Distributed Model Predictive Control for Platooning of Heterogeneous Vehicles with Multiple Constraints and Communication Delays

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In this paper, the vehicle platoon control problems for a group of heterogeneous vehicles are investigated, where the multiple constraints of the vehicles and the communication delays among the vehicles are taken into consideration. A distributed model predictive control (DMPC) scheme is proposed to drive the heterogeneous vehicles into the desired platoon. In this DMPC framework, the multiple constraints, including the control constraints, state constraints, and jerk constraints, are employed to describe the practical characteristics of vehicles and the communication delays are time-varying and bounded. In this framework, a group of platoon control schemes is proposed based on the DMPC techniques. Furthermore, the feasibility and stability of the proposed vehicle platoon control system are strictly analyzed. Finally, numerical simulation and experiment with TurtleBot3 mobile robots are provided to validate the effectiveness of proposed approaches.

1. Introduction

In recent years, since the dramatic increase of vehicles and the inferiority of human drivers, much more attention has been paid to the traffic problems (e.g., traffic congestion, road accidents, and air pollution) [1–3]. Vehicle platoon control is an effective way to increase the capability of roads and the fuel efficiency. It requires a leader vehicle in the platoon to follow a reference trajectory and the remaining vehicles to follow the leader vehicle with desired distances. Meanwhile, the autonomous vehicles in the platoon can reduce traffic jams and road accidents significantly.

The idea of vehicle platoon can be dated back to the Eighties when Partner for Advanced Transportation Technology (PATH) in California was established [4]. On-board cameras or laser sensors are used to measure the velocity, distance, and position of the surrounding vehicles and the vehicles in the platoon can cooperate with others via vehicle-to-vehicle (V2V) communication [5]. Then, various control techniques have been considered in the literature, including consensus-based control, sliding mode control, and model predictive control. For instance, the consensus formation control strategy is proposed for autonomous vehicular strings with V2V communication connections in [6]. In [7], the distributed adaptive control strategies based on the integral sliding mode control (ISMC) technique are proposed to maintain a rigid formation for a string of vehicle platoon in one dimension. A model predictive control system for a hybrid electric vehicle platoon considering the route information has been presented in [8]. A DMPC algorithm is proposed for the platoon of vehicles with nonlinear dynamics. Meanwhile, the stability and string stability are analyzed in [9]. In our previous work [10], a DMPC-based control scheme is proposed for a group of nonlinear vehicles. Due to the superiorities in dealing with constraints and optimizing control performance, the DMPC becomes more and more popular in vehicle platoon control [11–13].

More physical requirements, such as ride comfort, fuel economy, and velocity limit, have been imposed on vehicle platoon controllers to improve the driving performance. For instance, a smooth function \( \tanh(\cdot) \) is applied to restrict the control input of the vehicles in the platoon in [14]. In [15], a
neural adaptive sliding mode control algorithm is proposed to guarantee the string stability of the vehicle platoon, where the velocity constraints and input saturation are considered. A switched control strategy of heterogeneous vehicle platoon for multiple objectives is proposed in [16], which combines with the velocity constraints and acceleration constraints. Additionally, DMPC has a strong ability to explicitly take constraints into account [12, 17, 18]. In [19], time-varying control input constraints are considered for a group of autonomous underwater vehicles. In [20], a one-horizon model predictive control problem subject to acceleration, speed, and safety distance constraints is investigated for a platoon of connected autonomous vehicles. However, these results mainly consider one or two constraints in their control strategies. To the best of our knowledge, there are few studies taking multiple constraints such as input constraints, state constraints, and jerk constraints into consideration simultaneously. Hence, the vehicle platoon control with multiple constraints using DMPC is still an open problem in this field.

In addition, the V2V communication is an important part in vehicle platoon control. However, the most existing results about the vehicle platoon are developed based on the ideal communication links [21–23]. In practical applications, due to the hardware limitations or the network congestion, the communication delays may inhibit the designed control strategies and render the results invalid. To this end, a consensus-based control algorithm is implemented in the cooperative vehicle platoon system, where the impact of the heterogeneous V2V communication delay on the system performance is theoretically studied in [24]. In [25], a distributed consensus strategy is proposed for vehicle platoon with time-varying heterogeneous communication delays. In [26], a distributed $H_{\infty}$ control strategy is proposed for platooning of autonomous vehicles with switching and undirected topologies, which ensures the robust performance of platoon. In [27], a state predictor-based control strategy which can transmit the future information is proposed to compensate the information delay and an numerical method based on LMI is provided to find the required robust performance controller. A delay-involved DMPC scheme is proposed for a class of decoupled continuous-time systems by using a robustness constraint and a waiting mechanism in [28]. The authors further study the problem combining communication delays and external disturbances in [29]. Although the communication delay among the vehicle platoon has been investigated, there are few studies considering the multiple constraints and the communication delays simultaneously in vehicle platoon control.

Motivated by this fact, the platoon control problem of discrete-time vehicle systems with time-varying communication delays is investigated, in which multiple constraints and communication delays are considered to improve the driving performance. Moreover, compared with [28], we remove the robustness constraint and enlarge the upper bound of the communication delays. The main contributions are in three aspects as follows:

(i) A delay-involved DMPC strategy is proposed for the discrete-time vehicle platoon control problem subject to multiple constraints and communication delays. Multiple constraints, including control constraints, state constraints, and jerk constraints, are considered in each DMPC-based optimization problem. Time-varying and bounded communication delays are taken into consideration in the V2V communication, which can be dealt by the proposed DMPC strategy.

(ii) The feasibility and stability of proposed platoon control system are strictly analyzed. In detail, the feasibility of the proposed control strategy is proven by iteratively ensuring the multiple constraints and the terminal constraints. The stability of the vehicle platoon system is demonstrated through the Lyapunov stability theory. In addition, numerical simulation is presented to verify the theoretical results.

(iii) An experiment with three TurtleBot3 mobile robots on the Robot Operating System (ROS) platform is conducted to verify the proposed DMPC algorithm. In the experiment, these mobile robots are driven to the desired platoon by using the proposed DMPC algorithm and the experiment results validate the feasibility and effectiveness of proposed approaches in practical applications.

The remainder of this paper is organized as follows. In Section 2, preliminaries and problem formulation are presented. Section 3 introduces the DMPC-based vehicle platoon algorithm with multiple constraints and communication delays. Section 4 analyzes the feasibility and stability of the proposed control system. The simulation and experiment results are presented in Section 5. Section 6 concludes this paper.

