Low-Energy Supersymmetry: Prospects and Challenges

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Abstract

An introduction to the minimal supersymmetric Standard Model (MSSM) is given. The motivation for “low-energy” supersymmetry is reviewed, and the structure of the MSSM is outlined. In its most general form, the MSSM can be viewed as a low-energy effective theory parametrized by a set of arbitrary soft-supersymmetry-breaking parameters. A variety of techniques for reducing the parameter freedom of the MSSM are surveyed. The search for supersymmetry below and above the threshold for supersymmetric particle production presents a challenging task for experimentalists at present and future colliders.

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1. Motivation for Low-Energy Supersymmetry

The Standard Model of particle physics provides an extremely successful description of all particle physics phenomena accessible to present day accelerators. No unambiguous experimental deviation from the Standard Model have yet been confirmed. However, theorists strongly believe that the success of the Standard Model will not persist to higher energy scales. This belief arises from attempts to embed the Standard Model in a more fundamental theory. We know that the Standard Model cannot be the ultimate theory, valid to arbitrarily high energy scales. Even in the absence of grand unification of strong and electroweak forces at a very high energy scale, it is clear that the Standard Model must be modified to incorporate the effects of gravity at the Planck scale ($M_P \simeq 10^{19}$ GeV). In this context, it is a mystery why the ratio $m_W/M_P \simeq 10^{-17}$ is so small. This is called the hierarchy problem. Moreover, in the Standard Model, the scale of the electroweak interactions derives from an elementary scalar field which acquires a vacuum expectation value of $v = 2m_W/g = 246$ GeV. However, if one couples a theory of scalar particles to new physics at some arbitrarily high scale $\Lambda$, radiative corrections to the scalar squared-mass are of $O(\Lambda^2)$, due to the quadratic divergence in the scalar self-energy (which indicates quadratic sensitivity to the largest energy scale, $\Lambda$, in the theory). Thus, the “natural” mass for any scalar particle is $\Lambda$ (which is presumably equal to $M_P$). Of course, in order to have a successful electroweak theory, the Higgs mass must be of order the electroweak scale. The fact that the Higgs mass must not be equal to its natural value of $M_P$ is called the “naturalness” problem. It is instructive to consider the following historical precedent. In the 1920’s, quantum mechanics became the successful standard model of fundamental physics. But, this theory also possessed a disturbing hierarchy problem: why is $m_e/M_P \simeq O(10^{-22})$? A calculation of the electron self-energy using non-relativistic perturbation theory [see Fig. 1(a)] yields a mass shift that is linearly divergent. That is, the natural value for $m_e$ is the high energy scale $\Lambda$. This behavior is not surprising. After all, classically, the self-energy of an electron of radius $r$ is $e^2/r$ which diverges linearly as $1/r \rightarrow \infty$. The linear divergence persists in the relativistic single-electron quantum theory. How is this naturalness problem solved in quantum electrodynamics? The solution is remarkable. Invent a new symmetry called charge conjugation invariance ($C$). Now, double the known particle spectrum: for every particle, introduce a partner called an “antiparticle”. The $C$ symmetry guarantees that the antiparticle has the same mass and interaction strength as its partner. Now, let us reconsider the perturbation theory computation of the electron self-energy. Now, there is a second diagram to consider [see Fig. 1(b)], in which $e^+e^-\gamma$ is created from the vacuum, the $e^+\gamma$ annihilates the incom-
ing electron, while the $e^-$ just created continues to propagate. In old-fashioned time ordered perturbation theory, both time orderings must be included as shown in Fig. 1. Due to the C symmetry, the leading linear divergence cancels between the two graphs, leaving a logarithmic divergence. The mass shift of the electron is thus proportional to $e^2 m_e \ln \Lambda$. The naturalness problem is solved, since even for $\Lambda = M_P$, the radiative correction to the electron mass is of the same order as $m_e$. Of course, antiparticles were not invented to solve the naturalness problem of the single-electron quantum theory. Nevertheless, the cancelation of the linear divergence in Dirac’s theory of electrons and positrons, which was discovered by Weisskopf in 1934, was regarded as an important advance in the development of quantum electrodynamics.

