The Measure of Strong CP Violation

Zheng Huang

Department of Physics, Simon Fraser University, Burnaby, B.C., Canada V5A 1S6

We investigate a controversial issue on the measure of CP violation in strong interactions. In the presence of nontrivial topological gauge configurations, the $\theta$-term in QCD has a profound effect: it breaks the CP symmetry. The CP-violating amplitude is shown to be determined by the vacuum tunneling process, where the semiclassical method makes most sense. We discuss a long-standing dispute on whether the instanton dynamics satisfies or not the anomalous Ward identity (AWI). The strong CP violation measure, when complying with the vacuum alignment, is proportional to the topological susceptibility. We obtain an effective CP-violating lagrangian different from that provided by Baluni. To solve the IR divergence problem of the instanton computation, we present a “classically gauged” Georgi-Manohar model and derive an effective potential which uniquely determines an explicit $U(1)_A$ symmetry breaking sector. The CP violation effects are analyzed in this model. It is shown that the strong CP problem and the $U(1)$ problem are closely related. Some possible solutions to both problems are also discussed with new insights.

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I. INTRODUCTION

The discovery of instantons [1] has been associated with some of the most interesting developments in strong interaction theory. It has led to a resolution [2] of the long-standing $U(1)$ problem [3], and also pointed to the existence in QCD [4] of vacuum tunneling and of a vacuum angle $\theta$, which combining with the phase of the determinant of the quark mass matrix, signals the CP violation in strong interactions. The difficulty of understanding the very different hierarchies of the strong CP violation and weak CP violation in the standard model has been targeted as the so-called strong CP problem (for a review, see Ref. [5]).

The theoretical understanding of weak CP violation is well-established in the framework of Kobayashi-Maskawa mechanism [6] in spite of the challenge on the experiment measurement with higher precisions. It has been shown [7] that the determinant of the commutator of the up-type and down-type quark mass matrices $[M^u, M^d] \equiv iC$ given by

$$\det C = -2 J_{\text{weak}} (m_t - m_c)(m_c - m_u)(m_u - m_t)(m_b - m_s)(m_s - m_d)(m_d - m_b)$$ (1.1)

where

$$J_{\text{weak}} \equiv \sin^2 \theta_1 \sin \theta_2 \sin \theta_3 \cos \theta_1 \cos \theta_2 \cos \theta_3 \sin \delta$$ (1.2)

is the unique measure of the weak CP violation. All CP-violating effects in weak interaction must be proportional to $\det C$. Even though the CP-violating phase $\sin \delta$ can be of order 1, the physical amplitude is naturally suppressed by the product of Cabibo mixing angles.

However, the measure of CP violation in QCD, which we shall denote as $J_{\text{strong}}$, is not so clear. It has been long realized that $\theta_{QCD}$ and phases of quark masses are not independent parameters in QCD. In the presence of the chiral anomaly [8], they are related through the chiral transformations of quark fields. Thus $J_{\text{strong}}$ must be proportional to a combination

$$\bar{\theta} = \theta_{QCD} + \arg \det M$$ (1.3)

which is invariant under chiral rotations. It is well-known that if one of quarks is massless, $\bar{\theta}$ can be of an arbitrary value since one can make arbitrary rotations on the chiral field. This
suggests that the $\bar{\theta}$-dependence of $J_{\text{strong}}$ disappears in the chiral limit. Thus in the case of $L = 2$ where $L$ is the number of light quarks, $J_{\text{strong}}$ has a form

$$J_{\text{strong}} = m_u m_d K \sin \bar{\theta}$$

(1.4)

where we have written $\sin \bar{\theta}$ instead of $\bar{\theta}$ to take care of the periodicity of $\bar{\theta}$. Is there any other common factor that we can extract from strong CP effects? Or, is $K$ in (1.4) only a kinematical factor which varies with different physical processes.

To answer the question, we need to know whether there is another condition under which the strong CP violation vanishes. Recently, the reanalysis of strong CP effects has shed some light on this issue. Several authors have pointed out by studying an effective lagrangian that the conventional approach to estimating the strong CP effects is erroneous in concept though numerically it is close to the correct one. They believe that strong CP violation should vanish if the chiral anomaly is absent. We regard their work as constructive and enlightening. However, the connection of the effective theory with QCD is not apparent in their approaches. Indeed, if the chiral anomaly is absent in QCD, the phases of quark masses can be rotated away without changing the $\theta$-term. But it is not clear why $\theta_{QCD}$ does not lead to CP violation in strong interactions. In addition, the presence of the chiral anomaly in a gauge theory may not directly related to CP violation. One example is QED. It is well-known that QED is a CP-conserving theory even if it is chirally anomalous, and, in principle, could have a $\theta$-term and a complex electron mass term.

In this paper, however, we show that the measure of strong CP violation does acquire a factor referred to as the measure of the non-triviality of the non-abelian gauge vacuum. It is simply due to the fact that the $\theta$-term is a total divergence whose integration over space-time yields a surface term. It can be dropped off unless there are non-trivial gauge configurations at the boundary. $K$ in (1.4) will be shown to be the vacuum tunneling amplitude between different vacua characterized by the winding numbers

$$\nu = \int d^4 x F \tilde{F} \equiv \frac{g^2}{32\pi^2} \int d^4 x F_{\mu\nu} \tilde{F}^{\mu\nu}$$

(1.5)
where a semiclassical method makes most sense to deal with it. To probe the property of
the $K$-factor, we proceed to consider a classically gauged linear $\sigma$-model. A derivation of a $U(1)_A$ sector of the model can be made by taking into account the fermion zero modes in
the instanton fields. Contrary to the conventional result [10,12] where $K$ has a singularity
in quark masses such that $J_{\text{strong}}$ is a linear function of the quark mass, our model clearly
shows that $K$ is to be explained as the mass difference between the $U(1)$ particle and pions.
Thus, $J_{\text{strong}}$ has a form

$$J_{\text{strong}} = m_u m_d (m_q^2 - m_\pi^2) \sin \bar{\theta}.$$  (1.6)

In the context of the effective model, the strong CP effects can be explicitly calculated and
various solutions to the strong CP problem will be discussed with new insights.

The paper is organized as follows. In sect. 2, we discuss a long-standing problem raised
by Crewther [10,11] on whether the instanton is or not consistent with the anomalous Ward
identity (AWI). We find that the AWI does not put any constraint on the topological sus-
ceptibility $\langle\langle \nu^2 \rangle\rangle$ in QCD. The AWI is automatically satisfied by instanton dynamics if the
singularity in the chiral limit of some fermionic operator is taken care of. Sect. 3 deals
with a instanton computation of $\langle\langle \nu^2 \rangle\rangle$ in the dilute gas approximation. The vacuum align-
ment equations of the quark condensates are derived based on the path integral formalism.
Upon making alignment among strong CP phases, we rederive an effective CP-violating
lagrangian. In sect. 4 we present a classically gauged linear $\sigma$-model. In the semiclassical
approximation, the instanton fields are integrated out. An effective one-loop potential is
obtained by integrating over fermions in the instanton background where the fermion zero
modes are essential to yield an explicit $U(1)_A$ symmetry breaking. The strong CP effects
and the $U(1)$ particle mass are calculated in the model. Sect. 5 devotes to discussions on
various possible solutions to the strong CP problem. Sect. 6 reserves for conclusions.

II. DOES INSTANTON SATISFY THE AWI?
THE TOPOLOGICAL SUSCEPTIBILITY $\langle\langle \nu^2 \rangle\rangle$

Let us leave our discussion on $J_{\text{strong}}$ aside for a moment and turn to a problem which turns out to be a key to understand both strong CP violation and $U(1)$ problem. It has been long pointed out that the instanton physics, in some ways, suffers from some difficulties. It is well-known that the integration over the instanton size is of infrared divergence. It is further argued by Witten [13] that the semiclassical method based on the instanton solution of Yang-Mills equation is in conflict with the most successful idea of $\frac{1}{N_c}$ expansion in QCD. The reason is that instanton effects are of order $e^{-\frac{1}{g^2}}$, and for large $N_c$, $g^2$ is of order $e^{-N_c}$, which is smaller than any finite power of $\frac{1}{N_c}$ obtained by summing Feynman diagrams. These problems, as they stand now, indeed reflect various defects in the instanton calculation (we will come back to these points in later sections).

However, there was another type of objections initiated by Crewther [10] followed by others [11], which would be even more serious if they were correct. For many years Crewther has emphasized that the breakdown of $U(1)_A$ symmetry by the chiral anomaly and the instanton is related to the breakdown of the $SU(L) \times SU(L)$ symmetry. The relation is represented by the so-called anomalous Ward identity. He claimed that the instanton dynamics failed to satisfy the AWI and one would still expect the unwanted $U(1)_A$ goldstone boson. They further showed that the topological susceptibility defined as

$$\langle\langle \nu^2 \rangle\rangle = \int d^4x \langle T iF \tilde{F}(x) iF \tilde{F}(0) \rangle$$  \hspace{1cm} \text{(2.1)}$$

when satisfies the AWI must be equal to $m\langle \bar{\psi}\psi \rangle$ ($m$ is the quark mass, we have assumed that all quarks are of equal masses). As we shall see in sect. 3, $\langle\langle \nu^2 \rangle\rangle$ is to be identified as the measure of strong CP violation. If Crewther were right, it would seem that the strong CP is of no direct relation with the topological vacuum structure.

