Relative Position and Attitude Estimation of Non-cooperative Spacecraft Based on Stereo Camera and Dynamics

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Abstract. The relative position and attitude estimation of non-cooperative spacecraft is essential to high-precision guidance, navigation, and control systems. This paper mainly proposes a method to measure the relative position, velocity and attitude of non-cooperative spacecrafts using stereo camera and dynamics. First, using stereo camera observe the spacecraft, and extract the feature points of images. Then, calculate the 3D position of the feature points, and match the relationship between the feature points to obtain geometric constraint information between frames. Based on this, the relative position, velocity, attitude and angular velocity of the target spacecraft are calculated. According to relative orbit dynamics and relative attitude dynamics model, design extended Kalman filter to update the measurement results. The simulation results show that the estimation accuracy is better than 0.01m, the relative velocity accuracy is better than 0.001m/s, and the relative attitude accuracy is better than 0.01°. This method can provide technical support for the relative navigation of non-cooperative spacecraft.

1. Introduction

The precise measurement of the relative posture of the spacecraft is one of the core technologies for space junk clearance, rendezvous and docking, and on-orbit maintenance. The two spacecrafts are called chaser and target. Chaser spacecraft is generally the active party in on-orbit service missions, such as spacecraft or aircraft with fault repair capabilities, etc. Target spacecraft is generally a passive party, such as a failed satellite, the space telescope, or the space station. Target spacecraft are divided into cooperative and non-cooperative. Cooperative target spacecraft can transmit flight attitude information through inter-satellite links. However, non-cooperative target spacecraft lack prior knowledge and do not have manual markings that are easy to reliably extract and identify. So close relative navigation is difficult.

There have been research on the short-range relative navigation of non-cooperative targets. Because the stereo camera, with the advantages of low power consumption, low cost, simple structure, strong concealment, has been researched by various countries[1]. Since 2002, the United States has successively launched SUMO[2] and FREND[3]. They verify the feasibility of the integrated system of computer vision, mechanical equipment and autonomous control algorithms in future autonomous rendezvous and docking tasks. On this basis, the United States proposed the famous ‘PHOENIX’ project based on multiple cameras in December 2011. It demonstrates the reuse of failed satellite recovery technology, adopts a multiple-cameras system, and selects the pictures from the three best
angles to calculate the target pose \cite{4}. In 2004, the Japan Aerospace Exploration Agency (JAXA) implemented the ‘Space Debris Micro Remover’ (SDMR) project \cite{5}. This project uses stereo camera to measure the pose of the space debris, completes the circumnavigation, approach, and rendezvous of the target, and finally grabs the target with the roped flying claw.

In addition, scholars have carried out research on non-cooperative target motion estimation using visual sensors and filters. Pesce et al.\cite{6} established an attitude-orbit coupling motion model of feature points on surface and estimated attitude and principal axis inertia ratio by observing the feature points with stereo camera. Broida and Young used IEKF to estimate posture, position, and speed based on mono camera and stereo camera measurement respectively\cite{7}. Yoko\cite{8} used the EKF algorithm to obtain the posture estimation result by combining the measurement information obtained by the camera in the process of estimating the parameters such as the speed and position of the two aircraft. Son-Goo K\cite{9} established the relative position and relative attitude model. In addition, the measurements of the visual system are used as observation information to design EKF.

This paper proposes a general and effective relative pose measurement algorithm for non-cooperative targets whose surface feature points can be identified and can be tracked and matched between frames. The target feature points are acquired by binocular cameras and tracked by optical flow method. The measured values of attitude and angular velocity are calculated from the tracked feature points, and then the measured values of position and velocity are calculated. Then combined with the relative orbital dynamics, use the EKF to update the measurement results. So we can get the final result with less error. Finally, the effectiveness and practicability of the method are verified by simulation experiments. And the semi-physical simulation is expected to be completed in the future to verify the reliability of the algorithm.

2. Algorithm

2.1. State model

Assuming that chaser and target are in the earth orbit, as shown in Figure 1. The two orbits have a small angle, and the two spacecraft are relatively close. Their relative orbit is a small circular orbit. We define the state vector

\[
X = [q_{t}^{T} \quad \omega_{t}^{T} \quad \rho^{T} \quad \dot{\rho}^{T}]^{T}
\]  

(1)

where \(q_{t}\) denotes relative attitude quaternion of target spacecraft relative to chaser spacecraft, \(\omega_{t}\) denotes relative angular velocity, \(\rho\) denotes relative position, \(\dot{\rho}\) denotes relative velocity.

