Vibration of thermo lemv composite multilayered hollow pipes

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Abstract. The present paper is concerned the effects of thermo elastic waves in a composite multilayered hollow pipes which contain inner and outer zinc layers bonded by Linear Elastic Material with Voids (LEMV) is considered. The equation of movement is derived by means of the constitutive equations of linear thermo elasticity. The equation of movement and heat conduction models are decoupled by the displacement of potentials which are constructed based on equilibrium equations of elasticity. The dispersion equation are acquired by means of traction free boundary conditions and are numerically analyzed for zinc material. The enumerated frequency, phase velocity are presented graphically for LEMV and CFRP layers.

1. Introduction

The thermo pliant material contains various utilization in many extent of science, Engineering and Technology, by the propagation of waves namely study of modern engineering, thermal power station, sub marine framework, pressure thin and thick shells vessels, aircraft, oil and gas pipes and metallurgy. To replace components with classical materials such as, steel and concrete made by fiber reinforced materials. There are many attempts in engineering fields to use composites typically for the light-weight structures, on the modeling and analysis of multilayered composite. Denos Gazis[1] investigated three dimensional wave propagation in hollow circular cylinders. Jinyoung so and Leissa[2] discussed the free vibrations of thick hollow circular cylinders from three-dimensional analysis. Farhang Honarvar[3] analyzed the wave propagation in transversely isotropic cylinders. Chau[4] studied about the vibrations of transversely isotropic finite circular cylinders. Farhang Honarvar et al.[5] carried out the results for asymmetric and axisymmetric vibrations of finite transversely isotropic circular cylinders. Jai Lue Lai et al.[6] studied Propagation of harmonic waves in a composite elastic cylinder. Xi et al.[7] developed the study of dispersion of waves in immersed laminated composite hollow cylinders. Paul and Nelson [8] derived the frequency of equation and numerical solution for asymmetric vibration of piezoelectric composite cylinders. Nelson and Karthikeyan [9] analyzed the axisymmetric vibration of pyro composite hollow cylinders. Haines and Lee [10] carried out approximate theory of torsional wave propagation in elastic circular composite cylinders. Green and Lindsay [11] obtained an explicit version of the constitutive equations. Gei, Bigoni and Franceschin [12] discussed about longitudinal wave propagation in a generalized thermoelastic cylinder. They revealed that the elastic behavior dominates in mechanical and thermal modes. Eli Leinov et al.[14, 15] investigated guided wave propagation and attenuation in pipe buried in sand and guided wave propagation in pipes fully and partially embedded in concrete.
In the present analysis, we investigated the effects of thermo elastic waves in a multilayered composite hollow pipe consist of inner and outer zinc layers bonded together with the Linear Elastic Material with Voïds (LEMV). We got the equations of movement by means of constitutive equation of a linear thermo elastic material. The frequency of equations that includes both cylindrical layers and bonding material are derived with the presence of stress free boundary conditions. The calculations are executed for zinc material and the enumerated frequency, phase velocity are presented as the dispersal curves.

2. Conceptualization of problem

we deal with a homogeneous transversely isotropous thermally conducting composite multilayered hollow pipes of limitless length with constant temperature \(T_0\) in an undisturbed state at the beginning. Geometry of the problem shown in figure 1. The radial, tangential and axial displacement components represented by \(u_r, u_\theta\) and \(u_z\) respectively which are identified through the cylindrical axes \(r, \theta, z\). The guiding equation of movement is [8]

\[
\begin{align*}
&c_{11}(u'_{rr} + r^{-1}u'_r - r^{-2}u'_r) - r^{-2}(c_{11} + c_{66})v'_{\theta} + r^{-2}c_{66}u'_{\theta \theta} \\
+&c_{44}u'_{zz} + (c_{44} + c_{13})w'_r + r^{-1}(c_{66} + c_{12})u'_r - \beta_1 T'_r = \rho' u'_{tt} \\
(r^{-1})&(c_{12} + c_{66})u'_{r \theta} + r^{-2}(c_{66} + c_{11})u'_{\theta \theta} + c_{66}(v'_{rr} + r^{-1}v'_r - r^{-2}v'_r) \\
+&r^{-2}c_{11}u'_{\theta \theta} + c_{44}v'_r + r^{-1}(c_{44} + c_{13})w'_r - r^{-1}\beta_1 T'_\theta = \rho' w'_{tt} \\
c_{44}(u'_{rr} + r^{-1}u'_r - r^{-2}u'_r) + r^{-1}(c_{44} + c_{13})(u'_{zz} + v'_r) \\
+&(c_{44} + c_{13})u'_r + c_{33}w'_r - \beta_3 T'_z = \rho' w'_{tt}
\end{align*}
\]