To see where the problem lies, we carefully follow a path integral derivation of the AWI. Consider a fermion bilinear operator $\bar{\psi}_L \psi_R$ with chirality $+2$ (sum over flavor indices is understood). Its vacuum expectation value (VEV) is formally given
\[ \langle \bar{\psi}_L \psi_R \rangle = \frac{1}{V} \langle \int d^4x \bar{\psi}_L \psi_R(x) \rangle = \frac{1}{VZ} \int \mathcal{D}(A, \bar{\psi}, \psi) \int d^4x \bar{\psi}_L \psi_R(x) e^{-S[A, \bar{\psi}, \psi]} \] (2.2)

where the QCD action in Euclidean space is

\[ S[A, \bar{\psi}, \psi] = \int d^4x \bar{\psi} D\psi + m\bar{\psi}\psi + \frac{1}{4} F^2 - i\theta F \tilde{F} \] (2.3)

and \( Z \) is the normalization factor, \( V \) is the volume of space-time. Under an infinitesimal \( U(1)_A \) transformation

\[ \psi_R \rightarrow e^{i\alpha(x)} \psi_R ; \quad \psi_L \rightarrow e^{-i\alpha(x)} \psi_L \] (2.4)

the measure \( \mathcal{D}(A, \bar{\psi}, \psi) \) will change because of the chiral anomaly. However, the integral (2.2) will not change since (2.4) is only a matter of changing integral variables. (2.2) then becomes

\[ \langle \bar{\psi}_L \psi_R \rangle = \frac{1}{VZ} \int \mathcal{D}(A, \bar{\psi}, \psi) \int d^4x e^{2i\alpha(x)} \bar{\psi}_L \psi_R(x) \exp\{-S[A, \bar{\psi}, \psi] + i\alpha(x) \int d^4x [\partial_\mu J_\mu^5 - 2m\bar{\psi}\gamma_5 \psi - 2L F \tilde{F}]\} \] (2.5)

where the \( U(1)_A \) current \( J_\mu^5 = \bar{\psi} \gamma_\mu \gamma_5 \psi \). The independence of \( \langle \bar{\psi}_L \psi_R \rangle \) on \( \alpha(x) \) implies its vanishing of the first derivative which yields the AWI

\[ \int d^4x \partial_\mu \langle T J_\mu^5(x) \bar{\psi}_L \psi_R(0) \rangle = 2m \int d^4x \langle T \bar{\psi} \gamma_5 \psi(x) \bar{\psi}_L \psi_R(0) \rangle + 2L \int d^4x \langle T iF \tilde{F}(x) \bar{\psi}_L \psi_R(0) \rangle - 2i \langle \bar{\psi}_L \psi_R \rangle. \] (2.6)

Crewther’s arguments go as follows. If there is no \( U(1)_A \) goldstone boson coupling to \( J_\mu^5 \), the l.h.s. of Eq. (2.6) vanishes. In the chiral limit, the first term of the r.h.s. would vanish too. Thus one has when \( m \rightarrow 0 \)

\[ L \int d^4x \langle T F \tilde{F}(x) \bar{\psi}_L \psi_R(0) \rangle = \langle \bar{\psi}_L \psi_R \rangle. \] (2.7)

The instanton dynamics assumes that the integration over the gauge field is separated into a sum over gauge configurations characterized by the integer winding number \( \nu \) in (1.5), i.e. \( \int [dA] = \sum_\nu \int [dA]_\nu \) and \( \langle \bar{\psi}_L \psi_R \rangle = \sum_\nu \int \langle \bar{\psi}_L \psi_R \rangle_\nu \). Eq. (2.7) would then imply
(L\nu - 1)\langle \bar{\psi}_L \psi_R \rangle_\nu = 0. \quad (2.8)

By assuming the spontaneous chiral symmetry breaking caused by \langle \bar{\psi}_L \psi_R \rangle \neq 0, \quad (2.8) cannot be satisfied if \nu is an integer. Moreover, by noting that

\frac{d\langle \bar{\psi}_L \psi_R \rangle}{d\theta} = i \int d^4 x \langle T F \bar{F} (x) \bar{\psi}_L \psi_R (0) \rangle \quad (2.9)

one obtains

\langle \bar{\psi}_L \psi_R \rangle_\theta = \langle \bar{\psi}_L \psi_R \rangle_{\theta=0} e^{i \theta L} \quad (2.10)

which is unacceptable because the \theta-dependence of \langle \bar{\psi}_L \psi_R \rangle would have a wrong periodicity 2\pi L. Along the same line, one could derive the AWI for operator \bar{\psi}_R \psi_L and F \bar{F} and combine them with (2.6) to obtain

\langle \langle \nu^2 \rangle \rangle = \frac{m^2}{L^2} \int d^4 x \langle T \bar{\psi} i\gamma_5 \psi (x) \bar{\psi} i\gamma_5 \psi (0) + \frac{m}{L^2} \langle \bar{\psi} \psi \rangle \rangle. \quad (2.11)

By inspecting the first term in the r.h.s of (2.11) is of order \mathcal{O}(m^2), one would conclude that \langle \langle \nu^2 \rangle \rangle was a linear function of m, which, again, contradicts with the instanton computation.

We argue, however, that all these inconsistencies arise from dropping the first term of the r.h.s.of (2.9) in the chiral limit or treating it as a higher order term. The \textit{U}(1)_A fermion operator \bar{\psi} i\gamma_5 \psi, when the fermion fields are integrated out \textit{first} as they should be, may observe a \frac{1}{m} singularity in certain gauge configurations. To see this, we first calculate the VEV of \bar{\psi} i\gamma_5 \psi in a fixed background field A_\mu. Upon the fermion integration, one has

\langle \bar{\psi} i\gamma_5 \psi \rangle^A = Tr \frac{i\gamma_5}{D(A) + m} = \frac{1}{m} T(m^2) \quad (2.12)

where

T(m^2) = Tr \frac{i\gamma_5 m^2}{-D^2 + m^2} = Tr \frac{i\gamma_5 m^2}{-D^2 + \frac{1}{2} g \sigma_{\mu\nu} F_{\mu\nu} + m^2}. \quad (2.13)

It is easy to check that \frac{d}{dm^2} T(m^2) \equiv 0, i. e. T(m^2) is independent of \m^2. Thus it can be calculated in the limit \m^2 \rightarrow \infty.
\[
\lim_{m^2 \to \infty} T(m^2) = -i L \int d^4 x \, tr \gamma_5 (\frac{1}{2} \sigma_{\mu\nu} F_{\mu\nu})^2 \int \frac{d^4 p}{(2\pi)^4} \frac{m^2}{(p^2 + m^2)^3} = i L F \tilde{F}
\]

(2.14)

and therefore

\[
\langle \bar{\psi} i \gamma_5 \psi \rangle^A = -i L \frac{F \tilde{F}}{m}. \tag{2.15}
\]

It observes a pole at \( m = 0 \). It is clear that \( m \langle \bar{\psi} i \gamma_5 \psi \rangle^A \) may be finite in the limit \( m \to 0 \) if \( F \tilde{F} \) is nontrivial. Performing the fermion integration for the first term of r.h.s. of (2.6), we obtain

\[
m \int d^4 x \langle T \bar{\psi} i \gamma_5 \psi(x) \bar{\psi}_L \psi_R(0) \rangle = \int d^4 x \langle T Tr \left( \frac{im \gamma_5}{\not{D} + m} \right) (x) Tr \left( \frac{1 + \gamma_5}{2 (\not{D} + m)} \right) (0) \rangle - \langle Tr \left( \frac{im \gamma_5}{\not{D} + m} \frac{1 + \gamma_5}{2} \frac{1}{\not{D} + m} \right) \rangle \tag{2.16}
\]

\[
= -L \int d^4 x \langle T i F \tilde{F}(x) Tr \left( \frac{1 + \gamma_5}{2} \frac{1}{\not{D} + m} \right) (0) \rangle - i \langle Tr \left( \frac{1}{2} (1 + \gamma_5) \frac{m}{\not{D}^2 + m^2} \right) \rangle. \tag{2.17}
\]

Identifying the second term in (2.17) with \( \langle \bar{\psi} \psi \rangle^A \), we find that the r. h. s. of (2.6) vanishes identically for any \( m \). This is not surprising since if we had considered a global \( U(1)_A \) transformation instead of a local one in (2.4) at the beginning, we would have come up with the same conclusion immediately. Similarly, (2.11) is an identity to be satisfied (trivially) by any dynamics which respects the basic rule of the fermion quantization (and of course the anomaly relation. If there were no anomaly, the second term of r.h.s. of (2.6) would be absent. The cancellation would be incomplete indicating the existence of a massless excitation coupling with \( J^5_\mu \). Thus the chiral anomaly is essential to solve the \( U(1) \) problem.).

There is a delicate problem about taking the chiral limit. One may ask what if the quark mass term is simply absent in the lagrangian at the first place. Crewther’s problem seems to come back if the first term of the r. h. s. of (2.6) is not present. Actually this is where the puzzle comes about. In this case, however, a nonvanishing value of the quark condensate is not well-defined. It relates to a general feature of the spontaneous symmetry breaking mechanism. For example, in the \( \phi^4 \)-theory with spontaneous breaking of the reflection symmetry \( (\phi \to -\phi) \), the VEV of \( \phi \) is calculated
\[ \langle \phi \rangle = \frac{1}{Z} \int d\phi \, \phi e^{-\int d^4x (\partial_\mu \phi)^2 + \frac{1}{4}(\phi^2 - v^2)^2}. \]  
\hspace{2cm} (2.18)

Since the action is perfectly reflection-symmetric and \( \phi \) is an odd operator under reflection, we have \( \langle \phi \rangle \equiv 0 \). Mathematically this is true because of the equal weight of degenerate vacua. But what is of physical interest is a situation where one of the degenerate vacua is chosen as the ground state. The way to do it is to introduce a source term \( \int d^4x J \phi \) into the action which breaks the symmetry explicitly. The degeneracy of the vacua in the absence of the source implies that \( \langle \phi \rangle_J \) is a multi-valued function of \( J \) at \( J = 0 \). The VEV’s of \( \phi \) crucially depends on the way that \( J \) tends to zero. In particular, \( \langle \phi \rangle_{J \to 0^+} = -\langle \phi \rangle_{J \to 0^-} \neq 0 \).

