Investigating the Effect of Roughness and Stiffness in 2D Aeroelastic Around Oscillatory Airfoil

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Abstract: In this research, a software has been developed to investigate the effect of roughness and stiffness in two-dimensional aeroelastic in unsteady viscous flow around oscillatory airfoil. In this simulation to solve the Navier-Stokes equations, finite volume method has been used in the code with a high resolution scheme for fluid and structure simulation in transonic flows. For this purpose, fluid and structural behavior is solved separately at each time step and the effect of each one on the other is considered. For computing convection term in transonic unsteady compressible flow, high order SBIC (Second and Blending Interpolation Combined) scheme based on discretization of Normalized Variables Diagram (NVD) is used. Here the technique of inlet velocity vector oscillation which is a simpler method in comparison with rather complicated methods such as dynamic mesh is applied. The two-dimensional motion equations are obtained from the Lagrangian equations which are combined with the aerodynamic equations. The results of validation show that the extracted data has a desirable accuracy and had good agreement to experimental data. The FSI results show that, 1: Lift coefficient in smooth surface is more than the rough surface and also the drag coefficient in rough surface is more than the smooth surface, 2: Shock strength is weaker in the rough surface, 3: The shock’s place has moved to leading edge in the rough surface, 4: The number of oscillations in rough surface is reduced, 5: The structural stability of the airfoil when the surface of the airfoil is rough is much greater than smooth surface, 6: Because the density of the air and the amplitude of the oscillations are small and also small effect on the lift and drag coefficients, can be ignored the added mass in this simulation method.

Keywords: FSI, Aeroelastic, Oscillation, Transonic, Roughness, Stiffness, Unsteady

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1. Introduction

In recent years considerable attention has been paid to aeroelastic behaviors in various topics such as turbines, compressors and wing fighter aircraft. Nowadays linear aeroelastic constitutes a large part of aeroelastic studies but when the effects of roughness, shock waves, flow separation and viscosity effects are considered concurrently, a little research has been done. Particularly in the transonic flow non-linear parameters play an important role in the aeroelastic behavior which linear theories do not have the ability to study these behaviors accurately. One of the important problems in aeroelastic is the stability of the structure. By applying roughness to the airfoil surface, the distribution of the pressure and velocity on airfoil has changed and the structure is more stable.

Aerodynamic forces increase with increasing flow velocity, while elastic stiffness is independent of flow velocity, therefore the structure becomes unstable with speed of flow. This instability can cause finally lead to fail the structure. A significant phenomenon in aeroelasticity is flutter, in which fluctuations grow and produce great dynamical forces [1]. At first a summary of the work done in the field of aeroelastic is presented then the motivation and purpose of this research are given.

Lapointe and Dumas made a numerical simulation of the pitch-plunge forced oscillation on the NACA0012 airfoil and showed that in harmonic motion the oscillation amplitude and the pitch angle were immortal and constant [2]. Mowat used nonlinear aeroelastic modeling by using and coupling fluid-structure interaction. They used the Euler equations in the fluid region and in the solid region discrete equations by using Reduced Order Model (ROM). Their results show the effects of nonlinear terms in the governing equations [3]. Khairil and Zorkipli simulated the aeroelastic system by using nonlinear aerodynamics. Their results show a good trend with the experimental data [4]. Unger investigated oscillating airfoil for flapping wing propulsion and their results had a good agreement with the data of wind tunnel [5]. Riso investigated the numerical aeroelastic analysis of a flat plate in arbitrary motion with high amplitude. In their research, fluid is assumed to be a non-viscous, incompressible and potential flow [6]. Medjroubi used the spectral element method in Reynolds between 800 and 10,000 to discrete the Navier Stokes equations. They showed that when the frequency and amplitude of the oscillation are constant, the Reynolds number does not have a big effect on flow field and aerodynamic forces. Also they showed that if the amplitude of the oscillation increases, it wills greatly effect on the wake and the aerodynamic coefficients [7]. Yuan studied numerically an incompressible flow on airfoil NACA0012 in free oscillation and two degrees of freedom. They displayed that by increasing Reynolds, the number of the amplitude of the oscillation increases [8]. Banavara and Dimitrov proposed Reduced Order Model (ROM) in the aeroelastic behavior of the wings in the transonic flow because the linear aerodynamics theory did not have proper
predictions of detached flow due to nonlinear terms [9]. Akkala and Eslampahani examined the aerodynamic behavior of a flexible airfoil in plunge’s motion experimentally. They used PIV to determine the vortex and flow field. They showed that at low frequencies the flexible wing was under the slightest deformation and also the efficiency of flexible wing was higher than the rigid wing [10]. Razak studied aeroelastic behavior of a rectangular wing in pitch-plunge motion by using acceleration and pressure measurements in an experimental work. They showed that the phenomenon of stall flutter and flutter are related to the frequency of oscillations [11]. Walker and Patil examined the flexibilities of self-sustained oscillating and forced oscillation of rigid wing was higher than the rigid wing [10]. Taobai studied the effect of surface roughness on the turbine motion by using acceleration and pressure measurements in an experimental work. They showed that by increasing roughness height the intensity of turbulence near the wall increases [15].

