Computational Complexity of Continuous Variable Quantum Key Distribution

Yi-Bo Zhao, You-Zhen Gui, Jin-Jian Chen, Zheng-Fu Han, and Guang-Can Guo

Abstract—The continuous variable quantum key distribution has been considered to have the potential to provide high secret key rate. However, in present experimental demonstrations, the secret key can be distilled only under very small loss rates. Here, by calculating explicitly the computational complexity with the channel transmission, we show that under high loss rate it is hard to distill the secret key in present continuous variable scheme and one of its advantages, the potential of providing high secret key rate, may therefore be limited.

Index Terms—Computational complexity, continuous variable (CV), error correction, quantum key distribution (QKD), reconciliation.

I. INTRODUCTION

Due to its potential for achieving high modulation and detection speed, continuous variable (CV) quantum key distribution (QKD) has recently attracted more and more attention. Compared to single photon counting schemes, CVQKD does not require single photon sources and detectors which are technically challenging now. The CVQKD schemes typically use the quadrature amplitude of light beams as information carrier, and homodyne detection rather than photon counting. Some of these schemes use nonclassical states, such as squeezed states [1] or entangled states [2], while others use coherent states [3]–[6]. Because the squeezed states and entangled states are sensitive to losses in the quantum channel, coherent states are much more attractive for long distance transmission. To improve the performance of the CVQKD against the channel loss, Grosshans et al. proposed a reverse reconciliation (RR) protocol [11]. In the traditional direct reconciliation protocol, Alice sends Bob the quantum state and also sends the reconciliation (RR) protocol [11]. In the reverse reconciliation protocol, the quantum state is sent by Alice to Bob, but the reconciliation information is sent by Bob to Alice. Finally, Alice gets Bob’s received data with no error. However, in the reverse reconciliation protocol, the quantum state is sent by Alice to Bob, but the reconciliation information is sent by Bob to Alice. Finally, Alice gets Bob’s received data with no error.

Tabletop experimental setups that encode information in the phase and amplitude of coherent states have been demonstrated [7], [8], and recent experiments have shown the feasibility of CVQKD in optical fibers up to a distance of 55 km [9], [10], but without obtaining the final secret keys.

Unlike the single photon QKD schemes, many CVQKD schemes utilize the inertial quantum noise to protect information from Eve’s attack [7], [12]. However, at the same time the quantum noise also causes errors between two legitimate communicators, Alice and Bob. It is widely

Manuscript received January 14, 2007; revised November 25, 2007. This work was supported in part by the National Fundamental Research Program of China under Grant 2006CB921900, National Natural Science Foundation of China under Grants 60537020 and 60621064, the Innovation Fund of the University of Science and Technology of China under Grants KD2006005, and the Knowledge Innovation Project of the Chinese Academy of Sciences (CAS).

The authors are with the Key Lab of Quantum Information, University of Science and Technology of China (CAS), Hefei, Anhui 230026, China (e-mail: zhan@ustc.edu.cn).

Communicated by H. Imai, Guest Editor for Special Issue on Information Theoretic Security.

Digital Object Identifier 10.1109/TIT.2008.921889

1In the following, we use the conventional appellation. Alice is the quantum state sender and Bob is the quantum state receiver.
believed that through the classical error correction the errors between Alice and Bob can be corrected and partial information can still be kept secret. Nevertheless, in the QKD we should not only guarantee that the errors between Alice and Bob can be corrected, but also ensure that Eve cannot know all the information of the secret keys after the error correction. In the following, we will see that such requests actually pose a challenge to the error correction. After showing that the lower bound of the block size of the required error correction for the QKD is inversely proportional to the square of the secret information carried by per element, we illustrate that the reconciliation is a big hindrance to the CVQKD.

