Pushing Photons with Electrons: Observation of the Polariton Drag Effect

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We show the direct effect of free electrons colliding with polaritons, changing their momentum. The result of this interaction of the electrons with the polaritons is a change in the angle of emission of the photons from our cavity structure. Because the experiment is a photon-in, photon-out system, this is equivalent to optical beam steering of photons using a direct electrical current. The effect is asymmetric, significantly slowing down the polaritons when they move oppositely to the electrons, while the polariton momentum only slightly increases when electrons moving in the same direction.

We present a theoretical model which describes this effect.

It has long been known that an electron can impart momentum to an exciton-polariton through the scattering interaction with the exciton part of the polariton. Apart from the intrinsic interest in demonstrating this effect, it may have application in steering the direction of light emission by a direct electrical current, with the angle of deflection directly controlled by the applied current. However, observation of this effect has remained elusive. In this Letter we report observation of this effect with exciton-polaritons in a solid-state microcavity.

The physics of exciton-polaritons has been widely explored in recent years, and is well summarized in recent reviews, many of which have focused on Bose-Einstein condensation of polaritons [1–4]. In the present work, we use a polariton condensate, but the effect is not fundamentally one that only occurs for a condensate; rather, the condensate produces a spectrally narrow emission over a wide area that makes the polariton drag effect easy to observe. Superfluidity of the condensate does not prevent the drag effect, because it is equivalent to a body force, or longitudinal force. The structures we use also allow long-distance transport over hundreds of microns [5, 6], which allows good momentum resolution in our measurements.

Recent theoretical work taking into account the polaron effect on an exciton interacting with a Fermi sea [7] has predicted that applied direct current will give a drag force on exciton-polaritons, and that the effect will also be significant when the excitons are coupled to photons, as in an exciton-polariton system [8]. However, the drag effect can be understood even apart from the polaron effect as a purely collisional exchange of momentum due to Coulombic collisions between electrons and the excitonic part of a polariton, as discussed, e.g., in Ref. 9 and references therein. Other work [10–12] envisioned a drag effect between separated layers of excitons and free electrons, but this is not crucial for the effect, as free electrons can also move in the same quantum wells as the excitons.

**Experimental Method.** Exciton-polaritons in the strong-coupling limit were made by placing semiconductor quantum wells at the antinodes of a planar optical cavity. These semiconductor layers have excitons nearly resonant in energy with a cavity photon mode. If the Q-factor of the cavity is high enough, and the coupling of the photon and exciton states is strong enough, new eigenstates appear which are no longer purely photon or exciton, but a superposition of both. In other words, a photon in the cavity spends some fraction of its time as an excited electron-hole pair.

The sample used in this experiment consists of a 3A/2 microcavity formed by two distributed Bragg reflectors (DBRs), grown by molecular beam epitaxy (MBE). The DBRs were both made of alternating layers of Al0.2Ga0.8As and AlAs, with 32 periods in the top DBR and 40 in the bottom. 4 quantum wells (QWs), made of 7 nm GaAs layers with AlAs barriers, were placed at each of the 3 antinodes of the cavity. This microcavity design is the same used in previous work [5, 6, 13–22] (see the Supplemental Material for more details of the sample characteristics). Long wires (≈ 200 μm × 20 μm) were formed by etching away the top DBR, confining the polaritons within the wire and exposing the QWs.

To allow co-linear motion of free electrons in the same medium, two NiAuGe contacts were then placed upon the QWs at the ends of the wire, allowing electrical injection into the QWs (see Figure 1). Hall measurements show that the wire regions are intrinsically n-doped of the order of 10^{13} cm^{-3}, and the contacts are n-type with heavy doping of the order of 10^{19} cm^{-3}. A continuous-wave (CW) stabilized M Squared Ti:sapphire pump laser was used to excite a spot on the straight part of the wire, with a spot size of about 20 μm FWHM, in the same arrangement used in Ref. 17. The pump was non-resonant, with an excess energy of about 100 meV, and mechanically chopped at 400 Hz with a pulse width of about 60 μs. A source meter was used to sweep the applied voltage along the wires while measuring electrical current. The details of the fabrication and the electrical measurement are discussed in Ref. 16. The cavity detuning of the wire...
device presented in the main paper was $\delta = 6.8$ meV, corresponding to an exciton fraction $|X|^2 = 0.67$ for the lower polariton at $k = 0$. A full characterization of the sample is presented in the Supplementary Material.

