Abstract

We investigate the effects of the Chern-Simons coupling on the high energy behavior in the (2 + 1)-dimensional Chern-Simons QED with a four-Fermi interaction. Using the $1/N$ expansion we discuss the Chern-Simons effects on the critical four-Fermi coupling at $O(1/N)$ and the $\beta$ function around it. High-energy behavior of Green’s functions is also discussed. By explicit calculation, we find that the radiative correction to the Chern-Simons coupling vanishes at $O(1/N)$ in the broken phase of the dynamical parity symmetry. We argue that no radiative corrections to the Chern-Simons term arise at higher orders in the $1/N$ expansion.
Various phenomena peculiar in (2+1) dimensions happen when the Chern-Simons (CS) term [1, 2] is present. Combined with the ordinary Maxwell term, the CS term generates a gauge invariant mass for gauge fields. There has been a great interest in the so-called CS theory where the kinetic action for gauge fields is characterized only by the CS term [2]. In this case the gauge sector of the theory is renormalizable (though not super-renormalizable) in (2+1) dimensions, which makes it field theoretically interesting. While the CS term has been known to affect the long-distance behavior of the theory, our motivation in this paper is to study the CS effects on the short-distance behavior.

In this paper we investigate the effects of the CS term on the high energy behavior of the CS QED$_{2+1}$ with a four-Fermi interaction. It has been shown [3] that a class of theories with a four-Fermi interaction is renormalizable in the framework of the $1/N$ expansion in spite of its non-renormalizability in the weak coupling expansion. The $1/N$ technique also shows various nonperturbative phenomena among which is the interesting renormalization flow for the four-Fermi coupling. At leading order in $1/N$, the four-Fermi coupling has a nontrivial ultraviolet fixed point which survives beyond leading order in $1/N$.

We find the dependence of the $\beta$ function on $\theta$ (the coupling of the CS term) for the four-Fermi coupling at $O(1/N)$. The correction to the critical coupling is calculated. The nature of the ultraviolet fixed point at leading order strongly depends on $\theta$. It is drastically different from the model with the usual Maxwell term which has been previously considered in connection with the dynamical symmetry pattern and the critical behavior of the theory [4].

In this model another interesting issue is the radiative correction to $\theta$ which has been frequently discussed in the (2+1)-dimensional gauge theories [3] - [7]. By explicit calculation, we show that $\theta$ does not renormalize at $O(1/N)$. We also prove
that there are no radiative corrections to $\theta$ at all orders in $1/N$, thus extend the non-renormalization theorem by CoBleman and Hill [5] to the $1/N$ expansion.

The $(2+1)$-dimensional CS QED with the simplest four-Fermi interaction is given in the Euclidean version by

$$
\mathcal{L} = i\bar{\psi}_j D\psi_j + \frac{g^2}{2N} (\bar{\psi}_j \psi_j)^2 + i\theta \epsilon_{\mu\nu\rho} A_\mu F_{\nu\rho},
$$

(1)

where $D_\mu = \partial_\mu + ieA_\mu/\sqrt{N}$ and $j$ is summed over from 1 to $N$. The couplings $e$ and $\theta$ are dimensionless and $\psi_j$ are two-component spinors. The $\gamma$ matrices are defined as

$$
\gamma_1 = \sigma^3, \quad \gamma_2 = \sigma^1, \quad \gamma_3 = \sigma^2. \tag{2}
$$

Introducing an auxiliary field $\sigma$ to facilitate the $1/N$ expansion [3], we can rewrite Eq. (1) as

$$
\mathcal{L} = i\bar{\psi}_j D\psi_j + i\sigma \bar{\psi}_j \psi_j + \frac{N\sigma^2}{2g^2} + i\theta \epsilon_{\mu\nu\rho} A_\mu F_{\nu\rho} \tag{3}
$$

At leading order in $1/N$, the theory has a two-phase structure characterized by the order parameter $\langle \sigma \rangle$ as in the case without the gauge field [3]. For the weak coupling, $g^2 < g_c^2$, where $g_c^{-2} = 2\int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2}$, the parity is unbroken characterized by $\langle \sigma \rangle = 0$. If the coupling is larger than $g_c^2$, the auxiliary field $\sigma$ gets a non-zero vacuum expectation value. The parity symmetry is dynamically broken and the fermion acquires the mass equal to $\langle \sigma \rangle = M$. We consider this broken phase in this paper.

