Modified hyperbolic sine model for titanium dioxide-based memristive thin films

Raudah Abu Bakar1*, NurSyahirah Kamarozaman1, Wan FazlidaHanim Abdullah2, and Sukreen Hana Herman1, 2

1NANO-Electronic Centre (NET), Faculty of Electrical Engineering, UniversitiTeknologi MARA, Shah Alam, Selangor Darul Ehsan, Malaysia.
2Integrated Sensors Research Group, UniversitiTeknologi MARA, Shah Alam, Selangor Darul Ehsan, Malaysia.

E-mail: raudah088@salam.uitm.edu.my

Abstract. Since the emergence of memristor as the newest fundamental circuit elements, studies on memristor modeling have been evolved. To date, the developed models were based on the linear model, linear ionic drift model using different window functions, tunnelling barrier model and hyperbolic-sine function based model. Although using hyperbolic-sine function model could predict the memristor electrical properties, the model was not well fitted to the experimental data. In order to improve the performance of the hyperbolic-sine function model, the state variable equation was modified. On the one hand, the addition of window function cannot provide an improved fitting. By multiplying the Yakopcic’s state variable model to Chang’s model on the other hand resulted in the closer agreement with the TiO2 thin film experimental data. The percentage error was approximately 2.15%.

1. Introduction
In the past few years, memristor or memory resistor has been recognized as the fourth fundamental passive circuit element [1]. This newest device not only could relate the fluxes and charges relationship but also has the ability to remember its current state even without the external applied voltages[1-2]. Because of these, memristor can be incorporated in many applications including non-volatile memory device, neuromorphic computation system, artificial intelligence, analog computing and others [3-5].

To date, the memristive behavior a pinched-hysteresis current-voltage (I-V) characteristic has been discovered in many transition metal oxide materials such as titanium dioxide (TiO2), zinc oxide (ZnO), nickel oxide (NiO), tungsten oxide (WO3) and tantalum oxide (TaOx) [6-12]. Theoretically, the observed distinctive memristive properties in these materials are due to either the migration of oxygen vacancies or the formation of filamentary conducting path within the oxide layer [13].

Recently, a lot of studies on memristor modeling have been carried out to predict the performance of metal oxide-based memristive device and to allow successful memristor implementation in various applications [14-23]. Other than linear ionic drift model [14], models on hyperbolic sine function have been proposed to approximate the I-V relationship of metal-insulator-metal (MIM) configuration [21-23]. Unlike linear memristor model, this model suggests an exponential relationship between the current and applied voltage through a thin potential barrier [24]. Although using a hyperbolic sine function...
function in modeling the I-V relationship in MIM device satisfies both electronic measurements and material composition [25], only few of them were developed based on the experimental data. One of them is Chang et al. model [23]. In Chang’s model, both Schottky and tunneling effects were considered in the memristor I-V relationship. The state variable derivative on the other hand was included with the diffusion term.

Since the state variable provides the change in the resistance, modeling this function is therefore very important. Thus, in order to provide a well fitted model to the experimental data, the Chang’s state variable function was modified by adding the previously reported window functions [14-17]. Other than that, the inclusion of Yakopcic’s state variable function [25] was also examined. The model parameters were initially correlated to a sol-gel-derived spin-coated TiO₂ thin film annealed at 250°C for 20min and optimized. Various window functions were then added and the memristive behavior was analyzed. The characterization data were obtained from [26]. The simulation process was conducted using Linear Technology Simulation Program with Integrated Circuit Emphasis (LTspice).

2. Hyperbolic sine function model
The I-V relationship and state variable function for Chang’s model [23] are described by the following equations:

\[
I(t) = (1 - x(t))\left[1 - e^{V(t)}\right] + x(t)\gamma \sinh(\beta V(t))
\]

\[
\frac{dx}{dt} = 2[\eta_1 \sinh(\eta_2 V(t))]
\]

As given in (1), the I-V relationship is the combination of the Schottky barrier effect between the oxide layer and bottom electrode and the tunnelling effect through the MIM junction. The Schottky conduction becomes dominant if the state variable, \(x(t)\) is zero and vice versa when \(x(t)\) is 1. The state variable function, on the other hand, is the function of fitting parameters \(\lambda\), \(\eta_1\) and \(\eta_2\) which are used to shape the dynamics of the state variable equation [27]. The state variable function is then modified by adding an ion diffusion term to (2) to account for the natural oxygen vacancy (\(V_{ox}\)) diffusion [27]. The new equation is defined as:

\[
\frac{dx}{dt} = \lambda \left[\eta_1 \sinh(\eta_2 V(t)) - \frac{x(t)}{\tau}\right]
\]

where \(\tau\) is a time constant on the order of a few seconds.

