Generation of entangled photon-pairs from a single quantum dot embedded in a planar photonic-crystal cavity

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We present a formal theory of single quantum-dot coupling to a planar photonic crystal that supports quasi-degenerate cavity modes, and use this theory to describe, and optimize, entangled-photon-pair generation via the biexciton-exciton cascade. In the generated photon pairs, either both photons are spontaneously emitted from the dot, or one photon is emitted from the biexciton spontaneously and the other is emitted via the leaky-cavity mode. In the strong-coupling regime, the generated photon pairs can be maximally entangled, in qualitative agreement with the simple dressed-state predictions of Johne et al. [Phys. Rev. Lett. vol. 100, 240404 (2008)]. We derive useful and physically-intuitive analytical formulas for the spectrum of the emitted photon pairs in the presence of exciton and biexciton broadening, which is necessary to connect to experiments, and demonstrate the clear failure of using a dressed-state approach. We also present a method for calculating and optimizing the entanglement between the emitted photons, which can account for post-sample spectral filtering. Pronounced entanglement values of greater than 80% are demonstrated using experimentally achievable parameters, even without spectral filtering.

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I. INTRODUCTION

A source of polarization-entangled photon pairs has wide uses in quantum optics, leading to applications such as quantum computation, quantum information processing, quantum cryptography, and quantum metrology. Most of the experiments demonstrated to date employ entangled photons generated by parametric down conversion (PDC). A PDC is a “heralded” source of entangled photons in which the number of generated photon pairs is probabilistic. However, in many experiments, particularly in quantum information processing, a deterministic scalable source of entangled photons is essential. Recently, there has been considerable interest in developing an all-solid-state “on demand” source of entangled photon pairs using single quantum dots (QDs). In QDs, entangled photon pairs can be generated in a biexciton cascade decay via exciton states of angular momenta +1 and −1; single QDs are particularly appealing since they are fixed in place, scalable, and have long coherence times. However, a major difficulty for implementing these schemes is the naturally occurring anisotropic energy difference between the exciton states of different angular momentum. Specifically, a small anisotropic energy difference, can make the emitted x-polarized and y-polarized photon pairs distinguishable, and thus the entanglement between the photons is largely wiped out. There have been a few proposals to overcome this problem, e.g., by spectrally filtering indistinguishable photon pairs, by applying external fields to make the exciton states degenerate, and suppressing the biexciton binding energy in combination with time reordering, but these techniques have their own set of problems and are far from optimal.

Recently, Johne et al. proposed an interesting cavity-QED scheme in the strong coupling regime, where the exciton states become dressed with the cavity field and form polariton states. At the same time, a number of experimental groups have now demonstrated the strong-coupling regime using single QDs integrated with planar photonic-crystal cavities. These emerging “on-chip” cavity structures form an important breakthrough in the search for creating scalable sources of photons using single QDs, and much excitement is envisioned. However, the lack of appropriate theoretical descriptions becomes very challenging and the development of new medium-dependent models are required to properly describe the light-matter interactions and photon wave functions.

In quantum material systems such as solids, the interaction with the environment is inevitable. The biexcitons, excitons and cavity modes interact with their phonon and thermal reservoirs, which can have a substantial influence on the wavefunction of the emitted photon pairs. In the biexciton decay, the entanglement depends on the “indistinguishability” between x-polarized and y-polarized photon pairs, i.e. the overlap of their wavefunctions. Therefore, the precise form of the wave functions of the emitted photon pairs is ultimately required. Here we present rigorous, and physically-intuitive, analytical expressions for the wavefunction of the emitted photon pairs in the biexciton-exciton cascade decay using the Weisskopf-Wigner approximation for coupling to the environment. Extending previous approaches, we necessarily consider finite exciton and biexciton level broadenings and the damping of the leaky cavity mode. We further apply a method for optimizing the entanglement using a simple spectral filter, and find impressive entanglement values even with realistic parameters and a sizable anisotropic energy exchange.
II. THEORY

We consider a QD embedded in a photonic crystal cavity having two orthogonal polarization modes of frequency $\omega_x^c$ and $\omega_y^c$, which can be realized and tuned experimentally using e-beam lithography and, for example, AFM oxidization techniques. The exciton states, $|x\rangle$ and $|y\rangle$, have an anisotropic-exchange energy difference $\Delta_x$. The cavity modes are coupled with the exciton to ground-state transition, but spectrally decoupled from the biexciton state because of the relatively large biexciton binding energy, $\Delta_{xx} \gg$ cavity coupling. The schematic arrangement of the system is shown in Fig. 1.

