Non-Perturbative effects from orbifold constructions

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Abstract

We indicate how consistent heterotic orbifold compactifications, including non perturbative information, can be constructed. We first analyse the situation in six dimensions, $N = 1$, where strong coupling effects, implying the presence of five branes, are better known. We show that anomaly free models can be obtained even the usual modular invariance constraints are not satisfied. The perturbative massless sector can be computed explicitly from the perturbative mass formula subject to an extra shift in the vacuum energy. Explicit examples in $D = 4$, $N = 1$ are presented. Generically, examples exhibit non perturbative transitions leading to gauge enhancement and/or where the number of chiral generations changes.

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1 Introduction

The main aim of this talk, based on work presented in Ref.[1], is to indicate how consistent models, including non perturbative effects, can be constructed in the framework of heterotic string orbifolds. Interestingly enough, some of these models exhibit features like gauge symmetry enhancing or transitions changing the number of fermionic generations. Such kinds of phenomena are not possible in perturbation theory and imply a drastic change in our way of approaching string inspired phenomenology.

Due to their simplicity, perturbative heterotic orbifolds [2, 3] have proven to be a very powerful tool in building ”semirealistic stringy inspired”, effective low energy models (Standard model like, higher level StrinGuts etc.). Not only the gauge group and matter multiplets may be easily obtained (and somewhat controlled) but also the structure of Yukawa couplings, symmetry breaking patterns can be easily studied etc. The full partition function may be constructed in these cases.

We will not be able to go that far in the non perturbative case and we will concentrate on the structure of the massless spectrum.

Our first scope will be to incorporate non perturbative contributions in six dimensional orbifold compactifications of the heterotic, both $E_8 \times E_8$ and $SO(32)$, string theories. If non perturbative phenomena are to be included, this seems a good road before descending to the more involved and less known four dimensional case. In fact, we will show how relevant information in four dimensions may be obtained from six.

$D = 6$ is undoubtedly interesting in itself. $N = 1$ theories are chiral and consistency, that is, anomaly cancellation, constraints the allowed theories severely. Moreover, even if still incomplete, many non perturbative effects are quite well understood in six dimensions. In particular, examples have been derived from different approaches as Type IIB orientifolds F-theory and M-theory.

In an orbifold compactification $Z_M$ symmetry is divided out. Acting on the (complex) bosonic transverse coordinates, the $Z_M$ twist generator $\theta$ has eigenvalues $e^{2\pi i v_a}$. In $D = 6$ $v_a$ are the components of $v = (0, 0, \frac{1}{M}, -\frac{1}{M})$ and $M$ can take the values $M = 2, 3, 4, 6$. The embedding of $\theta$ on the gauge degrees of freedom is usually performed by a shift $V$, such that $MV$ belongs to the $E_8 \times E_8$ lattice $\Gamma_8 \times \Gamma_8$ or to the $Spin(32)/Z_2$ lattice $\Gamma_{16}$.

In perturbative string theory, this shift is restricted by the modular invariance constraint

$$M (V^2 - v^2) = \text{even} \quad (1.1)$$

The spectrum for each model is subdivided in sectors. There are $M$ sectors twisted
by $\theta^j, j = 0, 1, \cdots, M - 1$. Each particle state is created by a product of left and right vertex operators $L \otimes R$. At a generic point in the four-torus moduli space, the massless states follow from

$$
m_R^2 = N_R + \frac{1}{2}(r + j v)^2 + E_j - \frac{1}{2} ;
$$

$$
m_L^2 = N_L + \frac{1}{2}(P + j V)^2 + E_j - 1
$$

(1.2)

Here $r$ is an $SO(8)$ weight with $\sum_{i=1}^{4} r_i = \text{odd}$ and $P$ is a gauge lattice vector with $\sum_{I=1}^{16} P^I = \text{even}$. $E_j$ is the twisted oscillator contribution to the zero point energy and it is given by $E_j = j(M - j)/M^2$. The multiplicity of states satisfying Eq. (1.2) in a $\theta^j$ sector is given by the appropriate generalized GSO projections $[4, 5]$. The gravity multiplet, a tensor multiplet, charged hypermultiplets and 2 neutral hypermultiplets (4 in the case of $Z_2$) appear in the untwisted sector. Twisted sectors contain only charged hypermultiplets. The generalized GSO projections are particularly simple in the $Z_2$ and $Z_3$ case since all massless states survive with the same multiplicity.

