A new algorithm of solving harmonic balance equations

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Abstract. The harmonic balance method is widely used for simulation of nonlinear radio circuits in electronic CAD systems. The main problems of algorithms and programs based on HB methods are significant requirements for computer memory and huge computational costs when simulate a complex circuits containing thousands of electronic components and hundreds of thousands of model equations. Methods of model order reduction have become popular in recent years and can significantly reduce the dimension and memory required for electronic circuit models for dynamic mode analysis. The main problems of low-order methods for simulation of electronic circuits are associated with very small reductions in computational costs (with a significant reduction in the dimension of the equations and the required memory for the model). A new method and algorithm of solving the equations of harmonic balance method used in electronic CAD systems is presented. The new algorithm is based on applying the ideas of model order reduction methods to harmonic balance equations. A new method replaces the vector (matrix) of equation unknowns by two matrices of small dimensions that are solved by iteratively (as into the standard harmonic balance method). The first system of equations reduces the number of harmonics in balance equations, and the second system of equations reduces the number of circuit nodes. These equations of small dimensions are solved sequentially, so this algorithm allows significantly reduce the size of computer memory for storing of model equations and reduce of computational costs. The simulation program was developed in the Matlab/Simulink system. The results of simulation test circuit with different number of electronic circuit nodes and the number of considered harmonics showed that the gain was obtained only for dimensions of problems more than one hundred (the number of nodes and the number of considered harmonics).

1. Introduction
Harmonic balance (HB) methods are widely used for modeling nonlinear circuits in CAD systems in electronics [1]. The main problems of algorithms and software tools of CAD systems based on HB methods are significant memory requirements and huge computational costs for simulation of complex nonlinear electronic circuits containing thousands of electronic components and hundreds of thousands for circuit model equations [1].

In the article [2] a new method and algorithm for solving the HB equations was proposed based on the use of new idea for model order reduction (MOR). MOR methods have gained popularity in recent years [3-6]. These MOR methods allow to significantly reducing the number of unknowns and the size of memory for storing of electronic circuits model equations for dynamic mode analysis. The main problems of MOR methods for modeling electronic circuits are associated with very small reductions in computational costs (while reducing the dimensions of the equations and required memory for the
model). In the paper [2] we used the simple iteration algorithm. Here we propose the new equations and the algorithm for solving the balance equations using the Newton method.

The basic equations and methods for solving HB equations for electronic circuits are well known [1, 2], so here we give only a brief summary of them, which is necessary for the presentation of proposed method. Consider the HB equations in frequency domain as

$$ F(V) = I(V) + YV - I_E = 0. \quad (1) $$

Here $F(V)$, $I(V)$, and $V$ are vectors of dimension $[N \times (2K+1)]$ containing the spectrum at each node of the circuit; $I(V)$ describes nonlinear elements, vector of unknown $V$ is vector of nodal voltages of the circuit, matrix of nodal conductivities $Y$ is block matrix, $I_E$ is vector of input sources, $N$ is the number of nodes in the circuit, $K$ is the number of considered harmonics.

The solution of harmonic balance equations (1) in the frequency domain is most often performed by the Newton iterative method [1]

$$ J(V^j) \Delta V^{j+1} = -F(V^j), $$

where $J(V^j) = \frac{\partial F}{\partial V} |_{V^j}$ is a Jacobian; $j$ is iteration number.

In HB method the dimension of Jacoby matrix ($[(2K+1) \times N] \times [(2K+1) \times N]$) which for complex malleable components circuits becomes too large and needs excessive computer memory and computational costs.

The idea of the method proposed in [2] is to replace the vector of variables (unknown) $V$ of the HB equations in (1) to the new two matrices of reduced dimensions

$$ V = V_H \times V_N, \quad (2) $$

where the matrix $V_H$ reduces the number of harmonics and has dimension $[N \times R]$, the matrix $V_N$ reduces the number of circuit nodes and has dimension $[R \times (2K+1)]$. Here the $R$ is reduced dimensions of equations, where $R << N, R << (2K+1)$.

On the figure 1 the transformation of vector $V$ into two new matrices is illustrated. In this figure, the vector $V$ (with the dimension $[N \times (2K+1)]$) is represented as a matrix of the same dimension, for clarity [2].

![Figure 1](image)

Figure 1. Replacing the matrix $V$ with two new reduced-dimension matrices.

The reduction in memory and computational costs in the new method will be determinate by the smaller dimension of the new matrices and the fact that they are used sequentially.
2. Basic equations
To get the balance equations of reduced dimension, multiply equations (1) by the matrix \( V_N^T \), which reduces the dimension of equations to the value \([N \times R]\), we have reduced the number of harmonics taken into account in equations [2]

\[
F(V) = I(V) \cdot V_N^T + YV \cdot V_N^T - I_E \cdot V_N^T = 0 .
\] (3)

Then, replace the matrix \( V \) in (3) using the previously entered ratio (2)

\[
F(V_H V_N) = I(V_H V_N) \cdot V_N^T + YV_H V_N V_N^T - I_E \cdot V_N^T = 0.
\] (4)

Using the orthogonalize procedure to matrix \( V_N \), then

\[
F(V_N) = I(V_H V_N) \cdot V_N^T + YV_N V_N V_N^T - I_E \cdot V_N^T = 0 .
\] (5)

The dimension of (5) will be \([R \times (2K+1)]\), and we reduce the number of circuit nodes. Using (2) we can rewrite (5) as

\[
F(V_H V_N) = I(V_H V_N) \cdot V_H^T + YV_H V_H V_H^T - I_E \cdot V_H^T = 0
\] or (if \( V_H V_H^T = I \))

\[
F(V_N) = I(V_H V_N) \cdot V_H^T + YV_N - I_E \cdot V_H^T = 0 .
\] (6)

Equations (6) is balance equations [2] for finding \( V_N \) with known matrix of \( V_H \).

