How “cold” are superthin discs? An ‘MCMC’ approach.

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ABSTRACT

Superthin galaxies are a class of low surface brightness galaxies with strikingly high values of planar-to-vertical axes ratio \( b/a > 10^{-20} \) with little or no bulge component, possibly indicating the presence of an ultra-cold stellar disc, the origin, and evolution of which is not well understood. Using the 2-component model of gravitationally-coupled stars and gas in the external force field of a dark matter halo and assumed to be in vertical hydrostatic equilibrium, we determine the vertical velocity dispersion of stars and gas as a function of galactocentric radius for a sample of five superthin galaxies (UGC 7321, IC 5249, FGC 1540, IC2233 and UGC711) using observed stellar and atomic hydrogen gas (HI) scaleheights as constraints, and employing the Markov Chain Monte Carlo (MCMC) method. The mass models for our sample galaxies, constructed using stellar photometry and HI 21cm radio-synthesis observations were already available in the literature. We find that the central vertical velocity dispersion for the stellar disc in the optical band varies between \( \sigma_0 \sim 10^{-2} \) to \( 10^{-4} \) km/s and falls off with an exponential scale length of \( 2.6 \) to \( 3.2 R_D \) where \( R_D \) is the exponential stellar disc scale length. Interestingly, in the 3.6\( \mu \)m, the same, averaged over the two components of the stellar disc, varies between 5.9 to 11.8 km/s, mainly representative of the denser, thinner and smaller of the two-disc components. However, the dispersion of the more massive disc component varies between 15.9 - 24.7 with a scalelength of ~ 2.2 \( R_D \). Our calculated values of the N-component disc stability parameter \( Q_{\text{min}} \) lies between 1.7 - 5.7, thus confirming the dynamic stability of our model. Further, our model results are consistent with AGAMA (Action-based Galaxy Modelling Architecture).

Key words: galaxies: individual, galaxies:UGC7321, galaxies:IC5249, galaxies:FGC1540, galaxies:IC2233, galaxies: UGC00711, stability

1 INTRODUCTION

Superthin galaxies are a class of edge-on disc galaxies exhibiting strikingly high values of planar-to-vertical axes ratio 10-20, with no discernible bulge component. They are generally characterized by low values of central B-band surface brightness \( \mu_B = 23 - 26 \text{mag/arcsec}^2 \), gas richness as given by high values of the ratio of the total mass in neutral hydrogen gas (HI) to the blue band luminosity ratio \( M_{\text{HI}}/L_B = 1 \) (Uson & Matthews 2003) and dynamical dominance of dark matter at all galactocentric radii (Banerjee et al. 2010).

The term superthin was first introduced by Goad & Roberts (1981) who carried out a spectroscopic study of four edge-on galaxies: UGC 7321, UGC 7170, UGC 9242 and UGC 4278 (IC 2233). They were also studied as part of Flat Galaxy catalog (FGC) (Karachentsev et al. 1993) a survey which was an optical survey of flat and bulgeless galaxies in the local universe; 1150 out of 4000 FGC galaxies were found to be superthin (a/b > 10), as well as recent studies of very thin galaxies in SDSS (Bizyaev et al. 2016). Superthin galaxies are also subject of large HI surveys, see Matthews & van Driel (2000) and Giovanelli et al. (1997).

The origin of a superthin stellar disc in these galaxies is still not well understood, the vertical scaleheight of the stellar disc in galaxy is determined by a balance between the gradient of pressure in the vertical direction and the net vertical gravitational potential. Recent advances in IFU astronomy survey (MaNGA, SAMI, DISKMASS, CALIFA) have successfully estimated well resolved stellar velocity dispersion for face-on galaxies; however, due to the edge-on geometry of the superthin galaxies, the direct determination of the vertical velocity dispersion is not feasible. However the modeling the vertical velocity dispersion is crucial for understanding the origin and dynamical stability of the superthin discs. Zasov et al. (1991) showed that a massive dark mat-
ter halo was responsible for the stability of the superthin disc in UGC 7321, further Banerjee & Jog (2013) confirmed that a compact dark matter halo is responsible for the existence of superthin disc in UGC 7321. Interestingly (Banerjee & Bapat 2016) constructed mass models of three superthin galaxies (UGC 7321, IC 5249 and IC 2233) using stellar photometry and HI rotation curves and found out that $R_{e} / R_{d} \leq 1$, (Kurapati et al. 2018) have also found the same result for FGC 1540.

A plausible explanation for understanding the origins of the superthin structure might lie in the underlying disc heating mechanism. The vertical, radial, and the azimuthal velocity dispersion together constitute the stellar velocity ellipsoid, the shape of the stellar velocity strongly correlates with the morphological type of the galaxies, with early-type galaxies having isotropic velocity ellipsoid as compared to the late-type galaxies (Pinna et al. 2018). The superthin galaxies having isotropic velocity ellipsoid as compared to the morphological type of the galaxies, early-type galaxies having isotropic velocity ellipsoid as compared to the late-type galaxies (Pinna et al. 2018). The superthin galaxies are classified as late-type SD galaxies; thus we expect the ratio of the vertical velocity dispersion $\sigma_z$ to the radial velocity dispersion $\sigma_R$ to be highly anisotropic. Spiral arms, bars, Giant molecular clouds(GMCs), and satellite galaxies play an important role in heating the galaxy disc leading to anisotropy in velocity dispersion. Spiral arms in disc are an example of planar disc heating agent; they act to increase in the velocity dispersion in the radial direction but not in the vertical direction (Aumer et al. 2016), whereas GMC is an example of three-dimensional heating agent, whereas the increase in the vertical velocity dispersion is attributed to primarily to three-dimensional disc heating agents like GMCs (Jenkins & Binney 1990), GMCs heat the disc in both vertical and radial direction, so the ratio $\sigma_z / \sigma_r$ is an indicator of the relative importance of GMCs and spiral arms on disc heating. Grand et al. (2016) have shown that the time evolution of the bar strength in their N-body simulations correlates with the evolution of the global vertical energy of the star particles. On similar lines Saha (2014) have shown that galaxies hosting strong bars heat the disc very efficiently, leading to the formation of thick discs. In contrast, superthin galaxies possibly have weak bars that prevent strong vertical heating and help in preserving their superthin structure. Aumer et al. (2016) also finds that massive satellite galaxies and subhaloes act to heat the galaxies disc in the vertical direction significantly. Gerssen & Shapiro Griffin (2012) measure the value of $\sigma_z / \sigma_r = 0.3$ for late-type galaxies, and $\sigma_z / \sigma_r > 0.5$ for early-type galaxies.

In this paper we use our two-component model of gravitationally coupled stars and gas in the force field of the dark matter(Narayan & Jog 2002) to constrain the vertical stellar dispersion for five galaxies: UGC 7321, IC 5249, FGC 1540, IC2233 and UGC00711 in optical and 3.6 $\mu$m using observed vertical stellar scaleheight data using Markov Chain Monte Carlo method. The mass models for the above galaxies constructed using stellar photometry and HI radiosynthesis data available in the literature. Further, we check the consistency of our model using Action-based Galaxy Modelling Architecture (AGAMA)(Vasilyev 2018) by using our best-fit stellar dispersion as input and model the vertical scaleheights. Finally, we check the dynamic stability of our theoretical model by calculating two-component disc stability, as proposed by (Romeo & Wiegent 2011). We finally calculate the values of density-weighted average vertical velocity dispersion and compare the values thus obtained for the superthin with the value of vertical velocity dispersion for the Milkyway stars using GAIA data (Katz et al. 2018).

The paper is organized as follows: In section 2 we introduce the dynamical model describing the structure of the superthin galaxies and the modeling procedure, in section 3 we describe the basic structural properties of our sample superthin galaxies and outline the input parameters for our model in section 4, finally we present our results and conclude with a brief discussion of the results in section 5 and section 6, respectively.

