Coupled-channel picture of the core nucleus for the expansion of the nuclear size in the drip-line region

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Abstract. We study the energy and radius of neutron rich nuclei toward to the drip-line region. We employ the cluster-orbital shell model approach to calculate the radius of the oxygen isotopes. The result shows that the calculated radius underestimates the observed one at $^{23}$O, though the binding energy and the drip-line of the oxygen isotopes are reproduced. Therefore, we propose a coupled-channel approach to explain the sudden increase of the radius of the oxygen isotopes at $^{23}$O. As a result, the sudden change of the radius can be described in terms of the competition between the available valence neutron orbits and occupied orbits in the core nucleus. For the $^{23}$O case, we show that the excitation to the $1s_{1/2}$-orbit in the core nucleus is a key configuration to reproduce the radius.

1. Introduction

Nuclear size is one of an illustrative quantity for describing the nuclear systems. As the system comes close to the drip-line, and the binding energy becomes small, the observed radius shows a very large value. In general, such the phenomenon is called the “halo” structure of the nuclear system. The halo structure is described by a relatively stiff core nucleus and loosely bound valence nucleon(s) with a very small binding energy. The typical binding energy of the halo nuclei is a few hundred keV, which is small enough compared to the averaged separation energy of the normal nuclei such as ~ 8 MeV.

An exceptional case for the relation between the binding energy and radius is observed on the oxygen isotopes. The drip-line of the oxygen isotopes is determined as $^{24}$O and no bound state is observed for $A \geq 25$. For $^{23}$O and $^{24}$O, an abrupt increase of the r.m.s. radii has been observed [1]. However, recent experiment[2] shows smaller radius for $^{23}$O. In the theoretical study, it is not so straightforward to reproduce such the abrupt increase, since the one neutron separation energies of $^{23}$O and $^{24}$O are not so small compared to the typical “halo” nuclei such as $^{11}$Be and $^{11}$Li.

We have studied the oxygen isotopes by using the cluster-orbital shell model (COSM) approach [3, 4] in a model space of the $^{16}$O core plus valence neutrons. From the calculated results, we show that the sudden increase of the radius at $^{23}$O cannot be described by a model space with a closed-core configuration for $^{16}$O: $(0s)^4 (0p)^{12}$ plus valence neutrons, even if we adjust the nucleon-nucleon interaction for valence neutrons to reproduce drip-line at $^{24}$O [4]. On the other hand, the radii of $^{16}$–$^{22}$O are reproduced by using the model space described...
above. Hence, we consider that the model space with the closed-core configuration plus valence neutrons is not enough to reproduce the general tendency of the radius of the oxygen isotopes from \( A = 16 \) to 24 within the consistency to the observed binding energy.

In this paper, we propose a coupled-channel picture for the \(^{16}\text{O}\) core \([5]\), and the increase of the radius can be explained as the competition between the occupied orbit in the core-excited configuration and the Pauli-blocking effect to the valence neutrons.

2. Formalism and the results

For studying weakly bound systems with the model space of the core and many valence nucleons, we employ the cluster-orbital shell model (COSM) approach \([6]\). All the coordinates for the valence nucleons are defined so that the origin is the center of mass of the core nucleus and vectors are spanned to each valence nucleon. For the oxygen isotopes, the core nucleus is large enough and the recoil part due to the subtraction of the center of mass motion becomes small. Therefore, we consider the COSM approach is suitable to study the oxygen isotopes and other heavier nuclei in the drip-line region.

2.1. \( M \)-scheme cluster-orbital shell model

In the COSM approach, we transform the total \( A \)-body Hamiltonian by using the coordinate systems defined from the center of mass of the core as follows:

\[
\hat{H} = \sum_{i=1}^{A} \hat{t}_i - \hat{T}_G + \sum_{i<j}^{A} \hat{v}_{ij} = \hat{H}_C + \sum_{i \in V} (\hat{t}_i' + \hat{V}_i') + \sum_{i<j \in V} (\hat{T}_{ij} + \hat{v}_{ij}).
\]  

(1)

Here, \( \hat{t}_i' \) and \( \hat{V}_i' \) are one-body kinetic and potential terms for the \( i \)th valence nucleon. \( \hat{T}_{ij} \) is the recoil part, which comes from the subtraction of the center of mass motion, \( \hat{T}_G \). \( \hat{H}_C \) is the Hamiltonian for the core nucleus.

