Lepton electric and magnetic dipole moments
induced by a vector unparticle

A. Moyotl\textsuperscript{1}, A. Rosado\textsuperscript{1}, G. Tavares-Velasco\textsuperscript{2}

\textsuperscript{1}Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal 72570
Puebla, México
\textsuperscript{2}Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla,
Apartado Postal 1152, Puebla, Pue., México
E-mail: amoyotl@sirio.ifuap.buap.mx

Abstract. We calculate the magnetic dipole moment (MDM) and the electric dipole moment (EDM) of a lepton induced by a vector unparticle with both vector and axial-vector couplings to leptons. We consider the most general scenario in which the unparticle induces lepton flavor violation (LFV) and CP violation. Some specific scenarios are examined to obtain constraints on the LFV unparticle couplings from the current limits on the muon MDM and the decay $\tau \to 3\mu$. While the experimental limit on the muon MDM favors the scenario in which there is dominance of the unparticle vector couplings over the axial-vector couplings, the experimental limit on the $\tau \to 3\mu$ decay strongly constrains the unparticle LFV couplings. We use these constraints to estimate the EDMs of the electron and the muon, which are negligible and far from the current experimental limits.

1. Introduction
Motivated by the Banks and Zaks model [1], Georgi conjectured recently the existence of a hidden scale invariant sector in the high energy theory that could interact with the standard model (SM) [2, 3]. The effects of such a hidden sector could show up at an energy scale of the order of a few TeVs. Since physical particles cannot exist in such a scale invariant sector, it would interact with the SM fields through scale invariant stuff known as unparticles. The details of such a theoretical framework are unknown, but it is possible to study its potential low energy effects via an effective field Lagrangian. The appropriate theoretical framework to describe unparticle physics is the one introduced in Ref. [1]. The hidden sector is a Banks and Zaks ($BZ$) sector and the associated fields are described by renormalizable $O_{BZ}$ operators. It is assumed that these fields interact with the SM fields through the exchange of heavy particles at a very high energy $M_{U}$. Below this energy scale there emerge nonrenormalizable couplings between the fields of the $BZ$ sector and the SM fields, which generically can be written as $O_{SM}O_{BZ}/M_{U}^{d_{SM}+d_{BZ}-4}$. Dimensional transmutation occurs through the renormalizable couplings of the $BZ$ sector at an energy scale $\Lambda_{U}$ as scale invariance emerges. At low energies, an effective theory can be used to describe the interactions between the SM fields and the $BZ$ fields, which are associated with unparticles. The effective Lagrangian can be written as [2, 3]:

$$L_{UL} = C_{\mathcal{O}_{UL}} \frac{M_{U}^{d_{BZ}-d_{UL}}}{M_{UL}^{d_{SM}+d_{BZ}-4}} O_{SM}O_{UL},$$

(1)
where $C_{OU}$ stands for the coupling constant and the operator dimension $d_U$ is fractional. Unparticle operators $O_U$ can be constructed from the nature of the primary operator $O_{BZ}$ and its transmutation. The Lorentz structure of these operators can be of scalar, vector, spinor or tensor type.

In this work we are interested in the spin-1 unparticle contribution to the magnetic dipole moment (MDM) and the electric dipole moment (EDM) of leptons in the most general case when there is lepton flavor violating (LFV) interactions.

The effective interactions of a spin-1 unparticle with a fermion-antifermion pair can be written as \[ \mathcal{L}_{UL} = \frac{\lambda^{ij}_{V}}{A^{dU}_{Q}} \bar{f}_i \gamma_{\mu} f_j O^\mu_U + \frac{\lambda^{ij}_{A}}{A^{dU}_{Q}} \bar{f}_i \gamma_{\mu} \gamma^5 f_j O^\mu_U. \] (2)

where $i$ and $j$ stand for the family index and $\lambda^{ij}_{V,A}$ stands for the associated coupling constant.

