Weak Charm Decays with Lattice QCD

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In this paper I review the status of lattice QCD calculations of D and D_s meson decay constants and of D_s meson semi-leptonic decay form factors. I restrict my discussion to results obtained from simulations with $n_f = 2 + 1$ sea quarks.

1. Introduction and Motivation

Lattice QCD is the only systematically improvable calculational tool we have for quantitatively understanding nonperturbative QCD effects. Accurate theoretical calculations of nonperturbative QCD effects are essential for the experimental flavor physics program. One set of goals of the experimental program are accurate determinations of the CKM matrix elements. This is illustrated for the weak decay process $D \to K l^-$. The experimentally measured (di)essential decay rate can be written as

$$\frac{d}{dq^2} = (\text{known}) f_{\ell}^2(q^2)$$

where $f_{\ell}(q^2)$ is one of the hadronic form factors which parameterize the hadronic matrix element for this process. $Y$ is an element. Hence, to determine $Y_{cs}$ from experiment, we need a theoretical calculation of the form factor with matching precision. Another set of goals is to constrain beyond the standard model theories and to search for new physics signals. This means many experiments at the high energy frontier. Accurate theoretical calculations are again essential.

Since lattice QCD calculations are complicated and time consuming, a third important goal, namely of the charm physics program is to test lattice QCD methods. For example, we can use Eq. (1) to determine the form factors from experiment, in which the experimenters at the high energy frontier. Accurate theoretical calculations are again essential.

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1.1. Introduction to Lattice QCD

In lattice QCD theory, the space-time continuum is replaced by a discrete lattice. (For reviews of lattice QCD see Ref. [1].) This implies that derivatives are replaced by discrete differences, which in turn introduces discretisation errors into physical quantities. These errors generally vanish with a positive power of the lattice spacing ($a$).

Nonperturbative calculations in lattice QCD can be performed using Monte Carlo methods. Lattice artifacts can be removed by reducing the lattice spacing used in numerical calculations. However, the computational cost increases as $1/a^4$ (keeping the other parameters fixed). In general, one can reduce discretisation errors by adding higher-dimensional operators to the action. This is called improved actions. With improved actions the computational effort needed to perform reliable lattice QCD calculations can potentially be significantly reduced. This idea is behind much of the important progress made in lattice QCD calculations in recent years and has been an increasing part of research in lattice QCD theory.

The main obstacle for obtaining quantitative results (at the few percent level) from numerical simulations of lattice QCD has always been the computational effort associated with the proper inclusion of sea quark effects. Several years ago, substantial progress was made on this problem, in large part due to the development of an improved staggered fermion action [2]. For the first time, computationally efficient lattice simulations with realistic sea quark effects have become possible.

1.2. Light Quark Methods

The simplest lattice quark action replaces the covariant derivative in the continuum action by a discrete difference operator. This so-called naive action suffers from the doubling problem. For every continuum quark flavor, it has 15 additional unphysical flavors, called tastes. The staggered quark action combines four of these tastes into one Dirac–Pauli flavor, by staggering the quark fields on a hypercube. This leaves four unphysical flavors (tastes). This action suffers from large $O(a^2)$ lattice spacing artifacts due to taste.

[1] arXiv:0711.1616v1 [hep-ph] 12 Nov 2007
changing interactions. The A sq-tad action is an im-
proved staggered action where all tree-level discri-
isation errors are removed [2]. Its leading lattice spa-
cing errors are therefore of \( O \left( a^2 \right) \) and greatly reduced com-
pared to the original staggered action. The A sq-
tad action is the com putationally most e cient light
quark action available. However, in order to use it
for the sea quarks in numerical simulations, the un-
physical avor must be removed. The sea quarks are
present in the fermion determinant of the path inte-
gral. To simulate two degenerate light (up and down)
sea quarks, the M ILC collaboration simply takes the
square root of the light quark fermion determinant.
For the strange sea quark, they take the fourth root
of the determinant. This procedure is still somewhat
controversial, but there is a growing body of evidence
that its e ects are controllable and disappear in the
continuum limit [3]. The A sq-tad action with the
square root trick has been extensively tested in nu-
merical simulations, most prominently in Ref. [4].

