Studies to determine the value of air consumption and power of the device for automatic removal of the milking machine

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Abstract. Overexposure of the milking machines on the udder of the animal causes a negative effect on the mammary gland, which leads to illness and culling of cows. In this regard, during machine milking, it is necessary not only to ensure the complete milking of animals, but also to remove the teat cups from the udder in a timely manner. In order to solve this problem, the authors have developed a device for automatically removing the hanging part of the milking machine after milking. This device includes an air motor, a reducer and a flexible filament cylinder. In a theoretical consideration of the working process of removing the hanging part of the milking machine, analytical dependencies were obtained which make it possible to determine the airflow rate of the pneumatic motor, depending on its geometric dimensions, the rotor speed, the number of installed blades and the value of the working vacuum. It was found that with an increase in the above parameters, the air consumption increases. Also, expressions have been obtained that allow setting the required power on the cylinder of the device and its rotation frequency to ensure automatic removal of the hanging part of the milking machine at any time, excluding its hitting the floor of the stall.

1. Introduction
Milking is one of the most labor-intensive processes in milk production, which accounts for up to 30% of manual labor costs. Often during milking, overexposure of the milking machines on the udder of milked animals is observed, which leads to stress in cows and diseases. In cows, the milk flow reflex is inhibited, and they sometimes do not milk completely, which leads to a decrease in productivity and their premature culling. Elimination of the human factor, ensuring an adequate reaction of the body and the machine at all stages of the milking process and complete emptying of the udder will contribute to the rapid growth of milk yields, increase their use, improve herd reproduction and create conditions for the growth of the genetic potential of cows. Therefore, effective farming at the present stage is unthinkable without equipping enterprises with perfect milking equipment [1,2,3,4]. In our opinion, the most rational direction for improving the tethered keeping of animals during milking on linear installations with a milk pipeline is to equip them with portable milking machines.
containing in their design devices for tracking the milking process and timely removing the teat cups from the udder of the animal upon completion of the milk flow process.

We have developed and manufactured a device for automatically removing the hanging part of the milking machine (Figure 1).

![Figure 1. Device for automatic removal of the milking machine: a - developed scheme; b - manufactured sample: 1 - pneumatic motor; 2 - reducer; 3 – cylinder; 4 – cord; 5 - case; 6 – rotor shaft; 7 – curved blade; 8 – camera; 9 – cover; 10 – outlet branch pipe; 11 – inlet pipe.](image)

The device includes a pneumatic motor 1, a planetary reducer 2 and a cylinder 3 with a flexible thread 4. The pneumatic motor 1 contains a case 5, a rotor shaft 6, the outer surface of which is equipped with four slots in which curved blades are hinged 7. The rotor shaft 6 with blades 7 is installed in the case 5 and is closed by the cover 9, forming four separate chambers 9. These chambers 9 are sealed relative to the case 5 due to the wide contact surface of the curved blades 7 with the walls of the rotor chamber. The case 5 of the pneumatic motor 1 has an outlet 10 and an inlet 11 branch pipes for connection with a vacuum source and the atmosphere. The rotor shaft rotates due to the
difference in atmospheric and vacuum pressure in chambers 9, cylinder 3 rotates with it, winding a
flexible thread 4 on itself and pulling the hanging part of the milking machine from the udder of the
animal at the end of milking.

At the beginning of milking, the operator connects the milking machine to a vacuum source, pulls
the cord 4 of the cylinder 3 of the device and puts the teat cups on the udder of the cow. The milking
process begins. At the end of milking, the vacuum is supplied to the pneumatic motor through the
outlet pipe 10. Under the influence of the pressure difference acting on the curved blades 7, the rotor
shaft 6 and the cylinder 3 connected to it through the planetary reducer 2 begin to rotate. From which
the cord 4 is wound on its outer surface. The hanging part of the milking machine is pulled from the
teats of the udder, removed from under the cow, and hangs under the vacuum line. The use of the
device for automatic removal of the hanging part of the milking machine significantly reduces the
manual labor costs of the process of milking cows when they are tethered.

2. Purpose, objectives
Based on the foregoing, the object of the study was the working process of removing the hanging part
of the milking machine from the udder of the animal by the device. Theoretical studies, carried out
using the provisions of the sections of classical mechanics and mathematics, are aimed at determining
the amount of air flow and power by the pneumatic motor of the device for automatically removing
the milking machine from the udder of a cow.

