Complex configuration effects on $\beta$-decay rates

A. P. Severyukhin, V. V. Voronov, I. N. Borzov, N. N. Arsenyev, and Nguyen Van Giai

1Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

2Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, F-91406 Orsay Cedex, France

Abstract. The effects of the phonon-phonon coupling on the $\beta$-decay rates of neutron-rich nuclei are studied in a microscopic model based on Skyrme-type interactions. Making use of the finite rank separable approach for the quasiparticle random phase approximation enables one to perform nuclear structure calculations in very large configurational spaces. The inclusion of the tensor correlations and the $2p-2h$ configurations results in a redistribution of the Gamow-Teller strength and a consequent reduction of the $\beta$-decay half-life. Such an effect is discussed for some neutron-rich $N=82$ isotones below the doubly magic nucleus $^{132}$Sn. The calculations are performed by using the SGII Skyrme interactions with the tensor terms in the particle-hole channel and the surface-peaked interaction in the particle-particle channel. Predictions are given for $^{126}$Ru and $^{128}$Pd in comparison to $^{130}$Cd which is the r-process waiting-point nucleus.

The study of spin-isospin excitations in neutron-rich nuclei is presently an important problem not only from the nuclear structure point of view but also because of the special role they play in many astrophysical processes. Many fundamental issues depend on our quantitative understanding of the $\beta$-decay of atomic nuclei. It is desirable to have theoretical models which can describe the data wherever they can be measured and which can predict the properties related to spin-isospin excitations in systems too short-lived to allow for experimental studies. One of the successful tools for studying charge-exchange nuclear modes is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from a Skyrme-type energy-density functional (EDF), see e.g., [1, 2, 3, 4]. These QRPA calculations enable one to describe the properties of the ground state and excited charge-exchange states using the same EDF.

A comparison of such calculations with recent experimental data [5] demonstrates that QRPA approach cannot reproduce correctly the strength distribution of the spin-isospin resonances. It is necessary to take into account a coupling with more complex configurations that results in shifting some strength upward in excitation energy [6, 7, 8]. Using the Skyrme EDF and the RPA, such attempts in the past [9, 10] have allowed one to understand the damping of charge-exchange resonances and their particle decay. Recently, the damping of the Gamow-Teller (GT) mode was investigated using the Skyrme-RPA plus particle-vibration coupling (PVC) [11]. However, the size of the configuration space increases very rapidly and one has to work within limited number spaces.

It would be helpful to study the $2p-2h$ configuration effect on the $\beta^-$-decay rates of neutron-rich nuclei. It is somewhat simpler to include the PVC in QRPA calculations if one uses separable forces [7, 12]. Our tool is the QRPA with Skyrme interactions in the finite rank...
separable approximation (FRSA) [13, 14, 15, 16], allowing one to perform calculations in large configuration spaces. Successful applications of the method to study the electric low-lying states and giant resonances within and beyond the QRPA can be found in Refs [14, 15, 16, 17]. We use an extension of this approach proposed for the charge-exchange nuclear excitations [18] and also for accommodating the tensor correlations which mimic the Skyrme-type tensor interactions [19]. Taking into account the basic ideas of the quasiparticle-phonon model (QPM) [12], the Hamiltonian is then diagonalized in a space spanned by states composed of one and two QRPA phonons [20],

$$\Psi_{\nu}(JM) = \left( \sum_i R_i(J\nu)Q_{JiM_i}^+ + \sum_{\lambda_1i_1\lambda_2i_2} P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) \left[ Q_{\lambda_1i_1M_i}^+ \bar{Q}_{\lambda_2i_2M_i}^+ \right]_{JM} \right) |0\rangle,$$

(1)

where $$Q_{\lambda_1i_1M_i}^+|0\rangle$$ ($$\bar{Q}_{\lambda_2i_2M_i}^+|0\rangle$$) is the GT (electric) excitation having the QRPA energy $$\omega_{\lambda i}$$ ($$\bar{\omega}_{\lambda i}$$). The normalization condition for the wave functions (1) is

$$\sum_i R_i^2(J\nu) + \sum_{\lambda_1i_1\lambda_2i_2} (P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu))^2 = 1.$$  

(2)

The amplitudes $$R_i(J\nu)$$ and $$P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu)$$ are determined from the variational principle which leads to a set of linear equations

$$\left( \omega_{\lambda i} - \Omega_{\nu} \right) R_i(J\nu) + \sum_{\lambda_1i_1\lambda_2i_2} U_{\lambda_2i_2}^{\lambda_1i_1}(Ji) P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) = 0,$$