Figure 1

To solve the naturalness problem of electroweak theory, we mimic the steps just outlined. In this case, we “invent” a new symmetry called supersymmetry, which transforms fermions into bosons and vice versa. Next, we double the particle spectrum: for each particle we introduce a superpartner which differs in spin by half a unit. As in Dirac’s theory of electrons and positrons, the quadratic divergence of the scalar squared-mass is exactly cancelled when the virtual exchange of superpartners is added to the contributions of the Standard Model. Thus in a supersymmetric theory, the radiative corrections to the masses of both fermions and bosons are at most logarithmically sensitive to the high energy scale $\Lambda$. Of course, the historical precedent does not provide an exact analogy. Because CPT symmetry in quantum field theory must be exactly conserved, antiparticles must be mass-degenerate with their particle partners. In contrast, supersymmetry cannot be an exact symmetry of nature, since experimental data imply that supersymmetric particles are not mass degenerate with their partners. Nevertheless, if the scale of supersymmetry breaking is of order 1 TeV or below, then the naturalness problem of the Standard Model would be resolved. In such theories of “low-energy” supersymmetry, the supersymmetry breaking scale is tied to the scale of electroweak symmetry breaking.

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In addition to providing a potential solution of the naturalness problem of the Standard Model, supersymmetry provides an attractive theoretical framework that may permit the consistent unification of particle physics and gravity. It therefore deserves serious consideration as a theory of fundamental particle interactions.

2. The Minimal Supersymmetric Standard Model (MSSM)

The minimal supersymmetric extension of the Standard Model (MSSM) consists of taking the Standard Model and adding the corresponding supersymmetric partners. In addition, the MSSM contains two hypercharge $Y = ±1$ Higgs doublets, which is the minimal structure for the Higgs sector of an anomaly-free supersymmetric extension of the Standard Model. The supersymmetric structure of the theory also requires (at least) two Higgs doublets to generate mass for both “up”-type and “down”-type quarks (and charged leptons). All renormalizable supersymmetric interactions consistent with (global) $B – L$ conservation ($B =$baryon number and $L =$lepton number) are included. Finally, the most general soft-supersymmetry-breaking terms are added. If supersymmetry is relevant for explaining the scale of electroweak interactions, then the mass parameters associated with the soft-supersymmetry-breaking terms must be of order 1 TeV or below. Some bounds on these parameters exist due to the absence of supersymmetric particle production at current accelerators; see for a complete listing of supersymmetric particle mass limits. Additional constraints arise from limits on the contributions of virtual supersymmetric particle exchange to a variety of Standard Model processes. The impact of precision electroweak measurements at LEP and SLC on the MSSM parameter space is discussed briefly in section 4.

As a consequence of $B – L$ invariance, the MSSM possesses a discrete $R$-parity invariance, where $R = (-1)^{3(B–L)+2S}$ for a particle of spin $S$. Note that this formula implies that all the ordinary Standard Model particles have even $R$-parity, whereas the corresponding supersymmetric partners have odd $R$-parity. The conservation of $R$-parity in scattering and decay processes has a crucial impact on supersymmetric phenomenology. For example, starting from an initial state involving ordinary ($R$-even) particles, it follows that supersymmetric particles must be produced in pairs. In general, these particles are highly unstable and decay quickly into lighter states. However, $R$-parity invariance also implies that the lightest supersymmetric particle (LSP) is absolutely stable, and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle. In order to be consistent with cosmological constraints, the LSP is almost certainly electrically and color neutral. Consequently, the LSP is weakly-interacting in ordinary matter, i.e. it behaves like a heavy stable neutrino and will escape detectors without being directly observed. Thus, the canonical signature for $R$-parity conserving supersymmetric theories is missing (transverse) energy, due to the escape of the LSP. Some model builders attempt to relax the assumption of $R$-parity conservation. Models of the type must break $B – L$ and are therefore strongly constrained by experiment. In such models, the LSP is unstable and supersymmetric particles can be singly produced and
destroyed in association with $B$ or $L$ violation. These features lead to a phenomenology of broken-$R$-parity models that is very different from that of the MSSM.

