Abstract

In this work we suggest and consider an original, simple mathematical model of a "quasi-rapid" extinction population dynamics. It describes a decrease and final extinction of the population of one prey species by a "quasi-rapid" interaction with one predator species with increasing population. This "quasi-rapid" interaction means ecologically that prey species behaves practically quite passively (since there is no time for any reaction, i.e. defense), like an appropriate environment, in respect to "quasi-rapid" activity of the predator species that can have different "quasi-rapid" hunting abilities. Mathematically, our model is based on a non-Lotka-Volterraian system of two differential equations of the first order, first of which is linear while second, depending of a parameter that characterizes hunting ability is nonlinear. We compare suggested "quasi-rapid" extinction population dynamics and the global model of the overkill of the prehistoric megafauna (mammoths). We demonstrate that our "quasi-rapid" extinction population dynamics is able to restitute successfully correlations between empirical (archeological) data and overkill theory in North America as well as Australia. For this reason, we conclude that global overkill theory, completely mathematically modelable by "quasi-rapid" extinction population dynamics can consistently explain the Pleistocene extinctions of the megafauna.
1 Introduction

As it is well-known [1]-[3], Pleistocene extinctions of the megafauna (mammoths, etc.) represent a significant, open problem in the paleontology. There are two main mutually opposite attempts of the solution of given problem. First one, so-called (global) overkill (blitzkrieg) theory [4]-[6], supposes that human activity, precisely a rapid hunting or overhunting of the megafauna realized by initially relatively small but relatively quickly movable human groups - immigrants, has had dominant role in these extinctions. Second one, so-called climate changes theory [7]-[10], suggests that climate changes have had dominant role in the extinctions of the Pleistocene megafauna. Overkill theory points out that megafauna extinctions occurred in the different areas in the different times which, from overkill theory view point, eliminates climate changes as primary factors of these extinctions. Also, overkill theory point out that any megafauna extinction started ”immediately” after arrival of the human hunting groups in given area (eg. Clovis people immigration in North America) and correlated with relatively rapid migration (blitzkrieg or a wave like front) of given group over given area. All this in many cases corresponds to archeological data. Climate changes theory pointed out that in some cases, eg. in Australia, according to most recent archeological data [11], there is a relatively long-lasting period of the coexistence of the human hunting groups and Pleistocene megafauna. From the climate changes theory view point, it eliminates human hunting as primary factor of the megafauna extinctions. Also, there are observations [12], [13] that extinction interaction between human hunting groups as predators and megafauna as preys, supposed within overkill theory, contradicts to basic population dynamics, i.e. Lotka-Volterra equations [14]-[18] according to which both, predator species population and prey species population, oscillate but not disappear during time. Even if there are different computer simulations of the overkill theory in North America, eg. [19], they cannot remove given contradiction as well as they cannot explain Pleistocene megafauna extinction in Australia. Finally, there are different attempts of the explanation of the Pleistocene megafauna extinctions that suppose that these extinctions have been caused by a combination of the human hunting and climate changes [20], [21] or by hyperdisease [22], or by a very complex population dynamics based on the many species Lotka-Volterra equations [13], etc. Meanwhile, it seems that none of the mentioned attempts and theories is completely successful.

