Orbital maneuvers to form a constellation of small satellites from a single large spacecraft

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Abstract. The main goal of the present paper is to show how to form a constellation of small satellites when all small satellites leave from a larger spacecraft. It is assumed that they will all stay in a planar formation, in the same orbital plane of the mother spacecraft, just dispersed in terms of mean anomaly. Initially, it is considered that the larger satellite orbits the Earth in an almost circular orbit with 2000 km of altitude. Then, a search for initial conditions of the small satellite is performed to find the best ones to move it away from the larger satellite such that it is allocated in a co-orbital orbit with respect to the larger satellite. These initial conditions should minimize the consumption of an impulsive maneuver required to move away the small satellite from the large one to put it in course to its final orbit, trying to make the best use of the most relevant perturbations, such as the solar radiation pressure and the oblateness of the Earth. In a second study, we will analyze the influence of the solar radiation pressure, depending on the A/m ratio of the spacecraft, in the trajectories.

1. Introduction

The increasing demand for the use of satellites and other types of spacecraft over the years has been raising issues such as: settlement of certain orbital regions, high cost of construction of the spacecraft, spreading of new space debris from deactivated satellites and/or bodies fractionated by collisions around the Earth, among others [1]. Given this scenario, a new strategy is proposed that aims to reduce expenditures, the amount of mass launched in space and the time of development of the mission. Small satellites such as CubeSats are good examples of space vehicles that have low construction and launching costs, as well as small size [2] and fast development of the mission. Those are the reasons why they are becoming very popular and encouraging studies like the one made here. Therefore, in this paper we propose to use a large satellite that orbits Earth 2000 km high, so that atmospheric drag does not have to be considered [3], in an almost circular orbit to build a constellation of small satellites in a co-orbital orbit with the larger satellite. For this, a numerical search will be performed for initial conditions to be applied to the small satellites such that they move away from the larger satellite and are allocated in a co-orbital orbit with respect to the larger satellite separated by a given angle. The initial conditions will be selected to minimize the fuel consumption of an impulsive maneuver necessary to send the small satellites, making the best use of the perturbation coming from the flattening of the Earth and the solar radiation pressure. The mathematical model used is the elliptic restricted three body problem [4] where the Earth is considered to be the main body, the largest satellite the secondary body and the small satellite the body with a negligible mass. A study will also
be carried out on the influence of the solar radiation pressure on the orbits of the satellite, since this force depends on the area/mass ratio of each satellite and can be important in some situations [5].

2. Methodology and Dynamical Model

The motion of the small satellite can be described by Equation 1 shown next.

\[
\ddot{r} = \frac{-m_1}{r_1^3} r - \frac{m_2}{r_2^3} r + P_P + P_{SRP}
\]

(1)

where the first two terms refer to the restricted three body problem [4] that describes the gravitational interaction of the Earth, indicated by \(m_1\), and the largest satellite, indicated by \(m_2\), with the small satellite having negligible mass. \(G\) is gravitational constant and \(r_1\) and \(r_2\) are, respectively, the distances between the small satellite and the bodies \(m_1\) and \(m_2\). The second term \((P_P)\) corresponds to the acceleration due to the non-spherical shape of the Earth expressed by the usual term \(J_2\) of its gravity field [6]. The last term refers to the acceleration due to the solar radiation pressure \((P_{SRP})\), which depends on the small satellite specifications as: coefficient of reflectivity \((C_R)\), solar flux \((P_S)\), distance to the Sun and the relation between the area of the transversal section and the mass of the satellite \((A/m)\) [5].

The contribution of each force in the motion of the small satellite is calculated by integrating each acceleration over time, like shown in Equation 2 next.

\[
P = \frac{1}{N_f} \int_0^T |\alpha| dt
\]

(2)

where \(N_f\) is the normalization factor, \(\alpha\) is the acceleration vector of a force type and \(T\) is the final integration time of the trajectory. This type of integral measures the total acceleration on the small satellite. The absolute value of the acceleration is used, not to take into accounting compensations between opposite signals calculated at different times [7]. The reason is that the main idea is to compare forces. Other types of acceleration integrals that are used for other types of studies can be found in [8-16]. The numerical method used for integration is the eighth-order Runge-Kutta.

