Evolution of gas disc–embedded intermediate mass ratio inspirals in the LISA band

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ABSTRACT
Among the potential milliHz gravitational wave (GW) sources for the upcoming space-based interferometer LISA are extreme- or intermediate-mass ratio inspirals (EMRI/IMRIs). These events involve the coalescence of supermassive black holes in the mass range $10^5 M_\odot \lesssim M \lesssim 10^7 M_\odot$ with companion BHs of much lower masses. A subset of E/IMRIs are expected to occur in the accretion discs of active galactic nuclei (AGN). Previous work has shown that torques exerted by the disc can interfere with the inspiral and cause a phase shift in the GW waveform detectable by LISA. Here we use a suite of two-dimensional hydrodynamical simulations with the moving-mesh code DISCO to present a systematic study of disc torques. We measure torques on an inspiraling BH and compute the corresponding waveform deviations as a function of the binary mass ratio $q \equiv M_2/M_1$, the disc viscosity ($\alpha$), and gas temperature (or equivalently Mach number; $M$). We find that the absolute value of the gas torques is within an order of magnitude of previously determined planetary migration torques, but their precise value and sign depends non-trivially on the combination of these parameters, the GW-driven inspiral rate, and the accretion rate onto the satellite BH. The gas imprint is generally detectable by LISA for binaries embedded in AGN discs with surface densities above $\Sigma_0 \geq 10^{4-6} \text{ gcm}^{-2}$, depending on $q$, $\alpha$ and $M$. We find that the deviations are most pronounced in discs with higher viscosities, and for E/IMRIs detected at frequencies where LISA is most sensitive. Torques in colder discs exhibit a noticeable dependence on the GW-driven inspiral rate as well as strong fluctuations at late stages of the inspiral. Our results further suggest that LISA may be able to place constraints on AGN disc parameters and the physics of disc-satellite interaction.

Key words: black hole physics, gravitational waves, hydrodynamics

1 INTRODUCTION
In the 2030s we expect to detect binary mergers involving massive black holes (MBHs; in the mass range $M_{BH} \sim 10^4-10^7 M_\odot$) across the universe with the space-based gravitational wave (GW) detector LISA (Amaro-Seoane et al. 2017). LISA will also detect extreme and intermediate mass ratio mergers (termed EMRIs, $q \equiv M_2/M_1 \lesssim 10^{-4}$; or IMRIs, $q \approx 10^{-3}-10^{-4}$) up to a redshift $z \sim 4$.

Many near-equal mass MBH mergers in the LISA band may occur in gaseous environments, given that the galactic mergers that lead to the eventual formation of a MBH binary often carry a fresh supply of gas into the post-merger galactic nucleus (see Mayer 2013 for a review). Likewise, “gas-embedded” E/IMRIs may frequently occur in the accretion discs expected in active galactic nuclei (AGN) in which an MBH is surrounded by a thin, dense accretion disc. Compact objects in the nucleus can eventually align their orbits with the disc, provided a sufficient number of disc-crossing intersections (e.g. McKernan et al. 2012), or such events may arise from in-situ star formation in the disc that leaves compact remnants (Goodman & Tan 2004; Levin 2007; McKernan et al. 2014). Embedded stars and BHs can subsequently accrete, migrate, and merge with each other (Bellovary et al. 2016; Tagawa et al. 2020), before eventually merging with the central MBH. While the event rate for E/IMRIs has previously been estimated in dry nuclei (e.g. Barausse et al. 2015), the rate may be considerably higher when including...
formation pathways in accretion discs. The recent discovery of stellar-mass BH mergers by the ground based interferometer LIGO has stimulated work on such mergers that may be occurring in AGN discs, showing that they might contribute to LIGO events (Bartos et al. 2017; McKernan et al. 2018; Stone et al. 2017). Subsequent work has suggested that some of the LIGO events, based on their high chirp masses, may indeed have formed via this channel - large masses are expected both because of the preferential capture of heavier BHs by the disc (Yang et al. 2019b) and because repeated mergers are common and lead to hierarchical build-up (Yang et al. 2019a). The high effective spin provides additional support for this channel (Gayathri et al. 2020). These results suggest that BH mergers indeed occur in gas discs, and that they could constitute the building blocks of MBHBs (McKernan et al. 2012; Tagawa et al. 2020). Recent semi-analytical estimates of in-situ star formation suggest that the accretion of embedded compact objects may also contribute substantially to the growth of MBHBs (Dittmann & Miller 2020).

In general, whenever gas is present around a coalescing MBH binary, the torques exerted by the gas can influence the inspiral rate. For a near-equal mass binary in the late inspiral stage, these torques are noticeable only at extreme gas densities, comparable to those expected if the binary is embedded in a common envelope phase (Antoni et al. 2019; Chen & Shen 2019). For extreme- and intermediate- mass ratio systems, the GW torques are weaker and gas torques on the lower-mass companion are stronger. As a result, for a long-lived E/IMRI detected and monitored by several years with the low-frequency GW detector LISA, gas torques can become more comparable to (albeit still well below) GW torques, and impart a detectable imprint on the GW waveform (Yunes et al. 2011; Kocsis et al. 2011; Derdzinski et al. 2019). This presents a novel and unique opportunity for LISA to detect environmental influence in a MBH waveform and probe accretion disc physics.

A longstanding hurdle in numerically evaluating this effect is that gas torques are subtle, remain poorly understood, and show a complex dependence on system parameters (see § 2 below). The response of a disc to an embedded satellite can be highly nonlinear (Baruteau & Masset 2013), making simulations necessary to quantify it, and the resulting torque can be remarkably sensitive to small changes in the satellite mass or disc parameters (Duffell 2015). Solving for the disc torques requires especially careful considerations of numerical resolution, boundary effects and transients (i.e. achieving steady-state behaviour), as well as the accretion of both mass and momentum by the migrating BH (Tang et al. 2017; Muñoz et al. 2019; Moody et al. 2019; Muñoz et al. 2020). For a large range of mass ratios, simulations show that disc torques may either help or hinder the binary merger (Duffell et al. 2019).

In a previous paper (Derdzinski et al. 2019; hereafter Paper I), we simulated the gas response to an embedded IMRI (with mass ratio $q = 10^{-3}$) in order to measure its detectability in the GW signal. We found that the disc torques slow down, rather than speed up the inspiral, due to a critical contribution to the torque that comes from the asymmetry in the gas morphology near the migrator (near or within the BH’s Hill sphere). We estimated that the resulting deviation in the GW waveform, which comprises of a slow drift in the accumulated GW phase, is detectable if the IMRI resides in a disc with surface densities above $\Sigma_0 \gtrsim 10^{3-4}$ g cm$^{-2}$.

In Paper I, we considered only a single set of binary system and disc parameters. In order to extract meaningful information from environmental imprints on a GW waveform, we must understand the range of possible effects. In particular, we need to know how the torques and the corresponding waveform deviations depend on system parameters (mass ratio, eccentricity, inclination, spins) and the source environment (in our case, properties of the disc such as density, temperature or viscosity). In the present paper, we further explore the scaling of the torque with a subset of these parameters in the regime where GW-emission is dominant.

This work is motivated by the idea that, provided we understand how gas torques impact a GW inspiral, GWs can be used as a tool for providing measurements of AGN disc properties, and to improve our understanding of the physics of migration. To this end, we study disc torques as a function of the GW inspiral rate in order to isolate the effect of the inspiral, and to assess how the disc torques may evolve differently for different combinations of parameters. We restrict this study to include only three parameters - namely the binary mass ratio ($q$) and two of the disc parameters: temperature (or equivalently Mach number; $M$) and viscosity (parameterised by the Shakura-Sunyaev parameter $\alpha$). As in Paper I, we measure the torques directly in hydrodynamical simulations over a limited range of binary separations, and then extrapolate these measurements to cover the final coalescence of a physical IMRI, covering several years of LISA observations. Scaling our simulations to physical parameters, we calculate the detectability of the gas imprint on the GW signal across our set of simulated mass ratios, and place constraints on the minimum AGN disc density required to detect the gas-induced deviations on the GW waveform. We show that the detectability of these deviations depends on the mass ratio and also the stage at which the inspiral is observed; we also find that the time-evolution of the gas torques in the LISA window depends non-trivially on $q$, $M$, and $\alpha$.

This paper is organised as follows. In §2, we begin by summarising prior work. In §3, we describe our numerical approach, including the hydrodynamical simulations and the range of simulated parameters. In §4, we present our results, measuring and analysing the gas torques and their dependence on each parameter. In §5, we apply our results to LISA binaries, calculating the detectability of the gas-induced deviations in the waveform. Finally, we discuss our results in §6, and summarize our conclusions in §7.

## 2 PREVIOUS WORK

Here we summarize (i) applicable work on planetary torques for intermediate mass ratio systems, (ii) analytical work on disc torques on GW inspirals, and (iii) our previous paper that combines the two with simulations.

