We shall give dynamics to our spacetime manifold by first identifying the local affine symmetry as the characterizing symmetry for our geometry à la Felix Klein, this invariance is imposed on us by the Law of Inertia and the Law of Causality. And then by prescribing 16 gauge vector bosons to this symmetry à la Yang and Mills. Both disciplines give spectacular success in terms of experiments and physical observations, despite of the fact that they look very different.

There are, by now, many research works done in trying to put these two disciplines into one single footing. Some people try to visualize gauge vector boson interactions as geometrical manifestations in a higher dimensional sub-manifold [1, 2]. Other people try to consider the geometrical gravitation theory in the form of a local gauge vector boson approach pioneered by Yang and Mills. Both disciplines give dynamic success in terms of experiments and physical observations, despite of the fact that they look very different.

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**A SPACETIME MANIFOLD WITH A NON-DYNAMICAL BACKGROUND WORLD METRIC**

In this article we shall consider the local gauge theory approach of gravitation, albeit in a new context. A non-Minkowskian world metric for our spacetime is always regarded as what makes it curved. However, it is very difficult to regard the world metric (or more precisely its corresponding vierbein fields) as gauge vector fields because of the peculiar way it appears in the action that determines physics.

Here we shall assign the world metric of our spacetime with a much limited role. We shall assume that the only function of the world metric $g_{\mu\nu}$ is to give us world distance (and hence world volume element $\sqrt{-g}d^4x$), and will have no dynamical terms (terms that contain spatial or temporal derivatives of $g_{\mu\nu}$) in the action. This assumption frees us from taking the global Minkowskian metric as the de facto world metric and sets the notion that no particular metric is a priori world metric for physics. In this sense, the world metric for our spacetime serves just as an arbitrary background of measuring clock and stick in our discussion of physics.

With a given world metric $g_{\mu\nu}(x)$ at the point $x$ with world coordinates $x^a$, a set of vierbein fields $e^a_\mu(x)$ will follow. These vierbein fields are defined in a locally flat patch that is assumed to be equipped with an arbitrary but given local Minkowskian frame whose coordinates and metric are $x^a$ and $\eta_{ab}$ respectively. The differentials of these two coordinate systems define the vierbein fields as $dx^a = e^a_\mu dx^\mu$, and hence the world metric and the vierbein fields will then be related by

$$\eta_{ab}e^a_\mu e^b_\nu = g_{\mu\nu}. \quad (1)$$

Here, and in the following, the Latin indices will signify the Minkowskian components while the Greek indices will mean the world ones.

**SOME BASIC PHYSICAL PRINCIPLES THAT ARE REQUIRED TO BE INARIANT UNDER THE EXPECTED LOCAL COORDINATE TRANSFORMATIONS**

On a locally flat patch around a point of our spacetime is where we do our physics. Even though we have already had a local Minkowskian system $x^a$ and $\eta_{ab}$ on that patch, we may still have the freedom to re-label the points on that patch with different local coordinate systems, for example, by rotating and stretching these local Minkowskian coordinate axes.
The form of the admissible local coordinate transformations depends on what are the physical principles that, we hope, to remain invariant under these coordinate changes.

Here, we believe that the law of inertia should remain intact under these expected coordinate transformations. This means that the concept of straight lines should be preserved, as an object moving in straight line in one coordinate system should remain moving in straight line in another coordinate system. Also light should propagate in straight lines in whatever coordinate system we are using. Causality is also a very important concept in physics, and hence the order of points and the ratio of segment lengths in a straight line should not change with a change in coordinate system. And of course, the concept of parallelism should also be preserved because two parallel moving objects, as well as parallel light rays, should remain parallel under a change of coordinate system.

Those transformations which maintain collinearity, order of points and invariant segment ratios in straight lines, and parallelism are, in fact, the affine transformations of Euler [6]. Affine transformations are sometimes grouped together as dilations, rotations, shears and reflections. We shall call collectively transformations that are not rotations as strains.