In this work we shall suggest and consider an original, simple mathematical model of an extinction population dynamics. It describes a decrease and final extinction of the population of one prey species by a ”quasi-rapid” interaction with one predator species with increasing population. This ”quasi-rapid” interaction means ecologically that prey species behaves, practically, quite naively, precisely, passively or quasi-passively (since there is no sufficient time for any reaction, ”response”, i.e. defense tactic), like an appropriate environment, in respect to ”quasi-rapid” activity of the predator species. Also, different predator species can have different ”quasi-rapid” hunting abilities. Mathematically, our model is based on a non-Lotka-Volterraian system of two differential equations of the first order. This principal distinction between our and Lotka-Volterra equation system is ecologically quite reasonable. Namely, Lotka-Volterra equations system refers on a ”slow” interaction between one predator species and one prey species. This ”slow” interaction means ecologically that prey species, in respect to ”slow” predator species activity, behaves non-naively, precisely, actively (since there is sufficiently time for a significant reaction or ”response”, i.e. defense tactic modelable by corresponding Lotka-Volterra cross terms), unlike a naive and passive environment. Our first equation is linear while second, depending of a hunting ability parameter that, simply speaking, characterizes hunting ability, is nonlinear. (But in the limit
when hunting ability parameter tends to its maximal value 1, the second equation and whole equations system become linear too. Such linearity induces a population superposition principle. It means mathematically that a sum of two populations of one species, any of which satisfies given equations system, satisfies given equations system too. It characterizes a wave like population change. Our equations system describes in simple manner the populations homogeneously distributed over surface of given area, i.e. populations that do not depend of the space coordinates. But even in this case mentioned population superposition principle can implicitly support a wave like front of the migrations of the human hunting groups over surface of given area.) Since, in this way, whole our equations system depends, in fact, of the hunting ability parameter that can have different values, it can be concluded that this system can be applied in many different ecological situations any of which represents an especial case of the ”quasi-rapid” interaction between one predator species and one prey species. We shall compare suggested ”quasi-rapid” extinction population dynamics and the global model of the overkill of the Pleistocene megafauna (mammoths). We shall demonstrate that our ”quasi-rapid” extinction population dynamics is able to restitute successfully correlations between empirical (archaeological) data, i.e. estimated parameters and predictions or intentions of the global overkill theory, especially in North America and Australia. For this reason, we shall conclude that global overkill theory, completely mathematically modelable by ”quasi-rapid” extinction population dynamics, can consistently explain the extinction of the Pleistocene megafauna. In other word, instead of a ”mixture” of the overkill and climate change influences at the extinction of the Pleistocene megafauna, a ”mixture” of the ”quasi-rapid” extinction population dynamics for different values of the hunting ability parameter is completely sufficient for consistent explanation of the extinctions of the Pleistocene megafauna in full agreement with global overkill theory.

2 A ”quasi-rapid” extinction population dynamics

We shall suggest the following system of the differential equations

\[
\frac{dx}{dt} = -a \frac{dy}{dt}
\]

\[
\frac{dy}{dt} = by^k
\]  \[\text{for } 0 < k \leq 1\]

where \(x, y\) represent the real positive variables that depend of the time \(t\), while \(a, b\) and \(k\) represent the time independent real, positive constants, i.e. parameters.

We shall suppose that given system can be used for mathematical modeling of a population dynamics. Namely, we shall suppose that \(x\) represents the population of a prey species and that \(y\) represents the population of a predator species.

Further, it can be observed that for \(k\) equivalent to 1, i.e. for

\[
k = 1
\]

(3)

(2) turns in the following linear differential equation

\[
\frac{dy}{dt} = by
\]

(4)

with simple solution

\[
y = y_0 \exp(bt)
\]

(5)
where $y_0$ represents the initial value of $y$. It, in fact, corresponds to well-known [14]-[18] unlimited increase of the population of a species placed in an appropriate environment (that behaves quite naively or passively, without any reaction or "response", i.e. defense tactic, in respect to given species activity). Then $b$ corresponds to so-called birth rate of given species that represents simply the interaction between given (predator) species and environment. It suggests that prey species must implicitly correspond to given environment.

For this reason we shall suppose that even for $k$ smaller than 1 but relatively close to 1, i.e. for

$$ k \leq 1 $$

nonlinear differential equation (2), precisely its left hand, represents ecologically the "speed" of the predator species population corresponding to a species placed in an appropriate environment corresponding implicitly to prey species. Right hand of (2) points out that population "speed" is equivalent to a power function of the population so that this "speed" is larger and larger for $k$ closer and closer to 1. In this way $k$ can be considered as a degree of the population "speed" or degree of the "quasi-rapid" interaction between predator species and environment, or, implicitly, prey species. Also, it can be considered that $k$ expresses implicitly a hunting ability of the predator species. (Detailed ecological analysis of the hunting ability parameter goes over basic intentions of this work. Intuitively, it would be expected that this parameter depends not only of the characteristics of the predators, eg human, species, but also of the geographical characteristics, eg. magnitude of the surface of given Earth area, etc. It would be suspected that hunting ability parameter increases when surface of given are decreases which can explain extremely rapid extinction of a prey species at small islands.) Especially, in the limit (2), given interaction can be called "rapid". In other words we shall suppose that when interaction between one prey species and one predator species is "quasi-rapid" (which means that prey species behave, practically, quite passively or quasi-passively, without any reaction, "response", i.e. defense tactic, in respect to action of the predator species) predator species population dynamics can be presented by (2) for $k$ relatively close to 1. Simple solution of (2) in this case is

$$ y = (y_0^{1-k} + (1-k)bt)^{\frac{1}{1-k}}. $$

(7)

It can be observed and pointed out that (2) and (7) depend principally of the hunting ability parameter so that for different values of this parameter, any of which is relatively close to 1, there are different but "quasi-rapid" increase of the predator species population.