To understand the evolution of a low-mass satellite embedded in a gas disc, the large majority of the work to date has been done in the context of planetary migration in protoplanetary discs. The embedded satellite perturbs the disc non-axisymmetrically, and these perturbations back-react on the satellite’s orbit. The perturbations include spi-
ral density waves and gaps where streams of gas can flow on horseshoe orbits around the satellite (see e.g. Baruteau & Masset 2013 for a comprehensive review). Historically, two distinct regimes have been identified and known as Type I and Type II migration (Ward 1997), determined by the mass of the embedded satellite, as well as by the disc temperature and viscosity. In the Type I regime (for mass ratios $q \lesssim 10^{-4}$, for typical disc temperatures and viscosities), the disc response is linear, and the migration rate can be predicted analytically (Goldreich & Tremaine 1980) and described with simple formulae (such as in Tanaka et al. 2002) that have been confirmed and calibrated with two- and three-dimensional simulations (e.g., Dong et al. 2011; D’Angelo & Lubow 2010; Duffell & MacFadyen 2012) although these predictions assume that the disc is locally isothermal (Paardekooper & Mellema 2006). In the Type II regime (or gap-opening regime) (typically $q \gtrsim 10^{-4}$), the disc response becomes nonlinear and the secondary begins to carve a low-density annular gap. The migration rate typically scales with some fraction of the local viscous rate (e.g. Edgar 2007) and is proportional to the gas density in the gap (Kanagawa et al. 2018). However, the migration rate depends on disc parameters, and can even switch sign for certain combinations (Duffell et al. 2014; Duffell 2015). One distinct regime is when the local disc mass is much smaller than the satellite’s mass. Disc torques in this regime are lowered significantly, causing migration to slow down (Syer & Clarke 1995; Duffell et al. 2014). Note that the E/IMRIs considered in our study are in this regime. In summary, in both the Type I and Type II regimes, the disc response, especially in the co-orbital region of the satellite, and the resulting torques on the satellite, are sensitive to disc parameters and to the equation of state. This makes predictions for real systems (whose parameters are unknown) difficult to make (Kley & Nelson 2012).

Despite these caveats, analytical predictions are convenient and are often utilised in the literature. In the context of BHs embedded in gas discs, prior work has estimated the impact of gas torques on LISA sources in order to assess different outcomes in torque evolution and correspondingly more stringent constraints on detectability. We also then explore how these results depend on disc viscosity and Mach number. With a range of simulations we show that whether or not torques accelerate or hinder the GW-driven inspiral, and whether the resulting gas imprint is detectable in the GW data stream, will depend on the combinations of these parameters.

3 SIMULATION SETUP

We use the moving-mesh hydrodynamics grid code DISCO (Duffell 2016) to model a thin two-dimensional, viscous disc around a MBH with a low-mass satellite BH embedded in the disc, following a GW-driven inward migration (“inspiral”). In this section, we describe our scale-free numerical approach for the disc, the prescribed orbit of the migrator, as well as how we measure the torques.

3.1 Disc model

The simulation setup is the same as in Paper I, with slight modifications to the domain size and spatial resolution. We model a vertically-integrated, near-Keplerian disc, parameterised by a constant aspect ratio $h/r \equiv M^{-1}$, where $h$ is the disc scale height, $r$ is the distance to the central MBH held at the origin, and $M$ is the Mach number for the azimuthal velocity $v = \sqrt{GM/r}$ around the central MBH of mass $M_1$. For thin and cold discs, the latter is equivalent to specifying the local sound speed; $c_s = v/M$. We adopt a constant $\alpha$-law prescription for the viscosity, such that the kinematic viscosity is set by $\nu(r) = \alpha c_s(r) h(r)$. We force the disc to be locally isothermal by setting the pressure to $p = c_s^2 \Sigma(r)$, where $\Sigma(r)$ is the surface density (i.e. the vertically integrated density) at radius $r$. With the above constraints, the initial condition for the surface density profile becomes:

$$\Sigma(r) = \Sigma_0 \left( \frac{r}{r_0} \right)^{-1/2},$$

where $r_0$ is an arbitrary distance unit, and $\Sigma_0$ is a corresponding normalisation.

Paper I was the first to address how these torques may evolve during a GW-driven inspiral and to calculate their detectability directly from simulations. This demonstrated a proof-of-concept for an optimal case: a $10^{-3}$ mass-ratio inspiral is relatively loud (compared to EMRIs), and chirps substantially as it approaches merger. These two qualities are paramount to detect the corresponding waveform deviations, and to use the frequency-dependence of these deviations to distinguish them from variations in system parameters in order to securely identify them as environmental effects.

In the present work, we follow up on Paper I and explore how the detectability of disc torques in GW waveforms depends on system or disc parameters. Changing the mass ratio of the system will affect the gas dynamics (and the torque experienced by the secondary) as well as the GW evolution of the binary in the LISA band (and resulting detectability of the gas imprint). We expand on the IMRI parameter space over an order of magnitude of mass ratio, demonstrating that reducing the secondary mass leads to different outcomes in torque evolution and correspondingly more stringent constraints on detectability. We also then explore how these results depend on disc viscosity and Mach number. With a range of simulations we show that whether or not torques accelerate or hinder the GW-driven inspiral, and whether the resulting gas imprint is detectable in the GW data stream, will depend on the combinations of these parameters.

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radiation, for a circular GW inspiral (Peters 1964), the evolution of the separation of a binary driven together by GW emission follows
\[ i_{GW} = \frac{64 G^3}{5 c^5} \frac{M^3}{(1 + 1/q)(1 + q)} \frac{1}{r^3}, \]
where \( M = M_1 + M_2 \) is the total mass of the binary. Following this, the secondary BH is placed in a quasicircular, prograde orbit whose separation evolves according to
\[ r(t) = r_f (1 - 4R_f (t - t_{\text{tot}}))^{1/4}, \]
where \( t \) is the elapsed time, \( t_{\text{tot}} \) is the total simulation time, \( r_f \) is the final binary separation, and \( R_f = -(i_{GW}/r_f) \) is a scaled inspiral rate evaluated at the final separation.

The parameter \( R \) can also be considered an inverse residence time, which relates to the time spent at each orbital radius during a GW-driven decay. It is useful to express this quantity in terms of the orbital frequency, which provides a dimensionless inspiral rate:
\[ R = \frac{\dot{r}}{\Omega} = \frac{64}{20 \sqrt{2}} \frac{1}{(1 + 1/q)(1 + q)} \left( \frac{r_f(M_f)}{r} \right)^{3/2}, \]
where \( r_f = 2GM_1/c^2 \) is the Schwarzschild radius of the primary SMBH and \( \Omega \) is the orbital angular frequency. Note that \( \Omega/R \) is approximately the number of orbits a binary spends at each separation \( r \) (the total number of orbits from \( r \) to \( 0 \) is \( n_{\text{orb}} = 1/(5\pi)\Omega/R \)).

In general, for each simulation, we need to specify an initial and a final position \( [r_i, r_f] \) for the inspiraling binary BH. In practice, this choice is guided by two considerations. First, we wish to simulate E/IMRIs that are in the LISA frequency band, close to merger. Second, while ideally we would follow the inspiral for the entire duration of a LISA observation (4 years by default; see below), in practice, we are limited by numerical considerations to a fixed number of orbits \( (N_{\text{orb}} = 5000 \text{ unless specified otherwise}) \).

The parameterisation above allows us to conveniently implement these choices, by specifying values for \( R_f \) and \( t_{\text{tot}} \). These translate to ranges of \( [r_i, r_f] \) which depend on \( q \) and the total binary mass. Motivated by modeling potential LISA sources, we impose inspiral rates \( R_f \) corresponding to the final stages of a \( q = 10^{-3} \), \( M_1 = 10^5 M_\odot \), fiducial IMRI, reaching \( r_f = 3r_g \). Each of our simulations spans \( n_{\text{orb}} = 5,000 \) binary orbits. Provided a final separation \( r_f \) and a total number of orbits \( n_{\text{orb}} \), one can solve for the initial separation of the binary through the relation
\[ n_{\text{orb}} = \frac{1}{2\pi} \int_{r_f}^r \Omega \frac{\dot{r}}{r_{GW}} dr. \]

With our fiducial parameters listed above, our initial binary separation is \( r_i = 8r_g \).

While the simulation is scale-free (i.e. \( r_0 = 4\pi = 1 \) in code units), prescribing an inspiral rate with Eq. 3 implies a physical length scale (i.e. a physical value for \( r_0 \), or equivalently a value of \( r_0 \) in Schwarzschild units). We illustrate this conversion in Figure 2, where we show curves of constant \( i/\Omega r \) as a function of \( q \) and separation \( r \) (or corresponding GW frequency \( f_{GW} \) shown on the upper x-axis). We delineate the simulated ranges of binary separations (or equivalently frequency ranges) with straight horizontal lines. Light and dark lines correspond to six different simulations
we performed, each spanning 5,000 orbits and probing various physical regimes of coalescence. The dashed portion of the lines indicates overlap, where two different simulations probe the same inspiral rates.

As shown in Fig. 2, simulations with different $q$ cover the same range of dimensionless inspiral rates ($7 \times 10^{-6} \leq R_7 \leq 2 \times 10^{-4}$), despite the fact that when decreasing the mass ratio, such rates correspond to unphysically small separations inside the innermost stable circular orbit (ISCO). While these inspiral rates are not realised in nature for low-$q$ binaries, this academic exercise allows us to isolate the effect of the inspiral on the torque, changing one parameter at a time, since we expect torques to be sensitive to the inspiral speed in addition to mass ratio.

In addition to the fiducial simulation that starts at $8\Sigma$, and reaches the ISCO at $3 \Sigma$, we ran a set of "slow" simulations (lighter lines in Fig. 2) which probe the $q = 10^{-3}$ inspiral from $10 \Sigma$ to $6.5 \Sigma$, similar to the range covered in Paper I. Rather than simulating a full 10,000 orbits, we split our simulations into two runs that overlap in inspiral rate. This provides a sanity check that our results are physical, and not dependent on transients introduced by the initial conditions (in other words, we can test whether the end of the "slow run" yields the same torques as the beginning of the "fast" run). Moreover, this reduces the span of radii covered in each single simulation, allowing us to avoid the BH starting too close to the outer boundary. Each simulation exceeds a viscous time for the disc, which for reference we define as a function of orbital time $t_{\text{orb}}$ at the BH position as

$$t_{\text{visc}} = \frac{2r^2}{3\nu} = 1415 \left(\frac{M}{M_\odot}\right)^2 \left(\frac{\alpha}{\epsilon}\right)^{-1} t_{\text{orb}}. \quad (7)$$

We neglect relativistic effects and keep the potential Newtonian, despite the fact that we are simulating regions close to the ISCO, where relativistic effects will affect the dynamics. This choice was made primarily for simplicity and to maintain the scale-free nature of the simulation.

