Masses and Couplings of the Lightest
Higgs Bosons in the (M+1)SSM

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Abstract

We study the upper limits on the mass of the lightest and second lightest CP even Higgs bosons in the (M+1)SSM, the MSSM extended by a gauge singlet. The dominant two loop contributions to the effective potential are included, which reduce the Higgs masses by $\sim 10$ GeV. Since the coupling $R$ of the lightest Higgs scalar to gauge bosons can be small, we study in detail the relations between the masses and couplings of both lightest scalars. We present upper bounds on the mass of a ‘strongly’ coupled Higgs ($R > 1/2$) as a function of lower experimental limits on the mass of a ‘weakly’ coupled Higgs ($R < 1/2$). With the help of these results, the whole parameter space of the model can be covered by Higgs boson searches.

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1 Introduction

Curiously enough, the most model independent prediction of supersymmetric extensions of the Standard Model concerns a 'standard' particle: the mass $m_h$ of the (lightest CP even) Higgs boson. Within the minimal supersymmetric extension of the Standard Model (MSSM) its mass is bounded, at tree level, by

$$m_h^2 \leq M_Z^2 \cos^2 2\beta$$  \hspace{1cm} (1.1)

where $\tan \beta = h_1/h_2$ ($H_1$ couples to up-type quarks in our convention). It has been realized already some time ago that loop corrections weaken this upper bound [1]. These loop corrections depend on the top quark Yukawa coupling $h_t$ and the soft susy breaking parameters as the stop masses of $O(M_{susy})$. At the one loop level, given the present experimental errors on the top mass $m_t$ and assuming $M_{susy} \lesssim 1$ TeV, the upper limit on $m_h$ is $\lesssim 140$ GeV. Also two loop corrections to $m_h$ have been considered in the MSSM [2–4]; these have the tendency to lower the upper bound on $m_h$ by $\sim 10$ GeV.

The subject of the present paper is the next-to-minimal supersymmetric extension of the Standard Model ((M+1)SSM) [5–13] where a gauge singlet superfield $S$ is added to the Higgs sector. It allows to omit the so-called $\mu$ term $\mu H_1 H_2$ in the superpotential of the MSSM, and to replace it by a Yukawa coupling (plus a singlet self coupling):

$$W = \lambda SH_1 H_2 + \kappa S^3 + \ldots$$  \hspace{1cm} (1.2)

The superpotential (1.2) is thus scale invariant, and the electroweak scale appears only through the susy breaking terms.

In view of ongoing Higgs searches at LEP2 [14–16] and, in the near future, at Tevatron Run II [17], it is important to check the model dependence of bounds on the Higgs mass. In the (M+1)SSM, the upper bound on the mass $m_1$ of the lightest CP even Higgs differs from the one of the MSSM already at tree level: now we have [5,6]

$$m_1^2 \leq M_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right)$$  \hspace{1cm} (1.3)

where $g_1$ and $g_2$ denote the $U(1)_Y$ and the $SU(2)_L$ gauge couplings. Note that, for $\lambda < .53$, $m_1$ is still bounded by $M_Z$ at tree level. Large values of $\lambda$,

\footnote{As there are three CP even Higgs states in the (M+1)SSM, we denote them as $S_i$ with masses $m_i$, $i=1..3$, in increasing order.}
\( \lambda > .7 \), are in any case prohibited, if one requires the absence of a Landau singularity for \( \lambda \) below the GUT scale \([3, 4]\).

Loop corrections to \( m_1 \) have also been considered in the (M+1)SSM \([4]\). Given \( m_t \) and assuming again \( M_{\text{susy}} < 1 \) TeV, the upper limit on \( m_1 \) at one loop is then \( \sim 150 \) GeV. Within the constrained (M+1)SSM (the C(M+1)SSM), where universal soft susy breaking terms at the GUT scale are assumed \([5, 6]\), \( \lambda \) is always below \( \sim .3 \), and the upper limit on \( m_1 \) reduces to the one of the MSSM (at one loop) of \( \sim 140 \) GeV. Two loop corrections in the (M+1)SSM have recently been considered in \([13]\).

Within the (M+1)SSM this is, however, not the end of the story: It is well known \([7, 10, 11]\) that now the lightest Higgs scalar \( S_1 \) can be dominantly a gauge singlet state. In this case it decouples from the gauge bosons and becomes invisible in Higgs production processes, and the lightest visible Higgs boson is then actually the second lightest one \( S_2 \). Fortunately, under these circumstances \( S_2 \) cannot be too heavy \([7, 10, 11]\): In the extreme case of a pure singlet lightest Higgs, the mass \( m_2 \) of the next-to-lightest Higgs scalar is again below the upper limit designed originally for \( m_1 \). In general, however, mixed scenarios can be realized, with a weakly coupled (but not pure singlet) lightest Higgs, and a second lightest Higgs above the previous \( m_1 \) limits. Although analyses of the Higgs sector including these scenarios in the (M+1)SSM have been presented before \([14]\) we find that these should be improved: First, experimental errors on the top quark pole mass \( m_t^{\text{pole}} = 173.8 \pm 5.2 \) GeV \([18]\) have been reduced considerably, leading to stronger constraints on the top quark Yukawa coupling \( h_t \) which determines to a large extent the radiative corrections to \( m_{1,2} \). Second, at least the dominant two loop corrections to the effective potential should be taken into account, since they are not necessarily negligible. The purpose of the present paper is thus an analysis of the allowed masses and couplings to the gauge bosons of the lightest CP even Higgs scalars in the (M+1)SSM, including present constraints on \( m_t \) and a two loop improvement of the Higgs potential.