Equation (1) simply means that "speed" of the prey species population is proportional to negative "speed" of the predator species population. The same equation, independently of the value of $k$, can be simply transformed in

$$ \frac{d(x + ay)}{dt} = 0 $$

(8)

which yields

$$ x + ay = x_0 + ay_0 = \text{const} $$

(9)

where $x_0$ represents initial prey species population. In other words expression $x + ay$ represents a form that stands conserved during time so that (9) can be considered as a conservation law. Namely, $ay$ can be considered as the calibrated (dilated) predator species population and, in this sense, (9) can be considered as the law of the population conservation (prey species population turns in the predator species calibrated population, but whole population representing sum of the
prey species population and predator species calibrated population stands conserved during time). For the same reason $a$ can be called calibration parameter.

From (9) it follows

$$x = x_0 + ay_0 - ay$$

which, for (3),(5) yields

$$x = x_0 + ay_0 - ay_0 \exp(bt)$$

and, for (6),(7),

$$x = x_0 + ay_0 - a(y_0^{1-k} + (1-k)bt)^{1/k}$$

It is not hard to see that predator species population (5) or (7) represents a monotonously increasing time function while prey species population (11) or (12) represents a monotonously decreasing time function. It implies that there is a finite time moment, called extinction time, $T$, in which prey species population becomes equivalent to zero, i.e.

$$x(T) = 0$$

For (11) it yields

$$T = \frac{1}{b} \ln\left(\frac{x_0}{ay_0} + 1\right)$$

while for (12) it yields

$$T = \frac{1}{b(1-k)}\left((\frac{x_0}{a} + y_0)^{1-k} - y_0^{1-k}\right)$$

In this way we obtain an original, simple mathematical model of a population dynamics with decrease and final extinction of the population of one prey species in a finite time moment by "quasi-rapid" interaction with one predator species with increasing population. This model is principally different from the usual Lotka-Volterra equations system of two nonlinear differential equations that describes "slow" interaction between one predator and one prey species. Namely, "slow" interaction means ecologically that prey species, in respect to "slow" predator species activity, behaves nonpassively (nonnaively), i.e. actively (since there is sufficiently time for a significant reaction or "response", i.e. defense tactic modelable by corresponding Lotka-Volterra cross terms), unlike a passive environment. It, on the one hand, implies that hunting ability parameter $k$ in (2) becomes significantly smaller than 1, and, on the other hand, that system (1),(2) corresponding to "quasi-rapid" extinction population dynamics cannot be applied at all for description of the "slow" interaction between predator species and prey species. (In this work we shall not analyze it with more details, from ecological view point, what is meaning of the expression that $k$ is relatively close to 1 or that $k$ is significantly smaller than 1. We shall use the following rough or ad hoc criterion: $k$ is relatively close to 1 for $k > 0.5$ and vice versa $k$ is significantly smaller than 1 for $k \leq 0.5$. ) Since our equations system, in fact, depends of the hunting ability parameter, as well as birth rate parameter and calibration parameter, that can have different values, it can be concluded that this system can be applied in many different ecological situations any of which represents an especial case of the "quasi-rapid" interaction between one predator species and one prey species. Especially, it can be observed that our equations system (1), (2), i.e. its solutions (7), (12) have the following important characteristics. They are more sensitive in respect to variation of the hunting ability parameter than variations of other parameters and initial populations. Or, relative large variations of the other parameters and initial populations (corresponding to relatively large uncertainties of corresponding empirical, i.e. archeological data and estimations) can be relatively
simply compensated by relatively small variations of the hunting ability parameters. In other words, many relatively large uncertainties of the empirical (archeological) data are practically irrelevant for distinction between a "quasi-rapid" and "slow" interaction between one predator species and one prey species. All this opens a possibility that different Pleistocene megafauna extinctions, eg. Pleistocene mammoths extinction in North America and megafauna extinction in Australia, would be consistently mathematically modeled by "quasi-rapid" extinction population dynamics for different values of the hunting ability parameters (any of which is relatively close to 1). If given possibility would be affirmed then it can be concluded that basic suppositions, predictions and intentions of the global overkill theory, completely mathematically modelable by "quasi-rapid" extinction population dynamics, can consistently explain the Pleistocene extinctions of the megafauna. In other words, it would mean, according to basic supposition of the global overkill theory, that human hunting activities played dominant role while climate changes have had only secondary role in the Pleistocene extinctions of the megafauna.

