Electromagnetic Waves in a Model with Chern-Simons Potential

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We investigate the appearance of Chern-Simons terms in electrodynamics at the surface/interface of materials. The requirement of locality, gauge invariance and renormalizability in this model is imposed. Scattering and reflection of electromagnetic waves in three different homogeneous layers of media is determined. Snell’s law is preserved. However, the transmission and reflection coefficient depend on the strength of the Chern-Simons interaction, and parallel and perpendicular components are mixed.

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I. INTRODUCTION

Space-time homogeneity and isotropy are typical for usual quantum field theory models of elementary particles. It is a natural assumption in the study of various processes with simplest excitations of quantum vacuum. However, it is not suitable for modelling the interaction of quantum fields with macroscopic objects, changing essentially the vacuum properties. In this case, quantum macro-effects may appear in dynamics of material bodies which can not be explained in the framework of classical physics. Theoretically, this problem was first considered in 1948 by Casimir, who showed that quantum vacuum fluctuations cause the attraction between two perfectly conducting parallel plates of an uncharged capacitor. This phenomenon, called the Casimir effect (CE), is observed experimentally, and the results obtained empirically for perfectly conductive materials are with a high degree of accuracy in agreement with theoretical ones.

At typical distances of 10-1000 nm for the CE both quantum and classical features of the system become essential. Their combination forms a special nano-physical. Investigations of it are not only of general theoretical interest. They are important also for the development of new technical devices, in view of the increasing trend towards their miniaturization.

Although there are numerous papers devoted to the theoretical problems of the CE, they are based often on simplified models of a free scalar field theory with fixed boundary conditions, applying only to investigations of some particular aspect of the CE and ignoring usually specificity of quantum electrodynamics. Such models are not suitable for a complete description of a wide range of nano-physical phenomena occurring in the system as a result of the interaction of quantum degrees of freedom with the material body of a given shape (classical defect). The results presented in our paper were obtained within the Symanzik approach for construction of quantum field theory models when there are spatial inhomogeneities with sharp boundaries. They are described by an additional action functional (action of the defect) that is concentrated in the region of space where the macroscopic object is located. In quantum electrodynamics the interaction of photons with the defect modelling background field is completely determined by the requirements of the locality, gauge invariance, renormalizability, and is described by the Chern-Simons action functional with a dimensionless constant characterizing the material properties of the surface. It affects the Casimir force, which is non-universal and can be not only attractive, but also repulsive for a flat capacitor. It is shown also that in this model the static electric charge interacting with the surface defect generates a magnetic field, and stable straight-line current creates an electric field. The calculated Casimir-Polder potential for a neutral atom near a flat surface allowed to find the parity-violating corrections to the previously known results. Based on the earlier proposed model, we study in this paper the electromagnetic waves in three layers of matter with magnetic susceptibilities $\mu_1$, $\mu_2$, $\mu_3$ and permittivities $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ separated by two parallel material planes $x_3 = \pm l/2$ whose Chern-Simons interaction with the electromagnetic field is characterized by coupling constants $a_1$, $a_2$.

II. STATEMENT OF PROBLEM

For the formulation and investigation of the model it is convenient to use the notation $\alpha$, and $a$ for three- and two-component arrays correspondingly. We define also the scalar product and $*$-composition of them:

$$\alpha \beta = a_1 b_1 + a_2 b_2 + a_3 b_3, \quad a \ast b = (a_1 b_1, a_2 b_2, a_3 b_3), \quad a \ast b = (a_1 b_1, a_2 b_2).$$

Let us introduce the arrays

$$\theta_1 \equiv (\theta(-l/2 - x_3), \theta(l/2 - |x_3|), x_3 = l/2), \quad d_1 \equiv (\delta(x_3 + l/2), \delta(x_3 - l/2)).$$

Here $\theta(\alpha)$ and $\delta(\alpha)$ are Heaviside step-function and Dirac delta-function. The scalar products of $\theta_1$ with

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\[ \hat{\beta} = (\beta_1, \beta_2, \beta_3) \text{ and } \mathbf{d}_i \text{ with } \mathbf{c} = (c_1, c_2) \text{ are defined as} \]
\[ \mathcal{F}(\beta_1, \beta_2, \beta_3) = \mathcal{F}(\hat{\beta}) = \hat{\beta}\theta_i, \]
\[ D(c_1, c_2) = D(\mathbf{c}) \equiv \mathbf{c}\mathbf{d}_i. \]

Then one obtains
\[ \frac{\partial}{\partial \beta_3} \mathcal{F}(\hat{\beta}) = \mathcal{F} \left( \frac{\partial}{\partial \beta_3} \hat{\beta} \right) + D(s(\hat{\beta})), \]
\[ \mathcal{F}(\hat{\beta}) \mathcal{F}(\hat{\gamma}) = \mathcal{F}(\hat{\beta} \ast \hat{\gamma}), \quad \mathcal{F}(1, 1, 1) = 1. \]

where \( s(\hat{\beta}) \equiv (\beta_2 - \beta_1, \beta_3 - \beta_2) \). The model of the photon field \( A_\mu \) interacting with the two-dimensional material surface described by equation \( \Phi(x) = 0 \) can be generalized for the considered system by the definition of the action functional as
\[ S(A) = -\frac{1}{4} G_{\mu\nu} F^{\mu\nu} + S_0(A). \] (1)