Notation: $\mathbb{R}$ stands for the set of real numbers. $\mathbb{R}^n$ stands for the $n$-dimension real space. Given a matrix $M, M > 0$ $(M \geq 0)$ means the matrix is positive definite (positive semidefinite). $M_1 \geq M_2$ means that $M_1 - M_2 \geq 0$. For a given column vector $v$, $\|v\|$ represents the Euclidean norm. The $P$-weighted norm is defined as $\|v\|_P \triangleq \sqrt{v^T P v}$, where $P$ is a given matrix with appropriate dimension. Given matrix $Q$, $\lambda_1(Q)$ and $\lambda_\infty(Q)$ represent the minimum and maximum of the absolute values of the eigenvalues for $Q$.

2. Preliminaries and Problem Formation

2.1. Vehicle Modeling with Multiple Constraints. Consider a longitudinal vehicle platoon of $N_a + 1$ vehicles, which contains a leading vehicle (noted as leader, indexed by 0) and $N_a$ following vehicles (noted as followers, indexed from 1 to $N_a$).

The dynamics for the $i$th vehicle can be described as follows:

\[
\begin{aligned}
\dot{p}_i(t) &= v_i(t), \\
m_i \ddot{v}_i(t) &= F_i(t) + F_{\text{drag}}^i(t) + F_{\text{aero}}^i(t),
\end{aligned}
\]

(1)
where \( p_i(t), \dot{v}_i(t), \) and \( m_i \) are the position, velocity, and mass of the \( i \)-th vehicle, respectively. \( F_i(t) \) is the force generated by the vehicle engine with the derivative as follows:

\[
\dot{F}_i(t) = \frac{F_i(t) - c_i(t)}{\dot{v}_i(t) - c_i(t)},
\]

(2)

where \( c_i \) and \( \dot{v}_i \) denote time constant and throttle input of the vehicle. \( F_i^0(t) = -m_i g \sin(\theta_i(t)) \) represents the force due to gravity, where \( g \) is the acceleration of gravity and \( \theta_i(t) \) is the road slope. \( F_i^{\text{aero}}(t) = -\rho A_i c_{d_i} v_i^2(t) \) is the aerodynamic resistance with \( \rho \), \( A_i \), and \( c_{d_i} \) being the air density, cross-sectional area of the vehicle, and air drag coefficient. \( F_i^{\text{drag}}(t) = -c_i m_i g \cos(\theta_i(t)) \) represents the rolling resistance where \( c_i \) is the rolling coefficient.

Assumption 1. Vehicles run on a horizontal plane, i.e., \( \theta_i(t) = 0 \). Hence, it can be obtained that \( F_i^0 = 0 \) and \( F_i^{\text{drag}} = -c_i m_i g \).

Substituting (2) into (1), we have

\[
\dot{\dot{F}}_i(t) = -\frac{1}{c_i} \left( m_i a_i(t) + \frac{\rho A_i c_{d_i} v_i^2(t)}{2 m_i} + c_i \ddot{v}_i(t) \right) + \frac{c_i(t)}{c_i}.
\]

(3)

Combining the differentiation to (1) with (3), it results in

\[
\dot{a}_i(t) = -\frac{1}{c_i} \left( a_i(t) + \frac{\rho A_i c_{d_i} v_i^2(t)}{2 m_i} + c_i g \right)
- \frac{\rho A_i c_{d_i} v_i(t) a_i(t)}{m_i} + \frac{c_i(t)}{c_i m_i}.
\]

(4)

Therefore, the third-order dynamics for the \( i \)-th vehicle is formulated as follows:

\[
\begin{align*}
\dot{p}_i(t) &= \dot{v}_i(t), \\
\dot{v}_i(t) &= a_i(t), \\
\dot{a}_i(t) &= f_i(v_i(t), a_i(t)) + g_i c_i(t),
\end{align*}
\]

(5)

where

\[
f_i(v_i(t), a_i(t)) = -\frac{1}{c_i} \left( a_i(t) + \frac{\rho A_i c_{d_i} v_i^2(t)}{2 m_i} + c_i g \right)
- \frac{\rho A_i c_{d_i} v_i(t) a_i(t)}{m_i},
\]

(6)

\[
g_i = \frac{1}{c_i m_i}.
\]

In order to deal with the nonlinear model, we linearize it by employing precise feedback linearization [30];

\[
c_i(t) = u_i(t) m_i + \frac{\rho A_i c_{d_i} v_i^2(t)}{2} + c_i g + c_i \rho A_i c_{d_i} v_i(t) a_i(t).
\]

(7)

According to (5) and (7), we have

\[
\dot{a}_i(t) = -\frac{1}{c_i} a_i(t) + \frac{1}{c_i} u_i(t).
\]

(8)

After discretizing the system with sampling period \( T \), we obtain

\[
x_i(k + 1) = A_i x_i(k) + B_i u_i(k),
\]

(9)

where \( x_i(k) = [p_i(k), v_i(k), a_i(k)]^T \in \mathbb{R}^3 \); \( u_i(k) \in \mathbb{R} \) is the control input; \( t(k + 1) - t(k) = T \);

\[
A_i = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ 0 & 0 & -\frac{T}{c_i} + 1 \end{bmatrix},
\]

(10)

\[
B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{T}{c_i} \end{bmatrix}.
\]

In order to improve the driving performance, multiple constraints are considered for each vehicle in the platoon as follows:

1. Control constraints: \( u_{\text{min}} \leq u_i \leq u_{\text{max}} \), where \( u_{\text{min}} < 0 \) and \( u_{\text{max}} > 0 \) are bounds of control input for each vehicle. For the convenience of subsequent description, we define the compact set \( U_i \) to represent the control constraints.

2. Velocity constraints: \( v_{\text{min}} \leq v_i \leq v_{\text{max}} \), where \( 0 \leq v_{\text{min}} < v_{\text{max}} \) are bounds of longitudinal velocity for each vehicle.

3. Acceleration constraints: \( a_{\text{min}} \leq a_i \leq a_{\text{max}} \), where \( a_{\text{min}} > 0 \) and \( a_{\text{max}} < 0 \) are bounds of acceleration/deceleration for each vehicle.

4. Jerk constraints: \( \Delta a_{\text{min}} \leq \Delta a_i \leq \Delta a_{\text{max}} \), where \( \Delta a_{\text{min}} < 0 \) and \( \Delta a_{\text{max}} > 0 \) are bounds of jerk for each vehicle.

