Quantum reality with negative-mass particles

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Physical interpretations of the time-symmetric formulation of quantum mechanics, due to Aharonov, Bergmann, and Lebowitz are discussed in terms of weak values. The most direct, yet somewhat naive, interpretation uses the time-symmetric formulation to assign eigenvalues to unmeasured observables of a system, which results in logical paradoxes, and no clear physical picture. A top–down ontological model is introduced that treats the weak values of observables as physically real during the time between pre- and post-selection (PPS), which avoids these paradoxes. The generally delocalized rank-1 projectors of a quantum system describe its fundamental ontological elements, and the highest-rank projectors corresponding to individual localized objects describe an emergent particle model, with unusual particles, whose masses and energies may be negative or imaginary. This retrocausal top–down model leads to an intuitive particle-based ontological picture, wherein weak measurements directly probe the properties of these exotic particles, which exist whether or not they are actually measured.

In classical physics, knowing the initial state of a complete system is enough to infer any future state of that system, but due to the inherent indeterminism of measurement outcomes, this is not so in quantum physics. The time-symmetric, or two-state vector, reformulation of quantum mechanics was developed by Aharonov et al. (1) in order to promote a description of quantum mechanics with the same level of completeness as classical physics. For the quantum case, one needs not just the initial state of the system, but also its final state after a projective measurement, in order to make definite assertions about the quantum description of nature during the intervening time. They developed the ABL formula, which provides the time-symmetric conditional probability to have obtained a given outcome for a strong projective measurement performed in the time between a particular initial state preparation and a particular final measurement outcome. When these ABL probabilities are 1 or 0, they can be interpreted as describing simultaneous properties (eigenvalues) of the system between pre- and post-selection (PPS)—even if no intermediate measurement is actually performed. We call this the ABL interpretation, and it gives rise to various logical PPS-paradoxes, which are directly connected to Kochen–Specker-type (2) quantum contextuality. Because of these logical paradoxes, there is no consistent physical interpretation of such PPS situations, which has led to various ad hoc explanations of their meaning, as in the cases of the 3-box paradox (3, 4), the quantum Cheshire cat (5, 6), and the quantum pigeonhole effect (7–9), among others.

Sometime later, Aharonov et al. (3, 10, 11) developed a new idea of what the quantum description should look like in the two-vector formalism for the situation where a very weak measurement (or no measurement at all) is made during the time between an initial preparation and a final measurement outcome, in terms of a quantity they called the weak value, which is operationally more like an expectation value than an eigenvalue. The weak value of an observable $\hat{A}$ is defined as $A_w = \langle \psi | \hat{A} | \psi \rangle / \langle \psi | \psi \rangle$. Weak measurements were needed in the time-symmetric context in order to minimally disturb the pre- and post-selected states, thereby allowing us to answer questions which were not legitimate before. Analogously to in vivo studies, where biological experiments are performed on living organisms or cells, we would like to answer questions about freely evolving quantum systems without perturbing them much. When the postselection is not trivial, a strong measurement would alter either the pre- or post-selected state (or both), and hence we must employ weak measurements for keeping these quantum systems “alive” while being measured. This provides a richer description of quantum reality during the time interval between two projective measurements. The weak value is generally complex, making its physical interpretation somewhat subtle (12–15), but unlike the ABL interpretation, the weak values do not produce any PPS-paradoxes, and can rather be seen as resolving the paradoxes of the ABL interpretation. For a given PPS, the weak value of every
observable property of the system is fully defined, and they collectively form a coherent quasi-classical picture of the system during the intervening time between the pre- and post-selection, which we call the weak value interpretation (or weak reality) (16). As we will discuss, there is some physical motivation for this interpretation, since the weak value is observed in the limit of infinitesimally weak measurements with a PPS ensemble and can thus be thought to be an existent property of the system, even when no actual measurement is performed. In this sense, the weak value can be considered as more like an eigenvalue for a PPS, having a single definite value, but one that is operationally obscured by the spread of the pointer distribution in a weak measurement. We call this interpretation quasi-classical because all of the weak values of different observables are mutually consistent and noncontextual, regardless of whether the observables commute. Note that when we say the weak values provide a noncontextual value assignment, we mean only they are defined without reference to a measurement context. This is unrelated to the fact that projector weak values prove Spekkens-type quantum contextuality when they are complex, or have real parts outside the range [0, 1] (17–20).

