Varying G dynamics

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Abstract

In this paper it is shown that dynamics based on a variation of the
gravitational constant $G$ with time solves several puzzling and anomalous
features observed, for example the rotation curves of galaxies (attributed to
as yet undetected Dark matter). It is also pointed out that this provides an explanation for the anomalous acceleration of
the Pioneer space crafts observed by J.D.Anderson and co-workers.

1 Introduction

The Milky Way to which our sun belongs, contains about 100 billion stars.
On even larger scales, individual galaxies are concentrated into clusters of
galaxies. These clusters consist of the galaxies and any material which is in
the space between the galaxies. The force that holds the cluster together is
gravity. The space between galaxies in clusters is filled with a hot gas. The
cluster includes the galaxies and any material which is in the space between

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the galaxies. The gas is hot enough to make this space shine in X-rays instead of visible light. By studying the distribution and temperature of the hot gas we can measure the force of gravity from all the material in the cluster. This allows us to determine the total material content in that cluster.

It appears that there is five times more material in clusters of galaxies than the galaxies and hot gas which we can see add up to. That is most of the matter in clusters of galaxies is invisible and, since these are the largest structures in the Universe held together by gravity, many scientists then deduce that most of the matter in the entire Universe is invisible. This invisible matter is called ‘dark matter’. Over the years there has been a lot of research by scientists attempting to discover exactly what this dark matter is, how much there is, and what effect it may have on the future of the Universe as a whole. Fritz Zwicky was the first to note in 1933 that the outlying galaxies in the Coma cluster were moving much faster than mass calculated for the visible galaxies and this would indicate that there is dark matter. Vera Rubin used galactic rotation curves to deduce that there was dark matter in galaxies. These rotation curves showed how average velocity of stars change with distance from center of galaxy. The observed rotation curves differed from the theoretical ones based on Keplerian orbits as can be seen in Figures 1(a) and 1(b).

![Figure 1: Galactic Rotation Curves](image)

(a) Expected Keplerian curves           (b) Observed Rotation Curves
2 Varying G Dynamics

we would first like to observe that even after all these decades there is neither evidence for the hypothesized Dark matter, nor any clue to its exact nature, if it exists. Let us see how varying $G$ dynamics removes the need for Dark Matter thus providing an alternative explanation. Cosmologies with time varying $G$ have been considered in the past, for example in the Brans-Dicke theory or in the Dirac large number theory or in the model of Hoyle [1, 16, 15, 8, 17]. In the case of the Dirac cosmology, the motivation was Dirac’s observation that the supposedly large number coincidences involving $N$, the number of elementary particles in the universe had an underlying message if it is recognized that

$$\sqrt{N} \propto T$$

Equation (1) leads to a $G$ decreasing inversely with time in Dirac’s hypothetical development.

The Brans-Dicke cosmology arose from the work of Jordan who was motivated by Dirac’s ideas to try and modify General Relativity suitably. In this scheme the variation of $G$ could be obtained from a scalar field $\phi$ which would satisfy a conservation law. This scalar tensor gravity theory was further developed by Brans and Dicke, in which $G$ was inversely proportional to the variable field $\phi$. (It may be mentioned that more recently the ideas of Brans and Dicke have been further generalized.)

In the Hoyle-Narlikar steady state model, it was assumed that in the Machian sense the inertia of a particle originates from the rest of the matter present in the universe. This again has been shown to lead to a variable $G$. The above references give further details of these various schemes and their shortcomings which have lead to their falling out of favour.

Then there is fluctuational cosmology in which particles are fluctuationally created from a background dark energy, in an inflationary type phase transition and this leads to a scenario of an accelerating universe with a small cosmological constant. This 1997 work [22] was observationally confirmed a year later due to the work of Perlmutter and others [20]. Moreover in this cosmology, the various supposedly miraculous large number coincidences as also the otherwise inexplicable Weinberg formula which gives the mass of an elementary particle in terms of the gravitational constant and the Hubble constant are also deduced from the underlying theory rather than being ad
To quote the main result, the gravitational constant is given by

$$G = \frac{G_0}{T}$$

where $T$ is time (the age of the universe) and $G_0$ is a constant. Furthermore, other routine effects like the precession of the perihelion of Mercury and the bending of light, and so on have also been explained in this model. Moreover in this model, the cosmological constant $\Lambda$ is given by $\Lambda \leq 0(H^2)$ and shows an inverse dependence $1/T^2$ on time. We will see that there is observational evidence for (2).

With this background, we now give some tests for equation (2).

