3D Numerical Study on Magnetization Losses in Twisted Soldered-Stacked-Square (3S) HTS Wires Based on Homogenized Models

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Abstract. Magnetization loss is an important parameter in the application of high temperature superconductivity (HTS) power devices. In order to reduce magnetization losses, a novel soldered-stacked-square (3S) HTS wire with 1 mm width is firstly proposed and manufactured by our group. In previous work, numerical and experimental results have shown the magnetization loss in the 3S wire is one order smaller than that in the original 4 mm tape under perpendicular magnetic field. However, large eddy current loss and coupling loss will be generated under parallel field. Therefore, to further reduce the magnetization loss, the 3S wire has been twisted and investigated numerically in this paper. Firstly, two-dimensional (2D) numerical models are built for calculating the magnetization loss in untwisted 3S wires. To simplify the computation, a homogeneous bulk-like model is built which does not include the individual tapes of stack. The results show that the losses calculated by the homogenized model is similarly equal to the original results. Based on this, the homogeneous technique is further applied in three-dimensional (3D) numerical models of the twisted 3S wire. The results indicate that the homogenized model could make a good predict for the losses in twisted wires. What’s more, the magnetization losses in twisted wires is compared with those in untwisted wires to verify that twisting process could actually reduce the magnetization loss of the novel superconducting wire.

Index Terms—homogenized model, numerical model, twisted 3S wire

1. Introduction
When high temperature superconducting (HTS) wires are under external magnetic field, magnetization losses will be generated, which is a necessary consideration in superconducting power devices application. In order to reduce the loss, narrowing and twisting are the methods normally used[1]. Our group has proposed a novel HTS soldered-stacked-square (3S) wire, which has a stack of superconducting tapes[2]-[7]. In pervious work, some electrical and mechanical properties of the novel 3S wire have been studied. In this work, the magnetization losses in the twisted 3S wire are numerically investigated. On one hand, to verify whether twisting process could reduce the loss in 3S wires, the
losses in twisted wires are compared with those in untwisted wires. On the other hand, the three-dimensional (3D) simulation is a challenging task involving a large-scale computational problem. While setting up the 3D model for twisted 3S wires, we try to construct a bulk-like equivalent for the stack but the overall electromagnetic behaviour of the model should be kept\[8\]-[11]. The losses calculated by complete models considering the actual layout of a stack of superconducting tapes are compared with those calculated by homogeneous bulk model which does not include the individual tapes to confirm if homogeneous techniques could be reliable in 3D simulation models.

2. Numerical models

The numerical model in this paper is based on $H$-formulation which is composed of Faraday’s law and Ampere’s law,

\[
\nabla \times H = -\frac{\partial B}{\partial t} \\
J = \nabla \times H
\]

2.1. The 2D models for untwisted 3S wires

In the 2D problem, there are two dependent variables $H_x$ and $H_y$, respectively representing the components of the applied magnetic fields in the $x$ and $y$ directions. Also, the induced electric field and the current density in the superconducting domain only have the components in the $z$ direction, donated by $E_{sc,z}$ and $J_{sc,z}$\[12\]-[14]. Then, with $B = \mu_0 \mu_r H$, Equation (1) and (2) can be described as,

\[
\begin{align*}
\frac{\partial E_x}{\partial y} &= -\mu_0 \mu_r \frac{\partial H_x}{\partial t} \\
\frac{\partial E_y}{\partial x} &= -\mu_0 \mu_r \frac{\partial H_y}{\partial t} \\
J_{sc,x} &= \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}
\end{align*}
\]

In the superconducting domain, the electrical behavior of the superconducting material is represented by the $E$-$J$ power law,

\[
E_{sc,x} = E_0 \left(\frac{J_{sc,x}}{J_c}\right)^n
\]

where $J_c$ is the critical current density of the superconductor, and $E_0$ indicates the electrical field at which DC critical current is defined, $n$ is the power law exponent.

Figure 1 shows the schematic view and geometrical model of the single 3S wire. Figure 1(a) reveal the structure of the 3S wire composed of four superconducting tapes with the thickness of 10 µm and width of 1 mm each. Figure 1(b) shows the complete model that the features correspond to the actual stack where geometrical layout of the individual tapes fully copy that of Figure 1(a). Comparatively, Figure 1(c) represents the corresponding homogeneous bulk model which has a much simpler geometrical layout that does not include the individual tapes of stack. The size of the bulk is set to 1 mm $\times$ 0.5 mm to keep the overall electromagnetic properties of the stacking superconducting tapes. It is placed in the center of circular air domain and has the same critical current of the 3S wire in Figure 1(b).
Figure 1. (a) The schematic view of the theoretical structure for single 3S wire consisting of four superconducting tapes, (b) the complete model considering the actual layout of a stack of four superconducting tapes, (c) the homogenized model.