To solve the system described by the COSM Hamiltonian (1), we use the Gaussian basis function for the valence nucleons. For the case that the number of valence nucleons is \( N \), the radial part of the basis function is defined as follows:

\[
F(r_1, \ldots, r_N) \equiv g_1(r_1) \cdots g_N(r_N),
\]  

(2)

where \( g_i(r_i) \) is the Gaussian for the \( i \)th valence nucleon with a polynomial, which is proportional to the angular momentum \( l_i \), and its form is \( g_i(r) = N_i r^{l_i} \exp(-r^2/(2a_i)) \).

In order to simplify the treatment of the angular momentum part and the antisymmetrization, we employ an \( m \)-scheme approach for the angular momentum part of the basis function \([4]\). We only fix the \( z \)-components of the total spin and isospin described as \( |MM_T\rangle \) and solve the system using the basis functions with the same fixed \( z \)-components as follows:

\[
\Phi_V = \sum_m c^{(m)}_{MM_T} = \sum_m c^{(m)} A \left\{ F^{(m)}(r_1, \ldots, r_N) \cdot |MM_T^{(m)}\rangle \right\}.
\]  

(3)

If the basis size is sufficiently large, eigen vectors have a good quantum number for the total spin and isospin, and the coefficients \( c^{(m)} \) play the role of the Clebsch-Gordan coefficients.

To discuss the structure of the core and the motion of the valence nucleons on the same footing, we use a semi-microscopic treatment for the interaction between the core and a valence nucleon. The interaction between the core and the \( i \)th valence nucleon \( \hat{V}_i' \) is constructed by using the folding procedure of the wave function of the core nucleus \( \Phi_C \) as

\[
\hat{V}_i' |\Phi_V\rangle \equiv \sum_{k \in C} \left\langle \Phi_C \left| \hat{v}_{ik} \right| A' \left\{ |\Phi_C\rangle |\Phi_V\rangle \right\} \right\rangle \simeq \hat{V}_i + \hat{V}_{i}^{ex}.
\]  

(4)
Figure 1. Calculated and experimental (a) neutron separation energies and (b) r.m.s. radii of the oxygen isotopes. Open and solid circles are calculated results by using the parameter set (i) and squares are calculated with parameter set (ii), see text. In (a), open diamonds are experiment [9, 10]. In (b), crosses with bars are experiments [1] and Solid and open triangles with error bars are other experiments [2].

For the nucleon-nucleon interaction, we use the Volkov No.2 [8] interaction and modify the potential strength and parameters to reproduce the drip-line of the oxygen isotopes at $^{24}$O.

Using the $m$-scheme COSM approach, we calculated the binding energy and r.m.s. radius of the oxygen isotopes. We perform calculations using two types of the core-size with respect to the change of the mass number and two different exchange parameters for the nucleon-nucleon interactions of the valence part.

For the core part, we use ($\alpha$) the fixed core size and ($\beta$) the varied one, in which the size parameter has the mass number dependent as the $b_C \sim A^{1/6}$. Here, $b_C$ is the width parameter of the wave function of the core, which is described by using the harmonic oscillator. For the valence part, two exchange parameters are applied to reproduce (i) the energy of $^{18}$O ground state, and (ii) the drip-line of the oxygen isotopes at $^{24}$O.

The calculated r.m.s. radii using the four different parameter sets for the core and valence nucleons are shown in Figs. 1(a) and (b). As seen from Fig. 1(b), for $^{16-22}$O, the calculated radii using the fixed core size corresponds to the experimental one for both fixed-core cases, ($\alpha$-i) and ($\alpha$-ii). On the other hand, for $^{23}$O and $^{24}$O, the fixed core calculations do not show the sudden increase of the radius at $^{23}$O even for the ($\alpha$-ii) case, and the varied core-size case ($\beta$-ii) shows the better agreement compared to the other calculations.

This result suggests that a new mechanism is necessary to be introduced in order to describe the energy and radius of the oxygen isotopes consistently for $^{16-24}$O. Further, it can be considered that the configuration of the core changes drastically from $^{22}$O to $^{23}$O as the consequence for the valence neutrons added.