These constraints can be expressed as a set of linear inequalities:

\[
F_i x_i + G_i u_i \leq H_i,
\]

(11)

where
In practice, there exist some relationships among these constraints. Particularly, for the relationships between the vehicle’s velocity and acceleration, define $\bar{a}_i$ and $\underline{a}_i$ as the boundaries of the acceleration $a_i$. Then, for a certain velocity $v_i$, we have that

\[
a_i \in [\bar{a}_i(v_i), \underline{a}_i(v_i)],
\]

where $\bar{a}_i(v_i)$ and $\underline{a}_i(v_i)$ are both the functions of the velocity $v_i$. Since $v_i$ is always bounded by $[v_{\text{min}}, v_{\text{max}}]$, we can obtain that

\[
\bar{a}_i(v_i) \in [\overline{f}_1(v_i), \overline{f}_2(v_i)], \quad \forall v_i \in [v_{\text{min}}, v_{\text{max}}],
\]

\[
\underline{a}_i(v_i) \in [\underline{f}_1(v_i), \underline{f}_2(v_i)], \quad \forall v_i \in [v_{\text{min}}, v_{\text{max}}],
\]

where $[\overline{f}_1(v_i), \overline{f}_2(v_i)]$ and $[\underline{f}_1(v_i), \underline{f}_2(v_i)]$ are the boundaries of $\bar{a}_i(v_i)$ and $\underline{a}_i(v_i)$, respectively. On this basis, we choose the constraint of $a_i$ among the range $[a_{\text{min}}, a_{\text{max}}]$, $\forall v_i \in [v_{\text{min}}, v_{\text{max}}]$, where $a_{\text{min}} = \inf(\overline{f}_1(v_i))$ and $a_{\text{max}} = \sup(\underline{f}_2(v_i))$. The error model for the $i$th follower can be described as follows:

\[
\delta_i(k + 1) = A_i \delta_i(k) + B_i \delta_i(k),
\]

where $\delta_i(k) = x_i(k) - x_{i,\text{des}}(k) = [p_i(k) - p_0(k) + id_{k}, v_i(k) - v_0(k), a_i(k) - a_0(k)]^T$ and $\delta_i(k) = u_i(k) - u_{i,\text{des}}(k)$. $d_{k}$ is the desired distance between two adjacent vehicles.

It can be obtained that

\[
u_{i,\text{des}}(k) = \left(\frac{\xi}{\zeta_0} + 1\right) u_0(k) + \frac{\xi}{\zeta_0} u_0(k),
\]

where $x_{i,\text{des}}(k)$ and $u_{i,\text{des}}(k)$ are the desired state and control trajectories for the $i$th follower according to the leader.

For the error model, the set of linear inequalities representing the multiple constraints in (11) should be modified as follows:

\[
F_i \delta_i + G_i u_i \leq H_i(k),
\]

where

\[
F_i = \begin{bmatrix}
0 & -1 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \\
0 & 1 & 0 \\
0 & T & \xi \\
0 & -\Delta a_{\text{min}} & \Delta a_{\text{max}} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
G_i = \begin{bmatrix}
\xi \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix},
\]

\[
H_i = \begin{bmatrix}
-v_{\text{min}} \\
v_{\text{max}} \\
-a_{\text{min}} \\
a_{\text{max}} \\
-\Delta a_{\text{min}} \\
\Delta a_{\text{max}} \\
u_{\text{min}} \\
u_{\text{max}}
\end{bmatrix},
\]

\[
\tilde{H}_i(k) = \begin{bmatrix}
-(v_{\text{min}} - v_0(k)) \\
v_{\text{max}} - v_0(k) \\
-a_{\text{min}} - a_0(k) \\
a_{\text{max}} - a_0(k) \\
-\Delta a_{\text{min}} - \frac{T}{\zeta_0} a_0(k) + \frac{T}{\zeta_0} u_0(k) \\
\Delta a_{\text{max}} - \frac{T}{\zeta_0} a_0(k) + \frac{T}{\zeta_0} u_0(k) \\
u_{\text{min}} - u_{i,\text{des}}(k) \\
u_{\text{max}} - u_{i,\text{des}}(k)
\end{bmatrix}.
\]

Remark 1. In practice, there exist some relationships among these constraints. Particularly, for the relationships between the vehicle’s velocity and acceleration, define $\bar{a}_i$ and $\underline{a}_i$ as the boundaries of the acceleration $a_i$. Then, for a certain velocity $v_i$, we have that

The objectives of vehicle platoon control are to track the leader and maintain the desired distance between two consecutive vehicles subject to multiple constraints and communication delays. That is,
Lemma 1. The matrix \( P_i > 0 \) is the unique positive definite solution of the Lyapunov matrix equation:

\[
\begin{align*}
\lim_{k \to \infty} & \| v_i(k) - y_i(k) \| = 0, \\
\lim_{k \to \infty} & \| p_i(k) - p_i(k) - d_i \| = 0, \\
& \quad i = 1, \ldots, N_a.
\end{align*}
\]

2.2. Communication Topology. The communication topology among vehicles can be characterized as a weighted directed graph \( G = (V, E) \), where \( V = \{0, 1, 2, \ldots, N_a\} \) is the set of vehicles and \( E \subseteq V \times V \) is the collection of all directed edges between two connected vehicles. Let \( A = [a_{ij}] \in \mathbb{R}^{(N_a+1) \times (N_a+1)} \) be the adjacency matrix of \( G \). If there is a directed edge from node \( j \) to \( i \), then \( a_{ij} > 0 \); otherwise, \( a_{ij} = 0 \), \( i, j \in \{0, 1, 2, \ldots, N_a\} \). The neighbor follows set of the \( i \)th vehicle is denoted by \( N_i = \{ j | a_{ij} > 0, j \in \{1, 2, \ldots, N_a\} \} \). Meanwhile, we define the set \( C_i = \{ j | a_{ij} > 0, j \in \{1, 2, \ldots, N_a\} \} \), meaning that the \( i \)th vehicle can send its information to followers in the set.

Assumption 2. There is a spanning tree in graph \( G \), in which the root node of this spanning tree is the leader vehicle.

Assumption 3. Suppose that the leader can plan its desired trajectory in advance and send this trajectory to followers through communication topology.

Remark 2. Assumption 3 is necessary due to that the DMPC algorithm needs to make optimization by using the desired states in the receding horizon.

3. Delay-Involved DMPC-Based Vehicle Platoon Algorithm

This section introduces the formulation of DMPC for the vehicle platoon subject to multiple constraints and communication delays.

For the error model in (15), the pair \((A_i, B_i)\) is stabilizable. Hence, there exists a feedback control law \( \tilde{u}_i(k) = K_i \tilde{x}_i(k) \) to make sure that \( A_i + B_i K_i \) is stable.

