We derive a generalised concavity condition for potentials between static sources obtained from Wilson loops coupling both to gauge bosons and a set of scalar fields. It involves the second derivatives with respect to the distance in ordinary space as well as with respect to the relative orientation in internal space. In addition we discuss the use of this field theoretical condition as a nontrivial consistency check of the AdS/CFT duality.
1 Introduction

The AdS/CFT duality conjecture \([1, 2, 3]\) has passed an impressive number of consistency checks \([4]\). However, among these tests only few are not relying in one or another way on structures enforced by supersymmetry and/or conformal invariance. In this situation it appears worthwhile to further analyse any possible constraint set by the first principles of quantum field theory and to check, whether they are fulfilled by the corresponding dual partners in string theory/supergravity.

In this sense the present letter is devoted to the concavity of the potential between static sources in a gauge theory. In the Euclidean formulation Osterwalder-Schrader reflection positivity \([5]\) ensures this property for potentials derived from Wilson loops \([6, 11]\). In the AdS/CFT context the issue of concavity has been raised in ref.\([7]\). But the discussion so far has not taken into account the degree of freedom connected with the relative orientation of the static sources \((Q\bar{Q})\) in internal space.

We will fill this gap by analysing in some detail the consequences of OS reflection positivity for potentials derived in standard manner from Wilson loops for contours coupling both to the gauge bosons and to a set of scalar fields in the adjoint representation. We take the Wilson loop in the form suggested in \([8, 9]\) and analysed in various ways in \([10]\). For the case where the gauge bosons and the scalars are just the bosonic fields of \(D = 4, \mathcal{N} = 4\) super-Yang-Mills theory it has been characterised as an object of BPS type \([10]\).

Our discussion closely follows \([11]\). The new input in our presentation is the handling of the contour parameter dependent coupling to the scalars, which is described by a curve on \(S^5\). We also take care of the fact that the Wilson loop of refs.\([8, 10]\) is the trace of a generically non-unitary matrix.

The virtue of the arising concavity condition lies in its inequality property. It has to be fulfilled both for the classical SUGRA approximation and for the expressions obtained by adding successive corrections. Therefore, a violation at any level of approximation on the superstring side would indicate a breakdown of the corresponding duality.

2 Generalised concavity for potentials derived from BPS Wilson loops

We start with the functional \((A\dot{x} = A_\mu \dot{x}^\mu, \phi \theta = \phi_j \theta^j, \mu = 0, .., 3, \ j = 4, .., 9)\)

\[
U_{ab}[x, \theta] = \left(P \exp \int \{iA(x(s))\dot{x}(s) + \phi(x(s))\theta(s)|\dot{x}|\}ds\right)_{ab}.
\] (1)

The expectation value of its trace for a closed path \(x(s)\) yields the Wilson loop under investigation \([3, 11]\). \(\theta(s)\) specifies the coupling to the scalars \(\phi\) along the contour \(x(s)\).

A reflection operation \(\mathcal{R}\) is defined by

\[
(\mathcal{R}x)^1(s) = -x^1(s); \quad (\mathcal{R}x)^\alpha(s) = x^\alpha(s), \quad \alpha \neq 1
\]

\[
\mathcal{R}U_{ab}[x, \theta] = U_{ab}[\mathcal{R}x, \theta].
\] (2)
In addition, it is useful to define in connection with an isometry \( I \in O(6) \) of \( S^5 \) acting on the path \( \theta(s) \)

\[ IU_{ab}[x, \theta] = U_{ab}[x, I\theta] . \tag{3} \]

For linear combinations of \( U \)'s for different contours we extend \( R \) and \( I \) linearly.

