Effective potential of scalar–tensor gravity

Andrej Arbuzov\textsuperscript{1,2,} and Boris Latosh\textsuperscript{1,2,}\textsuperscript{*}

\textsuperscript{1} Bogoliubov Laboratory for Theoretical Physics, JINR, Dubna 141980, Russia
\textsuperscript{2} Dubna State University, Universitetskaya str. 19, Dubna 141982, Russia

E-mail: arbuzov@theor.jinr.ru and latosh@theor.jinr.ru

Received 1 July 2020, revised 14 October 2020
Accepted for publication 28 October 2020
Published 7 December 2020

Abstract
Effective potential of a scalar field induced by weak gravity is studied. The set of operators providing the leading contribution and preserving the second order of field equations is found. It is shown that only a mass term and a specific Brans–Dicke-like interaction are relevant within such a setup. An explicit form of the potential is found. The model has room for a natural inflationary scenario similar to the well-known case of the Starobinsky inflation. Possible implications for the standard model are highlighted.

Keywords: scalar–tensor gravity, effective field theory, effective action, Horndeski

1. Introduction
Scalar–tensor models of gravity occupy a special place in the modified gravity landscape. They extend the gravitational sector with an additional scalar degree of freedom providing, perhaps, the simplest alternative to general relativity. Horndeski theories are the most general class of models admitting second order field equations \[1,2\]. Despite their apparent simplicity, scalar–tensor theories have applications in many areas such as cosmology, inflation, black hole physics, etc \[3–6\].

The structure of a scalar field potential is crucial for a theory, as it provides room for some important physical phenomena. Its role is well-illustrated with two following examples. Firstly, within inflation theory the form of a potential defines whether a model admits the slow-roll inflation \[7–9\]. Secondly, some scalar–tensor models experience so-called Chameleon screening \[10–13\]. The screening requires an existence of of a specific non-minimal coupling between the scalar field and matter together with a potential of a special form. Due to the non-minimal coupling a scalar field potential receives an additional contribution proportional to the local matter density, develops a large effective mass, and ceases to propagate.

A scalar field potential is altered at the quantum level as the theory develops an effective potential that can radically change its features \[14,15\]. In the simplest case an effective poten-
tial develops a new independent energy scale in the infrared sector and breaks the conformal symmetry of a model [14]. Geometry of a curved spacetime also affects the effective potential [15]. This makes it essential to study the influence of quantum gravitational effects on a scalar field potential.

The effective field theory approach provides a framework capable to account for quantum gravitational effects and to avoid issues related to the non-renormalizable nature of quantum gravity [16, 17]. This approach was applied for gravity before and was found to be fruitful [18–22].

In this paper we address the influence of quantum gravitational effects on a scalar field potential. The paper is organized as follows. In section 2 we highlight the leading corrections relevant for an effective potential. We show that a massless scalar field minimally coupled to gravity does not develop an effective potential. The simplest non-minimal coupling affecting an effective potential belongs to Horndeski models and resembles the coupling from the Brans–Dicke theory. In section 3 we discuss possible implementations of these results. Namely, we point out that the effective potential may become relevant for inflation theory, as it can fit the slow-roll conditions. We also argue that the Chameleon screening can hardly be relevant within such a setup as quantum effects can be safely neglected in a matter dense environment. Finally, we point to some possible relations with a spontaneous conformal symmetry breaking and the cosmological constant problem. We conclude in section 4.

2. Scalar field effective potential

The most general class of scalar–tensor models admitting second-order field equations is given by the Horndeski Lagrangians [1, 2]:

\begin{align}
\mathcal{L}_2 &= G_2(\phi, X), \\
\mathcal{L}_3 &= G_3(\phi, X) \Box \phi, \\
\mathcal{L}_4 &= G_4(\phi, X) R + G_{4X} \left[ (\Box \phi)^2 - (\nabla_{\mu} \phi)^2 \right], \\
\mathcal{L}_5 &= G_5(\phi, X) G_{\mu\nu} \nabla_{\mu} \phi - \frac{1}{6} G_{5X} \left[ (\Box \phi)^3 - 3(\Box \phi)(\nabla_{\mu} \phi)^2 + 2(\nabla_{\mu} \phi)^3 \right].
\end{align}

(1)

Here \( X = \frac{1}{2} (\partial \phi)^2 \) is the standard kinetic term of a scalar field; \( G_i \) are arbitrary smooth functions; \( G_{iX} \) are the corresponding derivatives with respect to \( X \); \( R \) is the scalar curvature, and \( G_{\mu\nu} \) is the Einstein tensor. Horndeski Lagrangians (1) define the structure of a theory. Terms \( \mathcal{L}_2 \) and \( \mathcal{L}_3 \) constrain the spectrum of a suitable non-minimal interaction with gravity while \( \mathcal{L}_4 \) and \( \mathcal{L}_5 \) specify the structure of a scalar field self-interaction.

