Kaons and antikaons in asymmetric nuclear matter

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Abstract

The properties of kaons and antikaons and their modification in isospin asymmetric nuclear matter are investigated using a chiral SU(3) model. These isospin dependent medium effects are important for asymmetric heavy ion collision experiments. In the present work, the medium modifications of the energies of the kaons and antikaons, within the asymmetric nuclear matter, arise due to the interactions of kaons and antikaons with the nucleons and scalar mesons. The values of the parameters in the model are obtained by fitting the saturation properties of nuclear matter and kaon-nucleon scattering lengths. The pion-nucleon scattering lengths are also calculated within the chiral effective model and compared with earlier results from the literature. The density dependence of the isospin asymmetry is seen to be appreciable for the kaon and antikaon optical potentials. This can be particularly relevant for the future accelerator facility FAIR at GSI, where experiments using neutron rich beams are planned to be used in the study of compressed baryonic matter.

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I. INTRODUCTION

The study of the properties of hadrons in hot and dense matter is an important topic in strong interaction physics. This subject has direct implications for heavy-ion collision experiments, in the study of astrophysical compact objects (like neutron stars) as well as in the early universe. The in-medium properties of kaons have been investigated particularly because of their relevance for neutron star phenomenology as well as relativistic heavy-ion collisions. For example, in the interior of a neutron star the attractive kaon-nucleon interaction might lead to kaon condensation as originally suggested by Kaplan and Nelson [1]. The in-medium modification of kaon/antikaon properties can be observed experimentally primarily in relativistic nuclear collisions. Indeed, the experimental [2, 3, 4, 5, 6] and theoretical studies [7, 8, 9, 10, 11, 12, 13, 14, 15, 16] of $K^\pm$ production in A+A collisions at SIS energies of 1-2 A-GeV have shown that in-medium properties of kaons can be connected to the collective flow pattern of $K^+$ mesons as well as to the abundance and spectra of antikaons.

The theoretical research work on the topic of medium modification of hadron properties was initiated by Brown and Rho [17] who suggested that the modifications of hadron masses should scale with the scalar quark condensate $\langle \bar{q}q \rangle$ at finite baryon density. The first attempts to extract the antikaon-nucleus potential from the analysis of kaonic-atom data were in favor of very strong attractive potentials of the order of -150 to -200 MeV at normal nuclear matter density $\rho_0$ [18, 19]. However, more recent self-consistent calculations based on a chiral Lagrangian [20, 21, 22, 23] or coupled-channel G-matrix theory (within meson-exchange potentials) [24] only predicted moderate attraction with potential depths of -50 to -80 MeV at density $\rho_0$.

The problem with the antikaon potential at finite baryon density is that the antikaon-nucleon amplitude in the isospin channel $I = 0$ is dominated by the $\Lambda(1405)$ resonant structure, which in free space is only 27 MeV below the $\bar{K}N$ threshold. It is presently not clear if this physical resonance is an excited state of a ‘strange’ baryon or some short lived molecular state that, for instance, can be modelled in a coupled channel $T$-matrix scattering equation using a suitable meson-baryon potential. Additionally, the coupling between the $\bar{K}N$ and $\pi Y$ ($Y = \Lambda, \Sigma$) channels is essential to get the proper dynamical behavior in free space. Correspondingly, the in-medium properties of the $\Lambda(1405)$, such as its pole position...
and its width, which in turn strongly influence the antikaon-nucleus optical potential, are very sensitive to the many-body treatment of the medium effects. Previous works have shown that a self-consistent treatment of the $\bar{K}$ self energy has a strong impact on the scattering amplitudes \[14, 20, 22, 23, 24, 25\] and thus on the in-medium properties of the antikaons. Due to the complexity of this many-body problem the actual kaon and antikaon self energies (or potentials) are still a matter of debate.

The topic of isospin effects in asymmetric nuclear matter has gained interest in the recent past \[26\]. The isospin effects are important in isospin asymmetric heavy-ion collision experiments. Within the UrQMD model the density dependence of the symmetry potential has been studied by investigating observables like the $\pi^-/\pi^+$ ratio, the n/p ratio \[27\], the $\Delta^-/\Delta^{++}$ ratio as well as the effects on the production of $K^0$ and $K^+$ \[28\] and on pion flow \[29\] for neutron rich heavy ion collisions. Recently, the isospin dependence of the in-medium NN cross section \[30\] has also been studied.

In the present investigation we will use a chiral SU(3) model for the description of hadrons in the medium \[31\]. The nucleons – as modified in the hot hyperonic matter – have previously been studied within this model \[32\]. Furthermore, the properties of vector mesons \[32, 33\] – modified by their interactions with nucleons in the medium – have also been examined and have been found to have sizeable modifications due to Dirac sea polarization effects. The chiral SU(3)$_{flavor}$ model was generalized to SU(4)$_{flavor}$ to study the mass modification of D-mesons arising from their interactions with the light hadrons in hot hadronic matter in \[34\]. The energies of kaons (antikaons), as modified in the medium due to their interaction with nucleons, consistent with the low energy KN scattering data \[35, 36\], were also studied within this framework \[37, 38\]. In the present work, we investigate the effect of isospin asymmetry on the kaon and antikaon optical potentials in the asymmetric nuclear matter, consistent with the low energy kaon nucleon scattering lengths for channels I=0 and I=1. The pion nucleon scattering lengths are also calculated.

