Query complexity of unitary operation discrimination

Xiaowei Huang, Lvzhou Li*

Institute of Quantum Computing and Computer Theory, School of Computer and Engineering, Sun Yat-sen University, Guangzhou 510006, China

Abstract

Discrimination of unitary operations is fundamental in quantum computation and information. A lot of quantum algorithms including the well-known Deutsch-Jozsa algorithm, Simon’s algorithm, and Grover’s algorithm can essentially be regarded as discriminating among individual, or sets of unitary operations (oracle operators). The problem of discriminating between two unitary operations $U$ and $V$ can be described as: Given $X \in \{U, V\}$, determine which one $X$ is. If $X$ is given with multiple copies, then one can design an adaptive procedure that takes multiple queries to $X$ to output the identification result of $X$. In this paper, we consider the problem: How many queries are required for achieving a desired failure probability $\epsilon$ of discrimination between $U$ and $V$. We prove in a uniform framework: (i) if $U$ and $V$ are discriminated with bound error $\epsilon$, then the number of queries $T$ must satisfy $T \geq \left\lceil \frac{2\sqrt{1-4\epsilon(1-\epsilon)}}{\Theta(U^\dagger V)} \right\rceil$, and (ii) if they are discriminated with one-sided error $\epsilon$, then there is $T \geq \left\lceil \frac{2\sqrt{1-2\epsilon}}{\Theta(U^\dagger V)} \right\rceil$, where $\lceil k \rceil$ denotes the minimum integer not less than $k$ and $\Theta(W)$ denotes the length of the smallest arc containing all the eigenvalues of $W$ on the unit circle.

Keywords: Quantum Computing, Unitary Operation Discrimination, Quantum Query Complexity

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*Corresponding author
Email address: lilvzh@mail.sysu.edu.cn (Lvzhou Li)
1. Introduction

Discrimination of unitary operations plays a fundamental role in the field of quantum information science, as many quantum information processing tasks eventually involve discriminating among unitary operations. Specially, a quantum algorithm can be essentially regarded as a procedure to discriminate among individual, or sets of unitary operations. For example, the well-known Deutsch-Jozsa algorithm [1] is to distinguish two set of unitary operations \( S_B \) and \( S_C \) as follows:

\[
S_B = \{ O_x : x \in \{0,1\}^n, |x| = \frac{n}{2} \}, \\
S_C = \{ O_x : x \in \{0,1\}^n, |x| = 0 \text{ or } 1 \}
\]

where \( S_B \) stands for the balanced functions, \( S_C \) stands for the constant functions, and \( O_x = \sum_i (-1)^{x_i} |x_i\rangle \langle x_i| \) is the oracle operator. Discrimination among sets of unitary operations (oracle operators) standing for functions with different periodicities is central to Simon’s algorithm [2] and Shor’s algorithm [3]. Actually, in the quantum query complexity model, any quantum algorithm for computing a Boolean function is a procedure of discriminating between two sets of oracle operators.

Considering the significance of unitaries discrimination, it has been studied in depth (for a partial list [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]), and the discrimination problem has also been discussed for measurements [18], Pauli channels [19], oracle operators [20], and the general quantum operations [21, 22, 23, 24, 25].

Discrimination of unitary operations is generally transformed to discrimination of quantum states by preparing a probe state and then identifying the output states generated by different unitary operations. Two unitary operations \( U \) and \( V \) are said to be perfectly distinguishable (with a single query), if there exists a state \( |\psi\rangle \) such that \( U|\psi\rangle \perp V|\psi\rangle \). It has been shown that \( U \) and \( V \) are perfectly distinguishable if, and only if \( \Theta(U^\dagger V) \geq \pi \), where \( \Theta(W) \) denotes the length of the smallest arc containing all the eigenvalues of \( W \) on
the unit circle \[4, 5\]. The situation changes dramatically when multiple queries are allowed, since any two different unitary operations are perfectly distinguishable in this case. Specifically, it was shown that for any two different unitary operations \(U\) and \(V\), there exist a finite number \(N\) and a suitable state \(|\psi\rangle\) such that \(U^\otimes N|\psi\rangle \perp V^\otimes N|\psi\rangle\) \[4, 5\]. Such a discriminating scheme is intuitively called a parallel scheme. Note that in the parallel scheme, an \(N\)-partite entangled state as an input is required and plays a crucial role. Then, the result was further refined in Ref. \[6\] by showing that the entangled input state is not necessary for perfect discrimination of unitary operations. Specifically, Ref. \[6\] showed that for any two different unitary operations \(U\) and \(V\), there exist an input state \(|\psi\rangle\) and auxiliary unitary operations \(W_1, \ldots, W_N\) such that \(UW_NU \ldots W_1|\psi\rangle \perp VW_NV \ldots W_1|\psi\rangle\). Such a discriminating scheme is generally called a sequential scheme.