Since Horndeski models admit second-order field equations, they are free from instabilities associated with higher derivatives. Therefore they are highlighted by the stability reasoning. It should be noted that there is a generalization of Horndeski models known as beyond Horndeski which also admits second order field equations. These models obtained from Horndeski Lagrangians via disformal transformations [23–25]. These transformations map beyond Horndeski models with the minimal coupling to matter on Horndeski models with non-minimal couplings. In other words, beyond Horndeski models extend interactions with the regular matter. The main focus of this paper is on the gravitational influence on an effective potential, henceforward we will discuss only Horndeski models.

We pursue the goal to study the most universal effects taking place due to quantum gravitational contributions. The effective potential, alongside other quantum effects, strongly depends on the structure of a given model. To account only for the universal effects, we focus on
models without scalar field self-interactions and make only a brief comment on their account. For the sake of simplicity, we also separately discuss the cases with minimal and non-minimal couplings to gravity.

Application of effective field theory formalism for gravity models is widely covered in literature [15–17, 26] (see also [27] for a more general discussion of effective theories in the context of renormalization). An effective theory for gravity is defined in a narrow energy interval $0 \leq E \leq \mu$. Here $\mu$ is called the renormalization scale and it lies far below the Planck mass $m_P$. A theory is given by its microscopic action $\mathcal{A}$ which is defined at the renormalization scale $\mu$. The theory is extended in the low-energy area by the standard means of loop corrections. In such an approach the problem of divergent contribution is softened. First of all, an effective theory is defined in a narrow energy band and cannot be used for arbitrary large energies. Secondly, all divergent contributions can be normalized, the corresponding finite valued of parameters are taken either from the low-energy ($E \sim 0$) or from the high-energy region ($E \sim \mu$). To put it otherwise, the renormalization problem is softened by the price of a finite applicability domain.

For a given gravity model an effective theory is constructed as follows. First of all, the microscopic action $\mathcal{A}$ is defined at the normalization scale. It is naturally to expect that a model is gauge-invariant with respect to coordinate transformations. Therefore we assume that the gauge is fixed at this level. Secondly, a suitable background metric $g_{\mu\nu}$ that minimize $\mathcal{A}$ is chosen. In turn, gravity is described with small metric perturbations $h_{\mu\nu}$ propagating about the background metric $g_{\mu\nu}$, so the full spacetime metric $g_{\mu\nu}$ reads:

$$g_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}. \tag{2}$$

Therefore a given gravity model (that also may contain matter degrees of freedom) is described with the following generating functional [16, 17]:

$$Z = \int D[h] D[\psi] \exp \left[ i \mathcal{A}[\mathcal{F}, \Psi] + i \frac{\delta \mathcal{A}[\mathcal{F}, \Psi]}{\delta g_{\mu\nu}} h_{\mu\nu} + i \frac{\delta^2 \mathcal{A}[\mathcal{F}, \Psi]}{\delta g_{\mu\nu} \delta g_{\alpha\beta}} h_{\mu\nu} h_{\alpha\beta} + \mathcal{O}(h^3) \right]. \tag{3}$$

Here $\Psi$ notes matter degrees of freedom. The first term on the right-hand side corresponds to the matter sector of a theory. The second term describes interactions between matter and gravity. The third terms describes propagation of free gravitational field perturbations. Finally, $\mathcal{O}(h^3)$-terms describe self-interactions of gravitational perturbations and higher-order interaction with matter.

The main interest of this paper is corrections generated by quantum gravitational effects. Because of this, we will consider quantum perturbations of gravitational field (which we will call gravitons for the sake of simplicity). The corresponding effective theories are defined below the Planck scale, so $\mathcal{O}(h^3)$ are negligible and will be omitted in this paper. In other words, one may say, that within such an approach only gravity is quantized while matter fields remain classical.

This approach allows one to use well-developed tools of effective field theory [15, 17, 26]. Moreover, it allows one to avoid complex calculations occurring in mixed metric-scalar theories [28–31].

We use the simplest method based on direct perturbative calculations to find a one-loop effective potential [14, 15]. Firstly, one recovers one-loop connected irreducible $n$-point Green functions $\mathcal{G}_n$. Secondly, an $n$-point vertex function $\Gamma_n$ is obtained form $\mathcal{G}_n$ via an amputation of external lines. In $\Gamma_n$ all external momenta are set to zero to account only for potential interactions (i.e. interactions that do not depend on particles momenta). In such a setup, the effective
potential is given in terms of vertex functions $\Gamma_n$ as follows [15]:

$$V_{\text{eff}}(\phi) = i \sum_{n=1}^{\infty} \frac{1}{n} \Gamma_n \phi^n. \quad (4)$$

Let us address the simplest case of a scalar field without self-interactions coupled to gravity in the minimal way:

$$A_0 = \int d^4x \sqrt{-g} \left[ -\frac{\kappa^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{m^2}{2} \phi^2 \right], \quad (5)$$

with $\kappa^2 = 32\pi G_N$ begin related to the Newton gravitational constant $G_N$. The expansion of action $A_0$ in a series with respect to small metric perturbations $h_{\mu\nu}$ propagating over the flat spacetime $\eta_{\mu\nu}$ contains an infinite number of terms. In the low-energy area, where an effective theory takes place, it is essential to study only the leading order effects, as they themselves are suppressed by the Planck mass. Higher orders of perturbation theory are suppressed even stronger by higher powers of the Planck mass and can be omitted for the time being. It is worth noting that the same setup should be applied for strong gravity phenomena, like black holes or the early Universe, with caution, as higher-dimension operators may no longer be neglected and the effective theory setup may lost its relevance. Finally, we use the harmonic gauge throughout the paper with the following gauge-fixing term:

$$A_{gf} = \int d^4x \left[ \partial_\mu h_{\mu\nu} - \frac{1}{2} \partial^\mu h_{\sigma\sigma} \right]^2. \quad (6)$$

Feynman rules for models addressed in this paper are discussed in details in [32].

