Angle-Independent Nongyrotropic Metasurfaces

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Abstract – We derive a general condition for angle-independent bianisotropic nongyrotropic metasurfaces and present two applications corresponding to particular cases: an angle-independent absorber/amplifier and an angle-independent spatial gyrator.

I. INTRODUCTION

Metasurfaces are generally designed for a given specification, namely the incident, reflected and transmitted fields with specific angles, frequencies and polarizations [1, 2, 3]. Specifically, general bianisotropic metasurfaces have been shown to have intricate and diverse angular scattering properties [4]. It would be particularly desirable for many applications to devise metasurfaces that would exhibit the same response for different excitations.

Here, we theoretically derive bianisotropic nongyrotropic metasurfaces whose response is angle-independent. Based on this result, we study two potential applications of angle-independent bianisotropic metasurfaces: an absorber/amplifier and a spatial gyrator.

II. THEORETICAL DERIVATION

We consider the problem of an uniform bianisotropic metasurface surrounded by air placed in the \(xy\) plane \(z = 0\). Such a metasurface can be efficiently modelled as a zero-thickness discontinuity in space using generalized sheet transition conditions (GSTCs) and surface susceptibility tensors [5]. The tangential susceptibility-based GSTCs model of such a metasurface is

\[
\hat{z} \times \Delta H = j \omega \chi_{ee} \overline{E}_{av} + j k \chi_{em} H_{av},
\]

\[
\Delta E \times \hat{z} = j k \chi_{me} \overline{E}_{av} + j \omega \mu \chi_{mm} H_{av},
\]

where the symbol \(\Delta\) and the subscript ‘av’ represent the differences and averages of the tangential electric or magnetic fields at both sides of the metasurface, and \(\chi_{ee}, \chi_{mm}, \chi_{em}, \chi_{me}\) are the bianisotropic susceptibility tensors characterizing the metasurface.

We seek here a general condition for metasurface angle-independent transformation, with the only restriction of nongyrotropy to avoid excessive complexity at this point. For the sake of conciseness, we limit the investigation to the \(s\) polarization, and a similar treatment naturally applies to the \(p\) polarization. Under such conditions, Eqs. (1) reduce to

\[
\Delta H_x = j \omega \chi_{ee}^{sy} E_{y,av} + j k \chi_{en}^{xy} H_{x,av}, \quad (2a)
\]

\[
\Delta E_y = j k \chi_{lm}^{xy} E_{y,av} + j \omega \mu \chi_{mm}^{xy} H_{x,av}, \quad (2b)
\]

where the field differences and averages are related to the forward-incidence transmission and reflection coefficients \((T_1\) and \(R_1\)) as

\[
\Delta H_x = (T_1 - (1 + R_1)) H_{x,i}, \quad H_{x,av} = \frac{1}{2}(T_1 + R_1 + 1) H_{x,i}, \quad (3a)
\]

\[
\Delta E_y = (T_1 - (1 - R_1)) E_{y,i}, \quad E_{y,av} = \frac{1}{2}(T_1 + 1 - R_1) E_{y,i}, \quad (3b)
\]
The ratio of the tangential electric and magnetic fields in (3) as a function of the incidence angle \( \theta \) can be written as

\[
\frac{E_{y,1}}{H_{x,1}} = \frac{\eta_0}{\cos \theta},
\]

where \( \eta_0 \) is the wave impedance in free space. Inserting (3) into (2), respectively dividing the resulting equations by \( H_{x,1} \) and by \( E_{y,1} \) and substituting (4) yields the following expression for the forward transmission and reflection coefficients in terms of the susceptibilities:

\[
T_1 = \frac{(2 + jk\chi_{em}^{yy})(-2j + k\chi_{me}^{yy}) - j\epsilon\mu_0^2\chi_{ee}^{yy}\chi_{mm}^{xx}}{j\frac{\eta_0}{\cos \theta}2\epsilon\chi_{ee}^{yy}\omega + 2\mu\chi_{mm}^{xx}\cos \theta} - j(4 + 2k^2\chi_{em}^{yy}\chi_{me}^{xx} - \epsilon\mu_0^2\chi_{ee}^{yy}\chi_{mm}^{xx})^2,
\]

\[
R_1 = \frac{2(k(\chi_{ee}^{yy} - \chi_{em}^{yy}) + j\epsilon\mu_0\chi_{ee}^{yy}\cos \theta - \mu\chi_{mm}^{xx}\cos \theta)}{j\frac{\eta_0}{\cos \theta}2\epsilon\chi_{ee}^{yy}\omega + 2\mu\chi_{mm}^{xx}\cos \theta} - j(4 + 2k^2\chi_{em}^{yy}\chi_{me}^{xx} - \epsilon\mu_0^2\chi_{ee}^{yy}\chi_{mm}^{xx})^2.
\]

Following the same procedure for the backward case yields

\[
T_2 = \frac{(2 - jk\chi_{em}^{yy})(2j + k\chi_{me}^{yy}) - j\epsilon\mu_0^2\chi_{ee}^{yy}\chi_{mm}^{xx}}{j\frac{\eta_0}{\cos \theta}2\epsilon\chi_{ee}^{yy}\omega + 2\mu\chi_{mm}^{xx}\cos \theta} - j(4 + 2k^2\chi_{em}^{yy}\chi_{me}^{xx} - \epsilon\mu_0^2\chi_{ee}^{yy}\chi_{mm}^{xx})^2,
\]

\[
R_2 = \frac{2(k(\chi_{ee}^{yy} - \chi_{em}^{yy}) - j\epsilon\mu_0\chi_{ee}^{yy}\cos \theta - \mu\chi_{mm}^{xx}\cos \theta)}{j\frac{\eta_0}{\cos \theta}2\epsilon\chi_{ee}^{yy}\omega + 2\mu\chi_{mm}^{xx}\cos \theta} - j(4 + 2k^2\chi_{em}^{yy}\chi_{me}^{xx} - \epsilon\mu_0^2\chi_{ee}^{yy}\chi_{mm}^{xx})^2.
\]

