High-energy Emission from Nonrelativistic Radiative Shocks: Application to Gamma-Ray Novae

Indrek Vurm$^{1,2}$ and Brian D. Metzger$^1$

$^1$ Physics Department and Columbia Astrophysics Laboratory, Columbia University 538 West 120th Street New York, NY 10027, USA; indrek.vurm@gmail.com

$^2$ Tartu Observatory Tõravere, Tartumaa EE-61602, Estonia

Received 2016 November 14; revised 2017 November 15; accepted 2017 November 19; published 2018 January 5

Abstract

The observation of GeV gamma-rays from novae by Fermi/LAT demonstrates that the nonrelativistic radiative shocks in these systems can accelerate particles to energies of at least $\sim 10$ GeV. The low-energy extension of the same nonthermal particle distribution inevitably gives rise to emission in the hard X-ray band. Above $\gtrsim 10$ keV, this radiation can escape the system without significant absorption/attenuation, and can potentially be detected by NuSTAR. We present theoretical models for hard X-ray and gamma-ray emission from radiative shocks in both leptonic and hadronic scenarios, accounting for the rapid evolution of the downstream properties due to the fast cooling of thermal plasma. We find that due to strong Coulomb losses, only a fraction of $10^{-4}$–$10^{-5}$ of the gamma-ray luminosity is radiated in the NuSTAR band; nevertheless, this emission could be detectable simultaneously with the LAT emission in bright gamma-ray novae with a $\sim 50$ ks exposure. The spectral slope in hard X-rays is $\alpha \approx 0$ for typical nova parameters, thus serving as a testable prediction of the model. Our work demonstrates how combined hard X-ray and gamma-ray observations can be used to constrain properties of the nova outflow (velocity, density, and mass outflow rate) and particle acceleration at the shock. A very low X-ray to gamma-ray luminosity ratio ($L_X/L_{\gamma} \lesssim 5 \times 10^{-4}$) would disfavor leptonic models for the gamma-ray emission. Our model can also be applied to other astrophysical environments with radiative shocks, including SNe IIn and colliding winds in massive star binaries.

Key words: novae, cataclysmic variables – radiation mechanisms: nonthermal – shock waves

1. Introduction

There are multiple lines of evidence that show shocks play an important role in shaping the multiwavelength emission from nova outflows. Among these are the peculiar early-time radio light curves exhibiting significantly higher brightness temperatures than can be explained by passively cooling expanding outflows (Krauss et al. 2011; Metzger et al. 2014; Weston et al. 2016a; Yang et al. 2015; Vlasov et al. 2016). Among the most compelling evidence to date is the unexpected detection of GeV gamma-rays from three classical novae by Fermi/LAT in 2012–2013: V959 Monocerotis, V1324 Scorpii, and V339 Delphini (Ackermann et al. 2014); since then, at least three more novae, V1369 Cen, V5668 Sgr, and ASASSN-16ma, have been detected with high significance (Cheung et al. 2016; Li et al. 2017). The comparatively high GeV luminosities of novae suggest that shocks dissipate a sizable fraction of the total energy budget, and could in some cases even dominate the optical output (Metzger et al. 2015; Li et al. 2017).

Despite clear evidence for shocks, their origin and location within the nova outflow remains uncertain. Chomiuk et al. (2014) suggests a connection between the shocks and the bipolar geometry of the nova outflow (e.g., Ribeiro et al. 2013; Shore 2013), which in turn could be shaped by the orbiting binary companion (e.g., Livio et al. 1990). Unfortunately, this is challenging to confirm because the shock emission cannot be resolved when the gamma-rays are detected near the peak of the nova outburst.

Compared to the vast range of different astrophysical sources where shocks play a significant role, the shocks in nova outflows probe a fairly unique regime in parameter space, characterized by relatively high densities and low velocities. In this regime, the shocks giving rise to detectable gamma-rays are likely to be radiatively efficient, i.e., all of the dissipated energy is radiated as multiwavelength emission.

Several works have been dedicated to calculating the shock emission in different frequency bands, ranging from radio to TeV gamma-rays (O’Brien et al. 1994; Nelson et al. 2012; Martin & Dubus 2013; Metzger et al. 2014, 2015, 2016; Vlasov et al. 2016). In this work we focus on the hard X-ray band ($\gtrsim 10$ keV) accessible to, e.g., NuSTAR, and explore in particular the connection between the radiation properties in the X-ray and gamma-ray bands. Our work is motivated by recent NuSTAR observations of V339 Del and V5668 Sgr, which place rather stringent upper limits on the hard X-ray emission at the time of the LAT detections (K. Mukai et al. 2017, in preparation); within the framework of our model, these and future observations can be used to place constraints on the location and the electron/ion acceleration efficiencies of the shocks in novae.

Although our analysis is focused primarily on shocks in novae, our results are also applicable to other astrophysical sources of nonrelativistic radiative shocks. These include, for instance, the dense colliding stellar winds of massive binary stars (e.g., De Becker 2007) and SNe IIn, in which the supernova ejecta collides with a dense external shell of gas surrounding the progenitor star (e.g., Chevalier & Fransson 1994; Smith et al. 2007).

1.1. Nonrelativistic Shocks in Dense Media

Shocks taking place in, e.g., gamma-ray novae ($v_{\text{sh}} \approx 10^9$ cm s$^{-1}$) and SNe IIn ($v_{\text{sh}} \approx 10^8$ cm s$^{-1}$) heat the bulk of the gas to X-ray temperatures. In sufficiently dense media, the cooling time of the shocked gas is short compared to the dynamical...
time of the system. This results in strong compression of the gas in the shock downstream as it cools (e.g., Drake 2005).

Strong observational evidence also exists that nonrelativistic shocks can accelerate particles (either electrons and/or protons/ions) to ultrarelativistic energies, which emit broadband nonthermal radiation from radio to gamma-ray frequencies. However, in contrast to relativistic shocks, the cooling time of the relativistic particles in nonrelativistic radiative shocks can exceed the cooling/compression time of the thermal gas behind the shock. This has two effects on the nonthermal particles and their radiation: (1) the rising density in the downstream alters the relative importance of different radiative processes, most importantly, relativistic bremsstrahlung and inverse Compton (IC) emission, and (2) rapid adiabatic compression supplies additional energy to the nonthermal particles. Thermal cooling thus affects both the radiative efficiency and the spectral shape of the nonthermal emission.

1.2. Radiative Processes in Leptonic and Hadronic Scenarios

The main goal of this paper is to establish a theoretical framework that enables one to combine nonthermal X-ray and gamma-ray data into a diagnostic tool for the shock environment and the properties of nonthermal particle acceleration. Regardless of whether leptonic or hadronic processes are responsible for the gamma-ray emission, the same radiative processes inevitably also gives rise to X-ray radiation. The relative luminosity in the X-ray and gamma-ray bands, \( L_X / L_{\gamma} \), is most sensitive to the ratio of matter to radiation energy density, which in turn depends on the density of the shocked gas and on the location of the shock within the nova outflow. The ratio \( L_X / L_{\gamma} \) is also sensitive to whether hadrons or leptons dominate the accelerated nonthermal particle populations and to the injected particle spectra.

In the leptonic scenario, gamma-ray emission is the result of direct electron acceleration; the dominant nonthermal particle processes are relativistic bremsstrahlung and IC scattering. The observed gamma-ray luminosities require the injected energy spectrum to be almost logarithmically flat, i.e., \( q = 2 \) (where \( dN_{\text{inj}} / d\gamma \propto \gamma^{-q} \) and \( \gamma \) is the particle Lorentz factor), to avoid an energy crisis (if \( q > 2 \), most of the nonthermal energy is in low-energy electrons and is radiated below the LAT band; see Metzger et al. 2015). This is consistent with the approximately flat \( \dot{\nu} F_{\nu} \) spectra observed in gamma-ray novae (Ackermann et al. 2014). If \( q = 2 \), the bremsstrahlung and IC spectra are similarly flat in the gamma-ray range, thus limiting the diagnostic value of the gamma-ray spectrum alone in determining the density and radiation compactness of the shock. Nonthermal X-ray emission provides an independent diagnostic, which is comparatively more luminous at low densities and high (optical) luminosities, for which IC cooling dominates the nonthermal emission.

The high GeV luminosities of LAT-detected novae require shocks to occur in relatively dense environments. Nonthermal X-ray emission in such cases results from a combination of bremsstrahlung and IC emission, modified by Coulomb (and possibly synchrotron) losses. Indeed, we show here that electrons with Lorentz factors \( \gamma \lesssim 10^3 \) lose most of their energy via Coulomb collisions with the thermal population, which significantly (although not completely) suppresses their radiative output below the LAT band. As a result, the gamma-ray spectrum breaks to a steeper slope at energies below a few hundred MeV; a naive extrapolation of the LAT spectrum to the NuSTAR band would therefore grossly overestimate the X-ray flux by as much as three orders of magnitude.

In the hadronic scenario, when the energy in accelerated protons dominates the electron energy, gamma-rays are mainly generated by the production and decay of neutral pions (\( \pi_0 \)), created by proton–proton/ion collisions. These collisions also produce charged pions (\( \pi^\pm \)) that ultimately decay into relativistic electron–positron pairs, which carry energy comparable to that in gamma-rays from \( \pi_0 \) decay. As in the leptonic model, the created pairs radiate both X-rays and gamma-rays via bremsstrahlung and IC emission, which dominates the emission between \( \sim 10 \text{ keV} \) and \( \sim 100 \text{ MeV} \). The main difference from the leptonic scenario is the paucity of injected pairs with energies well below the pion rest mass \( \sim 100 \text{ MeV} \), which would otherwise make a significant contribution to the X-ray flux (despite Coulomb losses). Overall, the additional \( \pi_0 \) gamma-rays, coupled with fewer X-ray emitting leptons, result in a systematically lower ratio of X-ray to gamma-ray flux in the hadronic scenario.

This paper is organized as follows. In Section 2 we give an overview of the radiative processes relevant in nova shocks. The evolution of the heated plasma downstream of radiative shocks is discussed in Section 3. In Section 4 we describe the evolution of nonthermal particle distributions as they radiate and cool in the compressing downstream flow. A theoretical overview of the X- and gamma-ray emission from the cooling layer is given in Section 5. The numerical results and the constraints on parameter space from simultaneous hard X-ray and gamma-ray observations are presented in Sections 6 and 7. Our results are discussed and conclusions are summarized in Section 8.

2. Radiative Mechanisms and Post-shock Cooling

2.1. Thermal Processes

Consider the plasma downstream of a nonrelativistic shock. The bulk of the shock energy is transferred to thermal plasma, which provides the pressure support in the immediate downstream. At shock velocities \( v_{\text{sh}} \sim 10^8 \text{ cm s}^{-1} \) (see e.g., Shore et al. 2013), the post-shock temperature corresponds to soft X-rays,

\[
T = \frac{3m_p \mu v_{\text{sh}}^2}{16k} \approx 1.7 \times 10^7 v_{\text{sh},8}^2 \text{ K,}
\]

where \( v_{\text{sh},8} \equiv v_{\text{sh}} / 10^8 \text{ cm s}^{-1} \) and \( \mu = 0.76 \) is the mean molecular weight appropriate for nova composition (Schwarz et al. 2007; Vlasov et al. 2016). Here we have assumed a shock compression ratio of 4, which we use throughout this paper. The back-reaction of the accelerated particles on the shock upstream can modify the jump conditions (see e.g., Drury 1983; Blandford & Eichler 1987 for reviews), but the error made by neglecting this effect is \( \lesssim 10\% \) for acceleration efficiencies \( \varepsilon \lesssim 0.1 \) considered in this paper, consistent with typical efficiencies found in particle-in-cell (PIC) simulations of nonrelativistic shocks (Caprioli & Spitkovsky 2014). The
modification of the post-shock temperature due to part of the pressure being carried by nonthermal particles (e.g., Chevalier 1983; Tatischeff & Hernanz 2007) is taken into account in the numerical calculations below.