In the next section we present our method of obtaining the dominant two loop terms in the effective potential, and in section \([8]\) we give the resulting upper bound on the lightest Higgs mass. Albeit this upper limit can be obtained analytically, the mass of the second lightest Higgs in relation to the coupling to the gauge bosons requires a numerical analysis. Our methods of scanning the parameter space of the model in two different scenarios (constrained and general (M+1)SSM) are presented in section \([9]\). Results on the Higgs masses and couplings, and conclusions are presented in section \([10]\).
2 Two loop corrections

In order to obtain the correct upper limit on the Higgs boson mass in the presence of soft susy breaking terms, radiative corrections to several terms in the effective action have to be considered. Let us first introduce a scale $Q \sim M_{\text{susy}}$, where $M_{\text{susy}}$ is of the order of the susy breaking terms. Let us assume that quantum corrections involving momenta $p^2 \gtrsim Q^2$ have been evaluated; the resulting effective action $\Gamma_{\text{eff}}(Q)$ is then still of the standard supersymmetric form plus soft susy breaking terms. Assuming correctly normalized kinetic terms (after appropriate rescaling of the fields), the $Q$ dependence of the parameters in $\Gamma_{\text{eff}}(Q)$ is given by the supersymmetric $\beta$ functions (valid up to a possible GUT scale $M_{\text{GUT}}$).

Often one is interested in relating the parameters in $\Gamma_{\text{eff}}(Q)$ to more fundamental parameters at $M_{\text{GUT}}$. To this end one integrates the supersymmetric renormalization group equations between $M_{\text{GUT}}$ and $Q \sim M_{\text{susy}}$ to one or, if one wishes, to two loop accuracy. Note, however, that the limits on the Higgs boson mass depend exclusively on the parameters in $\Gamma_{\text{eff}}(Q)$ at the scale $Q \sim M_{\text{susy}}$; the two loop contributions to the effective potential considered below serve to specify this dependence more precisely. The accuracy to which one has (possibly) related the parameters at the scale $Q \sim M_{\text{susy}}$ to parameters at a scale $M_{\text{GUT}}$ is completely irrelevant for the relation between the Higgs boson mass and the parameters at the scale $Q \sim M_{\text{susy}}$.

One is left with the computation of quantum corrections to $\Gamma_{\text{eff}}$ involving momenta $p^2 \lesssim Q^2$. Subsequently the quantum corrections to the following terms in $\Gamma_{\text{eff}}$ will play a role:

a) Corrections to the kinetic terms of the Higgs bosons. Due to gauge invariance the same quantum corrections contribute to the kinetic energy and to the Higgs-Z boson couplings, which affect the relation between the Higgs vevs and $M_Z$;

b) Corrections to the Higgs-top quark Yukawa coupling;

c) Corrections to the Higgs effective potential. These corrections could, in principle, be decomposed into contributions to the Yukawa couplings $\lambda$ and $\kappa$ of eq. (1.2) and the soft terms (these contributions are the ones proportional to $\ln Q^2$ or, at two loop order, $\ln^2 Q^2$), and ”non-supersymmetric” contributions which are $Q^2$ independent. These latter contributions to the effective potential are of the orders $(\text{vev})^n$ with $n > 4$ and become small in the case of large soft terms compared to the vevs. Our results in section 5 are based on the effective potential including these contributions (which are not necessarily numerically
irrelevant), and there is no need to perform the decomposition of the radiative corrections to the effective potential explicitly.

Let us start with the last item: The Higgs effective potential $V_{\text{eff}}$ can be developed in power of $\bar{h}$ or loops as

$$V_{\text{eff}} = V^{(0)} + V^{(1)} + V^{(2)} + \ldots$$

(2.1)

Within the (M+1)SSM, we are interested in the dependence of $V_{\text{eff}}$ in three CP even scalar vevs $h_1$, $h_2$ and $s$ (assuming no CP violation in the Higgs sector). The tree level potential $V^{(0)}$ is determined by the superpotential (1.2) and the standard soft susy breaking terms [5–11]. For completeness, and in order to fix our conventions, we give here the expression for $V^{(0)}$:

$$V^{(0)} = m^2_{H_1}h_1^2 + m^2_{H_2}h_2^2 + m^2_2s^2 - 2\lambda A_\lambda h_1h_2s + \frac{2}{3}\kappa A_\kappa s^3$$

$$+ \lambda^2 h_1^2h_2^2 + \lambda^2(h_1^2 + h_2^2)s^2 - 2\kappa h_1h_2s^2 + \kappa^2s^4$$

$$+ \frac{g_1^2 + g_2^2}{8}(h_1^2 - h_2^2)^2.$$  

(2.2)

The one loop corrections to the effective potential are given by

$$V^{(1)} = \frac{1}{64\pi^2} \text{STr} M^4 \left[ \ln \left( \frac{M^2}{Q^2} \right) - \frac{3}{2} \right],$$

(2.3)

where we only take top and stop loops into account. The relevant field dependent masses are the top quark mass

$$m_t = h_th_1$$

(2.4)

and the stop mass matrix (in the $(T^c, T^L)$ basis)

$$\begin{pmatrix}
    m_T^2 + m_t^2 & m_t \tilde{A}_t \\
    m_t \tilde{A}_t & m_Q^2 + m_t^2
\end{pmatrix},$$

(2.5)

where $m_T, m_Q$ are the stop soft masses and

$$\tilde{A}_t = A_t - \lambda s \cot \beta$$

(2.6)

is the so-called stop mixing. In eq. (2.5) we have neglected the electroweak D terms which would only give small contributions to the effective potential in the relevant region $m_T, m_Q \gg M_Z$. The masses of the physical eigenstates $\tilde{t}_1, \tilde{t}_2$ then read
\[ m_{t_1t_2}^2 = M_{\text{susy}}^2 + m_t^2 \pm \sqrt{\delta^2 M_{\text{susy}}^4 + m_t^2 A_t^2} \]  
(2.7)

with \( M_{\text{susy}}^2 \equiv \frac{1}{2}(m_Q^2 + m_T^2) \) and \( \delta \equiv \frac{|m_Q^2 - m_T^2|}{|m_Q^2 + m_T^2|} \).  
(2.8)

Note that the top Yukawa coupling \( h_t \) in eq. (2.4) and below is defined at the scale \( Q \), cf. the discussion at the beginning of this section.