Let us come back to equation (1.1). This constraint ensures level matching. It corresponds to an orbifold version of the global consistency of the theory ensuring anomaly cancellation. This consistency may be understood as the vanishing of the total magnetic charge associated to the antisymmetric tensor field. Namely

$$
\int_X dH = \int_X (F^2 - R^2) = 0
$$

where $H$ is the three-form heterotic field strength with $dH = tr F^2 - tr R^2$ and $X$ is the compact space. For an orbifold $X = T^4/Z_M$ and, since curvature is localized at $Z_M$ the fixed points, we can restate this equation as

$$
Q_{\text{TOT}} = \sum_f Q_f = 0
$$

(1.3)

where integrals are taken around fixed points $f$. Equivalently, since the total Euler number of $X$ is $\int_X R^2 = 24$ we have

$$
I_{\text{TOT}} = \sum_f I_f = 24
$$

(1.4)

where $I_f$ is the instanton number at the fixed point.

The issue we want to stress here is that modular invariance requirement as stated in (1.1) and anomaly cancellation are equivalent. They are satisfied if there are 24 instantons at fixed points or equivalently if the magnetic charges at fixed points add up to zero. More explicitly, in Ref. [1] it is shown (for $SO(32)$ case) that $I_f = l + M E_{\Theta}$

Here $E_{\Theta} = \sum_{I=1}^{16} \frac{1}{2} V_I (V_I - 1)$, $E_{\theta} = \frac{(M - 1)^2}{M}$, $l$ is an integer and $V_I$ are the components of the shift $V$. Also, by computing the curvature at the fixed orbifold point it is found that

$$
Q_f = l' + M(E_{\Theta} - E_{\theta})
$$

(1.5)
with \( l' = l - M - 1 \). Thus, for \( Q_f = 0 \), Eq. (1.1) is obtained.

\( Z_M \) orbifolds corresponding to all possible embeddings allowed by equation (1.1) can be constructed. Indeed, their corresponding massless spectra may be reproduced by application of Index theorems on orbifold (ALE) singularities \([1]\) with \( I_{TOT} = 24 \) instantons.

The question to address now is: Could we still have a consistent theory if \( I_{TOT} < 24 \) is allowed, i.e. when \( n_B = 24 - I_{TOT} \) instantons become small? Since the dilaton is known to diverge \([3]\) in such a situation, non perturbative information is required to answer this question. In fact, small instantons in both \( SO(32) \) or \( E_8 \times E_8 \) have been studied \([4, 5]\) and may be identified as five branes, i.e., extended objects with their world volume filling six dimensional space time. They correspond to type I \( D5 − \) branes in the \( SO(32) \) case and to \( M−theory \) five branes for \( E_8 \times E_8 \). Five branes act as magnetic sources for the antisymmetric tensor field and therefore Eq. (1.3) must now read

\[
Q_{TOT} = \sum_f Q_f + n_B = 0 \quad (1.6)
\]

We see that, from our discussion about Eqns. (1.6, 1.4), when five branes are present the “modular invariance” constraint on the shift \( V \) must be abandoned. This is certainly troublesome, in perturbation theory, since only shifts complying with this constraint ensure anomaly cancellation. Other \( V \)’s would lead to anomalous spectra. On the other hand this should not be surprising when dealing with strong coupling effects, since modular invariance is a perturbative concept (associated to the expansion in terms Riemman surfaces spanned in string propagation).

Generically (we will be more precise about this), these five branes are expected to carry vector, hyper and tensor massless (six dimensional) multiplets on their world volumes and therefore are expected to contribute to the total, gauge and gravitational anomaly, of the spectrum.

