Non-Abelian dynamics in the resonant decay of the Higgs after inflation

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**Abstract.** We study the resonant decay of the Higgs condensate into weak gauge bosons after inflation and estimate the corrections arising from the non-Abelian self-interactions of the gauge fields. We find that non-Abelian interaction terms induce an effective mass which tends to shut down the resonance. For the broad resonance relevant for the Standard Model Higgs the produced gauge particles backreact on the dynamics of the Higgs condensate before the non-Abelian terms grow large. The non-Abelian terms can however significantly affect the final stages of the resonance after the backreaction. In the narrow resonance regime, which may be important for extensions of the Standard Model, the non-Abelian terms affect already the linear stage and terminate the resonance before the Higgs condensate is affected by the backreaction of decay products.

**Keywords:** particle physics - cosmology connection, physics of the early universe, inflation

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1 Introduction

The measured Higgs mass in the range $M_h = 125 - 126$ GeV [1, 2] ensures vacuum stability within the Standard Model (SM) up to very high energies — orders of magnitude above the electroweak scale. In particular, for top masses sufficiently below the best fit value the SM remains stable up to $\rho_{\text{inf}}^{1/4} \sim 10^{16}$ GeV. Assuming the detection of primordial gravitational waves by BICEP2 [3] will be confirmed, this is the energy scale of inflation. If the Standard Model is not significantly affected by new physics at these scales, one generically predicts that the Higgs is a light field during inflation ($m_h^2 \ll H^2$) with a negligible contribution to the total energy density [4, 5]. As a light field, during inflation the Standard Model Higgs acquires long-wavelength fluctuations which in general render the local Higgs value, averaged over the observable patch, different from the field value at the minimum of the potential.

Small modifications of the SM Higgs potential, such as adding a non-minimal Higgs-gravity coupling of the form $\xi H^\dagger HR$, are not expected to change the qualitative picture. On the other hand, it is well known that with an exceptionally large coupling, ($\xi \gg 1$), inflation might be realized with the Higgs field itself [6]. The predicted tensor-to-scalar ratio of the Higgs inflation, $r = 0.0033$ [6], is however significantly below the reported detection of $r = 0.20^{+0.07}_{-0.05}$ by BICEP2, although the level $r \sim 0.2$ could be achieved for a very specific choice of the SM parameters (see e.g. [7–10]).

After inflation the observable patch of the universe is filled by an effective Higgs condensate with small fluctuations around the homogeneous background value. For the non-minimally coupled Higgs inflaton the condensate dominates the energy density. If the non-minimal coupling is small (or zero), the energy density is dominated by a non-Standard Model inflaton(s) (or its decay products) while the Higgs condensate contributes only a tiny fraction of the total energy density. The Higgs fluctuations can however be imprinted on metric perturbations even in this case if the Higgs value affects the expansion history, for example through a modulation of the inflaton decay rate [11–13].

The dominant decay process of both the Standard Model Higgs and the non-minimally coupled Higgs at zero temperature is the non-perturbative production of weak gauge bosons
from the oscillating Higgs condensate [5, 14, 15]. However, so far the non-Abelian self-couplings of the SU(2) gauge bosons have been neglected. In investigating the resonant Higgs decay, the gauge fields have been treated as if they were Abelian. While this arguably could be justified at the onset of the resonance, the non-Abelian interactions eventually become important as the number density of the resonantly produced gauge field quanta grows large. This can have a significant impact on the duration and efficiency of the resonance.

In this work we investigate the non-perturbative decay of the Standard Model Higgs into weak gauge bosons accounting for the non-Abelian terms. Our primary goal is to estimate the time at which the non-Abelian terms become significant for the dynamics of the resonance. This should be compared with the time when the gauge field induced mass for the Higgs, given by $m_{\text{ind}}^2 \sim g^2 \langle A^2 \rangle$, becomes comparable to the effective mass $m_{\text{osc}}^2 \sim \lambda a^2 h^2$ due to the Higgs amplitude $h$. The time at which $m_{\text{ind}} \gtrsim m_{\text{osc}}$ yields an estimate for the beginning of backreaction of the resonantly produced gauge fields, which eventually shuts down the resonance [16, 17]. If the non-Abelian terms were to start to influence the gauge field dynamics when $m_{\text{ind}} \ll m_{\text{osc}}$, the Abelian approximation would fail already in describing the linear stage of the resonance before the onset of backreaction. We find that this is generically the case for the narrow resonance. In the regime of broad resonance the Abelian approximation fails and the non-Abelian terms grow large very soon after the linear stage of the resonance. The non-Abelian terms will therefore play a significant role in the subsequent non-linear regime after the backreaction. The non-Abelian terms can therefore significantly affect the decay of the Higgs condensate in the early universe.