Here, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad G_{\mu\nu} = \mathcal{E}(x_3) F_{\mu\nu}, \) if \( \mu = 0 \) or \( \nu = 0, \quad \text{and} \quad G_{\mu\nu} = \mathcal{M}^{-1}(x_3) F_{\mu\nu} \) if \( \mu \neq 0, \nu \neq 0 \) with \( \mathcal{E}(x_3) = \mathcal{F}(\varepsilon), \quad \mathcal{M}(x_3) = \mathcal{F}(\hat{\beta}). \)

The functional \( S_0(A) \) describes the interaction of the 2-dimensional material objects (defects) with the photon field. The defects lie in our case at two parallel planes \( x_3 = l_i \) with \( l = (-l/2, +l/2) \). Using the notation \( \Phi_j(x) = x_3 - l_i \), we can write the action of the defects as \( S_0(A) = S_1(A) + S_2(A) \) where
\[ S_j(A) = \frac{a_j}{2} \int \partial_\mu \Phi_j(x) A_\nu(x) \tilde{F}^{\mu\nu}(x) \delta(\Phi_j(x)) dx = \]
\[ = \frac{a_j}{2} \int A_\nu(x) \tilde{F}^{\mu\nu}(x) \delta(\Phi_j(x)) dx, \quad j = 1, 2. \]

In (2) \( \tilde{F}^{\mu\nu} \) is the dual field tensor \( \tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}, \) and \( \varepsilon^{\mu\nu\rho\sigma} \) is the totally antisymmetric tensor, \( \varepsilon^{0123} = 1. \)

The Euler-Lagrange equations for the action functional \( S(A) \) (1) are written as modified Maxwell’s equations,
\[ \delta S(A) \]
\[ \delta A_\nu \quad \partial_\xi G^{\xi\nu} + D(\mathbf{a}) J^\nu = 0. \] (2)

We use the notations \( J^\nu \equiv \varepsilon^{\nu\sigma\rho\sigma} F_{\tau\rho}, \quad \mathbf{a} \equiv (a_1, a_2). \) We construct the general solution of eqs. (2), analyze its properties and consider processes of plain wave scattering.

Action (1) and the Euler-Lagrange equations (2) are invariant under gauge transformation \( \text{A}_\mu(x) \rightarrow \text{A}_\mu(x) + \partial_\mu \varphi(x). \) Thus the solution of (2) is defined up to a gauge transformation. We fix it by choosing the temporal gauge \( A_0 = 0. \) Then the vector-potential \( A^\mu = (0, \vec{A}) \) yields the electric field \( \vec{E} = -\partial_0 \vec{A} \) and the magnetic induction \( \vec{B} = \hat{\rho} \times \vec{A}. \)

We solve the eqs. (2) using the Fourier transform over coordinates \( x_0 = ct, x_1, x_2 \) for the vector-potential \( A_\mu: \)
\[ A_\mu(x) = \frac{1}{(2\pi)^2} \int e^{ix_3} A_\mu(x_3, \vec{p}) d\vec{p} = \]
\[ = \frac{2\mathcal{R}}{(2\pi)^2} \int \theta(p_0) \left[ e^{ip_3} A_\mu(x_3, \vec{p}) \right] d\vec{p} \]

Here and later we use the notation \( \vec{p} \) for vector \( \vec{p} = (p_0, p_1, p_2), \quad \vec{p} = p_0 x_0 - p_1 x_1 - p_2 x_2. \) \( \mathcal{R} \) denotes the real part and \( \omega = cp_0 \) the frequency.

III. SOLUTION OF EULER-LAGRANGE EQUATIONS

With the gauge condition \( A_0 = 0, \) the eqs. (2) for \( \vec{A}(x_3, \vec{p}) \) are equivalent to the following ones
\[ (\partial_3 \mathcal{E} \mathcal{P}^{-2} \partial_3 + \mathcal{E}) \rho = \frac{2\mathcal{R}}{p_3^2} \mathcal{D}(\mathbf{a}) \tau, \] (3)
\[ (\partial_3 \mathcal{M}^{-1} \partial_3 + \mathcal{M}^{-1} \mathcal{P}^2) \tau = -2i p_0 \mathcal{D}(\mathbf{a}) \rho, \] (4)
\[ A_3 = -\mathcal{P}^{-2} \partial_3 \rho \] (5)

where \( \rho = i p_1 A_1 + i p_2 A_2, \quad \tau = i p_2 A_1 - i p_1 A_2, \quad \mathcal{P} \equiv \mathcal{F}(\kappa_1, \kappa_2, \kappa_3), \quad \kappa_i \equiv \sqrt{p_0^2 \epsilon_\mu \mu_4 - p_1^2 - p_2^2}. \)

