Relativistic, Causal Description of Quantum Entanglement and Gravity

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Abstract

A possible solution to the problem of providing a spacetime description of the transmission of signals for quantum entangled states is obtained by using a bimetric spacetime structure, in which quantum entanglement measurements alter the structure of the classical relativity spacetime. A bimetric gravity theory locally has two lightcones, one which describes classical special relativity and a larger lightcone which allows light signals to communicate quantum information between entangled states, after a measurement device detects one of the entangled quantum states. This theory would remove the tension that exists between macroscopic classical, local gravity and macroscopic nonlocal quantum mechanics.

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1 Introduction

One of the most important features of quantum mechanics is the Einstein-Podolsky-Rosen [1, 2] effect, in which strong correlations are observed between presently noninteracting particles that have interacted in the past. The problem of understanding the consequences of the EPR effect is still controversial [3]. Experiments on entangled particle states have verified the ‘nonlocal’ nature of quantum mechanics [4]. One disturbing feature of the standard interpretation of quantum mechanics is that the nonlocal nature of
the entanglement process has been divorced from our common intuitive ideas about spacetime events and causality. The standard interpretation asserts that for photons (or electrons) positioned at A and B, separated by a spacelike distance, there is no exchange of classical information and superluminal signals between A and B are impossible according to special relativity. With the advent of the possibility of constructing quantum computers and performing ‘teleportation’ experiments, the whole issue of the spacetime reality of the EPR process becomes more problematic.

There exists also the fundamental puzzle that contemporary quantum mechanics is nonlocal on a macroscopic level, whereas gravitation described by Einstein’s general relativity (GR) is a strictly local macroscopic theory. This causes a tension to exist between the two fundamental pillars of modern physics [5].

We can adopt three possible positions:

1. There is no problem. Quantum mechanics is nonlocal and we should accept that there is no possible causal phenomenon associated with a space and time interpretation of entanglement as dictated by classical special relativity and Bell’s inequality [6].

2. Quantum mechanics should be altered in some way to bring about a causal, space and time description of quantum mechanics.

3. Classical spacetime locally described by a flat, Minkowskian metric with one light cone, is not adequate to explain the physics of quantum entanglement. The standard, classical description of spacetime must be extended when quantum mechanical systems are measured.

According to (1) quantum entanglement is a purely quantum phenomenon and classical concepts associated with causally connected events in space are absent. This is the point of view advocated by practitioners of standard quantum mechanics. One should abandon any notion that physical space plays a significant role for distant correlations of entangled quantum states. For those who remain troubled by this abandonment of a causal spatial connection between entangled states, it is not clear how attempting to change quantum mechanics would help matters. This leaves the possibility (3) that classical special relativity is too restrictive to allow for a complete spacetime, causal description of quantum entanglement.

In the following, we adopt position (3) and propose a scenario based on a ‘bimetric’ description of spacetime. This kind of construction has been
successful in cosmology, in which it provides an alternative to the standard inflationary cosmologies \[7, 8, 9\]. In the present application of the bimetric description of spacetime to the quantum entanglement problem, we picture that a quantum mechanical metric frame is related to a gravitational metric frame by the gradients of a scalar field $\phi$. The gravitational metric describes locally a Minkowski light cone with constant speed of gravitational waves (gravitons) $c_g$, while the quantum mechanical metric describes locally a different light cone with an increased speed of light $c > c_0$, where $c_0$ is the currently observed speed of light. When the dimensionless parameter $\gamma = c/c_g = 1$, spacetime is described locally by a flat Minkowski metric with one fixed lightcone and we can choose units such that $c = c_g = c_0 = 1$.

The amount of entanglement of a quantum mechanical bipartite system is given by the density matrix of its von Neumann entropy. For a pure non-entangled state, the speed of transmitted signals travels with the standard classical, special relativity value $c_0$, but for entangled states, quantum mechanical superluminal signals can travel in the quantum mechanical metric frame, thereby providing a Lorentz invariant spacetime description of quantum entanglement phenomena. When an entangled quantum system suffers decoherence due to environmental effects, the system rapidly becomes a classical one with the spacetime structure determined by the gravitational metric $g_{\mu\nu}$, corresponding locally to Minkowski spacetime with a single lightcone.

