Aharonov-Anandan Effect Induced by Spin-Orbit Interaction and Charge-Density-Waves in Mesoscopic Rings

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We study the spin-dependent geometric phase effect in mesoscopic rings of charge-density-wave(CDW) materials. When electron spin is explicitly taken into account, we show that the spin-dependent Aharonov-Casher phase can have a pronounced frustration effects on such CDW materials with appropriate electron filling. We show that this frustration has observable consequences for transport experiment. We identify a phase transition from a Peierls insulator to metal, which is induced by spin-dependent phase interference effects. Mesoscopic CDW materials and spin-dependent geometric phase effects, and their interplay, are becoming attractive opportunities for exploitation with the rapid development of modern fabrication technology.

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Since Berry identified the importance of a "geometric phase" in adiabatic cyclic evolution, there have been numerous theoretical and experimental studies of this holonomy phenomenon termed the Berry phase. A fundamental generalization of the Berry phase was given by Aharonov and Anandan(AA). They removed the adiabatic condition and demonstrated the existence of the geometric phase in generic cyclic evolution. As is well known, the Aharonov-Bohm(AB) effect has led to a number of remarkable interference phenomena in mesoscopic systems, especially in rings. Based on the discovery of the geometric phase, it has been predicted that analogous interference effects can be induced by the geometric phases which originate from the interplay between an electron's spin and orbital degrees of freedom. Such an interplay can be maintained by an external electric field, which leads to a spin-orbit(SO) interaction.

Loss et al. first studied a textured ring in an inhomogeneous magnetic field. They found that the inhomogeneity of the field can results in a Berry phase, which can result in persistent currents. The effects of this Berry phase on conductivity were then discussed. On the other hand, the Aharonov-Casher(CA) effect in mesoscopic systems has also attracted much attention, since it specifically includes the spin degree of freedom. Meir et al. showed that SO interaction in one dimensional rings results in an effective magnetic flux. Mathur and Stone then pointed out that observable phenomena induced by SO interactions are essentially the manifestation of the AC effect and proposed an observation of the AC oscillation of the conductance on semiconductor samples. Balatsky and Altshuler and Choi studied the persistent current produced by the AC effect. Inspired by these studies of textured rings, the AC effect has also been analyzed in connection with the spin geometric phase. Aronov and Lyanda-Geller considered the spin evolution in conducting rings, and found that SO interaction results in a spin-orbit Berry phase which plays an interesting role in the transmission probability.

The charge-density-wave(CDW) broken-symmetry state induced by electron-phonon interaction has also been intensively investigated during the last decades. The dynamics of CDWs in materials such as NbSe$_3$, as well as their collective excitations, have received detailed study. Recently, it has been found that an external magnetic field has a pronounced effect on the CDW ground state. In sufficiently small mesoscopic rings, the AB flux induced by an external magnetic field can even destroy the CDW ground state: for the first time the instability of the CDW ground state with respect to the AB effect has been shown. Recently, it is further emphasized that, for spinless electrons, the AB effect depends on the parity of the number $N$ of electrons. Specifically, when the number of spinless electrons are even, the electronic polarizability, which in the absence of magnetic flux has a well known divergence at $2k_F$, can be compensated by magnetic flux, which then has a similar effect to temperature, inducing a transition from Peierls distortion to metal.

In this paper, we focus on the role of the electron spin in a cylindrical electrical field, which is the source of the SO interaction. This induces an AC phase, as the electrical field is dual to the magnetic field. We concentrate on the condition which has a destruction effect on the mesoscopic CDW system. We found that, when spin degree of freedom is explicitly taken into account, the parity effect is more complicated and $4n$($n$ is an integer) electrons has definite destruction effects, which is quite different from the spinless case, we will address the other filling case elsewhere. In the following, we focus on the $4n$ electron case, since our interest is mainly on the sector in which the spin-dependent geometric phase has a destruction effect. When the spin and the spin-dependent geometric phase are explicitly taken into account, we show that...
the Aharonov-Casher phase (composed of the nonadiabatic AA phase and the dynamical phase by SO interaction) induced by the cylindrical electric field can have a pronounced destruction effect on the CDW. We further propose novel observable consequences on the transport properties of mesoscopic CDW rings.

