Holographic Quantum Entanglement Negativity

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We propose a holographic prescription to compute the entanglement negativity for conformal field theories at finite temperatures. We show that our prescription exactly reproduces the entanglement negativity for (1+1)-dimensional conformal field theories at finite temperatures dual to (2+1)-dimensional bulk Euclidean BTZ black holes. We observe that the holographic entanglement negativity captures the distillable pure quantum entanglement and is related to the holographic mutual information. The application of our prescription to higher dimensional conformal field theories at finite temperatures within a $AdS_{d+1}/CFT_d$ scenario involving dual bulk $AdS$-Schwarzschild black holes is discussed to elucidate the universality of our conjecture.

Entanglement typically refers to the existence of both classical and quantum correlations for a composite extended quantum system at finite temperature. For a generic conformal field theory (CFT) describing a bipartite quantum system in (1+1)-dimensions the entanglement entropy may be computed through the replica method as described in [1]. Recently in the context of the $AdS/CFT$ correspondence there has been a surge of interest in studying the entanglement entropy of boundary conformal field theories at zero as well as finite temperatures which are dual to distinct bulk $AdS$ gravitational configurations, in a holographic framework given by Ryu and Takayanagi [2, 3]. Their conjecture states that the entanglement entropy $S_A$ for a region $A$ (enclosed by the boundary $\partial A$) in the $(d)$-dimensional boundary quantum field theory is proportional to the area of the extremal surface (a co-dimension two surface area denoted by $\gamma_A$) extending into the $(d + 1)$-dimensional bulk that is anchored on the boundary $\partial A$ of the region $A$ on the boundary of the $AdS$ space-time and is given as

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N^{(d+1)}}$$

where, $G_N^{(d+1)}$ is the gravitational constant of the bulk space-time.

Using this holographic prescription it was possible to obtain the entanglement entropy in the large central charge limit for strongly coupled boundary CFTs (for dimension $d \geq 2$) at both zero and finite temperatures which are dual to different bulk $AdS$ configurations [4-10]. However, the holographic entanglement entropy for such boundary CFTs at finite temperatures usually involves a mixing of both classical (thermal) and quantum correlations of the system [11, 12]. Naturally this renders the usual entanglement entropy unsuitable as a measure to identify the pure distillable quantum entanglement of the system at a finite temperature due to the thermal contributions. The idea of obtaining the pure distillable quantum entanglement for an extended quantum system in a mixed state through the removal of the contribution from the classical correlations is a well known problem in Quantum Information Theory (See [9] for references and reviews). Recently, in a classic work Vidal and Werner [13] have introduced a computable measure for such a mixed-state entanglement in bipartite quantum systems known as the “entanglement negativity” (logarithmic negativity). This measure involves a partial transpose of the full density matrix over one of the subsystems (say $A_2$) in a bipartite quantum system $(A_1 \cup A_2)$ [13]. Remarkably, the entanglement negativity provides a valid measure for computing the pure distillable quantum entanglement for an extended quantum system in a mixed state. Some of the salient features of entanglement negativity are that it is a basis independent measure and should be a monotonically decreasing function of the temperature. The monotonicity and non convexity of the entanglement negativity was later demonstrated in an important communication by [14]. Furthermore, In [16-18] authors Calabrese et. al have provided a systematic procedure for computing the entanglement negativity in quantum field theories and many-body systems. Following their prescription, the same authors in [18] were able to show that the entanglement negativity leads to the pure distillable quantum entanglement even at finite temperatures (See [17] for other applications of entanglement negativity).

The above discussion naturally indicates towards the plausibility of a corresponding holographic prescription for computing the entanglement negativity of boundary conformal field theories at finite temperatures in the context of the $AdS/CFT$ correspondence. In the recent past several attempts has been made to understand this critical issue which has led to interesting insights. In this context, the authors in [19] have computed the entanglement negativity of zero temperature conformal field theories dual to the bulk pure $AdS$ vacuum space time in various dimensions. Moreover, in the article [20] the authors have conjectured a holographic c-function which is identified with the entanglement negativity of the dual conformal field theory at a finite temperature. Although these attempts provide a deep insight in to the holo-
graphic interpretation of entanglement negativity, it is yet to be established a general and simple holographic prescription to compute the entanglement negativity of a large class of boundary CFTs at a finite temperatures.

