Decadal Mission for the New Physics Higgs/Flavor Era

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The LHC has not discovered any New Physics beyond the anticipated \( h(125) \) boson, and new ideas abound for out-of-the-box searches, or Effective Field Theories with multi-TeV cutoff. But, have we exhausted dimension-4 operators involving sub-TeV particles that are not exotic (non-XLP)? We advocate the existence of an extra Higgs doublet that carry extra Weinberg couplings, where emergent mass-mixing hierarchies and alignment have hidden their effects very well so far. \( \mathcal{O}(1) \) Higgs quartics can induce first order electroweak phase transition and imply sub-TeV \( H, A, H^\pm \) spectrum. The extra Weinberg couplings, led by \( p_{ttl} \) and \( p_{ltc} \) at \( \mathcal{O}(1) \), can drive electroweak baryogenesis, while \( |p_{eet}/p_{tlt}| \propto \lambda_e/\lambda_t \), the ratio of usual electron and top Weinberg couplings, can tame electron EDM. Finding these extra Higgs bosons via \( cg \to tH/A \to tt\bar{c}, tt\bar{t} \) and \( cg \to bH^+ \to b\bar{b} \) processes (and those allowed by Higgs boson splittings) at the LHC, and pushing the flavor frontier to break the flavor code, would usher in a new Higgs/flavor era. A new scale at 10–20 TeV, possibly related to the Landau pole of the scalar sector, may emerge.

Physics is an empirical science. For the Higgs and “Flavor” era unfolding before us, we have to thank the large LHC experiments and Belle II, plus smaller, more specialized experiments at the kaon, muon and electric dipole moment (EDM) frontiers.

I. PRELUDE: \( e, m_e, \mu, \) THE UNIVERSE

Modern physics share a common heritage. Electrodynamics, completed by Maxwell, is still the template of dynamics. Thomson’s electron, a hundred and twenty four years and going, remains fundamental: we still have not resolved any structure it may have. Rutherford discerned that Becquerel’s radioactivity separates into \( \alpha, \beta \) and \( \gamma \) rays. Demonstrating \( \alpha = \text{He}^{++} \), and lucking out that it is the tightest bound nucleus, he put it to good use and made the monumental discovery of the nucleus itself; and \( \gamma \) rays unraveled as but an energetic repeat of atomic spectra. However, the \( \beta \) ray, energetic electron emission from the nucleus, turned out to be the manifestation of a new fundamental interaction heretofore unknownst to man: the weak interaction.

With the backdrop of QED and the key provided by Chadwick’s neutron, Fermi took up Pauli’s audacious hypothesis of the nearly massless neutrino to formulate his theory of the weak interaction as a product of fermion bilinears, e.g. \((\bar{p}n)(\nu\bar{e})\). In turn, this inspired Yukawa’s short range nuclear force by massive pion exchange, which started nuclear physics. But it was the development of ultra-low temperature techniques by Kamerlingh Onnes, resulting in his discovery of superconductivity, that provided the seed of inspiration for the later development of the Standard Model (SM) of particle physics, namely spontaneous symmetry breaking (SSB) as the origin of mass. Who would have thought!

With atomic spectra explained by quantum mechanics, one could already ask:

“Why is the coupling constant \( e \) real?”
“Who ordered the Dirac’s theory of the electron settled by Anderson’s discovery of \( e^+ \), the anti-electron, who went on to discover the muon with \( m_\mu \gg m_e \) but otherwise electron-like, two further questions arose:

“Where have all the \( e^+ \) gone?”
“Who ordered the muon?”

where the latter was quipped by Rabi after Powell’s demonstration of the \( \pi-\mu-e \) decay chain.