The parameters of the MSSM are conveniently described by considering separately the supersymmetry-conserving sector and the supersymmetry-breaking sector. Supersymmetry breaking is accomplished by including the most general set of soft-supersymmetry breaking terms; these terms parametrize our ignorance of the fundamental mechanism of supersymmetry breaking. A careful discussion of the conventions used in defining the MSSM parameters can be found in. Among the parameters of the supersymmetry conserving sector are: (i) gauge couplings: $g_s$, $g$, and $g'$, corresponding to the Standard Model gauge group SU(3)$\times$SU(2)$\times$U(1) respectively; (ii) Higgs Yukawa couplings: $\lambda_e$, $\lambda_u$, and $\lambda_d$ (which are $3 \times 3$ matrices in flavor space); and (iii) a supersymmetry-conserving Higgs mass parameter $\mu$. The supersymmetry-breaking sector contains the following set of parameters: (i) gaugino Majorana masses $M_3$, $M_2$ and $M_1$ associated with the SU(3), SU(2), and U(1) subgroups of the Standard Model; (ii) scalar mass matrices for the squarks and sleptons; (iii) Higgs-squark-squark trilinear interaction terms (the so-called “$A$-parameters”) and corresponding terms involving the sleptons; and (iv) three scalar Higgs mass parameters—two diagonal and one off-diagonal mass terms for the two Higgs doublets. These three mass parameters can be re-expressed in terms of the two Higgs vacuum expectation values, $v_1$ and $v_2$, and one physical Higgs mass. Here, $v_1$ ($v_2$) is the vacuum expectation value of the Higgs field which couples exclusively to down-type (up-type) quarks and leptons. Note that $v_1^2 + v_2^2 = (246 \text{ GeV})^2$ is fixed by the $W$ mass, while the ratio

$$\tan \beta = v_2/v_1$$

is a free parameter of the model.

The supersymmetric constraints imply that the MSSM Higgs sector is automatically CP-conserving (at tree-level). Thus, $\tan \beta$ is a real parameter (conventionally chosen to be positive), and the physical neutral Higgs scalars are CP-eigenstates. Nevertheless, the MSSM does contain a number of possible new sources of CP violation. For example, gaugino mass parameters, the $A$-parameters, and $\mu$ may be complex. Some combination of these complex phases must be less than of order $10^{-2}$--$10^{-3}$ (for a supersymmetry-breaking scale of 100 GeV) to avoid generating electric dipole moments for the neutron, electron, and atoms in conflict with observed data. However, these complex phases have little impact on the direct searches for supersymmetric particles, and are usually ignored in experimental analyses.

Before describing the supersymmetric particle sector, let us consider the Higgs sector of the MSSM. There are five physical Higgs particles in this model: a charged Higgs pair ($H^\pm$), two CP-even neutral Higgs bosons (denoted by $h^0$ and $H^0$ where $m_{h^0} \leq m_{H^0}$) and one CP-odd neutral Higgs boson ($A^0$). The properties of the Higgs sector are determined by the Higgs potential which is made up of quadratic terms [whose squared-mass coefficients were mentioned above eq. (1)] and quartic interaction terms. The strengths of the interaction terms are directly related to the gauge couplings
by supersymmetry. As a result, \( \tan \beta \) [defined in eq. (1)] and one Higgs mass determine: the Higgs spectrum, an angle \( \alpha \) [which indicates the amount of mixing of the original \( Y = \pm 1 \) Higgs doublet states in the physical CP-even scalars], and the Higgs boson couplings. When one-loop radiative corrections are incorporated, additional parameters of the supersymmetric model enter via virtual loops. The impact of these corrections can be significant. For example, at tree-level, the MSSM predicts \( m_{h^0} \leq m_Z \). If true, this would imply that experiments to be performed at LEP-2 operating at its maximum energy and luminosity would rule out the MSSM if \( h^0 \) were not found. However, this Higgs mass bound can be violated when the radiative corrections are incorporated. For example, in, the following approximate upper bound was obtained for \( m_{h^0} \) (assuming \( m_{A^0} > m_Z \)) in the limit of \( m_Z \ll m_t \ll M_\tilde{t} \) [where top-squark \((\tilde{t}_L - \tilde{t}_R)\) mixing is neglected]

\[
m_{h^0}^2 \lesssim m_Z^2 + \frac{3g^2 m_Z^4}{16\pi^2 m_W^2} \left\{ \frac{2m_t^4 - m_t^2 m_Z^2}{m_Z^4} \ln \left( \frac{M_\tilde{t}^2}{m_t^2} \right) + \frac{m_t^2}{3m_Z^2} \right\}.
\]

(2)

More refined computations (which include the effects of top-squark mixing at one-loop, renormalization group improvement, and the leading two-loop contributions) yield \( m_{h^0} \lesssim 125 \) GeV for \( m_t = 175 \) GeV and a top-squark mass of \( M_\tilde{t} = 1 \) TeV. Clearly, the radiative corrections to the Higgs masses have a significant impact on the search for the MSSM Higgs bosons at LEP.

