An Analysis of Stapp’s “A Bell-type theorem without hidden variables”

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Abstract
H.P. Stapp has proposed a number of demonstrations of a Bell-type theorem which dispensed with an assumption of hidden variables, but relied only upon locality together with an assumption that experimenters can choose freely which of several incompatible observables to measure. In recent papers his strategy has centered upon counterfactual conditionals. Stapp’s paper in American Journal of Physics, 2004, replies to objections raised against earlier expositions of this strategy and proposes a simplified demonstration. The new demonstration is criticized, several subtleties in the logic of counterfactuals are pointed out, and the proofs of J.S. Bell and his followers are advocated.

1 Introduction
Henry Stapp’s article “A Bell-type theorem without hidden variables” \cite{Stapp1} is a reformulation of a project which has evolved over many years \cite{Stapp2, Stapp3, Stapp4, Stapp5, Stapp6, Stapp7, Stapp8} and has been the subject of numerous critical assessments, e.g., Clauser and Shimony \cite{Clauser}, Clifton, Butterfield, and Redhead \cite{Clifton}, Clifton and Dickson \cite{Clifton2}, Mermin \cite{Mermin}, Unruh \cite{Unruh}, Shimony and Stein \cite{Shimony, Shimony2}. The last four of these criticisms were directed specifically against Stapp \cite{Stapp6}. Concerning these four criticisms Stapp \cite{Stapp1} states on column 1 of p. 32, “I have answered these objections. However, the very existence of these challenges shows that the approach used . . . has serious problems, which originate in the fact that it is based on classical modal logic.” The purpose of the present paper is to assess carefully Stapp \cite{Stapp1}, restating previous criticisms when his new language leaves the scientific and philosophical theses of Stapp \cite{Stapp6} essentially unchanged, but also acknowledging and assessing innovations. Stapp’s seriousness and tenacity merit an attentive examination of his latest exposition.

Stapp \cite{Stapp1, Stapp6} expresses dissatisfaction with the alleged demonstrations of J.S. Bell \cite{Bell, Bell2} and his followers \cite{Bell3} that the predictions of Quantum Mechanics (QM) are inconsistent with the locality of the Special Theory of Relativity (STR). All of these demonstrations examine the possibility of recovering the predictions of QM by means of a local hidden-variables model, which assigns
definite values to all the observable quantities recognized by QM even when QM prohibits in principle their simultaneous measurability. In thus characterizing the entire class of local hidden variables theories Stapp argues that the difference between so-called “deterministic” and so-called “stochastic” theories is superficial, because the latter are equivalent to the former except for errors which tend to zero as the number of experiments goes to infinity; see especially Stapp [3]. Consequently, Stapp maintains, the demonstrations by Bell and his followers prove only the inconsistency of QM with the conjunction of locality, free choice of experiments, and the assumption of hidden variables, and since the assumption of hidden variables violates the philosophical viewpoint of QM, the conclusion reached by the standard derivations is neither surprising nor profound. Stapp aims at a stronger theorem throughout his entire series of investigations of the foundations of quantum mechanics: that QM is inconsistent with the conjunction of locality and the assumption that experimenters freely choose among incompatible experiments performable in a given space-time region, the latter being an assumption which he regards as implicit in the entire enterprise of experimental science. I wish to assert, at the commencement of my critical assessment, my agreement with Stapp that a successful demonstration of the inconsistency of QM with locality and free choice, without the explicit or implicit assumption of hidden variables, would indeed be a profound scientific and philosophical achievement.

