The Universe as a Nonuniform Lattice in Finite-Volume Hypercube.
I. Fundamental Definitions and Particular Features

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Abstract

In this paper a new small parameter associated with the density matrix deformation (density pro-matrix) studied in previous works of the author is introduced into the Generalized Quantum Mechanics (GQM), i.e. quantum mechanics involving description of the Early Universe. It is noted that this parameter has its counterpart in the generalized statistical mechanics. Both parameters offer a number of merits: they are dimensionless, varying over the interval from 0 to 1/4 and assuming in this interval a discreet series of values. Besides, their definitions contain all the fundamental constants. These parameters are very small for the conventional scales and temperatures, e.g. the value of the first parameter is on the order of $\approx 10^{-66+2n}$, where $10^{-n}$ is the measuring scale and the Planck scale $\sim 10^{-33} cm$ is assumed. The second one is also too small for the conventional temperatures, that is those much below the Plancks. It is demonstrated that relative to the first of these parameters the Universe may be considered as a nonuniform lattice in the four-dimensional hypercube with dimensionless finite-length...
edges. And the time variable is also described by one of the above-mentioned dimensions due to the second parameter and generalized uncertainty relations in thermodynamics. In this context the lattice is understood as a deformation rather than approximation.

1 Introduction

In the last decades the scientists have become aware that Quantum Mechanics of the early Universe should be different from the classical Quantum Mechanics. To illustrate, in the first the Generalized Uncertainty Relations (GUR) [1],[2] are valid, whereas in the second one the ordinary Uncertainty Relations (UR) of Heisenberg [3]are effective. Resultant from GUR is the fundamental length on the order of Planck’s [4] that is lacking in Quantum Mechanics with UR. Thus, GUR-involving Quantum Mechanics may be considered as a deformation of Quantum Mechanics with UR or in other words Quantum Mechanics with Fundamental Length (QMFL) is a deformation of well-known Quantum Mechanics. The deformation is understood as a theory extension by inclusion of one or several parameters so that the original theory be associated with the limit, where the indicated parameters are tending to some fixed values [5]. QM being a deformation of the Classical Mechanics presents a vivid example. The deformation parameter in this case is Planck’s constant $\hbar$. When $\hbar \to 0$, QM is transformed to the Classical Mechanics. The deformation in Quantum Mechanics at Planck scale takes different paths: commutator deformation or more precisely deformation of the respective Heisenberg algebra [6],[7],[8] and the density matrix deformation approach [9]–[15]. The first approach suffers from two serious disadvantages: 1) the deformation parameter is a dimensional variable $\kappa$ with a dimension of mass [6]; 2) in the limiting transition to QM this parameter goes to infinity and fluctuations of other values are hardly sensitive to it. The second approach is devoid of such limitations as in this approach the deformation parameter is represented by the dimensionless quantity $\alpha = l^2_{\text{min}}/x^2$, where $x$ is the measuring scale and the variation interval $\alpha$ is finite $0 < \alpha \leq 1/4$ [9]–[13]. Moreover, it gives a key to the solution of particular problems: an extra term in Liouville equation in the processes associated with black holes [11]–[13]; singularities and cosmic censorship [12],[13]; derivation of a semiclassical Bekenstein-Hawking formula for the black hole entropy and some others [13],[16]. In [16]–[18] it
has been shown that within the developing paradigm there is a possibility for the solution of Hawkings information Paradox problem [19]–[21], and \( \alpha \) may be interpreted as a new small parameter of quantum theory. Besides, over the dimensionless interval \( I_{1/4} = (0; 1/4) \) this parameter takes on a series of discrete values nonuniformly filling the indicated interval as distinct from the conventional lattice. This lattice may be considered in the ordinary cube, each edge of which is associated with a particular space dimension. This property of \( \alpha \) will be considered further together with similar feature for its counterpart in Statistical Mechanics - \( \tau, \tau \in I_{1/4} \) parameter. It will be demonstrated that due to the latter and Generalized Uncertainty Relations in thermodynamics the time variable may be also treated as a discrete and nonuniform series over the same interval \( I_{1/4} = (0; 1/4) \). Thus, any theory may be considered as a nonuniform lattice in the four-dimensional hypercube \( I_{1/4}^4 \). In this context the lattice is understood as a deformation rather than approximation.