The outline of the paper is as follows: In section II we shall briefly review the SU(3) model used in the present investigation. Section III describes the medium modification of the $K(\bar{K})$ mesons in this effective model. In section IV, we discuss the results obtained for the optical potentials of the kaons and antikaons and the isospin-dependent effects on these optical potentials in asymmetric nuclear matter. Section V summarizes our results and discusses possible extensions of the calculations.
II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

In this section the various terms of the effective hadronic Lagrangian used

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{\text{SB}}$$

are discussed. Eq. (1) corresponds to a relativistic quantum field theoretical model of baryons and mesons adopting a nonlinear realization of chiral symmetry [39, 40, 41] and broken scale invariance (for details see [31, 32, 33]) to describe strongly interacting nuclear matter. The model was used successfully to describe nuclear matter, finite nuclei, hyper-nuclei and neutron stars. The Lagrangian contains the baryon octet, the spin-0 and spin-1 meson multiplets as the elementary degrees of freedom. In Eq. (1), $\mathcal{L}_{\text{kin}}$ is the kinetic energy term, $\mathcal{L}_{BW}$ contains the baryon-meson interactions in which the baryon-spin-0 meson interaction terms generate the baryon masses. $\mathcal{L}_{\text{vec}}$ describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields. $\mathcal{L}_0$ contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential. $\mathcal{L}_{SB}$ describes the explicit chiral symmetry breaking.

The kinetic energy terms are given as

$$\mathcal{L}_{\text{kin}} = i \text{Tr} \bar{B} \gamma_\mu D^\mu B + \frac{1}{2} \text{Tr} D_\mu X D^\mu X + \text{Tr}(u_\mu X u^\mu X + X u_\mu u^\mu X) + \frac{1}{2} \text{Tr} D_\mu Y D^\mu Y$$

$$+ \frac{1}{2} D_\mu \chi D^\mu \chi - \frac{1}{4} \text{Tr} (\bar{V}^{\mu\nu}) - \frac{1}{4} \text{Tr} (F^{\mu\nu}F^{\mu\nu}) - \frac{1}{4} \text{Tr} (A_{\mu\nu}A^{\mu\nu}) .$$

In (2) $B$ is the baryon octet, $X$ the scalar meson multiplet, $Y$ the pseudoscalar chiral singlet, $\bar{V}^\mu$ ($A^\mu$) the renormalised vector (axial vector) meson multiplet with the field strength tensor $\bar{V}_{\mu\nu} = \partial_\mu \bar{V}_\nu - \partial_\nu \bar{V}_\mu$ ($A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$), $F_{\mu\nu}$ is the field strength tensor of the photon and $\chi$ is the scalar, iso-scalar dilaton (glueball) field. In the above, $u_\mu = \frac{i}{2} [u^\dagger \partial_\mu u - u \partial_\mu u^\dagger]$, where $u = \exp \left[ \frac{i}{\sigma_0} \pi^a \lambda^a \gamma_5 \right]$ is the unitary transformation operator. The covariant derivative of a field $\Phi \equiv B, X, Y, A_\mu, V_\mu$ reads $D_\mu \Phi = \partial_\mu \Phi + [\Gamma_\mu, \Phi]$ and $\Gamma_\mu = \frac{i}{4} [u^\dagger \partial_\mu u - \partial_\mu u^\dagger u + u \partial_\mu u^\dagger - \partial_\mu uu^\dagger]$.

The baryon-meson interaction for a general meson field $W$ has the form

$$\mathcal{L}_{BW} = -\sqrt{2} g_8^W (\alpha_W [B \bar{O} B W]_F + (1 - \alpha_W) [B \bar{O} B W]_D) - g_1^W \frac{1}{\sqrt{3}} \text{Tr} (\bar{B} \bar{O} B) \text{Tr} W ,$$

(3)
with \([\mathcal{B}^{BW}]_F := \text{Tr}(\mathcal{B}^{BW} - \mathcal{B}^{OBW})\) and \([\mathcal{B}^{OBW}]_D := \text{Tr}(\mathcal{B}^{OBW} + \mathcal{B}^{OBW}) - \frac{2}{3}\text{Tr}(\mathcal{B}^{OB})\text{Tr}W\). The different terms to be considered are those for the interaction of baryons with scalar mesons \((W = X, \mathcal{O} = 1)\), with vector mesons \((W = \tilde{V}_\mu, \mathcal{O} = \gamma_\mu\) for the vector and \(W = \tilde{V}_{\mu\nu}, \mathcal{O} = \sigma^{\mu\nu}\) for the tensor interaction), with axial vector mesons \((W = A_\mu, \mathcal{O} = \gamma_\mu\gamma_5)\) and with pseudoscalar mesons \((W = u_\mu, \mathcal{O} = \gamma_\mu\gamma_5)\), respectively.