## 2 Bimetric Gravity Theory

We postulate that four-dimensional spacetime is described by the bimetric structure \[7\]:

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \alpha \partial_\mu \phi \partial_\nu \phi,$$

where $\alpha > 0$ is a constant with dimensions of [length]$^2$ and we choose the scalar field $\phi$ to be dimensionless. The metric $\hat{g}_{\mu\nu}$ is called the matter quantum mechanical metric, while $g_{\mu\nu}$ is the gravitational metric. The scalar field $\phi$ belongs to the gravitational sector. We choose the signature of flat spacetime to be described by the Minkowski metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The inverse metrics $\hat{g}^{\mu\nu}$ and $g^{\mu\nu}$ satisfy

$$\hat{g}^{\mu\alpha} \hat{g}_{\nu\alpha} = \delta^{\mu}_{\nu}, \quad g^{\mu\alpha} g_{\nu\alpha} = \delta^{\mu}_{\nu}.$$  

We assume that only non-degenerate values of $\hat{g}_{\mu\nu}$ with $\text{Det}(\hat{g}_{\mu\nu}) \neq 0$ correspond to physical spacetime.
The action is given by
\[ S = S_G[g] + S_\phi[g, \phi] + \hat{S}_M[\hat{g}, \psi^I], \] (3)
where
\[ S_G[g] = -\frac{1}{\kappa} \int dt d^3x \sqrt{-g} (R[g] + 2\Lambda), \] (4)
and
\[ S_\phi[g, \phi] = \frac{1}{\kappa} \int dt d^3x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right). \] (5)
Moreover, the matter stress-energy tensor is
\[ \hat{T}^{\mu\nu} = -\frac{2}{\sqrt{-\hat{g}}} \frac{\delta \hat{S}_M}{\delta \hat{g}_{\mu\nu}}, \] (6)
\[ \kappa = 16\pi G/c^4, \Lambda \text{ is the cosmological constant and } \psi^I \text{ denotes matter fields.} \]
We have constructed the matter action \( \hat{S}_M \) using the matter quantum mechanical metric \( \hat{g}_{\mu\nu} \). The stress-energy tensor for the scalar field \( \phi \) is given by
\[ T_\phi^{\mu\nu} = \frac{1}{\kappa} \left[ g^{\mu\alpha} g^{\nu\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + g^{\mu\nu} V(\phi) \right]. \] (7)
An alternative choice of bimetric structure is [8, 9]:
\[ \hat{g}_{\mu\nu} = g_{\mu\nu} + \alpha \psi^I \psi^I, \] (8)
where \( \psi^I \) is a vector field and
\[ F_{\mu\nu} = \partial_\mu \psi^I - \partial_\nu \psi^I \] (9)
is the field strength tensor. In the following, we shall consider only the bimetric tensor-scalar spacetime structure defined by (1).
Variation of the action \( S \) in [3] gives the field equations
\[ G^{\mu\nu} = \frac{\kappa}{2} (s T^{\mu\nu} + T_\phi^{\mu\nu}) + \Lambda g^{\mu\nu}, \] (10)
\[ \nabla_\mu \nabla^\mu \phi + V'(\phi) - \kappa s a T^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu \phi = 0, \] (11)
where \( G^{\mu\nu} = R^{\mu\nu} - (1/2)g^{\mu\nu} R \) is the Einstein tensor and \( s = \sqrt{-g}/\sqrt{-\hat{g}} \). Moreover, \( \nabla_\mu \) and \( \hat{\nabla}_\mu \) denote the covariant differential operators associated with \( g_{\mu\nu} \) and \( \hat{g}_{\mu\nu} \), respectively, \( V(\phi) \) is the potential for the field \( \phi \) and
\( V'(\phi) = \partial V(\phi) / \partial \phi \). For a free scalar field \( \phi \) the potential will be 
\( V(\phi) = \frac{1}{2} m^2 \phi^2 \), where \( m \) is the mass of the particle associated with the scalar field \( \phi \). The energy-momentum tensor \( \hat{T}^{\mu \nu} \) satisfies the conservation laws

\[
\nabla_\nu \left[ \sqrt{-\hat{g}} \hat{T}^{\mu \nu} \right] = 0. \quad (12)
\]