3 "Quasi-rapid" extinction population dynamics and Pleistocene mammoths overkill in North America

Now we shall attempt to apply "quasi-rapid" extinction population dynamics (1),(2) for mathematical modeling of the global overkill theory [4] - [6]. Firstly, we shall attempt to apply "quasi-rapid" extinction population dynamics (1), (2) for mathematical modeling of the hypothesis on the Pleistocene mammoths overkill (blitzkrieg) in North America [1], [2], [4]-[6], [19], [23]. In other words we shall attempt to restitute, by equations system (1), (2) consistent correlations between empirical (archeological) data and estimated parameters that characterize Pleistocene extinction of the megafauna in North America.

So, suppose that Clovis population increased about 3% for one year. It, introduced in (5), yields

\[ b = \ln(1.03)(yr^{-1}) = 0.0295(yr^{-1}) \]  

(16)

Suppose, further, that a typical small group of about 50 Clovis people killed about 15 mammoths per year. It also means that a group of \( x_0 \) mammoths contacted during one year by given small group of Clovis people was reduced in the group of \( x_0 - 15 \) mammoths. Introduction of this supposition and (16) in (11), which implies "rapid" extinction population dynamics, yields

\[ x_0 - 15 = x_0 + 50a - 50a \cdot 1.03 = x_0 - 1.5a \]  

(18)

or

\[ a = \frac{15}{1.5} = 10 \]  

(19)

Suppose, finally, that mammoth extinction occurred in the extinction time interval that equals about 400 years, i.e.

\[ T = 400(yr) \]  

(20)
It, introduced, in common with (16) and (19), in (15) yields

\[ 400 = \frac{1}{\ln(1.03)} \ln\left( \frac{1}{10} \frac{x_0}{y_0} + 1 \right) \]  

or

\[ \frac{x_0}{y_0} = 10 \exp(400 \ln(1.03)) - 1 = 1.36 \cdot 10^6 \]  

where \( x_0 \) represents the initial population of the mammoths while \( y_0 \) represents the initial population of the Clovis people. From (22) it follows

\[ x_0 = 1.36 \cdot 10^6 y_0 \]  

which, for supposed initial Clovis population (17), yields

\[ x_0 = 68 \cdot 10^6 \]  

It represents a number comparable, precisely about 10 times greater than roughly estimated initial mammoths population before appearance of the Clovis people

\[ x_0 = 10 \cdot 10^6 = 10^7. \]  

In other words, "rapid" extinction population dynamics (1), (2) very roughly correlates the existing empirical (archeological) data and suppositions of the overkill theory in the case of the Pleistocene mammoths extinction in North America.

Suppose, meanwhile, that Pleistocene mammoths overkill in North America can be more successfully modeled by "quasi-rapid" extinction population dynamics.

Suppose, also,

\[ x_0 = 10^7 \]  

\[ y_0 = 10^2 \]  

\[ a = 10 \]  

\[ b = 2.5 \cdot 10^{-2} \]  

\[ T = 400 \text{(yr)} = 4 \cdot 10^2 \text{(yr)} \]  

comparable with (25), (17), (19), (16), (20). Introduction of (26)-(30) in (15) yields

\[ (1 - k) \simeq \frac{1}{10} \left( 10^6(1-k) - 10^2(1-k) \right) \]  

(First term on the right hand of (31) is obtained by neglecting of term \( y_0 = 10^2 \) relatively small in respect to term \( x_0/a = 10^6 \)). It represents a transcendent algebraic equation whose solution can be obtained in the following way. First of all it is well-known the following

\[ 10 = \exp(\ln(10)) \simeq \exp(2.3) \]  

which, introduced in (31), yields

\[ (1 - k) \simeq \frac{\exp(13.8(1 - k))}{10^1} - \frac{\exp 4.6(1 - k)}{10^1}. \]
Further, for "quasi-rapid" interaction, according to its definition, $k$ is close to 1 and $1 - k$ to 0. It implies that both hands of (33) are close to 0, while both exponential terms at right hand of (33) are close to 1. It admits that given terms can be Taylor expanded in the quadratic approximation which introduced in (33) yields

$$
(1 - k) \simeq 0.92(1 - k) + 16.92(1 - k)^2.
$$

(34)

It represents an algebraic quadratic equation whose unique solution, since $k$ must be close to 1 and $1 - k$ must be positive, is

$$
k \simeq 0.995
$$

(35)

$$
1 - k \simeq 0.005.
$$

(36)

So, "quasi-rapid" extinction population dynamics (1),(2), for $k$ (35) really close to 1, can successfully correlate all estimated significant data (26)-(30) that characterize Pleistocene mammoths extinction in North America according to overkill (blitzkrieg) theory. Also, it is not hard to see, according to previously mentioned characteristic of the equations system (1),(2), that possible relatively large uncertainties and variations of the empirical (archeological) data and estimated parameters (26)-(30), are practically irrelevant for final conclusion that $k$ must be close to 1.

