Mathematical creative thinking and problem posing: an analysis of vocational high school students’ problem posing

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Abstract. Science and technology subjects in schools are intended to let students acquire advanced competencies in science and technology and to cultivate critical and creative scientific thinking. Mathematical creativity is an important indicator of mathematics learning in different parts of the world. The importance of creative thinking is reflected in the last few decades which shows a great deal of research on creative thinking ability. The indicator of creative thinking ability, including fluency, flexibility, and novelty are reflected in learning problem posing. Problem posing scenario is categorized into three types, free problem posing, semi-structured problem posing, and structured problem posing. This study aims to explore students’ mathematical creative thinking ability by analyzing students’ problem posing test. The subjects of this study are vocational high school students of equal age with high school students in Indonesia. In addition, students have been given the material first before getting the problem-posing scenario. The measurement and instrumentation in this study include mathematical problem-posing test, creative thinking ability test, and interview. The differences of problems posed by students will also be analyzed based on indicators of fluency, flexibility, and novelty.

1. Introduction
Currently, the knowledge and technology has developed rapidly, which basically all of them can not be separated from the role of mathematics as the queen of all knowledge. People assume that mathematics is just a science of calculation and only deals with numbers only, and that mathematics is not interpreted as a king of any science that can be implemented in everyday life. Mathematics is considered as the most difficult lesson and even a determinant for the graduation. One of the important aspects in mathematics is creativity.

Mathematical creativity is an important indicator in the learning of mathematics in various parts of the world. Mathematical creativity is the most important aspect needed in the 21st century. It was evidenced from [1] about the key in learning and innovation of schools in Europe, one of which is the need to think creatively about new and useful ideas and work creatively, that is to be open and responsive to new and varied perspectives. In addition, research conducted by [2], [3], [4], and [5] show that in the last 10 years creativity in mathematics becomes an important discussion. Creativity has also become one of the indicators in the curriculum in various countries. National Council of Teacher of Mathematics (NCTM) reveals that “[m]athematical ideas are a triumph of human creativity, and framing mathematics in this way can motivate students to unleash their own creative potential” [4]. The 2013 curriculum in Indonesia also emphasizes that intelligence-based learning will not produce significant results (only 50% improvement) over creativity-based (up to 200%).
The creative thinking ability of the students can be seen in the learning process as well as in the final result. During the learning process, every student has different creative thinking ability. This is stated by Erynnick[3] that there are 3 levels in mathematical creativity: (1) preliminary technical stage (level 0), algorithmic activity (level 1), and (3) creative activity (level 2). At the preliminary level, the technical stage is still on the application and use of mathematical formulas and procedures. Then at the algorithmic activity level, learners have applied basic mathematical techniques, and work algorithmically. While at the creative activity level, learners can make decisions and solve problems which is not algorithm anymore. Meanwhile, a student is said to be creative if he/she meets the indicators of creative thinking ability. Torrance & Ball, Guilford mentions five indicators of creative thinking, including: (1) sensitivity, (2) fluency, (3) flexibility, (4) originality, and (5) elaboration [5]. While [6] says the characteristics of creativity can be measured through fluency, flexibility, and novelty. Further [7] classifies the five hierarchical levels of mathematical creative thinking in proposing and resolving problems: level 4 (very creative), level 3 (creative), level 2 (little creative), level 1 (almost non-creative), and level 0 (not creative).

Many things affect the creativity of student in mathematics. Mathematical creativity is influenced by how students solve mathematics problems. This is in line with the statement by Van Harpen & Sriraman [4] that "[i]n the literature, problem-posing abilities are reported to be an important aspect/indicator of creativity in mathematics". In line with that opinion, [6], [8], and [9] also revealed that effective problem posing learning is viewed from students’ creative thinking ability. The main step in learning problem posing is the students ask the problem. This is stated by [6] that the main thing in problem posing is the creation of new problems from the student's situation or experience. This indicates that an important step in learning is the process of posing a problem.

Problem posing is also categorized into three, including Free Problem Posing, Semi-Structured Problem Posing, and Structured Problem Posing [10]. These three types of problem posing will be analyzed based on indicators of students’ creative math ability. Problem posing stimulate students to create or ask their own questions in accordance with the problems faced daily related to the material being learned to be solved by the students themselves without a need to be answered directly by the teacher. The process of posing this question requires a high degree of mathematical creativity, in addition to understanding the material as well as creativity in posing problems.