The thermal plasma cools via free–free emission and line cooling. The latter dominates at $T \lesssim 10^8$ K; the shock is thus radiative if $t_{\text{line}} < t_{\text{exp}} = R/v_{\text{sh}},$ where

$$t_{\text{line}} \approx \frac{3kT}{8\mu m_{\text{H}} \lambda_{\text{line}}} \approx 3.1 \times 10^3 T_{10}^{1.7} n_0^{-1} \text{s} \approx 7.8 \times 10^3 v_{\text{sh}}^{3.4} n_0^{-1} \text{s},$$

where we have approximated $\lambda_{\text{line}} \approx 2.2 \times 10^{-22} (T/10^7)^{-0.7}$ erg cm$^3$ s$^{-1}$ (Schure et al. 2009; see also Vlasov et al. 2016). The upstream density $n$ has been normalized to the value corresponding to a typical mass-loss rate $10^{-5} M_\odot$ per week and a characteristic radius $R \approx 10^{14}$ cm on a timescale of a week as relevant for gamma-ray emission. On the same timescale, the shock is likely to be radiative if $v_{\text{sh}} \lesssim 2 \times 10^8$ cm s$^{-1}$ unless the density is very low, which would result in a gamma-ray luminosity that is too low to be detected by Fermi (Metzger et al. 2015), however.

If the shock is indeed radiative, cooling of the thermal plasma leads to strong compression in the downstream in order to maintain the required pressure. This compression is halted only when either nonthermal or magnetic pressure becomes dominant over thermal pressure, or when the gas cools to temperatures $\lesssim 10^4$ K, below which line cooling becomes less efficient due to recombination of the gas.

The luminous thermal $\sim$keV X-rays from the gamma-ray emitting shocks are not directly observable, as they are absorbed by bound-free processes in the material ahead of the shock. Instead, this energy is reprocessed to lower frequencies and released as optical/UV radiation (Metzger et al. 2014). In contrast, nonthermal X-rays with higher energy $\gtrsim 10$ keV are not significantly attenuated by bound-free absorption and only interact with the ejecta via Compton (Thomson) scattering. They could therefore potentially be detected at early times simultaneously with the gamma-ray emission.

### 2.2. Nonthermal Processes: Leptonic Scenario

In addition to thermal heating, a portion of the shock energy is used to accelerate a fraction of the electrons and/or baryons into a nonthermal distribution. In the leptonic scenario, the relativistic electrons cool via IC emission on (primarily) optical/UV photons, relativistic bremsstrahlung emission, Coulomb collisions with thermal electrons, and synchrotron emission if the downstream is appreciably magnetized. The interplay between these processes determines both the dominant radiative mechanism at hard X- and gamma-ray frequencies, as well as the partitioning of the nonthermal luminosity between different bands.

The key parameter that determines the dominant cooling mechanism of relativistic electrons is the ratio of soft (optical) radiation energy density (that determines the IC cooling rate) to matter density (that determines bremsstrahlung and Coulomb losses),

$$\chi \equiv \frac{u_{\text{opt}}}{\rho c^2}, \tag{3}$$

For typical parameters in gamma-ray novae, one obtains

$$\chi = 3.2 \times 10^{-5} L_{\text{opt},38} n_0 R_{14}^{-3}, \tag{4}$$

where $L_{\text{opt}} = 10^{38} L_{\text{opt},38}$ erg s$^{-1}$ is the optical luminosity, and $R = 10^{14} R_{14}$ cm is the shock radius. We have used $u_{\text{opt}} = L_{\text{opt}}/(4\pi c R^2),$ i.e., we neglected the $(1 + \tau_T)$ correction under the assumption that the Thomson optical depth $\tau_T \approx n R \sigma_T \approx 0.06 n_0 R_{14}$ of the shocks is $\lesssim 1.$

If the bulk of the optical luminosity is generated by reprocessed emission from the shock itself, then

$$L_{\text{opt}} \approx L_{\text{shock}} = \frac{9\pi}{8} m_p n_{\text{sh}} v_{\text{sh}}^3 R_{14}^2 f_{\text{ol}},$$

$$= 5.9 \times 10^{37} R_{14}^{-1} n_0 v_{\text{sh},8} f_{\text{ol}} \text{erg s}^{-1}, \tag{5}$$

where $f_{\text{ol}}$ is the fraction of the total solid angle subtended by the shock. Again expressing $u_{\text{opt}},$ one obtains from Equation (3)

$$\chi = 1.9 \times 10^{-5} v_{\text{sh},8}. \tag{6}$$

This represents the minimum value of $\chi$ that can be attained at a given shock speed.

Consider separately the cooling rates by different processes. Free–free emission from electrons of energy $\gamma \gtrsim 10$ receives comparable contributions from electron–electron and electron–proton bremsstrahlung. For analytical estimates we employ the approximate expression (accurate within $15\%$ for $\gamma = 10-10^4$)

$$\dot{\gamma}_{\text{br}} \approx \frac{5}{6} c \sigma_T \alpha_{fs} n_{\text{ds}} \gamma^{1.2} \sum_i \frac{X_i Z_i (1 + Z_i)}{A_i},$$

$$\approx \frac{5}{3} c \sigma_T \alpha_{fs} n_{\text{ds}} \gamma^{1.2}, \tag{7}$$

where $n_{\text{ds}} \approx 4 n = \rho/m_p$ is the downstream density of the shock, $\alpha_{fs} \approx 1/137$ is the fine structure constant, and the sum is taken over the atomic species of mass fraction $X_o,$ charge $Z_i,$ and atomic weight $A_i.$ We use the more accurate expression valid in both relativistic and nonrelativistic regimes given by Haug (2004) in our numerical calculations.

The IC cooling rate in the Thomson regime is

$$\dot{\gamma}_{\text{IC}} = \frac{4 \sigma_T u_{\text{opt}} (\beta/\gamma)^2}{3 m_e c}, \tag{8}$$

where $\beta = (1 - 1/\gamma^2)^{1/2}.$ Its ratio to the bremsstrahlung cooling rate is (in the $\gamma \gg 1$ limit)

$$\frac{\dot{\gamma}_{\text{IC}}}{\dot{\gamma}_{\text{br}}} = \left(\frac{\gamma}{\gamma^{*}}\right)^{0.8}, \tag{9}$$

where $\gamma^{*} = \gamma^2 (\beta/\gamma)^2.$

---

$^3$ Some novae show hard $\gtrsim$keV thermal X-ray emission of luminosity $L_x \sim 10^{35} - 10^{36}$ erg s$^{-1}$ within days to weeks of the outburst (e.g., V5589 Sgr: Weston et al. 2016b), consistent with being powered by adiabatic (nonradiative) shocks (Mukai & Ishida 2001; Osborne 2015). However, the kinetic power of these “fast” X-ray producing shocks are generally too low to explain the luminous LAT GeV emission, suggesting that they originate from a different location within the ejecta (e.g., Vlasov et al. 2016).

$^4$ Equivalently, $\chi$ can be defined as the ratio of radiation compactness $L_{\text{rad}} = \sigma_T u_{\text{opt}} R/(m_e c^2)$ to the Thomson opacity $\tau_T = \sigma_T n R,$ $\chi = L_{\text{rad}}/\tau_T.$
Here, the first case assumes that the optical luminosity is external to, and greater than, that generated at the shock. In the second case, the optical luminosity is given by Equation (5).

The Coulomb cooling rate is

$$\frac{\dot{\gamma}_{\text{Coul}}}{\dot{\gamma}_{\text{br}}} = \frac{3}{2} \ln \left( \frac{c\sigma T n_{\text{ds}}}{\beta} \right) \sum_i \frac{X_i Z_i}{A_i} \approx \frac{3}{2} \ln \left( \frac{c\sigma T n_{\text{ds}}}{\beta} \right),$$

(11)

The ratio of bremsstrahlung and Coulomb cooling is independent of the shock parameters,

$$\frac{\dot{\gamma}_{\text{Coul}}}{\dot{\gamma}_{\text{br}}} \approx \left( \frac{\gamma^*}{\gamma} \right)^{-1},$$

(12)

where

$$\gamma^* = \left( \frac{\ln \Lambda}{\alpha_{\text{fs}}} \right)^{0.83} \approx 900 \left( \frac{\ln \Lambda}{25} \right)^{0.83}$$

(13)

denotes the electron energy below which bremsstrahlung emission is affected by Coulomb losses.

Depending on the downstream magnetization, synchrotron radiation can also be significant. Although synchrotron emission is unlikely to be detectable at early times when gamma-rays are observed, due to free–free absorption (Metzger et al. 2014), it manifests indirectly by attenuating the power emitted in the LAT band. In complete analogy with IC cooling, the ratio of synchrotron to bremsstrahlung cooling rates is

$$\frac{\dot{\gamma}_{\text{syn}}}{\dot{\gamma}_{\text{br}}} = \left( \frac{\gamma}{\gamma^*} \right)^{0.8},$$

(14)

where

$$\gamma^* = \left( \frac{5\alpha_{\text{fs}}}{\chi_B} \right)^{1.25} \approx 1.8 \times 10^5 \varepsilon_{B,-4}^{-1.25} \varepsilon_{\text{sh},-8}^{-2.5},$$

(15)

and

$$\chi_B = \frac{u_B}{m_e c^2 n} = 2.3 \times 10^{-6} \varepsilon_{B,-4} \varepsilon_{\text{sh},-8}^{-0.4}. $$

(16)

Here $\varepsilon_B$ parametrizes the post-shock magnetic energy density in terms of the total energy density as $u_B = (9/8)m_p\varepsilon_B\varepsilon_{\text{sh},-8}^2$.

The above expressions for $\gamma^*$ and $\gamma_*$ are calculated for conditions immediately after the shock. However, when the plasma compresses further downstream, the bremsstrahlung and Coulomb cooling rates are enhanced proportionally to $n$, while the IC rate is unaffected by compression. As a result, the Lorentz factor above which IC dominates free–free cooling increases as $\gamma^* \propto \chi^{-1.25} \propto n^{1.25}$. Synchrotron losses are also enhanced by compression, to a greater extent than bremsstrahlung: $\dot{\gamma}_{\text{syn}}/\dot{\gamma}_{\text{br}} \propto n^{\alpha_{\text{syn}}-1}$, where $u_B \propto n^{\alpha_{\text{syn}}}$ and the adiabatic index $\alpha_{\text{syn}} = 4/3 - 2$ depending on the magnetic field configuration.

