An Environment for the Exploration of Non Monotonic Logic Programs

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February 1, 2008

Abstract

Stable Model Semantics and Well Founded Semantics have been shown to be very useful in several applications of non-monotonic reasoning. However, Stable Models presents a high computational complexity, whereas Well Founded Semantics is easy to compute and provides an approximation of Stable Models. Efficient engines exist for both semantics of logic programs. This work presents a computational integration of two of such systems, namely XSB and SMODELS. The resulting system is called XNMR, and provides an interactive system for the exploration of both semantics. Aspects such as modularity can be exploited in order to ease debugging of large knowledge bases with the usual Prolog debugging techniques and an interactive environment. Besides, the use of a full Prolog system as a front-end to a Stable Models engine augments the language usually accepted by such systems.

1 Introduction

Stable Model Semantics (STABLE) has been shown to be very useful in several applications in the field of non-monotonic reasoning. However, computing the stable models semantics is computationally expensive. In fact, it
has been shown that determining whether a given propositional program has
a stable model is already an NP-complete problem [7]. Well Founded Semantics (WFS) provides an interesting alternative for non-monotonic applica-
tions, presenting an approximation of stable models with a much lower com-
plexity. Efficient engines exist that implement both semantics for (classes of)
logic programs. XSB is a Prolog system that relies on a tabling mechanism to
implement the Well Founded semantics of normal logic programs. SMODEL-
ELS implements Stable Model semantics for a special class of programs,
called range-restricted programs, that impose restrictions on variables and
terms appearing in clauses.

In this paper, we present a computational integration of Well Founded
and Stable Model semantics by combining XSB and SMODELS into one
system called XNMR. XNMR aims at providing an interactive environment
for the exploration of non-monotonic semantics of logic programs. The goal
is to provide an easy-to-use environment to study new possibilities in the
joint use of STABLE and WFS.

One important feature of XNMR is its interactive nature. By allowing
the user to interact with the system and experiment with different queries
and semantics, she can obtain a better and deeper understanding of her
program or knowledge base. A related application of XNMR is in debugging
knowledge bases and systems. Additionally modularity can be exploited for
easy experimentation with large systems.

The remaining of the paper is divided into three parts. The next section
presents the definitions of both Stable Model semantics and Well Founded
semantics in the common framework of partial Stable Models. Section 3
presents implementation issues in XNMR, along with a brief introduction
of the characteristics of the systems it is based on, XSB and SMODELS.
Finally, Section 4 presents XNMR from the point-of-view of the user, pre-
senting its capabilities and features, along with some caveats resulting from
the integration of STABLE and WFS.

2 Basic Definitions

The system presented in this paper combines two engines that compute dif-
erent semantics for logic programs, namely Well Founded semantics (WFS)
and Stable Model semantics (STABLE). These semantics are related in that
they can both be defined in terms of Partial Stable Models. In order to
provide context for some considerations made in other parts of the paper
we present, in this section, the definitions for both semantics, in terms of
partial stable models.

The *model-theoretic* semantics presented here are defined in terms of fixed points of a particular operator.

### 2.1 Partial Stable Models

Partial Stable Models assign three-valued models to logic programs. That is, an atom can be assigned truth values of *true*, *false* or *undefined*.

Before introducing the definition of Partial Stable Models we show the Quotient operator, first introduced in [9]. The Quotient operator extends the Gelfond-Lifschitz transformation ([5]). Given a logic program $P$ and a partial interpretation $I_3$, it defines a unique non-negative program $P' = \frac{P}{I_3}$.

**Definition 2.1 (Quotient Operator)**

Consider a logic program $P$ and a partial interpretation $I_3$ of $P$. The *quotient of $P$ modulo $I_3$* is the new program $P' = \frac{P}{I_3}$ obtained from $P$ by replacing in every clause of $P$ all *negative* premises $\neg C$ which are true (resp. undefined; resp. false) in $I$ by their corresponding truth values $\mathsf{t}$ (resp. $\mathsf{u}$; resp. $\mathsf{f}$).

The Quotient operator allows one to obtain a non-negative program from any given logic program and an interpretation. It is possible to compute the least partial model of such a program by computing the least fixed point of the $T_3P$ operator. The $T_3P$ operator is an extension of the $T_P$ operator over three-valued interpretations.