MARRIAGE OF THE ERLANGEN PROGRAM WITH THE YANG-MILLS DOCTRINE-A WAY TO GIVE DYNAMICS TO A GEOMETRY

Here we want to emphasize that our choice of the affine transformations as our admissible local coordinate transformations comes from physics. It comes from our belief that these admissible transformations should leave the above said physical principles invariant. And if we are going to call such a chosen local coordinate system as a chosen local geometric setting, then we can say that physics is assumed to be invariant under a change of local geometric setting.

These transformations form a Lie group, called the local affine group. It was Felix Klein who first suggested of classifying geometries by their underlying symmetry groups, starting with the Projective Geometry (our affine geometry is a restriction of the Projective Geometry). Such a mathematical program is called the Erlangen Program [6].

Since matter, which are world objects, are described by local fields with the reference to a local coordinate system. These local fields could have structures that depend on the geometric setting chosen at that point. For example, if we want to describe the physics of an electron, it may be convenient to choose the local affine rotations as our local Lorentz transformations. Then an affine rotated setting will give a set of Lorentz transformed fields.

As we believe that the relative differences of the local fields of the same world object at two space-time points arising from different geometric settings are physically meaningless, we have to find some way to counteract such variations. Similar to what have been done by Yang and Mills [7], we shall introduce a set of vector bosons to do these counteractions.

Note that the introduction of vector bosons, as suggested by Yang and Mills, were originally used to facilitate the local identifications of internal quantum numbers for quantum systems. Here we extend their ideas to the local identifications of geometric settings in our spacetime.

At any point of our spacetime, these vector boson fields can be transformed away locally by a suitable choice of the coordinate system at that point. A more familiar way of saying this is that these vector fields are locally equivalent to a transformation of the coordinate system. One can then draw the strong analog between the above statement and the Principle of Equivalence of Einstein which states that "there is a complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system". The Principle of Equivalence is the founding principle taken by Einstein to build his General Theory of Relativity, and the gauge principle stated in the above will be shown, as given below, to give a theory of vector gravity.

In the following, these vector boson fields are regarded as dynamical variables. Their dynamics will be fabricated so as to insure that physics be invariant under local affine transformations. And this will be done, again by following Yang and Mills, by first constructing the Lagrangian that is locally affine symmetric.

THE AFFINE GROUP GL(4 R)

For a four dimensional patch, these affine transformations can be carried out by \(4 \times 4\) invertible real matrices, either actively or passively. All these \(4 \times 4\) invertible real matrices form a Lie group called the Real General Linear group of dimension 4 and is designated as \(GL(4 \mathbb{R})\). Hence the \(GL(4 \mathbb{R})\) will be synonymous with our affine group. The \(GL(4 \mathbb{R})\) has two sets of generators. The 6 anti-symmetric generators \(T^{ab}\) generate the rotations while the 10 symmetric generators \(T^{ab}\) generate the strains. They satisfy the following commutation relations [8].
These generators, when combined together as $M^{ab} = \frac{1}{2}(T^{ab} + J^{ab})$, and with the indices lowered by $\eta_{ab}$, give a compact commutation relation of the form

$$[M^a_b, M^d_c] = i \delta^b_c M^d_c - i \delta^d_a M^b_a$$

We know that the GL$(4\,\mathbb{R})$ with the defining Lie Algebra given in Eq. 3 has no presupposition of the existence of the Minkowskian metric on the locally flat patch of our spacetime. We are introducing GL$(4\,\mathbb{R})$ into physics in our way because we want to emphasize that there are some very fundamental laws, namely, the Law of Inertia and the Law of Causality, working together to impose the GL$(4\,\mathbb{R})$ symmetry onto our spacetime. “Invariance dictates interaction”, said C. N. Yang. This doctrine seems working well in the electroweak and the strong interactions. We propose in this article that this doctrine could apply to gravity too.

