Querying Geometric Figures Using a Controlled Language, Ontological Graphs and Dependency Lattices

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Abstract. Dynamic geometry systems (DGS) have become basic tools in many areas of geometry as, for example, in education. Geometry Automated Theorem Provers (GATP) are an active area of research and are considered as being basic tools in future enhanced educational software as well as in a next generation of mechanized mathematics assistants. Recently emerged Web repositories of geometric knowledge, like TGTP and Intergeo, are an attempt to make the already vast data set of geometric knowledge widely available. Considering the large amount of geometric information already available, we face the need of a query mechanism for descriptions of geometric constructions.

In this paper we discuss two approaches for describing geometric figures (declarative and procedural), and present algorithms for querying geometric figures in declaratively and procedurally described corpora, by using a DGS or a dedicated controlled natural language for queries.

Introduction

Dynamic geometry systems (DGS) distinguish themselves from drawing programs in two major ways. The first is their knowledge of geometry: from a initial set of objects drawn freely in the Cartesian plane (or maybe, on another model of geometry), one can specify/construct a given geometric figure using relations between the objects, e.g., the intersection of two non-parallel lines, a line perpendicular to a given line and containing a given point, etc. Another major feature of a DGS is its capability to introduce dynamics to a given geometric construction moving a (free) basic object always preserving the geometric properties of the construction [18].

That is, one uses a DGS by constructing a geometric figure with geometric objects and geometric relations between them, and not by placing points on specific Cartesian coordinates. Most (if not all) DGS possess a formal language for the specification of geometric constructions. In some systems this formal
language is explicit, in others it is hidden from the user by the graphical interface. The Intergeo project designed a common format, called I2G, for this formal language which is already accepted by some DGS [16,17]. Geometry automated theorem provers (GATP), being formal systems, need a formal language to describe geometric conjectures. GATPs are nowadays mature tools capable of proving hundreds of geometric conjectures [2,8]. The I2GATP formal language is an extension of the formal language used by the DGS. The I2GATP project goal is to define a common language, an extension of the I2G language, to the DGS/GATP tools [12].

The design of common languages, and the emergence of Web repositories of geometric knowledge is an attempt to make the already vast data set of geometric knowledge widely available. The Intergeo project [9], GeoThms [7] and TGTP [11] systems already meet some of these goals, having provided a large data set of geometric information widely available. In these systems the question of querying the geometric construction is not solved, that is, it is not yet possible to query the data set for a construction similar to some other construction, or to query for all constructions having some common geometric properties. The goal of our research is to develop a search mechanism for geometric constructions (done by a DGS or a GATP) using the different ways of geometric construction descriptions.

1 What You See and How to Get It: Declarative vs. Procedural vs. Analytic Figure Description

On Fig. 1 the reader can see (a visual representation of) the centroid theorem, a simple geometric figure taken from the TGTP corpus [11, Fig. 13].

There are many approaches for describing this figure:

– the **declarative** one: “we have points $A, E, B$ on the same line such that $AE = EB$, points $A, D, C$ on the same line such that $AD = DC$, points $C, F, B$ on the same line such that $CF = FB$, and displayed are $AC, BC, AB, AF, BD, CE$. The intersection of $BD$, $CE$, $AF$ is called $G$”;
– the **procedural** one: “draw segments $AB$, $AC$, $BC$, take their midpoints $E$, $D$, $F$, draw $AF$, $BD$ and $CE$, take the intersection of $BD$ and $CE$ and call it $G$”;
– the **analytic** one: “points $A$, $B$, $C$ have coordinates $(35, 40), (10,10), (40,10)$; points $D$, $E$, $F$, have coordinates $(37.5, 25), (22.5, 25), (25,10)$; segments have coordinates $((35, 40), (40,10)), ((35, 40), (10,10))$, etc.

In this paper we will concentrate on the **procedural** and **declarative** descriptions of a figure.

![Fig. 1. Construction 13 of the corpus.](image)
The declarative description is about “what the parts of the figure are and how they relate to each other,” while the procedural description is about “how to construct the figure.” In the former we can supply arbitrary (and potentially redundant) information about the figure; in the latter we provide only instructions that result into the given figure.

The first problem we encountered when querying figures was the fact that a given figure can often be constructed (and hence, procedurally described) in several ways. For example, Fig. 1 can be procedurally obtained in (at least) the following two ways (cf. [4] for the second):

| Start with points | Draw midpoints of | Call them | Draw the segments |
|-------------------|------------------|-----------|------------------|
| A, B, C           | AB, AC, BC       | D, E, F   |                  |
|                   |                   |           |                  |
|                   |                   |           |                  |

That is, we can start with the triangle and find the midpoints, or we can start with the midpoints and find the triangle.

