CASIMIR ENERGY IN THE SKYRME MODEL

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ABSTRACT

The so called Casimir energy embodies the $O((N_c)^0)$ contribution to the skyrmion mass according to the semi-classical expansion theory of solitons. We claim that this contribution can be accurately estimated despite the fact that some of the counterterms, provided in principle by chiral perturbation theory, are not known in practice. Using $\zeta$ function techniques we show that $E_{\text{cas}} = E_{\text{cas}}(\mu) + E_{\text{ct}}(\mu)$ where $E_{\text{cas}}(\mu) >> E_{\text{ct}}(\mu)$ because it incorporates all the zero-mode contributions and it can be exactly calculated. Our results confirm that the fourth order Skyrme lagrangian does not seem to provide a correct description of the lightest baryons as solitons. We show that a simple extension to order six gives, on the contrary, good results without tuning the parameters of the chiral lagrangian.

1. Introduction

It is a common experience to most skyrmion practitioners that the mass tends to come out too high. Even if one uses rather sophisticated chiral lagrangians, incorporated vector and axial vector mesons (e.g. [1]) one finds the predicted nucleon mass to lie nearly 50% too high and the situation becomes much worse in SU(3) extensions. Most other observables, in contrast to that, seem to be predicted with a much better accuracy. This has led to the belief that the mass may have a somewhat special status.

In this talk, I would like to try to convince you that this is not the case and that the problem with the mass has a very natural explanation once one tries to perform the semi-classical (which is also the $1/N_c$) expansion in a systematic way. This, in fact, is not usually done in the context of the skyrmion. Following ref. [2], one identifies collective coordinates and quantization is effected only at the level of these coordinates. If one does this for the mass, one finds besides the classical $O(N_c)$ contribution a $O(1/N_c)$ one. This procedure gives no contribution of order $N_c^0$. This contribution is precisely what one calls the ”Casimir energy” (CE) and its evaluation requires that one deals with the non-collective coordinates.

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What one has to do in principle can be inferred from the semi-classical soliton theory which was developed a long time ago (see e.g. [3]). In practice, however, one seems to face the difficulty that in early work, solitons were taken to live in 1+1 dimensions with a renormalizable lagrangian, while the skyrmion lives in 3+1 dimensions and the lagrangian is not renormalizable. A safe approach to this problem is to use a lagrangian involving only chiral field degrees of freedom. This is after all what Skyrme originally did and we nowadays have a theory, Chiral perturbation theory (ChPT) [4][5], which precisely gives us such a lagrangian and furthermore tells us how to make sense out of loops. Quite remarkably, recent determinations of the values of the parameters appearing in the lagrangian of ChPT turn out to be surprisingly close to those which Skyrme has guessed thirty years ago.

2. Basic ideas and formulae

Vibrational degrees of freedom for the skyrmion were first considered in ref. [6] who did not attempt, however, a complete evaluation of their role. One approximation scheme to actually calculate the CE was proposed by Schnitzer [7] who found a large positive value (of the order of 500 MeV). Later on, the presence of vector mesons in the lagrangian were found (in the same approximation) to considerably reduce this effect to a negligible 50 MeV [8]. Concurrently, an estimate was made in ref. [9] which makes use of the relationship between the CE and the effective action to one-loop. They recognized that an ultra-violet divergence was present and they used a derivative expansion approximation of the effective action. A somewhat better approximation was proposed later on [10] which seems nevertheless to yield a rather similar result of approximately $-200$ MeV.

How come that all these results are different from each other? The answer is that one must be very careful with the approximations that one makes. Most approximations will in fact kill the most important part of the effect. To show this, we will start from a formula which is exact and which relates the CE to the pion-skyrmion phase shifts. The correct size of the effect will be controlled by the fact that the phase-shifts are very large at the origin because there are six zero-modes (associated with three translations and three rotations). Any approximation of the Born type for the phase-shifts will be essentially incorrect. An approximation which respects the low-energy behaviour of the phases was proposed in ref. [11].

