Problem solution of optimal pathfinding for the movement of vehicles over rough mountainous areas

K Zhigalov¹, D K-S Bataev², E Klochkova³, O A Svirbutovich⁴ and G A Ivashchenko⁵

¹ V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences, Moscow, Russia
² Integrated Research Institute named after Kh.I. Ibragimov, Russian Academy of Sciences of the Chechen Republic, 21A, Starpromysovsksy ave., Grozny, 364051, Russia
³ Moscow University of the Ministry of Internal Affairs of the Russian Federation n.a. V. Ya. Kikotya, Moscow, Russia
⁴ Irkutsk National Research Technical University, 83, Lermontova str., Irkutsk, 664074, Russia
⁵ Bratsk State University, 40, Makarenko str., Bratsk, 665709 Russia

E-mail: kshakalov@mail.ru

Abstract. In recent years, a large number of studies were carried out in the field of finding and constructing the optimal path of movement. However, the task remains relevant due to the emergence of new aspects in this problem. The paper discusses and describes the existing algorithms for finding the shortest path on the graph. The material can be useful in building navigation systems and automated control in terms of the choice of optimal path.

1. Introduction
Solving the problem of finding the optimal route of movement has become more and more relevant in recent years [1]. The solution to the problem of finding the optimal path of movement has become increasingly popular in recent years [2, 3]. A sufficient number of navigation software has already been developed. The analysis showed that although a sufficient number of both international and national software products are presented on the market, they all a particular disadvantage. Most of the systems are designed to build a path only taking into account the extensive asphalt road network and without regard of the presence of unpaved, country roads, as well as the technical characteristics of the vehicle itself during the pathfinding.

The initial data are the coordinates of the initial and final points of movement, while the path of movement passes through mountainous area, as well as a description of the area, which includes its altitude, steepness, description of soil and rocks. Possessing information about the environment, as well as about the existing road network, it is necessary to build a possible path of movement, taking into account the technical characteristics of vehicles. This formulation of the problem may be especially relevant in the conditions of mining operations [4-6].

The criterion for the choice of a path will be the shortest time to overcome the path from the starting point to the destination. It is necessary to note that the shortest distance does not always mean
that this is the fastest way. When building a path, it is necessary to take into account that it is safe for
the movement of vehicles apart from all its other characteristics.

2. Pathfinding
To describe the area, we will use the graph model. The road network will be represented as a oriented
graph.

\[ G = (V, E) \]

- \(V\) - it is a set of vertices or nodes that represent intersections of roads;
- \(E\) - is a set of graph edges - road sections that connect their intersections.

Each edge of the graph will have its own “value”, i.e. some costs to overcome this section of the
road. Thus, we reduced the solution of the problem to find a path on the graph.

Our analysis of the existing algorithms to find the shortest path on a graph showed that all
algorithms have disadvantages.

| Algorithm                        | Brief description                                                                 |
|----------------------------------|----------------------------------------------------------------------------------|
| Breadth First Search             | the path found as a result of the algorithm is not always the shortest; the search is carried out evenly in all directions and therefore is not effective |
| Dijkstra algorithm               | the path found as a result of the algorithm is the shortest; the search is carried out evenly in all directions, and therefore is not effective |
| Depth first search algorithm     | as a result of the algorithm, the optimal path is not always found; the speed of the algorithm largely depends on the correct choice of the stop depth |
| Best-First Algorithm             | the found solution is not optimal; facing an obstacle, the path bends around it |
| Divide-and-conquer algorithm     | the found solution is not optimal, it does not allow overcoming major obstacles |
| Crash algorithm                  | a solution is not always found, obstacles are overcome, but the search is not effective |
| Algorithm A*                     | the optimal solution is always found, overcoming the obstacle, if there is such a possibility |

As an algorithm suitable to solve the problem, the authors proposed to use Algorithm A*.

The principle of the algorithm is based on a heuristic search, as a result of which all nodes are
sorted according to the approximation of the best path going through a given node.

In general, the heuristic function can be represented as follows:

\[ f(S) = g(S) + h(S), \]

where

\[ g(S) \] — real current value of the node \(S\),

\[ h(S) \] — a heuristic function that estimates the cost of the best sequence of actions starting at a node \(S\) and ending with a solution.