2 Stapp’s Explicit and Tacit Assumptions

The physical system S which Stapp envisages for analysis consists of two parts, located in two space-time regions R and L, which have space-like separation from each other. Einstein-Minkowski geometry, which is the geometry of the space-time of the Special Theory of Relativity (STR) implies that there exists a Lorentz frame F in which every point in L is earlier than every point in R, and Stapp prefers to speak of R and L in the frame F whereas I prefer to express the relation between R and L in a frame-invariant manner, even though the same geometrical facts can be equivalently expressed in both ways. The part of S located in L will be called λ, and the part located in R will be called ρ. λ is subjected to one of two measurements, labeled L1 and L2, freely chosen by an experimenter in L, while ρ is subjected to one of two experiments, labeled R1 and R2, freely chosen in R. Each measurement is bivalent, with possible results labeled + and −. The expression “L1+” ambiguously designates the + result of the measurement L1 or the proposition that the measurement L1 is performed with outcome +; and “L1−”, “L2+”, and “L2−” have analogous meanings. Context determines which of the possible meanings is intended. Likewise each of R1 and R2 has bivalent outcomes + and −. Following L. Hardy [19] and the slight modification by Eberhard and Rosselet [20] (the latter referred to here henceforth as “HER”, but Stapp calls it “Hardy”), Stapp ascribes to the composite system a quantum state which predicts the following correlations:

1. Upon the condition that L2 is performed, if R2+ then L2+.
2. If $R_1$ is performed and $L_2^+$, then $R_1^-$.  

3. If $R_2$ is performed and $L_1^-$, then $R_2^+$.  

4. If $R_1$ is performed and $L_1^-$, then $R_1$ has a non-zero probability of having outcome $+$ and a non-zero probability of having outcome $-$. (Actually HER choose these probabilities to be 50%, but I prefer not to work with quantitative probabilities for reasons to be stated later.)

Stapp considers probability to be a meaningful concept applicable to a single system, like the system $S$, even though empirical evidence about probabilities can be gathered only when one has an ensemble of similar systems, all prepared in the same quantum state. In the analysis of counterfactual conditionals at the heart of Stapp’s theorem a central concept is that of “possible worlds,” each of which is a maximal characterization of a system $S$ in terms of the experiments that can be performed upon it and their possible outcomes, subject to the constraints of internal consistency and obedience to general physical laws and specific constitution. In the case of the extremely simple system $S$ each candidate possible world is one of the following sixteen, each consisting of four bits of information: $(L_1^+, R_1^+)$, $(L_1^+, R_1^-)$, $(L_1^+, R_2^+)$, $(L_1^-, R_1^-)$, $(L_1^-, R_2^+)$, $(L_2^+, R_1^+)$, $(L_2^+, R_2^+)$, $(L_2^+, R_2^-)$, $(L_2^-, R_1^+)$, $(L_2^-, R_1^-)$, $(L_2^-, R_2^+)$, $(L_2^-, R_2^-)$. The qualification “candidate” is used because predictions (1), (2), and (3) have not yet been applied to winnow out impossibilities from the foregoing list. The list of candidates for possible worlds is similar to the set of state descriptions in the works of Carnap [21], except that the quantum mechanical restrictions against simultaneous performance of experiments $R_1$ and $R_2$ and simultaneous performance of experiments $L_1$ and $L_2$ have limited the catalogue. Predictions (1)-(3) of the HER model enforce the elimination of $(L_1^-, R_2^-)$, $(L_2^+, R_1^+)$, $(L_2^-, R_1^+)$, from the sixteen candidate possible worlds, leaving only thirteen possible worlds: $(L_1^+, R_1^+)$, $(L_1^+, R_1^-)$, $(L_1^+, R_2^+)$, $(L_1^-, R_1^-)$, $(L_1^-, R_2^-)$, $(L_2^+, R_1^-)$, $(L_2^+, R_2^-)$, $(L_2^-, R_2^-)$, $(L_2^-, R_1^-)$. There is quantitative information about probabilities in prediction (4), information which does not restrict the set of possible states for each system in the ensemble but only influences the statistics of their actualization in the ensemble (each member of which is assigned its own pair of space-like separated regions like the $R$ and $L$ assigned to $S$). The analysis of counterfactual conditionals for a single system does not depend upon the quantitative probabilities but only upon the more primitive matter of possibility or impossibility. Specifically, prediction (4) implies that $(L_1^-, R_1^+)$ and $(L_1^-, R_1^-)$ are both possible worlds.

