Fast Reconstruction and Iterative Updating of Spatial Covariance Matrix for DOA Estimation in Hybrid Massive MIMO

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ABSTRACT Due to the high resolution property, multiple signal classification (MUSIC) algorithm has been widely used for direction-of-arrival (DOA) estimation in wireless systems. To reduce the cost caused by radio frequency (RF) chains, hybrid structure has been adopted in massive MIMO systems operating at millimeter-wave bands. With hybrid structures, the received signals at the antennas are not fed directly to the receiver, and thus the spatial covariance matrix (SCM), which is essential to MUSIC algorithm, cannot be obtained using traditional sample average algorithm. Based on our previous works, we propose a fast beam sweeping algorithm in this article for SCM reconstruction in hybrid massive MIMO systems. Using multiple RF chains in hybrid structures, we find that beam sweeping can be conducted in a parallel manner so that the SCM can be reconstructed much faster than our previous works. To further accelerate the updating of the reconstructed SCM, an iterative updating procedure is also developed in this article. The iterative procedure can update the reconstructed SCM much faster than the batch-processing approach because it updates the SCM immediately as long as new beam sweeping results are available. Simulation results are also presented in this article to demonstrate the improvement of the proposed approach.

INDEX TERMS DOA estimation, massive MIMO, hybrid structure, MUSIC.

I. INTRODUCTION

Direction-of-arrival (DOA) estimation is an important topic in the field of array signal processing. Many algorithms have been proposed and investigated for DOA estimation in the past decades [1]. As one of the most important algorithms, multiple signal classification (MUSIC) algorithm gained a lot of attention due to its high resolution property [2], it has also been applied in many practical wireless systems.

To satisfy the future demand for high rate transmission, millimeter wave (mmWave) bands are being explored in the fifth generation (5G) and beyond 5G cellular systems [3], [4]. Massive multiple-input multiple-output (MIMO) is one of the most important enabling technologies in mmWave communication systems because the high path loss in the mmWave bands can be compensated effectively by the large array gain in massive MIMO systems [5]. Due to the high cost of radio frequency (RF) chains, digital beamformer can be hardly adopted in mmWave communication systems. Instead, analog beamformers are widely considered in massive MIMO at mmWave bands in order to reduce the number of RF chains. This leads to the hybrid analog-digital structure in massive MIMO systems [6]–[10]. For the hybrid structure, beamforming is conducted in the analog domain, and a group of phase shifters are used for analog beamforming. The received signals at the antennas are first combined in the analog domain before sent to the digital receiver via RF chains. In this way, the number of RF chains can be greatly reduced.

Although widely adopted, the hybrid structure makes it difficult to use MUSIC algorithm for DOA estimation in massive
MIMO systems. For the hybrid structure, the received signals at the antennas are unavailable to the digital receiver. In this case, the spatial covariance matrix (SCM), which is essential in MUSIC algorithm, cannot be obtained using traditional sample average approach [11]. As a consequence, MUSIC algorithm cannot be directly used in massive MIMO systems with hybrid structures. As the MUSIC algorithm is not applicable in hybrid systems, a simple method for DOA estimation is to search for the direction with the maximum received power [12]–[14]. However, these methods are restricted by the Rayleigh limitation, and thus the high resolution property cannot be sustained [2].

To utilize the high resolution property of MUSIC algorithm, we have developed a beam sweeping algorithm for SCM reconstruction in massive MIMO system with single RF chain [15]. Based on our previous works in [15], we will propose a fast beam sweeping algorithm in this article for SCM reconstruction in hybrid massive MIMO systems with multiple RF chains. In the presence of multiple RF chains, the predetermined DOA required in [15] are divided into several sub-groups, with each sub-group corresponding to one RF chain. Then, by beam sweeping simultaneously on all RF chains, the predetermined DOAs can be swept in a parallel manner, so that the SCM can be reconstructed much faster than our previous works. The parallel beam sweeping algorithm proposed in this article is designed to work in a batch-processing manner, and the reconstructed SCM cannot be updated until all the predetermined DOAs have been swept. To accelerate the updating of the reconstructed SCM, an iterative updating procedure is further developed in this article. Using the iterative procedure, the reconstructed SCM can be updated immediately as new beam sweeping results are available. As a result, the updating of reconstructed SCM can be accelerated because there is no need to wait for sweeping all the predetermined DOAs. Computer simulation is also conducted in this article to demonstrate the improvement of the newly proposed algorithms.

