Forward-backward asymmetry in $e^+e^- \rightarrow \nu\bar{\nu}Z$ from anomalous CP-odd $WWZ$ couplings

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Abstract

Anomalous CP-odd $W^+W^-Z$ couplings can give rise to a forward-backward asymmetry in the $Z$ angular distribution in the $e^+e^-$ centre-of-mass frame in the process $e^+e^- \rightarrow \nu\bar{\nu}Z$. Of the three CP-odd couplings possible, only the imaginary part of one of the couplings, $f^Z_4$, which is C odd and P even, contributes to the forward-backward asymmetry. It is found that a limit of the order of 0.1 can be placed on this coupling at a Next Linear Collider with centre-of-mass energy of 500 GeV and integrated luminosity of 50 fb$^{-1}$.

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The second phase of the Large Electron Positron collider (LEP) has begun operation. One of the tasks that this LEP2 phase will undertake is the investigation of anomalous $W^+W^-\gamma$ and $W^+W^-Z$ couplings through the process $e^+e^- \rightarrow W^+W^-$ \[1\]. This investigation would be able to improve limits on such anomalous couplings which might arise in scenarios beyond the standard model (SM). There have also been suggestions to study CP-violating vector boson couplings in $e^+e^- \rightarrow W^+W^-$ at LEP2 \[2\].

While $e^+e^- \rightarrow W^+W^-$ can test only a combination of $W^+W^-\gamma$ and $W^+W^-Z$ couplings, the process $e^+e^- \rightarrow \nu\bar{\nu}Z$ would be better suited to probe the $WWZ$ couplings separately. Though this latter process is not feasible of observation at LEP2, it would be significant at higher energies ($\sqrt{s} > 300$ GeV) of a future Next Linear Collider (NLC). Studies exist on the sensitivity of NLC to anomalous $WWZ$ couplings through the reaction $e^+e^- \rightarrow \nu\bar{\nu}Z$ \[3,4,5\]. However, these are restricted only to the CP-conserving couplings. We investigate in this work a simple CP-violating asymmetry, viz., the forward-backward (FB) asymmetry in the angular distribution of the $Z$ in the $e^+e^-$ centre-of-mass (cm) frame, as a signal for CP-odd $WWZ$ couplings.

In general, the CP-violating part of the most general Lorentz-invariant effective $WWZ$ coupling $i g_{WWZ} \Gamma_{\alpha\beta\mu}^{\gamma}(q, \bar{q}, p)$, representing the process $Z_{\mu}(p) \rightarrow W_{\alpha}^-(q) + W_{\beta}^+(\bar{q})$, is given by \[6\]

\[
\Gamma_{\alpha\beta\mu,\text{CP-odd}}^{\gamma}(q, \bar{q}, p) = i f_4 Z^\alpha g_{\mu\beta}^+ + p^\beta g^\mu_{\alpha}) - f_6 Z^{\mu\alpha\beta\rho} p_{\rho} - f_7 Z^2 Z^\mu (q - \bar{q})^\rho \epsilon^{\alpha\beta\rho\sigma} p_\rho (q - \bar{q})_\sigma
\]

The couplings $f_4, f_6$ and $f_7$ (we suppress the superscript $Z$ now onwards) are momentum dependent and complex in the general case. The overall coupling constant $g_{WWZ} = -g \cos \theta_W$, where $g$ is the usual semi-weak coupling, and $\theta_W$ is the weak mixing angle.

The forward-backward (FB) asymmetry of $Z$, related to an asymmetry in the variable

\[
\cos \theta_Z = \frac{(\bar{p}_e^+ - \bar{p}_e^-) \cdot \vec{p}_Z}{2|\vec{p}_e^-||\vec{p}_Z|}
\]

