Non-supersymmetric String Solitons

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We begin a search for non-supersymmetric/supersymmetric dual string pairs by constructing candidate critical non-supersymmetric strings as solitons in supersymmetric string theories. Using orbifold techniques, one can construct cosmic string solutions which lie in supersymmetric vacua but which do not fall in supermultiplets. We discuss two three-dimensional examples in detail. The effective worldsheet actions for the soliton strings have (0,2) and (1,1) supersymmetry and the correct numbers of massless bosons and fermions to be critical heterotic and type II strings, respectively.

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1. Introduction

It is a well-known fact in string theory that worldsheet supersymmetry alone does not imply spacetime supersymmetry. In this paper, we attempt to exploit this fact by constructing nonsupersymmetric strings as solitons in supersymmetric string vacua. In six dimensions, a solitonic solution of type IIA string theory compactified on \( K3 \) has been found with an effective worldsheet action identical to critical heterotic string theory compactified on \( T^4 \) \( [1] [2] \) (a similar construction has been given of the 10d \( SO(32) \) heterotic string as a soliton in the Type I theory \( [3] [4] \)). In this construction, the spacetime supersymmetry left unbroken by the soliton leads to worldsheet supersymmetry on the cosmic string, while the normalizable broken supercharges create the superpartners of the soliton \( [5] [2] \).

Our purpose here is to present a similar (though much weaker) construction of critical string solitons in supersymmetric theories, with unbroken spacetime supersymmetry leading to worldsheet supersymmetry, but without the broken supersymmetry generators which would have produced target space supersymmetry. The absence of broken supercharges for string solitons which nevertheless have finite action is a feature of the stringy cosmic string and orbifold cosmic string constructions of \( [6] \) and \( [7] \), and we use essentially the methods of \( [7] \) to construct our examples (a field theory example of this kind of phenomenon based on \( [8] \) was studied in \( [9] \)). In these constructions, the spacetime and internal degrees of freedom are tied together in such a way as to produce a noncompact manifold admitting some number of covariantly constant spinors. In order to have well-defined broken supersymmetries, there must be additional spinor fields which are not covariantly constant but which asymptote to covariantly constant spinors far away from the string core. Such spinors do not exist in the orbifold and noncompact Calabi-Yau constructions. Still, as explained by Bagger, Callan, and Harvey for the orbifold examples \( [7] \), far away from the string one recovers the full spectrum of the original vacuum. We will review this as it becomes relevant for us.

Recent work has demonstrated that the string-string duality relating 6d N=2 heterotic and type IIA vacua \( [10] [11] [12] \) (which implies the S-duality of 4d N=4 string theories \( [13] \)) extends also to 4d \( N = 2 \) examples \( [14] [15] \) and to some 4d \( N = 1 \) theories in which no superpotential is generated \( [16] [17] \). In some of the 4d \( N = 2 \) examples, the absence of neutral couplings between vector and hypermultiplets ensures that the duality allows one to compute quantum properties of one vacuum from the purely classical physics of
its dual. In $N < 2$ theories, one might hope that there is a duality in which the nonperturbative properties of one vacuum are manifest in the perturbative physics of another. The consequences of such a duality for dynamical $N=1$ theories would be extremely interesting. For four-dimensional $N=1$ models which break supersymmetry, as well as for three-dimensional supersymmetric models [8], such a structure would imply that we should expect non-supersymmetric duals for supersymmetric theories. This motivates us to search for candidate nonsupersymmetric dual strings in supersymmetric string vacua.

The construction presented here is preliminary and limited in several important respects. In the six-dimensional construction, the BPS mass formula for six-dimensional $N=2$ supersymmetry is sufficient to ensure that such a soliton becomes light at strong coupling and therefore provides a dual description of the original theory. In our case, as in any situation with $4d \, N \leq 2$ supersymmetry, the question is more delicate. It might require solving the theory at low energy to show convincingly that a soliton constructed at weak coupling becomes light anywhere. We also have little control over the effective worldsheet theory we obtain on the soliton beyond counting its modes (requiring a critical number) and counting the supersymmetries. However we believe that it is worth searching for circumstantial evidence for nonsupersymmetric/supersymmetric dual pairs through string solitons, since it is difficult to see what to compare in more direct approaches.