At leading order in $1/N$, the vacuum satisfies the following relation

$$
\frac{1}{g^2} - 2\int_0^\Lambda \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + M^2} = 0, \tag{4}
$$

where we introduce the high-momentum cutoff $\Lambda$. We can easily see from Eq. (4) that the dimensionless charge defined by $\lambda = 1/(g^2\Lambda)$ flows to the finite value $\lambda_c = 1/\pi^2$ in the continuum limit ($\Lambda \rightarrow \infty$). The $\beta$-function for $\lambda$ in the vicinity of $\lambda_c$, at leading order in $N$, is $\beta(\lambda) = -(\lambda - \lambda_c)$ [3].
The Feynman diagrams in the broken phase are depicted in Fig. 1. The propagator for the fermion $\psi$ is given by

$$S(p) = \frac{1}{p^2 + iM},$$

(5)

the propagator for the auxiliary field $\sigma$ is

$$D(p^2) = \frac{-4\pi\sqrt{p^2}}{(p^2 - 4M^2)\tan^{-1}\frac{\sqrt{p^2}}{2M}}.$$  

(6)

The photon propagator is given by

$$G_{\mu\nu} = \frac{1}{p^2} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi_1(p^2) + \epsilon_{\mu\nu\rho} \frac{p_\rho}{p^2} \Pi_2(p^2),$$

(7)

where $\Pi_1$ and $\Pi_2$ are given by

$$\Pi_1(p^2) = \frac{\Pi_\varphi}{\Pi_\varphi/p^2 + (\Pi_o + \theta)^2},$$

(8)

$$\Pi_2(p^2) = \frac{(\Pi_o + \theta)}{\Pi_\varphi/p^2 + (\Pi_o + \theta)^2}. $$

(9)

The resummation technique of the $1/N$ expansion results in the photon propagator and $\Pi_\varphi$ $[\Pi_o]$ in Eq. $[8]$ $[9]$ is the even (odd) part of the vacuum polarization:

$$\Pi_\varphi(p^2) = \frac{e^2}{8\pi} \left( 2M + \frac{p^2 - 4M^2}{\sqrt{p^2}} \tan^{-1}\frac{\sqrt{p^2}}{2M} \right),$$

(10)

and

$$\Pi_o(p^2) = \frac{Me^2}{2\pi} \frac{1}{\sqrt{p^2}} \tan^{-1}\frac{\sqrt{p^2}}{2M}. $$

(11)

As $p \to \infty$, the photon propagator behaves like $\frac{\kappa}{\kappa^2 + \varphi} p^{-1}$ (the symmetric part) and $\frac{\theta}{\kappa^2 + \varphi} p^{-1}$ (the antisymmetric part), where $\kappa = e^2/16$.

The vertex of $\sigma \bar{\psi} \psi$ is given by $-i\delta_{ij}/\sqrt{N}$ and the vertex of $A_\mu \bar{\psi} \psi$ is given by $-e\gamma_\mu \delta_{ij}/\sqrt{N}$. Notice that the graphs in Fig. 2 are forbidden to avoid double counting.
The renormalizability of the theory can be easily proved using a simple power counting and the Ward identity. We refer to Ref. [3] for detailed renormalization procedure. In terms of the renormalized quantities, the Lagrangian density (See Eq. (3).) can be rewritten as

$$\mathcal{L} = iZ_1 \overline{\psi}_j \gamma_\mu \gamma_5 \psi_j + iZ_2 \sigma \overline{\psi}_j \psi_j + \frac{NZ_3 Z_2^2}{2g^2} \sigma^2 + iZ_4 \delta \epsilon_{\mu\nu\rho} A_\mu F_{\nu\rho}$$

Note that the Maxwell term is not generated as a counter term, which assures the renormalizability of the theory. We keep $g^2$ as in Eq. (4), so that we are in the broken phase. Expanding $\sigma(x) = (Z_1/Z_2)(M + \sigma'(x)/\sqrt{N})$ in powers of $1/N$, we have at next-to-leading order

$$\frac{Z_2}{Z_1} \langle \sigma(x) \rangle = M + \frac{1}{\sqrt{N}} \langle \sigma'(x) \rangle,$$  \text{(13)}

where

$$\frac{\sqrt{N}}{D(0)} \langle \sigma'(x) \rangle = -\frac{MZ_3'}{g^2} + \text{Tadpole Diagrams in Fig. 3(c)},$$  \text{(14)}

with $Z_3 = 1 + Z_3'/N$.

