In this paper, I review to what extent we can understand the photometric properties of star clusters, and of low-mass, unresolved galaxies, in terms of population-synthesis models designed to describe ‘simple stellar populations’ (SSPs), i.e. groups of stars born at the same time, in the same volume of space and from a gas cloud of homogeneous chemical composition. The photometric properties predicted by these models do not readily match the observations of most star clusters, unless we properly take into account the expected variation in the number of stars occupying sparsely populated evolutionary stages, owing to stochastic fluctuations in the stellar initial mass function. In this case, population-synthesis models reproduce remarkably well the full ranges of observed integrated colours and absolute magnitudes of star clusters of various ages and metallicities. The disagreement between the model predictions and observations of cluster colours and magnitudes may indicate problems with or deficiencies in the modelling, and does not necessarily tell us that star clusters do not behave like SSPs. Matching the photometric properties of star clusters using SSP models is a necessary (but not sufficient) condition for clusters to be considered SSPs. Composite models, characterized by complex star-formation histories, also match the observed cluster colours.

Keywords: stellar evolution; population synthesis; spectral evolution; simple stellar populations; TP-AGB stars

1. Introduction

Simple stellar populations (SSPs), defined as groups of stars born at the same time, in the same volume of space, and from a gas cloud of homogeneous chemical composition certainly exist in Nature. A priori, we cannot say that all stellar groups, associations or star clusters are SSPs, however. Most galaxies certainly are not.

Conceptually, SSPs are appealing because they are easy to model theoretically and their temporal evolution can be followed accurately. All stars of an SSP should have the same initial metal content. At any given time, the stars composing an SSP describe an isochrone in the theoretical Hertzsprung–Russell (HR) diagram,
which can easily be transformed to an observational colour-magnitude diagram (CMD). Detailed observations of this kind are available only for resolved stellar populations, either Galactic clusters or star clusters in nearby galaxies, or the stars making up the satellites of the Milky Way. Only in these few cases can we reliably establish the simplicity of a stellar population by inspection of its CMD.

Integrated properties of stellar populations, such as their colours or spectral-energy distributions, are subject to degeneracies (e.g. old, metal-poor populations resemble younger, metal-richer ones) and statistical (stochastic) uncertainties (owing to the small number of stars present in low-mass systems) that, in most cases, prevent us from establishing with certainty if we are observing an SSP.

Of course, the question as to whether or not star clusters can be described by SSPs does not apply to those clusters that have been shown explicitly to host stellar populations of a composite nature, e.g. resolved clusters where a double or triple main sequence (MS) has been detected, such as NGC 2808 (Piotto et al. 2007; see also Kalirai & Richer 2010; van Loon 2010), or clusters showing evidence of prolonged star formation, such as ω Centauri (Villanova et al. 2007), nor to dwarf galaxies, such as Leo A (Cole et al. 2007), or other galaxies in the Local Group with well-established multiple episodes of star formation (Gallart et al. 2007).

2. Modelling simple stellar populations

Evolutionary population-synthesis models provide the most commonly used tool to study SSPs (e.g. Vazdekis 1999; Bruzual & Charlot 2003; Le Borgne et al. 2004; Maraston 2005). The basic ingredient of all these models is a complete set of stellar evolutionary tracks, which provide the evolution in the HR diagram of stars of different mass and metal content (e.g. Bertelli et al. 2008). From a set of tracks, we build isochrones in the theoretical HR diagram for any desired age using isochrone synthesis (Chiosi et al. 1988; Charlot & Bruzual 1991). The number of stars at each position along the isochrone is obtained from the assumed stellar initial mass function (IMF), which tells us how many stars are born with a given mass at time $t = 0$. To transform the isochrone from the HR diagram (i.e. effective temperature versus absolute luminosity) to observable CMDs, we must use an atlas of stellar spectra that are calibrated in terms of flux, effective temperature and metal content. This calibration is not a problem for libraries of theoretical model atmospheres (Westera et al. 2002; Lanz & Hubeny 2003, 2007; Martins et al. 2005; Rodríguez-Merino et al. 2005; Coelho et al. 2007), but represents a real challenge for empirical libraries (Le Borgne et al. 2003; Valdes et al. 2004; Heap et al. 2006; Sánchez-Blázquez et al. 2006).

In what follows, I will discuss some basic properties of SSPs derived from the Bruzual & Charlot (2003: BC03) and Charlot & Bruzual (in preparation; CB10) models. The CB10 models are based on the same principles as the older BC03 models, but include several important improvements. CB10 models use the tracks up to stellar masses of 15$M_\odot$ from the Bertelli et al. (2008) models with updated input physics. For stars in the range from 20 to 120$M_\odot$, CB10 models use the so-called Padova 1994 tracks (Alongi et al. 1993;
Bressan et al. 1993; Fagotto et al. 1994a,b; Girardi et al. 1996). In the CB10 models, the thermally pulsing asymptotic-giant-branch (TP-AGB) evolution of low- and intermediate-mass stars follows Marigo & Girardi (2007). Their semi-empirical prescription includes several important theoretical improvements over previous calculations, and was calibrated using carbon-star luminosity functions in the Magellanic Clouds and TP-AGB lifetimes (star counts) in Magellanic Cloud clusters. While the tracks used in CB10 cover 15 evolutionary stages in the TP-AGB (six in each of the oxygen-rich and carbon-rich phases and three in the superwind phase), the BC03 models include only one evolutionary stage for each of these phases.

