Experimental study of earthquake input energy of low-frequency structures equipped with a passive rate-independent damping device

Wei Liu1 | Kohju Ikago2

1School of Engineering, Tohoku University, Sendai, Japan
2International Research Institute of Disaster Science, Tohoku University, Sendai, Japan

Correspondence
Kohju Ikago, International Research Institute of Disaster Science, Tohoku University, Sendai 980-0845, Japan.
Email: ikago@irides.tohoku.ac.jp

Summary
The past two decades have witnessed the adverse effects of low-frequency components of severe earthquakes in low-frequency structures. The development of displacement control technologies for the seismic protection of low-frequency structures has been identified as a growing challenge in earthquake engineering. Related studies have suggested that rate-independent linear damping (RILD) could be a viable option for reducing excessive displacements in low-frequency structures. However, previous studies on RILD systems have not extensively investigated their input energy during an earthquake. The main objective of this study is to examine the earthquake input energy of low-frequency structures equipped with a passive rate-independent damping device. The differences in the input energy of the linear viscous damping (LVD) and RILD systems are investigated, based on real-time hybrid simulations and frequency-domain analysis.

KEYWORDS
earthquake input energy, low-frequency structures, passive rate-independent damping, rate-independent linear damping, real-time hybrid simulations

1 INTRODUCTION

With the increasing number of seismically isolated and high-rise buildings, extensive attention has been focused on the possibility of the excessive displacement of low-frequency structures induced by low-frequency components in an extreme seismic event. In this paper, a low-frequency structure represents a high-rise or seismically isolated building with a fundamental natural frequency below 1 Hz. During the 2011 Great East Japan Earthquake, many high-rise buildings in megacities were subjected to long-duration shaking, and some suffered excessive displacement, resulting in damage to their interior and exterior walls.

Rate-independent linear damping (RILD) is found to be an ideal linear model for simultaneously reducing excessive displacement and floor response accelerations because it achieves direct control of displacement via a resistive force that is proportional to the response displacement and in phase with the velocity. RILD is a non-causal element, which hinders its physical implementation. Nevertheless, many researchers have investigated the causal approximation of RILD. Boit proposed the first successful causal model of RILD, which comprised an infinite number of Maxwell...
elements. However, Caughey\textsuperscript{4} reported that Biot’s model can only be regarded as rate-independent at high frequencies. Inaudi and Kelly\textsuperscript{8} proposed a time-domain analysis method based on convergent calculations with the repeated use of the Hilbert transform. Makris\textsuperscript{9} proposed the addition of an adjustable real term to the complex-valued stiffness of an ideal RILD model to satisfy the causality requirement.

Makris and Zhang\textsuperscript{10} proposed two causal models that approximate the nearly frequency-independent cyclic behavior of soils and demonstrated that the causal hysteretic model is derived from the mathematical connection between the non-causal constant hysteretic model and Biot’s model.\textsuperscript{7} Researchers have also focused on mathematical means for the causal implementation of RILD.\textsuperscript{11–22} Inaudi and Makris\textsuperscript{23} proposed a time-domain analysis method to transform the integro-differential equations of structures incorporated with RILD into ordinary differential equations using analytic signals. They used backward integration over time to address the issue of unstable poles included in differential equations. For the case when the physical behavior of a system is closer to hysteretic than viscous, Deastra et al.\textsuperscript{24} proposed replacing the viscous element in inerter-based damping devices with RILD to more realistically capture the system performance. They also developed a time-domain integration method for the RILD-containing inerter-based device by modifying the method proposed by Inaudi and Makris.\textsuperscript{23}

Based on a signal processing approach, Keivan et al.\textsuperscript{3} proposed a first-order all-pass filter to approximate RILD causally. The digital filter they proposed has the form of a one-pole and one-zero transfer function. Keivan et al.\textsuperscript{25} examined the performance of a control algorithm based on a first-order all-pass filter in controlling a 14-story inter-story isolated building equipped with a magnetorheological damper. To ensure the reliability of a device that is free of failure, even in the case of power loss caused by a severe earthquake, Luo et al.\textsuperscript{26} proposed a mechanical configuration to realize RILD, in which the Maxwell element is arranged in parallel with a negative stiffness element to physically reproduce the first-order all-pass filter proposed by Keivan et al.\textsuperscript{3} Because the mass matrix derived in their study was singular, some explicit integral schemes could not be used in the numerical analysis.

Real-time hybrid simulation (RTHS) is a promising alternative to the dynamic testing of structural systems as it combines physical testing with numerical simulation\textsuperscript{27–33} and can significantly reduce the required time, labor, and cost. In the RTHS method, only the critical components, such as the rate-dependent component, are required to be physically tested, while the remaining parts of the structure can be simulated in the numerical domain.

A significant amount of research has been conducted on earthquake input energy\textsuperscript{34–37} because the energy input to a structure during an earthquake is an important measure of seismic demand, leading to better control of the dynamic responses of low-frequency structures.

