CP asymmetry in flavour-specific B decays\textsuperscript{a}

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I first discuss the phenomenology of \( a_{qfs} \) \((q = d, s)\), which is the CP asymmetry in flavour-specific \( B \) decays such as \( B_d \to X \ell \nu \ell \) or \( B_s \to D^- \pi^+ \). \( a_{qfs} \) can be obtained from the time evolution of any untagged \( B \) decay. Then I present recently calculated next-to-leading-order QCD corrections to \( a_{qfs} \), which reduce the renormalisation scheme uncertainties significantly.

For the Standard Model we predict \( a_{dfs} = - (5.0 \pm 1.1) \times 10^{-4} \) and \( a_{dfs} = (2.1 \pm 0.4) \times 10^{-5} \). As a by-product we determine the ratio of the width difference in the \( B_d \) system and the average \( B_d \) width to \( \Delta \Gamma_d / \Gamma_d = (3.0 \pm 1.2) \times 10^{-3} \) at next-to-leading order in QCD.

1 Preliminaries

The time evolution of the \( B_d - \bar{B}_d \) system is determined by a Schrödinger equation:

\[
\frac{i}{\hbar} \frac{d}{dt} \begin{pmatrix} |B_d(t)\rangle \\ |\bar{B}_d(t)\rangle \end{pmatrix} = \begin{pmatrix} M^d - i \frac{\Gamma^d}{2} \end{pmatrix} \begin{pmatrix} |B_d(t)\rangle \\ |\bar{B}_d(t)\rangle \end{pmatrix},
\]

which involves two Hermitian 2×2 matrices, the mass matrix \( M^d \) and the decay matrix \( \Gamma^d \). Here \( B_d(t) \) and \( \bar{B}_d(t) \) denote mesons which are tagged as a \( B_d \) and \( \bar{B}_d \) at time \( t = 0 \), respectively. By diagonalising \( M^d - i \Gamma^d / 2 \) one obtains the mass eigenstates:

Lighter eigenstate: \( |B_{d,L}\rangle = p|B^0_d\rangle + q|\bar{B}^0_d\rangle \)

Heavier eigenstate: \( |B_{d,H}\rangle = p|B^0_d\rangle - q|\bar{B}^0_d\rangle \) with \( |p|^2 + |q|^2 = 1 \).

We discuss the mixing formalism for \( B_d \) mesons, the corresponding quantities for \( B_s - \bar{B}_s \) mixing are obtained by the replacement \( d \to s \). The coefficients \( q \) and \( p \) in Eq. (2) are also different for the \( B_d \) and \( B_s \) systems. The \( B_d - \bar{B}_d \) oscillations in Eq. (1) involve the three physical quantities \( |M^d_{12}|, |\Gamma^d_{12}| \) and \( \phi_d = \arg(-M^d_{12}/\Gamma^d_{12}) \) (see e.g. [1]). The mass and width differences between \( B_{d,L} \) and \( B_{d,H} \) are related to them as

\[
\Delta M_d = M^d_H - M^d_L = 2|M^d_{12}|, \quad \Delta \Gamma_d = \Gamma^d_L - \Gamma^d_H = 2|\Gamma^d_{12}| \cos \phi_d,
\]

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where \( M_{L}^{d}, \Gamma_{L}^{d} \) and \( M_{H}^{d}, \Gamma_{H}^{d} \) denote the masses and widths of \( B_{d,L} \) and \( B_{d,H} \), respectively.