Both negative weak values and anomalously large weak values are ubiquitous in pre- and post-selected ensembles—whenever 0 < \(|\langle\psi|\psi\rangle|^{2} < 1\), there is always a positive amplified weak value for the projector onto \((|\psi\rangle + |\psi\rangle)/\sqrt{2}\) and a negative weak value for the projector onto \((|\psi\rangle - |\psi\rangle)/\sqrt{2}\) (these values switch their roles if \(-1 < \langle\psi|\psi\rangle < 0\). Complex weak values are similarly commonplace.

Although lying outside the spectrum of the weakly measured operator \(A\), weak values provide an effective description of the system between two strong measurements. This happens because the weak coupling to any operator \(A\) of the pre- and post-selected system, through the interaction Hamiltonian, can be replaced by the c-number \(A_w\) (21, 22). This means, for instance, that in the examples studied below, a negative weak value of the projection operator onto some location would lead to an effective interaction term with inverse sign between the particle there and the weak probe. In particular, this implies the possibility of gravitational repulsion rather than attraction within the weak reality. Moreover, not only the gravitational mass, but also the inertial mass will be shown to admit a negative sign.

Finally, because the weak values at a given moment in time are defined using both the preselected state, \(|\psi\rangle\), that propagates causally from past to future, and the postselected state, \(|\psi\rangle\) that propagates retrocausally from future to past, the weak value interpretation is also fundamentally nonclassical.

The weak reality leads to a top–down (23) model of the physical reality during the time interval between a pre- and post-selection. For an individual quantum particle, the model introduces additional positive–negative pairs (real or imaginary) of copies called counterparticles which are generated by the quantum particle during preselection and absorbed during postselection. Each positive–negative pair has exactly opposite values for all physical properties, and thus, their emergence from the vacuum obeys all conservation laws. The counterparties of a given system can interact with the counterparties of other systems, but not with each other.

The ontology is called top–down because it is the joint rank-1 projectors (outer products) of all \(N\) quantum particles in an entangled state that are most fundamental, and the states of individual physical systems emerge only as sums over these objects. The rank-1 projectors of an \(N\)-particle system correspond to delocalized \(N\)-point structures (\(N\)-structures), each made up of \(N\) counterparties of the same type—thus, the \(N\)-structures come in the same four types, \(+1, -1, +i, -i\) as the counterparties. For the special case of an isolated nonentangled quantum particle, the 1-structures are just single localized counterparties. Despite the retrocausal and top–down nature of the model, each of the counterparties follows a continuous trajectory (world-line) through space-time, providing us with a clear (if exotic) physical picture of nature during the PPS interval.

We begin in Section 1, where we introduce the general hierarchy of all PPS paradoxes. In Section 2, we develop the top–down \(N\)-structure/counterparticle model of the weak value interpretation and discuss its properties. In Section 3, we work through several significant examples to help develop a sense of the breadth of situations the model can be applied to. The quantum mirror is presented as a thought experiment to develop intuition about the ballistic properties of counterparties (1-structures). Several variations of the 3-box paradox are discussed, culminating with the Hardy paradox, (24–27) which is reviewed as an example of top–down 2-structures and their properties, and a possible resolution of the paradox is discussed. Finally, we conclude with some closing remarks.

SI Appendix begins with a review of quantum measurement using a continuous pointer system, from weak measurement to strong projective measurement. We also present a number of more general cases in SI Appendix, which flesh out the weak reality picture, along with the analysis of other known PPS paradoxes. Lastly, SI Appendix provides a generalization of top–down structures, which allows them to produce the weak values for all measurement bases at once, thereby demonstrating that the weak reality is noncontextual.