The main objective of this study is to investigate the input energy of low-frequency structures equipped with RILD and to identify further challenges for implementing RILD for the protection of low-frequency structures. A seismic isolated structure with a fundamental natural frequency of 0.25 Hz is exemplified in this paper. A Maxwell element and a negative stiffness are combined parallelly to implement a passive causal rate-independent linear damping (CRILD) device. The negative stiffness element in the CRILD is incorporated into the numerical domain in the RTHS loop, whereas the remaining components in CRILD are physically tested. This is because the most straightforward method to realize a negative stiffness element is by reducing the isolator stiffness in the numerical domain.

The remainder of this paper is organized as follows. Section 2 presents the numerical analyses conducted to compare the performance of RILD and LVD systems. Section 3 demonstrates that the earthquake input energy input to the RILD system is significantly smaller than that of the LVD system. In Section 4, a series of RTHS experiments are performed to verify the effectiveness of the passive CRILD in reducing the earthquake input energy of low-frequency structures. Furthermore, the results of the RTHS and frequency-domain analyses are compared to examine the validity of the numerical model of the CRILD device. Finally, Section 5 concludes the study.

## 2 | COMPARISON BETWEEN THE RILD AND LVD SYSTEMS

Let us consider a single-degree-of-freedom (SDOF) structure equipped with an LVD with a damping ratio of $\beta$ and subjected to harmonic ground excitation $\ddot{x}_g(t) = A_g e^{i\omega t}$, where $A_g$ and $\omega$ are the ground acceleration amplitude and excitation circular frequency, respectively. The equation of motion of the SDOF system, where the mass, stiffness, and damping coefficient are $m$, $k$, and $c = 2\beta\sqrt{mk}$, respectively, is as follows:
\[ m \ddot{x}(t) + c \dot{x}(t) + kx(t) = -mA_e e^{i\omega t} \quad (1) \]

If the response \( x(t) \) is harmonic, it can be represented by \( x(t) = X(i\omega)e^{i\omega t} \), where \( X(i\omega) \) is the displacement amplitude at frequency \( \omega \). Equation (1) can then be rewritten in the frequency domain as follows:

\[ (-\omega^2 + 2\beta \omega_0 \omega + \omega_0^2)X(i\omega)e^{i\omega t} = -A_e e^{i\omega t} \quad (2) \]

Subsequently, the frequency response functions of an SDOF structure equipped with an LVD can be obtained as follows:

1. Frequency response function of relative displacement:

\[ H_{LVD}^d(i\omega) = \frac{X(i\omega)}{A_e} = \frac{1}{\omega^2 - \omega_0^2 - 2\beta \omega_0 \omega} \quad (3) \]

2. Frequency response function of relative velocity:

\[ H_{LVD}^v(i\omega) = i\omega H_{LVD}^d(i\omega) = \frac{i\omega}{\omega^2 - \omega_0^2 - 2\beta \omega_0 \omega} \quad (4) \]

3. Frequency response function of floor response acceleration:

\[ H_{LVD}^a(i\omega) = 1 + (i\omega)^2 H_{LVD}^d(i\omega) = -\frac{\omega_0^2 + 2\beta \omega_0 \omega}{\omega^2 - \omega_0^2 - 2\beta \omega_0 \omega} \quad (5) \]

4. Frequency response function of dimensionless damping force coefficient:

\[ H_{LVD}^f(i\omega) = \frac{f_{LVD}}{mg} = \frac{2\beta \omega_0 \omega}{g} H_{LVD}^d(i\omega) \quad (6) \]

where \( \omega_0 \) represents the fundamental natural frequency of the undamped SDOF system.

Similarly, in the case of the SDOF structure equipped with an ideal RILD with a loss factor of \( \eta = 2h \), the governing equation of motion is expressed as follows:

\[ m \ddot{x} + kx + f_{RILD} = m \ddot{x} + kx + i\eta k \text{sgn}(\omega)x = -mA_e e^{i\omega t} \quad (7) \]

where \( h \) and \( \text{sgn}(\omega) \) represent the complex damping ratio and signum function, respectively. Therefore, the frequency response functions of the SDOF structure equipped with an ideal RILD are as follows:

5. Frequency response function of relative displacement:

\[ H_{RILD}^d(i\omega) = \frac{1}{\omega^2 - \omega_0^2 - \eta \omega_0 \text{sgn}(\omega)} \quad (8) \]
6. Frequency response function of relative velocity:

\[ H_{RILD}^R(i\omega) = i\omega H_{RILD}^d(i\omega) = \frac{i\omega}{\omega^2 - \omega_0^2 - \eta \omega_0^2 \text{sgn}(\omega)} \]  

(9)