Consider next the supersymmetric particle sector of the MSSM. The \emph{gluino} is the color octet Majorana fermion partner of the gluon with mass \( M_\tilde{g} = |M_3| \). The supersymmetric partners of the electroweak gauge and Higgs bosons (the \emph{gauginos} and \emph{higgsinos}) can mix. As a result, the physical mass eigenstates are model-dependent linear combinations of these states, called \emph{charginos} and \emph{neutralinos}, which are obtained by diagonalizing the corresponding mass matrices. The chargino mass matrix depends on \( M_2, \mu, \tan \beta \) and \( m_W \). The corresponding chargino mass eigenstates are denoted by \( \tilde{\chi}_1^\pm \) and \( \tilde{\chi}_2^\pm \), with masses

\[
M^2_{\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm} = \frac{1}{2} \left\{ |\mu|^2 + |M_2|^2 + 2m_W^2 \mp \left[ (|\mu|^2 + |M_2|^2 + 2m_W^2)^2 - 4|\mu|^2 |M_2|^2 - 4m_W^4 \sin^2 2\beta + 8m_W^2 \sin 2\beta \text{Re}(\mu M_2) \right]^{1/2} \right\},
\]

(3)

where the states are ordered such that \( M_{\tilde{\chi}_1^\pm} \leq M_{\tilde{\chi}_2^\pm} \). If CP-violating effects are ignored (in which case, \( M_2 \) and \( \mu \) are real parameters), then one can choose a convention where \( \tan \beta \) and \( M_2 \) are positive. (Note that the relative sign of \( M_2 \) and \( \mu \) is meaningful. The sign of \( \mu \) is convention-dependent; the reader is warned that both sign conventions appear in the literature.) The sign convention for \( \mu \) implicit in eq. (3) is used by the LEP collaborations in their plots of exclusion contours in the \( M_2 \) vs. \( \mu \) plane derived from the non-observation of \( Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \). The \( 4 \times 4 \) neutralino mass matrix depends on
$M_1$, $M_2$, $\mu$, $\tan \beta$, $m_Z$, and the weak mixing angle $\theta_W$. The corresponding neutralino eigenstates are usually denoted by $\tilde{\chi}^0_i$ ($i = 1, \ldots, 4$), according to the convention that $M_{\tilde{\chi}^0_1} \leq M_{\tilde{\chi}^0_2} \leq M_{\tilde{\chi}^0_3} \leq M_{\tilde{\chi}^0_4}$. Typically, $\tilde{\chi}^0_1$ is the LSP.

It is common practice in the literature to reduce the supersymmetric parameter freedom by requiring that all three gaugino mass parameters are equal at some grand unification scale. Then, at the electroweak scale, the gaugino mass parameters can be expressed in terms of one of them (say, $M_2$) and the gauge coupling constants:

$$M_3 = (g_2^2/g^2)M_2, \quad M_1 = (5g'^2/3g^2)M_2.$$  \hfill (4)

Having made this assumption, the chargino and neutralino masses and mixing angles depend only on three unknown parameters: the gluino mass, $\mu$, and $\tan \beta$. However, the assumption of gaugino mass unification could prove false and must eventually be tested experimentally.

