Relativistic model of hidden bottom tetraquarks

D. Ebert\textsuperscript{1}, R. N. Faustov\textsuperscript{2} and V. O. Galkin\textsuperscript{1,2}

\textsuperscript{1} Institut für Physik, Humboldt–Universität zu Berlin, Newtonstr. 15, D-12489 Berlin, Germany
\textsuperscript{2} Dorodnicyn Computing Centre, Russian Academy of Sciences, Vavilov Str. 40, 119991 Moscow, Russia

The relativistic model of the ground state and excited heavy tetraquarks with hidden bottom is formulated within the diquark-antidiquark picture. The diquark structure is taken into account by calculating the diquark-gluon vertex in terms of the diquark wave functions. Predictions for the masses of bottom counterparts to the charm tetraquark candidates are given.

PACS numbers: 12.40.Yx, 14.40.Gx, 12.39.Ki

During last few years a significant experimental progress has been achieved in meson spectroscopy. Many new heavy meson states have been discovered. Some of them are long-awaited states (such as $h_c$, $\eta_b$, etc.) while other states (such as $X(3872)$, $Y(4260)$, $Z(4430)$, etc.) cannot be easily fitted in the simple $q\bar{q}$ picture of mesons \cite{1}. These anomalous states and especially the charged ones can be considered as indications of the existence of exotic multiquark states which were predicted long ago \cite{2,3}. Very recently the Belle Collaboration \cite{4} observed an enhancement in $e^+e^- \to \Upsilon(1S)\pi^+\pi^-, \Upsilon(2S)\pi^+\pi^-$, and $\Upsilon(3S)\pi^+\pi^-$ production which is not well-described by the conventional $\Upsilon(10860)$ line shape. One of the possible explanations is a bottomonium counterpart to the $Y(4260)$ state which may overlap with the $\Upsilon(5S)$. New data on higher bottomonium excitations are expected to come in near future from KEKB, LHC and Tevatron. It is important to note that it is planned to search for bottom partners of anomalous charmonium-like states at LHC.

In papers \cite{5,6} we calculated masses of the ground and excited states of heavy tetraquarks in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. It was found that most of the anomalous charmonium-like states could be interpreted as the diquark-antidiquark bound states. Here we extend this analysis to the consideration of the excited tetraquark states with hidden bottom. As previously, we use the diquark-antidiquark picture to reduce a complicated relativistic four-body problem to the subsequent two more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed from light and bottom quarks in the colour antitriplet state. At the second step, a bottom tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that the diquark is not a point particle but its structure is explicitly taken into account by calculating the diquark-gluon vertex.

In the adopted approach the quark-quark bound state and diquark-antidiquark bound state are described by the diquark wave function ($\Psi_d$) and by the tetraquark wave function ($\Psi_T$), respectively. These wave functions satisfy the quasipotential equation of the
Schrödinger type \([7]\)

\[
\left( \frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R} \right) \Psi_{d,T}(p) = \int \frac{d^3q}{(2\pi)^3} V_{d,T}(p, q; M) \Psi_{d,T}(q),
\]

where the relativistic reduced mass is

\[
\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^2 - (m_1^2 - m_2^2)^2}{4M^3},
\]

and \(E_1, E_2\) are given by

\[
E_1 = \frac{M^2 - m_1^2 + m_2^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}.
\]

Here, \(M = E_1 + E_2\) is the bound-state mass (diquark or tetraquark), \(m_{1,2}\) are the masses of quarks (\(q\) and \(Q\)) which form the diquark or of the diquark (\(d\)) and antidiquark (\(\bar{d}\)) which form the heavy tetraquark (\(T\)), and \(p\) is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

\[
b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.
\]

The kernel \(V_{d,T}(p, q; M)\) in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. The explicit expressions for the corresponding quasipotentials \(V_{d,T}(p, q; M)\) can be found in Ref. [6].

The constituent quark masses \(m_b = 4.88 \text{ GeV}, m_u = m_d = 0.33 \text{ GeV}, m_s = 0.5 \text{ GeV}\) and the parameters of the linear potential \(A = 0.18 \text{ GeV}^2\) and \(B = -0.3 \text{ GeV}\) have been fixed previously and have values typical in quark models. The value of the mixing coefficient of vector and scalar confining potentials \(\varepsilon = -1\) has been determined from the consideration of charmonium radiative decays [7] and the heavy-quark expansion. The universal Pauli interaction constant \(\kappa = -1\) has been fixed from the analysis of the fine splitting of heavy quarkonia \(^3P_J\) - states [7]. In this case, the long-range chromomagnetic interaction of quarks vanishes in accordance with the flux-tube model.