The same procedure should follow for the spontaneous chiral symmetry breaking in QCD. In order to define the quark condensate \( \langle \bar{\psi}_L \psi_R \rangle \), one ought to add the source term \( \int d^4x J \bar{\psi}_L \psi_R(x) \) to the action. Then a \( U(1)_A \) transformation changes the source term as well

\[ \int d^4x J \bar{\psi}_L \psi_R \rightarrow \int d^4x J e^{2i\alpha} \bar{\psi}_L \psi_R. \]  
\hspace{2cm} (2.19)

We also need to take this change into account because \( \langle \bar{\psi}_L \psi_R \rangle \) defined by the way that \( J \to 0 \) would be different from the one defined by \( J e^{2i\alpha} \to 0 \). By differentiating \( \langle \bar{\psi}_L \psi_R \rangle \) with respect to \( \alpha \) we obtain a equation exactly the same as (2.4) except that \( m \) is replaced by \( J \). For the same reason as we have discussed, the r. h. s. of the equation is identically zero for any value \( J \) (even in the limit \( J \to 0 \)). There is no \( U(1)_A \) goldstone boson, and, in general, (2.7), (2.8) and (2.10) do not hold.

We have shown that the AWI for the isosinglet current \( J^5_\mu \) is trivially satisfied by QCD dynamics including the axial anomaly. (2.11) is an identity satisfied by any dynamics if the singularity of the singlet operator \( \bar{\psi}i\gamma_5\psi \) in the zero mass limit is appropriately handled. It does not put any constraint on how the topological susceptibility \( \langle \langle \nu^2 \rangle \rangle \) should behave as a function of the quark mass. Thus, it does not, from the context of the field theory, rule out the instanton computation. However, this should not be confused with the case of the AWI’s for non-singlet currents where the assumption on the lowest lying resonances have to be made. For a non-singlet axial current \( J^a_\mu = \bar{\psi} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} \psi \) (\( \lambda^a \)'s are generators of \( SU(L) \), \( a = 1, \cdots, L^2 - 1 \)), the corresponding AWI reads
\[ m^2 \int d^4 x \langle T \bar{\psi} i \gamma_5 \frac{\lambda}{2} \psi(x) \bar{\psi} i \gamma_5 \frac{\lambda}{2} \psi(0) \rangle - \delta^{ab} \frac{m}{L} \langle \bar{\psi} \psi \rangle = 0. \]  

(2.20)

It can be readily checked by integrating the fermion fields that (2.20) is satisfied in QCD. Unlike the singlet current in (2.12)

\[ \langle \bar{\psi} i \gamma_5 \frac{\lambda}{2} \psi \rangle = Tr \frac{\lambda}{2} i \gamma_5 \not{D} + m = 0 \]  

(2.21)

because \( \lambda^a \)'s are traceless. Assuming that pions are lowest lying resonances which dominant, one obtains

\[ m^2 \int d^4 x \langle T \bar{\psi} i \gamma_5 \frac{\lambda}{2} \psi(x) \bar{\psi} i \gamma_5 \frac{\lambda}{2} \psi(0) \rangle_{\text{res.}} = F_\pi^2 m_\pi^2 \delta^{ab} \]  

(2.22)

leading to \( F_\pi^2 m_\pi^2 = -\frac{1}{L} m \langle \bar{\psi} \psi \rangle \). Can we do the same analysis for the singlet operator

\[ m^2 \int d^4 x \langle T \bar{\psi} i \gamma_5 \psi(x) \bar{\psi} i \gamma_5 \psi(0) \rangle_{\text{res.}} =? \]  

(2.23)

such that we may get a phenomenological value for \( \langle \langle \nu^2 \rangle \rangle \) from (2.11) without resorting to instanton computations? This turns out to be of some difficulties. For the axial singlet operator, we cannot generally assume the pion dominance. In fact, \( m \bar{\psi} i \gamma_5 \psi \) does not couple to pions because \( \lambda^a \)'s commute with identity \[12\]. In addition, \( \bar{\psi} i \gamma_5 \psi \) has pole behavior at \( m = 0 \) whose residue is \( F \tilde{F} \). It may couple to a gauge ghost \[14\] as well as glue balls and \( U(1)_A \) particle. It may also exhibit a non-zero subtraction constant in the spectral dispersion representation \[15\], which by itself is not surprising in the presence of anomaly. All these factors may further fall into overlap, causing double countings. These have made an estimation on (2.23) extremely difficult if not impossible.

In summary, the AWI and the low energy phenomenology may not put a constraint on the topological susceptibility. Therefore, it leaves us a task of calculating \( \langle \langle \nu^2 \rangle \rangle \) and the measure of strong CP violation from instanton dynamics. To avoid the infrared divergence, we further relate \( \langle \langle \nu^2 \rangle \rangle \) to the \( U(1)_A \) particle mass in an effective theory.
III. THE EFFECTIVE CP VIOLATING LAGRANGIAN IN QCD

In Sect. 2 we have shown that the axial singlet operator $\bar{\psi}i\gamma_5\psi$ is related to $F\bar{F}$ in a fixed gauge background. When the gauge fields are integrated out, (2.13) becomes a relation on VEV’s. It can be easily proven that such a relation is true for each flavor. In general, when the quark mass is complex, one derives

$$-i(m_ie^{i\varphi_i}\langle \bar{\psi}_L^i\psi_R^i \rangle - m_ie^{-i\varphi_i}\langle \bar{\psi}_R^i\psi_L^i \rangle)$$

$$= -i(m_ie^{i\varphi_i}\langle Tr\frac{1}{2i} \frac{1 + \gamma_5}{\not{D} + m_ie^{i\varphi_i}\gamma_5} \rangle - m_ie^{-i\varphi_i}\langle Tr\frac{1}{2i} \frac{1 - \gamma_5}{\not{D} + m_ie^{i\varphi_i}\gamma_5} \rangle)$$

$$= \langle iF\bar{F} \rangle$$

(3.1)

where $\varphi_i$ is the phase of the ith quark mass ($i = 1, \cdots, L$), no sum over $i$ is understood in (3.1). Now define

$$\langle \bar{\psi}_L^i\psi_R^i \rangle \equiv -\frac{C_i}{2} e^{i\beta_i}; \quad \langle \bar{\psi}_R^i\psi_L^i \rangle \equiv -\frac{C_i}{2} e^{-i\beta_i}$$

(3.2)

or

$$\langle \bar{\psi}_L^i\psi_R^i \rangle \equiv -C_i \cos \beta_i; \quad \langle \bar{\psi}_R^i\gamma_5\psi_L^i \rangle \equiv C_i \sin \beta_i$$

(3.3)

where $C_i > 0$ and $\beta_i$ is the phase of the ith quark condensate. Eq. (3.4) yields

$$\langle iF\bar{F} \rangle = -m_iC_i \sin(\varphi_i + \beta_i).$$

(3.4)

which is to be referred to as the vacuum alignment equation (VAE) [17]. It can also be derived directly by taking vacuum expectation values on both sides of the anomaly relation [21]. Eq. (3.4) means that if the first moment of the topological charge is non-zero in the presence of instanton, the quark condensate develops a phase $\beta_i$ different from $-\varphi_i$. If the phase of the fermion mass $\varphi_i$ is zero as it can always be made so by making a chiral rotation, the fermion condensate has a non-trivial phase $\beta_i \neq 0$ i.e. develops an imaginary part which is determined by the topological structure of the theory. This of course would not happen
in a theory like QED where only the trivial topological configuration exists. We shall see
that it is the combination $\varphi_i + \beta_i$'s that determine the CP violating amplitude in strong
interactions.

$\langle F \bar{F} \rangle$ can be calculated from instanton dynamics in the dilute gas approximation (DGA)
[16]. The vacuum to vacuum amplitude in the presence of the $\theta$-term is given

$$Z(\bar{\theta}) = \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{n+n} (Z_+)^n (Z_-)^{n-} = e^{Z_+ + Z_-} \tag{3.6}$$

where $Z_+$ ($Z_-$) is the one single instanton (anti-instanton) amplitude

$$Z_+ = e^{\bar{\theta} \int d^4z \frac{d^\rho}{\rho^5} C_{N_c} \left( \frac{8\pi^2}{g^2(\rho)} \right)^2 N_c e^{-\frac{8\pi^2}{g^2(\rho)} d(M\rho)} \tag{3.7}$$

$$Z_- = Z_+^*$$

with

$$C_{N_c} = \frac{4.6 \exp(-1.68N_c)}{\pi^2(N_c - 1)!(N_c - 2)!}.$$ 

The factor $d(M\rho)$ in (3.7) is connected with the so-called fermion determinant, which intro-
duces important physics. It was first discovered by 't Hooft [18] that there exists a zero
mode of the operator $\mathcal{D}$ in the instanton field. Thus we expect $d(M\rho) \propto \det M$ ($M$ is the
quarks mass matrix). For small quark masses, $d(M\rho)$ is equal to [18,19]

$$d(M\rho) = \prod_{i=1}^{L} f(m_i \rho) \tag{3.8}$$

$$f(x) = 1.34x(1 + x^2 \ln x + \cdots), \quad x \ll 1.$$ 

Combining (3.8) and (3.7) with (3.6) one obtains

$$Z(\bar{\theta}) = \exp[2V \cos \bar{\theta} m_1 m_2 \cdots m_L K(L)] \tag{3.9}$$
where $K(L)$ is of dimension $4 - L$

$$K(L) \cong (1.34)^L \int \frac{d\rho}{\rho^{5-L}} C_N \left( \frac{8\pi^2}{g^2(\rho)} \right)^{2N_c} e^{-\frac{\pi^2}{g^2(\rho)}}.$$  \hspace{2cm} (3.10)