If \( \alpha \) bombardment utilized radioactivity provided by Nature, a Rutherford scattering experiment in the late 1960s utilized man-made high energy electron beams to probe inside the proton and discover another layer of constituents, the “reality” of \( u \) and \( d \) quarks, which stand with the electron as fundamental fermions. The Rabi question soon proliferated into the “flavor” problem: the electron, neutrino and \( u \) and \( d \) quarks each come in three copies in Nature, increasing in mass. Of these, the \( s \) quark echoes the muon, while the \( \tau \) lepton and the \( c, b \) and \( t \) quarks were discovered since November 1974. The latter four are called “heavy flavors” because they are supra-GeV in mass. But \( \mu \) and \( s \) share property features with these heavy flavors, even though \( m_\mu, m_s \sim 0.1 \) GeV. The span in mass, from \( m_e \simeq 0.511 \) MeV to \( m_\mu \simeq 173 \) GeV, is simply staggering.

With the development of gauge field theory, we now understand why \( e \) is real, but morphs the question into:

“Are there complex couplings?”

Together with how \( m_f \) arises (\( f \) now covering 9 charged fermions), where has antimatter gone, and the flavor problem, these are the issues we wish to touch upon in this targeted Review.

Start of Particle Physics. — Anderson’s discovery of the “positive electron” \([1]\), or anti-electron \( e^+ \), settled all worries of Dirac, but brought in the issue as big as the Universe itself:

“Where have all the antimitter gone?”

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1 The neutrinos, though having mass, are so light such that they belong in their own category, which we will not cover.
Since luminal matter is dominated by baryons, the fact that we see baryons but no equivalent amount of anti-baryons is called the Baryon Asymmetry of the Universe (BAU) problem. Remarkably, Anderson also discovered the muon \(2\) a few years later. But because of issues with the nature of cosmic rays and confusion with Yukawa’s pion, things were not clarified until the \(\pi^+ - \mu - e\) decay chain was established \(2\) by Powell, who coined \(\pi^+\) and \(\mu\). Pontecorvo then stepped in to show the absence of \(\mu \to e\gamma\), hence \(\mu \neq e^*\), i.e. it is not an excited electron.

The discovery of the positron and the muon, which are fundamental fermions to this day, can be viewed as the start of modern particle physics.

**From CP Violation to Baryogenesis.**— After Lee and Yang’s parity \((P)\) violation suggestion, which was quickly confirmed by Madame Wu, for 6–7 years, particle physicists embraced the conservation of \(CP\), the product of charge conjugation \(C\) and \(P\), in weak interactions. However, Cronin and Fitch showed \(3\) experimentally that \(CP\) is violated in \(K_L^0 \to 2\pi\) decays, albeit minutely. Armed with this and the concurrent observation \(4\) of the Cosmic Microwave Background (CMB), i.e. evidence of a hot early Universe, the Big Bang, during a hull of the nuclear arms race, the father of the Soviet H bomb, Sakharov, proposed \(7\) his conditions for BAU:

1. Baryon Number Violation (BNV);
2. \((C \& P)\) \(CP\) Violation;
3. Deviation from Equilibrium.

The BNV insight, ahead of its time, was injected by Sakharov.

**Weak Interaction and W/Z Bosons.**— The muon decays weakly, \(\mu \to e\nu\mu\bar{\nu}_e\), which follows Fermi theory, and it was shown experimentally that \(\nu_\mu \neq \nu_e\). With parity violation pointing the way to the \(V - A\) theory, it became clear that weak decay occurs via massive vector boson exchange. The \(SU(2)_L \times U(1)\) gauge group was proposed \(8\), where \(L\) corresponds to left-handedness of weak processes. *Something* the \(W\) and \(Z\) bosons gained mass, and only the \(U(1)\) symmetry of QED is left, with \(m_\nu = 0\) as observed. It was the exchange of virtual photons in Deeply Inelastic Scattering that unraveled the point-like partons, the \(u\) and \(d\) quarks, inside the nucleon. The observed “asymptotic freedom” led to the SU(3) group of Quantum Chromodynamics (QCD), the exquisite non-Abelian gauge dynamics that underlies nuclear binding. The gauge symmetry remains unbroken, so \(m_Q = 0\), although confinement means the gluon has an effective, dynamical mass.

The renormalizability, or control of divergences of non-Abelian gauge theories, was a big issue, but was resolved by ’t Hooft and Veltman, without or with SSB.