7. Frequency response function of floor response acceleration:

\[ H_{RILD}^a(i\omega) = 1 + (i\omega)^2 H_{RILD}^d(i\omega) = -\frac{\omega_0^2 + \eta \omega_0^2 \text{sgn}(\omega)}{\omega^2 - \omega_0^2 - \eta \omega_0^2 \text{sgn}(\omega)} \]  

(10)

8. Frequency response function of dimensionless damping force coefficient:

\[ H_{RILD}^f(i\omega) = \frac{f_{RILD}}{mg} = \frac{\eta \omega_0^2 \text{sgn}(\omega) \omega_0^2}{g H_{RILD}^d(i\omega)} \]  

(11)

As depicted in Figure 1, the relative displacement, relative velocity, and floor response acceleration of the RILD system (Figure 1a–c, respectively) are less than those of the LVD system at frequencies lower than the fundamental natural frequency for the same damping ratios.

**FIGURE 1** Transfer functions \((\omega_0 = 1.57 \text{ rad/s}, \beta = h = 0.2)\). (a) Relative displacement; (b) relative velocity; (c) floor response acceleration; (d) damping force
Figure 1d shows that the damping force of the LVD system is less than that of the RILD system at frequencies lower than the fundamental natural frequency. The opposite holds for a region of frequencies greater than the fundamental natural frequency. This indicates that an RILD system generates a high control force in the frequency region below the fundamental natural frequency of the primary structure, which is effective in mitigating excessive displacement. Another benefit of RILD is that its control force is low in the frequency region exceeding the fundamental natural frequency of the primary structure; this advantage prevents the generation of a high control force that is ineffective in mitigating displacement, causing high floor response accelerations.

3 | EARTHQUAKE INPUT ENERGY OF THE RILD AND LVD SYSTEMS

Uang and Bertero\textsuperscript{35} analyzed two types of energy equations: relative and absolute. They noted an advantage of the absolute energy equation in that it conveys a physical meaning, that is, the work done by the resistive force of the structure on the ground displacement. The energy-time histories obtained from the two equations significantly differed for a long-period (low-frequency) structure; nevertheless, they were identical when the ground velocity settled to zero after the end of the earthquake excitation. Moreover, the input energy and the energy dissipated by damping in an elastic system became equal when the structural vibration ended. Thus, we employed the commonly used relative input energy equation in the following discussion.

3.1 | Earthquake input energy in the frequency domain

Let us consider an SDOF structure, whose mass, stiffness, and damping coefficient are $m$, $k$, and $c$, respectively, when it is subjected to ground excitation $\ddot{x}_g$:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -m\ddot{x}_g$$  \hspace{1cm} (12)

On integrating Equation 12 with respect to $x$, we obtain the following:

$$\int m\ddot{x}dx + \int c\dot{x}dx + \int kxdx = -\int m\ddot{x}_gdx$$  \hspace{1cm} (13)

The right-hand side term of Equation 13 is the commonly used relative input energy\textsuperscript{35} $E_I$ (Equation 14), which was evaluated at the end of the ground motion duration.

$$E_I = -\int_{-\infty}^{\infty} m\ddot{x}_gdx$$  \hspace{1cm} (14)

Furthermore, the input energy normalized by the mass $m$ is as follows:

$$\frac{E_I}{m} = -\int_{-\infty}^{+\infty} \ddot{x}_gdx = -\int_{-\infty}^{+\infty} \dot{x}_g\dot{x}dt$$  \hspace{1cm} (15)

In the frequency domain, the relative velocity of the SDOF structure can be expressed as follows:

$$\tilde{X}(i\omega) = A(i\omega)H_V(i\omega)$$  \hspace{1cm} (16)

where $\tilde{X}(i\omega)$ is the Fourier transform of the relative velocity $\dot{x}(t)$, $A(i\omega)$ is the Fourier transform of the ground motion, and $H_V(i\omega)$ is the transfer function from the ground acceleration to the relative velocity.

An inverse Fourier transform of Equation 16 yields $\dot{x}(t)$ as follows\textsuperscript{37}:
By substituting Equation 17 into Equation 15 and changing the order of integration, we obtain

\[
\frac{E_l}{m} = -\int_{-\infty}^{\infty} \tilde{x}_g \tilde{x} dt = -\frac{1}{2\pi} \int_{-\infty}^{\infty} A(i\omega)H_V(i\omega)A^*(i\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} |A(i\omega)|^2 H_V(i\omega) d\omega
\]  

(19)

where \(\int_{-\infty}^{\infty} \tilde{x}_g e^{i\omega t} dt\) is the complex conjugate of the Fourier transform of ground motion \(\tilde{x}_g\), expressed as \(A^*(i\omega)\).

