Probing Exotic Higgs Sectors in $\ell^-\ell^-$ Collisions

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1. Introduction

If the origin of electroweak symmetry breaking lies in the existence of a Higgs sector, understanding the full nature of this sector will be one of the primary goals of future experimental programs. Although the Higgs sector could be quite simple, it is also possible that it will yield some real surprises. The purpose of this brief review is to assess the attractiveness of some exotic possibilities and the role of a $\ell^-\ell^-$ collider in exploring them. It will be convenient to divide the possible Higgs representations into: (a) singlets; (b) doublets; (c) triplets; (d) higher representations. The most important current aesthetic and experimental constraints on the Higgs representations, include: (i) naturality of $\rho = 1$; (ii) gauge coupling unification; and (iii) the branching ratio for $b \to s\gamma$. After reviewing these items as they affect various representations, I will turn to future experimental probes in high energy collisions, focusing on the importance of $\ell^-\ell^-$ collisions.

It is useful to list a selection of possible representations. For the moment, I restrict the discussion to the Standard Model (SM) SU(3)$\otimes$SU(2)$_L$$\otimes$U(1) gauge group, with $Q = I_3 + Y/2$. In the case of exotic representations, I will emphasize those which would lead to a doubly-charged Higgs boson eigenstate. A doubly-charged Higgs boson would constitute an incontrovertible signature for an exotic representation with non-standard U(1) hypercharge and/or SU(2)$_L$ weak isospin $I \geq 1$ and would very likely be of particular interest for $\ell^-\ell^-$ collisions. For simplicity, the Higgs sector will be assumed to be CP-conserving, with CP-even neutral states denoted by $h$ or $H$ and CP-odd states by $a$ or $A$. The notation will be $S_Y$.

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I will employ the notation $I$ for weak SU(2)$_L$ isospin.
for an $I = 0$ singlet, $D_Y$ for an $I = 1/2$ doublet, and $T_Y$ for an $I = 1$ triplet representation, respectively, where $Y$ denotes the absolute value of the hypercharge. All representations will be complex unless otherwise stated.

- **Doublet Models:**
  - The 1D$_1$ minimal Standard Model (MSM) Higgs sector with a single Higgs eigenstate, the $h_{SM}$.
  - A 2D$_1$ model, yielding the CP-even $h^0$ and $H^0$, a CP-odd $A^0$, and a $H^{+-}$ pair. In this review, only type-II two-doublet models (i.e. ones in which one doublet, $H_u$, gives mass to up-type quarks and the other, $H_d$, gives mass to down-type quarks and leptons) are considered. An important parameter of the model is the ratio $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$ of the neutral Higgs field component vev’s. If $m_{A^0}$ is large, it is natural, but not required, that the two-doublet model exhibit decoupling, according to which the light $h^0$ becomes SM-like while the $H^0$, $A^0$ and $H^{+-}$ decouple from electroweak symmetry breaking.
  - The 2D$_1$ minimal supersymmetric model (MSSM) [1]. Supersymmetry imposes restrictions on the Higgs sector which guarantee that the $h^0$ becomes SM-like when $m_{A^0} \sim m_{H^0} \sim m_{H^{++,--}}$ becomes large.
  - A 2D$_1$+1S$_0$ model, yielding states $H_{1,2,3}$, $A_{1,2}$, and $H^{+-}$.
  - A 1D$_1$+1Y$_{>1}$ model. The physical states depend on $|Y|$; e.g. for $|Y| = 3$ we have $h_{SM}$, $H^{+-}$, and $H^{++,-,-}$.
  - A 1D$_1$+1S$_4$ model, yielding $h_{SM}$ and $H^{++,-}$.

- **Triplet Models plus at least 1D$_1$ for fermion masses:**
  - 1D$_1$+1T$_y$ models; the physical states depend on the $Y$ of the triplet. For example, a $Y = 0$ (real representation) yields CP-even neutral states $H_{1,2}$ and a $H^{+-}$ pair. A $Y = 2$ (complex representation) yields $H_{1,2}$, $A$, $H^{+-}$, $H^{++,-,-}$.
  - A 1D$_1$+1T$_2$+1T$_0$(real) Higgs sector yields $H_{1,2,3}$, $A$, $H^{+-}_{1,2}$, $H^{++,-}$.
  - A 1D$_1$+2T$_2$ model yields $H_{1,2,3}$, $A_{1,2}$, $H^{+-}_{1,2}$, and $H^{++,-}$.

- **Higher Representations:**
  - The most interesting possibility is a 1D$_1$+1($I = 3$, $|Y| = 4$) Higgs sector, which yields $H_{1,2}$, $A$, $H^{+-}_{1,2}$, $H^{++,-,-}$, $H^{+++,-,-,-}$, $H^{++++,-,-,-,-}$, $H^{++++,--}$.

The 1D$_1$+1T$_2$+1T$_0$(real) Higgs sector with equal vev’s for the $T_2$ and $T_0$ triplet representation neutral members provides an especially useful benchmark. As summarized in Refs. [1,2,3,4,5,6,7], the Higgs eigenstates can be separated into a weak isospin five-plet ($H^{++,-,-}$, $H^{+-}_{5}$, $H^{0}_{5}$), a triplet ($H^{+-}_{3}$, $H^{+}_{3}$) and two singlets.
(H_{1}^{0}, H_{1}^{0}). Of these, only the H_{3}'s and the H_{0} have overlap with the doublet Higgs fields and, therefore, have fermionic couplings. An important parameter in this model is tan θ_{H} which is proportional to the ratio of the (common) vev of the neutral triplet members to the vev of the neutral doublet member.

2. Aesthetic/Experimental Constraints

• ρ = 1 at tree-level:

The first major consideration is the naturalness of ρ = m_{W}^{2}/[m_{Z}^{2} \cos^{2} \theta_{W}] = 1. The starting point is the well-known result:

$$\rho = \frac{\sum_{I,Y} [4I(I + 1) - Y^2] |v_{I,Y}|^2 c_{I,Y}}{\sum_{I,Y} 2Y^2 |v_{I,Y}|^2},$$  \hspace{1cm} (1)

where I specifies the SU(2)_{L} representation, v_{I,Y} is the vev of the neutral member of the representation if any, and c_{I,Y} = 1(1/2) for complex (real Y = 0) representations, respectively. The two lowest single representation solutions to ρ = 1 are I = 1/2, |Y| = 1 and I = 3, |Y| = 4. The minimal SM employs a single I = 1/2, |Y| = 1 doublet representation. As delineated earlier, a 1D_{1}+1(I = 3, |Y| = 4) Higgs sector would have an extensive array of eigenstates.

If a neutral member of a SU(2)_{L} triplet representation acquires a non-zero vev, ρ = 1 is never automatic. Even in the 1D_{1}+1T_{2}+1T_{0}(real) model with equal neutral vev’s for the triplets, which has ρ = 1 at tree-level, 1-loop corrections to ρ are infinite [4,6]. This means that ρ is a renormalizable quantity, the value of which must be inserted into the theory as an additional experimental input (just like α, G_{F}, ... in the SM) [6]. The precision electroweak data has been analyzed in this context [9] and it is found that there is no problem fitting all data, but this is hardly surprising given the loss of a (generally very constraining) prediction for ρ. In order to maintain predictability for ρ, it is tempting to favor models in which v_{L} = 0 for any L-triplet neutral member. (Note that v_{R} ≠ 0 in L-R symmetric models is possible without affecting ρ, and is necessary for m_{W_{R}} ≫ m_{W_{L}}. Of course, m_{W_{R}} is then a free renormalizable parameter.)

Among the exotic representations listed earlier, those which do not contain a neutral member are interesting in that ρ = 1 remains natural if such a representation is included in the Higgs sector.

• Gauge coupling unification:

The requirement that the gauge couplings unify with a desert between the TeV energy scale and the GUT unification scale M_{GUT} is widely regarded as being highly desirable. The resulting constraints on the Higgs representations that can be present are easily determined. At 1-loop, gauge coupling evolution depends only on the couplings themselves and the Higgs and other particle representations present at any given energy scale. Let us denote the number of |Y| = 1 doublets by N_{D_{1}},
the number of $|Y| = 2$ triplets by $N_{T_2}$, and so forth; $N_{34}$ denotes the number of $(I = 3, |Y| = 4)$ representations. I will not consider $|Y| \geq 6$ singlets, $|Y| \geq 5$ doublets, or $|Y| \geq 4$ triplets. The 1-loop evolution coefficients are then as follows (using $N_g$ to denote the number of complete generations).