in the cm frame, is odd under CP. This follows from the fact that under C, $\bar{p}_e^- \leftrightarrow \bar{p}_e^+$, $\vec{p}_Z \rightarrow \vec{p}_Z$, and under P, $\bar{p}_e^- \rightarrow -\bar{p}_e^-$, $\bar{p}_e^+ \rightarrow -\bar{p}_e^+$, $\vec{p}_Z \rightarrow -\vec{p}_Z$. However, $\cos \theta_Z$ is even under naive time-reversal $T_N$, which, like P, reverses all momenta. Unlike genuine time-reversal, however, $T_N$ does not interchange the initial and final states. $\cos \theta_Z$ is therefore CPT$_N$ odd.
Let us see what the CPT theorem implies for the dependence of an asymmetry in the variable \( \cos \theta_Z \) on the parameters of the effective Lagrangian. Observation of a CPT\(_N\)-odd quantity like \( \cos \theta_Z \) does not necessarily imply conflict with the CPT theorem since there is no interchange of initial and final states involved. However, in order to avoid conflict with the CPT theorem, quantities which are CPT\(_N\)-odd can have nonzero values only if the amplitude has an absorptive part. In its absence, the transition matrix is effectively Hermitian, and an interchange of initial and final states envisaged in genuine time reversal does not change the amplitude except for an overall phase factor. The transition amplitudes then have the same essential behaviour under T and TN. In an effective Lagrangian approach, where we calculate amplitudes only at tree level, this absorptive part could only arise if the couplings in the effective Lagrangian are complex. It can be seen from the effective couplings in eq. (1) that the absorptive contribution could come from \( \text{Im} f_4 \), \( \text{Re} f_6 \) and \( \text{Re} f_7 \), since without absorptive parts, \( f_4 \) would be real, and \( f_6 \), \( f_7 \) purely imaginary from Hermiticity. Thus, from considerations of the CPT theorem, an asymmetry in \( \cos \theta_Z \) can in principle get nonzero contributions only from \( \text{Im} f_4 \), \( \text{Re} f_6 \), and \( \text{Re} f_7 \).

We find, however, that if the azimuthal angle of \( Z \) is integrated over, as in the FB asymmetry, only \( \text{Im} f_4 \) contributes. A search for the FB asymmetry will therefore enable limits to be placed on the single parameter \( \text{Im} f_4 \).

We have calculated the squared matrix element for the process

\[
e^- (p_1) + e^+ (p_2) \to \nu(k_1) + \overline{\nu}(k_2) + Z(q)
\]

in the standard model, together with the anomalous CP-odd couplings \( \text{Im} f_4 \), \( \text{Re} f_6 \) and \( \text{Re} f_7 \), keeping only the linear terms in the anomalous couplings. The Feynman diagrams contributing to the process (3) are shown in Fig. 1. We drop contributions from the SM diagrams which involve \( Z \) exchange in the \( s \) channel (Fig. 1 (c)). These contributions in SM are known to be negligible for \( \sqrt{s} > 300 \text{ GeV} \). To ensure that this remains true in our case, we can also impose a cut on the \( Z \) energy which ensures that the \( \nu \overline{\nu} \) invariant mass is sufficiently far from the \( Z \) mass. We choose to apply a somewhat simpler cut, however. We apply a cut of \( |E_Z - \sqrt{s}/2| > 5 \Gamma_Z \), and

\[2\text{For a suggestion of an experiment to isolate the couplings } f_6^\gamma \text{ and } f_6^Z, \text{ see [7]. It should be noted that an analogous FB asymmetry in } e^+e^- \to \nu\overline{\nu}\gamma \text{ is absent, because an } f_4^\gamma \text{ term in the } W^+W^-\gamma \text{ couplings is forbidden because of electromagnetic gauge invariance.}\]
have checked that for $\sqrt{s} > 300$ GeV, this cut keeps the SM contribution to the cross section from the virtual $Z$ diagrams to less than 1%.

The phase space-integral is carried numerically using Monte Carlo integration. It is found that the only nonzero contribution to the forward-backward asymmetry comes from $\text{Im} f_4$. $\text{Re} f_4$ as well as the other CP-violating couplings give vanishing asymmetry because of the integration over the azimuthal angles.

