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Interaction-Driven Spontaneous Quantum Hall Effect on a Kagome Lattice

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Topological states of matter have been widely studied as being driven by an external magnetic field, intrinsic spin-orbital coupling, or magnetic doping. Here, we unveil an interaction-driven spontaneous quantum Hall effect (a Chern insulator) emerging in an extended fermion-Hubbard model on a kagome lattice, based on a state-of-the-art density-matrix renormalization group on cylinder geometry and an exact diagonalization in torus geometry. We first demonstrate that the proposed model exhibits an incompressible liquid phase with doublet degenerate ground states as time-reversal partners. The explicit spontaneous time-reversal symmetry breaking is determined by emergent uniform circulating loop currents between nearest neighbors. Importantly, the fingerprint topological nature of the ground state is characterized by quantized Hall conductance. Thus, we identify the liquid phase as a quantum Hall phase, which provides a “proof-of-principle” demonstration of the interaction-driven topological phase in a topologically trivial noninteracting band.

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Introduction.—Topological states of matter have led to an ongoing revolution of our fundamental understanding of quantum materials, as exemplified by the discoveries of the integer quantum Hall (IQH) effect [1,2], the quantum anomalous Hall (QAH) effect [3–5], and the quantum spin Hall effect [6–8]. Crucially, such topological phases emerge from noninteracting electronic structures with nontrivial topology, where a strong magnetic field or a large spin-orbit coupling is typically necessary. Despite great achievements, most of the materials that have exhibited unique topological properties to date can be understood based on noninteracting physics originating from nontrivial band physics. This limitation has inspired recent enthusiasm in exploring topological phases in “trivial” materials [9–13], without the requirement of a nontrivial invariant encoded in a single-particle wave function or band structure. Finding such novel phases can significantly enrich the class of topological materials and thus is of great importance.

The strong correlation between electrons can dramatically change the noninteracting physics and induce spontaneous symmetry breakings. Therefore, many-body interaction is expected to enable topological phases in strongly correlated systems by generating circulating currents and spontaneously breaking time-reversal symmetry (TRS), which act as an effective magnetic field or spin-orbit coupling [12,14,15]. This subject has attracted vigorous research due to support from mean-field studies, followed by low-energy renormalization-group analysis [13,16], and the existence of such topological phases has been suggested for the extended Hubbard model on various lattice models [17–30]. However, unbiased numerical simulations, such as exact diagonalization (ED) and density-matrix renormalization-group (DMRG) studies, found competing states other than topological phases as the true ground states in all previously proposed systems with Dirac points [31–34] or quadratic band touching points [35,36]. A major obstacle is that, instead of triggering the desired spontaneous TRS breaking, strong interactions tend to stabilize competing solid orders by breaking the translational or rotational lattice symmetry. Thus, the putative topological phase is usually preempted by various competing states [31–36]. Moreover, it is also technically challenging to detect such exotic phases with spontaneous TRS breaking, as the TRS partners usually tend to couple on finite-size systems. If the intrinsic energy gap generated by TRS breaking is small, numerical simulations can hardly distinguish a possible insulating topological phase from the semimetal phase [34]. Taken as a whole, the simple concept of realizing interaction-induced topological phases remains unsettled for realistic electron systems, and the types of physically attainable Hamiltonians that would exhibit these exotic ground states has yet to be established.

In this Letter, we provide unbiased numerical evidence for a QAH phase generated by electron-electron interactions in a fermion-Hubbard model on a kagome lattice. By engineering electron interactions, our ED calculations signal an incompressible liquid phase with a doublet degenerate ground state in torus geometry. To further investigate the nature of this gapped phase, we perform a DMRG calculation in cylinder geometry for larger systems. Remarkably, we show the TRS of the ground state is spontaneous TRS breaking, with compelling evidence from emergent long-range and uniform...
The topological nature of the ground state is identified by quantized Hall conductance, with the help of the idea similar to a Laughlin gedanken experiment performed numerically. Thus, we establish that the essence of the gapped phase is equivalent to the QAH phase. We also confirm that the QAH phase is robust against finite-size effects by accessing large systems up to the current computational limit. Moreover, we map out a phase diagram and identify two competing charge density wave phases by varying interactions, where transitions to the QAH phase are determined to be of the first order. Our results not only provide “fingerprint” evidence of the interaction-driven QAH phase but also open up a long-sought platform for exploring topological physics in strongly correlated systems.