Note that Bertelli et al. (2008) use a different set of TP-AGB tracks, also based on the Marigo & Girardi (2007) prescription, but extrapolated to different chemical compositions of the stellar envelope. Unfortunately, these sets of TP-AGB tracks are uncalibrated, as pointed out by the authors themselves, as no attempt was made to reproduce the available observations. CB10 models will discuss the differences introduced in the evolutionary models by using either the calibrated or uncalibrated TP-AGB tracks provided by these authors.

The resulting CB10 isochrones, constructed using the calibrated Marigo & Girardi (2007) TP-AGB prescription, are thus based on internally consistent sets of tracks that naturally obey the fuel-consumption theorem (Renzini & Buzzoni 1986) and provide other quantities necessary to consistently model galaxies, such as chemical yields and stellar remnant masses (Marigo et al. 2008). The CB10 models use the Miles atlas (Sánchez-Blázquez et al. 2006), combined with the theoretical libraries of Lanz & Hubeny (2003, 2007), Martins et al. (2005) and Rodríguez-Merino et al. (2005), to build integrated SSP spectral-energy distribution as a function of time.

From any isochrone synthesis code (e.g. BC03 or CB10), we obtain the spectral-energy distribution at time $t$ of a stellar population characterized by a star-formation rate $\psi(t)$ and a metal-enrichment law $\zeta(t)$,

$$\mathcal{F}_\lambda(t) = \int_0^t \psi(t - t') S_\lambda[t', \zeta(t - t')] \, dt',$$

where $S_\lambda[t', \zeta(t - t')]$ is the power radiated per unit wavelength per unit initial mass by an SSP of age $t'$ and metallicity $\zeta(t - t')$. This expression assumes that the IMF is time independent. For SSPs, the star-formation rate $\psi(t)$ in equation (2.1) reduces to an instantaneous burst described by a Dirac delta function, $\delta(t - t')$. Thus, from the point of view of population synthesis, the question as to whether or not star clusters can be considered as SSPs is equivalent to determining if their spectral and photometric properties behave as predicted by equation (2.1) for instantaneous bursts.

(a) Colour evolution of simple stellar populations

The temporal evolution of an SSP’s colours is one of the most straightforward predictions of population-synthesis models. Magnitudes and colours can be computed directly from the spectral-energy distribution, $\mathcal{F}_\lambda(t)$, in equation (2.1) using well-known synthetic photometry algorithms. When photometric (observed) properties of star clusters are compared with the predictions of population-synthesis models, one commonly finds that the scatter...
Figure 1. (a) $(U - B)$ versus $(B - V)$ and (b) $(V - K)$ versus $(B - V)$ colour diagrams. The solid dots represent Large Magellanic Cloud globular clusters. $(B - V)$ comes from van den Bergh (1981) and $(V - K)$ from Persson et al. (1983). The points with error bars correspond to the star clusters in NGC 7252 from Miller et al. (1997) and Maraston et al. (2001). The lines represent the evolution in this plane of CB10 SSP models for $Z = 0.2 Z_{\odot}$ (dot-short-dashed line), $0.4 Z_{\odot}$ (long-dashed line) and $1.0 Z_{\odot}$ (solid line), assuming the Salpeter (1955) IMF and the Westera et al. (2002) stellar atlas. The asterisk symbols along the $Z = Z_{\odot}$ line mark the model colours at the ages indicated by the labels, and can be used to roughly date the clusters. (Adapted from Bruzual 2002.)

among the observed colours and the difference between these colours and the model predictions are larger than allowed by the observational errors. This is particularly true for intermediate-age clusters in optical-infrared colours, but, to a lesser extent, the scatter is also large for bluer colours. Figures 1 and 2 illustrate this point.

Figure 1 shows the integrated, reddening-corrected $(U - B)$, $(B - V)$ and $(V - K)$ colours of Large Magellanic Cloud (LMC) clusters of different ages and the colour of young star clusters in the merger remnant galaxy NGC 7252. The predictions of the CB10 models for various metallicities are also shown in figure 1. The scatter in cluster colours in figure 1 is intrinsic (typical observational errors are indicated in each panel). The scatter is largest in $(V - K)$ colour, but is also present, although to a lesser extent, in $(U - B)$ and $(B - V)$. This scatter
cannot be accounted for by metallicity variations. The age-metallicity degeneracy implies that the evolution of SSPs with various metallicities is similar to that of the $Z = 0.008$ model in these colour–colour diagrams. Varying the IMF has almost no effect on the model predictions in this colour–colour plane.

To show the dependence of the population-synthesis predictions on model ingredients, in figure 2, I compare the temporal evolution of the $(V-K)$ and $(J-K)$ colours for SSP models of metallicity $Z = 0.008$ given by BC03, Maraston (2005), Marigo et al. (2008), CB10 and González-Lópezlira et al. (in press). The main difference between CB10 and BC03 models is found in the treatment of

\[ \text{Phil. Trans. R. Soc. A} (2010) \]
the TP-AGB phase of stellar evolution. While the CB10 models use the modern, Marigo & Girardi (2007) prescription for TP-AGB evolution, the BC03 models use a semi-empirical treatment of TP-AGB evolution based on an older and poorer empirical calibration of the lifetime of these stars, and an educated guess of the mass associated with TP-AGB stars of a given luminosity. The models of CB10 and Marigo et al. (2008) use the same evolutionary tracks, but differ in the stellar spectra used to compute colours. The models of González-Lópezlira et al. (in press) are identical to those of CB10, but include in a self-consistent fashion the effects of different amounts of mass loss and emission by a dusty envelope in the atmospheres of TP-AGB stars. The Maraston (2005) model uses different ingredients compared with the other models shown in figure 2.