It is instructive to compare the thermal cooling time behind the shock to the radiative and Coulomb loss times of the relativistic electrons/pairs. For bremsstrahlung, we find from Equations (2) and (7)

$$t_{\text{br}} = 2.6 \times 10^5 \gamma_3^{-0.2} n_{\text{sh},-8}^{-1} \text{s}, \quad t_{\text{br}} = \frac{33 \gamma_3^{-0.2} \varepsilon_{\text{sh},-8}^{-3.4}}{t_{\text{line}}} $$

(17)

where $\gamma_3 = \gamma/10^3$, and $n$ is the upstream density. Similarly, for Coulomb cooling, one obtains (for $\gamma \gg 1$)

$$t_{\text{Coul}} = \frac{3.3 \times 10^5 \gamma_3^{-1} \ln(\frac{\Lambda}{25})^{-1}}{t_{\text{line}}} \text{s},$$

(18)

Finally, for IC, we find

$$t_{\text{IC}} = \left\{ \begin{array}{ll}
1.2 \times 10^6 \frac{L_{38}^{-1}}{R_{44}^2} & L_{\text{opt}} > L_{\text{shock}} \\
2.0 \times 10^6 \gamma_3^{-1} \varepsilon_{\text{sh},-8}^{-0.4} & L_{\text{opt}} \approx L_{\text{shock}}
\end{array} \right.$$

(19)

and

$$t_{\text{IC}} = \left\{ \begin{array}{ll}
\frac{4.7}{\gamma_3 \varepsilon_{\text{sh},-8}} & L_{\text{opt}} > L_{\text{shock}} \\
\frac{150 \gamma_3^{-1} R_{44}^2 \varepsilon_{\text{sh},-8}^{-0.4}}{250 \gamma_3^{-1}} & L_{\text{opt}} \approx L_{\text{shock}}
\end{array} \right.$$

(20)

For shock velocities $\varepsilon_{\text{sh},-8} \lesssim 2 \times 10^8 \text{ cm s}^{-1}$, a range of $\gamma$ exists over which particles are unable to cool before the plasma has strongly compressed. In this regime, three different regions can be identified in electron energy space; using our fiducial parameter set as an example, one finds that (1) below $\gamma$ of a few tens, electrons rapidly share their energy with the thermal population via Coulomb interactions, (2) above $\gamma \sim 10^5$, the electrons lose most of their energy via IC before the plasma has time to significantly compress, and (3) in the intermediate range, the electrons undergo significant compression before cooling, and gain a moderate amount of additional energy from adiabatic heating. This intermediate range of $\gamma$ is broader for slower (more radiative) shocks. Conversely, in faster shocks, the nonthermal particles cool faster than the thermal plasma can cool, such that the adiabatic heating described above does not arise (note that $t_{\text{br}}/t_{\text{line}}$ is independent of density).

Figure 1 summarizes the different cooling regimes in the $\varepsilon_{\text{sh}}$–$\chi_B$ parameter space. The high observed luminosities of the Fermi/LAT emission from novae constrain the allowed region to lie not too far above the dotted line, which denotes where the total shock power is comparable to the optical luminosity (Metzger et al. 2015). In this region, the shock is radiative for velocities $\varepsilon_{\text{sh},-8} \lesssim 2 \times 10^8 \text{ cm s}^{-1}$ and thermal line cooling/compression is faster than either IC or bremsstrahlung cooling (red dashed line). In the radiative shock regime, IC losses are at most comparable to bremsstrahlung losses for the range of electron energies $\gamma \lesssim 10^3$ that are later shown to be responsible for most of the hard X-ray emission. Note that the relative dominance of bremsstrahlung losses over IC losses is further enhanced as the downstream plasma compresses.

2.3. Nonthermal Processes: Hadronic Scenario

Protons that undergo diffusive shock acceleration are injected into the downstream with a distribution $dN_p/d(\gamma_p\beta_p) \propto (\gamma_p\beta_p)^{-q_p}$, where $q_p = 2-2.5$ (Blandford & Ostriker 1978; Caprioli & Spitkovsky 2014), which places most of the...
nonthermal energy in relativistic protons ($E_p \gtrsim 1$ GeV). In dense media, they subsequently cool via hadronic collisions with thermal ions on a timescale

$$t_{pp} \approx \frac{1}{c \sigma_{pp} n_{ds}} = 2.5 \times 10^5 n_{-1} \, \text{s},$$

where $\sigma_{pp} \approx \sigma_T/20$, and we have again used $n_{ds} = 4n$. For $v_{th} < 3 \times 10^8$ cm s$^{-1}$, downstream compression due to thermal cooling occurs faster than hadronic losses,

$$t_{pp} \approx 32 \, \text{s}.$$

The cooling regimes in the $v_{th}$-$\chi$ parameter space are shown in Figure 1 (right panel). Note that if the shock is radiative, the thermal cooling/compression behind the shock is always faster than the losses due to nuclear collisions. The value of $t_{pp} \propto n^{-1}$ decreases as the post-shock gas compresses, leading to more efficient hadronic losses and extending the parameter range over which the energy of the accelerated protons can be efficiently tapped.

Mildly relativistic protons ($E_p \approx 1$ GeV) lose comparable fractions of their energy via elastic and inelastic collisions; the elastic fraction decreases at higher energies. The inelastic collisions produce both neutral and charged pions ($\pi^0$, $\pi^\pm$), which ultimately decay into GeV gamma-rays, $e_\pm$-pairs, and neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$, and $\bar{\nu}_e$). The spectra of the injected gamma-rays and pairs roughly mimic the slope of the primary protons (Kamae et al. 2006); for example, with $q_p = 2$, their distribution is flat in energy per logarithmic interval of $\nu$ ($\gamma$). Both the injected photon and electron spectra have a low-energy turnover at $\sim 100$ MeV.

The relativistic pairs cool via bremsstrahlung and IC emission (Section 2.2), generating both X-ray and gamma-ray radiation. Compared to the leptonic model, the fraction of the total nonthermal energy emerging as hard X-rays is lower, for two main reasons: (1) the fraction of energy of the injected $e_\pm$ pairs is only $\sim 10\%-20\%$ of the total energy dissipated via hadronic collisions, and (2) the pair injection spectrum has a turnover at $\gamma \sim 200$. The electrons that radiate in the hard X-ray band have Lorentz factors of $\gamma \approx 1$–1000; in leptonic models, the electrons are injected with comparable power throughout this range, while only $\gamma \sim 100$–1000 leptons from $\pi^\pm$ decay can make an appreciable contribution in the hadronic case.

3. Structure of the Cooling Layer

Consider the thermodynamic evolution of plasma in the downstream of a nonrelativistic shock. The gas pressure in the immediate downstream is dominated by thermal plasma. In addition, a fraction $\varepsilon_{nth}$, $\varepsilon_p$, and $\varepsilon_B$ of the energy is deposited into nonthermal electrons, baryons, and the magnetic field, respectively (over scales much smaller than the post-shock cooling length). The downstream plasma cools via radiation, both thermal and nonthermal, and compresses. If the radiative cooling timescale is shorter than the expansion time, the cooling layer is in a quasi-steady state with approximately constant total pressure.$^5$

Consider a small (Lagrangian) volume element of plasma as it propagates into the downstream. The first law of thermodynamics for this fluid element, $\delta (uV) = -p\delta V + V\delta Q_{rad}$,$^5$

$^5$ Immediately after the shock ($m_p n_{ds} u_{sh}^2 / \beta_{sh} = 1/3$, where the subscript “ds” refers to values in the immediate downstream and $u_{sh}$ is measured relative to the shock front (Metzger et al. 2014). In the steady-state, momentum conservation in the cooling layer imposes the condition $m_p n^2 v^2 + p = 4\beta_{sh}/3 = \text{constant}$. Thus, the post-shock pressure can increase at most by a factor 4/3 as the plasma cools ($v^2 \rightarrow 0$); we ignore this slight increase in our calculations.
can be rewritten as
\[ \delta u = h \delta \ln n + \delta Q_{\text{rad}}. \quad (23) \]

Here \( u \) and \( h = u + p \) are the total energy density and enthalpy per unit volume, respectively, \( n \) is the density, and \( \delta Q_{\text{rad}} \) is the total energy loss from the element (via radiation and neutrinos) per unit volume.

The assumption of constant downstream pressure imposes a constraint
\[ \delta p = \delta p_{\text{th}} + \delta p_{\text{nth}} + \delta p_p + \delta p_B = 0, \quad (24) \]

where \( p_{\text{th}}, p_{\text{nth}}, p_p, \) and \( p_B \) are partial pressures of the thermal plasma, nonthermal (accelerated) leptons, nonthermal protons, and the magnetic field, respectively.

Equations (23) and (24) must be complemented by an equation of state (EOS) for each component of the plasma, i.e.,
\[ h_i = \alpha_i u_i = \alpha_i p_i / (\epsilon_i - 1), \]
where \( \alpha_i \) is the adiabatic index. For the thermal component, \( \alpha_{\text{th}} = 5/3 \). The adiabatic index for the magnetic field depends on the field configuration; \( \alpha_B = 4/3 \) for a tangled field with a coherence length much smaller than the cooling length, while \( \epsilon_B = 2 \) for an ordered field perpendicular to the direction of compression\(^8\) (i.e., shock normal). The equations of state (EOS) of the nonthermal components of the plasma do not admit the above simple form, as they contain both nonrelativistic and relativistic particles, and hence must be found explicitly at each timestep from the solution of the nonthermal evolution (see below).

Equations (23) and (24) can be transformed to read
\[ \delta \ln n = \frac{(\alpha_{\text{th}} - 1)(\delta u_{\text{th}} + \delta u_p + \delta Q_{\text{rad}}) - (\delta p_{\text{th}} + \delta p_B)}{(\alpha_{\text{th}} - 1)h + (\alpha_B - \alpha_{\text{th}})h_B}, \]

where we have used the EOS described above for the thermal plasma and the magnetic field, as well as \( \delta u_{\text{th}} = h_{\text{th}} \delta \ln n \) for the adiabatically evolving B-field. Coupled with equations for the energy spectra of nonthermal electrons and protons (which provide \( \delta u_{\text{nth}}, \delta p_{\text{nth}}, \delta u_p, \) and \( \delta p_p \)), Equation (25) determines the downstream evolution of the shocked plasma. Here \( \delta Q_{\text{rad}} \) accounts for radiative (and neutrino) losses of both thermal and nonthermal plasma, but does not explicitly involve coupling (e.g., Coulomb) between the thermal and nonthermal particles. Once \( \delta n_B \) is known, the updated magnetic pressure is found from \( p_B \propto n^{\epsilon_B} \); condition (24) then yields the new \( p_{\text{th}} \). We neglect any reconnection or decay of the magnetic field, e.g., as could occur due to ambipolar diffusion once the temperature cools to \( \sim 10^4 \) K and the gas becomes neutral.

Figures 2 and 3 (left panel) show the evolution of the pressure and density in the cooling layer in the leptonic and hadronic models. Immediately behind the shock, the bulk of the dissipated energy is stored in thermal plasma; its (line-)cooling therefore determines the downstream evolution over the first thermal cooling time. The plasma initially compresses as \( n \propto T^{-1} \) at \( p_{\text{th}} \approx \) constant; the decreasing temperature and increasing density speed up the cooling \( t_{\text{cool}} \approx T^{-1.5} n^{-1} \), which leads to runaway loss of thermal pressure.

In the leptonic model, Figure 2 (left panel) shows that most relativistic electrons are unable to cool over the compression timescale of the plasma. Here the electron density relaxation occurs on a much longer timescale than in the case of hadronic collisions, Coulomb scattering, as well as heating by adiabatic compression. The right-hand side account for injection of pairs due to hadronic collisions; \( Q_{\text{mig}} = 0 \) in the proton equation.

4. Nonthermal Evolution

The distribution functions \( N(\gamma) \) of nonthermal leptons and protons evolve with distance \( z \) downstream of the shock according to (Appendix)
\[ \frac{\partial N(\gamma)}{\partial t} + \frac{\partial}{\partial \varepsilon}[\varepsilon N(\gamma)] + \frac{\partial}{\partial \gamma}[\gamma N(\gamma)] = Q_{\text{mig}}, \]

where \( \varepsilon \) is the downstream velocity, and \( \gamma \) is the energy loss/gain rate due to all interactions in units of the particle rest energy per second. Here \( N(\gamma) \) is defined such that \( n_{\text{nth}} = \int N(\gamma) d\gamma \). For leptons, \( \gamma \) accounts for IC, bremsstrahlung, and Coulomb losses, as well as adiabatic heating/cooling.\(^7\) For protons, \( \gamma \) represents losses due to nuclear collisions, Coulomb scattering, as well as heating by adiabatic compression. The right-hand side account for injection of pairs due to hadronic collisions; \( Q_{\text{mig}} = 0 \) in the proton equation.