## 2 Relevant Suggestions and Refinements

In this section we recall in short the introduction of \( \alpha \) and \( \tau \) parameters into the generalized Quantum and Statistical Mechanics. Here generalized means the Quantum and Statistical Mechanics describing the processes both in the current (conventional scales) and early (scales on the order of Planck’s) Universe. Now it is obvious that in the latter the notion of the fundamental (minimum) length \( l_{\text{min}} \sim l_p \) is a requisite [4], where \( l_p \) is the Plancks length. It has been demonstrated [9]–[13] that on retention of a well-known measuring procedure the density matrix becomes dependent on the additional parameter \( \alpha = l_{\text{min}}^2/x^2 \), where \( x \) is the measuring scale. In this way the density matrix is subjected to the deformation procedure (e.g. [2]), the resultant deformed object is referred to as density pro-matrix, whereas the conventional density matrix and exact definition will be as follows: [9]–[13]:

**Definition 1.** (Quantum Mechanics with Fundamental Length [for Neumann’s picture])

Any system in QMFL is described by a density pro-matrix of the form

\[
\rho(\alpha) = \sum_i \omega_i(\alpha) |i><i|, 
\]
where

1. Vectors \( |i> \) form a full orthonormal system;
2. \( \omega_i(\alpha) \geq 0 \) and for all \( i \) the finite limit \( \lim_{\alpha \to 0} \omega_i(\alpha) = \omega_i \) exists;
3. \( Sp[\rho(\alpha)] = \sum_i \omega_i(\alpha) < 1, \sum_i \omega_i = 1 \);
4. For every operator \( B \) and any \( \alpha \) there is a mean operator \( B \) depending on \( \alpha \):
   \[
   < B >_\alpha = \sum_i \omega_i(\alpha) < i|B|i >.
   \]
5. The following condition should be fulfilled:
   \[
   Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] \approx \alpha.
   \] (1)
   Consequently we can find the value for \( Sp[\rho(\alpha)] \) satisfying the above-stated condition:
   \[
   Sp[\rho(\alpha)] \approx \frac{1}{2} + \sqrt{\frac{1}{4} - \alpha}
   \] (2)
   and therefore
6. \( 0 < \alpha \leq 1/4 \).

It is no use to enumerate all the evident implications and applications of Definition 1., better refer to [12,13]. Nevertheless, it is clear that for \( \alpha \to 0 \) the above limit covers both the Classical or Quantum Mechanics depending on \( \hbar \to 0 \) or not.

It should be noted that according to Definition 1. a minimum measurable length is equal to \( l_{\text{min}}^* = 2l_{\text{min}} \) being a nonreal number at point \( l_{\text{min}}, Sp[\rho(\alpha)] \). Because of this, a space part of the Universe is a lattice with a spacing of \( a_{\text{min}} = 2l_{\text{min}} \sim 2l_p \). In consequence the first issue concerns the lattice spacing of any lattice-type model(for example [22, 23]): a selected lattice spacing \( a_{\text{lat}} \) should not be less than \( a_{\text{min}} \), i.e. always \( a_{\text{lat}} \geq a_{\text{min}} > 0 \). Besides, a continuum limit in any lattice-type model is meaning \( a_{\text{lat}} \to a_{\text{min}} > 0 \) rather than \( a_{\text{lat}} \to 0 \).

Proceeding from \( \alpha \), for each space dimension we have a discrete series of
rational values for the inverse squares of even numbers nonuniformly distributed along the real number line $\alpha \div 1/4, 1/16, 1/36, 1/64, \ldots$. A question arises, is this series somewhere terminated or, on the contrary, is it infinite? The answer depends on the answers to two other questions: (1) is there theoretically a maximum measurability limit for the scales $l_{\text{max}}$; and (2) is our Universe closed in the sense that its extension may be sometime replaced by compression, when a maximum extension precisely gives a maximum scale $l_{\text{max}}$? Should an answer to one of these questions be positive, we should have condition 6 Definition 1. rather than $0 < l_{\text{min}}^2/l_{\text{max}}^2 \leq \alpha \leq 1/4$.