These results show that by applying surface roughness, the lift coefficient decreased and the drag coefficient increased [17].

In this research, a simple, accurate, and time-saving way to simulate aeroelastic transient on the airfoil has been developed. Two-dimensional software has been developed to simulate transonic unsteady compressible flow passing an oscillatory airfoil and used a numerical finite volume method with a pressure base algorithm. To calculate convection terms high order SBIC scheme is used and the technique oscillation of flow boundary condition method is applied which is simpler method in comparison with other such as dynamic mesh. To simulate Navier Stokes equations a modified k-ε model for compressible flow is exerted. In this method the Normalized Variable Diagram (NVD) scheme has been used. The two-dimensional motion equations are obtained from the Lagrangian equations which are combined with the aerodynamic equations. This research sets out to study nonlinear aerodynamic effects on aeroelastic behavior of self-sustained oscillating and forced oscillation of rigid airfoil. For this purpose the structural stiffness and roughness and other effective parameters have been investigated.

2. Governing Equations and Discretization

2.1 Fluid Equations

The governing equations in this simulation include the equation of continuity, momentum equations and the energy equation.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v_i) = 0
\]  

To investigate the effect of roughness on the surface in equations, the wall functions are used.

\[
u^* = \frac{u^*}{\kappa} = \frac{1}{\kappa} \ln(Ey^*) + \beta - \Delta \beta
\]

\[
u^* = \sqrt{\frac{\nu_p}{\rho}}
\]

\[
\Delta \beta = \frac{1}{x} \ln(1 + 0.3h^*)
\]

\[
h^* = \frac{h}{u^*}
\]

Where \(y_p\) is distance to the wall and \(E = 9.793\) is empirical constant. As can be seen \(\Delta \beta\) has a relation with roughness height and in different references for different conditions, the coefficients used in these equations are different.

2.1.1 Discretization

The finite volume method is used to discrete equations Fig.1. For this purpose the Gaussian theorem has been used. For any variable \(\phi\) the result of the integration yield is:

\[
\frac{\partial (\rho \phi)}{\partial t} + \frac{\partial (\rho \phi v_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \frac{1}{\partial} \frac{\partial \phi}{\partial x_j} \right] + \frac{\partial}{\partial x_i} \left[ \frac{1}{\partial} \frac{\partial \phi}{\partial x_i} \right]
\]
The diffusion flux is approximated by central differences. A representation of the convective flux for cell-face $w$ is:

$$I_w = F_w \Phi_w = \Phi_w (\rho V_A)_w$$

(18)

To determine the value of $\Phi_w$ must interpolate between the neighboring points of the cell that is defined by the SBIC scheme that is based on the NVD technique. The $\Phi_w$ value is obtained under the following conditions from the SBIC scheme.

$$\Phi_w = \begin{cases} 
\Phi_p & \text{if } \Phi_p \notin [0,1] \\
\Phi_w = \left(1 + \frac{x_p-x_w}{k(x_p-1)}\right) \Phi_p - \frac{x_p-x_w}{k(x_p-1)} \Phi_p^2 & \text{if } \Phi_p \in [0,k] \\
\frac{x_w-1}{x_p-1} \Phi_p + \frac{x_p-x_w}{x_p-1} \Phi_p & 0 \leq k \leq 0.5 & \text{if } \Phi_p \in [0,k] 
\end{cases}$$

(19)

And the final form of discretized equation is:

$$\sum_i A_i \Phi_i + S'\Phi + S_{dc} = A_p \Phi_p \ i: E, W, N, S$$

(20)

### 2.2 Solid Equations

In the present research, motion equations in two degree of freedom have been used for aeroelastic modeling in pitch-plunge system. The equations of motion are:

$$[M][\ddot{X}] + [D][\dot{X}] + [K][X] = [F(t)]$$

(21)