All these elements suggest a possible positive answer to the above question. Perturbative contributions associated to ”fat” \( I \) instantons and to \( n_B \) five branes, with \( I + n_B = 24 \), would contribute to the massless spectrum such that the whole anomaly could cancel. In fact, we will see that this appears to be the case for the situations where non perturbative information at hand.

Let us first discuss the perturbative contribution to the massless spectrum. This spectrum corresponds to a number \( I_{TOT} < 24 \) of large instantons. As indicated, the instanton number is a function of the shift \( V \) in the gauge lattice. This \( V \) have to comply with a new constraint depending on the number of five branes since \( \sum I_f(V) + n_B = 24 \). For instance, assume that we have the same charge at each fixed point (this is expected
for $Z_3$ orbifold where all points are equivalent). Equation (1.6) tells us that $Q_f = -\frac{n_f}{3}$ where $n_f$ is the number of fixed points. Following the steps that lead us to (1.5) we now obtain

$$M(V^2 - v^2) + 2M E_B(f) = \text{even}$$

(1.7)

where we have defined, for further convenience, $E_B(f) = -M \frac{n_f}{2n_f}$. This offers us the result we expected. Moreover, recalling that (1.1) results by imposing level matching, our result suggests that masses of states could be obtained as in ordinary perturbative orbifolds by just modifying the mass of the left sector states to be

$$m_L^2 = N_L + \frac{1}{2}(P + j V)^2 + E_j + + E_B(j) - 1.$$  

(1.8)

In fact, $m_L^2 = m_R^2$ leads to (1.7) with $f$ a fixed point in twisted sector (j). We will propose this expression for computing the massless states in the perturbative sector of general orbifold models containing five branes. $E_B$ is interpreted as shift in the vacuum energy due to the flux of the antisymmetric field. Since in general there will be non-equivalent fixed points we do not expect in general a simple relationship as above between this energy shift and $n_B$. The untwisted sector is obtained by projecting onto invariant states as usual.

In order to illustrate how this proposal works, let us consider the case of smooth $Z_3$ compactifications. This is the simplest case. There is just one $\theta$ twisted sector with an energy shift to be considered and nine equivalent fixed points. Smooth compactification means that oscillator modes, needed to blow-up orbifolds singularities should be present, thus $N_L = 1/3$ in (1.8). For these modes (two at each fixed point) to be massless it is required that

$$V^2 = \frac{8}{9} - 2E_B$$

(1.9)

Thus the maximum shift in the vacuum energy will correspond to $E_B = \frac{4}{9}$ (obtained for $V = 0$). The other extreme case is $V^2 = \frac{8}{9}$, in which we have $E_B = 0$ corresponding to some modular invariant (perturbative) models.

Let us consider first the $SO(32)$ heterotic string with the class of shifts $V$ with $3V \in \Gamma_{16}$ of the form

$$V = \frac{1}{3}(1, \cdots, 1, 0, \cdots, 0)$$

(1.10)

and $m \leq 8$. The unbroken group is $U(m) \times SO(32 - 2m)$ and the untwisted sector contains hypermultiplets transforming as $(m, 32 - 2m) + (m(m - 1)/2, 1) + 2(1, 1)$. The twisted sectors need an extra vacuum shift $E_B = \frac{(8 - m)}{18}$ and the mass formula provides massless hypermultiplets in each twisted sector transforming as

$$(\frac{m(m - 1)}{2}, 1) + 2(1, 1)$$

(1.11)
for $m = 0, 2, 4, 6, 8$.

