Effective Field Theory and Isospin Violation in Few–Nucleon Systems

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Abstract. I discuss the leading and subleading isospin–breaking three–nucleon forces in the chiral effective field theory framework.

INTRODUCTION

Isospin violation has its origin within the Standard Model in the different masses of the up and down quarks and electromagnetic interaction. Its consequences for few–nucleon systems can be studied in a systematic way within chiral Effective Field Theory (EFT). This approach is based on the most general (approximately) chiral invariant Lagrangian for pions and nucleons which includes all possible interactions consistent with the isospin violation in the Standard Model. In particular, strong isospin–breaking terms are proportional to $\varepsilon M^2_\pi$, where $\varepsilon = (m_d - m_u)/(m_d + m_u) \sim 1/3$. Electromagnetic vertices due to exchange of (hard) virtual photons are proportional to the nucleon charge matrix $Q_{ch} = e/2(1 + \tau_3)$. In principle, one should also include explicit soft photons. Their contributions are, however, irrelevant for the present study and will not be considered. The effective Lagrangian has been applied to study isospin–violating two–nucleon (2N) forces [1, 2, 3, 4, 5, 6, 7, 8]. In these proceedings, I consider isospin–breaking three–nucleon forces (3NFs) within the chiral EFT approach, see also [9, 10].

POWER COUNTING AND THE EFFECTIVE LAGRANGIAN

In this work, I use the same counting rules for $e$ and $\varepsilon$ as in [9], namely:

$$\varepsilon \sim e \sim \frac{q}{\Lambda}; \quad \frac{e^2}{(4\pi)^2} \sim \frac{q^4}{\Lambda^4},$$

where $q (\Lambda)$ refers to a generic low–momentum scale (the pertinent hard scale). The N–nucleon force receives contributions of the order $\sim (q/\Lambda)^\nu$, where

$$\nu = -2 + 2N - 2C + 2L + \sum_i V_i \Delta_i.$$
Here, \( L, C \) and \( V_i \) refer to the number of loops, separately connected pieces and vertices of type \( i \), respectively. Further, the vertex dimension \( \Delta_i \) is given by

\[
\Delta_i = d_i + \frac{1}{2} n_i - 2 ,
\]

where \( n_i \) is the number of nucleon field operators and \( d_i \) is the \( q \)--power of the vertex, which accounts for the number of derivatives and insertions of pion mass, \( \varepsilon \) and \( \varepsilon/(4\pi) \) according to eqs. (1). Notice further that the nucleon mass is counted according to \( q/m \sim (q/\Lambda)^2 \), see [9] for more details.

The relevant isospin--symmetric terms in the effective Lagrangian in the nucleon rest frame are [11, 12]:

\[
\mathcal{L}^{(0)} = \frac{1}{2} \partial_\mu \pi \partial^\mu \pi - \frac{1}{2} M_\pi^2 \pi^2 + N^\dagger \left( i\partial_0 + \frac{g_A}{2F_\pi} \tau \vec{\sigma} \cdot \vec{\nabla} \pi - \frac{1}{4F_\pi^2} \tau \cdot (\pi \times \pi) \right) N \\
- \frac{1}{2} C_5 (N^\dagger N) (N^\dagger N) - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N) (N^\dagger \vec{\sigma} N)
\]

\[
\mathcal{L}^{(1)} = N^\dagger \left( - \frac{c_1}{F_\pi^2} M_\pi^2 \pi^2 + \frac{c_3}{F_\pi^2} (\partial_\mu \pi \cdot \partial^\mu \pi) - \frac{c_4}{2F_\pi^2} \epsilon_{ijk} \epsilon_{abc} \sigma_i \tau_a (\nabla_j \pi_b) (\nabla_k \pi_c) \right) N , \\
- \frac{D}{4F_\pi} (N^\dagger N) (N^\dagger \vec{\tau} \pi N) \cdot \vec{\nabla} \pi 
\]

where \( M_\pi \) and \( F_\pi \) refer to the pion mass and decay constant, \( g_A \) denotes the nucleon axial coupling and \( c_i, C_{S,T} \) and \( D \) are further low--energy constants (LECs). The relevant isospin--violating part of the Lagrangian reads [13]:

\[
\mathcal{L}^{(2)} = \frac{1}{2} \delta M_\pi^2 \pi^2 + N^\dagger \left( - \frac{1}{2} \tau_3 \epsilon m - \frac{c_5}{F_\pi^2} \epsilon M_\pi^2 (\pi \cdot \tau) \pi_3 \right) N \\
\mathcal{L}^{(3)} = N^\dagger \left( f_1 e^2 (\pi_3 - \pi^2) + \frac{1}{4} f_2 e^2 ((\pi \cdot \tau) \pi_3 - \pi^2 \tau_3) \right) \\
+ \frac{2d_{17} - d_{18} - 2d_{19}}{F_\pi} \epsilon M_\pi^2 \epsilon \vec{\nabla} \pi_3 \right) N + L \epsilon M_\pi^2 (N^\dagger \pi_3 N) (N^\dagger N) ,
\]

\( c_5, d_i, f_{1,2} \) and \( L \) are the LECs and \( \delta M_\pi^2 = M_\pi^2 - M_\pi^0 \). The LECs \( c_5 \) and \( f_2 \) are related to the proton--to--neutron mass difference via \( (\delta m)^{\text{str}} \equiv (m_p - m_n)^{\text{str}} = -4c_5 \epsilon M_\pi^2 \) and \( (\delta m)^{\text{em}} \equiv (m_p - m_n)^{\text{em}} = -f_2 e^2 F_\pi^2 \). Further, \( \delta m = (\delta m)^{\text{str}} + (\delta m)^{\text{em}} \). Notice that terms in the last line of the above equation lead to vanishing contributions to the 3NF and were not considered in [9]. Here I decided to keep them for the sake of completeness.

**ISOSPIN--BREAKING THREE--NUCLEON FORCE**

The diagrams contributing to the leading \((\nu = 4)\) and subleading \((\nu = 5)\) isospin--breaking 3NFs are depicted in Fig. 1. It should be understood that these diagrams only specify the topology and do not correspond to Feynman graphs. Clearly, the contributions to the 3NF do not include the pieces generated by the iteration of the 2N potential.
In [9] we have evaluated the corresponding 3NFs using the method of unitary transformation developed in [14]. For example, to calculate the contribution of the graph (a) in Fig. 1 one needs to evaluate the 3N matrix elements of the operator

\[ V_{2\pi} = \eta' \left[ \frac{1}{2} H_1 \frac{\lambda^1}{(H_0 - E_{\eta'})} H_1 \tilde{\eta} H_1 \frac{\lambda^1}{(H_0 - E_{\eta})} H_1 \right. \]

\[ - \frac{1}{8} H_1 \frac{\lambda^1}{(H_0 - E_{\eta'})} H_1 \tilde{\eta} H_1 \frac{\lambda^1}{(H_0 - E_{\eta})} H_1 \]

\[ + \frac{1}{8} H_1 \frac{\lambda^1}{(H_0 - E_{\eta'})} H_1 \tilde{\eta} H_1 \frac{\lambda^1}{(H_0 - E_{\eta})} H_1 \]

\[ - \frac{1}{2} H_1 \frac{\lambda^1}{(H_0 - E_{\eta'})} H_1 \frac{\lambda^2}{(H_0 - E_{\eta})} H_1 \frac{\lambda^1}{(H_0 - E_{\eta})} H_1 \]

\[ \left. \eta + \text{h. c.} \right] \quad (6) \]

where \( \eta, \eta' \) and \( \tilde{\eta} \) denote the projectors on the purely nucleonic subspace of the Fock space, while \( \lambda^i \) refers to the projector on the states with \( i \) pions. \( E_{\eta}, E_{\eta'} \) and \( E_{\tilde{\eta}} \) refer to the energy of the nucleons in the states \( \eta, \eta' \) and \( \tilde{\eta} \), respectively. Further, \( H_1 \) is the leading \( \pi NN \) vertex \( \propto g_A \) in eq. (4), and \( H_0 \) denotes the free Hamilton operator for pions and nucleons corresponding to the density

\[ H_0 = \frac{1}{2} \pi^2 + \frac{1}{2} (\bar{\psi} \pi)^2 + \frac{1}{2} M_\pi^2 \pi^2 + \frac{1}{2} N^\dagger \delta m \tau_3 N. \quad (7) \]

