Coexisting Charge-Ordered States with Distinct Driving Mechanisms
in Monolayer VSe$_2$

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Thinning crystalline materials to two dimensions (2D) creates a rich playground for electronic phases, including charge, spin, superconducting, and topological order. Bulk materials hosting charge density waves (CDWs), when reduced to ultrathin films, have shown CDW enhancement and tunability. However, charge order confined to only 2D remains elusive. Here we report a distinct charge ordered state emerging in the monolayer limit of 1T-VSe$_2$. Systematic scanning tunneling microscopy experiments reveal that bilayer VSe$_2$ largely retains the bulk electronic structure, hosting a tri-directional CDW. However, monolayer VSe$_2$ – consistently across distinct substrates – exhibits a dimensional crossover, hosting two CDWs with distinct wavelengths and transition temperatures. Electronic structure calculations reveal that while one CDW is bulk-like and arises from the well-known Peierls mechanism, the other is decidedly unconventional. The observed CDW-lattice decoupling and the emergence of a flat band suggest that the new CDW could arise from enhanced electron-electron interactions in the 2D limit. These findings establish monolayer-VSe$_2$ as a host of coexisting charge orders with distinct origins, and enable the tailoring of electronic phenomena via emergent interactions in 2D materials.

Keywords: two-dimensional materials; transition metal dichalcogenides; monolayer; VSe$_2$; charge density waves; scanning tunneling microscopy; band structure

Charge order in crystalline materials typically manifests as a static modulation of electron density, known as a charge-density wave (CDW), accompanied by periodic modulations of the atomic lattice [1]. The prototypical CDW arises in (quasi-)one-dimensional (1D) systems from the “nesting” of parallel Fermi surface (FS) regions connected by the CDW propagation vector $Q_{\text{CDW}}$. As real materials do not exhibit perfect nesting, CDW formation is supported by either electron-phonon coupling (EPC), other collective excitations, or electron-electron interactions [2–7]. In layered materials, CDWs often exist in proximity to other ordered phases, e.g. superconductivity and magnetism [8], due to a precarious balance between competing interactions. Approaching the two-dimensional (2D) limit enhances the potential for such interplay [9], while providing new knobs to tune electronic phases, such as electric fields and strain [10–13]. Notably, electron-electron interactions in the 2D limit are expected to induce competition among different CDW driving mechanisms as well as other ordered states [14–16]. In practice, however, a crossover towards electronic charge order driven by dimensional reduction remains to be discovered.

Transition metal dichalcogenides (TMDCs) are well-studied hosts of conventional and unconventional CDWs [4–6, 8, 17]. The tunability of CDWs in the ultrathin limit of several TMDCs is particularly relevant to practical electronic applications [11–13, 18, 19]. 1T-VSe$_2$ is a prototypical metallic TMDC with layered hexagonal crystal structure (1a). Bulk 1T-VSe$_2$ is paramagnetic, with a three-dimensional (3D) FS. Below temperature $T_{\text{bulk}} \sim 110$ K, it hosts a triple-$Q$ (triangular) CDW with 3D character. The CDW periodicity $\lambda_{\text{CDW}} \approx 4a \times 4a \times 3c + \delta$ is commensurate with the in-plane lattice constant $a$, but incommensurate with the inter-layer distance $c$, and corresponds to a weakly nested FS region, supported by a structured EPC [7, 20]. For thicknesses below 20 nm, the FS of 1T-VSe$_2$ transitions to 2D character, while maintaining triple-$Q$, 4a CDW order [21].

Meanwhile, monolayer (ML)-VSe$_2$, grown epitaxially in several recent works, purportedly hosts a ground state with concomitant charge and spin orders, the nature of which is controversial [22–24]. First, while some claim 4a CDW to be absent even at low temperatures [22, 24], others indicate its persistence to well above room temperature [23]. Second, several works report incommensurate superstructures with varying periodicities, viz. $\sqrt{3}a \times 2a$, $\sqrt{3}a \times \sqrt{7}a$, and $\sim 2a \times 3a$ [22–25], whose purported origins vary from structural distortions to nested CDWs. The relation of all these superstructures – identified via electronic density distributions over small real space re-
FIG. 1. Crystal structure and STM Imaging of Ultrathin VSe_{2}. (a) Top view (top) and side view (bottom) of the atomic structure of 1T-VSe_{2} on graphite substrate. (b) Large field-of-view STM topograph (150 × 150 nm^2, bias voltage, V_{tip} = 0.7 V, current setpoint, I_{set} = 100 pA) of MBE-grown epitaxial VSe_{2} on HOPG substrate (see Methods for details). The average thickness of 1.5 VSe_{2} layers resulted in monolayer (ML) and bilayer (BL) VSe_{2} regions identified within the image. Inset shows the height profile across layers, with the respective step heights indicated. (c-d) Atomically resolved STM topographs (5 × 5 nm^2, V_{tip} = −0.2 V, I_{set} = 200 pA) of BL (c) and ML (d) VSe_{2} at 78 K. Both topographs show a hexagonal lattice with visibly distinct superstructures.