In order to calculate the renormalization constants $Z_i$, we evaluate the diagrams in Fig. 3. We have

$$Z_1 = 1 - \frac{4}{3\pi^2 N} \left( 1 + \frac{4\kappa^2}{\kappa^2 + \theta^2} \right) \ln \left( \frac{\Lambda}{\mu} \right),$$  \text{(15)}

$$Z_2 = 1 + \frac{4}{\pi^2 N} \left( 1 + \frac{12\kappa^2}{\kappa^2 + \theta^2} \right) \ln \left( \frac{\Lambda}{\mu} \right),$$  \text{(16)}

and

$$\frac{Z_3}{g^2} = \frac{1}{\pi^2} \left[ 1 - \frac{2}{N} \frac{\theta^2 - \kappa^2}{\kappa^2 + \theta^2} \right] \Lambda$$

$$[B \omega B - \frac{M}{2\pi} \left( 1 + \frac{16}{3\pi^2 N} \left( 1 + \frac{10\kappa^2}{\kappa^2 + \theta^2} \right) \ln \left( \frac{\Lambda}{\mu} \right) \right].$$  \text{(17)}
Note that there is no dangerous $\ln \Lambda$ dependence in Eq. (17), which preserves the phase structure at leading order. The $\Lambda^2$ divergence from the tadpole diagram with internal photon line in Fig. 3(c) only shifts the gap equation and does not affect the phase structure of the theory.

The extra parameter $\mu$ in the above equations is an unphysical renormalization point. The $\mu$ dependence in Eq. (17) can be absorbed in $M_{\text{phys}}$, then $Z_3$ is rewritten as
\[
\frac{Z_3}{g^2 \Lambda} = \frac{1}{\pi^2} \left( 1 + \frac{2}{N} \frac{1 - x^2}{1 + x^2} \right) - \frac{1}{2\pi} (M_{\text{phys}}/\Lambda)^A
\]
where
\[
A = 1 - \frac{16}{3\pi^2} \frac{1}{N} \left( 1 + \frac{10}{1 + x^2} \right) > 0
\]
with $x = \theta/\kappa$. After rescaling $A_\mu \rightarrow A_\mu/e$, the theory has one gauge coupling $x$.

The exact $S$-matrix depends on just one scale $M_{\text{phys}}$, so the invariant charge $Z_3/g^2$ depends on $\Lambda$ and $M_{\text{phys}}$. The leading-order parameter $M$ is no longer equal to $M_{\text{phys}}$, and should be regarded as a function of $\mu$.

The $\beta$ function for $\lambda = Z_3/(g^2 \Lambda)$ around $\lambda_c$ is given by
\[
\beta(\lambda) = -A (\lambda - \lambda_c),
\]
where
\[
\lambda_c = \frac{1}{\pi^2} \left( 1 + \frac{2}{N} \frac{1 - x^2}{1 + x^2} \right).
\]
The finite ultraviolet fixed point still exists. When $x = 1$, it is not shifted at $O(1/N)$.

It moves downwards (upwards) when $x > (<) 1$. The slope of the $\beta$ function for $\lambda$ becomes $-A$. In the limit $x \rightarrow \infty$, the gauge degrees of freedom drop out and the theory has only the four-Fermi interaction. This can be immediately checked by taking the limit $x \rightarrow \infty$ in the Eqs. (19) and (21) and comparing them with the previous results in the four-Fermi interaction model [3]. In this limit, $A$ is always positive even when $N = 1$. Therefore the theory is consistent for any $N$. As we can
see from Fig. 4 in which we plot $x$ versus $N$, $N$ should be larger than 6 to make the theory consistent for any value of $x$.

From the renormalization constants in Eqs. (13) and (16) we can easily read the ultraviolet dimension of the fields $\psi$ and $\sigma$

$$[\psi] = 1 + \frac{2}{3\pi^2 N} \left( 1 + \frac{4}{1 + x^2} \right),$$  \hspace{1cm} (22)

$$[\sigma] = A. \hspace{1cm} (23)$$

Then the high energy behavior of the connected, truncated Green’s function with $n$ external fermion legs and $m$ external $\sigma$ legs is \( \sim E^p \) where $p = 3 - n [\psi] - m [\sigma]$. If the Maxwell term exists in the original theory, which we do not consider here, the QED sector is finite, thus it does not influence the high-energy behavior of the theory which is characterized by the four-Fermion interaction.