**Non-SUSY, MSM:**

$$
b_1 = \frac{4}{3}N_g + \frac{4}{9}(N_{S_2} + 4N_{S_4}) + \frac{1}{10}(N_{D_1} + 9N_{D_3}) + \frac{5}{3}N_{T_2} + \frac{28}{5}N_{34}
$$

$$
b_2 = \frac{4}{3}N_g + \frac{1}{9}(N_{D_1} + N_{D_3}) + \frac{2}{5}(N_{T_0} + N_{T_2}) + \frac{28}{5}N_{34} - \frac{22}{3}
$$

$$
b_3 = \frac{4}{3}N_g - 11. \quad (2)
$$

**SUSY, MSSM sparticle content only:**

$$
b_1 = 2N_g + \frac{4}{9}(N_{S_2} + 4N_{S_4}) + \frac{3}{10}(N_{D_1} + 9N_{D_3}) + \frac{9}{5}N_{T_2} + \frac{84}{5}N_{34}
$$

$$
b_2 = 2N_g + \frac{1}{9}(N_{D_1} + N_{D_3}) + 2(N_{T_0} + N_{T_2}) + \frac{28}{5}N_{34} - 6
$$

$$
b_3 = 2N_g - 9 \quad . \quad (3)
$$

Note that in the above equations there is no influence from $Y = 0$ singlets, but $Y \neq 0$ singlets affect $b_1$.

The requirement of unification (assuming “standard” SU(5) normalization of the U(1) coupling constant and a desert between $m_Z$ and $M_U$) can be written (at 1-loop) in the form

$$
\alpha_s(m_Z) = \alpha_{QED}(m_Z) \frac{5(b_1 - b_2)}{\sin^2 \theta_W (5b_1 + 3b_2 - 8b_3) - 3(b_2 - b_3)} \quad (4)
$$

Using $\alpha_s(m_Z) = 0.118$ and $\sin^2 \theta_W = 0.2315$, and the results of Eqs. (2) and (3), the constraint of Eq. (4) reduces to

**SM:**

$$
1 \approx -0.09N_{S_2} - 0.36N_{S_4} + 0.13N_{D_1} - 0.22N_{D_3}
+ 0.71N_{T_0} + 0.44N_{T_2} - 1.39N_{34} \quad (5)
$$

**SUSY:**

$$
1 \approx -0.33N_{S_2} - 1.31N_{S_4} + 0.49N_{D_1} - 0.82N_{D_3}
+ 2.61N_{T_0} + 1.63N_{T_2} - 5.11N_{34} \quad (6)
$$

in the non-SUSY and MSSM cases, respectively. The exact coefficients in the equations above are sensitive to the precise $\alpha_s$ and $\sin^2 \theta_W$ choices as well as to whether all the Higgs bosons have mass near $m_Z$, as assumed, or nearer to 1 TeV. Two-loop corrections also lead to small changes in the unification conditions. Thus, the following discussion of ‘solutions’ should be regarded as being a somewhat rough, but indicative, guide to the possibilities.

In assessing the constraints of Eqs. (2) and (3), it should be kept in mind that $N_{D_1} \geq 1 \text{ (} \geq 2\text{)}$ is required in the SM (MSSM) for Dirac fermion masses, and that in the SUSY context, all $|Y| \neq 0$ (complex) multiplets must come in $Y = \pm|Y|$ pairs (to cancel anomalies). Although there are many solutions to the equations, essentially all but the two-doublet MSSM solution are excluded if one requires a
large value for $M_U$,\footnote{This extends the result of \cite{9} where small $M_U$ was found for multi-doublet SM and $N_{D_1} \geq 4$ MSSM solutions.} as would be needed to ensure proton stability if the SU(3), SU(2) and U(1) groups merge to form a single group [such as SU(5)] containing gauge bosons that mediate proton decay. In what follows, a solution is defined as being any choice for the Higgs representations that yields $0.1 \leq \alpha_s \leq 0.13$. For simplicity, only values with $N_{D_1} \leq 16$ and all other $N$’s $\leq 8$ have been surveyed.

1. In the SM, the highest $M_U$ value is attained for the $N_{D_1} = 2$, $N_{T_0} = 1$ solution,\footnote{N’s not stated are all 0.} yielding $\alpha_s = 0.115, M_U \sim 1.6 \times 10^{14}$ GeV. There are some 35 additional solutions with $M_U \geq 10^{13}$ GeV, the next highest $M_U$ solutions being: $\alpha_s = 0.104, M_U \sim 1.2 \times 10^{14}$ GeV for $N_{D_1} = 1, N_{T_0} = 1$ and $\alpha_s = 0.120, M_U \sim 10^{14}$ GeV for $N_{D_1} = 3, N_{T_0} = 1$, which are obviously closely related to the best solution. Next on the list are $N_{D_1} = 2, N_{T_0} = 1, N_{S_2} = 1$, yielding $\alpha_s = 0.108, M_U \sim 7 \times 10^{13}$ GeV and $N_{D_1} = 8$, yielding $\alpha_s = 0.124, M_U \sim 5 \times 10^{13}$ GeV. The $N_{T_0} \neq 0$ solution with largest $M_U$ is $N_{D_1} = 3, N_{T_2} = 1$, yielding $\alpha_s = 0.105, M_U \sim 2 \times 10^{13}$ GeV. All $N_{D_1} \neq 0$ solutions have $M_U < 10^{13}$ GeV. The $N_{S_4} \neq 0$ solutions with highest $M_U$ are $N_{D_1} = 1, N_{S_4} = 1$ and $N_{S_2} = 1$ or 2, yielding $\alpha_s = 0.129, M_U \sim 3 \times 10^{13}$ GeV or $\alpha_s = 0.120, M_U \sim 1.5 \times 10^{13}$ GeV, respectively. The D$_1$ and ($I = 3, |Y| = 4$) representations are especially interesting since any combination of such representations would yield $\rho = 1$ naturally even when their neutral members have non-zero vev’s. However, the $N_{D_1} = 18, N_{34} = 1$ solution yields $M_U \sim 2.8 \times 10^9$ GeV; higher values of $N_{34}$ yield still smaller $M_U$ values.

2. In SUSY, the $N_{D_1} = 2$ solution to Eq. (3) with $\alpha_s = 0.115, M_U > 2 \times 10^{10}$ GeV is far and away the best. Next highest $M_U$ is achieved for the $N_{D_1} = 2, N_{T_0} = 1, N_{S_2} = 8$ and $N_{D_1} = 6, N_{S_2} = 6$ solutions, both of which yield $\alpha_s = 0.115, M_U \sim 3 \times 10^{13}$ GeV. Any $N_{D_1} \geq 2$ or $N_{S_2} \geq 2$ solution has much smaller $M_U$. Solutions with ($I = 3, |Y| = 4$) representations inevitably lead to very small $M_U$. For example, the $N_{D_1} = 22, N_{34} = 2$ solution yields $M_U \sim 5 \times 10^8$ GeV.

3. The L-R symmetric extension of the SM and the supersymmetric L-R symmetric model both require intermediate scale matter for full coupling unification \cite{14}. Many new possible solutions also emerge in the basic SM or SUSY contexts by including intermediate scale matter; in particular, solutions containing exotic Higgs representations that yield a large value of $M_U$ become possible. The intermediate-scale-matter solutions will not be pursued here.

- $\rho$ at one-loop:

   In models with $\rho = 1$ at tree level and for which $\rho$ is finitely calculable, parameters must still be chosen so that Higgs-loop corrections to $\rho$ are small. Too
large a mass separation between neutral and charged Higgs bosons with $W, Z$ couplings will lead to a large $\delta \rho$ at 1-loop. (See [8] for a brief discussion.) A large $\delta \rho$ is automatically avoided in supersymmetric models throughout all of Higgs parameter space by virtue of the fact that heavier Higgs bosons decouple from the $W, Z$ sector. For example, in the minimal two-doublet-Higgs-sector MSSM model, when $m_{A^0} \sim m_{H^0} \sim m_{H^\pm}$ is large the dangerous $W^+ \rightarrow H^+ h^0$ and $Z \rightarrow H^0 h^0$ couplings [proportional to the famous $\cos(\beta - \alpha)$ factor] are highly suppressed [8].

$\bullet \ b \rightarrow s\gamma$:

Any Higgs sector containing more than one $|Y| = 1$ doublet will have one or more singly-charged Higgs bosons with fermionic couplings. Any such $H^\pm$ will enter into a 1-loop $H^+\tilde{T}$ contribution to the $b \rightarrow s\gamma$ transition that must be added at the amplitude level to the SM $W^+\tilde{T}$ loop. In the case of a type-II two-doublet model, the $H^+\tilde{T}$ loop adds constructively to the $W^+\tilde{T}$ SM loop. Since the SM $W^+\tilde{T}$ loop alone leads to a $b \rightarrow s\gamma$ branching ratio that exceeds the measured value, the 95% confidence level limits on $m_{H^\pm}$ are large [12], roughly $m_{H^\pm} \gtrsim 300$ GeV for $\tan \beta > 1$, unless there are additional 1-loop graphs that can cancel the extra $H^+\tilde{T}$ loop contribution. The best-known example of such a cancellation arises in supersymmetric models. There, a stop-chargino graph can cancel an excessive top-charged-Higgs graph if $m_{\tilde{t}}$ and $m_{\tilde{\chi}^\pm}$ are small enough. Of course, charged Higgs that do not couple to quarks (of which there are many in the models listed earlier) are no problem.