There are two other $Z_3$ models with singlet moduli in the twisted sector. One of them, with shift $V = \left( \frac{2}{3}, 0, \cdots, 0 \right)$, has gauge group $SO(30) \times U(1)$. The other model has shift $V = \frac{1}{6}(1, \cdots, 1)$, $3V$ being a spinorial weight. The gauge group is $U(16)$. It is thus a $SO(32)$ embedding without vector structure, a $Z_3$ analogue to the $Z_2$ orientifolds constructed in \cite{[9, 10]}. Except for the $m = 8$ case, remaining models, as they stand, have gauge and gravitational anomalies and the corresponding shifts do not fulfill the perturbative modular invariance constraints. However, it turns out that the addition of an appropriate number of five-branes renders them consistent. Indeed, one can check that adding $3(8-m)$ five-branes to the vacua in Eq. (1.10) (12 five-branes in the other two cases) leads to anomaly-free results. The case of five-branes or small $SO(32)$ instantons was considered in \cite{[7]}. When $n_B$ branes coincide at the same point (and away from singularities) a non-perturbative gauge group $Sp(n_B)$ is expected to appear, along with hypermultiplets transforming in the fundamental, antisymmetric and singlet representations. We also assign these hypermultiplets into representations of the perturbative group. Thus, the massless matter content, transforming under the full $U(m) \times SO(32-2m) \times Sp(n_B)$ group is

$$
\frac{1}{2}(m, 1, 2n_B) + \frac{1}{2}(m, 1, 2n_B) + \frac{1}{2}(1, 32 - 2m, 2n_B) + (1, 1, \frac{2n_B(2n_B - 1)}{2} - 1) + (1, 1, 1)] \\
(1.12)
$$

It is straightforward to check that all non-Abelian gauge and gravitational anomalies do cancel. Thus, our construction provides a new class of consistent non-perturbative orbifold heterotic vacua.

Notice that the models obtained require the addition of $6s$, $s = 4, 3, 2, 1, 0$, five-branes. They contribute one unit of magnetic charge each. Thus, in order to achieve overall vanishing magnetic charge, each of the fixed points (which in these particular models are identical) must carry magnetic charge $q_f = -\frac{n_B}{9}$.

The $E_8 \times E_8$ case is to some extent similar but has some peculiarities. Consider the class of models generated by gauge shifts of the form

$$
V = \frac{1}{3}(1, \cdots, 1, 0, \cdots, 0) \times \frac{1}{3}(1, \cdots, 1, 0, \cdots, 0) \\
(1.13)
$$

with an even number $m_1$ ($m_2$) of $\frac{1}{3}$ entries in the first (second) $E_8$ and w $m = m_1 + m_2 \leq 8$. Models with appropriate oscillator moduli in the twisted sector have $(m_1, m_2) = (0, 0), (2, 0), (4, 0), (2, 2), (2, 4)$ and $(4, 4)$. Again, none of these models (except for $(m_1, m_2) = (4, 4)$) fulfill the perturbative modular invariance constraints and
are, therefore, anomalous. However, unlike the \( SO(32) \) case, they do not present non-Abelian gauge anomalies. We can check that they miss an equivalent of \( 3(8 - m) \times 30 \) hypermultiplets in order to cancel gravitational anomalies. But this is precisely the contribution corresponding to \( 3(8 - m) \) M-theory five branes, each one carrying a tensor multiplet and a gauge singlet hypermultiplet. Therefore, these missing modes match the non-perturbative spectrum corresponding to setting this same number of instantons to zero size in \( E_8 \times E_8 \). This is a nice check of our procedure. Simple addition of a shift in the vacuum energy automatically takes into account the difference between the \( SO(32) \) and \( E_8 \times E_8 \) heterotic strings, yielding no gauge anomalies in the second case. The \( Z_3 \) models under consideration are orbifold realizations of the \( E_8 \times E_8 \) vacua in the presence of wandering branes considered in refs.\[8, 11, 12\].

An interesting question is whether there is any shift \( V \) in \( E_8 \times E_8 \) (or \( Spin(32)/Z_2 \)) which admits both spectra, with and without five-branes. Such a situation, could indicate possible transitions between perturbative and non-perturbative vacua which proceed through the emission of five-branes to the bulk. Indeed, there is a unique case corresponding to the ‘standard embedding’, \( V = \frac{1}{3}(1, 1, 0, \ldots, 0) \times (0, \ldots, 0) \) (\( V = \frac{1}{3}(1, 1, 0, \ldots, 0) \) for \( Spin(32)/Z_2 \)) in which there are both a model without five-branes and a model with 18 five-branes. Both models have identical untwisted perturbative spectrum but differ in that the twisted spectrum of the perturbative model has extra hypermultiplets with respect to the non perturbative one.

