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To cite this article: Sivakumar Narsu and B Rushi Kumar 2017 IOP Conf. Ser.: Mater. Sci. Eng. 263 062016

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Diffusion thermo effects on unsteady MHD free convection flow of a Kuvshinski fluid past a vertical porous plate in slip flow regime

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Abstract. The main purpose of this work is to investigate the diffusion-thermo effects on unsteady combined convection magneto-hydromagnetic boundary layer flow of viscous electrically conducting and chemically reacting fluid over a vertical permeable radiated plate embedded in a highly porous medium. The slip flow regime is applied at the porous interface a uniform magnetic field is applied normal to the fluid flow direction which absorbs the fluid with suction that varies with time. The dimensionless governing equations are solved analytically using two terms harmonic and non-harmonic functions. The expressions for the fields of velocity, temperature and concentration are obtained. For engineering interest we also calculated the physical quantities the skin friction coefficient, Nusselt and Sherwood number are derived. The effects of various physical parameters on the flow quantities are studied through graphs and tables. For the validity, we have checked our results with previously published work and found good agreement with already existing studies.

Nomenclature.

*x*, *y* dimensional distances

*t* dimensional time

*C_p* specific heat at constant pressure

*u*, *v* dimensional velocity components

*C_s* concentration susceptibility

*T* dimensional temperature

*D_m* molecular diffusivity

*C* dimensional concentration

*D* mass diffusivity

*Du* Dufour number

*K* _dimensional chemical reaction parameter

*Sh* Sherwood number

*F* Skin friction

*Nu* Nusselt number

*V_s* suction velocity

Greek Symbols

*μ* coefficient of viscosity

*κ* thermal conductivity
1. Introduction

Convective flows with concurrent heat and mass transfer under the influence of a magnetic field, chemical reaction, and thermal radiation arise in many transport processes that has applications in many branches of science and engineering. This phenomenon plays a vital role in the chemical industry, chemical vapor deposition on surfaces, cooling of nuclear reactors, power and cooling industry for drying, and petroleum industries. Recently Anjalidevi and Kandasamy [1], Elbashbeshy [2], Shateyi and Motsa [3], Raju and Chamka et al. [4] are studied the Soret effect due to mixed convection on unsteady magnetohydrodynamic flow past a semi infinite vertical permeable moving plate in presence of thermal radiation, heat absorption and homogeneous chemical reaction. Ibrahim et al. [5] considered the effects of chemical reaction and radiation absorption on unsteady MHD free convection flow past a semi- infinite vertical permeable moving plate with heat source and suction. Vidya Sagar and Raju et al. [6] examined an unsteady magnetohydrodynamic (MHD) radiation absorption free convective boundary layer flow of Kuvshinski fluid in the presence of a porous medium.

At the macroscopic level, it is well known that the boundary condition due to viscous fluid at the solid wall is one of the no slip is said the fluid velocity of the solid boundary. While the no-slip condition has been treated experimentally to be accurate for a number of macroscopic flows, it remains an assumption that is not based on physical principles. The particle at the surface has a finite tangential velocity. In engineering applications, the study of magneto-micropolar fluid flows in the slip flow regimes with heat transfer, for example, the power generators, refrigeration coils, electric transformers, transmission lines, and heating elements. Khandelwal et al. [7] proposed the effects of permeability variation on the Magnetohydrodynamic (MHD) an unsteady flow of micropolar fluid through a porous medium in a slip flow regime over an infinite porous flat plate. Sharma and Chaudhary [8] analyzed free convective heat and mass transfer effect of variable suction on viscous incompressible fluid flows past a vertical porous plate in a slip flow regime. Recently, Sharma [9] demonstrated the effects of periodic temperature and concentration on unsteady free stream consisting of a mean velocity over an
exponentially varying with time. Chaudhary and Jha [10] discussed the numerical study of chemically reactive MHD micropolar fluid flow past a vertical plate in slip flow regime.