Figure 3 shows the \((V - K)\) and \((J - K)\) colour evolution for CB10 SSP models of various metallicities. Figures 1–3 show that, even though the model colours fall within the range of the observed colours, there are some clusters whose
colours are difficult to understand in terms of SSP models. The bluer colours of young clusters can be explained by statistical fluctuations in the number of blue supergiants contained in these clusters. However, clusters with ages in the range from 0.1 to 1 Gyr are all bluer than standard SSP models predict.

The model colours shown in figures 2 and 3 are sensitive to the contribution of TP-AGB stars to the integrated SSP spectrum. Bruzual (2007) showed that TP-AGB stars dominate the $K$-band luminosity in SSPs at an age of approximately 1 Gyr, and that models computed following the prescription by Marigo & Girardi (2007) have brighter $K$-band magnitudes and redder near-infrared colours than other models. Figures 2 and 3 suggest that either the contribution of TP-AGB stars predicted by Marigo & Girardi (2007) is too large, producing model colours that are too red, or the spectra assigned to these stars in the models are not realistic enough. This can be easily understood by examining the contribution of various stellar evolutionary phases to the total light of the galaxy in SSP models. Figure 4 shows the fraction of light contributed by stars at different evolutionary phases as a function of time in the $V$ and $K$ bands for the $Z = 0.008$ CB10 model. In the $V$ band, the TP-AGB stars never contribute more than a few per cent, but in the $K$ band, their contribution reaches close to 60–70% in the CB10 model, about a factor of two higher than the 40 per cent predicted by BC03.

Population-synthesis models are frequently used to estimate properties of stellar populations, such as the age and mass of the stars dominating the emitted light. The preceding discussion shows that mass estimates of star clusters and galaxies with dominant stellar populations in the age range around 1 Gyr depend critically on the treatment of TP-AGB stars in population-synthesis models. Masses derived from the CB10 models are about half the masses implied by the BC03 models.

An alternative interpretation of the colours of the clusters in figures 2 and 3 is that they are not genuine SSPs, i.e. we cannot use equation (2.1) to find $\psi(t)$, such that the model colours match the observations at the appropriate age and metallicity. At this point, it is difficult to decide in favour or against either one of these possibilities. It is likely that, as more and better data pertaining to the frequency of TP-AGB stars in resolved stellar populations become available, treatment of these stars in stellar evolution theory and population-synthesis models will improve. Thus, the current lack of agreement between model predictions and observations of cluster colours and magnitudes may be pointing us towards problems in the modelling, and do not necessarily imply that star clusters do not behave like SSPs.

### 3. Stochastic modelling of simple stellar populations

Population-synthesis models must reproduce the integrated colours of star clusters of various ages and metallicities, which are sensitive to the numbers of stars populating different phases along the isochrones. The standard population-synthesis models discussed so far assume that, at any given age, a large number of stars populate the isochrone. The stellar IMF is treated as an analytical expression, and we assume that, for any possible value of the stellar mass, the number of stars required by the IMF is always present in the stellar population.
Figure 4. Fraction of light as a function of time contributed by stars on the main sequence (MS; thin solid line), subgiant branch (SGB; dotted line), red-giant branch (RGB; short-dashed line), core-helium-burning phase and horizontal branch (CHeB/HB; long-dashed line), AGB (dot-dashed line), and TP-AGB (thick short-dashed/long-dashed line) in the (a) $V$ and (b) $K$ bands for the $Z = 0.008$ CB10 SSP model. The contribution of the TP-AGB in the $Z = 0.008$ BC03 model is also shown (thin short-dashed/long-dashed line). The TP-AGB stars in the CB10 models contribute close to a factor of two more light in the $K$ band than in the BC03 models. At maximum, the TP-AGB contributes close to 70% of the $K$-band light in the CB10 model, but only 40% in that of BC03. TP-AGB stars dominate an SSP’s $K$-band luminosity for ages of approximately 1 Gyr.

(c) The MS turnoff mass as a function of age for the $Z = 0.008$ CB10 SSP model.

Thus, the properties of standard models correspond to their limiting values for an infinite number of stars populating the stellar IMF. This is a good approximation for large stellar ensembles, such as massive galaxies but, depending on mass, it may not be appropriate to describe less numerous populations, like star clusters or dwarf galaxies.

(a) Algorithm

To describe low-mass populations, we must construct models that allow discrete sampling of the stellar IMF. The following algorithm, based on Santos & Frogel (1997), permits construction of such models while retaining the basic

Phil. Trans. R. Soc. A (2010)
properties of stellar populations (i.e. IMF shape, upper and lower stellar mass limits, stellar metallicity, etc.). See also Barbaro & Bertelli (1977), Chiosi et al. (1988), Girardi et al. (1995), Cerviño et al. (2001, 2002) and Bruzual (2002). The reader not interested in this algorithm may skip this subsection.

