CPT AND LORENTZ VIOLATION IN KAONS AND OTHER SYSTEMS

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ABSTRACT

This talk provides a status report on CPT violation in neutral-meson oscillations, focusing on implications of the CPT- and Lorentz-violating standard-model extension.

1 Introduction

The standard model of particle physics is invariant under CPT and Lorentz symmetry. However, small observable violations might emerge from a more fundamental theory. Sensitive CPT tests for these effects can be performed by taking advantage of the finely balanced natural interferometers provided by the neutral-meson systems.

It is possible to parametrize any indirect CPT violation in a neutral-meson oscillation with a complex quantity. Here, this quantity is denoted
by $\xi_P$, where $P$ represents one of the neutral mesons $K, D, B_d, B_s$. Assuming that $\xi_K$ is constant, experiments on kaons have determined that its real and imaginary parts are no greater than about $10^{-4}$.

In conventional quantum field theory, $\xi_K$ cannot be constant. The point is that the CPT theorem shows that $\xi_K$ must be zero unless Lorentz symmetry is broken, while using an explicit and general Lorentz-violating standard-model extension to calculate $\xi_K$ shows that it varies with the meson 4-momentum. Various CPT tests have also been proposed using the heavy mesons $D, B_d, B_s$. Recent experiments have obtained bounds on $\text{Re} \xi_{B_d}$ of order 1 and on $\text{Im} \xi_{B_d}$ of order $10^{-1}$ under the assumption of constant $\xi_{B_d}$.

This talk reviews the present theoretical situation for CPT violation in neutral-meson systems in the context of the CPT- and Lorentz-violating standard-model extension. Some experimentally accessible asymmetries are presented for both uncorrelated and correlated neutral-meson systems, and the implications of the variation of $\xi_P$ with meson 4-momentum are discussed.

2 Basics

Any linear combination of the Schrödinger wave functions for a meson $P_0$ and its antimeson $\bar{P}_0$ can be represented as a two-component object $\Psi(t)$. The time evolution of an arbitrary neutral-meson state is then controlled by a $2 \times 2$ effective hamiltonian $\Lambda$ according to $i\partial_t \Psi = \Lambda \Psi$. The eigenstates $|P_a\rangle$ and $|P_b\rangle$ of $\Lambda$ are physical states, in analogy with the normal modes of a classical oscillator. They evolve as $|P_a(t)\rangle = \exp(-i\lambda_a t)|P_a\rangle$, $|P_b(t)\rangle = \exp(-i\lambda_b t)|P_b\rangle$, where the complex parameters $\lambda_a \equiv m_a - \frac{1}{2}i\gamma_a$, $\lambda_b \equiv m_b - \frac{1}{2}i\gamma_b$ are the eigenvalues of $\Lambda$, with $m_a, m_b$ the physical masses and $\gamma_a, \gamma_b$ the decay rates. It is convenient to introduce the definitions $\lambda \equiv \lambda_a + \lambda_b = m - \frac{1}{2}i\gamma$, $\Delta \lambda \equiv \lambda_a - \lambda_b = -\Delta m - \frac{1}{2}i\Delta \gamma$, where $m = m_a + m_b$, $\Delta m = m_b - m_a$.

Without loss of generality, the effective hamiltonian $\Lambda$ can be adopted as

$$\Lambda = \frac{1}{2} \Delta \lambda \begin{pmatrix} U + \xi & VW^{-1} \\ VW & U - \xi \end{pmatrix},$$

(1)
where the parameters $UVW\xi$ are complex. The prefactor $\Delta \lambda/2$ ensures these parameters are dimensionless and eliminates some factors of 2 in subsequent equations. Imposing the trace as $\text{tr } \Lambda = \lambda$ and the determinant as $\det \Lambda = \lambda_a \lambda_b$ shows that $U \equiv \lambda/\Delta \lambda$ and $V \equiv \sqrt{1-\xi^2}$.

The independent complex parameters $W = w \exp(i\omega)$, $\xi = \text{Re } \xi + i\text{Im } \xi$ in Eq. (1) have four real components. However, one is physically unobservable: the argument $\omega$ changes under a phase redefinition of the $P^0$ wave function. The three others are physical. The parameter $w$ determines the amount of T violation, with T preserved if and only if $w = 1$. The two real numbers $\text{Re } \xi$, $\text{Im } \xi$ determine the amount of CPT violation, with CPT preserved if and only if both are zero. Note that in the standard notation specific to the $K$ system, in which the complex parameter for CPT violation is often denoted $\delta_K$, imposing small CP violation and making a suitable choice of phase convention yields the identification $\xi_K \approx 2\delta_K$.

