

PREFERRED OBSERVABLES, PREDICTABILITY, CLASSICALITY, AND THE ENVIRONMENT-INDUCED DECOHERENCE

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ABSTRACT

Selection of the preferred classical set of states in the process of decoherence – so important for cosmological considerations – is discussed with an emphasis on the role of information loss and entropy. Persistence of correlations between the observables of two systems (for instance, a record and a state of a system evolved from the initial conditions described by that record) in the presence of the environment is used to define classical behavior. From the viewpoint of an observer (or any system capable of maintaining records) predictability is a measure of such persistence. Predictability sieve – a procedure which employs both the statistical and algorithmic entropies to systematically explore all of the Hilbert space of an open system in order to eliminate the majority of the unpredictable and non-classical states and to locate the islands of predictability including the preferred pointer basis is proposed. Predictably evolving states of decohering systems along with the time-ordered sequences of records of their evolution define the effectively classical branches of the universal wavefunction in the context of the “Many Worlds Interpretation”. The relation between the consistent histories approach and the preferred basis is considered. It is demonstrated that histories of sequences of events corresponding to projections onto the states of the pointer basis are consistent.

1. Introduction

The content of this paper is not a transcript of my presentation given in course of the meeting: Proceedings contributions rarely are. However, in the case of this paper the difference between them may be greater than usual. The reason for the discrepancy is simple. Much of what I said is already contained in papers (see, for example, my recent Physics Today article (Zurek 1991) and references therein). While it would be reasonably easy to produce a “conference version” of this contribution, such an exercise would be of little use in the present context. Nevertheless, I have decided to include in the first part of this paper much of the same introductory material already familiar to these interested in decoherence and in the problem of transition from quantum to classical, but with emphasis on issues which are often glossed over in the available literature on the
subject, and which were covered in the introductory part of my presentation in a manner more closely resembling (Zurek 1991). The second part of the paper ventures into more advanced issues which considered only briefly during the talk, but are the real focus of attention here. In a sense, the present paper, while it is self-contained, may be also regarded by reader as a “commentary” on the material discussed in the earlier publications. Its union with my 1991 paper would be a more faithful record of my conference presentation, and a familiarity with that publication – while not absolutely essential – could prove helpful.

Decoherence is a process which – through the interaction of the system with external degrees of freedom often referred to as the environment – singles out a preferred set of states sometimes called the pointer basis. This emergence of the preferred basis occurs through the “negative selection” process (environment-induced superselection) which, in effect, precludes all but a small subset of the conceivable states in the Hilbert space of the system from behaving in an effectively classical, predictable manner. Here I intend to expand and enhance these past discussions in several directions:

(i) I shall emphasize the relation between the interpretational problems of quantum theory and the existence, within the Universe, of the distinct entities usually referred to as “systems”. Their presence, I will argue, is indispensable for the formulation of the demand of classicality. This will justify a similar split of the Universe into the degrees of freedom which are of direct interest to the observer – a “system of interest” – and the remaining degrees of freedom known as the environment. It is therefore not too surprising that the problem (in particular, the measurement problem, but also, more generally, the problem of the emergence of classicality within the quantum Universe) which cannot be posed without recognizing systems cannot be solved without acknowledging that they exist.

(ii) I shall discuss the motivation for the concept of the preferred set of “classical” states in the Hilbert space of an open system – the pointer basis – and show how it can be objectively implemented by recognizing the role of the interaction between the system and the environment. It will be noted that what matters is not a unique definition of the absolutely preferred set of states, but, rather, a criterion for eliminating the vast majority of the “wrong”, nonclassical states.

(iii) I shall consider the relation between the states which habitually diagonalize (or nearly diagonalize) the density matrix of an open system – states which can be found on or near the diagonal after a decoherence time has elapsed irregardless of what was the initial state – and the preferred states. It will be pointed out that diagonality alone is only a symptom – and not a cause – of the effective classicality of the preferred states. And, as all symptoms, it has to be used with caution in diagnosing its origin: Causes of diagonality may, on occasion, differ from those relevant for the dynamics of the process of decoherence (for example, “accidental” diagonality in any basis may be a result of complete ignorance of an observer) and the quantum to classical transition that decoherence precipitates.

(iv) Therefore, I shall regard the ability to preserve correlations between the records maintained by the observer and the evolved observables of open systems as a defining feature of the preferred set of the to-be-classical states, and
generalize it to the situations where the observer is not present. This definition, employed already in the early discussions of effective classicality (Zurek, 1981), can be formulated in terms of information-theoretic measures of the persistence of correlations, which leads one in turn to consider the role of predictability in the definition of the preferred basis.

(v) I shall analyse the relation of the decoherence approach with the consistent histories interpretation. In particular, I shall show that the histories expressed in terms of the events corresponding to the projections onto the pointer basis states are consistent. Thus, a successful decoherence resulting in a stable pointer basis implies consistency. By contrast, consistency alone does not constrain the histories enough to eliminate even very non-classical branches of the state vector. Moreover, consistency conditions single out not only the initial moment in which all the histories begin, but also the final time instant on a more or less equal footing. Furthermore, in the absence of a fixed set of pointer projectors, consistent histories cannot be expressed in terms of a “global” set of alternative events, which could be then used for all histories at a certain instant of time, but, rather, must be formulated in terms of “local” events which are themselves dependent on the past events in the history of the system, and which do not apply to all of the alternatives coexisting at a certain instant. Unless the records are a part of the physical Universe, this is bound to introduce an additional element of subjectivity into the consistent histories interpretation.

(vi) When decoherence is successful and does yield a preferred basis, the branches of the universal wavefunction can thus be traced out by investigating sequences of preferred, predictable sets of states singled out by the environment-induced superselection. In this setting the issue of the “collapse of the wavepacket” – formulated as a question of accessibility, through the existing records of the past events and the ability to use such records to predict a future course of events – is posed and discussed.

I shall “iterate” these themes throughout the course of the paper, returning to most of them more than once. Consequently, the paper shall cover the issues listed above only approximately in the order in which they were itemized in this introduction. Sections 2 – 5 cover the familiar ground but do it with a new emphasis. Sections 6 – 10 venture into a new territory and should be of particular interest to those working on the subject.

2. Motivation for Classicality, or “No Systems - No Problem”

The measurement problem – the most prominent example of the interpretational difficulties one is faced with when dealing with the quantum – disappears when the apparatus-system combination is regarded as a single indivisible quantum object. The same is true not just in course of measurements, but in general – the problems with the correspondence between quantum physics and the familiar everyday classical reality cannot be even posed when we refuse to acknowledge the division of the Universe into separate entities.
The reason for this crucial remark is simple. The Schrödinger equation:

\[ i\hbar \frac{d}{dt} |\Phi\rangle = H |\Phi\rangle , \]  

is deterministic. Therefore, given the initial state, it predicts, with certainty, the subsequent evolution of the state vector of any really isolated entity, including the joint state of the apparatus interacting with the measured system. Thus, in accord with Eq. (2.1), the initial state vector:

\[ |\Phi_{AS}(0)\rangle = |A_0\rangle |\phi_0\rangle = |A_0\rangle \sum_i a_i |\sigma_i\rangle , \]  

evolves into \(|\Phi_{AS}(t)\rangle\) in the combined Hilbert space \(\mathcal{H}_{AS}\). There is no hint of the interpretational problems in this statement until one recognizes that the expected outcome of the measurement corresponds to a state which – while it contains the correlation between \(A\) and \(S\) – also acknowledges the “right” of the apparatus to “a state of its own”. By contrast, a typical form of \(|\Phi_{AS}(t)\rangle\) is:

\[ |\Phi_{AS}(t)\rangle = \sum_i a_i |A_i\rangle |\sigma_i\rangle , \]  

where more than one \(a_i(t)\) are non-zero. Above, \(\{|A_i\rangle\}\) and \(\{|\sigma_i\rangle\}\) belong to separate Hilbert spaces of the apparatus and of the system, \(\mathcal{H}_A\) and \(\mathcal{H}_S\), such that

\[ \mathcal{H}_{AS} = \mathcal{H}_A \otimes \mathcal{H}_S. \]  

Let me emphasize at this point the distinction between quantum and classical measurements by adopting a somewhat Dirac-like notation to discuss the classical analog of a measurement-like evolution. The initial states of the system \(S\) and of the apparatus \(A\) can be denoted by \(|s_0\rangle\) and \(|A_0\rangle\). When these states are “pure” (completely known), and the evolution is classical, the final result of the measurement-like co-evolution is another “pure” state: Reversible evolutions preserve purity in both quantum and classical context, but with a final state which is simpler classically than for a typical quantum case:

\[ |\Upsilon_{tAS}\rangle = |A_t\rangle |s_t\rangle . \]  

The key difference between Eq. (2.3) and Eq. (2.5) is the absence of the superposition of the multiple alternatives corresponding to the possible outcomes. And even when the classical initial states are known incompletely (as is usually the case in the measurement situations) the initial state of the system is

\[ \sum p_i |s_i\rangle , \]  

where \(p_i\) are the probabilities of the various states. The final state of the interacting objects must be also described by a classical probability distribution:

\[ \sum_i p_i |\Upsilon_{tAS}^i\rangle = \sum_i p_i |A_i\rangle |s_i\rangle . \]
Nevertheless, and in contrast with the quantum case, the range of the final states – pointer positions of the apparatus – will continuously narrow down as the range of the initial states of the system is being restricted. This simultaneous constriction of the uncertainty about the initial state of the system and about the outcome – the final state of the apparatus – shall be referred to below as the complete information limit.

Its consequence – the expectation that a complete knowledge of the state of a classical system implies an unlimited ability to predict an outcome of every conceivable measurement – is the cornerstone of our classical intuition. This expectation is violated by quantum theory: Complete information limit in the sense described above does not exist in the quantum case. There, even in the case of the complete information about each of the two objects (i.e., the system and the apparatus) separately, one is still typically forced into a situation represented by Eq. (2.3) in which — following the measurement — neither the apparatus nor the system have “a state of their own”. Indeed, for every pure state of the quantum system there exists a corresponding “relative” state of the apparatus (see Everett, 1957). In fact, a correlation between the states of the apparatus and a measured system is, at this stage, analogous to the nonseparable correlation between the two particles in the Einstein-Podolsky-Rosen “paradox” (Zurek, 1981).

A closely related symptom of the quantum nature arises from the superposition principle. That is, pure quantum states can be combined into another pure state. There is no equivalent way of combining pure classical states. The only situation when their combinations are considered is inevitably tied to probability distributions and a loss of information. Thus, formally, classical states denoted above by $|·}$ correspond not to the vectors in the Hilbert space of the system, but, rather, to the appropriate projection operators, i.e.: $|·} \iff |·><·|$ (2.8)

Such projection operators can be combined into probability distributions in both quantum and classical cases. A pure classical state — a point in the phase space — is of course an extreme but convenient idealization. A more realistic and useful concept corresponding to a probability distribution — a patch in the phase space — would be then described by the density matrix with the form given by Eq. (2.6).