![Figure 1. Geometry of the problem showing the initial conditions, the Earth, the large and the small satellite.](image-url)
are aligned on the horizontal axis. In this system, the small satellite is a short distance from the large satellite. Figure 1 shows the positions of the Earth, the large and the small satellite, all with respect to the inertial frame, in the case where the three bodies are aligned in the horizontal axis. Other geometries can also be used, such as the one with the small satellite above the large one, to reduce the risk of a collision with the large satellite. It is also indicated the initial conditions of the small satellite used to identify the trajectories: the initial distance \( D \) between the small and the large satellite and the components of the initial velocity \( v_x \) and \( v_y \) of the small satellite. Then, several configurations of these initial conditions are integrated numerically, using Equation 1, for a certain time. Using this technique it is possible to find initial conditions that place the small satellite in a co-orbital orbit in front or behind the large satellite. For each set of initial conditions, the minimum, mean and maximum distances between the small and large satellite are calculated for a given time. To find the initial conditions a triple loop is made by varying the three variables that define the initial conditions (\( D \), \( v_x \), and \( v_y \)) in very large ranges. From the results it is possible to find the small ranges containing the most interesting trajectories. After that, the initial conditions sets are studied and it is possible to see the maximum, minimum and mean distances values between the large and small satellites, as well as the components of the velocity \( v_x \) and \( v_y \). With this, it is possible to choose the most adequate sets of initial conditions and then propagate the trajectory to calculate the angle between them during a time interval. Propagating the trajectory of the first small satellite give the conditions for the others. Finally, it is only necessary to choose the desired angle to separate the satellites and then to apply a small impulse when they reach this angle to keep the angle permanently. In this first study it will not be considered the influence of the solar radiation pressure. In the second part, it is analyzed the influence of the solar radiation pressure on the trajectories of the large and the small satellite. This will be done by varying the value of the A/m ratio, which is the factor that determines how large is this effect of the solar radiation pressure.

3. Results and Discussion

In the first study, it will be presented the results concerning the launching of small satellites from a large satellite in co-orbital orbits. The trajectories with respect to the set of initial conditions that presented the largest gap between the small and the large satellite during a time of 60 days. This study can be used by any other small satellite. The solar radiation pressure is not considered in this study, and it will be the reference case. In the second study, the solar radiation pressure is considered by varying the values of the A/m ratio of the large and the small satellite. The goal is to study if the variation of A/m can change the time histories of the angle between the small and the large satellite and try to find some relation between the different results. The A/m values that will be considered for the small satellite are: 0.0075, 0.075, 0.75, 7.5, 15, and 30 \( \text{m}^2/\text{kg} \). For the large satellite the A/m values will be: 0.0025, 0.025, 0.25, 2.5, 5, and 10 \( \text{m}^2/\text{kg} \). Table 1 shows the physical and orbital data of the Earth and the large satellite.

### Table 1. Physical and orbital data of the Earth and the large satellite

| Celestial body | Average radius (km) | Mass (kg) | \( J_2 \) | Semi-major axis | eccentricity |
|----------------|---------------------|-----------|----------|----------------|-------------|
| Earth          | 6378                | 5.97×10^{24} | 1082.63×10^{6} | 1.00 u.a.     | 0.0167      |
| Large Satellite| 3.75×10^{3}         | 6000      | -        | 8378 km        | 0.001       |
3.1. Launching of the small satellites

Figure 2 shows the trajectories of the small (in blue) and the large satellite (in red) with respect to different reference systems. It is important to remember that, initially, the small satellite is next to the large satellite in orbit around the Earth. Then, it is given an initial condition that slightly modifies the orbit of the small satellite such that it places it into a slightly different orbit compared to the large satellite, but co-orbital to it. The set of initial conditions which presented the largest distance between the small and the large satellite is given by: \( D = 5 \text{ m}, \, v_x = 0, \, v_y = -2 \text{ m/s}, \) and \( T = 60 \text{ d}. \) The Earth is represented in scale (in gray) and fixed according to the reference system.

![Trajectories](image)

(a) Small satellite trajectory around the Earth in the rotating reference system with origin fixed in the large satellite.

(b) Trajectory of the small satellite around the Earth in the fixed reference system with origin fixed to the Earth.

(c) Trajectory of the large satellite around the Earth in the fixed reference system with origin fixed in the Earth.

(d) Trajectory of the large and the small satellites around the Earth in the fixed reference system with origin fixed in the Earth.

