A Framework of Multilayer Multiphase Interleaved Converter for Electric Vehicle Based on Graph Theory

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1.Introduction

Battery management system is commonly known as battery steward, mainly for intelligent management and maintenance of each battery unit, preventing the battery from overcharging and overdischarging, extending the service life of the battery, and monitoring the state of the battery. Therefore, functional safety design of BMS is very important for the safety of the battery energy storage system. Lu et al. [1] analyzed the composition of BMS, summarized the evaluation methods of battery state, and concluded the basic methods and steps of BMS research as well as the key issues to be further explored. These results are of great help to the functional security design of BMS. Wang et al. [2] proposed a joint estimator based on particle filter to accurately evaluate the key indicators SOC and SOE in BMS and energy storage system. Currently, the most widely used chemical cells are fuel cells and electrochemical cells. To extend the life of battery systems in the future, Wang et al. [3] proposed a finite-state machine strategy for energy management in hybrid energy systems and an optimal PID-based OER excess oxygen ratio control method to maximize net power output from fuel cells. Compared with other electrochemical energy storage technologies, lithium-ion batteries have the largest cumulative installed capacity, accounting for 86.3% of electrochemical energy storage globally. Therefore, the present study focuses on the energy balance problem of lithium battery stacks, which limits the available capacity and cycle life of packs. Studies show that a variety of parameters, including manufacturing discrepancies, environmental conditions, and the operating conditions, can trigger substantial heterogeneities among cells [4]. Moreover, the complex chemical mechanism has certain adverse effects on the capacity, efficiency, and the service life of battery packs.
This causes further imbalance of the energy and thermal runway during the charging and discharging processes of the battery pack and increases the risk of catastrophic events [5, 6]. Therefore, equalization converters are necessary for series-connected battery packs to resolve the aforementioned disadvantage and prolong the overall lifetime of battery packs [7]. Tang et al. [8] proposed a battery equalization algorithm based on balanced current ratio (BCR), which solves the problem of weak observability in the voltage equalization system and the problem of real-time state estimation in the equalization management of lithium iron phosphate batteries when the series active equalized state information during the voltage equalization period.

Many scholars have conducted extensive research studies on battery equalization circuits, which can be divided into the active and passive equalizations [9–11]. Daowd et al. [12] showed that passive equalizations dissipate the excess energy of the highest voltage battery cell by discharging the cell through a resistor. This procedure is repeated until all cells reach the same charging level. They compared the modeled active and passive equalizations in the MATLAB software. The advantage of the passive equalization lies in the simplicity and high reliability of the circuit. On the other hand, the drawback of passive equalization is its direct electrical energy consumption in the form of thermal energy, which causes significant waste of energy. Yang et al. [13] investigated the heat dissipation of the system. Active equalization circuits often require peripheral circuits to detect the state of charge (SOC) so that a suitable equalization strategy can be selected to achieve battery energy transfer [14]. This increases the complexity of the equalization circuit, while achieves the fast equalization and efficient energy flow. Therefore, researching the circuit of the active equalization in balancing the aforementioned factors is highly interesting. At present, mainstream active equalization circuits can be divided into four types as the following: cell bypass, cell-to-cell, cell-to-pack, and hybrid balancing topology.

Cell bypass circuits are divided into the complete shunting [15, 16] and shunt resistor [17, 18] balancing methods, which achieve equalization by continuous switchings between the normal and bypassed operation for overcharged cells. The switch tube in the bypass circuit is repeatedly turned on and off, which complicates the estimation of the battery voltage level. The cell-to-cell circuit achieves energy flow between the overcharged and over-discharged cells. In order to achieve this equilibrium principle, a centralized equalization topology of bidirectional buck-boost circuits is usually adopted [19, 20]. Studies showed that as the switching frequency increases, the volume and quality of the system reduce effectively. Moreover, it was found that sharing the inductor or capacitor as an energy transfer medium reduces the system cost. Reviewing the literature indicates the recent improvements of the simple buck-boost and Cuk architectures [21, 22]. This cell-to-cell equalization mode can effectively reduce the repeated flow of energy. However, the energy of all abnormal batteries should be transferred through a shared energy storage inductor or capacitor. When the capacitor or inductor releases energy to a certain battery, other batteries can only be forced to wait. So it is a time-consuming procedure. Cell-to-pack returns the energy flow from an overcharged cell to the entire battery pack [23]. This architecture transfers the energy of the abnormal battery to the entire battery pack to reduce the energy inconsistency, which inevitably results in the secondary charge of the remaining overcharged batteries. Meanwhile, transferring the energy to the entire battery imposes high voltage stress on the switch [24, 25]. In summary, the hybrid equalization topology is a reasonable solution, which has a flexible energy transfer path and combines the advantages of the aforementioned circuits. An individual cell equalizer (ICE) is used to directly equalize some cells to achieve the cell-to-pack and pack-to-cell equalizations [26]. Wang et al. [27] and Mestrallet et al. [10] discussed the cell-to-pack and pack-to-cell equalization method in detail. It should be indicated that similar topologies that enable the hybrid equalization are attractive for researchers in terms of equalization speed efficiency and flexibility. The shortcoming of this kind of converter is complicated control logic, expanding challenges, and development limitations. It is anticipated that the highly integrated circuit and intelligent algorithms, such as deep learning, will resolve this problem in the recent future.

