A Skewness Fitting Model for Noise Level Estimation and the Applications in Image Denoising

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Abstract. In this paper, a novel noise level estimation technique is presented based on skewness fitting. The difference between exact skewness and observed value on each channel contributes to an objective function. The skewness concentration is helpful to guarantee the reasonability of the model. The optimal solution means an estimation of the noise variance and it is easy to be applied in some parametric denoising model. The accuracy and efficiency of proposed algorithm were verified by the numerical experiments.

1. Introduction

Image processing and computer vision is one of the most important and practical areas in past years [1-4]. Amounts of interesting and serviceable applications based on these have been developed in industry and engineering, such as face recognition, smart traffic, video retrieval and virtual realization. However, images are often polluted by different type noise (additive, multiplicative, mixture). It is important to erase or reduce the noise before image been used in those applications.

Many methods have been presented for different scenes or requirements, such as spatial-based filtering [5], transform-based filtering [6], statistical models [7-8], PDE models [2,3], nonlocal models [9] and hybrid models. Filtering is the most fundamental one among those and plays important role in practice. Most of them can work well with proper parameters setting. These can be modelled as optimization problems which should be solved by efficient fast algorithms [10].

The spatial-based filtering (such as Gaussian filtering) computes a weighted average of pixel values in the neighbourhood and the weights is determined by the distances to the centre of neighbourhood [1]. Noise can be averaged away while signal preserved when the image varies slowly over space. However, it is just reversed at edges and blurring can't be avoided. Anisotropic diffusion [3,11] can be adopted to prevent the undesired averaging across edges. Many anisotropic diffusions can be implemented by solve partial differential equations. For better and variety diffusions, tensors analysis and other technologies can be introduced to the PDE-based models [11,12]. As the correlation of image features between different regions taken into account, some nonlocal models can be obtained, such as nonlocal mean filter [13], block matching three dimensional filter (BM3D) [14].
The transform-based filtering means the image will be transformed from spatial domain to another domain such as frequency domain first. And then the part transformed from noise will be erased (or reduced) by some threshold methods or components analysis. It should be addressed that the features and noise are often mixed after the transform. And it is not simple to separate the noise part from the transform data. Many transform can be applied to construct filters for image denoising.

Some statistics technologies (such as principal components analysis (PCA), Bayesian estimation, cluster analysis, Singular value decompose (SVD), etc) can be applied to different denoising models. Those models work well as the noise is different from the image features in the statistical sense.

As an interesting technology, kurtosis has been introduced to some novel optimization models to estimate the noise variation in some polluted images [15,16]. It is reported that the kurtosis of marginal bandpass filter response distributions to be constant throughout scales, and the kurtosis values are lower for high frequency filters than for lower frequency ones. The local noise variances can be estimated based on the kurtosis values concentration in band-pass filtered domains, then the noise standard deviation in corrupted natural images can be proper estimated.

As another important statistic, skewness can also reveal the important distribution characteristic of the pixel values. Inspired by above works, we try to develop a novel model based on skewness fitting for noise variation estimation and apply it to image denoising. This paper is organized as follows. In Section 2, the related fundamentals of image denoising and the kurtosis model are prepared. In section 3, we give detail about the proposed method. Section 4 shows the experimental results and the conclusion is given in Section 5.

2. Noise Level Estimation and a Kurtosis-based model

In general, the observed model of a gray image data \( x \) can be described as

\[
y = Hx + n
\]

where \( H \) means an operator, \( n \) denotes the noise distribution. The image restoration is to achieve the recovery of unknown \( x \), and it will be classical image denoising task if \( H \) is set to be an identity operator. In many scenes, there is only limited information about \( H \) and \( n \), so it is difficult to solve such an ill-posed problem even if at the situation of image denoising.

The traditional solution for this obstacle is to add some proper assumption about \( H \) and \( n \), or adopt some regularization techniques. Even if this can help to acutely decrease the difficulty of the problem, some necessary parameters are still required to be set properly. Traditionally, the noise is assumed to yield Gaussian distribution with zero mean and variance \( \sigma^2 \). Lots of techniques have been developed to solve the problem in this situation. However, it is usually that the noise variation cannot be known in prior, so an accurate estimation is necessary to support the later efficient solving.

2.1. Noise Variance Estimation

In some traditional methods, with the application of some mathematical transformation or components analysis, the main part (high density) of the noisy image will be separated and the remainder part (low density) can be used to achieve the final estimation.

In [5], a filter is introduced to compute a low density part (noise estimation) and it can be denoted as

\[
\hat{n} = y \ast \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}
\]  

(2)

Then the noise variance can be estimated and well applied in some other denoising algorithms [19]. Principal components analysis has been widely applied in data mining and image processing [20-22]. [21] propose a new noise level estimation method based on principal component analysis of
image blocks and the noise variance can be estimated as the smallest eigenvalue of the image block covariance matrix if the noise-free image is redundant.

\[ E\left( \hat{\lambda}_{Y,M} - \hat{\lambda}_{Y,M-1} \right) = O\left( \sigma^2 / \sqrt{n} \right) \]  

[22] found that the noise level can be estimated from image patches using principal component analysis (PCA) if the image comprises only weak textured patches.

\[ \sigma_n^2 = \lambda_{\min}\left( \Sigma_y \right) \]

[20] applied the pca-based noise level estimation in image splicing detection.