For the current investigation the following interactions are relevant: Baryon-scalar meson interactions generate the baryon masses through coupling of the baryons to the non-strange \(\sigma(\sim \langle \bar{u}u + \bar{d}d \rangle)\) and the strange \(\zeta(\sim \langle \bar{s}s \rangle)\) scalar quark condensates. The parameters \(g_1^S\), \(g_8^S\) and \(\alpha_S\) are adjusted to fix the baryon masses to their experimentally measured vacuum values. It should be emphasized that the nucleon mass also depends on the strange condensate \(\zeta\). For the special case of ideal mixing \((\alpha_S = 1 \text{ and } g_1^S = \sqrt{6}g_8^S)\) the nucleon mass depends only on the non–strange quark condensate. In the present investigation, the general case will be used to study hot and strange hadronic matter \([32]\), which takes into account the baryon coupling terms to both scalar fields (\(\sigma\) and \(\zeta\)) while summing over the baryonic tadpole diagrams to investigate the effect from the baryonic Dirac sea in the relativistic Hartree approximation \([32]\).

In analogy to the baryon-scalar meson coupling there exist two independent baryon-vector meson interaction terms corresponding to the \(F\)-type (antisymmetric) and \(D\)-type (symmetric) couplings. Here we will use the antisymmetric coupling because, following the universality principle \([42]\) and the vector meson dominance model, one can conclude that the symmetric coupling should be small. We realize this by setting \(\alpha_V = 1\) for all fits. Additionally we decouple the strange vector field \(\phi_\mu \sim \bar{s}\gamma_\mu s\) from the nucleon by setting \(g_1^V = \sqrt{6}g_8^V\). The remaining baryon-vector meson interaction reads

\[
\mathcal{L}_{BV} = -\sqrt{2}g_8^V \left\{ [\bar{B}\gamma_\mu BV^\mu]_F + \text{Tr}(\bar{B}\gamma_\mu B)\text{Tr}V^\mu \right\}.
\]

(4)

The Lagrangian describing the interaction for the scalar mesons, \(X\), and pseudoscalar singlet, \(Y\), is given as \([31]\)

\[
\mathcal{L}_0 = -\frac{1}{2}k_0\chi^2I_2 + k_1(I_2)^2 + k_2I_4 + 2k_3\chi I_3,
\]

(5)

with \(I_2 = \text{Tr}(X + iY)^2\), \(I_3 = \text{det}(X + iY)\) and \(I_4 = \text{Tr}(X + iY)^4\). In the above, \(\chi\) is the scalar color singlet glueball field. It is introduced in order to mimic the QCD trace anomaly, i.e. the non-vanishing energy-momentum tensor \(\theta^\mu_\mu = (\beta_{QCD}/2g)\langle G^a_{\mu\nu}G^{a,\mu\nu}\rangle\), where \(G^a_{\mu\nu}\) is
the gluon field tensor. A scale breaking potential is introduced:

\[ \mathcal{L}_{\text{scalebreak}} = -\frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0} + \frac{\delta}{3} \chi^4 \ln \frac{I_3}{\det(X)_0} \]  

(6)

which allows for the identification of the \( \chi \) field width the gluon condensate \( \theta^\mu_\rho = (1 - \delta) \chi^4 \).

Finally the term \( \mathcal{L}_\chi = -k_4 \chi^4 \) generates a phenomenologically consistent finite vacuum expectation value. The variation of \( \chi \) in the medium is rather small \[31\]. Hence we shall use the frozen glueball approximation i.e. set \( \chi \) to its vacuum value, \( \chi_0 \).

The Lagrangian for the vector meson interaction is written as

\[ \mathcal{L}_{\text{vec}} = \frac{m_V^2 \chi_i^2}{2} \text{Tr}(\tilde{V}_\mu \tilde{V}^\mu) + \frac{\mu_V}{4} \text{Tr}(\tilde{V}_\mu \tilde{V}^{\mu\nu} X^2) + \frac{\lambda_V}{12} \left( \text{Tr}(\tilde{V}^{\mu\nu}) \right)^2 + 2(g_4)^4 \text{Tr}(\tilde{V}_\mu \tilde{V}^{\mu})^2. \]  

(7)

The vector meson fields, \( \tilde{V}_\mu \) are related to the renormalized fields by \( V_\mu = Z_V^{1/2} \tilde{V}_\mu \), with \( V = \omega, \rho, \phi \). The masses of \( \omega, \rho \) and \( \phi \) are fitted from \( m_V, \mu \) and \( \lambda_V \).

The explicit symmetry breaking term is given as \[31\]

\[ \mathcal{L}_{SB} = \text{Tr} A_p \left( u(X + iY)u + u^\dagger(X - iY)u^\dagger \right) \]  

(8)

with \( A_p = 1/\sqrt{2} \text{diag}(m_{\pi}^2 f_{\pi}, m_{\pi}^2 f_{\pi}, 2m_K^2 f_K - m_{\pi}^2 f_{\pi}) \) and \( m_{\pi} = 139 \text{ MeV}, \ m_K = 498 \text{ MeV}. \)

This choice for \( A_p \), together with the constraints \( \sigma_0 = -f_{\pi}, \ \zeta_0 = -\frac{1}{\sqrt{2}}(2f_K - f_{\pi}) \) on the VEV on the scalar condensates assure that the PCAC-relations of the pion and kaon are fulfilled.

