Mathematical Modelling of Jet Finishing Process for Hot-dip Zinc Coatings on Steel Strip

Pravin NAPHADE, Ananya MUKHOPADHYAY and Shantanu CHAKRABARTI

Research & Development, Tata Iron & Steel Company, Jamshedpur-831001, India.
1) Coated Product, Cold Rolling, Mill, Tata Iron & Steel Company, Jamshedpur-831001, India.

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A mathematical model has been developed to predict the zinc coating thickness, and hence coating weight on a steel strip after passing it through a molten zinc pot, and wiping excess zinc from the surface of the strip using a pair of air wiping jets. The developed model predicts the coating weight as a function of different operating parameters such as strip velocity, jet nozzle pressure, nozzle-to-strip distance and nozzle slit opening. The required pressure and the shear stress profile on the strip surface were calculated through numerical simulations, carried out using FLUENT, a finite volume based commercial code. These were then correlated to the plant operating parameters through statistical regression analysis. Having been incorporated the developed correlations; the model was validated with the actual coating weight measured in CGL#2 of Tata Steel and also with the experimental results of J. M. Buchlin et al. The validated model was used to carry out the sensitivity analysis to determine the favourable operating regime for the air jet wiping process. It has been proposed that favourable operating regime should be chosen such that nozzle-to-strip distance is as small as possible and the nozzle pressure is as high as possible.

KEY WORDS: air jet wiping; hot-dip galvanizing; computational fluid dynamics.

1. Introduction

Galvanized steel is a value added product, furnishing effective performance by combining the corrosion resistance of zinc with the strength and formability of steel. In a continuous hot-dip galvanizing line (CGL), the steel strip is coated by passing it through a pot of molten zinc and withdrawing it from the pot through a pair of air-wiping jets, to remove the excess liquid zinc, commonly known as air jet finishing process, as shown schematically in Fig. 1. The air jet wiping is always the preferred choice to other means like using rolls to remove the excess liquid zinc, since it permits the use of higher line speeds, lighter coating weights and thinner gage strip.1) The sole purpose of this process is to obtain uniform coating weight and uniformity on the hot dip coated strip as it leaves the molten zinc bath.2–4) The uniformity of coating is an important issue since many industries such as automobile and home appliances demand high quality zinc coated steel strips with excellent corrosion resistance.

The objective of this investigation is to find out the favourable operating regime for galvanizing line operation in which the coating thickness and hence coating weight can easily be achieved at uniform level. To accomplish this objective, following methodology is adopted.

1) To develop a mathematical model which relates the solidifying coating thickness \( t \) on the steel strip to the strip velocity \( V \), jet nozzle pressure \( P \), nozzle-to-strip distance \( p \) and nozzle slit opening \( d \). This model relates the solidifying coating thickness \( t \) to the strip velocity \( V \), jet nozzle pressure \( P \), nozzle-to-strip distance \( p \) and nozzle slit opening \( d \).

Fig. 1. Schematic of jet finishing process with Kohler type nozzle.
distance \( Z \), nozzle slit opening \( D \) and the physical properties of molten zinc such as viscosity and density.

2) To carry out numerical simulations of an impinging jet using the finite volume based commercial code and then correlate the obtained pressure and shear stress profile on the strip surface with the jet nozzle pressure, nozzle-to-strip distance, nozzle slit opening, and the distance of wiping jet form the molten zinc surface.

3) To compare the predicted coating weight through the developed model, with the actual measured coating weight in the plant and with the experimental results from literature.

4) To perform sensitivity analysis using the validated model to determine the operating window that would help to improve the galvanizing line operation of the plant.

2. Mathematical Modelling

The mathematical model is developed based on the following assumptions:

1) The fluid flow in the liquid coating layer can be described by the steady state, two dimensional equations of incompressible, constant viscosity creep flow.

2) The pressure across the relatively thin incompressible coating layer is assumed to be constant, since the velocity of the coating fluid flow in the direction perpendicular to the strip surface \( y \) is small compared to the flow velocity in the direction parallel to the strip \( x \).

3) The effects of surface tension, oxidation, substrate surface roughness and interface alloy formation are neglected.

