Abstract

In this chapter it is elucidated how the analytical solution was obtained to obtain the electromagnetic polarization profile of a completely polarized wave in the region of distant fields. The analytical solution was obtained from the interpretation of the physical phenomenon associated with the method of the linear component, which was adapted for the use of discrete and miniaturized elements, reducing the physical space for the measurement circuit and the expenses associated to the delay circuit. From the analytical solution it is possible to observe that with only one phase measurement in the delay circuit it is possible to obtain the polarization profile of the wave, with the axial ratio and phase, respectively.

Keywords: polarization, phased shifter, microfite, dipole

1. Introduction

Nowadays, in many industries, industries, offices, companies, supermarket chains among others, there is a very large diversity of radio frequency (RF) equipment. In these sectors, several RF equipment can coexist, operating simultaneously in a small space, even having different emission/reception characteristics. Because there are several devices, some of them may have their performances harmed or even compromised due to electromagnetic interference (EM) caused by other equipment that may be emitting an undesirable EM wave to the apparatus that is undergoing the interference process. In addition, it is also observed that, in the technological market of the present day, the size of the antennas existing in these
equipments becomes less and less. The miniaturization of antennas in many cases makes it difficult to locate the emitter system, which is possibly causing EM interference, since the antenna is often built into the equipment. Therefore, knowledge of the direction of propagation of the EM wave emitted by these antennas or EM field emitting elements will facilitate the discovery of the EM interference source(s) and the determination of possible engineering actions that help to attenuate the same. The study of techniques that aim to reduce the damage caused by EM interference between electronic devices has increased in recent years. Through a study of the polarization of electromagnetic waves, one can identify the behavior of the irradiated electric field. The knowledge of the amplitude and phase of the electric field [1] allows to identify the polarization pattern of the wave. The information of the phase of the electric field in the region of far fields (FFR) is of great value for the discovery and suggestions of overcoming problems caused by EM interference. An experimental technique was developed to confirm the linear component method, proposed by Kraus [2], for the reconstruction of the electromagnetic polarization in the FFR. The method was adapted using miniaturized discrete elements for the lagging circuit and an experimental and analytical confirmation was obtained in order to obtain the phase of the electric field radiated to electromagnetic waves that are completely polarized in the FFR using discrete components.

2. Methodology

The model for obtaining the polarization profile of an electromagnetic (EM) wave in the RCD was initially proposed by [2]. An adaptation in the phase-shift circuit to obtain the phase of polarization is proposed in an analytical way below.

The mathematical approach to gain access to the polarization phase of the EM wave in the FFR (Far Field Region) is made using the phasor analysis of the signal that is received by each dipole, and the resulting signal at the receiver of Figure 1(b). The discrepancy introduced by the discrete components does not identify the phase of the transmitted signal, and it allows to deduce this phase after a phasor sum of the two components. What would be simpler is the construction of a system to identify the phase of the emitted electric field.

In the delay circuit Figure 1(b) it was proposed a modification in the physical structure of the phase shift, replacing the split line with planar transmission lines at the end of the connection with the dipoles. And in a discrete way the delay was supposed from the interpretation of the associated physical phenomenon and the supposed theoretical formulation. The proposed delay circuit can be observed in Figure (2).

The particular cases were tested in order to validate the method. But for the general case, where the polarization phase of the signal emitted by the transmitting antenna is not known, the following reasoning is given. From the consideration that the fields captured by the dipoles are of the form

\[ E_v = E_v e^{j0/\hat{v}} \]

\[ E_h = E_h e^{j(\psi+\theta)/\hat{h}} \]

(1)
Where $\psi$ is the phase difference between the two signals picked up by the dipoles, i.e., the phase of the polarization of the transmitted signal, and $\theta$ is the delay intentionally inserted in order to find the phase of the emitted signal, and $\hat{v}$ and $\hat{h}$ are the unit vectors in the vertical and horizontal directions, respectively. The receiver is the one who performs a phasor sum of the signals at high frequencies, has its resulting field given by

$$\vec{E}_R = \vec{E}_v + \vec{E}_h$$  (3)

In phasor form one has to

Figure 1. Set up for measuring: (a) the axial ratio; (b) the phase difference $\psi$, using the method of linear components [1].

