Bulk viscosity driving the acceleration of the Universe

J. C. Fabris\textsuperscript{1}, S.V.B. Gonçalves\textsuperscript{2} and R. de Sá Ribeiro\textsuperscript{3}

Departamento de Física, Universidade Federal do Espírito Santo, 29060-900, Vitória, Espírito Santo, Brazil

Abstract

The possibility that the present acceleration of the universe is driven by a kind of viscous fluid is exploited. At background level this model is similar to the generalized Chaplygin gas model (GCGM). But, at perturbative level, the viscous fluid exhibits interesting properties. In particular the oscillations in the power spectrum that plagues the GCGM are not present. Possible fundamental descriptions for this viscous dark energy are discussed.

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1 Introduction

The observations of the dynamics of galaxies, cluster of galaxies and of the supernova type Ia indicate that about 95% of matter content of the universe is not composed of baryons \cite{1, 2}. A fraction of about 1/3 of this dark component appears in the agglomerated structures, and it is called dark matter, while the remaining 2/3 appears as a smooth component, driving the acceleration of the Universe, and is called dark energy. There is a large number of models trying to take into account the presence of the dark component. The most popular one is the so-called ΛCDM \cite{3, 4}, where dark energy is represented by the cosmological constant, while dark matter is composed of WIMPS, a cold dark matter, composed of weakly interacting massive particles which must be relics of a grand unified phase of the universe, like axions. The ΛCDM has achieved great success in explaining the observational data (even if there claims in the opposite sense \cite{5, 6}), but faces at same time many theoretical difficulties, like a huge discrepancy (of about 120 orders of magnitude) between the predicted and the observed values of the cosmological constant \cite{7, 8}.

Other models are competitive with ΛCDM, like quintessence \cite{9, 10} and K-essence \cite{11}. Another quite recent proposal is the Chaplygin gas \cite{12, 13}, which is phenomenologically represented by a fluid with negative pressure which varies with the inverse of the density. The Chaplygin gas model has been generalized by considering that the pressure, besides to be negative, depends on an arbitrary power of the inverse of the density. One of the great advantages of the Generalized Chaplygin gas model (GCGM) is the possibility of unifying the description of dark energy and dark matter \cite{14, 15}: a fraction of this exotic fluid can agglomerate locally, while the other fraction remains a smooth component. However, criticisms have been addressed to this proposal mainly due to its behaviour concerning density perturbations, which exhibits large oscillations in the resulting power spectrum which do not appear in the observed power spectrum of mass agglomeration \cite{16}.

In our point of view, the question of the oscillations in the matter power spectrum in the GCGM is controversial. The oscillations in the power spectrum of the GCG are not transferred to the baryonic power spectrum, and after all, the direct observation concerns baryons. But, there are claims that the oscillations in the dark component is reflected in the $\sigma_8$ normalization \cite{16}. In this sense, it should be interesting to find a way out to this problem, keeping at same time the advantages of the GCGM.

In this work we will explore the possibility that the present acceleration of the universe is due to a kind of viscous fluid. It is well known that bulk viscosity can generate an acceleration expansion \cite{17}. But, to our knowledge, such possibility has been investigated only in the context of the primordial universe,
concerning also the search of non singular models \[18,19\]. We will consider a simple bulk viscosity model, in the context of the Eckart formalism \[20\]. Naturally, this approach is phenomenological. Moreover, the Eckart formalism is not completely consistent, being a truncation of a causal theory \[21,22\]. Hence, everything that will be developed here must be later complemented by a fundamental model which can justify our phenomenological approach.

At background level, the description is equivalent to the GCGM: the viscous dark energy interpolates a matter dominate phase and a cosmological constant phase. Hence, all observational tests that concerns the background behaviour (like the supernova type Ia data) can be immediately used through the analysis already made for the GCGM \[23\]. However, at perturbative level, new features appear: the oscillations that plagues the GCG are absent here for a large range of the parameters. We make a simplified comparison with the 2dFGRS ignoring for the moment the presence of baryons. The goal is to show that it is possible qualitatively reproduce the general features of the mass power spectrum, with the absence of expressive oscillations.