The IMF,

$$\Phi(m) = \frac{dN}{dm} = C m^{-(1+x)}, \tag{3.1}$$

normalized as usual,

$$C = \frac{x}{m_l^{-x} - m_u^{-x}}, \tag{3.2}$$

obeys $$\Phi(m) \geq 0$$ and

$$\int_{m_l}^{m_u} \Phi(m') \, dm' = 1. \tag{3.3}$$

$$\Phi(m)$$ can be interpreted as a probability distribution function which gives the probability that a random mass, $$m'$$, is in the range between $$m$$ and $$m + dm$$. $$\Phi(m)$$ can be transformed into another probability distribution function, $$g(N)$$, such that the probabilities that the random variable $$N'$$ occurs within $$dN$$ and that $$m'$$ occurs within $$dm$$ are the same,

$$|\Phi(m) \, dm| = |g(N) \, dN|, \tag{3.4}$$

where $$N$$ is a single-valued function of $$m$$. From equation (3.1),

$$N(m) = \int_{m_l}^{m} \Phi(m') \, dm', \tag{3.5}$$

is a cumulative distribution function which gives the probability that the mass $$m'$$ is lower than or equal to $$m$$ and, using equation (3.4), it follows that

$$g(N) = 1 \quad (0 \leq N \leq 1). \tag{3.6}$$

g($$N$$) is thus a uniform distribution for which any value is equally likely in the interval $$(0 \leq N \leq 1)$$. If we sample $$N$$ using a random-number generator (e.g. Press et al. 1992), from equation (3.5), we can obtain $$m$$ as a function of $$N$$,

$$m = [(1 - N)m_l^{-x} + Nm_u^{-x}]^{-(1/x)}. \tag{3.7}$$

For each star of mass $$m$$ thus generated, we obtain its observational properties from the values of log $$T_{\text{eff}}$$ (effective temperature) and log $$L$$ (luminosity) corresponding to this mass on the isochrone at the age of interest. We repeat the procedure until the cluster mass, i.e. the sum of $$m$$ for all stars generated (including dead stars), reaches the desired value. Adding the contribution of each star to the flux in different photometric bands, we obtain the cluster magnitudes and colours in these bands.

(b) Results

The scatter observed in the integrated colours of star clusters (figure 1) is most probably caused by stochastic fluctuations in the numbers of stars populating different evolutionary stages. This is illustrated by generating random realizations of integrated cluster colours using the Monte Carlo technique outlined in §3a.
Figure 5. (a) Integrated $(U - B)$ versus $(B - V)$ and (b) $(V - K)$ versus $(B - V)$ star cluster colours. The different symbols represent LMC globular clusters in various age ranges according to the SWB classification scheme (classes I–III: filled circles; class IV: squares; class V: triangles; classes VI–VII: open circles). For data credit, see the caption of figure 2. The solid line shows the evolution of the standard BC03 SSP model for $Z = 0.008$ and ages from a few million years to 13 Gyr. The small dots show the results of 22 000 stochastic realizations of the integrated colours of clusters of mass $2 \times 10^4 M_{\odot}$, ages between $10^5$ yr and 13 Gyr, and for the same metallicity and IMF as for this SSP model. The thick dashed line shows the colours of the BC03 $Z = Z_{\odot}$ model and ages from 100 Myr to 1 Gyr. (c) Absolute $V$-band magnitude, $M_V$, versus $(V - K)$ colour. The data used are the same as in (a,b). Three models show the evolution of, from bottom to top, and $2 \times 10^4 M_{\odot}$ SSP of metallicity $Z = 0.008$, and $3 \times 10^6$ and $6 \times 10^6 M_{\odot}$ SSPs of solar metallicity. Small circles indicate the positions of the models at ages of 6, 7, 10, 100, 400 and 500 Myr and 1, 1.4, 2 and 10 Gyr (these marks can be used to roughly date the clusters). Stochastic realizations of integrated colours are shown only for the two less massive models, as the predicted scatter becomes smaller for increasing masses. Typical observational error bars are indicated at the bottom of each panel. (Adapted from Bruzual & Charlot 2003.)

Figure 5a,b shows the integrated, reddening-corrected $(U - B)$, $(B - V)$ and $(V - K)$ colours of LMC clusters, as well as the colours of young star clusters in the merger remnant galaxy NGC 7252 (using the same data points as in figure 1). The small dots in figure 5a,b show the results of 22 000 Monte Carlo realizations.
for clusters of mass $2 \times 10^4 M_\odot$ and metallicity $Z = 0.008$, and ages between $10^5$ yr and $13$ Gyr. The solid line shows the evolution of the BC03 standard SSP model for $Z = 0.008$ and ages ranging from a few million years at the blue end to 13 Gyr at the red end. For reference, the thick dashed lines in figure 5a,b show the colours of the standard BC03 SSP model for solar metallicity and ages from 100 Myr to 1 Gyr. It is clear that the models can account for the full ranges of observed integrated cluster colours, including the scatter of nearly 2 mag in $(V-K)$. The reason for this is that the $(V-K)$ colour is highly sensitive to the small number of bright stars populating the upper giant branch. Fluctuations are smaller in $(U-B)$ and $(B-V)$, which are dominated by the more numerous MS stars.

