PROSPECTS FOR CHARACTERIZING HOST STARS OF THE PLANETARY SYSTEM DETECTIONS PREDICTED FOR THE KOREAN MICROLENSING TELESCOPE NETWORK

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ABSTRACT

I investigate the possibility of constraining the flux of the lens (i.e., host star) for the types of planetary systems the Korean Microlensing Telescope Network is predicted to find. I examine the potential to obtain lens flux measurements by (1) imaging the lens once it is spatially resolved from the source, (2) measuring the elongation of the point-spread function of the microlensing target (lens+source) when the lens and source are still unresolved, and (3) taking prompt follow-up photometry. In each case I simulate the observing programs for a representative example of current ground-based adaptive optics (AO) facilities (specifically NACO on the Very Large Telescope), future ground-based AO facilities (GMTIFS on the Giant Magellan Telescope, GMT), and future space telescopes (NIRCAM on the James Webb Space Telescope, JWST). Given the predicted distribution of relative lens–source proper motions, I find that the lens flux could be measured to a precision of $\sigma_{\pi} \lesssim 0.1$ for $\gtrsim 60\%$ of planet detections $\geq 5$ yr after each microlensing event for a simulated observing program using GMT, which images resolved lenses. NIRCAM on JWST would be able to carry out equivalently high-precision measurements for $\sim 28\%$ of events $\Delta t = 10$ yr after each event by imaging resolved lenses. I also explore the effects of various blend components which would have on the mass derived from prompt follow-up photometry, including companions to the lens, companions to the source, and unassociated interloping stars. I find that undetected blend stars would cause catastrophic failures (i.e., $>50\%$ fractional uncertainty in the inferred lens mass) for $\lesssim (16 \cdot f_{\text{bin}})/\%$ of planet detections, where $f_{\text{bin}}$ is the binary fraction, with the majority of these failures occurring for host stars with mass $\lesssim 0.3$ $M_\odot$. 

Key words: gravitational lensing: micro – planets and satellites: detection – planets and satellites: fundamental parameters

1. INTRODUCTION

Microlensing is an indispensable tool for understanding exoplanet demographics due to its unique sensitivity to low-mass planets separated from their host stars by a few AU or more. This is underscored by the fact that this region roughly corresponds to the location of the snow line in protoplanetary disks, beyond which a higher surface density of solid material is thought to facilitate the growth of more massive protoplanets on shorter formation timescales (Lissauer 1987; Ida & Lin 2005; Kennedy & Kenyon 2008).

The current OGLE-IV (Udalski 2003) and MOA-II (Bond et al. 2001; Sumi et al. 2003) microlensing surveys collectively detect $\sim 15$ planets per year. However, converting the routinely measured mass ratio $q$ of the lens system (planet and host star) and the instantaneous projected angular separation $s$ into planet mass $M_p$ and instantaneous projected semimajor axis $a_\perp$ difficult and requires additional information beyond the standard microlensing light curve. Efficiently doing so will be even more important due to the influx of data from the Korean Microlensing Telescope Network (KMTNet; Kim et al. 2010, 2011; Kappler et al. 2012; Poteet et al. 2012; Atwood et al. 2012), a next-generation network of microlensing survey telescopes that is predicted to increase the annual microlensing planet detection rate by a factor of $\sim 5$ (Henderson et al. 2014a, hereafter H2014a). There are two primary methods by which to obtain $M_p$ and $a_\perp$ with minimal model dependence.

The first is by determining the microlens parallax $\pi_E$, which can be measured from the distortion in the observed light curve due to the acceleration of the Earth relative to the light curve expected for a constant velocity (Gould et al. 1994; Hardy & Walker 1995; Gould et al. 2009). If, for a given event, this resulting asymmetry and the angular size of the Einstein ring $\theta_E$ can be measured, the latter typically by combining multiband photometry with a detection of finite-source effects, then the mass of the lens system can be derived from these two observables via

$$M_l = \frac{\theta_E^2}{\kappa \pi_{\text{rel}}} = \pi_E \theta_E = AU \left(D_\ell^{-1} - D_s^{-1}\right),$$

where $\pi_{\text{rel}}$ is the relative lens–source parallax, $D_\ell$ and $D_s$ are the distances to the lens and source, respectively, and $\kappa \equiv 4G/(c^2AU) = 8.144$ mas/$M_\odot$. This has hitherto been accomplished for 12 planetary systems, including a two-planet system (Han et al. 2013) and a circumbinary planet with a mass twice that of Earth (Gould et al. 2014). There are three different ways to measure $\pi_E$, each with its own observational challenges. Satellite parallax can be measured by the combination of a ground-based observatory and a space telescope when the two are separated by a long spatial base line ($\sim$AU). Recently, Spitzer observations were combined with ground-based data to produce the first space-based measurement of the microlens parallax for an isolated star (Yee et al. 2014) and a microlensing exoplanet (Udalski et al. 2014). Orbital parallax can be measured for events with timescales that are a significant fraction of a year and requires good observational coverage. Finally, terrestrial parallax can be measured when multiple observatories at different longitudes monitor a high-magnification event simultaneously with extremely high cadences. In all cases the stringent observational requirements indicate that the fraction of events for which it is possible to measure $\pi_E$ is quite small.
The second is by constraining the flux of the primary lensing mass: the host star. When color information and finite-source effects provide \( \theta_E, M_\ell \) can be derived by measuring the lens flux, \( F_\ell \), and applying a mass–luminosity relation (Bennett et al. 2007) given a value of the extinction toward the lens. This method has only been applied to a few planetary microlensing events (e.g., Janczak et al. 2010; Batista et al. 2014), because it requires high-resolution follow-up photometry, which is typically in the near-infrared (NIR). However, it does not necessarily require waiting for the lens and source to be resolved. In fact, there are several channels through which \( F_\ell \) can be constrained: (1) imaging the lens after it is spatially resolved from the source, (2) inferring \( F_\ell \) by measuring the elongation of the point-spread function (PSF) of the unresolved microlensing target (lens+source) as the lens and source begin to separate, (3) promptly obtaining high-resolution follow-up photometry while the lens and source are unresolved, or (4) measuring a wavelength-dependent shift of the centroid of the unresolved microlensing target, stemming from the possibility that the lens and source have different colors. There is an array of current and planned ground-based and space telescopes that will have the NIR detectors and diffraction-limited resolution necessary to employ these methods.

Here I present the results of simulated observing programs that explore the ability to constrain \( F_\ell \) for predicted KMTNet planet detections. I specifically investigate only items 1–3 listed above, but note that measuring a color-dependent centroid shift is a useful tool and one that was successfully implemented for the first exoplanet discovered via microlensing (Bond et al. 2004; Bennett et al. 2006). I give a review of the simulations of H2014a in Section 2. In Section 3 I describe the specific facilities whose observational capabilities I consider. I provide an overview of the practical implementation of each of these three techniques and my approximated methodology in Section 4. In Section 5 I detail the results for each. I then discuss the effects that contaminating blend flux from different types of blend stars would have in Section 6. Finally, in Section 7 I explain the implications my findings have for deriving the masses of the planets that will be detected by KMTNet.

2. SUMMARY OF KMTNet SIMULATIONS

The simulations of H2014a were designed to optimize the observing strategy and predict the planet detection rates of KMTNet. There are four primary components to their methodology:

1. using Galactic models to generate populations of lens and source stars with physical properties that match empirical constraints,
2. populating each lens star with a single planetary companion and computing the magnification of the source star as a function of time,
3. creating realistic observed light curves, and
4. implementing a detection algorithm for each light curve.

Here I provide an overview of the details of each.

2.1. Galactic Model

H2014a use the luminosity function (LF) of Holtzman et al. (1998) to obtain the absolute \( I \)-band magnitude of each source star, \( M_I \). Their Galactic bulge and disk density models come from Han & Gould (1995a) and Han & Gould (1995b), respectively. They draw \( M_\ell \) from the mass function (MF) of Gould (2000), which assumes that all the main sequence stars in the range \( 1 < M_\ell/M_\odot < 8 \) have become white dwarfs (WDs), in the range \( 8 < M_\ell/M_\odot < 40 \) have become neutron stars (NSs), and in the range \( 40 < M_\ell/M_\odot < 100 \) have become black holes (BHs). All objects in the range 0.03 \( \leq M_\ell/M_\odot \leq 0.08 \) are assumed to be brown dwarfs (BDs). H2014a only consider main sequence stars to be host stars of planetary systems, excluding BDs and remnants (WDs+NSs+BHs) from the underlying lens mass distribution. The extinction map they use complements the \( I \)-band data of Nataf et al. (2013) with the NIR map of Majewski et al. (2011) and Nidever et al. (2012) for the inner bulge.