4 A "quasi-rapid" extinction population dynamics and overkill of the Pleistocene mega-fauna in Australia

Now, we shall attempt to apply "quasi-rapid" extinction population dynamics (1),(2) for mathematical modeling of the hypothesis on the Pleistocene megafauna extinction in Australia. In other words we shall attempt to restitute, by equations system (1),(2), consistent correlations between empirical (archeological) data and estimated parameters that characterize Pleistocene extinction of the megafauna in Australia. According to recent Roberts et al. data [11] human population arrived in Australia before $56000 \pm 4000$ yr, while extinction of the Pleistocene megafauna in Australia occurred before $46000 \pm 5000$ yr. It implies that Pleistocene megafauna extinction in the Australia occurred in an uncertainly determined time interval, i.e. extinction time $T$ that equals about $10000 \pm 9000$ yr. Suppose, meanwhile, that real value of $T$ is very close to its mean value 10000 yr, i.e.

$$
T = 10000(\text{yr}).
$$

(37)

Suppose that given megafauna extinction can be mathematically modeled by "quasi-rapid" extinction population dynamics (1),(2).

Suppose, also,

$$
x_0 = 10^7
$$

(38)

$$
y_0 = 10^2
$$

(39)

$$
a = 10
$$

(40)

$$
b = 2.5 \cdot 10^2
$$

(41)

Obviously, (38)-(41) are equivalent to (26)-(29). It represents a reasonable supposition. But, extinction time (37) is significantly larger than extinction time (30), which implies that here $k$ value must be significantly different from 0.995 (35).
Introduction of (37)-(41) in (15) yields

\[(1 - k)250 = 10^6(1 - k) - 10^2(1 - k) \tag{42} \]

It represents a transcendent equation. For reason of relatively large value of \( T \) (37), we shall solve (42) simply numerically, by fitting, which, with accuracy of 0.1% yields

\[ k = 0.680 > 0.5 \tag{43} \]

It can be considered as a value relatively close to 1 (in sense that it is greater than 0.5) so that supposition on the applicability of the "quasi-rapid" population dynamics (1),(2) can be considered consistent. On the other hand this value is relatively small which causes a relatively small "speed" of the megafauna population and relatively large extinction time (37).

So, "quasi-rapid" extinction population dynamics (1),(2), for \( k \) (42), can successfully to correlate all estimated significant data (26)-(30) that characterize Pleistocene extinction of the megafauna in Australia according to general (global) overkill (but not blitzkrieg) theory. Again it is not hard to see that possible uncertainties and variations of the empirical (archeological) data and estimated parameters (37)-(41), are practically irrelevant for final conclusion that \( k \) must be relatively close to 1.

5 Conclusion

In conclusion we can shortly repeat and point out the following. In this work we suggested and considered an original, simple mathematical model of a "quasi-rapid" extinction population dynamics. It describes a decrease and final extinction of the population of one prey species by a "quasi-rapid" interaction with one predator species with increasing population. This "quasi-rapid" interaction means ecologically that prey species behave, practically, quite passively (since there is no sufficient time for a significant reaction or "response", i.e. defense tactic), like an appropriate environment, in respect to activity of the predator species that can have different "quasi-rapid" hunting abilities. Mathematically, our model is based on a non-Lotka-Volterraian system of two differential equations of the first order, first of which is linear while second, depending of a parameter that characterizes hunting ability, is nonlinear. Global overkill scenario can be mathematically completely modeled and in this sense affirmed by suggested "quasi-rapid" extinction population dynamics even if in different cases (overkill in North America, Australia, etc.) corresponding hunting ability parameter can have different values (smaller than 1, but relatively close to 1). It implies that human hunting activities played dominant role while climatic changes have had only secondary role in the extinctions of the Pleistocene megafauna. In other word, instead of a "mixture" of the overkill and climate change influences, a "mixture" of the "quasi-rapid" extinction population dynamics for different values of the hunting ability parameter is completely sufficient for consistent explanation of the Pleistocene extinctions of the megafauna in full agreement with global overkill theory.
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