The third quantity to determine the mixing problem in Eq. (1) is

\[
a_{fs}^{d} = \text{Im} \frac{\Gamma_{12}^{d}}{M_{12}^{d}} = \frac{\Delta \Gamma_{d}}{\Delta M_{d}} \tan \phi_{d}. \tag{4}
\]

\( \alpha_{fs}^{d} \) is the CP asymmetry in flavour-specific \( B_{d} \rightarrow f \) decays, which means that the decays \( \bar{B}_{d} \rightarrow f \) and \( B_{d} \rightarrow \bar{f} \) (with \( f \) denoting the CP-conjugate final state) are forbidden [2]. Next we consider flavour-specific decays in which the decay amplitudes \( A_{f} = \langle f | B_{d} \rangle \) and \( \overline{A}_{f} = \langle \bar{f} | \bar{B}_{d} \rangle \) in addition satisfy

\[
|A_{f}| = |\overline{A}_{f}|. \tag{5}
\]

Eq. (5) means that there is no direct CP violation in \( B_{d} \rightarrow f \). Then \( \alpha_{fs}^{d} \) is given by

\[
\alpha_{fs}^{d} = \frac{\Gamma(\bar{B}_{d}(t) \rightarrow f) - \Gamma(B_{d}(t) \rightarrow f)}{\Gamma(\bar{B}_{d}(t) \rightarrow f) + \Gamma(B_{d}(t) \rightarrow f)}. \tag{6}
\]

Note that the oscillatory terms cancel between numerator and denominator. The standard way to access \( \alpha_{fs}^{d} \) uses \( B_{d} \rightarrow X\ell^{+}H \) decays, which justifies the name semileptonic CP asymmetry for \( \alpha_{fs}^{d} \). In the \( B_{s} \) system one can also use \( B_{s} \rightarrow D_{s}^{-}\pi^{+} \) to measure \( \alpha_{fs}^{s} \). Yet, for example, Eq. (6) does not apply to the flavour-specific decays \( B_{d} \rightarrow K^{+}\pi^{-} \) or \( B_{s} \rightarrow K^{-}\pi^{+} \), which do not obey Eq. (5).

\( \alpha_{fs}^{d} \) measures CP violation in mixing. Other commonly used notations involve the quantities \(|q/p|\) or \( \epsilon_{B} \); they are related to \( \alpha_{fs}^{d} \) as

\[
1 - \left| \frac{q}{p} \right| = \frac{\alpha_{fs}^{d}}{2}, \quad \frac{\text{Re} \epsilon_{B}}{1 + |\epsilon_{B}|^{2}} = \frac{\alpha_{fs}^{d}}{4}. \tag{7}
\]

Here \( \epsilon_{B} = (1 + q/p)/(1 - q/p) \) is the analogue of the quantity \( \tau_{K} \) in \( K^{0} - \bar{K}^{0} \) mixing. Unlike \( \alpha_{fs}^{d} \), it depends on phase conventions and should not be used. In Eq. (7) and future equations we neglect terms of order \((\alpha_{fs}^{d})^{2}\).

\( \alpha_{fs}^{d} \) is small for two reasons: First \( |\Gamma_{12}^{d}/M_{12}^{d}| = O(m_{c}^{2}/M_{W}^{2}) \) suppresses \( \alpha_{fs}^{d} \) to the percent level. Second there is a GIM suppression factor \( m_{c}^{2}/m_{b}^{2} \) reducing \( \alpha_{fs}^{d} \) by another order of magnitude. Generic new physics contributions to \( \arg M_{12}^{d} \) (e.g. from squark-gluino loops in supersymmetric theories) will lift this GIM suppression. \( \alpha_{fs}^{d} \) is further suppressed by two powers of the Wolfenstein parameter \( \lambda \approx 0.22 \). Therefore \( \alpha_{fs}^{d} \) and \( \alpha_{fs}^{s} \) are very sensitive to new CP phases [1, 3], which can enhance \( |\alpha_{fs}^{d}| \) and \( |\alpha_{fs}^{s}| \) to 0.01. |\( \alpha_{fs}^{d} \)| can be further enhanced by new contributions to \( \Gamma_{12}^{d} \), which is doubly Cabibbo-suppressed in the Standard Model.

