A displacement estimation method based on the magnetic field information of PMLSM

Chang Tian\textsuperscript{1,2} and Jinchun Hu\textsuperscript{1,2,3}

\textsuperscript{1} State Key Laboratory of Tribology, Department of Mechanical Engineering, Tsinghua University
\textsuperscript{2} Beijing Lab of Precision/Ultra-Precision Manufacture Equipment and Control, Tsinghua University
\textsuperscript{3} E-mail: hujinchun@tsinghua.edu.cn

Abstract. In this paper, a novel method for nonlinear displacement estimation in permanent magnet linear synchronous motor was proposed with concise formulations and high accuracy. In the proposed method, one-dimensional hall sensor array was fixed on the mover so that magnetic induction of the permanent magnet could be obtained while the mover was traveling. Utilizing the redundant measurement information, an original fixed point iteration method was designed. With additive white Gaussian noise, simulation results verified the feasibility of the method. The simulation results suggested that the computational error of displacement was only less than $4\times10^{-4}$mm.

1. Introduction
In recent years, linear motor driving linear motion instead of rotating motor and ball screw has been a new trend. Among linear motors, permanent magnet linear synchronous motor (PMLSM) has received growing attention for its excellent performance in precision, efficiency and reliability [1]. In the control system of PMLSM, displacement measurement is a key unit, which provides accurate position information for closed-loop control.

The most widely used typical sensors for displacement measurement are the eddy current sensor [2], the capacitance sensor [3] and the optical sensor [4]. The optical sensor, represented by the laser interferometer, exhibits excellent performance in resolution and accuracy. However, the header and the scale are still complicated and high-priced, and the sensitivity to environment limits its industrial applicability. In the case of the eddy current sensor and the capacitance sensor, although the demand of high accuracy and low price can be satisfied, a narrow measuring range remains a limitation and issue.

Since the magnetic field of PMLSM is readily translated into mathematical models, utilizing the magnetic information to detect the displacement has been a new trend. Hall sensors, converting the magnetic information into electrical signals, have the advantage of high accuracy, high efficiency, low cost and insensitivity to environment, which enhances the feasibility of the idea [5]. Songhan Pan presented an algorithm in which the position information was obtained through arctangent calculation with ideal sin and cos signals measured by two hall sensors which were electrically 90 degrees apart [6]. Jonghwa Kim then proposed a fixed point iteration based on linear equation with the use of 3 hall sensors which were electrically 60 degrees apart [7]. However, these methods have not solved the issue of nonlinear displacement measurement intrinsically. Thus, the accuracy could not be improved.
In this paper, a novel approach for nonlinear displacement estimation is proposed. The rest of this paper is organized as follows: Section 2 puts forward the novel algorithm and the proof of convergence. Section 3 verifies the effectiveness by simulation and Section 4 outlines conclusions.

2. Method Design
In PMLSM, the magnetic field of permanent magnets is of certain functional relation with displacement $x$, thus displacement measurement algorithm could be designed on the basis of the function. The research focuses on Ironless Permanent Magnet Linear Synchronous Motor (ILPMLSM), the magnets of which is structured in figure 1, where the hall sensor array moves along with the mover in Region I. The signal measured by hall sensors is the magnetic induction of permanent magnets in Region I, signed as $B$. It has demonstrated that the function between the magnetic induction in Region I and the displacement of coil could be formulated as the form of Fourier Series [1]:

$$B(x) = a_0 + \sum_{i=1}^{\infty} a_n \sin(n \pi x) + b_n \cos(n \pi x) \quad (1)$$

![Figure 1. The permanent magnets structure of ILPMLSM.](image)