3. Couplings and Decays

The phenomenology of exotic Higgs representations is a very complex topic and very model-dependent. In order to avoid too lengthy a discussion, I will focus on models containing triplet representations in addition to one or more doublet Higgs representations. A convenient summary of Higgs triplet phenomenology appears in [8]; more details can be found, for example, in [13,14,15,16,17,18,19,20]. I begin with a few preliminary reminders of well-known facts.

$\bullet$ There is never a $\gamma W^+ - H^-$ vertex. There is generally a non-zero tree-level $ZW^+ - H^-$ vertex if $v_L \neq 0$ (for a $|Y| \neq 0$ representation with non-zero SU(2)$_L$ isospin), even if $\rho$ is tuned to $\simeq 1$ at tree-level. However, if we require $v_L = 0$ in order to maintain naturality of $\rho = 1$, all $ZW^+ - H^-$ vertices will be zero at tree-level.

$\bullet$ Charged and neutral (non-singlet) Higgs bosons, triplet members or otherwise, have diagonal pair couplings to $\gamma \propto Q$ and $Z \propto I_3 - x W Q$.

$\bullet$ $W_L W_L$ couplings to Higgs bosons of an L-triplet representation are proportional to the vev $v_L$ of the neutral member of the triplet. This means that $\rho = 1$ is only natural if there are no $W_L W_L$ couplings at tree-level. In a L-R
symmetric model, $W_RW_R$ couplings to R-triplet members are $\propto v_R$, and are generally large.

- This same rule applies to multi-triplet models. For the $H_T$'s of such a model, $H_TVV$ couplings are proportional to an appropriate triplet vev (or zero if $H_T$ is CP-odd in nature). As an example, in the $1D_1+1T_2+1T_0$(real) Higgs sector with $\rho = 1$ at tree-level (as summarized in Ref. 8 and the Introduction) all such couplings are proportional to $\sin \theta_H$.

- For triplet $H_T$'s, most $H_TH_TV$ couplings are non-zero even if the triplet vev is zero. Again the $1D_1+1T_2+1T_0$(real) Higgs sector provides a useful example. Of particular interest are couplings of the type $H^+_T H^T_T \rightarrow W^-$ which can allow for $H_T^+ H^-_T$ production via $W^-$ exchange and $H_T^- \rightarrow H_T^- W^-$ decays (and charge conjugate versions thereof). These couplings are sizeable, independent of the magnitude of the neutral triplet vev.

- Non-zero trilinear couplings of three $H_T$'s require a triplet vev.

- $H_DH_T H_T$ couplings involving one doublet $H_D$ and two $H_T$'s are non-zero if the doublet vev is non-zero, even if the triplet vev is zero.

- When quantum-number allowed, $H_T$ couplings to $f(f')f$ require a non-zero triplet vev. Thus, if the triplet vev is zero, $H_T$ decays to fermion pairs will not be present at tree-level.

- There is a possibility of non-zero bi-lepton couplings of Higgs bosons. For example, for the standard SU(2)$_L$ case, with $Q = I_3 + Y = -2$, the allowed doubly-charged cases are:
  
  \begin{align*}
  e^{-}e^{-} &\rightarrow H^{-} (I = 0, I_3 = 0, Y = -4), \\
  e^{-}e^{-} &\rightarrow H^{-} (I = \frac{1}{2}, I_3 = -\frac{1}{2}, Y = -3), \\
  e^{-}e^{-} &\rightarrow H^{-} (I = 1, I_3 = -1, Y = -2). \\
  \end{align*}

  Note that the above cases do not include the $I = 3, Y = -4$ representation that yields $\rho = 1$, nor the $I = 1, Y = -4$ triplet with no neutral member, but do include the $I = 1/2, Y = -3$ doublet representation with no neutral member, and the $I = 1, Y = -2$ triplet representation. A $\nu_L \nu_L \rightarrow H^0 (I = 1, I_3 = +1, Y = -2)$ coupling also exists, but does not lead to neutrino mass if $v_L = \langle H^0 \rangle = 0$ (as preferred for $\rho = 1$ to be natural).

In the case of a $|Y| = 2$ triplet representation the lepton-number-violating coupling to (left-handed) leptons is specified by the Lagrangian form:

\[ \mathcal{L}_Y = i h_{ij} \psi_{iL}^T C_\tau_2 \Delta \psi_{jL} + \text{h.c.}, \]

where $i, j = e, \mu, \tau$ are generation indices, the $\psi$'s are the two-component left-handed lepton fields $[\psi_{iL} = (\psi, \ell^-)_{iL}]$, and $\Delta$ is the $2 \times 2$ matrix of Higgs fields:

\[ \Delta = \begin{pmatrix}
H^-/\sqrt{2} & H^- \\
H^0 & -H^-/\sqrt{2}
\end{pmatrix}. \]
In the supersymmetric model context, the supersymmetric analogues of the left-right symmetric models, we must separate $L$ from $R$ and use $Q_c$ where Bhabha scattering, $(g - 2)_{\mu}$, muonium-antimuonium conversion, and $\mu^- \to e^-e^+$. One finds rough limits of (for $m_{H^-}$ in GeV)

$$\begin{align*}
|h_{ee}^{\Delta^-}|^2 &\lesssim 10^{-5}m_{H^-}^2, \\
|h_{\mu\mu}^{\Delta^-}|^2 &\lesssim \text{few} \times 10^{-5}m_{H^-}^2, \\
|h_{\mu\mu}^{\Delta^-} - h_{ee}^{\Delta^-}| &\lesssim \text{few} \times 10^{-11}m_{H^-}^2
\end{align*}$$

where $h_{ij}^{\Delta^-}$ refers to the $h_{ij}$ couplings as they appear in $H^-\bar{H}^-$ interactions. The last limit suggests small off-diagonal couplings, as is assumed in what follows. It is convenient to write

$$|h_{\ell\ell}^{\Delta^-}| = c_{\ell\ell}m_{H^-}^2 \text{ (GeV)},$$

where $c_{ee} \lesssim 10^{-5}$ is the strongest of the limits. The only constraint on the $h_{ij}$ through couplings for the $H^0$ and $H^-$ that is potentially stronger than those above goes away for $\langle H^0 \rangle = 0$, as required in order to have $\rho = 1$ naturally.

In left-right symmetric models, we must separate $L$ from $R$ and use $Q = I_3(L) + I_3(R) + Y/2$, where $Y = B - L$, leading to

$$\begin{align*}
e_R e_R &\to H^- \bar{H}^- (I(R) = 1, I_3(R) = -1, Y = -2), \\
e_L e_R &\to H^- \bar{H}^- (I(R) = I(L) = \frac{1}{2}, I_3(R) = I_3(L) = -\frac{1}{2}, Y = -2), \\
c_L e_L &\to H^- \bar{H}^- (I(L) = 1, I_3(L) = -1, Y = -2).
\end{align*}$$

respectively. A $Y = -2$ bi-doublet, as required for the middle case, is not normally considered, but is a logical possibility. The $\mathcal{L}_Y$ for the 1st and 3rd cases above, would be analogous to Eq. (6) with $L$ fermions coupling to $\Delta_L$ and $R$ fermions to $\Delta_R$. In a large class of supersymmetric L-R symmetric models with automatic R-parity conservation, the doubly-charged $\Delta_L$ triplet Higgs bosons (and associated higgsinos) are naturally very light \[21\].

- In the supersymmetric model context, the supersymmetric analogues of the couplings listed above will all be present. In particular, any Higgs boson which couples to two fermions or two vector bosons, will couple to their sfermion or gaugino partners. Any Higgs boson which couples to another Higgs boson and a vector boson will couple also to the higgsino and gaugino partners.

Any Higgs boson which couples to two other Higgs bosons will couple to the corresponding higgsino partners. An $H^-\bar{H}^-$ that couples to $\ell^-\ell^-$ will couple to the corresponding $\bar{\ell}^-\ell^-$ channel. For example, the $H^-\bar{H}^-$ with $I(L) = 1, I_3(L) = -1, Y = -2$ that is usually included in a supersymmetric L-R symmetric model will couple to $\bar{\ell}_L^-\ell_L^-$. If R-parity is violated in a supersymmetric model, still more couplings of Higgs bosons emerge. The possible scenarios are quite complex. In what follows, all supersymmetric couplings and consequent decays of the Higgs bosons will be ignored. Modifications to the phenomenological discussions to come will, for the most part, be obvious.
It is now possible to outline the (non-supersymmetric) decays of triplet Higgs bosons and their implications. Obviously there is tremendous variation according to which couplings are present. A particular focus of the ensuing discussion will continue to be the exotic $H^{--}$ that would constitute an incontrovertible signature for an exotic Higgs representation and which could be produced directly in $s$-channel $\ell^--\ell^-$ collisions. Regardless of how it is produced, the decays of such a $H^{--}$ would be very revealing.

