Gaussian Attention Model and Its Application to Knowledgebase Embedding and Question Answering

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Abstract

We propose the Gaussian attention model for content-based neural memory access. With the proposed attention model, a neural network has the additional degree of freedom to control the focus of its attention from a laser sharp attention to blurred attention. It is applicable whenever we can assume that the distance in the latent space reflects some notion of semantics. We use the proposed attention model as a scoring function for the embedding of a knowledgebase into a continuous vector space and then train a model that performs question answering about the entities in the knowledgebase. The proposed attention model can handle both the propagation of uncertainty when following a series of relations and also the conjunction of thoughts in a natural way. On a dataset of soccer players who participated in the FIFA World Cup 2014, we demonstrate that our model can handle both path queries and conjunctive queries well.

1 Introduction

There is a growing interest in incorporating external memory into neural networks. For example, memory networks (Weston et al., 2014; Sukhbaatar et al., 2015) are equipped with static memory slots that are content or location addressable. Neural Turing machines (Graves et al., 2014) implement memory slots that can be read and written as in Turing machines (Turing, 1938) but through differentiable attention mechanism.

Each memory slot in these models stores a vector corresponding to a continuous representation of the memory content. In order to recall a piece of information stored in memory, attention is typically employed. Attention mechanism introduced by Bahdanau et al. (2014) uses a network (often called attention network) that outputs a discrete probability mass over memory items. A memory read can be implemented as a weighted sum of the memory vectors in which the weights are given by the attention network. Reading out a single item can be realized as a special case in which the output of the attention network is peaked at the desired item. The attention network may depend on the current context as well as the memory item itself. The attention model is called location-based and content-based, if it depends on the location in the memory and the stored memory vector, respectively.

Knowledgebases, such as WordNet and Freebase, can also be stored in memory either through an explicit knowledgebase embedding (Bordes et al., 2011; Nickel et al., 2011; Socher et al., 2013) or through a feedforward network (Bordes et al., 2015).

When we embed entities from a knowledgebase in a continuous vector space, if the capacity of the embedding model is appropriately controlled, we expect semantically similar entities to be close to each other, which will allow the model to generalize to unseen facts. However the notion of proximity may strongly depend on the type of a relation. For example Benjamin Franklin was an engineer but also a politician. We would need different metrics to capture his proximity to other engineers and politicians of his time.
In this paper, we propose a new attention model for content-based addressing. Our model scores each item \( v_{item} \) in the memory by the (logarithm of) multivariate Gaussian likelihood as follows:

\[
\text{score}(v_{item}) = \log \phi(v_{item} | \mu_{context}, \Sigma_{context}) \\
= -\frac{1}{2} (v_{item} - \mu_{context}) \Sigma_{context}^{-1} (v_{item} - \mu_{context}) + \text{const.} \quad (1)
\]

where \( \text{context} \) denotes all the variables that the attention depends on. For example, “American engineers in the 18th century” or “American politicians in the 18th century” would be two contexts that include Benjamin Franklin but the two attentions would have very different shapes.

Compare to the (normalized) inner product used in previous work (Sukhbaatar et al., 2015; Graves et al., 2014) for content-based addressing, the Gaussian model has the additional control of the spread of the attention over items in the memory. As we show in Figure 1, we can view the conventional inner-product-based attention and the proposed Gaussian attention as addressing by an affine energy function and a quadratic energy function, respectively. By making the addressing mechanism more complex, we may represent many entities in a relatively low dimensional embedding space. Since knowledgebases are typically extremely sparse, it is more likely that we can afford to have a more complex attention model than a large embedding dimension.

We apply the proposed Gaussian attention model to question answering based on knowledgebases. At the high-level, the goal of the task is to learn the mapping from a question about objects in the knowledgebase in natural language to a probability distribution over the entities. We use the scoring function (1) for both embedding the entities as vectors and calculating the output of the network.

In this context, the Gaussian attention model is ideal for representing the uncertainty in a multiple-answer question (e.g., name all the children of Abraham Lincoln) and also for propagating uncertainty for answering questions that require a traversal over the knowledge graph (e.g., name all the grand children of Abraham Lincoln) (see Guu et al., 2015).

