Confinement in Gauge Theories from the Condensation of World-Sheet Defects in Liouville String

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Abstract

We present a Liouville-string approach to confinement in four-dimensional gauge theories, which extends previous approaches to include non-conformal theories. We consider Liouville field theory on world sheets whose boundaries are the Wilson loops of gauge theory, which exhibit vortex and spike defects. We show that world-sheet vortex condensation occurs when the Wilson loop is embedded in four target space-time dimensions, and show that this corresponds to the condensation of gauge magnetic monopoles in target space. We also show that vortex condensation generates an effective string tension corresponding to the confinement of electric degrees of freedom. The tension is independent of the string length in a gauge theory whose electric coupling varies logarithmically with the length scale. The Liouville field is naturally interpreted as an extra target dimension, with an anti-de-Sitter (AdS) structure induced by recoil effects on the gauge monopoles, interpreted as D branes of the effective string theory. Black holes in the bulk AdS space correspond to world-sheet defects, so that phases of the bulk gravitational system correspond to the different world-sheet phases, and hence to different phases of the four-dimensional gauge theory. Deconfinement is associated with a Berezinskii-Kosterlitz-Thouless transition of vortices on the Wilson-loop world sheet, corresponding in turn to a phase transition of the black holes in the bulk AdS space.

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1 Introduction

The dazzling recent advances in the understanding of non-perturbative aspects of string theory have opened up new horizons in the quantum description of black holes, and now cast new light on gauge theories, through the conjectured equivalence between $d$-dimensional gauge theories in Minkowski (M) space and gravity theories in $d+1$-dimensional anti-de-Sitter (AdS) space. This may be regarded as an incarnation of the holographic principle, since $M_d$ is the boundary of $AdS_{d+1}$. The big hope is that this conjectured equivalence may offer new prospects for the understanding of infra-red properties such as confinement at low temperatures in three- and four-dimensional $U(N_c)$ gauge theories, at least in the large-$N_c$ limit, in terms of bulk supergravity or string theories in AdS$_{4,5}$.

These prospects have so far been obscured by a couple of obstacles: the original gauge theory/AdS gravity conjecture was formulated assuming not only supersymmetry but also conformal symmetry. Some progress has been made in relaxing the requirement of supersymmetry, but the hurdle of conformal symmetry has yet to be leapt. This appears to be essential if one is to incorporate the asymptotic freedom of non-Abelian gauge theories into the emerging AdS picture of confinement. So far, the AdS/gauge correspondence has been exploited at some fixed value of the gauge coupling $g_{YM}^2N_c$, in particular to calculate quantum properties of large-$N_c$ gauge theories in terms of classical solutions of the bulk AdS supergravity theories. However, one needs to verify that there are not different phases in different ranges of $g_{YM}^2N_c$, in order to establish that the short-distance ‘parton’ régime is in the same phase as the long-distance ‘hadron’ régime. This task appears to require an extension of the conjecture beyond the framework of conformal field theory.

Extending this framework has already been an issue in the formulation of non-critical string theory. Any such expansion should involve either non-conformal field theories on the two-dimensional world sheet and/or higher-dimensional theories. In the former case, one must introduce a renormalization scale on the world sheet, which becomes dynamical once higher-genus effects are taken into account, and may be identified as a Liouville field. Defects in this Liouville field theory have the characteristics of black holes, and this approach provides an alternative derivation of $D$ branes, thus making contact with the non-perturbative approach to string theory based on higher-dimensional extended objects.

The relevance of non-critical string theory to the Minkowski gauge theory/AdS supergravity correspondence arises in particular from the formulation of gauge-invariant quantities in terms of Wilson loops. These serve as the boundaries of surfaces that can be regarded as world sheets for non-critical strings. As we explain in more detail below, the necessary breakdown of conformal invariance can be accommodated in the presence of a Liouville field on this induced world sheet. The associated defects on the world sheet may be related to target-space $D$ branes, including monopoles, which we argue are related to those postulated by Mandelstam and ’t Hooft, and correspond to black holes in the bulk AdS theory.

We demonstrate the existence of a low-temperature phase in which the world-
sheet defects condense, leading to a non-zero string tension $\mu$ and hence the area law for the Wilson loop, that is a signature for confinement. Deconfinement at the critical temperature is described by a Berezinskii-Kosterlitz-Thouless (BKT) transition of the vortices \cite{9}. We show how AdS space emerges naturally when the string interaction with $D$ branes is considered \cite{10, 11}, and argue in favour of a correspondence between the higher-temperature phase structure of the gauge theory and the corresponding known results \cite{12} for black holes in AdS space.

The layout of the article is as follows: In Section 2, we review the Mandelstam-'t Hooft approach \cite{7, 8} to confinement in non-Abelian gauge theories, which emphasizes the rôle of magnetic monopoles, utilizing the Abelian projection hypothesis in order to simplify the discussion of the stringy description of Wilson loops à la Polyakov \cite{13, 14}. Here we also point out the emergence of 0 branes, and motivate the connection with non-critical Liouville string theory \cite{15, 16}. This is introduced in Section 3, with emphasis on the appearance of world-sheet vortex and ‘spike’ defects \cite{17}. In Section 4, we introduce the Awada-Mansouri approach \cite{18} to the connection between gauge theories and scale-invariant (super)strings in target space, which we use as the basis for our subsequent discussion of a dynamical scenario for the appearance of the string tension in conformal string theories.

In Section 5, we discuss aspects of Liouville theory in the intermediate region of the matter central charge, $1 < D < 25$, which is appropriate for the string-theory description of QCD. In particular, we show how world-sheet vortices or spikes may condense in different regions of $D$, corresponding to different ranges of the effective temperature. In Section 6, we discuss the rôle of world-sheet defects in the long-distance physics of quark confinement, arguing, in particular, that the presence of such defects on the world sheet makes redundant the rôle of supersymmetry in the approach of \cite{19, 20}. We demonstrate that world-sheet vortex condensation generates and effective string tension and corresponds to the condensation of target-space monopoles. In Section 7 we show how the encounter of closed strings (Wilson loops) with a world-sheet defect entails a treatment of the ‘recoil’ of the corresponding 0 brane, that naturally induces AdS space. Invoking a $T$-duality transformation within this framework, we demonstrate the Meissner effect in the confined phase where world-sheet vortices condense. In Section 8, we discuss the connection between the phases of gauge theory and black holes in AdS space, relating Berezinski-Kosterlitz-Thouless (BKT) transitions on the world sheet to the thermodynamics of AdS Black Holes, which has been argued \cite{19} to be relevant for the various phases of the gauge theory. Our conclusions and an outlook are presented in Section 9 \cite{21}.

\[\text{\footnotesize 1}\] Supersymmetry is also broken in the finite-temperature field theories which have recently been discussed \cite{1, 13}.

\[\text{\footnotesize 2}\] A preliminary version of this work was presented by N.E.M. \cite{19} at the ‘Workshop on Recent Developments in High-Energy Physics’, of the Hellenic Society for the Study of High Energy Physics, Democritos N.R.C., Athens, Greece, 8-11 April 1998.
2 Review of Confinement, Strings and Monopoles

The relevant observables that signal confinement are the Wilson loops

\[ W(C) = e^{ie \int_C A \cdot dl} \]  

(1)

where the curve \( C \) is space-like, and the Polyakov-Wilson loop,

\[ P(C') = e^{ie \int_{C'} A \cdot dl} \]  

(2)

where the closed curve \( C' \) extends in the time-like direction. The loop \( C' \) may conveniently be taken to be a rectangle \( R, t \), where \( R \) describes the distance between a quark-antiquark pair propagating for a time \( t \). In the confining phase of the theory, the time-like loop observable \( P \) obeys an area law

\[ P(R, T) \sim e^{-\mu RT} \]  

(3)

where \( \mu \) is the string tension, with an ‘effective potential’ \( V(R) \) being given by \( V(R) = \mu R \) in the confining phase [3]. At finite temperature, we expect that there is a confinement-deconfinement transition at some temperature \( T_c \), defined by the behaviour of the vacuum expectation values of the observables \( W \) and \( P \) as follows:

\[ < P(C') > = 0, \quad < W(C) > \sim e^{-\mu A}, \quad \text{for } T < T_c \]  

(4)

where \( A \) is the minimal area enclosed by the spatial loop \( C \), whereas \( < P(C') > \neq 0 \) for \( T > T_c \), taking a value in the centre of the gauge group.

