Communication via an entangled coherent quantum network

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Abstract

A quantum network (QN) is constructed via maximum entangled coherent states. The possibility of using this network to achieve quantum communication between multi-participants is investigated. We showed that the probability of the successful teleportation of an unknown state depends on the size of the used network. As the number of participants increases, the success probability does not depend on the intensity of the field. Implementing a quantum teleportation protocol via a noisy QN is discussed. The unknown state can be teleported perfectly with small values of the field intensity and larger values of the noise strength. The success probability of this suggested protocol increases abruptly for larger values of the noise strength and gradually for small values. For small-size QNs, the fidelity of the teleported state decreases smoothly, whereas it decreases abruptly for larger-sized networks.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Quantum teleportation is a way to teleport an unknown quantum state remotely between two users, where it is destroyed at the sending station and appears perfectly at the receiving station. Since the first quantum teleportation protocol was introduced by Bennett \textit{et al} \cite{Bennett}, a lot of attentions has been given to developing this phenomenon in many different directions. Among these directions is using quantum channels described by continuous variables \cite{Bergmann, Wolf, Milburn}. Some effort has been made to extend the use of quantum channel multiparities systems to perform quantum teleportation \cite{Wei, Wang, Ding}. In these types of quantum teleportation protocols, one needs larger numbers of participants collaborating together to transfer the unknown information between any two of them. This process is called remote information transfer over a quantum network (QN) \cite{AlAmri}. Some efforts have been made to generate an entangled QN. As an example, Nguyen \cite{Nguyen} has constructed a QN consisting of $2^N$ parties of coherent states and used it to implement quantum teleportation. Brougham \textit{et al} \cite{Brougham} have used a passive QN with logical bus topology to transfer information safely. Also, Ciccarello \textit{et al} \cite{Ciccarello} have proposed a physical model for the systematic generation of $N$-partite states for a QN. On the other hand, the presence of noise is associated with the deterioration of performance for quantum information tasks. Therefore investigating the dynamics of a QN and its usefulness for quantum information protocols is very important \cite{Zhang, Chen}.

This leads us to the aim of the current work, where we construct a maximum entangled coherent network, i.e. all the participants share maximum entangled coherent states (MECSs). The properties of these entangled coherent states have been investigated extensively in \cite{Metwally}. Figure 1 describes a QN consisting of four members: Alice, Bob, Clair and David, sending unknown information between Alice and David by the assistance of Bob and Clair. This network is generalized to include $m$ participants and we investigate the possibility of using this network to transfer unknown information between any two members by the co-operation of the others. The problem of remotely transferring information over a noisy QN is investigated, where it is assumed that during the construction of the network the traveling states from the source to their locations in the network is subject...
to noise. This type of noise is equivalent to employing a half mirror for the noise channel [19].

This paper is organized as follows. In section 2, a teleportation protocol is proposed to transfer unknown information between two distinct partners via a QN consisting of four members. The generalization of this protocol is discussed in section 2.2 Achieving quantum teleportation over a noisy QN is the subject of section 3. Finally, section 4 is devoted to discussing our results.

2. Teleportation via a perfect quantum network

Schrödinger cat states, which are superpositions of well-separated coherent states in optical systems, are useful for quantum teleportation [14]. They are defined as

$$|\alpha\rangle_{\text{cat}} = N_{\alpha}^\pm (|\alpha\rangle \pm |-\alpha\rangle),$$

where $N_{\alpha}^\pm = \sqrt{2 \pm e^{-2|\alpha|^2}}$ is a normalization factor and $|\pm \alpha\rangle$ are two coherent states with an equal complex amplitude $\alpha$ defined as

$$|\pm \alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{(|\pm \alpha|^n)}{\sqrt{n!}} |n\rangle.\quad (2)$$

