Coupling Constant Unification and LEP Data

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Abstract
The recent LEP data for gauge coupling constants constrain many grand unified models. In this paper, we study several possibilities for unification of gauge coupling constants. Without an intermediate scale, the minimal supersymmetric standard model with two Higgs doublets is the only possibility. For one intermediate scale, we present a few unification schemes without supersymmetry.
One of the best theoretical ideas in the last few decades has been to understand the strengths of gauge coupling constants. The first great advance in this direction has been the invention of grand unified theories\[1\]. The second advance has been four dimensional superstring models\[2\][3]. The string theory requires that the coupling must be the same at the string scale\[4\].

Among the predictions of GUT, the value \(\sin^2 \theta_W\), proton decay and fermion mass ratios have attracted most attention. Proton decay experiment constrained the unification scale\[4\]. In this regard, LEP data\[5]\ of three coupling constants have played the crucial role in testing this idea. Until recently, the strong coupling has not been measured accurately, and the value of \(\sin^2 \theta_W\) was the prime constraint for the unification models. Thus the accurate measurement of \(\alpha_c\) at LEP\[7]\ triggered an interest for the study of unification condition. For example, Giveon, Hall and Sarid \[8\] studied the criteria for unification of coupling constants recently. Our philosophy here is the same as their’s: *in search of greater number of possible unification models*. With a more accurate determination of \(\alpha_c\) available now\[7]\, it is timely to study this problem again. Furthermore, in this paper we go beyond the minimal unification models by introducing an intermediate mass scale.

The success of coupling constant unification originates from the observation that the apparent difference of coupling constants at low energy is attributed to the running of coupling constants\[4\]. If the coupling constants are unified to \(\alpha_X\) at some high energy scale, say \(M_X\), then it evolves to

\[
\alpha_i^{-1}(\mu) = \alpha_X^{-1} - \frac{b_i}{2\pi} \ln \left( \frac{\mu}{M_X} \right)
\]

where \(\mu\) is the scale in question, and \(b_i\) is the standard notation for the coefficient of \(\beta_i\). Coupling constants in Eq. (1) are defined for normalized generators. For the electroweak hypercharge, we use \(\alpha_Y\) for the usual coupling and \(\alpha_y\) for the normalized hypercharge; thus \(\alpha_y = \frac{2}{3} \alpha_Y\). Then the difference of coupling constants below \(M_X\) but above a new physics scale \(M_I\) satisfies

\[
\alpha_i^{-1} - \alpha_j^{-1} = -\frac{b_i - b_j}{2\pi} \ln \left( \frac{\mu}{M_X} \right)
\]

whence we obtain

\[
\frac{\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu)}{\alpha_j^{-1}(\mu) - \alpha_k^{-1}(\mu)} = \frac{b_i - b_j}{b_j - b_k}.
\]

\[1\]The ten dimensional heterotic string\[3\] unifies coupling constants, but one can say that it is unified because of the grand unification group \(E_8 \times E_8\).
Eq. (3), which must hold independent of scale $\mu$, is a one-loop criterion for successful unification of coupling constants at some scale. Suppose there are two interesting mass scales, the unification scale $M_X$ and the electroweak scale $M_Z$. Then the left-hand side can be evaluated by data at the scale $M_Z$. On the other hand, the right-hand side is calculated in a specific model for unification. If they turn out to be the same within experimental and theoretical errors, the model is not in conflict with low energy data. If they differ, the model is ruled out. The use of Eq. (3) is simplified since the right-hand side usually does not depend on the fermion content of the theory. It is mainly determined by the gauge group and the Higgs content corrects it by small amount. The specific role of the Higgs fields is due to the assumption on the split multiplet of Higgs fields for proton stability.