The paper is organized as follows. In section 2 we outline the initial conditions set by inflation and write down the relevant equations. In section 3 we describe resonant production of weak gauge bosons from an oscillating Higgs condensate for the Standard Model parameters and estimate the non-Abelian effects. In section 4 we focus on the narrow resonance which requires physics beyond the Standard Model. We conclude in section 5 with a discussion of the results and their implications.

2 Equations of motion and initial conditions

The tree-level potential of the Standard Model Higgs reads

$$V_0(h) = \lambda \left( h^2 - \frac{\nu^2}{2} \right)^2,$$

(2.1)

where $h = |\Phi|$ is the magnitude of the complex Higgs doublet and $\nu \approx 246 \text{ GeV}$. For large field values $h \gg \nu$ relevant for the early universe the renormalization group improved effective potential takes the form

$$V(h) \approx \frac{\lambda(h)}{4} h^4.$$

(2.2)

In the $\overline{MS}$ renormalization scheme and for the best fit SM parameter values the coupling becomes zero $\lambda(h_c) = 0$ at $h_c \approx 10^{10} \text{ GeV}$, indicating vacuum instability. The recent measurement of the CMB B-modes by the BICEP2 collaboration [3] has been claimed to suggest the existence of the background of primordial gravitational waves with a tensor-to-scalar ratio $r \sim \mathcal{O}(0.1)$, which implies an energy scale of inflation of $H_* \sim 10^{14} \text{ GeV}$ — well above the instability scale for the central values of the SM parameters. The Higgs effective potential exhibits a maximum near the instability scale, and in order for us to find ourselves in
the current electroweak vacuum, inflationary perturbations must not push the field over the maximum. This amounts to a consistency condition $H_* < V_{\text{max}}^{1/4}$ [18]. For the electroweak vacuum to remain stable up to the inflationary scale, the values of the SM parameters need to be at least $2\sigma$ away from their central values, or alternatively, new physics must come into play at high energies to stabilize the vacuum.

For field values below the instability scale the Higgs generically is a light field. Provided that inflation lasts at least a few tens of e-foldings longer that the $N_{\text{CMB}} \sim 60$ e-foldings corresponding to the presently observable patch, the Higgs fluctuations on superhorizon scales will settle down to an equilibrium state with the distribution given by $P(h) \sim e^{-8\pi^2 V/3H^4}$ [19, 20]. An estimate of the typical value of the Higgs condensate over a patch of the size of observable universe is then given by the root mean square of the fluctuations

$$h_* = \sqrt{\langle h^2 \rangle} \simeq 0.36 \lambda_*^{-1/4} H_* .$$

In the following we will use this estimate as the initial condition for the evolution of the Higgs condensate after the end of inflation.

After the end of inflation the Higgs condensate eventually starts to oscillate and decays through a non-perturbative resonance into weak gauge bosons [5]. Here we shall assume that there is not yet any thermal background due to the inflaton decay; in effect, we assume that the inflaton decays slowly as compared with the Higgs decay. In conformal time, the relevant part of the SM action containing the $SU(2) \times U(1)$ gauge fields and the Higgs sector reads

$$S = - \int d^4x \left\{ \frac{1}{4} \eta^{\mu\alpha} \eta^{\nu\beta} \left( F_{\mu\nu}^a F^{a}_{\alpha\beta} + G_{\mu\nu} G_{\alpha\beta} \right) + a^4 \left( D_\mu \Phi \right)^\dagger D^\mu \Phi + \frac{\lambda (\Phi^\dagger \Phi)^2}{4} \right\} ,$$

where the kinetic terms are given by

$$F_{\mu\nu}^a = \nabla_\mu A^a_\nu - \nabla_\nu A^a_\mu + g e^{abc} A^b_\mu A^c_\nu ,$$

$$G_{\mu\nu} = \nabla_\mu B_\nu - \nabla_\nu B_\mu ,$$

$$D_\mu \Phi = \left( \nabla_\mu - i g A^a_\mu \tau^a - i g' B_\mu \right) \Phi .$$