The rest of this article is organized as follows. In Section II, signal model for hybrid massive MIMO is introduced and then traditional MUSIC algorithm is reviewed briefly. In Section III, the fast beam sweeping algorithm is presented, including the sub-grouping of predetermined DOAs and parallel beam sweeping. The iterative updating procedure is presented in Section IV. Simulation results can be found in Section V and the conclusions are drawn in Section VI.

II. SYSTEM MODEL

In this section, we will first present the signal model used in this article. Then, we will review briefly the traditional MUSIC algorithm, and discuss its restriction in hybrid massive MIMO systems.

A. SIGNAL MODEL

As in Fig. 1, we consider a hybrid massive MIMO system composed of a uniform linear array (ULA) with $M$ antennas and $N$ RF chains. The received signals at the antennas are divided onto $N$ paths using analog power splitters. After going through the analog phase shifters, the signals are combined using the power splitters before sent to the RF chains.

Denote $y_m(t)$ to be the received signal at the $m$-th antenna, and $x_i(t)$’s ($i = 0, 1, \cdots, L - 1$) are $L$ narrow-band signals impinging from far field onto the array. Denote $\cdot^T$ to be the
transpose of a matrix or a vector, then the received signal vector \( y(t) = [y_0(t), y_1(t), \cdots, y_{M-1}(t)]^T \) can be represented as

\[
y(t) = \sum_{i=0}^{L-1} a(\theta_i)x_i(t) + z(t),
\]

where \( \theta_i \) is the DOA of \( x_i(t) \), \( z(t) \) denotes the additive Gaussian noise vector with \( \operatorname{E}[z(t)z^H(t)] = N_0 I_M \) where \( N_0 \) is the noise power and \( I_M \) is an \( M \times M \) identity matrix, \( (\cdot)^H \) indicates the conjugate transpose of a matrix or a vector and \( a(\theta_i) \) is an \( M \times 1 \) steering vector with the \( m \)-th entry

\[
a_m(\theta_i) = e^{j2\pi \frac{d}{\lambda} \sin \theta_i m},
\]

where \( d \) denotes the distance between adjacent antennas and \( \lambda \) indicates the wave length.

If \( L \) signals are mutually independent and the power of the \( i \)-th signal is \( \operatorname{E}[|x_i(t)|^2] = \sigma_i^2 \), then the \( M \times M \) SCM for the received signal vector in (1) can be obtained as

\[
R = \operatorname{E}[y(t)y^H(t)] = \sum_{i=0}^{L-1} \sigma_i^2 \cdot a(\theta_i)a^H(\theta_i) + N_0 I_M.
\]

As shown in (3), SCM is Hermitian, and therefore the eigenvalue decomposition of the SCM can be given by

\[
R = (U_s, U_n)\Lambda_s(U_s, U_n)^H,
\]

where \( U_s \) and \( U_n \) denote the orthogonal base vectors corresponding to the signal and the noise subspaces, respectively, and \( \Lambda_s \) is a diagonal matrix composed of the eigenvalues corresponding to \( R \).

### B. MUSIC ALGORITHM REVIEW

Denote \( y[k] = y(kT_s) \) to be the sample of the received signal vector where \( T_s \) denotes the sampling period. For the traditional MUSIC algorithm, the received signals are directly fed to the digital receiver after sampling, and thus \( y[k] \)'s are assumed to be available in the receiver. Under this situation, the SCM in (3) can be estimated by the sample average approach [2]

\[
\hat{R} \approx \frac{1}{K} \sum_{k=0}^{K-1} y[k]\hat{y}^H[k],
\]

where \( K \) denotes the number of samples. Accordingly, the eigenvalue decomposition of the estimated SCM in (5) can be given as

\[
\hat{R} = (\hat{U}_s, \hat{U}_n)\hat{\Lambda}_s(\hat{U}_s, \hat{U}_n)^H,
\]

where \( \hat{U}_s, \hat{U}_n, \) and \( \hat{\Lambda}_s \) denote the orthogonal base vectors, the noise subspaces, and the eigenvalue matrix corresponding to the estimated SCM. Then, unknown DOAs can be determined by searching the peak values of \( P(\theta) \),

\[
P(\theta) = \|\hat{U}_n^H a(\theta)\|^2, \quad \theta \in [-90^\circ, 90^\circ].
\]