Model and method.—We consider a spinless fermion-Hubbard model on a kagome lattice shown in the inset of Fig. 1(a) and described by the Hamiltonian:

\[ H = \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left[ t \mathbf{c}^\dag_{\mathbf{r}} \mathbf{c}_{\mathbf{r}'} + \text{H.c.} \right] + V_1 \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'} \]
\[ + V_2 \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_3 \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'}, \]

where \( \mathbf{c}^\dag_{\mathbf{r}} \) (\( \mathbf{c}_{\mathbf{r}} \)) creates (annihilates) a spinless fermion at site \( \mathbf{r} \). We set the nearest-neighbor hopping amplitude \( t = 1 \) as the energy unit here. \( V_1, V_2, V_3 \) denote the density-density repulsion strengths on first, second, and third nearest neighbors, respectively [37]. We focus on the total filling number \( \nu = N_e/N_s = 1/3 \) (\( N_e \) is the total electron number and \( N_s \) is the number of the lattice sites). In the non-interacting limit, this model supports three energy bands and the lowest flat band quadratically touches the second band at the \( \Gamma \) point \( [K = (0, 0)] \) [38]; thus, the system is gapless at \( \nu = 1/3 \) and topological trivial.

In order to study the ground state phase diagram in the \( \{V_1, V_2, V_3\} \) parameter space, we implement the DMRG algorithm [39,40] combined with ED, both of which have been proven to be powerful and complementary tools for studying realistic models containing arbitrary strong and frustrated interactions [41–47]. We study large systems up to \( L_y = 6 \) unit cells and keep up to \( M = 4800 \) states to guarantee a good convergence (the discarded truncation error is less than \( 2 \times 10^{-6} \)). We take advantage of a recent development in the DMRG algorithm by adiabatically inserting flux to probe the spontaneous TRS breaking and the topological quantized Hall conductance (see Ref. [38] for computational details) [43–46].

Phase diagram.—In the presence of strong interactions, our main findings are summarized in the phase diagrams shown in Figs. 1(a) and 1(b). In the intermediate parameter region (labeled in red), we find a robust QAH phase emerging with spontaneous TRS breaking. The QAH phase is featured by a twofold ground state degeneracy in torus geometry, arising from two sets of QAH states with opposing chiralities. The topological nature of the QAH states is characterized by the integer quantized Chern numbered \( C = \pm 1 \), respectively, for TRS breaking states with opposing chiralities. In addition, we show that the QAH phase is neighboring with several solid phases which all respect TRS: a stripe phase and a charge density wave phase, both demonstrating distinctive Bragg peaks in their density-density structure factors [Figs. 1(c) and 1(d)]. On the contrary, the QAH phase displays a structureless feature [Fig. 1(e)] in the structure factor, indicating the absence of space-group symmetry breaking. Finally, reducing \( V_1, V_2, \) and \( V_3 \) simultaneously, we find a parameter region shaded by light red in the left bottom corner in the phase diagrams shown in Figs. 1(a) and 1(b) which is likely a weaker QAH phase (labeled as QAH*); we will discuss further details below.

Energy spectrum and double degeneracy.—The emergent QAH phase in a finite torus system is expected to host a twofold ground state degeneracy, representing two TRS spontaneously breaking states with opposing chiralities as TRS partners with each other. To examine this property for the model systems, we first investigate the low-energy spectra based on the ED calculation. As shown in Fig. 2(a), we find two near degenerating ground states in energy spectra for both \( N_s = 27 \) and \( N_s = 36 \) clusters [51], which are separated from the excited levels by a finite energy gap.

FIG. 1. Phase diagram of an extended fermion-Hubbard model [Eq. (1)] plotted in (a) \( V_1 = V_2 \) and \( V_3 \) parameter space and in (b) \( V_1 \) and \( V_2 = V_3 \) parameter space, obtained by DMRG calculations on a cylinder of circumference \( L_y = 6 \). The QAH phase is characterized by the long-range current-current correlations and integer quantized Hall conductance. The phase boundary between the QAH phase and the other phase is determined by the emergent loop current which signals spontaneous TRS breaking. Contour plots of the static density structure factor are shown for (c) the charge density wave \( q = (0, 0) \) phase, (d) the stripe phase, and (e) the QAH phase. The white dashed line shows the first Brillouin zone.
Importantly, the ground states never mix with excited levels when varying the twisting boundary conditions, signaling the robustness of the excitation gap (see Ref. [38]). Moreover, a stable topological phase is expected to be protected by not only by the excitation gap but also the nonzero single-particle gap. In Fig. 2(b), we have also calculated the single-particle gap $\Delta(N_s)$ as a function of $1/N_s$ where the finite-size scalings indicate a nonzero $\Delta(\infty)$ for the QAH phase.

