Abstract—With the recent booming of open-source packages for scientific computing, power system simulation is being revisited to reduce the programming efforts for modeling and analysis. Existing open-source tools require manual efforts to develop code for numerical equations and sparse Jacobians. Such work would become repeated, tedious and error-prone when a researcher needs to implement complex models with a large number of equations. This paper proposes a two-layer hybrid library consisted of a symbolic layer for descriptive modeling and a numeric layer for vector-based numerical computation. The open-source library allows to implement differential-algebraic equation (DAE)-based models with descriptive equation strings, which will be transparently generated into robust and fast numerical simulation code and high-quality documentation. Thus, complex models and systems can be easily prototyped and simulated. These two layers are decoupled so that the symbolic layer is case-independent, and existing numerical programs can be reused. Implementation details in indexing, equation evaluation, and Jacobian evaluation are discussed. Case studies present a) implementation of turbine governor model TGOV1, b) the power flow calculation for MATPOWER test systems, c) the validation against commercial software using Kundur's two-area system with GENROU, EXDC2 and TGOV1 models, and d) the full eigenvalue analysis for Kundur's system.

Index Terms—Simulation, open-source, hybrid library

I. INTRODUCTION

POWER system modeling and transient simulation is a widely studied yet challenging topic. Digital computer-based simulation has been dominating in the industry and academia. Both closed-source tools [1] and open-source tools [2]–[8] are being widely used. Although simulation software comes with a set of built-in models, users will likely need to customize models for new devices or control algorithms.

To develop new models for a simulation software is to implement the model equations for interacting with the predefined software architecture. There are two main approaches that user-defined models (UDMs) can be implemented: programmatically or through a graphical user interface (GUI) [9], [10], which is unavailable in open-source tools due to complexity and lack of return. Still, open-source tools are crucial for scientific research, but they require programming proficiency to develop new models on top of a deep understanding of the tool [11].

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Two advanced UDM solutions exist in open-source tools: the cards in Dome [4] and the Function Mockup Unit (FMU) support in GridDyn [7]. Cards are plain-text files containing model descriptions in the card protocol. Using a symbolic library under the hood, cards can be used to generate code templates, which can be modified into final models. Although cards are quite flexible, they do not live with the simulation code and manual tweaks are often required. On the other hand, FMU is compiled directly from Modelica, an equation-based modeling language. FMU has excellent speed as compiled library and can be interfaced through the Functional Mockup Interface (FMI) for model exchange and co-simulation. Modelica libraries such as OpenIPSL [12] have been developed for power system simulation. While FMU is widely adopted in other industries, it does not suit the power system data structure in which devices are modeled separated and instantiated in multiple. As a solution, device combinations have to be enumerated and precompiled, causing significant overhead.

This work proposes a hybrid symbolic-numeric library aiming to reduce the efforts for modeling differential-algebraic equation (DAE) in power systems while maintaining numerical performance. In a general-purpose environment, the library allows writing DAE models in a descriptive manner as part of the library. This library has two layers: symbolic and numeric. The symbolic layer can generate Jacobian matrices, vector-based numerical code, and build documentations tailored for power system. The numeric layer can orchestrate function calls that can be used by numerical routines.

It is worth mentioning the prior works on symbolic modeling and discussing the advancements. Decades ago, symbolic approaches to power flow modeling [13], optimization [14], [15], and device transients modeling [16]–[18] were introduced. The pioneering works well proved the concept but exposed a remaining issue: scalability. As reported in [17], the symbolic modeling is instance-based, case-specific, and solved in the symbolic environment. For each specific instance of the same model, a new set of symbols and equations need to be created each time, causing verbosity for large systems and eventually slowing down the simulation. The proposed library preserves simplicity and modularity for modeling and performance for simulation by allowing developing models in individual and generating vectorial numerical programs.

This paper is organized as follows. Section II discusses the design philosophy and provides an overview of the work. Section III and Section IV explain the implementation techniques for the symbolic and numeric layers with an example. Section V presents case studies of the proposed library implemented in ANDES, including implementation of...
TGOV1 model, power flow computation for MATPOWER [19] test systems, time-domain simulation verification against DSATools TSAT, and full eigenvalue analysis results. Finally, Section VI concludes the proposed work.