## 2 DYNAMICAL MODEL OF THE GALAXY

### 2.1 Two-component-model

We model the galaxy as two concentric, co-planar axisymmetric disc of stars and gas which are gravitationally coupled to each other and are in the external force field of a rigid dark matter halo. We further assume that the discs are in vertical hydrostatic equilibrium and the velocity dispersion of the stars and HI-gas remains constant in the z-direction. The joint Poisson distribution in terms of galactic cylindrical coordinates $(R,\phi,z)$ is:

$$\frac{\partial^2 \Phi_{\text{total}}}{\partial z^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Phi_{\text{total}}}{\partial R} \right) = 4\pi G \sum_{i=1}^{2} \rho_i + \rho_{DM}$$

(1)

The angular terms vanish due to azimuthal symmetry, and assuming a constant rotation curve, the radial part disappears. Thus the Poisson’s equation reduces to

$$\frac{\partial^2 \Phi_{\text{total}}}{\partial z^2} = 4\pi G \sum_{i=1}^{2} \rho_i + \rho_{DM}$$

(2)

The equation of vertical hydrostatic equilibrium for the $i^{th}$ component of the disc ($i=$stars,gas) (Rohlfis 1977) is

$$\langle \sigma_z^2 \rangle = \frac{\rho_i}{\rho_{DM}} \frac{\partial \rho_i}{\partial z} + \frac{\rho_{total}}{\rho_{DM}} \frac{1}{\rho_i}$$

(3)

Combining the joint Poisson’s equation and the hydrostatic equation we get:

$$\frac{\partial^2 \rho_i}{\partial z^2} = \frac{4\pi G \rho_i}{\langle \sigma_z^2 \rangle} \left( \rho_i + \rho_{DM} \right) + \frac{\rho_{total}}{\rho_i} \frac{1}{\rho_i}$$

(4)

Where in $\rho_i$, $i$ stands for the density of stars and HI, $\langle \sigma_z^2 \rangle$ is the vertical velocity dispersion. At a given galacto-centric radius the above equation determines $\rho_i$ as a function of $z$.

The above equation describes the density fluid parcel consisting gravitationally coupled system of stars and HI gas in hydrostatic equilibrium. The equations are set of a coupled non-linear ODEs The above equation is solved iteratively using the Runge-Kutta method with initial conditions(Narayan & Jog 2002) at midplane $z=0$ given by:

$$\frac{dp_i}{dz}$$

(5)

and

$$\rho_i = (\rho_0)_{i}$$

(6)
The dark matter is modelled as a pseudo-isothermal profile (de Zeeuw & Pfenniger 1988)
\[ \rho_{\text{DM}}(r) = \frac{\rho_0}{(1 + \frac{r^2}{R^2})^{\frac{n}{2}}} \]  
(7)

\[ m^2 = R^2 + \frac{\zeta^2}{q^2} \]  
(8)

\[ \rho_0 \] is the central core density of the halo, \( R_c \) is the core radius, and \( q \) is planar to the vertical ratio of the halo. \( R_c \) and \( q \) are suitable parameters to mimic pseudo-isothermal dark matter.

We model superthin galaxies as consisting of a thin or ultra thin disc, a dark matter halo and twin \( \text{HI} \) disc describing double gaussian density profile. The stellar disc is modeled using a 'Disk' type potential with the scalelength(\( R_d \)), scaleheight(\( h_z \)), and surface density \( \Sigma_0 \) as given in table 1 and 2 respectively for optical and \( 3.6 \mu m \) respectively. The disk type potential is defined as:
\[ \rho = \Sigma_0 \exp\left(-\left[\frac{R}{R_d}\right]^2 - \frac{R_{\text{cut}}}{R}\right) \times \begin{cases} \delta(z) & \text{if} \quad h = 0 \\ \frac{1}{2}\exp\left(-\frac{h_z}{h}\right) & \text{if} \quad h > 0 \\ \frac{1}{2}\text{sech}^2\left(\frac{z}{2h}\right) & \text{if} \quad h < 0 \end{cases} \]  
(16)

Where, \( \Sigma_0 \) is the central surface brightness, \( R_d \) is the disc scalelength, \( h_z \) is the disc scalelength, \( R_{\text{cut}} \) is the disc inner cutoff and \( n \) is the sersic index.

The dark matter density is modeled using 'Spherial' type potential with \( a = 2 \), \( \gamma = 0 \), and \( \beta = 2 \). Where 'spherial' type density is given by
\[ \rho = \rho_0 \left(\frac{R}{a}\right)^{-\gamma} \left[1 + \left(\frac{R}{a}\right)^{-\frac{1}{2}}\right] \frac{2^{\frac{1}{2}\gamma}}{\sqrt{\pi}} \times \text{exp}\left[-\left(\frac{R}{r_{\text{cut}}}\right)^{\gamma}\right] \]  
(17)

Where \( \rho_0 \) is the central density, \( a \) is the core radius. The HI density is modeled using 'Disk' type density profile, with sersic index \( n = 0.5 \) and using the average HI scaleheight.

We add together the above densities to create a total density profile of the galaxy. Then we use the 'GalaxyModel' function to create a composite model of the galaxy using the total density profile and the quasi-isothermal distribution depicting the disk component. Using the tasks 'moments' and 'projectedMoments' we compute the radial and vertical stellar dispersion profiles and the corresponding scaleheight of the composite model of superthin galaxies.

2.2 Stellar kinematics with AGAMA

We use publicly available galactic dynamics code AGAMA Vasiliev (2018) \(^1\) for modelling our sample of galaxies, for comparing and checking consistency of our results. We model the stars as a double exponential disk with vertical and radial extent and HI as have a constant average scale-height and HI as have a constant average scale-height.

Mean vertical velocity dispersion

In 3.6\( \mu m \) the galaxy consists of two stellar disc; a thin disc with higher surface density and a thick disc with lower surface density. We define density averaged mean dispersion for depicting the average behaviour of both the galaxy discs in the following way:
\[ \sigma^2_{\text{avg}}(z) = \frac{\rho_1\sigma^2_{\text{in}} + \rho_2\sigma^2_{\text{out}}}{\rho_1 + \rho_2} \]  
(10)

2.3 Stability of the dynamical model.

Since an ultra thin disc structure implies an ultra cold disc, we need to check the dynamic stability of the galaxy model given the low values of stellar dispersion. The disc stability against local axis-symmetric perturbations is determined by the balance between the self gravity on one hand and combined effect of velocity dispersion as well as the centrifugal force due to coriolis spin up of the perturbations on the other hand.

\(^1\) https://github.com/GalacticDynamics-Oxford/Agama
The $Q$ parameter (Toomre 1964) for a one component rotating fluid disc is

$$Q = \frac{2\kappa \sigma^2}{\pi GM}$$

where $\kappa$ is the epicyclic frequency given by $\kappa^2 = -4B\Omega$, $B$ an $\Omega$ are the Oort constant and the angular frequency respectively, $\sigma$ is the radial velocity dispersion, and $\Sigma$ is the surface density at a given radius $R$. A value of $Q > 1$ implies a stable disc and $Q \leq 1$ is indicative of an unstable galactic disc which is characteristic of star forming regions.

Superthin galaxies are rich in gas, which may strongly regulate the disc dynamics closer to the midplane (Banerjee & Jog 2007). In addition, the dark matter dominates the dynamics at all radii. Hence, the galactic disc can no more be considered as a single component self-gravitating disc.

The galaxies in our sample consist of a stellar disc and HI disc in B-band and two stellar discs and a HI disc in 3.6 $\mu$m band, thus we rely on $Q_{RW}$ for explaining the stability of the galaxy in B-band and $Q_{N}$ for estimating the stability in 3.6 (Romeo & Falstad 2013), which is a generalization of $Q_{RW}$ to galaxy disc consisting of multiple components. The effective stability of two component system $Q_{RW}$ is defined as

$$\frac{1}{Q_{RW}} = \left( \frac{w_i}{T_1 Q_i} + \frac{1}{T_1 Q_i} \right), \text{if } T_i Q_i > T_1 Q_1$$

$$\frac{1}{Q_{RW}} = \left( \frac{w_i}{T_1 Q_i} + \frac{1}{T_1 Q_i} \right), \text{if } T_i Q_i < T_1 Q_1$$

The $Q_{RW}$ defined above is a modification to the effective two-component derived by Wang & Silk (1994) to include the effect of finite thickness of the galaxy disc. The $Q_{RW}$ is defined in a way as to provide less weightage to component with larger $Q$. Since the component with larger $Q$ is already stable so overall the stability condition depends on the less stable component, the weight function depends symmetrically on the ratio of the radial velocity dispersions and is defined as

$$W = \frac{2\sigma_s \sigma_g}{\sigma_s^2 + \sigma_g^2}$$

The finite thickness of the galaxy disc is a direct outcome of the velocity dispersion of the constituent stars and HI gas, the thickness of the galaxy disc embodies the stability of the galaxy disc and is a parametrized as the ratio of the vertical to the radial velocity dispersion of the component with a higher value of $Q$. The thickness correction is defined as

$$T = 0.8 + 0.7 \frac{\sigma_v}{\sigma_R}$$

A value of $Q_{RW} > 1$ indicates that the galaxy is stable against-axisymmetric perturbations. We take recourse to $Q_{N}$ Romeo & Falstad (2013) for studying the stability of the galaxy disc in 3.6$\mu$m where the galaxies are composed of two stellar discs and a HI, discs. The effective stability parameter $Q_{N}$ for a multi-component galaxy disc is defined as

$$\frac{1}{Q_{N}} = \sum_{i=1}^{n} \frac{W_i}{T_i Q_i}$$

The weight factor for the $Q_{N}$ is defined as

$$W_i = \frac{\sigma_m \sigma_i}{\sigma_m^2 + \sigma_i^2}$$

Where $i$ is the $i^{th}$ component of the galaxy and $m$ is the component with smallest $T_i Q_i$, i.e. $T_i Q_i = \min(T_i Q_i)$.

