Canals in Milky Way radio polarization maps

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ABSTRACT
Narrow depolarized canals are common in maps of the polarized synchrotron emission of the Milky Way. Two physical effects that can produce these canals have been identified: the presence of Faraday rotation measure (\(RM\)) gradients in a foreground screen and the cumulative cancellation of polarization known as differential Faraday rotation. We show that the behaviour of the Stokes parameters \(Q\) and \(U\) in the vicinity of a canal can be used to identify its origin. In the case of canals produced by a Faraday screen we demonstrate that, if the polarization angle changes by 90 across the canal, as is observed in all fields to-date, the gradients in \(RM\) must be discontinuous. Shocks are an obvious source of such discontinuities and we derive a relation of the expected mean separation of canals to the abundance and Mach number of supernova driven shocks, and compare this with recent observations by Haverkorn et al. (2003). We also predict the existence of less common canals with polarization angle changes other than 90. Differential Faraday rotation can produce canals in a uniform magneto-ionic medium, but as the emitting layer becomes less uniform the canals will disappear. We show that for moderate differences in emissivity in a two-layer medium, of up to \(1=2\), and for Faraday depth fluctuations of standard deviation \(1\ \text{rad}\), canals produced by differential rotation will still be visible.

Key words: radio continuum: ISM – magnetic fields – polarization – turbulence – ISM: magnetic fields

1 INTRODUCTION
Polarized (synchrotron) radio emission is a rich source of information about the relativistic and thermal plasmas and magnetic fields in the interstellar medium (ISM). Recent observations have revealed an abundance of unexpected features that arise from the propagation of the emission through the turbulent ISM (Wieringa et al. 1993; Uyaniker et al. 1998; Duncan et al. 1995; Gray et al. 1999; Haverkorn et al. 2000; Gaensler et al. 2001; Wolleben et al. 2006). Arguably the most eye-catching is the pattern of depolarized canals: a random network of dark narrow regions, clearly visible against a bright polarized background. These canals evidently carry information about the ISM, but it is still not quite clear how this information can be extracted. Two theories for the origin of the canals have been proposed; both attribute the canals to the effects of Faraday rotation, but one invokes steep gradients of the Faraday rotation measure (\(RM\)) across the telescope beam, in a Faraday screen (Haverkorn, Katgert & de Bruyn 2000, 2004), whereas the other relies on the line-of-sight effects producing differential Faraday rotation (Beck 1999; Shukurov & Berkhuijsen 2003). In order to use the canal properties to derive parameters of the ISM, one must correctly identify their origin.

In this letter we briefly discuss a few important aspects of the two theories describing the origin of canals. Detailed discussion of relevant depolarization mechanisms is presented in Fletcher & Shukurov (2006a and in preparation). Canals produced by Faraday rotation measure gradients are discussed in Section 2 where we show that these canals require a discontinuous distribution of free electrons and/or magnetic field; we further suggest an interpretation of the discontinuities in terms of interstellar shocks. The case of differential Faraday rotation is discussed in Section 3 in Section 4 we suggest an observational test that can be used to identify the specific mechanism that produces a given canal.

The defining features of the canals are as follows:

(i) the observed polarized emission \(P\) approaches the polarization noise level \(\nu P\);  
(ii) the canal is about one beam wide;
(iii) the canal passes through a region of significant polarized intensity, say \(P > 3 \nu P\);
(iv) the canal is not related to any structure in total intensity, and so cannot be readily explained by, e.g., an intervening gas or magnetic filament.

Polarized intensity vanishes when emission within the telescope beam consists of two equal parts with mutually orthogonal polarization planes. Thus, feature (iv) most often arises because the polarization angle changes by 90 across the canal. However, in Section 4 we predict a new type of canal across which the observed polarization angle does not change.
We only consider canals occurring in properly calibrated maps: [Haverkorn et al. (2004)] argue that the observations we discuss in Section 2.1 do not suffer from missing large-scale structure; [Reich (2006)] discusses the calibration of radio polarization maps in depth.

