RELATIVISTIC ACCRETION MEDIATED BY TURBULENT COMPTONIZATION

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ABSTRACT

Black hole and neutron star accretion flows display unusually high levels of hard coronal emission in comparison to all other optically thick, gravitationally bound, turbulent astrophysical systems. Since these flows sit in deep relativistic gravitational potentials, their random bulk motions approach the speed of light, therefore allowing turbulent Comptonization to be an important effect. We show that the inevitable production of hard X-ray photons results from turbulent Comptonization in the limit where the turbulence is trans-sonic and the accretion power approaches the Eddington limit. In this regime, the turbulent Compton $\gamma$-parameter approaches unity and the turbulent Compton temperature is a significant fraction of the electron rest mass energy, in agreement with the observed phenomena.

Key words: accretion, accretion disks – black hole physics – stars: neutron

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1. GENERAL ISSUE AND MOTIVATION

Like the envelopes of late-type dwarfs, accretion disks are thought to be turbulent and optically thick. Turbulence in stellar envelopes is driven by convection, while in accretion disks turbulence is commonly thought to be driven by the magnetorotational instability (MRI; Balbus & Hawley 1991).

Accretion disks found in proto-stellar and proto-planetary systems, cataclysmic variables, X-ray binaries, and quasars/active galactic nuclei (AGNs) all display coronal behavior as do late-type dwarfs. By “corona,” we mean the appearance, in a spectral energy distribution, of a relatively energetic population of particles that persistently emit radiation at temperatures in excess of the given object’s thermal photosphere.

In the case of black hole and neutron star accretion flows, the ratio of coronal to bolometric power $L_\lambda/L_{\text{bol}} \sim 0.1-1$. In other systems that display radiatively efficient accretion, the value of $L_\lambda/L_{\text{bol}}$ is smaller by orders of magnitude. For example, turbulent stellar envelopes achieve $L_\lambda/L_{\text{bol}} \lesssim 10^{-3}$ in the most extreme (rapidly rotating) case.

Here, we ask why black hole and neutron star accretion flows display such abnormally high levels of coronal power, observed in the form of hard power-law X-ray photons.

In the directly observable case of the solar corona, mechanical energy leaks out through the photosphere in some combination of waves and relatively steady magnetic structures. The two reservoirs of stellar mechanical energy are convective turbulence and differential rotation.

Since proto-planetary and white dwarf accretion disks as well as rapidly rotating late-type dwarfs display relatively modest levels of coronal power, differential rotation cannot be the sole key ingredient in explaining the extreme coronal phenomena seen in relativistic accretion flows. Furthermore, the difference between MRI-driven and convective turbulence must be ruled out as well because the non-relativistic accretion flows display only modest coronal phenomena.

An obvious feature that is unique to black hole and neutron star accretion flows is that close to the central sources of gravity, where most of the accretion power is generated, the bulk motions approach relativistic values. If the electrons possess bulk motions that are trans-relativistic, then they may readily amplify soft photons up to high energies, potentially forming a hard power-law X-ray spectra though the process of second-order bulk Comptonization.\footnote{As an aside, cosmologists refer to this radiative mechanism as the “second-order kinetic S-Z effect.”}

In the brief discussion that follows, we focus solely on the importance of turbulent Comptonization in relativistic accretion flows. Our approach differs from that of Socrates et al. (2004) and Thompson (2006) in that we completely ignore the relationship between the region from which soft thermal seed photons originate—presumably a cool thermal disk—with the region of Comptonization. We start with a description of an isolated eddy undergoing turbulent Comptonization.

2. THE BASIC UNIT: AN EDDY IN A BOX

First, we summarize the properties of an eddy in a box of dimension $L$. We assume that the eddy is embedded in an isotropic turbulent flow. The scale of the eddy $\lambda \lesssim L$ is of the order of the box size and the characteristic eddy velocity $v_\lambda \lesssim c$ is taken to be trans-relativistic. We assume that the box is fully ionized with a constant mass density $\rho$.

