The Study of Parametric Factor in Attic Space for Winter Season

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Abstract. This research investigates the structure of an attic space from the perspective of thermal engineering. It helps in understanding the variation of flow and field strength due to natural convection. A critical review concludes that heat transfer inside a building is most complex for the attic space. The study examines the effect of the parametric factor on natural convection. It includes analysis for the winter conditions. The numerical model is simulated for different values of Rayleigh Number \( (Ra = 10^4 - 10^6) \) using ANSYS 19 considering air with Prandtl number 0.71. With the assumptions of having the uniform temperature over the wall and no-slip condition for walls, Results are demonstrated with the use of Isotherm, Streamline and Nusselt number. With the finding of plots, the study argues that S/L ratio of 0.8 is recommended for the attic space having good thermal performance.

1. Introduction

With the appeal for the need of the energy profitable building by the society, the view of architects and builders is extended towards the human comfort. The designing of attic space inside a building plays an essential role in the variation of the temperature [1]. For the heat transfer inside an attic space, natural convection is dominated at low values for the Rayleigh Number (above Ra = 10^3) [2].

Nomenclature

\textbf{Dimensionless Numbers}

\begin{align*}
Gr & \quad \text{Grashof Number} \\
Nu & \quad \text{Nusselt Number} \\
Pr & \quad \text{Prandtl number}
\end{align*}

\textbf{Greek Symbols}

\begin{align*}
\beta & \quad \text{Coefficient of thermal expansion}
\end{align*}
Dimensionless temperature

**Physics Constants**

- $g$: Gravitational Constant
- $H$: Altitude of physical domain
- $h$: Convection coefficient
- $k$: Thermal Conductivity of fluid
- $L$: Length of the bottom wall for present study domain; Length of heater for reference study
- $L_H$: Total length of hot wall/s
- $P$: Pressure of fluid inside enclosure
- $S$: Length of the top wall for present study; Distance between center axis and mid-point of the heater located in reference study
- $T_H$: Temperature of hot wall
- $T_L$: Temperature of cold wall
- $U, V$: Velocity of Air
- $W$: Length of the half of bottom wall in reference study

Many studies are executed considering different heat load conditions and boundary heat fluxes for the rectangular, triangular and other possible attic space geometries.

In 1983, Bejan et. al. [3] considered half of the triangular domain with the bottom wall at low temperature, the inclined wall at high temperature, and the vertical wall as adiabatic. Rayleigh value in the range of $10^1$ - $10^5$ was investigated for the value of Aspect Ratio of 0.2, 0.4, and 1.0. Thymol blue pH indicator is used for the visualization of the flow and the study is concluded with the plot of Nusselt number and Rayleigh Number for the heat transfer. For the same year, Bejan et. al. [4] examined right-angle triangular enclosure for the night time attic space problem. The problem was focused on transient numerical simulation considering variable parameters as Rayleigh number, Prandtl number and aspect ratio.

Similarly, Salmun [5] and [6] investigated half of triangular domain considering vertical wall as insulated wall. Numerical simulation analysis for the time-dependent problem was investigated using two different techniques i.e. single convective cell and multi-cellular regime. Single convective cell was considered for the low value of the Rayleigh number which was resulted with conduction as mode of heat transfer. Multi-Cellular regime included higher value for the same parameter which was resulted with the convection flow regime in an enclosure.

In 2001, Asan [7] investigated numerical simulation for the triangular domain to check over transition from single cell to multi cell flow. The study was concluded by representing the influence of Rayleigh number and Aspect ratio on the temperature and the flow field.

In 2002, Haese and Teubner [8] investigated triangular enclosure with the consideration of realistic attic structure for night time duration. The study was concludes with the contradictory statement from Asan [7] that with the decrease in aspect ratio, the shift of multi-cellular flow is accelerated.

Various research work for the analysis of natural convection inside a square or rectangular shaped enclosure is conducted. Bejan et. al. [9] investigated the effect of heat input on natural convection inside a square cavity. Aydin et. al. [10] analyzed the effect of aspect ratio for the rectangular shaped enclosure. The sidewall was considered at high temperature and top wall at the lower temperature. Sarris et. al. [11] considered sinusoidal temperature profile in the rectangular shaped enclosure for upper wall, while others were considered as the adiabatic wall. He concluded with the observation over aspect ration that intensity of fluid circulation increases with increase in aspect ratio. Basak et. al. [12] stated in the reference of his observation for the
square cavity that overall heat transfer rate will be higher in case of the uniform wall heating case in comparison with the non-uniform heating case.

