Design and analysis of optical sensors composed of large arrays of nanowires using the surface impedance generating operator (SIGO) method

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Abstract
The design and analysis of large arrays of nanoscale scattering elements using existing computational tools is particularly difficult. Such difficulty mainly arises from large number of unknowns and the way existing CAD tools handle the analysis of complex nanostructures. In this work, a simple, reliable and fast analytical tool is introduced based on the surface impedance generating operator method, which not only reduces the number of unknowns through surface discretisation of the scatterers but can also be implemented in a parallel process, reducing simulation time. Circular and linear arrays of nanowires are designed using the proposed method to have a directive pattern and to be deployed in a rake-shaped optical probe head. The far-field scattering patterns of the arrays are investigated. In addition, the effects of array configuration as well as the number of elements and their properties on the directivity and beamwidth of the scattering patterns are also evaluated. Finally, high-resolution directive scanning probes made of a circular array of 31 gold nanowires and a linear array of 15 elements are designed at 800 nm with less than 3° beamwidth.

1 | INTRODUCTION

Nanostructures are used in diverse applications from biosensors to the food industry, solar energy harvesting, environmental monitoring, and so on [1–5]. Metallic nano-rods and nanoparticles are proliferated in recently introduced optical devices [6, 7]. Chip to chip interconnects and sensors are few examples of metallic nanostructures working at optical frequencies.

To design sophisticated devices, powerful and versatile simulation tools are required. Simulation of nanometre-sized metal particles at optical frequencies can be quite challenging [8–10]. This is because the numerical methods that discretise the whole volume, like finite difference time domain (FDTD) [11, 12] or finite element method (FEM) [13], have to solve for very fine meshes, especially around the boundary. This is mainly due to the fact that at optical frequencies, fields penetrate metals and decay very rapidly inside them. Therefore, mesh sizes should be selected cautiously in order to achieve convergence in solutions. By deploying very fine meshes, however, simulation may encounter round-off errors. Moreover, unlike differential-equation-based techniques, the integral equation-based tools leverage Green’s functions within which structural and material characteristics are embedded. Therefore, the latter techniques are generally more stable and accurate than say, finite difference (FD) based methods [14–16]. Integral equation-based techniques are generally classified into volume integral equations (VIEs) and surface integral equations (SIEs). VIEs are solved for finding polarisation currents, whereas SIEs are used to find surface equivalent electric and magnetic currents.

Unlike VIEs that discretise the volume, SIEs discretise boundaries, and thus, SIEs can have less unknowns and so are less processor exhaustive in numerical implementation. Different SIE formulations have been introduced, namely electric field integral equation (EFIE), magnetic field integral equation (MIE), Muller, PMCHW and combined field integral equation [17], some of which have been applied to metal optics problems. One of the major shortfalls of the mentioned techniques is that they are intrinsically unsuitable for parallel processing, and so, require long simulation times.
Design of large arrays with potential applications in optical sensors often call for iterative simulations and/or parametric study. Thus, it is of paramount importance that the utilised analytical tool is capable of performing fast and accurate simulations. The surface impedance generating operator (SIGO) formulation [18] is a numerically efficient method for solving scattering problems in which a scattering problem is divided into two separate interior and exterior subproblems, concerning the boundary of the scatterer. One can then use the method of moment (MoM) to solve the integral equations. Following the SIGO formulation, the moment matrices of all regions can be computed simultaneously. This feature makes the method suitable for parallel processing. Moreover, the solution of interior regions can be reused when multiple objects with the same shape and material are present. This ability can reduce the simulation time considerably, particularly for large arrays that may find applications in sensors or even in cloaking devices [19, 20].

In this work, we first derive the Green's function that is required for applying the SIGO formulation to a circular nanowire. Then, by combining equations of interior and exterior subproblems, a single-source integral equation is constructed, whose solution is the equivalent surface currents on the boundary of scattering objects. To analyse the performance of the introduced probe heads, arrays of circular nanowires in different topologies (linear and circular arrangements) are studied. It is shown that a directive scattering pattern can be produced through proper parameter selection. Moreover, front-to-back ratio (FBR) in both linear and circular arrangements is examined.