2.2. A coupled-channel picture for $^{16}$O

In order to reproduce the general tendency of the radius of the oxygen isotopes, and its sudden change at $^{23}$O, we introduce a coupled-channel picture to the $^{16}$O core [5]. We consider the essential point of this coupled-channel picture is to introduce an excitation of the neutron and proton ($S = 1$ and $T = 0$) in the core nucleus to the $1s_{1/2}$-orbit, see Fig. 2. The tensor induced core-excitation has been developed as the treatment of the tensor-optimized shell model (TOSM) approach [11]. With this condition, high-momentum components are taken into the model space as the size parameter of the orbit in the core, and the small size-parameter of the core is favored.
in the $2p2h$-excitation channel.

In this work, we use the same kind of mechanism of the TOSM to the $^{16}$O-core. To reduce the model space, we restrict ourselves to a two-channel model for describing the core configuration. Here, we consider a neutron and a proton in the $0p_{1/2}$-orbit go up to $1s_{1/2}$-orbit, and other

particle-hole excitations, for example the pairing-type one, are assumed to be renormalized into the $0p0h$- or $2p2h$-channel.

We parameterize the coupled-channel Hamiltonian as follows:

$$
\hat{H}_{ij} = \{\hat{T}_i + \hat{V}_j\}\delta_{ij} + \Delta_{ij}.
$$

(5)

The interactions in each channel and coupling parameter $\Delta_{ij}$ are adjusted to reproduce the energy and radius of the ground state of $^{17}$O. We use the parameter for the diagonal part $\Delta E = \Delta_{22} - \Delta_{11} = 10$ (MeV).

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**Figure 2.** Configurations of the $^{16}$O in this model. The core configurations are shown in (a) and (b). The configuration of $^{22}$O is shown in (c) and (d).

**Figure 3.** Calculated results of the coupled channel approach for (a) separation energy and (b) r.m.s. radius of the oxygen isotopes. Open and solid circles are our calculation. Crosses with a diamond in (a) are experimental values of the separation energy[9, 10]. Crosses with error bars are experiments[1]. Solid and open triangles with error bars are other experiments[2].
We calculate the energy and r.m.s. radius of the oxygen isotopes for \(^{17-23}\)O using the coupled-channel approach within the core+\(n\) model space. Results are shown in Fig. 3. Here, we adjusted the strength of the core-\(n\) interaction (4) so as to reproduce the \(^{17}\)O ground state. For \(^{19-23}\)O, we use the same potential parameters. Nevertheless, the calculated energies agree with the experimental ones, which is shown in Fig. 3(a).

The mechanism to make different binding energies for heavier oxygen isotopes is as follows. In our model, the core size parameter in the first channel is taken to be proportional to the mass number as \(\sim A^{1/6}\). The semi-microscopic potential (4) becomes weak for the large size parameter of the core wave function \(|\Phi_C\rangle\). Hence, the binding energy becomes small with the large mass number. For the second channel, the 1\(s_{1/2}\)-orbit of the valence neutron is occupied in the core configuration. The 1\(s_{1/2}\)-component in the second channel of valence neutron is hindered due to the Pauli principle for \(^{23}\)O, while the 1\(s_{1/2}\)-orbit is unoccupied in the first channel. Therefore, the dominant component becomes only the first channel, and binding energy becomes small because of the smallness of the core+\(n\) potential with a large core-size parameter.

The calculated radii show a promising result in the viewpoint for reproducing the sudden change of the radius at \(^{23}\)O, see Fig. 3 (b). As mentioned in the calculated energies, due to the Pauli blocking effect for the second channel, the first channel becomes dominant in \(^{23}\)O. Since the first channel has the mass number dependence for the core size, the calculated radius with the first channel dominance show the large value for \(^{23}\)O, while the second channel has the same amount of the contribution until \(^{21}\)O.

3. Summary and Discussions
We study the radius of the oxygen isotopes and its sudden increase at \(^{23}\)O. The \(m\)-scheme COSM approach indicates that the different core configurations are necessary to be introduced to reproduce the sudden increase of the radius at \(^{23}\)O. For this purpose, we proposed a coupled-channel approach with different core configurations.

The coupled-channel approach for the core configuration successfully reproduce the change of the radius. The essential point of the coupled-channel picture is the co-existence of a broad and small core-size in each channels and its competition with the presence of the valence neutrons.

For the future work, we will proceed to perform a microscopic treatment of the core excitation and quantitative estimation of the configurations of the core. The mechanism might be a general for neutron rich nuclei, and the radius and energies of these nuclei can be discussed in this point of view.

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