In the case of large susy breaking terms compared to the vevs \( h_i \), \( V^{(1)} \) can be expanded in (even) powers of \( h_i \). The terms quadratic in \( h_i \) will not affect the upper bound on the Higgs mass (and can be absorbed into the unknown soft parameters \( m_{H_1}, m_{H_2} \) and \( A_\lambda \) in (2.2)). In the approximation where the stop mass splitting \( \delta \) is small\(^2\), the quartic terms read

\[ V^{(1)} \bigg|_{h_i^4} = \frac{3 h_t^4}{16 \pi^2} h_i^4 \left( \frac{1}{2} \tilde{X}_t + t \right), \]  
(2.9)

where \( t \equiv \ln \left( \frac{M_{\text{susy}}^2 + m_t^2}{m_t^2} \right) \).  
(2.10)

and \( \tilde{X}_t \equiv 2 \frac{A_t^2}{M_{\text{susy}}^2 + m_t^2} \left( 1 - \frac{A_t^2}{12(M_{\text{susy}}^2 + m_t^2)} \right) \).  
(2.11)

In our computations, however, we used the full expression (2.3) for \( V^{(1)} \); we will use the quartic terms (2.9) in the next section only in order to compare our two loop result to those of refs. [4, 13].

Next, we consider the dominant two loop corrections. These will be numerically important only for large susy breaking terms compared to \( h_i \), hence we will expand again in powers of \( h_i \). Since the terms quadratic in \( h_i \) can again be absorbed into the tree level soft terms, we just consider the quartic terms, and here only those which are proportional to large couplings: terms \( \sim \alpha_s h_t^4 \) and \( \sim h_i^6 \). Finally, we are only interested in leading logs (terms quadratic in \( t \)). The corresponding expression for \( V^{(2)} \) can be obtained from the explicit two loop calculation of \( V_{\text{eff}} \) in [3] or, as we have checked explicitly, from the requirement that the complete effective potential has to satisfy the renormalization group equations also at scales \( Q < M_{\text{susy}} \), provided the non-supersymmetric \( \beta \) function for \( h_t \) is used. One obtains in both cases

\(^2\)This approximation is well motivated in the C(M+1)SSM where we take universal soft terms at the GUT scale. On the other hand, we have checked numerically that, in the general (M+1)SSM, the lightest Higgs mass takes its maximal value for \( \delta \sim 0 \).
\[ V_{LL}^{(2)} = 3 \left( \frac{h_t^2}{16\pi^2} \right)^2 h_1^4 \left( 32\pi\alpha_s - \frac{3}{2} h_t^2 \right) t^2. \]  

(2.12)

Now, we turn to the quantum corrections to the Higgs boson kinetic terms. They lead to a wave function renormalization factor \( Z_{H_1} \) in front of the \( D_\mu H_1 D^\mu H_1 \) term with, to order \( h_t^2 \),

\[ Z_{H_1} = 1 + 3 \frac{h_t^2}{16\pi^2} t \]  

(2.13)

Finally, the quantum corrections to the \( H_1 \)-top quark Yukawa coupling \( h_t \) have to be considered. After an appropriate rescaling of the \( H_1 \) and top quark fields in order to render their kinetic terms properly normalized, these quantum corrections lead to an effective coupling \( h_t(m_t) \) with, to orders \( h_t^2, \alpha_s \),

\[ h_t(m_t) = h_t(Q) \left( 1 + \frac{1}{32\pi^2} \left( 32\pi\alpha_s - \frac{9}{2} h_t^2 \right) t \right). \]  

(2.14)

In eqs. (2.13) and (2.14) the large logarithm \( t \) is actually given by \( \ln \left( \frac{Q^2}{m_t^2} \right) \) where \( Q^2 \) acts as a UV cutoff, cf. the discussion at the beginning of this section. In the relevant region \( M_{susy} \gg m_t \) the expression (2.10) for \( t \) can be used here as well. The (running) top quark mass is then given by

\[ m_t(m_t) = h_t(m_t) Z_{H_1}^{1/2} h_1 \]  

(2.15)

and the relation between the pole and running mass, to order \( \alpha_s \), reads

\[ m_t^{pole} = m_t(m_t) \left( 1 + \frac{4\alpha_s}{3\pi} \right). \]  

(2.16)

3 Upper bound on the lightest Higgs mass

In this section we derive an analytic upper bound on the mass of the lightest Higgs scalar. First, we summarize our contributions to the effective potential. As it is already known, in the \( (M+1)SSM \) the upper bound on the lightest Higgs mass \( m_1 \) is saturated when its singlet component vanishes \[4, 11, 14, 13\]. One is then only interested in the \( h_t \)-dependent part of the effective potential. Assuming \( h_t \ll M_{susy} \), i.e. up to \( O(h_t^4) \), one obtains from eqs. (2.2), (2.9) and (2.12)

\[ V_{eff}(h_1, h_2) = \tilde{m}_1^2 h_1^2 + \tilde{m}_2^2 h_2^2 - \tilde{m}_3^2 h_1 h_2 + \frac{g_1^2 + g_2^2}{8} (h_1^2 - h_2^2)^2 \]
\[ + \lambda^2 h_1^2 h_2^2 + \frac{3 h_1^2}{16 \pi^2} h_1^4 \left( \frac{1}{2} \bar{X}_t + t \right) + 3 \left( \frac{h_1^2}{16 \pi^2} \right)^2 h_1^4 \left( 32 \pi \alpha_s - \frac{3}{2} h_1^2 \right) t^2 \]  