We compute the torques exerted by the gas on the secondary BH, but we neglect their effect on the BH’s orbital evolution. This assumption is justified in the regime where the disc mass is insignificant compared to the mass of the secondary BH, and when the torque due to GW emission is far dominant. We demonstrate in §4 below that both of these criteria hold to high accuracy. This approach keeps the equations scale invariant and $\Sigma_0$ arbitrary, significantly reducing our computational costs and allowing us to run a full parameter study (see §3.5).

### 3.3 Sink prescription

Accretion onto the secondary is implemented with the same approach as in Derdzinski et al. (2019). The gas inside the smoothing radius $\epsilon$ is approximated as a mini-disc with the same $\alpha$ and $M$ as the global disc. The surface density within a distance $\epsilon$ of $M_2$ is decreased on the viscous timescale, $t_{\text{visc}}(\epsilon)$ at a rate

$$\frac{d\Sigma}{dt} = -\frac{\Sigma}{t_{\text{visc}}(\epsilon)} \exp\left(-\frac{r_2(\epsilon)}{\epsilon}\right) \epsilon. \quad (8)$$

This timescale, when converted to orbital times of the BH, corresponds to relatively “slow” accretion rate. We demonstrate the importance of the chosen timescale in Section 4, particularly for $q = 10^{-3}$. Unlike for more nearly equal-mass binaries (such as those studied in Duffell et al. 2019), we find here that the sink prescription affects the torques for a sufficiently massive gap-opening satellite.

### 3.4 Torque measurement

Here we describe how the gas disc torque is computed in the simulations and define other torques for comparison.

When comparing torques on binaries of different mass ratios, it is useful to use the Type I formula from Tanaka et al. (2002),

$$T_0 = \Sigma(r)r'\Omega^2q^2M^2, \quad (9)$$

where $\Sigma(r)$ is the initial surface density profile. As the BH migrates and the disc evolves from its initial conditions, it is useful to normalise the measured torque by $T_0$ in order to scale out the expected radial dependence of the torque. Analysing a dimensionless torque allows us to compare results for different mass ratios and isolate the effect of the inspiral.

The dominant mechanism for angular momentum loss on the secondary is the torque due to GW emission,

$$T_{\text{GW}} = \frac{1}{2}M_2r_2\Omega\Gamma_2. \quad (10)$$

Note that this is the torque on only one component of the binary.

The primary focus of this work is the gravitational torque (also referred to as migration or gas torque) $T$, which arises from the gravitational force exerted by the gas on the
secondary BH. We calculate this torque by summing up the \( \phi \)-component of the gravitational force \( g_\phi \) crossed with the binary lever arm \( r \) over all the grid cells in the disc,

\[
T_\phi = \sum g_\phi \times r
\]

where \( |r| \) is the distance from the secondary to the center of mass of the binary, which in our setup (and in the limit of \( q \ll 1 \)) is at the origin. We also discuss the torque density, defined by \( T = g_\phi \times r \) before summation, to analyse contributions from different patches of the disc to the total torque.

Torque can also be imparted via accretion of gas that directly adds both mass and momentum to the secondary BH (sometimes called accretion torque). For our parameters we find this torque to be significantly weaker than the gravitational torque (by several orders of magnitude, as in Derdzinski et al. (2019), so we refrain from discussing it further in the present work.

The magnitude of gas torques (Eqs. 9 and 11) all depend linearly on the normalisation of the surface density, which is arbitrary in our simulation setup. Rather than choosing a single value for \( \Sigma_0 \), we will use this freedom of normalisation to ask: at what value of the surface density do gas torques produce a detectable phase drift in LISA’s GW measurements? Additionally, is this density physically reasonable, given the densities expected in accretion disc models?

Estimates for accretion disc densities in the vicinity of the central SMBH vary by several orders of magnitude. We adopt two limiting estimates for the surface density normalisation \( \Sigma_0 \) (at the secondary’s final radius, taken to be \( r_f = 3r_S \) for our detectability estimates below) for the inner regions of thin, near-Eddington accretion discs. They depend on the presumed accretion rate \( M \) and the viscosity parameter \( \alpha \). The models differ in whether the viscosity scales with the gas pressure or with the total (gas + radiation) pressure, a choice that has a large impact when relating the accretion rate to \( \Sigma_0 \). Lower viscosity discs require much higher surface densities to maintain the same accretion rate.

We normalise our disc densities to represent AGN accreting at 10% of the Eddington rate with a radiative efficiency of 10%. The first estimate is obtained from the seminal model for a thin, viscous accretion disc (i.e. \( \alpha \)-disc; Shakura & Sunyaev 1973). In the inner, radiation-pressure dominated region,

\[
\Sigma_\alpha = 41.08 \left( \frac{\alpha}{0.03} \right)^{-1} \left( \frac{M}{0.1M_{\text{Edd}}} \right)^{-1} \left( \frac{r}{3r_S} \right)^{3/2} \text{g cm}^{-2}.
\]

In case the viscosity is proportional only to the gas pressure (i.e. for a so-called \( \beta \)-disc), the surface density at the same accretion rate is several orders of magnitude higher,

\[
\Sigma_\beta = 2.11 \times 10^7 \left( \frac{\alpha}{0.03} \right)^{-4/5} \left( \frac{M}{0.1M_{\text{Edd}}} \right)^{3/5} \times \left( \frac{M}{10^6M_\odot} \right)^{1/8} \left( \frac{r}{3r_S} \right)^{-3/5} \text{g cm}^{-2},
\]

see Haiman et al. (2009). While both of these models carry a different radial density scaling than our disc model, the values are meant to provide a reference for possible surface densities, which becomes important for our detectability estimates in §5 below. Note that in both these estimates the total disc mass inside the BH’s orbit is much less than the mass of the BHs. For example, even with the high-end estimate, an integral of the total enclosed mass within \( 10r_S \) yields a mass as low as

\[
M_{\text{encl}} = 2\pi \int_{3r_S}^{10r_S} \Sigma_\phi r dr = 0.16M_\odot
\]

for \( \alpha = 0.03, M = 10^6M_\odot \), and \( M = 0.1M_{\text{Edd}} \).

3.5 Simulation Suite

In Paper I, we performed a simulation for a single system with \( q, a, M = 10^{-3}, 0.03, 20 \), over a range of dimensionless inspiral rates between \( 7 \times 10^{-6} \) and \( 2 \times 10^{-5} \). We extend on that study by expanding the range of inspiral rates to higher values up to \( \dot{r}/\Omega r = 2 \times 10^{-4} \), and then varying \( q, M, \) and \( \alpha \).

Our fiducial system is a \( q = 10^{-3} \) binary embedded in a disc with \( \alpha = 0.03 \) and \( M = 20 \). We then run simulations for three different mass ratios \( q = 10^{-3}, 3 \times 10^{-4}, 10^{-4}, \) each with three different values of \( \alpha = 0.01, 0.03, 0.1 \). We also run simulations with a range of Mach numbers \( M = 10, 20, 30 \) around our fiducial model. In total, we present 15 different simulations, which are listed in Table 1, where each run is labelled with its mass ratio and viscosity (for example, ‘q1e3a03’ corresponds to a run with \( q = 10^{-3} \) and \( \alpha = 0.03 \)).

For computational feasibility, our study is limited to higher values of \( \alpha \) and lower Mach numbers, in order to avoid prohibitively long viscous times to reach steady state (see Eq. 7). Our fiducial choice of \( M = 20 \) corresponds to a much hotter and thicker disc than expected in thin, near-Eddington AGN discs, where continuum emission suggests Mach numbers exceeding \( M \sim 100 \) Krolik (1999). However, higher Mach numbers are numerically challenging to simulate, primarily because highly supersonic flows require increasingly high resolution as they develop complicated, small scale features. As we show in §4.1.3 below, increasing the Mach number to 30 already produces very noisy torques and a dense and unstable gas morphology.

4 RESULTS

Here we describe results from our simulations – measuring and analysing the torques exerted on the satellite BHs – before deriving estimates of the detectability of the corresponding imprints in the LISA GW waveforms.

4.1 Gas torques depend on parameters

As predicted by analytical estimates such as \( T_\phi(r, q, M) \), we expect that torques will not only depend on system parameters but also evolve with radius as the binary separation decreases. We find that the magnitude of gas torques generally agrees, within an order of magnitude, with the analytical predictions \( T_\phi(q, M, r) \) (Eq. 9). However, the direction of the torque is difficult to predict. Torques tend to oscillate around a value close to (typically less than) \( T_\phi \), but at the late stages of the inspiral, the evolution may deviate from the expected scaling. As we discuss in the following sections, whether torques are negative (inward) or positive (outward) depends on the combination of \( q, \alpha \), and \( M \), and we find...
there is no direct or obvious scaling in this intermediate mass ratio regime.

4.1.1 Mass ratio

In Fig. 3, we plot the measured $T_g/T_0$ as a function of inspiral rate (Eq. 5, itself a function of $r^*$) measured in simulations with different mass ratios in a disc with fiducial parameters $\alpha = 0.03$ and $M = 20$. As we expect from similar studies of torques on stationary (i.e. non-migrating) satellites, torques in this regime are sensitive to even small changes in system parameters. In fact, when increasing $q$ from $10^{-3}$ to $10^{-4}$, the figure shows that the direction of the torque changes. This particular behavior is, however, also dependent on the choice of $\alpha$ (see below).

