Backreaction of cosmological perturbations

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Cosmological Principle:

A sufficiently large volume, chosen arbitrarily in the Universe, should contain approximately the same amount of matter.

Scale of homogeneity: $70 - 370$ Mpc

Hercules-Corona Borealis Great Wall $\sim 2-3$ Gpc

At scales larger than the scale of homogeneity, the averaged evolution of the Universe is governed by the FLRW metric

$$ds^2 = a^2(\eta)[d\eta^2 - \delta_{\alpha\beta}dx^\alpha dx^\beta],$$

with the Friedmann equations ($\Lambda$CDM model)

$$\frac{3\mathcal{H}^2}{a^2} = \kappa \bar{\varepsilon} + \Lambda$$

and

$$\frac{2\mathcal{H}' + \mathcal{H}^2}{a^2} = \Lambda$$
Small-scale inhomogeneities, e.g. galaxies and groups of galaxies, perturb the averaged metric:

$$ds^2 = a^2(\eta) \left[ \left( 1 + 2\Phi + 2\Phi^{(2)} \right) d\eta^2 - \left( 1 - 2\Psi - 2\Psi^{(2)} \right) \delta_{\alpha\beta} dx^\alpha dx^\beta \right]$$

First-order perturbations

$$\Phi = \Psi$$

Second-order perturbations

Average values:

$$\overline{\Phi} = 0.$$

$$\overline{\Phi^{(2)}}, \overline{\Psi^{(2)}}, \overline{\Phi^2}, \overline{\rho\Phi} \neq 0 !!!$$

affect the averaged behavior of the Universe

How strong?
In the case of **strong influence:**

1. The expansion of perturbations in terms of the degree of smallness (see above) is not correct.

2. The Friedmann equations must be modified.

**Backreaction problem!**
I. Backreaction in Friedmann equations

Within the cosmic screening approach, the perturbed Friedmann eqs (ApJ 845 (2017) 153):

\[
\frac{3H^2}{a^2} - \frac{6H}{a^2} \left[ \mathcal{H}\Phi^{(2)} + (\Psi^{(2)})' \right] - 3\kappa \bar{\varepsilon} \bar{\Psi}^{(2)} = \kappa \bar{\varepsilon} + \Lambda + \kappa \bar{\varepsilon}^{(II)},
\]

\[
\frac{2H' + H^2}{a^2} - \frac{2}{a^2} \left[ \left( \bar{\Psi}^{(2)} \right)'' + 2\mathcal{H}(\bar{\Psi}^{(2)})' + \mathcal{H}(\Phi^{(2)})' + (2H' + H^2)\bar{\Phi}^{(2)} \right]
= \Lambda - \kappa \bar{p}^{(II)},
\]

effective average energy density and pressure:

\[
\kappa \bar{\varepsilon}^{(II)} = \frac{\kappa}{2} \bar{\varepsilon} \frac{\rho \Phi_0}{\bar{\rho}} - 5 (\kappa \bar{\varepsilon} + \Lambda) \frac{\Phi_0^2}{\bar{\rho}},
\]

\[
\kappa \bar{p}^{(II)} = \frac{\kappa}{6} \bar{\varepsilon} \frac{\rho \Phi_0}{\bar{\rho}} - \left( \frac{11}{6} \kappa \bar{\varepsilon} - \frac{5}{3} \Lambda \right) \frac{\Phi_0^2}{\bar{\rho}}.
\]
The first-order velocity-independent potential (ApJ 825 (2016) 84):

\[
\Phi_0 = \frac{1}{3} - \frac{\kappa c^2}{8\pi a} \sum_n \frac{m_n}{|r - r_n|} \exp \left( -\frac{a|r - r_n|}{\lambda} \right)
\]

The screening length:

\[
\lambda = \sqrt{\frac{2a^3}{3\kappa \bar{\rho} c^2}}.
\]

Inhomogeneities are considered in the form of a system of separate nonrelativistic point-like particles with masses \(m_n\) and comoving radius-vectors \(r_n\).

\[
\overline{\rho \Phi_0} = \frac{1}{3} \bar{\rho} - \frac{\kappa c^2}{8\pi a} \frac{1}{V} \sum_n \sum_{k \neq n} \frac{m_n m_k}{|r_k - r_n|} \exp \left( -\frac{a|r_k - r_n|}{\lambda} \right)
\]

A toy model: all particles (galaxies) have the same masses and are located at the same distance \(l\) from each other.

