Self-organization of vortices in type-II superconductors during magnetic relaxation

R. Prozorov
Loomis Laboratory of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana, IL 61801, U. S. A.

D. Giller
Institute of Superconductivity, Department of Physics, Bar-Ilan University, 52900 Ramat-Gan, Israel.

(submitted to Phys. Rev. B - 8 November 1998; accepted - 5 March 1999)

We revise the applicability of the theory of self-organized criticality (SOC) to the process of magnetic relaxation in type-II superconductors. The driving parameter of self-organization of vortices is the energy barrier for flux creep and not the current density. The power spectrum of the magnetic noise due to vortex avalanches is calculated and is predicted to vary with time during relaxation.

I. INTRODUCTION

The magnetic response of hard type-II superconductors, in particular magnetic flux creep, is a timely issue in contemporary research (see for review\cite{1}). In early 60s a very useful model of the critical state was developed to describe magnetic behavior of type-II superconductors\cite{2,3,4}. One of the distinguishing features of this behavior, observed experimentally, is that the density of flux lines varies across the whole sample. This model of the critical state remains in use, even though a significant progress has been made in understanding the particular mechanisms of magnetization and creep in type-II superconductors\cite{5,6,7,8,9,10,11,12,13,14,15,16}. It has also been noted that the magnetic flux distribution in type-II superconductors is, in many aspects, similar to a sandpile formed when, for example, sand is poured onto a stage\cite{17,18,19,20,21,22}. When a steady state is reached the slope of such a pile is analogous to the critical current density $j_c$ of a superconductor. Study of the dynamics (i.e. sand avalanches) of such strongly-correlated many-particle systems has led to a development of a new concept, called self-organized criticality (SOC), proposed originally by Bak and co-workers\cite{23,24,25,26}. Tang first analyzed direct application of SOC to type-II superconductors\cite{27}. Later numerous studies significantly elaborated on this topic\cite{28,29,30,31,32,33,34,35,36}.

In practice, especially in high-T_c superconductors, persistent current density $j$ in the experiment is much lower than the critical current density $j_c$ due to "giant" flux creep\cite{37,38,39}. The concept of SOC is strictly applied only to the critical state $j = j_c$ and it describes the system dynamics towards the critical state. Nevertheless, it is tempting to analyze magnetic flux creep in type-II superconductors during which the system moves out of the critical state, in a SOC context, because thermal activation can trigger vortex avalanches\cite{40,41}. However, it was found that modifications of the relaxation law due to vortex avalanches are minor and can hardly be reliably distinguished in the analysis of experimental data. Furthermore, flux creep universality has been analytically demonstrated in the elegant paper by Vinnokur et al.\cite{42}. Universality of the spatial distribution of the electric field during flux creep has also been found by Gurevich and Brandt\cite{43}. The direct application of SOC to the problem of magnetic flux creep thus meets a number of serious general difficulties. It is clear that critical scaling (power laws for vortex-avalanche lifetimes and size distributions) observed in the vicinity of the critical state\cite{44} must change during later stages of relaxation due to a time-dependent (or current-dependent) balance of the Lorentz and pinning forces.

In this paper we propose a new physical picture of self-organization in a vortex matter during magnetic flux creep in type-II superconductors. In this approach the driving parameter is the energy barrier for magnetic flux creep rather than the current density. We show that notwithstanding its minor influence on the relaxation rate, self-organized behavior may be observed by measuring magnetic noise during flux creep.

