The bandwidth-duration product of the received radar signal as a criterion for observing wind field inhomogeneity

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Abstract. The coherent mode meteorological radars applicability to recognizing weather hazard associated with wind fields inhomogeneity is analyzed. It is noted that for the aviation weather service the estimates of the received reflections spectral characteristics prove to be little informative. Problems that hinder the use of these phenomena detection results have been identified. It is proposed that new methods based on estimates of the reflected signal spectrum width and methods based on the detection of the complexity of the received signal, which is the sum of two reflections from different air masses with different characteristics, should be used for this purpose. As an indicator of this complication, it is assumed to use the bandwidth-duration product, which is a product of its energy spectrum width by the interval of its amplitude correlation. The theoretical calculation results of the bandwidth-duration product, which spectrum consists of two Gaussian form components, for different values of average frequency and/or width, are given. The received data allow asserting that the bandwidth-duration product practically does not change at small differences of Doppler shifts of signal components, but very sensitively reacts to a difference in a spectrum width, i.e. in the most interesting cases for a solved problem.

1. Introduction

The first net of the Doppler weather radars was deployed more than 30 years ago. The practical value of this big step in the technical development of the weather service is now widely recognized. The coherent mode makes it possible to measure the radial velocity of particles, to restore the vertical wind profile, to detect areas of squall, wind shear and tornados, i.e. to significantly expand the list of weather hazards detected by radar methods. It would seem that these additional capabilities should greatly facilitate the work of those involved in the meteorological maintenance of airports and space launch sites. However, the experience of the authors of the article shows that this information is not widely used among the mentioned specialists. It makes us consider the situation more carefully.

2. Analyzing the coherent mode applicability to the tasks of weather hazard detection

Let's consider the problems that arise when observing each of these events separately.

The vertical wind profile is interesting in two aspects – in itself and as a source material for wind shear detection. The wind shear will be considered below, and now let's focus on the profile as such. The profile of the speed and direction of air flows by height is necessary for the weather service of aviation flights. However, air speeds are of the greatest interest, either at aircraft flight altitudes or near the runway, when they are needed to ensure safe take-off and landing. Since radar for normal operation...
requires the presence of elementary reflective particles in the air (hydrometeors, as a rule), then to get
the information of interest at this altitude, which is now about 10 km, it is useless. The presence of
reflectors here is possible only in case of formation of powerful cumulonimbus (Cb), which implies a
large dispersion of particle velocities due to severe turbulence and localization of the reflection
formation zone. Both make it very difficult to apply known methods of restoring air velocity vector at a
selected height, such as VAD, UVT, VPP [1, 2] and others [3]. It is much more efficient to use
aerological data. Obviously, this is not the case for tasks such as military aircraft for landing planning
and similar problems. But these are specific tasks, which obviously require specific modes of operation
of the equipment in question [4], or the use of specific equipment.

On the ground in the area of the runway without wind shear, the wind parameters are well measured
by ground anemometers. Their information is much more accurate and cheaper than radar.

Detection of the squall was also possible in incoherent radars [5]. Indirect parameters were used and,
basically, everything was reduced to a probability estimation of how much the cloud is capable of
forming a squall. Coherence allows the radar to detect this event directly by estimating the radial velocity
of particles, but their movement is not necessarily directed to the radar or from it. This makes it
impossible to measure the real velocity and, if it is tangential, cannot be detected. Thus, there is no
radical solution to the problem or significant progress in this trend.

The wind shear is interesting for airports at altitudes up to 500 m [6] and cosmodromes at altitudes
up to several tens of kilometers [7]. In both cases it is useless to estimate the average radial velocity of
particles by radar. At the cosmodrome, because the operation at such altitudes requires waves of
decimeter and meter bands [8], and at airports because the meteorological radar parameters do not meet
the requirements for the wind shear gauge. Let us consider the last thesis in more detail.

The amount of shear is measured in m/s by 30 m (100 ft) of height. Therefore, the vertical size of the
antenna beam must not exceed 30 m, otherwise the wind speed estimates will be smoothed out and the
probability of passing the hazard is high. With a generally accepted beam width of 1 degree
meteorological radar antenna, the requirement is met up to a range of 1.8 km. But at about the same
range, the Fraunhofer zone is just beginning, where the directional diagram can be considered as formed.
Determining the boundary of this zone using the equation

\[ l_b = \frac{4D^3}{\lambda} \]

where \( D \) – antenna diameter;
\( \lambda \) – wavelength carrier wave,
we find that, for example, for the Russian radar DMRL-S \( l_b = 1.62 \) km. Therefore, measurements are
possible only in one distance sample, which should be scanned sequentially all heights of the interval of
interest (500 m) in 30 m steps. Obviously, these operations will take so much time that the main function
– every 10 minutes to update information about the prevailing weather conditions in the area of view –
will have to forget. For this reason, both at airports and on space platforms special devices – profilers
are used to measure the wind profile.

