Type I migration in optically thick accretion discs

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ABSTRACT
We study the torque acting on a planet embedded in an optically thick accretion disc, using global two-dimensional hydrodynamic simulations. The temperature of an optically thick accretion disc is determined by the energy balance between the viscous heating and the radiative cooling. The radiative cooling rate depends on the opacity of the disc. The opacity is expressed as a function of the temperature. We find the disc is divided into three regions that have different temperature distributions. The slope of the entropy distribution becomes steep in the inner region of the disc with the high temperature and the outer region of the disc with the low temperature, while it becomes shallow in the middle region with the intermediate temperature. Planets in the inner and outer regions move outward owing to the large positive corotation torque exerted on the planet by an adiabatic disc, on the other hand, a planet in the middle region moves inward toward the central star. Planets are expected to accumulate at the boundary between the inner and middle regions of the adiabatic disc. The positive corotation torque decreases with an increase in the viscosity of the disc. We find that the positive corotation torque acting on the planet in the inner region becomes too small to cancel the negative Lindblad torque when we include the large viscosity, which destroys the enhancement of the density in the horseshoe orbit of the planet. This leads to the inward migration of the planet in the inner region of the disc. A planet with 5 Earth masses in the inner region can move outward in a disc with the surface density of 100 g/cm² at 1 AU when the accretion rate of a disc is smaller than $2 \times 10^{-8} \, M_\odot/\text{yr}$. 

Key words: hydrodynamics - radiative transfer - planets and satellites: formation - planetary systems: gravitational interactions - planetary systems: accretion disc.

1 INTRODUCTION
Planets are thought to be formed in a protoplanetary disc around a young star. A planet growing by the accretion of planetesimals exchanges the angular momentum with disc gas and moves in the disc (Goldreich & Tremaine 1979, 1980). This is called the Type I migration of a planet. The Type I migration is caused by the torques acting on a planet by a disc. The torque is composed of the Lindblad torque and the corotation torque. The Lindblad torque is exerted by the disc gas at the Lindblad resonances, which are located inside and outside the orbit of the planet. The angular momentum of the planet is increased and decreased by the inner and outer Lindblad torques, respectively. The magnitude of the negative outer Lindblad torque is a little larger than that of the positive inner Lindblad torque (Ward 1986, 1997). The Lindblad torque becomes negative, leading to the inward migration of a planet.

The corotation torque exerted by the disc gas in the horseshoe orbit of a planet depends on the vortensity distribution and the entropy distribution of the disc gas. The vortensity related corotation torque is small compared with the Lindblad torque (Masset & Casoli 2011), on the other hand, the entropy related corotation torque can become larger than the magnitude of the negative Lindblad torque. Paardekooper & Mellema (2006) showed that the corotation torque and the corotation torque determines the total torque acting on the planet.

The Lindblad torque is exerted by the disc gas at the Lindblad resonances, which are located inside and outside the orbit of the planet. The angular momentum of the planet is increased and decreased by the inner and outer Lindblad torques, respectively. The magnitude of the negative outer Lindblad torque is a little larger than that of the positive inner Lindblad torque (Ward 1986, 1997). The Lindblad torque becomes negative, leading to the inward migration of a planet.

The corotation torque exerted by the disc gas in the horseshoe orbit of a planet depends on the vortensity distribution and the entropy distribution of the disc gas. The vortensity related corotation torque is small compared with the Lindblad torque (Masset & Casoli 2011), on the other hand, the entropy related corotation torque can become larger than the magnitude of the negative Lindblad torque. Paardekooper & Mellema (2006) showed that the corotation...
torque becomes positive in a disc with the high opacity, reducing the migration velocity of a planet. This process was recognized and intensively studied by many researchers (Baruteau & Masset 2008; Paardekooper & Mellema 2008; Paardekooper & Papaloizou 2008; Paardekooper et al. 2008; Masset & Casoli 2009; Paardekooper et al. 2010; Ayliffe & Bate 2010, 2011; Paardekooper et al. 2011; Yamada & Inaba 2011). The positive corotation torque cancels the negative Lindblad torque when the entropy distribution has steep negative slope. It was also shown that a planet in an adiabatic disc with the steeper negative slope starts to migrate even outward.

The corotation torque significantly decreases in an adiabatic disc after a few libration time of a planet because the entropy distribution becomes uniform within the horseshoe region (Masset & Casoli 2010; Paardekooper et al. 2011). Without the sustained corotation torque, only the Lindblad torque is eventually exerted on a planet in an adiabatic disc, leading to the inward migration of a planet with Earth mass in $10^5$ yrs (Ward 1997; Tanaka et al. 2002). This is a serious problem in the core accretion model of planet formation. The lifetime of a disc, $10^6-7$ yrs, is much longer than the timescale of the Type I migration, making the survival of planets in a disc difficult. However, recent radial velocity surveys of extrasolar planets show that a significant fraction of solar-type stars may harbor close-in super-Earths (Mayor et al. 2009; Lo Curto et al. 2010; Ségransan et al. 2011). Population synthesis models have great difficulties to reproduce the observed semimajor axis distribution of extrasolar planets once the Type I migration is included. The reduction of the migration velocity of a planet is required (Alibert et al. 2005; Mordasini et al. 2009).

The viscosity of a disc prevents the saturation of the corotation torque (Masset & Casoli 2010). Kley & Crida (2003) investigated the planet-disc interactions in an optically thick accretion disc. It is not clear if the positive corotation torque can always exceed the negative Lindblad torque when we include viscosity in a disc. We make global twodimensional hydrodynamic simulations to study the effect of the dissipation processes such as the viscosity and the radiation on the total torque.

This paper is organized as follows. In the section 2, we describe the basic equations and a disc model we presume in this study. The dissipative terms due to the viscosity and the radiation are included in the basic equations. The temperature of the disc is determined by the energy balance between the viscous heating and the radiative cooling. The rate of the radiative cooling is dependent on the opacity of the disc, which changes around the ice line. In the section 3, we show the results of the two-dimensional hydrodynamic simulations. We show the corotation torque decreases with an increase in the viscosity. The total torque acting on the planet depends on the radiative cooling rate and the viscous heating rate. We further derive the analytical formula to relate the viscosity and the opacity when the total torque exerted on the planet becomes zero. We summarize the results in the section 4.

## 2 BASIC EQUATIONS AND DISC MODEL

### 2.1 Basic Equations

A planet excites density waves in a disc and changes the density distribution of the disc. We examine the torque exerted on a planet by an optically thick accretion disc. A planet with 5 Earth masses rotates around a solar mass star in a fixed circular orbit. The position vector of the planet from the star is denoted by $r_p$. The problem is limited to a two-dimensional flow, where all physical quantities (e.g., the surface density) depend on $r$ and $\theta$, where $r$ is the distance from the star and $\theta$ is the angle between the $x$-axis and the position vector. Governing equations for the gas are the mass conservation, the Navier-Stokes equations, and the energy equation with dissipative terms due to the viscosity and the radiation.

