Fresh inflation with nonminimally coupled inflaton field

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Abstract

I study a fresh inflationary model with a scalar field nonminimally coupled to gravity. An example is examined. I find that, as larger is the value of $p \left( a \sim t^p \right)$, smaller (but larger in its absolute value) is the necessary value of the coupling $\xi$ to the inflaton field fluctuations can satisfy a scale invariant power spectrum.

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I. INTRODUCTION

The inflationary model is one of the most promising, for the early stage of the universe in modern cosmology [1,2]. It not only gives a natural explanation for the horizon, flatness, and monopole problems but also provides density perturbations as seeds for the large scale
structure in the universe. The standard inflationary period proceeds while a scalar field called an inflaton slowly evolves along a sufficiently flat potential.

The standard slow-roll inflation model separates expansion and reheating into two distinguished time periods. It is first assumed that exponential expansion from inflation places the universe in a supercooled second order phase transition. Subsequently thereafter the universe is reheated. Two outcomes arise from such a scenario. First, the required density perturbations in this cold universe are left to be created by the quantum fluctuations of the inflaton. Second, the scalar field oscillates near the minimum of its effective potential and produces elementary particles. These particles interact with each other and eventually they come to a state of thermal equilibrium at some temperature $\theta$. This process completes when all the energy of the classical scalar field transfers to the thermal energy of elementary particles. The temperature of the universe at this stage is called the reheating temperature $\theta$. From a viewpoint of quantum field theory in curved spacetime, it is natural to consider that the inflaton field $\phi$ couples nonminimally to the spacetime curvature $R$ with a coupling of $\xi R\phi^2/2$. For example, in the new inflationary model, the existence of nonminimal coupling prevents inflation in some cases, because the flatness of the potential of inflation is destroyed around $\phi = 0$. In the chaotic inflation model, the coupling $\xi$ in potentials like $V(\phi) = \frac{m^2}{2}\phi^2$ and $V(\phi) = \frac{\lambda^2}{4}\phi^4$ is restricted to $|\xi| < 10^{-3}$.

Very recently a new model of inflation called fresh inflation was proposed. As in chaotic inflation, in this model the universe begins from an unstable primordial matter field perturbation with energy density nearly $M_p^4$ and chaotic initial conditions. Initially the universe there is no thermalized $[\rho_r(t = t_0) = 0]$. Later, the universe describes a second order phase transition, and the inflaton rolls down slowly towards its minimum energetic configuration. Particles production and heating occur together during the inflationary expansion of the universe, so that the radiation energy density grows during fresh inflation ($\dot{\rho}_r > 0$). The Yukawa interaction between the inflaton field and other fields in a thermal bath lead to dissipation which is responsible for the slow rolling of the inflaton field. So, the slow-roll conditions are physically justified and there are not a requirement of a nearly flat
potential in fresh inflation. Furthermore, there is no oscillation of the inflaton field around the minimum of the effective potential due to the strong dissipation produced by the Yukawa interaction ($\Gamma \gg H$). This fact also provides thermal equilibrium in the last stages of fresh inflation.

The aim of this paper is the study of nonminimal coupling of the inflaton field in the fresh inflationary scenario. This topic will be studied in Sect. II. In Sect. III, I study the dynamics of inflaton fluctuations for $\xi \neq 0$ and, in Sect. IV, an example for this formalism is examined. Finally, in Sect. V, some final comments are developed.

II. FRESH INFLATION WITH NONMINIMAL COUPLING

We study a model of fresh inflation where an inflaton field $\phi$ is nonminimally coupled with a scalar curvature $R$

$$\mathcal{L} = \sqrt{-\text{g}} \left[ \frac{R}{16\pi G} - \frac{1}{2} (\nabla \phi)^2 - V(\phi) - \frac{1}{2} \xi R \phi^2 + \mathcal{L}_{int} \right],$$

where $G = M_p^{-2}$ is the gravitational constant, $M_p = 1.2\text{ GeV}$ is the Planckian mass and $\mathcal{L}_{int} = -g^2 \phi^2 \psi^2$ takes into account the interaction between the inflaton and the scalar field $\psi$. The Lagrangian (1) can be rewritten as