II. DESIGN PHILOSOPHY AND LIBRARY OVERVIEW

The design philosophy of the symbolic library is to make modeling as simple as describing equations and make simulation as fast as crafted code. Simplifying DAE modeling renders the library easy to use and modify for research and education. Maintaining a fast simulation speed makes the library capable of running large-scale studies.

Clearly, relying solely on the symbolic library cannot overcome the scalability issue, and a purely numerical approach cannot reduce the programming efforts. Therefore, a hybrid approach is proposed to take advantage of both symbolic and numeric approaches in one library. Different from Dome cards, symbolically defined models are coded as part of the library and distributed along with the program. The same code will be used for simulation and documentation to achieve consistency between description and simulation.

Since symbolically declared models should work for any input systems, the symbolic layer can be separated from the numeric layer. The symbolic layer handles all symbolic operations until the numerical code is generated. It is test-system independent and can be executed before the numeric layer. To avoid the overhead, generated functions are serialized to a file and reloaded before numerical simulations.

The numeric layer of the library provides orchestration functions to numerical functions defined in models and collect values into system-level matrices. At this layer, cases are loaded and variable addresses are populated so that vectorial operations can be performed. While this procedure is similar to existing numerical tools, special handling is required to work around the limitations of the generated code. For example, to implement a limiter, one can easily write an if-statement in numerical code. The hybrid library, however, has to provide a helper class implementing an element-wise comparer. The numeric layer handles the call to helper classes and ensures values are updated correctly. Nevertheless, the hybrid library allows to skip the symbolic layer and write models directly in numerical functions when necessary. More details will be discussed in Section IV.

A two-layer approach is taken to simplify the modeling and maintain the simulation speed. Fig. 1 illustrates the layer organization of the proposed hybrid symbolic-numeric library. The symbolic layer provides a declarative environment for formulating the model directly from the mathematical form. Developers can define model classes in both descriptive and numeric ways, although the former approach is recommended for simplicity and robustness. Symbolically described DAE models will be processed to derive symbolic equations and Jacobians, which, altogether, will be generated into loadable numerical functions for fast numerical simulation. The process to generate numerical functions is fully transparent, and previously generated code can be rapidly reloaded. The numeric layer organizes the numerical code for addressing, initialization, and equation evaluation. Methods exported from the numeric layer can be used to develop specific numerical routines. In Figure 1, equation generation, Jacobian derivation, and code generation employ an underlying symbolic engine [20], and the rest are developed in the proposed library.

III. SYMBOLIC LAYER IMPLEMENTATION

This section describes the implementation of the symbolic layer for the proposed library. The symbolic layer covers class-based declarative modeling, symbolic processing, code generation, and automated documentation. Methods discussed in this section are exemplified in the Python language with the SymPy library but can be extended to other environments.

A. MODEL CLASS DECLARATION

Model classes serve as containers for elements that describes the device models. Each element instance is a single sub-container for metadata such as variable name, description, unit, and equation strings. Element instances are class data attributes and can be rapidly loaded since they only contain basic data structure for lightweight metadata.

Three element types (parameters, variables, and services) are implemented as classes with the following functionalities:

1) Parameters are typically numerical and externally supplied data for defining specific devices.
2) Variables, either differential or algebraic, are values to be solved in the DAE system. Each variable is automatically associated with an equation.
3) Services are temporary values for simplifying expressions or fulfilling supplementary actions.

The proposed implementation is explained with a constant shunt capacitor model for power flow. Equations for the model is given as follows:

\[ p_h = -g v_h^2 \]
\[ q_h = b v_h^2 \]

(1)
where \( h \) is the connected bus index, \( v \) is the bus voltage, \( p \) and \( q \) are the power injections, and \( g \) and \( b \) are the conductance and susceptance, respectively. An implementation for the Shunt model is given in the following code snippet with the following annotations:

1) Lines 3-5 declares \texttt{bus}, \texttt{g}, and \texttt{b} parameters for bus index, shunt conductance, and susceptance value, respectively.
2) Lines 6-9 declares external algebraic variables \texttt{a} and \texttt{v} for voltage phase and magnitude, respectively, of the buses having indices of \texttt{bus}.
3) Lines 7 and Line 9, respectively, declares the active power load \((-v^2b)\) on the power balance equations associated with \texttt{a} and \texttt{v}.
4) The \\texttt{e_str} equation strings contain \texttt{a}, \texttt{v}, \texttt{b} and \texttt{g} strings, which are declared data attributes of the class.
5) The sequence of declarations does not matter as long as it follows the language syntax.

