Multi-Objective Optimization Design of Liftbody Aircraft Using Kriging Model

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Abstract. The evolutionary algorithm combined with Kriging surrogate model is applied to aerodynamic optimization design of high-altitude hypersonic aircraft. In this paper, a 3D liftbody aircraft in cruise flight state is designed which divided into forebody/inlet, engine and after-body/nozzle parts, and its variable parameterization method is studied by considering the multi-disciplinary coupling effect of airframe/engine integration; four disciplines analysis model, i.e. engine propulsion performance, the forebody/inlet air-breathing performance, aerodynamic force and thermodynamics, are established and the appropriate objective parameters are selected, then genetic algorithms are used to optimize the design. The single-objective optimization results show that the performance of aircraft is effectively improved; the multi-objective optimization results reveal that the total trade-off is achieved.

1. Introduction
In the century of human aerospace, a great leap from low speed, subsonic speed, supersonic speed to hypersonic speed has been achieved. Liftbody aircraft and hypersonic technology are becoming one of the key technologies actively explored and developed in the world [1-2]. At the same time, with the improvement of computer performance, the numerical simulation method of flow field is gradually emerging to replace a large part of wind tunnel experiment work. In the aerodynamic optimization design of three-dimensional aircraft, Kriging surrogate model is introduced into the optimization process to reduce the computational time of CFD (computational fluid dynamics) due to the slow convergence of evolutionary algorithm [3-5].

The optimization design of hypersonic liftbody aircraft has always been a hot topic, which attracts the interest of many researchers. Daines and Segal [6] studied a rocket and air-breathing combined cycle engines for earth-to-orbit applications. Birzer and Doolan [7] built a quasi-one-dimensional model to simulate the complex flow phenomenon in the hydrogen-fueled combustors. Li [8] employed a parameterized configuration modelling approach to develop a rigid-body model for hypersonic vehicle and applied it to the research of control method in the cruise state. Bolender [9] developed a nonlinear longitudinal dynamical model which captures a number of complex interactions between the different disciplines of hypersonic aircraft. Takashima and Lewis [10] optimized a wave-rider aircraft for hypersonic cruise state with considering some aspects of off-design performance. The engine and the airframe are usually designed separately in existing research, which results in the performance impact between each other being ignored, and the best solution cannot be effectively obtained.
The sub-disciplines of hypersonic aircraft are strongly coupled together, and only by taking a full consideration of the strong coupling effect between airframe and engine, can a globally optimal design be obtained. The parametric modelling and airframe/engine integrated optimization design of a hypersonic liftbody aircraft which is similar to X-43A is studied in this article. Single-disciplinary and multi-disciplinary optimization analysis models for the performance of engine system propulsion, the forebody/inlet air-breathing, aerodynamic force and aerodynamic heating are established. CFD numerical simulation, Latin hypercube sampling, Kriging surrogate model and GAs (genetic algorithms) are combined to optimize the above problems.

2. Kriging model

Kriging model is a mathematical function of unbiased estimation with a small amount of calculation and similar fitting results to the actual value, it expressed as

\[ y(x) = \mu + z(x) \]  

(1)

The dimension of \( x \) is \( m \), \( \mu \) is a constant global model and \( z(x) \) represents a local deviation from it. The correlation between the point \( x_i \) and \( x_j \) is expressed as

\[ R(x_i, x_j) = \prod_{k=1}^{n} \exp\left(-\theta_k |x_i^k - x_j^k|^2\right) \]  

(2)

Where \( \theta \) is the correlation vector. The Kriging function expressed as

\[ \hat{y}(x) = \hat{\mu} + r^T (x) R^{-1} (y - 1\hat{\mu}) \]  

(3)

Where \( R \) denotes the \( n \times n \) matrix whose \((i, j)\) entry is \( R(x_i, x_j) \), \( n \) is the number of samples, \( r \) denotes a vector whose \( i \)th element is \( R(x, x_j) \). \( \hat{\mu} \) is the estimated value of \( \mu \), expressed as

\[ \hat{\mu} = \left(1^T R^{-1} 1\right)^{-1} 1^T R^{-1} y \]  

