The Nature of Anomalous Period Increase in the Pulsating Variable V725 Sgr

Yu. A. Fadeyev1*

1Institute of Astronomy, Russian Academy of Sciences, Moscow, 119017 Russia

Received October 19, 2022; revised October 19, 2022; accepted October 26, 2022

Abstract—The evolutionary tracks of stars with masses on the main sequence $0.84 \, M_\odot \leq M_{\text{ZAMS}} \leq 0.95 \, M_\odot$ and initial metal abundances $Z = 0.006$ and $0.01$ were computed under various assumptions about the mass loss rate at the red giant stage as well as at the AGB and the post-AGB stages. Among 160 evolutionary sequences we selected nearly 30 sequences where the final thermal flash of the helium shell source occurs in the early post-AGB stage when the ratio of the hydrogen envelope mass to the stellar mass ranges from 0.01 to 0.08. Selected evolutionary sequences were used for calculation of initial and inner boundary conditions used in solution of the equations of radiation hydrodynamics and turbulent convection describing evolution of stellar pulsations after the helium flash. Among about three dozen hydrodynamic models we found the three ones demonstrating almost eightfold increase of the pulsation period observed in V725 Sgr during the last century as well as the gradual transformation of fairly regular pulsations with period $\Pi \approx 12$ day to semi–regular non-linear oscillations with period $80 \lesssim \Pi \lesssim 90$ day. We conclude that the anomalous growth of the pulsation period in V725 Sgr is due to the final thermal flash of the helium shell source that occurred in the early post-AGB star with mass $M \approx 0.53 \, M_\odot$ and the mass of the hydrogen envelope ranging from 0.013 to 0.019 $M_\odot$.

DOI: 10.1134/S1063773722100012

Keywords: stellar evolution, stellar pulsation, stars: variable and peculiar.

INTRODUCTION

The variable star HV 7642 was discovered by Swope (1937) and at present is known as V725 Sgr (Samus’ et al. 2017). Swope (1937) noted that the light curve of this variable resembles those of Cepheids but at the same time it is remarkably different from them due to the unusually high rate of period change. In particular, during almost ten years from 1926 to 1936 the pulsation period of V725 Sgr increased from 12 to 21 day (Swope 1937). The following photometric observations of this variable star were carried out in 1968 and 1969 by Demers (1973). He showed that V725 Sgr belongs to population II stars but should be classified as RV Tau or semi–regular type pulsating variable because of the significantly increased pulsation period ($45 \lesssim \Pi \lesssim 50$ day). The fact that V725 Sgr is the population II star was corroborated by Harris and Wallerstein (1984) who investigated the kinematic properties of type II cepheids. In 1973 the pulsation period of V725 Sgr was nearly 50 day (Demers and Madore 1974) and now is as high as $\Pi \approx 90$ day whereas the light variations became less regular (Percy 2020). Therefore, for the last century the pulsation period of V725 Sgr increased almost by a factor of eight so that this variable gradually transformed from the population II cepheid into the long-period semi–regular pulsating variable.

Percy et al. (2006) supposed that increase of the pulsation period observed in V725 Sgr is due to the thermal flash in the helium shell source. This assumption is based on comparison of the characteristic time of period change in V725 Sgr with theoretically computed rates of radius changes in AGB stars undergoing helium flashes (Vassiliadis and Wood 1993). Population II cepheids are the low-mass post-AGB stars so that the loop of the evolutionary track on the Hertzsprung–Russel (HR) diagram can cross the pulsation instability strip depending on the hydrogen envelope mass during the final thermal flash (Fadeyev 2020). Pulsation period increase observed in V725 Sgr is a unique phenomenon for population II cepheids and is of great interest to determine the fundamental parameters of the star by the methods of the stellar evolution and nonlinear stellar pulsation theories.

