Large Distance Modification of Newtonian Potential and Structure Formation in Universe

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In this paper, we study the effects of super-light brane world perturbative modes on structure formation in our universe. As these modes modify the large distance behavior of Newtonian potential, they effect the clustering of a system of galaxies. So, we explicitly calculate the clustering of galaxies interacting through such a modified Newtonian potential. We use a suitable approximation for analyzing this system of galaxies, and discuss the validity of such approximations. We observe that such corrections also modify the virial theorem for such a system of galaxies.

I. INTRODUCTION

We approximate the galaxies as point particles, and analyze the clustering of a system of such galaxies. This approximation will be valid as the distance between two galaxies is many orders of magnitude larger than the size of a single galaxy. Thus, we use techniques of standard statistical mechanical to analyze the clustering of a system of galaxies. It may be noted that such an analysis has already been performed using the usual Newtonian potential, and thus the techniques of statistical mechanics has already been used to analyze the clustering of galaxies. However, we have to either consider dark matter, or a modified Newtonian potential to explain the physics at large scales, we will analyze the clustering of galaxies using a Newtonian potential modified by super-light modes of a brane world model. We would like to point out that in this formalism a cosmic energy equation for a system of galaxies was obtained using the standard techniques of statistical mechanics. This was used to analyze the clustering of a system of galaxies using correlation functions. In this formalism, the correlation function and the power spectrum characterize the distribution of galaxies in clusters and superclusters.

So, in this paper, we analyze the clustering of galaxies using this formalism. In fact, we will use the large distance corrections to the Newtonian potential from super-light modes in brane world models. These cosmological models have been motivated from string theory due to extra dimensions in string theory. In these brane world models, our universe is a brane in a higher dimensional bulk. These models have been used for resolving the hierarchy problem and the weakness of gravitational force in comparison with other three fundamental forces. In fact, even though there are different models for brane world theories, a common feature of all of these different models is that the standard model fields are confined to the four dimensional brane and the gravitons propagate into the bulk confined to the brane.
and thus can propagate into the higher dimensional bulk \[16–18\]. Due to the propagation of gravity into higher dimensions, the Newtonian potential gets brane corrections. Furthermore, as the general relativity along with its Newtonian approximation have not been tested at very large or very small distances, it is possible that the Newtonian potential would get modified at such distances. Generally, Newtonian potential may be modified due to several effects like dark energy \[19, 20\]. So, usually, the corrections generated from brane world gravity modify the Newtonian potential at small distances \[13, 21\], and these modifications cannot produce any new astrophysical or cosmological effects. However, it is possible to obtain super-light perturbation modes in brane world models, and these super-light modes can modify gravity at large scales \[12, 22\]. The importance of the models with super-light perturbation modes is due to the fact that these predict a modification of the gravitational interaction for matter on the brane at astronomical scales. The corrections to Newton’s gravity due to such consideration may be promising for resolving the issue of dark matter in galaxies and galaxy clusters and even the cosmological dark energy problem \[12\]. The form of corrected Newtonian potential is given as \[23\]

\[
\phi = \phi_N \left(1 + \frac{k}{r^2}\right),
\]

where \(\phi_N\) is the standard Newtonian potential given by,

\[
\phi_N = -\frac{Gm^2}{r}.
\]

It may be noted that this long distance correction scales as \(1/r^3\), which is unlike the short distance correction which scales as \(1/r^2\) \[12\]. As this correction changes the Newtonian potential at large distances, this can be used in analyzing the dynamics of galaxies \[24–27\]. In fact, it has been demonstrated that the brane world models can explain the rotation curve of galaxies better than the models which are based on the existence of dark matter \[28\].

It may be pointed out that phenomenologically motivated modified theories of gravity (MOG) \[22, 30\] have been used as an alternative to dark matter. In fact, modified Newtonian dynamics (MOND) \[31\] and MOG \[32, 33\], as two possible modified theories of gravity have been used to obtain the correct rotation curves of galaxies. The MOG modifies the large distance behavior of Newtonian potential \[32, 33\], and this modification produces the correct rotation curve of galaxies. Thus, it is important to consider large distance correction to Newtonian potential for analyzing astrophysical phenomena. An advantage of using the corrections from super-light perturbation modes in brane world models is that such corrections are motivated from theoretical considerations and cosmological models motivated from string theory, but they can also have interesting phenomenologically applications \[24–27\]. In this paper, we will use this long distance correction to the Newtonian potential produced by super-light modes, and analyze its effect on the clustering of galaxies.

