Disentangling Strong Dynamics through Quantum Interferometry

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We present a new probe of strongly coupled electroweak symmetry breaking at the 14 TeV LHC by measuring a phase shift in the event distribution as a function of the decay azimuthal angles in massive gauge boson scattering. A large phase shift arises from an interference effect between the strongly interacting longitudinal modes and the transverse modes of the gauge bosons. We find that even very broad resonances of masses up to 900 GeV can be probed at 3σ significance with a 3000 fb−1 run of the LHC by using this technique. We also present the estimated reach for a future 50 TeV proton-proton collider.

INTRODUCTION

One of the most important goals of the Large Hadron Collider (LHC) is to find the nature of the mechanism of electroweak symmetry breaking (EWSB). The discovery of the 125 GeV Higgs-like object [1, 2] is a major milestone in this direction. However, in order to truly understand EWSB, we would like to learn the origin of the longitudinal components of massive electroweak gauge bosons (V). Broadly speaking, models of EWSB fall into two categories, those where the longitudinal components of the Vs are a) weakly interacting or b) strongly interacting. The most popular examples in the first category are the Standard Model (SM), with one or more elementary Higgs multiplets [3], and its supersymmetric extensions [4]. In the second category, one or more strongly interacting sectors appear at the TeV scale and are responsible for EWSB. The Higgs-like boson and the longitudinal Vs could arise as pseudo-Nambu-Goldstone bosons (PNGBs) in this scenario [5]. A definite distinction between these two cases at the LHC would be very important and serve as one of the first crucial steps towards a full understanding of EWSB mechanism.

One universal consequence of a strongly coupled EWSB sector is an enhancement of the longitudinal gauge boson scattering amplitude at high energies [6]. One strategy to confirm such an enhancement is to use the polar angle distribution of the gauge boson decay products to measure the longitudinal gauge boson polarization fraction [7]. In this paper, we would like to seek strategies to discover other possible universal consequences of strong dynamics.

Pion scattering in QCD provides a realistic example of strongly coupled PNGB scattering. By looking at the ππ scattering data [8], we can see that a large phase shift exists in the form-factor of both ππ → ππ and e+e− → ππ. In strongly coupled EWSB sectors, PNGBs are eaten into massive gauge bosons, and a large phase shift shows up in the scattering of the longitudinal Vs. In this article, we will use the azimuthal angle correlations of the Vs’ decay products to measure this phase shift as a new direct collider signal of EWSB from strong dynamics.

There are two methods to observe VV scattering at the LHC. One is the widely used weak boson fusion process pp → VVjj, where the two forward jets can be used to suppress the large SM backgrounds. The other method is the rescattering process pp → VV, which is not considered to be a promising channel for most studies due to the large SM backgrounds. However, this channel is not suppressed by the small effective-V luminosity and it also has better access to higher energies of the VV system. Since we are trying to observe the azimuthal angle correlation which arises from a quantum interference term, we do not have to suppress the SM backgrounds and we will consider the rescattering process in this paper.

PARAMETRIZATION

There are many different ways to parameterize strong dynamics in weak boson scattering, although some of them only give rise to real amplitudes. We will use a form factor to modify the scattering of pp → VV from its SM value. The entire form factor can be extracted from its phase by using analyticity arguments to define an Omnès function. In this formalism, the importance of the phase shift to the overall form factor is clearly demonstrated. For a given form factor, F(s), applying the subtracted dispersion relation to log(F(s)/s), we have

\[ F(s) = P(s) \frac{1}{\pi} \int_0^\infty ds' \delta(s') \left[ \frac{1}{s' - s - i\epsilon} - \frac{1}{s} \right] , \]

with P(s) = 1. When the phases δ(s') go beyond 2π (for instance, multiple resonances with additional branches), the additional 2πn phase factors can be recast into P(s) as a polynomial factor with P(0) = 1.
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The phase shift $\delta$ in $e^+e^- \to \pi\pi$ scattering versus energy $\sqrt{s}$. The experimental data (black points) are from Ref. [8]. The red solid line is the ansatz in Eq. (2) we have used to fit the phase shift. Using dispersion relations we can relate the phase shift to the magnitude of the form factor $|F(s)|$. For $P(s) = 1$, the magnitude is shown by the blue dashed line.