The eddy is stirred on the outer scale $\lambda \sim L$, and in the absence of any form of microphysical dissipation the energy injected into the scale $\lambda$ flows down to smaller scales via a self-similar turbulent cascade. At the dissipation scale, the cascade cuts off and the turbulent power is converted into heat.

We now discuss some of the relevant properties of our eddy.

2.1. Some Important Properties of the Eddy

The total kinetic energy $K_\lambda$ in the box at any given moment is given by

$$K_\lambda \sim M_\lambda v_\lambda^2 \sim \rho \lambda^3 v_\lambda^2.$$  \hspace{1cm} (1)

The total eddy power or luminosity $L_\lambda \sim K/t_\lambda$, where

$$t_\lambda \sim \lambda/v_\lambda.$$  \hspace{1cm} (2)

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is the eddy turnover time. We write the eddy luminosity as
\[ L_\lambda \sim \rho v^3_\chi \frac{2}{c^2}, \quad (3) \]
which is just the turbulent energy flux, multiplied by the eddy cross section. Also, we assume that in the absence of dissipation, the eddy velocity amplitude scales as a power law, i.e.,
\[ v_\chi \sim v_0 (\lambda/\lambda_0)^n. \quad (4) \]
For isotropic incompressible Kolmogorov turbulence, the index \( n = 1/3 \). We take \( v_0 \lesssim c \) and \( \lambda_0 \sim L \).

Since the velocity amplitude \( v_\chi \) of our eddy is trans-relativistic, the inverse-Compton effect on electrons may become important on timescales of interest, particularly when the eddy is optically thin. Now, we assess the importance of the various microphysical processes arising from turbulent Comptonization.

2.2. Eddy Comptonization: Cooling Rate and Compton Power

The eddy cooling rate is the rate at which photons remove energy from the turbulent electrons, which are “stuck” to the ions as result of—the assumed—tight Coulomb coupling. Therefore, inverse-Comptonization off of turbulent eddies may be viewed as a microscopic dissipation mechanism for the turbulence. Note however, that this dissipation does not necessarily occur on scales smaller than \( \lambda \sim L \). If the eddy optical depth \( \tau_\lambda \lesssim 1 \), i.e.,
\[ \tau_\lambda \equiv \kappa_{es} \rho \lambda \lesssim 1, \quad (5) \]
then the photons sample the turbulent velocities on the outer scale of the turbulent eddy, which is \( \sim v_\chi \sim v_0 \). Thus, the act of electrons, and thus the eddy, Compton up-scattering relatively cool photons, necessarily implies that turbulent Comptonization off an eddy is a damping mechanism.

Comptonization preserves the number of quanta, rather than the energy per quanta. If the radiation field is close to isotropic, the Kompaneets (1957; cf. Rybicki & Lightman 1979) equation, which expresses conservation of photon number, reads
\[ \frac{\partial n_\nu}{\partial t} \simeq \kappa_{es} n_\nu \epsilon \left( \frac{v_\chi^2}{3c^2} + \frac{k_B T_e}{m_e c^2} \right) \left( 4v \frac{\partial n_\nu}{\partial v} + v^2 \frac{\partial^2 n_\nu}{\partial v^2} \right), \quad (6) \]
where \( n_\nu \) is the photon occupation number. The above expression is accurate as long as \( v_\chi^2 + 3k_B T_e / m_e \gtrsim E_\gamma / m_e = h\nu / m_e \) (Psaltis & Lamb 1997). In other words, turbulent Comptonization is important as long as the contribution to the electron kinetic energy resulting from turbulence exceeds the contribution from random microscopic (e.g., thermal) motions as well as the average “seed” photon energy.

