Waveform Optimization for Non-orthogonal CP-FBMA System

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Abstract—Filter bank multiple access (FBMA) without subbands orthogonality has been proposed as a new waveform to better cater the requirements of future wireless communication systems and scenarios. In addition, the usage of cyclic prefix (CP) in FBMA (CP-FBMA) has been proved to have many advantages such as the ability to process complex signals, lower peak-to-average power ratio (PAPR) and bit error rate (BER). However, the non-orthogonality of the system has not been fully exploited for waveform design, which inspires us to propose a new manifold-based waveform optimization iteration algorithm on the full band for the non-orthogonal CP-FBMA system, where full band means that there is no restriction on frequency responses of the synthesis filter bank (SFB) to match channels more perfectly. Both general framework and detailed derivation processes of the proposed waveform optimization algorithm are presented in this paper. Numerical results show that the algorithm converges after only a few iterations and it can improve the sum rate of the system dramatically.

I. INTRODUCTION

Waveform design has become a hot topic in 5G communication system research to better satisfy the stringent requirements such as lower out-of-band radiation, robustness against carrier frequency offset (CFO) and narrow band interference. Among new waveforms that have been proposed, filter bank multi-carrier (FBMC) [1] is a popular one. It mainly consists of frequency-well-localized subband filters, and each subcarrier is filtered individually in frequency domain. It has been employed in some new scenarios like carrier aggregation and cognitive radio with spectrum sensing. Based on FBMC, universal-filtered multi-carrier (UFMC) [2] and Generalized frequency division multiplexing (GFDM) [3] are proposed. They are all candidates for 5G communication system. As an extension of orthogonal frequency division multiplex (OFDM), conventional FBMC also requires orthogonality between subbands. The orthogonality refers to the division of the input signal into real and imaginary parts to satisfy the orthogonality of the real domain. When used in multiuser communication, FBMC is referred to as filter bank multiple access (FBMA), which retains the orthogonality of subbands. However, only non-orthogonal schemes are eligible to achieve the entire capacity region of the channel. For this, [4] proposed a non-orthogonal scheme of FBMA system, which has better peak-to-average power ratio (PAPR) and bit error rate (BER) performance over conventional FBMC and OFDM. Besides, [5] introduced cyclic prefix (CP) and wide-banded subbands into non-orthogonal FBMA system (CP-FBMA). The benefits are that the former can facilitate the equalisation task and the latter can reduce the length of filters thus reducing PAPR and latency.

In [4, 5], non-orthogonality is exploited to improve the performance of PAPR and symbol detection. But the method of designing filters follows that in conventional FBMC system which are still orthogonal ones. In fact, non-orthogonality means that orthogonality between filters becomes unnecessary. A natural question is whether we can redesign the filters to further improving system performance. This is the motivation for our work. Here, we extend the idea of wide band to the full band which means that there is no restriction on the bandwidth of filters to match channels more perfectly.

The contribution of this paper is to extend the subband non-orthogonality nature of CP-FBMA system from wideband to full band and propose an algorithm to optimize the waveform. Specifically, we allow each user in the CP-FBMA system to occupy the full available bandwidth and propose a filter waveform design algorithm to maximize the sum rate according to the channel response. The time domain filter coefficients are selected as optimization variables and obtained in an iterative process. In addition, we propose a manifold-based gradient ascent algorithm to solve the suboptimization problem encountered in the iterative process. The proposed waveform optimization algorithm can apparently improve the sum rate of non-orthogonal wideband CP-FBMA system and has an excellent BER performance at the receiver compared with CP-FBMA with the traditional waveform. Simulation results show that the iterative algorithm converges fast. Besides, the sum rate of CP-FBMA with much shorter but optimized filters can be higher than that of the system with longer but non-optimized filters. This indicates that the proposed scheme could reduce the latency of the filter bank processing, which is desired for FBMC based systems since they involve additional subcarrier filtering as compared to OFDM and are often considered not suitable for low latency applications. In summary, the proposed FBMA with filter optimization is a promising
waveform candidate in future wireless communication.