The experimental world average for \( \alpha_{fs}^{d} \) is [4]

\[
\alpha_{fs}^{d} = 0.002 \pm 0.013.
\]

2 Measurement of \( \alpha_{fs}^{d} \)

2.1 Flavour-specific decays

We first discuss the flavour-specific decays without direct CP violation in the Standard Model. First note that the “right-sign” asymmetry vanishes:

\[
\Gamma(B_{q}(t) \rightarrow f) - \Gamma(\bar{B}_{q}(t) \rightarrow \bar{f}) = 0. \tag{8}
\]
Since we are hunting possible new physics in a tiny quantity, we should be concerned whether Eq. (5) still holds in the presence of new physics. Further no experiment is exactly charge-symmetric, and the efficiencies for $B \rightarrow \bar{f}$ and $B \rightarrow f$ may differ by a factor of $1 + \delta_c$. One can use the “right-sign” asymmetry in Eq. (8) to calibrate for both effects: In the presence of a charge asymmetry $\delta_c$ one will measure

$$a_{q,\delta_c}^{\text{right}} = \frac{\Gamma(B_q(t) \rightarrow f) - (1 + \delta_c)\Gamma(\bar{B}_q(t) \rightarrow \bar{f})}{\Gamma(B_q(t) \rightarrow f) + (1 + \delta_c)\Gamma(\bar{B}_q(t) \rightarrow \bar{f})} = \frac{|A_f|^2 - |\bar{A}_{\bar{f}}|^2}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2} - \frac{\delta_c}{2}. \quad (9)$$

Instead of the desired CP asymmetry in Eq. (6) one will find

$$a_{q}^{\text{fs}} = \frac{\Gamma(\bar{B}_d(t) \rightarrow f) - (1 + \delta_c)\Gamma(B_d(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_d(t) \rightarrow f) + (1 + \delta_c)\Gamma(B_d(t) \rightarrow \bar{f})} = a_{q,\delta_c}^{\text{fs}} + a_{q,\delta_c}^{\text{right}}. \quad (10)$$

Thus $\delta_c$ and the direct CP asymmetry $(|A_f|^2 - |\bar{A}_{\bar{f}}|^2)/(|A_f|^2 + |\bar{A}_{\bar{f}}|^2)$ enter Eq. (9) and Eq. (10) in the same combination and $a_{q}^{\text{fs}}$ can be determined. Above we have kept only terms to first order in the small quantities $1 - |\bar{A}_{\bar{f}}|^2/|A_f|^2$, $\delta_c$ and $a_{q}^{\text{fs}}$.

It is well-known that the measurement of $a_{q}^{\text{fs}}$ requires neither tagging nor the resolution of the $B_q-\bar{B}_q$ oscillations [2]. Since the right-sign asymmetry in Eq. (8) vanishes, the information on $a_{q}^{\text{fs}}$ from Eq. (6) persists in the untagged decay rate

$$\Gamma[f, t] = \Gamma(B_q(t) \rightarrow f) + \Gamma(\bar{B}_q(t) \rightarrow f). \quad (11)$$

At a hadron collider one also cannot rule out a production asymmetry $\delta_p = N_{\bar{B}_q}/N_{B_q} - 1$ between the numbers $N_{\bar{B}_q}$ and $N_{B_q}$ of $\bar{B}_q$'s and $B_q$'s. An untagged measurement will give

$$a_{q,\delta_c}^{\text{fs,unt}(t)} = \frac{\Gamma[f, t] - (1 + \delta_c)\Gamma[\bar{f}, t]}{\Gamma[f, t] + (1 + \delta_c)\Gamma[\bar{f}, t]} = a_{q,\delta_c}^{\text{right}} + \frac{a_{q,\delta_c}^{\text{fs}}}{2} + \frac{\delta_p}{2} \frac{\cos(\Delta M_q t)}{\cosh(\Delta \Gamma_q t/2)}. \quad (12)$$

The use of the larger untagged data sample to determine $a_{q,\delta_c}^{\text{fs}}$ seems to be advantageous at the $\Upsilon(4S)$ B factories, where $\delta_p = 0$. Then the time evolution in Eq. (12) contains enough information to separate $a_{q,\delta_c}^{\text{fs}}$ from $a_{q,\delta_c}^{\text{right}} = a_{q,\delta_c}^{\text{fs,unt}(t = 0)}$.

