Semantic-based Data Augmentation for Math Word Problems

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Abstract. It’s hard for neural MWP solvers to deal with tiny local variances. In MWP task, some local changes conserve the original semantic while the others may totally change the underlying logic. Currently, existing datasets for MWP task contain limited samples which are key for neural models to learn to disambiguate different kinds of local variances in questions and solve the questions correctly. In this paper, we propose a set of novel data augmentation approaches to supplement existing datasets with such data that are augmented with different kinds of local variances, and help to improve the generalization ability of current neural models. New samples are generated by knowledge guided entity replacement, and logic guided problem reorganization. The augmentation approaches are ensured to keep the consistency between the new data and their labels. Experimental results have shown the necessity and the effectiveness of our methods.

Keywords: Math word problem · Data augmentation

1 Introduction

Automatically solving Math Word Problem (MWP) has attracted more and more research attention in recent years. The MWP solvers are fed in with a natural language description of a mathematical question, and output a solution equation as the answer. In most cases, these questions are short narratives comprised of several known quantities and a query about an unknown quantity, whose value is the answer we desire. Table 1 shows a typical example of MWP, where \( x \) in the equation refers to the unknown quantity, and is calculated from the known quantities and specific constants such as \( \pi, 1, 2 \).

Numerous efforts have been devoted to solving this challenging task. Early studies relying on hand-crafted features \([14, 18, 19]\) and predefined patterns \([20]\) have limitations in generalization. Deep learning methods have become popular to solve the MWP task in recent years \([22, 24, 28, 32]\) due to their better capability of generalization. \([24]\) first modeled the MWP task as an equation generation task, and various works have followed this framework since then. Recent works in MWP mostly focus on designing complex generation models to capture more features from limited data. For example, \([28]\) proposed a tree-structured decoder...
Table 1: An example of math word problem and new samples generated by semantic-based data augmentation approaches.

| Original | Knowledge guided entity replacement | Logic guided problem reorganization |
|----------|------------------------------------|-----------------------------------|
| Question 1: There are 390 kilograms of pears in the store, which is 40% less than the weight of apples. \( x \) kilograms of apples are there in the store.  
Equation 1: \( x = 390 \div (1 - 40\%) \)  
Answer 1: 650 | Question 2: There are 390 kilograms of bananas in the kitchen, which is 40% less than the weight of watermelon. \( x \) kilograms of watermelon are there in the kitchen.  
Equation 2: \( x = 390 \div (1 - 40\%) \)  
Answer 2: 650 | Question 3: There are 390 kilograms of pears in the store, which is \( x \) less than the weight of apples. 650 kilograms of apples are there in the store.  
Equation 3: \( x = 1 - 390 \div 650 \)  
Answer 3: 0.4 |

...to imitate human behaviour, [32] utilized GCN, and [13] adopted the attention mechanism to better capture the relationship between quantities.

However, current MWP solvers still have weaknesses in terms of robustness and generalization. A superior MWP solver should understand a problem precisely in two ways. First, it is able to generate the same equation for a transformed question with only uninfluential entity replacement. For example, the equation generated for \( Q_1 \) in Table 1 should not be changed when the pears in the question is replaced with bananas. Second, excellent models should be capable of generating a different equation when the logic of the question changes even if the text of the question only changes slightly. For example, in Table 1 the only difference between \( Q_1 \) and \( Q_3 \) is the position of token \( x \). But their underlying logic is totally different and thus the corresponding equations are different. These two kinds of tiny local variances in questions lead to totally different results, one of which conserves the underlying logic, while the other one changes it completely. Humans are able to disambiguate these local variances easily while it’s hard for most neural models to deal with discrete local variances. Previous works on MWP hardly consider this challenge.

The limitation of existing datasets for MWP is a main reason for the above-mentioned weaknesses. Since labeling MWP data is time-consuming, existing MWP datasets are all too small compared to datasets for other natural language processing tasks. Besides, few challenging samples with similar questions but different equations can be found in MWP datasets, which makes it hard for neural models to learn to deal with tiny local variances. The most popular and largest single-equation MWP dataset Math23k contains only 23,161 prob-
lemns, which is rather limited compared to datasets in other field like sQuAD [17]
with 150,000 questions. The weaknesses of existing datasets, especially the lim-
ited coverage of challenging samples, motivated us to augment the dataset with
questions of minor variances leading to heterogeneous equations.