Therefore,

\[
\frac{E_l}{m} = -\int_{-\infty}^{\infty} \tilde{x}_g \tilde{x} dt = -\frac{1}{2\pi} \int_{-\infty}^{\infty} A(i\omega)H_V(i\omega)A^*(i\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} |A(i\omega)|^2 H_V(i\omega) d\omega
\]

(19)

As shown in Equations 4 and 9, a velocity transfer function \(H_V(i\omega)\) is a complex-valued even function, and \(|A(i\omega)|\) is the Fourier amplitude spectrum of the ground acceleration. Therefore, an equivalent expression for Equation 19 is

\[
\frac{E_l}{m} = -\frac{1}{\pi} \int_{0}^{\infty} |A(i\omega)|^2 \text{Re}[H_V(i\omega)] d\omega
\]

(20)

where \(\text{Re}[H_V(i\omega)]\) is the real part of \(H_V(i\omega)\).

Equation 20 can then be rewritten as

\[
\frac{E_l}{m} = -\frac{1}{2\pi} \int_{0}^{\infty} |A(i\omega)|^2 f_E(\omega) d\omega
\]

(21)

where \(f_E(\omega)\) is an energy transfer function, defined as follows:

\[
f_E(\omega) = \frac{2}{\pi} \text{Re}[H_V(i\omega)]
\]

(22)

By substituting Equation 4 into Equation 22, we can obtain the energy transfer function of the LVD system as follows:

\[
f_{E,LVD}(\omega) = \frac{2}{\pi} \text{Re} \left[ \frac{i\omega}{\omega^2 - \omega_0^2 - 2\beta\omega_0\omega} \right] = \frac{1}{\pi} \frac{4\beta\omega_0^2}{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega_0^2\omega^2}
\]

(23)

Similarly, the energy transfer function of the RILD system is

\[
f_{E,RILD}(\omega) = \frac{2}{\pi} \text{Re} \left[ \frac{i\omega}{\omega^2 - \omega_0^2 - \eta\omega_0^2\text{sgn}(\omega)} \right] = \frac{1}{\pi} \frac{2\eta\omega_0^2\omega\text{sgn}(\omega)}{(\omega^2 - \omega_0^2)^2 + \eta^2\omega_0^4}
\]

(24)

Figure 2 depicts the earthquake input energy transfer functions of the LVD and RILD systems. These two transfer functions have two intersections at approximately \(\omega/\omega_0 = 0.8\) and \(\omega/\omega_0 = 1.2\). The ratio of the two energy-transfer functions is shown in Figure 3. Despite the fluctuation around the frequency ratio of \(\omega/\omega_0 = 1.0\), the ratio decreased as
the frequency increased in most of the frequency ranges. This implies that if the fundamental natural frequency is lower than the dominant excitation frequency, the energy input is significantly reduced in the RILD system when compared with the LVD system, demonstrating the benefit of RILD in low-frequency structures.

### 3.2 Effect of ground excitation on the earthquake input energy

When we assume a white noise input, its power spectral density is constant irrespective of the excitation frequency.

\[ |A(\omega)|^2/T = S_0 \]  \hspace{1cm} (25)

where \( T \) and \( S_0 \) denote the duration and power spectral density of the ground motion, respectively.

Using a constant value \( V_0 \) that has the dimension of velocity, we obtain
Thus, the following equations hold for LVD and RILD systems.

\[ E_{L,LVD} = \frac{1}{2} \int_{0}^{\infty} |A(i\omega)|^2 f_{E,LVD}(\omega) d\omega = \frac{V_0^2}{2} \int_{0}^{\infty} f_{E,LVD}(\omega) d\omega \]  

(27)

\[ E_{L,RILD} = \frac{1}{2} \int_{0}^{\infty} |A(i\omega)|^2 f_{E,RILD}(\omega) d\omega = \frac{V_0^2}{2} \int_{0}^{\infty} f_{E,RILD}(\omega) d\omega \]  

(28)

When \( \eta = 2h = 2\beta = 0 \), \( f_{E,LVD}(\omega) \) and \( f_{E,RILD}(\omega) \) become Dirac's delta function, thereby resulting in Equation 29.

\[ \int_{0}^{\infty} f_{E,LVD}(\omega) d\omega = \int_{0}^{\infty} f_{E,RILD}(\omega) d\omega = 1 \]  

(29)

Consequently, from Equations 27 and 28, the following equation holds:

\[ \frac{E_{L,LVD}}{m} = \frac{E_{L,RILD}}{m} = \frac{V_0^2}{2} \]  

(30)

When \( \eta = 2h = 2\beta \neq 0 \), we obtain the following from Equation 20.

\[ \frac{E_{L}}{m} = -\frac{1}{2\pi} \int_{-\infty}^{\infty} |A(i\omega)|^2 H_V(i\omega) d\omega = -\frac{V_0^2}{2\pi} \int_{-\infty}^{\infty} H_V(i\omega) d\omega \]  

(31)

By applying the residue theorem to the integral in Equation 31, we obtain

\[ \int_{-\infty}^{\infty} H_V(i\omega) d\omega = 2\pi i \sum_{k=1}^{2} \text{Res}(H_V(i\omega), a_k) \]  

(32)

where \( \text{Res}[H_V(i\omega, a_k)] \) represents the residue at \( a_k \), which is the singularity of \( H_V(i\omega) \).