*E-mail: fadeyev@inasan.ru
The goal of the present study is that to verify the assumption proposed in Percy et al. (2006) on the basis of consistent evolutionary and stellar pulsation computations and to reproduce the period change observed in V725 Sgr. To solve this problem we computed the hydrodynamic models using the time-dependent inner boundary conditions describing evolutionary changes of the radius and the luminosity at the bottom of the stellar envelope model. In our preceding paper (Fadeyev 2022) we showed that in the framework of this approach the solution of the equations of radiation hydrodynamics and turbulent convection is completely consistent with results of stellar evolution calculations. It should be noted, however, that the necessary condition for the existence of the looped evolutionary track is that the thermal flash of the helium shell source should happen within the relatively short time interval (a few tens of thousands of the years) when the mass of the hydrogen envelope decreases from $\approx 8$ to $\approx 1\%$ of the stellar mass. The thermal flashes are not strictly periodic and for the average interflash interval $\langle \Delta t_{\text{ff}} \rangle \sim 2 \times 10^5$ yr the time interval between adjacent flashes varies by nearly 20\%. Therefore, not all the loops of evolutionary tracks may lead to observed changes in the pulsation period. Due to this fact in the present work we considered almost 160 theoretically computed evolutionary sequences of AGB and post-AGB stars. However, among the variety of initial conditions we succeeded to obtain only a few hydrodynamic models with period change which agrees with that observed in V725 Sgr.

EVOLUTIONARY SEQUENCES OF LOW-MASS POST-AGB STARS

Initial conditions required to compute the non-linear stellar oscillations were determined from evolutionary computations for stars with masses on the main sequence $0.84 \leq M_{\text{ZAMS}} \leq 0.95 M_\odot$. Metallicity of type II cepheids observed among field stars varies in a wide range and shows no strong dependence on the galactocentric distance (Harris 1981). Therefore, calculations of stellar evolution were carried out with two initial metal abundances: $Z = 0.006$ and 0.01 whereas the initial abundance of helium was assumed to be $Y = 0.28$.

To calculate the evolutionary sequences we employed the program MESA version r15140 (Paxton et al. 2019). Computational details of nucleosynthesis and convective mixing are discussed in our previous papers (Fadeyev 2020, 2021). Because the mass loss rates are very uncertain the evolutionary computations were carried out with various assumptions about the mass loss rate $\dot{M}$ on the stage preceding AGB (Reimers 1975) as well as on the AGB stage (Blöcker 1995). In particular, the Reimers mass loss rate formula was used with two values of the parameter ($\eta_R = 0.3$ and 0.5) whereas evolution on the AGB stage was computed with eight values of the parameter $\eta_B$ ranged within $0.03 \leq \eta_B \leq 0.1$ with step $\Delta \eta_B = 0.01$. In general we computed nearly 160 evolutionary sequences for the whole AGB and the early post-AGB stages.

Typical masses of stars at the end of the AGB stage are in the range $0.52 \leq M \leq 0.59 M_\odot$ whereas the rapidly decreasing mass of the hydrogen envelope $M_H$ is nearly one percent of the stellar mass. If the final thermal flash occurs when the ratio of the hydrogen envelope mass to the stellar mass is $M_H/M > 0.1$ the evolution of the post AGB star proceeds without loops at nearly constant luminosity between the red giant and the high-temperature region of the HR diagram ($T_{\text{eff}} \sim 10^5$ K). However, the post-AGB evolution is remarkably different if the final thermal flash occurs at smaller ratios $M_H/M$. Figure 1 shows the tracks of two evolutionary sequences $M_{\text{ZAMS}} = 0.88 M_\odot$, $\eta_R = 0.5$, $\eta_B = 0.08$ and $M_{\text{ZAMS}} = 0.92 M_\odot$, $\eta_R = 0.3$, $\eta_B = 0.07$ where the final thermal flash occurs for the ratios $M_H/M = 0.025$ and 0.013, respectively. The points of the evolutionary tracks corresponding to the maximum luminosity of the helium shell source $L_{3\alpha}$ are marked by the open circles. For a clearer presentation, the plots of the evolutionary tracks in Fig. 1 were deliberately truncated after the completion of the loop of the evolutionary track.

As seen in Fig. 1, decrease of the ratio $M_H/M$ at maximum $L_{3\alpha}$ is accompanied by displacement of the loop to higher effective temperatures. Of main interest in computation of hydrodynamic models of V725 Sgr is the point on the loop corresponding to the minimum stellar radius. In Fig. 1 these points are marked by filled circles. The period of radial pulsations is related to the stellar radius by $\Pi \propto R^{3/2}$ therefore the pulsation period reaches the minimum in the point of the minimal radius and then gradually increases during the following evolution. The main goal of the present study is to determine the theoretical dependence of the period of radial oscillations as a function of the star age $\Pi(t_{\text{ev}})$ therefore the necessary condition is that the star with minimal radius should reside within the instability strip.