(3.1)

with

\[ \tilde{m}_1^2 = m_{H_1}^2 + \lambda^2 s^2 + \text{rad. corrs.}, \]
\[ \tilde{m}_2^2 = m_{H_2}^2 + \lambda^2 s^2 + \text{rad. corrs.}, \]
\[ \tilde{m}_3^2 = 2 \lambda s (A_\lambda + \kappa s) + \text{rad. corrs.}. \]  

(3.2)

The radiative corrections in (3.2) stem from the contributions to \( V^{(1)} \) and \( V^{(2)} \) quadratic in \( h_i \). In the large tan\( \beta \) regime (which saturates the upper bound on the lightest Higgs in the MSSM), one is left with only one non-singlet light Higgs \( h_1 \) and (3.1) simplifies to

\[ V_{\text{eff}}(h_1) = \tilde{m}_1^2 h_1^2 + \tilde{\lambda} h_1^4 \]  

(3.3)

with

\[ \tilde{\lambda} = \frac{g_1^2 + g_2^2}{8} + \frac{3 h_1^2}{16 \pi^2} \left( \frac{1}{2} \bar{X}_t + t \right) + 3 \left( \frac{h_1^2}{16 \pi^2} \right)^2 \left( 32 \pi \alpha_s - \frac{3}{2} h_1^2 \right) t^2. \]  

(3.4)

(Note that in the large tan\( \beta \) regime \( \bar{A}_t = A_t \) and no dependence on the (M+1)SSM coupling \( \lambda \) is left in \( \lambda \).) Now, we can change the variable \( h_1 \) and replace it by a variable \( h'_1 \) in terms of which the kinetic term is properly normalized, so that we have

\[ M_Z^2 = \frac{g_1^2 + g_2^2}{2} h_1^2. \]  

(3.5)

From eq. (2.13) one finds

\[ h_1^2 \approx h_1^2 \left( 1 - \frac{3 h_1^2}{16 \pi^2} t \right). \]  

(3.6)

In terms of \( h'_1 \) the effective potential reads

\[ V_{\text{eff}}(h'_1) = \tilde{m}_1'^2 h'_1^2 + \tilde{\lambda}' h'_1^4 \]  

(3.7)

with

\[ \tilde{m}_1'^2 = m_1^2 \left( 1 - \frac{3 h_1^2}{16 \pi^2} t \right), \quad \tilde{\lambda}' = \tilde{\lambda} \left( 1 - \frac{3 h_1^2}{16 \pi^2} t \right)^2. \]  

(3.8)
Second, recall that $h_t$ in the one loop contribution to eq. (3.1) is given by the Yukawa coupling at the scale $Q$. Hence, we can replace $h_t(\equiv h_t(Q))$ in $\lambda'$ by $h_t(m_t)$ using eq. (2.14), which allows to relate it directly to the running top quark mass. Eq. (2.15) now reads $m_t(m_t) = h_t(m_t)h_1'$. 

From (3.7), one obtains the mass $m_h$ of the lightest non-singlet Higgs in the case where the singlet decouples (and in the large $\tan \beta$ regime)

$$m_h^2 = \frac{1}{2} \left. \frac{d^2 V_{\text{eff}}}{d h_1'^2} \right|_{\text{min}} = 4\bar{\lambda}' h_1'^2\right|_{\text{min}}. \quad (3.9)$$

This is just the correct running Higgs mass, but does not include the pole mass corrections, which involve no large logarithms and which we will neglect throughout this paper. Using (3.3) and expanding $\bar{\lambda}'$ to the appropriate powers of $t$, the expression for $m_h^2$ becomes

$$m_h^2 = M_Z^2 \left(1 - \frac{3h_t^2}{8\pi^2}t \right)$$

$$+ \frac{3h_t^2(m_t)}{4\pi^2} m_t^2(m_t) \left( \frac{1}{2} \bar{X}_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} h_t^2 - 32\pi \alpha_s \right) (\bar{X}_t + t)t \right) \quad (3.10)$$

which agrees with the MSSM result in [4]. (Note, however, that the coefficient of the term $\sim \bar{X}_t t$ on the right hand side of (3.10) is not necessarily correct, since we would obtain terms of the same order if we would take into account simple logarithms in the two loop correction $V^{(2)}$ to the potential.)

The same procedure can be applied for general values of $\tan \beta$. Then, one has to consider the 2x2 mass matrix $\frac{1}{2} \left( \partial_{h_i} \partial_{h_j} V_{\text{eff}} \right), i, j = 1, 2$ where the $h_i$ are properly normalized. Its smallest eigenvalue gives the following upper bound on the mass $m_1$ of the lightest Higgs boson for arbitrary mixings among the 3 states $(h_1, h_2, s)$ (which can be saturated if the lightest Higgs boson has a vanishing singlet component)

$$m_1^2 \leq M_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right) \left(1 - \frac{3h_t^2}{8\pi^2}t \right)$$

$$+ \frac{3h_t^2(m_t)}{4\pi^2} m_t^2(m_t) \sin^2 \beta \left( \frac{1}{2} \bar{X}_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} h_t^2 - 32\pi \alpha_s \right) (\bar{X}_t + t)t \right). \quad (3.11)$$