Finally I shall state two Auxiliary Assumptions which Stapp never makes explicit but seems to be implicit in the HER quantum state and in his proof of Property (II) (Stapp [1], second column of p. 31): (i) that the probability of $R_2^+$ is greater than zero, given that $R_2$ is chosen to be performed in $R$ and that either $L_1$ or $L_2$ is chosen to be performed in $L$; and (ii) that among the ensemble of systems considered there are some in which $L_2$ and $R_2$ are the
chosen experiments, with outcome + for R2, and some in which L1 and R2 are the chosen experiments, with outcome − for R2. These assumptions guarantee that in the actual world for the ensemble some systems have features which allow Stapp’s Property (II) to be proved rigorously in Sect. 3, though—as will be seen—the conjunction of Properties (I) and (II) does not suffice to complete the proof of the theorem at which he is aiming. The assumptions are of a kind which is commonplace in the construction of interesting quantum mechanical models. It should be emphasized that Assumption (ii) is not a tautology unless quantum mechanical probability is interpreted in a frequency sense, whereas a propensity interpretation is convincingly proposed by Popper \cite{22}.) In practice, however, Assumption (ii) is indispensable for the purpose of connecting probability in the propensity sense to observable frequencies.

3 Stapp’s Theorem

The first stage in Stapp’s demonstration of the theorem stated in italics in the last paragraph of Sect. 1 is a proof in column 2 of p. 32 of the quantum theoretical Property (I) of the (HER) state: that if $L_2$ is performed in an experiment of this type then $SR$ necessarily follows, where $SR$ asserts, “If $R_2$ is performed and gives outcome +, then if, instead, $R_1$ had been performed the outcome would have been −.”

There are two troublesome expressions in the definition of $SR$, namely “if, instead” and “would have been.” Stapp proposes a clarification of the first of these expressions in the course of proving Property (I): “The concept ‘instead’ is given an unambiguous meaning by the combination of the premisses of ‘free choice,’ and ‘no backward in time influence [NBITI]:’ the choice between $R_1$ and $R_2$ is to be treated, within the theory, as a free variable, and switching between $R_1$ and $R_2$ is required to leave any outcome in the earlier region $L$ undisturbed. But then statements (1) and (2) can be joined in tandem to give the result $SR$.” (Stapp \cite{1}, column 2 of p. 31.)

The last sentence of this quotation is cryptic, but it is illuminated by the discussion of “would have been” in column 1 of p. 32 of Stapp \cite{1}. “The previous argument rests heavily on the use of counterfactuals: the key statement $SR$ involves, in a situation in which $R_2$ is performed and gives outcome +, the idea ‘if, instead, $R_1$ had been performed...’ But then he says that Bell-type hidden variables assumptions are also counterfactual, and he takes pains to repeat that his assumptions are weaker than those of Bell and his followers. Later on this page, in Section IV, he goes on to say that his earlier exposition in Stapp \cite{6} has been criticized, and that even though he has answered these criticisms those challenges indicate there are serious problems in that exposition, “which originate in the fact that it is based on classical modal logic.” That logic, he says, has three drawbacks:

1. Although the symbolic proof is concise and austere, that brevity is based on a background that most physicists lack, which means that most physicists cannot fully understand it without a significant investment of time.
2. The question arises as to whether the use of classical modal logic begs the question by perhaps being based in implicit ways on the deterministic notions of classical physics.

3. Classical modal logic itself is somewhat of an open question, and it is not immediately clear to what extent these issues undermine the proof.

Drawback (1) should be disregarded for two reasons. One is the historical fact that physicists have learned to use much more intricate mathematics than modal logic when that turned out to be useful for physical problems. The other is that the formal proof on p. 302 of Stapp [6] is in fact not concise and austere, since its fourteen steps are susceptible to condensation—as shown in the first paragraph on p. 850 of Shimony and Stein [14] and the second column of p. 31 of Stapp [1].

