The magnetic dipole transitions in the \((c\bar{b})\) binding system

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Abstract

The magnetic dipole transitions between the vector mesons \(B^*_c\) and their relevant pseudoscalar mesons \(B_c\) (\(B_c, B^*_c, B_{c}(2S), B_{c}^{*}(2S), B_{c}(3S)\) and \(B_{c}^{*}(3S)\) etc, the binding states of \((c\bar{b})\) system) of the \(B_c\) family are interesting. To see the ‘hyperfine’ splitting due to spin-spin interaction is an important topic for understanding the spin-spin interaction and the spectrum of the the \((c\bar{b})\) binding system. The knowledge about the magnetic dipole transitions is also very useful for identifying the vector boson \(B^*_c\) mesons experimentally, whose masses are just slightly above the masses of their relevant pseudoscalar mesons \(B_c\) accordingly. Considering the possibility to observe the vector mesons via the transitions at \(Z^0\) factory and the potentially usages of the theoretical estimate on the transitions, we focus our efforts on calculating the magnetic dipole transitions, i.e. precisely to calculate the rates for the transitions such as decays \(B^*_c \rightarrow B_c\gamma\) and \(B^*_c \rightarrow B_c e^+ e^-\), and particularly work in the Bente-Salpeter framework. In the estimate, as a typical example, we carefully investigate the dependence of the rate \(\Gamma(B^*_c \rightarrow B_c\gamma)\) on the mass difference \(\Delta M = M_{B^*_c} - M_{B_c}\) as well.

PACS numbers: 13.40.Hg, 12.39.Ki
Comparing with the hidden flavored heavy quarkonia such as charmonia and bottomonia, the heavy meson $B_c$ and its family, being explicitly double flavored, have not been thoroughly studied yet. The reason is that not sufficient experimental data about $B_c$ meson are available and the experimental data about $B^*_c$ meson (the lowest one) still are unavailable at all so far. $B_c$ and $B^*_c$ are composed of two different heavy flavors, so that unlike the production of the hidden flavored heavy quarkonia, they cannot be produced via a simple QCD process even at the hadron colliders. At $e^+e^-$ colliders, the production is even more suppressed because of absence of gluon fusion. The earlier work [1] indicates that one cannot expect to find $B_c$ in the cases with a luminosity and collision energy as that of the LEP-I and II owing to the production rate is small. As estimated by the authors of ref. [1], the meson $B_c$ was first observed at a hadronic collider, TEVATRON [2]. It is natural that one would expect to make a detailed study on the $B_c$ family at the LHC, because the available energy and luminosity are much higher than that of TEVATRON and the $B_c$-involved events should be thousand times more. However, the messy QCD background of the hadron colliders and the fact, that one cannot control the total longitude momentum of the hadronic production, would contaminate the environment and make precise measurements on $B_c$ very difficult, and the observation on the other members of the $B_c$ family, i.e. the excited states of $B_c$, almost impossible. In this aspect, the proposed $Z^0$ factory possesses obvious advantage over the hadronic colliders.

$Z^0$ factory, an $e^+ - e^-$ collider with sufficiently high luminosity and running at $Z^0$-boson pole, now is, as a phase of ILC or independently, considered seriously. Even though the inclusive production $e^+e^-\rightarrow B_c(\bar{B}_c) + \cdots$ where two pairs of heavy quarks ($c\bar{c}$ and $b\bar{b}$) emerges from a hard gluon emission is suppressed, the high luminosity and the $Z$-pole effects would greatly enhance the event-accumulation rate so that the $B_c$ meson and its excited states (the other members of its family) may be expected to be observed. Thus if the luminosity is really high enough so the mesons $B_c$ and $B^*_c$ may be produced numerously, thus the magnetic dipole radiative transitions may be used to recognize the production of the excited states. In fact, LEP-I did search for the $B_c$ meson and could not make any definite conclusion, such as that the $B_c$ meson has been observed, due to 'low luminosity, so the small statistics for the events [3].