In a “hybrid” notation the measurement carried out by a classical apparatus on a quantum system would then involve a transition:

$$|A_0|\phi_0><\phi_0| \iff \sum_i p_i |A_i|\sigma_i><\sigma_i|$$ (2.9)

The difference between Eq. (2.3) and the right hand side of Eq. (2.9) is clear: Classicality of the apparatus prevents the combined object from existing in a superposition of states.

The need for a random “reduction of the state vector” arises as a result of the demand – based on the desire to reconcile consequences of Eq. (2.1) (i.e., the linear superposition, Eq. (2.3)) with the expectation based on the familiar experience (expressed above by Eqs. (2.5), (2.7) and (2.9)) that only one of the outcomes should actually happen (or, at least, appear to happen) in the “real world.”
There is, of course, a special set of circumstances in which the above distinction between quantum and classical disappears, and no reduction of the state vector following the interaction is required. This unusual situation arises when – already before the measurement – the to-be-measured quantum system was in an eigenstate (say, $|\sigma_k\rangle$) or in a mixture of the eigenstates of the measured observable. In the case of a single pure state the resulting state of the apparatus-system pair following the interaction between them is given by Eq. (2.3), but with only one non-zero coefficient, $|a_i|^2 = \delta_{ik}$. Hence, the distinction between Eq. (2.3) and Eq. (2.5) disappears. An obvious generalization of this effectively classical case for a mixture results in an appropriate probability distribution over the possible outcomes of the measurement. Then the correlation between the system and the apparatus has a purely classical character. Consequently, this situation is operationally indistinguishable from the case when a classical system in a state initially unknown to the observer is measured by a classical apparatus.

These considerations motivate the decoherence approach – an attempt to resolve the apparent conflict between predictions of the Schrödinger equation and perception of the classical reality. If the operationally significant difference between the quantum and classical characteristics of the observables can be made to continuously disappear, then maybe the macroscopic objects we encounter are “maintained” in the appropriate mixtures by the dynamical evolution of the quantum Universe. Moreover, one can show that purely unitary evolution of an isolated system can never accomplish this goal. Therefore, it is natural to enquire whether one can accomplish it by “opening” the system and allowing it to interact with the environment. The study of this and related questions defines the decoherence approach to the transition between quantum and classical.

3. Operational Goals of Decoherence

The criterion for success of the decoherence programme must be purely operational. The Schrödinger equation and the superposition principle are at its foundation. One must concede at the outset that they will not be violated in principle. Therefore, the appearance of the violation will have its origin in the fact that the records made and accessed by observers are subject to limitations, since they are governed by quantum laws with specific – rather than arbitrary – interactions and since their records are not isolated from the environment. One important criterion – to be discussed in more detail in the next section – refers to the fact that idealized classical measurements do not change the state of the classical system. That is, for example, a complete information limit – the classically motivated idea that one can keep acquiring information without changing the state of the measured object – should be an excellent approximation in the “everyday” classical context. The recognition of the role of decoherence and the environment allows one to show how this can be the case in a quantum Universe.

An objection to the above programme is sometimes heard that – in essence – the Universe as a whole is still a single entity with no “outside” environment, and, therefore, any resolution involving its division into systems is unacceptable. While I am convinced that much needs to be done in order to understand what constitutes a “system”, I have also little doubt that the key aspects of the resolution proposed here are largely independent
of the details of the answer to this question. As we have argued in the previous section, without the assumption of a preexisting division of the Universe into individual systems the requirement that they have a right to their own states cannot be even formulated: The state of a perfectly isolated fragment of the Universe – or, for that matter, of the quantum Universe as a whole – would evolve forever deterministically in accord with Eq. (2.1). The issue of the “classicality” of its individual components – systems – cannot be even posed. The effectively stochastic aspect of the quantum evolution (the “reduction of the state vector”) is needed only to restore the right of a macroscopic (but ultimately quantum) system to be in its own state.

Quantum theory allows one to consider the wave function of the Universe. However, entities existing within the quantum Universe – in particular us, the observers – are obviously incapable of measuring that wave function “from the outside”. Thus, the only sensible subject of considerations aimed at the interpretation of quantum theory – that is, at establishing correspondence between the quantum formalism and the events perceived by us – is the relation between the universal state vector and the states of memory (records) of somewhat special systems – such as observers – which are, of necessity, perceiving that Universe from within. It is the inability to appreciate the consequences of this rather simple but fundamental observation that has led to such desperate measures as the search for an alternative to quantum physics. One of the goals of this paper is to convince the reader that such desperate steps are unwarranted.

One might be concerned that the appeal to systems and a frequent mention of measurement in the above paragraphs heralds considerations with a character which is subjective and may be even inexcusably “anthropocentric”, so that the conclusions could not apply to places devoid of observers or to epochs in the prehistory of the Universe. A few remarks on this subject are in order. The rules we shall arrive at to determine the classical preferred basis will refer only to the basic physical ingredients of the model (that is, systems, their states, and the hamiltonians that generate their evolution) and we shall employ nothing but unitary evolution. These rules will apply wherever these basic ingredients are present, regardless of whether an observer is there to reap the benefits of the emerging classicality. Thus, the role of “an observer” will be very limited: It will supply little more than a motivation for considering the question of classicality.

Having said all of the above, one must also admit that there may be circumstances in which some of the ingredients appropriate for the Universe we are familiar with may be difficult or even impossible to find. For example, in the early Universe it might be difficult to separate out time (and, hence, to talk about evolution). While such difficulties have to be acknowledged their importance should not be exaggerated: The state vector of the Universe will evolve in precisely the same way regardless of whether we know what observables could have been considered as “effectively classical” in such circumstances. Therefore, later on in course of its evolution, when one can define time and discern the other ingredients relevant for the definition of classicality, the question of whether they have been present early on or what was or was not classical “at the time when there was no time” will have absolutely no observable consequences.

This last remark deserves one additional caveat: It is conceivable (and has been
even suggested, for example by Penrose (1989)) that the quantum theory which must be applied to the Universe as a whole will differ from the quantum mechanics we have become accustomed to in some fundamental manner (that is, for instance, by violating the principle of superposition for phenomena involving gravitation in an essential way). While, at present, there is no way to settle this question, one can nevertheless convincingly argue that the coupling with the gravitational degrees of freedom is simply too weak in many of the situations involving ordinary, everyday quantum to classical transitions (for example, the blackening of a grain of photographic emulsion caused by a single photon) to play a role in the emergence of the classicality of everyday experience.

4. Insensitivity to Measurements and Classical Reality.

The most conspicuous feature of quantum systems is their sensitivity to measurements. Measurement of a quantum system automatically results in a preparation: It forces the system into one of the eigenstates of the measured observable, which, from that instant on, acts as a new initial condition. The idealized version of this quantum “fact of life” is known as the projection postulate. This feature of quantum systems has probably contributed more than anything else to the perception of a conflict between the objective reality of the states of classical objects and the elusive subjective nature of quantum states which – it would appear – can be molded into a nearly arbitrary form depending solely on the way in which they are being measured. Moreover, the Schrödinger equation will typically evolve the system from the prepared initial state (which is usually simple) into a complicated superposition of eigenstates of the initially measured observables. This is especially true when an interaction between several systems is involved. Then the initial state prepared by a typical measurement involves certain properties of each of the separately measured systems, but quantum evolution results in quantum correlations – in states which are qualitatively similar to the one given by Eq. (2.3). Thus, when the measurement of the same set of observables is repeated, a new “reduction of the wavepacket” and a consequent restarting of the Schrödinger evolution with the new initial condition is necessary.

By contrast, evolutions of classical systems are – in our experience – independent of the measurements we, the observers, carry out. A measurement can, of course, increase our knowledge of the state of the system – and, thus, increase our ability to predict its future behavior – but it should have no effect on the state of the classical system per se. This expectation is based on a simple fact: Predictions made for a classical system on the basis of the past measurements are not (or, at least, need not be) invalidated by an intermediate measurement. In classical physics, there is no need to reinitialize evolution, a measurement does not lead to the preparation, only to an update of the records. This insensitivity to measurements appears to be a defining feature of the classical domain.

For a quantum system complete insensitivity to measurements will occur only under special circumstances – when the density matrix of the system commutes with the measured observable already before the measurement was carried out. In general, the state of the system will be influenced to some degree by the measurements. To quantify the degree to which the state of the system is altered by the measurement one can compare its density matrix before and after the measurement (idealized here in the way introduced by von
Neumann (1932)):  
\[ \rho_{\text{after}} = \sum_i P_i \rho_{\text{before}} P_i \]  
(4.1)

The projection operators \( P_i \) correspond to the various possible outcomes. A generalization to the situation when a sequence of measurements is carried out is straightforward:

\[ \rho_{\text{after}} = \sum_{i,j,\ldots,n} P_n \ldots P_j P_i \rho_{\text{before}} P_i P_j \ldots P_n \]  
(4.2)

Above, pairs of projection operators will correspond to distinct observables measured at consecutive instants of time. Evolution inbetween the measurements could be also incorporated into this discussion.

It is assumed above that even though the measurements have happened, their results are not accessible. Therefore, the statistical entropy;

\[ h(\rho) = -\text{Tr} \rho \ln \rho \]  
(4.3)

can only increase;

\[ h(\rho_{\text{after}}) \geq h(\rho_{\text{before}}) . \]  
(4.4)

The size of this increase can be used as a measure of the sensitivity to a measurement. Unfortunately, it is not an objective measure: It depends on how much was known about the system initially. For example, if the initial state is completely unknown so that \( \rho_{\text{before}} \sim 1 \), there can be no increase of entropy. It is therefore useful to constrain the problem somewhat by assuming, for example, that the initial entropy \( h(\rho_{\text{before}}) \) is fixed and smaller than the maximal entropy. We shall pursue this strategy below, in course of the discussion of the predictability in section 6.

The role of decoherence is to force macroscopic systems into mixtures of states – approximate eigenstates of the same set of effectively classical observables. Density matrices of this form will be then insensitive to measurements of these observables (that is, \( \rho_{\text{after}} \approx \rho_{\text{before}} \)). Such measurements will be effectively classical in the sense described near the end of Section 2, since the final, correlated state of the system and the apparatus will be faithfully described by Eqs. (2.7) and (2.9). No additional reduction of the state vector will be needed, as the projection operators \( P_i \) defining the eigenspaces of the measured observable commute with the states which invariably (that is, regardless of the initial form of the state of the system) appear on the diagonal of the density matrix describing the system.

This feat is accomplished by the coupling to the environment – which, as is well known, results in noise, and, therefore, in a degraded predictability. However, it also leads to a specification of what are the preferred observables – what set of states constitutes the preferred classical “pointer basis” which can be measured relatively safely, with a minimal loss of previously acquired information. This is because the rate at which the predictability is degraded is crucially dependent on the initial state of the system. The initial state is in turn determined by what was measured. We shall show that the differences in the rates at which
predictability is lost are sufficiently dramatic that only for certain selections of the sets of states – and of the corresponding observables – can one expect to develop the idealized but very useful approximation of classical reality.