In the case of the LVD system, where \( H_V(i\omega) = H_{LVD}^v(i\omega) \),

\[ \int_{-\infty}^{\infty} H_{LVD}^v(i\omega) d\omega = 2\pi \left[ \text{Res} \left( H_{LVD}^v, \beta \omega_0 + \sqrt{1-\beta^2} \right) + \text{Res} \left( H_{LVD}^v, \beta \omega_0 - \sqrt{1-\beta^2} \right) \right] = -2\pi \]  

(33)

Therefore,

\[ \frac{E_{L,LVD}}{m} = -\frac{V_0^2}{2\pi} \int_{-\infty}^{\infty} H_V(i\omega) d\omega = V_0^2 \]  

(34)

Similarly, in the case of RILD, where \( H_V(i\omega) = H_{RILD}^v(i\omega) \),
Thus,

\[
\frac{E_{I,RILD}}{m} = -\frac{V_0^2}{2\pi} \int_{-\infty}^{\infty} H_{RILD}^v(i\omega)d\omega = V_0^2
\]  

Equations 34 and 36 indicate that

\[
\frac{E_{I,LVD}}{m} = \frac{E_{I,RILD}}{m}
\]  

This implies that although there are differences in the input energy transfer function between the LVD and RILD systems, the earthquake input energy is identical for the two types of damping systems when they are subjected to white noise excitation.

As demonstrated in Figure 2, the areas enclosed by the input energy transfer function of the two damping devices and horizontal axis differ for the frequency ranges less than or exceeding the fundamental natural frequency of the primary structure. However, the area difference between the two damping systems in the two regions is canceled out when integrated over the entire frequency region.

We used the Kobe record (the NS component of the Japanese Meteorological Agency station during the Kobe earthquake of January 17, 1995), the peak ground acceleration (PGA) of which is adjusted to 2.2 m/s² to further investigate the earthquake input energy generated by a recorded ground motion. The numerical model represents an SDOF structure with a fundamental natural period of 4 s, following seismic design practice in Japan. Figure 4 depicts the dynamic responses of the SDOF system and the phase of the velocity transfer functions.

Figure 4 shows that the results of the relative displacement and velocity of the RILD and LVD systems are similar, whereas the floor response acceleration and damping force of the RILD system are less than those of the LVD system.

The differences between the velocity transfer functions of the LVD and RILD systems are minor in terms of amplitude and phase angle, as shown in Figure 1b. A closer look at the time history of the velocity response along the time axis shows that, although there is no significant difference in amplitude between these two damping systems, a slight phase differences can be observed in Figure 4e,g.

Figure 5 presents the earthquake input energy of the ideal RILD and LVD systems. The earthquake input energy of the former was reduced to approximately half that of the latter. As defined in Equation 15, when two SDOF systems are subjected to an identical ground motion, the differences in their earthquake input energies can be attributed to the response velocity of the two systems.

Figure 6 presents the power spectral density of the ground motion and earthquake input energy. This demonstrates that the two systems have very low power spectral and energy densities in the frequency range below the fundamental natural frequency. The opposite holds for a region of frequencies greater than the fundamental natural frequency. Furthermore, the LVD system suffered an earthquake input energy greater than that of the RILD system. Because the earthquake input energy of the structure corresponds to the area enclosed by the curve of the energy density function and the horizontal axis in Figure 6b, the difference in the earthquake input energy between the two systems can be attributed to the difference in energy densities on the high-frequency side.

Figure 7 presents the Nyquist plot of the velocity transfer functions for the LVD and RILD systems, wherein the frequency increases from zero to infinity. On this Nyquist diagram, according to Equation 21, the value of the input energy transfer function is obtained by multiplying the coordinates of the points corresponding to a specific frequency on the real axis (i.e., Re[Hv]) and −2/π.

For example, when the frequency equals the fundamental natural frequency (period = 4 s), the energy transfer functions for RILD and LVD are on the real axis (Figure 7). This indicates that the energy densities at ω/ω₀ = 1 are identical for RILD and LVD, as observed in Figure 6b. When the fundamental period of the structure is 2 s (ω/ω₀ = 2), the amplitudes (absolute values) of the velocity transfer functions (|HRILDv(iω)| and |HLVDv(iω)|) are similar. The angles ΦR

