Double-logarithmic asymptotics of the magnetic form factor of electron and quark

B.I. Ermolaev
CERN, 1211 Geneva 23, Switzerland
and
S.I. Troyan
St.Petersburg Institute of Nuclear Physics, 188350 St.Petersburg-Gatchina, Russia

The asymptotical behaviour of the magnetic form factor for electron and quark is obtained in the double-logarithmic approximation for the Sudakov kinematics, i.e. for the case when the value of the transfer momentum is much greater than the mass of the particle

I. INTRODUCTION

The interaction of electron and quark with the electromagnetic field is described in terms of two independent form factors $f$ and $g$:

$$\Gamma_\mu = \bar{u}(p_2)[\gamma_\mu f(q^2) - \frac{\sigma_{\mu\nu}q_\nu}{2m}g(q^2)]u(p_1)$$

where $\sigma_{\mu\nu} = [\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu]/2$, $q = p_2 - p_1$ is the momentum transferred to the electron or the quark, $m$ is the electron or the quark mass and both the electric form factor $f$ and the magnetic form factor $g$ depend on $q^2$.

In the Born approximation $f = 1$ and $g = 0$. One-loop radiative corrections [1] show that the form factor $f$ depends on the infrared cut-off as well as on the ultraviolet one. In contrast, the form factor $g$ in the one-loop approximation is both ultraviolet- and infrared- stable. As the magnetic form factor $g$ contributes to the value of the anomalous magnetic momentum, it has been calculated with great accuracy by direct graph-by-graph calculations in the case when $q^2 = 0$. The most recent review of such results is given in [2]. Meanwhile, the electric form factor $f$ was calculated many years ago in the “opposite” kinematical region of very large transferred momenta:

$$-q^2 \gg m^2, \quad (2)$$

in the leading logarithmic approximation (LLA), where the most important, double logarithmic contributions to all orders in the QED coupling $\alpha$ for the electron are taken into account. The sum of such contributions, the double logarithmic (DL) asymptotics for the electric form factor of the electron in the kinematical region (2) is

$$f = \exp\left[-\frac{\alpha}{4\pi} \ln^2(-q^2/m^2)\right] \quad (3)$$

The famous expression (3), obtained by V.V. Sudakov [3] was actually the birth of the approach that is so popular at present – the double logarithmic approximation (DLA) – where only the leading contributions $\sim (\alpha \ln^2(-q^2))^n$ are taken into account to all orders of the perturbation series. The generalization of the Sudakov form factor of Eq. (3) to quarks of QCD obtained in [4] amounts to replacing $\alpha$ by $\alpha_s C_F$ in Eq. (3), where $C_F = (N^2 - 1)/2N = 4/3$ for the colour group SU(3).

The exponential fall in Eq. (3) as $-q^2$ increases corresponds to a suppression of the non-radiative hard scattering of an electron by a virtual photon. The amplitude taking into account the bremsstrahlung of $n$ “soft” photons was shown in [5] to be the product of independent factors:

$$f_n = B_1 B_2 \ldots B_n f(q^2), \quad (4)$$

where the bremsstrahlung factors $B_i$ are given by (we drop the QED coupling here)

$$B_i = \frac{p_2 l_i}{p_2 k_i} - \frac{p_1 l_i}{p_1 k_i} \quad (5)$$

so that $l_i$ is the polarization vector and $k_i$ is the momentum of the $i$ -th emitted photon ($i = 1, \ldots, n$). As $f$ in Eq. (3) does not depend on $k_i$, and each of $B_i$ does not depend on $k_j$ with $j \neq i$, Eq. (3) leads to the Poisson energy spectrum

*Permanent address: A.F.Ioffe Physico-Technical Institute, 194021 St.Petersburg, Russia
for the bremsstrahlung photons in the DLA. The violation of the Poisson distribution in QCD for the emission of the “soft” gluons in the DLA was obtained in \[4\] by calculating the Feynman graphs up to order $\alpha_s^3$. The generalization of the form factor $f$ in Eq. (3) to QCD was given in \[3\].

