Chiral loop corrections and isospin violation effects in $\varepsilon'/\varepsilon$

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A complete analysis of isospin breaking in $K \to 2\pi$ amplitudes, including both strong ($m_u \neq m_d$) and electromagnetic corrections at next-to-leading order in chiral perturbation theory, has been achieved recently \[.\] We discuss the implication of these effects \[2\], together with the previously known chiral loop corrections \[3,4\], on the direct CP-violating ratio $\varepsilon'/\varepsilon$.

1. INTRODUCTION

The CP-violating ratio $\varepsilon'/\varepsilon$ constitutes a fundamental test for our understanding of flavour-changing phenomena within the Standard Model framework. The experimental status has been clarified by the recent KTEV \[5\], $\text{Re} (\varepsilon'/\varepsilon) = (20.7 \pm 2.8) \cdot 10^{-4}$, and NA48 \[6\], $\text{Re} (\varepsilon'/\varepsilon) = (14.7 \pm 2.2) \cdot 10^{-4}$, measurements. The present world average \[\text{KTEV, NA48}\].

$\text{Re} (\varepsilon'/\varepsilon) = (16.7 \pm 1.6) \cdot 10^{-4}$, \hspace{1cm} (1)

provides clear evidence for a non-zero value and, therefore, the existence of direct CP violation.

The CP violating signal is generated through the interference of two possible $K^0 \to \pi\pi$ decay amplitudes with different weak and strong phases,

$$\frac{\varepsilon'}{\varepsilon} = e^{\Phi} \frac{\omega}{\sqrt{2} |\varepsilon|} \left[ \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right].$$ \hspace{1cm} (2)

The isospin amplitudes $A_{0,2}$ are defined through

$$A(K^0 \to \pi^+ \pi^-) = A_0 e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2 e^{i\chi_2},$$

$$A(K^0 \to \pi^0 \pi^0) = A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2},$$

$$A(K^+ \to \pi^+ \pi^0) = \frac{3}{2} A_2^+ e^{i\chi_2^+}.$$ \hspace{1cm} (3)

In the limit of CP conservation, $A_0, A_2$, and $A_2^+$ are real and positive. In the isospin limit, $A_2 = A_2^+$, $\chi_2 = \chi_2^+$ in the Standard Model and the phases $\chi_i$ coincide with the corresponding $\pi\pi$ phase shifts at $E_{cm} = M_K$.

Owing to the well-known “$\Delta I = 1/2$ rule”, $\varepsilon'/\varepsilon$ is suppressed by the ratio $\omega = \text{Re} A_2/\text{Re} A_0 \approx 1/22$. The strong $S$–wave rescattering of the two final pions generates a large phase-shift difference between the two isospin amplitudes, making the phases of $\varepsilon'$ and $\varepsilon$ nearly equal. Thus,

$$\Phi \approx \chi_2 - \chi_0 + \frac{\pi}{4} \approx 0.$$ \hspace{1cm} (4)

The large $\pi\pi$ phase-shift difference clearly indicates that unitarity corrections (final state interactions) play a crucial role in $\varepsilon'/\varepsilon$ \[3,4\]. Moreover, this observable is very sensitive to isospin breaking effects, because the large ratio $1/\omega$ amplifies any potential contribution to $A_2$ from small isospin-breaking corrections induced by $A_0$.

The CP–conserving amplitudes $\text{Re} A_I$, their ratio $\omega$ and $\varepsilon$ are usually set to their experimentally determined values. A theoretical calculation is then only needed for the quantities $\text{Im} A_I$.

2. THEORETICAL FRAMEWORK

To obtain the Standard Model prediction for $\varepsilon'/\varepsilon$, one starts at the electroweak scale where the flavour–changing process, in terms of quarks and gauge bosons, can be analyzed in a rather straightforward way. Owing to the presence of very different mass scales ($M_\pi < M_K \ll M_W$), the gluonic corrections are amplified by large logarithms. The short-distance logarithmic corrections can be summed up using the Operator Product Expansion (OPE) and the renormalization group, all the way down to scales $\mu < m_\varepsilon$. One
Moreover, the full order in of any ratio of heavy mass scales includes all corrections of to-leading logarithmic order \[12,13\]. This in long–distance contributions, which are contained amplitudes are of course independent of renormalization scale [9,10]. The overall parameters: \[ C_i(\mu) \equiv z_i(\mu) - y_i(\mu) V_{td} V_{ts}^* / V_{ud} V_{us}^* \].