Immediately, we can read off a time scale from Equation (6)
\[ t_{\chi,\lambda}^{-1} \simeq \frac{4}{3} \kappa_{es} n_\nu \epsilon \frac{v_\chi^2}{c^2}, \quad (7) \]
where \( t_{\chi,\lambda}^{-1} \) is the time it takes for the eddy to boost the energy density of the photon field by a factor of 2. It follows that the turbulent Compton power of a given eddy is simply
\[ L_{c,\lambda} \simeq \frac{U_\gamma \Delta V}{t_{\chi,\lambda}} \simeq \frac{U_\gamma \lambda^3}{t_{\chi,\lambda}}, \quad (8) \]

If at any point along the cascade, the turbulent Compton power \( L_{c,\lambda} \) approaches the turbulent eddy power \( L_\lambda \), then turbulent Comptonization off of an eddy may be viewed as a viable microphysical dissipation mechanism for the turbulence.

2.3. Electron Heating Rate

There are really three particle distributions of concern in our box: the photons, and the bulk turbulent and microscopic thermal degrees of freedom for the electrons. So far, we have quantified the rate at which energy is transferred from the turbulence to the Comptonized radiation field. If the electrons are “cold” such that their thermal motions are small in comparison to their bulk motions, then the electrons stand a chance of heating up.

In a frame moving with velocity \( v_\chi \), co-moving with a given portion of an eddy, the thermal energy density of the gas \( U_g \) evolves as
\[ \frac{\partial U_g}{\partial t} \simeq \sigma_{es} n_\nu \epsilon \left( \frac{\bar{E}_\gamma}{m_e c^2} - \frac{4k_B T_e}{m_e c^2} \right) U_g. \quad (9) \]
In the above expression, \( \bar{E}_\gamma \) is the mean photon energy, averaged over the photon energy spectrum. The above expression conveys the action of thermal Comptonization.\(^6\)

We are interested in the regime where turbulent Comptonization boosts relatively few photons up to high energies, where \( E_\gamma \lesssim m_e c^2 \). Though few in number, their large energies allow them to carry away a significant—if not dominant—fraction of the photon power. If the gas is cold such that the bulk—and therefore, turbulent—motions are large in comparison to the particle’s thermal motions, then \( E_\gamma \gg k_B T_e \). In that case, we identify a Compton heating rate
\[ t_{\text{heat}}^{-1} \simeq \frac{U_\gamma}{t_{\chi,\lambda}} \frac{U_g}{U_g}, \quad (10) \]
which is the rate at which energetic Comptonized photons heat up the gas, increasing its thermal content per unit volume.

3. AN EDDY AROUND A BLACK HOLE

Now, we place our eddy near a black hole to get a feel as to how an ensemble of eddies—that collectively form an accretion flow—behaves. We place our eddy at a radius \( R = R_g / \epsilon \) from the hole, where \( R_g = GM_*/c^2 \) and \( \epsilon \) is the gravitational radius and radiative efficiency of the hole, respectively. The radiative efficiency \( \epsilon \) is roughly determined by the location of innermost stable circular orbit \( R_{iscr} \), i.e., \( \epsilon \sim R_g / R_{iscr} \). So, if \( \epsilon \sim 0.1 \), then roughly \( 100 \text{ MeV} \) worth of gravitational binding energy per baryon is available for conversion into other forms of energy. The discussion that follows applies to neutron star accretion flows as well. Since black holes do not have a surface, the overall physical system is simpler, and for this reason we restrict our discussion to black hole accretion flows.

3.1. Accretion Power and Accretion Timescale

We imagine that our eddy is surrounded by other eddies of comparable size and shape. The interaction between eddies not

\(^5\) Throughout, we assume that the electron and proton temperatures are equal to one another.

\(^6\) One of the most famous examples of thermal Comptonization maintaining thermal equilibrium between radiation and gaseous matter is in the cosmic background radiation, before and during the moments of recombination.
only contributes to the turbulent cascade of energy to small scales, but also transfers angular momentum between radially adjacent eddies.