Lemma 1. The matrix \( P_i > 0 \) is the unique positive definite solution of the Lyapunov matrix equation:

\[
P_i = (A_i + B_i K_i)^T P_i (A_i + B_i K_i) + Q_i + K_i^T R_i K_i,
\]

where \( Q_i \) and \( R_i \) are positive definite matrices. Then, there exists a constant \( \varepsilon_i \) such that \( \Omega_i (\varepsilon_i) \triangleq \| x_i(k) - \tilde{x}_i(k) \| \leq \varepsilon_i^2 \) is a control invariant set with the control law \( \tilde{u}_i(k) = K_i \tilde{x}_i(k) \) to ensure that \( \tilde{u}_i(k) \) and \( \tilde{x}_i(k+1) = (A_i + B_i K_i) \tilde{x}_i(k) \) satisfy with the multiple constraints in (17), where \( V_i (\tilde{x}_i(k)) \triangleq \| \tilde{x}_i(k) \|_2 \).

The detailed proof is shown in the Appendix.

Remark 3. For the error model, the multiple constraints in (17) represent the satisfaction of (11). The feedback control law \( u_i(k) = K_i \tilde{x}_i(k) + u_{i, \text{des}}(k) \in U_i \) is applied to the system in (9).

Assumption 4. Suppose that the communication delays are bounded, i.e., \( \tau = n^T, n^* \in \mathbb{R} \) and \( 0 \leq n^* \leq N - 1 \), where \( N \) is the length of predictive horizon used in DMPC. And the vehicle equipment has a storage function.

In order to describe the communication delays, the time domain is divided by the time instants \( k, k = 0, 1, \ldots \). Assume that at time \( k \), all the followers generate the control signals simultaneously and send their information to other followers that are connected with them. At the time \( k+1 \), each follower measures its system state. However, the communication delays occur in the process of transmitted information among the vehicle communication topology. As a result, each follower may not be able to receive its neighbor's information at the time \( k+1 \).

The communication delays are time-varying. For the \( i \)th vehicle, the communication delay of the transmitted information from its neighbor the \( j \)th vehicle to the \( i \)th vehicle is denoted as \( \tau_{ij}^k = n_i^* T \) at time \( k \). Due to the communication delay, the information of the \( j \)th vehicle is received by the \( i \)th vehicle at time \( k + 1 + n_i^* \). Then, define \( n_{ij}^k = \max \{n_{ij} | i, j \} \) as the maximum communication delay for the \( i \)th vehicle receiving all the neighbors' information and the time is \( k + 1 + n_{ij}^k \). We further define the communication delays as \( n_{ij}^k = \max \{n_{ij} | i, j \} \) to make sure the synchronization for all the followers, where \( n_{ij}^k \) denotes the smallest integer more than \( n_{ij}^k \). At time \( k + 1 + n_{ij}^k \), all the followers can receive their neighbors' information and generate the control signals simultaneously.

Remark 4. \( k \) is the time that can generate the control signal through the DMPC-based optimization problem. Note that due to the communication delays, the next time for the optimization problem is not \( k + 1 \), but \( k + 1 + n_{ij}^k \). Meanwhile, the last time for the optimization problem is not \( k - 1 \) similarly. Define the last time for the optimization problem as \( \tilde{k} \) and the communication delays as \( n_{ij}^k \); then it can be obtained that \( k = \tilde{k} + 1 + n_{ij}^k \).

Remark 5. At the synchronization time when all the followers can receive their neighbors' information, followers can also receive the trajectory plan for twice time length of the receding horizon from the leader. Then, the leader makes new plan for another twice time length of the receding horizon.

For the \( i \)th follower at time \( k \), the vehicle platoon with multiple constraints under DMPC scheme can be described in the following optimization problem.

Problem 1. The optimization problem is given by

\[
\min_{U_i(k)} J_i(x_i(k), u_i, x^0_i, k, x^{i+1}_i \cdot [k]).
\]
subject to (for \( p = 0, 1, \ldots, N - 1 \))
\[
x_i(k + p + 1 | k) = A_i x_i(k + p | k) + B_i u_i(k + p | k),
\]  
(21b)  
\[
x_i^a(k + p + 1 | k) = A_i x_i^a(k + p | k) + B_i u_i^a(k + p | k),
\]  
(21c)  
\[
x_i^a(k + p + 1 | k) = A_i x_i^a(k + p | k) + B_i u_i^a(k + p | k),
\]  
(21d)

\[
J_i(x_i(k), u_i, x_i^a, x_i^{a_i}) = \sum_{p=0}^{N-1} \left( \| x_i(k + p | k) - x_{i,des}(k + p) \|_{Q_i}^2 + \| u_i(k + p | k) - u_{i,des}(k + p) \|_{K_i}^2 + \| x_i(k + p | k) - x_i^a(k + p | k) \|_{M_i}^2 \right) + a_{ij} \sum_{p \notin K_i} \left( \| x_i(k + p | k) - x_j^a(k + p | k) - d_{ij} \|_{N_i}^2 \right) + \| x_i(k + N | k) - x_{i,des}(k + N) \|_{P_i}^2,
\]  
(22)

\( U_i(k) = [u_i(k | k), u_i(k + 1 | k), \ldots, u_i(k + N - 1 | k)] \) denotes the unknown variables to be optimized; \( U_i^*(k) = [u_i^*(k | k), u_i^*(k + 1 | k), \ldots, u_i^*(k + N - 1 | k)] \) is the optimal control trajectory; \( x_i(k | k) = x(k) \); \( d_{ij} = [(j - i)d_{\alpha}, 0, 0]^T \); \( N_i \) is the set of neighbors of the \( i \)th follower; \( \varepsilon_j \) is the constant determined in Lemma 1;

\[
u_i^a(k + p | k) = \begin{cases} u_i^*(k + p | \bar{k}), & p = 0, \ldots, N - 2 - n_k, \\ K_i x_i(k + p | k) + u_{i,des}(k + p), & p = N - 1 - n_k, \ldots, N - 1. \end{cases}
\]
(23)

where \( u_i^*(k + p | \bar{k}) \) and \( u_i^*(k + p | \bar{k}) \) are the optimal control trajectories at time \( \bar{k} \); \( x_i^a(k + p | k) = x_i^a(k + p | k) - x_{i,des}(k + p) \); \( x_i^a(k + p | k) = x_i^a(k + p | k) - x_{i,des}(k + p) \).

**Remark 6.** In (21a), \( x_i(k + p | k) - x_{i,des}(k + p) \) and \( u_i(k + p | k) - u_{i,des}(k + p) \) represent the state errors and input errors from the desired equilibrium. \( x_i(k + p | k) - x_i^a(k + p | k) \) means that vehicle \( i \) tries to maintain its assumed trajectory. \( x_i(k + p | k) - x_i^a(k + p | k) - d_{ij} \) is the error between vehicle \( i \) and the assumed trajectory of its neighbor \( j \).