These types of coherent states have been used to teleport a Schrödinger cat state [20]. The tripartite entangled coherent state of the form

$$|\phi^+\rangle = N_{\alpha}^+ (|\sqrt{2}\alpha, \alpha, \alpha\rangle \pm |\sqrt{2}\alpha, -\alpha, -\alpha\rangle),$$

where $N_{\alpha}^+ = [2(1 + e^{-|\alpha|^2})]^{-1/2}$, is used to teleport a two qubit entangled coherent state [21]. Recently, El-Allati et al. [15] introduced a teleportation protocol to teleport a tripartite coherent state via an entangled coherent state of four parties defined as

$$|\psi^\pm\rangle = N^\pm (|2\alpha, \sqrt{2}\alpha, \alpha, \alpha\rangle \pm |2\alpha, -\sqrt{2}\alpha, -\alpha, -\alpha\rangle),$$

where $N^\pm = \sqrt{2(1 \pm e^{-|\alpha|^2})}$. This protocol has been generalized to teleport a coherent entangled state of $m$ modes by using an entangled coherent state consisting of $m+1$ modes [15]. This class of states behaves as a maximum entangled state (MES) for $\rho^* = |\psi^\pm\rangle \langle \psi^\pm |$ where its concurrence [23] $C = 1$, and as partially entangled state (PES) for $\rho^* = |\psi^\pm\rangle \langle \psi^- |$.

Encouraged by the work of Nguyen [9], we employ this class of MES and its generalized version, which is described in [15], to teleport an unknown state over a QN. We expand upon this idea and its generalization in the following subsections.

2.1. Teleportation through a network of four participants

Assume that Alice is asked to send unknown qubit state $\rho_A$, to David, where

$$\rho_{A_1} = \frac{1}{N_A^+} (|\kappa_1|^2|2\alpha\rangle \langle 2\alpha | + \kappa_1^2|2\alpha\rangle \langle -2\alpha | + \kappa_2|2\alpha\rangle \langle -2\alpha | + \kappa_2|2\alpha\rangle \langle -2\alpha |),$$

where $\kappa_1, \kappa_2$ are unknown complex numbers, and $N_A = \sqrt{|\kappa_1|^2 + |\kappa_2|^2} \pm 2 e^{-|\alpha|^2} Re(\kappa_1 \kappa_2)$. In this context it is important to mention that the state vector of the density operator (5) is defined by $|\psi\rangle = |\kappa_1|\alpha\rangle + |\kappa_2|\alpha\rangle$. When $|\alpha| \to \infty$, the qubit can be encoded as $|\alpha\rangle = |1\rangle$ and $|\alpha\rangle = |0\rangle$ and the state vector $|\psi\rangle = |\kappa_1|\alpha\rangle + |\kappa_2|\alpha\rangle$, where in this limit $|\alpha\rangle$ and $|\alpha\rangle$ become orthogonal and $|\alpha\rangle = |\alpha\rangle = 0$. If $\kappa_1 = \kappa_2 = \kappa$ then the normalization factor becomes $N_A = 2\kappa$. However, $\alpha$ is very small, i.e., in the limit $|\alpha| \to 0$, $N_A \to \infty$. Therefore, the state vector $|\psi\rangle \to 0$ [16] represents classical information ‘0’. To teleport it we used the code in it in a pure state $\rho_{A_2} = |0\rangle \langle 0 |$ [17]. In this case, it is said that the users teleport classical information via a quantum channel [18].

Now, to teleport the state (5) to David, the users use a QN consisting of four MECSs:

$$|\psi\rangle_{ABCD} = N^-((2\alpha, \sqrt{2}\alpha, \alpha, \alpha)_{ABCD} - | -2\alpha, \sqrt{2}\alpha, -\alpha, -\alpha\rangle_{ABCD},$$

where $N^- = \sqrt{2(1 - e^{-16|\alpha|^2})}$. The total state of the system is given by $|\psi\rangle = \langle \psi\rangle_{A_1} \otimes |\psi\rangle_{ABCD}$. Where $A, B, C$ and $D$ refer to Alice, Bob, Clair and David, respectively. To implement this protocol the network’s members perform the following steps.

