Nuclear Forces from Lattice QCD

T. Hatsuda
Interdisciplinary Theoretical and Mathematical Sciences Program, RIKEN

- Lattice QCD
- Single Baryon
- Two Baryons
- Hyperon interactions
- Summary and Future

INPC 2019 (July 29, 2019)
Quantum Chromodynamics (QCD)

\[ \mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q} \gamma^\mu (i\partial_\mu - gt^a A^a_\mu) q - m\bar{q}q \]

\[ G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu \]

Quark Masses

| Quark Masses (from (2+1)-flavor lattice QCD + QED) | [MeV] (MS-bar @ 2GeV) |
|-----------------------------------------------|------------------------|
| \( m_u \)                                   | 2.27(9)                |
| \( m_d \)                                   | 4.67(9)                |
| \( m_s \)                                   | 92.0(9)                |

FLAG (Flavor-Lattice Averaging Group)
Review 2019   http://flag.unibe.ch/

Gauge Coupling

\[ \alpha_s(Q^2) \]

PDG (Particle Data Group)
Review 2018   http://pdg.lbl.gov/

\( QCD \alpha_s(M_Z) = 0.1181 \pm 0.0011 \)
Primordial form of matter
quark-gluon plasma

Origin of heavy elements
in explosive astrophysical phenomena

Dense stellar matter
neutron star, exotic matter, …

Search for “new physics”
0 νββ, proton decay, dark matter, …
Lattice QCD

\[ Z = \int [dU][dq d\bar{q}] \exp \left[ -\int d\tau d^3x L_E \right] \]

Huge integration variables
\( \sim 10^{9-10} \) (for 96\(^4\) lattice)

Importance Sampling
Hybrid MC = MD + Metropolis

Continuum & Thermodynamic Limits
\((a \to 0 \& L \to \infty)\)
Single Baryon
Flavor SU(3) classification

8 (Octet)

10 (Decuplet)
Hadron masses from LQCD

Review: Fodor and Hoelbling, Rev. Mod. Phys. 84 (2012) 449

Graph showing hadron masses from LQCD, with experimental, input, and prediction data points.
Percent-level determination of $g_A$ from LQCD

C.C. Chang (RIKEN iTHEMS/LBNL)+, Nature 558 (2018) 91

$(g_A)_{\text{LQCD}} = 1.271(13)$
Percent-level determination of $g_A$ from LQCD

C.C. Chang (RIKEN iTHEMS/LBNL)+, Nature 558 (2018) 91

\[
(g_A)_{\text{LQCD}} = 1.271(13) \quad \text{and} \quad (\tau_n)_{\text{LQCD}} = 884(15) \text{ s}
\]

\[
(g_A)_{\text{expt}} = 1.272(2) \quad \text{and} \quad (\tau_n)_{\text{PDG}} = 880.2(1.0) \text{ s}
\]
“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task.”

Y. Nambu, “Quarks – Frontiers in Elementary Particle Physics”, World Scientific (1985)
Nuclear Force: phenomenology vs. LQCD

- NN int.: about 4500 np and pp scattering data

| phenomenological NN interactions       | # of parameters |
|----------------------------------------|-----------------|
| CD Bonn (p space)                      | 38              |
| AV18 (r space)                         | 40              |
| EFT in N³LO (nπ+contact)               | 24              |

- NNN, YN, YY: data limited
- YYN, YNN, YYY: data very limited

Low energy QCD has only 4 known parameters: $g, m_{u,d,s}$

R. Machleidt, arXiv:0704.0807 [nucl-th]
Two methods to extract hadron interactions from LQCD

[1] Energy levels for finite $L \to$ observables at $L=\infty$

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}} \frac{1}{\vec{n}^2 - (kL/2\pi)^2}$$

Luescher, Nucl. Phys. B354 (1991) 531

[2] Spatial correlation for finite $L$
$\to$ non-local BB interaction $\to$ observables at $L=\infty$

$$\left\{ \frac{1}{4M_B} \frac{\partial^2}{\partial \tau^2} - \frac{\partial}{\partial \tau} - H_0 \right\} \mathcal{R}(r, \tau) = \int d^3 r' U(r, r') \mathcal{R}(r', \tau)$$

