From the Tully-Fisher relation to the Fundamental Plane through Mergers

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Accepted — . Received ——; in original form ——–

ABSTRACT
We set up a series of self-consistent N-body simulations to investigate the fundamental plane of merger remnants of spiral galaxies. These last ones are obtained from a theoretical Tully-Fisher relation at z=1, assuming a constant mass-to-light ratio within the ΛCDM cosmogony. Using a Sérsic growth curve and an orthogonal fitting method, we found that the fundamental plane of our merger remnants is described by the relation \( R_e \propto \sigma_0^{1.48 \pm 0.01} I_e^{-0.75 \pm 0.01} \), which is in good agreement with that reported from the Sloan Digital Sky Survey \( R_e \propto \sigma_0^{1.49 \pm 0.05} I_e^{-0.75 \pm 0.01} \). However, the \( R^{1/4} \)-profile leads to a fundamental plane given by \( R_e \propto \sigma_0^{1.79 \pm 0.01} I_e^{-0.60 \pm 0.01} \). In general, the correlation found in our merger remnants arises from homology breaking \( (V^2 \propto \sigma_0^2, R_g \propto R_e) \) in combination with a mass scaling relation between the total and luminous mass, \( M \propto M_l^\eta \). Considering an orthogonal fitting method, it is found that \( 1.74 \lesssim \nu \lesssim 1.79, 0.21 \lesssim \eta \lesssim 0.52 \) and \( 0.80 \lesssim \gamma \lesssim 0.90 \) depending on the adopted profile (Sérsic or \( R^{1/4} \)).

Key words: galaxies: formation – galaxies: fundamental parameters – galaxies: elliptical – methods: \( N \)-body simulations.

1 INTRODUCTION
The formation and evolution of elliptical galaxies continues to be an outstanding problem in astrophysics (e.g. Meza et al. 2003, Wright et al. 2003). In the current CDM models of hierarchical structure formation, mergers are the common process through which larger objects form from small ones (Frenk et al. 1988, Kauffmann & White 1993, Lacey & Cole 1993). In this scenario, ellipticals are a ‘natural’ outcome (Toomre 1977) and accumulating observational evidence seems to support this (Schweizer 1998 and references therein). However, an important challenge to the merger hypothesis is to explain how this seemingly random merging process can lead to the tight correlations found in ellipticals (White 1997).

One of these correlations found in ellipticals is the so-called Fundamental Plane (FP); a linear relation, in the logarithmic space, among their effective radius, \( R_e \), their effective surface brightness, \( I_e \), and their one-dimensional central velocity dispersion, \( \sigma_0 \) (Djorgovski & Davis 1987, Dressler et al. 1987). \( R_e \propto \sigma_0^\beta I_e^\alpha \). The FP exhibits a general trend that follows from the application of the virial theorem to ellipticals: \( M \propto V^2 R_e \), with \( M \) being the total mass, \( V^2 \) the 3-dimensional mean square velocity and \( R_e \) the gravitational radius of the system \( (V^2 = 2K/M \) and \( R_e = GM^2/W \), where \( K \) and \( W \) are the total kinetic and potential energy, respectively). Assuming that ellipticals are homologous systems in equilibrium \( (V^2 \propto \sigma_0^2, R_g \propto R_e) \) with a constant mass-to-light ratio, the virial theorem leads to

\[
R_e \propto \sigma_0^\beta I_e^\alpha \rightleftharpoons \log R_e \approx 2[\log \sigma_0 + 0.2 \mu_\sigma]
\]

where \( \mu_\sigma = -2.5 \log I_e \). Deviations of an observed FP from the theoretical expectation are attributed to variations of the mass-to-light ratio with luminosity and/or to the breaking of homology in galaxies (e.g. Trujillo, Burkert & Bell 2004).

The observed FP presents some differences depending on the wave-band spectrum and on the environment where galaxies reside. Jorgensen et al. 1996, Phare et al. 1998, Mohabers et al. 1999. Kelson et al. 2000, Bernardi et al. 2003. Recently, from a sample of about 9000 elliptical galaxies taken from the Sloan Digital Sky Survey (SDSS), Bernardi et al. (2003) obtained a FP relation given by \( R_e \propto \sigma_0^{1.49 \pm 0.05} I_e^{-0.75 \pm 0.01} \); where the \( R^{1/4} \)-profile was assumed.