In principle, if the mutual information between Alice and Bob is larger than that between Eve and Bob, Alice and Bob can establish unconditional secure secret keys by the RRCVQKD protocol [17]–[19]. However, in practice we should also consider feasibility. Before establishing the secret keys, the errors contained in Alice and Bob’s variables should be corrected. Then certain amount of error correction information should be publicly exchanged [12], [22], [23]. At the same time, while received this public error correction information, Eve can get more information about Alice and Bob. To guarantee the security we should ensure that Eve cannot totally know Alice and Bob’s key. Then the amount of publicly exchanged information should be below a certain threshold. Suppose that before the error correction the mutual information rate per key element between Alice and Bob is $I_{AB}$ and that between Eve and Bob is $I_{EB}$, after the error correction, the mutual information rate per key element between Alice and Bob is $I'_{AB}$, and the amount of the published effective error correction information rate per key element is $R$. Here we can see that before the error correction, the theoretical amount of the secret key rate they can generate is $I = I_{AB} - I_{EB}$-bit. After the error correction, the mutual information rate per key element between Alice and Bob becomes $I'_{AB} \leq I_{AB} + R$ and that between Eve and Bob becomes $I_{EB} + R$. Then if Alice corrects her errors and finally shares Bob’s bits, the secret key rate after the error correction becomes $\Delta I = (I - I'_{AB} + I_{EB})$. Here we can see that to generate pure secret keys finally, it should be guaranteed that $\Delta R = (I'_{AB} - I_{EB}) < \Delta I$. Set $\Delta I'_{EB} = \Delta I$. Therefore, in practice we should use less than $\Delta R + \Delta I$-bit information for per key element to correct errors that can only be corrected with more than $\Delta R$-bit information for per key element. While the $\Delta R + \Delta I$ exponentially decreases with the increase of the transmission distance, the practical error correction should exponentially approach the Shannon limit. In some literature, a reconciliation efficiency to the binary modulation variance is defined to evaluate the reconciliation [7], [11], [12]. Here $1 - \epsilon$ actually corresponds to $(R - \epsilon)/I_{AB}$. Also, in [13], Heid et al. have evaluated the influence of the reconciliation efficiency to the binary modulated CVQKD. From their result, it can be seen that if the reconciliation efficiency is not 1, the maximum achievable distance of the CVQKD will be reduced. In the following, we will show the relationship between the achievable distance and the computational complexity.

II. DIFFICULTY WITH REVERSE RECONCILIATION CVQKD: AN EXAMPLE

Here, we consider a specific example of Gaussian-modulated RRCVQKD [7]. In the reverse reconciliation protocol, the quantum state is sent by Alice to Bob but the reconciliation information is sent by Bob to Alice. Finally, the secret key rate is given by the difference between the mutual information rate between Alice and Bob and that between Eve and Bob. After the quantum communication, some CVQKD schemes directly provide the binary keys [3], while the Gaussian-modulated CVQKD scheme only provides the continuously distributed key elements [7] that should be converted into common binary bits. Here we only discuss the latter one. It has been proposed that this conversion can be achieved by reconciliation. Nevertheless, how to realize proper reconciliation is still an open problem. One existing reconciliation protocol was suggested by Van Assche et al. [12], in which they subtly combined the quantization with the sliced error correction. They quantize the continuous-variable into binary keys at first and then do the error correction to those keys. Different from single photon schemes, in the Gaussian-modulated RRCVQKD one, most of the transmitted information can be known by Eve and only a very small portion can be kept in secret. The signal-to-noise ratio (SNR) between Alice and Bob is only slightly higher than that between Eve and Bob, and the difference of the two may be overwhelmed by the fluctuation of noise. Then, distilling the secret key will amount to looking for a needle in a haystack. For example, let us take the line loss to be $20$ dB (100 km fiber with an attenuation coefficient of 0.2 dB/km), under which the maximum secret information per key element carries about $0.007$ bit [7] (i.e., on average, 1-bit secret key requires $1/0.007 = 143$ key elements). Suppose the variance of Bob’s measurement is $2N_0$, under which Alice’s modulation variance is $100N_0$, where $N_0$ denotes the variance of the vacuum noise. Then the mutual information between Alice and Bob is $0.5$ bit and that between Eve and Bob is $0.493$ bit. On estimating Bob’s results, the difference between Alice’s and Eve’s noise is about $0.01N_0$, which can be calculated from [7]. To distill the secret key, at first those associated key elements should be converted into binary keys. The quantization will certainly induce information loss, although the loss can be exponentially reduced. To ensure the lost information to be less than $0.007$ bit per key element, the reconciliation should be precise enough to distinguish the slight noise change of $0.01N_0$, so each key element should be converted into at least $0.5 \log_2(2N_0/0.01N_0) \approx 4$ independent binary digits. After the conversion, Alice and Bob at least get four binary digits from one key element and the secret information contained by them will be certainly less than $0.007$ bit. Since the information per element carry is $0.5$ bit, to correct bit errors, at least $3.5$-bit additional information should be exchanged for one key element. Then, on average, $143 \times 4 \approx 572$ binary digits contribute to less than one secret key and at the same time $143 \times 3.5 \approx 500$-bit additional information should be exchanged. Therefore, the distillation is equivalent to extracting a secret bit at least in 1072 binary digits, which is quite impractical as the error fluctuations in 572 digits may easily overwhelm the 1-bit secret key information to be distilled.