If no new physics scale is present between $M_X$ and $M_Z$, the unification mass is given by

$$M_X = M_Z \exp \left[ 2\pi \frac{\alpha_{w}^{-1}(M_Z) - \alpha_{c}^{-1}(M_Z)}{b_w - b_c} \right]$$

(4)

where $\alpha_w$ and $\alpha_c$ are $SU(2)_L$ and $SU(3)_c$ couplings, and $b_w$ and $b_c$ are coefficients of the respective $\beta$’s. For a successful unification, proton lifetime requires $M_X > 10^{15-16}$ GeV, which gives another constraint

$$b_w - b_c < (2\pi \log_{10} e) \frac{\alpha_{w}^{-1}(M_Z) - \alpha_{c}^{-1}(M_Z)}{(15 \sim 16) - \log_{10}(M_Z/\text{GeV})}$$

(5)

As the first example, let us consider the possibility of unification of the standard model. Let us define

$$r(\mu) = \frac{\alpha_{y}^{-1}(M_Z) - \alpha_{w}^{-1}(M_Z)}{\alpha_{w}^{-1} - \alpha_{c}^{-1}}, \quad R = \frac{b_y - b_w}{b_w - b_c}$$

(6)

From the LEP data,

$$\alpha_{y}^{-1}(M_Z) = 58.9 \pm 0.3$$
$$\alpha_{w}^{-1}(M_Z) = 29.7 \pm 0.2$$
$$\alpha_{c}^{-1}(M_Z) = 8.47 \pm 0.5$$

(7)

we obtain

$$r(M_Z) = 1.37 \pm 0.07$$

(8)
Let $h_2$ be the number of Higgs doublets and $n_g$ be the number of generations. Then, we have

$$
\begin{align*}
b_y &= \frac{4}{3}n_g + \frac{1}{10}h_2, \\
b_w &= -\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{6}h_2, \\
b_c &= -11 + \frac{4}{3}n_g.
\end{align*}
$$

(9)

Thus,

$$
R = \frac{\frac{22}{3} - \frac{1}{15}h_2}{\frac{11}{3} + \frac{1}{6}h_2} = 2 \ (h_2 = 0), \ 1.90 \ (h_2 = 1), \ 1.36 \ (h_2 = 8)
$$

(10)

Thus the coupling constant unification does not occur in the minimal ($h_2 = 1$) standard model. See Fig. 1 and 2(a). In Fig. 1, the horizontal lines correspond to $R$ and three curly lines correspond to experimentally determined $r(\mu)$’s within 1σ. The crossing point should be $M_Z$. Introduction of enough Higgs doublets ($h_2 = 8$) at low energy makes the theory unifiable, because they can meet at $\mu \sim M_Z$. But the proton lifetime constraint is not satisfied; the RHS of Eq. (5) is $4.44 \sim 4.13$ and the LHS of Eq. (3) is $11/3 + h_2/6 = 5$. In Fig. 2, we show the evolution of coupling constants in the standard model and in the minimal supersymmetric standard model.

As the second example, let us consider the supersymmetric standard model. For simplicity we assume that the supersymmetry breaking scale is comparable to the electroweak scale. Then, we can use

$$
\begin{align*}
b_y &= 2n_g + \frac{3}{10}h_2, \\
b_w &= -6 + 2n_g + \frac{1}{3}h_2, \\
b_c &= -9 + 2n_g.
\end{align*}
$$

(11)

and obtain

$$
R = 2 \ (h_2 = 0), \ 1.40 \ (h_2 = 2)
$$

(12)

Thus the coupling constant unification is successful in the minimal supersymmetric standard model with $h_2 = 2$. Because the supersymmetry breaking scale is very close to the electroweak scale, our study for the two scale physics for the supersymmetric standard model is approximately valid. For a more accurate calculation, we must use the three scale physics, which is shown in Fig. 2(b) for the supersymmetry breaking scale $M_S = 1$ TeV. Supersymmetric standard models[10] and $SU(5) \times U(1)$ models[11] from superstrings belong to this category. String theory gives the condition for equal couplings for each gauge group at the string scale.
Thus the criterion for unification of coupling constants can be satisfied by extending the minimal model, either by increasing the number of Higgs doublets or by supersymmetrizing the model. One may argue that the LEP data favors the supersymmetric standard model. However, if one is forced to introduce many Higgs doublets either from experimental discovery or from theoretical reasoning of understanding fermion mass matrix, this argument is no longer valid. But, in this case the proton stability must be explained by introducing a symmetry\[12\]. At present, we can conclude that there are a few paths toward coupling constant unification.