Only the \( y_i \) components are needed to determine the CP–violation decay amplitudes. The overall renormalization scale \( \mu \) separates the short– and long–distance contributions, which are contained in \( C_i(\mu) \) and \( Q_i \), respectively. The physical amplitudes are of course independent of \( \mu \).

The Wilson coefficients are known at the next-to-leading logarithmic order [12,13]. This includes all corrections of \( O(\alpha_t^2 t^n) \) and \( O(\alpha_t^{n+1} t^n) \), where \( t \equiv \ln(M_1/M_2) \) refers to the logarithm of any ratio of heavy mass scales \( M_1, M_2 \geq \mu \). Moreover, the full \( m_t/M_W \) dependence (at lowest order in \( \alpha_s \)) is taken into account.

In order to predict physical amplitudes, one is still confronted with the calculation of hadronic matrix elements of the four–quark operators. This is a very difficult problem, which so far remains unsolved. Those matrix elements are usually parameterized in terms of the so-called bag parameters \( B_i \), which measure them in units of their vacuum insertion approximation values.

To a very good approximation, the Standard Model prediction for \( \varepsilon'/\varepsilon \) can be written (up to global factors) as [14,15]

\[
\frac{\varepsilon'}{\varepsilon} \sim \left[ B_6^{(1/2)} (1 - \Omega_{IB}) - 0.4 B_8^{(3/2)} \right].
\]  

Thus, only two operators are numerically relevant: the QCD penguin operator \( Q_6 \) governs \( \Delta I = 1/2 \), while \( \Delta I = 3/2 \) is dominated by the electroweak penguin operator \( Q_8 \). The parameter \( \Omega_{IB} \) takes into account isospin breaking corrections, which get enhanced by the factor \( 1/\omega \). The value \( \Omega_{IB} = 0.25 \) was adopted in many calculations [14,16]. Together with \( B_1 \sim 1 \), this produces a large numerical cancellation in eq. [7] leading to unphysical low values of \( \varepsilon'/\varepsilon \) around \( 7 \times 10^{-4} \). The true Standard Model prediction is then very sensitive to the precise values of these parameters.

3. CHIRAL PERTURBATION THEORY

Below the resonance region one can use global symmetry considerations to define another effective field theory (EFT) in terms of the QCD Goldstone bosons \( (\pi, K, \eta) \). The chiral perturbation theory (\( \chi PT \)) formulation of the Standard Model [20,21,22] describes the pseudoscalar–octet dynamics, through a perturbative expansion in powers of momenta and quark masses over the chiral symmetry breaking scale \( \Lambda_{\chi} \sim 1 \) GeV.

Chiral symmetry fixes the allowed operators. At lowest order in the chiral expansion, the most general effective bosonic Lagrangian with the same \( SU(3)_L \otimes SU(3)_R \) transformation properties as the short–distance Lagrangian [15] contains three terms, transforming as \( (8_L, 1_R), (27_L, 1_R) \) and \( (8_L, 8_R) \), respectively. Their corresponding chiral couplings are denoted by \( g_{8_L} \), \( g_{27_L} \) and \( g_{8_W} \).

The tree–level \( K \to \pi \pi \) amplitudes generated
by the lowest–order $\chi$PT Lagrangian,
\[
A_0 = -\frac{G_F V_{ud} V_{us}^* \sqrt{2} f_\pi}{\sqrt{2}} \left\{ \left( g_8 + \frac{1}{9} g_{27} \right) (M_K^2 - M_{\pi}^2) - \frac{2}{3} f_\pi^2 e^2 g_{EW} \right\},
\]
\[
A_2 = -\frac{G_F V_{ud} V_{us}^* 2 f_\pi}{9} \left\{ 5 g_{27} (M_K^2 - M_{\pi}^2) - 3 f_\pi^2 e^2 g_{EW} \right\},
\]
(8)
do not contain any strong phases. From the measured decay rates one gets \[23\] $|g_8| \approx 5.1$ and $|g_{27}| \approx 0.29$. The $g_{EW}$ term is the low–energy realization of the electroweak penguin operator.

