Recent results on the cluster structure of light nuclei

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Abstract. A recently developed model (ACM) is introduced and applied to the study of $k\alpha$ structures with $k=2$ ($^8$Be), $k=3$ ($^{12}$C) and $k=4$ ($^{16}$O). Evidence for $Z_2$ ($k=2$), $D_3$ ($k=3$) and $T_d$ ($k=4$) symmetry is presented. An extension (ACFM) of the model to $k\alpha+x$ (neutrons, protons) structures is briefly mentioned and applied to the study of $^7$Be ($k=2, x=1$).

1. Introduction

The cluster structure of light nuclei has been the subject of many investigations since the seminal work of Wheeler [1]. Further work of Dennison [2] and others, culminated in the study of Brink [3] who suggested specific geometric configurations for $k\alpha$ nuclei ($k=2$-7) based on “microscopic” calculations, the so-called Brink-Bloch model. Since then, many theoretical approaches have been introduced and used to study cluster properties of light nuclei. Recently, renewed interest in clustering has arisen from new experiments, especially in $^{12}$C, where additional states, predicted by cluster models have been observed [4]. A review of recent experimental results is given in These Proceedings [5].

Open questions in cluster physics are: (i) what are the signatures of specific geometric configurations, (ii) are these configurations rigid or soft, (iii) how far clustering extends in excitation energy, and (iv) how far clustering extends in mass number. In order to answer these questions, especially the first two, we have recently developed an algebraic approach to clustering based on the algebraic theory of molecules introduced in 1981 [6]. For $k\alpha$ structures, the approach amounts to a bosonic quantization of the Jacobi variables in terms of the algebra of $U(3k-2)$, where $k$ is the number of constituents. Some results for $k=2$, will be reported in this contribution.

2. Algebraic cluster model (ACM)

This model describes cluster configurations of any type in terms of the Lie algebra $U(3k-2)$. An explicit construction of the algebra has been completed for $k=2,3,4$ and is summarized in Table 1.

| $k$  | Nucleus | $U(3k-2)$ | Jacobi variables | Discrete symmetry |
|------|---------|-----------|------------------|-------------------|
| $2\alpha$ | $^8$Be | $U(4)$ [6,7] | $\rho$ | $Z_2$ |
| $3\alpha$ | $^{12}$C | $U(7)$ [8,9] | $\rho, \lambda$ | $D_3$ |
| $4\alpha$ | $^{16}$O | $U(10)$ [10,11,12] | $\rho, \lambda, \eta$ | $T_d$ |

Table 1. Algebraic description of $k\alpha$ structures
A major advantage of ACM is that it is possible to derive analytical results that provide benchmarks for experiments.

2.1. Energy formulas. For rigid structures, explicit expressions for energy levels in terms of the angular momentum \( L \) and the vibrational quantum numbers \( v_i \) (i=1,2,\ldots), in a semi-classical approximation to ACM, are:

\[
E(v, L) = E_0 + \omega \left( v + \frac{1}{2} \right) + BL(L + 1)
\]

\[
E(v_1, v_2, L, K) = E_0 + \omega_1 \left( v_1 + \frac{1}{2} \right) + \omega_2 (v_2 + 1) + BL(L + 1) + (C - B)(K + 2f_2)^2
\]

\[
E(v_1, v_2, v_3, L) = E_0 + \omega_1 \left( v_1 + \frac{1}{2} \right) + \omega_2 (v_2 + 1) + \omega_3 \left( v_3 + \frac{3}{2} \right) + BL(L + 1)
\]

for \( 2\alpha \) (\( Z_2 \)), \( 3\alpha \) (\( D_3 \)) and \( 4\alpha \) (\( T_d \)) respectively, where the \( \omega \)'s, B and C are parameters.

Spectra are characterized by representations of the discrete group \( G \) and consist in a set of rotation-vibration bands, with specific values of the angular momentum and parity. Representations can be labeled either by \( G \) or by the isomorphic group \( S_n \), the permutation group. The conversion from \( G \) to \( S_n \) is: \( G=Z_2 \sim S_2 \sim P, A \equiv [2]; G=D_3 \sim S_3, A \equiv [3], E \equiv [21]; G=T_d \sim S_4, A \equiv [4], F \equiv [31], E \equiv [22] \).