At the first step, we take the previously calculated masses and form factors of the bottom diquarks [5]. The diquark interaction with the gluon field, which takes into account the diquark structure, is expressed through the form factor \(F(r)\) entering the vertex of the diquark-gluon interaction [8]. This form factor is determined through the overlap integral of the diquark wave functions. Our estimates showed that this form factor can be approximated with a high accuracy by the expression

\[
F(r) = 1 - e^{-\xi r - \zeta r^2}.
\]

The values of the masses and parameters \(\xi\) and \(\zeta\) for the bottom scalar diquark \([\cdots]\) and axial vector diquark \(\{\cdots\}\) ground states are given in Table II.

At the second step, we calculate the masses of heavy tetraquarks considered as the bound states of the bottom diquark and antidiquark. The explicit expression for the diquark-antidiquark interaction is given in [4]. In this picture of heavy tetraquarks both scalar \(S\) (antisymmetric in flavour \([Qq]_{S=0} = [Qq]\)) and axial vector \(A\) (symmetric in flavour
TABLE I: Masses $M$ and form factor parameters of bottom diquarks $\bar{S}$ and $A$ denote scalar and axial vector diquarks which are antisymmetric $\{\cdots\}$ and symmetric $\{\cdots\}$ in flavour, respectively.

| Quark content | Diquark type | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) |
|---------------|-------------|-----------|------------|----------------|
| $[b, q]$      | $S$         | 5359      | 6.10       | 0.55           |
| $\{b, q\}$   | $A$         | 5381      | 6.05       | 0.35           |
| $[b, s]$      | $S$         | 5462      | 5.70       | 0.35           |
| $\{b, s\}$   | $A$         | 5482      | 5.65       | 0.27           |

TABLE II: Masses of hidden bottom tetraquark ground (1$S$) states (in MeV) $\bar{S}$ and $A$ denote scalar and axial vector diquarks.

| State $J^{PC}$ | Diquark content | Mass $b\bar{q}b\bar{q}$ | Mass $b\bar{q}\bar{s}$ | Mass $b\bar{s}b\bar{s}$ |
|----------------|-----------------|--------------------------|-------------------------|--------------------------|
| 1$S$           | $S\bar{S}$      | 10471                    | 10572                   | 10662                    |
| $1^+\pm$       | $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ | 10492                    | 10593                   | 10682                    |
| 0$^+$          | $A\bar{A}$      | 10473                    | 10584                   | 10671                    |
| 1$^-$          | $A\bar{A}$      | 10494                    | 10599                   | 10686                    |
| 2$^+$          | $A\bar{A}$      | 10534                    | 10628                   | 10716                    |

$[Qq]_{s=1} = \{Qq\}$ diquarks are considered. As a result, a very rich set of tetraquark states emerges. However the number of states in the considered diquark-antidiquark picture is significantly less than in the genuine four-quark approach.

The previously calculated masses of the tetraquark ground (1$S$) states $\bar{S}$ and the corresponding open bottom thresholds are shown in Tables III IV. Note that most of the tetraquark states were predicted (in contrast to tetraquarks with hidden charm $\bar{S}$) to lie significantly below corresponding open bottom thresholds. The mass of the bottom counterpart to $X(3872)$ is predicted to be 10492 MeV. In Tables V VI we give predictions for the orbitally and radially excited tetraquark states with hidden bottom. Excitations only in the diquark-antidiquark system are considered.

Our model predicts three vector 1$^{--}$ tetraquark states with hidden bottom in the mass range 10807–10850 MeV. One of these tetraquarks is composed from a scalar diquark and antidiquark ($SS\bar{S}$). Two other 1$^{--}$ states contain an axial vector diquark and antidiquark with the total spin of the diquark and antidiquark $S$ equal to 0 and 2.