Figure 5c shows the absolute $V$-band magnitude as a function of $(V-K)$ colour for the same clusters as in figure 5a,b. The three models shown correspond to the evolution of, from bottom to top, a $2 \times 10^4 M_\odot$ SSP with metallicity $Z = 0.4 Z_\odot$, and $3 \times 10^6$ and $6 \times 10^6 M_\odot$ SSPs of solar metallicity. From figure 5c, it is apparent that the fluctuations in the colours become larger as the cluster mass decreases. Stochastic realizations of integrated colours are shown only for the two least massive models. The predicted scatter is smaller, in all colours, for clusters more massive than $2 \times 10^4 M_\odot$, as the number of stars in any evolutionary stage is larger. The number of stars in the simulated low-mass clusters may be unrealistically small, while some evolutionary stages are not sampled. There are not enough high-mass MS stars in the low-mass clusters to make them as blue in $(U-B)$ at early ages as the model based on the analytical IMF. For the higher mass cluster, the upper MS is well sampled and both models are equally blue in $(U-B)$. At intermediate ages, the models are redder in $(V-K)$ than the analytical IMF model because of a larger number of AGB stars, which appear naturally as a consequence of stochastic fluctuations in the IMF. As in figure 5a,b, random realizations at various ages of $2 \times 10^4 M_\odot$ clusters and a metallicity of $Z = 0.4 Z_\odot$ can account for the full observed range of LMC cluster properties in this diagram. The NGC 7252 cluster colours are consistent with them being very young (100–800 Myr) and massive ($10^6 - 10^7 M_\odot$) at solar metallicity, in agreement with the results of Schweizer & Seitzer (1998). See Bruzual (2002) for more details.

Marigo et al. (2008, fig. 9) performed similar Monte Carlo simulations to explore the scatter in the cluster colours shown in figures 2 and 3. We have repeated these simulations in more detail. Figure 6a,c shows the resulting colours for simulated clusters of varying metal content as a function of age based on the CB10 models. Overall, we find good agreement between the scatter predicted for the colours of the simulated clusters and that observed in real clusters. At ages younger than $10^8$ yr, a number of very blue clusters are predicted, but none are present in the observed sample. As pointed out by Marigo et al. (2008), the blue clusters are essentially those for which no red supergiants were predicted, which can be understood naturally from the fact that the red-supergiant phase is short-lived. They estimate that to recover the observed colours of the youngest clusters, the red-supergiant phase has to be significantly longer and cooler than implemented in the Bertelli et al. (1994) isochrones.

In the age range from $10^8$ to a few $\times 10^9$ yr, dominated by stars in the TP-AGB phase (see figure 4), the analytical IMF models are, on average, redder than the data. This is also evident in fig. 9 of Marigo et al. (2008). As these
Figure 6. Comparison of the \((V - K)\) and \((J - K)\) colour evolution for CB10 SSP models of various metallicities. For data credit, see the caption of figure 2. (a,c) The lines show the predicted colours for models computed on the basis of the analytical IMF for \(Z = 0.004\) (dot/long-dashed line), \(Z = 0.008\) (solid line), \(Z = 0.017\) (solar metallicity; short-dashed line), \(Z = 0.040\) (long-dashed line) and \(Z = 0.070\) (dot/short-dashed line). The small dots show the results of stochastic realizations of the integrated colours of clusters of mass \(1 \times 10^4\, M_\odot\) (light-grey dots; 6600 realizations for each metallicity) and \(1 \times 10^5\, M_\odot\) (dark-grey dots; 4400 realizations for each metallicity), for ages between \(10^5\) yr and 20 Gyr and adopting the same metallicity and IMF as for the standard SSP models. (b,d) Bluing effect for the colours of the \(Z = 0.008\) model (solid line) produced by a second burst of star formation amounting to 30\% of the initial cluster mass, occurring at different ages in the cluster’s history (thin vertical lines). The dashed lines show the colours of models undergoing continuous star formation. The star-formation rate decays exponentially with \(e\)-folding times \(\tau = 1, 2, 3, 4, 5\) Gyr and \(\infty\).

authors indicated, removing this feature from the analytical IMF models poses a theoretical difficulty, as it would also prevent the significant enrichment of helium and nitrogen in the most massive AGB stars, which is required to explain the observed abundances in type I planetary nebulae.

*Phil. Trans. R. Soc. A* (2010)
Conroy et al. (2009, in press) explored in detail the impact of key phases of stellar evolution, IMF variations and sampling, and other parameters on the derived properties of stellar populations using population-synthesis models. The reader is referred to these papers for enlightening discussions. Conroy et al. (2009) show that artificially increasing the effective temperature of TP-AGB stars by \( \Delta \log(T_{\text{eff}}/K) = 0.2 \) or decreasing their bolometric luminosity by \( \Delta \log(L_{\text{bol}}/L_\odot) = 0.4 \) makes the predicted \((V - K)\) colour bluer by 1 and 0.5 mag, respectively.