The two-dimensional motion equations are obtained from the Lagrangian equations which are combined with the aerodynamic equations. In this study the equations of motion in two cases were studied. In the first case the forced airfoil oscillation was studied under aerodynamic forces that the angle of attack is varied as a function of non-dimensional time $t$ as:

$$\theta = \theta_0 + \theta_{max} \sin(2\pi t)$$

(22)

In the second case free airfoil oscillation was studied under aerodynamic forces and only damping under the influence of fluid viscosity is achieved and no structural damping is considered. The equations under these conditions are given in the form below. In these test cases stiffness centers are located in the middle of cord.

$$X \cos(\theta) + F_0 \sin(\theta) = K_{dc} X$$

$$X \cos(\theta) \frac{C_1}{4} + F_0 \sin(\theta) \frac{C_2}{4} + M_c = K_{dc} \theta_0$$

(23)

By solving these equations in each time step airfoil pitch angle and vertical displacement is obtained.

### 3. Boundary Conditions

Boundary conditions in this research include:

1. velocity inlet at input boundary
2. pressure outlet at the exit boundary
3. far-field boundary on the upper and lower boundaries

Also the airfoil distance is 5C from the inlet and 15C from the exit. Here the wall functions are used to calculate the boundary layer velocity [22]. The airfoil distance from the boundaries is set, to have the least effect from the boundaries or the independence of the computing area is observed.

### 4. FSI Solution

At the fluid-structure interaction, first Navier Stokes equations are solved around the airfoil and gained pressure and viscosity forces on the airfoil surface then they are used as inputs of the structure equations. Then by solving the structural equations vertical displacement and pitch angle are obtained at each time step. In this simulation because the mesh is constant even with large time steps or large vertical displacement the solution does not diverge. This problem is very important in the dynamic mesh because by increasing the deformation, the mesh is distorted in most cases and solution is diverged. FSI solution algorithm is given below.

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**Fig2.** FSI solution algorithm

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**Fig1.** Finite volume and storage arrangement

$$\frac{\delta U}{\delta t} \left[ \left( \rho \Phi \right)_{i+1} - \left( \rho \Phi \right)_i \right] + I_e - I_w + I_n - I_s = S_q \delta U$$

(17)

The diffusion flux is approximated by central differences. A representation of the convective flux for cell-face $w$ is:

$$I_w = F_w \Phi_w = \Phi_w (\rho V_A)_w$$

(18)
5. Results and Discussion

First the forced airfoil oscillation was reviewed during the transonic flow in Mach 0.755. For this purpose first of all independence from the grid has been achieved.

The grid sizing is obtained by doing several different trials. The result of the independence of the computational grid is presented in Fig.4. As can be seen to simulate the flow, a grid with 115960 numbers of elements is utilized. After independence mesh is achieved, the validation of the solving method should be compared with valid laboratory results [23]. For this purpose, an airfoil with NACA 0012 section which undergoing forced sinusoidal pitching in flow at a Mach 0.755 is used. Where $\kappa = 0.0814$ is the reduced frequency and the primal and amplitude angles of attack are respectively $\theta_0 = 0.016^{\circ}$ and $\theta_{\text{max}} = 2.51^{\circ}$. As can be seen the numerical solution results in good agreement with experimental data Fig.5 [23].

As can be seen in the forced airfoil oscillation, the upper and lower limit of maximum lift coefficients is fixed and the results with passage of time are repeated cyclically Fig.5.

The FSI test case considered the transonic flow at $M_{\infty} = 0.8$ for all test cases. For this purpose first, steady solution is to be provided with an initial angle of attack around the airfoil that the initial angle of attack is 1 degree for all test cases. This steady solution produces the field of pressure and velocity and other relevant parameters. While the distribution of forces around the airfoil was achieved, you can leave airfoil and examine it in different conditions. It is clear that airfoil movements respond differently in different situations. For example with changing the initial amplitude will give different answers. In this research because the aim of the study is to investigate the motion of airfoil under different conditions, attention is paid to roughness and structural parameters. The meaning or the innovation point of these test cases is to provide a new method for the two-dimensional aeroelastic simulation and to reduce computational cost and convergence faster.

First, it is necessary to examine the effect of added mass in this simulation and then compare the results with the results of this method. For this purpose, free airfoil oscillation in Mach 0.85 has been simulated. As can be seen, since the air density is low, and the range of airfoil oscillations is small, when the effect of added mass is considered, there is no significant change in the lift and drag coefficients and vertical displacement airfoil also can ignore this effect in this simulation method [24].
As can be seen, when the effect of the added mass is considered the variation of oscillation amplitudes in comparison with the present method is very slight and can be ignored.

