On the Physical Interpretation of Malyskhin’s (2008) Model of Resistive Hall-MHD Reconnection

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A simple Sweet–Parker-like model for the electron current layer in resistive Hall magnetohydrodynamic (MHD) reconnection is presented, with the focus on the collisionless limit. The derivation readily recovers the main results obtained recently by Malyskhin [PRL, 101, 225001 (2008)] and others, but is much quicker and more physically transparent. In particular, it highlights the role of resistive drag in determining the electron outflow velocity. The principal limitations of any such approach are discussed.

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I. INTRODUCTION

Recently, Malyskhin [1] proposed an analytical model of reconnection in resistive Hall MHD regime without electron inertia (see also a recent paper by Simakov & Chacon 2008, Ref. [2]). Malyskhin’s derivation was based on the local Taylor expansion of the resistive Hall MHD equations at the center of the electron layer. Under several simplifying assumptions, he was able to obtain expressions for the main parameters of the reconnection layer, including the reconnection rate $E_z$, in terms of a few input parameters, namely, the length of the resistive electron layer $L$, the magnetic field just outside the layer $B_m$, the density $n$, and the magnetic diffusivity $\eta = \eta' c^2/4\pi$ (where $\eta'$ is the resistivity) [1]. One of the main results was that the reconnection rate becomes independent of the resistivity in the limit $\eta \rightarrow 0$ and scales inversely with $L$.

In this contribution we show that, under the assumptions of Malyskhin’s model, his main results can be obtained much quicker and easier, in a way that we believe is more transparent and lends itself more readily to a clear physical interpretation. Specifically, we point out that Malyskhin’s (2008) expressions for the reconnection layer parameters follow straightforwardly from (essentially) a Sweet–Parker-like analysis applied to the inner electron layer. To make a direct comparison with Malyskhin’s paper [1] easier, we adopt the same system of coordinates that he used, i.e., $x$ is the direction across the layer, $y$ is the outflow direction along the layer, and $z$ is the ignorable direction. The only major modification in the Sweet–Parker procedure that one needs to make for the problem under consideration, is in the (electron) equation of motion (generalized Ohm’s law) in the outflow ($y$) direction. Namely, since the electron mass is completely neglected here, the electron outflow velocity $v_{ey}$ is no longer controlled by their inertia, as it would be in the classical Sweet–Parker approach. Instead, it is determined by the balance between the outward acceleration due to the Lorentz force (and perhaps a comparable contribution from the pressure gradient force) and the collisional (resistive) drag exerted on the electrons by the much slower moving ions:

$$\frac{B_z j_{ez}}{c} \sim -n e \eta' j_{ey} = n^2 e^2 \eta' v_{ey} \Rightarrow v_{ey} \sim \frac{B_z j_{ez}}{c n^2 e^2 \eta'},$$

(1)

where the characteristic values for $v_{ey}$ and $B_z$ are taken at the outflow end of the electron layer, $x = 0, y = L$.

Next, using the $z$ component of the generalized Ohm law (the electron equation of motion) both at the end of the electron layer $(x = 0, y = L)$ and at the origin $O$ $(x = 0 = y)$, together with the stationarity condition $E_z(x,y) = \text{const}$, we can estimate $B_z \sim c E_z/v_{ey} = (c/v_{ey}) \eta' j_{ez}$. Substituting this into equation (1), we immediately obtain

$$v_{ey} \sim |v_{ez}| = j_{ez}/ne \sim c B_m/4\pi n e \delta \Rightarrow v_{ey} \delta = \frac{c B_m}{4\pi n e} = d_i V_A = d_i V_A,$$

(2)

consistent with the expectation that the out-of-plane magnetic field $B_z \sim B_m$ (e.g., Refs. [3, 4]). Note that the resistivity has dropped out, because similar resistive terms enter in both the $z$ and $y$ components of the electron equation of motion, balancing the electric force $-n e E_z$ and the Lorentz force $B_z j_{ez}/c$, respectively.

Combining equation (2) with the two remaining relationships of the standard Sweet–Parker analysis: the incompressibility condition, $|v_{ex}| L = v_{ey} \delta$, and the $z$-component of Ohm’s law considered at the origin $O$ and at the point $M = (x = \delta, y = 0)$ just above the resistive layer, $\eta B_m/\delta = c E_z = |v_{ex}| B_m$ imply $|v_{ex}| \delta = \eta$, we immediately get

$$\delta \sim \frac{\eta L}{d_i V_A} = \frac{L^2}{S d_i},$$

(3)

$$|v_{ex}| \sim \frac{d_i}{L} V_A,$$

(4)

$$v_{ey} \sim |v_{ez}| \sim \frac{d_i V_A}{\delta} \sim \frac{V_A S d_i^2}{L^2},$$

(5)

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\[ c E_z \sim \frac{d_i}{L} V_A B_m, \]

which coincide with the corresponding Malyskin’s expressions (20)-(26). Here, \( S \equiv LV_A/\eta \) is the Lundquist number. Note that \( E_z \) is actually independent of the ion mass \( m_i \). In addition, it is also independent of \( \eta \), and the fundamental reason for this is that resistive drag controls the electron flow velocity in both the \( z \) and \( y \) directions in a similar way.

We note that recently similar results in resistive Hall-MHD were independently obtained by Simakov & Chacón [2], and even earlier by Wang et al. [5]. Furthermore, the expression (6) for the collisionless reconnection rate dates at least as far back as Ref. [6] (Malyskin 2008, private communication).

Finally, we would like to stress that neither Malyskin [1], nor Simakov & Chacón [2] address what, to us, seems to be the most important question in collisionless reconnection research: what determines the length \( L \) (in the outflow direction \( y \)) of the electron and ion layers? This question is similar to the old question about the length of the central diffusion region in the Petschek reconnection model. Both Malyskin [1] and Simakov & Chacón [2] consider \( L \) (or \( w \) in Simakov & Chacón’s notation) just an input parameter whose determination is beyond the scope of their papers. In our view, however, the question of whether the layer is macroscopic (independent of the local plasma parameters such as \( \eta, d_i, d_e \), etc.) or microscopic, is of great importance, both fundamental and practical, since, for most space-and astrophysical systems, the global system size is usually by many orders of magnitude larger than \( d_i \), say. Thus, if \( L \) is comparable to the global system size, the resulting reconnection rate (6), even though independent of the resistivity, is too slow to explain many observable phenomena, such as solar flares. On the other hand, if \( L \) is microscopic (as seems to be indicated by numerical simulations [7, 8]; see, however, Refs. [8, 9]), it should be determined self-consistently as a part of the local analysis. Thus, until the issue of the length of the reconnection layer is resolved, achieving a final complete theory of collisionless reconnection cannot be claimed.

Likewise, neither the present analysis, nor the papers by Malyskin [1] and Simakov & Chacón [2], attempt to estimate the magnetic field \( B_m \) just outside the electron layer in terms of the global reconnecting magnetic field \( B_0 \) measured just outside the ion layer (R. Kulsrud 2008, private communication). In general, depending on the strength of the electron and ion currents in the ion diffusion region (\( \delta_e \ll x \ll \delta_i \)), \( B_m \) may be much smaller than \( B_0 \) or comparable to \( B_0 \). Until this issue is resolved, the solution of the reconnection problem remains incomplete.

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