We use a cylindrical coordinate where the star is located at the center of the coordinate. The mass of the planet is much smaller than that of the star and we neglect the indirect term. The mass conservation equation and the Navier-Stokes equations read

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \Sigma \nu_r \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \Sigma \nu_{\theta} v_{\theta} \right) = \Sigma v_r \Phi \frac{\partial \Phi}{\partial r} + f_r, \quad (1)$$

$$\frac{\partial}{\partial t} \left( r \Sigma v_r \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \Sigma v_r^2 + p \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \Sigma \nu_{\theta} v_{\theta} \right) = \frac{\Sigma \nu_r^2}{r} - \frac{p}{r} \Sigma \frac{\partial \Phi}{\partial r} + f_r, \quad (2)$$

where $\Sigma$ is the gas surface density, $p$ is the vertically integrated pressure; $\nu_r$ and $\nu_{\theta}$ are the radial and tangential velocities of the gas; $\Phi$ is the gravitational potential. We consider a less massive disc and neglect the self-gravity of the disc.
gas. The gravitational potential of the star and the planet is given by
\[ \Phi = -\frac{GM_{\odot}}{r} - \frac{GM_p}{\sqrt{r^2 + r_p^2 - 2rr_p\cos\psi + e^2H_p^2}} \] (4)
where \( M_{\odot} \) is the mass of the sun, \( M_p \) is the mass of the planet, \( \psi \) is the angle between \( r \) and \( r_p \), and \( H_p \) is the scale height of the disc at the location of the planet. The scale height is given by
\[ H_p = \frac{\sqrt{2}c_p}{\Omega_p}, \] (5)
where \( c_p \) and \( \Omega_p \) are, respectively, the isothermal sound velocity and the Keplerian angular velocity at \( r_p \). The smoothing length parameter, \( \epsilon \), is introduced to include the effect of the scale height of a disc. It is noted that the small softening parameter leads to the strong corotation torque (Baruteau & Masset 2008; Paardekooper et al. 2010; Yamada & Inaba 2011) examined the torque acting on a planet with several Earth masses in an optically thin disc and found the torques calculated by the simulations with \( \epsilon = 0.3 \) agree with that obtained by a linear analysis. We adopt \( \epsilon = 0.3 \) in this study. The last terms in the Navier-Stokes equations describe the radial and azimuthal components of the viscous forces:
\[ f_r = \frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma_{rr} \right) + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} \frac{\sigma_{\theta\theta}}{r} \] (6)
and
\[ f_\theta = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \sigma_{r\theta} \right) + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta}, \] (7)
where \( \sigma_{rr}, \sigma_{r\theta}, \) and \( \sigma_{\theta\theta} \) are the components of the stress tensor and are, respectively, given by
\[ \sigma_{rr} = 2\Sigma_v \frac{\partial v_r}{\partial r}, \] (8)
\[ \sigma_{r\theta} = \Sigma_v \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right\}, \] (9)
and
\[ \sigma_{\theta\theta} = 2\Sigma_v \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right). \] (10)
The gas is assumed to be ideal and the equation of state is given by
\[ p = \frac{\Sigma k_0 T}{\mu m_H}, \] (11)
where \( k_0 \) is the Boltzmann constant, \( T \) is the mid-plane temperature, \( m_H \) is the mass of a hydrogen atom, and \( \mu \) is the mean molecular weight of the gas. We set up \( \mu = 2.34 \).

The energy equation of the gas disc reads
\[ \frac{\partial (\Sigma e)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r v_r (\Sigma e + p) \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( v_\theta (\Sigma e + p) \right) = \frac{v_r}{r} \frac{\partial p}{\partial r} + \frac{v_\theta}{r} \frac{\partial p}{\partial \theta} + W - Q, \] (12)
where \( e \) is the specific energy of the gas given with the pressure by
\[ e = \frac{p}{(\gamma - 1) \Sigma}. \] (13)
In Eq. (13), \( \gamma \) is the ratio of the specific heats at constant pressure and volume. We use \( \gamma = 4/3 \) for a two-dimensional disc as given by Li et al. (2000). The energy of the gas is increased by the viscous heating term. The viscous heating term, \( W \), is given by
\[ W = \sigma_{rr} \frac{\partial v_r}{\partial r} + \sigma_{r\theta} \left\{ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right\} + \sigma_{\theta\theta} \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right). \] (14)
On the other hand, the energy of the gas is decreased by the radiative cooling term of the disc, \( Q \). The radiative cooling term, \( Q \), is given by
\[ Q = 2 \sigma_{SB} T_{\text{eff}}^4, \] (15)
where \( \sigma_{SB} \) is the Stefan-Boltzmann constant and \( T_{\text{eff}} \) is an effective temperature of a disc. The radiative flux in a radial direction is neglected because it is much smaller than \( Q \) by a factor of \( H_p/r_p \). The effective temperature is related to the mid-plane temperature of the disc as (Hubeny 1990)
\[ T = \frac{T_{\text{eff}}}{T_{\text{mid}}}, \] (16)
with
\[ T_{\text{eff}} = \frac{3\tau}{8} + \frac{\sqrt{3}}{4} + \frac{1}{4\tau}, \] (17)
where \( \tau \) is the optical depth of a disc. The optical depth of a disc is defined by
\[ \tau = \frac{1}{2} \xi_{\text{dust}} \kappa_0 \Sigma, \] (18)
where \( \kappa_0 \) is the best fit function of the opacity (Lin & Papaloizou 1985) and \( \xi_{\text{dust}} \) is the so-called grain content factor introduced by Mizuno (1980). Micron size dust particles are the main source of the opacity. Collisions between particles might produce a large number of small dust particles, increasing the opacity of a disc (Birnstiel et al. 2010). The opacity might decrease due to capture of small dust particles by large particles. It is valuable to study the torque acting on a planet in a disc with various values of \( \xi_{\text{dust}} \). We treat \( \xi_{\text{dust}} \) as a parameter of the simulations and consider the discs with \( 0.1 \leq \xi_{\text{dust}} \leq 100 \).

The opacity strongly depends on the temperature of a disc (Lin & Papaloizou 1982):
\[ \kappa_0 = \begin{cases} 5 \times 10^{-3} T & (210 \text{K} \leq T < 2000 \text{K}), \\ 2 \times 10^{16} T^{-7} & (170 \text{K} \leq T < 210 \text{K}), \\ 2 \times 10^{-4} T^2 & (T < 170 \text{K}). \end{cases} \] (19)
Equation (19) was derived from the available opacity data, which incorporates small particles found in interstellar medium. Dust particles are composed of rocks and metals in a region of disc with \( T \geq 210 \text{ K} \), while ice is added to form dust particles in a region of disc with \( T < 170 \text{ K} \). In the transition region between the cold and the hot region, the ice is evaporating. We divide a disc into three regions: the region 1 with \( T \geq 210 \text{ K} \), the region 2 with \( 170 \text{ K} \leq T < 210 \text{ K} \), and the region 3 with \( T < 170 \text{ K} \).