Figure 2. Circuit validation of adjustments made in the method of the linear component.
\[ E_R = E_v e^{i\theta} + E_h e^{(\psi + \theta)} \]  

In developing this expression one has to

\[ E_R = E_v \cos (\theta') + E_h \cos (\theta + \psi) + jE_h [\cos (\theta + \psi) + \sin (\theta + \psi)] \]

\[ E_R = E_v + E_h [\cos (\theta) \cdot \cos (\psi) - \sin (\theta) \cdot \sin (\psi)] + jE_h \sin (\theta) \cdot \cos (\psi) + \sin (\psi) \cdot \cos (\theta)] \]

Once \( E_R, E_h, E_v \) e \( \psi \) are known it can be adopted that

\[
\begin{align*}
A &= E_h \cos (\theta) \\
B &= E_h \sin (\theta) \\
C &= E_R
\end{align*}
\]

Is that \( z = a + bi \);

\[
\begin{align*}
a &= E_v + A \cos (\psi) - B \sin (\psi) \\
b &= A \cos (\psi) + B \sin (\psi) \\
C &= E_v + A \cos (\psi) - B \sin (\psi) + j[A \sin (\psi) + B \cos (\psi)].
\end{align*}
\]

The value of \( C \) in this equation is the electric field module \( E_R \) and this module is given by

\[
|z| = \sqrt{a^2 + b^2}
\]

Thus it is to develop the expression (9) in (10) that:

\[
|C| = \sqrt{[E_v + A \cos (\psi) - B \sin (\psi)]^2 + [A \sin (\psi) + B \cos (\psi)]^2}
\]

\[
C^2 = E_v^2 + A^2 + B^2 + 2E_v [A \cos (\psi) - B \sin (\psi)]
\]

In the case where the angle \( \psi \) it is unknown to have:

\[
\begin{align*}
C^2 - E_v^2 - A^2 - B^2 &= 2E_v [A \cos (\psi) - B \sin (\psi)] \\
\frac{C^2 - E_v^2 - A^2 - B^2}{2E_v} &= A \cos (\psi) - B \sin (\psi)
\end{align*}
\]

Doing

\[
D = \frac{(C^2 - E_v^2 - A^2 - B^2)}{2E_v}
\]

it has been

\[
D = A \cos (\psi) - B \sin (\psi)
\]

Using the expression
\[ \cos(\psi) = \pm \sqrt{1 - \sin^2(\psi)} \]  
\[ \text{(15)} \]

In (14) one has to 
\[ D = A \sqrt{1 - \sin^2(\psi)} - B \sin(\psi). \]  
\[ \text{(16)} \]

The geometric development of the analytic model is elucidated in the following expressions.

From the geometric analysis to Eq. (16) illustrated in the Figure 3 is possible to verify that:

\[ \alpha = \psi + \arctan\left(\frac{B}{A}\right) \]  
\[ \text{(17)} \]

\[ \overrightarrow{OP} = \overrightarrow{OP} \cdot \cos(\alpha) \]  
\[ \text{(18)} \]

\[ \overrightarrow{OP} = D \]  
\[ \text{(19)} \]

\[ D = \sqrt{A^2 + B^2 \cos\left(\psi + \arctan\left(\frac{B}{A}\right)\right)} \]  
\[ \text{(20)} \]

\[ \frac{D}{\sqrt{A^2 + B^2}} = \cos\left(\psi + \arctan\left(\frac{B}{A}\right)\right) \]  
\[ \text{(21)} \]

\[ \psi = \arccos\left(\frac{D}{\sqrt{A^2 + B^2}}\right) - \arctan\left(\frac{B}{A}\right) \]  
\[ \text{(22)} \]

Then it found a closed expression to find the value of phase difference $\psi$. The discovery phase can be analytically or by using the parameters measured experimentally as done at work.

Denoting $x = \sin(\psi)$, one can write Eq. (16) as 
\[ D = A \sqrt{1 - x^2} - B \cdot x. \]  
\[ \text{(23)} \]

In developing Eq. (23), one arrives at the expression that relates the unknown angle $\psi$, with known constants. In solving Eq. (23) have

\[ \text{Figure 3. This is achieved as the phase of the polarization.} \]
\[ \sin(\psi) = \frac{-D \pm \left[ (DB)^2 - (A^2 + B^2)(D^2 - A^2) \right]^{1/2}}{(A^2 + B^2)} \] (24)

An analysis was performed to validate Eq. (11) for elementary cases. The results can be seen in Table 1 and all the mathematical development that follows.

\[ \psi = \arcsin \left( \frac{-D \pm \left[ (DB)^2 - (A^2 + B^2)(D^2 - A^2) \right]^{1/2}}{(A^2 + B^2)} \right) \] (25)

To validate the cases where there is ambiguity in the polarization profile of the EM wave, which are those in which the axial ratio is unitary, Eq. (9) and tested a random phase in the delay circuit, in order to prove that it is enough to only insert a delay line to validate the adapted linear component method [3].

