Predicting the time of corrosion damage to a plate with a deep double-sided external undercut under stretching

L H Talibly 1,2, F B Imranov3, A M Jafarova4
1 Azerbaijan National Aviation Academy, Baku, Mardakan Prospect, 30, AZ1045, Azerbaijan
2 Institute of Mathematics and Mechanics of the National Science Academy of Azerbaijan (NSAA), Baku, B.Vagabzade street, 9, AZ1141, Azerbaijan
3 Institute of Physics of NSAA, Baku, G. Javid Avenue, 131, AZ1141, Azerbaijan
4 Institute of Systems Management of NSAA, Baku, B.Vagabzade Street, 9, AZ1141, Azerbaijan
ltalybly@yahoo.com

Abstract. The statement is formulated and the solution of the problem of determining the time of corrosion destruction of an elastic homogeneous plate with a deep bilateral external recess is given. A plate of corrosive material is subjected to bilateral stretching in a corrosive environment in a direction perpendicular to the axis of the recesses. The well-known solution of the problem of the stress state of this plate, obtained by H. Neuber, is used. The following cases are considered: a) the effect of stress, which occurs in the plate when it is stretched, on the corrosion process is negligible; b) the effect of plate voltage on the corrosion process is significant. In both cases, the dependences of the change in the area of the narrowest cross-section of the plate on time are found. The time of complete corrosion wear and the time of corrosion cracking of the plate before its complete wear are also determined.

Introduction
It is known that the combined effect of mechanical stress and aggressive environment on structural elements leads them to corrosion-mechanical destruction. The process of corrosive destruction of bodies has a significant effect on the stress that occurs in them under the influence of external factors (force, temperature, etc.) [1-6]. Corrosion cracks propagate perpendicular to the direction of action of the tensile stress during elastic and non-elastic deformations [7]. Significant works are devoted to the theoretical definition of the corrosion strength of structures, among which works of a general theoretical nature can be noted [2, 4, 5, 8-11], having applied values and works devoted to the determination of the corrosion strength of a specifically considered structure [12-16].

The study was carried out with the support of SOCAR in the framework of the project 12LR-MMA
This article attempts to theoretically predict the time of corrosion damage to a plate with a deep symmetrical two-sided undercut with its elastic stretching in an aggressive medium in the direction perpendicular to the axis of the undercut. Plate material is considered homogeneous and isotropic. Designs in the form of such plates are used in many industries - in engineering, construction, transport, shipbuilding, etc. Under the corrosion destruction of the plate, we understand its corrosion wear and corrosion cracking in an aggressive environment under stress. In this case, the corrosion wear of the plate will be called the process of reducing its thickness due to the separation
of corrosion products from the action of aggressive environment and tensile force. The corrosion cracking of the plate is the discontinuity of the plate under the conditions of an aggressive medium and the specified force. The process of corrosion wear of the plate can be continued until its thickness completely disappears, and its corrosion cracking can take place at any time. The purpose of this article is to determine the time of complete wear and the time of cracking of the considered plate from the properties of an aggressive medium, the effective tensile force, the mechanical and geometrical parameters of the plate.

Materials of scientific research

A plate of thickness \( h \) with a deep symmetrical two-sided undercut is subjected to bilateral stretching in a corrosive environment with a force \( P \) (Fig. 1).

![Fig. 1](image)

In this case, it deals with a construction with a stress concentrator. In [17] it is shown that with an increase in the depth of the undercut, the concentration coefficient approaches a limit value that does not depend on the depth of the undercut; only the curvature at the bottom of the undercut has a significant effect on the concentration coefficient. The shape of the undercut in the rest of the plate has little effect on the concentration ratio. Therefore, in order to facilitate computational procedures, the undercut is taken in the form of a hyperbola.

