Emission of Axions in strongly magnetized stars

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Abstract

We show that the axion decay constant does not get any correction at any order by external magnetic fields. On the other hand, in the context of the Wilsonian effective action under external magnetic fields, the axial currents get a finite correction. We then calculate the effect of strong magnetic fields $B > 10^{18} G$ on the axion-nucleon coupling and find that if $B \gtrsim 10^{20} G$ in strongly magnetized neutron and white dwarf stars, the emission rate of axions is enhanced by several orders of magnitude.

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The stellar evolution tightly constrains the emission of weakly interacting, low-mass particles such as neutrinos or axions [1] whose properties are otherwise hard to probe [2]. Recently, the emission rate of such particles from stars has been calculated more accurately by including the many-body effects [3], the effect of strong magnetic fields [4,5], and the thermal pions and kaons [6,7]. The many-body effects in the dense core of stars are found to suppress the axion or neutrino emission rate by a factor of about 2, while the thermal pions and kaons to increase the axion emission rate by a factor of 3-4 within a perturbative treatment. Under a strong magnetic field $B$ the cyclotron emission of neutrinos and axions offers a new cooling mechanism for magnetized white dwarfs and neutron stars. It puts a bound $g_{ae} \lesssim 10^{-10}$ on the axion electron coupling constant. Similarly from magnetic white dwarf (WD) stars $g_{ae} \lesssim 9 \times 10^{-13} (T/10^7 \text{K})^{5/4} (B/10^{10} \text{G})^{-2}$, where $T$ is the internal temperature of the white dwarf.

In this letter, we calculate the corrections to the emission rate of axions by nucleons in a strongly magnetized neutron or WD star whose magnetic field is stronger than the critical field, $B > B_c$, such that the Landau gap is larger than the rest mass energy or $\Lambda_{\text{QCD}}$. As we will see later, for $B \gtrsim 10^{20} \text{G}$ (or $\sqrt{|qB|} \gtrsim 4\Lambda_{\text{QCD}} \approx 800 \text{ MeV}$) the axion emission is enhanced quite a lot and gives a stringent bound on the axion-nucleon coupling. Large magnetic fields $B \sim 10^{12}-10^{14} \text{G}$ have been estimated at the surface of neutron stars from the synchrotron radiation spectrum [8] and fields $B \sim 10^{18}-10^{20} \text{G}$ are predicted in the core by scalar virial theorem [9].

Let us first consider how the axion decay constant is renormalized in QED under a strong magnetic field. The calculation will go in parallel in QCD also. Under a constant magnetic field oriented along the $z$ axis $\vec{B} = B \hat{z}$, the spectrum of charged particles of charge $q$ and mass $m$ is quantized by the Landau level

$$E_A = \alpha \sqrt{m^2 + k_z^2 + 2|qB|n},$$

where $\alpha = \pm$ is the sign of the energy and the quantum number $n$ labeling the Landau level

$$2n = 2n_r + 1 + |m_L| - \text{sgn}(qB)(m_L + 2\beta)$$

with $\beta = \pm \frac{1}{2}$ the spin component along the magnetic field. Here $n_r$ is the number of nodes of radial eigenfunction and $m_L$ is the angular momentum. (Note that there will be corrections to the energy eigenvalue Eq. (1) due to QED loop effects like the anomalous magnetic moment, but they are irrelevant since the corrections are smaller than the Landau gap.)

At energy lower than the Landau gap, $E < \sqrt{|qB|}$, the relevant degrees of freedom in QED are photons and the lowest Landau level (LLL) fermions. Their interaction can be described by an effective action derived by integrating out the modes at the higher Landau levels ($n > 0$) [10]. The effective action contains a (marginal) four-fermion interaction, which leads to chiral symmetry breaking even at weak attraction [11-13]. Using this effective action, we calculate the axion decay amplitude to two photons. The Wilsonian effective action of QED is given as [14]

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}(1 + a_1) F_{\mu\nu}^2 + (1 + b_1) \bar{\psi} (i \slashed{D} - m) \psi - (1 + c_1) \frac{ie^2}{2|eB|} \bar{\psi} A_{\mu} \gamma_\mu \partial_\mu \psi$$

$$- \frac{ie^2}{2|eB|} \bar{\psi} A_{\mu} \gamma_\mu \partial_\perp A\psi + \frac{g_1}{|eB|} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right] + \cdots,$$
where the coefficients $a_1, b_1$, and $c_1$ are of order $e^2$ while $g_1$ is of order $e^4$, determined by the one-loop matching conditions, and the ellipses are the higher order operators in momentum expansion. The components $\partial_\perp (\partial_{\parallel})$ are perpendicular (parallel) to the magnetic field and

$$\tilde{\gamma}^\mu = \begin{cases} i\gamma^\mu \gamma^1 \gamma^2 \ln 2 \text{sgn}(eB) & \text{if } \mu = 0, 3 \\ \gamma^\mu & \text{if } \mu = 1, 2. \end{cases} \quad (4)$$