The present study is organized as follows. The operation principle of the multiphase converter is explained in Section 2. Based on the graph theory and the corresponding mathematical descriptions, the modeling and evaluations are illustrated in Section 3. Moreover, the principle of the multi-layer optimization design is presented in Section 4. Subsequently, simulation and experimental analysis are performed in Section 5. Finally, conclusions and future work are provided in Section 6.

2. Principle of the Multiphase Interleaved Equalization Circuit

2.1. Multiphase Interleaved Converter Structure. In order to obtain the bidirectional energy transmission in the continuous current mode (CCM), the diode in the conventional buck-boost circuit structure is replaced by a MOSFET. To make it more suitable for the battery pack, a novel multiphase interleaved converter is proposed. For the multiphase interleaved converter, MOSFETs are driven in a complementary mode. Figure 1 shows the proposed converter. It indicates that the circuit is composed of \( n - 1 \) subconverters and \( n \) batteries, wherein the inductor of the \( i \)th equalization module is connected to the cathode of the \( i \)th battery. For each equalization circuit (EC) module, the drain of the upper switch tube and the source of the lower switch tube are connected to the positive and negative busses of the battery pack. Each subconverter is utilized to equalize the energy of the battery pack on both sides separated by the inductor.

The basic operating mode of the circuit is that each equalization module divides the battery components into two parts that complement each other, thereby realizing the energy transfer between two parts. The transistors of converter work in a complementary driving mode, and their
respective duty cycle is set by the function of input voltage and output voltage in equations (1) to (7). Table 1 is a symbol description table for equations (1) to (7).

\[ \alpha_i = \frac{U_{Hi}}{U_{Li} + U_{Hi}}, \quad (1) \]

\[ U_{Hi} = \sum_{x=i}^{i} U_{\text{cell}x}, \quad (2) \]

\[ U_{Li} = \sum_{x=i+1}^{n} U_{\text{cell}x}, \quad (3) \]

\[ R_{\text{INeq}} = \sum_{x=i}^{i} R_{\text{cell}x} + R_{\text{SHi}} + R_{\text{Li}}, \quad (4) \]

\[ R_{\text{OUTeq}} = \sum_{x=i+1}^{n} R_{\text{cell}x} + R_{\text{SLi}} + R_{\text{Li}}, \quad (5) \]

\[ \alpha_i U_{Li} - R_{\text{INeq}} \alpha_i I_{Lm} = (1 - \alpha_i) U_{Hi} + R_{\text{OUTeq}} (1 - \alpha_i) I_{Lm}, \quad (6) \]

\[ I_{Lm} = \frac{\alpha_i U_{Li} - (1 - \alpha_i) U_{Hi}}{\alpha_i R_{\text{INeq}} + (1 - \alpha_i) R_{\text{OUTeq}}}, \quad (7) \]

where the subscript \( i \) stands for the number of batteries on the input leg, \( i \in \text{int}[1; N-1] \), \( U_{Hi} \) is the input voltage of the equalizer, and \( U_{Li} \) is the output voltage of equalizer. This fixed duty cycle equalization mode produces an average voltage on one side of the inductor that equals the portion of the battery pack voltage. In one case where the energy of the two cells separated by the inductance is balanced, the potential generated voltage at both ends of the inductor is the same. In other words, the energy exchanged through the inductor from a part of the circuit is equal to the energy released from the other part of the circuit. In another case where the energy of two parts in the battery pack is unbalanced, the inductor will transfer the power donated by the

**Figure 1:** Configuration of the multiphase interleaving converter for balancing the battery.

**Table 1:** Symbol description.

| Symbol       | Meaning of symbol                        |
|--------------|------------------------------------------|
| \( \alpha_i \) | Voltage duty cycle of \( i \)th upper switch |
| \( U_{Hi} \)  | The input voltage of the equalizer        |
| \( U_{Li} \)  | The output voltage of the equalizer       |
| \( U_{\text{cell}x} \) | Battery terminal voltage                  |
| \( R_{\text{INeq}} \) | Equivalent input resistance               |
| \( R_{\text{cell}x} \) | The equivalent resistance of MOSFET at the upper end of the battery equalizer |
| \( R_{\text{SHi}} \) | Equivalent resistance at both ends of the battery |
| \( R_{\text{SLi}} \) | Battery equalizer energy storage inductance |
| \( R_{\text{OUTeq}} \) | The equivalent resistance of MOSFET at the lower end of battery equalizer |
| \( I_{Lm} \)  | Battery equalizer energy storage inductance at both ends of the current flow |

Complexity 3
overcharged batteries to the overdischarges. The duty cycle of each switching tube in equalization circuit is given in Table 2. Furthermore, in the natural spontaneous balancing method, these switches are actuated in a complementary mode.

