Recent neutrino data and a realistic tribimaximal-like neutrino mixing matrix

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A B S T R A C T

In light of the recent neutrino experimental results from the Daya Bay and RENO Collaborations, we construct a realistic tribimaximal-like Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix. Motivated by the Qin–Ma (QM) parametrization for the quark mixing matrix in which the CP-odd phase is approximately maximal, we propose a simple ansatz for the charged lepton mixing matrix, namely, it has the QM-like parametrization, and assume the tribimaximal mixing (TBM) pattern for the neutrino mixing matrix. The deviation of the leptonic mixing matrix from the TBM one is then systematically studied. While the deviation of the solar and atmospheric neutrino mixing angles from the corresponding TBM values, i.e. \( \sin^2 \theta_{12} = 1/3 \) and \( \sin^2 \theta_{23} = 1/2 \), is fairly small, we find a non-vanishing reactor mixing angle given by \( \sin \theta_{13} \approx \lambda/\sqrt{2} \) (\( \lambda \approx 0.22 \) being the Cabibbo angle). Specifically, we obtain \( \theta_{13} \approx 9.2^{\circ} \) and \( \delta_{CP} \approx \delta_{QM} \approx 0(90^{\circ}) \). Furthermore, we show that the leptonic CP violation characterized by the Jarlskog invariant is \( \vert f_{CP} \vert \approx \lambda/6 \), which could be tested in the future experiments such as the upcoming long baseline neutrino oscillation ones.

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1. Introduction

Recent analyses of the neutrino oscillation data [1,2] indicate that the tribimaximal mixing (TBM) pattern for three flavors of neutrinos [3] can be regarded as the zeroth order leptonic mixing matrix

\[
U_{\text{TBM}} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix} P_{\nu},
\]

where \( P_{\nu} = \text{Diag}(e^{i\delta_1}, e^{i\delta_2}, 1) \) is a diagonal matrix of phases for the Majorana neutrino. However, properties related to the leptonic CP violation remain completely unknown yet. The large values of the solar and atmospheric mixing angles, which may be suggestive of a new flavor symmetry in the lepton sector, are entirely different from the quark mixing ones. The structure of both charged lepton and neutrino mass matrices could be deduced by a flavor symmetry, for example, the \( A_4 \) discrete symmetry, which will tell us something about the charged fermion and neutrino mixings. If there exists such a flavor symmetry in nature, the TBM pattern for the neutrino mixing matrix may come out in a natural way. It is well known that there are no sizable effects on the observables from the renormalization group running for the hierarchical mass spectrum in the standard model [4]. Hence, corrections to the tribimaximal neutrino mixing from renormalization group effects running down from the seesaw scale are negligible in the standard model.

The so-called PMNS (Pontecorvo–Maki–Nakagawa–Sakata) leptonic mixing matrix depends generally on the charged lepton sector whose diagonalization leads to a charged lepton mixing matrix \( V_{\nu} \) which should be combined with the neutrino mixing matrix \( U_{\nu} \), that is,

\[
U_{\text{PMNS}} = V_{\nu}^\dagger U_{\nu}.
\]

In the charged fermion (quarks and charged leptons) sector, there is a qualitative feature which distinguishes the neutrino sector from the charged fermion one. The mass spectrum of the charged leptons exhibits a similar hierarchical pattern as that of the down-type quarks, unlike that of the up-type quarks which shows a much stronger hierarchical pattern. For example, in terms of the Cabibbo angle \( \lambda \equiv \sin \theta_C \approx \vert V_{us} \vert \), the fermion masses scale as \( (m_e, m_\mu) \approx (\lambda^2, \lambda^3) m_t \), \( (m_d, m_s) \approx (\lambda^4, \lambda^5) m_d \), and \( (m_u, m_c) \approx (\lambda^8, \lambda^4) m_t \). This may lead to two implications: (i) the Cabibbo–Kobayashi–Maskawa (CKM) matrix [6] is mainly governed by the

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1 This may not be true beyond the standard model. For example, for quasi-degenerate neutrinos and large \( \tan \beta \) in the minimal supersymmetric model, all three mixing angles may change significantly [5].
down-type quark mixing matrix, and (ii) the charged lepton mixing matrix is similar to that of the down-type quark one. Therefore, we shall assume that (i) \( V_{\text{CKM}} \) = \( V_{\text{L}}^d \) and \( V_{\text{L}}^u \) = \( \mathbf{1} \), where \( V_{\text{L}}^d \) (\( V_{\text{L}}^u \)) is associated with the diagonalization of the down-type (up-type) quark mass matrix and 1 is a \( 3 \times 3 \) unit matrix, and (ii) the charged lepton mixing matrix \( V_{\ell}^d \) has the same structure as the CKM matrix, \( V_{\text{L}}^e \) = \( \text{CKM} \).