With \( f_{\pi} = 93.3 \text{ MeV} \) and \( f_K = 122 \text{ MeV} \) we obtain \( |\sigma_0| = 93.3 \text{ MeV} \) and \( |\zeta_0| = 106.76 \text{ MeV} \).

We proceed to study the hadronic properties in the chiral SU(3) model. The Lagrangian density in the mean field approximation is given as

\[ \mathcal{L}_{BX} + \mathcal{L}_{BV} = -\sum_i \overline{\psi_i} \left[ g_{i\omega} \gamma_0 \omega + g_{i\phi} \gamma_0 \phi + m_i^* \right] \psi_i \]  

(9)

\[ \mathcal{L}_{\text{vec}} = \frac{1}{2} m_{\omega}^2 \chi_0^2 \omega^2 + g_{4\omega}^4 \omega^4 + \frac{1}{2} m_{\phi}^2 \chi_0^2 \phi^2 + g_{4\phi}^4 \left( \frac{Z_{\phi}}{Z_{\omega}} \right)^2 \phi^4 \]  

(10)

\[ \mathcal{L}_{\text{dilaton}} = -\frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2 + \delta^2) + k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 + k_2 (\frac{\sigma^4}{2} + \frac{\delta^4}{2} + \zeta^4) \]  

\[ + k_3 (\sigma^2 - \delta^2) \zeta - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0} + \frac{\delta}{3} \chi^4 \ln \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 \zeta_0} \right) \]  

(11)

\[ \mathcal{L}_{SB} = -\left[ m_{\pi}^2 f_\pi \sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}} m_{\pi}^2 f_{\pi}) \zeta \right], \]  

(12)

where \( m_i^* = -g_{i\sigma} \sigma - g_{i\zeta} \zeta - g_{i\delta} \delta \) is the effective mass of the baryon of type \( i \ (i = N, \Sigma, \Lambda, \Xi) \). In the above, \( \mathcal{L}_{\text{dilaton}} \) is the combined contribution from the Lagrangian densities \( \mathcal{L}_0, \mathcal{L}_{\text{scalebreak}} \).
and $L$ and $g_4 = \sqrt{Z_\omega} g_4$ is the renormalized coupling for $\omega$-field. The thermodynamical potential of the grand canonical ensemble $\Omega$ per unit volume $V$ at given chemical potential $\mu$ and temperature $T$ can be written as

$$\frac{\Omega}{V} = -L_{\text{vec}} - L_0 - L_{SB} - V_{\text{vac}} + \sum_i \frac{\gamma_i}{(2\pi)^3} \int d^3k \left( E_i^*(k) \left( f_i(k) + \bar{f}_i(k) \right) \right)$$

$$- \sum_i \frac{\gamma_i}{(2\pi)^3} \mu_i^* \int d^3k \left( f_i(k) - \bar{f}_i(k) \right).$$

(13)

Here the the potential at $\rho = 0$ has been subtracted in order to get a vanishing vacuum energy. In (13) $\gamma_i$ are the spin-isospin degeneracy factors. The $f_i$ and $\bar{f}_i$ are thermal distribution functions for the baryon of species $i$, given in terms of the effective single particle energy, $E_i^*$, and chemical potential, $\mu_i^*$, as

$$f_i(k) = \frac{1}{e^{\beta(E_i^*(k) - \mu_i^*)} + 1}, \quad \bar{f}_i(k) = \frac{1}{e^{\beta(E_i^*(k) + \mu_i^*)} + 1},$$

with $E_i^*(k) = \sqrt{k_i^2 + m_i^{*2}}$ and $\mu_i^* = \mu_i - g_i \omega$. The mesonic field equations are determined by minimizing the thermodynamical potential [32, 33]. They depend on the scalar and vector densities for the baryons at finite temperature

$$\rho_i^s = \gamma_i \int \frac{d^3k}{(2\pi)^3} \frac{m_i^*}{E_i^*} \left( f_i(k) + \bar{f}_i(k) \right); \quad \rho_i = \gamma_i \int \frac{d^3k}{(2\pi)^3} \left( f_i(k) - \bar{f}_i(k) \right).$$

(14)

The energy density and the pressure are given as, $\epsilon = \Omega/V + \mu_i \rho_i + TS$ and $p = -\Omega/V$.

III. KAON (ANTIKAON) INTERACTIONS IN THE CHIRAL SU(3) MODEL

In this section, we derive the dispersion relations for the $K(\bar{K})$ [43] and calculate their optical potentials in asymmetric nuclear matter [38]. The medium modified energies of the kaons and antikaons arise from their interactions with the nucleons and scalar mesons within the chiral SU(3) model.

