I Isovector Giant Dipole Resonance of Stable Nuclei in a Consistent Relativistic Random Phase Approximation

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A fully consistent relativistic random phase approximation is applied to study the systematic behavior of the isovector giant dipole resonance of nuclei along the $\beta$-stability line in order to test the effective Lagrangians recently developed. The centroid energies of response functions of the isovector giant dipole resonance for stable nuclei are compared with the corresponding experimental data and the good agreement is obtained. It is found that the effective Lagrangian with an appropriate nuclear symmetry energy, which can well describe the ground state properties of nuclei, could also reproduce the isovector giant dipole resonance of nuclei along the $\beta$-stability line.

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The investigation both experimentally and theoretically on various modes of nuclear collective giant resonances has become one of major research fields in the nuclear structure physics since the isovector electric (non-spin flip) giant dipole resonance (IV GDR) had been investigated firstly by Baldwin and Kläuger\textsuperscript{1} at the end of 1940’s. The isoscalar electric giant monopole resonance (IS GMR) in heavy nuclei is a unique direct source of experimental information on the compression modulus and the equation of state. The electric giant dipole resonance (GDR) and giant quadrupole resonance (GQR) are relevant to the symmetry energy coefficient and effective mass of nucleon, respectively. Much is already known about the electric giant resonance, especially for the isoscalar GMR, GQR and isovector GDR in cold nuclei, where exist extensive data sets on the excitation energy, strength distribution and width. The systematic behavior of the isovector GDR was reviewed by Berman and Fultz\textsuperscript{2} in 1975. The latest one was given by Speth\textsuperscript{3} in 1991. On the other hand, many theoretical works have been done to understand the physics mechanism of various collective vibrations. Furthermore, by comparing with the experimental data, one can test the microscopic theory and effective interactions which are used to reproduce the experimental data. One of the effective methods used to study properties of the giant resonance is the random phase approximation (RPA).

In recent years, the relativistic mean field (RMF) theory with non-linear meson self-interactions has achieved a great success in describing bulk properties of nuclei, not only spherical but also deformed nuclei and nuclei far from the $\beta$-stability line\textsuperscript{4}. In particular, the RMF theory has also been applied in studying the dynamical processes of nuclei. The linear response of a system to an external field can be calculated in the relativistic RPA. Early investigations using the relativistic RPA\textsuperscript{5, 6, 7} were based on Walecka’s linear $\sigma$-$\omega$ models, which provides considerably larger incompressibility\textsuperscript{6}. Therefore, one could not expect to obtain quantitative agreement with experimental data in those early calculations. Recently the meson propagators with non-linear self-interactions have been worked out numerically and the relativistic RPA calculations with the non-linear terms were performed\textsuperscript{8, 9, 10}. A fully consistent relativistic RPA has been established in the sense that the relativistic mean field wave function of nucleus and particle-hole residual interactions in the relativistic RPA are calculated from a same effective Lagrangian\textsuperscript{11, 12}. A consistent treatment of relativistic RPA within the RMF approximation requires the configurations including not only the pairs formed from the occupied Fermi states and unoccupied states but also the pairs formed from the Dirac states and occupied Fermi states. It has been formally proved\textsuperscript{11} that the fully consistent relativistic RPA is equivalent to the time dependent RMF (TDRMF) at the small amplitude limit\textsuperscript{11, 12}. The response functions for the closed shell nuclei\textsuperscript{11, 12} have been calculated using the fully consistent relativistic RPA. The isoscalar GMR, GQR and isovector GDR for some double magic nuclei, such as $^{208}$Pb, $^{144}$Sm, $^{114}$Sn, $^{90}$Zr were performed with different effective Lagrangian parameter sets NL3\textsuperscript{11}, NL1\textsuperscript{12}, NL-SH\textsuperscript{13}, and TM1\textsuperscript{14}. A good agreement with experimental data is obtained\textsuperscript{11}.

In this letter, we aim at the investigation on the systematic behavior of the isovector GDR in $\beta$-stable nuclei.
by focusing our attention to the centroid energies of response functions. By evaluating various moments of the response functions, we extract the centroid energies of response functions for each nucleus and compare with the available experimental data and discuss the systematic behavior. The method employed in our investigation is the fully consistent relativistic RPA \cite{10, 11}, which is a relativistic extension of non-relativistic RPA for studying microscopically nuclear dynamic excitations and giant resonances. The motivation of this work is to test the effective Lagrangians recently developed in a more wide case. We shall investigate the electric GDR relevant to the symmetry energy coefficient.

