Searching for the relativistic Sunyaev-Zeldovich effect with the PLANCK experiment

Torsten A. Enßlin\(^1\) and Steen H. Hansen\(^2\)

\(^1\) Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, Postfach 1317, 85741 Garching, Germany
\(^2\) University of Zurich, Winterthurerstrasse 190, 8057 Zurich, Switzerland

Abstract. Populations of relativistic electrons in clusters of galaxies should imprint a characteristic spectral distortion signature on the Cosmic Microwave Background (CMB) photons – the relativistic Sunyaev-Zeldovich (SZ) effect. We investigate how sensitive PLANCK and successor experiments will be in detecting or constraining the total optical depth \(\tau_{\text{rel}}\) of relativistic electrons. We find an expected sensitivity of PLANCK of \(\sigma_{\tau_{\text{rel}}} \sim 2 \cdot 10^{-5}\) for an individual cluster, which is too insensitive to detect even optimistic scenarios, which predict \(\tau_{\text{rel}} \sim 10^{-6}\) to \(10^{-7}\). However, by stacking the PLANCK-signal of all SZ detectable clusters the relativistic SZ effect can be statistically probed to a sensitivity level of \(\sigma_{\tau_{\text{rel}}} \sim 10^{-7}\). PLANCK successor experiments will be able to probe even conservative scenarios of relativistic electron populations in clusters of galaxies.

Key words. Cosmic microwave background – Intergalactic medium – Galaxies: cluster: general – Radiation mechanisms: non-thermal – Scattering – Submillimeter

1. Introduction

Clusters of galaxies contain a several keV hot thermal plasma, which is able to Comptonize the CMB while its photons are traversing the cluster. The resulting observable spectral distortions of the CMB – the SZ effect (Sunyaev & Zeldovich 1972) – are discriminated by the different nature of the energy forms of the scattering electrons: the thermal energy manifests itself in the thermal SZ (tSZ) effect, and the kinetic energy of a gas bulk motion in the kinematic SZ (kSZ) effect. The spectral characteristics of the tSZ and the kSZ effects are independent of the electrons’ energies as long as the electrons’ velocities are much smaller than the speed of light \(c\). However, the thermal electrons in hot galaxy clusters with temperatures of \(\approx 10\ \text{keV}\) have velocities of the order of \(0.2\ c\), and therefore relativistic corrections to the canonical tSZ spectral distortions have to be taken into account (Wright 1979).

In addition to the mildly relativistic thermal electrons there are also highly relativistic electrons present in galaxy clusters, which have their own characteristic relativistic SZ (rSZ) spectral distortion signature (Enßlin & Kaiser 2000). It is the aim of this work to investigate how future sensitive spectral measurements of CMB distortions caused by galaxy clusters can be used in order to investigate or constrain such electron populations.

We know about the existence of highly relativistic electrons in galaxy clusters since extended, cluster-wide radio emission, the so called cluster radio halos, has been detected in roughly one third of the X-ray brightest galaxy clusters (Giovannini et al. 1999). The radio halo emission is produced by radio synchrotron emitting ultra-relativistic electrons with energies of the order of 10 GeV. Relativistic electrons at energies far below a GeV have escaped our measurements so far since their synchrotron emission is at frequencies below the Earth ionosphere transparency cutoff, although their dim synchrotron self-Comptonized radiation should fall into our telescope’s frequency bands (Enßlin & Sunyaev 2002). However, large numbers of such relativistic electrons are expected, not only as a low energy continuation of the observed radio emitting electron spectra, but also due to numerous physical generation mechanisms. Such mechanisms are cluster merger shock waves, radio galaxy outflows, and electron production in inelastic interactions of relativistic protons with the thermal gas nucleons (Enßlin 1999b; Brunetti 2002, for reviews). There are two very different habitats of relativistic electrons in galaxy clusters. The first is the relatively dense thermal gas, in which Coulomb cooling should quickly thermalize any significant electron population below 100 MeV (Sarazin 1999). The second are the fossil remnants of former radio galaxy cocoons, the radio quiet ghosts cavities (Enßlin 1999a), in which even MeV electrons may survive for cosmological times due to the low gas density there. The physical origin of any high energy electrons is very interesting, and since our present
observational knowledge about these MeV to GeV electrons is poor (Enßlin & Biermann 1998, for a compilation of the available spectral information on Coma), any means to obtain additional information is highly desirable. In this paper we will analyze the ability of the PLANCK experiment or its successors to detect or constrain such electrons.