The proposed question answering model is able to handle not only the case where the answer to a question is associated with an atomic fact, which is called simple Q&A (Bordes et al., 2015), but also questions that require composition of relations (path queries in Guu et al. (2015)) and conjunction of relations. An example flow of how our model deals with a question “Who plays forward for Borussia Dortmund?” is shown in Figure 2 in Section 3.

This paper is structured as follows. In Section 2 we describe how the Gaussian scoring function (1) can be used to embed the entities in a knowledgebase into a continuous vector space. We call our model TransGaussian because of its similarity to the TransE model proposed by Bordes et al. (2013). Then in Section 3 we describe our question answering model that takes a question in natural language and outputs an attention over the knowledgebase entities as its prediction. We carry out experiments on WorldCup 2014 dataset we collected. The dataset is relatively small but it allows us to evaluate not only simple questions but also path queries and conjunction of queries. On the WorldCup 2014 dataset the proposed TransGaussian embedding with the question answering model achieves significantly higher accuracy than the vanilla TransE embedding or TransE trained with compositional relations (Guu et al., 2015) combined with the same question answering model.

### 2 Knowledgebase Embedding

In this section, we describe how the Gaussian attention model (1) can be used to train a knowledgebase embedding. While it is possible to train a neural network that computes the embedding
in a single pass as in [Bordes et al. (2015)] or over multiple passes as in [Li et al. (2015)], it is more efficient to offload the embedding as a separate step for question answering based on a large static knowledgebase.

2.1 The TransGaussian Model

Let $\mathcal{E}$ be the set of entities and $\mathcal{R}$ be the set of relations. A knowledgebase is a collection of triplets $(s, r, o)$, where we call $s \in \mathcal{E}$, $r \in \mathcal{R}$, and $o \in \mathcal{E}$, the subject, the relation, and the object of the triplet, respectively. Each triplet encodes a fact. For example, (Albert.Einstein, hasprofession, theoreticalphysicist). Although we don’t consider the temporal aspect of some relations in this paper, we note that such relations can also be reduced to the current setting by introducing an auxiliary entity corresponding to a distinct event (e.g., presidency).

All the triplets given in a knowledgebase are assumed to be true. However generally speaking a triplet may be true or false. Thus knowledgebase embedding aims at training a model that predict if a triplet is true or not given some parameterization of the entities and relations (Bordes et al., 2011; Nickel et al., 2011; Socher et al., 2013; Wang et al., 2014).

In this paper, we associate a vector $v_s \in \mathbb{R}^d$ with each entity $s \in \mathcal{E}$, and we associate each relation $r \in \mathcal{R}$ with two parameters, $\delta_r \in \mathbb{R}^d$ and a positive definite symmetric matrix $\Sigma_r \in \mathbb{R}^{d \times d}$.

Given subject $s$ and relation $r$, we can compute the score of an object $o$ to be in triplet $(s, r, o)$ using the Gaussian attention model (1) with

$$
\mu_{\text{context}} = v_s + \delta_r, \quad \Sigma_{\text{context}} = \Sigma_r
$$

as

$$
\text{score}(s, r, o) = \log \phi(v_o | \mu_{\text{context}}, \Sigma_{\text{context}}).
$$

Note that if $\Sigma_r$ is fixed to the identity matrix, we are modeling the relation of subject $v_s$ and object $v_o$ as a translation $\delta_r$, which is equivalent to the TransE model (Bordes et al., 2013). We allow the covariance $\Sigma_r$ to depend on the relation to handle one-to-many relations (e.g., profession has person relation) and capture the shape of the distribution of the set of objects that can be in the triplet. We call our model TransGaussian.

