Thermal emission in the ultrastrong-coupling regime

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Abstract

We have studied the thermal emission of a cavity quantum electrodynamic system in the ultrastrong-coupling regime, where the atom–cavity coupling rate becomes comparable with the cavity resonance frequency. In this regime, the standard descriptions of photodetection and dissipation fail. By following an approach that was recently put forward by Ridolfo \textit{et al} (2012 \textit{Phys. Rev. Lett.} \textbf{109} 193602), we were able to calculate the emission of systems with arbitrary strength of light–matter interaction, by expressing the electric-field operator in the cavity–emitter dressed basis. In this paper, we present thermal photoluminescence spectra, calculated for given temperatures and for different couplings, in particular, for available circuit quantum electrodynamics parameters.

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(Some figures may appear in color only in the online journal)

1. Introduction

The quantum theory of photodetection as originally formulated by Glauber [1] is a landmark in the field of quantum optics and has played a key role in understanding radiation–matter interactions. In recent years, a new regime of interaction cavity quantum electrodynamics (cavity QED) has been reached experimentally where the coupling strength between an emitter and the cavity photons becomes comparable with the cavity resonance frequency. In this regime, the standard descriptions of photodetection and dissipation fail. By following an approach that was recently put forward by Ridolfo \textit{et al} (2012 \textit{Phys. Rev. Lett.} \textbf{109} 193602), we were able to calculate the emission of systems with arbitrary strength of light–matter interaction, by expressing the electric-field operator in the cavity–emitter dressed basis. In this paper, we present thermal photoluminescence spectra, calculated for given temperatures and for different couplings, in particular, for available circuit quantum electrodynamics parameters.

2. Input–output relations

Applying Glauber’s idea of photodetection, we here introduce a full quantum theory to study the thermal emission in the USC regime [17]. This requires a proper generalization of input–output theory [18], since the standard relations would, for example, predict an output photon flux that is proportional to the average number of cavity photons, i.e. $\langle a_{\text{out}}^\dagger(t) a_{\text{out}}(t) \rangle \propto \langle a^\dagger(t) a(t) \rangle$ for vacuum input. Hence an unwary application of this standard procedure to the USC regime would predict an unphysical stream of output photons for a system in its ground state which contains a finite number of photons due to the counter-rotating terms in the Hamiltonian. It has been shown [17] that applying Glauber’s formulation...
of photodetection [1], it is possible to derive nth order correlation functions for the output fields which are valid for arbitrary degrees of light–matter interaction, by expressing the cavity electric-field operator in the atom–cavity dressed basis. An ideal detector absorbs a photon with a probability per unit time that is proportional to \((E^- (t) E^+ (t))\), where \(E^\pm (t)\) are the positive and negative frequency components of the electric-field operator of the output field [1, 19]. In the circuit QED the same quantities are measured with output voltages which are proportional to the electric fields. Following [17], the input–output relations for a cavity that is coupled to a one-dimensional output waveguide via an interaction between the cavity field \(X\) and the momentum quadratures \(\Pi_\omega\) of the waveguide field outside the cavity are

\[
a_{\text{out}}(t) = a_{\text{in}}(t) - i \frac{\epsilon_\omega}{\sqrt{8\pi \hbar \epsilon_\omega v}} \dot{X}^+, \tag{1}
\]

where \(\epsilon_\omega\) is a coupling parameter, \(\epsilon_\omega\) is a parameter describing the dielectric properties of the output waveguide, and \(v\) is the phase velocity. The input (output) field operators \(a_{\text{out}}(t)\) are defined as

\[
a_{\text{out(in)}}(t) = \frac{1}{2\pi} \int d\omega \sqrt{\omega} e^{-i\omega(t-t')} a_\omega(t'), \tag{2}
\]

where \(\tilde{t} = t_\text{l} \to +\infty\) for the output field and \(\tilde{t} = t_\text{0} \to -\infty\) for the input field and \(a_\omega\) \((a_\omega^\dagger)\) annihilation (creation) operators of the fields outside the cavity. In this way, \(\sqrt{\omega} (a_{\text{out}}(t) a_{\text{out}}^\dagger(t))\) is proportional to the measured \((E^- (t) E^+ (t))\) and describes an energy flux associated with the output light. The standard definition of output fields as used in many textbooks, see [20], is recovered if all frequencies of the field are very close to a carrier frequency \(\overline{\omega}\) and one may approximate \(\sqrt{\omega} \approx \sqrt{\overline{\omega}}\) in the integral kernel, which makes the observed energy fluxes proportional to photon number fluxes. One thus needs to find the positive frequency component of \(X\) according to its actual dynamical behavior, see [21], to compute the proper output fields. We do this by expressing \(X\) in the atom–cavity dressed basis. It is worth noting that in the USC regime, the positive frequency component of \(X\) is not proportional to the photon annihilation operator \(a\).