RESULTS AND DISCUSSION

STM Imaging Experiments

Thin films of VSe_{2} were grown using molecular beam epitaxy (MBE) on highly oriented pyrolytic graphite (HOPG) and MoS_{2} substrates under ultrahigh vacuum conditions (see Methods). Both substrates are known to stabilize the 1T polymorph of VSe_{2} [22] whose crystal structure is shown in 1a. The films were characterized in-situ using STM over temperatures of 77-200 K (see Methods). As shown in 1b, controlled growth of an average thickness of 1.5 layers resulted in the formation of both ML- and BL-VSe_{2} regions (on HOPG) within fields-of-view accessible to STM imaging. Topographic characterization of a terraced region at 78 K (1b: inset) reveals step heights of 0.9 nm and 0.6 nm for the first and second VSe_{2} layers respectively, in line with values reported previously [23].

1c-d display atomic resolution topographs obtained in the BL and ML regions, respectively. As expected, both cases show the expected hexagonal arrangement of atoms with lattice constant, a ≈ 0.34 nm [22, 23]. Meanwhile, the atomic-scale superstructures seen on ML- and BL-VSe_{2} appear starkly different. For BL-VSe_{2} (1c), the superstructure is tri-directional, i.e. it manifests along all three lattice directions with a single lengthscale. The overall phenomenology is remarkably similar to that of the triple-Q CDW reported in bulk and thinned 1T-VSe_{2} crystals [21]. In contrast, for ML-VSe_{2} (1d), the superstructure appears unidirectional, and has multiple lengthscales, consistent with recent results reported by other groups [22, 23]. Crucially, complementary imaging of the ML using non-contact atomic force microscopy
under similar conditions shows no corrugations beyond those of the atomic lattice (see Supporting Information (SI) §S2), which rules out structural distortions. Therefore, we conclude that the superstructures observed in STM imaging of ML-VSe$_2$ must be of electronic origin, and putatively regard them as CDWs.

In light of conflicting reports on the CDW phenomenology in ultrathin VSe$_2$, we systematically examine in 2 the Fourier space modulations from larger STM topographs obtained for both BL and ML cases. For BL-VSe$_2$, 2b shows the Fourier transform (FT) of a typical STM topograph. Here we find prominent peaks at $Q_1 \simeq 0.25a^\ast$ (green circles), where $a^\ast$ is the magnitude of the reciprocal lattice vector, with $C_6$ symmetry, i.e. along all three Bragg directions. Meanwhile, the anisotropy of Bragg peak intensities may indicate either local uniaxial strain within the sample, or asymmetry in the tip shape. Regardless, these observations are consistent with the triple-$Q$, $4a$ CDW reported in bulk and thinned $1T’-VSe_2$ crystals [21]. In contrast, the FT for ML-VSe$_2$ shown in 2d appears more complex, with only $C_2$ symmetry present. Firstly, c.f. the BL, the ML shows the persistence of the $Q_1 \simeq 0.25a^\ast$ peak (green circle) along a single Bragg direction, corresponding to a single-$Q$, $4a$ CDW. Secondly, the most prominent Fourier peak for the ML is seen at $Q_2 \simeq 0.36a^\ast$ at an angle $\theta_{12} \sim 30^\circ$ relative to the Bragg direction (magenta circle). As shown in 2f, a careful inspection of the FT for the ML suggests that all remaining Fourier peaks can be assigned to higher harmonics or Bragg reflections of $Q_1$ and $Q_2$, including previously reported multiplet superstructures [23–25]. While such superstructures may, in principle, be identified with several distinct wavelengths over small topographic regions, such identifications are not consistent over length scales above 5 nm in any of the reported data [23–25]. Instead, we propose that these apparent supercells are merely the result of superposing two single-$Q$ CDWs, one of which is aligned away from a high-symmetry direction and also incommensurate with the atomic lattice.

**Temperature Dependence**

To further establish the character of CDW(s), we studied the evolution of CDW peaks in BL- and ML-VSe$_2$ with temperature, across both substrates. Notably, the FT of ML-VSe$_2$ recorded at higher temperatures (2e) reveal only a single Fourier modulation with magnitude $Q_2$, as well as its harmonics and reflections. This further evidences the presence of only two principal CDWs – $Q_1$ and $Q_2$ – and suggests that they may have independent origins. At the same time, the slight thermal variation in the direction of $Q_2$ with respect to the lattice shows that the $Q_2$ CDW is not strongly coupled to the lattice. It also suggests a potential interplay between the two CDWs, which may lower the energetic cost of the charge ordered state when harmonics and reflections of $Q_2$ are connected by $Q_1$ (2f). Meanwhile, both BL- and ML-VSe$_2$ grown on MoS$_2$ substrate (see SI §S1) exhibit identical CDW phenomenology to their counterparts grown on HOPG (2), limiting the potential role of substrate-induced strain effects in driving CDW formation.