By definition the real part of \( \kappa_j \) is chosen to be non-negative, and if it vanishes, then \( \kappa_j = -i|\kappa_j|. \)

The fields \( \rho, \tau \) are found from eqs. (3) (4). The components \( A_1, A_2 \) of the vector-potential \( \vec{A} \) are expressed by \( \rho \) and \( \tau, \)
\[ A_1 = -i(\rho_1 p_1 + \tau p_2) p_2^{-2}, \quad A_2 = i(\tau_1 p_1 - \rho_2 p_2) p_2^{-2}, \] (6)

where \( p^2 = p_1^2 + p_2^2. \) The electromagnetic field \( \vec{A}(x_3, \vec{p}) \) in the considered medium is characterized by the mutually orthogonal vectors \( \vec{p}_\| = (p_1, p_2, 0), \vec{p}_\perp = (p_2, -p_1, 0), \)
\[ \vec{p}_\| = (0, 0, 1). \] The vectors \( \vec{p}_\|, \vec{p}_\perp \) define the plane of incidence. In virtue of (3), (4), the vector potential \( \vec{A} = (A_1, A_2, A_3) \) can be presented in the form \( \vec{A} = \vec{A}_\| + \vec{A}_\perp \)
where \( \vec{A}_\| \) is parallel to the plane of incidence, and \( \vec{A}_\perp \) is perpendicular to it,
\[ \vec{A}_\| (x_3, \vec{p}) = (-i p_0 \vec{p}_\|^2 - i \vec{p}_\|^2 \partial_3) \rho(x_3, \vec{p}), \] (7)
\[ \vec{A}_\perp (x_3, \vec{p}) = -i p_0 \vec{p}_\|^2 \tau(x_3, \vec{p}). \] (8)

Since in our gauge \( \vec{E}(\vec{p}, x_3) = -i p_0 \vec{A}(\vec{p}, x_3), \) the field \( \rho(x_3, \vec{p}), (\tau(x_3, \vec{p})) \) describe plane waves whose electric field vectors are parallel (perpendicular) to the plane of incidence. Eqs. (3) (4) show that the Chern-Simons defects mix parallel and transverse components of the phonon field.

Let us introduce the notations \( f(x_3) = (\rho(x_3), \tau(x_3)) \) and define
\[ \mathbf{K} = \begin{pmatrix} \mathcal{E} \mathcal{P}^{-2} & 0 \\ 0 & \mathcal{M}^{-1} \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & p_0^{-1} \\ -p_0 & 0 \end{pmatrix}, \]
\[ \mathbf{L}_i = \begin{pmatrix} e_i & 0 \\ 0 & m_i \end{pmatrix}, \quad e_i = \frac{\epsilon_\mu \mu_4}{\kappa_i}, \quad m_i = \frac{\kappa_i}{\mu_i}, \quad i = 1, 2, 3. \]

Then we can present (3) (4) in a compact form
\[ (\partial_3 \mathbf{K} \partial_3 + \mathbf{K} \mathbf{P}^2) \mathbf{f} = 2i \mathcal{D}(\mathbf{a}) \mathbf{C} \mathbf{f}. \] (9)
We conclude that $f$ is continuous at $x_3 = l_j$, 
\[
f_j(l_j) = f_{j+1}(l_j),
\]
(10) since a discontinuity would yield a $\delta'$-function on the l.h.s. of (9), which is absent on the r.h.s. Due to (9) $A_{1,2}$ is continuous at the defects. Thus the derivatives $\partial_0 A_{1,2}$ are continuous, which implies the continuity of the components $E_{1,2}$ and $B_3$.

Introducing $f(x_3) = \mathcal{F}(\tilde{f}(x_3))$ with $\tilde{f}(x_3) = (f_1(x_3), f_2(x_3), f_3(x_3))$ we integrate (9) from $x_3 = l_j - \eta$ to $x_3 = l_j + \eta$ with infinitesimal $\eta$ 
\[
\frac{L_{j+1}}{\kappa_{j+1}} \partial_3 f_{j+1}(l_j) - \frac{L_j}{\kappa_j} \partial_3 f_j(l_j) = 2ia_j \mathbf{Cf}(l_j).
\]
(11) Within the limits $x_3 = \pm l_j$ eq. (9) is written as $(\partial_3^2 + \kappa_3^2)f_3(x) = 0$ and yields 
\[
f_i = f_i^+ + f_i^-, f_i^\pm = (\rho_i^\pm, t_i^\pm) = c_i^\pm e^{\mp i\kappa_i x_3}.
\]
(12) For real $\kappa_i$ the solution with the upper (lower) sign describes a plane wave moving in positive (negative) $x_3$-direction.