This paper is organized as follows. In the next section we describe the viscous model and determine under which conditions it can account for the dark component of the universe. In section 3, a perturbative analysis is made, and the predicted power spectrum is compared with observations. In section 4, we present our conclusions, with some perspectives to a more fundamental motivation of this phenomenological model.

## 2 Background model

Let us consider a homogeneous and isotropic universe filled by a fluid with bulk viscosity. For simplicity, it will be supposed that the geometry is given by the flat Robertson-Walker metric,

\[
ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] ,
\]

where \(a(t)\) is the scale factor. The equations of motion are

\[
3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \rho ,
\]

\[
\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p^*) = 0 ,
\]

\[
p^* = p - \xi(\rho) u^\mu_\mu = p - 3\xi(\rho) \frac{\dot{a}}{a} .
\]

We will consider that \(p = \beta \rho\) and that the viscosity coefficient behaves as \(\xi(\rho) = \xi_0 \rho^n\). Hence, the equations of motion reduce to

\[
3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \rho ,
\]

\[
\dot{\rho} + 3 \frac{\dot{a}}{a} \left[ (1 + \beta)\rho - 3\xi_0 \rho^{n+1/2} \right] = 0 .
\]

From equation (5), we obtain the relation

\[
\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}} \rho^{1/2} ,
\]

so that the equation (6) can be rewritten as

\[
\dot{\rho} + 3 \frac{\dot{a}}{a} \left[ (1 + \beta)\rho - \tilde{\xi}_0 \rho^{n+1/2} \right] = 0 ,
\]

where

\[
\tilde{\xi}_0 = 3 \sqrt{\frac{8\pi G}{3}} .
\]
The equation (8) admits the solution

\[ \rho = \left[ \frac{\xi_0}{1 + \beta} + \frac{B}{1 + \beta} a^r \right]^{\frac{1}{2 - \nu}}, \tag{10} \]

where \( B \) is an integration constant and \( r = 3(\nu - 1/2)(1 + \beta) \).

In the case of the generalized Chaplygin gas, where there is no viscosity, the pressure is given by

\[ p = -\frac{A}{\rho^\alpha}. \tag{11} \]

The relation between the density and the scale factor is given by

\[ \rho = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}. \tag{12} \]

The GCGM and the viscosity model coincides, at background level, if \( \beta = 0 \) and if

\[ \nu = -\left( \alpha + \frac{1}{2} \right). \tag{13} \]

In this case, initially the Universe behaves as in the matter dominated phase, becoming later dominated by a cosmological term. In the case of the viscosity model, the initial phase is characterized by a domination of a fluid with equation of state \( p = \beta \rho \), when \( \nu < 1/2 \). For \( \nu > 1/2 \) the behaviour sketched above is inverted: initially there is a superluminal expansion followed by a subluminal expansion. The analysis based on the supernova type Ia data for the GCGM can be directly transferred to the dark viscous model, since it depends on the background only. In [23] an extensive analysis of the GCG parameters has been made. It has been found that the prediction for the parameter \( \alpha \) is \( \alpha = -0.75^{+1.04}_{-0.24} \). Hence, positive values of \( \nu \) ("normal" viscous behaviour) is preferred. But the dispersion is quite large.

3 Perturbative study

In principle, the most interesting situation in the model described above, in view of the present acceleration of the universe, concerns the choice \( \beta = 0 \) and \( \nu = -(\alpha + 1/2) \). These choices lead exactly to the same behaviour of the GCGM for the evolution of the background. However, here we have a more normal situation, where the viscosity grows with density when \( 0 < \nu < 1/2 \). On the other hand, most of the criticism on the GCGM concerns the fluctuations on the power spectrum which leads apparently to a \( \sigma_8 \) normalization that is not consistent with observation. Hence, in order to verify if the viscosity model can lead to improvements with respect to the GCGM, the scalar fluctuations must be studied. This study will be done here in the synchronous gauge.