2.2. Principle of Cell-to-Pack Architectures. When the upper switch \( S_{(i-1)} \) of EC\(_i\) is turned on, the battery \( B_i \) transfers the energy to the inductor \( L \). Then, the inductor divides the energy to the remaining cells, where each of the remaining \( n-1 \) batteries absorbs \( 1/(n-1) \) parts of the divided energy. Moreover, when the upper switch of the second equalization module EC\(_2\) is turned on, each of \( B_1 \) and \( B_2 \) batteries transfers \( 1/2 \) parts of the energy to the inductor. Then, the inductor releases the energy to the remaining batteries, which means the remaining \( n-2 \) batteries absorb \( 1/(n-2) \) parts of the energy. These equalization modules consist of \( 2n-2 \) switching tubes and \( n-1 \) inductors. To further reduce the number of components, the aforementioned and simplified circuit can be adopted, but at expense of loss of the equalization time.

The basic working principle of the submodule in the multiphase interleaved equalization circuit is illustrated in Figure 2. It is assumed that the battery \( B_i \) is overcharged. Therefore, it is necessary to transfer the excessive energy of \( B_i \) to other cells in the battery pack.

When the switch \( S_{(i-1)} \) is turned on, the current flows through the inductor \( L_{(i-1)} \) and switch \( S_{(i-1)} \) from battery \( B_i \) to battery \( B_n \). In the state I, the inductor \( L_{(i-1)} \) absorbs the energy from \( B_i \) to \( B_n \). Subsequently, the switch \( S_{(i-1)} \) is turned off and the current direction of the inductance stays the same due to the freewheeling. Then, the current flows from \( B_1 \) to \( B_{(i-1)} \) through the diode \( D_L \) and the batteries absorb the released energy from the inductor. The working principle of the state II is similar to that of the state I and will not be described here.

Combining the two states, the excessive energy of \( B_i \) can be transferred to the remaining cells with the following instructions.

It is assumed that the difference between \( B_i \) and the average of the battery pack is \( Q_{\text{extra}} \) for state I, the released energy by each battery from \( B_i \) to \( B_n \) is set to be \( \Delta \varepsilon_i \), which is temporarily stored in the inductor \( L_{(i-1)} \), and then it is released to batteries \( B_1 \) to \( B_{n-1} \). In state II, the released energy by the batteries from \( B_1 \) to \( B_i \) is set to \( \Delta \varepsilon_2 \), which is temporarily stored in the inductor \( L_{(n-i)} \) and then it is released to \( B_{n-i} \) to \( B_n \). It should be indicated that power delivery can be operated simultaneously in both EC\(_{n-1}\) and EC\(_n\).

Table 3 shows the switching of each state during the equalization process. During each switching cycle, the absorbed and released energies by the inductor are equal. Therefore, the governing equations can be derived as the following.

Equations (8)–(10) describe the energy exchange from \( B_i \) to \( B_{(i-1)} \), the released energy from the battery \( B_i \), and the exchanged energy from \( B_{(i+1)} \) to \( B_n \), respectively.

\[
\frac{(n-i+1)\Delta \varepsilon_i}{(i-1)\Delta \varepsilon_2} = \frac{Q_{\text{extra}}}{n}
\]  

\[
Q_{\text{extra}} - \Delta \varepsilon_i + \Delta \varepsilon_2 = \frac{Q_{\text{extra}}}{n}
\]  

\[
-\Delta \varepsilon_i + \frac{i\Delta \varepsilon_2}{N-i} = \frac{Q_{\text{extra}}}{n}
\]

The system of equations (11) and (12) can be reduced in the forms below:

\[
\Delta \varepsilon_1 = \frac{(i-1)Q_{\text{extra}}}{n}
\]  

\[
\Delta \varepsilon_2 = \frac{(n-i)Q_{\text{extra}}}{n}
\]

The proposed multiphase interleaved converter can easily transfer the energy of the overcharged battery to the remaining ones with only two switching tubes controlled. It should be indicated that this function is achieved by controlling only one switch tube when the overcharged battery is in the first or last position of the series battery pack. In actual situations, there are usually overdischarged batteries. In the above equalization process, controlling only one idle switch tube in the equalization module can realize transferring the energy of the remaining battery of the battery pack to an overdischarged battery.

![Table 2: Duty cycle rate of equalization circuit.](image)

\[
B_j L_i = \begin{bmatrix}
1 & 1 & 1 & 1 & \ldots & \frac{1}{n-2} & \frac{1}{n-1} \\
-\frac{1}{n-1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \ldots & \frac{1}{n-2} & \frac{1}{n-1} \\
-1 & -\frac{1}{n-2} & \frac{1}{3} & \frac{1}{4} & \ldots & \frac{1}{n-2} & \frac{1}{n-1} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
-\frac{1}{n-1} & -\frac{1}{n-2} & -\frac{1}{n-3} & -\frac{1}{n-4} & \ldots & \frac{1}{2} & \frac{1}{n-1} \\
-1 & -\frac{1}{n-2} & -\frac{1}{n-3} & -\frac{1}{n-4} & \ldots & \frac{1}{2} & \frac{1}{n-1} \\
\end{bmatrix}
\]

2.3. Principle of Cell-to-Cell Architectures. The multiphase interleaved converter has excellent performance in achieving energy exchange from any abnormal battery to the remaining batteries in the battery pack. In engineering applications, the number of abnormal batteries is not negligible so that
equalizing the abnormal batteries result in repeated loss of the energy. Therefore, it is a viable option to transfer the energy of the most overcharged batteries to the most overdischarged batteries. In response to this solution, the proposed circuit can achieve such a function without adding components or increasing the control complexity, where the function yields 1 for reachable path and 0 for unreachable path, respectively.