The thermal evolution of the CDW intensity in STM topographs is an established thermodynamic marker of the CDW transition [29, 30]. In 3, we show representative STM topographs for ML-VSe$_2$ on HOPG for different temperatures (extended dataset in SI §S3). While the data were recorded over varying fields-of-view, we emphasize that, within our experiments, none of the CDWs exhibit any macroscopic spatial variation across atomically smooth regions. For ease of comparison, the CDW peak intensities plotted in 3 are normalized to the corresponding Bragg peak intensities for each STM topograph. Consistently across BL- and ML-VSe$_2$, we find that the intensity of $Q_1$ ($4a$ CDW) drops sharply at $\sim 110$ K to a negligible magnitude, consistent with the thermal evolution of its bulk counterpart [21]. The small, finite magnitude of $Q_1$ in BL-VSe$_2$ at higher temperatures likely arises from small CDW pockets near defects, similar to defect-pinned CDWs at $T \gg T_{CDW}$ reported in other TMDCs [29, 30]. Meanwhile, for ML-VSe$_2$, the intensity of $Q_2$ – in sharp contrast to $Q_1$ – remains sizable well above $\sim 110$ K, and drops to nearly zero at $\sim 140$ K. Finally, no CDW signatures are observed in the 204 K topographs (3c,g), precluding the persistence of either CDW to room temperature [23].

Overall, our systematic analysis sheds much-needed light on the presence, character, and robustness of putative charge order in ML-VSe$_2$ in view of conflicting reports in literature [23, 24, 31–33]. First, our AFM-STM comparison confirms the purely electronic (CDW) origin of all observed superstructures on ML- and BL-VSe$_2$ (c.f. [23]). Second, $T$-dependent experiments conclusively establish the presence of two, and only two, independent single-$Q$ CDWs in ML-VSe$_2$ – $Q_1 \simeq 0.25a^\ast$ (i.e. $\lambda_1 \simeq 4a$) and $Q_2 \simeq 0.36a^\ast$ (i.e. $\lambda_2 \simeq 2.8a$), respectively. The $Q_1$ CDW is identical in magnitude, orientation, and transition temperature to the triple-$Q$ CDW observed in BL-VSe$_2$, and to (the in-plane projection of) the CDW reported in bulk crystals. Meanwhile, the $Q_2$ CDW persists at temperatures well beyond $Q_1$ and exhibits thermal variations in its orientation with respect to the atomic lattice. Finally, the observed consistency of $Q_1$ and $Q_2$ across distinct substrates (c.f.[32, 33]), and of BL-VSe$_2$ with bulk (c.f.[31]), strongly constrain the potential influence of substrate-induced strain effects on the CDW characteristics reported here. To understand the origin of this observed dichotomy in CDW characteristics within the same material, we conduct a detailed examination of the electronic structure of ultrathin VSe$_2$. 
FIG. 2. Comparison of CDWs in BL and ML-VSe$_2$. (a-d) STM topographs (a, c: 10 × 10 nm$^2$, $V_{\text{tip}} = -0.2$ V, $I_{\text{tip}} = 200$ pA) and their respective Fourier Transforms (FTs: b, d) acquired at 78 K on BL- (a-b) and ML- (c-d) VSe$_2$ from adjacent terraces with no observable grain boundary. Dashed colour-coded lines in (a, c) represent the real space CDW wavefronts, and corresponding circles in (b, d) denote the respective CDW wavevectors $Q_1$ (b, d: green) and $Q_2$ (d: magenta), whose magnitudes are indicated in reciprocal lattice units (rlu). Red circles denote atomic Bragg peaks in all FT images. (e) FT of STM topograph acquired on ML-VSe$_2$ at 116 K. Magenta circle denotes the $Q_2$ peak, while the $Q_1$ peak is absent. Dashed lines indicate the orientation of $Q_2$ and its harmonics with respect to the Bragg peak (red circle). (f) Annotated FT of ML-VSe$_2$ at 78 K (c.f. data in d). Green ($Q_1$) and magenta ($Q_2$) circles identify the primary CDW peaks. Color-coded arrows indicate the positions of harmonics with respect to primary and Bragg peaks. All peaks can be accounted for this way.