1. Pre- and Post-Selection Paradoxes

There are a number of well-known PPS paradoxes, including the 3-box, the quantum Cheshire cat, and the quantum Pigeonhole Effect, all of which are demonstrations of quantum contextuality (2, 8, 17, 18, 28–30). We show here that all PPS paradoxes belong to a single general family of extended \(N\)-box paradoxes. First, we introduce some tools. The weak value of any observable \(A\) is given by the formula,

\[
A_{w} = \frac{\langle \psi | A | \psi \rangle}{\langle \psi | \psi \rangle},
\]  

[1]

where the \(|\psi\rangle\) is the preselection and \(|\varphi\rangle\) is the postselection.

Noting the spectral decomposition \(A = \sum_{i} \lambda_i \Pi_i\), where \(\Pi_i = |\varphi_i\rangle \langle \varphi_i|\) is the projector onto a given eigenstate and \(\lambda_i\) is the corresponding eigenvalue, it is clear that \(A_{w} = \sum_{i} \lambda_i (\Pi_i)_{w}\).

The ABL probability formula gives the conditional probability to obtain a particular measurement result if a projective measurement was made during the time between the preselection of \(|\psi\rangle\) and the postselection of \(|\varphi\rangle\). The outcomes of a projective measurement of observable \(A\) are the projectors \(\Pi_i\), and the formula is,

\[
P_{\text{ABL}} (\Pi_i = 1 | \psi, \varphi, B) = \frac{|\langle \psi | \Pi_i | \psi \rangle|^{2}}{\sum_{k \in B} |\langle \psi | \Pi_k | \psi \rangle|^{2}} = \frac{|\langle \Pi_i | w \rangle|^{2}}{\sum_{k \in B} |\langle \Pi_k | w \rangle|^{2}}.
\]  

[2]

This formula shows that the key to understanding the general family of PPS paradoxes lies in the weak values of the projectors.
complex coefficients

{37x114}This has led to many counterintuitive physical examples. It also

PPS that will also produce these weak values.

{37x158}(preselection

N-box paradoxes require

additional projector weak values to zero, (e.g.,

be embedded into a higher-dimensional system, by setting all

These are the fundamental

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For example, this can be done by breaking a weak value 1 into

be generated in higher dimensions by taking any weak value in

with projectors of different ranks.

−

For a given rank-1 projector of an entangled

N-body state, with weak value \(x + iy\), there are, on average, \(|x|\) \(N\)-structures composed of real counterparticles of type \(\text{sign}(x)\), and, on average, \(|y|\) \(N\)-structures composed of imaginary counterparticles of type \(\text{sign}(y)\), with the \(N\) counterparticles in each structure spanning \(N\) sites/states of the rank-1 projector. On any given run there will always be an integer number of particles of various types, forming complete integer \(N\)-structures, and for cases where \(x\) and \(y\) are not integers, the average is obtained by a probabilistic mix of the two nearest integer values. This is a simplifying assumption of the model that we hope to prove in the future, which results in a unique and unambiguous ontology in the time between pre- and post-selection.

Each \(N\)-structure is a connected graph, but in general, it is not fully connected and to find the actual edges, we first define 2-structures for all pairs of subsystems by summing the rank-1
projects over all other systems, and then keep only the edges from these different pairwise 2-structures for the full $N$-structure. Once all $N$-structures are properly defined, they cleanly encode the system’s weak values for all projectors of all ranks. Specifically, $N$-structures for higher-rank projectors corresponding to $n < N$ subsystems are simply the subgraphs on the $n$ corresponding vertices of the $N$-structure.

If the momentum of a pointer system couples to a quantum system at just one location (the highest-rank projector of an entangled system) during the time between pre- and post-selection, the pointer receives a quasi-classical impulse (instead of entering an entangled superposition) corresponding to the sum of the counterparticles at that location. We begin with a few single-particle examples in order to highlight the simple ballistic properties of counterparticles during local interactions, and then we consider the Hardy paradox as an example with 2-structures.

