Interaction of nonuniform elastic waves with two-dimensional electrons in 
AlGaAs – GaAs – AlGaAs heterostructures

D.V. Fil
Institute for Single Crystals National Academy of Sciences of Ukraine, Lenin av. 60 Kharkov 310001 Ukraine
e-mail: fil@isc.kharkov.com
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An interaction of a double layer electron system realized in an AlGaAs – GaAs – AlGaAs heterostructure with nonuniform elastic waves localized in the GaAs layer is considered. The dependence of the coupling constant on the ratio between the thickness of the GaAs layer and the wavelength is calculated for the wave vector directed along the [110] axis, the displacement vector lying in the (110) plane and the interface boundaries parallel to the (001) plane. It is shown, that the coupling constant reaches the maximal value at the wavelength which is of order of the thickness of the GaAs layer. The renormalization of the velocity of the elastic modes is found for the case, when the electron system is in the fractional quantum Hall regime. It is shown, that for certain modes the dependence of the velocity shift on the wave vector is modified qualitatively under a transition of the electron system to the state, which corresponds to the Halperin wave function.

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Recently the surface acoustic waves (SAW) have been widely used for an investigation of the dynamical properties of two-dimensional electron layers in heterostructures AlGaAs [1, 2]. Since the AlGaAs is a piezoelectric material, the SAW generates an alternating electric field. The interaction of that field with the two-dimensional electrons results in a renormalization of the velocity and the absorption of the SAW. The frequency and the momentum dependence of the conductivity of the electron system can be obtained from the measurements of the velocity shift and the absorption coefficient. In particular, the power of the method has been demonstrated within the study of the quantum Hall systems for which the conductivity depends considerably on the external magnetic field.

Since the electron layer is placed at a certain distance from the surface of the sample, the coupling of the SAW with electrons depends on the parameter \( q d_0 \) (where \( q \) is the wave vector of the elastic mode). An exponential reduction of the matrix elements of the coupling takes place if the parameter \( q d_0 \) increases. Therefore, the SAW method is not applicable for a study of the dynamical properties of the electron system at large \( q \). In that case a nonuniform elastic wave, which is localized near the electron layer due to the acoustic nonhomogeneity of the heterostructure, can be used.

To specify the system, in which the elastic wave may be localized, we consider the heterostructure Al\(_x\)Ga\(_{1-x}\)As – GaAs – Al\(_x\)Ga\(_{1-x}\)As which incorporates two electron layers situated at the AlGaAs – GaAs interfaces. Similar heterostructures have been used for the investigation of the quantum Hall effect in the double-layer electron systems [3, 4].

Several types of the elastic modes localized in a central layer exist in the structure considered. The interaction of the two-dimensional electrons with the nonuniform transverse elastic wave with the displacement vector parallel to the interface boundaries has been studied in [3]. In this paper we continue the investigation of Ref. [3] and consider the nonuniform elastic waves for which the displacement vector is in the plane determined by the wave vector and the normal direction to the interface boundaries (these waves as well as the SAW are polarized elliptically). We found that the interaction of such waves with electrons may be much stronger then of the waves considered in Ref. [3].

I. GEOMETRY OF THE MODEL AND THE DISPERSION EQUATIONS

Let us consider the system for which the GaAs layer with the thickness \( 2a \) is situated between two \( Al_{0.3}Ga_{0.7}As \) layers with the thicknesses much larger than the wavelength of the elastic mode. We specify the case of the interface boundaries parallel to the (001) plane and the wave vector of the elastic mode directed along the [110] axis. We will use the fact that the elastic moduli are practically independent on the concentration of \( Al \) (we set them the same for the both media). The acoustical nonuniformity of the system is caused by the difference of the densities (\( \rho_1 = 5.3 \) g/cm\(^3\) for GaAs, \( \rho_2 = 4.8 \) g/cm\(^3\) for \( Al_{0.3}Ga_{0.7}As \)).

