Exploring CP Violation with $B^0_d \to D K_S$ Decays

Boris Kayser$^a$ and David London$^b$

\hspace{1cm}$^a$: Division of Physics, National Science Foundation, 4201 Wilson Blvd.,
\hspace{1cm}Arlington, VA 22230 USA
\hspace{1cm}$^b$: Laboratoire René J.-A. Lévesque, Université de Montréal,
\hspace{1cm}C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

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Abstract: We (re)examine CP violation in the decays $B_d^0 \to D K_S$, where $D$ represents $D^0$, $\overline{D}^0$, or one of their excited states. The quantity $\sin^2(2\beta + \gamma)$ can be extracted from the time-dependent rates for $B_d^0(t) \to \bar{D}^{\ast\ast 0}K_S$ and $B_d^0(t) \to D^{\ast\ast 0}K_S$, where the $D^{\ast\ast 0}$ decays to $D^{(*)}+\pi^{-}$. If one considers a non-CP-eigenstate hadronic final state to which both $\overline{D}^0$ and $D^0$ can decay (e.g. $K^+\pi^-$), then one can obtain two of the angles of the unitarity triangle from measurements of the time-dependent rates for $B_d^0(t) \to (K^+\pi^-)_D K_S$ and $B_d^0(t) \to (K^-\pi^+)_D K_S$. There are no penguin contributions to these decays, so all measurements are theoretically clean.
1 Introduction

In the coming years, the CP-violating angles $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle will be measured in $B$ decays in a number of different experiments $^{[1]}$. The hope, as always, is to find evidence of physics beyond the standard model (SM).

With this goal in mind, it is important to measure these three angles in as many different ways as possible. There are (at least) two reasons for this. First, it is possible to discover new physics by comparing values of the CP angles which are extracted in different $B$ decays. In fact, in this way one can often pinpoint this source of new physics $^{[2]}$. Second, regardless of what decay mode is used, there will always be some discrete ambiguities in the extraction of a CP angle. These discrete ambiguities make it difficult to confirm (or not) the predictions of the SM, and hence limit our ability to discover new physics. However, by using a variety of techniques, one can measure different functions of the CP angles, which allows us to remove the discrete ambiguities $^{[3, 4]}$.

In this paper, we (re)examine CP violation in the family of decays $B_d^0 \rightarrow DK_S$, where $D$ stands for $D^0$ or $\bar{D}^0$, as well as their excited states. Since $B_d^0$ and $\bar{B}_d^0$ mesons can each decay to both $D^0$ and $\bar{D}^0$, this makes $DK_S$ final states a particularly rich system to study.

For example, it has recently been pointed out that the weak phase $2\beta + \gamma$ is probed in CP asymmetries in the decay $B_d^0 \rightarrow D^- \pi^+$ $^{[5]}$. Here we show that this same phase can be extracted from $B_d^0 \rightarrow \bar{D}^{*0} K_S$, with the advantage that roughly one third as many $B$'s are needed.

In fact, $B_d^0 \rightarrow DK_S$ decays were studied many years ago $^{[3]}$. Then it was shown that one could extract two of the three angles of the unitarity triangle from the time-dependent rates for $B_d^0 \rightarrow D^0 K_s$, $B_d^0 \rightarrow \bar{D}^0 K_S$ and $B_d^0 \rightarrow D_{CP} K_S$, where $D_{CP}$ denotes a $D^0$ or $\bar{D}^0$ decay to a CP eigenstate. However, it was recently shown that this type of analysis runs into problems because it is virtually impossible to tag the flavor of the final-state $D$-meson $^{[7]}$, and so one cannot distinguish $B_d^0 \rightarrow D^0 K_s$ from $B_d^0 \rightarrow \bar{D}^0 K_S$ decays. In this paper we show that, despite these problems, it is still possible to obtain two CP angles from a study of $B_d^0 \rightarrow D^0 K_s$ and $B_d^0 \rightarrow \bar{D}^0 K_s$ if both $D^0$ and $\bar{D}^0$ decay to the same hadronic final state (e.g. $K^+ \pi^-$).

The paper is organized as follows. In Section 2 we examine how the CP angle $2\beta + \gamma$ is extracted from both $B_d^0 \rightarrow D^- \pi^+$ and $B_d^0 \rightarrow DK_s$ decays. Section 3 contains a discussion of how to obtain two angles of the unitarity triangle (e.g. $\beta$ and $\gamma$) from the time-dependent rates for $B_d^0(t) \rightarrow (K^+ \pi^-)_D K_s$ and $B_d^0(t) \rightarrow (K^- \pi^+)_D K_s$. We consider the question of discrete ambiguities in Section 4. We conclude in Section 5.

$^{1}$For example, if the value of $\beta$ as extracted via the CP asymmetry in the decay $B_d^0 \rightarrow \Psi K_s$ differs from that obtained in $B_d^0 \rightarrow \phi K_s$, this indicates the presence of new CP-violating physics in the $b \rightarrow s$ penguin amplitude $^{[3]}$. 

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2. $B_d^0 \rightarrow DK_S: 2\beta + \gamma$

It has been known for many years now that it is possible to cleanly extract weak phase information using CP-violating rate asymmetries in the $B$ system. The earliest studies of such rate asymmetries concentrated on final states which are CP eigenstates. However, it soon became clear that certain non-CP eigenstates can also be used. In fact, as Aleksan, Dunietz, Kayser and Le Diberder (ADKL) showed, clean phase information can be obtained in $B$ decays to almost any final state which is accessible to both $B^0_d$ and $\bar{B}^0_d$. We begin with a brief review of their method which, for the purposes of identification, we will refer to later in the paper as the ADKL method.

Consider a final state $f$ to which both $B^0_d$ and $\bar{B}^0_d$ can decay. Assume that $f$ is a two-body state, and that one weak amplitude dominates both the $B^0_d$ and $\bar{B}^0_d$ decays. (Both of these conditions hold for the decays studied in this paper – if one or both of these conditions is not satisfied, then a more complicated analysis is necessary.) We write

$$
\langle f|T|B^0_d \rangle = Me^{i\phi} e^{i\delta}, \quad \langle f|T|\bar{B}^0_d \rangle = \bar{M}e^{i\bar{\phi}} e^{i\bar{\delta}},
$$

where $\phi$ and $\bar{\phi}$ represent the weak phases of the decay, and $\delta$ and $\bar{\delta}$ are the strong phases.