In this model, the interactions of the kaons and antikaons to the scalar fields (non-strange, $\sigma$ and strange, $\zeta$), scalar–isovector field $\delta$ as well as a vectorial interaction with the nucleons (the so-called Weinberg-Tomozaawa interaction) modify the energies for K($\bar{K}$) mesons in the medium. In the following, we shall derive the dispersion relations for the kaons and antikaons, including the effects from isospin asymmetry originating from the scalar-isovector $\delta$ field as well as vectorial interaction with the nucleons and a range term arising from the interaction
The symmetry energy, $a_4$ plotted as a function of the baryon density, $\rho_B/\rho_0$.

with the nucleons. It might be noted here that the interaction of the pseudoscalar mesons to the vector mesons, in addition to the pseudoscalar meson–nucleon vectorial interaction leads to a double counting in the linear realization of the chiral effective theory [44]. Within the nonlinear realization of the chiral effective theories, such an interaction does not arise in the leading or sub-leading order, but only as a higher order contribution [44]. Hence the vector meson-pseudoscalar interaction will not be considered within the present investigation.

The scalar meson multiplet has the expectation value $\langle X \rangle = \text{diag}((\sigma + \delta)/\sqrt{2},(\sigma -$
FIG. 2: The kaon energies (for $K^+$ in (a) and for $K^0$ in (b)) in MeV plotted as a functions of the baryon density, $\rho_B/\rho_0$ for different values of the isospin asymmetry parameter, $\eta$. $
abla \delta/\sqrt{2}, \zeta$, with $\sigma$ and $\zeta$ corresponding to the non-strange and strange scalar condensates, and, $\delta$ is the third isospin component of the scalar-isovector field, $\vec{\delta}$. The pseudoscalar meson
field \( P \) can be written as,

\[
P = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} & \pi^+ & \frac{2K^+}{1+w} \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} & \frac{2K^0}{1+w} \\
\frac{2K^-}{1+w} & \frac{2K^0}{1+w} & 0
\end{pmatrix},
\]

(15)

where \( w = \sqrt{2}\zeta/\sigma \) and we have written down the terms that are relevant for the present investigation. From PCAC one gets the decay constants for the pseudoscalar mesons as \( f_\pi = -\sigma \) and \( f_K = -(\sigma + \sqrt{2}\zeta)/2 \).

The scalar meson exchange interaction term is determined from the explicit symmetry breaking term by equation (8), where \( A_p = 1/\sqrt{2} \) \text{diag} \( (m^2_\pi f_\pi, m^2_\pi f_\pi, 2m^2_K f_K - m^2_\pi f_\pi) \).

The interaction Lagrangian modifying the energies of the \( K(\bar{K}) \)-mesons is given as

\[
L_{KN} = -\frac{i}{8f_K} \left[ 3(\bar{N}\gamma^\mu N)(\bar{K}(\partial_\mu K) - (\partial_\mu \bar{K})K) + (\bar{N}\gamma^\mu \tau^a N)(\bar{K}\tau^a(\partial_\mu K) - (\partial_\mu \bar{K})\tau^a K) \right]
+ \frac{m_K^2}{2f_K} \left[ (\sigma + \sqrt{2}\zeta)(\bar{K}K) + \delta^a(\bar{K}\tau^a K) \right]
- \frac{1}{f_K} \left[ (\sigma + \sqrt{2}\zeta)(\partial_\mu K)(\partial^\mu K) + (\partial_\mu K)\tau^a(\partial^\mu K)\delta^a \right]
+ \frac{d_1}{2f_K^2} (\bar{N}N)(\partial_\mu K)(\partial^\mu K)
+ \frac{d_2}{2f_K^2} \left[ (\bar{p}p)(\partial_\mu K^+)(\partial^\mu K^-) + (\bar{n}n)(\partial_\mu K^0)(\partial^\mu \bar{K}^0) \right]
+ (\bar{p}n)(\partial_\mu K^+)(\partial^\mu \bar{K}^-) + (\bar{n}p)(\partial_\mu K^0)(\partial^\mu \bar{K}^-)
\]

(16)

In the above, \( K \) and \( \bar{K} \) are the kaon \((K^+, K^0)\) and antikaon \((K^-, \bar{K}^0)\) doublets. In (16) the first line is the vectorial interaction term obtained from the first term in (2) \text{(Weinberg-Tomozawa term)}. The second term, which gives an attractive interaction for the \( K \)-mesons, is obtained from the explicit symmetry breaking term \text{(8)}. The third term arises within the present chiral model from the kinetic term of the pseudoscalar mesons given by the third term in equation (2), when the scalar fields in one of the meson multiplets, \( X \), are replaced by their vacuum expectation values. The fourth and fifth terms in (16) for the KN interactions arise from the terms

\[
L_{(d_1)}^{BM} = d_1 Tr(u_\mu u^\mu \bar{B}B),
\]

(17)

and,

\[
L_{(d_2)}^{BM} = d_2 Tr(\bar{B}u_\mu u^\mu B).
\]

(18)
in the SU(3) chiral model \[37, 38\]. The last three terms in (16) represent the range term in the chiral model, with the last term being an isospin asymmetric interaction. The Fourier transformation of the equation-of-motion for kaons (antikaons) leads to the dispersion relations,

\[-\omega^2 + \vec{k}^2 + m_{K}^2 - \Pi(\omega, |\vec{k}|, \rho) = 0,\]

where \(\Pi\) denotes the kaon (antikaon) self energy in the medium.

