Loss of superfluidity in a Bose-Einstein condensate on an optical lattice via a novel classical phase transition

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Abstract

We predict the loss of superfluidity in a Bose-Einstein condensate (BEC) trapped in a combined optical and axially-symmetric harmonic potentials during a resonant collective excitation initiated by a periodic modulation of the atomic scattering length $a$, when the modulation frequency equals twice the radial trapping frequency or multiples thereof. This classical dynamical transition is marked by a loss of superfluidity in the BEC and a subsequent destruction of the interference pattern upon free expansion. Suggestion for future experiment is made.

Key words: Bose-Einstein condensation, Superfluid-insulator transition

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The detailed study of quantum phase effects on a macroscopic scale such as interference of matter waves [1] has been possible after the experimental loading of a cigar-shaped Bose-Einstein condensate (BEC) in a combined axially-symmetric harmonic plus a standing-wave optical lattice potential traps in both one [2,3] and three [4] dimensions. There have been several theoretical studies on different aspects of a BEC in a one- [5,6] as well as three-dimensional [7] optical-lattice potentials. The phase coherence among different optical-lattice sites of a trapped BEC on an optical lattice has been established in recent experiments [2,3,4,8,9] through the formation of distinct interference pattern when the traps are removed. This long-range phase coherence in the condensate along the entire optical lattice is a sign of communication among various sites which is necessary for developing superfluidity in the condensate.

The phase-coherent BEC on the optical lattice is a superfluid [4,10] as the atoms in it move freely from one optical site to another by quantum tunneling through the high optical potential barriers. It has been demonstrated in
a recent experiment by Greiner et al. [4] that, as the optical potential traps are made much too higher, the quantum tunneling of atoms from one optical site to another is stopped resulting in a loss of superfluidity in the BEC. Consequently, no interference pattern is formed upon free expansion of such a BEC which is termed a Mott insulator state [4,10], in which an individual atom is attached to a fixed optical site and its free mobility to a nearby site by tunneling is stopped as in an insulator. This phenomenon represents a superfluid-insulator quantum phase transition and cannot be properly accounted for in a mean-field model based on the Gross-Pitaevskii (GP) equation [11] where these quantum effects are mostly lost.

Following a suggestion by Smerzi et al. [12], Cataliotti et al. [13,14] have demonstrated in a novel experiment the loss of superfluidity in a BEC trapped in a one-dimensional optical-lattice and harmonic potentials when the center of the harmonic potential is suddenly displaced along the optical lattice through a distance larger than a critical value. Then a modulational instability takes place in the BEC. Consequently, it cannot reorganize itself quickly enough and the phase coherence and superfluidity of the BEC are lost. The loss of superfluidity is manifested in the destruction of the interference pattern upon free expansion. Distinct from the quantum phase transition observed by Greiner et al. [4], this modulational instability responsible for the superfluid-insulator transition is classical dynamical in nature [12,13,15]. This process is also different from the Landau dissipation mechanism [12,15], occurring when the fluid velocity is greater than local speed of sound. The present dynamical phase transition can be well described by the mean-field GP equation [6,12,15]. When Landau instability occurs, the system lowers energy by emitting phonons [15]. The GP equation can not simulate energy dissipation and hence the Landau dissipation mechanism.

The above modulational instability is not the unique dynamical classical process leading to a superfluid-insulator transition. Many other classical processes leading to a rapid movement or collective excitation in the condensate may lead to such a transition. The movement should be rapid enough so that the BEC cannot reorganize itself to evolve through orderly phase coherent states. Here we suggest that a collective resonant excitation of the BEC may also lead to the destruction of superfluidity due to a classical superfluid-insulator transition. There have been theoretical [16] and experimental [17] studies of collective excitation in the BEC in the absence of an optical trap initiated by a modulation of the trapping frequency. The study of such collective excitation in the presence of an optical trap has just began [18].