![CDW ring](image)

**FIG. 1.** The mesoscopic CDW ring in a cylindrically symmetric field with tilt angle $\chi$.

In the presence of an electric field $E = -\nabla V$, the one-particle Hamiltonian for non-interacting electrons confined to a mesoscopic ring is:

$$H = \frac{1}{2m_e} \mathbf{p}^2 + eV - \frac{e\hbar}{4m_e c^2} \sigma \cdot \mathbf{E} \times \mathbf{p}. \quad (1)$$

Where $\sigma$ is the Pauli matrix, $m_e$ is the effective mass of electrons and $\mathbf{p}$ represents the momentum of electrons. We consider a ring that is effectively one-dimensional (1D) and where the electric field which results in the SO interaction is cylindrically symmetric (see Fig. 1), i.e., $\mathbf{E} = E(\cos \chi \mathbf{e}_x - \sin \chi \mathbf{e}_z)$. For a ring lying in the $xy$ plane with its center at the origin, the Hamiltonian reads

$$H = \frac{\hbar^2}{2m_e a^2} [-i \frac{\partial}{\partial \theta} + \alpha (\sin \chi \sigma_r + \cos \chi \sigma_z)]^2 - \frac{\alpha^2 \hbar^2}{2m_e a^2}. \quad (2)$$

with $\sigma_r = \sigma_x \cos \theta + \sigma_y \sin \theta$ and $\alpha = -\frac{e a E}{4m_e c^2}$, where $a$ is the ring radius, and $\theta$ is the angular coordinate.

We adopt the geometric phase approach to identify the geometric and dynamical phases [8], which are responsible for the effects on the CDW broken-symmetry ground state in mesoscopic rings. The eigenstates of the Hamiltonian are of the form

$$\Psi_{n,\mu}(\theta) = \exp(i n \theta) \tilde{\psi}_{n,\mu}(\theta) / \sqrt{2\pi}, \quad (3)$$

in which $\mu = \pm$, $n$ are arbitrary integers, and the spin states are given by

$$\tilde{\psi}_{n,+}(\theta) = \begin{bmatrix} \cos \frac{\beta}{2} \\ e^{i \theta} \sin \frac{\beta}{2} \end{bmatrix}; \quad \tilde{\psi}_{n,-}(\theta) = \begin{bmatrix} -e^{-i \theta} \sin \frac{\beta}{2} \\ \cos \frac{\beta}{2} \end{bmatrix} \quad (4)$$

with $\tan \beta = \frac{2\alpha \sin \chi}{2\alpha \cos \chi - 1}$. The geometrical phase (AA phase) is given by

$$\int_0^{2\pi} \tilde{\psi}(\mu)^* (\theta) d\tilde{\psi}(\mu)(\theta) = -\mu \pi (1 - \cos \beta), \quad (5)$$

and the dynamical phase is

$$-\int_0^{2\pi} \tilde{\psi}(\mu)^* (\theta) H_{\mu\phi} \tilde{\psi}(\mu)(\theta) d\theta = -2\mu \pi \alpha \cos (\beta - \chi). \quad (6)$$

Then the AC phase is

$$\phi^\mu_{AC} = -\mu \pi (1 - \cos \beta) - 2\mu \pi \alpha \cos (\beta - \chi), \quad (7)$$

which satisfies $\sum_{\mu} \phi^\mu_{AC} = 0$, and the solution of the Hamiltonian in the mesoscopic system is

$$\varepsilon_{n,\mu} = \frac{\hbar \omega_0}{2} (n - \frac{\phi^\mu_{AC}}{2\pi})^2 - \frac{\alpha^2 \hbar^2}{2m_e a^2}, \quad (8)$$

where $\omega_0 = \hbar / ma^2$.