The three CP-violation parameters $w$, $\text{Re } \xi$, $\text{Im } \xi$ in this $w\xi$ formalism are dimensionless, can be used for arbitrary size CPT and T violation, and are independent of phase conventions. They are also independent of any specific model because they are phenomenological. However, it is unjustified a priori to suppose that they must be constant numbers. In fact, the assumption often found in the literature that $\xi$ is constant and nonzero is an additional strong requirement, which the CPT theorem shows is inconsistent with the basic axioms of Lorentz-invariant quantum field theory. If instead Lorentz violations are allowed, then in quantum field theory $\xi$ cannot be constant and is found to vary with the meson 4-momentum. This result is outlined in the next part of the talk.

3 Theory for CPT Violation

Lorentz-invariant quantum field theories are CPT-symmetric by virtue of the CPT theorem. In describing CPT violation at the level of quantum field theory, it is thus interesting to study the consequences of small Lorentz violations. A general CPT- and Lorentz-violating standard-model extension exists and could arise, for instance, as the low-energy limit of an underlying Planck-scale theory. In addition to the neutral-meson oscillations discussed here, signals in a variety of other types of experiment are predicted by the standard-model extension. These include, for example, tests of quantum electrodynamics with
trapped particles, measurements of muon properties, hydrogen and antihydrogen spectroscopy, clock-comparison experiments, studies of the behavior of a spin-polarized torsion pendulum, measurements of cosmological birefringence, and observations of the baryon asymmetry. However, none of these experiments involve flavor-changing effects, and as a result it can be shown that they leave unconstrained the sector of the standard-model extension relevant to experiments with neutral-meson oscillations.

The dominant CPT-violating contributions to the effective hamiltonian $\Lambda$ can be calculated as expectation values of interaction terms in the standard-model extension. It can be shown that the difference $\Delta \Lambda = \Lambda_{11} - \Lambda_{22}$ of the diagonal terms of $\Lambda$ is given by

$$\Delta \Lambda \approx \beta^\mu \Delta a_\mu,$$

where $\beta^\mu = \gamma(1, \vec{\beta})$ is the four-velocity of the $P$ meson in the laboratory frame and the coefficients $\Delta a_\mu$ are combinations of coefficients appearing in the lagrangian for the standard-model extension.

There are four independent components in $\Delta a_\mu$, which implies that a complete characterization of CPT violation requires four independent CPT measurements in each $P$-meson system. Moreover, the 4-velocity and consequent 4-momentum dependence in Eq. (2) shows explicitly that CPT violation cannot be described with a constant complex parameter in quantum field theory. Since the effects from CPT violation will typically vary with the momentum magnitude and orientation of the $P$ mesons, the experimental reach depends on the meson momentum spectrum and angular distribution. Among other consequences, this implies that experiments previously regarded as equivalent may in fact have different CPT reaches.

Another feature of experimental importance is the variation of some CPT observables with sidereal time, resulting from the rotation of the Earth relative to the constant vector $\Delta \vec{a}$. To exhibit directly the sidereal-time dependence, it is necessary to convert the result (2) for $\Delta \Lambda$ from the laboratory frame rotating with the Earth to a nonrotating frame. It is convenient to adopt a nonrotating frame compatible with celestial equatorial coordinates. Let the coefficient $\vec{a}$ for Lorentz violation in a $P$-meson system have nonrotating-frame components $(a^X, a^Y, a^Z)$. Take the unit vector $\hat{Z}$ to be aligned along the Earth's rotation axis, and let $\vec{\beta} = \beta(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ be the
laboratory-frame 3-velocity of a $P$ meson, where the angles $\theta$, $\phi$ are defined with respect to the laboratory-frame $\hat{z}$ axis. Define the momentum magnitude $p \equiv |\vec{p}| = \beta m_P \gamma(p)$, where $\gamma(p) = \sqrt{1 + p^2/m_P^2}$ as usual. Then, it can be shown that in any $P$ system and for arbitrary size CPT violation the complex CPT parameter $\xi$ is

$$\xi \equiv \xi(\hat{t}, \vec{p}) \equiv \xi(\hat{t}, p, \theta, \phi) = \frac{\gamma(p)}{\Delta \lambda} \left\{ \Delta a_0 + \beta \Delta a_Z (\cos \theta \cos \chi - \sin \theta \cos \phi \sin \chi) + \beta \left[ \Delta a_Y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) - \Delta a_X \sin \theta \sin \phi \right] \sin \Omega \hat{t} + \beta \left[ \Delta a_X (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) + \Delta a_Y \sin \theta \sin \phi \right] \cos \Omega \hat{t} \right\},$$

(3)

where $\hat{t}$ is the sidereal time. In the next part of the talk, some implications of the expression (3) for experiment are presented.