The supersymmetric partners of the quarks and leptons are spin-zero bosons: the squarks, charged sleptons, and sneutrinos. For a given fermion $f$, there are two supersymmetric partners $\tilde{f}_L$ and $\tilde{f}_R$ which are scalar partners of the corresponding left and right-handed fermion. (There is no $\tilde{\nu}_R$.) However, in general, $\tilde{f}_L$ and $\tilde{f}_R$ are not mass-eigenstates since there is $\tilde{f}_L - \tilde{f}_R$ mixing which is proportional in strength to the corresponding element of the scalar squared-mass matrix.

$$M^2_{LR} = \begin{cases} m_d(A_d - \mu \tan \beta), & \text{for “down”-type } f \\ m_u(A_u - \mu \cot \beta), & \text{for “up”-type } f, \end{cases}$$  \hfill (5)

where $m_d$ ($m_u$) is the mass of the appropriate “down” (“up”) type quark or lepton. Here, $A_d$ and $A_u$ are (unknown) soft-supersymmetry-breaking $A$-parameters and $\mu$ and $\tan \beta$ have been defined earlier. The signs of the $A$ parameters are also convention-dependent; see $\tilde{\nu}_R$. Due to the appearance of the fermion mass in eq. (5), one expects $M_{LR}$ to be small compared to the diagonal squark and slepton masses, with the possible exception of the top-squark, since $m_t$ is large, and the bottom-squark and tau-slepton if $\tan \beta \gg 1$. The (diagonal) $L$ and $R$-type squark and slepton masses are given by:

$$M^2_{u_L} = M^2_Q + m^2_u + m^2_Z \cos 2\beta(1 - \frac{2}{3} \sin^2 \theta_W)$$  \hfill (6)
$$M^2_{u_R} = M^2_U + m^2_u + \frac{2}{3} m^2_Z \cos 2\beta \sin^2 \theta_W$$  \hfill (7)
$$M^2_{d_L} = M^2_Q + m^2_d - m^2_Z \cos 2\beta(1 - \frac{1}{3} \sin^2 \theta_W)$$  \hfill (8)
$$M^2_{d_R} = M^2_D + m^2_d - \frac{1}{3} m^2_Z \cos 2\beta \sin^2 \theta_W$$  \hfill (9)
$$M^2_{\nu} = M^2_L + \frac{1}{2} m^2_Z \cos 2\beta$$  \hfill (10)
$$M^2_{e_L} = M^2_L + m^2_e - m^2_Z \cos 2\beta(1 - \sin^2 \theta_W)$$  \hfill (11)
$$M^2_{e_R} = M^2_E + m^2_e - m^2_Z \cos 2\beta \sin^2 \theta_W.$$  \hfill (12)

The soft-supersymmetry-breaking parameters: $M_Q$, $M_U$, $M_D$, $M_L$, and $M_E$ are unknown parameters. In the equations above, the notation of first generation fermions
has been used and generational indices have been suppressed. Further complications such as intergenerational mixing are possible, although there are some constraints from the nonobservation of flavor-changing neutral currents (FCNC).

3. Reducing the Supersymmetric Parameter Freedom

One way to guarantee the absence of significant FCNC’s mediated by virtual supersymmetric particle exchange is to posit that the diagonal soft-supersymmetry-breaking scalar squared-masses are universal in flavor space at some energy scale (normally taken to be at or near the Planck scale). Renormalization group evolution is used to determine the low-energy values for the scalar mass parameters listed above. This assumption substantially reduces the MSSM parameter freedom. For example, supersymmetric grand unified models with universal scalar masses at the Planck scale typically give $M_{\tilde{L}} \approx M_{\tilde{E}} \approx M_{\tilde{D}}$ with the squark masses somewhere between a factor of 1–3 larger than the slepton masses (neglecting generational distinctions). More specifically, the first two generations are thought to be nearly degenerate in mass, while $M_{\tilde{Q}}$ and $M_{\tilde{U}}$ are typically reduced by a factor of 1–3 from the other soft-supersymmetry-breaking masses because of renormalization effects due to the heavy top quark mass. As a result, four flavors of squarks (with two squark eigenstates per flavor) and $\tilde{b}_R$ will be nearly mass-degenerate and somewhat heavier than six flavors of nearly mass-degenerate sleptons (with two per flavor for the charged sleptons and one per flavor for the sneutrinos). On the other hand, the $\tilde{b}_L$ mass and the diagonal $\tilde{t}_L$ and $\tilde{t}_R$ masses are reduced compared to the common squark mass of the first two generations. In addition, third generation squark masses and tau-slepton masses are sensitive to the strength of the respective $\tilde{f}_L-\tilde{f}_R$ mixing as discussed below eq. (5).