To distill the secret key, errors in quantized bits should be corrected. In theory, the errors in 572 bits require at least 500-bit additional information to correct, but the secret information carried by 572 bits is less than 1 bit and Eve can use less than 501-bit additional information to know all of Bob’s information, which means that if 501-bit error correction information is exposed there may be no secret information kept from Eve, so to correct the errors in 572 bits and to ensure corrected bits still carry secret information, the publicly exposed reconciliation information should not exceed 501 bits. Then the practical error correction should use less than $501/500 = 1 + 0.2\%$ times the theoretical minimum information to correct the errors at a high bit error rate (BER). While in [22], [23], even at a BER as small as $3\%$, this ratio can be enhanced to $120\%$, much higher than $1 + 0.2\%$, but still pose a technical challenge. In fact, to improve the practical channel information capacity is a hot topic in classical communications [20]. One major goal is to correct the errors with as little redundant information
as possible. Up to now, even under ideal situations (e.g., in the standard channel, at fixed coding rate, under the optimized SNR and distinguishing the BER of each bit), present state-of-the-art error correction schemes, such as low-density parity-check (LDPC) [20] and turbo [21] codes, may in theory only get information with low BER from 1.001 times (0.0045 dB) of the theoretical minimum resource [20]. While in practice, this theoretical performance is still hard to reach at present. In the above example, Alice should get information from less than 572 + 501 = 1073 binary digits without any error, while in theory this information should be obtained from at least 1072 binary digits. The ratio of them is 1073/1072 = 1.0009 or 0.0040 dB, smaller than 0.0045 dB, which is a huge challenge to the error correction technique. In practice, the required performance of error correction will be much higher. To our knowledge there is no such error correction that can meet that requirement.

In [7], [12], the authors sorted those uncorrelated bits approximately with the same BER into one group and correct errors into another. It is possible that BERs of some groups are smaller than the average one, so their errors are easier to correct than what is suggested by the above analysis. However, there are certainly some groups the BER which are higher than the average one and require much more complex error correction. Then on average, its required error correction may be more difficult to realize than the above analysis. Actually, in [7] the authors directly expressed these bits with a high BER, whereas whether such operation affects the security requires further discussion.

In the above, we have discussed a limit case, under which the quantization can be ideal and the correlation as well as the asymmetry can be neglected. In fact, such limit case cannot be reached and each of the converted digits may be correlated with each other and the channel may be binary asymmetric, so one continuous element should be converted into many more digits. Also, the correlation and the asymmetry will greatly increase the complexity and reduce the efficiency of the error correction. Therefore, in practice, things will be much more difficult than what we have shown in the above example.