We consider the elastic mode with the displacement vector components

\[
u_i(\mathbf{r}, z, t) = u_i(z)e^{i(qr-\omega t)}, \quad (1)
\]

where \( \omega_q = \omega_q(q, \nu) \) is the elastic mode velocity, \( i = x, z \), the \( x \) axis is chosen along the [110] direction, the \( z \) axis, along the [001] direction, the \( \mathbf{r} \) vector is in the (001) plane. The wave equations for the \( u(z) \) components have the form

\[
(c_{44}\partial_z^2 - c'_{11}q^2 + \rho_0\omega^2)u_x + iq(c_{12} + c_{44})\partial_z u_x = 0
\]
\[ iq(c_{12} + c_{44}) \partial_u x + (c_{11} \partial_z^2 - c_{44}q^2 + \rho_0 \omega^2)u_z = 0, \]  
(2)

where \( c_{11}' = 0.5(c_{11} + c_{12} + 2c_{44}) \), \( \alpha=1,2 \) corresponds to the medium number. The solution of Eqs. (2) has the form

\[ u^\alpha(z) = \sum_k A^\alpha_k \exp y^\alpha_k zq, \]  
(3)

where \( y^\alpha_k \) are the roots of the equation

\[ y^4 + 2b_\alpha y^2 + c_\alpha = 0 \]  
(4)

with

\[ b_\alpha = \frac{1}{2c_{11}c_{44}}[(c_{12} + c_{44})^2 + c_{11}(\rho_0 v^2 - c_{11}) + c_{44}(\rho_0 v^2 - c_{44})], \]

\[ c_\alpha = \frac{1}{c_{11}c_{44}}(\rho_0 v^2 - c_{11})(\rho_0 v^2 - c_{44}). \]  
(5)

If we take into account the equivalence of the elastic moduli for two media, the boundary conditions are reduced to the requirement of the continuity of \( u_i \) and \( \partial_z u_i \) at the interfaces. The local mode corresponds to the solution for which the displacement approaches to zero at \( z \to \pm \infty \). The structure of the localized solution in the AlGaAs layers is similar to the structure of the SAW on the surface of the cubic crystal (see, for instance, Ref. [3]). Such a solution arranges if Eq. (3) at \( \alpha = 2 \) does not have imaginary roots. If the elastic moduli satisfy the inequality

\[ (c_{12} + c_{44})^2 - c_{11}(c_{12}' - c_{44}) < 0, \]  
(6)

the localized solution emerges at \( v < c_{44}/\rho_2 \). If the opposite inequality is satisfied, the velocity of the localized mode \( v < v_m \), where \( v_m \) is the root of the equation

\[ D_2(v) = b_2^2 - c_2 = 0 \]  
(7)

where

\[ \lambda = \sqrt{(\sqrt{c_2^2 - b_2^2})/2} \]

\[ \varphi = \sqrt{(\sqrt{c_2^2 + b_2^2})/2}. \]  
(8)

For the medium 1 the solution of Eq. (3) which satisfy the boundary conditions corresponds to the cases, for which Eq. (3) has two real and two imaginary roots or four imaginary roots

\[ y_{1,2} = \pm \kappa = \pm \sqrt{D_1 - b_1}, \]

\[ y_{3,4} = \pm i\xi = \pm i\sqrt{D_1 + b_1}, \]  
(9)

where \( D_1 = b_1^2 - c_1 \). The \( \kappa \) is a real value at \( v > c_{44}/\rho_1 \), and an imaginary value at \( v_m < v < c_{44}/\rho_1; v_m \) is the root of equation \( D_1(v) = 0 \).

There are two types of the solutions of the wave equations, which satisfy the boundary conditions (we will refer on them as I and II)

\[ u^P(z) = C^P f^P(z), \]

\[ u^\xi(z) = iC^P f^\xi(z), \]  
(10)

where \( C^P \) is the normalization factor, \( p=I,II \). In Eq. (10) \( f^P(z) \) is the odd function and \( f^\xi(z) \) is the even function, while \( f^H(z) \) is the even function and \( f^H(z) \) is the odd function.

