TRANSPORT THROUGH A SINGLE–BAND WIRE CONNECTED TO MEASURING LEADS

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Abstract. Transport through a one-dimensional wire of interacting electrons connected to semi infinite leads is investigated using a bosonization approach. The dynamic nonlocal conductivity is rigorously expressed in terms of the transmission. For abrupt variations of the interaction parameters at the junctions, an incident electron is transmitted as a sequence of partial charges: the central wire acts as a Fabry-Pérot resonator. The dc conductance is shown to be given by the total transmission which turns out to be perfect. When the wire has a tendency towards superconducting order, partial Andreev reflection of an incident electron occurs. Finally, we study the role of a weak barrier at one contact or inside the wire by a renormalization group method at finite temperature. We compute the conductance in the presence of localized or extended disorder. The results are compared to recent experiments.

1. Introduction

Quantum wires, obtained by lateral confinement of a two dimensional electron gas [1], provide a good candidate to test the well-developed theory of one-dimensional interacting systems which show the so-called Luttinger liquid behavior [2, 3]. According to [4, 5] the interactions should renormalize the conductance $g$ of a pure wire: $g = 2g_0 K$ where $g_0 = e^2/h$ is the conductance quantum and $K$ is a key parameter depending on interactions, with $K = 1$ for a non–interacting system. The reduction of the conductance in the presence of localized or extended disorder has a power-law dependence on temperature [6, 7, 8], or on the wire length [9], determined also by $K$. Recent experiments [10] on wires with a length up to $10\mu m$ show indeed a power law, and the authors extract $K = 0.7$. At high temperature, where
the effect of impurities is less pronounced, the conductance gets very close to the ballistic value \(2g_0\), i.e. it is different from the expected \(2Kg_0 = 1.4g_0\). The latter result is obtained by computing the current as a response to an electric field restricted to a finite segment of an infinite interacting wire \(5, 8\). However, in a mesoscopic device the boundaries intervene in a non-trivial way: one then has to change some standard concepts on resistance of macroscopic systems.

An intuitive approach to transport was pioneered by Landauer \([1]\) and developed by Büttiker \([2]\) and others: the conductance of a coherent sample ought to be proportional to the ease with which electrons can be transmitted through it:

\[
g = g_0 T, \tag{1}
\]

where \(T\) is the transmission coefficient. Then the finite resistance of a perfect system can be traced to its interface with the reservoirs \([3]\). Thus it is necessary to view the system formed by a mesoscopic sample and its contacts as a whole \([4]\). Furthermore, Büttiker \([5]\) stresses the importance of the inclusion of the nearby metallic bodies in any discussion of the a.c. conductance. When attempting to derive a Landauer formula rigorously, one has to face the problem of modeling the reservoirs properly, a point which has lead to different points of view in the literature. Many authors \([11, 12]\) include perfect leads between the sample and the reservoirs: this allows to define the incoming and outgoing scattering states, thus the transmission coefficients. However, this description becomes difficult if the electrons interact in the sample. The extension of Landauer’s formula to interacting electrons is mostly phenomenological \([13]\). Apel \([4]\) derived it if disorder is included on a finite segment of an infinite interacting one-dimensional wire, but this is not necessarily a realistic situation. For instance, a quantum wire typically opens smoothly into two wide leads formed by the same two-dimensional gas. While the electronic correlations are enhanced in the confined region, they may well be negligible outside of it.

It seems difficult to treat exactly interactions if one includes the complications of the experimental setup. Thus we suppose that the wire is connected to reservoirs via very long perfect one-dimensional leads where the electrons do not interact (see fig. \([1]\)). Neglecting the interactions in the leads might be appropriate as a primitive model of Fermi liquid behavior in the two-dimensional gas forming the contacts. The effect of the reservoirs is accounted for only through the voltage they impose in a conductance measurement, and we assume this voltage not to be affected by the current. The infinitely long leads insure that a transmitted electron never returns to the wire and therefore cannot interfere with either the wire or the incident current. Even though this is an oversimplification, it permits us to treat the wire and the leads on the same footing, and to treat exactly interactions in the wire using the bosonization method. In our model the contacts are per-
fect, and delimit the wire where short-range electron–electron interactions exist. Note that our approach is opposite to that of Fabrizio and Gogolin [19] who consider an isolated interacting wire, and disturb it by a weak tunneling into the ends. We investigate the effect of a weak backscattering potential at one contact, as well as an extended disorder. The extension of Landauer’s formula to the interacting dirty wire will be presented. Finally, we will see how our space-dependent interactions model can give rise to phenomena familiar from metal-superconductor interfaces.