Equation (1) indicates that the function is a nonlinear quasi-sinusoidal model. In an attempt to obtain $x$ from the nonlinear model, a novel method is put forward. The main idea is obtaining current displacement $x$ from the previous result by linearizing nonlinear model partially, based on continuity of the function and high sampling rate. Concretely, a series of identical hall sensors are arranged in one-dimension array along the X axis. Let $y_{k,i}$ be the signal of Sensor $i$ in the array at position $k$ and $\Delta d_i$ be the distance between Sensor $i$ and Sensor $i+1$, as shown in figure 2. By linearizing the nonlinear model between any two sensors at position $k+1$, the equation of $\Delta x$ and $\Delta d_i$ could be obtained:

$$\frac{\Delta x}{\Delta d_i} = \frac{y_{k+1,i} - y_{k,i}}{y_{k+1,i} - y_{k,i}} \quad (2)$$

$$\frac{\Delta d_i - \Delta x}{\Delta d_i} = \frac{y_{k+1,i} - y_{k,i+1}}{y_{k,i} - y_{k,i+1}} \quad (3)$$

![Figure 2. The diagram of the presented method.](image)
Sign the $\Delta x$ solved from Equation (2) or (3) as $\Delta x^{(i)}$. In order to improve the accuracy, using the $\Delta x^{(i)}$ establish another linear model, from which $\Delta x^{(2)}$ that is closer to the actual value can be obtained. Along this thought, continue the iteration until the solution is maintained in a small range. For different positions of hall sensors, the algorithm is of different forms as follow.

When $B'(x) \cdot B''(x) < 0$ (case 1), substitute $\Delta x^{(i)}$ solved from Equation (2) into the function model and then there is a new point $(x_k + \Delta x^{(i)}, B(x_k + \Delta x^{(i)}))$. Establish a new linear model with $(x_k + \Delta x^{(1)}, B(x_k + \Delta x^{(1)}))$ and $(x_k, y_k)$:

$$
\Delta x^{(2)} = \frac{y_{k+1,i} - y_{k,i}}{B(x_k + \Delta x^{(1)}) - y_{k,i}} \Delta x^{(1)}
$$

Rewrite Equation (4) into a general iterative formula:

$$
\Delta x^{(n+1)} = \frac{y_{k+1,i} - y_{k,i}}{B(x_k + \Delta x^{(n)}) - y_{k,i}} \Delta x^{(n)}
$$

Likewise, when $B'(x) \cdot B''(x) > 0$ (case 2), another iterative equation was as shown in Equation 6:

$$
\Delta d_i - \Delta x^{(n+1)} = \frac{y_{k+1,i} - y_{k,i+1}}{B(x_k + \Delta x^{(n)}) - y_{k,i+1}} \left(\Delta d_i - \Delta x^{(n)}\right)
$$

Mark the signal of hall sensor in case 1 as a and b for case 2, and define $B^{(n)} = B(x_k + \Delta x^{(n)})$.

Using least square method ultimate iterative equation for $\Delta x$ was shown as below:

$$
\Delta x^{(n+1)} = \frac{\sum_{r=1}^{M} \left[B^{(n)} - y_{k,a_r}\right] \left[y_{k+1,a_r} - y_{k,a_r}\right] \Delta x^{(n)}}{\sum_{r=1}^{M} \left[B^{(n)} - y_{k,a_r}\right]^2 + \sum_{i=1}^{N} \left[B^{(n)} - y_{k,b_i+1}\right]^2} + \sum_{i=1}^{N} \left[B^{(n)} - y_{k,b_i+1}\right] \Delta d_i
$$

Finally,

$$
x_{k+1} = x_k + \Delta x^{(n)}
$$

To ensure convergence, displacement $\Delta x$ between every two sampling points must be less than the minimum of $\Delta d_i$. Taking the case of $B'(x) > 0, B''(x) < 0$ as example, here shows the proof of convergence. Define $\Delta x^{(n+1)} = \phi(\Delta x^{(n)})$. According to theorems of concavity, there is an inequality as below:
\[
\frac{y_{k+i,j} - y_{k,j}}{\Delta x^{(e)}} > \frac{y_{k+i,i} - y_{k,i}}{\Delta d_i} = \frac{y_{k+i,j} - y_{k,i}}{\Delta x^{(l)}}
\]  

