Quantum Chromodynamics and the Z- width

N.D. Hari Dass†
*The Institute of Mathematical Sciences, Chennai - 600 113, INDIA

V. Soni ‡
*National Physical Laboratory, K.S. Krishnan Marg, New Delhi, INDIA

We show by explicit construction of an alternative theory for the strong interactions that it cannot be distinguished from QCD by any of the usual precision tests except on extending these theories to the full electroweak theory, by the Z particle width.

Quantum Chromodynamics (QCD) is today the best accepted theory of the strong interactions. This is based on the fact that,

1) Chiral Symmetry, which is the symmetry of the the hadronic interactions, and realised in a spontaneously broken phase with the pions as the the goldstone bosons, is also a symmetry of QCD.

2) QCD is asymptotically free (AF) : this translates experimentally into the phenomenon of scaling in deep inelastic scattering (DIS). So far DIS data has been consistent to good accuracy with the predictions of QCD. Even next to leading terms of QCD perturbation theory have been found necessary to explain certain high energy experiments. This is doubtless a very impressive record, hence the belief in QCD. While the relationship between AF and scaling is not straightforward, as argued by Coleman and Gross [1], hence the belief in QCD. Finally, it is the Z- width ($\Gamma_Z$) that selects between the two theories. Perhaps the most precise number in particle physics, $\Gamma_Z$, is the Z- width. It is then important to see how this theory fares experimentally.

Quantum Chromodynamics and the Z- width

We show by explicit construction of an alternative theory for the strong interactions that it cannot be distinguished from QCD by any of the usual precision tests except on extending these theories to the full electroweak theory, by the Z particle width.

The lagrangian is:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \bar{\sigma})^2 - \frac{1}{2}(\partial_\mu \bar{\pi})^2 - \lambda^2(\bar{\sigma}^2 + \bar{\pi}^2 - f_\pi^2)$$

$$- \bar{\Psi} \left[ D_\mu + g_y (\bar{\sigma} + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \right] \Psi - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a} \tag{1}$$

where $D_\mu = \partial_\mu - ig_3 A^a_\mu T^a$ and $G_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_3 f_{abc} A^b_\mu A^c_\nu$. $A^a_\mu$ is the gluon field and $T^a$ the SU(3) generator in the fundamental representation. $g_y$, $g_3$ and

A. The Theory

The lagrangian is:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \bar{\sigma})^2 - \frac{1}{2}(\partial_\mu \bar{\pi})^2 - \lambda^2(\bar{\sigma}^2 + \bar{\pi}^2 - f_\pi^2)$$

$$- \bar{\Psi} \left[ D_\mu + g_y (\bar{\sigma} + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \right] \Psi - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a} \tag{1}$$

where $D_\mu = \partial_\mu - ig_3 A^a_\mu T^a$ and $G_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_3 f_{abc} A^b_\mu A^c_\nu$. $A^a_\mu$ is the gluon field and $T^a$ the SU(3) generator in the fundamental representation. $g_y$, $g_3$ and
\( \lambda \) are the Yukawa, QCD, and scalar self-couplings respectively. \( \vec{\Psi} \) is the quark field and \((\vec{\sigma}, \vec{\pi})\) are the additional chiral scalar fields. Note that \( \vec{\pi} \) should not be confused with the traditional pions(\( \vec{\pi} \)) with mass \( \simeq 140 \text{ Mev} \), though it has the same quantum numbers. The \( \beta \) function for the QCD coupling, \( \alpha_s \), is

\[
\frac{\partial \alpha_s}{\partial t} = -\left( \frac{33 - 2N_F}{3} \right) \frac{\alpha_s^2}{8\pi^2} \left( g_\beta^2 = \alpha_s \right) \quad (2)
\]

This is for \( m_y = 0 \) and \( t = \ln(\rho/\mu) \).

To one-loop order the \( \beta \)-function for the QCD coupling does not receive any contribution from the Yukawa coupling, \( g_y \), or the scalar self coupling \( \lambda \), as the chiral multiplet is colour singlet.