Explicitly, the self energy \(\Pi(\omega, |\vec{k}|)\) for the kaon doublet arising from the interaction (16) is given as

\[
\Pi(\omega, |\vec{k}|) = -\frac{1}{4f_K^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \right] \omega + \frac{m_K^2}{2f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') \\
+ \left[ -\frac{1}{f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2}(\rho_s^p + \rho_s^n) \\
+ \frac{d_2}{4f_K^2} \left( (\rho_s^p + \rho_s^n) \pm (\rho_s^p - \rho_s^n) \right) \right](\omega^2 - \vec{k}^2),
\]

(19)

where the \(\pm\) signs refer to the \(K^+\) and \(K^0\) respectively. In the above, \(\sigma' (= \sigma - \sigma_0), \zeta' (= \zeta - \zeta_0)\) and \(\delta' (= \delta - \delta_0)\) are the fluctuations of the scalar-isoscalar fields \(\sigma\) and \(\zeta\), and the third component of the scalar-isovector field, \(\delta\), from their vacuum expectation values. The vacuum expectation value of \(\delta\) is zero (\(\delta_0=0\)), since a nonzero value for it will break the isospin symmetry of the vacuum (the small isospin breaking effect coming from the mass and charge difference of the up and down quarks has been neglected here). \(\rho_p\) and \(\rho_n\) are the number densities for the proton and the neutron, and \(\rho_s^p\) and \(\rho_s^n\) are their scalar densities.

Similarly, for the antikaon doublet, the self-energy is calculated as

\[
\Pi(\omega, |\vec{k}|) = \frac{1}{4f_K^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \right] \omega + \frac{m_K^2}{2f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') \\
+ \left[ -\frac{1}{f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2}(\rho_s^p + \rho_s^n) \\
+ \frac{d_2}{4f_K^2} \left( (\rho_s^p + \rho_s^n) \pm (\rho_s^p - \rho_s^n) \right) \right](\omega^2 - \vec{k}^2),
\]

(20)

where the \(\pm\) signs refer to the \(K^-\) and \(\bar{K}^0\) respectively.

After solving the above dispersion relations for the kaons and antikaons, their optical potentials can be calculated from

\[
U(\omega, k) = \omega(k) - \sqrt{k^2 + m_K^2},
\]

(21)
where \(m_K\) is the vacuum mass for the kaon (antikaon).

The parameters \(d_1\) and \(d_2\) are calculated from the empirical values of the KN scattering lengths for \(I=0\) and \(I=1\) channels, given by

\[
a_{KN}(I = 0) = \frac{m_K}{4\pi f_K^2(1 + m_K/m_N)} \left[ - \frac{m_K f_K}{2} \left( \frac{g_{\sigma N}}{m_\sigma^2} + \sqrt{2}\frac{g_{\zeta N}}{m_\zeta^2} - 3\frac{g_{\delta N}}{m_\delta^2} \right) + \frac{(d_1 - d_2)m_K}{2} \right]
\]

and

\[
a_{KN}(I = 1) = \frac{m_K}{4\pi f_K^2(1 + m_K/m_N)} \left[ - 1 - \frac{m_K f_K}{2} \left( \frac{g_{\sigma N}}{m_\sigma^2} + \sqrt{2}\frac{g_{\zeta N}}{m_\zeta^2} + \frac{g_{\delta N}}{m_\delta^2} \right) + \frac{(d_1 + d_2)m_K}{2} \right]
\]

These are taken to be \([35, 45, 46]\)

\[
a_{KN}(I = 0) \approx -0.31\ \text{fm}, \quad a_{KN}(I = 1) \approx -0.09\ \text{fm}.
\]

leading to the isospin averaged KN scattering length as

\[
\bar{a}_{KN} = \frac{1}{4}a_{KN}(I = 0) + \frac{3}{4}a_{KN}(I = 1) \approx -0.255\ \text{fm}.
\]

The pion nucleon scattering lengths given by

\[
a_{\pi N} \left( I = \frac{3}{2} \right) = \frac{m_\pi}{4\pi f_\pi^2(1 + (m_\pi/m_N))} \left[ - \frac{1}{2} - \frac{g_{\sigma N}}{m_\sigma^2}m_\pi f_\pi + \frac{(d_1 + d_2)m_\pi}{2} \right]
\]

and

\[
a_{\pi N} \left( I = \frac{1}{2} \right) = \frac{m_\pi}{4\pi f_\pi^2(1 + (m_\pi/m_N))} \left[ 1 - \frac{g_{\sigma N}}{m_\sigma^2}m_\pi f_\pi + \frac{(d_1 + d_2)m_\pi}{2} \right].
\]

are also calculated in the present work.