On the theoretical side, simulations of spiral mergers are numerous and have been extensively explored to study: the morphology of merger remnants (e.g. Mihos & Hernquist 1996), the line-of-sight velocity distribution (e.g. Bendo & Barnes 2000), and the effect of the orientation of the angular momentum of discs (e.g. Naab & Burkert 2003). Other works have considered the effect of mergers on the properties of the FP (e.g. Capelato et al. 1995, Levine & Aguilar 1996, Bekki 1998, Dantas et al. 2003, nipoti et al. 2003, González-García & van Albada 2003). A common feature of previous studies is that they have used models with properties chosen rather arbitrary resembling present day spiral galaxies. In partic-
ular, those works addressing the FP of merger remnants have not considered self-consistent models of disc galaxies.

Disc galaxies also define a tight correlation between the maximum of their rotation velocity \( V_m \) and their luminosity, termed the Tully-Fisher (TF) relation, that can be written as \( L = aV_m^2 \), where \( a \) and \( A \) are the ‘slope’ and the zero-point, respectively. The observed values for \( a \) lie between \( \approx 2.5-4 \), depending on the wave-band (e.g. Strauss & Willick 1995). In particular, the \( I \)-band TF relation is given by

\[
M_I - 5 \log h = -21.00 - 7.68(\log W - 2.5)
\]

where \( M_I \) is the absolute magnitude of the disc, and \( W \approx 2V_m \) is the 21-cm hydrogen line width resulting in a slope of \( \alpha \approx 3.1 \) (Giovanelli et al. 1997).

In this Letter we study whether merger remnants of self-consistent \( N \)-body spirals satisfying the TF relation can lead to the FP of ellipticals. Spirals properties are obtained from the Press-Schechter formalism (Press & Schechter 1974) in combination with the model of disc galaxy formation of Mo, Mao & White (1998, hereafter MMW) and the TF relation under the \( \Lambda CDM \) model. The rest of this work has been structured as follows: in §2 we summarise the model used to set up the theoretical TF relation, the initial conditions and the orbital parameters for our simulations. In §3 the results and a discussion are given.

2 NUMERICAL MODEL

In this section we summarise the method and properties of our models to study the FP associated to merger remnants.

2.1 Tully-Fisher relation of spirals

To construct a theoretical TF we use the Press-Schechter framework of hierarchical clustering and the model of MMW for the formation of galaxy discs. In this model five parameters are required to obtain the radial scale-length of a disc formed inside a spherical dark halo. These are the circular velocity, \( V_c \), the spin parameter, \( \lambda \), the concentration of the halo, \( c \), the fraction of disc to halo mass, \( m_d \) and the fraction of angular momentum in the disc, \( j_d \). To obtain \((V_c,\lambda,c,m_d,j_d)\) we have proceeded as in Shen, Mo & Shu (2002). The \( \Lambda CDM \) cosmology was adopted to obtain the particular properties of disc galaxies (with a mass density parameter of \( \Omega_0=0.3 \), a cosmological constant of \( \Omega_\Lambda=0.7 \), the Hubble’s constant \( h_0=0.7 \), and the perturbation power-spectrum normalization \( \sigma_8=1 \)).

We have used the theoretical TF at \( z=1 \), an epoch thought to be close to the one where formation of the discs occurred (Peebles 1993), as our fiducial redshift to construct the numerical disc galaxies. This allowed us to consider remnants resulting from binary encounters and to compare them with ellipticals that might have formed through these encounters. Present day massive ellipticals are probably the result of more than a binary merger, but their study is out of the scope of this work.

We have selected only disc galaxies with a stability criterion \( \varepsilon_m \geq 0.9 \), where \( \varepsilon_m=V_m(GM_d/R_d)^{-1/2} \), and \( V_m \) is the maximum rotation velocity (Efstathiou, Lake & Negroponte 1982; Syer, Mao & Mo 1997). Galaxies were evolved in isolation during 2 Gyr, about twice the time needed for the first pericentric passage in the encounter. Despite the fact that our galaxy models match the previous criterion some form a well defined bar (B) while in other cases they develop a transient bar (T) or remain stable (S) (see last column of Table 1).

