Dipole description of inclusive particle production

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Abstract

The effects of multiple interactions in colliding particles (e.g. in nucleus-nucleus collisions) are modeled using the light-cone dipole approach. Guided by the abelian analogue of multi-photon interactions in the production of a pair of charged particles, we relate the inclusive cross section of quark pair production with the cross sections of interaction of a QCD dipole with either the beam or the target.

1 Light-cone dipole representation

The light cone dipole description of hadronic interactions\cite{1,2} offers quite an effective phenomenology. The central quantity of this approach, the universal and flavor independent cross section of interaction of a colorless dipole (quark-antiquark, or glue-glue) with a target (proton) is fitted to data, and therefore it incorporates information on all possible gluonic exchanges and bremsstrahlung including also nonperturbative effects. This may be considered as an alternative to the parton distribution function, with the advantage that it includes by default all higher order corrections and higher twist effects. This approach is especially powerful for calculating nuclear effects and diffractive processes\cite{1,3,4,5}. Since QCD dipoles are eigenstates of interaction, multiple interactions effects for the elastic amplitude can be included via simple eikonalization.

The main difficulty of the dipole approach, unresolved so far, is modeling the distribution amplitude of QCD dipoles in the projectile high energy particle. This problem has only been solved in the lowest order of perturbative QCD for a photon projectile\cite{6,7}, and for radiation of photons and gluons by a color charge\cite{8,9,10}. However, once the multi-gluon exchange interactions with the target are important, one should not restrict oneself to a single parton density in the projectile, as is frequently done. Inclusion of higher order corrections and soft multiple interactions in the projectile particle remains a challenge. A model for simultaneous
inclusion of multiple interaction effects both in the beam and target was constructed in [11] for gluon radiation in nucleus-nucleus collision. Here we present another attempt to make progress in this direction.

The paper is organized as follows. In Sect. 2 we study the production of a pair of charge particles employing abelian dynamics. We start with the Born approximation (Sect. 2.1) and then include multi-photon exchanges with the target, and eventually with both colliding particles \( A \) and \( B \) (Sect. 2.2).

In Sect. 3 we develop the formalism for nonabelian dynamics. The Born approximation is described in Sect. 3.1 and the results are generalized to include multiple interactions in Sect. 3.2.

## 2 QED analogue

### 2.1 Born approximation in QED

We start with an abelian analogue for quark pair production, since this process has a simpler dynamics, but it contains many features of the nonabelian description. The process under discussion is production of a pair of particles, 1 and 2, in the collisions of two hadrons (or nuclei), \( A \) and \( B \),

\[
A + B \rightarrow A + B + 1 + 2 .
\]  

(1)

For the sake of simplicity we assume that the produced particles are spinless. The cross section of this process reads,

\[
d\sigma_{AB} = |M|^2 d\Gamma
\]  

(2)

where \( M \) is the Lorentz-invariant amplitude, and \( d\Gamma \) is a phase space element. In the lowest order of perturbative expansion corresponding to the graphs shown in Fig. 1a,b the amplitude

![Diagram](image)

**Figure 1:** Production of charged particles 1 and 2, in the Born approximation (a and b), and including multi-photon exchanges with the target or with both colliding particles (c and d respectively).
has the form,
\[ M = J^{(A)}_{\mu}(p_A, q_A) J^{(B)}_{\mu}(p_B, q_B) \frac{t_{\mu\nu}(q_A, q_B, p_1, p_2)}{Q_A^2 Q_B^2} , \quad (3) \]

where
\[ q_{A(B)} = p_{A(B)} - p'_{A(B)} ; \]
\[ Q_{A(B)}^2 = -q_{A(B)}^2 \approx q_{A(B)\perp}^2 + q_{A(B)_{\text{min}}}^2 ; \]
\[ q_{A(B)_{\text{min}}}^2 = \frac{q_{A(B)}^2}{\gamma_{A(B)}} ; \quad (4) \]

