What can we lean about GPDs from light meson leptoproduction experiments

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Abstract. On the basis of the handbag approach we study cross sections and spin asymmetries for leptoproduction of various vector and pseudoscalar mesons. Our results are in good agreement with high energy experiments. We analyse what information about Generalized Parton Distributions (GPDs) can be obtained from these reactions.

1 Introduction

The leptoproduction of light mesons at small momentum transfer and large photon virtualities $Q^2$ factorizes into a hard meson photoproduction subprocess off partons and GPDs [1]. GPDs are complicated nonperturbative objects which depend on 3 variables $x$ - the momentum fraction of proton carried by parton, $\xi$ - skewness and $t$-momentum transfer. GPDs contain the extensive information on the hadron structure. At $\xi = 0, t = 0$ GPD become equal to the corresponding parton distribution functions (PDFs). The form factors of hadron can be calculated from GPDs trough the integration over $x$. Using Ji sum rules [2] the parton angular momentum can be extracted.

In the light meson leptoproduction we can analyze effects of various GPDs. The vector meson production on the unpolarized target is sensitive to the gluon and quark GPDs $H$. Using GPDs $E$ we extend our analysis to the $A_{UT}$ asymmetry for a transversally polarized target [5].

The pseudoscalar meson leptoproduction is analysed in [6,7]. At leading-twist these reactions are sensitive to the GPDs $\tilde{H}$ and $\tilde{E}$. It was found that essential contributions from the transversity GPDs, $H_T$ and $E_T$, are required by experiment. Within the handbag approach the transversity GPDs are accompanied by a twist-3 pion wave function. It was shown that these transversity GPDs lead to a large transverse cross section for most reactions of pseudoscalar meson production.

Our results [3,4,5,6,7] on meson electroproduction at small and moderate $x$ are in good agreement with experimental data in the HERA [8,9] HERMES [10] and COMPASS [11] energy range.

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2 Light meson leptoproduction in handbag approach

In the handbag model, the amplitude of the light meson production off the proton reads as a convolution of the hard partonic subprocess $\mathcal{H}^{a}$ and GPDs $F_{\perp}^{a}$ ($\tilde{H}^{a}$)

$$\mathcal{M}^{a}_{\mu\perp,\mu\perp} = \sum_{a} \langle \mathcal{H}^{a} + ... \rangle ; \quad \langle \mathcal{H}^{a} \rangle \propto \sum_{\lambda} \int_{\xi_{1}}^{1} d\xi \mathcal{H}^{a}_{\mu\perp,\lambda\mu\perp}(Q^{2},\xi,\xi) \tilde{H}^{a}(\xi,\xi)$$

(1)

where $a$ denotes the gluon and quark contribution with the corresponding flavors; $\mu$ ($\mu'$) is the helicity of the photon (meson), and $\xi$ is the momentum fraction of the parton with helicity $\lambda$.

In contrast to other analyses the subprocess amplitudes are calculated within the modified perturbative approach (MPA) [12] where the quark transverse momenta $k_{\perp}$ are taken into account together with the Sudakov suppressions. The amplitude $\mathcal{H}^{a}$ is a convolution of the hard part calculated perturbatively, and the $k_{\perp}$- dependent meson wave function

$$\mathcal{H}^{a}_{\mu\perp,\mu\perp} = \frac{2\pi\alpha_{s}(\mu_{R})}{\sqrt{2N_{c}}} \int_{0}^{1} d\tau \int \frac{d^{2}k_{\perp}}{16\pi^{3}} \phi(\tau,k^{2}_{\perp}) f_{\mu\perp,\mu\perp}(Q,\xi,\xi,\tau,k_{\perp}).$$

(2)

Here $\phi$ is a meson wave function, $f_{\mu\perp,\mu\perp}$ is a hard subprocess amplitude where in the propagators we keep the $k_{\perp}^{2}$ terms. These terms are essential under integration near $\tau = 0, \tau = 1$ points.