2. General Constraints

In which dimensions should we expect to find cosmic string solutions which do not fall into supermultiplets? Orbifold cosmic strings are constructed using a discrete group which acts simultaneously on the internal compactification and on the transverse spacetime dimensions [7], i.e. by taking orbifolds of $T^8$ or $T^{8-d} \times R^d$. This prescription always has the property that no broken supercharges are defined, as discussed above. However, it only produces a cosmic string geometry in at most four dimensions. So fortunately this construction can only work in four or fewer dimensions, where it is possible to break supersymmetry.

Starting from heterotic or Type I string theory, one could obtain for example $(0,2)$ supersymmetry on the string worldsheet by constructing an eight-dimensional orbifold with holonomy $SU(4)$, or $(0,1)$ supersymmetry from an eight-fold of holonomy Spin(7) (in Table 1 we assemble a list of the surviving supersymmetries for the various string theories and holonomy groups). However a stable solution is not obtained with just
any eight-dimensional orbifold which preserves spacetime supersymmetry [18]. The eight-
dimensional effective action contains the Green-Schwarz term $\int B X_8$ where $X_8$ is an eight-
dimensional differential form involving the Riemann tensor and the gauge field strengths [19]. A term like this arises in the Heterotic, Type I, and Type IIA theories, destabilizing solitons for which it does not vanish. In particular it does not vanish for generic eight-
manifolds with $SU(4)$ or Spin(7) holonomy. There is another potentially destabilizing effect, discussed in [7]–the one-loop contribution to the two-dimensional vacuum energy, proportional to the difference between the number of massless bosons and fermions. We will consider one example which is unstable at one loop, and one in which neither of these effects arises. When such terms arise, one plausible possibility is that the configuration relaxes to a stable one with the same worldsheet structure as the unstable one [7].

| String Theory | SU(4) | G$_2$ | Spin(7) |
|---------------|-------|-------|---------|
| Heterotic/Type I | (0, 2) | (1, 1) | (0, 1) |
| Type IIA    | (2, 2) | (2, 2) | (1, 1) |
| Type IIB   | (0, 4) | (2, 2) | (0, 2) |

**Table 1:** Worldsheet SUSY of soliton strings coming from string compactifications on manifolds with various holonomy groups.

In order to guarantee the absence of supersymmetry in the dual theory, one can take the soliton worldsheet to have only (0,1) or (1,1) supersymmetry. The critical (0,2) and (2,2) cases are more delicate. We will consider a (0,2) example in section 3 and a geometrically puzzling (1,1) example in section 4. Because of the absence of broken supercharges and the corresponding lack of superpartners for the cosmic strings, one would expect the (0,2) and (2,2) examples also to fail to have target space supersymmetry. That would imply that the U(1) R-charges are not integrally quantized on such a soliton worldsheet. We do not yet have enough control over the worldsheet theory to check this for the example in section 3.

### 3. A (0,2) String in a Type IIB Vacuum

In this section we discuss an example of a (0,2) string soliton in a 3d Type IIB vacuum (with the string wrapped around one of the internal seven dimensions). We will find 8 nonchiral bosonic modes (along with their (0,2) superpartners) as well as 16 chiral
bosons on the soliton worldsheet. Begin with $Z_2 \times Z_2$ generators acting on the spacetime coordinates $x^1, \ldots, x^9$ as follows:

$$A : x \rightarrow (-x^1, -x^2, -x^3, -x^4, x^5, x^6, x^7, x^8, x^9) \quad (1)$$

$$C : x \rightarrow (-x^1, \frac{1}{2} - x^2, x^3, x^4, \frac{1}{2} - x^5, \frac{1}{2} - x^6, x^7, x^8, x^9) \quad (2)$$

