Two-phase flows in the formed tornado funnel

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Abstract. At present, it is obvious that the problem of the tornado is important not only for our planet to determine the conditions for the formation of a tornado, it is required to take into account a number of hydrodynamic and plasma processes [1 - 6]. Along to prediction of a tornado generation conditions [1 - 3] it is necessary to evaluate the characteristics of its quasi-stationary motion in a formed funnel: the mass of the moving moist air involved in the funnel and the size and form of the funnel. For a complete description of the phenomena, it is necessary to involve numerical calculations. We note that even for numerical calculations using powerful computers, the problem is very difficult because of the need to calculate multiphase turbulent flows with free, self-organizing boundaries [1, 6]. However, "strict" numerical calculations, it is impossible to do without the use of many, often mutually exclusive, models. For example, how to choose an adequate model of turbulence (algebraic, \(k-\varepsilon\) model, etc.) or the use of additional, often not accepted, hypotheses about certain processes used in calculations (mechanisms on the nature of moisture condensation, etc.). Therefore, along with numerical calculations of such flows, modeling problems that allow an exact solution and allow to determine the most important and observed characteristics of a tornado.

1. Characteristics of the turbulent flow of moist air in the funnel of a formed tornado

In the literature, two commonly observed forms of the tornado funnel are described: vague and dense. These forms, in turn, are divided into several types: snake-like, funnel-shaped, etc. Although in the course of its existence the tornado can change the shape of the funnel, often, before it disappears, the tornado funnel thins, becomes snake-like and then spreads out completely. In this paper, the stage of the formation of the funnel is not discussed, but the already existing form of the funnel is considered (for example, tornadoes, in which the funnel widens toward the cloud, leaving into it, and narrows to the ground). Some data on the factors that affect the generation of tornadoes can be found in [1 - 5]. The problem of turning a vortex in a humid atmosphere in a tornado capable of producing serious disruption on the path of its movement requires a separate analysis. An adequate analysis of the formation of a tornado should show how, due to the development of instability, a funnel will form. In this paper, we do not consider the stage of tornado formation, but investigate the mechanisms determining the shape of the formed tornado funnel.

It was assumed in [7] that a vortex flow of a two-phase (dry air and a water vapors) media takes place in tornado funnels. Funnel walls, according to the model [8], consist of hailstones, water droplets resulting from surface condensation of air moisture on wall surface. To describe the processes in the formed tornado funnel, stationary Navier-Stokes equations, written in a cylindrical coordinate system,
were used. Using the small parameter $\varepsilon = \frac{lr_1}{l_z} \ll 1$ ($lr_1, l_z \ll h$ are characteristic dimensions of the system in radius and height directions, respectively), these equations were reduced to zero approximation in the small-parameter:

$$\frac{\nabla^2 \phi}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r}, \quad \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \phi}{\partial r} - \frac{\nabla^2 \phi}{r} = 0, \quad (1), \quad (2)$$

$$\frac{1}{r} \frac{\partial}{\partial r} r \eta T \frac{\partial \xi}{\partial r} \frac{\partial}{\partial z} - \frac{\eta}{\partial r} r \lambda T \frac{\partial}{\partial r} \frac{\partial T}{\partial r} + q = 0. \quad (3), \quad (4)$$

Equation of state of air with water vapor:

$$P = P_r + P_w = R_\mu \rho(T(1 + \xi(T, z))) \quad P = P_r + P_w = R_\mu \rho(T(1 + \xi(T, z))). \quad (5)$$

The above equations were supplemented by the equation of the moisture diffusion in the funnel:

$$\frac{1}{r} \frac{\partial}{\partial r} r D_r \frac{\partial}{\partial r} \xi + \frac{D_z}{\partial z} \frac{\partial}{\partial z} \xi = V_z \frac{\partial}{\partial z} \xi. \quad (6)$$