Spontaneous time-reversal symmetry breaking and the emergent loop current.—To investigate the possible spontaneous TRS breaking of the ground states, we turn to larger systems in cylinder geometry and obtain the ground states by implementing a DMRG calculation. Indeed, we obtain two TRS breaking states $|\Psi^L(R)\rangle$, by random initializations of wave functions in DMRG simulations [41], which are degenerating in energy as expected (as TRS partners for each other). Here, we label different ground states by their chiral nature, where $L$ ($R$) stands for "left-hand" ("right-hand") chirality. The corresponding spontaneous TRS breaking of $|\Psi^L(R)\rangle$ can be obtained by measuring emergent currents $J_{ij} = i\langle \Psi^L(R)| \hat{c}_i^\dagger \hat{c}_j - \hat{c}_j^\dagger \hat{c}_i |\Psi^L(R)\rangle$ between two nearest-neighbor sites ($i,j$). As shown in Fig. 3(a), the local current pattern $J_{ij}$ uniformly distributes (an arrow represents the direction of the current), which excludes the possibility of bond modulated local orders. Most importantly, local current patterns form a loop structure circulating in the counterclockwise direction in each hexagon for $|\Psi^L\rangle$. (We have checked to see that the TRS partner $|\Psi^R\rangle$ hosts a clockwise loop current).

Interestingly, the staggered magnetic flux in each unit cell (enclosing one hexagon and two triangles) averages out to zero, exactly matching the expectation of the constructed model for the QAH effect [3,16].

Moreover, we also calculate the current-current correlation functions $\langle J_{ij} J_{i'j'} \rangle$ in Fig. 3(b) ($(ij)$ is the bond parallel to the reference bond $(i_0 j_0)$ and the distance measured by $R_{ij}$). We compare the $\langle J_{ij} J_{i'j'} \rangle$ from the QAH phase with different system widths. We find long-range correlations for the system widths $L_y = 4,5,6$. The current correlations remain stable with an increasing $L_y$, indicating that the spontaneous TRS breaking is robust against finite-size effects. By contrast, the current correlations decay exponentially for the stripe phase and the charge density wave phase, revealing the TRS preservation in solid phases. Last, we notice that the QAH* phase can develop a relatively weaker current correlation, although it shows a sharply decaying correlation at a short-range distance.

Quantized Hall conductance.—To uncover the topological nature of the QAH phase, we perform a numerical flux insertion simulation in a cylinder system [43,47] to determine the quantized Hall conductance $\sigma_H$. This simulation follows the idea of a Laughlin gedanken experiment for interpreting the IQH effect [2,52], where an integer quantized charge will be pumped from one edge to the other edge by inserting a $U(1)$ charge flux $\theta$ in the hole of the cylinder. On the DMRG side, we adiabatically increase the inserted flux $\theta$ and use the converged wave function for a smaller $\theta$ as the initial state for the increased $\theta$, achieving adiabatic evolution of the ground state [38,43]. The Hall conductance can be computed by $\sigma_H = (e^2/h) \Delta Q(\theta)$ [43,47], where the net charge transfer $\Delta Q(\theta)$ can be calculated from the net change of the total charge in the half-system: $\Delta Q(\theta) = Tr[\hat{\rho}_L (\theta) \hat{Q}]$ ($\hat{\rho}_L$ the reduced density matrix of left half system). As expected, in Fig. 4(a), the obtained $\sigma_H$ of the QAH phase takes a nearly quantized...
FIG. 4. Hall conductance $\sigma_H$ obtained from the Laughlin flux insertion gedanken experiment, where $\sigma_H$ is equal to the charge transfer $\Delta Q$ for one edge to the other edge. (a) Net charge transfer $\Delta Q$ for the QAH phase (the blue circle), the stripe phase (the navy triangle), the charge density wave (the green cross), and the QAH* phase (the purple dot). The system size is an $L_y = 4$ cylinder. (Inset) The adiabatic threading of a $U(1)$ charge flux in the hole of cylinder. (b) Net charge transfer $\Delta Q$ of the QAH phase for the system sizes $L_y = 4$ (the blue circle), $L_y = 5$ (the black square), and $L_y = 6$ (the red diamond).