Moreover, one may note that even though there are problems with certain distance based modifications of gravity, such as MOND, in order to explain the clustering of galaxies \[34–36\], it has been argued that other kind of modifications to gravity can explain clustering of galaxies \[37, 38\]. In fact, it is possible to modify MOND in such a way, that force law approximates MOND at large and intermediate accelerations, and gets further modified at ultra-low accelerations. Such ultra-low accelerations are relevant to the galaxy clusters, and such a modification has been observed to be consistent with the observations \[39–42\]. It has been demonstrated that MOG, which modifies the Newtonian law of gravitation, can consistently explain the clustering of galaxies \[43, 44\]. It is also possible to explain the clustering of galaxies without dark matter by using a modified theory of gravity based on covariant Galileon model \[46\]. So, even though the modification of gravity such as MOND cannot be used to analyze the clustering of galaxies, it is possible to have alternative theories of gravity, which may explain such a model.

So, even though there are problems with MOND in explaining clustering of galaxies, it is possible to consider other models of modified gravity, which do not have above discussed problems. Furthermore, as the clustering of galaxies has been studied using techniques of statistical mechanics with Newtonian potential \[47–52\], it would be both important and interesting to generalize such an analysis of modified law of Newtonian gravity. Even though there might still be problems with such an approach, it would
be a better approximation to explain the clustering of galaxies. We could improve this analysis further by incorporating dark matter, but the use of modified Newtonian potential would produce better results than the standard Newtonian potential. It has been argued that the modified theories of gravity are produced from dark matter models. It has also been discussed that clustering can be explained using brane world models. So, this motives us to use techniques of statistical mechanics, with brane world modified Newtonian potential, to galactic clustering.

II. CLUSTERING PARAMETER

In this section, we review the clustering of galaxy and exact equation of states in brane world corrected Newtonian potential. It is possible to consider super-light modes in a brane world models, and they modify the large distance behavior of the Newton’s law as

\[
\Phi_{i,j} = -\frac{Gm^2}{(r_{ij}^2 + \epsilon^2)^{1/2}} \left(1 + \frac{k_l}{(r_{ij}^2 + \epsilon^2)}\right),
\]

where relative position vector (between i and j particles) is \(r_{ij} >> \Lambda = |k_l|^{1/2}\), where \(\Lambda\) is considered as a typical length scale at which correction due to these super-light modes becomes dominant. The parameter \(\epsilon\) is a regularization parameter, which occurs due to the extended structure of galaxies. The reason for considering extended structure is as following. It is clear from expression (1) that the potential energy diverges for the point-mass (i.e., \(r = 0\)) nature of galaxies. This will lead to a divergence in the Hamiltonian and, consequently, to the partition function. This divergence can be removed by considering extended nature of galaxies (galaxies with halos) with the help of the softening parameter \(\epsilon\), which assures that the galaxies are of finite size. In units of the constant cell. It may be noted that at small enough distances this modified Newtonian potential reduces to the usual Newtonian potential. This is the limit in which the contribution from these super-light modes can be neglected. Furthermore, this is required from the physical constraints, as the Newtonian limit of general relativity has been well tested at such scales. Now it is possible to obtain the two-particle function form this modified potential as

\[
f_{i,j} = \exp \left[\frac{Gm^2}{T(r_{ij}^2 + \epsilon^2)^{1/2}} \left(1 + \frac{k_l}{(r_{ij}^2 + \epsilon^2)}\right)\right] - 1.
\]

This will further lead to the modification of configurational integrals \(Q_N\). For instance, the configurational integral for \(N = 1\), \(Q_1(T, V) = V\), and (for large \(r\) where the higher terms of \(\frac{k_l}{(r^2 + \epsilon^2)^{1/2}}\) can be neglected) the configurational integral for \(N = 2\),

\[
Q_2(T, V) = 4\pi V \int_0^{R_1} \left[r^2 + \left(Gm^2 \frac{T}{r_{ij}^2 + \epsilon^2}\right) \left(1 + \frac{k_l}{(r^2 + \epsilon^2)^{1/2}}\right)\right] dr.
\]

