Hardware Demonstrator of a Level-1 Track Finding Algorithm with FPGAs for the Phase II CMS Experiment

D. Cieri*1,2 on behalf of the CMS Collaboration and of the Time Multiplexed Track Trigger group
1 University of Bristol, Bristol, United Kingdom; 2 Rutherford Appleton Laboratory, Didcot, United Kingdom

*Supported by the EU FP7-PEOPLE-2012-ITN project nr. 317446, INFIERI, “Intelligent Fast Interconnected and Efficient Devices for Frontier Exploitation in Research and Industry”
E-mail: davide.cieri@stfc.ac.uk

Abstract. At the HL-LHC, proton bunches collide every 25 ns, producing an average of 140 pp interactions per bunch crossing. To operate in such an environment, the CMS experiment will need a Level-1 (L1) hardware trigger, able to identify interesting events within a latency of 12.5 µs. This novel L1 trigger will make use of data coming from the silicon tracker to constrain the trigger rate. Goal of this new track trigger will be to build L1 tracks from the tracker information.

The architecture that will be implemented in future to process tracker data is still under discussion. One possibility is to adopt a system entirely based on FPGA electronic.

The proposed track finding algorithm is based on the Hough transform method. The algorithm has been tested using simulated pp collision data and it is currently being demonstrated in hardware, using the “MP7”, which is a µTCA board with a powerful FPGA capable of handling data rates approaching 1 Tb/s.

Two different implementations of the Hough transform technique are currently under investigation: one utilizes a systolic array to represent the Hough space, while the other exploits a pipelined approach.

1. Introduction
The High Luminosity (HL–LHC) upgrade [1] to the Large Hadron Collider will operate at an increased instantaneous luminosity, up to 7 times the nominal value, in order to collect an integrated luminosity of 3000 fb−1 in the decade following 2025. Proton bunches at the HL–LHC will cross every 25 ns, producing an average of 140 pile-up pp collisions per bunch crossing (BX).

In order to meet these experimental challenges, the CMS collaboration [2] will need to address the aging of the present detector and to improve the ability of the apparatus to isolate and precisely measure the products of the most interesting collisions (CMS Phase II Upgrade [3]).

In fact, by 2025 the detector will have suffered significant radiation damage, therefore many components will need to be completely replaced. In particular, CMS will have a new tracker [4]
with an increased granularity in both pixel and outer tracker system, which will guarantee the required reconstruction performance.

A major upgrade will involve also the online L1 trigger system. Presently only the muon chambers and the calorimeters are part of the L1 trigger. Indeed, the combinatorial background at the HL–LHC will not allow to keep the current trigger system, since it will be impossible to maintain the trigger rate within the nominal range without degrading physics performance by rising substantially the trigger thresholds. Therefore, for the first time, data coming from the Tracker will be included in the L1 trigger of a high luminosity hadron experiment.

Before transmitting tracking information to the trigger processors, the amount of data needs to be reduced. This will be achieved through an on-detector selection on transverse momentum of particles $p_T$. Therefore the Outer Tracker system (Fig. 1a) will be formed by double sensor modules ($p_T$ modules [3]), capable of rejecting signals from particles with a $p_T$ lower than 2 GeV/c. A correlation between hits in sensors of one module, which are consistent with a high-$p_T$ track, is called a stub (see Fig. 1b).

The architecture that will be used to process the tracker data is still subject to discussion. An intriguing proposal is to use a time multiplexed system, similar to the one already adopted by the CMS Phase I calorimeter trigger [5], and composed entirely by FPGA boards.

2. The Time Multiplexed Track Trigger

The fundamental idea of a Time Multiplexed Trigger (TMT) [6] is that all data from a single event flow to a single destination (node) for processing. This requires two processing layers with a passive switching network between them, the Pre-Processor (PP) and the Main-Processor (MP). The PPs take data directly from the front-end modules, organise and format them, before distributing to the MPs, where the track finding algorithm will be implemented. Each MP receives data from many PPs belonging to a single event. Data from the next event flow to a second MP, out of phase by one LHC clock cycle. Each MP will then have a period of time (Time Multiplexed period) proportional to the total number of MP boards to run the algorithm.

This particular system is well matched to FPGA processing. Therefore, a conceptual architecture can be built by means of the best currently readily available. Our choice ended up being the µTCA board MP7 [7], originally designed for the CMS L1 calorimeter trigger upgrade, which contains a powerful Virtex 7 FPGA with 72 I/O optical links (total bandwidth $\sim$ 1 Tb/s).
3. L1 Track Finding with Hough Transform

The goal of the track trigger is to build L1 tracks from stub data. The Hough Transform (HT) \[8\] is the method chosen to build track candidates. The simplest version of this technique permits detection of straight lines. In the real space a line is defined a set of points \((x, y)\), which the HT represents in terms of the slope–intercept parameters \((m, q)\). In this way each point in the real space represents a line in the parameters space, so that a line in the real space will be identified by the crossing point in the parameter space.

Dividing the tracker into 5 trigger regions in pseudo–rapidity, each MP will receive around 3000 stubs per event. Each MP will bin then the stubs in 36 \(\phi_0\) segments of around 0.17 rad each, where \(\phi_0\) is the estimated azimuthal angle of the track at the production point. The Hough Transform is then performed in each of these \(\phi_0\) segments. Thanks to this \(\phi_0\) binning the algorithm will have to operate just with an average \(\sim 80\) stubs, with a maximum of 200 at a time. In the first stage of the algorithm the Hough Transform is applied to the stub coordinates \((r, \phi)\), which have the best resolution in CMS, independently for each \(\eta–\phi_0\) sector.