\[ p_{A(B)}, p'_{A(B)}, p_1 \text{ and } p_2 \] are the 4-momenta of particles \( A(B) \) in the initial and final states, and of the produced particles respectively; \( \gamma_{A(B)} \) are the Lorentz factors of the colliding particles; \( q_{A(B)} \) are the 4-momenta of the virtual photons emitted by the particles \( A(B) \). The electromagnetic currents in Eq. (3) have the form,
\[ J^{(A)}_{\mu}(p_A, q_A) = \sqrt{4\pi\alpha_{em}} Z_A F^{(A)}_{\mu}(Q_A^2)(2p_A - q_A)_{\mu} ; \quad (5) \]
\[ J^{(B)}_{\nu}(p_B, q_B) = \sqrt{4\pi\alpha_{em}} Z_B F^{(B)}_{\nu}(Q_B^2)(2p_B - q_B)_{\nu} , \quad (6) \]

where \( Z_{A(B)} \) are the charges of \( A \) and \( B \), and \( F^{A(B)}(Q_{A(B)}^2) \) are their form factors, respectively.

The tensor \( t_{\mu\nu} \) in (3) reads,
\[ t_{\mu\nu} = 4\pi\alpha_{em} \left[ D_1^{-1}(2p_A - q_A)_{\mu}(2p_2 - q_B)_{\nu} + D_2^{-1}(2p_2 - q_A)_{\mu}(2q_B - q_A)_{\nu} - 2q_{\mu\nu} \right] ; \quad (7) \]
\[ D_1 = m_2^2 - (p_1 - q_A)^2 = m_2^2 - (p_1 - q_B)^2 ; \quad (8) \]
\[ D_2 = m_2^2 - (p_2 - q_A)^2 = m_2^2 - (p_1 - q_B)^2 ; \quad (9) \]

Notice that,
\[ (q_A)_{\mu} t_{\mu\nu} = t_{\mu\nu} (q_B)_{\nu} = 0 . \quad (10) \]

The phase space factor in Eq. (2) is
\[ d\Gamma = \frac{d^3p_A' d^3p_B' d^3p_1 d^3p_2}{64 I(2\pi)^8 E_A' E_B' \epsilon_1 \epsilon_2} \delta^{(4)}(p_A + p_B - p'_A - p'_B - p_1 - p_2) \]
\[ = \frac{d^4q_A d^4q_B d^3p_1 d^3p_2}{64 I(2\pi)^8 \epsilon_1 \epsilon_2} \delta^{(4)}(q_A + q_B - p_1 - p_2) \delta(2p_Aq_A + Q_A^2) \delta(2p_Bq_B + Q_B^2), \quad (11) \]

where
\[ I = \sqrt{(p_A p_B)^2 - M_A^2 M_B^2} . \quad (12) \]

Here \( M_{A(B)} \) and \( E'_{A(B)} \) are the colliding hadrons or nuclei masses and energies; \( m \) and \( \epsilon_{1(2)} \) are the masses and energies of the produced particles.

To simulate QCD, we introduce a charge screening effect in what follows, i.e. consider the colliding particles as neutral dipoles with the screening potential,
\[ V(r) = \frac{Z\alpha_{em}}{r} e^{-\lambda r} . \quad (13) \]
This can be also simulated by an effective photon mass \( \lambda \), replacing in \( Q_{A(B)}^2 \Rightarrow \lambda^2 \).

Now, let us consider the collision of ultra-relativistic particles \( (\gamma_{A(B)} \gg 1) \) in the c.m. frame. Neglecting corrections of order \( O(\gamma_{A(B)}^{-2}) \) we have from \([2] - [12]\),

\[
\sigma = \int dq_A^+ dq_A^- dq_B^+ dq_B^- \frac{d^3p_1^+ d^3p_2^+}{\epsilon_1 \epsilon_2} \sigma_A^A(q_A) \sigma_B^B(q_B) |U(q_A, q_B, p_1, p_2)|^2 \\
\times \delta(q_A^-) \delta(q_B^+) \delta(q_A^+ + q_B^- - p_1^+ - p_2^+) \delta(q_A^- + q_B^- - p_1^- - p_2^-),
\]

where \( p_{1(2)}^\pm = (p_{1(2)})_0 \pm (p_{1(2)})_z \) are the light-cone momenta of particles 1 (2) (the z-axis is chosen along the momenta of \( A, B \)); \( \sigma_A^A(q_A) \) and \( \sigma_B^B(q_B) \) are the differential cross sections of elastic scattering of particles 1, 2 on hadrons (nuclei) \( A \) or \( B \) respectively,

\[
\sigma_A^A(q_A) \equiv \frac{d^2 \sigma[1(2) + A \rightarrow 1(2) + A]}{d^2 q_{A \perp}} = \left[ \frac{2Z_A \alpha_{em} F_A^A(Q_A^2)}{q_{A \perp}^2 + \lambda^2} \right]^2 ;
\]

\[
\sigma_B^B(q_B) \equiv \frac{d^2 \sigma[1(2) + B \rightarrow 1(2) + B]}{d^2 q_{B \perp}} = \left[ \frac{2Z_B \alpha_{em} F_B^B(Q_B^2)}{q_{B \perp}^2 + \lambda^2} \right]^2 .
\]