### THE YANG-MILLS ACTION FOR THE LOCAL GL$(4\,\mathbb{R})$ IN THE PRESENCE OF A BACKGROUND WORLD METRIC

The Yang-Mills gauge potentials for the GL$(4\,\mathbb{R})$ are

$$A_\mu = A^m_{\mu n} M^m_n.$$  

Note that there are totally 16 gauge bosons $A^m_{\mu n}$ appearing in our theory. The antisymmetric parts of $A^m_{\mu n}$ go with the generators $J^m_n$ while the symmetric parts go with the generators $T^m_n$. These 16 gauge bosons are world vector fields.

The Yang-Mills field strength tensor $F_{\mu\nu}$ is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu] = (\partial_\mu A^m_{\nu n} - \partial_\nu A^m_{\mu n} + A^m_{\mu p} A^p_{n \nu} - A^m_{\nu p} A^p_{m \nu}) M^m_n$$

$$= F^m_{\mu\nu} M^m_n.$$  

The Yang-Mills Lagrangian, which is invariant under the local GL$(4\,\mathbb{R})$ transformations, is

$$\mathcal{L}_{YM} = \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu}.$$

It is interesting to see how we can evaluate the trace of the products of the generators of GL$(4\,\mathbb{R})$. For GL$(4\,\mathbb{R})$, there exists a relation between the trace of the products and the product of the traces of the generators and the bilinear Killing form of the Special Linear group SL$(4\,\mathbb{R})$ (as denoted by a tilde), namely

$$\text{Tr}(M^a_b M^c_d) = \frac{1}{8} K(\tilde{M}_a^b \tilde{M}_c^d) + \frac{1}{4} \text{Tr}(M^a_b) \text{Tr}(M^c_d),$$

where

$$K(\tilde{M}_a^b \tilde{M}_c^d) = \text{Tr}(ad(\tilde{M}_a^b) ad(\tilde{M}_c^d)),$$

$K$ is the bilinear Killing form and $ad(\tilde{M}_a^b)$ is the Adjoint Representation of $\tilde{M}_a^b$ which is traceless. The calculation of the trace of the products of the Adjoint Representation is direct, and it is

$$\text{Tr}(ad(\tilde{M}_a^b) ad(\tilde{M}_c^d)) = 8 \delta^a_c \delta^b_d - 2 \delta^a_d \delta^b_c.$$  

The calculation of $\text{Tr}(M^a_b)$ is easy too. The generators of GL$(4\,\mathbb{R})$ differs from the generators of SL$(4\,\mathbb{R})$ by a dilatation part of $T^a_b$, which is $\frac{1}{4} \delta^a_b I$. In other words, $T^a_b = T^a_b + \frac{1}{4} \delta^a_b I$, and hence we have

$$\text{Tr}(M^a_b) = \delta^a_b.$$  

Combining Eq. 7, Eq. 8, Eq. 9 and Eq. 10 gives

$$\text{Tr}(M^a_b M^c_d) = \delta^a_c \delta^b_d.$$  

This is independent of the kind of representation chosen for GL$(4\,\mathbb{R})$. Note that the result given in Eq. 11 is not only from the structure constants of Eq. 3, but also comes from a judicious choice of the dilatation part of the GL$(4\,\mathbb{R})$ generators.

And hence the affine symmetric Yang-Mills action $\text{SYM}$, in the presence of the background world metric $g_{\mu\nu}$, will be

$$\text{SYM}[g, A, \partial A] = \kappa \int \sqrt{-g} d^4 x g^{\mu\nu} g^{\rho\sigma} (\delta^a_c \delta^b_d) F^a_{\mu\nu} F^c_{\rho\sigma}.$$  

\[\text{(12)}\]
\( \kappa \) is a dimensionless coupling constant of the theory.

