SYNTHESIS OF A ROBUST CONTROL SYSTEM FOR TWO-MASS ELECTRIC DRIVE BY $H_{\infty}$-THEORY

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The practice of designing and operating control systems for industrial facilities has shown that systems synthesized according to the criteria of modular and symmetric optima, as well as by quadratic quality criteria, are sensitive to changes in the parameters of the controlled object, incoming characteristics, disturbing influences, the structure and parameters of the object model changing, which used in control loops. Such systems can lose optimality as well as productivity, if information about the object and operating environment is known with some probability or uncertainty. For an industrial electric drive, changes in load torque, moment of inertia of rotating parts, supply voltage and environmental characteristics (temperature, vibration) are especially significant. These changes affect both the mechanical characteristics rigidity and the transients’ quality. Robust control systems provide a required quality of work when changing the characteristics of impacts and instability of the control object parameters in a wide range. In the presence of uncertainties, a robust controller provides robust stability and quality for all admissible uncertainties.

The synthesis of robust control systems with an $H_{\infty}$-speed controller of a two-mass electric drive by methods of $H_{\infty}$-theory is presented in order to prove the possibility of using algorithms for a given stabilization and speed control, as well as providing the necessary degree of sensitivity to parametric and coordinate disturbances acting on the control object. Application of the method of robust controllers’ synthesis taking into account the requirements for the quality of a controlled coordinate transient processes in the control system. The analysis of the operation of a synthesized robust system with an $H_{\infty}$-speed controller using digital modeling on mathematical models in the MATLAB / Simulink environment is carried out. The possibility of the speed independent control, the steady-state error and the nature of the transient processes of the controlled coordinate in the synthesized electric drive control system is shown.

Key words: robust control, uncertainty, $H_{\infty}$-controller, electromechanical object.

Introduction. One of the main problems of modern control theory is the dynamic objects control under uncertainty conditions. Uncertainty is caused by the lack of complete information regarding parameters or characteristics of the plant, in addition, the mathematical model of the plant itself, obtained analytically or as a result of identification, differ from the real technical system. In recent decades, an approach has been developed when, in the presence of uncertainty, the problem arises of controlling not a single object, but a family of objects belonging to a given set. In comparison with algorithms of the classical control theory, it is necessary to provide the closed system stability not only for a nominal object, but also for any object from the given class of uncertainty with a single controller - is the task of the robust control synthesis ($H_{\infty}$-theory). Robust control system is not adaptive, but provides the necessary quality of work when the plant characteristics are change.

Interest in the synthesis of robust controllers is associated with the need to reduce the required amount of a priori information about the plant, the aspiration for the universality of control systems and reduce the cost of setting them up. Despite serious theoretical advances [1], including in the automated electric drive (ED) field, $H_{\infty}$-optimization methods have not used in daily domestic and foreign practice. This is due to the dominance in the market of complete electric drives with "classical" control algorithms.

The goal of the paper is to improve the stabilization accuracy and the quality of an AC drive speed control with a synchronous motor as a two-mass electromechanical system operating under conditions of incomplete information about the object and considering the parametric uncertainty by the methods of $H_{\infty}$-theory of robust control.
Research materials. A two-mass electromechanical object, which includes a synchronous electric motor with permanent magnets with a controlled transistor converter, is adopted as a control object.

In many cases, in the mathematical description, an idealization of a plant is used, based on the concept of a rigid connection between the engine and the actuator. The assumption is correct provided that the frequency of the natural elastic oscillations of the mechanism is significantly higher than the frequency that determines the speed of an electric drive control system. If this condition is not met, then neglect of elasticity and the presence of gaps in the analysis and synthesis of a system can lead to erroneous results. The system of equations describing the two-mass electromechanical object in the Cauchy form [2-4]:

\[
\begin{align*}
 p\omega_2 &= \frac{1}{J_2} M_{12}; \\
 p M_{12} &= c_{12} (\omega_1 - \omega_2); \\
 p\omega_1 &= \frac{1}{J_1} M_{12} + \frac{k_e}{J_1} I; \\
 p I &= -\frac{k_e}{R_s T_e} \omega_1 - \frac{1}{T_e} I + \frac{1}{R_s T_e} E_c; \\
 p E_c &= \frac{1}{T_\mu} E_c + \frac{k_e}{T_\mu} U_c,
\end{align*}
\]

where \(\omega_1, \omega_2\) – the angular speed of the motor and the actuator, respectively, 1/s; \(J_1, J_2\) – the inertia moment of motor armature and the total inertia moment of various parts of a working body, reduced to the motor shaft, respectively, kg m²; \(M_{12}\) – elastic twisting moment of the shaft, Nm; \(M\) – electromagnetic torque of the motor, Nm; \(I\) – motor armature current, A; \(c_{12}\) – shaft stiffness coefficient, Nms; \(k_e = 3/2 p_0 \Psi_f\) – electromagnetic moment constant, a constant value that connecting an electromagnetic moment with an effective value of a stator current, Nm/A; \(k_e = p_0 \Psi_f\) – motor constant; \(\Psi_f\) – flux linkage of permanent magnets on the rotor; \(p_0\) – number of machine pole pairs; \(R_s\) – active resistance of a stator winding, Ohm; \(T_e\) – electromagnetic time constant, s; \(E_c\) – electromotive force of converter; \(T_\mu\) – time constant of converter, s; \(k_e\) – gain coefficient of converter; \(U_c\) – control voltage, V.

We assume that the active resistance of a stator winding is in a range of ± 40% and the moment of inertia reduced to the motor shaft is in the range of ± 50% of the nominal values.

Description of the above uncertainties, which are either not known exactly, or change during the operation of an electric drive, presented as a linear fractional transformation (LFT); the definition of the dynamics of the inputs / outputs of the system in a matrix representation, taking into account uncertainties as the transfer function matrix, as well as the sequence of transformation (LFT); the definition of the dynamics of the plant structural schemes with uncertain parameters, are considered by the authors in [5-7].

The creation of a generalized object \(P(s)\) taking into account the weight functions \(W_R(s), W_D(s)\) and \(W_f(s)\), which are used to ensure the desired quality of the system, as well as the detailed synthesis of the \(H_\infty\)-controller using the “Two-Riccati Approach” are published in [8-10].

The task of \(H_\infty\)-optimization is the controller selection \(K\) that would minimize the infinite norm \(T_{ZW}\). Selection of the optimal regulator \(K\) is carried out over the set of all regulators that have the property of making the closed system \(T_{ZW}\) internally stable, i.e. over a set of stabilizing controllers. The \(H_\infty\)-norm serves as a measure of the system strengthening. The \(H_\infty\)-norm of transfer function is the output energy of the system when a signal with unit energy is supplied to the input. If the output is an error and the input is a disturbance, then by minimizing the \(H_\infty\)-norm of the transfer function we minimize the error energy for the worst case of the input disturbance.

The robust \(H_\infty\)-suboptimal speed controller of synchronous electric motor with permanent magnets was synthesized using effective methods implemented in the Robust Control Toolbox extension package of the MATLAB system, which, using the presented algorithms [3-4], calculate the central \(H_\infty\)-suboptimal controller that minimizes \(H_\infty\)-closed-loop norm \(\|T_{ZW}\|_\infty\). The synthesized robust speed controller is a 7th order controller according to the \(H_\infty\)-norm criterion. The
achieved $H_{\infty}$-norm of the closed-loop system, obtained during the iterative process, was 0,814. With the help of algorithms for lowering the order of the system, the speed controller was reduced to the 5th order.

Fig. 1 shows the results of a two-mass electromechanical object operation with the synthesized $H_{\infty}$-speed controllers when processing closed robust control system of a given trajectory and an applied moment of resistance $0,5\cdot M_{\text{nom}}$.

In fig. 2 are shown the singular values of the sensitivity function $S$ and additional sensitivity $T$ which amplitude-limited by weight functions. These characteristics indicate the achieved quality of a robust control system with a synthesized $H_{\infty}$-controller.