In order to perform this perturbative study, the field equations are rewritten as

\[ R_{\mu\nu} = 8\pi G \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right), \tag{14} \]

\[ T^{\mu\nu} = 0, \tag{15} \]

\[ T^{\mu\nu} = (\rho + p^*) u^\mu u^\nu - p^* g^{\mu\nu}, \quad p^* = p - \xi(\rho) u^{\nu, \mu}. \tag{16} \]

The equations (14,15,16) are perturbed by introducing

\[ \tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} , \quad \tilde{\rho} = \rho + \delta \rho , \quad \tilde{u}^{\mu} = u^{\mu} + \delta u^{\mu} , \quad \tilde{p}^* = p^* + \delta p^* , \tag{17} \]

where \( g_{\mu\nu}, \rho, u^\mu \) and \( p^* \) are the background solutions described before, while \( h_{\mu\nu}, \delta \rho, \delta u^\mu \) and \( \delta p^* \) are small perturbations around them. The synchronous gauge condition \( h_{\mu 0} = 0 \) is imposed. A long but
We end up with the following perturbed equations:

\[
\hat{h} + \left( 2 \frac{\ddot{a}}{a} - 4 \pi G \xi \right) \hat{h} - 3 \left( \frac{\ddot{a}}{a} \right)^2 \left[ 1 + 3 \beta - 9 \left( \frac{\dot{a}}{a} \right)^2 \right] \Delta + 8 \pi G \xi \Theta = 0 ,
\]

\[
\hat{\Delta} + 9 \left( \frac{\ddot{a}}{a} \right)^2 \left[ \frac{\xi}{\rho} - \xi' \right] \Delta + \left( 1 + \beta - 6 \frac{\ddot{a}}{a} \frac{\dot{a}}{a} \right) \left( \Theta - \frac{h}{2} \right) = 0 ,
\]

\[
\left( 1 + \beta - 3 \frac{\ddot{a}}{a} \frac{\dot{a}}{a} \right) \hat{\Theta} + \left[ \frac{\ddot{a}}{a} \left( 1 + \beta - 3 \frac{\ddot{a}}{a} \frac{\dot{a}}{a} \right) \left( 2 - 3 \beta + 9 \frac{\ddot{a}}{a} \xi' - 3 \left( \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \xi \right) \Theta = \right.
\]

\[
\left. - \frac{\nabla^2}{a^2} \left[ \left( \beta - 3 \frac{\ddot{a}}{a} \xi' \right) \Delta - \frac{\xi}{\rho} \left( \Theta - \frac{h}{2} \right) \right] \right) .
\]

In these equations, the following definitions were used:

\[
h = \frac{h_{kk}}{a^2} , \quad \Delta = \frac{\delta p}{\rho} , \quad \Theta = \partial_i u_i ^ .
\]

To deduce these equations, the expression for the perturbation of the effective pressure has been used. From (16), the perturbation in the effective pressure (a crucial aspect for the results to be present later), is

\[
\delta p^* = \left( \beta - 3 \frac{\ddot{a}}{a} \right) \delta p - \xi (\rho) \left( \Theta - \frac{h}{2} \right) ,
\]

These equations can be rewritten in terms of the redshift variable \( z = -1 + 1/a \), where the scale factor has been made equal to unity today, \( a_0 = 1 \). Performing also a plane wave expansion in the perturbed quantities such that

\[
\delta(t, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \delta_k(t) e^{i \vec{k} \cdot \vec{x}} \, d^3 k , \quad k = \text{wavenumber of the perturbation} ,
\]

we end up with the following perturbed equations:

\[
h' - \frac{1}{2(1+z)} \left[ 4 - (1 + \beta)Af(z)^{2\nu^* - 1} \right] h = -3 \left( \frac{f(z)}{1+z} \right) \left[ 1 + 3 \beta - 3(1 + \beta) \nu Af(z)^{2\nu^* - 1} \right] \Delta + \frac{1 + \beta}{1 + z} f(z)^{2\nu^* - 1} \left( \Theta - \frac{h}{2} \right) ,
\]

\[
\Delta' - 3 \left( \frac{1 + \beta}{1 + z} \right) (1 - \nu) \frac{1 + \beta}{1 + z} f(z) \left( 1 - 2Af(z)^{2\nu^* - 1} \right) \left( \Theta - \frac{h}{2} \right) = 0 ,
\]

\[
(1 + \beta)(1 - Af(z)^{2\nu^* - 1}) \Theta' - \left( \frac{1 + \beta}{1 + z} \right) \left[ 1 - Af(z)^{2\nu^* - 1} \right] \left[ 2 - 3 \beta + 3 \nu (1 + \beta) f(z)^{2\nu^* - 1} \right] +
\]

\[
(1 + \beta)Af(z)^{2(\nu^* - 1)} f'(z) \Theta +
\]

\[
\frac{k^2}{k_0^2} \left( \beta - (1 + \beta) \nu f(z)^{2\nu^* - 1} \right) \Delta - \frac{1 + \beta}{3} Af(z)^{2(\nu^* - 1)} \left( \Theta - \frac{h}{2} \right) = 0
\]

The primes mean derivatives with respect to the redshift \( z \). The following definitions were also employed:

\[
f(z) = \left\{ A + (1 - A)(1+z)^{-r} \right\} \uparrow ,
\]

\[
r = 3 \left( \nu - \frac{1}{2} \right) (1 + \beta) ,
\]

\[
A = \frac{3}{1 + \beta} \sqrt{\frac{8 \pi G}{3} \xi_0 \nu^{1/2}} ,
\]
Figure 1: Behaviour for \( A = 0.1 \) and \( \nu = -0.5, 0.3, 0.0, 0.3 \) and \( 0.4 \). The ordinate represents \( \log_{10} P_k \) and the abscissa \( \log_{10} k h^{-1} \). As \( \nu \) grows, the theoretical curve approaches the observed curve.

The parameter \( k_0 \) is associated with the Hubble length, \( k_0 = 2\pi H_0/c \sim 2\pi/(3h) \times 10^{-3} Mpc^{-1} \), \( H_0 \) being the Hubble’s constant. The recent results from the WMAP measurements of the anisotropy of the cosmic microwave background radiation indicates \( h \sim 0.7 \) \[24\].

We allow the perturbed equations to evolve from \( z = 500 \) to \( z = 0 \), where the final spectrum is computed. The initial conditions are fixed by using the transfer function

\[
T(k) = \frac{B\sqrt{k}}{1 + \frac{8}{3\pi} k + 4\pi^2 k^2} .
\]

where \( \Omega \) is the total density fraction, with respect to the critical density, which in the present case is \( \Omega = 1 \). The amplitude \( B \) can also be fixed by using the normalization of the anisotropy of CMB. Following \[25\], we adopt \( B = (24h^{-1} Mpc)^4 \). At \( z = 0 \) we compute the power spectrum

\[
P_k = |\delta_k|^2 .
\]

The spectrum is computed for a large range of values of \( k \). The comparison with the observational results for the power spectrum of mass agglomeration obtained through the 2dFGRS program is displayed in figures 1, 2 and 3 for different values of the parameters \( \nu \) and \( A \). The main feature is the absence of oscillations in the power spectrum of the viscous dark fluid. The absence of oscillations occurs for positive and negative values of the parameter \( \nu \). In principle this seems to be surprising, since negative values of \( \nu \) should correspond to a positive sound velocity, which should drive oscillations in the power spectrum. The reason why these oscillations do not appear is that the spatial gradient term, which drives oscillations or instabilities depending on its overall sign, is now composed of three terms, containing not only the density contrast, but the velocity and the metric perturbations. The presence of this combination of terms avoids the appearance of strong oscillations or instabilities. This combination is due to the form of the effective pressure. Just to compare, the GCGM, at perturbative level, contains only the density contrast in the spatial gradient term.