IV. RESULTS AND DISCUSSIONS

The present calculations use the following model parameters. The values, \(g_{\sigma N} = 10.6\), and \(g_{\zeta N} = -0.47\) are determined by fitting vacuum baryon masses. The other parameters as fitted to the asymmetric nuclear matter saturation properties in the mean field approximation are: \(g_{\omega N}=13.3\), \(\gamma_{\rho N}=5.5\), \(g_4=79.7\), \(g_{N\delta}=2.5\), \(m_\zeta =1024.5\ \text{MeV}\), \(m_\sigma=\)
466.5 MeV and \(m_\delta=899.5\) MeV. The coefficients \(d_1\) and \(d_2\), calculated from the empirical values of the KN scattering lengths for \(I=0\) and \(I=1\) channels \([24]\), are \(5.5/m_K\) and \(0.66/m_K\) respectively. Using these parameters, the symmetry energy defined as

\[
a_4 = 2 \frac{d^2 E}{d\eta^2}\bigg|_{\eta=0}
\]

with the asymmetry parameter \(\eta = \frac{1}{2}(\rho_n - \rho_p)/\rho_B\), has a value of \(a_4 = 31.7\) MeV at saturation nuclear matter density of \(\rho_0=0.15\) fm\(^{-3}\). Figure 1 shows the density dependence of the symmetry energy, which increases with density similar to previous calculations \([47]\).

The values of the pion nucleon scattering lengths \([48, 49, 50, 51, 52, 53]\) are calculated in the present model, with the values of \(d_1\) and \(d_2\) as obtained by fitting the kaon nucleon scattering lengths. Their values for \(I = 3/2\) and \(I = 1/2\), given by equations \(26\) and \(27\) are obtained as \(a_{\pi N}(I = 3/2) = -0.1474\) fm and \(a_{\pi N}(I = 1/2) = 0.1823\) fm respectively. This determines the isoscalar and isovector scattering lengths for \(\pi N\) scattering \((a_+ = (a_{\pi N}(I = 1/2) + 2a_{\pi N}(I = 3/2))/3\) and \(a_- = (a_{\pi N}(I = 1/2) - a_{\pi N}(I = 3/2))/3\)\) to be \(a_+ = -0.0266/m_\pi\) and \(a_- = 0.078/m_\pi\). These may be compared with the results of \(a_+ = -0.0029/m_\pi\) and \(a_- = 0.0936/m_\pi\) derived from pionic atoms \([51]\), the values \(a_+ = -0.0012/m_\pi\) and \(a_- = 0.0895/m_\pi\) using the empirical values of the \(\pi^- p\) and \(\pi^-d\) scattering lengths \([52]\) and the values \(a_+ = -0.0001/m_\pi\) and \(a_- = 0.0885/m_\pi\) from pionic deuterium shift by the experimental PSI group \([53]\).

The \(\pi N\) and \(KN\) sigma terms are also calculated within the model. They turn out to be 44 MeV and 725 MeV for the set of parameters used in the present calculations.

The kaon and antikaon properties were studied in the isospin symmetric hadronic matter within the chiral SU(3) model in ref. [37]. The contribution from the vector interaction (Weinberg-Tomozawa term) leads to a drop for the antikaon energy, whereas they are repulsive for the kaons. The scalar meson exchange term arising from the scalar-isoscalar fields \((\sigma\) and \(\zeta\)) is attractive for both \(K\) and \(\bar{K}\). The first term of the range term of eq. \([16]\) is repulsive whereas the second term has an attractive contribution for the isospin symmetric matter \([37]\) for both kaons and antikaons. The third term of the range term has an isospin asymmetric contribution.

The contributions from (i) the last term of the Weinberg-Tomozawa term, (ii) the scalar-isovector, \(\delta\)-field as well as (iii) the \(d_2\) term in the interaction Lagrangian given by equation \([16]\) introduce an isotopic asymmetry in the \(K\) and \(\bar{K}\)-energies. For \(\rho_n > \rho_p\), in the kaon
sector, $K^+ (K^0)$ has negative (positive) contributions from $\delta$. The $\delta$ contribution from the scalar exchange term is positive (negative) for $K^+ (K^0)$, whereas that arising from the range term has the opposite sign and dominates over the former contribution.

In figure 2, the energies of the $K^+$ and $K^0$ at zero momentum, are plotted for different values of the isospin asymmetry parameter, $\eta$, at various densities. For $\rho_B = \rho_0$ the energy of $K^+$ is seen to drop by about 7 MeV at zero momentum when $\eta$ changes from 0 to 0.5. On the other hand, the $K^0$ energy is seen to increase by about 27 MeV for $\eta=0.5$, from the isospin symmetric case of $\eta=0$. The reason for this opposite behavior for the $K^+$ and $K^0$ on the isospin asymmetry originates from the vectorial (Weinberg-Tomozawa), $\delta$ meson contribution as well as from the isospin dependent range term ($d_2$-term) contributions. For $K^+$, the $\eta$-dependence of the energy is seen to be less sensitive at higher densities, whereas the energy of $K^0$ is seen to have a larger drop from the $\eta=0$ case, as we increase the density.

For the antikaons, the $K^- (\bar{K}^0)$ energy at zero momentum is seen to increase (drop) with $\eta$, as we increase the density as seen in figure 3. The sensitivity of the isospin asymmetry dependence of the energies is seen to be larger for $K^-$ with density, whereas it becomes smaller for $\bar{K}^0$ at high densities. The mass drop as modified in the isospin asymmetry in neutron star matter, will have relevance for the onset of antikaon ($K^-$ and $\bar{K}^0$) condensation.