The dispersion equation for \( v \) is obtained from the common linear problem on the factors \( A^\alpha_k \). For the mode I the dispersion equation reads as

\[ R_1 \tanh \zeta q + R_2 \tanh \zeta q + R_3 \tan \zeta q + R_4 = 0. \]  
(11)

For the mode II it is modified to

\[ R_1 \cot \zeta q - R_2 \cot \zeta q + R_3 \cot \zeta q - R_4 = 0, \]  
(12)

where

\[ R_1 = (km_\kappa + \xi m_\varphi)(\varphi \lambda_\varphi - \lambda \lambda_\kappa) \]

\[ R_2 = m_\varphi m_\kappa (\lambda^2 + \varphi^2) - \varphi (m_\varphi^2 + m_\kappa^2) - km_\kappa (\varphi \lambda_\kappa - \lambda \lambda_\varphi) \]

\[ R_3 = -m_\varphi m_\kappa (\lambda^2 + \varphi^2) + \varphi (m_\varphi^2 + m_\kappa^2) + \xi m_\kappa (\varphi \lambda_\kappa - km_\varphi) \]

\[ R_4 = (km_\xi - \xi m_\kappa)(\varphi \lambda_\varphi + \lambda \lambda_\kappa) \]

\[ m_\kappa = \frac{(c_{12} + c_{44})}{c_{11}^2 - c_{44} + \rho_1 v^2} \]

\[ m_\varphi = \frac{-c_{11}^2 - c_{44} + \rho_1 v^2}{\lambda c_{44}(R - 1)} \]

\[ m_\varphi = \frac{-c_{11}^2 - c_{44} + \rho_1 v^2}{\lambda c_{44}(R - 1)} \]

\[ m_\zeta = \frac{c_{12} + c_{44}}{c_{11}^2 - c_{44} + \rho_1 v^2} \]

\[ R = \frac{c_{11}(p_2 v^2 - c_{11})}{c_{44}(p_v^2 v^2 - c_{44})}. \]

Numerical solutions of Eqs. (11,12) for the parameters

\( c_{11} = 12.3, c_{12} = 5.7, c_{44} = 6.0 \) (all in 10^{11} dyn/cm^{2}) versus the ratio between the thickness of the central layer \( d = 2a \) and the wavelength are shown in Fig. 1. One can see from the dependences presented, there are two gapless modes (one of them is of the type I, while the other is of the type II) in the system. When the wavelength becomes shorter, additional modes with higher frequencies emerge. The interaction with the gapless modes is only considered below. The quantities \( f^P(z) \) can be written through elementary functions of \( z \), but the expressions contain a complicate dependence on \( q \) and \( v \). Therefore we will not present here the analytical expressions.
for $f^p(z)$. At an example, the plots of $u_x(z)$, $u_z(z)$ at $d/l = 1$ are shown in Fig. 2.

![FIG. 1. Dependences of the velocities of the elastic modes on the ratio between the thickness of the central layer $d$ and the wavelength $l$. Solid lines indicates the type I modes; dashed lines, the type II modes.](image)

![FIG. 2. Dependences of the displacement vector components (in relative units) on $z$ (in units of the wavelength) at $d/l = 1$; a, the type I mode; b, the type II mode.](image)

II. PIEZOELECTRIC COUPLING CONSTANTS

Let us calculate the renormalization the velocity of the nonuniform elastic mode, caused by the interaction with the double-layer electron system with the coordinates of the electron layers $z_1(z) = \pm a$. We use the approach similar to Ref. [7]. Let us write the Hamiltonian for the elastic waves in terms of the phonon creation and annihilation operators ($b^+$, $b$)

$$H_u = \sum_q \omega_q (b_q^+ b_q + \frac{1}{2}),$$

(13)

The Hamiltonian of the electron-phonon interaction is chosen in the form

$$H_{\text{int}} = \frac{1}{\sqrt{S}} \sum_{q,m} \int d^2r g_{qm} \Psi^{+}_{rm} \Psi_{rm} e^{iqr} (b_q + b_{-q}^+),$$

(14)

where $\Psi^+(\Psi)$ are the electron creation (annihilation) operators, $m$, the number of the electron layer, $S$, the area of the layer.