At time \( k + p, p = 0, \ldots, n_k \), the control input \( u_i(k + p) = u_i^*(k + p | k) \) is applied and then the optimization problem at \( k + 1 + n_k \) is constructed. For more details, the control process of the \( i \)th follower is shown in Figure 1. In this figure, the control inputs for time \( k \) to \( k + n_k \) are from the optimal control trajectory at time \( k \), and the next optimization problem is constructed at time \( k + 1 + n_k \).

For each follower, when the follower states are outside the terminal set \( \Omega_i(\varepsilon_i) \), the control input signal is applied according to the optimization problem; when the follower states enter the terminal set \( \Omega_i(\varepsilon_i) \), the stabilizing state feedback law \( u_i(k) = K_i x_i(k) + u_{i,des}(k) \) is applied.

The delay-involved DMPC algorithm is detailed in Algorithm 1.

### 4. Feasibility and Stability Analyses

#### 4.1. Feasibility Analysis
In order to prove the iterative feasibility by the induction principle, Problem 1 needs to be feasible at the initial time instant \( k = 0 \), i.e., there exists a control trajectory driving the initial state into the terminal set while satisfying all the constraints. This requirement can be fulfilled by choosing an appropriate prediction horizon \( N \).

**Assumption 5.** At time \( k = 0 \) with the initial state \( x_i(0) \), there exists a prediction horizon \( N \) such that Problem 1 has a solution.

**Theorem 1.** Suppose that Assumptions 4 and 5 hold. The proposed distributed DMPC-based vehicle platoon scheme
with multiple constraints and communication delays is iteratively feasible.

The key point of Theorem 1 is to show that $\tilde{u}_i(k + 1 + n_k + p | k + 1 + n_k), p = 0, \ldots, N - 1$ is a feasible control trajectory at time $k + 1 + n_k$ satisfying all constraints. The proof is provided in the Appendix.

When the follower states enter the terminal set, the local state feedback control $u_i(k) = K_i \tilde{x}_i(k) + u_{i,des}(k)$ is applied, which guarantees the multiple constraints.

Finally, the feasibility of the proposed approach is guaranteed.

\[ J_i(x_i(k + 1 + n_k), u^*_i(k + 1 + n_k), x^0_i, x^*_i) \leq J_i(x_i(k + 1 + n_k), \tilde{u}_i, x^0_i, x^*_i). \] (24)

4.2. Stability Analysis. The stability analysis is divided into two parts. When the follower states are outside the terminal set, the sum of the optimal control objective functions of all the followers will prove to be an appropriate Lyapunov function; when the follower states enter the terminal set, the local Lyapunov function in Lemma 1 can be used.

When the follower states are outside the terminal set, the feasible control trajectory is generated as (A.3) at time $k + 1 + n_k$. It can be obtained that
Define \( \Delta_i = f_i(x_i(k + 1 + n_k), u_{i}^{*}, x_{i}^{a}, x_{i}^{d}) - f_i(x_i(k), u_{i}^{*}, x_{i}^{a}, x_{i}^{d}) \); then it can be obtained that

\[
\Delta_i \leq f_i(x_i(k + 1 + n_k), \bar{u}_i, x_i^{a}, x_i^{d}) - f_i(x_i(k), u_{i}^{*}, x_{i}^{a}, x_{i}^{d})
\]

\[
= \sum_{p=0}^{N-1} \left( \|x_i(k + 1 + n_k + p) - x_{i,des}(k + 1 + n_k + p)\|_{\mathcal{Q}_i}^2 + \|\bar{u}_i(k + 1 + n_k + p) - u_{i,des}(k + 1 + n_k + p)\|_{R_i}^2 \right) \\
+ \|x_i(k + 1 + n_k + p) - x_{i,des}(k + 1 + n_k + p)\|_{M_i}^2 + a_{i,j} \sum_{j \in \mathcal{A}_i} \|x_j^*(k + p) - x_{j,des}(k + p)\|_{\mathcal{Q}_i}^2 - \|u_{i,des}(k + p)\|_{R_i}^2 \\
- \|x_i^*(k + p) - x_{i,des}(k + p)\|_{M_i}^2 - a_{i,j} \sum_{j \in \mathcal{A}_i} \|x_j^*(k + p) - x_{j,des}(k + p)\|_{N_i}^2 \\
+ \|x_i^*(k + 1 + n_k + p) - x_{i,des}(k + 1 + n_k + p)\|_{R_i}^2 - \|x_i^*(k + 1 + n_k + p)\|_{R_i}^2 \\
+ a_{i,j} \sum_{j \in \mathcal{A}_i} \|x_j^*(k + 1 + n_k + p) - x_{j,des}(k + 1 + n_k + p) - d_{i,j}\|_{N_i}^2 \\
- a_{i,j} \sum_{j \in \mathcal{A}_i} \|x_j^*(k + p) - x_{j,des}(k + p)\|_{\mathcal{Q}_i}^2 + \|u_{i,des}(k + p)\|_{R_i}^2 + a_{i,j} \sum_{j \in \mathcal{A}_i} \|x_j^*(k + p) - x_{j,des}(k + p) - d_{i,j}\|_{N_i}^2 \\
+ a_{i,j} \sum_{j \in \mathcal{A}_i} \|x_j^*(k + 1 + n_k + p) - x_{j,des}(k + 1 + n_k + p) - d_{i,j}\|_{N_i}^2 \\
- \sum_{p=0}^{N-1} \|x_i^*(k + p) - x_{i,des}(k + p)\|_{M_i}^2 \\
+ \|x_i^*(k + 1 + n_k + N) - x_{i,des}(k + 1 + n_k + N)\|_{P_i}^2 \\
- \|x_i^*(k + n) - x_{i,des}(k + N)\|_{P_i}^2
\]

(25)

**Lemma 2.** Due to \( P_i = (A_i + B_i K_i)^T P_i (A_i + B_i K_i) + Q_i + K_i^T R_i K_i \), it can be obtained that

\[
\varphi_i = \sum_{p=0}^{N-1} \left( \|x_i(k + 1 + n_k + p) - x_{i,des}(k + 1 + n_k + p)\|_{\mathcal{Q}_i}^2 + \|\bar{u}_i(k + 1 + n_k + p) - u_{i,des}(k + 1 + n_k + p)\|_{R_i}^2 \right) \\
+ \|x_i(k + 1 + n_k + p) - x_{i,des}(k + 1 + n_k + p)\|_{P_i}^2 - \|x_i^*(k + n + N) - x_{i,des}(k + N)\|_{P_i}^2 = 0.
\]