Very recently, a non-vanishing mixing angle \( \theta_{13} \) has been reported firstly from the Daya Bay and RENO Collaborations \cite{7,8} with the results given by

\[
\sin^2 2\theta_{13} = 0.092 \pm 0.016 \text{ (stat)} \pm 0.005 \text{ (syst)} \tag{3}
\]

and

\[
\sin^2 2\theta_{13} = 0.113 \pm 0.013 \text{ (stat)} \pm 0.019 \text{ (syst)}, \tag{4}
\]

respectively. These results are in good agreement with the previous data from the T2K, MINOS and Double Chooz Collaborations \cite{9}. The experimental results of the non-zero \( \theta_{13} \) indicate that the TBM pattern for the neutrino mixing should be modified. Moreover, properties related to the leptonic CP violation remain completely unknown yet.

In this work, we shall assume a neutrino mixing matrix in the TBM form,

\[
U_{\nu} = U_{\text{TBM}}. \tag{5}
\]

We will neglect possible corrections to \( U_{\text{TBM}} \) from higher dimensional operators and from renormalization group effects. Then we make a simple ansatz on the charged lepton mixing matrix \( V_{\ell}^d \), namely, we assume that \( V_{\ell}^d \) has the same structure as the Qin–Ma (QM) parametrization of the quark mixing matrix, which is a Wolfenstein-like parametrization and can be expanded in terms of the small parameter \( \lambda \). Unlike the original Wolfenstein parametrization, the QM one has the advantage that its CP-odd phase \( \delta \) is manifested in the parametrization and approximately maximal, i.e. \( \delta \sim 90^\circ \). As we shall see below, this is crucial for a viable neutrino phenomenology. It turns out that the PMNS leptonic mixing matrix is identical to the TBM matrix plus some small corrections arising from the charged mixing matrix \( V_{\ell}^d \) expanded in terms of the small parameter \( \lambda \). Schematically,

\[
U_{\text{PMNS}} = U_{\text{TBM}} + \text{corrections in powers of } \lambda. \tag{6}
\]

Consequently, not only the solar and atmospheric mixing angles given by the TBM remain valid but also the reactor mixing angle and the Dirac phase can be deduced from the above consideration.

The Letter is organized as follows. In Section 2 we discuss the parametrizations of quark and lepton mixing matrices and pick up the one suitable for our purpose in this work. After making an ansatz on the charged lepton mixing matrix we study the low-energy neutrino phenomenology and emphasize the new results on the reactor neutrino mixing angle and the CP-odd phase in Section 3. Our conclusions are summarized in Section 4.

2. Lepton and quark mixing

In the weak eigenstate basis, the Lagrangian relevant to the lepton sector reads

\[
-\mathcal{L} = \frac{1}{2} \overline{\nu}_L m_\nu \nu_L + \overline{\nu}^c L \ell_R + \frac{g}{\sqrt{2}} W^\mu L \overline{\nu}_L \nu_L^c + \text{H.c.} \tag{7}
\]

When diagonalizing the neutrino and charged lepton mass matrices, \( U_{\nu}^\dagger m_\nu U_{\nu} = \text{Diag}(m_1, m_2, m_3) \) and \( V_{\ell}^c \dagger m_\ell V_{\ell}^c = \text{Diag}(m_e, m_\mu, m_\tau) \), we can rotate the fermion fields from the weak eigenstates to the mass eigenstates, \( \nu_i \rightarrow U_{\nu i}^\dagger \nu_L, \ell_i \rightarrow V_{\ell i}^c \ell_L, \ell_R \rightarrow V_{\ell R}^c \ell_R \). Then we obtain the leptonic \( 3 \times 3 \) unitary mixing matrix \( U_{\text{PMNS}} = U_{\nu}^\dagger U_{\ell} \) as given in Eq. (2) from the charged current term in Eq. (7). In the standard parametrization of the leptonic mixing matrix \( U_{\text{PMNS}} \), it is expressed in terms of three mixing angles and three CP-odd phases (one \( \delta ' \) for the Dirac neutrino and two for the Majorana neutrino) \cite{10}