For the later convenience, we write all the quantities in a non-dimensional form using the unit length \( r_0 = 1 \text{ AU} \), the unit mass \( M_{\odot} \), and the unit time \( \Omega_0^{-1} \) where \( \Omega_0 \) is the Kepler frequency at 1 AU. The normalized quantities are denoted by a tilde (e.g., \( \tilde{r}_p \)).
2.2 Disc Model

Kitamura et al. (2002) obtained images of protoplanetary discs around T Tauri stars in Taurus using the thermal emission of dust. They found that the surface density distributions of protoplanetary discs are described by the power-law distribution with the power-law index of 0-1. The surface densities of the discs at 100 AU are in a range between 0.1 and 10 g/cm². In this study we adopt a disc model, of which the initial condition is given by the power-law distribution:

\[ \Sigma_{ini} = \Sigma_0 \tilde{r}^{-\alpha}, \]  

(20)

where \( \Sigma_0 \) is the surface density at \( r_0 \) and is set to be 100 g/cm².

Kitamura et al. (2002) also found disc radii expand with time, suggesting that most of the gas migrates toward inward a central star by transporting the angular momentum outward. We adopt an accretion disc model developed by Pringle (1981), in which the accretion rate of gas is constant throughout a disc. The viscosity \( \nu \) is expressed by a power-law function of the distance from the central star to satisfy the constant accretion rate of a disc as

\[ \nu = \xi v_0 \tilde{r}^{\alpha}, \]  

(21)

where \( v_0 \) is the kinematic viscosity at \( r_0 \) and we set \( v_0 = 4.2 \times 10^{13} \text{cm}^2/\text{s} \) and \( \xi \) is a viscous strength factor. The value of \( v_0 \) is approximately equivalent to 0.05 in terms of the alpha coefficient of Shakura & Sunyaev (1973). The accretion rate of a disc with the kinematic viscosity, \( v_0 \), and the surface density given by Eq. (20) becomes 6 \times 10^{-8} M_{\odot}/\text{yr}.

Observations of discs suggest that the accretion rates of discs are \( \sim 1 \times 10^{-8} M_{\odot}/\text{yr} \) (Hartmann et al. 1998; Kitamura et al. 2002). We treat \( \xi \) as another parameter in addition to \( \xi_{qr} \) to consider discs with various accretion rates of 0.1 \( \leq \xi \leq 1 \). Two parameters, \( \xi_{qr} \) and \( \xi_v \), determine the structure of a disc.

We derive the temperature distribution of an optically thick accretion disc following Kley & Crida (2008). The temperature distribution of the disc is determined by the balance between the viscous heating term, \( W \), and the radiative cooling term, \( Q \). Assuming that the viscous heating is due to the Keplerian motion, we have \( W \) given by

\[ W = \frac{9}{4} \tilde{r} \Omega^2. \]

(22)

Applying Eqs. (10) - (19) to \( Q \) with the assumption of \( \tau \gg 1 \), we obtain the temperature distribution of the disc from \( W = Q \) as

\[ T = \begin{cases} 210 \left( \frac{r_{12}}{1.4} \right)^{-(\alpha+3)/3} & (210 \text{ K} \leq T < 2000 \text{ K}), \\ 210 \left( \frac{r_{12}}{1.4} \right)^{-(\alpha+3)/11} & (170 \text{ K} \leq T < 210 \text{ K}), \\ 170 \left( \frac{r_{23}}{1.4} \right)^{-(\alpha+3)/2} & (T < 170 \text{ K}), \end{cases} \]  

(23)

where \( r_{12} \) and \( r_{23} \) are the distances from the star to the boundary of the regions 1 and 2 and that of the regions 2 and 3, respectively. For later convenience, the power-law indexes of the temperature distribution in the \( r \)-region are denoted by \( -\beta_i \) (e.g., \( \beta_1 = (\alpha+3)/3 \)).

Figure 1 shows the surface density distribution and the temperature distribution of the disc with \( \alpha = 1.0 \). It is seen from this figure that the temperature distribution has three different slopes. The boundary between the regions 1 and 2 and that between the regions 2 and 3 are located at 1.4 AU and 2.5 AU, respectively. The temperature distribution is nearly flat in the region 2 (\( \beta_2 = 4/11 \)). The power-law indexes of the temperature distributions are given by \( \beta_1 = 4/3 \) in the region 1 and \( \beta_3 = 2.0 \) in the region 3.

The larger viscosity and/or opacity increase the temperature of the disc. The boundary positions move outward with increases in the viscosity and/or opacity. The location of the boundary is expressed by the two parameters, \( \xi_{qr} \) and \( \xi_v \), and the boundary position between the regions 1 and 2 is given by

\[ r_{12} = 1.4 \left( \frac{\xi_v \xi_{qr}}{1.0} \right)^{1/4}. \]  

(24)

The boundary position between the regions 2 and 3 is given by \( r_{23} = 1.8r_{12} \) in the case of \( \alpha = 1.0 \). This boundary positions were derived by Hasegawa & Puurhtiz (2011) as well. It is noted that the boundary position also depends on the surface density. We plot the boundary positions of the regions 1 and 2, \( r_{12} \), as a function of \( \xi_{qr} \) and \( \xi_v \) in Fig. 2. The solid, dashed, dot-dashed, and dotted curves correspond to \( r_{12} = 2.4, 1.4, 1.2, \) and 0.8, respectively. It is found that the boundary position between the regions 1 and 2 ranges from 0.5 AU to 2.5 AU. The filled circles represent the parameter sets \( (\xi_{qr}, \xi_v) \) used in our numerical simulations.

3 NUMERICAL METHOD AND SIMULATION RESULTS

A planet generates density waves in a disc inside and outside of the orbit of the planet. The inner and outer density waves exert the positive and negative torques on the planet, respectively. The gravity of an outer density wave is a little stronger than that of an inner density wave because an outer density wave is closer to the planet due to the pressure gradient of a disc, leading to the negative Lindblad torque. Ward (1997). Another torque acts on the planet by a gas element in the horseshoe orbit of the planet (Baruteau & Masset 2008; Paardekooper & Papaloizou 2008). When a gas element in the horseshoe orbit approaches a planet, the angular momentum is exchanged between the gas element and the planet. This is called the corotation torque.

