Natural convection in an inclined parallelogrammic porous enclosure under the effect of magnetic field

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Abstract. A numerical study is conducted to understand the effect of magnetic field on the natural convection in a tilted parallelogrammic porous enclosure. The two opposing side walls are differentially heated with a temperature difference specified, while the upper and lower walls are adiabatic. Using the Darcy model to formulate the problem, a finite difference scheme consisting of the Alternating Direction Implicit and Successive Line over Relaxation methods are used to solve the coupled non-linear governing equations. Computations are carried out for a wide range of Rayleigh number ranging from 100 to $5 \times 10^3$, inclination of the walls of the parallelogram $\phi$ from -60° to 60°, angle of inclination of the enclosure $\alpha$ from -60° to 60°, aspect ratio from 0.5 to 2 and Hartmann number from 0 to 20. The flow structure and the corresponding heat transfer characteristics inside the enclosure are presented in detail. The results revealed that both the magnetic force and the inclination angle have significant effect on the flow field and heat transfer in porous medium. The reported results are in good agreement with the available published work in the literature.

1. Introduction

Natural convection of electrically conducting fluid in finite sized enclosures under the influence of a magnetic field applied in one or more directions have been studied by several investigators. In many industrial applications, such as materials processing and crystal growth techniques, the electrically conducting fluid the magnetic field is applied in one or more directions. In such situations, the fluid movement is influenced by an additional force known as Lorentz force which suppresses the fluid flow velocities which in turn affect the heat transport rates.

Due to enormous applications, natural convective heat transfer in the non-rectangular geometries has become an essential phenomenon. Among the non-rectangular enclosure, a parallelogrammic enclosure is an important type of enclosure and aptly describes the physical configuration of many applications. A detailed investigation of free convection in a parallelogrammic enclosure for engineering applications with special attention to the case of parallelogrammic diode cavity can be found in Bairi et al. [1]. Baytas and Pop [2] studied convective flow and heat transport in oblique enclosures filled with porous medium. They transformed the computational domain to a regular shaped domain using a standard transformation. Buoyancy-driven convection in a parallelogrammic...
enclosure is reported by Hyun and Choi [3] to analyse the transient effects. They analysed the influence of transient term on the flow pattern and the average Nusselt number for various tilt angles and aspect ratios. Later, Costa et al. [4] studied natural convection in parallelogrammic enclosures with variable geometry.

Revnici et al [5] examined the influence of inclination of a square porous enclosure on the convective flow and heat transport characteristics for different combinations of parameters. It has been observed that the flow and thermal patterns, the total heat transport rates are significantly modified by the cavity inclination and Rayleigh number. The influence of magnetic field on convection heat transfer is also investigated in inclined enclosures. Han [6] studied free convection of an electrically conducting fluid in an inclined enclosure under the influence of magnetic field. Ece and Buyuk [7] examined the effects of magnetic field on convective flow and heat transfer in an inclined rectangular enclosure, which is heated from one side and cooled from the adjacent side. This study reveals that the flow circulation became stronger as the Rayleigh number increases and the magnetic field suppresses the flow and the heat transport rates. For a tilted square enclosure, Pirmohammadi and Ghassemi [8] investigated the convection heat transfer by considering the effect of magnetic field. It is shown that for a given inclination angle, as the value of Hartmann number increases, the convection heat transfer reduces. Saleh et al. [9] numerically studied free convection in a porous parallelogrammic enclosure with an applied magnetic field. They concluded that, the variation of magnetic field strength and the cavity inclination angle, will affect the heat transfer performance.

A detailed analysis of the literature reveals that natural convection flow under the influence of magnetic field has not been investigated in an inclined parallelogrammic enclosure. Hence, in this paper, the natural convective flows and the associated heat transport characteristics are analysed in an inclined parallelogrammic enclosure with a magnetic field applied along the x-direction. The primary objective of this study is to analyse the effects of various controlling parameters, such as, angle of inclination of the enclosure, magnetic field strength, aspect ratio, Rayleigh number and the parallelogrammic angle, on the fluid flow and heat transfer.