Of course, the total action will also contain a piece coming from the matter fields which are supposed to couple gauge invariantly to the gauge fields \( A_{m
u} \), and thus making the total action \( S_{\text{total}} \) as

\[
S_{\text{total}} = S_{\text{YM}} + \int \sqrt{-g} d^4 x L_{\text{matter}}. \tag{13}
\]

**HOW THE 16 GAUGE VECTOR BOSONS SELECT THE BACKGROUND WORLD METRIC IN CLASSICAL PHYSICS**

Even though the world metric serves just as a background measuring clock and stick for our spacetime, it does contribute to the Feynman amplitudes in calculating physical processes because

\[
\text{Amplitude} = \int e^{\frac{iS_{\text{total}}}{\hbar}} D [g_{\mu\nu}, A^m_{\alpha \mu}, \text{matter fields}], \tag{14}
\]

where \( D [g_{\mu\nu}, A^m_{\alpha \mu}, \text{matter fields}] \) is the GL(4 \( R \)) invariant measure over the fields. The role played by \( g_{\mu\nu} \) in physics would be clear if we were able to integrate out the

\[
\begin{align*}
\frac{\delta S_{\text{total}}}{\delta g_{\theta \tau}} & = \sqrt{-g} (F_a^{\alpha \beta} F_c^{\alpha \beta} - \frac{1}{4} g_{\theta \tau} F_a^{\alpha \beta} F_c^{\alpha \beta}) \cdot \left( 1 - \frac{1}{4 \kappa} T_{\theta \tau} \right) = 0; \\
\frac{\delta S_{\text{total}}}{\delta A^m_{\mu \nu}} & = D_\rho (A) \left( \sqrt{-g} F_{m \rho}^{\mu \nu} \right) - \frac{1}{\kappa} \sqrt{-g} S_{m \nu} = 0; \\
\frac{\delta S_{\text{total}}}{\delta \text{matter fields}} & = 0.
\end{align*}
\tag{15}
\]

where \( D_\rho (A) \) denotes the Yang-Mills gauge covariant differentiation. The \( T_{\theta \tau} \) and \( S_{m \nu} \) are respectively the metric energy-momentum tensor and gauge current tensor of the source matter.

This is the set of equations that we are proposing to describe the spatial and temporal evolution of the classical GL(4 \( R \)) Yang-Mills and the matter fields, in our spacetime, which has a background world metric \( g_{\mu\nu} \).

It turns out that not all world metric can sustain a classical GL(4 \( R \)) Yang-Mills field, only some selected ones can do. Putting Eq. (13) in words: the solved \( A^m_{\mu \nu} \) from the second equation (which is the Yang-Mills equation) will be functionals of \( g_{\mu\nu} \). And when we plug the solved \( A^m_{\mu \nu} \) into the first equation, it will become an equation for \( g_{\mu\nu} \). And from this equation we shall select the world metrics for our classical world.

Up to this point, we have not used any sophisticated concepts in geometry such as the connection, parallelism and the curvature. The only thing we have used that may have something to do with geometry is that our spacetime should have a metric telling us how to measure distance and volume. So the geometry of our spacetime is not Riemannian, and is not even affine; it is just metrical.

From our point of view, a metric is fundamental and is needed if we want to construct an action from some fields. The concepts of connection, parallelism and curvature are, however, not. If we can show that the Schwarzschild metric and some other metric can follow from our Eq. (13) then we can claim that the description of gravity needs no sophisticated geometric ideas.

Before we embark on further discussions, we should clarify the role played by our dimensionless parameter \( \kappa \) and should also clarify the way how the Newtonian gravitational constant \( G \) makes its appearance in gravitation phenomena.

\( \kappa \) appears because there is an arbitrariness in fixing the relative scale between the matter and gauge parts of the Lagrangian. The value of \( \kappa \) should have no significance
in physics, and can be absorbed into the definition of the matter field.