It follows from (12), (7), (8) that \( \tilde{A} = \tilde{A}^+ + \tilde{A}^- \) and 
\[
\tilde{A}^\pm(x_3) = -\frac{i\rho_{\perp}^\pm(x_3)}{p^2}, \quad \tilde{A}_\perp(x_3) = -\frac{i\rho_{\perp}^\pm(x_3)}{p^2}
\]
(13) with 
\[
\rho_{\perp}^\pm \equiv \rho_{\perp}^\pm + p^2 \rho_{\perp}^\pm, \quad (\rho_{\perp}^\pm)^2 = p^2 \rho_{\perp}^2 - 2p^2, \quad (14)
\]
\[
\mathcal{P}_\perp \equiv \mathcal{P}_{p_1 = p_2 = 0} = p_0 \mathcal{F}(n_1, n_2, n_3), \quad n_i = \sqrt{\varepsilon_i \mu_i}. \quad (15)
\]
Since $\partial_3 f_j = ik_3 \tilde{f}_j$, where $\tilde{f}_j \equiv f_j^+ - f_j^-$, the conditions (11) can be written as 
\[
L_{j+1} \tilde{f}_{j+1}(l_j) - L_j \tilde{f}_j(l_j) = 2ia_j \mathbf{Cf}_j(l_j), \quad j = 1, 2. \quad (16)
\]
These eqs. describe the discontinuity of the components $H_{1,2}$ of the magnetic field and $D_3$ of the dielectric displacement due to the currents $a_j J^\alpha_j$ in (2).

\[
\begin{align*}
D_{3,j+1} - D_{3,j} &= -a_j J_j^\rho = -2a_j B_3, \\
H_{1,j+1} - H_{1,j} &= -a_j J_j^\rho = 2a_j E_1, \\
H_{2,j+1} - H_{2,j} &= a_j J_j^\rho = 2a_j E_2. \quad (17)
\end{align*}
\]

In order to solve the eqs. (10) it is convenient to introduce the following 2 × 2 matrices 
\[
T_{j}^{\alpha\beta} = 1 + \alpha \mathbf{L}_{j+1}^{-1} (\beta \mathbf{L}_j - 2a_j \mathbf{C}), \quad j = 1, 2, \quad \alpha, \beta = \pm 1
\]
and 4-component vectors $\mathbf{U}_j = (u_j^+, u_j^-)$, $\mathbf{V}_j = (v_j^+, v_j^-)$ with $u_j^\pm = f_j^\pm(l_j)$, $v_j^\pm = \tilde{f}_j^\pm(l_j)$. Then we obtain from (10) the relations between the $\mathbf{V}$ and $\mathbf{U}$ by means of the transfer matrices $T_{j}$, 
\[
\begin{align*}
\mathbf{V}_j &= T_j \mathbf{U}_j, \quad \mathbf{U}_2 = T_1 \mathbf{V}_1, \quad \mathbf{V}_2 = T_1 \mathbf{U}_1, \quad T_j = T_2 T_1, \\
T_{j} &= \begin{pmatrix} e^{-i\kappa_j} & 1 \\ 0 & e^{i\kappa_j} \end{pmatrix}, \quad T_j = \frac{1}{2} \begin{pmatrix} T_j^+ & T_j^- \\ T_j^+ & T_j^- \end{pmatrix}.
\end{align*}
\]
One has for nonactive media (real $\varepsilon$, $\mu$ and $\alpha$) 
\[
\begin{align*}
G_j &= T_j^1 G_{j+1} T_j, \quad T_j^1 G_j T_j = G_j, \quad T_j^1 G_j T_j = G_j, \quad (20)
\end{align*}
\]
Here $\dagger$, $\star$ denote the hermitian conjugation of matrix and the complex conjugation of vector components, 
\[
\mathbf{g}_j = \begin{pmatrix} \frac{\mathbf{g}_j}{|\mathbf{g}_j|} \\ \frac{\mathbf{g}_j}{|\mathbf{g}_j|} \end{pmatrix}, \quad \mathbf{g}_j \equiv \begin{pmatrix} p_0 e_{ij} \\ 0 \\ m_{ij}/p_0 \end{pmatrix}.
\]
(38) $\Re \alpha_j (\Im \alpha_j)$ is the real (imaginary) part of $\alpha_j$.

For a complete analysis of the propagation of waves in the considered medium it is enough to assume that in the region $x_3 > l/2$ there are no waves moving in the negative direction of $x_3$-axis. This restriction obeys $f_j^\pm(l/2) = v_j^\pm = 0$, since for real $\kappa_j$, $f_j^\pm(l/2)$ is the amplitude of the wave moving from $x_3 = -\infty$ to the plane $x_3 = l/2$, and for imaginary $\kappa_j$ the field must decay exponentially for $x_3 \rightarrow +\infty$. Then $\mathbf{V}_2 = 2U_1$ yields 
\[
\begin{align*}
\mathbf{T}^+ u_1^+ + \mathbf{T}^- u_1^- &= 0, \quad \mathbf{v}_2^\pm = (\mathbf{T}^+ + \mathbf{T}^-) u_1^\pm + \mathbf{T}^- u_1^- \quad (21)
\end{align*}
\]
where $\mathbf{T}^{\pm\pm}$ denote the corresponding $2 \times 2$ submatrices of the $4 \times 4$ - matrix $T$.