$B_c$ being the ground state, its decay characteristics are completely distinct from the hidden flavored heavy-quarkonia. Namely, the $B_c$ can only decay via weak interaction, and its lifetime has been carefully studied[4]. Whereas an excited state of $B_c$ meson must decay to a lower excited or the ground state via gluon (strong interaction) and/or photon (electromagnetic interaction) emissions, and it depends on the quantum number and the mass difference of the initial and final states. Moreover, it is known that of the electromagnetic decays, the magnetic dipole $M1$ transitions between the vector and pseudoscalar states play
an important role.

In another work [5], we discussed the possibility of observing the radially excited states \( B_c(ns) n > 1 \) via processes \( B_c(ns) \to B_c + \pi \pi \) at LHC and the \( Z^0 \) factory. Our calculation is based on the QCD multi-pole expansion method[6] and we find that at the \( Z^0 \) factory, it would be optimistic to observe the two-pion emission decays. On other aspect, the nearest member to \( B_c \) in the family is the vector-boson \( B_c^*(1s) \). With possible and precise spin-spin interaction, one may estimate the splitting between \( B_c(1s) \) and \( B_c^*(1s) \) explicitly as 30 \( \sim \) 50 MeV, so that \( B_c^* \to B_c + \pi^0(\eta, \eta') \) is forbidden by the energy-momentum conservation. Thus the dominant decay mode of \( B_c^* \) would be the magnetic dipole radiative decay \( B_c^* \to B_c + \gamma \). The decay \( B_c^* \to B_c + e^+ e^- \) is also governed by the electromagnetic process and the products \( e^+ e^- \) would be easily caught by the detector as a clear signal. Even though comparing with \( B_c^* \to B_c + \gamma \), its rate is suppressed by the three-body final phase space and an extra electromagnetic vertex, its observation may still be expected, because tracks of \( e^+ e^- \) would be easier to be identified than that of a photon. Definitely, we can gain more information about the \( B_c^* \) and determine the mass splitting \( \Delta M = M_{B_c} - M_{B_c^*} \) from the data which will be available at the \( Z^0 \) factory.

In Ref.[7] the authors explored radiative of \( M_1 \to M_2 \gamma \) in the Bhat-Salpeter(BS) framework[8]. Solving the BS equation one can obtain the wave functions and eigenvalues of the bound state. With the Mandelstam formula[9], we calculate the transition matrix elements between the bound states with appropriate BS wave functions. Concretely, in terms of the formula given in Ref.[7] we evaluate the transition matrix element of \( B_c^* \to B_c \) in the BS framework and extract the corresponding form factor \( F_{VP}(Q^2 = 0) \)[10]. With the form factor we are able to calculate the rate of \( B_c^* \to B_c + \gamma \). Then we go on to evaluate the rate of \( B_c^* \to B_c + e^+ e^- \) where the photon is virtual. In view of the progress in the experimental aspect we also evaluate the transitions \( B_c^*(2S) \to B_c + \gamma(e^+ e^-) \), \( B_c^*(3S) \to B_c + \gamma(e^+ e^-) \) and \( B_c(2S) \to B_c^* + \gamma(e^+ e^-) \).

Our strategy is follows: first we solve the BS equation using the parameters given in Ref.[11] and get the spectra and the wave functions of \( B_c^*(nS) \) and \( B_c(nS) \) respectively; then with the formula obtained in Ref.[7], we evaluate the transition matrix element of \( B_c^*(nS) \to B_c, B_c(2S) \to B_c^*, \) and \( B_c(3S) \to B_c^* \) and \( B_c(3S) \to B_c^* \) (2S) and extract the form factors \( F(Q^2 = 0) \); using these form factors the rates of \( B_c^*(nS) \to B_c + \gamma, B_c^*(nS) \to B_c + e^+ e^- \), \( B_c(2S) \to B_c^* + \gamma, B_c(2S) \to B_c^* + e^+ e^- \), \( B_c(3S) \to B_c^* + \gamma, B_c(3S) \to B_c^* + e^+ e^- \), \( B_c(3S) \to B_c^*(2S) + e^+ e^- \) are eventually obtained. After the introduction, we present the theoretical formulae for calculating the rates of \( V \to P + \gamma \) (\( P \to V + \gamma \)) and \( V \to P + e^+ e^- \) (\( P \to V + e^+ e^- \)), and then in sec.III, we list our numerical results, the last section is devoted to our discussion and conclusion.
II. THE FORMULA OF $V \rightarrow P$ AND $P \rightarrow V$ IN THE BS FRAMEWORK