The entropy of the gas is expressed as \( S = p/\Sigma^\gamma \propto r^\lambda \), where \( \lambda = (\gamma - 1)\alpha - \beta_i \). The corotation torque is dependent on the entropy distribution of a disc. It was shown that the corotation torque consists of linear and non-linear corotation torque. The non-linear corotation torque comes from the outgoing boundaries of the horseshoe region, that is the separatrix (Masset & Casoli 2010; Paardekooper et al. 2010). In a non-barotropic disc, the vortensity is changed after the horseshoe U-turn. The enhanced corotation torque is not due to the adiabatic compression, but comes from a singular streamline at the separatrix. Since the change of the vortensity is proportional to the radial entropy gradient of a disc, the large corotation torque is induced in a disc with the large magnitude of \( \lambda \). Yamada & Inaba (2011) showed that the positive corotation torque is comparable with the negative Lindblad torque when \( \lambda = -0.4 \) in an adiabatic disc. The total torque acting on a planet is determined by the sum of the negative Lindblad torque and the corotation torque.

The disc we consider in the section 2.2 has the surface
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3.1 Numerical Method

We use two-dimensional equidistant grids in r and θ with a resolution of \((N_r, N_θ) = (640, 3072)\). The inner and outer radii of the disc are given by \(r_{\text{min}}/r_p = 0.4\) and \(r_{\text{max}}/r_p = 2.0\), respectively. The damping boundary conditions (de Val-Borro et al. 2006), where all components are relaxed towards their initial state, are used in order to reduce wave reflection from these boundaries. All the quantities in the inner and outer boundaries are always fixed to be the initial values.

We develop a two-dimensional global hydrodynamic computer program with the gravitational forces of a star and a planet and a dissipative term (Yamada & Inaba 2011). The basic equations are solved simultaneously using the finite volume method with an operator splitting procedure. The source terms, which include the gravity, the viscous force, and the radiation, are computed with a second order Runge-Kutta scheme. The advection terms are calculated with a second order MUSCL-Hancock scheme and an exact Riemann solver (Toro 1999; Inaba et al. 2005).

The angular momentum of the disc gas is transferred to the planet. The transfer rate of the angular momentum from the disc gas at \(r\) to the planet is given by

\[
\Gamma_r = \int_0^{2\pi} \Sigma \frac{∂Φ}{∂θ} r dθ.
\]

By integrating the torque density over the radial distance, we obtain the total torque acting on the planet from the disc:

\[
\Gamma = \int_{r_{\text{min}}}^{r_{\text{max}}} \Gamma_r dr.
\]

The torque and the torque density are normalized by \(\Gamma_0\) and \(\Gamma_0/r_p\), where \(\Gamma_0 = (M_p/M₁)²(r_p/H_p)²\Sigma pr_p²Ω_p²\).

3.2 Simulation Results

3.2.1 The torque density acting on a planet in an optically thick accretion disc

We consider two adiabatic discs that have the temperature distributions with the power-law indexes 4/3 and 4/11, which correspond to that of the region 1 and the region 2 of the optically thick disc, respectively. The power-law index of the temperature distribution is expressed as \(-\beta\). Both discs have the initial surface density with \(\alpha = 1.0\). Hereafter we call the adiabatic disc with \((\alpha, \beta) = (1.0, 4/3)\) and that with \((\alpha, \beta) = (1.0, 4/11)\) the disc A and the disc B, respectively. The power-law index of the entropy distribution of the disc A is \(\lambda = -1.0\) and that of the disc B is \(\lambda = -3.0 \times 10^{-2}\). Yamada & Inaba (2011) found that the corotation torque increases with a decrease in the power index of the entropy distribution in an adiabatic disc.

The initial surface density with \(\alpha, \beta\) is the Earth mass and \(H_p\) is the scale height of the disc at \(r = r_p\):

\[
\frac{H_p}{r_p} = 4.8 \times 10^{-2} \left(\frac{r_p}{1.4}\right)^{(1-\beta)/2}.
\]

where \(M_E\) is the Earth mass and \(H_p\) is the scale height of the disc. Figure 6 shows the radial distribution of the torque density exerted on the planet in the adiabatic discs at \(t = 25t_p\), where \(t_p\) is the rotational period of the planet. The planet is located at 1 AU in the disc A and 2 AU in the disc B. The large corotation torque is exerted on the planet in the disc A due to the large entropy gradient, making the total torque positive. On the other hand, the corotation torque in the disc B is too weak to cancel the negative Lindblad torque. The normalized total torques become 3.0 and -2.4 in the disc A and the disc B, respectively.

The half width of the horseshoe region, \(δx_p\), is estimated as \(\delta x_p = 2.1 \times 10^{-3} \sqrt{\left(\frac{M_p}{M_E}\right) \left(\frac{r_p}{H_p}\right)}\).

(27)

where \(M_E\) is the Earth mass and \(H_p\) is the scale height of the disc at \(r = r_p\):

\[
\frac{H_p}{r_p} = 4.8 \times 10^{-2} \left(\frac{r_p}{1.4}\right)^{(1-\beta)/2}.
\]

Masset et al. (2006) showed that the half width of the horseshoe region of a planet scales with \(M_E^{1/2}\) as long as the flow around a planet remains linear. Figure 7 shows the half width of the horseshoe region given by the numerical simulations (the filled circles) together with the half width of the horseshoe region given by Eq. (27) (the curve). Both half widths agree well except the large planet mass. Masset et al. (2006) found the planet mass when the flow linearity breaks in a 2D simulation. The non-linearity of the flow around a planet becomes clear when the mass of the planet becomes larger than 10 Earth masses.

The ratios of the magnitude of the corotation torque to that of the Lindblad torque are 1.6 and 0.6 in the disc A and the disc B, respectively. Nearly the same Lindblad torques act on the planet in both discs. The corotation torque determines the magnitude and sign of the total torque exerted on the planet.

We perform the simulation to find the torque density exerted on the planet in the optically thick accretion disc with \((ξ_0, ξ_s) = (10, 0.1)\). The temperature distribution is the same as that of Fig. 1. The boundary position between the regions 1 and 2 is 1.4 AU and that between the regions 2 and 3 is 2.5 AU. Figure 8 shows the radial distributions of the torque density acting on the planets in the region 1 (panel (a)) and in the region 2 (panel (b)) at \(t = 25t_p\). The planets are located at 1 AU in the region 1 and 2 AU in the region 2. The radial distribution of the torque density in the region 1...
is similar to that in the disc A. The torque density has a local maximum and a local minimum at $|1 - r/r_p| = 3.3 \times 10^{-2}$. The distances from the planet to the local maximum and minimum can be approximated by the scale height of the disc in the region 1, indicating that this torque corresponds to the Lindblad torque.