Then, we can represent

\[
\frac{d^3 p_{1(2)}^+}{\epsilon_{1(2)}} = d^2 p_{1(2) \perp} \frac{dp_{1(2)}^+}{p_{1(2)}^+} = d^2 p_{1(2) \perp} \frac{dp_{1(2)}^-}{p_{1(2)}^-} ;
\]

\[
p_{1(2)}^+ p_{1(2)}^- = m^2 + \vec{p}_{1(2) \perp}^2 .
\]

It is convenient to introduce the fractions of light-cone momenta of the colliding virtual photons carried by the produced particle 1,

\[
\alpha_A = \frac{p_{1 \perp}^+}{q_A^+} ;
\]

\[
\alpha_B = \frac{p_{1 \perp}^-}{q_A^-} ,
\]

which are connected by the relation,

\[
\alpha_{A(B)} = \frac{(1 - \alpha_{B(A)})(m_1)^2 \perp}{(1 - \alpha_{B(A)})(m_1)^2 \perp + \alpha_{B(A)}(m_2)^2 \perp} .
\]

The amplitude \( U(q_A, q_B, p_1, p_2) \) in \([14]\),

\[
U(q_A, q_B, p_1, p_2) = \left[ \frac{p_{1 \perp}^+ p_{2 \perp}^+}{D_1} + \frac{p_{2 \perp}^- p_{1 \perp}^+}{D_2} - 1 \right] ,
\]

is function of three 4-momenta (since \( q_A + q_B = p_1 + p_2 \)). Therefore, we can choose as independent variables the three transverse momenta and one of the light-cone fractions \( \alpha_A \) (or \( \alpha_B \)). Selecting \( \vec{q}_{A \perp}, \vec{q}_{B \perp}, \vec{p}_{1 \perp} \) and \( \alpha_A \), we can represent \( U \) as,

\[
U(\vec{q}_{A \perp}, \vec{q}_{B \perp}, \vec{p}_{1 \perp}, \alpha_A) = \Phi(\vec{p}_{1 \perp} - \alpha_A \vec{q}_{A \perp}; \vec{q}_{A \perp}, \alpha_A) - \Phi(\vec{p}_{1 \perp} - \alpha_A \vec{q}_{A \perp} - \vec{q}_{B \perp}; \vec{q}_{A \perp}, \alpha_A) ,
\]

4
where
\[
\begin{align*}
\Phi(p^\perp, q^\perp, \alpha) &= \Phi^T(p^\perp, q^\perp, \alpha) + \Phi^L(p^\perp, q^\perp, \alpha) ; \\
\Phi^T(p^\perp, q^\perp, \alpha) &= 2\alpha(1 - \alpha) \frac{\vec{p} \cdot \vec{q}}{\vec{p}^2 + \epsilon^2} , \\
\Phi^L(p^\perp, q^\perp, \alpha) &= \frac{\alpha(1 - \alpha)(1 - 2\alpha)}{\vec{p}^2 + \epsilon^2} , \\
\epsilon^2(q^\perp, \alpha) &= m^2 + \alpha(1 - \alpha) \vec{q}^2 .
\end{align*}
\]

Thus, the cross section of pair production can be represented as,
\[
\sigma(A + B \rightarrow A + B + 1 + 2) = \int dq^+ d^2q_A \ n_A(q_A) \sigma(\gamma^* A + B \rightarrow 1 + 2 + B) ,
\]
where
\[
n_A(q_A) = \frac{\sigma_0^A(q_A) \vec{q}_A^2}{(2\pi)^2 \alpha_{em} q_A^+} = \left( \frac{Z_A F_A}{Q_A^2 + \lambda^2} \right)^2 \alpha_{em} q_A^+ ,
\]
is the density of equivalent photons in the projectile $A$.

