The hydraulic resistance of thermoviscous liquid flow in a plane channel with a variable cross-section

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Abstract. The problem of the possibility of adjusting the hydraulic resistance due to local thermal action is considered. The liquid flow with the temperature dependence of the viscosity in a plane channel with sudden contraction is investigated. The velocity profiles of the thermoviscous liquid flow and the dependence of the hydraulic resistance coefficient on various types of local action on the liquid flow with a temperature dependence of the viscosity in a plane channel are numerically obtained.

1. Introduction

Energy losses (reduction of the hydraulic head) can be observed in a moving liquid not only in comparatively long sections, but also in short ones. In some cases, head losses are distributed (sometimes evenly) along the length of the pipeline - these are linear losses; in others, they are concentrated on very short sections, the length of which can be neglected - on the so-called local hydraulic resistances: valves, all kinds of rounding, constriction, expansion, etc., i.e. everywhere where the flow undergoes deformation. The source of losses in all cases is the viscosity of the liquid. It should be noted that the pressure losses both along the length and in the local hydraulic resistances essentially depend on the so-called fluid motion regime.

As is known, local resistance is caused by different equipment of pipe-wire networks, which change the magnitude or direction of the velocity. Pressure losses on local resistances \( \Delta h \) for liquids with constant viscosity are well studied and determined by the Weisbach formula:

\[
\Delta h = \xi \cdot \frac{u_1^2}{2g},
\]

where \( u_1 \) – average velocity of liquid flow in the wide part of the channel, \( g \) – the acceleration of gravity, \( \xi \) – the coefficient of local resistance.

The coefficient of local resistance for sudden contraction for a turbulence liquid flow with constant viscosity can be determined by the following formula [1]:

\[
\xi = 0.5 \left( 1 - \frac{S_2}{S_1} \right),
\]
where $S_1$ and $S_2$ are the cross-sectional area in the wide and narrow parts of the channel, respectively. In contrary, this coefficient for sudden expansion in a channel has the different form:

$$\xi = \left(1 - \frac{S_1}{S_2}\right)^2,$$

where $S_1$ and $S_2$ are the cross-sectional areas in the narrow and wide parts of the channel, respectively.

Hydraulic losses are expressed either in losses of pressure in linear units of the column of the medium

$$\Delta h = \frac{\Delta p}{\rho g},$$

or in units of pressure

$$\Delta p = \xi \frac{\rho u_i^2}{2},$$

where $\rho$ is the density of the medium.

However, for laminar flow regime, viscosity cannot be neglected and the formulas for determining the coefficient of hydraulic coefficient are not applicable. In this case, this coefficient can only be determined experimentally [2] or using the following complicated formula [3]

$$\xi = A \cdot B \cdot \left(1 - \frac{S_2}{S_1}\right),$$

where

$$A = \sum_{i=0}^{7} a_i (\lg \text{Re})^i, \quad B = \sum_{j=0}^{2} \left( \sum_{j=0}^{2} a_{ij} \left(\frac{S_j}{S_1}\right) (\lg \text{Re})^j \right), \quad a_j = a_j (\text{Re}),$$

$$a_0 = -25.12458, \quad a_1 = 118.5076, \quad a_2 = -170.4147, \quad a_3 = 118.1949,$$

$$a_4 = -44.42141, \quad a_5 = 9.09524, \quad a_6 = 0.03408265, \quad a_7 = 0.03408265.$$

In real devices, as a rule, all flows are non-isothermal, therefore it is necessary to take into account the dependence of viscosity on temperature. Paper [4] contains a rather detailed numerical analyses of the influence of temperature dependent viscosity parameters on the flow regimes in plane channel.

In this work the problem of the possibility of regulating the hydraulic resistance due to local thermal effects.

2. The problem statement

Let us consider incompressible thermoviscous liquid flow in a plane channel with sudden contraction (figure 1). The direction of the flow is from left to right and it’s driven by the pressure gradient $\Delta p = p_{in} - p_{out}$. The width of the wide part of the channel is $h_1$ and the width of the narrow one is $h_2$. Three cross sections are also depicted in the figure: cross section A-A (red) is near to the channel inlet, cross section B-B (blue) is near to the channel contraction point, and cross section C-C (green) is near to the channel outlet. These cross sections are used below to plot and compare velocity profiles.