In (5), \( y[k] \) is necessary to estimate the SCM. In hybrid massive MIMO, however, Fig. 1 indicates that the received signals at the antennas cannot be obtained directly by the digital receiver. Only the weighted combination of the entries of \( y[k] \) is available in each RF chain. As a result, the sample average in (5) cannot be used in massive MIMO systems with hybrid structures.

To address this issue, we have developed an algorithm in [15] for SCM reconstruction in massive MIMO with single RF chain. As only one RF chain is considered in [15], the procedure for SCM reconstruction requires a long time duration because the predetermined DOAs has to be swept sequentially. In this article, we will introduce a fast beam sweeping algorithm in Section III so that the SCM can be reconstructed much faster.

### III. FAST BEAM SWEEPING ALGORITHM

For beam sweeping algorithm, we can define

\[
\Theta = \{\theta^{(0)}, \theta^{(1)}, \cdots, \theta^{(Q-1)}\},
\]

as a group of predetermined DOA angles. For the single RF chain case as in [15], the analog beamformers switch the beam directions to the predetermined DOAs in (8) sequentially. Then, by collecting the received power on each predetermined DOA, the SCM can be reconstructed through solving a set of linear equations.

For the fast beam sweeping algorithm, the predetermined DOAs are first divided into a set of sub-groups, with each sub-group corresponding to one RF chain. Then, the beam sweepings are conducted on all RF chains simultaneously.

#### A. SUB-GROUPING OF PREDETERMINED DOAS

In the case of \( N \) RF chains, define \( \Theta_n \) to be a sub-group of the predetermined DOAs corresponding to the \( n \)-th RF chain, then we have

\[
\Theta = \bigcup_{n=0}^{N-1} \Theta_n.
\]

Let \( |\Theta_n| \) indicate the cardinality of the set \( \Theta_n \), then we can obtain that \( |\Theta_n| \) is an integer with \( 0 \leq |\Theta_n| \leq Q \), and

\[
\sum_{n=0}^{N-1} |\Theta_n| = Q.
\]

If the beam sweeps are conducted simultaneously on all RF chains, the time duration for beam sweeping procedure is determined by the RF chain with the most predetermined DOAs. In other words, the required time duration should be proportional to

\[
\max_{n=0, \ldots, N-1} |\Theta_n|.
\]

Therefore, we should minimize (11) in order to achieve the fastest beam sweeping. Considering that \( 0 \leq |\Theta_n| \leq Q \) and the restriction in (10), it is easy to obtain that the minimization in (11) can be achieved when \( Q \) predetermined DOAs are uniformly divided among all the RF chains. Without loss of generality, we assume that \( Q/N \) is an integer to simplify the
notation in the rest of this article. Under this assumption, we can divide $Q$ predetermined DOAs evenly into $N$ subgroups, that is

$$\Theta_n = \{\theta^{nQ/N}, \cdots, \theta^{(n+1)Q/N-1}\},$$  \hspace{1cm} (12)$$

Accordingly, we have

$$\max_{n=0,\ldots,N-1} |\Theta_n| = Q/N,$$  \hspace{1cm} (13)$$

which means each sub-group has $Q/N$ predetermined DOAs.

**B. PARALLEL BEAM SWEEPING**

For the $n$-th RF chain, the analog beamformer switches the beam direction to the predetermined DOA angles in (12) sequentially. For the $q$-th sweeping beam on the $n$-th RF chain, the predetermined DOA is $\theta^{nQ/N+q}$ where $q = 0, 1, \ldots, Q/N - 1$. Accordingly, the analog beamforming vector on the $n$-th RF chain is

$$a_n(\theta^{q}) = a(\theta^{nQ/N+q}).$$  \hspace{1cm} (14)$$

In this case, the weighted combination of the received signals in the $n$-th RF chain can be represented by

$$c_n^{(q)}(t) = a_n^H(\theta^{q}) y(t).$$  \hspace{1cm} (15)$$

From Fig. 1, the signal combination is first down-converted to the baseband and then sampled before sent to the digital receiver. Therefore, the sampled signal can be expressed as

$$c_n^{(q)}(k) = c_n^{(q)}(kT_s) = a_n^H(\theta^{q}) y[k].$$  \hspace{1cm} (16)$$