Off-resonant optical pumping of this type of structure produces a cloud of excitons, which then lose energy and fall down into polariton states. As in previous experiments with similar structures [14, 17], there are two density thresholds for the optical excitation. At the lower threshold, a local quasicondensate is formed at the excitation spot, which can then ballistically expand away from the excitation region; above a higher critical threshold, the condensate jumps down dramatically into a much lower energy state, which is spectrally very narrow ($< 0.1$ meV width at half maximum), has high coherence length ($\gg 200 \mu$m), and fills most of the available space in a potential-energy minimum. We define the pump power needed to reach this second threshold as $P_{\text{thres}}$. The condensate was observed by recording the photons that leak out of the top mirror, using conventional imaging optics for both real-space (near field) and momentum-space (far field, Fourier plane) images, and a spectrometer for energy resolution. The leakage of photons out of the cavity was a tiny fraction of the population at any moment in time, because the $Q$ of the cavity is very high ($\sim 350,000$), so that a steady-state population was maintained [23]. The long polariton lifetime of 300-400 ps allows macroscopic flow over hundreds of microns [24] and demonstration of true equilibrium, confirmed by fits of the particle occupation number as a function of energy to an equilibrium Bose-Einstein distribution [25, 26]. In this steady state, the energy and spatial distribution of the polariton condensate is determined by the external potential profile it feels, which is a combination of the single-particle polariton dispersion and the repulsion of polaritons from slow-moving excitons with much higher mass, and the density-dependent, repulsive polariton-polariton interaction, which tends to flatten any external potential felt by the polaritons.

**Experimental Results.** The experimental assignment of the direction of positive $x$ and $k$ relative to the electrical contacts and pump spot is shown in Figure 1(b). Figure 2 shows the polariton distribution along the wire at two different pump powers above threshold and at zero applied voltage. The energy “hill” near 75 $\mu$m is due to the large population of excitons at the pump spot, which repulsively interact with the polaritons. A shallow cavity gradient along the $+x$-direction caused a small slope in the potential, with a total energy drop of about 0.5 meV across the wire. At the lower power, a large monoen-ergetic condensate forms on the side away from the pump, filling a region of the wire about 100 $\mu$m long. At higher power, the population on the right becomes large enough to have repulsive interactions that shift it up in energy, forming a single, nearly mono-energetic condensate.

![Figure 1](image1.png)

**Figure 1.** (a) The etched microcavity structure with the Ni-AuGe contacts at each end of the wire. (b) Optical image of a representative etched wire, with an overlay showing the experimental arrangement. The straight wire part is 200 $\mu$m long. The voltage connections of the source were connected to the device as shown. Thus, positive conventional current flowed in the direction of $-x$.

Figure 3 shows the energy vs. in-plane momentum ($k_\parallel$) of the polaritons in the level region, which is on the right side of pump spot (Figure 2), at three different applied voltages. With zero applied voltage (Figure 3(b)), an overall nonzero in-plane momentum is observed, which is due to flow in the main part of the wire, away from the pump spot, in the $+x$-direction. With negative applied voltage (Figure 3(c)), while all the other experimental conditions remain the same, the overall momentum is clearly reduced. This voltage corresponds to conventional current flowing in the $+x$-direction, and thus electron current flowing in the $-x$-direction, which opposes the polariton flow. With positive applied voltage (Figure 3(a)), the overall momentum is increased. This clearly shows an effect of drag upon the polaritons from the electrical current.

Figure 3(d) shows the polariton distribution vs. $k_\parallel$ for multiple applied voltages. A clear shift in the momentum is observed under applied voltages. Simple calculations indicate that the electrons move slower than the polaritons. The polariton velocity is measured directly from their momentum; 0.1 $\mu$m$^{-1}$ corresponds to $v = \hbar k/m = 1.2 \times 10^7$ cm/s. The electron velocity can be estimated from the mobility, which our measurements give as approximately 100 cm$^2$/V-s. For a voltage drop of 15 V over 200 $\mu$m, this gives electron velocities of the order of $7.5 \times 10^4$ cm/s.

**Theory.** We have developed a quantum Boltzmann
model showing the drag effect originating from the collisions between polaritons and electrons. We consider polariton-electron scattering where the two particles scatter from the state \( |\vec{k}_i\rangle = |k_1q_1\rangle \) to state \( |\vec{k}_f\rangle = |k_2q_2\rangle \) through a scattering interaction \( V_{\text{int}} \). The interaction potential is given by

\[
V_{\text{int}} = V_0 \sum_{k_1,k_2,\vec{q}_1} a_{k_2}^\dagger b_{\vec{q}_2}^\dagger b_{\vec{q}_1} a_{k_1},
\]

where \( a^\dagger \) and \( a \) are the polariton creation and annihilation operators respectively and \( b^\dagger \) and \( b \) are the electron creation and annihilation. The drag force that the electrons exert on a polariton with momentum \( k_1 \) is calculated using Fermi’s Golden rule

\[
F(k_1) = 2\pi V_0^2 \sum_{k_2,\vec{q}_2} (k_2 - k_1) N_e(|\vec{q}_1 - \vec{q}_0|)
\]

\[
\times (1 + N_p(k_2 - k_0)) \delta(E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1}).
\]

The wavevectors \( k_1 \) and \( k_2 \) correspond to the incoming and outgoing polariton with mass \( m_p \), respectively and \( \vec{q}_1 \) and \( \vec{q}_2 \) correspond to the incoming and outgoing electron with mass \( m_e \). The polaritons are taken to be 1-dimensional due to their light mass and the electrons are assumed to be 2-dimensional. The \( \delta \)-function accounts for the energy conservation in the polariton-electron scattering. \( N_p(k - k_0) \) is the occupation of the polaritons, and \( N_e(|\vec{q} - \vec{q}_0|) \) is the occupation of the electrons, where \( \vec{q}_0 \) is the drift wavevector of the electrons induced by the voltage difference and \( k_0 \) is the drift wavevector of the polaritons induced by ballistically expanding away from the excitation region. A detailed derivation of this model is discussed in the Supplemental Material.