\[
K_{11}(T'_{rr} + r^{-1}T'_{r} + r^{-2}T'_{\theta \theta}) + K_{33}T'_{zz} - \rho' c_\nu T'_{tt} = T_0[\beta_1(u'_{rt} + r^{-1}u'_{r} + r^{-1}v'_{\theta \theta}) + \beta_3 w'_{rt}]
\]

(1)

3. Solution of the field equation

Equation(1) is a collateral Partial differential equation along with displacement and heat conduction inherent. In order to separate the Equation(1) we followed the solutions by the tremor and displacements together with the axial direction is equal to zero, and speculate the solutions of Equation(1) in the form,

\[
\begin{align*}
&u^l = \sum_{n=1}^{\infty} \varepsilon_n[(\phi_{n,r} + r^{-1}\psi_{n,\theta}) + (\bar{\phi}_{n,\theta} + r^{-1}\bar{\psi}_{n,\theta})] \exp ikz + wt \\
v^l = \sum_{n=1}^{\infty} \varepsilon_n[(r^{-1}\phi_{n,r} - \psi_{n,\theta}) + (r^{-1}\bar{\phi}_{n,\theta} - \bar{\psi}_{n,\theta})] \exp ikz + wt \\
w^l = \frac{i}{\alpha} \sum_{n=1}^{\infty} \varepsilon_n(W + \bar{W}) \exp ikz + wt
\end{align*}
\]
\[ T^l = \frac{c_{44}}{\beta_0 a^2} \sum_{n=0}^{\infty} \varepsilon_n(T_n + \tilde{T}_n) \exp(i(kz + wt)) \]  

(2)

where \( \varepsilon_n = 0.5 \), if \( n \) is zero, \( \varepsilon_n = 1 \) if \( n \geq 0 \), \( i = \sqrt{-1} \), \( k \) denotes wave number, \( \omega \) denotes frequency. Further \( \phi_n(r, \theta), \ W_n(r, \theta) \) and \( \psi_n(r, \theta) \) are the displacement potentials, \( T_n(r, \theta) \) denotes the temperature change and \( a \) denotes geometrical parameter of the hollow pipes.

By launching the no conventionally measurable variables such as \( \bar{c}_{11}, \bar{c}_{13} = \frac{c_{44}}{c_{44}}, \bar{c}_{34} = \frac{c_{44}}{c_{44}}, \bar{c}_{66} = \frac{c_{44}}{c_{44}}, \bar{\beta} = \frac{\beta_1}{\beta_3}, \bar{k}_i = \frac{(pc_{44})^{\frac{1}{2}} K_1}{\beta_3 T_{0o} \alpha} \) \((i = 1, 3)\)

\[ d = \frac{pc_{44}}{(\beta_3 T_0)}, \ T_n = \frac{1}{2} \sqrt{\frac{c_{44}}{\rho}}, \ \bar{\varepsilon} = \frac{\bar{\varepsilon}}{a} \]

\[ [\bar{c}_{11}\bar{\nabla}^2 + (\Omega^2 - \bar{\varepsilon}^2)]\phi_n - \bar{\varepsilon} (1 + \bar{c}_{13}) W_n - \bar{\beta} T_n = 0 \\
\varepsilon (1 + \bar{c}_{13}) \bar{\nabla}^2 \phi_n + [\bar{\nabla}^2 + (\Omega^2 - \bar{c}_{33}\varepsilon^2)] W_n - \varepsilon T_n = 0 \\
\bar{\beta} \bar{\nabla}^2 \phi_n - \varepsilon W_n + (d + ik_1 \bar{\nabla}^2 - i\bar{k}_3 \varepsilon^2) T_n = 0 \]