In the \( E_8 \times E_8 \) case they transform as \( (56, 1) + 7(1, 1) \) while they organize as \( (2, 28) + 4(1, 1) \) under \( SO(28) \times U(2) \) for \( Spin(32)/Z_2 \). The corresponding non-perturbative model contains just three singlets per fixed point in both cases. In the non-perturbative model the fixed points have magnetic charge \( Q_f = -2 \). This suggests that there can be transitions by which, around a fixed point in the perturbative model, these hypermultiplets go into 2 five-branes producing the non perturbative model. The magnetic charge is conserved during the process since each fixed point has charge \( Q_f = -2 \) and each of the five-branes has charge +1.

In the \( Spin(32)/Z_2 \) case these transitions can be interpreted as an unhiggsing process where the rank is increased by two units, namely \( (2, 28) + 4(1, 1) \rightarrow Sp(2) + \text{matter} \). If this transition occurs at each of the 9 fixed points and all the branes are at the same (non singular point) an \( Sp(18) \) maximum enhanced group is obtained, with the matter content specify in \( (1.12) \). A similar enhancing is expected to occur in \( D = 4 \).

\( E_8 \times E_8 \) case is different. In the transition \( (56, 1) + 4(1, 1) \rightarrow 2(1 + \text{tensor}) \) there is no enhancing at all and a complete charged hypermultiplet disappears into the bulk. In terms of \( M - \text{theory} \) branes this corresponds to an \( E_8 \) instanton, living on one of
the “end of the world” nine branes, becoming pointlike and going into the bulk as a five M-brane. Interestingly enough, if an equivalent transition was possible in $D = 4$, for $N = 1$ it would imply a change in the number of generations. A chiral 27 generation (or 27) of $E_6$, contained in the 56 of $E_7$ would disappear from the spectrum. We will show an explicit realization below.

Transitions can happen at each fixed point independently so that there should exist similar models with any even number of five-branes in between 2 and 18. Thus, in this standard embedding models there is a discrete degree of freedom which corresponds to having pairs of zero size instantons.

Here we have concentrated in an certain class of $Z_3$ models with enough blowing up modes to resolve the singular points completely. A more general situation can be envisaged for cases where these modes are lacking ( $V^2 > \frac{8}{9}$ above) and for other $Z_M$ orbifolds. This is extensively discussed in [1]. Let us just recall that generically non-smooth models have five branes trapped at these non removable singularities. The dynamics associated to these stuck branes is different from that of the smooth case.

The behaviour of such five-branes for the $SO(32)$ heterotic string is better known. It can be extracted from type I D-five-branes on ALE spaces and F-theory analysis. [14, 15, 16, 17, 18]. For a bigger enough number $l_c$ of branes sitting at a singularity, further enhancements to unitary groups are expected. For instance, at a $Z_3$ orbifold point $S_p(l) \times U(2l + m)$ is obtained ($l_c = \frac{8-m}{2}$). Tensor multiplets associated to the missing blowing up modes do appear, somehow paralleling the $E_8 \times E_8$ case with wandering branes. Moreover, transitions where some hypermultiplets go into tensors are also suggested. For instance, when $m = 0$ and $l = 0$ in the $Z_3$ there is no enhancing at all and it is found that 28 + 1 $\rightarrow$ tensor, where 28 is a hypermultiplet transforming under a perturbative $U(8)(\times SO(16))$ group. This parallels the above $E_8 \times E_8$ example.