The order of the phase transition at \( T_c \) is still a matter of debate, and lattice simulations do not yet provide a conclusive answer. There are arguments based on effective field theory that the transition may be first order in pure gauge theory, since it is related to the value of the condensate \( < F_{MN}^2 > \), where \( F_{MN} \) is the gluonic field strength, whose formation breaks scale invariance and which has an effective potential of Coleman-Weinberg type [20], as seen in Fig. 1. The critical temperature \( T_c \) may lie between \( T_0 \) and \( T_2 \), taking the value \( T_c = T_1 \) under adiabatic conditions. The existence and order of one or more phase transitions is less clear in the presence of fermionic matter, where one might also expect a chiral phase transition at some temperature \( T_q \), associated with the disappearance of the quark condensate \( < \bar{q}q > \). This may be identified with the confinement transition, at least in the context of the Skyrme approach to baryons in QCD [20], and would be of second order if gluonic degrees of freedom are neglected. One possibility is that \( T_q < T_c \), so that there are two separate transitions [20]. However, it could also well be that the chiral phase transition is driven by the scale-breaking first-order gluonic transition at \( T = T_c \) [20]. Lattice evidence for the latter possibility has recently been reviewed in [21], where simultaneous rapid changes in the chiral condensate and the Polyakov loop observable were reported. We shall not be concerned here with the relation between \( T_q \) and \( T_c \), since we shall work with only gauge degrees of freedom.
Figure 1: Qualitative variation of the free energy with temperature in QCD\cite{20}. At temperatures below $T_0$, there is no stable state at the origin, the only stable state (with non-zero order parameter) corresponds to confinement, and there is a string tension. There is no stable confined state at temperatures above $T_2$. Between $T_0$ and $T_2$, there is the possibility of a mixed phase. The confined and unconfined phases have equal free energies at the temperature $T_1$, which would be $T_c$ under adiabatic conditions.

It has been suggested by Mandelstam\cite{7} and ’t Hooft\cite{8} that confinement in non-Abelian gauge theories may be understood in terms of a superconductor picture, where a duality interchanges the rôles of ‘electric’ and ‘magnetic’ variables, as compared to ordinary superconductivity. The idea is that the chromoelectric field in the region between a quark-antiquark pair is constrained by the dual Meissner effect into a flux tube, with constant energy per unit length, leading to a linear increase of the energy $E$ with the distance $R$:

$$E = \mu R$$

This dual superconductor picture necessitates the introduction of magnetic monopoles\cite{22}.

In this picture, the confining phase of QCD is that where the magnetic charges condense, and this rôle of magnetic monopoles in inducing confinement is a property not only of non-Abelian but also compact Abelian gauge theories. In fact, it was argued by ’t Hooft\cite{8} that, by fixing in the so-called Abelian-projection gauge, one can remove the non-Abelian degrees of freedom in such a way so as to break the symmetry of the non-Abelian gauge group $G$ to the maximal torus group $H$. In the case of the group $G=SU(N_c)$, for instance, $H = U(1)^{N_c-1}$. According to ’t Hooft, the Abelian-projected theory reduces to a $U(1)^{N_c-1}$ Abelian gauge theory supplemented by magnetic monopoles. In such a case, as mentioned above, confinement manifests itself as the phase where the monopoles condense. In addition to the Abelian projection hypothesis, a stronger statement has also been made in the literature, namely
the Abelian dominance hypothesis, according to which only the Abelian parts of a non-Abelian gauge group play an important rôle in the confinement of quarks \cite{23}.

The above discussion indicates the plausibility of the decoupling of the non-Abelian degrees of freedom at the expense of introducing magnetic monopoles into the theory. Hence, we now concentrate on compact Abelian gauge theories with magnetic monopoles, with the aim of describing the mechanisms associated with confinement.

An early analysis of the rôle of magnetic monopoles and their interactions with electric charges, which is crucial for the confining aspects, was made in \cite{29}, in the context of compact Abelian gauge theories on the lattice. In that analysis, the Villain form of the lattice action was adopted:

\[
S = \frac{1}{4e^2} \sum_{x,\alpha\beta} (F_{\alpha\beta}(x) + 2\pi \eta_{\alpha\beta})^2 + \int_{P^\mu} \eta_{\alpha\beta}(x) = j^\mu
\]

where \(\eta_{\alpha\beta}\) are matrices of integers, associated with lattice plaquettes, that may be related to integer-valued (monopole) currents when integrated over elementary cubes \(P^\mu\) of the lattice. Unfortunately, unlike the three-dimensional case, where such an action leads to a Coulomb-gas description of the gauge-theory vacuum \cite{13}, the four-dimensional case is more complicated. Consequently, the interaction between magnetic monopoles and electric currents is still the subject of speculation and intuitive arguments.

The main topic of this article is to argue that a quantitative description might be feasible within a string description of the gauge theories. To set the scene for this, we first recall aspects of the string approach to confining aspects of compact Abelian gauge theories \cite{14,25,26}, in particular the perspective advocated in \cite{26}. As observed in \cite{14,26}, monopole condensation with a dynamical scale \(\Lambda\) would imply that the low-energy physics of a compact Abelian gauge theory in the confining phase can be described by means of an effective action for the dual gauge field \(\varphi_\mu(x)\) \cite{14,27}, whose field strength we denote by \(f_{\mu\nu}\):

\[
S_{\text{eff}} = \int d^4x \{4e^2 f_{\mu\nu}^2 + z\Lambda^4 (1 - \cos \frac{\varphi_\mu}{\Lambda})\} \; ; \; z \propto \exp(-\text{const}/e^2)
\]

If one now integrates out \(\varphi_\mu\) and goes to the long-distance limit, one obtains the following effective action:

\[
S_0(B_{\mu\nu}) \simeq \int d^4x \left[ \frac{4}{3z\Lambda^2} H_{\mu\nu\alpha}^2 + \frac{1}{4e^2} B_{\mu\nu}^2 \right]
\]
where $H_{\mu\nu\alpha}$ is the antisymmetric-tensor field strength. Written in this form, the action is a special case of the Julia-Toulouse action \[28\], which describes confinement of $(p-1)$-branes by the condensation of $(D-p-3)$-branes in $D$ space-time dimensions in a rank-$p$ compact antisymmetric-tensor field theory.

The action (9) corresponds to the case $p = 1$, in which the antisymmetric-tensor theory has 0 branes (point-like defects) in $D = 4$ space-time dimensions, which is crucial for our subsequent analysis in Sections 6 et seq.. The framework for such a $D$-brane picture is provided by a stringy description of confinement \[14\], which involves coupling the action (9) to an antisymmetric-tensor string source:

$$e^{S_{cs}} = \frac{G}{Z} \int dB_{\mu\nu} e^{-S_0(B_{\mu\nu})} + \frac{i}{4} \int d^4x B_{\mu\nu} \int d^2\sigma \varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu \delta^4(x - X(\sigma))$$

where $G$ is a group-theoretical factor, and $Z$ is the partition function in the absence of the string source. The world sheet of the string is parametrized by $\sigma = (z, \bar{z})$, and corresponds to a two-dimensional space-like surface whose boundary is the Wilson loop $C$ (the string action will be derived later in more detail). The group-theoretical normalization factor $G = 1$ for compact Abelian gauge theories. For $SU(N)$ non-Abelian gauge theories, it has been argued \[14\] that $G = N^{-\chi}$, where $\chi$ is the Euler characteristic of the space-time manifold. Thus, in this formulation, the non-Abelian degrees of freedom simply introduce a multiplicative factor in front of the action $S_0$, a fact which is consistent with the generic argument of 't Hooft \[8\] on the rôle of the Abelian-projected gauge fixing, and the hypothesis of Abelian dominance \[23, 24\] described above.

In this paper we extend the above results so as to treat quantum fluctuations of the string world sheet that stretches across the Wilson loop, with the aim of providing a satisfactory string model incorporating both electric and magnetic degrees of freedom \[29\]. This will in turn lead to a better understanding of confining aspects of gauge theories. As we argue subsequently, the treatment of such quantum fluctuations requires a formulation of non-critical string theory. This is obtained most economically by introducing a Liouville field on the world sheet, which allows the appearance on the world sheet of topological defects that can be associated with monopoles.

### 3 Review of Liouville Strings and World-Sheet Defects

Fundamental string theory is usually formulated in the critical limit, in which conformal symmetry is respected. This limit may be realized in the so-called critical number of space-time dimensions - $D = 26$ for a bosonic string, or $D = 10$ for a supersymmetric string - or with additional fields on the world sheet. Clearly any string description of confining gauge theories in $D = 3$ or 4 dimensions must be non-critical, and hence requires some such additional field.
The minimal such possibility is a single Liouville field $\phi$ defined on a string world sheet. In a Liouville construction of non-critical string, the ‘matter’ central charge, $\mathcal{D}$, in general differs \[15\] from the number of space-time dimensions $D$ because of an extra screening charge $Q$:

$$\mathcal{D} = D + 12Q^2 \tag{11}$$

linked to a linear term $\propto Q\phi$ in the action. In our case, we assume $D = 3$ or 4, corresponding to the target-space dimensionality of ordinary gauge theories. In such a case, one may restore criticality by choosing the Liouville central charge deficit, $c_L = 26 - D - 12Q^2$, such that $c_{\text{total}} = \mathcal{D} + c_L = 26$. The Liouville mode is time-like \[14\] if $\mathcal{D} > 25$, space-like if $\mathcal{D} < 25$, and decouples in the critical case $\mathcal{D} = 26$.

