Model uncertainty in concrete breakout resistance of single anchors in shear – assessment of ACI 318 against available alternative

O B Olalusi1* and M Akinlolu1

1Department of Civil Engineering, University of KwaZulu-Natal, Durban, South Africa
dimejibenlusty@yahoo.co.uk, olalusio@ukzn.ac.za

Abstract. This contribution characterises the model uncertainty related to the shear breakout resistance of anchors located towards the edge of concrete members, considering experiments obtained from previous anchor test. The characterisation includes the computation of the sample moments (mean, standard deviation and skewness) required in reliability analysis. The study analysed the predictive models of current design guidelines (ACI 318 and the modified concrete capacity design method [MCCD] method). The assessment demonstrates that ACI 318 has a higher bias and variability compared to the MCCD method. The influence of the breakout resistance parameters on model uncertainty was evaluated, and significant parameters were identified. The edge distance and the anchor diameter are found to be the most influencing parameters. Also, a suitable theoretical probability distribution is required in reliability analysis for the model uncertainty random variable. Accordingly, it is shown that the log-normal distribution is the most appropriate probability distribution for the derived model uncertainty factors.

Keywords: Model uncertainty, Fasteners, Anchors, Modified CCD, ACI 318

1. Introduction
Fastening/anchorage technology is becoming increasingly important in the construction industry. The increased attention given to the use of fasteners in concrete is motivated by the growing interest in strengthening of existing structures, earthquake retrofitting, preservation of the historic built environment and the use of prefabricated structural elements [1]. Significant efforts are currently invested towards establishing a unified design framework concerning the design of anchors for use in concrete. The ACI 318 [3] and the EN 1992-4 [2] are the most recent and globally accepted design guidelines. Anchors loaded in shear may fail by steel rupture, pry-out failure, pull-out of the anchor, crushing of concrete and concrete breakout. The latter failure mode is investigated in this study.

Concrete related failures pose a critical safety issue since they are quasi-brittle, and as such, may develop swiftly without preceding signs of damage. There is a limited possibility for remediation action and avoidance of complete system failure. Hence, fastening design provisions should be reliable and accurate. This consequently leads to a particular interest in the adequate assessment of the reliability of the current fastening design provisions in the case of concrete breakout failure. To that end, proper characterisation and description of model uncertainty statistics is a necessary prerequisite, since these are required as input in reliability analysis [4]. The presented contribution focuses on the characterisation of model uncertainties in the breakout resistance prediction of shear-loaded anchors, towards the
reliability-based design of fastenings to concrete. The uncertainty related to the ACI 318 model [3] is evaluated and compared to that of the modified concrete capacity design (MCCD) method [5].

2. Model uncertainty
Model uncertainty can be understood as a description of the model's deficiency in accurately describing a physical phenomenon (in the present case, the anchor's resistance) due to lack of knowledge in the problem description, or mathematical simplifications [4]. In the realm of structural reliability, it is expressed as a random variable accounting for the abovementioned effects [4]. The model uncertainties related to the ACI 318 and MCCD concrete edge breakout models are investigated in this section, using the experimental database of 257 anchor tests presented in Section 4. The model uncertainty associated with a single experiment \( x \) is obtained as the ratio of the experimental \( V_{exp,x} \) to predicted shear capacity \( V_{pred,x} \), expressed in Equation 1 [4]. Apparently, model uncertainty \( R_x = 1 \) is a condition for an ideal model, \( R_x > 1 \) implies that the model underpredicts, and \( R_x < 1 \) that it overpredicts the actual concrete breakout resistance [6].

\[
R_x = \frac{V_{exp,x}}{V_{pred,x}(X)}
\]

where, \( V_{exp,x} \) is the failure load for an anchor test \( x \) in shear. \( X \) denotes the input variable. \( V_{pred,x} \) is the predicted mean breakout resistance for the same anchor test \( x \).

3. Methods for predicting the breakout capacity of anchors in shear
The mean shear capacity of single anchors located towards the edge of non-cracked concrete members according to ACI 318 and MCCD method is obtained based on Equation 2 [7, 8] and Equation 3 respectively [5].

\[
V_{pred}(X) = V_{ACI} = \min\left\{ \left( \frac{l_f}{d_o} \right)^{0.2} \cdot \sqrt{d_o} \sqrt{f_{cm}(c_1)}^{1.5}, 7.1 \cdot \sqrt{f_{cm}(c_1)}^{1.5} \right\}
\]

\[
V_{pred}(X) = V_{MCCD} = 3 \cdot d_o^\alpha \cdot l_f^\beta \cdot \sqrt{f_{cm} \cdot c_1^{1.5}}
\]

where

\[
\alpha = 0.1 \cdot \left( \frac{l_f}{c_1} \right)
\]

\[
\beta = 0.1 \cdot \left( \frac{d_o}{c_1} \right)^{0.2}
\]

where \( d_o \) is the diameter of the anchor, \( f_{cm} \) is the mean concrete strength, \( l_f \) is the influence length. \( c_1 \) is the edge distance.

