Neutron Star Properties with Hyperons

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In the light of the recent discovery of a neutron star with a mass accurately determined to be almost two solar masses, we carefully examine the maximum mass of a neutron star containing hyperons within a relativistic Hartree-Fock treatment. Using the quark-meson coupling model, which naturally incorporates hyperons without additional parameters, we find a maximum mass a little over 2.1 $M_\odot$.

Keywords: neutron stars, equation of state of dense matter, hyperons, quarks

The recent observation of a $1.97 \pm 0.04 M_\odot$ millisecond pulsar, PSR J1614-2230, by Demorest et al. [1] has set the most stringent limit on models of neutron star cores so far. This discovery has spurred a re-examination of the possibility of exotica such as hyperons, Bose condensates, and quark matter playing an important role in models of neutron star interiors, owing to a presumed softening of the equation of state (EoS) expected in the presence of additional degrees of freedom. Historically, this has led to expectations of reduced maximum neutron star masses for compact objects in hydrostatic equilibrium.

In this Letter we build on the earlier work of Stone et al. [2], who already predicted the existence of neutron stars containing hyperons with masses as large as 2 $M_\odot$ in 2007, to establish a conservative upper limit on the maximum mass of such a star. We work within the quark-meson coupling (QMC) model [3,4], which has the advantage of being derived from the quark level, with a very small number of adjustable parameters, while being consistent with a broad range of constraints derived from hypernuclei as well as normal nuclear properties. We find that the stability under variation of the very small number of adjustable parameters is such that if a star were discovered with a mass significantly above 2.1 $M_\odot$, we would need to consider more exotic physics, because it could not be accommodated within the QMC model.

We recall that QMC is based upon the self-consistent modification of the structure of a baryon embedded in nuclear matter. At Hartree level it involves only three adjustable parameters which describe the effective couplings of the $\sigma$, $\omega$ and $\rho$ mesons to the $u$ and $d$ quarks. These are fixed by adjusting them to fit the properties of symmetric nuclear matter, namely its saturation density and binding energy as well as its symmetry energy. We note that the $\sigma$ meson used here simply serves as a convenient representation of the scalar-isoscalar attraction arising from two-pion exchange.

In the most recent development of the QMC model [5], the self-consistent inclusion of the gluonic hyperfine interaction led to a very successful description of the binding energies of $\Lambda$-hypernuclei—as well as the observed absence of medium and heavy mass $\Sigma$-hypernuclei—with no additional parameters. We stress that this is achieved without any coupling of the strange quark to the $\sigma$, $\omega$ and $\rho$ mesons (which would be OZI suppressed) and without the need to introduce any further mesons. While the model could be supplemented with much heavier mesons containing strange quarks [6], Occam’s razor suggests that one should not introduce them if they are not needed.

A clear connection has been established between the self-consistent treatment of in-medium hadron structure and the existence of many-body [7] or density dependent [8] effective forces. Dutra et al. [9] critically examined a variety of phenomenological Skyrme models of the effective density dependent nuclear force against the most up-to-date empirical constraints. Amongst the few percent of the Skyrme forces studied which satisfied all of these constraints, the Skyrme model SQMC700, derived from the QMC model, was unique in that it incorporated the effects of the internal structure of the nucleon and its modification in-medium.

While the earlier study of Stone et al. [2] demonstrated the importance of exchange (Fock) terms in calculations of the EoS of dense baryonic matter in $\beta$-equilibrium, it included only the Dirac vector term in the vector-meson-nucleon vertices. In this work we include the full vertex structure which one might expect to enhance the pressure at high density. This is especially so in the case of the $\rho$ meson for which the tensor coupling is much larger than that of the $\omega$. Our calculation extends the work of Krein et al. [10], who considered nucleons only, by evaluating the full exchange terms for all octet baryons and adding them in the same way as Stone et al. [5]; as additional contributions to the energy density. It also extends and complements the important work of Miyatsu et al. who calculated the properties of neutron stars within an earlier version of the QMC model [11]. In particular, we employ the latest version of the model which reproduces key hypernuclear properties without the adjustment of...
coupling constants needed there. Furthermore, we carefully explore the limit on the maximum mass of a neutron star containing hyperons while ensuring consistency with critical nuclear properties, such as the incompressibility of nuclear matter.