![Figure 1. Theoretical Tully-Fisher relation for an ensemble of spirals (dots) at \( z=1 \). The dashed and solid lines correspond to the linear-square fit with a slope of \( \alpha = -8.0 \) and to the observed relation (2), respectively. The properties of discs of galaxies to be merged were selected from this ensemble. As an illustration, progenitors of mergers M01, M02 and M03 of Table 1 are shown.](image)

In Figure 1, the TF relation, at redshift \( z=1 \), is shown. The straight solid line indicates the observed slope of the TF in the \( I \)-band of equation (2) at the present epoch. A sample of twenty random points from this TF relation were selected in order to set up the self-consistent \( N \)-body disc galaxies.

2.2 Galaxy models

Our \( N \)-body disc galaxy models consist of a disc and a spherical dark halo component. Following Hernquist (1993), disc particle positions are randomly drawn from the axisymmetric density profile:

\[
\rho_d(R,z) = \frac{M_d}{4\pi R_d^2 z_d} \exp(-R/R_d) \sech^2(z/z_d) ,
\]

being \( R_d \) and \( z_d \) the radial and vertical scale length of the disc, respectively; \( z_d \) was chosen randomly in the range \((0.1-0.2)R_d \). For the halo we have adopted a modified version of the NFW–profile (Navarro, Frenk & White 1997) with an exponential cut-off given by:

\[
\rho_h(r) = \frac{M_h \alpha_h}{4\pi \sigma v^2} \exp \left[ -\left( \frac{r}{r_{\text{vir}}} + q \right)^2 \right] ,
\]

with

\[
\alpha_h = \frac{\exp(q^2)}{\sqrt{\pi q} \exp(q^2) \text{Erf}(q) + \frac{1}{2} \exp(q^2) \text{Erf}(q^2) - 1} ,
\]

being \( \text{Erf}(x) \) the complementary error function and \( \text{Erf}(x) \) the exponential integral, \( r_v \) and \( r_{\text{vir}} \) the scale and virial radii of the halo respectively, \( c = q^{-1} \approx (r_v/r_{\text{vir}})^{-1} \) the halo concentration, and \( M_h \) is the halo total mass. Finally, velocities are derived from Jeans equations.

The parameters characterizing our disc galaxy models are listed in Table 1 where \( f_d = M_d/(M_h + M_d) \). We have also in-

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cluded the radius \( R_\text{Q} \) at which Toomre’s \( Q \) parameter is normalized and the number of particles in the halo, \( N_\text{h} \), and the disc, \( N_\text{d} \). Notice the range of concentrations of haloes and galaxy disc sizes.

Our disc galaxy models do not include a central bulge component since, in the framework of MMW’s galaxy formation, they tend to produce smaller disc sizes which require a better spatial resolution and smaller timesteps and, hence, more demanding computational resources. It is not clear how this central component will affect the FP of the remnants; it will be addressed in a future work.

### 2.3 Initial conditions and experiments

We have considered only parabolic encounters where the initial separation between galaxy centres is \( R_\text{o} = 1.25(r_{200,1} + r_{200,2}) \). The pericentre for each encounter is randomly selected in the interval \( r_\text{P} \in (5,20)\text{kpc} \), consistent with the energies and eccentricities of cosmological N-body simulations (Navarro, Frenk & White 1995). The relative orientation between galaxy spins is also taken randomly.

To evolve the numerical simulations we use the parallel version of GADGET, an N-body/SPH code with individual timesteps (Springel, Yoshida & White 2001). Softening parameters of \( \epsilon_\text{d}=35\text{pc} \) and \( \epsilon_\text{h}=350\text{pc} \) were used for the disc and the halo particles, respectively. GADGET uses a spline kernel for the softening, so that the gravitational interaction between two particles is fully Newtonian for separations larger than twice the softening parameter (Power et al. 2003).

The typical time of arrival at pericentre is about 1 Gyr, and all simulations were run for a total time of about 8 Gyr; this corresponds to about the time span from \( z=1 \) to \( z=0 \) in a \( \Lambda \)CDM scenario. Remnants are already in a virial state by this time. All of our runs were performed on a cluster consisting of 32 Pentium processors running at 450 MHz (Velázquez & Aguilar 2003). Energy conservation was better than 0.25% in all cases.