![Figure 1. Relative motion of two satellites.](image-url)
The differential equation for the relative motion of spacecrafts at close range can be simplified to the CW equation[10]:

\[
\begin{cases}
\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = f_x \\
\ddot{y} + 2\omega\dot{x} = f_y \\
\ddot{z} + \omega^2 z = f_z
\end{cases}
\]  

(2)

where \(\omega\) is average orbital angular rate. Without considering the role of active control and perturbation force, there is an analytical solution.

Define \(I_t\), \(I_c\) is the inertia tensor of the target spacecraft and chaser spacecraft, respectively. \(T_t\), \(T_c\) is the total moment acting respectively. \(\dot{\alpha}_t, \dot{\alpha}_c\) denote the angular velocities of the target spacecraft and the chaser spacecraft with respect to the geocentric system, respectively. And the total disturbance moment is Gaussian white noise, denoted as \(w_a \sim N(0, \sigma_a^2)\). Derivative of relative angular velocity is

\[
\dot{\alpha}_t = I_t^{-1}\left(T_t - \left[(\alpha_t^i + R_t^i\dot{\alpha}_t^i) \times \left(I_t (\alpha_t^i + R_t^i\dot{\alpha}_t^i)\right)\right] + \alpha_t^i \times \alpha_t^i - R_t^{i-1}T_c I_c^{-1}\left(T_c - \left[\alpha_c^i \times \left(I_c \alpha_c^i\right)\right]\right) + I_t^{-1}w_a\right)
\]  

(3)

and derivative of quaternion is

\[
\dot{q}_{ic} = \frac{1}{2} \Theta(\dot{\alpha}_t^i, \dot{\alpha}_c^i)\alpha_{ic}^i
\]  

(4)

where \(\Theta(\dot{\alpha}_t^i, \dot{\alpha}_c^i) = \Omega(\dot{\alpha}_t^i) \cdot \Gamma(\dot{\alpha}_c^i)\) , \(\Omega(\dot{\alpha}_t^i) = \begin{bmatrix} -\dot{\alpha}_t^i \times & \dot{\alpha}_t^i \\ -\alpha_t^{i T} & 0 \end{bmatrix}\), \(\Gamma(\dot{\alpha}_c^i) = \begin{bmatrix} \dot{\alpha}_c^i \times & \dot{\alpha}_c^i \\ -\alpha_c^{i T} & 0 \end{bmatrix}\).

We can indicate the state equation of the EKF system software,

\[
\Delta \dot{X} = F \Delta X + Gw
\]  

(5)

where

\[
F = \begin{bmatrix} -\dot{\alpha}_t^i \times & I_{3\times 3} & 0_{3\times 6} \\ F_{eq} & F_{eq} & 0_{3\times 6} \\ 0_{6\times 3} & 0_{6\times 3} & F_1 \end{bmatrix}
\]  

(6)

\[
F_{eq} = 2I_t^{-1}\left[-\left(I_t \left(\alpha_t^i + R_t^i\dot{\alpha}_t^i\right)\right) \times + \left(\alpha_t^i + R_t^i\dot{\alpha}_t^i\right) \times I_t\right] \cdot \left[(R_t^i\alpha_c^i) \times \right] - 2\left[\alpha_t^i \times \left(R_t^i\alpha_c^i\right)\right] \times
\]  

(7)

\[
F_{eq} = I_t^{-1}\left[-\left(\alpha_t^i + R_t^i\dot{\alpha}_t^i\right) \times \right] \left[I_t + \left(I_t \left(\alpha_t^i + R_t^i\dot{\alpha}_t^i\right)\right) \times \right] - \left(R_t^i\alpha_c^i\right) \times
\]  

(8)

\[
G = \begin{bmatrix} 0_{3\times 3} & 0_{3\times 3} \\ I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix}
\]  

(9)

2.2. measurement model

The stereo camera is composed of two monocular cameras, which use two digital images acquired simultaneously from different locations to calculate the image corresponding point parallax based on
the parallax principle to obtain the 3-dimension geometric information of the observed object. The measurement principle is shown in Figure 2.

