MODEL OF THE EXPANSION OF H II REGION RCW 82

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ABSTRACT

This paper aims to resolve the problem of formation of young objects observed in the RCW 82 H II region. In the framework of a classical trigger model the estimated time of fragmentation is larger than the estimated age of the H II region. Thus the young objects could not have formed during the dynamical evolution of the H II region. We propose a new model that helps resolve this problem. This model suggests that the H II region RCW 82 is embedded in a cloud of limited size that is denser than the surrounding interstellar medium. According to this model, when the ionization–shock front leaves the cloud it causes the formation of an accelerating dense gas shell. In the accelerated shell, the effects of the Rayleigh–Taylor (R-T) instability dominate and the characteristic time of the growth of perturbations with the observed magnitude of about 3 pc is 0.14 Myr, which is less than the estimated age of the H II region. The total time \( t_\gamma \), which is the sum of the expansion time of the H II region to the edge of the cloud, the time of the R-T instability growth, and the free fall time, is estimated as 0.44 < \( t_\gamma \) < 0.78 Myr. We conclude that the young objects in the H II region RCW 82 could be formed as a result of the R-T instability with subsequent fragmentation into large-scale condensations.

Key words: H II regions – hydrodynamics – instabilities – ISM: individual objects (RCW 82) – stars: formation

1. INTRODUCTION

Observations by Pomares et al. (2009) show that the Galactic H II region RCW 82 is in a state of active star formation. They discussed the possible mechanism of formation of the observed young objects in the framework of a trigger model (Elmegreen & Lada 1977). According to this model, due to the expansion of the H II region, a complex of discontinuities called the ionization–shock front appears. It includes an ionization front and a prior shock wave (Spitzer 1978; Osterbrock 2006). The distance between the front and the shock increases in time. The mass of the gas compressed by the shock wave increases with time. When the integral mass of the layer between the ionization front and the shock wave exceeds the Jeans mass, fragmentation of the layer into smaller scale condensations occurs (Whitworth et al. 1994). In the case where the ionization–shock front propagates in a homogeneous gas, the estimated time of fragmentation will be approximately 1.6 Myr according to Pomares et al. (2009). At the same time, the estimated age of the H II region RCW 82 is approximately 0.4 Myr, thus the young objects could not have formed during the dynamical evolution of the H II region.

However, there is another possible way for large-scale condensations in the ionization–shock front layer to form. For this we need to assume that the medium is not homogeneous and the H II region is embedded in an interstellar cloud of limited size, which is denser than the surrounding interstellar medium. When the ionization–shock front leaves the cloud an accelerated dense gas shell is formed (Kotova & Krasnobaev 2009). The acceleration of the shell opens the possibility of Rayleigh–Taylor (R-T) instability developing and as a consequence makes the formation of large-scale condensations possible.

The influence of R-T instability on the appearance of non-uniform structures in the outer parts of H II regions and on the formation of gas clumps has been considered by many authors. Capriotti (1973) and Capriotti & Kendall (2006) showed that the origin of the drop-shaped condensations, which were observed in the nebula NGC 7293, could be due to R-T instability development. They considered the case where the scale of the condensations was smaller than the thickness of the shell. Malone et al. (1987) presented evidence that the origin of clumps around \( \lambda \) Orionis could be caused by R-T instability. According to Reipurth et al. (1997), the globules observed in the H II region IC 2944 could appear at the late stage of R-T instability development. Baranov & Krasnobaev (1977) and Krasnobaev (2004) investigated the stability of ionization fronts in accelerating gas. Giuliani (1979) found that the non-stationary ionization–shock front motion is unstable to long-wave perturbations. Schneps et al. (1980) showed that R-T instability can also cause fragmentation of the dense shell swept out by the stellar wind in the H II region. Mizuta et al. (2005, 2006) modeled the plane flow caused by the action of ionizing radiation on an isolated layer of neutral gas. These authors used models that do not take into account the cooling caused by inclusions of impurity elements, the heating by dissociation, and the spectral radiation transfer. The model of the two-dimensional motion of H II region formation in a spherical cloud, which took into account the contribution of the impurity elements in the heat balance and the dependence of the absorption coefficient on frequency, was considered by Kotova & Krasnobaev (2009, 2010). They found that in the thin layer between the shock and ionization fronts finger-shaped condensations, containing a large part of the initial perturbation masses, can appear. Krasnobaev & Tagirova (2013) constructed a linear stability theory of the self-gravitating layer, which is accelerated by the pressure difference on both of its sides, and gave the classification of the instability regimes, which depend on the relation between the gravitational force and the pressure force. Walsh et al. (2013) considered the evolution of a H II region formed in a molecular cloud with a given fractal dimension. It was found that either a few massive clusters or many objects with smaller masses—columns (pillars) and globules, depending on the fractal dimension, can possibly form.