2.2. A Kurtosis-based model
For a random variable \( x \), the kurtosis can be defined as

\[ \kappa = \frac{\mu_4}{\left( \sigma^2 \right)^2} - 3 \]  

where \( \mu_4 = E\left( \left[ x - E(x) \right]^4 \right) \) and \( \sigma^2 = E\left( \left[ x - E(x) \right]^2 \right) \). It can be easily get a Gaussian variable has kurtosis zero.

It has been reported in some works that for natural images, kurtosis values across different band-pass filter channels tend to be close to a constant, the phenomenon called kurtosis concentration [16,18]. The kurtosis concentration of two classical natural images is demonstrated in Figure 1.

![Figure 1. Kurtosis concentration of Lena and Peppers.](image)

And then an optimization model of minimization the difference of observed kurtosis and exact kurtosis on each channel can be denoted as

\[ \min_{\kappa, \sigma^2} \sum_{i=1}^{n} \sqrt{\kappa_i - \sqrt{\kappa \left( \frac{\sigma^2 - \sigma_i^2}{\sigma^2} \right)^2}} \]

whose optimal solution gives an estimation to the noise variance.

3. The Proposed Method
It is known that there are some statistics can be applied to describe the characteristic of a random variable. Kurtosis and skewness are both important statistics which give the feature of the random variable in different views. Inspired by the works above mentioned, this paper presents a novel optimization model to solve the noise variance estimation based on skewness fitting.

For a random variable \( x \), the skewness can be denoted as

\[ S = \frac{\mu_3}{\sigma^3} = E\left( \frac{X - \mu}{\sigma} \right)^3 \]

Similar to the kurtosis of a natural image, it can be also found that the skewness concentration on different channels.
Based on the simplified version (H is assumed to be an identity operator) of observed model (1), we can get the relation between observed skewness and exact skewness on each channel as follows.

\[ S(y)^2 = S(x)^2 \left( \frac{\sigma^2(y) - \sigma^2_x}{\sigma^2(y)} \right)^3 \]  

(7)

Then the minimization of the difference between two sides of above equation can be formulated as

\[ \min_{\hat{\sigma}^2, \sigma^2} \sum_{k=1}^{K} \left( \hat{S}_k^2 - S^2 \left( \frac{\sigma_k^2 - \sigma^2_x}{\sigma_k^2} \right)^3 \right)^2 \]

(8)

whose optimal solution gives an estimation to the noise variance. With some efficient numerical solving techniques applied, we can achieve the final solution and apply it in some image denoising model as shown in next section.

4. Experiments and Results
To verify the accuracy and efficiency of proposed model, we prepare several classical natural images with different degree noise. Then the noise level estimation results achieved by (8) are compared with some other traditional methods. Finally, the estimated noise variance is applied in a PDE-based image denoising model.

Figure 3 shows the prepared test images. In our first experiment, we will add different Gaussian noise (\( \sigma = 10, 15, 20, 25 \)) to each other and take them for the estimation tests.

Table 1. Noise variance estimation of test images

| Noisy image | Filter in [5] | Kurtosis fitting in [17] | Kurtosis fitting in [18] | Skewness fitting |
|-------------|---------------|--------------------------|--------------------------|------------------|
| Clock \( (\sigma=10) \) | 10.70 | 9.72 | 9.70 | 10.10 |
| Lena \( (\sigma=15) \) | 16.00 | 13.74 | 14.41 | 15.46 |
| Palace \( (\sigma=20) \) | 20.15 | 18.80 | 19.26 | 19.83 |
| Peppers \( (\sigma=25) \) | 25.84 | 23.64 | 24.30 | 24.92 |
After applying the gradient descend algorithm which is implemented on the computer (CPU i8550) with Windows 10 and Matlab R2017b, the final estimation results are given as Table 1 and Figure 4. It can be found better estimation values can be achieved by our proposed method in the experiments. And the accurate estimation will make a good role in some denoising models. Table 2 shows the final PSNR after Perona-Malik model [23] applied to some noisy images.

![Figure 4. Test images for noise estimation](image)

Table 2. Denoising results (Final PSNR) of polluted images

| image  | $\sigma=10$ | $\sigma=15$ | $\sigma=20$ | $\sigma=25$ |
|--------|------------|------------|------------|------------|
| Clock  | 32.96      | 31.15      | 29.92      | 29.06      |
| Lena   | 32.04      | 30.20      | 28.96      | 28.10      |
| Palace | 34.02      | 32.11      | 30.79      | 29.86      |
| Peppers| 31.56      | 30.06      | 29.09      | 28.27      |

5. Conclusions
In this paper, a novel noise level estimation technique is presented based on skewness fitting. The difference between exact skewness and observed value on each channel contributes to an objective function. The skewness concentration is help to guarantee the reasonability of the model. The optimal solution means an estimation of the noise variance and it is easy to be applied in some parametric denoising model. The accuracy and efficiency of proposed algorithm were verified by the numerical experiments.

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