An adaptive actuator failure compensation scheme for two linked 2WD mobile robots

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Abstract. This paper develops a new adaptive compensation control scheme for two linked mobile robots with actuator failures. A configuration with two linked two-wheel drive (2WD) mobile robots is proposed, and the modelling of its kinematics and dynamics are given. An adaptive failure compensation scheme is developed to compensate actuator failures, consisting of a kinematic controller and a multi-design integration based dynamic controller. The kinematic controller is a virtual one, and based on which, multiple adaptive dynamic control signals are designed which covers all possible failure cases. By combing these dynamic control signals, the dynamic controller is designed, which ensures system stability and asymptotic tracking properties. Simulation results verify the effectiveness of the proposed adaptive failure compensation scheme.

1. Introduction
Due to simplicity, efficiency and flexibility, two-wheel drive (2WD) mobile robots are widely used [1]. In some harsh conditions, resulting from a terrorist attack, a nuclear accident or nature disasters, 2WD rescue robots carrying instruments are employed to help rescuers. These adverse conditions may increase the probability of actuator (wheel motor) failures which will leads to the loss of the failed robots and the important instruments. To deal with this situation, several physical links may be used to connect the robots. For example, if one robot fails, then another robot carrying a mechanical arm can be sent to link the failed robot and to help it continue moving. This robot architecture provides actuator and sensor redundancies which improve the fault tolerance of the system. This paper is concerned with the fault tolerant control design for such a configuration with two linked 2WD robots.

There are a lot of research results on the motion control of 2WD mobile robots [2, 3] but without the consideration of actuator faults. For fault tolerant control, various effective design methods are proposed for different applications [4, 5]. As for the application to wheeled robots, a sensor fault accommodation scheme is presented in [6]; a fault-tolerant control design method is given in [7] for four-wheel drive (4WD) mobile robots; and a hybrid adaptive fault tolerant control scheme is proposed in [8] to accommodate partial faults and degradation for 2WD robots. Unlike 4WD robots, a 2WD robot has no redundant actuator, so that if one motor is lost, then the 2WD robot becomes uncontrollable.

To our knowledge, there is no research result on the fault tolerant control of 2WD robots with the motors (actuators) being totally faulty. To deal with such actuator failures, physical
links are used to connect several 2WD mobile robots as shown in Fig. 1 for two robots. For this configuration, if one or two motors are lost, the remaining motors can continue moving the two robots. In this paper, a multi-design integration based adaptive control scheme is developed. The main contributions are as follows.

(i) The kinematic and dynamic models of the two 2WD mobile robots with fixed link are proposed.

(ii) A new adaptive actuator failure compensation scheme is developed using a multi-design integration method, which ensures system stability and asymptotic tracking properties, despite the presence of actuator failures including simultaneous multi-failures.

![Figure 1: Two linked 2WD mobile robots](image)

The rest of this paper is as follows. In Section 2, system modelling is given and the actuator failure compensation problem is formulated. In Section 3, a multi-design integration based adaptive actuator failure compensation scheme is developed. In Section 4, a simulation study is presented to demonstrate the effectiveness of the proposed adaptive control scheme. Conclusions follow in Section 5.

2. System Modeling and Problem Formulation

In this section, the system models are given for the two linked 2WD mobile robots as shown in Fig. 1, and its actuator failure compensation problem is formulated.

As shown in Fig. 1, the orientation of robot 2 is consistent with the one of the physical link, but the orientation of robot 1 is independent, and for each robot: the front wheel is passive and the two rear wheels are actuated; $P_i$ ($i = 1, 2$) is the center between two actuated wheels, $C_i$ is the center of mass, $a_i$ is the distance between $P_i$ and $C_i$, $b_i$ is half of the distance between two actuated wheels, $r_i$ is the radius of wheels, $\theta_i$ is the orientation of the robot, and $\tau_{if}$ and $\tau_{ir}$ are the control torques applied to the left and right actuated wheels, respectively. Moreover, $d$ is the distance between $P_1$ and $P_2$, $OXY$ is the inertial frame, and $(x, y)$ denotes the position of $P_2$ in frame $OXY$.

**Kinematic model.** Let $q = [x, y, \theta_2, \theta_1]^T$ and $\eta = [v_2, \omega_1]^T$, where $v_2$ is the linear velocity of robot 2, and $\omega_1$ is the angular velocity of robot 1. The kinematic model of the two linked 2WD mobile robots in Fig. 1 is given by

$$\dot{q} = S(q)\eta. \quad (1)$$
where

\[
S(q) = \begin{bmatrix}
\cos \theta_2 & \sin \theta_2 & 1/2 \tan(\theta_1 - \theta_2) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}^T.
\]

Moreover, the system constraints may be expressed as

\[
A(q)\dot{q} = 0,
\]

where

\[
A(q) = \begin{bmatrix}
\sin \theta_1 & -\cos \theta_1 & -d \cos(\theta_1 - \theta_2) & 0 \\
\sin \theta_2 & -\cos \theta_2 & 0 & 0
\end{bmatrix}
\]

is the system constraint matrix. Then, we have the following property:

\[
S^T(q)A^T(q) = 0.
\]