The library also provides containers for discontinuous elements such as limiters and dead-band. Each discontinuous element exports binary variables for indicating the discontinuity status. These binary status variables, along with parameters and variables, can be used in descriptive equations. Details will be provided in the TGOV1 example in Section V.

Clearly, the efforts to develop DAE models have been significantly reduced in the proposed library. All that is required is to set up correct element containers and describe the equations as written in the mathematical form.

\begin{lstlisting}[language=Python] class Shunt(Model):
  def __init__(self):
    self.bus = IdxParam(info='bus_index')
    self.g = NumParam(info='conductance', unit='pu')
    self.b = NumParam(info='susceptance', unit='pu')
    self.a = ExtAlgeb(model='Bus', indexer=self.bus,
                      src='a', e_str='v**2*b/sqrt(2)*g')
    self.v = ExtAlgeb(model='Bus', indexer=self.bus,
                      src='v', e_str='-v**2*b/sqrt(2)*g')
\end{lstlisting}

Listing 1. Shunt model for power flow

Next, the symbolic library is utilized to generate the symbolic expressions and Jacobian matrices. The library performs the following essential steps:

1) Prepare all variable symbols into a vector \( xy \) in the declaration order so that each variable has a stable index.
2) Convert each equation string to a symbolic expression (using \texttt{sympy.simplify}).
3) Group differential and algebraic expressions into two vectors, \( f \) and \( g \), respectively in the declaration order.
4) Derive the expression vectors with respect to the ordered variable vector to obtain Jacobian matrices \( \frac{df}{dx} \) and \( \frac{dg}{dx} \) (using \texttt{sympy.Matrix.jacobian}).
5) Convert the Jacobian matrices to sparse to obtain non-zero triplets \((\texttt{row}, \texttt{column}, \texttt{value})\), where \texttt{row} is the index of the equation in the equation array, \texttt{column} the variable index, and \texttt{value} the derivative expression.

The symbolic processing tasks have the following performance characteristics. They are executed over each model, and thus the processing time scales linearly to the number of models. Each model only has a few to tens of equations, thus the processing time is fast. The processing is done in the symbolic layer before loading any system and thus is independent of system size.

The symbolic processing for \texttt{Shunt} is illustrated in Equations 2 to 4. The Jacobian derivation and triplet conversion shown in Equation 4 are automated with the symbolic library.

\[
xy = [a, v]
\]

\[
g = [v^2g, -v^2b]
\]

\[
\frac{dg}{dxy} = \begin{bmatrix} 0 & 2vg \\ 0 & -2vb \end{bmatrix} \rightarrow \begin{bmatrix} (0, 1, 2vg) \\ (1, 1, -2vb) \end{bmatrix}
\]

\[\text{dense} \rightarrow \text{sparse (row, column, value)}\]

C. Symbolic-to-Numeric Code Generation

The main task for the code generation to generate and store numerical function calls, which are are executable anonymous functions that return the value of expressions. The code generation feature of symbolic libraries are utilized through the following steps:

1) Generate numerical functions for each initialization, differential and algebraic equations, and each element in Jacobian Matrices (using \texttt{sympy.lambdify}).
2) For each Jacobian matrix, store the equation index \texttt{row}, variable index \texttt{column}, and the anonymous function for \texttt{value} correspondingly in lists.

It is important to note that \texttt{row} and \texttt{column} are local to each model and only depends on the number of declared variables. The following remarks are relevant:

1) In terms of performance, the generated numerical functions use the efficient NumPy library for vectorial computation and thus runs as fast as manually crafted code.
2) The overhead for symbolic processing and code generation can be eliminated by reusing the generated code through efficient serialization and de-serialization.

3) The library also takes manually written numerical function calls, as long as indices are provided and functions have the same signature as the generated code. This can be helpful to reuse existing numerical code.