Note that in the majority of cases all three space dimensions are equal, at least at large scales, and hence their associated values of $\alpha$ parameter should be identical. This means that for most cases, at any rate in the large-scale (low-energy) limit, a single deformation parameter $\alpha$ is sufficient to accept one and the same value for all three dimensions to a high degree of accuracy. In the general case, however, this is not true, at least for very high energies (on the order of the Plancks), i.e. at Planck scales, due to noncommutativity of the spatial coordinates [1],[2],[6]:

$$[x_i, x_j] \neq 0$$

In consequence in the general case we have a point with coordinates $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ in the normal(three-dimensional) cube $I_{1/4}^3$ of side $I_{1/4} = (\{0, 1/4\}$.

It should be noted that this universal cube may be extended to the four-dimensional hypercube by inclusion of the additional parameter $\tau$, $\tau \in I_{1/4}$ that is generated by internal energy of the statistical ensemble and its temperature for the events when this notion is the case. It will be recalled that $\tau$ parameter occurs from a maximum temperature that is in its turn generated by the Generalized Uncertainty Relations of energy time pair in GUR. The exact definition [14],[15] is as follows:

**Definition 2. (Deformation of Statistical Mechanics)**

Deformation of Gibbs distribution valid for temperatures on the order of the Planck’s $T_p$ is described by deformation of a statistical density matrix (statistical density pro-matrix) of the form

$$\rho_{\text{stat}}(\tau) = \sum_n \omega_n(\tau) |\varphi_n><\varphi_n|$$

having the deformation parameter $\tau = T^2/l_{\text{max}}^2$, where

1. The vectors $|\varphi_n>$ form a full orthonormal system;
2. \( \omega_n(\tau) \geq 0 \) and for all \( n \) at \( \tau \ll 1 \) we obtain \( \omega_n(\tau) \approx \frac{1}{Q} \exp(-\beta E_n) \).

In particular, \( \lim_{\tau \to 0} \omega_n(\tau) = \omega_n \).

3. \( Sp[\rho_{stat}(\tau)] = \sum_n \omega_n(\tau) < 1, \sum_n \omega_n = 1; \)

4. For every operator \( B \) and any \( \tau \) there is a mean operator \( B \) depending on \( \tau \)

\[ < B >_{\tau} = \sum_n \omega_n(\tau) < n|B|n > . \]

5. Finally, the following condition must be fulfilled:

\[ Sp[\rho_{stat}(\tau)] - Sp^2[\rho_{stat}(\tau)] \approx \tau. \]  

Hence we can find the value for \( Sp[\rho_{stat}(\tau)] \) satisfying the condition of Definition 2 (similar to Definition 1):

\[ Sp[\rho_{stat}(\tau)] \approx \frac{1}{2} + \sqrt{\frac{1}{4} - \tau}. \]

This implies that

6. \( 0 < \tau \leq 1/4 \)

So \( \tau \) is a counterpart (twin) of \( \alpha \), yet for the Statistical Mechanics. At the same time, originally for \( \tau \) nothing implies the discrete properties of parameter \( \alpha \) indicated above:

for \( \tau \) there is a discrete series (lattice) of the rational values of inverse squares for even numbers not uniformly distributed along the real number line: \( \tau \div 1/4, 1/16, 1/36, 1/64, ... \).

Provided such a series exists actually,

* the finitness and infinity question for this series amounts to two other questions:

(1) is there theoretically any minimum measurability limit for the average temperature of the Universe \( T_{min} \neq 0 \) and (2) is our Universe closed in a sense that its extension may be sometime replaced by compression? Then maximum extension just gives a minimum temperature \( T_{min} \neq 0 \).

The question concerning the discretization of parameter \( \tau \) is far from being idle. The point is that originally by its nature this parameter seems to be continuous as it is associated with temperature. Nevertheless, in the
following section we show that actually \( \tau \) is dual in nature: it is directly related to time that is in turn quantized in the end giving a series \( \tau \div \frac{1}{4}, \frac{1}{16}, \frac{1}{36}, \frac{1}{64}, \ldots \).