The continuity, Navier-Stokes and energy equations are the basic equations governing the flow of an incompressible Newtonian thermoviscous liquid. These equations in the dimensionless form can be written as:
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{1}{\text{Re}} \left[ \frac{\partial}{\partial x} \left( \mu(T) \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu(T) \frac{\partial u}{\partial y} \right) \right], \]

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \frac{1}{\text{Re}} \left[ \frac{\partial}{\partial x} \left( \mu(T) \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu(T) \frac{\partial v}{\partial y} \right) \right], \]

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\text{Pe}} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right], \]

where \( u \) is the longitudinal velocity, \( v \) is the transversal velocity, \( p \) is the pressure, \( T \) is the temperature, \( \mu(T) = e^{-\alpha T} \) is the temperature-dependent viscosity (non-dimensional, see also figure 2), \( \text{Re} \) is the Reynolds number, \( \text{Pe} \) is the Peclet number.

**Figure 1.** The scheme of the plane channel with sudden contraction

**Figure 2.** The exponential dependence of viscosity on the temperature for the different values of the thermostatic parameter \( \alpha \)
The boundary conditions for the velocity components and the pressure are standard: non-slip condition for velocity on the upper and lower rigid walls of the channel, the inlet and the outlet pressures are given.

To investigate the influence of temperature fields on the hydraulic resistance at thermoviscous liquid flows two different temperature regimes has been considered (figure 3). The temperature regime I is characterized with the different temperatures of the lower and upper walls of the channel. In this case the lower wall temperature $T_c$ differs from the upper wall temperature $T_h$. For the temperature regime II the walls of the wide and narrow part of the channel have different temperatures: the temperature of the wide part of the channel is $T_c$ and the walls temperature of the lower part is $T_h$. Notice that for the both regimes the temperature of the fluid flowing into the channel is $T_a$ and it is assumed that $T_c < T_h$.

![Figure 3. Two kind of temperature boundary conditions: a) temperatures of the lower and upper walls are different (regime I), b) temperatures of the wide and narrow parts are different (regime II)](image)

3. Results of the numerical analysis

The presented above equations of mathematical model is solved numerically with the original computer code based on the finite volume method and SIMPLE algorithm [5].

Figures 4 and 5 show the longitudinal velocity profiles at three different cross sections of the channel (A-A, B-B and C-C as shown in figure 1), viscosity distribution and velocity vector field for temperature regimes I and II. All the numerical results are obtained for the same value of the thermoviscous parameter $\alpha = 7$ (green curve in figure 2).

![Figure 4. Thermoviscous liquid flow in the plane channel with sudden contraction (temperature regime I): a) longitudinal velocity profiles at different channel cross section, b) viscosity and velocity fields](image)
For the temperature regime I (figure 4) cold liquid with high viscosity flows into the channel and along the channel liquid are heated near the upper wall. The viscosity of the heated liquid decreases and, as a result, the flow velocity increases. The velocity profiles have asymmetric form shifted to the upper wall that is the more intensive flow is observed in the upper part of the channel.

![Image](image_url)

**Figure 5.** Thermoviscous liquid flow in the plane channel with sudden contraction (temperature regime II): a) longitudinal velocity profiles at different channel cross section, b) viscosity and velocity fields

In the case of the temperature regime II (figure 5) the flow in the wide part of the channel is isothermal and temperature effect influences on the flow character only in the narrow part.

Figure 6 shows the dependence of the local hydraulic resistance coefficient on the ratio between the widths of narrow and wide parts of the channel for isothermal flow and temperature regimes I and II. For all cases the resistance coefficient decreases when the ratio increases (the narrow part width increases). The highest value of the resistance coefficient is observed for temperature regime I when the channel walls have different temperature. Therefore changing the temperature regime of the channel walls, it is possible to adjust the local resistance coefficient and, accordingly, to change the pressure losses of the fluid flow.

![Image](image_url)

**Figure 6.** The hydraulic resistance coefficient versus width of narrow to wide parts ratio for different temperature regimes
Conclusion
Thus, in this paper it is shown that the local hydraulic resistance coefficient for laminar flow in a channel with sudden contraction strongly depends on the temperature distribution in the channel. The smaller the width of the channel narrow part, the greater difference between the local hydraulic resistance coefficient obtained for isothermal and non-isothermal cases.

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