The response function of a quantum system to an external field is given by the imaginary part of the polarization operator, 

\[ R^L(Q, Q; k, k'; E) = \frac{1}{\pi} Im \Pi^L(Q, Q; k, k'; E), \]

where \( Q \) is an external field operator. The relativistic RPA polarization operator is obtained by solving the Bethe-Salpeter equation,

\[ \Pi(Q, Q; k, k', E) = \Pi_0(Q, Q; k, k', E) - \sum_i g_i^2 \int d^3k_1 d^3k_2 \Pi_0(Q, \Gamma_i; k, k_1, E) D_i(k_1, k_2, E) \Pi(\Gamma_i, Q; k_2, k', E), \]

where the \( i \) runs all mesons. In order to obtain the centroid energies of the response functions, we first calculate various moments of the response function in a given energy interval,

\[ m_k = \int_0^{E_{max}} R^L(E') E' k dE', \]

\[ E_{max} \] is the maximum excitation energy, which is carried out till 60 MeV in the present calculations. From those moments, then, we can define the centroid energy of the response function,

\[ \bar{E} = m_1/m_0. \]

We apply the fully consistent relativistic RPA to the collective excitations, especially in the isovector GDR mode. The isovector dipole operator is \( Q = \gamma^\lambda r Y_{10}^\lambda \), which excites an \( L = 1 \) type electric (spin-non-flip) \((\Delta T = 1 \) and \( \Delta S = 0 \)) giant resonance with spin and parity \( J^\pi = 1^- \). The response functions of the nuclear system to the external operator are calculated at the limit of a small momentum transfer. It is also necessary to include the space-like parts of vector mesons in the relativistic RPA calculations, although they do not play role on the ground state \cite{12}. The self-consistent treatment guarantees the conservation of the vector current.

First, we show nuclear matter properties: incompressibility \( K_\infty \), effective mass \( m^*/M \), and symmetry energy \( a_{sym} \) in Table I, which are calculated in the RMF with various non-linear parameter sets: NL-SH, TM1, NL3, and NL1. The calculated centroid energies of the isovector GDR in the relativistic RPA with those non-linear models for some double closed shell nuclei: \(^{208}\text{Pb}, \) \(^{116}\text{Sn}, \) \(^{90}\text{Zr}, \) and \(^{40}\text{Ca} \) are also listed in Table I. We also compare the calculated values with the corresponding experimental data. It is found that the relativistic RPA with those effective interactions can to a great extent give a good description of the experimental results for the centroid energies of the isovector GDR. According to the macroscopic hydrodynamics models \cite{21} the restoring force of the giant dipole mode is proportional to the symmetry energy of nuclear system. Although the nuclear symmetry energies for the four parameter sets vary from 43.6 MeV to 36.1 MeV, all of them give more or less reasonable centroid energies of the isovector GDR once they could well describe ground state properties of nuclei. The results of the centroid energy of the isovector GDR do not show a strong relationship between excited energies of GDR and the nuclear symmetry energy. Actually, even the symmetry energy at saturation density is not well constrained experimentally. Recently, it is reported theoretically that the nuclear matter symmetry energy at saturation density (volume asymmetry) can be located in the range of 32 MeV \( \leq a_4 \leq 36 \) MeV based on effective Lagrangians with density-dependent meson-nucleon vertex function in the framework of RMF theory. Up to now the density dependence of the nuclear symmetry energy is still under investigation by various theoretical approaches \cite{22, 24}.

|                  | NL-SH | TM1 | NL3 | NL1 | Exp. |
|------------------|-------|-----|-----|-----|------|
| \( K_\infty [\text{MeV}] \) | 355.36 | 281.0 | 271.76 | 211.29 |
| \( M^*/M \)      | 0.597 | 0.634 | 0.600 | 0.570 |
| \( a_{sym} [\text{MeV}] \) | 36.1 | 36.9 | 37.4 | 43.6 |
| \(^{208}\text{Pb}\) | 13.43 | 13.07 | 13.16 | 13.83 | 13.3±0.1 |
| \(^{116}\text{Sn}\) | 16.04 | 15.71 | 15.77 | 16.05 | 15.7±0.2 |
| \(^{90}\text{Zr}\) | 17.47 | 17.09 | 17.19 | 17.59 | 16.5±0.2 |
| \(^{40}\text{Ca}\) | 19.97 | 19.79 | 19.57 | 19.83 | 19.8±0.5 |

TABLE I: Nuclear matter properties: incompressibility \( K_\infty \), effective mass \( M^*/M \), and symmetry energy \( a_{sym} \), calculated in the RMF with various parameter sets. The centroid energies of the isovector GDR for \(^{208}\text{Pb}, \) \(^{116}\text{Sn}, \) \(^{90}\text{Zr}, \) and \(^{40}\text{Ca} \) are calculated in the relativistic RPA with the corresponding parameter set, respectively. The experimental data are taken from Ref. \cite{2}. All energy values in the table are in unit of MeV.

