Extended Nambu-Jona-Lasinio Model vs QCD Sum Rules

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Abstract

We argue that the extended Nambu-Jona-Lasinio model of hadrons can be probed and constrained in a nontrivial way via QCD sum rules. While there arise rather restrictive bounds on the strength of the effective four-fermion interaction in the vector channel, introduction of the four-fermion interaction in the pseudoscalar channel could resolve a long standing puzzle of QCD sum rules. We speculate also on possible connection between effective four-quark interactions describing the low-energy phenomenology and ultraviolet renormalons of the fundamental QCD.

1On leave of absence from Max-Planck Institute for Physics, Munich
1 Introduction

More than thirty years ago Nambu and Jona-Lasinio proposed a model of superconductivity type as an effective theory of hadrons at low energies [1]. Advantages of the model are its simplicity and elucidation of the mechanism of spontaneous breaking of the chiral symmetry. Since then the model has developed into a rich phenomenology of hadrons up to mass scale of order 1 GeV (for a review see, e.g., Ref.[2]). Moreover, the model has inspired introduction of similar ideas to describe hypothetical interactions at various mass scales (for a review see, e.g., Ref.[3]).

The basic feature of (the extended version) of the NJL model is the introduction of effective four-fermion interactions. The form of the interaction is constrained by the chiral invariance of the underlying fundamental Lagrangian and is parametrized in terms of two couplings $G_{S,V}$:

$$L_{\text{int}} = \frac{G_S}{2} \left( (\bar{q} \lambda^\alpha q)^2 + (\bar{q} i \gamma_5 \lambda^\alpha q)^2 \right) - \frac{G_V}{2} \left( (\bar{q} \gamma_\mu \lambda^\alpha q)^2 + (\bar{q} \gamma_\mu \gamma_5 \lambda^\alpha q)^2 \right),$$

(1)

where $\lambda^\alpha$ are the Gell-Mann SU(3) matrices in the flavour space and the colour indices are suppressed.

Loop integrations with Lagrangian (1) are allowed only up to an ultraviolet cutoff $\Lambda_{UV}$, so that (1) represents a low energy effective interaction. To get feeling of the parameters involved one may keep in mind the following estimates (see, e.g., Ref.[2]):

$$\Lambda_{UV} \approx 1.25 \text{ GeV}, \ G_S \approx 5 \text{ GeV}^{-2}, \ G_V \approx 10 \text{ GeV}^{-2}. \quad (2)$$

The fits may somewhat vary, however. It is worth noting that the scalar type interaction is responsible for the spontaneous breaking of the chiral symmetry, while the function of the vector type interaction is to generate spin-1 mesons. The modern way [4] to confront the NJL model (1) with the data is to calculate the parameters of the low-energy chiral Lagrangians classified in Ref.[5] at low energies.
A crucial problem is whether the effective Lagrangian (1) could be derived from the fundamental QCD Lagrangian. This problem has been addressed in a number of papers, see in particular review in Ref.[6]. One may not hope at the moment to derive analytically the effective Lagrangian, since the QCD coupling is strong at low energies. Thus the common lore does not extend too far beyond the one-gluon exchange.

Weakness of the standard one-gluon-exchange picture [6] is, to our mind, that it does not give any hint as to why the gluon line is harder than the quark ones so that introduction of a point like four-quark interaction could be a reasonable approximation. To our knowledge the only mechanism inherent to the perturbative QCD which makes gluon lines hard is the so-called ultraviolet renormalon [8] (for a review see also [9]). (For a possibility of the nonperturbative QCD to “induce” the $G_S$-type four-fermion interaction see Ref.[10].) In terms of the sum rules the ultraviolet renormalon results in $Q^{-2}$ corrections to the parton-model predictions and the possibility of such corrections has been brought to attention only recently [11, 12]. Moreover it was demonstrated that indeed the ultraviolet renormalon is related to four-quark operators [13]. However, there are no theoretical means to relate the renormalon contribution in different channels.