**Definition 2.2 (Immediate consequence operator $T_3P$ ([9]))**

Given $P$ a definite logic program, and $I_3$ a partial interpretation of $P$. $T_{3P}(I_3) = (\mathsf{Pos}, \mathsf{Neg})$ is the partial interpretation where:

- $\mathsf{Pos} = \{ A \iff \exists (A \leftarrow B_1, ..., B_n) \in P \mid \forall i, B_i \in \mathsf{Pos}(I) \}$
- $\mathsf{Neg} = \{ A \iff \forall (A \leftarrow B_1, ..., B_n) \in P, \exists i \mid B_i \in \mathsf{Neg}(I) \}$

The $T_{3P}$ operator can be used to obtain the least partial model (LPM) of a program.

**Definition 2.3 (Least Partial Model ([9]))**

If $P$ is a non-negative program, then the operator $T_{3P}$ has a least fixed point which coincides with the least partial model $\mathit{LPM}(P)$ of $P$, i.e., $\mathit{LPM}(P)$ is the least partial interpretation (w.r.t. the truth ordering $\preceq$) $I_3$ such that $T_{3P}(I_3) = I_3$. 

By combining the three previous operators, we define the Φ operator that, for a given logic program \( P \) assigns to each partial interpretation \( I_3 \) the least partial model of the quotient program \( \frac{P}{I_3} \).

**Definition 2.4 (Φ\(_P\) operator)** Given a program \( P \) and a partial interpretation \( I_3 \) of \( P \), we define \( Φ_P(I_3) \) as a new partial interpretation given by:

\[
Φ_P(I_3) = LPM(\frac{P}{I_3})
\]

Partial stable models are defined as fixed points of the Φ operator.

**Definition 2.5 (Partial Stable Models)** A Partial Stable Model \( PSM(P) \) of a logic program \( P \) is a partial interpretation \( I_3 \) which is a fixed point of the \( Φ \) operator.

\[
I_3 = LPM(\frac{P}{I_3})
\]

\[
PSM(P) = I_3
\]

### 2.2 Well Founded Semantics

The Well Founded Semantics assigns to a logic program \( P \) a three-valued model. The original presentation of WFS ([15]) provides a constructive definition based on an iterated least fixed point. However, Przymusinski [9] has shown that WFS can be equivalently defined in terms of Partial Stable Models. This is the approach used here.

Two orderings can be defined over partial interpretations. The *truth ordering* \( \preceq \) maximizes the number of true atoms, whereas the *information ordering* minimizes the amount of undefined atoms.

**Definition 2.6 ([10])** Given two partial interpretations \( I_3 = \langle Pos_I, Neg_I \rangle \) and \( J_3 = \langle Pos_J, Neg_J \rangle \), we define:

\[
I_3 \preceq J_3 \iff Pos_I \subseteq Pos_J \land Neg_I \supseteq Neg_J;
\]

and

\[
I_3 \subseteq J_3 \iff Pos_I \subseteq Pos_J \land Neg_I \subseteq Neg_J.
\]
The *information ordering* is equivalent to the set-theoretical *inclusion* ordering. The Well Founded Model of a program is given by the least Partial Stable Model with respect to the *information ordering*.

**Definition 2.7** ([]) Well-Founded Semantics] The well-founded model of a program $P$ is the $\subseteq$-minimal least fixed point of the $\Phi$ operator.

\[
WFS(P) = \min_{\subseteq} \{ I \mid \Phi(I) = I \}
\]

One important characteristic of Well Founded Semantics is that it provides a unique model for each program.

### 2.3 Stable Model Semantics

The Stable Model Semantics assigns, to a given logic program $P$, zero or more two-valued models. Programs that contain contradictions have no models; stratified programs have unique models, and non-stratified programs may have several models. $\text{STABLE}$ is defined as the fixed points of the $\Phi$ operator which are total, that is, those in which every atom occurs either in the $\text{true}$ set or in the $\text{false}$ set.