The energies of the kaons and antikaons, with respect to the isospin symmetric case, for different values of the isospin asymmetric parameter, $\eta$ are plotted as functions of the momentum in figures 4 and 5. The energies of the kaons and antikaons are plotted for densities $\rho_B = \rho_0$, $2\rho_0$ and $4\rho_0$ in the same figures. These are seen to be more sensitive to momentum as we increase the isospin parameter. The momentum dependence turns out to be stronger for higher densities, and in particular, the effect seems to be more significant for $K^0$ (as compared to $K^+$) and $K^-$ (as compared to $\bar{K}^0$).

The qualitative behavior of the isospin asymmetry dependencies of the energies of the kaons and antikaons are also reflected in their optical potentials plotted in figure 6 for the kaons, and in figure 7, for the antikaons, at selected densities. The different behavior of the $K^+$ and $K^0$, as well as for the $K^-$ and $\bar{K}^0$ optical potentials in the dense asymmetric nuclear matter should be observed in their production as well as propagation in isospin asymmetric heavy ion collisions. In particular an experimental study of the $K^-/\bar{K}^0$ ratio (and its dependence on the kaon momenta) might be a promising tool to investigate the isospin effects discussed here. The effects of the isospin asymmetric optical potentials could
thus be observed in nuclear collisions at the CBM experiment at the proposed project FAIR at GSI, where experiments with neutron rich beams are planned to be carried out.

V. SUMMARY

To summarize, within a chiral SU(3) model we have investigated the density dependence of the $K$, $\bar{K}$-meson optical potentials in asymmetric nuclear matter, arising from the interactions with nucleons and scalar mesons. The properties of the light hadrons – as studied in a SU(3) chiral model – modify the $K(\bar{K})$-meson properties in the hadronic medium. The model with parameters fitted to reproduce the properties of hadron masses in vacuum, nuclear matter saturation properties and low energy KN scattering data, takes into account all terms up to the next to leading order arising in chiral perturbative expansion for the interactions of $K(\bar{K})$-mesons with baryons. The $\pi N$ scattering lengths are also calculated for the fitted set of model parameters as $a_+ = -0.0266/m_\pi$ and $a_- = 0.078/m_\pi$ and have been compared with the other results in the literature [51, 52, 53]. The $\pi N$ and $K N$ sigma coefficients are calculated within the present chiral effective model and have values of $\Sigma_{\pi N} = 44$ MeV and $\Sigma_{KN} = 725$ MeV.

There is a significant density dependence of the isospin asymmetry on the optical potentials of the kaons and antikaons. The results can be used in heavy-ion simulations that include mean fields for the propagation of mesons [43]. The different potentials of kaons and antikaons can be particularly relevant for neutron-rich heavy-ion beams at the CBM experiment at the future project FAIR at GSI, Germany, as well as at the experiments at the proposed Rare Isotope Accelerator (RIA) laboratory, USA. The $K^-/\bar{K}^0$ ratio for different isospin of projectile and target is a promising observable to study these effects. Furthermore, the medium modification of antikaons due to isospin asymmetry in dense matter can have important consequences, for example on the onset of antikaon condensation in the bulk charge neutral matter in neutron stars. The effects of hyperons as well as finite temperatures on optical potentials of kaons and antikaons and their possible implications on the neutron star phenomenology as well as heavy ion collision experiments are the intended topics of future investigation.
Acknowledgments

We thank S. Kubis, C. Hanhart, S. Mallik, J. Reinhardt for many fruitful discussions. One of the authors (AM) is grateful to the Institut für Theoretische Physik Frankfurt for the warm hospitality where the present work was initiated. AM acknowledges financial support from Alexander von Humboldt stiftung. The use of the resources of the Frankfurt Center for Scientific Computing (CSC) is additionally gratefully acknowledged.

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FIG. 3: The energies of the antikaons (for $K^-$ in (a) and for $\bar{K}^0$ in (b)), at zero momentum as functions of the baryon density ($\rho_B/\rho_0$), are plotted for different values of the isospin asymmetry parameter, $\eta$. 

FIG. 4: The kaon energies (for $K^+$ in (a), (c) and (e) and for $K^0$ in (b), (d) and (f)) as compared to the isospin symmetric case, plotted as a functions of the momentum for various values of the baryon density, $\rho_B$ and for different values of the isospin asymmetry parameter, $\eta$. 
FIG. 5: The antikaon energies (for $K^-$ in (a), (c) and (e) and for $\bar{K}^0$ in (b), (d) and (f)) as compared to the isospin symmetric case, plotted as a functions of the momentum, for different values of the isospin asymmetry parameter, $\eta$ for various values of the baryon density, $\rho_B$. 
FIG. 6: The kaon optical potentials (for $K^+$ in (a), (c) and (e) and for $K^0$ in (b), (d) and (f)) in MeV, plotted as functions of the momentum for various baryon densities, $\rho_B$ and for different values of the isospin asymmetry parameter, $\eta$. 
FIG. 7: The antikaon optical potentials (for $K^-$ in (a), (c) and (e) and for $\bar{K}^0$ in (b), (d) and (f)) in MeV, plotted as functions of the momentum for various baryon densities, $\rho_B$ and for different values of the isospin asymmetry parameter, $\eta$. 