In [17], the problem of determining the stress components that arise in the plate under consideration in the absence of the influence of a corrosive environment was solved. In the process of corrosion, the plate sizes are subject to change. Let be \( \rho \)- undercut curvature radius, \( a \)- half the width of the narrowest part of the plate. In the process of corrosion values \( \rho, a, h \) depend from time \( t \): \( a = a(t), \rho = \rho(t) \) and \( h = h(t) \). As a characteristic size, we take the value \( s(t) = 2a(t)h(t) \). \( t \) characterizes the change of the narrowest section of the plate in the corrosion process. Wherein \( ds/dt < 0 \). We assume that in the process of corrosion the ratio \( a/\rho \) remains constant, which is the case, for example, in the case of \( \rho \), \( a \) in the process of corrosion changes one common parameter \( \lambda(t); a = a(t); \rho = \rho(t) = \rho_0 \lambda(t); \). Here \( \rho_0 = \text{constant} \). Function \( \lambda(t) \) satisfies the conditions \( \lambda(0) = 1 \), \( d\lambda(t)/dt < 0 \). Согласно [17], the maximum stress in the plate under consideration occurs at the bottom of the undercut and has the expression:

\[
\sigma_{\text{max}} = \frac{P}{s(t)} \frac{a}{\rho} \left(\frac{a}{\rho}\right).
\]
\[
F\left(\frac{a}{\rho}\right) = \frac{2\left(1 + \frac{a}{\rho}\right)\left[\frac{a}{\rho}\right]}{\left(1 + \frac{a}{\rho}\right)\arctg\left[\frac{a}{\rho} + \frac{a}{\rho}\right]}.
\]

The quantity \(a/\rho\) is called the undercut curvature [17]. At the initial moment of the corrosion process from the formula (1) follows:

\[
\sigma_{\text{max}}(0) = \frac{P}{s_0} F\left(\frac{a}{\rho}\right),
\]

where \(s_0\) - the initial area of the narrowest cross-section of the plate.

Consider the following independent cases of interest.

1. The case of insignificant influence on the corrosion process of a plate of mechanical stress. In this case, the influence of a mechanical stress on the rate of the corrosion process can be neglected and the equation for \(s(t)\) can be taken as [2]:

\[
\frac{ds(t)}{dt} = -k
\]

Here, the value \(k\) of the experimentally determined characteristic of the system “material-aggressive medium” is the corrosion rate. The solution of equation (4) with the initial condition \(s(0) = s_0\) is:

\[
s(t) = s_0 - kt.
\]

With full plate wear \(s(t)=0\). In this case, from equation (5) follows

\[
\frac{s_0}{k} = t_0.
\]

Here \(t_0\) - time of complete wear of the plate in the absence of the influence of mechanical stress on the corrosion process. This time is the largest of all times of existence of the plate.

Let \(t_0^b\) - time cracking plate in the case when you can neglect the effect of stress on its corrosion. It will be natural if we accept by the condition of plate cracking the condition of equality of the maximum stress and the strength of the plate material:

\[
\sigma_{\text{max}} \bigg|_{t_0^b} = \sigma_b.
\]

It is obvious \(t_0^b < t_0\) and volume \(s(t_0^b) = s_0 - k t_0^b\) is non-zero. Using (1) in (7), we obtain the formula for time \(t_0^b\):

\[
t_0^b = \frac{s_0}{k} - \frac{P}{k \sigma_b} F\left(\frac{a}{\rho}\right).\]

Using the condition of the existence of cracking time \((t_0^b > 0)\), from (8) define the force limit \(P\):

\[
P_{\text{lim}} < \frac{s_0 \sigma_b}{F\left(\frac{a}{\rho}\right)}.
\]

Calculate the ratio \(t_0^b / t_0\), where \(t_0^b\) and \(t_0\) determined by formulas (8) and (6) respectively:
\[
\frac{t_0^b}{t_0} = 1 - \frac{P}{s_0 \sigma_b}.
\]  

(9)

We take into account formulas (3) in (9). Will have a relationship \( \frac{t_0^b}{t_0} \), expressed through relationships \( \frac{\sigma_{\text{max}}(0)}{\sigma_b} \):

\[
\frac{t_0^b}{t_0} = 1 - \frac{\sigma_{\text{max}}(0)}{\sigma_b}.
\]

2. The case of a significant effect of mechanical stress on the corrosion process of the plate. In this case, according to [2], for the rate of the corrosion process we will use the equation:

\[
\frac{ds(t)}{dt} = -k - n\sigma_{\text{max}}(t).
\]

(10)

Here, the values of \( k \) and \( n \) are considered to be characteristics of the “metal-aggressive environment” system and, for each system, are determined by experiments on corrosion destruction in accordance with the method presented by [2].