3 A test

Let us first see the correct gravitational bending of light. In fact in Newtonian theory too we obtain the bending of light, though the amount is half that predicted by General Relativity\cite{14, 5, 23, 4}. In the Newtonian theory we can obtain the bending from the well known orbital equations (Cf.\cite{6}),

$$\frac{1}{r} = \frac{GM}{L^2}(1 + ecos\Theta)$$

where $M$ is the mass of the central object, $L$ is the angular momentum per unit mass, which in our case is $bc$, $b$ being the impact parameter or minimum approach distance of light to the object, and $e$ the eccentricity of the trajectory is given by

$$e^2 = 1 + \frac{c^2L^2}{G^2M^2}$$

For the deflection of light $\alpha$, if we substitute $r = \pm\infty$, and then use (4) we get

$$\alpha = \frac{2GM}{bc^2}$$

This is half the General Relativistic value.

We now observe that in this case we have,

$$G = G_o(1 - \frac{t}{t_o})$$

$$r = r_o\left(\frac{t_o}{t_o + t}\right)$$
We now observe that the effect of time variation of $r$ is given by equation (7) (cf. ref. [19]). Using this, the well known equation for the trajectory is given by,

$$u'' + u = \frac{GM}{L^2} + u \frac{t}{t_0} + 0 \left( \frac{t}{t_0} \right)^2$$

(8)

where $u = \frac{1}{r}$ and primes denote differentiation with respect to $\Theta$.

The first term on the right hand side represents the Newtonian contribution while the remaining terms are the contributions due to (7). The solution of (8) is given by

$$u = \frac{GM}{L^2} \left[ 1 + ecos \left\{ \left( 1 - \frac{t}{2t_0} \right) \Theta + \omega \right\} \right]$$

(9)

where $\omega$ is a constant of integration. Corresponding to $-\infty < r < \infty$ in the Newtonian case we have in the present case, $-t_0 < t < t_0$, where $t_0$ is large and infinite for practical purposes. Accordingly the analogue of reception of light for the observer, viz., $r = +\infty$ in the Newtonian case is obtained by taking $t = t_0$ in (9) which gives

$$u = \frac{GM}{L^2} + ecos \left( \frac{\Theta}{2} + \omega \right)$$

(10)

Comparison of (10) with the Newtonian solution obtained by neglecting terms $\sim t/t_0$ in equations (8) and (9) shows that the Newtonian $\Theta$ is replaced by $\frac{\Theta}{2}$, whence the deflection obtained by equating the left side of (10) to zero, is

$$cos\Theta \left( 1 - \frac{t}{2t_0} \right) = -\frac{1}{e}$$

(11)

where $e$ is given by (4). The value of the deflection from (11) is twice the Newtonian deflection given by (5). That is the deflection $\alpha$ is now given not by (5) but by the formula,

$$\alpha = \frac{4GM}{bc^2}$$

(12)

The relation (12) is the correct observed value and is the same as the General Relativistic formula which however is obtained by a different route [11, 2, 7].
4 Galactic Rotation Curves and Dark Matter

We now come to the problem of galactic rotational curves mentioned earlier (cf.ref.[14]). We would expect, on the basis of straightforward dynamics that the rotational velocities at the edges of galaxies would fall off according to

\[ v^2 \approx \frac{GM}{r} \]  \hspace{2cm} (13)

However as seen in Section (1), it is found that the velocities tend to a constant value,

\[ v \sim 300 \text{km/sec} \]  \hspace{2cm} (14)

as we approach the edges of the galaxies. This, as noted, has lead to the postulation of the as yet undetected additional matter alluded to, the so called dark matter. (However for an alternative view point Cf.[24]). We observe that from (7) it can be easily deduced that [20, 18]

\[ a \equiv (\ddot{r}_o - \ddot{r}) \approx \frac{1}{t_o} (t \ddot{r}_o + 2 \dot{r}_o) \approx -2 \frac{r_o}{t_o^2} \]  \hspace{2cm} (15)

as we are considering infinitesimal intervals \( t \) and nearly circular orbits. Equation (15) shows (Cf.ref[19] also) that there is an anomalous inward acceleration, as if there is an extra attractive force, or an additional central mass.

So, we now have

\[ \frac{GMm}{r^2} + \frac{2mr}{t_o^2} \approx \frac{mv^2}{r} \]  \hspace{2cm} (16)

From (16) it follows that

\[ v \approx \left( \frac{2r^2}{t_o^2} + \frac{GM}{r} \right)^{1/2} \]  \hspace{2cm} (17)