Eqs. (6), (9) and (10) still hold, when the time-dependent rates are integrated over $t$. The time-integrated untagged CP asymmetry reads (for $|A_f| = |\bar{A}_{\bar{f}}|$, $\delta_c = \delta_p = 0$):

$$A_{q,\text{fs,unt}}^{\text{fs}} = \frac{\int_0^\infty dt \left[ \Gamma[f, t] - \Gamma[\bar{f}, t] \right]}{\int_0^\infty dt \left[ \Gamma[f, t] + \Gamma[\bar{f}, t] \right]} = a_{q,\delta_c}^{\text{fs}} \frac{x_q^2 + y_q^2}{x_q^2 + y_q^2 + 1} \quad (13)$$

where $x_q = \Delta M_q/\Gamma_q$, $y_q = \Delta \Gamma_q/(2\Gamma_q)$ and $\Gamma_q$ is the average decay width in the $B_q$ system. In particular a measurement of $a_{q,\delta_c}^{\text{fs}}$ does not require to resolve the rapid $B_s-\bar{B}_s$ oscillations. In $\Upsilon(4S)$ B factories a common method to constrain $a_{q,\delta_c}^{\text{fs}}$ is to compare the number $N_{++}$ of decays $(B_d(t), \bar{B}_d(t)) \rightarrow (f, f)$ with the number $N_{--}$ of decays to $(\bar{f}, \bar{f})$, typically for $f = X\ell^+\nu_\ell$. Then one finds $a_{q,\delta_c}^{\text{fs}} = (N_{++} - N_{--})/(N_{++} + N_{--})$.

We next exemplify the measurement of $a_{q,\delta_c}^{\text{fs}}$ from time-integrated tagged $B_s \rightarrow f$ decays, having $f = X\ell^+\nu_\ell$ in mind. This approach should be feasible at the Fermilab Tevatron.

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1Direct CP violation requires the presence of a CP-conserving phase. In the case of $B_d \rightarrow D^+\ell^+\nu_\ell$ this phase comes from photon exchange and is small. Also somewhat contrived scenarios of new physics are needed to get a sizeable CP-violating phase in a semileptonic decay. Thus here one needs to worry about $|A_f| \neq |\bar{A}_{\bar{f}}|$ only, once $a_{q,\delta_c}^{\text{fs}}$ is probed at the permille level.
allow the detector to be charge-asymmetric ($\delta_c \neq 0$) and also relax Eq. (5) to $|A_f| \approx |\overline{A}_f|$. Let $N_f$ denote the total number of observed decays of meson tagged as $B_s$ at time $t = 0$ into the final state $f$. Further $\overline{N}_f$ denotes the analogous number for a meson initially tagged as a $\overline{B}_s$. The corresponding quantities for the decays $B_s(t) \rightarrow \overline{f}$ and $\overline{B}_s(t) \rightarrow f$ are $N_f$ and $\overline{N}_f$. One has

$$\overline{N}_f \propto \int_0^\infty dt \Gamma(\overline{B}_s(t) \rightarrow f), \quad N_f \propto (1 + \delta_c) \int_0^\infty dt \Gamma(\overline{B}_s(t) \rightarrow f)$$

with the same constant of proportionality. The four asymmetries

$$\frac{N_f - \overline{N}_f}{N_f + \overline{N}_f} = a_{\text{right}}^s \delta_c, \quad \frac{\overline{N}_f - N_f}{N_f + \overline{N}_f} = a_{\text{right}}^s + a_{\text{fs}}^s,$$

$$\frac{N_f - \overline{N}_f}{N_f + \overline{N}_f} = 1 - \frac{y_s^2}{1 + x_s^2}, \quad \frac{\overline{N}_f - N_f}{N_f + \overline{N}_f} = 1 - \frac{y_s^2}{1 + x_s^2} + \frac{a_{\text{fs}}^s}{2} (14)$$

then allow to determine $a_{\text{fs}}^s$ and $(1 - y_s^2)/(1 + x_s^2)$. In the second line of Eq. (14) terms of order $a_{\text{fs}}^s/x_s^2$ have been neglected. (Of course the last asymmetry in Eq. (14) is redundant.)