![Figure 2. Stereo camera measurement principle.](image)

$O_L - X_L Y_L Z_L, O_R - X_R Y_R Z_R$ respectively represent the left and right camera coordinate system. $o_L - x_L y_L, o_R - x_R y_R$ respectively represent the camera imaging planes. The imaging centre coordinates are $(u_{l0}, v_{l0}), (u_{r0}, v_{r0})$. Both the left and right camera focal lengths are $f$. Let the coordinates of the matched co-view feature point under the left camera be $P_L(X, Y, Z)$, the left and right camera pixels be marked as $P_L(u_l, v_l), P_R(u_r, v_r)$. From the triangular similarity between $\Delta P_L P_P$ and $\Delta P_L O_L O_R$, we can calculate $P_L(X, Y, Z)$:

$$\begin{align*}
X &= Z \frac{u_l - u_{l0}}{f} \\
Y &= Z \frac{v_l - v_{l0}}{f} \\
Z &= \frac{fb}{u_l - u_r}
\end{align*}$$

In the whole process, we first calculate the relative attitude and angular velocity, and then use them to calculate the rest of the state quantity in the camera system. More specifically, FAST algorithm is used to extract 120 corners, and optical flow method is used to track them. The feature point management is established. By using the matching relationship between the feature points of the front and back frames, the rotation between the adjacent frames is calculated by the least square method, and the relative attitude of the current moment can be obtained by multiplying the relative attitude of the previous moment. The relative angular velocity is calculated by the differential velocity of 120 feature points. The centre of rotation is the centre of mass, so the position and velocity of the centre of mass can be obtained by using the relative attitude and angular velocity. The calculation flow is shown in Figure 3.
Figure 3. Flow chart of measurement equations.

\( \{ \rho_i(t) \}_{i=1}^N \) denotes the set of 3-dimension coordinates of \( N \) feature points under the camera system. Based on the set of characteristic point velocities \( \{ \dot{\rho}_i(t) \}_{i=1}^N \) and \( \{ \rho_i(t) \}_{i=1}^N \), the relative angular velocity can be calculated:

\[
\vec{\omega}_c(t) = \left[ \begin{array}{c} \delta \rho_1(t) \times \\
\vdots \\
\delta \rho_{N-1}(t) \times \\
\delta \rho_N(t) \times \\
\end{array} \right] - \left[ \begin{array}{c} \delta \dot{\rho}_1(t) \\
\vdots \\
\delta \dot{\rho}_{N-1}(t) \\
\delta \dot{\rho}_N(t) \\
\end{array} \right]^T
\]

(11)

Where \( a \times \) is the cross-product matrix.

Inter-frame rotation matrix \( R(t,t-1) \) can be obtained from the set of the feature point vectors of the current frame and the previous frame.

\[
\rho_i(t) = R_i(t,t-1) \rho_i(t-1)
\]

(12)

This is a Wahba problem that can be solved by the q-method to find \( q_{ct} \).

Next, position and velocity can be calculated,

\[
\begin{align*}
\rho(t) &= \frac{1}{N} \sum_{i=1}^{N} [\rho_i(t) - [R_i^c(t)r_i^c] ] \\
\dot{\rho}(t) &= \frac{1}{N} \sum_{i=1}^{N} [\dot{\rho}_i(t) - [\vec{\omega}_c^i(t) \times [R_i^c(t)r_i^c]]]
\end{align*}
\]

(13)

where \( r_i^c \) is the position of \( \rho_i \) in the target coordinate system.

2.3. Filter design

In this paper, we predict the state vector of the next moment by dynamics, get the measured value by binocular vision, and use EKF to make corrections to the state quantity, and finally get a more accurate value of the result. Table 1 shows the summary of the EKF equations.
Table 1. EKF for relative position and attitude estimation.