Individual local measurements during a PPS reveal nothing about the top–down structure. Measurements of the lower-rank projectors which form this structure are discussed alongside an example of 3 entangled 2-level systems in SI Appendix.

### 3. Illustrative, Paradoxical, and Counterintuitive Examples

**A. 2-Level Systems.** First, consider the example of a single particle that is both pre- and post-selected in a superposition of two boxes, $|\psi\rangle = |\phi\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$, resulting in weak values $(\Pi_1)_w = (|1\rangle\langle 1|)_w = 1/2$ and $(\Pi_2)_w = (|2\rangle\langle 2|)_w = 1/2$. In the weak value ontology, the simplest possible distribution has the particle in each box with probability 1/2. To be clear, there are more complicated distributions involving additional positive–negative pairs of counterparticles which will produce the same weak values, and freely switching between any such distribution is a symmetry of the interpretation.

Next, consider an example of a 2-level system, with orthogonal projectors $\Pi_1$ and $\Pi_2$, having weak values $(\Pi_1)_w = -1/3 + 3i/2$ and $(\Pi_2)_w = 4/3 - 3i/2$ in a given PPS (there are infinitely many possible PPS choices that produce these values). We can work out the probability distributions of the real counterparticles and imaginary counterparticles separately, since the positive–negative pairs are independent for the two cases. Starting with the real part, the simplest distribution is that with probability 2/3 there is a positive particle in $\Pi_1$ and nothing in $\Pi_2$, and with probability 1/3 there are two positive particles in $\Pi_1$ and a negative one in $\Pi_2$. Then the simplest independent probability distribution for the imaginary part has one positive particle in $\Pi_1$, and a negative one in $\Pi_2$. The simplest independent probability distribution for the imaginary part has one positive particle in $\Pi_1$, and a negative one in $\Pi_2$ with probability 1/2, and also with probability 1/2, two positive particles in $\Pi_1$, and two negative particles in $\Pi_2$.

**B. The Quantum Mirror.** When a pointer interacts with a system in only one location, the impulse is due simply to the sum of impulses from the individual counterparticles at that location, which provides a very simple quasi-classical ballistic picture of these types of interactions during the PPS.

Consider a Mach–Zehnder interferometer (MZI) through which a single particle passes. On arm II of the MZI, we have the usual macroscopic mirror, but on arm I, we instead have a tiny mirror of mass $m$, which we will approximate as a single coherent superposition state with a small position uncertainty $\Delta x$, and a large momentum uncertainty $\Delta p$. We presume that the mirror still scatters the particle in the same direction as a macroscopic mirror and that $m$ is sufficiently large compared to the particle’s momentum so that the energy of the particle is not changed significantly. Furthermore, the particle must have an energy uncertainty, $\sigma$, which is large enough that this change in energy does not significantly reduce the visibility of interference at the second beamsplitter.

Ignoring the backaction on the particle’s energy, the coupling Hamiltonian is $\hat{H} = g(t)\hat{\Pi}_1^p\hat{X}$, where $g(t)$ is a time-dependent coupling strength, which integrates to $s = \int_{t_{\text{in}}}^t g(t)dt$ during the scattering interaction. This is the interaction for a measurement of $\hat{\Pi}_1^p$ using the momentum of the quantum mirror as the pointer system (SI Appendix). We are interested here in the limit $s \ll \Delta p$ where this becomes a weak measurement of the projector $\hat{\Pi}_1^p$ (refs. 35 and 36).