For real $\kappa_l$ the amplitude of the incident wave propagating in the region $x_3 < -l/2$ in the positive $x_3$-direction is $c_m = c_i^+ = u_1^+ e^{-ik_3 l/2}$. The amplitude of the reflected wave is $c_c = c_c^+ = v_1^+ e^{ik_3 l/2}$ and that of the transmitted wave is given by $c_l = c_c^+ = v_1^+ e^{ik_3 l/2}$ for real $\kappa_3$. The amplitudes $c_c$, $c_l$ are obtained from (21):

\[
c_c = e^{-ik_3 l/2} \chi^l c_m, \quad (22)
\]
\[
c_l = e^{ik_3 l/2} (\mathbf{T}^+ - \mathbf{T}^-) c_m. \quad (23)
\]
If $\kappa_j$ is imaginary, then $c_c$ yields again the amplitude of the reflected wave (total reflection), whereas $c_l$ describes the amplitude of the decaying wave.

If both $\kappa_1$ and $\kappa_3$ are imaginary, then the waves are totally reflected at both $x_3 = \pm l/2$. The waves obey $v_2^\pm = u_1^\pm = 0$. Then the equations (21) can have a nonzero solution only if $\kappa_3$ is imaginary (since in virtue of (20) $\mathbf{V}_2^T G_3 \mathbf{V}_2 = \mathbf{U}_1^T G_1 \mathbf{U}_1 = 0$), and det $\mathbf{T}^- = 0$ with 
\[
\begin{align*}
\mathbf{T}^- &= \frac{1}{4} (\mathbf{T}^+ e^{-ik_3 l/2} \mathbf{T}^- + \mathbf{T}^- e^{ik_3 l/2} \mathbf{T}^-) = \\
&= \frac{1}{4} (\mathbf{T}^- (\mathbf{T}^- e^{2ik_3 l/2} \mathbf{R}_1 - \mathbf{R}_2 \mathbf{R}_1) e^{-ik_3 l/2} \mathbf{T}^-), \\
\mathbf{R}_2 &= -(\mathbf{T}^-)^{-1} \mathbf{T}^+ , \quad \mathbf{R}_1 = \mathbf{T}^- (\mathbf{T}^-)^{-1}. \quad (24)
\end{align*}
\]
The matrices $\mathbf{R}_j$ describe the total reflection of the waves coming from the center to $l_j$, $\mathbf{v}_j^\perp = \mathbf{R}_j \mathbf{v}_j^\parallel$, $\mathbf{u}_j^\perp = \mathbf{R}_j \mathbf{u}_j^\parallel$. These matrices differ by a similarity transformation from unitary matrices $\mathbf{O}_j = \mathbf{g}_j^2/2 \mathbf{R}_j \mathbf{g}_j^{-1/2}$. Thus one obtains electromagnetic waves propagating in layer 2 as soon as one of the two eigenvalues $e^{\pm \xi}$ of the unitary matrix $\mathbf{O}_2 \mathbf{O}_1$ agrees with $e^{2ik_3 l/2}$.

If $\kappa_j$ is real, then the functions $f_j^\pm(x_3) e^{i\kappa_3 x_3}$ describe plane waves propagating in the medium with constants
\(\varepsilon_j, \mu_j\) in directions of vectors \(\vec{p}_j^\pm = (p_1, p_2, \pm \kappa_j)\) with velocity \(v_j = c p_0/|\vec{p}_j^\pm| = c/n_j\). For the angle \(\theta_j\) between \(\vec{p}_j\) and the \(x_3\)-axis it holds \(\sin \theta_j = p/|\vec{p}_j| = p/(p_0 n_j)\), and this equality yields Snell's law \(\sin \theta_j = \sin \theta_k = n_k/n_j\). The component \(v_j^\pm\) of the wave front velocity \(v_j\) is equal to \(v_j^\pm = \pm v_j \kappa_j / |\vec{p}_j^\pm| = \pm c \kappa_j / (p_0 n_j)^2\).