Additionally, the torque exerted on the $q = 10^{-3}$ satellite is significantly more noisy compared to the smaller mass ratios. This is attributed to the large pile-up of gas that accumulates very close to the BH in this mass regime. We discuss this behavior and its consequences in §4.2 below.

First we compare the strength of the gas torque to GWs. We compute the average torque from the last 2000 orbits in each of the slower-inspiral runs (where the scaled torques $T_g/T_0$ are constant to a good approximation), and compare these to the GW torques during the last stages of coalescence. These comparisons are shown in Fig. 4. The average torque measured in the simulation is extrapolated to earlier times for the two smaller mass ratios, assuming that the scaled torque remains constant (shown by the dashed curves). To scale the torques to physical values, we normalize the surface density with the $\beta$-disc estimate (Eq. 13), making these high-end estimates of the disc torques. In all cases, despite the high assumed disc density, gas torques are several orders of magnitude weaker than that due to GWs at this stage of coalescence. However, as we will show in §5, over an observation of many thousand cycles even weak gas impact can accumulate and produce detectable signatures in the measured GW signal.

In most cases we explore here, the average $T_g$ is within a factor of a few of the analytical estimate $T_0$, but deviates towards stronger torques (either negative or positive) as the inspiral rate increases. This happens at the final stages of the inspiral (the final ~ 1000 orbits), and in the case of the smallest mass ratio $q = 10^{-4}$, in reality the merger occurs before the inspiral has a chance to substantially affect the torque. However, as we describe below, when or whether this deviation occurs depends on other disc properties.

4.1.2 Viscosity

In this highly nonlinear regime, viscosity affects several important aspects to this system, each of which contribute to the torque in significant ways. First, viscosity affects the gap depth, which will affect the magnitude of the Lindblad torque excited in the disc. Secondly, viscosity sets the rate that angular momentum can be carried away from the corotation resonances, and whether corotation torques are in a “saturated” or “desaturated” state (Masset et al. 2006; Duffell 2015). Third, viscosity can suppress instabilities in the disc that might otherwise generate vortices which exert nontrivial time-dependent torques on the perturber. Finally, in this complicated nonlinear regime, it is fully possible there are comparable additional torque affects that have not been considered yet, as most analysis of viscous affects on torques are carried out in the linear or weakly nonlinear regimes. All of this adds up to the result that the torque has a nontrivial $\alpha$ dependence, and even small changes in $\alpha$ can change the sign of the torque. This is seen in simulations of a range of mass ratios from planetary scales (Duffell 2015) up to $q \sim 0.5$ in the context of MBH binaries (Duffell et al. 2019).

We have run additional sets of simulations for each mass ratio in discs with higher and lower viscosities ($\alpha = 0.1$ and

| Name | Mass Ratio | Viscosity | Mach Number | Separation [$r_\text{s}$] | Average Torque |
|------|------------|-----------|-------------|----------------|---------------|
| q1e3a03 (fiducial) | $10^{-3}$ | 0.03 | 20 | [10, 3, 6.5] | 0.19 |
| q3e4a03 | $3 \times 10^{-4}$ | 0.03 | 20 | [6.4, 4.0] | 0.01 |
| q1e4a03 | $10^{-4}$ | 0.03 | 20 | [4.1, 2.6] | $-0.29$ |
| q1e3a1 | $10^{-3}$ | 0.1 | 20 | [8.2, 3.0] | 0.34 |
| q3e4a1 | $3 \times 10^{-4}$ | 0.1 | 20 | [5.1, 1.9] | 0.38 |
| q1e4a1 | $10^{-4}$ | 0.1 | 20 | [3.3, 1.2] | $-1.26$ |
| q1e3a01 | $10^{-3}$ | 0.01 | 20 | [8.2, 3.0] | $-0.01$ |
| q3e4a01 | $3 \times 10^{-4}$ | 0.01 | 20 | [5.1, 1.9] | $-0.51$ |
| q1e4a01 | $10^{-4}$ | 0.01 | 20 | [3.3, 1.2] | $-0.25$ |
| q1e3a0m10 | $10^{-3}$ | 0.03 | 10 | [8.2, 3.0] | $-0.39$ |
| q1e3a0m30 | $10^{-3}$ | 0.03 | 30 | [8.2, 3.0] | $-0.03$ |
| q1e3a3 (no sink) | $10^{-3}$ | 0.03 | 20 | [8.2, 3.0] | see Fig. 9 |

Table 1. List of our 15 simulations and their parameters. Each binary is forced to inspiral at the GW-driven rate from its initial $r_f$ to its final separation $r_f$ (listed in Schwarzschild units). In those cases where we ran two simulations for a single binary to probe different inspiral rates, two ranges of radii are listed (first three rows). We also show the average torque value (measured over the last 2000 binary orbits) in simulations for which we calculate the SNR of the gas-induced deviations in the GW waveform.
\( \alpha = 0.01 \) to observe changes in the overall torque and its evolution with the inspiral rate. The results are shown in Fig. 5. We find that the magnitude \(|T_{\text{gas}}|\) typically increases in strength with \( \alpha \), as has been observed in other numerical studies (e.g. Robert et al. 2018), although the direction changes unpredictably, particularly for \( q = 3 \times 10^{-4} \).

Intuitively, we expect that for higher values of viscosity, the inward migration driven by GW emission should have less impact on the torque, simply because the disc can reorganize more quickly. Indeed, this is what we observe in the top panel of Fig. 5. For \( \alpha = 0.1 \) the torque is essentially constant throughout the entirety of the inspiral (modulo short time-scale oscillations) for all mass ratios, and its absolute value can be approximated by \( T_0 \) to within a factor of two. For the weaker viscosity, the lower panel of Fig. 5 shows that \( T_g/T_0 \) deviates from the nearly constant value during earlier stages, leading to an increasingly negative torque.

In the case of thin, fully ionised discs in near-Eddington AGN, observational evidence suggests that viscosity may reach values of \( \alpha \approx 0.1 - 0.4 \) (King et al. 2007). Our results therefore imply that E/IMRIs in such viscous discs may be subject to stronger torques, but are less likely to show significant changes in torque during the inspiral. However, it is the combination of \( \alpha \) and \( M \) that determines the disc dynamics (note the kinematic viscosity \( \nu = \alpha / \mathcal{M}^2 \)), and the values we consider here provide AGN-like characteristics (e.g. gap depth). Furthermore, in reality \( \alpha \) may scale with radius - thus it is possible that the torque evolution seen in lower \( \alpha \) simulations may still be relevant for AGN. In Section 5, we discuss the case in which torques follow a simple scaling with radius.

### 4.1.3 Mach number

The Mach number, a measure of disc temperature and thickness, is also a critical factor in determining the gap depth and disc dynamics near the BH (Duffell 2015). A low Mach number disc is subject to stronger pressure forces, resulting in shallower gaps, while a higher Mach number describes a dynamically colder disc that can consequently form deeper gaps. For high enough Mach number the scale height of the disc (recall \( h/r = 1/\mathcal{M} \)) can approach or fall within the Hill sphere of the perturber, leading to a more dynamic gas flow across the gap (see below).

For our fiducial system with \( q = 10^{-3} \) and \( \alpha = 0.03 \), we explore a range of Mach numbers from \( M = 10 - 30 \). While this range is limited, we are able to observe trends in gap depth and gain insight into the dependence of the gas dynamics close to the BH on \( M \), which gets increasingly complex for colder discs.

Fig. 6 shows surface density contours of gas close to the BH at the end of each simulation. For the lowest Mach number (\( M = 10 \), which, we note, is a value often adopted in binary simulations), pressure forces significantly smooth the flow. For the highest Mach number (\( M = 30 \)), gas flows more tightly around the BH (indeed, the scale height and corresponding smoothing length \( \epsilon \) is smaller). Gas morphology within the Hill radius is dynamic, with narrower streams that flow across a deeper gap and stark density contrasts that lead to instabilities in the gap edges and streams.

Fig. 7 shows the corresponding gas torques. In the warmer disc (\( M = 10 \); purple curve), the torque is smoother
and negative, rather than positive as in the fiducial $M = 20$ case (blue curve). For the colder disc ($M = 30$; blue curve), the net torque becomes positive, but the most striking feature is that the dynamic flow around the BH produces a highly variable torque. The variability increases dramatically as the BH migrates inward.

In order to verify that the large fluctuations are physical and not due to under-resolved gas flow near the BH, we performed higher-resolution runs as a test (up to 800 radial cells, corresponding to $\Delta r \gtrsim h/10$). These tests show that the variability persists, but the amplitude and timescale are not yet converged. In fact, a higher resolution leads to larger amplitude in torque fluctuations. This leads us to believe that the fluctuations are physical, and that highly supersonic discs are indeed more sensitive to the evolution of an embedded BH during a GW inspiral. Without stronger pressure forces to smooth out fluctuations in the flow, gas torques become more unstable. A more detailed investigation of the impact of such a variable torque on a GW inspiral will be investigated in future work.

### 4.2 Dissecting the torque

With our simulations we have the ability to assess contributions of different parts of the disc to the net gravitational torque. Of particular interest is being able to distinguish between gas very close to the satellite BH, i.e. near and within its Hill radius, and that from elsewhere in the disc (the inner and outer discs as well as streams crossing the gap). One might compare this to corotation torques discussed in the planetary migration literature, keeping in mind that these studies concern gas within the entire co-orbital region rather than just the Hill sphere. Similar to what we find here, numerical studies find that corotation torques can become positive in discs with steep density gradients (Paardekooper & Papaloizou 2008) or with sufficiently high viscosities (Masset et al. 2006; Duffell 2015). An important distinction is that the corotation torques include streams of gas in the horseshoe region, making a U-turn near but outside the secondary’s Hill radius, whereas here we find that torque from gas closer to the secondary dominates. In particular, we focus on the torques near and within the Hill radius, as most of the gas in the co-orbital region for gap-opening perturbers is confined close to the secondary, and the dominant contribution to the "corotation" torque in our case arises from back-to-front asymmetry in the gas distribution at or inside the Hill radius.