This article addresses the critical issue of providing an elegant general holographic prescription to compute the entanglement negativity of boundary conformal field theories at finite temperatures in arbitrary dimensions. In the context of $AdS_3/CFT_2$ following [18], this involves the holographic computation of the four point function of certain twist/anti-twist operators in the boundary conformal field theory. More precisely in this case, by employing the Ryu and Takayanagi conjecture [23] we will show that the the entanglement negativity of the $(1+1)$-dimensional boundary conformal field theory which is related to this four point function can be expressed in terms of a certain algebraic sum of the geodesic lengths in the bulk geometry. We then use this result obtained from $AdS_3/CFT_2$ to conjecture a general holographic prescription for computing the entanglement negativity of boundary conformal field theories in a generic $AdS_{d+1}/CFT_d$ setup. Particularly, in the $AdS_3/CFT_2$ setup our holographic prescription for the entanglement negativity captures the distillable pure quantum entanglement for the $(1+1)$-dimensional boundary CFT at zero or finite temperature and matches exactly with the results of [18]. More importantly, it is also observed that the entanglement negativity computed from the bulk is related to the holographic mutual information as we will elucidate later. This naturally indicates towards the universality of our proposed holographic conjecture for computing the entanglement negativity for finite temperature conformal field theories in arbitrary dimensions.

**Entanglement negativity in CFT$_{1+1}$:** Entanglement negativity may be defined as the measure which distills out pure quantum correlations of an extended quantum system in mixed state as shown by Calabrese et al. [18] for the case of $(1+1)$-dimensional CFTs at finite temperatures. In order to define entanglement negativity [18] first let us consider an extended one dimensional quantum system which is divided in to three parts $A_1, A_2$ and $B$ such that $A_1$ and $A_2$ correspond to finite intervals $[u_1, v_1]$ and $[u_2, v_2]$ of lengths $l_1$ and $l_2$ respectively whereas, $B$ represents the rest of the system. We consider $\rho_A$ as the reduced density matrix of system $A = A_1 \cup A_2$ such that $\rho_A = \rho_{A_1 \cup A_2}$ and $\rho_A$ is obtained by tracing out the full density matrix $\rho$ over the part $B$, i.e. $\rho_A = Tr_B \rho$. Moreover, if $|q_1^0 \rangle$ and $|q_2^0 \rangle$ represent the bases of Hilbert space corresponding to $A_1$ and $A_2$ respectively then the partial transpose with respect to $A_2$ degrees of freedom can be defined as

$$\langle q_1^0 q_2^0 | \rho_A^{T_2} | q_1^0 q_2^0 \rangle = \langle q_1^0 q_2^0 | \rho_A | q_1^0 q_2^0 \rangle.$$  

Using the above relation the entanglement negativity can be defined as

$$\mathcal{E} = \lim_{n \to 1} \ln(Tr[(\rho_A^{T_2})^n]).$$  

It is to be noted that the entanglement negativity which is related to $Tr(\rho_A^{T_2})^n$ shows different functional dependence on the eigenvalues of $\rho_A^{T_2}$ based on parity of $n$. However, one gets a sensible result for $Tr(\rho_A^{T_2})^n$ only if, $n = n_0$ (even) as suggested in [18]. Thus from now on we will work only with even values ($n_0$) of $n$. In the $(1+1)$-dimensional CFT taking transpose ($\rho_A^{T_2}$) of the reduced density matrix $\rho_A$ has the effect of exchanging upper and lower edges of the branch cut along the interval $A_2$ on an $n_0$-sheeted Riemann surface such that $Tr(\rho_A^{T_2})^n$ can be written in terms of four twist/anti-twist fields as

$$Tr(\rho_A^{T_2})^n = \langle T_{n_0}(u_1)\bar{T}_{n_0}(v_1)\bar{T}_{n_0}(u_2)T_{n_0}(v_2) \rangle.$$  