Due to $B^0_d$-$\bar{B}^0_d$ mixing, a state which is created as a $B^0_d$ or a $\bar{B}^0_d$ will evolve in time into a mixture of both states:

$$
|B^0_d(t)\rangle = e^{-im_B t} e^{-\Gamma_B t/2} \left[ \cos \left(\frac{\Delta m_B t}{2}\right) |B^0_d\rangle - e^{-2i\phi_M} \sin \left(\frac{\Delta m_B t}{2}\right) |\bar{B}^0_d\rangle \right],
$$

$$
|\bar{B}^0_d(t)\rangle = e^{-im_B t} e^{-\Gamma_B t/2} \left[ -e^{2i\phi_M} \sin \left(\frac{\Delta m_B t}{2}\right) |B^0_d\rangle + \cos \left(\frac{\Delta m_B t}{2}\right) |\bar{B}^0_d\rangle \right],
$$

where $\phi_M$ is the weak phase in $B^0_d$-$\bar{B}^0_d$ mixing. (In Eq. (2) the relative sign of the $B^0_d$ and $\bar{B}^0_d$ terms assumes, as indicated by lattice calculations, that the bag parameter, $B_{bd}$, is positive. Even if this assumption is incorrect, the analyses described below and in subsequent sections are largely, though not totally, unaffected. We will make several comments regarding the role of the bag parameter throughout the paper.) Using the $B$-decay amplitudes defined in Eq. (1), the time-dependent decay rates for $B^0_d(t)$ and $\bar{B}^0_d(t)$ to decay into the final state $f$ become

$$
\Gamma(B^0_d(t) \rightarrow f) = e^{-\Gamma_B t} \left[ M^2 \cos^2 \left(\frac{\Delta m_B t}{2}\right) + M^2 \sin^2 \left(\frac{\Delta m_B t}{2}\right) - M\bar{M} \sin(2\phi_M + \phi + \bar{\phi} + \delta - \bar{\delta}) \sin(\Delta m_B t) \right],
$$

$$
\Gamma(\bar{B}^0_d(t) \rightarrow f) = e^{-\Gamma_B t} \left[ M^2 \cos^2 \left(\frac{\Delta m_B t}{2}\right) + M^2 \sin^2 \left(\frac{\Delta m_B t}{2}\right) + M\bar{M} \sin(2\phi_M + \phi + \bar{\phi} + \delta - \bar{\delta}) \sin(\Delta m_B t) \right].
$$
while those involving decays to $f$ are

$$
\Gamma(B_d^0(t) \to f) = e^{-\Gamma_B t} \left[ M^2 \cos^2 \left( \frac{\Delta m_B t}{2} \right) + \overline{M}^2 \sin^2 \left( \frac{\Delta m_B t}{2} \right) - M\overline{M} \sin(-2\phi_M - \phi - \bar{\phi} + \delta - \bar{\delta}) \sin(\Delta m_B t) \right],
$$

(5)

$$
\Gamma(B_d^0(t) \to \bar{f}) = e^{-\Gamma_B t} \left[ \overline{M}^2 \cos^2 \left( \frac{\Delta m_B t}{2} \right) + M^2 \sin^2 \left( \frac{\Delta m_B t}{2} \right) + M\overline{M} \sin(-2\phi_M - \phi - \bar{\phi} + \delta - \bar{\delta}) \sin(\Delta m_B t) \right],
$$

(6)

Through measurements of the above time-dependent rates, it is possible to extract the amplitudes $M$ and $\overline{M}$, as well as the CP-violating quantities $S \equiv \sin(2\Phi + \Delta)$ and $\bar{S} \equiv \sin(2\Phi - \Delta)$, where $2\Phi \equiv 2\phi_M + \phi + \bar{\phi}$ and $\Delta \equiv \delta - \bar{\delta}$. The two sines can be combined to yield

$$
\sin^2 2\Phi = \frac{1}{2} \left[ 1 + S\bar{S} \pm \sqrt{(1 - S^2)(1 - \bar{S}^2)} \right].
$$

(7)

One of the signs gives the true value of $\sin^2 2\Phi$, while the other gives $\cos^2 \Delta$. This discrete ambiguity can be removed by repeating the analysis with another final state whose strong phases are likely to be different. Thus, the ADKL method allows one to obtain $\sin^2 2\Phi$ with no hadronic uncertainty.

Even if it turns out that, contrary to expectations, $B_{B_d}$ is in fact negative, the ADKL method will not be affected. The effect of $B_{B_d} < 0$ is to change the sign of the coefficient of the $\sin(\Delta m_B t)$ term in each of Eqs. (5)–(6). In this case the quantities $S$ and $\bar{S}$, as extracted from these rates, will have the wrong sign. However, the weak phase $\sin^2 2\Phi$ obtained from these quantities will be unaffected, since it depends only on the products $S^2$, $\bar{S}^2$ and $S\bar{S}$ [Eq. (7)].

It is amusing to note that that if the strong phase $\Delta$ is known independently, $S$ and $\bar{S}$ can be combined to yield the weak phase $2\Phi$ with no discrete ambiguity. However, if in fact $B_{B_d} < 0$, then the weak phase obtained in this way will be $2\Phi + \pi$. Thus, if one makes no assumption about the sign of $B_{B_d}$, in this scenario there is a twofold ambiguity in the extraction of $2\Phi$.

Recently, it has been noted that if one applies this technique to the final state $f = D^-\pi^+$ (or $D^{*-}\pi^+$, $D^-\rho^+$, etc.), one probes the weak phase $2\beta + \gamma$ 4. This can be seen as follows. The decays $B_d^0 \to D^-\pi^+$ and $\overline{B_d^0} \to D^-\pi^+$ are governed by the CKM matrix elements $V_{cd}V_{ud}$ and $V_{ub}V_{cs}^*$, respectively. In the standard Wolfenstein phase convention 4, $\beta = \text{Arg}(V_{ud})$ and $\gamma = \text{Arg}(V_{ub})$. Thus, for the final state $D^-\pi^+$ we have $\phi_M = \beta$, $\phi = 0$, and $\bar{\phi} = \gamma$. Therefore with the above method one extracts $\sin^2 (2\beta + \gamma)$.

