A New Variable Structure Multi-Model Target Tracking Algorithm

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Abstract. Maneuvering targets tracking technology has important research value in the fields of space target detection, civil aviation, ground traffic control and so on. To solve this problem, a new variable structure multi-model tracking algorithm based on fuzzy logic and strong tracking filter is proposed. The basic ideas of fuzzy logic inference and strong tracking filter are analyzed. An adaptive grid multi-model algorithm based on strong tracking filter and fuzzy interaction is established. The algorithm is also compared with generalized pseudo-Bayesian algorithm, interactive multi-model algorithm and traditional variable structure multi-model algorithm. The simulation results further prove the correctness and effectiveness of the proposed algorithm. Under the same simulation conditions, tracking performance of the proposed algorithm is significantly improved, and it has a higher cost-effectiveness ratio. Therefore, the proposed algorithm has a good application prospect in maneuvering target tracking.

1. Introduction
With the wide application of maneuvering target location and tracking technology in many important fields which affect the national economy and people's livelihood, it is of great significance to study a more rapid, accurate and robust target tracking algorithm. In 1992, Li XR proposed the concept of variable structure multi-model algorithm for the first time [1], which made a bold breakthrough on the fixed structure multi-model algorithm. At present, the research on multi-model algorithm is mostly based on Kalman filter and its improved algorithm. The matching degree of each model with the current motion pattern is calculated according to the prior probability and Markov transition probability of the model. However, we often know little about the target, and the Markov transition matrix also needs to be determined empirically, which makes the tracking performance of the traditional variable structure multi-model algorithm not ideal. Fuzzy logic inference can be used to implement rule-based control laws, especially when the system is uncertain or difficult to accurately describe by linear systems [2]. In addition, Zhou Donghua et al. proposed a Strong Tracking Filter (STF) [3], which can adjust the estimation bias adaptively and track the state changes rapidly.

In this paper, a new variable structure multi-model tracking algorithm is proposed based on fuzzy inference and strong tracking filtering algorithm. In the second part, the tracking problem of maneuvering target is analyzed, and the fuzzy logic inference process, strong tracking algorithm and adaptive grid method in the proposed algorithm are modeled concretely. In the third part, the proposed algorithm is compared with the generalized pseudo-Bayesian (GPB) algorithm, interactive multi-model (IMM) algorithm and traditional variable structure multi-model (VSMM) algorithm through simulation experiment in a real target motion scene. The simulation results prove that the theoretical
modeling is correct and the tracking performance of the proposed algorithm is improved compared with the traditional algorithm.

2. Adaptive grid multi-model algorithm based on strong tracking filter and fuzzy interaction

2.1. System Description

Consider a linear time-invariant system whose discrete state space model is expressed as follows:

\[
\begin{align*}
X_k &= \Phi X_{k-1} + G u_{k-1} \\
Z_k &= H X_k + v_k
\end{align*}
\]  
(Eq. 1)

In Eq.1, \( k \geq 1 \) is an integer, representing the \( k \)th observation time, \( X(k) \) is an \( n \) dimension state vector, \( Z(k) \) an \( m \) dimension measurement vector, \( \Phi \) an \( n \times n \) transition matrix, \( H \) an \( m \times n \) measurement matrix, \( G \) an \( n \times p \) process noise distribution matrix, \( u(k) \) and \( v(k) \) are process noise and measurement noise of the system, respectively, which are statistically independent. They are all zero-mean Gaussian white noise sequences with variances of \( Q(k) \) and \( R(k) \), respectively. Assuming that the state transition of the system follows a discrete-time Markov process, at the observation time \( k \), transition probability of the system from state \( i \) to state \( j \) can be expressed as follows:

\[
P^i_k = \frac{p_{ij}^k}{\sum_{j=1}^{M} p_{ij}^k}
\]  
(Eq. 2)

2.2. Algorithmic Design

The Adaptive Grid Multiple Model Based on Strong Tracking Filtering and Fuzzy Interacting algorithm is referred to as the AGMM_STF_FI algorithm. For the system described in the section 2.1, this paper uses a variable structure multi-model algorithm for tracking, and the adaptive grid method is selected as the model set adaptive strategy. To adapt to the strong maneuverability of the target, the strong tracking filter is selected when filtering. To solve the problem that the prior probability of the model and the Markov transition probability are difficult to determine, fuzzy logic inference is used to obtain the matching degree between the current motion pattern of the target and each model. The flow chart of the AGMM_STF_FI algorithm at the observation time \( k \) is shown in Figure 1.

