Laser Induced Condensation of Trapped Bosonic Gases

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We demonstrate that an appropriate sequence of laser pulses allows to condense a gas of trapped bosonic atoms into an arbitrary trap level. Such condensation is robust, can be achieved in experimentally feasible traps, and may lead to multistability and hysteresis phenomena.

Laser cooling has led to spectacular results in recent years [1]. So far, however, it has not allowed to reach temperatures for which quantum statistics become important. In particular, evaporative cooling is used to obtain Bose-Einstein condensation of trapped gases [2]. Nevertheless, several groups are pursuing the challenging goal of condensation via all-optical means [3–5].

In traps of size larger than the inverse wavevector, $k_L^{-1}$, the temperatures required for condensation are below, or of the order of, the photon recoil energy, $E_R = \hbar \omega_R = \hbar^2 k_L^2 / 2m$, where $m$ is the atomic mass. There exist several laser cooling schemes to reach such temperatures [6]. They exploit single atom “dark states”, i.e. states which cannot be excited by the laser, but can be populated via spontaneous emission. The main difficulty in applying dark state cooling for dense gases is caused by light reabsorption. Unfortunately, these states are not dark with respect to the photons spontaneously emitted by other atoms. Thus, at sufficiently high densities, dark state cooling may cease to work, since multiple reabsorptions can increase the system energy by several recoil energies per atom [7]. In particular, laser induced condensation is feasible only if the reabsorption probability is smaller than the inverse of the number of energy levels accessible via spontaneous emission processes [8].

Several remedies to the reabsorption problem have been proposed. First, the role of reabsorptions increases with the dimensionality. If the reabsorption cross section for trapped atoms is the same as in free space, i.e. $\approx 1/k_L^2$, the reabsorptions should not cause any problem in 1D, have to be carefully considered in 2D, and forbid condensation in 3D. Working with quasi-1D or -2D systems is thus a possible way to reduce the role of reabsorptions [9]. The most promising remedy against this problem employs the dependence of the reabsorption probability for trapped atoms on the fluorescence rate $\gamma$, which can be easily adjusted in dark state cooling [10].

In particular, in the so called Festina Lente limit, when $\gamma$ is much smaller than the trap frequency $\omega$, the reabsorption processes in which the atoms change energy and undergo heating are suppressed. However, neither collective cooling schemes in traps of realistic size have been investigated in this limit, nor has it been shown that laser induced condensation is possible.

In this Letter we present such investigation. First, we formulate the Master Equation (ME) that describes Raman cooling in the Festina Lente limit using coarse graining in time. Cooling is dynamical, and consists of sequences of pairs of laser pulses inducing stimulated and spontaneous Raman transitions between the two electronic levels of trapped atoms, $|g\rangle$ and $|e\rangle$. The stimulated absorption pulses induce the energy selective transition $|g\rangle \rightarrow |e\rangle$ that depopulates all motional states except the dark ones: the spontaneous emission pulses are non-selective, and repump the atoms from $|e\rangle$ to $|g\rangle$ populating all accessible motional states. We simulate the dynamics generated by the ME in 3D, and show that: i) laser induced condensation into an arbitrary trap level is possible; an arbitrary trap level may be made dark; ii) condensation is robust with respect to changes of physical parameters: dark states do not have to be completely dark; iii) multistability and hysteresis occur when the parameters undergo large changes. In the limit of large number of atoms analytic solutions of the ME are found.

We consider $N$ bosonic atoms with two levels $|g\rangle$ and $|e\rangle$ in a non-isotropic trap with the frequencies $\omega_{x,y,z}^g, \omega_{x,y,z}^e$ different for the ground and the excited states, and non-commensurable one with another. This assumption simplifies enormously the dynamics of the spontaneous emission processes in the Festina Lente limit. We use the coarse graining in time and describe variations of the atomic state after one absorption and one spontaneous emission pulse. After such cooling cycle all atoms are in the ground internal state described by the density matrix $\rho(t)$. This matrix is diagonal in the Fock representation corresponding to the bare trap levels. In order to derive the ME, we separate the effects of laser cooling from the ones due to atom–atom collisions. The latter can be described by a quantum kinetic ME, which has been studied in Ref. [11]. In this paper we concentrate on the laser cooling process. Thus, our results are valid in the case when collision processes are slow compared to laser cooling.

