BDS RTK High-precision Positioning Algorithm for Deformation Solution

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Abstract. In view of the increasing scope, scale and quantity of landslid e, debris flow and other geological disasters and without achieving long-term stable high-precision monitoring in the exist, a solution algorithm on RTK high-precision differential positioning deformation is presented in this paper. The algorithm uses high-precision board receiver as the satellite positioning reference station, the u-blox receiver as the monitoring station. The carrier phase double difference relative positioning model is based on the carrier phase observation and pseudo range observation, combined with the RTK unscented Kalman filter model and smooth filtering algorithm, and then more accurate position results are obtained. This method can be used to calculate the position of deformation monitoring body with high precision, which is simple, easy to realize and practical. The positioning accuracy of deformation monitoring is higher. After smoothing, the horizontal precision can reach 3mm, and the elevation precision can reach 5mm.

1. Introduction

In recent years, landslides, debris flows and other geological disasters occur frequently [1] due to the change of climate and the influence of man-made engineering activities. Especially in Southwest China, the scope, scale and quantity of landslides and other disasters are on the rise, which has become an important reason to restrict local economic development and threaten people's lives and property [2]. Meanwhile, with the completion and wide application of BDS and other satellite positioning systems, BDS/GPS positioning technology has become the mainstream trend of deformation monitoring for landslide, bridge, high-rise building, etc. [3]. In 2016, Shi Qiang and others of Central South University studied the single-frequency GPS/BDS combination system to test the deformation simulation device of steep slope, with the horizontal accuracy up to 2-3mm and elevation accuracy up to 5~6mm[4];In 2016, Peng Zhenzhong and others of Sun Yat-sen University used BDS/GPS technology to monitor the bridge and pointed out that the positioning accuracy of the combined technology can reach millimeter level [5].

At present, BDS/GPS based intelligent deformation monitoring system mainly uses BDS/GPS high-precision receiver to obtain Beidou multi-frequency data and other satellite navigation data and adopts RTK positioning algorithm to achieve real-time deformation monitoring. The traditional RTK algorithm has high real-time performance and deformation tracking performance, limited by centimeter level accuracy [6]. In literature [7], Kalman filter is used for iterative calculation in RTK algorithm to effectively reduce and separate the noise signals in satellite signals, eliminate errors and improve positioning accuracy. However, as single-frequency RTK used, it will be affected by other
errors so that the accuracy improvement is limited. In reference [8], sparse state transition matrix is used to improve Kalman filter algorithm, which improves the accuracy. However, the efficiency of the algorithm is limited because of the nonlinear system.

In the monitoring of landslide deformation monitoring, bridge monitoring and other scenes, the existing real-time dynamic differential positioning algorithm is difficult to meet such positioning requirements because of the need to achieve long-term and stable high-precision monitoring. In view of this situation, this paper presents a high-precision RTK differential positioning deformation solution method. Firstly, in the aspect of system design, the monitoring network uses the "1 +N" distribution mode to comprehensively monitor the monitoring body and master the overall deformation of the monitoring body. Secondly, in the algorithm, the least square method is used to estimate the position and clock difference of the monitoring station, and the ionospheric and tropospheric errors are transformed into distance information, which is added to the pseudo range parameter value to realize the single point pseudo range positioning and achieve the meter level positioning accuracy. Thirdly, the carrier phase double difference relative positioning model is constructed based on the carrier phase observation value and pseudo range observation value to eliminate the receiver clock difference, satellite clock difference, ionosphere error and troposphere error and achieve millimeter level positioning accuracy. Lastly, the tracking free Kalman filter is used for RTK positioning nonlinear system and the optimal estimation of floating-point solution is output based on the minimum variance. The Kalman filter algorithm is used to eliminate the disturbance phenomenon such as signal instability and get more accurate positioning results.

2. Deployment of RTK high precision differential positioning deformation monitoring

RTK high-precision differential positioning deformation monitoring system uses high-precision board receiver as the satellite positioning reference station for deformation monitoring and uses u-blox receiver as the satellite positioning monitoring station. Landslide monitoring site adopts the "1+N" distribution mode, 1 precision positioning reference station and N deformation monitoring stations, as shown in Figure 1.