The Yukawa coupling \( g_y \) for the pion and sigma to the quarks has the following \( \beta \) function (assuming 3 colours)

\[
\frac{\partial g_y^2}{\partial t} = \frac{g_y^2}{8\pi^2} \left[ 12 N'_g g_y^2 - 8 \alpha_s \right] \quad (3)
\]

where \( N'_g \) is the number of generations to which the chiral multiplet couples.

Following 3 we can now define the ratio \( \rho = g_y^2/\alpha_s \) and write the following equation for \( \rho \) using Eqs (2) and 3.

\[
\frac{\partial \rho}{\partial \alpha_s} = -\frac{\rho}{\alpha_s A} \left[ 12 N'_g \rho - 8 + A \right] \quad (4)
\]

where \( A = (33 - 2N_F)/3 \) and \( N_F \) is the number of flavours that effectively couple to the gluons. For the \( N_F = 6 \) case, there are two regimes. Calling \( \rho_c = 1/12 \), we have

**The Region** \( 0 < N'_g \rho < \rho_c \):

In this case \( \partial \rho/\partial \alpha_s > 0 \). This implies that \( \rho \) decreases as \( \alpha_s \) decreases, that is, \( \rho \) will decrease with increasing momentum scale. We can integrate the \( \rho \) equation to get

\[
\rho = \frac{\alpha_s^{1/7} K}{\left(1 + 12 N'_g \alpha_s^{1/7} K\right)} \quad (5)
\]

where \( K \) is a positive integration constant that is set by initial data on \( \alpha_s \) and \( g_y^2 \). It is therefore clear that there is a whole family of solutions corresponding to different \( K \)'s. Deep in the ultraviolet when \( \alpha_s \to 0 \)

\[
N'_g \rho \sim K \alpha_s^{1/7}
\]

which further implies that in the ultra violet

\[
N'_g g_y^2 \sim K \alpha_s^{8/7}
\]

This means that \( g_y^2 \) is asymptotically free and vanishes faster than \( \alpha_s \). Therefore, the leading behaviour of this theory in the ultraviolet is given by the QCD coupling with the Yukawa coupling contributing only in sub leading order.

The region \( N'_g \rho > 1/12 \) is also of interest, though from a different physical perspective. However, the theory is not AF in this region. Therefore we shall not consider it anymore in this paper. See however, refs 3 and 4 for further discussions of this region.

The above analysis is valid for \( q^2 \geq m^2 \). For the region \( m_b \leq q \leq m_t \), the relevant behaviours are : \( \rho_c = 1/36 \), \( N'_g \rho \sim K \alpha_s^{1/23} \). For \( N_F \leq 4 \), there is no AF regime. The solution for \( N_F = 4 \) is \( \rho/(\rho + 1/36) = K \alpha_s^{1/25} \).

For the purposes of our paper (in the context of the discussion on \( g - 2 \) for muons) it is important that regions where QCD is perturbatively treatable, our theory is too. AF for the scale \( > m_b \) implies \( \rho \leq 1/36 \). Thus at \( q = m_b \), the Yukawa couplings are small. Now the lack of AF for \( q < m_b \) has the desired effect of making \( \rho \) even smaller as we go to smaller energy scales. Thus \( g_y, \lambda \) are perturbatively treatable wherever QCD is.

The \( \beta \)-function for \( \lambda \) in our model is

\[
\frac{\partial \lambda}{\partial t} = \frac{1}{8\pi^2} \left[ 2\lambda^2 - 144N'_g \lambda^4 + 24N'_g g_y^2 \lambda^2 \right] \quad (6)
\]

By defining the ratio \( R = \lambda/g_y^2 \) we can convert to an equation for \( R \) that depends on the single variable \( \rho \) and find that

\[
\frac{\partial R}{\partial \rho} = \frac{1}{12 \rho - 8 + A} \left[ 2R^2 + R \left( 12 + \frac{8}{\rho} \right) - 144 \right] \quad (7)
\]

For related issues in the standard model see 10, 11, 14. It is found that only on a single trajectory in the \([R, \rho]\) parameter space, that is the so called invariant line \( \frac{\partial R}{\partial \rho} \), behaviour of \( R \) for the regime \( N'_g \rho < 1/12 \) is such that \( R \to 0 \) in the ultraviolet. It follows further that \( \lambda \to 0 \) in the extreme ultraviolet even faster than \( g_y^2 \).