**Dynamic model.** The dynamic equation of these two linked 2WD robots is obtained by using the Lagrange method as:

\[
M(q)\ddot{q} + E(q, \dot{q}) = B(q)\tau + A^T(q)\lambda,
\]

where \(M(q) \in \mathbb{R}^{4 \times 4}\) is the inertia matrix that is symmetric positive definite, \(E(q, \dot{q}) \in \mathbb{R}^4\) is the centripetal and coriolis vector, \(B(q) \in \mathbb{R}^{4 \times 4}\) is the input injection matrix, \(\tau = [\tau_1, \tau_1, \tau_2, \tau_2]^T\) is the vector of control torques, and \(\lambda \in \mathbb{R}^2\) is the vector of constraint forces. The matrices and vectors in (6) are given by

\[
M(q) = \begin{bmatrix}
m_1 + m_2 & 0 & -(a_2m_2 + dm_1) \sin \theta_2 & -a_1m_1 \sin \theta_1 \\
0 & m_1 + m_2 & (a_2m_2 + dm_1) \cos \theta_2 & a_1m_1 \cos \theta_1 \\
-(a_2m_2 + dm_1) \sin \theta_2 & (a_2m_2 + dm_1) \cos \theta_2 & m_2a_1^2 + m_1d^2 + I_{m_2} & a_1dm_1 \cos(\theta_1 - \theta_2) \\
-a_1m_1 \sin \theta_1 & a_1m_1 \cos \theta_1 & a_1dm_1 \cos(\theta_1 - \theta_2) & m_1a_1^2 + I_{m_1}
\end{bmatrix},
\]

\[
E(q, \dot{q}) = \begin{bmatrix}
-a_1m_1 \theta_1^2 \cos \theta_1 - (a_2m_2 + dm_1) \theta_2^2 \cos \theta_2 \\
-a_1m_1 \theta_1^2 \sin \theta_1 - (a_2m_2 + dm_1) \theta_2^2 \sin \theta_2 \\
a_1dm_1 \theta_1 \theta_2 (\sin(\theta_1 - \theta_2)) \\
a_1dm_1 \theta_2 \sin(\theta_1 - \theta_2)
\end{bmatrix},
\]

\[
B(q) = \begin{bmatrix}
\cos \theta_1 & \cos \theta_1 & \cos \theta_2 & \cos \theta_2 \\
\sin \theta_1 & \sin \theta_1 & \sin \theta_2 & \sin \theta_2 \\
-\theta_1 & -\theta_1 & \theta_2 & \theta_2 \\
\theta_1 & \theta_1 & -\theta_2 & -\theta_2
\end{bmatrix},
\]

where \(m_1\) and \(m_2\) are the masses of the robot 1 and robot 2, and \(I_{m_1}\) and \(I_{m_2}\) are the corresponding inertia parameters.

Substituting (1) into (6), and multiplying by \(S^T(q)\), \(A^T(q)\lambda\) is eliminated with (5). Then equation (6) becomes

\[
\dot{M}_1(q)\ddot{q} + \dot{M}_2(q)\eta + \dot{E}(q, \dot{q}) = \dot{B}(q)\tau,
\]

where \(\eta = [\omega_2, \omega_1]^T\), and

\[
\dot{M}_1(q) = S^T(q)M(q)S(q), \quad \dot{M}_2(q) = S^T(q)M(q)\dot{S}(q),
\]

\[
\dot{E}(q, \dot{q}) = S^T(q)E(q, \dot{q}), \quad \dot{B}(q) = S^T(q)B(q).
\]
Note that \( \tilde{M}_1(q) \) is also symmetric positive definite as well as \( M(q) \).

**Actuator failure model.** The considered actuator failure is that some motors totally lose power or are stuck which will introduce additional frictions. This type of actuator failure for one motor is modeled as

\[
\tau_j(t) = \bar{u}_j, \quad t \geq t_j
\]

where \( j = 1r, 1l, 2r, 2l \), \( \bar{u}_j \) is the friction value that is unknown but constant, and \( \bar{u}_j = 0 \) means that the motor loses its power but can rotate freely and \( \bar{u}_j > 0 \) means that the motor is stuck or can not rotate freely caused by some frictions in the bearing, and \( t_j \) is the unknown failure occurring time instant.

Consider that all actuators may be faulty. The control torque in (8) becomes

\[
\tau(t) = \sigma(t)u(t) + (I_4 - \sigma(t))\bar{u},
\]

where \( \tau = [\tau_{1r}, \tau_{1l}, \tau_{2r}, \tau_{2l}]^T \) is the vector of control torques generated by the wheel motors, \( u = [u_{1r}, u_{1l}, u_{2r}, u_{2l}]^T \) is the control signal vector to be designed, \( \bar{u} = [\bar{u}_{1r}, \bar{u}_{1l}, \bar{u}_{2r}, \bar{u}_{2l}]^T \) is the vector of constant frictions, \( I_4 \) is the identity matrix, and \( \sigma = \text{diag}\{\sigma_{1r}, \sigma_{1l}, \sigma_{2r}, \sigma_{2l}\} \) is the uncertain failure pattern matrix with

\[
\sigma_j(t) = \left\{ \begin{array}{ll}
0, & \text{if the } j\text{th motor fails,} \\
1, & \text{otherwise.}
\end{array} \right.
\]

**Actuation redundancy.** For the two linked 2WD mobile robots in Fig. 1, the following actuation redundancy condition needs to be satisfied:

\[
\text{rank}(B\sigma) = 2.
\]

for all possible failure pattern matrices \( \sigma \). This condition means that there are enough actuated motors to control \( \eta = [v_2, \omega_1]^T \).