In the H II regions isolated condensations surrounded by ionized gas are frequently observed. Their origin may be associated with the compression and ionization of pre-existing inhomogeneities (e.g., clumps, globules, columns, etc.). In particular,
the equilibrium configuration of globules has been studied by several authors (Kahn 1969; Dyson 1973). The dynamic influence of converging shocks on globules in one-dimensional motion was considered by Dibai & Kaplan (1964). Dyson (1975) proposed the model of axisymmetric flow around the globule. Lefloch et al. (1997) used the model of clump compression by the ionization–shock front to interpret the observations of the globule in the H II region IC 1848. Bodenheimer et al. (1979) and Mellema et al. (1998) calculated the interaction between an ionization–shock front and a dense cloud. They found that the clump has a complicated form, which differs from the drop shape. However, in the case of the H II region, the model of compression of the pre-existing clumps cannot sufficiently explain the typical location of the condensations, which are only on the periphery of the H II regions, and the absence of condensations inside them. Furthermore, there are no data indicating interaction between the gas of the H II regions and the gas heated by the radiation of the central star flowing away from the surface of the clump.

Since the structure of clumps depends on the conditions of their formation, it is possible to compare the observational data of the clump parameters with the theoretical models of the evolution of the dense layer on the boundary of the H II region. In our paper we propose a model of R-T instability in a shell that is generated by the ionization–shock front and accelerated as it goes out of a cloud with limited size and into the interstellar medium. Kotova & Krasnobaev (2009, 2010), using the framework of a complete system of equations for radiation gas dynamics, developed the following features of the evolution of H II regions in a spherical cloud. They found that if the cloud radius does not substantially exceed an initial Strömgren radius, the typical stages of expansion of the H II region are as follows.

In the first the ionization front rapidly propagates; hydrodynamic motion is insignificant. The second stage is characterized by the appearance of an ionization–shock front and a dense layer between these discontinuities. In the third stage the shock wave is going out of the cloud border and an accelerating shell is formed. In the latter stage a rarefaction wave appears in the shell, which causes the smothering of the density distribution and the reduction of the contrast between the density of the cloud and the environment with lower density.

It was found that the density of the expanding H II region decreases, the layer acceleration is maintained by the effect of the “rocket” force of the gas flowing from the ionization front. This conclusion agrees with the simple estimates of the acceleration value (Capriotti 1973; Capriotti & Kendall 2006; Krasnobaev & Tagirova 2008). Deceleration of this layer is important in the case when either the mass of the shocked matter is large (i.e., the cloud radius is large) or when the gas density outside the cloud is high enough.

The plan of this paper is as follows. In Section 2 a mathematical model of the accelerated motion of the ionization–shock front layer is described. Section 3 presents the results of the evolution of the perturbations of the layer. The morphology of the growing perturbations is studied, and the integral masses of the condensations are defined. The influence of self-gravitation and geometry of the motion on these condensations is estimated. The application of our model to the observations of the H II region RCW 82 (Pomares et al. 2009) is presented in Section 4.

### 2. MODEL

To formulate the model we make two simplifying assumptions. The first simplifying assumption made in our model is the adiabatic behavior of the flow. This approach is valid only in the case of perturbations that increase quickly enough.

The second simplifying assumption is the replacement of the ionization front by tangential discontinuity. This is possible because at the hydrodynamic stage of expansion of the H II region, the ionization front is a D-type front, which is characterized by small pressure changes at the sides of the front (Spitzer 1978).

We consider unsteady motions of the compressible self-gravitating gas. The equations expressing conservation of mass, momentum, and entropy and the Poisson equation for the gravitational potential are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p - \rho \nabla U,$$

$$\frac{d}{dt} \left( \frac{p}{\rho^2} \right) = 0,$$

$$\Delta U = 4\pi G\rho,$$

(1)

where \(\rho\), \(\mathbf{v}\), and \(p\) are the mass density, the velocity vector, and the pressure, respectively. Here \(\gamma\), \(G\), and \(U\) are the adiabatic index, the gravitational constant, and the gravitational potential, respectively.