Jyothi et al. [11] presently chemical and thermo-diffusion effects on MHD free convection flow through a porous medium in a slip flow regime. Effects on the magnetohydrodynamic free convective vertical porous plate due to diffusion-thermo and thermo-diffusion embedded in a non-Darcian porous medium have been examined by Ramachandra Prasad et al. [12]. Heat and mass transfer free convective MHD flow of a porous diffusion-thermo over an inclined surface has been demonstrated by Durga Prasad et al. [13]. Krishna Reddy et al. [14] studied heat and mass transfer MHD flow of Kuvshinski fluid with temperature gradient dependent heat source effects over a vertical porous surface in the presence of variable heat and mass flux. Varma et al. [15] developed an analysis of magnetohydrodynamic heat and mass transfer micropolar fluid with convective diffusion-thermo effects. Sekhar et al. [16] investigated the effects of heat and mass transfer MHD free convective diffusion-thermo and radiation absorption with a porous plate in a slip flow regime. Slip effects of viscous dissipation on unsteady MHD flow over a stretching sheet studied by Sekhar and Viswanatha Reddy [17].

As per the author’s knowledge, the interaction between the chemical reaction and diffusion-thermo in the presence of radiation absorption, porous medium effects has received little attention. Hence, an attempt is made to study the diffusion-thermo effects on an unsteady heat and mass transfer MHD free convection flow of a viscous incompressible electrically conducting Kuvshinski fluid through a porous medium from a vertical porous plate with varying suction velocity in slip flow regime.

2. Mathematical formulation

A two dimensional unsteady magnetohydrodynamic (MHD) flow past a vertical infinite plate embedded in a porous medium has been considered with the viscoelastic fluid model. A uniform transverse magnetic field $B_0$ is applied normal to the direction of the fluid flow direction, let $x^*$-axis taken along the flow in the vertical direction and $y^*$-axis is taken perpendicular to it. The presence of chemical reaction, Dufour effect, and a heat source or sink parameters are also considered along with buoyancy effects past a vertical porous plate with a dependent heat source in slip flow regime with uniform velocity $u_p^*$. The physical model of the problem as shown in Figure 1.

---

Figure 1. Physical Model
In the analysis, the following assumptions are made

1. The flow is unsteady and laminar.
2. The plate is taken that sufficiently long enough is very small, so the all the physical quantities are taken as the functions of $y$ and $t$ only.
3. The induced magnetic field is neglected because magnetic Reynolds number is very small.
4. The effects of viscous and Joule's dissipation are neglected.
5. Now assumed that all the fluid flow properties to be constant except that of the impact of density variation temperature.

\[
\frac{\partial v^*}{\partial y^*} = 0 \tag{1}
\]

\[
\left(1 + \lambda \frac{\partial}{\partial t^*}\right) \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^*^2} + g \beta_f \left(T^* - T_w\right) + g \beta_c \left(C^* - C_w\right) = \left(\frac{\sigma B_0^2}{\rho K} + \frac{\nu}{K}\right) \left(1 + \lambda \frac{\partial}{\partial t^*}\right) u^* \tag{2}
\]

\[
\left(1 + \lambda \frac{\partial}{\partial t^*}\right) \frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^*^2} - \frac{Q_b}{\rho C_p} \left(T^* - T_w\right) + \frac{Q_1}{C_p} \left(C^* - C_w\right) + \frac{D}{C_p} \frac{\partial^2 C^*}{\partial y^*^2} - K_1 \left(C^* - C_w\right) \tag{3}
\]

\[
\left(1 + \lambda \frac{\partial}{\partial t^*}\right) \frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^*^2} - K_1 \left(C^* - C_w\right) \tag{4}
\]

The appropriate boundary conditions are for the velocity, temperature and species diffusion fields are

\[u^* = u_p^* + L \frac{du^*}{dy^*}, \quad T^* = T_w + \varepsilon \left(T_w - T_w\right) e^{i\alpha y^*} \]

\[C^* = C_w + \varepsilon \left(C_w - C_w\right) e^{i\alpha y^*} \text{ at } y^* = 0 \]

\[u^* = 0, T^* \rightarrow T_w, C^* \rightarrow C_w \text{ at } y^* \rightarrow \infty \tag{5}\]

Where $n^*$ is constant velocity of the wall.

From Eq. (1) it is clear that the suction velocity at the plate is a function of time $t$ only. Let us assume that

\[v^* = -V_0 \left(1 + \varepsilon A e^{i\alpha y^*}\right) \tag{6}\]

Where $V_0$ is the suction velocity and $\varepsilon A$ $<< 1$. The negative sign indicated that the suction velocity directed towards the plate.