For simplicity, we consider the emission of $x$-polarized photon pair, but the formalism and results can easily be applied to $y$-polarized photons as well. The Hamiltonian for the emission of $x$-polarized photon pair, in the interaction picture, can be written as

$$H_I(t) = \hbar \left[ g|x\rangle\langle x|\hat{a}_t e^{i\Delta_x^t t} + \sum_{k \neq c} \Omega_{uk}|u\rangle\langle k|\hat{a}_k e^{i(\omega_{ux} - \omega_k)t} + \sum_{l \neq c} \Omega_{gl}|x\rangle\langle l|\hat{a}_l e^{i(\omega_{x} - \omega_l)t} + \sum_{m \neq c} \Omega_{cm}\hat{a}_m|c\rangle e^{i(\omega_{x} - \omega_m)t} \right] + \hat{H}_{\text{cav}}.$$  \hspace{1cm} (1)

plus a Hermitian conjugate term, where $\omega_{ux} = \omega_u - \omega_x$, $\Delta_x^c = \omega_x - \omega_x^c$, and $\hat{a}_t$ is the field operators with $\hat{a}_t$ the cavity mode operator. Here, $\Omega_{uk}, \Omega_{gl}, \Omega_{cm}$ represent the couplings to the environment from the biexciton, exciton and cavity mode; $g$ is the coupling between the exciton and cavity mode; and $\omega_k$, $\omega_l$, $\omega_m$, $\omega_u$, and $\omega_x$ are the frequency of the photon emitted from the biexciton and exciton, the frequency of photon leaked from cavity, and the frequency of the biexciton and exciton, respectively.

We consider a system that is pumped in such a way as to have an initially-excited biexciton, with no photons inside the cavity, thus the state of the system at any time $t$ can be written as

$$|\psi(t)\rangle = c_1(t)|u, 0\rangle + \sum_k c_{2k}(t)|x, 0\rangle|1_k\rangle + \sum_k c_{3k}(t)|g, 1\rangle|1_k\rangle + \sum_{k,l} c_{4kl}(t)|g, 0\rangle|1_k, 1_l\rangle + \sum_{k,m} c_{5km}(t)|g, 0\rangle|1_k, 1_m\rangle.$$  \hspace{1cm} (2)

The different terms in the state vector $|\psi\rangle$ represent, respectively, the dot is in the biexciton state with zero photons in the cavity, the dot is in the exciton state after radiating one photon, the dot is in ground state with one photon in cavity mode, the dot is in the ground state after radiating two photons, and the dot is in ground state after one photon is radiated from the biexciton and the other is leaked from the cavity mode.

By using the Schrödinger equation, the equation of motion for the probability amplitudes are

$$\dot{c}_1(t) = -i \sum_k \Omega_{uk} c_{2k}(t)e^{i(\omega_{ux} - \omega_k)t},$$ \hspace{1cm} (3)

$$\dot{c}_{2k}(t) = -i \Omega_{uk}^* c_k e^{-i(\omega_{ux} - \omega_k)t} - ig c_{3k}(t)e^{i\Delta_x^t t} - \sum_l \Omega_{gl} c_{4kl}(t)e^{i(\omega_x - \omega_l)t},$$ \hspace{1cm} (4)

$$\dot{c}_{3k}(t) = -ig c_{2k}(t)e^{-i\Delta_x t} - i \sum_m \Omega_{cm} c_{5km}(t)e^{i(\omega - \omega_m)t},$$ \hspace{1cm} (5)

$$\dot{c}_{4kl}(t) = -i \Omega_{gl}^* c_k e^{-i(\omega_x - \omega_l)t},$$ \hspace{1cm} (6)

$$\dot{c}_{5km}(t) = -i \Omega_{cm}^* c_k e^{-i(\omega - \omega_m)t}.$$ \hspace{1cm} (7)

Applying the Weisskopf-Wigner approximation, then Eqs. (3)-(5) simplify to

$$\dot{c}_1(t) = -\gamma_1 c_1(t),$$ \hspace{1cm} (8)

$$\dot{c}_{2k}(t) = -i \Omega_{uk}^* c_k e^{-i(\omega_{ux} - \omega_k)t} - ig c_{3k}(t)e^{i\Delta_x^t t} - \gamma_2 c_{2k}(t),$$ \hspace{1cm} (9)

$$\dot{c}_{3k}(t) = -ig c_{2k}(t)e^{-i\Delta_x t} - \kappa c_{3k}(t),$$ \hspace{1cm} (10)