In the massless case $m = 0$, all vertex functions $\Gamma_n$ vanish. This is due to the fact that the scalar field energy–momentum tensor is bilinear in derivatives. Because of this, if at least one external momentum in $\Gamma_n$ vanishes, so does the vertex. This leads to the first important result: a massless scalar field itself does not generate an effective potential due to quantum gravitational corrections at the one-loop level.

In the case of a non-vanishing mass $m \neq 0$, an effective potential is generated. In can be seen that vertex functions with an odd number of fields vanish due to the structure of the interaction with gravity. The gravity is coupled to an energy–momentum tensor which is bilinear both in derivatives and fields. Because of this the gravitational interaction cannot change the number of matter states. On the other hand, an arbitrary $n$-point diagram can be viewed as an amplitude of a scattering process. As it was just noted, the number of scalar particles is presented in such processes. Therefore all diagrams with an odd number of fields vanish. A vertex function with 2$n$ fields can be evaluated explicitly with the standard technique [15]:

$$\Gamma_{2n} = \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \left( \frac{\kappa}{2} \right)^{2n} \left( \frac{m^2}{k^2} \right)^{n} \left( \frac{m^2}{k^2} - m^2 \right)^n \left( 1 - \frac{d}{2} \right)^n d^n. \quad (7)$$

Dimensional regularization was used here to evaluate this expression with $\mu$ being the normalization energy scale. The corresponding effective potential reads

$$(V_0)_{\text{eff}} = i \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \left[ -\frac{1}{2} \ln \left( 1 - \frac{\kappa^2}{4} \frac{m^2}{k^2} \right) d \left( 1 - \frac{d}{2} \right) \phi^2 \right]$$

$$= \frac{m^4 \kappa^2 \phi^2}{16\pi^2} \left[ -\frac{1}{d-4} \frac{\gamma}{2} + \frac{1}{2} \ln(8\pi) - \frac{1}{2} \ln \frac{m^2}{\mu^2} - \frac{\ln(2)}{4\kappa^2 \phi^2} \right]$$
The expression is valid only for small values of $\phi$ for which $4\kappa^2 \phi^2 \ll 1$. The effective potential should be renormalized on the observable mass $m_{\text{obs}}$ measured at the low energy regime via the following counterterm:

$$\delta \mathcal{L} = \frac{m_{\text{obs}}}{2} \phi^2 - \frac{m^4}{16\pi^2} \left[ \frac{1}{d - 4} - \frac{\gamma}{2} \right] + \frac{1}{2} \ln \left( \frac{\mu}{m} \right).$$

(9)

It should be highlighted that terms containing log-functions provide a finite local contribution to the mass term, so their contribution is accounted in the counterterm. The renormalized effective potential is given by the following expression:

$$(V_0)_{\text{eff, ren}} = -\frac{m^4}{64\pi^2} \ln(2) + \frac{m_{\text{obs}}^2}{2} \left[ 1 + \frac{m^2}{m_{\text{obs}}^2} \frac{m^2\kappa^2}{32\pi^2} (1 + \ln(4)) \right] \phi^2$$

$$+ \frac{m^4}{128\pi^2} \left[ (1 - \sqrt{1 - 4\kappa^2 \phi^2}) \ln \left[ 1 - \sqrt{1 - 4\kappa^2 \phi^2} \right] \right.$$  
$$\left. + \left( 1 + \sqrt{1 - 4\kappa^2 \phi^2} \right) \ln \left[ 1 + \sqrt{1 - 4\kappa^2 \phi^2} \right] \right].$$

(10)

This renormalized effective potential (10) constitutes the second result of this paper. It is generated in the minimal model, so it appears universally in any other theory which contains a scalar of a non-vanishing mass. The potential contains two mass parameters. The first one we call the tree-level mass, as it is generated by the microscopic action. This mass parameter is measured at the normalization scale $\mu$ where the microscopic action is given. The second mass parameter $m_{\text{obs}}$ we call the observed mass. It is measured in the low energy regime, in $\phi \sim 0$. It corresponds to the mass of small perturbations existing far below the normalization scale.