FIG. 3. Temperature Dependence of CDW Intensities in ultrathin VSe$_2$. Thermal evolution of intensities of the CDW peaks, normalized to the averaged intensities of the six primary Bragg peaks at that temperature, as measured from FTs of 10 × 10 nm$^2$ STM topographs acquired on (a) BL- and (b) ML-VSe$_2$ respectively (dataset in SI §S3, Fig. S3). Error bars show the standard deviation, incorporating the variance in Bragg and CDW peak intensities at each temperature. Insets show STM topographs at selected temperatures for BL (left) and ML (right) respectively.
Band Structure Calculations

Density functional theory (DFT) calculations were performed to investigate the atomic and electronic structure of ultrathin 1T-VSe₂ using the Vienna Ab-initio Simulation Package (VASP, see Methods) [35]. ML-VSe₂ was simulated by requiring the interlayer distance to be 25 Å, and relaxing a 4 × 4 atomic supercell structure, both with and without the symmetry constraints of the underlying P3m1 space group [36]. In both cases, the resulting lattice is purely hexagonal, and free of any structural distortions (c.f. [23]). This further points to the electronic origin of superstructures observed in ML-VSe₂, in line with our experimental findings. Subsequently, the electronic structure was computed, both with and without including spin polarization. The resulting energies are nearly equal for both cases. This suggests, in conjunction with the absence of spin splitting in angle-resolved photoemission spectroscopy (ARPES) results [23, 24, 26, 27, 34, 37, 38], that magnetic order, even if present in ML-VSe₂, is unlikely to play a significant role in the energetics of charge ordered states.

The DFT band structure (4a) is broadly in agreement with the ARPES spectral function measured for ML-VSe₂ [34]. The data in ref. [34] provides a valuable benchmark given its high quality, large momentum range, and qualitative agreement with other ARPES reports, including data acquired on our samples (SI §S4, Fig. S4) [26]. Both techniques find a single band of predominantly d-orbital character crossing the Fermi energy \( E_F \). Previous works have emphasised the importance of the nesting of the sides of the FS lobes at the BZ edge [23, 24, 39]. The DFT electronic structure, however, underestimates \( k_F \) along \( M - K \) and suggests a “nesting vector” along \( a^* \) of length 0.21 rlu. This falls short of the vector extracted from ARPES data (0.54 ± 0.04 Å⁻¹), which corresponds to 0.25 ± 0.02 rlu [27, 34]. The DFT band along \( \Gamma - M \) also appears more dispersive than that in ARPES, while along \( \Gamma - K \) the DFT band is higher (50-200 meV) than the magnitude expected from the high photoelectron count around \( \Gamma \) [23, 24, 26, 27, 34, 37]. These discrepancies are likely due to the inability to duly account for electronic correlations [38]. As a result, our ab initio calculations may not capture the electronic structure near \( E_F \) with sufficient quantitative accuracy to describe CDW energetics.

We therefore complement the DFT calculation with a tight-binding (TB) fit to the ARPES data in ref. [34] (see Methods), the results of which are compared to the DFT in 4. In agreement with reported ARPES spectra, the TB fit shows a flat band region around the \( \Gamma \)-point, an indicator of strong correlations. The difference in topology between the DFT and TB FS (Fig. 4b) is due to the proximity of a van Hove singularity to \( E_F \) [34]. Overlaying the CDW vectors extracted from our STM data onto the FS visually suggests that \( Q_1 \) corresponds to nesting between the sides of neighbouring triangular FS pockets at the BZ edge, while \( Q_2 \) connects the flat-band region around \( \Gamma \) to the pocket corners around \( K \).

Nesting and Correlated Instabilities

A conventional CDW instability at wavevector \( Q_{CDW} \) results from a maximum in its electronic susceptibility \( D_2(q) \) for \( q = Q_{CDW} \) [2, 40]. In the weak electron-phonon coupling (EPC) limit (see Methods), \( D_2(q) \) can be expressed as [40]:

\[
D_2(q) = - \sum_{\mathbf{k} \in \text{BZ}} |g_{\mathbf{k}, \mathbf{k}+\mathbf{q}}|^2 \frac{f(E_{\mathbf{k}}) - f(E_{\mathbf{k}+\mathbf{q}})}{E_{\mathbf{k}} - E_{\mathbf{k}+\mathbf{q}} + i\delta}, \tag{1}
\]

Here, \( f(E) \) is the Fermi-Dirac function, \( E_{\mathbf{k}} \) is the bare (non-renormalized) electronic dispersion, and \( \delta \) is a small regulator (0.1 meV in this work). The EPC matrix elements, \( |g_{\mathbf{k}, \mathbf{k}+\mathbf{q}}| \), are often approximated to unity, resulting in the Lindhard, or bare susceptibility, \( \chi(q) \). However, for TMDCs whose near-\( E_F \) behaviour is governed by d-band(s), several works have established a more realistic approximation to \( |g_{\mathbf{k}, \mathbf{k}+\mathbf{q}}| \) via the electronic band structure [7, 41, 42] (see Methods). Here, we use the TB fit to calculate the bare (\( \chi(q) \)) and structured (\( D_2(q) \)) electronic susceptibilities, which are shown in 5a and b, respectively.