To obtain the matrix elements $g_{qm}$ we write the interaction of the elastic wave with the electrons in the form

$$H = \sum_m \int d^2r \varphi_{rm} \Psi^{+}_{rm} \Psi_{rm},$$

(15)

where $\varphi_{rm}$ is the scalar potential of the electric field, generated by the elastic wave in the layer $m$. The value of $\varphi$ is determined by the solution of the Poisson equation

$$\Delta \varphi = -(4\pi/\epsilon) \beta_{ijk} \partial_i u_{jk},$$

(16)

where $\epsilon$ is the dielectric constant, $\beta_{ijk}$ is the piezoelectric tensor, $u_{jk}$ is the strain tensor. Under the choice of the $x$, $y$ and $z$ axes along the [100], [010] and [001] directions correspondingly the $\beta$ tensor has nonzero (and all equal to the same value $\beta$) components for $i \neq j \neq k$.

Under substitution of Eq. (10) into Eq. (16) we obtain the following equation for the Fourier-component of the electric potential

$$(\partial_z^2 - q^2) \varphi_q(z) = i(4\pi \beta/\epsilon) C^p g^p(z),$$

(17)

where

$$g^p(z) = q^2 f^p_z(z) - 2q \partial_z f^p_z(z).$$

(18)

We assume that the $\epsilon$ and $\beta$ parameters are same for the whole system. Then the boundary conditions on $\varphi$ reduce to the requirement of continuity of $\varphi_q(z)$ and $\partial_z \varphi_q(z)$ at the interfaces. The solution of Eq. (17) with the boundary conditions has the form

$$\varphi_q(z) = i(4\pi \beta/\epsilon) C^p \chi^p(z),$$

(19)

where $\chi_1(z)$ is an even function and $\chi_1(z)$ is an odd function. We do not present the analytical expressions...
for the $\chi_p(z)$ functions here. As an example, the dependences $\tilde{\varphi}_q(z)$ at $d/l = 1$ are shown in Fig. 3 (we use the values of $C^p$, calculated below)

\[ \hat{D}(q, \omega) = (\hat{I} - \hat{\delta}^{(0)}(q, \omega)\hat{V}(q))^{-1}\hat{\delta}^{(0)}(q, \omega), \]

where

\[ V_{m,m'}(q) = \frac{2\pi e^2}{eq}(\delta_{mm'} + (1 - \delta_{mm'})e^{-qd}) - \]

the Fourier components of the Coulomb interaction, $D^{(0)}$ is the density-density response function for the system without the Coulomb interaction. The quantities $D^{(0)}$ can be expressed through the longitudinal conductivity of the electrons

\[ D^{(0)}_{mm'}(q, \omega) = -\frac{iq^2}{\omega^2 c^2}\sigma_{xx}^{m,m'}(q, \omega), \]

where $\sigma^{11} = \sigma^{22}$ are the diagonal with respect to the electron layers, $\sigma^{12} = \sigma^{21}$, nondiagonal with respect to the layers components of the conductivity. Here and below we consider the case of two equivalent electron layers. We should note, that the nondiagonal components of conductivity are equal to zero for the electron gas in the random phase approximation, while they may be nonzero for the composite fermion gas due to the interlayer statistical interaction (this case is considered in the next section).

Substituting Eqs. (23, 25, 27) into Eq. (24), we obtain

\[ \Delta v \frac{\Gamma}{\nu} = \alpha_+ \frac{-i\sigma_{xx}^{1+}(q, vq)/\sigma_{M}^{1+}}{1 + i\sigma_{xx}^{1+}(q, vq)/\sigma_{M}^{1+}} + \alpha_- \frac{-i\sigma_{xx}^{1-}(q, vq)/\sigma_{M}^{1-}}{1 + i\sigma_{xx}^{1-}(q, vq)/\sigma_{M}^{1-}}, \]

where $\sigma_{xx}^{1\pm} = \sigma_{zz}^{1\pm} \pm \sigma_{xx}^{1\mp}$, $\sigma_{M}^{1\pm} = \nu e/2\pi (1 \pm \exp(-qd))$.