\[
U_{\text{PMNS}} = \left( \begin{array}{ccc}
\frac{c_{13} c_{12}}{\text{NO}}, & -s_{13} c_{12}, & s_{13} e^{-i \delta} \\
c_{13} s_{12} - s_{23} c_{12} s_{13} e^{i \delta'}, & c_{13} s_{12} - s_{23} c_{12} s_{13} e^{i \delta'}, & s_{23} c_{13} e^{-i \delta'} \\
s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i \delta'}, & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i \delta'}, & c_{23} c_{13} e^{-i \delta'}
\end{array} \right) \times \rho \nu, \tag{8}
\]

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). The current best-fit values of \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) at 1\% (3\%) level obtained from the global analysis by Fogli et al. \cite{2} are given by

\[
\theta_{12} = 33.6^{+11.1(-13.2)}\% \tag{9}
\]

\[
\theta_{23} = \begin{cases}
38.4^{+1.4(-1.4)}_\circ & \text{NO}, \\
38.8^{+2.3(-1.3)}_\circ & \text{IO},
\end{cases} \tag{10}
\]

\[
\theta_{13} = \begin{cases}
8.9^{+0.5(-0.5)}_\circ & \text{NO}, \\
9.0^{+0.4(-0.5)}_\circ & \text{IO},
\end{cases} \tag{11}
\]

where NO and IO stand for normal mass ordering and inverted one, respectively. The analysis by Fogli et al. includes the updated data released at the Neutrino 2012 Conference.\footnote{For definiteness, we shall use the Jarlskog rephasing invariant as shown in Eq. (27) to define the Dirac CP-violating phase \( \delta_2 \). The Dirac phase defined in this manner is independent of a particular parametrization of the PMNS matrix. In general, \( \delta' \) may not be equal to \( \delta_2 \). It shall be shown that \( \delta_2 \) equals to the phase \( \delta \) defined in Eq. (18), up to the order of \( \lambda^4 \).}

In analogy to the PMNS matrix, the CKM quark mixing matrix is a product of two unitary matrices, \( V_{\text{CKM}} = V_{\ell}^c \dagger V_{\nu} \), and can be expressed in terms of four independent parameters (three mixing angles and one phase). Their current best-fit values in the 1\% range read \cite{12}

\[
\begin{align}
\theta_{12}^d &= (13.03 \pm 0.05)\% , \\
\theta_{23}^d &= (2.37^{+0.03(-0.07)}_\circ ) ,
\end{align} \tag{12}
\]

\[
\theta_{13}^d = (0.20^{+0.01(-0.01)}_\circ , \phi = (67.19^{+2.40(-1.76)}_\circ ) \tag{13}
\]

\[
A \text{ well-known simple parametrization of the CKM matrix introduced by Wolfenstein [15] is}
\]

\[
V_W = \left( \begin{array}{ccc}
1 - \frac{\lambda^2}{2} & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{array} \right) + O(\lambda^4). \tag{14}
\]

Hence, the CKM matrix is a unit matrix plus corrections expanded in powers of \( \lambda \).

Recently, Qin and Ma (QM) \cite{13} have advocated a new Wolfenstein-like parametrization of the quark mixing matrix
\[
V_{QM} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda e^{i\theta} & h\lambda^3 e^{-i\delta} \\
-\lambda & 1 - \lambda^2/2 & (f + h e^{-i\delta})\lambda^2 \\
f\lambda^3 & -(f + h e^{i\delta})\lambda^2 & 1 \\
\end{pmatrix} + \mathcal{O}(\lambda^4),
\]

(12)

based on the triminimal expansion of the CKM matrix.\(^5\) The parameters \(A, \rho, \) and \(\eta\) in the Wolfenstein parametrization\(^15\) are replaced by \(f, h, \) and \(\delta\) in the QM–Ma one. From the global fits to the quark mixing matrix given by\(^12\), we obtain

\[
f = 0.749 \pm 0.034, \quad h = 0.309 \pm 0.017, \quad \delta = (89.6_{-0.86}^{+2.94})^\circ.
\]

