Chapter 1

DECOHERENCE IN DISORDERED CONDUCTORS AT LOW TEMPERATURES, THE EFFECT OF SOFT LOCAL EXCITATIONS

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Abstract

The conduction electrons’ dephasing rate, \( \tau_{\phi}^{-1} \), is expected to vanish with the temperature. A very intriguing apparent saturation of this dephasing rate in several systems was recently reported at very low temperatures. The suggestion that this represents dephasing by zero-point fluctuations has generated both theoretical and experimental controversies. We start by proving that the dephasing rate must vanish at the \( T \to 0 \) limit, unless a large ground state degeneracy exists. This thermodynamic proof includes most systems of relevance and it is valid for any determination of \( \tau_{\phi} \) from linear transport measurements. In fact, our experiments demonstrate unequivocally that indeed when strictly linear transport is used, the apparent low-temperature saturation of \( \tau_{\phi} \) is eliminated. However, the conditions to be in the linear transport regime are more strict than hitherto expected. Another novel result of the experiments is that introducing heavy non-magnetic impurities (gold) in our samples produces, even in linear transport, a shoulder in the dephasing rate at very low temperatures. We then show theoretically that low-lying local defects may produce a relatively large dephasing
rate at low temperatures. However, as expected, this rate in fact vanishes when $T \to 0$, in agreement with our experimental observations.

1. Introduction

Electronic quantum effects in mesoscopic [1] and in disordered conductors [2] are controlled by the conduction electrons’ dephasing [3] rate, $\tau_\phi$, which is expected to vanish with the temperature [4, 5]. A very intriguing apparent saturation of this dephasing rate in several systems was recently reported [6] at very low temperatures. Serious precautions [6] were taken to eliminate experimental artifacts. It was speculated that such a saturation of the dephasing rate when $T \to 0$ might follow from interactions with the zero-point motion of the environment. These speculations have received apparent support from calculations by Golubev and Zaikin [7], which generated a major controversy [8] in the recent literature. Interestingly, however, this issue had appeared already in 1988, and good arguments against dephasing by the zero-point motion have already been given then [9]. Moreover, these results were in disagreement with other experiments, for example, by Khavin et al. [10]. More recent experiments [11] showed that in some cases the presence of trace magnetic impurities, even on the ppm level, caused the apparent saturation of the low-temperature dephasing. Similar effects may exist for models [12, 13] with low-lying two-level systems (TLS) [14], where an apparent saturation of the low-temperature dephasing rate may occur (which will, however, be eliminated at the $T \to 0$ limit). For the case of magnetic impurities, such an elimination of the dephasing rate will occur if and when Kondo screening of the magnetic moments or their freezing into a spin-glass state takes place.

In fact, it is physically clear that since dephasing must be associated with a change of the environment state [5], it cannot happen as $T \to 0$, except when a large ground-state degeneracy occurs. In that limit neither the electron nor the environment has any energy to exchange. This is a very general statement; the only physical input needed for it to hold true is that both the interfering particle and its environment are in equilibrium at the temperature $T$ (which we then let approach zero). This is so because the linear transport under consideration is rigorously determined by equilibrium dynamic correlation functions. Obviously, nothing prevents a high-energy particle far from equilibrium to thermalize with the $(T \to 0)$ bath by giving it energy, losing its phase coherence in the process. Therefore, dephasing of a particle which is far from equilibrium with a $T = 0$ bath is, of course, possible. As will be
discussed later in this article, an example for such a situation occurs when the energy of the conduction electrons exceeds the thermal one, due to a voltage bias larger than $k_B T$ and slow relaxation [15].

In the theoretical part of the present paper we do not attempt to settle the important question of where have the calculations leading to $T \to 0$ dephasing without magnetic impurities or TLS gone wrong [16]. We shall start by converting the above physical argumentation for the lack of $T \to 0$ dephasing into a more rigorous one. We shall note that, like many other physical properties, the dephasing rate can be expressed in terms of correlation functions of the conduction electron and those of the environment with which it interacts [1]. Using very general properties of these correlators [17, 18], which are almost always valid, we prove following Refs. [19, 20, 12] that the dephasing rate has to vanish at the $T \to 0$ limit, unless a large ground-state degeneracy exists. Such a degeneracy may be brought about, e.g., by free uncompensated magnetic impurities at a vanishing magnetic field. Because these magnetic moments will typically be screened or frozen when $T \to 0$, the proof encompasses most systems of relevance. Since it employs mainly the basic laws of thermodynamics, the proof is valid for any case in which $\tau_\phi$ is determined from linear transport measurements.