For the stimulated Raman transitions, we assume weak pulses of duration $\tau_{ab}$. Their effects can thus be described by second order perturbation theory (formally that corresponds to one atom excited at most),
\[
\rho(t + \tau_{abs}) = \rho(t) - \sum_{lm} \Gamma_{lm}^{abs} g_m g_m \rho(t) \\
- \sum_{lm} \Gamma_{lm}^{abs} \rho(t) g_m^\dagger g_m + 2 \sum_{lm} \Gamma_{lm}^{abs} \rho(t) e_l^m g_m^\dagger g_m,
\]
where \(g_m, g_m^\dagger (e_l^m, e_l^m)\) are bosonic annihilation and creation operators of atoms in the ground (excited) internal state and in the trap level \(m = (m_x, m_y, m_z)\) \([l = (l_x, l_y, l_z)]\). The absorption probabilities \(\Gamma_{lm}^{abs}\) describe transitions from the ground state level \(m\) to an excited state level \(l\), and depend on Raman laser pulses. For instance, if the Raman transition is characterized by the maximal (effective) Rabi frequency \(\Omega_0\), temporal envelope \(f(t)\), wavevector \(k_L\), frequency \(\omega_L\), and detuning \(\delta = \omega_L - \omega_a\), where \(\omega_a\) is the internal levels energy difference, the probabilities are given by
\[
\Gamma_{lm}^{abs} = \frac{\Omega_0^2}{8} \langle |\langle 1| e^{i k_L \cdot x} |m\rangle|^2 |\tilde{f}(\delta - \omega_L^+ - \omega_m^m)|^2, \tag{2}
\]
where the first term is Franck-Condon factor, and
\[
\tilde{f}(\delta) = \int_{-\infty}^{+\infty} f(t)e^{-i\delta t} dt,
\]
is a function peaked at \(\delta = 0\) with width of the order of \(1/\tau_{abs}\). In the following we use Gaussian pulses with \(f(t) = \exp(-t^2/\tau_{abs}^2)\) and \(\omega \tau_{abs} > 1\), so that only resonant \(\Gamma_{lm}^{abs}\) are relevant \((|\delta - \omega_L^+ + \omega_m^m| < \tau_{abs})\). On the other hand, \(\Omega_0 \tau_{abs} < 1\), so that only a small fraction of atoms is excited by the absorption pulse, which ensures the validity of the perturbative approach. In Eq. \(3\) two kinds of terms are omitted, since their contributions vanish after the repumping process: i) the first order terms, which correspond to coherences with respect to internal levels (off-diagonal matrix elements between the states with one excited, and no excited atoms); these terms are destroyed after spontaneous emission; ii) coherences between different excited trap levels; those terms are neglected since spontaneous emission in the Festina Lente limit is purely diagonal in the \(l\) index; thus \(l \neq l'\) coherences are destroyed in the quantum jump down to the ground state, and do not influence the dynamics of diagonal matrix elements \(\|\).