In [18] Calabrese et al. have given a systematic method to calculate the entanglement negativity in $(1+1)$-dimensional conformal field theories which involves computation of $Tr(\rho_A^{T_2})^n$ based on a replica trick. This method incorporates the fact that one has to go from a tripartite system $(A_1, A_2, B)$ to a bipartite configuration $(A, B, \emptyset)$ by first making the identification $u_2 \to v_1$ and $v_2 \to u_1$ in eq.(4) such that, the interval corresponding to the subsystem $A$ is now given by $[u, v] = [u_1, v_1] \cup [u_2, v_2]$ with $[u-v] = [-l, 0]$. With this identification the correct form of entanglement negativity of the subsystem $A$ is now given in terms of the four point function of twist/anti-twist fields as

$$\mathcal{E} = \lim_{L \to \infty} \lim_{n_0 \to 1} \ln \left[ \langle T_{n_0}(-L)\bar{T}_{n_0}^2(0)\bar{T}_{n_0}(L) \rangle \right]$$

where, $\rho = \rho_{A \cup B}$ and $T_n(L)$ corresponds to the twist field operator for the interval $B$ sitting at some large distance $L$ from the interval $A$. The scaling dimension $(\Delta_{n_0})$ of the operator $T_{n_0}^2$ can be related to the scaling dimensions $(\Delta_{n_0})$ of the operator $T_{n_0}$ as

$$\Delta_{n_0}^{(2)} = \frac{c}{6} \left( \frac{n_e}{2} - \frac{2}{n_e} \right), \quad \Delta_{n_0} = \frac{c}{12} \left( n_e - \frac{1}{n_e} \right).$$  

It is to be noted that in order to get correct result from eq.(5), the limit $(L \to \infty)$ should be applied only after taking the replica limit $(n_0 \to 1)$. In $(1+1)$-dimensional CFT computing the four point function given in the expression eq.(5) of negativity on the 2-dimensional complex plane one arrives at the result

$$\mathcal{E} = \frac{c}{2} \ln \left( \frac{l}{a} \right) + \text{constant}.$$  

From the above expression one may observe that at zero temperature the entanglement negativity is equal to the Rényi entropy of order 1/2 which is well known result for bipartite systems [18]. However, at finite temperatures one has to evaluate the four point function
in eq.\([5]\) on an infinitely long cylinder of circumference \(\beta = 1/T\). This cylindrical geometry can be obtained from the complex plane by the conformal transformation \((z \to \omega = \beta/2\pi \ln z)\) such that \(z\) denotes the coordinates on the complex plane and \(\omega\) denotes the coordinates on the cylinder. Under this transformation the four point function in eq.\([4]\) can be written in a generic form as

\[
\langle T_{n_a}(z_1) T_{n_b}(z_2) T_{n_c}(z_3) T_{n_d}(z_4) \rangle_C = \frac{c_{n_a} c_{n_b}/2}{2\Delta_{n_a} \Delta_{n_b}} \frac{\mathcal{F}_{n_a}(x)}{x^{\Delta_{n_a}}}, \quad x = \frac{z_1 z_4}{z_3 z_4} \quad (8)
\]

where, \(c_{n_a}\) and \(c_{n_b}/2\) are some arbitrary constants which may be set to unity by considering an appropriate normalization of the two point function of the twist fields. The \(z_i\)’s are arbitrary complex numbers and \(z_{ij} = |z_i - z_j|\) with \(\langle \rangle_C\) standing for expectation value on the cylinder. The cross ratio \(x\) of the four points has two limits \(x \to 0\) (for \(z_2 \to z_1, z_3 \to z_4\) and \(x \to 1\) (for \(z_4 \to z_3, z_1 \to z_2\)) which correspond to high and low temperature limits respectively. Moreover, following \([18]\) one can obtain the constraints on the arbitrary function \(\mathcal{F}_{n_a}(x)\) in the two limits \(x \to 1\) and \(x \to 0\) as follows

\[
\mathcal{F}_{n_a}(1) = 1, \quad \mathcal{F}_{n_a}(0) = C_{n_a}, \quad (9)
\]