The flow pattern within the horseshoe region is modified by the viscosity and the vortensity is no longer conserved during the U-turn. The vortensity is radially transferred by the viscosity. We compare the vortensity distributions of the inviscid disc, that is the adiabatic disc, and the accretion disc with non-vanishing viscosity. Figure 6a shows the contour of the vortensity within the horseshoe region of the planet. It is seen from Fig. 6 that the location of the minimum vortensity moves inward toward the central star when the viscosity is included. The viscosity plays an important role to diffuse the vortensity. In a disc with the low viscosity, the corotation torque is dominated by the non-linear corotation torque. The non-linear corotation torque decreases with the increasing viscosity.

The radiation restores the entropy distribution of a disc and is important to prevent the saturation of the corotation torque. Masset & Casoli (2010) derived the analytical expression of the corotation torque given by Eq. (27) in Fig. 7. We remove the contribution of thermal diffusivity in the analytical expression. Our results agree well with the analytical result. In this study, the heating only works to maintain the thermal structure of a disc. Our numerical results are compared with the analytical expression of the corotation torque given by Eq. (27) in Fig. 7. We remove the contribution of thermal diffusivity in the analytical expression. Our results agree well with the analytical result. In this study, the radiation affects the corotation torque exerted on a planet through the disc temperature profile. However, it is important to note that radiation significantly influences the corotation torque as pointed out by Masset & Casoli (2010).

The timescale for the gas to spread across the horseshoe width by the viscosity is defined by

$$\tau_{\text{visc}} = \frac{\delta x^2}{3\nu}. \quad (29)$$

We substitute Eqs. (24) and (27) into Eq. (29) to estimate the viscous timescale for the disc with $\xi_v = 0.1$ at $r_p = 1$ AU:

$$\tau_{\text{visc}} = 16 \left( \frac{\xi_v}{0.1} \right)^{-1} \left( \frac{H_p/r_p}{0.05} \right)^{-1} \left( \frac{M_p}{5M_0} \right) \Omega_0^{-1}. \quad (30)$$

We consider the pressure effect of the gas disc and obtain the turnover time along the horseshoe orbit in front of the planet at $r_p = 1$ AU (Baruteau & Masset 2008):

$$\tau_{\text{turn}} = 12 \left( \frac{H_p/r_p}{0.05} \right)^{3/2} \left( \frac{M_p}{5M_0} \right)^{-1/2} \Omega_0^{-1}. \quad (31)$$

The viscous timescale is nearly equal to the turnover time in the disc with $\xi_v = 0.1$. The effect of the viscosity on the corotation torque then starts to be important. Note that the cooling timescale by the radiation is the same with the viscous timescale in the disc.

The torque density near the orbit of the planet in the region 2 is smaller than that in the disc B. Morohoshi & Tanaka (2003) used a local shearing box calculation and studied the gravitational interactions between a planet and an optically thin disc, taking into account of energy dissipation by radiation. They found that the oval shape of the density contour profile near a planet is tilted with respect to the direction toward the central star. The asymmetry of the density structure increases the one-side torque. They showed that the density contour is less inclined with the decreasing opacity. The adiabatic disc can be considered to have much larger opacity than the region 2, yielding the larger torque density near the planet.

Paardekooper et al. (2011) have derived the torque formula for the Lindblad torque and the corotation torque exerted on a planet by a disc with viscosity and thermal diffusion. We find that the corotation and Lindblad torques obtained in our simulation agree with those calculated from their torque formula within 30%. The total torque is given by the sum of the Lindblad torque and the corotation torque. The total torque becomes $\tilde{\Gamma} = 2.1$ in the region 1 and it is slightly smaller than that in the disc A, $\tilde{\Gamma} = 3.0$. The density enhancement is reduced as the vortensity smears by the viscosity. However, the viscosity is too small to remove the corotation torque. Planets accumulate at the ice line as suggested by Hasegawa & Pudritz (2011).

We further examine the effect of the viscosity of a disc on the torque density exerted on the planet, using the large value of the viscosity, $\xi_v = 1.0$. We utilize the same surface density distribution and the temperature distribution given by Fig. 1. Figure 8 shows the radial distribution of the torque density exerted on the planet in the region 1 at $t = 25t_p$. The corotation torque is lost because the viscosity is large enough to smear the density enhancement. In this case, the viscous timescale becomes much shorter than the turnover time (see Eqs. (29) and (31)). The total torque becomes negative ($\tilde{\Gamma} = -2.4$), leading to the inward migration of the planet in the region 1.

The density contours around the planet are shown to make clear the effect of the viscosity on the torque density. Figure 3 shows the contours of the density fluctuations, $\Sigma(t = 25t_p)/\Sigma_{\text{ini}} - 1.0$, around the planet (a) in the disc A, (b) in the region 1 of the disc with $(\xi_{gr}, \xi_v) = (10.0, 1.0)$, and (c) in the region 1 of the disc with $(\xi_{gr}, \xi_v) = (1.0, 1.0)$. The enhancement of the density in the horseshoe orbit in the panel (b) is similar to that in the panel (a). On the other hand, the density in the horseshoe region is considerably reduced in the panel (c) because of the larger viscosity. The corotation torque decreases because the viscous timescale is much shorter than the turnover time in the case of $\xi_v = 1.0$. Moreover, the inner and outer spiral density waves are damped by the viscosity as well. This results in the small Lindblad torque.

### 3.2.2 Dependence of the total torque on the opacity and the viscosity of a disc

The corotation torque decreases in an adiabatic disc because the entropy of the gas in the horseshoe orbit of the planet tends to become uniform after the synodical period of the planet (Baruteau & Masset 2008; Paardekooper & Papaloizou 2008, 2009a). This is called the saturation of the corotation torque and happens in an adiabatic disc. The corotation torque does not saturate in a viscous disc because the vortensity is transferred from the horseshoe region to the outer region of the disc by viscosity
Figure [11] shows the time evolutions of the total torque acting on the planet in the adiabatic disc (the disc A) and in the optically thick accretion disc with $(\xi_{gr}, \xi_v) = (10, 0.1)$ (the region 1), which are represented by the solid and dotted curves, respectively. The total torque increases with time at the beginning of the simulation and reaches a steady state around $t = 10\tau_p$. The total torque increases in the adiabatic disc by the saturation because we consider the planet in a disc with the optically thick accretion disc. We do not have to worry about the saturation because we consider the planet in a disc with the viscosity.

We examine the effect of the boundary of the regions of the disc on the total torque by changing the position of the planet. Figure [11] shows the total torques acting on the planet at different positions in the optically thick accretion discs. The filled circles and triangles correspond to the total torques on the planet in the discs with $(\xi_{gr}, \xi_v) = (10, 0.1)$, respectively. The total torque on the planet in the region 1 is nearly independent of $\tilde{r}_p$. The effect of the boundary of the regions on the total torque is very small. The total torque in the region 2 decreases a little with $\tilde{r}_p$. The analytical formula for the normalized corotation torque [Paardekooper et al. 2011] depends on the disk scale height and the viscous coefficient, and the corotation torque decreases with increasing $\tilde{r}_p$ in the region 2. This agrees with our numerical results.