$$\mathcal{L} = \sqrt{-\text{g}} \left[ \frac{R}{16\pi G_{\text{eff}}(\phi)} - \frac{1}{2} (\nabla \phi)^2 - V(\phi) + \mathcal{L}_{int} \right],$$

where $G_{\text{eff}}(\phi) = \frac{G}{1 - \phi^2/\phi_c^2}$ with $\phi_c^2 = \frac{M_p^2}{8\pi\xi}$ and $R = 6(a\ddot{a} + \dot{a}^2)/a^2$, ($a$ is the scale factor of the universe). In order to connect to our present universe, $G_{\text{eff}}$ needs to be positive so that in the case $\xi > 0$ we require $\phi^2 < \frac{M_p^2}{8\pi|\xi|}$. In this paper I study only the case $\xi \neq 0$. The case $\xi = 0$ for a minimally coupled scalar field in fresh inflation was analyzed in [4].

The Einstein equations for a globally flat, isotropic and homogeneous universe described by a Friedmann-Robertson-Walker metric $ds^2 = -dt^2 + a^2(t)dr^2$, are given by

$$3H^2 = 8\pi G \left[ \left( \frac{1}{2} - 2\xi \right) \dot{\phi}^2 + V(\phi) + \rho_r \right],$$

$$3H^2 + 2\dot{H} = -8\pi G \left[ \left( \frac{1}{2} - 2\xi \right) \dot{\phi}^2 - V(\phi) + \rho_r \right].$$
where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $a$ is the scale factor of the universe. The overdot denotes the derivative with respect to the time. On the other hand, if $\delta = \dot{\rho}_r + 4H\rho_r$ describes the interaction between the inflaton and the bath, the equations of motion for $\phi$ and $\rho_r$ are

$$
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \xi R\phi + \frac{\delta}{\phi} = 0, \\
\dot{\rho}_r + 4H\rho_r - \delta = 0.
$$

(5)

(6)

As in previous papers \cite{7,8}, I will consider a Yukawa interaction $\delta = \Gamma(\theta) \dot{\phi}^2$, where $\Gamma(\theta) = \frac{g^4}{192\pi^2} \theta$ and $\theta \sim \rho_r^{1/4}$ is the temperature of the bath. Slow-roll conditions must be imposed to assure nearly de Sitter solutions for an amount of time, which must be long enough to solve the flatness and horizon problems. If $p_t = \frac{\dot{\phi}^2}{2} + \frac{\rho_r}{3} - V(\phi)$ is the total pressure and $\rho_t = \rho_r + \frac{\dot{\phi}^2}{2} + V(\phi)$ is the total energy density, hence the parameter $F = \frac{p_t + \rho_t}{\rho_t}$ which describes the evolution of the universe during inflation \cite{12}, is

$$
F = -\frac{2\dot{H}}{3H^2} = \frac{(1 - 4\xi) \dot{\phi}^2 + \frac{4}{3}\rho_r}{\rho_r + \left(\frac{1}{2} - 2\xi\right) \dot{\phi}^2 + V} > 0.
$$

(7)

When fresh inflation starts (at $t = G^{1/2}$), the radiation energy density is zero, so that $F \ll 1$.

From the two equalities in eq. \cite{7} one obtains the following equations

$$
\dot{\rho}_r^2 \left[ (2 - F) \left( \frac{1}{2} + 2\xi \right) \right] + \rho_r \left( \frac{4}{3} - F \right) - F V(\phi) = 0, \\
H = \frac{2}{3} \int F \, dt.
$$

(8)

(9)