### 3 Dual Nature of Parameter \( \tau \) and its Temporal Aspect

In this way when at point \( \tilde{\alpha} \) of the normal (three-dimensional) cube \( I_{1/4}^3 \) of side \( I_{1/4} = (0; 1/4) \) an additional temperature variable \( \tau \) is added, a nonuniform lattice of the point results, where we denote \( \tilde{\alpha}_\tau = (\tilde{\alpha}, \tau) = (\alpha_1, \alpha_2, \alpha_3, \tau) \) at the four-dimensional hypercube \( I_{1/4}^4 \), every coordinate of which assumes one and the same discrete series of values: \( 1/4, 1/16, 1/36, 1/64, \ldots \). (Further it is demonstrated that \( \tau \) is also taking on a discrete series of values.) The question arises, whether time falls within this picture. The answer is positive. Indeed, parameter \( \tau \) is dual (thermal and temporal) in nature owing to introduction of the Generalized Uncertainty Relations in Thermodynamics (GURT) [15],[24],[25]:

\[
\Delta \frac{1}{T} \geq \frac{k}{\Delta U} + \alpha' \frac{1}{T_p^2} \frac{\Delta U}{k} + \ldots,
\]

where \( k \) - Boltzmann constant, \( T \) - temperature of the ensemble, \( U \) - its internal energy. A direct implication of the latter inequality is occurrence of a maximum temperature \( T_{max} \) that is inversely proportional to minimal time \( t_{min} \approx t_p \): (11)):

\[
T_{max} = \frac{h}{2\sqrt{\alpha' t_p k}} = \frac{h}{\Delta t_{min} k}.
\]

However, \( t_{min} \) follows from the Generalized Uncertainty Relations in Quantum Mechanics for energy-time pair [14],[15],[24],[25]:

\[
\Delta t \geq \frac{\hbar}{\Delta E} + \alpha' t_p^2 \frac{\Delta E}{\hbar},
\]
Thus, $T_{max}$ is the value relating GUR and GURT together \cite{15,24,25}:

\[
\begin{align*}
\Delta x & \geq \frac{\hbar}{\Delta p} + \alpha L_p^2 \frac{\Delta p}{\hbar} + ... \\
\Delta t & \geq \frac{\hbar}{\Delta E} + \alpha t_p^2 \frac{\Delta E}{\hbar} + ... \\
\Delta \frac{1}{T} & \geq \frac{k}{\Delta U} + \alpha' \frac{1}{T_p^2} \frac{\Delta U}{k} + ..., \\
\end{align*}
\]

(5)

, since the thermodinamical value $T_{max}$ (GURT) is associated with the quantum-mechanical one $E_{max}$ (GUR) by the formula \cite{14,15,24,25}:

\[
T_{max} = \frac{E_{max}}{k}
\]

The notion of value $t_{min} \sim 1/T_{max}$ is physically crystal clear, it means a minimum time for which any variations in the energy spectrum of every physical system may be recorded. Actually, this value is equal to $t_{min}^* = 2t_{min} \sim t_p$ as at the initial points $l_{min}$ and $T_{max}$ the spurs of the quantum-mechanical and statistical density pro-matrices $\rho(\alpha)$ and $\rho_{stat}(\tau)$ are complex, determined only beginning from $2l_{min}$ $T_{max}^* = \frac{1}{2}T_{max}$ \cite{13}–\cite{15} that is associated with the same time point $t_{min}^* = 2t_{min}$. For QMFL this has been noted in the previous section.

In such a manner a discrete series $l_{min}^*, 2l_{min}^*, ...$ generates in QMFL the discrete time series $t_{min}^*, 2t_{min}^*, ...$, that is in turn associated (due to GURT) with a discrete temperature series $T_{max}^* \frac{1}{2}T_{max}^*, ...$. From this it is inferred that a temperature scale $\tau$ may be interpreted as a temporal one $\tau = t_{min}^2/t^2$.