The AC phase comprises the geometric AA phase and the dynamical phase, which is obtained by the spin cyclic evolution of the spin freedom of the electron. This AC phase will change the wave numbers of the two independent spin polarized cyclic electron gases. Thus, when the spin degree-of-freedom is explicitly taken into account, the accumulated spin-dependent geometric phase will have pronounced effects in a CDW mesoscopic ring, and result in an interesting effect on the transport properties of the CDW system, as we discuss in the following.

As is well known, electron-phonon interaction treated adiabatically in a quasi-one dimensional system leads to a CDW gap at wavevector $q = 2k_F$, so that the originally continuous energy band breaks into two bands: valence and conduction. Since Peierls first pointed out that a one-dimensional metal coupled to the lattice is unstable at low temperature, both theoretical and experimental studies have concentrated on the static and dynamical characters of charge-density-waves, including the frequency- and electric-field -dependent conductivity, current oscillation and pinning via defects or disorder [13]. These studies focused on the conductivity character in direct external electrical fields for macroscopic quasi-one dimensional CDW materials, such as $K_{0.3}MoO_3$, $NbSe_3$, etc. As fabrication techniques have become more mature, it is now promising that fabrication of mesoscopic CDW samples can be realized [13].

The logarithmetrical singularity of the dielectric response function at wavevector $2k_F$ makes the corresponding
phonon frequency soften drastically (Kohn anomaly). The modulated lattice structure also affects the electronic band structure. In second-quantized representation, with a cylindrical electric field in a CDW mesoscopic ring, we can use the Hamiltonian

$$ H = \sum_{k,\mu} (\varepsilon_{k,\mu} C_{k,\mu}^\dagger C_{k,\mu} + \Delta (C_{k+2k_F,\mu}^\dagger C_{k,\mu} + H. C.)). \quad (9) $$

Here the CDW gap is $\Delta = 2\gamma u$, with $\gamma$ the electron-lattice coupling constant and $u$ the displacement after dimerization, and $\varepsilon_{k,\mu}$ the eigenenergies in an external cylindrical electric field (Eq. 8). We consider the half-filled case, but extension to other band fillings for many real quasi-one dimensional CDW materials is straightforward.

After diagonalizing the reduced 2x2 matrix

$$ \begin{pmatrix} E_{k,\mu} - \varepsilon_{k,\mu} & \Delta \\ \Delta & E_{k+2k_F,\mu} - \varepsilon_{k+2k_F,\mu} \end{pmatrix} = 0, \quad (10) $$

we obtain the splitting into valence and conduction bands:

$$ E_{k,\mu}^{\text{vol}} = \frac{1}{2} (\varepsilon_{k,\mu} + \varepsilon_{k+2k_F,\mu}) - \frac{1}{2} \sqrt{(\varepsilon_{k,\mu} - \varepsilon_{k+2k_F,\mu})^2 + 4\Delta^2} $$

$$ E_{k,\mu}^{\text{con}} = \frac{1}{2} (\varepsilon_{k,\mu} + \varepsilon_{k+2k_F,\mu}) + \frac{1}{2} \sqrt{(\varepsilon_{k,\mu} - \varepsilon_{k+2k_F,\mu})^2 + 4\Delta^2}. \quad (11) $$

From Eq. (11), we see that when the spin degree-of-freedom is taken into account, the spin-dependent geometric phase affects the CDW ground state. First, we concentrate on the effect on the CDW gap. We will turn to the effect on transport properties later. As the $2k_F$ mode is dominant in the electron-phonon interaction, we can obtain the following total effective potential:

$$ E = \sum_{|k| \leq k_F} E_{k,\mu}^{\text{vol}} + \frac{1}{2} \omega_{2k_F}^2 u_{2k_F}^2, \quad (12) $$

where $\Delta = \gamma u_{2k_F}$. Adopting a standard minimization procedure, we now find the effect on the CDW gap by the spin-dependent geometric phases for the electron numbers $4n$ are:

$$ \prod_\mu \frac{|\phi_{\mu}^\text{AC}|}{2\pi} + \frac{1}{2} \frac{(\phi_{\mu}^\text{AC})^2}{2\pi} + \frac{1}{2} \frac{(\omega_{n_F})^2}{\hbar \omega_{n_F}} \prod_\mu \frac{|\phi_{\mu}^\text{AC}|}{2\pi} + \frac{1}{2} \frac{(\phi_{\mu}^\text{AC})^2}{2\pi} + \frac{1}{2} \frac{(\omega_{n_F})^2}{\hbar \omega_{n_F}} = \exp(-\frac{1}{g}), \quad (13) $$

with $g$ the dimensionless effective electron-lattice interaction, $g = \frac{\gamma^2}{\hbar \omega_{n_F} m \omega_{2k_F}^2}$. We have explored this AC phase effect on a CDW mesoscopic ring in various regions. We found that it is very sensitive to e-ph coupling, as illustrated in Figs. 2, 3. The effects of the spin-dependent geometric phases for different e-ph coupling constants shows that only when the e-ph coupling is weak enough, is a breaking effect on the CDW observable: the stronger the e-ph coupling, the more stable is the CDW. We emphasize that the breaking effect (4n electrons) by the AC phase is the result of the two spin-polarized electrons, which is produced by the external electric field, and these two spin polarized electron gases accumulate a spin-dependent geometric phase through the SO interaction. Hence, we have identified a new mechanism, different from that due to an external magnetic field.

We have concluded that when the external electrical field reaches a critical strength, the CDW ground state in a mesoscopic ring can be destroyed with the appropriate electron fillings (4n when spin is explicitly taken into account). Since the SO interaction is time-reversal-invariant, we expect that the effect of the spin-dependent geometric phase on a CDW mesoscopic ring will be manifested in transport processes. Here, it is induced by the cylindrical electric field. This is different from previous

FIG. 2. The CDW gap dependence on the spin-dependent geometric phase, for the strength of effective e-ph interaction $g=0.07$.

FIG. 3. The CDW gap dependence on the spin-dependent geometric phase, for the strength of effective e-ph interaction $g=0.064$.
studies in which the electrical field is parallel to the
direction of the quasi-one dimension of the CDW materials,
and hence has no corresponding topological effect on the
transport. We adopt the configuration with the ring
connected to two current leads in opposite directions, which
is the standard structure for transport studies and interference
effects in a mesoscopic field (see Fig. 4).

Our formulation is standard for that developed in the
study of quantum oscillations [19]. We obtain the trans-
mission probability in the presence of SO interactions(a
detailed derivation will be given elsewhere [20]) as:

\[ T = \frac{1}{2} \sum_{\mu} t(\phi_{AC}^{\mu} / 2\pi), \]  

(14)

with

\[ t(\phi_{AC}^{\mu}) = \frac{4e^2 \sin^2 \phi_s \cos^2 \phi_{AC}^{\mu}}{[a^2 + b^2 \cos 2\phi_{AC}^{\mu} - (1 - \epsilon) \cos 2\phi_s] \cos^2 2\phi_s + e^2 \sin^2 2\phi_s}. \]

Here \( \phi_s \) is the phase of the incident electron on the lead,
where \( a = \pm (\sqrt{1 - 2\epsilon} - 1)/2 \) and \( b = \pm (\sqrt{1 - 2\epsilon} + 1)/2 \)
with \( 0 \leq \epsilon \leq 1/2 \). In a mesoscopic ring with
radius \( a = 100 \mu m \), to make the above effect observable,
an electrical field with strength \( \sim 10^6 V/m \) is necessary,
so that the AC phase can be the order of unity.