Let us start with a simple 1+1 dimensional situation as a warm up. Many features are exactly similar to the 3 dimensional case. Consider, for instance, the action:

\[ S = \frac{1}{2} \int d^2 x \left( (\partial_{\mu} \phi)^2 + m^2 \phi^2 - \frac{\lambda}{2} \phi^4 \right) \]  

It is convenient to rescale the coordinates and the fields:

\[ \phi \rightarrow \frac{m}{\sqrt{\lambda}} \phi \quad x \rightarrow \frac{1}{m} x \]
so the action now looks like:

$$S = \frac{m^3}{2\lambda} \int d^2x \left( (\partial_\mu \phi)^2 + \phi^2 - \frac{1}{2} \phi^4 \right)$$  \hspace{1cm} (2)

We see that $1/\lambda$ appears in front of the action so the semi-classical expansion is identical to the weak coupling expansion. We have exactly the same situation in three dimensions with $N_c$ replacing $1/\lambda$. The classical solution (the "kink") and the classical mass are easily found:

$$\phi_{\text{class}} = \tanh(\frac{x - x_0}{\sqrt{2}}), \quad M_{\text{class}} = \frac{2\sqrt{2}m^3}{3\lambda}$$  \hspace{1cm} (3)

Next, one has to consider fluctuations around the classical solution and it is not difficult to show that the leading correction to the classical mass (i.e. of order $\lambda^0$) has the following formal expression \[3\]:

$$M^{(0)} = \frac{1}{2} \left[ \text{Tr} \left( -\partial_x^2 + 3 \tanh^2 \left( \frac{x}{\sqrt{2}} \right) - 1 \right)^{\frac{1}{2}} - \text{Tr} \left( -\partial_x^2 + 2 \right)^{\frac{1}{2}} \right] \equiv \frac{1}{2} (\text{Tr} H_{\frac{1}{2}} - \text{Tr} H_0^{\frac{1}{2}})$$  \hspace{1cm} (4)

Obviously if we rewrite the trace in terms of eigenvalues we obtain that

$$M^{(0)} = \sum \omega_n - \sum \omega_n^0$$  \hspace{1cm} (5)

which is why this contribution is called the Casimir energy, by analogy with the classic QED effect \[12\]. The operator involved in (4) is obtained by expanding the action to second order in powers of the fluctuation. The corresponding operator for the skyrmion is obtained by exactly the same procedure. A careful derivation can be found in ref.\[13\].

As was mentioned already, an extremely useful formula for practical purposes arises upon expressing (4) in terms of phase shifts\[14\]. A simple direct derivation, valid for any space dimension, is as follows. One first uses the identities:

$$\text{Tr} H_{\frac{1}{2}} = 2\text{Tr} \int_M^\infty dE E^2 \delta(E^2 - H) = -\frac{1}{i\pi} \text{Tr} \int_M^\infty dE E \left( \frac{E}{E^2 - H + i\epsilon} - \frac{E}{E^2 - H - i\epsilon} \right)$$  \hspace{1cm} (6)

where $M$ is the lower bound of the continuous spectrum (in the kink example $M = \sqrt{2m}$). Next one recognizes that:

$$\text{Tr} \left( \frac{E}{E^2 - H + i\epsilon} - \frac{E}{E^2 - H_0 + i\epsilon} \right) = \frac{d}{dE} \ln \Delta^+,$$  \hspace{1cm} (7)

where

$$\Delta^+ = \det \left( 1 - \frac{1}{E^2 - H_0 + i\epsilon} (H - H_0) \right)$$  \hspace{1cm} (8)

So we obtain a formula:

$$M^{(0)} = \frac{1}{4i\pi} \int_M^\infty dE E \frac{d}{dE} \left( \ln \Delta^+ - \ln \Delta^- \right)$$  \hspace{1cm} (9)
This takes care of the continuous part of the spectrum. Obviously, one must also add the discrete part in the trace if there is any. Now $\Delta^\pm$ is nothing but the Fredholm determinant. In one space dimension it is a well know fact from scattering theory that its phase is the phase-shift $[13]$

$$\Delta^\pm = |\Delta| \exp(\pm i\delta(E)) \quad (10)$$

so our expression (4) becomes:

$$M^{(0)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dE E \delta'(E) \quad (11)$$

In order to generalize to higher dimensions we must return to (9). If $H - H_0$ is a radial potential for instance one can again express the phase of the Fredholm determinant in terms of the phase-shifts of the radial operators $\delta(E) \equiv \text{phase}(\Delta^+) = \sum (2j + 1)\delta_j(E)$. The generalization to the skyrmion case is obvious: for every value of the grand-spin quantum number, $J$, we have three eigen-phase-shifts $[16]$ so we must sum over these before summing over $J$.

