Non-Perturbative Renormalization Group Analysis
of the Chiral Critical Behavior in QED

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We study the chiral critical behavior of QED using non-perturbative renormalization group (NPRG). Taking account of the non-ladder contributions, our flow equations are free from the gauge parameter dependence. We clarify the chiral phase structure and calculate the anomalous dimension of $\xi/\lambda$, which is enhanced compared to the ladder approximation. We find that the cutoff scheme dependence of the physical results is very small.

§ 1. Introduction

Dynamical chiral symmetry breaking in gauge theories is one of the most important subjects in particle physics. Many efforts have been devoted to this problem, particularly by solving the Schwinger-Dyson (SD) equation for the fermion self-energy, mainly in the ladder approximation with the Landau gauge. Adding the 4-fermi interactions, the chiral phase diagram and the critical behavior have been investigated, and its characteristic structures with the large anomalous dimension of the $\phi^4$ operator has been applied to various models beyond the standard model. In QED, the ladder SD with the running gauge coupling constant (the improved ladder) gives good results even quantitatively. However, the ladder SD is unsatisfactory in many aspects, including its strong gauge dependence, the difficulty involved in proceeding beyond the ladder, no firm base for the improved ladder.

Quite recently, numerical analysis using the non-perturbative renormalization group (NPRG) has drawn much attention as a promising new method to study non-perturbative phenomena in field theory. One of the essential features of the method is that it does not employ a series expansion and therefore is expected to give converging results when improving the approximation, in sharp contrast with other non-perturbative expansion methods such as $1/N$ or $\epsilon$ expansions.

Here we briefly explain the general aspects of the NPRG approach. The RG equation is the non-linear functional differential equation for the Wilsonian effective action, the exact form of which is explicitly written down. To approximate the RG equation, we project it onto a small dimensional subspace of the original full theory space. To improve the approximation, we enlarge the subspace step by step. As the lowest order, we take the local potential approximation (LPA). The sharp cutoff LPA NPRG has been applied to dynamical chiral symmetry breaking in gauge theories, and it successfully produces the critical behavior. When we further
artificially truncate the LPA down to its 'ladder parts', it reproduces the Landau
gauge ladder SD results, and even the improved ladder results when we adopt the
running gauge coupling constant. Thus the LPA NPRG should be regarded as a
non-ladder improvement of the ladder SD results, in the course of the systematic
approximation of NPRG.

In this article we proceed beyond the LPA analysis of the dynamical chiral
symmetry breaking in gauge theories, including anomalous dimensions of fields, which
brings about two new issues to be carefully dealt with. First, we must adopt a
smooth cutoff scheme to avoid the sharp cutoff singularities for derivative interac­tions. What we refer to as the cutoff scheme is the profile of a function used to
suppress the propagation of the low energy modes in the internal lines, and it is
introduced through the momentum dependent mass terms in the Lagrangian,
\[
\frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \phi(-q) \cdot \Lambda^2 \cdot C(A^2/q^2) \cdot \phi(q),
\]
where the function \( C \) defines the cutoff scheme. In the full space NPRG, physical
results do not depend on the cutoff scheme. However, the subspace approximation
breaks this independence,\(^{15}\) and the cutoff scheme dependence must be checked
explicitly.

Second, the NPRG is not manifestly compatible with the gauge symmetries due to
the momentum cutoff. Therefore we must introduce the gauge non-invariant opera­
tors to our subspace and constrain them by a certain identity, the so-called 'modified
Ward-Takahashi-Slavnov-Taylor identity'.\(^{16}\) This identity ensures the gauge invar­
iance of the total solutions of NPRG.

In the following sections, we formulate the NPRG flow equations beyond the LPA
and calculate critical behavior in QED. Further we investigate the cutoff scheme
dependence and the gauge dependence in the covariant \( a \)-gauge. These first results
beyond the LPA may give us a clue to the total reliability of the NPRG results.