1. Alice mixes the unknown state, $\rho_{A_1} = |\psi\rangle_{A_1} \langle \psi |$ with the quantum channel $\rho_{ABCD} = |\psi\rangle_{ABCD} \langle \psi |$ by applying a series of operations defined by beam splitters and phase shifters as [21, 22]

$$\rho_{A_1} \otimes \rho_{ABCD} \rightarrow R_{ij} \rho_{A_1} \otimes \rho_{ABCD} R_{ij}^* = \rho_{out}.$$

where $R_{ij}|\mu\rangle |\nu\rangle = (\frac{|i\nu\rangle}{\sqrt{|\mu|}})(\frac{|j\mu\rangle}{\sqrt{|\nu|}})\quad (15)$ and the output state, $\rho_{out}$ is

$$\rho_{out} = |\kappa_1|^2 |\psi_1\rangle \langle \psi_1 | - |\kappa_1|^2 |\psi_2\rangle \langle \psi_2 | - |\kappa_2|^2 |\psi_1\rangle \langle \psi_3 | + |\kappa_2|^2 |\psi_1\rangle \langle \psi_4 |$$

$$+ |\kappa_1|^2 |\psi_2\rangle \langle \psi_2 | + |\kappa_1|^2 |\psi_2\rangle \langle \psi_3 | - |\kappa_2|^2 |\psi_2\rangle \langle \psi_2 |$$

$$+ |\kappa_1|^2 |\psi_2\rangle \langle \psi_4 | + |\kappa_2|^2 |\psi_2\rangle \langle \psi_3 | + |\kappa_2|^2 |\psi_2\rangle \langle \psi_4 |$$

$$- |\kappa_2|^2 |\psi_4\rangle \langle \psi_1 | - |\kappa_2|^2 |\psi_4\rangle \langle \psi_2 | - |\kappa_2|^2 |\psi_4\rangle \langle \psi_3 |$$

$$+ |\kappa_2|^2 |\psi_4\rangle \langle \psi_4 |. \quad (8)$$
Alice's measurements are such that \( n_A = 0, n_A < 0 \). The participants, Alice, Bob and Clair, send their measurement outcomes to David via a public channel, where \( n_A + n_B + n_C = 0 \). In this case, the state at David’s hand collapses into

\[
\rho_D = \lambda_1 | -\alpha \rangle \langle -\alpha | - \lambda_2 | -\alpha \rangle \langle -\alpha | + \lambda_4 |\alpha \rangle \langle \alpha |,
\]

where

\[
\lambda_1 = \frac{|\kappa_1|^2}{N_1}, \quad \lambda_2 = \frac{\kappa_1 \kappa_2^*}{N_1} (\lambda_4 \sinh |\alpha|^2)
\]

and

\[
N_1 = |\kappa_1|^2 + |\kappa_2|^2 - 2 \langle e^{-|\alpha|^2} \rangle Re(\kappa_2^* \kappa_1)
\]

is the normalized factor. Finally, David applies the operator \( P(\pi) \) (phase shifter) to equation (10) to get the final state \( \rho_D = P(\pi) \rho_D P(\pi)^* \), which is exactly the state \( \rho_0 = |\psi\rangle_0 \langle \psi | \) where the probability of success is given by

\[
P_1 = \frac{e^{-3|\alpha|^2}}{4 \sinh(8|\alpha|^2)} \left[ \sinh(11|\alpha|^2) - \sinh(3|\alpha|^2) \right].
\]

Alc’s measurements are such that \( n_A = 0, n_A < 0 \). In this case David’s state collapses into

\[
\rho_D = \lambda_1 | -\alpha \rangle \langle -\alpha | - \lambda_2 | -\alpha \rangle \langle -\alpha | + \lambda_4 |\alpha \rangle \langle \alpha |
\]

where

\[
\lambda_1 = \frac{|\kappa_1|^2}{N_1}, \quad \lambda_2 = \frac{\kappa_1 \kappa_2^*}{N_1} (\lambda_4 \sinh |\alpha|^2)
\]

and

\[
N_1 = |\kappa_1|^2 + |\kappa_2|^2 - 2 \langle e^{-|\alpha|^2} \rangle Re(\kappa_2^* \kappa_1)
\]

is the normalized factor.