Denote $P_n^{(q)}$ to be the average power of $c_n^{(q)}[k]$, then it can be obtained, using sample average, as

$$P_n(\theta^{q}) = \frac{1}{K} \sum_{k=0}^{K-1} |c_n^{(q)}[k]|^2 = a_n^H(\theta^{q}) \frac{1}{K} \sum_{k=0}^{K-1} y[k] y^H[k] a_n(\theta^{q}).$$  \hspace{1cm} (17)$$

When the number of samples is large enough, the sample average in (17) can be replaced by the statistical average, and thus we can obtain

$$a_n^H(\theta^{q}) R a_n(\theta^{q}) = P_n(\theta^{q}).$$  \hspace{1cm} (18)$$

Using the vec($\cdot$) operator to (18), the left-hand-side of (18) can be given as

$$\text{vec}(a_n^H(\theta^{q}) R a_n(\theta^{q})) = [a_n^H(\theta^{q}) \otimes a_n^H(\theta^{q})] \text{vec}(R),$$  \hspace{1cm} (19)$$

where we have used equation (1.10.25) in [16], that is,

$$\text{vec}(BCD) = (D^T \otimes B) \text{vec}(C).$$  \hspace{1cm} (20)$$

Denote $b_n(\theta^{q}) = a_n(\theta^{q}) \otimes a_n^H(\theta^{q})$ and $r = \text{vec}(R)$, which are both $M^2 \times 1$ vectors, then (18) can be rewritten as

$$b_n^H(\theta^{q}) r = P_n(\theta^{q}),$$  \hspace{1cm} (21)$$

If taking all predetermined DOAs in $\Theta_n$ into account, (21) can be extended to a group of linear equations as

$$B_n^r r = p_n,$$  \hspace{1cm} (22)$$

where

$$B_n = [b_n(\theta^{(0)}), b_n(\theta^{(1)}), \ldots, b_n(\theta^{(Q/N-1)})],$$  \hspace{1cm} (23)$$

$$p_n = [P_n(\theta^{(0)}), P_n(\theta^{(1)}), \ldots, P_n(\theta^{(Q/N-1)})]^T$$  \hspace{1cm} (24)$$

are $M^2 \times Q/N$ matrix and $Q/N \times 1$ vector, respectively.

In above, we only consider the $n$-th RF chain. In fact, the procedure from (15) to (22) can be also conducted for all the other RF chains simultaneously. In other words, the beam sweepings over $\Theta_n$’s with $n = 0, 1, \ldots, N - 1$ can be conducted in a parallel manner. Therefore, (22) can be further extended, by taking the other RF chains into account, as

$$Br = p,$$  \hspace{1cm} (25)$$

where

$$B = \begin{pmatrix} B_0^T \\ \vdots \\ B_{N-1}^T \end{pmatrix}, \quad p = \begin{pmatrix} p_0 \\ \vdots \\ p_{N-1} \end{pmatrix},$$  \hspace{1cm} (26)$$

are $Q \times M^2$ matrix and $Q \times 1$ vector, respectively. Then, similar to [15], the SCM can be obtained, by solving (25), as

$$\hat{r} = (B^H B + \sigma^2 I_{M^2})^{-1} B^H p = C p.$$  \hspace{1cm} (27)$$

where $I_{M^2}$ denotes an $M^2 \times M^2$ identity matrix, $\sigma^2$ is a diagonal loading scalar to avoid ill-conditioned result, and

$$C = (B^H B + \sigma^2 I_{M^2})^{-1} B^H,$$  \hspace{1cm} (28)$$

is an $M^2 \times Q$ matrix. Finally, the desired SCM can be reconstructed through $\hat{R} = \text{unvec}(\hat{r})$.