The rest of the work is organised as follows: In Section 2, the SIGO formulation is derived for a circular nanowire. The required Green's function is obtained and the single source integral equation is formulated. In Section 3, the MoM implementations of the SIGO formulation are presented. The scattering patterns of probe heads with different array configurations and a large number of elements are calculated and demonstrated in Section 4. Concluding remarks are presented in Section 5.

2 FORMULATION OF THE PROBLEM

The structural arrangement of a large uniform array of circular nanowires in the form of a rake-shaped optical probe is shown in Figure 1, together with device under test (DUT), and an array of photodetectors. The idea is that at a certain arrangement of the nanowire uniform array, a directive pattern is obtained at the photodetectors’ array, which is altered only in the presence of an external object, namely DUT here. In other words, the optical sensor probe pronounces the presence of DUT when its directive pattern is distorted. Moreover, by rotating the rake head, a scanning effect of the probe head can be realised.

Two different uniform array configurations have been considered for the probe head. Figure 2a,b show linear array and circular array topologies, respectively. In order to use the SIGO formulation for the probe head design, we first analyse a typical array element. The cross-section of a general circular nanowire is illustrated in Figure 3. The nanowire can be made from any homogeneous, isotropic material with the constitutive parameters of relative permittivity and permeability of the nanowire \( \varepsilon_r \) and \( \mu_r \), which may be complex numbers considering the losses. Assuming that both the structure and the excitation are invariant with respect to the longitudinal direction, this 2D structure can support both TM and TE solutions, depending on the polarisation of the incident wave. In the case of TM polarisation, the existing cylindrical field components are \( E_z, H_y, \) and \( H_x \).

2.1 The SIGO integral equation

The SIGO formulation declares that each object enforces relation (1) between tangential electric and magnetic fields along its boundary, regardless of the composition of other scatterers and external excitations [18].

\[
\bar{E}(\vec{r}) = j\omega \mu_r \int_{C} \bar{G}(\vec{r}, \vec{r}') \cdot \hat{J}_i(\vec{r}') d\vec{r}'
\]  

(1)

where \( \bar{E}(\vec{r}) \) is the electric field, \( \hat{J}_i(\vec{r}') \) denotes the equivalent electric surface current defined as \( \hat{J}_i = \hat{n} \times \bar{H} \), \( \vec{r}' \) is the position vector, \( \bar{G}(\vec{r}, \vec{r}') \) is the electric dyadic Green's function of the second kind defined in [18], and \( \omega \) is the angular frequency. Although the exact values of the field distributions cannot be known without considering the exterior problem, the relation among field components on the boundary of each object can be identified directly from the properties of the object itself.

For the special case of a circular nanowire shown in Figure 3, the SIGO formulation, when written for TM polarisation, relates the \( E_z \) at each point to the \( H_y \) (or equivalently \( J_{xy} \)) of all points on the circumference. Therefore, only one out of nine components of \( \bar{G}(\vec{r}, \vec{r}') \) becomes non-zero and we have
between nanowire configurations

**Figure 2** Typical (a) linear and (b) circular uniform array configurations of the probe head, with \( d \) array element spacing, \( a \) radius of nanowire, and \( A \): radius of the circular array configuration (i.e. distance the between nanowire's centre and the focal point of the circular array)

\[
\tilde{G}(\hat{r}, \hat{r}') = G^{zz}(\varphi, \varphi') \hat{\imath} \hat{z}. \]

Consequently, (1) can be rewritten as follows:

\[
E_z(\varphi) = j \omega \mu_0 \int G^{zz}(\varphi, \varphi') J_z(\varphi') d\varphi' \tag{2}
\]

where \( G^{zz} \) is the desired Green's function which has to satisfy (3) the following:

\[
\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial G^{zz}}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 G^{zz}}{\partial \varphi^2} + k^2 G^{zz} = \frac{1}{\rho} \delta(\rho - \rho') \delta(\varphi - \varphi') \tag{3}
\]

under a homogeneous Neumann boundary condition

\[
\frac{\partial G^{zz}(\rho, \rho'; \varphi')}{\partial \rho} \bigg|_{\rho=a} = 0,
\]

where \( k = \omega \sqrt{\mu_0 \varepsilon_0} \) is the wavenumber and \( \delta \) denotes the Dirac delta function. One can solve this boundary value problem (BVP) and find the analytical solution of \( G^{zz}(\varphi, \varphi') \) in a single summation form: 

\[
G^{zz}(\varphi, \varphi') = \frac{-1}{2\pi ka} \sum_{p=-\infty}^{\infty} J_p(ka) e^{j p(\varphi - \varphi')} \tag{4}
\]

where \( J_p \) denotes the Bessel function of the first kind with order \( p \) and \( J'_p \) is its derivative.