The idea explored in the $D = 6$ case could be extended to $D = 4$, $N = 1$. One would construct heterotic orbifold vacua with perturbative and non-perturbative sectors in which the perturbative (but non-modular invariant) sector could be understood in terms of simple standard orbifold techniques. We should also add a non-perturbative piece, but we face the problem that non-perturbative phenomena in $N = 1$, $D = 4$ theories are poorly understood at the moment. However, we can concentrate [1] on certain restricted classes of $D = 4$ orbifolds in which much of the structure is expected to be inherited from $D = 6$. In particular, one can consider $Z_N \times Z_M$ orbifolds in $D = 4$ with unbroken $N = 1$ supersymmetry. Such type of orbifolds have two general classes of twisted sectors, those that leave a 2-torus fixed and those that only leave fixed points. The first type of twisted sectors is essentially 6-dimensional in nature,
the twist by itself would lead to an $N = 2, D = 4$ theory which would correspond to
$N = 1, D = 6$ upon decompactification of the fixed torus. For this type of twisted
sectors we can use our knowledge of non-perturbative $D = 6, N = 1$ dynamics. Twisted
sectors of the second type are purely 4-dimensional in nature and we would need extra
information about 4-dimensional non-perturbative dynamics. To circumvent this lack
of knowledge, one can restrict to a particular class of $Z_N \times Z_M$ orbifolds with gauge
embeddings such that these purely 4-dimensional twisted sectors are either absent or
else are not expected to modify the structure of the model substantially .

Let us present a specific example [1] based on $E_8 \times E_8$. Consider the $Z_3 \times Z_3$ orbifold
on $E_8 \times E_8$ with gauge shifts

$$
A = \frac{1}{3}(1, 1, 0, \cdots, 0) \times (0, \cdots, 0)
$$
$$
B = \frac{1}{3}(0, 1, 1, 0, \cdots, 0) \times (0, \cdots, 0)
$$

(1.14)

This leads to a perfectly modular invariant orbifold with gauge group $E_6 \times U(1)^2 \times E_8$.
However, we are going to consider the particular version of this orbifold with discrete
torsion first considered in Ref.[19]. This model has the special property that all particles
in the $(A + B)$ twisted sector, are projected out. In this way we get rid of the sector
which is purely 4-dimensional. The model has now three $27'$s in the untwisted sector
and nine $27'$s in each of the sectors $A, B$ and $A - B$. Hence, altogether the model has
twenty four net antigenerations. We can now consider a non-perturbative orbifold in
which the $D = 6$ subsectors $A, B$ and $A - B$ have a left-handed vacuum energy shifted
by $\frac{1}{3}$. This corresponds to a non-perturbative $D = 6$ vacuum with just singlets in the
twisted sectors and eighteen five-branes (leading to tensor multiplets) in each of the
three twisted sectors.

Therefore, a transition from perturbative to the non perturbative one implies

$$
3(27) + 27(\overline{27}) \rightarrow 3(27)
$$

(1.15)

The twenty seven antigenerations of the twisted sectors disappear into the bulk
and we are only left with three $E_6$ generations coming from the untwisted sector, plus
singlets. The $U(1)$'s will now be anomalous but there will be extra chiral singlets,
coming from the tensors, with non-universal couplings to the gauge fields which will
lead to a generalized version of the GS mechanism in $D = 4$.

Other examples undergoing chirality changes may be considered. A similar situation
is found in $Spin(32)/Z_2$ when there are branes stuck at a fixed point ([1]). A similar
$Z_3 \times Z_3$ orbifold projection applied to the $U(8) \times SO(16)$ model mentioned above leads
for instance to an $SU(6) \times SU(2)$ non abelian gauge group where a transition

$$(15, 1) + (6, 2) \rightarrow \text{singlets}$$

occurs. Notice that, an \textit{anomaly free} representation disappears from the spectrum.