here, \(C_{n_a}\) is an arbitrary constant. Replacing \(z_i\)’s by \((e^{-2\pi L/\beta}, e^{-2\pi L/\beta}, 1, e^{2\pi L/\beta})\) in the eq.\([3]\) the finite temperature result \([15]\) for the entanglement negativity in the \((1+1)\)-dimensional CFT may be written down as follows

\[
\mathcal{E} = c_2 \ln \left[ \frac{\beta}{\pi a} \sinh \left( \frac{\pi l}{\beta} \right) \right] - \frac{\pi c l}{2\beta} + f(x)
\]

\[
+ \beta l \ln(c^2_{1}c_{1}). \quad (10)
\]

\[
f(x) = \lim_{n \to 1} \mathcal{F}_{n_a}(x), \quad \lim_{L \to \infty} x = e^{-2\pi l/\beta}. \quad (11)
\]

Here, it is to be noted that entanglement negativity is a monotone for system going from pure to mixed state and thus decreases with the temperature as shown in \([14]\). Thus it may be said that the maximum value of the entanglement negativity should correspond to the zero temperature result given in the eq.\([7]\). This implies that the value of entanglement negativity at any finite temperature must be smaller than its zero temperature value which motivates us to write the following inequality using equations \([7]\) and \([10]\)

\[
\frac{c}{2} \ln \left( \frac{1}{a} \right) > \frac{c}{2} \ln \left[ \frac{\beta}{\pi a} \sinh \left( \frac{\pi l}{\beta} \right) \right] - \frac{\pi c l}{2\beta} + f(x)
\]

\[
f(x) < \frac{\pi c l}{2\beta} - \frac{c}{2} \ln \left[ \frac{\beta}{\pi l} \sinh \left( \frac{\pi l}{\beta} \right) \right]. \quad (12)
\]

Furthermore, the positivity of the entanglement negativity at finite temperatures puts a lower bound on the arbitrary function \(f(x)\) as follows

\[
f(x) > \frac{\pi c l}{2\beta} - \frac{c}{2} \ln \left[ \frac{\beta}{\pi l} \sinh \left( \frac{\pi l}{\beta} \right) \right]. \quad (13)
\]

The inequalities in eq.\([12]\) and eq.\([13]\) clearly suggests that the unknown function \(f(x)\) is bounded and small in the large central charge limit. Thus from the expression in eq.\([10]\) it may be said that the dominant contribution to the entanglement negativity in the large central charge limit comes from the first two terms whereas, the rest of the terms provide only the subleading contributions. However, the detailed mathematical argument for the above observation requires the full computation of the four point function in terms of the conformal blocks. In \([21]\) Zamolodchikov argued that in a 2d CFT, a four point function of primary operators in the large central charge limit receives an universal contribution which varies exponentially with the central charge as follows

\[
\langle \mathcal{O}_1(z_1) \mathcal{O}_2(z_2) \mathcal{O}_3(z_3) \mathcal{O}_4(z_4) \rangle \approx e^{-g(\Delta_4, z)} \quad (14)
\]

where, \(g\) is an unknown function to be determined by the solution of a certain monodromy problem \([22]\) and \(\Delta_4\) represents the scaling dimension of the primary operator \(\mathcal{O}_4\). Such computations involving a generic four point function which can be related to either entanglement entropy (in the case of two disjoint intervals) or negativity have already been studied for the case of 2d-CFTs. Specifically in \([22, 24]\), the authors have obtained a similar exponential dependence of the four point functions of twist fields on the central charge, which forms a universal result in the semiclassical limit (large central charge). Thus in the large central charge limit the dominant contribution to the entanglement negativity in eq.\([10]\) may be written down as follows

\[
\mathcal{E} = \frac{c}{2} \ln \left[ \frac{\beta}{\pi a} \sinh \left( \frac{\pi l}{\beta} \right) \right] - \frac{\pi c l}{2\beta} = S_{A}^{\text{ent}} - S_{A}^{\text{th}}. \quad (15)
\]