Equation (26) can be cast in a more convenient form by expanding the second term and using the continuity equation.

\(^7\) Here all processes are treated as continuous; this assumption breaks down for IC emission in the Klein–Nishina regime, which becomes relevant at \( \gamma \approx 10^3 \). However, the average energy loss rate over several scatterings is still accurate, provided that \( \gamma \) appropriately accounts for KN suppression.
The Astrophysical Journal, 852:62 (17pp), 2018 January 1
Vurm & Metzger

Figure 2. Evolution of downstream pressure. The parameters are as follows: pre-shock density $n = 3 \times 10^{8}$ cm$^{-3}$, shock velocity $v_{sh} = 10^{8}$ cm s$^{-1}$, and $\chi = 10^{-4}$ (which corresponds to $L = 10^{38}$ erg s$^{-1}$, $R = 10^{14}$ cm). Left panel: leptonic case. The nonthermal injection fraction is $\varepsilon_{nth} = 0.01$, and the magnetization is $\beta_{g} = 10^{-4}$. Nonthermal electrons are injected at the shock with a distribution $dN/d(\gamma\beta_{g}) = Q_{nth}(\gamma\beta_{g})$, where $\beta_{g} = 2$, and the distribution extends to $\gamma_{max} = 10^{5}$. Right panel: hadronic case. The nonthermal injection fraction is $\varepsilon_{nth} = 0.01$, and the magnetization is $\beta_{g} = 10^{-4}$. Nonthermal protons are injected with a distribution $dN_{p}/d(\gamma\beta_{p}) = Q_{nth}(\gamma\beta_{p})$, where $\beta_{p} = 2$, and the distribution extends to $\gamma_{p,max} = 10^{5}$. A weak nonthermal electron distribution with $\varepsilon_{nth} = 10^{-4}$ is also injected, with the same distribution as in the leptonic case. The adiabatic index for the magnetic field is $\alpha_{B} = 2$ in both panels. The Lagrangian time of a fluid element on the $x$-axis corresponds to coordinate $z = \int v(t') dt'$ (assuming $t < t_{exp}$).

\[ \partial n/\partial t + \partial (vn)/\partial z = 0, \]
\[ \frac{d}{dt} \left[ \frac{N(\gamma)}{n} \right] = -\frac{\partial}{\partial \gamma} \left[ \frac{\gamma N(\gamma)}{n} \right] + \frac{Q_{nth}}{n}, \]

(27)

where the time derivative is now taken along the path of the fluid element as it propagates into the downstream. Here $n$ represents the inverse of the comoving volume element (such that $nV = \text{constant}$), or equivalently, the density of any type of particle whose total number is conserved (e.g., baryons).

The rate of adiabatic heating/cooling of a particle of energy $\gamma$, as determined by considering the first law of thermodynamics for a monoenergetic particle distribution, is given by

\[ \Gamma_{\text{adiab}} = \frac{1}{3} \gamma^{2} \beta^{2} \frac{d \ln n}{dt}, \]

(28)

where $\beta$ is the particle velocity in units of $c$.

Given $N(\gamma)$, the nonthermal pressure and (kinetic) energy densities $p_{nth}$, $u_{nth}$, $P_{p}$, and $u_{p}$ are found from

\[ p = \frac{1}{3} mc^{2} \int_{1}^{\infty} N(\gamma) \gamma^{2} d\gamma, \]
\[ u = mc^{2} \int_{1}^{\infty} N(\gamma) (\gamma - 1) d\gamma, \]

(29)

where $m$ is the particle mass.

Equations (25) and (27) are coupled via $n_{p}$, $p_{nth}$, $u_{nth}$, $P_{p}$, and $u_{p}$ and are solved iteratively at each step. In the radiative regime, the downstream structure at any given radius is given by $n(z)$ and $N(\gamma, z)$, where $z = \int v(t') dt'$ and $v = v_{sh}n_{sh}/n$. Here $v_{sh}$ is the velocity relative to the shock front and $n_{sh}$ is the density, both measured in the immediate downstream. The solutions $N(\gamma, z)$ for both electrons and protons determine the emissivities due to different processes throughout the cooling layer; the emerging spectra are obtained by integrating the emissivities over $dz$.

Figure 4 shows snapshots of the cooling nonthermal electron distribution at different times/distances behind the shock ($p_{p} = 0$). The blue dashed line shows the distribution at a time $t = 0.8$ just before the thermal pressure is lost, i.e., before the fastest compression. The deficiency of electrons at low and high energies arises from their fast cooling relative to the compression/thermal cooling time, due to Coulomb and IC losses, respectively (Equations (18) and (20)). Rapid compression at $t = 1$ shifts the entire distribution toward higher energies by a factor $\propto n^{1/3} \sim 2$ (the apparent shift is larger at the low-energy end due to significant Coulomb losses between $t = 0.8$ and 1). At later times, the reenergized distribution rapidly cools owing to the increased rates $\Gamma_{br} \propto \gamma_{\text{Coal}} \propto n$; the cooling is slowest at $\gamma \approx \gamma_{b}$ where $\Gamma_{br} \approx \Gamma_{\text{Coal}}$ (Equation (13)). The nonthermal evolution is followed until $p_{nth}/n$ (approximately the energy per nonthermal particle) decreases by 4 orders of magnitude from its post-shock value.

5. High-energy Radiation from the Cooling Layer

Relativistic leptons, either accelerated directly at the shock or produced by $\pi_{0}$ decay, give rise to emission extending from the X-ray to gamma-ray band. In the hadronic scenario, however, the dominant source of $> 100$ MeV gamma-rays is the decay of neutral pions ($\pi_{0}$). The rates of these processes must be calculated self-consistently with the evolution of the post-shock plasma.

In broad terms, one can identify two main regions of the cooling layer that are relevant for the spectral formation: (1) the first thermal cooling length, where the plasma density and partial pressures are approximately equal to their immediate post-shock values, and (2) the high-density region farther downstream. As noted above, compression deposits additional energy into the nonthermal population; this is seen as a jump at $z \approx 2 \times 10^{11}$ cm in Figure 3 (right panel), where we show the nonthermal pressure normalized to unit density (which
approximately characterizes the energy per particle. Compression by a factor $\sim 10$ results in an approximate energy gain of $\mu \sim 13$ per particle. The increased density also enhances the bremsstrahlung and Coulomb cooling rates relative to IC ($\nu_{\text{bre}} / \nu_{\text{IC}} \propto \chi^{-1} \propto n$). Consequently, the additional energy deposited into the nonthermal leptons via adiabatic heating mainly enhances the bremsstrahlung spectral component.

5.1. Gamma-ray Spectrum

If the downstream magnetization is relatively weak, electrons/pairs of energy $\gamma \lesssim 10^3$ cool primarily by bremsstrahlung or IC radiation. The characteristic energies of the emitted photons are $E \approx \gamma m_e c^2 \approx 5 \gamma$ GeV (bremsstrahlung) and $E \approx (4/3) \gamma^3 E_{\text{opt}} \approx 0.3 \gamma^3 (E_{\text{opt}}/2$ eV) GeV (IC), where $E_{\text{opt}}$ is the average energy of optical seed photons, and $\gamma_{\text{IC}} \equiv \gamma / 4$.

Thus, absent significant synchrotron losses, gamma-ray emission above a few hundred MeV serves as a calorimeter for the particle acceleration efficiency (Metzger et al. 2015). For leptons injected with a distribution $Q_e \propto dN/d\gamma \propto \gamma^{-q}$, the high-energy spectrum in the fast-cooling regime follows (defining $x \equiv h\nu/m_\text{e} c^2$)

$$\nu F_e \propto \frac{dE_{\text{br}}}{d\ln x} = x^{-q+2}, \quad \frac{dE_{\text{IC}}}{d\ln x} = x^{-(q-2)/2}$$

in the bremsstrahlung and IC-dominated cases, respectively. The expected approximately flat injection spectrum ($q \approx 2$) results in a similarly flat GeV spectrum regardless of the emission mechanism, in broad agreement with observations of novae (Ackermann et al. 2014).

In the hadronic scenario, the spectrum of $\pi_0$-decay gamma-rays roughly mimics that of the accelerated protons, i.e., $\nu F_p \propto \nu^{-q+2}$; the same is true for injected pairs from $\pi_\pm$ decay, for which $q \approx q_p$. Thus a flat GeV spectrum is also expected in hadronic models.

One concludes that the shape of the GeV spectrum alone is insufficient to distinguish between leptonic and hadronic models, or between IC and bremsstrahlung origin of the GeV radiation in the former.

5.2. X-Ray Spectrum

Electrons/pairs of energy $\gamma \lesssim 10^3$ suffer significant Coulomb losses behind the shock, which dramatically suppresses the hard X-ray emission. Absent Coulomb losses, the relatively flat gamma-ray spectrum from $Q_e \propto \gamma^{-2}$ electrons in leptonic models would extend down into the X-ray band, in which case the hard X-ray luminosity would rival that in the LAT bandpass. In practice, this limit is attained only for unrealistically high values of $\chi \sim 1$, such that IC dominates all other cooling mechanisms. For more physical values of $\chi \ll 1$, Coulomb
losses suppress the X-ray emission by several orders of magnitude.

Let us estimate the energy radiated in a given frequency band, such as hard X-rays, by the rapidly cooling relativistic electrons behind the shock. The total energy loss rate of a relativistic electron is

$$\dot{\gamma} = \dot{\gamma}_{\text{br}} + \dot{\gamma}_{\text{IC}} + \dot{\gamma}_{\text{Cool}},$$

(31)

where the appropriate rates are given by Equations (7), (8), and (11), respectively.

The bremsstrahlung emissivity at frequency $x \equiv h\nu/(m_e c^2) \ll \gamma$ can be written as (e.g., Haug 1997)

$$j_{\text{br,eq}}(x) \approx \frac{2}{\pi} c \sigma_T \alpha_{\text{fs}} n \sum_i X_i Z_i^2 A_i \int \ln \left( \frac{1.2 \gamma_i^2}{x} \right) N(\gamma) d\gamma,$$

(32)

where hereafter we drop the sum over ion species in our estimates, assuming a hydrogen-dominated composition.

Similarly, using the delta-function approximation (as justified for a smooth electron distribution softer than $N(\gamma) \propto \gamma$), the IC emissivity can be written as

$$j_{\text{IC}}(x) \approx \frac{4 \sigma_T \alpha_{\text{fs}}}{3 \pi m_e c^2} \int \delta \left( x - \frac{4}{3} \gamma^2 \tau_{\text{opt}} \right) \gamma^2 N(\gamma) d\gamma \approx \frac{1}{2} c \sigma_T \alpha_{\text{fs}} N(\gamma_0),$$

(33)

where $\gamma_0 \equiv (3x/4\tau_{\text{opt}})^{1/2}$ and $\tau_{\text{opt}} \equiv E_{\text{opt}}/m_e c^2$ is the energy of the optical/UV seed photons.