Ishii, Aoki & Hatsuda, Phys. Rev. Lett. 99 (2007) 022001

Ishii+ [HAL QCD Coll.], Phys. Lett. B712 (2012) 437
Two-baryon observables
(phase shift, binding energy)

Theoretical and numerical relations between the two approaches have been eventually clarified by Iritani+ [HAL QCD Coll.], JHEP 1610 (2016) 101, PRD 96 (2017) 034521, PRD 99 (2019) 014514, JHEP 1903 (2019) 007
Hyperon Interactions
Large scale LQCD simulations for two baryons

- Small lattice spacing: \( a = 0.085 \) fm
- Large lattice volume: \( L = 8.1 \) fm
- (2+1)-flavor dynamical quarks
  \( M_n = 146 \) MeV, \( M_K = 525 \) MeV, \( M_N = 964 \) MeV
Large scale LQCD simulations for two baryons

- Small lattice spacing: $a = 0.085$ fm
- Large lattice volume: $L = 8.1$ fm
- (2+1)-flavor dynamical quarks
  - $M_n = 146$ MeV, $M_K = 525$ MeV,
  - $M_N = 964$ MeV

K computer (11 Pflops) FY2011–2019 ©RIKEN
Flavor dependence of the BB interactions

\[
8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a
\]
Flavor dependence of the BB interactions

T. Inoue+ [HAL QCD Coll.], arXiv:1809.08932 [hep-lat]

\[ 8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a \]

BB interactions at short distance:

(i) flavor dependent

(ii) consistent with constituent quark model

Park, Lee, Inoue, Hatsuda, arXiv:1907.06351 [hep-ph]
LQCD simulations for BB system

- Sasaki+ [HAL QCD coll.] in preparation
- Iritani+ [HAL QCD coll.]
  - PLB792 (2019) 284
- Gongyo+ [HAL QCD coll.]
  - PRL120 (2018) 212001

Jaffe (1977) | Goldman+ (1987) | Oka (1988) | Kopeliovich+ (1990)

- $S=0$: NN
- $S=-1$: $N\Lambda, N\Sigma$
- $S=-2$: $\Lambda\Lambda, \Lambda\Sigma, \Sigma\Sigma, N\Xi$
- $S=-3$: $\Lambda\Xi, \Sigma\Xi, N\Omega$
- $S=-4$: $\Xi\Xi$
- $S=-5$: $\Xi\Omega$
- $S=-6$: $\Omega\Omega$

Exp rich data → LQCD better S/N
LQCD simulations for $S=-2$ system

Jaffe (1977)

- Sasaki+ [HAL QCD coll.] in preparation
- Iritani+ [HAL QCD coll.] PLB792 (2019) 284
- Gongyo+ [HAL QCD coll.] PRL120 (2018) 212001

For $S=0$: NN
For $S=-1$: NA, NΣ
For $S=-2$: ΛΛ, ΛΣ, ΣΣ, ΣΞ
For $S=-3$: ΛΞ, ΣΞ, NΩ
For $S=-4$: ΞΞ
For $S=-5$: ΞΩ
For $S=-6$: ΩΩ

EXP rich data
LQCD better S/N
Coupled Channel S=-2 system

K. Sasaki+
[HAL QCD Coll.]
in preparation

\[
\begin{pmatrix}
\langle \Lambda \Lambda \rangle \\
\langle \Sigma \Sigma \rangle \\
\langle N \Xi \rangle
\end{pmatrix} =
\begin{pmatrix}
\sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\
-\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\
\sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}}
\end{pmatrix}
\begin{pmatrix}
\langle 27 \rangle \\
\langle 8_s \rangle \\
\langle 1 \rangle
\end{pmatrix}
\]

\( l=0, J^P=0^+ \)
Femtoscopic correlation analysis of $N\Xi$ at LHC

ALICE, Coll., arXiv:1904.12198 [nucl-ex]

$$C(k) = \int S(\vec{r}, k)|\psi(\vec{r}, k)|^2 d\vec{r}$$

[b) ALICE p-Pb $\sqrt{s_{NN}} = 5.02$ TeV

- $p-\Xi^- \oplus \bar{p}-\Xi^+$
- Coulomb + HAL-QCD
- Coulomb
- $p-\Xi^-$ sideband background

