On total $H$-irregularity strength of diamond ladder, three circular ladder, and prism graphs

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Abstract. Let $G$ be a graph with vertex set $V$ and edge set $E$. A total labeling $\varphi : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., \alpha\}$ is called a total $\alpha$-labeling of a graph $G$. For the subgraph $H \subseteq G$ under the total $\alpha$-labeling, $H$-weight is defined as $wt_{\varphi}(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e)$. A total $\alpha$-labeling is called an $H$-irregular total $\alpha$-labeling of the graph $G$ if $wt_{\varphi}(H') \neq wt_{\varphi}(H'')$ for any two distinct subgraphs $H'$ and $H''$ isomorphic to $H$. The minimum $\alpha$ for which the graph $G$ has a total $H$-irregular $\alpha$-labeling is called the total $H$-irregularity strength of $G$, denoted by $tHs(G)$. In this paper we initiate to study the total $H$-irregularity strength of $G$ and we have obtained the $tHs$ of diamond ladder, three circular ladder and prism graphs.

1. Introduction

We use a simple, connected, and finite graph, especially planar graph in this research. $G$ is a graph which has the vertex set is given as $V(G)$ and the edge set is given as $E(G)$. Graph labeling is mapping graph elements to positive or non-negative integers number. The most common choices of domain are the set of all vertices (vertex labellings), the only edge set (edge labellings), or the set of either vertices or edges (total labellings). Other domains are possible [5]. The graph $G$ contains $H$ includes each $H_j$ isomorphic subgraph which conditions each $E(G)$ edge included in every one of the $H_j$ subgraphs, $j = 1, 2, \ldots, s$ [4].

The total irregular vertex of $\alpha$-labeling on the graph $G$ is the assignment of the $1, 2, \ldots, \alpha$ for vertices and edges such that the weights calculated at the different vertices. The vertex weight $v \in V$ in $G$ is defined as the sum of label $v$ and labels all incident edges with $v$, that is $wt(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv)$ [9]. The vertex-irregularity strength of $G$ is the smallest $\alpha$ integer on the $H$-irregular label of $G$ and denoted by $vhs(G, H)$ [3]. Indriati et al. [7] obtain a for the total vertex irregularity strength of generalized helm graphs and prisms with outer pendant edges.

The irregular total edge $\alpha$-labeling of a graph $G = (V, E)$ is labeling $\phi : V \cup E = 1, 2, \ldots, \alpha$ so that the total edge weight of $wt(xy) = \phi(x) + \phi(xy) + \phi(y)$ is different for all different edge pairs. The minimum $\alpha$ where graph $G$ has an irregular total edge $\alpha$-labeling is called the total irregular edge strength of $G$ and denoted by $vhs(G, H)$ [2]. Baća and Siddiqui [6] investigate the total edge irregularity strength of generalized prism.
Ashraf et al. in [3] introduce total H-irregularity strength as a natural extension of the \( \text{tes}(G) \) and \( \text{tvs}(G) \) parameters. \( G \) is a graph that recognizes \( H \)-covering. For subgraph \( H \subseteq G \) under total \( \alpha \)-labeling \( \varphi \) associated with \( H \)-weight is defined as

\[
\text{wt}_\varphi(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e).
\]

The total \( \alpha \)-labeling is called \( H \)-irregular total \( \alpha \)-labeling of the graph \( G \) if \( \text{wt}_\varphi(H') \neq \text{wt}_\varphi(H'') \) for every two different subgraphs of \( H' \) and \( H'' \) isomorphic to \( H \).

The smallest integer \( \alpha \) for which an \( H \)-irregular total \( \alpha \)-labeling of exists is known as the total \( H \)-irregularity strength of \( G \) and denoted by \( tHs(G,H) \).