Thus we have classes of theories that are not only AF in all their couplings, but become increasingly indistinguishable from QCD at high energies. As far as AF is concerned one loop analysis is stable against higher loop corrections 1. Since these classes of theories are AF, they are all candidates for a consistent theory of strong interactions.

**B. The R parameter**

The \( R \) parameter measures the ratio of \( \sigma(e^+e^- \to \text{hadrons}) \) to \( \sigma(e^+e^- \to \mu^+\mu^-) \). In QCD, at high energies, the former is approximated by \( \sigma(e^+e^- \to \Sigma \bar{q}q) \). When the energies are well below the Z mass we can neglect the contribution of the Z mediated process to \( R \). To leading order, then, the \( R \) parameter measures the total number of operational flavours (of course, multiplied by the number of colors of the quarks), modulated by their charge squared.

As we move to higher generations the \( R \) parameter changes rapidly at quark mass thresholds. At thresholds
we also encounter many resonances which also give rapid changes in the R parameter. However, away from thresholds R can be quite stable. Therefore in the region above $bb$ threshold we expect R to be stable apart from QCD corrections. But as we approach $\sqrt{s} = M_z$, a new class of diagrams become operative and the R parameter has a steady rise to the Z peak.

There is thus an energy region $20 - 40$ GeV, where the effect of the Z is yet very small, where R is relatively stable. To leading order, that is, in the absence of QCD corrections, here $R_0 = 11/3$.

The R parameter will change for our theory as we have new scalar charged partons, the $\pi^+\pi^-$ that couple to the photon which will contribute to the hadronic cross section. For the the zero mass partons considered by us the contribution to the R parameter is exactly calculable and is given by $R = 1/4$. This additional contribution should be clearly visible, particularly in the region: $20 - 40$ GeV. The contribution of Z-exchanges are negligible in this region.

The R parameter receives QCD corrections and the QCD corrected R parameter in this region is: $R(s) = R_0(1 + \alpha_s/\pi + ...)$ The measured value of the R parameter in this region has a world average of 4.02. The difference between this number, 4.02, and $R_0 = 11/3$ is supposed to come from the QCD corrections and yield a value for $\alpha_s$ at this energy scale.

However if we add the extra contribution of our new partons, $R_0 = 11/3 + 1/4 = 3.91$, leaving only a deficit of 0.11 to be accounted for by the QCD corrections. The corresponding value of the QCD coupling will then be too low to be admissible. However presently the systematic errors at Amy, Topaz etc are not so small, of the order of 5%. Also, different groups have reported R in this region to be as high as 4.2 or even as low as 3.8. This circumstance means that our theory cannot be ruled out as the effect we are considering is $\Delta R/R = 6 \text{ %}$. It is worth pointing out that experimentally low multiplicity ($\lesssim 5$) jets are not counted as these are very unlikely in QCD. On the other hand, for the color neutral pionic partons of our theory, we expect to have only low multiplicity jets most of which would have been excluded by experimental cuts. Of course, two prong events would have been counted as $\mu^+\mu^-$ pairs and could have shown up as anomalies which could be distinguished by their different angular dependence as compared to leptons.

Given these facts the R parameter is not at present a definitive test for this theory versus QCD though reduced systematic errors could put things on the borderline.