$Q_{N}$ has been applied by (Romeo and Falstad, 2013) to examine the stability of large sample of spirals from THINGS galaxy sample. The analysis was able to account for significance of $H_2$ in explaining the stability of the galaxy disc. $Q_{RW}$ was used by Romeo & Wiegert (2011) to re-examine the spiral galaxy from THINGS survey analysed by Walter et al. (2008) taking into account the stabilizing effect of finite disk thickness of the spiral galaxies. $Q_{RW}$ was also incorporated by Westfall et al. (2014) to study the relationship of disk stability with star formation rates. Westfall et al. (2014) found out that the disk averaged star forming surface density doesn’t correlate with the Toomre $Q$ for just gas or star but rather with multi-component $Q$ for studying star formation activity.

3 DESCRIPTION OF SUPERTHIN GALAXIES SAMPLE

3.1 UGC7321

UGC 7321 is a prototypical nearby superthin galaxy with $D = 10Mpc$ (Matthews 2000), $i = 88^\circ$ (Matthews et al. 1999) and axial ratio $a/b = 10.30$. Its characterized by a steeply rising rotational curve with an asymptotic velocity $100 km/s$ (Uson & Matthews 2003). The value of deprojected central surface brightness in B-band is 23.5 mag/arcsec$^2$ (Matthews et al. 1999). Roberts & Haynes (1994) have inferred from their study that the galaxy has large dynamic mass $M_{dyn}/M_{HI} = 31$ and $M_{dyn}/L_B = 29$, which highlights the importance of the dark matter in these galaxies. Dark matter modelling of UGC 7321 (Banerjee et al. 2010) reveals a compact dark matter with $R_c = 2.99kpc$ and $\rho_0 = 0.039 M⊙pc^{-3}$ in B-band. The dark matter parameters in 3.6 $\mu$m taken from Banerjee & Bapat (2016), are $\rho_0=0.140 M⊙pc^{-3}$ and $R_c = 1.27kpc$.

3.2 IC 5249

IC 5249 is an edge on superthin galaxy observed at an inclination $i = 89^\circ$ (Abe et al. 1999) with planar to vertical axis ratio $a/b = 10.2$. The galaxy has an asymptotic rotational velocity of about 112 km/s. The ratio of dynamic mass to mass of HI in galaxy is given $M_{dyn}/M_{HI} = 9.5$ and $M_{dyn}/L_B = 9.5$ (Yock et al. 1999), (Van der Kruit et al. 2001). Dynamical mass modelling of IC 5249 (Banerjee & Bapat 2016) predicts dark matter halo with $R_c = 2.99kpc$ and $\rho_0 = 0.026 M⊙pc^{-3}$.

3.3 FGC 1540

FGC 1540 is an edge on superthin galaxy observed at an inclination of $i = 87^\circ$ at a distance of $D = 10Mpc$ with an axial ratio $a/b = 7.47$ and is classified under the Flat galaxy catalogue (Karachentsev et al. 1993). The ratio of the mass of HI to the total blue luminosity is $M_{HI}/L_B = 4.1$. It is characterized by an asymptotic rotational velocity of about 90
How “cold” are superthin discs? An 'MCMC’ approach.

The characteristic core radius and axial ratio $a/b = 7$, and observed at an inclination of $\text{IC 2233}$ is a superthin galaxy characterized by a planar to $\rho_0$ at a distance of 10 Mpc. The galaxies in optical and $\text{UGC00711}$ were taken from Querejeta et al. (2015). The galaxies in our sample are composed of two stellar discs. The structural parameters for the same galaxies in optical and in $\text{UGC00711}$ were taken from Mendelowitz et al. (2000), and $\text{UGC00711}$ was obtained by the fitting double gaussian. The HI scaleheight for the galaxy $\text{IC2232}$ and $\text{UGC00711}$ was obtained by the fitting the FWHM vs $D_{HI}$ from O'Brien et al. (2010). We obtain $\text{FWHM} = 2.4 R_{HI} + 0.244$, where $D_{HI}$ is the HI diameter.

The dark matter profile parameters i.e central density $\rho_0$ and the core radius $R_c$ for the UGC 7321, IC 5249 and $\text{UGC00711}$ in both 3.6 $\mu$m and optical were taken from Kurapati et al. (2018). The dark matter parameters for IC2233 in r-band and 3.6 $\mu$m were obtained by mass modeling using "rotnod" task in gipsy (Van der Hulst et al. 1992), the stellar photometry (B-band) and HI surface densities for the $\text{UGC00711}$ were taken from Mendelowitz et al. (2000), and stellar photometry in 3.6 $\mu$m taken from Querejeta et al. (2015). The stellar photometry and HI surface densities for constructing a mass model of IC2233 in r-band were taken from Bizyaev et al. (2016) and Matthews & Uson (2007) respectively.

4 INPUT PARAMETERS

We model the vertical stellar dispersion of the superthin galaxies in optical and in 3.6 $\mu$m using observed stellar and HI scaleheight as a constraint. The stellar disc appears superthin in optical and traces the young stellar population. 3.6 $\mu$m band constitutes of old stellar population and contributes significant dynamical stellar mass of the galaxy and is free of dust extinction. The structural parameters for the stellar disc, i.e., central surface density, disc scale length, and scaleheight for UGC 7321 in B-band, were taken from Uson & Matthews (2003). The structural properties of the galactic disc for UGC00711 in B-band were taken from $\text{IC2233}$ in r-band and FGC1540 in i-band, the properties of the stellar disc were obtained from Bizyaev et al. (2016) and Kurapati et al. (2018) respectively.

In 3.6 $\mu$m, the galaxies in our sample are composed of two stellar discs. The structural parameters for the same were taken from Querejeta et al. (2015). The galaxies in optical and 3.6$\mu$m are characterized by a constant stellar scaleheight and exponentially decreasing surface density, which translates into vertical velocity dispersion, which decreases exponentially with radius. The HI surface density for UGC 7321 was taken from Uson & Matthews (2003) and fitted with a double Gaussian profile, and the structure of HI surface density for IC 5249 was taken from Van der Kruit et al. (2001) by fitting a double gaussian to the observed profile. For FGC 1540, the parameters for HI surface density were extracted from the surface density profile in Kurapati et al. (2018) by fitting double gaussian. The HI surface density for IC2233 and UGC00711 were obtained from Matthews & Uson (2007) and Mendelowitz et al. (2000), by fitting double Gaussian profiles.

The HI scaleheight for UGC 7321 and IC 5249 were obtained from O’Brien et al. (2010) by converting the FWHM to scaleheight. For FGC 1540, we could not find accurate HI scale height data, so we used approximately constant HI scaleheight of 0.400kpc. HI scaleheight for the galaxy $\text{IC2232}$ and $\text{UGC00711}$ was obtained by the fitting the FWHM vs $D_{HI}$ from O’Brien et al. (2010). We obtain $\text{FWHM} = 0.5 D_{HI} + 0.44$, where $D_{HI}$ is the HI diameter.