Initialize \( \dot{X}(t_0) = \hat{X}_0, \ P(t_0) = P_0 \)

Propagation
\[
\dot{\hat{q}}_i = \frac{1}{2} \hat{\omega}^\top_i \hat{\omega}_i, \ \hat{\omega}_i = \int \hat{\omega}_i \ dt, \ x_p = \int (Fx_p) dt,
\]
\[
P_{k+1}^{-} = \Phi_{k+1,k} P_{k} \Phi_{k+1,k}^T + \Delta G G^T
\]

Gain
\[
K_k = P_k^{-} H^T \left( HP_k^{-} H^T + R_k \right)^{-1}, \ H = I_{12}
\]

Update
\[
P_k^{+} = (I - K_k H) P_k^{-}, \ \Delta X_k^{+} = \Delta Y,
\]
\[
\Delta X = \left[ \delta q_i \right]^T \left[ \delta \omega \right]^T \left[ \delta \rho \right]^T \left[ \delta \rho \right]^T, \ q_i = \hat{q}_i + \frac{1}{2} \Xi \delta \hat{q}_i,
\]
\[
\dot{\hat{q}}_i = \dot{\hat{q}}_i + \left( \dot{\hat{q}}_i \right)^{-1} \delta \dot{q}_i + \delta \dot{\omega}_i + \Delta \delta \rho_0 + \hat{p}_0 + \Delta \hat{p}_0
\]

3. Simulations
Based on the CAD model of the real satellite, the point cloud data is generated, and 120 points are down sampled as feature points for the simulation, as shown in Figure 4. The length of the satellite is about 6 meters.

![Figure 4. Simulation feature points.](image)

According to the dynamics, the feature points are made to do with-body motion, and finally the pixel point coordinates of each feature point are simulated. The baseline of the simulation camera is 0.92m, the focal length is 0.016m, and the resolution is 1280×1280. Initial parameters of the filter are shown in Table 2. Next, we can get the final result through the filter.

Table 2. Initial parameters of filter.

| Parameter                     | Initial Value                                    |
|-------------------------------|--------------------------------------------------|
| Relative position            | \[20 \ -25 \ -50\]                               |
| Relative velocity            | \[0 \ 0.0431345 84312064 \ -0.001\]               |
| Relative attitude            | \[1 \ 0 \ 0\]                                    |
| Angular velocity of target spacecraft | \[-0.04 \ -0.06 \ 0.05\]                           |
| Angular velocity of chaser spacecraft | \[0 \ 0 \ -0.001078364607802\]                  |
moments of inertia of target spacecraft

\[ I_t = \text{diag}[8 \ 5 \ 4] \text{kg} \cdot \text{m}^2 \]

moments of inertia of chaser spacecraft

\[ I_c = \text{diag}[500 \ 550 \ 600] \text{kg} \cdot \text{m}^2 \]

covariance matrix

\[ P_0 = \text{blkdiag}[\sigma_{\eta_1}^2, \sigma_{\eta_2}^2, \sigma_{\rho}^2, \sigma_{\rho}^2] \]
\[ \sigma_{\eta_1}^2 = \text{diag}[\frac{\pi}{180}^2, \frac{\pi}{180}^2, \frac{\pi}{180}^2] \]
\[ \sigma_{\eta_2}^2 = \text{diag}[(0.1)^2 \ (0.1)^2 \ (0.1)^2] \]
\[ \sigma_{\rho}^2 = \text{diag}[(0.5)^2 \ (0.5)^2 \ (0.5)^2] \]
\[ \sigma_{\rho}^2 = \text{diag}[(0.2)^2 \ (0.2)^2 \ (0.2)^2] \]

From Figure 5, we can understand that all determination errors can converge to a small value around zero after initialization.

![Graphs of errors](chart1.png)

a) Relative position error. 

b) Relative velocity error.

c) Relative attitude error. 

d) Relative angular velocity error.

Figure 5. Estimation errors.

4. Conclusion

We propose a theoretical algorithm for relative pose measurement of non-cooperative targets based on a stereo camera and CW equation. We take the visual measurements as the observation, the CW equation as the prediction and finally filter with the EKF to obtain an unbiased estimator. Finally, the
effectiveness and reliability of the algorithm are verified by experimental simulation. Our algorithm derivation uses a real sensor mathematical model. In the future, we will use stereo camera to photograph satellite models. The guide rail and mechanical arm are used to simulate the target capture process. We will carry out the semi-physical experiments to validate our algorithms.

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