Note that in these researches mentioned above, it was assumed by default that the unitary operations to be discriminated are under the complete control of a single party who can perform any physically allowed operations to achieve an optimal discrimination. A more complicated case is that the unitary operations to be discriminated are shared by several spatially separated parties. Then, in this case a reasonable constraint on the discrimination is that each party can only make local operations and classical communication (LOCC). Despite this constraint, it has been shown that any two bipartite unitary operations can be perfectly discriminated by LOCC, when multiple queries to the unitary operations are allowed \[7\] \[8\] \[9\] \[10\] \[11\] \[12\].

All the above mentioned works focus on the perfect discrimination of unitary operations. If a failure probability can be tolerated, then the problem can be discussed in two more general cases: minimum-error discrimination where a non-zero probability of the identification result being erroneous is tolerated, and unambiguous discrimination where the the identification result is always correct but we are not always guaranteed a conclusive result. Specifically, in the minimum-error discrimination case, two unitary operations \(U\) and \(V\) are said to be discriminated with bounded error \(\epsilon\), if each of them can be identified
correctly with probability more than $1 - \epsilon$; in the unambiguous discrimination case, $U$ and $V$ are said to be discriminated with one-sided error $\epsilon$, if for each one, either it is identified correctly, or the discrimination procedure output with probability less than $\epsilon$ the inconclusive result "I do not know".

Now suppose a unitary operation $X$ is selected from $\{U_1, U_2\}$ and can be accessed multiple times. Then a general procedure to identify $X$ (or discriminate between $U_1$ and $U_2$) is depicted in Fig. 3, where one adaptively performs a series of $X$ and some other auxiliary operations to an initial state and then performs a measurement with a result as the identifier of $X$. The problem considered in this paper is: How many queries are required for achieving a desired failure probability $\epsilon$ of discrimination between $U_1$ and $U_2$. We prove in a uniform framework that (i) if $U_1$ and $U_2$ are discriminated with bounded error $\epsilon$ (resp. with one-sided error $\epsilon$), then they need to be accessed by at least $\left\lceil 2\sqrt{1 - 4\epsilon(1 - \epsilon)} \right\rceil$ queries (resp. $\left\lceil 2\sqrt{1 - \epsilon^2} \right\rceil$ queries).

The rest of this paper is organized as follows. Section 2 present some preliminary knowledge especially for fidelity of two unitaries. The main result is presented in Section 3. A conclusion is made in Section 4.

2. Preliminaries

One can refer to an excellent textbook [26] for the details of quantum computation and quantum information. According to quantum mechanics, the evolution of a closed quantum system is described by a unitary operation (transformation). An operation $U$ is said to be unitary if $UU^\dagger = U^\dagger U = I$ where $U^\dagger$ denotes the adjoint of $U$. The fidelity of two unitary operations $U_1$ and $U_2$ is

$$ F(U_1, U_2) = \min_{|\psi\rangle} |\langle \psi | U_1^\dagger U_2 | \psi \rangle|, $$

(1)