Next we describe the main features of the QMC model and explore its parameters arranged into groups ("scenarios") to test the robustness of its predictions. Within the QMC model, the baryon energy density, \( \epsilon_B \), is given by

\[
\epsilon_B = \frac{2}{(2\pi)^3} \sum_B \int dp \sqrt{p^2 + M_B^2},
\]

where the effective, in-medium baryon masses, \( M_B^* \), are calculated self-consistently for an MIT bag immersed in (and in Ref. [5], parameterized as functions of) a mean scalar field, designated here by a barred symbol. At a given density, \( \bar{\sigma} \) is self-consistently expressed as

\[
\bar{\sigma} = -\frac{2}{m_\sigma^2 (2\pi)^3} \sum_B \int dp \frac{M_B^*}{\sqrt{p^2 + M_B^*}} \frac{\partial M_B^*}{\partial \bar{\sigma}}.
\]

An additional contribution, \( \delta \bar{\sigma} \), to the scalar field arises if we include the Fock terms in the minimization of the energy density. This provides only a small correction to the mean field, and its effect is included only in the scenario denoted \( \delta \bar{\sigma} \).

For the Yukawa propagator for meson \( \alpha \) with momentum \( \vec{k} = \vec{p} - \vec{p}' \). For the vector mesons, the full vertex structure is included in the manner of Ref. [10] as

\[
\Gamma_{\alpha B} = \frac{1}{2} \sum_{s,s'} \left[ \bar{u}_B(p') \Gamma_{\alpha} u_B(p, s) \right]^2 \Delta_{\alpha}(\vec{k}),
\]

As usual, the effect of short distance repulsion on the Fock terms is simulated by the replacement

\[
\frac{\tilde{q}^2}{(q^2 + m^2)} \rightarrow 1 - \frac{m^2}{(q^2 + m^2)}
\]

from which the unit term is subtracted, thus eliminating a \( \delta \)-function. The form factors \( F_{\alpha B} \) all have the same dipole form with the cutoff mass \( \Lambda \) varied from 0.9 to 1.3 GeV to test the sensitivity. As a further test of the model dependence, we consider two choices for the ratios of tensor to vector coupling constants \( \kappa_{\alpha B} = f_{\alpha B}/g_{\alpha B} \) (\( \alpha \in \{\sigma, \omega, \rho\} \)). In scenario \( \kappa_1 \) (consistent with values derived within QMC) we take these ratios from vector meson dominance (\( \kappa_{\sigma N} = f_{\sigma N}/g_{\sigma N} = 3.70 \)). Alternatively (scenario \( \kappa_{11} \)) we take these ratios from the Nijmegen potentials (Table VII of Ref. [1]), with a larger value of \( \kappa_{\rho N} = 5.7 \).

Of the baryon-meson coupling constants \( g_{\sigma B}(\bar{\sigma}), g_{\omega B}, \) and \( g_{\rho B}, \) only \( g_{\sigma B} \) is density dependent. Its model parameterisation [3] is dependent on the free nucleon radius, which is taken to be \( R_N^{free} = 1.0 \) fm - with an alternate scenario having \( R_N^{free} = 0.8 \) fm. The density dependence is given by

\[
g_{\sigma B}(\bar{\sigma}) = -\frac{\partial M_B^*}{\partial \bar{\sigma}} \equiv -\frac{\partial M_B^*(\bar{\sigma}, g_{\sigma N}, R_N^{free})}{\partial \bar{\sigma}}.
\]

Values of the coupling constants \( g_{\alpha N} \) for various mesons \( \alpha \) and a selection of scenarios considered in this work are presented in Table I. The couplings \( g_{\omega B} \) and \( g_{\rho B} \) are expressed in terms of the quark level couplings

\[
g_{\omega B} = n_{\omega d} g_{\bar{q}}^d; \quad g_{\rho B} = g_{\rho N} = g_{\bar{q}}^d,
\]

where \( n_{u,d} \) represents the number of light quarks in baryon \( B \). The \( \sigma, \omega \) and \( \rho \) couplings to the quarks are
the maximum stellar mass lies outside of the range 1–2 GeV. The diver-
gence between the baseline scenario (κI) and the ’Hartree
Only’ and ’Dirac Only’ scenarios highlights the importance of
the ρN tensor coupling in Hartree-Fock at high density.

In Table I we present the coupling constants, incom-
pressibilities of symmetric nuclear matter, and stellar
properties, for a number of variations of the QMC model,
in each scenario including the σ, π, ω and ρ Fock terms.
We note that in all scenarios Ξ− hyperons are present in
significant quantities in the the maximum mass stars.