Both the DGS to be used (GeoGebra [6]) and the controlled language we will define (§ 3) are procedural, and hence describe a figure by its construction. As there are many constructions resulting in the same figure, we concluded that our search system should better use a declarative approach. For this (see also [13]), we convert procedural descriptions into declarative ones and represent the information they contain by the use of ontological graphs. This operation is done both for the search corpus and for the queries, so that a figure query becomes the search of a graph pattern inside a corpus of ontological graphs.

The second problem we encountered is that procedural descriptions are sometimes lacunary, provided the correct visual result is obtained. For example, in the procedural description of Fig. 1, as it is included in our corpus, the creator of the figure has defined G as being the intersection of BD and CE, without going any further. Since the goal was to obtain the correct visual representation of the figure, it was not necessary to state that G is also the intersection of AF and BD as well as of AF and CE. This means that, after conversion into the declarative representation, the information provided in it will lack these facts.

Inference can fill some of the gaps and make a declarative description more complete. For example it can detect parallelisms or orthogonality relations that are not explicitly stated.¹

The way we propose to solve this problem is by going the “other way around”: instead of making the corpus richer, we can weaken the query. This method is called query reduction and is useful when the query contains too much information and cannot be found in the corpus.

¹ In a future development we plan to use the deductive database method to find all the fix-points for a given construction, finding in this way the missing facts [3].
The problem then is, how do we reduce the query? Indeed, when in front of an ontological graph query where all ingredients of the query figure have become nodes, and their relations have become edges, how do we choose the most suitable nodes or edges to remove?

It is the procedural description of the query that provides us with an answer to this question. From the procedural data, we build a dependency lattice of the query figure. The lattice structure provides us with the nodes to remove, and the order in which to remove them.

For these reasons, we have developed, and will discuss in this paper, both procedural and declarative descriptions of geometric figures. Thanks to their complementarity we obtain an efficient geometric figure search system.

2 Ontological Graphs

2.1 Describing a Geometric Figure by an Ontological Graph

In the following we will use an ontology specific to geometric figures on the plane. This ontology contains concepts:

- point: a point of the plane;
- segment: a segment, defined by two points. It has an attribute “length” which induces a relation of “ratio” among segment instances;
- line: a line, defined by two points or in some other way (for example, by a point and a property like perpendicularity);
- conic: a conic defined in various ways, and, in particular, a circle, defined by its center point and another point;
- angle: the angle of two segments/lines, it has an attribute value which can have a numeric value or the modal value “straight”.

The relations between instances will be:

- belongs_to: a relation whose domains are both points (belonging to segments, lines, circles and angles), and segments (belonging to lines);
- has_ratio: can be used for lengths and angle values. It is a 3-ary reified relation, the members of which are the nominator, denominator and ratio value;
- is_center_of: connects a point with the circle of which it is the center;
- is_parallel_to: connects two parallel lines (using inference, we will find all parallel lines);
- is_perpendicular_to: connects two perpendicular lines or segments (using inference, we will find all perpendicular lines or segments, knowing that the perpendicular of a perpendicular is a parallel);
- is_radius_of: connects a segment with the circle of which it is the radius.

Well understood, the answer is not unique since it strongly depends on the way the figure has been constructed, which is not unique.

The list is not exhaustive.
These concepts and relations have been inspired by the element types of DTD GeoCons.dtd [10] (the ontology does not cover XML elements towards, translation, rotation which are useful for drawing but do not affect the ontological graph of the figure) and of GeoGebra XML schema ggb.xsd [6].

Every figure becomes a graph of instances of concepts and of relations. Not only this approach is independent of the way the figure has been constructed, but it is also independent of instance names and allows to focus on the network of relations between the ingredients of the figure.

Our choice of concepts and relations makes some graphical constructions obtainable by a single relation, for example: “$AB \perp BC$”

\[
\begin{align*}
A & \rightarrow AB \\
B & \rightarrow BC \\
C & \rightarrow \angle ABC \\
\angle ABC & \rightarrow \text{value}=\perp
\end{align*}
\]

where solid arrows denote the belongs_to relation, and $\perp$ is the “right angle” value of the value attribute.