We also apply the relativistic RPA to make a systematic investigation on isovector dipole mode for stable nuclei along \( \beta \)-stability line. It is known that the best results have been obtained with the NL3 effective interaction for ground state properties as well as the collective giant resonance. Therefore, we shall employ the NL3 effective interaction to perform the relativistic RPA calculations for response functions of isovector dipole mode. In our investigation we consider even-even, odd and odd-odd nuclei, which are stable against \( \beta \)-decay in a wide
range of mass number $12 < A < 240$. There are more than 130 nuclei under our investigation, including spherical, deformed, and some stable isotope nuclei. In our calculations a spherical symmetry is assumed to simplify the relativistic RPA calculations, because most of nuclei we are dealing with are of closed shell or sub-closed shell, even are open shell. For some open-shell nuclei the last single particle level is partially occupied, the excitation of a deeper state to this partially occupied state is allowed. In order to take account of such excitations, an average assumption is adopted and each wave function is multiplied by its occupation factor $^{25}$.

The centroid energies of the isovector GDR for nuclei along $\beta$-stability line as a function of nuclear mass number. The solid-circles represent the experimental data. The theoretical results calculated with parameter set NL3 are indicated by stars. The lines (solid and dash) represent the results of the giant resonance energy as a function of nuclear mass number calculated with different systematic behaviors, see the text for details.

In a summary, we have studied the isovector GDR for some double closed shell nuclei: $^{208}$Pb, $^{116}$Sn, $^{90}$Zr, and $^{40}$Ca in a fully consistent relativistic RPA with different effective Lagrangians recently developed. By compare the centroid energies of those nuclei with experimental data, it is found that the parameter set NL3 can perform a good description on the properties of isovector GDR. The centroid energies of nuclei along $\beta$-stability line calculated with the parameter set NL3 are compared with corresponding experimental data and make a systematic investigation. The general conclusion is that the effective Lagrangian with an appropriate nuclear symmetry energy, which can well describe the ground state properties of nuclei, could also reproduce the isovector giant dipole resonances of nuclei along the $\beta$-stability line.

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FIG. 1: The centroid energies of the isovector giant dipole resonance for various nuclei along the $\beta$-stability line as a function of nuclear mass number. The solid-circles represent the experimental data. The theoretical results calculated with parameter set NL3 are indicated by stars. The lines (solid and dash) represent the results of the giant resonance energy as a function of nuclear mass number calculated with different systematic behaviors, see the text for details.

In Fig.1, we give the result calculated by eq(5), which is plotted by a solid curve. At the same time, we also show the curve calculated by the systematic behavior, $E_m = 31.2 A^{-1/3} + 20.6 A^{-1/6}$.

$$E_m = 31.2 A^{-1/3} + 20.6 A^{-1/6}.$$  \hspace{1cm} (5)

In a summary, we have studied the isovector GDR for some double closed shell nuclei: $^{208}$Pb, $^{116}$Sn, $^{90}$Zr, and $^{40}$Ca in a fully consistent relativistic RPA with different effective Lagrangians recently developed. By compare the centroid energies of those nuclei with experimental data, it is found that the parameter set NL3 can perform a good description on the properties of isovector GDR. The centroid energies of nuclei along $\beta$-stability line calculated with the parameter set NL3 are compared with corresponding experimental data and make a systematic investigation. The general conclusion is that the effective Lagrangian with an appropriate nuclear symmetry energy, which can well describe the ground state properties of nuclei, could also reproduce the isovector giant dipole resonances of nuclei along the $\beta$-stability line.

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[6] L’Huillier M et al 1989 Phys. Rev. C39 2022
[7] Shepard J R et al 1989 Phys. Rev. C40 2320
[8] Serot B D, Walecka J D 1986 Adv. Nucl. Phys. 16 1
[9] Ma Z Y et al 1997 Phys. Rev. C55 2385
[10] Ma Z Y et al 1997 Nucl. Phys. A627 1
[11] Giai V N et al 1999 Nucl. Phys. A649 37c
[12] Ma Z Y 1999 Commn. Theor. Phys. 32 493
[13] Ma Z Y et al 2001 Nucl. Phys. A686 173
[14] Ring P et al 2001 Nucl. Phys. A694 249
[15] Vretenar D et al 1999 Nucl. Phys. A649 29c
[16] Lalazissis G A et al 1997 Phys. Rev. C55 540

[17] Reinhard P -G et al 1984 Z. Phys. A323 13
[18] Sharma M M et al 1993 Phys. Lett. B312 377
[19] Sugahara Y et al 1993 Nucl. Phys. A575 557
[20] Ma Z Y et al 2002 Nucl. phys. A703 222
[21] Goldhaber M et al 1947 Phys. Rev. 74 1046
[22] Vretenar D et al nucl-th/0302070
[23] Satula W et al nucl-th/0211044
[24] Todd B G et al nucl-th/0301092
[25] Ma Z Y et al 1997 Prog. of Theor. Phys. 98 91