Fig. 2. Singular values $S$ and $T$ which amplitude-limited by weight functions (a) and frequency characteristics of the full order (7th) and reduced (5th order) regulator (b)

In fig. 3, 4 are shown the simulation results with a variation of the plant parameters (simultaneous decrease and increase $J_2$ and $R_s$ by 2 times from the nominal values).

Fig. 3. Graphs of transient processes in robust control system with a simultaneous decreasing $J_2$ and $R_s$ by 2 times

Fig. 4. Graphs of transient processes in robust control system with simultaneous increasing $J_2$ and $R_s$ by 2 times
The equation in a state space of the reduced rate $H_{\infty}$-controller of a two-mass ED:

$$K_n = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

where

$$A = \begin{bmatrix} -4258 & 339 & -472.7 & 0 & 0 \\ 59.5 & -4.9 & 6.6 & 0 & 0 \\ -44.8 & 6.2 & -4.7 & 0 & 0 \\ 0 & 0 & 0 & 2.4 \cdot 10^4 & 4783 \\ 0 & 0 & 0 & -1.2 \cdot 10^5 & 2.4 \cdot 10^4 \end{bmatrix}$$

$$B = \begin{bmatrix} 46.4 \\ 2.5 \\ 0.072 \\ 0.014 \end{bmatrix}$$

$$C = \begin{bmatrix} 47.62 & -2.72 & 5.2 & 10.2 & 2.08 \end{bmatrix}$$

$$D = [0].$$

Conclusions. For effective suppression of oscillation arising into two-mass visco-elastic electromechanical system "electric motor - actuator", the possibility of using the $H_{\infty}$-theory in the construction of control systems with such objects is shown. A robust $H_{\infty}$-suboptimal speed controller was synthesized for a two-mass electromechanical object under conditions of incomplete information about the object and taking into account its uncertainties. The resulting controller provides the control system with robust quality characteristics and a given accuracy of maintaining the speed.

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Руднев Е.Є., Тимофеева О.А., Брожко Р.Н. Синтез робастні систем управління двумасовим електроприводом методами $H_2$-теорії

Практика проектирования и эксплуатации систем управления промышленными объектами показала, что системы, синтезированные по критериям модульного и симметричного оптимума, а также по квадратичным критериям качества, чувствительные к изменениям параметров объекта управления, характеристик входящих, возмущающих воздействий, к изменениям структуры и параметров модели объекта, которая используется в контурах управления. Такие системы могут терять и оптимальность, и работоспособность, если информация об объекте и среде функционирования известна с некоторой вероятностью или неопределенностью. Для промышленного электропривода особенно значимы изменения момента нагрузки, момента инерции вращающихся частей, напряжения питания и характеристики окружающей среды (температура, вибрация). Эти изменения влияют как на жесткость механических характеристик, так и на качество переходных процессов.

Робастные системы управления обеспечивают необходимое качество работы при изменении характеристик воздействий и нестабильности параметров объекта управления в широком диапазоне. Робастный регулятор при наличии неопределенностей обеспечивает робастную устойчивость и качество для всех допустимых неопределеностей.

Приведен синтез робастных систем управления с $H_2$-регулятором скорости двумасового электропривода методами $H_2$-теории с целью доказательства возможности применения алгоритмов для заданной стабилизации и регулирования скорости, а также обеспечения необходимой степени чувствительности к параметрическим и координатным возмущениям, действующих на объект управления. Применение метода синтеза робастных регуляторов, учитывающих требования к качеству переходных процессов регулируемой координаты в системе управления. Проведен анализ работы синтезированной робастной системы с $H_2$-регулятором скорости с помощью цифрового моделирования на математических моделях в среде MATLAB/Simulink. Показана возможность независимого управления быстродействием, установившейся ошибкой и характером протекания переходных процессов регулируемой координаты в синтезированной системе управления электроприводом.

**Ключевые слова:** робастные управления, неопределенность, $H_2$-регулятор, электромеханический объект.

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