![Flow chart of AGMM_STF_FI algorithm](image)

- **Step1**: Model set adaptive adjustment
  The AGMM_STF_FI algorithm uses adaptive grid method to adjust the model set adaptively. Three Constant Turning (CT) models are selected as the basic models, and the turning velocity can be adaptively changed. Assuming that the turning rate of maneuvering target varies continuously in the interval \([-w_{\text{max}}, w_{\text{max}}]\). The model set used at the observation time \( k \) consists of three CT models with the turning rates of \( w^L_k \), \( w^C_k \) and \( w^R_k \), respectively, \( w^L_k, w^C_k, w^R_k \in [-w_{\text{max}}, w_{\text{max}}] \), and the requirement of \( w^L_k < w^C_k < w^R_k \) is satisfied. Then the model set is expressed as \( \text{Model set} (k) = \{ w^L_k, w^C_k, w^R_k \} \). The continuous interval of the turning rate is taken as the grid of a model set. The adaptive process is divided into grid center adjustments and grid interval adjustments. Assuming at time \( k = 1 \), the model set used is \( \text{Model set} (1) = \{ w^L_1 = -w_{\text{max}}, w^C_1 = 0, w^R_1 = w_{\text{max}} \} \), for the concrete adaptive adjustment strategy from time \( k \) to \( k + 1 \), we can refer to reference [4]:
- **Step2**: Input Interaction
Assuming that the model set used contains $M$ basic models, the prediction probability of model $i$ at time $k+1$ is:

$$
\mu_{k+1|i} = P\{X_{k+1}^i | Z_k\} = \sum_{j=1}^{M} \mu_{k}^j \cdot p_{k}^{ji}
$$

(Eq. 3)

The interaction probability between model $i$ and model $j$ is:

$$
\mu_{k+1|i}^{ji} = P\{X_{k+1}^i | X_{k+1}^j, Z_k\} = \frac{\mu_{k}^{ji} \cdot p_{k}^{ij}}{\sum_{j=1}^{M} \mu_{k}^j \cdot p_{k}^{ji}}
$$

(Eq. 4)

The interactive output state vector is:

$$
\bar{X}_{k|i}^0 = E\{X_k | X_{k+1}^i, Z_k\} = \sum_{j=1}^{M} \mu_{k+1|i}^{ji} \bar{X}_{k|i}^j
$$

(Eq. 5)

The covariance matrix of the interactive output state vector is:

$$
\bar{P}_{k|i}^0 = E\{[\bar{X}_{k|i}^0 - X_k] [\bar{X}_{k|i}^0 - X_k]^T | X_{k+1}^i, Z_k\} = \sum_{j=1}^{M} \mu_{k+1|i}^{ji} \left[ P_{k|i}^{ji} + [\bar{X}_{k|i}^0 - \bar{X}_{k|i}^j] [\bar{X}_{k|i}^0 - \bar{X}_{k|i}^j]^T \right]
$$

(Eq. 6)

Step3: Tracking Filtering

The AGMM_STF_FI algorithm adopts STF algorithm, which is modified on the basis of Kalman filtering to enhance the robustness of the filter to model uncertainties [5]. To some extent, the performance of the filter can be reflected by the mean and amplitude of the output residual sequence $\gamma_k$. The basic idea of STF is to adjust the gain matrix $K_{k+1}$ online so that the two basic conditions of the minimum mean square error criterion hold:

$$
\begin{align*}
E\left[ (X_{k+1}^i - \hat{X}_{k+1}^i) (X_{k+1}^i - \hat{X}_{k+1}^i)^T \right] & = \text{min} \\
E\left[ \chi_{k+1}^i \cdot \chi_{k+1}^i \right] & = 0 \\
& \text{for } k=0,1,2,\cdots; j=1,2,3,\cdots
\end{align*}
$$

(Eq. 7)