3. Window function
In this study, the window functions developed by Strukov [14], Joglekar [15], Biolek [16] and Prodromakis[17] were considered to examine the effects of window function inclusion to state variable derivative function. Table 1 summarized the window function used in this study. The window function was initially introduced by Strukov et al. in order to compensate the boundary problem in the linear ionic drift model. This can be achieved by multiplying the state-variable derivative \((dx/dt)\) with window function \((f(x))\) as in (4). The window function will not only ensure the proper state-variable shaping near the boundary \([0, D]\) but also capable to include the nonlinear behaviour effects close to the boundary [25, 28].

\[
\frac{dx}{dt} = k(t) f(x), \quad k = \frac{\mu_{r} R_{ON}}{D}
\]
Table 1. Window functions described by Strukov [14], Joglekar [15], Biolek [16], and Prodromakis [17]

| Models     | Window Function (f(w)) | References |
|------------|------------------------|------------|
| Strukov    | \( f(x) = x(1-x) \)   | [14] (5)   |
| Joglekar   | \( f(x) = 1-(2x-1)^2p \) | [15] (6)   |
| Biolek     | \( f(x) = 1-(x-stp(i))^2p \) | [16] (7)   |
| Prodromakis| \( f(x) = j \left[\left(1-(x-0.5)^2+0.75\right)^p \right] \) | [17] (8)   |

4. Results and discussions

Prior to the modification of state variable equation, the Chang’s model was correlated to the I-V characteristic of TiO\(_2\) thin films prepared by spin coating sol-gel method. The thin films were fabricated using titanium (IV) isopropoxide (Ti(OCH(CH\(_3\))\(_2\))\(_4\)), glacial acetic acid (CH\(_3\)COOH), Triton X-100 (C\(_{34}\)H\(_{62}\)O\(_{11}\)), absolute ethanol (C\(_2\)H\(_5\)OH) and deionized water, which acted as the precursor, stabilizer, surfactant, and solvent. The coated TiO\(_2\) layers were then subjected to annealing process at 250°C for 20 min to enhance the films crystallinity. The I-V characteristic of the memristive device structure consisted of TiO\(_2\) thin film sandwiched between platinum (Pt) electrode and indium-doped tin oxide (ITO) coated glass substrate (Pt/TiO\(_2\)/ITO glass) was then characterized by a two-point probe measurement method using a Keithley 4200 semiconductor characteristic system. The detailed fabrication method and results are reported in [26]. Figure 1 shows the annealed TiO\(_2\) thin film I-V characteristic. As apparent in Figure 1, the TiO\(_2\) thin film exhibits a pinched hysteresis loop. From the I-V characteristic, the low and high state resistances, \( R_{ON} \) and \( R_{OFF} \) were calculated as 51 and 137 \( \Omega \) respectively.

![Figure 1. I-V Characteristic of TiO\(_2\) thin film](image)

The Chang’s SPICE model was simulated using the circuit shown in Figure 2a. Figure 2b shows the comparison between the simulated I-V characteristic results and the experimental data for TiO\(_2\) thin film. Using the fitting parameters \( \alpha = 3.195, \beta = 6.505 \times 10^{-3}, \gamma = 2.45 \times 10^{-3}, \delta = 3.25 \times 10^{-2}, x_{max} = 0.95, x_{min} = 0.05, drift_bit = 150, \lambda = 0.06, \eta_1 = 12.5 \times 10^{-6}, \eta_2 = 4, \tau = 1 \) resulted in an asymmetrical I-V characteristic with a percentage error of 4.75%.
Figure 2. (a) Schematic circuit for parameters optimization in LTspice (b) Simulation result for TiO$_2$ thin film when modeled using Chang’s hyperbolic sine function model for a sinusoidal input of 5 V

Figure 3 a-d show the simulation results for the combination of Chang’s model and various window function types. As can be observed from the I-V characteristics, the addition of window function to the state variable derivative function leads to the un-fitted memristive behaviour especially during the applied negative voltages. The improvement on the positive voltages side however can be obtained by the inclusion of both Joglekar and Biolek window functions. Considering to Joglekar’s window function in (6), the improved characteristic is achieved because of the addition of control parameter, $p$ to their window function to control the linearity and non-linearity of the drift phenomenon [15]. Biolek on the hand, managed to address the boundary problem when the doped and un-doped layers reach either end of the device by introducing switching window function ($stp(i)$) [16]. Thus provide better correlated I-V characteristics than Joglekar.
Figure 3. Simulation result for the combination of Chang’s hyperbolic sine function model with (a) Stukov (b) Joglekar (c) Biolek and (d) Prodromakis window functions