In both cases the generated series has one and the same discrete set of values of parameter $\tau : \tau = 1/4, 1/16, 1/36, 1/64, ..., 1/4n^2, ...$. Thus, owing to time quantization in QMFL one is enabled to realize quantization of temperature in the generalized Statistical Mechanics with the use of GURT.

Using $Lat_{\tilde{\alpha}}$ we denote the lattice in cube $I_{1/4}^3$ formed by points $\tilde{\alpha}$, and through $Lat_{\tilde{\tau}}^{\tau}$ we denote the lattice in hypercube $I_{1/4}^4$, that is formed by points $\tilde{\alpha}_{\tau} = (\tilde{\alpha}, \tau)$. 

8
4 Quantum Theory for the Lattice in Hypercube

Any quantum theory may be defined for the indicated lattice in hypercube. To this end it is required to go from Neumann’s picture to Shrödinger’s picture. We recall the fundamental definition [13], [17], [18] with $\alpha$ changed by $\tilde{\alpha}$:

**Definition 1’ Quantum Mechanics with Fundamental Length (Shrödinger’s picture)**

Here, the prototype of Quantum Mechanical normed wave function (or the pure state prototype) $\psi(q)$ with $\int |\psi(q)|^2 dq = 1$ in QMFL is $\psi(\tilde{\alpha}, q) = \theta(\tilde{\alpha})\psi(q)$. The parameter of deformation $\tilde{\alpha} \in I^{3/4}$. Its properties are $|\theta(\tilde{\alpha})|^2 < 1$, $\lim_{|\tilde{\alpha}| \to 0} |\theta(\tilde{\alpha})|^2 = 1$ and the relation $|\theta(\alpha_i)|^2 - |\theta(\alpha_i)|^4 \approx \alpha_i$ takes place. In such a way the total probability always is less than 1: $p(\tilde{\alpha}) = |\theta(\tilde{\alpha})|^2 = \int |\theta(\tilde{\alpha})|^2 |\psi(q)|^2 dq < 1$ tending to 1, when $\|\tilde{\alpha}\| \to 0$. In the most general case of the arbitrarily normed state in QMFL (mixed state prototype) $\psi = \psi(\tilde{\alpha}, q) = \sum_n a_n \theta_n(\tilde{\alpha}) \psi_n(q)$ with $\sum_n |a_n|^2 = 1$ the total probability is $p(\tilde{\alpha}) = \sum_n |a_n|^2 |\theta_n(\tilde{\alpha})|^2 < 1$ and $\lim_{\|\tilde{\alpha}\| \to 0} p(\tilde{\alpha}) = 1$.

It is natural that Shrödinger equation is also deformed in QMFL. It is replaced by the equation

$$\frac{\partial \psi(\tilde{\alpha}, q)}{\partial t} = \frac{\partial [\theta(\tilde{\alpha}) \psi(q)]}{\partial t} = \frac{\partial \theta(\tilde{\alpha})}{\partial t} \psi(q) + \theta(\tilde{\alpha}) \frac{\partial \psi(q)}{\partial t},$$

where the second term in the right-hand side generates the Shrödinger equation as

$$\theta(\tilde{\alpha}) \frac{\partial \psi(q)}{\partial t} = -\frac{i}{\hbar} H \psi(q).$$

Here $H$ is the Hamiltonian and the first member is added similarly to the member that appears in the deformed Liouville equation, vanishing when $\theta[\tilde{\alpha}(t)] \approx const$. In particular, this takes place in the low energy limit in QM, when $\|\tilde{\alpha}\| \to 0$. It should be noted that the above theory is not a time reversal of QM because the combination $\theta(\tilde{\alpha}) \psi(q)$ breaks down this property in the deformed Shrödinger equation. Time-reversal is conserved only in the low energy limit, when a quantum mechanical Shrödinger equation is valid.