\footnote{In mathematical form, the fidelity should be defined as $F(U, V) = \min_{|\psi\rangle} |\langle \psi | U^\dagger V \otimes I | \psi \rangle|$, but it is easy to see that the minimum value is achieved in Eq. (1).}
If we let $U_1^U_2 = \sum_{j=1}^k e^{i\theta_j} |j\rangle \langle j|$, and $|\psi\rangle = \sum_j \lambda_j |j\rangle$ with $\sum_j |\lambda_j|^2 = 1$, then there is

$$F(U_1, U_2) = \min \left\{ \left| \sum_j |\lambda_j|^2 e^{i\theta_j} \right| : \sum_j |\lambda_j|^2 = 1 \right\}. \quad (2)$$

Denote the convex hull of $S = \{e^{i\theta_j}\}_j$ by

$$\text{conv}(S) = \left\{ \sum_j |\lambda_j|^2 e^{i\theta_j} : \sum_j |\lambda_j|^2 = 1 \right\}. \quad (3)$$

Then the fidelity can be represented by

$$F(U_1, U_2) = \min_{P \in \text{conv}(S)} ||O - P||, \quad (4)$$

which states that $F(U_1, U_2)$ corresponds to the minimum distance from the origin $O$ to the convex hull $\text{conv}(S)$.

In geometry, each $e^{i\theta_j}$ stands for a point on the unit circle in the complex plane. As shown in Fig. 1 let $P_j$ denote the point $e^{i\theta_j}$ with $j = 1, \ldots, k$. Without loss of generality, assume the counter-clockwise order of these points on the unit circle is $P_1, P_2, \ldots, P_k$. Denote by $\odot P_1 \ldots P_k$ the region enclosed by
the convex polygon with endpoints $P_1, \ldots, P_k$. Then $P_1 \ldots P_k$ is the convex hull $\text{conv}(S)$. By this geometry representation, we have

$$F(U_1, U_2) = \begin{cases} 
|OM| & O \not\in \text{conv}(S), \\
0 & O \in \text{conv}(S).
\end{cases}$$

(5)

Let $\Theta(W)$ denote the length of the smallest arc containing all the eigenvalues of unitary operation $W$ on the unit circle (depicted in Fig. 2). Then $F(U_1, U_2)$ can be rewritten as

$$F(U_1, U_2) = \begin{cases} 
\cos \frac{\Theta(U_1^\dagger U_2)}{2} & 0 \leq \Theta(U_1^\dagger U_2) < \pi, \\
0 & \Theta(U_1^\dagger U_2) \geq \pi.
\end{cases}$$

(6)

The trace norm of operator $A$ is defined as $\|A\|_{tr} = \text{Tr} \sqrt{A^\dagger A}$. In this way, $\|\langle \varphi | - |\phi \rangle\|_{tr}$ denotes the trace distance between $|\varphi\rangle$ and $|\phi\rangle$. For simplicity, we denote it by $\|\langle \varphi | - |\phi \rangle\|_{tr}$ throughout this paper, and it can be verified that

$$\|\langle \varphi | - |\phi \rangle\|_{tr} = 2 \sqrt{1 - (\langle \varphi | \phi \rangle)^2}.$$  

(7)

### 3. Lower bound on query complexity

A general procedure to discriminate two unitary operations $U_1$ and $U_2$ can be seen as a sequence of unitaries $W_T U_i W_{T-1} U_i \cdots W_1 U_i W_0$, where $W_k$’s are fixed unitaries and $T$ is called the query complexity to $U_i (i = 1, 2)$. As shown in Fig. 3 the discrimination process is as follows:
Figure 3: A $T$-query procedure to discriminate two unitary operations. $U_i (i = 1, 2)$ are the unitaries to be discriminated. $W_0, W_1, \ldots, W_T$ are some fixed unitaries. $T$ is called the query complexity.

1. Start with an initial state $|\varphi_0\rangle$.

2. Perform the operators $W_0, U_i, W_1, U_i, \ldots, W_T$ in sequence, and then obtain the following state corresponding to the chosen $U_i$:

$$|\Phi_i\rangle = W_T(U_i \otimes I)W_{T-1}(U_i \otimes I)\cdots W_0|\varphi_0\rangle. \quad (8)$$

3. A 2-outcome measurement is performed on $|\Phi_i\rangle$ with a result as the identifier of $U_i$.

Definition 1. (i) $U_1$ and $U_2$ are said to be discriminated with bounded error $\epsilon$, if there exists a procedure as shown in Fig. 3 and a POVM measurement $\{\Pi_1, \Pi_2\}$ such that $\langle \Phi_i | \Pi_i | \Phi_i \rangle \geq 1 - \epsilon$ for $i = 1, 2$.