To determine the conditions necessary for location of the model with minimal radius after the thermal flash within the instability strip we used the diagram shown in Fig. 2, where the effective temperature is plotted versus the ratio of the hydrogen envelope mass
Tracks of evolutionary sequences $M_{ZAMS} = 0.88$ $M_\odot$, $\eta_R = 0.5$, $\eta_B = 0.08$, $M_H/M = 0.025$ (a) and $M_{ZAMS} = 0.92$ $M_\odot$, $\eta_R = 0.3$, $\eta_B = 0.07$, $M_H/M = 0.013$ (b) for $Z = 0.006$ during the AGB and the early post-AGB stages. Open circles correspond to the maxima of $L_{3\alpha}$ and filled circles indicate the minimum radius of the star after the final helium flash.

Fig. 1. Tracks of evolutionary sequences $M_{ZAMS} = 0.88$ $M_\odot$, $\eta_R = 0.5$, $\eta_B = 0.08$, $M_H/M = 0.025$ (a) and $M_{ZAMS} = 0.92$ $M_\odot$, $\eta_R = 0.3$, $\eta_B = 0.07$, $M_H/M = 0.013$ (b) for $Z = 0.006$ during the AGB and the early post-AGB stages. Open circles correspond to the maxima of $L_{3\alpha}$ and filled circles indicate the minimum radius of the star after the final helium flash.

to the stellar mass $M_H/M$. For the sake of graphical clarity we present the plots for nearly 130 models with effective temperatures $T_{\text{eff}} < 4.5 \times 10^4$ K. As seen in Fig. 2, effective temperatures of models with minimal stellar radius do not show dependence on metallicity $Z$ (the plots corresponding to evolutionary sequences with $Z = 0.006$ and 0.01 are shown by circles and triangles, respectively) and depend only on $M_H/M$.

It follows from the calculations of non-linear pulsations of population II cepheids carried out by the author earlier (Fadeyev 2020) as well as in the present study that in the luminosity range $10^2 L_\odot \lesssim L \lesssim 10^3 L_\odot$ the edges of pulsation instability nearly correspond to effective temperatures $T_{\text{eff}} \approx 4 \times 10^3$ K (the red edge) and $T_{\text{eff}} \approx 6 \times 10^3$ K (the blue edge). These estimates of $T_{\text{eff}}$ agree with empirical results.
The effective temperature of the star with minimal radius versus the ratio of the hydrogen envelope to the stellar mass $M_{\text{H}}/M$. The circles and the triangles correspond to the evolutionary sequences computed with $Z = 0.006$ and $0.01$, respectively. Dashed lines indicate effective temperatures at the red ($t_{\text{eff}} = 4 \times 10^3 \text{ K}$) and at the blue ($t_{\text{eff}} = 6 \times 10^3 \text{ K}$) edges of the instability strip.

Therefore, to compute the hydrodynamic model we have to use the evolutionary sequences where the final thermal flash occurs for $0.02 \lesssim M_{\text{H}}/M \lesssim 0.08$. It should be noted that these values of the ratio $M_{\text{H}}/M$ provide with one of necessary conditions of applicability of the evolutionary sequence. Verification of another condition assuming that the pulsation period of the star with minimal radius is $\Pi \leq 12 \text{ day}$ can be obtained only from trial calculations. Moreover, evolutionary increase of the stellar radius after the radius minimum is accompanied by decrease of the effective temperature (see Fig. 1) so that many models locating near the lower (red) edge in Fig. 2 were found to be inapplicable for modeling of V725 Sgr since they show decaying oscillations after crossing of the red edge of the instability strip.