**Definition 2.8** ([]) The Stable Models of a program $P$ are given by a set of the interpretations that are fixed points of the $\Phi$ operator and in which no atom is undefined.

\[
\text{STABLE}(P) = \{ I \mid I = \Phi_P(I) \land \text{atoms}(P) \setminus (\text{Pos}(I) \cup \text{Neg}(I)) = \emptyset \}
\]

An interesting observation is that, if the set of Stable Models for a given program $P$ is not empty, than all models in the set are *consistent* with the Well Founded model of $P$. That is, all atoms that are assigned $\text{true}$ or $\text{false}$ in $\text{WFS}$ have the same assignments in all stable models; the different stable models differ only on the assignments to those atoms which are *undefined* under $\text{WFS}$.
3 Implementation of the XNMR System

In this section we present SMODELS [8], a Stable Models computation engine, and XSB [12], a Well Founded Semantics-based Prolog system, along with the issues regarding their integration. We describe how XSB can be used as a partial evaluator for programs before stable models are computed. The result of this partial evaluation, called the residual program, is the glue used to combine XSB and SMODELS. The architecture of the XNMR system, consisting of a two-layered model, is also presented.

3.1 SMODELS

SMODELS is an implementation of an evaluator for the stable model semantics for range-restricted programs. It is composed of two modules, namely a program grounder, and the stable models computation engine.

The program grounder, called lparse [14], computes a grounded version of a range-restricted normal program. All variables must have their domains specified by the introduction of special domain predicates in every clause of the program. This disallows the expression of more general programs, where values of variables are dependent on the input, thus known only at runtime. Also, complex structures and lists are not allowed, constraining the flexibility of the language. One evidence of this constraint is the impossibility to define a simple predicate like append/3, that appends two lists, creating a new one. The grounder is optimized so as to not create the whole set of ground instances of the program, but rather a subset of these that is sufficient to ensure that no stable models are lost.

The stable models computation engine uses a bottom-up backtracking search along with a pruning method to implement an efficient algorithm which has a polynomial space complexity. The pruning method is based on an approximation technique for stable models which is closely related to the well-founded semantics [8].

SMODELS also provides an application program interface (API) that allows it to be used as a library called from other programs. This API provides access to the functionalities of the SMODELS evaluation engine alone, without the grounder capabilities. This functionality is exploited in the system presented in this paper.
3.2 XSB

The XSB system is, arguably, the most efficient and well-known implementation of the well-founded semantics. XSB implements a fast WFS computation engine based on SLG resolution. SLG resolution is performed by successively applying specific operations which effectively implement WFS semantics for logic programs.

The implementation of XSB is based on the use of the tabulation, or memoization technique. For each tabled predicate found during the evaluation process, a table is created where the computed answers for this call are kept. This ensures that loops are easily detected. The fixpoint check needed to compute the well-founded semantics is realized by a completion procedure, which detects when all possible solutions to a predicate have already been computed.

The language accepted by XSB is an extension of full Prolog, thus allowing the use of the full power of logic variables and unrestricted structures. Standard predicates like, for instance, append/3, which appends two lists, can be easily programmed by using its usual definition. Also, the interactiveness of the system allows the user to work with predicates with infinite models by backtracking through the answers.

XSB, as a side effect of the well-founded model of a program, produces a residual program for the given query. The residual program is a concretization of the mutual interdependencies among the undefined atoms in the well-founded model.

3.3 Residual Program

In XSB, undefined atoms are obtained from conditional answers. An answer is said to be conditional if its set of delayed literals is not empty, and no operation is applicable. The concretization of the delayed literals into clauses results in a new program, called the residual program. The residual program is a representation of the interdependencies among the undefined atoms in the well-founded model of a program, where all true and false literals in the well-founded semantics are appropriately resolved away.

Example 3.1 Consider the program P given in Figure 4. The Well Founded model for P says that the set of positive atoms is \{s, t\}, the negative atoms are \{r\} and \{p, q, u\} are undefined. Suppose the user asks the query q(a). Figure 3 shows how the residual program for the query can be obtained by collecting the delayed literals for the query and recursively for the delayed literals themselves.
Figure 1: Example for residual program computation

Figure 2: Inspecting the residual program
The residual program, shown in Figure 3 is constructed by creating clauses associating each literal with its set of delayed literals.