By introducing the non-dimensional quantities are
By using the above non-dimensional quantities, the Eqs. (2)-(4) can be expressed in a dimensionless form as
\[
\begin{align*}
\frac{\partial u}{\partial t} + \alpha_2 \frac{\partial^2 u}{\partial y^2} - (1 + \varepsilon A e^{n u}) \frac{\partial u}{\partial y} = & \frac{\partial^2 u}{\partial y^2} + Gr \theta + GmC - \left( M + \frac{1}{K} \right) u - \alpha_2 \left( M + \frac{1}{K} \right) \frac{\partial u}{\partial t} \\
\frac{\partial \theta}{\partial t} + \alpha_2 \frac{\partial^2 \theta}{\partial y^2} - (1 + \varepsilon A e^{n u}) \frac{\partial \theta}{\partial y} = & \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \phi \theta + Q_C C + Du \frac{\partial^2 C}{\partial y^2} \\
\frac{\partial C}{\partial t} + \alpha_2 \frac{\partial^2 C}{\partial y^2} - (1 + \varepsilon A e^{n u}) \frac{\partial C}{\partial y} = & \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - \gamma C
\end{align*}
\]

The corresponding initial and boundary conditions in Eq. (5) in a non-dimensional form are given below:
\[
\begin{align*}
u = u_p + h \frac{du}{dy}, \quad \theta = 1 + \varepsilon e^{n u}, \quad C = 1 + \varepsilon e^{n u} \text{ at } y = 0 \\
u \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \text{ as } y \rightarrow \infty
\end{align*}
\]

3. Solution of the problem

The Eqs. (8)-(10) are coupled and non linear partial differential equations (PDEs) whose solutions in closed form are difficult to obtain, to solve these coupled non-linear partial differential equations, we assume that the unsteady flow, so that in the neighborhood of the plate, we have
\[
\begin{align*}
u(y,t) = & u_0(y) + \varepsilon e^{n u} u_1(y) + O(\varepsilon^2) \\
\theta(y,t) = & \theta_0(y) + \varepsilon e^{n u} \theta_1(y) + O(\varepsilon^2) \\
C(y,t) = & C_0(y) + \varepsilon e^{n u} C_1(y) + O(\varepsilon^2)
\end{align*}
\]

By substituting the Eq.(12) into the Eq.(8)-(10)the equations are reduced to harmonic and non-harmonic terms and neglecting the higher order terms in $\varepsilon$, we obtain
\[
\begin{align*}
u'' + u_0' - \left( M + \frac{1}{K} \right) u_0 = & - \left( Gr \theta_0 + GmC_0 \right) \\
u_1'' + u_0' - \left( n + \alpha_2 n^2 + \left( M + \frac{1}{K} \right) (1 + \alpha_2 n) \right) u_1 = & - \left( Gr \theta_1 + GmC_1 + Au_0' \right) \\
\theta_0'' + \Pr \theta_0' - \Pr \phi \theta_0 = & - \left( \Pr Q_0 C_0 + \Pr Du C_0'' \right)
\end{align*}
\]
\[ \theta''_i + \text{Pr} \theta'_i - \text{Pr} \left( \phi + n + \alpha_z n^2 \right) \theta_i = \text{Pr} Q_i C_i - \text{Pr} D \theta C''_0 - A \text{Pr} \theta'_0 \] (16)

\[ C''_0 + \text{Sc} C'_0 - \text{Sc} \gamma C_0 = 0 \] (17)

\[ C''_1 + \text{Sc} C'_1 - \text{Sc} \left( \gamma + n + \alpha_z n^2 \right) C_1 = -A \text{Sc} C'_0 \] (18)

Now, the corresponding boundary conditions are:

\[ u_0 = u_p + h_1 \frac{d u_0}{d y}, u_1 = h_1 \frac{d u_1}{d y}, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \] at \( y = 0 \)

\[ u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0, C_0 \to 0, C_1 \to 0 \] as \( y \to \infty \) (19)

\[ u(y, t) = \left( B_{10} e^{-m_{11} y} + B_{13} e^{-m_{11} y} + B_{16} e^{-m_{11} y} \right) \]
\[ + \varepsilon e^{m t} \left( B_{13} e^{-m_{11} y} + B_{14} e^{-m_{11} y} + B_{15} e^{-m_{11} y} + B_{16} e^{-m_{11} y} \right) \] (20)

\[ \theta(y, t) = \left( (1 - B_3) e^{-m_{11} y} + B_3 e^{-m_{11} y} \right) + \varepsilon e^{m t} \left( B_3 e^{-m_{11} y} + B_4 e^{-m_{11} y} + B_5 e^{-m_{11} y} + B_6 e^{-m_{11} y} \right) \] (21)

\[ C(y, t) = e^{-m_{11} y} + \varepsilon e^{m t} (1 - B_2) e^{-m_{11} y} + B_2 e^{-m_{11} y} \] (22)

