Structure Design and Performance Analysis of High-Speed Miniature Ball Bearing

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Abstract. The working performances of miniature ball bearings are obviously affected by its' geometric structure parameters. In this paper, quasi-static analysis theory is applied in the design of miniature ball bearings. Firstly, it is studied the influence of geometry structure, preload and rotating speed on the dynamic performance of bearing. Secondly, bearing dynamic characteristics are analyzed which include the bearing stiffness and Spin to roll Ratio. Lastly, the contact stress and bearing life are calculated. The results indicate that structure parameters play an importance role in bearing’s dynamic performances. Miniature ball bearings which have larger ball number, bigger ball diameter and smaller inner race groove radius can get better performances while velocity and preload have great impact on the bearing life. So that parameters of miniature bearing should be chosen cautiously.

1. Introduction
High speed miniature bearings are widely used in high speed instruments, which require high precision and long service life. In order to adapt to a variety of working conditions and satisfy interchangeability, the angular contact miniature ball bearings are often chose in these applications. The design of its geometric structure parameters and the selection of axial preload have close relationship with contact load, contract stress, service life and stiffness. The bearing performance can be affected by the subtle change of structure greatly due to its small geometry [1-2].

In structure design, machining method, performance analysis and measurement method of miniature bearing, many scholars have done a lot of researches. For calculating the critical speed of rotating machinery supported by miniature bearing, the stiffness of miniature bearings under various loads must be predicted first [3-5]. The friction torque is a very important performance of the miniature bearing, and the measurement of the torque is a difficult task in bearing test field Chen [6] has studied the miniature bearing and proposed that the better test data gathered must use more sophisticated test platform. In Gao’s [7] paper the factors which cause the friction torque and the measuring methods for the friction torque of the miniature bearing are summarized and analyzed by him. Li [8] established an analysis modal for calculating high speed miniature ball bearing friction torque based on thermal elastohydrodynamic lubrication (TEHL), which has proved by using high precision miniature bearing testing machine. Zhang [9] and his team mainly analysis the design and manufacture of miniature air bearing, and explore the influence of microscale effect and slip flow effect on lubrication characteristics respectively. A new type of grinding process is proposed in Gui’s [10] paper, which can effectively improve the wear resistant property of miniature bearing. A new kind of high strength alloy material is introduced by Tomasello [11], which can greatly improve the...
corrosion resistance and low temperature properties of miniature bearing. The damages due to contamination and high handling forces and preload options which are used to remove axial play and boost axial and radial stiffness are also presented [12]. According to Zhu’s [13] paper the structure and design scheme of a closed–type miniature bearing are introduced, and the causes of the vibration of the bearing are also analyzed. Based on the quasi-dynamics of high-speed miniature ball bearing and the principle of energy conservation and taking the characteristics of lubrication oil and friction mechanism into account, a mathematical model of friction torque is established in Li’s [14] paper. In this paper, based on the quasi-static model established by A B Jones [15], the calculation equations of miniature bearing parameters such as stiffness, life and contact stress are established.

In conclusion, it’s difficult to design and test the performance of miniature bearing, and it is of great value to study the influence of miniature bearing geometry parameters on the contact load, contact stress and bearing life, stiffness. In this paper, the effects of miniature ball bearing geometric structure parameters such as ball number, ball diameter, contact angle, groove radius on bearing working performances are analyzed. Moreover, some suggestions are put forward on bearing parameter selection. Undoubtedly, it can provide guidance for the design of miniature bearings.

2. Analytical calculation model
According to the quasi-static model established by A B Jones, it is known that combine bearing’s quasi-static equilibrium equation with displacement deformation compatibility equation, then iterate those equations can get many dynamic performance parameters, such as contact deformation, contact load, contact stress, ratio of revolution and rolling and bearing stiffness.