II. BARRIER FOR MAGNETIC FLUX CREEP AS THE DRIVING PARAMETER OF SELF-ORGANIZATION

We consider a long superconducting slab infinite in the $y$ and $z$ directions and having width $2w$ in the $x$ direction. The magnetic field is directed along the $z$ axis. In this geometry, the flux distribution is one-dimensional, i.e., $B(r,t) = (0,0,B(x,t))$. As a mathematical tool for our analysis we use a well known differential equation for flux creep\cite{45}:

$$\frac{\partial B}{\partial t} = -\frac{\partial}{\partial x}(Bv_0 \exp(-U(B,T,j)/T))$$

(1)

Here $B$ is the magnetic induction, $v = v_0 \exp(-U(B,T,j)/T)$ is the mean velocity of vortices in the $x$ direction and $U(B,T,j)$ is the effective barrier for flux creep. Note that we adopt units

\textbf{arXiv:cond-mat/9901344v3 [cond-mat.supr-con] 4 Mar 1999}
pressed before it arrives to the sample edge due to energy barrier $k m$ volume magnetization $m = M/V$ from Eq. [4]

$$\frac{\partial m}{\partial t} = -A \exp(-U(H,T,j)/T)$$

(2)

where $A \equiv H v_0/4\pi w$.

It is important to emphasize that we do not modify the pre-exponent factor $B v_0$ of Eq. [4] or $A$ of Eq. [2], as suggested by previous works on SOC (see e.g. [1]). Such modifications result only in logarithmic corrections to the effective activation energy, and they may be omitted in a flux creep regime [2]. Instead, we concentrate on the details of the spatial behavior of flux creep barrier $U(x)$, as analyzed in detail in our previous work [4]. In that work Eq. [2] was solved numerically and semi-analytically for different situations. We emphasize that, in general, the barrier for flux creep depends on magnetic field $B$, and persistent current density $j$ ($x$) is not uniform across the sample (see Fig. 1). Thus, $j$ cannot be used as a driving parameter for a SOC model. Instead the relevant parameter is $U$, which stays constant across the sample. Also, since experiments on magnetic relaxation are usually carried out at constant temperature and at high magnetic field, we can assume $U(B,T,j) = U(j)$. Central results of Ref. [1] are shown in Fig. 1 using a "collective creep"-type dependence $U(j) = U_0 (B/B_0)^n (j/j_c)^{1/2}$, with $n = 5$ and $\mu = 1$ as an example (see also Eq. [5] below (other models are analyzed in Ref. [1] as well and produce essentially similar results). Filled squares in Fig. 1 represent the distribution of the magnetic induction $B(x)/H$ at some late stage of relaxation (so that $j < j_c$), the solid line represents the normalized current density profile (note that $j_c$ is constant across the sample), and open circles show the profile of the effective barrier for flux creep $U(x)/T$. All quantities are calculated numerically from Eq. [1]. The important thing to note is that the energy barrier $M(x)$ is nearly independent of $x$, so that its maximum variation $\delta U$ is of order of $T$. As also shown from general arguments [5], such behavior means that the fluxon system organizes itself to maintain a uniform distribution of the barrier $U$ across the sample.

The vortex avalanches are introduced in an integral way. An avalanche of size $s$ causes a change in the total magnetic moment $\delta M = s$. This change is equivalent to a change of the average current density $\delta j = \delta M/\gamma = \gamma s$, where $\gamma = 2e/\mu V$. If the barrier for flux creep is $U(j)$, then the variation of current $\delta j$ leads to a variation of the energy barrier

$$\delta U = \left| \frac{\partial U}{\partial j} \right| \delta j = \gamma \left| \frac{\partial U}{\partial j} \right| s$$

(3)

As mentioned above, maximum fluctuation in the energy barrier $[\delta U]_{\text{max}}$ is of order of $T$ in the creep regime ($\delta U < U$). Any fluctuation $\delta U$ larger than $T$ is suppressed before it arrives to the sample edge due to exponential feedback of the local relaxation rate, which is proportional to $\exp(-U/T)$, (Eq. [5]). This means that only fluctuations $\delta U \leq T$ can be observed in global measurements of the sample magnetic moment. Thus,

$$s_m = \frac{T}{\gamma \left| \frac{\partial U}{\partial j} \right|} \propto VT$$

(4)

where we denote as $s_m$ the maximum possible avalanche, which depends on time via $\partial U/\partial j$. It is worth to note that Eq. [4] gives the correct dependence of $s_m$ on the system size and on temperature. It is clear that in a finite system the largest possible avalanche must be proportional to the system volume. Since it is thermally activated, it is proportional to temperature $T$, consistent with our derivation. The characteristic time-dependent upper cut-off of the avalanche size was experimentally observed by Field et al. [6] who studied magnetic noise spectra at different magnetic field sweep rates, i.e. at different time windows of the experiment.