This research focuses on vocational school students because the curriculum and learning process in vocational schools are different from regular high school. In Indonesia, one of the goals of vocational secondary education according to Law Number 20 Year 2003 is to develop the potential of learners to become citizens with noble character, healthy, knowledgeable, capable, creative, independent, democratic and responsible. Not only in Indonesia, the differences in curriculum in vocational schools and other schools are also expressed by Park [11], that is "[s]econdary vocational high schools in South Korea are facing imperative change because their programs need to be more flexible in response to the demands of changing economic and work-related structures. "It means that flexible thinking reflected in creative thinking skills is needed in vocational schools in Korea. On the basis of some of these thoughts then it became an initiative for the writers to examine the process of mathematical creative thinking of vocational students on problem posing learning.

2. Experimental Method

2.1. Subject

This research was conducted in Vocational High School (SMK) with age range of 15-17 years old. The study was conducted in one class in one vocational school with 31 students. Prior to learning with problem posing scenario, students first have obtained and reviewed the material being taught. This study proposed to focus only on the process of posing problems by students. After that, an interview was conducted with a representative of student chosen randomly at each level of creative thinking abilities.
2.2. Measurement
The measurement of this study was done by creative thinking test, problem posing test, and interview. Creative thinking test was done using 4 description questions based on indicators of creative thinking fluency, flexibility, and novelty. The test was used to measure and define descriptions of students' creative thinking ability. This test was done before and after learning problem posing. It is intended to discover the improvement of students' creative thinking ability.

There are three kinds of problem posing tests in this research, Free Problem Posing, Semi-Structured Problem Posing, and Structured Problem Posing. It is intended that students do not directly pose a problem because students are not accustomed to pose a problem as the main scenario in learning, so levels are needed in the problem posing tests. The test is a description and it is not only used to pose, but also to solve the problems.

Here is one of the problems posing scenarios applied in the study.

Task 1: Free problem posing
Make daily life issues related to arithmetic sequences!

Task 2: Semi-structured problem posing
Make math problems related to arithmetic sequences where two tribes are known!

Task 3: Structured problem posing
Given the sequence: -5, ..., ..., ...

Make a problem about the arithmetic sequence through the above situation.

The interview was done using 4 description questions based interviews which were conducted to obtain the power of data analysis of creative thinking skills reflected in problem posing scenarios. Each of the 3 students from the preliminary technical stage (level 0) category, algorithmic activity (level 1), and creative activity (level 2) would be interviewed. It aims to obtain more information about the process of posing a problem at different levels of creativity.

2.3. Data Analysis Technique
Data were analyzed using descriptive analysis to be classified into several levels of creative thinking. The data were also analyzed to determine the improvement of students' creative thinking ability after problem posing learning. The analysis was also done by categorizing the answer to the problem-posing test in raising the problem based on the criteria of creative thinking. Each response from students was grouped into three levels of problem posing test. After that, each level of the test was analyzed how deeply its creativity is according to fluency, flexibility, and novelty indicator. For example, on a free problem posing test, how is the score for its fluency indicator? As well as for other indicators. The results and interview data were typed and then compiled into a transcription. This interview data transcription was later used as data triangulation to reinforce the result of creative thinking analysis of students in problem posing scenario.

3. Result and Discussion
The data of students’ creative thinking ability consists of students' initial creative thinking ability and creative thinking ability after learning problem posing. The data of creative thinking ability of students was then converted into categories of highly creative, creative, creative enough, less creative, and not creative. Frequency distribution and percentage score of creative thinking ability before and after learning problem posing are presented in Table 1.

Table 1. Frequency Distribution and Percentage of Creative Thinking Ability in Problem Posing Learning.
Based on Table 1, it can be seen that the level of students’ creative thinking ability increases after given the problem posing learning. The data of creative thinking ability of students is also presented based on average, maximum score, and minimum score in Table 2 below.