At this point, executable numerical code is obtained from the symbolically defined class models.

D. Documentation

Code documentation is important for disseminating open-source research but is often under-emphasized. The situation is understandable because maintaining documentation can take as much as, if not more, efforts for development. All the existing power system simulators rely on manual efforts to document the implemented models.

The proposed library is able to automatically document the implemented equations for DAE models developed using declarative classes. Human-friendly equations can be generated from symbolic expressions by substituting in LNX-declarative classes. Human-friendly equations can be generated from symbolic expressions by substituting in LNX-declarative classes. The documentation feature completes the symbolic layer to ensure the same models are used for simulation and documentation. To the best knowledge of the authors, the proposed library is the first in power system tools capable of generating equation documentation directly from source code. For interested readers, the documentation is available online [21], and the automated model documentation is under Section “Model References”.

IV. NUMERIC LAYER IMPLEMENTATION

The main purpose of the numeric layer is to organize the generated numerical code for setting initial values, updating equations and Jacobian matrices. The organized numerical methods can be invoked in the desired sequence to implement a particular numerical routine. Methods discussed in this section are applicable to all numerical power system simulators.

A. Data Structure and Vectorial Storage

The implementation of the numeric layer relies on arrays and matrices to properly store data associated with declared elements. Storing the numerical values to the corresponding element instances is easy to understand and straightforward to implement. Depending on the type, an element instance may contain member attributes for addresses, values and equation values. Table I shows the correspondence between types and stored numerical attributes. Each of the address, value, and equation value attributes is stored as an array with its length equal to the number of devices of that model. For example, if a particular system contains three Shunt devices, attributes \( b \) and \( g \) will each contain a value array \( v \) with a length of three.

The data flow paths for the numerical attributes are outlined in Fig. 2. Parameter values are set after loading the data file and converting it to per unit under the system base. Service values are computed after parameters are set. Variable addresses are allocated after loading the test system, values are set by initialization calls, and equation values are updated by equation calls.

B. Variable Initialization

The purpose of variable initialization is to provide initial numerical values before a routine starts. It includes setting the starting point for power flow routine and initializing the rest of variables for dynamic routines. Although power flow initialization is simple, there could be value conflicts depending on the input data format. For example, default initial bus voltages are set by buses and overwritten by PV-generators. The library uses an additional flag to indicate if the values from one model overwrite the shared variables at the end.

Variable initialization for dynamics is mathematically a root-finding problem for the DAE system with all derivatives zeroed out. Two approaches can be used: sequential or iterative. Variables with an explicit solution can be initialized sequentially, while those without must be solved iteratively.

The library provides three entry points for initialization. First, an explicit-form equation can be specified for each variable if it can be initialized sequentially. A common technique is to set the initialization equation to a service that calculates from other services. Second, an optional, implicit-form equation with an initial value can be specified for each variable. All implicit equations will be gathered and solved iteratively from the given initial value. Third, a placeholder function is available if one decides to write numerical code. For best practice, sequential initialization should be used whenever possible. To ensure convergence, the initial values for the iterative initializers need to be carefully selected.

C. Numerical Equation Evaluation

After loading a test case and counting the total number of variables, four global numerical arrays are created for variables and equations (for both differential and algebraic). Each variable in a model receives an array of addresses indexing into the global array. The addresses can be used to access the corresponding numerical values of variables and equations.

To update the equation array, one approach is to call the update functions in each model and modify the global arrays.
in place. This approach is straightforward but has the following issues:

1) Parallelizing the function calls across models is infeasible because the global equation arrays are shared.
2) The numerical code from subsection III-C only returns the value for each expression. Additional handling is needed to load values into the global array.

Instead, a model-based local evaluation approach is proposed. The approach first evaluates all numerical equations within each model and then collect values into global arrays. It does not alter the DAE formulation but modifies the procedure for DAE construction. The approach has four steps and is illustrated in Fig. 3

1) Copy values from global arrays into model variables.
2) Call generated numerical functions with local values as inputs and store the outputs locally. Theoretically, this step can be parallelized since data are local to models.
3) Update equation values for equation-dependent limiters like anti-windup limiters.
4) Update the global equation values by summing up the equation values at the same address.