2. Physical model and basic equations
Consider a parallelogrammic porous enclosure, as shown in Fig.1 along with the coordinate system, containing an electrically conducting fluid. Let H and L be the height and width of the enclosure respectively, and the enclosure is tilted at an angle \( \alpha \) with respect to the \( x \)-axis. The inclined left wall is maintained at a higher temperature \( (T_a) \), while the right inclined wall is at lower temperature \( (T_c) \). However, the top and bottom walls are assumed to be perfectly insulated. The fluid is assumed to obey the Boussinesq approximation and is incompressible. Along the horizontal direction, a magnetic field of constant strength \( B_0 \) is applied. The inertial effects are assumed to be neglected and the heating causes by viscous, radiation and Joule heating are not taken into account. Further, the induced magnetic field is neglected on comparison with the applied magnetic field and hence the magnetic Reynolds number is considered to be small. Assuming that the Darcy’s law to be hold, the equations of conservation of energy and momentum can be written as,

\[
\frac{\partial T}{\partial t^*} + \frac{\partial \psi^*}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi^*}{\partial x} \frac{\partial T}{\partial y} = \alpha_m \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

\[
\frac{\partial^2 \psi^*}{\partial x^2} + \frac{\partial^2 \psi^*}{\partial y^2} = \frac{gK \beta}{\nu} \left( \frac{\partial T}{\partial y} \sin \alpha - \frac{\partial T}{\partial x} \cos \alpha \right) - \frac{\sigma B_0^2 K}{\mu} \frac{\partial^2 \psi^*}{\partial x^2}
\]

For the computational convenience, the parallelogrammic shaped computational domain is mapped on to a square domain as shown in Fig. 1 (bottom) using the transformation \( X = x - \tan \phi \), \( Y = y \) proposed by Baytas and Pop [2]. Using this transformation and by introducing the following dimensionless variables
\[ \xi = \frac{X}{L}, \eta = \frac{Y}{H \cos \phi}, t = \frac{t^* \alpha_m}{L H \cos \phi}, \psi = \frac{\psi^*}{\alpha_m}, \theta = \frac{(T - T_r)}{(T_h - T_c)}, \text{ where } T_r = \frac{(T_h + T_c)}{2}, \]

the equations (1) and (2) in dimensionless form can be written as

\[
\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial \theta}{\partial \eta} = \frac{A}{\cos \phi} \left( \frac{\partial^2 \theta}{\partial \xi^2} - 2 \frac{\sin \phi}{A} \frac{\partial^2 \theta}{\partial \xi \partial \eta} + \frac{1}{A^2} \frac{\partial^2 \theta}{\partial \eta^2} \right), \tag{3} \]

\[
\frac{\partial^2 \psi}{\partial \xi^2} \left[ 1 + Ha^2 \cos^2 \phi \right] - 2 \frac{\sin \phi}{A} \frac{\partial^2 \psi}{\partial \xi \partial \eta} + \frac{1}{A^2} \frac{\partial^2 \psi}{\partial \eta^2} = Ra \cos \phi \left[ \frac{\sin \alpha}{A} \frac{\partial \theta}{\partial \eta} - \cos (\phi - \alpha) \frac{\partial \theta}{\partial \xi} \right]. \tag{4} \]

**Figure 1.** Physical configuration and coordinate system.

The initial and boundary conditions in dimensional form are:

\[
t = 0: \quad \psi = 0, \theta = 0; \quad 0 \leq \xi \leq 1, 0 \leq \eta \leq 1
\]
\[
t > 0: \quad \psi = 0, \theta = +0.5; \quad \xi = 0, 0 \leq \eta \leq 1
\]
\[
\quad \psi = 0, \theta = -0.5; \quad \xi = 1, 0 \leq \eta \leq 1
\]
\[
\begin{cases}
\psi = 0, \\
\frac{\partial \theta}{\partial \eta} - A \sin \phi \frac{\partial \theta}{\partial \xi} = 0
\end{cases}
\quad \begin{cases}
\eta = 0 \text{ and } 1, \\
0 \leq \xi \leq 1.
\end{cases}
\]

