Entropy scaling and thermalization in hadron-hadron collisions at LHC

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The recent LHC data have once more brought the issue of the formation of thermalized state of matter in hadron-hadron collisions into the forefront. In this letter, we have shown that the scaling of the information entropy of the chaotically produced particles is valid up to recently available data from p+p collisions at $\sqrt{s} = 2.36$ TeV obtained by ALICE experiment. We predict that at the highest energies projected for LHC, almost all the particles will be produced chaotically, indicating that a collective behavior should be observed in hadronic collisions, as indicated by one of present authors quite some time ago.

The properties of strongly interacting matter at extreme conditions have been an intense area of research for quite some time now. The results from relativistic heavy-ion collision experiments [1, 2] have enriched our knowledge of matter at high temperatures and small densities. One of the main goals of these experiments is to test our understanding of Quantum Chromodynamics, the fundamental theory and look for the formation of deconfined quark-gluon matter.

The non-perturbative nature of QCD makes it difficult to understand all aspects of particle production in strongly interacting systems. The huge number of particles produced in heavy-ion collision experiments makes it even more difficult to decipher the proper sources of these particles. Under such circumstances, one can look at the simpler systems, such as hadron-hadron collision to gain better understanding of particle production mechanisms and their possible sources.

Though the research on the multi-particle productions in different systems, such as high energy hadron-hadron, hadron-nucleus and nucleus-nucleus collisions are being pursued by various groups for a long time (see e.g. [3] and references therein), a proper theory of multiparticle production remains elusive till date. More so, as non-perturbative nature of the interactions involved makes it difficult to calculate the soft processes. One of the possible routes to the understanding of multiplicity is to investigate the equation of state of the matter produced in these collisions [4, 5]. This, along with a hydrodynamic picture to take care of the overall energy-momentum conservation during the space-time evolution of the excited matter, can give some idea of the dependence of multiplicity on energy.

Multiplicity distribution of hadrons produced in high energy particle collisions has long been known to deviate from a Poisson distribution and has been regarded as a potentially useful source of information about the underlying production processes. Multiplicity distributions have been predicted using various models of the low-pT hadron production processes in hadron-hadron collisions [6–10].

The Feynman scaling hypothesis led to the idea of KNO scaling [11]. However it may be noted that the inelastic hadronic interactions do not strictly follow the KNO scaling [12, 13]. Inelastic cross section is usually made up of single diffraction dissociation and the non-single diffusive (NSD) part, the latter being the main source of particles produced. The collision energy dependence of the NSD multiplicity was nicely explained by this scaling up to the ISR energies. But this scaling law broke down once the collision energy reached the higher SPS energies.

Analysis of the multiplicity data using statistical moments [14, 13] and scaling laws provide new ideas to interpret the results. A new quantity, the entropy of multiplicity distributions [16, 17], was proposed to revive the scaling law in high energy hadronic collisions at the collider energies. An important observation was the dependence of the multiplicity distribution on energy and rapidity bins based on the assumption of the existence of two kinds of sources [15, 18]. Firstly, the chaotic source, which is concentrated at small rapidities and has the characteristics of thermally equilibrated system and secondly, the coherent source, that contributes to the whole rapidity region. It was also shown that the new scaling, based on entropy, holds good for the entire range of energy starting from ISR ($\sqrt{s} = 19$ GeV) up to the highest available collision energy at SPS ($\sqrt{s} = 900$ GeV) only if the entropy is calculated from the chaotically produced particles [19]. With increasing energy, the weight of the chaotic or thermally equilibrated source is expected to increase. This may also be indicative of the production of QGP, one of the main goal of all these investigations. Recently, deconfinement in hadronic collisions at TeV en-
ergies has been explored using 1-d hydrodynamic model by E735 collaboration \cite{20}.

In this letter, we will extend the results from this scaling law \cite{17} for the NSD multiplicity distributions in p-p collisions at the highest available collision energies at LHC and try to extrapolate to the highest energy projected at the LHC. We shall argue that even in hadron-hadron collisions, one can expect a thermalised state of matter at sufficiently high collision energies, as was already alluded to in \cite{14,15}.