where $\kappa = \pi |\Omega_{cm}|^2$ is the half width of the cavity mode and $\gamma_1$, $\gamma_2$ are the half widths of the biexciton and exciton levels, respectively. We note that $\gamma_1$ and $\gamma_2$ can include both radiative and nonradiative broadening, and for QDs, $\gamma_1 \approx 2\gamma_2$. Moreover, the radiative half width of biexciton will be sum of its spontaneous decay rates in the exciton states $|x\rangle$ and $|y\rangle$; if the decay rate of the biexciton in $|x\rangle$ and $|y\rangle$ are equal, the radiative half width of biexciton will be $2\pi |\Omega_{uk}|^2$. The radiative half width of the exciton $|x\rangle$ is given by $\gamma_x = \pi |\Omega_{gl}|^2$. We next solve Eqs. (6)-(10) for $c_{4kl}$ and $c_{5km}$ using the Laplace transform method. The probability amplitudes for two-photon
emission in long time limit are given by

\[ c_{4kl}(\infty) = \frac{\Omega_{uk}^*}{(\omega_k + \omega_l - \omega_u + i\gamma_1)} \times \frac{\Omega_{gl}^*}{(\omega_l - \omega_r + i\gamma_r)} \times \frac{1}{(\omega_l - \omega_x + ig_+)(\omega_l - \omega_x + ig_-)}, \]  

\[ c_{5km}(\infty) = \frac{\Omega_{uk}^* \Omega_{gl}^*}{(\omega_k + \omega_l - \omega_u + i\gamma_1)(\omega_l - \omega_x + i\gamma_2)}. \]

which is similar to the radiation-mode and cavity-mode emitted spectra reported by Cui and Raymer\textsuperscript{23}, and by Yao and Hughes\textsuperscript{24}. From Eqs. (16) and (17), the photon emitted from the spontaneous decay (second emitted photon) has a two-peak spectrum; these spectral peaks appear at the frequencies, \( \frac{1}{2}(\omega_x + \omega_c^\pm \pm \delta \omega) \), where \( \delta \omega \approx \sqrt{4g^2 + 4\Delta_{x}^2 - (\kappa - \gamma_2)^2} \) is the splitting between the peaks. In a dressed-state picture, these spectral peaks correspond to the two polariton states in the strong cavity regime, \( g \gg (\kappa, \gamma_2) \).

From the above discussion, the state of the “photon pair” emitted from both \( |x\rangle \)-exciton and \( |y\rangle \)-exciton branches is given by

\[ |\psi(\infty)\rangle = \sum_{k,l} c_{4kl}(\infty) |1_k, 1_l\rangle_{x,y} + \sum_{k,m} c_{5km}(\infty) |1_k\rangle_{x} |1_m\rangle_{y} + \sum_{k,l} d_{4kl}(\infty) |1_k, 1_l\rangle_{y} |0\rangle_{y} + \sum_{k,m} d_{5km}(\infty) |1_k\rangle_{y} |1_m\rangle_{y}, \]

where in each term the first ket represents the combined state of the biexciton and the exciton reservoirs, the second ket represents the state of the cavity reservoir, and the ket suffix labels the polarization. The coefficients \( c_{ijkl}(\infty) \) are given by Eqs. (11)-(12). For the same cavity coupling \( g \), the coefficients, \( d_{ijkl} \), are given by the Eqs. (11)-(12) after replacing \( \omega_{x, c}^\pm, \Delta_{x}^\pm \) with \( \omega_{y}, \omega_{c}^y, \) and \( \Delta_{x}^y = \omega_{y} - \omega_{c}^y \), respectively.

### III. RESULTS AND OPTIMIZING THE ENTANGLEMENT

There are two possible decay channels for generating a photon pair. In Figs. 2(a) and (b), we show the spectrum of the photon pair, when one photon is emitted from biexciton decay and the second is emitted (leaked) via the cavity mode. The spectra of photon pairs emitted in the biexciton and exciton radiative decay are shown in Figs. 2(c) and 2(d). Depending on the detunings between the frequency of the cavity field and the frequency of the excitons \( \Delta_{x}^y, y \)-polarized photon pair and \( y \)-polarized photon pair can be degenerate in energy. The spectra of the emitted \( x, y \) polarized photons, in the strong coupling regime, show peaks at the frequencies \( \omega_{x,y} \approx \omega_{u} - \omega_{x,y}^\pm \) and \( \omega_{y} \approx \omega_{x,y}^\pm \), where \( \omega_{x,y}^\pm = \frac{1}{2} \left( \omega_{c}^x + \omega_{c}^y \pm \sqrt{(\Delta_{x}^x)^2 + 4g^2} \right) \) are the frequencies of the polariton states. The polarization-entangled photon pairs can be generated by making the emitted \( x \)-polarized and \( y \)-polarized photon pairs degenerate. For the positive (negative) values of \( \Delta_{x}^y \), the peaks in the spectrum corresponding to \( \omega_{x,y} \approx \omega_{u} - \omega_{x,y}^+ \) and \( \omega_{x,y} \approx \omega_{u} - \omega_{x,y}^- \) and \( \omega_{y} \approx \omega_{x,y}^+ \) and \( \omega_{y} \approx \omega_{x,y}^- \) are stronger and the probability of generating photons for these frequencies is increased. Therefore, a large probability of generating degenerate photon pairs can be achieved by overlapping these stronger peaks in the spectrum. There are three possible coupling cases of