In the above equations, \( A = \frac{H}{L} \) is the aspect ratio, \( Ra = \frac{g K B \Delta T L}{\nu \alpha_m} \) is the Darcy-Rayleigh number and \( Ha = B_0 \sqrt{\frac{\sigma K}{\mu}} \) is the Hartmann number.
The measure of heat transfer rate is calculated through the local ($Nu$) and global ($\overline{Nu}$) Nusselt numbers given as

$$Nu = -\frac{1}{\cos \phi} \left( \sin \phi \frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \xi} \right),$$

where

$$\overline{Nu} = \int_0^1 Nu \, d\eta$$

3. Solution methodology

The non-dimensional governing equations (5) and (6) are solved numerically using finite difference based ADI and SLOR methods subjected to the chosen initial and boundary conditions. The discretized finite difference equations are in Tri-diagonal matrix form and are inverted using the Thomas algorithm. After a careful grid independence analysis, all the simulations are executed by choosing a different grid size with respect to the aspect ratio. A 201×101 grids for $A = 0.5$, 101×101 grids for $A = 1$ and 101×201 grids for $A = 2$ has been chosen. The FORTRAN code has been developed to perform the simulations, and successfully validated against the various benchmark solutions. mm.

4. Results and discussion

An in-depth analysis is made on natural convective flow and associated heat transport phenomena inside an inclined parallelogrammic porous enclosure under the influence of magnetic field applied in the direction perpendicular to gravity. The numerical results are discussed through the streamlines, isotherms and global heat transfer rates for vast range of the parameters of the investigation, namely the Darcy-Rayleigh number ($Ra$), the inclination angle of the walls of the enclosure ($\phi$), the tilt angle of the enclosure ($\alpha$), the aspect ratio ($A$) and the magnetic field strength ($Ha$). The range of dimensionless parameters considered is $0 \leq Ha \leq 20$, $-60^0 \leq \alpha \leq 60^0$, $-60^0 \leq \phi \leq 60^0$, $10^4 \leq Ra \leq 5 \times 10^3$ and $0.5 \leq A \leq 2.0$. A systematic study is made on the flow pattern and temperature distributions inside the enclosure under the influence of all the parameters considered. Further, the rate of overall heat transport is measured in terms of the average Nusselt number.

4.1. Effect of Darcy-Rayleigh number

The flow characteristics and thermal pattern in a tilted parallelogrammic enclosure is analysed for the Darcy-Rayleigh number through streamlines and isotherms. Figure 2 illustrates the flow and thermal pattern for different strengths of magnetic field, by fixing the other governing parameters. In the zero-magnetic field case ($Ha=0$), it is observed that the intensity of the flow is high and the streamline movement is observed mainly in the centre region of the cavity forming a single shallow cell. Further, the flow circulation rate is found to be higher for non-magnetic case, as can be noticed through the maximum stream function value. The isotherms show strong variations for non-magnetic case, but become less variant and stagnant in the core region as the value of $Ha$ increases. At higher value of the magnetic field strength ($Ha=20$), the intensity of the flow circulation is reduced, due to the retarding force exerted by the magnetic parameter $Ha$ and are accumulated at the bottom and top portion of the boundaries. Also, the direction of streamlines is changed from horizontal to vertical cell, and appears to be a parallel flow. As far the temperature lines are concerned with respect to the magnetic field, the isotherms reveal a conduction-like structure and are parallel to the inclined vertical walls. The effect of Darcy-Rayleigh number on the overall Nusselt number for four different magnetic field strengths ($Ha = 0, 5, 10$ and $20$), but fixing the other governing parameters at $\phi = 300$, $\alpha = 300$ and $A = 1$ is illustrated in Fig 3. When $Ha = 0$, the average $Nu$ curve is increasing monotonically with the increase in the Darcy-Rayleigh number. It can be observed that the average $Nu$ decreases with increasing the Hartmann number and the average $Nu$ reaches unity for a higher magnetic field ($Ha=20$).
4.2. Effect of the angle of inclination of the walls

In order to examine the combined effects of the angle of inclination of the walls and magnetic field strength, computations are carried out for three values of $Ha$ and fixing the other governing parameters at $Ra = 3000$, $\alpha = 30^0$ and $A = 1$. Figure 4 illustrates the effects of different values of magnetic field strength ($Ha = 0$, $Ha = 5$ and $Ha = 20$) at the inclination angle $\phi = 30^0$. In the zero-magnetic field case, it can be seen that the stream function attains its maximum value when compared to the presence of a magnetic field.

