QCD sum rule analysis of excited $\Lambda_c$ mass parameter

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Abstract

The mass parameter of orbitally excited $\Lambda_c$ baryons is calculated by using QCD sum rule in the framework of heavy quark effective theory. Two kinds of interpolating current for the excited heavy baryons are introduced. It is obtained that $\bar{\Lambda} = 1.08^{+0.095}_{-0.104}$ GeV for the non-derivative current and $\bar{\Lambda} = 1.06^{+0.090}_{-0.107}$ GeV for the current with derivative. These results are consistent with experimental data.

Key words: excited heavy baryon, QCD sum rules, heavy baryon mass

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1 Introduction

Study of heavy baryons widens the way to test the Standard Model. The experimental data on the heavy baryons are accumulating and waiting for reliable theoretical calculations. The heavy quark effective theory (HQET) [1] provides a model-independent method for analyzing hadrons containing one heavy quark. It simplifies the analysis greatly because of the heavy quark symmetry (HQS) and allows us to expand the physical quantity in powers of $1/m_Q$ systematically, where $m_Q$ is the heavy quark mass. To obtain detailed predictions, however, one needs to combine it with some nonperturbative methods.

QCD sum rule is a powerful nonperturbative method based on QCD [2]. It takes into account the nontrivial QCD vacuum, parametrized by various vacuum condensates. QCD sum rule has been used successfully to analyze the properties of light mesons [2] and light baryons [3]. Heavy mesons [4,5] and the ground state heavy baryons [6–8] have been analyzed in the framework of HQET sum rules with $1/m_Q$ corrections and $\alpha_s$ radiative corrections included.

With the discovery of the orbitally excited charmed baryons [9], it needs to investigate them theoretically [10]. This investigation will extend our ability in the application of QCD. It can also help us foresee other excited heavy baryons undiscovered yet. Within the framework of large $N_c$ HQET, we have studied the excited heavy baryon spectrum [11]. In this paper, we use QCD sum rule to calculate the mass parameter $\bar{\Lambda}$ of the excited heavy baryons in the leading order $\alpha_s$ expansion. $\bar{\Lambda}$ is defined in the heavy baryon mass expansion,

\begin{equation}
M = m_Q + \bar{\Lambda} + O\left(\frac{1}{m_Q}\right).
\end{equation}

It is independent of the heavy quark spin and flavor, and describes mainly the contribution of the light degrees of freedom in the baryon. The quantum numbers that describe the hadrons are angular momentum $J$ and isospin $I$.

For the heavy hadrons, because of HQS, the total angular momentum of the light degrees of freedom $J'$ becomes a good quantum number. In this case, the excited hadron spectrum shows the degeneracy of pair of states which are related to each other by HQS. For the baryons, the light degrees of freedom look like a collection of $N_c - 1$ light quarks. There are two kinds of excitation [11,12]. One is the symmetric representation of the $N_c - 1$ light quarks in the light quark spin-flavor space. The other is the mixed representation of the light quarks. In the constituent picture, the former kind corresponds to that the two light quarks with vanishing relative orbital angular momentum move around the heavy quark with orbital angular momentum $L = 1$. In the latter case, one light quark is $L = 1$ excited while the other light quark and the heavy quark are in ground state. Quark models [13] indicate that the former
lies about 150 MeV below the latter. In each representation, the states are not unique. The lowest lying pair of states in the symmetric representation are denoted as Λ_{c1}(\frac{1}{2}^{−}) and Λ_{c1}(\frac{3}{2}^{−}) with "−" standing for the parity. It is these states that we are going to calculate.

In Sec. 2, the interpolating fields of the excited heavy baryons are constructed. And the two-point correlation function is calculated. Sec. 3 presents the numerical results. A brief summary and discussion are made in Sec. 4.

2 Interpolating fields and the sum rules

The heavy baryon current is generally expressed as

\[ j(x) = \epsilon_{ijk} [q^T(x) C \tau q^j(x)] \Gamma h_v^k(x), \]

where \( i, j, k \) are the color indices, \( C \) is charge conjugation, and \( \tau \) is the isospin matrix. \( q(x) \) is a light quark field, while \( h_v(x) \) is the heavy quark field with velocity \( v \). \( \Gamma \) and \( \Gamma' \) are some gamma matrices which describe the structure of the baryons. We adopt two kinds of simple forms for \( \Gamma \) and \( \Gamma' \). Usually \( \Gamma \) and \( \Gamma' \) with least number of derivatives are used in the QCD sum rule method. The sum rules then have better convergence in the high energy region and often have better stability. For the ground state heavy baryons, the way to write down the \( \Gamma \) and \( \Gamma' \) was discussed in Refs. [6] and [7]. In our case, the angular momentum and parity of the light degrees of freedom are \( 1^{−} \). They are the same as that of \( \Sigma_Q(\frac{1}{2}^{+}) \) and \( \Sigma_Q^*(\frac{3}{2}^{+}) \) except the parity. Therefore the choice of \( \Gamma \) and \( \Gamma' \) without derivatives for \( \Lambda_{Q1}(\frac{1}{2}^{-}) \) and \( \Lambda_{Q1}(\frac{3}{2}^{-}) \) is