Figure 4(a) shows the force polaritons feel due to electron-polariton scattering as a function of the relative velocity between the electrons and polaritons, defined as \( v_{\text{rel}} = h (q_0/m_e - k_1/m_p) \). When the relative velocity is zero, the polaritons feel no drag force from the electrons. Depending on the sign of the relative velocity, the polaritons can either feel a positive force (push) or a negative force (drag) due to collisions with electrons.

To simulate the experimental data, we fit the three different steady-state \( N(k) \) curves in Fig. 3 with a drifted Bose-Einstein distribution \( N(k - k_0) = [e^{(E(k-k_0)-\mu)/k_B T} - 1]^{-1} \) and extracted the steady state.
FIG. 4. (a) The force the polaritons feel given by Eq. (2) as a function of the velocity of the electrons relative to the polaritons, defined as $v_{rel} = \hbar (q_0/m_e - k_1/m_p)$. (b) The average magnitude of the force the polaritons feel given by Eq. (3) as a function of total density of the polaritons. Blue curve: electrons move along the same direction as polaritons. Red curve: electrons are stationary. Yellow curve: electrons move opposite to polaritons. In all three cases, the polaritons move along the positive $x$-direction but with different speeds, which are extracted from the occupation of the polaritons in Fig. 3. The vertical dashed line corresponds to the density of the polaritons in Fig. 3.

Our model indicates that in the three different voltage cases in Fig. 3, the polaritons are slowed down since the polaritons are faster than the electrons. In the case where the electrons move along the same direction as the polaritons, the drag force is less than when they move in the opposite direction, which gives the shifts of the momentum shown in Fig. 3.

Because the photoluminescence intensity, which directly indicates the density of the condensate, was not found to be significantly different for different applied voltages, we can rule out the shifts in the condensate energy due to the polariton-polariton and polariton-exciton interactions. What remains is the effect of the electron drag in our model.

Conclusions. We have demonstrated that a direct current can directly alter the momentum of exciton-polaritons; this has the effect of changing the angle of photon emission. Since the polaritons are effectively renormalized photons, created by photon absorption and ending with photon emission, this polariton drag is, in effect, using a direct electric current to change the momentum of photons. As polariton structures move ever closer to practical room temperature devices [4], this basic effect may also be possible in those devices.

We used a simple kinetic theory to explain the effect of drag of a Bose gas interacting with a thermal, fermionic reservoir. We showed that the Bose gas experiences a drag force, which is a function of the relative velocity between the Bose gas and the fermionic reservoir. We further show in the Supplemental Material, when the superfluid velocity of the condensate exceeds the relative velocity between the Bose gas and the fermionic reservoir, the drag force decreases, but it is nonzero. This indicates that it is simplistic to say that condensates do not experience drag or dissipation. In steady state, when there is interaction with an incoherent reservoir of non-condensate particles, they experience both.

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Author Contributions: The experimental work was carried out by D.M.M and Q.Y. under the supervision of D.W.S. D.M.M and Q.Y. contributed equally to this work. The theoretical model was developed by H.A. with the help of S.M. The sample was grown by L.N.P and K.W., and was designed and characterized by J.B. D.M.M and Q.Y. designed the devices. The device in the main text was fabricated by Q.Y. with the help of B.O. The device in the Supplemental Material was fabricated by B.O. D.M.M, Q.Y., H.A. and D.W.S have done most of the writing.

See Supplemental Material at [URL TBD] for characteristics of the sample and fabricated devices, additional experimental data and details about the numerical model, comparison to other works showing apparently similar effects, and additional analysis of the influence of injected electrons.

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Pushing Photons with Electrons: Observation of the Polariton Drag Effect - Supplemental Material

D. M. Myers, Q. Yao, H. Alnatah, S. Mukherjee, B. Ozden, J. Beaumariage, L. N. Pfeiffer, K. West, and D. W. Snoke

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The sample has the same design as used in Ref. [1], where it was reported that the lower polariton lifetime was about 200 ps, and cavity Q was about 320,000. The quality of samples from different batches from the Princeton laboratory is stable. Ref. [2] uses another sample, and it indicates the lifetime of polariton is about 200 ps as well.