We choose to work in the unitary gauge where the three Goldstone modes vanish and the Higgs doublet takes the form

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \chi/a \end{pmatrix} ,$$

where we have denoted the rescaled Higgs modulus by $\chi \equiv ah$. The equation of motion for the Higgs then reads

$$\ddot{\chi} + \left[ \frac{\nabla^2}{a^2} - \frac{a}{a} - 2 \lambda' a^2 + \lambda \chi^2 \right] \chi = -\frac{1}{4} \left[ g^2 A^a_\mu A^{a\mu} - 2 g g' B_\mu A^{3\mu} + g^2 B_\mu B^\mu \right] \chi ,$$

where the dots denote derivatives with respect to conformal time $d\tau \equiv a^{-1} dt$ and the indices are raised and lowered with the Minkowski metric $\eta_{\mu\nu}$. For spatial components of the gauge fields the equation of motion is given by

$$\ddot{A}_i^a - \nabla^2 A_i^a = \partial_\mu \partial_\mu A_i^a + g^2 \chi^2 A_i^a = g e^{abc} \eta^{\mu\nu} \left[ \partial_\mu (A^b_\nu A^c_\tau) + A^b_\mu \partial_\nu A^c_\tau - A^b_\mu \partial_\nu A^c_\tau \right]$$

$$+ g^2 \eta^{\mu\nu} \left[ A^a_\mu A^b_\nu A^b_\tau - (A^b_\mu A^b_\nu) A^b_\tau \right] + \frac{g g' \chi^2}{4} \delta^{a3} B_i .$$

\footnote{In fact, in this case curved spacetime covariant derivatives $\nabla_\mu$ can be replaced by partial derivatives $\partial_\mu$.}
and the 0-components obey the constraint
\[ \nabla^2 A_0^a - \partial_0 (\partial_i A_i^a) - \frac{g^2 \chi^2}{4} A_0^a = -ge_{abc} \left[ \partial_i (A_i^b A_0^c) + A_i^b \partial_i A_0^c - A_i^b \dot{A}_i^c \right] - g^2 \left[ (\eta^{\alpha\mu} A_\mu^a A_0^a) A_0^b - (\eta^{\alpha\mu} A_\mu^b A_0^b) A_0^a \right] - \frac{gg'\chi^2}{4} \delta^{a3} B_0 . \] (2.9)

The kinetic part of the gauge field action has conformal symmetry. Therefore the mode functions of the gauge fields do not directly feel the expansion of spacetime during inflation and, consequently, the inflationary stage does not generate semiclassical long-wavelength gauge field fluctuations. The initial conditions for the gauge field dynamics in the post-inflationary epoch are thus given by the vacuum solution.

Having specified the equation of motion and the initial conditions for the system we now move on to study the dynamics and the decay of the Higgs condensate.

3 Non-perturbative decay of the Higgs condensate

3.1 Dynamics of the Higgs condensate

Neglecting interactions of the Higgs condensate with other fields, its equation of motion reads
\[ \ddot{\chi} + \lambda \chi^3 - \frac{\dot{a}}{a} \chi = 0. \] (3.1)