(3)

And

\[ (\bar{\nabla}^2 + \frac{(\Omega^2 - \varepsilon^2)}{\bar{c}_{66}}) \psi = 0 \]

(4)

The determinant of Equations\((3)\) is equal to zero, if a nontrivial solution of the algebraic equations systems \((1)\) exist.

\[ \begin{vmatrix} \bar{c}_{11}\bar{\nabla}^2 + (\Omega^2 - \varepsilon^2) & -\varepsilon (1 + \bar{c}_{13}) & -\bar{\beta} \\
\varepsilon (1 + \bar{c}_{13}) \bar{\nabla}^2 & \bar{\nabla}^2 + (\Omega^2 - \bar{c}_{33}\varepsilon^2) & -\varepsilon \\
-\bar{\beta} \bar{\nabla}^2 & -\varepsilon & (d + ik_1 \bar{\nabla}^2 - i\bar{k}_3 \varepsilon^2) \end{vmatrix} (\phi_n^l, W_n^l, T_n^l)^T = 0 \]

(5)

The above Equations\((5)\) is simplified and reduced as follows

\[ (A^l \bar{\nabla}^2 + B^l \bar{\nabla}^4 + C^l \bar{\nabla}^2 + D^l)(\phi_n^l, W_n^l, T_n^l)^T = 0 \]

(6)

The solution of Equation \((6)\) is obtained as

\[ \phi_n^l = \sum_{j=1}^{3} [A^l_j J_n(\alpha^l_j x) + B^l_j Y_n(\alpha^l_j x)] , \]

\[ W_n^l = \sum_{j=1}^{3} d^l_j [A^l_j J_n(\alpha^l_j x) + B^l_j Y_n(\alpha^l_j x)] , \]

\[ T_n^l = \sum_{j=1}^{3} e^l_j [A^l_j J_n(\alpha^l_j x) + B^l_j Y_n(\alpha^l_j x)] \]

(7)

where \( (\alpha^l_j a)^2 > 0 \) \((j = 1, 2, 3)\) are the solutions of the equation

\[ A^l (\alpha^l_j a)^6 + B^l (\alpha^l_j a)^4 + C^l (\alpha^l_j a)^2 + D^l = 0 \]

(8)

Here \( J_n \) denotes the Bessel function of the first kind of order \( n \). The constants \( d^l_j \) and \( e^l_j \) described in the Equation \((7)\) can be calculated from the given relation

\[ \varepsilon (1 + \bar{c}_{13}) d_j + \bar{\beta} e_j = -[\bar{c}_{11}(\alpha^l_j a)^2 - \Omega^2 + \varepsilon^2] \\
[\Omega^2 - \bar{c}_{33}\varepsilon^2 - (\alpha^l_j a)^2] d_j - \varepsilon e_j = (\alpha^l_j a)^2 (1 + \bar{c}_{13}) \varepsilon \]

Solving Equation\((4)\), the calculated solution is

\[ \psi_n^l = [A^l_{4n} J_n(\alpha^l_j x) + B^l_{4n} Y_n(\alpha^l_j x)] \]

(9)

where \( (\alpha^l_j a)^2 = \Omega^2 - \varepsilon^2 \). The Bessel function \( J_n \) is altered by the Modified Bessel function \( I_n \). If \( (\alpha^l_j a)^2 \leq 0 \).
4. Equations of Motion of Linearly Elastics Material with Void (LEMV)

The governing equations for isotropic LEMV materials are given as

\[ \nu^l \nabla^2 \bar{u}^l + (\lambda^l + \nu^l) \nabla \nabla \cdot \bar{u}^l = \rho^l \ddot{u}^l, \tag{10} \]

where \( \bar{u}^l \) is the displacement potential, \( \lambda^l = C_{12} \), \( \nu^l = C_{11} - C_{12} / 2 \) are lame’s constant, \( \rho^l \) denotes the mass density and \( t \) denotes time. The solution of above equation is taken as

\[ u^l = (u^l_r) \exp(kz + pt), \quad w^l = (\frac{i}{a}) w^l \exp(kz + pt) \tag{11} \]