One of the advantages of this method is that the branching ratios for $B_d^0(t) \to f$ and $\overline{B_d^0}(t) \to f$ ($f = D^-\pi^+$, $D^-\rho^+$, etc.) are relatively large, in the range $3\times10^{-3}$ to $8\times10^{-3}$. On the other hand, $|V_{ub}|$ is small, in the range $1\times10^{-3}$ to $3\times10^{-3}$, in the range $3\times10^{-3}$ to $8\times10^{-3}$. Therefore, the branching ratio will be $3\times10^{-6}$ to $8\times10^{-6}$. On

\[2\text{In fact, measurement of all four rates is not necessary. It suffices to measure one of the two rates in Eqs. (5)–(6), along with one of the two rates in Eqs. (5)–(6).}\]
the other hand, the disadvantage is that the interfering amplitudes $B^0_d \to D^-\pi^+$ and $\bar{B}^0_d \to D^-\pi^+$ are quite different in size, leading to a very small CP asymmetry:

$$A_{DKS} \equiv \frac{\Gamma(B^0_d(t) \to D^-\pi^+) - \Gamma(\bar{B}^0_d(t) \to D^+\pi^-)}{\Gamma(B^0_d(t) \to D^-\pi^+) + \Gamma(\bar{B}^0_d(t) \to D^+\pi^-)} \sim \left(\frac{M}{M}\right)_{D\pi} \sim \frac{|V_{ub}V_{cd}^*|}{V_{cb}^*V_{ud}|} \sim 0.02. \quad (8)$$

This is a serious problem since the number of $B$ needed to make the measurement is inversely proportional to the square of the asymmetry:

$$N_B \propto \frac{1}{BR(B^0_d \to f) A_{DKS}^2}. \quad (9)$$

In Ref. [10] it is estimated that one requires about $10^8$ tagged $B$ to measure $|\sin(2\beta + \gamma)|$ to an accuracy of ±0.1.

One can in principle improve this situation by considering instead the final state $f = \bar{D^0}K_s$. In this case the CKM matrix elements involved in the decays $B^0_d \to \bar{D^0}K_s$ and $\bar{B}^0_d \to \bar{D^0}K_s$ are $V_{cb}V_{us}$ and $V_{ub}V_{cs}$, respectively. (Technically, we should also include the CKM matrix elements involved in $K^0$-$\bar{K^0}$ mixing. However, in the Wolfenstein parametrization, these elements are real, and so do not contribute to CP violation.) The phase information is unchanged compared to the final state $D^-\pi^+$: $\phi_M = \beta$, $\phi = 0$, and $\tilde{\phi} = \gamma$. We therefore have the following time-dependent decay rates:

$$\Gamma(B^0_d(t) \to \bar{D^0}K_s) = e^{-\Gamma_{B^0_d} t} \left[ M^2 \cos^2 \left(\frac{\Delta m_B t}{2}\right) + 2M^2 \sin^2 \left(\frac{\Delta m_B t}{2}\right) - M\bar{M} \sin(2\beta + \gamma + \Delta) \sin(\Delta m_B t) \right], \quad (10)$$

$$\Gamma(\bar{B}^0_d(t) \to \bar{D^0}K_s) = e^{-\Gamma_{\bar{B}^0_d} t} \left[ \bar{M}^2 \cos^2 \left(\frac{\Delta m_B t}{2}\right) + 2\bar{M}^2 \sin^2 \left(\frac{\Delta m_B t}{2}\right) + M\bar{M} \sin(2\beta + \gamma + \Delta) \sin(\Delta m_B t) \right], \quad (11)$$

$$\Gamma(\bar{B}^0_d(t) \to D^0K_s) = e^{-\Gamma_{\bar{B}^0_d} t} \left[ M^2 \cos^2 \left(\frac{\Delta m_B t}{2}\right) + 2M^2 \sin^2 \left(\frac{\Delta m_B t}{2}\right) - M\bar{M} \sin(-2\beta - \gamma + \Delta) \sin(\Delta m_B t) \right], \quad (12)$$

$$\Gamma(B^0_d(t) \to D^0K_s) = e^{-\Gamma_{B^0_d} t} \left[ \bar{M}^2 \cos^2 \left(\frac{\Delta m_B t}{2}\right) + 2\bar{M}^2 \sin^2 \left(\frac{\Delta m_B t}{2}\right) + M\bar{M} \sin(-2\beta - \gamma + \Delta) \sin(\Delta m_B t) \right]. \quad (13)$$

Thus, with this final state, one can again extract $\sin^2(2\beta + \gamma)$.

The advantage of using this final state is that, since the two interfering amplitudes are of comparable size, the asymmetry is much larger:

$$A_{DKS} \sim \left(\frac{M}{M}\right)_{DKS} \sim \frac{|V_{ub}V_{cs}^*|}{V_{cb}^*V_{us}|} \sim 0.4. \quad (14)$$

The disadvantage is that the branching ratios for $B^0_d(t) \to \bar{D^0}K_s$ and $\bar{B}^0_d(t) \to \bar{D^0}K_s$ are considerably smaller: we estimate $B(B^0_d \to \bar{D^0}K_s) \approx \lambda^2 B(B^0_d \to \Psi K_s) \approx 2 \times 10^{-5}$. 

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However, the net effect is still an improvement over $B_d^0 \to D^- \pi^+$. Although the branching ratio to $\overline{D}^0 K_s$ is 150 times smaller than that to $D^- \pi^+$, the asymmetry is 20 times bigger. From Eq. (1), we see that one therefore requires roughly a factor of three fewer $B$’s to measure $\sin^2(2\beta + \gamma)$ using the final state $\overline{D}^0 K_s$.

Unfortunately, the final state $\overline{D}^0 K_s$ (or $D^0 K_s$) has its own problems, namely that tagging the flavor of the final-state $D$-meson is problematic [7]. Attempts to tag the $D^0$ via its semileptonic decay $\overline{D}^0 \to \ell^- \bar{\nu}_\ell X_s$ are hampered by the huge backgrounds from semileptonic $B$ decays. And hadronic tags of the $D^0$ such as $\overline{D}^0 \to K^+ \pi^-$ are not clean either, since the $D^0$ can also decay to that final state, though the amplitude is doubly Cabibbo-suppressed (DCS). We refer to this problem as “DCS contamination.”