Due to the invariant scale nature of unparticles, their propagators can be constructed by means of unitary cuts and the spectral decomposition formula. It can be shown that the propagator of a spin-1 unparticle is

\[ \Delta_{\mu\nu}(p^2) = \Delta_F(p^2) \left( -g_{\mu\nu} + a \frac{p^\mu p^\nu}{p^2} \right), \] (3)

with

\[ \Delta_F(p^2) = \frac{A_{dU}}{2 \sin(d_U \pi)} (-p^2 - i\epsilon)^{dU-2}. \] (4)

The $A_{dU}$ function, which is meant to normalize the spectral density, is given as follows:

\[ A_{dU} = \frac{16\pi^2 \sqrt{\pi}}{(2\pi)^{2dU} \Gamma(d_U - 1) \Gamma(2d_U)} \] (5)

If the spin-1 unparticle is taken as transverse, $p_{\mu} p^{\mu}(p) = 0$, which translates into the condition $a = 1$. However, in the context of a specific conformal invariant theory, $a = 2(d_U - 2)/(d_U - 1)$ [4].

\[ 2. \text{ Unparticle contribution to electric and magnetic dipole moments of leptons} \]

The spin-1 unparticle contribution to the electromagnetic vertex arises through the Feynman diagram shown in Fig. 1. The lepton MDM is given by [5]:

\[ d^{dU}_j = \sum_{J=V,A} \sum_{i=e,\mu,\tau} |\lambda^{ij}_{V,A}|^2 F_J(m_i, d_U), \] (6)

where $j$ is the flavor index and the $F_J$ functions can be written as

\[ F_J(m_i, d_U) = \frac{A_{dU}}{16\pi^2 \sin(\pi d_U)} \left( \frac{m_i^2}{\Lambda^2_U} \right)^{dU-1} f_J(\sqrt{r_i}, d_U), \] (7)

with $\sqrt{r_i} = m_j/m_i$ and

\[ f_{V}(z, d_U) = \frac{z}{2 - d_U} \int_{0}^{1} dx \left( (1 - x)^{d_U-1} - x^{2-dU} (1 - z^2 x)^{dU-3} \right) + z (3(d_U - 1)x + d_U - 3) + z^2 ((1 + x)^2 d_U + (x - 5)x - 2) - z^3 (3(d_U - 1)x + d_U - 3). \] (8)
also \( f_A(z, d_U) = f_V(-z, d_U) \).

As for the lepton EDM, which can only arise if both \( \lambda_{ij}^V \) and \( \lambda_{ij}^A \) are nonzero and have an imaginary phase, it is given by [5]:

\[
d_U^j = \sum_{i=e, \mu, \tau} \text{Im} \left( \lambda_{ij}^V \lambda_{ij}^A \right) G(m_i, d_U),
\]

with

\[
G(m_i, d_U) = \frac{e A_{d\ell}}{32\pi^2 \sin(\pi d_U) m_i} \left( \frac{m_i^2}{\Lambda_U^2} \right)^{d_U - 1} g(\sqrt{r_i}, d_U)
\]

and

\[
g(z, d_U) = \frac{1}{2 - d_U} \int_0^1 dx \left( 1 - x \right)^{d_U - 1} x^{2 - d_U} \left( 1 - z^2 x \right)^{d_U - 3} \left( 4(1 - z^2 x)(2 - d_U) + (1 - 3x) + z^2 ((x^2 - 1)d_U + (x - 1)x + 2) \right).
\]

Below we will analyze the constraints on the LFV unparticle couplings from the muon EDM and LFV decays.

3. Numerical analysis and discussion

3.1. Muon anomalous magnetic moment

The muon MDM, \( a_\mu \), has been measured with an impressive accuracy of 0.54 ppm. The current world average, which is dominated by the measurements of the E281 collaboration at BNL, is given by [6]

\[
a_\mu^{\text{Exp.}} = 116592089 (63) \times 10^{-11} \quad (0.54 \text{ ppm}),
\]

where the statistical and systematic errors, 0.46 ppm and 0.28 ppm, have been added in quadrature. On the other hand, the SM theoretical prediction, which is still under scrutiny as a large uncertainty arises from the hadronic contribution, is given by [7]

\[
a_\mu^{\text{SM}} = 116591834 (48) \times 10^{-11}.
\]

where the \( \sigma(e^-e^+ \rightarrow \text{hadrons}) \) data have been used to calculate the leading-order hadronic vacuum polarization contribution [8]. There is thus a discrepancy between the experimental
and theoretical predictions larger than 3.6 standard deviations. It may be however that such a discrepancy will reduce to an acceptable level once the hadronic contribution is determined with best accuracy. As long as this disagreement is attributed to new physics, the allowed region for this class of contributions, with 95% C.L., is $\Delta a_\mu = (255 \pm 1.96 \times 80) \times 10^{-11}$.