The only remaining problem is the calculation of the chiral couplings from the effective short–distance Lagrangian \[24,25\], because in this limit the four–quark operators factorize into currents which have well–known chiral realizations:

\[
g_8 = \frac{3}{5} C_2 - \frac{2}{5} C_1 + C_4 - 16 L_5 \left( \frac{\langle q\bar{q} \rangle (\mu)}{f_\pi^2} \right)^2 C_6,
\]
\[
g_{27} = \frac{3}{5} (C_1 + C_2),
\]
\[
g_{EW}^\infty = -3 \left( \frac{\langle q\bar{q} \rangle (\mu)}{e f_\pi^3} \right)^2 C_8.
\]
(9)

Together with eqs. \[8\], these results are equivalent to the standard large–$N_C$ evaluations of the $B_i$ factors. In particular, for $\varepsilon'/\varepsilon$ where only the imaginary part of the $g_i$ couplings matter eqs. \[6\] amount to $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$. Therefore, up to minor variations on some input parameters, the corresponding $\varepsilon'/\varepsilon$ prediction, obtained at lowest order in both the $1/N_C$ and $\chi$PT expansions, reproduces the published results of the Munich \[14\] and Rome \[18\] groups.

The large–$N_C$ limit is only applied to the matching between the 3–flavour quark theory and $\chi$PT, as indicated in Figure \[1\]. The evolution from the electroweak scale down to $\mu < m_c$ has to be done without any unnecessary expansion in powers of $1/N_C$; otherwise, one would miss large corrections of the form $\frac{e^2}{\mu} \ln (M/m)$, with $M \gg m$ two widely separated scales \[20\]. Thus, the Wilson coefficients contain the full $\mu$ dependence.

The large–$N_C$ factorization of the four–quark operators $Q_i$ ($i \neq 6,8$) does not provide any scale dependence, because their anomalous dimensions vanish when $N_C \to \infty$ \[20\]. To achieve a reliable expansion in powers of $1/N_C$, one needs to go to the next order where this physics is captured \[27\]. This is the reason why the study of the $\Delta I = 1/2$ rule has proved to be so difficult. Fortunately, these operators are numerically irrelevant in the $\varepsilon'/\varepsilon$ prediction.

The only anomalous dimensions which survive when $N_C \to \infty$ are precisely the ones corresponding to $Q_6$ and $Q_8$ \[17,20\]. These operators factorize into colour–singlet scalar and pseudoscalar currents, which are $\mu$ dependent. This generates the factors

\[
\langle q\bar{q} \rangle (\mu) = -\frac{f_\pi^2 M_\pi^2}{(m_u + m_d)(\mu)} = -\frac{f_\pi^2 M_\pi^2}{(m_s + m_d)(\mu)}
\]
in eqs. \[19\], which exactly cancel the $\mu$ dependence of $C_{6,8}(\mu)$ at large $N_C$ \[17,20,27,28\]. It remains of course a dependence at next-to-leading order. Thus, while there are large $1/N_C$ corrections to $\text{Re}(g_1)$, the large–$N_C$ limit can be expected to give a good estimate of $\text{Im}(g_1)$ \[27\].

4. CHIRAL CORRECTIONS

The strong phases $\chi_I$ originate in the final rescattering of the two pions and, therefore, are generated by chiral loops which are of higher order in both the momentum and $1/N_C$ expansions. Analyticity and unitarity require the presence of a corresponding dispersive effect in the moduli of the isospin amplitudes. Since the $S$–wave strong phases are quite large, specially in the isospin–zero case, one should expect large higher–order unitarity corrections.

The one–loop analyses of $K \to 2\pi$ \[13,14,29\] show in fact that pion loop diagrams provide an important enhancement of the $A_0$ amplitude, implying a sizeable reduction ($\sim 30\%$) of the fitted $|g_8|$ value. This chiral loop correction destroys the accidental numerical cancellation in eq. \[17\], generating a sizeable enhancement of the $\varepsilon'/\varepsilon$ prediction \[14\]. The large one–loop correction to $A_0$ is
associated with large infrared logarithms involving the light pion mass.

A complete one-loop calculation, including electromagnetic and isospin violation corrections, has been achieved recently \([1]\). It involves the \(O(p^6)\) strong \([21,30]\) and \(O(e^2p^2)\) \([31,32]\) electromagnetic chiral lagrangians, together with the non-leptonic \(O(G_Fp^4)\) \([33,34]\) and \(O(G_Fe^2p^2)\) \([35,36]\) electroweak lagrangians.