As mentioned above, the ladder SD results have been reproduced in the LPA by
further truncating the flow equations. Note that this truncation cannot be regarded
as any projection of the RG equation to a subspace. This is crucial for the gauge
independence. Therefore the ladder truncation necessarily breaks the gauge invar­
iance. In our system, for example, the crossed ladder diagrams are included as well
as the usual ladder diagrams, keeping the gauge independence of the flows (in some
special cutoff scheme), while the ladder SD takes only the ladder parts, and thus
suffers from a disastrous gauge dependence.\(^{6}\)

§ 2. RG flow equation for the massless QED

We start with the evolution equation for the effective action \( \Gamma[A, \bar{\psi}, \psi] \),\(^{9}\) where
\( \Lambda \) is the infrared momentum cutoff for the quantum corrections. The effective action
is nothing but the one particle irreducible parts of the Wilsonian effective action. The infrared cutoff is introduced by adding the following terms to the bare action:
\[
\Delta S_{\text{cut}} = \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{1}{2} A_\mu(-q) \cdot \Lambda^2 \cdot (\Lambda/q)^{2k} \cdot A_\mu(q) + (\Lambda/q)^{2k+2} \cdot q_\mu \cdot \bar{\psi}(-q) \gamma_\mu \phi(q) \right\},
\]
where the parameter $k$ labels the cutoff scheme, and we take the values $k=1, 2, 3, \cdots$. With larger $k$ the cutoff profile becomes sharper, and it reaches the $\delta$-function cutoff at $k=\infty$. Since the above cutoff terms maintain the chiral symmetry, the effective action $\Gamma_{\alpha}$ also respects it. The evolution equation with this cutoff scheme is written as

$$\Lambda \frac{\partial}{\partial \Lambda} \Gamma_{\alpha} = (k+1) \cdot \text{Str} \left\{ C^{-1} \left( C^{-1} + \frac{\delta}{\delta \Phi^T} \Gamma_{\alpha} \frac{\delta}{\delta \Phi} \right)^{-1} \right\}, \quad (3)$$

where $\Phi$ is shorthand notation for the fields $\Phi = \{A_{\mu}, \phi, \bar{\phi}\}$, and $C^{-1}$ is the matrix

$$C^{-1}(q) = \begin{pmatrix}
\Lambda^2 \cdot (\Lambda/q)^{2k} \cdot \delta_{\mu \nu} & 0 \\
0 & (\Lambda/q)^{2k+2} \cdot q_\mu \cdot \gamma_\mu^T
\end{pmatrix}. \quad (4)$$

To approximate the RG equation (3), we project it onto a finite dimensional subspace. Specifying a subspace defines an approximation. Here we take a subspace spanned by the seven chiral invariant operators

$$\Gamma_{A}[A_{\mu}, \bar{\psi}, \psi] = \int d^4x \left\{ \frac{1}{4} Z_3 F_{\mu \nu}^2 + \frac{1}{2\Lambda} (\partial \cdot A)^2 + \frac{1}{2} m^2 A_{\mu}^2 + Z_1 \bar{\psi} \gamma_\mu i\partial_\mu \psi + e A_{\mu} \bar{\psi} \gamma_\mu \psi - \frac{1}{2} \frac{G_s}{\Lambda^2} \left[ (\bar{\psi} \gamma_5 \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 \right] - \frac{1}{2} \frac{G_v}{\Lambda^2} \left[ (\bar{\psi} \gamma_\mu \gamma_5 \psi)^2 + (\bar{\psi} \gamma_5 \gamma_\mu \psi)^2 \right] \right\}, \quad (5)$$

where $F_{\mu \nu}$ is the field strength and $m^2$ is the photon mass which compensates for the lack of the gauge invariance of $\Gamma_{A}$.

Note that the chiral and parity invariant 4-fermi interactions without derivatives are spanned by the above two independent operators $G_s$ and $G_v$. Within the non-derivative (local potential) approximation for multi-fermi operators, the 4-fermi flow equations are not affected at all by higher dimensional multi-fermi operators which

![Fig. 1. Diagrams incorporated in our approximation. The diagrams in the solid box correspond to the ladder part, and the diagrams in the dashed box are crucial for the gauge invariance.](https://academic.oup.com/ptp/article-abstract/97/3/479/1839568)
are composed of at least 8-fermi operators, due to the chiral symmetry. In such a
case, the 4-fermi flows completely determine the critical behavior for the local
potential approximated multi-fermi system, although higher multi-fermi operator
flows are necessary when calculating the dynamically generated fermion mass and the
chiral condensates. 14)

Substituting the effective action (5) into Eq. (3), we obtain the RG flow equations
for the dimensionless coupling constants, i.e., \( Z_3, Z_l, \alpha, m^2, e, G_s, G_v \) (see Fig. 1).

