Spin accumulation induced anisotropic RKKY interaction

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Abstract. In a spin-valve structure, the magnetizations of the ferromagnetic layer are influenced by the RKKY exchange interaction. The RKKY exchange interaction is an indirect exchange interaction mediated by the conduction electron of the non-magnetic spacer. The RKKY exchange interaction has been widely studied both theoretically and experimentally. However, the effect of the spin accumulation in the spacer on the exchange interaction has not been studied yet. While the spin accumulation in the non-magnetic spacer is generally small, recent experiments show that non-magnetic metallic layers in magnetic multilayers can have a large spin accumulation. Utilizing linear response theory, we discuss the effect of the spin accumulation on the spin density of the conduction electron in the spacer and show that the spin accumulation induces an anisotropic exchange.

1. Introduction
Spintronics is the area of physics and technology that focuses on the manipulation of spin degree of freedom in nanoscale structure [1]. The development of spintronics started with the discovery of the giant magnetoresistance (GMR) [2,3] which was awarded with the Nobel Prize for Physics in 2007. The discovery started the era an upsurge of research on spin as an additional degree of freedom and potential to control by magnetic or electrical fields [4]. GMR is observed in spin-valve structure. A spin valve consists of two magnetic layers and a non-magnetic spacer sandwiched in between [5].

Because of its potential as magnetic storage devices, spin valve and other multi-layered magnetic nanostructures have attracted a lot of interest in recent years [6]. Furthermore, the exchange coupling between the two magnetic layers of a spin valve structure is among the most active areas of activity [7]. This interlayer exchange coupling can be mediated by the conduction electrons of the nonmagnetic metal spacer [8]. The conduction electron is spin-polarized by the exchange interaction with the localized spin of ferromagnetic layer. The spatial Friedel-oscillation of the perturbed spin density of the conduction electrons give rise to ferromagnetic or antiferromagnetic interlayer coupling as the indirect-exchange interaction changes sign (see Fig. 1) [9,10,11]. The indirect exchange coupling is well-known as RKKY interaction [12].

In the theory of RKKY interaction, the nonmagnetic metallic spacer is a paramagnetic that has no spin accumulation. However, the spacer can have a large spin accumulation [13,14]. The spin accumulation give rise to novel phenomenon, such as spin Hall effect [15]. In this article, we theoretically study the effect of the spin accumulation on the RKKY interaction. We show that the spin accumulation generates an anisotropic RKKY interaction. The anisotropic exchange interaction has similar form with those of Dzyaloshinskii–Moriya interaction.
2. Method

The origin of RKKY interaction is a perturbation from the paramagnetic ground state of metallic spacer due to the following exchange interaction between the spin density of the conduction electron \( \mathbf{\sigma}(r, t) \) and the localized spin of the first ferromagnetic layer \( \mathbf{S}_1(r) \).

\[
H_{\text{ex}} = -J \int d^3r \mathbf{S}_1(r, t) \cdot \mathbf{\sigma}(r, t)
\]

(1)

Here \( J \) is the exchange constant. In linear response regime, the exchange interaction dictates that \( \mathbf{\sigma}(r, t) \) response linearly to perturbation due to \( \mathbf{S}(r, t) \) via retarded susceptibility \( \chi(r, t) \).

\[
\sigma_a(r, t) = J \int d^3r' \chi_{ab}(r-r', t-t') S_{1b}(r', t')
\]

(2)

The space and time dependent susceptibility \( \chi_{ab} \) is given by the following Kubo formula

\[
\chi_{ab}(r-r', t-t') = \frac{i}{\hbar} \frac{1}{t-t'} \langle [\sigma_a(r, t), \sigma_b(r', t')] \rangle
\]

(3)

Here \( \theta(x) \) is the Heaviside step function and its derivation is delta function \( \delta(x) = \theta'(x) \) [16]. The RKKY interaction arises due to indirect interaction mediated by \( \mathbf{\sigma}(r) \).

\[
H_{\text{RKKY}} = -J \mathbf{\sigma}(r) \cdot \mathbf{S}_2
\]

(4)

We can evaluate the susceptibility by first writing \( \mathbf{\sigma}(r, t) \) in second quantization expression [17].

\[
\mathbf{\sigma}(r, t) = \sum_{a\beta} \int \frac{d^3p d^3q}{(2\pi)^6} e^{i p \cdot r} \sigma_{a\beta} a_{p+q}^\dagger(t) a_{p\beta}(t)
\]

(5)

Here \( \sigma_{a\beta} \) is the component of Pauli matrices [18].

\[
(\sigma_x, \sigma_y, \sigma_z) = \left( \begin{array}{ccc}
0 & 1 & 0 \\
1 & 0 & -i \\
0 & i & 0
\end{array} \right)
\]