The bluer colours observed in the data compared with the analytical IMF model predictions in figures 2, 3 and 6 can also be understood if we assume that clusters are not SSPs, and instead allow for the presence of composite stellar populations in these clusters. Figure 6b,d shows the bluing in the colours produced by a second burst of star formation amounting to 30 per cent of the initial cluster mass occurring at different ages throughout the cluster’s history. As indicated by the thin vertical lines, the colours become bluer than in the SSP model and remain blue for a short period of time. Successive bursts of star formation will maintain the bluer colours for a prolonged period of time. The dashed lines in figure 6b,d show the colours of models undergoing continuous star formation. The star-formation rate is assumed to decay exponentially in time, as indicated in the figure caption. In this case, the models do not become as blue as in the multiple-burst scenario.

(c) Discussion

Stochastic fluctuations in the number of stars populating the IMF thus provide a natural explanation for the LMC cluster colours in the age range from \(10^8\) to a few \(\times 10^9\) yr. In our simulations, the colours of the reddest younger clusters require supersolar metallicities, which may not be realistic for the LMC. At older ages, there is no problem reproducing the colours of the oldest LMC clusters, which have ages and metallicities typical of Galactic old globular clusters.

In a series of papers, Cerviño et al. (2000, 2001, 2002) examined the effects of the stochastic nature of the IMF in star clusters from a different perspective. They parameterized the distributions of the observed parameters in clusters of the same mass and age in terms of confidence limits in population-synthesis models. These confidence limits can be understood as the inherent uncertainties in the synthesis models owing to the fact that models use continuous functions that do not reproduce exactly the discontinuous nature of star formation, especially in systems containing small numbers of stars. This approach provides the theoretical basis for our Monte Carlo simulations, and reaches similar conclusions to ours.

The amplitude of the stochastic fluctuations depends critically on the number of stars present in the stellar population, i.e. on the cluster mass. For a given cluster mass, the fluctuations are smaller in colours determined by stars in evolutionary phases populated by large numbers of stars, e.g. the \((U - B)\) and \((B - V)\) colours in figure 5, which are dominated by MS stars. The fluctuations increase in those colours dominated by bright and short-lived stellar phases, e.g. the near-infrared colours dominated by TP-AGB stars.

The predictions of the standard population models should reproduce the behaviour of stellar populations with mass \(\geq 1 \times 10^6 M_\odot\). Stochastic fluctuations certainly dominate the predicted colours and magnitudes for clusters of mass \(\leq 1 \times 10^4 M_\odot\).
4. Conclusions

The magnitudes and colours predicted by stellar population-synthesis models do not readily match observations of unresolved star clusters, which are commonly expected to behave like ideal SSPs. This lack of agreement between the model predictions and observations may indicate problems or deficiencies in the modelling, and does not necessarily tell us that star clusters do not behave like SSPs.

In this review, I have briefly summarized the results of simple simulations that show how the range of colours observed in intermediate-age LMC star clusters can be understood on the basis of current stellar evolution theory, if we properly take into account the expected variation in the number of stars occupying sparsely populated evolutionary stages, owing to stochastic fluctuations in the IMF. In this case, population-synthesis models reproduce remarkably well the full ranges of observed integrated colours and absolute magnitudes of star clusters of various ages and metallicities. Some young clusters are described by supersolar-metallicity models, which may not be realistic for the LMC.

There is no need to introduce ad hoc assumptions into population-synthesis models (Maraston 1998; Maraston et al. 2001), representing a departure from our current understanding of stellar evolution theory, to explain the observed range of cluster colours and magnitudes. The predicted fluctuations in the integrated photometric properties of simulated clusters increase with decreasing cluster mass, as expected on the basis of the results of Cerviño et al. (2000, 2001, 2002).

It is worth pointing out that, because of the stochastic nature of the integrated-light properties of star clusters, single clusters may not be taken as reference standards of SSPs of a given age and metallicity. It should be emphasized that matching the photometric properties of star clusters using SSP models is a necessary condition for clusters to be considered SSPs, but not sufficient. So far, the single MS and unique MS turnoff required by SSPs can be established with certainty only for resolved stellar populations.

References

Alongi, M., Bertelli, G., Bressan, A., Chiosi, C., Fagotto, F., Greggio, L. & Nasi, E. 1993 Evolutionary sequences of stellar models with semiconvection and convective overshoot. I. \( Z = 0.008 \). Astron. Astrophys. Suppl. Ser. 97, 851–871.

Barbaro, G. & Bertelli, G. 1977 Integrated color synthesis of population I clusters. Astron. Astrophys. 54, 243–254.

Bertelli, G., Bressan, A., Chiosi, C., Fagotto, F. & Nasi, E. 1994 Theoretical isochrones from models with new radiative opacities. Astron. Astrophys. Suppl. Ser. 106, 275–302.

Bertelli, G., Girardi, L., Marigo, P. & Nasi, E. 2008 Scaled solar tracks and isochrones in a large region of the \( Z \)-\( Y \) plane. I. From the ZAMS to the TP-AGB end for 0.15–2.5 \( M_\odot \) stars. Astron. Astrophys. 484, 815–830. (doi:10.1051/0004-6361:20079165)

Bressan, A., Fagotto, F., Bertelli, G. & Chiosi, C. 1993 Evolutionary sequences of stellar models with new radiative opacities. II. \( Z = 0.02 \). Astron. Astrophys. Suppl. Ser. 100, 647–664.