To see how the phase shift $\delta$ depends on energy, we consider the old data for $e^+e^- \to \pi\pi$ scattering. When there is a resonant structure in this channel, the phase shift of $\pi\pi \to \pi\pi$ will pass through a pole which is located at $2\delta = \pi$. In the case of QCD, this pole is at the mass of the $\rho$ meson. For simplicity, we use an ansatz,

$$\delta = \begin{cases} \text{ArcTan}[s\Gamma/m(m^2 + \Gamma^2 - s)], & s < m^2 \\ \text{ArcTan}[s\Gamma/m(m^2 + \Gamma^2 - s)] + \pi, & s \geq m^2. \end{cases} \quad (2)$$

We can see that this ansatz can fit the QCD data very well, as shown in Fig. [1]. From Eq. (1), we can construct the full form factor from its phase,

$$F(q^2) = P(s) \frac{m^2 - i m \Gamma}{q^2 - m^2 + i m \Gamma} \quad (3)$$

In the figure we can also see the behavior of the absolute value of the form factor. At small $q^2$ we find that $F(q^2)$ falls off as $1/q^2$, this is similar to the behavior of QCD but in a general theory this depends upon the UV completion.

For composite Higgs models, we can use the generalized Adler-Weinberg sum rule (in the limit of vanishing gauge couplings) [9] to relate the Higgs coupling to $V$s to an integral sum of longitudinal gauge boson scattering cross-sections in various isospin channels,

$$1 - a^2 = \int_0^\infty \frac{u^2 ds}{6 \pi s} (2\sigma_{I=0}^{tot}(s) + 3\sigma_{I=1}^{tot}(s) - 5\sigma_{I=2}^{tot}(s)) \quad (4)$$

Here, $a$ parameterizes the scaling of the Higgs boson coupling to $V$s away from its SM value of unity. We will assume vector meson dominance, restricting us to the $I = 1$ channel. In the presence of a Higgs boson with a non-zero value for $a$, the total longitudinal $V$ cross-section must be rescaled by a factor of $1 - a^2$. Thus, we will rescale our form factor from Eq. (3) by $\sqrt{1 - a^2}$. From the latest fits [10] we can see that $a > 0.8$ at the 2σ level. Assuming $a = 0.8$, we show the currently excluded region for different form-factor parameters $m$ and $\Gamma$ in Fig. [2] using the latest LHC searches for $W' \to WZ$ resonances [11].

**ANGULAR CORRELATION**

Let us consider $W^+Z$ production from a $ud$ initial state modified by a form-factor as in Eq. (3). We consider this process at high energies where both the $W^+$ and the $Z$ decay leptonically. The kinematic dependence of the production and decay amplitudes are as follows: $M_1 : u(k_1,-) \to W^+(q_1,\lambda_1) Z(q_2,\lambda_2)$, $M_2 : W^+(q_1,\lambda_1) \to \nu(p_1,-) l_1^+ (p_2,+) \quad (5)$ and $M_3 : Z(q_2,\lambda_2) \to l_2^- (p_3,h) l_3^+ (p_4,-h)$. The parameters in the parentheses are the particle momentum and helicity respectively. The phase space of this process has five independent angles which are defined in the center-of-momentum frame as follows: the production angle ($\Theta$), two polar decay angles ($\theta_1, \theta_2$) and two azimuthal decay angles ($\phi_1, \phi_2$) which can be though of as the rotations of the $W^+/Z$ decay planes ($\tilde{n}_W, \tilde{n}_Z$) about the $W^+/Z$ momentum axis, and are measured relative to the production plane $\hat{n}$. The three planes are defined as $\tilde{n} \sim k_1 \times q_1, \tilde{n}_W \sim q_1 \times p_2$ and $\tilde{n}_Z \sim p_4 \times q_2$. All these kinematic variables are presented in Fig. [3].