(6)

The Pauli matrices satisfy \( \sigma_a \sigma_b = i \delta_{ab} + i \epsilon_{abc} \sigma_c \), where \( \delta_{ab} \) is the Kronecker delta and \( i \) is the \( 2 \times 2 \) identity matrix. Substituting Eq. (4) to Eq. (3), we can obtain the Fourier transformation relation between \( \chi_{ab}(r, t) \) and \( \chi_{ab}(\mathbf{p}, \mathbf{q}, t) \).

\[
\chi_{ab}(\mathbf{p}, \mathbf{q}, t) = \int d^3p d^3q e^{i \mathbf{p} \cdot \mathbf{r}} \chi_{ab}(\mathbf{p}, \mathbf{q}, t),
\]

(7)

\[
\chi_{ab}(\mathbf{p}, \mathbf{q}, t) = \frac{i\theta(t)}{\hbar} \sum_{a\beta a'\beta'} \int \frac{d^3p' d^3q'}{(2\pi)^6} (\sigma_a)_{a\beta} (\sigma_b)_{a'\beta'} \left( [a_{p+q}^\dagger(t) a_{p\beta}(t), a_{p+q}^\dagger(t) a_{p\beta}(t)] \right)
\]

(8)

\( \chi_{ab}(\mathbf{p}, \mathbf{q}, t) \) can be obtained by taking its time derivative and using Heisenberg equation [17].

\[
\frac{\partial}{\partial t} \sigma_{a\beta} a_{p+q}^\dagger(t) a_{p\beta}(t) = \frac{i}{\hbar} [\sigma_{a\beta} a_{p+q}^\dagger(t) a_{p\beta}(t), H]
\]

(9)

Here \( H \) is the unperturbed Hamiltonian of the nonmagnetic layer. In the rest of this section we review the derivation of \( \chi_{ab} \) for the case of simple metal without no spin accumulation i.e. the RKKY
susceptibility \([9,10,11]\). After that, we introduce the necessary component to add to the unperturbed Hamiltonian and discuss the effect of the spin accumulation in Section 3.

The unperturbed Hamiltonian of simple metal can be written in the following Hamiltonian.

\[
H_0 = \sum_{k\mathbf{y}} \varepsilon_k a_{k\mathbf{y}}^+ a_{k\mathbf{y}}. \tag{10}
\]

Here \(\varepsilon_k = h^2 k^2 / 2m\) is the energy dispersion of conduction electron with parabolic band and \(m\) is the effective mass of the conduction electron. Substituting Eq. (10) into Eq. (9)

\[
\frac{\partial}{\partial t} a_{p+q\mathbf{a}}^+ a_{p\mathbf{b}}(t) = \frac{i}{\hbar} \sum_k \varepsilon_k [a_{p+q\mathbf{a}}^+ a_{p\mathbf{b}}^-, a_{k\mathbf{y}}^+ a_{k\mathbf{y}}^-]. \tag{11}
\]

Since \(a^+\) and \(a^-\) is fermionic creation and annihilation operator, it follows the following anti-commutation relation

\[
\{a_i, a_j^+\} = a_i a_j^+ + a_j a_i^+ = \delta_{ij}. \tag{12}
\]

Eq. (11) can be used to derive the following commutation relation

\[
[a_i, a_j^+, a_k^-, a_l] = \delta_{jk} a_i^+ a_l - \delta_{il} a_k^+ a_j. \tag{13}
\]

We can use Eq. (13) to evaluate Eq. (11).

\[
\frac{\partial}{\partial t} a_{p+q\mathbf{a}}^+ a_{p\mathbf{b}}(t) = \frac{i}{\hbar} (\varepsilon_p - \varepsilon_{p+q}) a_{p+q\mathbf{a}}^+ a_{p\mathbf{b}}. \tag{14}
\]

In mean-field regime, we can use \(\sum_{\alpha} (a_{p\mathbf{a}}^+ a_{p\mathbf{b}}) = \delta_{pq} f_p\) to simplify the following expectation value.

\[
\sum_{\alpha'} (\sigma_{\alpha'}(\mathbf{a})_{\alpha'}(\mathbf{b})_{\alpha}) \rho_{\alpha'}(\mathbf{a})_{\alpha'}(\mathbf{b})_{\alpha}) = \delta_{\alpha p} \delta_{\beta p} \delta_{\gamma p} \delta_{\delta p} (f_p + q - f_p). \tag{15}
\]

Such that

\[
\frac{\partial^2 \chi_{ab}(\mathbf{p}, \mathbf{q}, \omega)}{\partial t^2} = \frac{i}{\hbar} (\varepsilon_{p} - \varepsilon_{p+q}) \chi_{ab}(\mathbf{p}, \mathbf{q}, t) + \frac{1}{\hbar} \delta_{ab}(f_p + q - f_p). \tag{16}
\]