Bruzual, A. G. 2002 Stellar populations in star clusters: the role played by stochastic effects. In Extragalactic star clusters, Proc. Int. Astron. Union Symp., vol. 207 (eds D. Geisler, E. K. Grebel & D. Minniti), pp. 616–624. San Francisco, CA: Astronomical Society of the Pacific.

Bruzual, G. 2007 On TP-AGB stars and the mass of galaxies. In Stellar populations as building blocks of galaxies, Proc. Int. Astron. Union Symp., vol. 241 (eds A. Vazdekis & R. F. Peletier), pp. 125–132. Cambridge, UK: Cambridge University Press.

Phil. Trans. R. Soc. A (2010)
Bruzual, G. & Charlot, S. 2003 Stellar population synthesis at the resolution of 2003. Mon. Not. R. Astron. Soc. 344, 1000–1028. (doi:10.1046/j.1365-8711.2003.06897.x)

Cerviño, M., Luridiana, V. & Castander, F. J. 2000 Confidence levels of evolutionary synthesis models. Astron. Astrophys. 360, L5–L8.

Cerviño, M., Gómez-Flechoso, M. A., Castander, F. J., Scherer, D., Mollá, M., Knödlseder, J. & Luridiana, V. 2001 Confidence limits of evolutionary synthesis models. III. On time-integrated quantities. Astron. Astrophys. 376, 422–433. (doi:10.1051/0004-6361:20011027)

Cerviño, M., Valls-Gabaud, D., Luridiana, V. & Mas-Hesse, J. M. 2002 Confidence levels of evolutionary synthesis models. II. On sampling and Poissonian fluctuations. Astron. Astrophys. 381, 51–64. (doi:10.1051/0004-6361:20011266)

Charlot, S. & Bruzual, G. 1991 Stellar population synthesis revisited. Astrophys. J. 367, 126–140. (doi:10.1086/169608)

Charlot, S. & Bruzual, G. In preparation. Population synthesis models: new tracks and stellar libraries.

Chiosi, C., Bertelli, G. & Bressan, A. 1988 Integrated colors and ages of LMC clusters. The nature of the bimodal distribution of the \((B-V)\) colors. Astron. Astrophys. 196, 84–108.

Coelho, P., Bruzual, A. G., Charlot, S., Weiss, A., Barbuy, B. & Ferguson, J. W. 2007 Spectral models for solar-scaled and \(\alpha\)-enhanced stellar populations. Mon. Not. R. Astron. Soc. 382, 498–514. (doi:10.1111/j.1365-2966.2007.12364.x)

Conroy, C., Gunn, J. E. & White, M. 2009 The propagation of uncertainties in stellar population synthesis modeling. I. The relevance of uncertain aspects of stellar evolution and the initial mass function to the derived physical properties of galaxies. Astrophys. J. 699, 486–506. (doi:10.1088/0004-637X/699/1/486)

Gallart, C. et al. 2007 Studying galaxy formation and evolution from Local Group galaxies. In First light science with the GTC, vol. 29 (eds R. Guzmán, C. Packham, J. M. Rodríguez-Espinosa & S. Torres-Peimbert), Rev. Mex. Astron. Astrophys. (Ser. Conf.), p. 158.

Gould, P., Goudfrooij, P., Gilmore, D., Kissler-Patig, M. & Maraston, C. 2006 Integrated-light VRI imaging photometry of globular clusters in the Magellanic Clouds. Mon. Not. R. Astron. Soc. 369, 697–704. (doi:10.1111/j.1365-2966.2006.10314.x)

Heap, S., Lanz, T. & Hubeny, I. 2006 Fundamental properties of O-type stars. Astrophys. J. 638, 409–432. (doi:10.1086/498635)

Kalirai, J. S. & Richer, H. B. 2010 Star clusters as laboratories for stellar and dynamical evolution. Phil. Trans. R. Soc. A 368, 755–782. (doi:10.1098/rsta.2009.0257)

Kyeong, J.-M., Tseng, M.-J. & Byun, Y.-I. 2003 Near-infrared color evolution of LMC clusters. Astron. Astrophys. 409, 479–484. (doi:10.1051/0004-6361:20031136)
Lanz, T. & Hubeny, I. 2003 A grid of non-LTE line-blanketed model atmospheres of O-type stars. Astrophys. J. Suppl. Ser. 146, 417–441. (doi:10.1086/374373)

Lanz, T. & Hubeny, I. 2007 A grid of NLTE line-blanketed model atmospheres of early B-type stars. Astrophys. J. Suppl. Ser. 169, 83–104. (doi:10.1086/511270)

Le Borgne, J.-F., Bruzual, G., Pelló, R., Lançon, A., Rocca-Volmerange, B., Sanahuja, B., Schaeerer, D., Soubiran, C. & Vílchez-Gómez, R. 2003 STELIB: a library of stellar spectra at $R \sim 2000$. Astron. Astrophys. 402, 433–442. (doi:10.1051/0004-6361:20030243)

Le Borgne, D., Rocca-Volmerange, B., Prugniel, P., Lançon, A., Fioc, M. & Soubiran, C. 2004 Evolutionary synthesis of galaxies at high spectral resolution with the code PEGASE–HR. Metallicity and age tracers. Astron. Astrophys. 425, 881–897. (doi:10.1051/0004-6361:200400044)