The electric field vector of the wave propagating in the \(j\)-th layer in the positive (negative) direction of the \(x_3\) axis is \(\vec{E}_j^\pm = -i p_0 \vec{A}_j^\pm (\vec{E}_j^- = -i p_0 \vec{A}_j^-)\), and the corresponding energy density is \(\varepsilon_j |\vec{E}_j^\pm|^2 = (\varepsilon_j |\vec{E}_j^-|^2)^2\). The energy current density propagating in the positive \(x_3\)-direction is \(I_j = I_j^+ - I_j^-\), \(I_j^\pm = v_j^\pm \varepsilon_j |\vec{E}_j^\pm|^2\). In virtue of \(I_1 = I_2^\pm = I_3^\pm = 0\),

\[
I_j^\pm = I_j^\pm + I_j^\pm, \quad I_j^\pm = \frac{p_0 q_j |\vec{p}_j^\pm|^2}{p^2}, \quad I_j^\pm = \frac{p_0 m_j |\vec{p}_j^\pm|^2}{p^2}.
\]

If we denote \(U_3 \equiv V_2\), then it holds \(I_j = p^2 U_j^* G_{Uj} / p_0\). The energy is conserved in the non-active medium, therefore the quantity \(I_j\) is independent of \(x_3\) and \(I_j = I_k\) (in agreement with \(I_1\)). In virtue of \(I_1\), the energy current \(I_j\) vanishes in case of total reflection, since \(V_2 G_{Uj} V_2 = 0\) by imaginary \(\kappa_3\) and \(\nu_2 = 0\).

If \(\kappa_j\) is imaginary, the waves propagate in the \(j\)-th layer parallel to the plane \(x_3 = 0\) in direction of vector \(\vec{p}_j\) similarly as in a wave-guide. Due to the boundary conditions given by the matrices \(O_j\), the relation between \(\vec{p}_j\) and \(\vec{p}_j\) will be changed.

**IV. CONCLUSION**

The Chern-Simons interaction at \(x_3 = l_i\) does not change Snell’s law. However, the reflection and transmission coefficients depend on the strengths \(a_i\) of these interactions. They lead to a mixing between the parallel and perpendicular components of the electromagnetic waves and they change the relation between frequency and wave-vector for waves between two totally reflecting media. Consequently such interactions will also modify the strength of the Casimir effect. A search for surfaces or layers showing such a behavior is of high interest.

The presented results may be verified experimentally. In this way, it is possible to determine the constant \(c\) describing the interaction of film with the electromagnetic field in our model. By finite \(a\) the Chern-Simons potential breaks the time and space parity. It holds also for interaction of photons with \((2+1)\)-dimensional Dirac field modelling two-dimensional materials.

In this paper we have considered only the case of inactive media (\(3 \alpha_2 = 3 \varepsilon_2 = 3 \mu_2 = 0\)). Using complex values of the model parameters and taking also into account the defect contribution of the \((3+1)\)-dimensional Dirac field, it is possible within Symanzik approach to construct in quantum electrodynamics a model for wide class of quantum macroscopic phenomena in systems with two-dimensional space inhomogeneities. In such a model one can investigate the Hall effect, plasmonics, nanophotonicics, topological insulators, properties of two-dimensional materials, doping, thin films and sharp interfaces.

Recently one places high emphasis on these problems, and many important results are obtained in studies of them. The comprehensive model built within the proposed approach and based on fundamental physical principles seems to be suitable for this research field. We expect that it provides an opportunity to obtain more accurate quantitative results, than those which have been achieved to date by use of other theoretical assumptions. Investigations of such models will enable us to understand more deeply the relationship between different nano-physical effects.

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1. H. B. G. Casimir, Proc. K. Ned. Akad. Wet. B 51, 793 (1948).
2. U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998).
3. A. Roy, C.-Y. Lin and U. Mohideen, Phys. Rev. D 60, 111101 (R) (1999).
4. B. W. Harris, F. Chen and U. Mohideen, Phys. Rev. A 62, 052109 (2000).
5. G. Bressi, G. Carugno, R. Onofrio and G. Ruoso, Phys. Rev. Lett. 88, 041804 (2002).
6. M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Adv. in the Casimir Effect (The International Series of Monographs on Physics, Oxford University Press, 2009), Vol. 145, p. 752.
7. K. A. Milton, J. Phys. A: Math. Gen. 37, R209 (2004).
8. K. Symanzik, Nucl. Phys. B 190, 1 (1981).
9. V. N. Markov and Yu. M. Pis'mak, J. Phys. A: Math. Gen. 39, 6525 (2006); ArXiv: hep-th/0505218 (2005).
10. V. N. Markov and Yu. M. Pis'mak, Phys. Rev. D 81, 065005 (2010).
11. V. Fialkovsky, V. N. Markov and Yu. M. Pis’mak, J. Phys. A: Math. Gen. 39, 6357 (2006); Int. J. Mod. Phys. A 21, 2601 (2006).
12. K. I. Kondo, T. Ebihara, T. Iizuka and E. Tanaka, Nucl. Phys. B 434, 85 (1995); T. Appelquist, M. J. Bowick, D. Darabali and L. C. Wijewardhana, Phys. Rev. D 33, 3774 (1986).
13. A. N. Grigorenko, M. Polini and K. S. Novoselov, Nature Photonics 6, 749 (2012); A. H. Castro Neto, F. Guinea, N. M. R. Torres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. 81, 109 (2009); I. V. Fialkovsky and D. V. Vassilevich, J. Phys. A: Math. Gen. 42, 442001 (2009); Wang-Kong Tse and A. H. MacDonald, Phys. Rev. B 84, 205327 (2011); Liang Chen and Shao-long Wan, Phys. Rev. B 84, 075149 (2011); J. González, JHEP 07, 175 (2013); V. N. Kotov, B. Uchoa, V. M. Pereira, F. Guinea and A. H. Castro Neto, Rev. Mod. Phys. 84, 1067 (2012); J. E. Moore, Nature 464, 194-198 (2010).
14. G. Bracco and B. Holst (Eds.), Surface Science Techniques, (Springer Series in Surface Science, Springer-Verlag Berlin Heidelberg, 2013), Vol. 51, p. 663; M. L. Brongersma and P. G. Kik (Eds.), Surface Plasmon Nanophotonics, (Springer Series in Optical Sciences, Springer, 2007), Vol. 131, p. 269; S. A. Maier Plasmonics: Fundamental and Applications, (Springer Science+Business Media LLC, 2007), p. 223.
Appendix: Detailed results and comments

