Dependence of magnetic cycle parameters on period of rotation in nonlinear solar-type dynamos

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ABSTRACT

The paper reports results of calculations of the magnetic cycle parameters, like the dynamo cycle period, amplitude of the magnetic flux and the Poynting flux from the surface for the solar analogs with rotation periods from 15 to 30 days. We employ the nonlinear mean-field axisymmetric dynamo models, which take into account the principal mechanisms of the nonlinear dynamo saturation. The study takes into account the magnetic helicity conservation, the magnetic buoyancy and the magnetic feedback on the angular momentum balance inside the convection zone. Also, we consider two types of the dynamo models. The D-type models employ the standard $\alpha$-effect distributed on the whole convection zone. The BL-type models employ the non-local $\alpha$-effect. Both the D- and BL-types of the dynamo models show the growth of the dynamo generated magnetic flux with the increase of the rotation rate. The magnetic helicity conservation is the most feasible effect for the dynamo saturation both for the D and BL-types dynamos. It is found that it is even more efficient for the BL-type dynamo. The D-type dynamo reproduces qualitatively the dependence of the cycle period on the rotation rate for the Sun analogs. For the Sun rotating with period 15 days we find regimes with multiply cycles which qualitatively reproduce the active branch stars in the dependence of the cycle period on the rotation period of the star.

Key words: stars:activity; stars:magnetic fields; dynamo: turbulence - magnetic fields

1 INTRODUCTION

To the date, there is an extensive data base about the magnetic activity on the main sequence stars (see, e.g., reviews by [Donati & Landstreet 2009] [Reiners 2012]). The cool stars with the external convective envelope have a particular interest because they are Sun-like. It is believed that the magnetic activity on the solar-like stars is resulted from the large-scale dynamo processes (Brandenburg & Subramanian 2005). The general idea of the dynamo mechanism was earlier proposed by [Parker 1955] who suggested that the dynamo operates in the depth of the convection zone and it results from a mutual interplay of the differential rotation and convection. Observations (e.g., [Böhm-Vitense 2007] [Donati & Landstreet 2009] [Katsanov et al. 2010] [Saar 2011] [Katsanov et al. 2013] [Marsden et al. 2014] [Vidotto et al. 2014]), as well as the mean-field models (Rüdiger 1989) [Kitchatinov & Rüdiger 1999] [Kitchatinov 2013] and the direct numerical simulations (Miesch & Toomre 2009) [Hotta & Yokoyama 2011] [Guerrero et al. 2013b] [Käpylä et al. 2014] shows that parameters of the differential rotation and convection, e.g., the typical size and turnover time of the convective flows, depend on the general parameters of the star, such as are the mass, the age, the spectral class and the rotation rate. The mass of a star and its Rossby number, which is the ratio between the period of rotation to the typical turnover time of convection, seems to be the most important parameters governing the stellar dynamo (Donati & Landstreet 2009) [Morin et al. 2013]. The diagram 3 from the paper by Donati & Landstreet (2009) shows the growth of the magnetic activity with the decrease the Rossby number and the mass of a star. These parameters determine the topology of the large-scale magnetic field, as well. It is found that the axisymmetric solar-type dynamo can operates in the stars with mass about $1M_\odot$ and periods of rotation larger than 10 days. The faster rotating suns show the substantial non-axisymmetric components of the large-scale magnetic field.

Interpretation of the stellar magnetic activity becomes more complicated as we have to take the nonlinear dynamo effects into account (Soon et al. 1993) [Saar & Brandenburg 1999] [Böhm-Vitense 2007] [Saar 2011] [Blackman & Thomas 2014].
Moreover, many details of the solar dynamo processes are poorly known (Charbonneau 2011). In particular, the origin of the large-scale poloidal magnetic field which forms the solar corona and polar fields is not well understood. Parker (1955) suggested that the large-scale toroidal magnetic field produces an ensemble of the small-scale magnetic loops, which have a small poloidal component. These loops are organized in the large-scales, because the cyclonic convective motions in a rotating star have the preferred sign for each hemisphere of the Sun. It is the so-called α-effect. In this picture, the drift of the sunspot formation latitude and the reversal of the polar magnetic field is explained by the diffusive propagation of the large-scale dynamo wave along the isolines of the angular velocity distribution (Yoshimura 1975, Brandenburg 2005, Käpylä et al. 2006, Kosovichev et al. 2013). Babcock (1961) suggested the alternative mechanism, where the poloidal magnetic field origins from the loop of the buoyant toroidal field which turned by the Coriolis force in floating through the convection zone from the tachocline to the top. This effect can be considered as the nonlocal α-effect (Brandenburg & Sokoloff 2002 Brandenburg & Käpylä 2007 Brandenburg et al. 2014). The dynamo with the nonlocal is concentrated to the bottom of the convection zone. It can operate with (Choudhuri et al. 1995; Dikpati & Charbonneau 1999) and without meridional circulation (Kitchatinov & Olemskoy 2011). Note, that information about the distribution of the meridional circulation on the Sun is rather controversial. Observations suggest the multiplicity structure of the meridional circulation (Zhao et al. 2013; Kholikov et al. 2014).

