THz radiation by amplitude-modulated self-focused Gaussian laser beam in ripple density plasma

SUBODH KUMAR, RAM KISHOR SINGH, MONIKA SINGH, AND R. P. SHARMA
Centre for Energy Studies, Indian Institute of Technology Delhi, New Delhi, India
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Abstract

The effect of self-focusing and defocusing on terahertz (THz) generation by amplitude-modulated Gaussian laser beam in rippled density plasma is investigated. A stronger transient transverse current is generated by transverse component of ponderomotive force exerted by laser on electrons that drives radiation at the modulation frequency (which is chosen to be in the THz domain) because of the variation in intensity in the direction transverse to the laser propagation. Numerical simulations indicate the enhancement of THz yield by many folds due to self-focusing of laser beam in comparison with that without self-focusing. The transient focusing of laser beam and its effect on the generated THz amplitude has also been studied.

Keywords: Amplitude modulated Gaussian laser beam; Filamentation; Ponderomotive nonlinearity; Self-focusing; THz radiation

1. INTRODUCTION

Terahertz (THz) frequency regime has become one of the thrust areas of research due to the variety of applications it offers. Application areas include information and communication technology (Hirata et al., 2006; Krumhholz et al., 2006), biology and medical sciences (Brucherseifer et al., 2000; Davis et al., 2002; Han et al., 2002; Wallace et al., 2006), nondestructive evaluation (Kawase et al., 2003; Zhong et al., 2005), homeland security and explosive detection (Shen et al., 2005; Karpowicz et al., 2005; Federici et al., 2005; Kemp et al., 2006; Ying et al., 2006), quality control of food and agricultural products, global environment monitoring (Tonouchi, 2007), ultrafast computing (Clery, 2002), remote sensing (Waters et al., 2006), material characterization (Jiang et al., 2000), and semiconductor studies (Xu, 1997). There is no strict definition for any wave to qualify as THz but the waves having frequencies in between microwave and far infrared of the electromagnetic spectrum are broadly considered as THz radiation. Typically, 0.1–10 THz is referred as THz region but the region is extending up to 40–50 THz (Wang, 2001). A number of schemes have been employed to produce THz radiation (Hafizi et al., 1992; Hashimshony et al., 1999; Muggli et al., 1998; Leemans et al., 2004; Sheng et al., 2005; Alekseev et al., 2006; Ma et al., 2006; Kukushkin, 2008). Currently, the powerful sources of THz radiation are large accelerator-based sources and the lack of compact high-power sources limits the exploitation of full potential of THz radiation. For the development of practical high-power THz sources, schemes based on laser–plasma interaction have been proposed (Hamster et al., 1993; Yugami et al., 2002; Gildenburg & Vvedenskii, 2007; Sharma et al., 2010; Jha et al., 2011; Chen 2013).

Filamentation (phenomena of periodic self-focusing and defocusing) is another process from which THz can be generated. THz radiation can be generated from the plasma filament created by short intense laser pulse. The mechanism of generation has been studied by some researchers (Wu et al., 2011). The generation of broadband THz within the plasma filaments formed by a femtosecond laser has been investigated by Ladouceur et al. (2001). Orders of magnitude enhancement of THz energy has been achieved because of filamentation of a femtosecond laser in air in the presence of a transverse static electric field (Houard et al., 2008). As the plasma is capable of handling high-power, these schemes generate THz radiation with high-electric fields without any physical damage to the medium used. In some studies, THz generation by focusing femtosecond bichromic laser pulses in a gas or plasma has been reported (Chizhov et al., 2013). Experimental studies have also been done to generate intense THz pulses using two color laser filamentation using...
TW lasers. High-energy (>1 μJ), high average power (>1 mW), intensity (>1 MV cm\(^{-1}\)), and broadband (0.01–60 THz) THz via two color filamentations have been reported (Oh et al., 2013). There are several techniques available for the study of self-focusing in the plasma. One of the simplest methods has been given originally by Akhmanov et al. (1968) for the paraxial regime and later developed by Sodha et al. (1980).

The role of density ripple in the plasma in THz generation has been discussed by several researchers (Tripathi et al., 2010; Singh et al., 2013). The importance of density ripple in the plasma has also been discussed by some authors in context of other phenomena taking place in the laser–plasma interaction (Singh et al., 2010; Xia & Xu, 2013; Xia, 2014). Significant amount of research is going into the fabrication of such kind of structure due to their other applications such as in the development plasma photonic devices.

Different spatial laser profiles (such as fundamental Gaussian, cosh-Gaussian, etc.) have been used to generate THz (Tripathi et al., 2010; Singh et al., 2013). As the propagation and focusing properties of Gaussian laser beam in plasma have been studied extensively (Sodha et al., 1974; 1979), it is one of the most preferred beam profiles used in laser– plasma interaction studies.