**Theorem 1.** [4] Let \( G \) be a graph that recognizes \( H \)-covering provided by the \( t \) isomorphic subgraph to \( H \). Then

\[
tHs(G,H) \geq \left[ 1 + \frac{t-1}{|V(H)|+|E(H)|} \right].
\]

Agustin et al [1] have conducted research and obtained results from \( tHs(G,H) \) of shackle and amalgamation graphs. Nisviasari [8] have conducted research and obtained results from \( tHs(G,H) \) of triangular ladder and grid graphs. Ashraf et al. [3] have conducted research and obtained results from \( tHs(G,H) \) of ladder and fan graphs. We use diamond ladder, three circular ladders, and prism graph to get \( tHs(G,H) \).

## 2. Results

In this paper, we provide the results of \( tHs(G,H) \) of diamond ladder, three circular ladders, and prism graphs, is as follows.

**Theorem 2.** Let \( Dl_m \) be a diamond ladder graph and subgraph \( H_1 \equiv Dl_n \). The total \( H_1 \)-irregularity strength of \( Dl_m \) graph for \( 2 \leq n < m \) is

\[
\left[ \frac{m+11n-3}{12n-3} \right].
\]

**Proof.** Let \( Dl_m \), \( m \geq 0 \), be a diamond ladder graph with the vertex set \( V(Dl_m) = \{x_j, y_j : j = 1, 2, 3, \ldots, m\} \cup \{z_j : j = 1, 2, 3, \ldots, 2m\} \) and the edge set \( E(Dl_m) = \{x_jy_j, x_jz_{2j-1}, x_jz_{2j}, y_jz_{2j-1}, y_jz_{2j} : j = 1, 2, 3, \ldots, m\} \cup \{x_ix_{i+1}, y_iy_{i+1} : j = 1, 2, 3, \ldots, m-1\} \cup \{z_{2j-2}z_{2j-1} : j = 2, 3, \ldots, m\} \). The diamond ladder graph \( Dl_m, m \) is positive integer, admits a \( Dl_n \) covering with exactly \((m - n + 1)\) diamond ladder \( Dl_n \), where \( n \) is a positive integer and \( 2 \leq n < m \). Based on Theorem 2, we have \( tHs((Dl_m), Dl_n) \geq \left[ \frac{m+11n-3}{12n-3} \right] \).

Put \( l = \left[ \frac{m+11n-3}{12n-3} \right] \). The following function of \( Dl_n \)-irregular total \( \alpha \)-labeling \( \varphi_n : V(Dl_m) \cup E(Dl_m) \rightarrow \{1, 2, \ldots, l\}, n = 2, 3, \ldots, m \) is prove that \( \alpha \) as an upper bound for the total \( Dl_n \)-irregularity strength of \( Dl_m \).

\[
\varphi_n(y_j) = \begin{cases} 
\left[ \frac{j + 20n - 6}{24n - 6} \right], & \text{for } j \text{ is even} \\
\left[ \frac{j + 16n - 5}{24n - 6} \right], & \text{for } j \text{ is odd}
\end{cases}
\]
\[ \varphi_n(x_j) = \left\lceil \frac{j + 11n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m], \quad \varphi_n(z_j) = \left\lceil \frac{j + 9n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m] \]
\[ \varphi_n(x_jx_{j+1}) = \left\lceil \frac{j + 2n - 2}{12n - 3} \right\rceil, \text{ for } j \in [1, m - 1], \quad \varphi_n(x_jy_j) = \left\lceil \frac{j + 3n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m] \]
\[ \varphi_n(z_jz_{j+1}) = \left\lceil \frac{j}{12n - 3} \right\rceil, \text{ for } j \in [1, m - 1], \quad \varphi_n(x_jy_{2j-1}) = \left\lceil \frac{j + 4n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m] \]
\[ \varphi_n(x_jy_{2j}) = \left\lceil \frac{j + 7n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m], \quad \varphi_n(y_{2j-1}z_j) = \left\lceil \frac{j + 5n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m] \]
\[ \varphi_n(y_{2j}z_{j}) = \left\lceil \frac{j + 6n - 3}{12n - 3} \right\rceil, \text{ for } j \in [1, m], \quad \varphi_n(y_{j}y_{j+1}) = \left\lceil \frac{j + 2n - 2}{24n - 6} \right\rceil, \text{ for } j \text{ is even.} \]

We get the upper bound from the function of \( Dl_{ir} \)-irregular total \( Dl_n \)-labeling. We take the largest label from \( \varphi_n(x_j) = \left\lceil \frac{j + 11n - 3}{12n - 3} \right\rceil \) for \( j = m \) \( \varphi_n(x_j) = \left\lceil \frac{m + 1n - 3}{12n - 3} \right\rceil \). We get to present the upper bound of the graph in the Theorem 2, \( tHs((Dl_m), Dl_n) \leq \left\lceil \frac{m + 11n - 3}{12n - 3} \right\rceil \).