### 2.2 Any decay

Since $q/p$ enters the time evolution of any neutral $B_q \rightarrow f$ decay, we can use any such decay to determine $a_{\text{fs}}^s$. The time dependent decay rates involve

$$\lambda_f = \frac{\langle f | B_q \rangle}{\langle f | B_q \rangle}.$$ 

In Eq. (1.73)-(1.77) of [1] $\Gamma(B_q(t) \rightarrow f), \Gamma(\overline{B}_q(t) \rightarrow f), \Gamma(B_q(t) \rightarrow \overline{f})$ and $\Gamma(\overline{B}_q(t) \rightarrow f)$ can be found for the most general case, including a non-zero $\Delta \Gamma_q$. For the untagged rate one easily finds

$$\Gamma[f,t] \propto e^{-\Gamma_q t} \left\{ 1 + \frac{a_{\text{fs}}^q}{2} \left[ \cosh \frac{\Delta \Gamma_q t}{2} + A^\Delta \Gamma \sinh \frac{\Delta \Gamma_q t}{2} \right] - \frac{a_{\text{fs}}^q}{2} \left[ A^\text{dir} \cos(\Delta M_q t) + A^\text{mix} \sin(\Delta M_q t) \right] \right\} (15)$$

with

$$A^\text{dir} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad A^\text{mix} = -\frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2} \quad \text{and} \quad A^\Delta \Gamma = -\frac{2 \text{Re} \lambda_f}{1 + |\lambda_f|^2}. (16)$$

Hence one can obtain $a_{\text{fs}}^q$ from the amplitude of the tiny oscillations in Eq. (15). Once $A^\text{dir}$ and $A^\text{mix}$ are determined from the $\cos \Delta M_q t$ and $\sin \Delta M_q t$ terms of the time evolution in the tagged $B_q(t) \rightarrow f$ decay. If $f$ is a CP eigenstate, $A^\text{dir}$ and $A^\text{mix}$ are the direct and mixing-induced CP asymmetries. For example, in $B_d \rightarrow J/\psi K_S$ one has $\lambda_f = -\exp(-2i\beta) + O(a_{\text{fs}})$, so that one can set $A^\text{dir} = 0$ and $A^\text{mix} = -\sin(2\beta)$ in Eq. (15). The flavour-specific decays discussed in the previous section correspond to the special case $\lambda_f = 0$. 

3 QCD corrections to $a_{fs}^q$

$a_{fs}^q = \text{Im} \, \Gamma_{12}^q / M_{12}^q$ is proportional to two powers of the charm mass $m_c$. A theoretical prediction in leading order (LO) of QCD cannot control the renormalisation scheme of $m_c$. Therefore the LO result $a_{fs}^q$ suffers from a theoretical uncertainty which is not only huge but also hard to quantify. While next-to-leading order (NLO) QCD corrections to $M_{12}^q$ are known for long [5], the computation of those to $\Gamma_{12}^q$ has been completed only recently. The LO and a sample NLO diagram are shown in Fig. 1. The NLO result for the contribution with two identical up-type quark lines (sufficient for the prediction of $\Delta \Gamma_s$) has been calculated in [6] and was confirmed in [7]. The contribution with one up-quark and one charm-quark line was obtained recently in [7] and [8]. In order to compute $\Gamma_{12}^q$ one exploits the fact that the mass $m_b$ of the $b$-quark is much larger than the fundamental QCD scale $\Lambda_{\text{QCD}}$. The theoretical tool used is the Heavy Quark Expansion (HQE), which yields a systematic expansion of $\Gamma_{12}^q$ in the two parameters $\Lambda_{\text{QCD}}/m_b$ and $\alpha_s(m_b)$ [9]. $\Gamma_{12}^q$ and $M_{12}^q$ involve hadronic “bag” parameters, which quantify the size of the non-perturbative QCD binding effects and are difficult to compute. The dependence on these hadronic parameters, however, largely cancels from $a_{fs}^q$.