Furthermore, due to $\dot{H} = H'\dot{\phi}$ (here the prime denotes the derivative with respect to the field), from que first equality in eq. \cite{4} it is possible to obtain the equation that describes the evolution for $\phi$

$$
\dot{\phi} = -\frac{3H^2}{2H'} F,
$$

(10)

and replacing eq. \cite{10} in \cite{8} the radiation energy density can be described as functions of $V$, $H$ and $F$ \cite{7}.
\[ \rho_r = \left( \frac{3F}{4-3F} \right) V - \frac{27}{4} \left( \frac{H^2}{H'} \right)^2 F^2 \left[ \frac{(2-F)}{(4-3F)} \left( \frac{1}{2} + 2\xi \right) \right] \]  

(11)

Finally, replacing (10) and (11) in eq. (3), the potential can be written as

\[ V(\phi) = \frac{3(1 - 8\pi \xi G \phi^2)}{8\pi G} \left[ \left( \frac{4 - 3F}{4} \right) H^2 + \frac{3\pi}{2} \frac{G F^2}{(1 - 8\pi \xi G \phi^2)} \left( \frac{H^2}{H'} \right)^2 \right] \left( 1 + 8\xi \right) \]  

(12)

Fresh inflation was proposed for a global group \( O(n) \), involving a single \( n \)-vector multiplet of scalar fields \( \phi_i \) [13], such that making \( (\phi_i \phi_i)^{1/2} \equiv \phi \), the effective potential \( V_{\text{eff}}(\phi, \theta) = V(\phi) + \rho_r(\phi, \theta) \) can be written as

\[ V_{\text{eff}}(\phi, \theta) = \frac{M^2(\theta)}{2} \phi^2 + \frac{\lambda^2}{4} \phi^4, \]  

(13)

where \( M^2(\theta) = M^2(0) + \frac{(n+2)}{12} \lambda^2 \theta^2 \) and \( V(\phi) = \frac{M^2(0)}{2} \phi^2 + \frac{\lambda^2}{4} \phi^4 \). Furthermore \( M^2(0) > 0 \) is the squared mass at zero temperature, which is given by \( M_0^2 \) plus renormalization counterterms in the potential \( \frac{1}{2} M_0^2 (\phi_i \phi_i) + \frac{1}{4} \lambda^2 (\phi_i \phi_i)^2 \) [13]. I will take into account the case without symmetry breaking \( M^2(\theta) > 0 \) for any temperature \( \theta \), so that the excitation spectrum consists of \( n \) bosons with mass \( M(\theta) \). Note that the effective potential (13) is invariant under \( \phi \rightarrow -\phi \) reflexions and \( n \) is the number of created particles due to the interaction of \( \phi \) with the particles in the thermal bath, such that

\[ (n+2) = \frac{2\pi^2}{3\lambda^2 g_{\text{eff}} \theta^2}, \]  

(14)

because the radiation energy density is given by \( \rho_r = \frac{\pi^2}{30} g_{\text{eff}} \theta^4 \) (\( g_{\text{eff}} \) denotes the effective degrees of freedom of the particles and it is assumed that \( \psi \) has no self-interaction).

**III. DYNAMICS OF THE INFILATON FIELD AND POWER SPECTRUM OF THE FLUCTUATIONS**

In this section I will study the dynamics of the inflaton field to make an estimation for the energy density fluctuations in a spatially flat Friedmann-Robertson-Walker (FRW) metric.
\[ ds^2 = -dt^2 + a^2(t)dx^2. \] (15)

The dynamics for the spatially homogeneous inflaton field \( \phi \) is given by

\[ \ddot{\phi} + (3H + \Gamma)\dot{\phi} + \xi R\phi + V'(\phi) = 0, \] (16)

where \( V'(\phi) \equiv \frac{dV}{d\phi} \). The term \( \Gamma \dot{\phi} \) is added in the scalar field equation of motion (16) to describe the continuous energy transferred from \( \phi \) to the thermal bath. This persistent thermal contact during fresh inflation is so finely adjusted that the scalar evolves always in a damped regime.