Evaluating the integrals, we obtain

\[
Q_2(T, V) = V^2 (1 + \alpha_1 x + \alpha_2 x),
\]

where \(x = \frac{3}{2}G^3m^6\rho T^{-3}\) and

\[
\alpha_1 = \sqrt{\frac{1 + \epsilon^2}{R_1^2} + \frac{\epsilon^2}{R_1^2}} \log \frac{\epsilon}{R_1 + \sqrt{R_1^2 + \epsilon^2}},
\]

\[
\alpha_2 = -2\frac{k_l}{R_1 \sqrt{R_1^2 + \epsilon^2}} + 2\frac{k_l}{R_1^2} \log \frac{R_1 + \sqrt{R_1^2 + \epsilon^2}}{\epsilon}.
\]
FIG. 1: Typical behavior of Helmholtz free energy in terms of $N$. We set unit values for all parameters. Blue dotted line represents the case of $\epsilon = k_l = 0$, green dashed line represents the case of $\epsilon = 1$, $k_l = 0$, orange dash dotted line represents the case of $\epsilon = 1$, $k_l = 0.6$, red solid line represents the case of $\epsilon = 1$, $k_l = 1$, violet space dash line represents the case of $\epsilon = 1$, $k_l = 1.4$.

Now, we write the most general form for the configurational integrals after including modified potential energy as

$$Q_N(T, V) = V^N (1 + \alpha_1 x + \alpha_2 x)^{N-1},$$

$$= V^N (1 + Ax)^{N-1},$$

(9)

where we have defined $A = \alpha_1 + \alpha_2$, and these are in turn obtained by solving the configuration integral. Using this expression, the partition function for the system of galaxies can be written as

$$Z_N(T, V) = \frac{1}{N!} \left( \frac{2\pi m T}{\lambda^2} \right)^{3N/2} V^N (1 + Ax)^{N-1}. $$

(10)

Here $\lambda$ refers the normalization factor resulting from integration over momentum space. It may be noted that this partition function is expressed as a sum over different order terms, and for a given system, we can restrict this to a certain order of accuracy.

We find that the partition function in increasing function of the super-light parameter, however at low temperature, variation with $k_l$ is infinitesimal. Furthermore, as this is the partition function of a system for galaxies interacting through a modified Newtonian potential, we can use it to analyze the effect of super-light modes on the thermodynamics of this system.

Thus, the Helmholtz free energy of a system of galaxies corrected by super-light modes in a brane world model can be written as

$$F = -T \ln \left( \frac{1}{N!} \left( \frac{2\pi m T}{\lambda^2} \right)^{3N/2} V^N (1 + Ax)^{N-1} \right).$$

(11)

In the Fig. 1 we can see typical behavior of Helmholtz free energy in terms of $N$ by variation of $k_l$. We find that it is decreasing function of $k_l$ in negative region, which means increasing net value of Helmholtz free energy under brane world parameter. There is special case where cases of $\epsilon = k_l = 0$ and $\epsilon = 1$, $k_l = 1.4$ yields to the same Helmholtz free energy. Now, let us consider a large number of galaxies, i.e., we take a large value of $N$, and use $N - 1 \approx N$. As the inter-galactic distance are very large, an collision...
FIG. 2: Typical behavior of internal energy in terms of $T$ for $N = 50$ and we set unit values for all parameters. Blue dotted line represents the case of $\epsilon = k_l = 0$, green dashed line represents the case of $\epsilon = 1$, $k_l = 0$, orange dash-dotted line represents the case of $\epsilon = 1$, $k_l = 0.6$, red solid line represents the case of $\epsilon = 1$, $k_l = 1$.

of galaxies will not usually occur, and can be neglected. Thus, we can write the entropy $S$ of a system of galaxies corrected by super-light modes as

$$S = N \ln(\bar{\rho}^{-1}T^{3/2}) + N \ln(1 + Ax) - 3NB + S_0,$$

(12)

where $S_0 = \frac{5}{2} + \frac{3}{2} \ln\left(\frac{2\pi m}{\lambda^2}\right)$ and clustering parameter is

$$B = \frac{Ax}{1 + Ax}.$$

(13)

Thus, the clustering parameter depends on $A$, which is obtained by solving the configurational integral for this system corrected by super-light brane modes. The internal energy $U$, which is defined as $U = F + TS$, can be written as

$$U = \frac{3}{2} NT(1 - 2B).$$

(14)

It may be noted that this internal energy depends on the clustering parameter, which in turn depends on $A$, and that is obtained as a solution to the configurational integral for a system corrected by these super-light modes. Thus, the internal energy of this system will also depend on these super-light modes. In the Fig. 2 we can see that internal energy is decreasing function of $k_l$. We can also see that there is a minimum with negative value of $U$. However, at high temperature and low temperature limit there is no effect with $k_l$.