We now discuss the radiative correction to $\theta$ at $O(1/N)$, which is given by

$$\Pi_o(0) = \lim_{p \to 0} \frac{1}{6} e^{\mu\nu\lambda} \frac{\partial}{\partial p^\lambda} \Pi_{\mu\nu}(p).$$

The leading order corrections to the vacuum polarization $\Pi_{\mu\nu}$ can be calculated from the diagram in Fig. 2 as in the weak coupling expansion. This is summed to the photon propagator in Eqs. (7)–(11). From Eq. (11) its contribution to $\Pi_o(0)$ is $-e^2/4\pi$ as is well known.

The corrections at $O(1/N)$ arise from the integrals depicted in the diagrams of Figs. 5 and 6. After taking the trace of the $\gamma$ matrices in the integrands, we find that there are no contributions to $\Pi_o(0)$ from the Feynman diagrams of Figs. 5(a). The diagrams Figs. 5(b) have been already discussed in the contexts of the Maxwell-Chern-Simons QED and the Chern-Simons QED in the weak coupling expansion scheme. Recalling the explicit calculations in Refs. [3, 7], we can deduce that they also have null contribution to the coefficient of the CS term. We may write their contributions to $\Pi_o(0)$ by

$$\Delta_1 \Pi_o(0) = \lim_{p \to 0} \frac{1}{6} e^{\mu\nu\lambda} \frac{\partial}{\partial p^\lambda} \Delta_1 \Pi_{\mu\nu}(p) \hspace{1cm} (24)$$
\[ \Delta_1 \Pi_{\mu\nu}(p) = \int \frac{d^3q}{(2\pi)^3} \Gamma_{\mu\nu\rho\sigma}(p, -p, q, -q) G^{\rho\sigma}(q), \] (25)

where \( \Gamma_{\mu\nu\rho\sigma} \) is the one-loop 4-photon function.

The calculations in Ref. 6 show that the same expression for \( \Delta_1 \Pi_{\rho}(0) \) exactly vanishes when the photon propagator in the Maxwell-Chern-Simons QED is given by

\[ G_{\rho\sigma}(q) = G^e_{\rho\sigma}(q) + G^o_{\rho\sigma}(q), \] (26)

where

\[ G^e_{\rho\sigma}(q) = \frac{1}{q^2 + \theta^2} \left( \delta_{\rho\sigma} - \frac{q_\rho q_\sigma}{q^2} \right), \] (27)

\[ G^o_{\rho\sigma}(q) = \frac{\theta}{q^2 + \theta^2} \epsilon_{\rho\sigma\lambda} \frac{q^\lambda}{q^2}. \] (28)

In fact, the contributions to \( \Delta_1 \Pi_{\rho}(0) \) with \( G^e_{\rho\sigma}(q) \) and \( G^o_{\rho\sigma}(q) \) separately vanish even before the gauge-field loop integration is performed. Since the induced photon propagator Eq. (7) which must be employed in Eqs. (24) and (25) has the same structure, it becomes clear that \( \Delta_1 \Pi_{\rho}(0) \) exactly vanishes.

The diagrams in Fig. 6 are of three loop order in the weak coupling expansion, but produce the same \( O(1/N) \) corrections as the diagrams in Fig. 5. We denote the one-loop \( m \)-photon \( n\)-\( \sigma \) function by \( \Gamma_{\mu_1...\mu_n(\sigma)} \). The corrections from the diagrams of Fig. 6(a) and those of Fig. 6(b) involve the one-loop 2-photon, 1-\( \sigma \) field functions \( \Gamma_{\mu\nu(1)} \) which is given by

\[ \Gamma_{\mu\nu(1)}(p, -q, q - p) = -\frac{e^2}{\sqrt{N}} \int \frac{d^3k}{(2\pi)^3} \text{tr} \left( \gamma_{\mu}S(k - p)\gamma_{\nu}S(k + q - p) \right. \]

