Standard-model coupling constants from compositeness

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Abstract

A coupling-constant definition is given based on the compositeness property of some particle states with respect to the elementary states of other particles. It is applied in the context of the vector-spin-1/2-particle interaction vertices of a field theory, and the standard model. The definition reproduces Weinberg’s angle in a grand-unified theory. One obtains coupling values close to the experimental ones for appropriate configurations of the standard-model vector particles, at the unification scale within grand-unified models, and at the electroweak breaking scale.
The coupling constants are the dimensionless numbers that measure the strength of nature’s interactions. Their values are fixed by experiment in the standard model (SM) of elementary particles, and depend on the energy scale. Clues to the origin of their values are suggested from the relations among the quantum numbers of the SM particles.

In general, the realization of unity among physical variables, originally thought as disconnected, has led to a new understanding and connections among additional ones. For example, by linking electric and magnetic phenomena, Maxwell’s theory showed that light is a phenomenon of the kind, and predicted its velocity in terms of likewise parameters. Indeed, recently proposed SM extensions including a unifying principle are able to provide information on the coupling constant values. Thus, grand unification[1] assumes that the gauge groups describing the interactions originate in a common group, and it predicts a single unified coupling, to which distinct couplings indeed appear to converge at high energy. It is also able to predict the coupling-constant ratios. In addition, compactification configurations of additional dimensions associated to interactions[2], and the dilaton-field ground state in string theory[3] predict their values, but, as yet, not uniquely. Information on the coupling constants may be also derived from extended-spin models[4]. Even if the underlying dynamics is not obvious, these connections may become manifest through symmetry arguments, which give additional information.

Composite models are another class of unifying theories that address the SM particle-multiplicity problem. Utilizing the connections among the quantum numbers of the 27 or so SM particles, these particles are constructed in terms of fewer elementary fields[5]. The SM Poincaré symmetry and gauge-invariant interactions provide the link.

In general, these symmetries dictate the few quantum numbers that describe a particle state. These are the configuration or momentum coordinates, the spin, the
gauge-group representation, and the flavor for quarks and leptons. Flavor characterizes only fermions. In the SM, fermions belong in the spin-1/2 Lorentz representation, and the gauge bosons are vectors. Similarly, fermions belong in the fundamental representation of the gauge group, while the vector bosons belong in the adjoint. This means that the gauge and spin quantum numbers of the latter can be constructed in terms of the former.

In the case of composite models, this facilitates their modelling in terms of simpler fields. However, it is difficult then to reproduce the SM dynamics without introducing additional fields and interactions, which, in turn, reduces the models’ predictability. Also, no additional substructure of the SM particles has been found. Another appealing idea is to assume that the vector bosons are composed of the SM fermions. A quantum electrodynamics model was proposed in which the photon is constructed from an electron and a positron[6]. This model requires an unobservable space asymmetry, and its renormalizability rules are unclear.

In this paper, we use the experimentally derived compositeness property of the SM particles to get information on the SM coupling constants. We focus on those vector quantum numbers that can be constructed in terms of those of the fermions. This is a remarkable SM property; fermions could otherwise belong to other representations transforming according to the Lorentz and gauge groups, without satisfying this property. As with grand unification, which assumes a connection among the quantum numbers of the vector bosons, this paper assumes a connection among those of the spin-1/2 particles and vector bosons. The associated symmetry provides the coupling information. In particular, the application of quantum mechanical rules leads to normalization constants, and Clebsch-Gordan coefficients that relate both representations, and ultimately relate to the coupling constants. We will also find that the grand-unified coupling ratio prescription is reproduced.

In addition, we show that this assumption is consistent with the SM. Indeed,
we apply an equivalent field-theory formulation that makes this kind of compositeness explicit, keeping the SM assumption that the fields are fundamental, unlike the composite-model case; all the SM predictions are therefore maintained. Thus, while composite models require additional fields in terms of which SM or new particles are constructed, this assumption is model independent. Hence, the putative problems associated with substructure compositeness are not encountered.