The virtual photoproduction cross section $\sigma(\gamma^* A + B \rightarrow 1 + 2 + B)$ in (27) can be expressed in terms of the dipole formalism as,
\[
\sigma(\gamma^* A + B \rightarrow 1 + 2 + B) = \int d^2r d\alpha_A \left| \Psi^T_A(r, \alpha_A) \right|^2 + \left| \Psi^L_A(r, \alpha_A) \right|^2 \sigma_B(r) .
\]
Here $\Psi^{T,L}_A$ are the light-cone wave functions of transversely or longitudinally polarized photons with 4-momentum $q_A$,
\[
\Psi^{T,L}_A(r, \alpha_A) = \sqrt{\frac{\alpha_{em}}{\alpha_A(1 - \alpha_A)}} \frac{1}{|\vec{q}_A^2|(2\pi)^2} \int d^2p_T \Phi^{T,L}(p^\perp, q^\perp, \alpha_A) e^{i\vec{p} \cdot \vec{r}} .
\]

The cross section of interaction of the dipole of particles $1 - 2$ with $B$ has the standard form
\[
\sigma_B(r) = 2 \int d^2q_B \sigma_0^B(q_B) \left( 1 - e^{i\vec{q} \cdot \vec{r}} \right) .
\]

The resulting representation, Eq. (29), which treats the production of particles $1 - 2$ as photoproduction by a virtual photon representing the electromagnetic field of the projectile $A$ interacting with the target $B$, looks asymmetric relative to the replacement $A \leftrightarrow B$. This is, however, an artifact of our choice of $\alpha_A$ as a variable. If our choice were $\alpha_B$, the same cross section would look as a result of interaction of a photon $\gamma^*_B$ with the target $A$. Thus, the choice of an independent variable, $\alpha_A$ or $\alpha_B$, leads to a breaking of the symmetry, $A \leftrightarrow B$.

### 2.2 Multi-photon exchanges

So far our considerations were restricted to the Born (one photon) approximation. It turns out, however, that the relation Eq. (29) is also correct if the dipole $1 - 2$ interacts with the target
\[ \sigma_B(r) = 2 \int d^2b \left\{ 1 - \exp[i\Delta \chi_B(\vec{b}, \vec{r})] \right\}, \tag{32} \]

where the phase shift
\[ \Delta \chi_B(\vec{b}, \vec{r}) = \chi_B(\vec{b}_+) - \chi_B(\vec{b}_-) ; \tag{33} \]
\[ \chi_B(\vec{b}_\pm) = \frac{Z_B \alpha_{em}}{\pi} \int \frac{d^2q_\perp}{q_\perp^2 + \lambda^2} e^{i\vec{q}_\perp \cdot \vec{b}_\pm} ; \tag{34} \]
\[ \vec{b}_+ = \vec{b} + (1 - \alpha_A) \vec{r}, \]
\[ \vec{b}_- = \vec{b} - \alpha_A \vec{r}. \]

At first glance the cross section Eq. (32) depends also on \( \alpha_A \). However, the change of integration variable, \( \vec{b} \rightarrow \vec{b} + (\alpha_A - 1/2) \vec{r} \) eliminates the \( \alpha_A \) dependence.

Another possible representation for the dipole cross section has the form,
\[ \sigma_B(r) = 2 \int d^2q_\perp \sigma_B^{Gl}(\vec{q}_\perp) \left( 1 - e^{i \vec{q}_\perp \cdot \vec{r}} \right), \tag{35} \]

where \( \sigma_B^{Gl}(\vec{q}_\perp) \) is the differential cross section of elastic scattering of one of the particles, 1 or 2, with the target \( B \), calculated in the eikonal (Glauber) approximation,
\[ \sigma_B^{Gl}(\vec{q}_\perp) = \left| f^\pm_B(\vec{q}_\perp) \right|^2 \tag{36} \]
\[ f^\pm_B(\vec{q}_\perp) = \frac{i}{2\pi} \int d^2b e^{i\vec{q}_\perp \cdot \vec{b}} \left[ 1 - e^{\pm i \chi_B(\vec{b})} \right]. \tag{37} \]

The signs \( \pm \) correspond to the opposite charges of particles 1, 2.

Thus, we conclude that the effect of all multi-photon exchanges with the target \( B \) (restricted to only one photon exchange with the projectile \( A \)) is equivalent to the replacement of the Born cross section \( \sigma_0^B(q_B) \) by the Glauber one, \( \sigma_B^{Gl}(q_B) \).