In the context of such a Liouville string theory, the presence of world-sheet defects is necessary for consistency of the theory in the dangerous region of the ‘matter’ central charge: $1 < \mathcal{D} < 25$ for a bosonic theory \[17\] ($1 < D < 9$ for a superstring theory), where the critical exponents are complex. This is because the non-critical string theory vacuum is stable only in the regions $\mathcal{D} < 1$ and $\mathcal{D} > 25(9)$ \[10\], where the critical exponents of the theory are real. On the other hand, there are suggestions \[17\] that the system undergoes a phase transition in the dangerous region, as we discuss in more detail below.

We first review some basic features of world-sheet defects, which we shall use later in our approach. A vortex defect on the world sheet \[30, 31\] is obtained as the solution $X_v$ of the equation

$$\partial_z \bar{\partial}_z X_v = \frac{i\pi q_v}{2} [\delta(z - z_1) - \delta(z - z_2)] \tag{12}$$

where $q_v$ is the vortex charge and $z_{1,2}$ are the world-sheet locations of a vortex and antivortex, respectively. If we map these to the origin and the point at infinity, the corresponding solution to (12) may be written as

$$X_v = q_v \text{Im} \ln z \tag{13}$$

and we see that the vortex charge $q_v$ must be integer. There are related spike defects which are solutions of the equation

$$\partial_z \bar{\partial}_z X_m = -\frac{\pi q_m}{2} [\delta(z - z_1) - \delta(z - z_2)] \tag{14}$$

The spike solution is given by

$$X_m = q_m \text{Re} \ln z \tag{15}$$

when the spike and antispark are located at the origin and the point at infinity.

We shall be interested in the partition function of a Liouville string with a gas of vortex and spike defects, in interaction with a heat bath of temperature $\beta^{-1}$, whose action may be written in the form \[17\]

$$Z = \int \mathcal{D}X \exp[-\beta S_{\text{eff}}(X)] : \ X = \beta^{1/2} \tilde{X} \tag{16}$$
where \( S_{\text{eff}} \) has the following sine-Gordon form:

\[
S_{\text{eff}}(\tilde{X}) = \int d^2z \{ 2\partial_z \tilde{X} \partial_{\tilde{z}} \tilde{X} + \frac{1}{4\pi r^2} \left[ g_v \varepsilon^{\alpha/2-2} (2\sqrt{|\gamma(z)|})^{1-\alpha/4} : \cos(2\pi \beta^{1/2} q_v [\tilde{X}(z) + \tilde{X}(\bar{z})]) : + g_m \varepsilon^{\alpha'/2-2} (2\sqrt{|\gamma(z)|})^{1-\alpha'/4} : \cos(\frac{q_m}{\beta^{1/2}} [\tilde{X}(z) - \tilde{X}(\bar{z})]) : \} \}
\]

(17)

We have assumed in writing (17) that the world sheet has a spherical topology, and we have used a stereographic projection of the sphere \( S_2 \), of radius \( r \) onto the complex plane, which induces a metric \( \gamma(z) \) with corresponding line element \( ds^2 = (1+|z|^2/4r^2)^{-2}dzd\bar{z} \). We use an angular cutoff \( \varepsilon \) in this projection, \( g_{v,m} \) are couplings of the vortex and spike defects, and \( \alpha \equiv 2\pi \beta q_v^2, \alpha' = q_m^2/2\pi \beta \).

It is easy to see \( [17] \) that, in the presence of both types of defect at finite temperature \( T \equiv \beta^{-1} \neq 0 \), the following quantization condition must be imposed:

\[
2\pi \beta q_v q_m = \text{integer} \quad (18)
\]

in order for the partition function to be single valued. This reflects a vortex-monopole duality, which is manifested as the invariance of the defect partition function \( [16] \) under the interchanges

\[
\pi \beta \leftrightarrow \frac{1}{4\pi \beta} \quad ; \quad q_v \leftrightarrow q_m \quad (19)
\]

It is the relevance, marginality or irrelevance (in a world-sheet renormalization-group sense) of the sine-Gordon deformations \( [17] \), which determines the phase structure of the non-critical string in the dangerous region of the central charge. This is because the sine-Gordon deformations are interpreted \( [17] \) as reflecting the dynamics of the world-sheet defects (vortices \( [13] \) or their dual spikes \( [15] \)). According to this picture, the effect that summation of vortices and spikes has on the stability of the vacuum of the string can be summarized by the conformal dimensions \( \Delta_{q_v}, \Delta_{q_m} \) of the two kinds of deformations in \( [17] \):

\[
\Delta_{q_v} = \frac{1}{4} \alpha \quad ; \quad \Delta_{q_m} = \frac{1}{4} \alpha' \quad (20)
\]

Relevant deformations: \( \Delta < 1 \) lead to an instability of the vacuum, and result in a plasma of free charges. On the other hand, irrelevant deformations with \( \Delta > 1 \) lead to a binding of the corresponding defects, and stability of the vacuum. Marginal deformations with \( \Delta = 1 \) correspond to a Berezinski-Kosterlitz-Thouless (BKT) transition \( [14] \) at some critical temperature.

In general, the effective temperature \( T = 1/\beta \) is related to the matter central charge by

\[
\beta = \frac{3}{\pi|\mathcal{D} - 25|} \quad (21)
\]
Hence, the conformal dimensions of the vortex and spike operators are seen from (20) to be

\[ \Delta_v = \frac{1}{2} \pi q^2, \quad \Delta_m = \frac{1}{8} \pi \beta q^2 \]

respectively. We see that the phase structure of the non-critical string vacuum will depend on the dimensionality of the target space-time, as we discuss in more detail later. First, however, we prefer to motivate the relevance of this phase structure to that of gauge theories.

## 4 General String Description of Abelian Gauge Theories

We proceed now to a more general formulation of the string description of Abelian gauge theories that does not rely \textit{a priori} on monopole condensation, and therefore provides a framework for determining when it may occur. Working with an Abelian gauge theory in \( D \) space-time dimensions, we consider a Wilson loop \( C \), parametrized by \( \tau \), whose exponent may be written as

\[ S_{\text{int}} = ie \int_C d\tau A(X(\tau)) \frac{\partial}{\partial \tau} X(\tau) \]  

where \( X^M : M = 1,\ldots, D \) denote the \( D \)-dimensional space-time coordinates. This may be rewritten using Stokes' theorem, in the form

\[ S_{\text{int}} = \frac{ie}{2} \int_{\Sigma(C)} d^{2}\sigma \epsilon^{ab} F_{ab}, \quad a, b = 1, 2, \]  

where \( \Sigma \) is a surface spanning \( C \), that can be regarded as its homotopic extension and plays the rôle of the world sheet of the string. We denote the two-dimensional space-like coordinates of the surface \( \Sigma \) by \( \sigma \), with lower-case Latin indices. The quantity

\[ F_{ab} = \partial_a X^M \partial_b X^N F_{NM} = \partial_a A_b - \partial_b A_a \]

in (24) is the pullback of the Maxwell tensor on the world sheet \( \Sigma \), with \( A_a \) the corresponding projection of the gauge field on \( \Sigma \):

\[ A_a = v_a^M A_M \]

We note that, from a two-dimensional view-point, (24) looks like a Chern-Simons term for a two-dimensional gauge theory on the world sheet \( \Sigma \).

It was observed in [18] that a \textit{second} string-like observable could be constructed, \textit{in a supersymmetric gauge theory}. This second observable is easily understood in the two-dimensional superfield formalism using the superspace representation

\[ Z^A \equiv (X^M, \theta^m, \bar{\theta}^\dot{m}) \]
where we use calligraphic indices: \( \mathcal{A}, \mathcal{B}, \ldots \) to denote superspace coordinates. The pullback basis \( v^M_a \) in (25) is now extended to \( v^A_a = E^A_B \partial_a z^B \), with the following components [18]:

\[
\begin{align*}
v^{\alpha\dot{\alpha}}_a &= \partial_a X^{\alpha\dot{\alpha}} - \frac{i}{2} \left( \theta^\alpha(\sigma) \partial_a \theta^{\dot{\alpha}}(\sigma) + \theta^{\dot{\alpha}}(\sigma) \partial_a \theta^\alpha(\sigma) \right) \\
v^\alpha_a &= \partial_a \theta^\alpha(\sigma) \\
v^{\dot{\alpha}}_a &= \partial_a \theta^{\dot{\alpha}}(\sigma)
\end{align*}
\]

in the standard notation [33], where Greek dotted and undotted indices denote superspace components, with \( x^{\alpha\dot{\alpha}} \equiv X^M \), etc.. Following [18], we now define:

\[
\begin{align*}
C_{ab}^{\alpha\beta} &= \frac{i}{2} v^{(\alpha}_a v^{\beta)}_b \\
C_{ab}^{\alpha} &= v^{\alpha}_a v^{\beta}_b \\
C^{\alpha\dot{\alpha}} &= \epsilon^{ab} C_{ab}^{\alpha\beta}
\end{align*}
\]

as well as the quantities \( C^{\alpha} \equiv \epsilon^{ab} C_{ab}^{\alpha} \), \( C^{\alpha\beta} \equiv \epsilon^{ab} C_{ab}^{\alpha\beta} \), and similarly for the corresponding dotted components of \( C \) [18].