Maraston, C. 1998 Evolutionary synthesis of stellar populations: a modular tool. Mon. Not. R. Astron. Soc. 300, 872–892. (doi:10.1046/j.1365-8711.1998.01947.x)

Maraston, C. 2005 Evolution of asymptotic giant branch stars. I. Updated synthetic TP-AGB models and their basic calibration. Astron. Astrophys. 469, 239–263. (doi:10.1051/0004-6361:20067772)

Marigo, P., Girardi, L., Bressan, A., Groenewegen, M. A. T., Silva, L. & Granato, G. L. 2008 Evolution of asymptotic giant branch stars. II. Optical to far-infrared isochrones with improved TP-AGB models. Astron. Astrophys. 482, 883–905. (doi:10.1051/0004-6361:20078467)

Martins, L. P., González-Delgado, R. M., Leitherer, C., Cerviño, M. & Hauschildt, P. 2005 A high-resolution stellar library for evolutionary population synthesis. Mon. Not. R. Astron. Soc. 362, 49–65. (doi:10.1111/j.1365-2966.2005.08703.x)

Miller, B. W., Whitmore, B. C., Schweizer, F. & Fall, S. M. 1997 The star cluster system of the merger remnant NGC 7252. Astron. J. 114, 2381–2401. (doi:10.1086/118655)

Persson, S. E., Aaronson, M., Cohen, J. G., Frogel, J. A. & Matthews, K. 1983 Photometric studies of composite stellar systems. V. Infrared photometry of star clusters in the Magellanic clouds. Astrophys. J. 266, 105–129. (doi:10.1086/160763)

Pessev, P. M., Goudfrooij, P., Puzia, T. H. & Chandar, R. 2006 A database of 2MASS near-infrared colors of Magellanic Cloud star clusters. Astron. J. 132, 781–800. (doi:10.1086/505625)

Piotto, G., Bedin, L. R., Anderson, J., King, I. R., Cassisi, S., Milone, A. P., Villanova, S., Pietrinferni, A. & Renzini, A. 2007 A triple main sequence in the globular cluster NGC 2808. Astrophys. J. 661, L53–L56. (doi:10.1086/518503)

Press, W. H., Teukolsky, S. A., Vetterling, W. T. & Flannery, B. P. 1992 Numerical recipes in Fortran, 2nd edn. Cambridge, UK: Cambridge University Press.

Renzini, A. & Buzzoni, A. 1986 Global properties of stellar populations and the spectral evolution of galaxies. In Spectral evolution of galaxies (eds C. Chiosi & A. Renzini), pp. 195–235. Dordrecht, The Netherlands: Reidel.

Rodríguez-Merino, L. H., Chávez, M., Bertone, E. & Buzzoni, A. 2005 UVBLUE: a new high-resolution theoretical library of ultraviolet stellar spectra. Astrophys. J. 626, 411–424. (doi:10.1086/429858)

Salpeter, E. E. 1955 The luminosity function and stellar evolution. Astrophys. J. 121, 161–167. (doi:10.1086/145971)

Sánchez-Blázquez, P., Peletier, R. F., Jiménez-Vicente, J., Cardiel, N., Cenarro, A. J., Falcón-Barroso, J., Gorgas, J., Selam, S. & Vazdekis, A. 2006 Medium-resolution Isaac Newton Telescope library of empirical spectra. Mon. Not. R. Astron. Soc. 371, 703–718. (doi:10.1111/j.1365-2966.2006.10699.x)

Santos Jr, J. F. C. & Frogel J. A. 1997 Integrated near-infrared colors of star clusters: analysis of the stochastic effects on the initial mass function. Astrophys. J. 479, 764–775. (doi:10.1086/303921)
Schweizer, F. & Seitzer, P. 1998 Ages and metallicities of young globular clusters in the merger remnant NGC 7252. *Astron. J.* **116**, 2206–2219. (doi:10.1086/300616)

Searle, L., Wilkinson, A. & Bagnuolo, W. G. 1980 A classification of star clusters in the Magellanic Clouds. *Astrophys. J.* **239**, 803–814. (doi:10.1086/158165)

Valdes, F., Gupta, R., Rose, J. A., Singh, H. P. & Bell, D. J. 2004 The Indo-US library of Coudé feed stellar spectra. *Astrophys. J. Suppl. Ser.* **152**, 251–259. (doi:10.1086/386343)

van den Bergh, S. 1981 *UBV* observations of globular clusters in the Magellanic Clouds. *Astron. Astrophys. Suppl. Ser.* **46**, 79–87.

van Loon, J. T. 2010 Chemical evolution of star clusters. *Phil. Trans. R. Soc. A* **368**, 801–828. (doi:10.1098/rsta.2009.0259)

Vazdekis, A. 1999 Evolutionary stellar population synthesis at 2 Å spectral resolution. *Astrophys. J.* **513**, 224–241. (doi:10.1086/306843)

Villanova, S. *et al.* 2007 The multiplicity of the subgiant branch of Centauri: evidence for prolonged star formation. *Astrophys. J.* **663**, 296–314. (doi:10.1086/517905)

Westera, P., Lejeune, T., Buser, R., Cuisinier, F. & Bruzual, G. 2002 A standard stellar library for evolutionary synthesis. III. Metallicty calibration. *Astron. Astrophys.* **381**, 524–538. (doi:10.1051/0004-6361:20011493)