The accessibility of the energy flow should be determined. For example, the excessive energy of $B_2$ cannot be transferred to $B_1$ through the $EC_1$ (i.e. $B_2L_1B_1 = 0$). The battery is capable of transferring the energy to a battery with a serial number greater than its equalization module, which is connected to its negative terminal.

Similarly, the battery can transfer energy to the battery with a serial number less than its equalization module, which is connected to its positive terminal.

According to Section 2.2, the excessive energy of each cell can be transferred to the remaining cells in the battery pack. In this process, the complementary switch tube in the control group is turned on so that the energy of the remaining batteries in the battery pack can be transferred to a battery. Combined with the aforementioned two processes, it is possible to transfer the excessive energy of an arbitrary battery in the battery pack to any battery.

Table 3: Switching states of the equalization.

| Switch       | Switch state in $EC_{i-1}$ | Switch state in $EC_i$ |
|--------------|----------------------------|------------------------|
|              | Counterclockwise arrow     | Clockwise arrow        | Counterclockwise arrow | Clockwise arrow |
| $S_{H(i-1)}$ | Off                        | Off                    | Off                     | Off            |
| $S_{L(i-1)}$ | On                         | Off                    | On                      | Off            |
| $S_{H(i)}$   | Off                        | On                     | Off                     | Off            |
| $S_{L(i)}$   | Off                        | Off                    | Off                     | Off            |

(14) \[
B_iL_iB_j = \begin{bmatrix}
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
1 & 0 & 0 & 0 & \ldots & 0 & 0 \\
1 & 1 & 0 & 0 & \ldots & 0 & 0 \\
1 & 1 & 1 & 0 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
1 & 1 & 1 & 1 & \ldots & 0 & 0 \\
1 & 1 & 1 & 1 & \ldots & 1 & 0 \\
\end{bmatrix}
\]

(15) \[
B_iL_{i-1}B_j = \begin{bmatrix}
0 & 1 & 1 & 1 & \ldots & 1 & 1 \\
0 & 0 & 1 & 1 & \ldots & 1 & 1 \\
0 & 0 & 0 & 1 & \ldots & 1 & 1 \\
0 & 0 & 0 & 0 & \ldots & 1 & 1 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & \ldots & 0 & 1 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
\end{bmatrix}
\]
which can directly achieve the effect of cross-stage equalization by calling the designated equalizer through the matrix method, the multi-layer multiphase interleaved converter can directly transfer the energy from an arbitrary battery to any battery in the battery pack. So, other batteries can be achieved by transferring the energy in both directions.

The process of transferring the energy from an arbitrary battery to the battery pack and the process of transferring the battery energy to other batteries can be achieved by transferring the energy from an arbitrary battery to any battery in the battery pack. So, the multi-layer multiphase interleaved converter can directly call the designated equalizer through the matrix method, which can directly achieve the effect of cross-stage equalization between nonadjacent batteries, thus reducing the energy circulation. Whereas in the battery equalization circuit with a specified path, the energy balance between nonadjacent batteries can only be transferred from one level to another by means of energy balance between adjacent batteries so as to achieve the purpose of cross-level equalization; therefore, different overlap energy circulation phenomena are caused by multiple nonadjacent cells due to the direction of energy transfer between and the equalization time will be very long when the path grows. In the battery equalization circuit with a specified path, when the battery number is large, level across a number of energy transfer between adjacent batteries needs to pass through the equalizer for many times. The total efficiency is the result of multiplying the single efficiency. However, the equalizer may cause energy loss each time, assuming that the efficiency of each time is equal to \( \eta \); therefore, the whole transfer efficiency of \( n \) times is \( \eta^n \). However, the energy transfer efficiency between one battery to another battery in multi-layer multiphase interleaved converter is at least \( \eta^n \), so when the number of batteries is large, the multi-layer multiphase interleaved converter is to reduce the energy loss. In short, compared with the conventional equalization circuit with specified paths between specific batteries, the multiphase interleaved converter has higher flexibility, thereby reducing the energy circulation, reducing the energy loss, and accelerating the equalization speed.

2.4. Principle of Hybrid Architectures. In the above equalization process, the equalization modules \( EC_i \) and \( EC_{i+1} \) connected to the battery \( B_i \) were controlled. Similarly, when the abnormal \( B_i \) is expanded into a plurality of adjacent batteries, an equalization subcircuit at the top of the partial series battery pack and an equalization subcircuit at the end were controlled. It was found that the energy can be transferred from an arbitrary part of the battery in the battery pack to another part of the battery, as shown in Figure 3. The battery energy in \( BP_1 \) is transferred to the remaining cells in the battery pack, and then the entire battery stack transfers their energy to \( BP_2 \). For the complete process, the energy of \( BP_1 \) is transferred to \( BP_2 \).