\[
H = \int_{-L}^{L} dx \frac{uK}{2\pi} \left[ uK (\partial_x \Theta)^2 + \frac{u}{K} (\partial_x \Phi)^2 \right] \tag{2}
\]

where the boson field \( \Phi \) is related to the particle density through: \( \rho - \rho_0 = -\partial_x \Phi / \pi \), and \( \partial_x \Theta / \pi \) is the conjugate momentum field to \( \Phi \): \([\partial_x \Theta(x), \Phi(y)] = i\pi \delta(x-y)\). There are no single-particle excitations, but sound wave excitations with velocity \( u \). Some intuition of the Luttinger liquid features can be gained by noting that an injected right-going electron decomposes into a charge \((1 + K)/2\) propagating at velocity \( u \) and a charge \((1 - K)/2\) propagating at \(-u\). The total resulting current is \( K \).

We now consider a finite interacting wire perfectly connected to two identical leads at its end points \( \pm a \) (see fig. (1)). We shall label the quantities pertaining to the leads (central wire) by the subscript 1 (2). In \( H \) (eq. (2)) the parameters \( u, K \) then vary from \( u_2, K_2 \) in the wire to \( u_1, K_1 \) outside; most of our results will concern abrupt variations at \( \pm a \), and the physically most relevant situation of noninteracting leads: \( K_1 = 1 \). We adopt periodic boundary conditions on the whole system. This is a mathematical convenience. At scales much less than the total length \( 2L \), the physics in the intermediate region does not depend on the boundary conditions at the end points of the leads, and is expected to be the same as for

Figure 1. An interacting one-channel wire connected perfectly to very long leads

2. The Model

For simplicity, we consider only spinless electrons with short-range interactions, which are described by the Hamiltonian [3]

\[
H = \int_{-L}^{L} dx \frac{uK}{2\pi} \left[ uK (\partial_x \Theta)^2 + \frac{u}{K} (\partial_x \Phi)^2 \right] \tag{2}
\]
an open system. We’ll henceforth discuss the results once $L$ has been taken larger than any other length.

3. The transmission process

We consider a thought experiment where one injects a right-moving electron on the left perfect lead (at a point $y \leq -a$) and places a detector on the right lead (at $x \geq a$) which measures the transmitted charge $M_{++}(x, y, t)$ at time $t$. According to the relation $\rho - \rho_0 = -\partial_x \Phi / \pi$, creating an electron amounts to induce a kink in $\Phi$: the propagation of our electron is determined once the equation of motion for $\Phi$ is solved. Using the Hamiltonian (2), this equation is

$$\partial_t \Phi - uK \partial_x \left( \frac{u}{K} \partial_x \Phi \right) = 0 . \tag{3}$$

The inverse of the differential operator acting on $\Phi$ is nothing but the correlation function $G(x, x', t) = i\theta(t) \langle [\Phi(x, t), \Phi(x', 0)] \rangle$. Its knowledge allows us to determine the time evolution of the operators of interest for us:

$$\tilde{\rho}_\pm = \frac{1}{2} (\rho \pm j/u) = \frac{1}{2\pi} \left( -\partial_x \pm \frac{1}{u} \partial_t \right) \Phi \tag{4}$$