The following boundary conditions were used for the components of the velocity vector, the temperature of the medium and the moisture content $\xi$:

$$V(r = 0) = 0, \quad V(r = r_1) = r_1 \Omega, \quad \frac{\partial}{\partial r} V_z(r = 0) = 0, \quad V_z(r = r_1) = 0, \quad (7)$$

$$\left(-\frac{\partial}{\partial r} T\right)_{r=0} = 0, \quad T(r = r_1) = TW. \quad (8)$$

$$\left(-\frac{\partial}{\partial r} \xi\right)_{r=0} = 0, \quad \xi(r = r_1) = \xi_w \xi(r, z = 0) = \xi_0 = \text{const}. \quad (9)$$

Considering the change of pressure $P_0(z)$ with altitude $z P(z) = P_0(z) + R_\mu \rho T(z), \rho g = -\frac{\partial P_0}{\partial z}$, presenting $\eta = \nu_r \rho$ ($\nu_r$ is the coefficient of turbulent viscosity) and assuming that in the presence of gravity the pressure gradient in the axial direction is determined by the humidity gradient of the air, one can find the vertical velocity profile: $V_z(r) = V_z(0) \left(1 - \frac{r^2}{r_1^2}\right)$.

In this equation $V_z(0) = \frac{r_1^2}{2r_1^2} \frac{\partial^{2} \xi}{\partial z^2}$ is the velocity on the axis of the tornado funnel. Velocity $V_z(0)$ is depending on the gradient of vertical humidity $\frac{\partial^{2} \xi}{\partial z^2}$, which could be found from the solution of the diffusion equation for water vapor (6). Some, more detailed calculations of processes in the tornado funnel, performed within the framework of the model proposed here, can be found in our works [9 - 12]. Here we shall consider the problem of the funnel shape in the formed tornado.

When solving the diffusion equation, moisture transfer in the radial direction was neglected $V_z \frac{\partial}{\partial z} \xi \gg \frac{1}{r \partial r} r D_r \frac{\partial}{\partial r} \xi$. The solution of equation (6) with boundary condition (7) can be represented in the form:
\[
\xi(z) = \xi_0 \exp \left( -\frac{zD_T}{r^2 V_z(0)} \right).
\]

Having found from equation (10) the gradient of vertical humidity \( \frac{\partial \xi}{\partial z} \), it is possible to present a vertical flow velocity profile in a tornado funnel in the form:

\[
V_z(r) = \left( \frac{\xi_0 D_T T(0) \mu}{4 V_T} \right)^{1/2} \left( 1 - \frac{r^2}{r_T^2} \right) \exp \left( -\frac{-\pi \rho_0 D_T z}{4 G} \right).
\]

The mass air flow in the tornado is found from the equation:

\[
G = 2\pi \int_0^{r_T} \rho_0 V_z r dr.
\]

Substituting solution (11) into this equation and integrating, we find the relationship between the flow rate, the funnel radius and the humidity change in height:

\[
G = \frac{\pi \rho_0 r_T^2}{2} \left( \frac{\xi_0 D_T \mu T(0)}{4 V_T} \right)^{1/2} \exp \left( -\frac{-\pi \rho_0 D_T}{4 G} \right).
\]

In the considered tornado funnel model it was assumed that the funnel walls are impermeable to air, so the flow rate should remain constant in height. As can be seen, formula (12) relates the air flow rate in a tornado and the funnel radius. So if the flow rate remains constant the funnel radius should vary with altitude. Next, we use this connection to represent the variation in the tornado funnel radius with height.