2. Alice performs two photon number measurements on modes \( A \) and \( B \) using two detectors \( D_A \) and \( D_B \) (see figure 1). Bob and Clair should carry out the local number measurement of modes \( B \) and \( C \) by their detectors \( D_B \) and \( D_C \), respectively. There are two different possibilities due to Alice’s operations:

- **Alice’s measurements are such that \( n_A = 0, n_A < 0 \).** The participants, Alice, Bob and Clair, send their measurement outcomes to David via a public channel, where \( n_A + n_B + n_C = 0 \). In this case, the state at David’s hand collapses into

\[
\rho_D = \lambda_1 | -\alpha \rangle \langle -\alpha | - \lambda_2 | -\alpha \rangle \langle -\alpha | + \lambda_4 |\alpha \rangle \langle \alpha |
\]

where

\[
\lambda_1 = \frac{|\kappa_1|^2}{N_1}, \quad \lambda_2 = \frac{\kappa_1 \kappa_2^*}{N_1} (\lambda_4 \sinh |\alpha|^2)
\]

and

\[
N_1 = |\kappa_1|^2 + |\kappa_2|^2 - 2 \langle e^{-|\alpha|^2} \rangle Re(\kappa_2^* \kappa_1)
\]

is the normalized factor. Finally, David applies the operator \( P(\pi) \) (phase shifter) to equation (10) to get the final state \( \rho_D = P(\pi) \rho_D P(\pi)^* \), which is exactly the state \( \rho_0 = |\psi\rangle_0 \langle \psi | \) where the probability of success is given by

\[
P_1 = \frac{e^{-3|\alpha|^2}}{4 \sinh(8|\alpha|^2)} \left[ \sinh(11|\alpha|^2) - \sinh(3|\alpha|^2) \right].
\]

However, if \( n_A + n_B + n_C = 0 \) is odd, then nothing should be done by David and the teleported state is obtained with a probability of success

\[
P_2 = \frac{e^{-3|\alpha|^2}}{4 \sinh(8|\alpha|^2)} \left[ \sinh(11|\alpha|^2) - \sinh(3|\alpha|^2) \right]
\]

which tends to \( \frac{1}{2} \) in the limit \( |\alpha| \to 0 \) and to \( \frac{1}{2} \) in the limit \( |\alpha| \to \infty \).

- **At the end, David performs some operations on the bases \( |\alpha\rangle \to |2\alpha\rangle \) and \( | -\alpha\rangle \to | -2\alpha\rangle \) using the modified beam splitter to obtain exactly the state of Alice \( \rho_D = \rho_\lambda \).**

Figure 2 describes the behavior of the success probability \( \mathcal{P} \) of achieving the quantum teleportation protocol via a network consisting of four members sharing a MECS defined by (8). It is clear that, for small values of the mean photon number, i.e. \( |\alpha|^2 \in [0, 0.7] \), the probability decreases as \( |\alpha|^2 \) increases. However, the minimum value of \( \mathcal{P} \) in the interval \( |\alpha| \in [0, 0.7] \) is almost \( \approx 0.43 \) and, in the limit \( |\alpha| \to 0 \), the probability tends to 0.5. On the other hand, the probability of success is independent of \( |\alpha|^2 > 1 \), where \( \mathcal{P} = 0.5 \).

Comparing our results with that depicted in [9], we can see that constructing a QN by using a MES defined by (3) is much better. It is clear that in the earlier study [9], \( \mathcal{P} \to 0.25 \) in the limit \( |\alpha| \to 0 \), while in the current investigation \( \mathcal{P} \to 0.5 \) as \( |\alpha| \to 0 \). Also, the probability of successful teleportation is independent from the intensity of the field, for \( |\alpha|^2 < 0.7 \), while in [9] this is \( |\alpha|^2 < 3 \). Additionally, although \( \mathcal{P} \) decreases in the interval \( |\alpha| \in [0, 0.7] \), the minimum probability is still much better than that depicted in the earlier work.