Parameterization For computational efficiency, we will restrict the covariance matrix $\Sigma_r$ to be diagonal in this paper. Furthermore, in order to ensure that $\Sigma_r$ is strictly positive definite, we employ the exponential linear unit (ELU, Clevert et al., 2015) and parameterize $\Sigma_r$ as follows:

$$
\Sigma_r = \text{diag} \left( \begin{array}{c} \text{ELU}(m_{r,1}) + 1 + \epsilon \\ \vdots \\ \text{ELU}(m_{r,d}) + 1 + \epsilon \end{array} \right)
$$

where $m_{r,j}, j = 1, \ldots, d$ are the unconstrained parameters that are optimized during training and $\epsilon$ is a small positive value that ensure the positivity of the variance during numerical computation. The ELU is defined as

$$
\text{ELU}(x) = \begin{cases} x, & x \geq 0 \\ \exp(x) - 1, & x < 0. \end{cases}
$$

Ranking loss Suppose we have a set of triplets $T = \{(s_i, r_i, o_i)\}_{i=1}^N$ from the knowledgebase. Let $\mathcal{N}(s, r)$ be the set of incorrect objects to be in the triplet $(s, r, \cdot)$.

The rank loss measure the margin between score of true answers and that of false answers. The objective is to minimize the following

$$
\min_{\{v_s, v_o \in \mathbb{R}^d\}, \{\delta_r, m_r: r \in \mathcal{R} \cup \mathcal{R}^{-1}\}} \frac{1}{N} \sum_{(s, r, o) \in T} \mathbb{E}_{t' \sim \mathcal{N}(s, r)} \left[ [\mu - \text{score}(s, r, o) + \text{score}(s, r, t')]_+ \right]

+ \lambda \left( \sum_{s \in \mathcal{E}} \|v_s\|^2 + \sum_{r \in \mathcal{R} \cup \mathcal{R}^{-1}} (\|\delta_r\|^2 + \|m_r\|^2) \right),
$$

(3)
where, \( N = |T| \), \( \mu \) is the margin parameter and \( M_r \) denotes the diagonal matrix with \( m_{r,j} = 1, \ldots, d \) on the diagonal. Here, we treat an inverse relation as a separate relation and denote by \( R^{-1} \) the set of inverse relations of \( R \). Moreover, \( \mathbb{E}_{t' \sim N(s, r)} \) denotes the expectation with respect to the uniform distribution over the set of incorrect objects, which we approximate with 10 random samples in the experiments. Finally, the last terms are L2 regularization terms for the embedding parameters.

### 2.2 Compositional relations

Guu et al. (2015) has recently shown that training TransE with compositional relations can make it competitive to more complex models, although TransE is much simpler compared to for example, neural tensor networks (NTN, Socher et al. (2013)) and TransH Wang et al. (2014). Here, a compositional relation is a relation that is composed as a series of relations in \( R \). For example, \( \text{grandfather_of} \) can be composed as first applying the \( \text{parent_of} \) relation and then the \( \text{father_of} \) relation, which can be seen as a traversal over a path on the knowledge graph.

TransGaussian model can naturally handle and propagate the uncertainty over such a chain of relations by convolving the Gaussian distributions along the path. That is, the score of an entity \( o \) to be in the \( \tau \)-step relation \( r_1/r_2/\cdots/r_{\tau} \) with subject \( s \), which we denote by the triplet \((s, r_1/r_2/\cdots/r_{\tau}, o)\), is given as

\[
\text{score}(s, r_1/r_2/\cdots/r_{\tau}, o) = \log \phi(v_o | \mu_{\text{context}}, \Sigma_{\text{context}}),
\]

with

\[
\mu_{\text{context}} = v_s + \sum_{t=1}^{\tau} \delta_{r_t}, \quad \Sigma_{\text{context}} = \sum_{t=1}^{\tau} \Sigma_{r_t},
\]

where the covariance associated with each relation is parameterized in the same way as in the previous subsection.

**Training with compositional relations** Let \( \mathcal{P} = \{ (s_i, r_{i_1}/r_{i_2}/\cdots/r_{i_{\tau_i}}, o_i) \}_{i=1}^{N'} \) be a set of randomly sampled paths from the knowledge graph. Here relation \( r_{i_k} \) in a path can be a relation in \( R \) or an inverse relation in \( R^{-1} \).