In this paper we will address ourselves to the problem of confronting the NJL model with the fundamental QCD. Our basic observation is that in $\rho$ meson channel the claimed regions of applicability of the effective theory and of perturbative QCD in fact overlap. Namely, the QCD sum rules [7] rely, on one hand, on the perturbative QCD and, on the other hand, are still valid at Euclidean mass scale as low as $m_{\rho}^2$. Since, according to the estimates [2], the ultraviolet cutoff $\Lambda_{UV}^2$ in the NJL model is substantially larger than $m_{\rho}^2$, the sum rules are sensitive to the introduction of the phenomenological piece (1).

It might worth emphasizing that generally speaking, the use of the effective Lagrangian and perturbative approach to fundamental QCD are justified in different kinematical regions. Namely, the language of (nearly) massless quarks and of gluons becomes rigorous at short distances where the running coupling is small, while the effective Lagrangian (1) describes
the large-distance, or the low-momentum dynamics. It is a very specific interplay of numbers that allows for a window in kinematical variables where the effective field theory approach to QCD confronts the perturbative approach to QCD. We use this to obtain independent information on $G_{S,V}$.

Our conclusion is that the value of $G_V$ is too big to be consistent with the sum rules. On the other hand, the effect of $G_S$ is just about what is needed to resolve a long standing puzzle of QCD sum rules, that is, the failure of the standard approach [7] in the pseudoscalar channel [14]. Pursuing this line of reasoning, we conclude that the phenomenological number (2) for $G_V$ is to be an overestimate. Careful analysis of the data from this point of view could serve as a crucial test of the proposed phenomenology. Without trying to surpass the results of such an analysis let us note that, to the best of our understanding, $G_V \neq 0$ is not needed for the NJL model to be successful in describing the low-energy pion physics (see [4]).

2 Sum rules in $\rho$ channel

We start our discussion with estimating the effect of the interaction (1) on the sum rules in $\rho$ channel. The sum rules [4] are formulated in terms of $\Pi^\rho(Q^2)$,

$$\Pi^\rho(Q^2)(q_\mu q_\nu - g_{\mu\nu} q^2) = i \int d^4 x e^{iqx} \langle 0 | T \{ j_\mu^\rho(x), j_\nu^\rho(0) \} | 0 \rangle, \quad Q^2 \equiv -q^2, \quad (3)$$

where $j_\mu^\rho$ is the quark current with quantum numbers of $\rho$:

$$j_\mu^\rho = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d).$$

The function $\Pi^\rho(Q^2)$ satisfies once subtracted dispersion relations

$$\Pi^\rho(Q^2) = \frac{Q^2}{\pi} \int \frac{R_{I=1}^I(s) ds}{s(s + Q^2)}, \quad (4)$$

where $R_{I=1}^I(s)$ is the ratio of the cross section of $e^+e^-\to$ annihilation into hadrons with total isospin $I = 1$ to that of annihilation into $\mu^+\mu^-$ pair; in particular, $R_{I=1}^I(s)$ is contributed by the $\rho$ meson.
The basic idea of the QCD sum rules [7] is to calculate $\Pi^\rho(Q^2)$ at large $Q^2$ using the perturbative QCD and then extrapolate the result to as low $Q^2$ as possible. Moreover, it turns out that the advance towards lowest $Q^2$ is checked by power corrections in $Q^{-2}$. The coefficients in front of the powers of $Q^{-2}$ are related to the quark and gluon condensates. Numerically the power like corrections set in around $Q^2 \sim 0.5$ GeV$^2 \sim m^2_\rho$. More precisely, the corrections are relatively small at such $Q^2$ but blow up fast at lower $Q^2$.

While the reader is referred to the original papers and reviews [7] for the details of the sum rules, here we only mention that the sum rules are most successful once applied to the Borel transform $\hat{\Pi}^\rho(M^2)$ of $\Pi^\rho(Q^2)$. The definition is

$$\Pi^\rho(M^2) \equiv \hat{\Pi}^\rho(Q^2) \equiv \lim_{n \to \infty} \frac{1}{(n-1)!}(-1)^n(Q^2)^n\left(\frac{d}{dQ^2}\right)^n\Pi^\rho(Q^2),$$

where the limit is understood in such a way that

$$n \to \infty, \quad Q^2 \to \infty, \quad Q^2/n \equiv M^2$$

and it is actually $M^2 \sim m^2_\rho$ rather than $Q^2 \sim m^2_\rho$ that can be reached starting from large $M^2$.