For a constant equation of state, after inflation the scale factor of the universe evolves as
\[ a = \left[ 1 + \frac{1}{2}(1 + 3w)H_* \tau \right]^{\frac{1}{1+3w}} , \]
where \( w = \rho/p \) and star denotes end of inflation. If the inflaton oscillates in a quartic potential or is assumed to decay instantaneously into radiation,\(^2\) \( w = 1/3 \), and the last term in equation (3.1) vanishes. If the inflaton oscillates in a harmonic potential, as is more commonly the case, the evolution is matter-like (\( w = 0 \)) and \( \chi \) grows initially until the last term becomes subdominant. This happens at \( \tau_{osc} \sim H_*^{-1} \). Afterwards the equation of motion is of the elliptic form and has a solution that can be expressed in terms of a Jacobi elliptic cosine:
\[ \chi = \chi_{osc} \text{cn} \left[ \sqrt{\lambda \chi_{osc}^2 (\tau - \tau_{osc})}, \frac{1}{\sqrt{2}} \right] . \] (3.2)

3.2 Abelian dynamics

Since the non-linear terms are small in the beginning, we treat the initial stage of particle production by considering the Abelian part of the equations of motion. Particle production is caused by the oscillating mass induced by the interaction with the Higgs condensate. Therefore we are interested in the mass eigenstates \( W_\mu^\pm \) (alternatively \( A_\mu^{1,2} \)) and
\[ Z_\mu = \cos \theta_W A_\mu^3 - \sin \theta_W B_\mu , \]
where \( \theta_W = \arctan \frac{g'}{g} \) is the weak mixing angle. The photon field \( A_\mu = \sin \theta_W A_\mu^3 + \cos \theta_W B_\mu \) does not acquire a mass through the Higgs mechanism so that there is no resonant production of photons.

Vector fields may be decomposed into divergence-free and curl-free parts, which in momentum space correspond to components that are transversal and longitudinal to the

\(^2\)However, if the inflaton decays before the Higgs condensate starts to oscillate then the subsequent dynamics will be affected by the thermal bath produced by the inflaton. In particular, the fields will acquire thermal masses which tend to block resonant production of particles \([21, 22]\).
momentum vector \( k \), satisfying respectively \( k \cdot A^T_k = 0 \) and \( k \times A^L_k = 0 \). Defining new variables as

\[
z \equiv \sqrt{\frac{\lambda \chi^2}{2 \pi} (\tau - \tau_{\text{osc}})}, \quad \kappa^2 \equiv \frac{k^2}{\chi_{\text{osc}}^2}, \quad q \equiv \begin{cases} \frac{g^2}{4} & \text{for the W bosons} \\ \frac{g^2 + g'^2}{4} & \text{for the Z boson} \end{cases} \tag{3.3}
\]

the transversal components obey the Lamé equation

\[
\frac{d^2 A^T}{dz^2} + \left[ \kappa^2 + q \left( z, \frac{1}{\sqrt{2}} \right) \right] A^T = 0. \tag{3.4}
\]

The setup is analogous to the case of massless preheating: the solutions get resonantly amplified within specific bands of momenta and the structure of resonance has been studied in detail in [17]. The longitudinal component obeys the equation

\[
\frac{d^2 A^L}{dz^2} + \frac{2\kappa^2}{\kappa^2 + q \left( \chi/\chi_{\text{osc}} \right)^2} \frac{d \ln \chi}{dz} \frac{d A^L}{dz} + \left[ \kappa^2 + q^2 \left( z, \frac{1}{\sqrt{2}} \right) \right] A^L = 0. \tag{3.5}
\]

Compared to the transversal components the longitudinal one acquires a friction term resulting from the time dependence of the effective mass induced by the interaction with the Higgs condensate. This prevents the resonance from occurring and the important contribution to the dynamics will come from the transverse modes. Thus in what follows we shall concentrate on the transverse modes alone.

In principle, the dynamics of the resonance are fully specified by the couplings of the Standard Model since the strength of amplification is determined only by their combination \( q \) (3.3). However, since the couplings run with energy, their values will depend on the energy scale of inflation.\(^3\) Figure 1 shows the structure of resonance together with the energy dependence of the resonance parameter \( q \) for weak gauge bosons. With the inflationary scale fixed by the amplitude of gravitational waves the dynamics are completely determined and taking into account the running of the couplings at two-loop order [23, 24], the system is in the intermediate to broad resonance regime, with \( q = 1 \ldots 100 \) unless we are extremely close to the instability scale. The energy scale at the end of inflation will determine the values of the couplings and by extension the strength of the resonant production, as well as the location of the resonance bands in momentum space. Note that the broader end of the resonance is sensitive to the uncertainties in the measurements for the masses of the Higgs and, in particular, the top quark. The Higgs also produces its own quanta; however, this process is comparatively inefficient because it is located right on the edge of a resonance band with \( q_h = 3 \).