In the present work we calculate the form factor $g$ for the electron and the quark in the kinematical region (3) in the double logarithmic approximation. Then, using results of \[3\] we obtain relations between the radiative (inelastic) electric and magnetic form factors of electron and quark. Apart from the simplicity and beauty of the expressions we obtain for the complete form of Eq. (1) in the kinematical region of Eq. (3), the reason for publication is that those expressions may be useful in the future for more precise descriptions of electron/quark scattering. In particular, it can be applied to the analysis of electron scattering off so-called magnetic walls, in the distant regions of our Universe, which together with related phenomena are at present under discussion. The paper is organized as follows: in Sect. 2 we calculate the magnetic formfactor of the electron. In Sec. 2 we generalize that result to QCD. Section 3 is devoted to concluding remarks.

II. THE MAGNETIC FORM FACTOR OF THE ELECTRON

In the lowest, one-loop approximation, the magnetic form factor of the electron was calculated long ago in \[4\]. The only Feynman graph yielding the main contribution to $g$ in the kinematics (3) is shown in Fig. 1. The result is \[5\]

$$g^{(1)}(q^2) = -\frac{m^2}{q^2} \frac{\alpha}{\pi} \ln(-q^2/m^2). \quad (6)$$

In order to obtain the leading-log approximation to higher loop contributions to the magnetic formfactor, we start by reproducing this result in a way that will make it easy to generalize.

The Feynman diagram in Fig. 1 corresponds to the expression

$$\bar{u}_2\Gamma^{(1)}_\mu u_1 = e^2 \int \frac{d^4k}{(2\pi)^4} \bar{u}_2\gamma_\mu \left[ \frac{m + \hat{p}_2 - \hat{k}}{m^2 - (p_2 - k)^2 - i\epsilon} \gamma_\mu \frac{m + \hat{p}_1 - \hat{k}}{m^2 - (p_1 - k)^2 - i\epsilon} \right] \gamma_\tau u_1 \frac{-g^{\lambda\tau}}{[k^2 - i\epsilon]}. \quad (7)$$

The integral in Eq. (7) is both ultraviolet- and infrared- divergent. To obtain a physically meaningful result, we must introduce the infrared cut-off and subtract from Eq. (7) its value at $q^2 = 0$.

As we want to obtain only the leading-log approximation (LLA) to the resulting expression, we may use the approach of quasi-real photons with the imposed cut-off $\mu$ on their transverse momenta, \[1\]. Choosing the cut-off parameter in the region

$$-q^2 \gg \mu^2 \gg m^2 \quad (8)$$

one can completely suppress the effects of electron mass, $m \to 0$, to obtain the LLA result as a function of $\ln(-q^2/\mu^2)$. Finally, we can shift $\mu^2$ down to $m^2$ to replace $\mu^2$ by $m^2$ in the obtained LLA expression.

Let us apply this procedure to Eq. (7). In the approach of quasi-real photons, the photon propagator is forced to be on-the-mass-shell,

$$\frac{1}{-k^2 - i\epsilon} \to 2\pi i\delta^+(k^2), \quad (9)$$

and the quark virtualities are restricted by the largest virtuality in the process:

$$2(p_2k) \ll -q^2, \quad 2(p_1k) \ll -q^2. \quad (10)$$

To perform the integrations we decompose the photon momentum in terms of the initial and final quark momenta $p_1$ and $p_2$:

$$k = \alpha p_2 + \beta p_1 + k^\perp, \quad k^2 = \alpha^2 s + (\alpha^2 + \beta^2)m^2 - k^\perp_2 \quad (11)$$

where $s = 2(p_1p_2) = -q^2 + 2m^2 \approx -q^2 \gg m^2$. Taking the integration over $k^\perp$ with the help of the $\delta$-function of Eq. (3) leads the remaining integral over $\alpha$, $\beta$ to have an integrand symmetrical in $\alpha$ and $\beta$. As the cut-off parameter $\mu$ is intentionally chosen to be in the region (5), we can neglect all terms $O(m^2/s)$ in the phase space

$$d^4k = d\alpha d\beta \frac{s}{2} d^2k^\perp$$

(12)
and in the denominators of the quark propagators. In the case of the electric form factor, we can also neglect the photon momentum $k$ in the spinor numerators of the propagators and immediately obtain the DL result:

$$f^{(1)} = -\frac{\alpha_e}{2\pi} \int \frac{d\alpha d\beta}{\alpha \beta} \Theta \left( \alpha \beta - \frac{\mu^2}{s} \right) \approx -\frac{\alpha_e}{4\pi} \ln^2 \frac{q^2}{\mu^2}. \quad (13)$$