The green circle in 5a(b) indicates the maximum of the bare (structured) susceptibility, which lies at \( Q = (0, 0.28) \approx Q_1 \). Its proximity to a commensurate value suggests that the corresponding CDW will lock to 0.25 rlu (\( \lambda = 4 \)) due to CDW-lattice interactions [4]. Although its periodicity is the same as that of the CDW observed in bulk and BL-VSe₂, the FS for the ML is strictly 2D, and the parts of the FS involved in CDW formation may be different. To elucidate the role of the FS in the observed CDWs, we plot in 5c, d the \( k \)-resolved contributions to \( \chi(q) \) and \( D_2(q) \) for \( q = Q_1 \). As anticipated in 4b, the dominant contributions to \( \chi(Q_1) \) arise from the parallel edges of the K-centred pockets, while the \( \Gamma \)-centred FS region plays a negligible role. The well-nested K-pocket edges with opposite group velocities are therefore inherently unstable to a Peierls-like CDW. The EPC matrix elements further enhance the contribution of these \( Q_1 \)-connected regions to \( D_2(Q_1) \), thereby confirming the conventional origin of the \( Q_1 \) CDW in ML-VSe₂.

In contrast, the phenomenology for \( q = Q_2 \) does not fit the conventional CDW framework. As highlighted by the magenta circles in 5a, b, this wavevector lies in the middle of a susceptibility plateau, and lacks a well-defined maximum. The dominant contribution to the bare susceptibility at \( q = Q_2 \) comes from the \( \Gamma \)-centered flat band region, with smaller contributions from the K-centered pockets (see 5c). However, the corresponding \( D_2(k, Q_2) \)
FIG. 4. **Electronic Structure of ML-VSe$_2$.** (a) Electronic band structure of ML-VSe$_2$ obtained from DFT calculations (red line), compared to published ARPES measurements of the spectral function for epitaxially grown ML-VSe$_2$ at $T = 170$ K (shaded yellow, crosses: peak positions, lines: full width at half maximum). The linewidth of the experimental data greatly exceeds the experimental resolution [34]. Blue line is a tight-binding (TB) fit to the ARPES-measured, near-$_E_F$ band structure, where $E_F$ is the Fermi energy. (b) Fermi surface (FS) of ML-VSe$_2$, obtained from the TB fit in (a) by plotting states within ±1 meV (dark blue) and ±10 meV ($\approx k_bT$ for $T = 100$ K, light blue) of $E_F$. Dotted red line shows the DFT FS, which qualitatively deviates from the TB fit. Hexagon shows the Brillouin zone (BZ), and the arrows indicate FS regions visually appearing to be nested by the experimentally determined CDW wavevectors.

FIG. 5. **Momentum Space Diagnostics of ML-VSe$_2$ CDWs.** (a) The normalised Lindhard susceptibility $\chi$ and (b) structured electronic susceptibility $D_2$ derived from the TB band structure. Circles highlight the positions of the susceptibility maximum $(0, 0.28) \approx Q_1$ (green) and experimentally determined $Q_2 = (0.182, 0.315)$ (magenta), with the latter being located on an intensity plateau. (c-f) Diagnostics indicating the BZ regions contributing to the two susceptibilities $\chi$ and $D_2$, given a chosen wavevector $Q_1$ (c,d) or $Q_2$ (e,f). Blue lines denote FS contours in the right half of the images, while black hexagons indicate the BZ. $Q_1$ (green) and $Q_2$ (magenta) arrows indicate the regions with prominent contributions to the susceptibilities as deduced from the diagnostics.
in 5f shows that the EPC matrix elements strongly suppress the intensity in these regions, and the remaining contributions are insufficient to drive the Q\textsubscript{2} CDW according to an EPC-assisted Peierls scenario. While the perturbative expansion used for the structured susceptibility calculations [7, 40–42] may not fully capture EPC in flat bands, that the origin of the Q\textsubscript{2} CDW lies beyond the Peierls description of CDWs is consistent with its empirical characteristics, viz. varying orientation with respect to the lattice, absence in BLs (and beyond), and the lack of a discernible peak in \( \chi(q) \).

In the 2D limit of layered TMDCs like 1T-VSe\textsubscript{2}, the screening of Coulomb interactions between electrons is much reduced [43]. The relative importance of the unscreened interactions is further enhanced within flat bands associated with a van Hove singularity, such as at the near-\( \Gamma \) region in ML-VSe\textsubscript{2} (4) [23, 34, 37, 38]. Indeed, the measured linewidth, or self-energy, of the band near \( E_F \) is much larger than the experimental resolution [34], supporting the presence of strong electronic correlations [44]. Such interactions can considerably renormalise electron and phonon properties, and enable CDW order at momenta that do not correspond to peaks in the conventional susceptibility (\( \chi(q) \) or \( D_2(q) \)). Indeed, such correlation-driven CDWs have been predicted to exist in TMDCs [8], including in ML-VSe\textsubscript{2} [39], and are consistent with the unusual characteristics of the Q\textsubscript{2} CDW. Crucially, a correlation-driven mechanism for the Q\textsubscript{2} CDW offers the only viable explanation of its prevalence over a well-nested counterpart (Q\textsubscript{1}), and the complete gapping of the FS [23, 34, 37, 38], despite the absence of any associated feature in susceptibility calculations based on models of non-interacting electrons. Further, we conjecture that the single-\( q \) character of the Q\textsubscript{2} CDW, which breaks the three-fold rotational symmetry of the lattice, makes it energetically favourable for the Q\textsubscript{1} CDW (nominally triple-\( q \)) to also order in a single-\( q \) configuration. The interplay of these CDWs could be examined in future theoretical works by iteratively incorporating the resulting lattice distortions.