5 RESULTS & DISCUSSION

5.1 UGC 7321

In figure (1), we present results obtained from dynamical modeling of the UGC 7321 using observed stellar and HI scaleheight constraint in B-band. The central stellar dispersion in B-band is $\sigma_0 = 10.23 \pm 0.64$ km/s which falls off ex-

| Parameters | UGC7321 | UGC0711 | IC2233 | FGC1540 |
|------------|---------|---------|--------|---------|
| $\mu_{01}$ | 23.5    | 25.5    | 22.90  | 20.60   |
| $\mu_{02}$ |        |         |        |         |
| $\Sigma_{01}$ | 34.7 | 15.0    | 17.58  | 88.79   |
| $\Sigma_{02}$ |        |         |        |         |
| $R_{11}$   | 2.1     | 1.6     | 2.47   | 1.29    |
| $R_{12}$   |        |         |        |         |
| $h_{01}$   | 0.150   | 0.317   | 0.332  | 0.185   |
| $h_{02}$   |        |         |        | 0.675   |

\[ a \] Central surface brightness of stellar disc (mag.arcsec$^{-2}$) 
\[ b \] Central surface brightness of stellar disc(mag.arcsec$^{-2}$) 
\[ c \] Central stellar surface density($M_\odot$pc$^{-2}$) 
\[ d \] Central stellar surface density($M_\odot$pc$^{-2}$) 
\[ e \] Disk scale length for exponentially falling disc (kpc) 
\[ f \] Disk scale length for exponentially falling disc(kpc) 
\[ g \] Stellar scale height (kpc) 
\[ h \] Stellar scale height (kpc)
Table 2. Stellar parameters of sample-galaxies in 3.6 $\mu$m-band.

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\mu_0$ | 21.73 | 21.73 | 22.23 | 21.67 | -- |
| $\Sigma_0$ | 7.165 | 5.44 | 3.37 | 5.59 | 14.6 |
| $R_{d1}$ | 2.39 | 5.24 | 1.85 | 2.16 | 2.14 |
| $h_1$ | 0.436 | 0.724 | 0.43 | 0.39 | 0.44 |
| $\mu_0$ | 19.9 | 20.53 | 21.39 | 20.53 | -- |
| $\Sigma_0$ | 37.26 | 15.97 | 8.167 | 12.2 | -- |
| $R_{d2}$ | 1.0 | 1.23 | 0.54 | 0.81 | -- |
| $h_2$ | 0.134 | 0.253 | 0.152 | 0.08 | -- |

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\Sigma_0$ | 4.912 | 3.869 | 4.09 | 2.236 | 30.83 |
| $\Sigma_0$ | 2.50 | 4.85 | 1.3 | 2.454 | -- |
| $\alpha_1$ | 3.85 | 5.92 | 2.48 | 2.52 | -- |
| $\alpha_1$ | 0.485 | 17.06 | 5.08 | 6.14 | -- |
| $R_0$ | 2.85 | 3.35 | 5.73 | 1.79 | 3.73 |
| $R_0$ | 1.51 | 4.05 | 1.02 | 1.69 | -- |

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\mu_0$ | 0.039 | 0.05 | 0.0457 | 0.308 |
| $R_c$ | 2.99 | 2.9 | 1.84 | 0.64 |

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\rho_0$ | 0.140 | 0.026 | 0.319 | 0.055 | 0.033 |
| $R_c$ | 1.27 | 2.99 | 0.63 | 1.83 | 2.95 |

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\Sigma_0$ | 4.912 | 3.669 | 4.09 | 2.236 | 30.83 |
| $\Sigma_0$ | 2.50 | 4.85 | 1.3 | 2.454 | -- |
| $\alpha_1$ | 3.85 | 5.92 | 2.48 | 2.52 | -- |
| $\alpha_1$ | 0.485 | 17.06 | 5.08 | 6.14 | -- |
| $R_0$ | 2.85 | 3.35 | 5.73 | 1.79 | 3.73 |
| $R_0$ | 1.51 | 4.05 | 1.02 | 1.69 | -- |

Table 3. Input parameters for HI disk.

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\rho_0$ | 0.039 | 0.05 | 0.0457 | 0.308 |
| $R_c$ | 2.99 | 2.9 | 1.84 | 0.64 |

Table 4. Input parameters of dark matter halo in optical-band.

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\rho_0$ | 0.140 | 0.026 | 0.319 | 0.055 | 0.033 |
| $R_c$ | 1.27 | 2.99 | 0.63 | 1.83 | 2.95 |

Table 5. Input parameters for HI disk.

| Parameters | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|------------|---------|--------|---------|--------|------|
| $\rho_0$ | 0.039 | 0.05 | 0.0457 | 0.308 |
| $R_c$ | 2.99 | 2.9 | 1.84 | 0.64 |

$^a$ Central HI surface brightness fitted to double gaussian ($M_0pc^{-2}$)
$^b$ Central HI surface density fitted to double gaussian ($M_0pc^{-2}$)
$^c$ Disk offset kpc
$^d$ Disk offset kpc
$^e$ HI disk scalelength kpc
$^f$ HI disk scalelength kpc

model comprising of a thin disc, a dark matter halo, and the HI distribution as a sersic profile with a constant scaleheight. Using the values of the stellar dispersion obtained from the two-component model in AGAMA, we note that the values of scaleheight calculated with AGAMA comply with that from the two-component model.

In figure (2) we present the results for UGC 7321 in 3.6 $\mu$m. UGC 7321 in 3.6 $\mu$m consists of a thick and thin stellar disc. The central value of vertical velocity dispersion for the thick disc is 24.66±0.88 km/s which falls off exponentially with disc scale length (2.15±0.60)$R_{d1}$, where $R_{d1}$ is the scalelength of the thick disc, then central dispersion for the thin disc is 9.02±0.8 km/s and falls off exponentially with scalelength (4.55±0.68)$R_{d2}$, where $R_{d2}$ is the scalelength of the thin disc. The value of density averaged vertical velocity dispersion is 11.58km/s, the average dispersion in the inner radius indicates mean behavior up to 3$R_d$ of the thick disc($R_d = 2.39kpc$), beyond which it falls off exponentially following the thick disc dispersion profile.

The value of central HI dispersion in B-band is 11.9±0.84 km/s, the steepness parameters are $a_{HI} = 0.29±0.14$ and $b_{HI} = 0.0$. We note that the central value of HI dispersion is higher than the central stellar dispersion of the thin disc. In contrast, the vertical dispersion of the thick disc exceeds the vertical dispersion of both HI and thin stellar disc. The density averaged stellar dispersion roughly equals the HI dispersion, which is constant at all galactocentric radius. In panel 2 (figure 2), we have calculated the multi-component $Q_N$, including the contribution of the thick and thin stellar disc and the HI disc, we note that the minimum value of $Q_N$ is 2.9 at 5$R_d$, beyond the physical extent of the stellar disc, indicating that the disc is stable against non-axis-symmetric perturbations at all radius. In panel 3 (figure 2) we compare the results obtained from the two-component model AGAMA, we set up an equilibrium distribution consisting of dark matter halo, thin and thick stellar disc and a rigid HI disc, we note that the scaleheights calculated from AGAMA agree with that from the two-component model.

We also note that the velocity dispersion of the thin disc in 3.6 $\mu$m and the stellar dispersion in B-band are comparable, and also the stellar disc in B-band has a scale length of about 2.1 kpc and the thin disc in 3.6 $\mu$ has a scale length

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$a$ Central surface brightness of stellar disc I in 3.6$\mu$m (mag arcsec$^{-2}$)
$b$ Central surface brightness of stellar disc II in 3.6$\mu$m (mag arcsec$^{-2}$)
$c$ Stellar scale height for disc I (kpc)
$d$ Stellar scale height for disc II (kpc)
$e$ Stellar scale height for exponentially falling disc for disc I (kpc)
$\alpha$ Stellar scale height for exponentially falling disc for disc II (kpc)
$f$ Stellar scale height for disc II (kpc)

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$a$ Dark matter density for pseudo-isothermal profile ($M_0pc^{-3}$)
$b$ Dark matter core radius for pseudo-isothermal profile (kpc)

dependently with scalelength of (2.58±0.613)$R_d$. The central value of HI dispersion 11.06±0.88 km/s and steepness parameters are $a_{HI} = 0.18±0.07$ and $b_{HI} = 0.047±0.02$. We note that the stellar dispersion and the HI dispersion are comparable. The value of $\sigma_v(r)$ falls off exponentially as a function of the galactocentric radius, whereas value of HI dispersion is almost constant at all radius. In panel 2 (figure 1) we have calculated the $Q_{RW}$ as function of galactocentric radius, we note that the minimum value of $Q_{RW}$ is 2.7 at about 5$R_d$, and typically the stellar disc doesn’t extend beyond 3$R_d$, indicating that UGC 7321 is stable against growth of non-axis symmetric perturbations. In panel 3 (figure 1) we compare the results from the two-component model with the publicly available stellar dynamics code AGAMA, we create a

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$a$ Dark matter density for pseudo-isothermal profile ($M_0pc^{-3}$)
$b$ Dark matter core radius for pseudo-isothermal profile (kpc)
How “cold” are superthin discs? An ’MCMC’ approach.