Before the particle in the MZI reaches the quantum mirror, the product state of the two systems is,

$$\int |dp\tilde{\psi}(p)(|I^p\rangle + |II^p\rangle)\rangle/\sqrt{2},$$

where $\tilde{\psi}(p)$ is the momentum-space wavefunction of the mirror, and $|\psi^p\rangle = |I^p\rangle + |II^p\rangle$ is the particle’s preselected state. After the particle scatters off of the quantum mirror, the two evolve into the entangled state,

$$\int |dp(\tilde{\psi}(p-s)|I^p\rangle + \tilde{\psi}(p)|II^p\rangle\rangle/\sqrt{2}.$$  \[4\]

Now, to allow a general postselection $|\psi^p\rangle$, we must allow that an arbitrary phase shifter be placed in one arm of the MZI, and that the second beamsplitter can be chosen with any transmissivity. Whatever the choice, the final state of the mirror will be $\tilde{\psi}(p-s)\langle \Pi_1^p\rangle_w$ (SI Appendix).

We consider two different cases, and because they have no effect on the weak values, we do not bother with normalization.

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**Fig. 1.** The quantum mirror for preselection, $|\psi^p\rangle = |I^p\rangle + |II^p\rangle$, and two different postselections, corresponding to different choices of transmissivity in the second (purple) beamsplitter and different setting of the phase shifter. (A) Counterparticle configuration for postselection, $|\psi^p\rangle = -|I^p\rangle + 2|II^p\rangle$. During preselection, the particle emits a positive-negative pair of counterparticles, and the negative one scatters off the quantum mirror, whose momentum wavefunction is depicted in orange. The counterparticles are reabsorbed during the postselection, and the postselected quantum mirror receives a negative impulse, as shown by the magenta subensemble. (B) Counterparticle configuration for postselection, $|\psi^p\rangle = i|I^p\rangle + (1-i)|II^p\rangle$. During preselection, the particle emits an imaginary positive-negative pair of counterparticles, and the imaginary positive one scatters off the quantum mirror, whose position wavefunction is depicted in black. The counterparticles are reabsorbed during the postselection, and the postselected quantum mirror receives translated in the positive direction, as shown by the green subensemble.
which, given the postselection, there is a weak trace in both arms Vaidman’s experiment (16, 37) with a nested pair of MZIs in 3-box is the key paradox. During the preselection and reabsorbed during the postselection. Fig. 2 illustrates how the extra positive–negative pair is created during the preselection and reabsorbed during the postselection. In general, the counterparticle representation provides a clear description, where there is only a positive imaginary particle on arm I, and on arm II there is a positive real particle and a negative imaginary particle. When the imaginary particle scatters off the quantum mirror, the mirror exerts an impulse on the imaginary particle, and in reaction, the imaginary particle translates the mirror with no impulse. This case serves as a guide for how we should conceptualize scattering interactions involving the imaginary counterparties in this quasi-classical picture.

These examples help to build intuition for this picture in the regime of observable weak measurements, but our general focus in this article is on the underlying ontology that is present during a PPS when no intermediate interaction occurs at all.

C. The 3-Box Paradox. The 3-box paradox is the simplest case (3, 4). For projector weak values \( \langle 1, 1, -1 \rangle \) there is a real positive particle in the first state, another in the second, and a real negative particle in the third. This example was the primary motivation for the development of the counterparticle model. Fig. 2 illustrates how the extra positive–negative pair is created during the preselection and reabsorbed during the postselection.

Next, we will discuss several other examples for which the 3-box is the key paradox.

D. Vaidman’s Nested Interferometer Paradox. Let us consider Vaidman’s experiment (16, 37) with a nested pair of MZIs in which, given the postselection, there is a weak trace in both arms of the inner interferometer, even though there is no trace in the only paths into, or out of, the inner interferometer. Let \( A \) be the arm of the outer interferometer that bypasses the inner interferometer, \( D \) be the arm that leads into the inner interferometer, \( E \) the same arm leading back out, and \( B \) and \( C \) the arms of the inner interferometer—as shown in Fig. 3

At time \( t_1 \), after the particle has entered the outer interferometer, but before it would reach the inner interferometer, the weak values of the relevant projectors are \( |A|_w = 1 \), \( |D|_w = 0 \)—thus there is no weak trace leading into the inner interferometer. At time \( t_2 \), when the particle would be inside the inner interferometer, the weak values of the relevant projectors are \( |A|_w = 1 \), \( |B|_w = 1 \), and \( |C|_w = -1 \), and thus there is a weak trace in both arms of the inner interferometer. Finally at time \( t_3 \), after the particle would exit the inner interferometer, the weak values of the relevant projectors are \( |A|_w = 1 \), \( |E|_w = 0 \), and there is no weak trace leading out of the interferometer.