Recently, Brun et al. (2010) and Karak et al. 2014 discussed results of simulations for the kinematic dynamo models which takes into account the non-local α-effect and the meridional circulation for the solar-type stars (1M⊙) for the range of the rotational period from 1 to 30 days. Those models qualitatively reproduce the growth of the magnetic activity with the increase of the rotation rate. The models fail to explain the decrease of the dynamo period with the increase of the rotation rate (and magnitude of the generated magnetic fields). Brun et al. (2010) found that the issue can be cured for the certain multiply-cell pattern of the meridional circulation.

The reverse correlation between the magnetic cycle amplitude and the cycle period is a common feature of the solar (Vitinsky et al. 1986) and stellar (Soon et al. 1994) magnetic cycles. It is fulfilled for the dynamo model with the distributed α-effect (Pipin & Kosovichev 2011) Pipin et al. 2012. Therefore it is interesting to examine the given relation for the range of the rotation rates in the nonlinear dynamo distributed in the convection zone (hereafter, the D-type dynamo).

In the study we consider the moderate range of the rotation rates covering the rotational periods 15 to 30 days. We assume that in the kinematic case the basic structure of the differential rotation corresponds to that suggested by helioseismology (Howe et al. 2011). Results of Karak et al. (2014) confirm our expectation for the given interval of the rotational periods. It is also supported by observations of the stellar differential rotation. Saar (2011) reported the linear growth of the latitudinal shear between the pole and equator for the given interval of the rotation rates.

The paper explore the set of the nonlinear dynamo models taking into account the magnetic feedback on the angular momentum balance, the magnetic helicity conservation and the magnetic buoyancy. The details of the meridional circulation distribution are unknown and we neglect it in the study. Two type of the dynamo model will be discussed. The D-type model that operates in the bulk of the convection zone. The BL-type model with the non-local α-effect (no meridional circulation) is concentrated to the bottom of the dynamo domain. Confronting two approaches allows us to explore how distribution of the turbulent sources of the dynamo impact the properties of the magnetic cycles. Also, it is interesting to confront the nonlinear saturation in the different types of the mean-field dynamos.

## 2 BASIC EQUATIONS

### 2.1 Dynamo model

In following to Krause & Rädler (1980), we explore the evolution of the induction vector of the mean field, \( B \), in the high conductive turbulent media with the mean flow \( \mathbf{U} \) and the mean electromotive force \( \mathbf{E} = \mathbf{u} \times \mathbf{B} \) (hereafter MEMF), where \( \mathbf{u}, \mathbf{b} \) are fluctuations of the flows and magnetic fields:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}).
\] (1)

Here, we use the decomposition of the axisymmetric field to the sum of the azimuthal (toroidal) and poloidal components:

\[
\mathbf{B} = \mathbf{e}_\theta \mathbf{B}_\theta + \nabla \times \frac{A e_\phi}{r \sin \theta} \mathbf{b},
\]

where \( \mathbf{e}_\theta \) is the unit vector in the azimuthal direction, \( \theta \) is the polar angle and \( A e_\theta \) is the vector-potential of the large-scale poloidal magnetic field. The mean electromotive force, \( \mathbf{E} \), is expressed as follows:

\[
\mathbf{E}_\iota = \left( \alpha_{\iota j} + \gamma_{\iota j}^{(H)} \right) \mathbf{B}_j - \left( \eta_{\iota jk}^{(H)} + \eta_{\iota jk} \right) \nabla_j \mathbf{B}_k + \mathbf{E}_{\iota}^{(H)}.
\] (2)

Tensor \( \alpha_{\iota j} \) models the generation of the magnetic field by the α-effect, \( \gamma_{\iota j}^{(H)} \) controls the mean drift of the large-scale magnetic fields in turbulent media, \( \eta_{\iota jk} \) governs the turbulent diffusion, and \( \eta_{\iota jk}^{(H)} \) models the generation of the magnetic fields by the \( \Omega \times \mathbf{J} \) effect (Rädler 1969). We take into account the effect of rotation and magnetic field on convection. The technical details and the complete expressions of the tensor coefficients can be found in our previous papers (Pipin 2008; Pipin et al. 2012 2013 Pipin & Kosovichev 2014).

The model of the α-effect takes into account the kinetic and magnetic helicities,

\[
\alpha_{\iota j} = C_\alpha \sin^2 \theta \psi_\alpha(\beta) \alpha_{\iota j}^{(H)} + \alpha_{\iota j}^{(M)}
\] (3)

where \( C_\alpha \) is a parameter to control the power of the α-effect. We introduced the latitudinal factor \( \sin^2 \theta \) to bring the model to better agreement with observations. This is supported by theoretical calculations of the α-effect for convective turbulent flows (Keevin & Rogachevskii 2003) and it is often employed by others in their dynamo models (see, e.g., Moss & Brook 2000). Function \( \psi_\alpha(\beta) \), where \( \beta = |B|/\sqrt{4\pi \rho u^2} \), \( u^\prime \) is the RMS of the convective velocity, controls the so-called algebraic quenching of the α-effect. It is like \( \beta^{-3} \) for the \( \beta \gg 1 \). The expression for the \( \alpha_{\iota j}^{(H)} \) and
dependence of magnetic cycle parameters on period

\[ \psi_a(\beta) \] can be found in [Pipin 2008] (hereafter P08). The \( a^{(M)}_{ij} \) takes into account the contribution of the mean small-scale magnetic helicity \( \chi = a \cdot \mathbf{b} \) (where \( a \) is the fluctuation of the magnetic vector potential). P08 found that:

\[ a^{(M)}_{ij} = f^2_2 \delta_{ij} \frac{\chi}{2 \pi \rho e^2} - f^1_1 e_i e_j \frac{\chi}{2 \pi \rho e^2}, \tag{4} \]

where \( e = \Omega / \Omega \) is the unit vector along the rotation axis, \( f^0, f^1, f^2 \) are function of the Coriolis number (see P08), \( \Omega^* = 4\pi \frac{p_{rot}}{P_{rot}} \), \( P_{rot} \) is the rotational period, \( \tau_c \) is the convective turnover time and \( \ell \) is the typical length of the convective flows (the mixing length). The last two parameters are determined by the mixing length model of the convection zone. We employ the model by [Stix 2002].