(4)

Then \( \theta \) can be estimated by maximizing the following function

\[ \ln \left( \text{Likelihood} \right) = -\frac{1}{2} \left(n \ln \sigma^2 + \ln |R| \right) \]  

(5)

Where

\[ \sigma^2 = \left(y - 1\hat{\mu}\right)^T R^{-1} \left(y - 1\hat{\mu}\right) / n \]  

(6)

For a series of given sampling points, \( y(x) \) can be obtained by equation (3) after determining the basic parameters of Kriging model. Latin Hypercube Sampling method is adopted in this paper. An example of Camel function is tested as shown in Fig. 1, the expression is:

\[ y = (4 - 2.1x_1^2 + x_1^4 / 3)x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2, \quad -2 \leq x_1 \leq 2, -1 \leq x_2 \leq 1 \]

We can see that as long as the sample size is sufficient, the Kriging fitting surface almost coincides with the original function. When combined with genetic algorithms for optimization (the process is showed in Fig. 2), the fitness function evaluation of individual populations is completed by Kriging surrogate model predictions rather than time-consuming high-precision numerical calculations.

3. Liftbody aircraft optimization design

An X-43A like medium slenderness ratio liftbody configuration is adopted in this paper and the optimization work is carried out under hypersonic cruise circumstances: \( Ma=8.0, \alpha=0\deg \) (angle of attack) and \( H=30\text{km} \).
3.1. Aircraft design with fuselage/engine integration

The aircraft is divided into three parts: forebody/inlet, isolator, combustor, and after-body/nozzle (2D components cross section are shown in Fig. 3). After passing through the lower wall of the forebody, the free air decelerates and pressurizes, flows into the isolator section in a horizontal state, then decelerates and reaches the inlet of the combustor after several reflected shock waves, flows out from the outlet of the combustor after burning, and finally expands and accelerates in the after-body/nozzle with thrust generation.

![Figure 3. Longitudinal view of aircraft configuration.](image)

The main role of the forebody/inlet is to capture sufficient air flow for the engine and provide a higher-pressure inlet flow field. As shown in Fig. 4, three oblique shock waves converge at the lower lip of the inlet, and the reflected shock waves converge at the lower shoulder point of forebody. For a given height \( H_0 \) of aircraft altitude, the shape parameters of aircraft can be obtained by the shock wave equation. Then three wedge angles \( \alpha_i \) \((i=1, 2, 3)\) are chosen to be the design variables of the forebody.

![Figure 4. Shape parameters of aircraft forebody/inlet](image)

In order to avoid the problem that the air inlet cannot be started, a constant area isolator is coupled with the forebody design, whose length is calculated by following empirical formula [11]:
$L_{iso} = 0.01(0.971875(H_0/h)^2 - 24.0875(H_0/h) + 162.3)$  \hspace{1cm} (7)

For quickly obtain the performance of the airflow parameters at the entrance and exit of the combustor, a theoretical model of the area-increasing combustor was adopted, the expansion angle of channel is 2 degree, and its length can be solved by

$$L_{cont} = (H_e - H_i) \times \cot 2^\circ$$  \hspace{1cm} (8)

Where $H_e, H_i$ represent the height of the combustor inlet and outlet.

The high temperature and high enthalpy gas which flows from the outlet of the combustor fully expand in the after-body/nozzle, then the greatest possible thrust can be obtained. As shown in Fig. 5, the after-body consists of an internal horizontal nozzle section and an external expanding nozzle section, the length $L_{sp}$ and the deflection angle $\theta_{sp}, \theta_{aft}$ are selected as the shape parameters. The cruise flight fuel/gas ratio $f_{cruise}$ could be gained through an iterative method shown in Fig. 6.