**CONCLUSIONS**

In summary, our systematic experimental and theoretical efforts elucidate that 1T-VSe\textsubscript{2} undergoes a dimensional crossover as its thickness is reduced to a single layer. While BL-VSe\textsubscript{2}, akin to bulk, hosts a conventional triple-\( Q \) CDW, ML-VSe\textsubscript{2} hosts two distinct single-\( Q \) CDWs with contrasting characteristics. One, with \( \lambda_1 \approx 4a \), behaves similarly to its BL/bulk counterpart, and arises from a weak-coupling Peierls mechanism utilizing nested FS regions. In contrast, the dominant CDW, with \( \lambda_2 \approx 2.8a \), cannot be explained within the conventional EPC-assisted Peierls framework. Instead, the observed thermal evolution and the calculated susceptibility suggest that this CDW – unique to the ML – arises from a flat region of the electronic band structure, where interactions and correlation effects are expected to dominate.

Monolayer VSe\textsubscript{2} stands apart in hosting two coexisting charge orders with distinct physical origins. Conventional electronic materials are typified by the mechanisms and phenomena they host. Our work suggests that ML-VSe\textsubscript{2} transcends such labelling, and hosts coexisting ordered states originating from contrasting coupling mechanisms. The prospect of such emergent electron correlations and ensuing ordered states presenting themselves in 2D TMDCs more generally is particularly promising given their predominance in the plethora of proposed designs for heterogeneous layered materials [10, 45, 46]. Their potential for tunability and their interplay with conventional charge and spin orders in the ultrathin limit is promising for realizing exotic ordered states on one the hand, and for applications in multifunctional electronics on the other.

**METHODS**

**Film Growth.** 1T-VSe\textsubscript{2} films were grown on HOPG substrates in a home-made, ultrahigh-vacuum molecular beam epitaxy (MBE) system, the growth chamber of which has a base pressure of \( 2 \times 10^{-9} \) mbar. The substrate was exfoliated \textit{ex-situ}, immediately transferred into the MBE chamber, and then outgassed at \( 420^\circ \)C for 3 h before MBE growth. The VSe\textsubscript{2} samples were grown \textit{via} simultaneously evaporating V and Se using an electron-beam evaporator and a Knudsen cell, respectively, onto the substrates maintained at \( 360^\circ \)C. The Se/V ratio was high, and Se was controlled to be in excess. A selenium capping layer was deposited onto the VSe\textsubscript{2} surface to prevent direct ambient contamination during \textit{ex-situ} transport to the varying temperature STM/nc-AFM system for subsequent measurements. The capping layer was removed by annealing at \( 240^\circ \)C for 30 min in the microscope chamber.

**STM & AFM Measurements.** STM/nc-AFM measurements were performed over 78-204 K in an Omicron UHV system interfaced to a Nanonis controller equipped with STM/qPlus sensor and an electrical local heater. To reduce thermal drift during data acquisition, the STM was first allowed to stabilise at each temperature. Electrochemically etched tungsten tips were used with bias voltage applied to the tip, while the sample holder was grounded. STM images were acquired using constant current mode. For nc-AFM imaging, the constant-height mode with an oscillation amplitude of 10 nm was used to record the frequency shift (\( \Delta f \)) of the qPlus resonator (sensor frequency \( f_0 \approx 24 \) kHz, \( Q \approx 8000 \)). A lock-in technique was used to measure \( dI/dV \) spectra, with a modulation of 625 Hz and 30 mV.

**DFT Calculations.** 1T-VSe\textsubscript{2} belongs to the space group \( P3m1 \), with the lattice parameters for the monolayer crystal being \( a = b = 3.33 \) Å, \( c = 120^\circ \) [36]. First-principle atomic and electronic structure calculations were performed within the density functional theory (DFT) framework as implemented in the Vienna Ab-initio Simulation Package (VASP) [35] with a plane-wave basis up to a cut-off of...
500 eV. To simulate the monolayer, we artificially set the distance between two layers of VSe$_2$ to 25 Å. The Perdew-Burke-Ernzerhof (PBE) [47] form was used for the exchange-correlation functional. The Γ-centred $k$-mesh was set to be $25 \times 25 \times 1$ in the Brillouin zone for the self-consistent calculation. The relaxation of atomic structure was done in two ways. First, a $4 \times 4$ supercell was relaxed under the symmetry constraints of the space group. This process was then repeated without any symmetry constraints applied.

