Study on the Resonant Torsion Vibration in Hourglass Specimens under VHCF Loading

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Abstract. The problem of cyclic loading under pure torsion is an important problem for numerous different industrial applications, such as shafts, springs, blades, and others. The loading conditions for aircraft, railway, automobile industries are often corresponding to long fatigue life. The rough estimation shows that the number of cycles for springs in car engine is exceeds $10^8$ cycle while blades for aircraft turbine may undergo more than $10^{10}$ loading cycles. Such, to provide a safety in service life for such elements a multiaxial fatigue failure criterion is needed to predict damage zone location and fatigue life. This paper is aimed to introduce the multiregime approach to predict fatigue damage zone evaluation and number of cycles to crack initiation and total fatigue life. The presented approach is based on damage theory supplemented by multiaxial failure criterion. The proposed model includes the algorithm for automatic determination the fatigue loading range (low-, high- or very-high cycle fatigue) and allows different crack opening modes. To show how to apply the proposed approach and verify the results of calculation the series of numerical simulation on hourglass specimen subjected to VHCF pure torsion is performed. The results of numerical simulation are compared to VHCF torsion tests.

1. Introduction

The problem of cyclic loading under pure torsion is widespread in different modern industries, such as automobile, railway, civil aircraft, energy productions, and others. All these industries are aimed to reach a long in service life for elements. The recent progress in material science and production technology lead to increasing the material characteristics. High strength of a new materials allows engineers to reduce the mass and cross-sections of some elements that lead to higher requirements for safety design. Nonetheless, the fatigue design of the most structural elements in based on results of low- and high cycle fatigue that are limited by $5 \times 10^4$ and $10^7$ cycles, respectively. Simple calculations shows that such laboratory tested durability is not matches with real in-service loading conditions. In the case of car industry, the typical number of turns per minute is about 2000 – 3000 times. The average velocity of the car in the city is about 40 – 50 km/h due to traffic jumps and speed limitations. Most of the modern citizen use a car about 3 – 5 years before sold with a typical mileage 100 000 – 150 000 km. The corresponding number of loading cycles is varying from 3 to $5 \times 10^8$ cycles that is in the range of very-high cycle fatigue. In the case of aircraft industry, the typical loading frequency can reach 4500 Hz [1] and corresponding number of cycles up to $10^{10}$. This estimated durability is far beyond the low- and high cycle fatigue range. The investigation on the very-high cycle fatigue is relatively new domain of material since with many currently developing subjects are linked to the experimental works. The scientists are mainly focused on the phenomena investigation, developing a new loading schemes and machines, deep investigation on the physics of VHCF crack initiation and early growth. However, there are only few results on fatigue life prediction in the VHCF range [2]. The difficulties of the VHCF behavior modeling are determining by high influence of the material microstructure on its fatigue properties at high number of cycles. Since the majority of scientist are working on phenomenological VHCF model it is assumed that a VHCF criterion should be a physically based one and contains some structural parameters. Nonetheless, it is important to develop an engineering criterion. The main idea
for such type of criterion is generalization of popular multiaxial criterion to the range of VHCF. Some pioneer works [2] in this direction have shown an appropriate result [3]. Such approach needs some axial experimental data under different stress ratios to determine the parameters of the model. The model should be verified by prediction the result of VHCF tests under complex, multiaxial loading such as torsion. Therefore, developing of a new multiaxial loading schemes is an important subject in the framework of an engineering criterion development.

The torsion VHCF tests are mainly developed in two laboratories: it is French school represented by C.Bathias [4], T.Palin-Luc [5] and Austrian school represented by H.Mayer [6] and S.Stanzl-Tschegg [7]. Also, there are two different schemes of torsion vibration excitation: (1) transformation of axial displacement into torsion vibration; (2) direct torsion loading. The first technology was developed by Swiss company and engaged by C.Bathias. The second technology is based on progress in piezo-electric industry and developed almost sinuously by H.Mayer and C.Bathias. Nowadays the direct torsion excitation is mainly used by all researchers. The result of torsion VHCF tests shows both surface and subsurface crack initiation. The first torsion tests were performed mainly on structural steels and internal crack initiation was associated with elongated nonmetallic inclusions, figure 1.