Polarized radiation is commonly described in terms of the complex polarization,

\[ P = p \exp(2i\phi) \]  

(1)

where \( p \), the degree of polarization, is the fraction of the radiation flux that is polarized. When polarized emission passes through magnetized and ionised regions, the local polarization angle \((\phi)\) changes by an amount depending on the wavelength due to the Faraday effect

\[ (\phi) = \frac{Z_1}{2K} n_e B_z dz^2 \]

where \( K = 0.1 \text{rad m}^{-2} \text{cm}^{-3} \text{G}^{-1} \text{pc}^{-1} \) is a constant, \( n_e \) is the number density of free thermal electrons, \( B_z \) is the component of the magnetic field along the line of sight (here aligned with the z-axis), and the observer is located at \( z \). \((\phi)\) is known as the Faraday depth at a position \( z \) and gives the change in polarization angle of a photon of wavelength as it propagates from \( z \) to the observer. The maximum amount of Faraday rotation in a given direction is called the Faraday depth

\[ F = (1) = \frac{Z_1}{2} n_e B_z dz^2 \]

The observed amount of Faraday rotation, determined by the rotation measure \( RM = d = d^2 \), cannot exceed \( F \), i.e., \( |RM| \leq F \).

The value of \( RM \) is related to \( F \), but often in a complicated manner (see, e.g., [Burn 1966, Sokoloff et al. 1998]). Simplest is the case of a Faraday screen, where the source of synchrotron emission is located behind a magneto-ionic region (e.g., because relativistic and thermal electrons occupy disjoint regions): then \( RM = F \).

In a homogeneous region, where relativistic and thermal electrons are uniformly mixed, \( RM = 0.5F \).

Observations of linearly polarized emission provide the Stokes parameters \( I, Q, U \) which are related to \( p \) and via \( P = Q + iU \):

\[ p = \frac{Q^2 + U^2)^{1/2}}{I} \]

(2)

\[ = \frac{1}{2} \arctan \frac{U}{Q} = \frac{1}{2} \text{sign} Q \text{ sign} U \]

(3)

and the polarized intensity is \( P = \frac{Q^2 + U^2)^{1/2}}{I} = p. \) The complex polarization can be written in terms of

\[ P = \frac{2}{I} \int V (\tau_x) \exp i2\tau_0(\tau_x + \tau) d\sigma \]

(4)

where integration extends over the volume of the telescope beam \( V, W (\tau_x) \) defines the shape of the beam, a function of position in the sky plane \( \tau_x = (\kappa y) \), and \( \tau \) is the synchrotron emissivity. The total intensity is similarly given by \( I = \int V (\tau_x) (\tau) d\sigma \); the Faraday depth is a function of \( \tau_x, F = F (\tau_x) \).\footnote{This terminology may cause confusion: many authors, including [Burn 1966] and [Sokoloff et al. 1998], define the Faraday depth as \( P = p \), in our notation. However, it is more convenient, and physically better motivated, to define the Faraday depth, similarly to the optical depth, as a dimensionless quantity, as used by [Spano et al. 1983] and [Eilek 1983].}

\section{2. CANALS PRODUCED BY A FARADAY SCREEN}

Consider polarized emission from a uniform background source passing through a Faraday screen—a layer where no further emission occurs, but which rotates the polarized plane. If \( F \) varies with \( \tau_x \), adjacent lines of sight within the beam are subject to different amounts of Faraday rotation and the observed degree of polarization decreases. If the variation of \( F \) within the beam, \( F_D = F(\tau_x) \), produces a 90\(^\circ\) difference in \( \phi \), one might expect that the depolarization will be complete as illustrated in Fig. 1, and a canal will be observed along contours defined by \( F_D = (n + 1=2) \) with \( n = 0,1,2, \ldots \).

The situation is more subtle, though. Figure 2(b) shows changing smoothly by 90\(^\circ\) across a beam, as a result of a monotonic change in Faraday depth by the same amount, \( F_D = F \) but in the range \( 2 \).

As shown in Fig. 1, adjacent lines of sight within the beam are subject to different amounts of Faraday rotation and the observed degree of polarization decreases.

Thus, gradients in \( F \) across the beam can produce complete depolarization in a Faraday screen if \( F \) changes by \( F_0 \), \( (n + 1=2) \) within a small fraction of the beam width. As shown in Fig. 2(b), the depth of a canal will only be less than 10\% of the surrounding polarized emission if the gradient in \( F \) occurs over one fifth or less of the beam:

\[ x = \frac{D}{2} \text{. } \frac{1}{2} \text{, where } x \text{ is the extent of the region where the gradient occurs and } D \text{ is the beamwidth. [Haverkorn et al. 2004] reached a similar conclusion in an analysis of the depth and profile of canals observed at 84 cm (see their Sect. 4.4).}