Figure 2. Trajectories of the small and large satellite for \( D = 5 \text{ m}, \, v_x = 0, \, v_y = -2 \text{ m/s} \) and \( T = 60 \text{ d}. \)
In Figure 2(a) the large satellite is fixed in the origin of the rotating system, so it is like if it were standing with respect to the small satellite that is initially next to it. After giving the initial conditions to the small satellite (\(D = 5 \text{ m}, v_x = 0, v_y = -2 \text{ m/s}, \) and \(T = 60 \text{ d}\)), it is observed that the small satellite moves with respect to the large one in a trajectory around the Earth. It is also possible to observe that, in the time of 60 days, the trajectory stops before returning to the origin of the system, presenting a final lag equal 147.73 degrees with respect to the large satellite. The final angle that the small satellite is shifted with respect to the large one is equal to 212.27 degrees. Figures 2(b), 2(c), and 2(d) have the Earth fixed at the origin of the fixed reference frame and show, respectively, the trajectories of the small satellite, the large satellite and the superimposed small and large satellite. They show that both satellites move in co-orbital orbits around the Earth. So it was shown that a small satellite can be launched from a large satellite in a different, but co-orbital, orbit when gaining or losing velocity. Considering the total shift of approximately 212 degrees, it can be concluded that it is possible to distribute approximately 10 small satellites approximately spaced in 21 degrees with respect to them and the large satellite during the time of 60 days. In 102 days we would have 17 small satellites separated by approximately 21 degrees. Table 2 presents the first five values of time found in the propagation of the trajectory considering angles closer to a separation of approximately 21 degrees specified to launch the first five small satellites.

### Table 2. Time values for the shift of the first 5 small satellites considering \(D = 5 \text{ m}, v_x = 0, v_y = -2 \text{ m/s}\) and \(T = 60 \text{ d}\).

| Small Satellite | 1   | 2   | 3   | 4   | 5   | 6   |
|-----------------|-----|-----|-----|-----|-----|-----|
| Total Angle (degree) | 21.00 | 42.00 | 63.01 | 84.00 | 105.00 | 126.01 |
| \(T (\text{d})\)  | 5.96 | 11.90 | 17.83 | 23.83 | 29.75 | 35.60 |

From Table 2 it is observed the correct times to release each satellite to get a separation of approximately 21 degrees one from each other. As the angles increase, the times also increase, as expected. But the table shows in details the times involved.

#### 3.2. Study of the solar radiation pressure in the trajectories

The study of the effects of solar radiation pressure on the time histories of the angle between the satellites is performed using the same set of initial conditions obtained in the previous section: \(D = 5 \text{ m}, v_x = 0, v_y = -2 \text{ m/s},\) and \(T = 60 \text{ d}\). The value of \(A/\text{m}\) equal to zero is used as the reference value because, in this case, the solar radiation pressure is not considered. Table 3 shows the respective total angle values for each \(A/\text{m}\) value of the small satellite. The \(A/\text{m}\) of the large satellite is considered to be equal to 0.0025 \(\text{m}^2/\text{kg}\) in all cases where the solar radiation pressure is considered.

### Table 3. Values of the angle between the small and the large satellite for each value of \(A/\text{m}\) of the small satellite considering \(T = 60 \text{ days}\).

| \(A/\text{m}\) (\(\text{m}^2/\text{kg}\)) | 0   | 0.0075 | 0.075 | 0.75 | 7.5 | 15  | 30  |
|-------------------------------|-----|--------|-------|------|-----|-----|-----|
| Total Angle (degree)          | 212.27 | 212.27 | 212.22 | 211.77 | 207.22 | 202.03 | 191.33 |

Observing Table 3, it is visible that, as the \(A/\text{m}\) value of the small satellite increases, the value of the total angle between the small and the large satellite decreases. For \(A/\text{m}\) equal to zero, the reference case, the highest total angle obtained was 212.27 degrees. The lowest values of \(A/\text{m}\) (0.0075, 0.075, and 0.75, \(\text{m}^2/\text{kg}\)) presented values close to the reference, 212.27, 212.22, and 211.77 degrees,
respectively. The higher values of $A/m$ (7.5, 15, and 30 $m^2/kg$) presented the lowest values for the angles, 207.22, 202.03, 191.33 degrees, respectively.

