Initial Nucleon Structure Results with Chiral Quarks at the Physical Point

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Outline

- Techniques: CG deflation, All-Mode Averaging
- Vector Form Factors
- Axial Vector Form Factors
- Quark Momentum Fraction
- Summary & Outlook
Dynamical Möbius (DW) Fermions

- RBC/UKQCD-generated ensemble:
  \[ a \approx 0.113 \text{ fm} = (1.75 \text{ GeV})^{-1} \quad 48^3 \times 96 = (5.4 \text{ fm})^3 \times 10.8 \text{ fm}, \quad m_\pi L_x \approx 3.84 \]

- 2+1 dynamical Möbius fermions [R.Brower et al, hep-lat/0409118] \[ L_5 = 24 \]

Deflation: \[ M_{eopc}^\dagger M_{eopc} \]

- deflate between \( m_u,d \) and \( m_s \)
- PARPACK with \( n=200 \) poly.acc.
- condition number \( \times 1/100 \),
- CG convergence speed \( \times 10 \)
- 1 evec=11.39GB; 500 evecs=5.7TB
- Limited by memory & IO; want 3 more evecs
Nucleon Matrix Elements

- Only connected quark contractions

\[ C_{3pt}^{\mathcal{O}} = \rho_{N} \]

- Matrix elements from \( C_{3pt}/C_{2pt} \) ratio

\[ R_{\mathcal{O}}(T, \tau; P, P') = \frac{C_{\mathcal{O}}(T, \tau; P, P')}{\sqrt{C_{2pt}(T, P)C_{2pt}(T, P')}} \cdot \sqrt{\frac{C_{2pt}(T - \tau, P)C_{2pt}(\tau, P')}{C_{2pt}(T - \tau, P')C_{2pt}(\tau, P)}} \cdot \langle P' | \mathcal{O} | P \rangle \]

- Employ summation method to “demote” transitional excited state contributions

\[ \mathcal{O} \left( e^{-\Delta E \cdot \frac{T}{2}} \right) \rightarrow \mathcal{O} \left( e^{-\Delta E \cdot T} \right) \]

\[ \sum_{\tau} R_{\mathcal{O}}(T, \tau) = \langle P' | \mathcal{O} | P \rangle \cdot T + O(e^{-\Delta E \cdot T}) \]
Improved Stoch. Estimation: All-Mode Averaging

- All-mode averaging [T. Blum et al, PRD88:094503 (arXiv:1208.4349)]:

\[
\langle \mathcal{O}\rangle_{\text{imp}} = \langle \mathcal{O}_{\text{approx}} \rangle_{N_{\text{approx}}} + \langle (\mathcal{O}_{\text{exact}} - \mathcal{O}_{\text{approx}}) \rangle_{N_{\text{exact}}}
\]

\[
(\delta \mathcal{O}_{\text{imp}})^2 \sim \frac{1}{N_{\text{approx}}} \text{Var}\{\mathcal{O}_{\text{approx}}\} + \frac{1}{N_{\text{exact}}} \text{Var}\{(\mathcal{O}_{\text{exact}} - \mathcal{O}_{\text{approx}})\} \tag{*}
\]

- Tune approximation \((n^{CG})\) and \((N_{\text{approx}}/N_{\text{exact}})\) for optimal cost

\[
\text{Cost}_{\text{imp}} \cdot (\delta \mathcal{O}_{\text{imp}})^2 \sim \left(1 + \frac{n^{CG}_{\text{approx}}}{n^{CG}_{\text{exact}}} \cdot \frac{N_{\text{approx}}}{N_{\text{exact}}}\right) \cdot \left[\text{Var}\{\Delta \mathcal{O}\} + \frac{N_{\text{exact}}}{N_{\text{approx}}} \text{Var}\{\mathcal{O}_{\text{approx}}\}\right]
\]

- \(\langle N(t)\bar{N}(0)\rangle\)

- \(G_E(Q_{\text{min}}^2) \sim \text{Re} \langle N(t) [\bar{q}\gamma_4 q]_{\tau,\bar{q}_{\text{min}}} \bar{N}(0)\rangle\)

- \(g_A \sim \text{Im} \langle N(t) [\bar{q}\gamma_3 \gamma_5 q]_{\tau,\bar{q}=0} \bar{N}(0)\rangle\)

- Select \(n^{CG}=400, N_{\text{approx}}/N_{\text{exact}}=32\); x2.5 - x3 noise reduction
Initial Results: Effective Mass

- 10 gauge configs, spaced to span 1/2 of the ensemble
- 320 approx("sloppy") samples, 10 exact (bias) samples
- Gaussian quarks source tuned to approximate nucleon ground state

\[ aE_{eff}(t) = \langle \log \frac{C_{2pt}(t)}{C_{2pt}(t + 1)} \rangle \]
Vector Charge $g_V$

- Source-Sink separations $T=8a,9a,10a,12a$
- Employ summation method to “demote” transitional excited states

$$\mathcal{O}(e^{-\Delta E \cdot \frac{T}{2}}) \rightarrow \mathcal{O}(e^{-\Delta E \cdot T})$$

Isovector $(u-d)$ vector charge

$$g_V^{\text{bare}} = \langle N | \int d^3x \bar{q} \gamma_4 q | N \rangle$$
Vector (u-d) Form Factors $F_{1,2}$

Comparison to phenomenology [J.J.Kelly, PRC70:068202 (2004)]

$$\langle P + q | \bar{q} \gamma^\mu q | P \rangle = \bar{U}_{P+q} \left[ F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] U_P$$
Axial Charge $g_A$

\[ \langle N(p) | \bar{q} \gamma^\mu \gamma^5 q | N(p) \rangle = g_A \bar{u}_p \gamma^\mu \gamma^5 u_p, \]

No excited state contribution seen (yet)
Axial Form Factors

Experiments with electroweak probes:

\( \nu \) scattering, \( \pi^\pm \) production, \( e^- \) & \( \mu^- \) capture

\[
\langle P + q | \bar{q} \gamma^\mu \gamma^5 q | P \rangle = \bar{U}_{P+q} \left[ G_A(Q^2) \gamma^\mu \gamma^5 + G_P(Q^2) \frac{\gamma^5 q^\mu}{2M_N} \right] U_P
\]

\[
G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{M_A^2})^2}
\]

\[
G_P(Q^2) = \frac{4M_N g_{\pi N} F_\pi}{m^2_{\pi} + Q^2} - \frac{2}{3} g_A M_N^2 \langle r_A^2 \rangle
\]
Quark Momentum Fraction

\[ \langle x \rangle_{u-d} = \int dx \, x \left( u(x) + \bar{u}(x) - d(x) - \bar{d}(x) \right) \]

Phenomenology: \( \langle x \rangle_{\overline{\text{MS}}(2 \text{ GeV})}^{\text{MS}} = 0.155(5) \)

\[ \langle N(p) | \bar{q} \gamma_{\mu} \overleftrightarrow{D_{\nu}} q | p \rangle = \langle x \rangle_{q} \bar{u}(p) \gamma_{\mu} p_{\nu} u_{p} \]

(*) renormalization from 24\(^{3}\) lattice with the same action & lattice spacing
Summary & Outlook

Summary

- Initial results (~1/4th - 1/6th of statistics) with chiral quarks at the physical point
- Promising results for vector, axial vector form factors
- Excited states clearly present in $G_P$

Outlook

- Increase statistics x6 in 2014-2015
- Improve exc.state analysis once statistics is sufficient for reliable fits
- Explore other approximations, e.g. Möbius with shortened L5
- Disconnected diagrams with hierarchical probing (S.Meinel)