The repumping pulses have constant amplitude and duration \(\tau_{sp}\) long enough to depopulate totally the excited states. In accordance with the Festina Lente limit, we will assume that (collective) spontaneous emission rates are small in comparison to the trap frequency \(\|\). This allows to perform a secular approximation in the master equation describing this process, which leads to the following equation after spontaneous emission has taken place \(\tau_{ch} = \tau_{abs} + \tau_{sp}\):
\[
\rho(t + \tau_{ch}) = \rho(t) - \sum_{lm} \Gamma_{lm}^{abs} g_m g_m \rho(t) + \rho(t) g_m^\dagger g_m \\
+ 4 \int_0^{\infty} dt' \sum_{mnl} \Gamma_{lm}^{sp} g_n g_n^\dagger e^{-\sum_k \Gamma_k^{sp} g_k g_k^\dagger} \rho(t) g_m^\dagger e^{-\sum_k \Gamma_k^{sp} g_k g_k^\dagger} \rho(g_m) + \sum_k \Gamma_k^{sp} g_k g_k^\dagger \rho(t) g_m^\dagger e^{-\sum_k \Gamma_k^{sp} g_k g_k^\dagger} \rho(g_m) + \sum_k \Gamma_k^{sp} g_k g_k^\dagger \rho(t) g_m^\dagger e^{-\sum_k \Gamma_k^{sp} g_k g_k^\dagger} \rho(g_m), \tag{3}
\]
The last term in Eq. \(3\) describes the integral over all possible times \(\tau\) in which the quantum jump from the excited state \(l\) to the ground state \(n\) occurs. The amplitude of the excited state is damped during the time \(\tau\) with the collective rate \(\sum_k \Gamma_k^{sp} g_k g_k^\dagger\), which evidently contain Bose enhancement factors, i.e. is proportional to occupation numbers plus one of the corresponding ground trap levels. The integral over \(\tau\) can be extended to \(\infty\) since \(\tau_{sp}\) is long enough. The spontaneous emission rates are
\[
\Gamma_{nm}^{sp} = \gamma \int d\Omega \langle \Omega | \rho e^{i k_n \cdot \Omega} \rangle |\langle |l|\rho e^{i k_l \cdot \Omega} \rangle|^2, \tag{4}
\]
where \(2\gamma\) is the single atom (effective) spontaneous emission rate (i.e. spontaneous Raman transition rate, controllable in experiments), integration extends over the solid angle \(\Omega\), \(W(\Omega)\) describes dipole radiation pattern, \(n(\Omega)\) is a unit vector in the \(\Omega\) direction, and \(k_n = \omega_n/c\).

Eq. \(3\) describes an elementary cooling step, i.e. maps a diagonal density operator \(\rho(t)\) onto a diagonal \(\rho(t + \tau_{ch})\), with all atoms in the ground state. It can thus be simulated using standard Monte Carlo procedures. We have performed such simulations for \(N = 1\) up to \(N = 500\) atoms, in various dimensions, and for various cooling strategies. Franck-Condon factors and trap frequencies can be efficiently approximated using the states of an isotropic trap of frequency \(\omega\). We concentrate on 3D cooling, beyond the Lamb-Dicke limit, i.e. for the traps for which the Lamb-Dicke parameter \(\eta = \sqrt{E_R/\hbar \omega}\) is larger than one. Due to memory storage and calculation times, our numerical simulation has to be restricted to values of \(\eta \lesssim 2\). In that case the atom (having initially an energy of the order of few \(E_R\)) may increase its trap energy level in the spontaneous emission process by energies \(\sim E_R\), and non-standard cooling schemes have to be used to avoid such heating effects \(7\). Generalization of the approach of Ref. \(7\) allows to cool dynamically (i.e. by changing absorption laser pulses appropriately) \(\text{individual atoms}\) to arbitrary trap levels \(8\).

The full dynamical cooling cycle must contain sequences of absorption pulses of appropriately chosen frequencies. The Fourier bandwidth of the pulses can be smaller than \(\omega\), so that various resonance conditions may be employed. We use the following types of pulses: \(1\) confinement pulses: spontaneous emission may increase each of the quantum numbers \(m_{x,y,z}\) by \(O(\eta^2)\). In \(D\)-dimensions pulses with detuning \(\delta = -D \eta^2 \omega\), where \(\eta^2\) is the closest integer to \(\eta^2\), have thus an overall cooling effect, and confine the atoms in the energy band of \(D\) recoils. The use of two slightly detuned confinement pulses is recommended; \(2\) dark-state cooling pulses: these pulses should fulfill dark state condition for a selected state to which the cooling should occur; \(3\) sideband and auxiliary cooling pulses: in general, dark state cooling pulses might lead to unexpected trapping in other levels. In order to avoid it, auxiliary pulses that empty undesired dark states and do not empty the desired dark state are needed. For cooling into the ground state, for
instance, the sideband cooling pulse with \( \delta = -\omega \) is used; 
iv) pseudo-confining pulses: with the use of pulses i)-iii) 
cooling is typically very slow. In order to shorten cooling 
time we use pulses with \( \delta = -3\eta^2\omega/2 \) and \( \delta = -\eta^2\omega \), 
which pseudo-confine the atoms below \( n = 3\eta^2/2 \) and \( n = \eta^2 \).