According to **Definition 1’ everywhere q is the coordinate of point at the three-dimensional space. As indicated in [9]–[18], for a density pro-matrix
there exists an exponential ansatz satisfying the formula

\[ \rho^*(\alpha) = \sum_i \omega_i exp(-\alpha)|i><i|, \]  

(8)

where all \( \omega_i > 0 \) are independent of \( \alpha \) and their sum is equal to 1. In this way \( Sp[\rho^*(\alpha)] = exp(-\alpha) \). Then in the momentum representation \( \alpha = p^2/p_{max}^2, p_{max} \sim p_{pl} \), where \( p_{pl} \) is the Planck momentum. When present in matrix elements, \( exp(-\alpha) \) damps the contribution of great momenta in a perturbation theory.

It is clear that for each of the coordinates \( q_i \) the exponential ansatz makes a contribution to the deformed wave function \( \psi(\tilde{\alpha}, q) \) the modulus of which equals \( exp(-\alpha_i/2) \) and, obviously, the same contribution to the conjugate function \( \psi^*(\tilde{\alpha}, q) \). Because of this, for exponential ansatz one may write

\[ \psi(\tilde{\alpha}, q) = \theta(\tilde{\alpha})\psi(q), \]  

(9)

where \( |\theta(\tilde{\alpha})| = exp(-\sum_i \alpha_i/2) \). As noted above, the last exponent of the momentum representation reads \( exp(-\sum_i p_i^2/2p_{max}^2) \) and in this way it removes UV (ultra-violet) divergences in the theory.

It follows that \( \tilde{\alpha} \) is a new small parameter. Among its obvious advantages one could name:

1) its dimensionless nature,
2) its variability over the finite interval \( 0 < \alpha_i \leq 1/4 \). Besides, for the well-known physics it is actually very small: \( \alpha \sim 10^{-66+2n} \), where \( 10^{-n} \) is the measuring scale. Here the Planck scale \( \sim 10^{-33}cm \) is assumed;
3) and finally the calculation of this parameter involves all three fundamental constants, since by Definition 1 of section 2 \( \alpha_i = l_{min}^2/x_i^2 \), where \( x_i \) is the measuring scale on \( i \)-coordinate and \( l_{min}^2 \sim l_{pl}^2 = G\hbar/c^3 \). Therefore, series expansion in \( \alpha_i \) may be of great importance. Since all the field components and hence the Lagrangian will be dependent on \( \tilde{\alpha} \), i.e. \( \psi = \psi(\tilde{\alpha}), L = L(\tilde{\alpha}) \), quantum theory may be considered as a theory of lattice \( Lat_\tilde{\alpha} \) and hence of lattice \( Lat^z_\tilde{\alpha} \).

5 Introduction of Quantum Field Theory and Initial Analysis

With the use of this approach for the customary energies a Quantum Field Theory (QFT) is introduced with a high degree of accuracy. In our context
customary means the energies much lower than the Planck ones. It is important that as the spacing of lattice \( \text{Lat}_{\tilde{\alpha}} \) is decreasing in inverse proportion to the square of the respective node, for a fairly large node number \( N > N_0 \) the lattice edge beginning at this node \( \ell_{N,N+1} \) will be of length \( \ell_{N,N+1} \sim 1/4N^3 \), and by this means edge lengths of the lattice are rapidly decreasing with the spacing number. Note that in the large-scale limit this (within any preset accuracy) leads to parameter \( \alpha = 0 \), pure states and in the end to QFT. In this way a theory for the above-described lattice presents a deformation of the originally continuous variant of this theory as within the developed approach continuity is accurate to \( \approx 10^{-66+2n} \), where \( 10^{-n} \) is the measuring scale and the Planck scale \( \sim 10^{-33} \) cm is assumed. Whereas the lattice per se \( \text{Lat}_{\tilde{\alpha}} \) may be interpreted as a deformation of the space continuum with the deformation parameter equal to the varying edge length \( \ell_{\alpha_1,\alpha_2} \), where \( \alpha_1, \alpha_2 \) are two adjacent points of the lattice \( \text{Lat}_{\tilde{\alpha}} \). Proceeding from this, all well-known theories including \( \phi^4 \), QED, QCD and so on may be studied based on the above-described lattice.

