On the predictability and robustness of Galileo disposal orbits

David J. Gondelach¹ · Roberto Armellin² · Alexander Wittig³

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Abstract
The end-of-life disposal of Galileo satellites is needed to avoid collisions with operational spacecraft and to prevent the generation of space debris. Either disposal into stable graveyard orbits or disposal into the atmosphere exploiting eccentricity growth caused by lunisolar resonances is possible. However, there is a concern about the predictability of medium Earth orbits because of possible chaotic behaviour caused by the overlap of resonances. In this work, we investigate if Galileo disposal orbits are predictable and robust, that is, if safe disposal is possible under uncertainties. For this, we employ finite-time Lyapunov exponents (FTLEs) to study the chaoticity of orbits. In addition, sensitivity analysis is used to quantify the effect of uncertainties on the orbital evolution and to determine if safe disposal is possible. Whether the disposal orbits are chaotic or not could not be concluded from the FTLE analysis, because the observed divergence between neighbouring orbits can also be caused by hyperbolicity of the dynamics. Nevertheless, because the resonance dynamics are perturbed and resonances may overlap, all disposal orbits are expected to be chaotic. Regarding robustness, we found that the majority of the investigated re-entry disposal trajectories (including low ΔV solutions) is robust. On the other hand, we find that the investigated graveyard orbits and a small portion of the assessed re-entry orbits are not robust under uncertainties in the disposal manoeuvre and in the dynamical model. Therefore, it is mandatory to assess the sensitivity of a disposal orbit to uncertainties to ensure safe disposal.

Keywords Galileo disposal · Stability · Chaos · Predictability · Finite-time Lyapunov exponent · Sensitivity analysis
1 Introduction

Most satellites of the four main global satellite navigation systems (GPS, GLONASS, Galileo and BeiDou) are located in the medium Earth orbit (MEO) region between 19,100 and 23,222 km altitude and with inclinations between 54.8° and 64.8° (Radtke et al. 2015). After their operational lifetime, it is important to put these satellites into stable graveyard orbits or to dispose them into the atmosphere to avoid collisions with operational spacecraft or between themselves, which would generate space debris.

The MEO region is also a location where lunisolar resonances occur, which mainly depend on the orbit inclination (Cook 1962; Hughes 1980; Ely and Howell 1997). These resonances can cause the eccentricity to grow secularly, and as a result an initially near-circular MEO orbit can re-enter the atmosphere within 200 years (Chao and Gick 2004; Rossi 2008; Deleflie et al. 2011). On the one hand, this instability in eccentricity is undesirable for safe disposal into graveyard orbits, because disposed satellites must keep a safe distance from the operational spacecraft. On the other hand, the instability can be exploited to lower the perigee and dispose of satellites by causing them to re-enter into the Earth’s atmosphere (Radtke et al. 2015; Sanchez and Yokoyama 2015; Alessi et al. 2016). So, depending on the type of disposal, we are looking for either minimum change in eccentricity for graveyard orbits or maximum change in eccentricity for re-entry disposal orbits.

In addition, due to the overlap of different lunisolar resonances, chaos can occur in the MEO region (Ely and Howell 1997; Rosengren et al. 2015; Daquin et al. 2016; Gkolias et al. 2016). This means that the evolution of MEO orbits can be very sensitive to the initial conditions, and a tiny perturbation of the initial state may cause the evolution of the orbit to be completely different. This makes it impossible to reliably predict the orbit and has lead to concern about the predictability of orbits in the MEO region (Daquin et al. 2016; Rosengren et al. 2017).

Unwanted orbital changes can be corrected by performing orbital manoeuvres; however, disposed satellites do not have this option. Therefore, to be sure that the disposal is successful in the presence of resonances and chaos, all possible orbital evolutions due to uncertainties must be considered beforehand. In the case of disposal orbits, we are especially interested in the evolution of the eccentricity that determines the perigee and apogee altitude of the disposal orbit and thus dictates the flight domain of a graveyard orbit and whether a spacecraft will re-enter. (Note that the semi-major axis remains nearly constant in the absence of drag, only slightly varying in a bounded range due to tesseral resonances.)

When designing a disposal orbit, we have to consider the practical aspects of carrying out a disposal. For example, we need to take into account the amount of fuel available to manoeuvre the spacecraft into the disposal orbit. This means that large parts of the phase space cannot be reached with a limited fuel budget and do not need to be considered during design. To save fuel, it has been suggested by, for example, Rosengren et al. (2017) to delay the disposal manoeuvre to reach certain parts of the phase space through natural precession of the orbit. One could, for example, wait until a favourable configuration with respect to the Moon is established. However, for Galileo orbits, the longitude of the ascending node changes by only −10°/year through natural precession. Therefore, waiting for a favourable configuration may take several years. This can be too long from an operational point of view, because the spacecraft may fail to operate.

Recently, Mistry and Armellin (2016) and Armellin and San-Juan (2018) approached the design of graveyard and re-entry disposal orbits for Galileo satellites by optimizing the orbits regarding the amount of ΔV required for the disposal manoeuvre. In this way, they found
practically feasible disposal options in terms of required propellant. In addition, Rosengren et al. (2019) and Skoulidou et al. (2019) carried out extensive numerical investigations to identify regions in the near-Earth space where passive and low-$\Delta V$ re-entry and graveyard disposal solutions are possible. However, these studies did not take into consideration the predictability and robustness of the disposal orbits, that is, the orbital evolution of the disposal options due to uncertainties was not considered.

The predictability of Galileo disposal orbits was addressed by Rosengren et al. (2017). In their work, they analysed the chaos and stability of orbits by studying the dynamics and computing the fast Lyapunov indicator (FLI) and Lyapunov time for different orbits in the initial phase space. They concluded that many orbits that seem stable within 200 years are actually unstable in 500 years, and the Lyapunov time is generally much shorter than the propagation time. However, as noted by Rosengren et al. (2017), the correlation between the Lyapunov time and the effective predictability time horizon is not always clear or can be hard to establish (see, e.g. Milani and Nobili (1992) and Siegert and Kantz (2016)) and was left open for future work. In addition, Daquin et al. (2018) showed that there can be discrepancies between the value of the FLI and the change in eccentricity, our main parameter of interest. In other words, strong divergence and large changes in eccentricity do not need to go hand in hand. Rosengren et al. (2017) suggests that the orbital evolution of chaotic orbits must be studied in statistical terms (see, e.g. Laskar and Gastineau (2009)), because a single trajectory is not representative and one must look at the evolution of ensembles of trajectories. This raises the question of how we should study the predictability of orbits? And how can we determine if an orbit is predictable on the timescale of interest?

In this paper, we study the predictability of Galileo disposal orbits with the goal to determine if Galileo disposal options are robust or not. Here, by robustness we mean that the uncertainty in eccentricity remains small enough such that re-entry of the satellite is guaranteed (re-entry disposal) or the satellite keeps a safe distance from operational orbits (graveyard disposal). Therefore, the main aspect of the disposal orbits that we are concerned with is how the orbits change as a result of uncertainty and if this affects disposal success. We want to know how initially nearby orbits diverge over time and are particularly interested in whether the divergence is exponentially fast. In such case, the orbit could be chaotic and its long-term evolution could be unpredictable, which would make it impossible to guarantee successful disposal. To investigate this, we compute the finite-time Lyapunov exponent and carry out sensitivity analysis. The sensitivity analysis is performed numerically while bounds of the uncertainty domain are also estimated using Taylor differential algebra (DA). The results are used to assess the predictability and robustness of the studied disposal orbits.

We focus mainly on four Galileo disposal orbits that were found by optimizing the disposal manoeuvre. Three of these orbits dispose the spacecraft via re-entry (Armellin and San-Juan 2018), and the other is a graveyard orbit (Mistry and Armellin 2016). These disposal orbits were designed to re-entry within 100 years or keep a safe distance for 100 years using minimum $\Delta V$. We investigate if these disposal criteria are also met under uncertainty and if the orbits are predictable. To gain additional insight, we will also investigate the robustness of higher $\Delta V$ disposal options.

