Development of a specialized mathematical model of heat transfer in a vacuum electric furnace

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Abstract. A specialized numerical model of the heat transfer process in a vacuum electric furnace is described. The model is based on neglecting factors weakly affecting the main technological processes, on development of specialized methods of the mesh construction, and on usage of effective algorithms of the radiative heat transfer calculation. A specialized mathematical model of the electric furnace allows reduction of the mesh size about three times in comparison with the initial model.

1. Introduction

A specialized mathematical model of heat transfer is developed for a vacuum electric furnace with a working space volume of about 15 m\(^3\), having ohmic heating elements in the form of a set of 36 columns, lining and internal equipment made of carbon materials. There are two retorts inside the furnace: external and internal (Figure 1). The main technological process takes place in the internal retort. The characteristic time of the furnace operation cycle is several days. The surface temperature of the outer retort achieves 1800 °C according to the pyrometer data. The pressure in the furnace is maintained at a level of several hundred Pa. The mechanism of heat transfer by radiation in the electric furnace is dominant. The flow of gases inside the electric furnace is laminar due to low density and low velocities (\(Re \sim 10\)).

A detailed mathematical model was initially developed to calculate heat transfer in a vacuum electric furnace. The model reproduced the whole geometry of the furnace and workpieces and included convective heat transfer. Cross-verification calculations with the ANSYS software complex were carried out for this model. The main disadvantage of the detailed mathematical model is the computation time due to the size of mesh (for simple workpieces at least 15 million cells) and the complexity of generating the mesh itself. On the base of the results obtained with the use of the detailed mathematical model a specialized model was developed. Mathematical models of heat transfer in a vacuum electric furnace, grid generator and a module for analyzing the results were developed on the basis of the software complex “SigmaFlow” of the Krasnoyarsk branch of IT SB RAS [1]. In the “SigmaFlow”, the control volume method is used to discretize the equations of hydrodynamics and heat transfer and parallel calculations based on the spatial decomposition of the computational domain are performed.
2. **Specialized mathematical model of heat transfer in a vacuum electric furnace**

FTn finite volume method (FVM) is used to calculate radiation heat transfer. It provides a more uniform discretization of the angular space into control (solid) angles in comparison with the standard approach using a uniform discretization over spherical angles [2]. Thus, the FTn FVM allows to use a smaller number of control angles than the standard angular discretization (typically, 24 control angles instead of 32).

Both standard methods used for solving the equations of hydrodynamics (BiCGSTAB, DILU, CS etc) and space-marching technique (Figure 2a) were considered to solve the system of linear algebraic equations (SLAE) that are the result of FVM discretization. The main advantage of the space-marching technique is that if all the control volumes (cells) are convex, then an effective bypass scheme can be constructed. An effective bypass scheme assumes that the radiation field is calculated in one pass if the emissivity is equal to unity for all the boundaries of the calculation domain and there is no dispersion. A negative feature of the space-marching technique is that it becomes iterative and its efficiency drops for geometrically complex objects, when using the error reduction procedure associated with control-angle overlap and for parallel computations based on spatial decomposition. Test calculations showed that if an effective bypass scheme is realized, then the advantage of calculation time over the standard methods of solving SLAE can achieve 11 times in conditions of optically thin medium [3]. In addition, it turns out that the BiCGSTAB method and other methods of the Krylov type (CRES, CG) can’t find the solution of the SLAE, when the medium is transparent. Therefore, the space-marching technique for calculating the radiative transfer in the electric furnace is comparable with DILU.

Numerical modelling of the radiation field for the electric furnace has shown that the space-marching technique provides 5.5 times acceleration of the calculation in comparison with DILU for single-threaded computing. When using multithreaded computing, this ratio varies and can both decrease and increase. This is because there are solid regions in the calculation domain, for that calculation of radiation field is not performed when using the space-marching technique (Figure 2b). The efficiency of multithreaded calculations is highly dependent on the decomposition of the calculated domain, i.e. on that how uniformly the gas region is decomposed. This can be seen from the comparison of the efficiency of the parallel calculations for the cases where solid opaque regions are absent or present. In the first case the efficiency of parallel calculations is close to unity whereas in the second it is much lower and varies markedly depending on the quality of the partitioning. For
example, the efficiency varies from 0.57 to 0.62 for decomposition of calculation domain into 2-6 parts.

**Figure 2.** Model radiative heat transfer: (a) Marching order; (b) The incident radiation in horizontal section, W/m$^2$.