To describe the two-dimensional motion we introduce the coordinates \((x_1, x_2)\), which are Cartesian \((x, y)\), cylindrical \((r, z)\), and spherical \((r, \theta)\) (where \(\theta\) is the polar angle). The velocity vector has two components \(u\) and \(v\) in the \(x_1\) and \(x_2\) directions, respectively.

We start our modeling at the time-moment \(t = 0\), which is when the shock wave reaches the boundary of the cloud. Though the distribution of the gas velocity in the H II region has a complex shape, the detailed distribution of the gas weakly influences the shell impulse in time (and consequently on the acceleration; Kotova & Krasnobaev 2009). Therefore we assume that the gas is at rest (i.e., \(v = 0\) at \(t = 0\)), and the density and the pressure are determined as follows:

$$a < x_1 < x_{int} : \rho = \rho_{hot}, \space p = p_{hot},$$

$$x_{int} < x_1 < x_{out} : \rho = \rho_{shel}, \space p = p_{shel},$$

$$x_{out} < x_1 < b : \rho = \rho_{cold}, \space p = p_{cold}.$$  

(2)

Figure 1 shows the initial condition. The surface \(x_1 = x_{int}\) corresponds to the ionization front, which is replaced in our model by the tangential discontinuity and therefore \(p_{hot} = p_{shel}\). The region \(a < x_1 < x_{int}\) is occupied by a hot gas, \(x_1 = x_{int}\) is the inner boundary of the high density layer. The surface \(x_1 = x_{out}\) is the outer boundary of the high density layer with initial thickness \(h = \text{constant}\), \(x_1 = x_{out}\) is the end of cloud (i.e., for \(x_1 > x_{out}\) the gas is rarefied) and the shock reaches this boundary at \(t = 0\). Therefore, we have an arbitrary
discontinuity there at time \( t = 0 \), where \( \rho_{\text{shell}} > \rho_{\text{cold}} \). The gas in the outer region \((x > x_{\text{out}})\) is assumed to have very low temperature and density. Both \( \rho_{\text{hot}} \) and \( \rho_{\text{cold}} \) are significantly smaller than \( \rho_{\text{shell}} \), which means that the gas in the shell has a higher density than the gas outside the shell. The distribution of the temperature between the shock and the ionization front has a complex shape due to the multiplicity and variety of cooling mechanisms (Bodenheimer et al. 1979; Mizuta et al. 2006; Whalen & Norman 2008; Kotova & Krasnobaev 2009; Iwasaki et al. 2011). However, the details of the temperature distribution do not change the integral characteristics of the resulting motion we are interested in. Therefore we assume that the gas in the shell has temperature of the same order of magnitude as the gas outside the shell \((i.e., \rho_{\text{shell}} / \rho_{\text{hot}} \sim \rho_{\text{cold}} / \rho_{\text{cold}})\).

We consider the periodic \((by the coordinate \( x_2 \))\) perturbed parameters of the shell. We set the type of perturbations, which are close to those considered by Zomenko & Chernyi (2003) in a study of the effects of mass accumulation in thin deformed shells. Therefore, at \( t = 0 \), for \( x_1 : x_{\text{int}} < x_1 < x_{\text{out}} \) we assume fluctuations of velocities:

\[
0 < x_2 < \lambda / 2 : u' = A \cos(2\pi x_2 / \lambda), \quad v' = A \sin(2\pi x_2 / \lambda),
\]

(3)

where \( A \) and \( \lambda \) are the amplitude and wavelength. In spherical coordinates \((r, \vartheta)\) the expression for \( x_2 \) in Equation (3) is changed by \( x_2 = \vartheta x_{\text{int}} \).

The boundary conditions are as follows: for the values \( \rho, \rho, \) and \( \vartheta \) in the system of Equation (1) we assume that the boundaries \( x_1 = a \) and \( x_1 = b \) are impermeable, i.e., \( u = 0 \) (Dudorov et al. 1999), and periodic by the coordinate \( x_2 \). For the potential \( U \) (or gravitational field) we set boundary conditions everywhere along the boundary surface of the computational domain. Those values are obtained analytically from the integral expression (Tikhonov & Samarskii 1990) for the external potential (or gravitational field). In the case of Cartesian coordinates at boundaries \( x_1 \) we set inhomogeneous Neumann boundary conditions for \( \partial U / \partial x_1 \); at penetrable boundaries \( x_2 \) we use symmetrical conditions, i.e., \( \partial U / \partial x_2 = 0 \). We consider the self-gravity for a plane problem, because in this case it is easy to obtain analytically the periodic boundary conditions for the gravitational field. For the other coordinates, self-gravity will be considered in future papers.