One might also be concerned that the produced gauge bosons would decay perturbatively into fermions before the resonance has a chance to build on them. This will happen if their decay rate is larger than the frequency of the Higgs oscillations. The time between two zero-crossings of the Higgs is given by \( \Delta \tau = 2Km_{\text{osc}}^{-1} \), where \( K \) is the complete elliptic integral of the first kind, while the total decay rate of the gauge bosons is \( \Gamma_Z \simeq \frac{2}{3 \cos^2 \theta_W} \Gamma_{W^\pm} \) [15]. Therefore, they will decay in the time interval between zero-crossings if

\[
q > \left( \frac{4 \pi \cos^2 \theta_W}{g^2 K} \right)^2 \sim O(10^2). \tag{3.6}
\]

\(^3\)The amplitude of the Higgs condensate in our local patch could also happen to be different from the equilibrium root mean square value (2.3), in which case the energy scale determining the effective couplings would not be directly related to the scale of inflation.
Thus, the decay of the produced gauge bosons is only relevant if the Higgs self-coupling is close to the instability scale and we ignore it in our analysis.

For definiteness, in the following we take the energy scale to be $H_\ast = 10^{14}$ GeV, as implied by BICEP2 [3], and choose $M_h = 125.7$ GeV, $M_t = 171.25$ GeV and $\alpha_s = .1191$ ($\lesssim 2\sigma$ away from central values) to ensure vacuum stability. The resonance parameters are then $q_W \simeq 18$ and $q_Z \simeq 29$. Note that typically either $W$ or $Z$ will be dominantly produced depending on which one lies within the strongest resonance band for a given energy scale and/or choice of parameters.

### 3.3 Non-Abelian corrections

As the modes get amplified, the non-linear interaction terms in equations (2.8) grow to a size comparable to the linear terms. We use the Hartree approximation to estimate the effect of these terms on the dynamics of the resonance, i.e., we take the vacuum expectation value of the non-linear terms using the linear solution

$$A_1^{(W,Z)}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_{\lambda=1}^3 \left[ A_{k,\lambda}^{(W,Z)} \epsilon_i(k,\lambda) \hat{a}_{k,\lambda}^{(W,Z)} e^{ik \cdot x} + A_{k,\lambda}^{(W,Z)*} \epsilon_i^*(k,\lambda) \hat{a}_{k,\lambda}^{(W,Z)\dagger} e^{-ik \cdot x} \right] ,$$

where $\hat{a}_k, \hat{a}_k^\dagger$ are the usual annihilation and creation operators, $\epsilon_i(k,\lambda)$ are the polarization vectors and $A_{k,\lambda}^{(W,Z)}$ are the solutions to the corresponding equations of motion. We choose the vector $\epsilon_i(k,3) = k_i/|k|$ corresponding to the longitudinal component and the remaining two vectors to be orthogonal to $k$, corresponding to transversal components, and satisfying

$$\sum_{\lambda=1}^2 \epsilon_i(k,\lambda) \epsilon_j^*(k,\lambda) = \delta_{ij} - \frac{k_i k_j}{k^2}.$$

**Figure 1**: Structure of resonance for the Lamé equation (3.4). Solutions are exponentially amplified within the blue bands. Also plotted the dependence of the resonance parameter $q$ on energy scale (initial value of the Higgs) for the $W$ (red) and $Z$ (orange) bosons. The curves correspond to central values $M_{\text{top}} = 173.1$ GeV, $M_{\text{Higgs}} = 125.66$ GeV and $\alpha_s = 0.1184$ while the shaded regions correspond to $1\sigma$ uncertainty in the top mass.
The usual quantization condition for the fields and their conjugate momenta then also imposes the Wronskian condition $AA^* - A^*A = i$. For the initial amplitudes we take the WKB solution $A = \frac{1}{\sqrt{2k}}e^{-i\int \omega_k d\tau}$ at $\tau_{osc}$.