**Meißner Effect and SSB.**— With the mention of the Meißner Effect, discovered in 1933 \(9\), levitated high speed trains come to mind: magnetic fields cannot penetrate a superconductor. Though it was explained phenomenologically by the London theory, it took the insight \(10\) of (Phil) Anderson to see through the cloud: the massless Goldstone boson of spontaneous breaking of charge symmetry (by condensation of Cooper pairs) became the longitudinal component of the photon, hence the photon gains mass inside the superconductor! And thus explains the attenuation length. This is exactly the Brout-Englert-Higgs (BEH) mechanism \(11,12\), worked out a couple of years later for relativistic fields.

Nature illustrated SSB with “squalid state” physics, which particle physics should be forever grateful. But why should the Universe be permeated by a Higgs field since it began, with a \(v.e.v\) that gives SSB, is a mystery.

The remainder of this Review is organized as follows. In Sec. II we cover the Standard Model, in particular mass generation by coupling to the Higgs field, as well as complex dynamics with three fermion generations. In Sec. III we put forth the Gell-Mann principle, “Everything not forbidden is compulsory.”, colloquially called “the Totalitarian Principle”, to discuss the possibility and implications for having a second Higgs doublet field, establishing what we call the *General* two Higgs doublet model (G2HDM). In Sec. IV we address big issues related to G2HDM in the new Higgs/Flavor era, namely: 1) baryogenesis confronts electron EDM; 2) spectrum of extra Higgs bosons that are fit for search at the LHC; 3) Nature’s design in regards flavor. After a very brief discussion in Sec. V, we offer our summary.

We will keep this Review as nontechnical as possible.

## II. STANDARD MODEL, MASS, AND HIGGS

The only pivotal event so far at the LHC was in 2012: observation \(13,14\) of the Higgs boson, \(h(125)\). The use of the lower case \(h\) would become clear in Sec. III. So why is it called the Higgs boson? Because it was Higgs who showed \(12\), after SSB of \(U(1)\), a remnant particle would receive mass from its own \(v.e.v\). This feature is retained in the Weinberg construction \(15\) of the spontaneous breaking of the \(SU(2)_L \times U(1)\) symmetry by the \(v.e.v\) of a complex doublet Higgs field.

Mass is the essence of matter, resulting in the clumpiness of the Sun, the Earth, etc. through gravitation. So, the Higgs field is the source of mass in the Standard Model (SM) of particle physics. Since all known massive fundamental particles receive mass by their coupling to the \(v.e.v\), these particles would have definite couplings to the remnant scalar particle, the Higgs boson \(h\). Before the advent of the LHC, the Higgs boson was whimsically dubbed “the God particle”, referring to its role as “Origin of Mass”. This all sounds simple, amazingly simple.

**Dynamical Origin of Mass.**— So, mass generation is dynamical in SM, by coupling to the universal condensate. This is reasonable for the \(W\) and \(Z\) vector bosons, the transmitter of the weak interaction, as it was the *raison d’être*. What may be surprising is that Nature used practically the same token to generate fermion masses:

\[
M_V = \frac{1}{2} g v \quad \iff \quad m_f = \frac{1}{\sqrt{2}} \lambda_f v. \tag{1}
\]
The photon and the gluons remain massless because they correspond to unbroken gauge symmetries. Weinberg introduced the complex scalar doublet $\Phi$,

$$\Phi = \left[ \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right].$$

The field develops a v.e.v. in the neutral component, $\langle \phi^0 \rangle = v$, and the SSB turns

$$\left[ \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right] \rightarrow \left[ \begin{array}{c} G^+ \\ v + h + iG^0 \end{array} \right],$$

where $G^\pm$ and $G^0$ are the (would-be) Goldstone bosons that become the longitudinal components of, i.e. “eaten” by, the $W^\pm$ and $Z^0$ bosons to become massive. But, as we will elucidate further in Sec. III, Weinberg also wrote down the other lepton-scalar interaction term, which gives the second, fermion part of Eq. (1). Though usually called the Yukawa interaction, we will refer to $\lambda_f$ as the Weinberg coupling in remembrance.

