Analytical and finite element method of calculation of multi-shell gas tanks

O Yu Sklemina, N A Tatus’ and A N Polilov
Blagonravov Mechanical Engineering Research Institute of Russian Academy of Science, 4 Maly Kharitonyevsky Pereulok, Moscow, 101990, Russia

sklemina97@yandex.ru

Abstract. The article considers the analytical and finite element method for calculating the stress-strain state of a multi-shell gas tank. The calculated dependences connecting the gas pressure inside the gas tank and strength are presented. The analytical calculation of a multi-shell container was verified using the finite element method in the ANSYS APDL software package.

1. Introduction
The development of multi-shell gas tanks (Figure 1) is actively conducted today at many enterprises in Russia and abroad. Multi-shell gas tanks is a set of shells enclosed in a common housing, which has a device for filling it with gas. Gas tanks of this type can be used in the aerospace industry, in land vehicles operating on gas fuels, as well as for storage and transportation of various gases. The main attention when designing gas tanks is paid to reducing the mass of gas tanks, increasing the volume of stored gas, increasing the durability and reliability of gas tanks.

Figure 1. Section of multi-shell gas tanks.

The traditional mono-shell vessels are widely known - high-pressure gas tanks, consisting of a metal body in the form of a cylindrical or spherical shell. The disadvantage of such gas tanks is the significant wall thickness and large mass even when using high-strength steel. There are also gas tanks
with multilayer shells: the inner one is made of a material with relatively high yield strength, and the outer ones are made of cheaper low-strength materials. The technology of manufacturing such gas tanks is also associated with significant difficulties. There are multi-shell gas tanks of high pressure, the design of which is based on the principle of unloading the shell, from high internal pressure by applying external pressure.

Common disadvantages of multi-shell gas tanks are:

- uncertainty of the shells number and their relative position;
- non-optimal pressure distribution over the inner volumes between the shells;
- complexity of the practical implementation of technical solutions;
- different shell efficiency of the material strength properties;
- need to use high strength materials;
- complexity of ensuring safety and reliability during operation.

The advantages include:

- ability to store various types of gases in common tank;
- large volume of injected gas for given external dimensions.

Thin-walled shells of revolution (spherical, cylindrical, conical, torus and their combinations) are an important component of the construction of aviation and rocket and space technology. The most common form of a casing operating under internal pressure of a liquid or gas is a cylindrical one. Spherical gas tanks are rarely used, despite a number of advantages. In a breathing apparatus - scuba gear with three spherical volumes, it is possible to reduce the position of the mass center relative to the waist belt, so it is more convenient to tilt with this apparatus. Spherical casings for gas storage have weight advantages in comparison with volumes of other geometric shapes, but they are inferior in the convenience of their arrangement in the device.

Multi-shell gas tank is a complex technical product, the manufacture of which is a sophisticated problem. Efficiency mark of such gas tanks can serve as the basis for the development of their production.

To determine the stress-strain state of a multi-shell gas tank, the ANSYS software package is used. ANSYS is based on efficient parallelization algorithms, which make it possible to significantly reduce the calculation time for demanding tasks in various industries.

So, the use of a multi-shell gas tank will allow:

- to provide for storage and transportation of light gases at very high temperatures (more than 300 ATM.) pressures;
- to increase the efficiency of using the strength properties of the material;
- to reduce material consumption with a constant gas volume;
- to reduce the cost of production.

2. The problem

A multi-shell gas tank is considered, consisting of thin-walled, concentric, spherical shells, between which there are cavities filled with gas (Figure 1).

The following conditions are specified [1]:

- \( \sigma^* \) - allowable stresses in the walls of the shells,
- \( \rho \) - material density,
- \( n \) - number of shell,
- \( i = 1, 2, 3, ..., n \) - serial number of shell,
- \( h \) - the wall thickness of the shells,
- \( r_i \) - the radius of the outer shell.

The law of variation of the average radii of the shells is adopted as follows:
The main steps for solving the problem are:
1. The calculation of the shells mass.
2. The calculation of the inner volumes between the shells.
3. The calculation of permissible pressures (in ATM.) in each inner volume.
4. The calculation of the accumulated gas total volume.
5. The calculation of the efficiency factor of a multi-shell gas tank.

2.1. The calculation of the shells mass is presented below.
We start the solution from the first, simplest stage of the task.
The mass of the outer shell is:
\[ M_i = 4\pi r_i^2 \rho \]  
\[ M_i = M_i \left( \frac{n-i+1}{n} \right)^2 \] - mass of \( i \)-th shell.
The total mass of the shells:
\[ \sum_{i=1}^{n} M_i = \frac{M_i}{n} \sum (n+1-i)^2 = M_i \left[ \sum (n+1)^2 - 2(n+1) \sum i + \sum i^2 \right] = \\
= M_i \left[ (n+1)^2 n - \frac{2(n+1)^2 n}{2} + \sum i^2 \right] = M_i \frac{(n+1)(2n+1)}{6n}; \tag{2} \]
\[ \sum i^2 = \frac{n(n+1)(2n+1)}{6}. \]
This problem can be solved, putting that \( n+1-i = j \).
\[ i = 1 \rightarrow j = n \]
\[ i = n \rightarrow j = 1 \] \[ \Rightarrow \sum_{i=1}^{n} (n+1-i)^2 = \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}. \]