\[
\Gamma(\frac{1}{2}) = (a + by) \gamma^5, \quad \Gamma'(\frac{1}{2}) = \gamma_\mu \gamma_5, \\
\Gamma(\frac{3}{2}) = (a + by) \gamma_\mu \gamma_5, \quad \Gamma'(\frac{3}{2}) = -g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu \equiv \Lambda^{\mu\nu},
\]

where a transverse vector \( A_t^\mu \) is defined to be \( A_t^\mu = A^\mu - v^\mu v \cdot A \), and \( g_{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu \). \( a, b \) are arbitrary numbers between 0 and 1.

On the other hand, the above discussed constituent picture as well as the experience from the excited heavy mesons [5], helps us to choose the following form of \( \Gamma \) and \( \Gamma' \) with one derivative for \( \Lambda_{Q1}(\frac{1}{2}^{-}) \) and \( \Lambda_{Q1}(\frac{3}{2}^{-}) \),

\[
\Gamma(\frac{1}{2}) = (a + by) \gamma_5, \quad \Gamma'(\frac{1}{2}) = \frac{\not{p}_t}{M} \gamma_5.
\]
\[
\Gamma(\frac{3}{2}) = (a + b\gamma_5), \quad \Gamma'(\frac{3}{2}) = \frac{1}{3M} \left( \bar{D}_\mu + \bar{D}_\mu \gamma_5 \right),
\]

where \( M \) in Eq. (4) is some hadronic mass scale to make \( \Gamma' \) dimensionless.

Furthermore it should be noted that the two possible choices for \( \Gamma \) are accounted through introducing parameters \( a \) and \( b \). The baryonic decay constants in the HQET are defined as follows,

\[
\langle 0| j_{1/2}^a | \Lambda_{Q1}(1/2) \rangle = f_{1/2} u,
\]

\[
\langle 0| j_{3/2}^\mu | \Lambda_{Q1}(3/2) \rangle = \frac{1}{\sqrt{3}} f_{3/2} u^\mu
\]

where \( u \) and \( u^\mu \) are the spinor and Rarita-Schwinger spinor, respectively. \( f_{3/2} \) is the same as \( f_{1/2} \) in the heavy quark limit. The decay constants corresponding to the two choices of the baryon interpolating currents in Eqs. (3) and (4) are not related to each other, although they are at the same order.

For analyzing the masses by QCD sum rule, the following two-point correlation function is evaluated,

\[
\Pi = i \int d^4x e^{ikx} \langle 0| \{ j(x), \bar{j}(0) \} | 0 \rangle.
\]

The hadronic representation of \( \Pi \) for \( \Lambda_{Q1}(1/2^-) \) at leading order of \( 1/m_Q \) is

\[
\Pi = \frac{f^2}{\Lambda - \omega} \frac{1 + \frac{v^2}{2}}{2} + \text{res.},
\]

where \( \omega = v \cdot k \). For \( \Lambda_{Q1}(3/2^-)^+ \), \( \Pi \) has the same form except the factor \( \frac{1 + \frac{v^2}{2}}{2} \) which is replaced by \( \Lambda^{\mu\nu} \). \( \Pi \) can be also calculated in terms of quark and gluon language with vacuum condensates. This establishes the sum rule. We use the commonly accepted assumption of quark-hadron duality for the resonance part of Eq. (7),

\[
\text{res.} = \frac{1}{\pi} \int_{\omega_c}^{\infty} d\nu \frac{\text{Im}\Pi_{\text{pert}}(\nu)}{\nu - \omega},
\]

where \( \Pi_{\text{pert}} \) is the perturbative contribution to \( \Pi \), and \( \omega_c \) is the continuum threshold. After the Borel transformation and dropping the common matrix
of \( \frac{1+\ell}{2} \) or \( \Lambda^{\mu\nu} \), we have

\[
\tilde{\Lambda} = \frac{d}{d(-1/T)} \ln \left[ \frac{1}{\pi} \int_0^{\omega_c} \Im \Pi_{\text{pert}}(\nu) e^{-\nu/T} d\nu + \hat{B}_T^{\omega} \Pi_{\text{cond}}(\omega) \right], \tag{9}
\]

