Effect of dust particles on kinetic Alfven wave in earth’s magnetoplasma

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Abstract. Kinetic Alfven waves are examined in the presence of density and charge of dust particles with bi-Maxwellian distribution function. The theory of particle aspect analysis is used to evaluate the dispersion relation and growth rate. It is assumed that a low $\beta$ (ratio of plasma pressure to magnetic pressure) plasma consist the resonant and non-resonant particles. The resonant particles participate in the energy exchange with the wave whereas non-resonant particles support the oscillatory motion of the wave. It is assumed that the dusty plasma model modify the scenario of the KAW. The density of dust particles enhanced the frequency of the KAW. The presence of charged dust grains gives rise to new kinds of waves. The finding may be applicable for the laboratory plasma and has wide applications in magnetosphere as well as space plasma which modify the propagational characteristics of KAW.

1. Introduction

Alfven’s waves play an important role in energy transport, in driving field-aligned currents, particle acceleration and heating, inverted–V structures in magnetosphere-ionosphere coupling, solar flares and the solar wind. Recently, observations [Klimushkin, 1997; Wygant, 2000] from the polar and cluster satellites have indicated that large amplitude kinetic Alfven waves are observed throughout the plasma sheet, particularly at the plasma sheet boundary layers, during substorms. It has also been observed simultaneously by polar and FAST satellites in the plasma sheet boundary layer at 4-6$R_E$ ($R_E$ is the earth’s radius). Measurements from the polar spacecraft show the existence of small scale Alfven waves that carry a large net poynting flux along magnetic field lines towards the earth. The small- scale spikes have electric field amplitudes up to 300 mV m$^{-1}$ and associated magnetic field variations between 0.5 and 5nT. The analysis has shown that the larger–scale Alfven waves have periods of ~20-60s and carry enough poynting flux to explain the generation of the most intense auroral structures observed in the polar ultraviolet imager data set [Wygant et al., 2002].

The study of collective effects in dusty plasmas is of the significant interest. Previously plasmas with constant (characteristic time scales of the processes under consideration) charges on the dust particles had been studied extensively. Recently [D’Angelo, 1995] the effect of variable charges on dust particles has been investigated and the influence is found to be strong, especially for low frequency waves.

2. Mathematical Considerations

The kinetic Alfven wave is assumed to start at t=0 when the resonant particles are undisturbed.

$$V_{Td} << V_{T||} \ll \omega / k_{||} \ll V_{Te} ; \omega \ll \Omega_i, \Omega_e, \Omega_d ; \omega^2 \rho_i^2 \ll k_{||}^2 \rho_e^2 \ll k_{||}^2 \rho_d^2 < 1 \ldots \ldots (1)$$
Where \( V_T\|, V_T\|, V_T\| \) are the mean velocities of dust, ions and electrons particles respectively along the magnetic field, \( \Omega_{\text{ion}}, \Omega_{\text{elec}}, \Omega_{\text{dust}} \) are gyration frequencies and \( \rho_{\text{ion}}, \rho_{\text{elec}}, \rho_{\text{dust}} \) are the mean gyro radii of the respective species. In this paper we have extended the particle aspect analysis for electromagnetic perturbations. We have adopted two ‘potential’ representations, which are commonly used to express electromagnetic perturbations in low - \( \beta \) plasmas [Hasegawa and Chen, 1975, 1976; Hasegawa and Mima, 1978; Kadomtsev, 1965]. The idea is to decouple the compressional Alfven mode by assuming \( (\nabla \times E)_|| = -(\partial B / \partial t)_z = 0 \), that is taking into account only the effect of the line bending.

3. Dispersion relation

To evaluate the dispersion relation of the KAW in dusty plasma we calculate the integrated perturbed density for non-resonant particles as

\[
\rho_j = \frac{2 \pi r_i^2}{2} \int_0^\infty \int_{-\infty}^\infty dV \rho_j(r,t)
\]  

(2)

Where \( \rho_j \) is the average density of the j\textsuperscript{th} species \( (j = i, e \text{ and } d) \) for the ion, electron and dust respectively. With the help basic equations of Shandilya et al., [2003, 2004] we obtain the dispersion relation for the kinetic Alfven wave in inhomogeneous dusty plasma as

