Multi-Fair Pareto Boosting

Arjun Roy¹,², Vasileios Iosifidis¹, and Eirini Ntoutsi²

¹ L3S Research Center (LUH), ² Free University Berlin (FUB)

Abstract. Fairness-aware machine learning for multiple protected attributes (referred to as multi-fairness hereafter) is receiving increasing attention as traditional single-protected attribute approaches cannot ensure fairness w.r.t. other protected attributes. Existing methods, however, still ignore the fact that datasets in this domain are often imbalanced, leading to unfair decisions towards the minority class. Thus, solutions are needed that achieve multi-fairness, accurate predictive performance in overall, and balanced performance across the different classes. To this end, we introduce a new fairness notion, Multi-Max Mistreatment (MMM), which measures unfairness while considering both (multi-attribute) protected group and class membership of instances. To learn an MMM-fair classifier, we propose a multi-objective problem formulation. We solve the problem using a boosting approach that in-training, incorporates multi-fairness treatment in the distribution update and post-training, finds multiple Pareto-optimal solutions; then uses pseudo-weight based decision making to select optimal solution(s) among accurate, balanced, and multi-attribute fair solutions.

1 Introduction

There are growing concerns about the potential discrimination and unfairness of Machine Learning (ML) models towards individuals or groups of people based on protected attributes like race or sex. A variety of methods for fairness-aware learning have been proposed, which however focus mainly on single protected attributes. In reality, though, bias cannot be attributed to a single attribute, rather multiple protected attributes (referred to as multi-fairness hereafter) can be the root causes of discrimination, e.g., a combination of race, gender, religion. Therefore, it is important to move from single-attribute solutions to the more realistic multi-fairness case. Existing legal studies define multi-fairness as compound, intersectional and overlapping. Of these, the intersectional definition seems to be the most promising as it requires fairness “compliance” for the intersection of groups (subgroups) defined by multiple protected attributes. However, it is hard to define the subgroups. Therefore, in this work, we focus on the more operational definition of overlapping fairness, in which discrimination is due to multiple factors that operate independently.
Recent studies [12,14] also showed that many datasets in the domain are imbalanced and typically, the \textit{class-imbalance} problem is more severe for the protected group (e.g. \textit{female}, \textit{black}, etc.), which is often underrepresented in the important minority class [13]. Despite high overall accuracy and fairness, methods ignoring imbalance may still perform poorly on the minority class, thus amplifying prevalent biases. The problem got unnoticed in previous works [2,15,17,24] mainly because group fairness measures [4] focus on performance differences and ignore the scale of the correct prediction.

Motivated by this problem, in this work, we investigate the combined problem of \textit{multi-fairness} and \textit{class-imbalance} which, to the best of our knowledge, has not yet been addressed in the literature. To better showcase the problem, let us assume a toy dataset with two protected attributes, Sex = \{M, F\} and Race = \{W, B\}, and a binary class attribute Class = \{+, -\}. The dataset is imbalanced with an imbalance ratio of 1 : 10 (+ : -). The exact split per group and class for each protected attribute is shown in Table 1. Moreover, let there be 4 different hypothetical classifiers (Cf1-Cf4) which differentiate on whether they treat multi-fairness and/or class-imbalance as summarized in Table 2. The performance of each of the hypothetical classifiers is shown in Fig. 1a in terms of false negative rates (FNR) and false positive rates (FPR) over a hypothetical test set following the distribution as in Table 1. Let fairness be defined as the difference between $FNR$ and $FPR$ across different groups, the so-called \textit{disparate mistreatment} [24]. Cf1 ensures high fairness for Sex (low difference in $FNR$ with $FNR$ between the groups ‘M’ and ‘F’) but discriminates for Race ($FNR_B > FNR_W$ and $FPR_W >> FPR_B$). Cf2 produces highly fair results for both Sex and Race, but under-performs in the minority (+) class (high $FNR$ scores). Cf3 achieves good predictive performance and fairness for both the protected attributes, but one may note that the entire unfairness comes from the minority (+) class by rejecting black females. This is because, the difference in $FNR_W$ with $FPR_W$, and $FNR_M$ with $FPR_M$ is very low. However, the difference in $FNR_F$ with $FPR_F$, and $FNR_B$ with $FPR_B$ is high due to high $FNR$ values. Cf4 achieves the most balanced performance across all classes and protected attributes and also ensures that discrimination is not biased towards any class or attribute.

Our main contributions in this work are as follows: i) We introduce the notion of \textit{Multi-Max Mistreatment} (MMM)\textsuperscript{3} that evaluates discrimination for

---

\textbf{Table 1: Toy dataset distribution}

| class | group |
|-------|-------|
|       | M     | F     | W     | B     |
| +     | 150   | 100   | 50    | 100   |
| -     | 1500  | 1000  | 500   | 1000  |

\textbf{Table 2: Hypothetical classifiers}

| tackles | Cf1 | Cf2 | Cf3 | Cf4 |
|---------|-----|-----|-----|-----|
| Race-fairness | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Sex-fairness | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| class-imbalance | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ |

---

\textsuperscript{3} The term ‘\textit{multi}’ here refers to both multiple protected attributes and multiple classes.
multiple protected attributes and across different classes. ii) We propose a multi-objective optimization problem (MOP) formulation that aims to learn an MMM-fair classifier that achieves high predictive performance in overall and minimizes performance differences across the classes. iii) We propose a boosting-based solution to the MOP, Multi-Fair Pareto Boosting (MFPB), which in-training, changes the data distribution to also account for fairness and post-training, finds multiple Pareto optimal solutions in the objective space, offering the flexibility to provide solutions per user needs. A single optimal solution can be selected based on user preferences. iv) We theoretically prove the fairness convergence of our method and experimentally, we demonstrate the superior performance and advantages of our method.

(a) Predictive performance  
(b) Fairness evaluation

Fig. 1: Performance of Cf1-Cf4 on different population groups

2 Related Work

Over the recent years, fairness-aware learning has emerged as an important topic in machine learning. Recent surveys provide a detailed overview on the plethora of fairness notions [4,6] and methods proposed to mitigate unfair outcomes [4,18]. However, the vast majority focuses on single-protected attributes and only a few approaches consider fairness concerning multiple protected attributes. Likewise, the problem of class-imbalance has mostly gone unnoticed in the related work. In the following, we summarize related work referring to multi-fairness and imbalanced learning. Moreover, we overview work on boosting, a popular sequential ensemble learning method that comprises our underlying model, in particular, boosting approaches for imbalance and/or fairness.

Multi-fairness-aware learning: Only a few existing approaches can handle multiple protected attributes; these can be categorized into supervised learning approaches [21,10,19,24,21] and unsupervised ones, e.g., [1]. We review henceforth only the multi-fairness supervised learning methods. The multi-fairness constraints approach [24] introduces fairness-related convex-concave constraints.
for each protected attribute (using the Disparate Mistreatment (DM) fairness notion) to a logistic regression classifier. A constraint-based idea is also proposed by [2] which employ an exponentiated-gradient reduction method combined with logistic regression. They tackle fairness-accuracy trade-off as a two-player game theory by imposing a set of linear DM fairness constraints for each protected attribute on accuracy. However, none of the approaches [2,24] tackles class-imbalance. Moreover, on the contrary to their complex algorithms needing to comply with many fairness constraints together, our mini-max view of fairness is much simpler still able to solve the complex problem of multi-fairness. 

Fairness-aware learning as a mini-max theory has been already used in the literature [17,21]. [21] impose predictor selection as a mini-max game between fairness constraints and worst case approximator. However, their method is tailored for a single protected attribute fairness learning, hence not considered here as a competitor. MiniMax [17] shares some conceptual similarity with us, as it tackles the fairness problem as a mini-max game, while searching for Pareto efficient solution of a multi-objective problem. Specifically they assign risk scores to the different protected groups as a separate objective, and then use an iterative algorithm (APStar) that alternates between different models to optimize the MOP as minimizing the maximum risk for any such group by employing a gradient descent approach, and henceforth tackling disparity between different groups. However, they do not tackle class-imbalance.