(3)

$$\left( \omega_{\lambda_1i_1} + \omega_{\lambda_2i_2} - \Omega_{\nu} \right) P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) + \sum_i U_{\lambda_2i_2}^{\lambda_1i_1}(Ji) R_i(J\nu) = 0.$$  

(4)

The rank of the set of linear equations (3) and (4) is equal to the number of one- and two-phonon configurations included in the wave function (1). Its solution requires to compute the matrix elements coupling one- and two-phonon configurations

$$U_{\lambda_2i_2}^{\lambda_1i_1}(Ji) = \langle 0|Q_{Ji}H \left[ Q_{\lambda_1i_1}^+ \bar{Q}_{\lambda_2i_2}^+ \right]_{J}|0\rangle.$$  

(5)

Equations (3) and (4) have the same form as the QPM equations [7, 12], but the single-particle spectrum and the residual interaction are derived from the same Skyrme EDF [20].

To calculate the half-lives, the same ansatz as Sec. II of Ref. [21] with the ratio of the weak axial-vector and vector coupling constants $$G_A/G_V=1.25$$ [22] is used. In the allowed GT approximation, the $$\beta^-$$-decay rate is expressed by summing the probabilities of the energetically allowed GT transitions (in units of $$G_A^2/4\pi$$) weighted with the integrated Fermi function

$$T_{1/2}^{-1} = \sum_m \lambda_{ij}^m = D^{-1} \left( \frac{G_A}{G_V} \right)^2 \sum_m f_0(Z, A, E_i - E_{i^+_m})B(GT)_m,$$

(6)

$$E_i - E_{i^+_m} \approx \Delta M_{n-H} + \mu_n - \mu_p - E_m,$$

(7)

where $$\lambda_{ij}^m$$ is the partial $$\beta^-$$-decay rate. $$D=6147$$ [22] is a constant, $$\Delta M_{n-H} = 0.782$$ MeV is the mass difference between the neutron and the hydrogen atom, $$\mu_n$$ and $$\mu_p$$ are the neutron and proton chemical potentials respectively, $$E_i$$ is the ground state energy of the parent nucleus, and $$E_{i^+_m}$$ denotes a state of the daughter nucleus $$(Z, A)$$. $$E_m$$ are the eigenvalues of the QRPA equations or Eqs. (3) and (4) taking into account the two-phonon configurations. The wave functions allow us to determine $$B(GT)_m$$. 

2
Figure 1. Energies and $B(E2)$ values for up-transitions to the $[2^{+}_1]_{QRPA}$ states in the neutron-rich $N=82$ isotones. Results of the calculations without the tensor interaction (open triangles) and with the tensor interaction (filled triangles) are shown. Experimental data (filled diamonds) are taken from Ref. [31].

There is a relation between the $N = 82$ shell closure and the $A \approx 130$ peak of the solar r-process abundance distribution [23]. The $N = 82$ isotones below $^{132}$Sn are important for stellar nucleosynthesis, see, e.g., Refs. [24, 25, 26, 27, 28]. It is interesting to study the influence of the coupling between one- and two-phonon terms in the wave functions and the tensor force effects on the strength distributions of the GT states in the neutron-rich $N = 82$ isotones, namely, $^{126}$Ru, $^{128}$Pd and $^{130}$Cd. As the parameter set in the particle-hole channel, we use the central Skyrme interaction SGII [29] and the same zero-range tensor interaction as that in the paper [30]. Since the SGII parametrization gives reasonable values for the Landau parameters $F'_0 = 0.73$ and $G'_0 = 0.50$, one obtains a successful description of the spin-dependent properties and, in particular, experimental energies of the GT resonances of $^{90}$Zr [29]. For the studied region of nuclei we use the surface-peaked pairing force fixed in Refs. [18, 19]. The single-particle continuum is discretized by diagonalizing the HF hamiltonian on a basis of 12 harmonic oscillator shells and cutting off the single-particle spectra at the energy of 100 MeV. This is sufficient to exhaust the Ikeda sum rule 3($N - Z$) [18, 19] as well as the sum rule for the electromagnetic excitation modes [16]. Having taking into account the tensor correlation effects within the $1p - 1h$ and $2p - 2h$ configurational spaces, we do not need any quenching factor [6].