Note that the second equation in (27) indicates that the vector form of $R$ can be obtained by projecting the beam sweeping result, $p$, onto the column space of $C$. Although the matrix inversion in $C$ causes heavy computational burden for real-time processing, $C$ can be pre-calculated off-line if the predetermined DOA angles and diagonal loading coefficient are fixed. In this case, the huge computational burden for real-time processing can be avoided, and only a linear matrix production is required for real-time processing, which needs $M^2 Q$ complex multiplications. Therefore, compared to the single RF chain case in [15], adopting multiple RF chains has no impact on the computation complexity.

**C. DISCUSSION**

For the single RF chain case as in [15], the predetermined DOAs have to be swept sequentially. If the number of predetermined DOAs is $Q$, then $Q$ beam sweeping operations are required with each beam sweeping operation corresponding to one predetermined DOA. As a result, the number of beam sweeping operations is $Q$. 

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In the case of multiple RF chains, $Q$ predetermined DOAs are divided into $N$ sub-groups, with each sub-group having $Q/N$ predetermined DOAs. For each beam sweeping operation, $N$ RF chains can sweep $N$ predetermined DOAs simultaneously, with each RF chain corresponding to one predetermined DOA. Therefore, the number of beam sweeping operations is $Q/N$, which is $N$ times lower compared to the single RF chain case. As a result, the number of beam sweeping operations can be reduced significantly, and thus the procedure for SCM reconstruction can be much faster.

### IV. ITERATIVE UPDATING

For continuous SCM reconstruction, the parallel beam sweeping algorithm presented in Section III can update the SCM in a batch-processing manner. In this case, all predetermined DOAs have to be swept before the reconstructed SCM can be updated. In this section, an iterative updating procedure will be presented so that the reconstructed SCM can be updated as long as the new beam sweeping results are available. Since there is no need to wait until all the predetermined DOAs are swept, the reconstructed SCM can be updated much faster.

#### A. BATCH-PROCESSING

To describe the iterative updating procedure, consider a sample sequence in the time domain as in Fig. 2 (a). In Fig. 2 (a), one sample bundle contains $K$ samples. Due to parallel beam sweeping, $N$ beam sweeping results can be obtained.
in one bundle, with each one corresponding to one RF chain. Therefore, for batch-processing, \(Q/N\) bundles are required to reconstruct the SCM because we need \(Q/N\) bundles to sweep all the predetermined DOAs, as in Fig. 2 (b).

### B. Iterative Updating Procedure

Denote \(P_n[i]\) to be the beam sweeping result during the \(i\)-th sample bundle on the \(n\)-th RF chain. Actually, \(P_n[i]\) is equal to \(P_n(\theta^{(q)})\) in (17) if the predetermined DOA, \(\theta^{(q)}\), is swept in the \(i\)-th sample bundle. Then, denote \(p[i]\) to be the overall beam sweeping results in the \(i\)-th bundle. Similar to (26), \(p[i]\) can be represented by

\[
p[i] = \begin{pmatrix} p_0[i] \\
\vdots \\
p_{N-1}[i] \end{pmatrix},
\]

where \(p_n[i]\) corresponds to the beam sweeping results on the \(n\)-th RF chain in the \(i\)-th bundle. For the \(n\)-th RF chain, the analog beamformer sweeps the predetermined DOAs in (12) sequentially. Since only one predetermined DOA can be swept in one sample bundle, \(p_n[Q/N + i]\) can be given as

\[
p_n[Q/N + i] = \begin{pmatrix} p_n[Q/N] \\
\vdots \\
p_n[Q/N + i] \\
p_n[i + 1] \\
p_n[i + 2] \\
p_n[Q/N - 1] \end{pmatrix},
\]

with \(i = 0, 1, \cdots, Q/N - 1\). For a given \(i\), the first \(i\) entries in (30) have been updated to new beam sweeping results, while the residual \(Q/N - i\) entries in (30) are not updated yet. Hence, the residual \(Q/N - i\) entries are kept to be the same with the last beam sweeping results. For illustration, the updating procedure of \(p_n[i]\) from \(i = Q/N - 1\) to \(i = 2Q/N - 1\) is shown in Fig. 3, where the red symbols indicate the newly updated entries while the black symbols indicate the entries that are not updated yet. For each sample bundle from \(i = Q/N\), the new beam sweeping results are stored in \(p_n[i]\) by updating the corresponding entries. The updating procedure proceeds until \(i = 2Q/N - 1\), and all the entries have been updated. Then, the entries at \(i = 2Q/N - 1\) are used as the old values again for the next \(Q/N\) sample bundles.