\[
\int_{-\infty}^{\infty} H_{RILD}^v(i\omega)d\omega = 2\pi i \left\{ \text{Res} \left[ H_{RILD}^v(\omega_0\sqrt{\frac{1}{1-\eta\text{sgn}(\omega)}}) + \text{Res} \left[ H_{RILD}^v(-\omega_0\sqrt{\frac{1}{1-\eta\text{sgn}(\omega)}}) \right] \right] \right\} = -2\pi
\]  (35)
and $\Phi_L$ shown in Figure 8 represent the phase angle of the energy transfer function at $\omega/\omega_0 = 2$, which is consistent with Figure 4g. In terms of the phase angle, the difference between $\Phi_R$ and $\Phi_L$ was minor. However, as depicted in Figure 8, when we compare the real part of the transfer function at $\omega/\omega_0 = 2$, there is a significant difference ($\text{Re}[H_R^E(i\omega)]/\text{Re}[H_V^T(i\omega)] = 0.53$) between the two damping systems.
To further investigate the effect of recorded ground motions on the earthquake input energy, four additional earthquake records—the El Centro 1940 NS, Tohoku 2011 NS, Tomakomai 2003 NS, and Hachinohe 1968 NS records—were used in the numerical analysis. Figure 9 presents the power spectral densities of the records.
The Tohoku 2011 NS record pertains to the 2011 Great East Japan Earthquake (Mw 9.0), which is the most powerful earthquake ever recorded in Japan. The Tomakomai 2003 NS record pertains to the 2003 Tokachi-Oki Earthquake (Mw 8.0), which comprised dominant low-frequency components, as shown in Figure 9c. Therefore, these two ground motion records can be categorized as low-frequency ground motions.

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As shown in Figure 9, the power spectral density of these seismic records is unevenly distributed in the region of frequencies greater than the fundamental structural natural frequency. Consequently, as shown in Figure 10, except for the case of the Tomakomai 2003 NS record, the earthquake input energy of the LVD system exceeds that of the RILD system on the high-frequency side.

Figure 11 presents the earthquake input energy of the SDOF systems. The earthquake input energy of the LVD system is significantly greater than that of the RILD system, except in the case of the Tomakomai 2003 NS record. Figure 11c indicates that the control effect of the RILD device is compromised when a low-frequency structure is subjected to a record dominated by low-frequency components.

4 | EXPERIMENTAL VERIFICATION

In this section, an RTHS is performed for further experimental investigation of the earthquake input energy in base-isolated structures equipped with CRILD. An LVD system was also examined using frequency-domain analysis as a control case.

4.1 | Mechanical realization of RILD

The damping force of an ideal RILD is expressed as follows:\(^38,39\):

\[
f(t) = \frac{\eta k}{\omega} \dot{x}(t) = \frac{i \eta k}{\omega} \omega x(t) = i \eta k \text{sgn}(\omega)x(t)
\]

where \(\eta\), \(k\), and \(\text{sgn}(\ )\) are the loss factor of the RILD, isolator stiffness, and signum function, respectively.

![Figure 10](energy_density.png)

**FIGURE 10** Energy density. (a) El Centro 1940 NS; (b) Tohoku 2011 NS; (c) Tomakomai 2003 NS; (d) Hachinohe 1968 NS
In Equation 38, $i\eta \text{sgn}(\omega)$ can be divided into two components: a real-valued coefficient $\eta k$ and Hilbert transform $i\text{sgn}(\omega)$. To realize the Hilbert transform, Keivan et al.\textsuperscript{3} proposed a first-order all-pass filter, as expressed in Equation 29, for the causal approximation of the Hilbert transform.

$$H_{AP}(io) = \frac{io - \omega_T}{io + \omega_T}$$

where $\omega_T$ is the target frequency of the filter.

As shown in Figure 12, the amplitude of the filter is unity over all frequencies and the phase advance is $90^\circ$ at the target frequency $\omega_T$. Therefore, the first-order all-pass filter is expected to favorably approximate the Hilbert transform around the target frequency.

Luo et al.\textsuperscript{26} proposed a mechanical realization of a first-order all-pass filter, as depicted in Figure 13, wherein $k_N$ is the negative stiffness. $k_M$ and $c_M$ are the stiffness and damping coefficients of the Maxwell-type damper, respectively. Hereafter, the mechanical configuration shown in Figure 13 is referred to as CRILD\textsubscript{NF} (“NF” indicates that no friction [NF] is considered). However, the limitation of this physical realization is that the passive causal method is only expected to approximate the Hilbert transform in the vicinity of the fundamental natural frequency.

A coil spring unit and an oil damper were designed and fabricated to build the Maxwell element in the CRILD device. Static and dynamic tests of the coil spring unit and oil damper were performed to identify the parameters of
each component. As depicted in Figure 14, a chain block was used to manually apply tension force on the coil spring, and a load cell was installed between the chain block and central plate of the coil spring unit to measure the restoring force. Four coil springs attached to the center plate from both sides were precompressed so that all springs generated restoring forces, even under tension. The stiffness and friction of the coil spring unit are 14.5 kN/m and 163.91 N, respectively. The maximum friction force can be attributed to the inherent mechanical friction of the spring support rod and slideway.