The scaling variable, information entropy has been calculated within the context of the two component model \cite{17} from multiplicity distribution. According to this model the emission of particles occurs from a convolution of a chaotic source with Planck-Polya distribution and a coherent source with Poisson distribution. The entropy for symmetric rapidity intervals $|\eta| \leq \eta_c$, for some arbitrary $\eta_c$, and at center of mass energy $\sqrt{s}$ is given by,

$$S(\eta_c, \sqrt{s}) = (n_{ch}(\eta_c, \sqrt{s}) + 1) \ln(n_{ch}(\eta_c, \sqrt{s}) + 1) - n_{ch}(\eta_c, \sqrt{s}) \ln(n_{ch}(\eta_c, \sqrt{s}))$$

where $n_{ch}$ is the chaotic fraction of the total multiplicity. (Similarly, $n_{co}$ and $n_{tot}$ denote the coherent and the total multiplicity, respectively, so that $n_{tot} = n_{ch} + n_{co}$.) $n_{ch}$ may be obtained as a product of the total multiplicity with the chaotic fraction $\tilde{P}$ as,

$$\langle n_{ch} \rangle = \tilde{P} \langle n_{tot} \rangle \tag{2}$$

where the chaotic fraction is given by,

$$\tilde{P} = \left[ C_2 - \left( 1 + 1/\langle n_{tot} \rangle \right) \right]^{1/2} \tag{3}$$

and $C_2 = \langle n_{tot}^2 \rangle / \langle n_{tot} \rangle^2$ is the second moment of multiplicity distribution. A scaling law then emerges if we plot the scaled entropy $S/\eta_{max}$ as a function of the scaled variable $\xi = \eta_c/\eta_{max}$ where

$$\eta_{max} = \ln((\sqrt{s} - 2m_x)/m_a) \tag{4}$$

In Fig. 1 we have plotted $S/\eta_{max}$ as a function of $\xi$ for different collision energies starting from SPS \cite{14,15} to the highest available energy at LHC \cite{21}. As can be seen, all the experimental data follow a single curve within the estimated statistical errors. A single scaling law is thus able to describe multi-particle productions up to the highest energy available for hadron-hadron collisions.

Expecting that the scaling law holds at even higher energies, an important consequence is that we can predict both the average multiplicity as well as the nature of the particles produced at higher energies. For this one has to extrapolate the available data to the desired $\sqrt{s}$ for both the average total multiplicity $\langle n_{tot} \rangle$ and the average chaotic multiplicity $\langle n_{ch} \rangle$. For the extrapolation in $\sqrt{s}$, we choose those three values of $\xi$ for which experimental data are available for the highest $\sqrt{s}=2.36$ TeV at ALICE. Experimental data are not available at exactly these values of $\xi$ at lower values of $\sqrt{s}$. They are first obtained through interpolation from the experimental data for $\langle n_{tot} \rangle$ and the model data for $\langle n_{ch} \rangle$.

![FIG. 1: Entropy scaling from two component model.](image1)

![FIG. 2: $\langle n_{ch} \rangle$ as a function of $\xi$. The squares and circles correspond to data and interpolated points respectively and the bands show the limits of statistical errors. The dotted line is from the fit described in the text.](image2)

In Fig. 2 we have plotted the $\langle n_{ch} \rangle$ as a function of $\xi$ for different collision energies. A good fit is obtained for different collision energies with the logistic function,

$$\langle n_{ch} \rangle = a + \frac{b-a}{1 + e^{-(\xi-c)/d}} \tag{5}$$

where $a$, $b$, $c$ and $d$ are parameters to be fitted. With this fit, we could obtain the value of $\langle n_{ch} \rangle$ at any value of $\xi$.
of $\xi$ for a given $\sqrt{s}$. Thus we could obtain the $\langle n_{ch} \rangle$ at the desired values of $\xi$.

The predicted values of both $\langle n_{ch} \rangle$ and $\langle n_{tot} \rangle$ are marked on the respective figures Fig. 3 and Fig. 4 and listed in Table II. The table also shows the contribution of the multiplicity of the coherent source $\langle n_{co} \rangle$. At these high energies the $\langle n_{co} \rangle$ is consistent with zero. The nature of $\langle n_{co} \rangle$ is shown in Fig. 5. We see that the $\langle n_{co} \rangle$, after an increase at the collision energies up to 900 GeV, indeed shows a dip towards higher energies. The four points are not enough to obtain a suitable functional form to fit these data. The line is just a guide to the eye.