![Figure 1. Internal crack initiation under pure VHCF torsion loading [7]](image)

The torsion VHCF crack is developing in the plane of maximal normal stress that is inclined about 45 degree by the specimen axis. However, the crack initiation site is not always being in the same plane. The results of experimental investigation have shown [8] that crack initiation and early growth is being in the plane of maximum shear stress orientated by 0 or 90 degree by the specimen axis. Therefore, there is a transition from crack initiation by the shear mechanisms to crack propagation by the normal crack opening mode. This is not only one bifurcation in the crack growth scenario. It is also observed that fatigue regime can also changes during the crack propagation. Due to crack length increments the local stress is increasing that may influence the change in fatigue regime. One’s crack initiated under VHCF loading the following crack growth could be conditioned by HCF. Therefore, the complex model should consider such feature of progressive fatigue damage.

2. Multi-regime model of fatigue damage accumulation

Taking into account the mention above features of fatigue crack initiation and growth the multi-regime model is proposed. The model combines the modern approach of SN-curve, special algorithm of SN-curve branch determination, kinetic equation for damage function evolution and multi-axial criterion as a closing equation. The scheme of the full S-N curve within multi-regime approach is presented on figure 2. There are three different ranges of fatigue behavior corresponding to LCF, HCF and VHCF. Each region is separated by the bifurcation area where both mechanisms of crack initiation/growth can be realized. The chose of the mechanisms is mainly describes by probability of certain mechanism. In order to realize the numerical procedure for calculation an average value was introduced where the probability is 50% by 50 %. Each branch of S-N curve is describing by similar low with its individual parameters for each branch.
2.1. Kinetic equation for damage in LCF-HCF mode

Various criteria use different stress combinations to calculate an equivalent stress value. Some of them based on normal stress components of a stress state while other based on shear components. In this paper we are going to implement two criteria simultaneously: one is based on a normal opening micro-cracks which is the stress-based Smith–Watson–Topper [9,10], the other one is based on a shear micro-cracks and implements the notion of a critical plane which is the stress-based Carpinteri–Spagnoli–Vantadori [11]. The considered model develops the damage model in case of cyclic loads, presented in [3] for the description of damages during dynamic loading.

The generic fatigue fraction criterion corresponding to the left branch of the bimodal fatigue curve in the following has the form:

$$\sigma_{eq} = \sigma_p + \sigma_f N^{-\beta_L}$$  \hspace{1cm} (1)

From the condition of repeated-static fracture up to values of $N \sim 10^3$ by the method [12] it is possible to obtain the value $\sigma_{eq} = 10^{\gamma \beta_L} (\sigma_p - \sigma_u)$ . In these formulas $\sigma_p$ is the static tensile strength of the material, $\sigma_u$ is the classic fatigue limit of the material during a reverse cycle (asymmetry coefficient of the cycle $R = -1$), $\beta_L$ is power index of the left branch of the bimodal fatigue curve.

In order to describe the process of fatigue damage development in the LCF-HCF mode, a damage function $0 \leq \psi(N) \leq 1$ is introduced, which describes the process of gradual cyclic material failure. When $\psi = 1$ a material particle is considered completely destroyed. Its Lame modules become equal to zero. The damage function $\psi$ as a function on the number of loading cycles for the LCF-HCF mode is described by the kinetic equation:

$$\frac{\partial \psi}{\partial N} = B_L \psi^\alpha / (1 - \psi^\beta)$$  \hspace{1cm} (2)

where $\alpha$ and $0 < \gamma < 1$ are the model parameters that determine the rate of fatigue damage development. The choice of the denominator in this two-parameter equation, which sets the infinitely large growth rate of the zone of complete failure at $\psi \rightarrow 1$, is determined by the known experimental data on the kinetic growth curves of fatigue cracks, which have a vertical asymptote and reflects the fact of their explosive, uncontrolled growth at the last stage of macro fracture.
An equation for damage of a similar type was considered in [13], its numerous parameters and coefficients were determined indirectly from the results of uniaxial fatigue tests. In our case, the coefficient $B_L$ is determined by the procedure that is clearly associated with the selected criterion for multiaxial fatigue failure of one type or another. It has the following form.

The expression for the coefficient $B_L$ has a form [4]:

$$B_L = 10^{-3} \left( (\sigma_{eq} - \sigma_0) / (\sigma_y - \sigma_0) \right)^{1/\beta} \frac{\alpha}{(1 + \alpha - \gamma) / (1 - \gamma)}$$

where the value $\sigma_{eq}$ is determined by the selected mechanism of fatigue failure and the corresponding multiaxial criterion.

