Nuclear physics in soft-wall AdS/QCD:
deuteron electromagnetic form factors

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We present a calculation of the deuteron electromagnetic form factors in a soft-wall AdS/QCD approach. The power scaling of the deuteron form factors is consistent with quark counting rules.

In this paper we present an extension of the soft-wall AdS/QCD approach1,3 to the deuteron and apply the formalism to the calculation of its electromagnetic (EM) form factors. The experimental and theoretical study of the deuteron is one of the main focuses of hadronic physics during the last decades (for detailed reviews see e.g. Refs. 4, 12).

Many theoretical approaches have been applied to the problem of the deuteron form factors: perturbative QCD, chiral effective and phenomenological approaches, quark models (see e.g. Refs. 4–12). One should stress the analysis of Ref. 9, where in the context of perturbative QCD the asymptotic large-momentum-transfer behavior of the deuteron form factor and the form of the deuteron distribution amplitude at short distances have been derived.

Our approach is based on an effective action, which in terms of the AdS fields $d^M(x, z)$ and $V^M(x, z)$, duals to the Fock component contributing to the deuteron with twist $\tau = 6$ and the electromagnetic field, respectively, is given by

$$ S = \int d^4x dz \ e^{-\psi(z)} \left[ -\frac{1}{4} F^{MN}(x, z) F_{MN}^{MN}(x, z) - D^M d^M (x, z) D_M d^N (x, z) - i c_2 F^{MN}(x, z) d^M (1) \right]$$

$$ + \frac{c_3}{4 M_5^2} e^{2A(z)} \phi M \ F^{NK}(x, z) \left( i D_K d^M (x, z) d^N (x, z) - d^M (x, z) i D_K d^N (x, z) \right) \right]$$

$$ + d^M (x, z) \left( \mu^2 + U(z) \right) d^M (x, z)$$

where $A(z) = \log(R/z)$; $F^{MN}(x, z) = \partial^M V^N (x, z) - \partial^N V^M (x, z)$ is the stress tensor of the vector field $V^M (x, z)$; $D^M = \partial^M - i e V^M (x, z)$ is the covariant derivative; $\mu R^2 = (\Delta - 1)(\Delta - 3)$ is the five-dimensional mass; $R$ is the AdS radius; $\phi(z) = \kappa^2 z^2$ is the background dilaton field; $\Delta = \tau + L$ is the dimension of $d^M (x, z)$ field; $L$ is the orbital angular momentum; $M_\Delta$ is the deuteron mass. $U(z)$ is the confinement potential with

$$ U(z) = \frac{\phi(z)}{R^2} U_0 ,$$

where $U_0$ is the constant fixed using the value of the deuteron mass. In the following we work in the axial gauge for both vector fields $d^2(x, z) = 0$ and $A^2(x, z) = 0$. In our consideration we have four free parameters: $c_2$, $c_3$, $U_0$ and $\kappa$.

First we perform a Kaluza-Klein (KK) decomposition for the vector AdS field dual to the deuteron

$$ d^\mu (x, z) = \exp \left[ \frac{\phi(z) - A(z)}{2} \right] \sum_n d^n_m (x) \Phi_n (z) .$$

where $d^n_m (x)$ is the tower of the KK fields dual to the deuteron fields with radial quantum number $n$ and twist-dimension $\tau = 6$, and $\Phi_n (z)$ are their bulk profiles.
Then we derive the Schrödinger-type equation of motion (EOM) for the bulk profile \( \Phi_n(z) \) with

\[
\left[ \frac{d^2}{dz^2} + \frac{4(L + 4)^2 - 1}{4z^2} + \kappa^4 z^2 + \kappa^2 U_0 \right] \Phi_n(z) = M_{d,n}^2 \Phi_n(z).
\] (4)

The analytical solutions of this EOM read

\[
\Phi_n(z) = \sqrt{\frac{2n!}{(n + L + 4)!}} \kappa^{L+5} z^{L+9/2} e^{-\kappa^2 z^2/2} L_n^{(5)}(\kappa^2 z^2),
\]

\[
M_{d,n}^2 = 4\kappa^2 \left[ n + \frac{L + 5}{2} + \frac{U_0}{4} \right],
\] (5)

where \( L_n^m(x) \) are the generalized Laguerre polynomials. Restricting to the ground state \((n = 0, L = 0)\) we get \( M_d = 2\kappa \sqrt{\frac{\kappa}{2} + \frac{U_0}{4}} \). Using the experimental value of the deuteron mass \( M_d = 1.875613 \text{ GeV} \) and \( \kappa = 190 \text{ MeV} \) (constrained by data on electromagnetic deuteron form factors), we fix \( U_0 = 87.4494 \). Note that the scale parameter \( \kappa = 190 \text{ MeV} \) is two times smaller than the corresponding parameter for the nucleon [3], which means that the size of deuteron is two times larger than the one of the nucleon.