Another logical possibility is the presence of three or more mass scales; namely we can introduce intermediate mass scales. The invisible axion idea requires an intermediate scale around $10^{12}$ GeV. Possibility of lepton number violation needs another intermediate mass scale. Grand unifications beyond $SU(5)$ require another scale in principle. Therefore, let us introduce intermediate scales for unification of gauge coupling constants. For a concrete study, let us introduce just one intermediate mass scale $M_I$ as the vacuum expectation value of some Higgs fields. If the vacuum expectation value in question is neutral under the gauge group at $M_I$, the conclusion is the same as the two scale case studied above. Therefore, for the study of three scale cases, let us consider at the intermediate scale $M_I$ a gauge group $G_I$ which contains the standard model as a proper subgroup.

A supersymmetric standard model with two Higgs doublets already satisfies the unification condition, and we will not consider supersymmetric cases with intermediate scales.

Without supersymmetry, one may wish to satisfy the unification condition with a small number of needed Higgs fields by introducing an intermediate mass scale. In the remainder of this paper, we will consider this case. As an example, consider $SO(10)$. If the symmetry breaking proceeds via $SO(10) \to SU(5)_{GG} \times U(1)$ where $SU(5)_{GG}$ is Georgi and Glashow’s $SU(5)$, we redefine the grand unification group as the $SU(5)_{GG}$. Then there is no intermediate scale. On the other hand, if the symmetry breaking proceeds via $SO(10) \to SU(5)_{fipped} \times U(1)$ or $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$, then we need an intermediate mass scale to obtain the standard model at $M_I$.

Let the gauge group be

\[
\begin{cases} 
G_3 \times G_2 \times G_1 & (M_I < \mu < M_X) \\
SU(3)_c \times SU(2)_w \times U(1)_Y & (M_Z < \mu < M_I)
\end{cases}
\]  

(13)
where
\[ G_3 \supset SU(3)_c, \ G_2 \supset SU(2)_w \] (14)

The electroweak hypercharge generator \( Y \) is a combination of a few generators above \( M_I \),
\[ Y = c_1 Y_1 + c_2 Y_2 + c_3 Y_3 \] (15)

where \( c_i (i = 1, 2, 3) \) are numbers and \( Y_i \) are normalized generators belonging to the group \( G_i \). Let \( \alpha_i \) be coupling constants of the group \( G_i \). Then, at \( M_I \) the hypercharge coupling satisfies at lowest order,
\[ \frac{5}{3} \alpha_y^{-1} = c_1 \alpha_1^{-1} + c_2 \alpha_2^{-1} + c_3 \alpha_3^{-1} \] (16)

For a unification, the following condition must be satisfied at \( \mu = M_I \),
\[ r(\mu = M_I) = R \] (17)

where
\[ r(\mu) = \frac{c_1^{-2} [\frac{5}{3} \alpha_y^{-1} - c_2 \alpha_1^{-1} - c_3 \alpha_2^{-1}]}{\alpha_w^{-1} - \alpha_c^{-1}} - \alpha_w^{-1} \] (18)

and
\[ R = \frac{b_1 - b_2}{b_2 - b_3}_{|_{\mu=M_I\sim M_X}}. \] (19)

The common point of \( r(\mu) \) and \( R \) determines \( M_I \). The proton lifetime constraint can be given as
\[ b_2 - b_3 < (2\pi \log_{10} e) \frac{\alpha_w^{-1}(M_I) - \alpha_c^{-1}(M_I)}{(15 \sim 16) - \log_{10}(M_I/\text{GeV})} \] (20)

As a successful example, let us consider the following symmetry breaking pattern of \( SO(10) \) model,
\[ SO(10) \xrightarrow{M_X} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \]
\[ \xrightarrow{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \] (21)