3. Modeling and Evaluation Based on the Graph Theory

3.1. Graph Theory Expression of the Multiphase Interleaved Converter. The graph theory method represents complex equalization circuits as complete nodes and incomplete nodes and edges. A complete node is a storage device that can store energy for a long time in an equalization circuit, such as a supercapacitor and a lithium-ion battery. Nodes that can only temporarily store energy (such as inductors and capacitors) are called incomplete nodes. Since the paper focuses more on the bidirectional converter, the edges not specifically stated in the text are the bidirectional edges that allow the energy to be transmitted bidirectionally. This distinction is captured by assigning directions to the different edges of the graph, thereby making it a directed graph or digraph. In this abstracted diagrammatic representation, only energy flow paths in the equalization circuit are taken
into account regardless of the actual components in these paths so that switching devices, relays, and diodes are only parts of the edges. Figure 4 illustrates the corresponding diagram of the multiphase interleaved circuit. It is observed that among all circuits, the equalization path, which is located in the middle of the battery pack is omitted.

3.2. Parameters and Design of the Model. In this section, we followed the methods of Chen et al. who proposed framework on battery equalization [28]. The consideration of the average efficiency of the system is further clarified and improved in this paper. Some expressions should be defined, as the following:

Scalability (S): it means that the circuit structure can accommodate any number of battery packs in series, while it is clearly adjusted to the equalization. The topology is not limited to the application of a determined number of battery packs. In the process of increasing or decreasing the battery cells, the equalization module can be simply adjusted to achieve the same function without requiring major changes to the circuit structure.

Longest router (LR): it is the path that causes the maximum energy loss or the longest equalization time in the extreme case, when the transfer process of the battery energy passes the maximum number of equalization modules. It is found that the longest path is four in the multiphase interleaved equalization.

Moreover, the longest path in cell bypass architectures is $n-1$.

Average efficiency (AE): the efficiency of each equalization circuit is assumed to $\eta$. In order to calculate the average efficiency of the system, efficiencies in various
equilibrium conditions should be considered and the cumulative sum of the product of path probability and efficiency should be calculated. For the multiphase interleaved equalization converter proposed in this paper, there are three types of equalization paths, as the following: (1) two abnormal batteries exist at ends of the battery pack. In this case, the efficiency of the equalizer is \( \eta^2 \). (2) One abnormal battery exists at the end of the battery pack, and the other abnormal battery is in the battery pack. In this case, the efficiency of the equalizer is \( \eta^3 \). (3) None of the abnormal batteries is located at the ends of the pack, and both of them are located inside the battery pack. In this case, the efficiency of the equalizer is \( \eta^4 \).

Subsequently, there are three cases of energy anomalous batteries in the battery pack for multiphase interleaved converters: both abnormal batteries are at the ends, only one of the abnormal batteries is at the end, and both abnormal batteries are inside the battery pack. When batteries with excessive energy are located at the ends of the battery pack, it is necessary to transfer the energy of the battery at one end to the other end. Equations (21) to (23) present the corresponding probabilities for each of the aforementioned three cases, respectively:

\[
P(x_1 = \eta^2) = \frac{1}{C_n^2}, \tag{21}
\]

\[
P(x_2 = \eta^3) = \frac{C_1^1 C_{n-2}^1}{C_n^3}, \tag{22}
\]

\[
P(x_3 = \eta^4) = \frac{C_2^2}{C_n^4}. \tag{23}
\]

The corresponding matrix is in the following form:

\[
E_{B_iB_j} = \begin{bmatrix}
0 & \eta^2 & \eta^3 & \ldots & \eta^3 & \eta^2 \\
\eta^3 & 0 & \eta^4 & \ldots & \eta^4 & \eta^3 \\
\eta^3 & \eta^4 & 0 & \ldots & \eta^4 & \eta^3 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\eta^3 & \eta^4 & \eta^4 & 0 & \ldots & \eta^3 & \eta^3 \\
\eta^2 & \eta^3 & \eta^3 & \eta^3 & \ldots & \eta^3 & 0
\end{bmatrix}. \tag{24}
\]

Therefore, the average efficiency of the multiphase interleaved converter can be calculated as

\[
AE = \sum_{k=1}^{3} x_k \cdot P(x_k) = \frac{\eta^2}{C_n^2} + \frac{\eta^3 C_1^1 C_{n-2}^1}{C_n^3} + \frac{\eta^4 C_2^2}{C_n^4}. \tag{25}
\]

The method of accumulating the equalization efficiency of each case to obtain the average efficiency can also be applied to other equalization circuits. It should be indicated that in most engineering practices, the income of each solution strategy is often weighted and averaged to obtain the average value of the problem.

3.3. Design of the Ideal Equilibrium Structure. It is possible to represent the efficiencies of different energy transmission paths in a balancing circuit by assigning these efficiencies as weights for the corresponding edges of the circuit’s graph. In the remaining sections of the present study, it is assumed that each edge has the same weight \( \eta \) for the same type of the equalizer. However, it should be indicated that the presented analysis methods are general and can be utilized even if this assumption is violated.