- If the triplet vev is non-zero, then $H_T \to VV$ (possibly virtual) decays and/or $H_T \to f^0f^0$ decays are usually most significant.

- If the triplet vev’s are zero, then many channels are eliminated. A type of mode that remains and is possibly important is $H_T \to H'_T V$. However, since many of the $H_T$’s of a typical model are approximately degenerate, it is possible for all such modes to be phase-space suppressed or virtual. In this case, bi-lepton decay modes could dominate if the bi-lepton couplings are non-zero. For example, in the case of a $I = 1, |Y| = 2$ $H^{--}$, we have

\[ \Gamma(H^{--} \to H^- W^-) = \frac{\alpha^2 m_{H^{--}}^3}{16 \pi m_{W^+}^2} \beta \left( \frac{m_{H^{--}}}{100 \text{ GeV}} \right)^3, \]
\[ \Gamma(H^{--} \to \ell^- \ell^-) = \frac{|h_{\ell\ell}^{--}|^2 m_{H^{--}}}{8\pi} \sim (0.4 \text{ GeV}) \left( \frac{c_{\ell\ell}}{10^{-16}} \right) \left( \frac{m_{H^{--}}}{100 \text{ GeV}} \right)^3. \]

(13)

where $\beta$ is the usual phase space suppression factor, and Eq. (11) has been used.

The implications for the detection of the $H^{--}$ have been considered in [3].

- If one or more of the $c_{\ell\ell}$’s is significantly larger than $10^{-5}$, the $\ell^- \ell^-$ channel with the largest $c_{\ell\ell}$ will dominate $H^{--}$ decays even if the $H^- W^-$ mode is allowed. Since there are currently no limits on $c_{\tau\tau}$, the $\tau^- \tau^-$ channel could easily have the largest partial width and be the dominant decay of the $H^{--}$.

- If the $H^- W^-$ mode is disallowed, then the $\ell^- \ell^-$ channel(s) will provide the dominant signal unless the $c_{\ell\ell}$’s are all extremely tiny.

- If all the $c_{\ell\ell}$’s are small and the $H^- W^-$ mode is allowed, then signatures would be more dilute and more events would be required for detection.

- If the $H^- W^-$ mode is virtual (i.e. the $H^-$ and/or the $W^-$ must be off-shell) and the $c_{\ell\ell}$’s are all very small, the $H^{--}$ could be sufficiently long-lived to yield a vertexable track or escape the detector entirely. (The latter requires that the $H^- W^-$ mode be doubly virtual and that $c_{\ell\ell} \lesssim 10^{-16}$.) A search for stable particle highly ionizing tracks is then appropriate.

- For zero triplet vev’s, the lightest $H_T$ will be stable unless it has bi-lepton couplings, in the absence of which it must be neutral and would be a candidate for cold dark matter.
4. Direct Production Probes

Specific production processes of interest that are especially sensitive to triplet Higgs representations can be separated into those that do not require bi-lepton couplings and those that do. Of course, the appropriate final state detection modes will depend upon the relative weight of bi-lepton vs. other couplings. In what follows, I once again focus on triplet representations. The emphasis will be on $\ell^−\ell^−$ and $\ell^−\ell^+$ processes that can take place at either an electron collider or a muon collider.

**Production processes independent of bi-lepton couplings.**

I give below an incomplete sampling of interesting processes that would probe for the presence of an exotic Higgs representation.

- Exclusive $H^−H^+$, $H^−H^+$, and $H^0H^0$ conjugate pair production in $\ell^+\ell^−$ or $pp$ collisions.

The cross section for pair production of a boson with weak isospin $I_3$ and charge $Q$ and its conjugate via annihilation of a fermion with $i_3$ and $q$ and its anti-fermion partner is given by:

$$\sigma_{\text{pair}} = \left( \frac{\pi\alpha^2\beta^2}{6} \right) \left\{ 2Q^2q^2P_{\gamma\gamma} + P_{\gamma Z} \frac{2QqA(a_L + a_R)}{x_W y_W} + P_{ZZ} \frac{A^2(a_L^2 + a_R^2)}{x_W y_W^2} \right\}, \quad (14)$$

where $x_W = \sin^2\theta_W$, $y_W = 1 - x_W$, $A = I_3 - x_W Q$, $a_L = i_3 - x_W q$, $a_R = -x_W q$, $P_{\gamma\gamma} = s^{-2}$, $P_{ZZ} = [(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2]^{-1}$, and $P_{\gamma Z} = (s - m_Z^2)P_{ZZ}/s$. A color averaging factor of 1/3 must be supplied in the context of $pp$ or $p\bar{p}$ collisions when convoluting this pair cross section with quark distribution functions; a factor of 1/2 is to be included for $H^0H^0$ production. Note that the pair cross sections do not rely on there being a non-zero vev for a neutral Higgs field.

Signatures of the $H^−H^+$ final state can be extremely striking. If $H^− \rightarrow \ell^−\ell^−$ decays are dominant, for $\ell = e$ or $\mu$ only a few spectacular events with two like-sign lepton pairs of equal mass are needed for an unambiguous signal. If $c_{\tau\tau}$ is the largest bi-lepton coupling, the $4\tau$ final state can be isolated using relatively simple cuts. The detection limits for $H^-H^+$ pair production in the case of a $T_2$ representation have been explored [22]. The Tevatron and LHC production cross sections are given in Fig. [1]. When the $H^−$ decays are dominated by $\ell^−\ell^−$ decay modes, Ref. [22] finds that detection of $H^−H^+$ pair production at the Tevatron (operating at $\sqrt{s} = 2$ TeV with $L = 30$ fb$^{-1}$) will be possible for $m_{H^-}$ up to 300 GeV for $\ell = e$ or $\mu$ and 180 GeV for $\ell = \tau$. The corresponding limits at the LHC are estimated by requiring the same raw number of events before cuts and efficiencies as needed at the Tevatron — $\sim 10$ for $\ell = e, \mu$ and $\sim 300$ for $\ell = \tau$ — yielding $m_{H^-}$ discovery up to roughly 925 GeV (1.1 TeV) for $\ell = e, \mu$ and 475 GeV (600 GeV) for $\ell = \tau$, assuming total integrated luminosity of $L =$
100 fb\(^{-1}\) \((L = 300 \text{ fb}^{-1})\). For \(\ell = e, \mu\), the reach of the LHC detectors will likely be even greater than this, due to the improved lepton acceptance and resolution anticipated over the current generation of hadron collider detectors. For \(\ell = \tau\), this simple extrapolation may not account for a different signal-to-background ratio in \(\tau\) selection at the LHC. A more complete study is necessary to evaluate this. If the \(H^{--} \rightarrow H^- W^-\) decay is dominant, then the signals will be more dilute than for the \(e^- e^-\) or \(\mu^- \mu^-\) cases. Using the leptonic \((\nu \ell, \ell = e, \mu)\) decays of the \(W^- W^+\) in the \(H^{--} H^{++} \rightarrow H^- W^- H^+ W^+\) final state suggests that one can hope to achieve \(S/\sqrt{B}\) levels similar to the \(\tau^- \tau^-\) case, in which case the discovery reaches for the latter channel discussed above would apply.

Although observation of \(H^{--} H^{++}\) pair production is certainly the clearest signal for an exotic Higgs representation, additional information and confirmation can be obtained from the \(H^+ H^-\) and \(H^0 H^0\) production rates, which will reveal non-doublet correlations between \(Q\) and \(A = I_3 - x_W Q\). Of course, in these latter cases, decays could be more complicated and would need to be understood in order to extract the underlying pair cross section.

In any case, the conclusion is that if there are exotic Higgs triplet bosons, they will be observed in pair production at the LHC, if not at the Tevatron, up to the highest mass \((\sim 500 \text{ GeV})\) for which they could be directly produced in the \(s\)-channel at a first \(\ell^- \ell^-\) collider with \(\sqrt{s} \lesssim 0.5 \text{ TeV}\).
• Exclusive production of non-conjugate Higgs pair.

Non-conjugate Higgs pairs can be produced via $s$-channel virtual $Z$ exchange in $e^+e^-$ collisions and via virtual $Z$ or $W$ exchange in $pp$ collisions. However, there has not been a careful study of the signals and backgrounds to the many possible final state detection channels. One background free channel would be $W^- \rightarrow H^- H^+$ production in which the $H^-$ decays to $\ell^- \ell^-$ ($\ell = e, \mu, \tau$). If the $H^+$ also belongs to the triplet representation, the required coupling is sizeable and independent of the triplet vev. This mode will generally allow confirmation of any signal seen for $H^- H^+$ pair production at the Tevatron or LHC (see above).