To extract the magnetic part of the amplitude $f^{(2)}$, we must not neglect the mass $m$ and the photon momentum in the numerators and also study the spinor structure of Eq. (3) in more detail. Substituting Eq. (13) in the denominators, and using the commutation rules and the Dirac equation, we obtain

$$2\tilde{u}_2 s(1-\alpha)(1-\beta)\gamma_\mu + m(p_1 + p_2)_\mu (\alpha - \alpha^2 + \beta - \beta^2) + m\gamma_\mu (\beta - \alpha + \beta^2 - \alpha^2) + O(m^2, mk_1^2/s) u_1. \quad (14)$$

Here the first term corresponds to the electric form factor. The second term, which might have been violating the charge conservation, vanishes after integration over $\alpha$, $\beta$ because of symmetry. As $k_2^2 \approx \alpha \beta s$, we can neglect the terms $O(k_2^2/s)$ as well as $O(\alpha \beta s)$ because they suppress both logarithmic integrals over $\alpha$ and $\beta$. Therefore for the one-loop contribution to the magnetic form factor we obtain

$$m\tilde{u}_2 (p_1 + p_2)_\mu u_1 = m\tilde{u}_2 \sigma_{\mu \nu} q^n u_1 + 2m^2 \tilde{u}_2 \gamma_\mu u_1, \quad (15)$$

determines the magnetic form factor. The third term, which might have been violating the charge conservation, vanishes after integration over $\alpha$, $\beta$ because of symmetry. As $k_2^2 \approx \alpha \beta s$, we can neglect the terms $O(k_2^2/s)$ as well as $O(\alpha \beta s)$ because they suppress both logarithmic integrals over $\alpha$ and $\beta$. Therefore for the one-loop contribution to the magnetic form factor we obtain

$$g^{(1)} = -\frac{\alpha_e m^2}{\pi q^2} \int \frac{d\alpha d\beta}{\alpha \beta} \Theta \left[ \alpha(1-\alpha) + \beta(1-\beta) \right] \Theta \left( \alpha \beta - \frac{\mu^2}{s} \right) \quad (16)$$

which reproduces the well-known result (3). The large logarithmic contribution to $g^{(1)}$ comes from two distinctly separate regions of the phase space in the integral of Eq. (16): one corresponds to the “hard”- photon radiated in a collinear way to the momentum $p_1$ ($\beta \sim 1, \alpha \ll 1$) and another – collinearly to the momentum $p_2$ ($\alpha \sim 1, \beta \ll 1$). The term “hard”- photon radiation implies that, apart from a “soft”- photon radiation, one cannot ignore the recoil effect of the quark or photon momentum in the spinor numerators of the quark propagators. When the photon is radiated along the momentum $p_1$, it is the integral over the quark virtuality of the line $p_1$, $2(p_1 k) \approx \alpha s$, yields the log contribution, whereas the quark virtuality of the line $p_2$, $2(p_2 k) \approx \beta s \sim q^2$, is of the largest scale.

For higher-loop diagrams in the approach of quasi-real photons, the LLA contributions come from graphs with all photons emitted from the line $p_1$ and absorbed on the line $p_2$. In the case of the electric form factor $f$, the leading DL contributions come from “soft”- photon emission when each photon contributes to both logarithmic integrals over $\alpha$ and $\beta$. But for the case of the magnetic form factor $g$, one of these photons turns out to be “hard” and collinear to $p_1$ or $p_2$. We argue below that, as soon as other photons are to be “soft”, each yielding a DL contribution, their contributions to the magnetic form factor turns out to be independent factors, similar to the case of the electric form factor, i.e.