Anuj et. al. [13] discussed simulative analysis for the study of natural convection inside the various shapes of the enclosure. The study has been executed for finding the most appropriate shape of the enclosure in natural convection with the maximum rate of heat transfer. Different values for the Grashof Number and Aspect Ratio has been considered which lies in the range of \(10^4 - 10^8\) and 0.2 - 0.5 respectively.

Maukalled et. al. [14], executed a numerical analysis over a trapezoidal shaped enclosure with the mounted baffles on the upper wall. Two conditions were examined as one with the short vertical wall at low temperature and long vertical wall at high temperature while the second condition will be vice-versa. The study has been executed for the four Rayleigh Numbers and three Prandtl Number.

Natural convection inside a partially divided trapezoidal has been examined by Emin et. al. [15]. The model has been simulated for the summer and winter condition separately. The horizontal divider has been placed to oppose the buoyancy which results in weaker flow strength while trapezoidal with two internal baffles and inclined top wall has been simulated by Silva [16]. The thermal model has been formed with the isothermal left heated and isothermal right cooled walls. Study has been executed for various values of Rayleigh Number (\(10^3 \leq Ra \leq 10^6\)), Baffles height \( (H_b = H/3, 2H/3 and H) \) and Prandlt No. (\(Pr. = 0.7, 10\) and 130).

Basak et. al. [17] [18] simulated a trapezoidal cavity for the study of natural convection. The model has been designed considering top wall as insulated wall and vertical wall as linearly heated while bottom wall as uniformly heated wall. Simulation has been carried out on various values of Rayleigh Number (\(10^3 - 10^5\)) and Prandtl Number (\(Pr. = 0.7-1\)). He also analyzes the domain for the study of entropy generation inside an enclosure and concluded that with the increase in Prandtl Number entropy generation increases. Basak concludes the best design on the basis of low value for the entropy generation instead of Nusselt Number.

In 2013, Ramakrishna et. al. [19] discussed entropy generation with the effect of fluid friction and heat transfer inside a trapezoidal shaped enclosure. A physical model for the study has been considered same as in Basak [17].

Baytas [20] executed a numerical study for the steady state condition of natural convection inside an inclined trapezoidal-shaped enclosure. The top surface has been kept at low temperature while bottom at high temperature. Darcy and energy equation are solved numerically using an Alternative Direction Implicit (ADI) finite different method. The study has been concluded with the flow and heat transfer characteristics by using streamlines, isotherm and Nusselt number for the various values of Rayleigh Number, Aspect Ratio and Inclined Angle.

B. V. R. Kumar et. al. [21] concluded that Nusselt number increases with the increase in the value of the Rayleigh Number and Grashof Number. The study also observed that Nusselt number increases with the increase in angle of inclination of the side wall.

Kuyper et. al. [22] investigated the influence of the angle of inclination in natural convection with respect to different value for the Rayleigh numbers.

Literature review on the topic brings out the gap to find the optimum shape of an attic corresponding to triangle and rectangle. Current article performs the numerical investigation of natural convection with variable parametric factor of attic space.

2. Problem Modeling

2.1. Geometry

The geometry is sketched in figure 2. The cavity is filled with air having initial temperature of 300K. Here, L is the length of the bottom wall, S is length of the top wall, h represents height of the cavity, \(T_c\) is the temperature uniformly applied on bottom wall while other wall at \(T_h\). \([T_c\ represents\ temperature\ at\ low\ value\ while\ T_h\ represents\ temperature\ at\ high\ value.]\)
For the analysis of the different shapes considering fixed value of H, ratio of the S and L is varies from 0 to 1. Here 0 represents pure triangular domain and 1 represents pure rectangle. While in between of 0 and 1, geometry represents trapezoidal shape with different inclination angles.

Figure 1: A schematic representation of geometry with operating and boundary conditions of the attic.

2.2. Governing Equations
For the modelling of problem, few assumptions are considered as:

(i) Two-dimensional laminar flow
(ii) Neglected viscous dissipation
(iii) Absence of internal heat source or sink
(iv) Newtonian Fluid.
(v) Zero Compressibility effects

With the help of Boussinesq approximation and considering assumptions, following governing equations are considered for the analysis:

Continuity equation

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  

(1)

x-momentum equation

\[ U \left( \frac{\partial U}{\partial X} \right) + V \left( \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \]  

(2)

y-momentum equation

\[ U \left( \frac{\partial V}{\partial X} \right) + V \left( \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \left( \frac{Ra}{Pr} \right) \theta \]  

(3)

Temperature field equation

\[ U \left( \frac{\partial \theta}{\partial X} \right) + V \left( \frac{\partial \theta}{\partial Y} \right) = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \]  

(4)
where, \( X = x/L \), \( Y = y/L \), \( U = uH/\nu \), \( V = vH/\nu \).