Example 3.1 shows what kind of information a residual program carries.

\[
\begin{align*}
q(a) & :- \texttt{nt}(p(a)). \\
q(a) & :- u(a). \\
p(a) & :- \texttt{nt}(q(a)). \\
u(a) & :- \texttt{nt}(u(a)).
\end{align*}
\]

Figure 3: Residual Program

3.4 XNMR

In Section 2.3 we noted that, when the set of Stable Models for a program is not empty, they differ only in those atoms which are undefined under WFS. A residual program represents the relationship amongst the undefined atoms in a program under WFS. So it seems natural to use the residual program of a given logic program in order to compute its stable models. That is exactly the feature exploited in XNMR.

XNMR provides an interactive system for the exploration of both Well Founded and Stable Model semantics of logic programs. It uses XSB to implement a front-end interface for SMODELS. XSB replaces the grounder of SMODELS, augmenting it with an interactive top-level system similar to normal Prolog environments. Also, XSB provides a strictly more powerful grounder \(^1\) for SMODELS, since it allows the use of complex structures in the program like, for example, lists. These structures are grounded on demand according to the query given. In most cases, results for queries in the well-founded semantics are already ground. In those cases in which that is not true, variables are skolemized by XNMR, which is a semantically sound operation.

One drawback is related to the use of variables in negative literals. In XSB, a negative goal with an unbound variable flounders, whereas in SMODELS, it is easy to deal with this problem since variables have their domains specified. The system also provides an application program interface (API) that can be used by Prolog programs to interact with SMODELS.

XNMR is implemented by exploiting XSB’s foreign predicate interface \(^1\) and SMODELS’ API. These are connected by a small amount of C glue

\(^1\)Provided that literals are ordered so as to be consistent with the left-to-right evaluation strategy of Prolog.
code layer that interfaces both systems. The basic functionality is complemented by a collection of Prolog predicates that automate the collection of the residual program from the internal information in XSB and convert it into a representation required by SMODELS. The user interface is implemented by a top-level shell very similar to those of normal Prolog systems, but with extended capabilities.

Figure 4 shows the conceptual architecture of the XNMR system. Both XSB and SMODELS are integrated by the Integration and API module which also provides a basic API to higher level layers. The Top-level shell is built upon this API and provides the main interface to the user, by means of an extended Prolog-like shell.

During the conversion phase, the residual program is collected from the conditional answers in the tables, and each atom is grounded and converted to a numerical value, as required by SMODELS. The grounding is necessary because conditional answers may contain variables. These variables are skolemized by binding them to new constants not appearing in the program otherwise. After this converted program is passed to SMODELS, and its stable models are received back, the conversion phase translates the results back in terms of the original program, so that they can be displayed to the user.
4 Functionalities of the XNMR System

As shown in the previous section, XNMR combines XSB and SMODELS using a two-layered architecture. In this section, we describe these layers from the point of view of the user. We focus mainly in the Top-level shell, which aims at providing an interface for the end-user. We also briefly discuss the programmatic interface provided by XNMR’s API.

4.1 The Interactive Top-Level Shell

The XNMR top-level shell module replaces the standard interactive mode of XSB with a specialized shell. The XNMR shell mimics standard Prolog environments, adding features to those common in such systems. These features include the displaying of delayed literals in undefined answers to queries, and the possibility of incremental exploration of stable models for the resulting residual programs.

The prompt works much like a standard Prolog top-level. The difference is that, when the user issues a query whose value under WFS is undefined, the system shows the delay list for the query. In this case, the user can either ignore the delay list and assume the query (with the given instantiations for the variables) is undefined under the well-founded semantics, or it can ask XNMR for further information on the query.

Three options are available, which are bound to different keys:

’s’ computes and displays all stable models of the residual program, one at a time, by backtracking through them;

’t’ computes and displays those stable models of the residual program where the query is true, also backtracking through them, if more than one exists; and

’a’ checks whether the query is true in all stable models of the residual program.