Since \( \alpha_L = \alpha_R \equiv \alpha_2 \) for \( M_I < \mu < M_X \), we can apply above formulae with \( G_2 = SU(2)_L \times SU(2)_R \). Then,
\[ R = \frac{22 - h_2 + 7h_3}{11 + h_2 + 2h_3} = 2 \ (h_2 = h_3 = 0), \ 2 \ (h_2 = h_3 = 1) \] (22)
where $h_2$ and $h_3$ are the numbers of Higgs doublets and triplets ($\in SU(2)_R$). The electroweak hypercharge is given by

$$Y = T^R_3 + \sqrt{\frac{2}{3}} T_0$$

(23)

where $T^R_3$ is a generator of $SU(2)_R$ and $T_0$ is the normalized generator of $U(1)_{B-L}$. Then we obtain the unification condition

$$\frac{5}{2} \cdot \frac{\alpha^{-1}_y - \alpha^{-1}_w}{\alpha^{-1}_w - \alpha^{-1}_c} \bigg|_{\mu=M_I} = R$$

(24)

from which $M_I$ calculated as

$$M_I = 1.81 \times 10^{10} \text{ GeV},$$

(25)

and the unification mass is given as

$$M_X = 3.73 \times 10^{15} \text{ GeV}.$$ 

(26)

These numbers are comparable to those obtained by Shaban and Stirling [13], but our method of testing the unification is simpler.

For $SO(10) \rightarrow SU(5)_{\text{flipped}} \times U(1)$, the situation is not better than the $SU(5)$ model. This can be easily understood from Fig. 2(a) ($h_2 = 1$) where the crossing point of $\alpha^{-1}_c$ and $\alpha^{-1}_w$ is higher than $\alpha^{-1}_1$. Let the mass scale of crossing point of $SU(3)_c$ and $SU(2)_w$ couplings be $M_I$. At $M_I$, $\alpha^{-1}_1$ jumps slightly due to the mixing $\alpha^{-1}_y = (24/25)\alpha^{-1}_1 + (1/25)\alpha^{-1}_5$, but this shift is not enough to overcome $\alpha^{-1}_5$. Therefore, there is no possibility of further unification of $SU(5)_{\text{flipped}} \times U(1)$ above $M_I$ with $h_2 = 1$.

As a final example, let us consider $SU(N)$ family unification models. There are many varieties for hypercharge assignments, but we will focus on the simplest generalization[14],

$$Y = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, q, \frac{1}{2}, \frac{1}{2}, -q, \cdots)$$

(27)

where $\cdots$ are zeros, $SU(3)_c$ is embedded in the $3 \times 3$ square matrix of the first three rows and columns, and $SU(2)_w$ is embedded in the $2 \times 2$ square matrix of the fifth and sixth rows and columns. More general hypercharge assignment is possible with $\Sigma_i q_i = 0$ for $q_i$ are the diagonal entries of $Y$ beyond $SU(5)$.
The simplest example is given for \(SU(7)\). Thus let us focus on \(SU(7)\). The
symmetry breaking pattern is assumed to be
\[
SU(7) \xrightarrow{M_X} SU(4) \times SU(3) \times U(1) \\
\xrightarrow{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y
\]
where \(SU(4)\) is embedded in the \(4 \times 4\) square matrix of the first four rows and
columns and \(SU(3)\) is embedded in the \(3 \times 3\) square matrix of the next three
rows and columns. Then,
\[
R = \frac{11 - \frac{1}{14} h_3}{\frac{11}{3} + \frac{1}{6} h_3} = 3(h_3 = 0), \quad 2.71(h_3 = 2), \quad 2.09(h_3 = 8)
\]
where \(h_3\) is the number of Higgs triplets (which will become eventually the
number of Higgs doublets of \(SU(2)_L\)) split from fundamental representations.