This paper is organized as follows. In Section II, the system model of non-orthogonal wideband CP-FBMA is presented. In Section III, the iterative algorithm based on manifold is proposed. In Section IV, simulation results are presented. Section V concludes this paper.

II. SYSTEM MODEL

In this section, we briefly review the system model of non-orthogonal wideband CP-FBMA for further analysis. Fig. 1 shows the transceiver structure of an $M$-subband non-orthogonal CP-FBMA system for $M$-user in the uplink, in which subband-$m$ is occupied by user-$m$.

A. Transmitter Structure

At the transmitter side, the input QAM-modulated symbol vector $x_m$ on subband-$m$ is assumed to be of length $N$. It is appended by a CP of length $L_g$ and fed to the (synthesis filter bank) SFB for filtering with upsampling factor $P \leq M$. The length of synthesis filter $f_m(l)$ is $N_f$. Generally, $N_f$ is a few times of $M$. When $P = M$, the system is called critically sampled. When $P < M$, it is called oversampled. The signal on subband-$m$ is transmitted through a fading channel $h_m(l)$ of length $L_h$, corrupted by i.i.d. additive white Gaussian noise (AWGN) $z(l)$ of zero mean and variance $N_0$. Note that the effective length of CP after upsampling is $L_gP$ which should cover both the length of the physical channel and the length of the filter at the transmitter.

B. Receiver Structure

At the receiver side, the received signal after removing the effective CP is denoted by $y$, which can be written as

$$y = \sum_{m=1}^{M} H_m F_m U x_m + z,$$

where $H_m \in \mathbb{C}^{NP \times NP}$ is a circular matrix whose first column is given by zero-padded version of $h_m = [h_m(0), ..., h_m(L_h-1)]^T$, $F_m \in \mathbb{C}^{NP \times NP}$ is a circular matrix whose first column is given by zero-padded version of $f_m = [f_m(0), ..., f_m(N_f-1)]^T$, $U \in \mathbb{R}^{NP \times N}$ is the upsampling operator for the input vector of the subband, and $z$ is AWGN. Then the symbol detection module recovers the transmitted signals from $y$. Regarding the symbol detection scheme for non-orthogonal CP-FBMA system, the most recent work is proposed in [6], which uses linear minimum mean square error (LMMSE) estimation iteratively until the process converges or reaches a predefined number of iterations. It totally avoids the usage of the analysis filter bank (AFB) in traditional CP-FBMA receivers and has additional advantages, including reduced complexity, improved spectral efficiency and detection performance. In this paper, to further improve the performance and reduce the computational complexity of the receiver for the system, we also consider more prominent designs applied in the symbol detection, namely approximate message passing (AMP) algorithm [7]. By using AMP, the computational complexity of detecting source symbols is $O(MN)$ in theory, which is much lower than LMMSE. For the receiver of CP-FBMA, due to most of matrices generated in detection process are block diagonal, the complexity can be even much lower. We will compare the performance of both LMMSE and AMP in simulation.

III. WAVEFORM OPTIMIZATION

In this section, we propose a filter waveform optimization algorithm to maximize the sum rate.

A. Problem Formulation

As indicated by [4], we will compute the achievable rate by observing $y$ to avoid potential information loss. From (1), the sum rate of CP-FBMA can be characterized by

$$R = \frac{1}{NP} \log_2 |I + \frac{1}{N_0} \sum_{m=1}^{M} H_m F_m U C_m U^H F_m^H H_m^H|,$$

in which $C_m$ denotes the covariance matrix of the user-$m$, and $| \cdot |$ denotes determinant. In CP-FBMA, the user
wise average power constraint $P_m$ is imposed directly on $x_m$ by $\frac{1}{NP} \text{tr} \{ F_m U C_m U^H F_m^H \} = P_m, \forall m$. To simplify the calculation, we assume that $C_m = P P_m I_N$, where $I_N$ is an identity matrix whose size is $N \times N$. With the constraint $f_m^H f_m = 1$, the filters will be optimized through the sum rate maximization problem defined by

$$f_m^* = \arg\max_{f_m} R, \quad \text{s.t.} \quad f_m^H f_m = 1, \quad m = 1, \ldots, M.$$  

