Anomalous charge tunneling in the fractional quantum Hall edge states at filling factor $\nu = 5/2$

M. Carrega$^1$, D. Ferraro$^{2,3,4}$, A. Braggio$^3$, N. Magnoli$^{2,4}$, M. Sassetti$^{2,3}$

1 NEST, Istituto Nanoscienze - CNR and Scuola Normale Superiore, I-56126 Pisa, Italy.
2 Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, 16146, Genova, Italy.
3 CNR-SPIN, Via Dodecaneso 33, 16146, Genova, Italy.
4 INFN, Via Dodecaneso 33, 16146, Genova, Italy.

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We explain effective charge anomalies recently observed for fractional quantum Hall edge states at $\nu = 5/2$ [M. Dolev, Y. Gross, Y. C. Chung, M. Heiblum, V. Umansky, and D. Mahalu, Phys. Rev. B. 81, 161303(R) (2010)]. The experimental data of differential conductance and excess noise are fitted, using the anti-Pfaffian model, by properly take into account renormalizations of the Luttinger parameters induced by the coupling of the system with an intrinsic $1/f$ noise. We demonstrate that a peculiar agglomerate excitation with charge $e/2$, double of the expected $e/4$ charge, dominates the transport properties at low energies.

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Introduction.—Since its discovery [1], the fractional quantum Hall (FQH) state at filling factor $\nu = 5/2$ has been subject of intense investigations. Many proposals have been introduced in order to explain this exotic even denominator, ranging from an Abelian description [2] to more intriguing ones which support non-Abelian excitations, like the Moore-Read Pfaffian model [3, 4] or its particle - hole conjugate, the anti-Pfaffian model [5]. The possible applications for the topologically protected quantum computation of non-Abelian excitations aroused even more interest for this FQH state [6].

In these models the excitations have a fundamental charge $e^* = e/4$ ($e$ the electron charge). This fact has been experimentally supported by bulk measurements [7] and with current noise experiments through a quantum point contact (QPC) geometry [8], successfully applied for other FQH states [9, 10]. Very recently, measurements were reported [11] for $\nu = 5/2$ where the $e/4$ charge value is observed at high temperatures, while at low temperatures the measured charge reaches the unexpected value $e/2$. Analogous enhancement of the carrier charge has been already observed [10, 12] and theoretically explained [13] in other composite FQH states belonging to the Jain sequence. However, there is still no interpretation of this phenomenon in the $\nu = 5/2$ state.

In this letter we propose an explanation for these puzzling observations, showing that a different kind of excitation, the 2-agglomerate, with charge double of the fundamental one dominates the transport at low energies. This excitation cannot be simply interpreted in terms of a bunching phenomena of single-quasiparticles due to the non-Abelian nature of the latter. We will consider the anti-Pfaffian model, despite the presented phenomenology could also be consistent with other models. In the anti-Pfaffian case three fields are involved, one charged and two neutral (one boson and one Majorana fermion). The key assumptions of our description are the finite velocity of neutral modes and the presence of renormalizations due to the interaction with the external environment. Among all the possible mechanisms leading to a renormalization of the Luttinger parameters [14, 15] we focus on the effects induced by the ubiquitous out of equilibrium $1/f$ noise in presence of a dissipative environment [16].

Our predictions show an excellent agreement with experimental data on a wide range of temperatures and voltages, demonstrating the validity of the proposed scenario.

Model.—The edge states of $\nu = 5/2$ in the anti-Pfaffian model are described as a narrow region at $\nu = 3$ with nearby a Pfaffian edge of holes with $\nu = 1/2$ [5]. Considering the second LL as the “vacuum”, the edge is modeled as a single $\nu = 1$ bosonic branch $\varphi_1$ and a counter-propagating $\nu = 1/2$ Pfaffian branch [4], composed of a bosonic mode $\varphi_2$ and a Majorana fermion $\psi$. The Lagrangian density is $L_{\text{edge}} = L_1 + L_2 + L_\psi + L_{12} + L_{\text{rdm}}$ with $\hbar = 1$

$$L_j = \frac{1}{2\pi\nu_j} \partial_x \varphi_j \left( \eta_j \partial_t - v_j \partial_x \right) \varphi_j \quad j = 1, 2$$