We perform a number of numerical simulations of gravitational interactions between the planet and the optically thick accretion discs. Figure [12] shows the magnitude and sign of the total torques exerted on the planet in the region 1 of the discs with various parameters of $\xi_{gr}$ and $\xi_v$. The planet is located at 1 AU when $\tilde{r}_{12} = 2.4$, 1.4, and 1.2, and at 0.7 AU when $\tilde{r}_{12} = 0.8$, respectively. The open and filled marks denote the positive and negative total torques acting on the planets, respectively. We obtain the largest magnitude of the total torques, $|\Gamma|_{\text{max}}$, from the simulations with the same boundary position of $\tilde{r}_{12}$. The total torques are plotted as circles, squares, and triangles when $|\Gamma| \geq 0.5|\Gamma|_{\text{max}}$, $0.5|\Gamma|_{\text{max}} > |\Gamma| > 0.1|\Gamma|_{\text{max}}$, and $0.1|\Gamma|_{\text{max}} > |\Gamma|$, respectively. The total torque increases with decreasing $\xi_{gr}$ and $\xi_v$ and its sign changes from negative to positive.

We find the small decrease in the corotation torque when the viscous timescale is nearly equal to the turnover time as shown in Fig. 5. The corotation torque on the planet decreases with the viscosity of the disc and vanishes when the viscous timescale becomes much shorter than the turnover time as shown in Fig. 8. We find that the positive corotation torque cancels the negative Lindblad torque to have the zero total torque when $\tau_{\text{visc}} \approx 0.5\tau_{\text{turn}}$. We use the temperature distribution of the region 1 of the disc to find the viscous timescale:

$$\tau_{\text{visc}} = 1.6 \left( \frac{\xi_{gr}}{1.0} \right)^{-1/6} \left( \frac{\xi_v}{1.0} \right)^{-7/6} \left( \frac{M_p}{5M_E} \right)^{2/3} \Omega_p^{-1}. \quad (32)$$

Using $\tau_{\text{visc}} = 0.5\tau_{\text{turn}}$, we obtain

$$\xi_{gr} = 3.8 \times 10^{-2} \tilde{r}_p^{11/5} \left( \frac{M_p}{5M_E} \right)^{18/5} \xi_v^{-17/5}. \quad (33)$$

The dashed and dot curves in Fig. [12] are drawn using Eq. (33). We consider the two positions of the planet, $\tilde{r}_p = 1.0$ and 0.7. Substituting $\tilde{r}_p = 1.0$ and 0.7 into Eq. (33), we draw the dashed and dotted curves, respectively. The whole region is separated into the two regions by the curves. The positive total torque and the negative total torque can be found in one of the regions. We conclude that the corotation torque is moderately damped by the viscosity when $\tau_{\text{visc}} \approx 0.5\tau_{\text{turn}}$ and cancels the negative Lindblad torque. The opacity of the disc increases with $\xi_{gr}$. Energy dissipates ineffectively in the disc with large $\xi_{gr}$, increasing the mid-plane temperature and the scale height. The viscous timescale of the disc with large $\xi_{gr}$ decreases because the width of the horseshoe region decreases with the increasing scale height (see Eq. (27)). In the disc with large $\xi_{gr}$, the viscosity quickly damps the corotation torque and the negative Lindblad torque becomes dominant. Moreover, it is valuable to mention the effect of the surface density at 1 AU, $\Sigma_0$, on Eq. (33). In this study, we set $\Sigma_0$ to be constant. Since the accretion rate and the optical depth are proportional to the surface density, $\tau_{\text{visc}}$ and $\tau_{\text{turn}}$ also increases with $\Sigma_0$. Hence, the coefficient of $\xi_v$ on the right-hand side of Eq. (33) changes by $\Sigma_0$.

Additionally, one can find from Eq. (33) that the planet mass has a large effect on the threshold at which the total torque becomes zero. We examine the effect of the planet mass on the total torque. Figure [13] shows the total torque exerted on the planet with 3 Earth masses and that with 7.5 Earth masses in the panels (a) and (b), respectively. The planet is located at $\tilde{r}_p = 1.0$ in both cases. The half width of the horseshoe region increases with the planet mass. It takes longer time to damp the density enhancement in the horseshoe region of the planet with large mass. The larger viscosity is required to reduce the corotation torque acting on the massive planet.

4 SUMMARY

We have studied the Type I migration of a planet in an optically thick accretion disc. The gravitational interactions between a planet and disc gas excite spiral density waves inside and outside of the orbit of the planet in the disc. The waves attract the planet gravitationally and exert torques on the planet. The negative torque by the outer density wave is a little larger than the positive torque by the inner density wave because the outer wave is closer to the planet than the inner wave due to the negative pressure gradient. The sum of the torques by the density waves (the Lindblad torque) becomes negative, leading to the inward migration of the planet [Ward 1997].

The corotation torque is very important to planetary migration because it might be able to halt the inward migration or reverse its direction [Baruteau & Masset 2008; Paardekooper & Papaloizou 2008]. In a non-barotropic disc, it is found that the entropy related non-linear corotation torque plays an essential role. The non-linear corotation torque comes from the density enhancement due to the generated vortensity at the outgoing separatrices [Masset & Casoli 2010; Paardekooper et al. 2010]. The entropy related corotation torque is proportional to the radial gradient of the entropy of a disc. A disc with the steep entropy gradient yields the large positive corotation torque. [Yamada & Inaba 2011] showed that the positive corota-
tion torque cancels the negative Lindblad torque when the power-law index of the entropy distribution of the disc becomes \( \lambda = -0.4 \), that is the critical power-law index of the entropy distribution.

The radiation from the central star cannot reach the mid-plane of an optically thick disc. The temperature of the mid-plane of an optically thick accretion disc is determined by the energy balance between the viscous heating and the radiative cooling. The rate of the cooling by the radiation is sensitive to the opacity of the disc. Micron size dust particles provide the sources of the opacity. The opacity changes with the temperature through the sublimation of ice. We utilize the opacity law frequently used in other researches and find that the disc is divided into three regions: from the region 1 with the highest temperature to the region 3 with the lowest temperature. The boundary positions of the regions are dependent on the viscosity and the opacity of the disc. The boundary between the regions 1 and 2 is located between 0.5 AU and 2.5 AU.