Using the hermiticity of the matrices \( A, \phi \) in the form \( \bar{A} = A^t, \bar{\phi} = \phi^t \) we can reformulate the r.h.s in the second line of (2) applying the following steps

\[ U_{ab}[x, \theta] = \left( P \exp \int_{s_i}^{s_f} \{-iA(t(x(s)))\dot{x}(s) + \dot{\phi}(x(s))\theta(s)|\dot{x}| \} ds \right)_{ab} \]

\[ = \left( \hat{P} \exp \int_{s_i}^{s_f} \{-iA(x(s))\dot{x}(s) + \phi(x(s))\theta(s)|\dot{x}| \} ds \right)_{ba} . \tag{4} \]

Here \( P, \hat{P} \) denote ordering of matrices from right to left with increasing/decreasing argument \( s \). \( \hat{P} \) applied to the path \( x \) yields the same result as \( P \) applied to the backtracking path

\[ (Bx)(s) = x(s_f + s_i - s), \quad (B\theta)(s) = \theta(s_f + s_i - s) . \tag{5} \]

Therefore, we get

\[ U_{ab}[x, \theta] = U_{ba}[Bx, B\theta] . \tag{6} \]

This, combined with (2),(3) yields finally

\[ RIU_{ab}[x, \theta] = U_{ba}[BRx, BI\theta] . \tag{7} \]

It is worth pointing out that for the result (6) the presence/absence of the factor \( i \) in front of the \( A \) and \( \phi \) term in \( U \) is crucial. One could consider this as another argument for the choice favoured by the investigations of ref. [10].

We now turn to a derivation of the basic Osterwalder-Schrader positivity condition in a streamlined form within the continuum functional integral formulation. All steps can be made rigorously by a translation into a lattice version with local and nearest neighbour interactions.

Let denote \( H_\pm = \{ x^\mu | \pm x^1 > 0 \}, \quad H_0 = \{ x^\mu | x^1 = 0 \} \). Then we consider for a functional of two paths \( x^{(1)}, x^{(2)} \in H_+ \)

\[ f[x^{(1)}, \theta^{(1)}; x^{(2)}, \theta^{(2)}] = U_{ab}[x^{(1)}, \theta^{(1)}] + \lambda U_{ab}[x^{(2)}, \theta^{(2)}], \quad \lambda \text{ real} , \tag{8} \]

\[ \langle f[x, \theta]RI[f[x, \theta]] \rangle = \int DAD\phi f[x, \theta]\bar{f}[Rx, I\theta] e^{-S} \]

\[ = \int DA^{(0)}D\phi^{(0)} e^{-S_0} \cdot \int_{(b.c.)} DA^{(+)}D\phi^{(+)} f[x, \theta] e^{-S_+} \cdot \int_{(b.c.)} DA^{(-)}D\phi^{(-)} \bar{f}[Rx, I\theta] e^{-S_-} . \tag{9} \]

\( \pm, 0 \) on the fields as well as on the action indicates that it refers to points in \( H_\pm, H_0 \). The index for the two paths has been dropped, and the boundary condition \( (b.c.) \) is

\[ A^{(\pm)}|_{\partial H_\pm} = A^{(0)}, \quad \phi^{(\pm)}|_{\partial H_\pm} = \phi^{(0)} . \]
With the abbreviation

\[ h[A^{(0)}, \phi^{(0)}, x, \theta] = \int_{(b.c.)} D\!A^{(+)} D\!\phi^{(+)} f[x, \theta] e^{-S_+}, \]  

(10)

the standard reflection properties of the action imply

\[ \langle f[x, \theta] R I f[x, \theta] \rangle = \int D\!A^{(0)} D\!\phi^{(0)} e^{-S_0} h[A^{(0)}, \phi^{(0)}, x, \theta] \cdot h[A^{(0)}, \phi^{(0)}, x, \theta] \]  

(11)

For \( I = 1 \) the integrand of the final integration over the fields in the reflection hyperplane \( H_0 \) is non-negative, hence

\[ \langle f[x, \theta] R I f[x, \theta] \rangle \geq 0. \]  

(12)

For nontrivial \( I \) the situation is by far more involved. If there would be no boundary condition, the result of the half-space functional integral in (10) would be invariant with respect to \( \theta \to I \theta \). A given boundary configuration in general breaks \( O(6) \) invariance on \( S^5 \). But due to the \( O(6) \) invariance of the action, the functional integration measure and the \( \phi \theta \) coupling in \( f \), we have instead

\[ h[A^{(0)}, I \phi^{(0)}, x, I \theta] = h[A^{(0)}, \phi^{(0)}, x, \theta]. \]  

(13)