$$g^{(n)} = f^{(n-1)} g^{(1)}. \quad (17)$$

Let us consider the two-loop diagrams with quasi-real photons in Fig. 3. To prove this, we fix the momentum of the “hard” photon, $k_1$, to be collinear to $p_1$ ($\beta_1 \sim 1, \alpha_1 \ll 1$). Then a “soft” photon $k_2$ can be emitted from the line $p_1$ either before $k_1$ or later, as shown in Figs. 3a,b,c,d however, to provide a DL contribution, it must be absorbed on the line $p_2$ after $k_1$, as shown in Figs. 3c,d. Indeed, the absorption part of the amplitude in the sum of two diagrams in Figs. 3a,c is

$$\tilde{u}_2 \left[ \gamma_{\lambda_2} \frac{m + \hat{p}_2 - \hat{k}_2}{m^2 - (p_2 - k_2)^2 - i\epsilon} \gamma_{\lambda_1} + \gamma_{\lambda_1} \frac{m + \hat{p}_2 - \hat{k}_1}{m^2 - (p_2 - k_1)^2 - i\epsilon} \gamma_{\lambda_2} \right] \frac{m + \hat{p}_2 - \hat{k}_1 - \hat{k}_2}{m^2 - (p_2 - k_1 - k_2)^2 - i\epsilon} \gamma_u \cdots \quad (18)$$

The above dots denote the remaining spinor part of the amplitude of photon radiation from the quark line $p_1$. Neglecting $k_2$ in the spinor numerators and the $O(m^2/s)$ terms in the denominators, and applying the commutation rules and the Dirac equation, this expression in LLA in the region

$$\alpha_2, \alpha_1 \ll 1, \quad \beta_2 \ll \beta_1 \sim 1 \quad (19)$$

can be simplified to
\[ \bar{u}_2 \left[ \frac{2p_2 \lambda_2}{\beta_2 s} \gamma_{\lambda_1} + \gamma_{\lambda_1}, \frac{2(p_2 - k_1) \lambda_2}{\beta_1 s} \right] \frac{m + \hat{p}_2 - \hat{k}_1}{(\beta_2 + \beta_1)s} + \frac{\gamma_{\lambda_1} \gamma_{\lambda_2}}{(\beta_2 + \beta_1)s} \gamma_{\mu} \ldots \]  

(20)

Only the first term in Eq. (20), which corresponds to Fig. 3d, provides logarithmic integration over \( \beta_2 \); the other terms, which correspond to Fig. 2, with the “soft” \( k_2 \) photon being absorbed before the “hard” \( k_1 \) photon, in can be neglected in LLA. Therefore expression (19) in LLA acquires the explicit factorized form:

\[ \left( \frac{2p_2 \lambda_2}{\beta_2 s} \right) \bar{u}_2 \gamma_{\lambda_1}, \frac{m + \hat{p}_2 - \hat{k}_1}{\beta_1 s} \gamma_{\mu} \ldots \]  

(21)

The emission part of the amplitude in the sum of Figs. 3a,b providing the LLA contribution,

\[ \cdots \gamma_{\mu} \frac{m + \hat{p}_1 - \hat{k}_1 - \hat{k}_2}{m^2 - (p_1 - k_1 - k_2)^2 - i\epsilon} \left[ \gamma_{\tau_1} m + \hat{p}_1 - \hat{k}_2 \gamma_{\tau_2} \gamma_{\tau_2} + \gamma_{\tau_2} m + \hat{p}_1 - \hat{k}_1 \right] \frac{\gamma_1}{\alpha_1 s}, \]  

(22)

in the LLA region (13), turns into

\[ \cdots \gamma_{\mu} \frac{m + \hat{p}_1 - \hat{k}_1}{\alpha_1 s + (1 - \beta_1)\alpha_2 s} \left[ \gamma_{\tau_1}, \frac{2p_1 \tau_2}{\alpha_2 s} + \frac{2(p_1 - k_1) \tau_2}{\alpha_1 s} \right] \gamma_{\tau_2} \frac{\gamma_1}{\alpha_1 s}, \]  