For engineering purpose, the friction factor, local Nusselt number and local Sherwood number are given by

\[ \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)^* \] at \( y^* = 0 \)

Where \( \tau_w = \rho V_0^2 u'(0) \)

\[ C_f = -m_2 B_{11} - m_1 B_9 - m_3 B_{10} \] + \( \varepsilon e^{m t} \left( -m_{11} B_{18} - m_1 B_{13} - m_3 B_{14} - m_2 B_{15} - m_7 B_{16} - m_5 B_{17} \right) \) (23)

\[ Nu_s = \frac{q_w}{T_w - T_x} \frac{x}{k} \]

\[ \frac{Nu}{Re_s} = -\theta'_0 = \left( 1 - B_3 \right) m_x + m_1 B_3 \] + \( \varepsilon e^{m t} \left( m_3 B_8 + m_1 B_9 + m_3 B_6 + m_5 B_7 \right) \) (24)

Where \( Re_s = \frac{V_0 x}{\nu} \)

\[ Sh = x \left( \frac{\partial c}{\partial y} \right)^* \] at \( y^* = 0 \)

\[ Sh = \frac{\partial c}{\partial y} \] at \( y = 0 \)

\[ = -m_1 + \varepsilon e^{m t} \left( 1 - B_2 \right) - m_1 B_2 \] (25)

4. Results and Discussion
The analytical solutions are transformed for velocity, temperature and concentration profiles for different values of physical parameters as Magnetic field parameter $M$, Dufour number $Du$, thermal Grashof number $Gr$, mass Grashoff number $Gm$, Schmidt number $Sc$, radiation absorption parameter $Q$, porosity parameter $K$, chemical reaction parameter $\gamma$, Visco-elastic parameter $\alpha_2$, Prandtl number $Pr$, and heat source parameter $\phi$ are depicted from figures 1-18. We assign values to the parameters as $n = 1, A = 1; Sc = 0.22, t = 1, U = 1.5, \varepsilon = 0.02, Pr = 0.71, Gr = 5, Gm = 5$.

The effect of magnetic field on velocity profile is plotted in Figure 5. It is clear that the increasing values of magnetic field parameter decrease the velocity profiles. This validates the general behavior of magnetic field. Physically, the rising values of magnetic field generate opposing force to the flow direction, this force is called Lorentz force. Due to this cause, we have seen a decrement in the velocity field.
Figure 6: Effects of $K$ on Velocity profiles

Figure 7: Effects of $\phi$ on Velocity profiles

Figure 8: Effects of $h_1$ on Velocity profiles

Figure 9: Effects of $Du$ on Velocity profiles

Figure 10: Effects of $\gamma$ on Velocity profiles

Figure 11: Effects of $Q_1$ on Velocity profiles
Figure 12. Effects of $\alpha_2$ on Temperature profiles

Figure 13. Effects of $\phi$ on Temperature profiles
Figures 3 and 4 show the effect of thermal and concentration buoyancy forces on velocity field. It is noted from that the increasing values of $Gr$ and $Gm$ improves the velocity field. Generally, raising the values of $Gr$ and $Gm$ keeps more pressure on the flow due to this we saw an increment in velocity field.
The porosity parameter on velocity distributions is depicted in Figure 6. It is observed that the enhancing value of porosity parameter increases the velocity distributions. The improving values of $K$ increases the permeability; this can lead to improving the velocity profiles.

Figures 2, 12 and 18 show the effect of $\alpha_2$ on velocity, temperature and concentration profiles. It is observed that the increasing values of $\alpha_2$ decreases the velocity, temperature, and concentration fields. This may happen due to the domination of viscous nature in the flow. The similar behavior was observed in the presence of Prandtl number it is shown in Figures 16. The effect of $\phi$ on velocity and temperature fields is shown in Figures 7 and 13. It is observed that the heat source parameter depreciates the velocity and temperature fields.