In summary, a unique two-dimensional geometric shape of liftbody aircraft corresponds to a given design variables $x = [\alpha_1, \alpha_2, \alpha_3, L_{sp}, \theta_{sp}, \theta_{aft}]^T$, and stretched laterally to a three-dimensional model shown in Fig. 7, we can see that the component design of aircraft is highly integrated and interacted.

![Figure 5. Shape parameters of after-body.](image1)

![Figure 6. The solution of $f_{cruise}$.](image2)

![Figure 7. Full view of aircraft.](image3)

### 3.2. Multiple objective analysis model

The integrated design of hypersonic liftbody aircraft involves many strongly coupled disciplines which interact and iterate continuously. In this section, four disciplines analysis model, i.e. engine propulsion performance, the forebody/inlet air-breathing performance, aerodynamic force and thermodynamics, are established and the appropriate objective parameters are selected and established.

An approximate simulation method [12] of combustor is adopted, the engine can be viewed as a pipeline flow contains the influence of area-varying and heat transfer. Let:

- $L_{iso}$
- $H_0$
- $H_i$
\[ a = \gamma (kc - 1) - kc; \quad b = 2kc - 1 \]  
\[ fm = (amA_e^2 + b) / (amA_e^2 + b); \quad gm = M_p^2 / Ma^2; \quad hm = \left[ 2 + (\gamma - 1)Ma^2 \right] / \left[ 2 + (\gamma - 1)Ma^2 \right] \]

Where, \( kc \) is a fixed parameter determined by the type of fuel (\( kc = 1.2 \) for liquid hydrogen), subscript \( i,e \) represent the entrance and exit of the combustor, and flow parameters can be solved:

\[ A_e / A_i = fm^{(k_1-1)/b} \cdot gm^{k_1/b}; \quad V_e / V_i = \left[ fm \cdot gm^{-1} \right]^{(k_1-1)/b} \cdot P_e / P_i = fm^{(k_1-1)/a} \]

\[ T_e / T_i = fm^{(1-b)/b} \cdot gm^{1/b}; \quad P_{se} / P_{ui} = hm^{(\gamma-1)/\gamma} \cdot P_e / P_i; \quad T_{se} / T_{ui} = hm \cdot T_e / T_i \]

With respect to the required heating value, there is:

\[ \eta \dot{m}_f \dot{H}_f = \dot{m}_a (h_{e} - h_{i}) + \dot{m}_j h_{e}; \quad f = \dot{m}_f / \dot{m}_a \]

The calorically perfect gas satisfied the following expression:

\[ h_{e} = c_p T_{e}; \quad h_{i} = c_p T_{i} \]

The fuel/gas ratio \( f \) is set to a fixed value, according to the forebody/inlet flow calculation, the flow characteristics can be obtained from the above expressions. From the momentum theorem, the thrust \( T_p \) and specific impulse of engine \( I_{sp} \) can be defined as:

\[ T_p = \dot{m}_e (V_e - V_i) + \dot{m}_i V_e + (P_e A_e - P_i A_i) + (P_e + P_i) (A_e - A_i) / 2 \]

\[ I_{sp} = T_p / \dot{m}_f \]

Here, \( I_{sp} \) is selected as the disciplinary objective for engine propulsion performance.

Aerodynamics and thermodynamics disciplines of liftbody aircraft are analyzed by CFD simulation, and the inviscid Euler equations are solved by using implicit second-order upwind finite volume scheme. The mesh distribution of aircraft surface is showed in Fig. 8, half model is used for calculations to save time. Then, the lift to drag ratio of aircraft \( C_{l} / C_{d} \) and the maximum temperature on aircraft surface \( T_{\text{max}} \) can be obtained which evaluate the aerodynamics and thermodynamics performance. Finally, the total pressure recovery \( \sigma \) is the optimization objective to evaluates the aerodynamic performance of forebody/inlet, and it reflects energy loss due to shock waves, which can be calculated by equations of shock wave theory.