Charged particles in CMS are bent by the strong magnetic field. It is possible in any case to assume for high–\(p_T\) particles \((p_T \geq 3\) GeV/c) a linear relation between \(\phi\) and \(r\).

\[
\phi(r) = \frac{cBQ}{2p_T} r + \phi_0,
\]

where \(c\) is the speed of light, \(B\) the CMS magnetic field, \(Q\) the electric charge of the particle, \(p_T\) its transverse momentum and \(\phi_0\) is the estimated azimuthal angle at production. Here the slope parameter \(m = cBQ/2p_T\) is bound by the lower \(p_T\) limit of 3 GeV/c, while the intercept \(q\) depends on the size of the \(\phi_0\) segment under consideration.

The algorithm histograms the stub data in \((m, q)\) bins in a 2-D array of \(32 \times 32\) cells. For each column in \(m\), a value of \(q\) is calculated,

\[
q = -mr + \phi - \phi_0^{\text{min}},
\]

where \(\phi_0^{\text{min}}\) is the lower bound of the current \(\phi_0\) segment. A track is found when stubs cluster in the 2-D histogram (see Fig. 2). In most cases the stubs present in a HT cell are not consistent with real tracks. Indeed they are usually associated to random combinations of hits belonging to pile–up tracks. Therefore several filtering stages are required to accept only cells compatible with true high–\(p_T\) particles.

**Bend Filter**

The first filter to be applied is the **Bend Filter**. The local bend of a stub can be easily calculated knowing the hits and sensors separation. The condition for a stub to be kept in a HT cell is that the stub bend \(\delta_s\) would be consistent with the predicted bend \(\delta_c\) based on the \(q/p_T\) of the cells and the \((r, z)\) coordinates of the stub.
\[ |\delta_x - \delta_c| < \sigma, \]

where \( \sigma \) is the assumed bend resolution.

**Z Filter**
Another filter exploits the use of a third coordinate (\( \eta \) or \( z \)), which has not been utilised so far. A possible solution would be to calculate the estimated value of \( z \) of the track at a chosen radius \( R \) per each stub, assuming that the tracks originate from the beam spot.

\[ z_R = \frac{R}{r} z. \]

Later stubs with a value of \( z_R \) far away from the cell average are then discarded.

**R Filter**
High-\( p_T \) tracks should be able to leave signals in all tracker layers they pass through. Therefore it is handy to require a HT cell to contain stubs belonging to 5 or more tracker layers, in order to be marked for readout.

**Algorithm Performance**
The algorithm has been tested using simulated data samples of different signal and pile-up content. Figure 3a shows the distribution of the algorithmic efficiency as a function of the pile-up for three different kinds of events (single muon, single electron and \( t\bar{t} \)). Results obtained show a generally good algorithmic efficiency, larger than 97% for muons and \( t\bar{t} \). In contrast, electron candidates have a lower efficiency (~85%), mainly because of the Bremsstrahlung effect that deviates the electron’s trajectory.

The distribution of the number of candidates per event as function of the pile-up content is displayed in Fig. 3b. The track filtering stages significantly reduce the number of track candidates, even if more improvements are still required to further diminish the fake rate.

![Algorithmic Efficiency vs. Pile-Up](image)

![Number of Track Candidates vs. Pile-Up](image)

**Figure 3:** (a) Track Finding Algorithmic Efficiency, shown as a function of the Pile-Up content, in different samples. (b) Distribution of number of track candidates per event found by the HT algorithm as a function of the pile-up content. Also the distributions of the number of real, duplicate and fake candidate tracks per event are shown.
4. Hardware Demonstration
Two different firmware implementations of the Hough Transform algorithm have been developed so far: the systolic array and the pipelined Hough Transform. The former reproduces in hardware the Hough Transform array in a bi-dimensional structure \((q \text{ cells} \times m \text{ cells})\). In this case the same histogram is used for all \(\phi\) sector, having then just one array per MP7. Nevertheless preliminary results have been obtained on a hardware demonstrator using the pipelined firmware. In this implementation each pair of I/O links of the MP7 is assigned to a single \(\phi_0\) segment. The Hough Transform algorithm is then run in parallel independently for each segment. Due to the resource limitations of the MP7, it is possible to implement a maximum of 9 segments per chip, therefore every board can process data belonging to one region in pseudo-rapidity and 9 segments in \(\phi_0\).

Preliminary hardware results show a good agreement with the floating point algorithm simulation. In fact the pipelined firmware has been able to find \(\sim 92\%\) of the candidate tracks identified by the simulation. Further work is therefore needed to assure an exact matching between the firmware and the simulation.

5. Conclusions
A Level–1 Track Finding algorithm for the proposed Time Multiplexed Track Trigger for the phase II CMS experiment has been presented herein. The proposed algorithm, based on the Hough Transform technique, is capable to detect tracks with transverse momenta above 3 GeV/c with an excellent efficiency, keeping the total number of track candidates within an acceptable level.

A hardware demonstrator has been set up, by means of MP7 boards. Two different implementations of the HT algorithm have been developed. Preliminary hardware results have shown a encouraging agreement with the software simulation. Further work is still required to obtain a perfect matching between the two.

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