Experiments were performed to examine the real-life validity of the above statement. Our experiments [15] demonstrate unequivocally that indeed when strictly linear transport is used, the apparent low-temperature saturation of $\tau_\phi$ is eliminated. Extremely small measurement currents had to be used in order to be in the linear-transport regime (see also Ref. [21]). These observations, along with the apparent lack of heating of the conduction electrons (see Ref. [6] and below), pose new and interesting basic questions.

Another novel result of the experiments is that introducing heavy nonmagnetic impurities (gold) into our samples produces, even in linear transport, an anomalously large dephasing rate at very low temperatures, but not at the $T \to 0$ limit. We show that low-lying local defects, as suggested for example in [12, 13], may produce a relatively large dephasing rate at low temperatures, which in fact vanishes when $T \to 0$.

2. **The vanishing of the dephasing rate as $T \to 0$: theory**

2.1 **A useful expression for the dephasing rate**

In this section, we shall derive a very useful expression for the dephasing rate of a “particle” coupled to the “environment”. The latter, which will also be referred to as “the bath”, represents all the degrees of freedom that the particle is coupled to and are not directly observed in the interference measurement.
As we show below, the dephasing rate can be expressed as

\[
1 / \tau_\phi = \frac{1}{\hbar^2 (2\pi)^3 V_{ol}} \int dq \int d\omega |V_q|^2 S_p(-q, -\omega) S_s(q, \omega), \tag{1.1}
\]

where \( V_q \) is the Fourier transform of the interaction \( V(r) \) between the conduction electron and the particles of the bath, and \( S_p(-q, -\omega) \) and \( S_s(q, \omega) \) are the dynamic structure factors of the conduction electron and the bath, respectively. These structure factors, which are the Fourier transforms of the density-density correlation functions [17], contain the necessary physical information on both the particle and the bath [18]. A subtle relevant example is provided by the case where the particle and the bath are identical fermions. The Pauli principle constraints are automatically taken into account by using the \( S_p(-q, -\omega) \) of the particle in the presence of the bath. These structure factors are in principle known for models of interest. They can, for example, be obtained from the dissipative part of the linear response function \( 1 / (\epsilon(q, \omega)) \), by using the fluctuation-dissipation theorem. The physical meaning of the expression in Eq. 1.1 is simply that the rate of creating (or annihilating) any excitation in the environment is the sum of these rates for all \((q, \omega)\) channels.

The dynamic structure factor is well known for a diffusing electron. In the classical limit, \( \hbar \omega \ll k_B T \), it is given by a Lorentzian of width \( Dq^2 \).

The low-temperature case [18] will be discussed later on. Replacing \( S_s(q, \omega) \) with the dynamic structure factor of the electron gas, given to leading order by \( \hbar q^2 \omega V_{ol} / (2\pi)^3 e^2 \sigma \) (see, e.g., Eqs. 3.28 and 3.44 of Ref. [22]), allows for an extremely simple calculation of the dephasing rate by electron-electron interactions, which reproduces the results of Ref. [4].

For a derivation of the basic equation 1.1, we start with a direct-product state of the particle and the environment, \(|im\rangle \equiv |i\rangle \otimes |m\rangle\), and evaluate the rate of transitions into all different possible states, \(|jn\rangle\), using the golden rule. In other words, at any later time \( t \), the state of the total system evolves into

\[
\Psi(t) = A \left[ |im\rangle + \sum_{j,n} \alpha_{jn}(t) |jn\rangle \right], \tag{1.2}
\]

where \( A \) is a normalization factor. The transition probability from \(|im\rangle\) is simply \(|A|^2 \sum_{j,n} |\alpha_{jn}(t)|^2\). At times larger than microscopic, the transition rate out of the initial state is well-known to be given by (see, for example, Ref. [23])

\[
1 / \tau_{\text{out}} = \frac{2\pi}{\hbar} \sum_{j,n} |\langle im | V |jn \rangle|^2 \int_{-\infty}^{\infty} d(\hbar \omega) \delta(E_{p,j} - E_{p,i} - \hbar \omega) \delta(E_{s,n} - E_{s,m} + \hbar \omega). \tag{1.3}
\]
Here the last integral represents the (joint) density of final states, \(|jn\rangle\), having the same energy as the initial one, \(|im\rangle\). The matrix elements in the last equation are easily evaluated from the Fourier representation of the interaction \(V(r)\):