In this paper, we propose a set of semantic-based data augmentation ap-
proaches suitable for MWP task, namely knowledge guided entity replacement
and logic guided problem reorganization. And two kinds of local variances are
provided accordingly. Neural MWP solvers can benefit from our augmentation
strategies in terms of generalization and the ability of dealing with tiny local
variances. Unlike other popular augmentation approaches [25,27,30], which may
cause inconsistency of the questions and equations in MWP task, our augmen-
tation methods are carefully designed for MWP task to ensure consistency.

Knowledge augmentation (knowledge guided entity replacement). As shown in Table 1, pears in the original question is replaced with another fruit bananas. And it is obvious that the replacement does not change the original logic, thus the new question conserves the original label. Our knowledge guided entity replacement method randomly replaces several entities in questions with other entities that belong to the same concept as the original ones. And we guide the replacement with knowledge base that contains much taxonomy knowledge.

Logical augmentation (logic guided problem reorganization). As the example shown in Table 1, quantities 390 and 40% are known while quantity 650 is unknown in Q1. In the generated Q3, we let 40% be the unknown one given 390 and 650. The equation is changed accordingly to keep consistency. x in Eq 3 is substituted for 650 and 40% is replaced with x. Afterwards, we transform the new equation to its equivalent equation of the form x = 1 − 390 ÷ 650. To enrich the problem types of the training data, our logical augmentation iteratively set the known quantities in the original question and equation to the unknown. And the new equation is further transformed to its equivalent equation with mathematical properties for normalization.

It is worth mentioning that previous works only learn the equation of the unknown, while our augmentation method helps the neural models to make full use of the limited data by learning all the possible equations of the quantities in the question.

Our contributions are summarized as follows:

– To the best of our knowledge, this is the first systematical study of data augmentation for MWP task. And is easy to be applied to any neural models and extend to other math-related tasks.

– Our methods can generate coherent questions with consistent labels, which largely diversify both textual descriptions and equation templates. It also brings in massive challenging samples that existing datasets lack.
Experimental results show the necessity and effectiveness of our methods. The performances of the additional evaluation also indicate that our methods largely enhances the generalization ability of neural models.

2 Related Work

2.1 Math Word Problem

Early works mostly utilized statistical methods or predefined rules. [18] utilized hand-crafted features to predict the lowest common ancestor operator for each quantity pair. [19] proposed a unit dependency graph based on [18], [14] predefined a group of logic forms and converted the math question into them. [24] first proposed to utilize a seq2seq model with recurrent neural network to generate equation template sequence. [21] proposed equation normalization to unify duplicated representations of equivalent expressions. [3] utilized a seq2seq model with the help of stack to align the semantic with the operator. [22] proposed a two-stage algorithm to predict a template tree. [13] applied group attention mechanism to extract more features.

[15, 28, 32] replaced the recurrent neural network based decoder with a tree-structured decoder and achieved satisfactory results. [31] leveraged the framework of knowledge distillation.

However, none of the works have tried to enhance the generalization ability of the neural models by data augmentation strategies.

2.2 Data Augmentation for Natural Language Processing (NLP)

We categorize data augmentation approaches for NLP into two types. The first type changes only the text while the second one changes both the text and labels. Some research adds random noise to input data [29] or hidden states [12] making the models less sensitive to small perturbations. [29] systematically examined some basic augmentation methods including random synonyms replacement, word insertion, etc. [27] utilized tf-idf to help determine which words to replace. [23] adopted k-nearest neighbors to find synonyms in word embedding space. The noise brought in by these methods could be tolerated in some tasks but not in MWP task due to the strict requirement of preciseness.

[9, 26] fine tuned a pre-trained language model with text and label to generate new sentences given specific labels. [4] replaced words in the source and target sentences with rare words. Other approaches mainly applied generative models like VAE [7], seq2seq model [6], GPT [1], etc., to generate new text given a specific label. All of the methods above generating new text given a specific label are inappropriate to MWP task, because the equation is a sequence and is not enumerable.
3 Methodology

3.1 Problem Statement

The MWP dataset contains training data $D_{\text{train}}$ and testing data $D_{\text{test}}$, both of which are comprised of numerous questions $Q$ and equations $Eq$. Neural models take $Q$ as the input and generate the $Eq$ sequence. In previous works, the neural model $\mathcal{M}$ is trained with the training data $D_{\text{train}}$ and tested with the testing data $D_{\text{test}}$. Due to the limitations of $D_{\text{train}}$ introduced in Sec. 1, we generate new samples $D_{\text{aug}}$ from $D_{\text{train}}$ by our augmentation approaches. And the neural model $\mathcal{M}$ is trained with both original and augmented data $D_{\text{train}} \cup D_{\text{aug}}$.