What have significance on physics are the integration constants that accompany the solutions to the equation of motion. Because $g_{\mu\nu}(x)$ is dimensionless, $x$ must be scaled by some integration constant $l$ which has the dimension of length. It is a different $l$ that gives a different strength in gravitational interaction. The fact that $l$ is proportional to the inertial mass $M$ strength in gravitational interaction. The fact that we shall display as

and then substitute the $A_{\mu}^m$ by the $\Gamma^\rho_{\tau\mu}$ by plugging $A_{\mu}^m$ into $F_{n\mu\nu}$ in Eq. 24. Note that $\Gamma^\rho_{\tau\mu}$ is defined by $A_{\mu}^m$ and has, so far, nothing to do with connections. Miraculously, the Yang-Mills field strength tensor can be re-expressed in the $\Gamma$ fields in a very simple way as

where we have used $R^\rho_{\sigma\mu\nu}$ to stand for $(\partial_\mu \Gamma^\lambda_{\sigma
u} - \partial_\nu \Gamma^\lambda_{\sigma\mu} + \Gamma^\lambda_{\kappa\mu} \Gamma^\kappa_{\sigma\nu} - \Gamma^\lambda_{\kappa\nu} \Gamma^\kappa_{\sigma\mu})$. Note that our $R^\lambda_{\sigma\mu\nu}$, though looks like the Riemann curvature tensor, is, in fact, a derived quantity coming from the Yang-Mills field tensor. Plugging the result in Eq. 17 into the Yang-Mills action in Eq. 12 the Yang-Mills action will look like

Now Eq. 16 looks like the famous geometric relation between the affine connections and the spin connections under the tetrad postulate, if we regard $\Gamma^\rho_{\tau\mu}$ as the affine connections and $A_{\mu}^m$ as the spin connections. And Eq. 18, Eq. 19 and Eq. 20 together, look like a geometric theory with a geometric Lagrangian and a set of geometric equations of motion that we have encountered frequently in talking about gravity. Hence it is natural for us to put all these in the following geometric jargons: that a parallel connection $\Gamma^\rho_{\tau\mu}$ is introduced into the spacetime, that a Riemann curvature tensor is constructed, that a gravitational Lagrangian is formed out of the product of the Riemannian curvature tensor, that we are trying to obtain the gravitational equations by varying the connections and the metric independently a la Palatini 11, and that gravity is a kind of Metric-Affine Gauge Theory 3. From the above discussions, it is now clear that what all these things that were done in the past, were done in pieces by following the doctrines laid down by Klein, Einstein, Yang and Mills and Feynman, either knowingly or unknowingly.

This form of the gravitational action has presented itself many times in the history of the development of the geometric theory of gravity, but is, in fact, carrying very different information at each one of the presentations.
The point of focus is on the relation between the metric and the connections.

In its very early version, as proposed by Hermann Weyl [12], the connections that appear in the theory are nothing but the Christoffel symbols which are of first derivatives in the metric. This will result into a theory in which the metric is the only dynamical variable, and the variation with respect to the metric will give an equation of motion of higher order derivatives. It is well known that such a theory will possess runaway solutions.

Later Yang [13], also regarded the connections as the Christoffel symbols at the start, but varied the connections instead in order to get the equation of motion. The final result is, again, an equation of higher derivatives in the metric.

Stephenson [14], put the anti-symmetric parts of the connections equal to zero, and regarded the symmetric parts of the connections and the metric as independent variables. And he obtained two equations of motion by varying both the metric and symmetric parts of the connections independently.

On the other hand, some people identify the symmetric parts of the connections as the Christoffel symbols and regard the anti-symmetric parts and the metric as independent variables. Those people working on the geometric theory of gravity. And of course, we also want to know how our known that such a theory will possess runaway solutions.

For us, the full connections, both the symmetric and the anti-symmetric parts, as well as the metric are independent variables. In this theory of gravity, the connections independently.

On the other hand, some people identify the symmetric parts of the connections as the Christoffel symbols and regard the anti-symmetric parts and the metric as independent variables. Those people working on the geometric theory of gravity. And of course, we also want to know how our known that such a theory will possess runaway solutions.