At each stage of the algorithm, a set of decisions is assessed, the result of which is the creation of
the best path that will have the minimum cost. In order to solve the set task of pathfinding of a vehicle
in mountainous area, it is necessary to compile a calculation function that takes into account the area and the capabilities of the existing vehicle.

Figure 1 shows a block diagram of the search for the optimal path using Algorithm A*.
Creation of graph: \( R = (E, A) \)
which specifies: \( S_0 \) and \( f(S_0) \)
1) initial situation and its assessment
2) set, which is divided into:
   - open set: \( F \)
   - closed set: \( \emptyset \).

\[ S = S_0 \]
\[ f(S) = \min \]

Calculation of the value of the set of possible actions for the current situation \( S_0 \):
\[ n_S = |AC| \]

\[ i = 1; \]

Situation development \( S_i \):
- situation creation \( S_i = S \cup AC \)
- calculation \( f(S_i) = g(S_i) + h(S_i) \)

Add new situation \( S_i \) to set \( O \):
\[ O = O \cup S_i \]

\[ i = i + 1; \]
\[ i > n_S \]
\[ \text{no} \]
\[ \text{yes} \]

Add the expanded situation \( S \) to the set \( F \):
\[ F = F \cup S \]
Eliminate the expanded situation \( S \) from the set \( O \):
\[ O = O \setminus S \]

Finish
3. **Creation of calculation function**

Let us compose a calculation function for a typical car, taking into account the general technical characteristics. Obviously, it is not possible to give an accurate assessment of the time of movement. Therefore, as an approximate assessment we will use the Naismith rule.

Vehicle speed in mountainous areas is selected depending on the condition of the road. On straight-line sections in conditions of good visibility and good road condition, it should not exceed an average of 45 km/h. It is recommended to drive at the same speed on sharp and closed turns. During the ascent of 6-8 degrees it is recommended to drive at 10 km/h, and at no more than 25 km/h on descents.

The most logical step towards the solution of this problem will be to compose a certain function of the assessment of time to overcome various parts of the area, which will satisfy the following requirements:

1) continuity - this will allow assessing the travel time of the sections for all possible descent angles
2) a certain level of sensitivity of the function to the factor - this will allow diversifying the areas of the area with slightly different descent angles;
3) the closeness of the values of the calculated assessments to the empirical results;
4) calculation ease.

A simple solution would be to create an interpolation function with nodes at the following points:

| Road descent | The time a car covers 1 km |
|--------------|---------------------------|
| $\alpha = 0^0$ | $T=1,2 \text{ (min)}$ |
| $\alpha = 10^0$ | $T=6 \text{ (min)}$ |
| $\alpha = 20^0$ | $T=6 \text{ (min)}$ |
| $\alpha = -10^0$ | $T=2,4 \text{ (min)}$ |

We use the Lagrange interpolation formula:

$$L_n(x) = \sum_{i=0}^{n} y_i \times \frac{(x-x_0)(x-x_1)...(x-x_{i-1})(x-x_{i+1})...(x-x_n)}{(x_i-x_0)(x_i-x_1)...(x_i-x_{i-1})(x_i-x_{i+1})...(x_i-x_n)},$$

where

- $L_n(x)$ – interpolating polynomial;
- $n$ – number of interpolation nodes;
- $x_i$ – the value of the argument at the interpolation node;
- $y_i$ – function value at interpolation node.

The resulting interpolation function is a heuristic assessment that allows assessing the complexity of the path.

4. **Calculation scheme of the assessment of path time**

During the description of the area, we will use a digital area map based on the use of a raster model of spatial data.