Since the I-V characteristics of TiO$_2$ thin film is asymmetrical, the function that is capable to model a switching state variable depending on the input voltage polarity is therefore essential. One of the memristor models which could provide this capability is the model created by Yakopcic [25]. In his model, the state variable $f(x)$ is modeled by dividing the state variable motion into two different regions as given in (9) and (10),

$$f(x) = e^{-\alpha_p(x-x_p)}w_p(x,x_p)$$
$$f(x) = e^{\alpha_n(x+x_n-1)}w_n(x,x_n)$$

where $\alpha_p$, $\alpha_n$, $x_p$ and $x_n$ are the parameters that determine where the state variable motion is no longer linear and the degree to which the state variable motion is dampened [25] while $w_p$ and $w_n$ are the window function which is given by the following formula

$$w_p(x,x_p) = \frac{x_p - x}{1 - x_p} + 1$$
$$w_n(x,x_n) = \frac{x}{1 - x_n}$$

As can be observed in Figure 4, the incorporation of Yakopcic’s state variable model to Chang’s model resulted in the improved correlated TiO$_2$ thin film I-V characteristic. The percentage error was calculated to be approximately 2.15%.
5. Conclusion

This study is carried out to examine the modified hyperbolic sine function model for titanium dioxide-based memristive thin film. The modification was performed by adding the various types of window functions and Yakopcic’s state variable function to Chang’s hyperbolic sine function model. The inclusion of window function however resulted in an un-fitted I-V characteristics. A better correlated characteristic was achieved when incorporating the Yakopcic’s model.

References
[1] L. Chua 1971 IEEE Trans. on Circuit Theory 18 507
[2] Williams R-S 2009 IEEE Spectrum 45 28
[3] Hamdou S, Xie L, Nguyen H A D, Taouil M, Bertels K, Corporaal H, Jiao H, Catthoor F, Wouters D, Eike L and Lunteren J V 2015 Design, Automation & Test in Euro. Conf. Exhibit. p 1718
[4] Potrebic M and Tosic D 2015 Radioeng. 24 409
[5] Tetzlaff R, Schmidt T 2012 IEEE Int. Symp. Circ. Sys. 1590
[6] Kasim S M M, Shaari N A A, Bakar R A and Herman S H. 2015 App. Mechan. Mats. 749 308
[7] Shaari N AA, Kasim SMM, Sauki NSM and Herman SH. 2015 4th Int. Conf. Electron Device Syst. Apps. 99 p 1
[8] Tripathi S K, Kaur R and Rani M 2014 Solid State Phenomena 222 67
[9] Lee S, Park J-B, Lee M-J and Boland J J 2016 AIP Advances 6 125010
[10] Pergament A, Stefanovich G, Malinenko V and Velichko A 2015 Adv. Condensed Matt. Phys. 654840 1
[11] Mohammad B, Jaoude M, Kumar V, Al Homouz D M, Nahla H A, Al-Qutayri M and Christoforo N 2016 Nanotechnol. Rev.
[12] Fauzi F B, Othman R, Mohamed M A, Herman S H, Ahmad A A Z and Ani M H 2015 Japan Ins. Metals Mats. 8 1302
[13] Saw A 2008 Mats. Today 11 28
[14] Strukov DB, Snider GS, Stewart D R, and Williams R S 2008 Nature 453 80
[15] Joglekar Y N and Wolf S J 2009 Euro. J. Phys. 30 661
[16] Biolek Z, Biolek D and Biolková V 2009 Radioengin. 18 210
[17] Prodromakis T, Peh BP, Papavassiliou C and Toumazou C 2011 IEEE Trans. Elect. Devices 58 3099
[18] Kvantinsky S, Friedman E G, Kolodny A and Weiser U C 2013 IEEE Trans. Cct. Systs – I: Regular Papers 60 211
[19] Abdalla H and Pickett MD 2011 *IEEE Int. Symp. Cct. Syst. (ISCAS)* 1832
[20] Laiho M, Lehtonen E, Russel A and Dudek P 2010 *Neuromorphic Engin.*
[21] Eshraghian K, Kavehei O, Cho K R, Chappel JM, Iqbal A, Al-Sarawi S F and Abbott D 2012 *Proc. IEEE* 100 1991
[22] Yakopcic C, Taha T M, Subramanyam G and Pino R E 2013 *Int. Joint Conf. Neur Net.* 1
[23] Chang T, Jo SH, Kim KH, Sheridan P, Gaba S, and Lu W 2011 *Appl Phys. A*, 102 857
[24] Merkel C.E Thermal profiling in CMOS/memristor hybrid architectures. Master thesis, Rochester Institute of Technology, Rochester, New York, 2011.
[25] Yakopcic C M S Memristor device modeling and circuit design for read out integrated circuits, memory architectures and neuromorphic systems. PhD thesis, University of Dayton, Dayton, Ohio. 2014.
[26] Kamarozaman N S, Mohamed Soder M F, Musa M Z, Bakar R A, Abdullah W F H, Herman S H and Rusop M 2014 *Adv-Maters. Research* 925 125
[27] Chang T. Tungsten oxide memristive devices for neuromorphic applications. PhD thesis, The University of Michigan, Michigan. 2012.
[28] Radwan A G and Fouda ME 2015 Memristor: models, types and applications *On the Mathematical Modeling of Memristor, Memcapacitor and Meminductor*, 1st ed. (Switzerland: Springer Int. Publishing) pp 13-23