This implies

\[ \langle f[x, \theta] R I f[x, \theta] \rangle = \int D\!A^{(0)} D\!\phi^{(0)} e^{-S_0} \]

\[ \cdot \frac{1}{2} \left( h[A^{(0)}, \phi^{(0)}, x, \theta] h[A^{(0)}, \phi^{(0)}, x, I \theta] + h[A^{(0)}, \phi^{(0)}, x, I \theta] \right) \]

which says us only (\( R \) real numbers)

\[ \langle f[x, \theta] R I f[x, \theta] \rangle \in R \quad \text{for} \quad I^2 = 1. \]  

(15)

The statements (12) and (13) are rigorous ones. Beyond them we found no real proof for sharpening (15) to an inequality of the type (12) for some nontrivial \( I \). For later application to the estimate of rectangular Wilson loops we are in particular interested in nontrivial isometries keeping the, by assumption common, \( S^5 \) position of the endpoints of the contours on \( H_0 \) fixed. Then \( I = I_\pi \), denoting a rotation around this fixpoint with angle \( \pi \), are the only candidates.

At least for boundary fields \( \phi^{(0)} \) in (14), which as a map \( R^3 \to S^5 \) have a homogeneous distribution of their image points on \( S^5 \), we can expect that for contours of the type discussed in connection with fig.1 below in the limit of large \( T \) the orientation of \( \theta \) relative to \( \phi^{(0)} \) becomes unimportant. Therefore, we conjecture for this special situation

\[ \langle f[x, \theta] R I_\pi f[x, \theta] \rangle \geq 0. \]  

(16)

From (12) and (16) for any real \( \lambda \) in (8) we get via the standard derivation of Schwarz-type inequalities

\[ \langle U_{ab}[x^{(1)}, \theta^{(1)}] R I U_{ab}[x^{(2)}, \theta^{(2)}] \rangle^2 \leq \langle U_{ab}[x^{(1)}, \theta^{(1)}] R I U_{ab}[x^{(1)}, \theta^{(1)}] \rangle \cdot \langle U_{ab}[x^{(2)}, \theta^{(2)}] R I U_{ab}[x^{(2)}, \theta^{(2)}] \rangle. \]  

(17)
Fig. 1 From left to right the contours $x^+ \circ x^-, \ x^+ \circ BRx^+, \ BRx^- \circ x^-$.  

This is a rigorous result for $I = 1$ and a conjecture for $I = \pi$.

Let us continue with the discussion of a Wilson loop for a closed contour which crosses the reflection hyperplane twice and which is the result of going first along $x^- \in H_-$ and then along $x^+ \in H_+$. In addition we restrict to cases of coinciding $S^5$ position at the intersection points with $H_0$ and treat in parallel $I = 1, \pi$

$$W[x^+ \circ x^-, \theta^+ \circ \theta^-] = \sum_{ab} \langle U_{ab}[x^+, \theta^+] \ U_{ba}[x^-, \theta^-] \rangle$$

$$= \sum_{ab} \langle U_{ab}[x^+, \theta^+] \ \mathcal{R}I U_{ba} [\mathcal{R}x^-, I^{-1} \theta^-] \rangle$$

$$\leq \sum_{ab} \langle U_{ab}[x^+, \theta^+] \ \mathcal{R}I U_{ab} [x^+, \theta^+] \rangle^{1/2} \langle U_{ba} [\mathcal{R}x^-, I^{-1} \theta^-] \ \mathcal{R}I U_{ba} [\mathcal{R}x^-, I^{-1} \theta^-] \rangle^{1/2}$$

$$\leq \left( \sum_{ab} \langle U_{ab}[x^+, \theta^+] \ \mathcal{R}I U_{ab} [x^+, \theta^+] \rangle \right)^{1/2} \left( \sum_{cd} \langle U_{cd} [\mathcal{R}x^-, I^{-1} \theta^-] \ \mathcal{R}I U_{cd} [\mathcal{R}x^-, I^{-1} \theta^-] \rangle \right)^{1/2}.$$

We have used (2), (3), $\mathcal{R} \mathcal{R} x = x$, (17) and the usual Schwarz inequality in the last step. Now with (3), (4) we get

$$W[x^+ \circ x^-, \theta^+ \circ \theta^-] \leq \left( W[x^+ \circ BRx^+, \theta^+ \circ BI \theta^+] \right)^{1/2} \cdot \left( W[BRx^- \circ x^-, BI^{-1} \theta^- \circ \theta^-] \right)^{1/2}.$$  

(19)

To evaluate the potential between two static sources ($Q\bar{Q}$) separated by the distance $L$ and located at fixed $S^5$-positions $\theta_Q, \theta_{\bar{Q}}$ we need Wilson loops for rectangular contours of extension $L \times T$ in the large $T$-limit. We choose the $S^5$-position on the two $L$-sides linearly
interpolating between $\theta_Q$ and $\theta_{\bar{Q}}$ on the corresponding great circle. For this restricted set of contours the Wilson loop becomes a function of $L$, $T$ and the angle between $\theta_Q$ and $\theta_{\bar{Q}}$, called $\Theta$.