For the BL-type models we employ the following expression for the mean-electromotive force related with the \( \alpha \)-effect [Brandenburg & Käpylä 2007; Brandenburg et al. 2014].

\[ \mathcal{E}^{(a)} \phi = C_S \frac{\psi^{(a)} (\Omega^*_\star)}{\psi^{(a)} (\Omega^*_\odot)} \cos \theta \Psi^{(-)} (r - r_s) \times \int_{r_b}^r \psi^{(+)} (r' - r_{MLT}) B (r', \theta) d r' + \frac{\nabla B (r, \theta)}{\rho^2}, \tag{5} \]

where \( C_S \) is a parameter to control the effect of the \( \alpha \)-effect, \( \psi^{(a)} \) are the Heaviside-like functions, \( \psi^{(+)}(r - r_{MLT}) \geq 0 \), with \( r_s = 0.95, r_{MLT} = 0.725 \). The function \( \Psi^{(-)} \) (see [Rüdiger & Kichatinov 1993]) controls the growth and saturation of the non-local \( \alpha \)-effect with the increase of the rotation rate. The \( \Omega^*_\star \) is the Coriolis number for the star and the \( \Omega^*_\odot \) is the same for the Sun. To determine them, it is common (see, [Noyes et al. 1984; Kim & Demarque 1996; Gunn et al. 1998]) to take the radial distance which equals to the half of the mixing length counting from the bottom of the convection zone. For the modern Sun we have \( \Omega^*_\odot = 2 \pi / (0.845 R_\odot) \Omega_\odot \approx 3.25 \), where \( \Omega_\odot = 2.87 \text{c} - 0.6 \text{~s}^{-1} \).

In the model we take into account the mean drift due to the magnetic buoyancy, \( \gamma_{ij}^{(boa)} \), the diamagnetic pumping, \( \gamma_{ij}^{(lp)} \), the effects due to the gradient of the mean density, \( \gamma_{ij}^{(Lp)} \), and due to the large-scale shear \( \gamma_{ij}^{(H)} \):

\[ \gamma_{ij}^{(a)} = \gamma_{ij}^{(Lp)} + \gamma_{ij}^{(lp)} + \gamma_{ij}^{(boa)} + \gamma_{ij}^{(H)} \]

\[ \gamma_{ij}^{(Lp)} = 3 \eta_T \left( f^3 (A^{(a)}_n + f^1 (\mathbf{e} \cdot A^{(p)}) \epsilon_n \right) \delta_{ij} \tag{6} \]

\[ \gamma_{ij}^{(lp)} = \frac{3}{2} C_v \eta_T \left( f^2 (A^{(a)}_n + f^1 ) e_j \epsilon_{inm} \epsilon_{lm} A^{(a)}_m \right) \tag{7} \]

\[ \gamma_{ij}^{(boa)} = -\frac{\alpha \Omega_{MLT}}{\gamma} \beta^2 K (\beta) g_n e_{inj}, \tag{8} \]

where \( A^{(p)} = \nabla \log p \) is the reverse scale of the mean density variations, \( A^{(a)} = \nabla \log (p^{(a)}_\eta) \) is the same for the turbulent diffusivity, the free parameter \( C_v \) is used to tune the BL-types models (see comments in [Kitchatinov & Olemskoy 2011]). In the D-type models we put \( C_v = 1 \). The \( \alpha_{MLT} \) is the parameter of the mixing length theory, \( \gamma \) is the adiabatic exponent and the function \( K (\beta) \) is defined in [Kitchatinov & Pipin 1993]. [Pipin 2013] analyzed the total effect of the \( \gamma_{ij}^{(H)} \) in details, where the expression for the \( \gamma_{ij}^{(H)} \) is given as well. In short, the \( \gamma_{ij}^{(H)} \) produce the downward pumping near the bottom of the convection zone, which is expected to increased by factor \( C_v \). Near the surface the pumping goes upward and to equator.