3. Dynamics of the open quantum system

We consider the Rabi model, which consists of a linear coupling between a single cavity mode and a TLS. In the following, we set for the sake of simplicity \(\hbar = 1\) and also the Boltzmann constant \(k_B = 1\). The Rabi Hamiltonian reads

\[
H_S = \omega_0 a^\dagger a + \omega_x \sigma^+ \sigma^- + g(a + a^\dagger) \sigma_x , \tag{3}
\]

where \(\omega_0, \sigma_x\) is the bare energy of the cavity mode (TLS), \(g\) is the coupling strength and \(\sigma_x\) is the standard Pauli operator. Figure 1 shows a plot of the spectrum of the eigenvalues as a function of the coupling \(g\) calculated for the parameters \(\omega_0 = \omega_x = 1\). In particular, it is worth noting that for a given value of \(g\), the system shows the characteristic ladder configuration like in the standard JC model. The possible transitions between the rungs are given by the selection rules due to the Hamiltonian structure, which enables these transitions owing to the non-zero values of the electric-field components. In order to describe a realistic system, dissipation induced by its coupling to the environment needs to be considered. Then, owing to the very high ratio \(g/\omega_0\), standard quantum optical master equations fail as it would, for example, predict that even zero-temperature environments could drive the system out of its ground state. An adequate description of the system’s coupling to its environment requires a perturbative expansion in the system–bath coupling strength. In order to perform this expansion, we write the Hamiltonian in a basis formed by eigenstates \(|j\rangle\) of \(H_0\), denote the respective energy eigenvalues by \(\omega_{0j}\), i.e. \(H_0 |j\rangle = \omega_{0j} |j\rangle\), and derive the Redfield equations [22] to describe the dissipative processes [23]. We choose the labeling of the states \(|j\rangle\) such that \(\omega_0 > \omega_k\) for \(k > j\) and focus on a single-mode cavity with \(T \neq 0\) temperature for the environment. Generalizations to a multi-mode cavity are straightforward. For our purposes, we neglect small Lamb shifts and dephasing contributions as they do not alter significantly the output photon, allowing for a simpler and lighter theoretical setup. We thus arrive at the master equation

\[
\dot{\rho}(t) = i[\rho(t), H_S] + L_\alpha \rho(t) + L_\xi \rho(t). \tag{4}
\]

The expressions \(L_\alpha\) and \(L_\xi\) are Liouvillian superoperators describing the losses of the system where \(L_\alpha \rho(t) = \sum_{j,k,j',k'} \Gamma_{jk}^{kj} (1 + \tilde{n}_j (\Delta_{jk}, T)) D [\langle j | k | \rho(t)] + \sum_{j,k,j',k'} \Gamma_{jk}^{kj} \tilde{n}_j \times \langle j | k | \rho(t)) D [\langle j | k | \rho(t)] \) for \(c = a, \sigma^+\) and \(D[\rho] = \frac{1}{2}(2\rho\rho^\dagger - \rho^\dagger \rho - \rho^\dagger \rho^\dagger \rho)\) and \(T\) is the temperature of the thermal bath. Here \(\tilde{n}_j (\Delta_{jk}, T)\) is the number of thermal photons that feed the system acting on all the possible \(|k\rangle \to |j\rangle\) transitions. Standard dissipators are recovered in the limit \(g \to 0\). The relaxation coefficients \(\Gamma_{jk}^{kj} = 2\pi d_j (\Delta_{jk}) a_j^2 (\Delta_{jk}) |C_{jk}^\alpha|^2\) depend on the spectral density of the baths \(d_j (\Delta_{jk})\) and the system–bath coupling strength \(\alpha_j (\Delta_{jk})\) at the respective transition frequency \(\Delta_{jk} = \omega_k - \omega_j\) as well as on the transition coefficients \(C_{jk} = -i (j \langle c - c' \rangle | k\rangle \) \(\langle c, a, \sigma^+\). These relaxation coefficients can be interpreted as the full-width at half-maximum of each \(|k\rangle \to |j\rangle\) transition. Since we consider a cavity that couples to the momentum quadratures of fields in one-dimensional output waveguides, it is possible

![Energy spectrum of \(H_0\) as a function of the coupling strength for the Rabi Hamiltonian. This plot is calculated for \(\omega_0 = \omega_x\). The level structure is analogous to that of the Jaynes–Cummings (JC) model. The ladders correspond to a coupling strength \(g = 0.2\omega_0\). The arrows indicate possible transitions of radiative decay due to the Hamiltonian structure.](image-url)
correspond to the values \( \omega / \omega_0 \sim 0.9 \) and \( \sim 1.1 \), are now separated roughly by twice the energy with respect to the previous case. This happens because the energy separation of the first two eigenmodes of (3), also in this range of interaction, grows linearly with respect to the coupling strength. Looking at figures 2 and 3, we can see that if the temperature increases, the number of resonances present in the system increases as well. Indeed, whenever the temperature rises, this increases the thermal occupancy \( \bar{n}_c(\Delta_{ij}, T) \) allowing to excite the \( |k \rangle \rightarrow |j \rangle \) transitions. A characterization of the resonances, e.g. in figure 3, is really straightforward if one looks at the ladder scheme (see figure 1). Sorting the energies from the lower value to the higher value, we identify \( |4 \rangle \rightarrow |3 \rangle, |2 \rangle \rightarrow |1 \rangle, |4 \rangle \rightarrow |2 \rangle, |5 \rangle \rightarrow |3 \rangle, |3 \rangle \rightarrow |1 \rangle \) and \( |5 \rangle \rightarrow |2 \rangle \) as shown by the rows in figure 1. It is worth noting that in all the spectra that we presented, there is an evident asymmetry also in the absence of detuning, in contrast with the results achievable within the standard model for dissipation and photodetection that naturally cannot apply to this regime of interaction. This is due to the fact that in the USC regime, (i) the effect of the thermal feeding of the reservoir acts differently on the different transitions and (ii) each resonance is characterized by a different damping rate.

5. Conclusions

In conclusion, we have presented a full description of the thermal emission in the USC regime, valid for arbitrary light–matter couplings. We showed thermal emission spectra calculated for the available circuit QED parameters. These results show that the recently proposed correlation functions are also able to describe correctly incoherent light emission from USC systems avoiding unphysical emission from the ground state. Generalizations of our study to multicavity devices [24] and to three-level in the USC regime [25–27] would form interesting perspectives for future research.

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