From the above expression it is straightforward to see that the entanglement negativity is a monotonically decreasing function of the temperature and for \(T = 0\) (zero temperature), one obtains the corresponding zero temperature result given in eq.\([7]\) for a \((1+1)\)-dimensional CFT. Moreover, the second term in the eq.\([15]\) corresponds to the thermal entropy \((S_{A}^{\text{th}})\) of the subsystem \(A\) in the \((1+1)\)-dimensional CFT and can be related to classical correlations. Remarkably, this clear separation of thermal entropy from the total entanglement entropy \((S_{A}^{\text{ent}})\) of the subsystem \(A\) suggests that the entanglement negativity forms the correct measure for pure quantum entanglement even at finite temperatures.

**Holographic entanglement negativity**- In order to formulate a holographic prescription for computing the finite temperature entanglement negativity in \(AdS_3/CFT_2\) setup we begin by first considering the \((1+1)\)-dimensional boundary of \(AdS_2+1\) space-time to be partitioned into the subsystem \(A\) and its complement \(A' = B_1 \cup B_2\) as shown in the fig.\([1]\). The two point function of twist operators \((T_{n_a}, T_{n_b})\) can be defined for
we set to unity in later computations. In the replica limit here, \( x \)
the (1+1)-dimensional boundary CFT as follows\[25\]

\[
\langle T_{n_{c}}(z_{i})T_{n_{c}}(z_{j}) \rangle_{C} = \frac{c_{n_{c}}}{z_{ij}^{\Delta_{n_{c}}}},
\]

(16)

From the AdS/CFT dictionary it is known that the two point function on the boundary \( CFT_{1+1} \) can be related to the length of the geodesic \( (L_{ij}) \) anchored on the points \( (z_{i}, z_{j}) \) and extending in to the bulk \( AdS_{2+1} \) space-time as follows

\[
\langle T_{n_{c}}(z_{i})\overline{T}_{n_{c}}(z_{j}) \rangle_{C} \sim e^{-\frac{\Delta_{n_{c}} \ln L}{R}},
\]

(17)

where, \( R \) is the \( AdS \) radius of the bulk \( AdS_{2+1} \) space-time. From fig.1 one can identify that

\[
L_{12} = L_{B_{1}}, \quad L_{23} = L_{A}, \quad L_{34} = L_{B_{2}},
\]

\[
L_{13} = L_{A \cup B_{1}}, \quad L_{24} = L_{A \cup B_{2}}, \quad L_{14} = L_{A \cup B}.
\]

(18)

Using the relations in (17) and (18) one can write the four point function of the twist fields given by eq.(8) in a suggestive form as

\[
\langle T_{n_{c}}(z_{i})T_{n_{c}}^{2}(z_{j})T_{n_{c}}(z_{k})T_{n_{c}}(z_{l}) \rangle_{C} = \alpha_{n_{c}} \frac{F_{n_{c}}(x)}{z_{ij}^{\Delta_{n_{c}}}} e^{-(\Delta_{n_{c}}L_{A \cup B} + \Delta_{n_{c}}^{(2)}L_{A})/R},
\]

(19)

\[
x = e^{(L_{B_{1}} + L_{B_{2}} - L_{A \cup B_{1}} - L_{A \cup B_{2}})/2R},
\]

(20)

here, \( x \) is the cross ratio and \( \alpha_{n_{c}} \) is some constant which we set to unity in later computations. In the replica limit \( n_{c} \to 1, \Delta_{n_{c}} \to 0 \) and \( \Delta_{n_{c}}^{(2)} \to \frac{\pi}{2} \) with the central charge 'c' of \( CFT_{1+1} \) being related to the \( AdS \) length \( R \) through the Brown-Hennama formula \( c = \frac{3R^{2}}{2G_{N}} \). Thus from the equations (5) and (19) the finite temperature entanglement negativity for \( CFT_{1+1} \) may be written down as follows

\[
\mathcal{E} = \lim_{L \to \infty} \left[ \frac{3}{16G_{N}} (2L_{A} + L_{B_{1}} + L_{B_{2}} - L_{A \cup B_{1}} - L_{A \cup B_{2}}) \right] + \lim_{L \to \infty} f(x), \quad f(x) = \lim_{n_{c} \to 1} \frac{F_{n_{c}}(x)}{z_{ij}^{\Delta_{n_{c}}}}
\]