The same cross section of pair production calculated in a single photon approximation for \( A \), but multi-photon with \( B \), can be represented differently if one chooses \( \alpha_B \) as a variable,
\[ \sigma(\gamma^* A + B \rightarrow 1 + 2 + B) = \int dq_{B-} d^2q_{B+} \tilde{n}_B(q_B) \sigma(\gamma^*_B + A \rightarrow 1 + 2 + A). \tag{38} \]

Here the process \( \gamma^*_B + A \rightarrow 1 + 2 + A \) is calculated in one-photon approximation, but the density function of "equivalent photons" is different from \( 28 \),
\[ \tilde{n}_B(q_B) = \frac{\sigma_B^{Gl}(q_B) q_{B+}^2}{(2\pi)^2 \alpha_{em} q_B} ; \tag{39} \]
with the replacement of single- to multi-photon exchange, \( \sigma_0^B(q_B) \Rightarrow \sigma_B^{Gl}(q_B) \).

It is natural to assume that the inclusion of multi-photon exchanges between the produced quark pair and both colliding nuclei (Fig. 11) can be done by replacing the "single-photon"
quantity $\sigma(\gamma_B^* + Z_A \to 1 + 2 + Z_A)$ in (33) by a multi-photon one. To do that we should replace the Born approximation for the cross section $\sigma_A(\vec{r})$ in (29) (with the interchange $A \leftrightarrow B$),

$$\sigma_A(\vec{r}) = 2 \int d^2q_A \sigma_0^A(q_A) \left(1 - e^{i\vec{q}_A \cdot \vec{r}}\right) \quad (40)$$

by the following Glauber form,

$$\sigma_A(\vec{r}) = 2 \int d^2b \left[1 - e^{i\Delta \chi_A(\vec{b}, \vec{r})}\right]. \quad (41)$$

The phase shifts $\Delta \chi_A(\vec{b}, \vec{r})$ are defined on analogy to $\Delta \chi_B(\vec{b}, \vec{r})$ in (33)-(34).

### 3 Quark production

#### 3.1 Born approximation in QCD

The Feynman graphs corresponding to $\bar{q}q$ pair production in a collisions of two hadrons $A$ (beam) and $B$ (target) in lowest order in $\alpha_s$ (double one-gluon approximation) are shown in Fig. 2. We assume that only one quark (antiquark) is detected, with transverse momentum $\vec{p}_T$ and rapidity $y$, while the accompanying antiquark (quark) is not observed, i.e. its momentum is integrated out. Then the cross section corresponding to the graphs in Fig. 2 has the form,

$$\frac{d\sigma(A B \to q X)}{d^2p_T dy} = \frac{4\pi\alpha_s}{3} \int \frac{dx_1}{x_q} \int d^2q_1 d^2q_2 \frac{q_1^2 \mathcal{F}_A(x_1, \vec{q}_1) \alpha}{(q_1^2 + q_{1\text{min}}^2)^2} \left(\frac{7|\Phi_1|^2 + 9|\Phi_2|^2}{16}\right) \frac{\mathcal{F}_B(x_2, \vec{q}_2)}{(q_2^2 + q_{2\text{min}}^2)^2}. \quad (42)$$

Here $\vec{p}_T$ and $y$ are the transverse momentum and rapidity of the produced quark (or antiquark); $x_q$ is the fraction of the plus component of the momentum of the hadron $h_1$ taken by the quark, which is related to the rapidity interval $\Delta y = \ln(1/x_q)$ between the hadron and the quark; and $\vec{q}_{1,2}$ are the transverse momenta of the gluons radiated by the hadrons $h_{1,2}$ with light-cone fractional momenta $x_{1,2}$ respectively. While we integrate over $x_1$, the value of $x_2$ is defined by the kinematics,

$$x_2 = \frac{1}{x_1s} \left[\frac{m_q^2 + p_T^2}{\alpha} + \frac{m_q^2 + (\vec{p}_T - \vec{q}_1 - \vec{q}_2)^2}{1 - \alpha}\right], \quad (43)$$