In the case of the vector field dual to the electromagnetic field we perform a Fourier transform with respect to the Minkowski coordinate

\[
V_\mu(x, z) = \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} V_\mu(q) V(q, z)
\] (6)

where \( V(q, z) \) is its bulk profile obeying the following EOM

\[
\partial_z \left( \frac{e^{-\varphi(z)}}{z} \partial_z V(q, z) \right) + q^2 \frac{e^{-\varphi(z)}}{z} V(q, z) = 0.
\] (7)

Its analytical solution [1] can be written in the form of an integral representation introduced in Ref. [13]

\[
V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} e^{-\kappa^2 z^2 x/(1-x)} x^a, \quad a = \frac{Q^2}{4\kappa^2}, \quad Q^2 = -q^2.
\] (8)

The gauge-invariant matrix element describing the interaction of the deuteron with the external vector field (dual to the electromagnetic field) reads

\[
M_{\mu \nu}^\mu(p, p') = -G_1(Q^2)\epsilon^\mu(p') \cdot \epsilon(p) - \frac{G_2(Q^2)}{2M_d^2} \epsilon^\mu(p') \cdot q \epsilon(p) \cdot q \left( p + p' \right)^\mu
\]

\[
- G_2(Q^2) \left( \epsilon^\mu(p) \epsilon^\lambda(p') \cdot q - \epsilon^\lambda(p') \epsilon^\mu(p) \cdot q \right)
\] (9)

where \( \epsilon^\lambda(p) \) and \( p(p') \) are polarization and four–momentum of the initial (final) deuteron, with \( q = p' - p \) being the momentum transfer. The three EM form factors \( G_{1,2,3} \) of the deuteron are related to the charge \( G_C \), quadrupole \( G_Q \) and magnetic \( G_M \) form factors by

\[
G_C = G_1 + \frac{2}{3} \tau_d G_Q, \quad G_M = G_2, \quad G_Q = G_1 - G_2 + (1 + \tau_d)G_3, \quad \tau_d = \frac{Q^2}{4M_d^2}.
\] (10)

These form factors are normalized at zero recoil as

\[
G_C(0) = 1, \quad G_Q(0) = M_d^2 Q_d = 25.83, \quad G_M(0) = \frac{M_d}{M_N} \mu_d = 1.714,
\] (11)

where \( M_d \) and \( M_N \) are deuteron and nucleon masses, \( Q_d = 7.3424 \text{ GeV}^{-2} \) and \( \mu_d = 0.8574 \) are the quadrupole and magnetic moments of the deuteron. Since the deuteron is a spin–1 particle it has three EM form factors in the one–photon–exchange approximation, due to current conservation and the \( P \) and \( C \) invariance of the EM interaction.
In our approach the deuteron form factors $G_i(Q^2)$, $i = 1, 2, 3$ are given by the analytical expressions

$$G_1(Q^2) = F(Q^2), \quad G_i(Q^2) = c_i F(Q^2), \quad i = 2, 3$$

(12)

where $F(Q^2)$ is the universal form factor predicted by soft-wall AdS/QCD, which is given by the overlap of the square of bulk profile dual to deuteron wave function and the confined electromagnetic current

$$F(Q^2) = \int_0^\infty dz \Phi^2(z) V(Q,z) = \frac{\Gamma(6) \Gamma(a+1)}{\Gamma(a+6)}$$

(13)

where $a = Q^2/(4\kappa^2)$. The form factor $F(Q^2)$ has the correct power-scaling $F(Q^2) \sim 1/(Q^2)^5$ at large $Q^2 \to \infty$. Also, it can be written in the Brodsky-Ji-Lepage form derived within perturbative QCD, which gives the factorization of the deuteron form factor in terms of the nucleon form factor $F_N(Q^2/4)$ and the so-called “reduced” nuclear form factor $f_d(Q^2)$ \[\footnote{F_d(Q^2) = f_d(Q^2) F_N^2(Q^2/4)}\]:

$$F_d(Q^2) \equiv F(Q^2) = \frac{\Gamma(6) \Gamma(a+1)}{\Gamma(a+6)} = \frac{5!}{(a+1)\cdots(a+5)} = f_d(Q^2) F_N^2(Q^2/4)$$

(14)

where our predictions for $f_d(Q^2)$ and $F_N(Q^2/4)$ are

$$f_d(Q^2) = \frac{30(a+1)(a+2)}{(a+3)(a+4)(a+5)}, \quad F_N(Q^2/4) = \frac{2}{(a+1)(a+2)}$$

(15)

where $a = Q^2/(4\kappa^2)$. Our predictions for the charge $G_C(Q^2)$, quadrupole $G_Q(Q^2)$ and magnetic $G_M(Q^2)$ form factors are in good agreement with data (see Figs.1-3). The data points are taken from Ref. \[\footnote{\cite{6,7}}\]. Also we would like to note that our result for the deuteron charge radius $r_C = (-6dG_C(Q^2)/dQ^2|_{Q^2=0})^{1/2} = \sqrt{\frac{137}{48\pi^2}} - Q_d = 1.846$ fm compares well with data $r_C = 2.130 \pm 0.010$ fm \[\footnote{\cite{4}}\].

In conclusion, we stress again the main result of this paper. As a further application of the soft-wall AdS/QCD model we calculated the deuteron electromagnetic form factors, which are given by analytical expressions in terms of a universal form factor $F(Q^2)$. In comparison with other theoretical approaches our framework gives a description of the deuteron form factors in a very simple form and with the use of four free parameters. Two of them, $c_2$ and $c_3$, are fixed by the normalization of the deuteron form factors, the parameter $U_0$ is fixed using the deuteron mass and the parameter $\kappa$ is related to the nucleon size.

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FIG. 1: Charge deuteron form factor.

FIG. 2: Quadrupole deuteron form factor.

FIG. 3: Magnetic deuteron form factor.