In the noninteracting leads, $\tilde{\rho}_\pm$ are simply the right and left-going electron density. Given the initial conditions $\langle \tilde{\rho}_+(x, 0) \rangle = \delta(x - y)$, $\langle \tilde{\rho}_-(x, 0) \rangle = 0$, the transmitted and reflected charge, $\langle \tilde{\rho}_+(x \geq a, t) \rangle = M_{++}(x, y, t)$ and $\langle \tilde{\rho}_-(y' \leq -a, t) \rangle = M_{--}(y', y, t)$, can be deduced from $G$. We can however predict $M_{++}$ and $M_{--}$ by simple arguments if $u$ and $K$ are step functions. In this case, eq.(3) reduces to a wave equation with discontinuous velocity. One can convince oneself that eq.(3) can’t lead to a discontinuity neither in $\Phi$ nor in $(u/K)\partial_x \Phi$. Thus we require the continuity of the current $j = \partial_t \Phi / \pi$ at the contacts $\pm a$, which is physically plausible, and the continuity of $(u/K) \rho$. Using the relation (4), this tells us how our electron will evolve: once it impinges on the contact at $-a$, it gets reflected with coefficient $\gamma = (1 - K^2) / (1 + K^2)$. The transmitted charge, which is a peak in $\langle \tilde{\rho}_+ \rangle$, will take a time $t_2 = 2a/u_2$ until it reaches $+a$, where a charge $1 - \gamma^2$ is transmitted, while the reflected part will continue to bounce back and forth so that after each period $t_2$ a partial charge leaves the wire. This process is illustrated in the figure (3). Thus the incident electron is decomposed into a sequence of spatially distributed charges. If we wait a long time compared to $t_2$, the series sums up to unity: $\sum_n \gamma^{2n}(1 - \gamma^2) = 1$. The total transmission is perfect.

This argument is confirmed by an exact computation of $G$ for any spatial arguments. The expression for $M_{++}(x, y, t)$ is given in [20]. We give here...
Figure 2. The transmission process of an incident electron on the wire in the case where $K_2 > 1$ and $u_1 = u_2$. We denote: $t_y = (−a − y)/u_1$. At $t_y + (2n + 1)t_2$ (resp. $t_y + 2nt_2$), a charge $γ^{2n}(1−γ)(γ^{2n}−1)(1−γ^2)$ comes out $a$ (resp. $−a$). The first reflected charge is of hole type, while the subsequent ones are of electron type. If $K_2 < 1$, the hole and electron type reflected charges are exchanged.

only its Fourier transform:

$$M_{++}(x, y, \bar{\omega}) = \exp(\bar{\omega}(t_x + t_y)) \frac{1 − γ^2}{\exp(−i\bar{\omega}t_2) − γ^2 \exp(i\bar{\omega}t_2)} \tag{5}$$

where $t_x$ is the time it takes for an electron to go from $x$ on the lead to the closest contact, i.e., $u_1t_x = |x| − a$. Recall that $t_2 = 2a/u_2$ is the traversal time of the central wire. We abbreviate $\bar{\omega} = \omega + i\delta$, where $\delta$ ensures the convergence in the thermodynamic limit: $e^{−\delta L} \ll 1$. It’s worth noting that $M_{++}(-a, a, \bar{\omega})$ coincides with the total transmission of a double scatterer obtained through the composition:

$$\begin{pmatrix} K_1 & 1 & γ \\ γ & 1 & 0 \end{pmatrix} \begin{pmatrix} e^{\bar{\omega}t_2} & 0 \\ 0 & e^{-\bar{\omega}t_2} \end{pmatrix} \begin{pmatrix} K_2 & 1 & −γ \\ −γ & 1 & 0 \end{pmatrix}$$