The question $Q$ consists of a sequence of tokens $W = \{w_i\}_{i=1}^{W}$ including known quantities $N = \{n_i\}_{i=1}^{N}$ and entities $E = \{e_i\}_{i=1}^{E}$. Since quantities are fairly sparse, we replace quantities in $Q$ with symbol $n_i$ according to the occurrence order of the quantities during preprocessing phase. Quantities in $Eq$ are replaced accordingly and we denote the answer of $Eq$ as $\hat{n}$.

The target of MWP is to generate the $Eq$ sequence which is composed of $N \cup O \cup C$. Among them, $O$ is the set of the operators (such as $\times$) and $C$ is the constant set.

3.2 Knowledge Augmentation

The category information of entities is introduced to generate new questions. Given a sample $(Q, Eq)$, we randomly choose $\theta$ entities mentioned in $Q$ to be replaced with other entities belonging to the same concept as the original ones. If an entity $e_i$ belonging to concept $c$ is selected, the alternative entities are $e_j \in E_c (i \neq j)$. Similar to [25], we set $\theta = \max(1, \alpha l)$, among which $l$ refers to the length of $Q$ and $\alpha$ is a hyper-parameter used to manage the replacement ratio. More entities are replaced for longer questions considering they tolerate noise better. We notice that an entity may appear more than once in a question, and all of them should be replaced if the entity is selected. For each question $Q$, the new questions generated by this means are denoted as $Q_K$. As an example, entities Ming Zhang and apples are replaced with a random entity of person and fruit accordingly. Since entities are less informative for MWP inference, the generated questions conserve the logic of the original question. And the new equation matched with $Q_K$ is still $Eq$.

| $Q$ | Ming Zhang ([PER]) bought $n_0$ apples ([FRU]) |
|-----|-----------------------------------------------|
| $Q_K$ | Hong Li ([PER]) bought $n_0$ blueberries ([FRU]). |

Recognize entities. There are two kinds of entities in MWP, namely real-world entities and named entities, and we take different strategies to recognize them. For detecting named entities like fake person names Alice, Bob, etc.
Table 3: An example of the question generated with logical augmentation.

|   |   |
|---|---|
| $Q$ | There are $n_1$ kilograms of pears in the store, which is $n_2$ less than the weight of apples. How many kilograms of apples are there in the store? |
| $Q'$ | There are $n_1$ kilograms of pears in the store, which is $n_2$ less than the weight of apples. $x$ kilograms of apples are there in the store. |
| $Q_L$ | There are $n_1$ kilograms of pears in the store, which is $x$ less than the weight of apples. $\hat{n}$ kilograms of apples are there in the store. |

which are common in MWP, there are numerous models and tools work very well on this task. And we follow the method of [2] to recognize named entities. Another kind of entities is real-world entities like apple, car, etc. These entities can be easily recognized by referring to KGs. In this paper, we use WordNet [16] to guide our knowledge augmentation. Entities are linked to the WordNet syn-sets, and the direct hypernyms are viewed as their concepts. Besides, we restrict that only physical entities in questions could be replaced to avoid possible semantic shifting. Abstract entities like time should not be replaced in our case.

3.3 Logical Augmentation

Given a sample $(Q, Eq)$, several $(Q_L, Eq_L)$ pairs are generated by setting a known quantity in the original question to the unknown one in the new question. And the new equation is normalized with mathematical properties. Since the textual description of $Q_L$ is similar to that of $Q$, neural models tend to generate similar equation sequences for them even though their ground truth solutions are totally different. With logical augmentation, neural models are forced to learn the different equations from the slight variations of input text, which to some extent enhances the inference ability of neural models.