The tornado was and is the most elusive object on the radar. Introduction of coherence opens up new
possibilities in this respect - a good predictor for it is considered to be a sharp difference in mean radial
velocities of particles in neighboring or nearby located elements of resolution [9], which is a
consequence of circular rotation of air in the considered space. However, to obtain this effect, it is
required that the horizontal size of the radar resolution volume is at least half the diameter of the tornado.
Since it can be only a few hundred meters, the range of such detection is very limited.

Let's sum it up. The introduction of a coherent mode in meteorological radars has provided new
information on particle velocities in cloud formations and precipitation. In some cases it can be very
useful, but its contribution to the practical tasks of radar maintenance of airfields and cosmodromes is
minimal. At the same time, it should be recognized that the main task of the radar in this field of
application is not to determine the wind speed and direction, but to detect various types of
inhomogeneity of spatial distribution of these parameters – shear, squall and tornado. From the radar
point of view, these are absolutely homogeneous events, differing only in orientation and size of the inhomogeneity corresponding to them.

As a result, the search for new methods of processing the information obtained by Doppler radar, capable of increasing the efficiency of this type of equipment becomes relevant. According to the authors, it should be conducted in the direction of the use of estimates of the spectrum width – spectral parameter, until now practically not used for the detection of hazardous events.

3. Application of spectrum width estimates to recognize wind field inhomogeneities

In previous publications, the authors have already described a method for detecting dangerous wind shear according to estimates of signal spectrum width [10, 11]. It is based on the concept of abnormally wide spectrum that occurs when the radar simultaneously observes two air masses, between which is directly inhomogeneous wind field as it takes place, for example, in the case of vertical wind shear presented in figure 1.

Since the magnitude and direction of the velocity vector of the particles in each of their masses is different, on most of the antenna azimuth values (except for two small areas where the radial components of these particles are equal) the energy spectrum of the received signal becomes two-modal (figure 2).

Figure 1. Scheme of observation by radar of vertical wind shear

The value of the width of such a spectrum depends on the difference between the Doppler average reflection frequencies formed by each mass and the power ratio of these reflections. At the distance where the power data are equalized, the average frequency of the spectrum appears to be located approximately in the middle between the maximums, and its width becomes abnormally large, i.e. a value that it cannot achieve with the difference in radial velocities existing in the resolution cell, if they are evenly distributed. As a result, the map of the conical section shows characteristic zones of abnormally wide spectra like those shown in figure 3.

On values of width of a spectrum in these zones and on some features of their arrangement in space the size of the wind shear, its character (the shear in value, in a direction or both) and altitude of an arrangement can be restored.

The main feature of the method is that it does not have any strict requirements for the antenna beam width, i.e. it is not afraid of data smoothing that takes place when restoring the wind profile. Information about wind behavior at different altitudes as if "moves" to the spectral area and becomes less sensitive to the characteristics of the radar.

A similar technique can be used to recognize vertical airflow [12]. But mechanically apply the method described above to this case is impossible. The main reason for this is the small difference between the Doppler mean shifts of signals reflected from the flow and the environment around it. This is due to the fact that the observation of these hazards usually occurs at small angles of the antenna. To overcome this obstacle, the authors proposed to analyze not the spectral width of the signal itself but its
envelope, which is known to be emitted at the output of an amplitude detector. In this work was determined not the spectral width, but the inverse value of the correlation distance of the function of the random process under study.

Figure 3. Scheme of observation by radar of vertical wind shear

It is suggested to consider the appearance of resolution cells in the antenna beam as an indicator of the fact that vertical flow has hit the antenna beam, in which there is a decrease in the correlation distance (or spectrum expansion) of the signal at the output of the amplitude detector with the invariability or much smaller change in the spectrum width of the original signal, i.e. the signal at the output of the phase detector. Actually it is necessary to find the multiplication of two values, the first of which is the spectrum width, and the second is the duration of the correlation function. This leads to the appearance of an analogy with what is called the bandwidth-duration product in radio engineering. Let us consider this thesis in more detail.

4. Applying the concept of the bandwidth-duration product to reflections from a weather object. In communication and radar, the bandwidth-duration product is defined as the multiplication of the pulse width by its spectrum width. If the pulse is simple, i.e. its high-frequency filling is a common sinusoid, it is close to one. This requirement is met, for example, by the width of a square pulse and the width of its spectrum in Hz at 0.5. In accuracy to the unit is equal to the multiplication of standard deviations of the correlation function and spectrum (in rad./sec) of the Gaussian pulse form.