The inspection of figures 1, 2 and 3 reveals that the theoretical curves approach the observational data when \( \nu \) \( \rightarrow \) \( 1/2 \) and \( A \) becomes small. A quite reasonable agreement is, for example, obtained when \( \nu = 0.4 \) and \( A = 0.1 \) (figure 1). Even if the fittings displayed in figures 1, 2 and 3 in general do not reveal a remarkable agreement between theory and observation, except in the limits stated above, this is not a serious problem in present context due to one fundamental reason: we have not considered baryons. In fact, what we should compute is the power spectrum for the baryonic component, perhaps with a bias factor which may take into account a contribution of a fraction of the viscous dark fluid. But, what we would like to stress is that there is no blow up in the perturbations of this viscous dark fluid, as it happens with, for example, the GCGM. Notice that there is a significant suppression of power in the spectrum for negative values of \( \nu \); this suppression is much less important for positive \( \nu \). Such suppression may be interesting if we remember that we are computing the power spectrum of the dark component,
Figure 2: Behaviour for $A = 0.4$ and $\nu = -0.5, 0.3, 0.0, 0.3$ and 0.4. The ordinate represents $\log_{10} P_k$ and the abscissa $\log_{10} kh^{-1}$. As $\nu$ grows, the theoretical curve approaches the observed curve.

Figure 3: Behaviour for $A = 0.7$ and $\nu = -0.5, 0.3, 0.0, 0.3$ and 0.4. The ordinate represents $\log_{10} P_k$ and the abscissa $\log_{10} kh^{-1}$. As $\nu$ grows, the theoretical curve approaches the observed curve.
since the dark component does not agglomerate completely. Hence, the suppression of power in the dark component may avoid serious discrepancies with the dynamics of clusters of galaxies.

4 Conclusions

In this work, we have developed a phenomenological model for dark energy based on a viscous dark fluid. The approach is very simplified since we consider the bulk viscosity in the Eckart formalism, ignoring consequently problems of causality. It has been showed that, with the hypothesis that the bulk viscosity depends on a power of the density, \( \xi = \xi_0 \rho^\nu \), interpolation between a matter dominated phase and a cosmological constant phase is achieved if \( \nu < 1/2 \). Hence, such interpolation can be obtained for a non exotic viscous fluid where the viscosity decreases with the decreasing of density. Moreover, the behaviour characteristic of the Chaplygin gas model is recovered for negative values of \( \nu \).

The evolution of density perturbations for this viscous dark fluid has been computed. There is no oscillation in the power spectrum, in opposition to what happens with GCGM. The spectrum is highly suppressed for negative values of \( \nu \) but reproduces qualitatively the observed power spectrum for mass agglomeration for \( \nu \) positive. The fitting of the observational data becomes quite good when \( \nu \rightarrow 1/2 \) and \( A \rightarrow 0 \). The model studied here contains just one fluid, the viscous dark energy. Hence, we can expect that the adding of baryons will allow to fit reasonably the observational data. The absence of oscillations is due to the fact that the spatial gradient of the pressure presents a competition between all perturbed quantities. This is dictated by the covariant representation of the bulk viscosity.

The phenomenological approach developed here must of course be supplemented by a fundamental description of this viscous fluid. To do this a specific fluid model must be considered, with some interaction between the particles composing this fluid. Topological defects (cosmic strings, domain walls, textures) can lead to cosmological fluids with negative pressure in the perfect fluid approximation. We can think for example on the evolution of domains wall with friction in an expanding universe \[26\]. However, the effective equation of state for these objects becomes more complex if interactions are taken into account. Since interactions are inevitable in a gas of topological defects, it can be expected that deviations from the simple perfect fluid approximation can appear. We intend to explore this possibility.

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