Centrality measures in transportation networks for Unpad campus route

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Abstract. Centrality measurement still becomes an interesting research topic recently, including its relation to transportation’s effectiveness and efficiency. A transportation network can be expressed in the form of a graph so that even a level of centrality for each vertex of a complex transportation network can be simply analyzed. There have been various developments/modifications of methods on centrality measures in transportation networks. The offered methods show the different points of view regarding which vertices are most important in the transport network. In this research, the study of some centrality measures in the transportation network is discussed. A numerical experiment of some centrality measures implemented to Universitas Padjadjaran (Unpad) campus route as a network is given. The comparisons are done to explore the performance of each method. The calculation is done using Python software with NetworkX library. The results for Unpad campus route show that some vertices (locations) are more prior than another.

1. Introduction
Centrality measures were firstly developed in Social Network Analysis (SNA). Research on centrality measures is widely used, currently also being done in determining priority goal points on SDGs ([1,2,3]). The relationship between goal points of Sustainable Development Goals is represented in the graph and then its centrality is calculated to see which goal is the most priority. The implementation of centrality measures to transportation networks is interesting, such as the study in [4] and [5]. Soh et al. in [4] contribute a complex weighted network analysis of travel routes on the Singapore rail and bus transportation systems. Their research is focused on topological and dynamical analyses and gives further evidence that a dynamical study gives to the information gained by the traditional topological analysis, giving a better sight of complex weighted networks. Paper [5] proposed a visual analytics system, TrajGraph, to study urban transportation patterns from taxi trajectories. TrajGraph uses a new graph model to reflect urban network structures and incorporate real traffic information carried by taxi trajectory data. Pagerank centrality and betweenness centrality are computed to show the importance of distinct city regions in transportation. Some visualizations and interactions give users to obtain priority in different stages of detail. Users can conduct tasks in analyzing the roles of city roads for developing urban planning and amending ([5]).

The meaning of priority in transportation networks can be seen from various points of view depending on the objectives to be achieved. We are interested in locating the priority location(s) in network transportation of Unpad Jatinangor Campus route using several types of centrality measures. Thus, the various literature review of centrality calculations is given in this research to spot the
different meaning of priority of location in that network. In this paper, degree centrality ([6,7,8]), betweenness centrality ([6,7,8,9]), closeness centrality ([2,6,7,8]), eigenvector centrality ([1,7,10]), and weighted eigenvector centrality ([5]) are studied and applied to a network transportation of Unpad Jatinangor Campus route (taken from [11]). The calculation of degree centrality and eigenvector centrality focus more on the degree of each vertex without regard to the weight of the graph while betweenness centrality and closeness centrality focus more on the weight of the graph in the calculation of the shortest path, while weighted eigenvector centrality focus to its weighted only.

Each offered method shows different points of view regarding which vertices are most important in the transport network. In this research, the comparison of some centrality methods in the transportation network is discussed. Numerical comparisons are given for these methods to see the performance of each method by using Python software with NetworkX library.

2. Method

2.1. Graph and Weighted Graph ([12])

A graph \( G = (V, E) \) consists of \( V \) and \( E \) respectively a nonempty set of vertices (or vertices), and a set of edges. Each Edge has either one or two vertices associated with it, namely endpoints. An edge is said to connect its endpoints. A weighted graph is a graph in which each edge is assigned a number, namely edge-weighted.

2.2. Centrality Measures

There have been various developments/modifications of methods on centrality measures in transportation networks. There are many theories and modifications of centrality measures with their weakness and features. In this paper, we discuss some of it and applied it to a graph of transportation route in Unpad Jatinangor campus. In this research, we compare some centrality measures of vertices that is: degree, betweenness, closeness, eigenvector, weighted eigenvector top rankings in each location.