chiral Luttinger liquid (cLL) with interaction parameters $\nu_j = 1/j$ and velocities $v_j$. The chiralities are $\eta_j = (−1)^{j+1}$ with $\eta = 1$ ($\eta = -1$) for a co-propagating (counter-propagating) mode. The interaction between the two bosonic modes is $L_{12} = -(v_{12}/2\pi) \partial_x \varphi_1 \partial_x \varphi_2$ with $v_{12}$ the coupling strength. The term $L_\psi = i \bar{\psi} (\partial_t + v_\psi \partial_x) \psi$ describes a Majorana fermion propagating with velocity $v_\psi$. We also need to include in the Lagrangian a disorder term $L_{\text{rdm}} = \xi(x) \psi e^{i \varphi_2 + i \varphi_1} + \text{h.c.}$ to describe the random electron tunneling processes which equilibrate the two branches. The complex tunneling amplitude $\xi(x)$ satisfies $(\xi(x) \xi^*(x')) = W \delta(x - x')$. These processes bring the edges to equilibrium, recovering the appropriate value of the Hall resistance, in analogy with what happen for $\nu = 2/3$ [5, 17].

When the disorder term $L_{\text{rdm}}$ is a relevant perturbation the system is driven to a disorder dominated phase [5]. At this fixed point the system naturally decouples in...
Green’s function are \( \langle \varphi_j(t)\varphi_i(0) \rangle = g_j|\nu_j| \ln (1 + i\omega, t) \) with \( g_j = g_j(F_1/\eta_1, F_2/\eta_2, F_3/F_2) \geq 1 \) \( (j = c, n) \). A detailed derivation of these facts will be given elsewhere [21]. Obviously the renormalizations affect only the dynamical properties of the excitations, without modifying universal quantities like their charge and statistics.

Excitations.--The generic operator destroying an excitation along the edge can be written as [3, 6]

\[
\Psi_{\chi,m,n}(x) \propto \chi(x)e^{i((m/2)\varphi_c(x)+(n/2)\varphi_n(x))}
\]

here, the integer coefficients \( m, n \) and the Ising field \( \chi \) define the admissible excitations. In the Ising sector \( \chi \) can be \( I \) (identity operator), \( \psi \) (Majorana fermion) or \( \sigma \) (spin operator). The operator \( \sigma \), due to the non-trivial operator product expansion \( \sigma \times \sigma = I + \psi \), leads to the non-Abelian statistics of the excitations [6]. The single-valuedness properties of the operators force \( m, n \) to be even integers for \( \chi = I, \psi \) and odd integers for \( \chi = \sigma \). The charge associated to the operator in Eq. (3) is \( e \chi_{m,n} = (m/4)e \) depending on the charged mode only. In the following we will indicate an \((m/4)e\) charged excitation as \( m\)-agglomerate [13].

The scaling dimension [22] of the operators in Eq. (3) is

\[
\Delta_{\chi,m,n} = \frac{1}{2} \delta_0 + \frac{g_c}{16} m^2 + \frac{g_n}{8} n^2 ,
\]

with \( \delta_0 = 0 \), \( \delta_c = 1 \) and \( \delta_n = 1/8 \) [6]. Inspection of Eq. (4) allows the determination of the more relevant excitations. Among all the single-quasiparticle (qp), with charge \( e^* = e/4 \), the most dominant are \( \Psi^{(1)} = \Psi_{\sigma,1,\pm 1} \) with scaling dimensions \( \Delta^{(1)} = \Delta_{\sigma,1,\pm 1} = (g_c + 2g_n + 1)/16 \). The other most relevant excitation is the \( 2\)-agglomerate with charge \( 2e^* = e/2 \) and operator \( \Psi^{(2)} = \Psi_{\sigma,2,0} \) with scaling dimension \( \Delta^{(2)} = \Delta_{\sigma,2,0} = g_c/4 \). It is worth to note that also the operator \( \Psi_{\psi,2,0} \) has a charge \( e/2 \), but is less relevant because its scaling dimension is increased by the Majorana fermion contribution. All other excitations are less relevant and will be neglected in the following.

