A quantum Otto engine with a spin-1/2 and an arbitrary spin coupled by Heisenberg exchange

Ferdi Altintas†
Department of Physics, Abant Izzet Baysal University, Bolu, 14280, Turkey

Özgür E. Müstecaplıoğlu‡
Department of Physics, Koç University, Istanbul, Sariyer, 34450, Turkey

We investigate a quantum heat engine with a working substance of two particles, one with a spin-1/2 and the other with an arbitrary spin (spin-\(s\)), coupled by Heisenberg exchange interaction, and subject to an external magnetic field. The engine operates in a quantum Otto cycle. Work harvested in the cycle and its efficiency are calculated using quantum thermodynamical definitions. It is found that the engine has higher efficiencies at higher spin values and can harvest work at higher exchange interaction strengths. The role of exchange coupling and spin-\(s\) on the work output and the thermal efficiency is studied in detail. In addition, the engine operation is analyzed from the perspective of local work and efficiency. The local work definition is generalized for the global changes and the conditions when the global work can be equal or more than the sum of the local works are determined.

PACS numbers: 05.70.Ln, 07.20.Pe

I. INTRODUCTION

The investigations of heat engines in the quantum regime, or quantum thermodynamics, has become an active area of research in the last decade [1–42]. A quantum heat engine (QHE) uses a quantum working substance to harvest work in a quantum thermodynamical cycle [2–4]. Three level masers can be considered as the first QHEs [1]. Prototype quantum systems, such as two level [2–6] and multilevel particles [7–9], coupled spins [10–30], and harmonic oscillators [31–34] are considered as quantum working substances. Circuit and cavity quantum electrodynamics systems [35–37], quantum dots [38], quantum Hall edge states [39], cold bosonic atoms [40], optomechanical systems [41], and a single ion [42] have been proposed to realize QHEs; while ultracold atoms are proposed for work measurements [43]. In addition to the studies focusing on the quantum properties, such as quantum coherence and correlations, of the working substance [10–12, 18, 20, 24, 26, 30], there are explorations of the quantum heat reservoirs as well [20, 33–36].

In the present contribution, we assume classical heat reservoirs, and consider two interacting particles, one with a spin-1/2 and the other with an arbitrary spin (spin-\(s\)), as our working medium. The particles are assumed to be in an external magnetic field and they interact with each other by Heisenberg exchange coupling. The two spin-1/2 case of this model has been a subject of much attention in quantum thermodynamics [10, 24, 30]. An appealing property of the Heisenberg model is that the quantum Otto engine efficiency can be enhanced at a critical exchange interaction between two spin-1/2 particles [11]. We consider the arbitrary spin value \(s\) as another control parameter next to the exchange coupling and explore its influence on the performance of the QHE. Such higher spin Heisenberg models could be implemented for QHE operations in nuclear magnetic resonance (NMR) systems [44]. Among typical quantum thermodynamical cycles [2–3], we choose to operate our QHE in the Otto cycle as it consists of less demanding processes to implement in comparison to other quantum cycles and proposed in various systems for implementations [33, 37, 41–42].

Using the quantum thermodynamical definitions of work and heat, we calculated the work output and efficiency of our model QHE. We found that at a certain Heisenberg exchange coupling, the QHE harvests more work with higher efficiency for higher spin-\(s\) values. In particular, the efficiency of our QHE can beat the upper bound of efficiency derived for two spin-1/2 particles [11]. Besides, contrary to the two spin-1/2 particles [11], our higher spin QHE can operate at the extreme limit of strong coupling regions. In addition, we analyze the global work and efficiency of the QHE in comparison to local work and efficiency [11]. We determine, similar to two spin-1/2 case [11], that the global work is equal to the sum of the local works unless both the exchange coupling and the magnetic field are varied simultaneously in the adiabatic stages of the cycle. It is also observed that even if the local efficiencies are the same and independent of the spin-\(s\), the local works are dramatically changed by the spin-\(s\). Moreover, it is found in terms of local work analysis that spin-\(s\) is solely responsible for the realization of our QHE in the strong coupling limit. In addition, we suggested a generalization of the local field concept which is applicable even in the case where only interaction parameters are changed in the engine cycle. Our idea is to use an effective mean field description to...
The paper is organized as follows. In Sec. II, we introduce our model QHE. The results for the global and local engine operations are given in Secs. III and IV respectively. A general discussion on the relation between global and local work is given in Sec. V. The conclusions are stated in Sec. VI.