**Remark 1** With \( B(q) \) in (9), the compensable failure cases satisfying this redundancy condition are: 1) fault free case with four actuated wheels; 2) one actuator fails with three remaining actuated wheels; 3) two actuators fail with two remaining actuated wheels. However, for case 3), if the two failed actuators are in robot 2, then the system is similar with a tractor-trailer system [9], and the failures may be tolerated; if each robot has one faulty actuator, then the failures are also compensable; but if the two failed actuators are in robot 1, then the failures are not compensable, because in this case, \( \text{rank}(B\sigma) = 1 \) with \( \sigma = \text{diag}\{0, 0, 1, 1\} \) and \( \omega_1 \) is uncontrollable.

**Control objective.** The control objective is to develop an actuator failure compensation scheme for two linked 2WD mobile robots in Fig. 1 to asymptotically track a reference trajectory, that is, to design a control signal \( u(t) \) to guarantee that all closed-loop system signals are bounded, and \( \lim_{t \to \infty} (x(t) - x_r(t)) = 0 \), \( \lim_{t \to \infty} (y(t) - y_r(t)) = 0 \) and \( \lim_{t \to \infty} (\theta_2(t) - \theta_r(t)) = 0 \) for system (3) and (8), in the presence of some actuator failures modeled as (10)-(12) that satisfies the actuation redundancy condition in (13), where \( x_r, y_r, \theta_r \) are the reference trajectories.

**Remark 2** In Fig. 1, the position of \( P_1 \) is determined by \( x, y \) and \( \theta_2 \), in this sense, the control objective contains the position for the whole system. In addition, this objective is implemented by the control of \( \omega_1 \) with the dynamic equation (8), as \( \dot{\theta}_1 = \omega_1 \), we can see that the orientation angle \( \theta_1 \) is an intermediate variable to be controlled.
A virtual robot is employed to generate the reference trajectories as follows:

\[ \dot{x}_r = v_r \cos \theta_r, \quad \dot{y}_r = v_r \sin \theta_r, \quad \dot{\theta}_r = \omega_r. \tag{14} \]

where \( v_r \) and \( \omega_r \) are the linear velocity and angular velocity. By choosing appropriate \( v_r \), \( \omega_r \) and initial values \( x_r(0), y_r(0) \) and \( \theta_r(0) \), the reference trajectories \( x_r, y_r \) and \( \theta_r \) are determined.

In this paper, we consider the tracking problem of a two-robot system. Then the following assumption is given for the reference trajectories.

**Assumption 1:** The reference trajectories \( x_r, y_r \) and \( \theta_r \), the velocities \( v_r \) and \( \omega_r \), and their derivatives are continuous and uniformly bounded, moreover \( v_r \neq 0 \).

**Design issues.** The structure of the proposed actuator failure compensation scheme is shown in Fig. 2. To design such a control scheme, the following technical issues need to be solved:

- to design a kinematic control law \( \eta_c = [v_{2c}, \omega_{1c}]^T \), such that when it is applied, the desired control performance can be ensured;
- to design a dynamic control law \( u \) using the designed kinematic control law, to achieve the control objective;
- to handle the failure uncertainties, for which, a multi-design integration based adaptive method will be employed;
- to evaluate the control performance.

3. Multi-Design Integration based Adaptive Failure Compensation Scheme

In this section, a multi-design integration based adaptive actuator failure compensation scheme is developed including a kinematic controller and a dynamic controller.

3.1. Kinematic Controller Design

3.1.1. Kinematic control law

Define the output tracking error vector as

\[ \tilde{e} = [\tilde{e}_x, \tilde{e}_y, \tilde{e}_\theta]^T = [x - x_r, y - y_r, \theta_2 - \theta_r]^T, \tag{15} \]

and a transformation matrix as

\[
T_e = \begin{bmatrix}
\cos \theta_r & \sin \theta_r & 0 \\
-\sin \theta_r & \cos \theta_r & 0 \\
0 & 0 & 1
\end{bmatrix}.
\tag{16}
\]
Then, a new error vector is defined as
\[ e = [e_x, e_y, e_\theta]^T = T_e \hat{e}. \] (17)

Since \( T_e \) is nonsingular with \( \text{det}[T_e] = 1 \), if \( \lim_{t \to \infty} e(t) = 0 \), then \( \lim_{t \to \infty} \hat{e}(t) = 0 \). It follows that
\[
\dot{e}_x = \omega_r e_y + v_2 \cos e_\theta - v_r, \\
\dot{e}_y = -\omega_r e_x + v_2 \sin e_\theta, \\
\dot{e}_\theta = \frac{v_2}{d} \tan(\theta_1 - \theta_2) - \omega_r.
\] (18) (19) (20)