Suppose that one attempts to tag the flavor of the final-state $D$-meson via the hadronic decay $\overline{D}^0 \to K^+ \pi^-$. What happens to the expressions for the time-dependent rates for $B_d^0(t)$ and $\overline{B}_d^0(t)$ into $\overline{D}^0 K_s$ and $D^0 K_s$? We define

$$
\begin{align*}
\langle K^- \pi^+ | T | D^0 \rangle &= d e^{i \phi_d} e^{i \delta_d}, \\
\langle K^+ \pi^- | T | D^0 \rangle &= \bar{d} e^{i \phi_d} e^{i \delta_d}, \\
\langle K^+ \pi^- | T | \overline{D}^0 \rangle &= \bar{d} e^{-i \phi_d} e^{i \delta_d}, \\
\langle K^- \pi^+ | T | \overline{D}^0 \rangle &= d e^{-i \phi_d} e^{i \delta_d}.
\end{align*}
$$

(15)

Note that, although these amplitudes have been written generally, in fact we can set $\phi_d = \bar{\phi}_d = 0$ since the CKM matrix elements involved in $D$ decays are essentially real in the Wolfenstein parametrization. CLEO has measured [10]

$$
\frac{BR(D^0 \to K^+ \pi^-)}{BR(\overline{D}^0 \to K^+ \pi^-)} = 0.0077 \pm 0.0025 \pm 0.0025.
$$

(16)

Taking the central value of this measurement, this gives

$$
\frac{\bar{d}}{d} \sim 0.09.
$$

(17)

Since $B_d^0$ and $\overline{B}_d^0$ can decay into both $\overline{D}^0 K_s$ and $D^0 K_s$, and since both $\overline{D}^0$ and $D^0$ can decay into $K^+ \pi^-$, the decay amplitudes must be added coherently, and the expressions for the time-dependent $B$ decay rates become considerably more complicated:

$$
\begin{align*}
\Gamma(B_d^0(t) \to (K^+ \pi^-)_D K_s) &= e^{-\Gamma_{Bt}} e^{-\Gamma_{Bt'}} \times \\
& \left\{ \cos^2 \left( \frac{\Delta m_B t}{2} \right) \left[ M^2 d^2 + \overline{M}^2 \bar{d}^2 + 2M\overline{M}d\bar{d} \cos(\gamma - \Delta - \Delta_d) \right] \\
& + \sin^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \overline{M}^2 d^2 + M^2 \bar{d}^2 + 2M\overline{M}d\bar{d} \cos(\gamma + \Delta - \Delta_d) \right] \\
& - \sin(\Delta m_B t) \left[ M\overline{M} d\bar{d} \sin(2\beta + \gamma + \Delta) + M\overline{M} \bar{d}d \sin(2\beta - \gamma - \Delta) \\
& + M^2 d\bar{d} \sin(2\beta + \Delta_d) + \overline{M}^2 d\bar{d} \sin(2\beta + 2\gamma - \Delta_d) \right] \right\}
\end{align*}
$$

(18)

$$
\Gamma(\overline{B}_d^0(t) \to (K^+ \pi^-)_D K_s) = e^{-\Gamma_{Bt}} e^{-\Gamma_{Bt'}} \times
$$
\begin{align}
\left\{ \cos^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \mathcal{M}^2 d^2 + M^2 \bar{d}^2 + 2M\mathcal{M} \bar{d} \cos(\gamma + \Delta - \Delta_d) \right] \\
+ \sin^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \mathcal{M}^2 d^2 + M^2 \bar{d}^2 + 2M\mathcal{M} \bar{d} \cos(\gamma - \Delta - \Delta_d) \right] \\
+ \sin(\Delta m_B t) \left[ M\mathcal{M} d^2 \sin(2\beta + \gamma + \Delta) + M\mathcal{M} \bar{d}^2 \sin(2\beta + \gamma - \Delta) \\
+ M^2 d\bar{d} \sin(2\beta + \Delta_d) + \bar{M}^2 d\bar{d} \sin(2\beta + 2\gamma - \Delta_d) \right] \right\}, 
\end{align}

where the $B$ and $D$ decays occur at times $t$ and $t'$, respectively ($\Gamma_D$ is the $D$ width), with $\Delta \equiv \delta - \bar{\delta}$ and $\Delta_d \equiv \delta_d - \bar{\delta}_d$. The rates for the CP-conjugate processes are obtained simply by changing the signs of the weak phases:

$$\Gamma(B_d^0(t) \to (K^- \pi^+) D) = e^{-\Gamma_{B_d} t} e^{-\Gamma_D t'} \times \left\{ \right.$$

\begin{align}
\cos^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \mathcal{M}^2 d^2 + M^2 \bar{d}^2 + 2M\mathcal{M} \bar{d} \cos(-\gamma - \Delta - \Delta_d) \right] \\
+ \sin^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \mathcal{M}^2 d^2 + M^2 \bar{d}^2 + 2M\mathcal{M} \bar{d} \cos(-\gamma + \Delta - \Delta_d) \right] \\
- \sin(\Delta m_B t) \left[ M\mathcal{M} d^2 \sin(-2\beta + \gamma + \Delta) + M\mathcal{M} \bar{d}^2 \sin(-2\beta - \gamma - \Delta) \\
+ M^2 d\bar{d} \sin(-2\beta + \Delta_d) + \bar{M}^2 d\bar{d} \sin(-2\beta + 2\gamma - \Delta_d) \right] \right\},
\end{align}

\begin{align}
\Gamma(B_d^0(t) \to (K^- \pi^+) D) = e^{-\Gamma_{B_d} t} e^{-\Gamma_D t'} \times \left\{ \right.