III. COMPUTATIONAL COMPLEXITY OF THE REQUIRED ERROR CORRECTION

A. Simple Model

In the following, we will calculate explicitly the computational complexity of the error correction including secret key information carried by a single digit. Here we will start from a binary symmetric model, and then extend it to a more general case. Later we will show that such model can be applied to the current CVQKD scheme. Suppose after many communications and processes, Alice and Bob get binary strings $M_A = (M_{A1}, \ldots, M_{Am})^T$ and $M_B = (M_{B1}, \ldots, M_{Bm})^T$ of length $m$, respectively. In the following, it is assumed that the channel is binary symmetric, i.e., the binary strings contain equal probabilities of $0$’s and $1$’s, the bits in the strings are uncorrelated. Also, it is assumed that the error correction information is sent by Bob to Alice and with this error correction information Alice can correct all of the errors only when the number of errors is below a threshold value [14]. We define $e_{AH} = \Pr(M_A \neq M_B)$ as the BER and $S(M_B[E])$ as the entropy rate per bit of Bob’s string conditioned on Eve’s state. Then the maximum amount of secret key that can be distilled by Alice and Bob is $mS(M_B[E] - H(e_{AH})$-bit, where $H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$ is the Shannon entropy. To share the common secret key, Alice and Bob should eliminate the errors in their final binary string. Then, they can use, for example, error correction, to tolerate certain amount of errors. Suppose Alice can correct at most $m\epsilon_{sec}$ errors with Bob’s error correction information, where $\epsilon_{sec}$ is an introduced parameter. Then $mH(\epsilon_{sec})$-bit additional information is required and after the error correction, the maximum amount of secret keys becomes $m[S(M_B[E] - H(\epsilon_{sec})]$-bit. While the average amount of errors between Alice and Bob is $m e_{AH}$, the actual amount of errors will fluctuate, and the fluctuation obey the binomial distribution with variance $m e_{AH}(1 - e_{AH})$. Here, we introduce the parameter $\lambda$ and suppose the number of practical errors is $m \lambda$. Then the binomial distribution of errors between Alice and Bob can be approximated by a Gaussian

$$P(\lambda) \approx \frac{m}{\sqrt{2\pi m e_{AH}(1 - e_{AH})}} \exp \left( - \frac{(m e_{AH} - m \lambda)^2}{2 m e_{AH}(1 - e_{AH})} \right) \quad (1).$$

Only when $\lambda \leq \epsilon_{sec}$ can the errors be corrected, so the probability that Alice corrects her errors is

$$\beta = \int_0^{\epsilon_{sec}} \frac{m}{\sqrt{2\pi m e_{AH}(1 - e_{AH})}} \exp \left( - \frac{(m e_{AH} - m \lambda)^2}{2 m e_{AH}(1 - e_{AH})} \right) \, d\lambda \quad (2).$$

To share a common secret key with high probability, Alice and Bob should guarantee that $\beta \rightarrow 1$. To distill the secret key, they should maintain $S(M_B[E]) > H(\epsilon_{sec})$. That requires $m$ to be sufficiently large. For given probability $\beta$, the error correction criteria $\epsilon_{sec}$ and BER $e_{AH}$, the length $m$ can be given by

$$m = \frac{e_{AH}(1 - e_{AH})}{(1 - \beta)} \frac{Q^2(1 - \beta)}{e_{AH}} \quad (3),$$

where $Q(x)$ is defined as the solution of the equation

$$1/\sqrt{2\pi} \int_{Q(x)}^{\infty} \exp(-y^2/2) \, dy = x$$

and we have supposed $\epsilon_{sec} > e_{AH}$.