2.1. Contact stress and contact load of ball with ring
Using the iterative method to solve the quasi-static equilibrium equation and displacement-deformation compatibility equation of the bearing can get the contact deformation $\delta_{ij}$ of every ball. According to Hertz contact theory, the contact load between ball and bearing ring is

$$Q_{ij} = K\delta_{ij}^{3/2}$$

(1)

$$K = \frac{1}{(1/K_1)^2 + (1/K_2)^2}$$

(2)

where $Q_{ij}$ is contact load between the ball number $j$ and raceway, for inner ring $i = 1$, for outer ring $i = 2$; $K$ is Load displacement coefficient; $K_1$ is outer ring stiffness; $K_2$ is inner ring stiffness; $\delta_{ij}$ is contact deformation between the ball number $j$ and rings.

The max contact stress between ball and bearing ring is

$$\sigma_{ij} = \frac{3Q_{ij}}{2\pi ab}$$

(3)

where $a$, $b$ is long and short radius of Hertz contact ellipse respectively, the calculation process provided by Wang [16] and Harris [17].

2.2. Bearing stiffness
Putting the contact load $Q_{ij}$, gyro torque $M_{gij}$ of balls and centrifugal force $F_{ci}$ into force balance equation (form a4-6), can get the total bearing loads, which include axial load $F_a$, radial load $F_r$ and angular bending moment load $M$.

$$F_a = \sum_{j=1}^{z} \left[ Q_{ij} \sin \beta_{ij} - \frac{2(1-\lambda)M_{gij}}{d} \cos \beta_{ij} \right]$$

(4)

$$F_r = \sum_{j=1}^{z} \left[ Q_{ij} \cos \beta_{ij} + \frac{2(1-\lambda)M_{gij}}{d} \sin \beta_{ij} \right]$$

(5)
\[ M = \sum_{j=1}^{\kappa} \left[ \frac{Q_{ij}}{d} \sin \beta_{ij} \right] - \frac{2(1-\lambda)}{d} \left( R \cos \beta_{ij} - f_{ij} D_b \right) \cos \phi_i \]  

where \( F_a, F_r \) is axial and radial load of bearing respectively; \( M_{bj} \) is gyro torque of \( j \) ball; \( \beta_{ij} \) is the contact angle between the \( j \) ball and the inner ring; \( \phi_i \) is position angle of \( j \) ball; \( D_b \) is ball diameter; for outer ring \( \lambda=1 \), for inner ring \( \lambda=2 \).

\[
[K] = \begin{bmatrix}
\frac{\partial F_x}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\
\frac{\partial F_x}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\
\frac{\partial M_x}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_z}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \\
\frac{\partial M_x}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_z}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \\
\frac{\partial M_x}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_z}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y}
\end{bmatrix}
\]

(7)

where \( x, y, z \) is the three coordinate axes of the bearing.

Using the variables in formula (3-5) to taking the derivative of the displacement (\( \delta_x, \delta_y, \delta_z, \theta_x, \theta_y \)) of the bearings on the five degrees of freedom, can get the relationship between bearing loads and displacement. The stiffness matrix of bearing on five degrees of freedom can be transformed into the above equation.

2.3. Spin to roll ratio
Spin to roll ratio on behalf the size of spin motion of ball on the ring. With the increasing of the ratio, the spin becomes more violent, the bearing generates more thermal, and also deteriorate the lubrication. According to A B Jones raceway control theory [18], the spin motion of the ball occurs only in the non-controlled raceway, so spin to rolling ratio of the ball in controlled raceway is 0, and in non-controlled raceway the ratio can be got through formula (8) and (9):

\[ \omega_{si} = -\omega_i \sin \beta_i + \omega_i \sin (\beta_i - \alpha) \]  

(8)

\[ \omega_{jol} = -\omega_i \frac{D_u}{d} \]  

(9)

where \( \alpha \) is the angle between the ball spin axis and the central axis of the bearing; \( \beta_i \) is the contact angle of the ball; \( D_u \) is bearing pitch circle diameter; \( \omega_i \) is the relative rotating speed between ring and cage, if the control ring is inner ring \( i = 1 \), if not \( i = 2 \).