The emissivity of a single electron of energy $\gamma$ is obtained using $N(\gamma') = \delta(\gamma' - \gamma)$ in Equations (32) and (33); the energy emitted by the electron at a given $x$ over its cooling history is then found by taking the ratio of Equations (32) or (33) and (31), and integrating over the energy $\gamma$ of the cooling electron.\footnote{Even though Equation (33) is a very poor representation of the IC spectrum for monoenergetic electrons, integration over the electron cooling history has the same effect as considering a broad electron spectrum and yields a sufficiently accurate result for our analytical estimates. Exact emissivities are used in the numerical calculations below.}

For concreteness, we focus on electron energies $\gamma \leq 10^3$, for which Coulomb losses dominate the total cooling rate. For bremsstrahlung emission, we obtain the spectrum of a single cooling electron

$$\frac{dE_{\text{br}}}{d\ln x} \bigg|_{\text{el.}} \approx \int_1^\gamma j_{\text{br,eq}}(x) d\gamma \approx \frac{4}{3 \pi} \alpha_{\text{fs}} \int_1^\gamma \ln \left( \frac{1.2 \gamma_i^2}{x} \right) d\gamma \approx \frac{4}{3 \pi} \alpha_{\text{fs}} \ln \left( \frac{1.2 \gamma_i^2}{x} \right),$$

(34)

where in the last equality we have used the fact that the integral is dominated by contributions from high $\gamma$. Now, considering emission from the entire injected electron distribution $Q_\gamma(\gamma)$, the total emitted energy per frequency interval $\ln x$ is given by

$$\frac{dE_{\text{br}}}{d\ln x} = \int_1^\gamma \frac{dE_{\text{br}}}{d\ln x} \bigg|_{\text{el.}} Q_\gamma(\gamma) d\gamma \approx \frac{4}{3 \pi} \alpha_{\text{fs}} \ln \left( \frac{1.2 \gamma_i^2}{x} \right) Q_\gamma(\gamma) \gamma d\gamma,$$

(35)

The upper boundary of the integral should be taken as the energy above which Coulomb collisions no longer dominate the cooling, i.e., $\tau_{\text{opt}}(\gamma) \approx \gamma^4$ (Equation (13)). Unless the injected distribution is strongly inverted\footnote{For instance, $Q_{\gamma}(\gamma) \propto \gamma^{-q}$, with $q \leq 1$, can be appropriate if the injected distribution has a low-energy cutoff, e.g., as a result of $\pi^0$ decay.}, the additional contributions from higher $\gamma$ (where cooling is dominated by either bremsstrahlung or IC) can be shown to be at most comparable to Equation (35). Equation (35) is notably independent of shock parameters, other than $Q_\gamma(\gamma)$.

For IC emission, a similar argument leads to

$$\frac{dE_{\text{IC}}}{d\ln x} \bigg|_{\text{el.}} = \int_1^\gamma \frac{j_{\text{IC}}(x)}{\gamma_{\text{Cool}}} d\gamma$$

$$= \frac{8}{9} \chi \ln x \int \delta \left( x - \frac{4}{3} \gamma^2 \tau_{\text{opt}} \right) \gamma^2 d\gamma \approx \frac{1}{3} \chi \ln x \gamma_0,$$

(36)

where again $\gamma_0 \equiv (3x/4\tau_{\text{opt}})^{1/2}$, but here $\chi = u_{\text{opt}}/(m_e c^2 n)$ is defined using the local density at the emission site (rather than the upstream density). The IC spectrum from the entire electron population is thus given by

$$\Delta \frac{dE_{\text{IC}}}{d\ln x} = \frac{1}{3} \chi \ln x \gamma_0 \int_0^\gamma Q_\gamma(\gamma) d\gamma,$$

(37)

where the lower integration limit follows because only electrons injected above $\gamma_0$ contribute to the flux at frequency $x$.

The analytical estimates given by Equations (35) and (37) should be compared with more detailed numerical calculations presented below, where the shock emission is computed by explicitly integrating the relevant emissivities through the post-shock cooling layer and the dense region farther downstream. Figure 5 shows the shock spectra for two different upstream densities, calculated assuming a logarithmically flat injected electron energy spectrum ($q = 2$). As expected from Equation (35), the bremsstrahlung component below the spectral peak is almost independent of density, whereas the IC spectrum scales as $\chi \propto n^{-1}$ (Equation 37). For both cases in Figure 5, the thermal plasma cools faster than $\gamma \sim 10^3$ electrons, i.e., $t_{\text{line}} < m_{\text{IC}, \text{IC}}$; the majority of the non-thermal emission therefore originates from the cooled and compressed layer, where the density is $\sim 100$ times higher than its value near the shock. The relevant value of $\chi$ one should use in the analytical estimate (37) is thus lower by the same factor compared to its value near the shock (Equation (4)).

For an injection slope $q = 2$, the bremsstrahlung X-ray emission receives approximately equal contributions per logarithmic electron energy interval up to $\gamma \approx \gamma_0 \approx 10^3$; the X-ray spectrum thus approximately follows $\nu F_\nu \propto \nu$ in both the IC- and bremsstrahlung-dominated cases. The IC component is slightly softer, as the hard X-ray band is not far below the smooth spectral break that occurs when the emitting electrons are no longer cooled by Coulomb collisions ($\gamma \sim 10^3$, which corresponds to IC photons of $\gamma_{\text{opt}} \approx 2 \sim 2$ MeV).

The shock spectrum in the hadronic model is shown in Figure 6. The arguments leading to Equations (35) and (37) apply equally well to hadronic models, except that the lepton injection $Q_\gamma(\gamma)$ now occurs in a volume rather than at the shock. The most significant difference is the lack of injected pairs below 100 MeV, which effectively introduces a lower...
integration limit of $\gamma_{\text{min}} \approx 200$ in Equation (35). The narrower integration range $\gamma \in [\gamma_{\text{min}}, \gamma_c]$ instead of $\gamma \in [1, \gamma_c]$ lowers the hard X-ray flux by a logarithmic factor of a few in the hadronic case (assuming $Q_\chi(\gamma) \propto \gamma^{-2}$).

6. X-Ray to Gamma-Ray Luminosity Ratio

The dimensionless ratio, $\chi$, of the radiation compactness and Thomson optical depth (Equation (3)) controls the partitioning of the nonthermal energy emitted in the hard X- and gamma-ray bands. Both thermal and nonthermal particles cool at a rate that is proportional to either the radiation energy density $u_{\text{opt}}$ (IC) or matter density $n$ (line cooling, bremsstrahlung, Coulomb, pp-collisions); thus $\chi \propto u_{\text{opt}}/n$ determines their relative importance.

Figures 7 and 8 show contours of constant $L_X/L_\gamma$ in the $\nu_{\text{ opt}}-\chi$ plane for leptonic and hadronic scenarios, respectively. The most obvious trend is that higher values of $L_X/L_\gamma$ are obtained at higher $\chi$. This can be understood from Equations (35) and (37), which show that IC becomes more dominant as $\chi$ increases. When IC dominates, the flux in the hard X-ray band is roughly proportional to $\chi$ for a fixed total nonthermal energy (i.e., constant $Q(\gamma)$ in Equation (37)). In simple terms, for a given electron energy $\gamma$, most of the IC power is emitted at lower frequencies ($\gamma \approx \gamma_{\pi 0}$) compared to bremsstrahlung ($\gamma \sim \gamma$), thus resulting in stronger X-ray emission in the former case. In the limit of complete IC dominance, the leptonic model spectrum approaches the fast-cooling shape of $\nu F_\nu \propto \nu^{2-\gamma}/\nu$, i.e., flat for $q = 2$. However, note that Equation (37) is no longer valid in this limit since Coulomb losses also become negligible at high $\chi$.

Another key feature is the existence of a lower limit of $L_X/L_\gamma \gtrsim 10^{-3}$ in the leptonic case and $\gtrsim 10^{-2}$ in the hadronic case. This corresponds to the complete dominance of bremsstrahlung over IC losses attained at low $\chi$. Because the low-energy tail of the bremsstrahlung spectrum of any given electron follows $\nu F_\nu \propto \nu^{2-\gamma}/\nu$, the relative power emitted in the X-ray and gamma-ray bands by leptonic emission must exceed $\nu F_\nu \propto 1/\nu \sim 10^{-3}$. In the leptonic scenario, the lower limit on $L_X/L_\gamma$ is actually somewhat higher than this because the X-ray emission receives additional contributions from electrons with energies $\gamma \lesssim 10^2$, which are too low to contribute in the gamma-ray band.

In the hadronic scenario, most of the gamma-ray flux arises from the decay of neutral pions; an appreciable contribution also comes from the IC and bremsstrahlung emission from $\pi_\pm$ pairs injected by $\pi_0$ decay. In contrast to the leptonic case, however, the decay of charged pions creates very few pairs below $\gamma \sim 10^2$ that would contribute to the X-ray flux. As a
Figure 7. Isocontours of relative fluxes at 30 keV and 1 GeV, \(n_{\text{e}}/\text{opt} 30\text{keV}/n_{\text{e}} 1\text{GeV}\) in leptonic models (blue solid lines), in the parameter space of shock velocity \(v_{\text{sh}}\) and the compactness to Thomson opacity ratio \(\chi = n_{\text{e}}/\text{opt} m_{\text{e}} c^2\) (Equation (3)), where \(n\) is the upstream density, and \(u_{\text{opt}}\) is the energy density of the soft (optical) radiation. Left and right panels correspond to assumed values of the post-shock magnetization of \(\eta_{\text{B}} = 10^{-6}\) and \(\eta_{\text{B}} = 10^{-4}\), respectively. The fluxes are computed assuming fast cooling for both thermal and nonthermal processes, and we have adopted characteristic values for the nonthermal injection fraction \(\varepsilon_{\text{nth}} = 10^{-2}\), injection index \(q = 2\), maximum energy of accelerated electrons \(E_{\text{max}} = 10^{12}\), and optical luminosity \(L_{\text{opt}} = 10^{38}\) erg s\(^{-1}\), where \(R = v_{\text{sh}} t\) and time \(t = 1\) week. However, note that the isocontours of \(n_{\text{e}}/\text{opt} 30\text{keV}/n_{\text{e}} 1\text{GeV}\) are independent of \(L_{\text{opt}}\) and \(t\), and depend weakly on \(\varepsilon_{\text{nth}}\). Black dashed lines show isocurves of constant upstream density for the chosen \(L_{\text{opt}}\) and \(t\), given by \(\chi \propto L_{\text{opt}}/(c^2 R^2)\). Red dashed lines show isocontours of constant gamma-ray to optical flux ratios, \(L_{\gamma}/L_{\text{opt}}\). Which scale linearly with \(\varepsilon_{\text{nth}}\) but are independent of \(L_{\text{opt}}\) and \(t\). The region to the right of the black dotted lines (\(L_{\text{shock}} > L_{\text{opt}}\)) is unphysical as the total shock-generated luminosity (a large fraction of which is absorbed and reprocessed to optical frequencies) cannot exceed \(L_{\text{opt}}\).

result, the minimum value of \(L_{X}/L_{\gamma} \sim 10^{-4}\) is a factor of several lower than in the leptonic case.

Post-shock magnetization affects the X-ray to gamma-ray ratio in two ways. First, it acts as an additional source of cooling for high-energy leptons, whereby a fraction of their energy is emitted as low-frequency synchrotron radiation. Strongly Compton-dominated regimes (\(\chi \gtrsim 10^{-2}\)) are unaffected since the cooling of relativistic leptons is dominated by IC even after strong compression in the cooling layer. Conversely, synchrotron losses can become the dominant cooling mechanism at smaller \(\chi\), corresponding to more (kinematically) luminous shocks expected to occur in novae. Since the highest energy particles are affected the most, increasing the magnetization mainly suppresses the leptonic gamma-ray emission; the X-ray to gamma-ray ratio increases as a result. Hadronic models are less sensitive to \(\varepsilon_{\text{B}}\) since the gamma-rays are produced predominantly via \(\pi_0\) decay, which is unaffected by magnetization.

Second, sufficiently strong magnetization can affect post-shock dynamics by halting the compression when the magnetic pressure \(u_{\text{B}} \propto n^2\) becomes dominant over both thermal and nonthermal plasma pressure. This alters the relative efficiency of bremsstrahlung and IC cooling in the compressed layer in favor of the latter, also increasing the X-ray to gamma-ray ratio.