The meson wave function is another nonperturbative object in the model which is chosen in the simple Gaussian form

$$\phi(k_{\perp},\tau) \propto a_{M}^{2} \exp \left[ -a_{M}^{2} \frac{k_{\perp}^{2}}{\tau(1-\tau)} \right].$$

(3)

The $a_{M}$ parameter determines the mean value of the quark transverse momentum $<k_{\perp}^{2}>$ in the meson. It can be seen that the wave function (3) integrated over $k_{\perp}^{2}$ has a form of asymptotic one $\propto 6\tau(1-\tau)$.

Together with the $<k_{\perp}^{2}>$ terms in the hard partonic subprocess amplitude $\mathcal{H}^{a}$ in the MPA we consider gluonic corrections in the form of the Sudakov factors. The Fourier transformation of the integrals is done from the $k_{\perp}$ to $b$ space where resummation and exponentiation of the Sudakov corrections can be performed [12]. Details of calculations can be found in [3].

To estimate GPDs, we use the double distribution (DD) representation [13]

$$H_{i}(\xi,\xi,\tau) = \int_{-1}^{1} d\beta \int_{-1-|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi \alpha - \xi) f_{i}(\beta,\alpha,\tau)$$

(4)

which connects GPDs with PDFs through the DD function $f_{i}$,

$$f_{i}(\beta,\alpha,\tau) = h_{i}(\beta,\tau) \frac{\Gamma(2n_{i}+2)}{2^{2n_{i}+1} \Gamma^{2}(n_{i}+1)} \frac{[(1-|\beta|)^{2}-\alpha^{2}]^{n_{i}}}{(1-|\beta|)^{2n_{i}+1}}.$$  

(5)

The functions $h_{i}$ are expressed in terms of PDFs and parameterized as

$$h(\beta,\tau) = N e^{b \tau} \beta^{-\alpha(\tau)} (1-\beta)^{n}.$$  

(6)

Here the $t$- dependence is considered in a Regge form and $\alpha(t)$ is the corresponding Regge trajectory. The parameters in (6) are obtained from the known information about PDFs [14] e.g., or from the nucleon form factor analysis [15].
From various meson productions at moderate HERMES and COMPASS energies we can get information about valence and sea quark effects. The quarks contribute to meson production processes in different combinations. For uncharged meson production we have the standard GPDs. We find quark contribution to $\rho$ production in the form: $\propto \frac{2}{3}H^u + \frac{1}{3}H^d$, to $\omega : \propto \frac{2}{3}H^u - \frac{1}{3}H^d$. For production of charged and strange mesons the transition GPDs contribute which using SU(3) symmetry can be connected with the standard one. For example, for $\rho^+ $ production the combination works: $\propto H^u - H^d$. For pseudoscalar mesons production similar combinations of polarized $\tilde{H}$ GPDs contribute. Thus, we can test various GPDs in the different reactions.

### 3 Vector meson leptoproduction

We apply now the results presented in section (2) for vector meson leptoproduction on the unpolarized target. All GPDs are modeled on the basis of the double distribution ansatz (4), (5) with using the CTEQ6 [14] parameterization of PDFs. We consider the gluon, sea and quark GPD contribution to the amplitude. The $a_M$ parameter in the wave function was determined from the best description of the cross section. Some more details together with other parameters of the model can be found in [3,4].

This approach was found to be successful in the analysis of data on the $\rho^0$ and $\phi$ leptoproduction [34]. In Fig.1a we show our results for the energy dependence of the $\rho$ cross section at different $Q^2$ in the HERA energy range which describe experimental data well.

![Graph](image)

**Fig. 1.** (a) The energy dependence of the $\rho$ production cross section at different $Q^2$ at HERA. Data: from ZEUS. (b) The ratio of longitudinal cross sections $\sigma_L(\phi)/\sigma_L(\rho)$ at HERA energies- full line and HERMES- dashed line. Data are from H1 -solid, ZEUS -open squares, HERMES solid circles.

In Fig. 1.b the ratio of the $\phi$ and $\rho$ longitudinal cross section is presented. If the sea GPDs are the flavor symmetric, this ratio should be independent of $Q^2$ and be not far from 2/9. The model we have the flavor symmetry breaking between $\bar{u}$ and $\bar{s}$ sea

$$H^u_{\text{sea}} = H^d_{\text{sea}} = \kappa_s H^u_{\text{sea}} \quad \text{and} \quad \kappa_s = 1 + 0.52 \ln(Q^2/Q^2_0)).$$

(7)
The symmetry breaking factor $\kappa_s$ is found from the CTEQ6M PDFs. Because of the flavor symmetry breaking $\kappa_s$, the ratio of $\sigma_\phi/\sigma_\rho$ becomes $Q^2$ dependent and very different from the 2/9 value. The valence quark contribution to $\sigma_\rho$ decreases this ratio at HERMES energies [3], Fig. 1b.