As simple examples of hypercharge assignments, let us consider
\(q = 0\), \(q = 1/3\) and \(q = 1/2\). We obtain the following electroweak hypercharge generator \(Y\)
and \(r(\mu)\) for each case,
\[
q = 0 : \quad Y = -\sqrt{\frac{7}{6}} T_0 + \sqrt{\frac{1}{3}} T_8 - \sqrt{\frac{1}{6}} T_{15} \\
r(\mu) = \frac{10}{7} \cdot \frac{\alpha_y^{-1} - \alpha_w^{-1} + \frac{1}{7}}{\alpha_w^{-1} - \alpha_c^{-1}}
\]
\[
q = 1/3 : \quad Y = -\sqrt{\frac{14}{27}} T_0 + \sqrt{\frac{25}{27}} T_8 - \sqrt{\frac{2}{3}} T_{15} \\
r(\mu) = \frac{45}{14} \alpha_y^{-1} - \frac{39}{14} \alpha_w^{-1} - \frac{9}{7} \alpha_c^{-1}
\]
\[
q = 1/2 : \quad Y = -\sqrt{\frac{7}{27}} T_0 + \sqrt{\frac{8}{3}} T_8 - \sqrt{\frac{25}{27}} T_{15} \\
r(\mu) = \frac{40}{7} \alpha_y^{-1} - \frac{39}{7} \alpha_w^{-1} - \frac{25}{7} \alpha_c^{-1}
\]

To see the unification condition explicitly, we show the \(r(\mu)\) and \(R\) plot in
Fig. 4(a) for a few \(q\)’s and for a few \(h_3\)’s. For \(q = 0\), the unification is possible
when \(h_3 \geq 8\). But then the proton lifetime constraint is not satisfied. For
\(q = 1/3\) and \(q = 1/2\), the unification is possible for any plausible value of \(h_3\).

In Fig. 4(b), we present the evolution of coupling constants for \(q = 1/2\). The
values of \(M_I\) and \(M_X\) for \(h_3 = 2\) are
\[
q = 1/3 : \quad M_I = 9.6 \times 10^6 \text{ GeV}, \quad M_X = 4.6 \times 10^{16} \text{ GeV}
\]
\[
q = 1/2 : \quad M_I = 1.1 \times 10^6 \text{ GeV}, \quad M_X = 4.2 \times 10^{16} \text{ GeV}
\]
Several extensions of the standard model for unification. \( h \) is the number of Higgs doublets needed. For unification, the requirement of supersymmetry or superstring are also shown. Cases for \( M_X < 10^{15} \) GeV are forbidden from the proton decay experiments. Cases \( M_I = M_Z \) correspond to no intermediate mass scale.

| GUT group         | Susy | String | \( h \) | \( M_I \) [GeV] | \( M_X \) [GeV] |
|-------------------|------|--------|--------|----------------|----------------|
| \( SU(5) \)      | No   | No     | 8      | \( M_Z \)     | \( 3.4 \times 10^{14} \) |
| \( SU(5) \)      | Yes  | No     | 2      | \( M_Z \)     | \( 1.0 \times 10^{16} \) |
| \( SU(5) \times U(1) \) | Yes  | Yes    | 2      | \( M_Z \)     | \( 1.0 \times 10^{16} \) |
| \( SO(10) \rightarrow LR model \) | No   | No     | 2      | \( 1.8 \times 10^{10} \) | \( 3.7 \times 10^{15} \) |
| \( SU(7) \ (c = 1/3) \) | No   | No     | 2      | \( 9.6 \times 10^9 \) | \( 4.6 \times 10^{10} \) |
| \( SU(7) \ (c = 1/2) \) | No   | No     | 2      | \( 1.1 \times 10^9 \) | \( 4.2 \times 10^{10} \) |

In \( SU(7) \), two generations can be accommodated in the spinor representation \( \mathbf{4} \) of \( SO(14) \), e.g. \( 1 + 7^* + 21 + 35^* \). To include the third generation, one must repeat the spinor representation. Other representations can be used for realistic unifications in \( SU(7) \).

The various possibilities for coupling constant unification studied in this paper are summarized in Table 1.

In conclusion, we showed the possibilities of coupling constant unification by extending the minimal standard model, either by introducing superpartners or by introducing an intermediate mass scale.

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