The transfer of angular momentum, the viscous generation of accretion power, and the release of gravitational binding energy are connected by the expressions that govern angular momentum and energy balance. The viscous dissipation rate per unit volume \( \epsilon^+ \) resulting from an effective turbulent viscosity is given by

\[
\epsilon^+ \simeq \rho v^2_\lambda \frac{d\Omega}{d\ln R} \sim \rho v^2_\lambda \Omega, \tag{11}
\]

which is just the kinetic energy density, multiplied by the shear rate. Note that in the above expression, we assume that each respective eddy is rotating at the Keplerian rate, with an angular velocity \( \Omega \simeq \sqrt{GM/R^3} \). The total accretion power, or accretion luminosity \( L_{\text{acc},\lambda} \) for the eddy, is given by

\[
L_{\text{acc},\lambda} \simeq \rho \lambda^3 v^3_\lambda \Omega \simeq K_\lambda \Omega. \tag{12}
\]

Of course, each eddy is a transient structure embedded in a continuum of matter that slowly drifts towards the hole. This drift, which results from the transport of angular momentum, is quantified by the specific angular momentum \( \ell \) and the mass accretion rate through the eddy, \( \dot{M}_\lambda \), in the following way:

\[
\dot{M}_\lambda \ell \simeq \lambda^2 \rho v^2_\lambda, \tag{13}
\]

where we have ignored the contribution from the inner boundary. As an interesting aside, we see that \( \dot{M}_\lambda \ell \simeq K_\lambda \), or the rate of angular momentum loss within an eddy is equal to the kinetic energy, \( K_\lambda \), of the eddy itself. The combination of the expression for viscous dissipation, Equation (12), and angular momentum conservation, Equation (13), allows us to connect the accretion power \( L_{\text{acc},\lambda} \) to the accretion rate \( \dot{M}_\lambda \) through the eddy, i.e.,

\[
L_{\text{acc},\lambda} \simeq K_\lambda \Omega \simeq \dot{M}_\lambda \ell \Omega \simeq \frac{GM_\bullet M_\star}{R}, \tag{14}
\]

analogous to the well-known result.

The characteristic time for an eddy to deplete its mass \( t_{\text{acc},\lambda} \) is obtained by rewriting the conservation of angular momentum, Equation (13), as

\[
\dot{M}_\lambda \ell \simeq \frac{M_\lambda}{t_{\text{acc},\lambda}} \Omega^2 \simeq M_\lambda v^2_\lambda \simeq K_\lambda
\]

\[
t_{\text{acc},\lambda}^{-1} \simeq \frac{v^2_\lambda}{R^2 \Omega}, \tag{15}
\]

which is often written in terms of an anomalous viscosity and a Shakura & Sunyaev (1973) \( \alpha \)-parameter.

### 3.2. An Eddington-limited Eddy

Now, we determine under what conditions the various dynamical, thermal, and radiative timescales become competitive with one another. Throughout, we specify that the turbulent motions on the stirring scale \( \lambda \) are trans-relativistic—a necessary condition for turbulent Comptonization to be a powerful effect. Another condition is that the optical depth must at least be of order unity, in which case the rate of photon escape is equal to the rate of energy gain, yielding a relatively flat (in \( v F_\gamma \)) and powerful Comptonized spectrum.

We take the limit where \( \lambda \sim R \sim R_g/\epsilon \) and \( v^2_\lambda \lesssim c^2 \lesssim \epsilon c^2 \).