2.2. Teleportation through a network of \( m \) participants

These results can be generalized to \( m + 1 \) participants on the QN by using maximally entangled coherent states. For this
aim, we assume that the network’s users share a maximally entangled coherent state of the form

$$\Psi_{\text{net}}^{(n)} = N_{m+1}(2^{\frac{n_1}{2}}|\alpha\rangle_m |\alpha\rangle_{m-1}\cdots|\alpha\rangle_2 |\alpha\rangle_1 - |\alpha\rangle_2\cdots|\alpha\rangle_m),$$

where $N_{m+1}$ is the normalization factor given by

$$N_{m+1} = [2(1 - e^{-2\alpha^2} |\alpha|^2)]^{-1/2}.$$

The entangled and separable properties of this class of states are investigated in [15]. Also, these states are employed to perform a teleportation between any number of parties. In this context, we use them to achieve quantum teleportation over a network. The suggested protocol is implemented as follows.

1. Assume that the user $m$, who shares a MES (15) with the remaining $m - 1$ members, co-operates to send a state $|\psi_0\rangle$ to any member of the network. In this case, the generator of this type of MES sends the mode $m$ to the emitter and the modes $m - 1$ to the remaining users. The total state of the system is defined by $\rho_{m} = \rho_{h_1} \otimes \rho_{\text{net}}$, where $\rho_{h_1} = |\psi_{\text{net}}\rangle |\psi_{\text{net}}\rangle^{\dagger}$ and $\rho_{\text{net}} = |\psi_0\rangle |\psi_0\rangle^{\dagger}$.

2. The user $m$ performs a sequence of local operations (7) on his own state and the state $|\psi_0\rangle$. Then, by using the two detectors $D_0$ and $D_m$, the user $m$ counts the photon numbers in modes 0 and $m$.

3. The other users carry out the local number measurement of modes $m - 1, m - 2, \ldots, 3, 2, 1$ by the local detectors $D_{m-1}, D_{m-2}, \ldots, D_1$, respectively. As a result of these measurements there are two possibilities.

   The first possibility is obtained for $n_0 = 0, n_m > 0$. In this case, the state $|\psi_0\rangle$ collapses at the receiver into

$$\rho_0' = (1 - \rho_{m} |\alpha\rangle |\alpha\rangle^{\dagger} - \rho_{m} |\alpha\rangle |\alpha\rangle^{\dagger} - \rho_{m} |\alpha\rangle |\alpha\rangle^{\dagger} + \lambda_4 |\alpha\rangle |\alpha\rangle^{\dagger}),$$

where

$$\lambda_1 = |\alpha|^2, \quad \lambda_2 = \frac{k^2 k^2}{N_m} \iota^{(1)}(1 - |\alpha|^2 |\alpha\rangle |\alpha\rangle^{\dagger} + \lambda_3 |\alpha\rangle |\alpha\rangle^{\dagger} - \lambda_4 |\alpha\rangle |\alpha\rangle^{\dagger},$$

and

$$N_m = |\alpha|^2 + 2 - (|\alpha|^2 |\alpha\rangle |\alpha\rangle^{\dagger} - \lambda_2 |\alpha\rangle |\alpha\rangle^{\dagger} Re(k_2^2 k_2^2)$$

is the normalized factor.

The remaining participants send their classical results via a public channel to the receiver, where $n_m + n_{m-1} + \cdots + n_1 = 0$, who applies the operator $P(\pi)$ and a series of operations $R_{i, j}$ defined by a modified beam splitter for replicating the original state on the state (17) to get the state $\rho_0 = P(\pi) \rho_0' P^*(\pi)$.