In summary, we have investigated a spin-dependent geo-
metric phase effect in mesoscopic CDW rings. When the
electron spin is explicitly taken into account in the pres-
ence of a cylindrical external field and under appropriate
filling electrons, the AC(AA) phase accumulated by the
two independent spin polarized electron gases can results
in frustration of the CDW on a mesoscopic scale. We thus
propose a new mechanism with which to probe meso-
scopic CDW materials, and associated spin-dependent
geometric phase consequences. As a novel consequence,
we suggest that a frustration effect of the spin-dependent
geometric phase consequences. As a novel consequence,
we suggest that a frustration effect of the spin-dependent
geometric phase will be observable in transport experi-
ments. There are natural extensions of our considerations
to competing spin-density-wave and CDW situations, as
well as the other filling electron case, which we will dis-
cuss elsewhere. These opportunities for studying meso-
scopic systems are becoming increasingly attractive with
the rapid progress of modern fabrication technology.

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[1] M. V. Berry, Proc. Roy. Soc. London A 392, 45(1984).
[2] Geometric phases in Physics, edited by A. Shapere and F. Wilczek, (World Scientific, Singapore, 1989).
[3] Y. Aharonov and J. Anandan, Phys. Rev. Lett. 58, 1593 (1987).
[4] Y. Imry, in Directions in Condensed Matter Physics, (World Scientific, Singapore, 1986); M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. 96A, 365 (1983);
[5] D. Loss, P. Goldbart, and A. V. Balatsky, Phys. Rev. Lett. 65, 1655 (1990).
[6] D. Loss and P. M. Goldbart, Phys. Rev. B 45, 13544 (1992). A. Stern, Phys. Rev. Lett. 68, 1022 (1992); D. Loss, H. Schoeller, and P. M. Goldbart, Phys. Rev. B 48, 15218 (1993).
[7] Y. Aharonov and A. Casher, Phys. Rev. Lett. 53, 319 (1984).
[8] Y. Meir, Y. Gefen, and O. Entin-Wohlman, Phys. Rev. Lett. 63, 798 (1989); O. Entin-Wohlman, Y. Gefen, Y. Meir, and Y. Oreg, Phys. Rev. B 45, 11800 (1992).
[9] H. Mathur and A. D. Stone, Phys. Rev. Lett. 68, 2964 (1992); H. Mathur and A. D. Stone, Phys. Rev. B 44, 10957 (1991).
[10] A. V. Balatsky and B. L. Altshuler, Phys. Rev. Lett. 70, 1678 (1993).
[11] M. Y. Choi, Phys. Rev. Lett. 71, 2987 (1993).
[12] A. G. Aronov and Y. B. Lyanda-Geller, Phys. Rev. Lett. 70, 343 (1993). Y. Lyanda-Geller, Phys. Rev. Lett. 71, 657 (1993).
[13] G. Grünner and A. Zettl, Phys. Rep. 119, 117 (1985). G. Grüner, Rev. Mod. Phys. 60, 1129 (1988).
[14] E. N. Bogacheck, I. V. Krive, I. O. Kulik, and A. S. Rozhavskii, Phys. Rev. B 42, 7614 (1990).
[15] B. Nathanson, O. Entin-Wohlman, and B. Mühlshlegel, Phys. Rev. B 45, 3499 (1992).
[16] M. I. Visscher, B. Rejaei, and G. E. W. Bauer, Europhys. Lett. 36, 613 (1996). J. Yi, M. Y. Choi, K. Park, and E. H. Lee, Phys. Rev. Lett. 78, 3523 (1997).
[17] G. Montambaux, Eur. Phys. J. 1, 377 (1998).
[18] H. J. van der Zant, O. C. Mantel, C. Dekker, J. E. Mooij and C. Traeholt, Appl. Phys. Lett. 68, 3823 (1996).
[19] Y. I. Latyshev, O. Laborde, P. Monceau and S. klau-muenzer, Phys. Rev. Lett. 78, 919 (1997). Yu. I. Latyshev, B. Pannetier and P. Monceau, [cond-mat/9711018].
[20] M. Büttiker, Y. Imry, and M. Ya. Azbel, Phys. Rev. A 30, 1982 (1984).
[21] Y. S. Yi, et al,(unpublished).