HYDRODYNAMIC MODELS OF V725 Sgr

Solution of the equations of radiation hydrodynamics and time—dependent convection describing radial stellar oscillations (Fadeyev 2013) was obtained on the finite difference grid consisting of 600 Lagrangian mass zones. Five hundred outer mass intervals increase inward from the upper boundary to the stellar center geometrically whereas 100 inner intervals reduce with another value of the common ratio. Such a distribution of Lagrangian mass zones allowed us to avoid the large approximation errors in the inner layers of the pulsating envelope where the gradients of pressure and temperature sharply increase. The inner boundary of hydrodynamic models is set in the layers with the gas temperature $T \sim 5 \times 10^6 \text{ K}$ and the radius $r_0 \sim 10^{-2}R$, where $R$ is the radius of the upper boundary of the evolutionary model. The boundary separating the regions with a different behavior of the mass interval locates in the layers with temperature $T \sim 5 \times 10^5 \text{ K}$ where the
mass of outer layers is $\approx 2/3$ of the stellar envelope mass.

During the helium flash the structure of the stellar envelope changes in the thermal time scale so that the solution of the Cauchy problem for equations of hydrodynamics was obtained with time—dependent inner boundary conditions explicitly describing temporal variations of the radius and the luminosity: $r_0(t_{\text{ev}})$ and $L_0(t_{\text{ev}})$. These dependences were determined from evolutionary computations for the fixed value of the Lagrangian coordinate whereas the continuous functions $r_0(t_{\text{ev}})$ and $L_0(t_{\text{ev}})$ needed in hydrodynamic computations were calculated using the cubic interpolating splines. This method was earlier employed for explanations of abrupt decrease of pulsation amplitude in RU Cam (Fadeyev 2021) and for modeling of the Mira-type pulsating variable T UMi undergoing the thermal flash in the helium shell source (Fadeyev 2022).

Evolution of stellar pulsations after the helium flash is illustrated in Fig. 3 by the plots of the upper boundary radius of the hydrodynamic model $Z = 0.01$, $M_{\text{ZAMS}} = 0.9 \ M_\odot$, $\eta_R = 0.5$, $\eta_B = 0.03$ in vicinity of the minimum radius of the evolutionary model $t_{\text{ev}} = 0$ (a) and after 70 yr (b).

As seen in Fig. 3, in vicinity of the minimum radius the stellar pulsations are characterized by sufficiently small amplitude of the radial displacement:

![Graph showing variation with time of the upper boundary radius of the hydrodynamic model](image-url)
ΔR/⟨R⟩ ≈ 0.12, where ⟨R⟩ is the average radius of the upper boundary of the hydrodynamic model. After 70 yr the relative amplitude increases up to ΔR/⟨R⟩ ≈ 0.6. Simultaneously, during the 70-yr time interval the period of radial pulsations increases from Π = 13.9 to 62 d, respectively, so that stellar oscillations become non-linear and less regular. Therefore, the hydrodynamic model shown in Fig. 3 qualitatively reproduces the main evolutionary features observed in V725 Sgr.

Figure 4 shows the observational estimates of the pulsation period of V725 Sgr obtained by Swope (1937), Demers (1973), Demers and Madore (1974), Wehlau et al. (2006) and Percy et al. (2006). Theoretical dependences Π(τ_{ev}) obtained from calculations of three hydrodynamic models are shown also in Fig. 4. It should be noted that for the sake of convenience the plots of Π(τ_{ev}) are shifted along the horizontal axis to fit the minimum period to the date \( t = 1926 \) yr when the pulsation period of V725 Sgr was 12 days. Main properties of hydrodynamic models shown in Fig. 4 are listed in Table 1.

### Table 1. Hydrodynamic models of V725 Sgr

| Z    | M_{ZAMS}/M_⊙ | η_R | η_B | M/M_⊙ | M_{H}/M | Π_{min}, d |
|------|-------------|-----|-----|-------|---------|------------|
| 0.006| 0.84       | 0.5 | 0.05| 0.527 | 0.037   | 11.3       |
| 0.006| 0.88       | 0.5 | 0.08| 0.534 | 0.025   | 14.2       |
| 0.01  | 0.90       | 0.5 | 0.03| 0.534 | 0.020   | 13.9       |