**Question generation** In MWP, the letter $x$ is usually assigned to represent the unknown. The value of it is the answer to the question $Q$ (denoted as $\hat{n}$). The unknown quantities in questions are usually indicated by question words such as how many. Considering both question words and letter $x$ refer to the same unknown, we replace the question words in $Q$ with the letter $x$. And then the original question could be viewed as an assertive sentence about the logical relationship between the known and unknown quantities $\mathcal{N} \cup \hat{n}$. Previous training without augmentation only learns how to get $\hat{n}$ given $\mathcal{N}$, for $Eq$ is in $x = f(\mathcal{N} \cup \mathcal{C})$ form, in which $x$ refers to $\hat{n}$ as $\hat{n}$ is the unknown of $Q$. However, the logical relationship between other quantity pairs remains unlearned. To make full use of each data sample, we make the neural models additionally learn how to get $n_i \in \mathcal{N}$ given the other quantities $n_j \in \mathcal{N} \cup \hat{n}(i \neq j)$. As the letter $x$ indicates the quantity we desire, quantities in $Q$ are iteratively set to $x$. The generated questions are denoted as $Q_L$.

Table 3 shows an example of questions generated with logical augmentation. Notably, when training with augmented data, original questions $Q$ are replaced with $Q'$ to ensure that all questions are expressed in an uniform form.
Fig. 1: The process of generating a new equation $Eq_L$. As illustrated in Table 3, the new unknown quantity of $Q_L$ and $Eq_L$ is $n_2$ with $n_1$ and $\hat{n}$ known. The equation tree in the initialization step is built from $Eq$ in Table 3. Equation trees from step 2 to step 4 are equivalent equations of $Eq_L$ in different forms. And the tree in the termination step is the normalized form of $Eq_L$.

Equation generation with equation tree conversion Since the known and unknown have been changed in the generated question $Q_L$, the corresponding equation $Eq_L$ should be changed accordingly to keep consistency. To normalize the form of equations, the generated equations are transformed to their equivalent equations of a specific form based on mathematical properties with the help of equation trees. In the final normalized state, the term $x$ is isolated on the left side of the equation.

Fig. 1 shows an example of how a consistent and normalized equation $Eq_L$ is generated from the original equation $Eq$.

Equation tree Equation tree is built from an equation whose root node is always the equal sign =. The left and right sub-trees of the root are expression trees of the left and right sides of an equation, thus the leaf nodes are operands and the inner nodes (except the root) are binary operators.

In the normalized state of an equation tree, the left sub-tree of the root only contains a leaf node $x$. The same equation tree in different states refers to equivalent equations of different forms. To normalize the equation, we transform the equation tree to its normalized state with two main actions based on the addition-subtraction property, the division property, and the multiplication property. 1) Move an operator node $o$ (an inner node) and its non-$x$ sub-tree (the sub-tree of $o$ that does not contain $x$) to another side. 2) Switch $o$ to its inverse operator, for example, switch $+$ to $-$ or $\times$ to $\div$.

The process of generating $Eq_L$ As shown in Fig 1 there are four steps to generate $Eq_L$. 1) Initialization: a normalized equation tree is built from the original $Eq$. 2) Swap: $x$ is substituted for $\hat{n}$, and the quantity $n_1$ which is the new unknown is replaced with $x_L$ (to distinguish with $x$). Since the new equation...
tree of $EQL$ is not in normalized state, we take the available actions to transform the new equation tree to its normalized state. 3) Recursively Move: All of the operator nodes and their non-$x$ sub-trees are recursively moved to the left of the root node in a top-down manner, leaving only $x_L$ in the right. Different actions are chosen in different situations, all based on the natural mathematical properties. 4) Termination: Simply swapping the left and right sub-trees of the root node will make the equation tree normalized. And $EQL$ could be restored from the equation tree.

4 Experiment

In this section, we conduct experiments to measure the scale of challenging samples in existing datasets. Besides, we evaluate our semantic-based augmentation strategies (denoted as s.based aug. for simplification) with three typical neural MWP solvers to show the improvements of the generalization ability brought in by s.based aug.. Extensive ablation studies are also conducted to verify the effectiveness of each augmentation strategy.