The paper is set up as follows. First, we introduce the methods for investigating the predictability and discuss some important considerations regarding Lyapunov exponents. We describe the dynamical models used in this work and introduce the test cases. In Sect. 5, the results of the FTLE and sensitivity analyses are presented. These are subsequently discussed in Sect. 6, and conclusions about the predictability and robustness of the investigated disposal orbits are drawn.

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2 Methods

In this section, the methods to compute the FTLE and to carry out sensitivity analysis are discussed. Before that, Taylor differential algebra is introduced that is used for calculating the FTLE and estimating the bounds of the uncertainty domain. We refer the reader to the provided references for further details on each technique.

2.1 Differential algebra

DA is an automatic differentiation method which enables the automatic computation of derivatives of functions in a computer environment (Berz 1999). Based on truncated power series algebra, the algebraic operations for floating-point numbers are replaced by operations for Taylor polynomials, such that any function $f$ of $v$ variables can straightforwardly be expanded into its Taylor polynomial up to arbitrary order. The differential algebra used in this work is the DA computational engine (DACE) (Rasotto et al. 2016) that includes all core DA functionality and a C++ interface.\(^1\)

One of the key applications of DA is the automatic high-order expansion of the solution of an ODE initial value problem with respect to the initial conditions (Berz 1999; Armellin et al. 2010). Using DA ODE integration, the Taylor expansion of the flow in $x_0$ can be propagated forward in time, up to any final time. The main benefit of the use of DA to expand the flow is that we can expand up to arbitrary order and there is no need to write and integrate variational equations.

The high-order Taylor expansions are only accurate in a domain close to the expansion point. To ensure that the computed Taylor polynomial is sufficiently accurate, we can estimate the truncation error of the expansion by estimating the size of the order $n + 1$ coefficients of the Taylor polynomial as explained in Wittig et al. (2015). Using the estimated truncation error, the distance from the expansion point where the Taylor series has a specific error can be estimated. For states inside the estimated domain, the high-order map has a truncation error that is approximately smaller than the specified error.

2.2 Lyapunov exponent

The Lyapunov exponent tells us the rate of divergence of nearby trajectories. Consider the distance $|\Delta x(t)|$ between two neighbouring orbits starting at $x_0$ and $x_0 + \Delta x_0$. This distance will change over time, and its evolution can be approximated by a linearization of the ODE around the reference orbit as:

$$|\Delta x(t)| \approx e^{\mu t} |\Delta x_0| = e^{t/T_L} |\Delta x_0|,$$

where $\mu$ is one of the Lyapunov exponents (LEs) and $T_L$ the corresponding Lyapunov time. The LE depends on the orientation of the initial $\Delta x_0$. If $\Delta x_0$ is in the direction of maximum growth, then the corresponding $\mu$ is the maximum Lyapunov exponent (MLE). The MLE can be computed as\(^2\) (Skokos et al. 2016):

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1 The open-source software package is available at https://github.com/dacelib/dace.

2 It is worth noting that the existence of these limits is not guaranteed in all systems (Ott and Yorke 2008). One common criterion ensuring existence is the existence of an invariant measure under the flow on a compact set: in this case the set of points where the limit does not exist has zero measure (Oseledec 1968). For symplectic systems, such as Hamiltonian dynamics, the first criterion is trivially satisfied as the phase space volume is.
\[
\mu = \lim_{t \to \infty} \lim_{\delta x_0 \to 0} \frac{1}{t} \ln \frac{|\delta x(t)|}{|\delta x_0|}.
\] (2)

If the MLE is positive, then neighbouring trajectories separate exponentially fast. In that case, a tiny deviation in the initial state could potentially result in a completely different orbital evolution after instabilities set in, depending on the uncertainty in the initial state. A positive MLE is therefore often considered as a definition of chaos; however, this is not strictly correct:

- “A positive Lyapunov exponent just indicates the presence of local hyperbolicity in the neighborhood of the orbit” (Morbidelli 2002, Chap. 5). A positive MLE does therefore not necessarily mean that the evolution of the orbit is unpredictable. For example, the hyperbolic fixed point of a pendulum has a positive MLE, while the flow is regular and predictable. In fact, the secular dynamics of the considered re-entry disposal orbits are dictated by the \(2\omega + \Omega\) resonance that (when isolated from other resonances) has a hyperbolic fixed point for circular orbits\(^3\); see Breiter (2001). Therefore, regardless whether the orbit is regular or chaotic, we can expect a positive MLE if the orbit is in resonance.

- A positive MLE only defines deterministic chaos if the motion is aperiodic and bounded. However, in our dynamics, periodic and unbounded motion is not expected to occur and was indeed not observed during the investigation. In addition, in the considered dynamics we can expect chaotic behaviour, because single resonance dynamics are perturbed, which results in separatrix splitting, and different resonances may overlap, which both result in chaotic motion (Morbidelli 2002, Chap. 4).

2.3 Finite-time Lyapunov exponent

Since it is in general not possible to propagate a trajectory for infinite time (as required to compute the MLE using Eq. 2), we can approximate the MLE in finite time. If the chosen finite time is sufficiently large, and all the limits in Eq. (2) exist, this procedure yields an accurate approximation of the MLE. In this work, we estimate the MLE using the finite-time Lyapunov exponent (FTLE)\(^4\) that is computed as follows (Shadden et al. 2005).

First, we compute the Jacobian of the flow map with respect to the initial condition: \(\frac{d\phi}{dx_0}\), where \(\phi = x(t, x_0)\) is the flow map. With the use of DA, this Jacobian can be computed straightforwardly using the first-order Taylor expansion of the flow with respect to the initial state without the need to write the variational equations. The values of the partial derivatives are simply given by the first-order coefficients of the Taylor polynomial.

Second, we compute the (right) Cauchy–Green deformation tensor \(\Delta\) by multiplying the Jacobian \(\frac{d\phi}{dx_0}\) by its transpose:

\[
\Delta = \left[ \frac{d\phi}{dx_0} \right]^T \left[ \frac{d\phi}{dx_0} \right].
\] (3)

Footnote 2 continued

preserved due to Liouville’s theorem. However, even then in many cases it is not easy to show that the relevant dynamics happen on a compact set.

\(^3\) Breiter (2001) associates the \(2\omega + \Omega\) resonance with the second fundamental model for second-order resonances (SFM2) that has a hyperbolic fixed point for circular orbits depending on the orbital configuration.

\(^4\) A good introduction to the FTLE and its use can be found online at http://shaddenlab.berkeley.edu/uploads/LCS-tutorial/contents.html.
where $T$ indicates that we take the transpose. This tensor provides a measure of the square of local change in distances due to expansion or contraction.

The maximum stretching is given by the square root of the maximum eigenvalue $\lambda_{\text{max}}$ of $\Delta$, and therefore the FTLE is defined as:

$$\text{FTLE}(t, x_0) = \frac{1}{t} \ln \sqrt{\lambda_{\text{max}}(\Delta)} = \frac{1}{2t} \ln \lambda_{\text{max}}(\Delta).$$

(4)

If we evaluate Eq. (4) for infinite time, we obtain the MLE. In addition, the different eigenvalues of $\Delta$ give the spectrum of Lyapunov exponents.

### 2.3.1 Coordinate and metric dependence

Lyapunov exponents are, by definition, independent of both the metric used to determine the distance between orbits and the choice of variables [given that the coordinate transformation is bounded, differentiable and invertible (Kuznetsov et al. 2016)]. However, since we cannot evaluate Eq. (4) for infinite time, the FTLE is not independent of the metric and choice of variables.

For the FTLE in particular, we find that the Jacobian can either be calculated in one set of coordinates $X$ directly, or it can be transformed from another set of coordinates $x$ a posteriori. The a posteriori coordinate transformation is achieved by multiplying the Jacobian of the flow by the Jacobian of the forward coordinate transformation at the initial point, and the Jacobian of the inverse transformation at the final point is as follows:

$$\frac{d\phi}{dX_0} = \left[ \frac{dX_f}{dx_f} \right] \left[ \frac{dx_f}{dx_0} \right] \left[ \frac{dx_0}{dX_0} \right].$$

(5)

Simple scaling of units can be handled in the same way.