The calculation process of STF is similar to that of Kalman filter, except that STF introduces a fading factor $\delta_{k+1}$ when calculating the co-variance of the predicted state. By adjusting co-variance matrix of the state prediction error and the corresponding gain matrix in real time, Eq.7 holds and system’s residual sequence is orthogonal, so that a better tracking performance of system state can be achieved in high maneuverability situations. Since the optimal fading factor has a large amount of calculation, the sub-optimal fading factor is adopted here, which has a high cost-effectiveness ratio. For the model $i$ of model set, the calculation process using strong tracking filtering algorithm is as follows [6]:

$$
\hat{X}_{k+1|i}^i = \Phi_k^i \hat{X}_{k|i}^i
$$

(Eq. 8)

$$
\gamma_{k+1|i} = Z_{k+1}^i - H_{k+1}^i \hat{X}_{k+1|i}^i
$$

(Eq. 9)

$$
V_{k+1|i}^i = E\left[ \chi_{k+1|i}^i \cdot \chi_{k+1|i}^i \right] = \begin{cases} 
\rho V_{k}^i + \gamma_{k+1|i}^i \gamma_{k+1|i}^i & \text{for } k \geq 1 \\
0 & \text{for } k=0
\end{cases}
$$

(Eq. 10)

$$
N_{k+1|i}^i = V_{k+1|i} - \beta R_{k+1|i} - H_{k+1}^i G_k^i Q_k^i G_k^i H_{k+1}^i
$$

(Eq. 11)

$$
M_{k+1|i}^i = \Phi_k^i P_{k|i}^i \Phi_k^i H_{k+1}^i H_{k+1}^i
$$

(Eq. 12)
\[ \eta_{k+1} = \frac{tr[N_{k+1}^i]}{\sum_{j=1}^{n} \alpha_{k+1}^i(j)M_{k+1}^i(j,j)} \]  
(Eq. 13)

\[ \lambda_{k+1}(j) = \begin{cases} 
\alpha_{k+1}^i(j)\eta_{k+1} & \text{if } \alpha_{k+1}^i(j)\eta_{k+1} > 1 \\
1 & \text{if } \alpha_{k+1}^i(j)\eta_{k+1} \leq 1
\end{cases} \]  
(Eq. 14)

\[ \delta_{k+1}^i = \text{diag}\{\lambda_{k+1}^i(1), \lambda_{k+1}^i(2), \cdots, \lambda_{k+1}^i(n)\} \]  
(Eq. 15)

\[ P_{k+1}^{i+1} = \delta_{k+1}^i, \Phi_{k+1}^i, P_{k+1}^i, \Phi_{k+1}^T + G_k^i Q_k^i G_k^T \]  
(Eq. 16)

\[ J_k^i = H_k^i, P_{k+1}^i, H_k^i, \cdots, R_k^i \]  
(Eq. 17)

\[ K_k^i = P_{k+1}^i H_k^i \cdots R_k^i S_k^i \]  
(Eq. 18)

\[ \hat{X}_{k+1}^{i+1} = \hat{X}_{k+1}^{i+1} + K_k^i J_k^i \]  
(Eq. 19)

\[ P_{k+1}^{i+1} = (I - K_k^i J_k^i) P_{k+1}^{i+1} \]  
(Eq. 20)

In Eq.10, \( \rho \in (0,1) \) is a forgetting factor, which is used to adjust the influence of the previous measurement residual on the current one. \( \rho = 0 \) means the previous measurement residuals are independent of the current one. The introduction of the forgetting factor can improve the adaptive ability of the filter in high maneuverability situations, but when the maneuver is too strong, it tends to cause the filter to diverge. In Eq.11, \( \beta \) is a weakening factor, which is used to adjust the influence of measurement errors on filter gain, and its value is generally determined empirically. In Eq.13, \( \alpha_{k+1}(i) \geq 1, i = 1,2,\cdots,n \), is a coefficient determined based on priori information. When \( \alpha_{k+1}(i) = 1, i = 1,2,\cdots,n \), the STF algorithm based on multiple fading factor is simplified to that based on a single fading factor. It can be proved that the STF algorithm can still guarantee a better tracking performance with less computation.

Step4: Computing the matching degree of each model

In AGMM_STF_FI algorithm, the fuzzy logic inference is used to calculate each model’s matching degree at the current moment. The following distance function is defined as the input of fuzzy logic reasoning.

\[ D_{k+1} = \gamma_{k+1}^T \gamma_{k+1} \]  
(Eq. 21)

Wherein, \( \gamma_k \) is the measuring innovation vector at time \( k \). It is apparent that the smaller the distance is, the higher the matching degree of the corresponding model. According to the distance’s characteristics, the inputs can be divided into three fuzzy subsets: Small, Medium and Large, and the Gauss membership function is selected [7].