where $K = (K_1 + K_2)/2$. The first matrix is the transfer matrix at $−a$, the third its inverse at $+a$, between which the propagation matrix is sandwiched. These matrices are acting on $(j_+ = u\rho_+, j_- = −u\bar{\rho}_−)$ and not on the wave function amplitudes as one is used to. Therefore each column of
a transfer matrix sums up to unity. The perfect transmission also appears in eq. (3) which gives \( M_{++}(x, y, \omega/2) \approx 1 \). Accordingly, the expression for the reflected charges gives \( M_{--}(y', y, \omega/2) \approx 0 \). Note also that \( M_{++}(-a, a, n\omega) = 1 \). The central wire acts as a Fabry-Pérot resonator with symmetric mirrors[20]. But the perfect transmission is not specific to the symmetric structure. We can show that for any internal shape of \( u \) and \( K \) within \([-a, a]\), varying smoothly or not, the total transmission is still perfect. This result holds even if we include interactions on the leads, i.e. for \( K_1 \neq 1 \). In this case, an incident or transmitted flux is recognized by a peak in \( \langle \tilde{\rho}_+ \rangle \), even if this does not correspond anymore to the original right-going electrons. Apart from an exact evaluation of \( \lim_{\omega \to 0} M_{++}(x, y, \omega) = 1 \) and \( \lim_{\omega \to 0} M_{--}(y', y, \omega) = 0 \), the following observation explains the perfect transmission: in the zero-frequency limit, not only the current \( j \) but also \( (u/K)\rho \) (see eq. (3)) are uniform along the system. We deduce that \( \langle \tilde{\rho}_+(\omega = 0) \rangle \), as well as \( \langle \tilde{\rho}_-(\omega = 0) \rangle \) take the same uniform values on the opposite leads since the latter have the same \( u/K \). In case an electron emerges initially from \( y \leq -a \), \( \langle \tilde{\rho}_+(x \leq -a, \omega = 0) \rangle = \int_0^{\infty} \delta [(x - y)/u_1 + t] dt = 1 \) while \( \langle \tilde{\rho}_-(x \geq a, t) = 0 \) (there are no electrons coming from the right lead). Thus the total density of transmitted (reflected) charge is unity (zero).

4. Transport in the pure wire

If we apply a time-dependent potential, the Hamiltonian acquires the supplementary term \(-\int V(x, t)\partial_x \Phi/\pi \). The equation (3) for \( \Phi \) has now a source term on the right hand side, \( E(x, t) \). Thus we get \( \langle \Phi(x, t) \rangle = \int \int G(x, y, t - t')E(y, t') \); differentiating this relation with respect to time, we find that the current is exactly linear in the electric field[20], and that \( \sigma(x, y, t) = -g_0 \partial_t G(x, y, t)/\pi \). This shows how transport is related to propagation [20].

For \( x, y \) on opposite leads, the last relation reads

\[
\sigma(x, y, t) = g_0 M_{++}(x, y, t)
\]  

(6)

This is a generalization of the Landauer formula, eq. (1), to a dynamic situation. When \( u \) and \( K \) are step functions, \( \sigma(x, y, \omega) \) can be computed for any spatial arguments. Its expression is reported in [20] for points inside the wire, while it is given by (3) for points on opposite leads, owing to the identity (3).

In order to exploit the identity (3) further, we make a digression. Suppose that the reservoirs set up a uniform oscillating potential \( V_R(\omega) \) and \( V_L(\omega) \) on the right and left leads. Since there are no scatters inside the wire that can generate an electric field, we expect the potential drop to be concentrated at the contacts. Taking the potential on the wire as a reference,
the resulting current at any point on the leads is:

$$j(x, \omega) = \sigma(x, -a, \omega)V_L(\omega) - \sigma(x, a, \omega)V_R(\omega) \quad (7)$$

Now let $\delta \ll \omega \ll \omega_2$; this is already a large frequency domain, of the order of some GHz, for wires with a length of some $\mu m$. Then the current on the leads oscillates with a spatial period much longer than the wire length, and we can take it as uniform on each lead (if measured at a reasonable distance from the contacts). It’s tempting to interpret eq.(7), taken for instance at $-a$ and $a$, as the relations that define the two conductance coefficients in a two-probe measurement [15]:

$$g_{LL}(\omega) = g_{RR}(\omega) = -g_0[1 + M_{-+}(-a, -a, \omega)] \simeq -g_0[1 + i\omega t_2(K_2^{-1} - K_2)/2]$$

$$g_{RL}(\omega) = g_0M_{++}(-a, a, \omega) \simeq g_0 \left[1 + i\omega t_2(K_2^{-1} + K_2)/2\right].$$

However, all this has to be taken with caution. The voltage profile cannot be known a priori. The interesting issues in a.c. transport developed by Büttiker [15] cannot be addressed here due to the lack of long-range Coulomb interactions. Things are less ambiguous in the zero-frequency limit, where we do not need to know the exact potential distribution within the wire (if we consider the total voltage drop as fixed by the reservoirs).