Two additional theoretical frameworks are often introduced to reduce further the MSSM parameter freedom. The first involves grand unified theories (GUTs) and the desert hypothesis (i.e. no new physics between the TeV-scale and the GUT-scale). Perhaps one of the most compelling hints for low-energy supersymmetry is the unification of SU(3)×SU(2)×U(1) gauge couplings predicted by supersymmetric GUT models (with the supersymmetry breaking scale of order 1 TeV or below). The unification, which takes place at an energy scale of order $10^{16}$ GeV, is quite robust (and depends weakly on the details of the GUT-scale theory). For example, a recent analysis finds that supersymmetric GUT unification implies that $\alpha_s(m_Z) = 0.129 \pm 0.010$, not including threshold corrections due to GUT-scale particles (which could diminish the value of $\alpha_s(m_Z)$). This result is compatible with the world average of $\alpha_s(m_Z) = 0.117 \pm 0.005$. In contrast, gauge coupling unification in the simplest nonsupersymmetric GUT models fails by many standard deviations. Grand unification can impose additional constraints through the unification of Higgs-fermion Yukawa couplings ($\lambda_f$). There is some evidence that $\lambda_b = \lambda_t$ leads to good low-energy phenomenology and an intriguing possibility that in the MSSM (in the parameter regime where $\tan \beta \approx m_t/m_b$) $\lambda_b = \lambda_t = \lambda_\tau$ may be phenomenologically viable. However, such unification constraints are GUT-model dependent, and do not address the
origin of the first and second generation fermion masses and the CKM mixing matrix. Finally, grand unification imposes constraints on the soft-supersymmetry-breaking parameters. For example, gaugino mass unification leads to the relations given in eq. (4). Diagonal squark and slepton soft-supersymmetry-breaking scalar masses may also be unified at the GUT scale (analogous to the unification of Higgs-fermion Yukawa couplings).

In order to further reduce the number of independent soft-supersymmetry breaking parameters (with or without grand unification), an additional simplifying assumption is required. In the minimal supergravity theory, the soft supersymmetry-breaking parameters are often taken to have the following simple form. Referring to the parameter list given above eq. (1), the Planck-scale values of the soft-supersymmetry-breaking terms depend on the following minimal set of parameters: (i) a universal gaugino mass $m_{1/2}$; (ii) a universal diagonal scalar mass parameter $m_0$ [whose consequences were described at the beginning of this section]; (iii) a universal $A$-parameter, $A_0$; and (iv) three scalar Higgs mass parameters—two common diagonal squared-masses given by $|\mu_0|^2 + m_0^2$ and an off-diagonal squared-mass given by $B_0\mu_0$ (which defines the Planck-scale supersymmetry-breaking parameter $B_0$), where $\mu_0$ is the Planck-scale value of the $\mu$-parameter. As before, renormalization group evolution is used to compute the low-energy values of the supersymmetry-breaking parameters and determines the supersymmetric particle spectrum. Moreover, in this approach, electroweak symmetry breaking is induced radiatively if one of the Higgs diagonal squared-masses is forced negative by the evolution. This occurs in models with a large Higgs-top quark Yukawa coupling (i.e., large $m_t$). As a result, the two Higgs vacuum expectation values (or equivalently, $m_Z$ and $\tan \beta$) can be expressed as a function of the Planck-scale supergravity parameters. The simplest procedure is to remove $\mu_0$ and $B_0$ in favor of $m_Z$ and $\tan \beta$ (the sign of $\mu_0$ is not fixed in this process). In this case, the MSSM spectrum and its interactions are determined by $m_0$, $A_0$, $m_{1/2}$, $\tan \beta$, and the sign of $\mu_0$ (in addition to the parameters of the Standard Model). However, the minimal approach above is probably too restrictive. Theoretical considerations suggest that the universality of Planck-scale soft-supersymmetry breaking parameters is not generic. In the absence of a fundamental theory of supersymmetry breaking, further progress will require a detailed knowledge of the supersymmetric particle spectrum in order to determine the nature of the Planck-scale parameters.