Figure (4) shows the results obtained from dynamical modeling of the superthin galaxy IC 5249 in 3.6 μm, using observed stellar and HI scaleheight data as constraints. We have modeled IC5249 in only 3.6 μm due to the unavailability of consistent photometry and mass modeling results in the optical band.

We find that the central value of the vertical velocity dispersion ($\sigma_{\mu_0}$) is 20.64±0.6 km/s, which falls off exponentially with scalelength (2.155±0.217)Rd, and the central dispersion of the thin disc is 9.32±0.39 km/s which falls off exponentially with scalelength (7.54±0.23)Rd. The density weighted velocity dispersion is 11.08 km/s/σ, σmean is almost constant at the inner radius, up to disc scale length of the thick disc (5.24 kpc), beyond which it falls off exponentially following the dispersion profile of the thick disc.

The HI dispersion is almost constant at all galactocentric radius, the central value is $\sigma_{HI}=12.4±0.53$, and steepness parameters are $a_{HI}=0.99±0.11$, and $b_{HI}=0.04±0.013$. We note the value of the stellar dispersion of the thick disc exceeds that of the HI disc and the thin stellar disc, whereas the density weighted mean stellar dispersion is slightly lower than the central HI dispersion. The value of stellar dispersion of the thick disc falls off exponentially with the galactocentric radius, although higher at the center as compared with the central HI dispersion. In panel 2, we have calculated $Q_N$ for the composite thick, thin, and the HI disc system, with the minimum value of 1.7, which is the lowest value of $Q_N$ in our galaxy sample. Unlike, UGC 7321 the (QN)min in IC5249 appears within 3Rd of the thick disc, indicating that the disc may be prone to the growth of local axis-symmetric instabilities.

In panel 3 we compare the results from the two-component model with AGAMA, we create a composite AGAMA model consisting of thin and thick stellar disc, HI disc and a dark matter density. We note that the AGAMA calculates lower scaleheights in the inner region and higher scaleheights in the outer region for the thick disc, when compared with the two-component model, and calculates higher scaleheight than the two-component model for the thin stellar disc.

The vertical velocity dispersion for the thin disc is 9.32 km/s, which is comparable to the density weighted dispersion value, i.e., 11.08 km/s possible, indicating the presence of cold young stars emitting in near-infrared. The velocity dispersion of the thin disc stars is slightly lower than HI dispersion, indicating the presence of thin and cold molecular clouds from which these stars were formed and hence have a value of vertical dispersion lower than of HI dispersion.

The value of velocity dispersion for the thin disc obtained from one component model is 10.3 km/s compared with 20.64 km/s obtained from two-component model, (Mstars/MHI)thick = 0.63, indicates that HI significantly contributes to the mass enclosed within the scaleheight of the thick disc, which is not being accounted for in the one-component calculation, and thus the disparity. Similarly, the value of vertical dispersion for the thin disc from the one-component calculation and two-component models are
10.4 km/s and 9.32 km/s, and the value of \((M_{\text{stars}}/M_{\text{HI}})_{\text{thin}}\) within the scaleheight of the thick disc is 1.87. Thus we can neglect the HI mass and explain the agreement of the one-component results with the values of dispersion obtained from two-component models.

Calculating the moments of the distribution function corresponding to equilibrium model of IC 5249, we find that \(\sigma_{01}\) and \(\sigma_{0R}\) corresponding to the thick disc are 18.51 km/s and 35.0 km/s and \(\sigma_{01}/\sigma_{0R}=0.52\). The value of \(\sigma_{01}\) and \(\sigma_{0R}\) for the thin disc are 8.36 km/s an 16.1 km/s and \(\sigma_{01}/\sigma_{0R}=0.51\). The equilibrium models constructed using AGAMA predict a slightly colder disc as compared to the two-component model. The values of stellar velocity are typical values observed for late-type galaxies.

We note that value of the dispersion for the thick disc and thin disc for IC5249 is comparable with that obtained for UGC 7321 in 3.6 \(\mu\)m, interestingly. However, the scaleheight thick disc in IC 5249 (0.724 kpc) is higher than the thick disc scaleheight of UGC 7321 (0.435kpc), the central dispersion of IC 5249 is slightly lower than that for UGC7321. In figure (5), we have shown the two-dimensional posterior distribution plots obtained from MCMC fitting. We note that posterior distribution for our parameters shows deviation from gaussianity, it can be seen from the contours that the almost all the parameters are independend of each other, except for \(\sigma_{01}\) and \(\alpha_2\) which show anti-correlation and \(\sigma_{02}\) and \(\alpha_2\) which are correlated.

### 5.3 FGC1540

In figure (6), we present results from dynamical modeling of the galaxy FGC 1540 in i-band. FGC 1540 in i-band consists of a thick and thin disc, with equal scalelength, 1.29 kpc. The value of the central stellar dispersion for the thick disc is 36.91±1.14 km/s and falls off exponentially with scale length 3.72±0.42\(R_{d1}\), where \(R_{d1}\) is the scalelength of the thick disc, the central dispersion for the thin disc is 13.08±1.17 km/s and the exponential scale length is 3.32±0.42\(R_{d2}\), where \(R_{d2}\) is the scalelength of the thin disc. We note that the average stellar dispersion is about 16.8 km/s, and closely follows the exponential fall-off of the thin stellar disc. Due to the unavailability of the HI scaleheight data, we use a constant HI scaleheight of about 400 kpc at all radius. The minimum value of multi-component \(Q_N\) is 1.9. See panel 2. In panel 3, we compare the results obtained from AGAMA with the two-component model; we note AGAMA calculates lower scaleheight than the observed, for the values of velocity-dispersion obtained from the two-component model. In fig-

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**Figure 1.** Panel 1 describes the vertical velocity dispersion profile UGC 7321 in B-band obtained from two component model. In panel 2 is rendered the stability parameter \(Q_{\text{RW}}\) appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

**Figure 2.** Panel 1 describes the vertical velocity dispersion profile UGC 7321 in 3.6\(\mu\)m obtained from two component model. In panel 2 is rendered the stability parameter \(Q_N\) appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.
How “cold” are superthin discs? An ‘MCMC’ approach.

Figure 3. MCMC best fit parameters for UGC 7321 in B-band (panel-1) and 3.6 µm (panel-2) indicated by the peaks of normal distribution, the contours indicates the all possible combinations of the parameter-space over which the model was optimised for finding the best fit parameters by MCMC.

Figure 4. Panel 1 describes the vertical velocity dispersion profile IC5249 in 3.6 µm obtained from two component model. In panel 2 is rendered the stability parameter $Q_N$ appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

In Figure (7), we discuss results pertaining to the modeling FGC 1540 in 3.6 µm. Like in i-band, FGC 1540 consists of a thin and thick disc in 3.6 µm, where the thick disc has a larger scale length (1.57 kpc), and the thin disc has smaller scale length (0.54 kpc). The central vertical dispersion of the thick disc is 16.2 ± 0.87 km/s and falls off exponentially with scale-length $3.77 \pm 0.42 R_d$, and the central vertical dispersion of the thin disc is 6.86 ± 0.57 km/s, and the exponential fall-off scale length is 6.0 ± 0.2. The density averaged velocity dispersion is 8.63 km/s; it is roughly constant within the inner radius up to 2 kpc, thereafter it falls off exponentially following the dispersion profile of the thick disc. The minimum value of $Q_N$ is 2.9, indicating the disc can resist the growth of axisymmetric instabilities, see panel 2. In panel 3, we compare the results obtained from the two-component model with the results from AGAMA; we note that AGAMA underestimates the scaleheight as compared with the two-component model, in the inner radius in case of both the thin and the thick disc. The scaleheight calculated from AGAMA agrees with the two-component model for the thin disc at the outer radius and is higher than the two-component model for the thick disc. Using one-component formula, we find the velocity dispersion of the thick disc is 6.25 for FGC 1540 in 3.6 µm and the same from two-component model is 6.2 km/s; we note that the value of $(M_{stars}/M_{HI})_{thick}$ is 0.62, thus when calculating the value of the stellar scaleheight we have effectively neglected the HI content within one scaleheight of the thick disc when using one component formula, and the same is taken care of when we use the two-component model. In case of the thin disc the value of the dispersion ob-
We note that the value of the stellar dispersion of the thick disc is higher in the i-band as compared to the 3.6 μm. Also the exponential fall-off length of the thin disc, is about 12 R_⊙, indicating the velocity dispersion of the thin disc falls off very slowly with the radius in 3.6μm. The minimum value of Q_N in i-band is comparable with that obtained for IC 5249 and indicates that the galactic disc locally might be susceptible to the growth of axis-symmetric instabilities. In figure (8), we present the two-dimensional posterior plots; we note that there are no strong correlations among the parameters.