We give an obvious form of matrices used in our calculations. They are functions of $\hat{\epsilon} = (\epsilon_1, \epsilon_2, \epsilon_3)$, $\hat{m} = (m_1, m_2, m_3)$ and can be written as

$$M(\hat{\epsilon}, \hat{m}) = \begin{pmatrix} f(\hat{\epsilon}, \hat{m}) & g(\hat{\epsilon}, \hat{m}) \\ -p^2 g(\hat{m}, \hat{\epsilon}) & f(\hat{m}, \hat{\epsilon}) \end{pmatrix}. \quad (A.1)$$

Thus, $M$ is completely defined by its elements $\{M\}_{11} = f(\hat{\epsilon}, \hat{m})$ and $\{M\}_{12} = g(\hat{\epsilon}, \hat{m})$.

The matrices $T_{j^\pm \pm}$ and their inverses read

$$\{T_{j^\alpha}^{\beta}\}_{11} = 1 + \alpha \beta \frac{\epsilon_j}{\epsilon_{j+1}}, \quad \{T_{j^\alpha}^{\beta}\}_{12} = -\frac{2a_j}{\epsilon_{j+1} p_0},$$

$$\{(T_{j^\beta}^{\alpha})^{-1}\}_{11} = 1 + \alpha \beta \frac{m_{j+1}}{m_{j+1}}, \quad \{(T_{j^\beta}^{\alpha})^{-1}\}_{12} = \frac{2a_j}{\alpha \beta m_{j+1}},$$

$$\det(T_{j^\beta}^{\alpha}) = \frac{4a_j^2 + (\epsilon_{j+1} + \alpha \beta \epsilon_j)(m_{j+1} + \alpha \beta m_j)}{\epsilon_{j+1} m_{j+1}}.$$

The matrices $T_{\pm \mp}$ obey

$$T_{\pm \mp} = (\cos(\kappa_2 l)Z_{\mp}^{\pm} + i \sin(\kappa_2 l)Z_{\pm}^{\mp}),$$

$$\{Z_{\pm}^{\mp}\}_{11} = \frac{\alpha \beta \epsilon_j + \epsilon_3}{2 \epsilon_3}, \quad \{Z_{\pm}^{\mp}\}_{12} = -\frac{(\alpha_1 + \alpha_2)}{\epsilon_3 p_0},$$

$$\{Z_{\mp}^{\pm}\}_{11} = \frac{4 \alpha_1 \alpha_2 m_2 - (\alpha \beta \epsilon_3)m_2}{2 \epsilon_2 m_3 \epsilon_3},$$

$$\{Z_{\mp}^{\pm}\}_{12} = \frac{\alpha \beta \epsilon_2 + \epsilon_3 m_2}{2 \epsilon_2 m_3 p_0}, \quad \alpha, \beta = \pm 1.$$

The relations (22) for the amplitudes $c_{\pm}$ can be written as

$$c_{\pm} = -e^{i\kappa l} T_{\pm} c_{\mp}, \quad c_{\mp} = e^{i(\kappa + \kappa_2)l/2} T_{\mp} c_{\pm}$$

with $T_{\pm} = (T^{\pm})^{-1} T^{\mp}$, $T_{\pm} = T^{\pm} + T^{\pm} - (T^{\mp})^{-1} T^{\mp}$.

