Supergravity Minimal Inflation
and its Spectral Index Revisited

Izawa K.-I.

Department of Physics, University of Tokyo,
Tokyo 113-0033, Japan

Abstract

Natural supergravity models of new inflation are reconsidered as minimal inflationary models within slow-roll approximation. Their running spectral index is derived in a revised form with recent observational results and future refinements in mind. This will possibly determine essential model parameters with respect to Planck-suppressed operators.
Primordial inflation \[1\] is expected to open a window into Planck-suppressed physics: Among possible ingredients beyond the standard model, supersymmetry roughly achieves natural realization of a flat potential for an inflaton. Discrepancy from the complete flatness may reflect physics suppressed by the gravitational scale \[2\]. In this respect, the detections of primordial fluctuations of cosmic microwave background radiation initiated by COBE and continued by WMAP, to name a few \[3\],\(^{1}\) might provide evidence for such basic objects as effective operators of higher dimensions with Planck-scale cutoff, which are naturally expected in the universal framework of effective field theories.

As a simplest example, we consider a single-field model for slow-roll inflation with a potential

\[ V(\varphi) = v^4 - \frac{\kappa}{2}v^4\varphi^2 - \frac{\lambda}{n!}\varphi^n \]  

(1)

for \(0 < \lambda, v, \kappa, \varphi \ll 1\) and \(n \geq 3\). Here and henceforth, we adopt the unit with the reduced Planck scale equal to one. The first term \(v^4\) yields vacuum energy for inflation; the second one \(\frac{\kappa}{2}v^4\varphi^2\) affects slow-roll dynamics during inflation; and the last one \(\frac{\lambda}{n!}\varphi^n\) eventually terminates inflation. Note that the initial condition for the inflaton field \(\varphi\) may be set by primary inflation \[1, 6, 7\].

This form is ubiquitous in supergravity inflation. For instance, let us take a minimal model considered in Ref.\[2\]. By means of a single chiral superfield \(\phi\), an inflaton \(\varphi\) can be provided by \(\sqrt{2}\) times the real part of its lowest component. We adopt a natural superpotential\(^2\)

\[ W = v^2\phi - \frac{g}{n + 1}\phi^{n+1} \]  

(2)

where \(g > 0\), and a generic Kähler potential

\[ K = |\phi|^2 + \frac{\kappa}{4}|\phi|^4 + \cdots \]  

(3)

where \(\kappa\) is generated, at least, radiatively of order \(10^{-2}\) due to suppression by a loop factor and the ellipsis denotes higher-order terms which may be disregarded.\(^3\) The tiny scale \(v^2\) can be generated dynamically \[2, 8\].

---

\(^1\)See also recent investigations on inflationary models in Refs.\[4, 5\].

\(^2\)This form is protected by nonrenormalization or \(R\) symmetry.

\(^3\)Concrete forms of radiative corrections lie beyond the scope of this paper.
The potential for the lowest component $\phi$ is given in supergravity by

$$V = e^K \left\{ \left( \frac{\partial^2 K}{\partial \phi \partial \phi^*} \right)^{-1} |DW|^2 - 3|W|^2 \right\},$$

(4)

where we have defined

$$DW = \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} W.$$  

(5)

Thus the potential is approximately given by Eq. (4) with $\lambda_n = \frac{g^2}{2} - v^2$.

The above example clearly shows that the $\varphi$-dependent terms in Eq. (4) directly originate from Planck-suppressed interactions with operators of higher dimensions.

This type of potentials also appears at the second stage of the double inflation [5, 6, 9]. Furthermore, $R$-invariant models may also yield such potentials effectively [10].

Let us investigate the inflationary dynamics of the above model Eq. (4) within slow-roll approximation [1].

The slow-roll inflationary regime is determined by the condition

$$\epsilon(\varphi) = \frac{1}{2} \left( \frac{V'(\varphi)}{V(\varphi)} \right)^2 \leq 1, \quad |\eta(\varphi)| \leq 1,$$

(7)

where

$$\eta(\varphi) = \frac{V''(\varphi)}{V(\varphi)}.$$  

(8)

For the potential Eq. (4), we obtain

$$\epsilon(\varphi) \simeq \frac{1}{2} \left( \frac{-\kappa v^4 \varphi - \lambda}{v^4} \varphi^{n-1} \right)^2 = \frac{\varphi^2}{2} \left( \kappa + \frac{\lambda v^{-4}}{(n-1)!} \varphi^{n-2} \right)^2,$$