\[ \alpha_{\pm} = \frac{4\pi \beta^2}{\nu e^2 I^p} \frac{|\chi_p(a) \pm \chi_p(-a)|^2}{1 \pm \exp(-qd)}. \]

The $\alpha_{\pm}$ functions play the role of the piezoelectric coupling constants, which are introduced for the consideration of the interaction of the SAW with the two-dimensional electrons. One can see from Eq. (29), that $\alpha_+$ coefficient is nonzero for the type I mode only, while the $\alpha_-$ coefficient is nonzero for the type II mode only.

The dependences of $\alpha_+$ for the I mode and $\alpha_-$ for the II mode versus the parameter $d/l$ are shown in Fig. 3. The parameters $\beta = 4.5 \times 10^4$ dyn cm$^{-1}/\sqrt{\text{cm}}$, $\epsilon = 12.5$ are used. The dependence of $\alpha_-$ on $d/l$ for the transverse mode considered in Ref. [3] is also shown in Fig. 3. One can see from the dependences presented, the interaction of the electrons with the elastic modes, elliptically polarized in the sagittal plane is much stronger then the interaction with the transverse mode. Note, that the case considered in [3] has the advantage, that there is only one transverse nonuniform mode (if the thickness of the central layer is not very large comparing to the wavelength), and its frequency is lower than the frequencies of the bulk modes. (For the case considered here the bulk

\[ D_{m,m'}(z) = (\hat{I} - \hat{\delta}^{(0)}(q, \omega)\hat{V}(q))^{-1}\hat{\delta}^{(0)}(q, \omega), \]

\[ \tilde{D}(q, \omega) = (\hat{I} - \hat{\delta}^{(0)}(q, \omega)\hat{V}(q))^{-1}\hat{\delta}^{(0)}(q, \omega), \]

\[ \hat{V}(q) = \int_0^\infty dz \rho(z)|f^p_2(z)|^2 + |f^p_2(z)|^2. \]

\[ I^p = 4q \int_0^\infty dz \rho(z)|f^p_2(z)|^2 + |f^p_2(z)|^2. \]

\[ g_{mm'} = i(4\pi \beta e/\epsilon \sqrt{v^p I^p})\chi_p(z_m). \]

\[ D_{m,m'}(z,t) = (\delta_{mm'} + (1 - \delta_{mm'})e^{-qd}) - V_{m,m'}(q, \omega) = -\frac{iq^2}{\omega^2 c^2}\sigma_{xx}^{m,m'}(q, \omega), \]
The third mode polarized along the [110] direction has the frequency, which is lower than the frequencies of the I and II modes.

Let us apply the results obtained in the previous section to the study of a possibility of the observation of phase transitions in the double-layer fractional quantum Hall systems. To describe the quantum Hall system we use the composite fermion approach (the Chern-Simons fermionic model, developed in Ref. [8] for the double-layer system). Within such an approach the fractional quantum Hall system is modelled as a gas of the composite fermions, which carry the auxiliary statistical charge and the flux of the statistical gauge field. For the double-layer model two types of the statistical charges corresponding to two layers and two types of the gauge fields are introduced. In general case the composite quasiparticles carry the fluxes of the both types, namely the even number \( s \) of the flux quanta of the statistical field corresponding to their statistical charges and the integer number \( n \) of the flux quanta of the field, which corresponds to the statistical charges in the opposite layer. This model at \( s \neq 0 \) corresponds to the states described by the Halperin wave function [5]. We refer the phase with \( s \neq 0 \) as the phase with the interlayer statistical interaction.