Properties of this potential should be discussed. Firstly, both (8) and (10) are regular about $\phi \sim 0$ and vanish at $\phi = 0$. This goes in line with the well-known case [14], as the scalar field mass $m$ serves as a natural regulator of the infrared sector. Secondly, before a normalization the potential (8) vanishes in the $m \to 0$ limit in full agreement with the first result found above. Finally, potential (10) is sensitive to the hierarchy of mass scales. This can be clearly seen from the following expression given in term of the tree-level mass $m$, the observed mass $m_{\text{obs}}$, and the Planck mass $m_P$:

$$(V_0)_{\text{eff, ren}} = \frac{m_{\text{obs}}^2}{2} \phi^2 + \frac{m^4}{m_P^4} \left( 64\pi^2 \frac{\phi^2}{m_P^4} \right) \phi^4 + \mathcal{O}(\phi^6).$$

(11)

If the tree-level mass $m$ is much smaller than the Planck mass $m_P$, then $\mathcal{O}(\phi^4)$ terms are strongly suppressed and do not influence the potential noticeably. However, if $m \sim m_P$, then $\mathcal{O}(\phi^4)$ terms can no longer be neglected, and the effective potential can develop new minima. However, this case hardly can be considered realistic, as the new minima are situated in the Planck region which lies beyond the model applicability domain. Therefore we only consider the case of the natural hierarchy $m \ll m_P$. 

\[ \text{Class. Quantum Grav. 38 (2021) 015012} \] 
A Arbuzov and B Latosh
Let us turn to a model with non-minimal interactions between gravity and the scalar field. According to the Horndeski Lagrangians (1), a model accounting for the simplest leading order interactions reads

\[
A_1 = \int d^4x \sqrt{-g} \left[ -\frac{2}{\kappa^2} R + \frac{1}{2} \left( g^{\mu\nu} + \beta G^{\mu\nu} \right) \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \left( m^2 + \lambda R \right) \phi^2 \right].
\]

(12)

Here \( R \) is the scalar curvature and \( G_{\mu\nu} \) is the Einstein tensor. Terms \( R \phi^2 \) and \( G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \) describe new non-minimal three-particle interactions. Term \( G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \) is known as the John interaction from the Fab four class [33], while \( R \phi^2 \) is typical for Brans–Dicke-like models with conformal symmetry [34, 35]. The mass term also provides an additive contribution to the effective potential that was found above, so we will not consider it further.

In full analogy with the minimal coupling case, the John interaction does not contribute to an effective potential which makes the third important result of the paper. As the John term is bilinear in derivatives, the corresponding interaction vertex vanishes if at least one scalar field carries a zero momentum. This makes a complete analogy to the minimal coupling case.

The Brans–Dicke-like interaction \( R \phi^2 \), on the contrary, contributes to the effective potential. The only part of the interaction relevant in the given setup reads

\[
\int d^4x \sqrt{-g} \left[ -\frac{\lambda}{2} R \phi^2 \right] \rightarrow \int d^4x \left[ -\lambda \kappa \eta^{\mu\nu} \phi(\partial_\mu \partial_\nu - \eta_{\mu\nu} \Box) \phi \right].
\]

(13)

As in the minimal coupling case, it is impossible to construct a vertex function with an odd number of fields, as the corresponding contributions vanish. An arbitrary vertex function with an even number of fields is given by

\[
\Gamma_{2n} = \left( \frac{1}{2} \lambda^2 \kappa^2 (3 - d)(d - 1) \right)^n \mu^{4-d} \int \frac{d^d k}{(2\pi)^d}.
\]

(14)

In contrast to the minimal coupling case, such vertex functions have a much simpler momentum structure, so the corresponding integral can be evaluated explicitly and it is equal to the regularized volume of the momentum space. Consequently, the effective potential reads

\[
(V_1)_{\text{eff}} = \frac{1}{2} \ln \left[ 1 + \frac{(d - 1)(d - 3)}{2} \lambda^2 \kappa^2 \phi^2 \right] \left( -i \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \right)
\]

\[
= \frac{1}{2} \ln \left[ 1 + \frac{3}{2} \lambda^2 \kappa^2 \phi^2 \right] \left( \int \frac{d^d k}{(2\pi)^d} \right).
\]

(15)

It should be noted that similar terms are usually omitted if evaluated in dimensional regularization. However, we believe that they should not be disregarded that easily, as they are relevant for the naturalness of a model [36, 37].

The presence of a non-vanishing mass term is crucial for the effective potential in full analogy with the previous case. The complete effective potential generated by \( A_1 \) consists of two parts:

\[
V_{\text{eff}} = (V_0)_{\text{eff}} + (V_1)_{\text{eff}}.
\]

(16)

In the massless case \( m = 0 \), the contribution \((V_0)_{\text{eff}} \) vanishes. The potential (15), on the contrary, develops a non-vanishing mass-like contribution that should be renormalized:

\[
(V_1)_{\text{eff}} = \frac{1}{2} \left( \frac{3}{2} \lambda^2 \kappa^2 \int \frac{d^d k}{(2\pi)^d} \right) \phi^2 - \frac{9}{16} \kappa^4 \lambda^4 \phi^4 \int \frac{d^d k}{(2\pi)^d} + O(\phi^6).
\]