The above solution and no conventionally measurable variable \( x \) and \( \epsilon \), in Equation(10) can be reduced as follows

\[ (\bar{\lambda}^l + 2\bar{\nu}^l) \nabla^2 + G_1^l \nabla^2 - G_2^l \nabla^2 + G_4^l (u^l, w^l) = 0 \tag{12} \]

The solution (13) are as follows

\[ u^l = \sum_{j=1}^{3} \left[ A^l_j J_n(\alpha^l_j x) + B^l_j Y_n(\alpha^l_j x) \right] \]

\[ w^l = \sum_{j=1}^{3} d^l_j \left[ A^l_j J_n(\alpha^l_j x) + B^l_j Y_n(\alpha^l_j x) \right] \]

where \( (\alpha^l_j)^2 \) is the nonzero roots of \( (\alpha^l_j)^4 + P^l(\alpha^l_j)^2 - Q^l = 0 \), and \( d^l_j \) is arbitrary constant. Its obtained from

\[ d^l_j = \frac{- (\bar{\lambda}^l + 2\bar{\nu}^l)(\alpha^l_j)^2 + G_1^l}{G_2^l} \]

5. Interface Boundary Conditions and Frequency Equations

The following confines and interface circumstance are used to obtain the frequency equation.

(i) On the traction free inner layer and outer layer

\[ \sigma^l_{rr} = \sigma^l_{rz} = T^l = 0 \quad \text{with} \quad l = 1 \text{ and } 5. \tag{14} \]

(ii) At the interaction between (outer and center layers and center and inner layers) hollow pipes

\[ \sigma^l_{rr} = \sigma^l_{rr}, \quad \sigma^l_{rz} = \sigma^l_{rz}, \quad \phi^l = \phi, \quad W^l = W, \quad T^l = 0, \quad \text{with} \quad l = 1, 2, 3, 4, 5. \tag{15} \]

Here mentioned system of equation by replacing the solutions in the boundary interface circumstances are enumerated by the frequency equations. It is denoted as follows,

\[ |(Y_{i,j})| = 0, \quad (i, j = 1, 2, 3, \ldots, 26) \tag{16} \]
By changing \( j \) from 1 to 3 and \( k \) from 1 to 2 , we got the following values:

\[
Y(1, j) = \left[ 2\mu \left( \frac{\alpha_j}{x_0} \right) J_{n+1}(\alpha_j x_0) + \left[ -(\lambda' + 2\nu)(\alpha_j)^2 \beta e_i - \lambda \varepsilon d_j \right] \right] + 2n(n - 1) \left( \frac{\nu}{x_0} \right) J_n(\alpha_j x_0)
\]

\[
Y(2, j) = (\varepsilon + d_j + \bar{k}_1 \varepsilon_j)(\alpha_j) J_{n+1}(\alpha_j x_0),
\]

\[
Y(3, j) = e_j J_{n+1}(\alpha_j x_0).
\]

At \( x_1 = \frac{a_1}{a} \),

\[
Y(4, j) = \left[ 2\nu \left( \frac{\alpha_j^1}{x_1} \right) J_{n+1}(\alpha_j^1 x_1) + \left[ -(\lambda + 2\nu)(\alpha_j^1)^2 \beta e_j^1 - \lambda \varepsilon e_j^1 \right] \right] + 2n(n - 1) \left( \frac{\nu}{x_1^1} \right) J_n(\alpha_j^1 x_1),
\]

\[
Y(4, k + 6) = - \left[ 2\nu \left( \frac{\alpha_j^2}{x_1} \right) J_{n+1}(\alpha_j^2 x_1) + \left[ -(\lambda + 2\nu)(\alpha_j^2)^2 \beta e_j^2 - \lambda \varepsilon e_j^2 \right] \right] + 2n(n - 1) \left( \frac{\nu}{x_1^2} \right) J_n(\alpha_j^2 x_1),
\]

\[
Y(5, j) = (\varepsilon + d_j^2 + \bar{k}_2 \varepsilon_j^2)(\alpha_j^2) J_{n+1}(\alpha_j^2 x_1),
\]

\[
Y(5, k + 6) = - \nu \left( -(\varepsilon + d_j^1) \right) \left( \alpha_j^1 \right) J_{n+1}(\alpha_j^1 x_1),
\]