(13)

Therefore, the CP-odd phase is approximately maximal in the sense that \(\sin \delta \approx 1\). Because of the freedom of the phase redefinition for the quark fields, we have shown in\(^16\) that the QM parametrization is indeed equivalent to the Wolfenstein one\(^6\) and pointed out that

\[
\delta = \gamma + \beta = \pi - \alpha,
\]

(14)

where the three angles \(\alpha, \beta, \) and \(\gamma\) of the unitarity triangle are defined by

\[
\alpha = \arg \left( -\frac{V_{ud}^\ast V_{ub}^\ast}{V_{td}^\ast V_{tb}} \right), \quad \beta = \arg \left( -\frac{V_{cd}^\ast V_{cb}^\ast}{V_{td}^\ast V_{tb}} \right),
\]

\[
\gamma = \arg \left( -\frac{V_{td}^\ast V_{tb}}{V_{cd}^\ast V_{cb}} \right),
\]

(15)

and they satisfy the relation \(\alpha + \beta + \gamma = \pi\). Since \(\alpha = (91.0 \pm 3.9)^\circ, \beta = (21.76_{-0.92}^{+0.93})^\circ\) and \(\gamma = (67.2 \pm 3.0)^\circ\) inferred from the current data\(^12\), the phase \(\delta\) in the QM parametrization is thus very close to 90°.

The rephasing invariant Jarlskog parameter \(J^q_{CP}\) in the quark sector is given by

\[
J^q_{CP} = \text{Im} \left[ V_{ud} V_{tb}^\ast V_{ub} V_{td} \right] = h f \lambda^3 (1 - \lambda^2/2) \sin \delta,
\]

(16)

implying that the phase \(\delta\) in Eq. (12) is equal to the rephasing invariant CP-violating phase. Numerically, it reads \(J^q_{CP} \approx 0.2 \lambda^3\) using Eq. (13). For our later purpose, we shall consider a particular QM parametrization obtained by rephasing \(u\) and \(d\) quark fields: \(u \rightarrow u e^{i\delta}\) and \(d \rightarrow d e^{i\delta}\)

\[
V_{QM} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda e^{i\theta} & h\lambda^3 e^{-i\delta} \\
-\lambda e^{-i\theta} & 1 - \lambda^2/2 & (f + h e^{-i\delta})\lambda^2 \\
f\lambda^3 e^{-i\theta} & -(f + h e^{i\delta})\lambda^2 & 1 \\
\end{pmatrix} + \mathcal{O}(\lambda^4),
\]

(17)

As we will show in the next section, it will have very interesting implications to the lepton sector.

3. Low energy neutrino phenomenology

Let us now discuss the low energy neutrino phenomenology with an ansatz that the charged lepton mixing matrix \(V^\ell_L\) has the similar expression to the QM parametrization given by Eq. (17):

\[
V^\ell_L = \begin{pmatrix}
1 - \lambda^2/2 & \lambda e^{i\theta} & h\lambda^3 e^{-i\delta} \\
-\lambda e^{-i\theta} & 1 - \lambda^2/2 & (f + h e^{-i\delta})\lambda^2 \\
f\lambda^3 e^{-i\theta} & -(f + h e^{i\delta})\lambda^2 & 1 \\
\end{pmatrix} + \mathcal{O}(\lambda^4),
\]

(18)

where the parameters \(\lambda, f, h\) and \(\delta\) in the lepton sector are \textit{a priori} not necessarily the same as those in the quark sector. Nevertheless, we shall assume that \(\lambda\) is a small parameter and that \(\delta\) is of order 90°. As we will see below, this matrix accounts for the small deviation of the PMNS matrix from the TBM pattern.