\[
\langle im|V|jn\rangle = (2\pi)^{-3} \sum_s \int dq V_q \langle i|e^{i\mathbf{q}\cdot\mathbf{r}_p}|j\rangle \langle m|e^{-i\mathbf{q}\cdot\mathbf{r}_s}|n\rangle,
\]

where the index \(s\) runs over the particles in the bath. The absolute value squared of this matrix element consists of “diagonal” terms (\(q = q'\)) which are positive, and “nondiagonal” terms (\(q \neq q'\)) whose phases are random.

An important step is now to average the result over, for example, the impurity ensemble. This will eliminate all the nondiagonal terms, leaving only the diagonal ones. We now introduce a thermal averaging over the initial state, \(|im\rangle\), by summing over \(i\) and \(m\), with the factorized weight of that initial state, \(P_{p,i}P_{s,m}\), in obvious notation. It is immediately recognized that the integral in Eq. 1.3 contains the product of the dynamic structure factors of the particle and the environment. As a result, we obtain Eq. 1.1 for the rate \(1/\tau_{out}\).

We emphasize that this result is exact within the golden-rule formulation, which fully captures the decay of a given initial state into a continuum. Based on a single-particle picture, it equally applies to low-energy quasiparticle excitations in a Fermi liquid. Hence this result extends well beyond perturbation theory for the bare electrons in the system. It does not apply to systems that develop a non-Fermi-liquid ground state.

Using fashionable terminology, \(1/\tau_{out}\) is the rate at which the particle gets “entangled” with the environment. In most situations, \(1/\tau_{out}\) is identical to the dephasing rate \(1/\tau_\phi\). Important exceptions having to do with the infrared behavior of the integral in Eq. 1.1, relevant at lower dimensions, were discussed in Refs. [5, 1].

### 2.2 Proof that the dephasing rate vanishes at the \(T \to 0\) limit

As discussed above, an important advantage of the present formulation is that all the relevant physical information is contained within the correct dynamic structure factors of both the particle and the environment. For example, at \(T = 0\), when the electron is diffusing on the Fermi surface, it can not lower its energy. Hence its dynamic structure factor automatically vanishes for positive frequencies [18], as does the dynamic structure factor of the electron gas at \(T = 0\) [22]. These facts are guaranteed by the detailed balance condition:

\[
S(-\mathbf{q}, -\omega) = e^{\beta\hbar\omega} S(\mathbf{q}, \omega).
\]

(1.5)
More generally, because of the occurrence of $\omega$ and $-\omega$ in the two dynamic structure factors, the integrand in Eq. 1.1 vanishes in general at any $\omega$ at the $T \to 0$ limit, see Fig. 1. In mathematical terms, it has “no support”.

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**Figure 1.1.** The two structure factors appearing in Eq. 1.1 as functions of $\omega$, for $T = 0$ (schematic sketch). Note that the product vanishes everywhere.

Therefore, except for the unusual case where the environment has a massive ground-state degeneracy, the dephasing rate must vanish at zero temperature [12, 19, 20]. This fact follows directly from Eq. 1.3 also prior to carrying out the impurity-ensemble averaging. If both the particle and the environment start at their lowest states, it is impossible that both $E_{p,j} > E_{p,i}$ and $E_{s,n} > E_{s,m}$. All this is guaranteed by basic thermodynamics. The only exception is when a large ground-state degeneracy exists in the environment, a situation which is very rare indeed, because such a degeneracy will typically be lifted by some perturbation that exists in the system.

As mentioned above, this formulation breaks down in the case of a non-Fermi-liquid ground state, when the elementary excitations of the system are not of single-particle nature. A notable example is provided by the two-channel Kondo effect, where single-particle–to–single-particle scattering is absent on the Fermi level at $T = 0$. In fact, the corresponding $S$ matrix has no matrix element to any outgoing state containing arbitrary finite numbers of particle-hole pair excitations [24]. This means that any scattering at $T = 0$ necessarily leaves its mark on the environment, resulting in a finite zero-temperature de-
phasing rate [25]. We emphasize, however, that the two-channel Kondo effect itself stems from a ground-state degeneracy that cannot be lifted. Hence a macroscopic ensemble of two-channel Kondo impurities likewise requires a macroscopic degeneracy of the ground state.