We notice that \( C^{\alpha} \) apparently vanishes in the absence of supersymmetry [18], whereas \( C^{\alpha\beta} \) exists in non-supersymmetric gauge theories as well. In fact, as we see later, \( C^{\alpha} \) can also have non-supersymmetric remnants in the presence of defects on the world sheet.

The first Wilson-loop observable (24) has a simple supersymmetric equivalent:

\[
W(C) = e^{\tilde{\mathcal{S}}^{(1)}_{\text{int}}}: \tilde{\mathcal{S}}^{(1)}_{\text{int}} \equiv \frac{i \epsilon}{2} \int_{\Sigma(C)} d^2 \sigma \epsilon^{ab} \mathcal{F}_{ab}
\]

where

\[
\mathcal{F}_{ab} \equiv \epsilon_{ab} \left\{ \frac{1}{2} C^{\alpha\beta}(\sigma) D_\alpha W_\beta(x(\sigma), \theta(\sigma)) + C^\alpha(\sigma) W_\alpha(x(\sigma), \theta(\sigma)) + \text{h.c.} \right\}
\]

and \( W_\alpha(x(\sigma), \theta(\sigma)) \) is the gauge superfield of the supersymmetric Abelian gauge theory. The second superstring observable [18] is constructed out of the \( C_{ab}^{\alpha\beta} \) components [28]:

\[
\Psi(\Sigma) \equiv e^{i \tilde{\mathcal{S}}^{(2)}_{\text{int}}}: \tilde{\mathcal{S}}^{(2)}_{\text{int}} \equiv \kappa \int_{\Sigma(C)} d^2 \sigma \sqrt{-\gamma} \gamma^{ab} C_{ab}^{\alpha}(\sigma) W_\alpha(x(\sigma), \theta(\sigma)) + \text{h.c.}
\]

where \( \gamma^{ab} \) is the metric on \( \Sigma \). This term is not a total world-sheet derivative, unlike the standard Wilson loop. Hence it lives in the ‘bulk’ of the world sheet \( \Sigma \), and depends on the metric \( \gamma \). The coupling constant \( \kappa \) is expected to be related non-perturbatively to the gauge coupling constant \( e \), as we discuss later.

The two observables (30,31) may be expressed [18] in terms of a local chiral current on \( \Sigma \):

\[
\tilde{\mathcal{S}}^{(1)}_{\text{int}} + \tilde{\mathcal{S}}^{(2)}_{\text{int}} = \int d^6 Z (\mathcal{J}^\alpha W_\alpha + \text{h.c.})
\]
where

\[ J^\alpha \equiv \kappa \int_{\Sigma(C)} d^2 \sigma \left( \frac{e}{2} C^{\alpha \beta}(\sigma) D_\alpha + e C^\alpha(\sigma) + \sqrt{-\gamma} \gamma^{ab} C^\alpha_{ab}(\sigma) \right) \delta^{(6)}(Z - Z(\sigma)), \quad (33) \]

where \( \delta^{(6)}(Z - Z(\sigma)) = \delta^{(4)}(Z - Z(\sigma)) (\theta - \theta(\sigma))^2 \), and the third term in (33) depends on the metric on the world sheet.

The Wilson loop integral \( S^{(1)}_{\text{int}} \) can be related to a confining string with an antisymmetric-tensor background, in the sense of [14], by considering [14] the appropriate loop equations which stem from considering the Wilson loop operator \( W(C) \) as a functional of the contour \( C \). These loop equations, which can be described by the dynamics of a string theory in an appropriate antisymmetric-tensor background, as discussed in Section 2, can be translated into the Schwinger-Dyson equations for the gauge field. Polyakov has argued [14] that the correct representation of the Wilson loop necessitates the absence of a target-space metric term in such a string, as seen in the first two terms of (33), corresponding to our previous remark that the world-sheet representation (24) of the first Wilson loop observable is topological.

In our case, the presence of world-sheet defects necessitates the introduction of the second superstring observable (31), which induces terms that involve the target-space metric \( \eta_{MN} \), and are non-topological from the world-sheet point of view. To see this, we integrate out the gauge-field components in (31), i.e., we consider the expectation value: \( < \Psi(\Sigma) > \), where \( \langle \ldots \rangle \) denotes an average with respect to the action for the Abelian gauge field. Simple power counting indicates that we obtain a string action [18] that, at the classical level, is scale invariant in target space, as well as on the world sheet:

\[ < \Psi(\Sigma) >_{\text{Maxwell}} = e^{\tilde{S}^{(2)}_{\text{int}}(\Sigma)} \tilde{S}^{(2)}_{\text{int}} = \int d^6 Z(S_0 + S_1) \delta^{(6)}(Z - Z(\sigma)) \quad (34) \]

where

\[ S_0 = \frac{\kappa^2_0}{16\pi} \int_{\Sigma} d^2 \sigma \sqrt{-\gamma} a^{ab} v^M_a v^N_b \sigma_M \sigma_N, \]

\[ S_1 = \frac{\kappa^2_1}{4\pi} \int_{\Sigma(C)} \sqrt{-\gamma} a^{ab} v^M_a v^N_b \eta_{MN} \sigma^K \sigma_K \quad (35) \]

and

\[ v^M_a = \partial_a X^M(\sigma) - i \theta^m(\sigma) \Gamma^M \partial_a \theta_m(\sigma) : \]

\[ \sigma^M = \frac{\sqrt{-\gamma} a^{ab}}{\sqrt{\det G}} \partial_a v^M_b, \quad G_{ab} \equiv v^M_a v^N_b \eta_{MN} \quad (36) \]

in standard four-component notation in a four-dimensional target space-time: lower-case indices \( m, n, \ldots \) denote spinor indices, and the \( \Gamma^M \) are Dirac four-dimensional matrices. The dimensionless couplings \( \kappa_{0,1} \) are expected to be related non-perturbatively to the gauge coupling \( e \) in the full quantum theory, as we discuss later.
5 Condensation of World-Sheet Defects

The important observation in the supersymmetric model of [18] was that the world-sheet action $\tilde{S}_{int}^{(2)}$ would resemble the classical Green-Schwarz superstring action in flat four-dimensional target space, if there were condensation of the ‘composite field’

$$\Phi \equiv \sigma^M \sigma_M,$$  \hspace{1cm} (37)

In this case, the string tension $\mu$ would be given by

$$\mu = \kappa^2 \frac{1}{4\pi} < \Phi >$$  \hspace{1cm} (38)

Condensation (37) in a critical theory would imply that a dimensionful scale could be obtained from a gauge theory without dimensionful parameters, leading via non-zero string tension (38) to a target-space metric term in the corresponding $\sigma$ model (35). The appearance of this string tension $\mu$ (38) would correspond to a confining area law for the Wilson loop observable $W(C)$.

In the approach of [18], this condensation was conjectured to arise from dynamical effects involving the $\theta$ and $\bar{\theta}$ components of $\nu^a_{\alpha}, \nu^\alpha_a$ and $\nu^a_{\dot{\alpha}}$ in (27). It was thought that the bosonic part could not contribute, since $\epsilon_{ab} \partial_a \partial_b X^M$ should vanish by simple antisymmetry. However, an essential point of our analysis is that supersymmetry is unnecessary once one includes world-sheet defects in the partition function, as one should in such a non-critical theory.

The reason is that, at the world-sheet location of such a defect, the target-space field $X(\sigma)$ diverges, as seen in (12,14). The quantity $\sigma^M$ (36) which appears in the composite field $\Phi$ (37) therefore acquires contributions also from the bosonic part of (36) that remains when we set $\theta^m = \bar{\theta}^m = 0$: $\sigma^M = \epsilon^{ab} \partial_a \nu^b_M = \epsilon^{ab} \partial_a \partial_b X^M$. This is because there is a non-zero vorticity:

$$\epsilon^{ab} \partial_a \partial_b X^M \neq 0$$  \hspace{1cm} (39)

as seen explicitly from (12,14). The quantity (39) is non-trivial for vortices, because the vortex angular variable is not differentiable at the origin, due to its non-trivial winding number around a closed loop. The quantity (39) is also not well defined at the spike core, because this also requires regularization, e.g., by cutting a small loop around the singularity. It is easy to see from (36) that vortex condensation implies

$$< \Phi > = < -\gamma \frac{1}{\det (\partial_a X^M \partial_b X^N \eta_{MN})} \left( \epsilon^{ab} \partial_a \partial_b X^M \right)^2 > \neq 0$$  \hspace{1cm} (40)

where $\gamma$ is the world-sheet metric. This demonstrates the importance of the condensation of vortices: it enables the emergence of an effective string tension to be calculated.