2 Comments and outlook

Since duality connects heterotic models with models derived from other formulations, heterotic string theory has lost the privilege of being the preferred theory for establishing links with phenomenological world. In fact, most of the models we have constructed have candidate duals (this is a further check of our proposal) obtained from $F-theory$, $M-theory$ and type I string formulations\[1\]. Nevertheless, it seems to us that, the heterotic orbifolds constructions we are proposing are particularly attractive for their simplicity, even if other construction, like $F-theory$, could be more powerful. This construction can be summarized in three steps: i. Find possible shifts (in general automorphisms) on the gauge lattice, both satisfying the constraint (1.1) or not . ii. Add an energy shift $E_B$ to ensure level matching. Then, compute the perturbative spectrum by using familiar orbifold techniques. iii. Add non perturbative physics information. In $D = 6$ this corresponds to branes wandering in the bulk or trapped at fixed points, their world volume fills space time . Even in $D = 6$, this last step is only partially known. As we stressed, only for a large enough number of branes $l \geq l_c$ on a fixed point and for $Spin(32)/\mathbb{Z}_2$ lattice, small instanton information is available. This information is not yet available for $E_8 \times E_8$. Furthermore, except for some cases, non-perturbative spectrum in not known in either of both lattices, when the number of small instantons on the singularity is smaller than the critical value. The situation for $D = 4$, $N = 1$ vacua is even more uncertain. Some insight can be obtained from recent type IIB orientifold constructions\[20, 21\]. Even though, we have seen that relevant non perturbative information can be derived, in certain cases, from $D = 6$ physics. The examples exhibiting chirality changing transitions are particularly interesting. They correspond to $D = 6$ transitions in which one tensor multiplet transmutes into twenty nine charged hypermultiplets. These transitions where also studied in Ref.[23] in another context. These examples show that the number of chiral generations is not invariant under non-perturbative effects. Something inconceivable in perturbative field theory and also in perturbative string theory where the net number of generations is a topological number associated to a given compactified internal manifold. Vacua with different number of generations can be connected.
Even if the processes involve strong coupling dynamics, quite presumably part or all the connected four dimensional models can be realized perturbatively in some region of moduli space thus, effectively reducing the excessively huge vacuum degeneracy. In the explicit examples we have sketched above, these transitions may occur at each fixed point independently. If these are achieved at all nine $Z_3$ fixed points we end up with three generations and this number, associated to the untwisted sector of the orbifold, cannot be reduced further. In perturbative string theory some effort has been dedicated to find appropriate compactifications leading to a small three, may be four (non vanishing) number of generations, hoping that non perturbative physics would privilege these realizations over infinitely many others. These transitions, indicate that, at least in some cases, the strongly coupled dynamics leading to models with few generations is available. Of course, other new phenomenological questions should be taken up now. For instance, since in other compactifications two or zero net generations are obtainable, which would be the preferred number? Other new, non perturbative, fact is that the gauge group may be significantly enhanced. This enhancement may be amazingly huge, and discouraging for predictivity, as it was found in some very singular F-theory compactifications[24]. The situation is much more bounded in the models we have discussed above. In particular non perturbative enhanced groups which contain factors like $SU(n) \times SU(n)$ with matter in $(n, \overline{n})$ are frequently found and they appear as specially apt for obtaining Grand Unified like models with adjoint representations (needed for GUT symmetry breaking). These could be achieved by giving adequate vev’s to one of the above representations to obtain the diagonal $SU(n)$ group. Interestingly enough in [21] we find that such kind of breaking can be achieved in a type IIB dual orientifold construction through a consistent inclusion of continuous Wilson lines. For instance, a $U(4)^3(\times U(4))$ gets broken to a diagonal $U(4)_{\text{diag}}(\times U(4))$, in a $Z_3$ orientifold example, through this mechanism (see also [22]). Let us remark that, unlike string GUTs constructions considered before (see for instance [25] and references therein), the unified group here would be non-perturbative, from the heterotic point of view. Also notice that it is not clear how, non perturbative, fermionic generations would arise. In particular, spinorial representations of $SO(2n)$ (and so for instance a generation $16$ of $SO(10)$ containing a $5 + \overline{10}$ of $SU(5)$ ) do not appear in the spectra associated to the presence of pointlike instantons, up to the present partial knowledge of the subject. Again, new phenomenological questions must be faced, not envisaged from perturbative heterotic string formulations. We conclude by recalling that extensions of our symmetric orbifold construction might be considered to asymmetric (non modular invariant) orbifolds or to arbitrary conformal field theories.
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