From (17) it is easily seen that at distances within the edge of a typical galaxy, that is \( r < 10^{23} \text{cms} \) the equation (13) holds but as we reach the edge and beyond, that is for \( r \geq 10^{24} \text{cms} \) we have \( v \sim 10^7 \text{cms} \) per second, in agreement with (14). In fact as can be seen from (17), the first term in the square root has an extra contribution (due to the varying \( G \)) which is roughly some three to four times the second term, as if there is an extra mass, roughly that much more. In fact the velocity at the edge of the galaxies as calculated
from equation (15) are tabulated in the following table, where the radius is in units of $10^{23}$ cm. The table shows that equation (15) is in agreement with the observed velocity given in equation (12).
| Velocity  | Radius |
|-----------|--------|
| $8.124038527 \times 10^6$ | 0.01 |
| $3.316635644 \times 10^7$ | 0.06 |
| $2.449539140 \times 10^7$ | 0.11 |
| $2.031135642 \times 10^7$ | 0.16 |
| $1.773059260 \times 10^7$ | 0.21 |
| $1.593679245 \times 10^7$ | 0.26 |
| $1.459778838 \times 10^7$ | 0.31 |
| $1.354963222 \times 10^7$ | 0.36 |
| $1.270085862 \times 10^7$ | 0.41 |
| $1.199589350 \times 10^7$ | 0.46 |
| $1.139877031 \times 10^7$ | 0.51 |
| $1.088505134 \times 10^7$ | 0.56 |
| $1.043747676 \times 10^7$ | 0.61 |
| $1.004346553 \times 10^7$ | 0.66 |
| $9.693603379 \times 10^6$ | 0.71 |
| $9.38068788 \times 10^6$ | 0.76 |
| $9.09910333 \times 10^6$ | 0.81 |
| $8.84439856 \times 10^6$ | 0.86 |
| $8.612994399 \times 10^6$ | 0.91 |
| $8.401975958 \times 10^6$ | 0.96 |
| $8.208942359 \times 10^6$ | 1.01 |
| $8.03189584 \times 10^6$ | 1.06 |
| $7.869158751 \times 10^6$ | 1.11 |
| $7.719310314 \times 10^6$ | 1.16 |
| $7.581138077 \times 10^6$ | 1.21 |
| $7.453599961 \times 10^6$ | 1.26 |
| $7.335794393 \times 10^6$ | 1.31 |
| $7.226936540 \times 10^6$ | 1.36 |
| $7.126339217 \times 10^6$ | 1.41 |
| $7.033397433 \times 10^6$ | 1.46 |
| $6.947575783 \times 10^6$ | 1.51 |
| $6.868398088 \times 10^6$ | 1.56 |
| $6.795438824 \times 10^6$ | 1.61 |
| $6.728315996 \times 10^6$ | 1.66 |
| $6.666685175 \times 10^6$ | 1.71 |
| $6.610234489 \times 10^6$ | 1.76 |
| $6.558680385 \times 10^6$ | 1.81 |
| $6.511764044 \times 10^6$ | 1.86 |
| $6.469248319 \times 10^6$ | 1.91 |
| $6.430915128 \times 10^6$ | 1.96 |
Thus the time variation of $G$ explains observation without invoking dark matter. It may be added that this also explains the latest studies by Metz, Kroupa and others of the satellite galaxies of the Milky Way galaxy, which also throw up the faster than expected rotational velocities, ruling out however, in this case dark matter, in addition.

5 Remarks

There could be other explanations, too. One of the authors and A.D. Popova have argued that if the three dimensionality of space asymptotically falls off, then the above can be explained [21].

Yet another prescription was given by Milgrom [8] who approached the problem by modifying Newtonian dynamics at large distances. It must be mentioned that this approach is purely phenomenological.

The idea was that perhaps standard Newtonian dynamics works at the scale of the solar system but at galactic scales involving much larger distances perhaps the situation is different. However a simple modification of the distance dependence in the gravitation law, as pointed by Milgrom would not do, even if it produced the asymptotically flat rotation curves of galaxies. Such a law would predict the wrong form of the mass velocity relation. So Milgrom suggested the following modification to Newtonian dynamics: A test particle at a distance $r$ from a large mass $M$ is subject to the acceleration $a$ given by

$$a^2/a_0 = MGr^{-2}, \quad (18)$$

where $a_0$ is an acceleration such that standard Newtonian dynamics is a good approximation only for accelerations much larger than $a_0$. The above equation however would be true when $a$ is much less than $a_0$. Both the statements in (18) can be combined in the heuristic relation

$$\mu(a/a_0)a = MGr^{-2} \quad (19)$$

In (19) $\mu(x) \approx 1$ when $x >> 1$, and $\mu(x) \approx x$ when $x << 1$. It must be stressed that (18) or (19) are not deduced from any theory, but rather are an ad hoc prescription to explain observations. Interestingly it must be mentioned that most of the implications of Modified Newtonian Dynamics or MOND do not depend strongly on the exact form of $\mu$.

It can then be shown that the problem of galactic velocities is now solved.
Finally it maybe mentioned that the above varying $G$ dynamics explains the puzzling anomalous acceleration of the Pioneer spacecrafts of the order of $10^{-7}$ to $10^{-8}$ cm/sec$^2$ observed by J.D.Anderson of the JPL Pasadena, and co-workers [25].

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