A transition from boundary- to bulk-driven acoustic streaming due to nonlinear thermoviscous effects at high acoustic energy densities

Jonas Helboe Joergensen,¹,* Wei Qiu,²,† and Henrik Bruus¹,‡

¹Department of Physics, Technical University of Denmark,
DTU Physics Building 309, DK-2800 Kongens Lyngby, Denmark
²Department of Biomedical Engineering, Lund University, Ole Rømers väg 3, 22363, Lund, Sweden

(Dated: 21 December 2021)

Acoustic streaming is studied in a rectangular microfluidic channel. It is demonstrated theoretically, numerically, and experimentally with good agreement, frictional heating can alter the streaming pattern qualitatively at high acoustic energy densities \( E_{\text{ac}} \) above 500 \( J/m^3 \). The study shows, how as a function of increasing \( E_{\text{ac}} \) at fixed frequency, the traditional boundary-driven four streaming rolls created at a half-wave standing-wave resonance, transition into two large streaming rolls. This nonlinear transition occurs because friction heats up the fluid resulting in a temperature gradient, which spawns an acoustic body force in the bulk that drives thermoacoustic streaming.

Microscale acoustofluidic devices are used to manipulate and control microparticles and cells. In such devices, two main forces act on the suspended particles, the acoustic radiation force and the drag force due to acoustic streaming, which is a time-averaged flow caused by the inherent nonlinearities of fluid dynamics. Recent work has clarified many subtle details pertaining to the radiation force on microparticles, including thermoviscous effects [1] and microstreaming [2]. Concurrently, similar progress has been made in the theory of acoustic streaming, especially regarding thermoviscous effects. The fundamental boundary-driven streaming caused by time-averaged forces in the oscillatory boundary-layer flow [3], and the fundamental bulk-driven streaming generated by the time-averaged dissipation of traveling waves [4], have recently been supplemented by bulk-driven baroclinic [5, 6] and thermoacoustic [7, 8] streaming, caused by an interplay between standing acoustic waves and steady temperature gradients. However, as noted in Refs. [7, 8], the validity of the conventional perturbation approach breaks down at moderately high, but experimentally obtainable acoustic energy densities above 100 \( J/m^3 \) in combination with moderate thermal gradients above 1 K/mm. This need for an extension of the theory beyond perturbation theory is addressed in this Letter and in the accompanying detailed presentation of the nonperturbative model in Ref. [9].

We introduce a nonperturbative iteration approach to investigate theoretically and numerically, the nonlinear effects appearing in a conventional acoustofluidic channel at high acoustic energy density \( E_{\text{ac}} \), and we validate experimentally the model predictions. We take as our generic acoustofluidic model system, the widely used rectangular channel driven at resonance with a transverse half-wave standing acoustic wave, for which the streaming at low \( E_{\text{ac}} \) is dominated by conventional boundary-driven streaming with four streaming rolls [10–13]. We show how nonlinear effects in the form of heating by viscous dissipation from the acoustic field inside the bound-
equation for mass, momentum, and energy in the fluid and solid. The independent fields are the pressure \( p \), the velocity \( \mathbf{v} \), and the temperature \( T \) in the fluid, and the displacement \( \mathbf{u} \) and \( T \) in the solid.

We study a fluid characterized by the following material parameters: density \( \rho \), isothermal compressibility \( \kappa_T \), thermal conductivity \( k^{th} \), specific heat \( c_p \), dynamic and bulk viscosity \( \eta \) and \( \eta_b \), thermal expansion coefficient \( \alpha_p \), the ratio of specific heats \( \gamma = c_p / c_v \), and the isentropic and isothermal compressibility \( \kappa_s \) and \( \kappa_T = \gamma \kappa_s \). The temperature dependence of the parameters for water are given by the polynomials derived in Ref. [12].

The boundary layers are localized near fluid-solid interfaces, and their dynamically-defined widths (jointly called \( \delta \)) are small compared to a typical device size or wavelength \( \lambda \) [1]. The stationary limit of the slow time scale and describe slow dynamics can be solved separately. Here, we study the stationary fields are the pressure fields \( p \) and \( v_0^d \), the isentropic and isothermal compressibility \( \kappa_T \) of the fluid.

A product of two acoustic fields will contain a steady 

\[ \text{stationary limit of the slow time scale and describe slow dynamics can be solved separately. Here, we study the stationary fields are the } \text{pressure fields } p \text{ and } v_0^d \text{, the isentropic and isothermal compressibility } \kappa_T \text{ of the fluid.} \]

\[ \text{A product of two acoustic fields will contain a steady} \]

\[ \text{stationary limit of the slow time scale and describe slow dynamics can be solved separately. Here, we study the stationary fields are the pressure fields } p \text{ and } v_0^d \text{, the isentropic and isothermal compressibility } \kappa_T \text{ of the fluid.} \]
The streaming flow \( \mathbf{v}_0^{\text{d}} \) can be driven either by the acoustic body force \( \mathbf{f}_\text{ac}^{\text{d}} \), called bulk-driven streaming, or by the effective boundary condition on \( \mathbf{v}_0^{\text{d}} \), called boundary-driven streaming.

