Polarization from a Strongly Magnetized Accretion Disks: Asymptotic Wavelength Behaviour

Gnedin Yu.N.\(^{(1)}\), Silant’ev N.A.\(^{(1,2)}\), Shternin P.S.\(^{(3)}\)

\(^{(1)}\) Central Astronomical Observatory at Pulkovo, Saint-Petersburg, Russia
\(^{(2)}\) Instituto National de Astrofisica, Optica y Electronica, Pue, Mexico
\(^{(3)}\) State Politechnical University, Saint-Petersburg, Russia

February 7, 2020

Abstract

We calculate the polarization of radiation from thick accretion disks with vertically averaged global magnetic field. The polarization arises as a result of the radiation scattering by free electrons in magnetized plasma of a disk. We consider as a basic effect the Faraday rotation of polarization plane along the photon propagation in a magnetized disk. The various models of optically thick accretion disk with a vertically averaged magnetic field are considered. The main goal of this paper is to obtain simple asymptotic formulae for the polarization of radiation when the Faraday rotation angle \(\Psi\gg 1\) at the Thomson optical length \(\tau \approx 1\). The results of our calculation allows us to estimate the magnetic field magnitude near the marginally stable orbit region of a black hole via the data of polarimetric observations including the expected future X-ray polarimetric observations. The wavelength dependence of polarization is strongly dependent on various models of an accretion disks and thus allows to choose a real model from the polarization data.

1 Introduction

An accretion disk is one of the basic elements for structure of accreting flows around the compact objects - neutron stars, black holes, quasars (QSO) and active galactic nuclei (AGN). Due to the absence of the axial symmetry with respect to a line of sight, the total integrated radiation emerging from accretion disks should be polarized via the electron scattering process. The presence of a magnetic field gives a new effect provided the Faraday rotation of a polarization plane along a photon mean free path in scattering medium. Namely, a nontrivial wavelength dependence of polarization arises when the Faraday rotation angle \(\Psi\) at the Thomson optical depth \(\tau\) (see Gnedin and Silant’ev, 1997)

\[
\Psi = 0.4 \left( \frac{\lambda}{1\mu m} \right)^2 \left( \frac{B}{1G} \right) \tau \cos \theta
\]

is sufficiently large. Here, \(\lambda\) is the radiation wavelength and \(\theta\) is the angle between the line of sight and magnetic field \(B\).

The key point is a solution of the Milne problem in the case of magnetized atmosphere. This problem was considered by many authors, who obtained the numerical solutions (Silant’ev, 1994, 2002, Agol and Blaes, 1996, Agol et al., 1998, Shternin et al. 2003).

The main goal of this paper is to obtain the analytical asymptotic formulae for the polarization of radiation and to use them for analysis of various models of an accretion disk with a global magnetic field. Here, it is more convenient to use the simple approximate formulae for the Stokes parameters of emerging radiation from magnetized, plane-parallel optically thick atmosphere, obtained by Silant’ev, 2002.
The Degree of Polarization from Optically Thick Accretion Disks with a Strong Magnetic Field

Silant’ev, 2002, obtained the quite simple analytical approximate formulae for the degree of polarization of radiation emerging from optically thick accretion disk which for considered case vertically averaged magnetic field acquires the form

$$p_l(\mu) = \frac{1 - g}{1 + g} \cdot \frac{1 - \mu^2}{J(\mu)} \cdot \frac{1}{\sqrt{(1 - k \mu)^2 + \delta^2 \mu^2}}. \quad (2)$$

where $\mu = \cos \theta$, $\theta$ is the angle between the magnetic field directed along the outward normal (z-axis) and the radiation emerging from a disk. The function $J(\mu)$ describes the angular distribution of emerging radiation. This function and the numerical parameters $g$ and $k$ are tabulated by Silant’ev, 2002. The parameter of the Faraday depolarization can be introduced from Eq. (1)

$$\delta = 0.8 \left( \frac{\lambda}{1\mu m} \right)^2 \left( \frac{B}{1G} \right). \quad (3)$$

We consider the case of dominant electron scattering process inside an accretion disk ($k = 0, g = 0.83255$).