II. MODEL QUANTUM HEAT ENGINE

The working substance of our QHE consists of two spins in a homogeneous magnetic field, coupled to each other with a Heisenberg exchange interaction and it is described by a Hamiltonian

\[ H = 8J\hat{s}_A \cdot \hat{s}_B + 2B (s^z_A + s^z_B), \]

where \( h = 1 \) is taken. \( \hat{s}_A = (s^x_A, s^y_A, s^z_A) \), \( \hat{s}_B = (s^x_B, s^y_B, s^z_B) \), \( s^i_A \) and \( s^i_B \) \((i = x, y, z)\) are the spin-1/2 and spin-\( s \) operators, respectively. We label the spin-1/2 and spin-\( s \) particles with \( A \) and \( B \), respectively. The factor \( B \) in the second term of the Hamiltonian denotes the external homogeneous magnetic field applied along the \( z \)-axis. We take \( \mu_B = 1 \) and assume there is no orbital angular momentum so that the gyromagnetic ratio \( \gamma \) is the same for both spins, \( \gamma = 2 \). \( J \) \((\geq 0)\) is the anti-ferromagnetic coupling constant. Here we restrict ourselves to \( s = 1/2, 1, 3/2, 2, 5/2, 3 \).

The eigenvalues \( E_n \) of the model Hamiltonian are tabulated in Appendix. In thermal equilibrium with a heat bath at temperature \( T \) the density matrix \( \rho \) of the working medium can be written as

\[ \rho = \sum_n P_n |\Psi_n\rangle \langle \Psi_n|. \]

The occupation probabilities of the eigenstates \( |\Psi_n\rangle \) are \( P_n = \exp (-E_n/T)/Z \) \((k_B = 1)\) and \( Z = \sum_n \exp (-E_n/T) \) is the partition function.

We consider the working medium described by the Hamiltonian in Eq. (1) undergoes a quantum Otto cycle which consists of two quantum adiabatic and two quantum isochoric processes. The adiabatic branches involve the change of magnetic field between two chosen values \((B_1 \rightarrow B_2 \rightarrow B_1)\) at a fixed coupling strength, \( J \). The details of the cycle are described below.

Stage 1. This stage is the quantum isochoric process, where the working medium with external magnetic field \( B_1 \) and coupling constant \( J \) interacts with a heat bath at \( T = T_1 \). The interaction takes long enough, so that the working substance falls into a steady state given by Eq. (2) with occupation probabilities \( P_n \) and energy levels \( E_n \). Stage 2. The working medium undergoes a quantum adiabatic process, in which the interaction between the system and the heat bath is turned off and the magnetic field is changed from \( B_1 \) to \( B_2 \). The quantum adiabatic theorem is considered to hold (provided the process is slow enough) \((1)\), so that the occupation probabilities remain unchanged, while the energy levels change from \( E_n \) to \( E'_n \) due to the change in the magnetic strength.

Stage 3. This process is almost the reverse of Stage 1, where the working medium is in contact with a cold heat bath at \( T = T_2 \) \((T_1 > T_2)\). Reaching equilibrium with the bath changes the energy probabilities to \( P'_n \) with \( B = B_2, T = T_2 \) and \( J \) in Eq. (2). Stage 4. The system undergoes another quantum adiabatic process with changing \( B_2 \) to \( B_1 (E'_n \rightarrow E_n) \), while keeping \( P'_n \) the same.

From the generalization of the first law of thermodynamics to quantum mechanical systems \((2, 4)\), the heat exchanges in Stages 1 and 3 are, respectively, given as

\[ Q_1 = \sum_n E_n (P_n - P'_n), \]

\[ Q_2 = \sum_n E'_n (P'_n - P_n). \]

The work is performed only in the adiabatic branches of the quantum Otto cycle. Due to the conservation of energy, the net work done by the QHE can be written as:

\[ W = Q_1 + Q_2 = \sum_n (E_n - E'_n) (P_n - P'_n), \]

where \( W > 0 \) signifies the work performed by the QHE with operational efficiency \( \eta = W/Q_1 \). To harvest positive work by the engine, we consider \( Q_1 > -Q_2 > 0 \) to conform to the second law of thermodynamics.