5.4 IC2233

In figure (9), we describe the results from the modeling of the IC2233 in r-band, the central velocity dispersion is 14.9±0.57 km/s, and the scalelength exponential fall-off is 2.36±0.36. In panel 2 we have plotted the Q_N as function of the galactocentric radius, we find that minimum value of Q_N is 2.24, which indicates that the disc is stable against growth of axisymmetric instabilities. In panel 3 we compare the results obtained from the two-component model with AGAMA, we calculate the scaleheight of the IC2233 using AGAMA, using the values of the velocity dispersion obtained from the two-component model, we note that AGAMA, predicts slightly higher scalelength than the two-component model. In figure (10), we present the results obtained from dynamically modeling of IC2233 in 3.6 μm, the vertical velocity dispersion of the thick disc is 15.97±0.54 km/s and falls off exponentially with scalelength (2.16±0.42)R_⊙. Whereas the values for the thin disc are 3.9±0.23 and (6.0±0.2)R_⊙. The density averaged velocity dispersion is about 5.98 km/s, and is roughly constant up to 2R_⊙. In panel 1 we have plotted the multi-component stability parameter as a function of galactocentric radius; minimum value of Q_N is 5.74, which implies that the disc is highly stable against axisymmetric instabilities. In panel 3 we compare our results, we find that the values of scaleheight values calculated using AGAMA agree with the two-component model, although AGAMA calculates slightly higher scaleheight in the and slightly higher scaleheights in the inner and the outer radius respectively.

The value of velocity dispersion from the one-component model for IC2233 in r-band is 14.9 km/s and from the two-component model is 12.5 km/s, the value of (M_{stars}/M_{HI}) is 3.7. Since the stellar mass dominates the HI mass, thus the vertical dispersion given by the one-component formula agrees with the value from the two-component model. In 3.6 μm the vertical velocity dispersion for the thick disc using one-component formula is 7.67 km/s, and the same from two-component model is 15.97, the value of (M_{stars}/M_{HI}) is 1.19, so we see that the mass of HI is equally important in determining the vertical velocity dispersion. Thus we can’t just resort to one component calculation for estimating vertical velocity dispersion. The vertical velocity dispersion for the thin disc calculated using the one-component formula is 5.13 km/s and that due to the two-component model is 3.9 km/s, (M_{stars}/M_{HI}) is 2.60, which implies the contribution of the stellar mass is more important in determining the vertical velocity dispersion. Thus we may in use the one component formula in principle for determining the vertical velocity dispersion when the stellar mass dominates HI.
Figure 6. Panel 1 describes the vertical velocity dispersion profile FGC1540 in i-band obtained from two component model. In panel 2 is rendered the stability parameter $Q_N$ appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

Figure 7. Panel 1 describes the vertical velocity dispersion profile FGC1540 in 3.6$\mu$m obtained from two component model. In panel 2 is rendered the stability parameter $Q_N$ appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

Figure 8. MCMC best fit parameters for FGC 1540 in i-band and 3.6$\mu$m
The value of vertical velocity dispersion of the thin disc in 3.6 $\mu$m is 3.9 km/s. The corresponding scaleheight is 0.08 kpc, the density weighted vertical dispersion is 5.98 km/s. The low values of vertical dispersion of the thin stellar disc in 3.6$\mu$m indicates possible near-infrared emissions from cold young stars. In optical(r-band) the vertical scaleheight(0.332 kpc) is comparable to that in 3.6 $\mu$m (0.39kpc). Also the vertical dispersion of the thick disc in optical is comparable to the same in 3.6$\mu$m, thus the optical(r-band) traces the old stellar population.

We find that the value of $\sigma_{\text{c}}$ and $\sigma_{\text{OR}}$ for the equilibrium models of IC2233 in r-band using AGAMA are 13.5 km/s and 25.04 km/s, with the values of stellar velocity ellipse $\sigma_{\text{c}}/\sigma_{\text{OR}}$=0.53. In 3.6 $\mu$m the value of $\sigma_{\text{c}}$ and $\sigma_{\text{OR}}$ for the thick disc are 16.11 km/s and 39.8 km/s respectively with the value of $\sigma_{\text{c}}/\sigma_{\text{OR}}$ being 0.4, where as in case of the thin disc the value of $\sigma_{\text{c}}$ and $\sigma_{\text{OR}}$ are 3.62 km/s and 13.02 km/s and the value of the stellar velocity ellipse being 0.28.

We note that the velocity dispersion in the r-band and the velocity dispersion of the thick disc in 3.6$\mu$m are comparable. The galaxy also shows exceptional stability in 3.6$\mu$m.

When compared with the other galaxies in our sample, in 3.6 $\mu$m, the thin disc usually has a higher value of central dispersion and a smaller value of exponential fall-off scalelength 1, whereas the disc thin has a larger exponential fall-off scale $\sigma_2$ and lower dispersion. The value of the central velocity dispersion of the thin disc is unusually small, indicating a very ‘cold’ stellar disc. In figure (11), we present the two-dimensional posterior plots.

### 5.5 U 711

In figure (11), we describe the results obtained from the dynamical modeling of the galaxy UGC00711 in B-band, unlike other galaxies in our sample UGC00711 in both B-band and in 3.6$\mu$m has only a single stellar disc. The central vertical velocity dispersion is 18.4± 0.87 km/s, and falls off exponentially with scale length (3.21±0.21)R$_d$. In panel 2 we have plotted $Q_N$ as a function of galactocentric radius, the minimum value of $Q_N$ is 4.46, which is much higher than the threshold $Q_N > 1$, indicating that the disc is stable against axis-symmetric systems. In panel 3 we compare our results with equilibrium model of UGC00711 created in AGAMA, using the values of values of velocity dispersion obtained from the two-component in AGAMA, we find that scaleheight calculated by AGAMA are slightly higher at the inner radius at higher in the outer radius, compared to the two-component model. In figure (12), we present the results for UGC00711 in 3.6 $\mu$m, the central velocity dispersion is 23.82±1.45 km/s, the dispersion falls off exponentially with scale length (2.42±0.28)R$_d$. From panel 2 where we have plotted $Q_N$ as a function of galactocentric radius, the minimum value of $Q_N$ is 4.31. In panel 3, we see that scaleheight calculated using AGAMA is higher than that due to the two-component model in the inner radius, but is lower in the outer radius.

The value of the velocity dispersion in B-band calculated using the one-component formula is 11.6 km/s and the value from two-component model is 18.4 km/s, the value of (M$_{\text{stars}}$/M$_{\text{HI}}$) is 0.48, so effectively we are neglecting the HI content inside the stellar scaleheight, when calculating the dispersion when using one component formula. Similarly in 3.6 $\mu$m, the value of the velocity dispersion from one component formula is 11.6 km/s and that from 18.4 km/s, the value of (M$_{\text{stars}}$/M$_{\text{HI}}$) is 0.47, so again we see the importance of including HI when calculating the vertical velocity dispersion of stars when HI makes up a considerable mass fraction.

UGC00711 in both B-band and 3.6$\mu$m consists of a single stellar disc, the scaleheight in both 3.6$\mu$m(0.44 kpc) and B-band(0.317 kpc) are comparable, and Similarly, the vertical velocity dispersion is also comparable; 18.4 km/s in B-band and 23.8 km/s in 3.6$\mu$m, the values of optical dispersion indicates lack of emissions from cold young stellar component in B-band.

The value of the vertical and the radial velocity dispersion estimated using AGAMA by computing the moments of the distribution function representing UGC00711 in B-band are $\sigma_{\text{c}}=15.84$ km/s and $\sigma_{\text{OR}}=29.04$ km/s and the value of the stellar velocity ellipse $\sigma_{\text{c}}/\sigma_{\text{OR}}$ is 0.54. Whereas in case of 3.6 $\mu$m the value of $\sigma_{\text{c}}$ and $\sigma_{\text{OR}}$ are 19.3 km/s and 34.68 km/s and the value of the stellar velocity ellipse $\sigma_{\text{c}}/\sigma_{\text{OR}}$ is 0.55.