Photons escape the optically thick disk basically from the surface layer with $\tau \approx 1$. If the Faraday rotation angle $\Psi$ corresponding to this optical length becomes greater than unity, then the emerging radiation will be depolarized as a result of the summarizing of radiation fluxes with very different angles of Faraday’s rotation. Only for directions that are perpendicular to the vertical magnetic field the Faraday rotation angle is too small to yield depolarization effect. Certainly, the diffusion of radiation in the inner parts of a disk depolarizes it even in the absence of magnetic field because of multiple scattering of photons. The Faraday rotation only increases the depolarization process. It means that the polarization of outgoing radiation acquires the peak-like angular dependence with its maximum for the direction perpendicular to magnetic field. The sharpness of the peak increases with increasing magnetic field magnitude. The main region of allowed angles appears to be $\sim 1/\delta$.

Another the very important feature characterizing the polarized radiation is the wavelength dependence of polarization degree that is strongly different from that for Thomson’s scattering.

For strong magnetic field magnitude (or large wavelength) when $\delta \mu \gg 1$ the simple asymptotic formulae takes place

$$p_l(B) \approx p_l(B = 0) \delta \mu, \quad (4)$$

where $p_l(B = 0)$ is the classical Chandrasekhar-Sobolev polarization due to pure electron scattering without magnetic field.

Silant’ev, 2002, has considered also the other forms of magnetic field distributions, namely, pure radial and toroidal magnetic fields. In both cases the Eqs. (2) and (3) take the same form with changing parameter $\delta \mu$ to $\delta (1 - \mu^2)^{1/2}$. For chaotic magnetic field the analytical asymptotic dependence does not radically change, i.e. $p_l(B) \sim 1/\delta$.

Here we restrict ourselves to the case of an accretion disk with vertically averaged magnetic field. Others distributions of a magnetic field will be considered in separate works.

Models of Thick Accretion Disks with Vertically Averaged Magnetic Field

First of all we start with a short review of present models of an accretion disk with vertically averaged magnetic field (see, for example, Campbell, 1997, Casse and Keppens, 2002, Li, 2002, Wang et al., 2003, Romanova et al., 2003, Pariev et al., 2003, Turner et al., 2003. Campbell, 1997, in his book "Magnetohydrodynamics in binary stars" considered the case of a simple dipole magnetic field. For radial distances from gravitation centre $r \gg z$ he presented the magnetic field distribution as

$$B_z = \frac{1}{2} B_p \left( \frac{R_p}{r} \right)^3, B_r \sim \frac{z B_z}{r}. \quad (5)$$
The processes of spin-up and spin-down of magnetized stars with accretion disks and outflows are considered by Romanova et al., 2003. They have made three-dimensional simulations of disk accretion to an inclined dipole. It was found the radial dependence of vertically averaged magnetic field as \( B_z \sim r^{-5/4} \). Using their result one can estimate the vertical magnetic field magnitude as

\[
B_z \approx 2.4 \cdot 10^4 \left( \frac{10^8 M_\odot}{M} \right) \left( \frac{\dot{M}}{10^{28} g/s} \right) \left( \frac{r}{10R_g} \right)^{-5/4} (G),
\]

where \( M \) is the mass of a compact object (a black hole) and \( \dot{M} \) is an accretion rate magnitude, \( R_g \) is a gravitational radius.

Li, 2002, obtained for self-gravitating magnetically support disks the following radial dependence of the vertically averaged magnetic field

\[
B_z \approx B_r \approx \left( \frac{2\pi G \Sigma M}{R_g^2} \right)^{1/2} \left( \frac{R_g}{r} \right),
\]

where \( \Sigma = \dot{M}/2\pi rv_r \) is the surface density of a disk. If the equipartition condition would be accepted it provides the following dependence

\[
B_z \approx 10^2 \left( \frac{10^8 M_\odot}{M} \right) \left( \frac{\dot{M}}{10^{28} g/s} \right)^{1/2} \left( \frac{r}{10R_g} \right)^{-3/2}. \tag{8}
\]

Casse and Keppens, 2002, developed a model depending on the central region of a disk, i.e., on \( \beta = 8\pi P_{gas}(z = 0)/B^2(z = 0) \). They obtain the following expression for vertically averaged magnetic field

\[
B_z \approx B_{10} \left( \frac{10R_g}{r} \right)^{5/2}. \tag{9}
\]

Pariev et al., 2003, extended the Shakura-Sunayev approach to strongly magnetized accretion disk model. They assume that the radial dependence of the vertically averaged magnetic field in the disk is described by the power law

\[
B_z = B_{10} \left( \frac{r}{10R_g} \right)^{-\eta}, \tag{10}
\]

where \( B_{10} \) is the strength of the magnetic field at \( 10R_g \) and \( \eta > 0 \) is some constant. They made numerical calculations and presented plots of radial structure and emission spectra from the disk in the region where it is optically thick for four choices of basic parameters: \( \eta = 5/4, B_{10} = 3 \cdot 10^3 G; \eta = 1, B_{10} = 3 \cdot 10^4 G; \eta = 1.4, B_{10} = 5 \cdot 10^4 G; \eta = 5/4, B_{10} = 700 G \) and for the mass of supermassive black hole \( M = 10^8 M_\odot \).