The first moment $\langle iF \tilde{F} \rangle$ is calculated by taking an average of the topological charge over 4-space

$$\langle iF \tilde{F} \rangle = \frac{1}{V} \langle \int d^4x iF \tilde{F} \rangle = \frac{1}{V} \frac{d}{d\bar{\theta}} \ln Z(\bar{\theta})$$

$$= -2m_u m_d \cdots m_L K(L) \sin \bar{\theta}$$ \hspace{2cm} (3.11)

and the topological susceptibility is equal to

$$\langle\{\nu^2\}\rangle = \frac{1}{V} \frac{d^2}{d\bar{\theta}^2} \ln Z(\bar{\theta}) = -2m_u m_d \cdots m_L K(L) \cos \bar{\theta}.$$ \hspace{2cm} (3.12)

Clearly enough, when $\bar{\theta}$ is small we have

$$\langle iF \tilde{F} \rangle = \langle\{\nu^2\}\rangle \bar{\theta}. \hspace{2cm} (3.13)$$

The vacuum alignment in QCD can be readily made through the VAE (3.4). By defining the quark field, one can change the phase of the quark mass $\varphi_i$ and phase of the quark condensate $\beta_i$. However, $\varphi_i + \beta_i$’s will not change under the redefinition. They are only functions of $\bar{\theta}$ as shown in (3.4). One may choose $\beta_i = 0 (i = 1, \cdots, L)$ such that the vacuum is CP-conserving

$$\langle \bar{\psi}_i i\gamma_5 \psi_i \rangle = 0. \quad (i = 1, 2, \cdots, L) \hspace{2cm} (3.14)$$

Then the phase of the quark masses are no longer arbitrary. They are uniquely determined by the vacuum alignment equation (3.4),

$$\varphi_i = -\frac{\langle\{\nu^2\}\rangle}{m_i C} \bar{\theta} \quad (i = 1, 2, \cdots, L) \hspace{2cm} (3.15)$$

$$\theta_{QCD} = \bar{\theta} - \sum_i \varphi_i = \left( 1 - \sum_i \frac{\langle\{\nu^2\}\rangle}{m_i C} \right) \bar{\theta}$$

where we have assumed $\varphi_i$’s are small and $C_i$’s are all equal to $C$. To be aligned with the vacuum, the strong CP phase $\bar{\theta}$ must be distributed among the $\theta$-term and the quark mass
terms according to their determined weights. The effective CP-violating part of the QCD lagrangian reads

\[ L_{CP}^{\beta_i=0} = i\theta_{QCD} F \tilde{F} - \frac{2}{C} m_u m_d \cdots m_L K(L) \bar{\theta} \bar{\psi} i\gamma_5 \psi. \]  

(3.16)

with \( \theta_{QCD} \) given in (3.15). (3.16) is different from that obtained by Baluni [23], which, as appearing in most literatures, lacks the topological factor \( K(L) \) and fails to observe the topological feature of the strong CP violation.

It is worth emphasizing that the effective CP-violating interactions in (3.16) are only valid in the CP-conserving vacuum where \( \beta_i \)'s are zero. One can alternatively choose a certain pattern of the phase distribution and ask in what direction the vacuum is to align with it. In general, the vacuum angles are not zero and should be determined by the VAE (3.4). For example, we can choose \( \varphi_i = 0 \ (i = 1, \cdots, L) \) such that \( L_{CP}^{\beta_i=0} = i\theta F \tilde{F} \). In this case, the vacuum condensates are complex \( \beta_i = -\frac{\langle \omega_i^2 \rangle}{m_i} \bar{\theta} \). A physical CP-violating amplitude is from both CP-violating part of the lagrangian and CP-violating part of the quark condensate. A proof of the equivalence of different chiral frames on strong CP effects is given in Ref. [22] where it is shown that the vacuum alignment equation (3.4) plays an essential role.

Does the left-over \( \theta \)-term in the effective lagrangians play any role in computing the strong CP effects? So far there have been only two CP violating processes available: \( \eta \to 2\pi \) and the electric dipole moment (EDM) of neutron. The latter process depends on a computation on the effective CP-odd \( \pi-N \) coupling [24]. Both of them would involve in an evaluation of the commutator \( [Q_5^a, F \tilde{F}] \) if the \( \theta \)-term were to contribute

\[ \langle \pi^a \pi^b | \theta_{QCD} F \tilde{F} | \eta \rangle = -\frac{i\theta_{QCD}}{F_\pi} \langle \pi^b | \left[ Q_5^a, F \tilde{F} \right] | \eta \rangle; \]

\[ \langle \pi^a N | \theta_{QCD} F \tilde{F} | N' \rangle = -\frac{i\theta_{QCD}}{F_\pi} \langle N | \left[ Q_5^a, F \tilde{F} \right] | N' \rangle \]  

(3.17)

where we have used the soft-pion theorem. It is obvious that \( [Q_5^a, F \tilde{F}] = 0 \) since \( Q_5^a \) is a non-singlet charge and thus the canonical commutation relation applies. It is at least safe to argue that the \( \theta \)-term in the effective lagrangian can be ignored. What really matters is the correlating feature of \( \varphi_i \)'s and \( \beta_i \)'s given by (3.4).
The above statement can be justified in the following example. For simplicity, let us assume \( m_u = m_d = \cdots = m_L = m \) and \( L = 3 \) where pions and \( \eta \) are all light pseudoscalars and the soft-pion theorem applies. The amplitude of \( \eta \to 2\pi \) is readily calculated when \( \beta_i \)'s are zero

\[
A(\eta \to 2\pi) = \left\langle \pi^0 \pi^0 | \mathcal{L}_{CP}^{\beta_i=0} | \eta \right\rangle = \bar{\theta} \left( \frac{-i}{F_\pi} \right)^3 \left\langle [Q^3_5, [Q^3_5, [Q^8_5, \bar{\psi} \gamma_5 \psi]]] \right\rangle 
\]

\[
= \frac{4}{\sqrt{3}} \frac{1}{F_\pi^3} m_u m_d m_s K \bar{\theta} 
\]

(3.18)

In deriving (3.18), we have dropped off \( F \bar{F} \) term. In a chiral frame where \( \phi_i \)'s are zero, we still drop off the \( \theta \)-term. But the CP-conserving part of the lagrangian will contribute because the vacuum condensates are CP violating

\[
A(\eta \to 2\pi) = -\left\langle \pi^0 \pi^0 | m \bar{\psi} \psi | \eta \right\rangle = -m \left( \frac{-i}{F_\pi} \right)^3 \left\langle [Q^3_5, [Q^3_5, [Q^8_5, \bar{\psi} \gamma_5 \psi]]] \right\rangle 
\]

\[
= -\frac{2}{\sqrt{3}} \frac{1}{F_\pi^3} m C \sin \beta = \frac{4}{\sqrt{3}} \frac{1}{F_\pi^3} m_u m_d m_s K \bar{\theta} 
\]

(3.19)

where \( \beta_i = -\frac{\langle \nu^2 \rangle}{m_i C} \bar{\theta} \). Both (3.12) and (3.19) yield the same result.

We conclude that the measure of strong CP violation is given by the topological susceptibility

\[
\mathcal{J}_{\text{strong}} = -\frac{1}{2} \langle \nu^2 \rangle \bar{\theta} = m_1 m_2 \cdots m_L K(L) \bar{\theta} 
\]

(3.20)

However, \( K(L) \) is still an unknown factor because the integral in (3.10) is simply divergent. This is the shortcoming of all instanton computations if one use the dilute gas approximation \[25\]. (3.20) can only make sense if one introduces a cutoff \( \bar{\rho} \) at large instanton density. this brings in an ambiguity of choosing \( \bar{\rho} \). Fortunately, as we shall show below, such an ambiguity can be removed by considering an effective model where \( K(L) \) can be naturally related to the mass of the \( U(1)_A \) particle.

IV. THE EFFECTIVE CHIRAL MODEL
A. The model and the Instanton Induced Quantum Corrections

We consider an effective chiral theory where meson degrees of freedom are explicitly introduced. The virtue of the model is that it reflects all flavor symmetries in strong interactions as described by QCD. Since the mesons as independent field excitations couple to fermions through Yukawa couplings, there is no need to saturate correlation functions of various currents in QCD with unclear assumptions on the lowest-lying resonances. Unlike a conventional effective theory [26] in which the nucleons are involved, the model that we will be discussing contains quarks, gluons and mesons. It is a linear version of the gauged sigma model suggested by Georgi and Manohar [27], which describes strong interactions in the intermediate energy region between the scale of the chiral symmetry breaking and the scale of the quark confinement.