• Exclusive $H^+ W^-, H^- W^+$ production.

The only diagram is $s$-channel $Z$ exchange. A non-zero $Z \rightarrow H^+, W^-$ coupling requires a (L) triplet representation with a non-zero vev. Signal and background rates for this reaction at an $e^+e^-$ collider have been explored in Refs. [19,20].

• $W^- W^+$ production from $W^- W^+$ fusion in $\ell^- \ell^+$ collisions.

The influence of triplet Higgs representations on standard $W^- W^+ \rightarrow W^- W^+$ fusion was considered at some length in [3], as summarized in [8]. In general, the $WW$ scattering processes, of which this is our first example, will exhibit strong unitary-violating behavior unless an appropriate set of Higgs bosons are present. Any Higgs boson belonging to a representation in which there is a non-zero vev for a neutral member will generally contribute to one or more of the amplitudes for $WW$ scattering. Contributions to the $W^- W^+ \rightarrow W^- W^+$ amplitude that do not arise from standard electroweak interactions include:

1. $s$-channel $H^0$ exchanges (appropriate vev required).
2. $t$-channel $H^0$ exchanges (appropriate vev required).
3. $u$-channel $H^{++}$ exchanges (vev and triplet required).

Note that only the $u$-channel diagram requires a triplet field; contributions to the $s$ and $t$ channel amplitudes are possible for both doublet and triplet neutral Higgs bosons, provided the appropriate doublet and triplet (respectively) vev’s are non-zero. In a general model, multiple exchanges of each type are possible. All Higgs bosons contributing to the amplitude must have mass $\lesssim 1$ TeV in order to avoid non-unitary high energy behavior. For lepton colliders, the draw-back of this scattering process and the other reactions summarized below, is that $\sqrt{s} > 1$ TeV is needed to fully explore the nature of the contributing diagram amplitudes.

• $W^- W^-$ production from $W^- W^+$ fusion in $\ell^- \ell^-$ collisions.

This process was considered, for example, in Refs. [3,15]. Contributions to the $W^- W^+ \rightarrow W^- W^-$ amplitude are:

1. $s$-channel $H^{-}$ exchanges (vev and triplet required).
2. $t$-channel $H^0$ exchanges (appropriate vev required).

3. $u$-channel $H^0$ exchanges (appropriate vev required).

The same comments as above apply. In particular, non-zero contributions require an appropriate non-zero vev for a neutral Higgs field.

Let us give a specific example in this case. In the SM, there are two $h_{SM}$-exchange graphs: one is a $t$-channel and the other a $u$-channel graph. They can be thought of as combining together and having effective strength $g^2m_W^2$, as required to cancel the bad high energy behavior coming from graphs involving standard electroweak boson exchanges. Now consider the 1D$_1$+1T$_2$+1T$_0$(real) Higgs sector detailed in the Introduction. In this model, there are three $u$-channel and three $t$-channel graphs for the $H^0$, $H^0'$ and $H^0_5$ neutral Higgs bosons and an $s$-channel graph for the $H^-^-$. An $s$-channel graph is equivalent to the sum of a $t$- and a $u$-channel graph except for an overall sign. Thus, the effective contributions of the different Higgs exchanges are:

$$H^0_1: g^2c_H^2m_W^2; \quad H^0_1': (8/3)g^2s_H^2m_W^2;$$
$$H^0_5: (1/3)g^2s_H^2m_W^2; \quad H^-^-: -2g^2s_H^2m_W^2,$$

(15)

where $s_H \equiv \sin \theta_H$ ($\theta_H$ was defined earlier). Note that these always sum to the SM weight of $g^2m_W^2$, thereby guaranteeing good high energy behavior. Also note that in the limit of $s_H \to 0$ (vanishing vev’s for the neutral triplet members) only the $H^0_1$ amplitudes survive; indeed, in this limit the $H^0_1$ has the same properties as the SM $h_{SM}$. However, for substantial $s_H$ the triplet Higgs fields play a very important role in guaranteeing good high-energy behavior for the $W^-W^- \to W^-W^-$ amplitude. Further, for sizeable $s_H$, the $H^-^-$ will create a clear resonance bump in the $W^-W^-$ invariant mass spectrum that should be easily detectable at an $\ell^-\ell^+$ accelerator of sufficient energy (and also in $W^-W^+$ fusion at the LHC).

- $H^-H^+$ from $W^-W^+$ fusion in $\ell^-\ell^+$ collisions.

This is our first example of a process involving a mixture of Higgs bosons and $W$ bosons. For such processes, not all amplitude contributions require that the neutral member of the Higgs boson representation have a non-zero vev. Thus, in what follows the exchanges which do and don’t require such a vev are indicated. For $W^-W^+ \to H^-H^+$, the contributing diagrams include:

1. $s$-channel $H^0$ exchanges (appropriate vev required);
2. $t$-channel $H^0$ and $A^0$ exchanges (vev not required);
3. $t$-channel $Z$ exchange (vev and triplet required);
4. $u$-channel $H^-^-$ exchange (triplet, but not vev, required);
5. quartic coupling.
The only standard electroweak contribution to \( W^- W^+ \rightarrow H^- H^+ \) is \( s \)-channel \( Z \)-exchange.

- \( H^- H^- \) from \( W^- W^- \) fusion in \( \ell^- \ell^- \) collisions.

The diagrams that contribute to \( W^- W^- \rightarrow H^- H^- \) include:

1. \( s \)-channel \( H^-\) exchange (vev and triplet required);
2. \( t \)-channel \( H^0 \) and \( A^0 \) exchanges (vev not required);
3. \( t \)-channel \( Z \) exchange (vev and triplet required);
4. \( u \)-channel \( H^0 \) and \( A^0 \) exchanges (vev not required);
5. \( u \)-channel \( Z \) exchange (vev and triplet required);
6. quartic coupling.

This reaction has been especially thoroughly studied in Ref. \[17\], where substantial sensitivity to the exact nature of the Higgs sector was demonstrated.

- \( H^+ W^- \) or \( W^+ H^- \) production via \( W^+ W^- \) fusion in \( \ell^+ \ell^- \) collisions.

Contributing diagrams include:

1. \( s \)-channel \( Z \) exchange (vev and triplet required);
2. \( t \)-channel \( H^0 \) exchange (vev required);
3. \( u \)-channel \( H^{++} \) exchange (vev and triplet required);

Even this partial list (which, for example, does not include processes with \( Z \)'s in the final state) should make clear that there are a very large number of processes to be considered. Because of the lurking possibility of unitarity-violating high-energy behavior for the processes considered above, the exact energy dependence and cross section magnitudes are quite sensitive to the contributing Higgs bosons through their masses and couplings. Thus, a full study of all such processes has an excellent chance of leading to a full understanding of a complicated Higgs sector. In a lepton collider environment, it will be absolutely crucial to have both \( \ell^- \ell^- \) and \( \ell^- \ell^+ \) collisions in order to be able to study all of the reactions of interest.

**Production processes requiring bi-lepton couplings.**

- Single \( (s \)-channel) \( H^{--} \) production in \( \ell^- \ell^- \) collisions \[10,23,24,25,26\].

Let us assume that the \( H^{--} \) has already been discovered, for example at the LHC via \( H^{--} H^{++} \) pair production as described earlier. If the \( H^- \) is detected in the \( \ell^- \ell^- \) final state, study of the \( H^{--} \) via \( \ell^- \ell^- \) collisions will have an extremely high priority. If the error in \( m_{H^{--}} \) as measured at the LHC, \( \Delta m_{H^{--}} \), is sufficiently
small, a narrow scan at the $\ell^-\ell^-$ collider will, in most cases, allow us to center on and study the resonance in a factory-like setting even if the coupling, $c_{\ell\ell}$, responsible for the production is very small.

The crucial feature of the $\ell^-\ell^-$ collider for narrow resonance production in this context is the width of and luminosity contained in the narrow Gaussian-like peak\(^{e}\) centered at the nominal $\sqrt{s}$ setting of the machine. We will denote the rms width of this Gaussian-like peak by $\sigma_\sqrt{s}$. It is related to the intrinsic \(\text{(i.e. before including beamstrahlung and bremsstrahlung)}\) machine beam energy resolution $R$ by

$$\sigma_\sqrt{s} \sim 0.2 \text{ GeV} \left( \frac{m_{H^-\ell^-}}{100 \text{ GeV}} \right) \left( \frac{R}{0.2\%} \right),$$

(16)

Values of $R$ that can be achieved at an electron collider are $R \sim 0.2\% - 0.3\%$; $R \sim 0.1\%$ is easily achieved at a muon collider, with values as small as $R \sim 0.003\%$ possible at reduced luminosity. As described below, the cross section for $H^-\ell^-$ production is maximized if $\sigma_\sqrt{s} \lesssim \Gamma_{H^-\ell^-}^{\text{tot}}$ can be achieved.