We employ the anisotropic diffusion tensor which is derived by [Pipin & Kosovichev 2014]:

\[ \eta_{jk} = 3 \eta_T \left( 2 f^2_1 - f^2_2 \right) \delta_{ij} \tag{9} \]

\[ + \frac{a}{3} \eta_T \left( g_n g_j \delta_{nk} - \epsilon_{ijk} \right), \tag{10} \]

where \( g \) is the unit vector in the radial direction, \( a \) is the parameter of the turbulence anisotropy. The reader can find the functions of the Coriolis number \( f^0_1 \) and \( \phi_1 \) in [Pipin & Kosovichev 2014]. \( \eta_T = C_v \eta_T^{(0)}, \eta_T^{(0)} \) is the magnetic diffusion coefficient which is determined by the MLT from the model of the convection zone, \( 0 < C_v < 1 \) is a free parameter to control the efficiency of the mixing of the large-scale magnetic field by the turbulence. It is usually employed to tune the period of the cycle. The \( \gamma_{ij}^{(H)} \) is defined as follows:

\[ \gamma_{ij}^{(H)} = 3 \eta_T C_v f^3_4 e_j \left( \sigma_{ij}^{(w)} \delta_{ik} + \tilde{\sigma}_{ij}^{(w)} \frac{B_i B_k}{B^2} \right), \tag{11} \]

where \( C_v \) controls the power of the \( \Omega \times J \) effect and the algebraic quotient functions \( \frac{\sigma_{ij}^{(w)}}{B^2} \) is the diffusive flux of the magnetic helicity, with the \( \eta_T = \frac{1}{\gamma} \left( f^2_1 - f^2_2 \right) \eta_T^{(0)} \) which is factor ten smaller than the isotropic part of the magnetic diffusivity [Mitra et al. 2010]. \( R_m = 10^6 \) is the magnetic Reynolds number.

The distribution of the turbulent parameters and the structure of the convection zone is kept for the same for the studied interval of the rotational periods. We employ the solar interior model of [Stix 2002] using \( \ell = \Omega_{MLT} \left( \Lambda^{(p)} \right)^{-1} \), where \( \Lambda^{(p)} = \nabla \log p \) is the reverse scale of the thermodynamic pressure and \( \alpha_{MLT} = 2 \). The profile of the turbulent diffusivity is given in form \( \eta_T^{(0)} = \frac{u^2}{3 f_{ov} (r)} \), where \( f_{ov} (r) = 1 + \exp (50 (r_{ov} - r)) \), \( r_{ov} = 0.725 R_\odot \) controls saturation of the turbulent effects near the bottom of the convection zone. The numerical integration is carried out from pole to pole and in radius from \( r_s = 0.715 R_\odot \) to \( r_e = 0.99 R_\odot \). On the bottom of the convection zone we employ the superconductor boundary condition. At the top of the convection zone the poloidal field is smoothly matched to the external potential field and the toroidal field is allowed to penetrate to the surface:

\[ \left( \delta + \frac{B}{B_{eq}} \right) \frac{\eta_T}{r_e} B (1 - \delta) \mathbf{E} = 0, \tag{12} \]
where $\delta = 0.99$, $B_{eq} = 10G$. For the $B \gg B_{eq}$ the condition (12) is close to the vacuum boundary conditions (Moss & Brandenburg 1992). Also it takes into account the dynamo generated Poynting flux $S_M = -\frac{1}{4\pi} \mathcal{E}_B B^*_{\varepsilon}$ through the top. For the BL-type models we employ the standard vacuum boundary conditions.

### 2.2 Angular momentum balance

We consider the mean-field equation of the angular momentum balance inside the convection zone. The mean balance is established by the dynamo generated peak stresses $T_{\phi \phi}$, $T_{\psi\phi}$ and the large-scale Lorentz force (Ruediger 1989):

$$
\sin \theta \frac{\partial \Omega}{\partial t} = \frac{1}{\rho r^4} \frac{\partial}{\partial r} \left( T_{\phi \phi} - \frac{B}{4\pi r^2} \frac{\partial A}{\partial \mu} \right) - \frac{1}{\rho r^2} \sin \theta \frac{\partial}{\partial \mu} \left( \sin^2 \theta T_{\phi \phi} - \frac{B}{4\pi \sin \theta} \frac{\partial A}{\partial \mu} \right)
$$

where the turbulent stresses take into account the turbulent viscosity and generation of the large-scale shear due to the $\Lambda$-effect (Kitchatinov & Rüdiger 1999):

$$
T_{\phi \phi} = \nu_\nu \left( \frac{\alpha_{MLT}}{\gamma} \right)^2 \left( V^{(0)} - \sin^2 \theta V^{(1)} \right),
$$

$$
T_{\psi \phi} = \nu_\nu \sin^2 \theta \left( \frac{\alpha_{MLT}}{\gamma} \right)^2 \left( H^{(0)} + \sin^2 \theta H^{(1)} \right),
$$

where $\nu_\nu = \frac{12}{15} \eta^{(0)}$. The viscosity functions $-\Psi_\phi, \Phi_\perp$ and the $\Lambda$-effect $V^{(0,1)}$ and $H^{(0,1)}$, are dependent on the Coriolis number and the strength of the large-scale magnetic field. They also depend on the anisotropy of the convective flows. In following to Kitchatinov (2004) and Pipin (2004) we employ the following expressions:

$$
\Phi_\perp (\Omega^*, \beta) = \Phi_\perp (\Omega^*) \left( \Phi_\perp - (1 - \Phi_\perp) \phi_{\perp V} (\beta) \right),
$$

$$
\Phi_\parallel (\Omega^*, \beta) = \Phi_\parallel (\Omega^*) \left( \Phi_\parallel - (1 - \Phi_\parallel) \phi_{\perp V} (\beta) \right),
$$

$$
V^{(0)} = \left( J_0 (\Omega^*) + J_1 (\Omega^*) + a (J_0 (\Omega^*) + J_1 (\Omega^*)) \right) \times \left( \Phi_\parallel K_1 (\beta) + (1 - \Phi_\parallel) \phi_1 (\beta) \right),
$$