2.2. The calculation of the inner volume between shells is presented below:
\[ V_i = \frac{4}{3} \pi \left( r_i^3 - r_i^3 \left( \frac{n-1}{n} \right)^3 \right). \] Let us set \( V_0 = \frac{4}{3} \pi r_1^3 \).
\[ V_i = \frac{V_0}{n^3} \left[ (n-i+1)^3 - (n-i)^3 \right] = \frac{V_0}{n^3} \left[ j^3 - (j-1)^3 \right] = \frac{V_0}{n^3} \left( 3j^2 - 3j + 1 \right). \tag{3} \]
The total volume of the gas tank through the sum of the inner volumes there is:
\[ \sum V_i = \frac{V_0}{n^3} \left[ 3 \sum j^2 - 3 \sum j + n \right] = \frac{V_0}{n^3} \left[ \frac{n(n+1)(2n+1)}{2} - \frac{3n(n+1)}{2} + n \right] = \\
= \frac{V_0}{n^3} \left[ 2n^3 + 3n^2 + n - 3n^2 - 3n + 2n \right] = V_0. \]
It is necessary to calculate not the sum of volumes, but the volume of accumulated gas.

2.3. The calculation of permissible pressures (in ATM.) in each inner volume is presented below.
The basic equation is: \( \sigma^* = \left( p_i - p_{i-1} \right) \frac{r_i}{2h} \).
The pressure in the \(i\)-th shell is determined by this following equation:

\[
p_i = \frac{2h}{r_i} \sigma^*;
\]

\[
p_2 = \frac{2h}{r_2} \sigma^* + p_1 = p_1 \left(1 + \frac{n}{n-i+1}\right) = p_i \left(1 + \frac{n}{j}ight) = p_i \left(1 + \frac{n}{n-1}\right) = p_i \frac{2n-1}{n-1};
\]

\[
p_3 = p_2 + p_i \frac{n}{n-2} = p_i \left(1 + \frac{n}{n-1} + \frac{n}{n-2}\right).
\]

The pressure in the \(i\)-th shell is determined by this following equation:

\[
p_i = p_i n \left(\sum_{k=0}^{i-1} \frac{1}{n-k}\right) = p_i n \sum_{j=n}^{n-i+1} \frac{1}{j}.
\]

### 2.4. The calculation of the accumulated gas total volume.

It is necessary to find the sum of the products of each inner volumes by the allowable pressure: \(\sum V_i p_i\). The problem reduces to double sequences when substituting for (3) from (4), therefore, the results of analytical calculations turned out to be rather cumbersome and are not presented. However, for small number of shells, the result is not difficult to obtain analytically based on the above general formulas (3), (4) (see Example 2 below).

### 2.5. The calculation of the efficiency factor of a multi-shell gas tank is presented below.

By efficiency factor \(E\) we mean the ratio of the accumulated gas possible volume (in terms of atmospheric pressure) to the gas tank mass. The analysis showed that this factor \(E\) does not depend on the number of shells, but depends only on the properties of the gas tank material: strength and density and the shape of the shells. For a spherical multi-shell gas tank, independently of the inner volumes number \(E\) is:

\[
E = \frac{\sum V_i p_i}{\sum M_i} = \frac{2 \sigma^*}{3 \rho}.
\]

Strength \(\sigma^*\) must be expressed in a dimensionless form in terms of the number of atmospheres since the pressures that each shell can withstand when calculating the total gas volume in (4) are expressed in atmospheres: for example, \(\sigma^* = 30\text{ kg}\cdot\text{mm}^{-2} = 300\text{ MPa} = 3000\text{ ATM}\). Dimension of efficiency factor is: \([E] = 1/\rho = \text{m}^3\text{kg}^{-1}\) - the number of gas cubic meters per 1 kg of gas tank weight (mass).

The factor of efficiency for the spherical shape of the shells is a property of the material of the gas tank. For another form gas tank, the numerical factor \(\left(\frac{2}{3}\right)\) in (5) will be different (see Example 3), but the ratio characteristic of the material \(\sigma^*/\rho\) will remain the same.

**Example 1.** Accept strength of the "isotropic" fiberglass \(\sigma = \frac{V}{S}\) from its strength along the fibers, i.e. 3000 ATM. density \(\rho = 2 \cdot 10^9\) kg\cdot\text{m}^{-3}. \(\rightarrow E = \frac{2}{3} \cdot \frac{3000}{2 \cdot 10^9} = 1\text{ m}^3\text{kg}^{-1}\). Fiberglass plastic allows you to accumulate 1000 m\(^3\) in containers weighing 1 ton.