Inspecting (5) and (6) reveals that a metasurface transformation generally has an angle dependence that is associated with the \( \cos \theta \) and \( \frac{1}{\cos \theta} \) coefficients in the above expressions of the reflection and transmission coefficients. However, these coefficients only affect the \( \chi_{ee}^{yy} \) and \( \chi_{mm}^{xx} \) susceptibilities. Therefore, imposing the restriction \( \chi_{ee}^{yy} = \chi_{mm}^{xx} = 0 \) removes the angular dependence. Practically, even if the \( \chi_{ee}^{yy} \) and \( \chi_{mm}^{xx} \) susceptibilities are not exactly zero but still negligible compared to the \( \chi_{em}^{yy} \) and \( \chi_{me}^{xx} \) susceptibilities, the response of the metasurface will be essentially independent of the angle, except towards grazing angles \( \frac{1}{\cos \theta} \to \infty \). Inserting this condition into (5) and (6) yields the angle-independent scattering coefficients

\[
T_1 = \frac{(2j - k\chi_{em}^{yy})(2j + k\chi_{me}^{yy})}{4 + k^2\chi_{em}^{yy}\chi_{me}^{xx}}, \quad R_1 = \frac{2jk(\chi_{em}^{yy} - \chi_{me}^{yy})}{4 + 2k^2\chi_{em}^{yy}\chi_{me}^{xx}},
\]

\[
T_2 = \frac{(2j - k\chi_{em}^{yy})(2j + k\chi_{me}^{yy})}{4 + k^2\chi_{em}^{yy}\chi_{me}^{xx}}, \quad R_2 = \frac{2jk(\chi_{em}^{yy} - \chi_{me}^{yy})}{4 + 2k^2\chi_{em}^{yy}\chi_{me}^{xx}}.
\]

### III. Angle-Independent Absorber/Ampplier

An extremely useful application of such an angle-independent metasurface would be that of an angle-independent absorber\(^1\).

Inspecting (7) shows that the reflection at both sides of the metasurface is suppressed by imposing \( \chi_{em}^{yy} = \chi_{me}^{xx} \) since this leads to \( R_1 = R_2 = 0 \). Note that such a condition implies nonreciprocity since as it violates the reciprocity condition \( \chi_{em}^{yy} = -\chi_{me}^{xx} \), which reveals that nonreciprocity is a fundamental condition for metasurface angle-independent absorption. The corresponding forward and backward transmission coefficients reduce then to

\[
T_1 = \frac{-k^2(\chi_{em}^{yy})^2 + 4jk\chi_{em}^{yy} + 4}{4 + k^2(\chi_{em}^{yy})^2},
\]

\[
T_2 = \frac{-k^2(\chi_{em}^{yy})^2 - 4jk\chi_{em}^{yy} + 4}{4 + k^2(\chi_{em}^{yy})^2}.
\]

Equations (8) are analysis equations giving the transmission coefficients through a metasurface of susceptibility \( \chi_{em}^{yy} \). For design, we need an inverse, synthesis equation. Such an equations is obtained by solving (8a) for a specific \( T_1, T_{1,\text{spec}} \), which yields

\(^1\)A bianisotropic metasurface was demonstrated as thin-absorber was theoretical and experimentally demonstrated in [6] and [7], respectively. However, this metasurface absorber is not angle-independent.
\[
\chi_{xy}^{me} = \chi_{yx}^{em} = \frac{2i}{k} \frac{1 - T_1}{1 + T_1}\tag{9}
\]

whose insertion in (8b) yields

\[
T_2 = \frac{1}{T_1},\tag{10}
\]

Equation (10) reveals that specifying \( T_1 \) via (9) results in having \( T_2 \) being the opposite of this \( T_1 \). This physically means that an angle-independent absorption corresponding to \( T_{1,\text{spec}} \) in the forward excitation implies a gain of the same level in the opposite direction. Therefore, the overall metasurface needs to be active. Such a metasurface may be realized by integrating transistors in the metasurface [6]. The topic of transistor-based metasurface absorber was previously discussed in [7].

IV. ANGLE-INDEPENDENT SPATIAL GYRATOR

Another interesting application would be an angle-independent spatial gyrator. Given the fundamental nature of the gyrator as nonreciprocal component [8], one may easily anticipate that an angle-independent spatial could enable many unique devices. Such a gyrator may be realized by specifying zero reflection and

\[
T_1 = e^{i\pi/2},
\]

so that

\[
T_2 = e^{-i\pi/2}
\]

according to (10), which indeed corresponds to a \( \pi \) phase difference between the forward and backward transmissions. The corresponding susceptibilities are found from (9) as

\[
\chi_{xy}^{me} = \chi_{yx}^{em} = \frac{2}{k}.	ag{11}
\]

V. CONCLUSION

We have derived a general condition for an angle-independent nongyrotropic metasurface, and theoretically derived two fundamental applications: an angle-independent absorber/amplifier and an angle-independent spatial gyrator.

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