The role of wetting-induced expansion of unsaturated soils in potential shallow landslides

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ABSTRACT

Unsaturated swelling soils expand in volume significantly during a wetting event (for example, rainfall infiltration, snow melting, or irrigation), which can lead to substantial change in the stress regime within and degradation of mechanical properties of the shallow soil mass. These changes can, though not completely, interpret many likelihoods of shallow landslides in swelling soils that has been frequently observed. In this work, an infinite slope formulation is proposed to address the mechanism of the shallow slope failure, where the unsaturated soil is described by the extended Mohr–Coulomb elasto-plastic constitutive law, in particular, the material parameters of the yield surface is altered according to the accumulation plastic deviatoric strain (extended to unsaturated soils from the classic softening), to simulate the degradation of mechanical properties. In this way, the influence of (i) the wetting induced swelling, (ii) the swelling induced stress change, and (iii) the associated softening behavior, on the stability of shallow layer is quantitatively examined. The proposed infinite slope formulation is applied to two typical swelling soil slopes: a synthetized one and a real field case. A good agreement between prediction and available experimental data from a published field study is obtained, illustrating the important implications for the engineering design of swelling soil slopes.

Keywords: swelling soils, shallow sliding, infinite slope, unsaturated soils

1 INTRODUCTION

Infiltration (rainfall, ground snow melting or other types) induced shallow failures of expansive (swelling) soil slopes are frequently reported in many countries around the world, including such as Canada (Fredlund and Widger 1979, Qi and Vanapalli 2015a,b,c), China (Hou et al. 2013; Xu et al. 2014, Qi 2017), Spain (Alonso et al. 2003; Azañón et al. 2010), and United States (Day 1994). The surficial layer of expansive soil slopes occurs in a state of unsaturated condition, and are likely to fail before attaining fully saturated condition. Most of the previous analyses were mainly conducted within the framework of saturated soil mechanics.

There is significant evidence of the strain softening behavior of unsaturated soil specimens in the literature from laboratory test results (e.g. Hoyos et al (2014), Miao (2002), Zhan et al. (2013) and Gui and Wu (2014)). Particularly, to simulate stress path within the slope soil subjected to rainfall infiltration, a different loading sequence (i.e. shearing infiltration: reducing the suction after shearing by increasing the net deviator stress to a prescribed level) was applied to unsaturated expansive soil specimens by Zhan et al. (2013) and Gui and Wu (2014) in a series of triaxial tests. The results also indicated a softening behavior.

The significant swelling of expansive soils upon wetting would be another important factor contributing to infiltration-induced shallow slope failures. This is because the swelling deformation can induce a constant and substantial change in the net stress regime of slope, especially within the surficial layer where the suction in soil element is reduced by water infiltration. Many previous laboratory measurements on expansive soils suggested that the lateral swelling stress can be several times higher than the vertical stress (Fourie 1989; Windal and Shahrour 2002; Boyd and Sivakumar 2011) under laterally confined condition. When the amount of non-uniform stress (in-situ stress plus the swelling-induced stress) state reaches the strength of soil element, local failure starts to occur at that particular point within the slope profile. The local failures within an unsaturated soil element can initiate the global failure of expansive soil slopes.

The swelling-induced stress and softening behavior observed from experimental results can likely contribute to the shallow sliding in in-situ expansive soils. The main objective of this study is to investigate their effect on expansive soil slope stability upon
infiltration in the framework of infinite slope formulation.

In this paper, the infinite unsaturated expansive soil slope is addressed through a general elasto-plastic constitutive relationship based on two stress state variables for unsaturated soils. The extended Mohr-Coulomb plasticity model that only needs some common soil parameters is adopted. The evolution of stress regime within the infinite slope profile upon infiltration is discussed. The infinite slope formulation is applied to two typical swelling soil slopes: a synthetized one and a real field case. A good agreement between prediction and available experimental data from a published field study is obtained, illustrating the important implications for the engineering design of swelling soil slopes.