Note that step 3 is needed for equation-dependent limiters. Anti-windup limiters, for example, need the equation values to update the limiter status. Step 3 updates the limiter status variables and sets the differential equation values to zero for binding anti-windup limiters.

D. Incremental Jacobian Building

Building Jacobian matrices involve steps to fill in sparse Jacobian matrices incrementally and efficiently. It is especially relevant for implicit numerical integration routine since Jacobian update takes up the most overhead. This subsection discusses how the Jacobian indexing is done with the local variable indices (from the symbolic layer) and variable addresses (assigned in the numeric layer).

It is worth noting the difference between the local variable indices and the assigned variable addresses. A local variable index is a scalar number based on the sequence of declaration and is independent of test cases. Variable addresses is an array assigned after loading a specific test case. Local indices are used to look up corresponding addresses in order to determine the positions of the values.

For a generic triplet \((\text{row}, \text{column}, \text{value}(\ast\text{args}))\) where \text{row} and \text{columns} are two scalars of the local indices, and \text{value}(\ast\text{args}) is the numerical function for the Jacobian value with \text{args} being a list of local values. Recall that \text{value} is the derivative of the \text{row}-th equation with respect to the \text{column}-th variable. Jacobian values, which have the same length as the \text{row} and \text{column} addresses should be summed at the positions defined by the case-specific addresses for the \text{row}-th and the \text{column}-th variables.

Fig. 4 illustrates the process with three \textit{Shunt} devices as an example. From the symbolic layer, there are two Jacobian triplets to be placed at local indices \((0, 1)\) and \((1, 1)\). In the numeric layer, the zeroth variable \(a\) is assigned addresses \([0, 1, 2]\) and the first variable \(v\) is assigned address \([5, 6, 7]\). Evaluate the numerical function \(2vg\) to obtain the Jacobian elements, for example, \([0.002, 0.002, 0.002]\). Next, these elements will be summed up at positions with the row number equal to the addresses of \(a\) \(\{(0, 1, 2)\}\) and the column number equal to the address of \(v\) \(\{5, 6, 7\}\). Repeat the process until all elements from all models are added.

For performance consideration, the library implements a two-step process that builds the sparsity pattern and fills in the values. It is well known that building sparse matrices incrementally can be time-consuming if repeated memory allocation is needed. Using the addresses of the non-zero elements, zero-filled sparse matrices can be constructed. The memory for the non-zero elements is allocated, and in-place modifications can happen. This technique is especially relevant for high-level languages without direct memory access.

V. CASE STUDIES

For verification and demonstration, this section presents a model implementation, power flow calculation, time-domain simulation, and eigenvalue analysis. The implementation of turbine governor model TG0V1 is demonstrated with source code developed in the proposed library. Next, power flow
results are reported and the time breakdown is analyzed. Finally, time-domain simulation is verified against DSATools TSAT, and eigenvalue analysis results are shown. This library is implemented in a custom package called ANDES [22]. All ANDES simulations are performed in CPython 3.7.6 with SymPy 1.5, NumPy 1.16.5, and CVXOPT 1.2.3 on Intel i9-9980H running macOS 10.15.3. TSAT simulations are performed with TSAT 19.0 on Windows Server 2012 R2.

A. Example Model: TGOV1

The TGOV1 turbine governor model [23] (shown in Fig. 5) is used as a practical example for modeling in the proposed library. This model is composed of a lead-lag transfer function and a first-order lag transfer function with an anti-windup limiter, which are sufficiently complex for demonstration. The corresponding differential equations and algebraic equations are given in (5) and (6).

\[
\begin{align*}
\dot{x}_{LG} &= \frac{z_{LG}^*}{(x_{LG} - x_{LL})/T_3} \\
\dot{x}_{LL} &= \frac{1 - \omega}{R \times \tau_{m0} - P_{ref}} \\
0 &= \begin{bmatrix}
1 - \omega - \omega_d \\
R \times \tau_{m0} - P_{ref} \\
(P_{ref} + \omega_d)/R - P_d \\
D \omega_d + y_{LL} - P_{OUT} \\
u (P_{OUT} - \tau_{m0})
\end{bmatrix}
\end{align*}
\]