An independent constraint on the parameter space is obtained from the gamma-ray to optical flux ratio, as shown by the red dashed lines in Figures 7 and 8. For a given observed value of \(L_{\gamma}/L_{\text{opt}}\), the allowed region on the \(v_{\text{sh}}-\chi\) plane lies between the black dotted line and the corresponding red dashed line. Combined with a measurement of \(L_{X}/L_{\gamma}\), this could in principle be used to lift the degeneracy between \(v_{\text{sh}}\) and \(\chi\) (or, equivalently, the density \(n\)) and to make an estimate of both. However, this assumes that the other uncertain parameters \(\varepsilon_{\text{nth}}\) \((\varepsilon_{\text{p}})\), \(\varepsilon_{\text{B}}\) are known or can be constrained.

In fact, the existence of an allowed parameter region for a measured \(L_{\gamma}/L_{\text{opt}}\) sets a lower limit on the particle acceleration efficiency (Metzger et al. 2015). For the flat acceleration spectra assumed in Figures 7 and 8 \((q, q_\text{p} = 2)\), the efficiencies \(\varepsilon_{\text{nth}}\) or \(\varepsilon_{\text{p}}\) are constrained to similar values in the two scenarios (note that we assume \(\varepsilon_{\text{nth}} = 10^{-2}\) in Figure 7, while \(\varepsilon_{\text{p}} = 0.1\) in Figure 8; \(L_{\gamma}/L_{\text{opt}}\) scales approximately \(\propto \varepsilon_{\text{nth}} \varepsilon_{\text{p}}\)). As pointed

---

Figure 8. Same as Figure 7, but for hadronic models. The parameters are as follows: fraction of shock energy injected into nonthermal protons \(\varepsilon_{\text{p}} = 0.1\), injection slope \(q_{\text{p}} = 2\), where \(dN_{\gamma}/dE_{\gamma} \propto E_{\gamma}^{-\alpha_{\gamma}}\cdot E_{\gamma}\), optical luminosity \(L_{\gamma} = 10^{38}\) erg s\(^{-1}\), and post-shock magnetization \(\eta_{\text{B}} = 10^{-4}\). The scalings of the different isocontours are the same as for the leptonic models, except that \(\varepsilon_{\text{p}}\) replaces \(\varepsilon_{\text{nth}}\).
out by Metzger et al. (2015), high observed values of $L_{\gamma}/L_{\text{opt}}$ (e.g., $\sim 10^{-2}$ in Nova V1324 Sco and $\sim 3 \times 10^{-4}$ in V399 Del) favor hadronic scenarios on both theoretical and observational grounds. Both particle-in-cell plasma simulations (e.g., Kato 2015; Park et al. 2015) and modeling of observed supernova remnants (e.g., Morlino & Caprioli 2012) suggest a relatively low electron acceleration efficiency of $\varepsilon_{\text{mh}} \lesssim 10^{-3}$ in nonrelativistic shocks.

Particle acceleration spectra with $q_1, q_2 = 2$ yield the most conservative (lowest) values for the ratio $L_{\gamma}/L_{\text{opt}}$, as well as for $\varepsilon_{\text{mh}}$: $\varepsilon_q$ at a given observed $L_{\gamma}/L_{\text{opt}}$. In light of this, it is worth noting that observations of supernova remnants as well as Galactic cosmic rays suggest somewhat steeper acceleration spectra in nonrelativistic shocks ($q_1 = 2.1 - 2.4$) (e.g., Ave et al. 2009; Caprioli 2011). The apparent discrepancy with the standard diffusive shock acceleration theory (which predicts $q_1, q_2 = 2$ in strong shocks) could be resolved by nonlinear feedback of the accelerated cosmic rays on the shock dynamics (e.g., Zirakashvili & Ptuskin 2008; Caprioli et al. 2009b; Caprioli 2012).

7. Constraints on Mass-loss Rate and Density

The allowed region in $v_{\text{sh}}$–$\chi$ parameter space obtained from simultaneous gamma-ray and hard X-ray observations can be used to constrain the mass outflow rate of the ejecta, as well as the density at the shock if one has an independent handle on the shock radius. In novae, one can visualize two scenarios for the shock formation: (1) a fast wind from the central object impacting upon a dense “external” shell (possibly ejected at an earlier phase of the same nova eruption), and/or (2) internal shocks within a single variable outflow. In both cases, one can express $\chi$ as

$$\chi = \frac{m_p L_{\text{w}} v_w}{m_e M c^3} = 2.1 \times 10^{-5} \frac{L_{\text{opt},38} v_{\text{w},8}}{M_5}$$

where $v_w$ is the outflow velocity and $M_5$ is the mass outflow rate in units of $10^{-5} M_\odot$ per week (a typical value in novae). It is important to note that Equation (38) does not explicitly depend on $R$, as the main unknown parameters that control $L_{\gamma}/L_{\text{opt}}$ are $M$ and $v_w$ in the shock upstream, and the shock velocity $v_{\text{sh}}$.

In either scenario, $v_w$ is unlikely to be very different from the shock velocity $v_{\text{sh}}$. In case of internal shocks, dissipating a substantial fraction of the outflow energy (as suggested by gamma-ray observations) requires $v_{\text{sh}} \sim v_w$. In the case of a fast tenuous wind impacting a slow dense shell, most of the energy is dissipated at the reverse shock, i.e., the shock running back into the wind material; as long as the velocity contrast between the two media is substantial, one again finds $v_{\text{sh}} \sim v_w$. We therefore parametrize $v_{\text{sh}} = \zeta v_w$, with $\zeta \lesssim 1$.

The isocontours of $L_{\gamma}/L_{\text{opt}}$, in $v_{\text{sh}}$–$M$ space are shown in Figure 9. As expected, the X-ray to gamma-ray luminosity ratio decreases with increasing mass outflow rate, which results in higher densities and stronger bremsstrahlung and Coulomb losses relative to IC.

To date, there has been no published detection of nonthermal X-rays from novae. Simultaneous hard X-ray and gamma-ray observations have been performed in two events, V339 Del and V5668 Sgr (K. Mukai et al. 2017, in preparation). Both novae were detected by Fermi/LAT days to weeks after the optical outburst (Ackermann et al. 2014; Cheung et al. 2016). The NuSTAR satellite observed V339 Del (24 ks) and V5668 Sgr (52 ks) approximately one and two weeks after the onset, respectively. The upper limits for the 20 keV flux were obtained (K. Mukai et al. 2017, in preparation) as $\nu F_\nu < 1.2 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$ (V339 Del) and $\nu F_\nu < 3.5 \times 10^{-14}$ erg cm$^{-2}$ s$^{-1}$ (V5668 Sgr). Comparison with the simultaneous LAT fluxes yielded the ratio of 20 keV to 100 MeV fluxes/luminosities as $L_{\gamma}/L_{\gamma} < 4.0 \times 10^{-3}$ (V339 Del) and $L_{\gamma}/L_{\gamma} < 1.7 \times 10^{-3}$ (V5668 Sgr).

The optical fluxes at the time of the X- and gamma-ray observations were approximately $10^{-7}$ erg cm$^{-2}$ s$^{-1}$ (V339 Del) and $6 \times 10^{-7}$ erg cm$^{-2}$ s$^{-1}$ (V5668 Sgr); the corresponding gamma-ray to optical flux ratios were $\sim 3 \times 10^{-4}$ and $\sim 3 \times 10^{-5}$, respectively (Skopal et al. 2014; Metzger et al. 2015; Munari et al. 2015).

The theoretical $L_{\gamma}/L_{\text{opt}}$ isocontours on the $v_{\text{sh}}$–$n$ plane for nova V339 Del and V5668 Sgr are shown in Figures 10 and 11, respectively. The allowed region as determined by the gamma-ray and optical observations lies between the red dashed and black dotted lines. Unfortunately, the present X-ray upper limits do not yield significant additional constraints on the parameter space in either nova. The leptonic case is somewhat more constraining, requiring $v_{\text{sh}} \lesssim 2 \times 10^8$ cm s$^{-1}$ and $n \gtrsim 10^8$ cm$^{-3}$ in both novae, which incidentally ensures that the shocks are radiative (see Figure 1).

It is worth noting that the NuSTAR limit for V5668 Sgr is sufficiently deep to give hope for more interesting constraints in future events. The LAT fluxes in several gamma-ray novae have exceeded $10^{-10}$ erg cm$^{-2}$ s$^{-1}$ (Ackermann et al. 2014), which could yield X- to gamma-ray ratios on the order of $\sim 3 \times 10^{-4}$. This is sufficient to either result in a detection or rule out leptonic models. Unfortunately, nova V5668 Sgr was intrinsically about an order of magnitude weaker in gamma-rays than more luminous events such as V1324 Sco (Cheung et al. 2016, their Figure 5).

7.1. Escape of the Shock Radiation from the Outflow

Our discussion so far has considered only the intrinsic emission from radiative shocks and assumed that the generated X-rays and gamma-rays can freely escape from the ejecta. However, if the shocks take place sufficiently deep in the outflow, the dense ambient material can leave a strong imprint on the escaping radiation. Above a few tens of keV, the main source of opacity in the outflow is Compton scattering. At $E \ll m_e c^2$, the average fractional energy loss of a photon in a scattering event is $\sim x = E/m_e c^2$. Emitted at $\tau_E \gg 1$, the photon experiences approximately $\tau_E$ scatterings before escaping. Thus if $x \tau_E^2 \gtrsim 1$, or equivalently $\tau_E \gtrsim 5(E/20$ keV)$^{-1/2}$, the photon energy is significantly degraded as it diffuses out of the ejecta. This can be seen in Figure 12: as $\tau_E$ is increased, the soft gamma-rays ($E \sim 1$ MeV) are depleted first, followed by hard X-rays at progressively lower energies. Note that if the primary spectrum is sufficiently hard ($F_\nu \propto \nu^{-\alpha}$ with $\alpha < 0$), the emission in a given band is initially enhanced as $\tau_E$ is increased, at the expense of higher-energy photons being downscattered into the band, before being suppressed at higher $\tau_E$.

At $E \gg m_e c^2$, the Klein–Nishina cross section approximately follows $\sigma_{\text{KN}} \approx (3 \pi r_T / 8 x) \ln(x)$, while a photon loses most of its energy in a single scattering event. At 100 MeV, $\tau_{\text{KN}} \approx \tau_E / 100$, i.e., the photons in the LAT band suffer significant recoil losses if $\tau_E \gtrsim 100$ (Figure 12).
Note that high $\tau_T$ also has the effect of enhancing the (optical) radiation density in the ejecta, as $\nu_{\text{opt}} \approx L_{\text{opt}}(1 + \tau_T)/(4\pi c R^2)$. The $\chi$ parameter is enhanced by the same factor $(1 + \tau_T)$, which has a positive effect on the hard X-ray emission (Figures 7 and 8).

In summary, for given shock parameters, the X-ray to gamma-ray ratio is somewhat increased if $\tau_T \sim$ a few. In the range $\tau_T \approx 10$–100, the X-rays are strongly suppressed, while LAT gamma-rays still escape unhindered. At even higher opacities, the GeV gamma-rays are also significantly degraded.

### 8. Discussion and Conclusions

Nonthermal emission from nonrelativistic shocks provides a wealth of information about the shock environment as well as about the physics of particle acceleration. In dense media characteristic of, e.g., nova eruptions during the first weeks, the heated plasma rapidly cools and compresses behind the shock due to line cooling and bremsstrahlung emission. The unique property of slow ($v_{\text{sh}} \lesssim 10^8$ cm s$^{-1}$) shocks is that the cooling time of the relativistic particles responsible for the hard X-ray and gamma-ray emission is longer than the thermal cooling/compression time. Thus the high-energy radiation samples a range of physical conditions behind the shock front, where the density as well as magnetization can change by a few orders of magnitude.