A. $V \rightarrow P\gamma$

In the BS framework the corresponding S-matrix element was formulated as

$$\langle M_2(P')\gamma(Q,\epsilon)|S|M_1(P)\rangle = \frac{(2\pi)^4e}{\sqrt{2\omega_1\omega_2}}\delta(P' + Q - P)\epsilon_\mu \langle M_2(P')|J^\mu_{em}|M_1(P)\rangle,$$  \hspace{1cm} (1)

where $M_1$, $M_2$ are the initial and daughter mesons, $P$, $P'$ are their four-momenta and $\omega_1, \omega_2$ are their energies. $Q$ is the momentum of the emitted photon, $\epsilon$ is its polarization vector and $\omega$ is its energy.

For the photon emission, the transition matrix element reads

$$\langle M_2(P')|J^\mu_{em}|M_1(P)\rangle = \langle M_2(P')|J^\mu_{em}|M_1(P)\rangle_1 + \langle M_2(P')|J^\mu_{em}|M_1(P)\rangle_2,$$

$$= \int \frac{d^3q_\perp}{(2\pi)^3} Tr \left\{ Q_1 \frac{P}{m_{M_1}} \left[ \bar{\varphi}^+_{P'}(q_\perp + \alpha_2 P_\perp)\gamma_\mu \varphi^+_{P}(q_\perp) \right] + Q_2 \left[ \bar{\varphi}^+_{P'}(q_\perp - \alpha_1 P'_\perp)\frac{P_{\perp}}{m_{M_1}}\varphi^+_{P}(q_\perp) \right] \right\}.$$  \hspace{1cm} (2)

where $Q_1(Q_2)$ is the charge carried by the quark/(antiquark) and other notations are listed in the Appendix. In the processes $B^{*}_c(nS) \rightarrow B_c(n'S) + \gamma$ ($n \geq n' = 1, 2, \cdots$), $B^{*}_c(nS)$ is a vector($V$) and $B_c(n'S)$ is a pseudoscalar($P$). Due to the quantum numbers of the initial and final states, the transitions must be the nature of magnetic dipole, and there is only one form factor for the current matrix elements, namely as that in Ref.[10], the general form factor $F_{VP}(Q^2)$ for $V \rightarrow P\gamma^*$ is related to the current matrix element as follows:

$$\langle P(P')|J^\mu_{em}|V(P)\rangle = i\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu(P)Q_\rho P_\sigma F_{VP}(Q^2),$$  \hspace{1cm} (3)

where $Q = P - P'$ is the four momentum of the virtual photon, $\epsilon_\nu(P)$ is the polarization vector of the initial meson. $F_{VP}(Q^2)$ can be extracted by evaluating the $\langle M_2(P')|J^\mu|M_1(P)\rangle$ in Eq.(2). For real photon case $Q^2$ is equal to 0 i.e. $F_{VP}(Q^2 = 0)$.

The decay width of $V \rightarrow P\gamma$ is

$$\Gamma(V \rightarrow P\gamma) = \frac{\alpha}{3} \left( \frac{m_V^2 - m_P^2}{2m_V} \right)^3 F_{VP}^2(0),$$  \hspace{1cm} (4)

where $\alpha$ is the fine-structure constant and $m_V$, $m_P$ are the masses of the $B^{*}_c$ and $B_c$ respectively.