(5)

\[
\begin{align*}
0 &= \frac{P_d - x_{LG}}{T_1} \\
0 &= \frac{2}{T_3} (x_{LG} - x_{LL}) + x_{LL} - y_{LL} \\
u &= \frac{y_{LL}}{P_{OUT} - \tau_{m0}}
\end{align*}
\]

(6)

where LG and LL denote the lag block and the lead-lag block, \( \dot{x}_{LG} \) and \( \dot{x}_{LL} \) are the internal states, \( y_{LL} \) is the lead output, \( \omega \) the generator speed, \( \omega_d \) the generator under-speed, \( P_d \) the droop output, \( \tau_{m0} \) the steady-state torque input, and \( P_{OUT} \) the turbine output that will be summed at the generator.

It is important to note on the binary flag \( z_{LG}^* \) indicating the anti-windup limiter status. The flag, along with other two flags, \( z_{LG}^* \) and \( z_{LG}^* \), are updated every time after differential equations are updated, based on the conditions given in (7).

Next, the equation will reset and variable values will be pegged using the flags. The Jacobians, however, will not need to be manipulated since the elements already contain the flag.

\[
\begin{align*}
\text{if } x_{LG} > V_{MAX} \text{ and } \dot{x} > 0 & \Rightarrow \dot{z}_{LG}^* = 1 \\
\text{if } x_{LG} < V_{MIN} \text{ and } \dot{x} < 0 & \Rightarrow \dot{z}_{LG}^* = 1 \\
\text{otherwise: } & \dot{z}_{LG}^* = 0, \text{ and } \dot{z}_{LG}^* = 1
\end{align*}
\]

(7)

An implementation of the TGOV1 model is given in Listing 2. The implementation consists of four types of declarations: parameters, external variables, external initial values, and internal variables and equations. Parameters are declared with special properties for data consistency and per-unit conversion. For example, Line 4 specifies that the droop parameter \( R \) must be non-zero and is an inverse-of-power quantity in the device base MVA. External variable \( \omega \) is retrieved for calculation and \( \tau_{m0} \) for power feedback to generators. Note that the equation associated with \( \tau_{m0} \) replaces the steady-state constant torque \( \tau_{m0} \) with the turbine output \( P_{OUT} \).

The initial value of the mechanical torque is retrieved for variable initialization. Finally, differential and algebraic variables are declared, followed by the mathematical equations in (5)–(6) written in a descriptive format, making it convenient to understand and troubleshoot.
### B. Power Flow Calculation

The proposed library is utilized to implement power flow models as the first proof of concept. Models for bus, PQ, PV, transmission line and shunt are developed, and a full Newton-Raphson algorithm is implemented using the direct linear equation solver UMFPACK shipped with CVXOPT. Unlike conventional power flow packages, the symbolically implemented line model does not implement an admittance matrix, although it is feasible to do so numerically. Instead, vectorial computation of power injections into the connected buses are used to maintain generality across models.

Power flow benchmarks are performed using seven test systems from MATPOWER 7.0. Using the same settings and start points, ANDES is able to solve the cases listed in Table II and obtain identical results to that from MATPOWER.

It is interesting to analyze the time breakdown. Updating the numerical equations and solving the linear equations is relatively fast and takes up less than 30% of the time. Most of the computation time is consumed for building Jacobian matrices. The Jacobian time, however, can be reduced by implementing a dishonest algorithm to avoid updating Jacobians at every iteration step. Lastly, the profiler to obtain the time breakdown slows down the computation by about 5% and is included in the reported time.

### C. Time-Domain Numerical Integration

To validate the numerical simulation results, Kundur’s two area system with four generators is simulated in ANDES and the commercial DSATools TSAT. DAE models in the test systems include four GENROU models, each with an EXDC2 exciter and a TGOV1 turbine governor. Parameters of the system are listed in the Appendix.