We first give a general coupling-constant definition based on the normalization and the compositeness property of some particle states with respect to other particle elementary states. Using the Wigner spinor classification of Lorentz representations, one may express SM fields in terms of their spinor components. It follows that the SM Lagrangian and its fields can be rewritten and reinterpreted in this way. Finally, we classify the configurations of the vector particles in relation to their SM and grand-unified theory content, calculate corresponding coupling values at the electroweak breaking and unification scales, and present final comments.

Quantum numbers characterize particles, and the normalized state $|w_i\rangle$ represents a particle with eigenvalue $w_i$ of the appropriate operator. The numbers $a_{ij}$ in the composite state

$$|W\rangle = \frac{1}{\sqrt{N}} \sum_{i,j} a_{ij} |w_i\rangle |w_j\rangle,$$

normalized with

$$N = \sum_{i,j} a_{ij}^* a_{ij},$$

fix $\langle w_i w_j | W \rangle$. The same amplitude is reproduced by the corresponding operator $\hat{W} = \frac{1}{\sqrt{N}} \sum_{i,j} a_{ij} |w_i\rangle \langle w_j|$, satisfying $tr \hat{W}^\dagger \hat{W} = 1$, through $\langle w_i | \hat{W} | w_j \rangle$. Thus, both structures keep the same information, and the same normalization prescription may be applied.

$\hat{W}$ is also the most general operator acting on the $|w_i\rangle$ states. Symmetry can determine the coefficients $a^\lambda_{ij}$, up to a constant, where $\lambda$ labels the representation
components of such symmetry. For example, the only (non-axial) vector operator that can be constructed out of spin-1/2 particle states is the Dirac matrix $\gamma_0 \gamma^\mu [7]$; $\partial^\mu$ stems from configuration space, and, when coupled to a vector field, it is not relevant in the SM vector-spin-1/2 interaction Lagrangian because it is neither renormalizable nor gauge invariant. For each $\mu$ (no sum) $tr\gamma_0 \gamma^\mu \gamma_0 \gamma^\mu = 4$ normalizes covariantly the operator, and fully determines it by providing the remaining constant; so is the case for the corresponding composite state $|W\rangle$. Hence, the matrix element between the spin states $|i\rangle$ and $|j\rangle$

$$\langle i | \hat{W}^\mu | j \rangle$$

(3)

is determined with $\hat{W}^\mu = \frac{1}{2} \gamma_0 \gamma^\mu$. The four-entry $\hat{W}^\mu$ acts on the space spanned essentially by the spin-1/2 particle, its antiparticle, and their two spin polarizations.

This procedure can be generalized to the case of greater number of degrees of freedom, using the rules for the direct product of vector spaces and the generalized operator that acts on such a space. The normalization for $M$ such operators, $\hat{W}^T = \hat{W}_1...\hat{W}_M$, is the product of the traces of each operator $\hat{W}_i$ in its space.

The vertex interaction Lagrangian $f \mathcal{L}_f$ with density $\mathcal{L}_f = -\frac{1}{2} g A^a_\mu \Psi^\dagger \gamma_0 \gamma^\mu G^a \Psi^\alpha$ is determined from Poincaré and gauge invariance. In general, the latter determines the interactions of the vector bosons with the other particles, and among themselves, up to the coupling constant $g$. In particular, $\mathcal{L}_f$ is the only boson-spin-1/2 vertex. In the SM the fermions belong in the fundamental representation. The vertex can be consistently viewed as the expectation value of the tensor-product operator $\hat{W}^{\mu a} = g \gamma_0 \gamma^\mu G^a 1_x 1_\alpha$, with vector components $A^a_\mu (x)$, acting upon the spin-1/2 particles $\Psi^\alpha (x)$; $\mu$ is the spin-1 index, $G^a$ the gauge-group representation matrix of the fermions, $a$ the group-representation index, $x$ the spacetime coordinate with the diagonal\(^{1}$ $1_x = |x\rangle \langle x|$, and $1_\alpha$ the unit matrix over the flavor $\alpha$. A composite state $A^a_\mu (x) |x\rangle |\mu\rangle |a\rangle$, with