By using the tabulated eigenvalues \( E_n \) of \( H \) in Appendix and the probabilities given by the thermal occupation numbers in Eq. (2), the work output and the efficiency of the engine can be calculated analytically. The analytical expressions are not very illuminating and will not be displayed here for brevity. We call the work done by the engine given by Eq. (4) and its efficiency \( \eta \) as the global work and global efficiency, respectively, to distinguish them from the local work and efficiency of individual spins, described later in the text.

III. GLOBAL WORK AND EFFICIENCY

Before presenting our results, we would like to review some of the main results in Ref. (11) where the authors investigated the same Hamiltonian in Eq. (1) but for two spin-1/2 particles. The conditions in which the coupled engine efficiency can be higher than the uncoupled one have been determined. Specifically, an upper bound \( \eta_b \) to the efficiency \( \eta \) of the quantum Otto engine has been obtained as

\[ \eta \leq \eta_b = 1 - B_2/B_1 \frac{1 - 4J/B_1}{1 - 4J/B_1} < \eta_c, \]

where the upper bound is always less than the classical Carnot efficiency \( \eta_c = 1 - T_2/T_1 \).
In Fig. 1, we investigate the role of spin-$s$ on the performance of the coupled quantum Otto engine. We plot the global work in Fig. 1(a) and global efficiency in Fig. 1(b), as a function of exchange coupling strength $J$ for $B_1 > B_2$ and $s = 1/2, 1, 3/2, 2, 5/2, 3$. For the uncoupled engine ($J = 0$), the engine efficiency can be calculated as $\eta_{J=0} = 1 - B_2/B_1$ which is independent of spin-$s$ as can be seen in Fig. 1(b). The coupled engine performance can be higher than the uncoupled one; both $W$ and $\eta$ first increase to certain maximums as a function of $J$ and then drop to zero. The role of spin-$s$ on the global work and efficiency is found to shift the maximums and the positive work conditions (PWCs) to the weak coupling regimes; accordingly the coupled Otto engine with high spin-$s$ can produce higher work with higher efficiency than the lower spin-$s$, below a certain sufficiently weak coupling strength (for instance, $J < \approx 0.12$ in Fig. 1). Especially, the engine with $s > 1/2$ can violate $\eta_b$ as indicated by the dashed line in Fig. 1(b).

The mutual relationship between the work output and efficiency is demonstrated by the characteristic curve in Fig. 2 for the same magnetic field and temperature values, and for the same coupling strength range as in Fig. 1. It can be deduced from Fig. 2 that the efficiency at maximum work output as well as the work at maximum efficiency are not notably affected by the spin-$s$ value of the working substance. It seems that the higher spin-$s$ values lead to higher efficiency and work output at the weak coupling regime. We should stress here that this is not the general conclusion; for differently tailored parameters, the maximum of work output and the efficiency can slightly be influenced by the spin-$s$.

In Fig. 1 we have restricted ourselves to the weak coupling regime, specifically $J \in [0, 0.5]$, and now we focus on the strong coupling region. It is possible to show that beyond this limit, i.e., $J > 0.5$, the working substance of spin-$1/2$ pairs cannot do positive work, since it violates the PWC given in Ref. [11]. This is also obvious from the $\eta_b < \eta_c$ inequality in Eq. (6) which puts a condition on $J$ as