In contrast to a thermal electron population\(^1\), the spectral shape of a relativistic electron population is poorly known. At GeV energies, they are well described by power laws. However their number density is generally dominated by the unknown lower end of their spectral distribution, which is expected to have a broad peak. Since the rSZ effect depends practically only on the total number of relativistic electrons and very little on their spectral distribution we approximate the spectrum by a delta function centered on that spectral peak. As will become clearer in the following, this idealized treatment is sufficient for our investigations of the sensitivity to detect the rSZ effect. A more realistic treatment would introduce several poorly constrained parameters like spectral index and low energy cutoff and its detailed shape, for which the natural choice is not obvious.

2. Spectra of CMB distortions

The PLANCK experiment should be able to identify \(\sim 4 \cdot 10^4\) clusters of galaxies via their tSZ effect (e.g. Bartelmann 2001). In order to be able to do spectral precision measurements of the CMB distortions caused by a galaxy cluster one has to suppress other contaminations as far as possible. A way to do this is optimal spatial filtering, which allows to remove efficiently foregrounds like galactic dust and synchrotron emission, but also CMB temperature fluctuations, which have their spatial power mostly on large angular scales (Haehnelt & Tegmark 1996). In the following we assume that such a filtering step was applied to the frequency maps of PLANCK, leading to galaxy cluster spectra which are mostly free of such contaminations. Since the galactic synchrotron and dust emission might have some power on small angular scales, which would partly survive such a filtering step, we further assume that the lowest and highest frequency channels of PLANCK are used to remove these residual contaminations.

The CMB-blackbody spectrum

\[
I_\nu = i_0 \frac{x^3}{e^x - 1},
\]

with \(x = h\nu/kT\) and \(i_0 = 2(kT_{\text{cmb}})^3/(hc)^2\) becomes distorted due to the inverse Compton scattering off the electrons in a galaxy cluster. The spectral distortions are described by

\[
\delta i(x) = g(x) y_{\text{gas}} \left(1 + \delta(x, T_e)\right) - h(x) \frac{\beta_{\text{gas}}}{\tau_{\text{gas}}} + (j(x, p) - i(x)) \tau_{\text{rel}}.
\]

The first rhs term describes the tSZ distortions, which have a shape given by

\[
g(x) = \frac{x^4 e^x}{(e^x - 1)^2} \left(\frac{e^x + 1}{e^x - 1} - 4\right),
\]

and a magnitude described by the Comptonization parameter \(y_{\text{gas}} = \sigma_T/(m_e c^2) \int dl n_{e,\text{gas}} kT_e\). For non-relativistic electrons the relativistic correction term is zero, \(\delta(x, T_e) = 0\), but for hot clusters even the thermal electrons are slightly relativistic, which will modify the thermal SZ effect (Wright 1979). These corrections are easily calculated (see e.g. Rephaeli 1995; Itoh & Nozawa 2003; Enßlin & Kaiser 2000; Dolgov et al. 2001), and can be used to measure the cluster temperature purely from SZ observations (Hansen et al. 2002). For the PLANCK channels this is most easily implemented using the simple fitting formulae presented in Diego et al. (2003).

The second rhs term in Eq. 2 gives the kSZ distortions, which have the spectral shape

\[
h(x) = \frac{x^4 e^x}{(e^x - 1)^2}
\]

and depend on \(\beta_{\text{gas}}\), the average line-of-sight streaming velocity of the thermal gas (\(v_{\text{gas}} = \beta_{\text{gas}} c\), \(\beta_{\text{gas}} > 0\) if gas is approaching the observer), and the Thomson optical depth \(\tau_{\text{gas}} = \sigma_T \int dl n_{e,\text{gas}}\).

Finally, the last rhs term in Eq. 2 describes the distortion by relativistic electrons, which we approximate to be mono-energetically distributed. We characterize the electron energy via the dimensionless momentum \(p = \beta_{\text{rel}}/(n_e c)\), where \(P_e\) is the usual electron momentum. The optical depth \(\tau_{\text{rel}} = \sigma_T \int dl n_{e,\text{rel}}\) determines the fraction of strongly scattered photons. Here, \(n_{e,\text{rel}}\) is the number density of relativistic electrons. The spectral distortions \(\delta i_{\text{rel}} = (j(x, p) - i(x))\tau_{\text{rel}}\) are described by two terms: The IC scattering redistributes the CMB photons, which means that photons disappear from the CMB spectrum according to \(-\tau_{\text{rel}} i(x)\), but reappear at different frequencies like \(\tau_{\text{rel}} j(x, p)\), which depends on the electron momentum. In general, \(\delta i_{\text{rel}}\) has to be calculated numerically (Enßlin & Kaiser 2000), however the non- and ultra-relativistic limits can be treated analytically.