We will consider some specific scenarios to obtain constraints on the LFV unparticle couplings. We will assume that there is a hierarchy in the LFV unparticle couplings as it is assumed in other models of LFV, i.e., $|\lambda^{e\mu}| < |\lambda^{e\tau}| < |\lambda^{\mu\tau}| \ll \lambda^\nu$ for all the unparticle couplings. Also, we will consider that there are no large cancellations between distinct new physics contributions. Since experimental data strongly constrains LFV between the muon and the electron, we will concentrate on the scenario in which LFV transitions occur dominantly between the muon and tau leptons.

We first evaluate the contribution of the loops with internal muon and tau leptons to $a_\mu^{dU}$. The results are shown in Fig. 2 as functions of the dimension $d_U$ and for $\Lambda_U = 1$ TeV. While the contributions from vector couplings are positive, the axial-vector contributions are negative, with the contributions from axial-vector couplings being slightly larger in magnitude than the vector contributions. It means that, as long the vector and axial-vector unparticle couplings are of similar order of magnitude, the total contribution to the muon MDM could be negative. This is a scenario disfavored by the current experimental data, which requires a positive contribution to $a_\mu^{dU}$ from new physics. The situation is similar to that observed in the case of the scalar and pseudoscalar contributions [5, 9], although in this case the scalar contributions are slightly larger in magnitude than the pseudoscalar contributions. We thus conclude that the most favored scenario is that in which the strength of the axial-vector couplings is smaller than that of the vector couplings. In this way, we obtain a positive contribution to $a_\mu^{dU}$.

![Figure 2](image_url)

**Figure 2.** Contribution from each internal lepton loop to the muon MDM due to vector (V) or axial-vector (A) unparticle couplings as a function of the $d_U$ dimension and for $\Lambda_U = 1$ TeV. The absolute values of the (negative) (A) contributions are shown. The allowed region with 95% C.L. for new physics lies between the horizontal lines. The contribution from vector and axial-vector tau loops are indistinguishable.

If the current discrepancy between the theoretical SM prediction and the experimental value of the muon MDM is assumed to be due entirely to a spin-1 unparticle, we obtain the allowed area on the $|\lambda^{\tau\mu}|$ vs $|\lambda^{\mu\tau}|$ plane shown in Fig. 3. For $\Lambda_U$ and $d_U$ we used values that are consistent with the bounds obtained on the scale $\Lambda_U$ from mono-photon production at LEP [10]. As inferred from Fig. 3, the experimental data allow a magnitude of the vector couplings as
large as unity for $d_U \simeq 2$ and $\Lambda_U = 1$ TeV, whereas the most strong constraints are obtained for $d_U$ close to unity. In general the limits on $|\lambda_V^{\mu\tau}|$ are slightly stronger than the limits on $|\lambda_V^{e\mu}|$, which is in agreement with our assumption. We also note that the bounds on the vector and axial-vector unparticle couplings are of similar order of magnitude than the bounds on the scalar and pseudoscalar unparticle couplings [5, 9].

![Graph](image)

Figure 3. Allowed area on the $|\lambda_V^{\mu\tau}|$ vs $|\lambda_V^{e\mu}|$ plane consistent with the experimental limit on the muon MDM with 95% C.L. We used the indicated values of $\Lambda_U$ and $d_U$.