$$J < \frac{B_2 - T_2 B_1 / T_1}{4(1 - T_2 / T_1)},$$

which becomes $J < 1/2$ for the parameters $B_2 = 3, B_1 = 4, T_2 = 1/2, T_1 = 1$ used in Figs. 1, 2. It is reasonable to assume that the change of energy gaps in the adiabatic stages by the change of magnetic field cannot contribute in the direction of total positive work gradient when $J > 0.5$. On the other hand, for the case of pairing spin-$1/2$ and spin-$s$ particles with $s > 1/2$, the role of energy...
gaps in the work extraction can be dramatically changed after a critical value of $J$ and the engine can reproduce useful work. This is shown in the inset of Fig. 2 where the global efficiency is plotted as a function of $J$ up to the very strong couplings. As shown in the inset, the positive work re-emerges after a critical value of coupling strength. Increasing the spin-$s$ value shifts the critical $J$ towards the weak coupling regime. The efficiency is less in the strong coupling regime. Since the corresponding thermodynamical quantities are invariant under uniform energy shifts, the coupled spin-$1/2$ and spin-$s$ model in the limit of very large coupling strengths (i.e., $J \to \infty$) can be mapped into a multilevel system with energy levels \{0, 2B, 4B, \ldots, (2s - 1)2B\} where $\eta = 0$ for $s = 1/2$, while $\eta = 1 - B_2/B_1$ for $s > 1/2$. This explains the behavior of the efficiency in the inset of Fig. 2 where $\eta$ converges to the spin independent value of $\eta = 1 - B_2/B_1$ for $s > 1/2$ and $\eta = 0$ for $s = 1/2$ in the deep strong coupling regime.

IV. LOCAL WORK AND EFFICIENCY

In this section, we investigate how the spin-$1/2$ and spin-$s$ individually undergo the engine operation. This can be done by the analysis of local heat exchanges between the local spin and the reservoir \cite{11}. The local heat exchanges in the isochoric branches of the Otto cycle can be expressed as the change in the local density matrix for a given local Hamiltonian. Let $q_1$ ($q_2$), with $i = A, B$, be the local heat transferred between the $i$th spin and the hot (cold) heat bath. Then the explicit expression of $q_1$ ($q_2$) reads as \cite{11}:

$$
q_1 = \text{Tr}[\{\rho_i - \rho'_i\}H_i], \\
q_2 = \text{Tr}[\{\rho_i - \rho'_i\}H'_i],
$$

where $\rho_i$ ($\rho'_i$) is the reduced density matrix for the $i$th spin at the end of stage 1 (3) and $H_i$ ($H'_i$) is the local Hamiltonian during the first (second) isochoric process. The local Hamiltonians can be written as $H_A = 2Bs_A^2$ and $H_B = 2BS_B^2$ for the spin-$1/2$ and spin-$s$, respectively. The local work done by the $i$th spin is then written as $w_i = q_1 + q_2$.

The local works $w_A$ and $w_B$, done by the spin-$1/2$ and spin-$s$ particles, respectively, are plotted as a function of coupling strength $J$ in Fig. 3 for $s = 1/2, 1, 3/2, 2, 5/2, 3$. The analytical calculation of the global and local works yields that $W = w_A + w_B$, the total work is the sum of local efforts. For further insight, it is possible to calculate the relation between the global and local heat exchanges, which is found to be

$$
Q_1 = q_1^A + q_1^B + \kappa_s J \mathcal{P}_s, \\
Q_2 = q_2^A + q_2^B - \kappa_s J \mathcal{P}_s,
$$

where $\kappa_s = (8s + 4)$ is a constant depending on the value of spin-$s$, and the factor $\mathcal{P}_s$ is related to the probabilities of certain energy levels at the end of stages 1 and 3, whose explicit expression depends on the spin-$s$ but not written here explicitly for brevity. The relations in Eq. \ref{8} suggest that only the local heat exchange is converted into total work output of the Otto cycle, as the last terms in $Q_1$ and $Q_2$ expressions reflect the collective heat intake and release which cancel each other. This is consistent with the extensive property of the work output of the cycle. We should stress here that same conclusion is reached for the case of spin-$1/2$ \cite{11} and spin-$3/2$ pairs \cite{22}. The extensive property is not a fundamental character of the work output and is not always true. Similar analysis in different conditions reveals that sum of the local works is not always equal to the global work \cite{13, 24}. We will present a more general discussion in the following section.