![Figure 1. Layout of landslide monitoring network](image)

2.1 Hardware system construction

The satellite positioning reference station and the satellite positioning monitoring station can adopt the single-mode positioning mode, double-mode positioning mode and multi-mode positioning mode. In the embodiments of this paper, the satellite positioning reference station and the satellite positioning monitoring station all adopt the multi-mode positioning method, so the satellite positioning reference station and the satellite positioning monitoring station are both BDS, GPS and GNSS multi-mode receiver[9]. The precise positioning reference station is located at the position with solid and stable foundation, wide vision and no high-power radio transmission source. According to the potential direction of landslide displacement and the possible position of surface subsidence and collapse, N monitoring stations are set up.
2.2 Communication system implementation
Satellite positioning reference station and satellite positioning monitoring station can measure the pseudo-distance, carrier phase and other data with high precision. The low-power 4g-dtu communication module is used to send monitoring station data to the server, and the communication network can switch between 2G, 3G and 4G according to the signal strength automatically. The location solution is carried out on the server to realize data analysis and get the landslide warning level.

3. RTK high-precision differential positioning deformation algorithm
The algorithm is mainly an RTK Positioning Algorithm of high-precision deformation monitoring with carrier phase combined with pseudo-distance pseudo range. The overall flow chart of the algorithm is shown in Figure 2. The single point position, datum station carrier phase calculation, the pseudo range residual are obtained by using least square method according to satellite position, clock error and pseudo range. The location of monitoring stations are determined by selecting the common satellites and the reference satellite, designing Kalman filter to update time and status, calculating fuzzy degree of the whole week, fixing ambiguity, obtaining fixed solution. These position results can be smoothed by designing the Kalman filter, improving the positioning accuracy, reducing the influence of outlier.

![Figure 2. Overall flow chart of carrier phase joint pseudo range high precision differential positioning algorithm](image)

3.1 Single point location algorithm based on least square method
The navigation data and observation data decoded at time k are obtained and classified and saved according to the reference station and monitoring station, including the pseudo-distance measurement value, carrier phase measurement value, ephemeris parameter and other data of each satellite from the reference station and monitoring station.

The ephemeris parameters are used to calculate the parameters such as satellite position and clock difference, and the least square method [10-11] is used to estimate the position and clock difference of the monitoring station according to the pseudo-distance value of the available satellites. The flow chart of the least square single point positioning algorithm is shown in Figure 3.
Set the initial position and the initial clock difference of the monitoring station. Generally, the initial clock difference $\delta t^{(t-1)}$ can be set to 0, and the position of the monitoring station can be set to a value close to the position of the monitoring station.

Set the distance between the satellite and the earth. According to the satellite position and the initial position of the monitoring station, the satellite ground distance can be obtained.

$$
\rho^{(N)} = \sqrt{(x^{(N)}-x)^2 + (y^{(N)}-y)^2 + (z^{(N)}-z)^2}
$$

where $x^{(N)}, y^{(N)}, z^{(N)}$ is N satellite location, and $(x, y, z)$ is the location of the monitoring station.

Seek broadcast ephemeris for ionospheric and tropospheric errors. The ionospheric and tropospheric errors can be calculated by the ionospheric delay correction model and the tropospheric delay correction model respectively. The ionospheric and tropospheric errors are reflected in the form of time and then converted into distance information, which is directly added to pseudo distance.

Construct geometric matrix $G$ and measurement matrix $b$. The details are as follows:

$$
G = \begin{bmatrix}
-l^{(N)}(x_{k-1}) & -l^{(N)}(y_{k-1}) & -l^{(N)}(z_{k-1}) & 1 \\
-l^{(N)}(x_{k}) & -l^{(N)}(y_{k}) & -l^{(N)}(z_{k}) & 1 \\
... & ... & ... & ... \\
-l^{(N)}(x_{m}) & -l^{(N)}(y_{m}) & -l^{(N)}(z_{m}) & 1
\end{bmatrix}
$$

$$
b = \begin{bmatrix}
\rho^{(N)}(x_{k-1}) - \delta t_{k-1} \\
\rho^{(N)}(x_{k}) - \delta t_{k} \\
... \\
\rho^{(N)}(x_{m}) - \delta t_{m}
\end{bmatrix}
$$

Where, $-l^{(N)}(x_{k})$ is represented as the distance between satellite N and the monitoring station $r^{(N)}$ between the satellite N and monitoring station to $x$ at the position $X_{k-1}$ of the monitoring station at the previous moment. $\rho^{(N)}$ is the Pseudo-range distance from monitoring station to satellite $N$. $\delta t_{k-1}$ is the clock bias at the previous moment.

$$
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta \delta t
\end{bmatrix} = (G^T G)^{-1} G^T b
$$

Figure 3. flow chart of the least square single point positioning algorithm
Where \((\Delta x, \Delta y, \Delta z)\) is the calculated difference of three axes of the monitoring station. \(\Delta \delta_t\) is the clock bias of the monitoring station. The last difference is added with the front location value to update station location and clock bias.

\[
x_i = x_{i-1} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}
\]

\[
\delta t_{u,i} = \delta t_{u,i-1} + \Delta \delta t_u
\]

(3)

(4)

Judge whether the difference \(\sqrt{\left(\Delta x + \Delta y + \Delta z\right)^2 + \left(\Delta \delta t\right)^2}\) is less than a preset threshold value. If so, the position at the present time is obtained; If not, repeat the difference until it is satisfied.