Introduce the diffeomorphism:
\[
z_1 = e_x, \\
z_2 = e_y, \\
z_3 = \tan e_\theta,
\] (21) (22) (23)

an additional signal:
\[ z_4 = \frac{\tan(\theta_1 - \theta_2)}{d \cos^3 e_\theta} - \frac{\omega_r}{v_r \cos^2 e_\theta} + e_y, \] (24)

and an input transformation:
\[
\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} v_2 \cos e_\theta - v_r \\ \dot{z}_4 \end{bmatrix}.
\] (25)

Then, the derivatives of \( z_1, z_2, z_3 \) and \( z_4 \) are
\[
\dot{z}_1 = \omega_r z_2 + \alpha_1, \\
\dot{z}_2 = -\omega_r z_1 + (v_r + \alpha_1) z_3, \\
\dot{z}_3 = v_r (z_4 - z_2) + \alpha_1 (z_4 - z_2 + \frac{\omega_r}{v_r} (1 + z_3^2)), \\
\dot{z}_4 = \alpha_2.
\] (26) (27) (28) (29)

From (24) and (25), we can obtain
\[ \alpha = T_\alpha \eta + f_\alpha, \] (30)

where
\[
T_\alpha = \begin{bmatrix} T_{\alpha 11} & T_{\alpha 12} \\ T_{\alpha 21} & T_{\alpha 22} \end{bmatrix}, \\
f_\alpha = \begin{bmatrix} f_{\alpha 1} \\ f_{\alpha 2} \end{bmatrix},
\] (31)

with \( T_{\alpha 11} = \cos e_\theta, T_{\alpha 12} = 0, \)
\[
T_{\alpha 21} = \frac{3 \tan^2(\theta_1 - \theta_2) \sin e_\theta}{d^2 \cos^4 e_\theta} - \frac{\tan(\theta_1 - \theta_2)}{d^2 \cos^3 e_\theta \cos^2(\theta_1 - \theta_2)} - \frac{2 \omega_r \tan(\theta_1 - \theta_2) \sin e_\theta}{d v_r \cos^3 e_\theta} \sin e_\theta, \\
T_{\alpha 22} = \frac{1}{d \cos^3 e_\theta \cos^2(\theta_1 - \theta_2)}, \\
f_{\alpha 1} = -v_r, \\
f_{\alpha 2} = -\frac{3 \omega_r \tan(\theta_1 - \theta_2) \sin e_\theta}{d \cos^4 e_\theta} - \frac{\dot{\omega}_r}{v_r \cos^2 e_\theta} + \frac{\omega_r v_r}{v_r \cos^2 e_\theta} + \frac{2 \omega^2_r \sin e_\theta}{v_r \cos^3 e_\theta} - \omega_r e_\theta.
\]
Define a virtual kinematic control signal as

\[ \alpha_c = T_\alpha \eta_c + f_\alpha, \]  

and design it as

\[ \alpha_{c1} = -k_1(z_1 + z_3(\frac{\omega_r}{v_r}(1 + z_3^2))), \]
\[ \alpha_{c2} = -k_2v_rz_3 - k_3z_4, \]

where \( k_1 > 0, k_2 > 0 \) and \( k_3 > 0 \) are chosen to be constant. Then, with (32), the kinematic control law is

\[ \eta_c = T_\alpha^{-1}(\alpha_c - f_\alpha). \]

3.1.2. Performance analysis Define the velocity tracking error as

\[ \eta_e = \eta - \eta_c. \]

Then, we have

\[ \alpha_e = [\alpha_{c1}, \alpha_{c2}]^T = \alpha - \alpha_c = T_\alpha \eta_e. \]

For the preliminary analysis of the designed kinematic control law, a positive-definite function is chosen as

\[ V_1 = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2 + \frac{1}{k_2^2}z_4^2). \]

With (26)-(29), the time derivative of \( V_1 \) is

\[ \dot{V}_1 = (z_1 + z_3(\frac{\omega_r}{v_r}(1 + z_3^2)))\alpha_{c1} + v_rz_3z_4 + \frac{z_4}{k_2}\alpha_{c2} \]
\[ + (z_1 + z_3(\frac{\omega_r}{v_r}(1 + z_3^2)))\alpha_{c1} + \frac{z_4}{k_2}\alpha_{c2}. \]

Letting \( f_\eta = [z_1 + z_3(\frac{\omega_r}{v_r}(1 + z_3^2)), \frac{z_4}{k_2}]^T \) and substituting (33), (34) and (37) into (39), we have

\[ \dot{V}_1 = -k_1(z_1 + z_3(\frac{\omega_r}{v_r}(1 + z_3^2)))^2 - \frac{k_3^2}{k_2^2}z_4^2 + f_\eta^T T_\alpha \eta_e. \]

If there is no \( f_\eta^T T_\alpha \eta_e \), then \( \dot{V}_1 \) is nonpositive, which means the system is stable. To eliminate it and ensure desired system performance, a dynamic controller is designed in the next section.