**Fairness-aware class-imbalance:** Recently it has been shown that skewed class distributions can affect the discriminatory behavior of a model [12,13,14], these studies however refer to single-protected attributes and none of the existing multi-fairness methods [10,19] considers class-imbalance. Although one could use decoupled classifiers [19] and tune fairness separately for the different classes. However, such an approach ignores the fact that one of the groups defined by one of the many protected attributes in the multi-fairness set-up, in one of the classes, maybe discriminated against more than the others in that class. As a result, such a group may require extra care at times. Our proposed multi-fairness notion MMM overcomes the limitation by considering both multiple-protected attributes and class membership of the instances within a mini-max framework.

**Boosting:** Boosting-based approaches have shown their effectiveness in tackling class-imbalance [5,23] and fairness [11,14]. However, neither SMOTEBoost [5] nor cost-sensitive boosting [23] consider fairness. The approach in [11] considers fairness but only for single-protected attributes and does not consider class-imbalance. AdaFair [14] tackles both fairness and class-imbalance but for a single protected attribute.

### 3 Basics and Multi-Max Mistreatment (MMM) fairness

We assume a dataset $D$ of $n$ instances drawn from the joint distribution $P(X,Y)$, where $X$ is the predictive attribute set, and $Y$ is the class attribute. The predictive attributes consist of a subset of protected attributes denoted by $S$, and
non-protected attributes denoted by $U$; it holds that $X = S \cup U$. For simplicity, we assume a binary problem: $Y \in C = \{+, -\}$ with ‘+’ being the minority class. The goal is to learn a fair classifier $f(\cdot) : X \rightarrow Y$ that minimizes discriminatory outcomes while maintaining good predictive performance. To evaluate fairness, we adopt Disparate Mistreatment ($DM$) \cite{24} originally proposed for single protected attributes. Let $S_j \in S = \{s_j, \bar{s}_j\}$ be such a (binary) attribute with $s_j$ ($\bar{s}_j$) denoting the protected (non-protected, respectively) group. $DM$ relies on the performance of the model for the different groups evaluated in terms of the $FPR$ and $FNR$: $FPR_{s_j} = P(\hat{y} = -|S_j = s_j, y = +)$, $FNR_{s_j} = P(\hat{y} = +|S_j = s_j, y = -)$. We denote the difference in $FPR$ and $FNR$ between $s_j$ and $\bar{s}_j$ by $\delta FPR_j = FPR_{s_j} - FPR_{\bar{s}_j}$ and $\delta FNR_j = FNR_{s_j} - FNR_{\bar{s}_j}$ respectively. $DM$ w.r.t. a single protected attribute $S_j$, can be then written as:

$$DM_j = |\delta FPR_j| + |\delta FNR_j| \tag{1}$$

Ideally, $f(\cdot)$ should result in equal treatment for both groups, i.e., $DM_j = 0$. Note that the $DM$ definition per se does not ensure good predictive performance, as low $\delta FPR, \delta FNR$ can be achieved even with high $FPR, FNR$ values.

It is the role of the classifier to ensure low $FPR$ and $FNR$ scores. However, in case class-imbalance, if the classifier does not tackle class-imbalance, it will typically end up with a low $FPR$ but high $FNR$ scores \cite{5}, i.e., it will not properly learn the minority (+) class. In \cite{14}, the authors showed that state of the art fairness methods were achieving low $DM$ scores by rejecting many of the minority instances (i.e., producing high $FNR$). The problem occurred because the class-imbalance problem was ignored. Therefore, we first propose a modification of $DM$ that also takes into account the across-classes discriminatory behavior of the model (Section 3.1) before moving to the multi-fairness case (Section 3.2).

### 3.1 Class-aware Disparate Mistreatment (CDM) measure

**Definition 1.** Class-aware Disparate Mistreatment (CDM) w.r.t a protected attribute $S_j$, denoted by $CDM_j$, is defined as the difference in the misclassification rates between the protected and non-protected groups for both classes.

$$CDM_j = | |FPR_{s_j} - FNR_{s_j}| - |FPR_{\bar{s}_j} - FNR_{\bar{s}_j}| | \tag{2}$$

**Lemma 1.** $CDM_j$ is a lower bound for $DM_j$, i.e $CDM_j \leq DM_j$, $\forall S_j \in S$.

Proofs of lemmas/theorems are provided in appendix. Lemma 1 shows that high $CDM_j$ is clearly not desirable. Now, if $CDM_j = DM_j$ and both are above 0, it is an indication that one group is favoured over the other in some class, i.e, the mistreatment for $S_j$ is class-biased.

**Lemma 2.** $CDM_j = DM_j > 0$ indicates class-biased discrimination for $S_j \in S$.

A relaxed version $CDM_j \approx DM_j > \delta$, $1 > \delta > 0$, can also be derived to show similar property. However, we should note that if one mitigates only $CDM_j$ but not $DM_j$, we get a combination of a low $CDM_j$ with a high $DM_j$, i.e., $CDM_j \leq \delta << DM_j$ which indicates that unfairness exists in both classes for $S_j$. Thus, fair predictions on $S_j$ in both classes requires mitigating both $CDM_j$ and $DM_j$. 
3.2 Multi-Max Mistreatment (MMM) measure

As already mentioned, and will also become evident from the experiments, mitigating discrimination for a single protected attribute does not ensure fair outcomes for other protected attributes [15]. But in real life, prejudicial treatment might be based on several factors e.g., sex, race, etc., rather than just a single factor. Furthermore, fair treatment should hold across all classes and should not be “dictated” by the majority class. Therefore, the aim is to mitigate \( CDM_j \) and \( DM_j \) w.r.t. multiple sensitive attributes \( \forall S_j \in S \). To this end, we introduce a new multi-fairness notion, called Multi-Max Mistreatment (MMM), that measures the maximum mistreatment among the protected attributes and for the different classes; this means focusing on the worst-case discriminatory behavior of the model w.r.t. a protected attribute and a class.

**Definition 2.** Given a set of \( k \) protected attributes \( S = \{S_j|S_j \in \{s_j,s_j,j = 1,\cdots,k\}\} \), fairness notions \( \delta FNR_j \) and \( \delta FPR_j \) measuring the mistreatment in \((+)\) class and \((-)\) class respectively w.r.t \( S_j \), then \( \text{MMM}_S \) the Multi-Max Mistreatment for the set of protected attributes \( S \) across all classes, is defined as:

\[
\text{MMM}_S = \max_{S_j \in S} \left( \max(|\delta FNR_j|,|\delta FPR_j|) \right)
\]

**Theorem 1.** \( CDM_j, DM_j \) for any protected attribute \( S_j \in S \) are upper bounded by \( 2 \times \text{MMM}_S \):

\[
CDM_j, DM_j \leq 2 \times \text{MMM}_S, \forall S_j \in S
\]

Theorem 1 shows that by minimizing \( \text{MMM}_S \), the upper-bound of \( CDM_j \) and \( DM_j \) for any \( S_j \in S \) can be minimized. Essentially it means, \( \text{MMM}_S \) can counter the worst possible discrimination for any \( S_j \in S \).

**Definition 3.** Given a tolerance threshold \( \mu \), a classifier is \( \text{MMM}\)-fair iff the maximum mistreatment w.r.t all protected attributes \( S_j \in S \) across all classes is less than \( \mu \) i.e., \( \text{MMM}_S \leq \mu \).