Thus, the “standard model” of linear transport in disordered metals (in which the defects are strictly frozen) gives, as expected, an infinite \( \tau_\phi \) at \( T = 0 \). On the other hand, there may be other physical ingredients that can make \( \tau_\phi \) relatively short at very low temperatures (but still divergent at the \( T \to 0 \) limit), without contradicting any basic law of physics. What is needed is an abundance of low-energy modes in the environment. A simple model for such modes was suggested in Ref. [12]. Its physics is reminiscent of models for \( 1/f \) noise, but the relevant frequencies here are in the gigahertz range and above. That type of model is a particular one, and its requirements may or may not be satisfied in real samples. However, other models with similar dynamics might exist as well. We reiterate that such models do not imply dephasing by zero-point fluctuations. The explanation for the large low-temperature dephasing rate is certainly not universal. In some cases this extra low-temperature dephasing depends on sample preparation, and on extremely small concentrations of stray magnetic impurities [11]. In other cases [15] “nonequilibrium” behavior, i.e., the sample not being in the linear-transport regime, is the relevant issue.

3. Experimental results

Recent experiments on the behavior of the phase coherent time in indium-oxide films showed some intriguing features. These results did not confirm the claim [6] that there is ‘inherent saturation’ of \( \tau_\phi \). In fact, in all cases \( \tau_\phi \) diverged when \( T \to 0 \). On the other hand, several aspects of the data reproduce the findings of Mohanty et al. Most importantly, over a considerable range of measurement conditions (in particular, the electric field \( F \) applied in the magneto-resistance measurements), the dephasing rate was \( T \)-independent below 1K, while the resistance was \( T \)-dependent. The latter suggests that heating is not a serious problem in this range of fields, a conclusion reached by Mohanty et al. based on the same observation. We also agree with these authors that external noise is not likely to be the source of the apparent saturation. In the indium-oxide films, however, it appears that the saturation is due to non-equilibrium effects, namely, when the conductance is no longer given by the standard second-order current-current correlation function. Indeed, it was shown that the problem of apparent saturation disappeared when sufficiently small bias conditions were employed. It was also shown that in order to be in the linear-response regime, the electric field used in the magneto-resistance (MR) measurements must be smaller than \( F_c = k_B T/e L_{er} \), where \( L_{er} \) is the energy relaxation length. The energy relaxation length \( L_{er} \) is the spatial scale
over which the electrons lose their excess energy (gained from their motion in an electric field) to the environment. This length should not be confused with $L_\phi$, which is the phase-coherent diffusion length. Except when dominated by electron-phonon scattering, $L_{ef}$ varies much faster with $T$ than $L_\phi$, and may attain macroscopic values at low temperatures [26]. In the pure In$_2$O$_{3-x}$ samples, for example, $L_{ef}$ reached values of a few mm’s below 1K.

Another intriguing finding in our studies is the behavior of the dephasing rate versus temperature of the Au-doped samples. Figure 3 illustrates this behavior for one such sample that was extensively studied. These measurements were all performed in the linear-response regime, which was much easier to achieve than in the pure In$_2$O$_{3-x}$ samples due to the relatively short $L_{ef}$. Note that the dephasing rate is well behaved for $T > 2$K, and vanishes as $T \to 0$ (based on data for $T < 0.6$K). The intermediate temperature regime reveals, however, an anomaly; $\tau^{-1}_\phi$ seems to be almost independent of temperature. In fact, if the measurements were carried out only down to $T = 0.6$K, one might have concluded that $\tau_\phi$ has saturated! This behavior was observed in all our Au-doped samples (with doping levels of 1-3%), and it illustrates a new type of an apparent saturation problem. The overall shape of $\tau^{-1}_\phi(T)$ is somewhat similar to the respective behavior observed in Au films doped with Fe [21], and in Cu films doped with Cr [27]. Both are well-known Kondo systems, and the "hump" observed in their $\tau^{-1}_\phi(T)$ data was indeed interpreted as extra dephasing due to the Kondo effect. When we tried to repeat the analysis of these authors on our data, we encountered a number of difficulties. In the first place, to fit the excess dephasing rate with the formulae used by these authors required a spin of the order of 10 (rather than $\frac{1}{2}$ in their case), which makes no physical sense. More importantly, we failed to detect any independent evidence for magnetic impurities (above 1ppm), either in the sample or in the Au material that was used for doping [15]. In addition, there is strong evidence against dephasing by magnetic impurities in the MR data themselves. Consider the MR data shown in Fig. 2. The values of $\tau^{-1}_\phi(T)$ are obtained by fitting MR data to weak-localization theory, which usually is based on data taken at small magnetic fields. In the graphs of Fig. 2, however, we deliberately extended the MR measurements to include much larger fields. Note that in both graphs data are shown up to fields that are high enough to cause significant spin polarization (the Zeeman energy exceeds $k_BT$). Yet, a nearly perfect fit to the theory (dashed black lines) is obtained using one value of $\tau_\phi$ for each temperature. If there were a contribution from a spin-flip mechanism (as one may expect from the presence of magnetic impurities), it would be impossible to fit the low-field data (namely, for $H < k_BT/\mu_Bg$) with the same $\tau_\phi$ as the one necessary for $H > k_BT/\mu_Bg$. The difference that might be expected is illustrated in the top graph of Fig. 2 by the dotted line. The latter represents the MR that ought to be observed when the extra contribution to dephasing by
the alleged Kondo impurities is suppressed by $H$. It would therefore appear that the anomaly represented by the “hump” around $T = 0.6$K in Fig. 3 is not due to the usual spin-flip scattering, resulting from the presence of magnetic moments.