The effective action (5) actually contains two redundant parameters, due to the
wave function renormalization. We set \( Z_l \) and \( Z_3 \) to unity by rescaling the fields \( A_\mu, \phi, \bar{\phi} \). Consequently, the RG equations read

\[
\frac{\partial}{\partial t} m^2 = \left( 2 - \frac{e^2}{6\pi^2} \right) m^2 - 4 I_1(k) e^2, \tag{6}
\]

\[
\frac{\partial}{\partial t} e = -\frac{1}{2} \left( 3 I_1(k, m^2) - I_5(k, m^2, \alpha) \right) e^3 - \frac{1}{12\pi^2} e^3 + 2e(G_s + 4 G_v) I_1(k), \tag{7}
\]

\[
\frac{\partial}{\partial t} G_s = -2G_s - (3 I_1(k, m^2) - I_5(k, m^2, \alpha)) e^2 G_s + 6 I_5(k, m^2) e^2 G_s
\]

\[+ 2 I_1(k) G_s (3 G_s + 8 G_v) - 6 I_5(k, m^2) e^4, \tag{8}\]

\[
\frac{\partial}{\partial t} G_v = -2G_v - 6 I_5(k, m^2) e^2 G_v - (3 I_1(k, m^2) - I_5(k, m^2, \alpha)) e^2 G_v
\]

\[+ I_1(k) G_s^2 - 3 I_5(k, m^2) e^4, \tag{9}\]

\[
\frac{\partial}{\partial t} \alpha = -\frac{1}{6\pi^2} e^2 \alpha, \tag{10}\]

where \( t \) is a cutoff scale parameter defined by \( \Lambda = \Lambda_0 e^{-t} \). There appear five independent functions of \( k, \alpha \) and \( m^2 \). These functions are defined by the following integrals:

\[
I_1(k) = \frac{k + 1}{8\pi^2} \int_0^\infty \! dx x^{2k+2} \left( \frac{1}{1 + x^{k+1}} \right)^3, \tag{11}\]

\[
I_5(k, m^2) = \frac{k + 1}{8\pi^2} \int_0^\infty \! dx x^{4k+2} \left( \frac{1}{1 + x^{k+1}} \right)^3 \left( \frac{1}{1 + x^{k+1} + m^2 x^k} \right)^2
\]

\[+ \left( \frac{1}{1 + x^{k+1}} \right)^2 \left( \frac{1}{1 + x^{k+1} + m^2 x^k} \right)^3, \tag{12}\]

\[
I_5(k, m^2) = \frac{k + 1}{8\pi^2} \int_0^\infty \! dx x^{3k+2} \left( \frac{1}{1 + x^{k+1}} \right)^2 \left( \frac{1}{1 + x^{k+1} + m^2 x^k} \right)^2
\]

\[+ 2 \left( \frac{1}{1 + x^{k+1}} \right)^3 \left( \frac{1}{1 + x^{k+1} + m^2 x^k} \right), \tag{13}\]

\[
I_5(k, m^2, \alpha) = \frac{k^2 - 1}{8\pi^2} \int_0^\infty \! dx \left[ x^{2k+1} \left( \frac{1}{1 + x^{k+1}} \right)^2 \left( \frac{1}{1 + x^{k+1}/\alpha + m^2 x^k} \right)
\]

\[+ \left( \frac{1}{1 + x^{k+1}} \right)^2 \left( \frac{1}{1 + x^{k+1}/\alpha + m^2 x^k} \right)^2 \right].
\]
Here we note some important features of these RG equations and integrals $I_x$. The gauge parameter $\alpha$-dependence of the RG equations comes solely from $I_5$, which vanishes with the special cutoff scheme $'k=1'$. Therefore under this cutoff scheme, our RG equations are completely free from $\alpha$-dependence. Of course this cancellation basically depends on the fact that our beta functions contain a gauge invariant set of diagrams, e.g., the crossed box as well as the box diagrams, and inclusion of the proper anomalous dimension of fields.