Another, complementary way of viewing the process of decoherence is to recognize that the environment acts, in effect, as an observer continuously monitoring certain preferred observables which are selected mainly by the system - environment interaction hamiltonians. Other physical systems which perform observations (such as the “real” observers) will gain little predictive power from a measurement which prepares the system in a state which does not nearly coincide with the states already monitored by the environment. Thus, the demand for predictability forces observers to conform and measure what is already being monitored by the environment. Indeed, in nearly all familiar circumstances, observers gain information by “bleeding off” small amounts of the information which has already disseminated into the environment (for example by intercepting a fraction of photons which have scattered off the object one is looking at while the rest of the photons act effectively as an environment). In such a case, the observer must be prepared to work with the observables premeasured by the environment. Moreover, even if it was somehow possible to make a measurement of some other very different set of observables with the eigenstates which are given by exotic superpositions of the states monitored by the environment, records of such observables would become useless on a timescale on which the environment is monitoring the preferred states. For, the “measurements” carried out by the environment would invalidate such data on a decoherence timescale – that is, well before the predictions about the future measurements made on its basis could be verified.

In what follows we shall therefore search for the sets of observables as well as the corresponding sets of states of open quantum systems which offer optimal predictability of their own future values. This requirement embodies a recognition of the fact that our measuring equipment (including our senses) used in keeping track of the familiar reality is relatively time-independent, and that the only measurements we are capable of refer to small fragments of the Universe – individual systems. We shall be considering sequences of measurements carried out on ensembles of identical open quantum systems. We shall return to the discussion of the practical implementation of this programme in Sections 6 and 7, having introduced – in the next section – a simple and tractable model in which decoherence can be conveniently studied.

5. Environment-Induced Decoherence in Quantum Brownian Motion.

The separation of the Universe into subsystems, indispensable in stating the problem, is also instrumental in pointing out a way to its resolution. Our experience proves that classical systems – such as an apparatus – have a right to individual states. Moreover, these states are not some arbitrary states admissible in $\mathcal{H}_A$ which can be selected on a whim by an external observer (as would be the case for an ordinary quantum system) but, rather, appear to be “stable” in that the state in which $A$ is eventually found is always one of the “eigenstates” of the same “menu” of options – i.e. the apparatus has a preferred “pointer observable.” Thus, even though the correlated system - apparatus state vector $|\Phi_{AS}(t)\rangle$, Eq. (2.3), can be in principle rewritten using any (complete) basis in $\mathcal{H}_A$
(Zurek, 1981; 1991), this formal application of the superposition principle and the resulting analogy with nonseparable EPR correlations is not reflected in “the familiar reality”. The familiar macroscopic systems tend to be well localized with respect to the usual classical observables – such as position, energy, etc.

Here we shall see how this tendency towards localization, as well as the choice of the preferred observables can be accounted for by the process of decoherence: A classical system (such as an apparatus) will be continuously interacting with its environment. Consequently, its density matrix will obey an irreversible master equation, valid whenever the environment is sufficiently large to have a Poincaré recurrence time much longer than any other relevant timescale in the problem – in particular, much longer than the timescale over which one may attempt to predict its behavior or to verify its classicality.

A useful example of such a master equation can be derived – under certain reasonable, although not quite realistic circumstances – for a particle of mass $m$ moving in a potential $V(x)$, and interacting with the external scalar field (the environment) through the hamiltonian

$$\mathcal{H}_{int} = \epsilon x \dot{\phi}(0).$$

In the high-temperature limit the reduced density matrix of the particle will obey the equation (written below in the position representation):

$$\frac{d}{dt} \rho(x,x') = -\frac{i}{\hbar} [H_R, \rho] - \gamma(x-x') \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) \rho - \frac{\eta k_B T}{\hbar^2} (x-x')^2 \rho. \quad (5.2)$$

Above $H_R$ is the hamiltonian of the particle with $V_R(x)$ renormalized by the interaction with the field, $\eta$ is the viscosity coefficient ($\eta = \frac{\epsilon^2}{2}, \gamma = \frac{\eta}{2m}$), and $T$ is the temperature of the field. Similar equations have been derived under a variety of assumptions by, for example, Caldeira and Leggett (1983; 1985), Joos and Zeh (1985), Unruh and Zurek (1989), and Hu, Paz and Zhang (1992).

The preferred basis emerges through the process of “negative selection”: In the “classical” regime associated with $\hbar$ small relative to the effective action the last term of Eq. (5.2) dominates and induces the following evolution of the density matrix:

$$\rho(x,x';t) = \rho(x,x';0) \exp\left(-\frac{\eta k_B T}{\hbar^2} (x-x')^2 t\right). \quad (5.3)$$

Thus, superpositions in position will disappear on the decoherence timescale of order:

$$\tau_D = \gamma^{-1} \left( \lambda T / \Delta x \right)^2 \quad (5.4)$$

where $\gamma^{-1}$ is the relaxation rate

$$\langle \dot{v} \rangle = -\gamma \langle v \rangle \quad (5.5)$$
This observation* marked an important shift from the idealized models of decoherence in the context of quantum measurements to the study of more realistic models better suited to the discussion of classicality in the “everyday” context. The conclusion is simple: Only relatively localized states will survive interaction with the environment. Spread-out superpositions of localized wavepackets will be rapidly destroyed as they will quickly evolve into mixtures of localized states.

This special role of the position can be traced to the form of the Hamiltonian of interaction between the system and the environment, Eq. (5.1): The dependence on \( x \) implies a sensitivity to position, which is, in effect, continuously monitored by the environment. In general, the observable \( \Lambda \) which will commute with \( H_{\text{int}} \);

\[
[H_{\text{int}}, \Lambda] = 0 ,
\]  

(5.6)

will play a key role in determining the preferred basis: Superpositions of its eigenspaces will be unstable in the presence of the interaction with the environment. Thus, the special role position plays in our description of the physical Universe can be traced (Zurek, 1986) to the form of the interaction potentials between its subsystems (including the elementary particles and atoms) which is almost invariably position dependent (and, therefore, commutes with the position observable).

The result of such processes is the “effective superselection rule,” which destroys superpositions of the eigenspaces of \( \Lambda \), so that the density matrix describing the open system will very quickly become approximately co-diagonal with \( \Lambda \). Thus, the classical system can be thought of – on a timescale of several \( \tau_D \) – as a mixture (and not a superposition) of the approximate eigenstates of position defined with the accuracy of the corresponding \( \Delta x \). This approximation becomes better when the self Hamiltonian of the system is less important when compared to \( H_{\text{int}} \). Thus, for example, overdamped systems have preferred states which are squeezed in position, while weakly damped harmonic oscillator translates position into momentum every quarter of its period and, consequently, favors still localized, but much more symmetric coherent states. Moreover, for macroscopic masses and separations the decoherence timescale is very small compared to the relaxation time (Zurek, 1984 and 1991).

The ability to describe the state of the system in terms of the probability distribution of the same few variables is the essence of effective classicality: It corresponds to the assertion that the system already has its own state (one of the states of the preferred basis) which is not necessarily known to the observer prior to the measurement, but is, nevertheless, already quite definite. It is important to emphasize that while this statement is, strictly

* See Zurek (1984) for the first discussion of this decoherence timescale; Zurek (1991) from the Wigner distribution function perspective; Hartle (1991) for a nice perspective based on effective action; Paz (1992) for a quick assessment of the role of the spectral density in the environment on the effectiveness of decoherence and Paz, Habib and Zurek (1992) for a more complete assessment of this important issue as well as for a discussion of the accuracy of the high temperature approximation, Eq. (5.2).
speaking, incorrect (there is actually a very messy entangled state vector including both the system and the environment) it cannot be falsified by any feasible measurement. Thus, the quantum Universe gives rise to an effectively classical domain defined through the operational validity of the assertions concerning “reality” of the states of the observables it contains.

6. Preferred States and the “Predictability Sieve”

In the preceding section we have confirmed that – at least for the model presented above – states localized in the familiar classical observable, position, will be among these selected by the decoherence process and, therefore, are effectively classical. Here I shall reverse the procedure. That is, I shall first formulate a test which will act, in effect, as a sieve, a filter accepting certain states in the Hilbert space of the system and rejecting others. The aim of this procedure will be to arrive at an algorithm capable of singling out preferred sets of states \textit{ab initio}, rather than just confirming the suspicions about classicality of certain observables. The guide in devising the appropriate procedure is a decade-old observation (Zurek, 1981; 1982) that – under some special assumptions – the absolutely preferred states remain completely unaffected by the environment. In the absence of these special conditions it is therefore natural to search for the states which are least affected by the interaction with the external degrees of freedom (Zurek, 1984; Unruh and Zurek, 1989). And a convenient measure of the influence of the environment on the state is – as was already mentioned – the ability to predict its future evolution, to use the results of the past measurements as initial conditions.

Let us consider an infinite ensemble of identical systems immersed in identical environments. The aim of the test will be to find which initial states allow for optimal predictability. In order to settle this question, we shall imagine testing all the states in the Hilbert space of the system \(\mathcal{H}_S\). To this end, we shall prepare the system in every conceivable pure initial state, let it evolve for a fixed amount of time, and then determine the resulting final state – which, because of the interaction with the environment, will be nearly always given by a mixture.

Entropy of the final density matrix \(\rho_k\) which has evolved from the initial state \(|k\rangle\):

\[ h_k = -Tr\rho_k \log \rho_k \]  

is a convenient measure of the loss of predictability. It will be different for different initial states. One can now sort the list of all possible pure states in order of the decreasing entropy \(h_k\) of the final mixed state: At the head will be the states for which the increase of entropy is the least, followed by the states which are slightly less predictable, and so on, until all of the pure states are assigned a place on the list.

The best candidates for the classical “preferred states” are, clearly, near the top of the list. One may, nevertheless, ask where should one put a cut in the above list to define a border between the preferred classical set of states and the non-classical remainder. The answer to this question is somewhat arbitrary: Where exactly the quantum-classical border is erected is a subjective matter, to be decided by circumstances. What is, perhaps, more
important is the nature of the answer which emerges from this procedure aimed at selecting the preferred set of states.

A somewhat different in detail, but similar in the spirit procedure for sorting the Hilbert space would start with the same initial list of the states and evolve them until the same loss of predictability – quantified by the entropy of the final mixture – has occurred. The list of states would be then sorted according to the length of time involved in the predictability loss, with the states which are predictable to the same degree for the longest time appearing on the top of the list.

It should be pointed out that the qualifying preferred states will generally form (i) an overcomplete set, and (ii) they may be confined to a subspace of $\mathcal{H}_S$. These likely results of the application of the “predictability sieve” outlined above will have to be treated as a “fact of life”. In particular, overcompleteness seems to be a feature of many a “nice” basis – for example, coherent states are overcomplete. Moreover, it is entirely conceivable that classical states may not exist for a given system in all of its Hilbert space.

It is reassuring to note that, in the situations in which an exact pointer observable exists (that is, there is a nontrivial observable which commutes with the complete hamiltonian, $H + H_{\text{int}}$, of the system of interest; see Zurek, 1981-1983) its eigenstates can be immediately placed on top of either of the predictability lists described above. The predictability sieve selects them because they are completely predictable; they do not evolve at all. Hence, there is no corresponding increase of entropy.