The only difference between the MSSM bound [4] and (3.11) is the 'tree level' term $\sim \lambda^2 \sin^2 2\beta$. This term is important for moderate values of $\tan \beta$. \footnote{In eq. (3.10) and below in eq. (3.11) we omit the argument of $h_t$ wherever its choice corresponds to a higher order effect.}
Hence, the maximum of the lightest Higgs mass in the (M+1)SSM is not obtained for large \(\tan\beta\) as in the MSSM, but rather for moderate \(\tan\beta\) (as confirmed by our numerical analysis, cf. section 5). On the other hand, the radiative corrections are identical in the (M+1)SSM and in the MSSM. In particular, the linear dependence in \(\tilde{X}_t\) is the same in both models. Hence, from eq. (2.11), the upper bound on \(m_1^2\) is maximized for \(\tilde{X}_t = 6\) (corresponding to \(\tilde{A}_t = \sqrt{6}M_{sushy}\) the 'maximal mixing' case), and minimized for \(\tilde{X}_t = 0\) (corresponding to \(\tilde{A}_t = 0\), the 'no mixing' case).

4 Parametrization of the (M+1)SSM

Eq. (3.11) gives an upper bound on the lightest Higgs mass \(m_1\) regardless of its coupling to the gauge bosons. In the extreme case of a pure singlet lightest Higgs, the next-to-lightest Higgs is non-singlet and the upper bound (3.11) actually applies to \(m_2\). On the other hand, it can occur that the lightest Higgs is weakly coupled to gauge bosons (without being a pure singlet) and \(m_2\) is above the limit (3.11). This case requires a numerical analysis, which will be performed in the next section. First, let us present our methods of scanning the parameter space of the (M+1)SSM.

Not counting the known gauge couplings, the parameters of the model are

\[
\lambda, \kappa, h_t, A_\lambda, A_\kappa, A_t, m_{H_1}^2, m_{H_2}^2, m_S^2, m_Q^2, m_T^2
\]

where \(h_t\) is eventually fixed by the top mass and an overall scale of the dimensionful parameters by the \(Z\) mass. Now, let us see how to handle this high dimensional parameter space in two different scenarios.

4.1 Constrained (M+1)SSM

In the C(M+1)SSM the soft terms are assumed universal at the GUT scale, the global minimum of the effective potential has to be the global minimum and present experimental constraints on the sparticle and Higgs masses are applied. The free parameters can be chosen as the GUT scale dimensionless parameters

\[
\lambda_0, \kappa_0, h_{t0}, \frac{A_0}{M_{1/2}}, \frac{m_0^2}{M_{1/2}^2}
\]

where \(A_0, M_{1/2}\) and \(m_0^2\) are the universal trilinear coupling, gaugino mass and scalar mass respectively. In order to scan the 5-dimensional parameter space of the C(M+1)SSM, we proceed as in refs. [4,5]:

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First, we scan over the GUT scale parameters (4.2) and integrate numerically the renormalization group equations \[5\] down to the susy scale in each case.

Then, we minimize the complete two loop effective potential in order to obtain the Higgs vevs \(h_1, h_2, s\). In principle, we could have followed the same procedure as in section 3 to obtain the dominant two loop corrections, i.e. replacing \(h_1\) by \(h_1'\) and \(h_t\) by \(h_t(m_t)\). However, in order to obtain numerically correct results also in the regime \(M_{\text{susy}} < 1\) TeV, we did not expand \(V^{(1)}\) in powers of \(h_i/M_{\text{susy}}\), i.e. we used the full expression (2.3) for \(V^{(1)}\). Then it becomes inconvenient to perform the field redefinition (3.6), which is implicitly non-linear due to the \(h_1\) dependence of \(t\) via \(m_t\). Therefore we proceed differently: For a given set of low energy parameters, which are implicitly obtained at the scale \(Q \sim M_{\text{susy}}\), we minimize directly:

\[
V_{\text{eff}} = V^{(0)} + V^{(1)} + V^{(2)}
\]

with \(V^{(0)}\) as in (2.2), \(V^{(1)}\) as in (2.3) and \(V^{(2)}\) as in (2.12). Points in the parameter space leading to deeper unphysical minima of the effective potential with \(h_i = 0\) or \(s = 0\) are removed.

The overall scale is then fixed by relating the vevs \(h_i\) to the physical \(Z\) mass through

\[
M_Z^2 = \frac{1}{2} (g_1^2 + g_2^2) (Z_{H_1}^2 h_1^2 + h_2^2)
\]

with \(Z_{H_1}\) as in eq. (2.13). Next, we throw away all points in the parameter space where the top quark mass (including corrections (2.14) to \(h_t\)) does not correspond to the measured \(m_t^{\text{pole}} = 173.8 \pm 5.2\) GeV. We also ask for sfermions with masses \(m_{\tilde{f}} \gtrsim M_Z/2\) and gluinos with masses \(m_{\tilde{g}} \gtrsim 200\) GeV.

Finally, the correct 3x3 Higgs mass matrix is related to the matrix of second derivatives of the Higgs potential at the minimum after dividing \(\frac{1}{2} \partial_{h_i}^2 V_{\text{eff}}\) by \(Z_{H_1}\), and \(\frac{1}{2} \partial_{h_1} \partial_{h_2} V_{\text{eff}}\) and \(\frac{1}{2} \partial_{h_1} \partial_s V_{\text{eff}}\) by \(Z_{H_1}^{1/2}\). For each point in the parameter space, we then obtain the two loop Higgs boson masses and couplings to gauge bosons. Then, we apply present constraints from negative Higgs search at LEP (cf. section 5 for details).