Here, we introduce a parameter $e_c$ and suppose $H(e_c) = S(M_B[E])$. Then from the requirement $S(M_B[E]) > H(\epsilon_{sec})$, we know that $e_c > e_{sec}$. In practice, $e_c$ and $e_{AH}$ are determined by the quantum channel, the BER is determined by practical requirements, so Alice and Bob need to choose proper $\epsilon_{sec}$ and $m$ to satisfy (3). Set $\delta I = H(e_c) - H(e_{AH})$, where $\delta I$ denotes the secret rate per digit. Then we have

$$m > e_{AH}(1 - e_{AH}) \left( \log_2 \frac{1 - e_{AH}}{e_{AH}} \right)^2 \left( \frac{1}{\delta I} \right)^2 Q^2(1 - \beta) \quad (4)$$

where we have used the result that $\epsilon_{sec} < e_c$ and

$$e_c - e_{AH} \approx \delta I / \log_2 \frac{1 - e_{AH}}{e_{AH}}.$$

The value of $m$ will determine the computational complexity of the error correction. For the common error correction code, the computational complexity is of order $O(m^2)$ [20].

In the preceding paragraph we have discussed the binary symmetric case. In the practical case, the channel may be asymmetric and the BER of different bit may be different. In such cases, whether the error can be corrected is not only determined by the number of errors, but also by the BER of each bit. There is still a threshold for such case, which is determined by the amount of the published error correction information, but now whether certain amount of errors can be corrected or not becomes probabilistic [20], [21], [25]. If the number of errors is larger than this threshold, all the errors can be corrected with high probability. On the contrary, if the number of errors is smaller than this threshold,

\footnote{Under the symmetric binary channel condition, the distribution of the number of errors obey the binomial distribution: $P(n) = \binom{m}{n} e_{AH}^n (1 - e_{AH})^{m-n}$, where $n$ describes the number of errors.}
the errors can be corrected with comparatively low probability. The larger the number of the errors, the smaller the probability that all errors can be corrected is. To guarantee that possible errors can be corrected with high probability, Alice and Bob should also ensure that the actual number of the errors is smaller than this threshold with a high probability. The upper bound of this threshold is determined by Alice’s conditional entropy conditioned on Bob and the secret key rate. Then the problem becomes similar to that under the binary symmetric case. According to the large numbers theory, the distribution of the number of errors can also be approximated by the Gaussian distribution in this case. Then in the same way used for the binary symmetric case, we can also prove that in the case that the channel is asymmetric or the BER of different bit is different, the minimum block size of the required error correction code satisfies

\[ m \approx \left( \frac{1}{\delta I} \right)^2. \]

(5)

Here we have not written (5) explicitly into terms of the BER and \( Q^2(1 - \beta) \), because under the asymmetric case it is difficult to give a general form to the variance of the Gaussian approximation and actually only the relationship between \( m \) and \( \delta I \) is essential for us. In the following, we will discuss the relationship between the minimum block size and the channel transmission.

In the reconciliation given in [12], the authors grouped the bits into several strings, so that in one string a different bit corresponds to a different key element. Then if Eve uses the individual attack\(^5\) or the collective attack, the bits in one string can be uncorrelated. Also, since Eve attacks each signal by the same method, the distribution of each key element is identical, so that the Gaussian approximation can be used to describe the distribution of the number of errors in one string. Therefore, under the individual attack or the collective attack case the above model can be applied to the reconciliation given in [12]. There may be other reconciliation methods, in which the bits in one string are totally correlated with each other, so that the above model cannot be applied. However, the efficiency of current error correction code will be much lower if the bits in one string are correlated. Actually, under the current technology, the computational complexity of the error correction will be largely increased if we make the bit correlated. Therefore, in the following we only discuss the uncorrelated case.

In the above, we have discussed the relationship between the minimum block size and the secret key rate. For the binary symmetric case, we gave the explicit expression for the minimum block size in (3) and (4). Since the binary symmetric case is the easiest condition to handle, using this result we can give a rough estimation of the minimum block size. We also considered the nonbinary symmetric case. However, we have not given the explicit expression for the block size under this case. Because under such case the minimum block size depends on the concrete properties of the channel, it is difficult for us to give a general form. At least using (5) we can give a rough estimation for the order of the computational complexity if some experimental results are given.