In this work, we investigate the use of two different element sets, namely:

1. Classical orbital elements (CEO): $(a, e, i, \Omega, \omega, M)$ with $a$ the semi-major axis, $e$ the eccentricity, $i$ the inclination, $\Omega$ the right ascension of the ascending node, $\omega$ the argument of pericentre and $M$ the mean anomaly.
2. Modified equinoctial elements (MEE): $(p, f, g, h, k, L)$ with $p = a(1 - e^2)$, $f = e \cos (\omega + \Omega)$, $g = e \sin (\omega + \Omega)$, $h = \tan (i/2) \cos \Omega$, $k = \tan (i/2) \sin \Omega$ and $L = \Omega + \omega + \nu$ where $\nu$ is the true anomaly (Walker et al. 1985).

Unless otherwise mentioned, we use the following units in this work: radian for $i$, $\Omega$, $\omega$ and $M$, the semi-major axis $a$ is scaled by its initial value ($a/a_0$) such that it is unitless and the eccentricity is dimensionless by default.

The coordinate transformation is carried out according to (5). The Jacobian of the coordinate transformation is obtained by computing the first-order Taylor expansion of the state in transformed coordinates $X$ with respect to the state in old coordinates $x$ using DA.

Another remark regarding the computation of the FTLE is that we only consider the first five orbital elements and neglect the behaviour of the fast angular variable (i.e. we do not consider the divergence in mean anomaly and true longitude). The averaged dynamics do not depend on the fast angle, and the long-term behaviour of the fast angle is not of interest. In addition, the evolution of the fast angle is sensitive to changes in the orbit size and therefore the amount of stretching between neighbouring orbits can be dominated by stretching in the fast angle while the difference in in-orbit position is not important. Furthermore, it can be noted that in our dynamics (see Sect. 3) the semi-major axis is constant and therefore there is no divergence in the semi-major axis. Still, for computing the FTLE the semi-major axis...
is considered, because initial deviations in the semi-major axis cause divergence in other parameters.

The FTLE tells us the rate of divergence of nearby trajectories. Instead, we can also compute the exponential divergence rate in a single or just two orbital elements. For this, we compute the Cauchy–Green deformation tensor, Eq. (3), using only the rows of the Jacobian corresponding to changes in the elements of interest. We then obtain the same number of nonzero eigenvalues as the number of elements of interest and compute the divergence rate using Eq. (4). For example, if we only want to consider the divergence in $e$, then we take the row of the Jacobian containing the partial derivatives of $e$, multiply this row with its transpose and compute the eigenvalue to obtain the rate of divergence in $e$.

### 2.3.2 Lyapunov time

The inverse of the MLE is the Lyapunov time $T_L$, which is the average time in which two nearby trajectories diverge by a factor $e$:

$$T_L(t, x_0) = \frac{1}{\text{MLE}} \approx \frac{1}{\text{FTLE}(t, x_0)} = \frac{t}{\ln \sqrt{\lambda_{\text{max}}(\Delta)}}. \quad (6)$$

This timescale is often interpreted as a limit of the predictability of the trajectory. This interpretation, however, is rather arbitrary and not always correct in practice as will be shown later. In particular, when approximating the MLE by the FTLE, additional complications arise due to the choice of $t$: during the finite time before the FTLE converges to the MLE, also the approximate Lyapunov time can vary greatly.

### 2.4 Sensitivity

The FTLE is based on the first-order variations of the flow. Therefore, it only provides an estimate of the sensitivity of an orbit with respect to infinitesimally small initial uncertainties. In practice, however, the uncertainties are not infinitesimal. If these finite uncertainties in the initial conditions are known, or can be estimated, we can study the orbital evolution of a set of orbits starting in the initial uncertainty domain to analyse the impact of the uncertainties on the orbit including nonlinear effects. In this way, we can analyse if the disposal requirements are satisfied in case of initial state errors or compute the probability of disposal success based on initial uncertainties. In addition, such a sensitivity analysis can reveal possible chaotic behaviour of the orbit.

In this work, the analysis is performed both numerically and by computing bounds of the uncertainty set using Taylor differential algebra. The numerical sensitivity analysis is carried out by numerically propagating the orbital evolution for many different initial conditions in the initial uncertainty domain. This is equivalent to a Monte Carlo simulation except for the fact that in this work the points in the uncertainty domain are selected on a grid and not picked randomly. A drawback of this technique is that to carry out a reliable analysis many initial conditions need to be propagated, which can be time-consuming.

Alternatively, we can estimate the bounds of the uncertainty domain over time using a single high-order Taylor expansion $T_\phi$ of the flow $\phi$. For this, the flow about the nominal initial condition is expanded with respect to the initial uncertainties using DA. The high-order expansion is computed such that the initial uncertainty domain corresponds with the domain $[-1,1]$ in the Taylor polynomial space (by scaling the expansion variables). An outer bound of the range of the Taylor expansion at different times is then estimated by applying interval
arithmetic (Moore et al. 2009). In this way, the estimated range is guaranteed to include all possible values of the polynomial, which, however, not necessarily includes all values of the function that is approximated. Hence, we obtain the bounds of the domain of possible trajectories due to initial uncertainties.

Once the Taylor expansion is computed, the estimation of the bounds using interval arithmetic is efficient, but the drawback is that the computed bounds overestimate the true range of the Taylor polynomial. In addition, the Taylor expansion needs to be accurate in the whole uncertainty domain to ensure that the estimated bounds are accurate. This can be checked heuristically by estimating the truncation error of the Taylor polynomial in the uncertainty domain.

In this work, we assume that the uncertainties in the initial state are due to manoeuvre errors, that is, errors in the magnitude and direction of the applied $\Delta V$. Therefore, we expand the flow with respect to the disposal manoeuvre magnitude $\Delta V$ and direction angles $\alpha$ and $\delta$, so we get: $T_{\phi} = T_{\phi}(\Delta V, \alpha, \delta)$. In this way, we can directly compute the orbital evolution of the disposal orbit due to manoeuvre errors. Moreover, the Taylor expansion has only three expansion variables, which reduces the computational effort compared to expanding with respect to six state variables. The assumed uncertainties are shown in Table 2 in Sect. 4.

2.5 Reference time scale

To obtain the MLE, we have to compute the FTLE for infinite time. However, the re-entry orbits considered in this work re-enter in 100 years, so we cannot compute their FTLE for more than 100 years. For orbits that do not re-enter, the FTLE can be computed for longer times.

To investigate the predictability of orbits, we shall define a reference timescale. Anything that happens beyond the reference time period is not considered in this study. For re-entry disposal, the reference timescale is simply the time to re-entry, which is 100 years. For a graveyard orbit, we set the reference timescale equal to 200 years. A timescale of 200 years may be considered short, since instabilities may only set in after 200 years, see e.g. Rosengren et al. (2017). However, a 200-year period was chosen, because the considered graveyard orbit was designed to satisfy safe disposal for only 100 years. Therefore, considering a longer timescale would not be comparable with the design timescale.

3 Dynamical model

The dynamical model used in this work is the one implemented in HEOSAT, a semi-analytical propagator developed to study the long-term evolution of satellites in highly elliptical orbits (Lara et al. 2018). The perturbations included in the HEOSAT model are zonal and lunisolar perturbations, solar radiation pressure (SRP) and drag. Short-periodic terms have been removed by expressing the gravitational terms in Hamiltonian form and averaging over the mean anomaly of the satellite using Deprit’s perturbation algorithm (Deprit 1969) based on Lie transformations. The averaging techniques applied for developing HEOSAT’s dynamical model are described in Lara et al. (2018).

In this work, we will only consider the effects due to the Earth’s, Sun’s and Moon’s gravitational pull and SRP, whereas drag is neglected because it only acts on the spacecraft for a short period of time before re-entry. The main characteristics of the perturbation model and averaging procedures are as follows:
The zonal-term Hamiltonians are simplified by removing parallactic terms (via elimination of the parallax (Deprit 1981; Lara et al. 2013, 2014)), and short-periodic terms are eliminated by Delaunay normalization. This is carried out up to the second order of the second zonal harmonic, $J_2$, and to the first order for $J_3 - J_{10}$.