4. Characteristics of the experimental database of anchors tests
The anchor database considered in this study was assembled from previous studies published in the literature [9-15]. It contains the observed experimental failure load from 257 anchor tests for different types of shear loaded single anchors in uncracked concrete. The database captured concrete strength \( f_{cm} \) in the range of 29 – 39 MPa, anchor diameters \( d_o \) in the range of 8 – 40 mm and influence length \( l_f \) in the range of 40 – 450 mm (Table 1). All anchors in the database failed due to concrete edge breakout. More elaborate details of the database of the anchor test can be found in [6].
Table 1. Range of design parameters for the database.

| Parameters                             | Minimum   | First Quartile (P25) | Median (P50) | Third Quartile (P75) | Maximum |
|----------------------------------------|-----------|----------------------|--------------|----------------------|---------|
| Concrete cylinder compressive strength $f_{cm}$ (MPa) | 29        | 30.3                 | 32.7         | 33.6                 | 39.5    |
| Influence length $l_f$ (mm)            | 50        | 110                  | 163          | 313                  | 450     |
| Diameter of anchor $d_o$ (mm)           | 8         | 16                   | 24           | 25                   | 40      |
| Concrete edge distance $c_1$ (mm)       | 50        | 72                   | 100          | 160                  | 320     |
| Shear breakout capacity $V$ (kN)        | 8.2       | 19.1                 | 35           | 66.9                 | 169     |

5. Statistical analysis of the model uncertainty

For each experiment, the best estimate/mean resistance is evaluated using Equation 2 and 3. The comparison of the models to experimental results, as shown by Figure 1 indicate the difficulty of the accurate prediction of the breakout capacity for anchors in shear. The resistance predictions of the different methods vary from the experimentally observed concrete breakout strength. Given the variations, quantification of model uncertainty for the various models is required. The model uncertainty is evaluated from Equation 1 and the main statistical properties such as the mean ($\mu_R$), the standard deviation (dispersion measures) ($\sigma_R$), coefficient of variation ($\Omega_R$) and skewness ($\eta_R$) are reported in Table 2. As shown in the table, the models generally overestimate the actual breakout resistance. The skewness coefficients of the models are nearer to what is anticipated from a positively skewed distribution such as log-normal distribution.
Figure 1. Comparison of resistance predictions from (a) ACI 318 model and (b) MCCD model to experimental values

Table 2. Statistical properties of model uncertainty

| Model Uncertainty symbol | ACI 318 | MCCD |
|--------------------------|---------|------|
| Mean ($\mu_R$)           | 0.87    | 0.93 |
| Standard Deviation ($\sigma_R$) | 0.18    | 0.13 |
| Coefficient of Variation | 0.21    | 0.14 |
| Skewness ($\eta_R$)      | 0.65    | 0.62 |
| Minimum ($R_{min}$)      | 0.55    | 0.59 |
| Maximum ($R_{max}$)      | 1.51    | 1.47 |

The trend in the performance of the various models is investigated by correlating the model uncertainty with model parameters (such as $f_{cm}$, $c_1$, $d_o$, $l_f$) [4]. The correlation coefficients for the relevant parameters are presented in Table 3. The edge distance and the anchor diameter are found to be the most influencing basic variables as indicated by the correlation coefficients.

Table 3. Linear correlation coefficient

|               | $R_{ACI}$ | $R_{MCCD}$ |
|---------------|-----------|------------|
| $d_o$ [mm]    | -0.49     | -0.19      |
| $l_f$ [mm]    | 0.04      | 0.18       |
| $c_1$ [mm]    | -0.62     | -0.25      |
| $f_{cm}$ [MPa]| 0.23      | 0.19       |
5.1 Probability distribution function

The type of probability distribution has been shown to play a key role in reliability performance assessment [16]. The choice of the appropriate distribution for the model uncertainty random variable was evaluated based on the chi-square goodness-of-fit test. The magnitude of the p-value obtained suggests that the log-normal distribution is a better fit with a p-value higher than 0.05. The Joint Committee on Structural Safety (JCSS) [17] also recommended the log-normal distribution as the most appropriate distribution function for model uncertainty. The histograms and cumulative distribution plots for the model uncertainty variables are presented in Figure 2.
Figure 2. Histograms and cumulative distribution plots for model uncertainty

6. Discussion
As presented in Table 2, the model uncertainty associated with the MCCD $R_{MCCD}$ has a bias (mean value) of $\mu_R = 0.93$, indicating that the MCCD model generally overpredicts the shear breakout resistance. MCCD has the closest bias to the value of 1. $R_{MCCD}$ shows small scatter with a standard deviation $\sigma_R = 0.13$. A mild correlation ($r = -0.25$) is observed between $R_{MCCD}$ and concrete edge distance $c_1$. MCCD did not have any significant correlation with other model parameters.

ACI 318 $R_{ACI}$ was found to have a model uncertainty mean value of $\mu_R = 0.87$ (with 13% unconservative bias) with a standard deviation of $\sigma_R = 0.18$. The model has the largest dispersion. As presented in Table 3, the $R_{ACI}$ has a strong decreasing safety bias with increasing anchor diameter $d_o$ ($r = -0.49$) and concrete edge distance $c_1$ ($r = -0.62$). The sensitivity analysis identified no significant trend between $R_{ACI}$ and the other basic variables (Table 3).

7. Conclusion
In order to conduct an efficient study of the reliability of fastening to concrete design provisions, characterisation of model uncertainty statistics is a required prerequisite. The model uncertainties related to the ACI 318 and MCCD [5] concrete-edge breakout models were investigated, using the experimental database of 257 anchor tests. The derived model uncertainties statistics were analysed, and suitable probability distribution was recommended. The assessment results in the following conclusions:

- MCCD model displayed the lowest bias (i.e. closest to the mean value of 1) and comparatively smallest dispersion; and mild correlation with shear breakout parameters.
- ACI 318 model displayed the largest dispersion and comparatively highest unconservative bias, with model sensitivity to anchor diameter and concrete edge distance.
- The log-normal distribution was found as the most suitable distribution for the model uncertainties.
The model uncertainty statistics presented in this contribution can be used as basic random variables in reliability investigation and also derive global partial factors for fastening design formulations.

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