Now, we consider the axion coupling to electrons:

$$L_{\text{int}} = \frac{1}{2f_{PQ}} \partial^\mu a \bar{\Psi} \gamma^\mu \gamma_5 \Psi, \quad (5)$$

where $a$ is the axion field and $f_{PQ}$ is the axion decay constant. We integrate out all the modes ($\Psi_{n \neq 0}$) except the LLL electron ($\psi$) to get the low energy effective interaction for $a$ and $\psi$, which is given as

$$L_{\text{eff}}^{\text{int}} = \frac{1}{2f_{PQ}} \partial^\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi - \frac{e}{2|eB|f_{PQ}} \bar{\psi} \gamma^\alpha \gamma_5 \partial_\alpha a \tilde{\gamma}^\mu i \partial^\nu a \gamma^\beta \psi A_\beta + \cdots, \quad (6)$$

where the ellipses denote the effective interaction terms containing the higher powers of axions and momenta, generated by the exchange interaction of $\Psi_{n \neq 0}$.

In the leading order, there are three diagrams that contribute to the axion decay amplitude (see Fig. 1). We find that the first two diagrams vanish but the third diagram gives

$$A_1 = -\frac{ie^2}{|eB|f_{PQ}} (2\pi)^4 \delta^4(k-P) \int_q (q+k)_{\mu} k_{\alpha} \varepsilon_{\mu}(p_1) \varepsilon_{\beta}(p_2) \text{Tr} \left[ \gamma^\mu \tilde{S}(q+p_1) \gamma^\alpha \gamma_5 \gamma^\nu \gamma^\beta \tilde{S}(q) \right] \quad (7)$$

$$= -\frac{e^2}{4\pi^2 f_{PQ}} \delta^4(k-P) \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \varepsilon_{\nu}(p_1) p_{2\alpha} \varepsilon_{\beta}(p_2), \quad (8)$$

where $P = p_1 + p_2$ and $\varepsilon_{\mu}(p)$ the photon wave function, and the LLL electron propagator

$$\tilde{S}(l) = \frac{i}{P_- - m} P_- e^{-i\tilde{p}_l^2/|eB|} \quad (9)$$

with $P_- = 1 - i\gamma^1 \gamma^2 \text{sgn}(eB)$. The effective Lagrangian for the axion decay amplitude is then

$$L_{a\gamma\gamma}^{\text{eff}} = \frac{e^2}{16\pi^2 f_{PQ}} a F \tilde{F}. \quad (10)$$

We find that the axion decay constant is not renormalized in the leading order under an external magnetic field, $f_{PQ}(B) = f_{PQ}(0)$, in contrast with the pion decay constant that increases under an external magnetic field \[\text{[citation]}\]. In fact, as is shown in the appendix A, the non-renormalization of the axion decay constant under external magnetic fields is exact simply because the axion couples to $\tilde{E} \cdot \tilde{B}$ not to magnetic fields $B$ alone.

Similarly, we calculate the decay amplitude of the axion into one photon (Fig. 2), using the effective Lagrangians Eqs. (3) and (6), to get
where one of two photons is replaced by the external photon, assuring that $f_{PQ}$ is not renormalized. We see that the external magnetic field does not change the axion-photon interaction. Therefore, the energy-loss processes involving only axions and photons such as the axion-photon conversion or the Primakoff process do not get affected by the dimensional reduction.