In the present study the collective excitation is initiated near a Feshbach resonance [19] by a periodic modulation of the repulsive atomic scattering length $a$ ($> 0$) via $a \rightarrow a + \tilde{A} \sin(\Omega t)$ where $t$ is time, $\tilde{A}$ an amplitude, and $\Omega$ the frequency of modulation. Such modulation of the scattering length can be re-
alized experimentally near a Feshbach resonance by manipulating an external background magnetic field. Although, the background magnetic field and the scattering length are nonlinearly related in general, for small modulations $A(<a)$ an approximate linear relation between the background magnetic field and the scattering length may hold which might make the implementation of the above modulation experimentally possible. When $\Omega = 2\omega$ or multiples thereof, resonant collective oscillation can be generated in the BEC, where $\omega$ is the radial trapping frequency [20]. This resonant oscillation destroys the superfluidity of the BEC provided that the condensate is allowed to experience this oscillation for a certain interval of time called hold time. We base the present study on the numerical solution of the time-dependent mean-field axially-symmetric GP equation [11] in the presence of a combined harmonic and optical potential traps. This transition involving collective excitation of the BEC and described by the mean-field GP equation is classical, rather than quantum, in nature.

The time-dependent BEC wave function $\Psi(\mathbf{r}; \tau)$ at position $\mathbf{r}$ and time $\tau$ is described by the following mean-field nonlinear GP equation [11]

$$\left[-i\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2 \nabla^2}{2m} + V(\mathbf{r}) + gN|\Psi(\mathbf{r}; \tau)|^2\right] \Psi(\mathbf{r}; \tau) = 0,$$  \hspace{1em} (1)

where $m$ is the mass and $N$ the number of atoms in the condensate, $g = 4\pi\hbar^2a/m$ the strength of interatomic interaction. In the presence of the combined axially-symmetric and optical lattice traps $V(\mathbf{r}) = \frac{1}{2}m\omega^2(r^2 + \nu^2z^2) + V_{\text{opt}}$ where $\omega$ is the angular frequency of the harmonic trap in the radial direction $r$, $\nu\omega$ that in the axial direction $z$, with $\nu$ the aspect ratio, and $V_{\text{opt}}$ is the optical lattice trap introduced later. The normalization condition is $\int d\mathbf{r} |\Psi(\mathbf{r}; \tau)|^2 = 1$.

In the axially-symmetric configuration, the wave function can be written as $\Psi(\mathbf{r}, \tau) = \psi(r, z, \tau)$. Now transforming to dimensionless variables $x = \sqrt{2}r/l$, $y = \sqrt{2}z/l$, $t = \tau\omega$, $l \equiv \sqrt{\hbar/(m\omega)}$, and $\varphi(x, y; t) \equiv x\sqrt{l^3/\sqrt{8}}\psi(r, z; \tau)$, Eq. (1) becomes [21]

$$\left[-i\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} - \frac{\partial^2}{\partial y^2} + \frac{1}{4} (x^2 + \nu^2 y^2)\right] + \frac{V_{\text{opt}}}{\hbar\omega} - \frac{1}{x^2} + 8\sqrt{2\pi n} \left|\varphi(x, y; t)/x\right|^2 \varphi(x, y; t) = 0,$$ \hspace{1em} (2)

where nonlinearity $n = Na/l$. In terms of the one-dimensional probability $P(y, t) \equiv 2\pi \int_0^\infty dx |\varphi(x, y; t)|^2/x$, the normalization is given by $\int_{-\infty}^{\infty} dy P(y, t) = 1$. 

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We use the parameters of the experiment of Cataliotti et al. [8] with repulsive $^{87}\text{Rb}$ atoms where the radial trap frequency was $\omega = 2\pi \times 92$ Hz. The optical potential created with the standing-wave laser field of wavelength $\lambda = 795$ nm is given by $V_{\text{opt}} = V_0 E_R \cos^2(k_L z)$, with $E_R = \hbar^2 k_L^2 / (2m)$, $k_L = 2\pi / \lambda$, and $V_0$ ($< 12$) the strength. For the mass $m = 1.441 \times 10^{-25}$ kg of $^{87}\text{Rb}$ the harmonic oscillator length $l = \sqrt{\hbar / (m \omega)} = 1.126 \, \mu\text{m}$ and the dimensionless time unit $\omega^{-1} = 1/(2\pi \times 92) \, \text{s} = 1.73 \, \text{ms}$. In terms of the dimensionless laser wave length $\lambda_0 = \sqrt{2\lambda / l} \simeq 1$ and the dimensionless energy $E_R / (\hbar \omega) = 4\pi^2 / \lambda_0^2$, $V_{\text{opt}}$ of Eq. (2) is $V_{\text{opt}} / (\hbar \omega) = V_0 (4\pi^2 / \lambda_0^2) \cos^2(2\pi y / \lambda_0)$.