![Graphical representation of MR for In$_2$O$_{3-x}$:Au sample](image)

Figure 1.2. MR for In$_2$O$_{3-x}$:Au sample (thickness 200Å with 2% Au). Dashed lines are fits to theory using a single $\tau_0$ for each of the temperatures shown (one above and one below the anomaly).

We shall now attempt to explain the restricted saturation below 2K in these samples based on the observation that this anomaly originates form the inclu-
sion of gold atoms in the In$_2$O$_{3-x}$ matrix, and is absent in the pure material. As noted elsewhere [28, 29, 15], the Au atoms probably reside in the oxygen vacancy (or di-vacancy) sites of the In$_2$O$_{3-x}$ (which is typically 10% oxygen deficient, see Ref. [30]). Given the chemical inertness of gold, it is not implausible that a sizeable portion of the Au atoms are loosely trapped in oxygen di-vacancies, thus acting as local scatterers with a low characteristic frequency. For simplicity, we model such a defect as a local TLS having a typical energy $\Delta$ (associated with two nearly equivalent positions of the Au in the di-vacancy). The dephasing rate versus temperature due to this model will be calculated in the next section. It is shown to be consistent with our experiment in Fig. 3.

4. A tunnelling model for loosely bound heavy impurities

In this section, we consider the inelastic scattering of the conduction electrons from loosely bound defects. The defects are taken for simplicity to be independent Born-approximation s-wave scatterers, having a scattering length $a$ and a total scattering cross section $4\pi a^2$. The differential cross-section for inelastic scattering of a particle with momentum $k$ into an element of solid angle $\Omega$ around the final momentum $k'$ is given by [17]

$$\frac{\partial^2 \sigma}{\partial \Omega \partial \omega} = a^2 S(q, \omega) = a^2 \sum_{i,f} P_i |\langle f| e^{i q \cdot x} |i \rangle|^2 \delta(\omega - \omega_{if}),$$

where $\hbar \omega$ is the energy transfer, $\hbar q$ with $q = k - k'$ is the momentum transfer, and $S(q, \omega)$ is the dynamic structure factor of the scatterer. Here $|i\rangle$ and $|f\rangle$ are the initial and final states of the scatterer (the former having a probability $P_{p,i}$), and $\hbar \omega_{if}$ is their energy difference.

In the tunnelling model we take the scatterer to reside in a double-minimum potential. The minima are separated by a vector $b$, the tunnelling matrix element between the two minima is $\Omega_0$, and their energy separation is $2B$. By diagonalizing the $2 \times 2$ problem, one easily finds [31, 12] that the separation $2\Delta$ between the ground state and excited state in the well, $|+\rangle$ and $|−\rangle$, respectively, is given by

$$2\Delta = 2\sqrt{\Omega_0^2 + B^2}.$$  \hspace{1cm} (1.7)