In the present paper, we have studied the effect of self-focusing on the THz generation by amplitude-modulated Gaussian laser beam propagating in rippled density plasma. Because of the variation in intensity in the direction transverse to the laser propagation, a component of ponderomotive force is exerted on electrons, which gives rise to a stronger transient current driving THz radiation. The ripple in plasma density provides the necessary condition for phase matching (Singh et al., 2013) and plays the main role for the extraction of THz radiation. In the absence of ripple density, the THz cannot be generated by the present scheme. In Section 2, we discuss the self-focusing of an amplitude-modulated Gaussian laser having frequency of modulation in the THz region. In the subsequent section, we derive the nonlinear current density, generated THz amplitude, and intensity. We discuss the results in Section 4 and then conclude.

2. SELF-FOCUSBING OF AMPLITUDE-MODULATED GAUSSIAN LASER BEAM

We consider an amplitude-modulated Gaussian laser (Gaussian in space) beam of frequency \(f_0\), propagating in the \(z\)-direction and polarized along the \(x\)-direction in a collisionless, unmagnetized, hot, and homogeneous plasma. The laser field, at \(z = 0\) is given by

\[
E_0 = \hat{x}E_0(1 + \mu \cos \Omega t) \exp(-x^2/2r_0^2) \exp(i\omega_0 t),
\]

where \(\Omega\) is the frequency of modulation and \(\mu\) is the index of modulation. \(r_0\) is the initial pulse width of the laser. Because of the nonuniform spatial intensity distribution of the laser pulse, a ponderomotive force is exerted on the electrons in the plane transverse to the direction of propagation (\(z\)-direction). The ponderomotive force in the presence of laser pulse is given by Sodha et al. (1976).

\[
F_P = -\frac{e^2}{4m\omega_0^2} \left( \nabla (E_0 \cdot E_0^*) \right).
\]

This ponderomotive force leads to the redistribution of electrons. Assuming \(\Omega \ll \omega_0\), the modified electron density can be written as (Sodha et al., 1980),

\[
N = N_0 \exp \left[ -\frac{e^2}{8m\omega_0^2} (E_0 \cdot E_0^*) \right],
\]

where, \(T_0\) is the temperature of the plasma and \(k_B\) is the Boltzmann constant.

The propagation of laser pulse in the plasma is governed by the wave equation:

\[
\frac{\partial^2 E_0}{\partial z^2} + \frac{\partial^2 E_0}{\partial y^2} + \frac{\partial^2 E_0}{\partial z^2} = \frac{1}{c^2} \frac{\partial^4 E_0}{\partial t^4} + \frac{4\pi e^2 J}{c^2} \frac{\partial J}{\partial t},
\]

where \(J\) is the total current density in the plasma in the presence of laser pulse. Assuming \(E_0\) to be varying in space and time as:

\[
E_0 = \hat{x}A_0(x, z, t) \exp[i(\omega_0 t - k z)],
\]

and

\[
A_0 = \frac{E_0}{f^{1/2}(x, z, t)} \exp \left[ \frac{-x^2}{2r_0^2} - ikS(x, z, t) \right],
\]

where \(f\) and \(S\) are dimensionless beam width parameter and eikonal of the laser beam, respectively. The eikonal shows the slightly converging/diverging behavior of the laser beam in the plasma (Sodha et al., 1974). The eikonal \(S\) is given by \(S = 1/(2\pi^2)\beta(x, z) + \Phi(z, t)\), where \([\beta(x, t)]^{-1}\) and \(\Phi(z, t)\) represent the radius of the curvature of the wavefront and initial phase, respectively. The current density in the plasma can be written as (Sodha et al., 1980)

\[
J \equiv \frac{x}{m\omega_0} \left( A_0 + i \frac{\partial A_0}{\omega_0 \partial t} \right) \exp[i(\omega_0 t - k z)].
\]

Substituting the expressions for \(E_0\) and \(J\) from Eqs. (5) and (7), respectively, in Eq. (4) and using the method adopted by Sodha et al. (1980), we obtain the equation governing the dimensionless beam width parameter \(f\) as

\[
\frac{d^2 f}{d\eta^2} = \left( \frac{\omega_0 n_0}{c} \right)^2 \alpha e^2 f^2 (1 + \mu \cos \Omega \xi \eta)^2 \times \exp \left[ -\frac{\alpha e^2}{f} (1 + \mu \cos \Omega \xi)^2 \right],
\]

where \(\xi = \eta k r_0^2, \xi = t - z/v_p v_e = c t_0^{1/2}\) and \(\alpha = (e^2/(8m\omega_0^2 T_0 r_0^2))\).