Based on the labeling above, we can show the all weights are different by the following equation:

\[
\text{wt}_{\varphi_n}(Dl_{i+1}^j) - \text{wt}_{\varphi_n}(Dl_{i}^j) = \varphi_n(x_{j+n}) + \varphi_n(y_{2j+2n-1}) + \varphi_n(y_{2j+2n}) + \varphi_n(z_{j+n}) + \varphi_n(x_{j+n-1}x_j) + \varphi_n(y_{2j}y_{2j+1}) + \varphi_n(z_jz_{j+1}) + \varphi_n(x_jz_j) + \varphi_n(x_jy_{2j-1}) + \varphi_n(x_jy_{2j}) + \varphi_n(y_{2j-1}z_j) + \varphi_n(y_jy_{2j-1}z_j) + \varphi_n(z_jy_{2j}) - \varphi_n(x_j) - \varphi_n(y_{2j-1}) - \varphi_n(y_{2j}) - \varphi_n(z_j) - \varphi_n(x_jx_{j+1}) - \varphi_n(y_{2j}y_{2j+1}) - \varphi_n(z_jz_{j+1}) - \varphi_n(x_jz_j) - \varphi_n(x_jy_{2j-1}) - \varphi_n(x_jy_{2j}) - \varphi_n(z_jy_{2j}) - \varphi_n(z_jy_{2j-1}) - \varphi_n(z_jy_{2j}) = 1
\]

We respect to \( \text{wt}_{\varphi_n}(Dl_{i}^j) < \text{wt}_{\varphi_n}(Dl_{i+1}^j) \), \( j = 1, 2, \ldots, m - n \) then \( \text{wt}_{\varphi_n}(Dl_{i}^{j+1}) = \text{wt}_{\varphi_n}(Dl_{i}^j) = 1 \). The all \( H_1 \)-weights are distinct. This matter concludes that \( tHs((Dl_m), Dl_n) = \left\lceil \frac{m + 11n - 3}{12n - 3} \right\rceil \). The example of total \( Dl_n \)-irregularity of diamond ladder graph labeling, we can see on Figure 1, and we get \( tHs((Dl_8), Dl_2) = 2 \).
**Figure 1.** The Example of Total $Dl_2$-Irregularity of $Dl_8$ labeling

**Theorem 3.** Let $TCl_m$ be a three circular ladder graph and subgraph $H_2 \equiv C_3$. The total $H_2$-irregularity strength of $TCl_m$ graph is $\left\lceil \frac{3m + 5}{6} \right\rceil$.

**Proof.** Let $TCl_m$, $m \geq 0$, be a three circular ladder graph with the vertex set $V(TCl_m) = \{x_j, z_j : j = 1, 2, 3, \ldots, m + 1\} \cup \{y_j : j = 1, 2, 3, \ldots, m\}$ and the edge set $E(TCl_m) = \{x_jy_j, x_{j+1}y_j, y_{j}z_{j}, z_{j}z_{j+1} : j = 1, 2, 3, \ldots, m\} \cup \{x_{j}z_{j} : j = 1, 2, 3, \ldots, m + 1\} \cup \{y_{j}z_{j+1} : j = 1, 2, 3, \ldots, m - 1\}$. The three circular ladder graph $TCl_m$, $m$ is positive integer, admits a $C_3$-covering with exactly $m$ cycles $C_3$. Based on Theorem 3, we have $tHs((TCl_m), C_3) \geq \left\lceil \frac{3m + 5}{6} \right\rceil$. Put $l = \left\lceil \frac{3m + 5}{6} \right\rceil$. The following function of $C_3$-irregular total $\alpha$-labeling $\varphi_3 : V(TCl_m) \cup E(TCl_m) \rightarrow \{1, 2, \ldots, l\}$ is prove that $\alpha$ as an upper bound for the total $C_3$-irregularity strength of $TCl_m$.