Our central idea is that in the vicinity of $j_c$ the system of fluxons, indeed, exhibits self-organized critical behavior, as initially proposed by Tang et al. [7] During flux creep, it maintains itself in a self-organized, however not critical state in the sense that it cannot be described by the critical scaling. The self-organization manifests itself by the appearance of almost constant across the sample $U$. Avalanches do not vanish, but there is a constrain on the largest possible avalanche, see Eq. [5]. Importantly, $s_m$ depends upon current density and, as we show below, decreases with decrease of current (or with increase of time), so their relative importance vanishes.

In order to calculate physically measured quantities let us derive the time dependence of $s_m$ assuming a very useful generic form of the barrier for flux creep, introduced by Griessen et al. [8]

$$U(j) = \frac{U_0}{\alpha} \left[ \left( \frac{j_c}{j} \right)^{\alpha} - 1 \right]$$

(5)

This formula describes all widely-known functional forms of $U(j)$ if the exponent $\alpha$ attains both negative and positive values. For $\alpha = -1$ Eq. [5] describes the Anderson-Kim barrier [9] for $\alpha = -1/2$ the barrier for plastic creep [10] is obtained. Positive $\alpha$ describes collective creep barrier [11]. In the limit $\alpha \to 0$ this formula reproduces exactly logarithmic barrier [12]. An activation energy written in the form of Eq. [5] results in an "interpolation formula" for flux creep, if the logarithmic solution of the creep equation $U(j) = T \ln(j/j_0)$ is applied [13] (for $\alpha \neq 0$):

$$j(t) = j_c \left( 1 + \frac{\alpha T}{U_0} \ln \left( \frac{j}{j_0} \right) \right)^{-\frac{1}{\alpha}}$$

(6)

For $\alpha = 0$, a power-law decay is obtained: $j(t) = j_c \left( t_0/t \right)^n$, where $n = T/U_0$.

Using this general form of the current dependence of the activation energy barrier, we obtain from Eq. [5]
the total power spectrum of magnetic noise during flux creep is

\[ S(\omega) = \int_0^\infty \rho(\tau) L(\omega, \tau) \, d\tau. \]  

Using Eq. 10 we find:

\[ S(p) \propto \frac{1}{2p^2} \left[ \cos \left( \frac{1}{p} \right) \text{Re} \left( Ei \left( \frac{i}{p} \right) \right) - \sin \left( \frac{1}{p} \right) \text{Im} \left( Ei \left( \frac{i}{p} \right) \right) \right]. \]  

Here \( p = \omega \tau_m (t) \) and \( Ei(x) = \int_0^\infty \exp(-x \eta) / \eta \, d\eta \) is the exponential integral. The power spectrum \( S(\omega, t) \) described by Eq. 13 is plotted in Fig. 2 using a solid line. Since there an upper cutoff for the avalanche lifetime at \( \tau_m \), the lowest frequency which makes sense is \( 2\pi/\tau_m \). Thus, only frequency domain \( 2\pi/\tau_m < \omega \) is important. In the limit of large \( p \), the spectral density of Eq. 13 has a simple asymptote:

\[ S(\omega) \propto \frac{\ln(p) - \gamma_e}{p^2} \]  

where \( \gamma_e \approx 0.577\ldots \) is Euler’s constant. This simplified power spectrum is shown in Fig. 2 by a dashed line. For \( p > 10 \) this approximation is quite reasonable. The usual way to analyze the power spectrum is to present it in a form \( S(\omega) \propto 1/\omega^{\nu} \) and extract the exponent \( \nu \) simply as \( \nu = -\partial \ln(S) / \partial \ln(\omega) \). In our case the parameter \( p = \omega \tau_m \) is a reduced frequency, so the exponent \( \nu \) can be estimated as

\[ \nu = -\frac{\partial \ln(S)}{\partial \ln(p)} = 2 - \frac{1}{\ln(p) - \gamma_e}. \]  

This result is very important, since it fits quite well the experimentally observed values of \( \nu \) which were found to vary between 1 and 2. As seen from Fig. 2, it is impossible to distinguish between real \( 1/\omega^{\nu} \) dependence and that predicted by Eq. 13 at large enough frequencies. Remarkably, in many experiments the power spectrum was found to deviate significantly from the \( 1/\omega^{\nu} \) behavior at lower frequencies, which fits, however, Eq. 13.