Equation (8) is solved numerically to study the self-focusing with initial condition, \(f = 1\) at \(z = 0\).
3. THz RADIATION GENERATION

We consider the propagation of amplitude-modulated Gaussian laser beam whose field is given by Eq. (1), through rippled density plasma. Plasma density is given by

\[ n = n_0 + n'_q, \]

\[ n'_q = n_0 e^{ikqz}, \]

where \( n_0 \) and \( k_q \) are the amplitude and the wave number of the density ripple. This kind of density ripple can be created by different techniques (Hazra et al., 2004; Pai et al., 2005; Layer et al. 2007; Liu et al., 2008). The laser propagates in the \( z \)-direction and polarized in the \( x \)-direction. Laser field is given by Eq. (5) and ponderomotive force is evaluated using Eq. (2) which comes out to be

\[ F_p = -\frac{e^2}{4m\omega_0^2} \nabla \left( \frac{E_{00}^2}{f} \right) (1 + \mu \cos \Omega k q \exp(-x^2/r_0^2^2)). \]  

Using Eqs. (11)–(13), we get

\[ J_{NL}^{\Omega} = \frac{i \mu_0 q e^3 E_{00}^2 e^{-2x^2/\sigma^2}}{4m^2 \omega_0^2} \left[ \frac{2x}{r_0^2} \frac{1}{\Omega} \hat{c} + \sqrt{\frac{\sigma}{r_0^2}} \right] \]

\[ \left\{ \frac{1}{\sqrt{e}} \left( \frac{2\omega^2}{\omega_0^2} - 1 \right) \frac{\hat{c}}{\hat{c}} \right\} \left[ 1 \left( \frac{\Omega}{\omega} - i\Omega \right) \right]. \]  

This nonlinear current gives rise to THz wave at frequency \( \Omega \) and wave number \( \Omega/\upsilon_g + k_q \), which is governed by the wave equation

\[ \nabla^2 E_T - \nabla (\nabla \cdot E_T) + \frac{\Omega^2}{c^2} \in E_T = 4\pi\Omega^2 J_{NL}^{\Omega}, \]

where

\[ \in = 1 - \frac{\omega^2}{\Omega^2}. \]

As the nonlinear current given by Eq. (14), is responsible for the THz generation, the THz field will also vary as \( e^{i\Omega z - kqz} \).

Using Eqs. (14) and (15), the \( x \)-component of Eq. (15) is obtained as

\[ 2ik_T \frac{\partial}{\partial z} E_{T, x} + \left[ \frac{\Omega^2}{c^2} \left( 1 - \frac{\omega^2_q}{\Omega^2} \right) - k_T^2 \right] E_{T, x} \]

\[ \equiv - \frac{1}{2} \left( \frac{n_q}{n_0} \right) \frac{\omega^2_q}{\omega_0^2} \frac{eE_{00}^2}{mc^2 \sigma^2} \left( \frac{r_0^2}{\sigma} \right) e^{-2x^2/\sigma^2} e^{i\Omega/\upsilon_g + k_q} \hat{c}. \]

Using \( k_T = (\Omega/c)(1 - (\omega^2_q/\Omega^2))^{1/2} \), and applying exact phase-matching condition \( k_T = \Omega/\upsilon_g + k_q \), Eq. (17) becomes

\[ \frac{\partial}{\partial z} \left( E_{T, x} \right) = \frac{i}{4} \left( \frac{n_q}{n_0} \right) \frac{\omega^2_q}{\omega_0^2} \frac{eE_{00}^2}{mc^2} \left( \frac{\mu x}{\sigma^2} \right) \left( \frac{r_0^2}{\sigma^2} \right) e^{-2x^2/\sigma^2} e^{i\Omega/\upsilon_g + k_q} \hat{c}. \]

Fig. 1. (a) Variation of beam width parameter with normalized distance along the direction of laser propagation for \( \mu = 0.05 \). (b) Variation of beam width parameter with normalized distance along the direction of laser propagation for \( \mu = 0.1 \).
4. RESULTS AND DISCUSSION

Equation (8) describes the variation in beam width parameter with normalized distance, $\eta$ where, $\eta = z/R_d$ and $R_d = \frac{\omega_0^2}{c}$ is the diffraction length. To solve Eq. (8), the following laser-plasma parameters were used: $\omega_0 = 1.78 \times 10^{14}$, $\omega_p = 2.5 \times 10^{13}$ rad s, $T_0 = 10^6$ K, $n_q = 0.3n_0$, intensity of laser beam, $I = 10^{14}$ W cm$^{-2}$, and initial radius of the laser beam, $r_0 = 40 \mu$m.