In the unrenormalized case \( (g_c = g_n = 1) \) the single-q (\( \Psi^{(1)} \)) and the \( 2\)-agglomerate (\( \Psi^{(2)} \)) have the same scaling dimension, equal to \( 1/4 \). Renormalization effects qualitatively change the above scenario. In particular, for \( g_c < (1 + 2g_n)/3 \), the \( 2\)-agglomerate becomes the most relevant excitation at low energies opening the possibility of a crossover between the two excitations, in agreement with experimental observations. Note that, due to the peculiar fusion rules of the \( \sigma \) operator, the \( 2\)-agglomerate cannot be simply created combining two single-q without introducing also an excitation with a Majorana fermion in the Ising sector. This fact suggests that, in the non-Abelian models, the \( 2\)-agglomerate is not simply given by a bunching of two quasiparticles, namely in general \( \Psi^{(2)} \neq (\Psi^{(1)})^2 \).

Transport properties.--In the QPC geometry tunneling of excitations between the two side of the Hall bar is allowed, and can be described through the Hamiltonian \( H_T = \)
\[ \sum_{m=1,2} t_m \Psi_R^{(m)\dagger}(0) \Psi_L^{(m)}(0) + \text{h.c.} \]
where \( R \) and \( L \) indicate respectively the right and the left edge, \( t_m \) \((m = 1, 2)\)
the tunneling amplitudes. Without loss of generality, we assume the tunneling occurring at \( x = 0 \). At lowest order in
\( H_{TB} \) [23] the backscattering current is \( I_B = \sum_{m=1,2} \langle I_B^{(m)} \rangle \)
with
\[ \langle I_B^{(m)} \rangle = me^* \left( 1 - e^{-\frac{\omega m}{2k_B T}} \right) \Gamma_m(m^* V) \] (5)
being \( V \) the bias, \( T \) the temperature and where \( \Gamma_m(E) \) indicates the first order Fermi’s Golden rule tunneling rate.
The differential backscattering conductance is given by \( G_B = \sum_{m=1,2} G_B^{(m)} \) with \( G_B^{(m)} = d\langle I_B^{(m)} \rangle /dV \).
Current noise [23, 24] is another relevant quantity in order to provide information on the \( m \)-agglomerate excitations.
The finite frequency symmetrized noise is \( S_B(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \{ \delta I_B(t), \delta I_B(0) \} \rangle \) with \( \langle \cdot, \cdot \rangle \) the anticommutator. At lowest order in the tunneling, is simply given by the sum of the two contributions
\[ S_B^{(m)}(\omega) = \sum_{m=1,2} S_B^{(m)}(\omega) \] (6)
where \( \omega_0 = e^* V \). A detailed analysis of this quantity will be given elsewhere [21], in this letter we will focus only on
the zero frequency limit. One can introduce the backscattering current excess noise \( S_{\text{exc}} = S_B(0) - 4k_B T G_B|_{V=0} \)
that, in the lowest order in the tunneling, can be directly compared with the current noise measured in the experiments.

Results—We will compare now the theoretical predictions with the raw experimental data for the differential conductance and the excess noise in the weak-backscattering regime, taken from Ref. [11]. In the shot noise regime \( k_B T \ll e^* V \) the current in Eq. (5) follows specific power-laws \( I_B \propto V^{\alpha-1} \). Being in the shot noise regime one has the same power-laws in the excess noise \( S_{\text{exc}} \propto V^{\alpha-1} \). The exponent \( \alpha \) changes varying the voltages and it is related to the scaling dimensions in Eq. (4). In particular it is \( \alpha = g_c \) at very low energy, where the 2-agglomerate dominates. At higher voltages, where the single-qp dominates, it is possible to distinguish two different regimes. For \( e^* V \ll \omega_n \), where the neutral modes contribute to the dynamics, one has \( \alpha = g_c/4 + g_n/2 + 1/4 \), while for \( e^* V \gg \omega_n \) the neutral modes are ineffective and the exponent reduces to \( \alpha = g_c/4 \). In thermal regime \( k_B T \gg e^* V \) the conductance is independent on the voltage and scales with temperature like \( G_B|_{V=0} \propto T^{\alpha-2} \) while \( S_{\text{exc}} \propto V^2 \). In Fig. 1 we show experimental data and theoretical predictions for the backscattering differential conductance (top) and excess noise (bottom) at different temperatures. All curves are obtained fitting with the same values for the renormalization parameters \( (g_c = 2.8, g_n = 8.5) \) and neutral mode bandwidth \( (\omega_n = 150 \text{ mK}) \). We also assume that the tunneling coefficients associated to the single-qp \( (\gamma_1) \) and the 2-agglomerate \( (\gamma_2) \) could vary with temperature. The fitting has been validated by means of the standard \( \chi^2 \) test and shows an optimal agreement with the whole sets of data. Notice that the value of the neutral mode bandwidth is lower than \( \omega_c = 500 \text{ mK} \), which is of the order of the gap, according with the Ref. [18].