This is because $\lim_{\omega t_2 \to 0} \sigma(x, y, \omega) = \sigma$ is independent of its spatial arguments, and thus yields the conductance $g = \sigma$ as may be checked from: $j(x) = \int \sigma(x, y)E(y)$. This relation shows also that the uniformity of $\sigma(x, y)$ is a constraint that ensures the uniformity of the current in a one-dimensional system. We can verify this explicitly in our model. It turns out that $g = \sigma = g_0$. This result appears from [1] which becomes a trivial Landauer identity in view of the perfect transmission (eq.[1] with $T = 1$). When the leads are interacting, $g = K_1g_0$; the conductance is renormalized by $K_1$, not $K_2$ as predicted by Kane and Fisher in a different geometry. These authors pointed out that their results are valid at temperatures or frequencies greater than $\omega_2$ so as to ensure that one is not measuring the properties of the external leads. At first sight, this condition does not come out of our analysis: all the previous results are independent of temperature, and there are oscillations at $\omega > \omega_2$. But our implicit assumption was that $\delta^{-1}$, the adiabatic turn-on time of the external field, is greater than the traversal time $t_2 = 2\pi/\omega_2$. It’s thus interesting to see what we get in the opposite limit $\delta t_2 \gg 1$. Then the expression of the conductivity [20] has different zero-frequency limits depending on its spatial arguments (the steady current gets homogenized only on scales of the order $\delta^{-1} \ll t_2$). Near the center of the wire, i.e. for $|x| \ll u_2/\delta$, we have $\sigma(x, y) = g_0K_2$, while near the contacts, i.e. $|x \pm a| \ll u_2/\delta$, $\sigma(x, y) = g_0K_a$, with $K_a = 1 - \gamma$. If we
can do a measurement which does not cause any additional scattering these values would yield respectively a conductance in the bulk (of the junction) equal to \( g_0 K_2 (g_0 K_a) \). Thus we recover the bulk result by inverting our assumption about \( \delta \), not the temperature. But can we identify the two quantities? An adiabatic switching of the external field is necessary if the system has no way to relax. But any real system is coupled to its environment. A simple way to describe this coupling is through a relaxation time \( \tau_p \). It is tempting to substitute \( \delta \rightarrow \tau_p^{-1} \): now \( \tau_p^{-1} \) is not necessarily less than \( \omega \) nor \( \omega_2 \). This depends on the underlying mechanism, which does not yield simply an imaginary part added to the external frequency \( \omega \). We make a final remark which will be useful for the sequel: the instantaneous correlation functions vanish exponentially at separations greater than the thermal length \( L_T = \mu/T \). Thus \( L_T \) plays the role of a coherence length.

5. **Andreev reflection**

At times \( t \) much less than \( t_2 \), the neighborhood of the origin and of the contacts behave differently. In particular, when \( K_2 > 1 \) and \( K_1 = 1 \), the local pairing correlation function decreases from \( t^{-2/K_2} \) on the bulk to \( t^{-2/K} \) at the contacts, where \( K = (1 + K_2)/2 \). Since \( K > 1 \), this indicates a tendency towards superconducting order which extends from the wire to the external leads. On the other hand, an incident electron on \(-a\) is reflected with a coefficient \( \gamma \) which is now negative, thus a partial hole is reflected back. These two facts are respectively the analogous of proximity effect and Andreev reflection\[21\]: it is known that an electron incident on a normal metal–superconductor interface needs to make a pair with an electron to enter the superconductor. Depending on whether its energy is less or greater than the gap, a total or a partial hole is reflected \[21\]. In our case, there is no gap, so we only get a partial hole reflected. However, in the limit \( K_2 \rightarrow \infty \), we get exactly one hole reflected.