Taking the Fourier interaction that limits \(\chi_{ab}(\mathbf{p}, \mathbf{q}, t \rightarrow \infty) = 0\),

\[
\chi_{ab}(\mathbf{p}, \mathbf{q}, t) = \lim_{\eta \rightarrow 0^+} \frac{\int d\omega}{2\pi} e^{(\eta-i\omega)t} \chi_{ab}(\mathbf{p}, \mathbf{q}, \omega) \tag{17}
\]

Eq. (18) in frequency domain become a linear equation

\[
(\eta - i\omega) \chi_{ab}(\mathbf{p}, \mathbf{q}, \omega) = \frac{i}{\hbar} (\varepsilon_{p} - \varepsilon_{p+q}) \chi_{ab}(\mathbf{p}, \mathbf{q}, \omega) + \frac{i}{\hbar} \delta_{ab}(f_p + q - f_p). \tag{18}
\]

\[
\chi_{ab}(\mathbf{p}, \mathbf{q}, \omega) = \delta_{ab} \chi_0(\mathbf{p}, \mathbf{q}, \omega) = \delta_{ab} \lim_{\eta \rightarrow 0^+} \frac{f_p - f_{p+q}}{\varepsilon_p - \varepsilon_{p+q} + \hbar \omega + i\eta}. \tag{19}
\]

\(\chi_{ab}(\mathbf{p}, \mathbf{q}, \omega) = \delta_{ab}\chi_0(\mathbf{p}, \mathbf{q}, \omega)\) In the static limit, we can obtain the spin density that responsible to the RKKY interaction is then

\[
\sigma(\mathbf{r}) = J \chi_0(\mathbf{r}) S_1 = J S_1 \int \frac{d^3 p d^3 q}{(2\pi)^6} e^{i \mathbf{q} \cdot \mathbf{r}} \rho_{\alpha p} - \rho_{\alpha q} = J S_1 \int \frac{d^3 m}{h^2 \pi^3} \sin 2k_{\mathbf{r}} - 2k_{\mathbf{r}} \cos 2k_{\mathbf{r}}. \tag{20}
\]

Substituting Eq. (20) to Eq. (4), we can see that this \(2k_{\mathbf{r}} \mathbf{r}\) term inside the trigonometric function gives the signare Friedel spatial oscillation of RKKY interaction.

\[
H_{\text{RKKY}} = -J^2 \chi_0(\mathbf{r}) S_1 \cdot S_2 - S_1 \cdot S_2 \frac{\int d^3 m \sin 2k_{\mathbf{r}} - 2k_{\mathbf{r}} \cos 2k_{\mathbf{r}}}{h^2 \pi^3} \tag{21}
\]

Since the spacer has been observed to have a large spin accumulation \([13,14]\), we consider the case of nonmagnetic spacer with spin accumulation. The spin accumulation act as a spin-dependent electric potential \(\mathbf{V}_0 = -\mathbf{\sigma} \cdot \mathbf{r}\) such that the unperturbed Hamiltonian can be written as follows.

\[
H_1 = H_0 + H_\mu = \sum_{k\mathbf{y}} \varepsilon_k a_{k\mathbf{y}}^+ a_{k\mathbf{y}} - \sum_{\mathbf{k} \alpha' \beta'} a_{\mathbf{k} \alpha'}^+ a_{\mathbf{y} \beta'} \sigma_{\alpha \beta} \cdot \mathbf{\mu} \tag{22}
\]
3. Result

We can then obtain the susceptibility $\chi_{ab}(p, q, \omega)$ using the same analytical step, but with $H_1$ as unperturbed Hamiltonian in the time derivation of $\chi_{ab}(p, q, \omega)$ instead of $H_0$. Using Eq. (13) and Pauli matrices identity, we obtain the following relation.

$$\sum_{ab} \sigma_{a\beta} \left[ a_{p+qa}^+ a_{p\beta}, H_1 \right] = 2i \sum_{ab} a_{p+qa}^+ a_{p\beta} \sigma_{a\beta} \times \mu$$

(23)

In this case, Eq. 14 should be modified as follows

$$\frac{\partial}{\partial t} \sigma_{a\beta} a_{p+qa}^+ a_{p\beta}(t) = \frac{i}{\hbar} (\epsilon_p - \epsilon_{p+q}) \sigma_{a\beta} a_{p+qa}^+ a_{p\beta} - \frac{2}{\hbar} \sum_{\alpha\beta} a_{p+qa}^+ a_{p\beta} \sigma_{a\beta} \times \mu$$

(24)