We have emphasized in\(^17\) that the phases of the off-diagonal matrix elements of \(V^\ell_L\) play a key role for a viable neutrino phenomenology. Especially, we have found that the solar mixing angle \(\theta_{12}\) depends strongly on the phase of the element \((V^\ell_L)_{12}\). This is the reason why we choose the particular form of Eq. (18). In the quark sector, there exist infinitely many possibilities of rephasing the quark fields in the CKM matrix and physics should be independent of the phase redefinition. The reader may wonder why we do not identify \(V^\ell_L\) first with the original QM parametrization in Eq. (12) and then make phase redefinition of lepton fields to get CP-odd phases in the off-diagonal elements. The point is that the arbitrary phase matrix of the neutrino fields does not commute with the TBM matrix \(U_{TB}\). As a result, the charged lepton mixing matrix in Eq. (2) cannot be arbitrarily rephased from the neutrino fields. Therefore, in the lepton sector, this particular form of Eq. (18) for the parametrization of \(V^\ell_L\) obtained by rephasing the \(u\) and \(d\) quark fields in Eq. (12) with a physical phase \(\delta\) is the only way for \(V^\ell_L\) consistent with the current experimental data, especially for \(\sin \theta_{12}\) (see Eq. (24) below).

With the help of Eqs. (2), (5) and (18), the leptonic mixing matrix corrected by the contributions from \(V^\ell_L\) can be written, up to the order of \(\lambda^3\), as

\[
U_{PMNS} = U_{TB} + \left( \begin{array}{ccc}
\frac{-\lambda e^{i\delta}}{\sqrt{2}} & \frac{\lambda e^{i\theta}}{\sqrt{2}} & \frac{\lambda e^{i(\theta + \delta)}}{\sqrt{2}} \\
\frac{f \lambda^3 e^{-i\theta}}{\sqrt{2}} & \frac{f \lambda^3 e^{-i\delta}}{\sqrt{2}} & \frac{f \lambda^3 e^{-i(\theta + \delta)}}{\sqrt{2}} \\
\frac{h \lambda^3 e^{-i(\theta + \delta)}}{\sqrt{2}} & \frac{h \lambda^3 e^{-i\delta}}{\sqrt{2}} & \frac{h \lambda^3 e^{-i(\theta + \delta)}}{\sqrt{2}} \\
\end{array} \right) \times P_{\nu} + \mathcal{O}(\lambda^4).
\]

(19)

By rephasing the lepton and neutrino fields \(e \rightarrow e e^{i\alpha_1}, \mu \rightarrow \mu e^{i\beta_1}, \tau \rightarrow \tau e^{i\gamma_1}\) and \(v_2 \rightarrow v_2 e^{i(\alpha_2 - \alpha_3)}\), the PMNS matrix is recast to

\[
U_{PMNS} = \begin{pmatrix}
|U_{e1}| & |U_{e2}| & |U_{e3}| e^{i(\alpha_1 - \alpha_3)} \\
U_{\mu1} e^{-i\beta_1} & U_{\mu2} e^{i(\alpha_2 - \alpha_3)} & |U_{\mu3}| \\
U_{\tau1} e^{-i\gamma_1} & U_{\tau2} e^{i(\alpha_2 - \alpha_3)} & |U_{\tau3}| \\
\end{pmatrix} P'_{\nu},
\]

(20)

where \(|U_{e1}|\) is an element of the PMNS matrix with \(\alpha = e, \mu, \tau\) corresponding to the lepton flavors and \(j = 1, 2, 3\) to the light neutrino mass eigenstates. In Eq. (20) the phases defined as \(\alpha_1 = \arg(U_{e1}), \alpha_2 = \arg(U_{e2}), \alpha_3 = \arg(U_{e3}), \beta_1 = \arg(U_{\mu3})\) and \(\beta_2 = \arg(U_{\tau3})\) have the expressions

\[
\alpha_1 = \tan^{-1} \left( \frac{-\lambda \sin \delta}{2 - \lambda \cos \delta - \lambda^2 + h \lambda^3} \right),
\]

\[
\beta_1 = \tan^{-1} \left( \frac{h \lambda^2 \sin \delta}{1 - \lambda^2 (f + h + \cos \delta)} \right),
\]

\[
\alpha_2 = \tan^{-1} \left( \frac{h \lambda^2 \sin \delta}{1 + \lambda \cos \delta - \frac{f^2}{\tau} + h \lambda^3} \right),
\]

\[
\beta_2 = \tan^{-1} \left( \frac{h \lambda^2 \sin \delta}{1 + \lambda \cos \delta} \right),
\]

\[
\alpha_3 = \tan^{-1} \left( \frac{-\sin \delta}{h \lambda^2 - \cos \delta} \right),
\]

\[
P'_{\nu} = \text{Diag}(e^{i\beta_1}, e^{i(\beta_2 + \alpha_1 - \alpha_3)}, 1).
\]