3. Methods and materials
The air flow required to drive the pneumatic motor is determined by the known relationship for rotary
pneumatic machines [5,6]:

\[
Q_v = \psi \cdot p_1 \cdot l \cdot S \cdot z \cdot n, \tag{1}
\]

where \( Q_v \) – free air flow, \( m^3/s \); \( \psi \) – correction factor for air leaks; \( p_1 \) – dimensionless quantity
characterizing the pressure drop acting on the blade; \( l \) – rotor length, \( m \); \( S \) – cross-sectional area
between adjacent blades at the time of cut-off, \( m^2 \); \( z \) – number of air motor blades, \( pcs \); \( n \) – rotor speed,
\( rps \).

\[
p_1 = \frac{p_0}{p_0 - p_v}, \tag{2}
\]

where \( p_0 \) is the barometric pressure of the environment, \( Pa \); \( p_v \) - vacuum value, \( Pa \).

To determine the cross-sectional area \( S \) enclosed between the blades at the time of air volume cut-
off, we will consider the design diagram of a rotary air motor (Figure 2).

The area enclosed between the curved blades at the moment of air cut-off (when the inlet and outlet
ports are closed) is designated by the letters ACDEFB. To simplify the task of finding the area of a
figure, we divide it into three component parts. Area \( S_1 \) corresponds to figure BCDF, area \( S_2 \) - to
figure DFE and area \( S_3 \) - to figure ABC.

The area of the ABFEDC figure will be:

\[
S = S_1 + S_2 + S_3. \tag{3}
\]

When solving this problem, we will assume that the blades have a thickness of zero, and the rotor
section is a circle with a radius \( r \), that is, we neglect the rotor protrusions in the rotor chamber and the
blade thickness. The arc length of the blades AB and ED is assumed to be \( l/4r \).
φ, φ₁ – the angles defining the position of the inlet and outlet ports (pipes); α, β – the angles defining the position of the blades and rotor at the moment of air cut-off; r – rotor radius; R – inner radius of the rotor chamber; e – eccentricity.

**Figure 2.** Design diagram of the pneumatic motor: a - general view of the section; b - design scheme (rotated).

Let the point O be the reference point. BFEG – circle with radius r centered at point O. The figure ACDG – circle with radius R>r centered at point O₁(r−R;0) and tangent to the first circle at the point G(−r;0), located on the axis OX. AB – quarter of the circle radius r, passing through the point B(r;0) and through the point A, lying on the second circle. ED – the quarter of the circle radius r, passing through the point E(0;r) and through the point D, lying on the second circle. Below are the results for determining the area of the figure ACDEFB.

Curved shape area \( S_1 \) will be:

\[
S_1 = \frac{1}{2} \left\{ \int_0^{\alpha_1} \left[ \rho_1(\varphi)^2 - \rho_2(\varphi)^2 \right] d\varphi \right\} = \frac{1}{2} \left\{ \int_0^{\alpha_1} [\rho_1(\varphi)^2] d\varphi - \int_0^{\alpha_2} [\rho_2(\varphi)^2] d\varphi \right\} = \frac{1}{2} (I_1 - I_2),
\]

where \( \rho = \rho_1(\varphi) \) – the polar line equation AC, and \( \rho = \rho_2(\varphi) \) – the polar line equation AB;

\[
I_1 = \int_0^{\alpha_1} [\rho_1(\varphi)^2] d\varphi, \quad I_2 = \int_0^{\alpha_2} [\rho_2(\varphi)^2] d\varphi.
\]

After determining the integrals in the expression (4), the formula for finding area \( S_1 \) (Figure 2) will take the form:

\[
S_1 = \frac{R^2 \alpha + (R-r)^2 \sin \alpha \cos \alpha}{2} + \frac{(R-r)^2}{2} \left[ \sin \alpha \sqrt{R^2 - (R-r)^2} \sin^2 \alpha + \frac{R^2}{(R-r)} \arcsin \left( \frac{R-r \sin \alpha}{R} \right) \right] +
\]


\[ S_2 = \frac{1}{2} \int_{\beta}^{\pi} (\rho_2(\phi))^2 - r^2 \, d\phi, \]

where \( \rho = \rho_1(\phi) \) – equation in polar coordinates line \( ED \), which is a quarter circle of radius \( r \) centered at point \((x_2; y_2)\).