Now we consider the process that electrons or quarks emit axions. For an extremely strong magnetic field $B > B_c$, all the charged particles are in the lowest Landau level and thus the Bremsstrahlung emission is more relevant than the cyclotron emission, unless the interior temperature of stars is comparable to the Landau gap. The one loop correction to the axion Bremsstrahlung amplitude is (see Fig. 3)

$$A_{BR} = \frac{i^2 g_1}{2f_{PQ} |eB|} \int_{x,y} \langle p', k | \bar{\psi} \gamma_5 \psi(y) \left[ (\bar{\psi} \gamma_5 \psi(x))^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right] | p \rangle$$

$$= \frac{-i g_1}{4\pi^2 f_{PQ}} (2\pi)^4 \delta^4(p - p' - k) \cdot \bar{u}(p') \, i k \gamma_5 u(p) \left( 1 + \frac{4}{3 m^2} + \cdots \right).$$

We see that the Bremsstrahlung process has an extra contribution coming from the effective four-fermion interaction. (Note that here we assume $k^2 \ll m^2$. But, if $m \to 0$, the contribution goes to zero.) In QED, $g_1 \propto e^4$ and therefore quite small [10], but in QCD, for a strong magnetic field $B > 10^{19} G$ (or $\sqrt{|qB|} > \Lambda_{QCD}$) such that the effective action for QCD is valid, $g_1^a \gtrsim \alpha_s^2$ and is no longer negligible (see the appendix B for details). Especially, for quarks under a strong external magnetic field $B \gtrsim 10^{20} G$, $g_1^a$ will be quite large. Thus the four-quark interaction will be stronger than the one-gluon exchange interaction and will break the chiral symmetry of QCD at energy above $\Lambda_{QCD}$.

The axion emission process of neutron stars was first calculated by Iwamoto and it is found that axion bremsstrahlung from nucleon-nucleon collision is the dominant energy-loss process in neutron stars [13]. Since the emission rate is proportional to the square of the axion-nucleon coupling $g_{an}^2$, any correction to this coupling will affect the emission rate. What we calculated above can be thought of as a finite correction to the axial singlet current of quarks and equally valid to the axial octet quark currents. Namely, both $\bar{q} \gamma_\mu T^a \gamma_5 q$ and $\bar{q} \gamma_\mu \gamma_5 q$ have the same correction $(1 + g_1^a/(2\pi^2))$. As was discussed in Ref. [16], the derivative couplings of the axion to the axial vector baryon current is

$$\frac{\partial^\mu a}{f_{PQ}} \left\{ 2\text{tr} (Q_A T^a) \left( F_{\mu\nu} B^\nu \right) + \frac{2}{3} \text{tr} Q_A \text{tr} B \gamma_\mu \gamma_5 \right\},$$

where $B$ is the baryon octet matrices and $Q_A$ the axion charge. According to the current algebra, $F$, $D$, and $S$ are defined to be proportional to the nucleon matrix elements of the axial quark currents [17]. Therefore we see that the axion-nucleon coupling gets a correction

$$\delta g_{an} = \frac{g_1^a}{2\pi^2} g_{an}.$$
Besides the correction due to the four-fermion interaction, the emission rate gets corrections since the axion-nucleon coupling and the nucleon mass will change due to the strong magnetic field. As a good approximation, we can take the nucleon mass as

\[ m_n \propto \langle \bar{q}q \rangle^{1/3}. \]  

(17)

In chiral perturbation theory, the quark condensate is given as [18]

\[ \langle \bar{q}q \rangle^B = -\frac{\partial \mathcal{E}_{\text{vac}}(m_q, B)}{\partial m_q} \bigg|_{m_q=0}, \]  

(18)

where \( \mathcal{E}_{\text{vac}} \) is the vacuum energy density. The one-loop vacuum energy under a magnetic field was calculated by Schwinger [19] as

\[ \mathcal{E}_{\text{vac}}(B) = \mathcal{E}_{\text{vac}}(0) - \frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-M^2_s} \left[ \frac{eBs}{\sinh(eBs)} - 1 \right]. \]  

(19)

Therefore, the condensate under an external magnetic field is, using the Gell-Mann-Oakes-Renner relation

\[ F_\pi^2 M_\pi^2 = \langle \bar{q}q \rangle (m_u + m_d), \]

\[ \langle \bar{q}q \rangle^B = \langle \bar{q}q \rangle^B=0 \left( 1 + \frac{|eB| \ln 2}{16\pi^2 F_\pi^2} + \cdots \right), \]  

(20)

where the ellipses denote the higher loop corrections [14]. Since the axion-nucleon coupling is proportional to the nucleon mass (\( g_{an} \propto m_n/f_{PQ} \)) and the axion decay constant does not change under magnetic fields,

\[ g_{an}(B) = g_{an}(0) \left( 1 + \frac{|eB| \ln 2}{48\pi^2 F_\pi^2} + \cdots \right). \]  

(21)

