MIMO Physical Layer Network Coding Based on VBLAST Detection

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Abstract—For MIMO two-way relay channel, this paper proposes a novel scheme, VBLAST-PNC, to transform the two superimposed packets received by the relay to their network coding form. Different from traditional schemes, which tries to detect each packet before network coding them, VBLAST-PNC detects the summation of the two packets before network coding. In particular, after firstly detecting the second layer signal in 2-by-2 MIMO system with VBLAST, we only cancel part of the detected signal, rather than canceling all the components, from the first layer. Then we directly map the obtained signal, summation of the first layer and the second layer, to their network coding form. With such partial interference cancellation, the error propagation effect is mitigated and the performance is thus improved as shown in simulations.

Index Terms—Multiple Input Multiple Output, Physical layer Network Coding, Two Way Relay Channel, VBLAST.

I. INTRODUCTION

In wireless networks, the use of relay is attracting increasing attention [1, 2] because of its many advantages. Among the relay channels, two-way relay channel (TWRC, as shown in Fig. 1) is a especially interested due to the almost double spectral efficiency with the physical layer network coding (PNC) [3] transmission scheme. It was further proved in [4, 5] that PNC can approach the capacity of TWRC in high SNR region.

Another spectral efficiency boosting technique is Multiple Input and Multiple Output (MIMO), which has been widely used in wireless systems. Therefore, it is of great interest to combine PNC and MIMO to further improve the wireless spectral efficiency. A straightforward way is to divide the MIMO transmission into parallel SISO streams by precoding, so that PNC can be implemented on each steam [6]. However, the precoding requires not only transmitter side channel state information (CSIT) but also strict time and carrier phase synchronization between the two end nodes.

MIMO NC scheme [7] is more practical, where only receiver side CSI (CSIR) is needed. In MIMO NC, the relay node detects each end node’s packet with traditional MIMO detection and then combines them with network coding. These schemes failed to exploit the fact that the relay does not need each end node’s individual information. Hence, the performance is limited by over-detection.

In [8], we have proposed a novel MIMO PNC scheme based on linear detection, which will be referred as linear MIMO PNC in this paper. Linear MIMO PNC tries to detect the summation and the difference of the two end node’s packets before transforming them to the network coding form. With similar complexity and CSIR requirement, it significantly outperforms MIMO NC. However, due to the performance limit of linear detection, linear MIMO PNC’s performance is poor under bad channel conditions.

Besides the linear detection, another popular MIMO detection method is VBLAST (Vertical Bell Laboratories Layered Space-time) which can achieve much better performance with an acceptable increase in complexity. The VBLAST architecture was first proposed in [9] where a code block is de-multiplexed into different layers and each is transmitted through a particular antenna. At the receiver, these layers are successively detected, where the detected interference are canceled and the unknown interferences are nulled by linearly weighting the residual signal vector with a ZF (zero-forcing) null vector (ZF VBLAST). A low complexity ZF VBLAST scheme is proposed in [10], where the channel matrix is rewritten in terms of the QR decomposition as $H = QR$. The inverse of unitary matrix $Q$ was then multiplied to the received signal before estimating the transmit information. In order to find the optimal detection order, [10] further proposed the sorted QR decomposition algorithm, ZF-SQRD.

In this paper, we combine the basic idea of PNC and the QR VBLAST MIMO detection scheme and propose VBLAST PNC. Our scheme only requires receiver side channel state information and symbol level synchronization between the end nodes, as in general virtual MIMO system. The basic idea of VBLAST PNC in a 2-by-2 MIMO system is as follows. With QR decomposition, the relay first detects the second layer (one end node) signal. Rather than canceling all the component of the second layer signal from the first layer as in traditional VBLAST detection, we only subtract a part of the second layer information and directly map the residual signal (including both second and first layer information) to the network coding form. With such partial interference cancellation, the error propagated from the incorrectly detected second layer is significantly decreased. Thus, the system performance is improved. We then extend our VBLAST PNC to a detection scheme with an optimal order as in ZF-SQRD, and even better performance is achieved. Numerical simulation is done to compare the performance of VBLAST PNC with linear MIMO PNC and MIMO NC schemes. The results show that VBLAST PNC can achieve much better BER performance than linear MIMO PNC and VBLAST MIMO NC.
II. System Model and Illustrating Example

A. System Model

This paper is mainly based on the TWRC in Fig. 1, where the relay is equipped with 2 antennas and the each end node is equipped with single antenna.