4. Challenges for Supersymmetry Searches

The verification of low-energy supersymmetry requires the discovery of the supersymmetric particles. Once superpartners are discovered, it is necessary to test their detailed properties to verify the supersymmetric nature of their interactions. Furthermore, one can explicitly test many of the additional theoretical assumptions of section 3 that were introduced to reduce the supersymmetric parameter freedom.

The search for supersymmetry at present and future colliders falls into two distinct classes. At colliders whose energies lie below supersymmetric particle production
threshold, indirect effects of supersymmetry may be observable. For example, in the Higgs sector, if $m_{A^0} \gg m_{h^0}$, then the properties of $h^0$ will be nearly indistinguishable from the Higgs boson of the minimal Standard Model. Small deviations from the Standard Model Higgs sector could signal the existence of additional Higgs states, as expected in the MSSM. One can also search for deviations from Standard Model predictions due to the effects of virtual supersymmetric particle exchange. Such effects could be revealed in the measurement of precision electroweak observables. In both cases, one is fighting the decoupling limit. That is, in the limit that soft-supersymmetry-breaking masses (collectively denoted by $M_{\text{SUSY}}$) become large, the MSSM below supersymmetric threshold precisely reproduces the predictions of the Standard Model. At colliders whose energies lie above supersymmetric particle production threshold, the direct effects of supersymmetric production and decay are detectable. In this case, once superpartners are discovered, one must elucidate the details of the low-energy supersymmetric theory.

The MSSM (with or without constraints imposed from the theory near the Planck scale) provides a framework that can be tested by precision electroweak data. The level of accuracy of the measured $Z$ decay observables at LEP and SLC is sufficient to test the structure of the one-loop radiative corrections of the electroweak model and is thus potentially sensitive to the virtual effects of undiscovered particles. Combining the most recent LEP and SLC electroweak results with the recent top-quark mass measurement at the Tevatron, a weak preference is found for a light Higgs boson mass of order $m_Z$, which is consistent with the MSSM Higgs mass upper bound noted in section 2. Moreover, for $Z$ decay observables, the effects of virtual supersymmetric particle exchange are suppressed by a factor of $m_Z^2/M_{\text{SUSY}}^2$, and therefore decouple in the limit of large supersymmetric particle masses. It follows that for $M_{\text{SUSY}}^2 \gg m_Z^2$ (in practice, it is sufficient to have all supersymmetric particle masses above 200 GeV) the MSSM yields an equally good fit to the precision electroweak data as compared to the Standard Model fit. On the other hand, there are a few tantalizing hints in the data for deviations from Standard Model predictions. Indeed, if $R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ is confirmed to lie above its Standard Model prediction due to the presence of new physics, then a plausible candidate for the new physics would be the MSSM with some light supersymmetric particles (e.g. a light chargino and top-squark and/or a light CP-odd scalar, $A^0$) close in mass to their present LEP bounds. Such a scenario would be tested by the search for supersymmetric particles at LEP-2 and the Tevatron.

If low-energy supersymmetry exists, it should be discovered at either upgrades of existing colliders or at the LHC. Due to its mass reach, the LHC is the definitive machine for discovering or excluding low-energy supersymmetry. Table 1 summarizes the supersymmetry mass discovery potential for hadron colliders. A variety of signatures are considered. Many of the supersymmetry searches rely on the missing energy signature as an indication of new physics beyond the Standard Model. Multi-leptonic signatures also play an important role in supersymmetry searches at hadron collid-
ers. (Such signals can also be exploited in the search for \( R \)-parity-violating low-energy supersymmetry.) A comprehensive analysis can be found in\(^{49}\).

Suppose that a signal is observed in one of the expected channels. This would not be a confirmation of low-energy supersymmetry, unless there is confirming evidence from other expected signatures. This presents a formidable challenge to experimenters at the LHC. Can they prove that a set of signatures of new physics is low-energy supersymmetry? Can they extract parameters of the supersymmetric models with any precision and test the details of the theory? These are questions that have only recently attracted serious study. It is in this context that a future \( e^+e^- \) collider (NLC) can be invaluable. If the lightest supersymmetric particles were produced at LEP-2 or the NLC, precision measurements could begin to map out in detail the parameter space of the supersymmetric model. In particular, beam polarization at the NLC provides an critical tool for studying the relation between chirality and the properties of supersymmetric particles\(^{52}\). One can then begin to demonstrate that there is a correlation between the left and right-handed electrons and their slepton partners as expected in supersymmetry. Moreover, the determination of superparticle masses allows one to test theoretical assumptions at various levels. For example, the universality of slepton masses can be tested at the 1% level. More experimentally challenging is the test of the