2.2. Microlensing Parameters

There are four parameters that specify a microlensing event due to a single lensing mass. The first is \( t_0 \), the time of closest approach of the source to the lens, which H2014a draw uniformly from a generic observing season. Second is \( \mu_0 \), the angular distance of the closest approach of the source to the lens, normalized to \( \theta_E \). H2014a set a maximum allowed impact parameter of three and draw its value uniformly. The Einstein crossing time \( t_E \) is computed via

\[
t_E \equiv \frac{\theta_E}{\mu_{rel}},
\]

where \( \mu_{rel} \) is the relative lens–source proper motion. Last is \( \rho \), the angular radius of the source star normalized to \( \theta_E \).

H2014a then populate each lens star with a planetary companion. The mass ratio \( q \) is given by

\[
q = \frac{M_p}{M_\ell}.
\]

H2014a assume a circular orbit for the planetary companion and compute \( s \) via

\[
s = \frac{\alpha}{R_E} \sqrt{1 - \cos^2 \zeta},
\]

where \( R_E \) is the physical size of the Einstein ring radius and \( \zeta \) is the angle between the plane of the sky and \( \alpha_\perp \) at the time of the microlensing event. Finally, \( \alpha \) gives the angle of the source trajectory relative to the star–planet binary axis and is drawn uniformly. H2014a use these parameters to compute the magnification of the source due to the static binary lens system as a function of time.

2.3. Light Curve Generation

H2014a then convert the magnification into an observed flux. Their weather data for each KMTNet site come from Peale (1997) and they compute the brightness of the Moon using the prescription of Krisciunas & Schaefer (1991). H2014a determine the photon rate normalization and the flux measurement uncertainties for KMTNet by calibrating to OGLE-III photometry and scaling accordingly.

2.4. Detection Algorithm

Lastly, H2014a subject each simulated microlensing event to several detection criteria to determine whether the planet is robustly detected. First, the \( \Delta \chi^2 \) of the observed light curve from its error-weighted mean flux must be greater than 500. Second, the light curve must have more than 100 data points and \( t_0 \) must fall within the time coverage of the light curve. Finally, the \( \Delta \chi^2 \) of the light curve from a best-fit single-lens model must
be greater than 160. The detection rates are then normalized according to a modified version of the cool-planet MF of Cassan et al. (2012) that has been leveled-off at $M_p = 5 M_{⊕}$.

3. HIGH-RESOLUTION FACILITIES AND SIMULATED OBSERVATIONAL PROGRAMS

3.1. Current Ground-based Adaptive Optics

There are several large telescopes (>8 m) with adaptive optics (AO) systems that are capable of achieving diffraction-limited resolution in the optical or NIR (see Section 4 of Henderson et al. 2014b for an overview). Of these, microlensing planet masses derived from $F_7$ have used $H$-band measurements made with NACO on Very Large Telescope (VLT; Janczak et al. 2010) or NIRC2 on Keck (Batista et al. 2014). I use the former here as a representative example and simulate its observing capabilities.

At $\lambda = 1.66 \mu m$ the FWHM of a diffraction-limited image on the 8.2m VLT—given by $1.22\lambda/D$, where $D$ is the telescope aperture—is $\theta_{\text{FWHM,VLT}} = 52.2$ mas. I use their exposure time calculator (ETC) to obtain the photon rate normalization, sky background, and scaling of the signal-to-noise ratio (S/N) with exposure time $t_{\text{exp}}$. For each method discussed in Section 4, I simulate an observing program for each lens system that H2014a predict KMTNet will detect, taking the aggregate sample to be characteristic of the types and variety of systems KMTNet will find. My assumed input instrumental parameters for the simulated observing program are:

1. $H$-band observations, balancing PSF sharpness and resolution with the sky background,
2. an input spectrum of an M0V star (though the choice of the template spectrum has little effect on the resulting S/N or photon rate normalization),
3. a laser guide star,
4. the VIS dichroic, which has high efficiency for NIR observations,
5. the S27 camera, which oversamples the $H$-band slightly, and
6. the FNS/HS instrument mode, which provides higher S/N for fixed $t_{\text{exp}}$ than does DCR/HD.

I set the minimum exposure time $t_{\text{exp,min}}$ to be 20 s, which is recommended for the $H$-band, or whenever $S/N = 100$ is reached, and limit each observation to a maximum of sixty 60 s exposures. Table 1 gives the parameters for the simulated observing program.

Table 1

| Facility         | $\theta_{\text{FWHM}}$ (mas) | $t_{\text{exp,min}}$ (s) | $t_{\text{exp,max}}$ (s) | Collecting Area (m$^2$) | Object Photon Rate (e s$^{-1}$) | Background Photon Rate (e s$^{-1}$) | Plate Scale (mas pixel$^{-1}$) |
|------------------|-------------------------------|--------------------------|--------------------------|-------------------------|----------------------------------|-----------------------------------|-------------------------------|
| GMTIFS on GMT    | 52.2                          | 20                       | 3600                     | 49.29                   | 49.0                             | 934                               | 27.0                          |
| NACO on VLT      | 16                            | 20                       | 3600                     | 368                     | 366                              | 239                               | 5.0                           |
| NIRC2 on JWST    | 68                            | 11                       | 3600                     | 25                      | 1290                             | 4.77                              | 31.7                          |

Note. 4 For a point source with $H = 18$.

3.2. Next-generation Ground-based Adaptive Optics

There are currently three planned extremely large telescopes (>20 m) that will each have an AO system and a NIR imager. I select GMTIFS on the Giant Magellan Telescope (GMT) as an example with which to simulate an observing program because South Korea is a 10% GMT partner, making this endeavor more feasible and probable.

The 24.5 m GMT will have a diffraction-limited resolution of $\theta_{\text{FWHM,GMT}} = 16$ mas in $H$-band (McGregor et al. 2012) and a collecting area of 368 m$^2$, ~7.5 times that of VLT. 3 To simulate an observing program on GMT I assume the same parameters as with VLT, but increase the photon rate normalization by the factor of 7.5 to account for the increase in aperture size. I also modify the sky background to include the increase in collecting area and the decrease in PSF area arising from the smaller pixel size. The parameters of the simulated observing program are listed in Table 1.

3.3. Next-generation Space-based Telescopes

Bennett et al. (2006) used optical Hubble Space Telescope ($HST$) observations to determine the mass of the first exoplanet discovered with microlensing (Bond et al. 2004). In the future, however, James Webb Space Telescope ($JWST$) will provide the largest aperture yet in space at 6.5 m and will use the NIR imager NIRCAM. The bigger aperture provides a smaller diffraction limit than for $HST$—$\theta_{\text{FWHM,JWST}} = 68$ mas for $JWST$’s $\lambda = 1.50$ $\mu m$ short-wavelength filter.

I use the $JWST$ ETC with the following instrumental parameters:
1. the F150W filter, which is a good approximation of $H$-band,
2. an M0V spectral distribution, and
3. average zodiacal and thermal backgrounds.

I set $t_{\text{exp,min}} = 11$ s (as suggested by the user’s manual, accessed via the ETC page) or whenever $S/N = 100$ is reached and again set $t_{\text{exp,max}} = 3600$ s. Table 1 shows the parameters for the simulated observing program.