2 ELASTO-PLASTIC CONSTITUTIVE MATRIX FOR UNSATURATED SOILS

For unsaturated soils, Fredlund et al. (1978) suggested using two independent stress state variables to describe the mechanical behavior and establish the constitutive relationships. The best combination of stress state variables for geotechnical applications is net stress and suction, s, which, respectively, refer to total stress in excess of pore air pressure, and pore air pressure in excess of pore water pressure. For most geotechnical problems, pore air pressure can be assumed to remain constant (usually atmospheric), the net stress is equivalent to the total stress, and suction, s, is equivalent to negative pore water pressure. The general simplified form of yield function for unsaturated soils, defined in terms of net stress, suction, and state parameters, can be expressed as

\[ F(\sigma_s, s, k) = 0 \]  

(2)

Correspondingly, the plastic potential function for unsaturated soils is of the form

\[ G(\sigma_s, s, g) = 0 \]  

(3)

where \( g \) is a vector of the state parameters.

The total incremental strains \( \{d\varepsilon\} \) for unsaturated soil includes four parts:

\[ \{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\} + \{d\varepsilon^s\} + \{d\varepsilon^i\} \]  

(4)

where \( \{d\varepsilon^e\} \) and \( \{d\varepsilon^p\} \) are the elastic and plastic incremental strains due to changes in net stress, and \( \{d\varepsilon^s\} \) and \( \{d\varepsilon^i\} \) denote the incremental elastic and plastic strains due to changes in suction, respectively.

The incremental net stress can be calculated as

\[ \{d\sigma\} = [D]\{d\varepsilon^e\} \]  

(5)

where \([D]\) is the elastic constitutive matrix.

The incremental elastic strain in response to suction change can be calculated as

\[ \{d\varepsilon^s\} = H^{-1}\{m\}ds \]  

(6)

where \( H \) is the elasticity parameter defined by Fredlund and Rahardjo (1993) for the soil structure with respect to incremental suction change, \( \{m\} \) is the vector \( \{1, 1, 1, 0, 0, 0\}^T \).

The incremental plastic strains are computed, via the flow rule, from the plastic potential function, as:

\[ \{d\varepsilon^p\} = \Lambda \left( \frac{\partial G}{\partial \sigma} \right) \]  

(7)

where \( \Lambda \) is the scalar plastic multiplier.

The plastic strain induced by change in suction is assumed to be equal to zero. Substituting Eq. (4) into Eq. (5) yields:

\[ \{d\sigma\} = [D]\{d\varepsilon^e\} + [D]\{d\varepsilon^s\} + \{d\varepsilon^i\} \]  

(8)

The consistency equation for unsaturated soil is written as:

\[ dF = \left( \frac{\partial F}{\partial \sigma} \right)^T \{d\sigma\} + \frac{\partial F}{\partial s} ds + \left( \frac{\partial F}{\partial k} \right)^T \{dk\} = 0 \]  

(9)

Follow similar procedure of derivation for saturated soils, the stress-strain relationship for elasto-plastic unsaturated soils reads:

\[ \{d\sigma\} = [D^\varepsilon]\{d\varepsilon\} + [D^\pi]ds \]  

(10)

where

\[ [D^\varepsilon] = \left[ D - \left[ D \left( \frac{\partial G}{\partial \sigma} \right) \left( \frac{\partial F}{\partial \sigma} \right)^T \right] \left[ D \left( \frac{\partial G}{\partial \sigma} \right) + A \right] \right] \]  

(11)

\[ [D^\pi] = -[D^\varepsilon]H^{-1}\{m\} - W^\pi \]  

(12)

\[ W^\pi = \left[ \frac{\partial F}{\partial s} \right] \left[ D \left( \frac{\partial G}{\partial \sigma} \right) + A \right] \]  

(13)

3 INFINITE SLOPE FORMULATIONS

3.1 Stress field

The problem in this study is idealized in Figure 1, where \( \eta \) and \( \xi \) are the axes rotated by a slope angle, \( \beta \), from the x and y axes in Cartesian coordinate system, respectively. The net normal stress, \( \sigma_\xi \), and shear stress, \( \tau_\eta \), at a certain depth can be determined by applying the static force equilibrium condition in the vertical direction for a unit width soil slice:

\[ \sigma_\xi = \gamma Z \cos^2 \beta \]  