### 3.2. Carrier phase double difference model

The geometric relationship between the satellite, the reference station and the monitoring station is shown in figure 4. The observation equation of the single-difference carrier phase model of satellite j, receiver b of reference station and monitoring station r can be described.

\[
\Phi_{rb} = \lambda^{-1}(r_{rb} - V_{rb} + \lambda T_{rb} + I_{rb} + N_{rb} + \delta \epsilon_{rb})
\]

(5)

Where, \(r\) is the satellite ground geometrical distance, and \(\lambda\) is the carrier wavelength, and \(T\) is the tropospheric error, and \(I\) is the ionospheric error, and \(\delta t\) is receiver clock bias, and \(N_{rb}\) is ambiguity of whole cycles, and carrier phase offset is measured in cycles, and \(\delta \epsilon_{rb}\) is carrier phase measurement noise.

In the case of short baseline, if the monitoring station and the reference station are at the same height, the influence of tropospheric delay and ionospheric delay can be eliminated by the single difference model. Formula (5) can be simplified as:

\[
\Phi_{rb} = \lambda^{-1}(r_{rb} - V_{rb} + \lambda T_{rb} + I_{rb} + N_{rb} + \delta \epsilon_{rb})
\]

(6)

(7)

For satellite j and satellite i, if satellite i is taken as the reference satellite, the dual-differential carrier phase model of the reference station receiver b and the monitoring station r can eliminate the receiver clock bias. The observation equation can be described.

\[
\Phi_{rb} = \lambda^{-1}(\rho_i - \rho_i') + N_{rb} + \delta \epsilon_{rb} \]  

\(j = 1,2,\ldots\)

(8)

In the monitoring of surface deformation, in order to ensure the accuracy and real-time performance of monitoring, this paper uses the algorithm of carrier phase and pseudo-distance joint solution to ensure the real-time performance of the solution and reflect the monitoring effect in real time. The observation equation of the pseudo-distance double-difference model can be described.

\[
\rho_{rb} = \rho_{rb} - \rho_{rb}' = \rho_i' - \rho_i' + \rho_i' - \rho_i'\]

(9)

Figure 4. Geometric relation diagram of the double-difference model of satellite reference station and monitoring station
3.3 RTK untracked kalman filter model design

RTK positioning system is a nonlinear system. The equation is described.

\[
\mathbf{x}_{k+1} = F(\mathbf{x}_k, \mathbf{u}_k, \mathbf{v}_k) \\
\mathbf{y}_{k+1} = H(\mathbf{x}_k, \mathbf{n}_k)
\] (10) (11)

In the formula of \( k \in \{1, \ldots, \infty\} \), \( \mathbf{x}_k \) is \( n_x \) dimensional vector, and \( \mathbf{y}_k \) is \( n_y \) dimensional vector, representing the system state and system observation quantity respectively. \( \mathbf{v}_k \) is the system noise, and \( \mathbf{n}_k \) is observed noise, with white gaussian noise and uncorrelation. The covariance matrices are respectively is \( P_v \) and \( P_n \). This is the specific value of the state vector of unsceneted Kalman filter.

\[
\mathbf{x}_k = (x_{k1}, x_{k2}, \ldots, x_{kn_x})^T
\] (12)

Where, \( (x_{k1}, x_{k2}, x_{k3}) \) is Coordinate values of monitoring stations, and \( N_{rb}^{lj} \) is ambiguity of whole cycles. This is the measured value of the unsceneted Kalman filter.

\[
\mathbf{y}_k = [\rho_{ib}^2 + \lambda \Phi_{rb}^2, \rho_{ib}^3 + \lambda \Phi_{rb}^3, \ldots, \rho_{ib}^m + \lambda \Phi_{rb}^m] ^T
\] (13)

Where, \( \rho_{ib}^l \) is the measurement value of the double difference pseudo range, and \( \lambda \) is wavelength, and \( \Phi_{rb}^{lj} \) is measurements of the double difference carrier phase.