After finding the integrals included in the expression (7), area formula \( S_2 \) will take the form:

\[ S_2 = \cos \beta \cdot \frac{2r^2(R-r)^2}{(R^2-2Rr+2r^2)^2} \left( R(R-2r) \sin \beta + 2r(R-r) \cos \beta \right) + \\ + \frac{1}{2} \left( x_2 \sqrt{r^2-x_2^2} - (x_2 \sin \beta + y_2 \cos \beta) \sqrt{r^2-(x_2 \sin \beta + y_2 \cos \beta)^2} \right) + \\ + \frac{r^2}{2} \left( \arcsin \left( \frac{x_2}{r} \right) - \arcsin \left( \frac{x_2 \sin \beta + y_2 \cos \beta}{r} \right) \right). \]

Angle definition \( \beta \) at the beginning of the release of air from the rotor chamber is carried out from the formulas:

\[ \cos \beta = \frac{4Rr^2 - 2r^2}{\sqrt{5R^4 - 16R^3 r + 24R^2 r^2 - 16Rr^3 + 4r^4}}; \quad \sin \beta = \frac{2R(R-r)}{\sqrt{5R^4 - 16R^3 r + 24R^2 r^2 - 16Rr^3 + 4r^4}}. \]

Figure area \( S_3 \) (Figure 2) is determined by the formula:

\[ S_3 = \frac{1}{2} \int_{\beta}^{0} (\rho_3(\phi))^2 - r^2 \, d\phi, \]

where \( \rho = \rho_1(\phi) \) – the polar line equation \( CD \), which is an arc of a circle of radius \( R \) centered at point \((R-r;0)\).

Determining the value of the integrals included in the expression (10), we finally get the formula for calculating the area \( S_3 \):

\[ S_3 = \frac{R^2-r^2}{2} \arcsin \left( \frac{2R(R-r)}{\sqrt{5R^4 - 16R^3 r + 24R^2 r^2 - 16Rr^3 + 4r^4}} \right) + \frac{(R-r)^2}{2} \sin \beta \cos \beta + \]

\[ + \frac{1}{2} \left[ \frac{2R^2 R^2 - R^2 r^2}{(R^2-2Rr+2r^2)^2} \left( R(R-2r) \sin \beta + 2r(R-r) \cos \beta \right) + \\ + \frac{1}{2} \left( x_2 \sqrt{r^2-x_2^2} - (x_2 \sin \beta + y_2 \cos \beta) \sqrt{r^2-(x_2 \sin \beta + y_2 \cos \beta)^2} \right) + \\ + \frac{r^2}{2} \left( \arcsin \left( \frac{x_2}{r} \right) - \arcsin \left( \frac{x_2 \sin \beta + y_2 \cos \beta}{r} \right) \right) \right]. \]
So, according to the formulas (5), (8) and (11) it is possible to determine the components of the cross-sectional area of the rotor chamber of the pneumatic motor. And the total cross-sectional area is determined by the formula (3).

Substituting the area value \( S \) into equation (1), we determine the air flow \( Q \) by the pneumatic motor.

From the observance of the working conditions of the pneumatic motor during shockless removal of the milking machine, the force developed by the pneumatic motor must be greater than or equal to the force for lifting the milking machine \( T \):

\[
T \geq \frac{T_y}{\cos \varphi_0} = \frac{mg}{\cos \varphi_0},
\]

where \( T \) – the power to lift the milking machine, \( N \); \( T_y \) – the vertical force component \( T \), \( N \); \( m \) – the weight of the hanging part of the milking machine, kg; \( \varphi_0 \) – the angle of initial deflection of the milking machine from the vertical.

In this case, the speed of rotation of the cylinder of the device must be greater than or equal to the critical \( n_{cr} \)

\[
n_{cr} = \sqrt{\frac{g(h-b)(1-\cos \varphi_0)}{2\cdot \pi \cdot r_{cr} \cdot \cos \varphi_0}}, \]

where \( n_{cr} \) – critical cylinder speed, \( s^{-1} \); \( h \) – installation height of the device above the stall floor, m; \( b \) – minimum formed distance from the stall floor to the hanging part of the milking machine during its work, m; \( r_{cr} \) – device cylinder radius, m.

The required power on the cylinder of the pneumatic motor depends on the weight of the hanging part of the milking machine and the speed of its lifting, to ensure the condition of shock-free removal from the stall floor:

\[
N = T \cdot \vartheta = T \cdot \omega \cdot r,
\]

where \( N \) – required power of the pneumatic motor, \( W \); \( \vartheta \) – milking machine linear speed, \( m/s \); \( \omega \) – meadow cylinder speed, \( rad/s \); \( r \) – cylinder radius, m;

Taking into account the required angular speed of the cylinder \( \omega_{cp} \ (rad/s) \):

\[
\omega_{cp} = \sqrt{\frac{g(h-b)(1-\cos \varphi_0)}{r \cdot \cos \varphi_0}},
\]

and the value of the force \( T \), the expression (14) takes the form:

\[
N \geq \frac{m\sqrt{3(h-b)(1-\cos \varphi_0)}}{\cos^2 \varphi_0}.
\]

According to the formula (16), it is possible to calculate the required power of the pneumatic motor for removing the suspended part from the udder of the animal upon completion of milking.
4. Results and discussion

Graphical interpretation of expression (1) for determining the airflow rate of the pneumatic motor from the design parameters of the device for automatic removal of the milking machine and its comparison with the obtained experimental dependence is shown in Figure 3.