The sharp cutoff (large $k$) limit makes the RG equations singular. The integrals $I_1$, $I_2$, $I_3$ approach finite functions of $m^2$, while $I_4$ and $I_5$ diverge, except for the case of vanishing photon mass ($m^2=0$). On the other hand, for $m^2=0$, the integrals $I_4(k, m^2=0)$ and $I_5(k, m^2=0, \alpha)$ vanish independently of $k$ and $\alpha$. The photon mass $m^2$ is to be determined as a function of the gauge coupling constant $\alpha$ by the requirement of the gauge invariance of the effective action $\Gamma_{A\rightarrow 0}$, and is the order of $\alpha^2$. Therefore, the $\alpha$-dependence of our RG flow equations is of order $\alpha^4$.

We should note the essential difference between our NPRG flows and the perturbative RG. Our RG equations describe the running of the 4-fermi and multi-fermi interactions as well, e.g., $\partial \bar{G}_s/\partial t \sim 2\bar{G}_s$, which finally gives the effective potential of the composite operator $\bar{\phi}\phi$, so that we can investigate the dynamical chiral symmetry breaking within our RG equations. Also, the beta function of the gauge coupling constant $\alpha$ contains the so-called quadratically divergent diagram contribution, i.e., $\partial\alpha/\partial t \sim \alpha G_s$, which will need special care in relation to the gauge invariance.

§ 3. The critical behavior of QED

We investigate the $G_s - G_v$ sub-system described by Eqs. (8) and (9) with the vanishing photon mass approximation ($m^2=0$). This system does not suffer from gauge parameter dependence, and also it has an infinite $k$ limit which completely reproduces the sharp cutoff LPA (Wegner-Houghton equation) results.\textsuperscript{13,14} In a more precise treatment with the running photon mass, a small gauge dependence exists, except for the special cutoff profile of $k=1$. The gauge coupling constant is fixed, not running. Complete analyses with the running gauge coupling constant will be described in a forthcoming paper.\textsuperscript{17} Our results here should be compared with the QED SD results with the fixed gauge coupling constant.\textsuperscript{2} Then the RG equations determining the critical behavior are the following coupled differential equations (note $I_3(k, 0)=1/(8\pi^2)$):

$$\frac{\partial}{\partial t}G_s = -2G_s + 2I_4(k)G_3(3G_5 + 8G_v) + \lambda G_s - 6\bar{G}_s(k, 0)\lambda^2,$$

(16)
\[
\frac{\partial}{\partial t} G_v = -2G_v - \lambda G_v + I_1(k) G_s^2 - 3 \bar{I}_2(k, 0) \lambda^2,
\]
(17)

where \( \lambda \) and \( \bar{I}_2 \) are defined by \( \lambda = 3e^2/4\pi \) and \( (4\pi^2/3)I_2 \) respectively.

In the RG approach, the critical behavior is governed by the structure of the fixed points. First we consider the case of vanishing gauge coupling constant, where the system is a pure fermi system and we are solving the Nambu-Jona-Lasinio model beyond the ladder (the large \( N \) leading) approximation. As is shown in Fig. 2, we have two fixed points, the Gaussian infrared fixed point at the origin and the modified Nambu-Jona-Lasinio (NJL) ultraviolet fixed point extended to the vector operator \( G_v \).\(^*)\) Also there appears another 3rd fixed point in the negative \( G_s \) region, which we will ignore here as a fake, resulting from the approximation.

There are two phases, the strong phase and the weak phase, and the renormalized trajectory is the straight line defined by \( G_s = 8G_v \). In the strong phase, the 4-fermi interactions grow and diverge at finite cutoff scale in this truncation. This divergence itself may be regarded as a signal of the limitation of our truncation, and we expect that in an enlarged subspace this singularity will disappear. However, in the local potential approximation, we can still estimate the dynamical fermion mass and the chiral condensate by taking account of the infinite number of higher dimensional fermi operators. In fact, investigating the effective potential of the composite operator \( \bar{\psi}\psi \) in either phase, we can conclude that in the strong phase the chiral symmetry is spontaneously broken, while in the weak phase the chiral symmetry is respected.\(^14\)

Switching on the gauge interactions (Fig. 3), these two fixed points move closer to each other and finally meet at some \( \lambda(=\lambda_c) \) to pair-annihilate. Beyond

\fig{2}{The flow diagram in the \( G_s - G_v \) plane with the vanishing gauge coupling constant and with the cutoff scheme '\( k=1 \).'}