An important adopted step is to simplify the aforementioned adjacency matrix by treating the balancing circuit’s energy storage elements as a part of a balancing pathway rather than treating them explicitly as nodes. This simplification represents the fact that the inductors in the above circuit simply act as a path, through which \( B_i \) and \( B_{i+1} \) can exchange energy bidirectionally.

One of the predominant benefits of the discussed framework is that it makes it possible to detect isomorphism between different balancing circuits. Two circuits are isomorphic if they have the same directed graph, the same weighted reachability matrix, and similar balancing characteristics. The other benefit of this framework is that graph theory makes it possible to describe the “ideal” characteristics of a balancing circuit. For example, one may choose the following properties as the desired characteristics:

1. The equalization structure should be bidirectional so that there should be an edge between any two solid points. In this case, the directed graph is complete and the longest and shortest energy transmission paths are equal.
2. The in-degree and out-degree of each solid points should both be \( n - 1 \) so that the total amount of the edges should be \( n(n - 1)/2 \).
3. The weighted reachability matrix of the ideal equalization structure should be symmetric, and all nondiagonal matrix elements should be equal.
4. The circuit structure should be easy to expand while enabling cell-to-cell and cell-to-pack equilibrium.

The direct cell-to-cell equalization method satisfies the above ideal requirements and has further advantages in terms of efficiency and balancing speed. In the next section, it is focused on balancing topologies, where each individual converter is a direct cell-to-cell converter. It is expected that combining two or more direct cell-to-cell converters into a larger balancing circuit does not automatically guarantee all the advantages of direct cell-to-cell pack balancing circuit.

4. Multi-Layer Optimization Principle Design Applied for the Multiphase Interleaved Converter

4.1. Multi-Layer Equalization of the Battery Pack. The connection of hundreds of batteries in series increases the device stress, while it improves requirements of the component parameter. Grouping and stratifying the battery pack is a reasonable choice to reduce the device stress. In order to
reduce this stress, the graph-theoretic representation of balancing circuits is proposed in the present study, and it is applied to multi-layer systems. The proposed scheme considers these devices as trees, with battery cells represented by leaves and lower equalization units represented by interior points. A full tree representation of a multi-layer balancing circuit can be constructed, which includes all the different layers and the interconnections between them. On the other hand, compact and reduced representations can be constructed, where each group of cells balanced by a lower-layer circuit is represented by a single vertex in a graph of the higher layers.

In order to simplify the analysis of the multi-layer balancing topology, the following assumptions are made:

1. Each converter in the battery equalization system is a direct cell-to-cell balancing converter
2. There are only two imbalanced batteries in the long series-connected battery string
(3) The excessive energy of the overcharged battery and the overdischarged battery is presented by $E$ and $-E$, respectively.

(4) When there are multiple layers in the system, the lower-layer equalizers operate first.

Appropriate hierarchical grouping of the battery pack can improve local equilibrium. Therefore, it can effectively improve the speed of the equalization, which is shown in Figure 5. However, the intensive grouping adversely affects the local equilibrium benefit. Moreover, it increases the cost of control.

4.2. Energy Path Optimization of the Hierarchical Equilibrium System. The optimization of the energy transfer path is required for the determined topology so that the equilibrium efficiency and speed are guaranteed in the case of any unbalanced battery packing. This problem is transformed into finding the shortest path problem in the search tree. The multi-layer equalization circuit energy path optimization problem can use the Floyd algorithm. The basic steps are as the following.

It is assumed that $A = (a_{ij}), d, R$ denote the weight matrix of the weight map, distance from point $i$ to point $j$, and the number of one of the shortest paths from point $V_i$ to $V_j$, respectively.

(1) Initializing: For all $i, j$, $d_{ij} = a_{ij}, r_{ij} = j, k = 1$.

(2) Updating $d_{ij}$, $r_{ij}$: For all $i, j$, if $d_{ik} + d_{kj} < d_{ij}$, $d_{ij} = d_{ik} + d_{kj}, r_{ij} = k$.

(3) Termination judgment: If $k = n$, then the process ends and $r_{ij}$ is the shortest path, else $k = k + 1$ and go to step two.

According to the aforementioned algorithm, it is clear that if the first and last battery energy status is abnormal, the equilibrium path is the longest in the cell bypass architectures. In hierarchical packet equalization, batteries that were previously able to be directly balanced in a single layer may also be grouped in different underlying groups in order to increase the path of equalization, which should be considered in the design. The benefits of the tiered grouping need to be weighed against the increase in the equalization path, which is a factor to consider when designing the equalization circuit in the future.

The Floyd algorithm is a dynamic programming algorithm, which is applicable to the “all pairs shortest paths” (APSP). The dense graph works best, and the edge weight can be positive or negative. This algorithm is simple and effective. Due to the compact structure of the triple loop, the efficiency of the dense graph is higher than that of the implementation of $|V|$ times Dijkstra algorithm and also higher than the execution of $|V|$ times SPFA algorithm.

5. Simulation and Experimental Analysis

5.1. Simulation Waveform and Analysis. In the experiment, $B_2$ is overcharged ($U_{B2} = 4\, V$, $U_{B1}, U_{B3}, U_{B4} = 3.6\, V, \, i = 2$). The process of transferring the excessive energy of $B_2$ to the other...
cells is simulated and verified. The equalization circuit composed of two batteries is equipped with an equalization module. Figure 6 indicates that simulation experiments of the four-cell battery are built in the PSIM11 software.