(21)

As discussed earlier in the large central charge limit the subleading contribution involving the function \( f(x) \) in the above expression for the entanglement negativity may be ignored \[24\]. Note that the large central charge corresponds to the large \( N \) limit of the boundary field theory \[28, 29\]. Thus in the \( AdS_{3}/CFT_{2} \) scenario being discussed here, we may express eq.(21) in terms of the holographic mutual information between the pair of intervals \( (A, B_{1}) \) and \( (A, B_{2}) \) as follows

\[
\mathcal{E} = \lim_{L \to \infty} \left[ \frac{3}{4} (\mathcal{I}(A, B_{1}) + \mathcal{I}(A, B_{2})) \right],
\]

(22)

\[
\mathcal{I}(A, B_{i}) = S_{A} + S_{B_{i}} - S_{A \cup B_{i}},
\]

\[
= \frac{(L_{A} + L_{B_{i}} - L_{A \cup B_{i}})}{4G_{N}^{3}}, \quad i = \{1, 2\}.
\]

(23)

Using the expression (22) one can now compute the finite temperature entanglement negativity of the boundary (1 + 1)-dimensional CFT purely in terms of bulk quantities by considering the points \( (z_{1}, z_{2}, z_{3}, z_{4}) = (e^{-2\pi L/\beta}, e^{-\pi/\beta}, e^{-\pi/\beta}, e^{2\pi L/\beta}) \) as shown in the fig.1. With the help of eq.(22), we will show in the later section that one can reproduce exactly both the finite and zero temperature results for the entanglement negativity in the large central charge limit for a (1+1)-dimensional CFT which are described in the previous section.

**Entanglement Negativity in AdS_{3}/CFT_{2}** According to AdS/CFT correspondence, a (1+1)-dimensional boundary field theory at a finite temperature (\( T = \beta^{-1} \)) is dual to a bulk Euclidean BTZ black hole. The metric of which is given by

\[
ds^{2} = (r^{2} - r_{+}^{2})d\tau_{E}^{2} + \frac{R^{2}}{(r^{2} - r_{+}^{2})} dr^{2} + r^{2}d\phi^{2},
\]

(24)

here, \( \tau_{E} \) is the compactified Euclidean time (\( \tau_{E} \sim \tau_{E} + \frac{2\pi}{\beta} \)) and \( \phi \) is identified with \( (\phi + 2\pi) \). Under the coordinate transformation \( r = r_{+} \cosh \rho, \tau_{E} = \frac{R}{r_{+}} \theta, \phi = \frac{R}{r_{+}} t \) the metric in (24) becomes

\[
ds^{2} = R^{2}(d\rho^{2} + \cosh^{2} \rho d\Omega^{2} + \sinh^{2} \rho d\phi^{2}).
\]

(25)

The lengths of the geodesics anchored to the boundary in these Euclidean Poincare co-ordinates are well known\[10\]. We denote the length of the geodesic to be \( \mathcal{L}_{\gamma} \) which may be given as

\[
\mathcal{L}_{\gamma} = 2R \ln \left[ \frac{\beta \sinh \left( \frac{\pi \gamma}{\beta} \right)}{\alpha} \right],
\]

(26)

here, \( a \) is the UV cut-off of the boundary field theory and \( R \) is the \( AdS_{3} \) length scale. The parameters \( L_{\gamma} \) represent the distance between the points on the boundary separating the subsystem \( (\gamma) \) from the rest of the system. In the \( AdS_{3}/CFT_{2} \) setup as shown in fig.1 the geodesic length \( \mathcal{L}_{\gamma} \) given by eq.(26) can be identified for \( \gamma = (A, B_{1}, B_{2}, A \cup B_{1}, A \cup B_{2}) \). Using the expression of the geodesic legth given by eq.(26) in eq.(22)
the holographic entanglement negativity for the (1+1)-dimensional boundary field theory may be obtained as follows

\[ \mathcal{E} = \frac{3R}{4G_N^{d+1}} \left( \ln \left[ \frac{\beta \cosh[\frac{\pi t}{\beta}]}{\beta} \right] - \frac{\pi t}{2\beta} \right), \]

\[ = S_{A}^{ent} - S_{A}^{th}. \tag{27} \]