The results in section 5 are based on scannings over \(\sim 10^6\) points in the parameter space. The essential effect of all constraints within the C(M+1)SSM is to further reduce the allowed range for the Yukawa coupling \(\lambda\) to \(\lambda \lesssim 3\).

### 4.2 General (M+1)SSM

In the general (M+1)SSM, we only assume that we are in a local minimum of the effective potential (4.3) and the running Yukawa couplings \(\lambda, \kappa, h_t\) are...
free of Landau singularities below the GUT scale. In order to scan the high dimensional parameter space (1.1) of the general (M+1)SSM we proceed as follows:

First, we use the three minimization equations of the full effective potential (1.3) with respect to $h_1$, $h_2$ and $s$ in order to eliminate the parameters $m^2_{H_1}$, $m^2_{H_2}$ and $m^2_S$ in favour of the three Higgs vevs. Using the relation (1.4), we replace $h_1, h_2$ by $\tan \beta$ and $M_Z$. Finally, eqs. (2.14), (2.15) and (2.16) allow us to express $h_t$ in terms of $m^\text{pole}_t$ and the other parameters.

We are then left with six 'tree level' parameters $\lambda$, $\kappa$, $A_\lambda$, $A_\kappa$, $s$, $\tan \beta$, and three parameters appearing only through the radiative corrections, which we choose as $\tilde{A}_t$, $M_{\text{susy}}$ and $\delta$, as defined in eqs. (2.6) and (2.8).

Requiring that the Yukawa couplings are free of Landau singularities below the GUT scale and using the renormalization group equations of the (M+1)SSM [5], one obtains upper limits on $\lambda, \kappa, h_t$ at the susy scale. The latter turns into a lower bound on $\tan \beta$ depending mainly on $m^\text{pole}_t$ and $M_{\text{susy}}$. As expected from eq. (3.11), we observe that upper limits on Higgs masses are obtained when $\lambda$ is maximal. From the renormalization group equations, one finds that the upper limit on $\lambda$ increases with decreasing $\kappa$, thus we choose $\kappa \sim 0$ and $\lambda = \lambda_{\text{max}} \sim .7$ (which still depends on $h_t$, i.e. on $\tan \beta$).

As already mentionned, one can see from eq. (3.11) that the lightest Higgs mass is maximized for moderate values of $\tan \beta$. Hence, except in fig. 4 where $\tan \beta$ varies, we fix $\tan \beta = 2.7$ which, as we shall see, maximizes the Higgs masses for $m^\text{pole}_t = 173.8$ GeV.

Unless stated otherwise, the upper limits on the Higgs masses presented in the next section are given in the maximal mixing scenario ($\tilde{A}_t = \sqrt{6}M_{\text{susy}}$). We have also found that Higgs masses are maximized for small values of $\delta$ and fixed $\delta = 0$ (thus $m_Q = m_T = M_{\text{susy}}$). In order to obtain the results presented in the next section, we have used numerical routines to maximize the Higgs masses with respect to the remaining three parameters $A_\lambda, A_\kappa, s$.

## 5 Reduced couplings versus mass bounds

Let us start with the mass $m_1$ of the lightest Higgs scalar, independently of its coupling to gauge bosons. The upper limit on $m_1$ in the general (M+1)SSM is plotted in fig. 1 (straight line) as a function of $M_{\text{susy}}$ (for $m^\text{pole}_t = 173.8$ GeV). This limit is well above the one of the MSSM because of the additional tree level contribution to $m^2_1$ proportional to $\lambda^2 M_Z^2$ (cf. eq. (1.3)). At $M_{\text{susy}} = 1$ TeV we have $m_1 \leq 133.5$ GeV (in agreement with the analytic approximation (3.11)); at $M_{\text{susy}} = 3$ TeV this upper limit increases only by $\sim 3$ GeV. This weak dependence on $M_{\text{susy}}$ is due to the
negative two loop contributions to $m_1$.

Within the C(M+1)SSM, the combined constraints on the parameter space require $\lambda$ to be small, $\lambda \lesssim 0.3$ \cite{[4,5]}. Accordingly, the upper limit on $m_1$ is very close to the one of the MSSM. It is shown as crosses in fig. 1, and reaches 120 GeV at $M_{\text{susy}} = 1$ TeV. In the following, we shall assume $M_{\text{susy}} = 1$ TeV.

A sideremark on the behavior for small $M_{\text{susy}}$ is in order: From eq. (2.7), it is obvious that, in the assumed limit $\delta \to 0$, the assumption of maximal stop mixing ($\tilde{A}_t = \sqrt{6}_{\text{susy}}$) cannot be maintained for

$$\frac{\sqrt{6} - \sqrt{2}}{2} m_t < M_{\text{susy}} < \frac{\sqrt{6} + \sqrt{2}}{2} m_t,$$

(5.1)

because it would imply a negative stop mass squared. Therefore, in the general (M+1)SSM, we choose $\tilde{A}_t$ in this regime such that the lightest stop mass squared remains positive. On the other hand, within the C(M+1)SSM, where soft susy breaking terms are related, the limit $M_{\text{susy}}$ small is not feasible since it would contradict the negative results on sparticle searches.

As discussed in the introduction, the upper limit on $m_1$ is not necessarily physically relevant, since the coupling of the lightest Higgs to the $Z$ boson can be very small. Actually, this phenomenon can also appear in the MSSM, if $\sin^2(\beta - \alpha)$ is small. However, the CP odd Higgs boson $A$ is then necessarily light ($m_A \sim m_h < M_Z$ at tree level), and the process $Z \to hA$ can be used to cover this region of the parameter space in the MSSM. In the (M+1)SSM, a small gauge boson coupling of the lightest Higgs $S_1$ is usually related to a large gauge singlet component, in which case no (strongly coupled) light CP odd Higgs boson is available. Hence, Higgs searches in the (M+1)SSM have possibly to rely on the search for the second lightest Higgs scalar $S_2$.

