Gate Effect Analysis and Compensation Method of the Multi-period Synchronous Frequency Discrimination

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Abstract: The Frequency Modulation (FM) mode sensor is widely used in the ultra-precision measuring fields since it has the characteristics of high sensitivity, strong ability of anti-jamming and digital output. The multi-period synchronous frequency discrimination circuit is in regular use to demodulate the output signal of the frequency modulation sensor because of its brief structure, low errors, wide linear range, and low drifting etc. In order to improve the accuracy of frequency discrimination, it generally makes the gate time of count long enough. The error of the demodulation becomes larger with the longer count time. Firstly, this paper takes a certain frequency resonant wave, i.e. the measured signal, as the research object in the time domain and constructs a universal transfer function of the multi-period synchronous frequency discrimination in a certain gate time and analyzes the principle and effect on the accuracy of gate effect based on this function. Then, this paper introduces the amplitude-frequency and phase-frequency characteristics of the multi-period synchronous frequency discrimination system according to the quantitative analysis of the gate effect. Finally, the paper presents a real-time compensation method based on the different frequency components. The experimental data shows that the theoretical calculation of the gate effect on accuracy is in agreement with experimental data and the real-time compensation method can eliminate 90% demodulation error caused by the gate effect.

1. Introduction
Frequency Modulation (FM) mode sensor has characteristics of high sensitivity, strong ability of anti-jamming and digital output. It is widely used in the ultra-precision measuring fields, such as nanometer displacement sensor of capacitance and inductance. To meet the accuracy of the system, the output signal demodulating unit of FM mode sensor must have excellent performance. In engineering application, pulse counting mode direct frequency demodulating method is employed for its simple structure of circuits, small error, wide linear range and non-drifting. Using this method can demodulate signal by measuring frequency of FM signal in certain gate time and get the modulating signal, i.e. measurand. The gate time is required to be long enough to guarantee the accuracy of frequency discriminating. During gate time, FM signal is always a transient variation, the gate time leads that the measuring result can not exactly discribe the measured signal. It may decrease the performance of the demodulator. The effect on demodulating performance will be enhanced while gate time is extending.

In this article, we study the dynamic characteristic of the direct pulse counting mode theory based multi-period synchronous frequency discriminating method, then propose a spectrum real-time compensating method and improve them by experiments.
2. Multi-period synchronous frequency discrimination theory

Multi-period synchronous frequency discrimination method is based on direct frequency discrimination theory. The principle is shown below. Firstly, transform the input signal to pulse sequence using preprocessing circuits. In fixed gate time, use clock pulse to measure the frequency of modulating signal. The reference gate signal is given by a micro processor. During period of reference gate time, the gate signal starts working when rising edge of the measured signal comes, then two counters are used to separately count the pulse of the measured signal and the reference clock. Also, when the reference gate signal is invalid, the counter doesn’t stop counting until the measured signal rising edge occurs. This method synchronizes the gate signal and the measured signal, eliminates the ±1 counting error brought by the counter.

Suppose the counting value of the measured signal and reference clock is \(N_x\) and \(N_0\). Reference clock frequency is \(f_c\), and then the frequency of the measured signal is

\[
f_x = \frac{N_x}{N_0} f_c\]  

According to analysis method of uncertainty, resultant error of \(n\) periods measurement is

\[
\frac{\Delta T}{T} = \pm \left(\frac{1}{nTf_c} + \frac{\Delta f}{f_c}\right)
\]

3. Characteristic analysis of multi-period synchronous frequency discrimination

Study the principle of multi-period synchronous frequency discriminating, we can know that the longer the gate time is the higher discriminating accuracy is. During gate time, FM signal frequency is always a transient variation, the gate time results that the measuring result is the mean value of the measured signal frequency during this period of time. Only when the input of the sensor is static or quasi-static signal, the modulating signal is approximated to fixed value. If modulating signal is dynamic, the discrimination result is just the correlated quantity of the signal during gate time. This introduces the measuring error. The correlated quantity brought in by gate time is called gate effect. Next, we will discuss what the gate effect does on the accuracy of frequency discriminating.