Each region has the different power-law indexes of the temperature and entropy distributions. The power-law indexes of the entropy distribution in the regions 1 and 3 are lower than the critical power-law index of \(-0.4\), while the power-law index in the region 2 is higher. We found that the total torque exerted on the planet by the adiabatic disc becomes positive in the regions 1 and 3 and becomes negative in the region 2, as expected. This means that the planet moves outward in the regions 1 and 3, while it moves inward toward the central star in the region 2. Planets might accumulate at the boundary between the regions 1 and 2 in the adiabatic disc.

Dissipative processes change the magnitude of the corotation torque \(^{[11]}\) Kley & Crida 2008, Paardekooper & Papaloizou 2008, 2009, Yamada & Inaba 2011). In a viscous disc, the vortensity is transferred radially as shown in Fig. 6. The viscosity is responsible for the decrease in the corotation torque. The radiation has an impact on the evolution of the entropy as well and keeps the entropy gradient within the horseshoe region. The corotation torque depends on these dissipative processes due to the viscosity and the radiation \(^{[12]}\) Masset & Casoli 2010). The positive total torque exerted on the planet by an adiabatic disc might become negative once some dissipative processes are included. We include the viscosity and radiation into the adiabatic disc and calculate the total torque exerted on the planet in an optically thick accretion disc. We focus on the planet in the regions 1 and 2. The magnitudes of the opacity and the viscosity are expressed with the two parameters, \( \xi_{gr} \) and \( \xi_{v} \), respectively. The opacity and the viscosity increase with \( \xi_{gr} \) and \( \xi_{v} \). The large mass accretion rate of the disc is found in the disc with large \( \xi_{v} \). The temperature of the disc increases when the energy of the disc dissipates ineffectively (large \( \xi_{gr} \)), resulting in the narrow horseshoe region. The viscosity decreases the enhancement of the density in the horseshoe region. The corotation torque gets smaller when the viscosity is larger. The total torque exerted on the planet by the optically thick accretion disc depends on the two parameters.

We have made a number of numerical simulations of gravitational interactions between the planet and the optically thick accretion disc with various parameter sets of \( \xi_{gr} \) and \( \xi_{v} \). The total torque always becomes negative in the region 2, leading to the inward migration of the planet. The total torque becomes positive in the region 1, if the effect of the dissipation is small. The dissipative processes work effectively and the total torque becomes zero when the timescale for the viscosity is half of the turnover time of the planet in the horseshoe orbit, \( \tau_{visc} = 0.5\tau_{turn} \). This equation is written with the parameters as \( \xi_{v} = 0.4\xi_{gr}^{1/17}(M_{p}/5M_{E})^{16/17}(\Sigma_{0}/100g/cm^2)^{2/17}\xi_{gr}^{-5/17} \), considering \( \Sigma_{0} \) as a parameter. In the optically thick accretion disc with \( \xi_{gr} \approx 1.0 \) and the surface density of 100 g/cm\(^2\) at 1 AU, the accretion rate of the disc is required to be smaller than \( 2.1 \times 10^{-6}M_{E}/yr \) for the planet to move outward. Our study suggests that planets with 5 Earth masses might accumulate and drive further growth of the planets at the boundary between the regions 1 and 2 in the optically thick accretion disc. The small accretion rate of gas is required for small planets to move outward.

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Type I migration in optically thick accretion discs

We examine the dependence of the total torque on the power-law index, \( \alpha \), of the surface density distribution of the disc. Figure A.1 shows the time evolutions of the total torque exerted on the planet by the disc with \( \alpha = 1.0 \) and the disc with \( \alpha = 0.5 \). Both discs have parameters of \((\xi_{\text{gr}}, \xi_{v}) = (10, 0.1)\). The solid and dashed curves represent the total torques acting on the planet by the discs with \( \alpha = 1.0 \) and \( \alpha = 0.5 \), respectively. The total torques increase at the beginning of the simulations and eventually reach steady states in both discs. The total torque by the disc with \( \alpha = 0.5 \) becomes approximately 1.5 times as large as that by the disc with \( \alpha = 1.0 \).

The torque densities by the disc with \( \alpha = 1.0 \) and by the disc with \( \alpha = 0.5 \) are plotted as the solid and dashed curves, respectively, in Fig. A.2. The inner and outer Lindblad torques increase and decrease with a decrease in \( \alpha \), respectively, leading to the smaller Lindblad torque. The analytical formula for the Lindblad torque, \( \tilde{\Gamma}_L \), is given by (Paardekooper et al. 2011)

\[
\gamma \tilde{\Gamma}_L = -2.5 - 1.7\beta_i + 0.1\alpha. \quad (A1)
\]

The temperature distribution of the region 1 of the optically thick accretion disc has the power-law index, \( \beta_1 = (3 + \alpha)/3 \). Hence, we have for the Lindblad torque in the region 1:

\[
\gamma \tilde{\Gamma}_L = -4.2 - 0.47\alpha. \quad (A2)
\]

\( \alpha \) ON THE TOTAL TORQUE ON A PLANET

We derive the relation between \( \xi_{\text{gr}} \) and \( \xi_v \) when the total torque becomes zero, using the same procedure explained in the section 3.2. Using \( \tau_{\text{visc}} = 0.5\tau_{\text{turn}} \), we have

\[
\xi_{\text{gr}} = 3.8 \times 10^{-2} \tilde{r}_{17}^{17/10} \left( \frac{M_p}{5M_E} \right)^{18/5} \left( \frac{\Sigma_0}{100g/cm^2} \right)^{7/5} \xi_v^{-17/5}. \quad (A3)
\]
Figure 1. (a) the surface density distribution and (b) the temperature distribution of the disc with $\alpha = 1.0$. The temperature distribution of the disc is determined by the balance between the viscous heating and the radiative cooling. The temperature distribution is described by the power-law distributions with three different gradients. The power-law index of the temperature distribution strongly depends on the opacity of the disc. The dependence of the opacity on the temperature changes due to the evaporation of ice. The power-law indexes of the temperature distribution change at 1.4 AU and 2.5 AU.

Figure 2. The boundary position between the regions 1 and 2, $\tilde{r}_{12}$, as a function of $\xi_{gr}$ and $\xi_v$. The solid, dashed, dot-dashed, and dotted lines correspond to $\tilde{r}_{12} = 2.4$, 1.4, 1.2, and 0.8, respectively. We use disc models with 20 parameter sets ($\xi_{gr}, \xi_v$) marked by the filled circles.
Figure 3. The radial distributions of the torque density acting on the planet with 5 Earth masses by the adiabatic disc at $t = 25t_p$. The power-law indexes of the temperature distributions of the disc are $\beta = 4/3$ and $\beta = 4/11$ in the panels (a) and (b), respectively. The planets are located at 1 AU and 2 AU in the panels (a) and (b), respectively. The horizontal axis is normalized by $r_p$. The strong corotation torque is found in the panel (a), leading to the outward migration of the planet.