4) There is no slip between the liquid zinc layer and the strip at the strip–fluid interface.

The coating thickness, based on the assumptions made, is determined by the following equations:

\[
\frac{du}{dy} = \tau \text{ at } y = t \quad \text{............................(4)}
\]

where \( V \) is the velocity of the strip and \( \tau \) is the shear stress.

The Eqs. (1) and (2) are solved subject to the above boundary conditions which will lead to the equation for zinc withdrawal rate as

\[
q = V t \left(1 - \frac{ST}{2} - \frac{GT^2}{3}\right) \quad \text{............................(5)}
\]

where the non-dimensional coating thickness \( T \), non-dimensional shear stress \( S \) and the effective gravitational acceleration \( G \) are given as

\[
T = \frac{\rho g}{\mu V}, \quad S = \frac{\tau}{\sqrt{\rho \mu V g}}, \quad \text{and} \quad G = 1 + \frac{1}{\rho g} \frac{dp}{dx} \quad \text{..........................(6)}
\]

Introducing the non-dimensional withdrawal rate flux \( Q \) in Eq. (5) as

\[
Q = \frac{q}{V} \sqrt{\frac{\rho g}{\mu V}} \quad \text{............................(7)}
\]

and by setting \( dQ/dT = 0 \), the non-dimensional coating thickness is obtained as

\[
T = \frac{S + \sqrt{S^2 - 4G}}{2G} \quad \text{...........................(8)}
\]

From Eqs. (6) and (8), it is clear that if the pressure profile and shear stress profile on the strip surface are known, the coating thickness can easily be calculated. The required data for pressure and shear stress profile are generated with the help of computational modelling.

3. Computational Modelling

Numerical simulations are carried out using FLUENT, a finite volume based commercial code, to generate pressure and shear stress profile on the strip surface due to an impinging air jet on a flat vertical moving strip. The Reynolds averaged transport equations along with the RNG based \( k-\varepsilon \) turbulence model and non-equilibrium wall functions are solved. The pressure and shear stress on a moving strip due to impinging high pressure air jet is obtained for different sets of operating parameters. The range of these operating parameters covers the regime in which the CGL#2 of Tata Steel operates.

3.1. Conservation Equations

The governing equations for a two dimensional steady state and incompressible turbulent flow in the Cartesian coordinate after the Reynolds averaging take the following forms:

Continuity Equation:

\[
\frac{\partial \rho}{\partial x} = 0 \quad \text{............................(9)}
\]

Momentum Equation:
\[
\frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij} + R_{ij}) \quad \text{(10)}
\]

where the stress tensor \(\tau_{ij}\) is given by

\[
\tau_{ij} = \left[ \mu_s \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} \mu_s \frac{\partial \bar{p}}{\partial x_j} \delta_{ij} \right] \quad \text{(11)}
\]

and turbulent stresses or Reynolds stresses \(R_{ij}\) can be expressed as

\[
R_{ij} = -\rho w_i w_j \quad \text{(12)}
\]

where \(\bar{u}_i\) and \(\bar{u}_j\) are the mean and fluctuating components of velocity, \(\bar{p}\) is the mean component of pressure, \(x_i\) is the Cartesian coordinates and \(\mu_s\) and \(\rho_s\) are the viscosity and density of air.

### 3.2. Turbulence Modelling

The turbulent stresses are modelled using Boussinesq hypothesis which relates the Reynolds stresses to the mean velocity gradients through turbulent viscosity \(\mu_s\). In the present study, turbulent viscosity is described by the high Reynolds-number renormalization group (RNG) based \(k-e\) model. This model has two equations, one for turbulence kinetic energy and other for its dissipation rate, respectively,

\[
\frac{\partial}{\partial t} \left( \rho \kappa \right) = \frac{\partial}{\partial x_i} \left( \alpha_s \mu_{eff} \frac{\partial \kappa}{\partial x_i} \right) + \mu_s S^2 - \rho_e e \quad \text{(13)}
\]

\[
\frac{\partial}{\partial t} \left( \rho \varepsilon \right) = \frac{\partial}{\partial x_i} \left( \alpha_s \mu_{eff} \frac{\partial \varepsilon}{\partial x_i} \right) + C_{1e} \frac{\varepsilon}{\kappa} \mu_s S^2 - C_{2e} \rho_e \frac{\varepsilon^2}{\kappa} - R \quad \text{(14)}
\]

where

\[
\mu_s = \rho_s C_s \frac{k^2}{\varepsilon} \quad \text{and} \quad \mu_{eff} = \mu_s + \mu_t \quad \text{(15)}
\]