It is remarkable that in all of the scenarios investigated,
the stellar properties are largely consistent, and similar
to those reported by Stone et al. [2]. Scenarios in which
the maximum stellar mass lies outside of the range 1.9–
2.14 M⊙ correspond to nuclear matter compressibilities
above the upper limit set in the recent comprehensive
analysis of giant monopole resonance data [13]. While
this cannot be true in general, it is certainly the case for
the QMC model.

Turning to the effects of the inclusion of the full ex-
change terms on stellar properties, we find that the
threshold density for Ξ− is lowered, while those of Λ and

| Scenario | gσN | gωN | gρ | K (MeV) | R (km) | Mmax (M⊙) | nmax (n0) |
|----------|-----|-----|----|--------|--------|------------|----------|
| κI       | 10.42 | 11.02 | 4.55 | 298 | 12.27 | 1.93 | 5.52 |
| κII      | 10.55 | 11.09 | 3.36 | 299 | 12.19 | 1.93 | 5.62 |
| Λ = 1.0  | 10.74 | 11.66 | 4.68 | 305 | 12.45 | 2.00 | 5.32 |
| Λ = 1.1  | 11.10 | 12.33 | 4.84 | 312 | 12.64 | 2.07 | 5.12 |
| Λ = 1.2  | 11.49 | 13.06 | 5.03 | 319 | 12.83 | 2.14 | 4.92 |
| Λ = 1.3  | 11.93 | 13.85 | 5.24 | 329 | 13.02 | 2.23 | 4.74 |
| R = 0.8  | 11.20 | 12.01 | 4.52 | 300 | 12.41 | 1.98 | 5.38 |
| Fock δσ  | 10.91 | 11.58 | 4.52 | 285 | 12.29 | 1.98 | 5.5 |
| Dirac Only | 10.12 | 9.25  | 7.83 | 294 | 12.56 | 1.79 | 5.2 |
| Hartree Only | 10.25 | 7.95  | 8.40 | 283 | 11.85 | 1.54 | 6.0 |
| Nucleon Only | 10.42 | 11.02 | 4.55 | 298 | 11.64 | 2.26 | 5.82 |

TABLE I. Meson-nucleon coupling constants determined for
our baseline scenario ’κI’ (for which Λ = 0.9 GeV, and K(0) =
1.0 fm) and subsequent scenarios in which differences from
κI are given in column 1. Also shown are the saturation
incompressibility, K; stellar radius; maximum stellar mass
and corresponding central density (units n0 = 0.16 fm−3).

For a compact object in β-equilibrium we solve the
familiar system of equations for the number densities
of the baryons and leptons [12]. The lepton energy
density and pressure are given by the usual formulas
for a degenerate Fermi gas. In order to obtain the
neutron star star properties shown in Table I, we solve the
Tollmann–Oppenheimer–Volkov equations for the grav-
itational mass and radius [12]. The resulting dependence
of the neutron star mass on radius, for a selection of the
variations of the model, is shown in Fig. 1.

![Fig. 1](image1.png)

In Fig. 2 we present the species fractions, and
Fock energy densities for our baseline (κI) scenario.

![Fig. 2](image2.png)

The Ξ− hyperons are raised, as demonstrated in Fig. 3. In all
scenarios there is a greater splitting between the thresholds of
the Ξ baryons than that found by Stone et al. [2]. In
our baseline scenario (κI), the Ξ− threshold occurs at
0.42 fm−3, followed by Ξ0 at 0.91 fm−3. We find that Λ
production is not energetically favoured at densities considered here, in agreement with Ref. [14]. Using the Nijmegen values of tensor coupling strength ('\(k_H\)'), the \(\rho N\) vector coupling is reduced as the tensor part of the interaction contributes significantly to the symmetry energy. The EoS is otherwise largely insensitive to this choice. Similarly, it is insensitive to the choice of free nucleon radius, despite a moderate impact on the couplings.

The correction (\(\delta \sigma\)) to the scalar mean field arising from the Fock terms decreases the incompressibility by 13 MeV, yet other observables remain largely unaltered by this addition. The cutoff, \(\Lambda\), used in form factors (which controls the strength of the Fock terms) exhibits a more pronounced relationship with the observables in Table I. Increasing \(\Lambda\) beyond 0.9 GeV raises the incompressibility at saturation density. The conceptual separation between the incompressibility at saturation density and the amount of pressure or ‘stiffness’ at higher densities is critical. It is the latter that leads to neutron stars with maximum masses ranging from 1.90 \(M_\odot\) to 2.14 \(M_\odot\), even when allowance is made for the appearance of hyperons. This suggests that hyperons are very likely to play a vital role as constituents of neutron stars.