The threshold \( \mu \) is 0 in the ideal case where the error prediction rates for all groups are equal. We showcase the effectiveness of the newly introduced fairness notions \( CDM \) and \( MMM \) in comparison to the existing \( DM \) measure on the toy example of Fig. 1a. The fairness behavior of the hypothetical classifiers is shown in Fig. 1b. We notice that \( Cf2 \) produces the fairest classifier unanimously identified by all the fairness measures. However, as already noticed in Fig. 1a, this is achieved only due to high \( FNRs \) (under-performing in the minority \((+)\) class). We also see that measuring fairness using \( DM \) evaluates both \( Cf3 \) and \( Cf4 \) as equally discriminating for both protected attributes. Using \( CDM \) instead reveals that \( Cf4 \) is better than \( Cf3 \) since the discriminatory behaviour is not class-biased. From Fig. 1a, we see that \( Cf3 \) has higher \( FNR \) values for \( F \) and \( B \) than for \( M \) and \( W \), while with equally low \( FPR \) values for all the groups, which shows that the entire discrimination is biased to the minority \((+)\) class. Finally, \( MMM \) is able to identify the multi-fairness behaviour of each classifier across the classes and judges \( Cf4 \) (our desired goal) as a fairer classifier than \( Cf1 \) and \( Cf3 \).
Multi-Fair Pareto Boosting

4 Multi-Fairness-aware Learning

Our goal is to learn a fair classifier \( f(\cdot) : X \rightarrow Y \) for multiple protected attributes \( S \) that takes into account class-imbalance. We formulate the problem as a multi-objective optimization problem (MOP) (Sec. 4.1), then solve it using a sequential learner (boosting), followed by a Pareto-based approach (Sec. 4.2).

4.1 Multi-Fairness Multi-Objective Formulation

We formulate the problem as a MOP with the following objectives for the fair-classifier \( f(\cdot) \): (i) high overall predictive performance \((O_1)\), (ii) similar rate of predictive performance across all classes \((O_2)\), (iii) mitigation of discriminatory outcomes for all protected attributes \((O_3)\).

The \( O_1 \) objective is defined as minimizing the classification loss (0-1 loss) to maximize overall predictive performance:

\[
O_1 : L(f) = \frac{1}{n} \sum_{x_i \in D} |y_i - \hat{y}_i| \quad (4)
\]

where \( \hat{y}_i \) is the predicted, \( y_i \) is the true class of \( x_i \in D \).

Objective \( O_2 \) explicitly targets class-imbalance by ensuring balanced performance across all classes. Motivated from \[9\], we define a balanced loss function to minimize the performance differences between the two classes:

\[
O_2 : B(f) = \frac{1}{|D_+|} \sum_{x_i \in D_+} |y_i - \hat{y}_i| - \frac{1}{|D_-|} \sum_{x_i \in D_-} |y_i - \hat{y}_i| \quad (5)
\]

where \( D_c \subset D \) denotes the instances belonging to class \( c \).

\( O_3 \) is the multi-fairness objective aiming to ensure fair behavior towards multiple protected attributes \( S_j \in S \) and across classes \( C = \{+,-\} \). We call it MMM\(_S\) loss, as on optimization it aims to mitigate MMM\(_S\) (c.f. Def. 2) as:

\[
O_3 : \Phi(f) = \max_{S_j \in S} \max_{c \in \{+,−\}} \left( \frac{1}{n_{ji}} \sum_{i=1}^{n} \right) \quad (6)
\]

where \( n_{ji} \) takes a value that depends on group membership of \( x_i \in D \) w.r.t the protected attribute \( S_j \in S \) and class label \( c \). In particular, \( n_{ji} \) takes values in \( \{\#(s_{j+}), -1\#(\bar{s}_{j+}), \#(s_{j-}), -1\#(\bar{s}_{j-})\} \), where \( \#(s_{j+}) \) is the cardinality of group \( s_j \) in class \(+\), and likewise. Note that for group \( \bar{s}_{j} \) it assigns a negative value to capture the difference in misclassification between \( s_j \) and \( \bar{s}_{j} \).

The MOP is then defined w.r.t to a parameter \( t \) as follows:

\[
\min_{t} O(t) := \min_{t} \{O_1(t), O_2(t), O_3(t)\} \quad (7)
\]

We propose a Pareto front approach to solve the MOP, because it allows to find multiple optimal solutions based on the different objectives \[7\]. This approach provides more flexibility to the end users who can select the best solution for their needs from the ones in the Pareto front.
4.2 The Multi-Fair Pareto Boosting (MFPB) Algorithm

We use a sequential learner (boosting) to get the solution set where we consider each partial ensemble as a member solution. In-training, we mitigate discrimination (objective $O_3$) by changing the data distribution. Post-training we investigate the Pareto front of the objective space to find the desired optimal solution from the obtained solution set based on user needs.

**In-training:** MMM-boosted weight distribution update. We assume a user-defined maximum number of boosting rounds $T$. We update the distribution $D_t$ in each round $1 \leq t \leq T$, based on the predictive performance of the previous weak learner $h_{t-1}$ as is typical in boosting but also by using its fairness-related performance as evaluated by the so-called *multi-fairness costs* $f_{c_t}$. The data distribution is updated as follows:

$$D_t(x_i) = \frac{D_{t-1}(x_i)f_{c_t}(x_i) \exp(-\alpha_t \text{sign}(y_i h_t(x_i)))}{Z_t}$$

where $f_{c_t}(x_i)$ is the multi-fairness cost for instance $x_i$ (explained hereafter). $\alpha_t$ is the weight of the weak learner $h_t$, function $\text{sign}(y_i h_t(x_i))$ returns $-1$ if $h_t(x_i) \neq y_i$ and 1 otherwise and $Z_t$ is the normalization factor which ensures that $D_t$ is a probability distribution. To calculate the fairness costs we extend to the multi-protected case the idea of cumulative fairness [14] which has been shown in the single protected setup to be advantageous over the fairness calculation of the individual weak learner at round $t$. In particular, we first define the discrimination cost of an instance $x_i \in D$ for a protected attribute $S_j \in S$:

$$cdc_{ij} = \begin{cases} 1 + |\delta FNR_{j,t}^{s_i,j}|, & \text{if } y_i = + \land \{ (\delta FNR_{j,t}^{s_i,j} \geq 0, \land x_i \in s_j) \lor (\delta FNR_{j,t}^{s_i,j} \leq 0, \land x_i \in \bar{s}_j) \}; \\ 1 + |\delta FPR_{j,t}^{s_i,j}|, & \text{if } y_i = - \land \{ (\delta FPR_{j,t}^{s_i,j} \geq 0, \land x_i \in s_j) \lor (\delta FPR_{j,t}^{s_i,j} \leq 0, \land x_i \in \bar{s}_j) \}; \end{cases}$$

where $\delta FNR_{j,t}^{s_i,j}$ and $\delta FPR_{j,t}^{s_i,j}$ are the cumulative discrimination of the partial ensemble $H_t(x_i) = \sum_{l=1}^{t} \alpha_l h_l(x_i)$ for $S_j$ as in [14]. Considering the set of protected attributes $S = \{S_1, \cdots, S_k\}, |S| = k$, multi-fairness cost for $x_i$ is now defined based on the group with the highest discrimination:

$$f_{c_t}(x_i) = \begin{cases} \max_{1 \leq j \leq k} (cdc_{ij}), & \text{if } H_t(x_i) \neq y_i; \\ 1, & \text{otherwise} \end{cases}$$

So $f_{c_t}(x_i)$ ensures that $x_i$, if misclassified by the partial ensemble $H_t(x_i)$, receives the boost for the group that is discriminated the most, among the groups it belongs to.

**Post-training:** Pareto Optimal Solution. In each boosting round $t$ we evaluate the partial ensemble $H_t$ in terms of the objectives $(O_1, O_2, O_3)$, and then collect the solution vector $f_t = [o_1, o_2, o_3]$, where $o_i = O_i(t)$ is a solution point of $H_t$ for the respective objective $O_i$. Typically, for a MOP there exist no single solution that simultaneously optimizes all objectives, rather several Pareto optimal solutions might exist that satisfy different objectives. Having a diverse set of
solutions allows for more flexibility in decision making. On the other side, if the end-user has specific preferences this can be explicitly modeled via a so-called user-preference vector $\tilde{U}$. The exact steps are presented as follows:

**Step 1** - Pareto front computation: among all solution vectors $\tilde{f}_t, \forall t \in \{1, \cdots, T\}$ we find the Pareto front $\mathbb{PF}$, containing the non-dominated set of Pareto optimal solutions. A solution $\tilde{f}_t'$ is said to be dominated by solution $\tilde{f}_t$ if 1) $O_1(t) \leq O_1(t') \forall i \in \{1, 2, 3\}$, and 2) $O_1(t) < O_1(t') \exists i \in \{1, 2, 3\}$.