We followed the process in Ref. [3] for sample characterization. The sample was characterized by examining different positions before it was etched. At one particular position of the sample, lower polariton energy (LP energy) was obtained directly from the experiment, while upper polariton energy (UP energy) was determined by photoluminescence excitation (PLE) experiment because the cavity is of very high Q factor and the upper polariton signal couldn’t be seen directly. With the growth sheet as reference, a model was set up to simulate the reflectivity of the cavity using transfer-matrix method. The model use several fitting parameters to fit the LP and UP dips of the reflectivity spectrum. These parameters are cavity thickness correction $r$, namely the ratio of the actual cavity layer thickness to the designed thickness (due to growth rate variation of the molecular beam epitaxy system), the exciton energy ($E_{ex}$), the exciton linewidth ($\gamma_{ex}$), and the imaginary part of the complex index of GaAs quantum well ($\kappa$), which is a measure of the intrinsic absorption of the material.

Figure S1 is the characterization result of one position where the LP PL is close to that in the main text. Figure S1(a) shows the simulated reflectivity curve (red) at normal incidence. The dips of the simulated curve aligned very well with experimental PL from the LP (blue) and the UP (green) data. Figure S1(b) shows the simulation on top of the experimental LP PL. The black bars label the simulated linewidth of the LP at different angles. The fitting parameters used in the simulation are: $r = 5.39\%$, $E_{ex} = 1604.40$ meV, $\gamma_{ex} = 0.71$ meV, $\kappa = 0.018$. Based on the those parameters, the exciton fraction of polariton at 1599.7 meV is 66.86%, and the coupling strength (half the LP-UP splitting) is 8.4 meV. With these parameters, the calculated $Q$ of the bare cavity is 335,000, which is comparable to the previous experiment.
FIG. S1. (a) simulated reflectivity curve (red), LP PL (blue) and UP PL (green) at normal incident. (b) Simulation on top of experimental LP PL. The black bars labels the simulated linewidth of LP.

CHARACTERISTICS OF THE DEVICE USED IN THE MAIN TEXT

The gradient of the bare lower polariton energy of the device can be obtained by comparing the LP energy at each end of the device for very low excitation density. In the experiment, lower polariton was generated by low power laser to avoid energy blue shift. The energy difference between two ends of the device used in these experiments was 0.5 meV. Since the wire device is 200 µm long, the energy gradient is 0.0025 meV/µm.

Figure S2 shows dependence of the photoluminescence (PL) intensity vs. the pump power.
for the device. Threshold power is defined as the power where the non-linearity of the curve begins. As it shows in Figure S2, the threshold power was 75 mW for this device. The data in Figure S2 was taken when pumping at the left side of the devices in reference to Figure 1(b) of the main text.

FIG. S2. Log-log plot of the photoluminescence (PL) intensity versus the pump power for the device in the main text. Data was taken by pumping on the left side of the devices in reference to Figure 1(b) of the main text. Threshold power, $P_{\text{thres}}$, and pump power, $10P_{\text{thres}}$ are labeled.

The dark current of the device was 0.8 nA. The current versus voltage characteristic is similar to what was observed in the square pillar devices of Ref. 4, and is shown in Figure S3. In Figure S3(a), (b), (c), an overall asymmetry is apparent when pumping on either end of the wire, and especially when pumping at the right. The asymmetry is nearly opposite for pumping on opposite ends, which indicates that it is primarily due to the pump spot location. We attribute this to greater illumination of the contact near the pump spot, creating free carriers that can carry current over the $n-i$ band bending barrier, as discussed in Ref. 4. Figure S3(d), (e), (f) include wider voltage range. Significant nonlinear rise of the current can be observed when applying voltages value above 10 V.
FIG. S3. Current versus voltage for various pump powers and with different pump spot locations. The locations are given in the upper left of each plot, and are defined in reference to Figure 1(b) of the main text. Specifically, they are the (a)(d) low energy end, (b)(e) middle, and (c)(f) high energy end of the wire, which correspond to the left, middle, and right sides of the x-axes used throughout this work. (a), (b), (c) focus on the linear part of (d), (e), (f).

ADDITIONAL DATA SHOWING THE DRAG EFFECT

Figure S4 shows the peak value of \( k_\parallel \) versus current for the device used in the main text, extracted from images similar to Fig. 3(b) in the main text. Figure S4(a) was taken with laser excitation power of 7.6 \( P_{thres} \), and Figure S4(b) was taken with laser excitation power of 10.3 \( P_{thres} \). Error bar labeled \( k_\parallel \) position at 95% of the peak.

The drag effect was also observed when pumping at the low energy end of a more photonic device, with detuning -3 meV on the left end and -2.1 meV on the right end. This device has a larger lower polariton energy gradient about 0.0035 meV/\( \mu \)m. Figure S5(a) shows the SEM image of the device, with the experimental arrangement same as Figure 1 in the main
FIG. S4. Peak value of $k_\parallel$ as a function of current, extracted from images similar to Fig. 3(b) for the device using in the main text. The threshold power for this device was 75 mW, as mentioned in the previous section. Error bar labels $k_\parallel$ at 95% of the peak. (a) laser excitation power of 7.6 $P_{\text{thres}}$. (b) laser excitation power of 10.3 $P_{\text{thres}}$.

text. When pumping at the left end of the wire, the threshold power, $P_{\text{thres}}$, is 130 mW. The current-voltage characteristic is presented in Figure S5(b). Although this device has no transition area at the end of the wire, like the one in the main text, it has much lower resistance (compare Figure S5(b) to Figure S3), because the metal contact and quantum well formed better ohmic contacts in this device. This allowed us to observe the drag effect on this device at lower pump power and lower voltage compared to the conditions in the main text. Figure S5(c) and (d) show energy-resolved real-space images at zero applied voltage, similar to Figure 2 in the main text. The dip at the end of the wire is due to the increased strain from the etching, similar to the corners of the square pillars discussed in Ref. 5.