\cos^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \mathcal{M}^2 d^2 + M^2 \bar{d}^2 + 2M\mathcal{M} \bar{d} \cos(-\gamma - \Delta - \Delta_d) \right] \\
+ \sin^2 \left( \frac{\Delta m_B t}{2} \right) \left[ \mathcal{M}^2 d^2 + M^2 \bar{d}^2 + 2M\mathcal{M} \bar{d} \cos(-\gamma + \Delta - \Delta_d) \right] \\
+ \sin(\Delta m_B t) \left[ M\mathcal{M} d^2 \sin(-2\beta + \gamma + \Delta) + M\mathcal{M} \bar{d}^2 \sin(-2\beta - \gamma - \Delta) \\
+ M^2 d\bar{d} \sin(-2\beta + \Delta_d) + \bar{M}^2 d\bar{d} \sin(-2\beta + 2\gamma - \Delta_d) \right] \right\}. 
\end{align}

Note that, in the limit where $\bar{d} \to 0$ and $d \to 1$ (i.e. no doubly Cabibbo suppressed $D$ decays), the above equations reduce to those of Eqs. (14)–(13).

From these expressions, it is clear that the fact that one cannot cleanly tag the final $D$ meson introduces a significant uncertainty into the extraction of $\sin^2(2\beta + \gamma)$. For example, if there were no DCS contamination, the quantities $M\mathcal{M} \sin(2\beta + \gamma + \Delta)$ and $M\mathcal{M} \sin(2\beta + \gamma - \Delta)$ could be respectively extracted from the coefficients of the $\sin(\Delta m_B t)$ terms in Eqs. (16) and (13). However, from Eqs. (18) and (20) above, we see that the presence of DCS contamination introduces an uncertainty in the extraction of these quantities:

$$\Delta \left( \frac{M\mathcal{M} \sin(2\beta + \gamma + \Delta)}{M\mathcal{M} \sin(2\beta + \gamma + \Delta)} \right) \sim \Delta \left( \frac{M\mathcal{M} \sin(2\beta + \gamma - \Delta)}{M\mathcal{M} \sin(2\beta + \gamma - \Delta)} \right) \sim \frac{M^2 d\bar{d}}{M\mathcal{M} d^2} \sim 22\%,$n

where we have used the estimates for $\mathcal{M}/M$ and $\bar{d}/d$ given in Eqs. (14) and (17). (Here and in the following equation, the symbol $\Delta$ used to indicate the error should not be
confused with the same symbol which denotes the strong phase \( \delta - \bar{\delta} \).) Furthermore, via a similar analysis, the extraction of amplitudes \( M \) and \( \bar{M} \) also has errors induced:

\[
\frac{\Delta M}{M} \sim \frac{M \bar{M} \bar{d} \bar{d}}{M^2 \bar{d}^2} \sim 4\% \\
\frac{\Delta \bar{M}}{\bar{M}} \sim \frac{M \bar{M} \bar{d} \bar{d}}{M^2 \bar{d}^2} \sim 22\% .
\]

Clearly, when all these errors are put together, there is a considerable systematic error in the extraction of the quantity \( \sin^2(2(\beta + \gamma)) \). Thus, the above analysis shows that, in fact, due to the problems of tagging the final \( D_0 \) meson, the final state \( \bar{D}^0 K_S \) cannot be used to cleanly obtain \( \sin^2(2(\beta + \gamma)) \) via the ADKL method. (Nevertheless, we can learn a great deal from the decays of \( B_d^0(t) \) and \( \bar{B}_d^0(t) \) to \( D^0 K_S \) and \( \bar{D}^0 K_S \) in a different way, as we will show in the next section.)

The problems with DCS contamination can be avoided if one uses instead a final state involving a self-tagging excited \( D^0 \) state such as \( D_1(2420)^0 \) or \( D_2^*(2460)^0 \), generically denoted as \( D^{**0} \). The \( D^{**0} \) decays to \( D^{(*)-} \pi^- \), while the CP-conjugate state decays to \( D^{(*)+} \pi^- \). The charge of the pion therefore tags the flavor of the decaying \( D \)-meson. Thus, the ADKL method can be used with the final state \( D^{**0} K_S \) or \( \bar{D}^{**0} K_S \) to extract \( \sin^2(2(\beta + \gamma)) \). This final state has no problems with DCS contamination, so the measurement is clean.

It is also possible in principle to use three-body final states such as \( D^+ \pi^- K_S, D_s^+ K^- K_S \), etc. in order to obtain \( \sin^2(2(\beta + \gamma)) \). However, there is a problem: such states will have nontrivial kinematic degrees of freedom due to the fact that the relative angular momenta of the final-state particles are not fixed. The ADKL method then applies only to a specific kinematical point, or in a small kinematical bin. Since one requires a huge number of \( B \)'s in order to accumulate an appreciable number of events in a small bin, the application of the ADKL method to such three-body final states is likely to be impractical.

Finally, we note that a measurement of \( \sin^2(2(\beta + \gamma)) = \sin^2(\beta - \alpha) \) does not, by itself, give any information about the angles \( \alpha, \beta \) and \( \gamma \). However, if \( \beta \) is measured in another \( B \) decay (e.g. in \( B_d^0(t), \bar{B}_d^0(t) \rightarrow \Psi K_S \)), then this information can be used in order to obtain \( \alpha \) or \( \gamma \), up to discrete ambiguities. And if two of the CP angles are measured elsewhere, then this method serves as an independent crosscheck. We will have more to say about this in section 4.

3 \( B_d^0 \rightarrow DK_S: \beta, \gamma \)

Gronau and London (GL) suggested another method for obtaining clean weak phase information from non-CP-eigenstate final states. It involves the decays \( B_d^0 \rightarrow D^0 K_S \),
\( B^0_d \to \bar{D}^0 K_S \) and \( B^0_d \to D_{CP} K_S \), where \( D_{CP} \) is the CP-even superposition

\[
D_{CP} = \frac{1}{\sqrt{2}} \left[ D^0 + \bar{D}^0 \right].
\]  

(24)

\( D_{CP} \) is identified by its decays to CP-even final states such as \( \pi^+ \pi^- \), \( K^+ K^- \), etc. (One can also use the CP-odd combination of \( D^0 \) and \( \bar{D}^0 \) – the CP asymmetry simply has an extra minus sign.)