At this point it would seem that we are running into trouble. A look at refs. $[16]$ reveals that (except for $J=0$) the phase-shifts are linearly diverging functions of $E$ and one can show that after performing the $J$ sum things become worse: one ends up with a cubic divergence. Even in one dimension, in fact, the integral (11) is logarithmically divergent. In that case, however, it is enough to remember that at order one in $\hbar$ there is an extra contribution to the energy which comes from the one-loop counterterm in the lagrangian. Once this is taken into account the divergence disappears. We will see later how one can generalize this mechanism to the skyrmion. To begin with, we must discuss the lagrangian.

3. Chiral perturbation theory and the skyrmion:

In the limit where the masses of the $N_f$ light quarks are set to zero the QCD lagrangian is invariant under the chiral $\text{SU}(N_f) \times \text{SU}(N_f)$ group. This invariance is spontaneously broken by the QCD vacuum down to $\text{SU}(N_f)$. The spectrum thus consists of $N_f$ zero-mass goldstone bosons and there is a mass gap of around $\Lambda = 1$ GeV above which one finds meson resonances, baryons, etc....ChPT is a systematic framework for describing low energy phenomena (with typical energy $E$) as an expansion in powers of $E/\Lambda$. In fact, it is a more general expansion which involves quark masses, external fields etc... but let us assume that these are zero for the moment. It is convenient to encode the goldstone bosons in a unitary matrix on which the chiral group operates linearly: $U = \exp(i\vec{\tau}.\vec{\pi})$ then the most general dynamics can be expressed with the aid of a chiral lagrangian:

$$L(U) = L_2 + L_4 + L_6 + \ldots \quad (12)$$

where the subscripts denote the numbers of derivatives of the matrix field $U$. As shown by Weinberg $[11]$ if one wants to expand an amplitude to order $E^n$ with $n > 2$
one must compute loops. There are simple counting rules, for example that one loop made out of two $L_2$ vertices contribute at order 4, one loop with one $L_2$ vertex and one $L_4$ contributes at order 6, a two-loop amplitude with $L_2$ vertices is also of order 6, etc... The chiral lagrangian contains the counterterms which render the loops finite.

If we are able to construct the skyrmion out of the chiral lagrangian (12) then, at least in principle, one should be able to renormalize the Casimir energy, which is a one-loop effect. Note that it is not clear that the chiral expansion does apply for the soliton but, after all, one might expect the average energy of a pion "inside" a skyrmion to be of the order of 200 MeV since the skyrmion size is expected to be of the order of 1 fm. So why shouldn’t we try?

The chiral lagrangian has so far been computed up to order four[5] and it reads (external sources being switched off):

$$L_{ChPT} = \frac{F^2}{2} \vec{A}^\mu \cdot \vec{A}_\mu + \frac{1}{96\pi^2} (\tilde{l}_1 - 1 + \ln \frac{M^2}{\mu^2}) (\vec{A}^\mu \cdot \vec{A}_\mu)^2 + \frac{2}{96\pi^2} (\tilde{l}_2 - 1 + \ln \frac{M^2}{\mu^2}) (\vec{A}^\mu \cdot \vec{A}_\nu)^2$$

(13)

where

$$i \vec{r} \cdot \vec{A}_\mu = U^\dagger \partial^\mu U, \quad M \simeq m_\pi, \quad F \simeq f_\pi.$$  
We see that a scale $\mu$ appears in (13). This is because it incorporates counterterms. If one uses a regularization prescription like dimensional regularization, one gets rid of the $1/(n-4)$ pole and one is left with a scale dependence. A similar but more convenient scheme for our purposes is the zeta function one[17] as we will see later.

In order to discuss solitons we must perform an $N_c$ expansion so we might start by assuming:

$$L^{(1)} = L_{ChPT}(\mu = m_\rho)$$

(14)

where the superscript designates the $N_c$ order. This is a meaningful assumption provided subleading terms in $N_c$ are very small for this particular scale value $\mu = m_\rho$. This seems indeed to be the case as follows from the recent work of ref.[18] who have analysed K14 decays and showed that this provides a quantitatively neat test for this suppression. Note that the fact that the terms of order $N_c^0$ in the chiral lagrangian are found to be small by no means implies that the correction of the same order to the skyrmion mass should also be small. However, this will turn out to be important for the accuracy of the CE determination, as we will see.