3.2. Dynamic Controller Design
3.2.1. Multi-design integration Substituting (11) into (8), we have the dynamic equations with actuator failures as follows:

\[ \dot{\eta} = -M_1^{-1}M_2\eta - \dot{M}_1^{-1}\dot{E} + \dot{M}_1^{-1}\dot{B}\sigma u + \dot{M}_1^{-1}\dot{B}(I_4 - \sigma)\ddot{u} \]
\[ = -M_1^{-1}M_2\eta - \dot{M}_1^{-1}\dot{E} + \dot{M}_1^{-1}B\sigma u + M_1^{-1}Bu_f, \]

where \( \ddot{u}_f = (I_4 - \sigma)\ddot{u} \), and \( u = [u_{1r}, u_{1t}, u_{2r}, u_{2t}]^T \) is the applied control signal to be designed.
Let $\sigma(k), k = 1, 2, \ldots, N$ denote the $k$th possible failure pattern matrix satisfying actuation redundancy condition (13), where $N$ is the number of all possible failure pattern matrices that are under consideration. For each $\sigma(k)$, a corresponding dynamic control signal $u(k)$ will be designed such that the desired system performance can be ensured if $u = u(k)$ and $\sigma = \sigma(k)$. Then, to cover all possible failure patterns, a nominal multi-design integration based dynamic control signal is constructed as

$$u^* = \sum_{k=1}^{N} \chi(k)u(k) = \sum_{k=1}^{N} \text{diag}\{\chi(k), \chi(k), \chi(k)\}u(k),$$  
(42)

where

$$\chi(k) = \begin{cases} 1 & \text{if } \sigma = \sigma(k) \\ 0 & \text{otherwise.} \end{cases}$$  
(43)

3.2.2 Dynamic control law Since the actual failure pattern $\sigma$ is uncertain, $\chi(k)$ is also uncertain for $k = 1, 2, \ldots, N$. To deal with such uncertainties, in this paper, the dynamic control law is designed as

$$u = \sum_{k=1}^{N} \tilde{\chi}(k)u(k),$$  
(44)

where $\tilde{\chi}(k) = \text{diag}\{\tilde{\chi}(k)_{1r}, \tilde{\chi}(k)_{1l}, \tilde{\chi}(k)_{2r}, \tilde{\chi}(k)_{2l}\}$ is the estimate matrix that will be described in the following.

With (41), the time derivative of the velocity tracking error $\eta_e = \eta - \eta_c$ in (36) is

$$\dot{\eta}_e = -\hat{M}_1^{-1}M_2\eta - \hat{M}_1^{-1}\hat{E} + \hat{M}_1^{-1}\hat{B}\sigma u + \hat{M}_1^{-1}\hat{B}\tilde{u}_f - \dot{\eta}_c.$$  
(45)

Let $\hat{u}_f$ denote the estimate of $\bar{u}_f$. Then, the dynamic control signal $u(k) = [u(k)_{1r}, u(k)_{1l}, u(k)_{2r}, u(k)_{2l}]^T$ is designed as

$$u(k) = (\hat{M}_1^{-1}B\sigma(k))^+(-k_4\eta_e - T_\alpha^Tf_\eta + \hat{M}_1^{-1}M_2\eta + \hat{M}_1^{-1}\hat{E} - \hat{M}_1^{-1}\hat{B}\tilde{u}_f + \dot{\eta}_c)$$  
(46)

for $\sigma(k), k = 1, 2, \ldots, N$, where $k_4 > 0$ is a chosen constant, and $(\hat{M}_1^{-1}\hat{B}\sigma(k))^+$ is a generalized inverse matrix satisfying $\hat{M}_1^{-1}B\sigma(k)(\hat{M}_1^{-1}\hat{B}\sigma(k))^+ = I_2$.

**Remark 3** Some elements of some $u(k)$ may be zero. For example, if $\sigma(k) = \text{diag}\{0, 1, 1, 1\}$ and $(\hat{M}_1^{-1}\hat{B}\sigma(k))^+ = \sigma(k)(\hat{M}_1^{-1}\hat{B})^T(\hat{M}_1^{-1}\hat{B}\sigma(k)(\hat{M}_1^{-1}\hat{B})^T)^{-1}$, then $u(k)_{1r} = 0$. In this sense, the estimate matrix $\tilde{\chi}(k)$ in (44) can be simplified for some $\sigma(k)$. For instance, for $\sigma(k) = \text{diag}\{0, 1, 1, 1\}$, $\tilde{\chi}(k) = \text{diag}\{0, \tilde{\chi}(k)_{1l}, \tilde{\chi}(k)_{2r}, \tilde{\chi}(k)_{2l}\}$; and for $\sigma(k) = \text{diag}\{0, 1, 1, 0\}$, $\tilde{\chi}(k) = \text{diag}\{0, \tilde{\chi}(k)_{1l}, \tilde{\chi}(k)_{2r}, 0\}$. This can reduce the number of estimates. □

Define the estimation errors as

$$\tilde{u}_f = \bar{u}_f - \hat{u}_f,$$

$$\tilde{\chi}(k)_{1r} = \chi(k) - \tilde{\chi}(k)_{1r},$$

$$\tilde{\chi}(k)_{1l} = \chi(k) - \tilde{\chi}(k)_{1l},$$

$$\tilde{\chi}(k)_{2r} = \chi(k) - \tilde{\chi}(k)_{2r},$$

$$\tilde{\chi}(k)_{2l} = \chi(k) - \tilde{\chi}(k)_{2l}.$$  
(47)
for \( k = 1, 2, \ldots, N \). Then, substituting (42), (44) and (46) into (45), we have

\[
\dot{\eta}_k = -M_1^{-1}M_2\eta - M_1^{-1}E + M_1^{-1}Bu^* + M_1^{-1}Bu_f - \dot{\eta}_k + M_1^{-1}B(\dot{u} - u^*)
\]