Using Eq. 13 or Eq. 14 one can find the temperature, magnetic field and time dependence of the power spectrum substituting \( p = \omega \tau_m = \omega s_m^{\gamma_e} \) and using values of \( s_m(H, T, t) \) derived in the previous section. Specifically, from Eq. 8 we obtain that any given frequency amplitude of a power spectrum increases with time in the collective creep regime, but saturates in the case of the logarithmic barrier and remains constant in the case of the Kim-Anderson barrier.

In general, we emphasize that the power spectrum of the magnetic noise during flux creep depends on time. Since parameter \( p \) decreases with the increase of time, the exponent \( \nu \) becomes closer to 1 during flux creep. At
these later stages of relaxation the effect of the avalanches is negligible and magnetic noise is mostly determined by thermally activated jumps of vortices with the usual (non-correlated) $1/\omega$ power spectrum. Thus, the manifestation of the avalanche-driven dynamics during flux creep is noise spectra with $1/\omega^n$ and decreasing $\nu(t)$ when sampled at different times during relaxation. This explains the experimental results obtained by Field et al. who measured directly vortex avalanches at different sweep rates. Those found that the exponent $\nu$ decreased from a relatively large value of 2 at a large sweep rate of $20 \text{ G/sec}$ to a smaller value of 1.5 for a sweep rate of $1 \text{ G/sec}$. This is in a good agreement with our model.

IV. CONCLUSIONS

In conclusion, self-organization of vortices in hard type-II superconductors during magnetic flux creep was analyzed. Using results of a numerical solution of the differential equation for flux creep, it was argued that the self-organized criticality describes the system dynamics at $j = j_c$. During flux creep, the vortex system remains self-organized, but there is no criticality in the sense that there are no simple power laws for distributions of the avalanche size, lifetime, and for the power spectrum. The driving parameter of the self-organized dynamics is the energy barrier $U(B, j)$ and not the current density $j$, as proposed by previous work. Using a simple model the power spectrum $S(\omega)$ of the magnetic noise is predicted to depend on time. Namely, fitting $S(\omega)$ to a $1/\omega^n$ behavior will result in a time-dependent exponent $\nu(t)$ decreasing in the interval between 2 and 1.

Acknowledgments: We acknowledge fruitful discussions with L. Burlachkov and B. Shapiro. We thank F. Nori for critical remarks. D.G. acknowledges support from the Clore Foundations. This work was partially supported by the National Science Foundation (DMR 91-20000) through the Science and Technology Center for Superconductivity, and by DOE grant DEFG02-91-ER45439.