Now our training objective (3) is generalized as

\[
\min_{\{v_o \in \mathbb{R}^d\}, \{\delta_r, M_r, r \in R \cup R^{-1}\}} \frac{1}{N'} \sum_{(s, p, o) \in \mathcal{T} \cup \mathcal{P}} \mathbb{E}_{t' \sim N(s, p)} \left[ \left[ \mu - \text{score}(s, p, o) + \text{score}(s, p, t') \right]_+ \right] \\
+ \lambda \left( \sum_{o \in \mathcal{E}} \|v_o\|^2 + \sum_{r \in R \cup R^{-1}} \left( \|\delta_r\|^2 + \|M_r\|^2 \right) \right),
\]

where \( p \in \mathcal{T} \cup \mathcal{P} \) can be an atomic relation or a compositional relation, \( N' = |\mathcal{T} \cup \mathcal{P}| \), and \( N(s, p) \) denotes the set of incorrect objects for the subject \( s \) and path \( p \). Note that we model the inverse of a relation as a distinct relation.

### 3 Question Answering

Given a set of question-answer pairs, in which the question is phrased in natural language and the answer is an entity in the knowledgebase, our goal is to train a model that learns the mapping from the question to the correct entity. Our question answering model consists of three steps, entity recognition, relation composition, and conjunction. We first extract a list of entities mentioned in the question. If the question is “Who plays Forward for Borussia Dortmund?” then the list would be [Forward, Borussia Dortmund]. The next step is to predict the path of relations on the knowledge graph starting from each entity in the list extracted in the first step. In the above example, this will be /Forward/position_played_by/ and /Borussia_Dortmund/has_player/. In general, we can have multiple relations appearing in each path. As we described in the previous section, we model the composition of relations along a path as a series of Gaussian convolutions. Finally, we take the conjunction of all the Gaussian, which is equivalent to the Bayes rule with a noninformative prior, to produce the final attention.
Figure 2: Schematic illustration of the proposed question answering model. The input to the system is a question in natural language, which is “Who plays forward for Borussia Dortmund?” in this case. The two entities Forward and Borussia_Dortmund are extracted from the question and associated with point mass distributions centered at the entity vectors $\mathbf{v}_{\text{Forward}}$ and $\mathbf{v}_{\text{Borussia_Dortmund}}$, respectively. An LSTM encodes the input into a sequence of output vectors of the same length. Then we take a weighted average of the output vectors for each recognized entity to predict the relation associated with it. We predict

### 3.1 ENTITY RECOGNITION

We assume that there is an oracle that provides a list containing all the entities mentioned in the question, because we believe that this is a relatively easy part of the task. Of course, there is an option to jointly train a model for this step. In general, the number of extracted entities can be different for each question.

### 3.2 RELATION COMPOSITION

We train a long short-term memory (LSTM, Hochreiter & Schmidhuber (1997)) network that emits a hidden state $\mathbf{h}_t$ for each token in the input sequence. Then we take a weighted sum over the hidden states as

$$ o_e = \sum_{t=1}^{T} p_{t,e} \mathbf{h}_t \quad (5) $$

where the weight $p_{t,e}$ is computed for each recognized entity $e$ as

$$ p_{t,e} = \text{softmax}(f(\mathbf{v}_e, \mathbf{h}_t)), $$

where $\mathbf{v}_e$ is the vector associated with the entity $e$. The function $f(\mathbf{v}_e, \mathbf{h}_t)$ is a multi-layer perceptron that has one output node. Finally, softmax denotes softmax over the $T$ locations.

We use a two-layer perceptron for $f$ in our experiments, which can be written as follows:

$$ f(\mathbf{v}_e, \mathbf{h}_t) = \mathbf{u}_f^T \text{ReLU}(W_{f,v} \mathbf{v}_e + W_{f,h} \mathbf{h}_t + b_1) + b_2, $$

where $W_{f,v} \in \mathbb{R}^{L \times d}$, $W_{f,h} \in \mathbb{R}^{L \times H}$, $b_1 \in \mathbb{R}^L$, $\mathbf{u}_f \in \mathbb{R}^L$, $b_2 \in \mathbb{R}$ are parameters. Here ReLU$(x) = \max(0, x)$ is the rectified linear unit.