That is, we approximate that the eddy is in dynamic balance with the black hole’s gravitational field at \( R/R_g \sim \epsilon^{-1} \) gravitational radii from the system’s gravitational center. In order to calculate the eddy optical depth, or alternatively the mean density, we further assume that the eddy is radiating at the Eddington limit for the eddy \( L_{\text{edd},\lambda} \), i.e.,

\[
L_{\text{acc},\lambda} \simeq \frac{GM_\bullet M_\star}{R} \simeq \epsilon M_\star c^2 \simeq L_{\text{edd},\lambda} \simeq \frac{GM_\bullet c}{K_{\text{es}}}. \tag{16}
\]

Note that \( L_{\text{edd},\lambda} \) differs from the Eddington limit \( L_{\text{edd}} \) for the entire flow by a factor of \( 1/4\pi \) since the eddy subtends \( \sim 1 \) sr of solid angle around the black hole. Putting together the expression above with Equation (15) we may write the eddy optical depth as

\[
\tau_\lambda \sim \epsilon^{-1/2} \gtrsim 1. \tag{17}
\]

At the Eddington limit, the Thomson optical depth across the eddy \( \tau_\lambda \) is of order unity, and therefore the photon field "samples" the energy bearing eddies on the outer scale \( \lambda \). Therefore, the rate for turbulent Compton heating \( t_{c,\lambda}^{-1} \) and the turbulent Compton luminosity \( L_{\lambda,\ell} \) should be evaluated by using the outer scale length \( \lambda \) and the outer scale velocity \( v_\lambda \). Finally, for an Eddington-limited eddy, we find

\[
t_{c,\lambda}^{-1} \simeq t_{\text{acc},\lambda}^{-1} \simeq t_{\text{dynamical}}^{-1}, \tag{18}
\]

where \( t_{\text{dynamical}} \sim c/L \sim 1/\Omega \). Furthermore,

\[
L_{c,\lambda} \simeq L_{\text{acc},\lambda}. \tag{19}
\]

for an Eddington-limited eddy.

The two expressions, Equations (18) and (19) above, are interesting results. Together, they indicate the release of gravitational energy resulting from turbulent viscous dissipation, which is equally matched by the Compton power that is generated by the turbulence as well. Therefore, it is possible that all of the fundamental physical mechanisms that mediate accretion, i.e., the transport of angular momentum, randomization of gravitational binding energy, and the conversion of that energy into photon power are entirely determined by a single physical phenomenon: turbulence.

However, our picture is not complete. Bathing the eddy in such an intense (self-generated) hard radiation field has consequences for the thermal motions of the gas. Consider the form of the Compton heating rate given by Equation (10). The act of turbulent Comptonization boosts the mean photon energy to a value \( \bar{E}_\gamma \lesssim m_e c^2/\epsilon \), which implies that

\[
t_{c,\text{heat}}^{-1} \simeq \frac{U_\gamma}{U_g} t_{c,\lambda}^{-1} \gg t_{c,\lambda}^{-1} \tag{20}
\]

for an Eddington-limited eddy since \( U_\gamma \gg U_g \) even if \( \epsilon k_BT_g \sim \bar{E}_\gamma \). This results from the fact that the radiation pressure is roughly a factor of \( m_e c^2/k_BT_p \) larger than the gas pressure, at the Eddington limit.

The large value of the heating rate \( t_{c,\text{heat}}^{-1} \) leads to the conclusion that in equilibrium the thermal energy per baryon equals the average energy of the hard Comptonized photons. In that case, the source term on the right-hand side of Equation (9) approaches zero and thermal equilibrium for the gas is reached.

Apparently, it seems unlikely that—very close to the hole—electrons have specific thermal energies that are significantly
smaller than $v_\lambda^2$. Therefore, turbulent Comptonization in black hole and neutron star accretion flows may also be viewed as an electron heating mechanism. This picture of turbulent Comptonization near black holes is sharply different from the idea put forth by Socrates et al. (2004). They found that the standard cool thermal thin disk approximation broke down near the Eddington limit since the thermal disk itself would be a site of turbulent Comptonization. Also they emphasized that the reason turbulent Comptonization operates within the disk is the large thermal radiation pressure. Furthermore, they noticed that the turbulent Compton $\gamma$-parameter was close to unity, without providing an explanation for this apparent coincidence. In what follows, we show that the turbulent Compton $\gamma$-parameter is generically close to unity for black hole accretion near the Eddington limit as long as a significant fraction of the accretion power is released in the form of turbulent Comptonized photons.