The model reads

\[
\mathcal{L} = -\bar{\psi} D\psi - \frac{1}{4} F^2 + i\theta F \tilde{F} - f \bar{\psi}_L \phi \psi_R - f \bar{\psi}_R \phi^\dagger \psi_L - Tr \partial_\mu \phi \partial_\mu \phi^\dagger - V_0(\phi \phi^\dagger) - V_m(\phi, \phi^\dagger)
\]

(4.1)

where \( \phi \) is a complex \( L \times L \) matrix, \( V_0(\phi \phi^\dagger) \) is the most general form of a potential invariant under \( U(L) \times U(L) \) (renormalizable)

\[
V_0(\phi \phi^\dagger) = -\mu^2 Tr \phi \phi^\dagger + \frac{1}{2} (\lambda_1 - \lambda_2) (Tr \phi \phi^\dagger)^2 + \lambda_2 Tr (\phi \phi^\dagger)^2
\]

(4.2)

and

\[
V_m(\phi, \phi^\dagger) = -\frac{1}{4} me^{ix} Tr \phi - \frac{1}{4} me^{-ix} Tr \phi^\dagger.
\]

(4.3)

(4.1) needs some explanations. Under \( U(L)_L \times U(L)_R \), the quark fields as well as the complex meson field transforms as

\[
\begin{align*}
\psi_L &\rightarrow U_L \psi_L, & \psi_R &\rightarrow U_R \psi_R; \\
\phi &\rightarrow U_L \phi U_R^\dagger, & \phi^\dagger &\rightarrow U_R \phi^\dagger U_L^\dagger.
\end{align*}
\]

(4.4)
In the absence of $V_m$, $\mathcal{L}$ is invariant \textit{classically} under (4.4) but broken down to $SU(L)_L \times SU(L)_R \times U(1)_V$ by the chiral anomaly. $V_m$, replacing the quark mass ($m$ now is of dimension 3), serves as an explicit symmetry breaking and must be treated as a perturbation. $f$ is the Yukawa coupling, chosen to be real by redefining $\phi$. Under $U(1)_A$ transformation

$$
\begin{align*}
\psi_L &\to e^{i\omega}\psi_L, \quad \psi_R \to e^{-i\omega}\psi_R; \\
\phi &\to e^{2i\omega}\phi, \quad \phi^\dagger \to e^{-2i\omega}\phi^\dagger.
\end{align*}
$$

(4.5)

the $\theta$-term and $V_m$ change as $\theta \to \theta - 2L\omega$, $\chi \to \chi + 2\omega$. But $\bar{\theta} = \theta + L\chi$ keeps unchanged.

Except the meson sector, the gauge interaction in (4.1) looks identical to QCD. One may wonder if we are doubly counting the degrees of freedom. This is explained in [27] that these quarks and gluons are not the same as in QCD. In particular, quarks are supposed to acquire \textit{constituent} masses about $360\text{MeV}$, which is huge compared to the current mass in QCD. The gauge coupling $g_s$ between quarks and gluons in the effective theory is found to be

$$
\alpha_s \cong 0.28 
$$

(4.6)

much less than its QCD counterpart. This may explain why nonrelativistic quark model works since the quarks inside a proton could be treated as weakly interacting objects.

However, the drawback of the model is that it has a very serious $U(1)$ problem. Indeed, if one calculates the physical spectrum from $V_0 + V_m$, one finds $L^2$ would-be goldstone modes. In addition, the nontrivial topological structure of the theory has been totally overlooked. The classical excitations such as instantons have not been accounted for in the model, which, according to the original idea of ’t Hooft [2], are crucial to solving the $U(1)$ problem.

We therefore consider the quantum correction to the lagrangian (4.1) in the presence of non-trivial classical gauge fields known as instantons. We argue that the effective gauge coupling $\alpha_s$ in (4.6) is obtained only if those classical extrema to the action have been effectively summed up by the semiclassical method. We find that the 1-loop quantum fluctuations around instantons lead to a dramatic change on the $U(1)_A$ sector of the model.
The $U(1)$ particle acquires an extra mass from the vacuum tunneling effects, which, in turn, results in the so-called strong CP problem.

The effective action of the meson field is calculated as

$$Z = \int D(\phi, \phi^\dagger) e^{-S_0[\phi, \phi^\dagger]} \int D(A, \bar{\psi}, \psi) e^{-S[\bar{\psi}, \gamma_5 A, \phi, \phi^\dagger]}$$

$$= \int D(\phi, \phi^\dagger) e^{-S_{eff}[\phi, \phi^\dagger]} \quad (4.7)$$

where

$$S_{eff}[\phi, \phi^\dagger] = S_0[\phi, \phi^\dagger] + \Delta S[\phi, \phi^\dagger] \quad (4.8)$$

and the quantum correction is given

$$\Delta S[\phi, \phi^\dagger] = -\ln \int D(A, \bar{\psi}, \psi) e^{-S[\bar{\psi}, \gamma_5 A, \phi, \phi^\dagger]} \equiv -\ln \tilde{Z}[\phi, \phi^\dagger]. \quad (4.9)$$

The calculation of $\tilde{Z}[\phi, \phi^\dagger]$ in the instanton background follows the standard derivation of the vacuum-to-vacuum amplitude as in [18]

$$\tilde{Z}[\phi, \phi^\dagger] = \sum_\nu \int DA e^{i\theta_\nu - S[A,\phi^\dagger]} (\text{Det} \mathcal{M}_A)^{-1/2} \text{Det} \mathcal{M}_\psi \text{Det} \mathcal{M}_{gh} \quad (4.10)$$

where

$$\mathcal{M}_A = -D^2 - 2F$$

$$\mathcal{M}_{gh} = -D^2 \quad (4.11)$$

$$\mathcal{M}_\psi = \slashed{D} + \frac{f}{2} (\phi + \phi^\dagger) + \frac{f}{2} (\phi - \phi^\dagger) \gamma_5.$$  

If only the effective potential is of concern, $\phi$ and $\phi^\dagger$ in $\mathcal{M}_\psi$ are to be taken as constant fields. The fermion determinant, as usual, needs special treatment:

$$\text{Det} \mathcal{M}_\psi = \text{Det} (0) \mathcal{M}_\psi \text{Det} \mathcal{M}_\psi.' \quad (4.12)$$

“$\text{Det} (0)$” denotes contributions from the subspace of zero modes of $\slashed{D}$. In a single instanton field, $\slashed{D}$ has a zero mode with chirality $-1 \ (\gamma_5 = -1)$ [21]. Thus we have

$$\text{Det} (0) \mathcal{M}_\psi = \text{det} \left[ \frac{f}{2} (\phi + \phi^\dagger) + \frac{f}{2} (\phi - \phi^\dagger) (-1) \right] = \text{det}(f\phi^\dagger)$$

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where “det” only acts upon flavor indices. The prime in $\text{Det}'M_\psi$ reminds us to exclude zero modes from the eigenvalue product. Since $[\mathcal{D}, \gamma_5] \neq 0$, $M_\psi$ cannot be diagonalized in the basis of eigenvectors of $\mathcal{D}$. The nonvanishing eigenvalues always appear in pair, i.e. if $\mathcal{D}\varphi_n = \lambda_n \varphi_n$ where $\lambda_n \neq 0$, then $\mathcal{D}\gamma_5\varphi_n = -\gamma_5 \mathcal{D}\varphi_n = -\lambda_n \gamma_5 \varphi_n$, namely both $\lambda_n$ and $-\lambda_n$ are eigenvalues of $\mathcal{D}$. In addition, $\gamma_5$ takes $\varphi_n$ to $\varphi_{-n}$. Therefore

$$\text{Det}'M_\psi = \det \prod_{\lambda_n > 0} \left( i\lambda_n + \frac{f}{2}(\phi + \phi^\dagger) \quad \frac{f}{2}(\phi - \phi^\dagger) \right)$$

$$= \det \prod_{\lambda_n > 0} (\lambda_n^2 + f^2 \phi\phi^\dagger) = \text{Det}^{1/2}(-\mathcal{D}^2 + f^2 \phi\phi^\dagger). \quad (4.13)$$

Now we are ready to make the DGA. We need to further assume a weak-field approximation of $\phi$ and $\phi^\dagger$. This can be justified by imagining that $\phi$ and $\phi^\dagger$ fluctuate about the VEV, which is about 300 MeV. The large fluctuations are exponentially suppressed by $\exp(-\lambda_1 |\phi|^4)$. In the DGA

$$\tilde{Z}[\phi, \phi^\dagger] = \text{Det}^{1/2}(-\partial^2 + f^2 \phi\phi^\dagger) \exp(\tilde{Z}_+ + \tilde{Z}_-) \quad (4.14)$$

where

$$\tilde{Z}_+[\phi, \phi^\dagger] = e^{i\theta} \det(f\phi^\dagger) \int dz d\rho C_{N_c} \left( \frac{8\pi^2}{g^2(\rho)} \right)^{2N_c} e^{-\frac{\alpha_s^2}{g^2(\rho)}} \det \left[ 1.34 \rho \left( 1 + f^2 \phi\phi^\dagger \ln f^2 \phi\phi^\dagger + \cdots \right) \right]$$

$$\cong VK(L) e^{i\theta} \det(f\phi^\dagger) \quad (4.15)$$

$$\tilde{Z}_-[\phi, \phi^\dagger] = \tilde{Z}_+[\phi, \phi^\dagger]$$

and $K(L)$ is given in (3.10).

Combining (4.14) with (4.9), and noticing that $\ln \text{Det}(-\partial^2 + f^2 \phi\phi^\dagger)$ contains terms which can be absorbed into the tree-level lagrangian by redefinition of bare parameters, we obtain the following effective lagrangian

$$\mathcal{L}_{\text{eff}} = -\bar{\psi} \mathcal{D}_x \psi - \frac{1}{4} F^2_x - (f \bar{\psi}_L \psi_R + \text{h.c.}) - \text{Tr}(\partial_\mu \phi \partial_\mu \phi^\dagger) - V_0(\phi\phi^\dagger) - V_m(\phi, \phi^\dagger) - V_k(\phi, \phi^\dagger) \quad (4.16)$$
where

\[ V_k(\phi, \phi^\dagger) = -K(L)f^L e^{i\theta} \det \phi^\dagger - K(L)f^L e^{-i\theta} \det \phi \]  \hspace{1cm} (4.17)

Several remarks on (4.16) are in order. The presence of \( V_k \) in (4.16) is the direct result of fermion zero modes in the instanton field. It is invariant under \( SU(L)_L \times SU(L)_R \times U(1)_V \) but not invariant under \( U(1)_A \). Under \( U(1)_A \) rotation (4.5), \( e^{i\theta} \det \phi \rightarrow e^{i(\theta - 2\omega L)} \det \phi \). Thus \( V_k \) takes over the role of the \( \theta \)-term and the anomaly. Again, \( \bar{\theta} = \theta + \chi L \) remains invariant.