As illustrated in Figs. 2(c) and 2(d), a variation in \( F \) of a magnitude \( F_0 = \text{across the FWHM of the Gaussian beam (i.e. } F = \text{and } x = D \text{) can also produce strong depolarization but now the change in polarization angle across the beam is } \frac{\pi}{2} \text{. Similarly other gradients in } F \text{ can significantly reduce the degree of polarization; for example } F = 0.7 \text{ (dashed line in Fig. 1) across a region that is about } 0.7 \text{ the beamwidth will produce complete depolarization and a change in angle of 54 across the beam.}

Larger gradients in \( F \) also result in strong depolarization and when \( F > \text{the degree of depolarization is less sensitive to the resolution. This is illustrated in Fig. 3 for the case where } F = 1 \pm \text{ (dash-dotted line); complete depolarization occurs when } x = D = 0; 1 \pm \text{ but in the range } 0 < x = D, 2 \text{ the degree}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Sketch showing the variation in Faraday depth \( F \) with respect to transverse distance in the sky plane \( x \). The Faraday depth changes by \( F \) across the distance \( x \) and by \( F_D \) within the beamwidth \( D \).}
\end{figure}
Figure 2. An illustration of depolarization in a Faraday screen: the shaded circle indicates the telescope beam, and dashes show the orientation of the polarization plane at various positions. (a) Abrupt change of the polarization angle by 90° in the middle of the telescope beam leads to complete cancellation of polarization. This can be caused by a discontinuous change in $F$ by 90° in a Faraday screen. (b) A similar but continuous change of $F$ does not result in strong depolarization. (c) A continuous change in $F$ by 180° can produce a canal.

![Image](image1.png)

Figure 3. The minimum degree of polarization in a canal produced by a gradient of Faraday depth in a foreground screen, $p_{\text{min}}$, as a function of the relative linear extent of the gradient $x=D$, where $D$ is the FWHM of a Gaussian beam and $x$ is the position in the sky-plane (see Fig. 2). Different curves represent different increments in Faraday depth, $F$ (indicated in the legend). For $x=D < 1$ (an unresolved gradient), $F$ changes over a distance smaller than the beamwidth and $F$ is effectively discontinuous for $x=D = 1$.

The minimum degree of polarization remains below the 10% level. Thus an increment of $F = 1.5x$ and more generally $F = (n+1=2)$, will produce one beam wide canals when the gradient occurs in a region roughly equal to or less than the beamwidth, i.e. where $x=D = 1$. Other large gradients in $F$, such as $F = n$, depolarize to around 10% of the background $p$ and therefore can also produce canals; however in this case one beam wide canals will only be formed when $x=D = 1$.

So, if $F$ is a continuous, random, function of position (with sufficiently strong fluctuations on the scale of the beam) we predict the existence of canals with every change in polarization angle $0 < 90$ across the beam. On the other hand a discontinuous change in $F$ by $(n+1=2)$, will produce canals across which the polarization angle changes by $90°$. This insight is important as to-date all observed canals have $90°$ only. The only other way in which Faraday screens can consistently produce canals with this change in angle is if all the gradients in Faraday depth on the approximate scale of the beamwidth are of the magnitude $F = (n+1=2)$, $n = 1; 2; ; (n=0$ in this case only produces weak depolarization, as noted above); this is extremely unlikely in a random medium.

If a system of observed canals has $90°$ and is caused by a foreground Faraday screen — see Section 3 for a diagnostic test for the origin of canals — then the distribution of rotation measure in the screen must be discontinuous on the scale of the beam.

Haverkorn & Heitsch (2004) used MHD simulations of Mach 10 turbulence to show that $RM$ gradients can be steep enough to produce depolarized canals on smoothing with a sufficiently large simulated beam. Note that a finite beam size is essential for the production of canals by this mechanism; with perfect resolution the width of the shock front will be resolved and no canal will appear. Furthermore, these canals will not fill up if the data are smoothed (solid and dash-dotted curves for $x=D < 0$ in Fig. 3) whereas canals produced by gradients other than $F = (n+1=2)$ will disappear under smoothing at a rate shown with dashed and dotted curves in Fig. 3.

2.1 The mean separation of canals produced by shocks

An obvious source of Faraday depth discontinuities are shocks in the ISM. Then the mean distance between the canals produced by a Faraday screen will be related to the distance between the shock fronts. The expected separation of suitable shock fronts can be derived using the model of interstellar shock-wave turbulence of Bykov & Toptygin (1987) and then compared with the separation of canals observed by Haverkorn, Katgert & deBruyn (2003) claimed to arise in a Faraday screen.