One finds the following charge–symmetry–breaking (CSB) 3NFs resulting from diagrams (a) and (b):

\[ V_{2\pi}^{3N} = \sum_{i \neq j \neq k} 2 \delta m \left( \frac{g_A}{2F_\pi} \right)^4 \frac{(\bar{\sigma}_i \cdot \bar{q}_i)(\bar{\sigma}_j \cdot \bar{q}_j)}{(\bar{q}_i^2 + M_\pi^2)(\bar{q}_j^2 + M_\pi^2)} \left[ [\bar{q}_i \times \bar{q}_j] \cdot \bar{\sigma}_k [\tau_i \times \tau_j]^3 \right] \]
where \( i, j \) and \( k \) denote the nucleon labels. Further, \( \bar{q}_i \equiv p_i' - p_i \) where \( p_i \) (\( p_i' \)) are initial (final) momenta of the nucleon \( i \). The \( 2\pi \)-exchange diagrams (c), (d) and (g) in Fig.1 lead to the 3NF

\[
V_{3\pi}^{3N} = \sum_{i \neq j \neq k} 2 \delta m C_F \left( \frac{g_A}{2F_\pi} \right)^2 \frac{\bar{\sigma}_j \cdot \bar{q}_i}{(\bar{q}_i^2 + M^2_{\pi})^2} [\bar{\sigma}_j \times \bar{\tau}_k] \bar{\tau}_j \cdot \bar{q}_i,
\]

Finally, the charge–symmetry–conserving 3NF resulting from diagrams (e) and (f) in Fig.1 reads:

\[
V_{2\pi}^{3N} = \sum_{i \neq j \neq k} \delta M^2 \left( \frac{g_A}{2F_\pi} \right)^2 \frac{(\bar{\sigma}_i \cdot \bar{q}_i)(\bar{\sigma}_j \cdot \bar{q}_j)}{(\bar{q}_i^2 + M^2_{\pi})(\bar{q}_j^2 + M^2_{\pi})} \left\{ \tau_i^3 \tau_j^3 \left[ \frac{4c_1 M^2_{\pi}}{F^2_\pi} + \frac{2c_3}{F^2_\pi} (\bar{q}_i \cdot \bar{q}_j) \right] \right\}
\]

These expressions should be used together with the corresponding isospin–symmetric 3NFs expressed in terms of the charged pion mass.

\[
\begin{align*}
V_{1\pi}^{3N} &= - \sum_{i \neq j \neq k} \delta M^2 \frac{g_A}{8F^2_\pi} D \left( \frac{\bar{\sigma}_i \cdot \bar{q}_i}{(\bar{q}_i^2 + M^2_{\pi})^2} \tau_i^3 \tau_j^3 (\bar{\sigma}_j \cdot \bar{q}_j) \right).
\end{align*}
\]

\[\text{FIGURE 2.} \quad \text{Leading (a–c) and subleading (d–i) isospin–violating contribution to the 3NF which vanish, as discussed in the text. For notation see Fig. 1.}\]

In addition to graphs shown in Fig. 1, diagrams (a)–(c) and (d)–(i) in Fig. 2 formally contribute to the leading (\( V = 4 \)) and subleading (\( V = 5 \)) isospin–breaking 3NF, respectively. Their pertinent contributions, however, vanish. In particular, for graphs (a), (b),
(d), (f) and (i) one observes similar cancellation between various time orderings as in the case of the corresponding isospin–invariant 3NFs, see e.g. [14]. Further, the diagram (c) is suppressed by a factor of $q/m$ due to the time derivative entering the Weinberg–Tomozawa vertex in eq. (4). Explicit evaluation of the contributions of graphs (e) and (f) can be performed along the lines of ref. [9], which leads to vanishing sum of these contributions.

In [9] we have estimated the relative strength of the leading and subleading corrections compared to the isospin–conserving 3NF at the same order. Isospin–violating 3NFs are expected to provide a small but non–negligible contribution to the $^3$He–$^3$H binding–energy difference.

**SUMMARY**

I have discussed the leading and subleading isospin–violating 3NFs. The leading contributions are generated by one– and two–pion exchange diagrams with their strength given by the strong neutron–proton mass difference. The subleading corrections are again given by one– and two–pion exchange diagrams, driven largely by the charged–to–neutral pion mass difference and also by the electromagnetic neutron–proton mass difference and the dimension two electromagnetic LEC $f_1$. In the future, these isospin–breaking forces should be used to analyze few–nucleon systems based on chiral EFT.

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