For two coupled spin-$1/2$ case, we have $w_A = w_B$ since $\rho_A = \rho_B$ and $H_A = H_B$ \cite{11}. Moreover, for $J = 0$, $w_A$ is independent of spin-$s$ value. $w_A$ depends on spin-$s$ value for $J = 0$, but this dependence is weak to be
visible in the scale of Fig. 3. On the other hand, these results are dramatically changed when \( s > 1/2 \) and \( J \neq 0 \). As shown in Fig. 3(a), \( w_A \) depends strongly on the spin-\( s \) value. In the region \( J < 0.5 \), increasing \( s \) shifts the PWCs and maximums of \( w_A \) and \( w_B \) to the weak coupling regions and increases (decreases) the maximums of \( w_A \) (\( w_B \)). The comparison of local works of both spins shows that, except a negligibly tiny range of \( J \), we have \( w_A > w_B \), that is spin-1/2 does more work than the spin-1/2. On the other hand, if we change our attention to the strong coupling regime where \( J > 0.5 \), this situation is completely reversed; as shown in the inset of Fig. 3(b), \( w_A \leq 0 \) for each spin-\( s \), while \( w_B \) can be non-zero for \( s > 1/2 \). From an analytical calculation of global and local works in the deep strong coupling regime (i.e., \( J \to \infty \)), it is possible to show that \( W = -(2s + 1)w_A = (2s + 1)(2s + 2)w_B \). This indicates that spin-\( s \) is solely responsible for the realization of our QHE in the strong coupling regime, where \( w_B > 0 \) and \( w_A < 0 \) in the regions \( W > 0 \).

Our final remark is on the local efficiencies of spins \( A \) and \( B \). In the local description, it would not always be possible to give a unique definition of local efficiencies. The global relation \( Q_1 > -Q_2 > 0 \) does not always imply \( q_1^i > -q_2^i > 0 \) in the local realm. Under certain conditions, the local heats and work will flow in the direction opposite to the global heat gradient, i.e., \( q_2^i > -q_1^i > 0 \) can be possible even when \( Q_1 > -Q_2 > 0 \). For the considered parameter regime in Fig. 3 we have \( q_1^i > -q_2^i > 0 \) when \( w_l > 0 \). For the local heat exchanges, we have the relation \( q_1^i = -(B_1/B_2)q_2^i \), so the individual spins undergo the cycle with the same local efficiency: \( \eta_A = \eta_B = w_l / q_1^i = 1 - B_2 / B_1 \), which is independent of spin-\( s \) and equals to the global uncoupled engine efficiency.

V. GENERAL RELATIONS BETWEEN GLOBAL AND LOCAL WORK

We have seen in Sec. III that the global work has an extensive property and can be written as a sum of the local works done by the individual spins. This conclusion strictly depends on the paths, or the methods, we choose to operate the engine Cycle. In the adiabatic stages of the quantum Otto cycle, we varied the homogeneous magnetic field acting on the spins. We can make a general statement that it is not possible break the extensive property of work output of a QHE by only making local changes in the adiabatic stages of the engine cycle. This simple fact can be quickly proven for a general Hamiltonian of a system of a collection of local subsystems, described in the form \( H = \sum H_{\text{loc}} + H_{\text{int}} \), where the non-interacting (local) and interacting (global) terms are denoted by \( H_{\text{loc}} \) and \( H_{\text{int}} \), respectively. The internal energy, \( U = \langle H \rangle = \text{Tr}(\rho H) \), of the system with density matrix \( \rho \) changes as \( dU = \text{Tr}(\rho dH) + \text{Tr}(H d\rho) \), where the first term can be defined as the work done on the system and denoted by \( dW := \text{Tr}(\rho dH) \). In a strictly quantum adiabatic process we have \( d\rho = 0 \). Accordingly, if \( dH_{\text{int}} = 0 \), the global work becomes extensive in terms of local works done by subsystems such that \( dW = \sum dw_{\text{loc}} \), with \( dw_{\text{loc}} := \text{Tr}_{\text{loc}}(\rho_{\text{loc}} dH_{\text{loc}}) \), where \( \rho_{\text{loc}} \) is the reduced density matrix of a particular subsystem found by tracing out the degrees of freedom of the other subsystems from the density matrix \( \rho \) of the whole system. While the global work is extensive under local changes, it can still be optimized by the interactions between the subsystems, through the interaction dependence of the reduced density matrices \( \rho_{\text{loc}} \), which is illustrated by our analysis in Sec. III and Sec. IV.