(14)

\[ \tau_\eta = \gamma Z \cos \beta \sin \beta \]  

(15)

where \( \gamma \) stands for the unit weight of the soil, \( Z \) for the...
element depth under the ground surface measured along
the y axis. The net normal stress, \( \sigma_\eta \), in the \( \eta \) direction
can be related to net normal stress, \( \sigma_\xi \), by equation below:
\[
\sigma_\eta = K \sigma_\xi = K \gamma Z \cos^2 \beta
\]  
(16)
where \( K \) is the stress ratio of the net normal stresses \( \sigma_\eta \) to \( \sigma_\xi \).

In the present study, the extended Mohr-Coulomb shear strength envelope is utilized as yield criterion for
unsaturated expansive soils, and is expressed, in terms of suction, major and minor principal net stresses, as:
\[
F([\sigma],[s],[k])=(\sigma_{\eta}-\sigma_{\xi})-2\left(c' + s \tan \phi' - (\sigma_{\eta} + \sigma_{\xi}) \sin \phi' \right)
\]  
(17)
where \( \phi' \) is the friction angle relative to suction. As can be seen, the net stress and suction constitute the
complete stress state in the yield function, and the vector of state parameters of unsaturated soil includes
three components, namely, effective cohesion, effective angle of internal friction and friction angle relative to
suction. When considering the stress condition within the infinite slope (Fig. 1), Eq. (17) can be rewritten as:
\[
F([\sigma],[s],[k])
= 2\sqrt{0.25(\sigma_{\eta}-\sigma_{\xi})^2 + \epsilon_{\eta}^2} - 2 \cos \phi \left[c' + s \tan \phi' \right] - (\sigma_{\eta} + \sigma_{\xi}) \sin \phi'.
\]  
(18)
For the infinite slope case, the soil element in Figure 1 will have a swelling trend in all directions with a
decreasing suction. In the \( \eta \) direction, the normal strain induced by swelling is restrained due to the unlimited
extension of the slope, so that the net normal stress \( \sigma_\eta \) will be increased such that there is no normal strain in
this direction during a wetting process. This condition is similar to that of constant volume test conducted for
measuring the swelling pressure of expansive soil.

In the \( \xi \) direction, the soil close to the ground surface can swell freely; however, at a deeper depth, it
can only swell partially because of the influence of the overburden pressure. This scenario is similar to that of
loaded-swell oedometer test for measuring the swelling pressure of expansive soil. Certain magnitude of normal
strain in this direction can be induced by soil swelling.

However, the soil is not able to sustain unlimited
increase in the net normal stress in the \( \eta \) direction with
decreasing suction, since yielding and subsequent strain
softening are likely to occur once a certain amount of
irreversible plastic strain is accumulated. The evolution
of stress state acting on a soil element within the
infinite slope during wetting process can be generally
divided into three phases: pure elastic phase, followed
by perfectly plastic phase, followed by strain softening
phase. This is described using a practical strain
softening model for unsaturated soils as shown in
Figure 2.

![Figure 1. Geometrical scheme of the infinite slope.](image)

3.2 Collapse of the infinite slope

The collapse of an infinite slope can be interpreted by
recalling the traditional Factor of Safety (FS). The
traditional FS for infinite slope case is defined as:
\[
FS = \frac{\sigma_{\eta} \tan \phi' + s \tan \phi'}{\tau_{\eta}}
\]  
(19)
When FS=1, any infinitesimal decrease in suction can result in an indefinite plastic straining along the straight
line at this depth that forms the slip surface, since it is
kinematically admissible now.