The disturbing potentials of the Sun and Moon (point-mass approximation) are expanded using Legendre series to obtain the Hamiltonians for averaging out the short-periodic terms (Lara et al. 2012). Second- and sixth-order Legendre polynomials are taken for the Sun and Moon potentials, respectively.

The averaged equations of motion due to SRP are obtained by analytically averaging Gauss equations over the mean anomaly using Kozai’s analytical expressions for perturbations due to SRP (Kozai 1963). For this, a spherical satellite and constant solar flux along the orbit (i.e. no shadow) are assumed.

The propagation is carried out in the True of Date reference system. Chapront’s analytical ephemeris is used to calculate the Moon’s orbit (Chapront-Touze and Chapront 1988; Chapront and Francou 2003), and Meeus’ algorithm is applied to compute the position of the Sun (Meeus 1991). The model has been validated against two-line element data for the Galileo satellite FM4; see Gondelach (2019).

### 3.1 Models

Three different models based on the described dynamics are used:

1. Simple gravitational model: includes only the first-order $J_2$ and second-order Legendre polynomials for both Sun and Moon potentials;
2. Full gravitational model: includes second-order $J_2$ and first-order $J_3 - J_{10}$, and second- and sixth-order Legendre polynomials for the Sun and Moon potentials, respectively;
3. Complete model: full gravitational model plus SRP;

According to Daquin et al. (2016), a single-averaged dynamical model considering only the second-order Legendre polynomial terms in the expansions of both the geopotential and lunisolar perturbations, i.e. the simple gravitational model, is sufficient to “capture nearly all of the qualitative and quantitative features of more complicated and realistic models (Daquin et al. 2016),” such as the full gravitational model. Therefore, we will use the simple model to compute the FTLE when studying the chaotic behaviour of many orbits in the phase space to reduce the required computational effort. On the other hand, for the sensitivity analysis we will use the full gravitational model that was also used to find the optimal disposal orbits (Armellin and San-Juan 2018). Finally, to analyse the impact of uncertainties in the dynamical model, we use the complete model for comparison.

### 3.2 Short periodics

The semi-analytical propagator propagates mean elements, which means that short-periodic motion is not considered. Neglecting the short-periodic motion affects the accuracy of the orbit propagation; however, Armellin and San-Juan (2018) showed that the impact on the accuracy is small. Furthermore, in practice, the osculating state of a spacecraft is known. Therefore, we need to convert the state to mean elements for propagation. This conversion is not error-free and thus introduces additional uncertainties in the initial state. To assess these uncertainties, we have computed the short-periodic terms for the initial mean elements of each test case (the initial states are shown in Table 2) considering the first-order $J_2$ and
Table 1 Average computation time for propagating an orbit for 100 years using the simple or full model numerically or in DA

| Model                          | Average computation time [s] | Numerical | First-order DA, 5 DA variables | Fifth-order DA, 3 DA variables |
|-------------------------------|-----------------------------|-----------|-------------------------------|-------------------------------|
| Simple gravitational model    | 1.82                        | 25.5      | 275                           |
| Full gravitational model      | 9.75                        | 460       | 5140                          |

lunisolar effects. [The conversion equations can be found in Armellin et al. (2018) and Lara et al. (2015).] For the re-entry cases, we found that the magnitude of the short-periodic terms is one order of magnitude smaller than the uncertainty in the initial state due to manoeuvre errors. For the graveyard orbit case, the short-periodic terms are of comparable magnitude. However, the uncertainties in the short-periodic terms are an order of magnitude smaller than the terms themselves such that any uncertainties due to osculating to mean elements conversion are much smaller than the considered orbit insertion uncertainties. Therefore, the uncertainties due to osculating to mean elements conversion are not considered in this work.

3.3 DA propagation

The propagator was implemented in Taylor Differential Algebra (Gondelach et al. 2017) to enable automatic high-order expansion of the flow. A DA version of a 7/8 Dormand-Prince Runge–Kutta scheme (eighth-order solution for propagation, seventh-order solution for step size control) is used for the numerical integration with an error tolerance of $10^{-10}$. The average computation times for propagating an orbit for 100 years using the simple and full gravitational model numerically and in DA is shown in Table 1. The simulations were run on a computer with an Intel Core i5-6500 processor running at 3.20 GHz using 16 GB of RAM. We either compute a first-order DA expansion w.r.t. the initial orbital elements $(a, e, i, \Omega, \omega)$ to compute FTLE or compute a fifth-order Taylor expansion w.r.t. the disposal manoeuvre parameters $(\Delta V, \alpha, \delta)$. Due to the singularity at zero eccentricity in the dynamical model, propagations take longer when the eccentricity becomes very small ($e < 10^{-5}$) as the stepsize decreases strongly to ensure accurate results. When propagating in the DA framework, the simple model is approximately 18 times faster than the full model. Finally, the complete model that also includes SRP requires about 2% more computation time than the full model.

4 Test cases

The main test cases in this study are three re-entry disposal orbits and one graveyard orbit for the Galileo satellites with NORAD IDs 37846, 40890 and 41175, and 38858, respectively. These disposal orbits were obtained through multi-objective optimization by minimizing the required $\Delta V$ for the disposal manoeuvre while also minimizing the time to re-entry (at most 100 years) (Armellin and San-Juan 2018) or maximizing the minimum distance from the Galileo constellation (at least 100 km) for 100 years (Mistry and Armellin 2016) for the re-entry and graveyard disposal, respectively. The disposal orbits considered here are the Pareto optimal minimum-$\Delta V$ solutions. It is worth noting that these disposal trajectories were computed without taking into account robustness as a design criterion and slightly higher
\[ \Delta V \] disposals could have been selected from the set of Pareto optimal solutions to account for robust disposal. This is an advantage of computing sets of Pareto optimal solutions. To demonstrate this benefit and to gain additional insight into disposal robustness, the robustness of all Pareto optimal solutions will be assessed.

The disposal manoeuvres are characterized by changes in semi-major axis, eccentricity and argument of perigee while leaving the inclination and ascending node almost unchanged, since plane changes are costly in terms of \[ \Delta V \]. The orbital states before and after the disposal manoeuvre are shown in Table 2. This table also shows the applied manoeuvre \[ \Delta V \] and its direction indicated by \( \alpha \) and \( \delta \), which are the in-plane and out-of-plane angles of the thrust vector with respect to the velocity vector, and the manoeuvre date. Note that the disposal into a graveyard orbit actually requires two manoeuvres: one that increases the semi-major axis and the other that inserts the spacecraft into the near-circular graveyard orbit. In Table 2, only the second manoeuvre is shown. For details about the optimization of the disposal orbits, the reader is referred to Mistry and Armellin (2016) regarding the graveyard orbit (38858) and Armellin and San-Juan (2018) for the re-entry disposal orbits (37846, 40890 and 41175).

To investigate the predictability of the orbits using sensitivity analysis, we consider the following uncertainties in the disposal manoeuvre execution: 1% uncertainty in the magnitude of the applied \[ \Delta V \] and 1° uncertainty in the applied thrust directions, \( \alpha \) and \( \delta \). These uncertainties represent realistic manoeuvre errors. For example, for the main engine of the Cassini–Huygens spacecraft\(^5\), the pre-launch estimate of the 1\(\sigma\) manoeuvre execution error was 0.35% in magnitude and 10 mrad in pointing per axis together with fixed errors of 10.0 mm/s for the magnitude and 17.5 mm/s for each pointing axis (Wagner and Goodson 2008). In addition, by in-flight calibration these errors can be reduced significantly. Table 2 shows the maximum absolute error in the initial orbital elements due to manoeuvre errors. The largest errors can be found in \( e \) and \( \omega \) and, noticeably, the errors in \( a \) and \( e \) increase with applied \[ \Delta V \].