Any signal is composed of orthogonal trigonometric sets. Suppose the carrier signal is \(u_c(t) = U_c \cos(2\pi f_C t)\), measured signal is \(x(t) = A_M \cos(2\pi f_M t)\), and modulating coefficient is \(m_f\). Then the modulating signal is

\[
u_{FM}(t) = U_c \cos[2\pi f_c t + m_f \sin(2\pi f_M t)]
\]

Use multi-period synchronous frequency discriminating method to demodulate FM signal, that is to determine how long \(n\) continuous periods of the FM signal is. Since the gate effect, discrimination result is just the average frequency of gate time. Suppose gate time is \(T_g\), gate frequency is \(f_g\). During the interval \([t-T_g, t]\), frequency integral mean value is

\[
\bar{f}_{FM}(t) = \frac{\Delta \phi_{FM}}{2\pi \Delta t} = \frac{\int_{t-T_g}^t f_{FM}(t) \, dt}{T_g} = f_c + K_f A_M \frac{f_M}{f_g} \frac{\sin(f_M \pi)}{f_g \pi} \cos(2\pi f_t t - \frac{f_M}{f_g} \pi)
\]

Remove carrier frequency element, the result is

\[
y(t) = \bar{f}_{FM}(t) - f_c = K_f \frac{\sin(f_M \pi)}{f_g} A_M \cos(2\pi f_M t - \frac{f_M}{f_g} \pi)
\]

Set \(\beta = f_M \cdot \pi / f_g\), \(A = K_f \cdot \sin \beta / \beta\), then
The transfer function of multi-period synchronous frequency discriminating system is

\[ h(t) = \frac{y(t)}{x(t)} = A \frac{x(t - \frac{T}{2})}{x(t)} \]  \hspace{1cm} (7)

Suppose gate frequency \( f_g \) is constant, modulating signal frequency \( f_M \) is variable, then the normalized amplitude frequency characteristics of gate effect is shown in Figure 1. During interval \([0, f_g]\), the system output amplitude decreases along with the increasing of the modulating signal frequency. Meanwhile, at point \( f_M = 0.443 f_g \), system output amplitude decrease to 70.7 percent of primary signal, i.e. system -3dB frequency point is 0.443\( f_g \). Figure 2 is the Phase frequency characteristics of gate effect. After analysis, we know that if the input signal \( x(t) \) is not single harmonic wave, corresponding to different frequency component of the measured signal \( x(t) \), discriminating system has various amplitude attenuation and phase delay, which may result in discriminating error, and decrease the accuracy of the measuring system.

![Figure 1. Normalized amplitude frequency characteristics of gate effect](image1)

![Figure 2. Phase frequency characteristics of gate effect](image2)

4. Spectrum compensating method of multi-period synchronous frequency discrimination

In this article, a real-time spectrum compensation method is presented to reconstruct modulating signal to improve the accuracy of multi-period synchronous frequency discriminator. Its principle is shown below. Analysis of the input FM signal spectrum by spectrum analysis unit of the frequency discriminating circuits, then compensate the amplitude \( A(f) \) and phase \( P(f) \) of all frequency components except carrier signal.

Suppose one frequency component of the modulating signal is \( x(t) = A \sin(\omega t) \), then its compensating function is

\[ z(t) = K_c(f) \cdot y(t) \cdot e^{\phi_c(f)} \]  \hspace{1cm} (8)

\( K_c(f) \) is the amplitude compensation coefficient

\[ K_c(f) = \frac{1}{K(f)} = \frac{B}{\sin B} \]  \hspace{1cm} (9)

\( P_c(f) \) is the phase compensation coefficient

\[ P_c(f) = -\arg[K(f)] \]  \hspace{1cm} (10)

Output after compensating is

\[ z(t) = K_c(f) \cdot y(t) \cdot e^{j\phi} = x(t) \]  \hspace{1cm} (11)
Because the compensation coefficient $K_c(f)$ is approximate to infinity at points $(f_0, 2f_0, 3f_0\ldots)$ where mean signal of the signal window decays to zero, it cannot be compensated in measurement. In this research, we take $[0, f_0)$ as frequency compensating interval.

5. Experiments and results
Frequency discriminating characteristic analyzing experiment and measured signal reconstructing experiment are done with Aglient33220A waveform generator, Hitachi V6545 digital storage oscilloscope and self-developed FPGA based frequency discriminating circuits.

Carrier signal frequency is 455kHz, amplitude is 2V, gate period is 1ms, the modulating signal starts from 0Hz, ends at 1kHz, and the step is 200Hz, amplitude is 3V, $K_f=1$. The measuring results on each frequency amplitude and phase shift with and without compensation is shown in Table 1.

| $f_M$ (Hz) | amplitude (V) | phase shift (°) | amplitude (V) | phase shift (°) |
|------------|---------------|----------------|---------------|----------------|
| 0          | 3.000         | 0              | 3.000         | 0              |
| 200        | 2.795         | 35.04          | 3.010         | -0.06          |
| 400        | 2.681         | 73.01          | 3.008         | 0.04           |
| 600        | 1.507         | 107.32         | 3.006         | 0.05           |
| 800        | 0.706         | 144.01         | 2.998         | -0.02          |
| 1000       | 0.002         | 179.05         | 3.002         | 0.03           |

Experiment results shows that multi-period synchronous frequency discrimination will lead to amplitude decay and phase shift of measured signal. And the compensation method which is presented in the paper can eliminate 90% demodulation error caused by the gate effect.

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