For \( 2\alpha \) with \( Z_2 \) symmetry, the spectrum consists of a set of rotation-vibration bands with A symmetry and \( L^P = 0^+, 2^+, 4^+, \ldots \)

For \( 3\alpha \) with \( D_3 \) symmetry, the spectrum consists of a set of rotation-vibration bands with A and E symmetry as shown in Fig.1. For A-representations, the rotational band has \( L^P = 0^+, 2^+, 3^-, 4^+, \ldots \) while for E-representations the rotational band has \( L^P = 1^-, 2^+, 3^+, \ldots \). Note the unusual angular momentum and parity content of the rotational bands and the parity doubling.

\[\text{Figure 1. Allowed states of } 3\alpha \text{ with } D_3 \text{ symmetry}\]

For \( 4\alpha \) with \( T_d \) symmetry the spectrum consists of a set of rotation-vibration bands with A, E and F symmetry as shown in Fig.2. For A-representations, the rotational band has \( L^P = 0^+, 3^-, 4^+, 6^+, \ldots \) while for E-representations it has \( L^P = 2^+, 4^+, 5^+, \ldots \) and for F-representations \( L^P = 1^-, 2^+, 3^+, \ldots \). Note again the unusual angular momentum and parity content and the parity doubling.
The occurrence of $D_3$ symmetry in $^{12}$C has been confirmed by recent experiments [4, 13] and is reported in These Proceedings [14]. The occurrence of $T_d$ symmetry in $^{16}$O was discussed long ago by Robson [15], and it has been emphasized recently in [12, 16]. The evidence is shown in Fig.3.

2.2. Electromagnetic transition rates. In the rigid limit, explicit expressions for B(EL) values along the ground state band can be derived. They are:
for $2\alpha$, $3\alpha$ and $4\alpha$ respectively. Here $\beta$ is the displacement of each $\alpha$ particle from the center of mass and $Z$ is the total charge. These expressions are compared with experiment in Tables 2, 3 and 4 for $^8$Be ($k=2$), $^{12}$C ($k=3$) and $^{16}$O ($k=4$). The measured B(EL) values provide strong evidence for close-packed clustering, $Z_2$, $D_3$, $T_4$, in the ground state bands of $^8$Be, $^{12}$C, $^{16}$O.

$$B(EL;0 \rightarrow L) = \left( \frac{Ze^2}{2} \right)^2 \frac{2L+1}{4\pi} \left[ 2 + 2P_l(-1) \right]$$

$$B(EL;0 \rightarrow L) = \left( \frac{Ze^2}{3} \right)^2 \frac{2L+1}{4\pi} \left[ 3 + 6P_l(-\frac{1}{2}) \right]$$

$$B(EL;0 \rightarrow L) = \left( \frac{Ze^2}{4} \right)^2 \frac{2L+1}{4\pi} \left[ 4 + 12P_l(-\frac{1}{3}) \right]$$

For $2\alpha$, $3\alpha$ and $4\alpha$ respectively. Here $\beta$ is the displacement of each $\alpha$ particle from the center of mass and $Z$ is the total charge. These expressions are compared with experiment in Tables 2, 3 and 4 for $^8$Be ($k=2$), $^{12}$C ($k=3$) and $^{16}$O ($k=4$). The measured B(EL) values provide strong evidence for close-packed clustering, $Z_2$, $D_3$, $T_4$, in the ground state bands of $^8$Be, $^{12}$C, $^{16}$O.

| B(EL;L\rightarrow 0^+ ) | Th   | Exp   | E(L\beta) | Th   | Exp   |
|------------------------|------|-------|-----------|------|-------|
| B(E2;2\rightarrow 0^+ ) | 20.4 | 21±2.3 | (23.0(2.5) W.u.) | E(2\beta) | 3060 | 3030 |
| B(E4;4\rightarrow 0^+ ) | 326.1|       |           | E(4\beta) | 10200 | 11350 |

Table 2. B(EL) values and energies in $^8$Be compared to those expected from $Z_2$ symmetry. B(EL) values in e$^2$fm$^{2L}$, $E$ in keV. The theoretical energies are calculated using $E$(keV)$=510L(L+1)$. The experimental value for $B(E2;2\rightarrow 0^+)$ is estimated from radiative capture [17].