The Hill radius, defined as the region where the orbital velocity of gas bound to the satellite BH matches the orbital velocity of the BH itself, is given by

$$n_H = (\frac{q}{3})^{1/3} r.$$  \hfill (15)

As discussed in Paper I already for the $q = 10^{-3}$ case, gas pile-up close to the BH becomes non-negligible. This can be seen in the density contrasts in Fig. 6, or in Fig. 8 where we show various distributions of torque density $T$ in our fiducial runs for each $q$. Excising the Hill radius (top panels) allows us to observe contributions to the torque from nearby streams, while zooming in on the Hill radius (bottom panels) allows us to analyse the dominant torque contributions (due to its proximity to the BH). Most notable is the high torque density for $q = 10^{-3}$ arising from the high-density atmosphere that accumulates around the BH. Despite such high values of $T$, the net torque in this region remains below $T_0$, implying that as density of the circum-BH gas increases, so does its degree of front-to-back symmetry.

Gas within the Hill radius contributes a substantial fraction to the total torque, as we show in Fig. 9. This figure shows separately the torque contributions from gas within and outside of the Hill radius for the fiducial $q = 10^{-3}$ run. Gas inside $n_H$ is responsible for the positive component of the torque and also shows the strongest dependence on the inspiral rate, increasing from $\sim 4n_H$ to $3n_H$ (see the solid lines in Fig. 9). This region is sometimes assumed to not contribute to the torque, or is damped by a manual "tapering" function (see, e.g. Dempsey et al. 2020), even though this gas is a crucial component of material flow across the gap and may exhibit non-negligible asymmetries. Indeed, in simulations by Crida et al. (2009), the migration rate of a live
In the present work, the Hill sphere torque is of particular importance as the asymmetry near the BH may be exacerbated during a sufficiently fast inspiral (here driven by GWs). We expect that any changes in the torque during a GW-driven inspiral will first arise from gas closest to the BH. This is shown more clearly by comparing versions of our fiducial run for which the sink prescription is turned off (shown as light curves in Fig. 9). Unsurprisingly, an accreting BH experiences less positive torque as density within the Hill radius is depleted (while it is nearly unaffected outside). Without accretion, torques from within $r_{\text{H}}$ substantially increases with the inspiral rate, more than doubling in comparison to $T_0$ within the final 5,000 orbits of the inspiral (from $8r_S$ to $3r_S$). If the evolution of the torque for more massive IMRIs (as we see for $q = 10^{-3}$) is sensitive to accretion efficiency of the secondary BH, this raises the hope that detecting the torque evolution with frequency may provide insight into the gas dynamics near the BH.

For satellites with mass ratios below $q = 10^{-3}$ accretion has a negligible impact on the torque — when comparing runs with and without accretion, we see no distinct difference in the surface density profile or the torque, given that smaller satellites accrete an insignificant amount of material according to our sink prescription.

### 4.3 Torque evolution

Does the torque on an embedded IMRI change in response to the increasingly rapid inspiral? This is important as a physics question about the system, as well as a practical question about interpreting any future LISA data. Additionally, it is important to know whether the numerous previous studies of disc torques for non-migrating satellites hold for GW-driven E/IMRIs, or if the torques are modified and need to be re-computed in simulations that include the GW-driven inspiral.

The short answer is: it depends. Whether the GW-driven inspiral produces a significant effect on the torque depends on the mass of the inspiraling BH, the disc viscosity, Mach number, and the BH accretion efficiency.

During the inspiral, we observe three effects that are possibly correlated with the nonzero inspiral rates. First, for all simulated binaries, the torque on a migrating BH shows long-term oscillations (on a timescale of ~ 100s of orbits). Although long-term modulations have been seen in the accretion rates of non-inspiraling, near-equal-mass binaries (e.g. see Fig.11 in D’Orazio et al. 2013), these modulations have not been seen for $q \ll 1$. In our case, the long-term modulations in the torque arise from global perturbations as they are present in the torque contribution from outside the Hill radius. The magnitudes are of order $\lesssim 10\%$, so for detectability purposes the torque can be approximated by an average, or possibly a stationary approximation for Type II torques.
However, analytical approximations neglect the ability for Type II torques to become positive. Second, depending on the mass ratio, a sufficiently fast inspiral rate can change the magnitude of the torque. These changes arise from changes in the gas distribution inside the Hill radius. For our fiducial viscosity changes arise from changes in the gas distribution inside the Hill radius. The torque begins to deviate from its steady-state value \( \alpha \) Hill radius. For our fiducial viscosity changes arise from changes in the gas distribution inside the Hill radius. In reality, this rate is only reached in a circular binary when \( q > 10^{-3} \). Lower mass ratio binaries merge before reaching this rate. However, this trend is dependent on accretion efficiency and disc viscosity. In discs with lower viscosity (or less efficient accretion onto the secondary), we expect this deviation to occur at earlier times. Overall we find that the most prominent torque evolution occurs for a relatively massive satellite with \( q > 10^{-3} \), due to the significant pile-up of gas in its Hill sphere. Interestingly enough, our \( q = 3 \times 10^{-3} \) simulation in the \( \alpha = 0.03 \) disc experiences a sign change in the torque as it approaches merger. In this case, gas torques would initially slow down the inspiral before accelerating it.

1 We note that the decreasing trend in the gas torque reported in Paper I was in fact a long-term transient due to the simulation having a closer outer boundary \( (r/n_\text{L} \leq 2.75, \text{rather than 6}) \). Our slower inspiral runs confirm that the normalized torque on a \( q = 10^{-3} \) IMRI is indeed constant (albeit with oscillations) until reaching separations near \( -4r_s \), although this depends on the accretion rate.

![Figure 8](image-url). 2D contours of torque surface density \( \mathcal{T} = g \times r \), equivalently a torque per unit disc surface area close to the satellite BH for discs with \( \alpha = 0.03 \) and \( M = 20 \) and three different mass ratios. In the top panels we have excised the gas in the Hill radius \( (r_H; \text{dashed circles}) \) to highlight the gas morphology in streams nearby. The bottom panels show zoom-in views of the torque contributed by gas within the Hill sphere. All contours are normalized by the maximum \( \mathcal{T} \), printed in each panel for reference. Note that gas pile-up for \( q = 10^{-3} \) is deep within the Hill radius and reaches significantly high densities. This results in much higher torque densities. The smoothing length of the gravitational potential is denoted with the solid purple circles.

| \( \zeta \) | Redshift | 1 |
|----------|----------|---|
| \( M \)  | Primary Mass | \( 10^3 M_\odot \) |
| \( \tau \) | LISA mission lifetime | 4 yrs |
| \( L \)  | LISA arm length | 2.5 million km |
| \( N \)  | Number of laser links | 6 |

Table 2. LISA parameters are used when computing detectability.

Finally, the inspiral has an impact on torque fluctuations. In particular for \( q = 10^{-3} \), for which gas pile-up on the BH is most significant, the Hill sphere torque shows an increase in fluctuation amplitude in the final \( \sim 1000 \) orbits. With a higher Mach number, these fluctuations are more extreme and arise at earlier times. In particular the spikes in the Hill torque for \( q = 10^{-3} \) and \( M = 20 \) may signify an interesting dynamical interaction between the BH’s orbit and the gas, given that they occur at the same radii regardless of viscosity or the BH sink rate. In simulations with varying boundaries \( (0.4 - 0.5 \leq r/n_\text{L} \leq 3.0 - 6.0) \) or higher resolution \( (800, \text{rather than 666 radial cells}) \), we find that these spikes in the torque still occur at the same physical radii.

5 SIGNIFICANCE FOR LISA INSPIRALS

For the rest of the paper we take the results from our simulations, primarily the average torques measured in the \( \alpha = 0.03 \)
and $a = 0.1$ runs, and analyse their detectability in the GW signal. First we introduce some relevant quantities that describe a GW event in the LISA band, with the goal of computing the detectability of the imprint of the gas torques.

As illustrated in Figure 2, the inspiral rate for each mass ratio corresponds to a physical separation and gravitational wave frequency. Recall that the GW frequency is twice the orbital frequency for a binary on a circular orbit: $f = 1/\pi \sqrt{GM/r^3}$, and we have chosen these inspiral rates to correspond to IMRIs in the LISA frequency band.

The GW amplitude of a circular inspiral depends on the source distance (or redshift $z$), its frequency, and its chirp mass $M_c = M_1^{3/5}M_2^{3/5}/(M_1 + M_2)^{1/5}$. The sky- and polarization-averaged GW strain amplitude of a source at co-moving coordinate distance $r(z)$ is given by

$$h = \frac{8\pi^{2/3}G^{2/3}M_c^{2/3}}{10^{17/2}} f^{2/3} r(z)^{1/2}, \quad (16)$$

where $f$ is the observed GW frequency and $f_r = f(1 + z)$ is the GW frequency in the source’s rest frame.