The prototype of \( V_k \) was suggested long time ago by several authors \[28\] and re-discussed by t’ Hooft \[29\]. It is different from a model originally proposed by Di Vecchia \[32\] and recently analyzed in Ref. \[9\], although physical contents of both models may be similar.

The gauge interactions between quarks and gluons are still present in (4.16) as required in the nonrelativistic quark model. However, they differs from QCD in that the gauge coupling \( g_s \) has a smaller value, and the most importantly, the gauge field \( A_s \) now possesses a trivial topology at infinity. The gauge interaction sector in (4.16) is very analogy to QED: the fermion chiral anomaly still exists, but any \( \theta \)-term \( \int d^4x \theta F_s \tilde{F}_s \) in the action would be simply a vanishing surface term and can be dropped off.

**B. The U(1) Particle Mass and Strong CP Violation**

We would like to discuss the physical spectrum of the model (4.16) (this part has been worked out in Ref. \[29\]) and show how the strong CP effects can be calculated effectively.

To simplify the problem, we take \( L = 2 \) and \( u \) and \( d \) quarks have a equal mass. In this case, \( \eta \) is identified as the \( U(1) \) particle and there will not be a mixing between \( \pi^0 \) and \( \eta \).

The complex meson field \( \phi \) contains eight particle excitations \( \sigma, \eta, \pi_a \) and \( \alpha_a \) (\( a = 1, 2, 3 \)):

\[ \phi = \frac{1}{2} (\sigma + i\eta) + \frac{1}{2} (\vec{\alpha} + i\vec{n}) \cdot \vec{\tau} \]  \hspace{1cm} (4.18)

where \( \tau^{1,2,3} \) are the Pauli matrices. In terms of physical fields, \( V_0, V_m \) and \( V_k \) can be rewritten as
\[
V_0(\phi \phi^\dagger) = -\frac{\mu^2}{2}(\sigma^2 + \eta^2 + \bar{\alpha}^2 + \bar{\pi}^2) + \frac{\lambda_1}{8}(\sigma^2 + \eta^2 + \bar{\alpha}^2 + \bar{\pi}^2)^2 + \\
\frac{\lambda_2}{2}[(\sigma \bar{\alpha} + \eta \bar{\pi})^2 + (\bar{\alpha} \times \bar{\pi})^2]
\]  
(4.19)

\[
V_m(\phi, \phi^\dagger) = -\frac{1}{4}me^{i\chi}(\sigma + i\eta) - \frac{1}{4}me^{-i\chi}(\sigma - i\eta)
\]  
(4.20)

\[
V_k(\phi, \phi^\dagger) = -\frac{1}{2}Kf^2(\sigma^2 - \eta^2 - \bar{\alpha}^2 + \bar{\pi}^2) \cos \theta - K(\sigma \eta - \bar{\alpha} \cdot \bar{\pi}) \sin \theta
\]  
(4.21)

Assuming, for convenience,

\[
\langle \phi \rangle = \frac{1}{2}\langle \sigma + i\eta \rangle = \frac{1}{2}ve^{i\varphi} \quad (v > 0).
\]  
(4.22)

we get, by taking the extremum of \(V_0 + V_m + V_k\) with respect to \(v\) and \(\varphi\)

\[
v^2 = \frac{2\mu^2}{\lambda_1} + \frac{2m}{\lambda_1v} \cos(\chi + \varphi) - \frac{2Kf^2}{\lambda_1} \cos(\theta - 2\varphi)
\]  
(4.23)

and

\[
m \sin(\chi + \varphi) - Kf^2v \sin(\theta - 2\varphi) = 0.
\]  
(4.24)

Eq. (4.24) plays a role of the vacuum alignment in the effective theory. If we take \(\varphi = 0\) as we wish, (4.24) implies a consistency constraint on \(\chi\) and \(\theta\): They are not separately independent parameters. They can expressed in terms of the physical parameter \(\bar{\theta} = \theta + 2\chi\) as

\[
\sin \chi \approx -\frac{Kf^2v}{m + 2Kf^2v} \sin \bar{\theta}
\]  
(4.25)

\[
\sin \theta \approx -\frac{m}{m + 2Kf^2v} \sin \bar{\theta}
\]  
(4.26)

where we have assumed that \(\sin \chi\) is very small (< < 1).

Rewriting \(\mathcal{L}_{\text{eff}}\) in terms of the shifted field \(\phi \rightarrow \langle \phi \rangle + \phi\), we get

\[
\mathcal{L}_{\text{eff}} = -\bar{\psi}(\not{D}_s + \frac{1}{2}f v)\psi - \frac{1}{4}F_s^2 - (f \bar{\psi}_L\psi_R + h.c.) - Tr(\partial_{\mu}\phi \partial_{\mu}\phi^\dagger) - \\
\frac{1}{2}(\sigma, \eta)M_{\sigma\eta}^2 \left(\begin{array}{c} \sigma \\ \eta \end{array}\right) - \frac{1}{2}(\bar{\alpha}, \bar{\pi})M_{\alpha\pi}^2 \left(\begin{array}{c} \bar{\alpha} \\ \bar{\pi} \end{array}\right) - \frac{\lambda_1v}{2} \sigma(\sigma^2 + \eta^2 + \bar{\alpha}^2 + \bar{\pi}^2) - \\
\lambda_2v \bar{\alpha} \cdot (\sigma \bar{\alpha} + \eta \bar{\pi}) - \frac{\lambda_1}{8}(\sigma^2 + \eta^2 + \bar{\alpha}^2 + \bar{\pi}^2)^2 - \frac{\lambda_2}{2}(\sigma \bar{\alpha} + \eta \bar{\pi})^2 - \frac{\lambda_2}{2}(\bar{\alpha} \times \bar{\pi})^2
\]  
(4.27)
where the meson mass matrices are given

\[ M_{\sigma\eta}^2 = \begin{pmatrix} \lambda_1 v^2 + \frac{m}{v} \cos \chi & -\frac{1}{2}K f^2 \sin \theta \\ -\frac{1}{2}K f^2 \sin \theta & \frac{m}{v} \cos \chi + 2K f^2 \cos \theta \end{pmatrix} \]

\[ M_{\alpha\pi}^2 = \begin{pmatrix} \lambda_1 v^2 + \frac{m}{v} \cos \chi + 2K f^2 \cos \theta & \frac{1}{2}K f^2 \sin \theta \\ \frac{1}{2}K f^2 \sin \theta & \frac{m}{v} \cos \chi \end{pmatrix}. \] (4.28)

The quark acquires a large constituent mass

\[ m_Q = \frac{1}{2}fv \approx \frac{f\mu^2}{\lambda_1} + \frac{fm}{\lambda_1 v} + \frac{Kf^3}{\lambda_1}. \] (4.29)

It is interesting to note that \( M_Q \) arises from three parts: the spontaneous chiral symmetry breaking (from \( V_0 \)), the explicit chiral symmetry breaking (from \( V_m \)) and the instanton induced symmetry breaking (from \( V_k \)). The instanton does *spontaneously* break chiral symmetry \( SU(L)_L \times SU(L)_R \) \(^{30}\). The mass spectrum of mesonic states can be read off from diagonalizing (4.28). The mixing probability is proportional to \((K f^2 \sin \theta)^2 = m^2 \sin^2 \chi\) which is of high order. It hardly affects the physical masses

\[ m^2_\eta = \frac{m}{v} \cos \chi + 2K f^2 \cos \theta, \quad m^2_\pi = \frac{m}{v} \cos \chi; \]

\[ m^2_\sigma = \lambda_1 + \frac{m}{v} \cos \chi, \quad m^2_\alpha = \lambda_2 v^2 + \frac{m}{v} \cos \chi + 2K f^2 \cos \theta. \] (4.30)

(4.30) clearly shows how the instanton induced \( V_k \) leads to a mass splitting between pions and the \( U(1) \) particle \( \eta \). When \( \bar{\theta} \) thus \( \theta \) is small,

\[ m^2_\eta - m^2_\pi = 2K f^2, \] (4.31)

and in the chiral limit \( m \to 0 \), \( m^2_\pi \to 0 \) but \( m^2_\eta \to 2K f^2 \). We conclude that the \( U(1) \) problem is solved in the framework of the effective theory if \( 2K f^2 \) is big enough.

CP-violating effects originates from the mixing between the scalar and pseudoscalars.