The phase shift from the strong dynamics only affects the longitudinal-longitudinal combination of $W^+/Z$ modes and shifts the corresponding amplitude by an energy dependent phase, $\delta$. This phase shift will enter into the azimuthal angle correlation of the $W^+/Z$ decay as an interference effect between the various polarizations of the vector bosons. To see this, recall that $W^+/Z$ decay...
produces an azimuthal angular dependence \( \exp(i s_2 \phi) \) in the amplitude, where \( s_2 \) and \( \phi \) are respectively, the spin projection and the azimuthal angle rotation about the \( W^+/Z \) direction. For a given interference term between a general helicity combination \( (\lambda_1, \lambda_2) \) and the \((0,0)\) combination, we find that the relevant terms in the differential cross-section are of the form \( |a_1 e^{i(\lambda_1 \phi_1 - \lambda_2 \phi_2)} + a_2 e^{i\delta}|^2 \sim \cos(\lambda_1 \phi_1 - \lambda_2 \phi_2 + \delta) \cos(\lambda_1 \phi_1 - \lambda_2 \phi_2) \sin \delta \). Thus, the \( \sin(\lambda_1 \phi_1 - \lambda_2 \phi_2) \) modes in the azimuthal angle correlation strongly suggest the existence of strong dynamics.

The production amplitudes can be separated by the different \( W^+/Z \) helicity combinations \((\lambda_1, \lambda_2)\). Among the nine different helicity combinations, there are three leading contributions from \((-,+),(0,0),\) and \((+,-)\). The four other significant ones are \((-,-) \approx (0,+)\) and \((+0) \approx (0,-)\) all of which have a relative suppression \( \sim m_W/\sqrt{s} \). The \((+-)\) and \((-,-)\) combinations are too small to affect the kinematic distributions. We note that (a): \((-+)/(+-)\) dominate as \( \Theta \to 0 \) or \( \pi \) because of the t-channel production. (b): Numerically the difference between \((-0),(0+)\) or \((+,0)\) and \((0,-)\) is negligible. We parameterize the production amplitudes as \( M_1(-,+)=A, M_1(0,0)=Be^{i\delta}, M_1(+-)=C,M_1(-0)=D,M_1(+,0)=E,M_1(0,+)=F \) and \( M_1(0,-)=G \). Here, all amplitudes depend on the center-of-mass energy and on \( \Theta \). Similar behavior has been pointed out in \( e^+e^-\rightarrow W^+W^- \).

The full differential cross-section can be obtained from the expression,

\[
\sum_{h} \left| \sum_{\lambda_1, \lambda_2} M^1_{h}(\lambda_1, \lambda_2)(\theta_1, \phi_1)M^2_{h}(\theta_2, \phi_2) \right|^2 . \tag{5}
\]

The \( W^+, Z \) decay amplitudes are given by,

\[
M^1_{h}(\theta_1, \phi_1) = g_W |q_1| d_{\lambda_1}(\theta_1)e^{i\lambda_1 \phi_1}, \tag{6}
\]

\[
M^2_{h}(\theta_2, \phi_2) = g_Z^2 |q_2| d_{\lambda_2}(\theta_2 + (h-1)\pi/2)e^{-i\lambda_2 \phi_2} .
\]

Here, \( g_Z^- \approx -g_Z^+ \) are the couplings of the \( Z \) to different lepton helicities. The polar angle dependent function \( d_\pm(\theta) = \sqrt{1/2} \cos(\theta) \), \( d_0(\theta) = \sin(\theta) \). If we integrate over both polar angles \( \theta_1 \) and \( \theta_2 \), there is an approximate cancelation in the \( \sin(\phi_1 + \phi_2) \) and \( \sin(\phi_2) \) correlation between \( \cos(\theta_2) > 0 \) and \( \cos(\theta_2) < 0 \) due to \( g_Z^- \approx -g_Z^+ \). Therefore, we only integrate the differential cross-section over either \( \cos(\theta_2) > 0 \) or \( \cos(\theta_2) < 0 \) to obtain:

\[
\frac{d^2 \sigma_\pm}{d \cos \Theta d \phi_1 d \phi_2} = \frac{1}{2} \left[ \left( H^2 + \frac{3 \epsilon}{4} (C^2 - A^2 + G^2 - F^2) \right) + B \sin \delta \left( \frac{3 \pi}{64} (\pm 4(A + C) - 3 \pi \epsilon(A - C)) \right) + \frac{3 \sqrt{2} \pi (E - D)}{8} \sin \phi_1 + \pm \frac{4(F + G)}{4 \sqrt{2}} \sin \phi_2 + \cdots \right] , \tag{7}
\]

where \( \epsilon = ((g_Z^-)^2 - (g_Z^+)^2))/((g_Z^-)^2 + (g_Z^+)^2) \approx 0.22 \) and the ellipse refers to the interference terms with \( \cos \phi \)-type dependence. The \( \sigma_\pm \) stands for \( \sigma(\cos(\theta_2 \geq 0)) \) and \( H \) is overall background \( H^2 = (A^2 + B^2 + C^2 + D^2 + E^2 + F^2 + G^2) \).