The second possibility is obtained for $n_{m+1} > 0, n_m = 0$. In this case, one gets the same probability of success as the previous case. So the information is transformed with an average probability given by

$$P = e^{-2 |\alpha|^2} \frac{2 \sinh(2^n |\alpha|^2)}{2 \sinh(2^n |\alpha|^2)} \sinh((3^n 2^{n-1} - 1) |\alpha|^2) - \sinh((2^n - 1) |\alpha|^2).$$

Figure 3 describes the dynamics of the successful probability of teleportation (18) for different sizes of QNs. It is clear that, for $m = 2$, i.e two partners co-operate to send the information to the third, the probability $P$ decreases smoothly in the interval $|\alpha|^2 \in [0, 0.25]$ and then increases gradually. However as $|\alpha|^2$ increases the probability increases and reaches its maximum value at $|\alpha|^2 \approx 1.6$. As one increases the numbers of partners, $P$ decreases abruptly in a small range of $|\alpha|^2$ and increases faster than that depicted for fewer partners. However the minimum value of the probability of success increases as one decreases the size of the network. Therefore, one can conclude that by increasing the number of partners in the QN the probability of successful teleportation is independent of $|\alpha|^2$. So, one can perform a quantum teleportation in the presence of low field intensity with probability 0.5 by increasing the size of QN. On the other hand, this figure confirms that the used quantum channel (15) is much better than that used in the earlier work given in [9].

3. Teleportation via a noisy quantum network

Decoherence represents the biggest challenge in the context of handling information. It arises during the interaction of systems with their environments [24], imperfect devices [25], noisy channels [26, 27], energy loss or photon absorption [19, 20, 28], etc. Therefore, it is very important to investigate the dynamics of information in the presence of noise [29].

The properties of quantum channels consisting of coherent states have received considerable attention. As an example, van Enk [20] has considered the decoherence of multi-dimensional entangled coherent states due to photon absorption losses, where he calculated how fast entanglement decays and how much entanglement is left. The entanglement degradation of entangled states suffering from photon absorption losses is investigated in [30].

In this section, we assume that we have a QN consisting of multi-entangled coherent states. We assume that we have a source supplying the partners, with a MECS given by (15). These entangled coherent states propagate from the source to the locations of the partners. Due to the interaction with the environment, the MECSs turn into PESs, where their degree of entanglement depends on the strength of the noise. Let us consider that the source produces a MECS defined by the
density operator $\rho^-$ (15). This entangled state turns into a PES:

$$\rho_{PE} = U_{AE} \otimes U_{BE} \rho^- U_{BE}^\dagger \otimes U_{AE}^\dagger,$$

where $U_{I[E]}|0\rangle_E = |\sqrt{\eta}|a\rangle_E |\sqrt{1-\eta}|a\rangle_E$, $I = A$, or $B$ and $|0\rangle_E$ refers to the environment state. This effect is equivalent to employing a half mirror for the noise channel [19]. In an explicit form, one can write the output density operator $\rho_{PE}$ as

$$\rho_{PE} = \frac{1}{N_\alpha} \left[ |2\sqrt{\eta}|a\rangle \sqrt{2} |\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle \right]$$

$$\times \left[ |2\sqrt{\eta}|a\rangle \sqrt{2} |\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle \right]$$

$$+ \left[ -2\sqrt{\eta}|a\rangle |\sqrt{2}|\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle \right]$$

$$\times \left[ -2\sqrt{\eta}|a\rangle |\sqrt{2}|\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle \right]$$

$$- e^{-8\langle (1-\eta)|a\rangle^2} |2\sqrt{\eta}|a\rangle \sqrt{2} |\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle$$

$$\times \left[ -2\sqrt{\eta}|a\rangle |\sqrt{2}|\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle \right]$$

$$- e^{-8\langle (1-\eta)|a\rangle^2} |2\sqrt{\eta}|a\rangle \sqrt{2} |\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle$$

$$\times \left[ 2\sqrt{\eta}|a\rangle \sqrt{2} |\sqrt{\eta}|a\rangle \sqrt{\eta}|a\rangle \right].$$