Figure 7 illustrates that when the equalization circuit is composed of four batteries, the process undergoes four modes. The dynamic voltage waveform is presented, where VP1 to VP4 denote the corresponding voltage of each battery. In order to simplify the parameter design in the simulation, the frequency of the switching is 1 kHz. In the subsequent experiments, the switching frequency is set to 20 kHz to reduce the loss of the device.

5.2. Experimental Waveform and Analysis. Experimental verifications have been completed to further verify the relevant results in the simulation experiment. The experiment system parameters are shown in Table 4.

Figure 8 shows that the experimental circuit is constructed without verifying the theoretical design and discusses effects of the simulation. In the experiment, eight batteries are used as the experimental objects to establish eight-cell single-layer equalization, three-layer dual-cell equalization, and four-layer bottom unit and two-layer upper unit structure. The experiment utilizes eight testing objects with model of 18650 lithium-ion battery. The flowchart of the equalization process is shown in Figure 9.

The voltage of the battery is recorded during the experiment, and it is plotted with the ORIGIN software. In order to verify the superiority of the above hierarchical packet equalization in comparison to the single-layer equalization, the following three battery pack hierarchical structures are designed. Figure 10(a) shows an eight-cell series single-layer equalization structure, in which seven equalizers sequentially divide the battery pack into eight complementary sections, enabling bidirectional flow of the energy between the sections. Figure 10(b) shows the eight-cell binary tree equalization structure. The last four equalization modules make the bi-directional transfer of the energy between the cells in the group for each of the eight batteries. The secondary equalization module realizes energy transfer between two batteries to two adjacent batteries, and the three-level equalization module achieves energy balance between the first four batteries and the last four batteries. Figure 10(c) shows a hybrid equalization structure. Figure 10(d) shows a bypass method equalization structure. In the last two equalization modules, four batteries are grouped to realize energy balance in the group, and the top layer equalization module realizes energy transfer between the two modules.

Figure 11 illustrates that in a single-layer multiphase interleaved converter of eight cells, the switching transistors in each equalization module operate in a continuous current mode of complementary conduction. In the experiment, the voltage of each battery in the battery pack gradually becomes uniform. The severely overcharged and overdischarged batteries have a faster energy transfer rate than the batteries with slight abnormal energy. At the end of the equilibrium, the battery energy transfer slows down.

Figure 12 indicates that the binary tree equalization structure can be equalized between two adjacent batteries at a lower level. During higher level equalization, there are potentially balanced batteries being charged or discharged, creating an unnecessary energy circulation. When the number of batteries in the pack increases, the power loss caused by the energy circulation will adversely affect the balance.

Figure 13 shows that hybrid hierarchical packet topology, to some extent, can reduce the generation of the energy circulation and as a result can improve the equilibrium speed. The increase in the number of cells in each group of the bottom layer reduces the offset of the local battery energy abnormality. It also reduces the total number
of layers of the battery pack and large voltage of stress on devices; therefore, the cost of system has been decreased. The continuous increase in the number of cells in the bottom cell pack increases the probability of the occurrence of local loops while reducing the difference in energy offset. Therefore, selecting the appropriate number of underlying packets in the case of a large number of battery packs in series is of significant importance.

5.3. Analysis and Comparison of the Results. According to the design and simulation discussed in the previous section, the experimental results are analyzed and processed and the average efficiency under ideal conditions is obtained as a reference value according to equations (26) and (27):

$$AE_{\text{Multiphase}} = f(n, \eta) = \frac{(n-2)(n-3)\eta^4 + 4(n-2)\eta^3 + \eta^4}{n(n-1)},$$  \hspace{1cm} (26)

$$AE_{\text{Bypass}} = g(n, \eta) = \frac{2\eta^2(\eta^{n-1} - 1) - 2(n-1)(\eta - 1)\eta}{n(n-1)(\eta - 1)^2}.$$  \hspace{1cm} (27)
According to these equations, the ideal equalization efficiency of multiphase and bypass circuit is obtained separately under different battery numbers \((n = 8\) and \(n = 16\)). The battery voltage array is randomly generated using a computer, and the average efficiency under different cycle times is separately counted. In order to avoid a large error caused by a single experiment, each cycle is performed ten times in the simulation and the result is plotted in Figure 14. The results show that the error between a single experiment and the ideal mean decreases as the number of
cycles increases. As the number of cells in the equalization circuit increases, the average efficiency decreases. When there are eight batteries in the circuit, the average efficiency of the multiphase converter is lower than that of the bypass converter. However, when the number of batteries increases to sixteen, the average efficiency of the multiphase converter is higher than that of the bypass converter.

Figure 15 illustrates the correlation between the average efficiency and the number of batteries. In order to further compare the two circuits, the average efficiency is compared between the two equalizer monomer efficiencies in the simulation with the number of battery cells as the independent variables. In this simulation, the efficiency of a single equalizer is \( \eta_1 = 0.8 \). When the number of batteries is less than sixteen knots, the average efficiency of the bypass circuit is always higher than that of the multiphase converter. However, when the number of batteries is more than sixteen, the result is reversed. As the number of the batteries increases, the average efficiency of the bypass circuit approaches zero, while the average efficiency of the multiphase converter approaches \( \eta^4 \).