In a somewhat simplified form the sum rules read

$$1 + \frac{\alpha_s(M^2)}{\pi} + \frac{\pi^2}{3} \frac{\langle \alpha_s/\pi \cdot (G^2_{\mu\nu})^2 \rangle}{M^4} + \frac{448\pi^3}{81} \frac{(\alpha_s^{1/2}\langle \bar{q}q \rangle)^2}{M^6} + O(M^{-8})$$

where $\alpha_s$ is the QCD coupling, $g_\rho$ the $\rho$ meson coupling ($= g_{\rho\pi\pi}$) related to the $e^+e^-$ width of the $\rho$ meson, $g_\rho^2/4\pi \approx 3$, and $\langle \bar{q}q \rangle$ and $\langle (G^2_{\mu\nu})^2 \rangle$ are the quark and the gluon condensates, respectively. The integral on the right-hand side represents contribution of the continuum.

The only observation concerning the sum rules which is important for our purposes here is that even at $M^2 \approx m^2_\rho$ the left-hand side of the sum rules [7] is calculable within the short-distance approach to QCD, i.e., is dominated by the unit while other terms can be
considered as small corrections. The sum rules agree with the data, or the right-hand side, to within about 10 per cent. Moreover, the $\rho$ contribution dominates over the continuum at such $M^2$ to about the same accuracy. Eq.(7) can be considered, therefore, as a refined form of duality derived within the fundamental QCD.

Now we come to the crucial point of the consistency of the effective interaction (1) with the QCD sum rules. As is emphasized in the previous section, it is far from being obvious that both the fundamental and the effective Lagrangians can be utilized simultaneously. In general, it is the opposite which is true. It is just a very specific result that the QCD sum rules apply even at $M^2$ numerically close to $m_\rho^2$, which allows for a check of self-consistency. Indeed, if the cutoff $\Lambda_{UV}$ is as high as indicated by eq.(2), then there exists region of so to say moderate $Q^2$,

$$0.5 \text{ GeV}^2 \leq Q^2_{\text{moderate}} \leq 1.5 \text{ GeV}^2,$$

where the both approaches claim their validity.

Thus at least superficially we are allowed to add at $Q^2 \sim m_\rho^2$ the contribution of interaction (1) to the bare quark loop to get

$$\Pi^\rho(Q^2)_{\text{modified}} \approx -\frac{1}{8\pi^2}lnQ^2 \left(1 - \frac{G_VQ^2}{2\pi^2}lnQ^2\right).$$

The first term on the right-hand side here represents the bare quark loop which is expected to dominate even at $Q^2 \sim m_\rho^2$, as explained above. As for the piece proportional to $G_V$, its evaluation is in fact not unique, because the effective interaction (1) is non-renormalizable. We could write, say, $G_V\Lambda_{UV}^2$ instead of $G_VQ^2lnQ^2$. However, as far as we stretch eq.(2) to $Q^2 \sim \Lambda_{UV}^2$, the two answers match each other smoothly. Moreover, we shall argue in the next section that in fact there is no much uncertainty in our estimates, provided that the $\rho$ meson is generated by the effective interaction (1).

If we use the Borel transformed $\Pi^\rho(M^2)$, then the effect of introduction of the new
interaction is enhanced numerically:

\[ \Pi^\rho(M^2)_{\text{modified}} \approx \frac{1}{8\pi^2} \left( 1 - \frac{G_V M^2}{\pi^2} \left( 1 - \gamma + \ln M^2 \right) \right), \quad (10) \]

or

\[ \frac{\delta \Pi^\rho(M^2)}{\Pi^\rho_0(M^2)} \approx \frac{G_V M^2}{\pi^2} \left( 1 - \gamma + \ln M^2 \right), \quad (11) \]

where \( \Pi^\rho_0(M^2) \) is the contribution of the bare loop graph with massless quarks and \( \gamma \approx 0.577 \) is the Euler constant. Note that the effect of the new interaction grows with \( M^2 \) as \( M^2 \ln M^2 \).