In the non-relativistic limit (\(p \ll 1\)), a mono-energetic electron population leads to spectral distortions which are identical to those of the well-known tSZ effect with

\[
\delta i_{\text{rel}} = \frac{p^2}{3} g(x) \tau_{\text{rel}}.
\]

In the ultra-relativistic limit (\(p \gg 1\)) CMB photons simply disappear from the spectral range of interest since they are scattered to very high frequencies, leading to

\[
\delta i_{\text{rel}} = -i(x) \tau_{\text{rel}}.
\]

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\(^1\) The reported high energy X-ray excess of the Coma galaxy cluster (Fusco-Femiano et al. 1999) triggered some speculations about the existence of a supra-thermal electron population in between 10 and 100 keV, which also would produce a unique SZ signature (Enßlin et al. 1999; Enßlin & Kaiser 2000; Blasi et al. 2000; Blasi 2000; Liang et al. 2002; Colafrancesco et al. 2003). However, such a population is questioned on theoretical grounds (Petrosian 2001), and even the high energy X-ray excess itself is under debate (Rossetti & Molendi 2004; Fusco-Femiano et al. 2004).
As long as all electrons are in one of these two limiting cases, the relative shape of the spectral distortion is completely independent of the details of the electron spectrum, only the distortion strength is either proportional to the electrons total energy content (non-relativistic limit) or their number density (ultra-relativistic limit).

Thus, only in the trans-relativistic regime ($p \sim 1$) the shape of the spectral distortions depends on details of the electron spectrum. Since we are interested in the ultra-relativistic case, our approximation of mono- tails of the electron spectrum. Since we are interested the shape of the spectral distortions depends on de-

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relativistic regime. For small $p$ the rSZ spectral distortion
approaches the one of the tSZ effect and thereby our sen-
sitivity to discriminate relativistic electrons decreases.

An order of magnitude estimate of the expected rSZ effect is in order. Any scattering of a CMB photon with an ultra-relativistic electron removes the photon from the typical CMB energy range. The relativistic electron optical depth $\tau_{\text{rel}}$ can be expressed in terms of the tSZ $y$-parameter, if a characteristic ratio of the relativistic electron to thermal gas energy density $X_{\text{rel, e}}$ can be assumed to hold approximately on macroscopic scales throughout the cluster:

$$\tau_{\text{rel}} = \frac{3 X_{\text{rel, e}} y_{\text{gas}}}{\gamma_e - 1}, \quad (7)$$

where $\gamma_e = E_e/(m_e c^2) = \sqrt{1+p^2}$. Inserting some optimistic values for a Coma-like galaxy cluster like $y_{\text{gas}} \sim 10^{-4}$, $X_{\text{rel, e}} \sim 0.03$, and $\gamma_e \sim 10$ for a relativistic electron population residing in radio quiet ghost cavities ($X_{\text{rel, e}} \sim 0.1$, and $\gamma_e \sim 300$ for relativistic electrons mixed with the thermal gas), one gets $\delta I_{\text{rel}}/I = -\tau_{\text{rel}} \sim -10^{-6}$ (or $\delta I_{\text{rel}}/I \sim -10^{-7}$ respectively).

### 3. Extracting the rSZ effect

Our goal is to extract the rSZ spectral signature from PLANCK experiment multi-frequency measurements. In order to estimate the significance of our measurement, we apply a $\chi^2$-statistics and search for the $\chi^2$-minimum with respect to the unknown parameters. Minimization of the parameters within the ranges given in table 1 is done through an extention of the publicly available code sasz (Hansen 2003), based on the technique of simulated annealing. In our case the vector in parameter space, $x$, is a 5 dimensional vector, $x = (y, T_e, v_{\text{gas}}, \tau_{\text{rel}}, p)$. We choose a simple simulated annealing exponential cooling scheme, $T_j = c T_{j-1}$, where $c \approx 0.8 - 1.0$, and we use $T_0 = 1$ and $T_{\text{final}} = 10^{-12}$.