Note that the dependence of the rate of the corrosion process on the mechanical stress, generally speaking, should be nonlinear. However, for a certain system “metal - aggressive environment” this dependence is close to linear [2]. At the same time, equation (10) can be used as a first approximation to the solution of the problem of the corrosion resistance of materials.

We take into account formula (1) in equation (10) and determine the differential \( dt \) through the differential \( ds \). After some transformations we get:

\[
dt = -\frac{s(t)}{ks(t) + nPF\left(\frac{a}{\rho}\right)}ds.
\]

(11)

Integrating equations (11), we arrive at a relationship between time and characteristic size — the area of the narrowest section of the plate in the process of corrosion:

\[
t = -\frac{1}{k}(s - s_0) + \frac{n}{k^2}PF\left(\frac{a}{\rho}\right)\ln\frac{s + \frac{n}{k}PF\left(\frac{a}{\rho}\right)}{s_0 + \frac{n}{k}PF\left(\frac{a}{\rho}\right)}.
\]

(12)

Let \( t_{0\sigma} \) there is a time of complete wear of the plate \( (s = 0) \) in the case when the effect of mechanical stress on the corrosion process is significant. Taking \( s = 0 \) in (12), we determine the time \( t_{0\sigma} \):

\[
t_{0\sigma} = \frac{s_0}{k} + \frac{n}{k^2}PF\left(\frac{a}{\rho}\right)\ln\frac{n}{k}PF\left(\frac{a}{\rho}\right)\left(\frac{a}{\rho}\right).
\]

(13)

Taking into account (3), formula (13) can be rewritten in the form:

\[
t_{0\sigma} = \frac{s_0}{k} - \frac{ns_0}{k^2}\sigma_{\text{max}}(0)\ln\left(1 + \frac{1}{\frac{n}{k}\sigma_{\text{max}}(0)}\right).
\]

(14)

For comparison, the time to complete wear of the plate in question in cases of dependence \( (t_{0\sigma}) \) and independence \( (t) \) corrosion process from mechanical stress, determine their attitude. For this purpose, we will use formulas (13) and (6). In this case, we obtain the ratio:
\[
\frac{t_{0\sigma}}{t_0} = 1 + \frac{n}{k s_0} PF \left( \frac{a}{\rho} \right) \ln \left( \frac{n}{k} PF \left( \frac{a}{\rho} \right) \right) - s_0 + \frac{n}{k} PF \left( \frac{a}{\rho} \right). \tag{14}
\]

Formula (14) using (3) turns to

\[
\frac{t_{0\sigma}}{t_0} = 1 - n \sigma_{\text{max}}(0) \left( \frac{s_0}{k} \right) \ln \left( 1 + \frac{1}{n k \sigma_{\text{max}}(0)} \right) . \tag{15}
\]

From the formula (15) follows the obvious inequality: \( t_{0\sigma} < t_0 \).

Now \( t_{0\sigma} \) there is a time cracking plate in an aggressive environment under the action of tensile force \( P \). Time \( t_{0\sigma} \) determine the condition:

\[
\sigma_{\text{max}}(t_{0\sigma}) = \sigma_b. \tag{16}
\]

Replace in the formula (1) \( t \) through \( t_{0\sigma} \). The resulting expression will be taken into account in (16) have:

\[
P \left( \frac{a}{\rho} \right) \left( \frac{s_0}{k} \right) = \sigma_b. \tag{17}
\]

Formula (17) when using (3) goes to: \( \sigma_{\text{max}}(0) \left( \frac{s_0}{k} \right) = \sigma_b \). From here we get:

\[
s(t_{0\sigma}) = \frac{\sigma_{\text{max}}(0)}{\sigma_b}. \tag{18}
\]

Now replacing in (12) \( s(t) \) on \( s(t_{0\sigma}) \) by the formula (18) determine the time \( t_{0\sigma} \):

\[
t_{0\sigma} = \frac{s_0}{k} \left( 1 - \frac{\sigma_{\text{max}}(0)}{\sigma_b} \right) + \frac{n s_0}{k^2} \sigma_{\text{max}}(0) \ln \left( \frac{\sigma_{\text{max}}(0)}{\sigma_b} \left( 1 + \frac{n}{k} \sigma_b \right) \right). \tag{19}
\]

Formula (19) determines the corrosion cracking time of the plate under consideration, depending on the characteristics of the “metal-aggressive environment” system, the plate material strength, the initial area of its narrow cross section and the maximum stress occurring at the initial moment of the corrosion process.