Discussion of the circumstances under which the condensation of world-sheet vortex and/or spike defects may occur may be made using the vortex and spike deformation operators [17] given earlier:

$$V_v =: \cos[2\pi q_v \beta^{1/2}(\tilde{X}(z) + \tilde{X}(\bar{z}))]:$$  \hspace{1cm} (41)
where $q_v$ is the vortex charge and $\tilde{X}$ is one of the rescaled target-space coordinates introduced in (16) in Section 3, and

$$V_m =: \cos\left[\frac{q_m}{\beta^{1/2}} (\tilde{X}(z) - \tilde{X}(\bar{z}))\right]:$$

where $q_m$ is the spike charge. The analysis of [17], which we have adopted and extended [4], indicates that it is possible to interpret the Liouville field theory [16] in the dangerous range of central charges $1 < D < 25$ ($1 < D < 9$ in the supersymmetric case) in terms of these defect configurations, recalling that the inverse temperature $\beta$ is related to the central charge deficit (11) associated with the space-time coordinates.

We first consider the non-supersymmetric case $1 < D < 25$, which is the most relevant for our low-energy description of gauge theories, as we discuss later. The simplest to consider are minimum charge defects $|q_v,m| = 1$. The various phases of such a deformed world-sheet theory are characterized by the different values of the matter central charge $D$. We see from (22) that the vortex deformation with minimal charge $|q_v| = 1$ is marginal when $D = 47/2$. Above this dimension, the vortex deformation is irrelevant, and the vacuum is stable. The quantization condition (18) tells us that the corresponding allowed charge for a spike defect, in the presence of a $|q_v| = 1$ vortex, is $|q_m| = \frac{|m|}{6}(D - 25)$, $m \in Z$. In this case, for $m = 1$ we find the three distinct regions shown in Fig. 2 [17, 31, 34, 4]:

\begin{align*}
47/2 < D < 25 & : \text{spike vacuum unstable, vortices bound} \\
15.48 < D < 47/2 & : \text{both spike and vortex vacua unstable} \\
1 < D < 15.48 & : \text{spikes bound, vortex vacuum unstable}
\end{align*}

Thus, in the $D < 25$ case the spike transition occurs at a lower value of $D$ than the corresponding transition for vortices.

We next consider the case $D > 25$. A similar discussion to that above yields the following phase diagram:

\begin{align*}
25 < D < 53/2 & : \text{spike vacuum unstable, vortices bound} \\
53/2 < D < 33.54 & : \text{both spike and vortex vacua unstable} \\
33.54 < D & : \text{spikes bound, vortex vacuum unstable}
\end{align*}

The first of these $D > 25$ cases includes critical bosonic string theories. We note that the spike BKT transition takes place at a higher value of $D$ that the corresponding transition for vortices, which is opposite from the situation encountered in the $D < 25$ case.

However, in both cases the critical temperatures (21) of the BKT transitions for vortex condensation are lower than for spikes:

\begin{align*}
T < T_{\text{vortex}} & : \text{spike vacuum unstable, vortices bound} \\
T_{\text{vortex}} < T < T_{\text{spike}} & : \text{both spike and vortex vacua unstable} \\
T_{\text{spike}} < T < \infty & : \text{spikes bound, vortex vacuum unstable}
\end{align*}

\footnote{Clearly, the critical values depend, in general, on the specific values of the charges $q_v,m$.}
where the temperatures are understood to be given in units of the corresponding string tension. In our case, as we discuss in more detail later on, the string tension arises dynamically via the condensation (38) of some form of defect (39). We note, that as a result of spike-vortex duality, there is no common region where both types of defect are stable.

In closing this Section, we remark that evidence for the dynamical significance of spike configurations in Liouville strings in the range $1 < \mathcal{D} < 25$ for the matter central charge has recently been found using the two-dimensional quantum Regge calculus, by showing [35] that $< l^n >$, where $l$ is a link length, is ill-defined for Liouville strings with matter central charges in the above region, when $n$ is sufficiently high. This was interpreted [35] as indicating the presence of world-sheet spike configurations in Liouville gravity.

6 World-Sheet Defects and Space-Time Monopoles

We now seek to relate the discussion of world-sheet defect condensation in the previous Section to the condensation of target-space monopoles. We consider an Abelian gauge theory in target space, whose gauge field $A_M(X)$ has a world-sheet vector field $A_a(z, \bar{z})$ as pullback, given by:

$$A_a(z, \bar{z}) = \partial_a X^M A_M(X)$$  \hspace{1cm} (46)

We use lower-case Latin indices to denote world-sheet variables, $\alpha = 1, 2$, and upper-case Latin indices $M = 1, \ldots D$ to denote target-space indices. Consider now a space-like Wilson loop $C$, homotopically extended to the world sheet $\Sigma(C)$. The world-sheet magnetic field corresponding to (46) is:

$$\mathcal{B} \equiv \varepsilon^{ab} \partial_a A_b = \varepsilon^{ab} \partial_a \partial_b X^M A_M(X) + \varepsilon^{ab} \partial_a X^M \partial_b X^N \partial_N A_M(X),$$  \hspace{1cm} (47)

in accordance with the general Hodge decomposition of an arbitrary gauge field $A_a$ on $\Sigma(C)$.

The first term in (47) includes a world-sheet vortex factor, whilst the second term that corresponds to target-space monopoles for the field $A_M$, since it is gauge invariant in target space, and enters directly into the computation of space-time fluxes. Consider the case where the world-sheet surface has the topology of a disc, whose boundary $C$ is to be identified with a Wilson loop for the gauge field $A_M$ in the target (embedding) space:

$$W(C) = e^{ie \int_C A.dl}$$  \hspace{1cm} (48)

Using Stokes’ theorem on the world sheet, it is clear that

$$\int_{\Sigma(C)} \mathcal{B}.dS = \int_C A.dl,$$  \hspace{1cm} (49)
Moreover, the flux of the world-sheet magnetic field $\mathcal{B}$ through $\Sigma(C)$ can be identified with the flux of the target-space gauge field $A_M$ in the embedding space. Substituting (47) into (49) we see that the contribution from the first term can be attributed to vortices $X_v$ on the world-sheet, whilst the second contribution can be attributed to spikes $X_m$. The geometrical relations between the loop $C$, the appearance of a world-sheet defect and its connection with a target-space monopole are displayed in Fig. 2.

Figure 2: World-sheet description of a string world sheet $\Sigma(C)$ whose boundary is a Wilson loop $C$, in the presence of a four-dimensional space-time magnetic field, as could be generated by a target-space monopole. The intersection of the field line with the string world sheet results in a world sheet vortex, which is related by world-sheet duality to a world-sheet spike.

A target-space gauge-field line may be regarded as a line defect that intersects the world sheet at a point. Around such points there is non-trivial vorticity $\epsilon_{ab} \partial_a \partial_b X^M \neq 0$. To see how the condensation of such world-sheet defects corresponds to target-space monopole condensation, we first note that one expects monopole condensation in target space to yield a non-zero condensate

$$\langle F_{MN}^2 \rangle = \frac{1}{4} \mathcal{L}_{\text{Maxwell}}(A) \neq 0,$$

where $\mathcal{L}_{\text{Maxwell}}(A) = -\frac{1}{4} F_{MN}^2$. We now show how such an expectation value $\langle F_{MN}^2 \rangle$ can be calculated using our string $\sigma$-model theory: $S^{(1)}_{\text{int}} + S^{(2)}_{\text{int}}$ in a gauge-field background. We consider such a string $\sigma$ model propagating in a gauge-field background $A_M(X)$. According to our previous discussion, the world-sheet partition function of such a $\sigma$ model is given by:

$$\langle \langle W(C) \rangle \rangle \equiv Z_\sigma = \int \mathcal{D}X e^{-\int d^2\sigma \left( S^{(1)}_{\text{int}} + S^{(2)}_{\text{int}} \right) + \int_C A_M \frac{\partial}{\partial \tau} X^M d\tau}$$

(51)
where $W(C)$ is the Wilson observable on a loop $C$, and $\Sigma$ is an open world sheet whose boundary is the loop $C$. The expectation value $\langle \langle W(C) \rangle \rangle$ is evaluated using the string tension $\mu$ given by (38), in the phase where world-sheet defect condensation occurs.

If one ignored the contribution from the $S^{(1)}_{\text{int}}$ part of the action, the target-space low-energy effective lagrangian corresponding to (51) would be of the Born-Infeld form (36):

$$L_{BI} = \sqrt{\det \left( \eta_{MN} - \frac{F_{MN}}{\mu} \right)} \quad (52)$$

where $\mu$ is the string tension. Summation over world-sheet topologies (WST) leads to canonical quantization of the background gauge field $A_M$, and thus

$$Z \equiv \sum_{\text{WST}} \langle \langle W(C) \rangle \rangle \simeq \int \mathcal{D}A e^{-\mu^2 \int d^4X L_{BI}(A)} \quad (53)$$

To lowest non-trivial order in derivatives, the Born-Infeld lagrangian reduces to the Maxwell kinetic term,

$$\int \mathcal{D}A e^{-\mu^2 \int d^4X L_{BI}(A)} \simeq \int \mathcal{D}A e^{-\frac{1}{4} \int d^4X F_{MN}^2} \quad (54)$$

Furthermore, $\langle \int d^4X F_{MN}^2 \rangle_{BI}$ may be expressed as

$$\int d^4X F_{MN}^2. \int \mathcal{D}A e^{-\mathcal{L}_{BI}(A)} \simeq \frac{\partial}{\partial \lambda} Z[\lambda] |_{\lambda=1} \quad (55)$$

where $Z[\lambda]$ corresponds to setting $A \rightarrow \lambda A$ in the argument of the Born-Infeld lagrangian in the exponent of the right-hand side of (53), and $\simeq$ signifies truncation to lowest non-trivial derivative order. Clearly, the relation (55) applies only to the case where there is vortex condensation on the world-sheet leading to a non-trivial string tension $\mu$ (38). Thus, target-space gauge-field condensation, in the sense of a non-trivial expectation value $\langle F_{MN}^2 \rangle$, is possible only in the case of world-sheet defect condensation.

