ON SOLAR FREQUENCY CHANGES

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ABSTRACT

The structure of the surface layers of the Sun is changed by magnetic activity which, in turn, changes the eigenfrequencies of the acoustic modes. These frequency shifts have been observed both in low- and high-degree data, and are found to be correlated with the solar cycle.

Time-distance helioseismology has shown that waves travelling through sunspots suffer time delays. Also it is known that sunspots induce phase shifts between inwardly and outwardly propagating waves.

In the present work we show how these phase shifts are related to the observed time delays by treating the scattering of acoustic waves off sunspots as a diffraction problem.

These time delays are then related to the frequency shifts of the modes. From this relation we obtain a lower limit for the contribution of sunspots to the observed frequency variations with the solar cycle. Moreover, we predict the frequency dependence of the time delays suffered by rays travelling through a sunspot.

Key words: helioseismology; solar cycle; sunspots.

1. INTRODUCTION

Since the mid eighties (Woodard & Noyes 1985) there has been clear evidence that the frequencies of acoustic oscillations in the Sun change with the solar cycle. These frequency shifts have now been observed over the period of a solar cycle (Elsworth et al. 1994, Jiménez-Reyes 1998), and frequency differences of the order of a few hundred of nanoHertz have been found between the two extremes of the cycle.

Since then, a significant amount of work has been undertaken in order to study the dependence of these shifts on many parameters, such as the frequency, degree and azimuthal order of the modes, as well as on different time-scales and on the different epochs of the solar cycle. As a result of these studies it was shown that the frequency shifts vary with frequency and degree of the modes in a way that is proportional to the inverse of the mode inertia (Libbrecht & Woodard 1990), suggesting that this effect is due to a change at the surface, rather than due to a change in the structure of the Sun's interior. The dependence on the azimuthal order, on the other hand, allowed for the study of the latitudinal dependence of the surface perturbation frequency shifts (Kuhn 1988, Gough 1988, Libbrecht & Woodard 1990, Woodard & Libbrecht 1993), showing that it follows the latitudinal variations of the effective temperature. Moreover, with another solar maximum coming up, new results are starting to appear (Dziembowski et al. 1997) which seem to agree well with those of the previous solar cycle.

The time-scales on which the frequency variations take place were also the subject of several studies. In addition to the long-term variations following the period of the solar cycle, Woodard et al. (1991) and Bachmann & Brown (1993) found short-term frequency changes, taking place over periods as short as one month. More recently, Jiménez-Reyes (1998) also found such short-term frequency shifts, this time using low-degree data.

The good correlation between the long-term frequency shifts and several solar-cycle indices (e.g. Bachmann & Brown 1993, Elsworth et al. 1994, Jiménez-Reyes 1998) supports the idea that these changes are due to either the direct effect of the magnetic field on the oscillations or its indirect effect, via changes in the Sun's thermal structure, or both. Exactly which of these effects is the most important is still unknown, although efforts have been made to reconcile the observed frequency shifts with the magnetic activity and the surface brightness variations (Goldreich et al. 1991, Balmforth et al. 1996, Kuhn 1998). Moreover, considering the effect of the magnetic field on the frequency shifts, it is still uncertain which magnetic regions contribute more significantly for these shifts. Is it the quiet sun, the sunspots, or other magnetically active regions? The theories that explore the direct effect of the magnetic field on the frequencies commonly use a regular perturbation method, which is valid only when the magnetic perturbations are small, breaking down in regions such as sunspots, where the magnetic field is strongly concentrated near the surface. As a result, the contributions arising from such regions are difficult to assess. Knowledge about the extent to which various regions and effects (thermal or magnetic) influence the solar frequencies would help us to know what solar cycle indices we should be correlating the frequency shifts with.

A possible way to study the effect of localized structure variation could be via time-distance helioseismology. In the present work we use the modal in combination with the time-distance approach to helioseismology in an at-
2. SUNSPOT SEISMOLOGY

Recently sunspots have been studied seismically by two completely different methods. The first one uses a Fourier-Hankel decomposition of the solar oscillations in the vicinity of the sunspot to calculate the complex amplitudes of inwardly and outwardly propagating \( p \) modes ("modal approach") (Braun, Duvall, & LaBonte 1988; Braun, LaBonte, & Duvall 1990, Braun et al. 1992; Bogdan et al. 1993; Braun 1995).