The energy-loss rate per unit volume due to the nucleon-nucleon axion bremsstrahlung is calculated in Ref. [15,20,7] and can be written as

\[ Q^1_a \propto \left( \frac{f}{M_\pi} \right)^4 m_n^{2.5} g_{an}^2 T^{6.5}, \]  

(22)

where \( f \) is the pion-nucleon coupling and \( T \) is the temperature. Therefore, we find the correction due to the strong magnetic field is

\[ Q^1_a(B) = Q^1_a(0) \left( 1 + \frac{g_{an}^s}{2\pi^2} + \cdots \right)^2 \left( 1 + \frac{|eB| \ln 2}{16\pi^2 F_\pi^2} + \cdots \right)^{2.5/3}, \]  

(23)

where we have used the Goldberg-Treiman relation \( m_n = f F_\pi \) together with the Gell-Mann-Oakes-Renner relation. Similarly, the lowest-order energy emission rate per unit volume by the pion-axion conversion \( \pi^- + p \rightarrow n + a [1] \) has corrections due to the strong magnetic field as

\[ Q^\pi_a(B) = Q^\pi_a(0) \left( 1 + \frac{g_{an}^s}{2\pi^2} + \cdots \right)^2 \left( 1 + \frac{|eB| \ln 2}{16\pi^2 F_\pi^2} + \cdots \right)^{-1/3}, \]  

(24)
Note that the above derivation of the change in the quark condensate relies on the chiral perturbation theory and the one loop vacuum energy calculation, which will not be valid presumably for an extremely strong magnetic field such that the Landau gap becomes bigger than the \( \rho \) meson mass \( \sqrt{|eB|} > m_\rho \). But, the condensate will still increase as \( B \) even for such a strong magnetic field because the increment of vacuum energy due to the external field will be balanced by forming larger condensates.

The correction to the emission rate due to the external magnetic field comes from two sources. First one is from the finite correction to the axial current due to the effective four-quark operator and the second one is by the change of the quark condensate under the external field. For \( B = 10^{18} \sim 10^{19} \text{ G} \), the correction by the second one is less than a few percent while the correction by the first one is of order one. For \( B = 10^{20} \text{ G} \) or \( \sqrt{|eB|} = 0.77 \text{ GeV} \), the correction by the second is about 24\% for the Bremsstrahlung and about \(-10\%\) for the pion-axion conversion, while the correction due to the first one is about 0.06\( \alpha_s \) for both processes, because the four-quark operator will evolve sufficiently. For \( B \gtrsim 10^{20} \text{ G} \) the perturbative calculation breaks down and we expect that the correction will be extremely large because the strong enhancement of \( g_s^4 \) at low energy. Therefore, if \( B \gtrsim 10^{20} \text{ G} \) in stars, they will emit axions too quickly and will not survive present. On the other hand, if we observe such a strongly magnetized star, the axion-nucleon coupling has to be smaller than the current upper bound by several orders of magnitude.

In conclusion, we have shown that the axion decay constant does not get a correction by the external magnetic field and we have calculated the correction to the axion-nucleon coupling due to external magnetic fields. We find that if \( B \gtrsim 10^{20} \text{ G} \) in strongly magnetized neutron and white dwarf stars, the axion emission rate is enhanced by several orders of magnitude.

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**APPENDIX A: QED VACUUM ENERGY**

The low energy effective action of QED, integrating out fermions and scalars, has been calculated for various external fields [22]. Here, we calculate the QED vacuum energy, integrating out the axion fields:

\[
e^{-\int_x \mathcal{E}_{\text{vac}}(E,B)} = \int d[a] \exp \left[ -\int_x \left( \frac{1}{4} F^2 - \frac{1}{2} (\partial \mu a)^2 + V_0 \cos \left( \frac{a}{f_{PQ}} \right) + \frac{ie^2}{16\pi^2} \frac{a F \tilde{F}}{f_{PQ}} \right) \right], \tag{A1}
\]

where we neglect other matter fields for simplicity but it does not change our argument. The vacuum energy density is then

\[
\rho_{\text{vac}} = \frac{1}{2} \int \mathcal{E}_{\text{vac}}(E,B) \cdot \mathrm{d}^{4}x.
\]
\[ \mathcal{E}_{\text{vac}}(E, B) = \frac{1}{2}(E^2 + B^2) + \mathcal{E}_{\text{vac}}^a(E, B), \]  

where the second term is due to axions and obviously a function of \( F \bar{F} \) or \( \vec{E} \cdot \vec{B} \). If \( \vec{E} \cdot \vec{B} = 0 \), the vacuum energy does not change and neither does the axion decay constant. But, if \( \vec{E} \cdot \vec{B} \neq 0 \), the vacuum energy increases by the axion loop contribution because the axion-photon coupling in Euclidean space is imaginary. Using Schwarz inequality, one can easily show that \( \mathcal{E}_{\text{vac}}^a(E, B) \geq 0 \). Thus under the external fields the condensate has to increase to balance the vacuum energy and the axion decay constant gets bigger. For general electromagnetic fields, we get \( f_{PQ}(E, B) = f_{PQ}(\vec{E} \cdot \vec{B}) \geq f_{PQ}(0) \).