In the first test case on airfoil NACA0012, the pitch ($K_\theta$) and vertical stiffness ($K_s$) values are:

$$K_s = 850470 \ (N/M)$$
$$K_\theta = 52500 \ (N \cdot M/\text{rad})$$

Below changes of lift and drag coefficients are given at the angle of attack (Figs. 9-10).

As you can see with the passage of time the range of lift and drag variations become smaller and converge to a steady value after several cycles. This state occurs when structural stiffness overcomes aerodynamic forces and causes domain variations to become smaller with time.
In the second test case on airfoil NACA 0012 the pitch and vertical stiffness values are:
\[
\begin{align*}
K_x &= 1700940 \text{ (N/M)} \\
K_\theta &= 105000 \text{ (N.M rad)}
\end{align*}
\]

In this test case the structural stiffness has doubled. Here the purpose of investigating is the behavior of airfoil in a specified Mach with increasing structural stiffness.

Changes of lift and drag coefficients are given on the angle of attack in figures 14 and 15 respectively. In this case since the structural stiffness is doubled, it causes the airfoil to fluctuate around its neutral axis and the lifting force in the upstroke and down stroke have positive and negative values. Also because the amplitude of airfoil angle is smaller, the amount of drag coefficient is lower than the previous one.

It is also observed here that when the value of the lift coefficient is locally maximal, the corresponding value of the drag coefficient is also in the maximum situation and as time passes the lift and drag coefficients are converted to steady values Fig. 16.

In the third test case on airfoil NACA 0012, the pitch and vertical stiffness values are:
\[
\begin{align*}
K_x &= 3401881.88 \text{ (N/M)} \\
K_\theta &= 210000 \text{ (N.M rad)}
\end{align*}
\]

In this test case the structural stiffness has doubled relative to the second case. Initially changes of lift coefficient to angle of attack are given Fig. 19.
It is observed that the magnitude of the lift coefficient is lower than previous cases and the reason is that with increasing structural stiffness, the amount of angular variations has been decreased. Also the airfoil arrives sooner to its steady state pose.

It is also clear that airfoil returns to its primary state after 2-3 fluctuations Fig 20. In this case the airfoil has come to its neutral state quickly it has had enough time to fluctuate around its neutral point so the drag coefficient has fluctuations around its neutral point. And it is clear that in this case fluctuations have a limit with the passage of time.

As shown in Figures 21 and 22 airfoil is almost constant after 1 second after release. It is also seen that the rate of decrease amplitude has increased relative to the previous cases.
Figure 23 shows the distribution of the Mach coefficient contours corresponding to the positions shown in Fig 14. As you can see the position of the shock wave in the upper and lower surface of the airfoil is distinct for different times or in other words for different angle of attack and by changing the angle of attack, the intensity of the shock wave changes. As time passes and decrease of the angle of attack, the position of the shock wave in upper and lower surface of the airfoil moves towards the trailing edge and has approximately identical values.

Here the results of the roughness effect on the aeroelastic are given. Due to the presence of roughness in the wall, the turbulence intensity near the rough wall is much greater than the smooth wall. In this simulation, the roughness height is 0.0002 meters.

In figures 24 and 25, Mach's contours around the oscillating airfoil are presented in two smooth and rough modes respectively.

As can be seen from the comparison of two smooth and rough surfaces it can be concluded that when roughness is applied on the surface, shock strength dropped and also the spot of the shock is moved to the leading edge.

Figures [26-27] show the distribution of the pressure coefficient on the smooth and rough surfaces of the oscillating airfoil respectively. It is also observed, in the rough surface the pressure distribution at the bottom surface is more uniform and there is no severe pressure gradient.
In Fig. 26, the pressure coefficient contours around the smooth airfoil are shown. As can be seen, the shock has occurred at both the upper and lower surfaces. In Fig. 27, the pressure coefficient contours around the rough airfoil are presented. The shock has occurred only at the upper surface on the rough airfoil. Fig. 28 shows the distribution of the pressure coefficient in two smooth and rough surfaces. On the smooth surface, the shock has occurred at both the upper and lower surfaces, while on the rough surface, the shock has occurred only at the upper surface. In this figure, it is easy to see that the shock's place in the rough airfoil has been relocated to the leading edge.