TABLE III: Thresholds for open bottom decays.

| Channel | Threshold (MeV) | Channel | Threshold (MeV) | Channel | Threshold (MeV) |
|---------|----------------|---------|----------------|---------|----------------|
| $BB$    | 10558          | $BB_s$  | 10649          | $B^+_sB^-_s$ | 10739           |
| $B\bar{B}^*$ | 10604       | $B^+B_s$ | 10695          | $B^+_sB^+_s$ | 10786           |
| $B^*B^*$ | 10650         | $B^*B^*_s$ | 10742          | $B^+_sB^-_s$ | 10833           |
**TABLE IV:** Masses of hidden bottom tetraquark excited $1P$, $2S$ states (in MeV). $S$ and $A$ denote scalar and axial vector diquarks; $S$ is the total spin of the diquark and antidiquark. ($C$ is defined only for neutral states).

| State $J^{PC}$ | Diquark content | $S$ | $bqbq$ | $bqb\bar{s}$ | $bsb\bar{s}$ |
|--------------|-----------------|-----|--------|-------------|-------------|
| $1P$         |                 |     |        |             |             |
| $1^{--}$     | $S\bar{S}$     | 0   | 10807  | 10907       | 11002       |
| $0^{±}$      | $(S\bar{A} ± SA)/\sqrt{2}$ | 1   | 10820  | 10917       | 11011       |
| $1^{±}$      | $(S\bar{A} ± \bar{S}A)/\sqrt{2}$ | 1   | 10824  | 10922       | 11016       |
| $2^{±}$      | $(S\bar{A} ± \bar{S}A)/\sqrt{2}$ | 1   | 10834  | 10932       | 11026       |
| $1^{--}$     | $A\bar{A}$     | 0   | 10850  | 10947       | 11039       |
| $0^{--}$     | $A\bar{A}$     | 1   | 10836  | 10934       | 11026       |
| $1^{--}$     | $A\bar{A}$     | 1   | 10847  | 10945       | 11037       |
| $2^{--}$     | $A\bar{A}$     | 2   | 10854  | 10952       | 11044       |
| $1^{--}$     | $A\bar{A}$     | 2   | 10856  | 10953       | 11046       |
| $3^{--}$     | $A\bar{A}$     | 2   | 10858  | 10956       | 11048       |
| $2S$         |                 |     |        |             |             |
| $0^{++}$     | $S\bar{S}$     | 0   | 10917  | 11018       | 11111       |
| $1^{±}$      | $(S\bar{A} ± \bar{S}A)/\sqrt{2}$ | 1   | 10939  | 11037       | 11130       |
| $0^{++}$     | $A\bar{A}$     | 0   | 10942  | 11041       | 11133       |
| $1^{--}$     | $A\bar{A}$     | 1   | 10951  | 11050       | 11142       |
| $2^{++}$     | $A\bar{A}$     | 2   | 10969  | 11067       | 11159       |

tetraquarks, with predicted masses 10807 MeV ($S\bar{S}$) and 10827 MeV ($A\bar{A}$), are bottom partners of $Y(4260)$ while the heavier one, with mass 10850 MeV ($A\bar{A}$), is the bottom partner of $Y(4360)$. Therefore a complicated structure of vector bottomonium states emerges in this mass range, which can be responsible for the anomalous production cross sections for $e^+e^- → \Upsilon(1S, 2S, 3S)\pi^+\pi^−$ observed by Belle. Their fit using a single Breit-Wigner resonance shape yielded a peak mass of $10889.6 ± 1.8 ± 1.5$ MeV. However, a more detailed experimental study is necessary to clarify this question. The bottom counterpart to the vector state $Y(4660)$ has predicted mass 11122 MeV. It is very important to search for the charged or strange states with hidden bottom. Their observation will be a direct proof of the existence of heavy tetraquarks. The masses of the bottom counterparts to charged $Z(4248)$ and $Z(4430)$ are predicted at around 10807 MeV and 10939 MeV, respectively.

It is necessary to emphasize that the observation of the bottom counterparts to the new anomalous charmonium-like states is very important since it will allow to discriminate between different theoretical descriptions of these states. Indeed theoretical models, such as the hybrid, molecular or diquark-antidiquark pictures etc., give significantly different results in the bottom sector (see also discussions in [9,10]).

In summary, we obtained predictions for the masses of tetraquarks with hidden bottom in the diquark-antidiquark picture. For the calculations we used the dynamical approach based on the relativistic quark model which was previously successfully applied in the charm sector.
TABLE V: Masses of hidden bottom tetraquark excited 1D, 2P states (in MeV). $S$ and $A$ denote scalar and axial vector diquarks; $S$ is the total spin of the diquark and antidiquark.