We can also consider the case where the central wire is finite but noninteracting \( (K_2 = 1) \), while the external leads have \( K_1 > 1 \): this is reminiscent of an S-N-S structure \[22\]. The internal reflections we found can be thought of as multiple Andreev reflections, and enhance the conductance of the wire: \( g = K_1 g_0 \).

To summarize, the physics is controlled by the local (external) \( K \) at time much less (larger) than the traversal time \( t_2 \) of the central wire. An incident electron from an external lead is partially reflected at the contacts to accommodate the interactions with the electrons inside the central wire. But there is no reason to expect only the change in interactions to cause scattering: any mismatch at the contact with the leads (e.g. geometrical) can reflect an incident electron. Impurities inside the wire have also to be
considered.

6. Effects of a backscattering potential

The first question we address in this section is the validity of Landauer formula in the presence of disorder. Next, the conductance is computed in a perturbative way for a Gaussian random disorder, as well as for a barrier at one contact. The results of the latter case are confirmed by a renormalization group approach, explicitly done at finite temperature.

6.1. LANDAUER FORMULA

In the presence of either local or extended disorder on the central wire, we add to $H$, eq.(2), the term

$$H_b = \int dx V(x) \Psi^\dagger(x) \Psi(x). \tag{8}$$

The equation of motion for $\Phi$, eq.(3) becomes nonlinear, with an additional force

$$F(x) = - \frac{\partial H_b}{\partial \Phi(x)} \tag{9}$$

We express the new Green function in terms of $G(x, y, t)$ and $F(x, t)$. The measurement procedure is the same as in the pure case. We skip the details, and give the exact expression for the conductance:

$$g/g_0 = 1 - \int \int dx dy \frac{d}{d \omega} \langle\langle F(x) F(y) \rangle\rangle_{\omega=0} \tag{10}$$

We have used the method of Götze and Wölfle [23], and have extended Apel’s procedure [4] to deal with our inhomogeneous system. The force-force correlation in eq.(10) is proportional to the $2k_f$ density response function. Following Apel, we interpret the integral appearing in $g$ as a reflection coefficient $R$ due to the impurities. Then eq.(10) says that $g = g_0 (1 - R) = g_0 T$, which is the Landauer identity (1) now extended to an interacting dirty wire. Note that if we impose the current and measure the potential drop as in [4], we find instead $g = g_0 (1 - R)/R$. Of course, $R$ is different from what we obtain in the absence of leads. Only a perturbative computation of $R$ will be performed.

6.2. PERTURBATIVE CORRECTION TO THE CONDUCTANCE

In an homogeneous interacting wire with parameters $u$ and $K$, the sensitivity to a backscattering potential is determined by $K$. Extended and local
disorder renormalize to zero or infinity depending on whether $K$ is greater or smaller than $3/2$ and 1, respectively \[8, 7, 5\]. Since the wire length $L$ is the largest length in the problem, the renormalization has a lower energy cutoff $uL^{-1}$. A widespread recipe to deduce the behavior at length $L$ from the properties in the thermodynamic limit consists in replacing $T \to uL^{-1}$ \[9, 5\]. It is thus an interesting issue to see if our model of a finite wire can yield a confirmation of this approach.

6.2.1. Gaussian disorder

We consider now a disordered wire, where backscattering impurities are randomly distributed on the segment $[-a, a]$. For simplicity and to compare with the known results we suppose that the potential in (8) has a uniform Gaussian distribution \[\langle V(x)V(y) \rangle = D\delta(x-y)\]

The exact expression in (10) has to be averaged. To first order in $D$, the $2k_f$ density response function can be computed with the pure Hamiltonian $H$ and is thus related to $G$. But the resulting expression is so complicated that we can evaluate $g$ only in the high and low temperature limits. Even then the calculation is involved, because one has to integrate over time and space.