The quite similar behavior was observed in Figures 17. It shows the effect of Schmidt number on concentration field. From Figures 11 and 14 show the increase in velocity and temperature profiles with increasing values of $Q_1$. The effect of refraction parameter $h_l$ on velocity profiles is shown in Figure 8. This validates the general behavior of slip. Basically, increasing values of slip parameter improve the thickness of the boundary layer this can help to enhance the velocity profiles. Figures 10 and 19 show the effect of chemical reaction parameter on velocity and concentration profiles. It is clear that the increasing the values of chemical reaction parameter depreciate the velocity and concentration profiles. The effect of Dufour number $Du$ on velocity and temperature profiles is plotted in Figures 9 and 15. It is observed that the increasing values of $Du$ increases the velocity as well as temperature profiles, due to the domination of interfacial mass transfer in the flow.

Table 1. The variations in Skin friction, Nusselt and Sherwood number for various values of $A = 1$, $Gr = 5$, $Gm = 20$, $Pr = 0.71$

| Sc | $\alpha_2$ | $Q_1$ | $\phi$ | $\gamma$ | $m$ | $K$ | Harinath Reddy et al. [18] | Present Work | ($Du = 0, h_l = 0$) |
|----|-------------|------|-------|---------|-----|-----|----------------------------|-------------|---------------------|
|    |             |      |       |         |     |     |                            |              |                     |
| 0.22 | 1           | 1    | 1     | 1       | 1   | 1   | 14.2065                     | -0.6478     | -0.7008             |
|      | 0.60        | 1    | 1     | 1       | 1   | 1   | 9.4799                      | -1.2384     | -0.7267             |
|      | 0.78        | 1    | 1     | 1       | 1   | 1   | 9.5134                      | -1.4851     | -0.7600             |
|      | 0.96        | 1    | 1     | 1       | 1   | 1   | 8.6703                      | -1.7218     | -0.7008             |
| 0.22 | 2           | 1    | 1     | 1       | 1   | 1   | 13.2966                     | -0.6541     | -0.7472             |
| 0.22 | 3           | 1    | 1     | 1       | 1   | 1   | 13.2425                     | -0.6597     | -0.7593             |
| 0.22 | 1           | 2    | 1     | 1       | 1   | 1   | 12.5446                     | -1.2443     | 3.7735              |
|      |             |      |       |         |     |     |                            |              |                     |

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5. Comparison of the results
In order to explain the accuracy of the present results of the study, it is considered that the analytical solutions obtained are computed the numerical results for friction coefficient, local Nusselt and Sherwood numbers respectively. These calculated and compared results are presented in Table 1. From this Table 1, it is interesting to watch that the present results in the absence of diffusion-thermo and refraction parameters \(h_i\) are in good agreement with the corresponding results obtained from Harinath Reddy et al. [18].

6. Conclusion
The velocity of the plate for the Kuvshinski fluid which is exposed to a transverse magnetic field can be maintained constantly, the obtained partial differential equations (PDEs) are expressed in the dimensional form using non-dimensional quantities. These equations are solved analytically by using perturbation technique through graphs are obtained by to study the effects of different physical

| 0.22 | 1  | 3  | 1  | 1  | 1  | 13.8542 | -1.2443 | 6.3093 | 13.8542 | -1.2443 | 6.3093 |
|------|----|----|----|----|----|---------|---------|--------|---------|---------|--------|
| 0.22 | 1  | 4  | 1  | 1  | 1  | 15.1638 | -1.2443 | 8.8451 | 15.638  | -1.2443 | 8.8451 |
| 0.22 | 1  | 1  | 2  | 1  | 1  | 10.2886 | -1.2443 | -0.3650 | 10.2886 | -1.2443 | -0.4650 |
| 0.22 | 1  | 1  | 3  | 1  | 1  | 9.6995  | -1.2443 | -1.6204 | 9.6985  | -1.2443 | -1.6294 |
| 0.22 | 1  | 1  | 4  | 1  | 1  | 9.5256  | -1.2443 | -1.8933 | 9.5256  | -1.2443 | -1.8833 |
| 0.22 | 1  | 1  | 1  | 2  | 1  | 9.2178  | -1.2443 | 1.2377  | 9.2178  | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 3  | 1  | 7.8009  | -1.2443 | 1.2377  | 7.8009  | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 4  | 1  | 6.7174  | -1.2443 | 1.2377  | 6.7175  | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 2  | 1  | 12.6597 | -1.2443 | 1.2377  | 12.6597 | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 3  | 1  | 13.2362 | -1.2443 | 1.2377  | 13.2354 | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 4  | 1  | 13.5496 | -1.2443 | 1.2377  | 13.5496 | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 1  | 5  | 11.2350 | -1.2443 | 1.2377  | 11.3350 | -1.2443 | 1.2377  |
| 0.22 | 1  | 1  | 1  | 1  | 10 | 8.6426  | -1.6980 | -0.9057 | 8.6926  | -1.7980 | -0.9057 |
| 0.22 | 1  | 1  | 1  | 1  | 15 | 7.6589  | -2.0473 | -0.9780 | 7.6789  | -2.9473 | -0.9780 |
| 0.22 | 1  | 1  | 1  | 1  | 20 | 6.9966  | -2.3424 | -1.0163 | 6.9866  | -2.3524 | -1.5163 |
parameters on temperature, velocity, and species diffusion profiles. In the current study, the following conclusions can be drawn:

1. Velocity decreases for an increase in Visco-elastic parameter $\alpha_2$, the radiation parameter $Q_1$, the Schmidt number $Sc$, the permeability parameter $K$, the magnetic field parameter $M$, chemical reaction parameter $\gamma$ and increases for an increase in mass Grashof number $Gm$.

2. Temperature profiles increased for an increase in $\alpha_2$, decreased due to in heat absorption coefficient $\phi$, radiation parameter $Q_1$ and chemical reaction parameter as well as Schmidt number $Sc$.

3. Velocity and temperature distributions are increased for an increase in diffusion-thermo effect $Du$.

**Appendix**

\[
\begin{align*}
m_1 &= \frac{Sc + \sqrt{Sc^2 + 4Sc\gamma}}{2} \\
m_3 &= \frac{Sc + \sqrt{Sc^2 + 4ScB_1}}{2} \\
m_5 &= \frac{Pr + \sqrt{Pr^2 + 4Pr\phi}}{2} \\
m_2 &= \frac{Pr + \sqrt{Pr^2 + 4PrB_1}}{2} \\
m_0 &= \frac{1 + \sqrt{1 + 4M_1}}{2} \\
m_1 &= \frac{1 + \sqrt{1 + 4B_{12}}}{2} \\
B_4 &= \gamma + n + \alpha_2 n^2 \\
B_2 &= \frac{AScm_1}{m_1^2 - Scm_1 - ScB_1} \\
B_3 &= \frac{-(PrQ_1 + Pr Du m_i^2)}{m_1^2 - Pr m_i - Pr \phi} \\
B_4 &= \phi + n + \alpha_2 n^2 \\
B_5 &= \frac{Ap r m_i B_1 + Pr Q_1 B_2 - Pr Du B m_i^2}{m_1^2 - Pr m_i - Pr B_4} \\
B_6 &= \frac{(Pr Q_1 - Pr Du m_i^2)(1 - B_2)}{m_1^2 - Pr m_i - Pr B_4} \\
B_7 &= \frac{APr m_i (1 - B_3)}{m_5^2 - Pr m_5 - Pr B_4} \\
B_8 &= 1 - (B_5 + B_6 + B_7) \\
B_9 &= \frac{-(Gr B_3 + Gm)}{m_1^2 - m_i - M_1} \\
B_{10} &= \frac{-Gr (1 - B_5)}{m_5^2 - m_5 - M_1} \\
B_{11} &= \frac{u_p - B_9 - B_{10} - B_6 h m_i - B_6 h m_5}{1 + h m_5} \\
B_{12} &= \frac{n + \alpha_2 n^2 + M_1 (1 + \alpha_2 n)}{m_1^2 - m_i - B_{12}} \\
B_{13} &= \frac{AB_6 m_i - Gr B_3 - Gm B_5}{m_1^2 - m_i - B_{12}} \\
B_{14} &= \frac{-Gr B_6 - Gm (1 - B_5)}{m_5^2 - m_5 - B_{12}} \\
B_{15} &= \frac{AB_{10} m_5 - Gr B_5}{m_5^2 - m_5 - B_{12}}.
\end{align*}
\]

**Acknowledgement:**

We are very much thankful to anonymous reviewers for their valuable comments and suggestions to improve this manuscript.

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