The model results for the cross section and spin observables of electroproduced $\rho$ and $\phi$ mesons are in good agreement with data on the unpolarized target at HERA [8,9], COMPASS [11], HERMES [10] energies [3,4]. Thus, we can conclude that our gluon, valence and sea quark GPDs $H$ reproduce adequately the vector meson lepto-production in a wide energy and $Q^2$ range.

To study spin effects on the transversally polarized target, the proton helicity flip amplitude is needed. It is expressed in terms of GPD $E$

$$M_{\mu'-\mu} \propto \frac{\sqrt{-t}}{2m} \int_{-1}^{1} d\tau E^a(\tau, \xi, t) F_{\mu',\mu}^a(\tau, \xi).$$  \hspace{1cm} (8)

We constructed the GPD $E$ from double distributions and constrained it by the Pauli form factors of the nucleon [15], positivity bounds and sum rules. The first moment of $e^a(x) = E^a(x, 0, 0)$ is proportional to quark anomalous magnetic moment

$$\int_{0}^{1} dx e^a_{val}(x) = \kappa^a.$$  \hspace{1cm} (9)

The $\kappa^u$ and $\kappa^d$ factors have different signs. This means that the GPD $E^u$ and $E^d$ should have different signs too. For the $\rho^0$ production, where the combination $\frac{4}{3}E^u + \frac{1}{3}E^d$ contributes, we have a compensation of quark contributions.

![Model results for the $A_{UT}$ asymmetry of the $\rho$ production at energy $W = 5\text{GeV}$ and $Q^2 = 3\text{GeV}^2$. HERMES data are shown. (b) Prediction for $A_{UT}$ asymmetry for various vector meson productions at COMPASS energy $W = 10\text{GeV}$. Dot-dashed line -$\rho^0$, full line -$\omega$, dotted line -$\rho^-$, dashed line -$K^*$.](image)

**Fig. 2.** (a) Model results for the $A_{UT}$ asymmetry of the $\rho$ production at energy $W = 5\text{GeV}$ and $Q^2 = 3\text{GeV}^2$. HERMES data are shown. (b) Prediction for $A_{UT}$ asymmetry for various vector meson productions at COMPASS energy $W = 10\text{GeV}$. Dot-dashed line -$\rho^0$, full line -$\omega$, dotted line -$\rho^-$, dashed line -$K^*$.  

The $A_{UT}$ asymmetry for transversally polarized protons is determined as an interference of the amplitudes connected with $E$ and $H$ GPDs.

$$A_{UT} \propto \frac{\text{Im} < E^* > < H >}{| < H > |^2}.$$  \hspace{1cm} (10)
The $H$ GPD is known from our analysis of the vector meson leptoproduction. Our results for the $\sin(\phi - \phi_a)$ moment of the $A_{UT}$ asymmetry of the $\rho^0$ production are shown in Fig. 2a [5] and describe HERMES data [16] quite well. The $A_{UT}$ asymmetry at COMPASS is predicted to be quite small and is in good agreement with the data [17]. Predictions for the $A_{UT}$ asymmetry at $W = 5\text{GeV}$ and $W = 10\text{GeV}$ were given in the model for the $\omega$, $\rho^+$, $K^{*0}$ mesons [5]. We show our results at COMPASS energy in Fig 2b. Our prediction for the $\omega$ production asymmetry is negative and not small. This is determined by enhancement of quark contributions which are in the $\frac{2}{3}E^u - \frac{1}{3}E^d$ combination there. Predictions for the $\rho^+$ asymmetry is positive and rather large $\sim 0.4$. In this reaction the contribution $E^u - E^d$ works and we have enhancement of quark contribution too. At the same time, smallness of the $\rho^+$ cross section does not give a good chance to measure this asymmetry. Good agreement with experimental data at HERMES and COMPASS of the $A_{UT}$ asymmetry found in the model shows that our estimations on GPDs $E$ are not far from reality. However, experimental errors are quite large now and additional experimental data for various reactions are needed to get more information about GPDs $E$. The analysis the $A_{UT}$ asymmetry for $\omega$ at HERMES and COMPASS can help here.