Here it is expedient to make the following remarks:

1. going on from the well-known energies of these theories to higher energies (UV behavior) means a change from description of the theories behavior for the lattice portion with high edge numbers to the portion with low numbers of the edges;
2. finding of quantum correction factors for the primary deformation parameter \( \tilde{\alpha} \) is a power series expansion in each \( \alpha_i \). In particular, in the simplest case (Definition 1′) means expansion of the left side in relation \( |\theta(\alpha_i)|^2 - |\theta(\alpha_i)|^4 \approx \alpha_i \):

\[
|\theta(\alpha_i)|^2 - |\theta(\alpha_i)|^4 = \alpha_i + a_0\alpha_i^2 + a_1\alpha_i^3 + ...
\]

and calculation of the associated coefficients \( a_0, a_1, ... \)

This approach to calculation of the quantum correction factors may be used in the formalism for density pro-matrix (Definition 1). In this case, the primary relation \( \square \) may be written in the form of a series

\[
Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0\alpha^2 + a_1\alpha^3 + ...
\]

As a result, a measurement procedure using the exponential ansatz \( \square \) may be understood as the calculation of factors \( a_0, a_1, ... \) or the definition of additional members in the exponent destroying \( a_0, a_1, ... \) \( \square, \square \). It is
easy to check that the exponential ansatz gives $a_0 = -3/2$, being coincident with the logarithmic correction factor for the Black Hole entropy [26].

Most often a quantum theory is considered at zero temperature $T = 0$, in this context amounting to nesting of the three-dimensional lattice $\text{Lat}_\tilde{\alpha}$ into the four-dimensional one: $\text{Lat}_\tau^\tau : \text{Lat}_\tilde{\alpha} \subset \text{Lat}_\tau^\tau$ and nesting of the cube $I^3_1$ into the hypercube $I^4_{1/4}$ as a bound given by equation $\tau = 0$. However, in the most general case the points with nonzero values of $\tau$ may be important as there is a possibility for nonzero temperature $T \neq 0$ (quantum field theory at finite temperature) that is related to the value of $\tau$ parameter, though very small but still nonzero: $\tau \neq 0$. To illustrate: in QCD for the normal lattice [27] a critical temperature $T_c$ exists so that the following is fulfilled:

$$T < T_c$$

case of the confinement phase occurs,

and for

$$T > T_c$$

case of the deconfinement is the case.

A critical temperature $T_c$ is associated with the critical parameter $\tau_c = T^2_c / T^2_{\max}$ and the selected bound of hypercube $I^4_{1/4}$ set by equation $\tau = \tau_c > 0$.

6 Conclusion

The principal issue of the present work is the development of a unified approach to study all the available quantum theories without exception owing to the proposed small dimensionless parameter deformation parameter: $\tilde{\alpha} \in \text{Lat}_\tau^\tau$ that is in turn dependent on all the fundamental constants $G, c, h$ and $k$.

Thus, there is a reason to believe that lattices $\text{Lat}_\tilde{\alpha}$ and $\text{Lat}_\tau^\tau$ may be a universal means to study different quantum theories. This poses a number of intriguing problems:

1) description of a set of lattice symmetries $\text{Lat}_\tilde{\alpha}$ and $\text{Lat}_\tau^\tau$.

2) for each of the well-known physical theories ($\phi^4$, QED, QCD and so on) definition of the selected (special) points (phase transitions, different symmetry violations, etc.) associated with the above-mentioned lattices.
These problems of current importance necessitate further investigation by the author.

References

[1] D. V. Ahluwalia, Quantum Measurement, Gravitation, and Locality, Phys. Lett. B339 (1994) 301; A. Kempf, G. Mangano, R. B. Mann. Hilbert Space Representation of the Minimal Length Uncertainty Relation. Phys. Rev. D52 (1995) 1108.

[2] D.V. Ahluwalia, Wave-Particle duality at the Planck scale: Freezing of neutrino oscillations, Phys. Lett. A275 (2000) 31; Interface of Gravitational and Quantum Realms, Mod. Phys. Lett. A17 (2002) 1135.

[3] W. Heisenberg. Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Zeitsch. fur Phys. 43, 172-184, (1927).

[4] R.J. Adler and D.I. Santiago. On Gravity and the Uncertainty Principle. Mod. Phys. Lett. A14, 1371-1377 (1999).