$$
V^{(1)} = \left( J_0 (\Omega^*) + a J_1 (\Omega^*) \right) \times \left( \Phi_\parallel K_1 (\beta) + (1 - \Phi_\parallel) \phi_1 (\beta) \right),
$$

$$
H^{(0)} = \left( J_0 (\Omega^*) \phi_0 (\beta) \right),
$$

$$
H^{(1)} = -V^{(1)},
$$

where $\Phi_q = \frac{\arctan \Omega^*}{\Omega^*}$, the $\phi_{\parallel, \perp}$ and $\psi_1$ can be found in (Kichatinov et al. 1994), the $J_{0,1}$ and $I_{0,1}$ in (Kichatinov 2004), the $K_1$ in (Kichatinov et al. 1994). The $\phi_{\perp, V}$ follows from the general expression for the $\Lambda$-effect and the turbulent viscosity for the fast rotating regime ($\Omega^* > 1$) given by Kueker et al. (1996) and by Pipin (2004). They are defined as follows:

$$
\phi_{\perp V} = \frac{4}{\beta^2 \sqrt{(1 + \beta^2)^3}} \left( \beta^4 + 19 \beta^2 + 18 \right) \sqrt{(1 + \beta^2)} - 8 \beta^4 - 28 \beta^2 - 18, \tag{22}
$$

$$
\phi V = \frac{2}{\beta^2} \left( 1 - \frac{1}{\sqrt{(1 + \beta^2)}} \right), \tag{23}
$$

$$
\phi_H = \frac{4}{\beta^2} \left( \frac{2 + 3 \beta^2}{\sqrt{(1 + \beta^2)^3}} - 1 \right). \tag{24}
$$

The given expression take into account the different regime of the magnetic $\Lambda$ and viscosity quenching in the cases of the slow ($\Omega^* < 1$) and the fast rotation ($\Omega^* > 1$). In particular the case $\Omega^* < 1$ the turbulent viscosity is quenched as $\beta^{-\gamma}$ when $\beta > 1$, and for the case $\Omega^* > 1$ there is a stronger quenching, $\beta^{-\gamma}$. It saturates the anisotropy of the viscosity along and perpendicular rotation axis for the large-scale toroidal magnetic field (Pipin 2004).

We skip the effects of the meridional circulation from the dynamo models and from the angular momentum balance. Currently the theoretical predictions (see Kitchatinov 2013) contradict to the helioseismological findings (see Zhao et al. 2013) and to direct numerical simulations which do not well reproduce the angular velocity profile given by Howe et al. 2011.

Thus the theoretical information about the stresses $T_{\phi \phi}$, $T_{\psi \phi}$ is not well known. We shall follow the practical approach suggested earlier by Malkus & Proctor (1975). It is assumed that in the absence of the large-scale magnetic activity the (13) is independent in time and describe the angular velocity profile by Howe et al. (2011). Then, we find the equation for the torsional oscillations, which appear due to the cyclic magnetic activity:

$$
\sin \theta \frac{\partial \Omega}{\partial t} = \frac{1}{\rho r^4} \frac{\partial}{\partial r} \left( T_{\phi \phi} (\Omega^*) + \delta \Omega, \Omega^*, \beta \right)
$$

$$
- T_{\phi \phi} (\Omega^*, \Omega^*, \beta = 0) - \frac{B}{4\pi r^2} \frac{\partial A}{\partial \mu} + \frac{1}{\rho r^2} \sin \theta \frac{\partial}{\partial \mu} \left( T_{\psi \phi} (\Omega^* + \delta \Omega, \Omega^*, \beta) - T_{\psi \phi} (\Omega^*, \Omega^*, \beta = 0) - \frac{B}{4\pi r^2} \sin \theta \frac{\partial A}{\partial \mu} \right),
$$

where $\Omega^*$ the angular velocity profile given by helioseismology, $T_{\phi (\psi \phi)} (\Omega^*, \Omega^*, \beta = 0)$ are the turbulent stresses ($14$) in the stationary state. The same is applied for the star rotating with the difference period than the Sun. We assume that the sum of the radial turbulent and the Maxwell stresses is zero at the boundaries. This guarantees the conservation of the angular momentum in the model.

### 3 RESULTS

The study employs the same set of the basic parameters as in our previous paper (see, e.g. Pipin et al. 2013, Pipin & Kosovichev 2014). The $C_\alpha = 0.04$ produces the $\alpha$ effect with magnitude about 1 m/s for the solar case. The given $C_\alpha$ is about 30% larger than the dynamo threshold.
value. The others parameters are as follows: $C_3 = C_2/3$ and $C_4 = 0.06$. For the given choice of the parameters the total magnetic diffusivity reaches values of about $10^{12}\text{cm}^2/\text{s}$ in the layer $0.05 - 0.95R_\odot$, which are consistent with the findings from observations (Abramenko et al. 2011). The given ratio between the $\alpha$ and the $\Omega \times J$ effect is supported by the direct numerical simulations (Käpylä et al. 2008). For the BL-type model we put $C_8 = 4$ which is supercritical for the given $C_2 = 0.06$ and $C_6 = 4$. To make the model closer to that discussed by Kitchatinov & Olemskoy (2011) we put $\gamma_{ij}^{(\lambda \phi),H} = 0$ and $C_8 = 0$ in the BL-type models. The model employs the pseudospectral approach for the numerical integration in latitude and the finite second-order differences in radius. The initial field was a weak dipole type poloidal magnetic field. The given numerical scheme preserve the parity of the initial field unless there are a real parity braking process, e.g., associated with the non-symmetric about equator fluctuations of the dynamo parameters. We don’t explore those effects in the paper.