At $T \gg \omega_2$ we find that $R$ contains the term found by Apel and Rice \[7\], \[\approx 2aT^2(K_2-1)\], but contains in addition infinitely many powers of $T$. If we keep only the three main terms, we have

$$ R = D \left[ c_1 \left( \frac{T}{\omega_2} \right)^{2K_2-3} + c_2 (\tau_0 T)^{2(K_a-1)} + c_3 (\tau_0 T)^{2K_2-3} \right] \quad (11) $$

$\tau_0$ is a time cutoff of the order of the inverse bandwidth and therefore much smaller than $\beta$ and $t_2$. The $c_i$ are constants depending only on $K_2$. For convenience, we call the above three terms $R_i$. We have always $R_3 \ll R_1$. If $K_2 < (>)K_c = (3 + \sqrt{17})/4$, then $R_2 \ll (>)R_3$. In particular, $R_1$ is the dominant term if $K_2 < K_c$. But if $K_2 > K_c$, the ratio $R_1/R_2$ depends explicitly on $T$ and $\omega_2$, and we have to consider $R = R_1 + R_2$. For $T \ll \omega_2$ the correction is independent of temperature because of the noninteracting leads, but depends on the length in the following way:

$$ R = D\xi [c (\tau_0 \omega_2)^{2K_2-3} + c' (\tau_0 \omega_2)^{2(K_a-1)}] \quad (12) $$

The first (second) power is the dominant term if $K_2$ is less (greater) than $K_c$. We see that the substitution $T \to \omega_2$ allows one to go from (11) to (12).
6.2.2. A weak barrier at \(-a\)

The simplest way to model an imperfection at a contact is through a point like backscattering potential. Thus \(V(x)\) in eq.(8) is now centered around \(-a\). In a spirit similar to Kane and Fisher [5], we develop the boson field \(\Phi\) around \(-a\) (separately on the left and right side of \(-a\) since \(\partial_x \Phi\) is discontinuous). Then

\[
H_b = \sum_{m=-\infty}^{+\infty} \frac{V_m}{\pi \tau_0} \exp 2im\Phi(-a)
\]

where we include the higher harmonics.

If we evaluate eq.(10) to first order in \(V_m\), we get, for \(T \gg \omega_2\)

\[
g/g_0 = 1 - \sum_m m^2 V_m^2 B \left( \frac{1}{2}, m^2 K_a \right) (\pi \alpha T)^{2(m^2 K_a - 1)}
\]

(13)

\(B\) is the beta function. The correction is the exactly the same as that for a homogeneous wire with \(K_a\) in the presence of a barrier. Note the independence of \(g\) on the wire length.

For \(T \ll \omega_2\) we have

\[
g/g_0 = 1 - \sum_m m^2 V_m^2 c_m B \left( \frac{1}{2}, m^2 \right) (\pi \tau_0 T)^{2(m^2-1)} (\tau_0 \omega_2)^{2(m^2 K_a - 1)}
\]

(14)

where \(c_m\) depends only on \(K_2\). As in the previous section, the substitution \(T \to \omega_2\) works well for the first harmonic contribution, but not for the higher ones. We can also carry out the renormalization procedure using imaginary time at finite temperature: we increase \(\tau_0\) to \(\tau_0 e^l\) and modify the parameters in order to keep the partition function invariant. Only the coefficients \(V_m\) need to be renormalized. If we neglect higher-order terms, we find that the renormalization equations integrate to

\[
V_m(l) = V_m(0) \exp \left[ l - m^2 U(-a, -a, l) \right]
\]

(15)

where \(U(x, x; l) = 2[G(x, x, \tau_0) - G(x, x, \tau_0 e^l)]\), with \(G\) the imaginary time Green function. When \(K\) is uniform:

\[
U(\tau) = K \log |\sin[\pi \tau T]/\pi \tau_0 T|
\]

(16)