To construct the wave functions (1) of the low-lying $1^+$ states we use only the $[1^+_i \otimes \lambda^+_j]_{QRPA}$ terms and all electric phonons with $\lambda > 2$ vanish. All one- and two-phonon configurations with the transition energies $|E_{1^+_i} - E_{1^+_j}|$ up to 10 MeV are included. We have checked that the inclusion of the high-energy configurations leads to minor effects on the half-life values. Since there is a clear influence of the $2^+_1$ phonon on the half-lives [20, 32], we examine the energies and transition probabilities of the $2^+_1$ QRPA states of the neutron-rich $N = 82$ isotones (see Fig.1). There is a remarkable increase of the $2^+_1$ energy of $^{132}$Sn in comparison with those in $^{134}$Te and $^{130}$Cd. It corresponds to a standard evolution of the $2^+_1$ energy near closed shells [16]. The
2$^+$ states in $^{126}$Ru, $^{128}$Pd and $^{130}$Cd have a non collective structure with a domination of the proton configuration \{1$g_{9/2}$, 1$g_{9/2}$\}. As shown in [16], the neutron transition density is shifted outwards as compared to the proton transition density due to the presence of the neutron skin. As can be seen from Fig.1, the FRSA model with the SGII+tensor interaction reproduces the experimental data [31] very well. Also, one can see that the inclusion of the tensor interaction does not change the energies and the transition probabilities.

The QRPA analysis within the one-phonon approximation can help to clarify the tensor correlation effect on the GT strength distributions. When the tensor terms are included, for the region of nuclei near $^{132}$Sn, the GT strength distributions calculated with the same parameter set are rather well studied in Refs. [19, 33]. As a general trend, there is a redistribution of the GT strengths by the inclusion of the tensor interaction. In particular, for $^{130}$Cd, the tensor correlations induces 3.6 MeV downward shift of the main peak of GT resonance and the part of the main peak strength is fragmented in the low-energy peaks and also the high-energy tail is formed [33]. Let us now focus on the properties of the low-energy spectrum of the $1^+$ states of the daughter nuclei, which influence $\beta$-decay half-lives. As expected, the largest contribution in the calculated $\beta^-$-decay half-life comes from the [1$^+_1$]$_{QRPA}$ state. To illustrate it, the $\beta$-transition rates $\lambda_{\beta}^m$ of $^{128}$Pd are shown in Fig. 2. The transition energies $|E_{1^+_m} - E_i|$ refer to the ground state of the parent nucleus. As pointed out in Ref. [25, 26], we find that the correct description of the two-quasiparticle configuration \{$1g_{3/2}\nu1g_{7/2}$\} is important for the half-life description in

\begin{figure}
\centering
\includegraphics[width=0.7\textwidth]{figure2.png}
\caption{The phonon-phonon coupling effect on the $\beta$-transition rates in $^{128}$Pd. The left and right panels correspond to the calculations within the QRPA and taking into account the $2p - 2h$ configurations, respectively. Results of the calculations without (resp. with) the tensor interaction are shown in the upper (resp. lower) panels.}
\end{figure}
all the cases considered. As can be seen from Fig. 3, the half-lives calculated with the tensor force are about 10 times shorter than those calculated without. It is worth to mention that the first discussion of the strong impact of the tensor correlations on the $\beta^-$-decay half-lives based on QRPA calculations with Skyrme forces has been done in Ref. [4].

The inclusion of $2p - 2h$ configurations results a further increase of the transition energies $|E_{1m}^+ - E_i|$ and partial rates of the main GT transitions (see, e.g., right bottom panel of Fig. 2). Fig. 3 shows that this effect produces an impact on the $\beta^-$-decay half-life which is reduced by a factor 2. The main two-phonon components of the $1^+_1$ wave function are the $[1^+_1 \otimes 2^+_1]_{QRPA}$ and $[1^+_1 \otimes 0^+_2]_{QRPA}$ configurations. The calculated value in the case of $^{130}$Cd overestimates the experimental data [34]. Moreover, for $^{132}$Sn, the $[1^+_1]_{QRPA}$ state of the daughter nucleus is above the parent ground state, i.e., this calculation gives stable $^{132}$Sn and one should seek for further improvements of the effective interaction. Finally, we predict the half-lives: 36 ms for $^{128}$Pd and 6 ms for $^{126}$Ru.

In summary, starting from the Skyrme mean-field calculations the GT strength in the $Q_{\beta^-}$-window has been studied within the extended FRSA model including both the tensor interaction effect and $2p-2h$ configurations. The suggested approach enables one to perform the calculations in very large configurational spaces. Choosing as an example the nuclei $^{126}$Ru, $^{128}$Pd and $^{130}$Cd, the $\beta^-$-decay rates are calculated. The inclusion of the tensor interaction leads to a redistribution of the GT strength. The low-energy GT strength is fragmented and the $1^+_1$ state is moved downwards in energy. Taking into account these effects results in a dramatic reduction of $\beta$-decay half-lives. At a qualitative level, we observe a improvement of the half-life description of $^{130}$Cd by the tensor force. The half-lives of $N = 82$ isotones are predicted.
Acknowledgments
We thank H. Sagawa for useful discussions. This work is partly supported by the IN2P3-JINR agreement.