Other representations, like eigen function expansion, are also possible; but this expression is preferred because it is numerically easier to compute. One should keep in mind that all representations of Green's function are semantically identical. Furthermore, note that although Green's function has been derived for circular elements in this work, the Green's function of other regular shapes can also be obtained (e.g. rectangular nanowires [18]) based on which associated arrays can be analysed.

Equation (2) expresses \( E_z(\varphi) \) in terms of \( J_z(\varphi') \) for an individual finite scatterer (nanowires of Figure 2), regardless of the exterior subproblems, that is, the composition of other scatterers and external excitations. As such, (2) can be solved for any number of nanowires as they are not interdependent. Thus, it can be claimed that the proposed method is well-adapted for parallel processing, which is vitally important in the optimisation of large arrays of nanowires.

### 2.2 The exterior problem

At this point, two unknowns are present in (2) and so, to solve for \( J_z(\varphi') \), one would need an extra equation which can come from the exterior problem. Using the equivalence principle,
one can easily formulate the integral equation in the exterior region based on equivalent surface electric and magnetic currents. For the present TM case, the EFIE formulation can be written as follows [21]:

\[
E_z^{\text{inc}}(\rho) = -P.V. \int_{\Gamma_0} E_z(\rho') \frac{\partial G_0(\rho, \rho')}{\partial \rho'} d\Gamma' + \frac{1}{2} E_z(\rho) + \frac{j \eta_0 \rho^3}{\pi} \int_{\Gamma_0} G_0(\rho, \rho') J_\text{sc}(\rho') d\Gamma' \tag{5}
\]

where \(E_z^{\text{inc}}\) is the incident electric field, \(\Gamma_0\) indicates the boundary of scatterers, \(k_0\) is the wavenumber in the background medium, \(\eta_0\) is intrinsic impedance of the surrounding medium, \(G_0 = \frac{1}{4\pi} H_0^1(k_0|\rho - \rho'|)\) and \(P.V.\) is the Cauchy principal value.

To find the unknown currents, one has to combine (2) with (5) after stating the equations in matrix form. Equations (2) and (5) are EFIEs that are written for the TM case. Using the duality, one can easily find the MFIEs for the TE polarisation [18].

3 | MoM IMPLEMENTATION

Discretisation of the circumference of each nanowire is illustrated in Figure 3. The unknown current \(J_z\) can be expanded by 1D pulse functions:

\[
J_z(\varphi') = \sum_{n=1}^{N} \alpha_n P_n(\varphi') \tag{6}
\]

where \(P_n\) is the pulse basis function, which is equal to one on the \(n\)th segment and zero elsewhere, and \(\alpha_n\) indicates the unknown expansion coefficients.

3.1 | The matrix equation of the interior problem

Substituting (6) and (4) in (2) gives the following:

\[
E_z(\varphi) = \frac{j \eta_0 \mu_0}{2\pi \rho} \sum_{p=-\infty}^{\infty} \sum_{n=1}^{N} P_n(\varphi') e^{j(k_0\rho - \rho')} d\varphi' \tag{7}
\]

Then both sides of (7) are point-matched to give the following:

\[
[E_z(\varphi_m)] = \left[ Z_{mn}^{\text{ego}}(\varphi_m, \varphi'_n) \right] [\alpha_n] \tag{8}
\]

where

\[
Z_{mn}^{\text{ego}} = S_0 + \sum_{p=1}^{\infty} U_p \tag{9}
\]

in which \(n\) and \(m\) are indices denoting the source and observation points:

\[
S_0 = \frac{j \Delta \varphi}{2\pi} J_0(ka) \tag{10}
\]

\[
U_p = -\eta \frac{j}{\pi} \frac{j}{J_p'(ka)} \sum_{p} \sin \left( \frac{p \Delta \varphi}{2} \right) \frac{2 \cos(p \varphi_m - \varphi'_n)}{p} \tag{11}
\]

To compute the terms \(U_p, J_p'(ka)\) needs to be calculated first. Two approaches may be used to find these derivatives: (a) using the recursive relation of Bessel functions, or (b) using the FD approximation.