**Tight-Binding Calculations.** A tight-binding fit was performed for the single $d$-orbital band crossing the Fermi level in the available ARPES data of [34]. To obtain the best fit, we used an expansion of the dispersion $E(k)$ in functions respecting the lattice symmetries. Including terms to fifth order, the fit can be expressed as:

$$E(k) = t_0 + t_1 \left( 2 \cos(\xi) \cos(\eta) + \cos(2\xi) \right)$$
$$+ t_2 \left( 2 \cos(3\xi) \cos(\eta) + \cos(2\eta) \right)$$
$$+ t_3 \left( 2 \cos(2\xi) \cos(2\eta) + \cos(4\eta) \right)$$
$$+ t_4 \left( \cos(\xi) \cos(3\eta) + \cos(5\xi) \cos(\eta) \right)$$
$$+ \cos(4\xi) \cos(2\eta) + \cos(4\xi) \cos(2\eta) + \cos(6\xi) ) ,$$

where $\xi = \frac{k_x}{a}$ and $\eta = \frac{k_y}{c}$, while $k_x, k_y$ are given in units of $\frac{\pi}{a},$ $\pi$ the lattice parameter. $t_i$ are the (in-plane) hopping amplitudes. The best fit to ARPES data based on this form of the dispersion is shown in 4.

**Susceptibility Calculations.** In the limit of weak electron-phonon coupling, the electronic susceptibility can be derived from a perturbative expansion of the phonon propagator, using the random phase approximation (RPA). Neglecting vertex corrections, which should be small [48], the renormalised phonon propagator is described by $D_{RPA} = (D_0^{-1} - D_2)^{-1}$, with bare phonon propagator $D_0$ and electronic susceptibility $D_2$, given by [5, 7, 40]:

$$D_2(q) = - \sum_{k \in \mathbb{BZ}} |g_{k, k+q}|^2 \frac{f(E_k) - f(E_{k+q})}{E_k - E_{k+q} + i\delta} .$$

Here, $f(E)$ is the Fermi-Dirac distribution function, $E_k$ is the non-renormalised electronic dispersion and we use a small regulator $\delta = 0.1$ meV. If the system has an intrinsic or electron-phonon driven CDW instability within the weak-coupling limit, the susceptibility will exhibit a maximum at the CDW wave vector $Q$. Generally, the electron-phonon coupling (EPC) matrix elements $|g_{k, k+q}|^2$ are difficult to compute exactly. For this reason, it is common to set them to unity, resulting in the Lindhard function:

$$\chi(q) = - \sum_{k \in \mathbb{BZ}} \frac{f(E_k) - f(E_{k+q})}{E_k - E_{k+q} + i\delta} .$$

In previous works, it has been shown that the EPC matrix elements can be approximated based purely on the electronic dispersion. This approximation has been well-tested for transition metal compounds with $d$-orbital character at $E_F$ [7, 41, 42]. In the case of a single band crossing $E_F$, the expression becomes:

$$g_{k, k+q} \propto \frac{\partial E_k}{\partial k} - \frac{\partial E_{k+q}}{\partial k} .$$

The orientation of $g_{k, k+q}$ indicates the direction of phonon polariation. We consider longitudinal CDWs, such that the relevant component of the EPC vector is parallel to the in-plane phonon momentum: $g_{k, k+q} = g_{k, k+q} \cdot q_i/|q_i|$. 

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**Supporting Information.** Supporting data on MoS$_2$ substrate, comparison of STM- and AFM-imaging of ML-VSe$_2$, full dataset for temperature-dependence presented in manuscript, and comparison of tight-binding fit to ARPES data.

**Author Contributions.** R.C., Y.L.H., J.G., and X.H. performed the experiments and analysed the data. S.S. and T.D. performed the ab-initio calculations. J.H. performed the tight-binding and susceptibility calculations. J.v.W., A.S., and A.T.S.W. coordinated and supervised the work. All authors discussed the results and provided inputs to the manuscript.

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Supplemental Information

S1. STM DATA ON MoS

FIG. S1. STM Imaging of Ultrathin VSe$_2$ on MoS$_2$ Substrate. (a) Large-scale STM topograph ($200 \times 200$ nm$^2$; $V_{tip} = 2.4$ V, $I_{tip} = 100$ pA) of ultrathin 1T-VSe$_2$ grown on 2H-MoS$_2$. Inset: line profile showing that the first VSe$_2$ layer is 7.8 Å above the substrate. (b-e) Atomic resolution low-temperature) STM topographs ($10 \times 10$ nm$^2$) of BL- (b) and ML-VSe$_2$ (c) at 78 K, and their respective Fourier transforms (FTs). $I_{tip} = 180$ pA, (b, d)$V_{tip} = 10$ mV (b), -10 mV (d).