In the average field approximation the partial screening of the external magnetic field \( B \) emerges due to the influence of the statistical fields

\[
B_{\text{eff}} = B (1 - \nu (\psi + s)),
\]

where \( \nu \) is the filling factor calculated for one layer. The fractional quantum Hall effect corresponds to the filling factors, for which the value of \( B_{\text{eff}} \) corresponds to the integer number \( N \) of the filled Landau levels:

\[
\nu = \frac{N}{N (\psi + s) \pm 1},
\]

where the upper sign describes the case of \( B_{\text{eff}} > 0 \), and the lower sign - the case of \( B_{\text{eff}} < 0 \). One can see from Eq. (31) that certain fixed filling factors may correspond to different sets of the \( \psi \) and \( s \) parameters (which describe different phases).

To analyze the incompressible states (which correspond to the filling factors (31)) it is convenient to express the quantities \( \sigma_{xx} \) through the polarization tensor components \( \Pi \):

\[
\sigma_{xx}^+(-) = -\frac{i}{\omega} \Pi_{xx}^+(--) = -\frac{i\omega}{q^2} \Pi_{00}^+-(-),
\]

where \( \Pi_{00}^+(-) = \Pi_{11}^+ \pm \Pi_{12}^+ \), \( \Pi_{11}^+ \), \( \Pi_{12}^+ \) are the diagonal and nondiagonal with respect to the layers components of the polarization tensor. The values of \( \Pi_{00}^+(-) \) are found to be

\[
\Pi_{00}^+(-) = -\frac{e^2 q^2}{2\pi \omega_c} \Delta_+\Delta_-(S_0),
\]

where

\[
S_0 = \Sigma_0 - \frac{m^* - m_b}{m^* N} (\Sigma_0 (\Sigma_2 + N) - \Sigma_2^2),
\]

\[
\Delta_+\Delta_- = \left(1 - (\psi \pm s) \Sigma_1 \right)^2 - (\psi \pm s)^2 \Sigma_0 (\Sigma_2 + N) - \frac{m^* - m_b}{m^* N} F,
\]

\[
F = \Sigma_2 + N + (\omega/\omega_c)^2 S_0,
\]

\[
e^{-x} \sum_{m=0}^{N-1} \sum_{n=0}^{\infty} \frac{\Sigma_j \lambda^j}{m! n!} \left[ x^m x^{n-1} \right] \left( m - n \right) \left( m^* - m_b \right) \left( \omega/\omega_c \right)^2 \left( x \right)^{2j} L_n^{m-n}(x) L_{m-n}^{n-n}(x) \left[ x \right]^d.
\]

In Eqs. (33) \( \omega_c = 2\pi n_0/m^* N \) is the effective cyclotron frequency, \( x = (q \lambda_{\text{eff}})^2/2 \), where \( \lambda_{\text{eff}} = (N/2\pi n_0)^{1/2} \) is the effective magnetic length, \( L_n^{m-n}(x) \) is the generalized Laguerre polynomial, \( m^* \) is the effective mass of the composite fermions, \( m_b \) is the band mass of the electrons, \( n_0 \) is the average electron density. We used the modified random phase approximation [10] for the calculation of \( \Pi_{00}^+(-) \).

Substituting Eqs. (33) into Eq. (28) we find

\[
\frac{\Delta v}{v} = \alpha_+ \frac{E^+_q S_0}{\Delta_+ - E^- q S_0} + \alpha_- \frac{E^- q S_0}{\Delta_- - E^- q S_0},
\]

where \( E^+_q = (e^2 q/\omega_c)(1 \pm \exp(-qd)) \) (the absorption coefficient \( \Gamma \) is equal to zero for the incompressible states).
Let us consider the filling factor \( \nu = 1/5 \). If there is no interlayer interaction, this filling factor corresponds to the parameters \( \psi = 4, s = 0, N = 1 \). When the interlayer distance becomes smaller a transition to the phase \( \psi = 2, s = 2, N = 1 \) may take place. One can see from Eqs. (24-25), the first term in Eq. (38) is not changed under such a transition. Therefore, the value of \( \Delta v \) for the type I mode remains unchanged. On the contrary the jump of the phase velocity takes place for the mode of the type II (in this case the dependence on \( \Delta \psi \) which is the function of \( \psi - s \) survives in Eq. (38)) The jump can be observed under the interlayer distance variation, but it can be hardly realized in experiments. But the effect can be observed indirectly if one measures the dependence of \( \Delta v \) on the wave vector. If one use the elastic mode velocity at \( \nu = 1 \) as the bare value (at \( \nu = 1 \) the bare shift is determined by the same Eq. (38) with the parameters \( m^* = m_e, \psi = s = 0, N = 1 \), the dependences of the relative shift on the wave vector distinguish qualitatively for the cases \( s = 0 \) and \( s \neq 0 \).