The effective cross section, denoted by $\bar{\sigma}_{H^-\ell^-}$, for $\ell^-\ell^- \rightarrow H^-\ell^-$ is obtained by convoluting the standard $s$-channel resonance form with the distribution in $\sqrt{s}$ of the machine luminosity. Results for the Gaussian approximation are easily summarized in the two limits of $\Gamma_{H^-\ell^-}^{\text{tot}} \gg \sigma_\sqrt{s}$ and $\Gamma_{H^-\ell^-}^{\text{tot}} \ll \sigma_\sqrt{s}$. Taking $\sqrt{s} = m_{H^-\ell^-}$, $\bar{\sigma}_{H^-\ell^-}$ is given by:

$$\bar{\sigma}_{H^-\ell^-} = \begin{cases} 
\frac{4\pi BF(H^-\ell^- \rightarrow e^-e^-)}{m_{H^-\ell^-}} & \Gamma_{H^-\ell^-}^{\text{tot}} \gg \sigma_\sqrt{s} \\
\sqrt{\pi} \frac{\Gamma_{H^-\ell^-}^{\text{tot}}}{2\sqrt{2} m_{H^-\ell^-}} & \Gamma_{H^-\ell^-}^{\text{tot}} \ll \sigma_\sqrt{s} 
\end{cases}$$

(17)

We compute rates as $L\bar{\sigma}_{H^-\ell^-}$ where $L$ is the luminosity within the Gaussian peak (after accounting for losses to the radiative tail). Two-year integrated luminosity of $L = 50$ fb$^{-1}$ is expected at an $e^-e^-$ collider. At a $\mu^-\mu^-$ collider, the expected $L$ depends strongly on both $R$ and $\sqrt{s}$. Conservative two-year expectations are $L = 0.1, 0.2, 1.2$ fb$^{-1}$ for $R = (0.003, 0.01, 0.1)\%$ at $\sqrt{s} \sim 100$ GeV rising to $L = (2.6, 14)$ fb$^{-1}$ at $\sqrt{s} \sim (200, 350, 400)$ GeV, assuming $R \sim 0.1\%$. As already noted, initially it will be necessary to scan over the interval $m_{H^-\ell^-} \pm \Delta m_{H^-\ell^-}$ in order to center on $\sqrt{s} \simeq m_{H^-\ell^-}$, implying that the resonance peak will receive only a fraction of the total $L$.

In fact, designing an appropriate scan is not necessarily straightforward. Even if the $H^-\ell^-$ is observed at the LHC in the $\ell^-\ell^-$ final state, the manner in which to perform a narrow scan at an $\ell^-\ell^-$ collider for centering on $\sqrt{s} \simeq m_{H^-\ell^-}$ can be delineated only if $\Gamma_{H^-\ell^-}^{\text{tot}}$ can be measured or estimated from the LHC observations. This is because the luminosity per scan point needed to observe or exclude the $H^-\ell^-$ depends on $\Gamma_{H^-\ell^-}^{\text{tot}}$, see Eq. (17). The most optimistic case is that the width of the $H^-\ell^-$ is larger than the intrinsic resolution (expected to be of order a few percent of

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\(^e\)In addition to the narrow Gaussian-like peak, there is a long tail at $\sqrt{s}$ values below the nominal central value coming from beamstrahlung and bremsstrahlung. When we refer to the luminosity in the Gaussian-like peak, this does not include the luminosity contained in this tail. The losses to this tail are larger for electron colliders than muon colliders.
mass) ≪ \( m_{H^--} \) for the \( e^-e^- \) and/or \( \mu^-\mu^- \) final state in which it is observed at the LHC. Then, \( \Gamma_{H^--}^{\text{tot}} \) can be read directly off the final state mass distribution(s). Another possibility is that the \( H^-- \rightarrow W^-H^- \) decay is observed. In this case, even if the mass resolution in all final states is inadequate for a direct \( \Gamma_{H^--}^{\text{tot}} \) determination, we can compute the \( W^-H^- \) partial width (using the presumably measured \( H^- \) mass) up to a Higgs-representation-dependent factor. Then an estimate for the total width is given by \( \Gamma_{H^--}^{\text{tot}} \sim \Gamma(H^-- \rightarrow H^-W^-)/BF(H^-- \rightarrow H^-W^-) \). In either case, \( BF(H^-- \rightarrow \ell^-\ell^-) \) would have been measured at the LHC and we could compute \( \Gamma(H^-- \rightarrow \ell^-\ell^-) = \Gamma_{H^--}^{\text{tot}} BF(H^-- \rightarrow \ell^-\ell^-) \); these are the crucial ingredients in Eq. (17). If \( \Gamma_{H^--}^{\text{tot}} \gg \sigma_{\sqrt{s}} \), we would place scan points within \( \pm \Delta m_{H^--} \) of \( m_{H^--} \) at intervals of roughly \( \Gamma_{H^--}^{\text{tot}} \) and devote the appropriate luminosity per point, as determined by \( BF(H^-- \rightarrow \ell^-\ell^-) \), to ensure that a signal is seen if present at that scan point. If \( \Gamma_{H^--}^{\text{tot}} \ll \sigma_{\sqrt{s}} \), we would place points at intervals of roughly \( \sigma_{\sqrt{s}} \) and the required luminosity per point would be determined by \( \Gamma(H^-- \rightarrow \ell^-\ell^-)/\sigma_{\sqrt{s}} \).

If, at the LHC, the \( H^-- \rightarrow H^-W^- \) decay is not observed and the bi-lepton mass distributions are consistent with resolution, it will not be possible to ab initio set up a scan procedure that is guaranteed to find the \( H^- \). The problem is that \( \Gamma_{H^--}^{\text{tot}} \) could be \( \ll \sigma_{\sqrt{s}} \), in which case knowledge of \( \Gamma(H^-- \rightarrow \ell^-\ell^-) \) is required [see Eq. (17)] to determine the luminosity per scan point needed to exclude or discover the \( H^- \). This situation applies also if the representation-dependent value for \( \Gamma(H^-- \rightarrow H^-W^-) \) computed (using an LHC measurement of the \( H^- \) mass) is \( \ll \sigma_{\sqrt{s}} \). This is because \( \Gamma(H^-- \rightarrow \ell^-\ell^-) \) cannot be estimated from the computed \( \Gamma(H^-- \rightarrow H^-W^-) \) value, there being no LHC measurement of \( BF(H^-- \rightarrow H^-W^-)/BF(H^-- \rightarrow \ell^-\ell^-) \). In both the above cases, the best that one could do is to spread a certain amount of luminosity (say one year’s worth) over the \( m_{H^--} \pm \Delta m_{H^--} \) mass interval at intervals of \( \sigma_{\sqrt{s}} \). As will become apparent later, this would still allow \( H^- \) detection in \( \ell^-\ell^- \) collisions for quite small \( c_{\ell\ell} \) values.

Although the need to scan in order to center on \( \sqrt{s} \approx m_{H^--} \) is an important ingredient in assessing our ability to study the \( H^- \) in \( \ell^-\ell^- \) collisions, a full discussion of the various scan procedures is quite involved. For simplicity, I discuss here only the event rates that would arise for a scan point placed successfully at \( \sqrt{s} \approx m_{H^-} \). It is worth re-emphasizing that the \( L \) per scan point might be a possibly small fraction of the full \( L \) available. The discussion divides naturally into the \( \Gamma_{H^--}^{\text{tot}} \gg \sigma_{\sqrt{s}} \) and \( \Gamma_{H^--}^{\text{tot}} \ll \sigma_{\sqrt{s}} \) cases.

**A:** \( \Gamma_{H^--}^{\text{tot}} \gg \sigma_{\sqrt{s}} \).

This scenario is typical if the \( H^-- \rightarrow W^-H^- \) decay is allowed or if one or more of the \( c_{\ell\ell} \)’s is above \( 10^{-5} \). For \( \sqrt{s} \approx m_{H^-} \), the number of \( H^- \) events in \( \ell^-\ell^- \) collisions is

\[
N(H^--) \sim 2.5 \times 10^9 \left( \frac{100 \text{ GeV}}{m_{H^-}} \right)^2 \left( \frac{L}{5 \text{ fb}^{-1}} \right) BF(H^-- \rightarrow \ell^-\ell^-). \tag{18}
\]
If the $H^{--} \rightarrow H^- W^-$ decay mode dominates the total width, then $BF(H^{--} \rightarrow \ell^- \ell^-) \sim 0.3 \beta_{W-H^-}^3 (c_{\ell\ell}/10^{-5})$. For $\beta_{H^- W^-} = 1$, $c_{\ell\ell} \sim 10^{-5}$ and $m_{H^{--}} = 500$ GeV, we predict $3 \times 10^7 H^{-}$ events (dominated by the $H^- W^-$ final state) for a typical scan point luminosity of $L = 5$ fb$^{-1}$. (This large an integrated luminosity per scan point is more easily achieved at an $e^- e^-$ collider than at a $\mu^- \mu^-$ collider, even though the latter would be operated with $R = 0.1\%$ when $\Gamma_{H^-}^{tot}$ is large.)