This analysis suggests that at the higher energies the chaotic part of multiplicity completely dominates over the coherent fraction. Data at higher collision energies that would be available shortly would test these predictions.

To summarise, we have established the scaling law for the information entropy obtained in a two source model, as a function of $\sqrt{s}$ upto the maximum possible range available at present. The multiplicity of chaotically produced particles has been evaluated from a two source model in hadronic collisions starting from lowest SPS energy to the highest available LHC energy. Our study leads to the prediction that at the highest energies projected for LHC almost all the particles are produced

FIG. 3: $\langle n_{ch} \rangle$ as a function of collision energy. The squares and circles correspond to data and predicted points respectively and the bands show the limits of statistical errors. The dotted line is from the fit described in text.

$$\langle n_{ch} \rangle = b \sqrt{s}^c$$

where $b$ and $c$ are parameters to be fitted. The fit parameters are given in the Table I. Using this fitting function one can now predict the value of $\langle n_{ch} \rangle$ at higher collision energies that are expected to be obtained at LHC in near future.

A similar exercise for the average total multiplicity $\langle n_{tot} \rangle$ (Figure. 4) gives the collision energy dependence of the average total multiplicity for different $\xi$. We again find the best fit to have a similar functional form as in eqn. 6. The fit parameters are shown in Table II.

![Image of graph showing $\langle n_{ch} \rangle$ vs. $\sqrt{s}$ with different $\xi$ values](image_url)

![Image of graph showing $\langle n_{tot} \rangle$ vs. $\sqrt{s}$ with different $\xi$ values](image_url)

TABLE I: Fit parameters of $\langle n_{ch} \rangle$ with $\sqrt{s}$ for different $\xi$

| $\xi$       | Parameters | Correlations |
|-------------|------------|--------------|
| $0.051362$  | $b = 0.10(1)$ | $(b, c) = -0.991$ |
|             | $c = 0.47(2)$ |              |
| $0.102724$  | $b = 0.24(2)$ | $(b, c) = -0.990$ |
|             | $c = 0.45(1)$ |              |
| $0.133542$  | $b = 0.35(5)$ | $(b, c) = -0.994$ |
|             | $c = 0.43(2)$ |              |

TABLE II: Fit parameters of $n_{tot}$ with $\sqrt{s}$ for different $\xi$

| $\xi$       | Parameters | Correlations |
|-------------|------------|--------------|
| $0.051362$  | $b = 0.24(3)$ | $(b, c) = -0.991$ |
|             | $c = 0.37(2)$ |              |
| $0.102724$  | $b = 0.53(4)$ | $(b, c) = -0.991$ |
|             | $c = 0.36(1)$ |              |
| $0.133542$  | $b = 0.8(1)$ | $(b, c) = -0.995$ |
|             | $c = 0.35(2)$ |              |
FIG. 5: $\langle n_{co} \rangle$ as a function of collision energy. Dotted line is to guide the eye.

| $\sqrt{s}$ (GeV) | $\xi$ | $\langle n_{tot} \rangle$ | $\langle n_{ch} \rangle$ | $\langle n_{co} \rangle$ |
|------------------|-------|-----------------|-----------------|-----------------|
| 7000             | 0.051362 | 6.7(2) | 6.6(2) | 0.1(3) |
|                  | 0.102724 | 13.4(4) | 12.5(4) | 0.9(6) |
|                  | 0.133542 | 17(1) | 16(1) | 1(1) |
| 14000            | 0.051362 | 8.7(4) | 9.1(4) | 0.4(6) |
|                  | 0.102724 | 17.3(6) | 17.0(7) | 0.3(9) |
|                  | 0.133542 | 22(2) | 21(2) | 1(1) |

TABLE III: Different multiplicities with varying $\xi$ for different $\sqrt{s}$.

chaotically *i.e.* the chaoticity fraction $\tilde{P}$, reaches almost unity, indicating the formation of a thermalized source and the possibility of observation of collective behaviour in hadronic collisions.

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