(23)

where we again neglected \( \hat{k}_2 \) in the spinor numerators, all \( O(m^2/s) \) terms in the denominators and other terms leading beyond the LLA. As the vector in square brackets is to be multiplied by the polarization vector \( p_2 \lambda_2 \) of expression (21), we can take the polarization vector \( p_1 \tau_2 \) out of the brackets. Although the denominators in expression (23) provide log integrations over \( \alpha_1 \) and \( \alpha_2 \) in two separate regions

\[ \alpha_1 \ll (1 - \beta_1)\alpha_2, \quad (1 - \beta_1)\alpha_2 \ll \alpha_1, \]  

(24)

the common numerator of the sum in square brackets exactly cancels the largest denominator and turns expression (23) into a factorized amplitude of the independent emission of photons:

\[ \cdots \gamma_{\mu} \frac{m + \hat{p}_1 - \hat{k}_1}{\alpha_1 s} \frac{u_1}{\alpha_1 s} \left( \frac{2p_1 \tau_2}{\alpha_2 s} \right). \]  

(25)

We would like to emphasize that deriving the factorized expressions (21), (25), we did not neglect \( m \) and any components of \( k_1 \) in spinor numerators, which are essential for the magnetic structure of the vertex \( \Gamma_{\mu} \). Comparing expressions (21) and (25) with the one-loop amplitude (6), we conclude that an additional “soft” photon loop does not spoil the spinor structure and yields only an additional DL factor as in the case of the electric form factor:

\[ g^{(2)} = f^{(1)} g^{(1)}. \]  

(26)

Now we are ready to submit arguments for Eq. (17) in a general case. Consider the emission of \( n \) quasi-real photons from the line \( p_1 \) and their absorption on the line \( p_2 \). Let us select one of them, e.g. \( k_1 \), to be “hard” and collinear to \( p_1 \) (\( \beta_1 \sim 1, \alpha_1 \ll 1 \)). This means that the photon \( k_1 \) must be absorbed first, i.e. it is the one closest to the vertex \( \gamma_{\mu} \) on the line \( p_2 \), as shown in Figs. 3a,b, as its momentum \( k_1 \) introduces the largest possible virtuality for the quark line \( p_2 \); \( \beta_1 s \ll -q^2 \). Then the factorization property of absorption amplitude of remaining “soft” photons on the final quark line \( p_2 \), the blob on the line \( p_2 \) in Figs. 3a,b, is evident and well-known. The same is true for the amplitude in Fig. 3b where all “soft” photons are emitted from the incoming quark before its “hard” decay and where the recoil factor \( (1 - \beta_1) \) does not influence the “soft” radiation in the blob on the line \( p_1 \). A more careful consideration is necessary for the amplitude in Fig. 3b, where “soft” photons are allowed to radiate off the quark line \( p_1 \) as before the “hard” one, \( k_1 \), as later.

We prove the factorization property of the emission amplitude for a general case with \( n \) quasi-real photons by induction. Let us take for granted that we have already proved the factorization property of the emission amplitude in case of \((n - 1)\) photons. Then momentum \( k_j \) of the last photon emitted from the line \( p_1 \) in Fig. 3b, which is the one closest to the vertex \( \gamma_{\mu} \), on the quark line \( p_1 \), enters only the quark propagator between the vertices \( \gamma_{\lambda_j} \) and \( \gamma_{\mu} \). One may consider, for a moment, \( \gamma_{\lambda_j} \) to play the role of \( \gamma_{\mu} \) and use the factorization property of the remaining amplitude with \((n - 1)\) photons:

\[ \cdots \gamma_{\mu} \gamma_{\tau_1} \frac{m + \hat{p}_1 - \hat{k}_1}{\alpha_1 s + (1 - \beta_1)(\alpha_2 + \ldots + \alpha_n)s} \gamma_{\tau_2} \frac{m + \hat{p}_1 - \hat{k}_1}{\alpha_1 s} u_1 \prod_{i=2}^n \left( \frac{2p_1 \alpha_i}{\alpha_i s} \right). \]  