We take $x_{7,8}$ to be noncompact – they will play the role of the conical geometry transverse to the string momentarily. If we compactify $x^9$, the direction along which the string lies, we have so far a 3d $N = 4$ vacuum. We will see later that the compactification of $x^9$ is necessary to obtain a critical number of worldsheet modes. To obtain a string, we include another $Z_2$ as follows:

$$B : x \rightarrow (x^1, x^2, x^3, x^4, -x^5, -x^6, -x^7, -x^8, x^9) \quad (3)$$

These three generators give an eightfold of $SU(4)$ holonomy considered by Joyce [20]. So far the worldsheet supersymmetry is $(0,4)$. The effective worldsheet theory is populated by the states tied to the string, namely the states from the $B$ and $AB$ twisted sectors (there are no massless states from the $BC$ sector). We can count these states using dimensional reduction on the blown up orbifold described by Joyce [20]. The (NS, NS) sector contributes the metric and antisymmetric tensor fields (as well as the dilaton, which does not lead to new states under dimensional reduction). As discussed in [20], the $B$ sector blow up introduces three extra metric deformations and one additional two-form per fixed torus. $B$ has four fixed tori to begin with but $C$ exchanges them in two pairs of two. So we have a total of eight modes on the string from the (NS, NS) $B$ sector. These modes have no potential on the worldsheet since they are (2,2) moduli of the original string theory.

The (R,R) sector contributes another two-index antisymmetric tensor field and a self-dual four-form. The vacuum in the (R,R) $B$ sector is an $SO(5,1)$ spinor from the fermions corresponding to $x^1, \ldots, x^4, x^9, x^{10}$. The GSO and BRST projections reduce this to states which have the same $SO(4)$ and $SO(1,1)$ chiralities on left and right, where the $SO(4)$ acts on $x^1, \ldots, x^4$ and the $SO(1,1)$ on $x^9$ and $x^{10}$. These are invariant under the $A$ projection, which acts the same on the left and right $SO(4)$ spinors. These massless states can be projected out by another $Z_2$ generated by $D$: $D$ acts similarly to $A$ except that it acts oppositely on the left and right-moving $SO(4)$ spinors. In order to avoid massless states in the $D$ twisted sectors, we include a shift on $x^9$. We also include a shift on $x^1$ in addition to the minus sign on that coordinate which will give us the desired degeneracy in the
$AB$ sector. $D$ projects out the massless states from the (R,R) and (R, NS) $B$ sectors, as well as projecting out the gravitinos coming from the left-movers. This projection thus leaves us with the 8 (NS, NS) $B$ sector bosons counted above while reducing the vacuum supersymmetry to $3d N = 2$ and the soliton worldsheet supersymmetry to (0,2).

What about the chiral modes on the heterotic soliton worldsheet? These come from the (R,R) $AB$ sector. Here the vacuum is unique, and before the $C$ and $D$ identifications there are $2^6 = 64$ fixed points. Because of their shifts, $C$ and $D$ reduce this by a factor of 4, leaving 16 bosons from the $AB$ sector. That these modes are chiral follows from the fact that the $AB$ NS sectors have positive vacuum energy ($E_{NS}^{AB} = 1/2$). So there are no supersymmetric partners of the 16 (R,R) bosons from the $AB$ sector, indicating that they are on the nonsupersymmetric side of the candidate critical heterotic soliton string worldsheet. These modes can also be understood from dimensional reduction: as explained in [20], blowing up an $AB$ fixed point introduces one additional self-dual four-form, leaving all the other betti numbers fixed. This leads to one additional scalar per fixed point upon dimensional reduction of Type IIB string theory.

