Effect of ideal flow conditions on springback

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Abstract. All sheet forming processes incorporate some bending. Various aspects of this process affect its design. One of the important aspects is springback. The present paper concerns with the effect of ideal flow conditions on springback after bending under tension. In general, this process is not an ideal flow process. An ideal flow path is produced if two inequalities are satisfied. A measure of springback is calculated for this case.

1. Introduction

The ideal flow theory is used as the basis of procedures for the direct preliminary design of metal forming processes. This theory has been fully developed for the rigid perfectly plastic constitutive equations comprising the Tresca yield criterion and its associated flow rule [1]. A comprehensive overview of the ideal flow theory has been provided in [2]. Recently, the existence of stationary planar ideal flow solutions has been proven for models of anisotropic and pressure – dependent plasticity [3, 4]. General methods for calculating planar ideal flows have been developed in [5 – 7]. Elasticity has been introduced into the theory in [8]. All sheet forming processes incorporate some bending [9]. Various aspects of this process affect its design. One of the important aspects is the geometric change of a part after forming (i.e. when the forces from forming toll are removed). It is known that springback is sensitive to various material and process parameters [10]. It is therefore of interest to predict springback after an ideal flow process. In general, the process of bending is not an ideal flow process. However, an appropriate combination of the bending moment and tensile (or compressive) force can make this process to be ideal. In the case of rigid plastic models this has been demonstrated in [11]. In order to evaluate the effect of ideal flow conditions on springback, it is necessary to extend this solution to elastic/plastic models. In the present paper, an elastic/plastic model of incompressible material is adopted. Using such a model a general solution for the process of pure plane strain bending has been given in [12]. Then, this solution has been extended to the process of bending under tension in [13] where quite a general model of strain hardening material has been adopted. This solution is used in the present paper to derive an ideal flow solution for the process of bending under tension.

2. General structure of the solution

Let us introduce two coordinate systems; namely, an Eulerian Cartesian coordinate system \((x, y)\) and a Lagrangian coordinate system \((\zeta, \eta)\). The transformation equations are...
Here $H$ is the initial thickness of the sheet and $a$ is a time-like parameter. This parameter monotonically increases with the time and vanishes at the initial instant. Also, $s$ is a function of $a$ and this function should be found from the solution. It has been shown in [12] that $x = H\zeta$ and $y = H\eta$ at the initial instant if

$$s = \frac{1}{4}$$

at $a = 0$. It has been shown in [13] that the transformation equations in (1) describe the solenoidal deformation of initial rectangular $A_1B_1C_1D_1$ into cross-section $ABCD$ where $AB$ and $CD$ are circular arcs (Fig. 1) satisfying the boundary conditions for the process of bending under tension. Since $H$ is the initial thickness of the sheet, $x = -H$ and $x = 0$ on $CD$ and $AB$, respectively, at the initial instant. Then, it follows from (1) and (2) that $\zeta = -1$ and $\zeta = 0$ on $CD$ and $AB$, respectively. In [13], quite a general model of elastic/plastic incompressible strain hardening material has been adopted.

![Figure 1. Geometric illustration of the transformation equations in (1).](image)

The deformation of $A_1B_1C_1D_1$ into $ABCD$ consists of several stages. At the first stage the whole strain is elastic everywhere. This stage ends when a plastic region starts to propagate from $AB$. During the second stage there is one elastic region and one plastic region. This stage ends when another plastic region starts to propagate from $CD$. During the third stage there are two plastic regions and one elastic region between the plastic regions. This stage is of special interest for the purpose of the present paper. It is illustrated in Fig. 2 where $\zeta = \zeta_1(a)$ and $\zeta = \zeta_2(a)$ are elastic plastic boundaries. It is worthy of note that $\zeta = 0$ on $AB$ and $\zeta = -1$ on $CD$ throughout the process of deformation because those are material lines. However, the elastic/plastic boundaries move through the material. Therefore, $\zeta_1(a)$ and $\zeta_2(a)$ are functions of $a$. The solution given in [13] is restricted by the following inequalities:

$$d\zeta_1/da \leq 0 \quad \text{and} \quad d\zeta_2/da \geq 0.$$  \(3\)

The physical sense of these inequalities is that the size of each plastic region increases with the time. In this case, the process is an ideal flow process. Therefore, ideal flow design is to find such a function $s(a)$ that the inequalities (3) are satisfied.

3. General solution for elastic perfectly plastic material

It is assumed that the material is perfectly plastic. Moreover, to simplify the further analysis, it is assumed that the tensile force vanishes during the first stage of the process, i.e. when the whole strain is elastic everywhere. This is the process of pure bending.
Figure 2. General structure of the solution.