Due to the usage of CP, $H_m$ and $F_m$ are circular matrices and can be transformed into diagonal ones, i.e., $H_m = W^H A_h m W, F_m = W^H A_f m W$, where $W$ is the $(NP)$-point discrete Fourier transform (DFT) matrix, $A_h m$ and $A_f m$ are diagonal matrices whose main diagonal entries are given by the $(NP)$-point DFT of $h_m$ and $f_m$ respectively. Now, (2) can be rewritten as

$$R = \frac{1}{NP} \log_2 \left( |I + \sum_{m=1}^{M} A_h m A_f m Q_m A_f m^H A_h m^H| \right),$$  

where $Q_m = \frac{P P_m}{NP} W \Omega \Omega^H W$ is a block matrix consisting of $P \times P$ subblocks. Each subblock is $\frac{NP P_m}{NP} \Omega \Omega^H \Omega$. $Q_m$ can be transformed into the sum of the product of a set of column vectors with their transposes, which is written as

$$Q_m = \sum_{i=1}^{N} q_{m,i} q_{m,i}^T.$$  

Here, $q_{m,i} = [q_{m,i}(0), \ldots, q_{m,i}(NP - 1)]^T$ is a sparse vector and only $q_{m,i}(kN + i - 1), k = 0, \ldots, P - 1$ is a nonzero constant $\sqrt{\frac{NP P_m}{NP}}$. Now, $A_f m Q_m A_f m^H$ becomes $A_f m \sum_{i=1}^{N} q_{m,i} q_{m,i}^T A_f m^H$. Changing the order of $A_f m$ and $q_{m,i}$, it can be rewritten as

$$A_f m \sum_{i=1}^{N} \Lambda_{q_{m,i}} \mathcal{F}_m \mathcal{F}_m^H \Lambda_{q_{m,i}}^T,$$

where $\Lambda_{q_{m,i}}$ is the $NP \times NP$ diagonal matrix whose main diagonal consists of $q_{m,i}(0), \ldots, q_{m,i}(NP - 1)$ and $\mathcal{F}_m$ is the $NP \times 1$ column vector of $(NP)$-point DFT of $f_m$, which can be written as $\mathcal{F}_m = WP f_m$ ($P$ denotes a zero complement matrix of size $NP \times N_f$). Therefore, (4) can be reformulated as

$$R = \frac{1}{NP} \log_2 \left( |I + \sum_{m=1}^{M} \sum_{i=1}^{N} G_{m,i} f_m f_m^H G_{m,i}^H| \right),$$

where $G_{m,i} = A_h m \Lambda_{q_{m,i}} WP$ is a $NP \times N_f$ matrix with only $(kN + i)$th rows are nonzero, $k = 0, \ldots, P - 1$.

It is difficult to maximize the sum rate in (7) by optimizing the filters on all subbands simultaneously. [8] showed that the sum-rate maximization problem could be solved efficiently using an iterative algorithm, where each step of the iteration was equivalent to a local maximization of one user’s data rate with multiuser interference treated as noise. When sum-rate converges, iteration stops. In other word, optimal $f_m$ can be found iteratively by regarding other filters as fixed, and in each iteration it can be written as

$$f_m^* = \arg\max_{f_m} \log_2 \left( \sum_{i=1}^{N} G_{m,i} f_m f_m^H G_{m,i}^H + \Phi_m \right).$$  