In Fig. 3 we plot the relative coefficients of the different \( \sin(\phi) \) correlations as a function of \( \cos \Theta \) for \( \sqrt{s} = 1 \) TeV using the expressions in Eq. (7). Measuring a non-zero coefficient for any of the \( \sin(\phi) \) modes is a positive indicator of strong dynamics. Integrating over the entire cos \( \Theta \) range would lead to cancellations that would dilute the significance of the probe. Thus, an optimal strategy is to use a maximum likelihood analysis on the measured \( \frac{d \sigma}{d \cos \Theta} \) distribution in the data to look for all non-zero \( \sin(\lambda_1 \phi_1 - \lambda_2 \phi_2) \) coefficients.

**PROCEDURE**

At the LHC, there are various kinematic ambiguities that must be incorporated into the analysis. We will choose to study only the \( \sin(\phi_1) \) mode, which will yield a much more transparent analysis that is robust to the kinematic ambiguities.

We simulate the process \( pp \rightarrow W^+Z \rightarrow l^+\nu l^+l^- \) with the form factor from Eq. (3) multiplying the \( J = 1 \)
channel of the (0, 0) helicity amplitude (We will rescale the form factor by $\sqrt{1-a^2}$ with $a = 0.8$). We scan over different form factor parameters $m$ and $\Gamma$, which will give rise to different phase shifts. We use the HELAS [15] program to simulate events at the parton level which is sufficient for the fully leptonic final state. The cuts used are as follows:

- $\Delta r > 0.4$ separation between leptons.
- $p_T > 20$ GeV and $|\eta| < 2.8$ cuts on the leptons.
- $Z$-reconstruction cut: We require that two opposite sign leptons reconstruct to give the $Z$ mass.
- Missing $E_T$ cut $> 20$ GeV.
- Invariant mass cut of the $W/Z$ system between $m-\Gamma$ and $m+\Gamma$.

When we reconstruct the events, there are two misidentification issues that arise that lead to a fourfold ambiguity in the kinematics:

- The $u$-quark direction is unknown.
- There is a two fold ambiguity in the neutrino momentum along the beam axis.

First, consider a misidentification of the $u$-quark direction. This leads to misidentifying $\Theta \rightarrow \pi - \Theta$, $\phi_1 \rightarrow \pi + \phi_1$, $\phi_2 \rightarrow \pi + \phi_2$. The azimuthal angle correlation, $\sin \phi_1$, is odd under such a misidentification. Note that from the solid green curve in Fig. 4 the coefficient of $\sin \phi_1$ is also approximately odd under $\Theta \rightarrow \pi - \Theta$. Thus, if we study the $\sin \phi_1$ mode for either $0.1 < \cos \Theta < 0.9$ or $-0.9 < \cos \Theta < -0.1$ we find that it is robust to misidentifications of the $u$-quark direction.

The presence of a false solution for the neutrino momentum would distort the azimuthal angle correlations that we seek for $\phi_1$. However, to study the $\sin \phi_1$ mode, we can simply measure the up-down asymmetry with respect to the $\phi_1 = 0$ (production) plane to find the sizes of such correlations. We will demonstrate that the up-down asymmetry is the same for both the true and the false solutions.

For a given $\phi_1$ azimuthal angle correlation we have,

$$\frac{d\sigma}{d\phi_1}_{\cos \Theta \geq 0} \simeq A_0 + A_1 \cos \phi_1 + A_2 \cos 2\phi_1 \pm B_1 \sin \phi_1 .$$

We define the events going “above” the plane for $\sin \phi_1 > 0$ and going “below” the plane for $\sin \phi_1 < 0$. Therefore, the up-down asymmetry can be defined as

$$AS|_{\cos \Theta \geq 0} = \frac{N_+ - N_-}{N_+ + N_-} = \pm \frac{2B_1}{\pi A_0} ,$$

where $N_+/N_-$ are the number of up/down events respectively.