If the tornado funnel has massive walls, then, as was believed in [8], their rotation should lead to an expansion of the funnel and a decrease in the air pressure inside it due to the action of centrifugal forces. The expansion occurs until the pressure drop \( \Delta p \) outside and inside balances the action of the centrifugal forces. Let the funnel wall surface is \( S \), then on the wall the force \( \Delta p S \) will act, and equilibrium between the centrifugal forces and the pressure drop will occur under the condition [8]

\[
\Delta p S = \frac{\sigma V^2}{r T} S.
\]

Using formula (13), we estimate the mass of the funnel walls for the previously considered tornadoes by specifying the pressure drop and the wall thickness from [8]. Then for a tornado of category F4 [13] with: \( r_{11} = 25 \text{ m} \), \( r_{12} = 62.5 \text{ m} \), \( h = 130 \text{ m} \), \( \delta = 2 \text{ m} \), \( V_0 = 116.5 \text{ m/s} \). The considered tornado is rather low, so the mass of the walls, if we use the pressure drop in the tornado adopted in [8] \( \Delta p = 0.5 \cdot 10^5 \text{ Pa} \), we obtain \( M_1 = 526 \text{ tons} \).

For a tornado of the same category (F4) [14] with parameters: \( r_{11} = 394.3 \text{ m} \), \( r_{12} = 1709 \text{ m} \), \( h = 1314 \text{ m} \), \( \delta = 2 \text{ m} \), \( V_0 = 116.5 \text{ m/s} \), and pressure drop \( \Delta p = 0.5 \cdot 10^5 \text{ Pa} \), we obtain \( M_2 = 128 \text{ thousand tons} \). If you use more real pressure drops of about 10-190 mm. These masses are several times less than the estimate given in [8] (the author of this work, in our opinion, used too large pressure drops).

2. Form of quasi-stationary funnel of a formed tornado

However, it is difficult to use photos of tornado that are given on the sites [13, 14] and presented in Figure1 for the analytical analysis of tornado funnel shapes. This is due to the lack of reliable data of the turbulent environment in the tornado. Recently in [15], the change of humidity with altitude (the city of Kirensk, Russia) were obtained. For getting the most appropriate tornado funnel shape, we
approximated a change of humidity with altitude by the next function corresponding to the data from [15]:

\[ \zeta(z) = \zeta_0 \exp\left(-b_a z\right). \]  

(14)

Here \( b_a = 2.839 \times 10^{-4} \) l/m is the value obtained by approximating the data of [15]. Further, solutions (11, 12) and the formula for the humidity change with altitude (14) will be used for finding a change of the funnel radius with altitude. It should be noted that the data given in [15] and approximated by formula (13) refer to the free atmosphere. Using the formula (14) suitable to the free atmosphere for a tornado funnel, it can be assumed that a more intense change of moisture along the height occurs in the tornado funnel (\( b_i > 2.839 \times 10^{-4} \) l/m). Therefore, to compare the shapes of the tornado showers shown in Figure 1 with the theoretical data, this circumstance should be taken into account. Using distributions of pressure, density, and moisture in the form (14) we represent the funnel radius change in height in the form

\[
r_T = \frac{\Omega \exp\left(\frac{b_i z}{2}\right)\left(\frac{8v_T \gamma G}{\pi \rho(0) \zeta_0 b_T}\right)^{1/2}}{a^2 M}.
\]

(15)

\( \Omega = 4.66 \) l/s, \( \zeta = 0.0144 \), \( M = 0.421 \), \( a = 276.4 \) m/s, \( T = 308.706 \) K, the value of the exponent in Formula (15), which gives good agreement with the observational data, was equal \( b_T = 5b_a \). For observations of 2b (the tornado is category F4 on the Fujita scale), the parameters of which were assumed to be equal: \( \Omega = 0.295 \) l/s, \( \zeta = 0.0145 \), \( M = 0.43 \), \( a = 270.9 \) m/s, \( T = 296.4 \) K, this parameter...
equal $b_T = 8\hbar a$. In our quantitative calculations we used data on the characteristics of tornadoes given in [1 - 4, 16].