\[
Y(6, j) = -(\alpha_j^1) J_{n+1}(\alpha_j^1 x_1),
\]

\[
Y(6, k + 6) = -(\alpha_j) J_{n+1}(\alpha_j x_1),
\]

\[
Y(7, j) = d_j^1 J_{n+1}(\alpha_j x_1),
\]

\[
Y(7, k + 6) = -d_j J_{n+1}(\alpha_j x_1),
\]

\[
Y(8, j) = e_j^1 J_{n+1}(\alpha_j^1 x_1)
\]

and the remaining elements are clarified by the same from the above relations. The frequency equation mentioned above are enumerated for distinct solid middle and outer multilayered hollow materials of zinc and arbitrary density of layers.

6. Numerical investigations

The free vibration of transversely isotropic thermo elastic composite multi layered hollow pipes is considered. Thus the frequency equation is numerically carried out for the material Zinc and their material properties are given below:

\[
c_{11} = 1.628 \times 10^{11} Nm^{-2}, c_{12} = 0.362 \times 10^{11} Nm^{-2}, c_{13} = 0.508 \times 10^{11} Nm^{-2}, c_{33} = 0.627 \times 10^{11} Nm^{-2}, c_{44} = 0.385 \times 10^{11} Nm^{-2}, \text{ and density } \rho = 7.14 \times 10^3 kgm^{-3}, \beta_1 = 5.75 \times 10^6 Nm^{-2} deg^{-1}, \beta_3 = 5.17 \times 10^6 Nm^{-2} deg^{-1}, C_v = 3.9 \times 10^2 Jkg^{-1} K^{-1}, K_1 = K_3 = 1.24 \times 10^2 Wm^{-1} deg^{-1}, T_0 = 296^o K.
\]

In this investigation, there are two types of basic liberated modes of wave propagation have been noted namely; the longitudinal and flexural modes. We can acquire the dimensionless frequencies of longitudinal and flexural modes of vibrations by choosing it respectively \( n = 0 \) and \( n = 1 \). Figure 2 and 3 presents the variations of the dimensionless frequency of a elastic multilayered elastic hollow pipes against to the wave number \( |k| \) for the different pasting materials such as LEMV (Linear elastic material with void) and CFRP (Carbon Fiber Reinforced Plastic). From figure 2, it is noticed that the dimensionless frequency of the thermo elastic hollow pipes shows nearly linear variation against to wave number. But in figure 3 some deviating nature
Figure 2. Dispersal of dimensionless frequency against dimensionless wave number of multilayered hollow pipe without thermal environment.

Figure 3. Dispersal of dimensionless frequency against dimensionless wave number of multilayered hollow pipe with thermal environment.

is noted between $0 \leq |k| \leq 2$ from these linear nature of frequency due to the damping effect of bonding layer and the thermal effects is more prominent in lower wave number. From figures 2 and 3, it is noticed that the dimensionless frequency mode is increasing and also travel in wave propagation in all the interfacial bonding layer.

The disparity of the phase velocity against the wave number of the multilayered elastic hollow pipes is put on display in figures 4 and 5. From this curves it is apparent that the phase velocity are dispersal only at the starting values of wave number in the territory $0 \leq |k| \leq 0.2$, but for highest values of wave number, these turn into non-dispersive for different interfacial material values.
7. Conclusion
The vibrating waves of thermo elastic multilayered hollow pipes with isotropic LEMV bonding layers by the frequency equations are derived. The equations of movement of the composite multilayered hollow pipes are built means of the constitutive equations of a liner thermo elastic material. The frequency equations that contain the combine between the composite hollow pipes and bonding layer are acquired for Zinc/LEMV/Zinc/CFRP/Zinc/CFRP/Zinc/LEMV/Zinc by the stress free boundary conditions. The numerical investigation are carried out for single layered thermo elastic hollow pipes with LEMV and CFRP cores and the calculated dimensionless frequency and phase velocity are plotted as the dispersion curves. It is detected from the graphical pattern that the frequency and phase velocity is high in CFRP core as compared with LEMV without voids. Also, the void core and thermal interactions are highly influencing the damping effect of the system.
8. References

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