Let us now rewrite the fourth order lagrangian using notations which are familiar in the skyrmion context:

$$L_4^{(1)} = \frac{1}{4e^2} (\vec{A}^\mu \wedge \vec{A}^\nu) \cdot (\vec{A}_\mu \wedge \vec{A}_\nu) + \frac{\gamma}{2e^2} (\vec{A}^\mu \cdot \vec{A}_\mu)^2$$

(15)

The first question that one might ask is whether the values of the parameters $e$ (Skyrme parameter) and $\gamma$ that are determined from ChPT are compatible with a stable soliton solution. Remember in this context that the $\gamma$ term tends to destabilize the soliton. An upper bound for stability was found to be $\gamma < 0.12$[19]. If we look at the 1984 values of Gasser and Leutwyler we find:

$$e = 6.8 \pm 4.15 \quad \gamma = -0.06 \pm 0.2$$
These numbers are not very conclusive because of the large error bars. Let us now consider the more recent determination of Riggenbach et al.\cite{18}:

\[ e = 7.1 \pm 2.30 \quad \gamma = 0.03 \pm 0.03 \]

The error bars are considerably smaller and we see that ChPT now seems perfectly compatible with a stable skyrmion. The results are in fact strikingly similar with the numbers proposed by Skyrme 30 years ago:

\[ e = 6.28 \quad \gamma = 0 \]

Unfortunately, chiral order four is not enough for a consistent description of the skyrmion. Firstly it is not consistent with the chiral expansion because the virial theorem implies that the contribution of \( L_2 \) is identical to that of \( L_4 \) instead of being much larger. Secondly it it is not consistent with the large \( N_c \) expansion either, because, as we will soon discover, the \( N_c \) contribution turns out to be nearly identical to the \( N_c^0 \) one instead, again, of being significantly larger.

The situation is perhaps not desperate. In fact I claim that a mere extension to chiral order six is sufficient to cure, apparently, all the difficulties. We need however to have a model in order to make a guess for the coefficients of the terms appearing in \( L_6 \) since they have not yet been worked out from ChPT. A reasonable starting point seems to be provided by the observation\cite{20} that all of the 10 parameters which appear in the chiral order four lagrangian can be saturated to a very good approximation by the contribution of the low-lying vector, axial vector, scalar and pseudo-scalar resonances. In particular since the rho meson saturates the Skyrme term it is natural to consider the omega meson as well which contributes at sixth order:

\[ L_{6,\omega} = -\frac{1}{2} M_\omega^2 B_\mu B^\mu, \quad B_\mu = \frac{\epsilon_{\mu\nu\lambda\sigma}}{12\pi^2} (\vec{A}_\nu, \vec{A}_\lambda, \vec{A}_\sigma) \]  

(16)

The value of the coupling parameter \( \beta \) can be estimated to be rather large \( \beta \approx 9.3 \)\cite{21} so it is tempting to assume that \( L_{6,\omega} \) is the dominant sixth order term. This is supported by estimates by Walliser\cite{22} who showed that the rho induced sixth-order term is much smaller than the omega one. One also notices that the rho and scalar contributions tend to cancel each other.

If one includes the contribution (16) in the chiral lagrangian and looks for a soliton solution, the result is found to be in much better agreement with the chiral expansion than before. Due to the repulsive effect of the omega induced sixth order term the profile size is much larger than before and, as a result, the contribution of \( L_2 \) becomes five times larger than that of \( L_4 \). We will see in the sequel that the \( N_c \) expansion also seems to become coherent, but before we can actually evaluate the CE in order to check that, we must discuss the \( \zeta \) function method of regularization which is a very important technical ingredient in the calculation.
4. \( \zeta \) function regularization method

The basic idea is to introduce instead of a sum like \( \sum (\omega^2)^{\frac{1}{2}} \) the sum \( \sum (\omega^2)^{\frac{1}{2} - s} \) depending on the complex parameter \( s \). One can eventually show that the function is analytic in \( s \) and further define the limit of interest, \( s = 0 \), by analytic continuation. The non trivial part is to actually perform this analytic continuation in a situation like ours where the eigenvalues \( \omega_n \) are known only numerically. As we will see, the phase-shift representation provides a simple solution to this problem.