\[
\overline{\rho \Phi_0} = \frac{1}{3} \bar{\rho} \left[ 1 - \frac{1}{4\pi \lambda^2} \sum_{k_1 = -\infty}^{+\infty} \sum_{k_2 = -\infty}^{+\infty} \sum_{k_3 = -\infty}^{+\infty} \frac{1}{\sqrt{k_1^2 + k_2^2 + k_3^2}} \times \exp \left( -\frac{\sqrt{k_1^2 + k_2^2 + k_3^2}}{\lambda} \right) \right],
\]
Renormalized screening length:

\[
\tilde{\lambda} = \frac{\lambda}{a_l} = \sqrt{\frac{2c^2}{9H_0^2\Omega_M}} \frac{1}{a_0l} \frac{1}{\sqrt{z+1}} \\
\approx \frac{3740 \text{ Mpc}}{a_0l} \frac{1}{\sqrt{z+1}},
\]

Figure 1: Behavior of $\frac{\rho \Phi_0}{\rho}$ as a function of the renormalized screening length $\tilde{\lambda}$.

$z < 100, \quad \frac{\rho \Phi_0}{\rho} \ll 10^{-3}$

$z = 100, \quad \tilde{\lambda} \approx 18.6$

$z = 0, \quad \tilde{\lambda} \approx 187$

$a_0l = 20 \text{ Mpc}$
Evaluation of $\bar{\Phi}_0^2$:

$$\bar{\Phi}_0^2 = \frac{1}{9} - \frac{\kappa c^2}{48\pi \rho \lambda} \frac{1}{\sqrt{V}} \sum_n \sum_k m_n m_k \exp \left( -\frac{a|\mathbf{r}_k - \mathbf{r}_n|}{\lambda} \right)$$

$$\bar{\Phi}_0^2 = -\frac{1}{9} \left[ 1 - \frac{1}{8\pi \hat{\lambda}^3} \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} \sum_{k_3=-\infty}^{+\infty} \exp \left( -\sqrt{\frac{k_1^2 + k_2^2 + k_3^2}{\hat{\lambda}}} \right) \right]$$

$z < 100 \Rightarrow \hat{\lambda} > 18.6$

$\Rightarrow \bar{\Phi}_0^2 \ll 10^{-6}$

Figure 2: Behavior of $\bar{\Phi}_0^2$ as a function of the renormalized screening length $\hat{\lambda}$. 
Conclusion 1:

The effective average energy density $\bar{\varepsilon}^{(\Pi)}(\eta)$ and pressure $\bar{p}^{(\Pi)}(\eta)$ have a negligible backreaction effect on the Friedmann equations.
II. Evaluation of the second-order term \( \Psi_0^{(2)} \). 

\[
\Psi_0^{(2)} = -\frac{3}{4} \Phi_0^2 + \frac{\Phi_0}{6} - \pi \bar{\lambda} \left( \frac{1}{12 \pi \tilde{\lambda}^2} \right)^2 \sum_k m_k e^{-a|r-r_k|/\lambda} \\
+ \frac{1}{2} \left( \frac{\kappa c^2}{8 \pi a} \right)^2 \sum_{k,k'} m_k m_{k'} e^{-a|r-r_k|/\lambda} e^{-a|r-r_{k'}|/\lambda} \frac{1}{|r-r_k|} \frac{1}{|r_{k'}-r_k|},
\]

\[
\Psi_0^{(2)} = -\frac{3}{4} \Phi_0^2 + \frac{\Phi_0}{6} - \pi \tilde{\lambda} \left( \frac{1}{12 \pi \tilde{\lambda}^2} \right)^2 \sum_k e^{-|\tilde{r}-\tilde{r}_k|/\tilde{\lambda}} \\
+ \frac{1}{2} \left( \frac{1}{12 \pi \tilde{\lambda}^2} \right) \left( \frac{1}{3} - \Phi_0 \right) \sum_q \frac{e^{-\tilde{r}_q/\tilde{\lambda}}}{\tilde{r}_q} \quad \Rightarrow \quad -\frac{3}{4} \Phi_0^2, \quad \tilde{\lambda} >> 1
\]

\[
|\Phi_0| \ll 1 \quad \Rightarrow \quad |\Psi_0^{(2)}| \ll |\Phi_0|.
\]
Conclusion 2:

Therefore, the second-order correction $\Psi_0^{(2)}$ is much less than the first-order quantity $\Phi_0$ as it should be!
Final conclusions:

1. The numerical evaluation shows that considered nonlinear perturbations have a negligible backreaction effect on the Friedmann equations.

2. The second-order correction to the gravitational potential is much less than the corresponding first-order quantity. Consequently, the expansion of perturbations into orders of smallness in the cosmic screening approach is correct.
THANK YOU!