Since the turbulent viscosity \(\mu_s\) is not a fluid property, but rather a property of the flow field, its value is added to the molecular viscosity \(\mu_t\) and yields the effective viscosity \(\mu_{eff}\) which is used in the computational model.

\(C_1, C_2\) and \(C_\mu\) are now analytically derived constants having values of 1.42, 1.68 and 0.085, respectively. For high Reynolds number flows, \((\mu_s/\mu_{eff} <<1)\), \(\alpha_s = \alpha_e = 1.393.\) The \(R\) term in the transport equation for \(\varepsilon\) represents the effect of the mean rate of strain on the dissipation rate and is defined as

\[
R = C_\mu \rho_s \varepsilon \left[ \frac{1 - \eta / \eta_0}{\eta} \right] \varepsilon^2 \quad \text{(16)}
\]

where \(\eta = S \kappa / \varepsilon, \eta_0 = 4.38\) and \(\beta = 0.012.\) The \(S\) term in Eqs. (13) and (14) is the modulus of the mean rate-of-strain tensors and is defined as

\[
S = \sqrt{2 S_{ij} S_{ij}} \quad \text{(17)}
\]

where the mean strain rate \(\dot{\gamma}\) is given by

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad \text{(18)}
\]

### 3.3. Numerical Procedure

The governing equations were discretized using the control-volume-based finite volume method proposed by Patankar.\(^\text{(11)}\) The velocity–pressure coupling in the momentum equations was resolved using a sequential procedure of the SIMPLE\(^\text{(11)}\) (Semi-Implicit Method for the Pressure-Linked Equations) algorithm. The SIMPLE algorithm indirectly imposes the principle of mass continuity \(\text{via the solution of pressure-correction equations. A staggered grid scheme was adopted for the location of dependent variables in which vectors such as velocities are calculated on the control-volume faces and the scalars such as pressures are calculated at the main grid points. The flow-property values at the control volume faces are approximated by the second order upwind scheme.\)\end{equation}

### 4. Results and Discussions

The regression analysis is carried out to develop the correlations which relate the pressure and shear stress profiles on a moving strip surface due to impinging high pressure air jet to the operating parameters such as jet nozzle pressure, the nozzle-to-strip distance and nozzle slit opening. The correlation for the pressure profile on the moving strip due to impinging air jet is given by

\[
p = P_{max} e^{-0.16 b} \quad \text{(19)}
\]

where the maximum pressure developed \(P_{max}\) is given by

\[
P_{max} = \left[ \left( C_P \left( \frac{Z}{D} \right) \right) + \left( 0.2 \left( \frac{Z}{D} \right)^2 \right) \left( \frac{\rho_a V_n^2}{2} \right) \right]
\]

\[
\text{..................(20)}
\]

where \(C_P\) is the model constant and \(V_n\) is the velocity of air at the exit of the nozzle which is correlated to the jet nozzle pressure as

\[
V_n = [(C_v \cdot P^2) + (0.0051 \cdot P)] + 9.42 \quad \text{(21)}
\]

where \(C_v\) is the model constant and \(\eta\) is expressed in terms of \(b,\) the value of \(x\) where \(P = P_{max}/2,\) as

\[
\eta = \frac{x}{b} \quad \text{..................(22)}
\]

The maximum dimensionless jet pressure gradient is defined as

\[
d P_{n} = \frac{1}{\rho_g} \left( \frac{d p}{d x} \right)_{\text{max}} \quad \text{..................(23)}
\]

where the maximum pressure gradient can be obtained by differentiating Eq. (19) with respect to \(x\) and realising the fact that \(b\) is the value of \(x\) where \(P = 0.5 P_{max}\) and \(P_{max}\) is the maximum pressure developed at \(x = 0.\) The maximum pressure developed on the strip surface predicted by the Eq. (20) is compared with that calculated...
through numerical simulations for a few $Z/D$ cases. It can be seen from Fig. 2 that the correlation developed for calculating maximum pressure developed on strip surface can predict the maximum pressure within $\pm 7\%$ of that calculated through numerical simulations.