One key ingredient of the first of the predictability sieves outlined above remains arbitrary: We have not specified the interval of time over which the evolution is supposed to take place. This aspect of the selection process will clearly influence the entropy generated in the system. We shall insist that this time interval should remain somewhat arbitrary. That is, the set of the selected preferred states for any time $t$ much longer than the typical decoherence timescale $\tau_D$, but much shorter than the timescale $\tau_{E\text{Q}}$ over which the system reaches thermodynamic equilibrium with its environment should be substantially similar. Similarly, for the second proposed sieve we have not specified the loss of predictability (increase of entropy) which must be used to define the timescale characterizing the location of the state on the list. Again, we expect that the list should not be too sensitive to this parameter: Indeed, the two versions of the sieve should yield similar sets of preferred states.

The distinction between the preferred set of states and the rest of the Hilbert space – the contrast indicated by the predictability sieve outlined above – should depend on the mass of the system or on the value of the Planck constant. Thus, it should be possible to increase the contrast between the preferred set of states and the other states which decohere much more rapidly by investigating the behavior of the entropy with varying $m$ and $\hbar$ in mathematical models of these systems. It is easy to see how such a procedure will work in the case of quantum Brownian motion described in the preceding section. In particular, reversible Newtonian dynamics can coexist with a very efficient environment-induced superselection process in the appropriate limit (Zurek, 1984; 1991): It is possible to increase mass, decrease the Planck constant, and, at the same time, decrease the relaxation
rate to zero, so that the master equation Eq. (5.2) yields reversible Liouville dynamics for the phase space probability distributions constructed from the sets of localized states (such as the coherent states), but eliminates (by degrading them into mixtures of the localized states) all other initial conditions.

A natural generalization of the predictability sieve can be applied to mixtures. Let us construct a list of all the mixtures which have the same initial entropy $h_m(0)$ using either of the two prescriptions outlined above. We can now consider their evolution and enquire about the entropy of the mixed state after some time $t$. An ordered list of the initial mixtures can be again constructed and the top of the list can be identified. An interesting question can be now posed: Are the mixtures on top of the initial mixture list diagonal in the pure states which are on top of the list of preferred states? When the exact pointer pointer basis in the sense of the early references on this subject (Zurek, 1981; 1982; 1983) exists, the conjecture expressed above can be readily demonstrated: Mixtures of the pointer basis eigenstates do not evolve at all. Therefore, their entropy remains the same. Any other kind of mixture will become even more mixed because superpositions of the pointer basis states decohere. Consequently, at least in this case the most predictable mixtures are diagonal in the basis of the most predictable states.

In this preliminary discussion we have neglected several issues which are likely to play a role in the future developments of the predictability sieve concept. In particular, we have ignored to enforce explicitly the requirement (stated in Section 4) that the measurements which have prepared the set of the candidate states should be suitable for the predictions of their own future outcomes. (For the closed systems this criterion naturally picks out complete sets of commuting observables corresponding to the conserved quantities.) Moreover, we have used only the statistical entropy to assess the cost of the predictability loss.

An addition of an algorithmic component might make for an even more interesting criterion. That is, the place of the pure state $|\ell>\$ on the list would be based on the value of the physical entropy (Zurek, 1989), given by the sum of the physical entropy with the algorithmic information content $K(|\ell>)$:

$$s(|\ell>) = h_\ell + K(|\ell>) .$$

(6.2)

Algorithmic contribution would discourage overly complicated states.

It is interesting to consider a possibility of using a similar predictability criterion to justify the division of the Universe into separate systems. The argument (which will be made more precise elsewhere) would start with the list of all of the possible states in some composite Hilbert space, perhaps even the Hilbert space associated with the Universe as a whole. The procedure would then be largely similar to the simpler version of the sieve outlined above, but with one important change: One will be allowed to vary the definition of the systems – that is, to test various projection operators, such as the projections corresponding to averaging process – to establish the most predictable collective coordinates, and, simultaneously, to obtain the preferred sets of states corresponding to such observables.

7. Predictability Sieve: An Example
A simple example of the dependence of the entropy production rate on the set of preferred states is afforded by a model of a single harmonic oscillator interacting with a heat bath we have analysed in Section 5. In this section we shall carry out a restricted version of the more ambitious and general “predictability sieve” programme for this case. In its complete version one would search for the set of states which minimize the entropy over a finite time duration of the order of the dynamical timescale of the system. Below we shall work with an instantaneous rate of change of linear entropy. A more detailed analysis can be found elsewhere (Zurek, Habib and Paz, 1992).

Linear entropy
\[ \varsigma(\rho) = Tr(\rho - \rho^2) \]  
(7.1)
is loosely related to the “real thing” through the expansion:
\[ h(\rho) = -Tr\rho \ln \rho \approx Tr\rho\{(1 - \rho) + (1 - \rho)^2 + \cdots = \varsigma(\rho) + \cdots \]  
(7.2)
Moreover, it is in its own right a recognized measure of the purity of a state. Consequently, in what follows I shall “cut corners” and consider the quantity:
\[ \dot{\varsigma} = \frac{d}{dt} Tr\rho^2 = Tr(\dot{\rho}\rho + \rho\dot{\rho}) \]  
(7.3)
for the initial pure states (that is, for all the density operators for which \( \rho^2 = \rho \)). By contrast, the more ambitious programme would deal with entropy increase over finite time of the order of the dynamical timescale of the system.

In the high temperature limit the equation which governs the evolution of the reduced density operator can be written in the operator form as (Caldeira and Leggett, 1983):
\[ \dot{\rho} = \frac{1}{i\hbar}[H_R, \rho] + \frac{\gamma}{2i\hbar}[[p, x], \rho] - \frac{\eta k_B T}{\hbar^2}[[x, [x, \rho]], [x, \rho]] - i\gamma\hbar([x, \rho p] - [p, \rho x]) . \]  
(7.4)
Above, square brackets and curly brackets indicate commutators and anticommutators, respectively, while \( H_R \) is the renormalized hamiltonian.

Only the last two terms can be directly responsible for generation of linear entropy; For, when the evolution of the density matrix is generated by the term of the form \( \dot{\rho} = [A, \rho] \), then the linear entropy is unaffected. The proof of the above assertion is simple: It suffices to substitute in the expression for \( \dot{\varsigma} \), Eq. (7.3), the above commutator form for the evolution generator of \( \rho \). Then it is easy to compute:
\[ \dot{\varsigma} = Tr(A\rho^2 - \rho^2 A) \]
which is obviously zero by the cyclic property of trace.

Consequently, after some algebra one obtains:
\[ \dot{\varsigma} = \frac{4\eta k_B T}{\hbar^2} Tr(\rho^2 x^2 - \rho x \rho x) - 2\gamma Tr\rho^2 \]  
(7.5)
When the system is nearly reversible ($\gamma \approx 0$) the second term in the above equation is negligible. This is especially true in the high temperature “macroscopic” limit (large mass, small Planck constant). It should be pointed out that the initial “decrease of entropy” predicted by this last term for sufficiently localized wavepackets is unphysical and an artifact of the high temperature approximation (Ambegaokar, private communication; Hu, Paz, and Zhang, 1992). Nevertheless, at late times the existence of that term allows for the eventual equilibrium with $\dot{\varsigma} = 0$.

For the pure states the first term can be easily evaluated:

$$\dot{\varsigma} = \frac{4\eta k_B T}{\hbar^2} (\langle x^2 \rangle - < x >^2), \quad (7.6)$$

where $< x >^2 = |\langle \varphi | x | \varphi >|^2$ and $< x^2 > = \langle \varphi | x^2 | \varphi >$. This is an interesting result. We have just demonstrated that for pure states the rate of increase of linear entropy in quantum Brownian motion is given by their dispersion in position, which is the preferred observable singled out by the interaction Hamiltonian. More generally, when the system is described by a density matrix so that Eq. (7.5) must be used, it is easy to see that the density matrix which has fewer superpositions of distinct values of $x$ will result in a smaller entropy production. Thus, the special role of the observable responsible for the coupling with the environment is rather transparent in both of the above equations.

In view of the preceding considerations on the role of predictability it is now easy to understand why localized states are indeed favored and form the classical preferred basis. It should be emphasized that this simple result cannot be used to infer that the preferred states are simply eigestates of position. This is because the self-Hamiltonian – while it does not contribute directly to the entropy production – has an important indirect effect. For a harmonic oscillator it rotates the state in the phase space, so that the spread in momentum becomes “translated” into the spread in position and vice versa. As a result, linear entropy produced over one oscillator period $\tau$ is given by:

$$\varsigma_{\tau} = \frac{4\eta k_B T}{\hbar^2} \int_0^\tau dt <\psi|(x - < x >)(\cos \omega t + (m\omega)^{-1}(p - < p >)\sin \omega t)^2|\psi >$$

$$= \frac{2\eta k_B T}{\hbar^2}(\Delta x_0^2 + \frac{\Delta p_0^2}{m^2\omega^2}). \quad (7.7)$$

Above, the dispersions are computed for the pure initial state $|\psi >$ and $\omega$ is the frequency of the oscillator.

It is now not difficult to argue that the initial which minimizes linear entropy production for a harmonic oscillator must be (i) a minimum uncertainty wavepacket, and (ii) its “squeeze parameter” $s$ – equal to the initial ratio between its spread in position and the spread of the ground state – must be equal to unity. This can be confirmed by more detailed calculations, which yield the value of $Tr\rho^2$ for the reduced density matrices which have evolved from various initial states with different $s$ as a function of time. Coherent states (minimum uncertainty wavepackets with $s = 1$) are clearly selected by the
predictability sieve acting on a dynamical timescale of the oscillator (see Fig. 1). This supplies a physical reason for the role they play in harmonic motion, including its examples encountered in quantum optics.

8. Insensitivity to Measurements and Consistent Histories

In 1984 Griffiths introduced a notion of history into the vocabulary used in the discussion of quantum systems. A history is a time-ordered sequence of properties represented by means of projection operators:

\[ \chi_{i,j,...,l} = P_i(t_i)E(t_i,t_j)P_j(t_j)\ldots P_l(t_l)E(t_l,t_0) \]  

where \( t_i > t_j > \ldots > t_l \), \( P \)'s stand for the projection operators (assumed to be mutually orthogonal at each of the discrete instants at which the corresponding properties are defined) and \( E(t,t') \) evolve the system in the time interval \( (t,t') \). The set of histories is called consistent when they do not interfere, that is, when the probabilities corresponding to different histories can be added. Griffiths (1984) and Omnès (1990,1992) (who emphasized the potential importance of the consistent histories approach to the interpretation of quantum theory) have demonstrated that a necessary and sufficient condition for the set of histories composed of the initial condition and just two additional events to be consistent is that the projection operators defining the properties at the instants 0, 1, and 2, when expressed in the Heisenberg picture, satisfy the algebraic relation:

\[ Re \, Tr P_1 P_0 \bar{P}_1 P_2 = 0 \]  

(8.2)

or, equivalently;

\[ Tr[P_1,[P_0,\bar{P}_1]]P_2 = 0 \]  

(8.3)

These equivalent consistency conditions must hold for every of the projection operators in the sets defined at the initial, intermediate, and final instants. Above, \( \bar{P}_i = 1 - P_i \) projects on the complement of the Hilbert space. Above, the explicit reference to the time instants at which different projection operators act has been abbreviated to a single index to emphasize that the time ordering is assumed to hold. There is also a stronger consistency condition (Gell-Mann and Hartle, 1990) with a physical motivation which draws on the decoherence process we have discussed above: It is possible to show that a sufficient condition for a family of histories to be consistent is the vanishing of the off-diagonal elements of the decoherence functional:

\[ D(\chi,\chi') = Tr(P_2P_1P_0P_1') \]  

(8.4)

Generalization to histories with more intermediate properties or with the initial condition given by a density matrix is straightforward. One of the appealing features of the Gell-Mann - Hartle approach is that it supplies a convenient measure of the degree of inconsistency for the proposed sets of histories.