In our model, \(U\) is a series of logarithmic functions we avoid to write down. The effective \(V_1\) obtained from eq.(15) when we stop the renormalization at the maximum allowed cutoff value \(\beta/2\) is plotted in fig.(3). It exhibits a crossover from a power law dependence at \(T > \omega_2\) to a “marginal” behavior at \(T < \omega_2\).
In order to obtain further information, we approach $U$ in the extreme limits of low or high temperature compared to $\omega_2$. Consider first $T \gg \omega_2$: the coherence between the two contacts is lost, and the effective $K$ in (16) is local: $K_a = 2K_2/(1 + K_2)$. If we stop the renormalization equation at $\beta/2$, eq.(15) and eq.(16) yield

$$V_m(\beta/2) = V_m(\pi T \tau_0)^{(m^2 K_a - 1)}$$

(17)

Thus the renormalized $V_m$ have the usual power-law behavior controlled by the local conductance $K_a$.

In the opposite limit of low temperature, there is coherence on a scale larger than the traversal time $\tau_2$. $U$ contains two parts: a temperature dependent term similar to eq.(16) with the external $K_1 = 1$, and an $\omega_2$ dependent term. Up to a constant depending only on $K_2$, the renormalized $V_m$ turns out to be, in view of eq.(15):

$$V_m(\beta/2) = V_m(\pi \tau_0 T)^{(m^2 - 1)}(\omega_2 \tau_0)^{(m^2 K_a - 1)}$$

(18)

In particular, $V_1$ doesn’t depend on the temperature, but acquires an $\omega_2$ dependence due to the integration of the energies higher than $\omega_2$.

From eqs.(17) and (18), it appears that it is $K_a$ that controls the power law behavior either with temperature or length, and this is true for the first harmonic contribution only. The perturbation approach is valid as long as $K_a > 1$, but this condition is equivalent to $K_2 > 1$. If $K_2 < 1$, one has...
to control the smallness of $V_m$ in eqs. (17) and (18). We recover the same correction to the conductance obtained in eqs. (13) and (14) replacing the $V_m$ by their renormalized values from eqs. (17) and (18), but computing the $2mk_f$ response functions at the cutoff $\beta/2$.

We note that we can equally treat a weak impurity localized at a point $x$ inside $[-a, a]$. The renormalized $V_m$ has the form of eq. (15), with $U(x, x, l)$ instead of $U(-a, -a, l)$. The conductance correction can be found, but there is no place to give more details here.

6.3. SUMMARY

In contrast to the pure case where the conductance does not depend on the interactions inside the wire, the backscattering potential generates a power law behavior determined by $K_2$. In the presence of a weak backscattering at one contact, the correction is similar to that predicted in a homogeneous wire with a parameter $K_\alpha$ and a barrier. If there is a Gaussian distributed disorder, one encounters again a contact exponent $2(K_\alpha - 1)$ in addition to the bulk term usually found. At $K_2 < K_c$, the latter dominates the former. This concerns in particular repulsive interactions, which seems to be the case in a quantum wire. But at $K_2 > K_c$, the boundary of the wire has a non-negligible contribution. From eq. (17), the Andreev reflection we talked about persists to a weak barrier at the contact because $K_\alpha > 1$. The computation can be carried out similarly for interacting leads: this might have an interest in an S-N-S structure. Note that when $K_1 \neq 1$, we cannot go from the high to the low temperature regime by the simple substitution temperature $\rightarrow \omega_2$.

7. Summary

We have investigated transport through a finite interacting wire connected to noninteracting leads. The conductance of the pure wire is not renormalized by the interactions for any spatial variation of the internal parameters $u, K$. If $K$ varies abruptly, the correlation parameter $K_2$ of the wire controls the decrease of the conductance in the presence of a backscattering potential. In quantum wires where interactions are believed to be repulsive, we recover the usual power law behavior. This seems to fit well the experiments of Tarucha [10]. The agreement of our results with experiment is surprising in view of our crude treatment of the opening up of the measuring leads into a two-dimensional electron gas.

Other authors [24, 25] adopted the same model and found the same result for the conductance of the pure wire. Nevertheless, we do not agree with the conductance correction as derived by Maslov in [26]: the inhomogene-
ity of the correlation functions is ignored and only the bulk contribution is found.

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