The H ISQ (highly improved staggered quark) ac-
tion is another version of an improved staggered ac-
tion [6]. Like the A sq-tad action, it removes all tree-
level \( O \left( a^2 \right) \) errors. The \( O \left( a^2 \right) \) errors in the A sq-tad action
are rather large, due to taste changing interactions
which appear at one-loop order. The H ISQ action re-
duces the \( O \left( a^2 \right) \) taste-changing e ects by roughly
a factor of three over the A sq-tad action. The H ISQ
action has not yet been used to generate \( n_f = 2 + 1 \) sea
quark ensembles. Its computational cost is expected
to be about a factor of two larger than the A sq-tad
action.

Other light quark methods include the W ilson ac-
tion and its improved version [7], Domain Wall Fem-
ions [8], and Overlap fermions [9], with increasing com-
putational cost. The W ilson action solves the doubling
problem by adding a dimension \( \eta \) to the problem which
breaks chiral symmetry. Domain Wall fermions solve
the doubling problem by adding a \( \eta \) dimension, while
keeping chiral symmetry exact. Overlap fermions have
exact chiral symmetry, but are computed with a com-
licated operator structure.

### 1.3. Heavy Quark Methods

On the lattice, heavy quarks with \( m_0 \) large, are
best treated within an effective field theory framework
(NRQCD or HQET). One can start with an effective
field theory, and discretise it as in Ref. [10], for ex-
ample Lattice QCD. Alternatively, one can start with a relativistic
lattice action and analyze the mass dependent discri-
ination errors using effective field theories. The chaim
quark is too light for a straightforward implementation of the former approach, so we will focus on the latter.

The Fermilab approach [11] starts with the im-
proved relativistic W ilson action [8] and the observa-
tion that the W ilson action has the same heavy quark
limit as QCD. With a simple prescription, the W ilson
action can be used for heavy quarks without errors
arising from the heavy quark mass, \( (m_0)^2 \). This
approach can be used for both charmed and beauty
quarks. With the improved W ilson action, the leading
discretisation errors are \( O \left( \frac{a}{m_0} \right) \) and \( O \left( \frac{a}{m_0} \right)^2 \).

The H ISQ action is so highly improved that it can be
used for charm quarks with an additional tuning of a pa-
rameter in the action, provided that the lattice
spacings are small enough [6]. The leading mass
dependent discretisation errors are of order
\( O \left( \frac{a}{m} \right)^2 \) and \( O \left( \frac{a}{m} \right)^4 \).

### 1.4. Systematic Errors

The most important source of systematic error in
lattice QCD calculations is sea quark e ects; using
unphysically large masses for the up and down
quarks; discretisation e ects; finite volume e ects; and renorm-
isation e ects.

In order to be phenomenologically relevant, a lattice
QCD calculation must use gauge configurations that
include the e ects of three light sea quarks. Since
the masses of the up and down quarks are generally
taken to be degenerate, this is also referred to as the
\( n_f = 2 + 1 \) case.

Until roughly 10 years ago, almost all lattice QCD
calculations used ensembles generated either in the
quenched approximation or with an incorrect number
of sea quarks (generally, \( n_f = 2 \)) because of the com-
putational cost associated with including sea quarks
in the simulations. The quenched approximation on its
sea quark e ects entirely, at the cost of adding a sys-

tematic error in the range of 10 % to 30 % for physical
quantities involving stable hadrons [10]. This error
must be determined on a case by case basis. Simu-
lations with an incorrect number of sea quarks carry
a similar systematic error, which is hard to estimate
a priori.

The computational cost increases with decreasing
sea quark mass as \( m_1 \rightarrow 0 \). All simulations to date use
masses for the light sea quarks which are larger than
the physical up and down quark masses. (Note, the
strange quark mass is large enough to be simulated
at its physical value.) We can use chiral perturbation
theory (ChPT) to guide the extrapolations from the
calculated sea quark masses used in the simulations to
the physical masses. ChPT is an effective theory of
QCD, which can be applied to lattice QCD calcula-
tions involving pions and kaons. It can be combined
with heavy quark effective theory and be applied to
heavy-light systems, such as D and B mesons. Fur-
thermore, it can be extended to include the leading
light quark discretisation errors. Indeed, this has been
done for the taste changing interactions of the A sq-
tad action and is called staggered ChPT (SChPT) [10].
2. Semileptonic D Meson Form Factors

The semileptonic decays \( D \rightarrow K^{(*)} l \) are mediated by weak vector currents. The hadronic matrix elements for semileptonic decays are parametrized in terms of form factors. In our case there are two form factors, conventionally \( f_1(q^2) \) and \( f_2(q^2) \). The form factors are functions of the virtual \( W \) boson momentum, \( q^2 \), or, equivalently, the recoil momentum of the daughter meson. This introduces additional lattice spacing errors:

\[
\mathcal{M}^\ell \mathcal{P}^{\,\ast} = \mathcal{M}^\ell \mathcal{P}^{\,\ast} + \mathcal{O}(\alpha_s) \quad (2)
\]