In fig. 1 we plot $\Delta \chi^2$ as a function of the unknown $\tau_{\text{rel}}$ for the PLANCK satellite, where we optimistically assume that only the first and last channels (30 and 857 GHz) are needed for removal of cluster-scale foreground contamination (like point sources, and galactic synchrotron and dust emission). We assume that the 'true' signal is $\tau_{\text{rel}} = 10^{-6}$ (the other unknown parameters are $y = 10^{-5}$, $T_e = 5$ keV, $v_{\text{gas}} = 600$ km/sec, $p = 200$), and we maximize over all the unknown parameters. We find 1$\sigma$ error-bars for the relativistic optical depth $\tau_{\text{rel}}$ just about $\sigma_{\tau_{\text{rel}}} \approx 2 \cdot 10^{-5}$. If one instead excludes also the 545 GHz channel, then the error-bases are increased by approximately a factor of 2. It is comforting to note that the best fit point is indeed very near the 'truth' $10^{-6}$.

We next have to test for parameter degeneracy. As is well known, parameter degeneracy is very important for the error-bars on the normal SZ parameters. E.g. a large negative peculiar velocity can mimic a higher temperature (and slightly changed Compton parameter). In this way a large negative peculiar velocity of a cluster results in larger error-bars on the determination of the peculiar velocity than in the case of a large positive peculiar velocity (Aghanim et al. 2003). We ran a sample of different parameters ($T_e = 2 - 10$ keV, $v_{\text{gas}} = -1000 + 1000$ km/sec, $y = 10^{-6} - 10^{-4}$), and we find both very little parameter degeneracy with $\tau_{\text{rel}}$, and that the error-bar on $\tau_{\text{rel}}$ remains $2 \cdot 10^{-5}$ within a factor of 2 for all the clusters in this sample. This also implies that systematic shifts of the other cluster parameters due to non-removable contamination should not strongly alter our conclusions.

As can be clearly seen on figure 2 the determination of $\tau_{\text{rel}}$ gives much larger error-bars for $p < 3$ (as explained in section 2), compared to the case $p > 3$ with small and constant error-bars. $\tau_{\text{rel}}$ is therefore practically non-determinable for $p < 1$. However, the unproblematic $p > 3$ case is exactly the range we are interested in.

We therefore conclude that the error-bar on the determination of $\tau_{\text{rel}}$ will be about $2 \cdot 10^{-5}$ for any single typical

### Table 1. The allowed parameter range.

| Parameter   | Allowed range                  |
|-------------|--------------------------------|
| $y$         | $10^{-5}$ to $10^{-2}$         |
| $T_e$       | 0 to 25 keV                    |
| $v_{\text{gas}}$ | -5000 to 5000 km/sec        |
| $\tau_{\text{rel}}$ | $-10^{-4}$ to $10^{-4}$       |
| $p$         | 0.01 to 1000                   |

![Fig. 1. $\Delta \chi^2$ as a function of $\tau_{\text{rel}}$, marginalized over the other parameters, ($y, T_e, v_{\text{gas}}, p$). $1\sigma$ error-bar of $\tau_{\text{rel}}$ is about $2 \cdot 10^{-5}.$](image-url)
clusters detectable by PLANCK, roughly independent of other cluster parameters.

4. Conclusion

We estimate the sensitivity of PLANCK to detect the optical depth $\tau_{\text{rel}}$ of relativistic electrons to be $\sigma_{\tau_{\text{rel}}} \sim 2 \times 10^{-5}$ for a single cluster. This is too insensitive to detect even optimistic scenarios for the relativistic electron population, which gives values of $\tau_{\text{rel}} \sim 10^{-7}$ to $10^{-6}$.

However, there is no significant parameter degeneracy with other cluster parameters for the measurement of the optical depth $\tau_{\text{rel}}$ of relativistic electrons. This allows one to stack the weak signals of the large number of galaxy clusters to be detected by PLANCK with the thermal SZ effect. Since PLANCK is expected to observe of the order of $N_{\text{cl}} \approx 4 \times 10^4$ individual clusters (e.g. Bartelmann & White 2002), the expected sensitivity for the statistical detection of the relativistic SZ signal should be of the order $\sigma_{\tau_{\text{rel}}} \sim N_{\text{cl}}^{-1/2} \sim 10^{-7}$. This is sufficient to confirm or refute optimistic scenarios.