For us, the full connections, both the symmetric and the anti-symmetric parts, as well as the metric are independent variables. In this theory, the connections are just the transformed $GL(4 \, R)$ Yang-Mills vector potentials $A_m^{\mu \nu}$, and can be taken as being independent of $g_{\mu \nu}$.

Here we feel obligatory for us to re-assert the reason that we are making excursion into the land of geometry is because we want to make use of some of the results known to the people working in the geometric theory of gravity. And of course, we also want to know how our proposed theory looks like in geometrical languages.

The different choices of the content coded in the Riemann curvature tensor give different stories for physics. For example, for the Weyl theory, the metric is the only dynamical variable, and hence the action will contain kinetic terms that have derivatives that are of orders higher than two. And when we look for the possible propagation modes in the theory, which can be obtained by looking at the inverse of the kinetic term, we will find that there will be propagators having the wrong signs, which will correspond to unphysical states called the ghosts or tachyons, and will end up into an unstable theory with the so called Ostrogradski instability [17].

For us, the metric is a non-dynamical background field [16], and the only dynamical variables are the connections which obey an equation second order in space and time derivatives. The propagating modes are the 16 vector bosons and nothing else. And hence our theory will contain no ghost and no Ostrogradski instability. We are not having the pathologies that are affecting quadratic curvature theories in which the metric is dynamical and is compatible with the connections.

**THE SCHWARZSCHILD METRIC IS INDUCED BY THE 16 GAUGE VECTOR BOSONS**

Let us now concentrate ourselves on the “vacuum” solutions of Eq. 18, Eq. 19 and Eq. 20. By “vacuum” here we shall mean the case where all the matter fields are absent, except possibly at the source point. A trivial solution with a global Minkowskian world metric and vanishing $GL(4 \, R)$ gauge potentials can be inferred immediately from the equations.

However, the $GL(4 \, R)$ gauge vector fields may not necessarily be vanishing. In the following, we shall search for solutions under the ansatz that the world metric $g_{\mu \nu}$ and the connections $\Gamma^\sigma_{\tau \iota}$ are compatible with each other. We shall call this ansatz the Compatibility Ansatz (CA). Then the “vacuum” version for Eq. 18, Eq. 19 and Eq. 20 can be written as

$$H^\theta \tau = \left[ \delta \int dx^4 \sqrt{g} \, g_{\mu \nu} \, g^{\mu \nu} \left( R^\tau_{\sigma \rho} \, R^{\sigma \tau}_{\rho} \right) \right]_{\text{CA + torsionless}}$$

And if we are searching for solutions that are torsionless, then $\Gamma^\sigma_{\tau \iota}$ will become the Levi-Civita Connection for $g_{\mu \nu}$ (this is guaranteed by the Fundamental Theorem of Riemannian geometry, and note that the substitution

$$\frac{\delta}{\delta g_{\mu \nu}} \int dx^4 \sqrt{-g} \, g_{\mu \nu} \, g^{\mu \nu} \left( R^\tau_{\sigma \rho} \, R^{\sigma \tau}_{\rho} \right) = 0,$$
of $\Gamma^\mu_{\nu\xi}$ by the Levi-Civita Connection is done only after the variation). A proper decomposition of the curvature tensor and the proper use of the Bianchi Identities will convert Eq. [21] and Eq. [22] into the Stephenson-Kilmister-Yang Equation [18–21] and the algebraic Stephenson tensor and the proper use of the Bianchi Identities will convert Eq. 21 and Eq. 22 into the Stephenson-Kilmister-Equation and the algebraic Stephenson

\begin{align*}
H_{\theta\tau} &= R^\lambda_{\sigma\theta\rho} R^\rho_{\lambda\tau} - \frac{1}{4} g_{\theta\tau} R^\lambda_{\sigma\mu} R^\sigma_{\lambda\mu} \\
&= \frac{1}{2} g_{\theta\tau} R_{\sigma\rho} R^{\sigma\rho} + \frac{5}{3} R_{\theta\tau} R - 2 R_{\rho\sigma} R_{\tau\sigma} - \frac{2}{5} g_{\theta\tau} R^2 + C_{\theta\tau}^\rho R^\sigma_{\rho} \\
&= 0.
\end{align*}

Here we have quoted the results given in Ref. [19], with $C_{\theta\tau}^\rho$ as the traceless Weyl conformal curvature.