In governing equations, \( U \) and \( V \) are air velocity component for \( X \) and \( Y \) respectively. \( P \) is the pressure of the fluid inside the enclosure, \( Ra \) is representing dimensionless Rayleigh number and \( Pr \) as Prandlt number, \( \theta \) is used for dimensionless temperature representation ( \( \theta = (T - T_c)/(T - T_h) \) ).

Rayleigh number is considered for the incorporation of buoyancy driven fluid i.e. Air inside an attic space. \( Ra \) is presented with the product of two other dimensionless parameters, Grashof Number (Gr) and Prandlt number (Pr). Grashof number is defined with the help of different parameters, includes thermal operating conditions, geometry of physical model and geographical parameter. It is formulated as:

\[
Gr = \frac{g\beta H^3(T_h - T_L)}{\nu^2} \tag{5}
\]

where \( g \) is the gravitational constant, \( \beta \) is the coefficient of thermal expansion and \( \nu \) as viscosity of the air.

3. NEED CORRECTION

Physical two dimensional model for the numerical simulation is designed with the help of ANSYS 19. Figure 2 explains the geometry of the model.

![Figure 2: Physical Model](image)

Above model is included with following assumptions:

(i) Two-dimensional laminar flow
(ii) Neglected viscous dissipation
(iii) Absence of internal heat source or sink
(iv) Newtonian Fluid.
(v) Zero Compressibility effects

For the model represented in figure 2, ratio value of "S" and "L" is considered in between of zero (0) and one (1). It provides the different shapes of trapezoidal enclosure in combination of rectangle and triangle. The heat load condition is considered with the consideration of sun rays normal to the surface of the earth, upper, left and right walls of the attic are considered as hot wall while lower wall as cold. This condition can also be defined as noon of the day. This condition is stated as Full Blown Heat Load Condition.
Mesh structure for the model is represented in figure 3 for the value of S/L ratio of 0, 0.5 and 1. Dense area of mesh structure is considered near all the walls of an attic space, for the generation of good quality boundary later.

Figure 3: Mesh structure for the physical domain (a) S/L = 0; (b) S/L = 0.5; (c) S/L = 1

From figure 2 and assumptions, dimensionless form of governing equations for the basic conservation of laws of mass, momentum and energy can be written as:

**Continuity equation**

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  \hspace{1cm} (6)

**x-momentum equation**

\[
U\left(\frac{\partial U}{\partial X}\right) + V\left(\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)
\]  \hspace{1cm} (7)

**y-momentum equation**

\[
U\left(\frac{\partial V}{\partial X}\right) + V\left(\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \left(\frac{Ra}{Pr}\right)\theta
\]  \hspace{1cm} (8)

**Temperature field equation**

\[
U\left(\frac{\partial \theta}{\partial X}\right) + V\left(\frac{\partial \theta}{\partial Y}\right) = \frac{1}{Pr}\left[\left(\frac{\partial^2 \theta}{\partial X^2}\right) + \left(\frac{\partial^2 \theta}{\partial Y^2}\right)\right]
\]  \hspace{1cm} (9)

where, \(X = x/L\), \(Y = y/L\), \(U = uH/\nu\), \(V = vH/\nu\), \(\theta = (T - T_C)/(T - T_H)\)

Boundary conditions for the domain introduces as:

Velocity on walls,

\[
U = V = 0
\]  \hspace{1cm} (10)

Temperature on adiabatic walls,

\[
\frac{\partial \theta}{\partial X} = 0
\]  \hspace{1cm} (11)
For the hot wall,

$$\theta = 1$$  \hspace{1cm} (12)

For the cold wall,

$$\theta = 0$$  \hspace{1cm} (13)

Dimensionless input parameter: Grashof Number,

$$Gr = \frac{g \beta H^3 (T_H - T_L)}{\nu^2}$$  \hspace{1cm} (14)

Dimensionless output or resulting parameter: Nusselt Number,

$$Nu = \frac{h H}{k}$$  \hspace{1cm} (15)

4. Validation of model

Vivek, Sharma and Balaji [23] investigated tilted enclosure of square shaped in the presence of air as fluid. To validate the integration model for radiation, square enclosure has been considered for the analysis of pure convection. Model for the Rayleigh Number $10^5$ has been simulated for different values of aspect ratio considering no slip on wall. Table 1 representing comparison of numerical simulation with the numerical simulation.