We note that the galaxy is highly stable in both 3.6 $\mu$m and in B-band with a stability parameter of about 4.7, the values of the central dispersion and exponential fall-off length are comparable. We also note that the value of vertical dispersion of UGC00711 in B-band and 3.6$\mu$m are comparable to the values of the vertical velocity dispersion that we have obtained for the thick disc in 3.6 $\mu$m. In figure 7 we have plotted the two-dimensional posterior distribution, we note that there are no strong correlations between the parameters and that the posterior distribution shows deviation from gaussianity.

### 6 CONCLUSION

Superthin galaxies are class of low surface brightness galaxies with strikingly high values ofplanar to vertical axes ratio(b/a)*10-20 with little or no bulge component possibly indicating the presence of ultra-cold stellar disc,thetorigin and evolution of which is not well understood.Using the 2-component model of gravitationally-coupled stars and gas in vertical hydrostatic equilibrium and in the external force field of the dark matter halo, we determine the radial profile of vertical velocity dispersion for both stellar and gas components for three superthin galaxies: UGC 7321,IC 5249,and FGC 1540 using Markov Chain Monte Carlo (MCMC)method,mass models using B-band,3.6 $\mu$m photometry and HI 21 cm radio-synthesis observations were already available in literature. We find that the central value of vertical velocity dispersion for stellar component B-band varies between 8.4 – 10.9 km/s and falls of with an exponential scalelength of 1.9to3.5R$_d$ where R$_d$ is the exponential stellar disc scalelength.In the 3.6$\mu$m band, for the thick disc the same varies between 5.6and9.7km/s with a scalelength of 5.5 –11.5R$_d$.Further our model results are consistent with AGAMA(Action-based Galaxy Modelling Architecture).Our calculated values of N-component disc stability parameter $Q_N$ lies between 1.7 - 5.7 thus confirming the dynamic stability of our model.

- **Effect of radial term in Poisson's equation.**

One of the underlying assumption in the two-fluid model is the constancy of the rotation curve in the outer radius, but
Table 6. Vertical stellar velocity dispersion in optical-band.

| Results | UGC7321$^B$ | UGC00711$^B$ | IC2233$^i$ | FGC1540$^j$ | Profile |
|---------|--------------|--------------|------------|-------------|---------|
| $\sigma_{\alpha}$ | 10.23 ± 0.64 | 18.4 ± 0.87 | 14.9 ± 0.57 | 13.08 ± 1.17 | $\sigma_z(r) = \sigma_{\alpha} e^{\frac{z}{z_{avg}}}$ |
| $\alpha_1$ | 2.58 ± 0.613 | 3.21 ± 0.40 | 2.36 ± 0.36 | 3.32 ± 0.42 | |
| $\sigma_{\beta}^c$ | – | – | – | 36.91 ± 1.14 | $\sigma_z(r) = \sigma_{\alpha} e^{\frac{z}{z_{avg}}}$ |
| $\alpha_2$ | – | – | – | 3.72 ± 0.4 | |
| $\sigma_{avg}^e$ | – | – | – | 16.78 | |

$^a$ Central vertical stellar velocity dispersion in optical.(km/s)
$^b$ steepness parameter of stellar dispersion profile.
$^c$ Central vertical stellar velocity dispersion in optical.(km/s)
$^d$ steepness parameter of stellar dispersion profile.
$^e$ Average stellar dispersion(km/s)

Table 7. Vertical HI velocity dispersion in optical-band.

| Results | UGC7321$^B$ | UGC00711$^B$ | IC2233$^i$ | FGC1540$^j$ | Profile |
|---------|--------------|--------------|------------|-------------|---------|
| $\sigma_{\alpha}$ | 11.06 ± 0.88 | 23.10 ± 1.11 | 12.52 ± 0.515 | – | $\sigma_z(r) = \sigma_{\alpha} + \alpha_{HI} R + \beta_{HI} R^2$ |
| $\sigma_{HI}$ | 0.18 ± 0.07 | 1.03 ± 0.145 | 1.03 ± 0.14 | – | |
| $\beta_{HI}$ | -0.047 ± 0.02 | -0.156 ± 0.05 | -0.141 ± 0.031 | – | |
| $\sigma_{\alpha}^d$ | – | – | – | 29.01 ± 1.16 | $\sigma_z(r) = \sigma_{\alpha} e^{\frac{z}{z_{avg}}}$ |
| $\alpha_{HI}^e$ | – | – | – | 4.27 ± 0.425 | |

$^a$ Central vertical HI dispersion in optical.(km/s)
$^b$ steepness parameter-1 of HI dispersion profile.
$^c$ steepness parameter-2 of HI dispersion profile.
$^d$ Central vertical HI velocity dispersion in optical(km/s).
$^e$ steepness parameter of HI dispersion profile.

Table 8. Vertical stellar velocity dispersion in 3.6$\mu$m.

| Results | UGC7321 | IC5249 | FGC 1540 | IC 2233 | U 711 | Profile |
|---------|---------|--------|----------|---------|-------|---------|
| $\sigma_{\alpha}^d$ | 24.66 ± 0.88 | 20.64 ± 0.63 | 16.20 ± 0.87 | 15.97 ± 0.54 | 23.82 ± 1.45 | $\sigma_z(r) I = \sigma_{\alpha} e^{\frac{z}{z_{avg}}}$ |
| $\alpha_1^b$ | 2.15 ± 0.607 | 2.155 ± 0.217 | 3.77 ± 0.42 | 2.16 ± 0.42 | 2.42 ± 0.28 | |
| $\sigma_{\alpha}^c$ | 9.02 ± 0.8 | 9.32 ± 0.39 | 6.86 ± 0.57 | 3.9 ± 0.23 | – | $\sigma_z(r) I = \sigma_{\alpha} e^{\frac{z}{z_{avg}}}$ |
| $\alpha_{avg}$ | 4.55 ± 0.68 | 7.54 ± 0.23 | 12.1 ± 0.59 | 6.0 ± 0.2 | – | |
| $\sigma_{avg}$ | 11.58 | 11.08 | 8.63 | 5.98 | – | |

$^a$ Central vertical stellar velocity dispersion in 3.6$\mu$m for thick disc.(km/s)
$^b$ steepness parameter of stellar dispersion profile 3.6$\mu$m for thick disc.
$^c$ Central vertical stellar velocity dispersion in 3.6$\mu$m for thin disc.(km/s)
$^d$ steepness parameter of stellar dispersion profile 3.6$\mu$m for thin disc.
$^e$ Average stellar dispersion(km/s)
Figure 9. Panel 1 describes the vertical velocity dispersion profile IC2233 in r band obtained from two component model. In panel 2 is rendered the stability parameter \( Q_N \) appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

Figure 10. Panel 1 describes the vertical velocity dispersion profile IC2233 in 3.6\( \mu \)m obtained from two component model. In panel 2 is rendered the stability parameter \( Q_N \) appraising the stability of the galactic disc. In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

the observed rotation curves are not always constant but are steeply or slowly rising starting from the inner radius itself. We model the effect of inclusion of the radial term on the vertical velocity dispersion by evaluating the term \( \frac{1}{R^4} \frac{\partial}{\partial R} \left(R^3 \frac{\partial \phi}{\partial R}\right) \), and noting that \( R \frac{\partial}{\partial R} = v_c^2 \), the value of \( v_c(r) \) is obtained by fitting the observed rotation curves to an exponential profile \( v_c(r) = v_\odot (1 - e^{-r/R}) \). In figure 18 are rendered the plots depicting the effect of inclusion of rotation velocity on the observed vertical velocity dispersion. UGC 7321 in B-band is characterized by a \( \sigma_0 = 10.23 \) and the \( \alpha = 2.58 \), inclusion of radial term lowers the value of central dispersion to \( \sigma_0 = 7.97 \) and increases the value of steepness parameter \( \alpha = 3.06 \). In 3.6 \( \mu \)m the values \( \sigma_0 \) for thick and thin disc change from 25.6 km/s and 9.02 km/s to 18.2 and 6.8 km/s. The value of \( \sigma \) changes from 2.15 and 4.55 to 2.45 and 5.24. The vertical velocity dispersion of the thick disc after the inclusion of the radial term is lowered by a maximum of 28 \% for IC2233 and 16 \% for IC5249, \( \sigma_0 \) changes by about 25 \% for all other galaxies in the sample. In case of the thin disc, the percentage reduction of vertical dispersion is lowest for UGC 7321: 24 \% and maximum for FGC1540: 33.5 \%. \( \sigma_0 \) is lowered by about 29 \% for all other galaxies. Steepness parameter of the thick disc increases by maximum of 34 \% for IC 2233, and a minimum change of 7.8 \% is seen in UGC00711, followed by a 13 \% increase in UGC7321. In case of a thin disc maximum increase of \( \alpha \) is seen in IC5249 about 54 \%, whereas the steepness parameter changes by 15 \% for UGC 7321 followed by 16 \% increase in IC2233. It is evident from table 11 that the inclusion of radial term lowers the values of the vertical velocity dispersion and increases the steepness parameter \( \alpha \). Thus the underlying assumption that the dynamics in the vertical direction is decoupled with the dynamics in the radial direction breaks down; we also note that that in case of cold two-fluid disc the velocity dispersion profile is parameterized by a large value of steepness parameter implying significant departure from exponential velocity dispersion profile.