(26)
The detailed proof is presented in the Appendix. In order to calculate the upper bound of $\Delta_i$, we have

$$\sum_{p=N-n_k}^{N-1} \left\| x_i(k + 1 + n_k + p | k + 1 + n_k) - x_i^0(k + 1 + n_k + p) \right\|_{N_i} \leq \sum_{p=N-n_k}^{N-1} \left( \left\| \bar{x}_i(k + 1 + n_k + p | k + 1 + n_k) - x_{i,des}(k + 1 + n_k + p) \right\|_{N_i} \right)$$

$$= \left( n_k + 1 \right) \left( \frac{\lambda(N_i^{0.5})}{\lambda(P_i^{0.5})} \xi_i + \frac{\lambda(N_j^{0.5})}{\lambda(P_j^{0.5})} \xi_j \right).$$

For $p = 0, \ldots, n_k$, $x_i^*(k + p|k) - x_{i,des}(k + p) \notin \Omega_i(\varepsilon_i)$, it can be obtained that

$$-\sum_{p=0}^{n_k} \left\| x_i^*(k + p|k) - x_{i,des}(k + p) \right\|^2_{Q_i} \leq -\left( n_k + 1 \right) \frac{\lambda(Q_i)}{\lambda(P_i)} \xi_i^2.$$

$$a_{ij} \sum_{p=0}^{n_k} \sum_{j \in N_i} \left\| x_i^*(k + 1 + n_k + p | k + 1 + n_k) - x_j^0(k + 1 + n_k + p | k + 1 + n_k) - d_{ij} \right\|_{N_i}$$

$$- a_{ij} \sum_{p=n_k+1}^{N-1} \sum_{j \in N_i} \left\| x_i^*(k + p | k) - x_j^0(k + p | k) - d_{ij} \right\|_{N_i}$$

$$= a_{ij} \sum_{p=n_k+1}^{N-1} \sum_{j \in N_i} \left\| x_i^*(k + p | k) - x_j^0(k + p | k) - d_{ij} \right\|_{N_i} - a_{ij} \sum_{p=n_k+1}^{N-1} \sum_{j \in N_i} \left\| x_i^*(k + p | k) - x_j^0(k + p | k) - d_{ij} \right\|_{N_i}$$

$$\leq a_{ij} \sum_{p=n_k+1}^{N-1} \sum_{j \in N_i} \left\| x_i^*(k + p | k) - x_j^0(k + p | k) \right\|_{N_i}.$$

**Theorem 2.** For all followers, suppose that Assumptions 1–5 hold. If $M_i > a_{ij} \sum_{j \in N_i} N_{ij}$ for $i = 1, \ldots, N_a$, then for $i = 1, \ldots, N_a$,

$$\frac{\lambda(Q_i)}{\lambda(P_i)} \xi_i^2 \geq a_{ij} \sum_{j \in N_i} \left[ \frac{\lambda(N_i^{0.5})}{\lambda(P_i^{0.5})} \xi_i + \frac{\lambda(N_j^{0.5})}{\lambda(P_j^{0.5})} \xi_j \right].$$

is guaranteed and the communication delays are bounded as $\tau \leq (N - 1)\tau_T$. The sum of all followers’ objective function is strictly monotonically decreasing. Then, the states of each follower outside the terminal set $\Omega_i(\varepsilon_i)$ will enter the set finally with the delay-involved DMPC algorithm.

We will prove that the optimal value of the cost function can be qualified as a Lyapunov function such that the followers’ states will enter the terminal set. The detailed proof is presented in the Appendix. When the follower states enter the terminal set, the stability of the vehicle platoon system is proven by the local Lyapunov function:

$$V_i(\bar{x}_i(k)) = \left\| \bar{x}_i(k) \right\|^2_{P_i}.$$

According to Lemma 1, we have

$$V_i(\bar{x}_i(k + 1)) - V_i(\bar{x}_i(k)) \leq -\bar{x}_i^T(k) (Q_i + K_i^T R_i K_i) \bar{x}_i(k).$$

By combining the two parts including outside the terminal set and in the terminal set, we prove the stability of the delay-involved DMPC strategy in vehicle platoon control.
5. Simulation and Experiment

5.1. Numerical Simulation. A simulation with 6 vehicles is provided to verify the proposed approach. Index the vehicles as 0, 1, . . . , 5, where 0 denotes the leader and 1, . . . , 5 are the followers. The communication topology is that each follower can communicate with its predecessor and the leader except follower 1, which can only communicate with the leader.

For heterogeneous vehicles, we choose different $c_i$ as follows: $c_0 = c_1 = 0.5$, $c_2 = 0.6$, $c_4 = 0.4$, $c_5 = 0.2$, and $c_3 = 0.8$. And the multiple constraints are bounded by $u_i(k) \in [-25, 25]$, $v_i(k) \in [0, 8]$, $a_i(k) \in [-6, 6]$, and $\Delta a_i(k) \in [-3, 3]$.

The parameters of objective functions are set as $Q_i = 4I$, $R_i = 0.1$, $N_i = 0.1I$, $M_i = I$, and $a_{ij} = 0.01$. The local state feedback gains for heterogeneous vehicles are designed as $K_1 = [-3.34, -6.59, -3.51]$, $K_2 = [-3.65, -7.26, -4.02]$, $K_3 = [-2.93, -5.76, -2.90]$, $K_4 = [-1.73, -3.37, -1.24]$, and $K_5 = [-4.10, -8.25, -4.84]$. According to Lemma 1, the related matrices $P_i$ are determined as follows:

$$
P_1 = \begin{bmatrix} 78.98 & 54.03 & 5.99 \\ 54.03 & 95.30 & 11.23 \\ 5.99 & 11.23 & 6.47 \end{bmatrix},
$$

$$
P_2 = \begin{bmatrix} 79.46 & 54.95 & 6.57 \\ 54.95 & 97.10 & 12.39 \\ 6.57 & 12.39 & 7.18 \end{bmatrix},
$$

$$
P_3 = \begin{bmatrix} 78.54 & 53.17 & 5.45 \\ 53.17 & 93.63 & 10.16 \\ 5.45 & 10.16 & 5.83 \end{bmatrix},
$$

$$
P_4 = \begin{bmatrix} 77.83 & 51.83 & 4.62 \\ 51.83 & 91.04 & 8.52 \\ 4.62 & 8.52 & 4.93 \end{bmatrix},
$$

$$
P_5 = \begin{bmatrix} 80.45 & 56.88 & 7.80 \\ 56.88 & 100.93 & 14.90 \\ 7.80 & 14.90 & 8.80 \end{bmatrix}.
$$

The sampling period is given as $T = 0.1$ s. The predictive horizon is $N = 20$, and the terminal set is determined as $\varepsilon_i = 0.3$. The desired spacing is set as $d_0 = 5$ m. The initial states of the leader are set as $p_0(0) = 25$ m, $v_0(0) = 5$ m/s, and $a_0(0) = 0$ m/s$^2$. The initial states of the followers are set as $x_1(0) = [22, 6, 1]^T$, $x_2(0) = [17, 7, 0]^T$, $x_3(0) = [12, 3, 2]^T$, $x_4(0) = [3, 6, 1]^T$, and $x_5(0) = [0, 2, 1]^T$.