\[
\varphi_3(x_i) = \left\lceil \frac{j + 1}{2} \right\rceil, \text{ for } j \in [1, m + 1], \quad \varphi_3(y_i) = \left\lceil \frac{j + 1}{2} \right\rceil, \text{ for } j \in [1, m] \\
\varphi_3(z_j) = \left\lceil \frac{j}{2} \right\rceil, \text{ for } j \in [1, m + 1], \quad \varphi_3(x_jy_j) = \left\lceil \frac{j}{2} \right\rceil, \text{ for } j \in [1, m] \\
\varphi_3(x_{j+1}y_j) = \left\lceil \frac{j}{2} \right\rceil, \text{ for } j \in [1, m], \quad \varphi_3(y_jz_j) = \left\lceil \frac{j}{2} \right\rceil, \text{ for } j \in [1, m] \\
\varphi_3(y_jz_{j+1}) = \left\lceil \frac{j + 1}{2} \right\rceil, \text{ for } j \in [1, m - 1], \quad \varphi_3(x_jz_j) = \left\lceil \frac{j}{2} \right\rceil, \text{ for } j \in [1, m + 1] \\
\varphi_3(z_jz_{j+1}) = \left\lceil \frac{j}{2} \right\rceil, \text{ for } j \in [1, m].
\]

We get the upper bound from the function of $C_3$-irregular total $TCl_m$-labelling. We take the largest label from $\varphi_3(x_i) = \left\lceil \frac{j + 1}{2} \right\rceil$ for $j = m + 1 \varphi_3(x_i) = \left\lceil \frac{m + 2}{2} \right\rceil$. We get to present the upper bound of the graph in the Theorem 3, $tHs((TCl_m), C_3) \leq \left\lceil \frac{3m + 5}{6} \right\rceil$. 


Based on the labeling above, we can show the all weights are different by the following equation:

\[
wt_3(C_3^{j+1}) - wt_3(C_3^j) = \varphi_3(x_j+1) + \varphi_3(y_j+1) + \varphi_3(z_j+1) + \varphi_3(x_{j+1}y_{j+1}) + \\
\varphi_3(x_{j+1}z_{j+1}) + \varphi_3(y_{j+1}z_{j+1}) - \varphi_3(x_j) - \varphi_3(y_j) - \\
\varphi_3(z_j) - \varphi_3(x_jy_j) - \varphi_3(x_jz_j) - \varphi_3(y_jz_j) = 3
\]

\[
wt_3(C_3^{j+1}) - wt_3(C_3^j) = \varphi_3(y_{j+1}) + \varphi_3(x_{j+2}) + \varphi_3(z_{j+2}) + \varphi_3(y_{j+1}x_{j+2}) + \\
\varphi_3(y_{j+1}z_{j+2}) + \varphi_3(x_{j+2}z_{j+2}) - \varphi_3(y_j) - \varphi_3(x_{j+1}) - \\
\varphi_3(z_{j+1}) - \varphi_3(y_{j+1}x_{j+1}) - \varphi_3(y_{j+1}z_{j+1}) = 3
\]

We respect to \( wt_\varphi(C_3^j) < wt_\varphi(C_3^{j+1}) \), \( j = 1, 2, \ldots, m \) then \( wt_\varphi(C_3^{j+1}) - wt_\varphi(C_3^j) = 3 \). The all \( H_2 \)-weights are distinct. This matter concludes that \( tHs((TCl_m), C_3) = \left\lceil \frac{3m + 5}{6} \right\rceil \). The example of total \( C_3 \)-irregularity of three circular ladder graph labeling, we can see on Figure 2, and we get \( tHs((TCl_m), C_3) = 5 \).