![Population of (0,0,0) as a function of number of cycles, each consisting of a sequence of absorption pulses with \( s=-12,-6,-4,0,-13,-7,-5,-1 \), and repumping pulses. For all the absorption pulses \( A_{x,y,z} = 1 \), except for \( s = 0 \) for which \( A_s = -2 \). The initial distribution is thermal with mean \( \langle n \rangle = 6 \), and \( \eta = 2.0 \). Dotted (solid) lines represent the case of \( N = 1 \) (500) atoms.](image)

Let us now consider possible dark state conditions, 
determined by the total probabilities of emptying a given 
level \( m \), \( \Gamma_m = \sum_l \Gamma^{obs}_{lm} \). We consider three Raman trans-
itions induced by laser pairs propagating in directions \( x \), \( y \), and \( z \) characterised by three different effective Rabi 
frequencies \( \Omega_p f(t) A_{j} \), where \( A_{j=x,y,z} \) account for differ-
ence of intensities or dephasing between the lasers. As 
we discussed in detail in Ref. [18] the dark states may ap-
pear due to two reasons: vanishing of all Franck-Condon 
coefficients for three lasers, or destructive interference 
effects between the three lasers. The first type of con-
ditions can be achieved by choosing appropriate detuning 
\( \delta = s\omega \) with integer \( s \): for instance, ground state 
\( m = (0,0,0) \) is dark with respect to side-band cooling 
pulses with \( s = -1 \). Since the Franck-Condon coeffi-
cient \( \langle m_x + s|e^{ikLz}|m_x \rangle \) vanishes for \( m_x = 1 \) provided 
\( \eta^2 = s + 1 \), and analogous property holds for \( y \) and \( z \), 
the state \((1,1,1)\) can be made dark provided \( \eta^2 \) is inte-
ger, and \( s = \eta^2 - 1 \). Similar conditions can be found for 
m = (2,2,2). The second type of dark state con-
ditions correspond to use of resonant absorption with 
\( s = 0 \). Chosing for instance \( A_{x} = 1 \) and \( A_{y,z} \) such that 
\( 1 + A_y + A_z = 0 \) makes all states \((m,m,m)\) dark.

In Fig. 1 we present our result for ground state cooling 
of 1 and 500 sodium atoms in 3D trap with \( \eta = 2 \), 
using 20 3D-energy shells (i.e. 1771 trap levels). The 
initial state of the system corresponds to mean energy 
\( 6\hbar\omega \), and is the same for all figures. The pulse sequence 
is \( s = -12,-6,-4,0,-13,-7,-5,-1 \). \( A_{x,y,z} = 1 \), except for 
\( s = 0 \), for which \( A_x,y = 1, A_z = -2 \). Pulses 1 and 5 are 
confining, 2,3,6 and 7 pseudo-confining, and 4 and 8 are 
dark state cooling pulses. The many body effects intro-
duce one very important element to the dynamics: Bose 
enhancement factors, that speed up the dynamics enor-
mously. The time scale is such that each cooling cycle must 
be, say few times longer than \( 2\pi/\omega \approx 10^{-4}s \) [19]. 
Then the function \( \tilde{f} \) is sufficiently narrow to neglect non-
resonant transitions. Therefore 1000 cycles correspond to 
about 1s. Keeping \( \Gamma \) fixed, cooling of one atom requires 
here few seconds, whereas collective cooling takes about 
0.1s. After achieving condensation with 500 atoms, con-
fining pulses can be avoided, a single dark state pulse can 
keep the atoms in the condensed state.