4. LENS FLUX MEASUREMENT METHODS

The feasibility of constraining $F_7$ for each technique explored here hinges on the relative lens–source proper motion, $\mu_{\text{rel}}$. The distribution of $\mu_{\text{rel}}$ for the predicted KMTNet planet detections is shown in Figure 1. It peaks at $\mu_{\text{rel}} = 5.5$ mas yr$^{-1}$ and falls off more steeply toward larger values of $\mu_{\text{rel}}$. Microlensing events with lenses located in the Galactic bulge generally have larger proper motions than do events arising from lenses in the Galactic bulge. The efficacy of a given observational facility to constrain

1. [http://www.eso.org/sci/facilities/paranal/instruments/naco.html](http://www.eso.org/sci/facilities/paranal/instruments/naco.html)
2. [http://www.eso.org/observing/etc/](http://www.eso.org/observing/etc/)
3. [http://www.gmto.org/resources/](http://www.gmto.org/resources/)
4. [http://jwstetc.stsci.edu/etc/input/nircam/imaging/](http://jwstetc.stsci.edu/etc/input/nircam/imaging/)
Figure 1. Distribution of relative lens–source proper motion $\mu_{\text{rel}}$ for the planet detections predicted for KMTNet (left) and the fraction of lenses that will be resolved from the source as a function of $\Delta t$ for each facility (right). The fraction of microlensing events for which a given facility will be able to constrain $F_{\ell}$ depends sensitively on the fraction of events that it can resolve a fixed time $\Delta t$ after each event. This population is similar for VLT and JWST, given their comparable diffraction-limited resolutions $\theta_{\text{FWHM}}$, but is shifted toward significantly shorter values of $\Delta t$ for GMT.

$F_{\ell}$ is set by the fraction of lens systems that are resolved from their accompanying sources a fixed time $\Delta t$ after the peak of the microlensing event, which is also shown in Figure 1. This, in turn, is fundamentally determined by what portion of the $\mu_{\text{rel}}$ distribution the facility is able to sample given its angular resolution.

There are two independent ways to obtain a relation that gives $M_{\ell}$ as a function of $D_{\ell}$. First, $\theta_{E}$ can be derived from a robust detection of finite-source effects from the observed microlensing light curve, which yields the angular size of the source star normalized to $\theta_{E}$, and multiband photometry from which the physical size of the source star can be determined. Assuming the source is in the bar, $D_{s}$ is known to a precision equivalent to the width of the bar. Then Equation (1) simplifies to a mass–distance relation for the lens. Second, a measurement of $F_{\ell}$ in conjunction with a mass–luminosity relation and an estimate of the extinction toward the lens provides another technique with which to compute the mass of the lens as a function of its distance. Coupling these two methods uniquely determines $M_{\ell}$ and, given an assumed $D_{s}$, $a_{\perp}$.

4.1. Imaging a Lens Spatially Resolved from the Source

4.1.1. Practical Implementation of Technique

A lens can be directly imaged after it is spatially resolved from the source. The wait time $\Delta t$ after the closest approach of the source to the lens is at least several years for typical Galactic microlensing events. This arises from the fact that it depends on the relative proper motion of the two systems, which is generally $<10$ mas yr$^{-1}$ (see Figure 1), as well as the angular resolution attained by the observational facility, which is $\sim 100$ mas for current facilities with the highest resolution. In principle, after a resolved lens is imaged using a high-resolution facility, its measured apparent magnitude can be combined with a mass–luminosity relation and an estimate of the lens extinction to provide $M_{\ell}$ and, given an assumed $D_{s}$, $a_{\perp}$.

4.1.2. My Approximated Methodology

Here I take a lens to be resolved from the source when its angular separation satisfies

$$\Delta \theta_{\ell,s} \equiv \sqrt{(\mu_{\text{rel}} \Delta t)^2 + (u_0 \theta_E)^2} \geq \theta_{\text{FWHM}}.$$  \hspace{1cm} (5)

In the case of imaging a resolved lens, it is possible to directly measure the vector proper motion from the angular separation of the lens and source, the time elapsed since the peak of the event, and $u_0$. When considering PSF elongation measurements and prompt follow-up photometry, I assume the magnitude of $\mu_{\text{rel}}$ is known from Equation (2). Then, in the case of the former, the elongation gives the flux ratio of the lens and source. The source flux is measured from the microlensing light curve, although typically in a different bandpass than is used for the high-resolution photometry, thereby requiring an estimate of the source color.

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Using H-band lens extinction, $A_{H,ℓ,t}$, using the relations of Cardelli et al. (1989) and assuming $R_V = 2.5$ (Nataf et al. 2013). Finally, I compute $H_ℓ$ from $M_{H,ℓ,t}$, $A_{H,ℓ,t}$, and $D_ℓ$ (see Section 3.1.2 of H2014a).

I then simulate an observing program for each lens system that would be resolved from its source for several values of $Δt$. For each facility I determine $t_{exp}$ and $S/N$ as described in their respective sections in Section 3. Then I compute the fractional precision to which $F_ℓ$ can be measured in the $H$-band, $ς_{H,ℓ}$, using each facility after each $Δt$ interval.

4.2. Elongation of the PSF of the Unresolved Microlensing Target

4.2.1. Practical Implementation of Technique

It is not necessary to wait until the lens and source are spatially resolved to constrain $F_ℓ$. As $Δt$ increases, the combined PSF of the unresolved lens and source will become distorted on a timescale dictated by $μ_{cel}$. In the regime in which $δθ_{ℓ,s} < θ_{FWHM}$, this elongation of the PSF of the microlensing target (lens+source) can be measured photometrically. But, the PSF elongation itself stems from two factors: the separation of the lens and the source as well as their brightness ratio. Thus in order to constrain $F_ℓ$ in this way, it is necessary to obtain an independent measurement of one of these two causal parameters. The lens–source separation can be determined by measuring $μ_{cel}$ as described in Section 4. The elongation of the PSF subsequently gives the flux ratio of the lens and source. Since the source magnitude is routinely derived from the ground-based light curve, $F_ℓ$ can be computed (see Bennett et al. 2007 for a complete discussion). Finally, as in the case of imaging a resolved lens, combining the inferred $F_ℓ$ with a mass–luminosity relation and an estimate of the lens extinction yields $M_ℓ$ and $a_⊥$, assuming a source distance.

It is important to note that this technique hinges sensitively on the precision to which the morphology of the PSF is known. Any distortion of a PSF whose shape is poorly characterized could lead to a false-positive elongation measurement. While I assume perfect knowledge of the PSF here, I concede that having sufficiently precise knowledge of the intrinsic PSF for a ground-based AO facility can be extremely challenging. This can be somewhat alleviated by the fact that typical bulge observing fields contain large samples of bright and isolated stars that can be used to model the PSF, but it may still be difficult to extensively model any spatial variations of the PSF.

4.2.2. My Approximated Methodology

In total there are four sources of uncertainty when using PSF elongation to constrain $F_ℓ$:

1. the statistical uncertainty of the source flux in the instrumental $I$-band, measured from the ground-based light curve,
2. the uncertainty in calibrating the instrumental $I$-band source brightness,
3. the uncertainty in transforming the $I$-band source brightness to the NIR filter of the high-resolution data, and
4. the uncertainty of the fractional lens flux.

The statistical uncertainty of the uncalibrated $I$-band magnitude of the source, determined from the modeling of the ground-based light curve, includes its co-variances with other model parameters and is typically $2%–5%$ (e.g., Dong et al. 2009; Janczak et al. 2010; Sumi et al. 2010; Batista et al. 2011; Yee et al. 2012). I take the typical fractional precision to be $2%$ to account for KMTNet’s aperture size and higher cadence. Then I take the sum of the uncertainty inherent to calibrating and transforming the uncalibrated ground-based $I$-band source brightness, items (2) and (3) from above, to be a conservative $0.03$ mag (Janczak et al. 2010). I note that it is possible to improve on this in cases for which ground-based $H$-band data were taken during the event when the source was magnified, allowing $I–H$ to be computed to $\sim 1%$ (Batista et al. 2014).