\[ S_c(\omega_k, \omega_m) = \frac{|\Omega_{uk}|^2}{(\omega_k + \omega_m - \omega_u + \gamma_1)} \times \frac{g^2|\Omega_{cm}|^2}{(\omega_m - \omega_x + ig_+)(\omega_m - \omega_x + ig_-)} \]

\[ S_r(\omega_l) = \frac{|\Omega_{gl}|^2}{(\omega_l - \omega_x + i\gamma_r)} \times \frac{1}{(\omega_l - \omega_x + ig_+)(\omega_l - \omega_x + ig_-)}. \]
interest that can do this. Case 1: by making both \(x\)-polariton states and \(y\)-polariton states degenerate, \(\omega^x_0 = \omega^\mp_0\) which can be achieved with \(\Delta^x_0 = -\Delta^y_0 = \delta_x\) (see Fig.2(a) and 2(c)); Case 2: by making one of the \(x\)-polariton state degenerate to the other \(y\)-polariton state \((\omega^x_0 = \omega^+_0, \text{see Figs. 2(b) and 2(d); or } \omega^x_0 = \omega^-_0)\) when \(\Delta^x_0 = \Delta^y_0\) are of opposite sign; Case 3: by making \(\omega^x_0 = \omega^+_0\) \((\omega^x_0 = \omega^-_0)\) when both \(\Delta^x_0\) and \(\Delta^y_0\) are positive (negative). Optimum entanglement is achieved from case-1 and case-2 above for \(\Delta^x_0 = -\Delta^y_0\), which we example in Fig. 2 and Fig. 3.

We stress that our calculated spectra are drastically different to those predicted previously using a dressed state picture, where the latter uses simple Lorentzian line widths for each state\(^{15}\). Moreover, in the strong coupling regime, the cavity-assisted generated photon pairs \((S_c)\) completely dominates the spontaneously emitted photons \((S_s)\), and by several orders of magnitude. This effect is similar the the cavity-feeding process that occurs for an off-resonant cavity mode\(^{20}\), where the leaky cavity mode emission dominates the spectrum.

The entanglement can be distilled by using frequency filters with a small spectral window \(w\) centered at the frequencies of degenerate peaks in the spectrum of \(x\)-polarized and \(y\)-polarized photons. Subsequently, the response of spectral filter can be written as a projection operator of the following form

\[
W(\omega,\omega') = \begin{cases} 
1, & \text{for } |\omega - \omega'| < w, \\
1, & \text{for } |\omega - \omega'| \geq w, \\
0, & \text{otherwise.}
\end{cases}
\]

After operating on the wave function of the emitted photons \((\ref{eq:wavefunction})\), by spectral function \(W(\omega,\omega')\) and tracing over the energy states\(^{33}\), we get the reduced density matrix of the filtered photon pairs in the polarization basis. We consider the photon pairs in which one photon is emitted from the biexciton decay and the other is emitted by the leaky cavity mode; in fact we can easily neglect the spontaneous emission of both biexciton and exciton photons as we have justified before. The normalized off-diagonal element of the density matrix of photons is given by\(^{13}\)

\[
\gamma = \frac{\int \int \vert c_{5kl}(\infty)\vert^2 d\omega_k d\omega_l + \int \int \vert d_{5kl}(\infty)\vert^2 d\omega_k d\omega_l}{\int \int \vert c_{5kl}(\infty)\vert^2 W d\omega_k d\omega_l + \int \int \vert d_{5kl}(\infty)\vert^2 W d\omega_k d\omega_l}.
\]