In this work we have computed the nonthermal emission from the cooling layer behind the shock front that is due to relativistic bremsstrahlung and IC upscattering of thermal (optical) radiation, as well as hadronic collisions leading to both gamma-ray and $e_\pm$ production. Additional losses due to Coulomb collisions with the thermal plasma and synchrotron cooling were also taken into account. The downstream compression was calculated using a simple prescription by keeping the total pressure constant, and by explicitly following the evolution of partial pressures of the thermal and nonthermal plasma as well as the magnetic field as the plasma cools.

The focus of our analysis was on using spectral information from flux ratios in different bands to constrain the physical conditions at the shock as well as particle acceleration mechanisms. In particular, we concentrated on the hard X-ray and GeV gamma-ray bands accessible to NuSTAR and Fermi/LAT, respectively. In contrast to soft X-rays, hard X-ray radiation is more likely to be representative of the intrinsic nonthermal emission from the shock: it is not impeded by bound-free absorption and can only be degraded by Compton recoil if $\tau_T \gtrsim 10$. Furthermore, the hard X-ray band is less likely to be contaminated by free–free emission from shock-heated thermal electrons, except if $v_{\text{sh}} \gtrsim 2 \times 10^8$ cm s$^{-1}$. The latter was probably the case in nova V5589 Sgr, where Swift/XRT observed unusually hard X-ray emission $(kT \gtrsim 30$ keV) 19 days after discovery (Weston et al. 2016b). Vlasov et al. (2016) interpreted this as thermal emission from shocks in the fast low-density (bi-)polar outflow, distinct from the slower but more energetic gamma-ray producing shocks at lower latitudes. Regardless, the different spectral shapes of the thermal and nonthermal X-rays make them readily distinguishable when data with reasonable quality are available.

In radiative shocks, the gamma-ray output above a few hundred MeV roughly traces the total energy placed into $>1$ GeV particles, even though the detailed spectrum depends on the particular model of particle acceleration as well as on the dominant radiative process. In contrast, emission at lower frequencies is more sensitive to the physical conditions near the shock. If the shock is embedded in an external radiation field with luminosity exceeding the shock-dissipated power (usually expected in novae), the fraction of nonthermal energy emerging in the X-ray band anticorrelates with density.

At very low densities ($\chi = i_{\text{rad}}/\tau_T \sim 1$, see Equation (3)), IC cooling dominates the electron (positron) energy loss, and comparable energy is radiated in the X-ray and gamma-ray bands by leptonic emission. This regime corresponds to low shock power. At high densities ($\chi \lesssim 10^{-4}$), relativistic bremsstrahlung and Coulomb collisions are the dominant cooling mechanisms for $\lesssim 1$ GeV electrons. In this regime,
the hard X-ray emission is a superposition of the low-energy tails of the bremsstrahlung spectra from relativistic leptons between $\gamma \approx 10^3$, attenuated by Coulomb losses. The high-density/low-$\chi$ regime corresponds to high shock power and is therefore most relevant for practical (detection) purposes, but the ratio of X-ray to gamma-ray energies is relatively low in this case, $L_X/L_\gamma \approx 10^{-4} - 10^{-3}$. In the extremely high-density limit, the X-ray to gamma-ray ratio approaches an asymptotic value in both leptonic and hadronic scenarios, which is approximately 3--4 times higher in the leptonic case. In this paper we have mostly concentrated on radiative shocks, which are of most interest in the nova context owing to their calorimetric nature. However, shocks in fast ($v_{sh} \gtrsim 2 \times 10^6 \text{ cm s}^{-1}$) outflows are likely adiabatic on a timescale when LAT emission is typically observed. In this case, the downstream plasma does not experience the strong compression due to loss of thermal pressure as in radiative shocks; the enhancement of bremsstrahlung and Coulomb rates, which has a suppressing effect on the hard X-rays, is also absent. Thus one generally expects a higher ratio of X-ray to gamma-ray luminosities. We defer a more detailed treatment of adiabatic shocks to future work.

Our simplified model of the cooling layer assumes that particle heating/acceleration takes place in an infinitely thin region around the shock. This is justified, for the purpose of this paper, as long as the particle diffusion length toward the shock from the downstream is much shorter than the post-shock cooling length. Assuming Bohm diffusion and typical nova shock parameters, this condition may break down at $E \gtrsim 1 \text{ GeV}$, particularly in slower shocks where the thermal cooling length is shorter. Nevertheless, the overall structure of the shock and the cooling layer should remain relatively unaffected, as these energetic particles make at most a subdominant contribution to the overall post-shock pressure.

8.1. Gamma-ray Novae

There is mounting evidence that strong shocks are commonplace in classical novae, which provide an independent avenue of constraining the the properties of nova outflows. Simultaneous *Fermi* LAT and optical observations of, e.g., V1324 and V399 Del already strongly limit the allowable parameter space (Metzger et al. 2015); in particular, they place a lower limit on the shock luminosity, which can be written as $L_{\text{shock}} = (9/32) \zeta M_{\text{sh}} v_{sh}^2$, where $\zeta = v_{sh}/v_\infty$. We have shown that the degeneracy between $M$ and $v_{sh}$ can be lifted, at least in principle, by a concurrent hard X-ray observation. The relevant observational measure is the ratio of X-ray and gamma-ray fluxes, which places an independent constraint on the allowed region $v_{sh}$--$M$ space, without explicit reference to, e.g., the shock radius or geometry.

Unfortunately, the currently available *NuSTAR* upper limits for two classical novae, V399 Del and V5668 Sgr, are not sufficiently deep to yield significant constraints, given their gamma-ray fluxes. There is reason for optimism, however, since the flux limits attainable by a $\sim 50$ ks *NuSTAR* observation (as performed for V5668 Sgr) of novae with higher gamma-ray fluxes such as V1324 Sco or V959 Mon would start pushing the theoretical limit of the $L_X/L_\gamma$ ratio, and likely result in a detection. Failing that, a deep upper limit could still be useful by ruling out leptonic models, for which $L_X/L_\gamma \gtrsim 5 \times 10^{-4}$ for any reasonable parameters (see Section 7 for a more detailed discussion, and Figures 7, 8).

A low X-ray luminosity could instead result from attenuation due to inelastic electron scattering by a high column of gas ahead of the shock with optical depth $\tau_\gamma \gtrsim 5$, in which case constraints on the shock properties from an X-ray nondetection would not be as strong; however, for $\tau_\gamma \gtrsim 100$ the $100 \text{ MeV}$ gamma-ray emission would itself be blocked, thus limiting the range of $\tau_\gamma$ over which this explanation would be viable to roughly one order of magnitude.
Acceleration of particles to $10^{10} \text{ GeV}$ required to produce the most energetic gamma-rays observed in novae implies significant magnetic field amplification at the shock (e.g., Metzger et al. 2016). However, aside from adiabatic compression, the downstream evolution (decay) of the magnetic field on the comparatively long radiative cooling scale is not well understood. In light of this, we chose to consider relatively weak magnetizations in this paper to allow a cleaner comparison of the radiative properties of hadronic and leptonic models, which are affected differently by the unknown magnetization. On the other hand, if the strong magnetic field ($B > 10^{-4}$) indeed persists throughout the post-shock cooling layer, strong synchrotron losses would tend to rule out leptonic models for gamma-ray bright novae, independently of other constraints (e.g., X-rays).

The model presented in this paper assumes a 1D planar shock and constant post-shock pressure. These simplifications may be questionable given the highly multidimensional thermal and thin-shell instabilities known to plague radiative shocks (e.g., Chevalier & Imamura 1982; Vishniac 1983). Nevertheless, insofar as the local thermodynamic conditions experienced by a cooling parcel of thermal and relativistic particles are reasonably captured by the simple processes of cooling and compression described here, these complications should not impact the qualitative features of our results.

Another caveat that should be kept in mind is that our model essentially provides a snapshot of emission for particular shock parameters, therefore comparison to data relies on simultaneous X-ray and gamma-ray measurements over a timescale shorter than the dynamical time. The \textit{NuSTAR} integration time in nova observations is typically a few tens of kiloseconds (compare with the expansion time $t = R/v_{sh} \approx 1 \text{ week}$), whereas the gamma-ray spectra published to date have been integrated over their whole duration. The comparison is still meaningful if the bulk of the gamma-ray fluence is accumulated over a timescale comparable to the expansion time. Furthermore, the framework developed in this paper can be straightforwardly extended to time-dependent calculations, given a model for the outflow evolution. This would yield a complete time-dependent evolution of the spectrum, but it requires additional assumptions about the structure and evolution of the ejecta (see e.g., Martin et al. 2017).

8.2. Colliding Wind Binaries

The colliding stellar winds of early-type stars (O, B, and Wolf–Rayet stars) in binary systems give rise to strong shocks that can accelerate both electrons and protons to high energies (De Becker 2007, and references therein). The existence of relativistic particles in these systems has been proven by the detection of radio synchrotron emission (e.g., Abbott et al. 1986; Chapman et al. 1999). To date, no gamma-rays have been detected in CWB (with the possible exception of $\eta$...
Carinae; Hamaguchi et al. 2014; Reitberger et al. 2015); upper limits for a sample of seven systems have been obtained by Fermi/LAT (Werner et al. 2013). Nonthermal X-rays have not yet been detected with Integral (e.g., De Becker et al. 2007), although Sugawara et al. (2011) have presented Suzaku observations showing evidence for a hard power-law X-ray component in WR140.

In tight binaries such as WR20a, or near the periastron passage of eccentric systems (e.g., WR 140), the particle densities at the shock are comparable to those expected in novae, and the shocks may become radiative.\(^{10}\) The shock/wind velocities are also similar, typically \((0.3-6) \times 10^9 \text{ cm s}^{-1}\) (see, e.g., Crowther 2007 for a review). However, the mass outflow rate, \(M \sim 10^{-4}-10^{-5} M_{\odot} \text{ yr}^{-1}\), is typically somewhat lower than in classical nova eruptions. Combined with comparable or higher optical/UV luminosities, \(L_{\text{opt}} \sim 10^5-10^6 L_{\odot}\), the cooling regime of the relativistic particles differs from novae. This can be seen by writing the \(\chi\) parameter as

\[
\chi = 0.04 \frac{L_{\text{opt}}}{10^6 L_{\odot}} \left(\frac{M}{10^{-5} M_{\odot} \text{ yr}^{-1}}\right)^{-1}.
\] (39)

Recalling Figure 1 (left panel), one concludes that the relativistic leptons cool predominantly by IC emission rather than bremsstrahlung (or Coulomb); furthermore, the IC cooling of the hard X-ray and gamma-ray emitting electrons is typically faster than the cooling of the thermal plasma (Equation (20)). Therefore, in leptonic models the X-ray and gamma-ray emissivities are not significantly affected by downstream compression, nor are the X-rays necessarily suppressed by Coulomb losses. As a result, the energy emitted in the hard X-ray and gamma-ray bands can be comparable (Figure 7).

In the hadronic scenario, the cooling time via pp-collisions relative to the compression time depends only on the shock velocity (Equation (22) and Figure 1, right panel). If \(v_{\text{sh}} \lesssim 3 \times 10^4 \text{ cm s}^{-1}\), the accelerated protons deposit most of their energy only after the thermal pressure has been lost and the downstream plasma has significantly compressed. This mainly affects the X-ray emission from secondary \(e^\pm\) pairs from \(\pi^\pm\) decay, which can experience both synchrotron losses in the compression-enhanced magnetic field and increased bremsstrahlung losses that give rise to harder spectra. Therefore the X-ray to gamma-ray ratio is expected to be over an order of magnitude lower than in the leptonic case.