Then we compute the weights $\alpha_{r,e}$ over all the relations as

$$ \alpha_{r,e} = \text{ReLU}(\mathbf{w}_r^T \mathbf{o}_e) \quad (\forall r \in \mathcal{R} \cup \mathcal{R}^{-1}). $$
Here the rectified linear unit is used to ensure the positivity of the weights. Note however that the weights should not be normalized, because we may want to use the same relation more than once in the same path. Making the weights positive also has the effect of making the attention sparse and interpretable because there is no cancellation.

For each extracted entity $e$, we view the extracted entity and the answer of the question to be the subject and the object in some triplet $(s, p, o)$, respectively, where the path $p$ is inferred from the question as the weights $\alpha_{r,e}$ as we described above. Accordingly, the score for each candidate answer $a$ can be expressed using (1) as:

$$\text{score}_e(v_a) = \log \phi(v_a | \mu_{e,a,KB}, \Sigma_{e,a,KB}),$$

with

$$\mu_{e,a,KB} = v_e + \sum_{r \in R \cup R^{-1}} \alpha_{r,e} \delta_r,$$

$$\Sigma_{e,a,KB} = \sum_{r \in R \cup R^{-1}} \alpha_{r,e}^2 \Sigma_r,$$

where $v_e$ is the vector associated with entity $e$.

3.3 CONJUNCTION

Let $E(q)$ be the set of entities recognized in the question $q$. The final step of our model is to take the conjunction of the Gaussian attentions derived in the previous step. This step is simply carried out by multiplying the Gaussian attentions as follows:

$$\text{score}(v_o | E(q), \Theta) = \log \prod_{e \in E(q)} \phi(v_o | \mu_{e,a,KB}, \Sigma_{e,a,KB})$$

$$= -\frac{1}{2} \sum_{e \in E(q)} (v_o - \mu_{e,a,KB})^T \Sigma_{e,a,KB}^{-1} (v_o - \mu_{e,a,KB}) + \text{const},$$

which is again a (logarithm of) Gaussian scoring function, where $\mu_{e,a,KB}$ and $\Sigma_{e,a,KB}$ are the mean and the covariance of the Gaussian attention given in (5). Here $\Theta$ denotes all the parameters of the question-answering model.

3.4 TRAINING THE QUESTION ANSWERING MODEL

Suppose we have a knowledge-base $(E, R, T)$ and a trained TransGaussian model $(\{v_e\}_{e \in E}, \{\delta_r, \Sigma_r\}_{r \in R \cup R^{-1}})$. During training time, we assume the training set is a supervised question-answer pairs $\{ (q_i, E(q_i), a_i) : i = 1, 2, \ldots, m \}$. Here, $q_i$ is a question formulated in natural language, $E(q_i) \subseteq E$ is a set of knowledge-base entities that appears in the question, and $a_i \in E$ is the answer to the question. For example, on a knowledge-base of soccer players, a valid training sample could be

(“Who plays forward for Borussia Dortmund?”,[Forward,Borussia,Dortmund],Marco_Reus).

Note that the answer to a question is not necessarily unique and we allow $a_i$ to be any of the true answers in the knowledge base. During test time, our model is shown $(q_i, s_i)$ and the task is to find $a_i$. We denote the set of answers to $q_i$ by $A(q_i)$.

To train our question-answering model, we minimize the loss function

$$\ell(\Theta) := \frac{1}{m} \sum_{i=1}^m \mathbb{E}_{t \sim N(q_i)} \left[ -\mu \cdot \text{score}(v_{a_i} | E(q_i), \Theta) + \text{score}(v_t | E(q_i), \Theta) \right]_{+}$$

$$+ \lambda \|\Theta\|_2^2 + \nu \sum_{i=1}^m \sum_{e \in E(q_i)} \sum_{r \in R} |\alpha_{r,e}|$$

where $\mathbb{E}_{t \sim N(q_i)}$ is expectation with respect to a uniform distribution over all incorrect answers to $q_i$. In the experiments, we approximate it with 10 random samples. We assume that the number of relations implied in a question is relatively smaller than the total number of relations in the knowledge-base. Hence the coefficients $\alpha_{r,e}$ output from the neural network are regularized by $\ell_1$ norm.
4 EXPERIMENTS

We first test our TransGaussian model on a knowledge-base completion task to show its potential capability of generalizing to unseen facts. Then, we present our experiments on question answering on Worldcup 2014 dataset which is a demonstration of the main objective of this work.