B. $V \rightarrow Pe^+e^-$

The S-matrix element for $V \rightarrow Pe^+e^-$ was given as

$$\langle M_2(P')e^+(p_a, s_a)e^-(p_b, s_b)|S|M_1(P)\rangle = \frac{(2\pi)^4e^2}{\sqrt{2^4\omega_1\omega_2}} \delta(P' + p_a + p_b - P) \frac{2m_e}{q^2}$$
where \( p_a, p_b \) are the four-momenta of \( e^+ \) and \( e^- \), \( \omega_a, \omega_b \) are their energies and \( U_e, \bar{U}_e \) are the corresponding spinors with spins \( s_a \) and \( s_b \). Using Eq. (3), \( \langle M_2(P') | J^\mu_{em} | M_1(P) \rangle \) can be parameterized into \( F_{VP}(Q^2) \) which can be calculated according to Eq. (2).

In terms of the formula given in Ref. [12] we obtain

\[
d\Gamma = \frac{16\alpha^2 F^2_{VP}(Q^2) (11m_e^2 + 4Q^2)}{4608m_\pi^2 \pi Q^6} \sqrt{Q^4 - 4m^2_eQ^2((-m_V^2 + m_P^2 + Q^2)^2 - 4m^2_PQ^2)^{3/2}} dQ^2, \tag{6}
\]

where \( Q = p_1 + p_2 \). Integrating out \( Q^2 \) in the expression\(^\text{[6]}\) one obtains the decay width of \( B_c^* \to B_c e^+e^- \).

For the transition of \( P \to V \), the form factor \( F_{PV}(Q^2) \) can be obtained as we did for \( F_{VP}(Q^2) \). We still can use Eq. (4) (Eq. (3)) to calculate the rate \( B_c(2S) \to B_c^* \gamma(e^+e^-) \) by replacing \( F_{PV}(Q^2) \) with \( F_{VP}(Q^2) \).

In principle, we can extend our computations to higher excited states and the P-wave states of the \( B_c \) family, but because their production rates are much lower and experimental measurements would be much more difficult, we do not intend to include them in this work.

III. NUMERICAL RESULTS

By solving the corresponding BS equations for \( B_c^*(nS) \) and \( B_c(nS) \), their wave functions and masses were evaluated in Ref. [11] where the authors systematically explored the spectra of mesons made of only heavy flavor quark-antiquark and all the free parameters in the theoretical model were fixed by fitting the data of heavy quarkonia and \( B_c \). The masses of \( B_c^*, B_c^*(2S), B_c^*(3S) \) and \( B_c(2s) \) are obtained as 6.3308 GeV, 6.9103 GeV, 7.2755 GeV and 5.7906 GeV respectively.
TABLE I: The theoretical predictions of the rates for several electromagnetic decay modes

| Decay Mode                  | $|F(Q^2 = 0)|$ (GeV$^{-1}$) | $\Gamma(M_1 \rightarrow M_2 \gamma)$ (keV) | $\Gamma(M_1 \rightarrow M_2 e^+ e^-)$ (keV) |
|-----------------------------|-----------------------------|--------------------------------------------|--------------------------------------------|
| $B_c^* \rightarrow B_c$    | 0.208                       | 17.1×10$^{-3}$                            | 8.64×10$^{-5}$                            |
| $B_c^*(2S) \rightarrow B_c$| 0.023                       | 0.28                                       | 1.59×10$^{-3}$                            |
| $B_c^*(3S) \rightarrow B_c$| 0.014                       | 0.37                                       | 2.11×10$^{-3}$                            |
| $B_c(2S) \rightarrow B_c^*$| 0.030                       | 0.38                                       | 1.65×10$^{-3}$                            |
| $B_c(3S) \rightarrow B_c^*$| 0.0072                      | 0.074                                      | 0.42×10$^{-3}$                            |
| $B_c(3S) \rightarrow B_c^*(2S)$ | 0.049                      | 0.25                                       | 1.41×10$^{-3}$                            |

FIG. 2: The dependence of $\Gamma(B_c^* \rightarrow B_c \gamma)$ to $\Delta M$. The shadowed region is centered at the mass of $M_{B_c^*} = 6.3308$ GeV and the area corresponds to the experimental errors which are taken as inputs to our numerical computation.