**Step 2** - Pseudo-weight calculation: we calculate the relative distance of each solution from the worst (maximum value) solution within the $\mathbb{PF}$ regarding each objective $O_i$ using the pseudo-weight vector method \([7]\). The pseudo-weight $w_{ji}$ for $o_i \in \tilde{f}_t$ is given by:

$$ w_{ji} = \frac{(o_{i}^{\text{max}} - o_{ji})/(o_{i}^{\text{max}} - o_{i}^{\text{min}})}{\sum_{i=1}^{3}(o_{i}^{\text{max}} - o_{ji})/(o_{i}^{\text{max}} - o_{i}^{\text{min}})} \quad (11) $$

This way, for each solution $\tilde{f}_j = [o_1, o_2, o_3] \in \mathbb{PF}$ we compute the corresponding pseudo-weight vector $\tilde{w}_j = [w_{j1}, w_{j2}, w_{j3}]$.

**Step 3** - User preference vector: Let $\tilde{U} = [u_1, u_2, u_3]$ be a user preference vector over the objectives such that: $u_1 + u_2 + u_3 = 1$. In the absence of such a preference vector, we can assume all weights are equal.

**Step 4** - Optimal solution according to preferences: We choose the Pareto optimal solution $\tilde{f}_t'$, whose corresponding pseudo-weight vector $\tilde{w}_{t'}$ is closest, according to L1 distance, to the preference vector $\tilde{U}$.

Finally, we output $H_{t'} = \sum_{t=1}^{t'} \alpha_t h_t$ as the optimal classifier.

### 4.3 Fairness Convergence Properties

We derive the effectiveness of our boosting algorithm in producing $MMM$-fair outcomes. We assume that the boosting based part of training the partial ensembles of our algorithm, will converge w.r.t. the classification loss ($O_1$) when trained for sufficient number of weak learners; the proof for this assumption can be based on the convergence proof of vanilla AdaBoost \([20]\).

**Lemma 3.** As classification (0-1) loss ($O_1$) of a partial ensemble $H_{t'} = \sum_{t=1}^{t'} \alpha_t h_t$ trained for a protected attribute $S_j$ via the in-processing of MFPB Algorithm (Eq. \([8]\)) converges, cumulative fairness cost $\delta f_{j}^{t'} \in \{[\delta FPR_{j}^{1, t'}, |\delta FNR_{j}^{1, t'}|]\}$ (Eq. \([9]\)) of each partial ensemble w.r.t $S_j$ also tends to converge to a low value.

$$ P_t(\hat{y}_i \neq y_i) - P_t(\hat{y}_i \neq y_i) \to 0; t' \to \infty \implies \delta f_{j}^{t'} \to \mu; \forall H_{t'}, \text{ as } t' \uparrow \text{; where } \delta f_{j}^{t'} \in \{[\delta FPR_{j}^{1, t'}, |\delta FNR_{j}^{1, t'}|]\}; \mu \text{ is a tolerable discrimination for } S_j \text{ in any class} $$

Lemma 3 guarantees that if the assumption of convergence on $O_1$ holds, then the MFPB algorithm trained for any protected attribute $S_j$ will always converge to a very low $MMM_S$ loss ($O_3$).
Theorem 2. A final ensemble \( H^* \) given as output by the MFPB algorithm trained w.r.t a set of \( k \) protected attributes \( S = \{S_1, ..., S_k\} \), with any choice of \( \hat{U} \), upon being chosen from a set of partial ensembles converged w.r.t \( O_1 \) and \( O_3 \), will be a MMM-fair classifier with a threshold \( (\mu + \epsilon) \).

\[
\text{MMM}_S \leq (\mu + \epsilon); \ S = \{S_1, ..., S_k\}
\]

where \( \epsilon \) is trade-off for \( O_3 \) w.r.t \( O_1 \) and \( O_2 \).

Lemma 3 and Theorem 2 together characterize the conditions by which in-training MFPB algorithm upon being trained for sufficiently large number of weak learners, will converge in multi-fairness with class-balanced fair outcomes \( CDM_j, DM_j \leq 2\mu \), where \( \mu \) is a tolerable discrimination in any class \( \forall S_j \in S \).

5 Experiments

We evaluate MFPB against state-of-the-art approaches in terms of predictive performance \((O_1 \text{ and } O_2 \text{ objectives})\) and discriminatory behavior \((O_3 \text{ objective})\). In particular, we demonstrate the per-protected attribute fairness behavior and the per-class predictive performance of the model (Section 5.2). Moreover, we analyze the Pareto front solutions in terms of the \( O_1-O_3 \) objectives (Section 5.3) and demonstrate the behavior of the model on a dataset.

5.1 Experimental settings

Datasets: We choose 4 real-world datasets (c.f., Table 3) that vary in terms of class imbalance, dimensionality, cardinality and number of protected attributes.

| Data | #Attr | n  | Protected attribute (protected group) | + class | IR (+:+) |
|------|-------|----|-------------------------------------|---------|----------|
| Adult | 14    | 45K | Race (non-white), Sex (Female)       | > 50k   | 1.3      |
| Bank  | 16    | 40K | Marital (Married), Age \((\leq 25 \& \geq 60)\)) | subscription | 1.89  |
| Compass | 9   | 5K  | Race (non-white), Sex (Female)       | recidivism | 1:1.2    |
| Credit | 23   | 30K | Sex (Female), Age \((\leq 25 \& \geq 60)\), Marital (Married) | default pay, | 1:4:3 |

Evaluation Measures: Regarding fairness, we report on the proposed MMM-fairness and CDM, as well as on DM for each protected attribute. Regarding predictive performance, we report on accuracy \((\text{Acc}) \) for overall prediction, on worst class accuracy \( WC_{\text{Acc}} \) to evaluate the minimum predictive rate in any class, as well as on area under the curve \((\text{AUC}) \) and geometric mean \((\text{G.M}) \).

We compare against four state-of-the-art fairness-aware methods: Zafar et al. [24], FairLearn [2], Minimax [17] and AdaFair [14]. The first three approaches tackle multi-fairness but do not consider class-imbalance, the last one tackles class-imbalance but for single-protected attributes.
Hyperparameters and evaluation setup: We set the number of weak learners $T = 500$. We follow the same evaluation setup as in [14,24] by splitting each dataset randomly into train (50%) and test set (50%) and report on the average of 10 random splits. We perform an ablation analysis of the solutions in Pareto front along with the associated pseudo-weight vectors for choosing the user preference vector.

5.2 Predictive and fairness performance

In Fig. 2 we depict the predictive and fairness performance of the different methods on different datasets. For our experiments we set $\vec{U} = [0.43, 0.3, 0.27]$ for Adult, $\vec{U} = [0.35, 0.44, 0.21]$ for Bank, $\vec{U} = [0.23, 0.34, 0.4]$ for Compas and $\vec{U} = [0.02, 0.52, 0.46]$ for Credit dataset.

![Performance evaluation](image)

Fig. 2: Performance evaluation: lower fairness, higher predictive scores are better.

Adult data (Income prediction): As shown in Fig. 2a, MFPB has the most class balanced multi-fairness performance ($MMM = 0.04$), while achieving the
most balanced predictive performance ($AUC=0.77$, $G.M=0.76$). Interestingly, FairLearn also achieves the same $DM=0.06$ for Race-fairness, and slightly outperforms in Sex-fairness with $DM=0.035$ compared to $DM=0.042$ for MFPB. However, one may notice that FairLearn has $CDM$ values equal to $DM$ values for both Race and Sex, indicating that the discrimination is class-biased. FairLearn also has the least $WC_{Acc}=0.58$ compared to $WC_{Acc}=0.64$ by MFPB and AdaFair, but comparable $Acc=0.84$, which indicates that it achieves fairness by under-performing in the minority (+) class. Zafar et al. and MiniMax having a similar fairness objective that requires solving multiple objectives $\forall S_i \in S$, manages to achieve better balanced performance across classes compared to FairLearn, but both fail to counter discrimination for Race. Surprisingly, AdaFair albeit trained only on sex also achieves low $MMM=0.052$, with equally good balanced performance as MFPB across classes; we comment on this later.