The transmission consists of two phases. In the uplink phase, both the end nodes transmit their packets to the relay node simultaneously using QPSK modulation. We assume that the two packets arrive at the relay node in a symbol level synchronization. In that way, the superimposed signal received by the relay is:

\[ y_i = h_{i1}x_1 + h_{i2}x_2 + n_i \]
\[ y_2 = h_{21}x_1 + h_{22}x_2 + n_2 \]  
(1)

where \( y_i \) denotes the received signal at the \( i \)-th antenna of the relay node; \( h_{i,j} \) is zero mean complex Gaussian random variable, which denotes the channel coefficient from the end node \( N_j \) to the \( i \)-th antenna of the relay node; \( x_i \in \{ \pm 1 \pm \sqrt{-1} \} \) is transmitted signal with QPSK modulation of the end node \( N_i \); and \( n_i \) denotes the complex Gaussian noise with zero mean and \( \sigma^2 \) variance for each dimension.

In this phase, the full channel information is available at the relay node. Rewriting the received signal in the vector form as

\[ Y = HX + N \]  
(2)

Then, the relay node tries to extract some useful information from \( Y \) and transforming it to the network coded form \( x_1 \oplus x_2 \). The detailed processing will be illustrated later, which is also the focus of our paper.

In the downlink phase, the relay broadcasts the network coded packet to both end nodes. After receiving the packet from the relay, the end nodes extract their target packets with the help of their own information.

B. Illustrating Example

Now, we present an example to illustrate the basic idea of VBLAST PNC and its superiority over VBLAST NC, with a simple channel realization.

In the first phase, the transmission in (2) can be regarded as a 2x2 MIMO system (two transmit antennas and two received antennas). The relay node’s goal is to acquire an estimate of \( x_1 \oplus x_2 \). In the traditional MIMO NC scheme, the relay node decodes \( x_1 \) and \( x_2 \) explicitly before network encoding them. Nevertheless, this scheme is suboptimal.

Consider an ill-conditioned channel matrix \( H \)

\[ H = \begin{bmatrix} 1 & 1 + \Delta \\ 0 & \Delta \end{bmatrix} \]  
(3)

where \( \Delta \) is a small quantity. Then eq.(1) can be rewritten as

\[ y_1 = x_1 + (1 + \Delta)x_2 + n_1 \]
\[ y_2 = \Delta x_2 + n_2 \]  
(4)

With QR ZF VBLAST detection, \( x_2 \) is first detected as

\[ \tilde{x}_2 = \frac{1}{\Delta}y_2 = x_2 + \frac{1}{\Delta}n_2 \]  
(5)

According to the processing of VBLAST, we cancel the interference \( x_2 \) in \( y_1 \) and obtain the estimate of \( x_1 \) as

\[ \tilde{x}_1 = y_1 - (1 + \Delta)\tilde{x}_2 \]
\[ = x_1 + (1 + \Delta)x_2 + n_1 - (1 + \Delta)(x_2 + \frac{1}{\Delta}n_2) \]
\[ = x_1 + n_1 - \frac{1}{\Delta}n_2 \]  
(6)

From (5) and (6), we can find that the SNR of \( x_1 \) tends to zero when \( \Delta \) is very small. It means that the VBLAST NC scheme can’t accurately estimate the target signal \( x_1 \oplus x_2 \) in this case.

Based on the basic idea of PNC, the relay can estimate the target signal \( x_1 \oplus x_2 \) from \( x_1 \oplus x_2 \). Therefore, we can first estimate \( x_1 \oplus x_2 \) rather than estimate individual information of \( x_1, x_2 \), from \( Y \). The particular processing is as follows. After estimating \( x_2 \), we subtract \( \Delta \tilde{x}_2 \) rather than \( (1 + \Delta)\tilde{x}_2 \) from \( y_1 \). Then, we can obtain

\[ y_1' = y_1 - \Delta \tilde{x}_2 = x_1 + x_2 + n_1 - n_2 \]  
(7)

With the PNC mapping [3], we can directly map \( y_1' \) in (7) to the target information \( x_1 \oplus x_2 \). Since the noise in (7) is small (independent of \( \Delta \)), the performance of this scheme is much better than the MIMO NC scheme.

This example indicates that MIMO PNC may significantly outperform the traditional MIMO NC. In the following sections, we introduce the proposed VBLAST PNC scheme in detail.