\[ + \gamma_{\mu}S(k - p)S(k - q)\gamma_{\nu}S(k) \bigg), \] (29)

and the one-loop 3-photon function \( \Gamma_{\mu\nu\lambda} \) which is given by

\[ \Gamma_{\mu\nu\lambda}(p, -q, q - p) = -\frac{e^3}{\sqrt{N}} \int \frac{d^3k}{(2\pi)^3} \text{tr} \left( \gamma_{\mu}S(k - p)\gamma_{\nu}S(k + q - p)\gamma_{\lambda}S(k) \right. \]

\[ + \gamma_{\mu}S(k - p)\gamma_{\lambda}S(k - q)\gamma_{\nu}S(k) \bigg), \] (30)
Their contributions to the vacuum polarization may be written as
\[
\Delta_2 \Pi_{\mu\nu}(p) = \int \frac{d^3 q}{(2\pi)^3} \Gamma_{\mu\rho(1)}(p, -q, q - p) D(q - p) G^{\rho\sigma}(q) \Gamma_{\nu\sigma(1)}(-p, q, p - q), \quad (31)
\]
\[
\Delta_3 \Pi_{\mu\nu}(p) = \int \frac{d^3 q}{(2\pi)^3} \Gamma_{\mu\rho\beta}(p, -q, q - p) G^{\alpha\beta}(q - p) G^{\rho\sigma}(q) \Gamma_{\nu\sigma\alpha}(-p, q, p - q). \quad (32)
\]

Their null contributions to the coefficient of the CS term can be discussed in the same spirit of the non-renormalization theorem [5] in the weak coupling expansion. The vanishing of the corrections in \( \Delta_2 \Pi_{\mu\nu} \) and \( \Delta_3 \Pi_{\mu\nu} \) to the coefficient of the CS term follows from the observation that the CS term is of order of \( p \) while \( \Delta_2 \Pi_{\mu\nu} \) and \( \Delta_3 \Pi_{\mu\nu} \) are of order \( p^2 \) in the limit where \( p \to 0 \). We can see that the gauge invariance
\[
p^{\mu} \Gamma_{\mu\rho(1)}(p, -q, q - p) = 0 , \quad p^{\mu} \Gamma_{\mu\rho\beta}(p, -q, q - p) = 0 \quad (33)
\]
and the analyticity of the one-loop functions yields that
\[
\Gamma_{\mu\rho(1)}(p, -q, q - p) = O(p) , \quad \Gamma_{\mu\rho\beta}(p, -q, q - p) = O(p) \quad (34)
\]
as \( p \to 0 \). Similarly we also find that
\[
\Gamma_{\nu\sigma(1)}(-p, q, p - q) = O(p) , \quad \Gamma_{\nu\sigma\alpha}(-p, q, p - q) = O(p). \quad (35)
\]
It follows from this that the integrands in Eqs. (31) and (32) may be of order \( p^2 \). But it is yet to be examined whether the integration over \( q \) may change the order in \( p \); that is possible if the integrands have singularities as \( q \to p \).

The singular behavior of the photon propagator \( G_{\alpha\beta}(q - p) \) depends on whether \( \theta \) is cancelled by \( \Pi_\alpha(0) \) which is the one-loop corrections to \( \theta \). When \( \Pi_\alpha(0) + \theta \neq 0 \), the photon propagator \( G_{\alpha\beta}(q - p) \) behaves as \( q \to p \)
\[
G_{\alpha\beta}(q - p) \to \begin{cases} 
\frac{1}{(e^2/4\pi)^2} \frac{2}{3M} & \text{(symmetric part)} \\
\frac{1}{(e^2/4\pi)^2} \frac{1}{|q - p|} & \text{(antisymmetric part)}.
\end{cases} \quad (36)
\]
When $\Pi_\alpha(0) + \theta = 0$, the antisymmetric part of the photon propagator vanishes and the symmetric part introduces a singularity

$$G_{\alpha\beta}(q - p) \to \frac{3M}{2} \frac{1}{(q - p)^2}. \quad (37)$$

But the gauge invariance and the analyticity ensure that the two one-loop photon functions, Eqs. (30) and (30), introduce a factor of $(q - p)^2$. Thus the integrand in Eq. (31) is nonsingular where $q = p$.