4.1 KNOWLEDGE BASE COMPLETION

Large-scale knowledge base are known to be incomplete. Knowledge-base completion is a common task that test if a model is able to determine the validity of an unseen fact. Here, we build our model on the triplets and paths extracted from WordNet by Guu et al. (2015) and used the same train / dev / test splitting of the dataset. We only tested on atomic triplets. The same task has been performed by Socher et al. (2013); Wang et al. (2014) and many other researchers. It is also known as the link prediction task. Two versions of training were tested: SINGLE training with atomic triplets only and COMP training with atomic triplets and paths.

We did not incorporate word embedding in this task and each entity is assigned its individual vector. Without tuning the parameters too much, we obtain results comparable to the one in Guu et al. (2015). See Table 1 for our results.

4.2 QUESTION AND ANSWERING

We perform question and answering on a dataset of soccer players.

In this work, we consider two types of questions. A path query is a question that contains only one named entity from the knowledgebase and its answer can be found from the knowledge-graph by walking down a path consisting of a few relations. A conjunctive query is a question that contains more than one entities and the answer is given as the conjunction of all path queries starting from each entity.

4.2.1 WORLD CUP 2014 DATASET

We created this dataset of the players that participated in FIFA Worldcup 2014 based on the data available from Datahub. The dataset consists of 1127 entities, which consists of 736 players, 297 professional soccer clubs, 51 countries, 39 numbers, and 4 positions. There are 6 relations

- plays_in_club: PLAYER → CLUB,
- plays_position: PLAYER → POSITION,
- is_aged: PLAYER → NUMBER,
- wears_number: PLAYER → NUMBER,
- plays_for_country: PLAYER → COUNTRY,
- is_in_country: CLUB → COUNTRY,

where PLAYER, CLUB, NUMBER, etc, denotes the type of entities that can appear as the left or right argument for each relation. Some relations share the same type as the right argument, e.g., plays_for_country and is_in_country.

We treat these relations as atomic relations and transformed the dataset into a set of 3977 triplets. See Appendix for a list of sample triplets.

Table 1: Accuracy of knowledge-base completion on WordNet.

| Accuracy          | TransE (COMP) | TransGaussian (SINGLE) | TransGaussian (COMP) |
|-------------------|---------------|------------------------|----------------------|
| 80.3%             | 58.4%         | 76.4%                  |                      |

1https://datahub.io/dataset/fifa-world-cup-2014-all-players
2This is players’ jersey numbers in the national teams.
Table 2: Some statistics of the worldcup2014 dataset.

| # entity | # atomic relations | # atomic triplets | # path query Q&A (train / dev / test) | # conjunctive query Q&A (train / dev / test) |
|----------|-------------------|-------------------|--------------------------------------|-------------------------------------------|
| 1127     | 6                 | 3977              | 5149 / 952 / 1902                    | 1523 / 219 / 466                         |

In addition to these atomic relations, we randomly sampled 50000 paths with length 1 or 2 from the knowledge-graph. We built a TransGaussian model by using all these triplets and paths by the formulation introduced in Section 2. The dimension of embedding space was set to 30.

We created two sets of question answering tasks which we call path query and conjunctive query respectively.

**Path query.** Among the paths on the knowledge graph, there are some natural composition of relations, e.g., `plays_in_country` (PLAYER → COUNTRY) can be decomposed as the composition of `plays_in_club` (PLAYER → CLUB) and `is_in_country` (CLUB → COUNTRY). We manually picked a few meaningful composition of relations and created a set of path based question-answer pairs. See Table 6 for the list of composed relations, sample questions and answers. Note that all atomic relations in this dataset are many-to-one while these composed relations can be one-to-one, one-to-many or many-to-many.