The stationary temperature \( T_0^{\text{d,f}} \) in the fluid is governed by the heat equation (energy conservation) [9],

\[
0 = -\nabla \cdot \left[ k_0^{\text{th}} \nabla T_0^{\text{d,f}} \right] - c_p \rho_0 \mathbf{v}_0 \cdot \nabla T_0^{\text{d,f}} + P_\text{ac}^{\text{d}}\tag{5a}
\]

\[
P_\text{ac}^{\text{d}} = -\nabla \cdot \left( k_1^{\text{th},d} \nabla T_1^{d} \right) - \langle \rho_1^{d,p} \mathbf{v}_1^{d,p} \rangle + \langle \mathbf{v}_1^{d,p} \cdot \mathbf{\tau}_1^{d} \rangle - c_p \langle \rho_1^{d,p} \mathbf{v}_1^{d,p} \rangle \cdot \nabla T_0^{\text{d,f}},\tag{5b}
\]

and similarly for \( T_0^{\text{d,sl}} \) in the solid [9],

\[
0 = -\nabla \cdot \left[ k_0^{\text{th}} \nabla T_0^{\text{d,sl}} \right] + P_\text{ac}^{\text{d}}\tag{6a}
\]

\[
P_\text{ac}^{\text{d}} = -\nabla \cdot \left( k_1^{\text{sl},d} \nabla T_1^{d} \right)\tag{6b}
\]

Here, \( P_\text{ac}^{\text{d}} \) is the power density delivered by the acoustic wave through frictional dissipation and energy flux. \( T_0^{\text{d,f}} \) and \( T_0^{\text{d,sl}} \) are connected at the fluid-solid interface by the two effective boundary conditions taking the boundary layers into account analytically: continuity of temperature and of heat flux, applied respectively as a Dirichlet condition on \( T_0^{\text{d,f}} \) and a flux condition on \( \mathbf{n} \cdot \nabla T_0^{\text{d,sl}} \) [9],

\[
T_0^{\text{d,f}} = T_0^{\text{d,sl}} - T_0^{\text{d,0}} - \frac{1}{2} \Re \left\{ \mathbf{u}_1 \cdot \nabla T_1^{\text{d},s} - k_0^{\text{th}} (\mathbf{u}_1 \cdot \mathbf{n}) T_0^{\text{d,0}a} \right\},\tag{7a}
\]

\[
k_1^{\text{th},d} \mathbf{n} \cdot \nabla T_0^{\text{d,sl}} = k_0^{\text{th}} \mathbf{n} \cdot \nabla T_0^{\text{d,0}} + k_0^{\text{th}} \frac{\partial}{\partial \mathbf{n}} T_0^{\text{d,0}} - \frac{1}{2} \Re \left\{ k_0^{\text{th},d} (\mathbf{u}_1 \cdot \mathbf{n}) T_1^{\text{d},s} \right\} \tag{7b}
\]

In summary, the bulk temperature \( T_0^{\text{d}} \) is governed by the heat equations (5) and (6) together with the effective boundary conditions (7).

**Experimental method.**—The experiments were performed using a long straight microchannel of width \( W = 375 \mu m \) and height \( H = 135 \mu m \) in a glass-silicon chip with a piezoelectric transducer glued underneath. The transducer was driven at a frequency of 1.97 MHz at input power \( P_{\text{in}} \) of 6.1, 86.8, and 182.5 mW, resulting in the energy density \( E_\text{ac} \approx 27.2 \pm 1.1, 388.7 \pm 15.9 \), and 817.3 \pm 33.5 J/m\(^3\), respectively, as measured from the focusing of 5.0-\mu m-diameter particles at 140 fps using confocal micro-particle image velocimetry (µPIV) at the low \( P_{\text{in}} \) [27]. At higher \( P_{\text{in}} \), \( E_\text{ac} \) is estimated using the proportionality \( E_\text{ac} \propto P_{\text{in}} \). The confocal µPIV technique only captures the particle motion near the focal plane (channel mid-height), excluding particles near the top and bottom walls influenced by hydrodynamic and acoustic particle-wall interactions, and as a result, \( E_\text{ac} \) is measured accurately. The acoustic streaming for each \( E_\text{ac} \) was measured at 10 to 60 fps by tracking the motion of 0.5-\mu m-diameter particles using a defocusing-based 3D particle tracking technique [28–30]. To avoid the resonance frequency shift due to the temperature rise of the transducer under moderate (86.8 mW) and high (182.5 mW) \( P_{\text{in}} \), each measurement was run for 2 s and repeated 100 times to improve the statistics, resulting in 7800-12000 recorded frames for each driving condition.