\[
\begin{aligned}
&\left( 1 - \frac{\omega_i^2}{c^2 \Omega_{i}^2} \right) \left( 1 - \frac{\omega_e \Omega_{e}}{V_A} \right) = \frac{k_i^2 \Omega_{i}^2}{c^2} + \frac{\omega_i^2 \Omega_{i}^2}{2 \rho_{\text{ion}}} \left( \frac{T_{i,i}}{m_i} + \frac{\omega_p \Omega_{p}}{\omega_e \Omega_{e}} \frac{T_{i,i}}{m_i} \right) + \frac{\omega_e \Omega_{e}}{c^2 \Omega_{i}^2} \left( \frac{T_{i,i}}{m_i} + \frac{\omega_p \Omega_{p}}{\omega_e \Omega_{e}} \frac{T_{i,i}}{m_i} \right)
&\left( 1 - \frac{\omega_e \Omega_{e}}{V_A} \right) \left( 1 + \frac{\omega_i^2}{c^2 \Omega_{i}^2} \right) = \frac{k_i^2 \Omega_{i}^2}{c^2} + \frac{\omega_i^2 \Omega_{i}^2}{2 \rho_{\text{ion}}} \left( \frac{T_{i,i}}{m_i} + \frac{\omega_p \Omega_{p}}{\omega_e \Omega_{e}} \frac{T_{i,i}}{m_i} \right) + \frac{\omega_e \Omega_{e}}{c^2 \Omega_{i}^2} \left( \frac{T_{i,i}}{m_i} + \frac{\omega_p \Omega_{p}}{\omega_e \Omega_{e}} \frac{T_{i,i}}{m_i} \right)
&\left( 1 - \frac{\omega_d \Omega_{d}}{V_A} \right) \left( 1 + \frac{\omega_i^2}{c^2 \Omega_{i}^2} \right) = \frac{k_i^2 \Omega_{i}^2}{c^2} + \frac{\omega_i^2 \Omega_{i}^2}{2 \rho_{\text{ion}}} \left( \frac{T_{i,i}}{m_i} + \frac{\omega_p \Omega_{p}}{\omega_e \Omega_{e}} \frac{T_{i,i}}{m_i} \right) + \frac{\omega_d \Omega_{d}}{c^2 \Omega_{i}^2} \left( \frac{T_{i,i}}{m_i} + \frac{\omega_p \Omega_{p}}{\omega_e \Omega_{e}} \frac{T_{i,i}}{m_i} \right)
\end{aligned}
\]

(3)

Where

\[
\omega_i = \omega - k_{||} V_{Te}, \quad \omega_e = \omega - k_{||} V_{Te}, \quad C_d^2 = \frac{\omega_d^2 \Omega_{d}^2}{\omega_p^2}
\]

Is the square of dust-acoustic speed and

\[
V_A^2 = \frac{c^2 \Omega_{i}^2}{\omega_p^2}
\]

Is the square of Alfven speed in dusty magneto plasma, \( V_{Te} \) is the beam velocity of respective species.

4. Energy balance and growth rate

Using the law of conservation of energy, we calculate the growth rate of kinetic Alfven wave in dusty magneto plasma by Shandilya et al. [2003, 2004]

\[
\frac{d}{dt}(W_e + W_i) = 0
\]

(4)

Which provides
In the case $V_{D_e} = V_{Di} = 0$, we recover the growth rate as derived by Dwivedi et al., [2001]. The growth is possible when $k_{||} V_{D_e} < \omega$ and $T_{e,||} k_{||} V_{e}^{e} \left( \omega - k_{||} V_{D_e} \right) \geq 1$. The computational results are explained with the justification to the physical process.

5. Results and discussion

The dispersion relation, growth rate parallel and perpendicular current for the kinetic Alfvén wave in dusty magnetized plasma have been evaluated. The following dusty plasma parameters for the auroral acceleration region are used to calculate the dispersion relation, growth rate, [Shandilya et al., 2003, 2004; Dwivedi et al., 2001; Das et al., 1996; Tiwari and Rostoker, 1984; Varma and Tiwari, 1992]. The results are presented by fig. 1 to 4.

$$B_0 = 4300 \text{ nT}; \quad \Omega_i = 412 \text{ s}^{-1}; \quad \Omega_d = \frac{Z_e e B_0}{m_d c} = 6.88 \times 10^{-10} Z_d ; \quad N_0 = 10 \text{ cm}^{-3}; \quad m_d = 10^{-12} \text{g}$$

$$\frac{\omega_{pe}}{\Omega_i} = 10^4; \quad kT_i = 100\text{eV}; \quad kT_e = 10\text{ KeV}; \quad V_{T,\perp i} = 3.5 \times 10^4 \text{ m s}^{-1}; \quad E_i = 100 \text{ mV m}^{-1}$$

$$E_{\perp} = 200\text{mVm}^{-1}.$$
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Figure 1. Frequency ($\omega$) Vs perpendicular wave number ($k$) for different values of $N_{do}$. 
Figure 2. Growth rate ($\gamma/\omega$) Vs perpendicular wave number ($k_{\perp}$) for different values of $N_{do}$.
Figure 3. Frequency ($\omega$) Vs perpendicular wave number ($k_{\perp}$) for different values of $Z_d$. 

Fig.3
Figure 4. Growth rate ($\gamma/\omega$) Vs perpendicular wave number ($k_\perp$) for different values of $Z_d$. 