where \( \hat{B}_T^{\omega} \) means the Borel transformation, \( T \) is the Borel parameter, and \( \Pi_{\text{cond}} \) is the condensate contributions to \( \Pi \). Here the right-handed side must be understood as such that \( \frac{1+\ell}{2} \) or \( \Lambda^{\mu\nu} \) is dropped. Relevant Feynman diagrams contributing to Eq. (9) are listed in Figs. 1 and 2 corresponding to the choice of Eqs. (3) and (4), respectively. The fixed point gauge is used as in [14]. For Fig. 1, the dependence of \( \tilde{\Lambda} \) on \( a \) and \( b \) is canceled in Eq. (9). We obtain

\[
\frac{1}{\pi} \Im \Pi_{1a}(\nu) = \frac{N_c!}{480\pi^4} \nu^5 \Tr(\tau^\dagger \tau) \Tr(\psi \Gamma \psi \bar{\Gamma}) \left( \Gamma^{\nu} \frac{1+\psi}{2} \bar{\Gamma}^{\nu} \right),
\]

\[
\hat{B}_T^{\omega} \Pi_{1b}(\omega) = \frac{N_c!}{24\pi^2} \Tr(\tau^\dagger \tau) \left( \langle \bar{q}q \rangle T^3 - \frac{1}{16} \langle \bar{q}g \sigma \cdot Gq \rangle T \right) \Tr(\Gamma \psi \bar{\Gamma} - \bar{\Gamma} \psi \Gamma)
\times \left( \Gamma^{\nu} \frac{1+\psi}{2} \bar{\Gamma}^{\nu} \right),
\]

\[
\hat{B}_T^{\omega} \Pi_{1c}(\omega) = -\frac{N_c!}{144} \langle \bar{q}q \rangle^2 \Tr(\tau^\dagger \tau) \Tr(\Gamma \bar{\Gamma}) \left( \Gamma^{\nu} \frac{1+\psi}{2} \bar{\Gamma}^{\nu} \right),
\]

\[
\hat{B}_T^{\omega} \Pi_{2a}(\omega) = \frac{N_c!}{384\pi^2} T^2 \Tr(\tau^\dagger \tau) \Tr(\psi \gamma_5 \Gamma \psi \gamma_5 \bar{\Gamma} - \gamma^\alpha \gamma_5 \Gamma \gamma_\alpha \gamma_5 \bar{\Gamma}) \left( \Gamma^{\nu} \frac{1+\psi}{2} \bar{\Gamma}^{\nu} \right),
\]

\[
\hat{B}_T^{\omega} \Pi_{2b}(\omega) = \frac{N_c!}{1536\pi^2} T \Tr(\tau^\dagger \tau) \Tr(\sigma_{\mu\nu} \Gamma \left\{ \sigma_{\mu\nu}, \psi \right\} \bar{\Gamma} - \sigma_{\mu\nu} \bar{\Gamma} \left\{ \sigma_{\mu\nu}, \psi \right\} \Gamma)
\times \left( \Gamma^{\nu} \frac{1+\psi}{2} \bar{\Gamma}^{\nu} \right), \tag{10}
\]

where the subscripts of \( \Pi \) denote the figures, \( N_c \) is the number of colors, and \( \bar{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0 \). The quark condensate \( \langle \bar{q}q \rangle \), gluon condensate \( \langle \alpha_s GG \rangle \), and quark-gluon mixed condensate \( \langle \bar{q}g \sigma \cdot Gq \rangle \), which are of dimension less than 6 and the dimension 6 condensate \( \langle \bar{q}q \rangle^2 \) are included. Note that the term of \( \left( \Gamma^{\nu} \frac{1+\psi}{2} \bar{\Gamma}^{\nu} \right) \) appears commonly. This is proportional to \( \frac{1+\ell}{2} \) or \( \Lambda^{\mu\nu} \) for Eq. (3) and (4), respectively, which is dropped in Eq. (9). For Fig. 2, we obtain the following results,

\[
\frac{1}{\pi} \Im \Pi_{2a}(\nu) = \frac{3N_c!}{4\pi^2} \nu^7 \Tr(\tau^\dagger \tau) (24a^2 + 40b^2) \left( \frac{1+\psi}{2} \right),
\]

\[
\hat{B}_T^{\omega} (\Pi_{2b}(\omega) + \Pi_{2f}(\omega)) = \frac{N_c!}{2\pi^2} \Tr(\tau^\dagger \tau) [\langle \bar{q}q \rangle T^5(16ab) - \langle \bar{q}g \sigma \cdot Gq \rangle T^3ab] \left( \frac{1+\psi}{2} \right),
\]
\[ \hat{B}_T^2 \Pi_{2c}(\omega) = \hat{B}_T^2 \Pi_{2g}(\omega) = 0 , \]
\[ \hat{B}_T^2 \Pi_{2d}(\omega) = \frac{\langle \alpha GG \rangle}{32\pi^3} T^4 \text{Tr}(\tau^1\tau)(-a^2 + b^2) \left( \frac{1 + \psi}{2} \right) , \]
\[ \hat{B}_T^2 \Pi_{2e}(\omega) = -\frac{\langle \bar{q}g\sigma \cdot Gq \rangle}{4\pi^2} T^3 \text{Tr}(\tau^1\tau)(3ab) \left( \frac{1 + \psi}{2} \right) , \]

(11)

for spin 1/2. For spin 3/2 case, the results are the same except the factor \( \frac{1 + \psi}{2} \) replaced by \( \Lambda^\mu \).