**Table 2.** The Average, Minimum Score and Criteria of Creative Thinking Ability Before and After Learning Problem Posing.

| Description               | Pre-test | Post-test |
|---------------------------|----------|-----------|
| Number of Respondents     | 31       | 31        |
| Average                   | 45.21    | 74.79     |
| Standard Deviation        | 12.95    | 11.91     |
| Ideal Minimum Score       | 0        | 0         |
| Ideal Maximum Score       | 100      | 100       |
| Minimum Score Achieved    | 26.67    | 50        |
| Maximum Score Achieved    | 73.33    | 93.33     |
| Category                  | Not creative | Creative |

Table 2 shows that there is an increase in the average score of students' creative thinking ability. The aspects exist in the ability to think creatively are fluency, flexibility, and novelty. The result of creative thinking ability in each aspect is presented in Table 3.

**Table 3.** The Average and Maximum Score Ideal Before and After Learning Problem Posing in terms of aspects of Creative Thinking Ability.

| Creative Thinking Aspects | Ideal Maximum Score | Pre-test | Post-test |
|---------------------------|---------------------|----------|-----------|
| Fluency                   | 40                  | 21.08    | 30.32     |
| Flexibility               | 40                  | 16.88    | 29.78     |
| Novelty                   | 20                  | 5.48     | 13.87     |
| **Total**                 | 100                 | **43.44**| **73.98** |

Based on Table 3, all aspects of students' creative thinking ability increased after problem posing learning. Nevertheless, the flexibility aspect is still not optimal. Based on previous explanation, it can be concluded that students’ creative thinking ability increases after problem posing learning, either from fluency aspect, flexibility, and novelty. This is supported by the test results of the difference in the average paired sample pre-test and post-test score.

**Table 4.** The Average Difference Test Results of Two Paired Samples.

| t Value  | Sig. Value. | Decision |
|----------|-------------|----------|
| -19.089  | 0.000       | Ho is rejected |

Based on SPSS output results in Table 4, it can be concluded that there is a significant difference of creative thinking ability between before and after problem posing learning. By looking at their mean, then it can be said that creative thinking ability after problem posing learning is better than before problem posing learning.

### 3.1. Result of Problem Posing Test Answers

The analysis of students' answers to each problem posing learning steps below.
Students perform free problem posing after analyzing the given problem in the reviewing the material activity. Based on Figure 1, it can be concluded that students’ creative thinking ability is not so optimal because the statement of the question is not clear.

Figure 2(a) shows that the student is more creative in posing problems. It also relates well to students in giving ideas so that it becomes a good question. It’s just that in Figure 2(b) the students still have not make sure that the problem is an example of a problem about geometry sequences.
Figure 3 shows an increase in creative thinking ability. It is seen from the fluency and flexibility of students in solving problems. In addition, students' creative thinking also can be seen from illustrative drawings created to illustrate the problems to be solved.

The result of mathematical problem posing test are also presented for analyses of fluency, flexibility, and novelty.

There are some viable and nonviable problems posed by the students. It was determined by the researcher. Problem posing scenario was done in five meetings, so the average percentage of nonviable problems generated by the students in each meeting was calculated with the following division:

\[
\frac{\text{number of nonviable problems}}{\text{number of nonviable problems} + \text{number of viable problems}} \times 100\%
\]

It was found that 38.7% of first meeting’s problems, 19.35% of second meeting’s problems, and 6.45 % of third meeting’s problems were nonviable problems. It can be seen by the problems that very lack in information.

After nonviable problems were eliminated, all the viable problems were analyzed for their triviality. It can be seen by the problems posed by students are at a low or high level for vocational high school. It was found that 29.03% of first meeting’s problems, 16.12% of second meeting’s problems, and 6.45% of fourth meeting’s problems were trivial problems.

The problems posed by students in every meeting are various. There are some students who posed the problems in different ways. There is diagram, picture and table that makes the problems are clearer to understand. It can be found that 38.7% problems in first meeting, 58.06% problems in second meeting, 61.3% problems in third meeting, 70.96% problems in fourth meeting, and 74.19% problems in last meeting are packaged in different ways. It can be concluded that students’ flexibility increase in every meeting.

The novelty can be seen in every meeting. As the first researcher planning, a problem was designated as not novelty if it was posed by 10% or more students. In reality, there was found that some problems are new and not mentioned before in textbook or teacher’s example, but it was more than 10% make that the problem with that criteria. So, in this research, the novelty problem was
designated if it was new and not mentioned before in textbook or teacher’s example. It can be found that 9.68% problems in first meeting, 16.13% problems in second meeting, 19.35% problems in third meeting, 35.48% problems in fourth meeting, and 48.39% problems in last meeting are novelty.