The time-varying communication delays are generated as follows:

$$
[1.3, 1.0, 1.7, 1.1, 0.9, 1.5, 1.6, 1.2, 1.9, 0.9, 1.4, 1.8, 1.1, 1.2],
$$

with the bound $r \leq (N - 1)T$ during the simulation time. Based on these parameters, the simulation results are as follows.

The states (including position, velocity, and acceleration) and the related errors between the real and desired values for followers are illustrated in Figures 2–4. The jerks of followers are shown in Figure 5 and the control inputs are presented in Figure 6.

From Figures 2–4, it can be obtained that all states converge to the desired values and the related errors converge to 0. Meanwhile, the velocity constraints ($v_i(k) \in [0, 8]$) and acceleration constraints ($a_i(k) \in [-6, 6]$) are guaranteed. In detail, Figure 2 depicts that the convergence time of the position states is about 3.6 s. The velocities of followers converge to the leader’s velocity (5 m/s) at about 3.8 s in Figure 3 and the accelerations converge to 0 at about 0.4 s according to Figure 4. From Figure 5, the jerk constraints ($\Delta a_i(k) \in [-3, 3]$) are satisfied apparently. As shown in Figure 6, the control input curves reveal that the control inputs converge to 0 and the input constraints ($u_i(k) \in [-25, 25]$) are satisfied. In addition, the feasibility of the proposed strategy is guaranteed. According to the simulation results, the proposed delay-involved DMPC-based vehicle platoon algorithm can drive all vehicles with multiple constraints and communication delays to the desired platoon effectively.

5.2. Experiment with TurtleBot3 Mobile Robots. The TurtleBot3 is a ROS standard platform robot, which can use enhanced 360° LiDAR, 9-Axis Inertial Measurement Unit, and precise encoder for research and development (see Figure 7). The hardware specifications of the mobile robots are shown in Table 1.
For the experiment, three real TurtleBot3 mobile robots are used to verify the practicality and effectiveness of the proposed approach. The initial states of the three mobile robots are set as 
\[ x_0 = [0.90, 0, 0]^T \], \[ x_1 = [0.40, 0, 0]^T \], and \[ x_2 = [0, 0, 0]^T \], which is presented in Figure 8. The sampling period and the desired spacing are set as \( T = 0.2 \) s and \( d_0 = 25 \) cm. The prediction horizon is set as \( N = 5 \). Then the leader runs at 0.05 m/s in the experiment.

Figure 3: Velocities and velocity errors of the vehicles.

Figure 4: Accelerations and acceleration errors of the vehicles.

Figure 5: Jerks of the vehicles.

Figure 6: Control inputs \( u \).
The experiment results are shown in Figures 9–11, where the tb3_0, tb3_1, and tb3_2 denote the leader, follower 1, and follower 2 in the platoon, respectively. Figures 9 and 10 show the positions and velocities of the TurtleBot3 mobile robots. They illustrate that the mobile robots can be driven to the desired platoon using the proposed algorithm. In Figure 10, although the velocities have a little oscillation, they can converge to the leader’s velocity 0.05 m/s at about 20 s. Figure 11 reveals that the distances between each follower and the leader satisfy the desired spacing (0.25 m), which shows the stability of the TurtleBot3 platoon. In summary, the experiment results validate the feasibility and effectiveness of the proposed approaches.

The oscillation of velocity curves might be caused by the measuring errors of the speed sensors or external disturbances. To solve this problem, we will apply a Kalman filter in the experiment to acquire a better performance in our future work.

Table 1: Hardware specifications of TurtleBot3 mobile robots.

|                         | Maximum translational velocity | Maximum rotational velocity | Size (L × W × H) | Weight |
|-------------------------|--------------------------------|-----------------------------|-------------------|--------|
|                         | 0.22 m/s                       | 2.84 rad/s                  | 138 mm × 178 mm × 192 mm | 1 kg   |

Figure 7: TurtleBot3 mobile robot.

Figure 8: TurtleBot3 platoon in the experiment.

Figure 9: Positions of TurtleBot3 mobile robots in the experiment.
6. Conclusion

The platoon control problem for vehicles with multiple constraints and communication delays has been studied by using the dual-mode DMPC strategy in this paper. The heterogeneous decoupled vehicle platoon with multiple constraints constructs an optimization problem for each vehicle. In addition, a delay-involved DMPC algorithm is proposed to deal with the time-varying and bounded communication delays by using the waiting mechanism. The iterative feasibility of the proposed scheme is proven and the stability conditions are provided. Finally, numerical simulation and experiment are presented to verify the feasibility and effectiveness of proposed approaches.

Appendix

Proof of Lemma 1

Proof. According to the definition of $\Omega_i(\varepsilon_i)$, there exists $\varepsilon_i$ such that the set is not nonempty and satisfies (17). Then, it is necessary to prove that $\Omega_i(\varepsilon_i)$ is an invariant set with the control law $\bar{u}_i(k) = K_i\bar{x}_i(k)$:

$$
\bar{u}_i(k + 1 + n_k + p | k + 1 + n_k) = \begin{cases} 
  u_i^*(k + 1 + n_k + p | k), \\
  K_i\bar{x}_i(k + 1 + n_k + p | k + 1 + n_k) + u_{i,\text{des}}(k + 1 + n_k + p),
\end{cases}
$$

where $\bar{x}_i(k + 1 + n_k + p | k + 1 + n_k) = \bar{x}_i(k + 1 + n_k + p | k + 1 + n_k) - x_{i,\text{des}}(k + 1 + n_k + p)$. And the corresponding feasible state trajectory candidate is generated as $\bar{x}_i(k + 1 + n_k + p + 1 | k + 1 + n_k) = A_i\bar{x}_i(k + 1 + n_k + p | k + 1 + n_k) + B_i\bar{u}_i(k + 1 + n_k + p | k + 1 + n_k)$, where the initial state is given as $\bar{x}_i(k + 1 + n_k | k + 1 + n_k) = x_i^*(k + 1 + n_k | k)$.

According to Lemma 1, it can be obtained that

$$
\bar{x}_i(k + N | k + 1 + n_k) = \bar{x}_i^*(k + N | k) \in \Omega_i(\varepsilon_i),
$$

where $\bar{x}_i^*(k + N | k) = x_i^*(k + N | k) - x_{i,\text{des}}(k + N)$.