Before we turn to that, however, we must make sure that the regularization procedure that we use for the Casimir energy is the same as the one which is used to regularize Green’s functions (leading to the counterterms as they appear in (13)). Now Green’s functions to one loop are generated by an effective action which can be written as a trace log of a four dimensional operator. If we call \( O \) such an operator then the corresponding \( \zeta \) function is defined as

\[
\zeta_O(s) = \frac{1}{\Gamma(s)} \text{Tr} \int_0^\infty d\tau \, \tau^{s-1} \exp\left(-\frac{\tau O}{\mu^2}\right)
\]

(17)

the scale \( \mu \) is introduced at this level in order to make \( \tau \) dimensionless. The regularized form of the trace log is then given by:

\[
\text{Tr} \log(O) \equiv -\zeta'(s = 0)
\]

(18)

we can relate the Casimir energy to a four dimensional operator via the identity:

\[
\text{Tr}(H^{\frac{1}{2}}) = \lim_{T \to \infty} \frac{1}{T} \text{Tr} \log (-\partial_t^2 + H)
\]

(19)

which holds for a time independent operator \( H \). If we let \( O = (-\partial_t^2 + H) \) in formula (17) a simple calculation shows that the \( \zeta \) function regularization of the Casimir energy which matches the one used for the effective action is:

\[
\text{Tr}(H^{\frac{1}{2}}) \equiv -\zeta'(0) \quad \text{with} \quad \zeta(s) = -\frac{\mu^2 \Gamma(s - \frac{1}{2})}{\Gamma(s) \Gamma(\frac{1}{2})} \frac{1}{2} \text{Tr} H^{\frac{1}{2} - s}
\]

(20)

The same derivation as before yields the appropriate phase-shift formula:

\[
\text{Tr} H^{\frac{1}{2} - s} - \text{Tr} H_0^{\frac{1}{2} - s} = \frac{1}{\pi} \int_0^\infty dp \, \delta'(p)(p^2 + M^2)^{\frac{1}{2} - s}
\]

(21)

where a finite pion mass \( M \) was introduced. It can be shown that the large momentum behavior of the phase function is of the form:

\[
\delta(p) = a_0 p^3 + a_1 p + \frac{a_2}{p} + ...
\]

(22)

where the \( a_i \)’s are numbers which are simply related to the heat kernel expansion of the operator \( H \). One can subtract and add this leading asymptotic behavior and perform the analytic continuation in \( s \) by using the well-known integrals

\[
\int_0^\infty dp \, p^m (p^2 + M^2)^{\frac{1}{2} - s} = \frac{1}{2} M^{m+2-2s} \Gamma\left(\frac{m+1}{2}\right) \Gamma(s-1-\frac{m}{2}) \Gamma\left(s-\frac{1}{2}\right)
\]

(23)
Finally, one obtains the finite and closed form formula for the regularized CE:

$$E_{\text{cas}}(\mu) = \frac{1}{2\pi} \left\{ \int_0^\infty dp \left[ -\frac{p}{\sqrt{p^2 + M^2}} (\delta(p) - \bar{a}_0 p^3 - \bar{a}_1 p) + \frac{\bar{a}_2}{\sqrt{p^2 + \mu^2}} \right] - \frac{3\bar{a}_0}{8} M^4 \left( \frac{3}{4} + \frac{1}{2} \ln \frac{\mu^2}{M^2} \right) + \frac{\bar{a}_1}{4} M^2 \left( 1 + \ln \frac{\mu^2}{M^2} \right) - M\delta(0) \right\}$$

(24)

Note that it is well defined for zero as well as finite pion mass $M$. It is also clear
that, by construction, only the low-energy behavior of the phase shift is important
in the integration. Now, in a way similar to the 1+1 dimensional case we must add
counterterms to (24), i.e. the $O((N_c)^0)$ part of the chiral lagrangian:

$$E_{\text{ct}}(\mu) = -(\mathcal{L}_2^{(1)} + ... + \mathcal{L}_n^{(1)} - \mathcal{L}_\text{ChPT})$$

(25)

Actually, since $\mathcal{L}^{(1)}$ is being truncated at chiral order $n$ (in practice $n = 4$ or 6) $E_{\text{ct}}$
contains terms of order $O(N_c)$ and of chiral order $n + 2, ..., 2n$. Consistency requires
that these should be of the same order in magnitude (or smaller) that the terms of
lower chiral order but subleading in $N_c$. In this respect already, $n = 6$ is more
satisfactory than $n = 4$. Next, when we add the two pieces:

$$M^{(0)} = E_{\text{cas}}(\mu) + E_{\text{ct}}(\mu)$$

(26)

the scale dependence should disappear (at least up to $O(1/N_c)$ terms). In practice,
of course, it does not since we do not know enough terms in $\mathcal{L}_\text{ChPT}$. One would
therefore like to argue that

$$E_{\text{ct}}(\mu) << E_{\text{cas}}(\mu)$$

(27)