4 DESCRIPTION OF THE ILLUSTRATIVE EXAMPLE

4.1 The seepage analysis for input of the infinite
slope formulation

The input of the infinite slope formulation analysis
is the variation of pwp or suction over time, which is
obtained from a seepage analysis in this study. In the
seepage analysis, the water flow through a domain of
interest can be modeled via SEEP/W (GeoSlope
International Ltd., 2012) by solving the well-known
Richards’ equation (1931) with Darcy’s Law
\[
\frac{\partial \theta}{\partial t} = \nabla (k \nabla H)
\]  
(20)
where \( \theta \) is volumetric water content, \( \nabla \) is the gradient
operator, \( t \) is time, \( k \) is the hydraulic conductivity of
soils, \( H \) is the total hydraulic head and equal to \( u_w / \gamma_w + y \),
where \( \gamma_w \) is the unit weight of water, \( y \) is elevation
and \( u_w \) is pore water pressure. \( u_w \) is positive in saturated

![Figure 2. Variation of mobilised material parameters
with plastic deviatoric shear strain](image)
zone and negative in unsaturated zone. Both the $\theta_w$ and $k$ are designated as nonlinear functions of negative pore water pressure in unsaturated zone, namely soil water characteristic curve (SWCC) and hydraulic conductivity function (HCF).

The closed-form van Genuchten (1980) and van Genuchten and Nielsen (1985) models are used to generate the two functional hydraulic properties (i.e. SWCC and HCF), which are summarized thru Eqs (21) to (24):

$$\theta_e = \frac{\theta_w - \theta_r}{\theta_w - \theta_r} \left[ \frac{1}{1 + (as)^n} \right]^m$$  \hspace{1cm} (21)

$$S_e = \frac{S_w - S_r}{1 - S_r} \left[ \frac{1}{1 + (as)^n} \right]^m$$  \hspace{1cm} (22)

$$k_e = \left( S_w - S_r \right) \left[ 1 - \left( S_w/S_r \right)^m \right]$$  \hspace{1cm} (23)

$$k = k \cdot k_e$$  \hspace{1cm} (24)

in which, both the effective volumetric water content $\theta_e$ and effective degree of saturation $S_e$ (calculated as function of respective current values ($\theta_w$, $S$), saturated values ($\theta_r$, 1) and residual values ($\theta_r$, $S_r$)) are related to suction $s$ using the same types of function with three fitting parameters $a$, $n$, $m$. The parameter $m$ is usually related to $n$ with $m = (n-1)/n$.

4.2 A hypothetical case based on Regina clay, Canada

The city of Regina is situated on a glacial deposit basin consisting of over-consolidated expansive clays, which is commonly referred to as Regina clay in the literature. This case analysis is performed with reference to slopes using Regina clay soil properties. The infinite slope analyzed in this study is assumed to have a thickness of 3 m. The infinite slope is assumed to be relatively gentle with a slope angle, $\beta = 18^o$, and have an initial stress ratio, $K = 1.5$. Table 1 summaries the hydraulic parameters used for this case. Table 2 summaries the mechanical parameters used herein.

| Parameters                              | Value |
|----------------------------------------|-------|
| Saturated volumetric water content     | 50.1  |
| Residual volumetric water content      | 10    |
| * Fitting parameter                    | 1     |
| * Fitting parameter                    | 1.062 |
| Saturated degree of saturation         | 95.4  |
| Residual degree of saturation          | 30    |
| † Fitting parameter                    | 1     |
| † Fitting parameter                    | 1.059 |
| Saturated permeability                 | 5×10⁻⁷|

Note: * van Genuchten fitting parameters for SWCC in terms of volumetric water content; † in terms of degree of saturation.

| Parameters                              | Value |
|----------------------------------------|-------|
| Slope angle                            | 18    |
| Initial stress ratio                   | 1.5   |
| Soil unit weight                       | 18.04 |
| Peak effective cohesion                | 0     |
| Residual effective cohesion            | 0     |
| Peak effective frictional angle        | 20    |
| Residual effective frictional angle    | 13    |
| Peak angle with suction                | 13.33 |
| Peak angle with suction                | 8.67  |
| Plastic deviatorial strain at peak     | 3.2   |
| Plastic deviatorial strain at residual | 12    |
| Poisson’s ratio                        | 0.4   |

The predicted evolution of net stress in the sloping direction, $\sigma_n$, at the several depths with time (i.e. 0.9 m at failure surface, and two other depths) are shown in Figure 3. Also, illustrated along with these are the timely evolution of two other parameters, namely, angle of internal friction and FS (see Figure 4 and Figure 5).