value $\sigma_H \approx -1.00 e^2/h$ (for $|\Psi^L\rangle$) by threading a flux quantum $\theta = 0 \to 2\pi$. We also checked to see that the TRS partner $|\Psi^R\rangle$ hosts $\sigma_H \approx 1.00 (e^2/h)$. By comparison, neither the stripe phase nor the charge density wave respond to the inserted flux; therefore, they have an exactly zero Hall conductance $\sigma_H = 0$ [Fig. 4(a)] consistent with the trivial topology of these states. Furthermore, we examine the stability of the topological quantization on finite-size systems. In Fig. 4(b), we show the Hall conductance $\sigma_H$ of the QAH phase in a cylinder system with widths $L_y = 4, 5, 6$, all of which give the nearly quantized value $\sigma_H \approx -1.00 (e^2/h)$, supporting the fact that the QAH phase is stable in the thermodynamic limit.

Phase transition.—We address the nature of the quantum phase transitions between the QAH phase and other phases (see Ref. [38]). We utilize several quantities, such as ground state wave function fidelity, which are expected to signal the sensitivity of the wave function with varying interacting parameters. Moreover, we have also inspected several order parameters related to TRS and translational symmetry spontaneous breakings [38], respectively. Based on these studies, we find that transitions between the QAH phase and the stripe phase as well as the charge density wave phase are the first order ones, with evidence coming from the steplike change in wave function overlap and order parameters [38].

Even though the QAH phase is shown to be remarkably robust in the phase diagram, we are less certain about the QAH* phase (sitting at the left bottom corner of Fig. 1). We do not observe TRS breaking (or long-range current correlations) in the QAH* phase [Fig. 3(b)]. Interestingly, if we perform flux insertion (inducing TRS breaking explicitly), we observe a nearly quantized Hall conductance [Fig. 4(a)], while the evolution of the pumped charge versus the flux is not as smooth as the QAH phase. Hence, we believe that the ground state of the QAH* phase is a QAH state with, however, a strong finite-size effect [as indicated by the relatively small energy gap in Fig. 2(b)]. In addition, going closer to the weak interaction limit, using ED calculations, we find that the ground state continues to evolve adiabatically from the QAH phase to the QAH* phase without an additional quantum phase transition [38], which is consistent with the mean-field [16,38] and perturbative renormalization-group [25] prediction of an infinitesimal interaction inducing QAH effect.

Conclusion and outlook.—We have presented convincing evidence for an interaction-driven spontaneous QAH phase in an extended fermion-Hubbard model on a kagome lattice at one-third filling through engineering interactions. Our complete characterization of the universal properties of the QAH phase includes ground state degeneracy, spontaneous TRS breaking, and a quantized Hall conductance, all of which provide an unambiguous diagnosis of a QAH phase. Such an exotic state has been sought after for a long time; however, its existence in a microscopic model has remained elusive until now. Our current results offer a “proof-of-principle” demonstration of the spontaneous QAH effect purely driven by interactions, without the need for an external magnetic field or another mechanism of explicit TRS breaking. We believe our work will stimulate future research in a number of directions. For example, introducing additional degrees of freedom in simple models usually results in richer behavior; hence, our current model, including spin or orbital degrees, will provide a promising playground for synthesizing and engineering other exotic states, such as an emergent quantum spin Hall effect [6–8,53] without spin-orbital coupling. Moreover, the lowest energy band on a kagome lattice is perfectly flat [38,48–50]; thus, one could imagine a nearly flat band with a nonzero Chern number after the gap opening through interactions. This would be quite significant since it may provide a platform to realize the spontaneously fractional QAH phase or fractional Chern insulators [54–61] when such a flat band is partially filled. On the experimental side, based on the recent experimental development of artificial kagome systems [62], we anticipate activities for realizing and detecting the QAH state in ultracold atomic systems [63].

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Note added.—Recently, we became aware of a work claiming a QAH phase on a checkerboard lattice based on an ED calculation on small sizes [64] (see also Ref. [65]).

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We emphasize that our study is distinct from previously proposed Chern insulators or fractional Chern insulators [54–58], where nontrivial band structures in a noninteracting limit play a vital role in stabilizing the topological phases. In the current model [Eq. (1)], the noninteracting band structure is trivial (see Ref. [38]), and the resulting topological nature of the ground states comes completely from the strong correlation.