**Conjunctive query.** To generate question-and-answer pairs of conjunctive queries, we first picked three pairs of relations:

- `plays_position and plays_in_club`,
- `plays_position and plays_for_country`,
- `plays_in_club and is_in_country`.

For a pair of relations $r_1$ and $r_2$, we enumerated all entities that can be their object and created queries of the form

$$\text{Find } e \in E, \text{ such that both } (e, r_1, o_1) \text{ and } (e, r_2, o_2) \text{ are true}.$$  

Each of the query is formulated in natural language. See Table 7 for a list of sample questions and answers.

### 4.2.2 Experimental results

**Evaluation metric** During test time, our model receives a question in natural language and a list of knowledge-base entities contained in the question, and then we ranks all entities in the knowledge-base by their scores according to Eq. (7). For each entity which is a correct answer, we check its rank relative to all incorrect answers and call this rank the filtered rank. For example, if a correct entity rank above all negative answers, it has filtered rank one. If there is one negative answer ranked above it, it has filtered rank two. We compute this rank for all true answers and report $H@1$ and mean filtered rank, where $H@1$ is the percentage where a true answer has filtered rank 1.

**Results** We list our results in Table 3 and 4. In addition to the embedding trained from our TransGaussian model, we embedded the Worldcup2014 dataset by using TransE (SINGLE) and TransE (COMP) introduced in Guu et al. (2015). As baselines, we used these embeddings in path query task and include the results in Table 3.

### 5 Related work

The work of Vilnis & McCallum (2014) is similar to our Gaussian attention model. They discuss many advantages of the Gaussian embedding; for example, it is arguably a better way of handling asymmetric relations and entailment. However the work was presented in the word2vec (Mikolov et al., 2013)-style word embedding setting. Our Gaussian attention model can be considered as
Table 3: Experimental results of path queries on Worldcup2014.

| Relation                  | TransE (SINGLE) | TransE (COMP) | TransGaussian (COMP) |
|---------------------------|-----------------|---------------|----------------------|
|                           | H@1(%) Mean     | H@1(%) Mean   | H@1(%) Mean          |
|                           | Filtered Rank   | Filtered Rank | Filtered Rank        |
| plays_in_country          | 94.44           | 88.27         | 99.38                |
| plays_position            | 99.43           | 92.33         | 99.43                |
| is_aged                   | 90.2            | 80.39         | 100                  |
| wears_number              | 35.29           | 72.79         | 100                  |
| plays_for_country         | 94.24           | 88.49         | 100                  |
| is_in_country             | 100             | 98.41         | 96.83                |
| plays_in_country^−1       | 82.86           | 74.29         | 98.1                 |
| plays_for_country^−1      | 39.61           | 35.75         | 72.46                |
| plays_position^−1         | 100             | 73.29         | 100                  |
| is_in_country^−1          | 30.81           | 25.58         | 46.51                |
| plays_in_country / is_in_country | 44.83 | 56.55 | 74.48 |
| plays_for_country^−1/plays_in_country | 14.02 | 26.17 | 43.93 |
| overall                   | 70.77           | 68.3          | 86.7                 |

Table 4: Experimental results of conjunctive queries on Worldcup2014.

| Relations                        | H@1(%) | H@10(%) | Mean Filtered Rank |
|----------------------------------|--------|---------|-------------------|
| plays_position and plays_in_club | 92.14  | 100.00  | 1.09              |
| plays_position and plays_for_country | 93.89 | 99.24  | 1.22              |
| plays_in_club and plays_for_country | 91.78 | 100.00 | 1.13              |

| Overall | 92.57 | 99.76 | 1.14 |

an extension of their work to a more general setting in which any memory item can be addressed through a concept represented as a Gaussian distribution over the memory items.