The STM data reported in the manuscript is acquired on VSe$_2$ grown on a HOPG substrate. Samples were grown similarly (by MBE, see Methods) on a 2H-MoS$_2$ substrate [28]. A large-scale STM topograph, shown in S1a, indicates that the first VSe$_2$ layer lies 7.8 Å above the substrate. The remaining panels in S1 show atomic resolution zoom-ins of defect-free regions corresponding to BL- and ML-VSe$_2$. The slight lattice mismatch between VSe$_2$ and the substrate generates a hexagonal moiré superstructure, which is clearly visible in the topographs (Fig. S1b-c). The new supercell consists of 17 $\times$ 17 VSe$_2$ unit cells atop 18 $\times$ 18 MoS$_2$ unit cells [28]. In contrast, no moiré patterns was observed for ultrathin VSe$_2$ on HOPG for any setpoint. Meanwhile, the Fourier transforms (FTs) of the zoomed-in topographs demonstrate, however, that aside from the difference in moiré intensity, the same CDW peaks are present for VSe$_2$ grown on 2H-MoS$_2$ as seen for VSe$_2$ grown on HOPG (see manuscript Fig. 1-2). That is, the BL hosts a triple-$Q$, $4a$ CDW phase just like the bulk (green circles in S1c), while the ML hosts two unidirectional CDWs with $Q_1 = 0.25a^*$ and $Q_2 = 0.36a^*$ (green and magenta circles, respectively).
Supplemental Information

S2. COMPARISON OF STM AND AFM IMAGING OF ML-VSe$_2$

FIG. S2. STM and AFM topographs of ML-VSe$_2$. Comparison of atomic resolution topographs (4 × 4 nm$^2$) of ML-
VSe$_2$ on HOPG at $T = 78$ K acquired by (a) STM ($V_{tip} = -0.7$ V, $I_{tip} = 220$ pA), and (b) non-contact AFM techniques.
Superstructures are visible in (a), but not in (b).

In ref. [23], it was suggested that the additional superstructures seen in STM FTs beyond $Q_1$ (identified in our
case as $Q_2$) was purely due to structural distortions of the atomic lattice. If this is the case, one would expect the
deformation to also be visible in non-contact atomic force microscopy (nc-AFM) topographs acquired under similar
sample conditions as STM (see Methods). As we demonstrate in S2, however, ac-AFM measurements at 78 K show
ML-VSe$_2$ to be atomically flat within the experimental resolution. This, in combination with the thermal rotation
of the orientation of $Q_2$, and the lack of evidence for structural instabilities in our $ab$-initio atomic relaxation (see main
text), allow us to interpret $Q_2$ as a charge density wave (CDW). Meanwhile, the expected magnitude of the Peierls
distortion associated with CDWs is $\sim 10^{-2} a$, $i.e.$ well below the resolution of available microscopy techniques.
S3. TEMPERATURE-DEPENDENT STM DATA OF BL- AND ML-VSe$_2$

FIG. S3. Temperature-dependent STM imaging of BL- and ML-VSe$_2$. Atomically resolved STM topographs (approx. 10 $\times$ 10 nm$^2$) and their respective FTs of BL- (top half) and ML-VSe$_2$ (bottom half), acquired at temperatures varying over 78-204 K. Imaging conditions for each topograph are indicated. The data shown is used in Fig. 3 of the main text.
S4. COMPARING TIGHT-BINDING FIT TO ARPES

FIG. S4. Comparing TB fit to ARPES spectrum. Azimuthally averaged ARPES spectrum acquired on an ultrathin VSe$_2$ sample grown on HOPG, reported previously [26]. The sample is from the same batch as that used for the STM measurements reported in the manuscript. Lines show TB fits for a sequence of $k$-directions in steps of 10$^\circ$, from along $\Gamma - M$ (dark blue) to $\Gamma - K$ (white).

The tight-binding (TB) model used in the susceptibility calculations (main text) was fitted using a least-squares fit procedure to the peak positions of the ARPES spectrum of ML-VSe$_2$ on bilayer graphene reported in ref. [34]. To ensure that the fitted spectrum is also consistent with the samples used in the present study, we compare our tight-binding fit to the ARPES spectrum of a sample from the same batch of ML-VSe$_2$ grown on HOPG, previously reported in ref. [26]. As the macroscopic ARPES beam averages over ML-VSe$_2$ grains of multiple orientations, the resulting spectrum corresponds to an azimuthal average over $k(x,y)$-space. While this limits a direct experimental determination of the $k$-resolved Fermi surface for the samples studied by STM, here we compare the tight-binding fit band structure with the azimuthally averaged ARPES data. To account for the azimuthal averaging of this spectrum, we plot the tight-binding spectrum (S4) for a sequence of different $k$-directions, rotated 10$^\circ$ from one another, lying along $\Gamma - M$ (dark blue) to $\Gamma - K$ (white). As can be seen in the figure, the tight-binding spectra and ARPES spectra are in good agreement.