(21)
Up to the order of $\lambda^3$, the elements of $U_{PMNS}$ are found to be

$$|U_{e1}| = \sqrt{\frac{2}{3}} \left( 1 - \frac{\lambda \cos \delta}{2} - \frac{\lambda^2 (3 + \cos^2 \delta)}{8} + \frac{\lambda^3}{16} \left( \cos \delta - 8h - \cos^3 \delta \right) \right),$$

$$|U_{e2}| = \frac{1}{\sqrt{3}} \left( 1 + \lambda \cos \delta - \frac{\lambda^2}{2} \cos^2 \delta + \frac{\lambda^3}{2} \left( 2h - \cos \delta + \cos^3 \delta \right) \right),$$

$$|U_{e3}| = \frac{\lambda}{\sqrt{2}} \left( 1 - h \lambda^2 \cos \delta \right),$$

$$U_{\mu 1} = -\frac{1}{\sqrt{6}} \left( 1 + 2\lambda e^{-i\delta} - \frac{\lambda^2}{2} \left( 1 + 2f + 2he^{-i\delta} \right) \right),$$

$$U_{\mu 2} = \frac{1}{\sqrt{3}} \left( 1 - \frac{\lambda^2}{2} \left( 1 - 2f - 2he^{i\delta} \right) \right),$$

$$U_{\mu 3} = \frac{1}{\sqrt{2}} \left( 1 - \frac{\lambda}{2} \left( 1 + 2f + 2he^{i\delta} \right) \right),$$

$$U_{\tau 1} = -\frac{1}{\sqrt{6}} \left( 1 - \lambda \left( f + he^{i\delta} \right) - 2f \lambda^3 e^{-i\delta} \right),$$

$$U_{\tau 2} = \frac{1}{\sqrt{3}} \left( 1 - \frac{\lambda^2}{2} \left( f + he^{i\delta} \right) + f \lambda^3 e^{-i\delta} \right),$$

$$U_{\tau 3} = \frac{1}{\sqrt{2}} \left( 1 + \frac{\lambda^2}{2} \left( f + h \lambda \cos \delta \right) \right).$$

From Eq. (20), the neutrino mixing parameters can be displayed as

$$\sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{13} = |U_{e1}|^2.$$ (23)

From Eq. (22), the solar neutrino mixing angle $\theta_{12}$ can be approximated, up to the order of $\lambda^3$, as

$$\sin^2 \theta_{12} \simeq \frac{1}{3} \left( 1 + 2\lambda \cos \delta + \frac{\lambda^2}{2} + 2h \lambda^3 \right),$$

which indicates, interestingly enough, a tiny deviation from $\sin^2 \theta_{12} = \frac{1}{3}$ when $\cos \delta$ approaches to zero. Since it is the first column of $V_{\ell}^T$ that makes the major contribution to $\sin^2 \theta_{12}$, this explains why we need a phase of order $90^\circ$ for the element $(V_{\ell}^T)_{13}$.\(^7\) Likewise, the atmospheric neutrino mixing angle $\theta_{23}$ comes out as

$$\sin^2 \theta_{23} \simeq \frac{1}{2} \left( 1 - \frac{\lambda^2}{2} (4f + 4h \cos \delta + 1) \right),$$

which shows a very small deviation from the TBM angle $\sin^2 \theta_{23} = 1/2$. The reactor mixing angle $\theta_{13}$ can be obtained by

$$\sin^2 \theta_{13} = \frac{\lambda}{\sqrt{2}} \left( 1 - h \lambda^2 \cos \delta \right).$$

Thus, we have a non-vanishing large $\theta_{13}$.