The proportionality coefficient taking into account air leaks was determined from the expression:

$$\psi = \frac{Q_{o}}{Q_{r}}, \quad (17)$$

where $Q_{r}$ – air flow value obtained from theoretical relationship (1), m³/h;
$Q_{o}$ – air flow value obtained experimentally, calculated by the regression equation, m³/h

$$Q_{o} = -0.00165327 + 0.00006117 \cdot p_{s} + 0.0003425 \cdot D + 0.0017174 \cdot d - 0.00000037 \cdot p_{s}^2 - 0.0000002 \cdot D^2 - 0.00000066 \cdot d^2 - 0.00000017 \cdot p_{s}D + 0.00000016 \cdot p_{s}d - 0.00000029 \cdot Dd, \quad (18)$$

where $D$ – cylinder diameter, mm; $d$ – inner diameter of the outlet, mm.

![Figure 3](image_url)

**Figure 3.** Graphical dependences of comparing the convergence of research results (air consumption of the pneumatic motor on the vacuum value)

The theoretical and experimental airflow rates for the air motor of the automatic milking machine remover are presented in Tables 1 and 2.

The final value of the empirical coefficient taking into account air leaks $\psi$, in expression (1) should be taken equal to $\psi = 2.51$. When comparing the theoretical and experimental results of the values of the airflow rate versus the vacuum value, their difference under vacuum 35…60 kilopascals averages 3.2%. This suggests that the theoretical model has a high convergence with experimental data.
Table 1. Theoretical air flow rates

| Vacuum value $P$, kPa | Rotor diameter $2r$, m | Rotary diameter cameras $2R$, m | Rotor length $l$, m | Angle $\phi$, degree | Angle $\phi_1$, degree | Number of blades $z$, pcs. | Rotor speed $n$, rpm | Coefficient $\Psi$ | Air consumption $Q$, m$^3$/h |
|-----------------------|------------------------|-------------------------------|------------------|-------------------|-------------------|----------------------|-----------------|---------------|------------------|
| 35                    | 0.45                   | 0.056                         | 0.06             | 120               | 60                | 4                    | 1000            | 2.51          | 5.0              |
| 40                    | 0.45                   | 0.056                         | 0.06             | 120               | 60                | 4                    | 1000            | 2.51          | 5.51             |
| 45                    | 0.45                   | 0.056                         | 0.06             | 120               | 60                | 4                    | 1000            | 2.51          | 6.01             |
| 50                    | 0.45                   | 0.056                         | 0.06             | 120               | 60                | 4                    | 1000            | 2.51          | 6.48             |
| 55                    | 0.45                   | 0.056                         | 0.06             | 120               | 60                | 4                    | 1000            | 2.51          | 6.88             |
| 60                    | 0.45                   | 0.056                         | 0.06             | 120               | 60                | 4                    | 1000            | 2.51          | 7.50             |

Table 2. Experimental values of air flow

| Vacuum value $P$, kPa | Cylinder diameter $D$, m | Inner diameter of the outlet $d$, m | Weight of the lifted load $m$, Kg | Air consumption $Q$, m$^3$/h |
|-----------------------|--------------------------|------------------------------------|---------------------------------|-----------------------------|
| 35                    | 0.022                    | 0.007                              | 3.4                             | 5.18                        |
| 40                    | 0.022                    | 0.007                              | 3.4                             | 5.73                        |
| 45                    | 0.022                    | 0.007                              | 3.4                             | 6.22                        |
| 50                    | 0.022                    | 0.007                              | 3.4                             | 6.65                        |
| 55                    | 0.022                    | 0.007                              | 3.4                             | 7.01                        |
| 60                    | 0.022                    | 0.007                              | 3.4                             | 7.3                         |

5 Conclusion

Theoretical studies have established the required power on the cylinder of the pneumatic motor and the cylinder rotation frequency to ensure the process of shockless removal of the milking machine, depending on the radius of the cylinder, the height of the device installation, the initial angle of deflection of the milking machine from the vertical and the weight of the suspension part. With an increase in the value of the initial angle of deflection of the milking machine from the vertical and the height of the suspension of the pneumatic motor, the rotation frequency and power increase. With an increase in the radius of the cylinder, the rotational speed decreases and the power increases. The airflow rate of the pneumatic motor depends on its geometric dimensions, rotor speed, the number of installed blades and the vacuum value of the milking machine. With an increase in the above parameters, the air consumption increases.

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