Liu et al., 2003, developed also the model that appears quite close to Pariev et al. model. Liu et al., 2003, obtained the following expression for the averaged magnetic field

\[
B_z = 7.2 \cdot 10^4 \alpha_{01}^{-9/20} \beta_1^{-1/2} \left( \frac{M}{10^8 M_\odot} \right)^{-9/20} \left( \frac{\dot{M}}{0.1 M_E} \right)^{2/5} r_{10}^{-51/40}. \tag{11}
\]

Here, \( \alpha \) is the Shakura-Sunayev viscosity coefficient and \( M_E \) is the Eddington accretion rate, \( \dot{M}_E = 1.4 \cdot 10^{26} (M/10^8 M_\odot) g s^{-1} \). One can see that their radial dependence of the averaged magnetic field is practically the same as in Pariev et al. model with \( \eta = 5/4 \).

Turner et al., 2003, developed the model of radiation pressure supported accretion disk. In their model the vertically averaged magnetic field was presented by the relation

\[
B_z \gtrsim 10^8 \left( \frac{M_\odot}{M} \right)^{1/2} \left( \frac{r}{R_g} \right)^{-\eta}, \tag{12}
\]

where \( \eta = 3/4 \).

The magnetic field was also included in the standard models of an accretion disk developed by Shakura and Sunayev 1973 and by Narayan and Yi, 1995 (ADAF model), without detail specification of
its geometry. For these both models the index $\eta = 5/4$. For example, Shakura and Sunayev gave the following expression for the magnetic field strength

$$B_z = B_{10} \left( \frac{10R_g}{r} \right)^{51/40} \lesssim 6 \cdot 10^7 \left( \frac{M}{M_\odot} \right)^{-17/20} \left( \frac{\dot{M}}{10^{17} \text{g/s}} \right)^{2/5} \left( \frac{10R_g}{r} \right)^{51/40}. \quad (13)$$

It means that $B_{10} \equiv B_{10}(M, \dot{M})$ and $\eta = 51/40 \approx 5/4$.

The effective temperature of a disk is determined by

$$\sigma T_e^4(r) = \frac{3GM\dot{M}}{8\pi r^3} \left( 1 - \sqrt{\frac{3R_g}{r}} \right). \quad (14)$$

The disk radiates as a black body. It means that the peak spectral wavelength is $\lambda = 0.29/T_e$. The last expression allows to get the relation between radial distance from a compact object and the peak spectral wavelength corresponding to this distance

$$\frac{r}{10R_g} = 1.3 \cdot 10^2 \left( \frac{\lambda_{\text{m}}}{1\mu\text{m}} \right)^2 M_8^{1/3} \left( \frac{\dot{M}}{M_\odot} \right)^{1/3}. \quad (15)$$

As a result one obtains

$$\delta = 0.8 \cdot (1.3 \cdot 10^2)^{-\eta} \left( \frac{\lambda_{\text{m}}}{1\mu\text{m}} \right)^{\eta - 4/3} M_8^{\eta/3} \left( \frac{\dot{M}}{M_\odot} \right)^{-\eta/3} B_{10}. \quad (16)$$

The Eq. (16) can also be rewritten in the form

$$\delta = 0.8 \cdot (0.92)^\eta \cdot 10^{-13\eta/3} \left( \frac{\lambda_{\text{m}}}{1\mu\text{m}} \right)^{\eta - 4/3} \left( \frac{M}{M_\odot} \right)^{2\eta/3} \left( \frac{\dot{M}}{10^{17} \text{g/s}} \right)^{-\eta/3} B_{10}. \quad (17)$$

The strong depolarization takes place only for those wavelengths which yield $\delta > 1$.