The characteristic strain $h_c$ of a source whose frequency evolves during a LISA observation of time $\tau$ (i.e., the LISA lifetime) is given by $h_c = h(\tau)$, where $n = f^2/f_r$, which is a measure of the total number of cycles the source spends at each frequency (see Sesana et al. 2005 for a more detailed discussion). In Fig. 10 we plot the characteristic strain of IMRIs at each simulated mass ratio for two different possible observations of duration $\tau = 4$ years, the currently planned nominal LISA mission lifetime (Amaro-Seoane 2018). We assume a fiducial primary mass $M_1 = 10^6 M_\odot$ and place the source at redshift $z = 1$. These parameters are listed in Table 2. The dashed lines correspond to the final 4 years of a binary prior to reaching ISCO at $r_{ISCO} = 3r_S$, and the shorter, solid lines correspond to binaries observed for 4 years and ending up at 10$r_S$ (in their rest frame).

The signal to noise ratio (SNR) of the event is a measure of its “loudness”, or a way to characterise its detectability compared to the LISA noise. It is an integral of the strain amplitude over the noise, given in the stationary phase approximation by

$$\rho^2 = 2 \times 4 \int_{f_{\text{min}}}^{f_{\text{max}}} df \frac{|\tilde{h}(f)|^2}{S_n(f)/f^2}. \quad (17)$$

The pre-factor of two assumes the currently planned configuration of six links (effectively two independent interferometers), and $S_n(f)$ is the spectral density of LISA’s noise per frequency bin, adopted from Klein et al. (2016), where we assume an arm length of 2.5 million km and include an estimate of confusion noise produced by foreground sources (galactic binaries).

For a fixed primary mass, binaries with lower mass ratios (smaller $M_c$) emit weaker gravitational waves, and thus span shorter frequency windows during a fixed observation time $\tau$. This reduces the total SNR as well as the chances of detecting a deviation in the signal.

### 5.1 The imprints of gas on GWs

#### 5.1.1 Phase drift

Depending on the mass ratio and viscosity, gas torques either slow down or speed up the inspiral. For a gravitational wave event in the LISA data stream, this produces a phase drift in the waveform compared to that in vacuum, and a shift in the total phase accumulated during the event. If the accumulated phase (often defined in Fourier space) due to...
GW emission alone is $\Phi_{\text{GW}}(f)$, the phase of an event experiencing gas torques will be $\Phi_{\text{GW}}(f) + \delta\phi(f)$, i.e. the underlying vacuum signal plus some small deviation which can also depend on frequency. If the phase deviation $\delta\phi(f)$ is significant (and unique), the gas imprint is potentially distinguishable from the vacuum waveform (and from other deviating effects). This depends on (i) the strength of the gas torque compared to GWs, which changes with radius; (ii) the frequency window that is observed, and (iii) the signal to noise ratio (SNR) of the event itself. Thus $\delta\phi$ is not only a function of frequency, since torques evolve with radius, but also scales with the disc surface density as this determines the strength of the torque.

For calculating the phase shift induced by gas we take the same approach as in Paper I (see Derdzinski et al. 2019), but we implement the updated result for the simulated gas torque and apply this to all simulated mass ratios.

Given that gas torques are much weaker than GWs (Fig. 4) and assuming the insipr remains circular, the phase drift (in radians) can be integrated by

$$\delta\phi = 2\pi \int_{r_{1}}^{r_{2}} \frac{f_{\text{gas}}}{f_{\text{GW}}} \left[ 1 + \frac{O\left(\frac{f_{\text{gas}}}{f_{\text{GW}}}\right)^{2}}{r} \right] \, dr,$$

where $f_{\text{gas}}$ is the change in separation due to gas torques.

Note that $f_{\text{gas}}$ can be a function of radius, as we define explicitly below, but that this function can vary for systems with different mass ratios or disc parameters.

As shown in our fiducial runs in Fig. 3, the magnitude of the gas torque when normalized by $T_{0}$ is approximately constant throughout the inspiral for all mass ratios. To compute the phase shift we use the average of these torques, calculated over the last 2000 orbits of the slower-inspiral simulations. We neglect the short time-scale oscillations in the torques, as well as the deviations from the average that occur at the highest inspiral rates. Given that normalised torques either stay constant or increase in absolute value with inspiral rate, this provides a lower limit on detectability.

If $T_{\ell}/T_{0}$ is constant throughout the inspiral, then torques scale with the radial dependence of $T_{0}$, which is dependent on the initial disc density profile. We can define the average gas torque analytically as

$$\langle T_{\text{gas}} \rangle = C_{\text{fit}} T_{0}(q, r, M, \alpha, \Sigma(r)),$$

where $C_{\text{fit}}$ is the (constant) average of the torque before it begins to deviate due to rapid inspiral. These fits are shown as horizontal dashed lines in Fig. 3 and also provided in Table 1.

The gas torque on the satellite BH can be expressed in terms of the rate of change of specific angular momentum $\ell_{\text{gas}} = T_{\text{gas}}/M_{2}$, which relates to the rate of change of separation as $\ell_{\text{gas}}(r) = 2\sqrt{GM_{1}r}/C_{\text{fit}}^{1/2}T_{\text{gas}}(r)$. Plugging this expression into Eq. 18 allows us to solve for the shift in GW phase due to the gas torque on each binary, provided an observed frequency window (and corresponding range of separations). Note that in converting the torque from code units to physical units, these quantities must be scaled with our fiducial parameters by $T_{\text{gas}} = T_{\text{code}} \times GM_{1}\Sigma_{0} r_{0}$ and $\ell_{\text{gas}} = \ell_{\text{code}} \times G^{2} \rho_{0} / \rho_{q}$.

We integrate over the two different frequency windows for each mass ratio, defined by the 4-yr observations shown in Fig. 10. Since our simulated inspiral does not cover the entire observed frequency range, we extrapolate the torque fit from Eq. 19 to the earlier stages, which implicitly (and reasonably) assumes that torques continue to scale with $T_{0}$ at earlier times.

In Fig. 11, we plot the total accumulated phase shift for each observation as a function of the disc surface density. $\delta\phi$ scales linearly with $\Sigma_{0}$, but accumulates to different values depending on the strength of the torque and the frequency window observed (both of which depend on $q$). For reference, we mark the surface densities in the $\alpha$-disc and $\beta$-disc models at $r_{0} = 3r_{g}$ by vertical lines. In the low-density limit, gas torques only impart a phase shift of $\delta\phi \sim 10^{-3}$, which is likely undetectable. For higher densities ($\Sigma_{0} \gtrsim 10^{5}$ g cm$^{-2}$), the phase shift can exceed several radians.

### 5.1.2 Detectability of waveform deviation

The detectability of a deviation can be estimated by calculating the SNR of the difference between the two waveforms ($\delta\phi$). Similarly to Eq. 17, we calculate the SNR of the deviation as

$$\rho_{\delta\phi}^{2} = 2 \times 4 \int_{f_{\min}}^{f_{\max}} df \frac{|\delta\tilde{h}(f)|^{2}}{S_{\tilde{h}}^{2}(f)f^{2}}.$$

where instead of the strain amplitude we integrate the strain deviation in Fourier space,

$$|\delta\tilde{h}|^{2} = |\tilde{h}|^{2} \left( 1 - \rho_{\delta\phi}^{2} \right) = 2|\tilde{h}|^{2} (1 - \cos (\delta\phi))$$

This assumes the chirp is slow and that gas only imparts a difference in GW phase and not amplitude, also known as
the stationary phase approximation (see Yunes et al. 2011 and Kocsis et al. 2011).

We show the accumulated SNR of the gas-induced deviation for the $a \simeq 0.03$ runs in Fig. 12 as a function of disc surface density normalisation $\Sigma_0$. This allows us to assess, given a disc density, how distinguishable the phase-shifted waveform will be from the vacuum waveform. Just as the phase shift depends linearly on the surface density, $\rho_{\delta \phi}$ initially scales linearly with $\Sigma_0$. This can be interpreted from Eq. 21, where for small $\delta \phi$ the strain deviation becomes linear with the phase shift, as $|\delta h| \propto (1 - \cos \delta \phi)^{1/2} \approx \delta \phi$. However, at high enough surface densities, where torques shift the phase by a whole period ($\delta \phi \gtrsim 2\pi$), this linear dependence disappears and the SNR saturates. This behavior is observed in binaries that are essentially monochromatic in frequency, an inevitable feature of circular, extreme mass ratio inspirals that coalesce very slowly. The exception is for binaries that are approaching merger, whose phase shift accumulates past $2\pi$ as they sweep through higher frequencies. If an IMRI is embedded in a disc with surface densities reaching that of the $\beta$-disc model, and we observe the late stages of coalescence, its waveform may be significantly altered.

Ultimately the detectability of a deviation relies on the event accumulating substantial total SNR. Indeed, the observations for which a phase drift accumulates the highest $\rho_{\delta \phi}$ are those that accumulate highest total SNR, as indicated in Fig. 10. In Fig. 13, we show the SNR of the deviation divided by the total SNR of the event (a relative SNR, $\rho_{\delta \phi}/\rho$), as a function of disc surface density. This illustrates that the accumulated deviation for inspirals that are chirping faster (dashed lines) is weaker than for the inspirals that are observed at earlier times (solid lines). Nevertheless, we see in Fig. 12 these weaker deviations are more detectable due to the larger overall SNR. Figure 10 clearly illustrates the reason: these tighter binaries are observed at frequencies close to the minimum of LISA’s sensitivity curve. Louder events – ideally intermediate mass ratios at low redshift – are the most promising candidates for detecting gas imprints.

Adopting a detectability threshold of $\rho_{\delta \phi} \gtrsim 8$, we conclude that the gas imprint is detectable for all simulated mass ratios if the disc density exceeds $\Sigma_0 \gtrsim 10^{4-6} \text{g cm}^{-2}$. The surface density required for detectability depends on the strength of the torque, which varies for each value of the mass ratio. Given that the gas torque on the $q = 3 \times 10^{-4}$ binary is an order of magnitude weaker than for the higher mass ratios, it requires a correspondingly higher $\Sigma_0$ for detectability. The detectability of gas for lower mass ratios at earlier stages suffers from weaker GW emission and more modest frequency evolution, and gas torques are less detectable even for the highest disc densities.