Other constructions, although trivial, are more difficult to encode. For example: “$AB$ is tangent at circle $c$ at point $B$”

\[
A \rightarrow AB \rightarrow c \rightarrow B \rightarrow BO
\]

cannot be encoded by a single relation. We need to use the radius $BO$ and say that $B \in c \land AB \perp BO$

\[
A \rightarrow AB \rightarrow BO \rightarrow B \rightarrow c
\]

and the graph of relations will be
where the dotted arrow represents the relation \textit{is\_center\_of} and the dashed one, the relation \textit{is\_radius\_of}.

The ontological graph of a geometrical figure can rapidly increase in size. Its generation is done in a two-step process:

1. every XML element of the figure description is converted into a set of concepts and relations;
2. then, inference is applied to generate additional relations:
   (a) we calculate the transitive closure of parallelism and orthogonality relations \( a \parallel b \land b \parallel c \vdash a \parallel c \) and \( a \perp b \land b \perp c \vdash a \perp c \);
   (b) nodes having equal lengths or equal angle values by construction\footnote{We emphasize the fact that equality is explicitly given in the construction and is not the result of measurements between objects of the figure.} obtain a \textit{has\_ratio} relation with value attribute equal to 1;
   (c) if necessary, angles are instantiated for every pair of segments with a common point.

Our corpus of 134 figures, encoded as an XML file of 3,137 elements resulted into graphs of a total of 5,282 concept instances and 10,211 relation instances.

2.2 Example

Take Fig. 1 representing Figure 13 of the corpus (illustrating the fact that medians intersect at the barycenter of the triangle). The construction, as given in the XML file, takes arbitrary points \( A, B, C \), defines \( D \) (resp. \( E, F \)) as the midpoint of \( AC \) (resp. \( AB, BC \)), and \( G \) as the intersection of \( BD \) and \( CE \). Furthermore, the segment \( AF \) is drawn.

The ontological graph will contain concepts for points \( A, B, C, D, E, F, G \), and segments \( AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF \). The relations will all be of type \textit{belongs\_to}, except for some 3-ary \textit{has\_ratio} relations representing equal lengths. In Fig. 2, unlabelled arrows denote the \textit{belongs\_to} relation.

2.3 Querying Ontological Graphs

To be able to search in a corpus, we convert all figures of the corpus into ontological graphs and we store them in a graph database (we use a neo4j database \cite{15}). The user query is a figure drawn by using a DGS, or a query using the controlled query language (§3). This figure or CQL statement is converted into an ontological graph on-the-fly, and then into a Cypher query (Cypher is the query language of the neo4j graph database system). The query is send to the database and returns graph instances containing the query as sub-graph.

At this step, ranking is performed to present the results to the user in a pertinent way. Our ranking criterion (which we will try to improve in the future) is the ratio between number of nodes and relations of the query and the number of nodes and relations of the matched graphs. Using this criterion we obtain first
the smallest figure possible figure containing the query subgraph. We intend to use graphical mechanisms to highlight the matched pattern in the resulting graphs by using, for example, a different color.

3 The Controlled Query Language

In some cases the user may not wish to use the DGS to build the query, either because it is cumbersome to use or because it does not provide the necessary abstractions. We propose, as an alternative to the DGS, a controlled query language that allows the (procedural) description of a figure in a way that is simple and close to natural language.

3.1 Description of the controlled query language

Here is the grammar of the controlled query language:

\[ S \rightarrow \text{query} \]
\[ \text{query} \rightarrow \text{sents drawvp PERIOD} \]
\[ \text{query} \rightarrow \text{sents PERIOD} \]
\[ \text{drawvp} \rightarrow \text{DRAW ents} \]
\[ \text{sents} \rightarrow \text{sent SEMICOLON sents} \]
\[ \text{sents} \rightarrow \text{sent} \]
\[ \text{sent} \rightarrow \text{nps vrb pps} \]
\[ \text{sent} \rightarrow \text{nps vrb} \]
\[ \text{ent} \rightarrow \text{INST LABEL} \]
\[ \text{ent} \rightarrow \text{LABEL} \]
and here are its rules:

1. Every query is of the form:
   [list of sentences separated by ;] draw [list of instances separated by ,].
2. An instance consists of a type and a name (or just a name, if there is no ambiguity). It is written in the form “type [name]”.
3. A primitive instance is an instance of a point, a line or a circle.
4. The name of a primitive instance matches the regular expression `[A-Z]([-]?[0-9]+)?`. Names of non-primitive instances are composite: they are written by joining names of points using the hyphen character (for example: segment A-B).
5. An instance type can be followed by more than one instance, in that case it is written in plural form and the instances are separated by commas (for example: points A, B, C).
6. The first part of a query defines (and draws) new instances, the second draws already known instances.
7. The following sentences can be used:
   (a) line ? intersects line ? at point ?
   (b) point ? is the midpoint of segment ?
   (c) line ? is perpendicular to line ? at point ?