## 4 The Faraday Depolarization Parameter $\delta$ for Various Disk Models

We calculate now the value of the Faraday depolarization parameter $\delta$ for various models of an accretion disk. We are starting with the situation of an accretion disk in close X-ray binaries and consider two cases of magnetic field power law: $\eta = 3$ (dipole field) and more popular case when $\eta = 5/4$. For $\eta = 3$ one obtains from Eq. (17)

$$\delta = 0.5 \cdot 10^{-11} \left( \frac{M}{10M_\odot} \right)^2 \left( \frac{\dot{M}}{10^{17} \text{g/s}} \right)^{-1} \left( \frac{\lambda_{\text{m}}}{1\mu\text{m}} \right)^{-2} B_{10}. \quad (18)$$

For optical (V band) photons $\delta \approx 2 \cdot 10^{-11} \left( \frac{M}{10M_\odot} \right)^2 \left( \frac{\dot{M}}{10^{17} \text{g/s}} \right)^{-1} B_{10}$. It means that for depolarization of optical photons too much magnetic strength $B > 10^{11}(G)$ is required. For X-rays with the energy $E = 1\text{keV}$ ($\lambda = 1.2\mu\text{m}$) the magnetic field strength $B \gtrsim 10^4(G)$ provides the depolarization of X-ray photons from optically thick accretion disk.

For $\eta = 5/4$ and for V-band of radiation the depolarization parameter becomes equal to $\delta = 1.35 \cdot 10^{-5} \left( \frac{M}{10M_\odot} \right)^{5/6} \left( \frac{\dot{M}}{10^{17} \text{g/s}} \right)^{-5/12} B_{10}$, and magnetic field strength at the inner radius of a disk $B > 10^5(G)$ is quite sufficient for strong depolarization. The result of calculation parameter $\delta$ for other values of the index $\eta$ are presented at Table 1 (V-band photons) and Table 2 (X-ray photons with the energy 1keV). The most surprising result arises when $\eta = 3/2$. In this case the depolarization parameter $\delta$ does not depend on the wavelength (energy) of photons from a disk. It means that the polarization will not depend on the wavelength (energy) of photons as it happens in the classical Sobolev-Chandrasekhar case. However, the value of polarization will be less due to the depolarization effect.

One can predict also that, if $\eta > 3/2$ the hard energy photons will be tested by most strong depolarization than soft energy photons, and the polarization spectrum will drop in the hard energy region.
On the other way, if $\eta < 3/2$, the soft energy photons will be stronger depolarized and the spectrum will drop in the soft energy region.

5 The Asymptotic Wavelength Dependence of Polarization from the Magnetized Standard Accretion Disk

Now we present the asymptotic formulae for the polarization degree using the result of analytical calculations by Silant’ev, 2002. (see Eqs. and ).

According to Eq.(4) the asymptotic wavelength dependence of the optical polarization is determined by

$$p_l \approx p_l (B = 0) / \delta \mu \sim 10^{2\eta} \left( \lambda / 1\mu m \right) \left( \frac{M}{M_E} \right) \left( \frac{\dot{M}}{\dot{M}_E} \right) \frac{\eta}{3} \mu / \mu, \quad (19)$$

or

$$p_l \sim p_l (B = 0) \left( \frac{M}{10^{17} g/s} \right) \left( \frac{\lambda / 1\mu m}{3} \right) \left( \frac{\dot{M}}{\dot{M}_E} \right) \left( \frac{\eta}{3} \right) B_{10} \sim \lambda^{4\eta-5} / \mu. \quad (20)$$

Thus the asymptotic wavelength behaviour of the polarization allows to derive the value of the index of the power-law radial dependence of the vertically averaged magnetic field for the standard magnetized accretion disk. The results of the calculations for the various models of the standard magnetized disk are presented in the Table 3.

The basic result from the Table 3 is the radical change of the wavelength dependence of the polarization at the region of the index values $\eta = 1.5$. For the more strong radial dependence of the magnetic field ($\eta > 3/2$) the polarization is increasing with the increase of the wavelength. For the more weak radial dependence ($\eta < 3/2$) the polarization is increasing in the short wavelength region, i.e. $p_l \sim \lambda^{-n}$, where $n = f(\eta)$. It means that for standard magnetized accretion disk the only fact of the qualitative wavelength dependence of polarization (increase or decrease the wavelength) gives an evidence of the radial dependence of the magnetic field inside an accretion disk.

6 Summary and Conclusions

We presented the formulae for the asymptotic wavelength dependence of the polarization of radiation from thick accretion disks with vertically averaged magnetic field. The polarization arises as a result of the scattering of radiation by electrons in magnetized plasma of a disk. The effect of the Faraday rotation of polarization plane in a magnetized disk is taken into account. We considered the different models of the thick accretion disk with the vertically averaged magnetic field. The asymptotic wavelength behaviour of the net polarization displays very strongly the power-law radial dependence of the vertical magnetic field. It appears that for more strong dependence (the index of the power-law radial dependence $\eta > 1.5$) the hard energy photon will be tested by more strong depolarization than soft photons. For more smooth radial dependence of the magnetic field the soft energy photons will be stronger depolarized and spectrum of polarized radiation will drop in the soft energy region. This fact displays the qualitative estimation the power-law radial dependence of the magnetic field using only the simple observations of the super scale dependence of polarization respect to wavelength of photon (increase or decrease depending on the wavelength).