To diagonalize the quadratic terms in (4.27), we define the physical meson fields (the prime fields)

\[ \sigma = \sigma' \cos \gamma + \eta \sin \gamma, \quad \eta = -\sigma' \sin \gamma + \eta \cos \gamma; \] (4.32)

\[ \bar{\alpha} = \bar{\alpha}' \cos \gamma' + \bar{\pi} \sin \gamma', \quad \bar{\pi} = -\alpha \sin \gamma' + \bar{\pi} \cos \gamma'. \] (4.33)
such that the off-diagonal elements in (4.28) vanish. The mixing angles $\gamma$ and $\gamma'$ are determined

$$
\begin{align*}
\gamma &= \frac{Kf^2 \sin \theta}{m_\sigma^2 - m_\eta^2} = \frac{1}{2} \frac{m_\pi^2}{m_\sigma^2 - m_\eta^2} \left(1 - \frac{m_\pi^2}{m_\eta^2}\right) \bar{\theta} \quad (4.34) \\
\gamma' &= -\frac{Kf^2 \sin \theta}{m_\sigma^2 - m_\eta^2} = -\frac{1}{2} \frac{m_\pi^2}{m_\sigma^2 - m_\eta^2} \left(1 - \frac{m_\pi^2}{m_\eta^2}\right) \bar{\theta} \quad (4.35)
\end{align*}
$$

which meet the criteria that the mixing therefore strong CP violation must disappear as $m_\pi^2 \to 0$ or $m_\eta^2 = m_\pi^2$ or $\bar{\theta} = 0$. In terms of the physical fields, the CP-violating part of the effective potential is identified (for simplicity we drop the prime notations)

$$
V_{CP} = \frac{\lambda_1 v}{2} \sin \gamma \eta (\sigma^2 + \eta^2 + \bar{\alpha}^2 + \bar{\pi}^2) + \lambda_2 v \cos \gamma' \sin(\gamma - \gamma') \bar{\alpha} \cdot (\eta \bar{\alpha} - \sigma \bar{\pi}) + \lambda_2 v \sin \gamma \cos(\gamma - \gamma') \bar{\pi} \cdot (\sigma \bar{\alpha} + \eta \bar{\pi}) (4.36)
$$

and the Yukawa coupling between quarks and mesons contains CP-violating part too

$$
\mathcal{L}_{\text{yukawa}} = -\frac{1}{2} \bar{\psi} (\sin \gamma + i \gamma_5 \cos \gamma) \psi \eta - \frac{1}{2} \bar{\psi} (\sin \gamma' + i \gamma_5 \cos \gamma') \bar{\tau} \psi \cdot \bar{\pi}. \quad (4.37)
$$

The Feynman rules for CP-violating vertices and the typical CP-violating $qq \to qq$ amplitude are shown in Fig. 1.

The amplitude of $\eta \to 2\pi$ decays reads from (4.36)

$$
A(\eta \to 2\pi) = \frac{1}{4} \frac{m_\pi^2}{F_\pi} \left(1 - \frac{m_\pi^2}{m_\eta^2}\right) \bar{\theta} \quad (4.38)
$$

where $F_\pi = \frac{v}{2}$. (4.38) does not have a direct comparison with the QCD calculation (3.18) and (3.19) where we worked in the case $L = 3$ and $\eta$ is one of the would-be goldstone bosons. However, in (4.38), $\eta$ has been referred to as the $U(1)$ particle.

### C. The EDM for the Constituent Quark

The CP-violating Yukawa coupling in (4.37) results in an important strong CP effect: the EDM of the constituent quark. It can be examined by introducing an external electromagnetic field $A_\mu^{\text{em}}$ and computing the effective interaction of the type
The coefficient $\mu_{EDM}$ is defined as the EDM of the quark. Since (4.39) is not invariant under the chiral rotation, we have to check the phase of the constituent quark mass $m_Q$. In our convention, $m_Q$ is real at tree-level. At higher level, the mass acquires infinite renormalization. The renormalizability of our model guarantees that the renormalized mass will not develop a $\gamma_5$-dependent counterpart. It is still possible that $m_Q$ acquires a finite renormalization which may contain a $\gamma_5$-part at higher order. But that phase is too small to cancel (4.39).

In the background of EM field, the charged quarks and pions coupling to $A_{\mu}^{em}$ through the covariant derivative $D_{\mu}^{em}$

$$\bar{\psi} Q \frac{D_{\mu}^{em} \psi Q - |D_{\mu}^{em} \pi^+|^2}{2}$$

(4.40)

where

$$D_{\mu,Q}^{em} = \partial_\mu + eQA_{\mu}^{em}$$

(4.41)

and $Q$ is the electric charge of the particle. Following Schwinger’s formalism [31] on the derivation of the anomalous magnet moment of electron, we obtain the effective interactions

$$\int d^4x L_{eff}^{em} = - \int d^4x \sum_{Q=u,d} \bar{\psi}_Q (D_{\mu}^{em} + m_Q \psi_Q$$

$$- \frac{f^2}{2!} \int d^4x d^4y \sum_{Q=u,d} \bar{\psi}_Q(x) e^{i\gamma_5 S_\pi^0 \pi^0 S_F^Q(x,y) e^{i\gamma_5 \psi_Q(y)}$$

$$- \frac{f^2}{2!} \int d^4x d^4y \bar{u}(x) e^{i\gamma_5 S_{\pi^+\pi^-} S_F^d(x,y) e^{i\gamma_5 u(y)}$$

$$- \frac{f^2}{2!} \int d^4x d^4y \bar{d}(x) e^{i\gamma_5 S_{\pi^+\pi^-} S_F^Q(x,y) e^{i\gamma_5 d(y)}}$$

(4.42)

where $S_{\pi^0}$'s and $S_F^Q$'s are pion and quark propagators in the background of $A_{\mu}^{em}$,

$$S_{\pi^0} = \frac{1}{\partial^2 - m^2_{\pi}}$$,

$$S_{\pi^+\pi^-} = \frac{1}{(D_{\mu}^{em})^2 - m^2_{\pi}}$$,

$$S_F^Q = \frac{1}{D_{\mu}^{em} + m_Q}$$

(4.43)
Because $\frac{e^2}{4\pi} << 1$, we can expand these propagators perturbatively in $e$

$$S_F^Q = \frac{D_{em}^Q - m_Q}{(D_{em}^Q)^2 - m_Q^2} \left( 1 + \frac{1}{2} e Q \sigma_{\mu\nu} F_{\mu\nu}^{em} \right) + \cdots$$ (4.44)

$$S_{\pi^+\pi^-} = \frac{1}{\partial^2 - m_\pi^2} \left( 1 + \frac{e A_{\mu}^{em} \partial_\mu + e \partial_\mu A_{\mu}^{em}}{\partial^2 - m_\pi^2} \right) + \cdots$$ (4.45)

where the elliptic notation denotes $O(e^2)$. The extraction of the effective interaction of (4.39) is done with the aid of Feynman diagrams in Fig. 2. The contributions from the second term in (4.42) correspond to Fig. 2(a), the third to Fig. 2(b) and the fourth to Fig. 2(c). Summing them up, we get

$$\mu_{EDM}^u = \mu_{EDM}^d = \frac{e f^2}{32\pi^2} \sin 2\gamma' m_Q \left[ -\frac{2}{3} \frac{1}{m_Q^2 - m_\pi^2} + \frac{m_Q^2}{(m_Q^2 - m_\pi^2)^2} \ln \frac{m_Q^2}{m_\pi^2} \right].$$ (4.46)

The EDM of neutron is obtained by applying the $SU(6)$ quark model,

$$\mu_{EDM}^{neutron} = \frac{4}{3} \mu_{EDM}^d - \frac{1}{3} \mu_{EDM}^u \simeq \frac{e f^2}{2m_Q} \frac{1}{16\pi^2} \sin 2\gamma' \ln \frac{m_Q^2}{m_\pi^2}$$ (4.47)

where we have used $m_Q^2 \ll m_\pi^2$ and $\gamma'$ is given in (4.35).

**V. POSSIBLE SOLUTIONS TO THE STRONG CP PROBLEM**

In above, we have studied extensively the measure of strong CP violation and its physical effects from viewpoints of QCD and an effective chiral theory. $J_{\text{strong}}$ is a product of quark masses, $\bar{\theta}$ and the instanton amplitude $K(L)$. It should vanish when any one of them vanishes. The most stringent experiment constraint on $J_{\text{strong}}$ comes from the EDM of neutron, which has been measured at a very high precision $35$

$$\mu_{EDM}^{\text{neutron}} < 1.2 \times 10^{-25} \text{ecm.}$$ (5.1)

this implies

$$J_{\text{strong}} < 10^{-16} \text{GeV}^4.$$ (5.2)

At a typical hadron energy scale, one would suspect $J_{\text{strong}} \simeq \Lambda_{QCD}^4 \simeq 10^{-4} \sim 10^{-6} \text{GeV}^4$, enormously larger than the upper limit. This is so-called strong CP problem. It has puzzled us for more than a decade, ever since the instanton was discovered.
A. The strong CP Problem or the U(1) Problem?

If the instanton is to solve the U(1) problem as we have seen in Sect. 4, the vacuum-to-vacuum amplitude $K(L)$ is related to the mass of the U(1) particle. (5.2) then implies $\bar{\theta} < 10^{-10}$, a very unnatural value since the CP symmetry is violated in weak interactions since $J_{weak} \neq 0$. The strong CP problem and the U(1) problem are so closely related that a solution to one actually repels its resolution to another one. In the context of QCD, there is no theoretical bias to decide which one of them is solved and the other keeps mysterious. Both of them are equally serious in the sense that any solution would be incomplete if it fails to solve both.

However, it may be more natural to argue that $K(L)$ is as small as $10^{-10} \text{GeV}^2$. In the instanton computation

$$K(L) \propto \bar{\rho}^{L-4} e^{-\frac{8\pi^2}{g^2(\bar{\rho})}}$$

(5.3)

where $\bar{\rho}$ is the average density of the instanton gas. The exponential behavior in (5.3) is a standard factor for quantum tunneling and other non-perturbative amplitude. When the instanton density is small as required by the validity of the DGA, (5.3) is exponentially small and can naturally provide a suppression factor of $10^{-10}$ while only requiring a reasonable small value of $\alpha_s(\bar{\rho}) = \frac{g^2(\bar{\rho})}{4\pi} \simeq 0.2 \sim 0.3$. The extreme smallness of $K(L)$ can also be observed in the large $N_c$ limit [13] where it behaves like $e^{-N_c}$. If this indeed is true, $\sin \bar{\theta}$ can be of order 1. There is no strong CP problem.