Figure 4. The half width of the horseshoe region as a function of the planet mass. The horizontal axis is normalized by the Earth mass. The optically thick disc with the accretion rate of $\xi_v = 0.1$ is used. The numerical half width of the horseshoe region (filled circles) is evaluated at $t = 25t_p$. The planet is located at $\tilde{r}_p = 1.0$. The curve is drawn using the analytical expression for the half width of the horseshoe region given by Paardekooper et al. (2010).
Figure 5. Same as Fig. 3, but the optically thick accretion disc with \((\xi_{gr}, \xi_v) = (10, 0.1)\). The normalized total torque acting on the planet is 2.1 in the panel (a), while it becomes \(-1.5\) in the panel (b). The planet migrates outward in the region 1 and inward in the region 2. Planets might accumulate at the boundary of the regions.

Figure 6. Contour of the vortensity in (a) the inviscid adiabatic disc (disc A) and (b) the accretion disc with the non-vanishing viscosity \((\xi_v, \xi_{gr})=(0.1, 10)\) at \(t = 10t_p\). The initial distribution of vortensity is subtracted from the vortensity for better contrast. The horizontal and vertical axes correspond to \(r/r_p\) and \(\theta/\pi\), respectively. The planet with 5 Earth masses is located at 1 AU.
Figure 7. The corotation torques acting on the planet with 5 Earth masses given by the analytical expression of Masset & Casoli (2010) (solid line) and the numerical results (filled circles). The horizontal and vertical lines represent the viscous strength factor, $\xi_v$, and the normalized corotation torque, $\tilde{\Gamma}_c$, respectively. In deriving the numerical corotation torque, we used Eq. (27) for the half width of the horseshoe region. The planet is located at 1 AU.

Figure 8. The radial distribution of the torque density acting on the planet with 5 Earth masses by the region 1 of the disc with $(\xi_{gr}, \xi_v) = (1.0, 1.0)$. The planet is located at 1 AU. The total torque acting on the planet becomes negative, $\tilde{\Gamma} = -2.4$. Planets move inward toward the central star even in the region 1.
Figure 9. The contour plots of the surface density enhancement $\Sigma(t = 25t_p)/\Sigma_{ini} - 1.0$ around the planet with 5 Earth masses (a) in an adiabatic disc (disc A), (b) in the region 1 of the disc with $(\xi_{gr}, \xi_v) = (10, 0.1)$, and (c) in the region 1 of the disc $(\xi_{gr}, \xi_v) = (1.0, 1.0)$, respectively. The horizontal axis is normalized by $r_p$ and the vertical axis is divided by $\pi$. The planet is located at $(r, \theta) = (r_p, \pi)$. The density contour in the panel (b) is similar to that in the panel (a) because the viscosity is too small to decrease the density enhancement. On the other hand, the density enhancement is greatly reduced by the viscosity in the panel (c).
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Figure 10. The time evolution of the total torque acting on the planet with 5 Earth masses in the adiabatic disc (solid curve) and in the region 1 of the disc with \((\xi_{gr}, \xi_v) = (10, 0.1)\) (dashed curve). In both cases, the planet is located at 1 AU. The total torque decreases around \(t = 25t_p\) in the adiabatic disc due to the saturation of the corotation torque. On the other hand, the positive total torque is maintained in the disc with \((\xi_{gr}, \xi_v) = (10, 0.1)\).

Figure 11. The total torques exerted on the planet by the disc with \((\xi_{gr}, \xi_v) = (1.0, 1.0)\) (filled circle) and \((10, 0.1)\) (filled triangle) as a function of \(\tilde{r}_p\). The boundary position between the regions 1 and 2 is shown by the dotted line. In the region 1, the total torque seems to be nearly independent of \(\tilde{r}_p\). In the region 2, the total torque decreases a little with increasing \(\tilde{r}_p\).

Figure 12. The diagram of the total torques exerted on the planet with 5 Earth masses by the disc with various parameter sets of \((\xi_{gr}, \xi_v)\). The open and filled marks correspond to the positive and negative total torques acting on the planets, respectively. We find the largest magnitude of the total torques, \(|\tilde{\Gamma}|_{\text{max}}\), from the simulations with the same boundary position of \(\tilde{r}_{12}\). The total torques are plotted as circles, squares, and triangles when \(|\tilde{\Gamma}| > 0.5|\tilde{\Gamma}|_{\text{max}}\), \(0.5|\tilde{\Gamma}|_{\text{max}} > |\tilde{\Gamma}| > 0.1|\tilde{\Gamma}|_{\text{max}}\), and \(0.1|\tilde{\Gamma}|_{\text{max}} > |\tilde{\Gamma}|\), respectively. The dotted and dashed curves are drawn using \(\tau_{\text{visc}} = 0.5\tau_{\text{turn}}\) when \(\tilde{r}_p = 0.7\) and 1.0, respectively. It is noted that the curves are truncated because \(\tilde{r}_{12}\) is smaller than \(\tilde{r}_p\). The sign of the total torque changes when \(\tau_{\text{visc}} \simeq 0.5\tau_{\text{turn}}\). The planet with 5 Earth masses moves outward in the region 1 of the optically thick accretion disc with the surface density of 100 g/cm\(^2\) at 1 AU when the accretion rate is smaller than \(2.1 \times 10^{-8}M_\odot/\text{yr}\). The planets are then expected to accumulate at the boundary of the regions 1 and 2 in the optically thick disc.
Figure 13. Same as Fig. 12, but the planet with (a) 3 Earth masses and (b) 7.5 Earth masses. The open and filled marks correspond to the positive and negative total torques, respectively. The dotted curve is drawn using $\tau_{\text{visc}} = 0.5\tau_{\text{turn}}$ when $\tilde{r}_{12} = 1.0$. The planets with 3 and 7.5 Earth masses moves outward in the region 1 of the optically thick accretion disc with the surface density of 100 g/cm$^2$ at 1 AU when the accretion rate becomes smaller than $1.2 \times 10^{-8}$ and $3.2 \times 10^{-8} M_{\odot}/\text{yr}$, respectively.

Figure A1. The time evolution of the total torque exerted on the planet with 5 Earth masses by the disc with $(\xi_{\text{gr}}, \xi_{\text{v}}) = (10, 0.1)$. The solid and dashed curves correspond to the total torques acting on the planet by the discs with $\alpha = 1.0$ and $\alpha = 0.5$, respectively. The planet is located at 1 AU. The total torque by the disc with $\alpha = 0.5$ is larger than that by the disc with $\alpha = 1.0$. 

Figure A2. The torque densities exerted on the planet by the discs with $\alpha = 1.0$ (the solid curve) and $\alpha = 0.5$ (the dashed curve). Both discs have the parameters of $(\xi_{gr}, \xi_v) = (10, 0.1)$. The planet is located at 1 AU.