Defining the fractional lens flux as $f_ℓ = F_ℓ/F_{tot}$, where $F_{tot} = F_s + F_ℓ$ and $F_s$ is the flux of the source, the fractional precision of $f_ℓ$ is given by

$$ς_{f_ℓ} = \frac{2}{N_{tot}} \left( \frac{r_0}{δθ_{ℓ,s}} \right)^2 \frac{1}{\left| 1 - 2f_ℓ \right|}$$

where $N_{tot}$ is the total number of photons of the lens and source in the combined high-resolution PSF and $r_0$ is its Gaussian width (Bennett et al. 2007). This implies that $r_0$ is given by

$$r_0 = \frac{θ_{FWHM}}{2\sqrt{2\ln(2)}}$$

Computing both $N_{tot}$ and $f_ℓ$ requires $F_ℓ$. More specifically, it requires the apparent $H$-band magnitude of the source, $H_s$. To determine $H_ℓ$ for each microlensing event I first use the absolute $I$-band magnitude of the source $M_{I,s}$ (see Section 3.1.1 of H2014a) and the same isochrone as in Section 4.1.2 to determine $M_{I,ℓ,s}$, the absolute $H$-band magnitude of the source. While rare, it is possible that $M_{I,ℓ,s} < 2.67$, which is the bright limit of the isochrone, in which case I assume the source is a red clump giant. I then use the absolute $I$-band and $H$-band magnitudes of the red clump, $M_I = -0.12$ (Nataf et al. 2013) and $M_H = -1.49$ (Laney et al. 2012), to derive its intrinsic $I–H$ color, $I–H = 1.37$, from which I compute $M_{H,ℓ,s}$. $H_s$ and $H_ℓ$ are then determined from their respective absolute magnitudes using the procedure described in Section 4.1.2. I similarly compute $H_{ℓ,s}$, which is the apparent magnitude of the lens and source combined in the single PSF, from which I obtain $N_{tot}$. Although both $f_ℓ$ and $N_{tot}$ could be affected by the contaminating flux of a blend star, for these computations I assume no such contribution.

I subsequently simulate an observing program for each lens and source pair that would not be spatially resolved for several values of $Δt$. The unresolved microlensing target is treated as a single point-source object whose brightness is the combined flux of the lens and the source, $F_{ℓ,s} = F_ℓ + F_s$. I then use $H_{ℓ,s}$ to determine $t_{exp}$ and $S/N$ for the respective facilities as described in Section 3. Finally, I compute $ς_{H,ℓ}$ by adding Equation (6) in quadrature with the statistical uncertainty on $F_s$ (item 1 from above), and the uncertainty in calibrating and transforming $F_s$ (items 2 and 3 from above).

4.3. Prompt High-resolution Follow-up Photometry

4.3.1. Practical Implementation of Technique

For cases in which the lens and source are unresolved and the PSF elongation is minimal, it is still possible to constrain $F_ℓ$. Both $F_s$ and $F_{tot}$ are routinely measured from the ground-based microlensing light curve. Then, a high-resolution image of the microlensing target will, to a high probability, resolve out all stars that are not dynamically associated with the microlensing event. For reference, at the distance of the center of the Galactic bulge, $D_{GC} = 8.2$ kpc (Nataf et al. 2013), an angular separation of $θ_{FWHM,WST} = 68$ mas corresponds to a physical separation of...
560 AU. By assuming no companions to the lens or the source, any difference between the flux of the target measured in the high-resolution image and \( F_t \) can be attributed solely to the lens.

In practice, this requires taking high-resolution observations of the unresolved microlensing target after the peak of the event, typically in the NIR, and calibrating them. The \( I \)-band flux of the source, which is routinely measured from the ground-based observed microlensing light curve data, must be transformed to the filter of the high-resolution data and also calibrated. Then, the calibrated source flux can be subtracted from the high-resolution flux of the unresolved target, and any excess light can be attributed to the lens (see Section 6 for a discussion of the contaminating blend flux). Each of these steps—calibrating the high-resolution NIR data and transforming and calibrating the ground-based optical data—introduces uncertainty that propagates through to the excess flux measurement. A detection of the lens flux is secure only when the total uncertainty of the measured excess flux is small compared to the computed flux difference. If \( F_t \) is indeed robustly detected, \( M_t \) and \( a_\perp \) can be derived from a mass–luminosity relation and known values for the lens extinction and \( D_s \).

4.3.2. My Approximated Methodology

A secure detection of \( F_t \) via prompt follow-up photometry crucially requires careful treatment of the five sources of uncertainty involved in matching the ground-based and high-resolution data. In addition to items (1)–(3) discussed in Section 4.2.2 there is

4. the statistical uncertainty of the instrumental brightness of the unresolved microlensing target \((\text{lens} + \text{source})\) in the high-resolution data, and

5. the uncertainty in calibrating the high-resolution measurement.

As in Section 4.2.2, I take the statistical uncertainty of the source flux to be 2% and the sum of the uncertainty inherent to calibrating and transforming the ground-based \( I \)-band source brightness to be 0.03 mag. The final fractional precision of the calibrated \( H \)-band magnitude of the source, \( \sigma_{H_s} \), is computed as the quadrature sum of these two uncertainties.

With regard to the high-resolution data, I compute the statistical uncertainty of the flux of the unresolved microlensing target via the methods described in Section 3, wherein I assume the target to be a point source with a flux equal to the combined flux of the lens and the source. I conservatively assign a constant 0.03 mag uncertainty to the calibration process (Batista et al. 2011) and add the two in quadrature to obtain \( \sigma_{H_{\text{tot}}} \). Finally, I define a lens flux detection via prompt follow-up photometry to occur when

\[
\Delta H \equiv H_e - H_{\text{filt}} \geq N_{\text{sig, pfp}} \times \sigma_{H_{\text{tot}}},
\]

where \( N_{\text{sig, pfp}} \) represents the number of standard deviations at which the lens flux is detected and

\[
\sigma_{H_{\text{tot}}} \equiv \sigma_{H_s} + \sigma_{H_{\text{tot}}}. \tag{9}
\]

5. RESULTS

5.1. Imaging a Lens Spatially Resolved from the Source

Figure 2 shows a cumulative distribution function (CDF) of \( \sigma_{H_e} \) for observing programs that simulate imaging resolved lens systems \( \Delta t = 1, 5, 10, \) and 25 yr after the microlensing events using NACO on VLT, GMTIFS on GMT, and NIRCAM on JWST. Although JWST has the smallest aperture of the three facilities, its extremely low background allows it to achieve \( \sigma_{H_e} < 0.1 \) for all lenses that are resolved from their source after a fixed \( \Delta t \). Furthermore, the smaller background means the total exposure time required to do so is reduced compared to VLT and GMT. For example, given the assumptions of my simulated observing programs and using the normalized planet detection rates computed by H2014a, after \( \Delta t = 10 \) yr it would take VLT \( \sim 31 \) hr to image \( \sim 25 \) planetary systems, whereas it would take JWST only \( \sim 3.8 \) hr to image \( \sim 18 \) planetary systems, and the majority of those imaged with VLT (about two-thirds) would have \( \sigma_{H_e} > 0.1 \). The total fraction of the events that can be observed after a fixed \( \Delta t \) with VLT or JWST rises as \( \Delta t \) increases from 5 to 10 to 25 yr, stemming from the fact that their values of \( \theta_{\text{FWHM}} \) are sampling the high proper motion tail (\( \gtrsim 10 \) mas yr\(^{-1}\)), the peak (\( \gtrsim 6 \)), and the low proper motion tail (\( \gtrsim 2 \)), for those respective \( \Delta t \) intervals.

JWST, on the other hand, will have a collecting area \( \sim 7.5 \) times bigger than that of VLT and \( \sim 15 \) times bigger than that of JWST. Additionally, if GMT is able to achieve diffraction-limited imaging in the \( H \)-band it will have a \( \theta_{\text{FWHM}} \) that is \( \sim 3 \) and \( \sim 4 \) times smaller than that of VLT and JWST, respectively. Figure 2 shows the result of the confluence of these two factors. After \( \Delta t = 5 \) yr, GMT’s diffraction-limited resolution of \( \theta_{\text{FWHM}} = 16 \) mas allows it to resolve all events with \( \mu_{\text{tot}} \gtrsim 3 \) mas yr\(^{-1}\), or \( \sim 79\% \) of the total planet detection rate, as shown in Figure 1. GMT would be able to measure the flux of three-fourths of those events (or \( \sim 60\% \) of the total planet detection rate) to a precision of \( \sigma_{H_e} < 0.1 \). Again using the normalized planet detection rates of H2014a, after \( \Delta t = 5 \) yr GMT would be able to image the host star for \( \sim 51 \) planetary systems whose lens is resolved from the source, \( \sim 38 \) of those to a precision better than 10%, and would be able to do so in \( \sim 39 \) hr given my assumptions for the simulated observing programs. On the other hand, after \( \Delta t = 5 \) yr VLT and JWST could image only \( \sim 3 \) and \( \sim 2 \) total planetary systems with spatially resolved lenses, respectively.