4.1 Dataset Analysis

Existing MWP datasets such as AI2 [5], SingleEQ [10], and AllArith [19] only have hundreds of samples. While relatively large-scale MWP datasets such as MAWPS [11] contains very few challenging samples as it is constructed with low lexical and template overlap. So we conduct our experiments on the largest and most popular MWP dataset Math23k [24], a Chinese MWP dataset with 23,161 pairs of $(Q, Eq)$.

To make the neural models able to deal with discrete tiny local variances, it’s necessary for the dataset to contain a great ratio of challenging samples that have similar questions but different equations. In this section, we will analyze Math23k from the amount and the quality of challenging samples in the training and the testing set.

Notably, the analysis in this section are all based on equation templates, which means only the structures of the equations are considered rather than the actual values. For example, equations $x = 3 + 2 + 1$ and $x = 5 + 4 + 2$ are viewed as the same in template.

The amount of challenging samples in the training set For a question $Q_i \in D_{train}$, if $\exists Q_j \in D_{train}(i \neq j)$ with different equation but similar text, $Q_i$ and $Q_j$ are considered as challenging samples. The similarity of two questions are measured with three similarity scores, namely BLEU, ROUGE-L, and normalized reverse edit distance defined below (referred to as ED-DIST).

$$ED - DIST(Q_i, Q_j) = 1 - \frac{edit - dist(Q_i, Q_j)}{max(l_{Q_i}, l_{Q_j})}$$
Fig. 2: (a) the amount of challenging samples filtered with different similarity score thresholds in the training set. (b) the quality of existing challenging samples.

Table 4: Examples of challenging samples.

| Challenging samples in $D_{\text{train}}$ | $Q_i$ | $Q_j$ |
|-----------------------------------------|-------|-------|
| The divisor is 8 and the quotient is 2, how about the dividend? | The dividend is 24 and the divisor is 3, how about the quotient? |
| 3 times a number is 300, this number is equal to ? | A number is 7 times 21, this number is equal to ? |
| Number A is equal to 150, and number B is 20% more than A. B = ? | Number A is 10.78, and number B is 3 more than B. B = ? |

| Challenging samples in $D_{\text{test}}$ | In a parking lot, totally 48 cars and motorcycles are parked. Each car has 4 wheels, and each motorcycle has 3 wheels. If there are 20 motorcycles in the parking lot, how many wheels are there in total? | In a parking lot, totally 48 cars and motorcycles are parked. Each car has 4 wheels, and 172 wheels are there in total. If there are 20 motorcycles in the parking lot, how many wheels does a motorcycle have? |

$l$ refers to the length of the question and $\text{edit} - \text{dist}(\cdot)$ is the Levenshtein distance of the given pair of questions. The more similar $Q_i$ and $Q_j$ are, the closer $\text{ED} - \text{DIST}(Q_i, Q_j)$ is to 1. Afterwards, we count the amount of challenging samples with the similarity thresholds set to different values as shown in Fig. 2(a). The amount of samples in Fig. 2(a) is normalized with respect to the size of the training set $|D_{\text{train}}|$. It’s obvious that the amount of challenging samples in Math23k is rather limited as no more than 15% of the questions meet the condition when the threshold score is 0.9.

The quality of the challenging samples High-quality MWP questions are supposed to be longer sentences with rich background descriptions. Thus we analyze the average length and words diversity of challenging samples in $D_{\text{train}}$. The threshold of similarity scores is set to 0.9 to filter the challenging samples.
In Fig. 2(b), $Q$ refers to all questions in $D_{train}$, while $Q_{\text{metric}}$ refers to the challenging samples filtered by certain metrics. e.g. $Q_{\text{BLEU}}$ are challenging questions that have another similar question with BLEU score higher than 0.9. As shown in Fig. 2(b), the challenging samples filtered by all metrics are much shorter than the average question length, and so is the diversity of words. Table 4 shows some examples of existing challenging samples in $D_{train}$. The experimental results indicate that most of the challenging samples in $D_{train}$ are too short and lack background descriptions.

Since both the amount and quality of existing challenging samples are not quite satisfactory, we propose knowledge and logical augmentation to generate questions with tiny local variances, which brings in massive high-quality challenging samples.