(9)

Then

\[\Delta x^{(e)} < \Delta x^{(l)} < \Delta d_i\]  

(10)

\[y_{k+i,i} < B\left(x_k + \Delta x^{(n)}\right) < y_{k,i+1}\]  

(11)

Continue iteration and there was a new inequality:

\[\frac{y_{k+i,j} - y_{k,j}}{\Delta x^{(e)}} > \frac{B\left(x_k + \Delta x^{(l)}\right) - y_{k,i}}{\Delta x^{(l)}} = \frac{y_{k+i,j} - y_{k,i}}{\Delta x^{(2)}}\]  

(12)

Thus,

\[\Delta x^{(e)} < \Delta x^{(2)} < \Delta x^{(l)}\]  

(13)

Along this method it could be proved:

\[\Delta x^{(e)} < \Delta x^{(n+1)} < \Delta x^{(n)}\]  

(14)

3. Simulation

Assume that 13 identical hall sensors are arranged 3mm apart in one-dimensional array along the X-axis. For universality, the displacement of the mover is generated by a stochastic acceleration, with the initial values of velocity and displacement both zero, as shown in figure 3.

![Figure 3. The displacement curve of the mover in simulation.](image)

The iterative algorithm is shown in Equation 7. Setting the iteration to 8, the computational errors of displacement could be seen in figure 4. In the traveling range of more than a cycle of quasi-sinusoidal model, the max error is $7.1 \times 10^{-15}$ mm and the RMS error is $2.2 \times 10^{-15}$ mm. The simulation results revealed that the presented method could reach a considerable high level in accuracy.

Taking the noise interference from hardware and environment into account, adding white Gaussian noise of 70dB on the quasi-sinusoidal model shown in Equation (1), simulation result is shown in figure 5. The max error is less than $4 \times 10^{-4}$ mm and the max error is only $2.4 \times 10^{-4}$ mm, demonstrating that LIM is of feasibility and effectiveness for nonlinear displacement estimation.
Figure 4. The computational errors of displacement without noise.

Figure 5. The computational errors of simulation with Gaussian white noise.

4. Conclusions
In this research, a novel iteration method for nonlinear displacement measurement in PMLSM system has been proposed. Based on continuity of the function and high sampling rate, current displacement \( x \) can be obtained from the previous result by linearizing the nonlinear model. The accuracy is further improved with multiple iterations and redundant measurement information. Compared to existing studies, which use hall sensors to detect the displacement utilizing arctangent method or fixed point iteration method, the method resolves the problem of nonlinear displacement measurement from the perspective of solving nonlinear equations. The simulation results confirm that the precision of the method achieves the level of submicron.

References
[1] Dong S X 2016 The research of position estimation for permanent magnet synchronous linear motor based of sensorless control (Hefei, Anhui University Press) pp 1-4
[2] Gao W, Kim S W, Bosse H, et al 2015 Measurement technologies for precision positioning CIRP ANN-MANUF. TECHN. 64 773-796
[3] Zhang J, Sun B, Dai H and Hu X 2006 High linear non-contact displacement capacitance sensor Instr. Techn. Sensor 1 6-7
[4] Cho K, Kim J, Choi S and Oh S 2015 A high-precision motion control based on a periodic
adaptive disturbance observer in a PMLSM *IEEE/ASME Trans. Mechatronics* **20** 2158-2171

[5] Ralf W, Florian S, Christian J and Stephan S 2007 Low cost position sensor for permanent magnet linear drive *IEEE PEDS* 1367 -1371

[6] Songhan P, Philip A C and Haiping D 2015 Tubular linear motor position detection by hall-effect sensors *IPEC* **34** 1-5

[7] Kim J, Choi S, Cho K and Nam K 2016 Position estimation using linear hall sensors for permanent magnet linear motor systems *IEEE Trans. Ind. Electron* **63** 7644-7652