4 Experiment

Consider for simplicity the case of semileptonic decays into a final state $f$ or its conjugate state $\bar{f}$, neglecting any violations of the $\Delta Q = \Delta S$, $\Delta Q = \Delta C$, or $\Delta Q = \Delta B$ rules. The basic transition amplitudes can be taken as $\langle f | T | P^0 \rangle = F^*, \langle \bar{f} | T | P^0 \rangle = \bar{F}$, $\langle f | T | \bar{P}^0 \rangle = F$, $\langle f | T | P^0 \rangle = \langle \bar{f} | T | \bar{P}^0 \rangle = 0$. Time-dependent decay amplitudes and probabilities can then be calculated as usual. In addition to the proper-time dependence, there is now also sidereal time and momentum dependence from $\xi(\hat{t}, \vec{p})$. Note that $\xi$ is independent of $t$ to an excellent approximation because the meson decays are rapid on the scale of sidereal time.

As a simple example for uncorrelated mesons, consider the case where $F^* = F^*$, i.e., negligible direct CPT violation. In terms of decay probabilities, a CPT-sensitive asymmetry is

$$A^{\text{CPT}}(t, \hat{t}, \vec{p}) = \frac{F^+(t, \hat{t}, \vec{p}) - F^-(t, \hat{t}, \vec{p})}{F^+(t, \hat{t}, \vec{p}) + F^-(t, \hat{t}, \vec{p})} = \frac{2 \text{Re} \xi \sinh \Delta \gamma t/2 + 2 \text{Im} \xi \sin \Delta m t}{(1 + |\xi|^2) \cosh \Delta \gamma t/2 + (1 - |\xi|^2) \cos \Delta m t},$$

(4)

which depends implicitly on $\hat{t}, \vec{p}$ through the dependence on $\xi(\hat{t}, \vec{p})$.

Careful averaging over one of more of the variables $t, \hat{t}, \vec{p}, \theta, \phi$ either before or after constructing the asymmetry (4) yields independent bounds on the four
coefficients $\Delta a_\mu$. In particular, inspection of Eq. (3) reveals that binning in $\hat{t}$ gives information on $\Delta a_X$ and $\Delta a_Y$, while binning in $\theta$ separates the spatial and timelike components of $\Delta a_\mu$. An example that has already given two independent CPT bounds of about $10^{-20}$ GeV each on different combinations of the coefficients $\Delta a_\mu$ in the $K$ system is provided by the special case of mesons highly collimated in the laboratory frame, for which the 3-velocity can be written $\vec{\beta}=(0,0,\beta)$ and $\xi$ simplifies. Binning in $\hat{t}$ provides sensitivity to the equatorial components $\Delta a_X$, $\Delta a_Y$, while averaging over $\hat{t}$ eliminates them altogether. More generally, note that the variation with sidereal time can provide clean CPT bounds even using observables that mix T and CPT effects, such as the standard rate asymmetry $\delta_l$ for $K_L$ semileptonic decays.

As another example, consider the case of the decay into $ff$ of a correlated meson pair created by quarkonium decay. The probability for the double decay is a function of the sidereal time $\hat{t}$ and of the proper decay times $t_1$, $t_2$ and momenta $\vec{p}_1$, $\vec{p}_2$ of the two mesons. Note that according to Eq. (3) the CPT-violating parameters $\xi_1$ and $\xi_2$ for each meson typically differ. Experimentally, the time sum $t = t_1 + t_2$ is unobservable and so the relevant probability $\Gamma_{ff}$ is found by integrating over $t$. An asymmetry $A_{ff}^{\text{CPT}}$ sensitive to the sum $\xi_1 + \xi_2$ and the difference $\Delta t = t_1 - t_2$ can then be defined as

$$A_{ff}^{\text{CPT}}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) = \frac{\Gamma_{ff}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) - \Gamma_{ff}(-\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2)}{\Gamma_{ff}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) + \Gamma_{ff}(-\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2)} = \frac{-\text{Re}(\xi_1 + \xi_2) \sinh \frac{1}{2} \Delta \gamma \Delta t - \text{Im}(\xi_1 + \xi_2) \sin \Delta m \Delta t}{\cosh \frac{1}{2} \Delta \gamma \Delta t + \cos \Delta m \Delta t},$$

(5)

where the sidereal-time and momenta dependences are implicit in $\xi_1$, $\xi_2$. As before, different experiments using this asymmetry may have different CPT reach. If the quarkonium is produced at rest in a symmetric collider, for instance, then the sum $\xi_1 + \xi_2 = 2\gamma(p)\Delta a_0/\Delta \lambda$ is independent of $\Delta \hat{u}$, so a direct fit to the variation with $\Delta t$ provides a bound on $\Delta a_0$. In contrast, quarkonium production in an asymmetric collider implies $\xi_1 + \xi_2$ is sensitive to all four components of $\Delta a_\mu$, and so suitable binning permits the extraction of four independent CPT bounds. These experiments are feasible, for example, at the existing asymmetric $B_d$ factories BaBar and BELLE.
Acknowledgments

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