As $\beta_{H^- W^-}$ decreases below 1, the number of events grows rapidly.$^7$ A total of 100 $H^{-}$ events are produced for

$$c_{\ell\ell}|_{100\text{ events}} = 1.3 \times 10^{-12} \left( \frac{m_{H^{--}}}{100 \text{ GeV}} \right)^2 \beta_{H^- W^-}^3 \left( \frac{5 \text{ fb}^{-1}}{L} \right), \quad \Gamma_{H^-}^{tot} \gg \sigma_{\sqrt{s}}; \quad (19)$$

with $L = 5$ fb$^{-1}$ per scan point an observable signal occurs for very small $c_{\ell\ell}$ values.

If the $\tau \tau$ decay mode dominates $\Gamma_{H^-}^{tot}$, then $BF(H^{--} \rightarrow \ell^- \ell^-) \sim c_{\ell\ell}/c_{\tau\tau}$. For $L = 5$ fb$^{-1}$, $10^7 H^{-}$ events (almost entirely $\tau^- \tau^-$) would be obtained for $c_{\ell\ell}/c_{\tau\tau} = 0.1$ and $m_{H^{--}} = 500$ GeV. In this case, 100 $H^{-}$ events would correspond to $c_{\ell\ell}/c_{\tau\tau} = 4 \times 10^{-8}(m_{H^{--}}/100 \text{ GeV})^2$, again a very respectable sensitivity. Note that the phenomenology of this latter case of $\tau^- \tau^-$ dominance of $H^{--}$ decays is essentially independent of $\Gamma(H^{--} \rightarrow H^- W^-)$.

Extracting the actual magnitudes of the $c_{\ell\ell}$ coupling strengths from the $H^{--}$ observations would be a very important goal. In assessing our ability to perform this task, we shall presume that the efficiencies for the observed final states after cuts can be computed with sufficient reliability that the total number of $H^{--}$ events before cuts (summing over all modes) and the relative branching fractions $BF(H^{--} \rightarrow X)$ for the different observed final states $X$ can all be determined with reasonable accuracy. The following procedures would then be appropriate in the present $\Gamma_{H^-}^{tot} \gg \sigma_{\sqrt{s}}$ case.

(a) The partial width for any final state $X$ can be computed from $\Gamma_{H^-}^{tot}$ (which would have been directly measured as part of the scan in this case, if not already known from LHC measurements) as $\Gamma(H^{--} \rightarrow X) = \Gamma_{H^-}^{tot} BF(H^{--} \rightarrow X)$. If $X = H^- W^-$ is observed, then a Higgs-representation-dependent value of $\Gamma(H^{--} \rightarrow H^- W^-)$ can be computed and compared to the direct determination.

(b) The value of $BF(H^{--} \rightarrow \ell^- \ell^-)$ determined from $N(H^{--})$ [see Eq. (18)] can be used to compute $\Gamma(H^{--} \rightarrow \ell^- \ell^-) = BF(H^{--} \rightarrow \ell^- \ell^-) \Gamma_{H^-}^{tot}$ and, thence, $c_{\ell\ell}$, even if the $\ell^- \ell^-$ final state cannot be detected.

(c) If the $\ell^- \ell^-$ decay mode can be seen but is not dominant, then the above $\Gamma(H^{--} \rightarrow \ell^- \ell^-) = BF(H^{--} \rightarrow \ell^- \ell^-) \Gamma_{H^-}^{tot}$ determination can be used to compute $\Gamma(H^{--} \rightarrow X) = \Gamma(H^{--} \rightarrow \ell^- \ell^-) BF(H^{--} \rightarrow X)/BF(H^{--} \rightarrow \ell^- \ell^-)$ for the other observed $H^{--}$ final states $X$ as a cross check of the determinations in (a).

$^7$This growth occurs until $\Gamma_{H^-}^{tot}$ approaches $\sigma_{\sqrt{s}}$. 
If $H^{--} \rightarrow H^- W^-$ is either highly suppressed or virtual, then $\Gamma_{H^{--}}^{\text{tot}}$ can be very small if all of the $c_{\ell\ell}$'s are relatively small. At a $\mu^- \mu^-$ collider, $\sigma_{\sqrt{s}} = 3 \text{ MeV} \times \left( \frac{m_{H^{--}}}{100 \text{ GeV}} \right)$ can be achieved for $R = 0.003\%$, which implies [see Eq. (13)] that all $c_{\ell\ell}$'s would have to be below $10^{-7} \times \left( \frac{m_{H^{--}}}{100 \text{ GeV}} \right)^3$ before this situation would be forced upon us.

Using Eq. (16) for $\sigma_{\sqrt{s}}$ and the earlier result for $\Gamma(H^{--} \rightarrow \ell^- \ell^-)$, Eq. (17) predicts an event rate of

$$N(H^{--}) \sim 3 \times 10^9 \left( \frac{c_{\ell\ell}}{10^{-5}} \right) \left( \frac{0.2\%}{R} \right) \left( \frac{L}{5 \text{ fb}^{-1}} \right). \quad (20)$$

Thus, an enormous event rate results if $c_{\ell\ell}$ is within a few orders of magnitude of its upper bound. In this $\Gamma_{H^{--}}^{\text{tot}} \ll \sigma_{\sqrt{s}}$ case, the total number of events depends only on $c_{\ell\ell}$. From Eq. (20), we predict $100$ $H^{--}$ events for

$$c_{\ell\ell}|_{100 \text{ events}} \sim 3.3 \times 10^{-13} \left( \frac{R}{0.2\%} \right) \left( \frac{5 \text{ fb}^{-1}}{L} \right), \quad \Gamma_{H^{--}}^{\text{tot}} \ll \sigma_{\sqrt{s}}. \quad (21)$$

How many events are needed for observation will depend upon the final state.

What final states will be important? Assuming that $H^- W^-$ is two-body forbidden [as typically required for small $\Gamma(H^{--} \rightarrow H^- W^-)$], see Eq. (13), one or several of the $\ell^- \ell^- $ modes would dominate unless all the $c_{\ell\ell}$'s are extremely small, in which case the $H^- W^-$, $H^- W^-$ semi-virtual three-body modes would be dominant. The precise cross-over point between the $\ell^- \ell^-$ modes and the semi-virtual modes depends on details. Of course, if the $H^{--}$ is observed at the LHC or NLC, we will know ahead of time what final state to look in and its detailed characteristics, even if the semi-virtual final state is dominant. Only the latter semi-virtual modes and the $\ell^- \ell^-$ final state would have significant backgrounds at an $\ell^- \ell^-$ collider.

How many events are required for observation of the $H^{--}$? The semi-virtual $H^- W^-$ final state would probably have the least distinctive signature, and we could pessimistically assume that 1000 events would be required for $H^{--}$ detection if this is the dominant final state mode. The $\ell^- \ell^-$ final state (same $\ell$ as for the initial state) would also have a large background. Presumably at least 100 events would be needed before applying cuts to tame the direct $\ell^- \ell^- \rightarrow \ell^- \ell^-$ background. Perhaps a similar number would be required for the $H^- W^-$ final state when it is not virtual and for the $\tau^- \tau^-$ final state. Fewer than 10 events would presumably suffice for the $\mu^- \mu^-$ final state in $e^- e^-$ collisions and the $e^- e^-$ final state in $\mu^- \mu^-$ collisions. If one adopts the prejudice that $c_{ee} \ll c_{\mu\mu}$ and $c_{\tau\tau}$, then the $\mu^- \mu^-$ and $\tau^- \tau^-$ final states would probably dominate $H^{--}$ decays and 10 to 100 events would suffice in $e^- e^-$ collisions.

Even if 1000 events are required, Eq. (21) implies dramatic sensitivity. For $L = 5 \text{ fb}^{-1}$ per scan point and requiring 1000 events, in $e^- e^-$ collisions we are able to observe a signal for $c_{ee}$ values that are nearly 7 orders of magnitude smaller.
than current limits. We can probably do even better in the case of \( c_{\mu\mu} \) using \( \mu^-\mu^- \) collisions, depending upon just how low in \( R \) one can go without losing too much luminosity in this latter case.

Determination of the basic partial widths of the \( H^{--} \) follows a slightly different strategy in this \( \Gamma_{H^{--}}^\text{tot} \ll \sigma \sqrt{s} \) case as compared to the \( \Gamma_{H^{--}}^\text{tot} \gg \sigma \sqrt{s} \) case. As before we presume that the total event rate and the branching ratios for observable final states can be obtained with reasonable accuracies.