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Table 1. Discovery reach of various options of future hadron colliders\(^{22}\). The numbers are subject to \( \pm 15\% \) ambiguity. Also, the clean trilepton signals are sensitive to other model parameters; representative ranges from Ref\(^{51}\) are shown where \(|\mu|\) is typically much larger than the soft-breaking electroweak gaugino masses. For \( \mu > 0 \), the leptonic decay of \( \tilde{\chi}^0_2 \) may be strongly suppressed so that \( 3\ell \) signals may not be observable even if charginos are just above the LEP bound.

| Signal | Tevatron I | Tevatron II | Main Injector | Tevatron* | DiTevatron | LHC |
|--------|------------|-------------|---------------|-----------|------------|-----|
|        | 0.01 fb\(^{-1}\) | 0.1 fb\(^{-1}\) | 1 fb\(^{-1}\) | 10 fb\(^{-1}\) | 1 fb\(^{-1}\) | 10 fb\(^{-1}\) |
| \( B_T(\tilde{q} \gg \tilde{g}) \) | \( \tilde{g}(150) \) | \( \tilde{g}(210) \) | \( \tilde{g}(270) \) | \( \tilde{g}(340) \) | \( \tilde{g}(450) \) | \( \tilde{g}(1300) \) |
| \( \ell^\pm \ell^\pm (\tilde{q} \gg \tilde{g}) \) | \( \tilde{g}(160) \) | \( \tilde{g}(210) \) | \( \tilde{g}(270) \) | \( \tilde{g}(320) \) | \( \tilde{g}(1000) \) |
| \( all \rightarrow 3\ell (\tilde{q} \gg \tilde{g}) \) | \( \tilde{g}(150-180) \) | \( \tilde{g}(150-260) \) | \( \tilde{g}(150-430) \) | \( \tilde{g}(150-320) \) |
| \( B_T(\tilde{q} \sim \tilde{g}) \) | \( \tilde{g}(220) \) | \( \tilde{g}(300) \) | \( \tilde{g}(350) \) | \( \tilde{g}(400) \) | \( \tilde{g}(580) \) | \( \tilde{g}(2000) \) |
| \( \ell^\pm \ell^\pm (\tilde{q} \sim \tilde{g}) \) | \( \tilde{g}(180-230) \) | \( \tilde{g}(325) \) | \( \tilde{g}(385-405) \) | \( \tilde{g}(460) \) | \( \tilde{g}(1000) \) |
| \( all \rightarrow 3\ell (\tilde{q} \sim \tilde{g}) \) | \( \tilde{g}(240-290) \) | \( \tilde{g}(425-440) \) | \( \tilde{g}(550) \) | \( \tilde{g}(550) \) | \( \tilde{g}(1000) \) |
| \( \tilde{t}_1 \rightarrow c \tilde{\chi}^0_1 \) | \( \tilde{t}_1(80-100) \) | \( \tilde{t}_1(120) \) |
| \( \tilde{t}_1 \rightarrow b \tilde{\chi}^+_1 \) | \( \tilde{t}_1(80-100) \) | \( \tilde{t}_1(120) \) |
| \( \Theta(\tilde{t}_1 \tilde{t}_1^*) \rightarrow \gamma \gamma \) | \( \tilde{t}_1(250) \) |
| \( \tilde{\ell}^* \) | \( \tilde{\ell}(50) \) | \( \tilde{\ell}(50) \) | \( \tilde{\ell}(100) \) | \( \tilde{\ell}(250-300) \) |
GUT-relation among gaugino masses [eq. (4)]. However, one can still test eq. (4) at the few percent level by combining slepton and chargino signals, based on the measured masses and polarization dependence of the cross sections. See[4] for further details.

Many theorists believe that the prospects for supersymmetry are excellent. Nevertheless, the search for supersymmetry at future colliders may reveal many surprises and raise new challenges for both theorists and experimentalists. If low-energy supersymmetry is discovered it will have a profound effect on the development of 21st century theories of fundamental particles and their interactions.

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