### 5 Results

#### 5.1 FTLE analysis

To investigate if the disposal orbits are chaotic, we checked if the disposal orbits are located in a chaotic region of the phase space by computing the FTLE of different orbits in the initial phase space (using the simple gravitational model). For this, a section of the phase space is investigated by varying the initial \( \Omega \) and \( \omega \) to generate FTLE plots, also called stability maps. In addition, the FTLE is computed at different times and using different coordinates to analyse if and how the FTLE depends on time and coordinates. The results are compared with the behaviour of the eccentricity, which is the key orbital parameter for successful re-entry or graveyard disposal.

##### 5.1.1 Re-entry

The FTLEs for different initial \( \Omega \) and \( \omega \) for case 40890 are shown in Fig. 1. The red cross indicates the initial condition of the disposal orbit, and the black dots indicate orbits that have entered the Earth, such that the dynamics are not valid any more. The resolution of all plots is 3° in \( \Omega \) and 5° in \( \omega \). The re-entry orbit is located in a region of high FTLE, as expected due to

\(^5\) The main engine was generally used for all manoeuvres larger than 0.4 m/s.
the hyperbolicity of the dynamics. Figure 2 shows the maximum eccentricity reached by the orbits within 100 and 200 years. We can see that regions with large maximum eccentricity correspond to regions with high FTLE values. When the FTLE is computed using COE after 100 years, we also see high FTLE values in regions where the eccentricity growth is small; see the yellow and green dots in Fig. 1a where $\omega \in [40, 120]^\circ$ and $\omega \in [220, 300]^\circ$. These high FTLE values are caused by divergence in the argument of periapsis and are therefore not present when MEE are used (see Fig. 1c). After longer times, both the FTLEs computed using COE and MEE converge to the MLE and therefore the FTLE plots computed using COE and MEE after 200 years look more similar (compare Figures 1b, d). Figure 1e, f shows the FTLE computed using different units for the semi-major axis $a$ and angles $i$, $\Omega$ and $\omega$. Clearly, the choice of units also has an impact on the value of the FTLE before convergence. Figure 3 shows the exponential divergence rate in $e$ and $i$ combined. The divergence rate in $e$ and $i$ is smaller than the total divergence rate that is given by the FTLE. This shows that the values of the FTLE in Fig. 1 are dictated by divergence in $\Omega$ and $\omega$.

Figure 4a shows the Lyapunov time for different initial $\Omega$ and $\omega$ for cases 40890, 37846 and 41175 computed after 100 years using MEE. All re-entry orbits are located in a region with very short Lyapunov times, and the estimated Lyapunov times for the re-entry orbits are all smaller than 20 years. On the other hand, Fig. 2a shows that after 100 years neighbouring orbits have similar values of maximum eccentricity, which suggests that the evolution of the eccentricity is not very sensitive to changes in the initial conditions. In addition, all orbits near the re-entry orbit have re-entered after 200 years; see Fig. 2b.

| Object          | $a$ (km)       | $e$ (−) | $i$ (°) | $\Omega$ (°) | $\omega$ (°) | $M$ (°)  |
|-----------------|----------------|---------|---------|-------------|--------------|---------|
| Manoeuvre: $\Delta V = 3.44$ m/s, $\alpha = 5.23^\circ$, $\delta = 17.67^\circ$, JD = 2,457,273.22175 | 38858  | Before $\Delta V$ | 29,650.06 | 0.001869 | 54.974 | 210.010 | 221.901 | 41175 | Graveyard | After $\Delta V$ | 29,702.84 | 0.000395 | 54.988 | 210.020 | 293.465 | 99.847 |
| Max error       | 0.92           | 2.54E−5 | 1.17E-3 | 9.40E−4     | 4.466 | 4.465   |
| Manoeuvre: $\Delta V = 86.7$ m/s, $\alpha = 1.87^\circ$, $\delta = -3.0^\circ$, JD = 2,457,517.89887 | 40890  | Before $\Delta V$ | 29,601.77 | 0.000426 | 57.256 | 323.093 | 40.862 | 292.704 |
| Re-entry        | After $\Delta V$ | 31,086.33 | 0.047913 | 57.183 | 323.137 | 333.904 | -0.369 |
| Max error       | 18.49          | 0.000566 | 0.030   | 0.018       | 0.709 | 0.372   |
| Manoeuvre: $\Delta V = 128.5$ m/s, $\alpha = 3.83^\circ$, $\delta = -18.75^\circ$, JD = 2,457,535.41865 | 41175  | Before $\Delta V$ | 29,598.90 | 0.000163 | 54.943 | 202.432 | 270.326 | 288.351 |
| Re-entry        | After $\Delta V$ | 31,737.42 | 0.067472 | 55.529 | 202.672 | 196.333 | 1.908   |
| Max error       | 39.77          | 0.001719 | 0.046   | 0.019       | 0.699 | 0.607   |
| Manoeuvre: $\Delta V = 173.3$ m/s, $\alpha = -23.87^\circ$, $\delta = -1.13^\circ$, JD = 2,457,540.57102 | 37846  | Before $\Delta V$ | 29,601.79 | 0.000526 | 55.560 | 82.396  | 1.597   | 314.777 |
| Re-entry        | After $\Delta V$ | 32,479.72 | 0.089906 | 56.074 | 81.807  | 307.488 | 7.635   |
| Max error       | 57.77          | 0.001573 | 0.050   | 0.057       | 0.807 | 0.692   |

Table 2 Manoeuvre magnitude and direction, orbital state before and after manoeuvre, and maximum error in state after manoeuvre assuming manoeuvre execution uncertainties of 1% in $\Delta V$ and 1° in $\alpha$ and $\delta$. JD means Julian date.
Fig. 1 FTLE computed using COE and MEE and using different units for $a$, $i$, $\Omega$ and $\omega$ for different initial $\Omega$ and $\omega$ for case 40890 after 100 (left) and 200 years (right)
(a) Maximum eccentricity after 100 years
(b) Maximum eccentricity after 200 years

Fig. 2 Maximum eccentricity for different initial $\Omega$ and $\omega$ for case 40890 after 100 (left) and 200 years (right)

(a) Exponential divergence rate in $e$ and $i$ after 100 years
(b) Exponential divergence rate in $e$ and $i$ after 200 years

Fig. 3 Exponential divergence rate in $e$ and $i$ for different initial $\Omega$ and $\omega$ for case 40890 after 100 (left) and 200 years (right)

(a) 40890
(b) 37846
(c) 41175

Fig. 4 Lyapunov time computed using MEE for different initial $\Omega$ and $\omega$ for cases 40890, 37846 and 41175 after 100 years. The estimated Lyapunov times for the re-entry orbits 40890, 37846 and 41175 are 16.7, 19.4 and 17.9 years, respectively
Fig. 5  FTLE computed using COE and MEE, exponential divergence rate in $e$ and $i$, and maximum eccentricity for different initial $\Omega$ and $\omega$ for case 38858 after 200 (left) and 500 years (right).
5.1.2 Graveyard

Figure 5 shows the FTLE, the divergence rate in $e$ and $i$ and maximum eccentricity for different initial $\Omega$ and $\omega$ for the graveyard orbit case. First of all, we can see that all FTLEs in the investigated domain of the phase space are larger than zero. This indicates that nearby orbits diverge exponentially fast and that the orbits may be chaotic, which is very undesirable for a graveyard orbit. Also, the Lyapunov time of the graveyard orbit computed using COE and MEE after 500 years was found to be only 62.8 and 53.6 years, respectively. In addition, from Fig. 5g, it is clear that the graveyard orbit has an initial $\Omega$ (equal to 210°) that results in large eccentricity growth within 200 years for most initial values of $\omega$ except in two narrow blue valleys at $\omega = 120^\circ, 300^\circ$. This means that a small perturbation in the initial argument of perigee will result in a significantly larger eccentricity on the long term. This sensitivity of the eccentricity to changes in the initial $\omega$ is undesirable, because the error in initial $\omega$ due to manoeuvre uncertainties may be as large as 4.5°; see Table 2. The sensitivity of the eccentricity to the initial $\omega$ can be reduced by changing the initial $i$ or $\Omega$, e.g. changing $\Omega$ to around 160° (see Fig. 5h). This is, however, impractical in terms of $\Delta V$ required or waiting time needed for $\Omega$ to change to 160° due to natural precession. (It takes approximately 5 years for $\Omega$ to change from 210° to 160° by natural precession.)