Since the fields are treated as independent the antisymmetric interaction terms vanish in the Hartree approximation and the only contribution comes from the trilinear terms so that the right hand side of equation (2.8) for the component $A_a^i$ will then give

$$-g^2 r^{\mu
u} \sum_{b \neq a} \left[ \left\langle A^b_a A^b_b \right\rangle A^a_a - \left\langle A^b_a A^b_b \right\rangle A^a_a \right].$$

(3.9)

The temporal component is tied to the longitudinal one and so is likewise not amplified. The spatial two-point function for the mass eigenstates is given by

$$\left\langle A^{(W,Z)}_i A^{(W,Z)}_j \right\rangle = \frac{\delta_{ij}}{3\pi^2} \int_0^\infty dk k^2 \left( |A^{T}_{W,Z}|^2 + \frac{1}{2} |A^{L}_{W,Z}|^2 \right) \simeq \frac{\delta_{ij}}{3\pi^2} \int_0^\infty dk k^2 |A^{T}_{W,Z}|^2$$

The transversal components therefore acquire an effective mass term induced by the non-linear interactions. The induced masses for $W$ and $Z$ bosons are respectively

$$m^2_W = \frac{2g^2 \lambda_{osc}^2}{3\pi^2} \int_0^\infty d\kappa \kappa^2 \left( |X_W|^2 + \cos \theta_W^2 |X_Z|^2 \right);$$

(3.10)

$$m^2_Z = \frac{4g^2 \lambda_{osc}^2 \cos \theta_W}{3\pi^2} \int_0^\infty d\kappa \kappa^2 |X_W|^2,$$

(3.11)

where $X \equiv (\lambda_{osc}^2)^{1/4} A$ are independent of initial field values. The integrals are in principle infinite and need to be regularized but we are only interested in the particles produced by the resonance, which are in the IR regime, so we can integrate up to the cutoff $\kappa \sim q^{1/4}$ [16, 17]. This induced mass terminates the resonance once it becomes comparable to the mass coming from the interaction with the Higgs condensate. Figure 2 compares the number of produced particles $n_k$ in the Abelian case and the case with a mass induced by non-Abelian terms, where

$$n_k = \omega_k \left( \frac{|A|^2}{\omega_k^2} + \frac{|\dot{A}|^2}{2} \right) - \frac{1}{2}.$$  

(3.12)
Note that the $Z$ boson contributes only to the effective mass of the $W$ bosons but not to its own mass. Therefore, in principle, if the charged weak bosons are in a less efficient resonance band then the produced $Z$ particles will shut down the $W$ production before it can affect the production of $Z$ particles leaving the latter unimpeded. However, we expect higher order loop effects, which are not captured by the Hartree approximation, to come into play at that point. In that case a more detailed analysis using lattice simulations is needed and we leave this for a future publication. We concentrate instead on the case where $W$ production is more efficient.

### 3.4 Back-reaction to the Higgs

The interactions among the weak gauge bosons also induce an effective mass for the Higgs condensate. From equation (2.7) this is given by

$$m_{\chi}^2 = \frac{g^2}{4} \left[ 2\langle W_{\mu}^+ W_{\mu}^- \rangle + (\cos \theta_W)^{-2} \langle Z_{\mu} Z_{\mu} \rangle \right].$$

(3.13)

Since the induced mass for the gauge bosons is $m_{W,Z}^2 \sim q m_{\chi}^2$, in the broad resonance regime the correction to the Higgs equation of motion will become important earlier than the correction to the dynamics of gauge bosons themselves. However, since the resonance is not extremely broad for the Standard Model parameters, with a typical resonance parameter given by $q \sim \mathcal{O}(10)$, the difference is rather small and the non-Abelian corrections may still require attention even during the final stages of the linear regime of the resonance. Of course, this very much depends on the actual values one adopts for the SM parameters. Figure 3 shows the comparison between different contributions to the effective masses of the fields.