(27)
Again we have neglected here all “soft” $k_j$ in the numerators and the $O(m^2/s)$ terms in denominators. Let us apply to this expression the commutation rules and Dirac equation, and take into account the fact that the polarization vector of the $k_j$ photon must be multiplied to the polarization vector $p_2^\mu$ of its absorption in the blob on the line $p_2$ in Fig. 3. Summing over all “soft” photons we get

$$
\sum_{j=2}^{n} \frac{(1-\beta_j)2p_1\tau_j}{(\alpha_1+(1-\beta_1)(\alpha_2+\ldots+\alpha_n))s} \frac{1}{\alpha_1s} \left( \prod_{i=2}^{n} \frac{2p_1\tau_i}{\alpha_is} \right) = \prod_{i=2}^{n} \frac{2p_1\tau_i}{\alpha_is} \frac{[1-\beta_j](\alpha_2+\ldots+\alpha_n)s}{\alpha_1+(1-\beta_1)(\alpha_2+\ldots+\alpha_n)s} \frac{1}{\alpha_1s} \quad (28)
$$

Adding this expression to the evident resulting factor for the graph in Fig. 3,

$$
\prod_{i=2}^{n} \frac{2p_1\tau_i}{\alpha_is} \frac{1}{(\alpha_1+(1-\beta_1)(\alpha_2+\ldots+\alpha_n))s} \quad (29)
$$

we finally prove that the “soft” photons contribution in DLA is just a scalar factor to the spinor structure of the one-loop graph, which is the same for both the electric and magnetic form factors.

Summing over the number of photons $n$ then leads to the final generalization of the DLA relation Eq. (17) of magnetic and electric form factors:

$$
g = f g^{(1)}, \quad f = \sum_{n=0}^{\infty} f^{(n)} = \exp(f^{(1)}). \quad (30)
$$

Taking into account the explicit expressions for $f^{(1)}$ and $g^{(1)}$, Eqs. (23) and (24), this relation can be read as

$$
g = -2 \frac{\partial}{\partial \rho} f \quad (31)
$$

with $\rho = s/\mu^2 \approx -q^2/m^2$ (as $\mu^2$ is shifted down to $m^2$). For Eq. (20) we therefore obtain in DLA the following formula:

$$
\bar{u}_2 \Gamma_{\mu} u_1 = \bar{u}_2 \left[ \gamma_\mu + \frac{\sigma_{\mu\nu} q_\nu}{m} \frac{\partial}{\partial \rho} \right] u_1 \exp \left[ -\frac{\alpha}{4\pi} \ln^2 \rho \right] \quad (32)
$$

or, in a different form,

$$
\bar{u}_2 \Gamma_{\mu} u_1 = \bar{u}_2 \left[ \gamma_\mu - \frac{1}{2} m \sigma_{\mu\nu} \frac{\partial}{\partial q_\nu} \right] e^{-\frac{\alpha}{4\pi} \ln^2 (-q^2/m^2)} u_1 \quad (33)
$$

III. THE MAGNETIC FORM FACTOR OF THE QUARK

The reason for the simplicity of relation (31) between $f$ and $g$, which we obtained in the previous section, is that they differ in only one respect: compared to $f$, $g$ misses only one logarithmic contribution coming from any of the photon propagators attached to the uppermost vertices in Fig. 3. Otherwise, they are identical. So, $g$ can be regarded as the result of a convolution of the single-logarithmic first-loop contribution with the infinite number of double-logarithmic contributions from the other loops. Such a picture is helpful to get a generalization of (32) to QCD. The main technical difference between calculating the form factor $g$ for electron and for quark is that, in QCD, the three-gluon vertices also are essential in the DLA, increasing considerably the number of involved Feynman graphs in each order in $\alpha_s$. However, they contribute to both $f$ and $g$, and the result for $f_q$, electric form factor of the quark, is

$$
f_q = \exp[-\frac{\alpha_s C_F}{4\pi} \ln^2 \rho]. \quad (34)
$$

where $C_F = (N^2 - 1)/2N = 4/3$.