4. THE TURBULENT COMPTON $\gamma$-PARAMETER AND THE MEANING OF ITS VALUE NEAR A BLACK HOLE

We continue our assessment of the Eddington-limited eddy near a black hole introduced in the last section. The spectral features of saturated and mildly unsaturated Comptonization are well described by the slope of a self-similar Comptonized spectral feature and a high energy cutoff. The slope is often expressed in terms of the Compton $\gamma$-parameter, given by the fractional energy shift per scattering multiplied, by the average number of scatterings. When $y \sim 1$, the “input” spectrum, responsible for providing the soft seeds, is deformed by the addition of a hard self-similar power-law component. In such an event, the ratio of hard Comptonized to soft seed power is also of order unity. When the eddies are optically thin such that $\tau_\lambda \lesssim 1$, we define the turbulent Compton $\gamma$-parameter as

$$y_\gamma = \frac{4}{3} \frac{v_\lambda^2}{c^2} \tau_\lambda \sim \frac{4}{3} \epsilon^{1/2} \lesssim 1. \quad (21)$$

The important result above, that the turbulent Compton $\gamma$-parameter is close to unity, is not an accident.

Consider the simple representation of extreme quasi-spherical black hole accretion given by Figure 1. For illustrative purposes, the hypothetical case where accretion is 100% efficient, i.e., $\epsilon = 1$, is depicted. Thus, dynamical effects arising from general relativity, such as the presence of the last stable circular orbit, are completely ignored. Critical accretion, which implies $t_{\text{acc},\lambda}^{-1} \sim t_{\text{dyn}}^{-1}$, while unit efficiency, $\epsilon = 1$, implies that $\lambda \sim R \sim R_G$. For this case, the number of protons $N_p$ responsible for fueling an Eddington’s worth of accretion power is roughly given by

$$N_p \sim \frac{A_\epsilon}{\sigma_T} \sim \frac{R_G^2}{r_e^2},$$

where $A_\epsilon$ and $r_e$ are the surface area of the black hole and the classical electron radius, respectively.

It follows that Comptonized photons remove the accretion power of each individual proton on average scatter once with an electron—as long as the number of electrons equals the number of the protons. Or, in other words, $\tau_\lambda \sim 1$ for highly efficient critical accretion. At the same time, the value of randomized proton velocity squared $\Delta v^2 \sim v_\lambda^2 \sim c^2$, since of order $m_p c^2$ worth of binding energy is removed per proton. The combination of these two attributes of critical efficient accretion onto a black hole yields the result that $y_\gamma \sim 1$. Hence, *Eddington-limited relativistic accretion is a natural turbulent Compton amplifier.*

5. CONTACT WITH OBSERVATIONS AND DISK PHENOMENOLOGY

The picture described above for black hole accretion, mediated by turbulent Comptonization, while providing a natural and simple explanation for hard Comptonized photons, differs greatly from current orthodoxy and belief. Below, we attempt to place our turbulent Compton accretion model in context of what is known both observationally and theoretically of black hole accretion.

A thin cool optically thick flow, where the accretion torque is mediated by highly subsonic ($\alpha \ll 1$) turbulence, is the picture most commonly invoked when describing black hole accretion at or near the Eddington limit. Of course, this makes sense. Of the order of half of the observed power for both luminous stellar and supermassive black accretion arrives in the form of relatively cool thermal photons. However, as previously stated, up to half of the radiated power is due to Comptonization.