Another effect that may be relevant for the dynamics of the resonance is the interaction of the produced gauge bosons with the oscillating field $\chi$, often referred to as ‘rescattering’. The oscillating field $\chi$ may be thought of as a condensate of particles at rest. A crude estimate for the importance of rescattering is given by [16], using the usual methods of scattering between $\chi$ and gauge boson particles. The time for each boson to scatter once with a particle in the condensate is

$$\tau > \sqrt{q m_{\chi}^{-1} m_{\text{osc}}^2 \left( \frac{m_{\text{osc}}}{m_{\chi}} \right)^2}$$

(3.14)

where $m_{\chi}$ is the effective mass of $\chi$ induced by the produced gauge bosons. When backreaction is negligible and $q \sim \mathcal{O}(10)$ this appears to be much larger than the oscillation time $m_{\text{osc}}^{-1}$ and rescattering becomes important only after backreaction sets in. Therefore rescattering does not appear to significantly affect our estimates for the onset of backreaction.

While it seems justified to neglect the non-Abelian terms as a first approximation under the linear stage of the resonance their impact is essential when considering the non-linear regime after the back-reaction. We plan to address the dynamics after the back-reaction in a future work using a full lattice simulation. A detailed investigation of the resonance dynamics is indeed required to determine the time when the Higgs condensate eventually has fully decayed.

### 4 Narrow resonance

As we have shown above, for a broad resonance, the produced gauge bosons backreact on the Higgs before their own evolution dynamics is significantly affected. Here we consider
Figure 3: Comparison between the different contributions to the effective masses. Blue and red curves show the effective masses induced by non-Abelian interactions for W and Z bosons respectively. The green lines correspond to the initial masses contributed from interaction with the Higgs condensate for W (dot-dashed) and Z (dotted). The purple line shows the contribution to the effective Higgs mass from produced gauge bosons while the dashed orange line is the original mass of the Higgs condensate $m_{\text{osc}} \equiv \sqrt{\lambda \chi_{\text{osc}}^2}$.

The narrow resonance regime where the opposite effect occurs. Narrow resonance may be relevant if some physics beyond the Standard Model affect the running of the couplings or if the decaying field is other than the Higgs. In these cases the couplings are no longer determined by the SM beta functions so that we may choose arbitrary values. For simplicity, let us consider $g' \ll g$ so that $q_Z \simeq q_W$ and $\cos \theta_W \simeq 1$.

Approximating $\text{cn} \left( z, 1/\sqrt{2} \right) \simeq \cos \left[ \frac{\pi z}{2K} \right]$, where $K$ is the complete elliptic integral of the first kind, and defining

$$\tilde{\kappa}^2 \equiv \frac{4K^2}{\pi^2} \left( \kappa^2 + \frac{q}{2} \right) \quad \tilde{q} \equiv \frac{K^2}{\pi^2} q \quad x \equiv \left( \frac{1}{2} + \frac{z}{2K} \right) \pi \quad (4.1)$$

we can write the equations of motion as the Mathieu equation

$$X'' + \left( \tilde{\kappa}^2 - 2\tilde{q} \cos 2x \right) X = 0. \quad (4.2)$$

Solutions are known to get amplified in a narrow bands $\tilde{\kappa}^2 \sim l^2 \pm \tilde{q} l$, where $l = 1, 2, 3, \ldots$, with the dominant contribution coming from the first such band [16]. The center of the band is at