C. g-2 for the muon

The contribution to g-2, the muon magnetic moment, can be potentially disturbed by the presence of additional charged particles. Our theory has particles with zero or light masses coupling to photons thereby contributing additionally to the photon propagator. Such a contribution to the propagator can be related to the contribution to the R parameter we have just considered. This is precisely how g-2 is calculated in the literature \cite{14}. In this work the contribution to g-2 for the muon, $a_\nu$, is given by

$$a_\nu = \int dt K(t)R(t)$$

where $R(t)$ is the R-ratio and $K(t)$ is a known function.

However, the manner in which zero mass scalar partons enter the low energy description of R(t) (where $\sqrt{s}$ is the centre of mass energy) is subtle. At low energy there is mixing between our zero mass pionic and the pseudoscalar channel. This will generate a higher mass state with quantum numbers of the pion in addition to the usual ‘goldstone pion’ associated with the spontaneous breaking of chiral symmetry. Since we cannot calculate the mass of this non-perturbatively generated extra state we can only use experiment to glean its mass. The particle data book lists the first additional state with pion quantum numbers at 1.3 GeV. At low energies then we must use this state to calculate the extra contribution to the photon self energy or the R parameter as the usual pion’s contribution is already accounted for in the standard treatment of hadronic contributions to g-2. The threshold for the contribution of this state then starts at $\approx 2.6$ GeV. This puts us in the perturbative QCD regime.

Before computing the extra contributions a few remarks are in order: 1) The low energy regime, $0.8 - 2.0$ GeV, has to be gleaned from experiment as perturbative QCD cannot be used. The contribution to $a_\nu$ from this region as listed in Table 2 of \cite{14} is $(1404 \pm 100) \times 10^{-11}$. As this is evaluated from experimental data, one cannot differentiate the contributions from QCD and our theory. 2) Perturbative QCD is used for $t > 2$ GeV (Table 1 of \cite{14}) except for the threshold regions which are populated by numerous resonances, where again one has to only rely on experimental data, and can not differentiate between our theory and QCD. These regions are: $3.3 - 3.6$ GeV, $3.6 - 4.9$ GeV and $9 - 14$ GeV.

Thus it is only for regions for which estimates are made via perturbative QCD that comparison between this theory and QCD is possible. In respect of the foregoing discussion this region for us must begin at $\sqrt{s} > 2.6$ GeV. We briefly sketch how the additional contribution can be evaluated for our theory in the region $2.6 - 3.1$ GeV: i) $K(t)$ goes as $1/t^2$. Assuming $R(t) = R_0$ and using eqn(10) we can get the two contributions for 1.4 - 2.6 GeV and $2.6 - 3.1$ GeV for QCD. The partial contribution for the region $2.6 - 3.1$ GeV is found to be 1/10 of the total. ii) The extra contribution to R assuming zero mass pionic partons is $\delta R = 1/4$ whereas the QCD contribution is $R_0 = 2$. Thus the fractional extra contribution...
is 1/8. iii) This is further down when we take into account the mass of the excited state (1.3 Gev). A rough estimate is provided by multiplying by the phase space factor $(1 - 4M^2/t) = 0.17$ taken at the average value 2.85 Gev for $\sqrt{t}$. The total extra contribution for the region 2.6-3.1 Gev works out to $1.2 \cdot 10^{-11}$. Below we display the contributions and errors for various regions:

| Region   | QCD multiplet error | Chiral Theo. error (5%) |
|----------|---------------------|-------------------------|
| 2.6-3.1  | 56                  | 1.2                     |
| 4.9-9    | 67.5                | 4                       |
| > 14     | 13                  | 0.9                     |

It should be noted that: i) The theoretical error in [14,15] is arbitrarily estimated as half the $\alpha_s^2$ correction to $R$. ii) we have taken the systematic error to be 5% of $R$ (see Sec 3).

The extra contribution of our theory falls within the sum of the theoretical and the systematic errors. The error in the low energy region, 0.8-2 Gev is roughly 100. 10^{-11}. By comparison, all our extra contributions are negligible.