3 Numerical results

Fig. 3 shows the numerical result of the mass parameter \( \bar{\Lambda} \) as a function of the Borel parameter \( T \). The following values for the condensates have been used,

\[ \langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3 , \]
\[ \langle \alpha GG \rangle = 0.04 \text{ GeV}^4 , \]
\[ \langle \bar{q}g\sigma \cdot Gq \rangle \equiv m_0^2 \langle \bar{q}q \rangle , \quad m_0^2 = 0.8 \text{ GeV}^2 . \]

(12)

Five curves with various continuum threshold \( \omega_c \) are plotted, and the best value of \( \omega_c \) is

\[ \omega_c^{\text{best}} = 1.40 \pm 0.20 \text{ GeV} . \]

(13)

This number is reasonable considering the quark model results [13]. The sum rule window is fixed in the following. The lower bound of the sum rule window is chosen such that the condensate contribution is less than 30%, while the upper bound is determined from the requirement that the pole contribution is at least 30%. The window is determined as

\[ 0.188 \text{ GeV} \leq T \leq 0.310 \text{ GeV} \] for baryon current without derivative,
\[ 0.114 \text{ GeV} \leq T \leq 0.222 \text{ GeV} \] for baryon current with derivative. (14)

From the windows, \( \bar{\Lambda} \) is obtained as

\[ \bar{\Lambda} = \begin{cases} 
1.08^{+0.095}_{-0.104} \text{ GeV} & \text{(non-derivative current),} \\
1.06^{+0.090}_{-0.107} \text{ GeV} & \text{(derivative current).}
\end{cases} \]

(15)
For the case of derivative current, the best stability is at $a = 1$ and $b = 0$. From
the experimental value of the spin-averaged mass $\frac{1}{3}(M_{\Lambda_{c\bar{c}}(1/2)} + 2M_{\Lambda_{c\bar{c}}(3/2)}) \simeq 2616$ MeV [9], we have

$$m_c = \begin{cases} 
1.54^{+0.104}_{-0.095} \text{ GeV} & \text{(nonderivative current)}, \\
1.56^{+0.107}_{-0.090} \text{ GeV} & \text{(derivative current)}. 
\end{cases}$$

These values are consistent with those obtained from the sum rules for the heavy mesons [4,5] and the ground state heavy baryons [7].

4 Summary and discussion

We have calculated the mass parameter of the excited $\Lambda_{c}$ baryons using QCD sum rules in the framework of HQET. Two kinds of interpolating current for the excited heavy baryons are introduced. It has been obtained that $\bar{\Lambda} = 1.08^{+0.095}_{-0.104}$ GeV for the non-derivative current and $\bar{\Lambda} = 1.06^{+0.090}_{-0.107}$ GeV for the current with derivative. These result in $m_c \simeq 1.54 \pm 0.1$ GeV and $1.56 \pm 0.1$ GeV, respectively, which are consistent with those from the analyses of other heavy hadrons. Continuum threshold is at $\omega_{c}^{\text{best}} = 1.40 \pm 0.20$ GeV. Due to HQS, our result is also valid for the excited $\Lambda_{c}^*$ baryons.

This work can be improved through considering $1/m_Q$ and $\alpha_s$ corrections. The $1/m_Q$ correction which keeps the heavy quark spin symmetry will affect the spin-averaged mass. It is at the 15% level for the ground state $\Sigma_{c}$ and $\Sigma_{c}^*$ baryon [7]. The spin symmetry violating correction contributes to the mass difference of $\Lambda_{Q_1}(1/2^-)$ and $\Lambda_{Q_1}(3/2^-)$. While the $\alpha_s$ correction can be significant for the decay constant, it is expected to be small for the mass parameter $\bar{\Lambda}$ because of the cancelation in Eq. (9).

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FIGURE CAPTIONS

Fig. 1
Diagrams for calculating $\Lambda_{Q1}$ mass with interpolating current without derivative. Double line denotes the heavy quark.

Fig. 2
Diagrams for calculating $\Lambda_{Q1}$ mass with interpolating current with derivative. Double line denotes the heavy quark.

Fig. 3
HQET sum rule for the mass parameter $\bar{\Lambda}$ by using baryon interpolating current with derivative (a) and without derivative (b). $T$ is the Borel parameter.
Fig. 1.
Fig. 2.
Fig. 3.