Time relation \( t_{0\sigma} \) and \( t_0 \) by using (19) and (6) in:

\[
\frac{t_{0\sigma}}{t_0} = 1 - \frac{\sigma_{\text{max}}(0)}{\sigma_b} + \frac{n}{k} \sigma_{\text{max}}(0) \ln \left( \frac{\sigma_{\text{max}}(0)}{\sigma_b} \left( 1 + \frac{n}{k} \sigma_b \right) \right). \tag{19}
\]

The last relation implies an explicit inequality \( t_{0\sigma} < t_0 \).

Note that the time to cracking of the plate under consideration can also be determined by the phenomenological formula derived in [9] using the concept of accumulated corrosion damage.

**Conclusions**
The problem of determining the time of corrosion damage of a plate with a deep two-sided external undercut when it is stretched in an aggressive medium in a direction perpendicular to the axis of the undercut is set and solved. In cases where a) the effect of the stress that occurs in the plate when it is stretched, the corrosion process is negligible and b) the effect of the plate voltage on the corrosion process is significant, the time of complete corrosion wear and the time of corrosion cracking of the plate are determined.

References

[1] Evans Yu R 1962 Corrosion and oxidation of metals. Theoretical foundations and their practical application Moscow: Mashgiz 256 p
[2] Gutman E M 1981 Mechanochemistry of metals and corrosion protection Moscow: Metallurgy, 281 p
[3] Ulig G G, Revie R U 1989 Corrosion and fight with it. Introduction to corrosion science and technology Leningrad: Chemistry 456 p
[4] Ovchinnikov I G, Naumova G A 2000 Strength calculations of complex core and pipeline structures, taking into account corrosion damage Saratov: SSTU 207 p
[5] Lokoshchenko A M 2016 Creep and durable metals in corrosive environments Moscow: Fizmatlit 504 p
[6] Semenova I V, Florianovich G M, Khoroshilov A V 2002 Corrosion and corrosion protection Moscow: Fizmatlit 336 p
[7] Zhukov A M 1954 Strength and plasticity of brass in ammonia medium Izv. Academy of Sciences of the USSR No 4 pp 67–72
[8] Ovchinnikov I G 1988 On the methodology for constructing models of structures interacting with aggressive media Durability of materials and structural elements in aggressive and high-temperature environments. Interuniversity scientific collection pp 7-21
[9] Talybly L Kh 2003 On determining the time to corrosion fracture of metals Transactions of National Academy of Sciences of Azerbaijan, ser. of physical-technical and mathematical sciences. issue mathematical and mechanics pp 239–46
[10] Talybly L Kh 2011 About one case of derivation of the simplest formula of corrosive destruction under constant load Technology of machines, mechanisms, from nano-to macro-level. Materials of the 13th International Conference St. Petersburg V 2 pp 423-26
[11] Talybly L Kh, Narimanov S V 2012 On one formula for corrosion damage under voltage Heavy engineering No 8 pp 39-41
[12] Dolinsky V M 1967 Calculation of loaded pipes prone to corrosion Chemical and petroleum engineering No 2 pp 9-10
[13] Lokoshenko A M, Agakhi K A, Fomin L V 2013 Bending creep in bending in aggressive media Problems of mechanical engineering and machine reliability No 4 pp 70-75
[14] Pronina Yu G 2009 Uniform mechanochemical corrosion of a hollow sphere made of Prandtl material under the action of constant pressure Vestnik SPBPU Issue 1 pp 113-22
[15] Sedova O S, Pronina Yu G, Kabrits S A 2018 Corrosion of spherical elements under the action of pressure and uneven heating Deformation and destruction of materials No 1 pp 2-6
[16] Talybly L Kh, Gunbataliev E Z 2013 About one theoretical solution of the problems of corrosion destruction of a ring sector fixed at the end when bending by shearing force Mechanics - mechanical engineering No 2 pp 23-25
[17] Neuber G 1947 Stress Concentration Moscow - Leningrad: Gostekhizdat 204 p