It can be seen there is a significant initial increase in the value of net stress, $\sigma_n$, at depth of 0.9 m, when suction is decreasing with time. The net stress reaches its maximum value when soil yields at 2.24 days, which is more than 3 times its initial value. After yielding, the net stress decreases with further decrease in suction until failure at 6.16 days. In the meantime, plastic straining develops within the soil at this depth. Once the accumulated plastic strain reaches a certain amount, soil softening occurs with reductions in the material parameters, for example, the effective internal friction angle starts to decrease from its peak value at 3.19 days. It should be noted that the net stress decreases at a slightly faster rate after softening than before. This is because, it is only the suction decrease that contributes to reduction in net stress before softening, in order to keep the stress state on the yield surface (i.e. the extended Mohr-Coulomb failure criterion), while after softening, stress is decreasing not only due to suction reduction but the change in the position of yield surface (reduction in the material parameters).

![Figure 3](image-url)
Similar trends can be observed for the net stress evolution at the depth of 2.0 m and 3.0 m, but they reach their maximum values and the point of softening later than those at 0.9 m depth, which is mainly due to the slower suction decrease rate at deeper depth. When the slope failure occurs at the depth of 0.9 m, soils at 2.0 and 3.0 m depths are in the softening phase. However, the FSs at these two depths are approximately around 1.4 (see Figure 5), indicating a fairly stable state at those depths.

4.3 A real case based on a field study, China

In order to better understand the failure mechanisms of expansive soil slopes upon wetting, a comprehensive field test was carried along an open channel in Zaoyang, Hubei, China Ng et al. (2003), of which the most interesting measurements would be the variation of the displacements induced by soil swelling upon infiltration in the shallow layer. Similarly, the seepage analysis is carried out for input of the infinite slope analysis, and the displacement is calculated by integrating the corresponding strain over the depth of infinite slope layer at several particular locations.

Fig. 6 compares the vertical swelling predicted using infinite slope model to those measured at three depths in the middle of the slope by Ng et al. (2003). The delayed soil swelling in response to the first rainfall are observed in both the predictions and measurements. It appears that the field study registered a longer duration of delayed response than predictions, particularly at the depth of 1.0 m. This may be attributed to instrumentation response delays. Nevertheless, the general trends that deeper embedded movement point registered a longer delayed swelling (Ng et al. 2003) are reflected in the simulation. The accumulated swelling increases rapidly from Day 2 to Day 3 after the soil starts to swell at all three depths, and remains at a relative stable value. The predicted stable values are in good agreement with the measurements for all three depths. It should also be noted that the recorded value of accumulative swelling is quite small, for example, in the order of a few millimeters at depth of 1.0 m. This illustrates the applicability of the infinite slope formulation.

5 CONCLUSIONS

The following conclusions can be drawn from the analysis of the results presented in this study:

(1) During wetting, the net stress in the sloping direction keeps increasing and reaches its maximum value at yielding, after which it decreases due to suction decrease and possible softening. The maximum value of net stress in the sloping direction can be several times the net stress perpendicular to the sloping direction. These phenomena are similar to those observed from the published experimental tests.

(2) The failures depths are not necessarily at the base of surface layer, this is different from those assumed for shallow landslide analysis based on saturated soil mechanics.

(3) Reasonable agreements in displacements between prediction and measurement are achieved using the infinite slope framework. This also conforms the practical applicability of model despite its simplicity.
ACKNOWLEDGEMENTS

This work is sponsored by the National Key R&D Program of China (2017YFC1501102), the National Basic Research Program of China (2014CB047005), and the Zhejiang Provincial NSF of China (LY12E08011). The first author gratefully acknowledges the China Scholarship Council and the University of Ottawa, Canada for funding his PhD research program.

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