If the frequency distribution of interstellar shocks of Mach number $M$ is $G(M)$, the mean number of shocks of a strength exceeding $M$ that cross a given position in the ISM per unit time can be obtained as

$$N(M) = \frac{G(M) \, dM}{\frac{4}{3} \pi R^3},$$

and the mean separation between the shocks in three dimensions follows as

$$L \left(\frac{c_s}{N(M)}\right) = \frac{c_s}{N(M)};$$

where $c_s$ is the sound speed. The separation in the plane of the sky is reduced by $\frac{1}{\sin^2 \theta \, d} = 1=2$ due to projection effects and multiplied by a further factor $L=1$ if the shocks occur in a region of depth $d$. At an average distance to the shocks of $d=2$, the average angular separation in the sky plane is then

$$L \left(\frac{1}{d^2}\right);$$

For supernova-driven shocks, Bykov & Toptygin (1987) derived

$$G(M) = G_0 \left[ \frac{1}{M^4} + 3C(M) \, e_0 \, (M-1)^4 \right]$$

allowing for both primary and secondary shocks, where $M$ is a numerical factor ($=2$ for a supernova remnant in the Sedov phase, and $=4.5$ for a three-phase ISM); $C(M) = 2 \, 10^2, 4 \, 10^3$; allowing for $e_0 = 2, 4.5$ respectively; $e_0$ is the volume filling factor of diffuse clouds that reflect the primary shocks to produce the...
secondary ones; \( G_0 = \frac{S}{r_0^2} \) with \( S \) the supernova rate per unit volume and \( r_0 \) the maximum radius of a primary shock. We then obtain the following expression for the separation of shock fronts with \( M > M' \) in three dimensions:

\[
L(M') = \frac{9pc \frac{D}{10km s^{-1}} \frac{M}{0.02yr^{-1}}}{15kpc \frac{h}{50pc} \frac{R}{100pc} \left( 1 + \frac{C}{M} \frac{r}{17} \right)}
\]

where \( D \) is the supernovae rate and \( R \) and \( h \) are the radius and scale height of the star-forming disc. The term in square brackets is approximately 1 for \( D \approx 45 \) and \( M = 1.2 \).

If the magnetic field is frozen into the gas and the gas density increases by a factor \( F \) at the shock then so will the field strength (we neglect a factor \( F^2 \) due to the relative alignment of the shock and field). A canal forms where \( F \) increases discontinuously by \( F_D = (n + 1/2) \). So the required density compression ratio is

\[
s = \frac{F_D}{\frac{HF}{1} + 1};
\]

where \( HF \) is the mean value of \( F \). The gas compression ratio depends on the shock Mach number \( M \) as (e.g., Landau & Lifshitz 1964)

\[
= \left( \frac{1 + 1M^2}{M^2 + 2} \right)^{1/2};
\]

where we have used \( M = 3 \) for the ratio of specific heats, which yields the value of the shock Mach number required to produce a canal:

\[
M > 4 \frac{1}{3} \left( \frac{1}{3} \frac{F_D}{HF} \frac{1}{1} + 1 \right)^{1/2};
\]

In the field of canals observed by Haverkorn et al. (2003) at 84 cm, \( \mu = 1 \) \( \mu = 3 \) mrad m \( ^{-1} \) and a by-eye estimate of their mean separation gives \( L = 45kpc \). The most abundant canals are produced by the shocks which can generate \( F_D = =2 \) and from Eq. (10) this requires \( M = 1.2 \). Using \( F_D = 0.25 \), taking \( = 45 \) and the parameter values used to normalize Eq. (7), we obtain \( L = 10pc \) for shocks with \( M > M' \). Our estimate of \( L \) includes shocks with \( M > M' \) that will not produce canals, but the strong dependence of \( G \) in Eq. (8) on Mach number means that \( L \) will underestimate the separation of canal generating shocks insignificantly. Using Eq. (7), with \( L = 10pc \) and \( L = 45kpc \), we find that the canals observed by Haverkorn et al. (2003) are compatible with a system of shocks occurring in a Faraday screen with a depth of \( d = 100pc \). The maximum distance in this field, beyond which emission is completely depolarized, is estimated to be \( 600pc \) by Haverkorn et al. (2003). Thus this model for the canals’ origin requires that the nearest 100pc in the direction (12b) = (61.1 16.) is effectively devoid of cosmic ray electrons; if the cosmic rays normally spread over a distance \( v_0t \), \( 1kpc \), where \( v_0 \approx 20km s^{-1} \) is the Alfvén speed and \( t \approx 10^6yr \) their lifetime, such a condition is difficult to explain.