As discussed in [7], away from the string we recover the original vacuum spectrum. Given a vertex operator $V$ which is antiinvariant under $B$, there is an associated invariant vertex operator $\tilde{V} = (e^{i(p_7x_7+p_8x_8)} - e^{-i(p_7x_7+p_8x_8)})V$. Because $x^7$ and $x^8$ are noncompact, the momenta $p_7$ and $p_8$ can chosen to yield a massless state ($p^2=0$). The vacuum in this theory perturbatively has only abelian gauge symmetry with no charged matter, and therefore uninteresting dynamics. However, as discussed in [8], in three dimensions massive particles do not fall into supermultiplets in any case. Therefore our candidate duality is possible.

4. A (1,1) String in a Heterotic Vacuum

We are now going to consider a candidate critical type II string, obtained from compactification of the heterotic string on a manifold of $G_2$ holonomy. While it is impossible to find cosmic strings in four spacetime dimensions by compactification on an orbifold of $G_2$ holonomy, we will now briefly explain that we can obtain three dimensional cosmic strings through compactification on $X \times S^1$ with $X$ a (noncompact) manifold of $G_2$ holonomy. It is important to make a distinction between this construction and the previous one, in which the string wound around a circle in the internal space but the transverse geometry consisted of a two-dimensional cone. In the present construction, the string will be intrinsically 2+1 dimensional with the transverse space being one dimensional.
We must first understand the classical geometry of a string in 2+1 dimensions. This was studied by Deser and Jackiw [21]. They found that a delta-function localized closed string with tension produces a geometry consisting of a capped half-infinite cylinder, with the string at the boundary of the cap. With tension, the mass is fixed at \( \mathcal{M} = G^{-1} \) (where \( G \) is Newton’s constant), yielding the 2\( \pi \) deficit angle.

The half-cylinder part of this spacetime, including the localized cosmic string, can be produced by the orbifold technique. Consider a compactification down to three dimensions on a Calabi-Yau manifold \( K \) times a circle, accounting for dimensions \( x^0 \) through \( x^6 \). Orbifold by a \( Z_2 \) which maps \( x^7 \rightarrow -x^7 \) and simultaneously acts by an involution on \( K \) such that the resulting eight-fold has holonomy \( G_2 \) (such examples were introduced by Joyce [22] and discussed in the context of conformal field theory in [23]). If we compactify \( x^8 \), we obtain a half-cylinder geometry. Furthermore, we have localized stress-energy at \( x^7 = 0 \) from the internal components of the metric, which appears to the three-dimensional observer as gradient energy from the moduli. So we have recovered the part of the Deser/Jackiw solution external to and including the string.

Classically, we could now patch on the flat geometry internal to the string (the cap of the half-cylinder), completing the geometry to a solution of the classical equations of motion. Quantum mechanically, the situation is less clear: In nonrelativistic quantum mechanics, propagation through a delta-function wall occurs perturbatively in Planck’s constant, and it is hard to see how this would be mirrored in our orbifold conformal field theory describing the half-cylinder. But it is not completely clear that this is the behavior in our case. If in our situation the tunneling through the string is nonperturbative, then the patching construction is valid, and we have a closed string in three dimensions in line with the solution of [21]. Without the patching, we can still interpret our construction as a closed string, but in a non-simply connected spacetime. In either case, we obtain a closed string instead of the infinitely extended cosmic strings which occur in the higher-dimensional dual string soliton constructions of [1] [2] [3] [4].