2.2.1. Degree Centrality ([6,7,8])

Degree centrality is used in locating the importance of a direct connection between a vertex and another vertex in the transportation network. Let \( d(v) \) is the number of degrees in a graph or the number of connections of vertex \( v \) to other vertices in a network and \( n \) is the number of vertices. The degree of centrality, \( C_D(v) \), is defined by:

\[
C_D(v) = \frac{d(v)}{n-1}
\]  

(1)

2.2.2. Betweenness Centrality ([6,7,8,9])

Betweenness centrality calculates how often a vertex is passed by another vertex to go to a particular vertex in the network. The Dijkstra algorithm is used in determining the shortest path. The formula in calculating betweenness centrality, \( C_B(v) \), is:

\[
C_B(v) = \frac{2}{(n-1)(n-2)} \sum_{s \neq t \neq v \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}
\]  

(2)

With \( \sigma_{st} \) is the number of shortest paths from vertex \( s \) to \( t \) and \( \sigma_{st}(v) \) the number of shortest paths from vertex \( s \) to \( t \) passing through vertex \( v \).

2.2.3. Closeness Centrality ([2,6,7,8])

Closeness centrality is used to calculate the average distance between a vertex and all other vertices in the network, in other words measure the closeness of a vertex with other vertices either directly or indirectly. Let \( n \) be a number of vertices on the network and \( d(v,t) \) be the shortest path distance from vertex \( v \) to vertex \( t \). The closeness centrality is defined by:

\[
C_C(v) = \frac{n-1}{\sum_{t \in V} d(v,t)}
\]  

(3)
Dijkstra algorithm is one of the simple and usually used methods that provide optimum value in determining the shortest path in closeness and betweenness centrality.

2.2.4. Eigenvector Centrality ([1,7,10])

Eigenvector centrality performs measurements that give higher weight to vertices that are connected to other vertices with high centrality values. Let the adjacency matrix $A$ contain entries $A_{v,t}$ and $x$ is the eigenvector with the dominant eigenvalue $\lambda$. One of the simplest iterative methods in determining the dominant eigenvector of adjacency matrix $A$ is the Power Method. The eigenvector point centrality of a vertex $v$, namely $x_v$, can be computed by:

$$C_E(v) = \frac{1}{\lambda} \sum_{t \neq v \in V} A_{v,t}x_t$$

(4)

2.2.5. Weighted Eigenvector Centrality ([4])

Soh et al. ([11]) use the extension of eigenvector centrality concept to weighted networks by noting that weights should affect the importance of edges. The formula given as follows

$$C^{w}_E(v) = \frac{1}{\lambda} \sum_{t \neq v \in V} A_{v,t}w_{v,t}x_t$$

(5)

3. Result and Discussion

3.1. Results and Discussion

In this study, the data used is the Unpad Jatinangor campus vehicle routes, where the vertex location was taken from the Unpad Map ([11]) and the distance between locations is taken from Google Map data, as presented in Figure 1.

(a) [Unpad Jatinangor Campus Plan ([1])](#)

(b) [Graph of vehicle route representation in Unpad Jatinangor](#)

It is assumed that the vehicle route is two-way, so it uses an undirected graph. The location lists with its corresponding vertex is given in Tabel 1, which follows the location stated on the Unpad plan/map. Edge states that there is a direct vehicle route between locations while the vertex states the location of the vehicle to stop, and side weights represent the distance between locations in meters.

| $v$ | Location Name | $v$ | Location Name | $v$ | Location Name | $v$ | Location Name |
|-----|---------------|-----|---------------|-----|---------------|-----|---------------|
| 1   | REKTORAT      | 8   | PERPUSTAKAAN  | 15  | FPIK          | 21  | FAPSI         |
| 2   | BALE SANTIKA  | 9   | FIB           | 16  | FTIP          | 22  | FKEP          |
| 3   | PPBS          | 10  | FISIP         | 17  | FAPET         | 23  | FKU           |

**Figure 1.** (a) Unpad Jatinangor Campus Plan ([1]); (b) Graph of vehicle route representation in Unpad Jatinangor with side weights is the distance between locations in meters