In this case, the calculation scheme for the assessment of travel time of a given path can be presented in the following form (Figure 2):

1) to choose the calculated path;
2) to divide the path into elementary components with reference vertices in accordance with the scale of the digital map;
3) to compose a “relief profile” of the path.
4) When compiling a profile, the height at each reference vertex is determined;
5) to calculate the descent value \( \alpha_i \) for each segment:

\[
\alpha_i = \arctg \left( \frac{h_{i+1} - h_i}{d_i} \right),
\]

where

\( h_i \) and \( h_{i+1} \) - height of \( i^{th} \) and \( (i+1)^{th} \) heights respectively;

\( d_i \) – distance on the map between vertices, calculated by the formula:

\[
d_i = k_i \times t,\]

\( k_i = \begin{cases} 1, & \text{if movement is horizontal or vertical;} \\ \sqrt{2}, & \text{if movement is diagonal;} \end{cases} \)

\( t \) – scale accuracy

5) to calculate the total travel time:

\[
T = \sum_{i=1}^{n} T(\alpha_i) \cdot \frac{d_i}{1000} M.
\] (2)

The proposed calculation scheme does not take into account all the aspects related to the characteristics of the area. However, to solve the problem in the first approximation, this is quite enough, since it will be quite simple to include them in the calculation system as additional parameters in the future and the obtained results are quite close to real ones.

In order to solve the problem, it is necessary to optimize the algorithm \( A^* \), taking into account the proposed principle of the calculation of objective function.

The optimization of the algorithm \( A^* \) is in the fact that in order to create a travel path, it is necessary to set not only the values of the costs of moving along the corresponding sections of the path, but also the description of these sections for the possibility of overcoming them by vehicles with different technical characteristics. According to the available information, the algorithm will calculate the function and select the optimal path.

Our system is developed exclusively for mountainous areas, without taking into account vegetation, type of soil and the passability of mountains depending on the location of the system of ridges, spurs, valleys and gorges. During the travel in mountainous area, the main obstacles that arise in front of the vehicles are descents (more than 20°) and ascents (more than 30°), as well as the angle of lateral static stability. Possessing information about the area, the algorithm will be able to determine the passability of a specific obstacle for a given technical means.

Thus now let us determine exactly how the algorithm will determine the possibility of overcoming an obstacle using the area data and taking into account the characteristics of a particular car.

The calculation of the ascent / descent angle (influencing the choice of the path, determining whether the obstacle is overcome) depends on the map scale. The calculation of it is more accurate, when the scale is larger. It follows that when using small-scale maps, the object (car) will be represented as a point object, and in this case, during the movement, only the possibility of overcoming ascents/descents will be taken into account, and the influence of the angle of inclination (the angle of cross-passability) will be ignored. In order to solve this problem, we propose to transform the car from a point object view into a larger object, that is, to use a larger scale.
5. **Determination of the angle of lateral static stability**

Let us consider the type of obstacle that depends on the angle of lateral static stability. The angle of lateral static stability is included in the technical characteristics of a car and is defined as the angle by which a car must be bended around the longitudinal axis in order to tip it over (Fig. 3). At the same time, various lateral forces are constantly acting on any vehicle, contributing to its deviation from the rectilinear direction.

![Figure 3. Lateral static stability angle](image)

To find this angle, we can use the methods of analogy and similarity. To simplify the task, a car can be represented as a parallelepiped.

\[
\alpha = \arctg \left( \frac{h_{i+1} - h_i}{d_i} \right)
\]

\[
T = \sum_{i=1}^{\infty} T(\alpha_i) \left( \frac{d_i}{1000 \, M} \right)
\]
in this case, the coordinates of the point of intersection of the diagonals and parallelogram are known (Figure 4a).

It is at this point that the coordinates of the current position of a car will be calculated, which are necessary for next calculations.

Accordingly, when a car is bended at an angle $\beta$, the position of the KO, and, consequently, the coordinates and the value of heights at point K and point O will change (Figure 5).

In this case, the intersection of the straight lines KO and KM (KM is parallel to the line of the Earth's plane, from which the angles of ascent and descent are calculated) will create an angle equal to the angle to which the car was bended.

As a result, all the calculations are reduced to find the value of the angle $\beta$ in the triangle OKM, in which OK is calculated as ½ of the width of a car that is ½ $l$; the values of heights at the point O and K of a car are known (using a height map of large scale). Therefore, the angle $\beta$ will be calculated as:

$$\beta = \arcsin \left( \frac{h_o - h_k}{A_1D_1/2} \right)$$
It is necessary to make a note about the used scale once again. The larger the scale, the more reliable the angle $\beta$ will be calculated. Since in its calculations, we proceeded from the idea that a car is not represented as a point object, but as a parallelogram, which takes into account the characteristics of a car, one of which (namely the width of a car), we used in the calculations. Otherwise, the algorithm will not see the height difference between point O (a fixed point for pathfinding) and point K (the intersection of the perpendicular dropped from point O to the side plane of a car, the length of which is $1/2l$).