In other words, the whole effect of replacement of the electromagnetic gauge group $U(1)$ by $SU(3)$ results in the replacement of $\alpha$ in Eq. (31) by $\alpha_s C_F$ ($C_F = (N^2 - 1)/2N = 4/3$) as the DL contributions of diagrams with three-gluon vertices turn to cancel each other in the total sum of graphs for the amplitude (3). For the case of $g_q$ form
factor this observation was made in two-loop calculation in \cite{10}. In order to prove the exponentiation of DL radiative corrections for the electric and magnetic form factors, \( f_q \) and \( g_q \), one might follow the approach developed in \cite{8}, which is based on generalization of the Gribov bremsstrahlung theorem \cite{11} to QCD, for the case of the electric form factor. Repeating that proof here would lead us far beyond the scopes of the present paper. Thus we restrict ourselves just with the statement that with all orders in \( \alpha_s \) taken into account, we arrive at the following expression for the vertex \( \Gamma^{(q)}_\mu \) of the quark in the external electromagnetic field in the kinematical region \cite{2} of large transfer momenta:

\[
\Gamma^{(q)}_\mu = \left[ \gamma_\mu + \frac{2}{m} \frac{q_\mu}{m} \frac{\partial}{\partial \rho} \right] \exp \left[ -\frac{\alpha_s C_F}{4\pi} \frac{\ln^2 \rho}{\rho} \right].
\]  

(35)

IV. CONCLUSION

We have calculated the magnetic form factor of electron and quark in the asymptotical regime where the momentum \( q \) transferred to the electron or the quark is much greater than their mass. Eqs. (32) and (35) that we have obtained predict an exponential fall for the magnetic form factors when \( q^2 \) increases. This corresponds to the suppression of non-radiative scattering, without photon bremsstrahlung, at high energies. When the photon bremsstrahlung is taken into account in the DLA, in the expressions for such radiative (inelastic) form factors Eqs. (32) and (35) are multiplied by the bremsstrahlung factors \( B_i \) defined in Eq. (5). All the form factors in Eqs. (32) and (35) do not depend on the features of the emitted photons. It means that taking into account the magnetic formfactor does not violate the Poisson energy spectrum for the photon bremsstrahlung in the double logarithmic approximation. On the contrary, the inelastic electric form factor that accounts for the gluon bremsstrahlung depends, in the DLA, both on the emitted gluon momenta and on the structure of each of the gluon cascades \cite{7,8}. This makes possible to obtain the simple expressions for the inelastic formfactor in QCD only separately for every kinematical region fixed by a certain ordering of the emission energies and angles \cite{4}. Still, in the DLA, even with that complication, the relation (31) holds also for the inelastic electric and magnetic formfactors.

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[1] J.Schwinger. Phys. Rev. 73 (1948) 416.
[2] A.Czarnecki and W.J.Marciano. BNL-HET-98/43; hep-ph/9810512.
[3] V.V.Sudakov. ZhETP 3 (1956) 65.
[4] J.J.Carazzone, E.C.Poggio and H.R.Quinn. Phys. Rev. D11 (1975) 2286; J.M.Cornwall and G.Tiktopoulos. Phys. Rev. D13 (1976) 3370.
[5] V.G.Gorshkov. ZhETP 56 (1969) 598.
[6] V.S.Fadin and E.A.Kuraev. Sov. J. Nucl. Phys. 27 (1983) 587.
[7] B.I.Ermolaev and V.S.Fadin.JETP Lett. 33 (1981) 269; V.S.Fadin. Sov. J.Nucl.Phys. 37 (1983) 2145.
[8] B.I.Ermolaev, V.S.Fadin and L.N.Lipatov. Sov. J. Nucl. Phys. 45 (1987) 508.
[9] L.N.Lipatov. Phys. Lett. B116 (1982) 411.
[10] R.Barbieri and E.Remiddi. Nuovo Cimento 11A (1972) 824.
[11] V.N.Gribov. Sov. J. Nucl. Phys. 5 (1967) 280.
FIG. 1. One-loop diagram for $\Gamma_\mu$.

FIG. 2. Two-loop diagrams for $\Gamma_\mu$ in DLA.

FIG. 3. General graph for of $\Gamma_\mu$ in DLA.