![Figure 2. The Example of Total C₃-Irregularity of TClₘ labeling](image)

**Theorem 4.** Let \( Pr_m \) be a prism graph and subgraph \( H_3 \equiv C_4 \). The total \( H_3 \)-irregularity strength of \( Pr_m \) graph for \( m \geq 3 \), \( m \equiv 0 \mod 4 \) and \( m \equiv 1 \mod 4 \) is \( \left\lfloor \frac{n + 7}{8} \right\rfloor \).

**Proof.** Let \( Pr_m \), \( m \geq 3 \), be a prism graph with the vertex set \( V(Pr_m) = \{x_j, y_j : j = 1, 2, 3, \ldots, m\} \) and the edge set \( E(Pr_m) = \{x_jx_{j+1}, y_jy_{j+1} : j = 1, 2, 3, \ldots, m - 1\} \cup \{x_jy_j : j = 1, 2, 3, \ldots, m\} \cup \{x_my_1\} \cup \{y_my_1\} \). The prism graph \( Pr_m \), \( m \geq 3 \), contains a \( C_4 \)-covering with exactly \( m \) cycles \( C_4 \). Based on Theorem 4, we have \( tHs(Pr_m), C_4) \geq \left\lfloor \frac{m + 7}{8} \right\rfloor \). Put \( \alpha = \left\lfloor \frac{m + 7}{8} \right\rfloor \). The following function of \( C_4 \)-irregular total \( \alpha \)-labeling \( \varphi_3 : V(Pr_m) \cup E(Pr_m) \to \{1, 2, \ldots, \alpha\} \) is prove that \( \alpha \) as an upper bound for the total \( C_4 \)-irregularity strength of \( Pr_m \).
A $C_4$-irregular total $\alpha$-labeling $\varphi_4 : V(Pr_m) \cup E(Pr_m) \to \{1, 2, \ldots, \alpha\}$ is as follows:

for $j = 1, 2, \ldots, \left\lfloor \frac{m}{2} \right\rfloor$

$$\varphi_4(x_j) = \left\lceil \frac{j + 2}{4} \right\rceil,$$
$$\varphi_4(y_j) = \left\lceil \frac{j + 2}{4} \right\rceil,$$
$$\varphi_4(x_jx_{j+1}) = \left\lceil \frac{j + 1}{4} \right\rceil,$$

for $j = \left\lceil \frac{m}{2} \right\rceil + 1, \ldots, m - 1$, and $m \equiv 0 \text{ mod } 4$

$$\varphi_4(x_j) = \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j}{4} \right\rceil + 2,$$
$$\varphi_4(y_j) = \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j + 1}{4} \right\rceil + 2,$$
$$\varphi_4(x_jx_{j+1}) = \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j + 3}{4} \right\rceil + 2,$$

for $j = \left\lceil \frac{m}{2} \right\rceil + 1, \ldots, m - 1$, and $m \equiv 1 \text{ mod } 4$

$$\varphi_4(x_j) = \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j + 1}{4} \right\rceil + 2,$$
$$\varphi_4(y_j) = \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j + 2}{4} \right\rceil + 2,$$
$$\varphi_4(x_jx_{j+1}) = \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j}{4} \right\rceil + 1,$$

for $i = m$

$$\varphi_4(x_m) = 2, \quad \varphi_4(x_my_m) = 1,$$
$$\varphi_4(y_m) = 1, \quad \varphi_4(x_mx_1) = 1,$$
$$\varphi_4(y_my_1) = 1.$$

We get the upper bound from the function of $C_4$-irregular total $Pr_m$-labeling. We take from the largest label of graph. We get to present the upper bound of the graph in the Theorem 4,
$$tHs(Pr_m, C_4) \leq \left\lceil \frac{m + 7}{8} \right\rceil.$$
Based on the labeling above, we can show the all weights are different by the following equation:

for every \( j = 1, 2, \ldots, \left\lfloor \frac{m}{2} \right\rfloor \), we have

\[
wt_{\phi_n}(C_4^{j+1}) - wt_{\phi_n}(C_4^j) = \varphi_4(x_{j+1}) + \varphi_4(x_{j+2}) + \varphi_4(x_{j+1}x_{j+2}) + \varphi_4(y_{j+1}) + \varphi_4(y_{j+2}) + \varphi_4(y_{j+1}y_{j+2}) + \varphi_4(x_{j+1}y_{j+1} + y_{j+1}y_{j+2}) - \varphi_4(x_j) - \varphi_4(x_{j+1}) - \varphi_4(x_{j}x_{j+1}) - \varphi_4(y_j) - \varphi_4(y_{j+1}) - \varphi_4(y_{j}y_{j+1}) - \varphi_4(x_{j}y_{j}) - \varphi_4(x_{j+1}y_{j+1})
\]