B. Computational Complexity of Present Schemes

In single photon schemes, after the quantum communication, almost all the information of the distributed binary string is kept in secret from Eve, i.e., \( \delta I \rightarrow 1 \), so the required \( m \) can be very small. However, in CVQKD, most of the information may be tapped by Eve, and consequently, the \( \delta I \) is very close to 0, so that the required \( m \) will be very large. We can give a rough estimation to the example given in Section II. We suppose the channel is binary symmetric. At first we roughly estimate the BER and the secret key rate per binary digit of a typical string. Alice’s conditional entropy rate conditioned on Bob’s variable is 3.5 bits per key element. Since Alice and Bob map their key element into four digits, we estimate BER \( \epsilon_{AB} \) of a typical converted binary string by \( 4H(\epsilon_{AB}) = 3.5 \). Thus, \( \epsilon_{AB} \approx 0.29 \). Also, we suppose the secret information each digit carries is 0.007 bit.\(^6\) If we set 1 - \( \beta \) = 10\(^{27} \), then \( Q^2(1 - \beta) \approx 27 \). Using (3), we obtain \( m > 10^7 \). That means, to share the common secret key with high probability and to restrain the probability that Eve gets their key, Alice and Bob should set the criteria of the error correction at least to 3.1 dB digits. Then, even with an ideal error correction technique, the required block size should be larger than 10\(^7\). For present LDPC coding, only under the simulation condition and with a very simple code, can this block size be realized\([20]\). Because we suppose that the channel is binary symmetric and many other factors have not been taken into account, the estimated result may be far from the practical one. In the following, we will give an estimation based on current experiment.

From the [7] we know that for the RRCVQKD if there is no excess noise and Eve is classical, the secret key rate per key element is proportional to \( 1/\eta \), where \( \eta \) denotes the channel transmission. Then we can safely assume that after the conversion, the secret key information carried by single binary bit is proportional to \( 1/\eta \). Since the computational complexity generally is proportional to the square of the block size\([20]\), from (5) we conclude that the minimum computational complexity of the error correction is at least of order \( O(m^2) \approx O(1/\delta I^2) \approx O(1/\eta^4) \approx O(\exp(0.1 \delta I \eta)) \), where \( L \) denotes the transmission distance in units of kilometers, and the fiber attenuation efficiency has been chosen to be 0.2 dB/km. The authors of [7] can distill the secret key to within 3.1-dB loss (\( \eta = 0.5, 15 \) km fiber), even though high performance error correction was used and Eve was assumed to be classical. While under the condition of 10-dB loss (\( \eta = 0.1, 50 \) km fiber), the required error correction will be at least 600 times more complex. Under the 100-km fiber case, it will be at least 6\(^{6}\)\(^{10}\) times more complex.\(^7\) Then, whether the secret key can be distilled becomes doubtful. Moreover, as the error correction becomes very complex, the practical computation speed of the algorithm will certainly become very low. Therefore, even though the secret key can be distilled and the physical modulation and detection rate can be very high, the practical secret key rate, which will be limited by computational complexity, cannot be actually improved.

IV. Remarks

The purpose of this correspondence is to show a hindrance of the secret key distillation for the CVQKD under the high loss condition, so in many sections we just discussed an ideal case. In the example of Section II we supposed an ideal quantization that can convert continuous variables into symmetric and independent bit string which is the easiest to handle. Consequently, if we show that even under this ideal condition, the error correction is prohibitive, then it will be much more prohibitive under the practical case. Moreover, here we not only discussed the error correction, but also considered the security. Although it is widely believed (the Gilbert–Varshanov conjecture) that general decoder can only correct half the errors which can be corrected by an

\(^5\)Generally, Eve’s attack can be separated into three classes, individual attack, collective attack, and coherent attack. The individual attack means that Eve attacks each signal system separately with the same method and later measures her quantum state right after the sifting step. The collective attack means that Eve attacks each signal system separately with the same method but can do arbitrary measurement after all of the steps, including the reconciliation, privacy amplification, and the encryption. The coherent attack is the most general attack, where Eve can attack all of the signals together and do the measurement at the end of the protocol.