Thus, at \( t_2 \) we have obtained exactly the 3-box paradox, with a real positive particle in arm \( A \), another in arm \( B \), and a real negative particle in arm \( C \). Now, since the positive–negative counterparticle pair must be emitted during the preselection, and reabsorbed during the postselection, and each must follow a complete world-line through the PPS, we interpret \( |D|_w = 0 \)
and $|E_{w}| = 0$ as the positive–negative counterpart particle pair from $B$ and $C$ moving together, so that they apparently mask one another. However, there are operators whose weak values are not zero in arms $D$ and $E$. If we describe the states at arms $B$ and $C$ as the two eigenstates of $\sigma_z$, then $\sigma_z$ has a nonzero weak value within the inner interferometer, but not in arms $D$ and $E$. Nevertheless, the weak value of $\sigma_z$ (or $\sigma_x$) which corresponds to a flip from arm $B$ to arm $C$ (up to a phase) does not vanish in arms $D$ and $E$. These two operators, which are sensitive to the relative phase between the arms, are nonlocal and cannot be instantaneously measured in $D$ and $E$, but thanks to the Heisenberg equations they change in time into locally measurable operators.

It is interesting to note the momentum exchange of the particle-counterparticle pair with the inner beamsplitters in the case that the latter are heavy but not held fixed. The net momentum at the entrance to the inner interferometer is zero, because the particle and counterpart particle have opposite momenta. However, within the interferometer they gain a net momentum to the left, which means that the first inner beamsplitter must receive a net momentum to the right in order to conserve the total momentum. Also, the second inner beamsplitter must receive from the pair a net momentum to the right. The proposed ontology can therefore give rise to predictions which are not otherwise obvious. This is yet another way for us to see that the arms $D$ and $E$ are not empty, but rather contain a positive–negative counterpart particle pair.

In the ABL interpretation, and noting that $|B_{w}| + |C_{w}| = 0$, we can conclude that the particle is definitely in path $A$ at all three times $t_1$, $t_2$, and $t_3$, and that it was never in the inner interferometer at all, nor in the paths leading into or out of it. However, if at $t_2$ a measurement were performed in the basis $\{|A, B, C\}$ the ABL formula predicts probability $1/3$ to find the particle in $B$, and probability $1/3$ to find it in $C$. The appearance of the particle in the inner interferometer, despite it having zero probability to be found entering or exiting, is then the same sort of paradox as the appearing and disappearing particle (38).

### E. The Quantum Cheshire Cat as a 3-Box Paradox

Consider a spin in cavity with a spin-sensitive mirror in the center. The spin begins on the left in the state $2|L \uparrow\rangle + |L \downarrow\rangle$. The mirror is transparent to $|\downarrow\rangle$, and acts as a 50/50 beam splitter for $|\uparrow\rangle$, and thus after striking the mirror, the state has evolved to $|\psi\rangle = |L \uparrow\rangle + |R \uparrow\rangle + |R \downarrow\rangle$. After a second pass through the mirror, the state is measured as $2|R \uparrow\rangle + |L \downarrow\rangle$. Counteringparting this through the mirror gives us $|\varphi\rangle = |L \uparrow\rangle - |R \uparrow\rangle + |R \downarrow\rangle$. $|\psi\rangle$ and $|\varphi\rangle$, are, respectively, the pre- and post-selection during the time between the first and second pass of the mirror. The weak values of the four rank-1 projectors are then,

$$
|L \uparrow\rangle_{w} = |L \uparrow\rangle + |L \downarrow\rangle = 1, \quad |L \downarrow\rangle_{w} = 0, \quad |R \uparrow\rangle_{w} = -1, \quad |R \downarrow\rangle_{w} = 1.
$$

and for the rank-2 projectors they are,

$$
|L |_{w} = |L \uparrow\rangle + |L \downarrow\rangle = 1, \quad |R |_{w} = |R \uparrow\rangle + |R \downarrow\rangle = 0,
$$

and

$$
| \uparrow\rangle_{w} = |L \uparrow\rangle + |R \uparrow\rangle = 1, \quad | \downarrow\rangle_{w} = |L \downarrow\rangle + |R \downarrow\rangle = 1.
$$