Important dynamical aspects of the Wilson loop $W(C)$ in this condensation phase are also encoded in the $S^{(1)}_{\text{int}}$ part of the world-sheet action (32), which is effectively described by the 0-brane Julia-Toulouse action (10), resummed over world-sheet genera. Therefore, such contributions result in the appearance in the target-space effective action of terms depending on the antisymmetric-tensor field strength $H_{MNP}$. It is in this subtle form that the connection of target-space monopole condensation and $D$ particles is manifested. As we shall see in the next section, the presence of such $D$-brane configurations is very important, since their quantum fluctuations induce a five-dimensional AdS geometry in our Liouville framework. As we now discuss, these quantum fluctuations appear as recoil effects, arising from the encounter of the $D$ brane with the Wilson loop $C$, as seen in Fig. 2. This analysis also enables us to demonstrate the Meissner effect for external electric fields, as expected in the dual superconductor picture of confinement, in the phase where world-sheet vortex condensation occurs.
7 Derivation of AdS Space Times from Liouville String

We saw in the previous Section how a magnetic monopole in the embedding space may be coupled to a defect on the world-sheet surface $\Sigma(C)$. We recall that world-sheet defects had earlier been mapped \[31, 4\] into topologically non-trivial two-dimensional target space times containing Schwarzschild black holes \[37\], and that this correspondence could be generalized to black holes in higher dimensions. This generalization relied on the correspondence of world-sheet defects to $D$-branes, and the use of the latter as string representations of black holes. The phase structure of gauge theories has been related to that of black holes in AdS space. In our approach, this emerges because of an association between the physics of world-sheet monopoles and that of black holes in AdS_5. In this Section, we review the way in which AdS space-times emerge naturally from Liouville string, via our treatment of $D$-brane recoil \[38\]. The analysis in \[38\] was motivated primarily by the search for a Liouville formulation of M theory \[11\]. Here we limit ourselves to summarizing aspects of the string-$D$-brane interaction \[38\] that are relevant to the present work. We discuss in a later Section how the world-sheet phase structure discussed in Section 5 can be related to the corresponding phases of gravity and black holes in AdS space.

We assume the existence of a suitable conformal closed-string theory in $D$ dimensions which admits $D$-brane solutions described by such world-sheet defects. We now consider configurations combining a closed-string state and such a world-sheet defect, which induces a distortion (recoil) of the corresponding $D$-brane. The combined system is characterized by a homotopic ‘evolution’ parameter $T$. We look for a consistent description of the coupled system in a maximally-symmetric background space. This can be described by a pair of logarithmic deformations \[39\], that correspond to the $D$-dimensional location $y_i$ of the recoiling $D$ brane and homotopic ‘velocity’ $u_i \equiv \partial_T y_i$ \[10\]. These two operators are slightly relevant \[10\], in a world-sheet renormalization-group sense, with anomalous dimensions $\Delta = -\epsilon^2/2$ where $\epsilon \to 0^+$ is a regularization parameter. This is independent of the ‘velocity’ $u_i$, but is related \[10\] to the world-sheet size $L$ and a world-sheet short-distance cut-off $a$ via

$$\epsilon^{-2} \sim \eta \ln(L/a)^2, \tag{56}$$

where $\eta = \pm 1$ for a Euclidean- (Minkowski-)signature homotopic parameter $T$. Thus, the recoiling $D$ brane is no longer described by a conformal theory on the world sheet, despite the fact that the theory was conformally invariant before the encounter that induced the recoil.

To restore conformal invariance, one may again invoke Liouville dressing \[10\], this time by a mode $\varphi$ that can be identified \[38, 10\] with a time-like homotopic variable $T$. This is a second Liouville field, which restores conformal invariance in a critical string theory $c = 26$ that was perturbed to supercriticality by the interaction \[4\].

\[5\]We emphasize that this dressing by the time-like $\varphi$ is independent of the dressing of the subcritical gauge-theory string by the previous space-like Liouville mode $\phi$. 

The dressing by the *time-like* Liouville mode \( \varphi \equiv T \) leads to an effective curved space-time manifold in \( D + 1 \) dimensions. We find a consistent solution to the world-sheet \( \sigma \)-model equations of motion which is described \([10]\) by a metric of the form:

\[
G_{00} = -1, \quad G_{ij} = \delta_{ij}, \quad G_{0i} = G_{i0} = f_i(y_i, T) = \epsilon(\epsilon y_i + u_i T), \quad i, j = 1, \ldots, D \quad (57)
\]

We restrict ourselves to the case where the recoil velocity \( u_i \to 0 \), as occurs if the \( D \)-brane is very heavy. This is formally justified in the weak-coupling limit for the string, since the \( D \)-brane mass \( M \propto 1/g_s \), where \( g_s \to 0 \) is the string coupling. From the world-sheet point of view \([31]\), such a very heavy \( D \) brane corresponds to a strongly-coupled defect. Indeed, the coupling \( g_v \) of the world-sheet defect is related to the string coupling \( g_s \) via

\[
g_v \propto \frac{1}{\sqrt{g_s}} \quad (58)
\]

which implies a world-sheet/target-space strong/weak-coupling duality. When one views the world sheet as the area enclosed by a Wilson loop of the gauge theory in target space, the world-sheet defect coupling \( g_v \) becomes proportional to the target-space gauge-theory coupling \( g_g \). Thus, this approach allows us to study a *strongly-coupled gauge theory* by using a *weakly-coupled* string theory.

In the limit \( u_i \to 0 \), the only non-vanishing components of the \( D \)-dimensional Ricci tensor are \([10]\):

\[
R_{ii} \simeq -\frac{(D - 1)/|\epsilon|^4}{(1/|\epsilon|^4 - \sum_{k=1}^D |y_k|^2)^2} + O(\epsilon^8) \quad (59)
\]

where we have taken \((56)\) into account, for the appropriate Minkowskian signature of the Liouville mode \( T \). In this limiting case, when \( T \gg 0 \), the Liouville mode decouples, and one is effectively left with a maximally-symmetric \( D \)-dimensional manifold. Hence, we may write \((59)\) as

\[
R_{ij} = \mathcal{G}_{ij} R \quad (60)
\]

where \( \mathcal{G}_{ij} \) is a diagonal metric corresponding to the line element:

\[
ds^2 = \frac{|\epsilon|^{-4} \sum_{i=1}^D dy_i^2}{(1/|\epsilon|^4 - \sum_{i=1}^D |y_i|^2)^2} \quad (61)
\]

This metric describes the interior of a \( D \)-dimensional ball, which is the Euclideanized version of an AdS space time. In its Minkowski version, one can easily check that the curvature corresponding to \((61)\) is

\[
R = -4D(D - 1)|\epsilon|^4, \quad (62)
\]

which is *constant* and *negative*. The radius of the AdS space is \( b = |\epsilon|^{-2} \).
The Ricci tensor (59) corresponds to the low-energy: $\mathcal{O}(\alpha')$, $\alpha' \ll 1$ equation of motion for a world-sheet $\sigma$ model, as obtained from the vanishing of the $\beta$ function in this background. It might seem at first sight that the Ricci tensor (59), describing a constant-curvature space time, cannot be a consistent string background compatible with conformal invariance to order $\alpha'$. However, as shown in [42], this conclusion is false if one includes string-loop corrections. These induce a target-space cosmological constant, corresponding to a dilaton tadpole, which renders the constant-curvature backgrounds consistent with the conformal-invariance conditions.

This discussion of the appearance of the AdS structure of the five-dimensional effective space-time in the limit $u_i \to 0$ enables us to demonstrate the Meissner effect for external electric fields in the phase of vortex condensation (38). To see this, we recall the relevant parts of the open world-sheet action for the $\sigma$ model describing a single $D$-particle:

$$S_{\sigma} = \mu \left( \int_{\Sigma} \partial X^M \partial X^N \eta_{MN} + 2 \int_{\partial \Sigma} Y_j(X^0) \partial_n X^j \right)$$  \hspace{1cm} (63)

where the $\partial_n$ denote normal world-sheet derivatives, the $Y_j(X^0)$ are $D$-particle collective coordinates that obey Dirichlet boundary conditions on the boundary of the open world sheet, and $X^0$ is the target-time coordinate, that obeys Neumann boundary conditions. The upper-case indices $M, N$ include the time, whilst lower-case latin indices are spatial. We assume for simplicity uniform motion with a velocity $u_i$: $Y_j(X^0) = y_j + u_j X^0$, where the $y_j$ are the initial spatial coordinates of the $D$ particle.