2.4. Bearing fatigue life
The fatigue life of bearing is discrete, according to literature [3], it can be got from the following formula:

\[ L_{ci} = \left( \frac{Q_{ci}}{Q_{ci}} \right)^3 \]  

(10)

\[ Q_{ci} = 98.1 \left( \frac{f_i}{f_i - 1} \right)^{0.41} \left( \frac{l_{i3}}{1} \right)^{0.43} \left( \frac{\chi}{8 \beta_i} \right)^{0.41} D_b^{1.3} Z \]  

(11)

\[ Q_{ci} = \left[ \frac{1}{z} \sum_{j=1}^{z} Q_{ci}^{0.41} \right] \]  

(12)

where \( Q_{ci} \) is the rated static load of the ring; \( Q_{ci} \) is working load of the ring; \( f_i \) is curvature radius of the ring; \( Z \) is the number of the ball; for the rotating ring \( e=1/3 \), for static ring \( e=0.3 \).
2.5. Model verification

In order to verify the correctness of the model, it is used to calculate the bearing parameters in Li’s paper and compare the result with it. Bearing parameters shown in table 1, bearing structure shows in figure 1 and the results are as table 2.

Table 1. Bearing parameters in Li’s [19] paper

| Parameters                          | Value |
|-------------------------------------|-------|
| Number of balls                     | 11    |
| Diameter of ball (D_b/mm)           | 3.175 |
| Pitch diameter of bearing (d_m/mm)  | 16    |
| Contact angle (β/°)                 | 15    |
| Outer curvature coefficient (f_1)   | 0.55  |
| Inner curvature coefficient (f_2)   | 0.53  |
| Material                            | GCr15 |
| Axial force P_a (N)                 | 70    |

where \( f_1 = R_1 / D_b \) \( f_2 = R_2 / D_b \)

Figure 1. Schematic diagram of parameters

| Parameters                          | Result of Li’s paper | Result of this paper |
|-------------------------------------|----------------------|----------------------|
| Contact stress \( \sigma \) (MPa)   | 1200                 | 1372                 |
| Stiffness \( K \) (N·mm)            | 22                   | 21                   |
| Spin to roll Ratio                  | 0.27                 | 0.25                 |
| Life \( L \)/h                       | 6000                 | 6600                 |

It can be seen from table 2 that the results of this model are close to Li’s study. So, it indicates that the modal in this paper is credibility.

3. Design calculation and result analysis

Using computer programmes above formula, take a miniature bearing for example whose parameters are shown in table 3. Material of the bearing is GCr15. When it’s working, the rotate speed of outer ring is 36000r/min and the load is 4N, using constant pressure preload. Because of the radial load the stress condition of each ball is inconsistent, and the contact load, contact deformation, contact stress, Spin to roll ratio changes with ball position angle, so it is necessary to analyze and calculate the force of all balls. According to paper [20] the reasonable range of bearing parameters is shown in table 4.

Table 3. Geometric structures parameters of this example.

| Parameter                          | Value |
|-------------------------------------|-------|
| Number of balls                     | 6     |
| Diameter of bal (D_b/mm)            | 1.2   |
| Pitch diameter of bearing (d_m/mm)  | 1.47  |
| Contact angle (β/°)                 | 15    |
| Outer curvature coefficient (f_1)   | 0.5337|
| Inner curvature coefficient (f_2)   | 0.5667|

Table 4. Reasonable range of each bearing parameter

| Parameter                          | Value |
|-------------------------------------|-------|
| Number of balls                     | 5~9   |
| Diameter of bal (D_b/mm)            | 1.0~1.3|
| Contact angle (β/°)                 | 0~45  |
| Outer curvature coefficient (f_1, f_2) | 0.52~0.59 |
| Rotational speed (n/rpm)            | \(10^3~10^6\) |
| Axial preload \( F_a/N \)           | 2~7   |

3.1. The max contact stress between ball and outer inner raceway

When the bearing in a high speed, centrifugal force makes the contact stress in outer raceway much bigger then it in inner raceway. But because the curvature coefficient in inner and outer raceway is different, the contact stress is related to rotate speed and geometric construction. Figure 2 shows the max contact stress change trend, out ring of the bearing rotate speed is 36000/r/min while inner ring is static. As shown in Figure 1a and Figure 1b, when the pitch circle of bearing is constant, increasing the number of ball properly can reduce the contact stress, but if too many balls increased will decrease the ball diameter. It is not conducive to reducing ring contact stress. According to Figure 2c, the contact stress decreases with the increasing of contact angle. According to Figure 2d and Figure 2e, contact
stress in raceway is increased with the increase of its curvature radius. As shown in Figure 2f and Figure 2g, contact stress is increased with the increasing of axial preload. Meanwhile, as the rotate speed increasing, the contact stress in inner ring is reduced.