Let us now define $R_i$ as the square of the coupling $ZZS_i$ divided by the corresponding standard model Higgs coupling:

$$R_i = (S_{i1} \sin \beta + S_{i2} \cos \beta)^2$$

(5.2)

where $S_{i1}, S_{i2}$ are the $H_1, H_2$ components of the CP even Higgs boson $S_i$, respectively. Evidently, we have $0 \leq R_i \leq 1$ and unitarity implies

$$\sum_{i=1}^{3} R_i = 1.$$ 

(5.3)

Fortunately, as it was already mentioned, in the extreme case $R_1 \to 0$ the upper limit on $m_2$ is the same as the above upper limit on $m_1$. On the other hand, scenarios with, e.g., $R_1 \sim R_2 \sim 1/2$ are possible. In the following we will discuss these situations in detail.
We are interested in upper limits on the two lightest CP even Higgs bosons $S_{1,2}$. These are obtained in the limit where the third Higgs, $S_3$, is heavy and decouples, i.e. $R_3 \sim 0$ (This is the equivalent of the so called decoupling limit in the MSSM: the upper bound on the lightest Higgs $h$ is saturated when the second Higgs $H$ is heavy and decouples). Hence, we have $R_1 + R_2 \simeq 1$.

In the regime $R_1 \geq 1/2$ experiments will evidently first discover the lightest Higgs (with $m_1 \leq 133.5$ GeV for $M_{s_{\tiny{susy}}} = 1$ TeV). The 'worst case scenario' in this regime corresponds to $m_1 \simeq 133.5$ GeV, $R_1 \simeq 1/2$; the presence of a Higgs boson with these properties has to be excluded in order to test this part of the parameter space of the general (M+1)SSM.

The regime where $R_1 < 1/2$ (and hence $1/2 < R_2 \leq 1$) is more delicate: here the lightest Higgs may escape detection because of its small coupling, and it may be easier to detect the second lightest Higgs. In fig. 2 we show the upper limit on $m_2$ as a function of $R_2$ in the general (M+1)SSM as a thin straight line. For $R_2 \to 1$ (corresponding to $R_1 \to 0$) we obtain the announced result: the upper limit on a Higgs boson with $R \to 1$ is always given by the previous upper limit on $m_1$, even if the corresponding Higgs boson is actually the second lightest one. The same applies, of course, to the C(M+1)SSM where the upper limit on $m_2$ is also indicated as crosses in fig. 2. In the following we will discuss this 'delicate' regime, $R_1 < 1/2$ and $1/2 < R_2 \leq 1$, in some detail:

Fortunately, one finds that the upper limit on $m_2$ is saturated only when the mass $m_1$ of the lightest Higgs boson tends to 0. Clearly, one has to take into account the constraints from Higgs boson searches which apply to reduced couplings $R < 1/2$ – i.e. lower limits on $m_1$ as a function of $R_1 \simeq 1 - R_2$ in order to obtain realistic upper limits on $m_2$ vs $R_2$.

Lower limits on $m_1$ as a function of $R_1$ (in the regime $R_1 < 1/2$) have been obtained at LEP [15]. We use the following analytic approximation for the constraints on $R_1$ vs $m_1$ in this regime:

$$\log_{10} R_1 < \frac{m_1}{45 \text{GeV}} - 2$$ (5.4)

The resulting upper limit on $m_2$ is shown in fig. 2 as a thick straight line. This constraint is automatically included in the C(M+1)SSM results (crosses). Present and future Higgs searches at LEP will lead to more stringent constraints in the regime $1 < R_1 < 1/2$ [15]. We approximate the possible constraints from a run at 198 GeV c.m. energy and 200 pb$^{-1}$ by

$$\ln R_1 < 2 \left( \frac{m_1}{98 \text{GeV}} \right)^4 - 3$$ (5.5)

The resulting upper limit on $m_2$ is shown in fig. 2 as a thick dashed line.
It would be desirable to have the upper limit on $m_2$ in the general (M+1)SSM for arbitrary lower limits on $m_1$ as a function of $R_1$. To this end we have produced fig. 3. The different dotted curves show the upper limit on $m_2$ as a function of $R_2$ for different lower limits on $m_1$ (as indicated on each curve) as a function of $R_1$ (as indicated at the top of fig. 3).

In practice, fig. 3 can be used to obtain upper limits on the mass $m_2$, in the regime $R_1 < 1/2$, for arbitrary experimental lower limits on the mass $m_1$: For each value of the coupling $R_1$, which corresponds to a vertical line in fig. 3, one has to find the point where this vertical line crosses the dotted curve associated to the corresponding experimental lower limit on $m_1$. Joining these points by a curve leads to the upper limit on $m_2$ as a function of $R_2$. We have indicated again the present LEP limit (5.4), already shown in fig. 2, which excludes the shaded region ($m_2 > 172.5$ GeV for $R_2 = .5$, $m_2 > 150$ GeV for $R_2 = .75$, etc). We have also shown again the possible LEP2 constraints on $m_2$ arising from (5.5) as a thick dashed line.