Making a $T$-duality transformation, which is a canonical transformation [43] of the $\sigma$-model path integral, the problem (63) is mapped, for $u_i \neq 0$, into a $\sigma$ model describing a string with Neumann boundary conditions for all the target space-time coordinates, moving in a background gauge field $A_M(X^0)$ that corresponds to a constant electric field $E_i \leftarrow u_i$. In such a case, the target-space effective action of the string is the Born-Infeld action [36] (52): $\mu^2 \int d^4 X \sqrt{1 - (E_i/\mu)^2}$, which under $T$ duality corresponds to the $D$-particle action [3]: $\mu^2 \int d^4 X \sqrt{1 - u_i^2}$. In the case where the recoil induces a five-dimensional AdS space-time, as discussed in this section, the recoil velocity $u_i \to 0$. By $T$ duality, this limiting case corresponds to a vanishing electric field $E_i \to 0$. This is consistent with the Meissner phenomenon, as expected in the dual superconductor picture of confinement [7, 8]. We emphasize that this correspondence is valid providing string tension $\mu$ can be defined, which in our approach occurs only in the phase with world-sheet vortex condensation occurs as in (38).

We have shown in this section how AdS string backgrounds arise naturally in our approach, using the Liouville $\sigma$-model approach to $D$-brane recoil advocated in [38]. In the next section, we describe how this analysis can be combined with that of the previous sections to yield a holographic Liouville-string approach to confinement in gauge theories.
8 Holographic Liouville-String Approach to Confinement in Four-Dimensional Gauge Theories

We are now in a position to bring together the various elements in our string approach to four-dimensional gauge theories. We have stressed that such a description must be based on non-critical string theory, which we formulate using appropriate Liouville fields, one of which (\(\phi\)) is space-like, and the other (\(\varphi\)) is time-like. Liouville field theories incorporate defects on the string world sheet, which can be associated with target-space background magnetic fields. As seen in Sections 5 and 6, these defects may condense, in which case the string would have a tension and confinement would follow. This condensation may be understood from a higher-dimensional point of view, using the AdS construction developed in the previous Section. The discussion of [3, 4] relies on the important property of AdS space times that a classical field theory on the boundary of the space has a unique extension to the bulk [3, 4]. This is the main point of the holographic nature of field/string theories in AdS space times, according to which all the information about the bulk AdS theory is 'stored' in its boundary. Conversely [1, 3], information about quantum aspects of gauge theories on the boundary of the AdS space is encoded in classical properties of gravity in the bulk of the AdS space. This is determined by the properties of black holes in AdS space, which are related to world-sheet defects in our Liouville-string point of view, as we now show.

Our starting point is a compact Abelian gauge theory in a four-dimensional Minkowski space \(M_4\), parametrized by coordinates \(X_M\), of which one is the target time \(t\). Including the space-like Liouville field \(\phi\) elevates \(M_4\) to a generic five-dimensional manifold \(M_5\). The consideration of interactions given in the previous Section requires the introduction of second Liouville field \(\varphi\), which is time-like. As we saw there, the consistent maximally-symmetric solution of the one-loop \(\sigma\)-model equations of motion is that the generic five-dimensional manifold \(M_5\) is in fact AdS\(_5\).

In order to discuss the confinement/deconfinement transition, we shall want to put the four-dimensional gauge theory at finite temperature \(T\). This may be achieved by Euclideanizing the original time variable: \(t \equiv i\tau\), compactifying it on \(S_1\) with a radius \(R = 1/T\), and imposing the appropriate periodic (antiperiodic) boundary conditions on other bosonic (fermionic) fields [4]. In the limit of large \(R\), \(M_5\) retains its essential AdS\(_5\) character. However, as \(R\) decreases, i.e., \(T\) increases, a simple thermodynamical analysis of classical black holes in AdS\(_5\) indicates non-trivial phase structure. Our task will be to display this in our Liouville-string approach.

The world-sheet \(\sigma\)-model action for our Liouville string model may be written in the form

\[
S_\sigma = \int_{\Sigma(C)} d^2\sigma \left\{ \eta_{MN} \partial X^M \partial X^N + V_m(X) + V_v(X) + (\partial \phi)^2 + Q(\phi)R^{\alpha(2)} + V_v(\phi) + V_m(\phi) + \ldots \right\}
\]

(64)

---

At this stage, our formalism resembles the conventional real-time formalism of field theory at finite temperature, in which time becomes a complex variable with two real components corresponding to \(\tau\) and \(\varphi = T\), but we do not pursue this connection further in this paper.
where the dots denote appropriate supersymmetrizations or other deformations that do not concern us for now. In view of the AdS$_5$ structure derived in the previous Section, we may treat $X_M, \phi$ symmetrically, replacing the metrics in their kinetic terms by the AdS$_5$ metric $G$ (60). We have already stressed the rôles of the central-charge deficit $Q$ and the sine-Gordon deformations $V_m(X), V_v(X)$ (42,41). In particular, we recall the values (14,13,15) of $\mathcal{D}(Q)$ (11) and the corresponding effective temperatures (21) for which these deformations condense. The new features of (64) above are the corresponding deformations in the space-like Liouville field $\phi$:

$$V_m(\phi) =: \cos\left[\frac{q_m}{\beta^{1/2}}(\phi(z) - \phi(\bar{z}))\right]: \quad (65)$$

and

$$V_v(\phi) =: \cos[2\pi q_v \beta^{1/2}(\phi(z) + \phi(\bar{z}))]: \quad (66)$$

whose rôles we discuss next.

In the maximally-symmetric AdS$_5$ space, these defects exhibit patterns of condensation similar to those of $V_m(X), V_v(X)$ (42,41) as the effective temperature $T = 1/\beta$ is varied. As we saw in Section 6, world-sheet defects (41) are linked to target-space magnetic fields. We recall that the target-space gauge magnetic monopoles are singular 0-brane classical solutions of the compact $U(1)$ gauge-theory equations of motion in Maxwell theory. This action is equivalent at low energies to those of the Born-Infeld effective action (52), which solves the $\sigma$-model conformal-invariance conditions. Therefore, we may represent the target-space monopole as an appropriate solitonic string background (D-brane), living on the boundary of AdS$_5$. Analogous $D$-brane solutions in the bulk of AdS$_5$ may be interpreted as the horizons of AdS$_5$ black holes, which are also consistent solutions of the conformal invariance conditions of the $\sigma$ model to $\mathcal{O}(\alpha')$, i.e, the Einstein equations of motion.

We now review briefly relevant properties of AdS space-times and their black holes [12, 3]. The topology of the regular AdS$_{d+1}$ space time is

$$X_1 = B_d \times S_1 \quad (67)$$

and the line element for this metric, in the absence of black holes, is given by:

$$ds^2_{x_1} = (1 + \frac{r^2}{b^2})dt^2 + \frac{1}{1 + \frac{r^2}{b^2}}dr^2 + r^2 d\Omega^2 \quad (68)$$

where the AdS radius $b = |\epsilon|^{-2}$ in our approach. We have already shown that vortices of the target-space coordinates $X^M$ are expected to condense in the vacuum at sufficiently low temperature, leading to non-zero string tension (68). Because of the symmetry between $\phi$ and the $X^M$ in the AdS space (67), we expect the same to be true in AdS$_{d+1}$ at sufficiently low temperatures.

On the other hand, the Minkowskian-signature AdS$_{d+1}$ black-hole solution of [12, 3] corresponds to a metric element of the form:

$$ds^2 = -V(dt)^2 + V^{-1}(dr)^2 + r^2 d\Omega^2 \quad (69)$$
where $d\Omega^2$ is the line element on a $(d - 1)$-dimensional sphere of volume $V$, $r$ is the radial coordinate of the AdS$_{d+1}$ space, and $t$ is its time coordinate. This AdS black-hole space time, which is a consistent classical solution of Einstein’s equations in a space with cosmological constant $\Lambda < 0$, corresponds to a smooth geometry if one Euclideanizes and compactifies the time direction with a special period $\beta$, defining a corresponding AdS black-hole temperature $T = 1/\beta$:

$$\beta = \frac{4\pi b^2 r_+}{(d - 2)b^2 + d r_+^2}, \quad b = \sqrt{-\frac{3}{\Lambda}}$$  

Here

$$V \equiv 1 - \frac{w_d M}{m_P^{d-1} r_d^{-2}} + \frac{r^2}{b^2},$$  

where $m_P$ is the effective Planck mass in AdS$_{d+1}$, $w_d = 16\pi G^{(d)}_N / ((d - 1)\text{Vol}(S_{d-1})$ where $G^{(d)}_N$ is the $d+1$-dimensional Newton’s constant, and $r_+$ is the larger of the two solutions of the equation $V = 0$. The topology of this Euclideanized finite-temperature space time is

$$X_2 = B_2 \times S_{d-1}$$  

We note that this black hole corresponds to a world-sheet vortex defect in the extra space-like Liouville dimension $\phi$. This may be seen as a generalization of the two-dimensional target-space black-hole case [37], where the ‘cigar’ geometry in target space can be mapped onto a world sheet with a vortex defect [4]. The AdS black holes are also spherically symmetric, and one may generalize the mapping of [4] to include this case.