- **MCMC model with dark matter halo parameters free** Finally, we use MCMC to optimize the two-component model keeping the dark matter core radius and central density as free parameters along with the stellar and Hi velocity dispersion to match the observed stellar and Hi scaleheight. The best-fit values of dark matter density and core radius obtained by optimizing the two-fluid model are \( \rho_0 = (0.033 \pm 0.007)M_\odot pc^{-3} \) and \( R_c = (2.42 \pm 0.260)kpc \), as compared to 0.039\( M_\odot pc^{-3} \) and 2.99\( kpc \) obtained from mass modelling. We find that the estimate of the dark matter profile parameters obtained by optimizing the two-fluid model...
How “cold” are superthin discs? An ‘MCMC’ approach.

Figure 11. Panel 1 describes the vertical velocity dispersion profile UGC00711 in B-band obtained from two component model. In panel 2 is rendered the stability parameter $Q_N$ appraising the stability of the galactic disc, In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

Figure 12. Panel 1 describes the vertical velocity dispersion profile UGC00711 in 3.6$\mu$m obtained from two component model. In panel 2 is rendered the stability parameter $Q_{NW}$ appraising the stability of the galactic disc, In panel 3, we compare the scaleheight obtained from the two-component model with action based dynamical model of galaxies studied using AGAMA.

agrees with the values from the mass modeling within the error-bars.

- **How cold are the superthin galaxies?** In figures 13, 14 and 15, we compare the ratio of the vertical velocity dispersion to the total rotation velocity obtained for the superthin galaxies, with the corresponding ratio for the Milky way stars. We obtain the radial profile of the vertical velocity dispersion at different thickness for the Milkyway stars from Katz et al. (2018). We note that in the case of UGC 7321 in B-band and IC 2233 in r-band, the ratio $\sigma_z/V_{rot}$ is lower than that for the Milky way stars at any thickness, indicating the disc is indeed very cold. Whereas in the case of IC 5249 and FGC 1540 in 3.6$\mu$m, the ratio $\sigma_z/V_{rot}$ is higher than that of Milkyway stars which might indicate that the disc has been heated up in the vertical direction. We note that in the case of UGC 7321 and IC2233 in 3.6 $\mu$m, and FGC 1540 in i-band, $\sigma_z/V_{rot}$ is comparable with that of Milkyway stars in 200 to 600 pc. In the case of UGC00711 in B-band and 3.6 $\mu$m $\sigma_z/V_{rot}$ is slightly higher than the stars in -200pc to 200pc thickness section of Milkyway, and falls off to a much lower value in the outer radius, lower than $(\sigma_z/V_{rot})_{max} = 0.1$, corresponding to the stars in -200pc to 200pc section of Milkyway. We see the stellar disc is truly cold and devoid of strong vertical heating in case of UGC 7321 in B-band and IC 2233 in r-band, whereas for other galaxies in our sample $\sigma_z/V_{rot}$ is at least comparable with Milkyway stars lying in the thickness section 200pc to 600pc.

**SOFTWARES/PACKAGES**

We have used publicly available R and python packages in this work. We have used FME (Soetaert et al. 2010) for MCMC modelling, for analysis of results we relied on packages ggmcmc(Fernández-i Marín 2016), BayesianTools (Hartig et al. 2017), and for purpose of plotting we have used ggplot2(Wickham 2011), Matplotlib(Hunter 2007),tonic (Vaughan et al. 2016).
Figure 13. Panel 1 and panel 2 depicts the ratio of the vertical stellar dispersion to the total rotational velocity for UGC7321 in B-band and 3.6 μm. Panel 3 indicates the same for IC5249 in 3.6 μm.

Figure 14. Panel 1 and panel 2 depicts the ratio of the vertical stellar dispersion to the total rotational velocity for FGC1540 in i-band and 3.6 μm. And panel 3 depicts the same for IC2233 in r-band.

Figure 15. Panel 1 and panel 2 depicts the ratio of the vertical stellar dispersion to the total rotational velocity for UGC00711 in B-band and 3.6 μm. And panel 3 indicates the same for IC2233 in 3.6 μm.
How “cold” are superthin discs? An ‘MCMC’ approach.

Table 9. Vertical HI velocity dispersion in 3.6µm.

| Results | UGC7321 | IC5249 | FGC1540 | IC2233 | U711 |
|---------|---------|--------|---------|--------|------|
| $\sigma_{0\text{HI}}$ | $11.19 \pm 0.84$ | $12.4 \pm 0.53$ | – | $12.0 \pm 0.56$ | $22.03 \pm 1.07$ |
| $a_{\text{HI}}$ | $-0.29 \pm 0.14$ | $-0.99 \pm 0.11$ | – | $0.53 \pm 0.23$ | $0.92 \pm 0.16$ |
| $\beta_{\text{HI}}$ | 0.0 | $0.04 \pm 0.0013$ | – | $-0.055 \pm 0.026$ | $-0.1 \pm 0.054$ |
| $\sigma_{0\text{HI}}^d$ | – | – | $17.75 \pm 0.83$ | – | – |
| $a_{\text{HI}}^e$ | – | – | $6.85 \pm 0.56$ | – | – |

$\sigma_{0\text{HI}}$ $^a$ Central vertical HI dispersion in 3.6µm (km/s).

$\beta_{\text{HI}}$ steepness parameter-1 of HI dispersion profile.

$\beta_{\text{HI}}$ steepness parameter-2 of HI dispersion profile.

$\sigma_{0\text{HI}}^d$ Central vertical HI velocity dispersion in 3.6µm for FGC 1540(km/s).

$\sigma_{0\text{HI}}^e$ steepness parameter of HI dispersion profile 3.6µm for FGC 1540.

Figure 16. Panel 1 depicts the best fit parameters for IC2233 in r-band, and panel 2 are rendered the MCMC results for IC 2233 in 3.6 µm.
Figure 17. MCMC best fit parameters for UGC00711 in B-band (panel-1) indicated by the peaks of normal distribution, the contours indicates the all possible combinations of the parameter-space over which the model was optimised for finding the best fit parameters by MCMC. Panel 2 indicates the MCMC best-fit model of U 711 in 3.6μm-band.
How “cold” are superthin discs? An ‘MCMC’ approach.

Table 10. The value of average vertical dispersion of thin disc and thick disc computed at $R_d$, $2R_d$, $3R_d$ computed as

$$\sigma_{(\text{avg})}^2 = \frac{\rho_1^2 \sigma_1^2 + \rho_2^2 \sigma_2^2}{\rho_1 + \rho_2}$$

| Name            | $\sigma_{(\text{avg})}(R_d)$ | $\sigma_{(\text{avg})}(2R_d)$ | $\sigma_{(\text{avg})}(3R_d)$ |
|-----------------|-------------------------------|-----------------------------|-----------------------------|
| UGC7321 $B$-band | 7.14                          | 4.85                        | 3.29                        |
| UGC7321 $3.6\mu$m | 8.44                          | 6.31                        | 4.82                        |
| IC5249 $3.6\mu$m  | 8.8                           | 7.26                        | 6.15                        |
| FGC1540 $3.6\mu$m  | 7.38                          | 6.4                         | 5.64                        |
| FGC1540 $i$-band | 12.6                          | 9.46                        | 7.11                        |
| IC2233 $3.6\mu$m  | 4.31                          | 3.25                        | 2.54                        |
| IC2233 $r$-band  | 9.75                          | 6.38                        | 4.17                        |
| UGC00711 $3.6\mu$m | 15.75                         | 10.42                       | 6.89                        |
| UGC00711 $N$-band | 13.47                         | 9.86                        | 7.22                        |
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