Leptonic CP violation at low energies could be detected through neutrino oscillations that are sensitive to the Dirac CP phase, but

\(^7\) In [17] we have considered three different scenarios for the matrix $V_{\ell}^T$. We obtained the constraint $0.17 \leq \cos \delta \leq 0.64$ in two of the scenarios in order to satisfy the quark-lepton complementarity (QLC) relations $\theta_{12} + \theta_{23} = \pi/4$ and $\theta_{23} + \theta_{13} = \pi/4$. In this work, we will not impose these QLC relations from the outset.

\[\sin^2 \theta_{12} = 0.307^{+0.018}_{-0.016} \quad \sin^2 \theta_{23} = 0.450, \quad \sin^2 \theta_{13} = 0.159, \quad f_6^CP \simeq -\frac{\lambda}{6}.\] (31)
have found a non-vanishing reactor mixing angle $\theta_{13}$ and giving a relatively large $\sin^2 2\theta_{13} = 0.057$ (best-fit value) corresponding to $\theta_{13} = 8.8^\circ$. On the theoretical ground, we have proposed a simple ansatz for the charged lepton mixing matrix, namely, it has the QM-like parametrization in which the CP-odd phase is approximately maximal. Then we have proceeded to study the deviation of the PMNS matrix from the TBM one arising from the small corrections due to the particular charged lepton mixing matrix we have proposed. We have obtained the analytic results for the mixing angles expanded in powers of $\lambda$: the solar mixing angle $\sin^2 \theta_{12} \simeq \frac{1}{2}(1 + 2\lambda \cos \delta + \lambda^2 \frac{\delta}{\lambda})$, the atmospheric mixing angle $\sin^2 \theta_{23} \simeq \frac{1}{2}(1 + O(\lambda^2))$, the reactor mixing angle $\sin \theta_{13} = \frac{1}{\sqrt{2}}[1 + O(\lambda^2 \cos \delta)]$ and the Dirac CP-odd phase $\delta_{CP} \simeq \delta$. Therefore, while the deviation of solar and atmospheric mixing angles from the TBM values are fairly small, we have found a non-vanishing reactor mixing angle $\theta_{13} \simeq 9.2^\circ$ and a very large Dirac phase $\delta_{CP} \simeq \delta_{QM} \simeq O(90^\circ)$. Furthermore, we have shown that the leptonic CP violation characterized by the Jarlskog invariant is $|J_{CP}| \simeq \lambda/6$, which could be tested in the future experiments such as the upcoming long baseline neutrino oscillation experiments.

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**References**

[1] M.C. Gonzalez-Garcia, M. Maltoni, Phys. Rept. 460 (2008) 1, arXiv:0704.1800 [hep-ph]; T. Schwetz, AIP Conf. Proc. 981 (2008) 8, arXiv:0710.5027 [hep-ph]; G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, Phys. Rev. Lett. 101 (2008) 141801, arXiv:0806.2649 [hep-ph]; G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, arXiv:0809.2936 [hep-ph].

[2] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, arXiv:0905.3549 [hep-ph].

[3] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167, arXiv:hep-ph/0202074;

[4] P.H. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163, arXiv:hep-ph/0203209;

[5] P.F. Harrison, W.G. Scott, Phys. Lett. B 557 (2003) 76, arXiv:hep-ph/0302025; See also L. Wolfenstein, Phys. Rev. D 18 (1978) 958.

[6] Y.H. Ahn, H.Y. Cheng, S. Oh, Phys. Rev. D 83 (2011) 076012, arXiv:1102.0879.

[7] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[8] N. Qin, B.Q. Ma, Phys. Lett. B 695 (2011) 194, arXiv:1011.6412 [hep-ph];

[9] F.P. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163, arXiv:hep-ph/0203209;

[10] P.F. Harrison, W.G. Scott, Phys. Lett. B 557 (2003) 76, arXiv:hep-ph/0302025; See also L. Wolfenstein, Phys. Rev. D 18 (1978) 958.

[11] Y.H. Ahn, H.Y. Cheng, S. Oh, Phys. Rev. D 83 (2011) 076012, arXiv:1102.0879.

[12] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, arXiv:0905.3549 [hep-ph].

[13] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno, arXiv:1005.2294 [hep-ph].

[14] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167, arXiv:hep-ph/0202074;

[15] P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163, arXiv:hep-ph/0203209;

[16] P.F. Harrison, W.G. Scott, Phys. Lett. B 557 (2003) 76, arXiv:hep-ph/0302025; See also L. Wolfenstein, Phys. Rev. D 18 (1978) 958.

[17] Y.H. Ahn, H.Y. Cheng, S. Oh, Phys. Rev. D 83 (2011) 076012, arXiv:1102.0879.