The concurrence, which is a quantitative measure of entanglement, for the state of the filtered photon pair is given by \(C = 2|\gamma|\)\(^{22}\). The photons are thus maximally entangled when \(C = 0.5\). In Fig. (3), the value of \(\gamma\) is plotted for two different cases of degenerate \(x\)-polarized and \(y\)-polarized photon pairs, corresponding to Figs. 2(a) and 2(b); \(\delta_x\) and \(\Delta^x_0 - \Delta^y_0\) are fixed, while \(\Delta^x_0 + \Delta^y_0\) is changed, e.g., by temperature of gas tuning; both unfiltered (a) and filtered (b) values are shown. The spectral filter has negligible effect on case 1, but it improves the concurrence of case 2 significantly. After filtering, the generated entangled photons, when both polariton

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**FIG. 2:** The spectrum of the generated photon pair in the biexciton-exciton cascade decay, for \(\delta_x = 0.1\ \text{meV}, \Delta_{xx} = 1.0\ \text{meV}, \gamma_1 = 2\gamma_2 = 0.004\ \text{meV}, \gamma_6 = \pi \vert \Omega_{uk} \vert^2 = \pi \vert \Omega_{yl} \vert^2 = 0.05\ \text{meV}^2, \kappa = 0.05\ \text{meV}, \gamma = 0.11\ \text{meV},\) and \(\omega_0 = 0.5(\omega_x + \omega_y)\). In (a) and (b), one photon is emitted from biexciton decay and the other is leaked via cavity mode, and in (c) and (d), both photons are radiated from the biexciton and exciton states via spontaneous decay. The other parameters are as follows: for (a) and (c), \(\Delta^x_0 = -\Delta^y_0 = \delta_x\), and for (b) and (d), \(\Delta^x_0 = -\Delta^y_0 = -0.175\ \text{meV}\). The \(x\)-polarized photons are shown in blue and \(y\)-polarized are shown in red; the solid curves are for photons generated in the excitation decay and dotted curves are for photons generated in the biexciton decay.

**FIG. 3:** The amplitude of the off-diagonal element of the density matrix for filtered photon pairs, when keeping \(\delta_x = 0.1\ \text{meV}\) fixed. In (a) and (b) is shown the unfiltered and filtered cases, respectively. The black line represents \(\Delta^x_0 - \Delta^y_0 = 2\delta_x\) (case 1), and the red line represents \(\Delta^x_0 - \Delta^y_0 = -0.35\ \text{meV}\) (case 2); the other parameters are the same as in Fig. 2. The filter function corresponds to two spectral windows of width \(w = 0.2\ \text{meV}\) centered at \(\omega_x^\pm\) and \(\omega_x - \omega_x^\pm\). Note that for \(\Delta^x_0 + \Delta^y_0 = 0\) corresponds to the optimal conditions for generating entangled photon pair as shown in Fig. (2).
The black curve represents $\Delta \omega$ filter function corresponds to two spectral windows of width $x$. The degenerate photons pairs with an $\omega$ polariton state and an $\omega$ polarized po-
larized photons are de-
generate (in Fig. 2(a)), have a smaller entanglement than states of $x$–polarized and $y$–polarized photons are de-
generate (in Fig. 2(a)), have a smaller entanglement than the degenerate photons pairs with an $\omega$ –polarized polariton state and an $\omega$ –polarized state (see Fig. 2(b)). However, the photon source operating under the conditions of Fig. 2(a) is a deterministic entangled photon source, while the photon source operating under the conditions of Fig. 2(b)–and using a spectral filter–is a probabilistic photon sources, as there is some probability of generating non-degenerate photon pairs. In both cases, we get significant concurrence values greater than 0.9.

Finally, we discuss the general criteria for achieving efficient entanglement. In general, one desires to be in the strong coupling regime to overcome the exchange splitting, thus the required conditions are $g > \kappa$ and $g > \delta_x/2$. To gain insight into a smaller $g$ situation, we show in Fig. (4) the spectra and entanglement that occurs for $g = \kappa$ and for smaller values of $\delta_x$. For the spectra (a-b), it is clear that the indistinguishability of the $x$–polarized and $y$–polarized pairs is increased, yet in (c-d) we see that impressive entanglement values can still be achieved, even without a filter. In addition, use of a spectral filter can not improve entanglement significantly in these conditions. Thus we believe that the general cavity-improvement could be significant in the context of generated entangled photon pairs, and that these values are achievable using realistic experimentally parameters.

**IV. CONCLUSION**

In conclusion, we have derived and exploited analytical results for the wave functions of the emitted photon pairs from a QD embedded in a photonic crystal cavity that supports quasi-degenerate cavity modes. In particular, we have necessarily included finite exciton and biexciton level broadening, and damping of the leaky cavity mode, and show that these relaxation mechanism must be included to connect to realistic experiments. Finally, we have also discussed the method for optimizing and measuring the entanglement between the emitted photons using a simple spectral filter.

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