Formula (15) allows us to calculate the average radius of the funnel of a tornado

$$
\langle r_T \rangle = \frac{1}{h} \int_0^h r_T(z) dz = \frac{2\Omega}{b_T ha^2 M} \left[ \exp \left( \frac{b_T h}{2} \right) - 1 \right] \left( \frac{8\nu_T \gamma G}{\pi \rho(0) \xi_0 b_T} \right)^{1/2}
$$

and establish a scaling between this radius and other parameters of the steady tornado.

If the inequality $\frac{b_T h}{2} \gg 1$ is satisfied, then the complex

$$
\frac{2\Omega}{a^2 M} \left( \frac{8\nu_T \gamma G}{\pi \rho(0) \xi_0 b_T} \right)^{1/2} = b_T h \langle r_T \rangle \exp \left( \frac{-b_T h}{2} \right).
$$

This allows us to present a change in the radius of the funnel of a steady tornado with a height in the form:

$$
r_T(z) = b_T h \langle r_T \rangle \exp \left( \frac{b_T (z-h)}{2} \right). \quad (16)
$$

Using the observational data on the shape of the tornado funnel with height, we can calculate, using formula (16), a parameter characterizing the change in humidity inside the tornado funnel.

3. Conclusion

1. An equation that describes the shape of the tornado funnel, depending on the turbulent air viscosity, the turbulent moisture diffusion coefficient, and the flow rate is obtained.
2. We have obtained the tornado funnel profiles and their dependence on the flow rate using the modified data of humidity changes with altitude in the free atmosphere. This approach allows eliminating some unknown turbulent characteristics of flow inside the funnel. These results give the funnel shape and its dependence on the tornado intensity that is closest to the observation data [7, 8].
3. Using the tornado forms obtained in [7, 8], the rate of change of humidity in the funnel, 5-9 times higher than the analogous value in free atmosphere.
4. The rate of moist air through the funnel and its dependence on the intensity of the tornado are estimated using pictures of tornadoes presented in [9, 10] and information about their power on the Fujita scale.
5. It is shown that estimates of the mass enclosed in the wall of the tornado funnel, obtained in [4], are based on pressure drops, which are orders of magnitude higher than those observed in the Earth's atmosphere.
6. According to our estimates, for a tornado recorded in [10], the mass enclosed in the wall of the tornado funnel was 175 tons.
Figure 2. Comparison of the shape of the tornado funnel, calculated from formula (17) with the data of photographs shown in Figure 1: a) the data of http://tornadotitans.com; B) data (https://fineartamerica.com/profiles/ethan-schisler.html.

List of used symbols

$V(0, V_\phi, V_z)$ is the velocity field, $r$, $z$ are the radial and vertical coordinates, respectively, $V_\phi$ is the angular velocity component, $V_z$ is the axial velocity component, $\Omega=\text{const}$ is an angular velocity of the medium rotation, $G$ is a flow rate of moist air in the funnel, $a$ is the speed of sound, $\eta_r$-is the coefficient of turbulent dynamic viscosity, $r_\tau$ are the funnel radius and its height, respectively, $P$ is the air pressure, $\rho$ is the air density, $\rho_0$ is the air density at the funnel axis, $T$ is the temperature, $T_w$ is the temperature, that closes to the condensation temperature of water vapor, $\xi$ is the humidity of air, $\xi_0$ is the humidity at the base of the tornado, $g$ is an acceleration of gravity, $\gamma_0 = 176.846$ J/kgK is the gas constant of the mixture consisting of air and water vapor, $\lambda_r$ is the turbulent thermal conductivity of air in the tornado funnel, $q_r$ is the volumetric heat release, $D_r, D_z$ are the turbulent radial and axial coefficients of diffusion of moisture in the tornado funnel, $M$ is the Mach number for the flow in the funnel, $\gamma$ is the adiabatic index of moist air, $m$ is the ratio of the average air density inside the funnel to the air density outside the tornado funnel, $\nu$ is the ratio of the funnel mass, including the wall mass, to the mass of the displaced from this volume of air, $\sigma$ is the mass per unit area of the wall.

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