We can also write an expression for the pressure of this system, using the following thermodynamics relation,

$$P = -\left(\frac{\partial F}{\partial V}\right)_{N,T},$$

(15)
FIG. 3: Typical behavior of chemical potential in terms of $T$. We set unit values for all parameters. Blue dotted line represents the case of $k_l = 0$, green dashed line represents the case of $k_l = 0.6$, red solid line represents the case of $k_l = 1$.

which yields to the following expression [61]

$$P = \frac{NT}{V} (1 - B). \quad (16)$$

The chemical potential of this system of galaxies can be expressed as [61],

$$\frac{\mu}{T} = \ln(\bar{\rho}T^{-3/2}) - \ln(1 + Ax) - \frac{3}{2} \ln \left( \frac{2\pi m}{\lambda^2} \right) - B. \quad (17)$$

In the Fig. 3, we can see that chemical potential is a decreasing function of $k_l$. We can also see that there is a minimum with negative value of $\mu$. However, at high temperature and low temperature limit there is no effect with $k_l$.

The inclusion of the super-light modes modifies the behavior of $B$, and this in turn modifies the thermodynamics of the system.

III. VIRIAL THEOREM

In this section, we will analyze the effect of super-light brane modes, on the virial theorem for galaxies. This has been studied for the usual Newtonian potential by analyzing the adiabatic growth of gravitational clustering [63]. So, for such a system, the first law of thermodynamics can be written as

$$\frac{d(uR^3)}{dt} + P \frac{dR^3}{dt} = 0, \quad (18)$$

where $u$ is the energy density of considered matter, $P$ is the pressure and $R(t)$ is the scale factor. Now it is possible to consider a spherical system of volume $V$ which contains $N$ galaxies. It is possible to associate an thermodynamic temperature $T$, energy $U$, and pressure $P$ with this system of galaxies. This has been done for a system of galaxies interacting through the usual Newtonian potential [64], and here
we obtain this for a system of galaxies interacting thought a potential modifies by super-light brane world modes, and we get

\[ U = \frac{3}{2} NT + \frac{N \bar{\rho}}{2} \int_V \Phi(r) \xi(r) 4\pi r^2 dr, \]  

(19)

\[ P = \frac{NT}{V} - \frac{\bar{\rho}^2}{6} \int_V r \frac{d\Phi(r)}{dr} \xi(r) 4\pi r^2 dr. \]  

(20)

Now using the explicit expression for the interaction large distance modified Newtonian potential between two galaxies, we can write expression for energy and pressure for this system as

\[ U = \frac{3}{2} NT + W_N + W_M, \]  

(21)

\[ P = \frac{3NT + W_N + \epsilon^2 W'_N + 3W_M + 3\epsilon^2 W'_M}{3V}. \]  

(22)

where the large distance corrected gravitational correlation energies can be written as

\[ W_N = \frac{GN \bar{\rho} m^2}{2} \int_V \frac{\xi(r)}{(r^2 + \epsilon^2)^\frac{3}{2}} 4\pi r^2 dr, \]

(23)

\[ W'_N = -\frac{GN \bar{\rho} m^2}{2} \int_V \frac{\xi(r)}{(r^2 + \epsilon^2)^\frac{3}{2}} 4\pi r^2 dr, \]

\[ W_M = \frac{k_l GN \bar{\rho} m^2}{2} \int_V \frac{\xi(r)}{(r^2 + \epsilon^2)^\frac{5}{2}} 4\pi r^2 dr, \]

\[ W'_M = -\frac{k_l GN \bar{\rho} m^2}{2} \int_V \frac{\xi(r)}{(r^2 + \epsilon^2)^\frac{5}{2}} 4\pi r^2 dr. \]

(23)

As the total energy for a spherical volume of radius \( R \) can be written as \( U = (4/3)\pi u R^3 \), and so from (21), we get

\[ \frac{4}{3} \pi u R^3 = \frac{3}{2} NT + W_N + W_M = K + W, \]  

(24)

where \( W = W_N + W_M \). Exploiting (22), the relation \( \frac{d}{dt} R^3 = \frac{\dot{R}}{R}(3R^3) \) reduces to

\[ \frac{d}{dt} (R^3) = \frac{\dot{R}}{R} \left[ \frac{3}{4\pi} \frac{2K + W_N + \epsilon^2 W'_N + 3W_M + 3\epsilon^2 W'_M}{P} \right], \]  

(25)

where we have used \( V = \frac{4}{3} \pi R^3 \).