Parameters of the models and some results are listed in the Table 1.

Figure 1 illustrate the time-latitude diagrams for the radial magnetic field at the surface and the near-surface toroidal magnetic field for the model D0, when the dynamo saturates only due the nonlinear $\alpha$-effect, for period of rotation 25 and 15 days.

Figure 2 shows the same as Figure 1 for the model BL0. The butterfly diagrams of the model BL0 for the period of rotation 25 days is rather similar to those by Kitchatinov & Olemskoy (2011). We bring the similarity between our results and the results of the cited paper by the increase of the diamagnetic pumping (see the table 1) and switching off some of the turbulent effects, which are included in the distributed dynamo model.

The Figures 1-2 illustrate the principle differences between the models with the «local» and non-local $\alpha$-effect. The first is the decrease of the dynamo period in the model D-type when the period of rotation decreases (the amplitude of the dynamo wave grows in the same time). While the opposite is for the BL-type models. The second feature is that the radial propagation of the dynamo wave is outward for the D-type models and it is opposite for the BL-type models. It seems that the downward propagation of the dynamo wave is a typical feature of the Babcock-Leighton types dynamo models (see, e.g., Dikpati & Charbonneau 1999). The meridional circulation increase this effect further. In fact, in this types of models the dynamo wave propagates to the region of the fast rotation (at the latitudes higher than $45^\circ$) and smaller turbulent diffusivity. This is likely results to increase of the dynamo period when the amplitude of the dynamo grows (see the Table 1).

The nonlinear effects due to magnetic buoyancy, magnetic helicity conservation and the $\alpha$- quenching results to saturation both the D-type and BL-type dynamos. Contrary to the claims by Kitchatinov & Olemskoy (2011) magnetic helicity conservation reduce the amplitude of the BL-types dynamo. Recently, Brandenburg et al. (2014) re-considered this effect for the two different kinds of the dynamical quenching. Here we confirm their results for the 2D spherical dynamo models. The magnetic buoyancy results to considerable saturation of the BL-type dynamo, as well. This effect is often neglected for this type of the dynamo model. However

the concept of the non-local $\alpha$- effect is based on the consideration of the buoyant magnetic flux tube which is rising through the convection zone and it is turned by the Coriolis force. Mathematically, the part of the poloidal flux which is produced by the Coriolis force is given by the expression of the mean electromotive force in the Eq. (5). The rest of the toroidal flux should be either considered as the flux escaping the convection zone (take into account the magnetic buoyancy effect) or as the flux returning in the dynamo region.

Currently the models like those in the paper by Dikpati & Charbonneau (1999) or Kitchatinov & Olemskoy (2011) consider the second situation. This seems unrealistic. Results in the Table 1 (see the BL1) shows that the magnetic buoyancy saturates the magnetic flux produced by the dynamo by factor 2 (periods around 25 days) to factor 4 for the period of rotation 15 days. The magnetic buoyancy is less essential for the D-type models (Kitchatinov & Pipin 1993). Here we did not present the results of the separate runs for the case when the magnetic buoyancy saturates the D-type dynamo without effect of magnetic helicity conservation and the $\alpha$-quenching. For this case the results are close to the model D2.

Saturation of the dynamo by the magnetic helicity conservation make a little change to the structure of the butterfly diagram of the BL-type dynamo shown in Figure 2. The changes in the D-type models are considerable. They are illustrated in the Figure 3. We find that the polar branch of the toroidal magnetic field becomes more and more pronounced when the period of the rotation is decreased. Also the character of the polar reversals is eventually changed from the smooth and quite monotonic kind as in the case of the 25 day period to the surge like pattern when the period of rotation is 14.5 days.

The further saturation of the dynamo process can be seen when we take into account the back reaction of the large-scale magnetic field on the angular momentum transport. Here we found the different effect of the dynamo on the angular velocity distributions inside the convection zone for the D-type and BL-types dynamo models. In the second case the torsional oscillations are concentrated to the bottom of the dynamo domain. Note that the situation can be different for the BL-type model with regards for the meridional circulation and the magnetic effect to the heat transport (see, Rempel 2006).

Figure 4 shows the snapshots of the torsional oscillations and the mean states of the angular velocity distributions for the set of the periods: 25.3 and 14.5 days for the D2 and BL3 models. The D2 and BL3 type of the dynamo models show the strongest deviation of the rotation profiles from the unperturbed state. The magnetic helicity conservation saturates the amplitude of the torsional oscillations damping the magnitude of the dynamo generated magnetic fields. For the D2 and D3 type models the distribution of the angular velocity deviates strongly from the radial-like profiles for the star rotating with the period of 14.5 days (see, e.g., Figure 4 (top,left)). In the model D2 and D3 the rotation profiles in the mid part of the convection zone becomes more and more cylinder-like when the period of the rotation decreases. Also, we find the smaller rotation period - the smaller latitudinal shear (compare to the unperturbed state). This effect is not profound for the BL-type dynamos. Figure 5 summarizes our findings about modulation of the
surface differential rotation and the magnitude of the torsional oscillations in the dynamo modes D2, D3, BL3 and BL4.