Figure 2: One gluon approximation to the central production of a $\bar{q}q$ pair.
where \( \alpha = x_q/x_1 \). The function \( \Phi_1 \) in [42] corresponds to the sum of diagrams Fig. 2a,b, while \( \Phi_2 \) corresponds to the difference of the amplitudes Fig. 2a,b plus the graph in Fig. 2. These functions are expressed in terms of the usual LC wave functions \( \Psi_{q\bar{q}}(k_T, \alpha, q^2) \) of the \( \bar{q}q \) Fock state in a gluon, where \( \vec{k}_T \) is the relative transverse momentum of the \( \bar{q}q \),

\[
\Psi_{q\bar{q}}(k_T, \alpha, q^2) = \sqrt{\frac{4\alpha_x}{3}} \frac{x_q}{\epsilon^2 + k_T^2},
\]

where

\[
\epsilon^2 = \alpha(1-\alpha)Q_1^2 + m_q^2;
Q_1^2 = q_1^2 + q_{1\min}^2 \quad (q_{1\min} \sim \Lambda_{QCD});
\hat{O} = m\vec{\sigma} \cdot \vec{e} + (1-2\alpha)(\vec{\sigma} \cdot \vec{n})(\vec{p} \cdot \vec{e}) + i(\vec{p} \times \vec{n}) \cdot \vec{e};
\]

and the gluon polarization vector is related to its transverse momentum, \( \vec{e} = \vec{q}_1/q_1 \). Then, we have

\[
\Phi_1 = \Psi_{q\bar{q}}^G(p_T - \alpha \vec{q}_1, \alpha, q_1^2) - \Psi_{q\bar{q}}^G(p_T - \alpha \vec{q}_1 - \vec{q}_2, \alpha, q_1^2);
\Phi_2 = \Psi_{q\bar{q}}^G(p_T - \alpha \vec{q}_1, \alpha, q_1^2) + \Psi_{q\bar{q}}^G(p_T - \alpha \vec{q}_1 - \vec{q}_2, \alpha, q_1^2) - 2\Psi_{q\bar{q}}^G(p_T - \alpha \vec{q}_1 - \alpha \vec{q}_2, \alpha, q_1^2).
\]

### 3.2 Multiple interactions

In the light-cone approach employed in the rest frame of the target (the bottom hadron in Fig. 2) the process depicted in Fig. 2 looks like the interaction of a \( \bar{q}q \) fluctuation of the projectile gluon with the target. Although the interaction is mediated by one gluon exchange, one can make it more realistic using the phenomenological dipole cross section with the target. Although the interaction is mediated by one gluon exchange, one can make it more realistic using the phenomenological dipole cross section \( \sigma_{q\bar{q}}^{h}(r, x) \) of interaction of a \( \bar{q}q \) dipole of transverse separation \( \vec{r} \) with a hadron \( h \) at energy \( s \sim (\pi r^2)^{-1} \). This cross section fitted to data incorporates the unknown dynamics of soft multi-gluon exchanges and radiation. Applying to [42] a Fourier transformation we get,

\[
\frac{d\sigma(AB \rightarrow qX)}{d^2p_T \, dy} = \frac{1}{8\pi^2} \int \frac{dx_1}{x_1} \int \frac{d^2q_1}{q_1^2} \frac{d^2q_2}{q_2^2} \frac{\mathcal{F}_{A}(x_1, \vec{q}_1)}{(q_1^2 + q_{1\min}^2)^2} \times \int d^2r_1 \, d^2r_2 \, \exp[i(p_T - \alpha \vec{q}_2)(\vec{r}_1 - \vec{r}_2)] \Psi_{q\bar{q}}^* \Psi_{q\bar{q}} \Sigma^B(\vec{r}_1, \vec{r}_2, x_2, \alpha),
\]

where

\[
\Sigma^B(\vec{r}_1, \vec{r}_2, x_2, \alpha) = \frac{1}{16} \left\{ g \left[ \sigma_{q\bar{q}}^B(\vec{r}_1 - \alpha \vec{r}_2, x_2) + \sigma_{q\bar{q}}^B(\vec{r}_2 - \alpha \vec{r}_1, x_2) + \sigma_{q\bar{q}}^B(\alpha \vec{r}_1, x_2) + \sigma_{q\bar{q}}^B(\alpha \vec{r}_2, x_2) \right] + \left[ \sigma_{q\bar{q}}^B(\vec{r}_1, x_2) + \sigma_{q\bar{q}}^B(\vec{r}_2, x_2) - 8\sigma_{q\bar{q}}^B(\vec{r}_1, x_2) - 8\sigma_{q\bar{q}}^B(\alpha \vec{r}_1, x_2) \right] \right\},
\]