3.3. Optimization model and results

Optimization work can be carried out based on the analysis models proposed in previous section. Here, MIGA (multi-islands genetic algorithm) is used to solve single-objective optimization problems, and NSGA-II (non-dominated sorting genetic algorithm II) is used to solve multi-objective optimization problem. 200 sample points are picked up to construct the Kriging surrogate model before optimization, so the design objective can be obtained by this response surface instead of time-consuming flow calculations.

3.3.1. Definition of optimization design. For a set of design variables: \( x = [\alpha_1, \alpha_2, \alpha_3, \theta_{ip}, L_{sp}, \theta_{ip}]^T \), the definition of optimization problem expressed as:
3.3.2. Single-objective optimization results. The comparison of four single-objective optimization results and initial configuration are shown in Table 1, we can see that the pressure recovery has increased by 5.55% for model I; the lift to drag ratio has increased by 27.75% for model II; the specific impulse of engine has increased by 34.85% for model III; and the maximum temperature on aircraft surface has decreased by 12.60% for model IV. The pressure cloud diagram of the flow field is shown in Fig. 9, all models have reached the integrated design standard, and the optimized configurations of different targets have different shapes and flow characteristics.

Table 1. Single-objective optimization results.

| Model     | Design Variables | \( f_{\text{cruise}} \) | \( \sigma \) | \( C_l/C_d \) | \( I_{sp} \) (N\( \cdot \)s/kg) | \( T_{\text{max}} \) (K) |
|-----------|------------------|----------------|---------|---------------|----------------|------------------|
| Benchmark | [5.45,6.94,5.26,6.60,6.00,13.24] | 0.00612 | 0.4218 | 2.0066 | 27992 | 1350 |
| Model I   | [4.89,5.86,6.32] | 0.4452 | — | — | — | — |
| Model II  | [4.64,8.76,3.31,160,24.98,10.78] | 0.00686 | 0.4235 | — | — | — |
| Model III | [8.34,9.98,3.38,496,14.74,2.65] | 0.00536 | 0.2858 | — | — | — |
| Model IV  | [5.05,4.61,7.38,447,2.93,12.96] | 0.00368 | 0.4402 | 0.3283 | — | — |

Figure 9. The comparison of pressure contour.
3.3.3. **Multi-objective optimization results.** The Pareto front of multi-objective optimization result is split into two 3-3 sub-fronts as shown in Fig. 10, it can be seen clearly that each point of Pareto front has the property of global relative optimum, but not all the target values of this point are better than the initial values. The trade-off information between objectives is very important for aircraft designers to make choices according to the actual situation, in order to comprehensively consider the influence and weight of various disciplines, a compromise scheme on the Pareto front is often selected.

![Figure 10. Pareto front and feasible solution set.](image)

4. **Conclusion**
In this paper, Kriging surrogate model is constructed to replace the extremely time-consuming CFD calculation, and applied to aerodynamic optimization design of a three-dimensional hypersonic liftbody aircraft combined with genetic algorithms. The shape of the aircraft is parameterized with the consideration of multi-disciplinary coupling effects between the airframe and the engine. In cruise flight status, the total pressure recovery coefficient of forebody, the lift to drag ratio of aircraft, the specific impulse of engine and maximum surface temperature are selected as the optimization targets, and four single-objective optimization models are proposed for optimization and analyzation. The results show that the objective of each optimization model is better than the initial configuration, and the performance of the aircraft is effectively improved. Then a multi-objective optimization model is proposed and the non-inferior solution set of the Pareto front is obtained, every point on the Pareto front reveals the trade-off information between integrated disciplines, which enable the designer to make the best choice based on the actual situation.

The research in this paper provides a set of efficient and feasible methods for the airframe/engine integration design of hypersonic liftbody aircraft and multi-disciplinary design optimization, and some valuable conclusions can be used for reference and consideration. Meanwhile, further research in model accuracy and algorithm efficiency improving is still needed.

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