So the initial value of \( \mathbf{x}_k \) is set.

\[
\hat{\mathbf{x}}_0 = E[\mathbf{x}_0] \\
P_0 = E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T]
\] (14) (15)

Compute each time of Sigma points.

\[
\mathbf{x}_{a,i} = \left[ \hat{\mathbf{x}}_{a,i} - \gamma \sqrt{P_{a,i}}, \hat{\mathbf{x}}_{a,i}, \hat{\mathbf{x}}_{a,i} + \gamma \sqrt{P_{a,i}} \right]
\] (16)

The basic principle is that \( 2n + 1 \) Sigma points \( \chi_i' \) are obtained by the UT transformation. Details are as follows.

\[
\chi_i' = \chi \\
\chi_i' = \chi + (\sqrt{1 + \lambda P_{a,i}} \chi) \\
\chi_i' = \chi - (\sqrt{1 + \lambda P_{a,i}} \chi) \quad i = n + 1, \ldots, 2n
\] (17) (18) (19)

Where, \( \chi \) and \( P_{a,i} \) are mean and variance of the state vector \( x \). The corresponding weight \( W_i \) of each Sigma is calculated as follows.

\[
W_{a,i} = \lambda/(n+\lambda) \\
W_i = \lambda/(n+\lambda) + (1-\alpha^2 + \beta) \\
W_{a,i} = 1/(2n+\lambda) \quad i = 1, \ldots, 2n
\] (20) (21) (22)

Where, \( \lambda \) is higher order impact factor, and its size can be adjusted so that the distribution of Sigma points around the mean \( \chi \) is minimized by the influence of higher order terms. \( \lambda = \alpha^2(n+\lambda) - n \) is the scaling factor, and \( K \) is the semi-definite factor. Size of semidefinite factor can be adjusted so that \( (n+\lambda)P_{a,i} \) is a semidefinite matrix. \( n = n_x \cdot \beta \) is variance precision factor, and the accuracy of variance can be improved. The mean value and variance of \( y_k \) can be obtained by the following formula.
The basic principle of Unscented Kalman Filter time updating is as follows.

\[ \chi_{k|k} = F \chi_{k-1|k-1} \]  

(26)

\[ \hat{\chi}_k = \sum_{i=0}^{2^n} \bar{W}^{(i)} \chi_{i,k} \]  

(27)

\[ \hat{P}_k = \sum_{i=0}^{2^n} \bar{W}^{(i)} [\chi_{i,k} - \hat{\chi}_k] [\chi_{i,k} - \hat{\chi}_k]^T + \mathcal{Q} \]  

(28)

Among them, \( \mathcal{Q} \) is the noise variance of the system, and \( E \) is time interval used by the BDS/GPS receiver, and \( \sigma_v, \sigma_a, \sigma_m \) is standard deviation of velocity noise at the monitoring station in east, north, and elevation. The basic principle of measurement update of unscented Kalman filter is as follows.

\[ \chi_{k|k} = I \chi_{k|k} + \chi_{k|k} \]  

(29)

\[ \hat{\chi}_k = \sum_{i=0}^{2^n} \bar{W}^{(i)} y_{i,k} \]  

(30)

This is receiver dynamic positioning mode.

\[ F_{k|k} = \begin{pmatrix} I_{3\times3} & I_{3\times3} & I_{3\times3} \\ \tau & 1 & 0 \\ 0 & \tau & 1 \end{pmatrix} \]  

(31)

\[ Q_{k|k} = \begin{pmatrix} \mathcal{Q} & 0 \end{pmatrix} \begin{pmatrix} I_{3\times3} & \mathcal{Q} \end{pmatrix} \]  

(32)

3.4. Smooth filter design

Because there may be interference phenomena such as unstable signals, the above RTK positioning algorithm based on UKF mentioned above may have low positioning accuracy, unstable phenomenon, even appeared estimating outliers. In order to get the more accurate positioning results, remove estimating outliers, Kalman filter can be designed to smooth the solution result. Use the calculated 3D coordinates of the landslide point as the observation value of Kalman filter, and use observations at neighboring times and time intervals to observe landslide speeds. The monitoring station motion model can be described below.

\[ s_{j+1} = s_j + v_j \tau + \frac{1}{2} a_j \tau^2 \]  

(33)

\[ y_{j+1} = y_j + v_j \tau + \frac{1}{2} a_j \tau^2 \]  

(34)

\[ \begin{pmatrix} s_{j+1} \\ v_{j+1} \end{pmatrix} = \begin{pmatrix} A_{j+1} & B_{j+1} \\ 0 & I \end{pmatrix} \begin{pmatrix} s_j \\ v_j \end{pmatrix} + \begin{pmatrix} w_{j+1} \\ \tau \end{pmatrix} \]  