This case study compares the generator speed, mechanical power inputs, terminal voltages, and excitation voltages following a line trip event. For the time-domain simulation, the trapezoidal method is used with a fixed step size of 1/120 second. All PQ loads are converted to constant impedances after power flow calculation. At \( t = 2 \)s, one of the two lines between Bus 8 and Bus 9 is disconnected. Simulation results are depicted in Fig. 6, Fig. 7, Fig. 8. Clearly, the proposed hybrid symbolic-numeric library achieves almost the same time-domain simulation results, and the proposed methodology is thus validated.

#### D. Eigenvalue Analysis

Finally, the numerical routine for eigenvalue analysis is developed by reusing existing eigenvalue programs. Eigenvalues are obtained for the state matrix after the time-domain initialization. The eigenvalues of Kundur’s two-area system presented in the last section are plotted in Fig. 10, which...
Fig. 9. Excitation voltages of generators on Buses 1 and 3

Fig. 10. Relevant eigenvalues in the S-domain for Kundur’s system

also shows two dotted lines for the loci with 5% damping. From the calculation, the system has a pair of eigenvalues \( \lambda = -0.1918 \pm 4.2247 \) with 4.54% of damping at 0.67 Hz.

VI. CONCLUSIONS

In conclusion, this paper presents a hybrid symbolic-numeric library for DAE-based power system modeling and numerical simulation. This paper presented the design philosophy for a two-layer library that brings together the advantages of both symbolic and numeric approaches. The symbolic layer is case-independent and handles descriptive modeling, symbolic processing, code generation, and automated documentation. The numeric layer organizes the generated code for symbolic and numeric approaches. The symbolic layer of the library is demonstrated with a TGOV1 turbine governor model. The library is verified for power flow calculation against MATPOWER, and the computation time is analyzed. It is also verified for time-domain simulation using a Kundur’s two area system with three dynamic models. Numerical simulation from this library obtains almost the same result as from DSATools TSAT.

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APPENDIX

[Model Parameters for Kundur’s Two Area System]

**TABLE A1: Bus Data**

| idx | Vn | v0  | a0  | area |
|-----|----|-----|-----|------|
| 0   | 1  | 20  | 1.000 | 0.570 | 1 |
| 1   | 2  | 20  | 0.998 | 0.369 | 1 |
| 2   | 3  | 20  | 0.963 | 0.185 | 2 |
| 3   | 4  | 20  | 0.817 | 0.462 | 2 |
| 4   | 5  | 230 | 0.979 | 0.480 | 1 |
| 5   | 6  | 230 | 0.958 | 0.284 | 1 |
| 6   | 7  | 230 | 0.936 | 0.127 | 1 |
| 7   | 8  | 230 | 0.879 | -0.081 | 2 |
| 8   | 9  | 230 | 0.891 | 0.094 | 2 |
| 9   | 10 | 230 | 0.830 | 0.337 | 2 |

**TABLE A2: Line Data**

| idx | bus1  | bus2  | r     | x     | b     | tap | phi |
|-----|-------|-------|-------|-------|-------|-----|-----|
| 0   | Line_0 | 5     | 6     | 0.005 | 0.050 | 0.075 | 1   | 0   |
| 1   | Line_1 | 5     | 6     | 0.005 | 0.050 | 0.075 | 1   | 0   |
| 2   | Line_2 | 6     | 7     | 0.002 | 0.020 | 0.030 | 1   | 0   |
| 3   | Line_3 | 6     | 7     | 0.002 | 0.020 | 0.030 | 1   | 0   |
| 4   | Line_4 | 7     | 8     | 0.022 | 0.220 | 0.330 | 1   | 0   |
| 5   | Line_5 | 7     | 8     | 0.022 | 0.220 | 0.330 | 1   | 0   |
| 6   | Line_6 | 7     | 8     | 0.022 | 0.220 | 0.330 | 1   | 0   |
| 7   | Line_7 | 8     | 9     | 0.002 | 0.020 | 0.030 | 1   | 0   |
| 8   | Line_8 | 8     | 9     | 0.002 | 0.020 | 0.030 | 1   | 0   |
| 9   | Line_9 | 9     | 10    | 0.005 | 0.050 | 0.075 | 1   | 0   |
| 10  | Line_10| 9     | 10    | 0.005 | 0.050 | 0.075 | 1   | 0   |
| 11  | Line_11| 1     | 5     | 0.001 | 0.012 | 0.000 | 1   | 0   |
| 12  | Line_12| 2     | 6     | 0.001 | 0.012 | 0.000 | 1   | 0   |
| 13  | Line_13| 3     | 9     | 0.001 | 0.012 | 0.000 | 1   | 0   |
| 14  | Line_14| 4     | 10    | 0.001 | 0.012 | 0.000 | 1   | 0   |