This argument suggests that there is a correspondence between the world-sheet vortex/spike phase structure discussed earlier and the analysis of the AdS black-hole system in [12]. The latter has three critical temperatures. For $T$ below the first critical temperature $T_0$, there is only radiation, but the specific heat of a gas of AdS black holes changes sign when $T = T_0$. This first critical temperature $T_0$ corresponds to the maximum of $\beta$ (70), which has the following value in AdS$_4$:

$$T_0 = (2\pi)^{-1} \sqrt{3} b^{-1}$$  

Here and subsequently, we quote the numbers for the case $d = 3$ which was discussed in [12]. The generic analysis for arbitrary $d$ is straightforward, leading only to different proportionality coefficients in front of the critical temperatures. Qualitatively, the behaviour is similar to the $d = 3$ case. At temperatures above $T_0$, the topology of the space time may change to (72), so as to include black holes, but in the region $T_0 < T < T_1$, where:

$$T_1 = \frac{1}{\pi} b^{-1}$$

the free energy of the black hole is positive, so the black hole is unstable and tends to evaporate. When $T > T_1$, the free energy of the configuration with both black hole
and thermal radiation is lower than the corresponding configuration with just thermal radiation, so the radiation tends to form black holes. Finally, at temperatures $T$ greater than a third value $T_2$:

$$T_2 = \frac{m_{\mu}^{1/2} \cdot 3^{1/4}}{b^{1/2}}$$  \hspace{1cm} (75)

there is no equilibrium configuration without a black hole.

This phase structure corresponds to that suggested for gauge theories in Fig. 1: the radiation-dominated (confined) state corresponds to the minimum of the free energy at a non-zero value of the order parameter, the black-hole dominated (non-confined) state to that at the origin of the order parameter. The black-hole minimum becomes a local maximum of the free energy when $T < T_0$. The stability or otherwise of the black-hole state is linked the relative heights of the two local minima of the free energy, which cross at $T_1$. The radiation state ceases to exist when $T > T_2$, where the corresponding minimum becomes a point of inflection.

The discussion of Sections 5 and 6 in the context of the gauge theory on the boundary of AdS$_5$ related $T_0$ to the vortex BKT temperature $T_{\text{vortex}}$, and $T_2$ to the spike BKT temperature $T_{\text{spike}}$, but did not give a derivation of $T_1$. The discussion in this Section establishes a BKT transition temperature for the condensation of $\phi$ vortices in the bulk AdS$_5$ theory, also at $T_{\text{vortex}} = T_0$, corresponding to the condensation of black holes in the bulk, as illustrated in Fig. 3. Analogously, $\phi$ spikes have a BKT transition temperature $T_{\text{spike}} = T_2$, above which they condense. Combining these results, we obtains the following AdS/gauge-theory phase diagram:

(i) $T < T_{\text{vortex}} = T_0$: vortices bound, no AdS$_5$ black holes, confining phase
(ii) $T_{\text{vortex}} = T_0 < T < T_{\text{spike}} = T_2$: vortices unbound, AdS$_5$ black holes unstable, mixed phase
(iii) $T > T_{\text{spike}} = T_2$: spikes bound, stable AdS$_5$ black holes, unconfined phase

(76)

to be compared with (73).

Space-like Wilson loops $C$ obey an area law below $T_0$, with the corresponding string tension $\mu_{\text{AdS}}$ given in the AdS picture by the square of the (large) AdS radius of curvature [3]:

$$\mu_{\text{AdS}} \propto b^2 \propto (-\Lambda)^{-1}$$

(77)

It was the topology change in AdS$_5$ at $T = T_0$ from $X_1$ (77) to $X_2$ (72) that prompted the conjecture [3], based on the conformal field theory/AdS correspondence [1, 3], that there exists a connection between the bulk AdS phase transitions [12] and the finite-temperature confinement/deconfinement phase transition of the large-$N_c$ $U(N_c)$ gauge theory on the boundary. In our case, we first saw the string tension arising in the boundary gauge theory as a result of world-sheet $X^M$-vortex condensation [38] corresponding to the condensation of target-space monopoles. In
the bulk AdS picture, it is the equivalent condensation of world-sheet $\phi$ vortices, corresponding to AdS black holes, which is responsible for (38).

In the picture advocated in Section 3 and 5, where both vortex and spike defects appear, one has a nice correspondence to the phase diagram of AdS black holes presented above, with the the higher BKT temperature $T_2$ corresponding to world-sheet spike condensation. From the world-sheet duality (19) we have:

$$T_2^{-1} = \frac{T_0 \alpha'}{4\pi^2}$$

(78)

where $\alpha' = 1/\mu$ is the Regge slope for the string, which is thus given by:

$$\alpha' = \frac{1}{\mu} = \frac{8\pi^3}{m_P^{1/2} (-\Lambda)^{3/4}}$$

(79)

The expression for the string tension in (77) is consistent with (79), provided one identifies $m_P \sim (-\Lambda)^{-7/2}$ in fundamental string units. Notice that, in this way, $m_P^2 >> |\Lambda|$, (79) is small, and the entire approach is consistent. In this picture, then, one has a higher-dimensional D-brane analogue of the correspondence between world-sheet spike/anti-spike pairs and Schwarzschild black holes in two-target-space dimensional strings [31].

We have the following comment on the intermediate AdS phase-transition temperature $T_1$. As we saw above [12], AdS black holes are unstable at temperatures between $T_0$ and $T_1$, with a tendency to evaporate. It is natural to conjecture that this instability is due to some other relevant world-sheet operator, which is neither
the vortex nor the monopole/spike discussed at length above. Just such an operator is known in the two-dimensional black-hole case, namely an instanton that causes the renormalization-group flow in the model $[4, 45]$, reducing the level of the non-linear $\sigma$ model, and hence its central charge and the mass of the black hole. It may well be that an analogous instanton operator is relevant for temperatures between $T_0$ and $T_1$ in the AdS case, but this remains to be demonstrated.

We now recall that in the above picture the string tension is associated with the AdS$_5$ radius $b^2$, which is given in our ‘recoil’ approach to AdS space by $b^2 \propto |\epsilon|^{-4}$. Furthermore, $|\epsilon|^{-2}$ is proportional $[50]$ to the size of the world-sheet disc $[40]$. According to the approach of $[3]$, and identifying the string tension with the squared radius of the AdS$_5$, one obtains a logarithmic scaling violation of the area law, manifested as a dependence of the string tension on the logarithm of the area of the large quark loop:

$$\mu \sim \kappa_1^2 \ln^2 A,$$

(80)

where $A$ is the minimal loop area. Since the string tension should be independent of $A$, one expects the explicit $A$ dependence in (80) to be cancelled by a logarithmic variation $\kappa_1 \sim 1/\ln A$.

This argument suggests that the coupling $\kappa_1$ should vary with the scale/size of the world-sheet loop. As we mentioned previously, this coupling is expected to be proportional to the gauge coupling strength of the original theory. We are not yet in a position to determine independently the sign of the logarithmic dependence of $\kappa_1$, and hence verify asymptotic freedom, but this analysis does suggest that there need be no barrier between confined and asymptotically-free phases.

9 Conclusions and Prospects

We have shown in this paper how Liouville string theory may be used to gain insight into confinement in gauge theories in four dimensions. In the presence of a Liouville field, the requirements of conformal symmetry are relaxed, and four-dimensional gauge theories with a logarithmically-varying coupling and without supersymmetry can be treated using string techniques. We have used this approach to set up a quantum string model of Wilson loops, demonstrated that world-sheet defects on the world sheet condense, and argued that this corresponds to the condensation of gauge-theory monopoles in the underlying target space. We have extended this analysis to the AdS$_5$ extension of four-dimensional Minkowski space $M_4$. In this way, we have established the relation between world-sheet vortex condensation, gauge monopole condensation and black-hole condensation in AdS space in a theory that exhibits asymptotic freedom. The confinement-deconfinement transition in gauge theory is related to a BKT transition for vortices on the effective string world sheet.

This Liouville-string approach may open the way towards more quantitative string calculations in gauge theories. There has recently been exciting progress in the calculation of glueball masses in some limit of the bulk AdS theory $[46]$, and it would be interesting to extend this to more realistic models and limits. It would
also be interesting to explore the calculation of other properties of gauge theories that may be related to experiment, such as behaviour close to the confinement-deconfinement transition. We believe that the Liouville-string approach espoused here may be able to contribute significantly to such an ambitious programme.

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