Let's use Newton's iterative method to solve the balance equations (4) and (6). For (4) we obtain a system of linear algebraic equations (SLAE) of the form

\[
J(V_H^j) \cdot \Delta V_H^{j+1} = -F(V_H^j) ,
\] (7)

where \( J(V_H^j) = \frac{\partial F}{\partial V_H} \big|_{V_H^j} \) is Jacobian; \( \Delta V_H^{j+1} = V_H^{j+1} - V_H^j \).

For equations (4), the expression for the Jacobian will look like this

\[
J(V_H^j) = \frac{\partial I(V_H V_N V_N^T)}{\partial V_H^j} + Y .
\] (8)

Usually, in circuit analysis (design) programs, the form of the Newton method equations with respect to the new value \( V_H^{j+1} \) is more convenient instead of (7)

\[
J(V_H^j) \cdot V_H^{j+1} = \frac{\partial I(V_H V_N V_N^T)}{\partial V_H^j} V_H^j - I(V_H V_N V_N^T V_H^j + I_E V_N^T .
\] (9)
Similarly, we obtain relations for the Newton method for solving equations (6). Omitting the almost repeated conclusions (8-9), we give the final relations similar to (9)

\[ J(V_N^j) \cdot V_N^{j+1} = \frac{\partial I(V_H V_N^j) V_N^T}{\partial V_N^j} V_N^j - I(V_H V_N^j) V_H^T + I_E V_H^T. \] (10)

One of the features of harmonic balance type methods is that the basic balance equations are solved in the frequency domain. At the same time, most models of nonlinear elements are described by equations in the time domain. Therefore, at each iteration of Newton method, when solving equations, it is necessary to make transformations from the frequency domain to the time domain and back using the fast Fourier transform (FFT) [1].

To simplify the recording of balance equations using the FFT, a relationship is introduced between the instantaneous values of the function in the time domain and harmonics in the frequency domain in matrix form [1]. Let the samples in the time domain have the form

\[ u(t) = [u(t_0), u(t_1), ..., u(t_n)], \]

where \( n = 2K + 1 \); the coefficients of the Fourier series (amplitudes, spectrum) are

\[ V = [V_0, V_1^c, V_1^n, ..., V_K^n]. \]

Then by entering the direct Fourier transform matrix

\[ T = \begin{bmatrix}
1 \cdot \cos \omega t_0 \cdot \sin \omega t_0 & \cdots & \cos K \omega t_0 \cdot \sin K \omega t_0 \\
1 \cdot \cos \omega t_1 \cdot \sin \omega t_1 & \cdots & \cos K \omega t_1 \cdot \sin K \omega t_1 \\
\vdots & \ddots & \vdots \\
1 \cdot \cos \omega t_{n-1} \cdot \sin \omega t_{n-1} & \cdots & \cos K \omega t_{n-1} \cdot \sin K \omega t_{n-1}
\end{bmatrix}, \]

we obtain a relationship between the representation of the signal in the time and frequency domains as

\[ V = Tu \quad \text{and} \quad u = T^{-1}V, \]

where \( T^{-1} \) is the inverse Fourier transform.

In this case, in equation (10), the nonlinear transformations will look like

\[ \frac{\partial I(V_H V_N^j) V_N^T}{\partial V_N^j} = T \cdot \frac{\partial i(u_H u_N^j)}{\partial u_N^j} T^{-1}. \] (11)

and

\[ I(V_H V_N^j) = T \cdot i(u_H u_N^j) T \] (12)

3. The Algorithm of solving equations and simulation results

The simulation program was developed in the Matlab/Simulink system, which is an open source upgrade of the SMORES program [7].

The algorithm for solving the balance equations will look like this:

**Algorithm 1**

1: \( V_N = \) initial value or 0;  
   \( V_H = \) initial value or 0;  
   \( j = 1. \)

2: Solution of SLAE (9);  
   Solution of SLAE (10).

3: if \( (V_H^{j+1} - V_H^j \leq \varepsilon) \land (V_N^{j+1} - V_N^j \leq \varepsilon) \) stop.
Example from previously published works was taken as test for checking the developed algorithms and program (figure 1 [4], figure 3 [3]). Since the convergence in all the examples coincided with the calculations by the standard HB method, we do not provide graphs of the results.

The table 1 below shows the results of comparing the new method and the standard HB method by calculation time without taking into account the time of entering the circuit topology.

The results of simulations of different electronic circuit nodes and the number of considered harmonics showed that the gain was obtained only for dimensions of problems more than one hundred (the number of nodes and the number of considered harmonics).

**4. Conclusion**

The following conclusions can be drawn from the presented results:

1. The new method and algorithm is suitable for solving large-dimensional harmonic balance equations.
2. Further studies of this algorithm are related to the research on finding the optimal value of $R$.

**Table 1.** Comparison of methods for computation time.

| Calculation options: the number of repeated cascades of circuit, the number of nodes, the number of considered harmonics | New method | The standard HB |
|---|---|---|
| 3, 11, 3 | 5 s. | 4 s. |
| 3, 11, 99 | 9 s. | 9 s. |
| 14, 42, 3 | 20 s. | 22 s. |
| 14, 42, 99 | 119 s. | 135 s. |
| 45, 135, 3 | 176 s. | 206 s. |
| 45, 135, 99 | 267 s. | 342 s. |

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