In Fig. 29, the comparison of wall shear stress around the smooth and rough airfoils is shown. When the surface is rough, it causes turbulence intensity near the wall, and the shear stress of the rough wall increases. Here it is also observed that when a shockwave occurs, the wall shear stress is suddenly dropped.

In Figure 30, the lift coefficient in two smooth and rough conditions is compared. It is observed that in a rough surface, the lift values are less than smooth surface, and the airfoil has reached a steady state earlier. Figure 31 shows that at a rough surface, the amount of drag is much larger than the smooth surface because the shear stress of the smooth surface is less than the rough surface.
Fig 31. Comparison of drag coefficient versus number of iteration around the smooth and rough airfoils

Fig 32. Comparison of vertical displacement around the smooth and rough airfoils

Fig 33. Comparison of airfoil angle around the smooth and rough airfoil

In figures 32 and 33 vertical displacement and airfoil angle are brought. Because in a rough surface, aerodynamic forces are smaller, structural stiffness will play a more effective role in the movement of airfoil and the rough airfoil will reach its steady state earlier, and it will have a smaller fluctuation range.

6. CONCLUSION

In this article the static mesh technique is used instead of a dynamic mesh that is most costly. Also to simulate Navier Stokes equations, a modified model for compressible flow and for computing convection term high order SBIC scheme based on discretization of Normalized Variable Diagrams used. The results of validation and simulation show that the extracted data has a desired accuracy and had good agreement to experimental results. Some principal points can be explained as follow.

- The new numerical method provides acceptable results of 2D aeroelastic.
- The concurrence between experimental data and numerical results are considerable.
- The present technique is considered to provide effective method in aeroelastic simulation in which the mesh is fixed and it does not distort.
- This simulation method is much simpler and less costly than other methods.
- The results show that with increasing structural stiffness the number of oscillations and drag coefficient decreases.
- This method takes less time to converge than other methods.
- The results show that with increasing structural stiffness, oscillations are eliminated more quickly.
- By applying surface roughness it causes the shock strength to drop and the shock’s place moves to the leading edge.
- Also the roughness causes the airfoil to reach a steady state earlier and the number of fluctuations is lower.

Nomenclature

| Symbol | Description |
|--------|-------------|
| FSI | Fluid-Structure Interaction |
| SBIC | Second and Blending Interpolation Combined |
| NVD | Normalized Variable Diagram |
| CFD | Computational Fluid Dynamic |
| SIMPLE | SemiImplicit Method For Pressure Linked Equation |
| AOA | Angle of Attack (in degree) |
| K | SBIC parameter |
| Re | Reynolds number |
| c | Chord Length |
| δ | Cell Face Area |
| C_μ, C_1, C_2 | Empirical coefficients |
| u_τ | Friction velocity |
| υ | Von karman constant |
| τ_w | Wall shear stress |
| h_s | Roughness height |
| F | Mass flux |
| l | Flux |
| K | Kinetic Energy of Turbulence |
| M_∞ | Free stream Mach number |
| q | Scalar flux vector |
| T | Stress tensor |
| u, v | Velocity components in X and Y directions |
| Γ | Diffusivity coefficient |
| Γ_τ | Turbulent diffusivity coefficient |
| φ | Cell volume |
| ε | Volumetric rate of dissipation |
| μ | Dynamic viscosity |
| μ_t | Turbulent viscosity |
| ρ | Density |
| σ_k | Turbulent Prandtl number for turbulent kinetic energy |
| σ_ε | Turbulent Prandtl number for dissipation rate |
| φ | Scalar quantity |
\[ \phi \sim \text{Normalized scalar quantity} \]

\[ A, D \sim \text{Finite difference coefficients} \]

\[ \kappa \sim \text{Reduced frequency} \]

\[ X \sim \text{Vertical displacement} \]

\[ K_\theta \sim \text{Spring matrix} \]

\[ D \sim \text{Dynamic damper matrix} \]

\[ M \sim \text{Mass of the airfoil} \]

\[ K_s \sim \text{Plunging spring coefficient} \]

\[ K_\theta \sim \text{Pitching spring coefficient} \]

\[ M/c_\theta \sim \text{Moment about the aerodynamiccentre} \]

\[ F_L \sim \text{Lift force} \]

\[ F_D \sim \text{Drag force} \]

\[ \theta \sim \text{Pitch angle} \]

\[ f \sim \text{Physical frequency} \]

\[ \omega_a \sim \text{Circular frequency} \]

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