We solve the equations of gas dynamics (Equation (1)) numerically with the so-called total variation diminishing Lax–Friedrichs scheme (Hoffmann & Chiang 2000; Kulikovsky et al. 2001). In order to obtain the potential in the inner computational domain we solve the Poisson equation by means of the discrete Fast Fourier Transform method.

3. TWO-DIMENSIONAL PERTURBATIONS AND MASS ACCUMULATION

In the works of Krasnobaev & Tagirova (2008) and Kotova & Krasnobaev (2010), the sufficient conditions for the appearance of the regular finger-shaped condensations with mass accumulation were established. It is shown that the condensations appear when the density of the shell significantly exceeds the density of the hot gas (more exactly, when the ratio \( \rho_{\text{shell}} / \rho_{\text{hot}} \geq 10 \)) in the presence of long wave perturbations \( h \ll \lambda \). In our calculations the initial conditions (Equations (2) and (3)) were chosen to satisfy these criteria. We also chose the parameters of the shell in such a way that the ratio of the mass of the hot and the cold gas roughly corresponds to that which is observed in a typical \( \text{H}\alpha \) region.

To estimate the role of self-gravity, it is convenient to introduce the parameter \( \beta \), defined as the ratio of the characteristic value of the gravitational acceleration in the layer \( 2\pi G \rho_{\text{shell}} h \) to the acceleration \( W = (\rho_{\text{hot}} - \rho_{\text{cold}}) / (d \rho_{\text{shell}} h) \) caused by the pressure difference on two sides of the dense shell:

\[
\beta = 2\pi G \rho_{\text{shell}} h / W.
\]

The one-dimensional calculations (Krasnobaev & Tagirova 2013) show that for \( \beta < 1 \) and \( \rho_{\text{hot}}, \rho_{\text{cold}} \to 0 \) self-gravity effects are not essential, and the motion of the gas is characterized by the formation of several discontinuities with a subsequent acceleration of the layer. In our problem the density \( \rho_{\text{shell}} \) considerably exceeds \( \rho_{\text{hot}} \) and \( \rho_{\text{cold}} \); nevertheless the mass of the hot ionized gas contained in the Strömgren zone is large and is comparable with the mass of the layer. This fact, as well as the presence of cold gas before the layer, makes gravitational effects important for the gas motion.

In the problem considered it is convenient to normalize the pressure, the density, the velocity, and the temperature \((\rho_{\text{hot}}, \rho_{\text{hot}}, \rho_{\text{hot}}, u_{\text{hot}} = \sqrt{\rho_{\text{hot}} / \rho_{\text{hot}}}, \text{ and } T_{\text{hot}}\) respectively) by their values in the region of the hot gas. The spatial scale is normalized by some arbitrary value \( L \). We assume that the typical values of the \( \text{H}\alpha \) region are \( \rho_{\text{hot}} = 1.67 \times 10^{-22} \; \text{g cm}^{-3}, T_{\text{hot}} = 10^4 \; \text{K}, \rho_{\text{hot}} = 2.76 \times 10^{-16} \; \text{erg cm}^{-3}, \text{ and } u_{\text{hot}} = 12.9 \; \text{km s}^{-1}. \) The scale \( L = 1 \; \text{pc} \) is chosen to be the average size of the condensations on the borders of the \( \text{H}\alpha \) region, and the characteristic time is equal to \( t_0 = L / \sqrt{\gamma} - T_{\text{hot}} = 0.06 \; \text{Myr}. \)

General properties of the two-dimensional motion of the self-gravitational layer are shown in Figure 2 with the initial conditions \( \rho_{\text{shell}} / \rho_{\text{hot}} = 25, \rho_{\text{shell}} / \rho_{\text{hot}} = 1, \rho_{\text{cold}} / \rho_{\text{hot}} = 1, \rho_{\text{cold}} / \rho_{\text{hot}} = 0.04, \text{ and } b / L = 0.5, \gamma = 5 / 3. \) In this simulation the boundaries of the computational domain are \( a = 0, b / L = 10 \) and the initial position of the shell is \( x_{\text{int}} / L = 4.75. \)

Numerical modeling of two-dimensional motions shows that at the nonlinear stage of evolution of the perturbation, the effects of R-T instability dominate for \( \beta < 1 \) (Figure 2(a)).