If we keep only the axion mass term in the potential in Eq. (A1) and integrate over \( a \), we get for slowly varying fields

\[ \mathcal{E}_{\text{vac}}^a(E, B) \simeq 8 \left( \frac{e^2}{16\pi^2} \right)^2 \left( \frac{\vec{E} \cdot \vec{B}}{\Lambda_{QCD}^2} \right)^2, \]  

where we used the axion mass \( m_a = \Lambda_{QCD}^2/f_{PQ} \).

**APPENDIX B: QCD UNDER A CONSTANT EXTERNAL MAGNETIC FIELD**

Since quarks have electric charges, magnetic catalysis will operate for quarks as well when the external magnetic field is sufficiently strong, \( B > 10^{19} \) \( G \) or \( \sqrt{|qB|} > \Lambda_{QCD} \simeq 200 \text{MeV} \). By the renormalization group analysis, we will see that if the magnetic field is stronger than \( 10^{20} \) \( G \), the four-quark interaction is stronger than the one-gluon exchange interaction and thus the chiral symmetry will break at scale higher than \( \Lambda_{QCD} \). The derivation of low energy effective Lagrangian will be same as that of QED except the group theoretical factors. At low (but quite higher than \( \Lambda_{QCD} \)) energy the relevant degrees of freedom are LLL quarks, gluons, and photons. But, since the gluons couple to quarks more strongly than photons, we will neglect the photon interactions. As in QED, the exchange of quarks at the higher Landau level will generate a tree-level interaction of quarks and gluons, which will induce a four-quark interaction at one-loop matching (see Fig. 4);

\[ \mathcal{L}_{\text{eff}} \ni \frac{g_s^4}{4|qB|} \left[ (\bar{Q}_0 Q_0)^2 + (\bar{Q}_0 i\gamma_5 Q_0)^2 \right], \]  

where \( Q_0 \) is the LLL quark and \( q \) is the electric charge of quark. The one-loop RG equation for the four-quark interaction is given as

\[ \frac{d}{d\mu} g_s^4 = -\frac{40}{9} \alpha_s^2 (\ln 2)^2, \]  

where \( \alpha_s \) is the coupling constant of strong interaction. Unlike QED, the strong coupling constant runs at scale below the Landau gap \( \Lambda_L = \sqrt{|qB|} \) as \[21\]

\[ \frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\Lambda_L)} + \frac{11}{2\pi} \ln \frac{\mu}{\Lambda_L}. \]
Note that quarks do not contribute to the running coupling since the quark loop is finite due to the dimensional reduction. Combining the above RG equations (B2) and (B3), we find how the four-quark coupling changes as scale:

\[ g_1^s(\mu) = 1.1424 (\alpha_s(\mu) - \alpha_s(\Lambda_L)) + g_1^s(\Lambda_L), \]  

(B4)

where \( g_1^s(\Lambda_L) \) is of the order of \( \alpha_s(\Lambda_L)^2 \). We see that the four-quark coupling becomes stronger than the strong coupling constant at a scale larger than the QCD scale \( \mu > \Lambda_{\text{QCD}} \) if \( \Lambda_L \gtrsim 4\Lambda_{\text{QCD}} \) or \( B \gtrsim 10^{20} \, G \). For such a strong magnetic field, the chiral symmetry of QCD will be broken at scale larger than \( \Lambda_{\text{QCD}} \), the usual chiral symmetry breaking scale of QCD.
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FIG. 1. Axion decay amplitude. Wiggly lines denote photons, solid lines the LLL fermions, and broken lines axions.

FIG. 2. Axion conversion into a photon. Solid line denote electrons.

FIG. 3. Axion Bremsstrahlung emission. Solid lines denote electrons or quarks.
FIG. 4. Matching conditions. Double lines denote quarks at the higher Landau levels \((n > 0)\), single line LLL quarks, and curled lines gluons. (a) tree level, (b) one loop level