On the other hand, at large \( M^2 \) the effect should disappear because of the onset of the asymptotic freedom. This emphasizes once more that the effective interaction (1) can be valid only at relatively low momenta. Numerically, once we accept the estimates (2), we can extrapolate (11) up to \( M^2 \approx \Lambda_{\text{UV}}^2 \) and allow then for a form factor which would describe softening of the effective interaction.

For the new interaction to be consistent with the sum rules we expect that (11) represents a small correction. However, using eq.(3), we conclude that introduction of interaction (1) results in fact in a considerable change of \( \Pi^\rho(Q^2) \):

\[ \frac{\delta \Pi^\rho(Q^2 = m^2_\rho)}{\Pi^\rho_0(Q^2 = m^2_\rho)} \approx - \frac{G_V m^2_\rho}{2\pi^2} \ln m^2_\rho \sim -0.25 \ln m^2_\rho, \quad (12) \]

where \( \ln m^2_\rho \approx 2 \) if we take typically \((250 \text{ MeV})^2 \) for the infrared scale of the logarithm. This can hardly be compatible with the strong dominance of the quark loop established within the sum rules. The effect is even more dramatic, if we turn to the Borel transform \( \Pi^\rho(M^2) \) pertinent to the sum rules (see eq.(11)):

\[ \frac{\delta \Pi^\rho(M^2 = m^2_\rho)}{\Pi^\rho_0(M^2 = m^2_\rho)} \sim 0.5 \left( 1 - \gamma + \ln m^2_\rho \right). \quad (13) \]

We see that the new interaction cannot be considered as a small correction at all but is to be rather iterated and summed up in the spirit of the NJL model.

Thus, the conclusion is that the condition of consistency with the QCD rules does not allow us to have \( G_V \) as big as indicated by (2). Because of the importance of this conclusion
we are going to discuss next to which extent it depends on the particular parameterization (2).

3 Vector meson dominance vs sum rules

In this section we will argue that the notion of $\rho$ meson dominance at (Euclidean) momenta $Q^2 \sim m_\rho^2$ is not indeed consistent with the QCD sum rules, so that the numerical contradiction found in the previous section is significant.

The point we wish to emphasize is in fact very simple. Namely, consider $\Pi^\rho(Q^2)$ at $Q^2 \sim m_\rho^2$. One can try to apply either $\rho$ meson dominance to evaluate $\Pi^\rho(Q^2)$ or just approximate it by the bare quark loop graph. The former approximation assumes the distances to be large, coupling large and the use of the language of strongly bound states. The latter approximation rests on the assumption that the coupling is already negligible and the language of free quarks is the appropriate one. The both pictures cannot be right when applied to the same quantity, $\Pi^\rho(Q^2)$ at the same values of $Q^2$. However, since the sum rules also imply a kind of vector meson dominance (VMD), we proceed to reiterate the argument on a more technical level.

Let us start with the VMD picture, as it arises in the extended NJL model. In the limit of a large $N_c$ the correlator (3) is approximated by the chain of the loop graphs ("bubble sum"):

$$\Pi^\rho(Q^2) \approx \frac{1}{8\pi^2} \ln \frac{Q^2 + m_q^2}{m_\rho^2} \left( 1 + \frac{G_\rho}{2\pi^2} Q^2 \ln \frac{Q^2 + m_q^2}{Q^2 + m_\rho^2} \right), \quad (14)$$

where $m_q$ is the constituent quark mass, $m_q \approx 300$ MeV, generated within the same NJL approach through the $G_S$ interaction in (I).