(ii) $U_1$ and $U_2$ are said to be discriminated with one-sided error $\epsilon$, if there exists a procedure as shown in Fig. 3 and a POVM measurement $\{\Pi_0, \Pi_1, \Pi_2\}$ such that $\langle \Phi_2 | \Pi_1 | \Phi_2 \rangle = \langle \Phi_1 | \Pi_2 | \Phi_1 \rangle = 0$ and $\langle \Phi_i | \Pi_0 | \Phi_i \rangle \leq \epsilon$ for $i = 1, 2$.

The problem considered in this paper is: How many queries are required for achieving a desired error $\epsilon$ of discrimination between $U_1$ and $U_2$. Our main result is given in Theorem 1. In order to prove that, some intermediate results (i.e., Lemma 1, 2 and 3) are required.

Lemma 1. (i) If $U_1$ and $U_2$ can be discriminated with bounded error $\epsilon$, then $|\langle \Phi_1 | \Phi_2 \rangle| \leq 2\sqrt{\epsilon(1-\epsilon)}$; (ii) If $U_1$ and $U_2$ can be discriminated with one-sided error $\epsilon$, then $|\langle \Phi_1 | \Phi_2 \rangle| \leq \epsilon$. 

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Proof 1. Item (i) is essentially a known and widely used result. For completeness, we give a proof in the following. If we denote 
\[ P_s = \langle \Phi_1 | (I - \Pi_2) | \Phi_1 \rangle + \langle \Phi_2 | \Pi_2 | \Phi_2 \rangle, \]
then we have
\[ P_s = 1 + \text{Tr} (\Pi_2 (|\Phi_2 \rangle \langle \Phi_2 | - |\Phi_1 \rangle \langle \Phi_1 |)). \tag{9} \]
Similarly, we have
\[ P_s = 1 - \text{Tr} (\Pi_1 (|\Phi_2 \rangle \langle \Phi_2 | - |\Phi_1 \rangle \langle \Phi_1 |)). \tag{10} \]
Thus there is
\[ P_s = 1 + \frac{1}{2} \text{Tr} ((\Pi_2 - \Pi_1) (|\Phi_2 \rangle \langle \Phi_2 | - |\Phi_1 \rangle \langle \Phi_1 |)). \tag{12} \]
\[ \leq 1 + \frac{1}{2} \|\Phi_2\|_2 (|\Phi_2 \rangle \langle \Phi_2 | - |\Phi_1 \rangle \langle \Phi_1 |) \|_\text{tr} \tag{13} \]
\[ = 1 + \sqrt{1 - |\langle \Phi_1 | \Phi_2 \rangle|^2}. \tag{14} \]
where the inequality holds by observing that 
\[ |\Phi_2 \rangle \langle \Phi_2 | - |\Phi_1 \rangle \langle \Phi_1 | = Q - S \]
for two positive operators and \[ \|\Phi_2\|_2 (|\Phi_2 \rangle \langle \Phi_2 | - |\Phi_1 \rangle \langle \Phi_1 |) \|_\text{tr} = \text{Tr}(Q) + \text{Tr}(S). \]
On the other hand, by item (i) of Definition 1, there is \( P_s \geq 2(1 - \epsilon) \). Thus we get 
\[ |\langle \Phi_1 | \Phi_2 \rangle| \leq 2\sqrt{\epsilon(1 - \epsilon)}. \]