1. G. Blatter, M. V. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994).
2. E. H. Brandt, Rep. Prog. Phys. 58, 1465 (1995).
3. Y. Yeshurun, A. P. Malozemoff, and A. Shaulov, Rev. Mod. Phys. 68, 911 (1996).
4. C. P. Bean, Phys. Rev. Lett. 8, 250 (1962), Y. B. Kim, C. F. Hempstead, and A. R. Strand, Physical Review 129, 528 (1963).
5. P. W. Anderson, Phys. Rev. Lett. 9, 309 (1962), Y. B. Kim, C. F. Hempstead, and A. R. Strand, Phys. Rev. Lett. 9, 306 (1962); Y. B. Kim, C. F. Hempstead and A. R. Strand, Phys. Rev. 129, 528 (1963); P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. 36, 39 (1964); M. R. Beasley, R. Labush, and W. W. Webb, Phys. Rev. 181, 682 (1969).
6. A. M. Campbell and J. E. Evetts, “Critical currents in superconductors” (Taylor Francis Ltd., London, 1972); A. I. Larkin and Y. N. Ovchinnikov, J. Low Temp. Phys. 73, 109 (1979); H. Ullmaier, “Irreversible properties of type-II superconductors” (Springer-Verlag, Berlin, Heidelberg, New York, 1975); E. H. Brandt and M. V. Indenbom, Phys. Rev. B 48, 12893 (1993); E. Zeldov, J. R. Clem, M. McElfresh, and M. Darwin, Phys. Rev. B 49, 9802 (1994); E. H. Brandt, Phys. Rev. Lett. 74, 3025 (1995).
7. C. Tang, Physica A 194, 315 (1993).
8. P. G. de Gennes, “Superconductivity of metals and alloys” (W. A. Benjamin, New-York, 1966).
9. P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. 59, 381 (1987); P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A 38, 364 (1988); C. Tang and P. Bak, Phys. Rev. Lett. 60, 2347 (1988).
10. V. M. Vinokur, M. V. Feigelman, and V. B. Geshkenbein, Phys. Rev. Lett. 67, 915 (1991).
11. O. Pla and F. Nori, Phys. Rev. Lett. 67, 919 (1991); Z. Wang and D. Shi, Phys. Rev. B 48, 4208 (1993); Z. Wang and D. Shi, Phys. Rev. B 48, 9782 (1993).
12. S. Field, J. Witt, F. Nori, and X. Ling, Phys. Rev. Lett. 74, 1206 (1995).
13. W. Pan and S. Doniach, Phys. Rev. B 49, 1192 (1994).
14. E. Bonabeau and P. Lederer, Physica C 256, 365 (1996).
15. C. J. Olson, C. Reichhardt, and F. Nori, Phys. Rev. B 56, 6175 (1997).
16. A. Gurevich, Int. J. of Mod. Phys. 9, 1045 (1995); E. H. Brandt, Phys. Rev. Lett. 76, 4030 (1996).
17. L. Burlachkov, D. Giller, and R. Prozorov, Phys. Rev. B 58, 15067 (1998).
18. V. B. Geshkenbein and A. I. Larkin, Zh. Eksp. Teor. Fiz. 95, 1108 (1989).
19. E. Zeldov, N. M. Amer, G. Koren, A. Gupta, R. J. Gambino, and M. W. McElfresh, Phys. Rev. Lett. 62, 3093 (1989).
20. R. Griessen, A. F. T. Hoekstra, H. H. Wen, G. Doornbos, and H. G. Schnack, Physica C 282-287, 347 (1997); H. H. Wen, A. F. Th. Hoekstra, R. Griessen, S. L. Yan, L. Fang, and M. S. Si, Phys. Rev. Lett. 79, 1559 (1997).
21. Y. Abulafia, A. Shaulov, Y. Wolfus, R. Prozorov, L. Burlachkov, Y. Yeshurun, D. Majer, E. Zeldov, H. Wühl, V. B. Geshkenbein, and V. Vinokur, Phys. Rev. Lett. 77, 1596 (1996).
22. M. J. Ferrari, M. Johnson, F. C. Wellstood, J. Clarke, P. A. Rosenthal, R. H. Hammond, and M. R. Beasley, Appl. Phys. Lett. 53, 695 (1988).
Figure captions

Fig. 1 Results of numerical solution of Eq. 1 for $U(j) = U_0 (B/B_0)^5 (j_c/j - 1)$ at $j < j_c$. Spatial distribution of magnetic induction $B(x)/H$ (filled squares); corresponding profile of the normalized current density (solid line) and the corresponding profile of the effective barrier for flux creep $U(x)/T$ (open circles).

Fig. 2 The power spectrum $S(\omega, t)$ described by Eq. 13 (solid line) and approximate asymptotic solution (dashed line).
$S = \frac{(\ln(p) - \gamma_e)}{p^2}$

Fig. 2 Prozorov and Giller