### 4.1. \(O(p^4)\) \(\chi PT\)

It is convenient to decompose the isospin amplitudes \(A_I = A_I e^{i\chi I}\) in their different \(SU(3)_L \otimes SU(3)_R\) components. The \(O(p^4)\) correction to a given lowest-order amplitude \(a_I^{(X)}\),

\[
A_I^{(X)} = a_I^{(X)} \left[ 1 + \Delta_L A_I^{(X)} + \Delta_C A_I^{(X)} \right],
\]

contains a one-loop contribution \(\Delta_L A_I^{(X)}\) which is completely fixed by chiral symmetry plus a local contribution generated by the corresponding higher-order chiral lagrangian. The most relevant loop corrections take the values \([13]\):

\[
\begin{align*}
\Delta_L A_0^{(8)} &= (0.27 \pm 0.05) + 0.47i, \\
\Delta_L A_0^{(27)} &= (1.02 \pm 0.60) + 0.47i, \\
\Delta_L A_0^{(ew)} &= (0.27 \pm 0.05) + 0.47i, \\
\Delta_L A_2^{(27)} &= (-0.04 \pm 0.05) - 0.21i, \\
\Delta_L A_2^{(ew)} &= (-0.50 \pm 0.20) - 0.21i.
\end{align*}
\]

The dispersive components depend on the chiral renormalization scale \(\nu_\chi\), which has been fixed at \(\nu_\chi = 0.77\) GeV. The quoted uncertainties reflect the changes under a variation of \(\nu_\chi\) between 0.6 and 1 GeV plus a small contribution from varying the short-distance scale \(\mu\) between 0.77 and 1.3 GeV. Notice that the most relevant correction \(\Delta_L A_0^{(8)}\) has a very small uncertainty because it is dominated by the non-polynomial part, which is associated with the large isoscalar absorptive contribution and does not depend on \(\nu_\chi\).

The local contributions \(\Delta_C A_I^{(X)}\) have been computed at leading order in the \(1/N_C\) expansion. At this order, there is matching ambiguity because we do not know at which value of the chiral scale the estimates apply. This is taken into account by the \(\nu_\chi\) uncertainty incorporated in \(\Delta_L A_I^{(X)}\). Whenever the absorptive loop correction is large, the final prediction for \(A_I^{(X)}\) is quite insensitive to the values of the low-energy chiral couplings adopted in \(\Delta_C A_I^{(X)}\).

The CP-conserving parts of the low-energy couplings \(\text{Re}(g_8)\) and \(\text{Re}(g_{27})\) have been fitted to the data, together with the phase-shift difference \(\chi_0 - \chi_2\).

### 4.2. Isospin breaking in \(\varepsilon'/\varepsilon\)

There are two sources of isospin breaking: the light quark mass difference \(m_u - m_d\) and electromagnetic corrections. Accounting for isospin violation via the general parametrization \((3)\), the ratio \(\omega\) differs from \(\omega_+ = \text{Re}A_2^\prime /\text{Re}A_0\) by a pure \(\Delta I = 5/2\) effect:

\[
\omega = \omega_+ (1 + f_{5/2}) \quad ; \quad f_{5/2} = \frac{\text{Re}A_2}{\text{Re}A_2^\prime} - 1.
\]

Since \(\omega_+\) is directly related to branching ratios, it proves useful to keep \(\omega_+\) in the normalization of \(\varepsilon'\) \([37]\). The formula for \(\varepsilon'\) takes then the form:

\[
\frac{\varepsilon'}{\varepsilon} = \frac{\varepsilon' e^{i\chi_+}}{\sqrt{2|\varepsilon|}} \left[ \frac{\text{Im}A_0^{(0)}}{\text{Re}A_0^{(0)}} (1 + \Delta_0 + f_{5/2}) - \frac{\text{Im}A_2^{(0)}}{\text{Re}A_2^{(0)}} \right] = \frac{\varepsilon' e^{i\chi_+}}{\sqrt{2|\varepsilon|}} \left[ \frac{\text{Im}A_0^{(0)}}{\text{Re}A_0^{(0)}} (1 - \Omega_{\text{eff}}) - \frac{\text{Im}A_2^{\text{emp}}}{\text{Re}A_2^{(0)}} \right],
\]

where

\[
\Delta_0 = \frac{\text{Im}A_0^{(0)}}{\text{Im}A_0^{(0)}} \cdot \frac{\text{Re}A_0^{(0)}}{\text{Re}A_0^{(0)}} - 1,
\]