6. **Determination of the angle of ascent / descent of a car**

The same method will be used to determine the ascent / descent angle of a vehicle. Then, we present the side surface of a car as a rectangle $DD_1C_1C$ (Fig. 3c) with the point O (the point of intersection of the diagonals) and the perpendicular ON (equal to $\frac{1}{2}D_1C_1$, where $D_1C_1$ is the length of a car).

Then, all the calculations for the determination of the ascent / descent angle will be reduced to the determination of the angle $\alpha$ from the triangle NOP, in which $NP$ is the height difference, and NO is $\frac{1}{2}D_1C_1$ (Fig. 6). Therefore, the angle $\alpha$ will be calculated as follows:

$$\alpha = \arctg \frac{D_1C_1 / 2}{NP}$$

![Figure 6. Determination of the angle of ascent/descent of a car](image)

Thus, at the moment we know all the parameters and calculation methods by which the best path will be chosen by the algorithm A*.

At the same time, the support points are rigid, not elastic, due to the use of pneumatic tires, in order to exclude factors that change the stability of the machine with different conditions of the tires during operation [7, 8].

7. **Conclusion**

The problem of optimal pathfinding for the movement of vehicles over rough mountainous area is relevant. To solve this problem it is proposed to use the algorithm for finding the shortest path A*, which has proven itself during pathfinding on rough area with possible obstacles in the movement path. In order to improve the efficiency of the algorithm, the authors proposed optimization, which allows taking into account not only the distinctive features of the area, but also the technical characteristics of vehicles, on which their passability depends during pathfinding. To assess the obtained paths, a computational function is proposed, which depends on the area.

The proposed solution to the problem of pathfinding over rugged mountainous area may be more effective than the use of other algorithms.
References
[1] Mazakov E B, Matrokhina K V and Trofimets V Y 2021 Traffic management at the enterprises of the mineral industry. Advances in raw material industries for sustainable development goals (London: CRC Press) p 528
[2] Safiullin R N, Afanasyev A S and Rezchenko V V 2019 The Concept of Development of Monitoring Systems and Management of Intelligent Technical Complexes J. of Mining Institute 237 322-330. DOI: 10.31897/PMI.2019.3.322
[3] Afanasyev A S, Egoshin A M and Alekseev S V 2018 Justification of application of the logistic approach in the models of road safety management IOP Conf. Ser.: Earth Env. 194 218
[4] Klyuev R, Bosikov I, Gavrina O, Madaeva M and Sokolov A 2021 Improving the energy efficiency of technological equipment at mining enterprises Advances in Intelligent Systems and Computing 1258 262-271. DOI: 10.1007/978-3-030-57450-5_24
[5] Klyuev R, Fomenko O, Gavrina O, Turluev R and Marzoev S 2021 Energy indicators of drilling machines and excavators in mountain territories Advances in Intelligent Systems and Computing 1258 272-281. DOI: 10.1007/978-3-030-57450-5_25
[6] Kosarev O V, Tcvetkov P S, Makhovikov A B, Vodkaylo E G, Zulin V A and Bykasov D A 2019 Modeling of industrial IOT complex for underground space scanning on the base of Arduino platform Topical Issues of Rational Use of Natural Resources – Proc. Of The Int. Forum-Contest of Young Researchers pp 407-412
[7] Klyuev R V, Bosikov I I, Mayer A V and Gavrina O A 2020 Comprehensive analysis of the effective technologies application to increase sustainable development of the natural-technical system Sustainable Development of Mountain Territories 2 283-290. DOI: 10.21177/1998-4502-2020-12-2-283-290
[8] Abdulaev E K, Makharatkin P N and Pumpur E V 2019 A priori ranking and an analysis of factors affecting tire wear IOP Conf. Ser.: Earth Env. 378(1) 012001. DOI: 10.1088/1755-1315/378/1/012001