Here,

$$\Phi_m = I + \sum_{j=1,j\neq m}^{M} \sum_{i=1}^{N} G_{j,i} f_j f_j^H G_{j,i}^H$$

is a fixed $NP \times NP$ Hermitian matrix whose structure is similar with that of $Q_m$ (as $\Phi_m$ is equal to the sum of products of multiple $Q_m$ and diagonal matrices). Then, (8) can be further deduced as

$$f_m^* = \arg\max_{f_m} \log_2 \left( |I + \sum_{i=1}^{N} G_{m,i} f_m f_m^H G_{m,i}^H + \Phi_m^{-1/2} \sum_{i=1}^{N} f_m^H G_{m,i} f_m| \right).$$

Here, $\Phi_m^{-1/2}$ is also a block Hermitian matrix with $P \times P$ subblocks which are $N \times N$ diagonal matrices. Considering the matrix structure of $G_{m,i}$, we can find that $\tilde{G}_{m,i} = \Phi_m^{-1/2} G_{m,i}$ has a similar structure as $G_{m,i}$, i.e., only $(kN + i)$th rows are nonzero, $k = 0, \ldots, P - 1$. This inspires us to remove zero rows of $\tilde{G}_{m,i}$. Define an interleaving matrix $\Omega$ of size $NP$. For any $\omega \in [0, \ldots, NP - 1]$, if $\omega = \omega_1 P + \omega_2$ where $\omega_1$ and $\omega_2$ are non-negative integers, the $\omega$th row of $\Omega$ is all zeros except for the $(\omega_1 + \omega_2 N)$th entry. The trick is that $\Omega \sum_{i=1}^{N} \tilde{G}_{m,i} f_m f_m^H G_{m,i}^H \Omega^T$ is a block diagonal matrix whose $m$th subblock is $G_m f_m f_m^H G_m^H$, where $G_m$ evolves from $G_{m,i}$ by removing its zero rows and the size of $G_m$ is $P \times N_f$. Now, noting that $\Omega^T \Omega = I$, (10) can be reformulated as

$$f_m^* = \arg\max_{f_m} \sum_{i=1}^{N} \log_2 \left( |I + \tilde{G}_{m,i} f_m f_m^H \tilde{G}_{m,i}^H| \right).$$

where $\tilde{B}_{m,i} = \tilde{G}_{m,i}^H \tilde{G}_{m,i}$ is a $N_f \times N_f$ Hermitian matrix. The problem now is to find optimal $f_m$ with the constraint of $f_m^H f_m = 1$.