4 Leptoproduction of pseudoscalar mesons.

Hard exclusive pseudoscalar meson leptoproduction was studied on the basis of the handbag approach too. These reactions are sensitive to the polarized GPDs $\hat{H}$ whose parameterization can be found in [4] and $\hat{E}$. The pseudoscalar meson production amplitude with longitudinally polarized photons $M^{0+}_{0^+,0^+}$ dominates at large $Q^2$. The amplitudes with transversally polarized photons are suppressed as $1/Q$. The pseudoscalar meson production amplitude can be written as:

$$M^{P}_{0+0+} \propto \left[ \left( \hat{H}^P \right) - \frac{2\xi m Q^2}{1 - \xi^2} \frac{\rho_P}{t - m_P^2} \right]; \quad M^{P}_{0-0+} \propto \frac{\sqrt{-t}}{2m} \left[ \xi \langle \hat{E}^P \rangle + 2m Q^2 \frac{\rho_P}{t - m_P^2} \right].$$

The first terms in (11) represent the handbag contribution to the pseudoscalar (P) meson production amplitude [4] calculated within the MPA with the corresponding transition GPDs. For the $\pi^+$ production we have the $p \to n$ transition GPD where the combination $F(n, \rho) = F^{(a)} - F^{(d)}$ contributes.

The second terms in (11) appear for charged meson production and are connected with the P meson pole. Here we use the fully experimentally measured electromagnetic form factor of P meson.

In addition to the pion pole and the handbag contribution which in the leading twist is determined by the $\hat{H}$ and \hat{E} GPDs a twist-3 contribution to the amplitudes $M_{0-,\mu^+}, M_{0+,\mu^+}$ is required by the polarized data at low $Q^2$. To estimate this effect, we use a mechanism that consists of the transversity GPD $H_T, \hat{E}_T$ in conjugation with the twist-3 pion wave function. For the $M_{0-,\mu^+}$ amplitude we have

$$M^{0-,\mu^+}_{\text{twist-3}} \propto \int_1^{-1} d\tau H_{0-,\mu^+}(\tau, ...) \left[ H_T^P + \ldots \right].$$

The $H_T$ GPD is connected with transversity PDFs as

$$H_T^P(x, 0, 0) = \delta^n(x); \quad \delta^n(x) = C N^n_{\rho} x^{1/2} (1 - x) \left[ q_a(x) + \Delta q_a(x) \right].$$

Here we parameterize the PDF $\delta$ using the model [18]. The DD form [18] is used to calculate GPD $H_T$. 

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The twist-3 contribution to the amplitude $\mathcal{M}_{0+,\mu^+}$ has a form similar to (14)

$$\mathcal{M}_{0+,\mu^+}^{P,\text{twist-3}} \propto \sqrt{-t} \int_{-1}^{1} d\tau \mathcal{H}_{0-\mu^+}(\tau, \ldots) \tilde{E}_T^P.$$  \hspace{1cm} (14)

The information on $\tilde{E}_T$ was obtained only in the lattice QCD [19]. The lower moments of $\tilde{E}_u^T$ and $\tilde{E}_d^T$ were found to be quite large, have the same sign and a similar size. At the same time, $H^T_u$ and $H^T_d$ are different in the sign. This means that we have an essential compensation of the $\tilde{E}_T$ contribution to the $\pi^+$ amplitude: $\tilde{E}_T^{(3)} = \tilde{E}_u^T - \tilde{E}_d^T$. $H_T$ does not compensate in this process. For the $\pi^0$ production we have the opposite case. We find here a large contribution from $\tilde{E}_T^{(3)} = 2/3\tilde{E}_u^T + 1/3\tilde{E}_d^T$. $H_T$ effects are not so essential here.