It is easy to see item (ii) by observing the following:
\[ |\langle \Phi_1 | \Phi_2 \rangle| = |\langle \Phi_1 | (\Pi_0 + \Pi_1 + \Pi_2) | \Phi_2 \rangle| \]
\[ \leq |\langle \Phi_1 | \Pi_0 | \Phi_2 \rangle| + |\langle \Phi_1 | \Pi_1 | \Phi_2 \rangle| + |\langle \Phi_1 | \Pi_2 | \Phi_2 \rangle| \]
\[ \leq \sqrt{\langle \Phi_1 | \Pi_0 | \Phi_1 \rangle} \sqrt{\langle \Phi_2 | \Pi_0 | \Phi_2 \rangle} \]
\[ \leq \sqrt{\epsilon} = \epsilon \]
where the second inequality follows from \[ |\langle \Phi_1 | \Pi_1 | \Phi_2 \rangle| = |\langle \Phi_1 | \Pi_2 | \Phi_2 \rangle| = 0 \] (implied in item (ii) of Definition 1) and the application of Cauchy-Schwarz inequality to \[ |\langle \Phi_1 | \Pi_0 | \Phi_2 \rangle|. \]

For a T-query discrimination procedure as shown in Fig. 3 denote
\[ |\varphi^k_i \rangle = W_k (U_i \otimes I) W_{k-1} (U_i \otimes I) \cdots W_0 |\varphi_0 \rangle. \tag{15} \]
for \( i = 1, 2 \), and \( k = 0, 1, \ldots, T \). Note that \(|\varphi_i^0\rangle = W_0|\varphi_0\rangle\) and
\[ |\varphi_i^{k+1}\rangle = W_{k+1}(U_i \otimes I)|\varphi_i^k\rangle. \] (16)

Denote
\[ D_k = \| |\varphi_1^k\rangle - |\varphi_2^k\rangle \|_{\text{tr}} = 2\sqrt{1 - |\langle \varphi_1^k | \varphi_2^k \rangle|^2}. \] (17)

Then we obtain the following crucial result.

**Lemma 2.** \( D_0 = 0 \) and \( D_{k+1} \leq D_k + 2\sqrt{1 - F^2(U_1, U_2)} \) for \( k = 0, 1, \ldots, T - 1 \).

**Proof 2.** First it is easy to see that \( D_0 = 0 \). For \( D_{k+1} \) we have
\[
D_{k+1} = \| |\varphi_1^{k+1}\rangle - |\varphi_2^{k+1}\rangle \|_{\text{tr}} \\
= \| W_{k+1}(U_1 \otimes I)|\varphi_1^k\rangle - W_{k+1}(U_2 \otimes I)|\varphi_2^k\rangle \|_{\text{tr}} \\
= \| (U_1 \otimes I)|\varphi_1^k\rangle - (U_2 \otimes I)|\varphi_2^k\rangle \|_{\text{tr}} \\
= \| (U_1 \otimes I)|\varphi_1^k\rangle - (U_1 \otimes I)|\varphi_2^k\rangle + (U_1 \otimes I)|\varphi_2^k\rangle - (U_2 \otimes I)|\varphi_2^k\rangle \|_{\text{tr}} \\
\leq \| |\varphi_1^k\rangle - |\varphi_2^k\rangle \|_{\text{tr}} + \| (U_1 \otimes I)|\varphi_2^k\rangle - (U_2 \otimes I)|\varphi_2^k\rangle \|_{\text{tr}} \\
= D_k + 2\sqrt{1 - |\langle \varphi_2^k | U_1^\dagger U_2 \otimes I|\varphi_2^k \rangle|^2} \\
\leq D_k + 2\sqrt{1 - F^2(U_1, U_2)},
\]
where the first inequality follows from the triangle inequality and the unitary invariance of trace distance. The last inequality follows from the fact that \( F(U, V) \leq |\langle \Phi | U^\dagger V \otimes I |\Phi \rangle| \) for any \(|\Phi\rangle\) which is implied by the definition in Eq. (3).

**Lemma 3.** Suppose \( U_1, U_2 \) are discriminated by using \( T \) queries. Then we have:
(i) if \( U_1, U_2 \) are discriminated with bounded error \( \epsilon \), then \( D_T \geq 2\sqrt{1 - 4\epsilon(1 - \epsilon)} \);
(ii) if \( U_1, U_2 \) are discriminated with one-sided error \( \epsilon \), then \( D_T \geq 2\sqrt{1 - \epsilon^2} \).