We present below the squared matrix element for (3) in the approximation mentioned above. The expression for the SM contribution is rather long, and we refer the reader to ref.[4], where full expressions may be found. Our results agree with those of [4]. The interference between the matrix elements of the $WWZ$ diagram (that in Fig. 1(a)) from SM and the same diagram from anomalous CP-violating couplings is (we present only the part proportional to $\text{Im} f_4$):

$$2 \sum \text{Re} \left( M_{SM}^{WWZ} M_{anom}^{WWZ*} \right) = \frac{\text{Im} f_4 \, g^6 \cos^2 \theta_W}{32 (2p_1.p_1' + m_W^2)(2p_2.p_2' + m_W^2)^2} \left[ (p_1 - p_1').k \right.$$

$$\times \left\{ \frac{8}{m_Z^2} \left( p_1.p_2.p_1'.k + p_1.p_2.p_2'.k - p_2.p_2'.p_1.k \right) + \left\{ \begin{array}{l}
-4(p_1.p_1'.p_2 + p_1.p_2.p_2' - p_1.p_2'.p_2') \\
4(p_1.p_1'.p_1' + p_1.p_2.p_2' - p_1.p_2'.p_2') \\
2m_Z^2 (p_1.p_1'.p_2 + p_1.p_2.p_1' - p_1.p_2'.p_2') \end{array} \right. \right\}$$

$$\left. + \left\{ \begin{array}{l}
-4p_1.p_1'.p_2 + p_1.p_2.p_2' - p_1.p_2'.p_2' \\
4p_1.p_1'.p_1' + p_1.p_2.p_2' - p_1.p_2'.p_2' \\
-2m_Z^2 (p_1.p_1'.p_2 + p_1.p_2.p_1' - p_1.p_2'.p_2') \end{array} \right\} \right\} \right\} \right) \right).$$

The interference of the matrix element from the SM involving $Z$ bremsstrahlung from $e^\pm, \nu, \bar{\nu}$ lines (diagrams in Fig. 1(b)) and the anomalous $WWZ$ diagram (that of Fig. 1(a)) is:

$$2 \sum \text{Re} \left( M_{SM}^{Zbrem} M_{anom}^{WWZ*} \right)$$

$$= \frac{\text{Im} f_4 \, g^6}{64(2p_1.p_1' + m_W^2)(2p_2.p_2' + m_W^2)} \left[ (-1 + 2 \sin^2 \theta_W) \right.$$

$$\times \left\{ \begin{array}{l}
\frac{T_1}{(m_Z^2 - 2p_1.k)(2p_2.p_2' + m_W^2)} + \frac{T_2}{(m_Z^2 - 2p_2.k)(2p_1.p_1' + m_W^2)} \\
\frac{T_3}{(m_Z^2 + 2p_1'.k)(2p_2.p_2' + m_W^2)} + \frac{T_4}{(m_Z^2 + 2p_2'.k)(2p_1.p_1' + m_W^2)} \end{array} \right\} \right]. \quad (5)$$

4
where $T_1$ are given by

$$T_1 = 4 \left[ (p_1', p_2' - p_2', k)(p_1' p_2 - p_1 p_2') - p_1' p_2' p_1 k (p_2 + p_1, k) \right] + \frac{8}{m_Z^2} \left[ (p_1, k)^2 (p_1' p_2' p_2 - p_1' k p_2 p_2') \right] + \frac{8}{m_Z^2} \left[ (p_1', k p_2 p_2' - p_1' k p_2, k) \right], \quad (6)$$

$$T_2 = 4 \left[ - (p_1', p_2 - p_1', k) (p_2 p_2' p_2 - p_1 p_2' p_2') + p_1' p_2' p_2 k (p_2 + p_1, k) \right] - \frac{8}{m_Z^2} \left[ (p_1', k)^2 (p_1' p_2' p_2 - p_2 k p_1' p_1') \right] + \frac{8}{m_Z^2} \left[ (p_1' k p_2 - p_2' k p_1') \right], \quad (7)$$

$$T_3 = 4 \left[ (p_1' p_2 + p_2') (p_1' p_2 p_2' - p_1' p_2 p_2') + p_1 p_2' p_2 k (p_2' - p_1', k) \right] - \frac{8}{m_Z^2} \left[ (p_1' k)^2 (p_1' p_2' p_2 - p_1 k p_2' p_2') \right] + \frac{8}{m_Z^2} \left[ (p_1' k p_2 - p_2' k p_1') \right], \quad (8)$$

$$T_4 = 4 \left[ - (p_1', p_2' + p_1 k) (p_2 p_2' p_2' - p_1' p_2' p_2') - p_1 p_2' p_2 k (p_2' + p_1', k) \right] + \frac{8}{m_Z^2} \left[ (p_1' k)^2 (p_1' p_2' p_2 - p_2 k p_1' p_1') \right] + \frac{8}{m_Z^2} \left[ (p_1' k p_2 - p_2' k p_1') \right]. \quad (9)$$