In Fig. 3a, we show our results for unseparated cross section of the $\pi^+$ production which describes fine HERMES data. The $\sigma_L$ and $\sigma_T$ are shown as well. The longitudinal cross section determined by leading-twist dominates at small momentum transfer $-t < 0.2\text{GeV}^2$. At larger $-t$ we find an essential contribution from the transverse cross section. Effects of $\tilde{E}_T$ are small here. In Fig. 3b, we show our results for the cross section of the $\pi^0$ production which are quite surprising. The transverse cross section where the $\tilde{E}_T$ contributions are important dominates. The longitudinal cross section which is expected to play an essential role is much smaller with respect to the transverse cross $\sigma_T$. The $\sigma_T$ cross section is determined by the twist-3 $\tilde{E}_T$ contributions and decreases quickly with $Q^2$ growing, Fig. 3b.

In the same way we calculate the strange particle production. The proton- hyperon transition GPDs which contribute here are contracted by using the SU(3) flavor symmetry. For example, for the $\gamma p \rightarrow K^+ A$ reaction we find:

$$F_{p\rightarrow A} \sim \frac{1}{\sqrt{6}} [2F_u^s - F_d^s - F_s^s].$$  \hspace{1cm} (15)

Details of calculations can be found in [7]. Our results for the cross section of various processes are shown in Fig. 4a. It is important that we predict at HERMES for $-t > 3$.
0.3GeV\(^2\) not small and closed cross section for the \(\pi^+, \pi^0\) and \(K^+\Lambda\) production. This prediction is a result of large transversity \(\bar{E}_T\) contributions in the two last processes. In Fig. 4b, we show our results for the beam-spin \(A_{LU}\) asymmetries at HERMES for different meson channels. Predicted asymmetries are not small except the \(\pi_0\) production. Analysis of this asymmetry at HERMES is very important because at CLAS the \(A_{LU}\) asymmetry is not small [20]. This analysis gives information about non-pole \(\bar{E}^{\pi_0}\) GPD.

**Fig. 4.** (a)The cross sections at COMPASS energies. (b) Predicted beam-spin asymmetries for at HERMES.- Both for various pseudoscalar meson productions

## 5 Conclusion

We analysed the meson leptoproduction within the handbag approach where the amplitude at high \(Q^2\) factorizes into a hard subprocess and GPDs. The hard subprocess amplitude is calculated using the MPA, where the transverse quark momenta and the Sudakov factors were taken into account.

In the vector meson production on unpolarized target the gluon, sea and valence quarks \(H\) GPDs contribute which are calculated using the CTEQ6 parameterization. Our results are in good agreement with experiment from HERMES to HERA energies and we know \(H\) GPDs quite well. The GPDs \(E\) can be studied in experiments with a transversely polarized target. We model the \(E\) GPDs using information on the Pauli form factors of the nucleon and sum rules. We predict the cross sections and \(A_{UT}\) asymmetries for various meson leptoproductions [5]. The experimental data available now only for the \(\rho^0\) production at HERMES and COMPASS are described well. However, the experimental uncertainties are quite large and more experimental data are needed to study GPDs \(E\). We hope that analysis of the \(A_{UT}\) asymmetry in the \(\omega\) production at HERMES and COMPASS can get more constraints on the GPDs \(E\).

The amplitudes of pseudoscalar meson production in the leading twist are sensitive to \(\bar{H}\) and \(\bar{E}\) GPDs. However, we have now experimental data at quite low \(Q^2\). It was shown that in this region the twist -3 effects determined by the transversity GPDs \(H_T\) and \(E_T\) are very important in understanding the cross section and spin asymmetries
of the pseudoscalar mesons production. There are some model estimations of GPD $H_T$ [18]. For GPDs $E_T$ only the information from the lattice is available [19]. At HERMES and COMPASS energies the twist-3 $E_T$ effects produce a large transverse cross section $\sigma_T$ [7] which exceeds substantially the leading twist longitudinal cross section. Similar behavior of $\sigma_T$ is observed for most reactions of the pseudoscalar meson production with the exception of channels with $\pi^+$ and $\eta'$. This important prediction of the model can be tested experimentally and shed light on the role of transversity effects in these reactions.

We describe fine the cross section and spin observables for various meson productions in a wide energy range. Thus, we can conclude that information on GPDs discussed above should not be far from reality.

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