If the gravitational force has the same order of magnitude as or larger than the pressure difference on both sides of a layer,
In Figure 3 the time $t$ is normalized by $t_0 = L/u_0$ and the value of the initial position of the ionization front $x_{\text{int}}/L$ is equal to 0, 8.0, and 1.5 for the plane layer, and the cylindrical and the spherical layers, respectively.

The results presented in Figure 3 show that mass accumulation takes place for all types of motion (plane, cylindrical, or spherical) for the case when the density of the gas in the shell significantly exceeds the density of the hot gas. The area of the mass concentration is narrow enough and considerably smaller than the wavelength of the perturbations. Therefore we have shown here that the structure of the condensations and the effect of mass accumulation in the shell slightly depend on the geometry of the problem.

4. INTERPRETATION AND CONCLUSIONS

Numerical simulations allow us to estimate the characteristic time of layer destruction, which depends on the scale of perturbations and the parameter $\beta$. We will use the results of our calculations to identify the influence of the acceleration on the appearance of large-scale condensations (clumps) in the external parts of the H$^{\text{ii}}$ region RCW 82. According to the observations of Pomares et al. (2009), the present radius of the H$^{\text{ii}}$ region $R_i$ is equal to 3 pc. The scale of condensations near the boundary separating the neutral and the ionized gas is also on the order of 3 pc. The mass of the nebula $M_0$ is approximately $2520 M_\odot$, and the mass of the shell $M_e$ is approximately $2260 M_\odot$. Pomares et al. (2009) suggested that the H$^{\text{ii}}$ region RCW 82 could have been formed under the influence of flux ionizing radiation of $\Phi = 9 \times 10^{48}$ s$^{-1}$ in a homogeneous medium with a concentration of neutral particles $n_0 = 10^3$ cm$^{-3}$ (additionally the authors of this work give other values for the parameters of the gas and flux $\Phi$ which differ slightly from previous values). Then the age of the H$^{\text{ii}}$ region $t_R$ is approximately 0.4 Myr. This is less than the time of the fragmentation, which is equal to 1.6 Myr. Therefore, the young objects (which are possible candidates for star formation regions) observed in the H$^{\text{ii}}$ region RCW 82 are not formed according to the model by Pomares et al. (2009), since due to this model the objects have to appear much later than $t_R$.

However, if we assume that the H$^{\text{ii}}$ region was not formed in a homogeneous medium but in a cloud with mass $M_c$ and with number density $n_0 = 10^3$ cm$^{-3}$, then the accelerating neutral shell could be formed during the time $t_\gamma$, which is considerably less than $t_R$. Indeed, let $R_c$ be the initial cloud radius, which is considered to be spherical and to consist of atomic hydrogen for simplicity. Then we have $4\pi m_1 n_0 R_c^3/3 = M_c$ and, therefore, $R_c = 2.92$ pc (where $m_1$ is the mass of a hydrogen atom). The corresponding Strömgren radius $R_S$ is equal to

$$R_S = \left(\frac{3\Phi}{4\pi n_0^2 \alpha_{\text{H}}^2}\right)^{1/3} = 0.71 \text{pc},$$

where $\alpha_{\text{H}} = 2 \times 10^{-13}$ cm$^3$ s$^{-1}$ is the coefficient of photo recombination at all levels of a hydrogen atom, except the ground level. The radius $R_S$ of the static H$^{\text{ii}}$ region is reached during the time $(n_0 \alpha_{\text{H}})^{-1} \ll t_R$ (Spitzer 1978). Therefore the time $t_\gamma$, when the ionization front passes the distance $R_c$ (i.e., reaches the boundary of the cloud, forming an accelerated moving shell) can be defined by the approximate formula (Baranov & Krasnobaev 1977; Spitzer 1978)

$$\frac{R_c - R_S}{R_S} = \left(1 + \frac{7}{4} \frac{ct_\gamma}{R_S}\right)^{4/7}. \quad (4)$$