Drawback (2) is troublesome on first inspection, since classical modal logic does rely upon causal analysis, and the shift from the deterministic laws of classical physics to the indeterministic laws of QM would presumably require a modification of causal analysis. However, an examination of Stapp’s proofs of Property (I) and Property (II) shows that only special cases of quantum mechanical probabilities are used: impossibility and necessity in property (I), that is to say, probability zero and unity, and necessity and intermediate probability (neither zero nor unity) in property (II). The ubiquitous statistical character of quantum mechanical predictions, permitting the entire range of probabilities, has not been invoked.

Drawback (3) is troublesome because of an important contribution by Stapp himself. In the often-cited work on the logic of counterfactuals by D. Lewis [23] there is indeed an open question: how to give a reasonable criterion of the comparative closeness of possible worlds \( w' \) and \( w'' \) to actual world \( w \), a criterion that is needed to implement Lewis’s truth condition for a counterfactual conditional proposition. The discussion in Lewis’s book sounds quite scholastic, and one has a feeling that his entire program is endangered by the likelihood that the question of comparative closeness is not well posed. Stapp cuts through the question by proposing that a counterfactual conditional “if \( p \) were true, then \( q \) would be true” is true if and only if \( q \) is true in every possible world \( w' \) that differs from the actual world \( w \) only by the consequences of the action described by \( p \); according to STR this condition means every possible world \( w' \) in which \( p \) is true and which agrees with \( w \) everywhere outside the future light-cone of the set of space-time points where the experimenter’s action is localized (paraphrase of paragraph 5 in column 1 on p. 855 of Stapp [8]; also Shimony and Stein [14], paragraph 2 on p. 502). A clear criterion from STR has thus replaced the elusive criterion of “closeness” of one world to another. Stapp has thus admirably solved the problem that has made him wary, in drawback (3), of using classical modal logic.

With these clarifications of counterfactual conditional propositions we can prove Stapp’s Property (I), which asserts (in column 2 of p. 31 of Stapp [1]), “if an experiment of the Hardy-type is performed then \( L_2 \) implies \( SR \), where \( SR = \) ‘If \( R_2 \) is performed and gives outcome +, then if instead \( R_1 \) had been
performed the outcome would have been \( - \).'' This conclusion is correct even if "implies" is taken in its strongest sense, the sense of strict implication (i.e., \( p \) strictly implies \( q \) is true if and only if in every possible world in which \( p \) is true \( q \) is also true). The possible worlds, as stated in Section 2, are maximal characterizations of the system \( S \) consistent with each other and with the obedience by \( S \) of the general laws of physics (the relevant ones being the laws of quantum mechanics) and the specific constitution of \( S \) (the HER state): The antecedent of the implication asserted by property (I) is that \( L2 \) is performed in \( \mathbf{L} \), which is a premiss that picks out six of the thirteen possible worlds: \((L2+, R1-), (L2+, R2+), (L2+, R2-), (L2-, R1+), (L2-, R1-), (L2-, R2-).\) The antecedent of \( SR \) is false in all of these except \((L2+, R2+), \) and therefore \( SR \) itself is trivially true in these five cases by the logic of material implication. But in the case of the world \((L2+, R2+)\) the consequent of \( SR \) is true, since the only possible world in which \( R1 \) is performed instead of \( R2 \) and which is in agreement with the part of \((L2+, R2+)\) in region \( \mathbf{L} \) is \((L2+, R1-).\) Hence \( SR \) is true in all possible worlds in which \( L2 \) is performed, ensuring the truth of Property (I).

