Saturation effects in low-x DIS structure functions and related hadronic total cross sections

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Abstract. High-energy nucleon total cross sections are related to low-x DIS structure functions by using the additive quark model.

In the additive quark model, the hadron-hadron total cross section can be written as a product of the cross sections of the constituents, $\sigma_{qq}$ [1, 2], e.g.

$$\sigma(s)_{pp}^{T} = \sigma_{qq}[n_{V} + n_{S}(s)]^{2},$$

where $n_{V}$ is the number of valence quarks and $n_{S}(s)$ is that of sea quarks, their number increasing with energy.

It was suggested in Refs. [1, 2] that the increasing number of sea quarks is related to the Bjorken scaling-violating contribution to the deep inelastic lepton-hadron structure function (DIS SF), namely to the momentum fraction of the relevant quarks given by the integral over the DIS structure function $F_{2}(x, Q^{2})$. In Ref. [1] a simple model for the DIS structure function, known at those times, was used, resulting in the following expression for the total cross section, compatible with the data

$$\sigma(s)_{pp}^{T} = \sigma_{qq}n_{V}^{2}(1 + 0.016 \ln(s/Q_{0}^{2})),$$

where $\sigma_{qq}$ is a free parameter, $Q_{0}^{2}$ was fitted to the DIS data, and $n_{V} = 3$.

In Ref. [2] the DIS SF was related to hadronic cross sections by means of finite-energy sum rules in $Q^{2}$. The number of quarks in a reaction can be calculated from the SF by means of sum rules. see e.g. [3, 4].

In Ref. [5] following ansatz for the small-$x$ singlet part (labelled by the upper index $S, 0$) of the proton structure function, interpolating between the soft (VMD, Pomeron) and hard (GLAP evolution) regimes was proposed:

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Figure 1. Slope of the structure function $F_2(x, Q^2)$.

\[ F_2^{(S,0)}(x, Q^2) = A \left( \frac{Q^2}{Q^2 + a} \right)^{1+\tilde{\Delta}(Q^2)} e^{\Delta(x, Q^2)}, \]  

(3.1)

with the "effective power"

\[ \tilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ln \left( 1 + \gamma_2 \ln \left( 1 + \frac{Q^2}{Q^2_0} \right) \right), \]  

(3.2)

and

\[ \Delta(x, Q^2) = \left( \frac{\tilde{\Delta}(Q^2) \ln x_0}{x} \right)^{f(Q^2)}, \]  

(3.3)

where

\[ f(Q^2) = \frac{1}{2} \left( 1 + e^{-Q^2/Q_1^2} \right). \]  

(3.4)

At small and moderate values of $Q^2$, the exponent $\tilde{\Delta}(Q^2)$ (3.2) may be interpreted as a $Q^2$-dependent "effective Pomeron intercept", as shown in Fig. 1.

The function $f(Q^2)$ has been introduced in order to provide for the transition from the Regge behaviour, where $f(Q^2) = 1$, to the asymptotic solution of the GLAP evolution equation, where $f(Q^2) = 1/2$.

In Ref. [5] the above singlet SF was appended by a non-singlet part, important at large values of $x$. The parameters were fitted to the DIS data in a wide range of $x$ and $Q^2$. The values of the fitted parameters are: $A = 0.1623$, $a = 0.2916$ GeV$^2$, $\gamma_1 = 0.01936$, $Q^2_0 = 0.1887$ GeV$^2$, $Q^2_1 = 916.1$ GeV$^2$, $x_0 = 1$, $\epsilon = 0.08$, $\gamma_1 + 2.4$ were fixed (by QCD-related arguments). The resulting fits and more details can be found in Ref. [5].

The proton-proton total cross section is cast by integrating Eqs. (3) between $x = 0$ and $x = 1$. At high energies, only the singlet part of the SF, Eqs. (3) (the "Pomeron") is relevant. Integration can be
Figure 2. HHT (top) and HERAPDF2.0 (bottom) NLO predictions of the $F_2$ structure function at $Q_{\min}^2 = 3.5 \text{ GeV}^2$, compared to extracted values. For more details, see Ref. [7].
performed numerically. The result is in reasonable agreement with the data on $pp$ total cross sections, including those from the LHC.

The operator-product expansion beyond leading twist has diagrams in which two, three or four gluons may be exchanged in the $t$-channel such that these gluons may be viewed as recombining. This recombination could lead to gluon saturation. The colour-dipole framework also inspired a phenomenological model of saturation by Golec–Biernat–Wüsthoff (GBW), in which the onset of saturation is characterised as the transition from a “soft” to a “hard” scattering regime. This occurs along a “critical line” in the $x_{Bj}$, $Q^2$ plane.

Recently new results on low-$x$ DIS parton distributions (PDF and SF) have appeared [6, 7]; they show an intriguing change of regime towards smallest values of $x$ (Fig. 2) - possible saturation effect?

We intend to investigate its impact on the asymptotic behaviour of the total cross sections.

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