In addition it is useful to restrict ourselves to contours which are situated in planes orthogonal to the reflection hyperplane and with $T$-sides running parallel to it in a distance $L \pm \delta$, see fig.1. Then $\mathcal{I} = \mathcal{I}_x$ reflects $\theta^\pm(s)$, which both lie on the great circle through $\theta_Q$ and $\theta_{\bar{Q}}$, with respect to the common $S^5$-position of the points $A$ and $B$, see fig.1. As a consequence, (19) implies

$$W(L, T, \Theta) \leq \left( W(L - \delta, T, \frac{L - \delta}{L} \Theta) \right)^{\frac{1}{2}} \left( W(L + \delta, T, \frac{L + \delta}{L} \Theta) \right)^{\frac{1}{2}}, \quad (20)$$

which by standard reasoning yields for the static potential

$$V(L, \Theta) \geq \frac{1}{2} \left( V(L - \delta, \frac{L - \delta}{L} \Theta) + V(L + \delta, \frac{L + \delta}{L} \Theta) \right). \quad (21)$$

The last inequality implies the local statement $\frac{\partial^2}{\partial s^2} V(L + \delta, \frac{L + \delta}{L} \Theta) \leq 0$, i.e.

$$\left( L^2 \frac{\partial^2}{\partial L^2} + 2L\Theta \frac{\partial^2}{\partial L \partial \Theta} + \Theta^2 \frac{\partial^2}{\partial \Theta^2} \right) V(L, \Theta) \leq 0. \quad (22)$$

It means concavity on each straight line across the origin, in the relevant part of the $(L, \Theta)$-plane, $0 < L < \infty$, $0 < \Theta \leq \pi$.

Both (21) and (22) rely on the conjecture (19). From the rigorous point of view we are allowed to use (19) for $\mathcal{I} = 1$ only. Then the paths generated on the r.h.s. are, with respect to their $S^5$ properties, no longer of the type with which we started on the l.h.s. On the part of the space time contour orthogonal to $H_0$ we go e.g. from $\theta_Q$ to the common $S^5$ position of the points $A$ and $B$ and then back to $\theta_Q$. Since in the large $T$-limit, relevant for the extraction of the $Q\bar{Q}$-potential, only the behaviour on the large $T$-sides matters, we get

$$V(L, \Theta) \geq \frac{1}{2} \left( V(L - \delta, 0) + V(L + \delta, 0) \right). \quad (23)$$

This means standard concavity at $\Theta = 0$ and

$$V(L, \Theta) \geq V(L, 0). \quad (24)$$

If the same steps are repeated for rectangles with large $T$-sides still parallel to $H_0$, but spanning a plane no longer orthogonal to $H_0$ one finds

$$V(L, \Theta) \geq \frac{1}{2} \left( V(\alpha(L - \delta), 0) + V(\alpha(L + \delta), 0) \right), \quad 0 \leq \alpha \leq 1. \quad (25)$$

The only new information gained from (25) is that $V(L, 0)$ is monotonically non-decreasing in $L$. 