The differential cross section for the process to first order in $\text{Im} f_4$ is given by

$$\frac{d\sigma}{d\cos\theta_Z} = \frac{1}{2\pi} \sum \left[ |M_{SM}^{WWZ} + M_{SM}^{Z\text{brem}}|^2 \right] + 2 \text{Re} \left( (M_{SM}^{WWZ} + M_{SM}^{Z\text{brem}}) M_{anom}^{WWZ*} \right) dE_Z d\cos\tilde{\theta} d\phi, \quad (10)$$

where $E_Z$, $\theta_Z$ and $\phi$ are respectively the $Z$ energy, polar and azimuthal angles in the c.m. frame, and $\tilde{\theta}$ is the polar angle of the neutrino in the $Z$ rest frame.

The forward-backward asymmetry of the $Z$ with respect to the beam direction, with $\theta_Z$ integrated over the ranges $\theta_Z > \theta_0$ and $\theta_Z < \pi - \theta_0$, is defined by

$$A_{FB}(\theta_0) = \frac{1}{\sigma(\theta_0)} \left[ \int_{\cos\theta_0}^{\cos\theta_0} \frac{d\sigma}{d\cos\theta_Z} d\cos\theta_Z - \int_{-\cos\theta_0}^{\cos\theta_0} \frac{d\sigma}{d\cos\theta_Z} d\cos\theta_Z \right], \quad (11)$$
where

\[ \sigma(\theta_0) = \int_{-\cos\theta_0}^{\cos\theta_0} \frac{d\sigma}{d \cos \theta_Z} d \cos \theta_Z. \]  

(12)

As discussed before, \( A_{FB}(\theta_0) \) is proportional to \( \text{Im} f_4 \). A cut on \( \theta_Z \) is imposed because in practice, it would be difficult to observe a \( Z \) near the forward or backward directions.

We have evaluated \( \sigma(\theta_0) \) and \( A_{FB}(\theta_0) \) for different energies and \( \theta_0 = 20^\circ \). Fig. 2 shows \( \sigma(\theta_0) \) as a function of \( \sqrt{s} \). In Fig. 3, we have plotted \( A_{FB}(\theta_0)/\text{Im} f_4 \) as a function of \( \sqrt{s} \).

We find that the asymmetry is of the order of \( 10^{-1} \) in units of \( \text{Im} f_4 \). This information can be converted into a 95% confidence level (CL) limit on \( \text{Im} f_4 \) that can be placed if an asymmetry is not seen, by demanding that in the limiting case, the difference in the number of forward and backward events is 1.96 times the statistical error in the total number of events:

\[ L\sigma(\theta_0)A_{FB}(\theta_0) = 1.96 \sqrt{L\sigma(\theta_0)}, \]  

(13)

where \( L \) is the integrated luminosity. Writing

\[ A_{FB}(\theta_0) = \text{Im} f_4 \cdot C(\theta_0), \]  

(14)

eqq.(13) leads to the 95% CL limit on \( \text{Im} f_4 \) of

\[ \text{Im} f_4 < \frac{1.96}{C(\theta_0) \sqrt{L\sigma(\theta_0)}}. \]  

(15)

Table 1 shows the 95% CL limit that could be achieved on \( \text{Im} f_4 \) with integrated luminosities of 10 \( \text{fb}^{-1} \), 50 \( \text{fb}^{-1} \) and 100 \( \text{fb}^{-1} \) for different energies, and an angular cut of \( \theta_0 = 20^\circ \).

We thus see that it is possible to achieve a sensitivity of 0.24 to \( 7.5 \times 10^{-2} \) in \( \text{Im} f_4 \) for an NLC operating at \( \sqrt{s} = 500 \text{ GeV} \) with luminosity in the range 10 to 100 \( \text{fb}^{-1} \). For somewhat higher \( \sqrt{s} \), the sensitivity is improved further, though not drastically. For very large \( \sqrt{s} \) (\( \sqrt{s} \approx 2000 \text{ GeV} \)), the sensitivity starts to decrease.