On the other hand, the two one loop 2-photon 1-$\sigma$ field function in Eq. (32) does not introduce the factor $(q - p)^2$, since the fermion-$\sigma$ vertex is not associated with the gauge invariance. Therefore if the $\sigma$ propagator $D(q - p)$ has a singularity at $q = p$, the integration over $q$ may change the order of $p$. Fortunately the $\sigma$ propagator $D(q - p)$ is regular as $q \to p$

$$D(q - p) \to \frac{2\pi}{M}. \quad (38)$$

The above arguments also apply to the diagrams of Fig. 6(c), therefore they lead us to conclude that there are no corrections to the coefficient of the CS term at $O(1/N)$.

Our explicit calculation confirms that there are no infinite radiative corrections at $O(1/N)$.

We can extend the arguments discussed above and apply them to the corrections from the diagrams at higher orders in $1/N$. In general, higher order diagrams for the vacuum polarization consist of the (fermion) one-loop functions, the internal photon and $\sigma$-field lines. The internal lines represent the induced propagators (See Eqs. (6), (7), (8)). and connect the legs of the one-loop diagrams together all but two photon legs which are the external ones which carry the momenta $p$ and $-p$ respectively. Since the $1/N$ expansion respects the gauge invariance,

$$p^{\mu_1} \Gamma_{\mu_2, \ldots, \mu_m} \mu_{m+1, \ldots, n}(p_1, \ldots, p_i, \ldots, p_m, p_{m+1}, \ldots, p_n) = 0 \quad (39)$$
as $p^{\mu_i} \to 0$ where $\sum_{j=1}^{n} p_j = 0, (i = 1, \ldots, m)$ and the analyticity of the one-loop functions implies that, for $m \geq 3$,

$$\Gamma_{\mu_1 \ldots \mu_i \ldots \mu_m} (p_1, \ldots, p_i, \ldots, p_m, p_{m+1}, \ldots, p_n) = O(p_1, \ldots, p_i, \ldots, p_m). \quad (40)$$

The two external photon legs may be attached to the same one-loop diagram or to two separate one-loop diagrams. Then the integrand for the vacuum polarization tensor contains $\Gamma_{\mu \nu \ldots (m)}(p, -p, \ldots)$ in the former case and $\Gamma_{\mu \ldots (m)}(p, \ldots)\Gamma_{\nu \ldots (n)}(-p, \ldots)$ in the latter case. In either case, Eq. (40) shows that the one-loop functions introduce $p^2$ in the integrand. The photon propagators may introduce singularities when their momenta vanish. But the gauge invariance and the analyticity again assure that the one-loop functions, corresponding to the one-loop diagrams where the photon propagators are attached to, vanish precisely such that the integrands are free of singularities. As we discussed in the corrections at order $1/N$, the induced $\sigma$ propagator does not introduce any singularity in the infrared region. Therefore we can conclude that the integrations over the internal momenta do not change the order in $p$ and the resultant corrections to vacuum polarization are of order $p^2$, i.e., no higher order corrections to the CS term in the $1/N$ expansion.

However, these arguments on the vanishing radiative corrections do not apply to the unbroken phase in which the fermions remain massless. The absence of the analyticity in the massless case indeed results in a correction to $\theta$ in the weak coupling expansion $[7,9]$. This may happen in the unbroken phase. In this case, we can still argue that there will be no infinite radiative correction to $\theta$ but only finite one, since the mass of the fermions would not change the ultraviolet structure of the theory.

We find that the CS term affects the high energy behavior of the (2+1)-dimensional Chern-Simons QED with a four-Fermi interaction. The nature of the critical four-Fermi coupling at $O(1/N)$ is investigated under the CS influence. The $\beta$ function
around the critical coupling is calculated at $O(1/N)$. By explicit calculation, we find that the radiative correction to $\theta$ vanishes at $O(1/N)$ in the dynamically broken phase of the parity symmetry. We also prove that there is no radiative corrections to $\theta$ at higher orders in $1/N$.

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Figure Captions

Figure 1: Feynman rules in the phase of the broken parity.
Figure 2: Forbidden diagrams.
Figure 3: Diagrams contributing to $Z$’s.
Figure 4: $N$ versus $x$.
Figure 5: Two-loop diagrams at $O(1/N)$ contributing to $Z_4$.
Figure 6: Three-loop diagrams at $O(1/N)$ contributing to $Z_4$. 