Further optimization of the model aimed at reducing the computational cost is carried out by selecting constant or rarely changed elements of the electric furnace and describing the properties of these elements with the help of economical submodels derived from the analysis of the problem in its complete formulation. A part of the considered problem, which is subject to change relatively rarely, is the space and equipment of the furnace in the interval from the outer walls cooled by water to the wall of the outer retort (the dimensions of the latter are fixed, while the dimensions of the inner retort can vary from one operating mode to the other). The composition of this part includes the thermal insulation of the furnace, the internal lining of its walls, the columns of heaters and the current leads to them. The submodel for its description is chosen based on an analysis of spatial and temporal scales. Hereafter the most significant results of the analysis are given.

The heat transfer coefficient at the furnace surfaces is estimated via thermal radiation relations: $\alpha = 4\sigma T^3 = 5 \div 2000$. The Biou number for the heater columns $Bl = 0.001 \div 0.1$, therefore the temperature differences between the heaters and other elements of the furnace are much greater than the temperature differences in the heaters themselves. The characteristic time of establishment of the heaters’ temperature as a result of heat exchange with other elements of the furnace $T_h = 10^2 \div 10^4$ s, therefore the temperature field in the heaters can be considered homogeneous and unsteady. Therefore, heaters can be described by a non-stationary balance thermal model based on the energy conservation equation

$$C_h(T_h)m_h\frac{dT_h}{dt} = Q_{El} + Q_{WH} + Q_{RH}. \quad (1)$$

where $C_h$ is the specific heat capacity of the heater material, $m_h$ is the heater mass, $T_h$ is the heater temperature, $Q_{El}$ is the Joule power, $Q_{WH}$ is the power of radiative heat exchange with the inner surface of furnace lining, $Q_{RH}$ is the same for the wall of outer retort.

In electric furnaces, an assigned time dependence of temperature (measured in a certain point or somehow averaged) is achieved by regulation of the heating power. PID regulators have wide use, in that the required heating power is calculated via the expression

$$Q_{El}(t) = C(T_a(t) - T_m(t)) + C_d\frac{d}{dt}(T_a(t) - T_m(t)) + C_l\int_0^t(T_a(t') - T_m(t'))dt', \quad (2)$$

where $C$, $C_d$, $C_l$ are coefficients depending on the parameters of furnace and regime (loaded mass, response time etc.), $T_a$ is the assigned temperature and $T_m$ is the measured temperature. The same dependence $Q_{El}(t)$ is assumed for the numerical model.
The temperature difference in the thermal insulation and lining is inevitable. During the formulation of the model it is necessary to determine whether the temperature field in the heat-insulating layer can be considered steady. At specified values of the characteristic time, thermophysical properties of the heat-insulating material (soot), and the thickness of the heat-insulating layer (0.3 m) the Fourier number is of the order of 0.1, which means an essentially unsteady nature of the temperature field in the heat-insulating layer. Consequently, the maximally simplified model should assume a one-dimensional nonstationary temperature field in the outer wall of the furnace, described by the equation of thermal conductivity. Its domain covers both the internal graphite lining of the furnace and the thermal insulation layer. A boundary condition for the heat conduction equation at one of the boundaries corresponding to the external, water-cooled surface of the furnace wall is the known temperature value. At the second boundary, corresponding to the inner surface of the lining, the boundary condition is the given value of the heat flux generated by the emitted and falling radiation flux; the latter is created by the radiation of the heater and the wall of the outer retort.

The boundary of the domain of the spatial heat transfer problem is the outer wall of the outer retort. The boundary condition on it is the magnitude of the heat flux – emitted and falling, formed by the heater and the inner wall surface of the furnace. The heat flux from the heater depends on the temperature of the heater (as a fourth power) and on the coordinate of the point on the surface of the outer retort:

\[ q(r) = R_w(r)(T_w(0)^4 - T(r)^4) + R_h(r)(T_h^4 - T(r)^4) \] (3)

To clarify the form of the spatial dependence, it was necessary to calculate the heat transfer in a complete geometric setting without simplification, including the heater columns and other elements outside the retort space. In this case, the temperature of the heater is artificially set equal to a certain value, much greater than the temperature of all other elements, as a result of which the radiative heat flux on the remaining elements of the problem is related only to the radiation of the heater. After solving the heat radiation transfer problem, the dependence of the radiation flux incident on the external retort on the coordinate was determined. For the lateral surface of the retort, the obtained dependence was averaged over the intervals of the vertical coordinate. In this way we obtain a tabulated distribution of the radiation flux incident on the side of the heater, which is used in solving the problem in the reduced formulation (taking into account the proportionality of the heat flux to \( T_h^4 \)).

