Neutron Pairing Energy of Finite Nuclei

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Abstract

Neutron pairing energies of some of the finite nuclei have been calculated to understand how the neutron pairing changes as the neutron and proton numbers change from odd to even values in isotopes. How its value changes for even (N) - even (Z), even (N) - odd (Z), odd (N) - even (Z) and odd (N) - odd (Z) nuclei has been brought out. The values of pairing energies ($P_n$) have been calculated for light nuclei ($20 \leq A \leq 55$), medium nuclei ($100 \leq A \leq 140$) and heavy nuclei ($190 \leq A \leq 238$) and found to lie between: $-16 \leq P_n \leq +14$ MeV, $-6 \leq P_n \leq +6$ MeV and $-5 \leq P_n \leq +5$ MeV respectively. Positive pairing energies only occurred in odd (N) - even (Z) nuclei and this indicated the most stable isotopic nuclei.

Subject Areas

Applied Physics, Mechanical Engineering

Keywords

Neutron Pairing

1. Introduction

One of the purposes of nuclear physics is to investigate the interaction between the nucleons in the nucleus. This will give insight into the strong nuclear forces and pairing among the nucleons. In this study, the neutron pairing energy for odd-odd (o-o), even-even (e-e), even-odd (e-o) and odd-even (o-e) nuclei has been explored.

Nuclear forces act between pairs of nucleons only, the presence of other nucleons nearby does not influence the force law between any given two nucleons. The force is always attractive and independent of the type of nucleons interacting with each other. The nuclear binding energies and forces are on the order of a million times greater than electron binding energies of light atoms like...
hydrogen. Nuclear force is a close-range force (its strongly attractive at a distance of 1.0 fm and becomes extremely small beyond a distance of 2.5 fm), and virtually no effect of this force is observed outside the nucleus [1] [2]. Binding energy of the nucleus is given by the Einstein’s relation:

\[ B = \Delta c^2 \]  

where \( \Delta \) is the mass defect discovered in 1905 by Albert Einstein and it’s the difference between the mass of an object and the sum of the masses of its constituent particles [3].

In the periodic table of elements, the series of light elements from hydrogen up to sodium is observed to exhibit generally increasing binding energy per nucleon as atomic masses increases. This increase is generated by increasing forces per nucleon in the nucleus, as each additional nucleon is attracted by other nearby nucleons, and thus more tightly bound to the whole. At the peak of binding is nickel and iron nuclei which is the most stable and abundant [4].

The separation energy \( S_a(X) \) is the energy necessary to remove a particle \( a \) to infinity from nucleus \( X \) in its ground state, leaving residual nucleus \( Y \) also in its ground state i.e.

\[ X = Y + a \]  

The particle could be a neutron or a proton. The separation energy in terms of masses can be written as:

\[ S_a(X) = m_x c^2 = (m_p + m_n) c^2 \]  

In light nuclei the number of protons and neutrons is the same \( N = Z \) but the number of neutrons increases fast as go up to balance the repulsive force between positively charges protons so that the nucleus remains a bound system. When the number of neutrons becomes larger than protons \( N > Z \) then neutron excess parameter is given by:

\[ \eta = N - Z / A \]  

This study investigates the neutron pairing energy due to separation energy of neutrons based on if the number of neutron to proton in nuclei is e-e, o-o, o-e or e-o.

2. Theoretical Derivations

The pairing energy is estimated as the difference between the binding energy of the second neutron \( N \)(proton \( Z \)) minus that of the first neutron \( N \)(proton \( Z \)) in the case when two neutrons (protons) are successively added to the nucleus with even (neutron) or (proton) number [5]. So far it is known that for the even \( N \)- even \( Z \) nucleus in general, the pairing energy depends on the kind of particles that compose the pair, and the state occupied by the pair; and for odd—A nucleus it is smaller compared to the even—even nucleus by a factor of 1/2 to 2/3 when the pair is in the same shell and the mass number \( A \) values very close to one another [6]. The difference between two kinds of particles pairing energy
may depend on the character of the neutron-neutron, neutron-proton, or the so-called nucleon-nucleon potential.

We now derive an expression for the pairing energy of the neutron pair, denoted by: \( P_n(A,Z) \) in terms of the separation energy \( S_n \) of the neutron and the binding energy \( B(A,Z) \) of the nucleus. Consider a nucleus \( X(A+1,Z) \) and its separation energy for the neutron can be written as \( S_n(A+1,Z) \) i.e.

\[
S_n(A+1,Z) = B(A+1,Z) - B(A,Z)
\]  

Similarly the separation energies \( S_n(A,Z) \) and \( S_n(A-1,Z) \) can be written as,

\[
S_n(A,Z) = B(A,Z) - B(A-1,Z)
\]

\[
S_n(A-1,Z) = B(A-1,Z) - B(A-2,Z)
\]

Now, by definition, the pairing energy \( P_n(A,Z) \) of the neutron pair can be written as,

\[
P_n(A,Z) = S_n(A+1,Z) - 2S_n(A,Z) + S_n(A-1,Z)
\]

Equations (5)-(7) in Equation (8) gives

\[
P_n(A,Z) = B(A+1,Z) - 3B(A,Z) + 3B(A-1,Z) - B(A-2,Z)
\]

Equation (9) is used to calculate \( P_n(A,Z) \) for different isotopes.