The law of conservation of energy for this system of galaxies can be written as

\[ \frac{d}{dt} (K + W) + \frac{\dot{R}}{R} (2K + W_N + \epsilon^2 W'_N + 3W_M + 3\epsilon^2 W'_M) = 0. \]  

(26)

In the limit case, when we neglect the expansion, i.e., \( \dot{R} = 0 \), we obtain

\[ 2K + W_N + \epsilon^2 W'_N + 3W_M + 3\epsilon^2 W'_M = 0. \]  

(27)

This is the virial theorem for the system of galaxies interacting by the large distance modified Newtonian potential. It can be used to understand the effects of super-light brane modes on the virial theorem. The general equation for the law of conservation of energy is given by Eq. (26), and it can lead interesting limiting cases. This equation for an extended structures with Newtonian potential can be written as

\[ \frac{d}{dt} (K + W) + \frac{\dot{R}}{R} (2K + W_N + \epsilon^2 W'_N) = 0. \]  

(28)
Now, if we neglect the effects of an extended structure of galaxies, i.e. \( \epsilon = 0 \), and so in this limit, a system interacting by the Newtonian potential, can be described by

\[
\frac{d}{dt}(K + W) + \frac{\dot{R}}{R}(2K + W_N) = 0. \tag{29}
\]

By neglecting the extended structure of galaxies, and only considering the modification by super-light brane modes, we obtain,

\[
\frac{d}{dt}(K + W) + \frac{\dot{R}}{R}(2K + W_N + 3W_M) = 0. \tag{30}
\]

Thus, we have analyzed the virial theorem for a system of galaxies using the large distance modification to the Newtonian potential due to super-light modes in the brane world models.

### IV. VALIDITY OF EXTENSIVITY

In this paper we have used extensivity, and this approximation is valid for infinite systems whose thermodynamic functions are extensive. An requirement for such an approximations is that the correlation energy between the cells should be less than the correlation energy within an average cell \[65\]. We assume the size of the cells is much larger than the correlation length, and a power law behavior for the two-point correlation function, which is given by \[66\]

\[
\xi(r) = \xi_0 r^{-\gamma}, \tag{31}
\]

where \( \gamma \sim 1.8 \) is a constant parameter.

The correlation energy for an extended mass in a spherical volume \( V \) is given by

\[
W_M(V) = -\frac{Gm^2 \bar{\rho}^2 V}{2} I_1, \tag{32}
\]

where

\[
I_1 = \int_0^{R_1} \left[ \frac{1}{(r^2 + \epsilon^2)^{1/2}} \left( 1 + \frac{k_l}{r^2 + \epsilon^2} \right) \right] \frac{\xi_0}{r^\gamma} 4\pi r^2 dr. \tag{33}
\]

Thus, for a system of galaxies interacting by a Newtonian potential modified by super-light brane modes, we obtain

\[
W_M(V) = W_N(V) \left[ 1 + \left( \frac{2 - \gamma}{2\gamma} + \frac{2\gamma}{\gamma + 2R_1^2} \right) \frac{\epsilon^2}{R_1^2} + \left( \frac{2 - \gamma}{-\gamma} \frac{k_l}{R_1^2} \right) \right], \tag{34}
\]

where

\[
W_N(V) = -2\pi Gm^2 \bar{\rho}^2 V \xi_0 \frac{R_1^{2-\gamma}}{2 - \gamma}. \tag{35}
\]

Now using \( 2V \), this expression for \( W_N \) can be written as

\[
W_N(2V) = -2\pi Gm^2 \bar{\rho}^2 (2V) \xi_0 \frac{(2^{1/3}R_1)^{2-\gamma}}{2 - \gamma}. \tag{36}
\]