(35)

\[ \hat{\chi}_k = \hat{x}_k + K_k (y_k - \hat{y}_k) \]  

(36)

\[ P_k = P_k - K_k P_k \tau K_k^T \]  

(37)

The output optimal estimate of floating point solution is

\[ \hat{x}_k = (x_n, x_e, x_d, N_{1\theta}, N_{2\alpha}, N_{3\phi})^T \]  

(38)

Among them, \( s_{j+1} \), \( v_{j+1} \) and \( a_{j+1} \) respectively represents displacement, velocity, acceleration. This is the equation of state.

\[ x_{j+1} = A_{j+1} x_j + B_{j+1} u_j \]  

(39)
As for it, this is state quantity. \( x \in \mathbb{R}^n \) and \( s \in \mathbb{R}^m \) respectively represent the amount of displacement in three axes; \( v \in \mathbb{R}^m \) respectively represent the speed of three ax es; These are state transition matrix and input relation matrix of state equation.

\[
A = \begin{bmatrix}
1 & t & 0 & 0 & 0 \\
0 & 1 & t & 0 & 0 \\
0 & 0 & 0 & 1 & t \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
n & 0.5 \tau^2 & n & 0 & 0 \\
1 & t & 0 & 0 & 0 \\
0 & 0 & t & 0.5 \tau^2 & 0 \\
0 & 0 & 0 & 0 & t \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

As result, the relationship (measurement equation) of equation of state \( x \) with observation vector \( y \) is:

\[
y = C x + v
\] (40)

Among them, the measurements is \( y = [s, v] \), and this is the relation matrix of the measurement equation.

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
\]

The prediction process of Kalman filter is as follows.

\[
\begin{align*}
\hat{x}_{k+1}^{(\text{predict})} &= A \hat{x}_{k}^{(\text{predict})} \\
P_{k+1}^{(\text{predict})} &= A P_{k}^{(\text{predict})} A^T + B Q_{\text{pin}} B^T
\end{align*}
\] (41) (42)

The Kalman filter correction process is followed.

\[
\begin{align*}
K_{k} &= P_{k}^{(\text{predict})} C^T (C P_{k}^{(\text{predict})} C^T + R_{\text{pin}})^{-1} \\
\hat{x}_{k}^{(\text{correct})} &= \hat{x}_{k}^{(\text{predict})} + K_{k} (y_{k} - C \hat{x}_{k}^{(\text{predict})}) \\
P_{k}^{(\text{correct})} &= (I - K_{k} C) P_{k}^{(\text{predict})}
\end{align*}
\] (43) (44) (45)

As for it, the process noise is \( W_{\text{pin}} \), \( Q_{\text{pin}} = \text{Cov}(W_{\text{pin}}) = E(W_{\text{pin}} W_{\text{pin}}^T) \), and \( Q_{\text{pin}} \) is symmetric matrix. The measurement noise vector is \( V_{\text{pin}} \), and solving process noise covariance matrix is \( R_{\text{pin}} \), which is symmetric matrix \( R_{\text{pin}} = \text{Cov}(V_{\text{pin}}) = E(V_{\text{pin}} V_{\text{pin}}^T) \). As for it, processing noise \( W_{\text{pin}} \) and measuring noise \( V_{\text{pin}} \) are both controllable amount, and this parameter can be adjusted according to the actual application to obtain the optimal filtering result during filtering. The flow chart of the smoothing algorithm for positioning results KF is shown in Figure 5.
4. Conclusion
In this paper, an RTK high-precision differential positioning deformation algorithm is proposed. In the case of hardware distribution, the least square method is used to realize single point location. The carrier phase relative positioning model is constructed by using the observed carrier phase value and the pseudo-distance observed carrier phase value. The unscented Kalman filter is used to output the optimal estimate of floating-point solution based on the minimum variance. In addition, the landslide velocity is added to the Kalman filter algorithm to eliminate the signal instability and other interference phenomena, and more accurate positioning results are obtained. The horizontal accuracy can reach $\pm 3\text{mm}$ after smoothing, and elevation accuracy can reach $\pm 5\text{mm}$. The algorithm is simple to implement, high positioning accuracy, and long-term stable high-precision monitoring.

Acknowledgments
Foundation Items: The Foundation of The Innovation Project of Guet Undergraduate Education(201810595045), Key Laboratory of Cognitive Radio and Information Processing, Ministry of Education (Guilin University of Electronic Technology) (CRKL190105), Guangxi Natural Science Foundation (2018JJA170154)

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