$$\kappa_{\text{center}}^2 = \frac{x^2}{4K} - \frac{q}{2}. \quad (4.3)$$

The mass induced by the production of particles shifts the location of the band as $\kappa^2 \to \kappa^2 + \frac{m_{\text{ind}}^2}{m_{\text{osc}}}$ so as the induced mass grows, the resonance band moves towards the origin until it reaches it after which no modes are excited any more. This happens when $\frac{m_{\text{ind}}^2}{m_{\text{osc}}} \sim \frac{x^2}{4K}$ which holds true when $m_{\text{ind}}$ is of order $m_{\text{osc}} \equiv \sqrt{\lambda \chi_{\text{osc}}^2}$. However, the resonance becomes inefficient even before that because the width of the resonance band is $\sim q$ so that the band moves away from its original location when the induced mass reaches $\sim \sqrt{q} m_{\text{osc}}$ (i.e. the initial gauge boson mass). After that the resonance can no longer build on the particles that it has been producing until then. Figure 4 shows the growth of the effective mass induced by non-Abelian terms and the resulting termination of particle production. As in the broad resonance regime, the effective mass induced by non-Abelian interactions shuts down the resonance, but
Figure 4: Left: mass induced by non-Abelian interactions for the narrow resonance case $q = 0.1$ (blue solid line). The green dot-dashed line is the original gauge boson mass due to interaction with the Higgs $m_0$ and the dashed orange line is the original mass of the Higgs condensate $m_{osc} \equiv \sqrt{\lambda \chi^2_{osc}}$. Right: termination of the production of particles for $\kappa = \kappa_{center}$ by the non-Abelian effective mass.

in contrast to broad resonance this happens before the dynamics of the Higgs are affected. Therefore, non-Abelian terms should be taken into account in the narrow resonance regime already in investigating the dynamics before the onset of back-reaction.

5 Conclusions

We have studied the effects of non-Abelian interactions in the resonant decay of the Higgs condensate after inflation. We found that the self-interactions of the weak gauge fields induce an effective mass term which grows quickly and shuts down the resonance. However, for the Standard Model parameters, the non-perturbative decay of the Higgs condensate is in the intermediate to broad regime, and in this case the gauge fields backreact on the Higgs dynamics before their non-Abelian terms will come to dominate their own equations of motion. This suggests that, as a first approximation, the non-Abelian terms can be justifiably neglected before the backreaction and the system can be approximated by an Abelian one. Nevertheless, since the resonance is only moderately broad for the Standard Model couplings (unless the energy is very close to the instability scale), the time difference between the produced particles backreacting on the Higgs dynamics and on the gauge fields themselves is rather short. Therefore, the effect of non-Abelian terms may still play a role during the final stages of the linear regime before the backreaction. Moreover, the non-Abelian terms appear to play a significant role during the non-linear stage after the backreaction which eventually determines the duration and efficiency of the resonance.

We have also considered the case of narrow resonance, which can be relevant if physics beyond the Standard Model intervenes at high energies and changes the running of the couplings. This case is also pertinent if the decaying field is taken to be something other than the Higgs. For narrow resonance non-Abelian terms become important before the Higgs dynamics are significantly affected and therefore they need to be taken into account already in the linear regime before the backreaction. The effect of the non-Abelian terms is to shut down the resonance by shifting the resonance band in momentum space so that the resonance can no longer build on previously produced particles.
Since inflation inevitably generates a Higgs condensate, unless the Standard Model Higgs sector is significantly modified, a detailed understanding of the Higgs decay is an integral part in completing the knowledge of the hot big bang epoch after inflation. The decay of the Higgs condensate after inflation can have significant consequences for subsequent physics, such as phase transitions either within the SM or its modifications. Details of the Higgs decay are also highly important for scenarios where the Higgs sources curvature perturbations for example by modulating the decay of the inflaton [4, 13]. In such scenarios the Higgs should survive at least until the decay of the inflaton and termination of the resonance by non-Abelian interactions could facilitate this. Non-perturbative decay of the Higgs condensate, as well as other spectator fields, would also generate gravitational waves whose spectral features depend on structure of the decay [25].

We have considered the non-Abelian corrections in the Hartree approximation. A more detailed analysis using lattice simulations is needed in order to gain a detailed understanding of the decay process and in particular to address the dynamics after the backreaction. We leave this for a future work. We have assumed the inflaton decays slowly compared to Higgs and therefore we have not considered possible thermal effects arising from the thermal bath produced by the inflaton decay. If present, the buildup of the thermal bath typically blocks the resonance by inducing large thermal masses for the fields [21, 22]. Other thermal effects, such as dissipation of the field due to scattering off of particles in the thermal bath may also become important [26, 27]. However, previous studies have focused on scalar fields alone, and it might be interesting to consider thermal effects also within the context of resonant production of non-Abelian gauge fields.

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