Hence, discretization errors are smallest, when \( p_K \) is small and \( q^2 \) is finite. The lattice volume provides an infrared cutoff, and therefore a minimum value for the recoil momentum, \( p_{\text{in}} = \frac{m_D}{2} \). Lattice three-point functions can be written in terms of \( p_{\text{in}} \) as \( p = \frac{m_D}{2} (n_K n_\pi n_\pi) \), where \( n_K n_\pi n_\pi \) are integers. For example, for \( a = 0.1 \) \( \text{fm} \), \( L = 20, p_{\text{in}} = 620 \) \( \text{MeV} \).

To date, the only lattice results for semileptonic \( D \) meson form factors with \( n_f = 2 + 1 \) are from the Fermilab Lattice and MILC collaborations \( [13] \). They use the MILC action for the light valence quarks and the Fermilab action for the charm quark. Staggered chiral perturbation theory is used to extrapolate to the physical light quark masses and to remove the leading discretization errors due to taste violations.

Figure 2 shows a comparison of the lattice QCD result for the normalization \( f_1^D(0) \) for \( D \rightarrow K \ell \) with experimental determinations (where \( v_{\pi} \) is taken from other sources). The results are in very good agreement; however, the lattice QCD result has much larger errors than the experimental determinations. The comparison between lattice theory and experiment for \( f_1(0) \) is similar \( [14] \).

The shape of the form factor can also be determined in lattice QCD. However, in Ref. \( [13] \) the form factors were calculated at a few values of recoil momentum. Then the BK \( [16] \) parametrization was used to determine the \( q^2 \) dependence of the form factors. Since the errors increase with recoil momentum, the shape of the form factors is fixed mainly by the form factors near \( q^2_{\text{ax}} \) and from using the BK parametrization \( [16] \). The lattice QCD result appeared before the new measurements by the FOCUS \( [17] \) and Belle \( [18] \) collaborations were announced, so it is one of very few lattice QCD predictions. Figure 3 \( [15, 16] \) shows a comparison of the lattice prediction for the \( q^2 \) dependence with experimental data from the Belle collaboration \( [16] \). The agreement is excellent. However, a quantitative comparison between the BK shape parameter determined from experimental and lattice theory is difficult to interpret, as eloquently argued by Richard
3. Leptonic Decay Constants $f_D$ and $f_{D_s}$

Charm leptonic decays provide another important test of lattice QCD. The lattice methods for calculating decay constants in the charm and beauty meson system are the same. Indeed, with the Fermilab approach one uses the same heavy quark action in both systems and the heavy quark discretisation errors are expected to be larger for D mesons than for B mesons.

There are now results from two groups (FNAL/MILC and HPQCD). Both use the MILC ensemble at $a = 0.09 \text{ fm}$, $0.12 \text{ fm}$, $0.15 \text{ fm}$.

The first FNAL/MILC results came out in 2005 [25] just days before CLEO announced its precise determination of $f_{D_s}$ [26]; the two results were in good agreement.

The HPQCD collaboration announced their results for decay constants with much reduced errors this summer [27] and FNAL/MILC presented updated results at the Lattice 2007 conference [28], also with reduced errors. The new FNAL/MILC analysis was done "blind", where an overall unknown parameter was added to the lattice data. The final results were unblinded shortly before they were presented at the Lattice 2007 conference, making this the first (intentionally) blind lattice analysis. Table I compares the main
features of the two calculations. More details about the HPQCD and FNAL/ILC calculations, including discussion of the error analysis and plots of chiral and lattice spacing extrapolations can be found in Refs. [25] and [26] respectively. The FNAL/ILC analysis includes three lattice ensembles, three valence quark masses per ensemble, and uses staggered chiral perturbation theory (Staggered ChPT) to remove the leading light quark discretization errors. The HPQCD collaboration considers only the case \( m_\text{q} = m_1 \), where \( m_\text{q} \) denotes the light valence quark mass and \( m_1 \) denotes the light sea quark mass. They use continuum ChPT with generic \( O(a^2) \) terms added in the chiral expansion.