### 3 RESULTS AND DISCUSSION

To analyse the merger remnants a constant mass-to-light ratio, \( \Upsilon_b \), is assumed; hence \( I_\text{d} \) is proportional to the effective surface luminous-mass density \( \Sigma_\text{e} \). We first compute their ‘photometric’ properties \( (R_\text{e}, \Sigma_\text{e}) \) and then its central velocity dispersion \( \sigma_0 \) inside an aperture of radius \( R_{200}/8 \). The surface density profiles \( \Sigma(R) \) were fitted using a Sérsic profile (Sérsic 1968; Ciotti & Bertin 1999; Caon, Capaccioli & D’Onofrio 1993) and the \( R^{1/4} \) profile, we also fitted the corresponding growth curves of the luminous part of the remnants: \( M_L(R) = 2\pi \int \Sigma(R)RdR \).

To check how the photometric properties are affected by the adopted region to be fitted, we have considered an aperture with outer radius of 17.5 kpc as in Wright et al (2003) and for the inner radius of the fit, \( \xi \), we use two different values: (1) the resolution of the luminous component in our simulations, \( \xi \approx 2.8\epsilon_\text{d} \approx 100 \text{pc} \), and (2) \( \xi=300 \text{pc} \). This also allows us to test the effects of the boundaries of the fitting region on the values of the exponents \( a \) and \( b \) of the FP \( (R_\text{e} \propto \sigma_0^a \Sigma_\text{e}^b) \). Each remnant was “observed” from 100 random line-of-sights. A \( \chi^2 \)-minimisation by Levenberg-Marquart method (Press et al. 1992) was used to obtain \( R_\text{e} \) and \( \Sigma_\text{e} \) for each projection. The exponents \( (a, b) \) of the FP were fitted to the complete set of projected values of the remnants, using both orthogonal and direct fits.

The resulting values for these exponents are plotted in Fig-
The values of parameters \((a, b)\) are sensitive to the fitting function (profile or growth curve), to the adopted profile (Sérsic or \(R^{1/4}\)), to the fitting method (direct or orthogonal) and to the slope of the radius \(\xi\). The observed values in the \(r^+\)-band of the SDSS have also been included (star symbols). In particular, the dotted lines refer to the exponents of the FP of the SDSS (Bernardi et al. 2003). Perhaps, a more general relation of the form \(M/L \propto r_{0}^{\gamma} \sigma_{e}^{\delta} L_{e}^{\mu}\) need to be explored. A more extensive study of the scaling relations of merger remnants, their non-homology, the distribution of luminous and dark matter, and their kinematics is under way and it will be presented in a future work.

Our results support the idea that mergers of spirals, with properties obtained from the Tully-Fisher relation within a ΛCDM cosmogony, lead to the tight correlation expressed by the FP. In dissipational numerical simulations of mergers, it is found that a gas component can yield important differences in the stellar component of the remnants in comparison with their collisionless counterparts (e.g., Barnes & Hernquist 1996). For instance, a gas component deepens the potential well, affecting more the remnant kinematics than its luminous profile. However, it is not clear from these results how the FP is going to be altered; in particular, if its slope is going to be changed or just the dispersion of values around it. This remains an open issue.

**ACKNOWLEDGMENTS**

We thank Luis Aguilar and Simon White for their helpful comments on this work. This research was funded by CONACyT-México Project 37506-E. An anonymous referee is thanked for useful comments that helped to clarify some points of this work.

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Figure 2. (Left) Parameters \((a, b)\) for the FP \((R_e \propto \sigma^4_0 \Sigma^0_e)\) of our galaxy merger remnants. For comparison purposes, the values of the SDSS (star symbols) in the \(r^*\)-band have been included. Horizontal dotted lines indicate the FP relation, \(R_e \propto \sigma^1_0 \Sigma^{-0.75}_e\), quoted by Bernardi et al. (2003). To the left, these parameters were obtained using an orthogonal fitting method while to the right using a direct one. Here, \(S=\)Sérsic profile, \(S_{GC}=\)Sérsic’s growth curve, \(R_{1/4} = \)de Vaucouleurs profile, and \(R_{1/4}^{GC}\) its growth curve. Also, the effect introduced by two values of the inner radii \(\xi\) is illustrated. (Right) Edge-on view of the FP. Remnants are indicated by filled squares using the mean values listed in Table 2. The solid and dotted lines correspond to the SDSS FP \((R_e \propto \sigma^4_0 \Sigma^0_e)\) and to our FP model. As a reference, the dashed line with slope \(\alpha_V=2\) corresponding to virialised homologous systems has been plotted. Remnant mergers have been artificially displaced to coincide with the ‘zero-point’ of the FP of the SDSS.