In all of the above consistency requirements above \( P_0 \) acts as an initial condition and \( P_2 \) (or, more generally, the set of projection operators corresponding to the “last” instant)
acts as a final condition. The two sets of projection operators determine the initial and final sets of alternatives for the system. (It is of course possible to start the system in a mixture of different $P_0$'s.) This special role of the initial and final sets of projection operators was emphasized already by Griffiths (1984). It is particularly easy to appreciate in the version of the consistent histories approach due to Gell-Mann and Hartle (1990; 1991; see also their contribution in this volume).

Mathematically, the similarity between the role of the initial and final projections in the formula for $D(\chi, \chi')$ follows from the cyclic property of the trace. It is reflected in the special physical function of these two instants, which is usually defined by appealing to the physical content of the situation to which the consistent histories approach is applied. For example, in the context originally considered by Griffiths the initial condition and the set of final alternative properties were suggested by the detectors contained inside the closed system under investigation (see especially section 3 of Griffiths, 1984). That is, the consistent histories approach is capable of disposing of the outside intervention at the intermediate instants, but not at the initial or final moments. Of course, in search for consistency one can explore a variety of selections for the final set of projectors. Thus, if there was just a single, unique set of consistent histories, one would be still guaranteed to find it by employing the search procedure involving first a selection of the preferred sets of initial and final projectors, and then enquiring whether intermediate projectors consistent with these initial and final conditions can be found. Unfortunately, consistent histories can be found rather easily simply by evolving initial (or final) sets of projectors. Moreover, these histories, while they are perfectly consistent, seem to have little in common with the classical “familiar reality”. Indeed, the demand for strict consistency will most likely have to be relaxed if the quantummechanical histories are to be consistent with the reality we perceive. In this context, where the demand for consistency is easily satisfied, and where the consistent histories are crucially influenced by the initial and final selections, there is reason for concern with the ability of such “outside” selections to unduly influence the results.

In the cosmological context the initial density matrix could be perhaps supplied by quantum cosmological considerations but the final set of alternatives is more difficult to justify. This distinction between the role of the intermediate and final projections is worth emphasizing especially because it is somewhat obscured by the notation which does not distinguish between the special nature of the “last” property and the associated set of the projection operators and the intermediate properties. (The initial time instant is usually explicitly recognized as different by notation, since density matrices are typically considered for the initial condition in the majority of the papers on the subject (Omnès, 1992; Gell-Mann and Hartle, 1990).) Yet, the final set of options should not be – especially in the cosmological context, where one is indeed dealing with the closed system – determined “from the outside”. The question therefore arises: How can one discuss consistency of histories without appealing to this final set of “special” events?

Evolving density matrix can be, at any instant $t$, always written as:

$$\rho(t) = \sum_{K} \sum_{K'} \chi(K) \rho(0) \chi(K').$$  \hspace{1cm} (8.5)
Above $\chi(\mathcal{K})$ is a collection of the projection operators with the appropriate evolution operators sandwiched inbetween, and can be thought of as a hypothetical history of the system. A composite index $\mathcal{K}$ designates a specific time-ordered sequence. Projectors corresponding to $\mathcal{K}$ and $\mathcal{K}'$ differ, but the time instants or evolutions do not.

Equation (8.5) is simply an identity, valid for arbitrary sets of orthocomplete projection operators defined at arbitrary intermediate instants $t_i$, $0 < t_i \leq t$. There is an obvious formal similarity between the expression for the decoherence functional, Eq. (8.4), and the identity, Eq. (8.5). If one were to perform a trace on the elements of $\rho(t)$ corresponding to pairs of different histories written in the above manner, one would immediately recover the expression for the elements of the decoherence functional. The questions of consistency of the sequences of events should be therefore at least partially testable at the level of description afforded by the density matrix written in this “historical” notation. In particular, additivity of probabilities of the distinct sequences of events corresponds simply to the \textit{diagonality} of the above expression for $\rho(t)$ in the set of considered histories. That is, the set of histories $\{\chi\}$ will be called \textit{intrinsically consistent} when they satisfy the equality:

$$\rho(t) = \bar{\rho}(t), \quad (8.6)$$

where:

$$\bar{\rho}(t) = \sum_{\mathcal{K}} \chi(\mathcal{K}) \rho(0) \chi(\mathcal{K}). \quad (8.7)$$

The physical sense of the equality between the double sum of Eq. (8.5) (which is satisfied for every complete set of choices of the intermediate properties) and the single sum immediately above (which can be satisfied for only very special choices of $\chi$’s) is that the interference terms between the properties defining the intrinsically consistent sets of histories do not contribute to the final result:

$$\sum_{\mathcal{K} \neq \mathcal{K}'} \chi(\mathcal{K})\rho(0)\chi(\mathcal{K}') = 0. \quad (8.8)$$

An equivalent way of expressing the physical content of this equivalence condition, Eqs. (8.6) or (8.8), is to use the terminology of Section 4 and note that the system is insensitive to the measurement of the properties defined by such intrinsically consistent histories. An obvious way in which the above equation can be satisfied, and the only way in which it can be satisfied exactly without demanding cancellations between the off-diagonal terms, is to require that histories should be expressed in terms of the properties which correspond to the operators which commute with the density matrix. This simple remark (which, on a formal level, implies the consistency condition, Eq. (8.4)) is, I believe, essential in understanding the connection between the consistent histories approach and the discussions based on the process of decoherence.

An obvious exact solution suggested by the above considerations – a set of histories based on the evolved states which diagonalize the initial density matrix – does supply a formally acceptable set of consistent histories for a closed system, but it is very unattractive in the cosmological, or indeed, any practical context. The resulting histories do not seem
to have much in common with the familiar classical evolutions we have grown accustomed
to. It is clear that additional constraints will need to be imposed, and the search for such
constraints is under way (Gell-Mann and Hartle, 1990; 1991; Omnès, 1992).

In the decoherence approach presented in this paper the additional constraints on the
acceptable sets of consistent histories are supplied through insistence on the observables
which correspond to separate systems. This assumption significantly constrains sets of the
“properties of interest” (see the contribution of Albrecht to these proceedings as well as
Albrecht (1992) for a discussion of this point). Indeed, the restriction is so severe that
one can no longer hope for the exact probability sum rules to be satisfied at all times.
Thus, instead of the ideal additivity of the probabilities (or ideal diagonality of the density
matrix in the set of preferred pointer states) one must settle for approximations to the
ideal which violate it at a negligible level almost all the time, and which may violate it at
a significant level for a negligible fraction of time.

Insensitivity of the system to measurements proposed in Section 4 as a criterion for
classicality emerges as a useful concept unifying these two approaches – the environment-
induced decoherence and the consistent histories interpretations. The condition of equality
between the evolved density matrix, with its double summation over the pairs of histories,
and the restricted sum over the diagonal elements only, Eq. (8.7), can be expressed by
noting that the measurements of the properties in terms of which the consistent histories
are written does not perturb the evolution of the system. This suggests a physically
appealing view of the consistent histories approach in the context of the environment-
induced decoherence: Measurements of the preferred observable do not alter the evolution
of the system which is already monitored by the environment!

9. Preferred Basis, Pointer Observables, and the Sum Rules

In the last section we have discussed a connection between the consistency of histories
and the notion of insensitivity of a quantum system to measurements. Such insensitivity
underlies much of our classical intuition. It has served as an important conceptual input in
the discussion of the environment induced superselection, decoherence, and the preferred
pointer basis. If the conceptual link between the foundations of the “many histories inter-
pretation” and the preferred basis of an open system does indeed exist, one should be able
to exhibit a still more direct relationship between the formalisms of these two approaches.

Let us consider a system interacting with a large environment. In accord with the
preceding discussion, I shall focus on its reduced density matrix \(\rho(t) \equiv \rho_S(t)\) which can
be always computed by evolving the whole, combined system unitarily from the initial
condition, and then tracing out the environment. Throughout much of this section we
shall also work under a more restrictive assumption: We shall consider only situations
in which the reduced density matrix of the system suffices to compute the decoherence
functional. It should be emphasized that this is not a trivial assumption, as the consistent
histories formalism was introduced for closed quantum systems and its extension to the
open quantum systems (required in essentially all practical applications, since macroscopic
systems are nearly impossible to isolate from their environments) leads to difficulties some
of which have not yet been even pointed out in the literature until now, let alone addressed.
The nature of this demand is best appreciated by noting that (given a split between the system and the environment and the usual tracing over the environment degrees of freedom) we are demanding that the consistency of the histories for the whole Universe with the events defined through \( P_k \otimes \mathbf{1}_E \) must be closely related to the consistency of the histories with the events \( P_k \) in the history of the open system. Using the decoherence functional, the strongest form of such equivalence could be expressed by the equality:

\[
D_{SE} \equiv \text{Tr}_{SE} \{ \mathbf{1}_E \otimes P_1 \ldots \mathbf{1}_E \otimes P_i \ldots \rho_{SE} \ldots P'_i \otimes \mathbf{1}_E \ldots \} = \text{Tr}_S \{ P_1 \ldots P_i \ldots \rho \ldots P'_i \ldots \} \equiv D_S
\]  

(9.1)

Above, “. . .” stand for the appropriate evolutions inbetween events (which is unitary for \( SE \) considered jointly and non-unitary for \( S \) alone). While the above equation has the advantage of stating a requirement with brevity, one should keep in mind not only that the diagonality of the decoherence functional is a more restrictive condition than the one necessary to satisfy the sum rules, but also that exact diagonality is not to be expected in the physically interesting situations. Moreover, when the approximate consistency is enough (as would have to be almost always the case) the equality above could be replaced by an approximate equality, or could hold with a very good accuracy almost always, but be violated for a fraction of time which is sufficiently small not to be worrisome. Finally, it is conceivable that the approximate equality could hold only for a certain sets of projection operators, and not for others. We shall not attempt to analyse these possibilities here. For the time being, we only note that whenever the density matrix of the open system obeys a master equation which is local in time (such as Eq. (5.2)) the reduced density matrix will suffice to express the decoherence functional. These issues will be discussed in a separate paper (Paz and Zurek, 1992).

To begin the discussion of the connection between the pointer basis and the consistent histories approach I shall assume – guided by the experience with the environment - induced superselection – that after a decoherence time the density matrix of the system settles into a state which, irregardless of the initial form of the state of the system ends up being nearly diagonal in the preferred basis \( P_i \). I shall, for the moment, demand exact diagonality and show that it implies exact consistency.