3. Results

Tables 1-3 are obtained by using Equation (9) and the values of binding energies were obtained from [7] [8] [9].

Table 1. Neutron pairing energy for 20 ≤ A ≤ 55.

| \( Z \) | Nucleus | \( A \) | \( N \) | \( P_n \) (MeV) | Type of Pairing (\( N - Z \)) |
|------|--------|------|------|--------------|-----------------|
| 10   | Ne     | 20   | 10   | -15.333      | e – e |
| 10   | Ne     | 21   | 11   | +13.706      | o – e |
| 10   | Ne     | 22   | 12   | -8.767       | e – e |
| 11   | Na     | 23   | 12   | -6.811       | e – o |
| 12   | Mg     | 24   | 12   | -12.587      | e – e |
| 12   | Mg     | 25   | 13   | +12.963      | o – e |
| 12   | Mg     | 26   | 14   | -8.412       | e – e |
| 13   | Al     | 27   | 14   | -7.025       | e – o |
| 14   | Si     | 28   | 14   | -12.570      | e – e |
| 14   | Si     | 29   | 15   | +10.842      | o – e |
| 14   | Si     | 30   | 16   | -6.158       | e – e |
| 15   | P      | 31   | 16   | -5.367       | e – o |
| 16   | S      | 32   | 16   | -8.392       | e – e |
| 16   | S      | 33   | 17   | +9.178       | o – e |
| $Z$ | Nucleus | $A$ | $N$ | $P_n$ (MeV) | Type of Pairing |
|-----|---------|-----|-----|------------|----------------|
| 44  | Ru      | 100 | 50  | −5.073     | e – e           |
| 44  | Ru      | 101 | 57  | +5.290     | o – e           |
| 44  | Ru      | 102 | 58  | −5.406     | e – e           |
| 44  | Ru      | 104 | 60  | −5.534     | e – e           |
| 45  | Rh      | 103 | 58  | −4.198     | e – o           |
| 46  | Pd      | 102 | 56  | −5.170     | e – e           |
| 46  | Pd      | 104 | 58  | −5.300     | e – e           |
| 46  | Pd      | 105 | 59  | +5.382     | o – e           |
Continued

| Element | Atomic Number | Spin | Multiplicity | Orbital | Spin | Multiplicity | Orbital |
|---------|---------------|------|--------------|---------|------|--------------|---------|
| 46 Pd   | 106           | 60   | -5.490       | e – e   |
| 46 Pd   | 108           | 62   | -5.755       | e – e   |
| 46 Pd   | 110           | 64   | -5.711       | e – e   |
| 47 Ag   | 107           | 60   | -3.833       | e – o   |
| 47 Ag   | 101           | 62   | -4.287       | e – o   |
| 48 Cd   | 106           | 58   | -5.373       | e – e   |
| 48 Cd   | 108           | 60   | -5.414       | e – e   |
| 45 Cd   | 110           | 62   | -5.531       | e – e   |
| 48 Cd   | 111           | 63   | +5.350       | o – e   |
| 48 Cd   | 112           | 64   | -5.273       | e – e   |
| 48 Cd   | 113           | 65   | +5.396       | o – e   |
| 48 Cd   | 114           | 56   | -5.405       | e – e   |
| 48 Cd   | 116           | 68   | -5.480       | e – e   |
| 49 In   | 113           | 64   | -3.952       | e – o   |
| 49 In   | 115           | 66   | -4.016       | e – o   |
| 50 Sn   | 112           | 62   | -5.664       | e – e   |
| 50 Sn   | 114           | 64   | -5.316       | e – e   |
| 51 Sb   | 115           | 65   | +4.780       | o – e   |
| 51 Sb   | 116           | 66   | -4.639       | e – e   |
| 50 Sn   | 117           | 67   | +5.005       | o – e   |
| 50 Sn   | 118           | 68   | -5.234       | e – e   |
| 50 Sn   | 119           | 69   | +5.466       | o – e   |
| 51 Sb   | 120           | 70   | -5.557       | e – e   |
| 51 Sb   | 122           | 72   | -5.490       | e – e   |
| 50 Sn   | 124           | 74   | -5.299       | e – e   |
| 51 Sb   | 123           | 72   | -4.646       | e – o   |
| 52 Te   | 120           | 68   | -5.712       | e – e   |
| 52 Te   | 122           | 70   | -5.505       | e – e   |
| 52 Te   | 123           | 71   | +5.408       | o – e   |
| 52 Te   | 124           | 72   | -5.352       | e – e   |
| 52 Te   | 125           | 73   | +5.400       | o – e   |
| 52 Te   | 126           | 74   | -5.369       | e – e   |
| 52 Te   | 128           | 76   | -5.196       | e – e   |
| 52 Te   | 130           | 78   | -4.825       | e – e   |
| 53 I    | 127           | 74   | -4.319       | e – e   |
| 54 Xe   | 124           | 70   | -5.399       | e – e   |
| 54 Xe   | 126           | 72   | -5.201       | e – e   |
Table 3. Neutron pairing energy for $190 \leq A \leq 238$.