Furthermore, the fluctuations \( \delta \phi(x, t) \) are described by the equation of motion

\[ \ddot{\delta \phi} - \frac{1}{a^2} \nabla^2 \delta \phi + (3H + \Gamma)\dot{\delta \phi} + \left[ \xi R + V''(\phi) \right] \delta \phi = 0, \] (17)

where \( R = 12H^2 + 6\dot{H} \). Here, the additional second term appears because the fluctuations \( \delta \phi \) are spatially inhomogeneous. The equation for the modes \( \chi_k(x, t) = \xi_k(t)e^{i\vec{k}.\vec{x}} \) and \( \chi_k^*(x, t) = e^{-i\vec{k}.\vec{x}}\xi_k^*(t) \), of redefined fluctuations \( \chi = a^{3/2}e^{\frac{1}{2}\int \Gamma dt} \delta \phi \) (which can be written as a Fourier expansion as)

\[ \chi(x, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ a_k \chi_k(x, t) + a_k^\dagger \chi_k^*(x, t) \right], \] (18)

is

\[ \ddot{\xi}_k + \omega_k^2 \xi_k = 0, \] (19)

where \( \omega_k^2 = a^{-2} [k^2 - k_0^2] \) is the squared frequency for each mode and \( k_0^2 \) is given by

\[ k_0^2(t) = a^2 \left\{ \frac{9}{4} (H + \Gamma/3)^2 - 12\xi H^2 + 3 \left[ (1 - 2\xi) \dot{H} + \Gamma/3 \right] - \left[ 12\xi H^2 + V''[\phi(t)] \right] \right\}. \] (20)

Here, the time-dependent wave number \( k_0(t) \) separates the infrared (IR) and ultraviolet (UV) sectors. The IR sector includes the long wavelength modes \( (k < k_0) \) and the UV sector takes into account the short wavelength modes \( (k > k_0) \). Furthermore \( (a_k, a_k^\dagger) \) are respectively the annihilation and creation operators, which complies with the commutation
relations \[a_k, a_{k'}^\dagger = \delta^{(3)}(\vec{k} - \vec{k}')\] and \[a_{k_k}, a_{k_{k'}}^\dagger = [a_k, a_{k'}] = 0\]. If we take \(\xi_k = \xi_k^{(0)} e^{\int g dt}\), the equation for \(\xi_k^{(0)}\) can be approximated to

\[\ddot{\xi}_k^{(0)} + a^{-2} \left[ k^2 - \tilde{k}_0^2 \right] \xi_k^{(0)} = 0, \tag{21}\]

where \(\tilde{k}_0^2\) is given by

\[\tilde{k}_0^2 = a^2 \left\{ H^2 \left( \frac{9}{4} - 12\xi \right) + 3 \left( 1 - 2\xi \right) \dot{H} + \dot{\Gamma} - V''[\phi(t)] \right\}. \tag{22}\]

The function \(g(t)\) only takes into account the thermal effects. The differential equation for \(g\) is

\[g^2 + \dot{g} = \frac{3}{2} H \Gamma + \frac{1}{4} \Gamma^2, \tag{23}\]

with initial condition \(g(t = t_0) = 0\), since the temperature when fresh inflation starts is zero.

The squared fluctuations for super Hubble scales \((k^2 \ll \tilde{k}_0^2)\), are given by

\[\langle (\delta\phi)^2 \rangle = a^{-3} \frac{2}{2\pi^2} \mathcal{F}(t) \int_0^{k_0(t)} dk k^2 \xi_k^{(0)} \left( \xi_k^{(0)} \right)^*, \tag{24}\]

where the asterisk denotes the complex conjugate and the function \(\mathcal{F}\) is given by \(\mathcal{F}(t) = e^{\int [2g(t) - \Gamma] dt}\).