III. VBLAST PNC Scheme

In this section, we first briefly review the MIMO NC scheme based on QR VBLAST detection algorithm [9, 10] for a comparison. After that, we elaborate the proposed VBLAST PNC.

A. MIMO NC based on QR VBLAST

Consider the 2-by-2 MIMO system in (2). The channel matrix can be decomposed with QR decomposition so that \( H = QR \), where the matrix \( Q \) is a unitary matrix (orthogonal columns with unit norm) and \( R \) is an upper triangular matrix. Multiplying the received signal \( Y \) by \( Q^H \), we can obtain the calculated signal

\[ W = Q^H Y = RX + U \]  
(8)

where \((\cdot)^H\) denotes the matrix conjugate transpose and the new noise vector \( U = Q^H N \) has the same distribution as \( N \). The scalar form of \( W \) is

\[ w_1 = r_{11}x_1 + r_{12}x_2 + u_1 \]
\[ w_2 = r_{22}x_2 + u_2 \]  
(9)

Owing to the upper triangular characteristic of \( R \), we can easily estimate the second layer signal \( (x_2 \) here) as:

\[ \tilde{x}_2 = \frac{1}{\Delta}y_2 = x_2 + \frac{1}{\Delta}n_2 \]  
(5)
\[ \hat{x}_2 = \frac{w_2}{r_{22}} = x_2 + \frac{w_2}{r_{22}} \]  
(10)

Since the QPSK modulation is adopted, the above signal can be demodulated with hard decision as

\[ \hat{x}_2 = \text{sign}(\hat{x}_2) \]  
(11)

Note that the hard decision in (11) is performed for the real part and the imaginary part respectively. According to the basic idea of VBLAST, after the second layer signal \( x_2 \) is detected, we can then detect the first layer signal \( (x_1) \) here by canceling the interference of \( x_2 \) as:

\[ \hat{x}_1 = \text{sign}(\hat{x}_1) = \text{sign} \left( \frac{w_1 - r_{12}\hat{x}_2}{r_{11}} \right) \]  
(12)

After obtaining the individual decisions of \( x_1 \) and \( x_2 \), a straightforward way to calculating \( x_1 \oplus x_2 \) is to combine the estimates of \( x_1 \) and \( x_2 \) (obtained in (11) and (12) respectively):

\[ x_1 \oplus x_2 = \hat{x}_1 \oplus \hat{x}_2 \]  
(13)

Hereafter, we refer to this scheme as VBLAST NC. VBLAST NC detects the signals separately with VBLAST algorithm, and then encodes them into the network-coded form. However, it may perform poorly as shown in our illustrating example.

As in the VBLAST processing above, exchange of the detection order between \( x_1 \) and \( x_2 \) (first detect \( x_1 \) and then cancel it before detecting \( x_2 \)) is also workable. As shown in [10], the detection sequence is crucial to the performance of VBLAST because of the risk of error propagation. To obtain a better performance, we can permute the columns of the VBLAST NC detecting sequence is crucial to the performance of VBLAST because of the risk of error propagation. To obtain a better performance, we can permute the columns of the VBLAST because of the risk of error propagation. To obtain a better performance, we can permute the columns of the VBLAST. In particular, we use QPSK modulation and set \( n = 1 \) for each dimension (real part or imaginary part) signal, the error propagation effect is Mitigating the error propagation effects. The detail of VBLAST PNC is as follows.

We rewrite \( w_1 \) in (9) as

\[ w_1 = r_{11}(x_1 + kx_2) + (r_{12} - kr_{11})x_2 + u_1 \]  
(14)

where \( k \) is an integer to be determined later. In (14) we regard \( x_1 + kx_2 \) as the signal to be estimated and \((r_{12} - kr_{11})x_2\) as interference to be cancelled.

In order to decrease the effect of error propagation, we must minimize \((r_{12} - kr_{11})\). For example, if \( k = r_{12}/r_{11} \), no interference needs to be cancelled and there will never exist error propagation during the detection of the signal in (14).

Taking the integer requirement of \( k \) into account, we can determine the value of \( k \) as

\[ k = \lfloor \text{real}(r_{12})/r_{11} \rfloor \]  
(15)

where \( \lfloor \cdot \rfloor \) means the integer nearest to \( t \). In (15), \( r_{12} \) is a complex variable and \( r_{11} \) is a real variable, and we only take account the real part to calculate \( k \) in this paper.