**Proof 3.**
\[
D_T = \| |\varphi_1^T\rangle - |\varphi_2^T\rangle \|_{\text{tr}} = \| |\Phi_1\rangle - |\Phi_2\rangle \|_{\text{tr}} \\
= 2\sqrt{1 - |\langle \Phi_1 | \Phi_2 \rangle|^2}.
\]
Combining the above formula with Lemma 2, we get the result.
Now we are in the position to present our main result and its proof.

**Theorem 1.** Suppose $U_1, U_2$ are discriminated by using $T$ queries. Then we have:

(i) if they are discriminated with bounded error $\epsilon$, then $T \geq \left\lceil \frac{2\sqrt{1 - 4\epsilon(1 - \epsilon)}}{\Theta(U_1^\dagger U_2)} \right\rceil$;

(ii) if they are discriminated with one-sided error $\epsilon$, then $T \geq \left\lceil \frac{2\sqrt{1 - \epsilon^2}}{\Theta(U_1^\dagger U_2)} \right\rceil$.

**Proof 4.** Below we prove the result for case (i): $U_1, U_2$ are discriminated with bounded error $\epsilon$. The other case can be proved similarly. First, by Lemma 2 we have

$$D_T = (D_T - D_{T-1}) + (D_{T-1} - D_{T-2}) + (D_1 - D_0) + D_0 \leq 2T \sqrt{1 - F^2(U_1, U_2)}.$$ 

Then by Lemma 3, there is

$$2\sqrt{1 - 4\epsilon(1 - \epsilon)} \leq D_T \leq 2T \sqrt{1 - F^2(U_1, U_2)},$$

which leads to

$$T \geq \sqrt{\frac{1 - 4\epsilon(1 - \epsilon)}{1 - F^2(U_1, U_2)}}. \quad (19)$$

If $F(U_1, U_2) = 0$, then $U_1$ and $U_2$ can be perfectly discriminated with one query. Thus, we need only consider the case of $F(U_1, U_2) \neq 0$. Then by Eq. (6), we have $F(U_1, U_2) = \cos \frac{\Theta(U_1^\dagger U_2)}{2}$, which leads to

$$\sqrt{1 - F^2(U_1, U_2)} = \sqrt{1 - \cos^2 \frac{\Theta(U_1^\dagger U_2)}{2}} = \sin \frac{\Theta(U_1^\dagger U_2)}{2} \leq \frac{\Theta(U_1^\dagger U_2)}{2}. \quad (21)$$

Combining (21) and (19), we obtain

$$T \geq \frac{2\sqrt{1 - 4\epsilon(1 - \epsilon)}}{\Theta(U_1^\dagger U_2)}. \quad (23)$$

**Remark 1.** If $U_1, U_2$ are required to discriminated perfectly (that is, $\epsilon = 0$), then both the two cases lead to $T \geq \frac{2}{\Theta(U_1^\dagger U_2)}$. This is consistent with the result
given in [6] that two unitaries $U_1, U_2$ can be discriminated perfectly by making queries to the unitaries $\left\lceil \frac{\pi}{\Theta(U_1 U_2)} \right\rceil$ times.

Compared with a related work [15], two main differences are as follows: (i) We have obtained the query complexity lower bounds in a uniform framework for both the bounded error case and the one-sided error case, whereas Ref. [15] considered only the former case. (ii) We have presented a lower bound for an arbitrary $\epsilon$, but the proof in [15] has dependence on the specific value $\epsilon = \frac{1}{3}$.

4. Conclusion

We have considered the query complexity of discrimination between two unitary operations $U$ and $V$. It is proved that (i) for minimum-error discrimination, at least $\left\lceil \frac{2\sqrt{1-4\epsilon(1-\epsilon)}}{\Theta(U V)} \right\rceil$ queries are required to discriminate between $U$ and $V$ with bounded error $\epsilon$, and (ii) for ambiguous discrimination, at least $\left\lceil \frac{2\sqrt{1-\epsilon^2}}{\Theta(U V)} \right\rceil$ queries are required to discriminate between them with one-sided error $\epsilon$. The proof is presented in a uniform framework.

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