Table 1: Comparison of present pure convection Nusselt Number with published result for $Ra = 10^5$

| Aspect Ratio | Vivek et. al. [23] | Present |
|-------------|---------------------|---------|
| 1           | 4.56                | 4.55    |
| 2           | 4.15                | 4.27    |
| 3           | 3.85                | 4.10    |
| 5           | 3.50                | 3.81    |

5. Results and Discussion

For the different shapes (defined with S/L Ratio) and for the different values of Rayleigh Number (defined with the difference of temperature between walls) simulation is discussed in three forms isotherm lines, streamlines and Nusselt Number.

For the heat flow analysis, isotherm lines are represented for the different value for S/L Ratio and Rayleigh Number. Isotherm line is used to represent the region with the same temperature inside the enclosure. It also represents the strength of the thermal field inside the enclosure.

5.1. Winter Condition

This section will discuss the conditions for the days of the winter. It is defined in reverse of the summers. Top and Side walls are to be considered at low temperature i.e. cold wall and bottom wall is considered at high temperature i.e. hot wall.
5.1.1. Fluid Flow Analysis. With the rise of flow in the center of the attic and falling down with inclined/side wall at low temperature, figures 4, 5, 6 are representing streamlines for Rayleigh number $10^4, 10^5$ & $10^6$ and S/L Ratio of 0, 0.5 and 1.0.

![Streamlines for different S/L Ratios](image)

**Figure 4: Streamlines at Rayleigh Number $10^4$ for different S/L Ratio**

From the mirror images of streamline, clockwise and anti-clockwise rotational vortexes are observed which are formed due to natural convection. Following observations are pointed at Rayleigh Number $10^4$, figure 4:

(i) At S/L = 1, Flow pattern is observed very similar to the symmetric pattern in attic space.
(ii) With the rise of S/L Ratio, symmetry is disturbed for triangular shaped enclosure, which can also relate from the experimental analysis performed by Holtzman [24] in 2000.
With the increase in Rayleigh number, thermal current flow increases which effects the flow field strength inside an attic and heat transfer rate between the walls. With the buoyancy effect, particle velocity increase with the rise in temperature. So with the rise in flow field strength, number of clockwise and anti-clockwise loops increases.

Similar to the Ra $10^4$, symmetry is observed for the rectangular domain (S/L ratio = 1.0) and triangular domain (S/L ratio = 0) is not having any sequence of symmetry inside an attic space.
5.1.2. Heat Flow Analysis  Figure 7 represents stable lines for the isotherm at low value of Rayleigh Number. It indicates towards the presence of heat transfer with conduction to level of observation, which is further dominated with convection as we increases Rayleigh number (to be observed in figure 8 and figure 9).

With the observation of pattern for the formations of isotherm lines in figure 7 attic space structures near to the triangular domain are not symmetric in nature but with the increase in ratio of S/L symmetry is obtained for the trapezoidal and rectangular shaped attic structure.

![Figure 7: Isotherm Lines at Rayleigh Number 10^4 for different S/L Ratio](image)

Figure 7: Isotherm Lines at Rayleigh Number 10^4 for different S/L Ratio
Figure 8: Isotherm Lines at Rayleigh Number $10^5$ for different S/L Ratio

Figure 9: Isotherm Lines at Rayleigh Number $10^6$ for different S/L Ratio
In figure 8 and 9, isotherm lines are dense adjacent to the walls and expanded in between space. It is due to increase in heat current flow between the walls with the increase in temperature, which is representing the mode of transfer to be convection.

In figure 10, variation of Nusselt number with the change in Rayleigh number is represented. For the low difference of temperature, Ra=10^4, very obvious variation is observed with the higher value for triangular domain and lower value for the rectangular domain. As we increase value for the Rayleigh number, value for S/L at 0.8 is observed as lowest point of Nu.

6. Conclusion
This section elaborates the finding of present study for the shape of trapezoidal under three different conditions at different Grashof Numbers.

- Symmetrical pattern for the isotherm and streamline are observed for the rectangular domain and some of trapezoidal domains. But its symmetry disturbs with the decrease in ratio of S/L.
- With the rise of Rayleigh Number, number of loops for the fluid flow increases due to increase in strength of fluid flow.
- At S/L ratio of 0.8, minimum value for Nu is observed. which drives optimum value for the attic space as parametric factor will be S/L =0.8.
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