B. Manifold-based Algorithm

As a sum of logarithms of multiple quadratic forms, (11) is difficult to be solved by using Lagrangian multiplier method directly. Here, we use a manifold-based gradient ascent algorithm mentioned in [9], which is tailored to be applicable to (11) to find near-optimal filters. The manifold-based algorithm
uses iteration to get the current solution. Define
\[ \tilde{R}_m = \sum_{i=1}^{N} \log_2 (1 + f_{m,i}^H \mathbf{B}_{m,i} f_{m,i}), \]
(12)
which is a function of \( f_{m} \). Different from the traditional gradient ascent algorithm, at the \((t+1)\)th iteration, the solution \( f_{m}^{(t+1)} \) in the manifold-based algorithm is written as
\[ f_{m}^{(t+1)} = \text{Proj}[f_{m}^{(t)} + \rho_t \nabla f_{m} \tilde{R}_m],\]
(13)
where \( \rho_t \) is the stepsize. \( \text{Proj}[\cdot] \) is the projection of \( f_{m}^{(t)} + \rho_t \nabla f_{m} \tilde{R}_m \). Actually, provided that \( f_{m} f_{m}^H = 1 \) can be satisfied after each iteration. \( \nabla f_{m} \tilde{R}_m \) is the gradient of \( \tilde{R}_m \) on the manifold \( \mathcal{M} = \{ f_{m} f_{m}^H f_{m} = 1 \} \), it can be regarded as the projection of \( \nabla f_{m} \tilde{R}_m \) on the tangent space \( T_{f_{m}}(\mathcal{M}) = \{ \zeta : \zeta^H f_{m} + f_{m}^H \zeta = 0 \} \) at \( f_{m} \), where \( \nabla f_{m} \tilde{R}_m \) is the gradient of \( \tilde{R}_m \) with \( f_{m} \) and can be calculated as
\[ \nabla f_{m} \tilde{R}_m = \frac{1}{\ln 2} \sum_{i=1}^{N} \mathbf{B}_{m,i} f_{m,i} \frac{1}{1 + f_{m,i}^H \mathbf{B}_{m,i} f_{m,i}}. \]
(14)
Then \( \nabla f_{m} \tilde{R}_m \) can be expressed by
\[ \nabla f_{m} \tilde{R}_m = \text{argmin} \| \zeta - \nabla f_{m} \tilde{R}_m \|^2. \]
(15)
Using the standard Lagrangian multiplier method
\[ L(\zeta, \lambda) = (\zeta - \nabla f_{m} \tilde{R}_m)^H (\zeta - \nabla f_{m} \tilde{R}_m) - \lambda (\zeta^H f_{m} + f_{m}^H \zeta), \]
(16)
and let \( \frac{\partial L(\zeta, \lambda)}{\partial \zeta} = 0 \) and \( \frac{\partial L(\zeta, \lambda)}{\partial \lambda} = 0 \), we can get
\[ \begin{cases} \zeta - \nabla f_{m} \tilde{R}_m - \lambda f_{m} = 0, \\ \zeta^H f_{m} + f_{m}^H \zeta = 0. \end{cases} \]
(17)
Solving (17), we can get
\[ \begin{cases} \zeta = \nabla f_{m} \tilde{R}_m - \text{Re}[(\nabla f_{m} \tilde{R}_m)^H f_{m}]f_{m}, \\ \lambda = -\text{Re}[(\nabla f_{m} \tilde{R}_m)^H f_{m}]. \end{cases} \]
(18)
Thus, the closed form of \( \nabla f_{m} \tilde{R}_m \) is
\[ \nabla f_{m} \tilde{R}_m = \nabla f_{m} \tilde{R}_m - \text{Re}[(\nabla f_{m} \tilde{R}_m)^H f_{m}]f_{m}, \]
(19)
where \( \text{Re}[\cdot] \) denotes the real part. Noting \( \tilde{R}_m \) has an upper bound \( \sum_{i=1}^{N} \log_2 (1 + \lambda_{m,i}) \), where \( \lambda_{m,i} \) is the maximum eigenvalue of \( \mathbf{B}_{m,i} \), the manifold-based algorithm for (11) converges. The stepsize \( \rho_t \) in (13) can be taken as a constant, i.e., \( \rho_t = 0.01 \). When \( \| \nabla f_{m} \tilde{R}_m \|^2 \) is smaller than \( \epsilon \) which is an extremely small constant, iteration stops, and \( f_{m} \) after the last iteration is taken as the result of (11).

A general framework of our proposed waveform optimization algorithm is shown in Algorithm 1. 

Algorithm 1 Waveform Optimization Algorithm
1: Initialize \( f_{m}, m = 1, ..., M \). Set \( \rho_t \) and \( \epsilon \).
2: Compute \( \tilde{R} \) in (2).
3: repeat
4: for \( m = 1 \) to \( M \) do
5: Compute \( \mathbf{B}_{m,i}, i = 1, ..., N \) in (11).
6: repeat
7: Compute \( \nabla f_{m} \tilde{R}_m \) at \( f_{m}^{(t)} \) by (19).
8: Update \( f_{m}^{(t)} \) to \( f_{m}^{(t+1)} \) by (13).
9: until \( \| \nabla f_{m} \tilde{R}_m \| \leq \epsilon \)
10: until \( \tilde{R} \leq \epsilon \)

IV. NUMERICAL RESULTS

In this section we provide simulation results to show the performance of the proposed waveform optimization algorithm. Different conditions including various lengths of filters and oversampling factors are simulated to expose the behavior of the algorithm. The results are compared with those of the CP-FBMA system whose waveform is designed on a traditional manner. The channel model is typical urban (ETU) extended from long term evolution (LTE). Some of the key configurations and parameters are given as follows. We assume there are totally \( M = 4 \) users, thus the number of subbands/subcarriers is 4. The number of symbols per subband is \( N = 32 \). All users share a same \( \gamma_m \), where \( \gamma_m = \frac{P_m}{N_0} \) denotes the transmitted signal-to-noise ratio (SNR) of user-\( m \). For oversampling factors, we select three cases: for \( P = 1 \), it is maximally oversampled; for \( P = M/2 \), it is 2x oversampled; for \( P = M \), it is critically sampled.