References
[1] Bender M, Dobaczewski J, Engel J and Nazarewicz W 2002 Phys. Rev. C 65 054322
[2] Fracasso S and Colò G 2007 Phys. Rev. C 76 044307
[3] Bai C L, Zhang H Q, Sagawa H, Zhang X Z, Colò G and Xu F R 2011 Phys. Rev. C 83 054316
[4] Minato F and Bai C L 2013 Phys. Rev. Lett. 110 122501
[5] Ichimura M, Sakai H and Wakasa T 2006 Prog. Part. Nucl. Phys. 56 446
[6] Bertch G F and Hamamoto I 1982 Phys. Rev. C 26 1323
[7] Kuzmin V A and Soloviev V G 1984 J. Phys. G 10 1507
[8] Drożdż S, Osterfeld F, Speth J and Wambach J 1987 Phys. Lett. B 189 271
[9] Colò G, Nguyen Van Giai, Bortignon P F and Broglia R A 1994 Phys. Rev. C 50 1496
[10] Colò G, Sagawa H, Nguyen Van Giai, Bortignon P F and Suzuki T 1998 Phys. Rev. C 57 3049
[11] Niu Y F, Colò G, Brenna M, Bortignon P F and Meng J 2012 Phys. Rev. C 85 034314
[12] Soloviev V G 1992 Theory of Atomic Nuclei: Quasiparticles and Phonons (Bristol and Philadelphia: Institute of Physics)
[13] Nguyen Van Giai, Stoyanov Ch and Voronov V V 1998 Phys. Rev. C 57 1204
[14] Severyukhin A P, Stoyanov Ch, Voronov V V and Nguyen Van Giai 2002 Phys. Rev. C 66 034304
[15] Severyukhin A P, Voronov V V and Nguyen Van Giai 2004 Eur. Phys. J. A 22 397
[16] Severyukhin A P, Voronov V V and Nguyen Van Giai 2008 Phys. Rev. C 77 024322
[17] Severyukhin A P, Arsenyev N N and Pietralla N 2012 Phys. Rev. C 86 024311
[18] Severyukhin A P, Voronov V V and Nguyen Van Giai 2012 Prog. Theor. Phys. 128 489
[19] Severyukhin A P and Sagawa H 2013 Prog. Theor. Exp. Phys. 2013 103D03
[20] Severyukhin A P, Voronov V V, Borzov I N, Arsenyev N N and Nguyen Van Giai arXiv 1410.0614v1
[21] Engel J, Bender M, Dobaczewski J, Nazarewicz W and Surman R 1999 Phys. Rev. C 60 014302
[22] Suhonen J 2007 From Nucleons to Nucleus (Berlin: Springer-Verlag)
[23] Burbidge E M, Burbidge G R, Fowler W A and Hoyle F 1957 Rev. Mod. Phys. 29 547
[24] Martínez-Pinedo G and Langanke K 1999 Phys. Rev. Lett. 83 4502
[25] Borzov I N 2003 Phys. Rev. C 67 025802
[26] Dillmann I et al. (ISOLDE collaboration) 2003 Phys. Rev. Lett. 91 162503
[27] Jungclaus A et al. 2007 Phys. Rev. Lett. 99 132501
[28] Cuenca-García J J, Martínez-Pinedo G, Langanke K, Nowacki F and Borzov I N 2007 Eur. Phys. J. A 34 99
[29] Nguyen Van Giai and Sagawa H 1981 Phys. Lett. B 106 379
[30] Bai C L, Zhang H Q, Zhang X Z, Xu F R, Sagawa H and Colò G 2009 Phys. Rev. C 79 041301(R)
[31] Pritychenko B, Birch M, Singh B and Horoi M 2014 Nuclear Data Sheets 120 (2014) 112
[32] Severyukhin A P, Voronov V V, Borzov I N and Nguyen Van Giai 2013 Rom. J. Phys. 58 1048
[33] Severyukhin A P and Sagawa H 2012 Eur. Phys. J. Web of Conf. 38 06001
[34] Hannawald M et al. (ISOLDE collaboration) 2001 Nucl. Phys. A 688 578c