### 3 Bimetric Special Relativity and Quantum Mechanics

The local special relativity metric is given by 
\( g_{\mu \nu} = \eta_{\mu \nu} \) with

\[
ds^2 \equiv \eta_{\mu \nu} dx^\mu dx^\nu = c_g^2 dt^2 - \left( dx^i \right)^2, \quad (13)
\]

where \( (i, j = 1, 2, 3) \). The quantum mechanical metric for the choice \( \Box \) is

\[
ds^2 \equiv \hat{g}_{\mu \nu} dx^\mu dx^\nu = (\eta_{\mu \nu} + \alpha \partial_\mu \phi \partial_\nu \phi) dx^\mu dx^\nu. \quad (14)
\]

The latter can be written as

\[
ds^2 = c_0^2 dt^2 \left( 1 + \frac{\alpha}{c_0^2} \dot{\phi}^2 \right) - (\delta_{ij} - \alpha \partial_i \phi \partial_j \phi) dx^i dx^j, \quad (15)
\]

where \( \dot{\phi} = d\phi / dt \). We see that the speed of light in the quantum mechanical metric is space and time dependent. If we assume that \( \partial_i \phi \approx 0 \), then we have \(^1\)

\[
c(t) = c_0 \left( 1 + \frac{\alpha}{c_0^2} \dot{\phi}^2 \right)^{1/2}. \quad (16)
\]

The null cone equation \( ds^2 = 0 \) describes gravitational wave signals moving with the constant measured speed \( c_g \), whereas \( d\hat{s}^2 = 0 \) cannot be satisfied along the same null cone lines, but determines an expanded null cone with the speed of light \( c > c_0 \). The bimetric null cone structure is described in Fig.1.

\(^1\)In GR, with one local lightcone, we can always perform a diffeomorphism transformation to remove the time dependence of \( c \). This corresponds to being able to choose units in which rigid ruler and clock measurements yield \( \Delta c / c = 0 \). In the bimetric gravity theory, we cannot simultaneously remove the time dependence of \( c \) and \( c_g \) by performing a diffeomorphism transformation. If we choose \( c_g \) to be constant, then the time dependence of \( c \) is non-trivially realized.

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Fig. 1. Bimetric light cones showing the timelike communication path between the two entangled states at A and B in the quantum mechanical metric $\hat{g}_{\mu\nu}$.

Both metrics (13) and (14) are separately invariant under local $^2$ Lorentz transformations

$$x'{}^\mu = \Lambda^\mu_\nu x^\nu,$$  \hspace{1cm} (17)

where $\Lambda^\mu_\nu$ are constant matrix coefficients which satisfy the orthogonality condition

$$\Lambda^\mu_\nu \Lambda^\nu_\sigma = \delta^\sigma_\sigma.$$  \hspace{1cm} (18)

The ‘physical’ matter metric $\hat{g}_{\mu\nu}$ describes the geometry in which quantum matter propagates and interacts. Because all quantum matter fields are coupled to the same metric $\hat{g}_{\mu\nu}$ in the same way, the weak equivalence principle is satisfied in this metric frame. However, because $\hat{g}_{\mu\nu}$ does not couple to matter in the same way as in GR unless $\phi = 0$, the strong equivalence principle is not satisfied.

If we choose a spacelike surface for each individual metric $\hat{g}_{\mu\nu}$ and $g_{\mu\nu}$, then a Cauchy initial value solution of the field equations for given initial

$^2$We consider a small patch of spacetime in which $g_{\mu\nu} \approx \eta_{\mu\nu}$, so that we can restrict our attention to local Lorentz transformations.
data on these spacelike surfaces can be obtained, which preserves the causal evolution of the fields and field equations [9]. If we consider a vector field \( V^\mu \), which is timelike (or null) with respect to both metrics \( \hat{g}_{\mu\nu} \) and \( g_{\mu\nu} \), then an observer would see a normal, causal ordering of timelike events. However, we could picture a situation in which an observer at \( A \), in Fig.1, observes that the vector field \( V_\mu \) is timelike with respect to the smaller light cone, but is spacelike with respect to the second larger lightcone. In this sense, causality in the bimetric gravity theory is observer dependent. Thus, by performing a Lorentz transformation to a boosted frame in the second, larger light cone, events connected by a ‘superluminal’ quantum information signal would appear to be acausal with respect to the observer at \( A \), i.e. the emission of a light signal would appear to take place later than its reception. However, in the ‘physical’ matter metric frame described by \( \hat{g}_{\mu\nu} \), an observer at \( B \) would see a proper timelike and causal ordering of events, provided the observer is restricted to observing a timelike vector field. Thus, the emission of quantum matter information signals would be causal with respect to the observer at \( B \) in the metric frame described by \( \hat{g}_{\mu\nu} \).