The signature of the sunspot manifests itself in travel-time anomalies relative to their quiet-sun counterparts. The lateral dependence of the new boundary condition... (see Fig.1) (Duvall et al. 1993; Duvall 1995; Korzennik, Noyes, & Ziskin 1995; D’Silva, & Duvall 1995; Duvall et al. 1996; D’Silva et al. 1996; Kosovichev 1996; Braun 1997).

These studies generally find an attenuated amplitude of the outwardly propagating wave and a positive phase shift relative to the inwardly propagating wave.

The second method measures the temporal cross-correlation between oscillations recorded at two diametrically opposite points on a circle centred on the sunspot (see Fig.1) (Duvall et al. 1993; Duvall 1995; Korzennik, Noyes, & Ziskin 1995; D’Silva, & Duvall 1995; Duvall et al. 1996; D’Silva et al. 1996; Kosovichev 1996; Braun 1997).

The signature of the sunspot manifests itself in travel-time anomalies relative to their quiet-sun counterparts. Assuming geometrical acoustics to be valid, Duvall (1995) deduced a phase shift of 30° induced by one particular sunspot (corresponding to a time delay of 0.56 min at 3 mHz). Various models have been invoked to explain the observed phase shifts and the attenuation of the amplitude, including downflows beneath sunspots or conversion to slow magnetoacoustic waves within the sunspot. However, to date, no conclusive explanation has been found. In fact, in most cases the interpretation of the observations is still subject to speculation. The idea behind the work we present here is to relate these travel-time anomalies to changes in eigenfrequencies of acoustic modes.

3. TRAVEL-TIME DIFFERENCE VS FREQUENCY SHIFT

3.1. Normal modes

We first concentrate on the analysis of normal modes, and choose a radius \( R^* \) in the region of propagation of the eigenmode which is below the region perturbed by the sunspot but still close to the surface (Fig. 3).

Note that, even if the sunspot extends considerably into the Sun’s interior, its effect is important only near the surface, due to the rapid downward increase in the gas pressure, and, therefore, it is always possible to choose a suitable value for \( R^* \). In the interior of the Sun, defined here as the region between the centre and \( R^* \), the oscillations obey the usual equations of hydrodynamics. Therefore, we can express any perturbation above \( R^* \) as a change in the boundary condition applied to these equations at \( r = R^* \). This change in the boundary condition at \( r = R^* \) generates a shift in the eigenfrequencies of the modes, and it is precisely this shift that we want to obtain. In principle, the change in the boundary condition will not only change the eigenfrequencies of the modes but also the eigenfunctions, which will no longer be proportional to single spherical harmonics, as result of the latitudinal dependence of the new boundary condition. Fortunately, however, it is possible to use a variational principle to relate the change in the boundary condition to the frequency shifts, while avoiding the tedious work of calculating the perturbed eigenfunctions. The variational principle can be expressed as:

\[
\frac{\Delta \omega}{\omega_0} = \frac{\pi \Delta \phi}{\int_{R^*}^{R^* \ast} \frac{\omega_0^2 c^2}{\kappa^2} \kappa^{-1} \mathrm{d}r},
\]

(1)

where \( \omega_0 \) is the unperturbed frequency, \( c \) is the sound speed, \( \kappa \) is the vertical wavenumber and \( \pi \Delta \phi \) is the change in the phase of the local eigenfunction at \( r = R^* \), which constitutes the change in the boundary condition to which we referred above. The bar over the phase shift stands for the average over latitude, i.e.

\[
\bar{\pi} \Delta \phi = \int_{0}^{2\pi} \int_{-1}^{1} \pi \Delta \phi \left( Y_l^m \right)^2 \mathrm{d}(\cos \theta) \mathrm{d}\phi,
\]

(2)

where \( Y_l^m \) is a spherical harmonic. So, equation (3) can be used to convert a perturbation in the local phase of the eigenmode at \( r = R^* \) into a shift in its eigenfrequency.