Figure 5 shows a simple session of the XNMR shell. A simple program which consists of two atoms that depend negatively on each other is consulted, and the truth value of one of these atoms is queried. First, XNMR responds with a delay list, which means that the atom is undefined under WFS. The user is then prompted for a subsequent action, and the three possibilities are shown. The first returns the two stable models of the relevant program, one at a time. The second option displays only that model
\$ xsb xnmr
[xsb_configuration loaded]
[sysinitrc loaded]
[packaging loaded]
[sModels loaded]

XSB Version 2.4 (Bavaria) of July 13, 2001
[i686-pc-linux-gnu; mode: optimal; engine: chat; gc: copying; scheduling: local]
nmr| ?- [user].
[Compiling user]
:- table p/0, q/0.
p :- tnot(q).
q :- tnot(p).
end_of_file.
[user compiled, cpu time used: 0.2210 seconds]
[user loaded]

yes
nmr| ?- p.

DELAY LIST = [tnot(q)] ? s

Stable Models:
   \{q\} ? ;
   \{p\} ? ;
   no

no
nmr| ?- p.

DELAY LIST = [tnot(q)] ? t

Stable Models:
   \{p\} ? ;
   no

no
nmr| ?- p.

DELAY LIST = [tnot(q)] ? a
   no

no
nmr| ?- halt.

Figure 5: Simple session of XNMR
in which the atom queried is true. And, finally, the system says *no* for the third option, meaning that the atom is not true in all stable models of the relevant program.

```
xsb version 2.4 (bavaria) of july 13, 2001
[i686-pc-linux-gnu; mode: optimal; engine: chat; gc: copying; scheduling: local]
nmr| ?- [user].
[compiling user]
:~ table p/0, q/0, r/0.
p :- tnot(q).
q :- tnot(p).
r :- p, tnot(r).
end_of_file.
[user compiled, cpu time used: 0.2590 seconds]
[user loaded]

yes
nmr| ?- p.

delay list = [tnot(q)] ? s

stable models:
{q} ? ;
{p} ? ;
no

no
nmr| ?- halt.

end xsb (cputime 0.47 secs, elapsetime 54.24 secs)
```

Figure 6: Relevancy of XNMR results

Figure 6 shows an example of a possibly unexpected result of the XNMR system. The shown program has only one model under STABLE, namely the one in which `q` is true, and `p` and `r` are false. But by computing the residual program of the query `p`, only the relevant part of the program is considered, so two stable models are returned. It is important to note that XNMR computes partial stable models of the residual program relevant to a given query. That means that, as shown above, it is not guaranteed that every model computed by XNMR corresponds to a total stable model of the original program. For instance, the program may contain *inconsistencies* that force it to have no stable models. Still, if these inconsistencies are not relevant to a given query, XNMR may return stable models for the residual
program that correspond to partial stable models of the whole program in which the atoms involved in the inconsistencies are deemed undefined. Another trivial example is provided in Figure 7. The program has only one stable model, namely the one shown for the query \( r \). That is due to the inconsistency created by the third clause if we take \( p \) to be true. This effectively forces \( p \) to be false in all stable models. But if we ask the query \( p \) to XNMR, as shown in Figure 8 for the same program, the third clause is not considered when collecting the residual program, thus two models are computed.

```
nmr| ?- [user].
[Compiling user]
:- table p/0, q/0, r/0.
p :- tnot(q).
q :- tnot(p).
r :- p, tnot(r).
end_of_file.
[user compiled, cpu time used: 0.0610 seconds]
[user loaded]

yes
nmr| ?- r.

DELAY LIST = [p,tnot(r)] ? s

Stable Models:
{q} ? ;
  no
no
nmr| ?-
```

Figure 7: Non-relevant inconsistency

This issue of non-relevancy in Stable Model Semantics is well known, and partly responsible for the non-existence of goal-directed stable model computation engines. A class of programs has been devised that is guaranteed to not suffer from these non-relevant inconsistencies. Programs in this class are called *call-consistent programs* [6, 13]. All models computed by XNMR for queries over such programs correspond to total stable models of the original program.