\(^{1}$\(1_x\) only connects local fields, without compositeness. Formally, $a^x_{x,x'} = \delta_{x,x'} \delta_{xx'}$. $A^a_\mu (x)$ normalizes in $x$ space for $tr1_x = 1.$
\(|\mu\rangle, |a\rangle\) elements as in Eq. 1, underlies the operator association leading to \(\hat{W}^{\mu a}\): 

\[ |\mu\rangle \rightarrow (\gamma_0 \gamma^\mu)_{\sigma\eta}, |a\rangle \rightarrow G^a_{bc}, |x\rangle \rightarrow |x\rangle\langle x|; \]

the fermion state is \(\Psi^a_{\eta c}(x)\). All are written explicitly in terms of \(\sigma, \eta\) spin-1/2 indices, \(b, c\), gauge-group representation indices, and the flavor. \(A^a_\mu \hat{W}^\mu a\) is also the expression for the vector field in spin space, treated, e. g., in Ref. [8] (the same generalization is applied to the gauge degrees of freedom).

In that reference, a spinor description of the Lorentz representations is given. At each spacetime point, tensor spinorial objects are defined. In particular, a real basis of (bi)spinorial objects is constructed that spans the Lorentz vector representation. The component elements of such a basis are essentially constructed out of the unit and the Pauli matrices. A map is defined between these bispinor objects and vectors. Their identification follows from the fact that they have the same transformation properties. In fact, Maxwell’s equations can be equivalently formulated in terms of such objects, as two Dirac equations[9]. The other Lagrangian terms can also be reinterpreted and formulated in terms of spin-projected fields.

Canonical quantization in quantum field theory normalizes \(A^a_\mu\); the compositeness assumption further imposes such condition on the \(\hat{W}_i\) operators, which fully normalizes \(A^a_\mu \hat{W}^\mu a\). In general, \(A^a_\mu\) can be understood as an element in a polarization or group basis \(A^a_\mu = trn^a_\mu A^b_\nu n^{\nu b}\), where in our case \(n^{\nu b} = \hat{W}^{\nu b}\), and it is assumed to be normalized. Indeed, we recognize in the vertex

\[ \mathcal{L}_f = -\Psi^a_{\sigma b}(x)^\dagger A^a_\mu(x) \Psi^a_{\eta c}(x) \langle \sigma | \gamma_0 \gamma^\mu | \eta \rangle \frac{1}{2} g \langle b | G^a | c \rangle \]  

the matrix elements in Eq. 3, and the gauge-group ones. Within the compositeness assumption, we equate each matrix element in Eq. 4 with that of the composite vector in Eq. 3, and similarly for the group-representation matrices, all of which contain operators acting upon the spin-1/2 particles, which leads to the identification

\[ g \rightarrow 2 \sqrt{\frac{1}{N}}. \]  

(5)
The normalization $N$ is calculated as in Eq. 2, with the convention for the $\gamma$-matrices

$$tr\gamma_\mu\gamma_\nu = 4g_{\mu\nu},$$

and irreducible representations

$$trG_iG_j = 2\delta_{ij}.$$  

Essentially, we are setting normalization constants for the matrix elements in Eq. 4, which connect representations, and can be viewed as Clebsch-Gordan coefficients. $\mathcal{L}_f$ contains sums over matrix elements for each $\mu$ and $a$, which determine the coupling constant; only two polarizations $\mu$ have a physical-state interpretation, while gauge and Lorentz invariance demand a unique value. Quantum field theory admits arbitrary coupling constants for a vertex, which are obtained experimentally. The theoretical assignment of $g$ complements this theory.