\[
= -k_4\eta_k - T^T_\sigma \dot{f}_\sigma + M_1^{-1}\dot{B}u_f
\]

\[
- \sum_{k=1}^N (M_1^{-1}\dot{B})_{c1}\sigma_{1r}\dot{\chi}(k)_{1r}u(k)_{1r} - \sum_{k=1}^N (M_1^{-1}\dot{B})_{c2}\sigma_{1l}\dot{\chi}(k)_{1l}u(k)_{1l}
\]

\[
- \sum_{k=1}^N (M_1^{-1}\dot{B})_{c3}\sigma_{2r}\dot{\chi}(k)_{2r}u(k)_{2r} - \sum_{k=1}^N (M_1^{-1}\dot{B})_{c4}\sigma_{2l}\dot{\chi}(k)_{2l}u(k)_{2l},
\]

where \((M_1^{-1}\dot{B})_{ci} (i = 1, 2, 3, 4)\) denotes the \( i \)th column vector of matrix \( M_1^{-1}\dot{B} \).

### 3.2.3. Adaptive laws

To construct the control signal \( u(k) \) in (46), the adaptive law of \( \dot{u}_f \) is chosen as

\[
\dot{u}_f = \Gamma_f(M_1^{-1}\dot{B})^T\eta_e,
\]

where \( \Gamma_f = \Gamma_f^T \in \mathbb{R}^{4 \times 4} \) is the positive-definite adaptation gain matrix that is chosen to be constant.

To construct the dynamic control law \( u \) in (44), the adaptive laws of \( \dot{\chi}(k)_{1r}, \dot{\chi}(k)_{1l}, \dot{\chi}(k)_{2r} \) and \( \dot{\chi}(k)_{2l} \) are chosen as

\[
\dot{\chi}(k)_{1r} = -\gamma_{k1r}u(k)_{1r}\eta_e^T(M_1^{-1}\dot{B})_{c1} + f_{k1r},
\]

\[
\dot{\chi}(k)_{1l} = -\gamma_{k1l}u(k)_{1l}\eta_e^T(M_1^{-1}\dot{B})_{c2} + f_{k1l},
\]

\[
\dot{\chi}(k)_{2r} = -\gamma_{k2r}u(k)_{2r}\eta_e^T(M_1^{-1}\dot{B})_{c3} + f_{k2r},
\]

\[
\dot{\chi}(k)_{2l} = -\gamma_{k2l}u(k)_{2l}\eta_e^T(M_1^{-1}\dot{B})_{c4} + f_{k2l},
\]

for \( k = 1, 2, \ldots, N \), where \( \gamma_{k1r} > 0, \gamma_{k1l} > 0, \gamma_{k2r} > 0 \) and \( \gamma_{k2l} > 0 \) are chosen to be constant, and \( f_{k1r}, f_{k1l}, f_{k2r} \) and \( f_{k2l} \) are the projection signals.

Let

\[
p_{k1r} = -\gamma_{k1r}u(k)_{1r}\eta_e^T(M_1^{-1}\dot{B})_{c1}, \quad p_{k1l} = -\gamma_{k1l}u(k)_{1l}\eta_e^T(M_1^{-1}\dot{B})_{c2}, \quad p_{k2r} = -\gamma_{k2r}u(k)_{2r}\eta_e^T(M_1^{-1}\dot{B})_{c3}, \quad p_{k2l} = -\gamma_{k2l}u(k)_{2l}\eta_e^T(M_1^{-1}\dot{B})_{c4}.
\]

The projection signals are given as follows:

\[
f_{kj} = \begin{cases} 
0, & \text{if } \dot{\chi}(k)_{ij} \in (0, 1), \text{ or } \dot{\chi}(k)_{ij} = 0 \text{ and } p_{kj} \geq 0, \text{ or } \dot{\chi}(k)_{ij} = 1 \text{ and } p_{kj} \leq 0, \\
-p_{kj}, & \text{otherwise},
\end{cases}
\]

for \( j = 1r, 1l, 2r, 2l \).

**Lemma 1** The adaptive laws in (50) with the projection signals in (51) for \( k = 1, 2, \ldots, N \) guarantee that: i) \( \dot{\chi}(k)_{1r}, \dot{\chi}(k)_{1l}, \dot{\chi}(k)_{2r}, \dot{\chi}(k)_{2l} \in [0, 1] \); and ii) \( \dot{\chi}(k)_{1r}f_{k1r} \geq 0, \dot{\chi}(k)_{1l}f_{k1l} \geq 0, \dot{\chi}(k)_{2r}f_{k2r} \geq 0 \) and \( \dot{\chi}(k)_{2l}f_{k2l} \geq 0 \).

**Proof:** The proofs for the four adaptive laws are similar. Here the details for \( \dot{\chi}(k)_{1r} \) are given as an example.