6.8623 GeV respectively. The forms of the BS wave functions for the vector and pseudoscalar mesons are listed in the Appendix and the corresponding parameters can be found in Ref. [11].

Using the wave functions of the initial and daughter mesons, we calculate the transition matrix element in Eq. (2) and extract the form factor as $|F_{VP}(Q^2 = 0)| = 0.208$ GeV$^{-1}$. Substituting the value of $|F_{VP}(Q^2 = 0)|$ into Eq. (4) we get the width $\Gamma(B_c^* \rightarrow B_c \gamma) = 17.1 \times 10^{-3}$ keV. In Ref. [13] the authors used light-front quark model to study $B_c^* \rightarrow B_c \gamma$ and obtained $\Gamma(B_c^* \rightarrow B_c \gamma) = 22.4(19.9) \times 10^{-3}$ keV for $\Delta M = 50$ MeV which is consistent with our result.

$\Gamma(B_c^* \rightarrow B_c \gamma)$ is sensitive to the mass of $m_{B_c^*}$ (or $\Delta M = m_{B_c^*} - m_{B_c}$ as $m_{B_c}$ has already been experimentally determined) since the rate is proportional to $\Delta M^3$. Fig. 2 shows the dependence of $\Gamma(B_c^* \rightarrow B_c \gamma)$ on $\Delta M$. In our calculation the form factor $|F_{VP}(Q^2 = 0)|$ hardly changes for the different values of $m_{B_c^*}$ and it is nearly equal to 0.208 GeV$^{-1}$.

With the wave functions of $B_c^*$ and $B_c$, $m_{B_c} = 6.276$ GeV we also can obtain the form factor $F_{VP}(Q^2)$ for $Q^2 \neq 0$ i.e. the emitted photon is an off-shell virtual one. For the
transition $B_c^* \to B_c e^+ e^-$, $Q^2$ varies from $Q^2_{\min} = (2m_c)^2$ to $Q^2_{\max} = (m_{B_c} - m_{B_c})^2$. We find the $|F_{VP}(Q^2)|$ is almost a constant for our calculation accuracy, thus we set $|F_{VP}(Q^2)| = |F_{VP}(Q^2_{\min})| = 0.208$ GeV$^{-1}$. Integrating $d\Gamma$, we eventually obtain the width $\Gamma(B_c^* \to B_c e^+ e^-) = 8.64 \times 10^{-5}$ keV. The decay rates including the transitions: $B_c^*(2S) \to B_c$, $B_c^*(3S) \to B_c$, $B_c(3S) \to B_c^*$ and $B_c(3S) \to B_c^*(2S)$ are listed in Tab.I.

IV. CONCLUSION

The family of $B_c$ meson is composed by two different heavy flavors: $b\bar{c}$, the members’ production and decays are different from those for hidden-flavored heavy quarkonia, and the study of the $b\bar{c}$ system must be helpful in gaining insights into the hadron structure and the governing physical mechanisms. The two heavy quarks with different flavors cannot annihilate into gluons, in addition to that the physics is rich, the influence of the relativistic effects is alleviated, and in the BS framework the instantaneous approximation seems to work well and the results are more reliable. Even though the ground state, $B_c$ of $J^P = 0^-$, was found several years ago, its partner $B_c^*$ of $J^P = 1^-$ has not been seen yet. Fortunately LHC begins running and a $Z^0$ factory is proposed, both of them will offer us optimistic opportunity to explore $b\bar{c}$ family, especially, $B_c^*$.