Thus, for a fixed \( \Delta t \), GMT will be able to obtain direct lens flux measurements for a significantly larger fraction of predicted KMTNet planet detections than VLT or JWST. The primary benefit of VLT is that it exists and so could start observing lens systems shortly after KMTNet comes online, as early as the 2015 Galactic bulge observing season, which begins in early February. The advantage of JWST rests in its ability to obtain \( \sigma_{H_e} < 0.1 \) for all lenses that are resolved after a given \( \Delta t \), that its diffraction-limited capabilities do not hinge on favorable weather conditions or guide star characteristics, and the resulting shorter observing program required to image a fixed number of resolved lens systems.

5.1.1. Physical Properties of Imaged Lens Systems with High-precision Flux Measurements

I also examine the physical properties of the planet detections whose host star fluxes can be measured to a precision of \( \sigma_{H_e} < 0.1 \) by an example observing program. Figure 3 shows the distributions of \( M_t \) and \( D_t \) for spatially resolved lenses whose flux can be measured to \( \leq 10\% \) for my simulated observing program using GMTIFS on GMT after \( \Delta t = 5 \) yr. Approximately 58% of such planetary systems that will be accessible by such an example observing program reside in the Galactic disk, whereas the remaining 42% of lens systems will be bulge lenses. In contrast, only 45% of the predicted KMTNet planet detections
Figure 2. Cumulative distribution functions (CDFs) of $\sigma_{H_\ell}$ for imaging resolved lenses or measuring PSF elongation. The left column is for NACO on VLT, the middle shows GMTIFS on GMT, and the right shows NIRCAM on JWST. Each row represents a fixed time $\Delta t$ after the peak of the microlensing event. In each figure, the curves are color-coded according to technique. GMT will be able to image a majority of resolved lenses after $\Delta t = 5$ yr and JWST will be able to obtain $\sigma_{H_\ell} \leq 0.1$ for all lenses that are resolved after a given $\Delta t$ interval. Regarding PSF elongation, GMT can constrain $F_\ell$ to 10% or better for approximately one-seventh of predicted KMTNet detections after only $\Delta t = 1$ yr, and VLT and JWST can do so for $\sim 35\%$ and $\sim 17\%$ of planetary systems, respectively, after $\Delta t = 5$ yr.

Figure 3. Distributions of $M_\ell$ and $D_\ell$ for resolved lens systems whose flux can be measured to a precision of $\sigma_{H_\ell} \leq 0.1$ using GMTIFS on GMT $\Delta t = 5$ yr after each event. The fraction of disk lenses (58%) is substantially higher than the 45% of all predicted KMTNet planet detections that reside in the disk. This stems from the fact that closer lenses will be brighter for fixed $M_\ell$ and extinction, which is confirmed by the fact that the average lens distance is 0.5 kpc closer than the mean distance of 6.1 kpc for the full sample of predicted planetary systems. Furthermore, the disk lens population that can be imaged in this way will probe down to lower masses than will the bulge lens sample.
are expected to arise from disk lenses. This increase in the fraction of disk lenses intuitively stems from the fact that disk lenses are generally closer, facilitating flux measurements that can be obtained with better precision. This is corroborated by the distribution of \( D_\ell \) for spatially resolved lenses versus that for the overall KMTNet planet detection sample. The former has an average lens distance of \( D_\ell = 5.6 \) kpc, while the latter has a mean distance of \( D_\ell = 6.1 \) kpc. Thus, an observing program for imaging spatially resolved lenses will preferentially select for closer lens systems that are more likely to reside in the Galactic disk rather than the bulge.

Furthermore, the distributions of \( M_\ell \) for the disk and bulge lenses differ at the low-mass end. The simulated observing program predicts that precise flux measurements will be possible for stars down to the Hydrogen burning limit at \( \sim 0.08 \, M_\odot \) for lenses in the Galactic disk. However, the least massive bulge lenses able to be imaged in this way have masses that are \( \sim 50\% \) higher, around \( 0.13 \, M_\odot \). While this is to be expected, because more massive stars will be brighter, this indicates another implicit bias in the properties of lens systems whose masses are derived from photometric flux measurements. Those at larger distances that generally reside in the bulge will have, on average, higher masses than nearby disk lenses, for which it will be possible to probe planetary systems whose host stars have lower mass. Understanding and accounting for these underlying selection effects when undertaking studies of the global properties of exoplanet detections will be of critical importance.

### 5.2. Elongation of the PSF of the Unresolved Microlensing Target

Figure 2 also shows a CDF of \( \sigma_H \) for observing programs that estimate measuring the PSF elongation of the unresolved microlensing target for \( \Delta \tau = 1, 5, 10, \) and 25 yr, again using NACO on VLT, GMTIFS on GMT, and NIRCAM on JWST. For GMT, after \( \Delta \tau = 25 \) yr essentially all lenses and sources will be resolved given \( \theta_{\text{FWHM}, \text{GMT}} \), precluding PSF elongation measurements. However, the first two terms of Equation (6) contain the implicit scaling \( \sigma_k \propto D^{-3} \), where \( D \) is the diameter of the telescope aperture. Thus, not only will the elongation of the PSF of the microlensing target be more pronounced (accounting for \( D^{-3} \)), but significantly more photons of the target will be collected for a fixed \( t_{\exp} \) (yielding the remaining \( D^{-1} \)). GMT is consequently able to measure \( F_\ell \) to \( \lesssim 10\% \) for about one-seventh of planet detections merely one year after the event. This presents a huge advantage for future ground-based microlensing surveys, particularly KMTNet, and their ability to convert mass ratios \( q \) to planet masses \( M_p \) on expeditious timescales.

### 5.3. Prompt High-resolution Follow-up Photometry

In Figure 4 I show the fraction of lens systems for which it will be possible to securely detect the flux of the lens as a function of planet mass \( M_p \) for three different \( N_{\text{sig,pfp}} \) detection thresholds using NIRCAM on JWST \( \Delta \tau = 3 \) months after the peak of each event. I consider lens flux detections with significances as low as \( N_{\text{sig,pfp}} = 1 \) to explore cases in which an upper limit on the brightness of the lens can be established, even if the flux measurement itself is less secure. For \( N_{\text{sig,pfp}} = 1 \), indicating that \( F_\ell \) is detected at the one-sigma level according to Equation (8), it will be possible to measure \( F_\ell \) for \( \sim 42\% \) of planet detections in each mass bin. This decreases to \( \sim 31\% \) for \( N_{\text{sig,pfp}} = 2 \) and \( \sim 26\% \) for \( N_{\text{sig,pfp}} = 3 \). In all cases, though, it is approximately constant as a function of \( M_p \), indicating no preference for or against certain planetary systems (as expected). Furthermore, these fractions are essentially equivalent for NACO on VLT and GMTIFS on GMT, stemming from the fact that \( \sigma_H \) and \( \sigma_{\text{H,sys}} \) are dominated by the uncertainties in calibrating and transforming the ground-based optical data and calibrating the high-resolution NIR data, respectively, rather than the photometric precision of each facility.

### 6. POTENTIAL SOURCES AND EFFECTS OF CONTAMINATING BLEND LIGHT

In the above scenarios I have ignored the possible contributions of additional flux from stars blended with the lens and/or source. However, their presence could affect the measured fluxes and the masses ultimately derived from them. Here I investigate the three most likely scenarios and the effect each would have.

#### 6.1. Lens Companion

In principle, each lensing system could contain an additional stellar component whose flux could interfere with the derived value of \( M_\ell \). To test this, I begin by populating the lens system of each planet detection from H2014a with such a companion, all of whose parameters I will designate using the subscript \( \ell_2 \). Here I explore the impact for prompt follow-up photometry taken \( \Delta \tau = 3 \) months after the time of the microlensing event. Even with GMT, which has the smallest \( \theta_{\text{FWHM}} \) of the facilities
Figure 5. Detectability of lens (top left) and source (top right) companions and their respective resulting $\delta M_\ell$ CDFs (bottom). Assuming every lens or source has a companion, the companions for approximately half of the events with robust non-source flux detections will go undetected. However, the fraction of catastrophic failures in the derived values of $M_\ell$ is low, not exceeding $(16 \cdot f_{\text{bin}})\%$ of all planet detections, where $f_{\text{bin}}$ is the binary fraction. This is then suppressed by empirically determined binary fractions, which are a steep function of spectral type and can be as low as $f_{\text{bin}} \sim 25\%$ for M stars (Lada 2006).