[5] L. Faddeev. Mathematical View on Evolution of Physics. Priroda. 5, 11-18, (1989).

[6] M. Maggiore, The algebraic structure of the generalized uncertainty principle, Phys. Lett. B319 (1993) 83.

[7] M. Maggiore, Quantum Groups, Gravity and Generalized Uncertainty Principle, Phys. Rev. D49 (1994) 5182.

[8] S. Capozziello, G. Lambiase and G. Scarpetta, The Generalized Uncertainty Principle from Quantum Geometry, Int. J. Theor. Phys. 39 (2000), 15.

[9] A.E. Shalyt-Margolin, Fundamental Length, Deformed Density Matrix and New View on the Black Hole Information Paradox, gr-qc/0207074.

[10] A.E. Shalyt-Margolin and A. Ya. Tregubovich, Generalized Uncertainty Relations, Fundamental Length and Density Matrix, gr-qc/0207068.

[11] A.E. Shalyt-Margolin and J.G. Suarez, Density Matrix and Dynamical aspects of Quantum Mechanics with Fundamental Length, gr-qc/0211083.
[12] A.E.Shalyt-Margolin and J.G.Suarez, Quantum Mechanics of the Early Universe and its Limiting Transition, gr-qc/0302119

[13] A.E.Shalyt-Margolin and J.G.Suarez, Quantum Mechanics at Planck’s scale and Density Matrix, Intern. Journ. of Mod. Phys. D. 12(2003)1265

[14] A.E.Shalyt-Margolin, Density Matrix in Quantum and Statistical Mechanics at Planck-Scale gr-qc/0307057

[15] A.E.Shalyt-Margolin, A.Ya.Tregubovich, Deformed Density Matrix and Generalized Uncertainty Relation in Thermodynamics, Mod. Phys. Lett. A, Vol.19, No.1(2004)pp.71-81, hep-th/0311034.

[16] A.E.Shalyt-Margolin. Deformed density matrix, Density of entropy and Information problem, gr-qc/0307096.

[17] A.E.Shalyt-Margolin, Non-Unitary and Unitary Transitions in Generalized Quantum Mechanics and Information Problem Solving, hep-th/0309121

[18] A.E.Shalyt-Margolin, Non-Unitary and Unitary Transitions in Generalized Quantum Mechanics, New Small Parameter and Information Problem Solving, (to be published in Mod. Phys. Lett. A), hep-th/0311239.

[19] S.Hawking, Breakdown of Predictability in Gravitational Collapse, Phys. Rev. D14(1976)2460

[20] S.Giddings, The Black Hole Information Paradox, hep-th/9508151

[21] A.Strominger, Les Houches Lectures on Black Holes, hep-th/9501071.

[22] H.Grosse, Models in Statistical Physics and Quantum Field Theory, Springer-Verlag, 1988

[23] C.Itzykson, J-M.Drouffe, Statistical Field Theory, Vol.1,2., Cambridge University Press, Cambridge, 1991.

[24] A.E.Shalyt-Margolin and A.Ya.Tregubovich, Generalized Uncertainty Relations in a Quantum Theory and Thermodynamics From the Uniform Point of View gr-qc/0204078
[25] A.E. Shalyt-Margolin and A.Ya. Tregubovich, Generalized Uncertainty Relations in Thermodynamics [gr-qc/0307018].

[26] P. Majumdar, Black hole entropy: classical and quantum aspects, Expanded version of lectures given at the YATI Conference on Black Hole Astrophysics, Kolkata, India, April 2001, hep-th/0110198; S. Das, P. Majumdar and R. K. Bhaduri, General Logarithmic Corrections to Black Hole Entropy hep-th/0111001; E. C. Vagenas, Semiclassical Corrections to the Bekenstein-Hawking Entropy of the BTZ Black Hole via Selfgravitation, Phys. Lett. B533 (2002) 302, hep-th/0109108.

[27] Adriano Di Giacommo, Confinement of Color: A Review, Talk at XII International Conference on Selected Problems of Modern Physics, Dubna, June 2003, hep-lat/0310023.