(17)
There are two possible ways to perform the renormalization. The first one is to respect the structure of the tree-level theory and to normalize \((V_1)_{\text{eff}}\) on the vanishing mass. In this case the whole potential vanishes in full analogy with the previous case. The second option is to assume a spontaneous dynamical generation of a new energy scale which produces the scalar field mass at the level of radiation corrections similarly to [14]. Therefore the mass term generated by \((V_1)_{\text{eff}}\) should be normalized on a non-vanishing value of the mass \(m_{\text{obs}}\) measured in the low-energy regime \(E \sim 0\). The corresponding counterterm is
\[
\delta \mathcal{L} = \frac{m_{\text{obs}}^2}{2} \left[ 1 - \frac{3}{2} \frac{\kappa^2 \lambda^2}{m_{\text{obs}}^2} \int \frac{d^4k}{(2\pi)^4} \right] \phi^2.
\]
(18)
So, the renormalized potential reads
\[
(V_1)_{\text{eff, ren}} = \frac{1}{2} \ln \left[ 1 + \frac{3}{2} \frac{\kappa^2 \lambda^2}{\sqrt{m_{\text{obs}}^2}} \right] \left( \frac{2m_{\text{obs}}^2}{3\kappa^2 \lambda^2} \right).
\]
(19)
We discuss in detail the role of these scenarios in the next section. In the rest of this section we focus on the case of a non-vanishing mass. The complete effective potential should be normalized in a cut-off regularization scheme since \((V_1)_{\text{ren}}\) is ill-defined within dimensional regularization. The complete effective potential reads
\[
V_{\text{eff}} = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left[ 1 + \frac{\kappa^2 \phi^2}{m^2} \left( \frac{\Lambda^2}{m^2} + 1 \right) \right] + \frac{1}{2} \ln \left[ 1 + \frac{3}{2} \frac{\kappa^2 \lambda^2}{\sqrt{m_{\text{obs}}^2}} \right] \int \frac{d^4k}{(2\pi)^4}
= \frac{1}{32\pi^2} \left[ \frac{m^4}{\Lambda^2} \int_0^{\Lambda^2/m^2} dx \ln \left[ 1 + \frac{\kappa^2 \phi^2}{\sqrt{x(x+1)}} \right] + \frac{1}{2} \ln \left[ 1 + \frac{3}{2} \frac{\kappa^2 \lambda^2}{\sqrt{m_{\text{obs}}^2}} \right] \Lambda^4 \right]
= \frac{m^4}{32\pi^2} \left\{ \frac{1}{2} \Lambda^4 \left( \ln \left[ 1 + \frac{m^4 \kappa^2 \phi^2}{\Lambda^2 (\Lambda^2 + m^2)} \right] + \ln \left[ 1 + \frac{3}{2} \frac{\kappa^2 \lambda^2}{\sqrt{m_{\text{obs}}^2}} \right] \right) \right\}
+ \frac{\kappa^2 \phi^2}{2} \ln \left[ 1 + \frac{\Lambda^2 (\Lambda^2 + m^2)}{m^4 \kappa^2 \phi^2} \right] - \frac{\kappa^2 \phi^2}{\sqrt{1 - 4\kappa^2 \phi^2}}
\times \ln \left[ \frac{2\Lambda^2/m^2 + 1 - \sqrt{1 - 4\kappa^2 \phi^2}}{2\Lambda^2/m^2 + 1 - \sqrt{1 - 4\kappa^2 \phi^2}} \right] \frac{1 + \sqrt{1 - 4\kappa^2 \phi^2}}{1 - \sqrt{1 - 4\kappa^2 \phi^2}}
+ \frac{\kappa^2 \phi^2}{\sqrt{1 - 4\kappa^2 \phi^2}} \frac{1}{\sqrt{1 - 4\kappa^2 \phi^2}} \ln \left[ \frac{m^2}{\Lambda^2 + m^2} \left( 1 + \frac{2\Lambda^2/m^2}{1 - \sqrt{1 - 4\kappa^2 \phi^2}} \right) \right]
+ \frac{\kappa^2 \phi^2}{\sqrt{1 - 4\kappa^2 \phi^2}} \frac{1}{\sqrt{1 - 4\kappa^2 \phi^2}} \ln \left[ \frac{m^2}{\Lambda^2 + m^2} \left( 1 + \frac{2\Lambda^2/m^2}{1 + \sqrt{1 - 4\kappa^2 \phi^2}} \right) \right],
\]
(20)
with \(\Lambda\) being the cut-off scale.

The potential (20) can be renormalized via standard methods. As it was highlighted above, it is a direct sum of two contribution \(V_0\) (8) and \(V_1\) (15) describing minimal and non-minimal interaction correspondingly. Therefore it requires two normalization conditions. For the sake of simplicity we normalize the potential on the observed mass \(m_{\text{obs}}\) and on the observed four-particle interaction coupling \(g_{\text{obs}}\). The normalization point \(\phi_0\) should be set below the
normalization scale. Therefore the normalization conditions read:

$$\frac{d^2 V_{\text{eff}}}{d \phi^2} \bigg|_{\phi_0} = m_{\text{obs}}, \quad \frac{d^4 V_{\text{eff}}}{d \phi^4} \bigg|_{\phi_0} = g_{\text{obs}}. \quad (21)$$

Neither (8) nor (15) develops new minima in the effective theory applicability domain, so it appears that $\phi = 0$ can be used as a suitable normalization point. However, this is not so. Potential (20) is regular a proximity of $\phi \sim 0$ and vanishes in $\phi = 0$ limit. Due to the presence of a non-vanishing tree-level mass $m$, its second derivative is also regular in a proximity of $\phi = 0$. But the fourth derivative diverges in $\phi = 0$ limit. This behavior is typical for effective models and it appears even in simple cases [14].