Neglecting $Q^2$ in the argument of the logarithm, one readily derives the VMD for the correlator (3):

$$\Pi^\rho(Q^2)_{\text{VMD}} = \frac{1}{g_\rho^2} \frac{m_\rho^2}{Q^2 + m_\rho^2}, \quad (15)$$
with
\[
m^2_\rho = \frac{2\pi^2}{G_V \ln \Lambda_{UV}^2 / m_q^2}
\] (16)
and
\[
g^2_\rho = \frac{8\pi^2}{\ln \Lambda_{UV}^2 / m_q^2}
\] (17)
Upon applying the Borel transform (5) to (15), we obtain
\[
\frac{1}{\hat{L}} \Pi^\rho(Q^2)_{\text{VMD}} = \frac{1}{g^2_\rho M^2} \exp \left( -\frac{m^2_\rho}{M^2} \right).
\] (18)
Eqs. (15) and (18) are expected to hold at low \( Q^2, M^2 \), i.e., much smaller than \( \Lambda_{UV}^2 \). Experimentally, eq.(18) holds at \( M^2 \leq m^2_\rho \) and this could be claimed a success of the VMD model.

Now, the perturbative QCD proceeds in the following way. The correlator \( \Pi^\rho(Q^2) \) is approximated by the bare loop graph:
\[
\Pi^\rho(Q^2)_{\text{pert QCD}} \approx -\frac{1}{8\pi^2} \ln Q^2,
\] (19)
where \( Q^2 \) is assumed to be “large”, in contradistinction from the VMD model. Applying the Borel transform, one finds
\[
\Pi^\rho(M^2)_{\text{pert QCD}} \approx \frac{1}{8\pi^2}
\] (20)
and this again holds experimentally to a reasonable accuracy at \( M^2 \approx m^2_\rho \), since numerically (18) and (20) are close to each other. This calculation could be claimed a success of the short-distance approach to QCD.

Thus, the perturbative QCD derives (20) and treats (18) as a phenomenological fit to the integral over \( R^{I=1}(s) \), see eq.(4). In other words, one derives duality from the first principle, provided that the existence of resonances is granted. Within the effective Lagrangian approach, on the other hand, one derives resonances. The sum rules (7) become then a triviality, since both \( \Pi^\rho(Q^2) \) in the Euclidean region and \( Im \Pi^\rho(s) \sim R(s) \) are dominated by one and the same \( \rho \) meson.
In terms of \( \Lambda_{UV} \) the equation (15) is valid at \( \Lambda_{UV}^2 \gg M^2 \) (VMD case), while (19) is true if \( \Lambda_{UV}^2 \ll M^2 \) (perturbative QCD case). Moreover, if one evaluates the simplest quark graph as it is prescribed by the VMD, then it does not contribute to \( \Pi^\rho(M^2) \) at all, since

\[
\hat{L}(\frac{1}{8\pi^2} \ln \Lambda_{UV}^2) = 0,
\]

while in case of perturbative QCD this graph dominates (see eq.(20)).

Thus, the success of the QCD sum rules implies necessity to modify the VMD model around \( M^2 \sim m_\rho^2 \) so that the composite nature of the vector mesons would become manifest at such \( M^2 \).

To see whether the effect is dramatic numerically we should have worked out an interpolation between (20) and (18). A rigorous treatment of this transitive region seems to be out of reach of any known framework. We can, however, suggest a simple-minded interpolation which might reproduce gross features of the reality.

Namely, let us keep the \( \ln Q^2 \) factor in eq.(14). Using the Borel transform (see eq. (5)),

\[
\hat{L}\{Q^{-2k}(\ln Q^2/\Lambda^2)^{-\epsilon}\} \approx \frac{1}{\Gamma(k)} M^{-2k}(\ln M^2/\Lambda_{UV}^2)^{-\epsilon},
\]

one obtains from (14)

\[
\Pi^\rho(M^2) \approx \frac{1}{g_\rho^2 M^2} \exp \left( -\frac{m_\rho^2 \ln \Lambda_{UV}^2 - \ln m_q^2}{M^2 \ln \Lambda_{UV}^2 - \ln M^2} \right),
\]

where \( \Lambda_{UV}^2 > M^2 \).

Eq.(23) demonstrates a remarkably sharp dissolution of \( \rho \) as \( M^2 \) approaches \( \Lambda_{UV}^2 \). For the purpose of eliminating the disagreement with the sum rules one needs at \( M^2 \sim m_\rho^2 \)

\[
\frac{\ln \Lambda_{UV}^2 - \ln m_q^2}{\ln \Lambda_{UV}^2 - \ln m_\rho^2} \sim 3,
\]

so that the effect of “elementary” \( \rho \) meson generated via the interaction (II) goes away at Euclidean \( Q^2 \sim m_\rho^2 \). Although this conclusion contradicts naive expectations based on the
VMD, let us note that the VMD could still be a valid approximation at Minkowski momenta and down to, say, $Q^2 \sim 0$.