= \( 2 \)

for every \( j = \left\lfloor \frac{m}{2} \right\rfloor + 1, \ldots, m - 1 \), we have

\[
wt_{\phi_n}(C_4^{j+1}) - wt_{\phi_n}(C_4^j) = \varphi_4(x_{j+1}) + \varphi_4(x_{j+2}) + \varphi_4(x_{j+1}x_{j+2}) + \varphi_4(y_{j+1}) + \varphi_4(y_{j+2}) + \varphi_4(y_{j+1}y_{j+2}) + \varphi_4(x_{j+1}y_{j+1} + y_{j+1}y_{j+2}) - \varphi_4(x_j) - \varphi_4(x_{j+1}) - \varphi_4(x_{j}x_{j+1}) - \varphi_4(y_j) - \varphi_4(y_{j+1}) - \varphi_4(y_{j}y_{j+1}) - \varphi_4(x_{j}y_{j}) - \varphi_4(x_{j+1}y_{j+1})
\]

= \( -2 \)

for every \( j = m \), we have

\[
wt_{\phi_n}(C_4^m) = \varphi_4(x_m) + \varphi_4(y_m) + \varphi_4(x_my_m) + \varphi_4(x_mx_1) + \varphi_4(y_my_1) + \varphi_4(x_1y_1)
\]

= \( 2 + 1 + 1 + 1 + \left\lfloor \frac{j + 2}{4} \right\rfloor + \left\lfloor \frac{j + 2}{4} \right\rfloor + \left\lfloor \frac{j}{4} \right\rfloor \)

= \( 6 + \left\lfloor \frac{1 + 2}{4} \right\rfloor + \left\lfloor \frac{1 + 2}{4} \right\rfloor + \left\lfloor \frac{1}{4} \right\rfloor \)

= \( 9 \)

We respect to \( wt_{\phi_4}(C_4^j) < wt_{\phi_4}(C_4^{j+1}) \), \( j = 1, 2, \ldots, m \). If every \( j = 1, 2, \ldots, \left\lfloor \frac{m}{2} \right\rfloor \) then

\[
wt_{\phi_4}(C_4^{j+1}) - wt_{\phi_4}(C_4^j) = 2. \text{ If every } j = \left\lfloor \frac{m}{2} \right\rfloor + 1, \ldots, m - 1 \text{ then } wt_{\phi_4}(C_4^{j+1}) - wt_{\phi_4}(C_4^j) = -2.
\]

If every \( j = m \) then \( wt_{\phi_4}(C_4^m) = 9 \). The all \( H_3 \)-weights are distinct. This matter concludes that \( tHs((Pr_m), C_4) = \left\lfloor \frac{m + 7}{8} \right\rfloor \). We know that example of total \( C_4 \)-irregularity of prism graph on Figure 3, and we get \( tHs((Pr_{16}), C_4) = 2 \) which \( j \) is even. But we can see the example of total \( C_4 \)-irregularity of prism graph labeling on Figure 4, and we get \( tHs((Pr_{17}), C_4) = 2 \) which \( j \) is odd.
3. Concluding Remarks
In this research we have obtained of the total $H$-irregularity strength of prism graphs, diamond ladder graphs, and three circular ladder graphs. We recognize $H$-covering on prism graphs and three circular ladder graphs for which $H$ is cyclical. But, we recognize $H$-covering on diamond ladder graph that $H$ is a diamond ladder graph.

Open Problem 1 Find the total $H$-Irregularity Strength $(tHs)$ of the $Pr_m$, $m \geq 3$.

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