The Fermi upper limits (Werner et al. 2013) are at odds with theoretical predictions for the gamma-ray flux by several groups (Benaglia & Romero 2003; Pittard & Dougherty 2006; Reimer et al. 2006). The discrepancy has not yet been resolved. Strong synchrotron losses could provide a possible explanation if the downstream plasma is able to compress and amplify the magnetic field before the relativistic particles have cooled. This requires the colliding winds to be dense and relatively slow, however. Note also that gamma-ray emission in hadronic models is not significantly affected by synchrotron losses, as the gamma-rays are produced predominantly via \(\pi^0\) decay. On the other hand, the general lack of observed hard X-rays (De Becker et al. 2007) could be explained in the hadronic scenario through the abovementioned suppression via synchrotron and bremsstrahlung losses.

We thank Andrei M. Beloborodov, Guillaume Dubus, Pierrick Martin, and Koji Mukai for helpful conversations. I.V. acknowledges support from Estonian Research Council grant PUT1112. B.D.M. gratefully acknowledges support from NASA grants NNX15AU77G (Fermi), NNX15AR47G, NNX16AB30G (Swift), and NNX16AB30G (ATP), NSF grant AST-1410950, the Alfred P. Sloan Foundation, and the Research Corporation for Science Advancement through the Scialog Program (Grant number RCSA 23810).

Appendix

Kinetic Equation for Nonthermal Particles

Here we show that the particle evolution Equation (26) is equivalent to the more “standard” form typically used in the cosmic-ray literature (see e.g., Blasi 2004; Caprioli et al. 2009a), namely

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} - \frac{1}{3} \frac{\partial v}{\partial p} p \frac{\partial f}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \dot{\rho}_{\text{rad}} f \right) = \dot{Q}_{\text{mj}}.
\] (40)

where \(f\) is the (isotropic) phase-space density (so that \(n=4\pi \int f dp\)), \(p = \gamma \beta\) is the dimensionless particle momentum, and \(\dot{\rho}_{\text{rad}}\) accounts for radiative cooling. The third term on the left-hand side accounts for adiabatic cooling/heating of particles if the plasma is (de)compressed as it propagates (i.e., if \(\partial v / \partial z \neq 0\)). We have neglected particle diffusion in Equation (40); see the discussion in Section 8. Equation (40) can be transformed as

\[
\begin{align*}
\frac{\partial f}{\partial t} &+ \frac{v}{3} \frac{\partial f}{\partial z} - \frac{1}{3} \frac{\partial v}{\partial p} p \frac{\partial f}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \dot{\rho}_{\text{rad}} f \right) \\
&= \frac{\partial f}{\partial t} + \frac{\partial}{\partial z} \left( \nu f \right) - \frac{\partial v}{\partial p} p \frac{\partial f}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \dot{\rho}_{\text{rad}} f \right) \\
&= \frac{\partial f}{\partial t} + \frac{\partial}{\partial z} \left( \nu f \right) - \frac{10}{p^2} \frac{\partial}{\partial p} \left( \frac{1}{3} \frac{\partial p}{\partial z} f \right) \\
&+ \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \dot{\rho}_{\text{rad}} f \right) = \dot{Q}_{\text{mj}}.
\end{align*}
\] (41)

Multiplying by \(4\pi p^2 dp / d\gamma\) and observing that \(4\pi p^2 dp = N(\gamma) d\gamma\) (or equivalently, \(4\pi p^2 f = N(\gamma)\)), one obtains

\[
\begin{align*}
\frac{\partial N(\gamma)}{\partial t} + \frac{\partial}{\partial z} \left[ \nu N(\gamma) \right] - \frac{\partial}{\partial \gamma} \left[ \frac{1}{3} \frac{\partial p}{\partial \gamma} N(\gamma) \right] \\
&+ \frac{\partial}{\partial \gamma} \left[ \dot{\gamma}_{\text{rad}} N(\gamma) \right] = \dot{Q}_{\text{mj}},
\end{align*}
\] (42)

where \(Q_{\text{mj}} = 4\pi p^2 \dot{Q}_{\text{mj}}\), and we have used \(p \dot{\rho}_{\text{rad}} = \dot{\gamma}_{\text{rad}} \) in the last term on the left-hand side. Using the identity

\[
\frac{d \ln n}{dt} = \frac{1}{n} \left[ \frac{\partial n}{\partial t} + \nu \frac{\partial n}{\partial z} \right] = \frac{1}{n} \left[ \frac{\partial n}{\partial t} + \nu \frac{\partial (\nu n)}{\partial z} \right] \\
- \frac{\partial \nu}{\partial z} = \frac{\partial}{\partial z}
\] (43)
and defining $\gamma_{\text{radab}} = -(p^2/3\gamma) \partial v/\partial z = (\gamma\beta^2/3) d \ln n/dz$ (Equation (28)), Equation (42) becomes

$$\frac{dN(\gamma)}{dt} + \frac{\partial}{\partial z}[vN(\gamma)] + \frac{\partial}{\partial \gamma}[(\gamma_{\text{radab}} + \gamma_{\text{rad}})N(\gamma)] = Q_{\text{ej}}$$

(Equation (44)

Equation (44) is identical to Equation (26) when one identifies $\dot{\gamma} = \gamma_{\text{radab}} + \gamma_{\text{rad}}$.

References

Abbott, D. C., Beiging, J. H., Churchwell, E., & Torres, A. V. 1986, ApJ, 303, 239

Ackermann, M., Ajello, M., Albert, A., et al. 2014, Sci, 345, 554

Ave, M., Boyle, P. J., Höppner, C., Marshall, J., & Müller, D. 2009, ApJ, 697, 106

Banerjee, D. P. K., Srivastava, M. K., Ashok, N. M., & Venkataraman, V. 2016, MNRAS, 455, L109

Benaglia, P., & Romero, G. E. 2003, A&A, 399, 1121

Blandford, R., & Eichler, D. 1987, PhR, 154, 1

Blandford, R. D., & Ostriker, J. P. 1978, ApJL, 221, L29

Blasi, P., & Spitkovsky, A. 2004, ApPh, 21, 45

Caprioli, D. 2011, JCAP, 5, 026

Caprioli, D. 2012, JCAP, 7, 038

Caprioli, D., Blasi, P., & Amato, E. 2009a, MNRAS, 396, 895

Caprioli, D., Blasi, P., Amato, E., & Vietri, M. 2009b, MNRAS, 395, 895

Caprioli, D., & Spitkovsky, A. 2014, ApJ, 783, 91

Chapman, J. M., Leitherer, C., Koiralski, B., Bouter, R., & Storey, M. 1999, ApJ, 518, 890

Cheung, C. C., Jean, P., Shore, S. N., et al. 2016, ApJ, 826, 142

Chevalier, R. A. 1983, ApJ, 272, 765

Chevalier, R. A., & Fransson, C. 1994, ApJ, 420, 268

Chevalier, R. A., & Imamura, J. N. 1982, ApJ, 261, 543

Chomiuk, L., Linford, J. D., Yang, J., et al. 2014, Natur, 514, 339

Crowther, P. A. 2007, ARA&A, 45, 177

De Becker, M. 2007, A&ARv, 14, 171

De Becker, M., Rauw, G., Pittard, J. M., et al. 2007, A&A, 472, 905

Drake, R. P. 2005, Ap&SS, 298, 49

Drury, L. O. 1983, RPPh, 46, 973

Hamaguchi, K., Corcoran, M. F., Takahashi, H., et al. 2014, ApJ, 795, 119

Haug, E. 1997, A&A, 326, 417

Haug, E. 2004, A&A, 423, 793

Kamae, T., Karlsson, N., Mizuno, T., Abe, T., & Koi, T. 2006, ApJ, 647, 692

Kato, T. N. 2015, ApJ, 802, 115

Krauss, M. I., Chomiuk, L., Rupen, M., et al. 2011, ApJL, 739, L6

Li, K.-L., Metzger, B. D., Chomiuk, L., et al. 2017, NatAs, 1, 697

Livio, M., Shankar, A., Burkert, A., & Tsuran, J. W. 1990, ApJ, 356, 250

Martin, P., & Dubus, G. 2013, A&A, 551, A37

Martin, P., Dubus, G., Jean, P., Tatischeff, V., & Dossev, C. 2017, arXiv:1710.05515

Metzger, B. D., Caprioli, D., Vurm, L., et al. 2016, MNRAS, 457, 1786

Metzger, B. D., Finzell, T., Vurm, L., et al. 2015, MNRAS, 450, 2739

Metzger, B. D., Hascoët, R., Vurm, L., et al. 2014, MNRAS, 442, 713

Morlino, G., & Caprioli, D. 2012, A&A, 538, A81

Mukai, K., & Ishida, M. 2001, ApJ, 551, 1024

Munari, U., Henden, A., Banerjee, D. P. K., et al. 2015, MNRAS, 447, 1661

Nelson, T., Donato, D., Mukai, K., Sokoloski, J., & Chomiuk, L. 2012, ApJ, 748, 43

O’Brien, T. J., Lloyd, H. M., & Bode, M. F. 1994, MNRAS, 271, 155

Osborne, J. P. 2015, JHEAp, 7, 117

Park, J., Caprioli, D., & Spitkovsky, A. 2015, PhRvL, 114, 085003

Pittard, J. M., & Dougherty, S. M. 2006, MNRAS, 372, 801

Reimer, A., Pohl, M., & Reimer, O. 2006, ApJ, 644, 1118

Reitberger, K., Reimer, A., Reimer, O., & Takahashi, H. 2015, A&A, 577, A100

Ribeiro, V. A. R. M., Munari, U., & Valisa, P. 2013, ApJ, 768, 49

Schure, K. M., Kosenko, D., Kaasstra, J. S., Keppens, R., & Vink, J. 2009, A&A, 508, 751

Schwarz, G. J., Shore, S. N., Starrfield, S., & Vanlaningham, K. M. 2007, ApJ, 657, 453

Shore, S. N. 2013, A&A, 559, L7

Shore, S. N., De Gennaro Aquino, I., Schwarz, G. J., et al. 2013, A&A, 553, A123

Skopal, A., Drechsel, H., Tarasova, T., et al. 2014, A&A, 569, A112

Smith, N., Li, W., Foley, R. J., et al. 2007, ApJ, 666, 1116

Sugawara, Y., Maeda, Y., Tsaoi, Y., et al. 2011, BSRSL, 80, 724

Tatischeff, V., & Hernanz, M. 2007, ApJL, 663, L101

Vishniac, E. T. 1983, ApJL, 274, 152

Vlasov, A., Vurm, L., & Metzger, B. D. 2016, MNRAS, 463, 394

Werner, M., Reimer, O., Reimer, A., & Egberts, K. 2013, A&A, 555, A102

Weston, J. H. S., Sokoloski, J. L., Chomiuk, L., et al. 2016b, MNRAS, 460, 2687

Weston, J. H. S., Sokoloski, J. L., Metzger, B. D., et al. 2016a, MNRAS, 457, 887

Williams, P. M., van der Hucht, K. A., van Wyk, F., et al. 2012, MNRAS, 420, 2526

Yang, J., Paragi, Z., O’Brien, T. J., Chomiuk, L., & Linford, J. D. 2015, in Proc. of the 12th European VLBI Network Symposium, Supernovae and late stages of stellar evolution, ed. A. Tarchi, M. Giroletti, & L. Feretti (Trieste: SISSA), 53

Zirakashvili, V. N., & Ptuskin, V. S. 2008, in AIP Conf. Ser. 1085, High Energy Gamma-ray Astronomy, ed. F. A. Aharonian, W. Hofmann, & F. Rieger (Melville, NY: AIP), 336