The equivalent LVD coefficients represent the energy dissipation in the oil damper and friction in the coil spring unit, which are 12.09 kN·s/m and 1.83 kN·s/m, respectively. Consequently, an additional viscous damping component $c_c$, which represents the energy dissipated by friction, was arranged parallel to the coil spring, as shown in Figure 15.
4.2 Real-time hybrid simulation

Table 1 lists the earthquake records used for the RTHS.

Because reducing the isolator stiffness is a straightforward approach to realizing negative stiffness, the negative stiffness component in the CRILD device is incorporated into the numerical domain, while the remaining components are in the physical domain.

Figure 16 presents a schematic of an SDOF structure equipped with a CRILD. The mass and stiffness of the SDOF structure are 12.60 tons and 18.13 kN/m, respectively. The loss factor of CRILD is set as 0.4, which is a common practice in the design of seismic isolation in Japan. Therefore, the damping ratio was $\beta = 0.2$.

After performing preliminary numerical analyses, the maximum amplification factors of the earthquake records were 84%, 49%, 48%, 37%, and 79% for the El Centro 1940 NS, Tohoku 2011 NS, Kobe 1995 NS, Tomakomai 2003 NS, and Hachinohe 1968 NS records, respectively.

When performing RTHS, the amplitudes of all the earthquake records start at a small percentage and gradually increase to a maximum value. This is to avoid the potential instability of the RTHS, which may cause damage to the shaking table. Thus, only the data obtained using the maximum amplitudes are used in the following discussion.

The layout of the RTHS experiment is presented in Figure 17. Two laser displacement transducers were used to measure the displacements of the shaking table and oil damper.

Because the time delay of the restoring force of a linear elastic element leads to the introduction of energy, which results in instability, time-delay compensation is crucial for ensuring stability during testing. Band-limited white noise was used to identify the transfer function from the command displacement to the measured actuator displacement. The transfer function of the equipment is calculated as follows:

| Earthquake          | Duration (s) | Peak ground acceleration (cm/s²) | Peak ground velocity (cm/s) |
|---------------------|--------------|---------------------------------|-----------------------------|
| El Centro 1940 NS   | 53.74        | 341.65                          | 33.88                       |
| Tohoku 2011 NS      | 49.46        | 332.83                          | 47.53                       |
| Kobe 1995 NS        | 19.98        | 817.98                          | 91.26                       |
| Tomakomai 2003 NS   | 290.00       | 86.68                           | 32.04                       |
| Hachinohe 1968 NS   | 50.98        | 180.34                          | 37.85                       |
Here, we assume that the dynamics of the equipment are expressed as a two-pole and no-zero transfer function of the form expressed in Equation 41.

\[
TF(s) = \frac{b_0}{a_2s^2 + a_1s + a_0} \tag{41}
\]

where \( b_0 = a_0 = 3948.0 \), \( a_1 = 125.7 \), and \( a_2 = 1 \).

Figure 18 compares the measured and identified transfer functions. To cancel out the dynamics of the testing equipment, its inverse transfer function (ITF) is applied to the desired signal to yield a command signal. The ITF, which is the reciprocal of Equation 41, does not satisfy the causality because it is improper; that is, the order of the numerator is greater than that of the denominator. To address this, we employed a feedforward actuator controller developed by Phillips et al.\(^4\)\(^1\) Therefore, the command displacement \( u_i \) at the \( i \)th step is yielded by the ITF discretely, as expressed by Equation 42.

\[
u_i = Ax_i + Bx_{i-1} + Cx_{i-2} + Dx_{i-3} \tag{42}
\]

where \( x_i \) is the desired displacement at the \( i \)th time step.

\[
A = \frac{1}{b_0} \left( a_0 + \frac{3a_1}{2\Delta t} + \frac{2a_2}{\Delta t^2} \right), B = \frac{1}{b_0} \left( -2a_1 + \frac{5a_2}{\Delta t^2} \right), C = \frac{1}{b_0} \left( \frac{a_1}{2\Delta t} + \frac{4a_2}{\Delta t^2} \right), D = \frac{1}{b_0} \left( -\frac{a_2}{\Delta t^2} \right), \tag{43}\]

and \( \Delta t = 1/2000 \text{ s} \) is the sampling time.
4.3 Results of RTHS and frequency-domain analyses

The velocity responses and earthquake input energy of the SDOF systems equipped with three types of damping devices—CRILD, ideal RILD, and LVD—were compared to investigate the difference in the input energy between the LVD, RILD, and CRILD systems. The amplification factors of the earthquake records were set as 80%, 45%, 45%, 35%, and 35% for the El Centro 1940 NS, Tohoku 2011 NS, Kobe 1995 NS, Tomakomai 2003 NS, and Hachinohe 1968 NS records, respectively.

Figure 19 presents the relative velocity response of the SDOF systems equipped with the three damping devices. The results obtained from the RTHS and frequency-domain analyses showed no significant differences, except in the case of the Tomakomai 2003 NS record.