Obviously the above two equations are satisfied simultaneously by the vanishing of the Ricci curvature tensor, and hence satisfied by the Schwarzschild metric. Note that we didn’t solve Eq. 14 directly, but routed ourselves into the domain of the theory of quadratic gravity and borrowed the results from her. Our statement that the Schwarzschild metric is induced by a configuration of the 16 vector bosons is thus verified.

It is also worth mentioning that there exists another metric, different from the Schwarzschild metric, that is also a simultaneous torsionless solution to Eq. 23 and Eq. 24. This new metric, together with the Schwarzschild metric, different from the Schwarzschild metric, that is torsion. This cosmic situation is a reflection of the solutions to Eqs. 18, 19, and 20 with torsion could also be simultaneously by the vanishing of the Ricci curvature tensor, and hence satisfied by the Schwarzschild metric. Note that we didn’t solve Eq. 15 directly, but routed ourselves into the domain of the theory of quadratic gravity and borrowed the results from her. Our statement that the Schwarzschild metric is induced by a configuration of the 16 vector bosons is thus verified.

Solutions to Eqs. 18 and 19 with torsion could also play a role in physics. For example, an exponentially inflating cosmic metric could be generated by a primordial torsion. This cosmic situation is a reflection of the solutions of the form of $R^\lambda_{\sigma\mu\nu} = 0$.

This means that our Universe is taking up the Weitzenbock geometry in the course of its late time evolution, and in virtue of Eq. 17, it is taking up the pure gauge potential of $GL(4 R)$. This cosmic solution is displayed in the following,

\begin{equation}
ds^2 = (1 + \frac{G' M'}{r})^{-2} dt^2 - (1 + \frac{G' M'}{r})^{-2} dr^2 - r^2 d\Omega^2,
\end{equation}

may have something to do with the forces that give the galactic rotation curves and the intergalactic lensings as are observed in astronomy. The reader is referred to [22] for more details.

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This means that our Universe is taking up the Weitzenbock geometry in the course of its late time evolution, and in virtue of Eq. 17, it is taking up the pure gauge potential of $GL(4 R)$. This cosmic solution is displayed in the following,

\begin{equation}
ds^2 = dt^2 - \rho_0^2 e^{2q(t)} (d\rho^2 + \rho^2 d\Omega^2),
\end{equation}

\begin{equation}
\Gamma_{101} = \Gamma_{202} = \Gamma_{303} = \xi.
\end{equation}

For more details, see [22].

\section*{The Spontaneous Breakdown of the Local Affine Symmetry to the Local Lorentz Symmetry at the Gravitational Level}

A very thorny problem facing physicists when they try to develop a local affine symmetric gauge theory for gravity is the inexistence of finite dimensional spinor representations for the gauge group $GL(4 R)$ [4]. The observed finite dimensional spinor fields with definite spins and masses will invalidate our claim that we are having $GL(4 R)$ symmetries in our laboratories.

This dilemma is resolved here in the following way: in spite of the fact that the Lagrangian of the theory is having the full symmetry of the affine group, the solutions of the equations of motion (for example, the solutions that induce the Schwarzschild metric and the new metric and the accelerating cosmic metric) which describe our gravitational phenomena, retain far less symmetries.