In this case \[ s = \left( 2a + \sqrt{1 + 4a^2} \right)/4. \] (4)

Plastic yielding initiates at \( \zeta = 0 \) and \( \zeta = -1 \) simultaneously (Fig. 2). The first stage ends when \[ s = s_e = \exp\left(\sqrt[3]{3k}\right)/4 \quad \text{and} \quad a = a_e = \sinh\left(\sqrt[3]{3k}\right)/2. \] (5)

Here \( k = \sigma_0 / (3G) \), \( G \) is the shear modulus of elasticity and \( \sigma_0 \) is the yield stress in tension. The latter is a material constant. In what follows, it is assumed that \( a \geq a_e \). In general, the tensile force does not vanish in the range \( a \geq a_e \) and therefore (4) is not valid in this range. The general solution for strain hardening material in the range \( a \geq a_e \) has been given in [13]. A particular case of this solution corresponding to perfectly plastic material is outlined in this section.

In what follows, the equivalent plastic strain is used as an independent variable. This strain is defined as \( \varepsilon_{eq}^p = \sqrt[3]{3}\int_0^t \varepsilon_{\eta\eta}^p \varepsilon_{\eta\eta}^p \, d\tau. \) Here \( \varepsilon_{\eta\eta}^p \) are the plastic components of the strain rate tensor, \( t \) is the time and integration in this equation should be performed over the strain path.

3.1. Solution in plastic region 1 (Fig. 2)

In this region
\[ \varepsilon_{eq}^p = \left(1/\sqrt[3]{3}\right)\ln\left[4(\zeta a + s)\right] - k \] (6)

and \( (\varepsilon_{eq}^p, a) \) can be used as independent variables instead of \( (\zeta, a) \). Let \( \varepsilon_1 \) be the value of the equivalent strain at \( \zeta = 0 \). Then,
\[ s = \frac{\exp\left[\sqrt[3]{3}(\varepsilon_1 + k)\right]}{4}, \quad \zeta_1 = \frac{1}{4a}\left(\exp\left(\sqrt[3]{3k}\right) - \exp\left[\sqrt[3]{3}(\varepsilon_1 + k)\right]\right). \] (7)

The through thickness distribution of the normal stresses in the Lagrangian coordinates (these stresses are the principal stresses) can be found from
\[ \sigma_\zeta / \sigma_0 = \varepsilon_{eq}^p - \varepsilon_1, \quad \sigma_\eta / \sigma_0 = \varepsilon_{eq}^p - \varepsilon_1 + 2/\sqrt[3]{3}. \] (8)
3.2. Solution in plastic region 2 (Fig. 2)
In this region
\[ \varepsilon_{eq}^{pl} = -\left(1/\sqrt{3}\right)\ln\left[4(\zeta a + s)\right] - k. \] (9)
As before, \((\varepsilon_{eq}^{pl}, a)\) can be used as independent variables instead of \((\zeta, a)\). Let \(\varepsilon_2\) be the value of the equivalent strain at \(\zeta = -1\). Then,
\[ s - a = \frac{\exp\left[-\sqrt{3}(\varepsilon_2 + k)\right]}{4}, \quad \zeta_2 = \frac{1}{4a}\left\{\exp\left(-\sqrt{3}k\right) - \exp\left[-\sqrt{3}(\varepsilon_2 + k)\right]\right\} - 1. \] (10)
The through thickness distribution of the normal stresses in the Lagrangian coordinates can be found from
\[ \sigma_{\zeta}/\sigma_0 = \varepsilon_{eq}^{pl} - \varepsilon_2 - fa/\sqrt{s - a}, \quad \sigma_y/\sigma_0 = \varepsilon_{eq}^{pl} - 2\sqrt{3} - fa/\sqrt{s - a}. \] (11)
Here \(f\) is the dimensionless tensile force defined as
\[ f = F/(\sigma_0 H) \] (12)
where \(F\) is the tensile force.

3.3. Solution in the elastic region (Fig. 2)
The through thickness distribution of the normal stresses in the Lagrangian coordinates can be found from
\[ \frac{\sigma_{\zeta}}{\sigma_0} = \frac{1}{6k}\ln^2\left[4(\zeta a + s)\right] + C, \quad \frac{\sigma_y}{\sigma_0} = \frac{1}{6k}\ln^2\left[4(\zeta a + s)\right] + \frac{2}{3k}\ln\left[4(\zeta a + s)\right] + C. \] (13)
Here \(C\) is constant of integration.