The above labelling of the states reflects their spatial symmetry for $B = 0$. The transition matrix element is given in turn by

$$\langle + | e^{i q \cdot x} |i \rangle = 2i \alpha \beta \sin(q \cdot b/2) \cong i \alpha \beta (q \cdot b),$$

where $\alpha$ and $\beta$ are the normalized weights in the two wells. Their product is $\alpha \beta = \Omega_0/(2\Delta)$. The combination $2|\alpha \beta|$ is a symmetry parameter, ranging from unity for a symmetric well to zero for a very asymmetric one, rendering the latter ineffective for the inelastic scattering. To get the second equality in
Figure 1.3. Dephasing rate versus temperature for the same In$_2$O$_{3-x}$:Au sample as in Fig. 2. The dotted line is a fit to Eq. 13 with a symmetric well, using $n_s v_F \sigma_0 = 3.4 \cdot 10^{10} \text{sec}^{-1}$, $\Delta = 0.3$K, and adding the standard 2d result [4] for the present situation: $	au_{\phi}^{-1} = 5 \cdot 10^9 \cdot T \cdot \text{sec}^{-1}$, where $T$ is in degrees K. A better fit to the data (squares) may be obtained by using the result that would apply were the film behaving as in 3D [4]: $	au_{\phi}^{-1} = 4 \cdot 10^9 \cdot T^{3/2} \cdot \text{sec}^{-1}$ (where $T$ is again in degrees K), for the high-temperature regime (dashed line).

Eq. 1.8, we used the dipole approximation $q \cdot b/2 \ll 1$, which is appropriate for $k_F b \ll 1$. For simplicity, we took $b$ to be sufficiently large as compared to the characteristic length of each well. We shall also assume $\epsilon_F \gg \Delta, k_B T$.

The inelastic cross section for scattering between the two levels of the tunnelling center is given by

$$\sigma_{\text{in}}(q, \omega) = 4 \alpha^2 \beta^2 \frac{a^2}{\pi} \sum_{\gamma=\pm} P_{\gamma} \sin^2\left(\frac{q \cdot b}{2}\right) \delta(\omega + 2\gamma \Delta). \quad (1.9)$$
Here $P_{\pm}$ are the thermal populations of the $|\pm\rangle$ states:

$$P_{\pm} = \frac{e^{\pm \Delta / (k_B T)}}{2 \cosh(\Delta / (k_B T))}.$$  \hfill (1.10)

For simplicity we consider an electron with an initial momentum $k$ very close to the Fermi sphere, i.e., $\epsilon_k \ll k_B T$, where the kinetic energy $\epsilon_k$ is measured relative to the chemical potential. The total inelastic cross sections for an upwards/downwards excitation of the TLS are given by

$$\sigma_{\pm} = \alpha^2 \beta^2 a^2 P_{\pm} \int d\Omega_{k'} \int d\epsilon_{k'} ((k - k') \cdot b) \frac{2}{1 - f(\epsilon_{k'} - \epsilon_k \mp 2\Delta)},$$  \hfill (1.11)

where $d\Omega_{k'}$ is an element of solid angle around the final wave vector $k'$, and $\epsilon_k = \hbar^2 k^2 / 2m$. For clarity we take the initial wave vector $k$ to be parallel to $b$ for the time being. Averaging over the direction of $k$ will introduce a numerical factor $\lambda$, which we shall reinstate later on.

To proceed with Eq. 1.11, we note that $(k - k') \cdot b$ equals $2k_F b \sin^2(\theta/2)$, where $\theta$ is the angle between $k'$ and $b$. Performing the angular integration and the integral over the energy, we obtain

$$\sigma_{\text{in, tot}} = \frac{16 \sigma_0 (\alpha \beta)^2}{\cosh^2(\Delta / (k_B T))} \left[ P_+(1 - f(\epsilon_k - 2\Delta)) + P_-(1 - f(\epsilon_k + 2\Delta)) \right] =$$

$$= \frac{4 \sigma_0 (\alpha \beta)^2}{\cosh^2(\Delta / (k_B T))}. \hfill (1.12)$$

The prefactor $\sigma_0$ in Eq. 1.12 is given by $\sigma_0 \equiv \frac{\pi}{4} \lambda a^2 (k_F b)^2$, and is expected to be of the order of the square of a small fraction of an Angstrom. For a concentration $n_s$ of the soft impurities, the rate for inelastic scattering is thus given by

$$\frac{1}{\tau_{\text{in}, s}} = \frac{4(\alpha \beta)^2 n_s v_F \sigma_0}{\cosh^2(\Delta / (k_B T))}, \hfill (1.13)$$

where $4(\alpha \beta)^2 = 1$ in the symmetric case ($B = 0$). Note that the situation here is rather distinct from the one for the electron-electron scattering with disorder, where the scattering is dominated by small $q$'s (the infrared regime). Since the scatterers are short ranged, the important range of $q$ is $q \ell \gg 1$ for $k_F \ell \gg 1$, as in Ref. [12]. In this range, the dynamics of the electrons is effectively ballistic. For the same reason, the inelastic rate and dephasing rate are essentially equal [32].