The simulation model is figured by the partial differential equations (PDE) modes based on the finite element method (FEM). The Dirichlet boundary conditions are set to fit external field, with \( H_y = B_m \sin(\omega t) / \mu_0 \). The final magnetization loss (\( Q \) in J/m/cycle) is defined as,

\[
Q = 2J^{1/2}_f \iint E_{sc,x} J_{sc,z} \, ds \, dt
\]

where \( f \) is the frequency of external field and \( s \) is the area of the HTS domain. The parameters above are listed in Table 1.

Table 1. Parameters of the 2D model for untwisted 3S wires.

| Parameters | \( I_c \) (A) | \( J_c \) (A/m²) | \( n \) | \( E_0 \) (µV/cm) | \( f \) (Hz) | Width (mm) | Height (mm) |
|------------|---------------|-----------------|-------|-----------------|------------|------------|-------------|
| Values     | Figure 1(b)   | 3.5x10⁹         | 140   | 3.5x10⁹         | 1          | 50         | 0.01x4      |
|            | Figure 1(c)   | 2.8x10⁸         | 17    | 1               | 1          | 0.5        |

Figure 2. (a) The schematic view of the theoretical structure for three-stacked 3S wires, (b) the complete model considering the actual layout of three-stacked 3S wires, (c) the homogenized model.

To further confirm the reliability of the 2D models, Figure 2 reveals the three-stacked untwisted 3S wires. Figure 2(a), (b) and (c) respectively show the theoretical structure, the complete model and the homogenized model. To enhance the article rigor and persuasiveness, the critical current in the wires of Figure 2(b) and Figure 2(c) should also retain the same. The geometrical size and simulation method of the three-stacked wires are both identical to those of single wire in Figure 1.

2.2. The 3D models for twisted 3S wires

To verify whether twisted 3S wires could have good performance under external field, models for twisted wires are put forward. However, twisted wires can hardly be presented in a 2D model, so the 3D techniques are detailedly described in this part.

In the 3D problem, there are three independent variables of magnetic field, denoted as \( H_x, H_y, \) and \( H_z \). The current density and electric field also have three components in the \( x, y \) and \( z \) directions, defined as \( J_x, J_y, \) and \( J_z \), and \( E_x, E_y, \) and \( E_z \), respectively[15]. Equation (1) and (2) can be described as,

\[
\begin{bmatrix}
\frac{\partial E_z}{\partial y} & \frac{\partial E_y}{\partial z} & \frac{\partial E_x}{\partial z} & \frac{\partial E_z}{\partial x} & \frac{\partial E_x}{\partial y} & \frac{\partial E_y}{\partial x}
\end{bmatrix} = -\mu_0 \mu_r \begin{bmatrix}
\frac{\partial H_x}{\partial t} & \frac{\partial H_y}{\partial t} & \frac{\partial H_z}{\partial t}
\end{bmatrix}
\]

(7)
The $E$-$J$ power law indicating the nonlinear electrical behaviour of the superconducting materials is defined as,

$$
E^* + E^3 = E_0 \left[ \begin{array}{c}
J_x \\
J_y \\
J_z
\end{array} \right] = \left[ \begin{array}{c}
\frac{\partial H_x}{\partial y} \\
\frac{\partial H_y}{\partial z} \\
\frac{\partial H_z}{\partial x}
\end{array} \right] \left[ \begin{array}{c}
J_{norm} \\
J_{norm} \\
J_{norm}
\end{array} \right]^{n} \left[ \begin{array}{c}
I_{norm} \\
I_{norm} \\
I_{norm}
\end{array} \right]^{n}
$$

(8)

The $E$-$J$ power law indicating the nonlinear electrical behaviour of the superconducting materials is defined as,

$$
\left[ \begin{array}{c}
E_x \\
E_y \\
E_z
\end{array} \right] = E_0 \left[ \begin{array}{c}
J_x \\
J_y \\
J_z
\end{array} \right] \left[ \begin{array}{c}
J_{norm} \\
J_{norm} \\
J_{norm}
\end{array} \right]^{n} \left[ \begin{array}{c}
I_{norm} \\
I_{norm} \\
I_{norm}
\end{array} \right]^{n}
$$

(9)

where $J_{norm} = \sqrt{J_x^2 + J_y^2 + J_z^2}$.