I investigate, fewer than 0.01% of lens systems will be resolved from their source after three months.

6.1.1. Implementation

As described in H2014a and Section 2.1, the lens masses are derived from the MF of Gould (2000). I draw the mass of the companion, $M_{\ell,2}$, from the same MF. If the companion is a stellar remnant or BD, which I do not exclude as viable companions, I assume it to be dark and set its apparent magnitude accordingly, $H_{\ell,2} = 30$. Otherwise, I determine $H_{\ell,2}$ via the procedure described in Section 4.1.2.

Next I determine the parameters of the binary system composed of the primary lens mass and this companion, which I designate as $\ell_2$. I compute the orbital period $P_{\ell,2}$ from the log-normal Gaussian distribution of Raghavan et al. (2010), which has a mean of $\log P = 5.03$ and $\sigma_{\log P} = 2.28$, where $P$ is in days. From $P_{\ell,2}$ and $M_{\ell,2}$ I compute $a_{\ell,2}$. I assume a circular orbit and compute $a_{\ell,2,\perp}$ according to

$$a_{\ell,2,\perp} = a_{\ell,2} \sqrt{1 - \cos^2 \zeta}. \quad (10)$$

For randomly oriented orbits, $\cos \zeta$ is uniformly distributed, so I draw $\cos \zeta$ from a uniform random deviate in the range $[0-1]$. The mass ratio $q_{\ell,2}$ is simply $M_{\ell,2}/M_{\ell,1}$.

6.1.2. Occurrence Probability and Effect on Derived Lens Mass

In many cases it will be possible to detect the presence of a lens companion aside from its flux contribution, circumventing errors introduced by an unseen companion. The two primary ways that this can be achieved are if the lens companion is spatially resolved from the unresolved microlensing target or if the source trajectory passes near enough to the central caustic that the perturbations to the caustic induced by the presence of this additional lensing mass are then observed in the light curve. To determine the former I compute the angular separation of the lens companion and the unresolved microlensing target, $\Delta \theta_{\ell,2}^\text{res} + \theta_{\ell,2}$, after $\Delta t = 3$ months and presume the companion would be detected if $\Delta \theta_{\ell,2}^\text{res} \geq \theta_{\text{FWHM}}$.

Regarding the latter, I assume that the lens companion would be detected if the source passes over or very near the central caustic perturbation induced by the companion’s presence, specifically if $u_0 \leq u_{\text{cc},2}$. For a two-body lens system there exists a set of 1–3 closed caustic curves, depending on the angular separation of the lensing masses, that identify the locations in the plane of the source where the magnification of a point-like source diverges to infinity. There is one central caustic that is located near the center of mass of the lens system and 1–2 planetary caustics. I first compute the topology of the $\ell_2$ binary (Erdl & Schneider 1993), which determines the total number of caustics. For all topologies I take $u_{\text{cc},2}$ to be the size of the central caustic along its longest dimension. In the case of a resonant topology for which the central caustic is the sole caustic I compute $u_{\text{cc},2}$ numerically. If the topology is close or wide, I find the approximate dimensions of the central caustic analytically using Equations (22) and (23) or (9) and (10) of Bozza (2000), respectively.

I show the fraction of lens companions whose presence would be detected via the above methods in Figure 5. Here I have also excluded events whose lens and source would be resolved after $\Delta t = 3$ months with JWST (as a conservative

The third term in Equations (9) and (10) of their manuscript contains $\rho^2$, where $\rho$ is their nomenclature for projected separation, when they should instead read $\rho^3$ as a result of their perturbative analysis. I corrected this prior to using these analytic approximations.
the precision with which it is currently possible to obtain
dependence on flux contamination from a lens companion. There is a strong
lens companions as a function of the true lens mass. On average,
undetected companions. The bulk of detections with
the companion will manifest itself in the light curve.

For the ∼32% of lens systems in which ∈ is undetected, I estimate the fractional uncertainty that the blend flux from this undetected companion introduces to the derived lens mass. I compute $M_{\text{H2014a}}$, the absolute magnitude of the combination of the lens and its companion, from $D_1$, $A_{\text{H2014a}}$, and $H_{\text{ext}}$. Using the same isochrone as in Section 4.1.2 I determine $M_{\text{blend}}$, which is the mass of the primary lens that would be inferred if the blend flux contributed by the companion were undetected and otherwise attributed to the lens. I then compute the absolute fractional difference between the true primary lens mass and the mass determined when including blend flux from the undetected lens companion,

$$\delta M_L = \frac{|M_L - M_{L,\text{blend}}|}{M_L}. \quad (11)$$

Figure 5 shows the resulting CDF of $\delta M_L$ for three different values of $N_{\text{sig,pfp}}$.

The CDF was truncated at $\delta M_L = 0.01$, which explains why the rightmost limit is lower than the height of the “Undetected” bin, for two reasons. First, this threshold is comparable to the finest steps in stellar mass of the isochrone. Second, of the 32 planets that have hitherto been detected via microlensing, all mass values have fractional uncertainties greater than 4%, indicating that a ∼1% uncertainty is an appropriate floor for the precision with which it is currently possible to obtain $M_p$ for microlensing planet detections. I take $\delta M_L > 0.5$ to define the lens systems for which there is a catastrophic failure in the determination of $M_L$ due to contaminating flux from an undetected companion to the lens. After excluding systems whose lens and source are resolved, whose non-source light is not detected at one sigma or better, and lens companions whose presence would be otherwise noticed, either by being spatially resolved or via perturbing the central caustic, I find that $\lesssim 16\%$ of lens systems will have their masses severely mis-estimated with $\delta M_L > 0.5$.

Figure 6 shows the fraction of lens systems with undetected lens companions as a function of the true lens mass. On average, ∼30% of lenses with a given mass will potentially be subjected to flux contamination from a lens companion. There is a strong dependence on $M_L$ when considering only systems whose mass derivations are subject to catastrophic failures from these undetected companions. The bulk of detections with $\delta M_L > 0.5$ have $M_L \lesssim 0.3\, M_\odot$ and none have $M_L \gtrsim 0.7\, M_\odot$. Lenses with the lowest masses will have more massive companions that are thus more luminous, leading to a higher probability of significant contamination from the blend flux. Conversely, lenses near the high-mass end of the MF will generally have lower-mass companions. The mass–luminosity relation is sufficiently steep that the light from the lower-mass companions to these massive lenses will not significantly skew the derived lens mass.

For this calculation I assumed that each primary lens star has exactly one companion (in addition to the planet). However, not only is the binary fraction $f_{\text{bin}} < 100\%$, it also depends steeply on spectral type (e.g., Duquennoy & Mayor 1991; Fischer & Marcy 1992; Lada 2006; Raghavan et al. 2010, and references therein). In fact, M stars, which comprise the bulk of the Galactic lens population, can have a binary fraction as low as $\sim 25\%$ (Lada 2006). While Figure 6 shows the contamination fraction as a function of $M_L$, I allow for lens companions that are more massive than the planet host star. In these cases the host star would not be the primary star of the stellar binary. Thus, although the higher fraction of catastrophic lens mass derivation failures for low-mass lenses is caused by brighter, higher-mass lens companions, it is these companions—not the lens host stars themselves—that are the primary bodies in the stellar binaries, and as such they will have different values of $f_{\text{bin}}$. Digesting any potential bias in the distribution of photometrically derived lens masses is thus quite complicated and will likely require a global approach rather than being addressed system-by-system.

Whatever the value of $f_{\text{bin}}$, it is crucial to also note that the CDF in Figure 5 will be suppressed by that same factor. This establishes the results presented here as an upper limit. Therefore, while individual systems may yet experience catastrophic failures in their mass determinations, this indicates that undetected lens companions will have a small net effect on the derived values of $M_L$ for the statistically large samples of planet detections that H2014a predict KMTNet will find.