The example presented here is based upon modifications of one of the \( G_2 \) orbifolds constructed by Joyce [22], and we use some of his notation. We begin with the \( SO(32) \) heterotic string compactified on \( S^1 \times K \) where \( K \) is a Calabi-Yau manifold realized as a toroidal orbifold. Let \( x^0 \) be the coordinate along the circle and \( x^1, \ldots x^6 \) the coordinates on the \( T^6 \). The group elements producing \( K \) act on \( T^6 \) as follows:

\[
\alpha : x \rightarrow (-x^1, -x^2, -x^3, -x^4, x^5, x^6)
\] (4)
\[ \beta : x \rightarrow (-x^1, \frac{1}{2} - x^2, x^3, x^4, -x^5, -x^6) \]  
\[ s_1 : x \rightarrow (\frac{1}{2} + x^1, \frac{1}{2} + x^2, x^3, x^4, x^5, x^6) \]  
\[ s_2 : x \rightarrow (x^1, x^2, \frac{1}{2} + x^3, \frac{1}{2} + x^4, x^5, x^6) \]  
\[ s_3 : x \rightarrow (x^1, x^2, x^3, x^4, \frac{1}{2} + x^5, \frac{1}{2} + x^6) \]  

We use the standard embedding of these group elements into the gauge degrees of freedom. So far we have a three-dimensional \( N = 2 \) compactification on \( S^1 \times K \).

To produce the cosmic string, we include another group element \( \gamma \), which acts on one noncompact transverse dimension \( x^7 \) as well as on \( K \).

\[ \gamma : x \rightarrow (\frac{1}{2} - x^1, x^2, -x^3, x^4, -x^5, x^6, -x^7) \]  

We use the standard embedding for \( \gamma \) as well. The dimensions \( x^8 \) and \( x^9 \) form the cosmic string worldsheet. The orbifold produced by these group elements (which can be blown up to a manifold of \( G_2 \) holonomy [22]) leaves (1,1) supersymmetry on the string worldsheet. Our construction will leave 8 bosonic zero modes on the string worldsheet, consistent with the worldsheet action for a critical type II string. As explained above, we will compactify \( x^8 \) (and possibly insert a flat disc at \( x^7 = 0 \)) to obtain a closed string solution in 2+1 dimensions. The only group elements with fixed points (and therefore candidate massless states) are \( \alpha, \beta, \) and \( \gamma \).

The modes tied to the string come from the \( \gamma \) twisted sector [4]. We will later introduce one more group element, \( g \), to reduce the degeneracy of \( \gamma \)-twisted states to a critical number. As discussed in [4], despite the \( \gamma \) projection we recover the full vacuum spectrum away from the string in the untwisted, \( \alpha \), and \( \beta \) sectors. This is because the direction \( x^7 \) is noncompact. Given a vertex operator \( V \) with \( \gamma \) eigenvalue -1, we can construct another vertex operator \( \tilde{V} \equiv V(\exp(ip_7x^7) - \exp(-ip_7x^7)) \) for \( x^7 \neq 0 \) which is \( \gamma \)-invariant. Because \( p_7 \) is not quantized, we can arrange that \( p^2 = p_7^2 + p_8^2 - p_9^2 = 0 \) for a massless state.

To understand the string worldsheet action, we are interested in the states from the \( \gamma \) sector. Denote the left-moving real fermions \( \eta^I \) where \( I = 1, \ldots, 32 \) and the right-moving real fermions \( \psi^i \). For the bosons (right-moving Neveu-Schwarz sector) the right and left-moving vacuum energies in the \( \gamma \) sector are

\[ E_{R}^{NS} = 0 \]
\[ E_L^A = -1/2 \] (11)

where \( E_L^A \) is the left-moving vacuum energy for the sector in which the \( \eta^I \) are naturally antiperiodic. The periodic sector has positive vacuum energy and yields no massless states. Both left and right have 4 real fermion 0-modes in this sector. Group them into complex fermions \( \lambda_1 = \eta_1 + i\eta_3 \) and \( \lambda_2 = \eta_5 + i\eta_7 \) on the left and \( \chi_1 = \psi_1 + i\psi_3 \) and \( \chi_2 = \psi_5 + i\psi_7 \) on the right. There are also 4 twisted spacetime bosons yielding creation operators \( \alpha^{-1,3,5,7}_\lambda \).