In classical physics, information is communicated through space with a limiting speed \( c = c_0 \). Information can affect events only in the forward light cone. We are concerned with the transmission of ‘quantum information’ (QI), which is transported through space at some speed \( v_{QI} \). Let us consider a smallest cone with \( \theta_{QI} \) the angle of the cone to the vertical. Then \( v_{QI} = 0 \) corresponds to \( \theta_{QI} = 0 \), \( v_{QI} = c_0 = c_g \) to \( \theta_{QI} = \pi/4 \), and \( v_{QI} = \infty \) corresponds to \( \theta_{QI} = \pi/2 \) [10]. We can also define an inverse speed of QI:

\[
\frac{1}{v_{QI}} = \cot \theta_{QI} = \left( \frac{c_0}{v_{QI}} \right).
\]

Then, \( v_{QI} = c_g = c_0 \) and \( \infty \) correspond to \( w_{QI} = 1 \) and \( 0 \), respectively. The case \( w_{QI} = 1 \) is related to the classical special relativity metric frame with the constant values \( c = c_0 = c_g \), while \( 0 \leq w_{QI} \leq 1 \) corresponds to the quantum mechanical metric lightcone swept out by the time dependence of \( c = c(t) \) for \( \alpha \neq 0 \).

We require a local relativistic description of quantum mechanics. To this end, we introduce the concept of a general spacelike surface in Minkowski spacetime, instead of the flat surface \( t = \text{constant} \). We demand that the normal to the surface at any point \( x \), \( n_\mu(x) \), be time-like: \( n_\mu(x)n^\mu(x) > 0 \). We denote a spacelike surface by \( \sigma \). A local time \( t(x) \) is assigned, so that in the limit that the surface becomes plane, each point has the same time.
\( t = \text{constant. We can now define the Lorentz invariant functional derivative } \frac{\delta}{\delta \sigma(x)} \text{ and the Tomonaga-Schwinger equation } \text{[11] [12] [13]} \)

\[
i \hbar c \frac{\delta \psi(\sigma)}{\delta \sigma(x)} = \mathcal{H}_{\text{int}}(x) \psi(\sigma), \tag{20}\]

where

\[
H_{\text{int}}(t) = \int d^3x \mathcal{H}_{\text{int}}(x), \tag{21}\]

is the Hamiltonian operator in the interaction representation, and \( \mathcal{H}_{\text{int}}(x) \) is the Lorentz invariant Hamiltonian density. Eq. \( (20) \) is a relativistic extension of the Schrödinger equation

\[
i \hbar \partial_t \psi(t) = H(t) \psi. \tag{22}\]

It is essential that the domain of variation of \( \sigma \) is restricted by the integrability condition

\[
\frac{\delta^2 \psi(\sigma)}{\delta \sigma(x) \delta \sigma(x')} - \frac{\delta^2 \psi(\sigma)}{\delta \sigma(x) \delta \sigma(x)} = 0. \tag{23}\]

This equation in turn implies that

\[
[\mathcal{H}_{\text{int}}(x), \mathcal{H}_{\text{int}}(x')] = 0, \tag{24}\]

for \( x \) and \( x' \) on the spacelike surface \( \sigma \). In quantum field theory, it is usual to work in the interaction picture, so that the invariant commutation rules for the field operators automatically guarantee that \( (24) \) is satisfied for all interacting fields with local nonderivative couplings.

The Tomonaga-Schwinger equation evolves unitarily in the special relativity metric \( g_{\mu\nu} = \eta_{\mu\nu} \) with \( \alpha = 0 \), before a measurement of a quantum mechanical state is performed and before the collapse of the state wave function. After a measurement is performed on a quantum state and \( \alpha \) is ‘switched on’, then depending upon the spatial distance of the causal communication between two entangled states \( A \) and \( B \), and the size of \( \alpha \) required to make \( A \) and \( B \) timelike separated, the Tomonaga-Schwinger equation and its non-relativistic counterpart – the Schrödinger equation – will have a spacelike region outside the quantum mechanical metric lightcone to evolve unitarily. For \( \alpha \to \infty \) this spacelike region will shrink to zero and \( A \) and \( B \) will be timelike separated by an infinite spatial distance and \( v_{QI} = \infty \).
We introduce the concept of a locally Lorentz invariant density matrix:

\[ \rho(\sigma(x)) = |\psi(\sigma(\sigma))\rangle\langle \psi(\sigma(\sigma))|. \]  