This, of course, cannot hold for arbitrary values of $\mu$. Now as we have seen, it follows
from ref.13 ( and also from ref.20 ) that for the particular value $\mu = m_\rho$ terms of
order $N_c^0$ are strongly suppressed in the chiral lagrangian (by a factor of 10 or so) so
it seems a good idea for us to choose $\mu = m_\rho$. According to formula (25) this will
suppress the contributions to $E_{\text{ct}}$ up to chiral order $n$. Those of order $n + 2, n + 4, ...$
are of order $N_c$ so one must assume that they are suppressed because of they high chiral
order. The main reason why (27) should hold, though, is that $E_{\text{cas}}(\mu)$ is enhanced
because it incorporates the zero-mode contributions. In our formalism, they show up
via Levinson’s theorem forcing the phase-shift at the origin to be fairly large ($=6\pi$).

An approximate way to estimate the CE by singling out the zero-mode contributions
was imagined recently by Holzwarth[25]. Let us now illustrate these points on some
examples.

5. Some results and conclusions

Let us first consider the Skyrme lagrangian $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{4,sk}$ (see (15)) with physical
values of $f_\pi$ and $m_\pi$ and
a)e=5.5 (by analogy with ref.2). Following the method described above one finds:

$$M^{(0)} = -957 + (-72) = -1029 \text{ MeV}$$

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where the number in parenthesis is the counterterm contribution (which is indeed small). This is to be compared with the leading $N_c$ contribution:

$$M^{(1)} = 1263 \text{ MeV}$$

Clearly, the $1/N_c$ expansion seems to be in trouble and furthermore, the nucleon mass is found to be much too small. In fact, better phenomenological results are expected from the Skyrme lagrangian if one takes a smaller value for $e$. Let us consider then: b) $e=4.0$ (which gives for example a reasonable delta-nucleon splitting of 250 MeV). In this case, one finds:

$$M^{(0)} = -805 + (-452) = -1297 \text{ MeV}$$

while the leading contribution is

$$M^{(1)} = 1761 \text{ MeV}$$

This is slightly better than before and one could eventually arrive at a reasonable nucleon mass, but an unsatisfactory feature now is that the counterterm contribution is rather large. This is because of the large mismatch between the value of $e$ and the one compatible with ChPT (see sec. (3)). In this situation one can no longer argue that the unknown contributions of $E_{ct}$ are necessarily small.

The only way out of this dilemma seems to include higher order terms in the chiral lagrangian. Let us add an omega induced sixth order term then, and consider:

$$L^{(1)} = L_2 + L_{4,sk} + L_{6,\omega}$$

with the parameters $e = 7.22$ (from the meson saturation fit of [20]) and $\beta = 9.3$ from [21]. The calculation gives, in that case:

$$M^{(0)} = -604 + (+153) = -451 \text{ MeV}$$

while the classical value

$$M^{(1)} = 1553 \text{ MeV}$$

The striking feature is the strong reduction of the Casimir contribution. Now the large $N_c$ expansion looks much more reasonable than before (the correction is a factor of 3 smaller than the classical mass). We argued in sec. (3) that the chiral expansion was also more justified. The two things are in fact related: the reason why $E_{cas}$ is smaller is that the phase-shift function $\delta(p)$ drops faster as a function of the momentum, and this is because the classical profile function has a larger extension in space. Note that the counterterm contribution is positive now (because the choice of $e$ is slightly larger than that of ChPT) but the Casimir energy itself is always found to be negative. If we add $M^{(1)}$ and $M^{(0)}$ we find that the nucleon mass is correctly predicted to within 20% while we did not attempt to fit any parameter in the chiral lagrangian.

In conclusion, a reasonable picture seems to emerge provided one incorporates besides the fourth order rho induced term a sixth order omega meson one. This is not really a surprise. In fact, the most succesfull skyrmion phenomenology seems to
require that one has these resonances (and a few others) explicitly in the lagrangian.
The rationale for including resonances is that this should extend the range of validity
of the effective lagrangian from $E << \Lambda$ to $E \approx \Lambda$. It is likely that one could
extend the calculation of the CE to that case but the value of the "optimal scale"
should perhaps be larger. An open question at the moment is how to estimate loop
corrections for other nucleon observables. In particular, it is not clear (to me) whether
the zero-mode enhancement which lead to a sizeable effect in the case of the mass
will also operate for some other observables and how.

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