\fig{3}{The motion of the fixed points with the various cutoff schemes. The ladder part LPA results\(^{13,14}\) giving the Landau gauge ladder SD equivalents are plotted by the dotted line.}

\(^*)\) To be precise, without gauge interactions, there is no ultraviolet fixed point since the 8-fermi operators do not have a fixed point solution.
this critical gauge coupling constant $\lambda_c$, no fixed point exists. The Gaussian fixed point at $\lambda=0$ moves to some finite values of $G_s$ and $G_v$, which represents the scale invariant infrared effective 4-fermi interactions generated by the gauge exchange interactions.

We now understand the total phase structure. For the region $0 \leq \lambda < \lambda_c$, there are two fixed points, and two phases divided by the ultraviolet fixed point, the modified NJL fixed point. For $\lambda > \lambda_c$, no fixed point appears, and the whole theory space belongs to the strong phase of the dynamical chiral symmetry breaking. These structures generate a peculiar phase diagram on the $\lambda-G_s$ plane. We show the $k=1$ gauge independent results in Fig. 4, where the arrows show the RG flows.

The critical surface obtained in various cutoff schemes are displayed in Fig. 5. In Figs. 3 and 5, we see that the fixed point position and the critical surface depend strongly on the cutoff scheme. This is seen in the RG equations, where the criticality is directly determined by the cutoff scheme dependent coefficients ($I_1$ and $I_2$). For example, the non-trivial fixed point on the $\lambda=0$ plane is given by $G_s^\infty = 8G_v^\infty = 1/(4I_1(k))$. It should be mentioned that these quantities of the position of the criticality are not the physical quantities at all, that is, they cannot be measured. Hence these large cutoff scheme dependences do not cause any difficulty in our approach.

Our chiral phase criticality qualitatively coincides with the Landau gauge ladder SD results. The quantitative differences come from two sources. Our results take account of the non-ladder contributions as well as the ladder parts, which guarantees the gauge independence. Also our cutoff profile is smooth, while the SD uses the $\theta$-function sharp cutoff. Taking a large value of $k$, say $k=100$, then our results are almost the same as the LPA sharp cutoff results within the line width in every figure.
§ 4. Critical exponent and anomalous dimension

Here, we evaluate the physical quantities, the critical exponent of the ultraviolet fixed point ($\nu_4$), and the anomalous dimension of the fermion mass operator ($\gamma_m$). The critical exponent $\nu_4$ is obtained by linearizing the RG flow equations around the ultraviolet fixed point, and it is related to the anomalous dimension of the chiral invariant 4-fermi operators $\gamma$ through $\nu_4 + 2 = \gamma_4$. On the other hand, the fermion mass operator $m_f \bar{\psi} \psi$ explicitly breaks the chiral symmetry, and the vanishing mass point is its ultraviolet fixed point. Thus we may evaluate its anomalous dimension $\gamma_m$ from the linearized RG equation for the operator $m_f \bar{\psi} \psi$,

$$\frac{\partial}{\partial t} m_f = (1 + \gamma_m) m_f + O(m_f^3). \quad (18)$$

The anomalous dimension $\gamma_m$ is evaluated in terms of the gauge coupling constant $\lambda$ and the critical scalar 4-fermi interaction $G^s(t, \lambda)$ as

$$\gamma_m(\lambda) = \frac{\lambda}{2} + 8 I_1(k) G^s(t, \lambda), \quad (19)$$

where the corresponding diagrams are shown in Fig. 6.

In Fig. 7, we plot the critical exponent $\nu_4$ and the anomalous dimension $\gamma_m$ as a

![Diagrams](https://example.com/diagram.png)

**Fig. 6.** Diagrams contributing to the anomalous dimension of the fermion mass operator $\bar{\psi} \psi$.

![Graphs](https://example.com/graph.png)

**Fig. 7.** The critical exponent $\nu_4$ and the anomalous dimension $\gamma_m$ in various cutoff schemes, compared to the result of the Landau gauge ladder SD equation.
Table I. The critical exponent $\nu_4$ and the anomalous dimension $\gamma_m$. Our gauge independent results ($k=1$) are compared with the Landau gauge ladder SD\textsuperscript{2} and the sharp cutoff LPA\textsuperscript{3} results.