Let us first consider an initial state which is a projection operator \( Q_o \), and a historical property which corresponds to a pointer observable:

\[
\Lambda^i = \sum_i \lambda_i P_i , \tag{9.2}
\]

which commutes with the reduced density matrix:

\[
\rho_{Q_o} (t^i) = \text{Tr}_E \rho_{SE} (t^i) , \tag{9.3}
\]

at the first historical instant;

\[
[\Lambda^i , \rho_{Q_o} (t_i)] = 0 . \tag{9.4}
\]

Above, the lower case “\( i \)” plays a double role: When in subscript, it is a running index which can be summed over (as in Eq. (9.2)). When in the superscript it is used to label the instant of time (i. e. \( t^i , t^j \), etc.) or a pointer observable (\( \Lambda^i \)) at that instant.
Given Eq. (9.2), the density matrix can be written as:

$$\rho_{Q_o}(t^i) = \sum_i p(i|o) \rho_{ii|o} = \sum_i p_i \rho_{Q_o}(t^i) P_i .$$  \hspace{1cm} (9.5)

In other words, the system is insensitive to the measurement of the pointer observable at that instant. Above, $p(i|o) = Tr_S P_i \rho_{Q_o}(t^i) P_i$ is the conditional probability that a pointer projector $P_i$ would have been measured at a time $t^i$ given that the system started its evolution in the state $Q_o$ at the initial instant. The conditional density matrix;

$$\rho_{ii|o} = P_i \rho_{Q_o}(t^i) P_i / p(i|o)$$  \hspace{1cm} (9.6)

which appears in Eq. (9.5) will, in general, contain additional information about the measurements of higher resolution which could have been carried out. It would reduce to unity when $Tr P_i = 1$.

Given this form of the reduced density matrix at $t^i$ it is not difficult to evaluate the contribution of that instant to the decoherence functional for histories including other properties $\{P_k\}$ at the corresponding instants $t^k$:

$$D_{Q_o}(\chi_{i,j,...,l}, \chi_{i',j',...,l}) = Tr\{...(P_i \rho_{Q_o}(t^i) P_i')...\} = p(i|o)D_{\rho_{ii|o}}(\chi_{j,...,l}, \chi_{j',...,l}) \delta_{ii'}$$  \hspace{1cm} (9.7)

Above, the projection is followed by the (nonunitary) evolution which continues until the next projection, and so on, until the final instant $t_l$. The off-diagonal elements of the decoherence functional are always zero because of our simplifying assumption of existence of a perfect pointer basis at each “historical” instant.

Such computation can be carried out step by step until finally one can write;

$$D_{Q_o}(\chi, \chi) = p(l|k...j|i|o)...p(j|i|o)p(i|o) ,$$  \hspace{1cm} (9.8)

and;

$$D_{Q_o}(\chi \neq \chi') = 0$$  \hspace{1cm} (9.9)

for the case when pointer observables existed and were used to define properties at each historical time instant.

When the initial state was mixed and of the form;

$$\rho(t^o) = \sum_o p(Q_o) Q_o ,$$  \hspace{1cm} (9.10)

the elements of the above formula for the decoherence functional would have to be weighted by the probabilities of the initial states, so that;

$$D_p = \sum_o p(Q_o) D_{Q_o} .$$  \hspace{1cm} (9.11)

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Conditional probabilities in Eq. (9.8) can be simplified when the “historical process” is Markovian, so that the probability of the “next” event depends only on the preceding historical event. Delineating the exact set of conditions which must be satisfied for this Markovian property to hold is of interest, but, again, beyond the scope of this paper: Markovian property of the underlying dissipative dynamics is not enough. One can, however, see that the process will be Markovian when the projections are onto rays in the Hilbert space. By contrast, conditional probabilities will be determined to a large extent by the properties in the more distant past when such events have had a better resolution – that is, if they projected onto smaller subspaces of the Hilbert space of the system – than the more recent ones.

We have thus seen than when (i) the density matrix of the system alone is enough to study consistency of histories and (ii) when the system evolves into density matrix which is exactly diagonal in the same set of pointer states each time historical properties are established, then the histories expressed in terms of the pointer projectors are exactly consistent. In the more realistic cases the reduced density matrix will be only approximately diagonal in the set of preferred states. It is therefore of obvious interest to enquire how the deviations from diagonality translate into a partial loss of consistency.

To study this case we suppose now that the reduced density matrix of the system at a time \( t^i \) does not satisfy Eq. (9.4), but, rather, is given by;

\[
\rho_{Q_o}(t^i) = \sum_i p(i|o) \rho_{ii|o} + \sum_{i \neq i'} P_i \rho_{Q_o}(t^i) P_i'.
\]  

(9.12)

Below, we shall set \( \rho_{ii'|o} = P_i \rho_{Q_o}(t^i) P_i'. \) Following the similar procedure as before one obtains (for the diagonal contribution at the instant \( t^i \) ) the same formula as before. By contrast, an off-diagonal term also contributes;

\[
D_{Q_o}(\chi_{i,j,...,l}, \chi_{i',j',...l}) = D_{\rho_{ii'|o}}(\chi_{j,...,l}, \chi_{j',...l}) .
\]  

(9.13)

The new “initial density matrices” for such off-diagonal terms are non-hermitean. Nevertheless, they can be evolved using – for example – the appropriate master equation and will contribute to the off-diagonal terms of the decoherence functional. Moreover, the size of their contribution to the decoherence functional can be computed by writing them as a product of a hermitean density matrix and an annihilation operator;

\[
\rho_{ii'} = q_{ii'|o} \rho_{ii'|o} \Pi_{ii'} ,
\]  

(9.14)

where;

\[
P_i = \Pi_{ii'} P_i' \Pi_{ii'}^\dagger ,
\]  

(9.15)

and \( q(ii'|o) \) is the trace of this component of \( \rho_{ii'} \) which remains after separating out \( \Pi_{ii'} \).

The simplest example of this procedure obtains in the case when the projectors are one-dimensional. Then Eq. (9.14) simplifies and the off-diagonal element of the decoherence functional can be evaluated by a formula similar to the one for diagonal elements:

\[
D_{Q_o}(\chi_{i,j,...,l}, \chi_{i',j',...l}) = q(ii'|o) D_{\Pi_{ii'}}(\chi_{j,...,l}, \chi_{j',...l}) .
\]  

(9.16)
Expressing the decoherence functional in this form allows one to estimate the decrease of the potential for interference – for the violations of the sum rules – from the relative sizes of the numerical coefficients such as \( p(i|o) \) and \( q(ii'|o) \), products of which establish the “relative weight” of the various on-diagonal and off-diagonal terms. Similar procedure applies also more generally and leads to a formula:

\[
D_{Q_o}(\chi_{i,j,...,l}, \chi_{i',j',...l}) = q(ii'|o)D_{\rho_{ii'|o}}\Pi_{ii'}(\chi_{j,...,l}, \chi_{j',...l}). \tag{9.17}
\]

Equations (9.7), (9.8) and (9.17) – representing individual steps in evaluation of the decoherence functional – can be repeated to obtain “chain formulas” of the kind exhibited before in this section. We shall not go into such details which are relatively straightforward conceptually but rather cumbersome notationally.

These considerations establish a direct link between the process of decoherence with its preferred observables, corresponding sets of states, etc., and the requirement of consistency as it is implemented by the decoherence functional. It is now clear that – given the assumptions stated early on in this section – the environment induced superselection implies consistency of the histories expressed in terms of the pointer basis states. It should be nevertheless emphasized that the issue of the applicability of the sum rules and the consistency criteria address – in spite of that link – questions which differ, in their physical content, from those which are usually posed and considered as a motivation for the environment - induced decoherence and the resulting effective superselection.

The key distinction arises simply from the rather limited goal of the consistent histories programme – the desire to ascribe probabilities to quantum histories – which can be very simply satisfied by letting the histories follow the unitary evolution of the states which were on the diagonal of the density matrix of the Universe at the initial instant. This obvious answer has to be dismissed as irrelevant (Gell-Mann and Hartle, 1990) since it has nothing to do with the “familiar reality”. The obvious question to ask is then whether the approximate compliance with the sum rules (which have to be relaxed anyway to relate the histories to the “familiar reality”) would not arise as a byproduct of a different set of more restrictive requirements, such as these considered in the discussions of environment-induced decoherence. In the opinion of this author, the division of the Universe into parts – subsystems – some of which are “of interest” while other act as “the environment” is essential. It is required in stating the problem of interpretation of quantum theory. Moreover, it automatically rules out trivial solutions of the kind discussed above, while, at the same time, allowing for the consistency of histories expressed in terms of the preferred basis. In this context the approximate validity of the sum rules which is of course an important prerequisite for classicality, arises naturally as a consequence of the openness of the systems through the dynamics of the process of decoherence.

10. Preferred Sets of States and Diagonality

Diagonality of the density matrix in some basis alone is not enough to satisfy the physical criteria of classicality. Pointer states should not be defined as the states which just happen to diagonalize the reduced density matrix at a certain instant. What is
crucial (and sometimes forgotten in the discussions of these issues) is the requirement of stability of the members of the preferred basis (expressed earlier in this paper in terms of predictability). This was the reason why the preferred observable was defined in the early papers on this subject through its ability to retain correlations with other systems in spite of the contact with the environment (Zurek, 1981). Hence, the early emphasis on the relation of the pointer observable to the interaction hamiltonian and the analogy with the monitoring of the “nondemolition observables” (Caves et al., 1980).

An additional somewhat more technical comment – especially relevant in the context of the many-histories approach – might be in order. One might imagine that the exact diagonality of the density matrix in some basis (rather than existence of the stable pointer basis) might be employed to guarantee consistency. After all, the property of the density matrix we have used earlier in this section to establish consistency of the histories expressed in terms of the pointer basis at a single instant, Eqs. (9.7) - (9.9) – was based on diagonality alone. This might be possible if one were dealing with a single instant of time, but would come with a heavy price when the evolution of the system is considered: The projection operators which diagonalize the reduced density matrices which have evolved from certain “events” in the history of the system will typically depend on these past events! That is, the set of projectors on the diagonal of $\rho_{Q_o}(t_i)$ will be different from the set of projectors on the diagonal of $\rho_{Q'_o}(t_i)$ even when the initial conditions – say, $Q_o$ and $Q'_o$ – initially commute, or even when they are orthogonal. This is because the evolution of the open system does not preserve the value of the scalar products. Thus, for instance, the implication:

$$[\rho(0), \rho'(0)] = 0 \iff [\rho(t), \rho'(t)] = 0,$$

which is valid for unitary evolutions, does not hold for the evolution of open systems. This can be established, for example, for the dissipative term of the master equation (5.2).