The GL method works as follows. From the time-dependent rates for \( B^0_d(t) \) to decay into \( \bar{D}^0 K_S \) and \( D^0 K_S \) [Eqs. (10) and (13)], one can extract the quantities \( M, \bar{M}, \sin(2\beta + \gamma - \Delta) \) and \( \sin(2\beta + \gamma + \Delta) \), as before. But there is important, additional information to be obtained by considering also the decay \( B^0_d(t) \to D_{CP} K_S \). The time-dependent rate is given by

\[
\Gamma(B^0_d(t) \to D_{CP} K_S) = \frac{1}{2} e^{-\Gamma_B t} e^{-\Gamma_{D'} t} \times 
\left\{ \cos^2 \left( \frac{\Delta m_t}{2} t \right) \left[ M^2 + \bar{M}^2 + 2M\bar{M} \cos(\gamma - \Delta) \right] 
+ \sin^2 \left( \frac{\Delta m_t}{2} t \right) \left[ \bar{M}^2 + M^2 + 2M\bar{M} \cos(\gamma + \Delta) \right] 
- \sin(\Delta m_t) \left[ M^2 \sin(2\beta) + \bar{M}^2 \sin(2\beta + 2\gamma) 
+ M\bar{M} \sin(2\beta + \gamma - \Delta) \right] + M\bar{M} \sin(2\beta + \gamma + \Delta) \right\}. \tag{25}
\]

The measurement of this rate yields the additional quantities \( \cos(\gamma - \Delta) \) and \( \cos(\gamma + \Delta) \).

From these four trigonometric quantities — \( \sin(2\beta + \gamma - \Delta) \), \( \sin(2\beta + \gamma + \Delta) \), \( \cos(\gamma - \Delta) \) and \( \cos(\gamma + \Delta) \) — it is straightforward to show that one can obtain \( \sin(2\beta) \) and \( \sin(2\beta + 2\gamma) = -\sin(2\alpha) \). Thus, two angles of the unitarity triangle can in principle be extracted with no hadronic uncertainty from the time-dependent measurements of \( B^0_d(t) \to \bar{D}^0 K_S \), \( D^0 K_S \), and \( D_{CP} K_S \).

This technique was adapted by Gronau and Wyler (GW) to the decays \( B^\pm \to \bar{D}^0 K^\pm \), \( D^0 K^\pm \) and \( D_{CP} K^\pm \) as a probe of the angle \( \gamma \) [13]. However, it was recently pointed out by Atwood, Dunietz and Soni (ADS) that this method runs into the problems of DCS contamination mentioned in the previous section [7]. Specifically, although the branching ratio for \( B^+ \to \bar{D}^0 K^+ \) can be measured, obtaining \( B(B^+ \to D^0 K^+) \) is extremely difficult due to the problems of tagging the final state D-meson. Nevertheless, ADS were able to save the GW method. They pointed out that one can still obtain the angle \( \gamma \), up to a fourfold discrete ambiguity, by studying decays such as \( B^+ \to (K^+ \pi^-)_D K^+ \) and \( B^+ \to (K^+ \rho^-)_D K^+ \), along with their CP-conjugates. Note that final states involving \( D_{CP} \) are not necessary.

This raises the following questions. First, is the GL method affected by DCS contamination? And second, if so, can it be rescued in a similar manner to the ADS modification of the GW method?
The answer to both of these questions is yes. Including the DCS contamination, the time-dependent decay rates of $B_d^0(t)$ into $\bar{D}^0K_S$ and $D^0K_S$ are given in the previous section in Eqs. (18)–(21). As discussed there, due to DCS contamination, it is not possible to obtain the quantities $\sin(2\beta + \gamma - \Delta)$ and $\sin(2\beta + \gamma + \Delta)$ precisely [see Eq. (22)], so the GL method breaks down.

Fortunately, the method can be saved in a fashion analogous to the ADS modification of the GW method. Referring again to Eqs. (18)–(21), we make the following two observations. First, these four time-dependent rates depend on four amplitudes ($M, \bar{M}, d, \bar{d}$) and four phases ($\gamma, \beta, \Delta, \Delta_d$). Of these eight quantities, the two amplitudes $d$ and $\bar{d}$ have been measured [Eq. (16)]. Second, because of the time dependence, six independent quantities can be extracted from the measurements of these rates. These can be taken to be the coefficients of the $\cos^2(\Delta m_B t/2)$, $\sin^2(\Delta m_B t/2)$ and $\sin(\Delta m_B t)$ terms in the rates $\Gamma(B_d^0(t) \to (K^+\pi^-)_B K_S)$ and $\Gamma(B_d^0(t) \to (K^-\pi^+)_B K_S)$.

Thus, we are left with six measurements in terms of six unknowns: $M, \bar{M}, \beta, \gamma, \Delta, \Delta_d$. Although we cannot solve the equations analytically, once the measurements are made one will be able to perform a fit and extract the unknown amplitudes and phases, up to discrete ambiguities. As in the ADS modification of the GW method, final states involving $D_{CP}$ are not used.

Note that one is not constrained to use $K^+\pi^-$ as the state to which $D^0$ and $\bar{D}^0$ decay. One can equally use another state such as $K^+\rho^-$. In this case, only the parameter $\Delta_d$ is changed; the remaining five parameters $M, \bar{M}, \beta, \gamma$ and $\Delta$ are the same as in the $K^+\pi^-$ case. It is therefore possible in principle to use a variety of hadronic states to tag the $D$ mesons. By fitting to all these measurements simultaneously, the experimental error on the CP angles can be reduced.

In the above method, we assume that the $D$-decay amplitudes $d$ and $\bar{d}$ are known. However, if one wants to play very arcane games, one can imagine that none of the quantities are known. If two final hadronic states are used (say $K^+\pi^-$ and $K^+\rho^-$), then one ends up with twelve measurements in eleven unknowns (two weak phases, three strong phases, two $B$-decay amplitudes, four $D$-decay amplitudes). In principle all of these unknown quantities can be extracted from a fit to the data, up to discrete ambiguities.