We solve Eq. (2) numerically using a split-step time-iteration method with the Crank-Nicholson discretization scheme described recently [21]. The time iteration is started with the known harmonic oscillator solution of Eq. (2) with $n = 0$: $\varphi(x, y) = [\nu / (8\pi^3)]^{1/4} x e^{-\left(x^2 + \nu y^2\right)/4}$ [21]. The nonlinearity $n$ as well as the optical lattice strength $V_0$ are slowly increased by equal amounts in 10000$n$ steps of time iteration until the desired nonlinearity and optical lattice potential are attained. Then, without changing any parameters, the solution so obtained is iterated 50 000 times so that a stable solution is obtained independent of the initial input and time and space steps.

The one-dimensional pattern of BEC on the optical lattice for a specific nonlinearity and the interference pattern upon its free expansion have been recently studied using the numerical solution of Eq. (2) [6]. Here we study the destruction of this interference pattern after the application of a periodic modulation of the scattering length resulting in a similar modulation of nonlinearity $n$ in Eq. (2) via

$$n \rightarrow n + A \sin(\Omega t),$$

where $A$ is an amplitude. In the present model study we employ nonlinearity $n = 5$, the axial trap parameter $\nu = 0.5$, and the optical lattice strength $V_0 = 6$ throughout. First we calculate the ground-state wave function in the combined harmonic and optical lattice potentials.

When the condensate is released from the combined trap, a matter-wave interference pattern is formed in a few milliseconds as described in Ref. [6]. The atom cloud released from one lattice site expand, and overlap and interfere with atom clouds from neighboring sites to form the robust interference pattern due to phase coherence. The pattern consists of a central peak and two symmetrically spaced peaks, each containing about 10% of total number of atoms, moving in opposite directions [8,9,6,22].

If we introduce the modulation of nonlinearity (3) after the formation of the BEC in the combined trap, the condensate will be out of equilibrium and start to oscillate. As the height of the potential barriers in the optical lattice...
Fig. 1. One-dimensional probability $P(y,t)$ vs. $y$ and $t$ for the BEC on optical lattice under the action of modulation (3) with $n = 5$, $\Omega = 2\omega$ and $A = 3$ and upon the removal of the combined traps after hold times (a) 17 ms, (b) 35 ms, and (c) 52 ms.

is much larger than the energy of the system, the atoms in the condensate will move by tunneling through the potential barriers. This fluctuating transfer of Rb atoms across the potential barriers is due to Josephson effect in a neutral quantum liquid [8]. The phase coherence between different optical sites of the condensate may be destroyed during this rapid oscillation with large amplitude and no matter-wave interference pattern will be formed after the removal of the joint traps.

However, resonant oscillation arising from modulation (3) is not the only classical dynamical mechanism for the destruction of superfluidity. It can also be destroyed via a periodic modulation of the radial trapping potential [23] or by giving the BEC a large displacement along the optical trap [9]. For a weak modulation or small displacement, the superfluidity is preserved independent of hold time in the modulated trap. For a stronger modulation and longer hold
time, there is destruction of superfluidity via a classical dynamical process.

Now we explicitly study the destruction of superfluidity in the condensate upon application of the modulation (3) leading to a resonant oscillation. The loss of superfluidity only takes place if the BEC is allowed to experience the resonant oscillation for a certain interval of time (hold time). The resonant oscillation is excited for \( \Omega = 2\omega \) or multiples thereof. Strictly speaking, this should happen when \( \Omega \) equals the natural frequency of oscillation of the system or multiples thereof. This natural frequency is \( 2\omega \) for a noninteracting system with \( n = 0 \). In the presence of nonlinear mean-field interaction this frequency is expected to change. This change is found to be small in numerical simulation in the present context of small nonlinearity. However, some deviation is found to take place for large nonlinearity [21].