\[
\Omega_{\text{IB}} = \frac{\text{Re}A_0^{(0)}}{\text{Re}A_2^{(0)}} \cdot \frac{\text{Im}A_2^{\text{non-emp}}}{\text{Im}A_0^{(0)}}
\]

and the superscript (0) denotes the isospin limit. The quantity \([2]\)

\[
\Omega_{\text{eff}} = \Omega_{\text{IB}} - \Delta_0 - f_{5/2}
\]

includes all effects to leading order in isospin breaking and it generalizes the more traditional parameter \(\Omega_{\text{IB}}\). We have adopted the usual (but scheme dependent) separation of the electroweak penguin contribution to \(\text{Im}A_2\), \(\text{Im}A_2^{\text{emp}}\), from the effects of the other four-quark operators.
Table 1
Isospin violating corrections for $\varepsilon'$ in units of $10^{-2}$ [2]. LO and NLO denote leading and leading plus next-to-leading orders in $\chi$PT.

|       | $\alpha = 0$ | $\alpha \neq 0$ |
|-------|--------------|-----------------|
|       | LO | NLO | LO | NLO |
| $\Omega_{\text{IB}}$ | 12 | 16 $\pm$ 5 | 18 $\pm$ 7 | 22.7 $\pm$ 7.6 |
| $\Delta_0$ | $\approx$ 0 | $-0.4 \pm 0.1$ | 9 $\pm$ 3 | 8.3 $\pm$ 3.6 |
| $f_{5/2}$ | 0 | 0 | 0 | 8.3 $\pm$ 2.4 |
| $\Omega_{\text{eff}}$ | 12 | 16 $\pm$ 5 | 9 $\pm$ 6 | 6.0 $\pm$ 7.7 |

Although $\Omega_{\text{IB}}$ is enhanced by the ratio $1/\omega^{(0)}$, the numerical analysis shows all three terms in [19] to be relevant when both strong and electromagnetic isospin violation are included. The different corrections are shown in Table 1 where the first two columns refer to strong isospin violation only ($m_u \neq m_d$) and the last two contain the complete results including electromagnetic corrections. Taking $\alpha = 0$, the isospin breaking is completely dominated by the $\pi^0 - \eta$ mixing contribution $\Omega_{\text{IB}}^{\pi^0 - \eta} = 0.16 \pm 0.03$ [38]. Electromagnetic effects give sizeable contributions to all three terms, generating a destructive interference and a smaller final value [2]

$$\Omega_{\text{eff}} = (6.0 \pm 7.7) \cdot 10^{-2}$$

for the overall measure of isospin violation in $\varepsilon'$.

5. DISCUSSION

The infrared effect of chiral loops generates an important enhancement of the isoscalar $K \to \pi\pi$ amplitude. This effect gets amplified in the prediction of $\varepsilon'/\varepsilon$, because at lowest order (in both $1/N_C$ and the chiral expansion) there is an accidental numerical cancellation between the $I = 0$ and $I = 2$ contributions. Since the chiral loop corrections destroy this cancellation, the final result for $\varepsilon'/\varepsilon$ is dominated by the isoscalar amplitude. The small value obtained for $\Omega_{\text{eff}}$ [2] reinforces the dominance of the gluonic penguin operator $Q_6$. Taking this into account, the Standard Model prediction for $\varepsilon'/\varepsilon$ [3] turns out to be

$$\text{Re}(\varepsilon'/\varepsilon) = (1.8 \pm 0.2 ^{+0.5}_{-0.6} \pm 0.5) \cdot 10^{-3},$$

in excellent agreement with the experimental measurement [1]. The first error has been estimated by varying the renormalization scale $\mu$ between $M_\rho$ and $m_c$. The uncertainty induced by $m_s$ [39], which has been taken in the range [3] $(m_s + m_d)(1 \text{ GeV}) = 156 \pm 25 \text{ MeV}$, is indicated by the second error.

The most critical step is the matching between the short and long–distance descriptions, which has been done at leading order in $1/N_C$. Since all next-to-leading ultraviolet and infrared logarithms have been taken into account, our educated guess for the theoretical uncertainty associated with subleading contributions is $\sim 30\%$ (third error). While a better determination of $m_s$ can be expected soon, the control of these nonlogarithmic corrections at the next-to-leading order in $1/N_C$ remains a challenge for future investigations [40].

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