From the rank-2 projector weak values, we would conclude that the particle must be on the left ($|L |_{w} = 1$), with spin down ($| \downarrow\rangle_{w} = 1$), but this directly contradicts the rank-1 projector weak values ($| \uparrow\rangle_{w} = 0$). Furthermore, a weak measurement of the mass on the right arm will detect nothing, while a weak measurement of the spin on the right arm will detect twice the expected value down, so we seem to have a disembodied spin, as in the original quantum Cheshire cat.

The counterpart particle description is quite straightforward. There is a standard spin-up particle on the left, a standard spin-down particle on the right, and a negative spin-up particle on the right—which produces the same magnetic field as a positive spin-down particle, due to its opposite charge. Instead of a disembodied spin on the right, we have two particles whose masses sum to zero, but whose magnetic fields add constructively (and to an anomalously large value).

Interestingly, there appears to be a way to experimentally validate this description. The two counterparts in the right side of the cavity have opposite mass and charge and are in opposite spin states. The opposite charge and spin result in identical magnetic moments, which is why we observe zero mass and charge (if they are charged particles), but nonzero magnetic moment. Now, if an electric field were turned on within the right cavity, the particles would experience opposite Coulomb force, but they also have opposite inertial mass, and their accelerations are identical. However, if a magnetic field gradient were turned on instead, they would have opposite acceleration, because they would experience identical magnetic force.

Applying such a magnetic field would allow us to separate the two counterparts within the Right side of the cavity, and then their individual masses and magnetic moments could be weakly measured, before reversing the direction of the magnetic field to put the particles back together, such that the postselection is not disturbed. Thus, a suitably modified experiment could show that what appears to be a spin without a mass is really two counterparts partially masking each other.

Note that the inertial mass of counterparts on the right side is indeed negative since the effective momentum associated with these particles is negative (the weak value of the corresponding projector is negative), but their velocity is positive (it was not pre- and post-selected at all).

### F. The Hardy Paradox

Hardy’s paradox (24, 25) has been widely studied in connection with quantum nonlocality. It features two MZIs, one traversed by an electron and the other by a positron. There is a point of intersection between an arm of one interferometer and an arm of the other, such that if the electron and positron both travel down that arm, they are annihilated and two gamma-ray photons are produced. This creates effectively a 5-level system—the positron in one of two arms, together with the electron in one of two arms give four states, and the photons produced by the annihilation are the fifth.

Let the arms of the interferometer traversed by the positron be $|L^+\rangle$ and $|R^+\rangle$, and the arms of the interferometer traversed by the electron be $|L^-\rangle$ and $|R^-\rangle$, with the arms $|R^+\rangle$ and $|L^-\rangle$
intersecting such that $|R^+\rangle|L^-\rangle \rightarrow |γ\rangle|γ\rangle$. The interferometers have bright ports $b^\pm$ and dark ports $d^\pm$ and are aligned such that if the two interferometers were moved apart so that there could be annihilation, then no particles would be detected at either dark port $d^\pm$. For the present configuration, the possibility of annihilation alters the wavefunction such that there is a $1/16$ probability to detect both the electron and the positron at their dark ports, which can be interpreted as indicating that the two particles detected one another without ever interacting, i.e., interaction-free measurement (39).