This matches exactly with the finite temperature result given by eq. (15) for the entanglement negativity of a (1+1)-dimensional CFT obtained by the authors Calabrese et al. in [18] with the central charge \( c \) being given by the Brown-Hennaux formula \( c = \frac{3\pi}{2G_N^{d}} \). The expression in eq. (27) clearly suggest that the holographic entanglement negativity only captures the pure quantum correlations for the (1+1)-dimensional boundary field theory at finite temperatures.

The above observations motivate us to propose that our holographic prescription of computing entanglement negativity in \( AdS_3/CFT_2 \) can be also be extended to a more generic \( AdS_{d+1}/CFT_d \) setup. The generalization of our holographic prescription can be understood in the following manner, first we consider an extended bipartite system \((A \cup B)\) in \( d \)-dimensions such that subsystem \( A \) is formed by partitioning the whole system along one spatial dimension whereas the rest of the system corresponds to the complement \( B = (B_1 \cup B_2) \). If \( A_A, A_B \) and \( A_B \) denote the co-dimension two static minimal surface areas in \( AdS_{d+1} \) which are anchored to the boundary of subsystems \( A, B_1 \) and \( B_2 \) then in the large central charge limit the holographic entanglement negativity for the subsystem \( A \) in the \( d \)-dimensional boundary CFT at finite temperature is given by the following expression

\[ \mathcal{E} = \lim_{B \to A^c} \left[ \frac{3}{4} (\mathcal{I}(A, B_1) + \mathcal{I}(A, B_2)) \right], \]

\[ \mathcal{I}(A, B_i) = S_A + S_{B_i} - S_{A_iB_i}, \]

\[ = \frac{\mathcal{A}_A + \mathcal{A}_{B_i} - \mathcal{A}_{A_iB_i}}{4G_N^{d+1}}, \quad i = \{1, 2\}, \tag{28} \]

where, \( G_N^{d+1} \) is the \((d+1)\)-dimensional Newton constant and the limit \((B \to A^c)\) in eq. (28) corresponds to taking the end points of subsystems \( B_1 \) and \( B_2 \) to infinity along the partitioning direction such that \( B = (B_1 \cup B_2) \) becomes infinitely large.

**Discussion** - To summarize we have proposed a holographic conjecture in the context of the \( AdS/CFT \) correspondence for computing the finite temperature entanglement negativity for conformal field theories in arbitrary dimensions which are dual to bulk \( AdS \) black holes. Using our conjecture we have established that the entanglement negativity captures the pure distillable quantum entanglement at finite temperatures and is related to the holographic mutual information. In the context of the \( AdS_3/CFT_2 \) our holographic prescription exactly reproduces the finite temperature entanglement negativity of the \((1+1)\)-dimensional boundary conformal field theory which is dual to an Euclidean BTZ black hole. Remarkably, it is also possible to show that in higher dimensions \((d > 2)\) the entanglement negativity of CFTs dual to the \( AdS_{d+1} \)-Schwarzschild black holes once again captures just the distillable pure quantum entanglement and is related to the holographic mutual information which may be expressed as an algebraic sum of the area of extremal co-dimension two surfaces in the bulk geometry (The reader should refer to [30] for detailed computation of holographic entanglement negativity in \( AdS_{d+1}/CFT_d \) setup). To this end we would like to emphasize that our holographic conjecture for computing the entanglement negativity may found applications in understanding some of the difficult questions related to condensate formation, quantum phase transitions, non-equilibrium transport phenomena etc. in strongly coupled condensed matter systems such as the High \( T_c \) superconductors. Moreover, the holographic prescription for computing the entanglement entropy has played a crucial role in the investigation of the information loss paradox [31][32]. Thus it would be interesting to explore the implications of the holographic entanglement negativity in the context of the information loss paradox and the associated black hole firewall problem.

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