3.2. Travel times

Having obtained the relation between phase shift and frequency shift, the next step is to infer this phase shift from the travel-time anomalies produced by the sunspots. To do that, let us go back to the ray paths and consider again...
a wave travelling between any two surface points A and B. The starting point A can be regarded as a source point of waves, each travelling in a different direction. Therefore, at point B the wave travelling horizontally will interfere with the wave travelling along the ray path joining A and B. The condition for constructive interference is that \( (\text{Gough } 1984) \):

\[
\int_A^B k_s ds - \int_A^B k_h Rd\theta + \Phi = 2\pi n, \tag{3}
\]

where \( k_s \) is the wavenumber along the ray path, \( k_h \) is the horizontal wavenumber and \( \Phi \) is a phase which accounts for any phase jump that the ray may suffer on reflection.

But \( \Phi \) is yet another phase.

\[
\Delta \Phi = \omega_0 \Delta \tau - \frac{\omega_0 \Delta \tau}{I}\frac{A_{\text{sp}}}{A_{\odot}}, \tag{9}
\]

where \( A_{\text{sp}}/A_{\odot} \) is the ratio between the sunspot area and the surface area of the Sun.

At this point it is probably a good idea to stop and explore equation \( (9) \) more carefully. As was mentioned in section 1, it is known from observations that the frequency shifts of global modes follow the inverse of the mode inertia. The contributions to the frequency shifts from the sunspot regions have a similar dependence. Hence, we can predict the dependence of \( \Delta \tau \) on the frequency of the rays considered, by writing,

\[
\Delta \tau = -C \Delta \omega \int_{r_1}^{r_2} \frac{\omega_0^2 e^{-2\kappa^{-1}} dr}{\omega_0 A_{\text{sp}}}, \tag{10}
\]

where \( C \) is a constant. We hope that in the near future it will be possible to test this frequency dependence of the time delays against observational data.

Another point that deserves some attention is the dependence of the frequency shifts on the degree \( l \) of the modes. Aside from the dependence on \( l \) due to the mode inertia, the frequency shift varies with the degree of the mode also due to the asphericity of the surface perturbations. Equation \( (9) \) was written for the case \( l = 0 \), but it is straightforward to write an equivalent expression for other degrees simply by using the definition \( (2) \) to take the average of equation \( (8) \), and substituting it in equation \( (9) \). Having done that, it is possible to compare the outcome with frequency shifts suffered by modes of different degrees, and, in principle, extract information about the asphericity of the perturbations, since perturbations at different locations in the sun affect modes of different degrees in different ways.

In this work we use the time delays published by Duvall (1995). The area covered by sunspots at a given time was obtained from the international relative sunspot number, where, the area in millionths of the visible hemisphere is given by \( A_{\text{sp}} = 16.7 R_s \) and \( R_s \) is the relative sunspot number. Using those data we obtain for a mode of degree \( l = 0 \) and a frequency of 3 mHz, a frequency shift of about one tenth of the observed frequency difference between the two extremes of the solar cycle for a mode with the same characteristics. We must emphasize, however, that this value is only a lower limit to the contribution of the sunspots to the variations in the eigenfrequencies. In our calculation we have ignored two effects that will increase this contribution: Firstly, in the upper layers of the sun geometric acoustics breaks down and, consequently, we must allow for diffraction effects when relating the observed time delay with the phase jump \( \Delta \phi \) induced by the sunspots. As we shall see in section 4, this phase jump is actually larger than the value deduced from geometrical acoustics; secondly, the area cover by sunspots (as far as interaction with acoustic waves is concerned) might be larger than the observed penumbral area.

4. SCATTERING OF SOUND WAVES BY SUNSPOTS

When a wave is incident on an obstacle in its path, in addition to the undisturbed wave, there is a scattered wave...
which spreads out from the scatterer in all directions and which distorts and interferes with the original wave. If the obstacle is very large compared with the wavelength, i.e. in the limiting case of geometrical acoustics, the scattered wave interferes with the undisturbed wave in such a way as to create a sharp-edged shadow behind the object.

However if the size of the obstacle is comparable with the wavelength, a variety of diffraction phenomena occur.

Here our scattering body is the sunspot, which we model as a patch on the solar surface whose effect is to induce a fixed phaseshift $\delta$ upon all incident rays.

Notwithstanding the fact that the asymptotic ray description breaks down at the upper turning point and that the linearised adiabatic wave equation is not even valid there, let us consider a ray originating at a point $A$ as shown in Fig.1.