Figure 3 indicates the geometrical model of the twisted 3S wire which is derived from the wire in Figure 1 after twisting process. Figure 3(a) shows the complete model considering the actual layout of the stack of four superconducting tapes. The twisting pitch is 10 cm. Also, to simplify the computation of calculating magnetization losses based on FEM, homogenization techniques has also been applied in the 3D model. To verify whether it could be well suited for providing a satisfying description of the twisted wire, Figure 3(b) presents the homogenized model with the cross-section of $1 \text{ mm} \times 0.5 \text{ mm}$ and twisting pitch of 10 cm. In PDE mode, the two models both keep the same critical current and Dirichlet boundary conditions as those of the 2D model in Figure 1. The cross sections at both ends all have been set to satisfy the periodic condition in $H_x$, $H_y$, and $H_z$. The mesh is predefined coarsely except that the HTS domain is mapped much more finely with distributions of $20 \times 10$ (respectively for the $x$ direction and $y$ direction). Finally, the magnetization loss ($Q$ in $J/m/cycle$) in the 3D model is defined as,

$$
Q = 2 \int_{-l/2}^{l/2} \int \int \left( E_x J_x + E_y J_y + E_z J_z \right) dv dt
$$

(10)
Figure 4. (a) The complete model of three-stacked twisted 3S wires, (b) its corresponding homogenized model.

In addition, the complete and corresponding homogenized model of three-stacked twisted wires are also shown in Figure 4. The geometrical size and simulation method are the same as those of single-twisted wire in Figure 3.

3. Results and discussions
In order to keep the logic consistency, all the 3S wires in this paper have the same geometrical layout of the stack of four superconducting tapes with the critical current of 140 A as set up in the numerical models above.

3.1. The magnetization losses in 3S wires

Figure 5 shows the magnetization losses numerically calculated by the complete model and homogenized model under perpendicular magnetic fields.

Figure 5. The magnetization losses in 3S wires calculated by the complete model and homogenized model under perpendicular magnetic fields.

As shown in Figure 5, generally good agreement over the entire range is achieved between the losses calculated by the complete model and homogenized model for both single wire and three-stacked wires, which verifies the reliability of the homogenized model. That is the homogenization techniques could also simplify the geometrical structure and reduce the computing time while well predicting the losses in 3S wires.

3.2. The magnetization losses in twisted 3S wires

Figure 6 indicates the magnetization losses calculated by two ways of simulation technique in single 3S wire. The red line represents the loss predicted by the complete model described in Figure 3(a) in single twisted 3S wire while the blue one is calculated by the homogenized model described in Figure 3(b). The twisting pitch is 10 cm. The results show two curves have similar tendency, but appear a little disparity as the field gets larger. The error between two models might be caused by the complex mesh computing in the 3D complete model of twisted wires and this problem needs further work to figure.
The green and purple lines are for the losses in untwisted 3S wire which have been shown in Figure 5. Through comparing these two curves and the last two curves, it can be indicated that the twisted wire has much smaller magnetization losses than the untwisted wire, where exists a difference of one magnitude order in low field region. This means twisting process decreases the magnetization losses in 3S wires.

**Figure 6.** The magnetization losses in single 3S wire (left).

**Figure 7.** The magnetization losses in three-stacked 3S wires (right).

Similarly, the results for three-stacked wires have also been shown in Figure 7. Firstly, the losses calculated by the complete model (red line) coincide well with those calculated by the homogenized model (blue line) in most areas which further verifies the reliability of the homogenization technique applied in 3D models. What’s more, the losses in twisted stacking wires are smaller than those in untwisted stacking wires. The results above could confirm that twisting process does help reduce the losses in 3S wires under external magnetization field.

4. Conclusion

In order to investigate the magnetization loss of the twisted 3S wire with 1 mm width, the 3D numerical models based on FEM have been built. On one hand, the complete model considering the actual layout of a stack of superconducting tapes is compared with its corresponding homogeneous bulk model which does not include the individual tapes. On the other hand, the twisted wire is compared with the untwisted wire to verify whether twisting could reduce the loss in the novel superconducting wire.

Firstly, the 2D models for untwisted wires are set up. The results calculated by the complete model correspond well with those of the homogenized model whether in single wire or in three-stacked wires, which verifies the homogenization techniques are reliable in the 2D model for 3S wires.

Due to the bulk model is much simpler and time-saving, the homogenization techniques has also been applied in the 3D model for calculating the losses in twisted wires. The results show that the losses calculated by the complete model and the bulk-like model have similar tendency in most areas, but appear a little disparity as the field gets larger which might be caused by the complex mesh computing in the 3D complete model and this problem still needs to be further studied. What’s more, the twisted wire has much smaller magnetization losses than the untwisted wire whether in single wires or three-stacked wires, where exists the difference of one magnitude order in low field region. It can be confirmed that twisting could actually reduce the magnetization losses in 3S wires through the above results.

In addition, the magnetization losses usually include the hysteresis loss, eddy loss and coupling loss. The former is generated in superconducting layers which is detailedly considered in numerical models. The latter two are generated by other materials, such as the copper layer, buffer layer and substrate layer which not exist in our numerical models but exist in real 3S wire samples. Therefore, our next work will
prepare these 3S wire samples and make practical experiments on these samples to further verify the above conclusions. Also the coupling and eddy losses will be discussed in our future work because the influence of the losses is significant in power applications.

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