A. Convergence and Waveforms

We consider three different lengths \( (N_f = 16, N_f = 24, N_f = 32) \) of filters and two different transmitted SNRs \( (\gamma_m = 10 \text{ dB}, \gamma_m = 20 \text{ dB}) \) to reveal their effects on the convergence behavior of the sum rate. Fig. 2 and Fig. 3 show the change of sum rate with the number of the outer iterations in Algorithm 1 while Fig. 4 and Fig. 5 show the frequency response of the filters with and without optimization, respectively. We can observe that: 1) the proposed waveform optimization algorithm converges after only a few iterations. The length of filters and transmitted SNR have little impact on the speed of convergence; 2) in Fig. 4, frequency responses of filters in different subcarriers share a same waveform shape, given that synthesis filters are derived from a same prototype filter. In Fig. 5, such regularity of waveforms in frequency domain is broken after filters are optimized, and the frequency responses of filters are expanded to the full band.

B. Impacts of Length and Oversampling

Now we consider the impacts of different values of \( N_f \) and \( P \) on the sum rate respectively. Fig. 6 and Fig. 7 show the sum rates of the system with traditional waveform and that with optimized waveform, respectively. We can see that:
1) by optimizing the waveform, the sum rate of CP-FBMA is increased significantly; 2) $N_f$ has no obvious impact on the sum rates. For CP-FBMA with optimized waveform, even in the case that the filter length is the shortest, its sum rate is still higher than the system with longer filters without optimization; 3) critically sampled CP-FBMA has the shortage compared with oversampled CP-FBMA. But if the waveform is optimized by the proposed algorithm, the critically sampled system will perform better than the oversampled one without waveform optimization.

### C. BER Performance

Fig. 8 presents BER performance of the non-orthogonal wideband CP-FBMA system different waveforms and symbol detection schemes. Specially, we also present BER results of the following systems: CP-FBMA with optimized waveform and the receiver based on LMMSE, CP-FBMA with traditional waveform and the receiver based on AMP, CP-FBMA with traditional waveform and the receiver based on LMMSE. The results clearly show that: 1) AMP outperforms LMMSE when other parameters are fixed; 2) the proposed optimized waveform can bring lower BER than the traditional waveform and its role is even more pronounced than AMP; 3) combining the optimization algorithm with AMP produces an excellent performance of BER for CP-FBMA system.

### V. Conclusions and Future Works

This paper presents a waveform optimization algorithm for synthesis filter bank of non-orthogonal wideband CP-FBMA on the full band to maximize the sum rate of the system. The problem is formulated as an iterative optimization one which is solved by manifold-based gradient ascent algorithm. Through simulation results, several key factors affecting the performance are investigated, and advantages of the optimized waveform compared with the traditional one are illustrated. It is found that the optimization algorithm converges dramatically fast and can significantly improve the sum rate of...
Moreover, the sum rate of CP-FBMA with the optimized waveform can be higher than that of the traditional CP-FBMA system, even in the case of shorter length of filters. It indicates the reduction of latency in the communication link, which is desired for future communication system. In addition, the usage of AMP is also proved to be a progressive attempt for the system and its combination with the aforementioned waveform optimization algorithm can produce an excellent performance of BER. In summary, the proposed waveform is promising in future wireless communication. As a preliminary work, the mask of frequency response of the filter bank is not considered in this paper, which will be studied as a future work. Other potential works include combination with multiple-input multiple-output (MIMO) and statistical channel state information (CSI).

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