The beam width parameter $f$ is plotted with the distance at different times. Plots are shown in Figures 1a and 1b corresponding to modulation indices $\mu = 0.05$ and 0.1, respectively. The beam self-focuses and defocuses in the course of its propagation and the degree of focusing changes with time. Equation (18) was solved numerically and THz amplitude was evaluated at different times and compared with result obtained without considering the effect of self-focusing (i.e., $f = 1$). The amplitude was found to be enhanced significantly when self-focusing occurs. Further the enhancement was found to be time dependent as expected because of transient focusing of laser beam. Figure 2 shows schematic representation for THz radiation generation in the presence of ripple density plasma. Figures 3a and 3b exhibit the radial profile of the generated THz field amplitude (corresponding to values of modulation indices $\mu = 0.05$ and 0.1, respectively) at different times for $n_q = 0.3n_0$. The peak THz field is obtained at different values of transverse distances at different times. THz amplitude depends upon the generated nonlinear current density which in turn depends on the gradient of intensity of the laser beam. Since the laser beam has the spatial Gaussian profile, the gradient of intensity at the propagation axis is zero. It increases along the radial direction and after attaining the maximum value again starts decreasing. Thus the spatial profile of generated THz amplitude will follow the same trend as shown in Figures 3a and 3b. The line shown in black corresponds to the case without self-focusing [$f(z) = 1$, i.e., without focusing/defocusing], whereas colored lines correspond to the case with self-focusing at different times. Figures 4a and 4b show the power spectra of generated THz radiation at different times corresponding to the values of modulation indices 0.05 and 0.1, respectively. The lines shown in black and in colors correspond to the cases without and with self-focusing as before. The enhancement in the generated THz amplitude is primarily due to three factors: Modulation index ($\mu$), ripple density amplitude ($n_q$), and dimensionless beam width parameter ($f$).

Figures 3a and 3b. The line shown in black corresponds to the case without self-focusing [$f(z) = 1$, i.e., without focusing/defocusing], whereas colored lines correspond to the case with self-focusing at different times. Figures 4a and 4b show the power spectra of generated THz radiation at different times corresponding to the values of modulation indices 0.05 and 0.1, respectively. The lines shown in black and in colors correspond to the cases without and with self-focusing as before. The enhancement in the generated THz amplitude is primarily due to three factors: Modulation index ($\mu$), ripple density amplitude ($n_q$), and dimensionless beam width parameter ($f$).

The efficiency of THz generation that is, the ratio of the energy of the generated THz to the energy of the laser used for the generation, which comes out to be of the order of $\sim 10^{-4}$ for the parameters used. As amplitude of the generated THz depends on $\mu$, $n_q$, and $f$, the choice of their values will

![Fig. 2. Schematic representation for THz radiation generation in the presence of ripple density plasma.](https://doi.org/10.1017/S026303461500035X)

![Fig. 3. (a) Radial profile of THz amplitude for the parameters same as those used for Figure 1a. (b) Radial profile of THz amplitude for the parameters same as those used for Figure 1b.](https://doi.org/10.1017/S026303461500035X)
play an important role in optimizing the conversion efficiency to the best possible value. It is a matter of choosing these parameters as realistically as possible. Some of the best conversion efficiencies reported so far are $\sim 10^{-5}$ (Hamster et al., 1993), $\sim 5 \times 10^{-4}$ (Wang et al., 2011), $\sim 10^{-4}$ (Chen, 2013b), and $\sim 10^{-5}$ (Wu et al., 2007). Thus, the efficiency achieved in the proposed scheme is of the order of those achieved by other researchers using other mechanisms.

5. CONCLUSION

We can conclude that amplitude-modulated Gaussian laser beam self-focuses and defocuses when it propagates in the plasma. In the absence of density ripple, the phase-matching condition for the generated THz wave is not satisfied and hence the generated wave will not be able to propagate even if the dispersion relation is satisfied. The appropriate phase-matching condition is provided by the ripple density plasma. Accordingly, focusing changes with time and THz amplitude get enhanced. We observe significant enhancement in the THz amplitude in comparison with that generated without using self-focusing of the laser. Comparison of the predicted conversion efficiency with the best reported values shows that the THz generation by self-focusing of amplitude-modulated Gaussian laser beam can be a potential scheme for the efficient generation of the radiation.

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Fig. 4. (a) Power spectra of THz radiation for the parameters same as used for Figure 1a. (b) Power spectra of THz radiation for the parameters same as used for Figure 1b.
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