![Figure 2. The Max Contact Stress Change Trend Between Ball and Rings.](image)

3.2. Bearing stiffness
According to Figure 3a and Figure 3b, stiffness increases with the increasing of ball numbers and diameter, especially radius stiffness. That because the more balls, the stronger bearing capacity. From Figure 3c, it can be known that with the increasing of contact angle, $K_a$ and $K_\theta$ increase obviously, but $K_r$ showed a downward trend. According to Figure 3d and Figure 3e, the bigger curvature radius coefficient is, the smaller the bearing stiffness becomes. According to Figure 3f and 3g, the bearing stiffness increases with the increasing of axial preload and the stiffness decreases with the increasing of rotational speed.

![Figure 3. Stiffness Change Trend.](image)

3.3. Spin to roll ratio
The influence of bearing’s parameters to spin to roll ratio is showed in Figure 4. According to raceway control theory, spin exists only on the non-control raceway. Figure 4a and Figure 4b shows that ratio increases with ball numbers and ball diameter. According to the Figure 4c, ratio increases with the contact angle significantly. According to the Figure 4d and 4e, $f_1$ has little effect on the ratio, and with
the increasing of \( f_2 \) ratio will decrease. According to the Figure 4f and 4g, decrease radial force can lower the ratio.

Figure 4. Spin to Rolling Ratio Change Trend.

3.4. Bearing fatigue life

Figure 5 is the influence of the variation of parameters on the fatigue life of bearing \( L \). According to Figure 5a and 5b, increasing ball numbers and ball diameter can prolong the fatigue life of the bearing. According to Figure 5c, increasing contact angle \( \theta \) bearing life will be extended. According to Figure 5d and 5e, increasing \( f_2 \) will lead to the reduction of bearing life, while \( f_1 \) has little effect on fatigue life. According to Figure 5f and 5g, increasing axial preload and rotate speed can reduce the fatigue life of bearing.

Figure 5. Bearing Fatigue Life Change Trend.

4. Bearing design

(1) It is hoped that a working bearing has small contact load and contact stress, low Spin to roll ratio, high stiffness and longer service life. According to the above analysis, to achieve those targets the ball number and the diameter should be larger, however, Spin to roll ratio will be higher and heat generation will be intensify. Because of the significant influence of ball numbers on the stiffness and life, as well as the ball diameter’s significant influence on Spin to roll ratio, it should be appropriate when increases the number of balls and decreases ball diameter.
(2) The bigger contact angle and the smaller contact stress, the longer bearing life will be. But small angle can reduce the bearing ratio and increase the radial stiffness. According to Shun’s [21] paper, when the contact angle is 15 degrees, the friction torque of the bearing is the smallest. According to the study of Yang [22], when the miniature bearing is loaded, the contact angle will increase significantly compared with the normal bearing, so bearing contact angle should be a small value, the range from 10 degrees to 25 degrees will be better.

(3) Raceway curvature radius coefficient changes will lead to the change of control ring, and the increase of the curvature radius will enlarge the contact stress, reduces the stiffness and shortens the life. According to Deng [23] and Wang’s [24] research, a smaller radius of curvature of the inner raceway contributes to the formation of a thicker lubricant film, and bearing’s friction is small, which contributes to the formation of elastohydrodynamic lubricating film in its interior. Therefore, it is necessary to select a relatively small raceway curvature radius, and curvature radius of the inner raceway should be smaller than the outer.