(a) When \( \Gamma_{H^{--}}^\text{tot} \ll \sigma \sqrt{s} \), Eq. (20) shows that the total event rate is a direct measure of \( c_{\ell\ell} \) (equivalently \( \Gamma(H^{--} \rightarrow \ell^-\ell^-) \)).

(b) If the \( \ell^-\ell^- \) final state is observable, we can then compute \( \Gamma(H^{--} \rightarrow X) = \Gamma(H^{--} \rightarrow \ell^-\ell^-)BF(H^{--} \rightarrow X)/BF(H^{--} \rightarrow \ell^-\ell^-) \) for other observable channels \( X \). If \( X = \mu^+\mu^- \), \( \tau^+\tau^- \) is observed, this yields \( c_{\mu\mu}, c_{\tau\tau} \). If \( X = H^-W^- \) is observed, the value of \( \Gamma(H^{--} \rightarrow H^-W^-) \) can be compared to predictions for different Higgs representation choices. Finally, \( \Gamma_{H^{--}}^\text{tot} \) can be computed by summing all the partial widths determined as above. If \( \Gamma_{H^{--}}^\text{tot} \) is not a great deal smaller than \( \sigma \sqrt{s} \), a very precise scan of the \( H^{--} \) resonance might allow a direct measurement of \( \Gamma_{H^{--}}^\text{tot} \) which could then be compared to the value computed above.

(c) If the \( \ell^-\ell^- \) final state is not detected, and \( \Gamma_{H^{--}}^\text{tot} \) is so much smaller than \( \sigma \sqrt{s} \) that any sort of scan determination of \( \Gamma_{H^{--}}^\text{tot} \) is impossible, then a direct determination of the partial widths for \( X \neq \ell^-\ell^- \) channels will not be possible. In this case, the best that one can hope for is that the \( H^-W^- \) final state is detected, in which case the \( \Gamma(H^{--} \rightarrow H^-W^-) \) partial width can be computed using an assumed representation for the \( H^{--} \) and other partial widths can be obtained from the ratio of branching ratios.

Aside from direct \( s \)-channel resonance production, there are several other potentially interesting processes.

- **Exclusive \( H^{--}Z \) and \( H^{--}\gamma \) production in \( \ell^-\ell^- \) collisions.**

  The reaction \( \ell^-\ell^- \rightarrow H^{--}\gamma \) is essentially equivalent to using the bremsstrahlung tail in \( \sqrt{s} \) at the \( \ell^-\ell^- \) collider to self-scan for the \( H^{--} \). The \( H^{--}\gamma \) rate is proportional to \( c_{\ell\ell} \) as in the \( \Gamma_{H^{--}}^\text{tot} \ll \sigma \sqrt{s} \) limit of on-resonance production, but observable rates are only possible if \( c_{\ell\ell} \) is not too far below current bounds. If the \( H^{--} \) is discovered in this way, first measurements of \( m_{H^{--}} \) and \( c_{\ell\ell} \) would be obtained and one could then proceed to the \( s \)-channel scan for centering on \( \sqrt{s} \approx m_{H^{--}} \) as described above and a factory-like study of the \( H^{--} \).

- **Exclusive \( H^-W^- \) production in \( \ell^-\ell^- \) collisions.**

  This process can occur via a diagram with \( \nu_t \) exchanged in the \( t \)-channel, provided there is a \( H^- \rightarrow \ell^-\nu_t \) coupling. If the \( H^- \) is part of a triplet representation,
such a coupling would be automatically present in Eq. (8). The $\ell^-\ell^- \rightarrow H^-W^-$ production would be both an interesting way of observing any $H^-$ with a sizeable value for such a coupling and a way of determining the magnitude of the coupling.

- The value of polarization.

For all the above triplet Higgs production mechanisms mediated by a bi-lepton coupling, it would be very valuable to have the ability to polarize one or both of the lepton beams. The polarization dependence of the rates can allow a model-independent determination of the representation to which the triplet Higgs boson belongs. In particular, Eqs. (11) and (12) show the specific polarization choices for the incoming $\ell^-$ beams that yield a non-zero rate for $\ell^-\ell^- \rightarrow H^-s$-channel production for different choices of the $H^-$ representation. For example, we see that the production rate for a $Y = -2$ triplet $H^-$ would be suppressed if either of the $\ell^-$ beams is given right-handed polarization, since such an $H^-$ couples only to $\ell^-_L\ell^-_L$.

5. Conclusions

The Higgs sector remains one of the big unknowns of particle physics. Although, it could consist of only doublets and singlets, more exotic Higgs sectors remain a distinct possibility and are even required in some popular extensions of the Standard Model, in particular left-right symmetric models. In this review, we have delineated the essential role that an $\ell^-\ell^-$ collider would play in fully revealing the nature of a typical exotic Higgs sector, focusing on the case in which Higgs triplet fields are present. It would appear that construction of appropriate $\ell^-\ell^-$ accelerators (both $\ell = e$ and $\ell = \mu$) could become a very high priority in order to ensure our ability to fully explore the exact nature of an exotic Higgs sector that is first revealed by LEPII, Tevatron or LHC experiments.

References

1. J.F. Gunion and H.E. Haber, Nucl. Phys. B272, 1 (1986)[E: B402, 567 (1993)]; B278, 449 (1986).
2. H. Georgi and M. Machacek, Nucl. Phys. B262, 463 (1985).
3. R.S. Chivukula and H. Georgi, Phys. Lett. B182, 181 (1986).
4. M. S. Chanowitz and M. Golden, Phys. Lett. B165, 105 (1985).
5. J.F. Gunion, R. Vega and J. Wudka, Phys. Rev. D42, 1673 (1990).
6. J.F. Gunion, R. Vega and J. Wudka, Phys. Rev. D43, 2322 (1991).
7. P. Bamert and Z. Kunszt, Phys. Lett. B306, 335 (1993).
8. J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, The Higgs Hunter’s Guide (Addison-Wesley Publishing Company, Redwood City, CA, 1990).
9. T. Blank and W. Hollik, hep-ph/9703392.
10. U. Amaldi, W. de Boer and H Furstenau, Phys. Lett. B260, 447 (1991).
11. A sampling of references is: M. Bando, T. Sato and T. Takahashi, Phys. Rev. D52, 3076 (1995); B. Brahmachari, Phys. Rev. D52, 1 (1995); B. Brahmachari and R.N. Mohapatra, Phys. Lett. B357, 566 (1995); E. Ma, Phys. Rev. D51, 236 (1995); D.-G. Lee and R.N. Mohapatra, Phys. Rev. D62, 4125 (1995).
12. For a review and references, see J.L. Hewett, S. Thomas and T. Takeuchi, in *Electroweak Symmetry Breaking and New Physics at the TeV Scale*, edited by T.L. Barklow, S. Dawson, H.E. Haber and J.L. Siegrist (World Scientific, Singapore, 1996) pp. 548–649.

13. R. Vega and D.A. Dicus, Nucl. Phys. B329, 533 (1990).

14. Phenomenological considerations for Higgs triplet representations in left-right symmetric models are outlined in J.F. Gunion, J. Grifols, B. Kayser, A. Mendez, and F. Olness, Phys. Rev. D40, 1546 (1989); N.G. Deshpande, J.F. Gunion, B. Kayser, and F. Olness, Phys. Rev. D44, 837 (1991); K. Huitu and J. Maalampi, Phys. Lett. B344, 217 (1995).

15. T. Han, Int. J. Mod. Phys. A11, 1541 (1996).

16. J.F. Gunion, Int. J. Mod. Phys. A11, 1551 (1996).

17. T. Rizzo, Int. J. Mod. Phys. A11, 1563 (1996).

18. K. Huitu, J. Maalampi, A. Pietila and M. Raidal, Nucl. Phys. B487, 27 (1997).

19. K. Cheung, R.J.N. Phillips and A. Pilaftsis, Phys. Rev. D51, 4731 (1995).

20. R. Godbole, B. Mukhopadhyaya and M. Nowakowski, Phys. Lett. B352, 388 (1995).

21. Z. Chacko and R.N. Mohapatra, [hep-ph/9712359](https://arxiv.org/abs/hep-ph/9712359).

22. J.F. Gunion, C. Loomis and K.T. Pitts, Proceedings of the 1996 DPF/DPB Summer Study on High-Energy Physics, June 25—July 12, 1996, Snowmass, CO, edited by D.G. Cassel, L.T. Gennari, and R.H. Siemann (Stanford Linear Accelerator Center, 1997) pp. 603–607.

23. P. Frampton, Int. J. Mod. Phys. A11, 1621 (1996).

24. F. Cuypers, Nucl. Phys. B510, 3 (1997).

25. G. Barenboim, K. Huitu, J. Maalampi and M. Raidal, Phys. Lett. B394, 132 (1997).

26. R.A. Alanakyan, [hep-ph/9706383](https://arxiv.org/abs/hep-ph/9706383).