| $Z$ | Nucleus | $A$ | $N$ | $P_n$ (MeV) | Type of Pairing |
|-----|---------|-----|-----|-------------|-----------------|
| 76  | Os      | 190 | 114 | $-3.905$    | e – e           |
| 77  | Os      | 192 | 116 | $-3.775$    | e – e           |
| 77  | Ir      | 191 | 114 | $-3.480$    | e – o           |
| 78  | Ir      | 193 | 116 | $-3.279$    | e – o           |
| 78  | Pt      | 190 | 112 | $-4.933$    | e – e           |
| 78  | Pt      | 192 | 114 | $-4.597$    | e – e           |
| 78  | Pt      | 194 | 116 | $-4.335$    | e – e           |
| 78  | Pt      | 195 | 117 | $+0.064$    | o – e           |
| 78  | Pt      | 196 | 118 | $-3.924$    | e – e           |
| 79  | Au      | 198 | 120 | $-3.709$    | e – e           |
| 80  | Hg      | 197 | 118 | $-2.990$    | e – o           |
| 80  | Hg      | 196 | 116 | $-4.080$    | e – e           |
| 80  | Hg      | 198 | 118 | $-3.521$    | e – e           |
| 80  | Hg      | 199 | 119 | $+3.187$    | o – e           |
| 80  | Hg      | 200 | 120 | $-3.162$    | e – e           |
Continued

| 80  | Hg  | 201 | 121 | +3.320 | e – e |
|-----|-----|-----|-----|--------|-------|
| 80  | Hg  | 202 | 122 | –3.282 | o – e |
| 81  | Hg  | 204 | 124 | –3.320 | e – e |
| 81  | Tl  | 203 | 122 | –2.177 | e – o |
| 82  | Tl  | 205 | 124 | –2.249 | e – o |
| 82  | Pb  | 204 | 124 | –3.137 | e – e |
| 82  | Pb  | 206 | 124 | –2.704 | e – e |
| 82  | Pb  | 207 | 125 | +1.979 | o – e |
| 82  | Pb  | 208 | 126 | –4.068 | e – e |
| 83  | Bi  | 209 | 126 | –3.430 | e – o |
| 84  | Po  | 209 | 125 | +2.118 | e – o |
| 85  | At  | 210 | 125 | +1.911 | e – o |
| 86  | Rn  | 222 | 136 | –4.075 | e – e |
| 87  | Fr  | 223 | 136 | –2.455 | e – o |
| 88  | Ra  | 226 | 138 | –3.326 | e – e |
| 89  | Ac  | 227 | 138 | –2.273 | e – o |
| 90  | Th  | 232 | 142 | –2.976 | e – e |
| 91  | Pa  | 231 | 140 | –2.296 | e – o |
| 92  | U   | 234 | 142 | –2.632 | e – e |
| 92  | U   | 235 | 143 | +2.795 | o – e |
| 92  | U   | 238 | 146 | –2.374 | e – e |

**Table 1** gives the calculated values of pairing energies for light nuclei $20 \leq A \leq 55$ with values of pairing energies between $-16$ MeV and $+14$ MeV.

**Table 2** gives the calculated values of pairing energies for medium nuclei $100 \leq A \leq 140$ with values of pairing energies between $-6$ MeV and $+6$ MeV.

**Table 3** gives the calculated values of pairing energies for heavy nuclei $190 \leq A \leq 238$ with values of pairing energies between $-5$ MeV and $+5$ MeV.

### 4. Discussion and Conclusion

In summary, the neutron pairing energies of finite nuclei $20 \leq A \leq 238$ are calculated and the following are conclusions:

1) For light nuclei ($20 \leq A \leq 55$), the $P_n$ was found to lie between: $-16 \leq P_n \leq +14$ MeV.

2) For medium nuclei ($100 \leq A \leq 140$), the $P_n$ was found to lie between: $-6 \leq P_n \leq +6$ MeV.

3) For heavy nuclei ($190 \leq A \leq 238$), the $P_n$ was found to lie between: $-5 \leq P_n \leq +5$ MeV.

4) Positive pairing energies only occurred in odd (N) - even (Z) nuclei that indicate the most stable isotopic nuclei.
5) Negative pairing energies occurred in even \((N)\) - even \((Z)\), odd \((N)\) - odd \((Z)\) and even \((N)\) - odd \((Z)\).

6) The magnitude of the values of pairing energies decreases as you move from light to heavy nuclei.

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**Conflicts of Interest**

The author declares no conflicts of interest regarding the publication of this paper.

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