The entirety of the accretion power is not generated within a single gravitational radius of the event horizon. Immediately outward from the innermost stable circular orbit, the flow becomes optically thick, the turbulent velocities decrease, and thus turbulent Comptonization becomes increasingly inefficient. In these relatively opaque outer regions, where another significant and perhaps dominant, fraction of the accretion power is generated, the flow releases its energy via thermal radiation. For AGNs/quasars, the characteristic photon energies are measured
in tens of eV, while for X-ray binaries, the thermal photons are of order 1 keV. These relatively soft photons presumably serve as the seeds for the turbulent Comptonization, which most effectively takes place at the inner edge of the flow. It follows that the geometry for thermal reprocessing of hard X-rays is fundamentally different for a turbulent Comptonization model than for a slab-coronal model (e.g., Svensson & Zdziarski 1994).

Turbulent Comptonization is not responsible for mediating the entirety of the accretion power onto a given black hole. However, it provides a simple explanation for spectral behavior that is unique to relativistic accretion, i.e., a sizeable Comptonized power-law component. We warn the reader that our discussion of turbulent Comptonization is only meant to reproduce qualitative features of the hard power-law X-rays produced from black hole accretion, e.g., slope and normalization.

5.1. Quasars/AGNs

Integral studies, based on the quasar/AGN luminosity function, conclude that supermassive black hole mass in the universe is accrued during short-lived epochs of radiatively efficient optically luminous accretion (Soltan 1982; Yu & Tremaine 2002). The characteristic luminosity is close to the Eddington value during the mass buildup phase, with radiative efficiencies characteristic of spinning or even rapidly spinning Kerr black holes (Yu & Tremaine 2002; Fabian & Iwasawa 1999; Elvis et al. 2002). Furthermore, a significant (≤50%) fraction of the bolometric power is radiated in hard X-rays, presumably a result of Comptonization of some form.

The X-ray spectral shape of individual accreting supermassive black holes is characterized by a power law with a flat spectral index (νFν ∼ 1a const.) that originates from Comptonizing layer of electron optical depth τ ∼ 1 and characteristic kinetic energies ∼ 100 keV (Haardt & Maraschi 1991, 1993; Madejski et al. 1995; Magdziarz & Zdziarski 1995). The spectral properties of individual sources coincide with the spectral shape and normalization of the cosmic hard X-ray background as observed by the International Gamma-Ray Astrophysics Laboratory (INTEGRAL; Churazov et al. 2007).

The broadband variability properties of AGNs are difficult to explain by the standard disk-corona slab model. Edelson et al. (1996, 2000) simultaneously monitored the optical to hard X-ray emission from NGC 3516 for 3 days. They found that the variability in thermal disk emission was not correlated with variability in the hard X-rays. The X-rays varied with large amplitude on timescales of a few hours, corresponding to a few light crossing times at the innermost stable circular orbit. Whereas the optical varied on day timescales with relatively low amplitude. Nandra et al. (1998) found similar behavior for the Seyfert Galaxy NGC 7469 when examining the contemporaneous variability properties of the UV and X-ray.

Altogether, the observational picture outlined above for the X-ray emission of supermassive black holes is roughly consistent with the turbulent Compton model described in this work. The integrated history of supermassive black hole accretion in the universe indicates that black holes accrete near their Eddington limit, with high radiative efficiency and a significant fraction of their energy release is in form of hard Comptonized X-rays. The spectral shape of well-studied individual sources as well as the (unresolved) cosmic hard X-ray background corresponds to a Comptonized power law with a flat spectral index—implying a γ-parameter ∼ 1—with a high energy cutoff near 100 keV, which is consistent with Eddington-limited Comptonizing eddies near the innermost stable circular orbit. The lack of correlation between X-ray and UV/optical variability as well as the relatively strong variability in the X-ray implies that the region of X-ray Compton power is located close to the hole—where turbulent Comptonization is most important—and thermal UV and optical disk emission is generated further away from the hole—where turbulent Comptonization is least important.