**Testing set analysis** A good testing set requires low lexical and template overlap between the testing and training set [10]. Besides, as the ability to disambiguate tiny local variances in questions largely reflects the generalization ability of neural models, the testing set is supposed to have more challenging samples to better evaluate the ability of neural models. As shown in Table 5, we count the number of equation templates that only appeared in the $D_{test}$ (New eq. template in Table 5) to measure the template overlap between the testing and training set. Meanwhile, the textual similarity between questions in the testing set and the training set is calculated with ED-DIST as introduced above. Specifically, for each $Q_{te}$ in the testing set, we calculate a similarity score with $s = \max\{ED - DIST(Q_{te}, Q_{tr}), \forall Q_{tr} \in D_{train}\}$. The challenging samples in the testing set are counted as Sec. 4.1 with the threshold set to 0.9. Results in Table 5 are normalized with respect to the size of the testing set.

As shown in Table 5, the original testing set shares a high template (low new eq. templates) and lexical (high q. similarity) overlap with the training set, and contains limited challenging samples (low num. challenging samples). It indicates that the testing set is to some extent similar to the training set, making it hard to evaluate the actual inference ability of neural models perfectly. Not to mention the ability of neural models dealing with challenging samples, the limited scope of challenging samples indicates $D_{test}$ is not able to evaluate neural models from this aspect.

Considering the weaknesses mentioned above, we manually labeled an additional testing set $D_{test}^*$ with 380 samples in total which contains a great deal of high-quality challenging samples (Table 4 shows a pair of example challenging sample in $D_{test}^*$). Besides, the additional testing set $D_{test}^*$ holds lower lexical and template overlap with $D_{train}$, and also contains more challenging samples as shown in Table 5.

### 4.2 Experimental Setup

**Dataset** We conduct our experiments on Math23k [24]. Besides training with the whole training set $D_{train}$, to evaluate the performance of our augmentation
Table 5: Comparison of the two testing sets.

| New eq. template | Mean q. similarity | Num. challenging samples | BLEU | ROUGE-L | ED-DIST |
|------------------|-------------------|--------------------------|------|---------|---------|
| $D_{test}$       | 0.0401            | 0.094                    | 0.0211 | 0.0361  | 0.0201  |
| $D'_{test}$      | 0.155             | 0.593                    | 0.574 | 0.282   | 0.0421  |

approaches on different data sizes, we randomly picked three training sets of sizes 500, 2k, and 20k (the whole training set) from the $D_{train}$ as what have done in [25]. The neural models are evaluated on both original testing set $D_{test}$ and the manually labeled $D'_{test}$.

**Neural model $\mathcal{M}$** We evaluate s.based aug. strategies with three most typical MWP neural models.

- **Vanilla seq2seq** (marked as seq2seq). In this paper, we adopt a seq2seq model as [24] whose encoder is a 2-layer BiGRU and the decoder is a 2-layer LSTM. Besides, to help the model learn the slight variation of our augmented data, we utilize the attention mechanism before the feed-forward network.

$$
\alpha_i = \frac{e^V \tanh(W_1 h_{n_i} + W_2 h_t + b)}{\sum_{j=1}^{\left|N\right|} e^V \tanh(W_1 h_{n_j} + W_2 h_t + b)}
$$

$$
h_N = \sum_{i=1}^{\left|N\right|} \alpha_i h_{n_i}
$$

$$
P(y_t|y_0, ..., y_{t-1}) = f(\hat{h}_t \bigoplus h_N)
$$

$V, W_1, W_2, b$ are all parameters, and $h_{n_i}$ is the encoder hidden vector of the quantity $n_i$, while $h_t$ refers to the decoder hidden vector of the time step $t$.

- **GTS** [28] is a tree-based neural model which has outperformed previous works significantly.

- **Graph2Tree** [32] is the state-of-the-art neural model for MWP.

**Baseline models for comparison** We compare the performances of the three neural models trained with s.based aug. strategies with an extensive set of related work. **Math-EN** [21] proposed equation normalization based on a vanilla seq2seq model to effectively reduce the target space. **TRNN** [22] applied a seq2seq model to predict a tree-structured template in a bottom-up manner. **GROUP-ATT** [13] proposed a group attention mechanism to extract intra-relation features. **AST-Dec** [15] proposed to generate abstract syntax tree of the equation in a top-down manner.