**Tabel 1.** Vertices ($v$) and its corresponding location
The syntax of the centrality measurements using Python 3.7 program with library package used named NetworkX. Following are the calling function from NetworkX for the calculation, the full syntax program not given in this article but the main function library only. At the beginning of the program, the module is called with the command: import network as nx, and the graph named as $G$. The weighted eigenvector centrality is calculated by a standard eigenvector routine in Python, by locating its maximum eigenvalue first. The command for calculating eigenvalue and vectors is import numpy as np.

```
# Routine for calculating centrality
nx.degree_centrality(G)
mx.closeness_centrality(G)
nx.betweenness_centrality(G)
x.eigenvector_centrality(G)

# Routine for calculating weighted eigenvector
A = nx.to_numpy_matrix(G)
eigvals, eigvecs = np.eig(A)
eigvals = eigvals.real
eigvecs = eigvecs.real
SUM = 0
for i in range(26):
    SUM = SUM + eigvecs[i,0]**2
EC = (eigvecs[:,0])/SUM**0.5
```

### Table 2. Centrality values for each vertex $v$

| $v$ | Location | $C_D(v)$ | $C_C(v)$ | $C_B(v)$ | $C_E(v)$ | $C_E^w(v)$ |
|-----|----------|----------|----------|----------|----------|------------|
| 1   | REKTORAT | 0.28000  | 0.43860  | 0.23290  | 0.32715  | 0.55471    |
| 2   | BALE SANTIKA | 0.20000  | 0.40984  | 0.05673  | 0.30141  | 0.13320    |
| 3   | PPBS     | 0.28000  | 0.46296  | 0.17609  | 0.37504  | 0.10487    |
| 4   | FEB      | 0.12000  | 0.36765  | 0.07378  | 0.14373  | 0.01887    |
| 5   | FIKOM    | 0.12000  | 0.34722  | 0.08028  | 0.05109  | 0.00361    |
| 6   | FH       | 0.08000  | 0.29762  | 0.02872  | 0.01198  | 0.00028    |
| 7   | PUSAT RISET BERSAMA | 0.16000  | 0.37879  | 0.05194  | 0.10288  | 0.01111    |
| 8   | PERPUSTAKAAN | 0.16000  | 0.39683  | 0.14166  | 0.09024  | 0.01028    |
| 9   | FIB      | 0.12000  | 0.33784  | 0.06761  | 0.02248  | 0.00237    |
| 10  | FISIP    | 0.12000  | 0.32895  | 0.07317  | 0.01398  | 0.00487    |
| 11  | ASRAMA 5 | 0.12000  | 0.30488  | 0.03789  | 0.00957  | 0.00100    |
| 12  | FKG      | 0.16000  | 0.35714  | 0.14408  | 0.03871  | 0.05779    |
| 13  | F. FARMASI | 0.24000  | 0.39062  | 0.05112  | 0.32198  | 0.16600    |
| 14  | ASRAMA 2 DAN 4 | 0.16000  | 0.33333  | 0.00964  | 0.19735  | 0.39820    |
| 15  | FPIK     | 0.16000  | 0.33333  | 0.01222  | 0.21324  | 0.17874    |
From the calculation results, we can see that location 3-PPBS has 3 highest value of centrality, that is degree, closeness, and eigenvector centrality. This means that this location is the most importance in the direct connection between one location to another, the mostly passed by location, and most connected to other vertices with high centrality values in Unpad Campus route.

Followed by it, location 1-Rektorat and location 20-FMIPA has 2 highest values of centrality. Location 1-Rektorat has the most central based on degree and weighted eigenvector centrality. While location 20-FMIPA is prior in degree and betweenness centrality. As we can see from Table 2 that the results for eigenvector centrality compared to weighted eigenvector centrality is quite different. The priority location obtained by using standard eigenvector is located in location 3-PPBS while for weighted eigenvector centrality is located in location 1-Rektorat, thus the weight of the graph may affect the dominant eigenvalue of the adjacency matrix of graph G.