6 From http://exoplanet.eu as of 2014 September 8.
6.2. Source Companion

Another possibility hitherto unaccounted for is the contribution of additional flux from a companion to the source star. As with a lens companion, if the presence of such a star goes undetected, its flux will skew the derived value of $M_t$. Here I populate the source star of each detected planetary microlensing event predicted for KMTNet from H2014a with a companion, designating all of its parameters with the subscript $s, s$. I again investigate the resulting effect only for prompt follow-up photometry $\Delta t = 3$ months after the time of each microlensing event.

6.2.1. Implementation

I determine the mass of the source companion, $M_{s}$, and its apparent $H$-band magnitude, $H_{s}$, following the prescription described in Section 6.1.1. However, prior to obtaining the parameters of the binary source $s_{bin}$, I must determine the mass of the source star itself. I draw its absolute $I$-band magnitude $M_{I,r}$ from the LF of Holtzman et al. (1998) (see Section 3.1.1 of H2014a), which I use in conjunction with the same isochrone as previously to obtain $M_t$. If $M_{I,r} < 2.67$, the bright end of the isochrone, $M_{s}$ is taken to be 1.1 $M_{\odot}$, which typical for G and K giants. With the masses of both components of $s_{bin}$ in hand, I compute the binary parameters $P_{s,s}, a_{s,s}, q_{s,s}$, and $g_{s,s}$ as laid out in Section 6.1.1.

6.2.2. Occurrence Probability and Effect on Derived Lens Mass

I investigate three channels through which a companion to the source can be detected. As with a lens companion, the simplest is if the source companion and the microlensing target are spatially resolved. If $\Delta t_{c,c,s} \geq \theta_{WHM}$ after $\Delta t = 3$ months I assume the companion to be detected.

Otherwise, I assume that the source companion would be detected if the source passes over the central caustic created by the lens host star and planet. In this regime $q \ll 1$, so I make use of analytic approximations for the size of the central caustic, depending on the topology. If it is a resonant topology, I compute $u_{cc,t}$ numerically. Otherwise, if it is a close or wide topology, I use Equations (10) and (11) of Chung et al. (2005) (or, equivalently, Equations (24) and (25) of Han 2006). If $u_{0} \leq u_{cc,t}$, I assume that the source companion would be detected via additional features in the light curve.

In many cases, however, the trajectory of the source will cause it to maintain a wide separation from the lensing star throughout the duration of the event (see Figure 23 of H2014a). The detection of the planet then arises from the source passing near or over (at least) one of the planetary caustics. Just as the presence of a source companion would manifest itself through the extra magnification structure in the light curve as it passes over the central caustic, so would it if it were to pass over a planetary caustic, if $a_{s,s,\perp}$ were sufficiently small. There are no planetary caustics for a resonant topology. For a close topology I approximate the size of the planetary caustic $u_{pc,t}$ using Equations (3), (15), and (18) of Han (2006), noting that the caustic width along the axis parallel to the planet-star separation vector is always larger than the width along the perpendicular direction, obviating the computation of the latter. If it is a wide topology I compute $u_{pc,t}$ from Equation (8) of Han (2006). If the angular separation of $s_{bin}$ normalized to $\theta_{E}$ is smaller than the size of the planetary caustic, $\Delta \theta_{s,s,\perp}/\theta_{E} \leq u_{pc,t}$, I assume that the source companion would induce detectable perturbations on the light curve.

In principle it is also possible to detect the presence of a source companion due to a shift in the observed color of the microlensing event. Gravitational microlensing is itself achromatic. However, in the case of a binary source it is likely that the flux ratio of the two components will deviate from one, indicating a difference in color between the two stars. Then, a microlensing target whose total observed flux is color-dependent evinces the binarity of the source, which is more readily detectable as the binary source passes over or near the caustics. While this effect has been measured previously (Hwang et al. 2013), I do not consider it here and instead mention it as an additional tool with which source companions can be detected.

Figure 5 shows a histogram of the fraction of events whose companions to the source would be detected by combinations of the methods described above. Similar to the consideration for lens companions, I include only events that remain unresolved after $\Delta t = 3$ months and for which the lens flux would be detected at the one-sigma level or better via prompt follow-up photometry using NIRCAM on JWST. This represents a more conservative approach, given that the smaller values of $\theta_{WHM}$ for VLT and GMT cause them to spatially resolve more companions from sources. I again note that the sum of the histogram bins gives a fraction that is larger than the one-sigma fraction shown in Figure 4, arising from the increase in $\Delta H$ (Equation (8)) that is due to the increase in brightness of the non-source term. Approximately 25% of source companions would go undetected, which is smaller than the ~32% of lens companions.

If the source companion is indeed undetected, then its contributed blend flux will influence the derived value of $M_t$. I then follow the same procedure as in Section 6.1.2 to compute $\delta M_t$. The resulting CDF is shown in Figure 5, which is again truncated at $\delta M_t = 0.01$. Even fewer undetected source companions would induce catastrophic failures in the eventual lens mass determination, with $\lesssim 9\%$ of detections having $\delta M_t > 0.5$. Additionally, the same caveat regarding the binary fraction $f_{bin}$ applied to lens companions holds here, which would only further reduce the fraction of planetary systems for which prompt follow-up photometry would ultimately produce values of $M_t$ that would be catastrophically skewed. However, it is important to note that the source stars of microlensing events have spectral types that are, in general, earlier than those of lens stars, leading to different binary fractions between the two populations.

6.3. Ambient Interloping Star

I lastly investigate the probability that a star that is not dynamically associated with the microlensing event could be blended with the microlensing target, even in a high-resolution image. This was previously estimated on a case-by-case basis for individual planet detections (Dong et al. 2009; Sumi et al. 2010; Janczak et al. 2010; Batista et al. 2011). The approach is to count the number of stars on the high-resolution image within (e.g., $3\sigma$ of the detected excess flux) and estimate the probability that there could be one within the PSF of the microlensing target. While the probability of such an occurrence has been $\lesssim 5\%$ in all cases, a blend contribution from an ambient interloping star could be more insidious. Rather than the possibility that all of the excess flux could be due to an interloper, there exists the possibility that only some of it is. Depending on the magnitude of the contribution, this could affect the derived lens mass in the same way as an undetected companion to the lens or source.
I begin by appending the LF of Zheng et al. (2004) to that of Holtzman et al. (1998), normalizing the former to the latter using data in the range 6.5 \( \leq M_I \leq 9.0 \), where the two overlap. Then I convert the combined LF, which now extends to \( M_I = 13.5 \), to the \( H \)-band using the same isochrone as in Section 4.1.2, again using \( I-H = 1.37 \) for stars with \( M_I < 2.67 \). I take \( A_H = 2.0 \) to be typical of the proposed KMTNet fields (see Figure 13 of H2014a) and convert to the \( H \)-band using the Cardelli et al. (1989) relations and \( R_V = 2.5 \) (Nataf et al. 2013) to obtain \( A_H = 0.8 \). Assuming a uniform distance to all interloping stars that is equivalent to the Galactocentric distance, \( D_{\text{int}} = R_{\text{GC}} = 8.2 \) kpc (Nataf et al. 2013), I then compute the CDF for \( \theta_{\text{FWHM,JWST}} \) because it has the smallest aperture and thus largest \( \theta_{\text{FWHM}} \) of the facilities explored here. I also multiply the stellar number density of the LF by a factor of 1.25 to account for the average increase in the surface density of stars of the KMTNet fields compared to that of Baade’s Window, for which the LF was derived, using the Galactic density models described in Section 3.1.2 of H2014a.

6.3.1. Results

Figure 7 shows a CDF of the probability of the presence of an ambient interloping blend star as a function of the \( H \)-band magnitude of the interloper \( H_{\text{int}} \) as computed above. The apparent magnitude distribution extends to \( H_{\text{int}} \sim 26 \), which is equivalent to the faint limit of lenses that could be detected by any of the methods discussed here. The probability of such a star falling within a seeing disc with \( \text{FWHM} = \theta_{\text{FWHM,JWST}} \) is \( \leq 3\% \) integrated over the full magnitude range. Decreasing the assumed values of \( A_H \) and \( D_{\text{int}} \) only acts to shift the distribution to brighter magnitudes, leaving the maximum probability unaffected. In assuming \( \theta_{\text{FWHM}} = 200 \) mas as a worst-case scenario approximation, the CDF reaches a maximum probability of \( \sim 23\% \). While this is undoubtedly more significant, it is still low and otherwise improbable, and even for smaller values of \( D_{\text{int}} \) and \( A_{H,\text{int}} \) the net effect (i.e., the resulting \( \Delta M_I \)) is likely non-catastrophic. More optimistically, using \( \theta_{\text{FWHM,GMT}} \) yields a maximum probability of \( \sim 0.15\% \).