**Baseline augmentation approach** We also compare the specifically designed s.based aug. with one of the most popular text augmentation strategy back
Table 6: **Answer accuracy (%)** of neural models evaluated on the two testing sets. We evaluate s.based aug. on three most typical neural models. The improved performances w/ augmentation are shown in bold.

| Model       | $D_{test}$ | $D_{test}^*$ | All samples | Challenging samples |
|-------------|------------|--------------|-------------|---------------------|
|             | w/o aug.   | w/ s.based aug. | w/ back trans. | w/ s.based aug. |
| Math-EN     | 66.7       | -             | -           | -                   |
| TRNN        | 66.9       | -             | -           | -                   |
| AST-Dec     | 69.0       | -             | -           | -                   |
| GROUP-ATT   | 69.5       | -             | -           | -                   |
| seq2seq     | 66.1       | **71.2** ↑5.1 | **66.4** ↑0.3 | 30.0 53.2 ↑23.2 11.4 27.3 ↑15.9 |
| GTS         | 75.6       | **76.1** ↑0.5 | 75.5 ↓0.1 33.2 53.2 ↑20.0 11.4 36.4 ↑25.0 |
| Graph2Tree  | 77.4       | 77.0 ↓0.4     | 76.9 ↓0.5 30.3 52.4 ↑22.1 11.4 **40.9** ↑29.5 |

translation [30], which is used to generate paraphrase of original sentence. The original sentence is first translated into a pivot language, and back to its original language. Other augmentation approaches such as synonym replacement [25], generation-based methods [7], etc. are omitted here due to the intolerable noise for mathematical scenario.

**Implementation details** The baseline models are trained with $D_{train}$ while the augmentation models are trained with $D_{train} \cup D_{aug}$. All models are evaluated on the same testing set $D_{test}$. To verify the ability of dealing with challenging samples, the three neural models trained with s.based aug. are additionally evaluated on $D_{test}^*$. Our evaluation metric is answer accuracy, which is calculated by comparing the answer of the predicted equation and the ground truth one rather than comparing the equation sequence. The parameters of the vanilla seq2seq model are set as [24]: The hidden units of both the encoder and decoder are 512. The word embedding dimension is set to 50 and the dropout for GRU and LSTM are set to 0.5. The number of epochs and mini-batch size are 80 and 32 respectively. We adopt an early stop policy after the accuracy of the validation set not increasing for 10 epochs. Adam optimizer [8] is used with learning rate set to 0.001, $\beta_1 = 0.9$ and $\beta_2 = 0.999$, and the learning rate is halved every 10 epochs. Settings of GTS and Graph2Tree are the same as that in [28] and [32].

**4.3 Results**

**Comparison results** Table [3] shows the answer accuracy of neural models trained with s.based aug. strategies compared with back translation and various baseline models. It’s obvious that specifically designed s.based.aug. performs much better than back translation on MWP task. Since MWP has a strict requirement of precision, the noises brought in by back translation are sometimes unacceptable. We notice that many of the generated questions by the means of
Table 7: The ablation study on datasets with different sizes. The best performance on each dataset is shown in bold.

| Model           | Training Set Size | 500 | 2k  | 20k |
|-----------------|-------------------|-----|-----|-----|
| seq2seq         |                   | 9.52| 31.6| 66.1|
| seq2seq + knowledge |             | 12.3| ↑2.78| 33.5| ↑1.90| 67.0| ↑0.90|
| seq2seq + logic  |                   | 13.0| ↑3.48| 34.1| ↑2.50| 67.3| ↑1.20|
| seq2seq + s.based aug. |          | 16.7| ↑7.18| 36.0| ↑4.40| 71.2| ↑5.10|

back translation are inconsistent with their equations. Trained with s.based aug., the accuracy of the seq2seq model increases by 5.1%, which is able to beat many other complex neural models with 71.2% answer accuracy. Performance gains of simple neural networks like seq2seq are much more than that of complex neural models like GTS and Graph2Tree. Our explanation is that complex models have better inference ability, making them able to learn more useful features from limited data, while simpler models have to learn these features from more diverse data. However, as analysed in Sec. 4.1, considering the original testing set $D_{test}$ holds a high overlap with the training set, we reasonably suspect that the performance decrease is caused by the complex models likely to be overfitting and learns some dataset-specific features before augmentation. And the massive challenging samples brought in by our augmentation strategies may confuse the neural models, since none of them have specially designed to handle the tiny local variances. We’ll further verify our hypothesis in Sec. 4.4.