At each node there are not one but two $B_L$ values, namely $B_L^1$ and $B_L^2$. They have the forms:

$$B_L^1 = 10^{-3} \left( (\sigma^n - \sigma_0) / (\sigma_y - \sigma_0) \right)^{1/\beta} \frac{\alpha}{(1 + \alpha - \gamma) / (1 - \gamma)}$$

$$B_L^2 = 10^{-3} \left( (\sigma^\tau - \sigma_0) / (\sigma_y - \sigma_0) \right)^{1/\beta} \frac{\alpha}{(1 + \alpha - \gamma) / (1 - \gamma)}$$

It means there are 2 damage values $\psi^r = f(B_L^1)$ and $\psi^\tau = f(B_L^2)$. We will assume that the choice is determined by the mechanisms of microcrack development and fatigue fracture criteria SWT and CSV. For microcracks of normal opening $\sigma_{eq} = \sigma^r$, for shear $\sigma_{eq} = \sigma^\tau$. The resulting formulas for the coefficients of the kinetic equation for damage operate in the range $\sigma_y < \sigma_{eq} < \sigma_y$.

### 2.2. Kinetic equation for damage in VHCF mode

The criterion for multiaxial fatigue failure in the VHCF mode corresponding to the right branch of the bimodal fatigue curve has the form:

$$\sigma_{eq} = \sigma_y + \sigma_v N^{\beta_v}$$

We will assume that the choice $\sigma_{eq}$ is determined by the same mechanisms of microcrack development and fatigue fracture criteria SWT and CSV as in the HCF mode, $\sigma_{eq} = \sigma^r$ or $\sigma_{eq} = \sigma^\tau$.

From the condition of similarity of the reference points for the left and right branches of the bimodal fatigue curve [14], one can obtain the formula $\sigma_v = 10^{\beta_v} (\sigma_y - \sigma_y)$. Here $\sigma_y$ is the fatigue limit of the material in the reverse cycle for the VHCF mode, $\beta_v$ is the power exponent of the right branch of the bimodal fatigue curve.

For the VHCF mode, it is possible to determine the coefficient in the evolutionary equation for damage:

$$d\psi / dN = B_v \psi^r / (1 - \psi^r), \quad 0 < \alpha, \gamma < 1$$

As in the previous section, we can obtain expressions for the coefficients of the kinetic equation of damage in the VHCF mode:

$$B_v^1 = 10^{-3} \left( (\sigma^r - \sigma_y) / (\sigma_y - \sigma_y) \right)^{1/\beta_v} \frac{\alpha}{(1 + \alpha - \gamma) / (1 - \gamma)}$$

$$B_v^2 = 10^{-3} \left( (\sigma^\tau - \sigma_y) / (\sigma_y - \sigma_y) \right)^{1/\beta_v} \frac{\alpha}{(1 + \alpha - \gamma) / (1 - \gamma)}$$

The resulting formulas for the coefficients of the kinetic equation for damage operate in the range $\sigma_y < \sigma_{eq} < \sigma_y$. 
2.3. Multiaxial criterion for multi-regime model
The presented model can be used with different multiaxial criteria depending on material’s fatigue behavior. It is well known that some materials can carry an important tensile load but has a low resistance again shear. For others material the damage evolution is mainly realize due to normal stress or certain combination. Such, the model should be capable to describe such macroscopic features of the materials. We propose to use the multiaxial criterion as ‘close equation’ for damage function. Moreover, sometimes different crack opening mechanisms can be in competition as it is for torsion loading. Here we present the way how to model such complex behavior. We introduce the example when normal and shear crack opening are in competition.

Smith–Watson–Topper criterion
The criterion of multiaxial fatigue failure in the LCF-HCF mode with the development of normal-stress micro-cracks (stress-based SWT) corresponding to the left branch of the bimodal fatigue curve has the form:

\[
\sqrt{\left(\frac{\sigma}{\sigma_{\text{um}}}ight)\Delta \sigma / 2} = \sigma_u + \sigma_1 N^{-\beta_c}
\]

(10)

where \(\sigma\) is the largest principal stress, \(\Delta \sigma\) is the spread of the largest principal stress per cycle, \(\Delta \sigma / 2\) is its amplitude. According to the chosen criterion only tensile stresses lead to failure, so it has the value

\[
\left\langle \sigma_{\text{um}} \right\rangle = \sigma_{\text{um}} H (\sigma_{\text{um}}).
\]

Let us put down the following designation:

\[
\sigma^n = \sqrt{\left(\frac{\sigma}{\sigma_{\text{um}}}ight)\Delta \sigma / 2}
\]

(11)

Here the upper index \(n\) stands for denotation and should not be considered as a power.