The wavefield $\psi$ observed at point $B$ due to all singly reflected rays from a source at point $A$ can be represented by the undisturbed reflected ray (continuous curve) plus the ray that is scattered off the sunspot (dashed curve):

$$\psi(B) \propto 1 + ae^{i\delta}e^{i\omega(\tau_s - \tau_r)},$$

where $\omega$ is the angular frequency of the acoustic waves, $\tau_r$ is the wave travel time of the reflected ray and $\tau_s$ that of the scattered wave. The amplitude $a$ is essentially the scattering strength of the sunspot, which depends on the physical properties of the sunspots as well as the exact shape of the ‘trapping potential’, $k(r) = (\omega^2 - \omega_c^2)/c^2$ ($\omega_c$ is the acoustic cut-off frequency) at the upper turning point. We omitted here the $1/r^2$ dependence of the amplitudes because the lengths of the paths of the reflected and scattered ray are nearly equal (identical for plane-parallel polytrope).

Since the waves propagate nearly vertically at the upper turning point, we are neglecting any angular dependence of the scattered wave. Also we are ignoring any possible angular variation in the amplitude of the incident wave field.

In the Born approximation to scattering, this scattering strength is proportional to the area of the sunspot. Thus, by observing sunspots of different sizes, one might be able to constrain $a$.

The time delay caused by a sunspot as a function of the offset from the spot of the origin of the circle over which the signal is averaged (see Fig.1) has been measured by Duvall (1995), and is shown in Fig.2.

Fig.2 can be reproduced by averaging equation (11) over all azimuthal angles for a particular offset of the sunspot from the centre of the observation ring and the result is shown by the dotted line in Fig.2.

It is important to notice that different sets of pairs of scattering strengths, $a$, and phase shifts, $\delta$, can yield the same time delays in Fig.2. In Fig.5 we have plotted those scattering strengths versus those phase shifts that are compatible with the measurement of Fig.2.

We note that in order to produce a time delay of 0.56 minutes the scattering strength has to lie between 0.6 and 1.0, causing the phase shift induced by the sunspot to lie between 1.05 and 1.55 rad.

5. DISCUSSION

As mentioned in the previous section, it is not possible to evaluate the phase shift induced by a sunspot from the observed time delay alone, unless we know the scattering strength of the sunspot. However, a simple model suggests that this phase shift lies between 1.05 and 1.55 radians. The correction to the value obtained for the contribution from sunspots to the frequency shifts depends on this phase shift. Moreover, it depends on the effective area of the sunspots (i.e., the area of interaction with the acoustic waves), which might be larger than that observed. Therefore, using the minimum value for the phase shift obtained from the observed time delays, we conclude that the contribution to frequency shifts coming from sunspots is greater than one fifth. It might be significantly greater if the effective area of the sunspots is larger than that observed, or if the phase shifts induced by the sunspots is larger than the minimum value required to reproduce Duvall’s measurements.

6. CONCLUSION

We have shown how the changes of the frequencies of acoustic modes during the solar cycle can be related to the time delays induced by sunspots.

We have demonstrated that the time delays found in the presence of a sunspot cannot be converted unambiguously into a phase shift due to a sunspot without the knowledge of its scattering strength. We can, however, set limits on the maximum and minimum values of the phase shift, those being 1.05 and 1.55 radians, respectively.

Accordingly, at the present, we can give only a lower limit to the contribution from the sunspots to the frequency variations of global modes; this lower limit is about one fifth of the observed frequency shifts. Moreover, by relating the frequency shifts to the time delays as determined by the time-distance analysis of sunspots, we predict the frequency dependence of these time delays to follow equation (10).

We hope that in the near future time-distance observations will be able to test the latter prediction, as well as to provide sufficient data on sunspots of different sizes in order to allow for the determination of the scattering strength of the sunspots as a function of their effective area. This will yield a more accurate estimate of the contribution of sunspots to the frequency shifts of global modes.

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Figure 2. Travel time difference in the presence of a sunspot as compared to the quiet sun as a function of the offset of the origin from the spot (from Duvall 1995). The marks on the bottom indicate the extent of the penumbra. Superimposed (dotted line) is the theoretically expected time delay for those parameters shown in Fig.5.

Figure 3. Diagram of the propagating region of an eigenmode; $r_1$ and $r_2$ are the lower and upper turning points, respectively.

Figure 4. Scattering of acoustic rays at sunspot.
Figure 5. Scattering strength versus phase shift compatible with the observation by Duvall (1995).