To calculate the term \(S_0\) in (9), we use the recursive relation, which is expressed for \(p = 0\) as \(J_0(x) = -J_1(x)\). For \(p > 0\), however, the ratio of \(J_p'(ka)/J_p(ka)\) can be calculated easily without using the recursive formula. Note that

\[
\frac{J_p(x)}{J_p'(x)} = \left[ \ln \left( J_p(x) \right) \right]' \tag{12}
\]

The above equation holds true for all values of \(x > 0\), except the roots of \(J_p(x)\). Hence, the ratio of \(J_p(x)/J_p'(x)\) can be described using the central difference approximation formula:

\[
\frac{J_p(x)}{J_p'(x)} = \frac{2\Delta x}{\ln(J_p(x + \Delta x)) - \ln(J_p(x - \Delta x))} \tag{13}
\]

After finding the \(U_p\) terms, we have to compute the sum of an infinite series, as expressed in (9), in order to find \(Z_{mn}^{\text{ego}}\). For numerical computations, however, this series needs to be truncated by properly selecting an upper bound of \(P\). Depending on the values of \(a\) and \(k\), this series may not converge rapidly; therefore, achieving acceptable accuracy may require \(P\) to be at least in the order of a few thousand. This causes another problem. For large orders of Bessel functions, calculations of \(J_p'(ka)/J_p(ka)\) become erroneous and may lead to 0/0 ambiguity due to the limited machine precision. To overcome this difficulty in series computation, we select a so-called decision parameter \(D\) at the point of 0/0 ambiguity in computations. We calculate the Bessel functions as usual when the order \(p < D\). Otherwise, the asymptotic expansion of Bessel functions of large orders is used, when \(p > D\).

The Bessel functions can be expanded for large orders using the following:

\[
J_{\nu}(z) \approx \frac{1}{\sqrt{2\pi\nu}} e^{\nu(1 - \frac{2\nu}{z} + \frac{z^2}{8\nu^2})} \tag{14}
\]
where \( l_n \) is the length of the \( n \)th segment and \( \gamma \) and \( \epsilon \) are the Euler and Napier constants, respectively. Moreover, \( \mathbf{\vec{H}}_0 \) is the outward normal unit vector, defined at the source point (see Figure 3), and \( \mathbf{\vec{H}}_1(x) \) is the Hankel function of the order \( \nu \). The boldface letters indicate the position vectors.

Using (8) that relates \([E_z] \) and \([\alpha_n] \), (16) can be restated as follows:

\[
[V_m] = 
\begin{bmatrix}
Z^{A}_{mn} \end{bmatrix}
- 
\begin{bmatrix}
Z^{M}_{mn} \\
Z^{\text{no go}}_{mn} \\
Z^{\text{volgo}}_{mn}
\end{bmatrix}
[\alpha_n]
\]  

(19)

which is obviously a single-source matrix equation that can be solved to find the unknown coefficients \([\alpha_n] \).

In the case of an array formed of identical elements, like that shown in Figure 2, \([Z^{\text{no go}}_{mn}] \) can be obtained once and reused again. This is very advantageous in large arrays as it significantly saves simulation time and processing resources. Assuming that equal number of segments and numbering patterns (sequences) are used in the meshing of designed elements (scatterers) and that similar discretisation technique is employed, one can write the following:

\[
[Z^{\text{no go}}_{mn}] = 
\begin{bmatrix}
\mathbf{A} & \mathbf{0} & \ldots & \mathbf{0} \\
\mathbf{0} & \ddots & \mathbf{0} \\
\mathbf{0} & \ldots & \mathbf{A}
\end{bmatrix}
\]  

(20)

where \([\mathbf{A}] \) is the SIGO matrix of one typical element of the array. After finding \([\mathbf{A}] \), calculating the effects of scatterers on the total fields through (20) is very fast compared with methods like FEM which need to discretise the volume of all scatterers one by one and solve for the fields inside the objects.