Choose the initial estimate as \( \dot{\chi}(k)_{1r}(0) \in [0, 1] \). The projection signal in (51) with \( j = 1r \) ensures \( \dot{\chi}(k)_{1r}(t) \in [0, 1] \), and

\[
(\dot{\chi}(k)_{1r} - \dot{\chi}(k)_{1r})f_{k1r} = \dot{\chi}(k)_{1r}f_{k1r} \geq 0
\]

(52)
that is analyzed for the following three cases: (i) if $\dot{x}(k)_{1tr} = 0$ and $p_{k1tr} < 0$, then $\chi(k)_{1tr} - \dot{x}(k)_{1tr} \geq 0$ and $f_{k1tr} = -p_{k1tr} > 0$, which means (52) is ensured; (ii) if $\dot{x}(k)_{1tr} = 1$ and $p_{k1tr} > 0$, then $\chi(k)_{1tr} - \dot{x}(k)_{1tr} \leq 0$ and $f_{k1tr} = -p_{k1tr} < 0$, which also means (52) is ensured; and (iii) otherwise, we have $f_{k1tr} = 0$, then (52) is ensured.

Similarly, we can also obtain the same properties for the other three adaptive laws. The proof is completed.

### 3.3. System performance

Choose the global Lyapunov function candidate as

$$
V_2 = V_1 + \frac{1}{2}\eta^T_l \eta_l + \frac{1}{2}\bar{u}^T_f \Gamma_f^{-1} \bar{u}_f + \frac{1}{2} \sum_{k=1}^{N} \sigma_1 r \gamma_{k1r}^2 \hat{x}(k)_{1r}^2 + \frac{1}{2} \sum_{k=1}^{N} \sigma_1 r \gamma_{k2r}^2 \hat{x}(k)_{2r}^2
$$

$$
+ \frac{1}{2} \sum_{k=1}^{N} \sigma_2 r \gamma_{k2r}^2 \hat{x}(k)_{2r}^2 + \frac{1}{2} \sum_{k=1}^{N} \sigma_2 r \gamma_{k2r}^2 \hat{x}(k)_{2r}^2.
$$

Then, the derivative of $V_2$ is

$$
\dot{V}_2 \leq -k_1(z_1 + z_3(z_4 + \frac{\omega_r}{v_r}(1 + z_3^2)))^2 - \frac{k_3}{k_2} \bar{z}_1 z_2^2 - k_4 \eta^T_l \eta_l \leq 0,
$$

which indicates: $z_1, z_2, z_3, z_4, \eta_l, z_1 + z_3(z_4 + \frac{\omega_r}{v_r}(1 + z_3^2)) \in L^\infty$ and all estimates are bounded, and $z_4, \eta_l, z_1 + z_3(z_4 + \frac{\omega_r}{v_r}(1 + z_3^2)) \in L^2$. It follows from (23) and (24) that $\cos \theta \neq 0 \quad \tan(\theta_1 - \theta_2) \in L^\infty$ meaning $\cos(\theta_1 - \theta_2) \neq 0$. Then, with (18)-(35) and (41)-(46), we have: $T_\alpha$ is nonsingular and bounded, and $f_\alpha, \alpha_\epsilon, \alpha_\epsilon^T, \eta, \dot{z}_1, \dot{z}_2, \dot{z}_3, \dot{z}_4, \dot{\eta}_l, \dot{u}(k), \dot{\eta}, \dot{\eta}_l, \dot{\alpha}_l, \dot{\alpha}_r \in L^\infty$, which also implies that the derivative of $z_1 + z_3(z_4 + \frac{\omega_r}{v_r}(1 + z_3^2))$ is bounded.

According to Barbalat’s lemma, we can conclude that all closed-loop signals are bounded, and $\lim_{t \to \infty} z_4 = 0$ and $\lim_{t \to \infty} \eta_l = 0$, which also means $\lim_{t \to \infty} \alpha_\epsilon = 0$, $\lim_{t \to \infty} \alpha_\epsilon^T = 0$, and $\lim_{t \to \infty} \eta_l = 0$ with (37) and (33).

From (29), $\dot{z}_4 = \dot{\alpha}_2 + \dot{\alpha}_e + \dot{\alpha}_c \in L^\infty$ with $\dot{\alpha}_2, \dot{\alpha}_e \in L^\infty$, which means that $\dot{z}_4$ is uniformly continuous, together with $\lim_{t \to \infty} \int_0^t \dot{z}_4(\tau) d\tau = z_4(\infty) - z_4(0) = -z_4(0)$, we have $\lim_{t \to \infty} \dot{z}_4 = 0$. Therefore, we can conclude that the derivative of $\alpha_\epsilon$ is bounded, and $\lim_{t \to \infty} \alpha_\epsilon = 0$, $\lim_{t \to \infty} \alpha_\epsilon^T = 0$, $\lim_{t \to \infty} \eta_l = 0$, and $\lim_{t \to \infty} \eta_l = 0$ with (37) and (33).

Finally, we can conclude that: all closed-loop signals are bounded, and $\lim_{t \to \infty} z_i(t) = 0$ ($i = 1, 2, 3, 4$) and $\lim_{t \to \infty} (\eta(t) - \eta_l(t)) = 0$, which also means $\lim_{t \to \infty} (x(t) - \hat{x}(t)) = 0$, $\lim_{t \to \infty} (y(t) - \hat{y}(t)) = 0$, and $\lim_{t \to \infty} (\theta_2(t) - \theta_r(t)) = 0$ with the diffeomorphism in (21)-(23) and the transformation in (17).

In summary, we have the following theorem.