This feature of the theory, however, only shows that $\phi_0 = 0$ is not a suitable normalization point, so one should use some non-vanishing $\phi_0$ instead. In the rest respect the standard normalization procedure can be performed. It is possible to provide explicit expressions for the normalization conditions (21). However, such expressions are pretty massive and inconclusive, so we omit them for the sake of brevity.

Potential (20) constitutes the fourth result of this paper. In full analogy to the previous case, this potential is regular in $\phi \sim 0$ and it develops no new minima in the relevant domain $4 \kappa^2 \phi^2 \ll 1$. Behavior of this potential can be analyzed. First of all, higher-order terms in $\kappa \phi$ are irrelevant for an effective theory, as they can only modified the potential in the Planck region, which lies beyond the effective theory applicability domain. Secondly, as it was discussed, potentials (8) and (15) do not develop new minima in the low energy area. Finally, the fourth derivative of the potential diverges in $\phi = 0$ limit, so the theory has peculiar infrared behavior. Therefore gravitational corrections described by the effective potential (20) vanishes in $\phi = 0$ and can only introduce small corrections to scalar field behavior in $\phi \sim 0$ region.

This Section can be summarized as follows. Firstly, it was found that the scalar field mass plays an important role in a generation of an effective potential. If a model admits only a minimal coupling, then the effective potential can be generated only if $m \neq 0$. If a model admits a non-minimal coupling, then there is an opportunity to generate the scalar field mass dynamically. Still, a scalar field of a non-vanishing mass generates effective potential (20). In the next section we discuss various implications of these results.

3. Implementations

The role of a scalar field mass found in the previous section agrees with previous findings [14]. Firstly, the mass provides a natural scale for regularization of the infrared sector of a theory. Secondly in the considered models, mass is the only dimensional parameter relevant for the problem.

The role of the Brans–Dicke-like interaction $R \phi^2$ can be understood in a similar way. This interaction is quadratic in derivatives which should be attributed to the gravitational sector rather than to the scalar one. In other words, the interaction describes a momentum-independent coupling of a scalar field to a metric gradient. This makes it conceivable to explain how this interaction can generate a non-vanishing effective potential in the massless case. The complete energy–momentum tensor of a scalar field, which admits a momentum-dependent part, generates a non-vanishing curvature of the spacetime. The corresponding metric gradient couples to the momentum-independent part of the scalar field energy–momentum tensor and generates an interaction that does not depend on the scalar field momentum explicitly. This reasoning gives grounds to believe that the effective potential (15) can be indeed generated in the massless case.
It may appear that a new energy scale associated with the scalar field mass will be generated, but we would argue that this is not so. It is well-known that the following action similar to $A_1$ admits the conformal symmetry [34, 35]:

$$A_{\text{conformal}} = \int d^4x \sqrt{-g} \left[ \frac{1}{6} R \phi^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right].$$

(22)

The non-minimal interaction present in (12) is similar to the interaction in the conformal-invariant model. In contrast to the conformal model, the symmetry in $A_1$ is explicitly violated by a dimensional parameter $\kappa$ related to the Planck mass. Therefore, $A_1$ admits at least one dimensional parameter which, in term, defines the divergent mass term.

Based on this reasoning we argue the following. Firstly, in the case of a non-vanishing scalar field its mass plays the role of an energy scale regularizing the infrared sector. Secondly, in the massless case a scalar field with a specific non-minimal coupling to gravity can develop a non-vanishing effective potential. The non-minimal coupling resembles a model with conformal symmetry, so the corresponding effective potential resembles a potential that may dynamically break the symmetry. However the conformal symmetry of the considered model is explicitly broken at the tree-level, so the similarity can hardly point to a deeper relation to the symmetry breaking. Finally, in both of these cases the effective potentials remain regular in the infrared sector. They do not develop new minima within the effective theory applicability domain and do not alter the vacuum energy content of the model. These facts show that at the one-loop level the fundamental physical properties of the model is hardly altered.

It is important to point to possible implications of these effects in the context of inflation and of the Chameleon screening mentioned in the Introduction. Firstly, we would like to comment upon the Chameleon screening. It requires a certain form of a scalar field potential and a non-minimal coupling to matter of the following form [10–13]:

$$L_{\text{int}} \sim g^{\mu\nu} T_{\mu\nu} \exp[\beta \kappa \phi].$$

(23)

For the models addressed in this paper no such couplings were introduced, therefore there is no room for the screening.