The implications of this phenomenon for both of the approaches considered here remains to be explored. It seems, however, that consequences of such environment-induced noncommutativity for the consistent histories approach could be quite important: The standard formalism underlying the many-histories interpretation of quantum mechanics requires an exhaustive set of exclusive properties which can be defined for the system under study as a whole (see papers by Griffiths, Omnès, Gell-Mann, and Hartle cited earlier). This requirement is perfectly reasonable when the system in question is isolated and does not include observers who decide what questions are being asked – what sets of projections operators will be selected – as is surely the case for this Universe (see Wheeler’s “game of twenty questions” in his 1979 paper). It could be perhaps satisfied for macroscopic quantum systems of the sort considered in the context of quantum measurements for finite stretches of time. It is, however, difficult to imagine how it could be satisfied for all the branches of the Universe and “for eternity”, especially since the effective hamiltonians which decide what are the sensible macroscopic observables are more often than not themselves determined by the past events (including the dramatic symmetry breaking transitions which are thought to have happened early on in the history of the Universe, as well as much less dramatic, but also far less hypothetical transitions in macroscopic systems). A similar comment can be made about the selection of the same “historical”
instants of time in a multifarious Universe composed of branches which specifically differ in the instant when a certain event has occurred.

The above remark leads one to suspect that such global (or, rather, universal) orthogonal sets of events at reasonably simultaneous instants of time cannot and satisfy the demands of consistency “globally”. Only “local” properties defined in a way which explicitly recognizes the events which have occurred in the past history of the branch have a chance of fulfilling this requirement. This suggests the interesting possibility that the consistent history approach can be carried out with the same set of the event-defining properties and for the same sequence of times only for a relatively local neighborhood of some branch defined by the common set of essential ingredients (which can be in turn traced to the events which contributed to their presence). Such branches would then also share a common set of records – imprints of the past events reflected in the “present” states of physical systems. It should be noted that while the standard formalism does not call for the branch dependence, consistent histories framework can be generalized to accommodate it. Indeed, both Omnès (1990) as well as Gell-Mann and Hartle (1990) have mentioned such branch-dependent histories in their papers. Environment-induced noncommutativity may force one to consider this possibility even more seriously.

11. Records, Correlations and Interpretations

The discussion near the end of the last section forces us to shift the focus from the histories defined in terms of “global” projection operators, abstract “events”, and equally abstract “records” related to them and existing “outside” of the Universal state vector to far more tangible “local” events which have shaped a specific evolution of “our” branch, and which are imprinted as the real, accessible records in the states of physical systems. It also emphasizes that even an exceedingly “nonperturbative” strategy of trying to identify consistent causally connected sequences of events appears to have inevitable and identifiable physical consequences.

In a sense, we have come back full circle to the essence of the problem of measurement understood here in a very non-anthropocentric manner, as a correlation between the states of two systems, as an appearance of records within – rather than outside – of the state vector of the Everett - like Universe. Moreover, if our conclusions about the need to restrict the analysis of consistent histories to local bundles of branches derived from the records which are in principle accessible from within these branches are correct, and the emphasis on the coarse - grainings which acknowledge a division of the Universe into systems (existence of which will undoubtedly be branch - dependent) is recognized, the difference between the two ways of investigating classicality considered above begins to fade away.

The central ingredient of the analysis is not just a state of the system, but, rather, a correlation between states of two systems – one of which can be regarded as a record of the other. The states of the systems we perceive as an essential part of classical reality are nothing but a “shorthand” for the existence of a reliable correlation between these states and the corresponding records – states of our memory (possibly extended by such external devices as notebooks, RAM’s, or – on rare occasions – papers in conference proceedings).
This “shorthand” is sufficient only because the states of physical system used as memories are quite stable, so that their reliability (and effective classicality) can be taken for granted at least in this context. Nevertheless, the only evidence for classicality comes not so much from dealing with the bundles of histories which may or may not be consistent, but, rather, from comparisons of these records, and from the remarkable ability to use the old records as initial conditions which – together with the regularities known as “the laws of physics” – allow one to predict the content of the records which will be made in the future.

If our discussion was really about the stability of correlations between the records and the states (rather than just states) it seems appropriate to close this paper with a brief restatement of the interpretation problem from such “correlation” point of view. The entity we are concerned with is not just a to-be-classical system $S$ interacting with the environment, but, rather, a combination of $S$ with a memory $M$, each immersed in the environments $E_S$ and $E_M$. The density matrix describing this combination must have – after the environment has intervened – a form:

$$\rho_{MS} = \text{Tr}_{E_S} \text{Tr}_{E_M} \rho_{MSE} = \sum p(i,j) P^M_i P^S_j,$$

where $p(i,j)$ must be nearly a Kronecker delta in its indices:

$$p(i,j) \approx \delta_{ij},$$

so that the states of memory $\{P^M_i\}$ can constitute a faithful record of the states of the system.

One might feel that these statements are rather transparent and do not have to be illustrated by equations. Nevertheless, the two equations above clarify some of the aspects of the environment-induced decoherence and the emergence of the preferred sets of states which were emphasized from the very beginning (Zurek, 1981; 1982) but seem to have been missed by some of the readers (and perhaps even an occasional writer) of the papers on the subject of the transition from quantum to classical. The most obvious of these is the need for stability of the correlations between the preferred sets of states selected by the processes of decoherence occurring simultaneously in the memory and in the system, and the fact that memory is a physical system subjected to decoherence inasmuch as any other macroscopic and, therefore, coupled to the environment degrees of freedom, open system.

The second point is somewhat more subtle (and, consequently, it has proved to be more confusing): The form of the classical correlation expressed above by the two equations precludes any possibility of defining preferred states in terms of the density matrix of the system alone. The key question in discussing the emergence of classical observables is not the set of states which are on the diagonal after everything else (including the memory $M$) is traced out, but, rather, the set of states which can be faithfully recorded by the memory – so that the measurement repeated on a timescale long compared to the decoherence time will yield a predictable result. Hence the focus on predictability, on the form of the interaction hamiltonians, and on the minimum entropy production, and the relative lack of concern with the states which remain on the diagonal “after everything but the system...
is traced out”. The diagonality of the density matrix – emphasized so much in many of the papers – is a useful symptom of the existence of the preferred basis when it occurs (to a good approximation) in the same basis. For, only existence of such stable preferred set of states guarantees the ability of the to be classical systems to preserve the classical amount of correlations. I shall not belabor the discussion of this point any further: The reader may want to go back to the discussion of the role of the pointer basis in preserving some of the correlations in measurements (Zurek, 1981; 1982; 1983), as well as to the earlier sections where some of the aspects of this issue were considered.

Finally, the most subtle point clarified – but only to some extent – by the above two equations is the issue of the perception of a unique reality. This issue of the “collapse of the wave packet” cannot be really avoided: After all, the role of an interpretation is to establish a correspondence between the formalism of a theory describing a system and the results of the experiments – or, rather, the records of these results – accessible to the observer. And we perceive outcomes of measurements and other events originating at the quantum level alternative by the alternative, rather than all of the alternatives at once.

An exhaustive answer to this question would undoubtedly have to involve a model of “consciousness”, since what we are really asking concerns our (observers) impression that “we are conscious” of just one of the alternatives. Such model of consciousness is presently not available. However, we can investigate what sort of consequences about the collapse could be arrived at from a rather more modest and plausible assumption that such higher mental processes are based on information processing involving data present in the accessible memory. “To be conscious” must than mean that the information processing unit has accessed the available data, that it has performed logical operations including copying of certain selected records. Such copying results in redundant records: The same information is now contained in several memory cells. Therefore, questions concerning the outcome of a particular observation can be confirmed (by comparing different records) or verified (by comparing predictions based on the past records with the outcomes of new observations) only within the branches.

In a sense, the physical state of the information processing unit will be significantly altered by the information-processing activity and its physical consequences – its very state is to some degree also a record. Information processing which leads to “becoming conscious” of different alternatives will result in computers and other potentially conscious systems which differ in their physical configuration, and which, because their state is a presently accessible partial record of their past history, exist in different Universes, or rather, in different branches of the same universal state vector. These differences will be especially obvious when the machinery controlled by the computer reacts to the different outcomes in different ways. A particularly dramatic example of different fates of the machinery is afforded by the example of Schrödinger’s Cat: The cat in the branches of the Universe in which it is dead cannot be aware of any of the two alternatives because its “machinery” was significantly modified by the outcome of the quantum event – it became modified to the extent which makes it impossible for the cat to process information, and, hence, to be aware of anything at all.

I believe that similar, but (fortunately) much more subtle modification of both the
records and the identity – the physical state – of the recording and information processing
“observer” (animated or otherwise) are the ultimate cause of the “collapse”. This mecha-
anism for the collapse is closely coupled to the phenomenon of decoherence, which makes
the changes of the machinery irreversible. The selection of the alternatives arises because
of the split into the machinery (including the memory and the information processing unit)
and “the rest of the Universe”, through the environment induced superselection. The pos-
sible set of alternatives of both “what I know” and “what I am” is then fixed by the same
process and at the same time. An observer carries in its state either an implicit or an
explicit record of its branch. It should be emphasized that these last few paragraphs are
considerably more speculative than the rest of the paper, and should be treated as such
by the reader.

The interpretation that emerges from these considerations is obviously consistent with
the Everett’s “Many Worlds” point of view. It is supplemented by a process – decoherence –
which arises when the division of the Universe into separate entities is recognized. Its
key consequence – emergence of the preferred set of states which can exist for time long
compared to the decoherence timescale for a state randomly selected from the Hilbert
space of the system of interest – is responsible for the selection of the individual branches.
As reported by an observer, whose memory and state becomes modified each time a “split-
ting” involving him as the system takes place, the apparent collapses of the state vector
occur in complete accord with the Bohr’s “Copenhagen Interpretation”. The role of the
decoherence is to establish a boundary between quantum and classical. This boundary is in
principle movable, but in practice largely immobilized by the irreversibility of the process
of decoherence (see Zeh, 1991) which is in turn closely tied to the number of the degrees
of freedom coupled to a macroscopic body. The equivalence between “macroscopic” and
“classical” is then validated by the decoherence considerations, but only as a consequence
of the practical impossibility of keeping objects which are macroscopic perfectly isolated.

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Figure Caption

Fig. 1. Predictability sieve for an underdamped harmonic oscillator. The plot shows $Tr\rho^2 \sim 1/\text{area}$ (which serves as a measure of purity of the reduced density matrix $\rho$) for mixtures which have evolved from the initial minimum uncertainty wavepackets with different squeeze parameters $s$ in a damped harmonic oscillator with $\gamma/\omega = 10^{-4}$. Coherent states ($s = 1$) which have the same spread in position and momentum, when measured in the units natural for the harmonic oscillator, are clearly favored as a preferred set of states by the predictability sieve. Thus, they are expected to play the crucial role of the pointer basis in the transition from quantum to classical.
Discussion

Halliwell: You have the nice result that the off-diagonal terms of the density matrix (i.e., parts representing interferences) become exponentially suppressed in comparison with the on-diagonal terms. But why does this mean that you have decoherence? Why is it the peaks of the density matrix that matter? In short, what exactly is your definition of decoherence? In the approach of Gell-Mann and Hartle, decoherence is defined rather precisely by insisting that the probability sum rules are satisfied by the probabilities of histories. These rules are trivially satisfied, always, for histories consisting of one moment of time. You seem to have only one moment of time. How is your notion of decoherence reconciled with theirs?

WHZ: Very simply. I prefer to use the word “consistency” employed in the earlier work to describe histories which satisfy probability sum rules (i.e., Griffiths, 1984) and adopted by the others (Omnès, 1990; 1992, and references therein).