To summarize, success of the sum rules in the $\rho$ channel implies that the VMD is to be replaced by the fundamental QCD around $M^2 \sim m^2_\rho$, which means in turn a change in the fitting parameters (4). Our eq.(24) above is an attempt to change $\Lambda_{UV}$. However, as we shall see in the next section, the low value of $\Lambda_{UV}$ is not favoured by consideration of the pseudoscalar, or the pion channel. Therefore it is worth emphasizing that the most straightforward solution to the problem is to assume that $G_V$ is substantially smaller than that indicated by (4). This would imply, however, that light vector mesons ($m^2_\rho \ll \Lambda^2_{UV}$) are not generated by the effective interaction (1).

4 Sum rules in pseudoscalar channel

In this section we will argue that introduction of effective interaction (1) with values of $G_S$ and $\Lambda_{UV}$ as indicated by (2) could resolve a long standing problem of QCD-based phenomenology.

Namely, one can consider sum rules in the $\pi$ channel in exactly the same way as in the $\rho$ channel outlined above. The corresponding current is defined as

$$j^\pi = \frac{1}{2}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d) \quad (25)$$

and the sum rules take the form [14]:

$$\left(\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right)^{8/9} \left(1 + \pi^2 \frac{\alpha_s/\pi \cdot (G^a_{\mu\nu})^2}{3M^4} + \frac{896\pi^3}{81} \frac{\langle \alpha_s^{1/2} \bar{q}q \rangle^2}{M^6} + O(M^{-8})\right)$$

$$= \frac{16\pi}{3M^4} \int ds \exp(-s/M^2) Im\Pi^\pi(s), \quad (26)$$

where $Im\Pi^\pi(s)$ is the imaginary part of the correlator of two currents $j^\pi$ and the factor $\left(\alpha_s(M^2)/\alpha_s(\mu^2)\right)^{8/9}$ is due to a non-vanishing anomalous dimension of $j^\pi$. As far as $Im\Pi^\pi(s)$
is concerned, the only experimentally known contribution to it comes from the pion:

\[ Im\Pi_{\text{pole}}^{\pi} = \pi f_\pi^2 m_\pi^4 (m_u + m_d)^{-2} \delta(s - m_\pi^2), \]

(27)

where \( f_\pi \approx 93 \text{ MeV} \) and \( m_{u,d} \) are the current quark masses.

Now, it has been demonstrated that the sum rules (26) do not hold experimentally as they are stated [14]. The point is that the pole contribution (27) alone, with negligence of the rest of \( Im\Pi^{\pi}(s) \) which is positive definite, is large enough to break asymptotic freedom in the pion channel at

\[ (M_\text{crit}^{\pi})^2 \approx 2 \text{ GeV}^2. \]

(28)

Moreover, the actual number could be even higher, since we neglected all other states with the same quantum numbers. Note that the asymptotic freedom is represented by the unit in the left-hand side of eq. (26). The corresponding scale in the \( \rho \) channel discussed in the previous section is numerically \( (M_\text{crit}^{\rho})^2 \approx 0.6 \text{ GeV}^2 \) and this was the basis for the whole success of the sum rules in describing the \( \rho \) meson.

In this way the sum rules reveal that the pion is not dual to the quark loop, in contradistinction from the \( \rho \) meson case. More specifically, one can prove existence of a new contribution for \( M^2 \) in the window \( 0.5 \text{ GeV}^2 < M^2 < 2 \text{ GeV}^2 \). At \( M^2 > 2 \text{ GeV}^2 \) the sum rules can be dominated by the bare quark loop, while at \( M^2 < 0.5 \text{ GeV}^2 \) the power like corrections to the sum rules become important and the sum rules are no longer sensitive to new physics. At \( M^2 = 0.5 \text{ GeV}^2 \) it is not less than the pion contribution, while at \( M^2 = 2 \text{ GeV}^2 \) the new contribution is still larger than 10% of the pion contribution.