7. DISCUSSION

Here I have explored the potential of current and future high-resolution facilities to obtain flux measurements of the host stars of planetary systems predicted to be detected by KMTNet (H2014a). GMTIFS on GMT provides a powerful tool with which to constrain lens fluxes. It will be able to measure \( F_I \) to \( \leq 10\% \) for \( \sim 60\% \) of KMTNet’s predicted planet detections \( \Delta t = 5 \) yr after each event by imaging lenses spatially resolved from the source, and for roughly one-seventh of detections after \( \Delta t = 1 \) yr by measuring the elongation of the PSF of the unresolved microlensing target (lens+source). Furthermore, NIRCAM on \( \text{JWST} \) would be able to carry out high-precision (\( \sigma_{H_I} \leq 0.1 \)) measurements for \( \sim 28\% \) of events \( \Delta t = 10 \) yr after each event by imaging resolved lenses, and NACO on VLT could obtain lens flux measurements via prompt follow-up photometry for the \( \sim 42\% \) of planet detections accessible to it at the one-sigma level within \( \Delta t = 3 \) months of the events and could be used as soon as KMTNet comes online. These are exciting prospects for increasing the number of well-constrained microlensing planet detections, which are integral to our understanding of their underlying demographics and formation mechanisms.

I additionally explore the effects that contaminating flux from possible blended objects would have on \( F_I \). Undetected companions to the lens would lead to catastrophic failures in the derived lens mass, \( \delta M_I > 0.5 \), for \( \leq \langle 16\% \rangle \) of predicted KMTNet planet detections. The same fraction for undetected companions to the source drops to \( \lesssim \langle 9\% \rangle \). In both cases I assumed 100\% binarity, so these fractions would be further suppressed by the underlying distribution of stellar multiplicity. The integrated probability of blend flux contributions from interloping stars not dynamically associated with the event is even lower, reaching a maximum of \( \sim 3\% \) for \( \theta_{\text{FWHM,JWST}} = 68 \) mas.

7.1. Measuring Lens Masses with Parallax and Proper Motion

In this paper I focused on methods to constrain lens fluxes. By combining measurements of \( F_I \) and \( \theta_H \) with a mass–luminosity relation and an estimate of the extinction toward the lens, \( M_I \) can be derived via Equation (1). It is possible to also obtain an independent measurement of the lens mass from information obtained when imaging a resolved lens. Because the lens and source are resolved, their angular separation \( \Delta \theta \) can be computed from the photometric images. The time elapsed since the peak of the microlensing event \( \Delta t \) is also known. From these two parameters the vector heliocentric proper motion can be measured. The direction of proper motion is parallel to the parallax vector. Thus, a measurement of the vector proper motion combined with a one-dimensional measurement of the component of the microlens parallax that is parallel to the direction of Earth’s acceleration, \( \pi_{E,\parallel} \), yields a direct measurement of \( M_I \) (Gould 2014).

There are several advantages to this method. It does not rely on a detection of finite-source effects, it does not require multiband data, it is not subject to the systematic uncertainties inherent in the conversion of a source color to a physical radius, and the presence of a companion to the lens and/or source does not introduce additional uncertainties (in fact, it is precisely the opposite if said companions are bright).
Here I explore the ability of a simulated observing program on GMT $\Delta t = 5$ yr after each event to compute the vector proper motion and $\pi_{E,\parallel}$, which ultimately give a direct measurement of $M_\ell$. There are three quantities involved in this process: the magnitude of the proper motion, the direction of the proper motion, and the one-dimensional parallax. Given that $\Delta t$ will be known to extremely high precision, the two primary sources of uncertainty will be from the vector proper motion and $\pi_{E,\parallel}$. As shown in Gould et al. (2003), the asymmetry induced by parallax can be encapsulated in a single parameter $\gamma$ that is proportional to $\pi_{E,\parallel}$. They provide an analytic scaling relation\footnote{Their manuscript indicates that $\sigma_\gamma \propto f$ when it should read $\sigma_\gamma \propto f^{-1}$ (i.e., a higher observational cadence acts to improve the precision to which $\gamma$ can be measured). I have corrected this in the above equation.} to estimate the fractional precision to which $\gamma$ can be determined from a dedicated ground-based microlensing observational campaign,

$$\frac{\sigma_\gamma}{\gamma} = \frac{1}{12} \left( \frac{\sigma_{\text{ph}}}{0.01} \right) \left( \frac{f}{144 \text{ day}^{-1}} \right)^{-1} \left( \frac{S}{3} \right) \left( \frac{\tilde{v}}{800 \text{ km s}^{-1}} \right) \times \left( \frac{t_E}{20 \text{ days}} \right)^{-3/2} \left( \frac{|\cos \psi \cos \phi|}{0.5} \right)^{-1}, \quad (12)$$

where $\sigma_{\text{ph}}$ is the photometric precision of the target, $f$ is the cadence of observations, $S$ will vary monotonically between 2.1 and 4.4 for typical KMTNet observations, $\tilde{v}$ is the relative lens–source velocity projected onto the observer plane, $\cos \psi$ gives the length of the Earth–Sun separation projected onto the plane of the sky, and $\phi$ is the angle between the source trajectory and said projected separation. I determine $\sigma_{\text{ph}}$ for each event as described in Section 3.3.2 of H2014a (excluding noise due to the Moon and to unresolved stars) and estimate $f$ to be 54 day$^{-1}$, assuming an average nine-hour observing night and a 10 minute cadence for KMTNet’s three telescopes. I take $S$ to be three and compute $\tilde{v}$ for each event via

$$\tilde{v} = \frac{\theta_E D_s}{t_E} \left( \frac{D_s - D_t}{N} \right). \quad (13)$$

Then I set the final term equal to the fiducial value of 0.5 for each event.

I assume that the fractional precision of the magnitude of $\mu_{\text{rel}}$ is the quadrature sum of the precisions to which the centroids can be determined for both the lens and the source, which I approximate as the ratio of the FWHM (in pixels) to the $S/N$, divided by $\Delta t$. The uncertainty in centroiding both the lens and the source is included twice, once for each component axis. The fractional precision of the direction of $\mu_{\text{rel}}$ is similar to the fractional precision of its magnitude, so I multiply the latter by $\sqrt{2}$ to obtain the fractional precision of the vector proper motion. Because the vector proper motion measurement comes from the high-resolution data, the $S/N$ for both the lens and source is computed as described in Section 3.2.

In practice, the fractional precision of a lens mass derived in this manner requires careful treatment of the covariances between all input parameters. Furthermore, the measured proper motion vector is derived in a heliocentric reference frame whereas $\pi_{E,\parallel}$ and $\Delta t$ are in a geocentric frame. Transforming between the two requires solving a quadratic equation and can thus lead to a potential two-fold degeneracy in certain cases (see Gould 2014 for a complete discussion). However, as shown in Figure 8, the fractional precision of the vector proper motion from the high-resolution data is generally much better than that of $\gamma$. I thus assume that the latter will set the minimum fractional precision of $M_\ell$ and show its distribution in Figure 8. Even taking this to be a conservative lower limit, only $\sim 14\%$ of events would have $\sigma_\gamma/\gamma \lesssim 0.1$, and $\sim 40\%$ would have $\sigma_\gamma/\gamma > 0.5$.\footnote{Their manuscript indicates that $\sigma_\gamma \propto f$ when it should read $\sigma_\gamma \propto f^{-1}$ (i.e., a higher observational cadence acts to improve the precision to which $\gamma$ can be measured). I have corrected this in the above equation.}
Nevertheless, photometric follow-up of microlensing events by groups such as the Microlensing Follow-Up Network (Gould et al. 2006) and RoboNet (Tsapras et al. 2009) provide high-cadence coverage that can increase $f$ in Equation (12) by up to an order of magnitude.

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