At \( \vec{r}_1 = \vec{r}_2 \) this becomes the familiar combination \( \sigma_3(\vec{r}, \alpha, x) = \frac{g}{8} \left\{ (\sigma_{q\bar{q}}^h(\alpha \vec{r}, x) + \sigma_{q\bar{q}}^h((1-\alpha)\vec{r}, x) - \frac{1}{8} \sigma_{q\bar{q}}^h(2\alpha - 1)\vec{r}, x) \right\} \) which is the dipole cross section for a three-body system \( \bar{q}qG \) interacting with a hadron \( h \). In particular, it enters the total cross section of \( \bar{q}q \) pair production by a gluon [14, 8, 15].
The dipole cross section vanishes at small \( \bar{q}q \) separations, \( \sigma_{\bar{q}q}(r, x)_{r \to 0} = r^2 G^h(x, 1/r^2) \pi^2 \alpha_s/3 \), where \( G^h(x, 1/r^2) = x g^h(x, 1/r^2) \) is the gluon distribution function in hadron \( h \). In this limit the dipole cross section corresponds to one gluon exchange (in the inelastic amplitude) and Eq. (42) is recovered. At the same time, it is usually assumed that at large separations \( \sigma_{\bar{q}q}(r_T, x) \) saturates at some constant value \( \sigma_0^h \). This may be motivated by either saturation of the gluon density, or shortness of the gluon interaction radius.

Let us introduce a function

\[
\omega^h(\vec{r}, x) = \sigma_0^h - \sigma_{\bar{q}q}^h(\vec{r}, x) ,
\]

which has the properties \( \omega^h(\vec{r}, x)_{r \to 0} \to \sigma_0^h(x) \) and \( \omega(\vec{r}, x)_{r \to \infty} \to 0 \). Therefore its Fourier transform,

\[
\omega^h(q, x) = \frac{1}{(2\pi)^2} \int d^2 \vec{r} \omega^h(\vec{r}, x) e^{i\vec{q} \cdot \vec{r}} ,
\]

is defined for any \( \vec{q} \). Then, taking into account that \( \int d^2 q \omega^h(q, x) e^{-i\vec{q} \cdot \vec{r}} = \sigma_0^h(x) \), we can represent the function \( \Sigma^B(\vec{r}_1, \vec{r}_2, \alpha) \), Eq. (48), in the form,

\[
\Sigma^B(\vec{r}_1, \vec{r}_2, x_2 \alpha) = \int d^2 q \omega^B(q, x_2) \left\{ \frac{7}{16} \left[ 1 - e^{i\vec{q} \cdot \vec{r}_1} \right] \left[ 1 - e^{-i\vec{q} \cdot \vec{r}_2} \right] + \frac{9}{16} \left[ 1 + e^{i\vec{q} \cdot \vec{r}_1} - 2 e^{i\alpha \vec{q} \cdot \vec{r}_1} \right] \left[ 1 + e^{i\vec{q} \cdot \vec{r}_2} - 2 e^{i\alpha \vec{q} \cdot \vec{r}_2} \right] \right\} .
\]

Using this expression we can rewrite Eq. (47) as

\[
\frac{d\sigma(AB \to qX)}{d^2 p_T dy} = \frac{1}{2} \int dx_1 \frac{\alpha}{x_1} \int d^2 q_1 \frac{q_1^2 \mathcal{F}_A(x_1, \vec{q}_1)}{(q_1^2 + q_{1min}^2)^2} \omega^B(\vec{q}, x_2) \left[ \frac{7}{16} \left| \Phi_1 \right|^2 + \frac{9}{16} \left| \Phi_2 \right|^2 \right] (52)
\]

Finally, using the relation \( \omega(\vec{q}, x) = \sigma_0(x) \delta(\vec{q}) - \sigma_{\bar{q}q}(\vec{q}, x) \) we arrive at,

\[
\frac{d\sigma(AB \to qX)}{d^2 p_T dy} = -\frac{1}{2} \int dx_1 \frac{\alpha}{x_1} \int d^2 q_1 \frac{q_1^2 \mathcal{F}_A(x_1, \vec{q}_1)}{(q_1^2 + q_{1min}^2)^2} \left( \frac{7}{16} \left| \Phi_1 \right|^2 + \frac{9}{16} \left| \Phi_2 \right|^2 \right) \sigma_{\bar{q}q}^B(\vec{q}, x_2)
\]