If any frequency or phase changes are introduced into the pulse carrier wave, the spectrum width increases and the bandwidth-duration product value increases accordingly. In this case the impulse is called complex or broadband. The most common changes in the carrier are linear frequency modulation and phase code modulation.

Let us return to reflections from weather formations, more precisely those that contain heterogeneity of the vector field of the wind. The received signal is the sum of two reflections formed by each of the air masses in the resolution cell. These reflections have different Doppler frequency shifts and different values of spectrum width. In fact, a complex signal is created, similar to what is called a multi-frequency signal in radio engineering. The essential difference between the former and the latter is that it does not have a strictly defined duration, as it is a sequence of samples with the period of probing radar pulses. Therefore, it is necessary to find an analogue for this parameter. The burst time is not suitable, because it is in no way related to the spectrum width of the received reflection. The most suitable candidate for this role is seen as the already mentioned correlation distance of the signal at the output of the amplitude detector, which, as was established in previous studies of the authors [12], reacts to the complexity of the signal, regardless of the behavior of its spectrum. To confirm this assumption, we will find mathematical expressions for each of the parameters involved in the formation of the bandwidth-
duration product in the assumption that the normalized correlation function of the signal reflected by the "calm" air mass, which does not contain inhomogeneous wind, has a Gaussian form, i.e.

\[ r(\tau) = \exp \left( -\frac{\tau^2}{2\sigma^2} + j\omega_0 \tau \right), \]  

(2)

where \( \sigma_\tau \) – correlation distance;

\( j \) – imaginary unit;

\( \omega_0 \) – Doppler mean frequency deviation.

The energy spectrum width of the signal, including two components, under the assumptions made is determined by the equation \([13]\)

\[ \sigma_\text{se} = \left[ \frac{1}{I + \varepsilon} \left( \frac{1}{\sigma_{\tau_1}} \right)^2 + \frac{1}{I + \varepsilon} \left( \frac{1}{\sigma_{\tau_2}} \right)^2 + \varepsilon \frac{\omega_0}{(I + \varepsilon)^2} (\omega_{01} - \omega_{02})^2 \right]^{0.5}, \]

(3)

where

\( \varepsilon = \frac{P_2}{P_1} \) – power ratio;

\( \sigma_{\tau_1} \) and \( \sigma_{\tau_2} \) – correlation distance of each of the components, defined as a standard deviation of the correlation function;

\( \omega_{01} \) and \( \omega_{02} \) – Doppler mean deviation of the component frequency.

To find the second co-factor required to calculate the bandwidth-duration product (correlation distance) of the signal at the output of the amplitude detector we will use the determination of the correlation function of the process received at the output of the amplitude detector after passing through a Gaussian random process with the correlation function \( R(\tau) \) \([13]\). In case of linear-law detector it is equal to

\[ R_{\text{out}}(\tau) = \frac{\pi}{d(4 - \pi)} \left[ R_0^2(\tau) + \sum_{n=2}^{\infty} \left[ \left( \frac{2n - 3}{2} \right) \right]^2 R_n^2(\tau) \right], \]

(4)

where

\[ R_0(\tau) = \sqrt{\left[ \text{Re}(R_0(\tau)) \right]^2 + \left[ \text{Im}(R_0(\tau)) \right]^2} \]

(5)

in the case of square-law detection

\[ R_{\text{out}}(\tau) = R_0^2(\tau). \]

(6)

Obviously, there is no principal difference in calculations in both cases. For the sake of certainty, we will further assume that the detection is square-law.

In the case we are interested, when the total signal consists of two processes with correlation functions \( R_1(\tau) \) and \( R_2(\tau) \)

\[ R_{\text{max}}(\tau) = \frac{1}{(I + \varepsilon)^2} \left[ \text{Re}(r_1(\tau)) + \text{Re}(r_2(\tau)) \right]^2 + \left[ \text{Im}(r_1(\tau)) + \text{Im}(r_2(\tau)) \right]^2, \]

(7)

where \( r_i(\tau) = \frac{R_i(\tau)}{P_1} \) – the normalized correlation function of the i-th random process.