Thus the problem is that the power corrections accounted for in (7) and (24) fail to reproduce this change of scales with the switching from the \( \rho \) channel to the \( \pi \) channel [14]. What we propose here is to ascribe this difference to the presence of the effective interaction (1) in the \( \pi \) channel.

At \( M^2 < \Lambda_{\text{UV}}^2 \) the estimates of the effect of the new interaction can be made similar to
\begin{align}
\Pi_\pi(Q^2)_{\text{modified}} & \approx \frac{3}{16\pi^2}Q^2\ln Q^2 \left( 1 + \frac{3G_sQ^2}{4\pi^2}\ln Q^2 \right), \\
\Pi_\pi(M^2)_{\text{modified}} & \approx \frac{3}{16\pi^2}M^2 \left( 1 - \frac{3G_SM^2}{\pi^2}\left( \frac{3}{2} - \gamma + \ln M^2 \right) \right),
\end{align}

or
\begin{align}
\frac{\delta \Pi_\pi(Q^2)}{\Pi_0^\pi(Q^2)} & \approx \frac{3G_sQ^2}{4\pi^2}\ln Q^2, \\
\frac{\delta \Pi_\pi(M^2)}{\Pi_0^\pi(M^2)} & \approx -\frac{3G_SM^2}{\pi^2}\left( \frac{3}{2} - \gamma + \ln M^2 \right),
\end{align}

where by \( \Pi_0^\pi(Q^2) \) and \( \Pi_0^\pi(M^2) \) we understand the contribution of the bare quark loop for the sake of normalization.

Eq.\((32)\) is supposed to be valid at \( M^2 \) much smaller than \( \Lambda_{UV}^2 \). Applying it at \( M^2 = 0.5 \text{ GeV}^2 \), we find:
\begin{equation}
\frac{\delta \Pi_\pi(M^2 = 0.5 \text{ GeV}^2)}{\Pi_0^\pi(M^2 = 0.5 \text{ GeV}^2)} \approx 2.5.
\end{equation}

Literally, the analysis of the sum rules suggests a new contribution of order one in the same units (see above). The factor we get now is in rough agreement with that estimate.

To get a better estimate we should have iterated the effect of the new interaction, since its effect \((33)\) turned out to be large. The change brought by the iterations of the effective interaction is remarkably simple and well known within the NJL model. Namely, within the NJL model the summation of the chain of the graphs generated by the interaction \((\mathbb{I})\) produces a pion. In this way we reproduce the contribution of the pion into the right-hand side of the sum rules \((26)\) and explain the failure of the sum rules which do not account for interaction \((\mathbb{I})\) at \( M^2 \approx 0.5 \text{ GeV}^2 \).

Thus the effective interaction in the pseudoscalar channel with the parameters indicated in \((2)\) provides a natural explanation to the phenomenon found in Ref.\(\cite{14}\). Namely the four-quark interaction gives rise to the pion as is proposed in the original papers \((\mathbb{I})\). At \( M^2 \sim \Lambda_{UV}^2 \sim 1.5 \text{ GeV}^2 \) the effective interaction is dissolved and its contribution is replaced by the bare quark loop which dominates in the region of small effective coupling of QCD.
The estimate of the mass scale of the onset of asymptotic freedom in the pion channel as 2 GeV (see above) turns out to be in reasonable agreement with the estimate of $\Lambda_{\text{UV}}^2$ within the NJL model.

Of course this picture does not explain by itself why the vector and the pseudoscalar channels look different. Since the sum rules are derived within the fundamental QCD, there should be new corrections, not accounted for in the standard approach. This conclusion has been reached long time ago [14]. There were also speculations on possible new sources of corrections [14]. What is common for these corrections is that they depend on a high inverse power of $M^2$. In conclusion of this section we would like to make a comment that in fact the matching of the effective interaction (1) with fundamental QCD looks most natural, if $M^{-2}$ corrections are introduced. Indeed, if we invoke $M^{-2}$ terms, then the difference between the channels is least dramatic; namely, the difference between $M_{\text{crit}}^2$ in the $\pi$ and $\rho$ channels (2 GeV$^2$ and 0.5 GeV$^2$, respectively) is a factor of 4, which does not look so drastic by itself. What makes the situation especially difficult for the standard sum rules is that they operate with $M^{-4}, M^{-6}$ corrections which boost the difference to the factor $\sim 10^{-10}$ and, moreover, these corrections are well fixed numerically and do not allow for speculations on numbers. In Ref.[14] an attempt is made to ascribe the difference to the contribution of direct instantons. But that contribution depends on $M^2$ even more drastically, like $M^{-9}$ and the difference between the two channels is difficult to explain.