Figure S6(a), (b), (c) show the energy versus in-plane momentum ($k_\parallel$) of the polaritons at three different applied voltages, similar to Figure 3 in the main text. The data was collected with the pump region blocked by a real space filter. A population near $k_\parallel = 0$ and at slightly lower energy is visible, and is emitted from the end trap. At zero voltage, the condensate has positive momentum due to flowing towards the $+x$ direction. At negative voltage, the overall momentum is clearly reduced. Figure S6(d) shows the polariton distribution versus $k_\parallel$ for multiple applied voltages. A clear shift in the distribution toward lower $k_\parallel$ is visible with applied negative voltage, indicating drag upon the polaritons. However, the increase of the condensate momentum was not observed in this device. This is because the polariton
FIG. S5. (a) Scanning electron microscope (SEM) image of a more photonic device, with an overlay showing the same experimental arrangement as Figure 1 in the maintext. (b) Current-voltage characteristic of this device when pumping on the left side of the wire. (c), (d) PL intensity versus energy and position along the length of the wire at zero applied voltage. The white dotted lines give the outline of the potential felt by the polaritons. The threshold power \( P_{\text{thres}} \) was about 130 mW, and the powers used were (c) 1.2 \( P_{\text{thres}} \) and (d) 1.9 \( P_{\text{thres}} \). The PL intensity was normalized separately for each image.

The velocity in this device is much higher than the device discussed in the main text. In this device, the polaritons move approximately 6 times faster in this device than the device shown in the main text. In addition, the electrons move much slower in this device. Using the same calculations as in the main text, the mobility 100 cm\(^2\)/V-s and voltage drop of 1 V over 100 \( \mu \)m, the electron velocity is approximately 5 \( \times \) 10\(^3\) cm/s, an order of magnitude slower than the electrons for the device in the main text. Our theoretical model (discussed below) indicates that the drag force becomes constant when the relative velocity is high > 3 \( \times \) 10\(^7\) cm/s (see orange curve in Fig. 4 of the main text).
In our experiment, a non-resonant pump was applied on one side of the wire, making an exciton reservoir. The excitons fell into the lower polariton states due to scattering with the lattice, and relaxed to the lower polariton states near $k_{\parallel} = 0$ as a result of the polariton-polariton interaction [6]. The polariton-electron interaction can also enhance polariton relaxation process and increase the population near $k_{\parallel} = 0$ on the lower polariton branch at low polariton density, as shown theoretically [7] and by direct measurement in experiments [8].

One might imagine a mechanism by which the enhanced cooling of the excitons and polaritons changed the potential-energy profile at the injection spot, which could change the speed of the escaping polaritons. From general considerations, this is unlikely, because the cooling rate is a function of the density of the carriers, not the speed of the current, to first order. However, we can look into this possibility in more detail by examining the spatial
FIG. S7. Additional experimental data for the device in the main text. (a) RGB data from energy-resolved real space images with the pump region. The pump power was 10.1 \( P_{\text{thres}} \). Red, green, and blue curves correspond to data with applied voltage -14 V, 0 V, and 14 V. Yellow, cyan, and magenta represent overlap of the -14 V and 0 V curves, 0 V and 14 V curves, and -14 V and 14 V curves, respectively. White corresponds to all three overlapping. (b) Energy vs. Current at two positions: left-end trap \((x = -4.1 \mu m)\), and near the right end of the wire \((x = 178 \mu m)\). (c) Energy difference between \(x = 60.5 \mu m\) and \(x = 177.7 \mu m\), i.e., in the range of the wire in which the change of momentum was measured.

profile of the pump spot as voltage was changed.

Figure S7(a) shows data for the energy versus spatial position with three applied voltages for the device in the main text. False color has been used with red, green, and blue corresponding to data with applied voltages -14 V, 0 V and 14 V, respectively; regions of overlap are indicated by other colors as given in the caption. As seen in this figure, the energy position of the condensate at the top of the profile (1602.5 meV) is not observably
changed by the applied voltage, indicating that the injection velocity from the top of the excitation profile will be unaffected by voltage.

Another observation is that the polariton energy changed due to the interaction between exciton reservoir and the injected electrons. Figure S7(b) gives energy change due to current on the left ($x = -4.1\mu m$), and near the end of the wire on the right ($x = 178\mu m$). However, the energy gradient of the region of interest ($x = 60 - 180\mu m$) was almost constant. Figure S7(c) shows the energy difference between two positions with various applied voltages. Also, the energy change was not observed in the more photonic device, which was discussed in the previous section.