Before the \( \alpha \) and \( \beta \) projections there are states of the following form

\[ S_+ \alpha^{-1,3,5,7}_- | -1/2 \rangle \otimes \tilde{S}_- |0\rangle \] (12)

and

\[ S_- \eta^I_- | -1/2 \rangle \otimes \tilde{S}_- |0\rangle \] (13)

where \( S_\pm \) is a \( \pm \)-chirality spinor from the \( \lambda_{1,2} \) 0-modes and \( \tilde{S}_- \) is a negative-chirality spinor from the \( \chi_{1,2} \) 0-modes. The \( S_+ \) spinor is obtained by acting with \( \bar{\lambda}_1 \) or \( \bar{\lambda}_2 \) on the vacuum \( | -1/2 \rangle \) killed by \( \lambda_{1,2} \); the \( S_- \) spinor has components \( | -1/2 \rangle \) and \( \bar{\lambda}_1 \bar{\lambda}_2 | -1/2 \rangle \). The I index runs over the 28 \( \eta \)s which are antiperiodic in the gamma sector. It is easy to see that these states are all \( \gamma \)-invariant.

It follows from (11) and (5) that \( \alpha \) acts with -1 on \( \bar{\lambda}_1 \) and \( \bar{\chi}_1 \) and with +1 on \( \bar{\lambda}_2 \) and \( \bar{\chi}_2 \), while \( \beta \) acts by anticonjugating all the \( \lambda \)s and \( \chi \)s. In order to form \( \alpha \) and \( \beta \)-invariant states we first need to find combinations of the left and right-moving spinor components that are eigenstates of both \( \alpha \) and \( \beta \). For states of the form (12), these are provided by the following combinations:

\[ V_\pm \equiv \bar{\lambda}_1 \otimes 1 \pm \bar{\lambda}_2 \otimes \bar{\chi}_1 \bar{\chi}_2 \] (14)

which has \( \alpha = -1 \) and \( \beta = \mp 1 \), and

\[ W_\pm \equiv \bar{\lambda}_1 \otimes \bar{\chi}_1 \bar{\chi}_2 \pm \bar{\lambda}_2 \otimes 1 \] (15)

which has \( \alpha = +1 \) and \( \beta = \mp 1 \). Using these we find invariant states \( \alpha^1_{-1/2} V_+, \alpha^2_{-1/2} V_-, \alpha^7_{-1/2} W_-, \) and \( \alpha^5_{-1/2} W_+ \). This gives four states on the string so far. (Recall from the shifts in (3), (4), and (5) that there is only one \( \gamma \) fixed point.)

What about states of the form (13)? First of all, let’s project a lot of them out by the following extra \( Z_2 \) generated by \( g \). Let \( \lambda_3 = \eta_2 + i\eta_4 \) and \( \lambda_4 = \eta_6 + i\eta_8 \). Have \( g \) act with -1 on \( S_- \) and on \( \lambda_3 \) and \( \lambda_4 \), and also a \( Z_2 \) shift on \( x^0 \) to avoid massless states in any
$g$-twisted sector. The action on $\lambda_3$ and $\lambda_4$ is included for level matching. Before the $\alpha$ and $\beta$ projections, this leaves states of the form

$$S_-(\lambda_{3,4}^{3,4} - 1/2) \otimes \tilde{S}_-(|0\rangle)$$

(16)

By making the analogous combinations as described above for the (12) states, we find another 4 states on the string from these. We have thus a total of 8 massless bosonic modes (along with their (1,1) fermionic superpartners) on our closed string world sheet.