[Bordes et al. 2014; 2015] proposed a question-answering model that embeds both questions and their answers to a common continuous vector space. In [Bordes et al. 2015], they show that their method can combine multiple knowledgebases; they have shown that their model can even generalize to a knowledgebase that was not used during training. However, their method is limited to the simple question answering setting in which the answer of each question associated with a triplet in the knowledgebase. In contrast, our method can handle both composition of relations and conjunction of concepts, which are both naturally enabled by the proposed Gaussian attention model.

[Neelakantan et al. 2015] proposed a method that can combine relations to deal with compositional relations for knowledgebase completion. Their key technical contribution is to use recurrent neural networks to model the sequence of relations. Note that the weighted convolution we use in our model (see Figure 2) cannot handle non-commutative composition of relations well. It would be fruitful to combine their RNN-based composition with our Gaussian attention model in order to deal with both the propagation of uncertainty and the non-commutativity.

6 Conclusion

In this paper, we have proposed the Gaussian attention model which can be used in a variety of contexts where we can assume that the distance between the memory items in the latent space is compatible with some notion of semantics. We have shown that the proposed Gaussian scoring function can be used for knowledgebase embedding achieving competitive accuracy. We have also shown that our embedding model can naturally propagate uncertainty when we compose relations together. Our embedding model also benefits from compositional training proposed by [Guu et al. 2015]. Furthermore, we have demonstrated the power of the Gaussian attention model in a challenging question answering problem which involves both composition of relations and conjunction of thoughts. Future work includes experiments on natural question answering datasets and end-to-end training including the entity extractor.
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A  Wordcup2014 Dataset

Sample triplets:

| Subject             | Relation                  | Object          |
|---------------------|---------------------------|-----------------|
| david_villa         | plays_for_country         | spain           |
| lionel_messi        | plays_in_club             | fc_barcelona    |
| antoine_griezmann   | plays_position            | forward         |
| cristiano_ronaldo   | wears_number              | 7               |
| fulham_fc           | is_in_country             | england         |
| lukas_podolski      | is_aged                   | 29              |

Table 5: Statistics of the worldcup2014 dataset.

| metric                          | value           |
|---------------------------------|-----------------|
| # entity                        | 1127            |
| # atomic relations              | 6               |
| # atomic triplets               | 3977            |
| # relations (atomic and compositional) | 12            |
| # question and answer pairs in path queries | 5149 / 952 / 1902 |
| # types of questions in conjunctive queries | 5             |
| # question and answer pairs in conjunctive queries | 1523 / 219 / 466 |
| size of vocabulary             | 1730            |

Table 6: Path queries and sample questions.

| relation                  | type        | sample question                              | sample answer                          |
|---------------------------|-------------|---------------------------------------------|----------------------------------------|
| plays_in_club             | many-to-one | which club does alan pulido play for ?     | tigres, uanl                            |
| plays_in_club             | one-to-many | who plays for liverpool fc ?                | steven gerrard, masoud shojaei         |
| plays_in_club             | one-to-many | which soccer club is based in mexico ?     | cruz azul fc                            |
| plays_in_club / is_in_country | one-to-one | where is the club that edin dzeko plays for ? | england, crystal palace fc               |
| is_in_country             | one-to-many | which country is the soccer team fc porto based in ? | portugal                               |
| plays_in_club             | many-to-one | who plays forward for fc barcelona ?      | per_mertesacker, ezquiel lavezzi        |
| is_in_country             | one-to-many | which soccer team do players from australia play for ? | paris saint-germain, fc argentina      |
| plays_in_club             | many-to-many| which player in paris saint-germain fc is from argentina ? | lionel messi                            |

Table 7: Conjunctive queries and sample questions.

| relations                  | sample questions                              | sample answer                  |
|----------------------------|----------------------------------------------|---------------------------------|
| plays_position and plays_in_club | who plays forward for fc barcelona ?            | forward, fc_barcelona           |
| plays_position and plays_for_country | who are the defenders on germany national team ? | defender, germany              |
| plays_in_club and plays_for_country | which player in paris saint-germain fc is from argentina ? | paris_saint-germain, fc argentina |
| plays_in_club and plays_for_country | which soccer team do players from australia play for ? | cristian royal, cruz azul fc |