3.4. Complete solution
The stress \(\sigma_{\zeta}\) must be continuous across the elastic/plastic boundaries. It is worthy of note that \(\varepsilon_{eq}^{pl} = 0\) on both elastic/plastic boundaries. Therefore, it follows from (7), (8), (10), (11) and (13) that
\[ k + 2C = -2\varepsilon_1, \quad k + 2C = -2\varepsilon_2 - 2fa/\sqrt{s - a}. \] (14)
Eliminating \(C\) between these equations yields
\[ \varepsilon_1 = \varepsilon_2 + fa/\sqrt{s - a}. \] (15)
Using the definition for \(\varepsilon_1\) and \(\varepsilon_2\) together with (6) and (9) results in
\[ \sqrt{3}\varepsilon_1 = \ln(4s) - \sqrt{3}k, \quad \sqrt{3}\varepsilon_2 = -\ln\left[4(s - a)\right] - \sqrt{3}k. \] (16)
Equations (15) and (16) combine to give
\[ \ln\left[16s(s - a)\right] = \sqrt{3}fa/\sqrt{s - a}. \] (17)
If \(f\) is a prescribed function of \(a\) then this equation determines \(s\) as a function of \(a\). In particular, (4) follows from (17) if \(f = 0\). On the other hand, if a desirable path of strain is prescribed (i.e. \(s\) is a prescribed function of \(a\)) then (17) determines the variation of the tensile force with \(a\) that produces this path.
In order to verify that the solution is an ideal flow solution, it is necessary to consider the inequalities (3). It follows from (7) and (10) that

\[ 4a\zeta_1 = \exp(\sqrt{3}k) - 4s, \quad 4a\zeta_2 = \exp(-\sqrt{3}k) - 4s. \quad (18) \]

Differentiating with respect to \( a \) gives

\[ \frac{d\zeta_1}{da} = \frac{4(s - a ds/da) - \exp(\sqrt{3}k)}{4a^2}, \quad \frac{d\zeta_2}{da} = \frac{4(s - a ds/da) - \exp(-\sqrt{3}k)}{4a^2}. \quad (19) \]

Having \( s \) as a function of \( a \) and using (19) it is possible to check whether or not (3) is satisfied. It is seen from (19) that (3) is equivalent to

\[ g_1(a) = 4(s - a ds/da) - \exp(\sqrt{3}k) \leq 0, \quad g_2(a) = 4(s - a ds/da) - \exp(-\sqrt{3}k) \geq 0. \quad (20) \]

4. Ideal flow path and springback

It is seen from (17) that (4) is valid in the elastic/plastic regime if \( f = 0 \). Substituting (4) into (20) shows that the process of pure bending satisfies the ideal flow conditions in the range

\[ a_e \leq a \leq a_c = \frac{1}{2} \sqrt{\exp(2\sqrt{3}k) - 1}. \quad (21) \]

It follows from (4) that

\[ s = s_e = \frac{1}{4} \left[ \exp(\sqrt{3}k) + \sqrt{\exp(2\sqrt{3}k) - 1} \right] \quad (22) \]

at \( a = a_e \). In order to choose an ideal flow path in the range \( a > a_e \), consider a family of linear functions:

\[ s = s_0 + s_1 a. \quad (23) \]

Substituting (23) into (20) gives

\[ \exp(\sqrt{3}k) \geq 4s_0 \geq \exp(-\sqrt{3}k). \quad (24) \]

Since the function \( s(a) \) is continuous, the value of \( s_1 \) is determined from (23) as

\[ s_1 = (s_e - s_0)/a_e. \quad (25) \]

So, (23) provides an ideal flow path in the range \( a > a_e \) if \( s_0 \) is chosen from the interval (24) and \( s_1 \) is then found from (25).

The measure of stringback chosen in [13] is \( \rho = R/R_0 \) where \( R_0 \) is radius of curve CD (Fig. 1) at the end of loading and \( R \) is its radius after unloading. Assuming that no reversed yielding occurs the general solution for \( \rho \) has been found in [13]. Using the solution given in Section 3 at \( k = 0.003 \) the distribution of stress has been determined at \( 4s_0 = \exp(\sqrt{3}k) \) and \( 4s_0 = \exp(-\sqrt{3}k) \). It has been found that no reversed yielding occurs if \( R_0 \geq 6.45H \) in the former case and if \( R_0 \geq 5.55H \) in the latter case. The variation of \( \rho \) with \( R_0 \) for the case \( 4s_0 = \exp(-\sqrt{3}k) \) is depicted in Table 1.

| \( R_0 \) | \( 5.55H \) | \( 6H \) | \( 7H \) | \( 8H \) | \( 9H \) |
|---------|---------|---------|---------|---------|---------|
| \( R_0 \) | \( 5.55H \) | \( 6H \) | \( 7H \) | \( 8H \) | \( 9H \) |

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5. Conclusions
A new ideal flow solution for elastic plastic material has been derived. The solution describes the process of bending under tension/compression. The strain path consists of two smooth parts. The first part, \(0 \leq a \leq a_c\), corresponds to the process of pure bending. This process is not an ideal flow process at \(a > a_c\). Therefore, in this range the strain path is determined by the linear function given in (23). The coefficients of this function should satisfy (24) and (25). Using this function the measure of springback, \(\rho\), has been calculated and its variation with \(R_0\) is depicted in Table 1. The force \(f\) required to produce the ideal flow process can be found from (15) and (16).

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