**Bank data (Telemarketing sell prediction):** The dataset is highly imbalanced (IR=1:89) which as shown in Fig. 2b, affects all baselines except AdaFair which tackles imbalance. In particular, all baselines perform poorly in the minority (+) class with $WC_{Acc} < 0.5$. FairLearn, which delivers the fairest performance ($MMM=0.01$) here, noticeably has the poorest $WC_{Acc} = 0.22$ and $Acc=0.60$, implying that gain in fairness was achieved by lower predictive performance for both the classes. MFPB bags the second spot in multi-fairness ($MMM=0.07$), outperforms all the other baselines in balanced performance ($AUC=0.79$, $G.M=0.79$) by a good margin ($WC_{Acc} 22\% \uparrow$) with nearly 0.8 accuracy in both the classes. MiniMax has the 3rd best performance both in balanced performance and multi-fairness outperforming Zafar et al. and AdaFair in $MMM$ and Zafar et al. and FairLearn in $WC_{Acc}$, $AUC$, $G.M$.

**Compas data (Recidivism prediction):** Compas is the most balanced dataset with $IR = 1 : 1.2$. Even though imbalance which is one of our objectives is almost absent here, but still we use the dataset to show that to achieve fairness the baselines still fails to deliver class-balanced performance. As we see in Fig. 2c MFPB manages to generate the fairest predictions ($MMM=0.04$) along with FairLearn, while managing to produce equal predictive performance (0.66) in both the classes, hence achieving the best $WC_{Acc}$, $Acc$, $AUC$, and $G.M$ (all 0.66). Zafar et al. along with MiniMax also achieves the same $Acc$, $AUC$ as MFPB but outperforms MiniMax in multi-fairness ($MMM 72\% \downarrow$). AdaFair trained on protected attribute Race achieves same $WC_{Acc}$ as MiniMax, but both fail on multi-fairness ($MMM=0.19$) with high $CDM=0.16$, $DM=0.36$ for Sex by AdaFair, and $CDM=0.26$, $DM=0.31$ for Race by MiniMax.

**Credit data (Default payment risk prediction):** Credit has the largest number (3) of protected attributes and comprises therefore the most challenging dataset regarding multi-fairness objective; a decent imbalance is also present ($IR = 1.4 : 3$). We see in Fig. 2d all the competitors including AdaFair (trained for only fairness on Sex) achieves good multi-fairness with $MMM \leq 0.06$. Still MFPB along with FairLearn outperforms the other baselines with $MMM=0.017$. The biggest challenge in this data is the difficulty to learn class-balanced performance while maintaining fairness. We see that all the baselines
performs poorly in this aspect with $WC_{Acc} \leq 0.37$. MFPB is able to outperform all the baselines by a good margin delivering the best balanced performance with $WC_{Acc}=0.65$ (75% ↑), $AUC=0.70$ (6% ↑) and $G.M=0.70$ (16% ↑). Interestingly MiniMax, again achieves the second best balanced performance, outperforming AdaFair on $WC_{Acc}$ (3% ↑), $AUC$ (2% ↑), and $G.M$ (3% ↑) while having equal multi-fairness with $MMM=0.05$, behind MFPB, and FairLearn.

**Conclusion:** To conclude, our MFPB outperforms existing multi-fairness methods by producing the best $MMM$ values in three out of four datasets and achieving the best $WC_{Acc}$, $AUC$, $G.M$ in all of them, while marginally compromising on the overall $Acc$ in Adult and Credit datasets. FairLearn also tackles the multi-fairness problem in all the datasets, but it does so by consistently underperforming in the minority (+) class. Interestingly other multi-fairness methods namely, Zafar et al. in Adult and Bank datasets, and MiniMax in Adult and Compas datasets, fail to mitigate discrimination in the multi-attributed setup. MiniMax lacks in multi-fairness convergence while trying to attain their ‘no unnecessary harm’ policy, but manages to produce balanced predictive performance per class. For Zafar et al. the difficulty in finding the optimal parameters is probably the main contributor to its poor multi-fairness and predictive performance. However, as shown for Compas, the method performs well, whenever the proper parameter settings are found. AdaFair which has a similar underlying boosting method, albeit trained for single protected set-up is seen to tackle multi-fairness in Adult, and Credit datasets. A closer look at the data reveals that in the Adult data for 65% of the instances, the protected-group memberships per instance are either all protected (i.e. \{black, female\}) or non-protected (i.e. \{white, male\}) ∀$S_j \in S$. Likewise, in the Credit dataset, this holds for 45% of the data.

### 5.3 Behaviour analysis

The goal of this section is to investigate MFPB’s ability to produce state-of-the-art balanced performance while dealing with multi-fairness. Due to space limitations, we present the results for only one dataset. We select the Bank dataset, as this is the only dataset in our experiments where we got outperformed in multi-fairness by FairLearn. Bank is also the most imbalanced dataset in our batch (IR=1:89).

**Pareto front and pseudo-weights.** In Fig. 3, we present the Pareto optimal solutions of MFPB with respect to the three objectives. In particular, we show the solutions with respect to $O_1$ vs $O_2$ loss (Fig. 3a), $O_1$ vs $O_3$ loss (Fig. 3b) and $O_2$ vs $O_3$ loss (Fig. 3c). We highlight the best (minimum value) solutions with respect to a single objective with different symbols, as well as the selected

---

4 For interested readers the analysis associated with other (Adult, Compas, Credit) datasets are illustrated in Appendix F.
solution. Moreover, we also display the associated pseudo-weight vectors ($\vec{W}_j$) of these solutions ($\vec{f}_j$) as well as of a random solution $\vec{f}_r$ from the Pareto front.

![Fig. 3: Analysis of solutions and pseudo-weights in Pareto front: Bank data](image)

The selected solution (⋆) based on which the optimal classifier is derived has a loss $O_1=0.2$, $O_2=0.01$ and $O_3=0.06$ and a good performance on cross-validation ($Acc=0.79, WC_{Acc}=0.78, MMM=0.07$). Therefore we use its associated weight vector as the user preference vector, i.e., $\vec{U}=[0.35, 0.44, 0.21]$ for our experiments. Coincidentally this solution is very close to the 'Best $O_2$' solution, which has a marginal improvement of 0.01 in $O_2$ and $O_3$ with a compromise of 0.01 in $O_1$. In Fig. 3b we see that there exists a solution $\vec{f}_r$ with $\vec{W}_r=[0.2, 0.33, 0.47]$ which has the $MMM$ loss $O_3=0.01$. If one sets the $\vec{U}$ equal to this pseudo-weight vector, then the resultant Pareto optimal MFPB classifier can achieve the same multi-fairness as the state-of-the-art baseline FairLearn. However, from Fig. 3a, we see that this outcome comes at a heavy compromise on the balanced loss $O_2$ by a value 0.39, and on $0-1$ loss by a value 0.19. This indicates that such multi-fairness improvement can be met by underperforming in both classes. The solution ‘Best $O_1$’ also improves on $O_3$ by 0.03 and $O_1$ by 0.1 but compromises on $O_2$ by 0.7, indicates under-performance in the minority (+) class. One can also notice in Fig. 3 that MFPB learns a solution labelled as ‘Best $O_3$’ with associated pseudo-weight=$[0.49, 0.0, 0.51]$, which is the Pareto optimal with best accuracy-fairness trade-off with $O_1=0.11$ and $O_3=0.0$. But from Fig. 3b and 3c we see that the $O_2$ loss for this solution is 1.0. This again asserts that where class-imbalance is high, the fairest classifier with the best accuracy trade-off is the one that predicts everything as majority (-) class. Thus, such solutions without considering class imbalance have no practical use.