The second stage in Stapp’s demonstration of the theorem stated in Sect. 1 is to prove Property (II), which asserts, “Quantum theory predicts that if an experiment of the Hardy-type is performed then \( L1 \) implies \( SR \)’ is false.” This assertion is correct if “implies” is interpreted in the sense of strict implication. The premises of Property (II) are the correctness of quantum mechanics, the HER characterization of \( S \), and the performance of \( L1 \) on \( \lambda \). By Assumption (ii) there are members of the ensemble in which \( L1 \) is actually performed. One must check that \( SR \) holds of the part in \( \mathbf{R} \) of any such member. If \( R2 \) is either not performed on the part of the system in \( \mathbf{R} \) or is performed with outcome—then \( SR \) is trivially true by the logic of material implication, because its antecedent is false. On the other hand, if in the actual world \( R2 \) is performed on this system with outcome +, then \( SR \) is true of the system if and only if the consequent of \( SR \) is true, i.e., if and only if \( R1 \) has outcome—in all possible worlds in which \( R1 \) is performed on the part of this system in \( \mathbf{R} \) but which agree with the actual world outside the forward light cone of \( \mathbf{R} \), in particular in region \( \mathbf{L} \). By Assumption (ii) there are systems in the ensemble for which \( L1 \) is actually performed on the members in \( \mathbf{L} \) with outcome—while experiment \( R2 \) is freely chosen to be performed on the corresponding systems in \( \mathbf{R} \). By prediction (3) \( R2 \) has outcome + for these systems, thus guaranteeing the truth of the antecedent of \( SR \). By prediction (4) in each possible world in which \( R1 \) is performed instead of \( R2 \) for the systems just mentioned there is a non-zero probability that the outcome will be \( R1+ \). Hence \((L1-, R1+)\) is a possible world. Consequently there is a case in which the consequent of \( SR \) is false even though the antecedent of \( SR \) is true. Hence we have proved that there is a case in which \( L1 \) is the experiment chosen in \( \mathbf{L} \) but \( SR \) is false, thus establishing Property (II).

Stapp then combines Properties (I) and (II) to conclude that the free choice between \( L1 \) and \( L2 \) has a causal relation to the truth of the proposition \( SR \). He claims that “in any theory or model in which the three assumptions (free choice, NBITI, certain predictions of quantum theory) are valid, the statement \( SR \) must always be true if the free choice in region \( \mathbf{L} \) is \( L2 \), but must sometimes
be false if that free choice in $L$ is $L_1$. But the truth or falsity of SR is defined by conditions on the truth or falsity of statements describing possible events located in region $R$. The fact that the truth of $S$ depends in this way on a free choice made in region $L$, which is space-like-separated from region $R$, can reasonably be said to represent the existence within the theory or model of some sort of faster-than-light influence.” (Stapp [1], bottom of p. 31, top of p. 32.)

The error in Stapp’s argument is his claim that SR is a statement about region $R$ alone. To be sure, the only events mentioned explicitly in SR are choices between experiments in $R$ and outcomes of the chosen experiments. But SR is not simply a statement about actually occurring events. It is a counterfactual conditional, and its truth condition is (by substituting in the italicized passage in Sect. 2) the following: $R_1$— is true in every possible world $w'$ that differs from the actual world $w$ only by the consequences of the action described by $R_1$; according to STR this means every possible world $w'$ in which $R_1$ is true and which agrees with $w$ everywhere outside the forward light-cone of $R$. The phrase “outside the forward light-cone of $R$” applies to $L$ because of the assumption that $R$ and $L$ are space-like separated. Hence SR does refer to parts of space-time outside the region $R$. And once this characteristic of SR is recognized one sees immediately that the choice between $L_1$ and $L_2$ in region $L$ is not a superluminal cause of an effect in the space-like separated region $R$, causing SR to be true if the latter choice is made and SR to be false if the former choice is made. SR is an intricate proposition involving three different kinds of entities: events in $R$, namely the actual choice between experiments $R_1$ and $R_2$ for $\rho$ and the outcome of the chosen experiment; events in $L$, namely the actual choice between performing experiment $L_1$ or $L_2$ on $\lambda$ and the outcome of the chosen experiment; and a set of possible worlds satisfying the requirements spelled out in the truth condition for the counterfactual conditional that constitutes the consequent of SR, this set being an entity determined jointly by the aforementioned events in $L$ and the aforementioned events in $R$. Stapp’s assertion that SR is localized in region $R$ is clearly incompatible with the intricacy of SR.