Figure 2. Isotherms and streamlines for different values of $Ha$ and $Ra$ at $A = 1, \phi = 30^0, \alpha = 30^0$.

Figure 3. Effect of $Ra$ and $Ha$ on the average Nusselt number for $\phi = 30^0, \alpha = 30^0, A = 1$.
of magnetic field. Also, the direction of streamlines greatly depends on the inclination of enclosure. In general, as the magnetic field is applied, the flow is remarkably suppressed due to the retarding force generated from the magnetic field force. As the magnetic field strength is increased, the main vortex at the core gets elongated along the entire enclosure. Also, for a positive inclination of side walls ($\phi = 30^0$), flow circulation rate is higher compared to the negative inclination. This can be observed through the $|\psi_{\text{max}}|$ values of these two cases. A meticulous examination of the change of streamlines and isotherms pattern exhibits an important fact that the magnetic field strength and the inclination angle of sidewalls plays a key role in altering the flow and thermal pattern.

![Figure 4. Effect of Hartmann number on isotherms and streamlines for $A = 1, Ra = 3 \times 10^3, \alpha = \phi = 30^0$.](image)

The effect of inclination angle of the sidewalls on the average Nusselt number for different $Ha$ is shown in Fig. 5. When $\phi = 0^0$, the maximum heat transfer occurs for all the values of magnetic field, i.e., $Ha = 0, 5, 10$ and 20. The overall heat transport rates are comparatively higher for $Ha = 0$ compared to the other values of $Ha$. With the introduction of magnetic field, the overall heat transfer is suppressed drastically and can be observed from the figure. Further, the variation in average $Nu$ is very minimal as the magnetic field is introduced ($Ha = 5, 10$ and 20). This indicates that the suppression of heat transport is due to both magnetic field and inclination angle of the sidewalls.
4.3. Effect of the angle of inclination of the walls

Figure 6 exhibits the influence of the cavity tilt angle on streamlines and isotherms for three different parametric values of $Ha$. For zero- magnetic field case, a strong a parallelogram shaped cell is formed within the enclosure with diagonally rotating eddies. The flow intensity is high in the absence of magnetic field. When a magnetic field is applied, the axis of the streamlines is changed and an elongated cell is observed with a compressed flow in the core region. This is due to the impeding effect of the Lorentz force. As the strength of the magnetic field is further increased, it can be observed that the fluid movement becomes slow and forms a weak parallel flow in the enclosure. The flow is compressed in the core region of the enclosure. The isotherms are initially stratified and became straight lines with the increase in the magnetic field.

$\phi$

**Figure 5.** Effect of $\phi$ and $Ha$ on the average Nusselt number for $Ra = 3 \times 10^3$, $\alpha = 15^0$, $A = 1$.

$Ha = 0, |\psi_{\max}| = 46.9 \quad Ha = 5, |\psi_{\max}| = 10.2 \quad Ha = 20, |\psi_{\max}| = 0.9$

**Figure 6.** Isotherms and streamlines for different values of $Ha$ at $A = 1, Ra = 3 \times 10^3, \alpha = 45^0, \phi = 30^0$. 
The variation of average Nusselt number for different values of $\alpha$ and Ha is shown in Fig. 7. In the absence of magnetic field the average Nusselt number increases with an increase in $\alpha$ from $-60^0$ to $60^0$. The maximum heat transfer occurs in this case is at $\alpha = 30^0$. With the introduction of magnetic field, the rate of heat transfer reduces. This shows that the convection in the enclosure is suppressed due to the introduction of magnetic field. As $Ha$ is increased, the heat transfer rate dropped down abruptly due to the suppression of convective flow caused by the Lorentz force. It has been noted that the effect of magnetic field is to reduce the overall heat transport considerably for all parameters considered in this analysis.

5. Conclusions
Numerical simulations are carried out to understand the influences of tilt angles of cavity and side walls, magnetic field on buoyancy-driven convection in a porous parallelogrammic enclosure. The flow circulation and heat transfer rates are suppressed by the Hartmann number, but are augmented by the Darcy-Rayleigh number, enclosure inclination. Through the systematic analysis, it has been observed that the heat transfer rate can be effectively controlled by the careful choices of tilt angles of the enclosure and sidewalls.

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