We conclude with a few remarks.

We have studied a CP-violating asymmetry which depends on the imaginary part of a single effective coupling, namely, \( f_4 \). While our approach has been to purely phenomenological, where we do not assume any definite
Table 1: Cross sections and 95% CL limits on \( \text{Im} f_4 \) which could be obtained for integrated luminosities (a) 10 fb\(^{-1}\), (b) 50 fb\(^{-1}\) and (c) 100 fb\(^{-1}\). An angular cut-off of 20° has been imposed on the forward and backward Z directions.

| \( \sqrt{s} \) (GeV) | Cross section (fb) | \( 95\% \) CL limits (a) | (b) | (c) |
|-----------------------|-------------------|-----------------|-----|-----|
| 300                   | 48.0              | .40             | .18 | .13 |
| 400                   | 114               | .27             | .12 | .087|
| 500                   | 186               | .24             | .11 | .075|
| 600                   | 256               | .23             | .10 | .072|
| 700                   | 323               | .22             | .10 | .071|
| 800                   | 385               | .23             | .10 | .071|
| 900                   | 441               | .23             | .10 | .073|
| 1000                  | 494               | .24             | .11 | .074|
| 1500                  | 692               | .27             | .12 | .087|
| 2000                  | 824               | .32             | .14 | .10 |

model, it would be worthwhile to give a little thought to possible scenarios which could give rise to non-negligible \( \text{Im} f_4 \). In gauge models, such anomalous couplings are expected to arise only at the loop level, and therefore expected to be extremely small. Possible contributions to \( f_4 \) have been investigated in a general scenario in [8]. While their investigation is for LEP 200 energies, and restricted to the dispersive contributions below new thresholds, we might extrapolate their reasoning to conclude that there could be large scalar or fermion-loop contributions at \( \sqrt{s} = 500 \) GeV provided there are new particle thresholds in that region. These contributions can arise in models with exotic fermions or extra Higgs with masses in the region of a few hundred GeV, which are not ruled by present experiments. Even then, the numerical value of \( \text{Im} f_4 \) is still expected to be at least an order of magnitude smaller than the limit we quote as feasible. If the signal we discuss is discovered, it would indeed signal very new physics. This situation perhaps calls for a search for more sensitive tests for studying CP-violating couplings like \( f_4 \).

We have assumed ideal conditions, viz., 100% efficiency for energy and
angle determination of $Z$. While a leptonic decay channel of $Z$ can lead to an accurate determination of the energy and angle, there would be some loss of efficiency because of angular cuts in the forward and backward directions on the leptons. Effects of initial-state radiation and off-shell $Z$ effects can also lead to some amount of deterioration in the sensitivity. These effects and the effects of cuts to remove the SM background need further study. However, we expect the quantitative conclusions to remain valid to a good deal of accuracy.

We have thus demonstrated the possibility of measuring separately (the imaginary part of) the CP-odd coupling $f_4$ at NLC by means of a simple forward-backward asymmetry.

**Acknowledgements** One of the authors (S.D.R.) thanks Prof. J.W.F. Valle for his warm hospitality at the University of Valencia where most of this work was done. He also thanks Debajyoti Choudhury for discussions. The other author (J.P.S.) thanks colleagues and authorities of Jiwaji University, Gwalior, where most of the work was done. The authors also thank Prof. R. Rajaraman for seeding an initial collaboration which evolved into this work. The work of S.D.R. has been supported in part by DGICYT under Grant Ns. PB95-1077 and SAB95-0175, as well as by the TMR network ERBFMRXCT960090 of the European Union. J.P.S. acknowledges financial assistance from D.S.T., New Delhi, during the first phase of this work.
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Fig. 1 Feynman diagrams contributing to the process $e^+e^- \to \nu\overline{\nu}Z$. 
Fig. 2: Cross section $\sigma(\theta_0)$ for $e^+e^- \rightarrow \nu\bar{\nu}Z$, with an angular cut $\theta_0 = 20^\circ$ on the forward and backward production angles of the $Z$, as a function of $\sqrt{s}$.

Fig. 3: The forward-backward asymmetry for unit $\text{Im}f_4$ as a function of $\sqrt{s}$, with the same angular cut as for Fig. 1.