Even though XNMR may compute more models than there are stable models for a given program, it has been shown in [2] that all stable models of the program are always represented among the answers computed. So, if an
expected answer is not computed by XNMR, the user is guaranteed that is also does not appear in any stable model of the original program. This is an important feature that can be applied in the debugging of large knowledge bases, specially when combined with the modularity that is usually necessary to maintain such large systems. Individual modules can be inspected separately, forcing external predicates to be undefined, for instance.

```
:- table win/2, move/3.
win(X,L) :- move(X,Y,L), tnot(win(Y,L)).
move(X,Y,L) :- member(m(X,Y),L).
```

Figure 8: Win – not win program

The XNMR top-level allows the use of full Prolog in conjunction with Stable Models computation. As a result of that, complex structures can be used that will be instantiated and grounded during WFS computation, before being passed to SMODELS. The integration of STABLE computation in the query-based Prolog top-level allows the inspection of programs that generate possibly infinite structures (or lists) by backtracking, for instance, interactively until the user feels confident that the program is computing the desired result. Additionally, the interactivity of the query-based shell, with unrestricted variables, allows for much more flexible programs. For instance, consider the classic win - not win program over a graph. In XNMR, it can be modeled to have the graph of possible moves encoded as a list, as shown in Figure 8. The size and shape of the graph can be changed by asking different queries, without any modifications to the program. Figure 8 shows some possible queries.

In the cases where the user knows enough about the non-relevant parts of the program to know that they do not interfere in the stable models of the relevant part, she can use XNMR as a fast Stable Models computation system. Otherwise, XNMR computes supersets of the stable models of the original program, restricted to the relevant part with respect to the query given. Alternatively, XNMR can be viewed as a program generator for SMODELS. The program in Figure 8 is an example of such application. The code dynamically builds programs according to the size and shape of the graph passed to it as an argument. This program is then passed to SMODELS for further evaluation.
Figure 9: Different queries against fixed program
4.2 Application Program Interface

The top-level shell described above is implemented as an application of the API provided by the XNMR architecture. The API provides predicates to initialize the XSB/SMODELS interface, automatically extracting the residual program for a given predicate, encoding and sending it to SMODELS. Besides, several control and state-checking predicates are defined. The most significant predicates provided by XNMR’s API are shown below.

- **init_smodes(+Query)** — initializes SMODELS with the residual program for Query; the residual program is collected from the table for Query, grounded if necessary, encoded and sent to SMODELS for further computation.

- **atom_handle(?Atom, ?AtomHandle)** — is set by init_smodes/1 to be true of the set of atoms in the residual program (and thus the Herbrand Base of the Stable Models to be computed). AtomHandle is an integer uniquely identifying the atom. It is also used when decoding results returned from SMODELS.

- **a_stable_model** — invokes SMODELS to find a stable model of the residual program set by the previous invocation of init_smodes/1. Fails if there are no (more) Stable Models. It will compute all stable models through backtracking. Atoms true in a stable model can be obtained by the next predicate.

- **in_current_stable_model(?AtomHandle)** — true of handles of atoms that are true in the current stable model (set by an invocation of a_stable_model/0).

- **current_stable_model(-AtomList)** — constructs a list of all atoms that are true in the current stable model (set by a_stable_model/0).

5 Conclusions

In this work we introduce XNMR, a system that integrates two computation engines for two different semantics of logic programs. These engines are XSB, that computes the Well-Founded semantics of logic programs, and SMODELS, that computes Stable Model semantics of a class of syntactically-restricted logic programs.

The XNMR system provides an interactive environment for the exploration of different semantics of logic programs. The interactive nature invites
uses such as debugging of knowledge bases and programs, and the possibility to work with partial information provided by the Well Founded semantics allows for the management of large systems, exploiting the modularity of such systems.

The combination of the goal-directedness of XSB with the global nature of Stable Model semantics introduces some caveats with respect to the semantics computed by XNMR. There exists a class of programs, called call-consistent programs, for which XNMR computes only models that have a one-to-one correspondence to the Stable Models of the program. Still, we show that, even for programs not in this class, the models computed by XNMR correspond to a class of Partial Stable Models of the whole program, and can be useful in several applications. A major advantage is the greater flexibility provided by the unrestricted variables provided XSB. This allows, for instance, the interactive exploration of predicates with infinite models, and the creation of meta-programs, that created ground programs to be evaluated under the Stable Models semantics.

XNMR is distributed as part of the XSB System in the form of a package that can be compiled when SMODELS is also present in the system. XNMR can be obtained, along with XSB, from http://xsb.sourceforge.net/.

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