\[\text{Figure 2. Morphology of the condensations for the plane layer without self-gravity (a), (c) and taking it into account self-gravity effects (b), (d) with $\beta = 3.1$. Panels (a) and (b) show density isolines at time $t/t_0 = 5$. Panels (c) and (d) show the function $m_e(x, t)$ at $t/t_0 = 2, 3, 4, 5$ (curves 1–4). The perturbation parameters are $A\sqrt{T}/L = 0.1$ and $\lambda/L = 5$.}\]
Here $c$ is the isothermal sound speed in a fully ionized hydrogen gas at the temperature $T_e = 10^4$ K. Note that Equation (4) agrees well with the results of the modeling of an ionization–shock front leaving the cloud (Kotova & Krasnobaev 2009). Using Equation (4) we find that $t_c \approx 0.23$ Myr. Assuming that the luminosity of the star is invariable we see that the number density of the ionized gas $n_e$ and the radius $R_i$ of the H II region are related by $n_e R_i^{3/2} = \text{const.}$ Thus

$$n_e(t_c) = n_0 \left(\frac{R_i}{R_i(t_c)}\right)^{3/2} \approx 120 \text{ cm}^{-3},$$

which agrees with the value given in the work of Pomares et al. (2009). Assuming that $n_e = 120$ cm$^{-3}$, we estimate the acceleration of the shell $W$ and the parameter $\beta$

$$W = \frac{8\pi R_i^2 n_e kT_e}{2260 M_\odot} = 8 \times 10^{-8} \text{ cm s}^{-2},$$
$$\beta = \frac{2\pi G\rho h}{W} = \frac{GM_e}{2WR_i^2} = 0.02$$

(where $\rho h$ is the surface density of the shell and $k$ is the Boltzmann constant).

During the acceleration of the shell the effects of R-T instability dominate and the characteristic time $\tau$ of the growth of perturbations with scale $R_i = 2\pi/\kappa$ is $\tau = (W\kappa)^{-1/2} \approx 0.14$ Myr.

Thus, at the time $t_c + \tau = 0.37$ Myr the condensations could be formed in the layer whose integrated density exceeds the integrated density of the unperturbed layer several times. The time of development of Jeans instability in these clumps has the same order as the free fall time $t_{ff} = \sqrt{3\pi/(32G\rho_0)}$ (Spitzer 1978), where $\rho_0$ is the average value of the layers density. If the layer mass $M_e$ is known, then the $\rho_{av}$ is determined by the detailed structure of the layer—i.e., its thickness and the spatial distribution of the density. Numerous calculations (Tenorio-Tagle et al. 1979; Mizuta et al. 2005; Kotova & Krasnobaev 2009) show that the thickness and the density distribution depend considerably on the abundances of the impurity elements in the neutral gas, on its degree of ionization, and on the spectra of the star (or the stars group) that excite the H II region. However, we can estimate the minimum and maximum values of $\rho_{av}$ and therefore the value of $t_{ff}$.

The maximum value of $\rho_{av}$ corresponds to isothermal flow in the layer and it can be roughly estimated from the condition of equality between the pressure in the layer and the pressure in the H II region. Assuming that in the H II region we have $n_e = 120$ cm$^{-3}$ and $T_e = 10^4$ K we obtain that $2n_e kT_e = \rho_{av}a_{av}^2$, where $a_{av} = 0.2 \text{ km s}^{-1}$ is the chaotic average particle velocity in the layer (Pomares et al. 2009). Then the minimum of $t_{ff}$ is equal to $0.07$ Myr.

To estimate the minimum value of $\rho_{av}$ we assume that after the shock wave goes out of the border of the cloud, the value of the density of the compressed neutral gas is not less than four times the unperturbed density $\rho_0$, since the flow is adiabatic with the index $Y = 5/3$. This value $4\rho_0$ is the minimum value, since after the shock front gas radiative cooling takes place that results in a density increase. If we take into account that the clumps density increased approximately four times due to the development of R-T instability (referring to Figure 3, one can see that $m_e$ may reach the value of $m_e \sim 4$), it is reasonable to take $\rho_{av} = 16\rho_0$. Then we have the maximum of $t_{ff} \approx 0.407$ Myr.
Consequently, the time of formation of young objects $t_\Sigma = t_c + \tau + t_{ff}$ is estimated as $0.44 < t_\Sigma < 0.78$ Myr.

Thus the surface density of the shell increases and the time of gravitational compression decreases. Therefore, the young objects in the Galactic H\textsc{ii} region RCW 82 could be formed as the result of the development of R-T instability with subsequent fragmentation into large-scale condensations. For a detailed analysis of the process of fragmentation one needs to make a more complete assessment of the effects of radiation heating and cooling and examine an essentially nonlinear stage of growth of perturbations in the self-gravitating gas.

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