![Population of (1,1,1) as a function of number of cycles, each consisting of a sequence of absorption pulses with \( s=-12,-6,-3,3,-13,7,-4,-2 \), and \( A_{x,y,z} = 1 \). The initial distribution is thermal with mean \( \langle n \rangle = 6 \). The cases of \( \eta = 2.0 \) and 2.05, \( N = 1 \) and 500 are depicted.](image)

In Fig. 2 we show results for cooling into the state 
\((1,1,1)\) using the sequence \( s=-12,-6,-3,3,-13,7,-4,-2 \), with 
\( A_{x,y,z} = 1 \). Here the pulse 4 is the dark state pulse 
\( (\eta^2 = s + 1) \), the other are either confining or auxi-
liary. First, note that when dark state condition is ful-
filled exactly \( (\eta = 2) \), cooling of a single atom to \((1,1,1)\), 
although slow, is possible. This cooling mechanism is, 
evertheless fragile. A tiny perturbation of the dark 
state \( (\eta = 2.05) \) makes efficient cooling impossible. 
This conclusion does not hold for many atoms though. 
Quantum statistics helps to achieve 100% condensation that 
is robust with respect to parameter changes; the results 
for \( \eta = 2 \) or 2.05 are undistinguishable, cooling is much 
shorter than in the 1 atom case, and takes about 1s [20].

The existence of various stationary states in our sys-
tem suggest the possibility of multistability and hystere-
sis effects [21]. Indeed, in Fig. 3 we investigate the same 
cooling sequence as in Fig. 1, except that \( s_k = -2 \), and for 
pulse 4 \( A_z \) varies adiabatically from \(-1.94 \) to \(-0.08 \) and 
back during \( 37200 \) cycles. For \( A_z \) close to \(-2 \), the ground 
state is in this case dark for \( s = 0 \). As \( A_z \) grows, at some 
point \( (A_z = -2/3) \) the states \((1,0,1)\) and \((0,1,1)\) become 
dark. This occurs when the destructive interference 
\( \langle 0_z|e^{ikLz}|0_z \rangle + \langle 1_y|e^{ikLz}|1_y \rangle = -A_z\langle 1_z|e^{ikLz}|1_z \rangle \) 
takes place. The system shows multistability and hystere-
sis: the transfer from \((0,0,0)\) to \((1,0,1)\) and \((0,1,1)\) 
occurs for higher values of \( A_z \) than vice versa.
If most atoms are condensed, confining pulses are no more needed. One can study then the stationary limit of the ME with cooling pulses with fixed $s$. Amazingly, using the Glauber’s $P$ representation and systematic $1/N$ expansion the ME can be solved analytically. The results can be summarized as follows: i) conditions for absorption and spontaneous emission rates that lead to cooling into an arbitrary state are analytically obtained: the system condenses into $n_0$-state iff $\Gamma_n \equiv \sum_m (\Gamma_{nm} - \Gamma_{mn} \Gamma_{nm} / \Gamma_{nn}) > 0$ for all $n \neq n_0$. ii) the dynamics exhibits two time scales: on a fast scale (several cooling cycles), noncondensed modes behave as independent quantum harmonic oscillators that approach thermal equilibrium (quantum Ornstein-Uhlenbeck processes). The condensate mode is correlated to that dynamics through the atom number conservation. The cooling time is of the order of $\max_n (\tau_{ch}/\Gamma_n)$. On a slow time scale (that is $N$ times slower than the fast one) the dynamics is dominated by self-transitions from the condensate and back. That produces slow phase diffusion and slow decay of the two-time correlation function of the condensed mode with the rate $\simeq \sum_m \Gamma_{nm} / 2 N \tau_{ch}$. The scattered photon statistics is Poissonian.

Sumarizing, using the quantum ME in the Festina Lente limit, we have demonstrated that properly designed sequences of laser pulses allow to condense a gas of trapped bosonic atoms into an arbitrary state. Such condensation is robust, can be achieved in experimentally feasible traps, and leads to multistability and hysteresis phenomena. We have neglected in our approach atom–atom collisions. Collisionally induced population redistribution should not affect condensate in the (collisionally modified) ground state $(0,0,0)$, since this state corresponds to thermal equilibrium at very low temperatures. Thermalisation mechanism might destroy condensates in excited states, but that depends on time scales. Condensation requires seconds, but we have not attempted to optimize this time. Optimal times become shorter than collisional thermalisation time if $N$ is not too large, and $\eta$ not too small.

We acknowledge support from Spanish Dirección General de Investigación Científica y Técnica (Grant PB95-0955), Junta de Castilla y León (Grant SA 16/98), and Deutsche Forschungsgemeinschaft (SFB 407).

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