(20)

where $N_\alpha = 2(1 - e^{-16\langle |a\rangle^2} )$ is the normalized factor (for more details see [15]). We assume that the aim of Alice is to send unknown state $\rho_{AI}$ (5) to David through the noisy QN (20). The partners follow the same steps as described in section 2.1 to fulfill this task. If the members Alice, Bob and Clair who performed the number measurement send their outcomes through a classical channel to David such that $n_A = 0$ and $n_B > 0$ and $n_A + n_B + n_C$ is odd, then the final state at David’s hand is

$$\rho_D = \lambda_1 | -\sqrt{\eta}|a\rangle |\chi\rangle |\sqrt{\eta}|a\rangle |\chi\rangle - \lambda_2 |\sqrt{\eta}|a\rangle |\chi\rangle | -\sqrt{\eta}|a\rangle |\chi\rangle$$

$$\times \left[ \lambda_3 |\sqrt{\eta}|a\rangle |\chi\rangle |\sqrt{\eta}|a\rangle |\chi\rangle \right]$$

$$+ \lambda_4 |\sqrt{\eta}|a\rangle |\chi\rangle |\sqrt{\eta}|a\rangle |\chi\rangle ,$$

(21)

where

$$|\chi\rangle = \left[ |2\sqrt{\eta}|a\rangle_E |\sqrt{2\sqrt{\eta}|a\rangle_E \sqrt{\eta}|a\rangle_E \sqrt{\eta}|a\rangle_E \right]$$

$$\lambda_1 = \frac{|k_1|^2}{N_x}, \quad \lambda_2 = \frac{k_1 k_2^*}{N_x} (-1)^a \eta^m a^s c, \quad \lambda_3 = \frac{k_2 k_3^*}{N_x} (-1)^a \eta^m a^s c,$$

$$\lambda_4 = \frac{|k_2|^2}{N_x},$$

$$N_x = |k_1|^2 + |k_2|^2 + e^{-2\langle |a\rangle^2} e^{-16\langle \eta|a\rangle^2} Re(k_2^* k_1),$$

$$\eta' = 1 - \eta.$$

The probability of finding odd number of photons in this case is given by

$$P = \frac{N_x e^{-11|\sqrt{\eta}|a|^2}}{N_A^2 N_C^2} \left[ \sinh(11|\sqrt{\eta}|a|^2) - \sinh(3|\sqrt{\eta}|a|^2) \right],$$

(23)

where $N_A$ is the normalization factor of the unknown state, equation (5), and $N_c$ is the normalization of quantum channel state (6).

In figure 4, the dynamics of the total success probability, $P_s = 2P$, is investigated for different values of the noise strength, $\eta$. It is clear that as one increases $\eta$ the probability $P_s$ increases faster, while it increases gradually for small values of $\eta$. For larger values of $|\alpha|^2$, $P_s \rightarrow \frac{1}{2}$, namely, the probability of success is independent of noise. However for small values of the field intensity, one can increase the probability of success by increasing the noise strength. Therefore, to send coded information through a noisy cavity with a large probability, one reduces the intensity of the field and increases the noise strength or increases the field intensity and reduces the noise strength.

The fidelity $F$ of the teleported state (5), via a noisy QN defined by (20), is given by [31]

$$F = tr(\rho_D \rho_{AI}) = \frac{F_N + 1}{N_H + 1},$$

(24)

where

$$f = \frac{(1 - e^{-16\langle |a\rangle^2}) (1 + e^{-16\langle (1-\eta)|a\rangle^2})}{2(1 - e^{-16\langle |a\rangle^2})},$$

(25)

is the fidelity of the QN (8) and $N_H$ is the dimension of the Hilbert space.

Figure 5(a), describes the dynamics of the fidelity $F$ in a small interval of the field intensity, i.e. $|\alpha|^2 \in [0, 1]$. It is clear that, for small values of $\eta$ and $\alpha$, the fidelity of the teleported state increases very fast and it increases smoothly for larger value of $\alpha$. Therefore, to send information with a unit fidelity for a low intensity field, one has to increase the noise strength. In figure 5(b), the fidelity $F$ is plotted for different values of the noise strength $\eta$. It is shown that, for large values of $\eta$, $\eta = 0.95$ (uppermost curve), the fidelity decreases smoothly to reach its lower limit ($F = \frac{1}{2}$). As one decreases $\eta$, the fidelity decreases faster and reach its lower limit at $|\alpha|^2 \approx \frac{3}{2}$. However, for small values of $\eta \in [0, 0.5]$, the fidelity increases gradually and reaches its upper limit for larger values of the field intensity. On the other hand, the fidelity is independent of the noise strength in the limit of $|\alpha|^2 \rightarrow \infty$ for $\eta > 0.5$, while for any value of $\eta \leq 0.5$, the fidelity approaches the fidelity of classical teleportation, $F = \frac{1}{2}$.