An answer to the foregoing argument is given in Stapp [3], p. 857, column 2, paragraph 2: “My input-output analysis makes the following point: The fact that (1), the truth or falsity of this statement SR is, for any fixed choice made by the experimenter in $L$, determined explicitly by whether or not a certain conceivable event, $R_1$—, must occur in $R$ under conditions defined in $R$, coupled with the agreed-upon fact that (2) the truth of SR depends upon which choice is made by the experimenter in $L$, means that whether or not this conceivable event in $R$ must occur depends upon which choice is made by the experimenter in $L$.” Stapp misstates the conditions under which SR is true or false. They are indeed determined by whether or not a certain conceivable event, $R_1$—, must occur in $R$ under certain conditions, but it is not the case that these are “conditions defined in $R$.” Yes, two of the conditions are indeed defined in $R$: the actual performance of $R_2$ with outcome $+$, and the restriction of relevant possible worlds in which the outcome $R_1$— is required to those in which $R_1$ is performed instead of $R_2$; but the further restriction upon the relevant possible worlds, that they agree with the actual world outside the
forward light-cone of \( R \), is not a condition defined in \( R \), because the region in which agreement is enforced includes \( L \).

An analogy may provide some relief from the logical complexity of the preceding paragraph. Some properties can be attributed to a local region \( R \) of space-time intrinsically, without any reference to other space-time regions. Others, however, cannot be attributed intrinsically. For instance, a comparative or superlative property \( p \) like “locus of the fastest hundred meter free-style swim” cannot be attributed to \( R \) without making a comparison with other regions. Indeed, if the choice is freely made to schedule a hundred meter free-style swimming match in a region \( R' \) in the forward light-cone of \( R \) and the outcome is a world’s record, then \( p \) will not hold of \( R \), but it cannot be (note NBITI) that an event in \( R' \) is the cause of an event in \( R \). This remark, of course, is a banality. But some sophistication and reflection is required in order to understand that a modal property like SR has a relational character just as much as the manifestly superlative property \( p \).

4 Conclusion

We can now see the virtue of the demonstrations of Bell’s theorem offered by Bell [16, 17] himself and by his followers [18], which assume a hidden-variables model as a premiss. Such a model—as Einstein, Podolsky, and Rosen [24] understood—explains correlations between regions \( R \) and \( L \) by means of attributions of intrinsic properties (their “elements of physical reality”) to each region. Consequently the correlations cannot be dismissed as banalities stemming from the relational character of the attributed properties. The premisses of demonstrations offered by Bell and his followers are indeed stronger than those offered by Stapp, but the reward of the stronger premisses is a logically impeccable conclusion that certain experimentally demonstrated quantum mechanical correlations violate relativistic locality.

In column 2 of p. 32 of Stapp [1] one finds an interesting historical remark, “The EPR argument rests strongly on counterfactual ideas.” This statement is widely held, and I myself believed for a long time (Shimony [25]) that a commitment to counterfactual reasoning is a corollary of EPR’s physical realism. Recently, however, I have been convinced by an old thesis of d’Espagnat [26] that EPR could reach their conclusions by ordinary inductive logic, without any invocation of counterfactual conditionals. Shimony [27] presents an elaboration of d’Espagnat’s position and argues, in addition, that EPR’s “elements of physical reality” would suffice to provide a grounding for counterfactual conditionals. Of course, the preceding sentence is itself conditional, and it is undermined by Bell’s theorem and the related experiments. These throw doubt on the assumption of relativistic locality needed by EPR to reach their conclusion that there exist independent but correlated elements of physical reality in spacelike separated regions.
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