The interaction required for the screening is typically found in $f(R)$-gravity models given in the Einstein frame [38, 39]. It can be argued that there is room for the Chameleon screening dynamically generated at the level of radiation corrections, but this problem lies far beyond the scope of this paper and should be discussed elsewhere. We would only like to note that such a scenario looks unrealistic. Firstly, potentials (10) and (19) do not admit a form suitable for the screening. Secondly, dimensional considerations disfavor such an opportunity. A characteristic energy scale generated by the earth mean density $\rho_\odot \simeq 5 \times 10^3$ kg m$^{-3}$ is $\epsilon \simeq 3$ MeV. Effects associated with quantum gravity, including the considered effective potential, can hardly be considered relevant at such low energies. Thus the influence of discussed effective potentials can safely be considered negligible in a terrestrial environment.

Secondly, possible implications for inflation should be noted. It is natural to expect an inflationary expansion to occur in the strong field regime $\kappa \phi \sim 1$ which puts it on the edge of an effective theory applicability. Nonetheless, formal expressions for the potentials are well-defined up to $\kappa \phi \simeq 1/2$, so the corresponding effects may be correctly estimated by the effective theory.

For a certain range of parameters both (10) and (19) admit small slow-roll parameters $\eta$ and $\epsilon$ [7–9]. As we mentioned before, the first effective potential (8) is sensitive to the mass-scale hierarchy and the effective model is applicable if and only if $m_{\text{obs}} \sim m$ and $m \ll m_P$. In that
case $\epsilon$ and $\eta$ are small in a proximity of $\kappa \phi \sim 1$. The second effective potential (19) is free from such hierarchy dependence and it also admits $\epsilon, \eta \sim 0$ about the Planck scale.

We would like to highlight ones more that the discussed models admit small slow-roll parameters at the edge of effective theory applicability domain. Nonetheless, we suppose that inflation within this models should be studied, as they may describe a natural appearance of an inflationary cosmology in the spirit of [40].

4. Discussion and conclusions

The following results are found in this paper. The first result clarifies the structure of an effective scalar field potential within the minimal model (5). In the massless case a scalar field does not receive additional contributions to the effective potential at the one-loop level due to the structure of the energy–momentum tensor. The second result specifies the structure of the minimal-model effective potential with a scalar field of a non-vanishing mass. In that case the effective potential is given by (8) and it can be renormalized on the observed mass (10). The third result shows that the John interaction does not contribute to the effective potential in a way similar to the mass term. The interaction itself provides a minimal beyond general relativity three-particle interaction. However, the interaction is bilinear in scalar field derivatives so it fails to generate a contribution to the effective potential in full analogy with the massless case. The fourth result is the effective scalar field potential (20) obtained within a model admitting second-order field equations which accounts for all possible three-particle interactions (12). This potential accounts both for the scalar field mass term and for the non-minimal interaction.

The following features of these potentials are important in the context of realistic models. Firstly, all potentials do not alter the vacuum energy content of a model. They do not develop new minima within the applicability domain and also vanish in $\phi \to 0$ limit. Therefore the ground state together with the vacuum energy remains unchanged. Secondly, the Chameleon screening, mentioned in the introduction, can hardly be realized in such models. The effective potentials have a form which is not suitable for the screening and the studied models lack a non-minimal interaction essential for the screening. Inflationary scenarios, on the contrary, may be successfully realized. Both parts of the effective potential generated by a mass term (8) and by the interaction term (15) have areas with small slow-roll parameters. An opportunity to describe an inflationary expansion within such a setup requires an additional investigation and will be discussed elsewhere.

Finally, possible implications of these results for the standard models should be highlighted. The standard model contains the Higgs field which should be affected by gravitational corrections in a way similar to the case presented. The part of a scalar field effective potential generated by a mass term (8) is sensitive to the hierarchy of the Higgs and Planck scales. Consequently, the contribution can be omitted in models with a realistic hierarchy $M_{\text{Higgs}} \ll M_{\text{Pl}}$. The contribution (15) associated with the non-minimal interaction $R \phi^3$, on the contrary, may not be easily neglected. It is defined by the value of the corresponding coupling $\lambda$ (12). For large $\lambda \sim 10^{17}$ the characteristic dimensionless parameter $\lambda \kappa \phi \sim 1$ at $\phi \sim M_{\text{Higgs}}$ and the corresponding influence cannot be neglected. The value of $\lambda$, on the other hand, is not completely free, as it can be constrained by the empirical Solar system data on PPN parameters. A more detailed discussion of such a combined constraint on the model lies beyond the scope of this paper and will be discussed elsewhere alongside other formal developments of the model.

This discussion shows perspective directions for the further studies. Namely, an application of the effective potential technique to the Higgs sector of the standard model in the context of
gravity should be analyzed. Results of this paper show a qualitative behavior of simple scalar field, but they can hardly be considered sufficient in the particular case of the standard model. To proceed with this goal it is required to evaluate the gravitational contribution to the Higgs effective potential generated by possible non-minimal interactions with gravity. It allows one to constrain the corresponding corrections with the empirical particle physics data. The non-minimal interaction with gravity, on the other hand, should be independently constrained by the Solar system data. Therefore combined constraints on the model can be found and a more comprehensive conclusion can be drawn about the role of non-minimal gravitational interaction within Higgs physics.