The simulation data results are imported into ORIGIN software to draw an image of the average efficiency, the number of batteries, and the number of cycles. Figures 16 and 17 show the results, respectively. The average efficiency of the system is positively correlated with the individual cell efficiency, but an increase in the number of batteries leads to a decrease in the average efficiency. According to the actual
requirements of the project, the appropriate number of batteries in the series is selected and the improvement of individual equilibrium efficiency is considered for optimization.

Moreover, the relationship between the average efficiency of the system and a single equalizer is considered. Furthermore, the efficiency of the two equalization circuits in the case of eight batteries and sixteen batteries is compared. Figure 18 indicates that when there are eight cells in series, the average efficiency of the bypass converter is always greater than that of the multiphase converter. However, when the number of cells increases to thirty two, the cell equalization module is greater than fifty percent. As a result, the multiphase converter has a higher system efficiency than the bypass converter. As the number of batteries increases, the average efficiency of the bypass circuit is significantly reduced. However, the average efficiency of the multiphase converter is only slightly reduced.

Proper hierarchical packet equalization improves the overall efficiency of the system as a result of an increase in the number of battery cells. The average efficiency of the system is positively correlated with the efficiency of a single equalizer. Increasing the efficiency of a single equalizer can significantly increase the average efficiency of the equalizer system. It is found that for sixteen batteries, if the efficiency of a single equalizer is more than sixty percent, the average efficiency of the multiphase converter is higher than that of the bypass topology. However, when the efficiency of a single equalizer is less than sixty percent, the average efficiency of the multiphase converter is lower than that of the bypass converter. It is observed that for the same number of batteries, the average system efficiency of the two equalizers may also have advantages due to the difference in monomer efficiency, and this problem should be considered in engineering practice.

A comparison with existing technology for the circuit complexity is made in Table 5. Compared with traditional bypass circuits, the MMIC has fewer components and lower voltage stress, which will reduce the cost of the system and improve system stability. To illustrate the systems thoroughly, we performed an analysis of efficiency and power loss as shown in Figure 19.

As the number of batteries increases, the energy and circuit losses transferred by the equalizer increase, but the efficiency of the system increases slightly in MMIC while different in bypass circuit. It means the bypass circuit is more efficient than the MMIC in small scale, but lower when the scale is expanded. As the number of batteries increases, the efficiency of the bypass circuit is significantly reduced, but the efficiency of the MMIC is slightly improved. By deeply illustrating the energy loss of each part in the eight-cell series equalization, the main power loss of the system is clearly
found by the switching and inductor, whereas applications of low on-resistance SiC devices will significantly reduce system losses in the future.

6. Conclusions and Future Work

Multiphase interleaved converters make flexible energy balance among series battery packs. Considering the energy flow process, the graph theory is used to simplify the analysis of the circuit and to optimize the path. In view of the diversity of the equilibrium state caused by multcell series connection, the current study proposes a novel calculation method for the average efficiency of the system and compares the efficiency of the multiple aspects of the multiphase interleaved converter and the bypass topology. Experiments show that the average efficiency of the system is related to the efficiency of a single equalization module. Appropriate hierarchical packet equalization effectively improves the average efficiency of the system.

Compared with the existing equalizers, multiphase interleaved converters have significant advantages in the large-scale battery energy storage equalization systems. The average efficiency index can affect the system efficiency under multiple states and reduce the evaluation of the system performance due to the particularity of the initial state. This method of simplifying complex problems into graphs and using graph theory knowledge to solve related problems has been applied to a wide range of engineering practical problems.

Data Availability

The PSIM11 simulation data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Shuailong Dai proposed a framework of evaluation of the equalization circuit by graph-theoretic. Zhifei Shan designed and implemented the balancing circuit and its control. Dawei Song set up an experimental platform and performed a physical simulation. Teng Li drafted the manuscript. Mengfan Li finalized and polished the manuscript, and Pinduan Hu helped correct galley proofs.

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Table 5: Typical equalization device comparison.

| Topology  | Component | Voltage stress | Inductor | Transformer | Speed | Scalability | Control |
|-----------|-----------|----------------|----------|-------------|-------|-------------|---------|
| #1        | #S 0 | #D 0 | #S 0 | #D 0 | 0 | Low | Easy | Easy |
| #2        | #S 1 | #D n | #S V | #D V | 0 | Medium | Hard | Hard |
| #3        | #S V | #D V | #S V | #D V | 0 | Medium | Medium | Hard |
| #4        | #S 2n | #D 2 | #S V | #D V | 0 | High | Easy | Medium |

#1: flying-capacitor equalization structure; #2: multioutput winding transformer centralized equalization structure; #3: buck-boost centralized equalization structure; #4: multi-layer multiphase interleaved converter structure; #S: switch; #D: diode.

Figure 19: Power loss comparison between bypass and MMIC.
Supplementary Materials

Floyd algorithm main program. (Supplementary Materials)

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