### 3.3 Far-field computations and post processing in the SIGO approach

To find the scattering pattern of the probe head (Figure 1), which is an array of nanowires, the far-field radiations of equivalent currents should be considered. This is essential to ensure the optimum performance of the probe in terms of narrow beamwidth and acceptable side-lobe level (SLL). The scattering pattern is defined as follows [21]:

\[
\sigma_{2D}(\varphi) = 
\lim_{\rho \to \infty} \left[ 2\pi \rho \frac{|E^\text{sc}|^2}{|E_m|^2} \right]
\]  

(21)

where \( E^\text{sc} \) is the scattered electric field. To find \( E^\text{sc} \) at the far-field region (\( \rho \to \infty \)), (5) is solved using the large argument approximation of Hankel functions. Thus \( \sigma_{2D} \) derived from the SIGO method becomes the following:
\[ \sigma_{2D} = \frac{4}{k_0} \left| \frac{1}{k_0} \oint_{\Gamma^0} \frac{1}{4} \left( E_z(r') e^{j k_0 \hat{n} \cdot r'} \right) d\Gamma' \right|^2 \]

Expressing (22) in matrix form gives

\[ \sigma_{2D} = \frac{4}{k_0} \left( \left[ Z_{MN}^{Far} \right] \left[ Z_{MN}^{SIGO} \right] - \left[ Z_{MN}^{Far} \right] \right) [\alpha_n] \right|^2 \]

where

\[ Z_{mn}^{Far} = \frac{k_0}{4} a \Delta \psi \cos(\varphi_m - \varphi_n) e^{j k_0 (x' \cos \varphi_m + y' \sin \varphi_m)} \]  
\[ Z_{mn}^{Afar} = \frac{k_0}{4} a \Delta \psi e^{j k_0 (x' \cos \varphi_m + y' \sin \varphi_m)} \]

\[ \varphi_m \] is the angle at which the far-field is observed. Once \( \alpha_n \) is known, (23) is solved to give the scattering pattern as a function of \( \varphi \).

**FIGURE 5** Flowchart of the array design procedure using the SIGO method. SIGO, surface impedance generating operator
It is worth mentioning that to obtain the scattering pattern using methods like FEM, one needs to calculate field distributions at the boundary of the numerical window first and then obtain the far-field quantities [22, 23]. Therefore, correct calculation of far-field characteristics requires the simulation set-up parameters to be selected very cautiously, meaning that often very fine meshing is required, the parameters of the PML layer need to be correctly adjusted, the numerical window need to be chosen large enough and so on. Such considerations generally make the simulation numerically intensive and time consuming. The situation becomes worse when solving metal optic problems. Paradoxically, although relaxing these requirements helps with the simulation time, it leads to erroneous results with incorrect nulls or grating lobes.

Interestingly, in the proposed method the above-mentioned simulation set-up parameters are inherently satisfied in the Green's functions of the problem. As shown in (23), the scattering pattern in our method is found right from the equivalent currents on the radiating elements. From the EM theory, we know that the radiation pattern is a variational expression with respect to the equivalent current sources. Therefore, first-order errors in current calculations lead to second-order errors in the computed radiation pattern, Thus offering superior performance compared to FEM or FDTD.

The above-described optimisation procedure based on the SIGO method is graphically illustrated as a flowchart depicted in Figure 5. Evidently, inherent independence and parallel processing of each block is the winning factor for the speed and distribution of the computational burden. To further illustrate this, in the following section scattering patterns are obtained for different arrays made of various structures and materials.

4 | ANALYSIS OF NUMERICAL RESULTS

In this section, different arrays of nanowire structures are studied and their application as optical probe heads is investigated. Before delving into array analysis, we start with a single element.