### 3 Canals produced by differential Faraday rotation

In Section 2 we discussed the depolarization effects of Faraday rotation measure gradients transverse to the line of sight acting on smooth polarized background emission. Now we will consider a uniform layer in which both emission and Faraday rotation occur. This gives rise to the well known effect of differential Faraday rotation (Braun 1966; Sokoloff et al. 1998), where polarized emission from two positions along the line of sight separated by a rotation of \( \phi = =2 \) exactly cancel, thus reducing the degree of polarization. When a line of sight has a total rotation of \( \phi = 0 \) there is total cancellation of all polarized emission in the layer and the degree of polarization is \( p = 0 \). Canals produced by this mechanism are discussed by Shukurov & Berkhuijsen (2003).

#### 3.1 Differential Faraday rotation in a non-uniform medium

We now investigate how canals produced by differential Faraday rotation are affected by deviations from uniformity in the magnetized medium. First we will discuss the case of a two-layer medium in which the synchrotron emissivity is different in each layer. Then we allow for random fluctuations of \( F' \) (the latter case produces what is known as Faraday dispersion).

For a two-layer medium the fractional polarization can be written as

\[
p = \frac{F_D^2}{F'} \frac{I_1 \sin^2 F_1 + I_2 \sin^2 F_2 - 2I_1 I_2 \sin F_1 \sin F_2 \cos F}{F_1 F_2} \cos F;
\]

where we have assumed \( I_0 = 0 \), \( I_1 \) and \( I_2 \) are the synchrotron fluxes from the furthest and nearest layers to the observer respectively, \( I = I_1 + I_2 \), and \( F_1 \) and \( F_2 \) are the Faraday depths through the two layers. [This is similar to Eq. (10) in Sokoloff et al. (1998), but with typos corrected.] We assume that \( F = F_1 + F_2 \), the condition for canals to form due to differential Faraday rotation, and investigate what will happen to the canals if \( I_1 \neq I_2 \). We parametrize the difference in the synchrotron emissivity in the two layers as \( \phi = (\xi = 1) = 1 \), choose equal Faraday rotation through each layer for simplicity so \( F_1 = F_2 = =2 \), and substitute these into Eq. (13). We then obtain the dependence of the minimum degree of polarization on:

\[
\frac{p}{p_0} = \frac{\frac{1}{2} \frac{1}{L + 1 \approx 2} \frac{\frac{1}{2}}{L + 1 \approx 2} \frac{1}{2}}{O \left( \frac{1}{2} \right)};
\]

Thus, for a moderate difference in emissivity of \( = 1 \approx 2 \) between two layers the canals will have a depth of \( p/p_0 = 1 \approx 6 \) compared to \( p = 0 \) for a strictly uniform medium; these canals will still be clearly visible and can be interpreted as e.g. contours of \( F' \) or \( R \) (Shukurov & Berkhuijsen 2003). Where the difference in emissivity is greater, say \( > \) or \( \approx 2 \) or more as one might expect viewing distant bright spiral arm emission near the Galactic plane, the canals will more readily fill up, become much less distinct and more difficult to interpret confidently.

Now let us consider the effect of random fluctuations in Faraday rotation, sometimes called internal Faraday dispersion. We start with Eq. (34) of Sokoloff et al. (1998):

\[
P = \frac{p_0 \exp \left( \frac{S}{S} \right)}{S};
\]

where \( S = 2 \frac{2}{F} \frac{2}{F} \frac{2}{F} \) and \( F \) is the standard deviation of the
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(iii) If shocks produce the discontinuities in a foreground Faraday screen that generate canals, the mean separation of the canals can provide information about the Mach number and separation of shocks in the screen (Section 4.1).

(iv) Canals produced by differential Faraday rotation are sensitive to non-uniformity in the medium along the line of sight, systematic or random. However, they will remain recognisable if the synchrotron emissivity varies by less than a factor of about 2 or if the standard deviation of the Faraday depth is $F < 1$ (Section 4.1).

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