**THEORY OF POLARITON DRAG**

We consider the polariton-electron scattering to derive the drag force that the electrons exert on the polaritons. The out-scattering rate for a polariton with momentum $k_1$ follow Fermi’s golden rule

$$\frac{\partial N_{k_1}}{\partial t} = \frac{2\pi}{\hbar} \sum_{k_f} \left| \langle \vec{k}_f | V_{\text{int}} | \vec{k}_i \rangle \right|^2 \delta (E_f - E_i).$$

(S1)

Here, we consider the polariton-electron scattering where the two particles scatter from the state $| \vec{k}_i \rangle = | k_1 \vec{q}_1 \rangle$ to state $| \vec{k}_f \rangle = | k_2 \vec{q}_2 \rangle$ through a scattering interaction $V_{\text{int}}$. The wavevectors $k_1$ and $k_2$ correspond to the incoming and outgoing polariton with mass $m_p$, respectively and $\vec{q}_1$ and $\vec{q}_2$ correspond to the incoming and outgoing electron with mass $m_e$. We take the electrons to be 2-dimensional but the polaritons are treated to be 1-dimensional. This is because the confinement energy of the wire is inversely proportional to the mass, making polaritons feel much more confinement than the electrons since the polaritons are much lighter. The interaction potential for the polariton-electron scattering is given by

$$V_{\text{int}} = V_0 \sum_{k_1,k_2,\vec{q}_1} a_{k_2,\vec{q}_2}^\dagger b_{\vec{q}_1}^\dagger b_{\vec{q}_2} a_{k_1},$$

(S2)

where $a^\dagger$ and $a$ are the polariton creation and annihilation operators respectively and $b^\dagger$ and $b$ are the electron creation and annihilation. Substituting Eq. (S2) into Fermi’s Golden rule, we obtain

$$\frac{\partial N_{p}(k_1)}{\partial t} = \frac{2\pi}{\hbar} V_0^2 \sum_{k_2,\vec{q}_2} N_e(|\vec{q}_1|) (1 - N_e(|\vec{q}_2|)) N_p(k_1) (1 + N_p(k_2)) \delta (E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1}),$$

(S3)
where $N_e(|\vec{q}|)$ is the occupation number of the electrons and $N_p(k)$ is the occupation number of the polaritons. Converting the sum to an integral gives

$$\frac{\partial N_p(k_1)}{\partial t} = \frac{2\pi}{\hbar} V_0^2 \frac{A}{(2\pi)^2} \frac{L}{2\pi} \int dq_x \int dq_y \int dk_2 \frac{N_e(|\vec{q}_1|)}{N_p(k_1)} \frac{N_e(|\vec{q}_2|)}{1 + N_p(k_2)} \delta(E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1}),$$

(S4)

where we have taken the polaritons to be moving along the $x$-direction. Here, $L$ is the length of the wire and $A = L \times W$ is the area of the wire, $q_{2x}$ and $q_{2y}$ are the $x$ and $y$ components of the outgoing electron wavevector and $k_2$ is the outgoing 1-dimensional polariton wavevector, which is taken to be along the $x$-direction. Rewriting Eq. (S4) gives

$$\Gamma(k_1) = \frac{1}{N_p(k_1)} \frac{\partial N_p(k_1)}{\partial t} = \frac{g^2}{\hbar(2\pi)^2} \int dq_x \int dq_y \int dk_2 \frac{N_e(|\vec{q}_1|)}{1 + N_p(k_2)} \delta(E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1}),$$

(S5)

where $\Gamma_{k_1}$ is the out-scattering rate and $g^2 = V_0^2 LA$. Since the goal is to compute the shift of polariton momentum, we weigh each scattering by the change of momentum of the polariton.

$$\hbar \Delta k \Gamma(k_1) = \frac{g^2}{\hbar(2\pi)^2} \int dq_x \int dq_y \int dk_2 \frac{h}{2\pi} \delta(E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1}).$$

(S6)

The term $\hbar \Delta k \Gamma(k_1)$ gives the the force the electrons are exerting on a polariton with momentum $k_1$, which we will denote with $F(k_1)$. Taking the low density limit of the electrons implies $N_e(|\vec{q}_1|) (1 - N_e(|\vec{q}_2|)) \simeq N_e(|\vec{q}_1|)$, which gives

$$F(k_1) = \frac{g^2}{\hbar(2\pi)^2} \int dq_x \int dq_y \int dk_2 \frac{h}{2\pi} \delta(E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1}).$$

(S7)

The argument of the delta function can be written as

$$\Delta E = E_{k_2} + E_{\vec{q}_2} - E_{k_1} - E_{\vec{q}_1} = E_{\text{bog}}(k_2) - E_{\text{bog}}(k_1) + \frac{\hbar^2}{2m_e} \left( q_{2x}^2 + q_{2y}^2 - q_{1x}^2 - q_{1y}^2 \right).$$