From Fig. 3, the updating procedure of \(p_n[Q/N + i]\) in (30) can be better demonstrated if we consider the following differential form, that is

\[
p_n[Q/N + i] = p_n[Q/N + i - 1] + \Delta_n[i]e_i, \tag{31}
\]

with \(i = 1, 2, \cdots, Q/N - 1\), where \(e_i\) is a \(Q/N \times 1\) vector with the \(i\)-th entry being 1 while the other entries being zeros, and \(\Delta_n[i]\) describes the difference between the beam sweeping result in the \((Q/N + i)\)-th bundle and the \(i\)-th bundle on the \(n\)-th RF chain, that is

\[
\Delta_n[i] = p_n[Q/N + i] - p_n[i]. \tag{32}
\]

Equation (31) indicates that, from the \((i - 1)\)-th bundle to \(i\)-th bundle, only the \(i\)-th entry of \(p_n[Q/N + i]\) is updated while the other entries remain unchanged. This feature can be utilized to develop the iterative updating procedure.

Denote \(\hat{r}[i]\) to be the vector form of the reconstructed SCM in the \(i\)-th bundle. Then, using (27), we can obtain

\[
\hat{r}[Q/N + i] = Cp[Q/N + i]. \tag{33}
\]

Using the identity in (27), \(C\) can be divided into

\[
C = (C_0, C_1, \cdots, C_{N-1}), \tag{34}
\]

where \(C_n = (B^\dagger B + \sigma^2 I_{M^2})^{-1} B_n^\dagger \) is an \(M^2 \times Q/N\) matrix, then (33) can be rewritten as

\[
\hat{r}[Q/N + i] = \sum_{n=0}^{N-1} C_n p_n[Q/N + i]. \tag{35}
\]

By substituting (31) into (35), the iterative updating procedure can be obtained as

\[
\hat{r}[Q/N + i] = \sum_{n=0}^{N-1} C_n p_n[Q/N + i - 1] + \sum_{n=0}^{N-1} \Delta_n[i]e_i
\]

\[
= \hat{r}[Q/N + i - 1] + \sum_{n=0}^{N-1} \Delta_n[i]e_{n,i}. \tag{36}
\]
where \(c_{n,i}\) denotes the \(i\)-th column of \(C_n\), that is

\[
c_{n,i} = (B^{H}B + \sigma^{2}I_{M^2})^{-1}b_n^*(\theta^{(i)})
\]

(37)

Since \(c_{n,i}\)'s can be pre-calculated once the predetermined DOAs are fixed, each iteration in (36) needs \(NM^2\) complex multiplications.

Equation (36) shows that the vector form of SCM reconstructed in the \(i\)-th sample bundle can be obtained as the SCM reconstructed in the \((i-1)\)-th sample bundle plus an correction term. The correction term is a linear combination of \(c_{n,i}\), where the coefficients depends on the difference between of the beam sweeping results in the \((Q/N + i)\)-th bundle and the \(i\)-th bundle. The iterative updating procedure in (36) is demonstrated in Fig. 2 (c).

C. DISCUSSION

The iterative updating procedure in (36) indicates that the reconstructed SCM can be updated every sample bundle. As a comparison, if the batch-processing is used to update the reconstructed SCM, we can obtain

\[
\hat{R}[i_0Q/N - 1] = CP[i_0Q/N - 1],
\]

(38)

where \(i_0\) is an integer. Equation (38) indicates that the batch-processing cannot update the reconstructed SCM every sample. Instead, the SCM is updated every \(Q/N\) sample bundles because the batch-processing has to wait until all the predetermined DOAs have been swept. Therefore, the iterative procedure can achieve a faster updating than the batch-processing, as shown in Fig. 2 (c).

Note that the iterative procedure cannot work without initialization although it can accelerate the updating of SCM. In this sense, the batch-processing can be used jointly with the iterative procedure. The batch-processing can be used for initialization within the first \(Q/N\) sample bundles, while the iterative procedure can be used for consecutive updating in subsequential sample bundles. The combination of iterative updating and batch based initialization is shown in Fig. 2 (c).