The main difference between the two calculations is the valence quark actions. The HPQCD collaboration uses the HQE action for all (charm, strange and light) valence quarks, whereas the FNAL/ILC collaboration uses the FermiLab action for the charm quarks and the A-sqcd action for the strange and light valence quarks. Since the HQE action is more improved than the Fermilab action, the HPQCD result has much smaller heavy quark discretization errors. This is the main reason for the difference in the total errors between the two results.

Table I compares the error budgets for the 2005 FNAL/ILC calculation with the FNAL/ILC Lattice 2007 one. The error reduction is mainly due to including three M ILC ensembles at \( a = 0.09 \) fm (and 8-12 different valence masses). This reduces the heavy quark and light quark discretization errors, and better constrains the staggered ChPT.

Figures 5, 6, and 8 compare the lattice results for \( f_{D^+} \), \( f_{D^-} \), and \( f_{D^0} = f_{D_{s0}} \), respectively, to the corresponding experimental averages. The experimental averages are from Ref. [14]. The new CLEO-c result \( f_{D^0} = 275 \pm 10 \pm 5 \) presented at this conference by Steven Bink[31], is very similar to Ref. [14].

The FNAL/ILC results agree with the experimental averages at the one sigma level. The HPQCD results agree very well with the FNAL/ILC results. There is a hint of disagreement between the HPQCD result for \( f_{D^0} \), and the experimental average at the two sigma level. However, the experimental determinations of the decay constants must assume a value for the CKM angle \( \theta_{\text{eff}} \) from other sources. We are approaching a level of precision, where tests of lattice QCD should be performed on CKM free quantities such as the ratio of semi-leptonic to leptonic decay rates suggested in Ref. [31].

4. Conclusions and Outlook

With the generation of the M ILC ensembles, the stakes for lattice QCD calculations have risen. We are now able to calculate the simplest quantities to a few percent accuracy. As always, repetition is desirable to test different lattice methods against each other. To date, all lattice calculations that include realistic sea quark effects use the M ILC ensembles with rooted A-sqcd sea quarks. As mentioned in section 1.2, the A-sqcd action carries the smallest computational cost of any light quark action. Nevertheless, recently other collaborations have started to generate ensembles with different sea quark actions. An overview is given in Figure 8. It shows that the other ensembles are being generated at similar values of lattice spacing and light quark masses as the M ILC ensembles. The M ILC collaboration continues to generate new ensembles at even smaller lattice spacings. They are also generating additional calculations for the existing ensembles to further reduce statistical errors. As in experiment, in lattice QCD smaller statistical errors give better con-
Table I Comparison of the main features of the HPQCD and Fermilab Lattice/M ILC calculations.

| FNAL/ILC | HPQCD |
|----------|-------|
| Femlab action for charm quark | H ISQ action for charm and light valence quarks |
| A quartet action for strange and light valence | |
| a (fm) m_{s} sea quark | a (fm) m_{s} sea quark |
| 0.09 | 0.09 |
| 0.12 | 0.12 |
| 0.15 | 0.15 |
| 8 | 1/5, 2/5 |
| 12 | 1/5, 2/5 |
| D_s/D_{+} decay constant ratio | |
| Partial nonperturbative renormalisation | Nonperturbative renormalisation from PCAC |
| Staggered ChPT to all valence and sea quark ensembles together | Continuum ChPT + O(a^2) terms to all ensembles together |
| Blind analysis for Lattice 2007 | |

Table II Comparison of the error budget of the 2005 FNAL/ILC results with the Lattice 2007 results. Numbers are given in percent.

| Source | PRL 2005 [25] | Lattice 2007 [28] |
|--------|---------------|-------------------|
| Statistics | 1.5 | 3.6 |
| HQ discretisation | 4.2 | 1.0 |
| Light quark + Chiral ts | 6.3 | 2.7 |
| Inputs (a, m_c, m_s) | 2.8 | 3.4 |
| Higher order PT | 1.3 | 0.3 |
| Other sm small sources (nir volu m e, ) | 2.0 | |
| Total system atic | 8.5 | 5.4 |

Figure 7: Comparison of lattice QCD results for $f_0 = f_{0,}$ with experiment.

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Figure 8: Simulation parameters for ensemble b with n_{c} = 2 + 1 showing m_{c}, m_{s} vs. lattice spacing a. Filled symbols denote existing ensembles. Unfilled symbols denote ensembles which are currently being generated or planned. Red squares: MILC [29], blue diamonds: BMW (improved Wilson) [33], pink right triangles: PACS-CS (nonperturbatively improved Wilson) [24], green circles: JLQCD (Overlap) [34]. The physical point is at m_{c}, m_{s} = 1-25 (pink burst).

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