As shown in Figure 17, because the load cell was installed at the moving part (central plate of the coil spring), its inertial force effect was examined during the post-processing of the experimental results. The maximum relative acceleration of the load cell ranged from 0.10 to 3.80 m/s² during RTHS. The mass of the load cell was 4 kg. Thus, the inertial force of the load cell ranged from 0.4 to 15.19 N, which is significantly smaller than the damping force of the Maxwell-type damper. Therefore, the inertial force of the load cell was neglected in this study.

In the case of the Tomakomai 2003 NS record, the relative velocity obtained from the RTHS is significantly different from that obtained from the frequency-domain analyses after 120 s. After 190 s, the relative velocities yielded by the RTHS are approximately zero, which can be attributed to the existence of friction in the coil spring unit and the friction between the piston and cylinder of the hydraulic actuators of the shake table.

Figure 20 demonstrates that the results of the CRILD system are lower than those of the LVD system. This indicates that CRILD can achieve a good reduction in earthquake input energy, which is consistent with the theoretical results, as discussed in Section 3.2.

A comparison of the earthquake input energy of the CRILD and ideal RILD systems imply that the performance of CRILD in reducing the earthquake input energy is slightly compromised in the majority of cases from the selected

![Figure 19](image_url) Velocity response of the SDOF system. (a) El Centro 1940 NS (amplification factor: 80%); (b) Tohoku 2011 NS (amplification factor: 45%); (c) Kobe 1995 NS (amplification factor: 45%); (d) Tomakomai 2003 NS (amplification factor: 35%); (e) Hachinohe 1968 NS (amplification factor: 75%)
CONCLUSIONS AND DISCUSSION

The objective of this study was to experimentally investigate the earthquake input energy of low-frequency structures equipped with a passive rate-independent damping device. Both RTHS and frequency-domain analyses were performed to investigate the earthquake input energy of the LVD and RILD systems. In this study, RILD was passively realized using a CRILD device, a parallel layout of a Maxwell-type damper, and negative stiffness. The primary contributions of this study are as follows:

1. The relative displacement, relative velocity, and floor response acceleration of the RILD system are lower than those of the LVD system at frequencies below the fundamental natural frequency. The damping force of the LVD system is lower than that of the RILD system at frequencies lower than the fundamental natural frequency. The opposite holds for a region of frequencies exceeding the fundamental natural frequency.
2. The input energy transfer function is proportional to the real part of the velocity transfer function. The energy transfer function of the RILD system is lower than that of the LVD system on the frequency region exceeding the fundamental natural frequency. The opposite holds for a region of frequencies less than the fundamental natural frequency. The differences in the earthquake input energy between the LVD and RILD systems can be attributed to the differences in the phase angles of the complex velocity transfer functions for the two different damping systems.
3. Except for the case of the Tomakomai 2003 NS record, the results of the RTHS and frequency-domain analyses confirmed that CRILD could achieve a similar reduction effect in the earthquake input energy in low-frequency records because of the limitation in physical realization, except in the case of the Tomakomai 2003 NS record, which is dominated by low-frequency components.

FIGURE 20  Earthquake input energy of the SDOF system. (a) El Centro 1940 NS (amplification factor: 80%); (b) Tohoku 2011 NS (amplification factor: 45%); (c) Kobe 1995 NS (amplification factor: 45%); (d) Tomakomai 2003 NS (amplification factor: 35%); (e) Hachinohe 1968 NS (amplification factor: 75%)
structures, as compared to the ideal RILD. In the case of the Tomakomai 2003 NS record, CRILD achieves a better control effect than the ideal RILD. The slight differences in performance between CRILD and ideal RILD can be attributed to limitations in physical realization. CRILD is only expected to favorably approximate the Hilbert transform in the vicinity of the fundamental natural frequency. Furthermore, friction is present in the coil spring unit. Therefore, an RILD-based device must be designed meticulously, considering its benefits and notable drawbacks.

4. The quadratic backward difference method was used to address the non-causality of the ITF of the shaking table. Overall, the experimental results suggest that RTHS was successful. Real-time hybrid testing confirmed the feasibility of a passive CRILD device, comprising a negative-stiffness element in the numerical domain and a small-scale Maxwell-type damper in the physical domain.

The present study, for the first time, attempted to investigate the earthquake input energy of the RILD system and experimentally verify the feasibility of the passive implementation of RILD. This study clarifies why the earthquake input energy of the RILD structure is lower than that of the LVD structure during earthquakes. Furthermore, this study identified the challenges of realizing RILD as a passive physical device, which include eliminating unwanted friction from the device and phase difference in the dynamic stiffness of the CRILD device. The scope of this study, however, is limited to an SDOF structure. Therefore, further research on multi-degree-of-freedom systems is required to promote the practical implementation of RILD.

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ORCID
Wei Liu  https://orcid.org/0000-0003-4151-7738
Kohju Ikago  https://orcid.org/0000-0003-0350-0142

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