Including corrections of order $\alpha_s$ [6–8] and $\Lambda_{\text{QCD}}/m_b$ [7, 8, 10] we predict [8]

$$a_{fs}^d = 10^{-4} \left[ -\frac{\sin \beta}{R_t} (12.0 \pm 2.4) + \left( \frac{2 \sin \beta}{R_t} - \frac{\sin 2\beta}{R_t^2} \right) (0.2 \pm 0.1) \right].$$

Here $\beta$ is the angle of the unitarity triangle measured in the CP asymmetry of $B_d \to J/\psi K_S$. If $(\bar{\rho}, \bar{\eta})$ denotes the apex of the usual unitarity triangle, then $R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}$ is the length of one of its sides. For the Standard Model fit to the unitarity triangle with $\beta = 22.4^\circ \pm 1.4^\circ$ and $R_t = 0.91 \pm 0.05$ [11] one finds:

$$a_{fs}^d = -(5.0 \pm 1.1) \cdot 10^{-4}$$

The impact of a future measurement of $a_{fs}^d$ on the unitarity triangle is shown in Fig. 2. The result for the $B_s$ system is

$$a_{fs}^s = (12.0 \pm 2.4) \cdot 10^{-4} |V_{us}|^2 R_t \sin \beta = (2.1 \pm 0.4) \cdot 10^{-5}.$$

From Eq. (3) one finds that $\Delta \Gamma_q / \Delta M_q = -\text{Re}(\Gamma_{12}^q / M_{12}^q)$. This ratio was predicted to NLO in [6] for the $B_s$ system. With the new result of [7, 8] we can also predict $\Delta \Gamma_d / \Delta M_d$. Due to a numerical accident, the Standard Model prediction for the ratio $\Delta \Gamma_q / \Delta M_q$ is essentially the same for $q = d$ and $q = s$:

$$\frac{\Delta \Gamma_q}{\Delta M_q} = (4.0 \pm 1.6) \times 10^{-3}, \quad \frac{\Delta \Gamma_d}{\Gamma_d} = (3.0 \pm 1.2) \times 10^{-3}. \quad (17)$$
Figure 2: Constraint in the $(\bar{\rho}, \bar{\eta})$ plane from $a_d^{fs}$. Area between solid pair of curves: NLO, for the cases $a_d^{fs} = -5 \times 10^{-4}$ (left) and $a_d^{fs} = -10^{-3}$ (right). Area between dashed curves: LO for $a_d^{fs} = -5 \times 10^{-4}$. The current best fit to the unitarity triangle [11] is also shown.

The precise values for the quark masses, “bag” factors and $\alpha_s$ used for our numerical predictions can be found in Eq. (7) of [8].

We close our discussion with a remark about the $B_s$ system. It is possible that new physics contributions render the $B_s - \bar{B}_s$ oscillations so large that a measurement of $\Delta M_s$ will be impossible. In general such new physics contribution will affect the CP phase $\phi_s$ and suppress $\Delta \Gamma_s$ in Eq. (3). Different measurements of $\Delta \Gamma_s$ can then determine $|\cos\phi_s|$ despite of the unobservably rapid $B_s - \bar{B}_s$ oscillations [12]. A measurement of the sign of $a_s^{fs} \propto \sin\phi_s$ (which will then be enhanced, unless $\Delta M_s$ is extreme) through e.g. Eq. (13) will then reduce the four-fold ambiguity in $\phi_s$ from the measurement of $|\cos\phi_s|$ to a two-fold one.

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