The fields scattered by a circular nanowire are analytically known [21]. So a single nanowire scatterer can be a good indicator of the validity and accuracy of the proposed approach (the SIGO method), as it offers the possibility of comparison of the obtained results. Referring to the structure of a single nanowire in Figure 3, a TM, plane wave, propagating in the x-direction, impinges the nanowire and scatters into the background medium. The nanowire is supposed to be made of gold with the radius of \( a = 0.1 \lambda_0 \). The wavelength of the incident wave in vacuum is 800 nm. This is the operating wavelength that is assumed constant in all proceeding examples as well. The complex permittivity of gold at this wavelength can be extracted from [24]. For this problem, the induced current on the nanowire can be computed both analytically or using the SIGO method described in Section 2. Figure 6 compares the results of the SIGO method applied to the above-mentioned single nanowire with the analytical solutions. Real and imaginary parts of the calculated equivalent electric currents are depicted in Figure 6a,b, respectively, at different points on the circumference of the scatterer.

Figure 6 also shows the results of the SIGO method for different numbers of segments \( N \). Evidently, for as few as 36 segments, the currents calculated by the SIGO approach are in very good agreement with the analytical solutions. For \( N = 360 \), the difference between the currents at different points on the boundary is barely noticeable, indicating that the accuracy of the method, (9) and (in Section 2.2.1) led to such good accuracy. At this stage, the scattering pattern of this single nanowire is calculated using (24). It is observed in Figure 7 that the electric field is scattered in all directions almost evenly. To improve directivity, one can increase the number of scatterers and deploy proper array configuration. It is also worth mentioning that the good accuracy obtained from the SIGO analysis of a single element (as seen in the example) has offered
reliability of $[\Lambda]$ to be used in the array design through (19), (20) and finally (24).

The first array structure that is considered here is the linear array configuration of Figure 2a formed of gold nanowires. The number of elements, material property at the frequency of interest, array spacing and so on. are the influential parameters determining the scattering pattern of the array. For $a/\lambda_0 = 0.1$ and $d/\lambda_0 = 0.3$, the scattering pattern is computed for the three different cases of number of arrays $N_{\text{Array}} = 5, 15$ and 25. Figure 8 illustrates the effect of the number of elements on the scattering pattern. Clearly, increasing the number of elements not only decreases the beamwidth but also increases the grating lobes and may slightly deteriorate the SLL. The same structure is simulated in ANSYS HFSS, the result of which is also included in Figure 8 for comparison. Note that although HFSS is inherently developed for the analysis of 3D scatterers, with some tweaks (i.e. defining excitation, absorbing boundary condition, PML layers and the numerical window) it can be used for the analysis of 2D structures as well. It is suggested that the meshing is controlled manually rather than using automatic mesh generation, in order to model the wave behaviour correctly near the metal-air interface. It takes about 15 min for the solver to find the pattern of Figure 2a with 15 elements. Even if only 10 iterations are required to optimise this array in HFSS, it takes about 150 min for the process to be completed. The time required for the SIGO method to solve this problem is about 1 min (i.e. 10 min for 10 iterations) as depicted in Table 1. Furthermore, as illustrated in Figure 6, the SIGO method is capable of offering very accurate results even with very coarse meshing (e.g. $N = 36$ segments), meaning that the simulation time of the SIGO method can be even further reduced while maintaining accuracy.

Figure 9 shows the influence of the element spacing $d$ on the scattering pattern. Setting $a/\lambda_0 = 0.1$ and $N_{\text{Array}} = 5$, the scattering width is computed for $d/\lambda_0 = 0.5, 0.7$ and 0.9. As expected, the larger $d$ increases the array dimension and, consequently, reduces the beamwidth and increases the grating lobes. For the 15 elements array with $d/\lambda_0 = 0.3$ and $a/\lambda_0 = 0.1$, the FBR of 0.5 dB is obtained as depicted in Figure 10. As shall be seen, better FBRs are achievable if circular arrays are deployed.