What are the residual symmetries that are left unbroken? This can be answered by noting that the $\Gamma^\mu_{\tau\rho}$ functions that are coming from the classical solutions, and that we are now working with are the Levi-Civita Connections which are compatible with the background metric. This compatibility will require that the observed Yang-Mills vector fields $A^m_{n\mu}$ be anti-symmetric in their $m, n$ indices. For this case, the ten symmetric generators $T^{ab}$ of the $GL(4 R)$ will not be used. The remaining six anti-symmetric generators $J^{ab}$ are just the generators of the local Lorentz Group. The geometric picture for the Compatibility Ansatz is that we are going to identify the defining Minkowskian frame for the vierbein fields and its rotations as our admissible geometric settings in the discussions of classical physics.

Hence what is left unbroken by our solutions is the local Lorentz symmetry. The changes in the gauge potentials are compensated by the changes in the vierbein fields when the Minkowskian axes are rotated, so as to leave our metrics and Levi-Civita Connections unchanged. That explains why we are now seeing particles of definite spins
and masses in our laboratories. In summary, classical gravity is expressed by a spontaneously broken Erlangen program [24].

DISCUSSIONS

We are fully aware that some of the terms and ideas used here have already appeared in the literatures. But we want to emphasize that they are appearing here in very different contexts. For example, we do not regard the world metric in our theory as a fundamental variable. Instead, the world metric is just an arbitrary background of measuring clock and stick for our spacetime. The observed metric takes a particular form (for example the Schwarzschild metric) simply because the affine gauge bosons require that particular metric in order to exist as a solution. As a result, we will not have spin-2 gravitons in our theory; the 16 spin-1 vector bosons are the only propagating particles for gravitation. And the dynamics of gravity is fully governed by the $GL(4)$ Yang-Mills equation of motion. Our theory is a theory of vector gravity.

It might be interesting to point out that there had been many works done in the direction called the gauge theories of gravity. Yet most of them are not gauge theories in the sense of Yang and Mills. People are either reluctant to give up the metric (or the vierbein) as gauge potential because they might think that the metric is too important for gravity theory to be put in an auxiliary position. Or they might be afraid to take up the gravitational Lagrangian that is quadratic in the field strength tensor as Yang and Mills did, because of the fear that there could be spurious solutions that will upset the known gravitational observations [22]. But an important factor that is preventing people to arrive earlier at a true gauge theory of gravitation based on $GL(4)$, as we believe, is the wrong perception that the particle contents of the theory will not fit into experiments. It is one of our observations, as we have explained in the above, that our observed gravitational world is, in fact, a solution to the affine symmetric theory and is in a state of spontaneously broken symmetry. The remaining symmetry is the local Lorentz symmetry.

Finally, a gravity theory with the Lorentz group (which is a subgroup of $GL(4)$) as the local gauge group can be obtained from our theory by simply restricting ourselves to the anti-symmetric components of the $A_{\mu}^{\alpha\beta}$. Only the generators $J^{ab}$ and the first commutation relation of Eq. 2 will be used. And we shall have a Yang-Mills theory of 6 gauge vector bosons. All the results given in this article will then remain valid, with the exceptions that the local Lorentz symmetry will now be honored by both the Lagrangian and the solutions to the equations of motion, and that the metric and the connections will now be compatible automatically. One price has to be paid, though, by gauging the Lorentz group instead of the $GL(4)$. The Riemannian tensor will then contain terms which are of second derivatives in the metric, if we are going to regard the metric and the torsion (which are functions of the vierbeins and the gauge potentials) as independent variables. And gravity theories basing on Lagrangians quadratic in the Riemannian tensor may have to face the affections by many of the pathologies of higher derivative theories. From the solutions of the equation of motion, it seems that gauging the Lorentz group is good enough to describe the present day physical phenomena of gravity. Apart from staying away from the pathologies that we have just mentioned, inherited from gauging the Lorentz group, we are venturing into $GL(4)$ because we believe that there might be something in physics, in addition to gravity, that can be explained by the full affine symmetric gauge theory [1, 25].

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* Electronic address: yi.yang@cern.ch
† Electronic address: phwyueung@phys.sinica.edu.tw
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