**Results and discussion.**—The simulation and experimental results shown in Fig. 2 reveal the dominant nonlinear behavior of the stationary streaming \( \mathbf{v}_0^{\text{d}} \) and temperature \( T_0^{\text{d}} \) in a standard acoustofluidic device. In the linear regime at low \( E_\text{ac} \ll 30 \text{ J/m}^3 \), \( \mathbf{v}_0^{\text{d}} \) is dominated by the boundary-driven streaming entering the model through the slip-velocity condition (4), and the usual four boundary-driven streaming rolls appear, see Fig. 2(a). Due to friction in the viscous boundary layers, heat is generated both at the top and bottom of the channel. At the bottom, this heat is removed efficiently because of the high heat conductivity of silicon. At the top, however, the heat is removed less efficiently by the lower heat conductivity of glass, and a steady temperature gradient \( \nabla T_0 \) is established, which explains the temperature \( T_0^{\text{d}} \) seen in Fig. 2(f).

The gradient \( \nabla T_0 \) created by the acoustic frictional heating results in gradients in \( \nabla \rho_0 \) and \( \nabla \kappa_0 \), thus inducing a thermoacoustic body force (3c) \( \mathbf{f}_\text{ac}^{d} \) [7, 8],

\[
\mathbf{f}_\text{ac}^{d} = -\frac{1}{4} |\mathbf{p}_1|^2 \mathbf{n} \nabla \kappa_{s,0} + \frac{1}{4} |\mathbf{v}_1|^2 \mathbf{n} \nabla \rho_0\tag{8}
\]

\[
= -\frac{1}{4} |\mathbf{p}_1|^2 \left( \frac{\partial \kappa}{\partial T} \right)_{T_0} \mathbf{n} \nabla T_0 - \frac{1}{4} |\mathbf{v}_1|^2 \left( \frac{\partial \rho}{\partial T} \right)_{T_0} \mathbf{n} \nabla T_0.
\]

Since \( |\mathbf{p}_1|^2 \propto |\mathbf{v}_1|^2 \propto E_\text{ac} \) and \( |\mathbf{v}_0^{\text{d}}| \propto E_\text{ac} \), we have \( |\mathbf{f}_\text{ac}^{d}| \propto E_\text{ac}^2 \) and \( \mathbf{f}_\text{ac}^{d} \) will become important in the bulk at high \( E_\text{ac} \) and cause qualitative nonlinear changes of the streaming pattern. According to Eq. (8), \( \mathbf{f}_\text{ac}^{d} \) is pointing toward high temperature at the top and is strongest at the pressure antinodes at the sides [7, 8]. Consequently, \( \mathbf{f}_\text{ac}^{d} \) pushes liquid from the sides up toward the top center, with a back-flow down along vertical center axis, thus creating a streaming pattern that consists of two streaming rolls in each side of the channel. This pattern is seen at the high \( E_\text{ac} \approx 5300 \text{ J/m}^3 \) in Fig. 2(d), where the streaming is completely dominated by the thermoacoustic streaming. The transition from boundary-driven streaming at low \( E_\text{ac} \) to bulk-driven streaming at high \( E_\text{ac} \) is studied qualitatively in Fig. 2(a)-(d) and quantitatively in Fig. 2(e). During the transition, the two bottom streaming rolls expand and the two top rolls shrink, see Fig. 2(b)-(c) at \( E_\text{ac} = 380 \) and 800 J/m\(^3\), respectively. The bottom rolls expand because they rotate the same way as the two thermoacoustic streaming rolls.
This transition is studied quantitatively in Fig. 2(e) by plotting the maximum streaming velocity \( v_0^{\text{max}} \) and the vertical distance \( \Delta_v \) (thick white line) from the bottom of the channel to the position of the maximum horizontal streaming velocity \( v_0^{\text{max}} \) toward the center occurs. In the log-log plot (dark blue), the perturbative result \( v_0^{\text{max}} \propto E_{\text{ac}} \) is valid up to \( E_{\text{ac}} \approx 1000 \text{ J/m}^3 \), but at higher values \( v_0^{\text{max}} \) increases faster. A stronger signal is seen in the log-log plot (dark red), where the perturbative result \( \Delta_v \propto E_{\text{ac}}^0 \) only holds for \( E_{\text{ac}} \lesssim 30 \text{ J/m}^3 \), after which point \( \Delta_v \) increases with increasing \( E_{\text{ac}} \).