Carpinteri–Spagnoli–Vantadori criterion
The criterion of multiaxial fatigue failure in the LCF-HCF mode, including the concept of a critical plane (stress-based CSV), corresponding to the left branch of the bimodal fatigue curve has the form:

\[
\sqrt{\left(\frac{\Delta \sigma_n}{2}\right)^2 + k_3^2 \left(\frac{\Delta \tau_u}{2}\right)^2} = \sigma_u + \sigma_1 N^{-\beta_c}
\]

(12)

where \(\Delta \tau_u / 2\) is the amplitude of the tangential stress on the plane (critical), where it reaches its maximum value, \(\Delta \sigma_n / 2\) is the amplitude of the normal (tensile) stress on the critical plane, \(\left\langle \Delta \sigma_n \right\rangle = \Delta \sigma_n H (\sigma_{\text{um}})\). Here, the shear fatigue limit \(\tau_u\) for a pulsating cycle is additionally introduced at a cycle asymmetry coefficient of \(R = -1\). In a simplified formulation, we can approximately accept \(k_3 \approx \sigma_u / \tau_u\) and \(k_3 \approx \sqrt{3}\). This criterion includes the mechanism of fatigue fracture with the formation of shear micro-cracks.

Let us put down the following designation:

\[
\sigma^3 = \sqrt{\left(\frac{\Delta \sigma_n}{2}\right)^2 + 3 \left(\frac{\Delta \tau_u}{2}\right)^2}
\]

(13)

Here the upper index \(\tau\) stands for denotation and should not be considered as a power.

3. Experimental methods and results
The VHCF tests were performed by using a direct piezoelectric fatigue torsion machine \([5]\). The loading frequency is about 20 kHz. All the tests were performed in laboratory environment at room temperature. The tests were design to reach fatigue life beyond \(10^7\) cycles. The specimen geometry is hourglass with smooth gage section. The material for torsion tests is titanium alloy with following characteristics:
Young’s modulus is 115 GPa, density 4500 kg/m\(^3\), Poisson ratio is 0.3, yield stress 980 MPa, fatigue limit 440 MPa, VHCF fatigue limit is 385 MPa. The results show a permanent decreasing of cyclic strength with number of cycles. The main objective of this investigation within the present paper is to
study the crack path. The example of crack path developed at the lateral surface of the specimen is presented on figure 3.

![Figure 3](image)

Figure 3. The crack path at lateral surface of hourglass specimen under pure VHCF torsion loading

The analysis of the result has shown that at there is a change in crack propagation direction. At the early stage, the crack developed in the plane of maximum shear stress that is orientated by 0 or 90 degree by the specimen axis. When the crack reaches a certain length, the further developing is being in the plane or planes of maximal normal stresses. The same scenario was observed as for specimens with surface crack initiation, as well with subsurface. This branching is also observed in the bulk of material. The fracture surface contains some wing-like structures inclined by 45 degree by the main crack pattern. Therefore, there is a clear competition between two crack opening mechanisms.

4. Results of numerical simulation

The results of damage zone development in hourglass specimen under pure VHCF loading are presented on figure 4. The macroscopic vie shows a typical for torsion loading a zigzag crack path, figure 4-a. The details of this crack are presented on figure 4-b. The special algorithm allows to count the number of elements degraded by a given mechanisms. It was found that at the initial stage of damage development the elements were mostly degraded due to shear mechanisms. This is corresponding to vertical crack along the specimen axis. Later, the dominant mechanism for mechanism degradation is normal crack opening. This period is corresponding to inclined part of crack path in the plane of maximum normal stress.

Comparison these results of numerical simulations with experimental results, figure 4-c shows a qualitative coincidence of crack paths. It is good result since there is not any numerical switches inside the algorithm. Therefore, the model shows nice agreement with experimental results under multiaxial loading, allow to estimate the number of cycles for crack initiation and duration of crack growth, predict the shape of fatigue damage zone at different stages. This is powerful instrument to investigate the structural integrity of engineering components subjected to cyclic loading.
5. Conclusion

The paper introduces the multi-mode model of fatigue damage accumulation based on damage theory and multiaxial criteria. The proposed approach does not need to introduce an initial crack before the numerical simulation. The model can be combined with different multiaxial criteria with different driving mechanisms of failure such as normal crack opening, shear cracks. This makes the proposed approach suitable for numerous different materials even with clear anisotropy. The model can simultaneously consider several mechanisms of damage accumulation (tension, shear) and indicate the principal mechanism of fatigue failure. The model can be used to describe the degradation of the material under arbitrary stress state.

The model was applied to predict the crack shape under VHCF pure loading in the smooth hourglass specimens. The results of calculation show good agreement with experimental results. The model capable to predict the crack initiation under share condition and sporadic transition to the normal crack opening growth. Therefore, the proposed approach is powerful instrument to investigate the structural integrity of complex engineering elements.

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