In this work we mainly study the transitions $B_c^* \to B_c \gamma$ and $B_c^* \to B_c e^+ e^-$ in the BS framework. Writing the transition matrix element in the form of Eq.(2), we determine the form factor $F_{VP}(Q^2)$. In the calculation, we substitute the BS wave functions of initial and daughter mesons which are obtained by solving the BS equation. With the form factor $F_{VP}(Q^2)$, we evaluate $\Gamma(B_c^* \to B_c \gamma)$ and $\Gamma(B_c^* \to B_c e^+ e^-)$. When the mass of $m_{B_c}$ is 6.3308 GeV and $m_{B_c^*} = 6.276 \pm 0.004$ GeV (the measured value [12] i.e. $\Delta M = 55 \pm 4$ MeV), we obtain $\Gamma(B_c^* \to B_c \gamma) = 17.1 \times 10^{-3}$ keV and $\Gamma(B_c^* \to B_c e^+ e^-) = 8.64 \times 10^{-5}$ keV. The branching ratio of $B_c^* \to B_c \gamma$ is three orders larger than that of $B_c^* \to B_c e^+ e^-$, it means that the chance of observing $B_c^* \to B_c \gamma$ seems to be superior to $B_c^* \to B_c e^+ e^-$, however the positron and electron are charged and their tracks would be easier caught by the detector than a single photon, so that $B_c^* \to B_c e^+ e^-$ may still have its advantage for detection. We will rely on the Monte-Carlo simulation made by our experimental colleagues to make a judgement if at the $Z^0$ factory it is a possible process to be measured.

Since the value of $B_c^* \to B_c \gamma$ is sensitive to the vary of $\Delta M$ we study the dependance of $\Gamma(B_c^* \to B_c \gamma)$ to $\Delta M$. Our calculation indicates that the value of $F_{VP}(Q^2)$ is not sensitive to the $m_{B_c^*}$ but $\Gamma(B_c^* \to B_c \gamma)$ is rather sensitive to the mass splitting $\Delta M$. We calculate the decay width based on the BS framework, the obtained $\Gamma(B_c^* \to B_c \gamma)$ is accordant with that obtained in terms of the light-front-quark model by the authors of Ref.[13]. In the work of Ref.[13], the authors used the variational method to fix the free parameters, whereas we determine the parameters by fitting the data for quarkonia. Application of both the
light-front quark model and the BS framework seems to be reasonable to deal with the radiative process, however, the difference of the two theoretical evaluated values may hint the feasibility of their application in this case. Fortunately, the future experiments at the $Z^0$ factory will make a more accurate measurement on $\Gamma(B_c^* \to B_c \gamma)$ and $\Gamma(B_c^* \to B_c + e^+ e^-)$, and the data would judge which model to be more reasonable. We are expecting the new data.

It is worth emphasizing again, even though at LHC, a large database on $B_c$ and $B_c^*$ will be available, but the complicated background makes a precise observation of $\Gamma(B_c^* \to B_c \gamma)$ rather difficult, so we lay our hope on the proposed $Z^0$ factory.

Acknowledgments

This work is supported by the special grant for new faculty from Tianjin University. This work is partially supported by the National Natural Science Foundation under the contract No. 10775073, No. 10875032, No.10875155, No.10847001 and the Special grant for the PH.D program of the Education Ministry of China.

Appendix A: Notations

Concerning how to solve the BS equation the readers are suggested to refer Ref. [7, 11, 14]. Here we only present some notations appearing in this paper for readers’ convenience.

For a bound state of two constituents with the total momentum $\mathcal{P}$ and relative momentum $q$, $\mathcal{P}$ and $q$ are defined as:

\[ p_1 = \alpha_1 \mathcal{P} + q, \quad \alpha_1 = \frac{m_1}{m_1 + m_2}, \]
\[ p_2 = \alpha_2 \mathcal{P} - q, \quad \alpha_2 = \frac{m_2}{m_1 + m_2}. \]

The relative momentum $q$ is divided into two parts, $q_{\mathcal{P}_\parallel}$ and $q_{\mathcal{P}_\perp}$ and they are longitudinal and transverse to $\mathcal{P}$, respectively:

\[ q^\mu = q_{\mathcal{P}_\parallel}^\mu + q_{\mathcal{P}_\perp}^\mu, \]

where $q_{\mathcal{P}_\parallel}^\mu \equiv (\mathcal{P} \cdot q/M^2)\mathcal{P}^\mu$, $q_{\mathcal{P}_\perp}^\mu \equiv q^\mu - q_{\mathcal{P}_\parallel}^\mu$, and $M$ is the mass of the bound state.