(S8)

Momentum conservation along $x$ and $y$ directions implies that $q_{1x} = q_{2x} + k_2 - k_1$ and $q_{1y} = q_{2y}$, which gives

$$\Delta E = E_{\text{bog}}(k_2) - E_{\text{bog}}(k_1) + \frac{\hbar^2}{2m_e} \left( q_{2x}^2 - (q_{2x} + k_2 - k_1)^2 \right).$$

(S9)
where $E_{bog}(k)$ is the Bogoliubov dispersion of the polaritons. Integrating over the delta function to eliminate $q_{2x}$ gives

$$F(k_1) = \frac{g^2 m_e}{2\pi^2 \hbar^2} \int dq_{2y} \int dk_2 \frac{k_2 - k_1}{|k_2 - k_1|} N_e \left( \sqrt{q_{1x}^2 + q_{2y}^2} \left(1 + N_p(k_2)\right) \right),$$

(S10)

where $q_{1x} = q_{2x}^{(0)} + k_2 - k_1$. Here, $q_{2x}^{(0)}$ is given by the roots of Eq. (S8), which is given by

$$q_{2x}^{(0)} = \frac{1}{2} (k_1 - k_2) + \frac{m_e}{\hbar^2} \left( \frac{E_{bog}(k_1) - E_{bog}(k_2)}{k_1 - k_2} \right),$$

(S11)

and therefore

$$q_{1x} = \frac{1}{2} (k_2 - k_1) + \frac{m_e}{\hbar^2} \left( \frac{E_{bog}(k_1) - E_{bog}(k_2)}{k_1 - k_2} \right).$$

(S12)

We assume the electrons are in a drifted Maxwell-Boltzmann distribution, given by

$$N_e \left( \sqrt{q_{1x}^2 + q_{2y}^2} = e^{\mu_e/k_B T_e} e^{-\hbar^2 q_{2y}^2/2m_k B e} e^{\hbar^2 (q_{1x} - q_0)^2/2m_k B T_e},$$

(S13)

where $q_0$ is the momentum of the electrons induced by the voltage across the wire, $\mu_e$ is the chemical potential and $T_e$ is the temperature of the electron gas. Integrating over $q_{2y}$ gives

$$F(k_1) = e^{\mu_e/k_B T_e} \frac{g^2 m_e}{2\pi^2 \hbar^2} \int dk_2 \frac{k_2 - k_1}{|k_2 - k_1|} e^{-\frac{\hbar^2 (q_{1x} - q_0)^2}{2m_k B T_e}} (1 + N_p(k_2)).$$

(S14)

To account for the fact that polaritons are ballistically expanding away from the excitation region, we replace $N_p(k_2)$ with $N_p(k_2 - k_0)$, where $k_0$ is the momentum induced by exciton cloud. We then have

$$F(k_1) = e^{\mu_e/k_B T_e} \frac{g^2 m_e}{2\pi^2 \hbar^2} \int dk_2 \frac{k_2 - k_1}{|k_2 - k_1|} e^{-\frac{\hbar^2 (q_{1x} - q_0)^2}{2m_k B T_e}} (1 + N_p(k_2 - k_0)).$$

(S15)

The average force the polaritons feel can then be defined as

$$F_{avg} = \frac{\int_{-\infty}^{\infty} dk_1 F(k_1) N_p(k_1 - k_0)}{\int_{-\infty}^{\infty} dk_1 N_p(k_1 - k_0)}.$$

(S16)

Simulating the experimental data

To simulate the experimental data shown in Fig. 3(d), we fit the PL intensity with the Bose-Einstein distribution $N(k - k_0)$ to extract the drift momentum $k_0$ and the chemical potential of the polaritons. We fixed the temperature of the polaritons to $T = 5$ K and we vary $\mu_p$ and $k_0$ to find the best fit to the Bose-Einstein distribution, given by

$$N_p(k - k_0) = \frac{A}{e^{(E(k - k_0) - \mu_p)/k_B T} - 1},$$

(S17)
FIG. S8. Circles: experimental time integrated average momentum distribution of the condensate under different applied voltages (Fig. 3(d) of main text). Dashed lines: the best fit to the drifted Bose-Einstein distribution given by Eq. (S17).

where $A$, $\mu_p$ and $k_0$ are fit parameters and $E(k - k_0) = \hbar(k - k_0)^2/2m_p$. Since our model does not take into account reflection from the edge of the wire, we exclude the negative momentum PL in Fig. 3(d) from the fit, which comes from reflection of the polaritons off the far end of the wire.

Figure S8 shows the fit to the Bose-Einstein distribution for the positive, zero and negative voltages shown in Fig. 3(d) of the main text. The extracted drift momenta $k_0$ from the best fit are $8.71 \times 10^4 \mu m^{-1}$, $7.79 \times 10^4 \mu m^{-1}$ and $5.83 \times 10^4 \mu m^{-1}$ for the $+14$ V, $0$ V and $-14$ V cases respectively. These values are used in the numerics to find the average drag force given by Eq. (S16) for these three different voltage cases. Since the magnitude of current for the $+14$ V and $-14$ V is not the same experimentally, we assume that the electrons move $143.4 \mu A/115.4 \mu A \approx 1.24$ times faster for the negative voltage case.