The correlation for the shear stress on the vertical moving strip due to the impinging air jet is given by

$$\tau = \left[ C_z \left( \frac{Z}{D} \right) + 0.01 \left( \frac{Z}{D} \right)^2 \right] - 0.00003 \left[ \rho_s V_s^2 \right] / 2 $$

...............................................(24)

where $C_z$ is the model constant.

The shear stress developed on the strip surface predicted by Eq. (24) is compared with that calculated through numerical simulations for a few $Z/D$ cases. It can be seen from Fig. 3 that the correlation developed for calculating shear stress developed on strip surface can predict the shear stress within $\pm 7\%$ of that calculated through numerical simulations.

The predicted coating weight, using the correlations described in Eqs. (19)–(23) in the developed mathematical model, is compared with the actual coating weight measured in CGL#2 of Tata Steel. As Fig. 4 clearly indicates, the model seems to adequately predict the coating weight within $\pm 15\%$ of the measured coating weight.

The non-dimensional zinc withdrawal rate flux, $Q$ versus maximum non-dimensional jet pressure gradient, $dPn$ as predicted by mathematical model and the experimental data of J. M. Buchlin et al.\textsuperscript{12) are plotted in Fig. 5 The mathematical model results are found to be in very good agreement with the experimental data.

The validated model is used to study the effect of different operating parameters such as strip speed, nozzle pressure and $Z/D$ ratio on the coating weight. The coating weight dependence on $Z/D$ for a different jet nozzle pressure is shown in Fig. 6. It can be clearly seen that for a fixed pressure and strip velocity, as the $Z/D$ increased the coating weight also increases. Also it can be observed that slope of the curve for low $Z/D$ (<19 in this case) is less as compared to steep slope of the curve for high $Z/D$ ratio. This indicates that high sensitivity coating weight region occurs at very high $Z/D$ which is difficult to control since small changes would result in large variations in coating weight. So it is better to carry out the air jet wiping operation in relatively low $Z/D$ region which is easy to control in
view to achieve uniform coating weight.

The dependence of coating weight on nozzle pressure for different strip velocities is shown in Fig. 7. It can be clearly seen that for a fixed velocity and $Z/D$ ratio, coating weight decreases as the nozzle pressure increases. For low pressure the slope of the curve is very steep which indicate the sensitive region. The choice of high pressure would be effective.

The dependence of coating weight on velocity of strip for different operating pressures is shown in Fig. 8. It can be seen that for a fixed pressure and $Z/D$, as the velocity of strip increases coating weight increases. The operating strip speed is always dictated by the thickness of strip and hence the processing time required in the annealing furnace for the specific product.

5. Conclusion

A mathematical model was developed to predict the coating thickness and hence the coating weight as a function of the operating parameters such as strip velocity, the jet nozzle pressure, the nozzle-to-strip distance and nozzle slit opening. The pressure and shear stress profile required in model were calculated through numerical simulations, carried out using the finite volume based commercial code. These profiles were correlated to the operating parameters through regression analysis.

The predicted coating weight is in good agreement with the actual coating weight measured in CGL#2 of Tata Steel. The coating weight predictions are within $\pm 15\%$ of the measured coating weight. The non-dimensional zinc withdrawal rate flux versus maximum non-dimensional jet pressure gradient as predicted by mathematical model and the experimental data are found to be in very good agreement.

The validated model was used to provide the sensitivity information which can be used to determine the appropriate operating regime for a given coating weight. It is proposed that favourable operating regime should be chosen such that nozzle-to-strip distance is as small as possible and a nozzle pressure as high as possible. This model thus helps to improve the consistency of zinc coating on the steel strip, from the knowledge of operating parameters like nozzle pressure, nozzle-to-strip distance and nozzle slit opening for a particular line speed.

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