Of course, this would leave the U(1) problem unsolved. As is argued by Witten and Veneziano [14], the instanton may not be fully responsible for the mass of the U(1) particle although it does break $U(1)_A$ symmetry. The amplitude of the symmetry breaking may be far too small to produce an enough mass for $\eta$ ($L = 2$) or $\eta'$ ($L = 3$). They further point out that based on a reconciliation with the quark model, $m^2_\eta$ is of order $\frac{1}{N_c}$ in the $\frac{1}{N_c}$ expansion. In this case, the mass of the U(1) particle is related to the topological susceptibility in pure Yang-Mills theory.
\[ m_0^2 \approx \frac{4\langle \nu^2 \rangle_{YM}}{F_\pi^2}. \] (5.4)

It is necessary to have a Kogut-Susskind\[33\] type of a gauge ghost in order to realize this scenario. It is not clear whether this is or not a separate solution to the U(1) problem without imposing the strong CP violation. But it is worth noting that the strong CP problem in QCD may not be as serious as we thought if we do not insist on a solution to the U(1) problem by the same mechanism.

**B. \( m_u = 0 \) Scenario**

When \( m_u = 0 \) thus \( J_{\text{strong}} = 0 \), the strong CP problem is most neatly and elegantly solved. In the meantime, the U(1) problem can be solved by instanton without resorting to other assumptions. There is an additional \( U(1)_A \) symmetry associated with \( u \) quark. Thus \( m_u = 0 \), unlike setting \( \bar{\theta} = 0 \), does increase the symmetry of the system and does not violate 't Hooft’s naturalness principle. However, that \( m_u = 0 \) seems to contradict with the phenomenology where \( m_u^{\text{exp}} \approx 5 \sim 10 \text{ MeV} \)[36].

However, there is a loophole in this argument\[34\]. The instanton *explicitly* breaks \( U(1)_A \), as well as \( U(1)_{u_A} \) associated with the massless \( u \) quark if all other light quarks are *massive*. The instanton is acting as a flavor-changing force, as a result, \( u \) quark acquires a radiative mass from other flavors! This is again due to the existence of the zero modes of \( \mathcal{D} \) in the nontrivial instanton field. In the presence of a massless fermion, the vacuum tunneling effect is suppressed unless we insert an operator that contains enough grassmann fields to ‘kill’ all the zero modes. In the \( \nu = \pm 1 \) sector, the only operator which survives is \( \bar{u}u \). To see how it works, let’s recall the partition function \( Z(\bar{\theta}) \) in \[33\]. \( \langle \bar{u}u \rangle \) is calculated by taking the average over space-time

\[
\langle \bar{u}u \rangle_{\text{instanton}} = \frac{1}{V} \langle \int d^4 x \bar{u}u(x) \rangle = -\frac{1}{V} \frac{d}{dm_u} \ln Z(\bar{\theta})
\]

\[= -2m_d \cdots m_L K(L) \] (5.5)
where we have rotated $\bar{\theta}$ to zero as we can when $m_u = 0$. (5.3) implies that $U(1)_A^u$ symmetry is broken by instanton. Of course we would not have the goldstone boson since it is referred to as an explicit breaking. We should not confuse the condensate $\langle \bar{u}u \rangle$ caused by the spontaneous symmetry breaking with $\langle \bar{u}u \rangle_{\text{instanton}}$. The former can be non-zero even if all quarks are massless while the latter vanishes if $d$ quark mass is zero. The instanton induced $u$ quark mass can be roughly estimated $[31]$ in the case $L = 2$ where $K(2)$ is related to $m_\eta^2$

$$m_{u_{\text{instanton}}} \approx -\pi \alpha_s(\bar{\rho}) C_F \bar{\rho}^2 \langle \bar{u}u \rangle_{\text{instanton}}$$

$$= \frac{4}{3} \pi \alpha_s(\bar{\rho}) \bar{\rho}^2 F^2 \frac{m_\eta^2 - m_\pi^2}{m_\eta^2} m_d \cong 4 \text{ MeV}$$

(5.6)

where we take $\bar{\rho} \simeq \left( \frac{1}{3} \Lambda_{QCD} \right)^{-1}$, $K = -\frac{1}{27} (m_\eta^2 - m_\pi^2)$ and $f = \frac{2m_\eta}{F_\pi}$. $m_{u_{\text{instanton}}}$ must be viewed as an explicit mass because of its proportionality to $m_d$. What seems remarkable is that the order of magnitude of $m_{u_{\text{instanton}}}$ is in consistence with the phenomenological value. The massless $u$ quark is still the most favorable solution to the strong CP problem.

C. Peccei-Quinn Symmetry

Another possibility of rendering $J_{\text{strong}} = 0$ is that $\bar{\theta} = 0$ for some dynamical reason. This is realized if the phase of the quark masses $\theta_{QFD} = \sum_i \varphi_i$ is equal to $-\theta_{QCD}$. A decade ago, Peccei and Quinn $[38]$ suggested that the strong CP problem may be naturally solved if one or more quarks acquire the current masses entirely through the Higgs mechanism where the lagrangian of quarks and scalars exhibits an adjoint chiral symmetry: the Peccei-Quinn symmetry.

For simplicity, let us examine a toy model of a single quark

$$\mathcal{L}_{\text{toy}} = -\bar{\psi} D\psi - \frac{1}{4} F^2 + i \theta \bar{F} F - (f \bar{\psi}_L \psi_R \phi + h.c.) - \partial_\mu \phi \partial_\mu \phi^* - V_0(\phi, \phi^*)$$

(5.7)

where

$$V_0(\phi, \phi^*) = -\mu^2 \phi \phi^* + \frac{1}{4} \lambda (\phi \phi^*)^2.$$

(5.8)
(5.7) is invariant under the PQ symmetry

\[ \psi_R \rightarrow e^{i\alpha} \psi_R \ , \ \psi_L \rightarrow e^{-i\alpha} \psi_L; \]

\[ \phi \rightarrow e^{-2i\alpha} \phi \ , \ \phi^* \rightarrow e^{2i\alpha} \phi^*. \] (5.9)

The PQ symmetry is quantumly broken by the chiral anomaly, and effectively

\[ \mathcal{L}_{toy} \rightarrow \mathcal{L}_{toy} - 2i\alpha F\tilde{F}. \] (5.10)

Choosing \( \alpha = \frac{\theta}{2} \) yields \( \bar{\theta} = 0 \).

The effective potential of the scalar fields can be calculated in a similar way to (4.16)

\[ V_{eff}(\phi, \phi^*) = -\mu^2 \phi \phi^* + \frac{1}{4}\lambda(\phi \phi^*)^2 - K f^* e^{-i\theta} \det \phi^* - K f e^{i\theta} \det \phi \] (5.11)

where \( K \) is the instanton amplitude. The last two terms in the effective potential breaks the PQ symmetry. The VEV’s of \( \phi \) and \( \phi^* \) are found to be

\[ \langle f \phi \rangle = ve^{-i\theta} ; \ \langle f^* \phi^* \rangle = ve^{i\theta} \] (5.12)

and

\[ v^2 = \frac{2\mu^2 |f|^2}{\lambda} + \frac{2K|f|^4}{\lambda v}. \] (5.13)

Thus the fermion mass reads from the Yukawa interaction \( m = fve^{-i\theta} \) and

\[ \bar{\theta} = \theta + \arg \langle f \phi \rangle = 0. \] (5.14)

The axion [39] mass is readily derived from (5.11) by diagonalizing the quadratic terms

\[ m_{axion}^2 = \frac{2K|f|^2}{v}. \] (5.15)

Unfortunately, we have not been able to discover this particle yet so far.
VI. CONCLUSIONS

We have studied the measure of CP violation in strong interactions. It arises from the nontrivial topological structure of Yang-Mills fields, a non-zero vacuum angle $\bar{\theta}$ as well as nonvanishing quark current masses. The instanton dynamics makes most sense in dealing with the topological gauge configurations where the semiclassical method applies. It has been shown that the instanton dynamics, as a consistent field theory, automatically satisfies the so-called anomalous Ward identity. Crewther’s original complaints on the topological susceptibility and $\theta$-periodicity of the fermion operator are a result of inconsistently handling the singularities in some fermion operators. We conclude that QCD theory itself does not put any constraint on the instanton computation.

In the presence of the chiral anomaly, there is no would-be goldstone particle. By studying an effective chiral theory, we find that the instanton leads to an explicit $U(1)_A$ symmetry breaking. If the instanton is to solve the $U(1)$ problem, the measure of the strong CP violation is connected to the mass of the $U(1)$ particle. It may be natural to think that strong CP problem is the side effect of the $U(1)$ problem and both problems cannot be solved simultaneously in the context of QCD.

However, we point out that the massless $u$ quark scenario to solve the strong CP problem may not be such a silly idea. $u$ quark may acquire a mass from $d$ quark through the instanton interaction in which the fermion zero modes plays an essential role. In any case, with the failure to observing axions experimentally, the strong CP problem is wide open to new mechanisms.

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FIGURES

FIG. 1. Feynman rules for $\eta^3$ and $\eta\pi^2$ couplings. The CP-violating $qq \rightarrow qq$ scattering. We have assumed that $m_\sigma^2 \gg m_\eta^2$, $m_\alpha^2 \gg m_\pi^2$ and $v = 2F_\pi$.

FIG. 2. Diagramatic representations of Schwinger’s formulation on the EDM’s for constituent quarks.