The conclusion is therefore that, even in the presence of DCS contamination, it is still possible to obtain two of the angles of unitarity triangle, say $\beta$ and $\gamma$, from time-dependent measurements of $B_d^0(t)$ into $\bar{D}^0K_S$ and $D^0K_S$. It must be acknowledged, however, that such measurements will be difficult, and will require $O(10^9)$ tagged $B_d^0$ decays. Therefore, in all likelihood this method can only be carried out at a hadron collider.

\[\text{In fact, it may be possible to measure } \Delta_d \text{ at a charm factory [14].}\]
4 Discrete Ambiguities

In the previous two sections we have seen that (i) one can obtain \( \sin^2(2\beta + \gamma) \) from a study of the time-dependent decays \( B_d^0(t) \to D^{*0}K_S \) and \( B_d^0(t) \to \bar{D}^{*0}K_S \), and (ii) two of the angles of the unitarity triangle can be extracted from the rates for \( B_d^0(t) \to (K^+\pi^-)_dK_S \) and \( B_d^0(t) \to (K^-\pi^+)_dK_S \). In this section we discuss the subject of discrete ambiguities. Specifically, we are interested in two questions. First, what are the discrete ambiguities inherent in these methods? And second, can these measurements be used to remove some of the discrete ambiguities which remain if the CP angles are measured in other decays?

Consider first the decays \( B_d^0(t) \to \bar{D}^{*0}K_S \) and \( B_d^0(t) \to D^{*0}K_S \). From the time-dependent rates [Eqs. (10) and (13)], one can extract the quantities \( \sin(2\Phi + \Delta) \) and \( \sin(2\Phi - \Delta) \), where \( 2\Phi = 2\beta + \gamma \). This means that \( 2\Phi \) and \( \Delta \) can be obtained with a fourfold ambiguity: if \( 2\Phi_0 \) and \( \Delta_0 \) are the true values, then the following four sets of angles all reproduce the measured values of \( \sin(2\Phi + \Delta) \) and \( \sin(2\Phi - \Delta) \):

\[
(2\Phi_0, \Delta_0), \quad (\pi + 2\Phi_0, \pi + \Delta_0), \quad (-2\Phi_0, \pi - \Delta_0), \quad (\pi - 2\Phi_0, -\Delta_0).
\]  
(26)

(As indicated in the discussion following Eq. (7), there is also a discrete ambiguity between \( \sin^2 2\Phi \) and \( \cos^2 \Delta \). This discrete ambiguity can be removed by repeating the analysis with another \( D^{*0}K_S \) final state.)

Now consider the decays \( B_d^0(t) \to (K^+\pi^-)_dK_S \) and \( \Gamma(B_d^0(t) \to (K^-\pi^+)_dK_S \). The time-dependent rates [Eqs. (18) and (21)] depend on ten independent trigonometric functions of \( \beta, \gamma, \Delta \) and \( \Delta_d \). It is straightforward to show that these four angles can be extracted up to a 16-fold ambiguity:

\[
(\beta_0, \gamma_0, \Delta_0, \Delta_d), \quad (\beta_0, \pi + \gamma_0, \pi + \Delta_0, \Delta_d), \quad (\pi + \beta_0, \gamma_0, \Delta_0, \Delta_d), \quad (\pi + \beta_0, \pi + \gamma_0, \pi + \Delta_0, \Delta_d),
\]

\[
(\pm \frac{\pi}{2} + \beta_0, \pi + \gamma_0, \Delta_0, \pi + \Delta_d), \quad (\pm \frac{\pi}{2} + \beta_0, \gamma_0, \pi + \Delta_0, \pi + \Delta_d),
\]

\[
(\pm \frac{\pi}{2} - \beta_0, -\gamma_0, -\Delta_0, -\Delta_d), \quad (\pm \frac{\pi}{2} - \beta_0, \pi - \gamma_0, \pi - \Delta_0, -\Delta_d),
\]

\[
(\pi - \beta_0, -\gamma_0, \pi - \Delta_0, \pi - \Delta_d), \quad (\pi - \beta_0, \pi - \gamma_0, -\Delta_0, \pi - \Delta_d),
\]

\[
(-\beta_0, -\gamma_0, \pi - \Delta_0, \pi - \Delta_d), \quad (-\beta_0, \pi - \gamma_0, -\Delta_0, \pi - \Delta_d).
\]  
(27)

In the above, we have assumed that the bag parameter, \( B_{B_d} \), is positive. Suppose, however, that this assumption is wrong, and that, in fact, \( B_{B_d} < 0 \). How is the above analysis affected? As far as the weak phases are concerned, the answer is: not at all. Changing the sign of \( B_{B_d} \) has the effect of changing the signs of all the \( \sin(\Delta m_{d} t) \) terms in Eqs. (18)–(21). This in turn implies that the extracted angles will be the negatives of those listed in the above solutions. However, note that, for every candidate set of the weak phases \( (\beta, \gamma) \), there is another (discretely ambiguous) solution which contains the
angle set \((-\beta, -\gamma)\). In other words, as long as we have no information about the strong phases, the extraction of weak phases is independent of the actual sign of \(B_{\Delta d}^\dagger\). (As per the discussion following Eq. (7), this is completely analogous to what happens in the original ADKL method.)

On the other hand, if we had some information about the strong phases, then the actual sign of \(B_{\Delta d}^\dagger\) would be important for extracting the weak phases. For example, suppose we knew the true value of \(\Delta_d\). Then, assuming that \(B_{\Delta d}^\dagger > 0\), this method allows one to extract the CP phases up to a fourfold ambiguity consisting of the four angle sets in the first two lines of Eq. (27). However, if we assume instead that the sign of \(B_{\Delta d}^\dagger\) is unknown, then the four additional solutions in the fourth line of Eq. (27) are also permitted, leading to an eightfold ambiguity in the extraction of the weak phases.

From Eq. (27), we see that, although one can extract CP angles with these techniques, one is left with an uncertainty due to discrete ambiguities. In fact, discrete ambiguities plague all methods of obtaining the angles of the unitarity triangle. This is a serious problem. There are a variety of ways of testing for the presence of new physics in CP asymmetries: seeing if \(\alpha\), \(\beta\) and \(\gamma\) do indeed add up to 180 degrees, comparing independently-measured values of the same CP angles, checking the consistency between the measured values of these angles and the ranges allowed by other measurements of non-CP-violating quantities, etc. [15]. However, if there are discrete ambiguities, then it is often the case that one of the values is consistent with the SM (particularly when experimental error is taken into account), while the others are not. Thus, in general, if one hopes to find new physics, it is necessary to be able to remove the discrete ambiguities [1].