We find that in the D-type models the magnetic activity can reduce the latitudinal shear at the surface up to 10% of the $\Delta \Omega_\odot$ in compare to the kinematic case in the model D3 and it is about 5% for the fully nonlinear dynamo D4. For the BL-type models the mean deviation of the latitudinal shear can reach magnitude of 5% of the $\Delta \Omega_\odot$ in the case BL3. The impact of the fully nonlinear BL-type dynamo on the latitudinal shear is rather small. It is less than 1% of the $\Delta \Omega_\odot$. The magnitude of the torsional oscillations behave similarly. The maximum of the torsional oscillations magnitude is found for the model D2 (about 70 m/s) for the period of rotation 14.5 days.

We find that for the fast rotation case the nonlinear dynamos of D-type can show the multiply periods. The regime of the multiply dynamo periods is not well developed for the given choice of the dynamo parameters $(C_0, C_T$ and $R_m$) and for the investigated interval of the rotational periods. Figure 6 illustrates variations of the total magnetic flux generated by the dynamo in the model D3 for the rotational periods 25.3 and 14.5 days and for the models D1 and D2 for the rotational period 14.5 days. For the period of rotation 14.5 days the D1 model for the shows the «long-term» variations with period about 5-6 of the basic periods. The long-term variations saturates by the nonlinear $\Lambda$-quenching effects and the large-scale Lorentz forces and the model D3 (with the rotational period 14.5 days) seems at the threshold of the nonlinear regime with the «long-term» variations.

Figure 7 illustrates our findings about dependence of the magnetic cycles parameters on the period of rotation of the star. In the given interval of the rotational periods the magnetic flux generated by the dynamo grows for the D-type models. The rate of the growth is reduced in the fully nonlinear model in compare with the models that exclude the magnetic helicity conservation. Together with the magnetic buoyancy, the latter factor seems to be the most important nonlinear effect for the dynamo saturation. The dynamo generated magnetic flux grows, when the star is rotating faster, for the BL0 type dynamo model, as well. The BL/(1-4) types of the dynamo models shows the sign of saturation when the period of rotation reaches 15 days.

The D-type models shows the decrease of the dynamo period with the decrease of the rotational period of the star. The period of magnetic cycles changes from about 11 years for the solar case to the value which is about 3 year for the star rotating with the period about 15 days. The opposite tendency is found for the BL-types dynamo models, where the dynamo period increases from 11 to 17 years with the decrease of the rotational period.
**Figure 2.** The same as Figure 1 for the model BL0. The toroidal field in the time-latitude diagram is taken from the bottom of the convection zone.

**Figure 3.** The time-latitude diagrams for the radial magnetic field (contours) at the surface and the near-surface toroidal magnetic field for the model D1 for the set of the periods of rotation 25.3, 20.3, 16.9 and 14.5 days clockwise starting from the upper left.

**Figure 4.** Top line: left panel shows the isolines of the mean distribution of the angular velocity in the star \( \Omega(r, \theta)/\Omega_\star \) for the periods of rotation 25.3 (blue color) and 14.5 days (red color), black colors show the unperturbed distribution; middle panel shows snapshots of the magnetic field distribution (contours) and the torsional oscillation (color image) for the model D2 with the period of rotation 25.3 days; the right panel shows the same as the middle panel for the period of rotation 14.5 days. The bottom line shows the same as the top line for the model BL3.
4 DISCUSSION AND CONCLUSIONS

In the paper we have studied the nonlinear effects in the large-scale magnetic solar-type dynamo on the parameters of the dynamo cycles for the range of the rotation periods from 14.5 to 30 days. This a tentative study. The dynamo model lacks the self-consistent description of the angular momentum in the convection zone of a star. We also exclude the evolutionary changes of the stellar convection zone structure, which could occur together with the angular momentum loss. The model also lacks the effects of the meridional circulation to the dynamo action, which is found in the direct numerical simulations (Miesch et al. 2011; Guererro et al. 2013a; Käpylä et al. 2014) and it is extensively used in the advection BL-type solar dynamo models (Brun et al. 2014). Also, the self-consistent kinematic model of the angular momentum balance inside the convection zone is not possible with out regards for effects of the meridional circulation. The current situation with the meridional circulation is rather uncertain. Observations suggest the case of the multiply circulation cells (Zhao et al. 2013) and the mean-field theory insists on the one-cell case (Kitchatinov 2013). By this reason we disregard the effects of the meridional circulation in our study. Also, it is helpful to see the dependence of the dynamo period vs the period of rotation in the BL-type dynamo model without the meridional circulation. Bear in mind the inconsistent character of the model.

The top line show the dependence of the total magnetic flux generated by the dynamo versus the period of rotation. The bottom line shows the variations of the dynamo period. The filled symbols shows results for the models D1 and D3 with the multiply periods. In those cases, the results for the sum of the principal periods are shown as well.

In the given sample of the dynamo model the Rossby number varies according to increase of the rotation rate because the other parameters of the convection zone are fixed.