Finally, we find that the FTLE has not converged after 200 years, since the FTLEs computed using COE and MEE are very different. On the other hand, after 500 years the FTLEs computed using COE and MEE are similar, which suggests near-convergence, and regions with high FTLE correspond to regions of large eccentricity growth. In addition, after 200 years we again find high values for FTLE computed using COE in regions of low eccentricity growth as a result of divergence in the argument of perigee; see Fig. 5a.

5.2 Sensitivity analysis

To determine if the disposal orbits satisfy the disposal requirements when subject to manoeuvre uncertainties, we investigate the effect of manoeuvre errors on the evolution of the disposal orbits. This also allows us to see if the orbits display chaotic behaviour and to explicitly compute the divergence between orbits due to manoeuvre errors. The sensitivity analysis is carried out by sampling 225 different manoeuvre errors (by combining nine different $\Delta V$ errors $\in [-1, 1]\%$ and five different errors $\in [-1, 1]^\circ$ in both $\alpha$ and $\delta$) and propagating the resulting initial conditions for 110 years for re-entry orbits and for 200 years for the graveyard orbit (using the full gravitational model).

5.2.1 Re-entry

Figure 6 shows the orbital evolution of 225 orbits (black curves) that start in the initial uncertainty domain due to manoeuvre uncertainties for re-entry case 37846. Close-ups of the perigee altitude around 100 years for all three re-entry cases are shown in Figure 7. (Trajectories are not shown after entering the Earth.) The red curve is the nominal orbit, and the dashed blue line shows the re-entry altitude of 120 km. The green curves indicate the bounds of the uncertainty set computed using a fifth-order DA expansion. All orbits in the uncertainty set are within the bounds computed using the fifth-order DA expansion. The bounds overestimate the domain of the uncertainty set, but are a good estimate of size of the domain. In addition, the figures show that neighbouring orbits diverge from the nominal trajectory and that the distance between orbits grows faster over time. However, up to re-entry
the behaviour seems regular, since all orbits in the uncertainty domain evolve similarly to the nominal orbit.

For case 37846, not all orbits in the uncertainty domain reach the re-entry altitude of 120 km; see Fig. 7a. This re-entry orbit is therefore not robust in case of manoeuvring errors. The fifth-order DA bounds correctly indicate that part of the uncertainty set does not reach 120 km altitude. On the other hand, for cases 40890 and 41175, all orbits in the uncertainty set do re-enter within 101 years; therefore, these trajectories are robust disposal options.

It has to be remarked that the selected re-entry trajectories were designed for re-entry in 100 years with minimum $\Delta V$. For case 37846, this resulted in a nominal trajectory whose maximum eccentricity is just enough for re-entry; however, consequently a small perturbation in the initial conditions can result in failure to re-enter. To meet the robustness requirements, one could select another solution from the available Pareto set or include robustness in the design of the disposal by, for example, setting a lower value for the minimum altitude or forcing a steeper atmospheric re-entry. Therefore, we analysed the robustness of all Pareto solutions corresponding to the three re-entry cases; see Fig. 8. On the one hand, for each re-entry case, the Pareto set contains non-robust trajectories. This highlights the necessity to assess the robustness of re-entry trajectories. On the other hand, the majority of the solutions (including low $\Delta V$ solutions) is robust (84–92% of the solutions is robust), such that a Pareto set can be used to find robust solutions. In addition, for case 37846, we find that robust re-entry is possible within 99 years at the cost of 10.4 m/s extra $\Delta V$ (183.7 m/s instead of 173.3 m/s).

**Fig. 6** 110-year evolution of 225 different orbits (black lines) in the uncertainty domain of object 37846. The red curve is the nominal orbit, and the green curves are the bounds of the uncertainty domain computed using a fifth-order DA expansion. The blue dashed line is the re-entry altitude.
To improve the robustness of the Pareto solutions, we designed new re-entry orbits that target a perigee altitude of 0 km instead of the original 120 km. This was achieved by repeating the optimization procedure as described by Armellin and San-Juan (2018). By ensuring that the nominal trajectory reaches a lower altitude than required for re-entry (successful re-entry is still assumed to occur at 120 km), the likelihood of robust disposal is supposed to increase. Figure 9 shows the robustness of the new Pareto optimal solutions. Targeting a lower altitude than required clearly improved the proportion of robust solutions (91-97% of the solutions is robust). Furthermore, for cases 40890 and 41175, the new low-ΔV solutions require only 2–4 m/s additional ΔV, while for 37846 the new solutions need about 5–10 m/s extra ΔV (for the same time to re-entry). Therefore, embedding robustness requirements in the design of the disposal orbits (in this case simply targeting a lower altitude) can improve the robustness of the solutions. Finally, Figures 8 and 9 show that non-robust solutions only appear in specific parts of the Pareto fronts. This is probably due to the similar initial conditions of these solutions, but the exact cause needs to be investigated.

5.2.2 Graveyard

The evolution of the uncertainty domain of the graveyard disposal orbit for object 38858 is shown in Fig. 10, and a close-up of the perigee radius for the first 110 years is shown in Fig.
11. All orbits in the initial domain are within the bounds computed using the DA expansion. Only when the eccentricity becomes very small around 70 years the bounds underestimate the range of $\omega$; see Fig. 10. This can be contributed to strong nonlinearities close to the singularity in the dynamical model at zero eccentricity that are not approximated well by the Taylor expansions. For all other orbital elements, the limits of the uncertainty set were estimated accurately by the DA bounds for 200 years.

The dashed blue line in Fig. 11 indicates the limit of the protected GNSS region, which is 100 km above the Galileo operational altitude of 23,200 km. As expected, the nominal orbit remains above the limit altitude for 100 years, since that was the requirement during design. However, several orbits in the uncertainty set cross the limit altitude before 100 years. This graveyard orbit is therefore not robust and should be improved such that all orbits in the uncertainty domain remain above the safe-distance altitude for at least 100 years.

To analyse if other graveyard orbit solutions in the Pareto set are robust, we assessed their robustness for 200 years; see Fig. 12. Almost all orbits are robust for 100 years, but only 4
Fig. 10 200-year evolution of 225 different orbits (black lines) due to manoeuvre uncertainties for object 38858. The red curve is the nominal orbit, and the green curves are the bounds of the uncertainty domain computed using a fifth-order DA expansion. The blue dashed line is the limit altitude for safe disposal. The middle panels only show the bounds; the nominal and all sensitivity propagations are within these bounds.

of the 75 orbits are robust for 200 years. This robustness comes, however, at the cost of a significantly higher \(\Delta V\) of at least 37.2 m/s. Moreover, the four robust solutions are expected to violate the safe-distance requirement within 300 years as the instability in eccentricity has already set in (not shown here).

5.2.3 DA expansion accuracy

To check the accuracy of the DA expansions, the estimated truncation errors of the fifth-order Taylor expansions of the eccentricity for the four disposal orbits are shown in Fig. 13a. The truncation error is at most \(10^{-4}\) in the entire uncertainty set for all disposal orbits for the first 100 years. In addition, for the graveyard orbit the error is never larger than \(10^{-5}\) for 200 years. Considering that the semi-major axes of the orbits are constant and equal to approximately 30,000 km, an error in the eccentricity of \(10^{-4}\) results in a 3 km error in perigee altitude. Furthermore, the size of the domain where the estimated truncation error of the eccentricity expansion is smaller than \(10^{-5}\) is shown in Fig. 13b. For the graveyard orbit, the size of this domain is always larger than the uncertainty domain. These results show that Taylor expansions computed using DA can be used to accurately compute the evolution of orbits in the entire uncertainty domain due to manoeuvre errors. In addition, the small truncation errors suggest that only weak exponential divergence of neighbouring orbits takes place, because strong exponential divergence would result in large high-order coefficients and increase the truncation error of the Taylor expansions.
On the predictability and robustness of Galileo disposal orbits

5.3 Sensitivity to dynamical model

To verify that the simple model is sufficiently accurate for investigating the predictability of Galileo disposal orbits and to analyse the sensitivity to model uncertainties, two sensitivity analyses were repeated using different dynamical models. First, the sensitivity analysis for the re-entry orbit 41175 was carried out using the simple, full and complete model. Figure 14a shows the evolution of the perigee altitude according to the three different dynamical models over 110 years for 45 orbits in the initial uncertainty domain. At 100 years, the maximum difference between the sets is 157 km in perigee altitude. This difference is significant regarding the strict re-entry altitude of 120 km and in comparison with the effect of manoeuvre uncertainties. On the other hand, the difference is small compared to the total change...
in perigee altitude, which is more than 23000 km. Also, the divergence in perigee altitude due to uncertainties is similar for all models. The simple model can therefore be used to both qualitatively and quantitatively study the orbit (which is in agreement with Daquin et al. (2016)) if highly accurate results are not required.