Finally, the opportunity to describe the inflation a la [40] within such an approach must also not be forgotten. As it was highlighted above, effective potentials admit areas with small slow-roll parameters, so possible inflationary scenarios should be studied. Some formal developments also should be discussed. We used the direct perturbative calculations to evaluate the effective potential which put some constraints on the studied spectrum of models. More sophisticated techniques similar to those discussed in [15] and those based on renormalization group approach [41–46] may provide a simple way to analyze models with more intricate structures.

Acknowledgments

The work (BL) was supported by the Foundation for the Advancement of Theoretical Physics and Mathematics ‘BASIS’.

ORCID iDs

Andrej Arbuzov https://orcid.org/0000-0001-9326-6905
Boris Latosh https://orcid.org/0000-0001-7099-0861

References

[1] Horndeski G W 1974 *Int. J. Theor. Phys.* **10** 363–84
[2] Kobayashi T, Yamaguchi M and Yokoyama J 2011 *Prog. Theor. Phys.* **126** 511–29
[3] Ishak M 2019 *Living Rev. Relativ.* **22** 1
[4] Berti E et al 2015 *Class. Quantum. Grav.* **32** 243001
[5] Clifton T, Ferreira P G, Padilla A and Skordis C 2012 *Phys. Rep.* **513** 1–189
[6] Tretyakova D and Latosh B N 2018 *Universe* **4** 26
[7] Linde A D 1983 *Phys. Lett. B* **129** 177–81
[8] Senatore L 2017 Lectures on inflation Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (arXiv: 1609.00716)
[9] Gorbunov D S and Rubakov V A 2011 *Introduction to the Theory of the Early Universe: Cosmological Perturbations and Inflationary Theory* (Singapore: World Scientific)
[10] Khoury J and Weltman A 2004 *Phys. Rev. D* **69** 044026
[11] Khoury J and Weltman A 2004 *Phys. Rev. Lett.* **93** 171104
[12] Brax P, van de Bruck C, Davis A C and Shaw D J 2008 *Phys. Rev. D* **78** 104021
[13] Barrage C and Sakstein J 2018 *Living Rev. Relativ.* **21** 1
[14] Coleman S and Weinberg E 1973 *Phys. Rev. D* **7** 1888–910
[15] Buchbinder I, Odintsov S and Shapiro I 1992 *Effective Action in Quantum Gravity* (New York: Routledge)
[16] Burgess C P 2004 *Living Rev. Relativ.* **7** 5–56
[17] Donoghue J F 1994 *Phys. Rev. D* **50** 3874–88
[18] Bjerrum-Bohr N E J, Donoghue J F, Holstein B R, Planté L and Vanhove P 2015 Phys. Rev. Lett. 114 061301
[19] Heisenberg L, Noller J and Zosso J 2020 arXiv:2004.11655
[20] Calmet X and Latosh B 2018 Eur. Phys. J. C 78 205
[21] Latosh B 2018 Eur. Phys. J. C 78 991
[22] Arbuzov A B and Latosh B N 2017 Eur. Phys. J. C 77 702
[23] Zumalacárregui M and García-Bellido J 2014 Phys. Rev. D 89 064046
[24] Bekenstein J D 1993 Phys. Rev. D 48 3641–7
[25] Kobayashi T 2019 Rep. Prog. Phys. 82 086901
[26] Barvinsky A O and Vilkovisky G A 1985 Phys. Rep. 119 1–74
[27] Rivat S 2019 Stud. Hist. Phil. Sci. B 68 23–39
[28] Barvinsky A O, Kamenshchik A Y and Karmazin I P 1993 Phys. Rev. D 48 3677–94
[29] Shapiro I L and Takata H 1995 Phys. Rev. D 52 2162–75
[30] Steinwachs C F and Kamenshchik A Y 2011 Phys. Rev. D 84 024026
[31] Kamenshchik A Y and Steinwachs C F 2015 Phys. Rev. D 91 084033
[32] Latosh B N 2020 Phys. Part. Nucl. 51 859–78
[33] Charmousis C, Copeland E J, Padilla A and Saffin P M 2012 Phys. Rev. Lett. 108 051101
[34] Chernikov N and Tagirov E 1968 Ann. Inst. H. Poincare Phys. Theor. A 9 109
[35] Deser S 1970 Ann. Phys. 59 248–53
[36] Jack I and Jones D R T 1990 Nucl. Phys. B 342 127–48
[37] Jack I and Jones D R T 1990 Phys. Lett. B 234 321–3
[38] De Felice A and Tsujikawa S 2010 Living Rev. Relativ. 13 3
[39] Sotiriou T P and Faraoni V 2010 Rev. Mod. Phys. 82 451–97
[40] Starobinsky A A 1980 Phys. Lett. B 91 99–102
[41] Elizalde E and Odintsov S 1994 Russ. Phys. J. 37 25–9
[42] Elizalde E and Odintsov S D 1994 Phys. Lett. B 333 331–6
[43] Elizalde E, Kirsten K and Odintsov S D 1994 Phys. Rev. D 50 5137–47
[44] Elizalde E and Odintsov S D 1994 Phys. Lett. B 321 199–204
[45] Elizalde E and Odintsov S D 1994 Z. Phys. C: Part. Fields 64 699–708
[46] Elizalde E, Odintsov S D and Romeo A 1995 Phys. Rev. D 51 1680–91