\(B\)-decay modes likely to be used for the extraction of \(\alpha\), \(\beta\) and \(\gamma\) are \(B_d^0 \rightarrow \pi^+\pi^-\), \(B_d^0 \rightarrow \Psi K_s\) and \(B^\pm \rightarrow D K^\pm\), respectively [1]. These decays probe the functions \(\sin 2\alpha\), \(\sin 2\beta\) and \(\sin^2 \gamma\) (or equivalently \(\cos 2\gamma\)). Each of the three CP angles can be obtained from these functions with a fourfold ambiguity. However, if one assumes that the three angles form the interior angles of the unitarity triangle, then one is left with only a twofold discrete ambiguity [1]. The form of the discrete ambiguity depends on the signs of \(\sin 2\alpha\) and \(\sin 2\beta\). Denoting the true values of the CP angles by \(\alpha_0\), \(\beta_0\) and \(\gamma_0\), the various twofold discrete ambiguities are summarized in Table 1.

Can the methods described in the previous chapters be used to remove the final twofold discrete ambiguity? Unfortunately, the answer to this question is no. First, as regards the method of Sec. 2, it is obvious that \(\sin^2(2\beta + \gamma)\) is the same for both angle sets in any line of Table 1. And second, for the technique described in Sec. 3, we see that the discrete ambiguities in Table 1 are among those found in the fourth line of Eq. (27). Thus, the twofold discrete ambiguity cannot be resolved by the \(B_d^0 \rightarrow D K_s\) studies described in Secs. 2 and 3.
Table 1: The twofold discrete ambiguity in $(\alpha, \beta, \gamma)$ remaining following measurement of $\sin 2\alpha$, $\sin 2\beta$ and $\cos 2\gamma$.

Still, the methods described in the previous sections may turn out to be useful for other reasons. First, they give independent ways of measuring the CP angles. By comparing the values of these angles obtained in these ways with those extracted from other decay modes, it is conceivable that a discrepancy will be found, revealing the presence of new physics. Second, due to penguin contributions, there may be difficulties in measuring the angle $\alpha$ using $B_d^0 \to \pi^+\pi^-$. In principle it is possible to remove the penguin “pollution” by either an isospin analysis or a Dalitz-plot analysis of the decays $B_d^0 \to \rho\pi$, but these techniques are difficult as well. The methods described above can be used to get at $\alpha$. In Sec. 2 the phase $2\beta + \gamma = \pi + \beta - \alpha$ is probed. If $\beta$ is known, this gives information about $\alpha$. And in Sec. 3, two angles of the unitarity triangle can be obtained. One of these can be taken to be $\alpha$. Furthermore, note that there is no penguin pollution in these methods. Thus, it is possible that $B_d^0 \to DK$ decays will be useful for cleanly measuring $\alpha$.

Finally, it is important to note that there is in fact a way to remove the twofold discrete ambiguity of Table 1 through measurements similar to $B_d^0 \to DK$. Recently, Charles et al. have proposed looking at Dalitz-plot asymmetries in the decay $B_d^0 \to D^{*+}K_S$. This final state is fed by several intermediate resonant channels: $B_d^0 \to D^{*+}K_S$, $B_d^0 \to D^{*++}\pi^-$, and $B_d^0, \bar{B}_d^0 \to D^{*0}K_S$. The measurement of this Dalitz-plot asymmetry enables one to extract $\sin 2(2\beta + \gamma)$. This knowledge in turn removes the discrete ambiguity of Table 1.

## 5 Conclusions

We have examined the prospects for observing CP violation in the decays $B_d^0 \to DK$, where $D$ represents $D^0, \bar{D}^0$, or any of their excited states. Since $B_d^0$ and $\bar{B}_d^0$ mesons can each decay to both $D^0$ and $\bar{D}^0$, there are a number of different CP-violating possibilities.

For example, $\sin^2(2\beta + \gamma)$ can be extracted from the time-dependent rates for $B_d^0(t) \to D^{*0}K_s$ and $\bar{B}_d^0(t) \to D^{*0}K_s$, where the $D^{*0}$ decays to $D^{(*)+}\pi^-$. This same quantity can also be obtained using the final states $D^-\pi^+$ and $D^+\pi^-$. However, although the
branching ratio to $\bar{D}^0 K_s$ is 150 times smaller that to $D^-\pi^+$, the asymmetry is 20 times bigger. Thus, the $D^{*0} K_s$ state requires roughly a factor of three fewer $B$’s to measure $\sin^2(2\beta + \gamma)$. Assuming $O(1)$ detection efficiencies, we estimate that about $3 \times 10^7$ tagged $B$’s are needed to make this measurement.

In principle, the final state $\bar{D}^0 K_s$ can also be used to probe $2\beta + \gamma$. However, in practice it is very difficult to tag the flavor of the final-state $D$-meson, so that one cannot distinguish $\bar{D}^0 K_s$ from $D^0 K_s$. Nevertheless, one can obtain a great deal of information from such decays. If one considers a non-CP-eigenstate hadronic final state to which both $\bar{D}^0$ and $D^0$ can decay (e.g. $K^+\pi^-$), then one can obtain two of the angles of the unitarity triangle from measurements of the time-dependent rates for $B^0_d(t) \to (K^+\pi^-)_D K_s$ and $B^0_d(t) \to (K^-\pi^+)_D K_s$. These measurements are admittedly difficult, and we estimate that $O(10^9)$ tagged $B^0_d$ decays will be required.

Note that both of these methods are theoretically clean: there are no penguin contributions to the decays. In addition, these two methods give independent ways of measuring the CP angles. If one compares these values of the angles with those extracted from other decay modes, one may find a discrepancy. This would be a clear signal of new physics.

Finally, suppose that $\alpha$, $\beta$ and $\gamma$ are measured via the standard decays $B^0_d \to \pi^+\pi^-$, $B^0_d \to \psi K_s$ and $B^\pm \to DK^\pm$, respectively. Then, assuming that the three angles form the interior angles of the unitarity triangle, one is still left with a twofold discrete ambiguity. Unfortunately, the two methods described in this paper do not resolve this discrete ambiguity. However, this discrete ambiguity can be removed by examining Dalitz-plot asymmetries in the $B^0_d \to DK_s$-like decay $B^0_d \to D^\pm \pi^\mp K_s$. Such asymmetries allow one to extract, among other things, $\sin 2(2\beta + \gamma)$. Knowledge of this quantity is sufficient to remove the remaining discrete ambiguity.

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