**The effect of Bogoliubov branches**

In this section, we discuss the effect of Bogoliubov branches, i.e., superfluidity of the polaritons on the drag force. We consider the case where the polaritons are stationary...
FIG. S9. The average force the polaritons feel as a function of the electron velocity divided by the polariton superfluid velocity.

\( k_0 = 0 \) and the electrons are moving in the negative \( \hat{x} \)-direction. Thus, we have

\[
F(k_1) = e^{\mu_e/k_B T_e} \frac{g_p^2 m_e}{2\pi^2 \hbar^2} \left( \frac{2\pi m_e k_B T_e}{\hbar} \right) \int dk_2 \frac{k_2 - k_1}{|k_2 - k_1|} e^{-\frac{k_2^2 (q_{1x} - q_0)^2}{2m_e k_B T_e}} (1 + N_p(k_2)), \tag{S18}
\]

where

\[
q_{1x} = \frac{1}{2} (k_2 - k_1) + \frac{m_e}{\hbar^2} \left( \frac{E_{\text{bog}}(k_1) - E_{\text{bog}}(k_2)}{k_1 - k_2} \right). \tag{S19}
\]

We assume that the polariton occupation is given by the Bose-Einstein distribution, which is given by

\[
N_p(k_2) = \frac{1}{e^{(E_{\text{bog}}(k_2) - \mu_p)/k_B T} - 1} \tag{S20}
\]

and

\[
E_{\text{bog}}(k_2) = \sqrt{E(k_2) (E(k_2) + 2g_p n)}, \tag{S21}
\]

where \( E(k_2) = \hbar^2 k_2^2 / 2m_p \), \( g_p \) is the polariton-polariton interaction strength and \( n \) is the total density of the polaritons. The superfluid velocity of the polaritons is given by \( v_s = \sqrt[1/2]{g_p n / m_p} \). The average drag force that the polaritons with an occupation \( N(k_1) \) feel is then given by

\[
F_{\text{avg}} = \frac{\int_{-\infty}^{\infty} dk_1 F(k_1) N_p(k_1)}{\int_{-\infty}^{\infty} dk_1 N_p(k_1)}. \tag{S22}
\]

Figure S9 shows the average drag force the polaritons feel as a function of \( v_e / v_s \), where \( v_e \) is the electron velocity. In this calculation, the polariton density was fixed, i.e. constant \( v_s \).
and the electron velocity was varied. We find that when the electron velocity exceeds the polariton superfluid velocity, the drag force increases significantly and eventually saturates.

COMPARISON TO OTHER WORK

A series of papers [9–16] reported observation of a “photon drag” effect, also called “optical rectification” or a “photogalvanic effect.” In these works, a high-intensity laser beam was directed at a semiconductor system, at an optical resonance, and a voltage difference, i.e., an electric polarization, was measured in the material. This can be interpreted as, and was analyzed as, a nonlinear optical effect which is essentially the inverse of the electro-optic effect. In the electro-optic effect, an applied DC electric field leads to a change of the index of refraction of a medium felt by an electromagnetic wave; in these “optical rectification” experiments, a nonlinear $\chi^{(3)}$ term led to shift in the index of refraction that then led to a DC electric field. The effect seen in these papers relied on high-intensity lasers to obtain nonlinear effects, while in our experiments, the condensate is a very low photon density. They also relied the presence of a strong absorption resonance, and saw only a voltage in response to a total intensity, not any shift of the momentum of the photons in response to current.

A theoretical paper, Ref. [17], proposed a mechanism by which a current could lead to a drag effect on photons. In that work, it was proposed that a current could shift the subband energies of electron states in a quantum well, which would then shift the index of refraction, which then would have an effect on photons passing through the medium. For that effect to occur, there would have to be substantial shift of the exciton subband states. In our experiments, as seen in Figure S6, at the point of maximum exciton density, there is no shift of the exciton energy with applied voltage.

In Ref. [18], an electron density gradient was used to accelerate polaron-polaritons, by creating a voltage-dependent potential energy gradient for the polaritons. Again, as discussed above, the fact that in our experiments the polariton energy at the point of maximum exciton density does not shift at all with voltage, as seen in Figure S6(a), shows that the free electrons are having negligible effect on the reservoir exciton spatial distribution. There is therefore no effect of the electron current on the static potential energy felt by the polaritons.

There is a sense in which all of the above experiments and ours may be considered different
limits of a general effect, which is that photons and electrons can exchange momentum in
a medium with electron-photon resonances. In the experiments reviewed in this section,
however, there is an intrinsic time lag, as there must be a macroscopic rearrangement of the
electron density in the medium to create an electric polarization. In the theoretical model
we have presented here, which perfectly describes our experimental results, the response is
instantaneous as a result of direct polariton-electron collisions.

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