The possibility of existence of $M^{-2}$ corrections has been realized only recently [11, 12]. In the most direct way they arise from the so-called ultraviolet renormalons [8]. Ultraviolet renormalons are associated with insertions of vacuum bubble insertions into a gluon line. An extra bonus of such interpretation is that the gluon line carries a large virtual momenta $k^2$,

$$k^2 \sim e^n Q^2,$$  \hspace{1cm} (34)

where $n$ is the order of perturbative calculation considered and $e$ is the base of natural logs. In general, gluon exchanges generate various effective interactions but it was demonstrated
that four-fermion operators (1) dominate at large $n$ and induce $M^{-2}$ terms [13]. Once we take $M^2$ to be low enough, the language of asymptotic freedom is no longer valid and the four-fermion operators governing the renormalon become effective interaction (1).

5 Conclusions

To summarize, confronting the NJL model with QCD sum rules uncovers a nontrivial interplay of the two approaches. In the $\rho$ channel the success of the QCD sum rules [7] calls for a revision of the NJL model for vector mesons. In the $\pi$ channel, to the contrary, the QCD sum rules badly need a new contribution [14] and we have demonstrated that the NJL effective interaction could well be used for this purpose.

The only way how this effective interaction could be accommodated into the perturbative QCD seems to be through ultraviolet renormalon [13]. (Such an interaction might also be induced by the nonperturbative effects of QCD [10].) Such a hypothesis assumes, however, that the renormalon contribution is large numerically. Let us note that similar assumption in fact underlies the original QCD sum rules as well. Indeed, one can argue that $M^{-4}$ corrections arise from divergencies of perturbation theory in large orders and are related to the so-called infrared renormalon [15]. However, it is not obvious at all that these corrections are numerically important and are not screened by lower order perturbative terms. The basis of the phenomenology of the sum rules is the assumption that the $M^{-4}$ corrections are numerically large.

For the ultraviolet renormalon to be relevant, there should be a large numerical factor as well. However, what makes the phenomenology of $M^{-2}$ corrections still much less definite is that there is no way to relate the $M^{-2}$ terms in different channels, even if one is prepared to assume that these terms are enhanced numerically. It is at this point that the phenomenology of the NJL model could play a crucial role.
Indeed the chain of the arguments above gets closed through the prediction

\[ G_V \ll G_S, \tag{35} \]

where \( G_{V,S} \) are the constants of the effective four-four-fermion interactions (1). We need (35) to ameliorate the sum rules in the pseudoscalar channel without destroying them in the vector channel.

Although eq.(35) contradicts the spirit of the extended NJL model (see, e.g., review in Ref.[2]), it is not obvious that the prediction (35) can be ruled out phenomenologically. In particular, the most elaborated comparison of the NJL model with the experimental data performed in Ref.[4] reveals that the solution with \( G_V = 0 \) gives a very satisfactory fit to all known parameters of the low-energy pion interactions. Moreover, any \( G_V \neq 0 \) drives the predicted value of the constant \( g_A \) governing the beta-decay of neutron off its experimental value. Less dramatically, taking \( G_V = 0 \) improves fits to some other parameters as well (see [4]). Furthermore, as is noted in [4], the NJL model with \( G_V = 0 \) is equivalent to the effective QCD Lagrangian of Ref.[16] which turned out to be successful in other phenomenological applications [17].

Thus it seems fair to say that the NJL model with \( G_V = 0 \) and \( G_S \neq 0 \) results in a sound phenomenology, although it puts pseudoscalar and vector meson on different footing, as is required by the QCD sum rules.

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