Such that

$$(\eta - i\omega) \chi_{ab}(p, q, \omega) = \frac{i}{\hbar} (\epsilon_p - \epsilon_{p+q}) \chi_{ab}(p, q, \omega) - \frac{2}{\hbar} \sum_{cd} \epsilon_{ac} \mu_d \chi_{cd}(p, q, \omega) + \frac{i}{\hbar} \delta_{ab} (f_p - f_p - f_p).$$

(25)

To solve this, we consider the relation of $\chi_{xx}(p, q, \omega), \chi_{yx}(p, q, \omega)$ and $\chi_{zx}(p, q, \omega)$

$$\begin{pmatrix}
\epsilon_{p+q} - \epsilon_p + \hbar(\omega + i\eta) \\
-2i\mu_x \\
2i\mu_y
\end{pmatrix} = \begin{pmatrix}
2i\mu_x \\
-2i\mu_y \\
2i\mu_y
\end{pmatrix} \begin{pmatrix}
\chi_{xx} \\
\chi_{yx} \\
\chi_{zx}
\end{pmatrix}$$

(26)

Similarly, the equation for all combination of $\chi_{ab}$ is.

$$\begin{pmatrix}
\epsilon_{p+q} - \epsilon_p + \hbar(\omega + i\eta) \\
-2i\mu_x \\
2i\mu_y
\end{pmatrix} = \begin{pmatrix}
2i\mu_x \\
-2i\mu_y \\
2i\mu_y
\end{pmatrix} \begin{pmatrix}
\chi_{xx} \\
\chi_{xy} \\
\chi_{xz}
\end{pmatrix}$$

(27)

For $\mu$ that is much smaller than Fermi energy $\mu \ll \epsilon_F$, the leading order of $\chi_{ab}(p, q, \omega)$

$$\begin{pmatrix}
\chi_{xx} \\
\chi_{xy} \\
\chi_{yx} \\
\chi_{yy} \\
\chi_{zx} \\
\chi_{zy}
\end{pmatrix} \approx \begin{pmatrix}
\chi_0(p, q, \omega) - 2\mu_1 \chi_1(p, q, \omega) \\
-2\mu_1 \chi_1(p, q, \omega) \\
\chi_0(p, q, \omega) - 2\mu_1 \chi_1(p, q, \omega) \\
2\mu_1 \chi_1(p, q, \omega) \\
-2\mu_1 \chi_1(p, q, \omega) \\
2\mu_1 \chi_1(p, q, \omega)
\end{pmatrix}$$

(28)

We can see that the susceptibility is no longer isotropic. The isotropic term $\chi_0(p, q)$ is the same as when there is no spin accumulation and the anisotropic term is

$$\chi_1(p, q) = \lim_{\eta \to 0^+} \left( \frac{f_p - f_p - f_p}{\epsilon_{p+q} - \epsilon_p + \hbar(\omega + i\eta)} \right)^2.$$ 

(29)

We can see that additional term appear in the spin density that responsible to the RKKY interaction

$$\sigma(r) = J \chi_0(r) S_1 + 2J \chi_1(r) \mu \times S_1$$

(30)

where

$$\chi_1(r) = \frac{m^2}{2\pi^4 \hbar^2} \left( \frac{\sin \frac{k_F r}{\hbar}}{k_F r} \right)^2$$

(31)
Substituting Eq. (31) to Eq. (4),

\[ H_{RKKY} = -J^2 \chi_0(r) S_1 \cdot S_2 + H_{RKKY}^{\text{anisotropic}} \]  

(32)

we found that there is a new term in RKKY interaction that arise from the anisotropic susceptibility.

\[ H_{RKKY}^{\text{anisotropic}} = -2J^2 \chi_4(r) \mu \cdot (S_1 \times S_2) \]  

(33)

4. Conclusion
To summarize, we studied the effect of spin accumulation on the indirect exchange interaction in spin valve structure. The RKKY interaction arises when we apply linear response theory to the spin density of conduction electron \( \sigma(r) \). We derived the expression for \( \sigma(r) \) in term of susceptibility \( \chi_{ab} \). We utilize Kubo formula for magnetic susceptibility in mean-field approximation to theoretically derive the expression of \( \chi_{ab} \) under a finite spin accumulation \( \mu \). When we have a finite spin accumulation, we showed that \( \sigma(r) \) has isotropic and anisotropic term (see Eq. (30)). The anisotropic term give rise to anisotropic exchange interaction \( H_{RKKY}^{\text{anisotropic}} \). This anisotropic exchange interaction is antisymmetric and has the same form with Dzyaloshinskii–Moriya interaction (DMI). Since DMI is basic interaction for magnetoelectric effects in multiferroic materials, we expect that this spin-accumulation-induced anisotropic RKKY interaction can contribute to the application of spin valve for functional multiferroic-based device.

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