**In-training behaviour.** To understand how the algorithm in-training handles multi-fairness, we look into the changes in weight distributions of the different groups in Fig. 4a. The initial weights (Marital$_{in}$, Age$_{in}$), which represent the actual distribution of instances in the Bank data, show that the protected ‘Age’ group in the minority (+) class is extremely under-represented. The final weights
show notable changes in the distribution, with under-represented groups being boosted.

Fig. 4: In-training analysis: Bank data. P., Np. are protected, non-protected groups respectively

These changes are due to the inherit boosting of misclassified instances in vanilla Adaboost but also due to the extra boosting, via the fairness costs, of misclassified instances that are discriminated. To this end in Fig. 4b, we also plot the gradual change of the fairness costs (Eq. 10) over the boosting rounds for each protected attribute. We observe that the protected ‘Age’ group in the minority class (P.Pos), receives higher costs (Eq. 9) indicating that this group is discriminated most. With respect to the Marital status attribute, we observe that non-protected positive instances are discriminated more and therefore receive higher weights in-training. The fairness cost for all the protected attributes shows high initial fluctuations, but later stabilizes as MFPB in-training reaches fairness convergence, confirming our theoretical results in Section 4.3.

6 Conclusion and Future works

In this work, we focused on multi-fairness under class-imbalance. To this end, we proposed the Multi-Max Mistreatment(MMM) fairness measure, a MOP formulation for learning a MMM-fair classifier and a Pareto boosting approach (MFPB) for selecting the best solution in terms of overall performance, balanced performance and multi-fairness. Our experiments showed that our method provides Pareto optimal solutions with the state-of-the-art multi-fairness and balanced performance, without a big compromise on accuracy, thus outperforming the existing approaches that ignore class-imbalance and sometimes achieve multi-fairness by under-performing in the minority (+) class. Further, we also showed that MFPB is flexible enough to produce a variety of diverse solutions and allows for the selection of the best according to user preferences. In our future work, we plan to integrate MOP in the training process.
References

1. Abraham, S.S., Sundaram, S.S., et al.: Fairness in clustering with multiple sensitive attributes. arXiv preprint arXiv:1910.05113 (2019)
2. Agarwal, A., Beygelzimer, A., Dudík, M., Langford, J., Wallach, H.M.: A reductions approach to fair classification. In: ICML (2018)
3. Barocas, S., Selbst, A.D.: Big data’s disparate impact. Calif. L. Rev. 104, 671 (2016)
4. Caton, S., Haas, C.: Fairness in machine learning: A survey. arXiv preprint (2020)
5. Chawla, N.V., Lazarevic, A., Hall, L.O., Bowyer, K.W.: Smoteboost: Improving prediction of the minority class in boosting. In: ECML PKDD. pp. 107–119. Springer (2003)
6. Corbett-Davies, S., Goel, S.: The measure and mismeasure of fairness: A critical review of fair machine learning. arXiv preprint (2018)
7. Deb, K.: Multi-objective optimization using evolutionary algorithms, vol. 16. John Wiley & Sons (2001)
8. Fredman, S.: Intersectional discrimination in eu gender equality and non-discrimination law. Brussels, UK: European Commission (2016)
9. García, V., Mollineda, R.A., Sánchez, J.S.: A new performance evaluation method for two-class imbalanced problems. In: SSPR. pp. 917–925. Springer (2008)
10. Hébert-Johnson, U., Kim, M., Reingold, O., Rothblum, G.: Multicalibration: Calibration for the (computationally-identifiable) masses. In: ICML. pp. 1939–1948 (2018)
11. Hickey, J.M., Di Stefano, P.G., Vasileiou, V.: Fairness by explicability and adversarial SHAP learning. In: ECML PKDD 2020. pp. 174–190. Springer (2020)
12. Hu, T., Iosifidis, V., Liao, W., Zhang, H., YingYang, M., Ntoutsi, E., Rosenhahn, B.: Fairnn-conjoint learning of fair representations for fair decisions. In: DS. pp. 581–595. Springer (2020)
13. Iosifidis, V., Fetahu, B., Ntoutsi, E.: Fae: A fairness-aware ensemble framework. In: 2019 IEEE Big Data. pp. 1375–1380 (2019)
14. Iosifidis, V., Ntoutsi, E.: Adafair: Cumulative fairness adaptive boosting. In: CIKM. p. 781–790. CIKM ’19 (2019)
15. Kearns, M., Neel, S., Roth, A., Wu, Z.S.: Preventing fairness gerrymandering: Auditing and learning for subgroup fairness. In: ICML. pp. 2564–2572 (2018)
16. Makkonen, T.: Multiple, compound and intersectional discrimination: bringing the experiences of the most marginalized to the fore (2002)
17. Martinez, N., Bertran, M., Sapiro, G.: Minimax pareto fairness: A multi objective perspective. In: ICML. pp. 6755–6764. PMLR (2020)
18. Mehrabi, N., Morstatter, F., Saxena, N., Lerman, K., Galstyan, A.: A survey on bias and fairness in machine learning. arXiv pp. arXiv–1908 (2019)
19. Morina, G., Ollinyk, V., Waton, J., Marusic, I., Georgatzis, K.: Auditing and achieving intersectional fairness in classification problems. arXiv preprint (2019)
20. Mukherjee, I., Rudin, C., Schapire, R.E.: The rate of convergence of adaboost (2013)
21. Rezaei, A., Fathony, R., Memarrast, O., Ziebart, B.D.: Fairness for robust log loss classification. In: AAAI. vol. 34, pp. 5511–5518 (2020)
22. Schapire, R.E.: A brief introduction to boosting. In: Proc. IJCAI, 1999 (1999)
23. Sun, Y., Kamel, M.S., Wong, A.K., Wang, Y.: Cost-sensitive boosting for classification of imbalanced data. Pattern Recognition 40(12), 3358–3378 (2007)
24. Zafar, M.B., Valera, I., Gomez-Rodriguez, M., Gummadi, K.P.: Fairness constraints: A flexible approach for fair classification. JMLR 20, 1–42 (2019)
Appendix

A. Proof of lemma 1

Proof. Let us denote $FPR_{a_j} = a$, $FPR_{b_j} = \bar{a}$, $FNR_{a_j} = b$, $FNR_{b_j} = \bar{b}$, where $a, b, \bar{a}, \bar{b} \in [0, 1]$. Then from definition of $CDM_j$ (Eq. 2) and $DM_j$ (Eq. 1) we can write:

$$CDM_j = |a - b| - |\bar{a} - \bar{b}|$$
$$DM_j = |a - \bar{a}| + |b - \bar{b}|$$ (12)

The proof of $DM_j \geq CDM_j$ can be established by proving $DM_j - CDM_j \geq 0$. Thus, formally to prove,

$$|a - \bar{a}| + |b - \bar{b}| - |a - b| - |\bar{a} - \bar{b}| \geq 0$$ (13)

From definition we know, $CDM_j$, and $DM_j$ are positive real numbers, in the range $[0, 1]$, and $[0, 2]$ respectively. Now, as $DM_j \geq 0$, we get:

$$DM_j = 0 \implies \text{Case 1: } |a - \bar{a}| = 0, \ |b - \bar{b}| = 0;$$
$$DM_j > 0 \implies \begin{cases} \text{Case 2: } |a - \bar{a}| > 0, \ |b - \bar{b}| = 0; \\ \text{Case 3: } |a - \bar{a}| = 0, \ |b - \bar{b}| > 0; \\ \text{Case 4: } |a - \bar{a}| > 0, \ |b - \bar{b}| > 0; \end{cases}$$ (14)