7 Acknowledgements

One of the authors (P.S.S.) is grateful to the Dynasty Foundation and ICFPM for financial support.

This work was supported by the RFBR grant 03-02-17223, the Program of the Presidium of RAN "Nonstationary Phenomena in Astronomy", the Program of the Department of Physical Sciences of RAN "The Extended Structure..., and by the Program of Russian Education and Science Department.
References

[1] Gnedin Yu.N., Silant’ev N.A. 1997, Basic Mechanisms of Light Polarization in Cosmic Media (Hartwood Academic Publ., Amsterdam).

[2] Shternin P.S., Gnedin Yu.N., Silant’ev N.A. 2003, Astrofizika, v.46, p.434.

[3] Agol E., Blaes O. 1996, MNRAS, v.282, p. 965.

[4] Agol E., Blaes O., Ionescu-Zanetti C. 1998, MNRAS, v.293, p.1?.

[5] Casse, Keppens, 2002, astro-ph/0208459

[6] Li Z.-Y. 2002, astro-ph/0207014

[7] Wang D.-X., MA R.-Yi., Lei W.-H. 2003, Astrophys. J., v. 595, p.1?.

[8] Romanova M.M., Ustyugova G.V., Koldova A.V., Wick J.V. and Lovelace R.V.E. 2003, Astrophys J., v.595, p.1009.

[9] Pariev V.I., Blackman E.G., Boldyrev S.A. 2003, Astron. Astrophys., v.407, p.403.

[10] Turner N. 2003, astro-ph/0304511

[11] Shakura N.I., Sunyaev R.A. 1973, Astron. Astrophys., v.24, p.3.

[12] Narayan R., Yi I. 1995, Astrophys. J., v.452, p. 710.

[13] Silant’ev N.A. 2002, Astron. Astrophys., v.383, p.326.

[14] Campbell C.G. 1997, Magnetohydrodynamics in Binary Stars, Kluwer Academic Publs., Dordrecht/Boston/London.

[15] Liu B.F., Mineshige S., Oshuga K. 2003, Astrophys. J., v.587, p.561.
Table 1.
The values of the magnetic field strengths $B_{10}$ for various magnitudes of the power law index of the magnetic field distributions in the standard accretion disk for V-band photons, if $\delta > 1$.

| $\eta$ | 3   | 5/2 | 3/2 | 1.4 | 5/4 | 1   | 3/4 | 0  |
|--------|-----|-----|-----|-----|-----|-----|-----|----|
| $B_{10}$ | $10^{14}$ | $1.2 \cdot 10^9$ | $5 \cdot 10^8$ | $2.5 \cdot 10^9$ | $10^9$ | $10^4$ | $5 \cdot 10^4$ | 5  |

Table 2.
The values of the magnetic field strengths $B_{10}$ for various magnitudes of the power law index of the magnetic field distributions in the standard accretion disk for X-ray photons ($E_x \approx 1keV$), if $\delta > 1$.

| $\eta$ | 3   | 5/2 | 3/2 | 1.4 | 5/4 | 1   | 3/4 | 0  |
|--------|-----|-----|-----|-----|-----|-----|-----|----|
| $B_{10}$ | $10^4$ | $1.3 \cdot 10^4$ | $5 \cdot 10^6$ | $8 \cdot 10^5$ | $1.2 \cdot 10^6$ | $3 \cdot 10^6$ | $6 \cdot 10^6$ | $10^8$ |

Table 3.
The asymptotic wavelength dependence of the polarization for the standard magnetized accretion disk.

| $\eta$ | 3   | 5/2 | 3/2 | 1.4 | 5/4 | 1   | 3/4 | 0  |
|--------|-----|-----|-----|-----|-----|-----|-----|----|
| $p_{\lambda}(\lambda) \sim$ | $\lambda^2$ | $\lambda^{4/3}$ | $\lambda^{1/2}$ | $\lambda^{-0.133}$ | $\lambda^{1/3}$ | $\lambda^{-2/3}$ | $\lambda^{-1}$ | $\lambda^{-2}$ |