varphi_x - \varphi_y \\
\tilde{r} &= \frac{E'_x}{E'_y} = \frac{e^{i\delta} + \tan^2 \alpha \cos \delta}{(e^{i\delta} - 1) \tan \alpha} \\
\alpha &= \frac{1}{2} \tan^{-1} \left( \frac{1}{\text{Re}(\tilde{r})} \right) \\
\delta &= 2 \tan^{-1} \left( \frac{1}{\sin(2\alpha) \text{Im}(\tilde{r})} \right) \quad (15)
\end{align*}$$

The calculated $\alpha$ has a smooth value in the range from 0.4 to 1.2 THz and the averaged value is 44.3°, which is close to the expected angle 45°. We convert the phase retardation ($\delta$) into the difference in refractive index ($\Delta n = n_e - n_o$) using the relation $\Delta n(\omega) = c \delta / (\omega d)$, where $d$ is the thickness of sample. Figure 6(c) shows the frequency dependent $\Delta n$ using the data with or without the Jones matrix calibration described in Section 3.2. The calibration has significantly improved the birefringence extracted from TDP measurements compared to the uncalibrated results. The calibrated value $\Delta n = 0.34$ is consistent with earlier studies [4,60,61].

![Fig. 6.](image)

The birefringence measurements presented in Fig. 6(a) were obtained using a 2f setup (shown in Fig. 1), where the THz emitter and detector are located at the focal point of two off-axis parabolic mirrors, and the sample was placed near the THz emitter. This transmission measurement layout differs from the 4f configuration, where two pairs of parabolic mirrors are incorporated. This 4f configuration is advantageous as the THz waves are focused when transmitting through the sample. For comparison and to test the robustness of our Jones matrix calibration approach, we extended our transmission setup to a 4f arrangement (Fig. 7). We obtained THz-TDP measurements from
the same sapphire sample placed at the transmission focal position. Using the Jones matrices to describe the THz waves as they propagate through optical components and the birefringent sample, the measured THz field vector is given by,

$$E' = T_2 S T_1 E_0,$$  
(16)

where $T_1$ and $T_2$ are the Jones matrices of the two pairs of parabolic mirrors, as labeled in Fig. 7, and $S$ is the Jones matrix of sample which is described by Eq. (13).

Fig. 7. Schematic of the 6f arrangement of the THz-TDP system. Samples can be placed in either transmission mode or reflection mode focus position. For calculating the Jones matrix of 4f arrangement, the polarimetry block is placed at the focal point of PM4. The inset shows the schematic of ellipsometry measurement used in reflection configuration.

The polarization of THz source $E_0$ is adjusted so that $E_0 = T_1^{-1}[1, 0]$. The calibrated THz electric vector $T_1^{-1}E'$ is then used to extract the sample properties. As shown in Fig. 6(c), the birefringence measured is in agreement with the 2f data, and the impact of different arrangements of parabolic mirrors can be accounted for using the Jones matrix calibration procedure.

3.4. Terahertz ellipsometry

THz ellipsometry can measure the refractive index of samples in reflection mode without the need for a reference measurement. In an ellipsometry setup, the two orthogonal polarizations of the THz pulses reflected from the sample are measured at an incident angle. If we denote the incident light as $E_0$ and the reflected light as $E'$, the complex reflectivity ratio $\rho$ is defined by the ratio between the reflectivity of the s- and p- components:

$$\rho = r_p/r_s = \frac{E'_p/E_0}{E'_s/E_0}.$$  
(17)
Importantly, here $\rho$ is independent of the polarization state of $E_0$. In the case of an isotropic sample, the complex refractive index $\tilde{n}$ can be obtained from $\rho$, using [62]

$$\tilde{n} = n - i\kappa = \sin \theta_i \sqrt{1 + \tan^2 \theta_i \left( \frac{1 - \rho}{1 + \rho} \right)^2},$$

(18)

where $\theta_i$ is the incident angle. We demonstrate this principle on three different samples. The first sample is a 1.0 mm thick wafer made by high resistivity silicon (Si). The second sample is a single hexagonal crystal of aluminum nitride (AlN), with (0001) orientation and 0.44 mm thickness. The third sample is a 2.5 mm thick c-cut quartz window. The schematic of the reflection mode THz-TDP is shown in Fig. 7. The THz waves are focused onto the sample at an incident angle of 45°. The reflected polarization state is first modified by the complex reflectivity ratio $\rho$ of the sample. Subsequently, this ratio is slightly altered by the components such as PM5 and PM6. Again, we can use Jones matrix representation to describe the polarization changes of THz waves:

$$E' = T_3 \begin{bmatrix} \rho & 0 \\ 0 & 1 \end{bmatrix} T_2 T_1 E_0.$$  

(19)

Here $T_1, T_2$ and $T_3$ are the Jones matrices of the three pairs of parabolic mirrors (as labeled in Fig. 7). The calibrated THz electric vectors $T_3^{-1}E'$ and $T_2 T_1 E_0$ are used to extract the sample properties. Figure 8 shows the measured refractive indices for the three samples. For comparison, we have presented the results from both calibrated and uncalibrated values of $\rho$. For statistical analysis, we used three different polarization of incident THz pulses on the sample, which are oriented at $\gamma = 30^\circ, 45^\circ, 60^\circ$, respectively. The measurements are conducted five times for each angle of $E_0$. The mean value and standard deviation of all the 15 measurements are thus presented (Fig. 8). Importantly, while the uncalibrated results show a considerable error, the mean value of the calibrated results are in excellent agreement with the results from traditional THz-TDS transmission measurements [61,63].

![Fig. 8. Ellipsometry measurements on the refractive indices of isotropic wafers made by (a) high-resistivity silicon, (b) aluminum nitride and (c) quartz. The results are extracted from both the uncalibrated (red) and calibrated (blue) THz-TDP measurements. The values obtained from transmission THz-TDS (black) are shown as references.](image-url)
4. Conclusion

We have presented a time-domain THz polarimetry system based on a rotating electro-optic detector. We showed that the polarization angle of THz pulses can be extracted at a fixed time delay with a precision of approximately $1.3^\circ$, and the frequency-dependent rotation angle with a precision of about $0.5^\circ$. We have used this system to calibrate the polarization change induced by optical components and proved that the measurement error can be suppressed to within $0.5^\circ$ using the Jones matrix representation of the propagation optics. Subsequently, we have measured the birefringence of a sapphire crystal in transmission mode, using both 2f and 4f configurations. Also, we have extracted the refractive indices of three isotropic materials (HR-Si, AlN and quartz) in reflection mode, using a 6f configuration. We have demonstrated that the same calibration procedure using Jones matrices can reduce the measurement error in various experiment geometries. Since our THz-TDP system does not rely on external polarizers, it is not affected by the error of manual rotation or the bandwidth limitation of wire grid polarizers. In addition, this system features the ability to trace the amplitude and the angle of the electric field vector in the time-domain simultaneously, whereas previous techniques usually required separate sets of orthogonal polarization measurements. The THz-TDP system would be very useful in the broadband characterization of anisotropic materials, terahertz ellipsometry and polarization-sensitive biomedical imaging applications.

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