The parameters of the various TLS’s within the system, are often distributed. Reasonable distributions are [12]: a uniform distribution for $B$ in the range $0 \leq B \leq B_{\text{max}}$, and a $1/\Omega_0$ distribution for $\Omega_0$, between $\Omega_{\text{min}}$ and $\Omega_{\text{max}}$. The latter distribution follows by taking $\Omega_0$ to be the exponential of a large
negative, uniformly distributed quantity in the corresponding range. One generally expects $\Omega_{\text{max}} \ll B_{\text{max}}$. Thus, the combined distribution function reads

$$P(B, \Omega_0) = \frac{1}{\Omega_0 B_{\text{max}} \ln(\Omega_{\text{max}}/\Omega_{\text{min}})}.$$  

(1.14)

Figure 1.4. The inelastic cross-section of the TLS as a function of $k_B T$. (a) A single TLS (Eq. 1.12) with $B = 3$ and $\Omega_0 = 1$ (all energies in the same units). (b) The cross section averaged over the distribution of Eq. 1.14, with $B_{\text{max}} = 20$, $\Omega_{\text{min}} = 0.2$, and $\Omega_{\text{max}} = 2$. Note the qualitative similarity between these results and the hump of Fig. 3. Adding the electron-electron contribution as in Ref. [4] produces a reasonable fit of the experimental results with a TLS model, see Fig. 3.

The inelastic cross section of a single TLS, and the one averaged over the distribution of Eq. 1.14, are depicted in Fig. 1.4. For the TLS distribution of Eq. 1.14, the following qualitative behavior of the averaged cross section $\langle \sigma_{\text{in}} \rangle$ is found:

$$\langle \sigma_{\text{in}} \rangle \propto e^{-2\Omega_{\text{min}}/(k_B T)} \quad \text{for} \quad k_B T \ll \Omega_{\text{min}};$$

$$\langle \sigma_{\text{in}} \rangle \propto T \quad \text{for} \quad \Omega_{\text{min}} \ll k_B T \ll \Omega_{\text{max}};$$

$$\langle \sigma_{\text{in}} \rangle \propto \text{const.} \quad \text{for} \quad k_B T \gg \Omega_{\text{max}}.$$  

(1.15)

The behaviors of Eq. 1.15 are in agreement with curve (b) of Fig. 1.4.

Strictly speaking, these results hold only for temperatures sufficiently low so that the higher levels of the double-minimum well are thermally inaccessible. The constant nature of the inelastic rate for $k_B T \gg \Delta_{\text{min}}$ was invoked in
Ref. [12] to explain the apparent saturation of the dephasing rate. This necessi-
tates $\Delta_{\text{max}} \approx 0.1K - 0.5K$, as would seem appropriate for heavy defects. As pointed out in Ref. [12], the dephasing rate will then vanish linearly with $T$ at lower temperatures. If the lower cutoff $\Omega_{\text{min}}$ exists and is attainable, the TLS dephasing rate should eventually vanish exponentially, as specified above. To remove any doubt, we reemphasize that models without a large enough ground-state degeneracy do not have a saturation of $\tau_\phi$ at the $T \to 0$ limit.

5. Conclusions

We showed that in normal systems, that do not have large ground state de-
generacies, the quasiparticle dephasing rate must vanish at the $T \to 0$ limit. Abundance of low-energy excitations can, however, produce a relatively large dephasing rate at low nonzero temperatures. An appropriate TLS model can explain an intriguing feature of our experimental results below 1K, obtained by controlled addition of heavy impurities. An apparent low-temperature sat-
uration of the dephasing rate can also be due to magnetic impurities, as long as their magnetic moments are uncompensated and unfrozen. We also find experimentally that the condition to be in the linear-transport regime at very low temperatures is much more strict then ordinarily expected. Not reaching the linear-transport regime might also produce an apparent “nonequilibrium” saturation. More theoretical work is necessary in order to fully understand this last result.

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