Improving monoid concept understanding ability through problem posing learning model at abstract algebra course

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Abstract. This study aims to improve the ability to understand monoid concepts through Problem Posing learning. The method used is a quasi-experiment. The research design used was pre-test post-test group design. The research sample was 32 undergraduate students of the Mathematics Education Study Program FKIP Universitas Bengkulu at the Algebra Abstract course of even semester of 2018-2019 academic year. The instrument used was a 4 item test about the concept of monoid. Data were analysed qualitatively and quantitatively. The results of the study are as follows: 1) an increase in the ability to understand the monoid concept of students by 0.67, including the medium category; 2) the achievement of the ability to understand the monoid concept is 74.75 in the high category. The results of this study have implications for the importance of the role of questions in learning mathematics.

1. Introduction
Abstracting and relating process in concepts understanding of abstract algebra still have some difficulties [1]. Like relation among monoid component, which are relation among “grupoid”, associative attitude, and unity element. The weakness of this aspect is that it makes students fail in solving the problems about monoid. The students’ weak understanding of monoid concept is caused by learning strategy done by the lecturer [2]. Because of that, the learning strategy needs to be fixed, with the implementation of problem posing learning model. Problem posing learning model is very important, especially to improve students’ thinking ability [3]. Problem posing is a learning which adapt the assignment with question that need a high cognitive also develop the students’ mathematics thinking more [4]. The questions express a certain concept [5].

The problem in this research is how are achievement and improvement of student’s monoid concept understanding ability which is taught by using problem posing learning model?

2. Methods
This research is a quasi-experiment research with one group pre-test post-test design [6]. The population in this research is 60 students that taking Abstract Algebra course at even semester of 2018-2019 academic year in mathematics study program of FKIP Universitas Bengkulu. The samples were 34 students which is taken purposively. Instrument about monoid consists of 4 items in essay form. Validity of item 1 with $r = 0.72$, item 2 with $r = 0.81$, item 3 with $r = 0.69$, and item 4 with $r = 0.76$. Instrument reliability is 0.85 (high reliability). The data was analysed qualitatively and quantitatively.
Research design is presented with the following model:

\[ O1 \times O2 \]

Note:
- \( O1 \) = pre-test,
- \( O2 \) = post-test
- \( \times \) = Treatment of problem posing learning model

Analysis of achievement and improvement data about monoid concept understanding ability is determined through post-test and N-gain score. There are two measurements to know the monoid concept understanding ability after implementing problem posing learning model, they are achievement and improvement. The achievement of monoid concept understanding ability is determined by post-test score, while the improvement of monoid concept understanding ability is determined by N-gain or normalized gain. The meaningfulness testing of achievement and improvement is by using t-test.

3. Results and discussion

Problem posing, learning that gives a wide opportunity for students to ask questions or problems about context that is related to the materials [7]. Some questions (problems) that is proposed by students in “monoid” learning topic are as follow: 1. What is the nature of monoid?, 2. What is unity element of monoid, have to fulfil left or right element of unity?, 3. How is the position of associative nature and commutative or a monoid?, 4. Is a monoid, a grupoid?, and 5. What is the relation of semigroup and monoid? Questions (problems) that is asked by the students is a closed problem. The problems can also be an opened problem if the problems have variation way of solving. If it is seen from the component arrangement, the problems can be classified to be well-structured problem and less-structured problem [8].

The monoid problem that is completed by the students is: if \( R \) is a set of real number and \( G = R \times R \). The operation \( \circ \) is defined by \( (a,b) \circ (c,d) = (ad + bc) \) for each \( (a,b), (c,d) \in G \). Is \( (G, \circ) \) a monoid? One of the works from the students can be seen at Figure 1:

![Figure 1. Completion of monoid problem by students.](image-url)
Students’ completion of the monoid problem is correct. The students are able to correctly show that \((G, \ast)\) is a grupoid, by showing there are a set and a correct composition. Next, the students showed that the grupoid is associative and has unity element. The relation of parts from that concept will give a strong and meaningful understanding to the main concept [9]. Therefore, the students conclude that \((G, \ast)\) is monoid. According to Mas’oed [10], a grupoid \((G, \ast)\) can be said as monoid to multiplication if it meet three conditions: 1. \((G, \ast)\) is closed to multiplication, 2. Is associative to multiplication, and 3. Have unity element (identity) to multiplication.

Students’ ability to correctly understand monoid concept is influenced by the lecturer used of problem posing learning model in learning abstract algebra. The implementation of a learning model has an impact in students’ competence [11]. The average of students’ achievement and improvement in monoid concept understanding ability is explained in table below.

| Data     | Pre-test | Post-test | N-gain |
|----------|----------|-----------|--------|
| N        | 32       | 32        | 32     |
| Min score| 0        | 50        | 0.38   |
| Max score| 0        | 90        | 0.88   |
| Average  | 21.25    | 74.75     | 0.67   |
| STD      | 13.08    | 11.29     | 0.14   |

Based on N-gain criteria, the improvement of students’ score (0.67) is in medium category. Besides, the achievement of monoid score is 74.75 which is in high category. The number of students for each category in post-test (achievement) and N-gain (improvement) of monoid score is showed in figure 2 below:

![Figure 2](image.png)

**Figure 2.** Monoid concept understanding ability based on post-test category and N-gain.

Based on figure 2 above, it can be seen that both of achievement and improvement monoid score is not in low category, only categorized in medium and high. Similarly, the percentage of students in high category of achievement and improvement is more than students in medium category.

Test result of pre-test and post-test score is presented in Table 2 below:
Table 2. Hypothesis testing with paired samples test.

| Paired Differences | Mean  | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | t     | df  | Sig. (2-tailed) |
|--------------------|-------|----------------|-----------------|------------------------------------------|-------|-----|----------------|
|                    | -53.50| 15.486         | 2.738           | -59.083 - -47.917                          | -19.543 | 31  | .000           |

The information that is obtained from the table is that the difference between pre-test and post-test is very significant with 0.00 significance level. Therefore, it can be concluded that score of the achievement and the improvement of monoid concept understanding ability that is taught through problem posing learning model is very significant. It means, problem posing learning model is effective to improve concept understanding ability in mathematics. This shows that the problem posing learning model can provide support for mathematics learning [12], especially towards the ability of students to understand concepts in mathematics is part of academic ability [13].

4. Conclusion
The monoid concept understanding ability may improve through problem posing learning model. The achievement of monoid concept understanding ability of students is 74.75 (high category), while the improvement is 0.67 (medium category).

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