**TABLE A3: PQ Data**

| idx | bus | p0  | q0  |
|-----|-----|-----|-----|
| 0   | PQ_0 | 7   | 11.59 | -0.735 |
| 1   | PQ_1 | 8   | 15.75 | -0.899 |

**TABLE A4: PV Data**

| idx | bus | p0  | q0  | v0  | ra | xs |
|-----|-----|-----|-----|-----|----|----|
| 0   | 2   | 2   | 7   | 3.0 | 1  | 0  | 0.25 |
| Continued on next page |
### TABLE A4: PV Data

| idx | bus | p0 | q0 | v0 | ra | xs |
|-----|-----|----|----|----|----|----|
| 1   | 3   | 3  | 7  | 5.5| 1  | 0  | 0.25 |
| 2   | 4   | 4  | 7  | -1.0| 1  | 0  | 0.25 |

### TABLE A5: Slack Data

| idx | bus | p0 | q0 | v0 | ra | xs | a0 |
|-----|-----|----|----|----|----|----|----|
| 0   | 1   | 1  | 7.459| 1.436| 1  | 0  | 0.25 | 0.57 |

### TABLE A6: GENROU Data

| idx | bus | gen | D  | M  | xl | xq | xd | xd1 | xd2 | xq1 | xq2 | Td10 | Td20 | Tq10 | Tq20 |
|-----|-----|-----|----|----|----|----|----|-----|-----|-----|-----|------|------|------|------|
| 0   | 1   | 1   | 0  | 13.00| 0.06| 1.7 | 1.8 | 0.3  | 0.25 | 0.55| 0.25 | 8    | 0.03 | 0.4  | 0.05 |
| 1   | 2   | 2   | 0  | 13.00| 0.06| 1.7 | 1.8 | 0.3  | 0.25 | 0.55| 0.25 | 8    | 0.03 | 0.4  | 0.05 |
| 2   | 3   | 3   | 0  | 12.35| 0.06| 1.7 | 1.8 | 0.3  | 0.25 | 0.55| 0.25 | 8    | 0.03 | 0.4  | 0.05 |
| 3   | 4   | 4   | 0  | 12.35| 0.06| 1.7 | 1.8 | 0.3  | 0.25 | 0.55| 0.25 | 8    | 0.03 | 0.4  | 0.05 |

### TABLE A7: EXDC2 Data

| idx | syn | TR  | TA  | TC  | TB  | TE  | TF1 | KF1 | KA  | KE  | VRMAX | VRMIN |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|-------|
| 0   | 1   | 0.02| 0.02| 1   | 1   | 0.83| 1.246| 0.075| 20  | 1   | 5.2   | -4.16 |
| 1   | 2   | 0.02| 0.02| 1   | 1   | 0.83| 1.246| 0.075| 20  | 1   | 5.2   | -4.16 |
| 2   | 3   | 0.02| 0.02| 1   | 1   | 0.83| 1.246| 0.075| 20  | 1   | 5.2   | -4.16 |
| 3   | 4   | 0.02| 0.02| 1   | 1   | 0.83| 1.246| 0.075| 20  | 1   | 5.2   | -4.16 |

### TABLE A8: TGOV1 Data

| idx | syn | R   | VMAX | VMIN | T1  | T2  | T3  | Dt |
|-----|-----|-----|------|------|-----|-----|-----|----|
| 0   | 1   | 1   | 0.05 | 33   | 0.4 | 0.49| 2.1 | 7  | 0  |
| 1   | 2   | 2   | 0.05 | 33   | 0.4 | 0.49| 2.1 | 7  | 0  |
| 2   | 3   | 3   | 0.05 | 33   | 0.4 | 0.49| 2.1 | 7  | 0  |
| 3   | 4   | 4   | 0.05 | 33   | 0.4 | 0.49| 2.1 | 7  | 0  |