In comparing the fermion states with the vector ones, we find that the latter are composite only in the Lorentz and the gauge groups, whereas the configuration variable $x$ is elementary for both types of field. In general, an additional fermion index $\beta$ independent of $A_\mu^a$ corresponds to $\hat{W}_F = \sum |\beta\rangle\langle\beta| = 1_F$, a unit operator present in the vertex, not contributing to the coupling constant. This is the flavor’s case. However, there are two consistent coupling definitions when such a kind of operator acts in a fermion subspace. Thus, e.g., $SU(2)_L$ generators in a grand-unified theory such as $SU(5)$ are constructed with their lepton ($l$) and baryon ($b$) components as $G_{SU(2)_{LL}} + (G_{SU(2)_{Lb}} \times 1_{SU(3)})$, with $1_{SU(3)}$ a projection operator in color space (leptons are color singlets); $1_{SU(3)}$ does not commute with some $SU(5)$ generators, and the associated vector-field components interact with the other unified-group ones. Physically, this full case corresponds to active degrees of freedom. In a lower energy regime, the symmetry is broken, and the interactions are truncated to the weak $SU(2)_L$ and the other SM interactions, while $1_{SU(3)}$ commutes with these generators. Then, in this reduced case, $1_{SU(3)}$ drops out of the calculation.
Grand-unified theory predicts coupling-constant ratios under the condition that the SM generators belong to the same unified-group representation, which determines Weinberg’s angle at the unification energy scale[1], and the running of the coupling of each interaction gives values at lower energies.

Similarly, the configuration of the fields’ group representations $G_i$ gives a clue to the energy scale. To obtain unified and SM couplings we specify the normalized vector-field polarizations and gauge-group generators. The couplings are calculated using the fermion quantum numbers, which are the generators’ eigenvalues, and make the generators themselves (the Cartan subset). A generation of SM left-handed [quarks; leptons] is classified by $[Q, u^c, d^c; L, e^c]$, with $L = (e, \nu)$, $Q = (u, d) \, SU(2)_L$ doublets, and $u^c, \, d^c, \, e^c$, charge-conjugate singlets, according to their color-weak-hypercharge $SU(3) \times SU(2)_L \times U(1)_Y$ groups; the latter can be viewed as subgroups of the $SU(5)$ grand-unified theory. The multiplets are $[(3, 2, 1/3), (3, 1, -4/3), (3, 1, 2/3); (1, 2, -1), (1, 1, 2)]$. The fermions fit neatly into the 5 and 10 representations of this group. The hypercharge $Y$ and the weak interaction have different $\sigma_{\mu \pm} = \frac{1}{2}(1 \pm \gamma_5)\gamma_0\gamma_\mu$ components, with the pseudoscalar $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$, which uses $L_f$ with possibly different $\hat{W}_a^\mu = \sigma_{\mu \pm} G^a_{\pm}$ components, in an obvious notation.

One gets for $Y$ in the full configuration, with the above quantum numbers, the rules in Eqs. 2 and 5, and conventions in Eqs. 6 and 7, $g' = 2/[2(2 + 2^2 + 6(\frac{1}{3})^2 + 3(\frac{2}{3})^2 + 3(\frac{4}{3})^2)]^{1/2}$ $= \frac{1}{2}\sqrt{\frac{2}{5}}$, where the 2 in the denominator normalizes each chiral component $\sigma_{\mu \pm}$, to which corresponds one massless fermion polarization. The first two terms in the parenthesis $2 + 2^2 = 1^2 + 1^2 + 2^2$ are the lepton hypercharges and, the last three are the quark hypercharges; their multiplicity is taken into account. $Y$ may be also viewed as a generator of the $SU(5)$ interaction.

Two coupling definitions apply for the weak $SU(2)_L$ interaction, one of whose generators has diagonal components $I_{(t,b)} = (1, -1)$. For the full configuration, $g'^{nni} = 2/[2(1 + 3)(1^2 + 1^2)]^{1/2} = \frac{1}{2}$, where the second factor in the denominator counts the
lepton and quark doublets, which in turn give the third factor. Non-supersymmetric unified models\cite{10} give experimentally consistent unification couplings of $g_{ex}^{uni} \sim .52 - .56$, at $10^{14} - 10^{16}$ GeV. From the SM\cite{11}-\cite{13}, $\tan(\theta_W) = g'/g_{uni}$, and one reproduces the $SU(5)$ unification result for Weinberg’s angle\cite{14} $\sin^2(\theta_{W}^{uni}) = 3/8$. In general, the coupling definition in Eq. 5 is consistent with the grand-unified prescription for such a coupling ratio.