Based on the above design principle, the improved parameters are shown in table 5. The table 6 shows the comparison of bearing performance parameters before and after improvement. Despite the slight increase of the contact stress and the Spin to roll ratio, the other parameters have been improved considerably.

| Parameters                  | Value                  | Scheme 1 | Scheme 2 |
|-----------------------------|------------------------|----------|----------|
| Number of balls             |                        |          |          |
| Diameter of ball (D_b/mm)   | 1.2                    | 1.0      |          |
| Pitch diameter of bearing (d_u/mm) | 4.16            | 4.16    |          |
| Contact angle (β/°)         | 15                     | 15       |          |
| Outer curvature coefficient (f_i) | 0.58                 | 0.58    |          |
| Inner curvature coefficient (f_i) | 0.53                | 0.53    |          |

| Parameters                  | Before improved | After improved |
|-----------------------------|-----------------|---------------|
| Contact stress σ/(MPa)      | Outer 859.9     | Outer 940.6   |
|                             | Inner 1060.5    | Inner 952.8   |
| Stiffness K/(N·mm)          | Axial 1662.0    | Axial 1972.1  |
|                             | Radial 9166.5   | Radial 10832  |
|                             | Angle 3916.9    | Angle 4457.7  |
| Spin to roll Ratio          | 0.15            | 0.16          |
| Life L/(h)                  | 12751           | 14033         |

5. Conclusions

(1) The performance of miniature bearing is greatly influenced by its internal geometry parameters, so the geometric parameters of each part should be selected reasonably. When the outer ring rotates, the appropriate increase of the number of balls can be considered, and the appropriate contact angle should be chosen.

(2) The stiffness of bearing can be improved by increasing the axial preload. At the same time, the contact load and contact stress will increase, bearing life will reduce. So when design bearings the requirement of bearing stiffness and life must be taken into account.

(3) Because of the ball centrifugal force, high rotating speed makes the bearing contact stress in outer ring increases obviously, and decreases in the inner ring which reduces bearing stiffness and life. The spin motion of the ball on the raceway is aggravated and the heating is serious, which should pay attention to when design bearings.
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References
[1] Yu L M, Chen X Y and Gu J M 2011 J. Machinery 5 71
[2] Beatty R F and Rowan B F 1982 Determination of Ball Bearing Dynamic Stiffness
[3] Okayasu A, Ohta T, Azuma T and Fujita T 1990 Vibration Problem in the LE-7 LH2 Turbo Pump. AIAA 90 2250
[4] Butner M F, Murphy B T and Akian R A 1991 J ASME Rotating Mach. Veh. Dyn 35 155
[5] Chen C Y, C S Liu and Y C Li 2015 J. Microsystem Technologies 21 1
[6] Bugra H and Vance J M 2015 J. Journal of Propulsion & Power 20 634
[7] Gao X 2003 J. Shanghai Measurement & Testing 6 13
[8] Li S S, Chen J and Lin J 2013 J. Lubrication Engineering 8 32
[9] Zhang W M, Meng G and Di C 2008 J. Journal of Vibration and Shock 5 27
[10] Gui L F, Tang R J and Zhu Z L 1986 Failure Analysis of Miniature Bearings J. ASM 167
[11] Tomaszello C M, Maloney J L and Ward P C. 1998 J. ASTM Special Technical Publication 1327 437
[12] Vol N 2003 J. Machine Design 75 54
[13] C D Zhu 2002 J. Bearing 12 7
[14] Li S S, Lin J and Chen J 2014 J. Journal of Shanghai University(Natural Science Edition) 4 429
[15] Jones A B 1978 Rotor Bearing Dynamic Technology Design Guide, Part II: Ball Bearings (New York: Shaker Research Corporation Ballston Lake) pp 96-115
[16] Wan C S 1978 Rolling bearing analysis method (Beijing: China Machine Press) p 113
[17] Harris T A 1991 Rolling Bearing Analysis (New York: A Wiley-Interscience Publication) p 231
[18] Jiang X Q 2001 D. Study on thermal characteristics of spindle bearing and its effect on speed and dynamics Zhejiang University
[19] Li S S 2006 D. Study on dynamic characteristics of ball bearing –rotor system in ultra high speed spindle Shanghai University
[20] Liu C H, Wei M Q and Chen L 2000 J. Bearing 10 6
[21] Shun Z C and Z K Wang 1979 J. Bearing 1 28
[22] Yang C G, Jia H G and B Liu 2010 J. Computer Simulation 27 291
[23] Deng S E, Li X L and J G Wang 2011 J. Journal of Mechanical Engineering 47 114
[24] Wang J G and Hong Y F 2002 J. Machinery Design & Manufacture 1 59