Comparing this expression with Eq. (42) we conclude that the transformations done above are equivalent to the replacement

\[
\frac{q_1^2 \mathcal{F}_B(x_2, \vec{q}_2)}{(q_2^2 + q_{2min}^2)^2} \Rightarrow -\frac{3}{4\pi\alpha_s} \sigma_{\bar{q}q}^B(\vec{q}_2, x_2)
\]

in Eq. (42). This observation leads to the natural assumption that the same procedure should be performed with the contribution to Eq. (42) of the upper part of the graphs in Fig. 2, namely,

\[
\frac{q_1^2 \mathcal{F}_A(x_1, \vec{q}_1)}{(q_1^2 + q_{1min}^2)^2} \Rightarrow -\frac{3}{4\pi\alpha_s} \sigma_{\bar{q}q}^A(\vec{q}_1, x_1)
\]

Switching back to coordinate representation,

\[
\sigma_{\bar{q}q}^A(\vec{q}, x) = \frac{1}{(2\pi)^2} \int d^2 \rho e^{-i\vec{q} \cdot \vec{r}} \hat{\nabla}_\rho \sigma_{\bar{q}q}^A(\vec{p}, x) ,
\]
we eventually arrive at the cross section in a form which includes multi-gluon exchange,

\[
\frac{d\sigma (A B \rightarrow q X)}{d^2 p_T \, dy} = \frac{6}{(4\pi)^3 \alpha_s^2} \int \frac{dx_1}{x_1} \int \frac{1}{x_q} \frac{1}{x_q} \int d^2 q \int d^2 \rho \, d^2 r_1 \, d^2 r_2 \, \exp \left[ i (\vec{p}_T - \alpha \vec{q}) \cdot (\vec{r}_1 - \vec{r}_2) - i \vec{q} \cdot \vec{\rho} \right] \\
\times \bar{\psi}_q (\vec{r}, x_1) \, \bar{\psi}_q (\vec{r}, x_2) \, \sigma (\vec{r}_1, \vec{r}_2, x_1, x_2) .
\]

(57)

This is the central result of this paper. It looks asymmetric, while the graphs in Fig. 2 are symmetric relative to beam-target interchange. This expression, however, has been derived in the rest frame of the target. In the beam rest frame one should just interchange \( A \leftrightarrow B \) and \( x_1 \leftrightarrow x_2 \).

The total yield of quarks integrated over transverse momentum has the simple form,

\[
\frac{d\sigma (A B \rightarrow q X)}{dy} = \frac{6}{(4\pi)^3 \alpha_s^2} \int \frac{dx_1}{x_1} \int \frac{1}{x_q} \int d^2 q \int d^2 \rho \, d^2 r_1 \, e^{-i \vec{q} \cdot \vec{\rho}} \, \bar{\psi}_q (\vec{r}, x_1) \, \psi_q (\vec{r}, x_1) \, \left| \psi_q (\vec{r}, x_1) \right|^2 \, \sigma (\vec{r}, x_1) .
\]

(58)

Thus the production cross section is expressed in terms of the dipole cross section either on one (A), or another (B) colliding particles or nuclei. Unfortunately, it has an asymmetric form which is related to our choice of \( \alpha_A \). Switching to \( \alpha_B \) we will get an equivalent cross section, but having a different form.

4 Summary

Guided by the abelian analogue of particle production in QED, we suggested an approach incorporating multiple interactions both in the beam and target, within the dipole formalism. The proposed procedure of replacing the unintegrated gluon density in the target by a combination of cross sections of dipole-target interaction is assumed to be valid also for the beam. The main result, Eq. (57), still needs needs to be tested through numerical calculations and comparison with data. This expression looks asymmetric relative to the dipole cross sections of interacting with the colliding particles or nuclei, A and B. However, effectively it is symmetric provided that a proper replacement of variables is done. It is possible to rewrite it in an explicitly symmetric form, however, in this case the light-cone distribution function \( \psi_q \) in (57) should be replaced by a more complicated function. Further development of this formalism will be published elsewhere.

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