We find that the D-type dynamo models (the dynamo distributed over the convection zone) satisfactory explains the dependence of the dynamo magnitude and the dynamo periods on the periods of rotation of the star (see, e.g., Soon et al. 1994; Saar 2011). The BL-type models fails to explain the latter fact. In mean-field models of the stellar dynamo it was seen earlier by Jouve et al. (2010) and Karak et al. (2014). Here we confirm this effect on the BL-type model without meridional circulation. Note, that the proportional growth of the dynamo period and the dynamo magnitude was reported earlier by Ruediger & Brandenburg (1995). In their case the dynamo wave propagate inward to the bottom of the overshoot layer, see Fig.4 (right) in Ruediger & Brandenburg (1995). The similar is found in our study for the completely different model design. This proves the generic character of this effect for this type of the large-scale dynamo. Figure 2 shows that in the BL-type models the dynamo wave propagates to the bottom of the convection zone. Which is promoted by the diamagnetic pumping. The effect can be amplified in the model with the meridional circulation. The concentration of the dynamo wave to the bottom of the convection zone, where the magnetic diffusivity is low, is amplified for the fast rotating star. This enlarges the am-

Figure 7. The top line show the dependence of the total magnetic flux generated by the dynamo versus the period of rotation. The bottom line shows the variations of the dynamo period. The filled symbols shows results for the models D1 and D3 with the multiply periods. In those cases, the results for the sum of the principal periods are shown as well.
Dependence of magnetic cycle parameters on period

Figure 5. Top panel shows the deviation of the surface differential rotation from the case of the kinematic dynamo, bottom panel shows the magnitude of the torsional oscillations at the 30\degree latitude.

Figure 6. Variation of the total flux in D-type models

...plitude of the dynamo and the dynamo period as well. The certain set of the multiply meridional circulation cells can help to solve this issue (Jouve et al. 2010).

The D-type models have the boundary conditions which produce the non-zero large-scale toroidal field at the top of the convection zone. This allows us to compute the dynamo generated Poynting flux from the surface. It is found that (see the last column of the Table 1), that the flux reaches the magnitude 10^{-3} F_\odot, where F_\odot = \frac{L_\odot}{4\pi R_\odot^3}. It seems that the flux is around to saturation level. The result is in the agreement with the observational findings (Katsova & Livshits 2006). However the results for the total magnetic flux generated by the dynamo do not show the saturation at all. Thus we can conclude that the dynamo generated Poynting flux grows as \( B_{\perp}^{2-1} \) where the \( \delta < 1 \) and \( B_{\perp} \) is the near surface toroidal magnetic field strength. This was earlier predicted by Kleeorin et al. (1995) (cf., to Blackman & Thomas 2015) for the dynamo models with magnetic helicity conservation as the principal dynamo non-linearity. It is confirmed by VIlotto et al. (2014) who found \( \delta \approx 1.61 \). We postpone a more detailed comparison of our results with observations for a future.

Surprisingly, the BL-types models shows the sign of saturation of the dynamo at the period of rotation about 15 days. This is not found in observations where the dynamo start to saturate for the period of rotation less than 10 days. We find that the nonlinear saturation goes mostly because of the magnetic helicity conservation. Here we confirm the results about saturation of the BL-types dynamo which were reported earlier by Brandenburg & Käpylä (2007) and Brandenburg et al. (2014). The magnetic helicity conservation quenches the D-type dynamos as well. However in the given range of the rotational periods the saturation level is not reached yet.

The large-scale Lorentz force and the A- quenching reduce the surface differential rotation and produce the torsional oscillations. The effects are stronger in the D-type dynamo models than in the BL-type models. This seems due the distributed character of the dynamo generated effects on the angular momentum balance in the dynamo operating in the whole convection zone. It is found that correction of the magnetic feedback to the surface differential rotation can be about 10 % of the solar value. Thus in the given range of the rotational period the magnetic saturation of the differential rotation by the dynamo (see, e.g., Saar 2011) is not considerable. Following to hints from observations it could be found for the higher rotation rates.

Analysis of observations of stellar magnetic activity (see, e.g., Soon et al. 1993, Baliunas et al. 1995, Saar & Brandenburg 1999, Böhm-Vitense 2007) reports the multiply branch of the dynamo activity for the cool stars. The set of restrictions which was assumed in our study does not allow to discuss the issue in details. However we can use our sample to study if those branches results from the different nonlinear dynamo regimes for the D-types models. The models D1(saturation due to magnetic helicity) and an D3(the fully nonlinear regime) shows the multiply periods for the period of rotation about 15 days and produce the second branch in the period-period (hereafter PP) dependence. The active branch is more profound for the model D1 which is less saturated than the D3. The results on the Figure 7 agree qualitatively with those from the papers by Saar & Brandenburg (1999) (Fig.2(left)) and by Böhm-Vitense (2007) (Fig.1). In the study we don’t take into account the breaking of the equatorial symmetry of the large-scale magnetic field. This effect could contribute to the active branch on the PP diagram as well. Another interesting feature of the PP dependence which is illustrated by the Figure 7 is that the models with the higher saturation level (e.g. D3 cf. D0) are above the models which are less saturated by the nonlinear feedback of the magnetic activity on the dynamo. This put the question about the origin of the active and quite branches in the stellar activity. We can guess that it has the stochastic origin, i.e., it results from fluctuations of the dynamo parameters. The difference between two distinct branches of magnetic activity is decreased with the increase of the rotation rate. This means that the nonlinear effects,
e.g. those which have studied in the paper saturates the effects of fluctuations.

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