From the preceding discussions, one can notice that the advantage of this current teleportation protocol is the probability of success is much better than that obtained by van Enk and Hirota [30]. Also, for $\eta = \frac{1}{2}$, the fidelity $F = \frac{3}{2}$.
while in the former work \[30\] (\(F = \frac{1}{3}\)). On the other hand, for any value of \(\alpha \in [0, 1]\), the fidelity \(F\) is maximal for any value of \(\eta\).

Finally, let us assume that the sender and receiver are members of a noisy QN consisting of \(m\) participants. The members follow the same steps as described in section 2.2 and they complete their protocol with a final state at the receiver’s hand with a fidelity given by

\[
F_n = \frac{1}{3} \left[ \frac{1 + e^{-2\alpha(1-\eta)|\alpha|^2}}{(1 - e^{-2\eta|\alpha|^2})} + 1 \right]. \tag{26}
\]

In figure 6, we investigate the dynamics of the fidelity \(F_n\) for different network sizes, where we assume two different values of the noise strength. In figure 6(a), we set a larger value of the noise strength, \(\eta = 0.9\), and a different QN size. It is clear that, for small size (\(m = 3\)), the fidelity \(F_n\) decreases smoothly and reaches its minimum bound for larger values of the field intensity. However for larger values of \(m\), \(F_n\) decreases faster and reaches its minimum bound for smaller values of the field intensity. Figure 6(b) describes the dynamics of \(F_n\) for a smaller value of \(\eta \in [0, 0.5]\) where, for small values of \(m\), the fidelity increases gradually and reaches its upper bound for larger values of \(|\alpha|^2\). However for larger size of the network, i.e. larger \(m\), the fidelity increases very fast and reaches its upper value for small values of \(|\alpha|^2\).

4. Conclusion

A QN consisting of multiple participants is constructed by using a MECS. This network is used to send information between any two members, where the other members co-operate with the sender to achieve their common aim with a high success probability. We show that as the size of the network increases, i.e. the number of members grows, the probability of successful teleportation does not depend on the intensity of the field. This type of entangled coherent state is much better than that used in the earlier work of Nguyen \[9\] where, in his proposal, the probability of success reaches its maximum value for larger values of field intensity (\(|\alpha|^2 \geq 3\)) while for the current work (\(|\alpha|^2 \geq 0.7\)).

The possibility of using this network to perform communication between its members in the presence of noise is investigated, where we assume that the traveling MES from the sources to the locations of the members is subject to noise. Our results show that the probability of achieving successful quantum teleportation increases faster for small values of the noise strength and reaches its maximum value for small values of the field intensity. However, for larger value of the noise strength the successful probability increases gradually and needs a larger value of field intensity to be maximum.

The effect of the noise strength and the field intensity on the fidelity of the teleported state is investigated. It is shown that the unknown state can be teleported between any two members of the network with a unit fidelity for small values of the intensity of the field. This type of entangled coherent state is much better than that used in the earlier work of Nguyen \[9\] where, in his proposal, the probability of success reaches its maximum value for larger values of field intensity (\(|\alpha|^2 \geq 3\)) while for the current work (\(|\alpha|^2 \geq 0.7\)).
This protocol is generalized to be used between any two members of a network consisting of \( m \) users, where \( m - 2 \) members should co-operate with the sender to teleport unknown state safely to the receiver. Also, the possibility of implementing this generalized protocol in the presence of noise is discussed. The fidelity of the teleported state decreases smoothly for small network sizes and abruptly for larger sizes.

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