Now when Case 1 of Eq. (14) holds true, we get $a = \bar{a}$ and $b = \bar{b}$. Thus, L.H.S of Eq. (13) implies,

$$|a - \bar{a}| + |b - \bar{b}| - |a - b| - |\bar{a} - \bar{b}| = |a - a| + |b - b| - |a - \bar{a}| - |b - \bar{b}| = 0$$ (15)

Thus, the proof holds true for Case 1 of Eq. (14). From Case 2 we get, $b = \bar{b}$, and $|a - \bar{a}| > 0$. Then, from L.H.S of Eq. (13) it implies,

$$|a - \bar{a}| - |a - b| - |\bar{a} - \bar{b}| \geq |a - \bar{a}| - |a - b - \bar{a} + \bar{b}|$$
$$\implies |a - \bar{a}| - |a - b| - |\bar{a} - \bar{b}| \geq 0$$ (16)

Thus, the proof also holds true for Case 2 of Eq. (14). Similarly, Eq. (13) can also be proved for Case 3 following the proof of Case 2. Now, for Case 4 of Eq. (14) we have $|a - \bar{a}| > 0$ and $|b - \bar{b}| > 0$. Again, following the inequality property $|m| - |n| \leq |m - n|$ we get from L.H.S of Eq. (13)

$$|a - \bar{a}| + |b - \bar{b}| - |a - b| - |\bar{a} - \bar{b}| \geq |a - \bar{a}| + |b - \bar{b}| - |a - \bar{a} + \bar{b} - \bar{b}|$$
$$\implies |a - \bar{a}| + |b - \bar{b}| - |a - \bar{a} + \bar{b} - \bar{b}| \geq 0$$

Thus, it is shown that for all cases when $DM_j \geq 0$, the Eq. (13) holds true. Hence, $CDM_j \leq DM_j$ proved. ■
B. Proof of lemma 2

Proof. Given condition \( CDM_j = DM_j > 0 \). Assuming, \( FPR_{s_j} = a, FPR_{s_j} = \bar{a}, FNR_{s_j} = b, FNR_{s_j} = \bar{b} \), where \( a, b, \bar{a}, \bar{b} \in [0, 1] \). Then, with the given condition implying the statement that discrimination is class-biased, can be formalized as:

\[
|a - \bar{a}| + |b - \bar{b}| = |a - \bar{b}| - |\bar{a} - b| > 0 \quad \Longrightarrow \quad \begin{cases} a - \bar{a} > b - \bar{b}; \text{if bias on - class} \\ b - \bar{b} > a - \bar{a}; \text{if bias on + class} \\ b - \bar{b} = \bar{a} - a; \text{if bias on - and + class} \end{cases}
\] (18)

The last case in Eq. (18) means that discrimination is biased in + class, while a equally reverse discrimination biased in - class (a visa-versa situation also holds true). Alternatively, the statement implies that \( a - \bar{a} \neq b - \bar{b} \) (or, \( \bar{a} - a \neq \bar{b} - b \)) when the given condition in the lemma holds. We begin our proof by method of contradiction. We assume that \( a - \bar{a} = b - \bar{b} \) holds true, when the condition of \( CDM_j = DM_j > 0 \) holds. Now, from L.H.S of Eq. (18) we get,

\[
|a - \bar{a}| + |b - \bar{b}| > 0
\] (19)

Applying the assumption on L.H.S of the condition in Eq. (18) we get,

\[
0 < |a - b| - |\bar{a} - \bar{b}| \leq |a - b - \bar{a} + \bar{b}| = |a - \bar{a} - (b - \bar{b})| = 0 \quad \text{[contradiction]}
\] (20)

Hence, given the condition \( CDM_j = DM_j > 0 \), the assumption \( a - \bar{a} = b - \bar{b} \) is proved false. Similarly, it can be also proved false for the assumption \( \bar{a} - a = \bar{b} - b \). Therefore, we conclude that given \( CDM_j = DM_j > 0 \) implies \( a - \bar{a} \neq b - \bar{b} \) (or, \( \bar{a} - a \neq \bar{b} - b \)) i.e discrimination is class-biased. ■

C. Proof of theorem 1

Proof. Given a \( f(\cdot) \) and a set of \( k \) protected attributes \( \{S_1, \cdots, S_k\} \). Let \( S_j \in S \) be a protected attribute for which \( f(\cdot) \) has the disparate treatment \( DM_j \). By definition (Eq. (1)) it holds that: \( DM_j = |\delta FNR_j| + |\delta FPR_j| \). Let us assume that for prediction given by \( f(\cdot) \) we get:

\[
DM_j \leq DM_l, \quad \forall S_j \in S, l \neq j
\]

\[
\Longrightarrow |\delta FNR_j| + |\delta FPR_j| \leq |\delta FNR_l| + |\delta FPR_l| \quad \text{(21)}
\]

Using definition (2) and Eq. (21) we get:

\[
MMMS = \max(|\delta FNR_j|, |\delta FPR_j|) \quad \text{(22)}
\]

Now by using the property of arithmetic mean for any protected attribute \( S_j \in S \) we get:

\[
DM_j \leq 2 \times \max(|\delta FNR_j|, |\delta FPR_j|) \quad \text{(23)}
\]

Combining Eqs. (21) and (22) we get:

\[
DM_j \leq 2 \times MMMS, \forall S_j \in S \quad \text{(24)}
\]

By applying Lemma 1 i.e \( CDM_j \leq DM_j \), we conclude that:

\[
CDM_j \leq 2 \times MMMS, \forall S_j \in S \quad \text{(25)}
\]
D. Proof of lemma 3

Proof. Using the weight update step in [22] we can derive,

\[ D_t(x_i) = \frac{1}{Z_t} \prod_{t} f_{c_t}(x_i) \exp(-\sum_{t} \alpha_t \text{sign}(y_i h_t(x_i))) \]

where, \( Z_t = \sum_{i} D_{t-1}(x_i) f_{c_t}(x_i) \exp(-\alpha_t \text{sign}(y_i h_t(x_i))) \)

\[ \leq \sum_{i} D_{t-1}(x_i) 2 \exp(-\alpha_t \text{sign}(y_i h_t(x_i))) \quad \because 1 \leq f_{c_t}(x_i) \leq 2 \]

Using Eq. 9 and 10 we can write the fairness cost \( f_{c_t}(x_i) \) at round \( t \) for a single protected attribute \( S_j \) as:

\[ f_{c_t}(x_i) = \begin{cases} 1 + \delta f_t, & \text{if } H_t(x_i) \neq y_i; \\ 1, & \text{otherwise} \end{cases} \]

where, \( \delta f_t = \{ \delta FNR^{l,t}, \text{ if } y_i = + \land ((\delta FNR^{l,t} \geq \delta FPR^{l,t} \leq 0) \land \delta FPR^{l,t} \leq 0) \land x_i \in \bar{s}_j) \}; \]

\[ \{ \delta FPR^{l,t}, \text{ if } y_i = - \land ((\delta FPR^{l,t} \geq \delta FPR^{l,t} \leq 0) \land x_i \in \bar{s}_j) \}; \]

Following 23, the training error bound for any \( H_t \) can be reduced by reducing the upper bound of \( Z_t \) (Eq. 26), from which \( \alpha_t \) for respective \( h_t \) can be induced as:

\[ \alpha_t = \frac{1}{2} \log \left( \frac{\sum D_t(x_i) | h_t(x_i) = y_i |}{\sum D_t(x_i) | h_t(x_i) \neq y_i |} \right) \]  

(28)

Thus, for training error to be less than random guessing, each \( \alpha_t \) for respective weak learner \( h_t \) needs to be positive. This implies for any partial ensemble \( H_t = \sum_1^t \alpha_t h_t, \)

\[ \sum D_t(x_i) | H_t(x_i) = y_i | > \sum D_t(x_i) | H_t(x_i) \neq y_i | \]  

(29)