The elementary cases were tested in Eq. (9), the whole mathematical development for these cases is next

- For the linear case at 45°, it is possible to write:

\[
\begin{align*}
E_h &= 1 \text{(horizontal dipole amplitude)} \\
E_v &= 1 \text{(vertical dipole amplitude)} \\
\psi &= 45^\circ \text{(phase difference)} \\
\theta &= 45^\circ \text{(generic phase of the line)}
\end{align*}
\]

Substituting this data into Eq. (9) we have that:

\[
E_R = 1, 0 + 1, 0 \cdot \left[ \cos \left( 45^\circ + 45^\circ \right) \right] + j, 1, 0 \cdot \left[ \sin \left( 45^\circ + 45^\circ \right) \right] \\
E_R &= 1 + j \\
|E_R| &= \sqrt{2}
\] (26)

- For the linear case at 135°, we have that:

\[
\begin{align*}
E_h &= 1 \text{(horizontal dipole amplitude)} \\
E_v &= 1 \text{(vertical dipole amplitude)} \\
\psi &= 135^\circ \text{(phase difference)} \\
\theta &= 45^\circ \text{(generic phase of the line)}
\end{align*}
\]

Substituting this data into Eq. (C.4.3) gives:

\[
E_R = 1, 0 + 1, 0 \cdot \left[ \cos \left( 135^\circ + 45^\circ \right) \right] + j, 1, 0 \cdot \left[ \sin \left( 135^\circ + 45^\circ \right) \right] \\
E_R &= 0 \\
|E_R| &= 0
\] (27)
For the left circular case, we have:

- $E_h = 1$ (horizontal dipole amplitude)
- $E_v = 1$ (vertical dipole amplitude)
- $\psi = 90^\circ$ (phase difference)
- $\theta = 45^\circ$ (generic phase of the line)

Substituting this data into Eq. (C.4.3) gives:

$$E_R = 1, 0 + 1, 0 \cdot [\cos (90^\circ + 45^\circ)] + j, 0 \cdot [\sin (90^\circ + 45^\circ)]$$

$$E_R = 1 - \frac{\sqrt{2}}{2} + j \left( \frac{\sqrt{2}}{2} \right)$$

$$|E_R| = \sqrt{2 - \sqrt{2}}$$

For the right circular case, we have that:

- $E_h = 1$ (horizontal dipole amplitude)
- $E_v = 1$ (vertical dipole amplitude)
- $\psi = 270^\circ$ (phase difference)
- $\theta = 45^\circ$ (generic phase of the line)

Substituting this data into Eq. (C.4.3) gives:

$$E_R = 1, 0 + 1, 0 \cdot [\cos (270^\circ + 45^\circ)] + j, 0 \cdot [\sin (270^\circ + 45^\circ)]$$

$$E_R = 1 + \frac{\sqrt{2}}{2} + j \left( -\frac{\sqrt{2}}{2} \right)$$

$$|E_R| = \sqrt{2 + \sqrt{2}}$$

Therefore what will be done is a comparison of the values of $\theta$, with the possible results of $E_R$, and we have:

| Polarization profile       | $E_h$ | $E_v$ | $\theta$ | $|E_R|$ | $\psi$ |
|----------------------------|-------|-------|-----------|---------|--------|
| Linear to 45°              | 100   | 100   | 45°       | $\sqrt{2}$ | 45°    |
| Linear to 135°             | 100   | 100   | 45°       | 0        | 135°   |
| Left circular              | 100   | 100   | 45°       | $\sqrt{2 - \sqrt{2}}$ | 90°    |
| Right circular             | 100   | 100   | 45°       | $\sqrt{2 + \sqrt{2}}$ | 270°   |

Table 1. Polarization patterns tested.
- Linear at 45°:
  - \( E_h = 1 \);
  - \( E_v = 1 \);
  - \( \psi = 45^\circ \);
  - \( |E_R| = \sqrt{2} \);
  - \( \psi = ? \)