As the streaming velocity increases, convection becomes increasingly important for the heat transport (5) and strongly affects the temperature field, see Fig. 2(f–i) for \( E_{\text{ac}} = 380, 800, 5300, \) and \( 12,600 \text{ J/m}^3 \). Convection becomes important at a Pécel number \( Pe = |v_0|H_f/D^\text{th} \approx 1 \) corresponding to \( |v_0| \approx 1 \text{ mm/s} \), consistent with Fig. 2(f–j). Qualitatively, we see that for \( E_{\text{ac}} \gtrsim 800 \text{ J/m}^3 \), the two flow rolls pull the temperature profile down along the vertical center axis. We quantify this effect by the maximum temperature \( T_0^{\text{max}} \) and the vertical distance \( \Delta_T \) along the center axis from the bottom edge to the point where \( T_0 = \frac{1}{2}T_0^{\text{max}} \). The thermoacoustic streaming increases the heat transport from the fluid-glass interface to the silicon wafer, thus \( T_0^{\text{max}} \) increases less steeply than the perturbative result, \( T_0^{\text{max}} \propto E_{\text{ac}} \), as seen in the log-log plot (blue) of \( T_0^{\text{max}} \) vs. \( E_{\text{ac}} \) for \( E_{\text{ac}} \gtrsim 5000 \text{ J/m}^3 \) in Fig. 2(j). A stronger signal is seen in the log-log plot (dark red), where the perturbative result \( \Delta_T \propto E_{\text{ac}}^0 \) only holds for \( E_{\text{ac}} \lesssim 500 \text{ J/m}^3 \), after which point \( \Delta_T \) decreases with increasing \( E_{\text{ac}} \).

Conclusion.—In this Letter we have shown numerically and experimentally that the acoustic streaming in a standard microscale acoustofluidic device is changed qualitatively for moderately high acoustic energy densities \( E_{\text{ac}} \gtrsim 500 \text{ J/m}^3 \). We have explained this effect
by a nonperturbative model [9], in which a transition from boundary- to bulk-driven acoustic streaming occurs, as the acoustic body force $f_{ac}$ begins to dominate the streaming at increased $E_{ac}$ due to the internal heating generated in the viscous boundary layers. We have shown good qualitative and quantitative agreement between our model predictions and experimental data.

$$E_{ac} \gtrsim 500 \text{ J/m}^3$$ can easily be obtained in standard acoustofluidic devices, where $E_{ac} \approx 10 - 50 \text{ J/m}^3 \times [U_{pp}/(1 \text{ V})]^2$ has been reported in the literature, $U_{pp}$ being the applied voltage on the piezoelectric transducer [11, 31–33]. The physical understanding of how such acoustofluidic devices behave at high $E_{ac}$ is important for the continued development of high throughput devices in particular for biotech applications.

We thank R. Barnkob and M. Rossi for providing the software DefocusTracker, defocustracking.com/. WQ was supported by MSCA EF Seal of Excellence IF-2018 from Vinnova, Sweden’s Innovation Agency (Grant No. 2019-04856). HB and JHJ was supported by Independent Research Fund Denmark, Natural Sciences (Grant No. 8021-00310B).
[27] W. Qiu, J. T. Karlsen, H. Bruus, and P. Augustsson, Experimental characterization of acoustic streaming in gradients of density and compressibility, Phys. Rev. Appl. 11, 024018 (2019).

[28] R. Barnkob, C. J. Kähler, and M. Rossi, General defocusing particle tracking, Lab Chip 15, 3556 (2015).

[29] R. Barnkob and M. Rossi, General defocusing particle tracking: fundamentals and uncertainty assessment, Exp. Fluids 61, 110 (2020).

[30] M. Rossi and R. Barnkob, A fast and robust algorithm for general defocusing particle tracking, Meas. Sci. Technol. 32, 014001 (2020).

[31] R. Barnkob, P. Augustsson, T. Laurell, and H. Bruus, Measuring the local pressure amplitude in microchannel acoustophoresis, Lab Chip 10, 563 (2010).

[32] P. Augustsson, R. Barnkob, S. T. Wereley, H. Bruus, and T. Laurell, Automated and temperature-controlled micro-PIV measurements enabling long-term-stable microchannel acoustophoresis characterization, Lab Chip 11, 4152 (2011).

[33] R. Barnkob, I. Iranmanesh, M. Wiklund, and H. Bruus, Measuring acoustic energy density in microchannel acoustophoresis using a simple and rapid light-intensity method, Lab Chip 12, 2337 (2012).