It turns out that we can learn more than just the number of massless modes. We must further check that our purported zero modes are gauge-neutral and have a flat potential on the worldsheet. The gauge group in our three-dimensional vacuum given by the $\alpha$, $\beta$, $s_{1,2,3}$, and $g$ orbifold is

$$G = U(1)^2 \times SO(24) \times SO(2)^4.$$  

(17)

The first $U(1)^2$ factor comes from the dimensional reduction of the 4$d$ metric and antisymmetric tensor field on the circle $x^0$. Our $\gamma$-sector worldsheet states (12) and (16) are neutral under the $SO(24)$ factor here. The $SO(2)^4$ gauge generators arise from the worldsheet currents $\eta^1\eta^2$, $\eta^3\eta^4$, $\eta^5\eta^6$, and $\eta^7\eta^8$. Since these operators are odd under $\gamma$, the corresponding gauge boson vertex operators are constructed using the momentum $p_7$ and only exist away from the string. We will now demonstrate that our worldsheet modes are flat. If we leave the $\alpha$ and $\beta$ fixed points untouched (i.e. not turning on the blowing up modes from these sectors) this follows from the fact that the untwisted and $\gamma$-sector scalars form a subset of the scalar spectrum of heterotic string theory compactified on $K3$. That is, $\gamma$ and $g$ alone produce a compactification on $K3$. The states from the untwisted and $\gamma$ sectors which survive the further projections onto $\alpha$, $\beta$, and $s_{1,2,3}$-invariant states form a subset of the $K3$ spectrum. These states inherit their correlation functions from the $Z_2$ orbifold $K3$ given by $\gamma$ and $g$, which is governed by $N = 2$ supersymmetry in four dimensions. The scalar potential arises from a highly constrained superpotential and from $U(1)$ $D$-terms, in $N = 1$ language. The superpotential arises for gauge-charged hypermultiplets and is a cubic coupling of the adjoint chiral multiplet $a$ with the two chiral multiplets $\phi$ and $\tilde{\phi}$ in the hypermultiplet. Our spectrum is not the full $K3$ spectrum but is truncated by the $\alpha$ and $\beta$ projections. This means in particular that if $\phi$ is an invariant chiral multiplet then its $N = 2$ partner $\tilde{\phi}$ will not appear in the spectrum, as $\alpha$ removes one of the two supersymmetries. Therefore as long as $a = 0$, there is no superpotential preventing us from giving the soliton worldsheet modes arbitrary VEVs.
We should also make sure that the worldsheet modes are not obstructed by $D$-terms. With just the $\gamma$ and $g$ projections, one has a gauge group $SO(4)^2 \times SO(24)$ in the $K^3$ theory under discussion. (Note that this spectrum of gauge bosons has nothing to do with that of our vacuum theory generated by $\alpha$, $\beta$, $s_{1,2,3}$ and $g$ given in (I7); we are considering it simply as a shortcut to determining the flat directions among the untwisted and $\gamma$-sector states). All our $\gamma$-sector scalars are invariant under the $SO(24)$ factor. Consider a $U(1)^4$ subgroup of $SO(4)^2$ which is generated by the worldsheet currents $\eta_1 \eta_3$, $\eta_5 \eta_7$, $\eta_2 \eta_4$, and $\eta_6 \eta_8$. Turning on our worldsheet modes leads to nonzero $D$-terms only for these generators; this is also the case for the VEVs we will turn on for vector representations in the untwisted sector. Our set of worldsheet states (12) and (16) includes states charged under each of these $U(1)$s. Before the $\beta$ projection, in the untwisted sector we find scalars of positive and negative charge under each $U(1)$. For example we find the 16 states

$$(\eta^1 \pm i\eta^3)^{-\frac{1}{2}} (\eta^6 \pm i\eta^8)^{-\frac{1}{2}} | -1 \rangle \otimes (\psi^1 \pm i\psi^3)^{-\frac{1}{2}} | -\frac{1}{2} \rangle$$

(18)

$$(\eta^5 \pm i\eta^7)^{-\frac{1}{2}} (\eta^2 \pm i\eta^4)^{-\frac{1}{2}} | -1 \rangle \otimes (\psi^1 \pm i\psi^3)^{-\frac{1}{2}} | -\frac{1}{2} \rangle$$

(19)