Thus, we can write \( W_M \) as

\[
W_M(2V) = 2(2^{1/3})^{2-\gamma} W_N(V) I_2, \tag{37}
\]
where
\[ I_2 = \left[ 1 + \left( \frac{2 - \gamma}{2\gamma} + \frac{2\gamma}{\gamma + 2} \frac{k_l}{(2^{1/3} R_1)^2} \right) \frac{\epsilon^2}{(2^{1/3} R_1)^2} \right. \]
\[ + \left. \left( \frac{2 - \gamma}{\gamma - 2} \frac{k_l}{(2^{1/3} R_1)^2} \right) \right]. \] (38)

So, the correlation energy for an extended mass in a spherical volume \( V \), can be expressed as
\[ W_M(V) = W_N(V) \left[ 1 + f(\gamma, 2, \frac{\epsilon}{R_1}) \right], \] (39)
where
\[ f(\gamma, 2, \frac{\epsilon}{R_1}) = \left( \frac{2 - \gamma}{2\gamma} + \frac{2\gamma}{\gamma + 2} \frac{k_l}{(2^{1/3} R_1)^2} \right) \frac{\epsilon^2}{R_1^2} + \left( \frac{2 - \gamma}{\gamma - 2} \frac{k_l}{R_1^2} \right). \] (40)

However, the correlation energy for an extended mass in \( {2V} \) can be written as
\[ W_M(2V) = 2(2^{1/3})^{2-\gamma} W_N(V) \left[ 1 + h(\gamma, 2, \frac{\epsilon}{R_1}) \right], \] (41)
with
\[ h(\gamma, 2, \frac{\epsilon}{R_1}) = \left( \frac{2 - \gamma}{2\gamma} + \frac{\gamma}{\gamma + 2} \frac{6k_l}{(2^{1/3} R_1)^2} \right) \frac{\epsilon^2}{(2^{1/3} R_1)^2} \]
\[ - \left( \frac{2 - \gamma}{\gamma} \frac{k_l}{(2^{1/3} R_1)^2} \right). \] (42)

For extensivity to be a valid approximation, we should have \( |W_{2V}/W_V| \sim 1 \). Thus, we obtain
\[ \left| \frac{W_{M2V}}{2W_{MV}} \right| = (2^{1/3})^{2-\gamma} g(\gamma, 2, \epsilon/R_1), \] (43)
where
\[ g(\gamma, l, \epsilon/R_1) = \frac{1 + f(\gamma, 2, \epsilon/R_1)}{1 + h(\gamma, 2, \epsilon/R_1)}. \] (44)

Here, we observe that the validity of the extensivity approximation depends upon the values of \( \gamma \).