The up or down events for $\phi_1$ can be defined by the sign of the scalar triple product

$$SGN \equiv \text{sgn} (\hat{n} \cdot p_2) = \text{sgn} ((k_1 \times q_1).p_2) .$$

For a particular event if $SGN > 0(0)$ then we increment $N^+(N^-)$. The normal vector to the production plane $\hat{n} = k_1 \times q_1 = k_1 \times (p_1 + p_2)$ is independent of the $\nu$ momentum along the $u$-quark direction and hence $SGN$ is insensitive to the difference between the true and false solution. In addition to this, the asymmetry variable has the advantage of being insensitive to a number of cuts such as rapidity and $p_T$ cuts that would otherwise distort the angular distribution.

**RESULTS**

If the background fluctuation is Gaussian, the statistical significance of the nonzero asymmetry is given by,

$$S \equiv \frac{|N^+ - N^-|}{\sqrt{N}} = |AS|\sqrt{N} .$$

where $N = N^+ + N^-$ is the total number of events. As a rule of thumb, we found that choosing an invariant mass window between $m \pm \Gamma$ seemed to optimize the tradeoff between picking up a large $|AS|$ by being close to the resonance, while still keeping a sizeable number of events.

In Tab. 4 we show the significance of the asymmetry measurement at the 14 TeV LHC with 3000 fb$^{-1}$ of data for different choices of form factors by varying over the parameters $m$ and $\Gamma$. We show the results using expected statistics including both the $W^+Z$ and $W^-Z$ fully-leptonic modes. We find that new wide resonances can be probed at the 3$\sigma$ level for masses up to more
from results show that a simple up-down asymmetry in leptons correlations in massive gauge boson scattering. Our reassuring a phase shift in the the decay azimuthal angle dynamics of a strongly coupled EWSB sector by broad resonances from strong dynamics. This strongly event reconstruction ambiguities and is a good probe of resonances from strong dynamics. An even higher energy at a future collider would make the sin(φ1 + φ2) interference term the dominant piece and would require a different analysis strategy.

There are several theory and analysis issues that would require a different analysis strategy. Multiple resonances in a narrow mass window could also yield an enhancement in the longitudinal scattering cross-section which would show up as the P(s) factor mentioned earlier.

CONCLUSIONS

We have proposed a novel technique to disentangle the dynamics of a strongly coupled EWSB sector by measuring a phase shift in the the decay azimuthal angle correlations in massive gauge boson scattering. Our results show that a simple up-down asymmetry in leptons from W decay in pp → W±Z is robust to a number of event reconstruction ambiguities and is a good probe of broad resonances from strong dynamics. This strongly motivates a high luminosity run of the 14 TeV LHC. A future 50 TeV pp collider could yield conclusive evidence of resonant behavior in the presence of an excess of WZ events at the LHC. Furthermore, we have outlined several analysis strategies and theoretical issues which would significantly increase the reach of searches based upon this technique and could lead to a promising signal at the next run of the LHC.

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| 14 TeV LHC | 50 TeV pp collider |
|------------|-------------------|
| Γ/m → 0.3 | Γ/m → 0.2 |
| m (GeV)    | m (GeV)          |
| 800        | 1000             |
| 900        | 1100             |
| 1000       | 1200             |

TABLE I: Table showing the significance of the asymmetry variable, from Eq. (11), using both the W±Z fully-leptonic modes for different form-factor parameters in a composite Higgs model (assuming a = 0.8). The results are shown for an integrated luminosity of 3000 fb⁻¹ at the 14 TeV LHC and at a future 50 TeV pp collider.

than 900 GeV. This further motivates an extended run of the LHC should an excess in W±Z be discovered. A future 50 TeV proton-proton collider could probe resonances up to around 1.2 TeV with the same luminosity. An even higher energy at a future collider would make the sin(φ1 + φ2) interference term the dominant piece and would require a different analysis strategy.

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