**IV. AN EXAMPLE**

We consider the case in which \(F\) is a constant during inflation and the Hubble parameter is given by

\[H(\phi) = 4M(0) \sqrt{\frac{\pi G}{3(4 - 3F)}} \phi, \tag{25}\]

From the eq. (19) one obtains the time dependence for the inflaton field

\[\phi(t) = \frac{1}{6FM(0)} \sqrt{\frac{3(4 - 3F)}{\pi G}} t^{-1}. \tag{26}\]

This is a very interesting case because \(\phi(t)\) never holds zero, as in the model initially proposed for fresh inflation [7].
The initial value of $\phi$ being given by the equation $\rho_r(\phi_i) = 0$:

$$\phi_i = \sqrt{\frac{4 - 3F}{\pi G \left[ F(36\xi + 12) + 16\xi - F^2(9 + 33\xi) \right]}}. \quad (27)$$

Furthermore, since $V(\phi) = (\mathcal{M}^2(0)/2) \phi^2 + (\lambda^2/4) \phi^4$, replacing eq. (25) in eq. (12), one obtains the expression for $\lambda$ as a function of $\mathcal{M}(0)$

$$\lambda^2 = \frac{16\pi G}{3(4 - 3F)} \left[ \frac{9}{4} F^2(1 + 3\xi) - 3\xi(4 - 3F) \right] \mathcal{M}^2(0), \quad (28)$$

so that the scalar potential can be written as

$$V(\phi) = \frac{\mathcal{M}^2(0)}{2} \phi^2 + \frac{4\pi G}{3(4 - 3F)} \left[ \frac{9}{4} F^2(1 + 3\xi) - 3\xi(4 - 3F) \right] \mathcal{M}^2(0) \phi^4. \quad (29)$$

In this paper I will consider $\mathcal{M}^2(0) \simeq 10^{-12} G^{-1}$.

Notice that for $\xi = 0$ one obtains $\lambda^2 = \frac{12\pi G F^2}{(4 - 3F)} \mathcal{M}^2(0)$ [7]. The scale factor evolves as $a \sim t^{2/(3F)}$ and the temperature is given by the equation [see eq. (11)] where $\delta = \Gamma(\theta) \dot{\phi}^2$, $\Gamma(\theta) = \frac{g_{eff}^4}{192\pi} \theta(t)$ and $\rho_r$ is given by eq. (11)

$$\theta(t) = \frac{192\pi}{g_{eff}^4 \phi^2} \left[ \dot{\rho}_r + 4H\rho_r \right], \quad (30)$$

which, for $t \gg G^{1/2}$ gives

$$\theta(t) \simeq \frac{768\pi \mathcal{M}^2(0)}{(4 - 3F) g_{eff}^4} t. \quad (31)$$

The number of created particles for $g_{eff} \simeq 10^2$ is [see eq. (13)]

$$n \simeq 2.2 \times 10^{-7} \frac{\mathcal{M}^4(0)}{(4 - 3F)^2 t^4}, \quad (32)$$

which increases with time. So, during fresh inflation the expansion is accompanied by intense particle creation. Hence, the decay width of the inflaton field for $F \ll 1$ is

$$\Gamma[\theta(t)] \simeq \mathcal{M}^2(0) \cdot t. \quad (33)$$

To obtain $\frac{\dot{\kappa}^2}{a^2}$ in eq. (22), we make $p = 2/(3F)$

$$\frac{\dot{\kappa}^2}{a^2} = \left[ \left( \frac{9}{4} + 24\xi \right) \dot{\phi}^2 - 3 \left( 1 + 4\xi \right) \phi \dot{\phi} - 3 \left( 1 + 3\xi \right) \right] t^{-2}. \quad (34)$$
Hence, the equation for $\xi_k^{(0)}(t)$ is

$$\ddot{\xi}_k^{(0)} + \left(\frac{k^2 t^{-2 p}}{a_0^2 t_0^{-2 p}} - \left[\frac{9}{4} + 24 \xi\right] p^2 - 3 \left[1 + 4 \xi\right] p - 3 \left[1 + 3 \xi\right]\right) t^{-2} \right) \xi_k^{(0)} = 0. \quad (35)$$