(9)

$$\eta(\varphi) \simeq \frac{-\kappa v^4 - \frac{\lambda}{(n-2)!} \varphi^{n-2}}{v^4} = -\kappa - \frac{\lambda v^{-4}}{(n-2)!} \varphi^{n-2}.$$  

The slow-roll condition Eq. (7) is satisfied for $\varphi \leq \varphi_f$ where

$$\varphi_f^n \simeq \frac{(n-2)! (1 - \kappa)}{\lambda v^{-4}},$$

(10)

which provides the value of the inflaton field at the end of inflation.
The value $\varphi_N$ of the inflaton field corresponding to the $e$-fold number $N$ is given by

$$N = \int_{\varphi_f}^{\varphi_N} d\varphi \frac{V(\varphi)}{V'(\varphi)} \simeq \int_{\varphi_f}^{\varphi_N} d\varphi \frac{\varphi}{-\kappa v^4 \varphi - \frac{\lambda}{(n-1)!} \varphi^{n-1}}, \quad (11)$$

which implies

$$\varphi_N^{-2} \simeq \frac{(n-1)!\kappa}{\lambda v^{-4}} \left( \frac{1 + (n-2)\kappa}{1 - \kappa} e^{(n-2)\kappa N} - 1 \right)^{-1}. \quad (12)$$

Hence the scalar spectral index $n_s$ of the primordial density fluctuations is given by

$$n_s(N) \simeq 1 - 6\epsilon(\varphi_N) + 2\eta(\varphi_N) \simeq 1 - 2\kappa \left\{ 1 + (n-1) \left( \frac{1 + (n-2)\kappa}{1 - \kappa} e^{(n-2)\kappa N} - 1 \right)^{-1} \right\}. \quad (13)$$

This expression is intended for comparison with anticipated precise data. Note that it is independent of the coupling $\lambda$ and the scale $v$, and crudely $n_s \simeq 1 - 2\kappa$. Sizable deviations of the value $n_s$ from one thus directly indicate the presence of Planck-suppressed effects stemming from higher-dimensional operators in the Kähler potential, in particular.

The amount of running with respect to the $e$-fold number $N$ or the logarithm of the comoving wavenumber can be seen immediately from Eq.(13). For the sake of convenience, numerical values of the running spectral index $n_s(N)$ for the cases $n = 3, 4, 5$ are given in Tables 1, 2, 3, respectively.

That is, in the realm of precision cosmology, we might obtain $\kappa$, $n$, and so on with some fundamental meaning from analyses of observational results, if the class Eq.(1), for instance, is realized in Nature. The values $n_s(N)$ in the Tables are largely consistent with the recent WMAP data, though the class Eq.(1) may well turn out to be, at least, incomplete in no distant future, since its predictions are quite restricted due to its minimality. Then, more elaborate models with two or more fields in collaboration might be adequate. Even in such a case, it is conceivable that some effects suppressed by the gravitational scale will be revealed through detailed analyses of observational data in the near future. This, if true, confirms our view that the framework of effective field theories is universal.

---

4Numerical correspondence between these two variables depends on the reheating process, which we do not specify in this paper.
Acknowledgements

We would like to thank T. Watari and T. Yanagida for valuable discussions.

References

[1] For reviews, D.H. Lyth and A. Riotto, arXiv:hep-ph/9807278
    W.H. Kinney, arXiv:astro-ph/0301448
    S.F. King, arXiv:hep-ph/0304264
[2] Izawa K.-I. and T. Yanagida, arXiv:hep-ph/9608359
[3] C.L. Bennett et al., arXiv:astro-ph/9601067
    S. Hannestad, S.H. Hansen, and F.L. Villante, arXiv:astro-ph/0012009
    C.L. Bennett et al., arXiv:astro-ph/0302207
    D.N. Spergel et al., arXiv:astro-ph/0302209
    E. Komatsu et al., arXiv:astro-ph/0302223
    H.V. Peiris et al., arXiv:astro-ph/0302225
    S.L. Bridle, A.M. Lewis, J. Weller, and G. Efstathiou, arXiv:astro-ph/0302306
    V. Barger, H.-S. Lee, and D. Marfatia, arXiv:hep-ph/0302150
    C.R. Contaldi, H. Hoekstra, and A. Lewis, arXiv:astro-ph/0302435
    U. Seljak, P. McDonald, and A. Makarov, arXiv:astro-ph/0302571
    P. Mukherjee and Y. Wang, arXiv:astro-ph/0303211
    W.A. Chiu, X. Fan, and J.P. Ostriker, arXiv:astro-ph/0304234
[4] B. Feng, M. Li, R.-J. Zhang, and X. Zhang, arXiv:astro-ph/0302479
    B. Kyae and Q. Shafi, arXiv:astro-ph/0302504
    J. Ellis, M. Raidal, and T. Yanagida, arXiv:hep-ph/0303242
    L. Pogosian, S.-H.H. Tye, I. Wasserman, and M. Wyman, arXiv:hep-th/0304188
    Q.-G. Huang and M. Li, arXiv:hep-th/0304203
    W.H. Kinney, E.W. Kolb, A. Melchiorri, and A. Riotto, arXiv:hep-ph/0305130
[5] M. Kawasaki, M. Yamaguchi, and J. Yokoyama, arXiv:hep-ph/0304161
[6] Izawa K.-I., M. Kawasaki, and T. Yanagida, arXiv:hep-ph/9707201
[7] Izawa K.-I., arXiv:hep-ph/9710479