Future experiments with increased sensitivities may be able to test for even smaller optical depth of relativistic electron populations. An experiment with 4 observing frequencies at 90, 150, 217 and 270 GHz, and with South-Pole-Telescope-like sensitivity of 0.1 $\mu$K would allow to probe relativistic electrons down to $\sigma_{\tau_{\text{rel}}} \sim 2 \times 10^{-6}$ for an individual cluster. Thus optimistic theoretical scenarios could be tested using a few individual clusters, while statistical measurements would probe even deep into conservative scenarios of relativistic electron populations in galaxy clusters.

We conclude that sensitive CMB experiments like PLANCK allow to constrain or examine the presently poorly known low energy part of the existing relativistic electron populations in galaxy clusters. Therefore they provide a valuable vehicle to explore this terra incognita in electron energies in between MeV and GeV.

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References

Aghanim, N., Hansen, S. H., Pastor, S., Semikoz, D. V., 2003, J. of Cosmology and Astro-Particle Physics 5, 7
Bartelmann, M., 2001, A&A 370, 754
Bartelmann, M., White, S. D. M., 2002, A&A 388, 732
Blasi, P., 2000, ApJL 532, L9
Blasi, P., Olinto, A. V., Stebbins, A., 2000, ApJL 535, L71
Brunetti, G., 2002, in S. Bowyer & C.-Y. Hwang (eds.), Matter and Energy in Clusters of Galaxies, ASP Conference Series, astro-ph/0208074
Colafrancesco, S., Marchegiani, P., Palladino, E., 2003, A&A 397, 27
Diego, J. M., Hansen, S. H., Silk, J., 2003, MNRAS 338, 796
Dolgov, A. D., Hansen, S. H., Pastor, S., Semikoz, D. V., 2001, ApJ 554, 74
Enßlin, T. A., 1999a, in P. S. H. Böhringer, L. Feretti (ed.), Ringberg Workshop on ‘Diffuse Thermal and Relativistic Plasma in Galaxy Clusters’, Vol. 271 of MPE Report, p. 275, astro-ph/9906212
Enßlin, T. A., 1999b, in IAU Symp. 190: ‘The Universe at Low Radio Frequencies’, astro-ph/0001433
Enßlin, T. A., Biermann, P. L., 1998, A&A 330, 90
Enßlin, T. A., Kaiser, C. R., 2000, A&A 360, 417
Enßlin, T. A., Lieu, R., Biermann, P. L., 1999, A&A 344, 409
Enßlin, T. A., Sunyaev, R. A., 2002, A&A 383, 423
Fusco-Femiano, R. et al., 1999, ApJL 513, L21
Fusco-Femiano, R. et al., 2004, ApJL in press, astro-ph/0312625
Giovannini, G., Feretti, L., Govoni, F., 1999, in IAU Symp. 199: ‘The Universe at Low Radio Frequencies’, astro-ph/0006380
Haehnelt, M. G., Tegmark, M., 1996, MNRAS 279, 545
Hansen, S. H., 2003, New Astronomy in press, astro-ph/0310149
Hansen, S. H., Pastor, S., Semikoz, D. V., 2002, ApJL 573, L69
Itoh, N., Nozawa, S., 2003, ApJ Suppl. submitted, astro-ph/0307519
Liang, H., Dögel, V. A., Birkinshaw, M., 2002, MNRAS 337, 567
Petrosian, V. , 2001, ApJ 557, 560
Rephaeli, Y., 1995, ApJ 445, 33
Rossetti, M., Molendi, S., 2004, A&A in press, astro-ph/0312447
Sarazin, C. L., 1999, ApJ 520, 529
Sunyaev, R. A., Zeldovich, Y. B., 1972, Comments on Astrophysics 4, 173
Wright, E. L., 1979, ApJ 232, 348

\[ p = P_\mu / (m_\mu c) \]

Fig. 2. The error-bar on $\tau_{\text{rel}}$ as a function of the dimensionless electron momentum, $p$. The error-bar on $\tau_{\text{rel}}$ is independent of $p$ as long as $p > 3$. 

\[ 0.001 \]

\[ 0.0001 \]

\[ 10 \]

\[ 100 \]

\[ 1000 \]

\[ 7 \times 10^{-9} \]

\[ 2 \times 10^{-6} \]