Using the notations

$$\varphi(a, b) = a \cos(\kappa_2 l) + i b \sin(\kappa_2 l),$$

$$\psi(a, b, c) = b(a + c) \cos(\kappa_2 l) + i (a c + b^2) \sin(\kappa_2 l),$$

$$\epsilon_i^\alpha = \varphi(\alpha \epsilon_2, \epsilon_1), \quad m_i^\beta = \varphi(\beta m_2, m_1), \quad \phi_i^{\alpha \beta} = \epsilon_i^\alpha m_i^\beta,$$

$$\epsilon^{\alpha \beta} = \epsilon^\alpha m^\beta, \quad \alpha, \beta = \pm 1,$$

one can present the matrices $T_{\pm}, T_{\mp}$ in the following form

$$\{T_{\pm}\}_{11} = \frac{2a_1(\epsilon_2 m_3 + 4i \alpha_1 \alpha_2 m_2 \sin(\kappa_2 l))}{\epsilon_2}, \quad \{T_{\pm}\}_{12} = \frac{-4m_1(\alpha_2 \epsilon_2 m_3^2 + \alpha_1 \epsilon_2 m_3^2)}{p_0^2 z},$$

$$\{T_{\mp}\}_{11} = \frac{1}{z} \left(8a_1 \alpha_2 \epsilon_2 m_2 + \psi^{\pm \mp} + 4(\alpha_1^2 \phi_3^{\pm \mp} - \alpha_2^2 \phi_3^{\mp \pm} - 4 \alpha_2 \epsilon_2 m_3^2 \sin^2(\kappa_2 l)) \right),$$

$$\{T_{\mp}\}_{12} = \frac{4m_1(\alpha_2 \epsilon_2 m_3 + \alpha_1 (\epsilon_2 \phi_3^{\mp \pm} + 4 \alpha_2 \epsilon_2 m_3^2 \sin^2(\kappa_2 l)))}{p_0^2 z},$$

where $z = 4a_2 m_2 \epsilon_3 \sin(\kappa_2 l)$.

The reflection matrices $R_{\pm}$ defined by (24) are

$$R_{1} = T_{1^+}^- (T_{1^-}^+)^{-1}, \quad R_{2} = -(T_{2^+}^-)^{-1} T_{2^-}^+,$$

$$r_{1}^{\alpha \beta} = 4a_2^2 + (\epsilon_1 + \alpha \epsilon_2)(m_1 + \beta m_2), \quad r_{2}^{\alpha \beta} = 4a_2^2 + (\epsilon_3 + \alpha \epsilon_2)(m_3 + \beta m_2), \quad \alpha, \beta = \pm 1.$$

Multiplication and the inverse of matrices of the form (A.1) yield matrices of the same type. Because $g_2$ does not belong to this class of matrices, this is also the case for the matrices $O_{\pm}$ with

$$O_{0} = O_{0} O_{1} = \frac{1}{R} \left( \begin{array}{cc} P & Q \end{array} \right)^* \left( \begin{array}{cc} P & Q \end{array} \right). \quad (A.2)$$

The relations (22) obey

$$r_{1}^{+} r_{2}^{+} + 16a_2^2 \epsilon_2 m_2 = r_{1}^{-} r_{2}^{-}. \quad (A.3)$$

If $a_1, a_2, a_2, m_2$ are real, and $e_1, e_3, m_1, m_3$ are imaginary, then $(r^{+})^* = r^{-}, \quad (r^{-})^* = r^{+}$, and it follows from (A.2, A.3) that the matrices $O_{1}, O_{2}$ and $O_{0}$ are unitary.

The eigenvalues $\lambda_{1,2}$ of the matrix $O$ read

$$\lambda_{1,2} = \frac{-P - P^* \pm \sqrt{(P - P^*)^2 - 4QQ^*}}{2R} = e^{i(\zeta + n_2)},$$

$$\tan(\zeta) = \frac{3R}{\Re P}, \quad \tan(n_{1,2}) = \mp \sqrt{(3P)^2 + |Q|^2}. \quad (R, \Re P)$$

They coincide for $3P = 0, Q = 0$. In this case $n_{1,2} = 0,$

$$r_{2}^{+} = \frac{-a_2}{a_1^2}, \quad P = \frac{a_2}{a_1} r_{1}^{+}, \quad r_{1}^{-} = P^*.$$

The boundary conditions (17, 19) can be proved directly from (50). Using the relations $\bar{D} = \varepsilon \bar{E}, \bar{B} = \bar{\delta} \bar{A}, \bar{E} = -\delta_0 \bar{A}, \bar{B} = \bar{\delta} \bar{A}, \bar{p} = \kappa^2 \bar{p}^2 \bar{\epsilon} \mu$ and notations $\varepsilon \bar{\kappa} = e, \kappa/\mu = m$, we obtain $D_{\bar{3}} = -p_0 \bar{\epsilon} \mu r$.

$$H_{1} = \frac{p_1 m^2 - p_2 \epsilon_1 p_0^2}{p^2}, \quad H_{2} = \frac{p_1 \epsilon_1 p_0^2 + p_2 m^2}{p^2}.$$

It follows from $J^0 = e^{i\omega \sigma} F_{\sigma \rho}$ that $J^0 = 2r$,

$$J^1 = \frac{2p_0 (p_1 \tau - p_2 \rho)}{p^2}, \quad J^2 = \frac{2p_0 (p_1 \rho + p_2 \tau)}{p^2}. \quad (p^2)$$

Thus in virtue of (10), the equalities (17, 19) are fulfilled.