The flow to the upper surface of the retort is averaged and tabulated along the intervals of the coordinate \( r \) - the distance from the axis of the retort (Figure 3). A similar procedure is performed to find and further use the heat flux incident on the surface of the retort and heater from the inner surface of the furnace lining.

![Figure 3. Coefficients of radiative heat transfer for the external retort.](image)
One of the essential elements of the model is the dependence of thermal conductivity of the heat-insulating material (soot) on temperature. There are no reliable measurement data on this issue. In addition, the change in material properties during the operation of the furnace is largely uncertain. In this connection, it is expedient to determine the required dependence $\lambda(T)$ using the data of the measurements carried out during the operation of the furnace. The most convenient for this are the data on the time dependence of the retort temperature during the cooling of furnace, i.e. in the absence of electrical heating. As the results of preliminary methodological calculations show, the retort and its contents can be approximately regarded as a single isothermal body. Considering the energy balance of the retort, multiple calculations of the retort cooling were performed, varying the constant parameters of the temperature dependence of the thermal conductivity of heat-insulating material

$$\lambda(T) = LT^p, \ L, p = \text{const}$$

(4)

Time dependence of the calculated retort temperature was compared with the measured one. The integral square difference is considered as a function of the parameters $L, p$ in (4)

$$\int \left( T_{R}^{\text{calc}}(t) - T_{R}^{\text{exp}}(t) \right)^2 dt = f(L, p).$$

(5)

the integration time interval is the period of the retort cooling, in that the electrical power is zero. Minimization of this function results in the following values of the parameters: $L=1.41\cdot10^{-5}, p=1.72$.

The space between the retorts was considered as a solid body with the mass equal to the aggregated mass of the retorts and with the thermal conductivity proportional to $T^3$. It allows to avoid a detailed mesh in the regions of thin walls of retorts and to reduce the mesh size.

3. Construction of the geometry and computation mesh

The calculation domain (geometry and mesh), adapted for the use in a specialized mathematical model of heat exchange in the electric furnace is constructed in a separate program unit. The geometry of the calculation domain consists of three types of elements: constant, parametrized and arbitrary ones. The constant elements are common for all technological regimes. The parametrized elements are certain elements of the furnace equipment that can change only in certain scales. Arbitrary elements of the geometry are described in the STL format.

The grid generation is based on the preliminary octal partition with various discretization of a computational domain (according to geometry), otherwise called the octree mesh generation. A final grid is created from an octree by projecting vertexes of a preliminary grid to the boundaries of the numerical domain and forming cells that connect the areas with different discretization (transition areas). As a result, we obtain an unstructured hybrid grid (figure 2), which includes mostly hexahedral elements (cubes) and polyhedral elements (pyramids, prisms, and tetrahedrons) in the transition areas and on the boundaries of the object. The most often encountered problems of the mesh generation on the basis of an octree are connected with the projection of the preliminary nodes on the boundaries of objects, when singular edges of cells can emerge. At present, the maximum effort is put towards minimization of frequency of these events and correction of bad cells. It must be mentioned that the quality of initial geometry, certainly, the size of polygons, is also of big importance.

An example of the temperature field calculation in the specialized model is given in Figure 4.
Figure 4. Construction of geometry for a specialized mathematical model. Mesh and temperature field after 16 hours of heating, T=930÷1030 K.

4. Conclusions
A model of heat exchange in a vacuum electric furnace is developed and implemented. In strict conditions of the geometrical complexity and wide spread of time scales, the model permits mass calculations during development of the technological process. The processes of conductive and radiation heat transfer in the outer space of the furnace’s retorts are described in a separate submodel. The submodel is based on the energy balance equations for the elements of the furnace equipment and 1d equation of heat transfer in the material of the heat insulation of the furnace. Due to that, essential simplification of the geometry of considered cases and of the model as a whole is achieved. In particular, the dimensions of the calculation mesh are reduced 2-3 times, depending on the complexity of the inner equipment of the retort. Applying the space-marching technique for the calculation of the radiation heat transfer additionally reduces the computational cost. As a result, the model, specialized in accordance with features of the considered problem requires an order less computational resources as compared with a non-specialized model considering the same physical process. It becomes possible to perform numerical analysis of the technological process with use of a modern workstation with ~10 processor cores during a day.

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