$k^* (\text{MeV}/c)$
Femtoscopic correlation analysis of $\Lambda\Lambda$ at LHC

ALICE Coll., arXiv:1905.07209 [nucl-ex]
Elusive H-dibaryon in physical world

K. Sasaki+
[HAL QCD Coll.]
in preparation

\[ I=0, J^P=0^+ \]

\[ m_{\Sigma\Sigma} = 2380 \text{MeV} \]

\[ m_{N\Xi} = 2260 \text{MeV} \]

\[ m_{\Lambda\Lambda} = 2230 \text{MeV} \]
LQCD simulations for S=-3, -6 systems

Goldman+ (1987)  Oka (1988)
Kopeliovich+ (1990)

Sasaki+ [HAL QCD coll.]
in preparation

Iritani+ [HAL QCD coll.]
PLB792 (2019) 284

Gongyo+ [HAL QCD coll.]
PRL120 (2018) 212001

EXP rich data

LQCD better S/N
Both are Pauli allowed
S=-3
Iritani+ [HAL QCD Coll.], Phys.Lett. B792(2019) 284

\[ N\Omega \left( ^5S_2 \right) \]

\[ V_c(r) \text{[MeV]} \]

| t/a | Color |
|-----|-------|
| 14  | Red   |
| 13  | Green |
| 12  | Blue  |
| 11  | Purple|

S=-6
Gongyo+ [HAL QCD Coll.], PRL 120 (2018) 212001

\[ \Omega\Omega \left( ^1S_0 \right) \]

\[ V(r)\text{[MeV]} \]

\[ \delta_{\text{deg}} \text{[deg]} \]

[Graphs showing potential and phase shift for different t/a values]
New dibaryons near unitarity?

HAL QCD Coll., PRL 120 (2018) 212001
PLB 792 (2019) 284

Bound state energy [MeV]

Root mean square distance [fm]

$\Omega^-\Omega^- (^{1}S_{0})$
$pn (^{3}S_{1})$
$p\Omega^- (^{5}S_{2})$

RHIC, LHC
J-PARC, FAIR
Summary and Future

2011-2019: past supercomputer(s), e.g. “K” (11 Pflops)

- Proof of concept and realistic simulations
- LQCD prediction vs. experimental data

| S=0  | S=−1 | S=−2 | S=−3 | S=−4 | S=−5 | S=−6 |
|------|------|------|------|------|------|------|
| NN   | NΛ, NΣ | ΛΛ, ΛΣ, ΣΣ, NΞ | ΛΞ, ΣΞ, NΩ | ΞΞ | ΞΩ | ΩΩ |

EXP
rich data

LQCD
better S/N

2021-: new supercomputer(s), e.g. FUGAKU (100 x “K”)

| S=0  | S=−1 | S=−2 | S=−3 | S=−4 | S=−5 | S=−6 |
|------|------|------|------|------|------|------|
| NN   | NΛ, NΣ | ΛΛ, ΛΣ, ΣΣ, NΞ | ΛΞ, ΣΞ, NΩ | ΞΞ | ΞΩ | ΩΩ |

LQCD

- Precision data for S=0, -1
- Spin-orbit force, three body force, …
Backup slides
Flavor SU(3) classification of BB systems

\[ 8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a \]

\[ \text{NN}(^1S_0) \quad H_{\Lambda\Lambda-N\Xi-L\Sigma}(^1S_0) \quad \text{NN}(^3S_1) \]

Jaffe (1977)

\[ 8 \times 10 = 35 + 8 + 10 + 27 \]

\[ \text{N}\Omega( ^5S_2) \]

Goldman+ (1987), Oka (1988)

\[ 10 \times 10 = 28 + 27 + 35 + 10^* \]

\[ \Omega\Omega( ^1S_0) \quad \Delta\Delta( ^7S_3) \]

Kopeliovich+ (1990), Dyson+ (1964)
Nuclear Force: $V_C(r)$ and $V_T(r)$

Central force

Tensor force

$V_C(r)$ for $^1S_0$

$V_T(r)$ for $^3S_1$ and $^3D_1$

Parameters:
- $a = 0.085$ fm
- $L = 8.1$ fm
- $m_n = 146$ MeV
- $M_K = 525$ MeV

T. Inoue et al. [HAL QCD Coll.]