The general solution (for $\nu \neq 0, 1, 2, \ldots$) is

$$\xi_k^{(0)}(t) = C_1 \sqrt{\frac{t}{t_0}} H_\nu^{(1)} \left[\frac{k t^{1-p}}{a_0 t_0^{-p} (p-1)}\right] + C_2 \sqrt{\frac{t}{t_0}} H_\nu^{(2)} \left[\frac{k t^{1-p}}{a_0 t_0^{-p} (p-1)}\right], \quad (36)$$

where $\nu = \sqrt{\left(9 + 96 \xi\right) p^2 - 12 \left[1 + 4 \xi\right] p - \left[11 + 36 \xi\right]/[2(p-1)]}$, which, for a given value of $\xi$, tends to a constant as $p \to \infty$. For $\xi = 0 \nu \to 3/2$ as $p \to \infty$. Furthermore, $(H_\nu^{(1)}, H_\nu^{(2)})$ are the Hankel functions. These functions take the small-argument limit $H_{\nu}^{(2)}[x] \bigg|_{x \ll 1} \simeq \frac{(x/2)^\nu}{\Gamma(1+\nu)} \pm \frac{i}{\pi} \frac{\Gamma(\nu)}{x^{\nu}}$. We can take the Bunch-Davis vacuum such that $C_1 = 0$ and $C_2 = \sqrt{\pi/2}$ \[16\].

Notice that $\xi_k^{(0)}$ is the solution for the modes when the interaction is negligible ($\Gamma \propto \theta \simeq 0$).

The function $g(t)$ only takes into account the thermal effects. Taking into account the small-argument limit for the Hankel functions and the Bunch-Davis vacuum, we obtain

$$\xi_k^{(0)}(\xi_k^{(0)})^* \simeq \frac{2^{2\nu}}{\pi^2} \Gamma^2(\nu) \left[\frac{a_0 (p-1)}{t_0^p} t^{(p-1)}\right]^{2\nu} k^{-2\nu}, \quad (37)$$

so that the integral controlling the presence of infrared divergences in eq. \[24\] is $\int_0^k dk k^{2(1-\nu)}$, with a power spectrum $P_{<\delta \phi^2>} \sim k^{3-2\nu}$. Hence, the condition $n_s = 3/2 - \nu$ gives a spectral index $n_s \simeq 1$ according with the experimental data \[15\] for $\nu \simeq 1/2$. This implies $p \simeq 2$ for $\xi = 0$. On the other hand, the condition $N = \int_{G^{1/2}}^{G^{1/2}} H(t) \ dt \geq 60$ (that implies $F \leq 1/3$) assures the solution of the horizon problem to give a sufficiently globally flat universe. This condition implies $p \geq 2$. On the other hand, for $1/2 \leq \nu < 3/2$ there is no infrared divergence. These conditions implies $\xi \leq 0.3174$ and $p \geq 2$. The experimental data \[15\] obtained from BOOMERANG-98, MAXIMA-1 and COBE DMR, is consistent with $\nu \simeq 1/2$ to obtain a spectral index $n_s \simeq 1$. Such a condition constrains the possible values for $p$ and $\xi$ to $\xi \leq 0$ and $p \geq 2$.

V. FINAL COMMENTS

We have investigated the dynamics of a fresh inflationary scenario with a inflaton field nonminimally coupled to gravity. Fresh inflation attempts to build a bridge between the
standard and warm inflationary models, beginning from chaotic initial conditions which provides naturality. In this sense, this model can be viewed as an unification of both chaotic and warm inflation scenarios. In our study the inflaton field coupled nonminimally to a spacetime curvature \( R \) by means of an additional term \( -\xi R \phi^2 / 2 \) in the Lagrangian. In the example here studied, I find that the possible values for the coupling are restricted to \( \xi \leq 0 \), for \( p \geq 2 \). These values becomes from experimental data obtained from BOOMERANG-98, MAXIMA-1 and COBE DMR, which are consistent with a spectral index \( n_s \approx 1 \), related to \( \nu \approx 1/2 \) in the example here studied. The most interesting here, is that, as larger is the value of \( p (a \sim t^p) \), smaller (but larger in its absolute value because the permitted values of \( \xi \) are negative) is the value of \( \xi \) necessary to satisfy \( \nu \approx 1/2 \), which is consistent with a scale invariant power spectrum \( (n_s \approx 1) \) for the inflaton fluctuations.
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