3 Test of the generalised concavity condition for potentials derived via AdS/CFT duality

The simplicity of the calculation recipe for Wilson loops in the classical SUGRA approximation via AdS/CFT duality allows to make statements on universal properties of the arising $Q\bar{Q}$-potential for a large class of SUGRA backgrounds [12, 13]. We now enter a discussion of (22) within this framework. The metric of the SUGRA background is assumed in the form

$$G_{MN}dx^M dx^N = G_{00}(u)dx^0dx^0 + G_{||}(u)dx^m dx^m + G_{uu}(u)dudu + G_\Omega(u)d\Omega^2. \quad (26)$$

Then with

$$f(u) = G_{00}G_{||}, \quad g(u) = G_{00}G_{uu}, \quad j(u) = G_{00}G_\Omega \quad (27)$$

we get along the lines of [9, 12, 13, 14]

$$L(\Lambda) = 2\sqrt{f_0\sqrt{1-l^2}} \int_{u_0}^{\Lambda} \sqrt{\frac{gj}{f}} \frac{du}{\sqrt{j(f-f_0) + (jf_0 - j_0f)^2}} ,$$

$$\Theta(\Lambda) = 2l\sqrt{j_0} \int_{u_0}^{\Lambda} \sqrt{\frac{gfj}{j}} \frac{du}{\sqrt{j(f-f_0) + (jf_0 - j_0f)^2}} ,$$

$$V(\Lambda) = \frac{1}{\pi} \int_{u_0}^{\Lambda} \sqrt{gfj} \frac{du}{\sqrt{j(f-f_0) + (jf_0 - j_0f)^2}} . \quad (28)$$

We defined $f_0 = f(u_0)$ etc. $\Lambda$ is a cutoff at large values of $u$. In the following our discussion will be restricted to values of $L$ and $\Theta$ for which all expressions appearing under square roots above are positive and where the inversion $u_0 = u_0(L, \Theta), \quad l = l(L, \Theta)$ is well defined. (28) implies

$$V(\Lambda) = \frac{1}{\pi} \int_{u_0}^{\Lambda} \sqrt{gfj} \frac{du}{\sqrt{j(f-f_0) + (jf_0 - j_0f)^2}}$$

$$+ \frac{1}{2\pi} \sqrt{f_0\sqrt{1-l^2}} L(\Lambda) + \frac{1}{2\pi} \sqrt{j_0 l} \Theta(\Lambda) . \quad (29)$$

Now we differentiate with respect to $u_0$ and $l$. After this $\Lambda$ can be sent to $\infty$ ending up with a relation for the renormalised potential $V$:

$$\frac{\partial V}{\partial u_0} = \frac{1}{2\pi} \sqrt{f_0\sqrt{1-l^2}} \frac{\partial L}{\partial u_0} + \frac{1}{2\pi} \sqrt{j_0 l} \frac{\partial \Theta}{\partial u_0} ,$$

$$\frac{\partial V}{\partial l} = \frac{1}{2\pi} \sqrt{f_0\sqrt{1-l^2}} \frac{\partial L}{\partial l} + \frac{1}{2\pi} \sqrt{j_0 l} \frac{\partial \Theta}{\partial l} . \quad (30)$$

For $V$ defined by (28) implicitly as a function of $L$ and $\Theta$ this means

$$\frac{\partial V}{\partial L} = \frac{1}{2\pi} \sqrt{f_0\sqrt{1-l^2}}, \quad \frac{\partial V}{\partial \Theta} = \frac{1}{2\pi} \sqrt{j_0 l} . \quad (31)$$
i.e. \( V \) is monotonically nondecreasing both in \( L \) and \( \Theta \). The monotony in \( \Theta \) is in agreement with our rigorous result (24).

Calculating now second derivatives one arrives at \((f'_0 = \frac{df(u_0)}{du_0} \text{ etc.})\)

\[
\left( L^2 \frac{\partial^2}{\partial L^2} + 2L\Theta \frac{\partial^2}{\partial L \partial \Theta} + \Theta^2 \frac{\partial^2}{\partial \Theta^2} \right) V(L, \Theta) =
\]

\[
= \frac{1}{4\pi \sqrt{f_0 j_0}} \left( L j'_0 \sqrt{j_0 (1 - l^2)} + \Theta j'_0 \sqrt{f_0 l} \right) \left( L \frac{\partial u_0}{\partial L} + \Theta \frac{\partial u_0}{\partial \Theta} \right)
\]

\[
+ \frac{1}{2\pi \sqrt{1 - l^2}} \left( \Theta \sqrt{j_0 (1 - l^2)} - L \sqrt{f_0 l} \right) \left( L \frac{\partial l}{\partial L} + \Theta \frac{\partial l}{\partial \Theta} \right).
\]

Neglecting for a moment the issue of internal space dependence by restricting oneselfs to the case \( \Theta = l = 0 \), one finds usual concavity in \( L \) from (32) if \( f'_0 \frac{\partial u_0}{\partial L} \leq 0 \). The last inequality is for \( f' > 0 \) guaranteed by theorem 1 of ref. [13].