The output of fuzzy logic is the posterior probability \( \mu \) of each model. Similarly, the model’s posterior probability is also divided into three fuzzy subsets: Small, Medium and Large, and the triangle membership functions are selected. According to the above analysis, the corresponding relationship between input and output is shown in Tab.1:

| Input       | Small | Medium | Large |
|-------------|-------|--------|-------|
| Output      | Large | Medium | Small |

Tab.1 Input-output correspondence in the calculation of model matching degree

Assuming that the model set contains \( M \) models, there are \( M^3 \) fuzzy logic rules in total. They are as follows:

IF \( \{ (D^1 \text{ is Small}) \text{ AND } (D^2 \text{ is Small}) \cdots \text{AND } (D^M \text{ is Small}) \} \), THEN \( \{ (\mu^1 \text{ is Large}) \text{ AND } (\mu^2 \text{ is Large}) \cdots \text{AND } (\mu^M \text{ is Large}) \} \);
IF \{(D^1 \text{ is Small}) \land (D^2 \text{ is Medium}) \land \cdots \land (D^M \text{ is Small})\},
THEN \{(\mu^1 \text{ is Large}) \land (\mu^2 \text{ is Medium}) \land \cdots \land (\mu^M \text{ is Large})\};

According to the above inference rules, the matching degree of each model can be obtained according to its distance input value. Here we use the centroid method as the defuzzification method. And then we can get the matching degree of each model, that is, the posterior probability \( \mu_{k+1}^{i}, \) \((i = 1,2,\cdots,M)\).

*Step 5: Output interaction*

The output state vectors after models' interaction are as follows:

\[
\hat{X}_{k+1|k+1} = \sum_{i=1}^{M} \hat{X}_{k+1|k+1}^{i} \mu_{k+1}^{i}
\]  \hspace{1cm} (Eq. 22)

The covariance of the output state vector is:

\[
P_{k+1|k+1} = \sum_{i=1}^{M} \mu_{k+1}^{i} \times \left[ P_{k+1|k+1}^{i} \right. + \left( \hat{X}_{k+1|k+1}^{i} - \hat{X}_{k+1} \right) \left( \hat{X}_{k+1|k+1}^{i} - \hat{X}_{k+1} \right)^{T} \]
\]  \hspace{1cm} (Eq. 23)

Above five steps are the calculation process of AGMM_STF_FI algorithm at the time \( k \).

### 3. Simulation Results and Discussions

#### 3.1. Simulation test

In order to prove the correctness and effectiveness of the AGMM_STF_FI algorithm proposed in this paper, we establish the following target motion scene to simulate and verify it. Assume that target’s initial position is \((340000, 2500)\), its initial speed is \((300, 50)\), and the sampling time of the system is \(1\) \(s\). Assume that the system noise of the target motion is Gaussian white noise with a mean of 0 and a variance of 0.1, and the measured noise is also Gaussian white noise with a mean of 0 and a variance of 10,000. The target’s maneuvering process is shown in Tab.2:

| Time (s) | Motion pattern | Motion parameters |
|----------|----------------|------------------|
| 0~30     | Constant Velocity | \( a_x = 0 \) m/s, \( a_y = 0 \) m/s |
| 30~45    | Constant Acceleration | \( a_x = 20 \) m/s, \( a_y = 30 \) m/s |
| 45~55    | Constant Turning | \( w = 0.2 \) rad/s |
| 55~70    | Constant Acceleration | \( a_x = -50 \) m/s, \( a_y = -10 \) m/s |
| 70~75    | Constant Turning | \( w = 0.3 \) rad/s |
| 75~90    | Constant Velocity | \( a_x = 0 \) m/s, \( a_y = 0 \) m/s |

When the AGMM_STF_FI algorithm is simulation verified, it is compared with the first-order generalized pseudo-Bayesian (GPB1) algorithm, the second-order generalized pseudo-Bayesian (GPB2) algorithm, the interactive multi-model (IMM) algorithm and the traditional variable structure multi-model algorithm (VSMM). The model sets of GPB1, GPB2 and IMM are all composed of CV, CA and CT model. The traditional VSMM uses adaptive grid method to adjust the model set, and the model interaction is based on fuzzy logic inference, while Kalman filter is used in tracking filtering. Assume that the initial probabilities of the three models are \( P_0 = \begin{bmatrix} 0.9 & 0.05 & 0.05 \end{bmatrix} \) and that their transition follows Markov property, and the transition probability matrices are:

\[
P = \begin{bmatrix} 0.95 & 0.025 & 0.025 \\ 0.025 & 0.95 & 0.025 \\ 0.025 & 0.025 & 0.95 \end{bmatrix}
\]
Several important parameters of AGMM_STF_FI algorithm are set as follows: impossible model probability threshold \( t_1 = 0.1 \), important model probability threshold \( t_2 = 0.9 \), minimum grid interval \( \delta_{\text{rad}} = 0.5 \text{ rad/s} \). In STK algorithm, forgetting factor \( \rho = 0.8 \), weakening factor \( \beta = 1 \), and \( \alpha_{k+1}(i) = 1, i = 1,2,\cdots,n \). Using the simulation tool of MATLAB, 100 Monte Carlo simulations were performed on GPB1, GBP2, IMM, VSMM and AGMM_STF_FI. Fig.2 is the tracking trajectories of each algorithm. Fig.3 and Fig.4 are position error curves and position root mean square error curves, respectively. And the root mean square error of the position, velocity and acceleration of these five algorithms are shown in Tab.3.

### Tab.3 Root mean square error of position, velocity and acceleration of each tracking algorithm

| Tracking algorithm | RMSE_position | RMSE_velocity | RMSE_acceleration |
|--------------------|---------------|---------------|-------------------|
| GPB1               | 493.5193      | 227.0306      | 25.3512           |
| IMM                | 340.0900      | 189.7573      | 24.4953           |
| GBP2               | 214.1165      | 175.3341      | 24.7263           |
| VSMM               | 136.5576      | 186.1367      | 25.3546           |
| AGMM_STF_FI        | 21.7077       | 1.6398e+05    | 25.3546           |

#### 3.2. Results analysis

By analyzing the curves in Figs. 2-4 and data in Table 3, following conclusions can be drawn:

1. In the actual motion scene designed in this paper, target’s motion is divided into six different stages (CV→CA→CT→CA→CT→CV). It is clear in figs. 3-4 that when the target motion model is maneuvering, tracking error will change abruptly and the error curve will appear a peak value. This is because when the target is maneuvering, the tracking algorithm needs a certain time to adaptively adjust the model set, and the shorter the duration of the error peak, the stronger the tracking algorithm’s adaptability.
As the tracking process proceeds, the tracking error will increase significantly; this is because the target lasts for a short period of time at each stage, while the adaptive adjustment of the model set takes a certain time. From Table 3, it can be seen that for the same target motion scene, under the same simulation conditions, the tracking performance is improved in the order of GPB1, IMM, GPB2, VSMM, AGMM_STF_FI, and the position average error of the proposed AGMM_STF_FI algorithm is minimal.

Output information fusion and conditional filtering information fusion in GPB algorithm are the same operation, while they are different in IMM. Therefore, IMM algorithm has a better information interaction and fusion effect among models. IMM’s tracking accuracy is much better than GPB1, and its computation amount is only $1/M$ of GPB2 when the tracking accuracy is near to that of GPB2, wherein $M$ is the amount of models contained in the model set.

When comparing AGMM_STF_FI with VSMM, STF is better than Kalman filter in robustness to target model and in adaptability to noise signal, so STF is an effective method for tracking high maneuvering targets.

Because the model set used in AGMM_STF_FI consists of CT models whose acceleration are 0 in both $x$ and $y$ directions, its tracking effect of CA motion is not ideal, and there is no significant improvement in the root mean square error of acceleration on the basis of other algorithms.

In summary, the AGMM_STF_FI algorithm proposed in this paper is obviously superior in tracking accuracy to other interactive multi-model algorithms, and by introducing SKF, the tracking error is also significantly reduced on the basis of the traditional VSMM algorithm. Simulation results show that AGMM_STF_FI algorithm, which use adaptive grid method, strong tracking filter and fuzzy logic inference, can match the target motion law much better and improve the tracking performance.

4. Conclusion
In order to improve the tracking performance of maneuvering targets, AGMM_STF_FI algorithm is proposed. By introducing fuzzy logic inference and strong tracking filter into variable structure multi-model tracking algorithm, the tracking problem of maneuvering target can be effectively solved. Simulation results show that the proposed algorithm can match the model well with the actual motion of the target by using only three models, which effectively reduce the amount of computation and has a better tracking performance. The cost-effectiveness ratio is also satisfactory. Therefore, the proposed algorithm has good application value in maneuvering target tracking.

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