(25)

The density matrix operator \( \rho(\sigma) \) satisfies the invariant Heisenberg equation of motion

\[ i\hbar c_0 \frac{\delta \rho(\sigma)}{\delta \sigma} = [H_{\text{int}}(x), \rho(\sigma)]. \]  

(26)

A local relativistic measure of the entanglement of a bipartite quantum state is given by

\[ S(\rho_m)(\sigma) = -\text{Tr}_m \rho(\sigma) \log \rho(\sigma), \]  

(27)

where \( S(\rho_m)(\sigma) \) is the relativistic entropy of the subsystems A and B. Moreover, \( \rho_A = \text{Tr}_B |\psi\rangle\langle \psi| \) is the reduced density matrix obtained by tracing the whole system’s pure state density matrix \( \rho_{AB} = |\psi\rangle\langle \psi| \) over A’s degrees of freedom, while \( \rho_B = \text{Tr}_A |\psi\rangle\langle \psi| \) is the partial trace over B’s degrees of freedom. In the non-relativistic limit, (27) reduces to the pure bipartite entropy measure of entanglement [14, 15].

For a non-entangled state for which the \( \psi(\sigma) \) can be expressed as a tensor product \( \psi_A(\sigma) \otimes \psi_B(\sigma) \), we have \( S(\rho)(\sigma) = 0 \), \( \hat{g}_{\mu\nu} = g_{\mu\nu} \) and there is no signal transmitted between the bipartite states A and B by light signals associated with the special relativity metric \( c_\mu \) light cone with \( c_\mu = c_0 \), for they are now spacelike separated.

For an entangled state \( \alpha \neq 0 \) the spacetime is described by the local metric (14). Since the speed of light can become much larger in the quantum mechanical metric frame, it is now possible to transmit signals at ‘superluminal’ speeds without violating the spatial causality notions that prevail in the familiar classical special relativity frame. In this way, by introducing a bimetric spacetime, we have incorporated the notions of spacetime events and causality for quantum mechanical entangled states.

4 Conclusions

An observer who detects a quantum mechanical system with some measuring device and subsequently observes an entangled state with \( \alpha \neq 0 \) in the

\[ ^3\text{We consider in the following only the simple physical system of two photons (or two electrons). The entanglement measure for a mixed state and a multiparticle state is controversial and no consensus has been reached on how to define it.}\]
quantum mechanical metric frame, will also observe an exchange of quantum information with the spatially distant other component of the entangled state. In the quantum mechanical metric frame, the speed of the light signal emitted, say, at a counter at A and received at a distant counter B will be finite but large compared to the measured value of the speed of light $c_0$ in the classical, local gravitational metric frame, and A and B are no longer ‘spacelike’ separated.

A possible experiment to verify the bimetric spacetime scenario would be to attempt to measure a decrease in the communication signals associated with the solutions of the fields $\phi$ with $\alpha \neq 0$. Since electromagnetic (photon) communication can take place over large cosmological distances in the universe, then given enough transmission energy the two entangled states $A$ and $B$ could communicate at large spatially separated distances without an observer detecting a significant decrease in the amount of entanglement and correlation of A and B. The current correlation experiments have a separation distance $\sim 11$ kilometres and it is estimated in a preferred frame that $c \sim 10^4c_0$ [4]. It is possible that when experiments are performed at larger spatial separations, a dilution of the correlation between entangled states will be observed. Until then, we cannot exclude our alternative proposal, based on an extended spacetime structure using a bimetric geometry.

In our bimetric scenario, the Bell inequalities would not exclude our extension of special relativity theory and gravitation theory. The observed experimental violations of Bell’s inequality tell us that the assumption of classical locality, based on one metric and one Minkowski light cone, is not compatible with quantum mechanics. But the observed violation of Bell’s inequality does not exclude the extended concept of locality possessed by the bimetric two-lightcone structure and quantum mechanics.

In our generalized bimetric gravity theory, we no longer have the peculiar tension that exists between GR and quantum mechanics, caused by classical gravity theory being a strictly local theory at macroscopic distances (equivalence principle) and the nonlocal behavior of spatially separated entangled states in quantum mechanics. This may lay the groundwork for a consistent quantum gravity theory.

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