\(^6\)The secret key rate 0.007 bit per key element is obtained under the assumption that Eve is classical.

\(^7\)This relationship is of the minimum block size. It is possible that the efficiency of the reconciliation used in [7] is low and after some improvement the practical computational complexity under 3.1-dB loss can be largely reduced. Therefore, this relationship does not accurately represent the practical one.
ideal error correction code [25], in the security analysis we should assume that Eve has the most powerful error correction and her ability should only be restricted by the theoretical limit rather than the wide belief. We can see that even under the condition that Alice can correct all the errors that an ideal error correction code can correct, the computational complexity is still very exaggerated. If the practical error correction is the widely believed one, the practical reconciliation will be much more prohibitive.

It should also be noted that we supposed Eve’s attack is an individual attack or a collective attack, so by the reconciliation method given in [12] we can ensure that the bits in Alice and Bob’s strings are uncorrelated. If we take the most powerful attack, coherent attack, into account, the computational complexity will be much more complex. Here we were going to show the impossibility of the secret key distillation at long distance transmission, so we only considered the individual and collective attack cases. If the secret key cannot be distilled under this limited condition, it certainly cannot be distilled under the coherent attack condition.

In the examples of Sections II and III we were discussing the pure RRCVQKD that is without any other tactic, such as post-selection, assisted. For this pure RRCVQKD, it has been shown that the mutual information between Alice and Bob is certainly larger than that between Eve and Bob and thus it is expected to provide high secret key rate. Here we showed that if other tactics are not employed, the minimum computational complexity is certainly exponentially proportional to the transmission distance.

V. CONCLUSION

In conclusion, we have discussed in detail the computational complexity of the error correction required by the CVQKD. It has been shown that if other tactics, such as post-selection [5], are not introduced, the minimum computational complexity of the RRCVQKD is exponentially proportional to the transmission length. For the single photon schemes, the single photon source and detector are the main limitations to the transmission distance and the secret key rate [24], while for the present CVQKD we have shown that the computational complexity may be the key hurdle. Although the CVQKD solves the problem of the light source and detection, it brings a new problem, computational complexity. With the progress of the computer and new error correction techniques, the computational complexity of the RRCVQKD may no longer be a problem, but under existing techniques, its strict requirements are difficult to satisfy.

ACKNOWLEDGMENT

The authors wish to thank Ling-An Wu, H. Pu, and P. Grangier. We also wish to thank F. Zhang and J. Ma for the discussions on the error correction.

REFERENCES

[1] D. Gottesman and J. Preskill, “Secure quantum key distribution using squeezed states,” Phys. Rev. A, vol. 63, p. 022309, 2001.

[2] C. Silberhorn, N. Korolkova, and G. Leuchs, “Quantum key distribution with bright entangled beams,” Phys. Rev. Lett., vol. 88, p. 167902, 2002.

[3] R. Namiki and T. Hirano, “Practical limitations for continuous-variable quantum cryptography using coherent states,” Phys. Rev. Lett., vol. 92, p. 117901, 2004.

[4] T. Hirano, H. Yamanaka, M. Ashikaga, T. Konishi, and R. Namiki, “Quantum cryptography using pulsed homodyne detection,” Phys. Rev. A, vol. 68, p. 042331, 2003.

[5] C. Silberhorn, T. C. Ralph, N. Lütkenhaus, and G. Leuchs, “Continuous variable quantum cryptography: Beating the 3 dB loss limit,” Phys. Rev. Lett., vol. 89, p. 167901, 2002.

[6] C. Weedbrook, A. M. Lance, W. P. Bowen, T. Symul, T. C. Ralph, and P. K. Lam, “Quantum cryptography without switching,” Phys. Rev. Lett., vol. 93, p. 170504, 2004.

[7] F. Grosshans, G. Van Assche, J. Wenger, R. Brouil, N. J. Cerf, and P. Grangier, “Quantum key distribution using Gaussian-modulated coherent states,” Nature, vol. 421, pp. 238–241, Jan. 2003.