This allows us to balance the $D$-terms while leaving the $\gamma$-sector scalars free to vary. Now, $\beta$ projects out half of each $U(1)$ representation, preserving only the real or imaginary part from the scalar component of each chiral multiplet. The $D$-terms can still be made to vanish while allowing the worldsheet scalars to vary freely by adjusting the VEVs of the untwisted charged fields. So we have 8 worldsheet scalars which appear to lie on some real subspace of the moduli space of $K3$. It is not clear what the global structure of this worldsheet sigma model is. If it flows to a nontrivial conformal field theory in the infrared, then we have constructed a critical nonsupersymmetric type II string as a soliton in a three-dimensional $N = 2$ compactification of heterotic string theory.

As mentioned in §2, we need to discuss the stability of our cosmic string when quantum corrections are included. In [7] the one-loop correction to the vacuum energy for such orbifold cosmic strings, which should be interpreted as a loop correction to the string tension (since it is independent of the size of the transverse dimensions), was evaluated and found to be proportional to the difference between the numbers of bosons and fermions in two dimensions. Because our compactification leaves (1,1) supersymmetry in two dimensions, this vanishes.\footnote{In order to use the computation of [7], we should strictly speaking compactify $x_7$ as well. This would introduce another string soliton, but there would still be a critical number of worldsheet modes on both string solitons.} So at least at one loop we do not need to worry about string decay through
dilaton emission. Also the part of $X_8$ depending only on the Riemann curvature tensor vanishes for our solution, since there is one transverse flat circle ($x^0$). Thus our solution is stable to one loop.

Now that we have found a candidate critical nonsupersymmetric string soliton in a $3d \ N = 2$ heterotic vacuum, we should discuss in a bit more detail the physics of this vacuum. As discussed above, the standard embedding of $\alpha$, $\beta$, and $\gamma$ combined with the extra $g$-projection reduces the gauge group to $U(1)^2 \times SO(24) \times SO(2)^4$. All states in the $\alpha$ and $\beta$ sectors are uncharged under the $SO(24)$ factor. The states (12) and (16) are invariant under $SO(24)$. The extra projection onto $g$-invariant states removes the fundamental matter representation of $SO(24)$ from the untwisted spectrum. The four-dimensional vacuum obtained by taking $x^0$ large consists of pure $SO(24) \ N = 1$ supersymmetric Yang-Mills theory with some additional neutral chiral multiplets, a theory which we do not expect to spontaneously break SUSY. This 4d theory is a limit in which our soliton string becomes a two-brane whose properties are not well understood. Compactifying $x^0$ to an $S^1$ produces scalars in the adjoint representation of $SO(24)$, yielding a $3d \ N = 2$ $SO(24)$ Yang-Mills theory with some additional neutral fields. We also have no reason to expect this three-dimensional theory to spontaneously break supersymmetry [24]. Nevertheless [8], the massive states in such a theory do not fall into supermultiplets and our candidate duality between this theory and a nonsupersymmetric one is possible.

5. Conclusions

It is interesting that we naturally find ourselves working in 2+1 dimensions when we search for candidate critical nonsupersymmetric string solitons in supersymmetric string vacua. In particular, it has recently been suggested that a nonsupersymmetric world in four dimensions could be dual to a supersymmetric world in three dimensions [8]. Such a relation would provide a natural explanation of the vanishing of the cosmological constant.

It is possible that our construction is providing a realization of such ideas, but in order to verify this one would need a better understanding of the worldsheets theories on the soliton string. To begin with, one would need to gain better control over (1,1) conformally invariant sigma models in general, as well as (0,2) models with fractional R-charges. Independent of any relation to the ideas of [8], it seems to us that the nonsupersymmetric string solitons discussed here should be interpreted as evidence that there are indeed nonsupersymmetric/supersymmetric dual pairs of string vacua awaiting further exploration.
A related issue is the role in supersymmetry breaking, if any, of the plethora of noncritical nonsupersymmetric solitons in string theory.

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