V. COSMIC ENERGY EQUATION

The cosmic energy equation has been studied for a system of galaxies interacting through a Newtonian potential \([6, 67]\). In this section, we will analyze the effect of super-light modes on the cosmic energy equation. Thus, we can start from the expression
\[ \frac{d}{dt} (K + W) + \frac{R}{R} (2K + W_N + \epsilon^2 W'_N + 3W_M + 3\epsilon^2 W'_M) = 0. \] (45)
This can be written as
\[ \frac{d}{dt} (K + W) + \frac{R}{R} (2K + W_N + \eta W_N) = 0, \] (46)
where $\eta$ is a constant defined by
\begin{equation}
\eta = \epsilon^2 \frac{W'_N}{W_N} + 3 \frac{W_M}{W_N} + 3 \epsilon^2 \frac{W'_M}{W_N}.
\end{equation}
(47)
The ratio of gravitational correlation energy to the kinetic energy, for this system of interacting extended mass galaxies can be written as
\begin{equation}
B = -\frac{W}{2K}.
\end{equation}
(48)
This has been analyzed for a system of galaxies interacting through a usual Newtonian potential [6, 67], and we will analyze the correction to that from super-light brane modes. Thus, for a system of galaxies interacting thought a large distance modified brane world Newtonian potential, we obtain
\begin{equation}
\frac{dy_1}{dt} - \frac{2}{W} \frac{dW}{dt} - 2 \frac{\ddot{R}}{R} (1 - y_1 + \eta) = 0,
\end{equation}
(49)
where $y_1 = 1/B$. Now, using a relation
\begin{equation}
y_1 = \frac{y + A - 1}{A},
\end{equation}
(50)
where $A = \alpha_1 + \alpha_2$ and $y = 1/B_0$, where $B_0$ is the ratio of correlation energy to the kinetic energy for point mass galaxies. In fact, these values are obtained by solving the configurational integrals. The evolution, of this system of galaxies interacting through large distance modified Newtonian potential is given by
\begin{equation}
\frac{dy}{dt} - \frac{1 + A - y}{W} \frac{dW}{dt} - 2 \frac{\ddot{R}}{R} (1 - y + A\eta) = 0.
\end{equation}
(51)
The expansion factor $R(t)$ depends on time $t$ as
\begin{equation}
R(t) \propto t^s.
\end{equation}
(52)
This is a general expression valid for any cosmological model for a constant $s$. Now, we can write $W_N + W_M$ as
\begin{equation}
W(t) \propto t^\beta.
\end{equation}
(53)
This equation can be obtained by using $\frac{4}{dt} = \dot{R} \frac{dR}{dt}$, and the BBGKY hierarchy method [65]. Here we have considered $(\ddot{\rho} \propto R^{-3})$. Thus, we can use such a power law dependence for $R(t)$ and $W(t)$ to solve (51). Using (54) and (55), Eq. (51) reduces to
\begin{equation}
\frac{dy(t)}{dt} = \frac{(1 + A - y)\beta}{t} + \frac{2s(1 - y + A\eta)}{t},
\end{equation}
(54)
where $s = 1/2, 2/3$ and 1 corresponds to Dirac, Einstein-de Sitter and Mihe Universes, respectively. The above equation can be solved for $y(t)$, and so we obtain
\begin{equation}
y(t) = y_c + (y_0 - y_c) t^{-\beta - 2s},
\end{equation}
(55)
where $y_0$ corresponds to $y$ at $t = 0$. The asymptotic value (or critical value) of $y(t)$ is given by:
\begin{equation}
y_c = \frac{1}{B_c} = \frac{A(\beta + 2\eta s) + \beta + 2s}{\beta + 2s}.
\end{equation}
(56)
This has been done for a system of galaxies interacting through a usual potential [6], and here we will analyze the effects of large distance corrected brane world Newtonian potential on this system. Thus,
again $B = B_c$ will not depend on the present value $B$, and can be expressed in terms of $\beta$. The value of $\beta$ is sensitive to the primordial power spectrum law, i.e., $n$, and is roughly fitted, by simulations, to

$$\beta \sim \frac{1 - n}{3}. \quad (57)$$

For this value of $\beta$, $B_c$ in (56) becomes,

$$B_c \sim \frac{(1 - n) + 6s}{A(1 - n + 6\eta s) + 1 - n + 6s}. \quad (58)$$

So, the value of $B_c$ depends on $A$, but the value of $A$ depends on the super-light brane modes. Thus, the super-light brane modes would modify the clustering properties of a system of galaxies interacting.

VI. CONCLUSION

In this paper, we have analyzed the thermodynamics of gravitational clustering of galaxies in brane world models. We have analyzed the modification to the Newtonian potential produced by super-light brane modes. We have used an adiabatic approximation for performing this analysis. We calculated partition function and found that is increasing function of super-light parameter, analyzing of Helmholtz free energy shows that is decreasing function of super-light parameter and yields to negative infinity for the large super-light parameter. We also found that the internal energy as well as chemical potential is decreasing function of super-light parameter. We observed that the clustering parameter gets modified in the brane world models. We calculate the corrections to the virial theorem from brane world models. Further, we have discussed the correction to the validity of extensivity, which is influenced by $\gamma$. The corrections to the clustering parameter modify the cosmic energy equation. We also solved the corrected cosmic energy equation and analyzed the asymptotic behavior. It was observed that the large scale modification of Newtonian potential directly affects the solutions of the cosmic energy equation.

The modification to the Newtonian potential considered in this paper, occurred due to the super-light modes in the brane world models. It may be noted that the Newtonian potential also gets modified from various different approaches. These include non-commutative geometry \cite{69, 70}, minimal length in quantum gravity \cite{71, f(R) gravity \cite{72, 73}, dark energy \cite{74} and the entropic force \cite{75}. It would be possible to analyze super-light modes in such deformed theories, and this will have an effect on the large scale behavior of Newtonian potential. Thus, we can use such a large scale corrected Newtonian potential, and analyze its effects on clustering of galaxies. So, it would be interesting to analyze the clustering using these modifications to the Newtonian potential. It would also be interesting to analyze the effect of these modifications to the Newtonian potential on the cosmic energy equation. It is expected that the virial theorem will get corrected due to the deformation of the Newtonian potential. The correction to the virial theorem will effect the cosmic energy equation.

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