5 We will use sans serif font for illustrating the controlled language.
6 All type names are optional except for center in (h).
(d) point ? is the foot of point ? on line ?
(e) line ? is the mediatrix of segment ?
(f) line ? is parallel to line ? at point ?
(g) line ? connects points ?, ?
(h) circle ? is defined by center ? and point ?
(i) points ?, ? are the intersections of circles ?, ?
(j) points ?, ? are the intersections of circle ? and line ?
(k) line ? is the bisector of angle ?
(l) angle ? is right

9. All sentences have plural forms where arguments are distributed at all positions and separated by commas (for example: lines L_1, L_2, L_3 connect points A_1, B_1, A_2, B_2, A_3, B_3, which means that \{A_i, B_i\} \subset L_i).

10. In all sentences except (8e), the terms segment and line are synonymous, with the syntactic difference that segment must be followed by a composite name (for example A-A_1), while line must be followed by a primitive name, since “line” is a primitive instance.

11. Some variation is allowed, for example and is a synonym of the comma, articles the in front of nouns are optional.

12. Queries end by a period ‘.’.

Here is, for example, a description of Fig. 1 in the controlled query language:

\[ D, E, F \] are midpoints of \( A-C, A-B, B-C \); \( C-E \) intersects \( B-D \) at \( G \); draw \( A-C, A-F, A-B, B-C, B-D, C-E \).

The query language is compiled, producing a Cypher query, which is then submitted to the graph database exactly as when using the DGS. The compiler has been developed using the Python PLY package [1].

### 3.2 Future plans for the controlled language

In future versions of the controlled language, we plan to introduce the possibility of extending the query ontology by introducing new concepts and/or new relations. For example, it may be interesting to define a type square as

Points A, B, C, D form a square A-B-C-D when we draw equal segments A-B, B-C, C-D, D-A where angles A-B-C, B-C-D, C-D-A, D-A-B are right.

This would allow queries of the form (which will draw the notorious figure of the Pythagorean theorem)

Angle A-B-C is right; A-C-C_1-A_2, A-A_1-B_1-B, C-B-B_2-C_2 are squares.

### 4 Reduced Queries

The algorithms we describe in this paper can be quite successful in finding exact matches of queries in the corpus. But what happens when the figures in the corpus match only partially the query?
4.1 Ontological Graphs

For example, let us consider Fig. 2 anew. The ontological graph of the figure has been built solely using the XML data of Figure 13 of the corpus (cf. Fig. 1). What is not visible on Fig. 1 is the fact that $G$ has not been defined as lying on $AF$, and hence the $\text{belongs_to}$ edge between $G$ and $AF$ is missing in the ontological graph.

This is also reflected in the CQL query example we gave in §3.1, where we request that $C-E \text{ intersects } B-D$ at $G$ but not that $A-F \text{ intersects } B-D$ at $G$, probably because this could be inferred from the previous one, if we had the external Euclidean Geometry knowledge of the fact that the three medians of a triangle have a common intersection.

Nevertheless, the user seeking Fig. 1 is not necessarily aware of this subtlety, and will search for “a triangle with three medians,” which will result in an ontological graph similar to the one of Fig. 2 but containing an additional edge $G \rightarrow AF$, and this graph, of course, will not match Figure 13 of the corpus, since it is not a sub-graph of it.

To solve this problem, as long as a query does not return any results, we retry with reduced queries, in the sense of the same query graph with one or more instances (or relations) removed.

But how do we decide which nodes and edges to remove from a query, and in what order? The answer to this question is provided by dependency lattices, described in the next section.

4.2 Dependency Lattices

Let us return to the procedural approach of describing geometric figures. How do we describe a figure using the operations that led to its construction?

Strictly speaking, such a description would require a Berge-acyclic hypergraph [5, §3], where each operation would be a hyper-edge, connecting the input (the set of known nodes) and the output (the set of new nodes), for example, in the case of the midpoint operation on segment $AC$, the hyper-edge would connect $\{A, C\}$ (input) and $\{B\}$ (output).