After cancelling the interference with the hard estimate of \( x_2 \) in (11), we can obtain the estimate of \( x_1 + kx_2 \) as

\[ x_1 \oplus x_2 = \frac{w_1 - (r_{12} - kr_{11})x_2}{r_{11}} \]  
(16)

Finally, the relay node estimates \( x_1 \oplus x_2 \) only from \( x_1 + kx_2 \) as long as \( k \) does not equal to 0. When \( k > 0 \), intuitively, \( abs(x_1 + kx_2) \) is larger for \( x_1 \oplus x_2 = 0 \), while it is smaller for \( x_1 \oplus x_2 = 1 \). When \( k < 0 \), the situation reverse. Then, for each dimension (real part or imaginary part) signal, the corresponding decision rule is

\[ x_1 \oplus x_2 = \text{sign}(k)\text{sign}(abs(x_1 + kx_2) - \gamma) \]  
(17)

where \( \gamma \) is the decision threshold and its optimal value can be calculated as in [3]. In high SNR, we can simplify the calculation of \( \gamma \) and set it to \( abs(k) \). Then we have

\[ x_1 \oplus x_2 = \begin{cases} \text{sign}(abs(x_1 + kx_2) - k) & \text{when } k > 0 \\ \text{sign}(-k - abs(x_1 + kx_2)) & \text{when } k < 0 \end{cases} \]  
(18)

To further improve the performance, we can extend our scheme to the sort VBLAST PNC, where the optimal detection order is chosen as the tradition VBLAST. In particular, we exchange the columns of \( H \) to obtain a larger \( r_{22} \). Then, the VBLAST algorithm is performed on the new \( H \).

B. VBLAST PNC Scheme

In VBLAST, the diversity order of the first layer should be larger than the second layer in theory under the assumption of clean interference cancellation. In practice, however, the diversity order of both layers is the same [11]. The main reason is the error propagated from the erroneously detected second layer signal. To mitigate the error propagation and better the performance, we propose VBLAST PNC scheme which only cancels a part of the detected second layer information. After the cancellation, we require the remaining signal to be in the form of \( x_1 + kx_2 + u_1 \) where \( k \) is an integer. We can then directly map this signal to the target signal \( x_1 \oplus x_2 \) by applying PNC mapping, without explicitly detecting \( x_1 \). Since the PNC mapping has similar performances as the ordinary point-to-point transmission, our VBLAST PNC could achieve a better performance by mitigating the error propagation effects. The detail of VBLAST PNC is as follows.

We rewrite \( w_1 \) in (9) as

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IV. NUMERICAL SIMULATION

This section, we present the simulation results for the proposed VBLAST PNC. To compare its performance, we also show the simulation performance of linear MIMO PNC, the performance of linear MIMO NC and VBLAST NC.

The simulation setting is mainly based on the system model in section II. In particular, we use QPSK modulation and set the packet length to \( 10^6 \). The wireless channels are assumed to be block fading with each entry of the channel matrix to be independently complex Gaussian distributed over \( N(0, 1) \). The noise is Gaussian distribution with \( N(0, \sigma^2) \) and the SNR of the system is defined as \( 1/\sigma^2 \). Simulation results are measured in terms of bit error rate (BER) of \( x_1 \oplus x_2 \) at the relay node since the broadcast phase is straightforward.

In Figure 2, we plot the BER performance of different schemes. As shown in the figure, the proposed MIMO PNC schemes always outperform their counterparts. Specifically, our VBLAST PNC outperforms VBLAST NC by about 0.5
Fig. 2: BER performance of MIMO PNC and MIMO NC schemes

dB at a BER of $10^{-3}$; the sorted VBLAST PNC outperforms sorted VBLAST NC by about 1dB. In sorted VBLAST, the more performance improvement mainly comes from the fact that the average value of $r_{11}$ is smaller and the interference to be cancelled, $(r_{12} - kr_{11})x_2$, is smaller.

V. Conclusion

In this paper, a novel signal detection and network encoding scheme, VBLAST PNC, is proposed to extract $x_1 \oplus x_2$ at the relay node in MIMO TWRC. The basic idea is that the relay node first uses partial interference cancellation to obtain $x_1 + kx_2$ during VBLAST detection process and then converts it to $x_1 \oplus x_2$ with PNC mapping. With partial interference cancellation, error propagation effect is mitigated and the performance is significantly improved. The simulation results verified the performance advantages of VBLAST PNC under the setting of random Rayleigh fading channel coefficients. Our scheme is of great interest in practice since only CSIR, symbol lever synchronization and low complexity are needed.

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