[8] Izawa K.-I. and T. Yanagida, arXiv:hep-th/9602180, arXiv:hep-ph/9809366, arXiv:hep-ph/9904426;
T. Hotta, Izawa K.-I., and T. Yanagida, arXiv:hep-ph/9606203;
Izawa K.-I., Y. Nomura, K. Tobe, and T. Yanagida, arXiv:hep-ph/9705228;
Izawa K.-I., arXiv:hep-ph/9708315

[9] M. Kawasaki, N. Sugiyama, and T. Yanagida, arXiv:hep-ph/9710259;
M. Kawasaki and T. Yanagida, arXiv:hep-ph/9807544;
T. Kanazawa, M. Kawasaki, N. Sugiyama, and T. Yanagida, arXiv:hep-ph/9908350, arXiv:astro-ph/0006445;
T. Kanazawa, M. Kawasaki, and T. Yanagida, arXiv:hep-ph/0002236

[10] Izawa K.-I., M. Kawasaki, and T. Yanagida, arXiv:hep-ph/9810537;
T. Asaka, K. Hamaguchi, M. Kawasaki, and T. Yanagida, arXiv:hep-ph/9906366, arXiv:hep-ph/9907559;
M. Kawasaki, N. Sakai, M. Yamaguchi, and T. Yanagida, arXiv:hep-ph/0005073
Table 1: The running spectral index $n_s$ for $n = 3$.  

| $e$-fold $N$ | 60  | 55  | 50  | 45  | 40  | 35  | 30  |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $\kappa = .01$ | .933 | .928 | .921 | .913 | .903 | .891 | .874 |
| .02          | .927 | .922 | .916 | .909 | .899 | .887 | .871 |
| .03          | .918 | .914 | .908 | .901 | .892 | .881 | .866 |
| .04          | .905 | .902 | .897 | .891 | .883 | .873 | .858 |
| .05          | .891 | .888 | .884 | .879 | .872 | .863 | .849 |
| .06          | .874 | .872 | .869 | .865 | .859 | .851 | .839 |
| .07          | .856 | .855 | .852 | .849 | .844 | .837 | .827 |

Table 2: The running spectral index $n_s$ for $n = 4$.  

| $e$-fold $N$ | 60  | 55  | 50  | 45  | 40  | 35  | 30  |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $\kappa = .01$ | .955 | .951 | .947 | .941 | .934 | .924 | .912 |
| .02          | .949 | .946 | .942 | .938 | .932 | .924 | .912 |
| .03          | .935 | .934 | .931 | .928 | .924 | .917 | .908 |
| .04          | .918 | .917 | .916 | .914 | .911 | .906 | .899 |
| .05          | .899 | .899 | .898 | .897 | .895 | .892 | .887 |
| .06          | .880 | .880 | .879 | .879 | .877 | .875 | .872 |
| .07          | .860 | .860 | .860 | .859 | .859 | .857 | .855 |

Table 3: The running spectral index $n_s$ for $n = 5$.  

| $e$-fold $N$ | 60  | 55  | 50  | 45  | 40  | 35  | 30  |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| $\kappa = .01$ | .965 | .962 | .958 | .953 | .947 | .939 | .929 |
| .02          | .956 | .954 | .952 | .949 | .945 | .940 | .931 |
| .03          | .939 | .938 | .938 | .936 | .934 | .930 | .925 |
| .04          | .920 | .920 | .919 | .919 | .918 | .916 | .912 |
| .05          | .900 | .900 | .900 | .900 | .899 | .898 | .896 |
| .06          | .880 | .880 | .880 | .880 | .880 | .879 | .878 |
| .07          | .860 | .860 | .860 | .860 | .860 | .860 | .859 |