Therefore, for \( \Theta = 0 \) standard concavity of \( Q\bar{Q} \) potentials with respect to the distance in usual space is guaranteed for the wide class of SUGRA backgrounds covered by theorem 1 of ref. [13].

However, due to the more complicated structure of the l.h.s. of (32) for \( \Theta \neq 0 \) we did not found a similar general statement in the generic case. We can only start checking (22) case by case.

As our first example we consider the original calculation of Maldacena [9] for the \( AdS_5 \times S^5 \) background. The result was \((R^2 = \sqrt{2g^2_{YM}N})\)

\[
V(L, \Theta) = - \frac{2R^2}{\pi} \frac{F(\Theta)}{L},
\]

with

\[
F(\Theta) = (1 - l^2)^{\frac{3}{2}} \left( \int_1^\infty \frac{dy}{y^2 \sqrt{(y^2 - 1)(y^2 + 1 - l^2)}} \right)^2,
\]

\[
\Theta = 2l \int_1^\infty \frac{dy}{\sqrt{(y^2 - 1)(y^2 + 1 - l^2)}}.
\]

Due to this special structure \((L \frac{\partial V}{\partial L} = -V, L^2 \frac{\partial^2 V}{\partial L^2} V = 2V, \frac{\partial \Theta}{\partial u_0} = 0)\), (22) is equivalent to

\[
\Theta^3 \frac{d^2}{d\Theta^2} \left( \frac{F}{\Theta} \right) \geq 0.
\]

A numerical calculation of \( F \) confirms (24) clearly, see fig.2.

\[\text{\textsuperscript{2}}\text{Our} f \text{ and } g \text{ are called } f^2 \text{ and } g^2 \text{ in that paper.}\]
Next we discuss the large $L$ confining potential including internal space dependence and $\alpha'$ corrections of the background derived in [14]. It has the form ($\gamma = \frac{1}{8} \zeta(3) R^{-6}$, $T$ temperature parameter)

$$V(L, \Theta) = \frac{\pi R^2 T^2}{2} (1 - \frac{265}{8} \gamma) \cdot L + \frac{R^2}{4\pi} (1 + \frac{15}{8} \gamma) \frac{\Theta^2}{L} + O(1/L^3).$$ (36)

Although this potential for $\Theta \neq 0$ violates naive concavity $\frac{\partial^2 V}{\partial L^2} \leq 0$, there is no conflict with the correctly generalised concavity (22). Applied to (36) the differential operator just produces zero.

4 Concluding remarks

The $QQ$-potential derived [1] from the classical SUGRA approximation for the type IIB string in $AdS_5 \times S^5$ fulfils our generalised concavity condition at $\Theta \geq 0$. This adds another consistency check of this most studied case within the AdS/CFT duality.

Potentials have been almost completely studied only for $\Theta = 0$ in other backgrounds. At least partly, this might be due to the wisdom to approach in some way QCD, where
after all there is no place for a parameter like this angle between different orientations in $S^5$. However, one has to keep in mind that this goal, in the approaches discussed so far, requires some additional limiting procedure. Before the limit the full 10-dimensionality inherited by the string is still present. Fluctuation determinants in all 10 directions have to be taken into account for quantum corrections \cite{4,15} and the $\Theta$-dependence of the potentials is of course not switched off.

Although we proved in classical SUGRA approximation monotony in $L$ and $\Theta$ as well as concavity at $\Theta = 0$ for a whole class of backgrounds, we were not able to get a similar general result on concavity for $\Theta > 0$. Further work is needed to decide, whether at all general statements for $\Theta > 0$ are possible. Alternatively one should perform case by case studies for backgrounds derived e.g. from rotating branes \cite{16}, type zero strings \cite{17} or nonsupersymmetric solutions of type IIB string theory \cite{18}.

On the field theory side further work is necessary to really prove the conjectured inequality (14), otherwise the available set of rigorous constraints on the $L$ and $\Theta$ dependent potential, beyond the standard concavity at $\Theta = 0$, would contain only the very mild condition (24).

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