Let us now consider a more general situation where both the magnetic field and the exchange interaction between the spins could change. In such a case, Eq. (8) directly shows that the extensive behavior of the global work is violated by the simultaneous change of magnetic field strength \( (B_1 \to B_2 \to B_1) \) and the exchange coupling strength \( (J_1 \to J_2 \to J_1) \) in the adiabatic stages such that

\[
W = w_A + w_B + \kappa_s (J_1 - J_2) P_s, \tag{9}
\]

where \( \kappa_s \) and \( P_s \) are defined in Eq. (3).

A curious result of Eq. (9) is that when \( B_1 = B_2 \) and \( J_1 \neq J_2 \), the system can harvest positive work in a purely collective manner, as no local work can be done by the local systems in constant magnetic field. Since there is no change in local Hamiltonians, the total local heat exchange is zero. If we take the ratio \( W / w_{\text{loc}} \), where \( w_{\text{loc}} = w_A + w_B \) is the total local work, as a figure of merit measuring the cooperativity in work extraction, it is infinite. On the other hand, we can still consider a possible generalization of the local work definition in Ref. [11] to scrutinize them in a purely interacting cycle without explicit local variations. We suggest that a mean field Hamiltonian can always be introduced to describe a local Hamiltonian of a subsystem.

To make our discussion concrete let us take a pairwise interaction Hamiltonian of the form \( H = gAB \), where \( A \) and \( B \) are operators for two subsystems, and \( g \) is their coupling constant. The work done on the system in an adiabatic stage by the \( dg \) variation of the coupling constant can be written as \( dW = dg \langle AB \rangle \), where \( \langle AB \rangle = \text{Tr}(\rho AB) \). If we use mean field Hamiltonians \( H_A = g\langle B \rangle A / 2 \) and \( H_A = g\langle A \rangle B / 2 \) for the local Hamiltonians then the corresponding local work contributions become \( w_A = w_B = g\langle A \rangle \langle B \rangle \). Accordingly the global work can be expressed as \( dW = dw_A + dw_B + dw_{\text{coop}} \), where we introduced a cooperative work term \( dw_{\text{coop}} := dg \langle A, B \rangle \). Here, the notation \( (A, B) := \langle AB \rangle - \langle A \rangle \langle B \rangle \) stands for the covariance of \( A \) and \( B \) as a measure of correlations between the subsystems. The net work done in the cycle then becomes

\[
W = w_A + w_B + w_{\text{coop}}, \tag{10}
\]
where the local and cooperative works are given by
\[ w_A = w_B = \frac{1}{2}(g_1 - g_2)(\langle A \rangle_1 \langle B \rangle_1 - \langle A \rangle_2 \langle B \rangle_2), \]
\[ w_{\text{coop}} = (g_1 - g_2)(\langle A, B \rangle_1 - \langle A, B \rangle_2). \]  
(11)

Here \( g_1 \) and \( g_2 \) are the coupling constants at the end points of the adiabatic stages, and the expectation values \( \langle X \rangle_i = \text{Tr}(\rho^i_X X) \) are evaluated with the reduced density matrix \( \rho^i_X \) of the subsystem \( X = A, B \) in the adiabatic stage labeled by \( i = 1, 2 \). With this generalized definition of the local work, the cooperativity of the work extraction can be characterized by the ratio
\[ \frac{W}{w_{\text{loc}}} = 1 + \frac{\langle A, B \rangle_1 - \langle A, B \rangle_2}{\langle A \rangle_1 \langle B \rangle_1 - \langle A \rangle_2 \langle B \rangle_2}. \]  
(12)

Applying the generalized local work formalism to our Heisenberg exchange model QHE, we find the local Hamiltonians
\[ H_A = 2BS^z_A + \frac{1}{2}8J\vec{s}_A.\vec{S}_B, \]
\[ H_B = 2BS^z_B + \frac{1}{2}8J\vec{s}_B.\vec{S}_A, \]  
(13)

which gives the relation between global and local works as \( dW = dw_A + dw_B + dw_{\text{coop}} \), where
\[ dw_A = 2dB\langle \vec{s}^z_A \rangle + \frac{1}{2}8dJ\langle \vec{s}_A \rangle.\vec{S}_B, \]
\[ dw_B = 2dB\langle \vec{S}^z_B \rangle + \frac{1}{2}8dJ\langle \vec{s}_B \rangle.\vec{S}_A, \]  
(14)
and \( dw_{\text{coop}} = 8dJ\langle \vec{s}_A, \vec{S}_B \rangle \). From this result we conclude that the extensive property of the global work can be violated by changing the interaction parameter in the adiabatic stages, if the covariance of the interacting spins changes as well. If the covariance remains the same, then the global work can be expressed as the sum of efficient local works of the individual spins under the mean field description.