The reduced configuration of the normalized weak vector implies that the color components drop from the calculation. It gives the same weight to quarks as to leptons, as is necessary if one omits unification-group information. We should get information on the electroweak-breaking scale to the extent that these weak and hypercharge configurations describe on-shell Z and W vector bosons. We find $g^{l,e} = 2/[2(2)(1^2 + 1^2))]^{1/2} = \frac{1}{\sqrt{2}} \approx .707$, while at the $M_Z$ scale\cite{15}, $g_{ex} = .649519(20)$, where one standard-deviation uncertainty for the last digits is given in parenthesis.

Each isospin doublet component corresponds to a different hypercharge isospin singlet; this suggests, extending the rule to color components, that only the full configuration need be considered for $Y$. Thus, $g' \approx .387$ is between $g_{ex}^{uni}(M_Z) = .35603(6)$ and the unified hypercharge values $\sqrt{\frac{3}{5}} g_{ex}^{uni} \sim .40 - .43$; the relatively narrow range provides a test of the prediction. From $\tan(\theta_W) = g'/g^{l,e}$, we find $\sin^2(\theta_W) = 3/13 \approx .23078$, while at $M_Z$ $\sin^2(\theta_{Wex}) = .23113(15)$.

One may also interpret the reduced weak configuration within the minimal supersymmetric model, with a unified\cite{16} $g_{ex}^{suni} = .69(4)$ at $10^{15.8 \pm .4}$ GeV. $g^{l,e}$ reproduces a value also within a narrow low and high energy range. For the gluons’ coupling in the $1_{SU(2)_{L}}$-reduced case we use the $\lambda_3$ Pauli-matrix fundamental component of the $SU(3)$ (any other generator would also do) with the convention of Eq. 7 $g_s = 2/[2(2)(1^2 + 1^2))]^{1/2} = 1/\sqrt{2} \approx .707$, or $\alpha_s = \frac{g_s^2}{4\pi} \approx .040$, while $\alpha_{s(ex)}(M_Z) = .1172(20)$. Then $g_s$ provides a lower limit around the unification scale.

While only the fermion-vector vertex has been examined, the results are valid for
a more general Lagrangian. The coupling constants in the other Lagrangian terms get a unique value, because gauge invariance demands it for each gauge group. All along, flavor is assumed to belong to the reduced configuration, for it does not influence interactions.

In a grand-unified theory and in the SM, the electroweak-field components at the unification scale, and at the symmetry-breaking scale, are determined, respectively, through the ratios of the electroweak couplings, namely, Weinberg’s angle. The SM fermions and bosons, and their simple interactions conform to a compositeness assumption. Under this assumption, the allowed vertices and the fields’ normalized polarization generate the coupling constants. Specifically, these are obtained by associating composite-field configurations both to the unification scale and the W and Z particle regime. Already at tree level, Weinberg’s angle is reproduced for the $SU(5)$ unified theory, and a value close to the experimental one is obtained at the $M_Z$ electroweak-breaking scale, which validates the ascribed configuration in each regime. This set of two coupling constant or Weinberg angle values provides a connection between the two energy scales through, e.g., the renormalization group equations, which have to be supplemented with boundary conditions. Although the low-energy ratio $\tan(\theta_W)$ does not contain couplings at precisely the same energy, it contains information on the group-generator structure, stemming from the compositeness assumption; this is not in contradiction with the coupling running that should be applied, and whose corrections cancel among the two couplings. The couplings are also interpreted consistently within the minimal supersymmetric model. The calculation of $\theta_W$ can be viewed as complement to, or as alternative to, that of $\theta_W^{\text{uni}}$. In the first approach, the coupling constants relate energies in the unified and symmetry-breaking scales. In the second approach, one obtains information on the $M_Z$ scale, understood as fundamental[17].

The paper’s approach may also be applied in other extensions, which require
only the consideration of reducible representations. The compositeness hypothesis is supported with coupling constants obtained among a limited number of allowed configurations, and that reproduce experimental values, which are within a narrow range at different energy scales.

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