After substitution (2) in (7) and simple transformations we get

\[ r_{\text{out}}(\tau) = \frac{1}{(I + \varepsilon)^2} \exp \left( -\frac{\tau^2}{\sigma_{\tau_1}^2} \right) + \varepsilon^2 \exp \left( -\frac{\tau^2}{\sigma_{\tau_2}^2} \right) + 2\varepsilon \exp \left( -\frac{\sigma_{\tau_1}^2 + \sigma_{\tau_2}^2}{2\sigma_{\tau_1}^2\sigma_{\tau_2}^2} \tau^2 \right) \cos((\omega_{01} - \omega_{02})\tau), \]

(8)
Process correlation distance at the output of the amplitude detector search according to the equation

$$\sigma^2_{\text{out}} = \frac{\int_{-\infty}^{\infty} \tau^2 r_{\text{out}}(\tau) d\tau}{\int_{-\infty}^{\infty} r_{\text{out}}(\tau) d\tau},$$

(9)

By substituting (8) in (9) we find the expression for the required parameter

$$\sigma^2_{\text{max}} = 0.5 \frac{\sigma^2_{\tau_1} + \varepsilon^2 \sigma^2_{\tau_2} + 2\varepsilon(1 - (\Delta \omega)^2 \sigma^2_{\tau_1} \sigma^2_{\tau_2})}{\sigma_{\tau_1} + \varepsilon \sigma_{\tau_2} + 2\sqrt{2\varepsilon} \sqrt{\sigma^2_{\tau_1} + \sigma^2_{\tau_2}}} \exp\left[\frac{(\Delta \omega)^2 \sigma^2_{\tau_1} \sigma^2_{\tau_2}}{2(\sigma^2_{\tau_1} + \sigma^2_{\tau_2})}\right]$$

(10)

where \(\Delta \omega = \omega_{01} - \omega_{02}\).

We study the behavior of the bandwidth-duration product for different ratios of parameters of the total signal components. In this case, take into account that in practice, as a rule, the direct assessment is not the correlation distance and spectrum width itself, but the squares of their values. Therefore, we will not study the bandwidth-duration product itself, but its second degree.

$$B^2 = \sigma^2_{\text{tot}} \sigma^2_{\omega \Sigma},$$

(11)

and relative magnitude

$$\mu(\eta, \Delta \omega, \varepsilon) = \frac{B^2(\eta, \Delta \omega, \varepsilon)}{B^2(1,0,1)},$$

(12)

where \(\eta = \frac{\sigma_{\tau_2}}{\sigma_{\tau_1}}\).

First of all, we find the bandwidth-duration product value in the absence of wind field inhomogeneity, when power and spectral width of the components are equal and Doppler shifts are the same, i.e. \(B^2(1,0,1)\). As might be expected, it is equal to 0.5, because the total signal in this case is simply the result of squaring of the correlation function of the initial process. Further, we consider the most interesting case of the Doppler shift average but different values of the correlation distance to solve the problem. The figure 4 presents graphs of the relative bandwidth-duration product \(\mu\) dependence on the ratio of the signal components power in the set conditions.

It follows that the bandwidth-duration product reacts to the difference in correlation distances of its components, even if their average Doppler frequency shifts fully coincide. At its value of 60%, the bandwidth-duration product reaches the value of 1.2, at 100% – 1.5. This is for the case of equal power \(P_1\) and \(P_2\). If their ratio changes by 4 times, the decrease of the bandwidth-duration product value do not exceed 10%, i.e. the effect is quite resistant to this parameter.

The reaction of the bandwidth-duration product becomes even more intense if in addition to the difference in the value of correlation distances there is a difference in Doppler frequency shifts. The figure 5 shows its dependence on the relative value of \(\Delta \omega \sigma_{\tau_1}\) at different values of the correlation distances ratio \(\eta\).
Vertical strokes on the graphs mark the values of the difference in frequency corresponding to the sum of the spectral widths of the components, i.e. the points that determine the moment of resolution of two spectra. It can be seen that already at correlation distances of 1.5, the bandwidth-duration product response to the shift of spectra along the frequency axis begins much earlier than reaching the moment when they can be resolved, and reaches the boundary value of 1.5. As \( \eta \) increases, the steepness curve of the function increases.

5. Conclusion
From the given results of research it is clear that the possibilities of such parameter of reflections received by the meteorological radar as the signal spectrum width (or the spectrum width of radial velocities of observed particles) are far from exhausted and can be successfully applied in tasks of detection of hazardous events associated with the wind. A special effect in this respect should be expected from the application of estimates of the bandwidth-duration product, understood as the multiplication of the width of its energy spectrum by the correlation distance of the process obtained after passing the amplitude detector. The ability of this new parameter to respond to the appearance of two air masses in the antenna beam, giving reflections with the same Doppler frequency shift and differing only by the spectrum width, makes its application for radar detection of microbursts and tornados very promising.

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