The results of all calculated centrality measures also displayed in Figure 3 with vertex sizes are arranged related to the value of each centrality measure. The yellow vertices indicate some top rank of vertices in each centrality measure. We can see clearly that each type of centrality might resulting the same priority location, but also might fail in locating the priority vertex with a different type of centrality.
Figure 3. Graph display with vertex size arranged related to the value of each centrality measures: (a) Degree Centrality (b) Betweenness Centrality (c) Closeness Centrality (d) Eigenvector Centrality, and (e) Weighted Centrality

3.2. Recommendation
This model of campus route is quite simple, and it is possible to be modified into a directed graph so that the results might be more accurate and realistic. For a directed graph, a different variant of calculation is applied, for example in degree centrality, there are two types of degree centralities to be calculated, namely indegree and outdegree. For the eigenvector centrality, it is quite interesting also to analyze a PageRank centrality, a variant of eigenvector centrality which is used by a google search engine, to the analyzed directed graph.

4. Conclusion
Based on the results obtained, it can be concluded that based on the ranking of the top tree centrality values, locations 3-PPBS, 1- Rektorat, and 20-FMIPA has the greatest number of highest centrality values. From the calculation results, we can see that location 3-PPBS has 3 highest value of centrality, that is degree, closeness, and eigenvector centrality. This means that this location is the most important in the direct connection between one location to another, the most passed by location, and most connected to other vertices with high centrality values in Unpad Campus route.

These results can be a recommendation regarding the route, for example, the capacity of bus/transport inside the campus which to be reproduced across the location, road maintenance should be prioritized over priority locations, security capacity around the location, and so on.

The results of this study are still very simple, according to the author's point of view, research involving a directed graph model will be closer to the real problem than the undirected graph assumption as in this study.

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References
[1] Zhou X and Moinuddin M 2016 Review of the SDG Index and Dashboards: An example of Japan’s global ranking results IGES Work. Pap. 32
[2] Allen C, Metternicht G and Wiedmann T 2019 Prioritising SDG targets: assessing baselines, gaps and interlinkages Sustain. Sci. 14 421–38
[3] El-Maghrabi M H, Gable S, Osorio Rodarte I and Verbeek J 2018 Sustainable Development Goals Diagnostics: An Application of Network Theory and Complexity Measures to Set Country Priorities 22
[4] Huang X, Zhao Y, Ma C, Yang J, Ye X and Zhang C 2016 TrajGraph: A Graph-Based Visual Analytics Approach to Studying Urban Network Centralities Using Taxi Trajectory Data *IEEE Trans. Vis. Comput. Graph.* 22 160–9

[5] Soh H, Lim S, Zhang T, Fu X, Lee G K K, Hung T G G, Di P, Prakasam S and Wong L 2010 Weighted complex network analysis of travel routes on the Singapore public transportation system *Phys. A Stat. Mech. its Appl.* 389 5852–63

[6] Zhang J and Luo Y 2017 Degree Centrality, Betweenness Centrality, and Closeness Centrality in Social Network 132 300–3

[7] Golbeck J 2013 Analyzing the Social Web, Ebook: https://doi.org/10.1016/C2012-0-00171-8 (Retrieved: 10 April 2020)

[8] Freeman L C 1978 Centrality in social networks conceptual clarification *Soc. Networks* 1 215–39

[9] Dekker A 2008 Centrality in social networks: Theoretical and simulation approaches *SimTect, Melbourne, Aust.* 33–8

[10] Choi J H, Barnett G A and Chon B S 2006 Comparing world city networks: A network analysis of Internet backbone and air transport intercity linkages *Glob. Networks* 6 81–99

[11] Unpad Jatinangor Campus Map: http://www.unpad.ac.id/pengumuman/panduan-perjalanan-dan-dedenah-universitas-padjadjaran-kampus-jatinangor/ (Retrieved: 10 April 2020)

[12] Gross J L 2019 Graph Theory and Its Applications, 3rd Edition. Boca Raton: CRC Press.