Fig. 4 illustrates this behavior. The dependences \( \Delta v/v \) on the inverse wavelength (where \( \Delta v \) is the difference between the velocities at \( \nu = 1/5 \) and \( \nu = 1 \)) are shown. We use the parameters \( n_0 = 10^{11} \text{ cm}^{-2}, \ d = 500 \text{Å}, \ m_b = 0.07 m_e, \ m^* = 4 m_b \) (\( m_e \) is the electron mass) for the calculations.

![Fig. 5](image.png)

**FIG. 5.** Dependence of the velocity shift on the inverse wavelength for the type II mode at the filling factor \( \nu = 1/5 \); 1, the phase \( \psi = 4, s = 0 \); 2, the phase \( \psi = 2, s = 2 \).

One can see from Eq. (25), that the phase transition at certain fixed filling factors may be accompanied by the change of the sign of \( B_{\text{eff}} \). For instance, at \( \nu = 2/7 \) the phase without the interlayer statistical interaction corresponds to the parameters \( \psi = 4, s = 0, N = 2 \) (\( B_{\text{eff}} < 0 \)), while the phase with the interlayer statistical interaction may correspond to the parameters \( \psi = 2, s = 1, N = 2 \) (\( B_{\text{eff}} > 0 \)). The transition between these phases results in a jump of the velocity for the modes of the both types. But the qualitative behavior of \( \Delta v(q) \) remains practically unchanged.

The mode of the type I, for which the maximum of the coupling is shifted to the long wave region, can be used for the indirect observation of the dependence of the effective magnetic length on the filling factor. The dependences \( \Delta \nu(d/l) \) (the value of \( \Delta v \) is calculated relative the velocity for the mode with the same \( l \) at \( \nu = 1 \)) are shown in Fig. 5 at \( \nu = 1/3, 2/5, 3/7 \) (\( \psi = 2 \) and \( N = 1, 2, 3 \) correspondingly). Here specify the case of \( s = 0 \). One can see from Fig. 5 the value of \( \Delta v \) oscillates as the function of \( 1/l \). The period of the oscillations is reduced when \( \nu \) approaches to the 1/2 value. It reflects that the effective magnetic length is increased for this sequence of the filling factors.

![Fig. 6](image.png)

**FIG. 6.** Dependence of the velocity shift on the inverse wavelength for the type I mode; 1, \( \nu = 1/3 \); 2, \( \nu = 2/5 \); 3, \( \nu = 3/7 \).

Thus, the interaction of the double-layer electron system which is realized at the interfaces of the wide quantum well in the AlGaAs—GaAs—AlGaAs heterostructure with nonuniform elastic modes, localized in the central layer of the heterostructure and elliptically polarized in the sagittal plane, is studied theoretically. The dependence of the piezoelectric coupling constant on the ratio between the thickness of the quantum well and the wavelength is found. It is shown, that the coupling constant increases under the decreasing of the wavelength and it reaches the maximal value at the wavelength which is of order of thickness of the GaAs layer (the concrete value of this wavelength is determined by the type of the nonuniform mode). The effect considered can be used for an experimental study of the dynamical properties of two-dimensional electron layers at large wave vectors, for which the interaction of the electrons with the surface acoustic wave is suppressed exponentially due to the finite value of the distance between the electron layer and the surface of the sample. The renormalization of the phase velocity of the nonuniform elastic modes coupled to the double-layer fractional quantum Hall system is calculated. It is shown, that the transition of the Hall system into the state described by the Halperin wave function results in that the dependence of the velocity shift on the wave vector is modified quantitatively for...
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