5.2. Luminous States of Black Hole X-ray Binaries

The hard X-ray phenomenology of accretion onto stellar mass black holes is complicated in comparison to that of AGNs. At high accretion rates, when the luminosity is close to the Eddington limit, the observed photon spectral energy distribution of the accretion flow resides in the so-called very high state (Esin et al. 1997). At these high luminosities, at least half of the photon power is released in the form of heavily Comptonized photons. It follows that Eddington-limited Comptonizing eddies may operate in this accretion regime near the black hole, where a significant fraction of the power is released.

In the very high state, black hole accretion flows display a unique variability signature known as high-frequency quasi-periodic oscillations (HFQPOs). The photons responsible for these oscillations are heavily Comptonized and the associated oscillation frequencies correspond to the dynamical frequency near the innermost stable circular orbit (Remillard & McClintock 2006). An interesting application of turbulent Comptonizing eddies near the inner-edge of the accretion flow is the possible excitation of internal oscillation modes, analogous to turbulent convection within the context of helioseismology. A detailed study of resonant wave excitation by Comptonizing eddies will be the subject of future work.

As formulated in this work, turbulent Comptonization generates a hard X-ray photon spectrum, which to leading order resembles a Comptonized spectrum produced by a thermal distribution of electrons. However, there is no evidence of a cutoff in photon energy in the very high state (Zdziarski & Gierlinski 2004). Equation (10) informs us that near the black hole, the electrons will be heated by the Comptonized radiation field to thermal energies of order the electron rest mass energy. Therefore, near the black hole, the electron Coulomb depth is of order the photon Thomson depth, which is of order unity for Eddington-limited eddies. In this event, the region near the black hole is an ideal site for second-order Fermi acceleration of the electrons by the turbulence itself.

6. SOME OPEN QUESTIONS AND CONCLUSIONS

At radii immediately further out from the innermost stable circular orbit, the ability for trans-sonic turbulence to Comptonize soft photons diminishes since ντ increases, while τ increases. The latter implies that the input photons can only “sample” eddies of lower energy, on smaller scales (Socrates et al. 2004). Also at larger radii, the ability of the flow to absorb a photon via free–free and bound–free processes increases (Socrates et al. 2005). We suspect that these adjacent outer regions serve as the source of—presumably thermal—soft input photons for the centrally located Comptonizing eddies. In the case of accretion onto a neutron star, thermal emission from the stellar surface is an additional source of soft seeds.

We are yet to specify the actual source of trans-sonic turbulence itself. In the familiar language of the Shakura & Sunyaev (1973) thermal disk model, we have assumed throughout that...
\( \alpha \lesssim 1 \), i.e., \( v_\lambda^2 \lesssim c_s^2 \). It is now widely accepted that the MRI (Balbus & Hawley 1991) generically leads to a turbulent accretion stress, allowing for the transfer of angular momentum to occur in highly conducting fluids. However, simulations of the MRI typically find low values for the Shakura & Sunyaev (1973) \( \alpha \)-parameter such that \( \alpha \sim 0.1\text{--}0.01 \), though a clear understanding for the value of the MRI’s \( \alpha \) is not currently in

Our main results for Eddington-limited trans-sonic turbulence near a relativistic source of gravity are as follows.

1. Turbulence is responsible for transporting angular momentum while simultaneously converting gravitational power of protons into Comptonized photon power.
2. The dynamical, accretion, Compton cooling and eddy overturn time are all comparable to one another.
3. The turbulent Compton power of each individual eddy is comparable to the accretion power of that eddy.
4. Electrons will inevitably heat up to a significant fraction of the electron virial temperature. In other words, turbulent Comptonization is an important electron heating mechanism.
5. The \( \gamma \)-parameter due to turbulent Comptonization is close to unity. Therefore, the outgoing hard X-ray spectrum is a power law with flat spectral index \( \Gamma \sim 2 \), in agreement with observations.
6. The results above are independent of black hole mass.

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