Lower experimental limits on a Higgs boson with $R > 1/2$ restrict the allowed regime for $m_2$ (for $R_2 > 1/2$) in fig. 3 from below. The present lower limits on $m_2$ from LEP are not visible in fig. 3, since we have only shown the range $m_2 > 130$ GeV. Possibly Higgs searches at the Run II of the Tevatron push the lower limits on $m_2$ upwards into this range. This would be necessary if one aims at an exclusion of the 'delicate' regime of the (M+1)SSM: Then, lower limits on the mass $m_2$ – for any value of $R_2$ between 1/2 and 1 – of at least 133.5 GeV are required; the precise experimental lower limits on $m_2$ as a function of $R_2$, which would be needed to this end, will depend on the achieved lower limits on $m_1$ as a function of $R_1$ in the regime $R_1 < 1/2$.

In principle, from eq. (5.3), one could have $R_2 > R_1$ with $R_2$ as small as 1/3. However, in the regime 1/3 < $R_2 < 1/2$, the upper bound on $m_2$ as a function of $R_2$ for different fixed values of $m_1$ can only be saturated if $R_1 = R_2$. Then it is sufficient to look for a Higgs boson with a coupling $1/3 < R < 1/2$ and a mass $m \simeq 133.5$ GeV to cover this region of the parameter space of the (M+1)SSM.

Finally, we consider the dependence of the upper bounds on $m_{1,2}$ on tan$\beta$ and the top quark pole mass. In fig. 4 we plot the upper limit on $m_{1,2}$ (for $R_{1,2} = 1$) against tan$\beta$ for $m_t^{pole} = 173.8$ GeV as a thick straight line. Remarkably, as announced before, this tan$\beta$ dependence is very different from the MSSM: the maximum is assumed for tan$\beta \simeq 2.7$ (with $m_{1,2} \simeq 133.5$ GeV in agreement with figs. 2, 3). The origin of this tan$\beta$ dependence is the tree level contribution $\sim \lambda^2 \sin^2 \beta$ to (3.11). The height and the location of the maximum varies somewhat with $m_t^{pole}$; the thick dashed and dotted curves correspond to $m_t^{pole} = 173.8 \pm 5.2$ GeV, respectively. The absolute maximum
is at $\tan \beta \simeq 3$ with $m_{1,2} \simeq 135$ GeV.

In the 'delicate' regime, where one has to search for the second lightest Higgs with $R_2$ between $1/2$ and 1, one could worry whether the $\tan \beta$ dependence of the upper limit on $m_2$ is different. This is not the case: As a thin straight line we show the upper limit on $m_2$ in the extreme case $R_2 = 1/2$ and $m_t^{pole} = 173.8$ GeV (where the LEP constraint (5.4) is taken into account), which assumes again its maximum for $\tan \beta \simeq 2.7$ (now with $m_2 \simeq 172.5$ GeV in agreement with figs. 2,3). As above, the thin dashed and dotted curves correspond to $m_t^{pole} = 173.8 \pm 5.2$ GeV, respectively, and the absolute maximum is at $\tan \beta \simeq 3$ with $m_2 \simeq 175.5$ GeV. Within the C(M+1)SSM, where $\lambda$ is small, the dependence of the upper limit on $m_2$ on $\tan \beta$ resembles more to the one of the MSSM as shown as crosses in fig. 4.

To conclude, we have studied the CP even Higgs sector of the general (M+1)SSM and the C(M+1)SSM including the dominant two loop corrections to the effective potential. We have emphasized the need to search for Higgs bosons with reduced couplings, which are possible within this model. Our main results are presented in fig. 3, which allows to obtain the constraints on the Higgs sector of the model both from searches for Higgs bosons with weak coupling ($R < 1/2$), and strong coupling ($R > 1/2$). The necessary (but not sufficient) condition for testing the complete parameter space of the (M+1)SSM is to rule out a CP even Higgs boson with a coupling $1/3 < R < 1$ and a mass below 135 GeV. The sufficient condition (i.e. the precise upper bound on $m_2$ vs $R_2$) depends on the achieved lower bound on the mass of a 'weakly' coupled Higgs (with $0 < R < 1/2$) and can be obtained from fig. 3. At the Tevatron this would probably require an integrated luminosity of up to 30 fb$^{-1}$ [17]. If this cannot be achieved, and no Higgs is discovered, we will have to wait for the results of the LHC in order to see whether supersymmetry beyond the MSSM is realized in nature.
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Figure Captions

**Figure 1:** Upper limits on the mass $m_1$ of the lightest CP even Higgs boson versus $M_{susy}$ in the general (M+1)SSM (straight line) and the C(M+1)SSM (crosses).

**Figure 2:** Upper limits on the mass $m_2$ of the second lightest CP even Higgs (in the regime $R_2 > 1/2$) against $R_2$ in the general (M+1)SSM (thin straight line); the general (M+1)SSM with LEP constraints (5.4) (thick straight line); the general (M+1)SSM with expected LEP2 constraints (5.5) (thick dashed line); the C(M+1)SSM with LEP constraints (5.4) (crosses).

**Figure 3:** Upper limits on the mass $m_2$ against $R_2$, for different lower limits on the mass $m_1$ (as indicated on each line in GeV) of the lightest Higgs boson for $1/2 < R_2 < 1$. $R_1 = 1 - R_2$ is shown on the top axis. The boundary of the shaded area corresponds to the thick line in fig. 4; also the dashed line is the same as in fig. 4.

**Figure 4:** Upper limits on $m_{1,2}$ with $R_{1,2} = 1$ (thick lines), and upper limits on $m_2$ with $R_2 = 1/2$ (thin lines) versus tan $\beta$ in the general (M+1)SSM for $m_t^{pole} = 173.8$ GeV (straight), 179 GeV (dashed) and 168.6 GeV (dotted); upper limit on $m_2$ in the C(M+1)SSM (crosses) for $m_t^{pole} = 173.8 \pm 5.2$ GeV. The LEP constraints (5.4) are taken into account in each case.
Figure 1
Figure 2
Figure 4