It results in that when $p = N - 1 - n_k, \ldots, N - 1$, we have $K_i\bar{x}_i(k + 1 + n_k + p | k + 1 + n_k) + u_{i,\text{des}}(k + 1 + n_k + p)$
\[ \varepsilon \in U_i \text{ and } F_i \tilde{x}_i (k+1+n_k+p \mid k+1+n_k) + G_i \tilde{u}_i (k+1+n_k+p \mid k+1+n_k) \leq H_i. \]

It is obvious that \( u_i^*(k+1+n_k+p \mid k) \) and \( x_i^*(k+1+n_k+p \mid k) \) satisfy the multiple constraints when \( p = 0, \ldots, N-2-n_k \). Hence, the feasible control trajectory and its corresponding state trajectory satisfy the constraints in (21e).

Due to (A.4) and the set \( \Omega_i (\varepsilon_i) \) is a control invariant set with the control law \( u_i (k) = K_i \tilde{x}_i (k) + u_{i, \text{des}} (k) \in U_i \), it is obtained that \( \tilde{x}_i (k+1+n_k+N \mid k+1+n_k) \in \Omega_i (\varepsilon_i) \), which means that the terminal constraint (21f) is guaranteed. At time \( k+1+n_k \), there exists a feasible solution to Problem 1.

In conclusion, Problem 1 is iteratively feasible. \( \square \)

Proof of Lemma 2

Proof. According to (A.3), we have

\[ \varphi_i = \sum_{p=0}^{n_k} \left( \| (A_i + B_i K_p)^{T} \tilde{x}_i^*(k+N|k) \|_{Q_i}^2 + \| K_i (A_i + B_i K_p)^{T} \tilde{x}_i^*(k+N|k) \|_{R_i}^2 \right) \]

In order to prove Lemma 2, we need to show that

\[ \sum_{p=0}^{n_k} \left[ (A_i + B_i K_p)^{T} (Q_i + K_i^T R_i K_i) (A_i + B_i K_p) + \left[ (A_i + B_i K_p)^{T} \right] P_i (A_i + B_i K_p)^{T} \right] = 0. \] (A.6)

Due to \( P_i = (A_i + B_i K_p)^{T} P_i (A_i + B_i K_p) + Q_i + K_i^T R_i K_i \), it can be obtained that

\[ \left[ (A_i + B_i K_p)^{T} (Q_i + K_i^T R_i K_i) (A_i + B_i K_p) \right] = \left[ (A_i + B_i K_p)^{T} P_i (A_i + B_i K_p)^{T} \right] + \left[ (A_i + B_i K_p)^{T} \right] P_i (A_i + B_i K_p)^{T} + (A_i + B_i K_p)^{T} P_i (A_i + B_i K_p)^{T} \]

which results in

\[ \sum_{p=0}^{n_k} \left[ (A_i + B_i K_p)^{T} (Q_i + K_i^T R_i K_i) (A_i + B_i K_p) \right] = P_i - \left[ (A_i + B_i K_p)^{T} \right] P_i (A_i + B_i K_p)^{T} + (A_i + B_i K_p)^{T} P_i (A_i + B_i K_p)^{T} \] (A.8)

Hence, (A.6) is guaranteed and then Lemma 2 is proven. \( \square \)

Proof of Theorem 2

\[ \Delta_i \leq \sum_{p=0}^{n_k} \left( \| u_i^*(k+p \mid k) - u_{i, \text{des}} (k+p) \|_2^2 + a_{i,j} \sum_{j \in N_i} \| x_j^* (k+p \mid k) - x_j^* (k+p \mid k) \|_{M_i} \right) \]

\[ + a_{i,j} \sum_{p=n_k+1}^{N_k} \sum_{j \in N_i} \| x_j^* (k+p \mid k) - x_j^* (k+p \mid k) \|_{M_i} - \sum_{p=n_k+1}^{N_k} \sum_{j \in N_i} \| x_j^* (k+p \mid k) - x_j^* (k+p \mid k) \|_{M_i}. \] (A.9)
It can be obtained that

\[
\sum_{i=1}^{N_i} \Delta_i \\
\leq - \sum_{i=1}^{N_i} \sum_{p=0}^{N_i-1} \| \hat{u}_i^* (k+p|k) - u_{i,k} (k+p) \|_{\mathcal{R}} + a_{ij} \sum_{j\in\mathcal{N}_i} \| \hat{x}_j^* (k+p|k) - x_j^a (k+p|k) - d_{ij} \|_{\mathcal{N}_i} + \| \hat{x}_i^* (k+p|k) - x_i^a (k+p|k) \|_{\mathcal{M}} \\
+ \sum_{i=1}^{N_i} a_{ij} \sum_{p=0}^{N_i-1} \sum_{j\in\mathcal{N}_i} \| \hat{x}_j^* (k+p|k) - x_j^a (k+p|k) \|_{\mathcal{N}_i} - \sum_{p=0}^{N_i-1} \| \hat{x}_i^* (k+p|k) - x_i^a (k+p|k) \|_{\mathcal{M}},
\]

(A.10)

where

\[
\sum_{i=1}^{N_i} a_{ij} \sum_{p=0}^{N_i-1} \sum_{j\in\mathcal{N}_i} \| \hat{x}_j^* (k+p|k) - x_j^a (k+p|k) \|_{\mathcal{N}_i} - \sum_{p=0}^{N_i-1} \| \hat{x}_i^* (k+p|k) - x_i^a (k+p|k) \|_{\mathcal{M}} \\
= \sum_{p=0}^{N_i-1} \sum_{j\in\mathcal{N}_i} \| \hat{x}_j^* (k+p|k) - x_j^a (k+p|k) \|_{\mathcal{N}_i} - \| \hat{x}_i^* (k+p|k) - x_i^a (k+p|k) \|_{\mathcal{M}}.
\]

(A.11)

According to \( M_i > a_{ij} \sum_{j\in\mathcal{N}_i} N_j \), it can be obtained that

\[
\sum_{i=1}^{N_i} \Delta_i < - \sum_{i=1}^{N_i} \sum_{p=0}^{N_i-1} \| \hat{u}_i^* (k+p|k) - u_{i,k} (k+p) \|_{\mathcal{R}} + a_{ij} \sum_{j\in\mathcal{N}_i} \| \hat{x}_j^* (k+p|k) - x_j^a (k+p|k) - d_{ij} \|_{\mathcal{N}_i} + \| \hat{x}_i^* (k+p|k) - x_i^a (k+p|k) \|_{\mathcal{M}} \leq 0.
\]

(A.12)

Finally, the sum of optimal control objective functions of all followers is monotonically decreasing and the follower states with the initial states outside \( \Omega_i (e_i) \) can enter the terminal set. \( \Box \)

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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