The efficiency of a cylindrical shape shell was calculated too. The numerical coefficient before \(\sigma^*/\rho\) in formula 5 turned out to be equal \(\frac{1}{2}\), but the ratio \(\sigma^*/\rho\) remained the same.
Example 2. A multi-shell gas tank with the number of shells $n = 3$ is presented below.

$\sigma^* = 300$ MPa $\approx 3000$ ATM.

Gas tank wall thickness is $h = 2.5$ mm $= 0.0025$m.

The radius is determined by the formula (1):

$$r_i = 500 \text{ mm} = 0.5 \text{ m};$$
$$r_2 = 333 \text{ mm} = 0.333 \text{ m};$$
$$r_3 = 167 \text{ mm} = 0.167 \text{ m}.$$

1) The calculation shells mass are:

$$M_1 = 4\pi \cdot r_1^2 \cdot h \cdot \rho = 0.00785 \rho;$$
$$M_2 = 4\pi \cdot r_2^2 \cdot h \cdot \rho = 0.00348 \rho;$$
$$M_3 = 4\pi \cdot r_3^2 \cdot h \cdot \rho = 0.00088 \rho;$$
$$\sum M_i = M_1 + M_2 + M_3 = 0.01221 \rho.$$

2) The calculation of permissible pressures (in ATM.) is presented below:

$$p_1 = \frac{2h\sigma^*}{r_1} = 0.01\sigma^*;$$
$$p_2 = p_1 + \frac{2h\sigma^*}{r_2} = 0.025\sigma^*;$$
$$p_3 = p_2 + \frac{2h\sigma^*}{r_3} = 0.055\sigma^*.$$

3) The calculation of cavities between the shells volume and the accumulated gas presented below:

$$V_{g1} = \frac{4}{3} \pi \cdot \left( r_3^3 - r_1^3 \right) \cdot p_1 = 0.0037 \sigma^*;$$
$$V_{g2} = \frac{4}{3} \pi \cdot \left( r_2^3 - r_1^3 \right) \cdot p_2 = 0.0034 \sigma^*;$$
$$V_{g3} = \frac{4}{3} \pi \cdot r_3^3 \cdot p_3 = 0.0011 \sigma^*.$$

4) The calculation of accumulated gas total volume is:

$$\sum V_{g} = V_{g1} + V_{g2} + V_{g3} = 0.0082 \sigma^*.$$

5) The calculation of the efficiency factor of the shells as the ratio of the largest possible amount of gas in a multi-shell gas tank to the mass of the shells is presented below:

$$E = \frac{\sum V_{g}}{\sum M_i} = 0.67 \frac{\sigma^*}{\rho} \approx 2 \frac{\sigma^*}{3 \rho}.$$
Example 3. The calculation of the factor of efficiency for a cylindrical gas tank with a length \( L \) and radius \( r \) is presented below.

The efficiency factor of a cylindrical gas tank while maintaining a characteristic ratio \( \frac{\sigma^*}{\rho} \) is 25% lower than that of a spherical gas tank made of the same material:

\[
\frac{\sigma_0}{\rho} = \frac{p \cdot r}{h} \Rightarrow p = \frac{\sigma^* h}{r}; \\
V = \pi r^2 L; \\
M \approx 2 \pi r h L \rho; \\
E = \frac{V \cdot p}{M} = \frac{\sigma^*}{2 \rho} = \text{(for fiberglass)} = \frac{3000}{2 \cdot 2 \cdot 10^3} = \frac{3}{4} \text{m}^3 \text{kg}^{-1}.
\]

The above analytical calculation of a multi-shell gas tank was verified by the finite element method using the ANSYS software package. In each shell, the pressure was set, which was calculated analytically by the formula (4). The circumferential stresses in each shell, as expected for thin-walled shells, turned out to be approximately 300 MPa.

Conclusions

1. The efficiency factor of a spherical gas tank as the ratio of the quantity of injected gas to the multi-shell gas tanks shells mass is a constant value for this material and does not depend on the number of shells.
2. The advantage of multi-shell gas tanks is the ability to store more gas in the same external dimensions.
3. The design of winding composite gas tanks should be optimized by selecting the winding angles. The fibers in the gas tanks must be loaded with the same tensile stresses. Any rational design of a gas tank with equally strained fibers has the same mass at a given gas tank size and gas pressure, and the best option can be chosen only from technology.
4. The calculation of stresses in a multi-shell gas tank using the finite element method coincided with analytical calculations.
5. The efficiency factor of a cylindrical gas tank is 25% lower than that of a spherical one; therefore, in some applications it is more advantageous to use spherical gas tanks.

References

[1] Polilov A N, Tatus N A and Sklemina O Yu Three etude problems on composite fuel gas tank  
IOP Conf. Series: Materials Science and Engineering 525 (2019) 012086. doi:10.1088/1757-899X/525/1/012086.

Acknowledgments

This work was carried out as part of the Program of Fundamental Scientific Research of the Russian State Academies of Sciences for 2013-2020.