After passing through the first beamsplitter of their MZIs, the positron is in the state $(|L^+\rangle + i|R^+\rangle)/\sqrt{2}$ and the electron is in the state $(|R^-\rangle + i|L^-\rangle)/\sqrt{2}$ which result in the joint product state,

$$|ψ_0\rangle = (|L^+\rangle|R^-\rangle + i|L^+\rangle|L^-\rangle + i|R^+\rangle|R^-\rangle - |R^+\rangle|L^-\rangle)/2.$$  

Once the two pass the intersection point, this evolves into the entangled state,

$$|ψ\rangle = (|L^+\rangle|R^-\rangle - i|L^+\rangle|L^-\rangle - i|R^+\rangle|R^-\rangle - |R^+\rangle|L^-\rangle)/2.$$  

which is to say that the possibility of annihilation has created an entanglement correlation between the electron, positron, and gamma-ray photons. This entangled state is the preselection we will consider here.

For the postselection, we of course take the paradoxical case where the detectors at both dark ports $d^\pm$ have clicked. After propagating them retrocausally back through the second beamsplitter of their MZIs, the positron is postselected in the state $(|L^+\rangle - i|R^+\rangle)/\sqrt{2}$ and the electron in the state $(|R^-\rangle - i|L^-\rangle)/\sqrt{2}$, resulting in the product state,

$$|ψ\rangle = (|L^+\rangle|R^-\rangle - i|L^+\rangle|L^-\rangle - i|R^+\rangle|R^-\rangle - |R^+\rangle|L^-\rangle)/2.$$  

With this pre- and post-selection, both valid during the time interval between the possible annihilation event and the arrival of the particles at the second beamsplitter of each interferometer, we construct the weak values $|L^+R^-\rangle_w = -1$, $|L^+L^-\rangle_w = 1$, $|R^+R^-\rangle_w = 1$, $|R^+L^-\rangle_w = 0$, and $|γγ\rangle_w = 0$, using compact notation $|R^+\rangle|L^-\rangle \equiv |R^+\rangle^L\langle L^-|$, which correspond to the 3 (nonzero) 2-structures in Fig. 5.

The weak values of the localized rank-2 projector corresponding to individual particle states are,

$$|L^+\rangle_w = |L^+R^-\rangle_w + |L^+L^-\rangle_w = 0,$$  

$$|R^+\rangle_w = |R^+R^-\rangle_w + |L^+L^-\rangle_w = 1,$$  

$$|L^-\rangle_w = |L^+L^-\rangle_w + |R^+L^-\rangle_w = 1,$$  

and

$$|R^-\rangle_w = |L^+R^-\rangle_w + |R^+R^-\rangle_w = 0.$$  

These correspond to the sum of the counterparties at each location.

In the ABL interpretation, these weak values force us to conclude that the electron and positron both took the inner arms of their respective interferometers, and yet they both reached the detectors, and so must have passed without annihilating, which is the original Hardy paradox.

The $N$-structure picture for this scenario is shown in Fig. 5. Weak measurements of the rank-2 projectors corresponding to the individual arms probe the corresponding counterparties, while weak measurements of the rank-1 projectors corresponding to arm-products between different systems probe the corresponding 2-structures, with the pointer receiving a quasi-classical impulse in each case.

In this description, we still have a positron and electron on the inner arms where they would annihilate, but because they are not truly isolated particles, but rather the component ends of different 2-structures, the annihilation is prevented. We are essentially positing a physical rule that both ends of these 2-structures would need to meet simultaneously to produce an annihilation, and this rule provides us with an elegant resolution of the paradox.

4. Discussion

We hope that we have clearly conveyed the top-down weak reality as a retrocausal model with quasi-classical particles that move on well-defined trajectories during the time between pre- and post-selection, and that we have convinced the reader that this model provides an intuitively useful particle-based picture of the underlying physics of unmeasured systems.

The examples in SI further emphasize the generality and versatility of the weak reality model, which may, we believe, form the foundation for a retrocausal formulation of quantum physics based on particles rather than waves. We plan to develop the model further to see where this insight leads.

Data, Materials, and Software Availability. There are no data underlying this work.

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