The backscattering differential conductance always presents a minimum at zero bias which is the signature of the ‘mound-like’ behavior generally observed for the transmission in the QPC geometry at very weak-backscattering [8]. For low enough temperatures, i.e. blue (short dashed) and cyan (long dashed) lines, one can see the dominance of the 2-agglomerate for low bias \( V \lesssim 50 \text{ mV} \) and a crossover region related to the dominance of the single-qp increasing voltages. At higher temperatures, where the single-qp contribution becomes relevant, the curves appear quite flat and voltage independent (dotted magenta and solid red lines). This is a signature of the ohmic behaviour reached in the thermal regime \( e^* V \ll k_B T \). Notice

\[ G_B(10^{-6} \text{S}) \]

\[ S_{\text{exc}}(10^{-30} \text{A}^2/\text{Hz}) \]

**FIG. 1.** Differential conductance \( G_B \) (top) and excess noise \( S_{\text{exc}} \) (bottom) as a function of voltage. Symbols represent the experimental data, corresponding to the sample indicated with the full circles in Fig. 5 of Ref. [11], with courtesy of M. Dolev. Different styles indicate different temperatures: \( T = 27 \text{ mK} \) (asterisks, short-dashed blue), \( T = 41 \text{ mK} \) (triangles, dashed-dotted cyan), \( T = 57 \text{ mK} \) (crosses, long-dashed green), \( T = 76 \text{ mK} \) (squares, dotted magenta), \( T = 86 \text{ mK} \) (circles, solid red). Fitting parameters are: \( g_c = 2.8, g_n = 8.5, \omega_c = 500 \text{ mK}, \omega_n = 150 \text{ mK} \) and \( (k_B = 1), \gamma_1 = |t_1|^2/(2\pi \nu_c)^2 = 3.1 \cdot 10^{-2}, \gamma_2 = |t_2|^2/(2\pi \nu_c)^2 = 1.2 \cdot 10^{-2}, 4.9 \cdot 10^{-2}, 4.2 \cdot 10^{-2} \) and \( 2 \cdot 10^{-2} \). Notice that the value of the neutral mode bandwidth is lower than \( \omega_c = 500 \text{ mK} \), which is of the order of the gap, according with the Ref. [18].
that the presence of renormalizations for the charged and neutral modes is crucial in the fit. Let us discuss now the excess noise curves. At high temperature (low bias) they present an almost parabolic behavior as expected for the thermal regime. Nevertheless this behavior is also present for $e^*V \gg k_BT$. This effect is not universal and it is due to the peculiar scaling dimension of the 2-agglomerate and to the value of the charge mode renormalization. At high bias ($V \approx 100 \mu V$) the lowest temperature curve deviates from the quadratic behavior as a consequence of the single-qp contribution.

In Fig. 2 we compare the effective charge $e_{\text{eff}}$ (triangles), calculated from our theoretical curves using a single parameter fitting procedure, with the results of Ref. [11] (circles with error bars). This result reinforces the idea that the evolution of the effective charge, as a function of the temperature, is essentially due to the crossover between the single-qp and the 2-agglomerate contributions.

**Conclusions.**—We fit recent experimental data on differential backscattering conductance and excess noise in a quantum point contact geometry for filling factor $\nu = 5/2$ in the weak back-scattering regime, demonstrating that the tunneling excitation has a charge double of the fundamental one at very low temperatures. In order to fit the experimental data, it is essential to assume the presence of interactions which renormalize the scaling behavior. We present a model for them in terms of the coupling of the system with the ubiquitous non-equilibrium $1/\hbar$ noise. This external coupling only affects the dynamical properties of the system, such as the scaling dimension, but does not change universal quantities, i.e. charge and statistics of the excitations. The presented phenomenology is also consistent with other models for the $\nu = 5/2$ state and it is not restricted to the considered anti-Pfaffian model.

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