\section*{VI. CONCLUSIONS}

We consider a pair of spin-1/2 and spin-s particles coupled via Heisenberg exchange interaction under a homogeneous magnetic field as the working medium of a quantum Otto engine. The influence of exchange coupling and spin-s value on the work output and efficiency of the quantum Otto engine is investigated in detail. The global engine operation is also analyzed in comparison to local work contributions of the individual spins. It is found that increasing spin-s value at a certain exchange coupling strength can make the QHE to produce more work with higher efficiency, which can violate the upper bound of efficiency for two coupled spin-1/2 particles\(^\text{[1]}\). Moreover, spin-s makes it possible to realize the QHE at the strong coupling regimes. From the local work analysis, it is found that global work is equal to the sum of the local works by the individual spins. Although in local realm, the spin-1/2 and spin-s operate with the same efficiency, their local works are found to be significantly influenced by the spin-s. The local work definition is generalized to examine global changes from the local work perspective, too. The general conditions for which the global work is not equal to the sum of the local works are discussed.

\section*{Acknowledgments}

F. A. thanks R. Eryigit, G. Thomas, and R. S. Johal for fruitful discussions. Ö. E. M. acknowledges illuminating comments by N. Allen and support of Lockheed Martin Corporation.

\section*{Appendix: The Eigenvalues of the Working Medium}

Here we report the eigenvalues of the Hamiltonian \([1]\) for \( s = 1/2, 1, 3/2, 2, 5/2, 3 \). The corresponding orthonormal eigenstates can also be calculated. We should stress here that the eigenstates are system parameter (i.e., \( J \) and \( B \)) independent. Since the discussion of text does not require the explicit form of the eigenstates, we do not report them here for brevity.

The eigenvalues for \( \{(\frac{1}{2}, s)\} \) system with \( s = 1/2 \) are \([1]\):
\[ \{-6J, 2J - 2B, 2J + 2B\}. \]

The eigenvalues for \( \{(\frac{3}{2}, s)\} \) system with \( s = 1 \) are:
\[ \{-B - 2J, -B - 2J, -3B - 4J, -B + 4J, B - 4J, 3B + 4J\}. \]

The eigenvalues for \( \{(\frac{5}{2}, s)\} \) system with \( s = 3/2 \) are:
\[ \{-2B - 10J, -2B - 10J, -2B + 6J, -4B + 6J, 6J, 2B + 6J, 4B + 6J\}. \]

The eigenvalues for \( \{(\frac{7}{2}, s)\} \) system with \( s = 2 \) are:
\[ \{-3B - 12J, -B - 12J, B - 12J, 3B - 12J, -5B + 8J, -3B + 8J, -8J, -B + 8J, B + 8J, 3B + 8J, 5B + 8J\}. \]

The eigenvalues for \( \{(\frac{9}{2}, s)\} \) system with \( s = 5/2 \) are:
\[ \{-4B - 14J, -2B - 14J, -14J, 2B - 14J, 4B - 14J, -2B + 10J, -4B + 10J, -6B + 10J, 10J, 2B + 10J, 4B + 10J, 6B + 10J\}. \]

The eigenvalues for \( \{(\frac{11}{2}, s)\} \) system with \( s = 3 \) are:
\[ \{-5B - 16J, -3B - 16J, -B - 16J, B - 16J, 3B - 16J, 5B - 16J, -3B + 12J, 3B + 12J, -7B + 12J, -5B + 12J, -B + 12J, B + 12J, 5B + 12J, 7B + 12J\}. \]

---

\[\text{[1]}\] H. E. D. Scovil and E. O. Schulz-DuBois, Phys. Rev. Lett. 2, 262 (1959).