In this calculation we take \( n = 5 \) in Eq. (3) and for the effective nonlinearity after modulation to remain positive (repulsive condensate) we restrict to \( A < 5 \). Negative values of nonlinearity corresponding to atomic attraction may lead to collapse and instability [11] and will not be considered here. We shall present results with \( A = 3 \) in this study, although any other \( A \), which is not negligibly small, leads to similar result. For \( \Omega = 2\omega \) the BEC executes rapid oscillation exciting collective resonant modes [17,18] resulting is a destruction of superfluidity.
For numerical simulation we allow the BEC to evolve on a lattice with $x \leq 20 \mu m$ and $20 \mu m \geq y \geq -20 \mu m$ after the modulation (3) is applied and study the system after different hold times. The one-dimensional probability $P(y, t)$ is plotted in figures 1 (a), (b) and (c) for hold times 17 ms, 35 ms and 52 ms, respectively. For hold time 17 ms, prominent interference pattern is formed upon free expansion. In figure 1 (a) three separate pieces in the interference pattern corresponding to three distinct trails can be identified. The interference pattern is slowly destroyed at increased hold times as we can see in figures 1 (b) and (c). As the hold time increases the maxima of the interference pattern mix up upon free expansion and finally for the hold time of 52 ms the interference pattern is completely destroyed as we find in figure 1 (c). As the BEC is allowed to evolve for a substantial interval of time after the application of the periodic modulation in the scattering length, a dynamical instability of classical nature sets in which destroys the superfluidity [12,13].

The superfluidity reappears rapidly as frequency $\Omega$ of the modulation (3) is changed to a nonresonant value. We demonstrate this for $\Omega = \omega$ and $3\omega$ in the following. In figures 2 (a) and (b) we present the evolution of probability $P(y, t)$ after the application of the modulation for $n = 5$, $A = 3$ and $\Omega = \omega$ and $3\omega$. We see from figures 2 (a) and (b) that in both cases the interference pattern is obtained after a hold time of 69 ms.

Fig. 3. One-dimensional probability $P(y, t)$ vs. $y$ and $t$ for the BEC on optical lattice under the action of modulation (3) with $\Omega = 4\omega$ with $n = 5$ and $A = 3$ upon the removal of the combined traps after hold time 69 ms.

The resonant collective oscillation also appears for higher multiples of $\Omega = 2\omega$ thus leading to a destruction of superfluidity. This is illustrated in Fig. 3 where we present the evolution of probability $P(y, t)$ after the application of the modulation with $n = 5$, $A = 3$ and $\Omega = 4\omega$ for a hold time of 69 ms. We see that no interference pattern is formed in this case upon release of the BEC from the joint traps. In this case the superfluidity is maintained for a hold time of 52 ms whereas for $\Omega = 2\omega$ it was destroyed at 52 ms as can be seen in figure 1 (c).

In the absence of the optical trap, a cigar-shaped BEC can be excited to collective resonant states for modulation (3) when $\Omega$ is an even multiple of $\omega$. 
or $\nu \omega$ [20]. The present study shows that in the presence of an optical trap, only the former possibility leads to a loss of superfluidity, at least for the small nonlinearity ($n = 5$) considered here. The possibility $\Omega = 2\nu \omega$ does not seem to lead to prominent collective resonant excitation and hence does not easily lead to a loss of superfluidity as one can see from figure 2 (a) with $\nu = 0.5$. The reason for the absence of collective resonant excitation in this case is not clear. It is possible that for large nonlinearity $n$ and amplitude $A$ in (3) there is breakdown of superfluidity in this case.

In conclusion, we have studied the destruction of superfluidity in a cigar-shaped BEC loaded in a combined harmonic and optical lattice traps upon the application of a periodic modulation of the scattering length leading to resonant collective excitation when the frequency of modulation equals twice the radial trapping frequency or multiples thereof. In the absence of modulation, the formation of the interference pattern upon the removal of the combined traps clearly demonstrates the phase coherence [7,6]. At these resonance frequencies the phase coherence is destroyed signaling a superfluid-insulator classical phase transition, provided that the BEC is kept in the modulated trap for a certain hold time. Consequently, after release from the combined trap no interference pattern is formed. The superfluidity in the BEC is quickly restored when the frequency of modulation of the scattering length is changed to a nonresonant value away from $\Omega = 2\omega$ or multiples. It is possible to study this novel superfluid-insulator classical phase transition experimentally and a comparison of those results with mean-field models will enhance our understanding of matter wave BEC. After the recent experiments of Cataliotti et al. [13] and Müller et al. [22] that study seems possible in a not too distant future.

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