3.2. Decay $\tau \to 3\mu$

Apart from the muon MDM, constraints on LFV couplings can also be obtained from LFV processes involving the muon, which are strongly constrained by the experiment. The current constraints on LFV muon decays are $\text{BR}(\mu \to e\gamma) < 2.4 \times 10^{-12}$ [11] and $\text{BR}(\mu \to 3e) < 1.0 \times 10^{-12}$ [12]. On the other hand, less stringent constraints exist for LFV $\tau$ transitions: $\text{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ [13], $\text{BR}(\tau \to 3e) < 3.6 \times 10^{-8}$ [14], etc. We will now show that the $\tau \to 3\mu$ decay places strong constraints on the $\lambda_V^{\mu\tau}$ and $\lambda_V^{e\mu}$ couplings. If we neglect the axial-vector contributions, the $\tau \to 3\mu$ branching ratio can be written as follows [5]:

$$
\text{BR}(\tau \to 3\mu) = \frac{m_\tau m_\tau}{2^8 \pi^3} \left| \frac{A_{dU}}{\sin(d\pi)} \right|^2 \left( \frac{m_\tau}{\Lambda_U} \right)^{4(dU-1)} |\lambda_V^{\mu\tau}|^2 |\lambda_V^{e\mu}|^2 \eta_1 \left( \frac{m_\mu}{m_\tau}, d_U \right),
$$

with $\tau_\tau$ the tau mean life, while the $\eta_1$ function is

$$
\eta_1(z, d_U) = \frac{1}{2} \int dx_1 \int dx_2 \left( 2q_1^{2(dU-2)} ((z+1)(3z-1)x_1 + x_2^2 + 2(x_1 + x_2 - 1)x_2 \\
+ (6z-1)x_2^2) + q_1q_2^{dU-2} (x_1 + x_2 + z - 1)(x_1 + x_2 + (3z+1)z) \\
+ (x_1 \leftrightarrow x_2) \right),
$$

(14)
where \( q_i = x_i + z^2 \). The integration area on the \( x_1 \) vs. \( x_2 \) plane is

\[
3s^2 \leq x_1 \leq 1 - 2s, \\
x_2 \leq \frac{1}{2} \left( 1 - x_1 \pm \sqrt{\frac{(1 - x_1)^2 - 4s^2}{x_1 + s^2}} \right),
\]

with \( s = m_\tau / m_\mu \).

The \( \tau \to 3\mu \) branching ratio is constrained to be lower than about \( 10^{-8} \). When we combine this bound with the one on the muon MDM, we find strong constraints on the LFV unparticle coupling constants. We show in Fig. 4 the allowed area on the \( \lambda^V_\tau \) vs \( \lambda^V_\mu \) plane for several values of \( \Lambda_U \) and \( d_U \). We observe that the \( \lambda^V_\tau \) parameter is tightly constrained for \( d_U \) close to unity, where \( \lambda^V_\mu \) is allowed to be considerably larger. There is also an area in which the opposite is true (\( |\lambda^V_\tau| \gg |\lambda^V_\mu| \)), but we will not consider it as conflicts with our previous assumption. Again, our bounds on the vector unparticle couplings are of similar order of magnitude than the ones obtained on the scalar unparticle couplings [5, 9].

3.3. Electric dipole moment

In the SM, the electron EDM is predicted to be negligibly small as it arises up to the three-loop level via the Cabbibo-Kobayashi-Maskawa (CKM) phase. A large fermion EDM, \( d_f \), would thus be a clear indication of new sources of CP-violation [15]. The current experimental limit on the electron EDM with 90% C.L. is [16]:

\[
|d_e| \leq 1.6 \times 10^{-27} \text{ e cm},
\]

Figure 4. Allowed area on the \( |\lambda^V_\tau| \) vs \( |\lambda^V_\mu| \) plane consistent with the experimental constraints on the muon MDM and the \( \tau \to 3\mu \) decay when the dominant contribution to these observables arise from the vector unparticle couplings.
whereas the experimental limits on the positive and negative muon EDMs with 95% C.L. are:

\[ |d^+_\mu| \leq 2.1 \times 10^{-19} \text{ e-cm}, \]  
\[ |d^-_\mu| \leq 1.5 \times 10^{-19} \text{ e-cm}. \]  