But there is a simpler way. In fact, it suffices for our needs to represent dependencies as edges of a directed graph. For example, in the midpoint example, $B$ is dependent on $A$ and $C$, since the latter have been used to calculate the former:

\[ \text{A} \rightarrow \text{m} \rightarrow \text{B} \rightarrow \text{m} \rightarrow \text{C} \]

By adding a “global source node” (located above all source nodes) and a “global sink node” (underneath all “final results”), this graph becomes a lattice, the partial order of which is the dependency relation.

On Fig. 3, the user can see the dependency lattice of Fig. 1. We have used only nodes that are used in calculations, so that, for example, segments $AB$, $AE$, etc. do not appear in the lattice. $S$ and $T$ are the global source and global sink.
nodes, they are connected to nodes of the lattice by dashed arrows. Full arrows represent operations and are labelled by their initial letters (m = midpoint, s = segment drawing, i = intersection).

4.3 Using Dependency Lattices for Reduced Queries

Let us now see how the dependency lattice would be affected if the XML description of Fig. 1 had an additional instruction, saying that \( G \) is (also) the intersection of \( BD \) and \( AF \). On Fig. 4 one can compare the two graphs, on the right side one can see the one with the additional instruction.

**Fig. 3.** The dependency lattice of Fig. 1, as it is procedurally described in the corpus.

**Fig. 4.** The dependency lattice of Figure 13 of the corpus (left) and the dependency lattice of Figure 13 plus an additional instruction segment \( A-F \) intersects segment \( B-D \) at point \( G \) (right).
**Data:** A corpus of declaratively described figures, a query

**Result:** One or more figures matching the query

```plaintext
if using controlled query language then
    write the query in controlled query language;
else
    draw the query in a DGS;
end
```

convert query into ontological graph;
apply inference to ontological graph;
convert ontological graph to Cypher;
submit to neo4j database;

```plaintext
if no results returned then
    convert query into dependency lattice;
    while no results returned do
        extract node or relation from bottom of dependency lattice;
        remove that node or that relation from the ontological graph;
        convert ontological graph to Cypher;
        submit to neo4j database;
    end
end
```

**Algorithm 1:** The Query Algorithm for a Declaratively Described Corpus.

Indeed, the new dependency lattice has one additional edge $AF \rightarrow G$. On the other hand, the edge $AF \rightarrow T$ disappears since there is a path from $F$ to $T$, and $AF$ is not a sink anymore.

As dependencies have to be respected, if we remove a node from the upper part of the lattice, we will have to remove all descendants of it. For this reason, the only reasonable query reduction strategy would be to remove nodes or edges from the lower part of the lattice.

If we remove, for example, the node $G$ (and hence the edges $CE \rightarrow G$, $AF \rightarrow G$ and $BD \rightarrow G$), then we obtain a triangle with three medians but where the barycenter has not been explicitly drawn. (Interestingly, we still obtain a figure that is visually identical to Fig. 1.)

If we go further and remove one of the relations among $CE \rightarrow G$, $AF \rightarrow G$ and $BD \rightarrow G$ (for symmetry reasons it doesn’t matter which relation we remove), then the query will succeed, while the visual representation of the figure still has not changed.

Algorithm 1 is a synthesis of the geometric figure query algorithm we propose.

### 5 Evaluation

We plan to evaluate the algorithms described in this paper, in the following ways:
Querying a sub-figure in a corpus of declaratively described figures gives a binary result: either the figure is matching the sub-figure or it is not, so evaluation is simply counting the number of successes.

An interesting parameter to observe is the number and nature of query reductions that were necessary to obtain results, correlated with the number of results obtained.

We will proceed as follows: after visually inspecting the corpus (and hence with no knowledge about the procedural and declarative descriptions of the figures) we will formulate 20 queries and manually annotate the figures we expect to find.

After using the algorithm, we will count the number of successes and study the number of results vs. the parameters of query reduction.

6 Future Work

As future work, besides extending the controlled natural language (§ 3.2), we plan to integrate this search mechanism in repositories such as TGTP and Intergeo, and in learning environments like the Web Geometry Laboratory [14]. In a more generic approach, we should use a common format and develop an application programming interface that will allow to integrate the searching mechanism in any geometric system in need of it.

7 Conclusion

In this paper we have presented algorithms for querying geometric figures in either declaratively or analytically described corpora, by using either a DGS or a dedicated controlled query language.

At the time of submission of the article, evaluation was not completed, hence it is presented as a plan.

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