As mentioned, to have an estimation of the required simulation time in the SIGO method, one can refer to Table 1, which outlines the duration of calculation of matrices for each step

**Figure 7** The far-field scattering pattern calculated for the gold single nanowire with $a/\lambda_0 = 0.1$

**Figure 8** The scattering pattern of the linear array for different number of elements for $a/\lambda_0 = 0.1$ and $d/\lambda_0 = 0.3$ obtained from the SIGO method for 5, 15 and 25 elements, compared against the HFSS result for the same structure with 15 elements. SIGO, surface impedance generating operator
TABLE 1  The computation time recorded at different stages of the SIGO method for a linear array with \( N_{\text{array}} = 15, \ a/\lambda_0 = 0.1, \ d/\lambda_0 = 0.3 \) and 360 segments on each element

|       | \( Z_{\text{sig}} \) | \( Z_{\text{inv}}^2 \) | \( Z_{\text{inv}}^4 \) | Inverse calc. | \( Z_{\text{inv}}^6 \) | \( Z_{\text{inv}}^{12} \) |
|-------|---------------------|-----------------------|----------------------|----------------|-----------------|------------------|
| Time (s) | 3.72               | 21.1                  | 23.07                | 12.76            | 4.4             | 5.45             |

Abbreviation: SIGO, surface impedance generating operator.

FIGURE 9  Scattering pattern of the linear array for different element spacings for \( a/\lambda_0 = 0.1 \) and \( N_{\text{array}} = 5 \)

FIGURE 10  The far-field scattering pattern of the linear array shown in Figure 2a: with \( N_{\text{array}} = 15, \ a/\lambda_0 = 0.1 \) and \( d/\lambda_0 = 0.3 \)
involved in the process of simulation of the array. These times are reported for the above-mentioned case of $N_{\text{Array}} = 15$. The simulations are performed on a DESKTOP machine with Core i7, 8th generation CPU, and 32G RAM. The scattered electric field at the far-field is computed at 1440 uniformly distributed points.

Designing an optimum array for target detection applications (probe head design here) requires several simulations and parametric sweeps. This process could be formidable without having a swift, accurate, and reliable tool at hand. The SIGO approach can come to rescue here as it separates the interior and exterior subproblems. This means that for the same array elements, changing $N_{\text{Array}}$ or $d$ does not change $Z_{mn}$. Therefore, only $Z_{mn}$ and $Z_{mn}^\text{pole}$ matrices need to be updated for finding the unknown coefficients $z_n$ when $d$ is swept or $N_{\text{Array}}$ is changed. This proves to be beneficial in saving simulation time by reusing previously obtained data.

Finally, the circular array shown in Figure 2b is considered. Here, in addition to $N_{\text{Array}}$ and $a$, the radius of the circular array configuration, $A$, can be changed to obtain the best form of scattering pattern for the probe head. Figure 11 illustrates the effect of change of $A$ on the scattering pattern. As expected, by increasing $A$, the dimension of the array increases and, consequently, the grating lobes increase. Similarly, small $A$ means that the wires are close to each other and have considerable mutual coupling. The scattering patterns depicted in Figures 11 and 12 are the result of the nanowire array designed using the SIGO method, obtained in only one minute per iteration. This is significantly faster than the elapsed simulation time in other commercially available tools such as COMSOL. Figure 12 shows the scattering pattern of the circular array with $N_{\text{Array}} = 31$ elements. In this array, an FBR better than 15 dB and beamwidth lower than 3° has been achieved. The far-field scattering pattern depicted in Figure 12 is a clear indication of the success of the proposed method in achieving good design in terms of narrow beamwidth, acceptable SLL and good FBR.

5 | CONCLUSION

In this work, the SIGO method was applied to the problem of large arrays of 2D circular nanowires. The required Green's function was derived. Thanks to the proposed analytical
techniques that were used to compute the elements of the SIGO matrix, very good accuracy was obtained between the analytical results and those obtained from the SIGO method. More specifically, it was illustrated that for a single circular nanowire, with only 36 segments on its circumference, the SIGO results follow the theoretical solutions quite well. Both linear and circular arrays with a large number of elements were analysed, with potential applications in optical probe head design. The effect of different structural parameters was studied. It was shown that the SIGO method could be a fast and accurate simulation tool for the design and optimisation of large arrays. Using this tool, the design of a circular array of gold nano wires with 31 elements was proposed with SLL better than 13 dB, FBR greater than 15 dB, and beamwidth lower than 3°.

CONFLICT OF INTEREST
The authors declare that there are no conflict of interest.

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