\[\text{[2]}\] H. T. Quan, Yu-xi Liu, C. P. Sun and F. Nori, Phys. Rev. E 76, 031105 (2007).
1. H.T. Quan, Phys. Rev. E 79, 041129 (2009).
2. T.D. Kieu, Phys. Rev. Lett. 93, 140403 (2004).
3. J. Wang, Z. Wu and J. He, Phys. Rev. E 85, 041148 (2012).
4. R. Wang, J. Wang, J. He and Y. Ma, Phys. Rev. E 87, 042119 (2013).
5. R. Uzdin and R. Kosloff, EPL 108, 40001 (2014).
6. H.T. Quan, X. Zhang and C.P. Sun, Phys. Rev. E 72, 056110 (2005).
7. A.E. Allahverdyan, R.S. Johal and G. Mahler, Phys. Rev. E 77, 041118 (2008).
8. F. Altintas, A.U.C. Hardal and O.E. Mustecaplioglu, Phys. Rev. E 90, 032102 (2014).
9. G. Thomas and R.S. Johal, Phys. Rev. E 83, 031135 (2011).
10. G.-F. Zhang, Eur. Phys. J. D 49, 123-128 (2008).
11. T. Feldmann and R. Kosloff, Phys. Rev. E 68, 016101 (2004).
12. T. Feldmann and R. Kosloff, Phys. Rev. E 65, 055102 (2002).
13. M.J. Henrich, G. Mahler and M. Michel, Phys. Rev. E 75, 051118 (2007).
14. T. Wang, W.-T. Liu, P.-X. Chen and C.-Z. Li, Phys. Rev. A 75, 062102 (2007).
15. G. Thomas and R.S. Johal, Eur. Phys. J. B 87, 166 (2014).
16. X.L. Huang, T. Wang and X.X. Yi, Phys. Rev. E 86, 051105 (2012).
17. W. Hubner, G. Lefkidis, C.D. Dong, D. Chaudhuri, L. Chotorlishvili and J. Berakdar, Phys. Rev. B 90, 024401 (2014).
18. M. Azimi, L. Chotorlishvili, S.K. Mishra, T. Vekua, W. Hubner and J. Berakdar, New J. Phys. 16, 063018 (2014).
19. E. Albayrak, Int. J. Quantum Inform. 11, 1350021 (2014).
20. R. Dillenschneider and E. Lutz, EPL 88, 50003 (2009).
21. Y. Rezek and R. Kosloff, New J. Phys. 8, 83 (2006).
22. J. Robnagel, O. Abah, F. Schmidt-Kaler, K. Singer and E. Lutz, Phys. Rev. Lett. 112, 030602 (2014).
23. X.Y. Zhang, X.L. Huang and X.X. Yi, J. Phys. A: Math. Theor. 47, 455002 (2014).
24. M.O. Scully, M.S. Zubairy, G.S. Agarwal and H. Walther, Science 299, 862 (2003).
25. H.T. Quan, P. Zhang and C.P. Sun, Phys. Rev. E 73, 036122 (2006).
26. F. Altintas, A.U.C. Hardal and O.E. Mustecaplioglu, Phys. Rev. A 91, 023816 (2015).
27. B. Sothmann and M. Buttiker, EPL 99, 27001 (2012).
28. B. Sothmann, R. Sanches and A.N. Jordan, EPL 107, 47003 (2014).
29. P. Faliko and D.W. Hallwood, Phys. Rev. Lett. 108, 085303 (2012).
30. K. Zhang, F. Bariani and P. Meystre, Phys. Rev. Lett. 112, 150602 (2014).
31. O. Abah, J. Robnagel, G. Jacob, S. Delfner, F. Schmidt-Kaler, K. Singer and E. Lutz, Phys. Rev. Lett. 109, 203006 (2012).
32. A.J. Roncaglia, F. Cerisola and J.P. Paz, Phys. Rev. Lett. 113, 250601 (2014); G. Chiara, A.J. Roncaglia and J.P. Paz, arXiv:1412.6118.
33. S. Sinha, J. Emerson, N. Boulant, E.M. Fortunato, T.F. Havel, and D.G. Cory, Quantum Information Processing 2, 433 (2003).
34. S.-S. Li, T.-Q. Ren, X.-M. Kong and K. Liu, Physica A 391, 35 (2012).