We are therefore led to the conclusion that g-2 for the muon despite being a very high precision measurement of the charged-particle content of theories cannot differentiate between QCD and our theory - a rather non trivial result.

D. Z width

The Z-width data on the other hand is known with great accuracy. The minimal coupling of the chiral multiplet to $Z_\mu$ is [3,4]:

$$L^{\text{lin}} = e(A_{\mu} - \frac{\gamma}{2}Z_{\mu})(\vec{\pi} \times \partial_{\mu}\vec{\pi})_3 - e\frac{c}{2c_s}Z_{\mu}(\vec{\sigma}_0\partial_{\mu}\vec{\sigma} - \partial_{\mu}\vec{\sigma}_0)$$ (9)

where $\gamma = (1 - 2s^2)/cs$ with $s$ being $\sin\theta_W$ and $c^2 = 1-s^2$. The contribution to the hadronic width of Z-boson due to the extra scalars can be calculated easily:

$$\frac{\Delta \Gamma_Z}{\Gamma_Z^{\text{had}}} = \frac{9((1 - 2\sin^2\theta)^2 + 1)}{N_c(90 - 168\sin^2\theta + 176\sin^4\theta)}$$ (10)

At $\sin^2\theta \approx 0.25$, this works out to roughly 4.5% of the total width $\Gamma_Z$. The high precision LEP data only allows total uncertainty of about 3%.

This more or less immediately rules out the extended version where the chiral symmetry in the extended sector is spontaneously broken or where the chiral symmetry in the extended sector is manifest with low mass for the multiplet.

A different version of this theory where the chiral multiplet mass of over one half of the Z width will be exempt from this problem. This will be considered separately.

E. CONCLUSION

By explicit construction of an alternative theory to QCD for the strong interactions we have found that all precision tests for QCD except for the Z width cannot select between the two theories. Only by extending the theories to the FULL electroweak standard model do we find an unambiguous support in favour of QCD from the Z width. This underscores the fact that most tests and vindications of QCD that are to be found in archival references in the literature are just not adequate. From this work we find that only the Z width is the final arbiter.

† electronic address : dass@imsc.ernet.in
‡ electronic address : vsoni@ren.nicnet.in

[1] S. Coleman and D. J. Gross, Phys. Rev. Lett. 31, 851(1973).
[2] For example, M. Derrick et al, hep-ex 9505011
[3] V. Soni, Mod. Phys. Letts A 11, No. 4331(1996)
[4] N. D. Hari Dass, V. Soni, H. S. Sharat Chandra, R. Anishetty and R. Basu, “Asymptotic freedom and compositeness in Yukawa, Yang-Mills chiral theory”
[5] B. Schrempp and F. Schrempp, Phys. Lett. B299, 3221 (1993).
[6] B. Pendelton and G. G. Ross, Phys. Lett. 98B, 291(1981).
[7] C. T. Hill, Phys. Rev. D 24 (1981) 691
[8] W. Zimmermann, Comm. Math. Physics 97 (1985) 211
[9] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. B 259 (1985) 331
[10] V. A. Miransky, M. Tanabashi and K. Yamawaki, Phys. Lett. B221, 177(1989), W. A. Bardeen, C. T. Hill and M. Lindener, Phys. Rev. D41 , 1847(1990).
[11] K. I. Kondo, M. Tanabashi and K. Yamawaki, Prog. Theo. Phys. 91 (1994) 541
[12] J. Kubo, M. Mondragon and G. Zoupanous, Nucl. Phys. B 259 (1989) 291
[13] M. Harada, Kikkukawa, Kugo and Nakano, Prog. Theo. Phys. 92 (1994) 1161
[14] J.A. Casas, C. Lopez and F.J. Yndurain Phys Rev D 32(1985) 736
[15] N. D. Hari Dass and V. Soni,”Asymptotically Free Alternatives to QCD”
[16] N. D. Hari Dass and V. Soni,”Asymptotically Free Alternatives to QCD and ALEPH Four Jet events” hep-ph/9709399