I feel that the term decoherence is best reserved for the process which destroys quantum coherence when a system becomes correlated with the “other” degrees of freedom, its environment (which can include also internal degrees of freedom as would certainly be the case for the environment of a pointer in a quantum apparatus). In spite of this “conservative” attitude to the nomenclature, I am partial to the “decoherence functional” Gell-Mann and Hartle have employed to formulate their version of consistency conditions. This is because much of the motivation for their work goes beyond simple consistency, and in the direction which, when pursued for open systems, will result in a picture quite similar to the one emerging from the recognition of the role of the process of decoherence (see Sections 8 and 9 above).

Now as for your other queries; (i) Decoherence is a process, hence it happens in time. In recognition of this, the discussions of the effects of decoherence have always focused on correlations and on the interaction Hamiltonians used, for example, to define preferred basis, rather than on the instantaneous state of the density matrix, which can be always diagonalized. (ii) Consistency rules are “trivially satisfied” at a single moment of time because consistency conditions are always formulated in a manner which effectively results in a “collapse of the state vector” onto the set of alternatives represented by projectors at both the beginning and the end of each history. And applying a formalism which was developed to deal with histories to an object defined at an instant is likely to be a bad idea — and yield results which are either wrong or trivial. (iii) Nevertheless, the effect of decoherence process can be studied at an instant through the insensitivity of the system to measurements of the preferred observables. This insensitivity can be related to the existence of classical correlations and seems to be a natural criterion for the classicality of the system. It sets in after a decoherence time irregardless of the initial state of the system.

Griffiths: The results you have discussed are very interesting, and add to our understanding of quantum mechanics. On the other hand, I think that they could be stated in a much more clearer way if you would use the “grammar” of consistent histories. For example, in the latter interpretation the choice of basis is not left to the “environment”; it is a choice
made by the theoretician. In fact, you choose the basis that interests you and, from the consistent histories point of view, what you then did was to show that in this choice of basis, and after a suitably short time, etc., the questions you want to ask form a consistent history. But in order to think clearly about a problem, it is useful to use a clear language, rather than talking about “fairies”, which is what one tends to do if one carries over the traditional language of quantum measurement.

**WHZ:** I tend to believe that even though theoreticians have more or less unlimited powers, they cannot settle questions such as the one posed by Einstein in a letter to Born.* Moreover, I find it difficult to dismiss questions such as this as “fairies”. Furthermore, I feel rather strongly that it is the openness of the macroscopic quantum systems which – along with the form of the interactions – allows one to settle such issues without appealing to anything outside of physics.

I agree with you on the anticipated relationship between the consistent histories and decoherence, and on the need to establish correspondence between these two formalisms. I believe that they are compatible in more or less the manner you indicate, but I do not believe that they are equivalent. In particular, consistency conditions alone are not as restrictive as the process of decoherence tends to be. They can be, for example, satisfied by violently non-classical histories which obtain from evolving unitarily projection operators which initially diagonalize density matrix. The relation between decoherence and consistency clearly requires further study.

**Leibowitz:** I agree with much of Griffiths’ comment although I disagree with him about “fairies”. I believe that all questions are legitimate and, as pointed out by Bell and fully analyzed in a recent paper by Durr, Goldstein, and Zangi (*J. Stat. Phys.*, to appear) the Bohm theory which assigns complete reality to particle positions gives a completely clear explanation of the non-relativistic quantum mechanics which is free from both problems of measurement and from “having to talk to one’s lawyer” before answering some of the questions. Whether Bohm’s theory is right I certainly do not know, but it does have some advantages of clarity, which should be considered when discussing “difficulties” of quantum mechanics.

**WHZ:** I am always concerned with whether the relativistic version of the Bohm - de Broglie theory can even exist. (How could photons follow anything but straight lines? Yet, in order to explain double - slit experiments a lá Bohm - de Broglie one tends to require rather complicated trajectories!) Moreover, having grown accustomed to the probabilities in quantum mechanics, I find the exact causality of particle trajectories unappealing.

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* The quote from a 1954 letter from Albert Einstein to Max Born “Let \( \Psi_1 \) and \( \Psi_2 \) be solutions of the same Schrödinger equation. ... When the system is a macrosystem and when \( \Psi_1 \) and \( \Psi_2 \) are ‘narrow’ with respect to the macrocoordinates, then in by far the greater number of cases this is no longer true for \( \Psi = \Psi_1 + \Psi_2 \). Narrowness with respect to macrocoordinates is not only independent of the principles of quantum mechanics, but, moreover, incompatible with them...” was on one of the transparencies shown during the talk, quoted after the translation of E. Joos from *New Techniques and Ideas in Quantum Measurement Theory*, D. M. Greenberger, ed., (New York Acad. Sci., 1986)
Omnès: Concerning the basis in which the reduced density operator becomes diagonal: For a macroscopic system this question is linked with another one, which is to define the “macrocoordinates” from first principles. My own guess (relying on the work of the mathematician Charles Fefferman) is that one will find diagonalization in terms of both position and momentum, as expressed by quasiprojectors representing classical properties.

WHZ: I agree with your guess and I can base it on the special role played by coherent states in a quantum harmonic oscillator (as it is shown in Section 7 of the paper in more detail), but I believe that decoherence is sufficient to understand similar spreads in position and momentum. The symmetry between $x$ and $p$ is broken by the interaction Hamiltonian (which depends on position) but partially restored by the rotation of the Wigner function corresponding to the state in the phase space by the self-Hamiltonian. I am, however, not familiar with the work of Fefferman. Therefore, I cannot comment on your motivation for your guess.

Bennett: If the interaction is not exactly diagonal in position, do the superpositions of position still decohere?

WHZ: This will depend on details of the interaction, self-Hamiltonian, etc., as discussed in the paper and references. It is, however, interesting to note that when the interaction Hamiltonian is periodic in position (and, therefore, diagonal in momentum) and the particles are otherwise free (so that the self-Hamiltonian is also diagonal in momentum) diagonalization in momentum (and delocalization in position) is expected for the preferred states. Such situation occurs when electrons are traveling through a regular lattice of a crystal, and the expectation about their preferred states seems to be borne out by their behavior.

Hartle: I would like to comment on the connection between the notion of “decoherence” as used by Wojciech in his talk and the concept of “consistent histories” discussed by Bob Griffiths and Roland Omnès that was called “decoherent histories” in the work by Murray Gell-Mann and myself. As Griffiths and Omnès discussed yesterday, probabilities can be assigned to sets of alternative coarse-grained histories if, and only if, there is nearly vanishing interference between the individual members of the set, that is, if they “decohere” or are “consistent”.

For those very special types of coarse grainings in which the fundamental variables are divided into a set that is followed and a set that is ignored it is possible to construct a reduced density matrix by tracing the full density matrix over the ignored variables. Then one can show that typical mechanisms that effect the decoherence of histories (as in the Feynman - Vernon, Caldiera - Leggett type of models) also cause the off-diagonal elements of the reduced density matrix to evolve to small values (see, e.g., my lectures, Hartle 1991). However, the approach of the reduced density matrix to diagonality should not be taken as a fundamental definition of decoherence. There are at least two reasons: First, realistic coarse-grainings describing the quasiclassical domain such as ranges of averaged densities of energy, momentum, etc. do not correspond to a division of the fundamental variables into ones that are distinguished by the coarse-graining and the others that are ignored.
Thus, in general and realistic cases there is no precise notion of environment, no basis associated with coarse-graining, and no reduced density matrix.

The second reason is that decoherence for histories consisting of alternatives at a single moment of time is automatic. Decoherence is non-trivial only for histories that involve several moments of time. Decoherence of histories, therefore, cannot be defined at only one time like a reduced density matrix. It is for these reasons that the general discussion of decoherence (consistency) is important.

**WHZ:** I have a strong feeling that most of the disagreement between us that you outline in your comment is based on a difference in our vocabularies. You seem to insist on using words “decoherence” and “consistency” interchangeably, as if they were synonymous. I believe that such redundancy is wasteful, and that it is much more profitable to set up a one-to-one correspondence between the words and concepts. I am happy to follow the example of Griffiths and Omnès and use the term “consistency” to refer to the set of histories which satisfy the probability sum rules. I would like, on the other hand, to reserve the term “decoherence” to describe the process which results in the loss of quantum coherence whenever (for example) two systems such as a “system of interest” and an “environment” are becoming correlated as a result of a dynamical interaction. This distinction seems to be well established in the literature (see, in addition to Omnès, also Albrecht, Conradi, DeWitt (in these proceedings) Griffiths (including a comment after DeWitt’s talk), Fukuyama, Habib, Halliwell, Hu, Kiefer, Laflamme, Morikawa, Padmanabhan, Paz, Unruh, and Zeh, and probably quite a few others). I see little gain and a tremendous potential for confusion in trying to change this existing usage, especially since the term “decoherence” is well-suited to replace phrases such as “loss of quantum coherence” or “dephasing” which were used to describe similar phenomena (although in more “down to earth” contexts) for a very long time. Thus, I am in almost complete agreement with much of your comment providing that we agree to follow Griffiths and Omnès and continue to use the word “consistent” when referring to histories which satisfy the sum rules. And since you indicate that the two words can be used interchangeably, I assume that you will not object to this proposal. Having done so (and after re-stating your comment with the appropriate substitutions) I find only a rather minor items which require further clarification.

The procedure required to distinguish the system from the environment is one of them. You seem to insist on such distinction appearing in the *fundamental* variables (which, I assume, would probably take us well beyond electrons and protons, to some version of the string theory). I do not believe that such insistence on fundamental variables is necessary or practical. Indeed, the questions addressed in the context of transition from quantum to classical usually concern variables such as the position and momentum of the center of mass, which are not fundamental, but are nevertheless coupled to the rest of the Hilbert space, with the environment consisting in part of the “internal variables”. For the hydrodynamic variables such split is accomplished by defining appropriate projection operators which correspond to the averages (see, e. g., “Equilibrium and Nonequilibrium Statistical Mechanics” by Radu Balescu, as well as papers by Zwanzig, Prigogine and Réstoibois, and others). Similarly, in Josephson junctions (which motivated the work of Caldeira and Leggett on the influence functional) the environment is “internal” and at
least one of the macroscopic quantum observables (the current) is hydrodynamic in nature.

Finally, as to the question whether (1) decoherence and (2) consistency can be studied at a single moment of time, I would like to note that: (i) Decoherence is a process, which occurs in time. Its effects can be seen by comparing the initial density matrix with the final one. The corresponding instantaneous rate of the entropy increase, as well as the discrepancy between the instantaneous density matrix and the preferred set of states can be also assessed at an instant, as discussed in Section 7 above. (ii) I agree with you that it is pointless to study consistency of histories which last an instant, since the first and the last set of the projection operators are “special” and, in effect, enforce a “collapse of the wave packet” (see Sections 8 and 9 for details), and instantaneous histories are rather degenerate. Moreover, (iii) I could not find a better example than the last part of your comment to illustrate why “decoherence” and “consistency” should be allowed to keep their original meanings, so that we can avoid further linguistic misunderstandings and focus on the physics instead.