Now, at any given round \( t' \), we have \( t' \) number of partial ensembles (\( \because H_t = \sum_1^t \alpha_t h_t \)). Let \( D \) be set of \( n \) data instances, \( M_1 \subset D \) be the set of \( m_1 \) instances that gets misclassified by every partial ensemble \( H_t \in \{ H_1, \cdots, H_t \} \), \( M_2 \) be the set of \( m_2 \) instances also misclassified by the partial ensemble \( H_t \) but not by every \( H_t \). Then for the partial ensemble \( H_{t'}, \) using Eq. 26 and 29 we get,

\[ \begin{align*}
\sum_{i} f_{c_t}(x_i) \exp(-\sum_{i} \alpha_t \text{sign}(y_i h_t(x_i))) \\
+ \sum_{i} f_{c_t}(x_i) \exp(-\sum_{i} \alpha_t \text{sign}(y_i h_t(x_i))) \\
< \sum_{i} f_{c_t}(x_i) \exp(-\sum_{i} \alpha_t \text{sign}(y_i h_t(x_i))) 
\end{align*} \]  

(30)

Now let us suppose \( \exists \delta \gg \mu (\mu \approx 0) \) which approximates the aggregated costs of instances for \( H_{t'}, \) s.t \( \prod_{t} f_{c_t}(x_i) \approx (1 + \delta)^l \), where \( l \) is the number of partial
ensembles that misclassified instance \( x_i \). Let’s also for simplicity assume that on correct classification the weights do not change. Now, for instances belonging to set \( M_2 \) and \( D - M_1 - M_2 \), the combination of partial ensemble which misclassifies a subset of such instances, can be expressed as a polynomial sum. For simplicity, without loss of generality let the sets of partial ensemble \( t \subset \{ H_1, \cdots, H_{t'} \} \) and \( t2 \subset \{ H_1, \cdots, H_{t'-1} \} \) bounds the error for the \( m2 \) instances and \( n - m1 - m2 \) instances of Eq. 30 respectively, such that:

\[
(1 + \delta)^{\lceil t1 \rceil} \sum_{i}^{m2} \exp \left( \sum_{t,H_i \in t} \alpha_t \right) \leq \sum_{i}^{m2} \prod_{t} f_{c_t}(x_i) \exp \left( - \sum_{t} \alpha_t \text{sign}(y_i h_t(x_i)) \right) ;
\]

\[
\sum_{i}^{n - m1 - m2} \prod_{t} f_{c_t}(x_i) \exp \left( - \sum_{t} \alpha_t \text{sign}(y_i h_t(x_i)) \right) \leq (1 + \delta)^{\lceil t2 \rceil} \sum_{i}^{n - m1 - m2} \exp \left( \sum_{t,H_t \in t2} \alpha_t \right)
\]

Then, from Eq. 30 we get,

\[
m1(1 + \delta)^{t' - \lceil t2 \rceil} \exp \left( \sum_{t} \alpha_t - \sum_{t,H_t \in t2} \alpha_t \right) + m2(1 + \delta)^{\lceil t1 \rceil - \lceil t2 \rceil} \exp \left( \sum_{t,H_t \in t1} \alpha_t - \sum_{t,H_t \in t2} \alpha_t \right) < (n - m1 - m2)
\]

Now, as the algorithm approaches classification loss convergence, the variance of \((n - m - n1)\) tends to decrease, and \( t' - \lceil t2 \rceil, \lceil t1 \rceil - \lceil t2 \rceil \) tends to increase. But, \((1 + \delta)\) grows exponentially with \( t' \), whereas size of dataset is fixed \((n)\). Thus, if \( \delta >> \mu \), then this contradicts the condition in Eq. 29. Hence, to fulfill the requirement stated in Eq. 29, the approximation \( \delta \) approaches to a small value \( \mu \approx 0 \) till convergence. This can only be satisfied when \( \delta f_j \rightarrow \mu \), henceforth proving the statement of lemma 3.

\[\blacksquare\]

E. Proof of theorem 2

Proof. From Lemma 3 we get to know that the in-training part of MFPB algorithm trained w.r.t protected attribute \( S_j \), upon classification loss convergence will give a set of fair partial ensembles \( \{ H_t | \delta f_j \approx \mu \} \) w.r.t \( S_j \in S \). Now, let \( S_j \in S \) be a protected attribute that dominates \( f_{c_t}(x_i) \) (Eq. 10), i.e:

\[
\delta f_j \geq \delta f_k, \forall S_k \in S
\]

From Def. 1 and Eq. 33 we get that,

\[
CDM_b \leq 2\delta f_j; \forall S_k \in S
\]

Now, following Lemma 3 we get that as \( H_t \) approaches convergence, \( \delta f_j \rightarrow \mu \), where \( \mu \) is a small value \((\approx 0)\). Again, from Eqs. 25 and 34 we get,

\[
MMMS \leq \mu
\]

Assuming \( o^\text{min}_3 = \mu \), where \( o^\text{min}_3 \) is the best available solution value for \( O_3 \) after convergence of \( H_t \). Using the post-training selection steps (Step1 – Step4) with
any \( \vec{U} \), we get \( H^* \), which gives a solution vector \([o_1, o_2, o_3] \in \mathbb{F} \). Assuming the given condition of \( H^* \) being chosen from the set of the partial ensemble converged in \( O_1 \) and \( O_3 \), we get that worst possible value for \( o_3 = o_3^{\min} + \epsilon \), where \( \epsilon \) is small divergence from the optima \( o^*_3 \). This implies that for predictions by \( H^* \),

\[
M \cdot M S \leq (\mu + \epsilon)
\]  

(36)

F. Analysis of Pareto front with associated pseudo-weights

![Fig. 5: Analysis of solutions and pseudo-weights in Pareto front: Adult data](image)

![Fig. 6: Analysis of solutions and pseudo-weights in Pareto front: Compas data](image)

**Adult data:** We notice that the fairest classifier learned by MFPB on this data achieves ideal value 0 in \( O_3 \), by rejecting the entire minority (+) class. The plot also reveals that MFPB is capable of offering more fair solutions if \( U \) is
set [0.06, 0.38, 0.56] instead of the currently selected (⋆) [0.43, 0.3, 0.27], but also compromises the performance on $O_1$ and $O_2$. Another solution achieving equally balanced performance across classes with ideal value (0) in balanced loss ($O_2$) is also offered by MFPB with $U = [0.35, 0.44, 0.21]$. However, this comes with a compromise in 0-1 loss ($O_1$) and MMM loss ($O_3$).

**Compas data:** In Fig. 6, the Pareto front of MFPB upon training on Compas data, shows that with $\vec{U} = [0.42, 0.39, 0.19]$ instead of ($\vec{U} = [0.23, 0.34, 0.4]$), the one selected (⋆) for experimental evaluation, can deliver a classifier which remarkably betters in objectives $O_1$, with a compromise in $O_2$ and $O_3$. Further, setting $\vec{U} = [0.0, 0.03, 0.97]$ fairness objective $O_3$ can be further improved while compromising on $O_1$ and $O_2$.

**Credit data:** Looking into the Pareto front in Fig. 7, we notice from the solution point of selected (⋆) classifier with $\vec{U} = [0.02, 0.52, 0.46]$, it is very close to the one with best $O_2$ in Fig 7a and $O_3$ in Fig 7b and 7c, naturally because of high preference to the objectives. One may notice that there also exists a cloud of solutions in Fig. 7b, which manages to have low $O_3$ loss, while improving on $O_1$. But, from Fig. 7c we can see that these solutions compromises on $O_2$. Although, the training loss suggests that such classifiers (each w.r.t each of these solution points) can still outperform the baselines in balanced performance.

**Conclusion:** As a conclusion, we find that the analysis shows that in imbalanced data, accuracy-fairness trade-off alone is not always the best solution. MFPB is capable of producing a range of diverse solutions which can meet any preferred need. Improving on *multi-fairness* beyond a certain data specific threshold often comes by under-performing in one or more classes. However, in situations where such solutions have higher priority, MFPB is flexible enough to meet such need. In imbalanced data, protected groups are often under-represented. MFPB is able to counter such situation by changing weight distribution such protected groups positively, and converge in *multi-fairness* with a low fairness loss.