In order to compare the time lag for different values of $A/m$, the mean square error was calculated between each value of $A/m$ of the small satellite with the reference value, $A/m = 0$. The results showed that the mean square error increases with the value of $A/m$ and it corroborates with the trend observed in Table 3. Figure 3 shows the distribution of the mean square error as a function of $A/m$ and also the linear regression of the data.

![Figure 3](image)

**Figure 3.** Linear regression of the mean squared error values (RMSE) of the angles as a function of $A/m$ of the small satellite.

In Figure 3 it is observed that the mean square error increases with the increase of $A/m$ of the small satellite. The errors are below 0.1 and the first two values with respect to $A/m$ equal to 0.0075 and 0.075 overlap, being $1.17 \times 10^{-5}$ and $1.64 \times 10^{-4}$, respectively. Through linear regression we obtained a mathematical equation of the error distribution as a function of $A/m$ of the small satellite: $y = 2.37 \times 10^{-3}x - 2.12 \times 10^{-4}$.

Finally, the contribution of each force in the movement of the small satellite was also calculated. It has been observed that the Earth and its flattening term $J_2$ are the ones that most disturb the trajectories and their values are of the order of $10^{-3}$ and $10^{-6}$, respectively. The disturbance due to the solar radiation pressure ranges from $10^{-11}$ to $10^{-7}$ according to the increase in $A/m$. The lowest perturbative term is due to the perturbation of the largest satellite, as expected, which is of the order of $10^{-17}$.

Table 4 shows the respective total angles for each value of $A/m$ of the large satellite. The $A/m$ value of the small satellite is considered equal to 0.0075 $m^2/kg$ in all cases, except where the solar radiation pressure is not included.

| $A/m$ (m$^2$/kg) | 0  | 0.0025 | 0.025 | 0.25 | 2.5 | 5   | 10   |
|------------------|----|--------|-------|------|-----|-----|------|
| Total Angle (degree) | 212.27 | 212.27 | 212.28 | 212.42 | 213.84 | 215.47 | 218.87 |

Observing Table 4, it is visible that when $A/m$ of the large satellite increases, the value of the total angle between the small and the large satellite also increases. It means that a given angle is obtained in short times. When the solar radiation pressure is not considered, the reference case and the case where 0.0025 $m^2/kg$ (the lowest value) the angle obtained is 212.27 degrees. The next lowest values of $A/m$, 0.025 and 0.25 $m^2/kg$, presented angles close to the reference, 212.28 and 212.42 degrees,
respectively. The higher values of A/m (2.5, 5 and 10 m^2/kg) presented the higher angles, 213.84, 215.47, and 218.87 degrees, respectively.

In order to compare the time shift for the different values of A/m, the mean square error was calculated between each value of A/m of the large satellite with the reference value, with no solar radiation pressure. The results showed that the mean square error increases with A/m and it corroborates with the trend observed in Table 4. Figure 4 shows the distribution of the mean square error as a function of A/m and also the linear regression of the data.

![Figure 4. Linear regression of the mean squared error values (RMSE) of the angles as a function of A/m of the large satellite.](image)

In Figure 4 it is observed that the mean square error increases with increase of A/m of the small satellite. The errors are below 0.03 and the first two values with respect to A/m (0.0025 and 0.025 m^2/kg) overlap. They are 1.17×10⁻⁵ and 3.57×10⁻⁵, respectively. Through linear regression we obtained a mathematical equation of the error distribution as a function of A/m of the small satellite: y = 2.24×10⁻³x – 1.02×10⁻⁴. The error for the larger values of A/m of the large satellite is less than the error observed for the higher values of A/m of the small satellite, 0.025 and 0.080, respectively.

Finally, the perturbations of each force in the motion of the small satellite due to the variation of A/m of the large satellite are of the same order of magnitude as those found for the variation of the A/m of the large satellite.

4. Conclusion

In this paper it was presented a new technique to form a constellation of small satellites where all members are launched from a larger satellite in co-orbital orbits that are angularly spaced. The technique consists in finding a set of initial conditions for launching a small satellite and then the propagation of its trajectory is made, from where it is possible to calculate the time histories of the angle between the small and the large satellite. The shift required for this angle can be maintained through a small impulse applied to the small satellite. Making those studies for different values of A/m, it has been found that the solar radiation pressure has an order of magnitude of 10⁻¹¹ to 10⁻⁷, very small values. In this way, the major perturbing effects of the trajectory are the Earth and its term J₂, respectively in the order of 10⁻³ and 10⁻⁶.
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