| State $J^{PC}$ | Diquark content | $S$ | $bqbq$ | $bqb\bar{s}$ | $bsb\bar{s}$ |
|----------------|-----------------|-----|--------|-------------|-------------|
| 1D 2$^+$       | $SS$            | 0   | 11021  | 11121       | 11216       |
| 1$^+$          | $(S\bar{A} \pm S\bar{A})/\sqrt{2}$ | 1   | 11040  | 11137       | 11232       |
| 2$^+$          | $(S\bar{A} \pm S\bar{A})/\sqrt{2}$ | 1   | 11042  | 11139       | 11235       |
| 3$^+$          | $(S\bar{A} \pm S\bar{A})/\sqrt{2}$ | 1   | 11045  | 11142       | 11238       |
| 2$^+$          | $AA$            | 0   | 11064  | 11162       | 11255       |
| 1$^-$          | $AA$            | 1   | 11060  | 11158       | 11251       |
| 2$^-$          | $AA$            | 1   | 11064  | 11161       | 11254       |
| 3$^-$          | $AA$            | 1   | 11066  | 11164       | 11257       |
| 0$^+$          | $AA$            | 2   | 11054  | 11152       | 11245       |
| 1$^+$          | $AA$            | 2   | 11057  | 11155       | 11248       |
| 2$^+$          | $AA$            | 2   | 11062  | 11159       | 11252       |
| 3$^+$          | $AA$            | 2   | 11066  | 11164       | 11257       |
| 4$^+$          | $AA$            | 2   | 11067  | 11165       | 11259       |
| 2P 1$^-$       | $SS$            | 0   | 11122  | 11221       | 11316       |
| 0$^-$          | $(S\bar{A} \pm S\bar{A})/\sqrt{2}$ | 1   | 11134  | 11232       | 11326       |
| 1$^-$          | $(S\bar{A} \pm S\bar{A})/\sqrt{2}$ | 1   | 11139  | 11236       | 11330       |
| 2$^-$          | $(S\bar{A} \pm S\bar{A})/\sqrt{2}$ | 1   | 11148  | 11245       | 11340       |
| 1$^-$          | $AA$            | 0   | 11163  | 11260       | 11353       |
| 0$^+$          | $AA$            | 1   | 11151  | 11248       | 11342       |
| 1$^+$          | $AA$            | 1   | 11161  | 11259       | 11351       |
| 2$^-$          | $AA$            | 1   | 11168  | 11265       | 11358       |
| 1$^-$          | $AA$            | 2   | 11143  | 11241       | 11333       |
| 2$^-$          | $AA$            | 2   | 11169  | 11266       | 11359       |
| 3$^-$          | $AA$            | 2   | 11172  | 11269       | 11362       |

sector. The tetraquark masses were obtained by numerical solution of the quasipotential wave equation with the corresponding relativistic potentials. The diquark structure was taken into account in terms of diquark wave functions. In our analysis we did not introduce any free adjustable parameters but used their values fixed from the previous considerations of heavy and light meson properties [7]. The correspondence between bottom and charm tetraquark candidates was discussed.

The authors are grateful to V. Matveev, G. Pakhlova and V. Savrin for support and discussions. One of us (V.O.G.) thanks M. Müller-Preussker and the colleagues from the particle theory group for kind hospitality. This work was supported in part by Deutscher Akademischer Austauschdienst (DAAD) (V.O.G.), the Russian Science Support Foundation (V.O.G.) and the Russian Foundation for Basic Research (RFBR), grant No.08-02-00582
(R.N.F. and V.O.G.).

[1] G. V. Pakhlova, arXiv:0810.4114 [hep-ex]; S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. 58, 51 (2008); E. S. Swanson, Phys. Rept. 429, 243 (2006); L. Maiani, A. D. Polosa and V. Riquer, New J. Phys. 10, 073004 (2008).

[2] R. L. Jaffe, Phys. Rev. D 15, 267 (1977); Phys. Rev. Lett. 38, 195 (1977); V. A. Matveev and P. Sorba, Lett. Nuovo Cim. 20, 443 (1977).

[3] A. M. Badalyan, B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 281, 85 (1987); A. B. Kaidalov, Surveys in High Energy Physics 13, 265 (1999).

[4] I. Adachi et al. [Belle Collaboration], arXiv:0808.2445 [hep-ex].

[5] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B 634, 214 (2006).

[6] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 58, 399 (2008).

[7] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 67, 014027 (2003).

[8] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 72, 034026 (2005); Phys. Lett. B 659, 612 (2008).

[9] W. S. Hou, Phys. Rev. D 74, 017504 (2006).

[10] M. Karliner and H. J. Lipkin, arXiv:0802.0649 [hep-ph].