However, for graveyard orbits the effect of SRP is significant and should not be neglected. Figure 15 shows the evolution of the perigee radius for 225 orbits in the initial uncertainty domain of the graveyard orbit according to the full gravitational model and the complete model that also includes SRP. When SRP is included, the nominal graveyard orbit crosses the safe-distance altitude after just 16.8 years due to oscillations in the eccentricity caused by SRP. This shows that for accurate predictions the effects of SRP and higher-order gravitational perturbations due to the Earth and Moon cannot be neglected and should at least be considered as an additional uncertainty in the orbit’s evolution.

A more dramatic example of the influence of the dynamical model is shown in Fig. 16. The graveyard orbit in this example is very stable in the full gravitational model; it stays 58 km above the safe distance for 200 years, and the eccentricity remains smaller than 0.00046. In addition, in case of manoeuvre errors the safe-distance altitude is not crossed for 146 years. However, when SRP is included in the model, the safe-distance requirement is violated by the nominal orbit after only 81 years (see pink curve) and the eccentricity grows to 0.05 in 200 years. This large difference in orbital evolution is related to behaviour of the argument of perigee. The SRP perturbs the orbit, and due to the sensitivity of \( \omega \), \( \omega \) diverges strongly from the evolution it followed in the full gravitational model. As a consequence of the different evolution of \( \omega \), the growth in eccentricity is much larger. This also happens when the simple model is considered instead of the full model (see green curve), but the effect on the eccentricity growth is smaller. This means that in the full gravitational model the orbit is stable in the sense of eccentricity change, but due to the sensitivity of the argument of perigee the orbital evolution is very sensitive to perturbations, such as SRP. Indeed, in Fig. 5g, we have seen that the region of low eccentricity growth can be very small and a small perturbation can therefore move the orbit into a region of large eccentricity growth. For that
Fig. 14  Evolution of the perigee altitude for object 41175 according to different dynamical models: simple gravitational model, full gravitational model and complete model

Fig. 15  110-year evolution of perigee radius of 225 different orbits in the uncertainty domain of object 38858 computed using the full gravitational model (black lines) and complete model (orange lines). The blue dashed line is the limit altitude

reason, a large region of low eccentricity growth is required to ensure that a graveyard remains stable when subject to uncertainties in the dynamical model.

6 Discussion

6.1 Predictability analysis

The FTLE of all considered disposal orbits is positive. This can be explained by the hyperbolicity in the $2\omega + \Omega$ resonance dynamics. On the other hand, the positive FTLE values suggest that the orbits are chaotic, because chaotic behaviour is expected due to perturbed resonance dynamics and the overlap of resonances. Therefore, from the FTLE analysis it is difficult to draw conclusions about the predictability of the disposal orbits. Still, regardless
of whether the orbits are chaotic or not, the high FTLE values and short Lyapunov times indicate strong divergence between nearby orbits, which is undesirable for robust disposal.

The sensitivity analysis showed no chaotic behaviour on the timescales of interest. Orbits that start close the nominal orbit as a result of manoeuvre errors diverge away for the nominal orbit, but this happens smoothly until re-entry for re-entry orbits and for 200 years for the graveyard orbit. We can therefore conclude that the orbits may behave chaotically on the long term, but do not exhibit chaotic behaviour during the time period considered for safe disposal.

The FTLE and Lyapunov time quantify of the rate of separation of nearby orbits. However, as discussed, these measures cannot be used to distinguish between regular and chaotic orbits in the presence of strong local hyperbolicity. Still, the rate of divergence is of interest, because divergence between orbits can cause failure of disposal under uncertainties. However, the interpretation of FTLE and Lyapunov time values as a measure of the rate of divergence for analysing the effect of uncertainties on the evolution of the orbit is complicated, because:

1. In our study, we computed the FTLE for finite times, which introduces dependence on coordinates and units if the FTLE has not yet converged to the MLE;
2. The FTLE assumes infinitesimal deviations, whereas in reality deviations are finite;
3. The FTLE looks at the direction of maximum growth, whereas in finite time a finite deviation in another direction may grow more in absolute terms;
4. The FTLE considers all orbital elements, whereas only the behaviour of the eccentricity is of interest.

For the re-entry orbits, the FTLEs were computed after 100 years, because the dynamics are not valid once orbits enter the Earth. However, the FTLE after 100 years was shown to be dependent on the choice of coordinates and units. On the long term, this dependency should disappear, because the FTLE converges to the MLE independent of the coordinates or metric. Indeed, after 200 and 500 years, the FTLE plots computed using different coordinates or units look similar. This also corresponds with Daquin et al. (2016, Fig. 15) that shows how features in a FLI map change when the time span is increased.
To check if the FTLEs of the disposal orbits have converged, we computed their evolution over time. Figure 17 shows the evolution of the FTLE for the four disposal orbits computed using COE and MEE. From these plots, it is clear that the FTLEs computed using COE and MEE converge to the same value on the long term. In addition, the FTLEs for the re-entry disposal orbits (37846, 40890 and 41175) seem to have already converged after 100 years, whereas the FTLE of the graveyard orbit keeps decreasing for the first 500 years. The figures also contain a plot of the inverse of the time $T^{-1}$ that indicates the boundary between exponential divergence and slower divergence or contraction. The FTLEs of all orbits are larger than $1/T$ except for the first few years, so all orbits appear to be chaotic. Furthermore, the FTLE of the graveyard orbit is smaller than the FTLEs of the re-entry orbits, which indicates that nearby orbits diverge more slowly in the graveyard case than in the re-entry cases.

Another aspect that makes it difficult to interpret the FTLE for our predictability and robustness study is the fact that the FTLE considers infinitesimal deviations, while in reality we deal with finite uncertainties. This means that the growth of a finite deviation can be larger or smaller than computed by the FTLE.

In addition, the FTLE considers the direction of maximum growth, whereas actual uncertainties may not be aligned with this direction. The FTLE and Lyapunov time are based on the maximum stretching between neighbouring orbits, which is computed via the maximum eigenvalue of the Cauchy–Green deformation tensor. The eigenvector that corresponds to the maximum eigenvalue is the direction of maximum expansion. This means that an initial deviation will grow most (relative to its initial size) when it points in the direction of maximum growth. Deviations that are not aligned with the direction of maximum expansion will grow slower or may even shrink over time.
Deviations from the nominal initial state due to manoeuvre uncertainties may only have a small component in the direction of the maximum growth. On the long term, this component in the direction of the maximum growth will become the largest component of the deviation, because it grows fastest. However, in finite time this component does not necessarily become the largest component of the deviation vector. Instead, another component of the deviation vector that grows relatively slower but is initially larger can increase more in absolute size.

Finally, the FTLE considers growth in all orbital elements (except the fast angle), whereas we are mainly interested in the deviation in eccentricity. Therefore, we can find a high FTLE due to strong divergence in one of the state variables while the deviation in eccentricity only grows little. This was indeed the case for the FTLEs of both re-entry and graveyard orbits whose value was dominated by the deviation in \( \Omega_1 \) and \( \omega \).