For the final state with the total momentum $\mathcal{P}'$, the momentum $\mathcal{P}'$ is also divided into two parts, $\mathcal{P}'_{\mathcal{P}_\parallel}$ and $\mathcal{P}'_{\mathcal{P}_\perp}$, longitudinal and transverse to the momentum $\mathcal{P}$ of initial state, respectively:

\[ \mathcal{P}'^\mu = \mathcal{P}'_{\mathcal{P}_\parallel}^\mu + \mathcal{P}'_{\mathcal{P}_\perp}^\mu, \]

where $\mathcal{P}'_{\mathcal{P}_\parallel}^\mu \equiv (\mathcal{P} \cdot \mathcal{P}'/M^2)\mathcal{P}'^\mu$, $\mathcal{P}'_{\mathcal{P}_\perp}^\mu \equiv \mathcal{P}'^\mu - \mathcal{P}'_{\mathcal{P}_\parallel}^\mu$. 

8
Let us introduce several important notations:

\[
\varphi_{\pm\pm}(q_{p_\perp}) \equiv \Lambda_{i_p}^\pm(q_{p_\perp}) \frac{\not{p}}{M} \varphi_p(q_{\mu}) \frac{\not{p}}{M} \Lambda_{2_p}^\pm(q_{p_\perp}),
\]

\[
\bar{\varphi}_{\pm\pm}(q_{p_\perp}) \equiv - \gamma_0 \left[ \varphi_{\pm\pm}^\dagger(q_{p_\perp}) \right]^\dagger \gamma_0, \tag{A3}
\]

and

\[
\Lambda_{i_p}^\pm(q_{p_\perp}) = \frac{1}{2\omega_{i_p}} \left[ \frac{\not{p}}{M} \omega_{i_p} \pm J(i)(m_i + q_{p_\perp}) \right],
\]

\[
\omega_{i_p} = \sqrt{m_i^2 + q_{p_\perp}^2}, \quad q_{p_T} = -q_{p_\perp}^2, \quad \tag{A4}
\]

where \(i = 1, 2\) correspond to the quark and anti-quark, respectively, and \(J(i) = (-1)^{i+1}\).

The relativistic wave function for the mesons with the quantum numbers \(J^P = 0^-\) and \(J^P = 1^-\) can be generally written as

\[
\varphi_0^-(q_{p_\perp}) = \left[ f_1(q_{p_\perp}) \not{p} + f_2(q_{p_\perp}) M + f_3(q_{p_\perp}) \not{p}_{p_\perp} + f_4(q_{p_\perp}) \frac{\not{p}}{M} \right] \gamma_5,
\]

\[
\varphi_1^-(q_{p_\perp}) = \not{p} \cdot \gamma_5 \left[ g_1(q_{p_\perp}) + g_2(q_{p_\perp}) \frac{\not{p}}{M} + g_3(q_{p_\perp}) \not{p}_{p_\perp} + g_4(q_{p_\perp}) \frac{\not{p}}{M} \right] \not{p} + g_f(q_{p_\perp}) \not{p}^\Lambda - q_{p_\perp} \cdot \gamma_5 \not{p}^\Lambda - q_{p_\perp} \cdot \not{p}^\Lambda \not{p}^\Lambda + q_{p_\perp} \left( \frac{\not{p} \not{p}^\dagger_{p_\perp} - \not{p} \not{p}^\dagger_{p_\perp} \gamma_5 \not{p}^\Lambda - q_{p_\perp} \cdot \gamma_5 \not{p}^\Lambda}{M} \right), \tag{A5}
\]

where \(f_i(g_i)\) are scalar functions and can be obtained by solving the BS equation.

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