V. COMPUTER SIMULATION

Computer simulation is adopted in this section to demonstrate the efficiency of the fast reconstruction algorithm. In the simulation, we consider a ULA composed of \(M = 16\) antennas with \(d/\lambda = 0.5\). Three independent signals from \(\theta = -20^\circ, 20^\circ, 40^\circ\) are impinging onto the array with unit powers, and each sample bundle contains \(K = 3000\) samples. The predetermined DOAs are uniformly distributed from \(-90^\circ\) to \(90^\circ\). Without special specification, the signal-to-noise ratio (SNR) is set as \(-5\) dB in the simulation. Similar to [15], normalized squared error (NSE) is adopted in this article to evaluate the accuracy of SCM reconstruction, which is defined by

\[
\text{NSE} = \|\hat{R} - R\|_2^2 \cdot \|R\|_2^{-2}.
\]

(39)

In the simulation, the NSE is obtained by averaging over 2000 runs to obtain a mean value. MUSIC algorithm is adopted for DOA estimation once reconstructed SCM is available, and the grid for exhaustive searching in (7) is set to be \(-90^\circ\) to \(90^\circ\) with \(1^\circ\) as the grid size. Note that MUSIC algorithm may suffer from the off-grid DOA estimation issue [17], interpolation between neighboring DOA grids can be used in this case to improve the accuracy of DOA estimation [18].

![Figure 4](image)

FIGURE 4. NSE performances versus time duration in the cases of different numbers of RF chains. By exploiting multiple RF chains in hybrid massive MIMO systems, the reconstruction of SCM can be accelerated significantly.
can be conducted to $i = \infty$. In practice, the iteration can be stopped as long as the SCM reconstruction is not required any more.

Fig. 6 shows the reconstruction accuracy with different SNRs. From Fig. 6, it is interesting to find that the NSE will get worse as the rising of SNR. This is especially the case for $Q = 32$. When SNR is small, the desired SCM is close to an identity matrix except for a scaling factor determined by the SNR. In this sense, the reconstruction of SCM is similar to estimate this single scaling factor. The reduction of the numbers of unknown variables leads to improvement of reconstruction accuracy. When SNR is large, however, there exist much more unknown variables in the desired SCM, and thus the reconstruction accuracy gets worse. When $Q = 40$, the reconstructed SCM is already accurate enough, and thus changing the SNR has little impact on the NSE. Fig. 6 also indicates that the NSE can be improved by using more RF chains. By adopting more RF chains, the number of beam sweeping operations can be reduced, and the overall noise power included by beam sweeping operations can be also reduced accordingly. Therefore, NSE can be improved by adopting more RF chains.

Fig. 7 shows the performance improvement on the DOA estimation, where we have adopted the mean-square-error (MSE) as an indicator to evaluate the accuracy of DOA estimation, that is

$$\text{MSE} = E[(\hat{\theta} - \theta)^2].$$  \hspace{1cm} (40)

As expected, the accuracy of DOA estimation can be improved as the rising of the time duration. This is because more predetermined DOAs can be swept as the rising of time duration and the reconstructed SCM is thus more close to the true SCM. Similar to Fig. 4, the time duration can be greatly reduced by exploiting multiple RF chains, so that the DOA estimation procedure can be conducted much faster than the single RF chain case.

VI. CONCLUSION

In this article, we have developed a fast beam sweeping algorithm by exploiting multiple RF chains in hybrid massive MIMO system. In the presence of multiple RF chains, the predetermined DOA required are divided into several subgroups, with each subgroup corresponding to one RF chain. Then, by beam sweeping simultaneously on all the RF chains, the predetermined DOAs can be swept in a parallel manner, so that the SCM can be reconstructed much faster than our previous works. To accelerate the updating procedure, an iterative updating procedure has also been developed in this article. Using the iterative procedure, the reconstructed SCM can be updated as long as the new beam sweeping.
results are available, and there is no need to wait until all the predetermined DOAs are swept. Simulation results have also been presented in this article, demonstrating that the proposed approach can greatly accelerate the reconstruction and updating procedures for SCM reconstruction.

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