[8] A. M. Lance, T. Symul, V. Sharma, C. Weedbrook, T. C. Ralph, and P. K. Lam, “No-switching quantum key distribution using broadband modulated coherent light,” Phys. Rev. Lett., vol. 95, p. 180503, 2005.

[9] M. Legre, H. Zbinden, and N. Gisin, “Implementation of continuous variable quantum cryptography in optical fibers using a go-&-return configuration,” 2005, arXiv: quant-ph/0511113.

[10] I. Lodewyck, T. Debuisschert, R. Tuylle-Broui, and P. Grangier, “Controlling excess noise in fiber-optics continuous-variable quantum key distribution,” Phys. Rev. A, vol. 72, p. 050303, 2005.

[11] F. Grosshans and P. Grangier, “Reverse reconciliation protocols for quantum cryptography with continuous variables,” 2002, arXiv: quant-ph/0204127.

[12] G. Van Assche, J. Cardinal, and N. J. Cerf, “Reconciliation of a quantum-distributed Gaussian key,” IEEE Trans. Inf. Theory, vol. 50, no. 2, p. 394–400, Feb. 2004.

[13] M. Heid and N. Lütkenhaus, “Efficiency of coherent state quantum cryptography in the presence of loss: Influence of realistic error correction,” Phys. Rev. A, vol. 73, pp. 052316–1–052316–7, 2006.

[14] C. E. Shannon, “Certain results in coding theory for noisy channels,” Inf. Contr., vol. 1, pp. 6–25, 1957.

[15] C. E. Shannon, “A mathematical theory of communication,” Bell Syst. Tech. J., vol. 27, pp. 379–423, 623–656, 1948.

[16] C. H. Bennett, G. Brassard, C. Crepeau, and U. M. Maurer, “Generalized privacy amplification,” IEEE Trans. Inf. Theory, vol. 41, no. 6, pt. 2, pp. 1915–1923, Nov. 1995.

[17] U. M. Maurer and S. Wolf, “Secret-key agreement over unauthenticated public channels—Part III: Privacy amplification,” IEEE Trans. Inf. Theory, vol. 49, no. 4, pp. 839–851, Apr. 2003.

[18] U. M. Maurer and S. Wolf, “Secret-key agreement over unauthenticated public channels—Part I: Definitions and a completeness result,” IEEE Trans. Inf. Theory, vol. 49, no. 4, pp. 822–831, Apr. 2003.

[19] U. M. Maurer, “Secret key agreement by public discussion from common information,” IEEE Trans. Inf. Theory, vol. 39, no. 3, pp. 733–742, Mar. 1993.

[20] S. Y. Chung, G. D. Forney, T. J. Richardson, and R. Urbanke, “On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit,” IEEE Commun. Lett., vol. 5, no. 2, pp. 58–60, Feb. 2001.

[21] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error-correcting coding and decoding,” in Proc. Int. Conf. Communications (ICC’93), Geneva, Switzerland, May 1993, pp. 1064–1070.

[22] C. Elliott, A. Colvin, D. Pearson, O. Pikalo, J. Schlafer, and H. Yeh, “Current status of the DARPA quantum network,” 2005, arXiv quant-ph/0503058.

[23] D. Pearson, “High-speed QKD reconciliation using forward error correction,” in Proc. 7th Int. Conf. Quantum Communication, Measurement and Computing (QCMC), Glasgow, Scotland, U.K., Jul. 2004, pp. 299–302.

[24] X. F. Mo, B. Zhu, Z. F. Han, Y. Z. Gui, and G. C. Guo, “Faraday-Michelson system for quantum cryptography,” Opt. Lett., vol. 30, pp. 2632–2634, 2005.

[25] D. J. C. MacKay, Information Theory, Inference, and Learning Algorithm, Cambridge, U.K.: Cambridge Univ. Press, pp. 212–213.