| B(EL;L\rightarrow 0^+ ) | Th   | Exp   | E(L\beta) | Th   | Exp   |
|------------------------|------|-------|-----------|------|-------|
| B(E2;2\rightarrow 0^+ ) | 9.3  | 7.6±0.4 | (4.65(26) W.u.) | E(2\beta) | 4400 | 4439 |
| B(E3;3\rightarrow 0^+ ) | 84   | 103±17 | (12(2) W.u.) | E(3\beta) | 9640 | 9641 |
| B(E4;4\rightarrow 0^+ ) | 68   |       |           | E(4\beta) | 14670 | 14080 |

Table 3. B(EL) values and energies in $^{12}$C compared to those expected from $D_3$ symmetry. B(EL) values in e$^2$fm$^{2L}$, $E$ in keV. The theoretical energies are calculated using $E$(keV)$=730L(L+1)$. The value of $\beta=1.9$fm is estimated from the elastic form factor measured in electron scattering.

| B(EL;L\rightarrow 0^+ ) | Th   | Exp   | E(L\beta) | Th   | Exp   |
|------------------------|------|-------|-----------|------|-------|
| B(E3;3\rightarrow 0^+ ) | 181  | 205±10 | (13.6(7) W.u.) | E(3\beta) | 6132 | 6130 |
| B(E4;4\rightarrow 0^+ ) | 338  | 378±133 | (3.7(1.3) W.u.) | E(4\beta) | 10220 | 10356 |
| B(E6;6\rightarrow 0^+ ) | 8245 |       |           | E(6\beta) | 21462 | 21052 |

Table 4. B(EL) values and energies in $^{16}$O compared to those expected from $T_4$ symmetry. B(EL) values in e$^2$fm$^{2L}$, $E$ in keV. The theoretical energies are calculated from $E$(keV)$=511L(L+1)$. The value of $\beta=2.0$fm is extracted from the elastic form factor in electron scattering.

2.3. Form factors in electron scattering. In the rigid limit and semi-classical approximation, the form factors are given by $F_{L}(0 \rightarrow L)=c_{L}(\beta \beta)$. A discussion of these form factors and comparison with experiment is given in the accompanying contribution of Bijker [18].

3. Current work
An important question is to what extent clustering persists when additional particles are added to $k\alpha$ structures. To this end we have initiated a study of $k\alpha+x$ neutrons (protons) structures (ACFM model). These structures were suggested by von Oertzen [19] and have been the subject of many investigations. The ACM offers a simple way to calculate single particle levels in a field with discrete symmetry on top of which rotation-vibration bands are built. Calculations are currently being done [20] for $^7$Be, $^7$B.
(2α+1neutron, proton); $^{13}$C, $^{13}$N (3α+1neutron, proton); and $^{17}$O, $^{17}$F (4α+1 neutron, proton). Preliminary results for cluster rotational bands in $^9$Be are shown in Fig.4. It should be noted that no additional parameter is required in the calculation except for the value $β=2.6$ fm. This value is larger than the value $β=2.0$ fm obtained from the moment of inertia of $^8$Be due to the finite extent of the nucleon-alpha interaction.

Figure 4. Cluster rotational bands in $^9$Be

4. Summary and conclusions
A model based on symmetry has been developed (ACM) within which one can derive analytic expressions for energy levels, electromagnetic transition rates and form factors in kα nuclei. Evidence has been presented for k=2, 3, 4 ($^8$Be, $^{12}$C, $^{16}$O). Clustering appears to be robust for the ground state band and somewhat soft for vibrational bands. A model (ACFM) is currently being developed to derive analytic expressions for energy levels and electromagnetic transition rates in kα+x (neutrons, protons). Evidence has been presented for k=2, x=1. The development of ACM and ACFM opens the way for a renewed study of clustering in light nuclei.

Current answers to the questions posed in the introduction are: (i) signatures of clustering are properties of spectra and electromagnetic transition rates, especially parity doubling, unusual structures of rotational bands, and enhanced E2, E3, E4, E5, ... transition rates; (ii) clustering appears to extend at least to angular momentum J=6 in k=2, 3, 4 systems and up to excitation energies of the order of 25MeV; (iii) clustering appears to extend to A=16; (iv) clustering appears to survive the coupling of fermions, at least up to x=1.

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