| $\lambda$ | $\nu_4$ | $\gamma_m$ |
|-----------|---------|------------|
| $ladder SD$ | $LPA$ | $our result$ | $ladder SD$ | $LPA$ | $our result$ |
| 0.0 | 2.000 | 2.000 | 2.000 | 2.000 | 2.000 | 2.000 |
| 0.1 | 1.897 | 1.918 | 1.918 | 1.949 | 1.987 | 1.987 |
| 0.2 | 1.789 | 1.828 | 1.827 | 1.894 | 1.969 | 1.969 |
| 0.3 | 1.673 | 1.729 | 1.726 | 1.837 | 1.945 | 1.944 |
| 0.4 | 1.549 | 1.619 | 1.614 | 1.775 | 1.914 | 1.912 |
| 0.5 | 1.414 | 1.497 | 1.487 | 1.707 | 1.874 | 1.872 |
| 0.6 | 1.265 | 1.359 | 1.342 | 1.632 | 1.825 | 1.820 |
| 0.7 | 1.095 | 1.200 | 1.173 | 1.548 | 1.763 | 1.754 |
| 0.8 | 0.894 | 1.013 | 0.968 | 1.447 | 1.684 | 1.667 |
| 0.9 | 0.632 | 0.776 | 0.699 | 1.316 | 1.575 | 1.542 |
| 1.0 | 0.000 | 0.418 | 0.177 | 1.000 | 1.396 | 1.274 |
| at $\lambda_c$ | 0.000 | 0.000 | 0.000 | 1.000 | 1.170 | 1.177 |
| ($\lambda_c$) | (1.0000) | (1.0409) | (1.0069) | (1.0000) | (1.0409) | (1.0069) |

function of the gauge coupling constant $\lambda$ obtained in various cutoff schemes, with those obtained by the ladder SD equation. Although there is a large ‘scheme dependence’ in the position of criticalities, the exponent and anomalous dimension are almost independent of the cutoff scheme. This indicates that our method and approximation are fairly stable with respect to the cutoff profile dependence.

Comparing with the ladder SD results, the anomalous dimension $\gamma_m$ is greatly enhanced, while the critical exponent $\nu_4$ is almost the same. This large corrections certainly come from the contribution of the non-ladder diagrams. Note that our results are gauge independent due to the inclusion of these non-ladder contributions. In the ladder approximation, these two quantities are not independent and satisfy a relation, $\nu_4 + 2 = 2\gamma_m$, which is given by the simple ladder relation of $\gamma_4 = 2\gamma_m$, where only the scalar 4-fermi operator is taken into account. In our non-ladder extension, this relation is broken, generating the large deviation of $\gamma_m$. The numerical values of our gauge independent results ($k=1$) are listed in Table I, comparing them with the Landau gauge ladder SD\textsuperscript{2} and the sharp cutoff LPA\textsuperscript{3} results. It should be noted that the value of the critical gauge coupling constant $\lambda_c$ depends on the approximation adopted.

§ 5. Discussion and comments

We solved the next to LPA NPRG and obtained the critical behavior of the dynamical chiral symmetry breaking in QED. Our RG system with a special cutoff scheme ($k=1$) exhibits gauge parameter independence, which is assured by the inclusion of the non-ladder contributions like the crossed box diagrams. The NPRG approach to the dynamical chiral symmetry breaking in gauge theories has great
potential. It turns out that the NPRG with our subspace approximation gives a beyond-the-ladder approximation, restoring the gauge independence. It should be noted that the $k=1$ cutoff scheme does not work if we go to higher orders of the derivative expansion, since the beta function integrals do not converge. However, we expect that the gauge dependence should not be large since the NPRG approach may easily incorporate the 'gauge invariant' set of diagrams.

We have also confirmed the stability of our method. The cutoff scheme dependences in physical quantities are found to be negligible. Also we obtain rather small deviations of physical results compared to the LPA. For example, the anomalous dimension $\gamma_m$ at $\lambda_c$ is 1.170 in the LPA and 1.177 in our next to LPA, respectively. In our level of approximation in this article ($m^2=0$), this stability is actually equivalent to the cutoff scheme independence, since the infinite $k$ limit corresponds to the sharp cutoff LPA.

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