3.2. Interview Result

The result of the interview analysis is used to find out the interest and the difficulties of the students in posing the problem. One student who has a high level of creative thinking ability is asked the question "What makes you interested in asking questions? Why do you think this is creative? ". The student's answer is "I feel that by asking questions independently and I can explore new problems related to the majors I am working on." This shows that the tendency of vocational school students to ask questions according to their majors. The question posed by majority of students is a matter related to the building, which is their major in vocational high school. This is evident from some of the questions students have drawn up as seen in Figure 4 below.

![Figure 4. Students’ Problem Posing Based on Their Major.](image)

Another finding is that a student with low creative thinking ability is asked the question "Why do you find it difficult to ask questions and what causes them?". The answers from students are because they are not used to asking questions and they often just solve problems.

Based on the result of the data analysis, it can be said that mathematics learning with problem posing stimulates students to make or ask their own questions in accordance with the problems faced daily related to the material being studied to be solved alone by the students without needed to be answered directly by the teacher. The process of requesting this question requires a high degree of mathematical creativity, in addition to understanding the material as well as creativity in composing questions. This is what causes the improvement of students' creative thinking ability through learning problem posing. These findings are in line with the results of the studies which revealed that posing problems can improve students' creative thinking ability [3][6][8][9].

Based on creative thinking indicators (fluency, flexibility, and novelty), it can be said that in every meeting, these indicators increased. Students' novelty also increased although the increasing was not high. It means that problem posing gave positive impact to creative thinking ability by increasing the creative thinking indicators. Problem posing activity demands the students to create their own ideas in posing the problems. This was in line with the opinion said that creativity has the same characteristic with problem posing [8].

The test of creative thinking ability in this study is more to the problems that require students to think differently. In fact, creative thinking can not only be measured by using an open question only. However, learning of problem posing can improve divergent thinking ability in this research. This is in
line with the opinion by Haylock who said that the problem posing can improve the ability to think divergent [12].

4. Conclusion
Based on the result of the research and discussion, it can be concluded that the teachers need to give an attention to the students in posing mathematics problems. Problem posing learning should be well designed in order to develop mathematical creativity, especially for schools that demand creative thinking ability such as vocational high school. This is indicated that students’ fluency, flexibility, and novelty can be improved significantly by posing the problems.

The result of the study also indicated that the problems posed by students might be related to their major in vocational high school. It was related to their background of mathematical knowledge. This was supported by the opinion that basic knowledge and basic skills in mathematics could be highly related to creativity in mathematics [13]. A larger goal of bringing problem posing to the mathematical learning might be the alternative to improve students’ creativity.

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6. References
[1] Eurydice 2011 Key Data on Learning and Innovation through ICT at School in Europe (Brussels: The Education, Audiovisual and Culture Executive Agency)
[2] Tenzin D, Paul B, David J L, Anoop S and Ram B B 2017 J. Comput. Educ. 44 355-369
[3] Xianwei Y, Van H, Bharath S 2013 Educ. Stud. Math. 82 201-221
[4] NCTM 2000 Principal and Standards for School Mathematics (VA: NCTM) p. 52-60
[5] Munandar U Mengembangkan Kreativitas Anak Berbakat (Jakarta: Gramedia) p. 52-54
[6] Edward A S 1994 On Mathematical Problem Posing (Canada: FLM Publishing) p. 20
[7] Tatang Y E S 2010 J. Math. Educ. 11 17-40
[8] Ali M 2007 Pythagoras 31 43-50
[9] Maria F A, Isabel A G and Julio B C 2016 Propositos y Representaciones 41 169-218
[10] Reda A E E S 2002 J. Sci. Math. Educ. Rev 251 56-69
[11] Joo-ho P 2012 Asia Pasific Educ. Rev 131 89-102
[12] Theodore L, Stephen P and Anne M H 1999 Journal of Industrial Teacher Education 361
[13] Dianzhou Z 2005 The “two basics”: Mathematics Teaching in Mainland China (China: Shanghai Educational Publishing House)