To compare the maximum growth with the growth of finite deviations, let us consider the FTLE computed using COE for the re-entry orbit of object 40890 after 100 years. The value of the FTLE is 0.064, which corresponds to a growth factor of 603.3 (computed using the simple model). The Jacobian of the flow map corresponding to this FTLE is:

\[
\frac{d\phi}{dx_0} = \begin{bmatrix}
\frac{\partial a}{\partial a} & \frac{\partial a}{\partial e} & \frac{\partial a}{\partial i} & \frac{\partial a}{\partial \Omega_1} & \frac{\partial a}{\partial \omega} \\
\frac{\partial e}{\partial a} & \frac{\partial e}{\partial e} & \frac{\partial e}{\partial i} & \frac{\partial e}{\partial \Omega_1} & \frac{\partial e}{\partial \omega} \\
\frac{\partial i}{\partial a} & \frac{\partial i}{\partial e} & \frac{\partial i}{\partial i} & \frac{\partial i}{\partial \Omega_1} & \frac{\partial i}{\partial \omega} \\
\frac{\partial \Omega_1}{\partial a} & \frac{\partial \Omega_1}{\partial e} & \frac{\partial \Omega_1}{\partial i} & \frac{\partial \Omega_1}{\partial \Omega_1} & \frac{\partial \Omega_1}{\partial \omega} \\
\frac{\partial \omega}{\partial a} & \frac{\partial \omega}{\partial e} & \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial \Omega_1} & \frac{\partial \omega}{\partial \omega}
\end{bmatrix}
\]

(7)

here the angles \( i \), \( \Omega \) and \( \omega \) are in radian and the semi-major axis is scaled by its nominal value \( a_{\text{nom}} = 31086 \text{ km} \). The Jacobian shows that the largest growth in deviation occurs in \( \Omega_1 \) and \( \omega \) (see values in the fourth and fifth row). Consequently, the value of the FTLE after 100 years is dominated by divergence in \( \Omega_1 \) and \( \omega \).

The corresponding direction of maximum expansion, that is, the eigenvector \( u \) corresponding to the maximum eigenvalue \( \lambda_{\text{max}} \) of \( \Delta \), is:

\[
u_{\lambda_{\text{max}}} = [0.206190, 0.977595, -0.041636, 0.002684, 0.007248].
\]

The largest possible deviation due to manoeuvre errors for object 40890 (see Table 2) that is aligned with the direction of maximum expansion is:

\[
\Delta x_u = [0.0001193 a_{\text{nom}}, 0.0005658, -2.410 \times 10^{-5} \text{ rad}, 1.553 \times 10^{-6} \text{ rad}, 4.195 \times 10^{-6} \text{ rad}].
\]

According to the Jacobian (7), this deviation vector grows by a factor 603.3 and the magnitude of the deviation in \( e \) grows to 0.001762 after 100 years. If, however, we apply this deviation to the nominal initial state and propagate the orbit for 100 years using the simple model, then the deviation grows by a factor 608 and causes a divergence from the nominal orbit of 3.7 km in \( a \) and 0.000853 in \( e \) after 100 years, which is equivalent to a deviation of 25.8 km in perigee altitude.

On the other hand, the error in the disposal manoeuvre that causes the largest change in eccentricity after 100 years is an error of \(-1\%\) in \( \Delta V \) and \(-1^\circ\) in \( \alpha \) and \( \delta \), which corresponds
to a deviation $\Delta x_e$ in the initial orbital state of the disposal orbit of:

$$
\Delta x_e = [-0.0005389 a_{\text{nom}}, -0.0005139, -7.737 \times 10^{-5} \text{ rad},
4.591 \times 10^{-5} \text{ rad}, -0.01219 \text{ rad}].
$$

This deviation grows by a factor of only 34, but causes a change in $a$ and $e$ of 16.8 km and 0.00426, respectively, resulting in a deviation in perigee altitude of 129 km after 100 years according to propagation in the simple gravitational model. Although the deviation is clearly not aligned with the maximum expansion direction and has only a small component in the direction of maximum expansion, it causes a larger change in eccentricity and perigee altitude. In addition, it can be noted that according to the Jacobian (7) the divergence in $e$ due to this initial perturbation $\Delta x_e$ is only 0.00287, which shows that nonlinear terms cannot be neglected.

This example shows that it is difficult to interpret the FTLE for analysing the effect of uncertainties on the evolution of the orbit. On the other hand, in the sensitivity analyses we did not look at the maximum possible growth of a deviation, but considered the growth of the whole uncertainty domain and focused on the uncertainty in eccentricity. The advantage is that here nonlinear terms are taken into account and the magnitude of the actual divergence due to initial uncertainties is computed directly. So, instead of having an estimate of the divergence and chaotic behaviour, we directly calculate the effect of initial uncertainties on the disposal orbit.

### 6.2 DA-based bounding of uncertainty domain

In addition to point-wise propagation of uncertainties, we have used a high-order Taylor expansion of the flow to estimate the bounds of the uncertainty domain. The estimated bounds of the Taylor polynomials contained the entire set of orbits due to initial uncertainties. However, the DA bounds were also found to significantly overestimate the uncertainty domain. On the other hand, the truncation error of a fifth-order DA expansion was estimated to result in a 3 km error in perigee altitude after 100 years. This error is small compared to the 157 km difference in perigee altitude as result of the use of different dynamical models. Depending on the order, the computation of the Taylor expansion can be expensive (see Table 1), but once computed the orbits for many different initial conditions and the bounds can be computed extremely fast.

### 6.3 Robustness of disposal orbits

We have investigated the robustness of several hundreds of disposal orbits. We found that tens of re-entry disposal orbits and the majority of the considered graveyard orbits are not robust. This underlines that sensitivity analysis is mandatory to assess the robustness of both re-entry and graveyard disposal orbits.

The majority of the considered re-entry disposal orbits and two of the three minimum-$\Delta V$ re-entry orbits were found to be robust, which shows that robust disposal via re-entry is feasible. Furthermore, the uncertainties for all considered re-entry orbits were found to grow smoothly, such that with a small modification of a non-robust orbit re-entry can be guaranteed. Indeed, for the third non-robust orbit, we showed that one can easily find a robust alternative by taking a nearby solution from the same Pareto optimal set (at the cost of a small increase in
Moreover, robustness can be considered during orbit design, e.g. by targeting a lower altitude, to obtain a higher proportion of robust Pareto optimal solutions.

Finally, it was found that uncertainties in the dynamical model have little effect on the orbital evolution of the re-entry orbits, which gives additional confidence that robust re-entry disposal is feasible in practice.

For the investigated minimum-$\Delta V$ graveyard orbit, we found that there is only a very narrow range of initial argument of perigee that results in safe disposal for 100 years. Manoeuvre errors can cause deviations in the initial argument of perigee of several degrees that result in increased eccentricity growth and violation of the safe disposal requirements. In addition, it was shown that neglected perturbations in the dynamical model may cause the orbit to evolve differently and cross the safe-distance altitude much earlier than required. To guarantee safe disposal of this satellite, a higher $\Delta V$ budget is needed to move the satellite further away from the operational altitude and/or to change the initial inclination or longitude of the node to reduce the sensitivity to initial $\omega$ (i.e. moving the satellite outside of the unstable domain of the resonance). Indeed, we found four graveyard solutions from the Pareto optimal set, located at a much higher altitude, that are robust for 200 years. However, these solutions require a significantly higher $\Delta V$ and are only robust for 200 years but not for longer timescales.

In conclusion, this study shows that robust Galileo re-entry disposal options exist and that sensitivity analysis is mandatory to assess the robustness of disposal orbits. To obtain a stable and robust graveyard orbit, large changes in the initial conditions at the cost of increased $\Delta V$ are required. In case the $\Delta V$ required for reaching a robust graveyard orbit is high, re-entry can be considered as alternative. In future, robustness should be included in the design of disposal orbits. In that way, one can directly find solutions that are robust and require minimum $\Delta V$.

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**Compliance with ethical standards**

**Conflict of interest** The authors declare that they have no conflict of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

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