For higher viscosities, where torques are stronger for each mass ratio, the detectability of a deviation is improved (see Fig. 14), and an SNR deviation of $\rho_{\delta \phi} \gtrsim 8$ can be reached with lower disc densities, $\Sigma_0 \sim 10^4 \text{g cm}^{-2}$.

One may wonder how our choice of primary mass affects the detectability of the gas imprints. In principle, IMRIs can occur for more or less massive primary MBHs while still emitting GWs within the LISA frequency band. We demonstrate the effect of our choice of primary mass $M_1$ for a fixed mass ratio $q = 10^{-3}$ in Fig. 15. Higher mass binaries emit louder gravitational waves, but they merge at lower frequencies due to their increasingly large ISCO ($r_S \propto M$). Taking the mass ratio $q = 10^{-3}$, we plot the strain and corresponding detectability of the phase shift, adopting the dimensionless gas torque from our fiducial run ($q \geq 3\text{a}_0 \beta_3$), where $T_{\text{gas}} / T_0 = 0.21$. We fix each observation window to the binary reaching $3r_S$ in a 4 year observation. In this case the lower-mass binary can exhibit more detectable gas torques, given that the coalescence occurs at frequencies where LISA is most sensitive. However, the overall SNR of the event (and consequently the SNR of the deviation) will depend on the range of observed frequency of the binary and its relation to the peak sensitivity—notice that the coalescence of the $10^6 M_0$ IMRI attains the highest total SNR (dashed purple line, $\rho_B$), while the $10^7 M_0$ merger occurs at higher frequencies, reducing the total SNR (dashed orange line, $\rho_B$). Additionally, lower-mass binaries span a larger frequency range in a fixed observation time, simply because at fixed $r/r_S$ the frequency evolution rate $\dot{f}$ scales more steeply with frequency than chirp mass ($f \propto M_0^{5/3} \dot{f}^{1/3}$). In summary, the detectability of the gas deviation is tied to the stage of the coalescence we observe — binaries that are chirping in the
Relative SNR (i.e. the SNR of the deviation divided by the total SNR of the event) for the $\alpha = 0.03$ runs, with the same color key as in Fig. 10. This shows that a faster chirp signal is relatively less affected by the gas torques, but the gas imprint is still more detectable because of the higher total SNR of the event (seen in Fig. 10).

Figure 15. Top panel: Characteristic strain vs. observed frequency for 4-year observations of $q = 10^{-3}$ binaries at $z = 1$, varying the primary mass $M_1$ from $10^5 M_\odot$ to $10^7 M_\odot$. Solid lines correspond to early stages of evolution where the binary reaches a rest-frame separation of 10$R_S$. Dashed lines correspond to observing the final coalescence, ending when the binary merges at $r_{\text{ISCO}} = 3R_S$. In the legend we provide the accumulated SNR for the early (‘A’) and late (‘B’) observations, respectively. For lower binary mass, the merger occurs at a smaller $r_{\text{ISCO}}$ and correspondingly higher frequencies. This affects the detectability of the event. Bottom panel: SNR of gas deviation as a function of disc surface density given the observation windows depicted above, and using the average dimensionless gas torque from the $q = 10^{-3}$ and $\alpha = 0.03$ runs. A binary with $M_1 = 10^7 M_\odot$ accumulates a stronger waveform deviation as it spans a larger frequency range. Note that the vertical line for the $L_g$ estimate corresponds to a disc around an $M_1 = 10^6 M_\odot$ BH; see Eq. 13 for the weak scaling of the disc surface densities with BH mass.

A critical feature for distinguishing between various environmental effects and system parameters is the respective scaling of each effect with binary separation or GW frequency. For example, some proposed modifications to general relativity predict waveform deviations that would increase as the EMRI coalesces, scaling with a predictable

\[ \frac{\rho_{\text{obs}}}{\rho} \frac{\delta \phi}{\delta \phi} \]

\[ \tau = 4 \nu t \]

\[ \alpha = 0.03 \]

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5.2 Uniqueness and Degeneracies

It is important to consider whether the deviations we find in the waveform are degenerate with changes in the chirp mass or other system parameters, and/or with other possible environmental effects, and if such degeneracies may hinder parameter estimation or chances of detection.

cusp of LISA’s sensitivity are the most promising candidates for extracting a phase shift.

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**Figure 13.** Accumulated SNR of the deviation due to average torques in the $\alpha = 0.1$ runs, shaded up to our detectability threshold of $\rho_{\text{det}} \geq 8$. Higher viscosity generally produces stronger torques; hence the imprint is detectable at lower disc densities.
power of frequency\(^3\) (Yunes 2009). In principle a deviation to the waveform could be interpreted as a binary with different system parameters that also determine the frequency evolution, such as chirp mass \(M_c\). This can be distinguished by how various parameters affect the waveform as the frequency evolves. This dependence is often quoted in Fourier space, where deviations can be compared to Post-Newtonian (PN) terms in the Fourier phase. The scaling of changes in parameters with frequency can then be compared amongst system parameters, external effects, or modifications of general relativity.

Neglecting short-timescale fluctuations, our simulated gas torque scales with the analytical estimate \(T_0\), which is a function of the disc density profile. Given our disc model, \(T_{\text{gas}} \propto r^4 \Omega^2 \Sigma(r) \propto r^{1/2}\). The viscous torque also carries the same scaling with radius, given by \(T_v \propto r^2 \Omega v(r) \Sigma(r) \propto r^{1/2}\). This predicts that gas torques get progressively weaker as the binary coalesces: if \(f_{\text{GW}} \propto r^{-3/2}\), then the gas torque initially scales with GW frequency as \(T_{\text{gas}} \propto f_{\text{GW}}^{-1/3}\), and the integral for the deviation scales as \(\delta \phi \propto f^{-3/2}\). This frequency-scaling can be compared to the phasing function for a circular inspiral around a non-spinning BH, where 1.5 PN terms scale with frequency as \(\phi \propto A f^{2/3} + B f\) (where \(A\) and \(B\) are constants that depend on system parameters or GR corrections). However, confirming a phase drift to be of gas-disc origin, rather than other possible environmental effects, may only be possible for events that span a sufficiently large range of frequencies.

These considerations exclude evolutions in the dimensionless torque that can occur at the fastest inspiral rates, particularly for low-viscosity discs, or if the BH migrates through a disc where parameters (e.g. \(a, M\)) vary with radius. It also neglects the oscillations which occur in the torque which we see for all simulated inspiral rates. If measurable, these time-variable fluctuations could be a smoking-gun signature of disc response to an embedded IMRI. Ultimately the frequency dependence of the effect varies with disc physics, and changes in \(\alpha, M\), or accretion efficiency will lead to different scalings.

5.3 Caveats

This study provides a crucial first step in determining how gas torques respond to a GW-driven inspiral, but much work must be done to make more accurate observational predictions. These simulations are numerically challenging in that they require a large boundary (to avoid transient effects) and high spatial resolution (to resolve the gas in the immediate vicinity of the satellite BH), they must model the global disc (the small-scale gas dynamics near the satellite depends on the global disc), and they must be evolved for several thousand dynamical times for each set of parameters (to avoid transient behaviour reflecting the initial conditions). In the interest of computational efficiency, we have neglected several physical processes that are important for modeling a realistic system. We summarize some of our limitations here.

Our simulations do not resolve 3-dimensional gas morphologies which may be important for resolving accurate torques (Tanigawa et al. 2012; Szulágyi et al. 2014; Morbidelli et al. 2014), although in some parameter regimes 2D simulations are sufficiently accurate (e.g. see Lega et al. 2015; Urbe et al. 2011). In particular, the asymmetric density distribution that determines the gravitational torques is in some cases concentrated near or even inside the smoothing length. The dense pile-up is only a factor of ~two inside the smoothing length in the most extreme case (i.e. for \(q = 10^{-3}\); see Fig. 8). It is useful to keep in mind that unlike in N-body or smoothed particle hydrodynamics simulations, where smoothing is done purely for numerical stability, the smoothing we employ here is physically motivated: the smoothing of the potential mimics a vertical averaging of the gravitational forces. Nevertheless, the small-scale density distribution may be impacted by our choice of smoothing prescription, and 3-dimensional simulations will be necessary to fully understand how this morphology arises and if this gas distribution and the resulting torques are modified in 3D.

Our disc model is isothermal and does not include radiative cooling, heating, nor more sophisticated physics such as magnetohydrodynamics or radiation pressure, all of which may alter the gas dynamics near the BH. Future work must consider how gas morphology near the BH is affected by accretion rate and feedback from the BH itself, which may heat the gas in its vicinity and dampen the torque (Szulágyi et al. 2016).

The choice of accretion prescription and sink timescale of the embedded BH should also be considered carefully. Our estimate assumes that accretion occurs on the viscous timescale via a thin disc around the BH. However, given that the specific angular momentum of the gas with respect to the gap-opening perturber is low (and the resulting accretion torque is negligible, as discussed in Paper I), perhaps a quasi-spherical accretion prescription (i.e., Bondi accretion; Edgar 2004) would be more appropriate. The possibility for super-Eddington accretion rates should be considered, which can result in feedback that further affects the orbital properties of the BH (Gruzinov et al. 2020).