As described in Sec. 4.1, the manually labeled testing set $D^*_{test}$ holds lower overlap with the training set and contains 380 samples with a significant ratio of challenging samples. The three neural models trained w/ and w/o s.based aug. are tested on $D^*_{test}$ to further evaluate the generalization ability of them. As shown in Table 6, we calculate the answer accuracy for all the questions and the challenging samples respectively. The challenging samples are filtered as 4.1 with the threshold set to 0.9. For the accuracy of the challenging samples, a challenging sample $Q_i$ is viewed to be correctly answered only if all its similar questions $Q_j$ has been correctly answered.

As shown in Table 6, s.based aug. strategies significantly boost the performances of all the three neural models with more than 20% increments. Besides, the ability of neural models dealing with discrete tiny local variances has also been largely improved. The results suggest that the s.based aug. strategies successfully benefit the generalization ability of the neural models. However, since existing MWP neural models hardly considered the discrete local variances which lead to respectively low accuracy, there is still a large space for future work to improve the ability of neural models dealing with such challenges.
### Table 8: Examples of error cases that are predicted wrongly after augmentation.

| $Q_{te} \in D_{train}$ | $Q_{te} \in D_{test}$ | pred. Eq of $Q_{te}$ | tgt. Eq of $Q_{te}$ |
|-------------------------|------------------------|----------------------|---------------------|
| A dictionary is priced at $n_1$ yuan, and after $n_2$ of the sale, the price is still $n_3$ higher than the purchase price. The purchase price of this dictionary is ? | A dictionary is priced at $n_1$ yuan, and after $n_2$ of the sale, it will earn $n_3$. The purchase price of this dictionary is ? | $x = n_1 \times (1+n_3) \div n_2$ | $x = n_1 \times n_2 \div (1+n_3)$ |
| A project can be completed in $n_1$ days if $n_2$ people come to do it. If $n_3$ people do it, how many days can it be done? | $n_1$ workers will complete a project within $n_2$ days. If it takes $n_3$ days to complete, how many workers are needed? | $x = 1 \div (n_1 \times n_2) \div n_3$ | $x = (n_1 \times n_2) \div n_3$ |
| The original staff of a certain agency is $n_1$, and there are currently $n_2$ staff. How many percent of the staff are reduced compared to the original staff? | The original staff of a certain agency is $n_1$, and there are currently $n_2$. How many percent does it reduce? | $x = n_2 \div n_1$ | $x = n_1 - n_2 \div n_1$ |

**Ablation study** As illustrated in Table 7, both knowledge and logic guided augmentation methods contribute to the performance gains. And logical augmentation performs better than knowledge augmentation on all three datasets. This gap is more obvious on smaller datasets. We guess that on smaller datasets, the lack of problem types is more severe, thus enriching the equation templates is more needed. Besides, the results have shown that smaller datasets benefit more from the augmentation strategies than larger ones.

### 4.4 Case study

In this section, we’ll analyze the questions that are predicted correctly before the neural models trained with augmented data but are predicted wrongly afterwards. For each error case, we search for a most similar question in $D_{train}$ to see whether these problems have occurred during training phase. And it turns out that most of the error cases have a nearly the same question in $D_{train}$ as shown in Table 8. The $Q_{te} \in D_{test}$ are error predicted questions in the testing set. $Q_{tr} \in D_{train}$ are their similar questions found in the training set. In the first row of Table 8, the target Eq of the $Q_{tr}$ is $x = n_1 \times n_2 \div (1+n_3)$. According to the logical augmentation described in Sec. 3.3, one of the $E_{L}$ could be $(n \times (1+n_3)) \div n_2$, which is equivalent to the mistakenly predicted equation in template. This result further support our hypothesis in Sec. 4.3 and explain the performance decreases of complex models on $D_{test}$.
5 Conclusion & Discussion

In this paper, we argue that discrete tiny local variances are a big challenge for neural models which previous works have ignored. And we propose a set of novel semantic-based data augmentation methods to supplement existing datasets with challenging samples. Both augmentation methods we proposed are able to generate coherent questions with consistent labels and largely diversify both textual descriptions and equation templates. Extensive experimental results have shown the necessity and effectiveness of the approaches we proposed. Besides, the idea we proposed could also be transferred to other math-related tasks like MWP generation.

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