We stress that the QMC model does not predict the appearance of \(\Sigma\) hyperons at any density where the model can be considered realistic. This is in contrast to a number of other relativistic models which do predict the \(\Sigma\) threshold to occur, even prior to that of the \(\Lambda\) [15, 16]. We note that Schaffner-Bielich [15] considered a phenomenological modification of the \(\Sigma\) potential with additional repulsion, which significantly raised its threshold density. In the case of the QMC model the physical explanation of the absence of \(\Sigma\)-hyperons is very natural, with the mean scalar field enhancing the repulsive hyperfine force in the bound \(\Sigma\). Recall that the hyperfine splitting is due to one-gluon-exchange, which determines the free \(\Sigma\)-\(\Lambda\) mass splitting in the MIT bag model.

For comparison purposes, we also include a ‘Nucleon Only’ scenario, in which hyperons are artificially excluded. In this case the EoS is increasingly stiffer at densities above 0.4 fm\(^{-3}\), leading to a large maximum stellar mass of 2.26 \(M_\odot\), consistent with many other nucleon-only models.

It is worth remarking that upon inclusion of the tensor coupling, the proton fraction increases more rapidly as a function of total baryon density. This is likely to increase the probability of the direct URCA cooling process in proto-neutron stars. As a further consequence, the maximum electron chemical potential is increased in this case, which may well influence the production of \(\pi^-\) and \(\bar{K}\) condensates. Changes to the \(\Lambda\) threshold (occurs at higher density with lower maximum species fraction) reduces the possibility of H-dibaryon production as constrained by \(\beta\)-equilibrium of chemical potentials.

In summary, taking into account the full tensor structure of the vector-meson-baryon couplings in a Hartree-Fock treatment of the QMC model gives increased pressure at high density—largely because of the \(\rho N\) tensor coupling—while maintaining reasonable values of incompressibility at saturation density. The conceptual separation between the incompressibility at saturation density and the amount of pressure or ‘stiffness’ at higher densities is critical. It is the latter that leads to neutron stars with maximum masses ranging from 1.90 \(M_\odot\) to 2.14 \(M_\odot\), even when allowance is made for the appearance of hyperons. This suggests that hyperons are very likely to play a vital role as constituents of neutron stars.

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[1] P. Demorest et al., Nature 467 (2010) 1081.
[2] J. Rikovska Stone et al., Nucl. Phys. A 792 (2007) 341
[3] F. Weber, Prog. Part. Nucl. Phys. 835 (2007) 341
[4] P. A. M. Guichon, Phys. Lett. B 200, 235 (1988).
[5] P. A. M. Guichon et al., Nucl. Phys. A 601, 349 (1996)
[6] P. A. M. Guichon, A. W. Thomas and K. Tsushima,
Nucl. Phys. A 814, 66 (2008).
[7] S. Weissenborn, D. Chatterjee and J. Schaffner-Bielich,
Phys. Rev. C 85, 065802 (2012) [arXiv:1112.0234 [astro-ph.HE]]
[8] P. A. M. Guichon and A. W. Thomas, Phys. Rev. Lett.
93, 132502 (2004)
[9] P. A. M. Guichon et al., Nucl. Phys. A 772, 1 (2006)
[10] M. Dutra et al., Phys. Rev. C 85, 035201 (2012).
[11] G. Krein, A. W. Thomas and K. Tsushima, Nucl. Phys.
A 650, 313 (1999)
[12] T. A. Rijken, M. M. Nagels and Y. Yamamoto, Prog.
Theor. Phys. Suppl. 185, 14 (2010).
[13] N. K. Glendenning, “Compact stars: Nuclear physics, particle physics, and general relativity,” New York, USA: Springer (1997)
[14] J. R. Stone, S. A. Moskowski and N. J. Stone, to be published.
[15] T. Miyatsu, T. Katayama and K. Saito, Phys. Lett. B
709, 242 (2012)
[16] F. Weber, Prog. Part. Nucl. Phys. 54, 193 (2005)
[17] G. Bayn, C. Pethick and P. Sutherland, Astrophys. J.
170, 299 (1971).