**Theorem 1** The developed multi-design integration based adaptive actuator failure compensation control scheme, constituted by the kinematic control law in (35), and dynamic control law in (44) with multiple control signals in (46) and updated by the projection adaptive laws in (49) and (50), applied to two linked 2WD mobile robots modeled as (1) and (8), guarantees that all closed-loop signals are bounded, and $\lim_{t \to \infty} (x(t) - \hat{x}(t)) = 0$, $\lim_{t \to \infty} (y(t) - \hat{y}(t)) = 0$ and $\lim_{t \to \infty} (\theta_2(t) - \theta_r(t)) = 0$, despite the presence of actuator failures modeled as (10)-(12).
4. Simulation Study
To verify the effectiveness of the developed multi-design integration based adaptive actuator failure compensation scheme for two linked 2WD robots shown in Fig. 1, the following simulation study is presented.

4.1. Simulation Conditions
In this simulation, the physical parameters of each robot are $a_1 = a_2 = 0.3 \text{ m}$, $b_1 = b_2 = 0.75 \text{ m}$, $r_1 = r_2 = 0.15 \text{ m}$, $m_1 = m_2 = 30 \text{ kg}$, $I_m1 = I_m2 = 15.625 \text{ kg m}^2$. The length of the link is assumed to be $d = 1.7 \text{ m}$.

To verify the system tracking property, a circle reference trajectory is considered. To generate such reference a trajectory, the velocities $v_r$ and $\omega_r$ are chosen as: $v_r = 0.5 \text{ m/s}$ and $\omega_r = 0.5 \text{ rad/s}$. Then, $x_r$, $y_r$ and $\theta_r$ are generated by (14) with $x_r(0) = y_r(0) = 0$ and $\theta_r(0) = 45 \text{ deg}$.

In order to verify the failure compensation effectiveness of the developed adaptive control scheme, the following failure cases are simulated:

- no failure, \( \sigma_{(1)} = \text{diag}\{1,1,1,1\} \), \( 0 \leq t < 10\text{s} \).
- $\tau_{1r}$ fails, \( \sigma_{(2)} = \text{diag}\{0,1,1,1\} \), $\tau_{1r} = 0$, \( 10\text{s} \leq t < 20\text{s} \).
- $\tau_{1r}, \tau_{2l}$ fail, \( \sigma_{(3)} = \text{diag}\{0,1,1,0\} \), $\tau_{1r} = 0$, $\tau_{2l} = -2 \text{ Nm}$, \( 20\text{s} \leq t < 30\text{s} \).
- $\tau_{2l}$ fails, \( \sigma_{(4)} = \text{diag}\{1,1,1,0\} \), $\tau_{2l} = -2 \text{ Nm}$, \( 30\text{s} \leq t < 40\text{s} \).
- $\tau_{2r}, \tau_{2l}$ fail, \( \sigma_{(5)} = \text{diag}\{1,1,0,0\} \), $\tau_{2r} = -1 \text{ Nm}$, $\tau_{2l} = -2 \text{ Nm}$, \( t \geq 40\text{s} \).

There are 5 failure pattern matrices which satisfy the actuation redundancy condition in (13), covering the cases of fault free, one actuator fails, both two actuators of robot 2 fail, and one actuator of each robot fails.

The initial conditions are chosen as: $x(0) = 0$, $y(0) = 1 \text{ m}$, $\theta_2(0) = 10 \text{ deg}$, $\theta_1(0) = 0$, $v_2(0) = 0$, and $\omega_1(0) = 0$. The adaptation gains are chosen as: $\Gamma_f = I_4$, and $\gamma_{k1r} = \gamma_{k1i} = \gamma_{k2r} = \gamma_{k2i} = 1$ for $k = 1, 2, 3, 4, 5$. The control gains are chosen as: $k_1 = 1$, $k_2 = 1$, $k_3 = 0.2$ and $k_4 = 20$.

4.2. Simulation Results

![Figure 3: Robot trajectories in (X, Y) plane.](image)

![Figure 4: Control torques.](image)

Fig. 3 shows the positions of the robot 2, the reference robot and the robot 1. Fig. 4 shows the control torques generated by the four wheels, which are consistent with the faulty cases in simulation conditions. Fig. 5 shows the tracking errors. Fig. 6 shows the orientation error.
between two robots. From them, we can see that the desired system stability and asymptotic tracking properties are ensured by the developed multi-design integration based adaptive failure compensation scheme, despite the presence of some actuator failures.

5. Conclusions
This paper developed a multi-design integration based adaptive actuator failure compensation scheme for two linked 2WD robots. The kinematics and dynamics of this robot configuration were modeled, for which, the proposed adaptive failure compensation scheme was designed. The developed control scheme ensures desired system stability and asymptotic tracking properties, which was verified by simulation results. Extending the proposed method for \( n(n > 2) \) linked 2WD mobile robots is our interest in the future work.

ACKNOWLEDGMENT
This work was partially supported by the Regional project SUCRé (Sûreté de fonctionnement et résilience pour la gestion et le contrôle coopératifs des systèmes sociotechniques: Coopération Homme(s)-Robot(s) en milieu hostile) of the Hauts de France region.

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Figure 5: Tracking errors.
Figure 6: Orientation error of two robots.