Finally, our simulations are purely Newtonian, neglecting any relativistic effects which can alter gas dynamics in the inner regions of the accretion disc closer to the primary BH. We assume the binary inspiral remains circular, when in fact gas discs may excite non-negligible eccentricity in the orbit (e.g. Goldreich & Sari 2003; D’Angelo et al. 2006), and this eccentricity may also produce additional modulations in the torque. Additionally, our estimates of binary evolution may be slightly inaccurate due to our use of the Peters (1964) quadrupole formula for the inspiral rate, which is lacking PN terms that become important near the ISCO (Zwick et al. 2019).

6 DISCUSSION

In the present work we analyse torque evolution during GW-driven inspirals in the intermediate mass ratio regime.
These sources, while their rates are less certain, provide the tantalizing possibility of probing nonlinear binary+gas-disc physics. At lower mass ratios, they evolve more slowly and quietly than near equal-mass MBH mergers, yet they maintain the ability to accumulate significant SNR.

In all cases, we observe short time-scale modulations in the torque throughout the inspiral. For most cases, the strength of the torque is on the order of simple analytical predictions (some fraction of $T_0$), but the precise value and the sign of the torque changes nontrivially with $q$ and $\alpha$. For our highest simulated mass ratio $q = 10^{-3}$, where gas pile-up on the BH is significant, torques are positive (outward), noisy, and show a distinct increase in variability at the fastest inspiral rates. The strength and evolution of the torque in this case is sensitive to accretion efficiency: less efficient accretion can lead to an increasingly positive torque as the binary coalesces. This effect may be amplified in more massive IMRIs, but this must be confirmed with future simulations. For lower-$q$ inspirals, we find that torques are smoother and become more negative at fast inspiral rates, depending on the viscosity. Overall, IMRIs with different system parameters can experience different torque evolution.

Unsurprisingly, the dependence of torque with inspiral rate itself depends on disc parameters, namely $M$ and $\alpha$. We find that inspirals in low $\alpha$ discs show deviations in the torque (compared to a constant dimensionless value) at earlier times. We interpret this as the disc’s inability to respond to the satellite BH’s increasing inspiral speed. Given the current understanding of viscosity in AGN discs (from estimates of $\alpha$ in MHD simulations), such low viscosities are unlikely. In the case of higher $\alpha$, torques may hold a relatively steady dependence on radius that scales with the viscous torque.

We find that fluctuations in the torque are highly dependent on the Mach number, or disc temperature. Hotter (low $M$) discs produce smoother, "well-behaved" torques, while thinner, colder discs exert torques that are stronger (scaling on average with the $T_0$ prediction) and more variable. Our simulations of a $M = 30$ disc yields a torque with strong variations that oscillate between positive and negative values. This suggests that IMRIs in thin, supersonic discs may experience the most dynamic gas effects.

The variability in torques throughout the inspiral may be attributed to interesting gas dynamics that warrants further investigation. For example, our $q = 10^{-3}$ simulations show peaks in the noise amplitude at $r \approx 8.6$ and $4R_S$ (most clearly seen in Fig. 9). These peaks occur at the same physical radii regardless of the simulation resolution, boundary location, or sink prescription. This raises the possibility that torques may show coherence with binary separation. We defer this analysis, as well as a closer look at the underlying cause of torque variability, to future work.

Assuming a gas-embedded IMRI event has sufficiently high SNR and spans a large enough frequency range that a frequency-dependent waveform deviation $\delta \phi(f)$ can be measured, it will not only confirm the deviation to be of gas origin (at the least ruling out other possibilities), it can also provide an invaluable measure of disc properties as a function of radius. While a measure of accumulated phase shift can place a constraint on the disc density $\Sigma_0$ in the most optimistic case a phase drift $\delta \phi(f)$ could reveal how the surface density changes with radius $r$, which is a distinguishing factor amongst several accretion disc models. Understanding the complex physical processes at play in the inner regions of AGN accretion discs is an active field of research. Recent radiation, magneto-hydrodynamic simulations by Jiang et al. (2019) predict that inner disc regions may have lower densities than predicted by the $\beta$-disc model, although the densities should increase with accretion rate and may change with central MBH mass.

We note that these results are all subject to the limitations of our simplistic disc model, where the Mach number and aspect ratio do not vary with radius. This approach allows us to investigate whether changes in the torque are truly due to the GW inspiral, and not due to encounters with varying disc dynamics. Previous works suggest that torques in response to an artificially imposed migration may change sign at fast migration rates (Duffell et al. 2014), but those simulations utilise a disc model with constant surface density, implying a radially dependent aspect ratio. Comparison between these types of studies highlights the importance of considering how different disc models may affect an inspiralling BH. On this basis, IMRIs embedded in more physically-motivated discs – in which the Mach number and disc structure changes with radius – may show more extreme changes in the torque, although this remains to be confirmed with more sophisticated models.

The presence or absence of a phase drift should be considered in conjunction with other characteristics of the source, particularly any tell-tale signs of gas discs. A likely signature of a gas-embedded E/IMRI would be the combination of a phase drift with low eccentricity: gas-embedded E/IMRIs should be distinctly less eccentric than those expected to occur in dry galactic nuclei. They may be close to circular, but with mild gas-driven eccentricity ($\epsilon \lesssim 0.2$, Ragusa et al. 2018, D’Orazio et al., in prep, Zrake et al. in prep) or eccentricity induced by other embedded perturbers. Additionally, if the accretion disc is aligned with the spin of the central MBH (this may only be true in some cases to varying degrees, see Volonteri et al. (2013) and references therein), the spins of the binary components may be closely aligned. However, this will depend on the history and nuances of accretion onto the disc-born BH, for which there are many uncertainties.

Our estimates of detectability of the deviation implicitly rely on the assumption that we have a waveform catalog of all possible I/EMRIs. Currently, the catalog of intermediate mass ratio waveforms is incomplete (Mandel & Gair 2009). A lack of available waveforms will affect the accuracy with which we can extract binary parameters from the signal, let alone detecting subtle environmental deviations (Cutler & Vallisneri 2007). To further complicate the picture, the range of possible AGN environments means that deviations may vary from system to system. In terms of data analysis, we suggest methods that search for generic deviations in a waveform, which can be informed by studies such as that presented here. In the optimistic case, one hopes that gas imprints can be traced back to constraints on the source environment. However, challenges will arise with circular, gas-embedded I/EMRIs that are in early stages of evolution, as these will arise in GW data as quasi-stationary sources. In these cases, assessing degeneracies, or disentangling the phase-shifted waveform between system parameters and var-
ious environmental effects, will not be possible if one cannot measure $f$ of the source within a limited observation time. Gas also provides the opportunity for electromagnetic emission that may coincide with the GW event. We do not address this issue here, but remind the reader that combining a phase shift with any associated EM signatures would be invaluable for confirming the presence of circumbinary gas, and for learning about AGN discs. For example, the coalescence of a gap-opening secondary may accompany a change in AGN continuum that correlates with the mass ratio (e.g. Gültekin & Miller 2012). These events may also be correlated with the fading observed in changing-look quasars, where the characteristic timescales suggest the change in emission is due to an abrupt change in the structure of the innermost accretion disc (Stern et al. 2018). Such changes could be driven by thermal or magnetic instabilities and possibly triggered by embedded perturbers.

7 CONCLUSIONS

In this paper, we analyse the gas torques on an intermediate mass ratio binary insular embedded in an accretion disc. We present a suite of simulations of IMRIs embedded in two-dimensional, near-Keplerian, isothermal accretion discs, where the satellite BH is modeled as a smoothed point-mass with a sink prescription. We analyse the torque exerted by the gas on the inspiraling BH for a range of mass ratios ($10^{-4} < q < 10^{-3}$), disc viscosities ($0.03 < \alpha < 0.1$), and Mach numbers ($10 < M < 30$). We also consider binaries at different states of the inspiral, and with and without allowing the satellite BH to accrete. Here we summarize our conclusions:

- As in similar numerical studies, we find that torques in the intermediate mass ratio, gap-opening regime have a nonlinear scaling with disc properties. Torques either slow down or speed up the inspiral; their strength is some fraction (~1%–120%) of the Type I torque $T_{\text{I}}$ (Tanaka et al. 2002), but these values are sensitive to small changes in $q$, $\alpha$, and $M$.

- During the inspiral, the torques exerted by the gas on the satellite BH show weak fluctuations, but the average strength of the torque ($T_{\Sigma}$ normalised by $T_{\text{I}}$) remains constant for the majority of inspiral rates in the LISA band. For the fastest inspiral rates, particularly for $q \approx 10^{-3}$ approaching the ISCO, torques exhibit an increase in variability originating from the gas flow within the BH’s Hill radius.

- We scale our simulation setup to a fiducial binary with primary mass $M = 10^6 M_\odot$ at $z = 1$ in order to compute the detectability of gas-induced deviations in the GW waveform. Using the average of $T_{\Sigma}$ for each mass ratio, we compute the accumulated phase shift and the corresponding SNR of the deviation as a function of disc density normalisation. We find that the phase shift is detectable (with relative SNR $\rho_{0\Delta} > 8$) when the source is embedded in a disc with surface density $\Sigma_0 \gtrsim 10^{-6}$ g cm$^{-2}$, depending on the mass ratio and disc viscosity. Detectability is maximized for the lowest events that are chirping significantly throughout a LISA observation, and coalesce at frequencies near ~5 x 10^{-5} Hz where LISA is most sensitive.

- This work is an important step towards understanding the scope of environmental impact on LISA sources due to circumbinary gas. Ultimately the strength, direction, and evolution of the torque exerted on a gas-embedded IMRI is dependent on the mass ratio and disc parameters, and the resulting waveform deviations can manifest in a variety of ways. A measure of a phase drift can provide a constraint on the disc surface density or disc structure, provided we have the tools to extract a variable deviation from the GW signal.

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