We will get an estimate of the order of magnitude for the electron and muon EDMs induced by a spin-1 unparticle. Apart from the magnitude of the $|\lambda_{ij}^{V,A}|$ parameters, the EDM depends on the associated CP-violating phases, so its analysis is even more complicated. Depending on the relative sign of the CP violating phases, the partial contributions can add coherently or destructively. We will content ourselves with considering a somewhat restrictive scenario, which will allow us to get an estimate of the order of magnitude of the EDM. First of all, as was the case for the muon MDM, we will assume that there is no large cancellation between different contributions to the EDM, so we can analyze each contribution separately. Also, to be consistent with the previous discussion, we will assume the following hierarchy $|\lambda_{ij}^{V}| \gg |\lambda_{ij}^{A}|$ and $|\lambda_{ij}^{T}| \gg |\lambda_{ij}^{\mu}|$. It means that the EDM would be entirely driven by the tau loop contributions. In view of this scenario, we numerically evaluate the tau loop contributions to Eq. (9). The results are shown in Fig. 5. We observe that the contribution from the tau loop to the electron and muon EDMs are indistinguishable, which means that the EDM is entirely controlled by the magnitude of the coupling constants and the imaginary phases.

![Figure 5. Contribution from the tau lepton loop to the electron EDM due to vector-axial unparticle couplings as a function of the $d_U$ dimension and for $\Lambda_U = 1$ TeV. This contribution is negative and thus its absolute value is shown. There is no distinction from the tau contribution to the muon EDM.](image-url)

We now consider the constraints on the coupling constants obtained above to estimate the muon EDM in our working scenario. The spin-1 unparticle contribution is of the order of $10^{-17}$ e-cm for $d_U \simeq 1.1$ and $10^{-21}$ e-cm for $d_U \simeq 1.6$. The allowed magnitude of the $|\lambda^{\mu T}_V|$ parameter is of the order of $10^{-5}$ for $d_U = 1.1$ and $10^{-3}$ for $d_U = 1.6$. A good assumption for the $|\lambda^{\mu T}_A|$ coupling is that its magnitude is about one or two orders of magnitude below $|\lambda^{\mu T}_V|$, which is consistent with our previous discussion. Therefore the muon EDM is about $|d^\mu_U| \simeq |\sin \delta_{(V,A)}^{\mu T}| \times 10^{-20}$ e-cm for $d_U \simeq 1.1$, whereas $|d^\mu_U| \simeq |\sin \delta_{(V,A)}^{\mu T}| \times 10^{-29}$ e-cm for $d_U \simeq 1.6$. Here $\delta_{(V,A)}^{\mu T}$ is the relative CP-violating phase between the vector and axial-vector couplings. These results are several
orders of magnitude smaller than the experimental limit on the muon EDM. The magnitude of the coupling constants for LFV transitions involving the electron are expected to be more suppressed as long as the hierarchy discussed above is respected. Then we can estimate that the electron EDM induced by vector unparticles is well below the $10^{-30}$ e-cm level and far from the experimental measurement.

4. Concluding remarks
The lepton magnetic and electric dipole moments were calculated in the framework of unparticle physics assuming interactions mainly driven by a spin-1 unparticle with both vector and axial-vector couplings to leptons. Analytical expressions were found that agree with previous calculations for the muon MDM in the limit of flavor conserving couplings.

The muon MDM was numerically evaluated to obtain bounds on the coupling constants from the experimental measurements assuming that LFV is mainly dominated by LFV couplings involving the muon and tau leptons. It was found that the latest experimental data for the muon MDM favor a scenario in which the spin-1 unparticle contribution is dominated by the vector couplings since the contribution from axial-vector couplings is negative. It is also found that the decay $\tau \to 3\mu$ can constrain considerably the allowed magnitude of the $\lambda_V^{\mu\mu}$ and $\lambda_V^{\tau\tau}$ couplings.

As far as the EDM is concerned, since it depends on several free parameters, we content ourselves with estimating its order of magnitude. It is found that the unparticle contributions are well below the experimental limits of the electron and muon EDMs.

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