**CONCLUSIONS**

Results of stellar evolution and non-linear stellar pulsation calculations presented above confirm the hypothesis by Percy et al. (2006) that the period change observed in V725 Sgr is due to the thermal flash of the helium burning source of the
population II low-mass post-AGB star. Moreover, a satisfactory agreement between three theoretical dependences \( \Pi(t_{\text{ev}}) \) and the observed secular period change in V725 Sgr allowed us to obtain approximate theoretical estimates of the stellar mass and the mass of the hydrogen envelope: \( M \approx 0.53 \, M_\odot \), \( 0.013 \leq M_H \leq 0.019 \, M_\odot \). It should be noted that uncertainties in estimates of the stellar mass are due to both the significant scatter and the small number of observational estimates of the pulsation period of V725 Sgr. The minimum value of the pulsation period of V725 Sgr in the beginning of the 20th century remains unknown because all available results of observations show the period growth. Nevertheless, results of our computations allow us to assume that the minimum value of the period can only slightly differ from the value \( \Pi = 12 \, \text{day} \). This is due to the fact that the common feature of hydrodynamic models is that the characteristic time of period growth increases with increasing \( \Pi_{\text{min}} \). This is illustrated in Fig. 5 where we plot three dependences of period change \( \Pi(t_{\text{ev}}) \) obtained from computation of hydrodynamic models \( Z = 0.006, M_{\text{ZAMS}} = 0.84 \, M_\odot, \eta_R = 0.3, \eta_B = 0.05 \) (dashed line, \( \Pi_{\text{min}} = 22.5 \, \text{day} \)), \( Z = 0.006, M_{\text{ZAMS}} = 0.88 \, M_\odot, \eta_R = 0.5, \eta_B = 0.08 \) (solid line, \( \Pi_{\text{min}} = 14.3 \, \text{day} \)), and \( Z = 0.006, M_{\text{ZAMS}} = 0.84 \, M_\odot, \eta_R = 0.5, \eta_B = 0.06 \) (dotted line, \( \Pi_{\text{min}} = 10.0 \, \text{day} \)). The evolutionary time is set to zero at the minimum of the stellar radius.

**REFERENCES**

1. T. Blöcker, Astron. Astrophys. 297, 727 (1995).
2. S. Demers, J. R. Astron. Soc. Canada 67, 19 (1973).
3. S. Demers and B. F. Madore, Inform. Bull. Var. Stars 870, 1 (1974).
4. S. Demers and W. E. Harris, Astron. J. 79, 627 (1974).
5. Yu. A. Fadeyev, Astron. Lett. 39, 306 (2013).
6. Yu. A. Fadeyev, Astron. Lett. 46, 734 (2020).
7. Yu. A. Fadeyev, Astron. Lett. 47, 765 (2021).
8. Yu. A. Fadeyev, Mon. Not. R. Astron. Soc. 514, 5996 (2022).
9. H. C. Harris, Astron. J. 86, 719 (1981).
10. H. C. Harris and G. Wallerstein, Astron. J. 89, 379 (1984).
11. B. Paxton, R. Smolec, J. Schwab, A. Gautschy, L. Bildsten, M. Cantiello, A. Dotter, R. Farmer, J. A. Goldberg, A. S. Jermyn, S. M. Kanbur, P. Marchant, A. Thoul, R. H. D. Townsend, W. M. Wolf, M. Zhang, and F. X. Timmes, Astrophys. J. Suppl. Ser. 243, 10 (2019).
12. J. R. Percy, J. Am. Assoc. Var. Star Observ. 48, 162 (2020).
13. J. R. Percy, A. Molak, H. Lund, D. Overbeek, A. F. Wehlau, and P. F. Williams, Publ. Astron. Soc. Pacif. 118, 805 (2006).
14. D. Reimers, in Problems in Stellar Atmospheres and Envelopes, Ed. by B. Baschek, W. H. Kegel, and G. Traving (Springer, New York, 1975), p. 229.
15. N. N. Samus’, E. V. Kazarovets, O. V. Durlevich, N. N. Kireeva, and E. N. Pastukhova, Astron. Rep. 61, 80 (2017).
16. H. H. Swope, Ann. Harvard College Observ. 105, 499 (1937).
17. E. Vassiliadis and P. R. Wood, Astrophys. J. 413, 641 (1993).
18. A. Wehlau, T. Atcheson, and S. Demers, J. Am. Assoc. Var. Star Observ. 35, 187 (2006).

Translated by Yu. Fadeyev