When applying in (9) we have:

\[
\sqrt{2} = 1 + \left( \frac{\sqrt{2}}{2} \right)^2 + \left( \frac{\sqrt{2}}{2} \right)^2 + \cos(2\psi) \left[ \left( \frac{\sqrt{2}}{2} \right)^2 - \left( \frac{\sqrt{2}}{2} \right)^2 \right] + 2.1 \left[ \frac{\sqrt{2}}{2} \cdot \cos(\psi) - \frac{\sqrt{2}}{2} \cdot \text{sen}(\psi) \right] \\
2 = 2 + 2 \cdot \frac{\sqrt{2}}{2} \left[ \cos(\psi) - \text{sen}(\psi) \right] \\
\text{sen}\left( \frac{\pi}{2} - \psi \right) = \text{sen}(\psi), \\
\frac{\pi}{2} - \psi = \psi + 2k\pi \\
\psi = \frac{\pi}{4} + 2k\pi. 
\]

Which certifies the phase difference of 45°.

- Linear at 135°:
  - \( E_h = 1 \);
  - \( E_v = 1 \);
  - \( \psi = 45^\circ \);
  - \( |E_R| = 0 \);
  - \( \psi = ? \)

When replacing in (9) we have:

\[
0 = 1 + \left( \frac{\sqrt{2}}{2} \right)^2 + \left( \frac{\sqrt{2}}{2} \right)^2 + \cos(2\psi) \left[ \left( \frac{\sqrt{2}}{2} \right)^2 - \left( \frac{\sqrt{2}}{2} \right)^2 \right] + 2.1 \left[ \frac{\sqrt{2}}{2} \cdot \cos(\psi) - \frac{\sqrt{2}}{2} \cdot \text{sen}(\psi) \right], \\
0 = 2 + 2 \cdot \frac{\sqrt{2}}{2} \left[ \cos(\psi) - \text{sen}(\psi) \right], \\
\text{sen}\left( \psi - \frac{\pi}{4} \right) = \text{sen}\left( \frac{\pi}{2} \right) + 2k\pi, 
\]
\[
\psi = \frac{3\pi}{4} + 2k\pi.
\] (31)

- Which validates the phase difference of 135°.
- Circular left
  - \(E_h = 1\);
  - \(E_v = 1\);
  - \(\psi = 45°\);
  - \(|E_R| = \sqrt{2 - \sqrt{2}}\);
  - \(\psi = ?\)

When applying in (9), we have that:

\[
\left( \sqrt{2 - \sqrt{2}} \right)^2 = 1 + \left( \frac{\sqrt{2}}{2} \right)^2 + \left( \frac{\sqrt{2}}{2} \right)^2 + \cos (2\psi) \left[ \left( \frac{\sqrt{2}}{2} \right)^2 - \left( \frac{\sqrt{2}}{2} \right)^2 \right] + 2.1 \left[ \frac{\sqrt{2}}{2} \cos (\psi) - \frac{\sqrt{2}}{2} \sin (\psi) \right]
\]

\[
2 - \sqrt{2} = 2 + 2 \sqrt{2} \left[ \cos (\psi) - \sin (\psi) \right],
\]

\[
\sin (\psi - \frac{\pi}{4}) = \sin \left( \frac{\pi}{4} \right) + 2k\pi,
\]

\[
\psi = \frac{\pi}{2} + 2k\pi.
\] (32)

Which gives the phase difference of 90°.

- Circular right:
  - \(E_h = 1\);
  - \(E_v = 1\);
  - \(\psi = 45°\);
  - \(|E_R| = \sqrt{2 + \sqrt{2}}\);
  - \(\psi = ?\)

When replacing in (9), we have that:
\[
\left( \sqrt{2 + \sqrt{2}} \right)^2 = 1 + \left( \frac{\sqrt{2}}{2} \right)^2 + \left( \frac{\sqrt{2}}{2} \right)^2 + \cos(2\psi) \left[ \left( \frac{\sqrt{2}}{2} \right)^2 - \left( \frac{\sqrt{2}}{2} \right)^2 \right] + 2.1 \left[ \frac{\sqrt{2}}{2} \cos(\psi) - \frac{\sqrt{2}}{2} \sin(\psi) \right]
\]

\[
2 + \sqrt{2} = 2 + \frac{\sqrt{2}}{2} [\cos(\psi) - \sin(\psi)],
\]

\[
\sin \left( \psi - \frac{\pi}{4} \right) = \sin \left( \frac{5\pi}{4} \right) + 2k\pi,
\]

\[
\psi = \frac{3\pi}{2} + 2k\pi. \quad (33)
\]

Which certifies the phase difference of 270°.

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