The vector boson part of Eq. (1) reflects the prediction of the Glashow-Weinberg-Salam theory, which was affirmed by experiment, and the $W$ and $Z$ bosons were directly observed at the SppS collider at CERN in 1983. But turning further to the empirical nature of physics in the LHC era may be even more stunning. First, the dynamical coupling of $g \simeq 2M_V/v$ was affirmed by the observation of $h \rightarrow WW^*, ZZ^*$ final states themselves, and further confirmed in the observation of Vector Boson Fusion (VBF) production of $VV \rightarrow h$ via the $W$ and $Z$ bosons. This may not be too surprising. Second, during 2018–2020, a chain of measurements highlighted the prowess of the ATLAS and CMS experiments: both were able to measure the $tth$, $bbh$ and $\tau\tau h$ Weinberg couplings in 2018, and all agreed with SM expectations. Most impressive is the evidence for $h \rightarrow \mu\mu$ decay [17], announced summer 2020, where the measured couplings vs mass share the common slope in the “linear plot” (see Fig. 1), agreeing with

$$\frac{1}{2}g = M_V/v,$$

$$\frac{1}{\sqrt{2}}\lambda_f = m_f/v.$$ 

That this covers 6 particles, and from $\lambda_t \simeq 1$ to $\lambda_\mu \simeq 0.0006$, spans over more than 3 orders in coupling strength (or mass), is truly eye-popping. The dynamical nature of mass generation is now empirical.

If Nature chooses this very simple scheme for generating particle masses, the way she generates SSB, i.e. non-vanishing v.e.v. is also deceivingly simple. One writes the very simple Higgs potential,

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4,$$

first for a complex scalar $\Phi$, but works also for a complex doublet. Demand now $\mu^2 < 0$, one gets (the bottom of) a “wine bottle” shape with minima at $|\langle \phi^0 \rangle|^2 = v^2 = |\mu^2|/\lambda$. The purist would say we do not really know the shape of the Higgs potential, which is true. But Occam’s razor would favor the simplicity of Eq. (6).

**Chiral Fermions and Complex Couplings.**—One really curious aspect for fermion mass generation in SM is that, the weak interaction is left-handed. Had the weak interaction been vector-like as in the Dirac theory of the electron (QED), or like QCD, then gauge dynamics would dictate all coupling constants be real!

Now take the weak coupling,

$$i g V_{ij} \bar{u}_i \gamma_\mu Ld_j W^-_\mu,$$

where $V$ is the CKM matrix, and only left-handed quarks participate. Replacing $W^-_\mu$ by $k_\mu/M_W^2$ and imposing on-shell condition for external quarks $u_i$ and $d_j$, after simple manipulations [18], one finds

$$i V_{ij} \bar{u}_i \left( \frac{m_{i} L - m_{j} R}{v} \right) d_j G,$$

where the gauge coupling $g$ cancels out the $g$ in $M_W = gv/2$. Note that $k_\mu k_\nu/M_W^2$ propagates the would-be Goldstone boson, we find that the would-be (i.e. eaten) Goldstone boson $G$ couples with Weinberg coupling, and hides behind, or is veiled by, the vector gauge coupling. We have thus reverse engineered the Weinberg coupling $\lambda_f$ using empirical knowledge; it has more to do with the left-handed doublet, right-handed singlet nature of fermions under the SU(2)$_L$ gauge group, rather than the Higgs representation used for SSB.

We will elucidate in Sec. III why Weinberg couplings, unlike gauge couplings, are intrinsically complex. For now, recall the textbook example of the $3 \times 3$ matrix

\[2\text{ Nomenclature preferred by Peter Higgs.}\]
V. Motivated by the observation \[^{[3]}\] of CPV, Kobayashi and Maskawa showed \[^{[19]}\] there is a unique, irremovable phase. In Eq. \[^{[7]}\], one may be confused to think that the KM phase is associated with the gauge sector, but it arises from diagonalization of quark masses, which arise dynamically from Weinberg couplings. The matrix \(V\) is the difference of the \(u\) and \(d\)-type left handed transformations of the respective bi-unitary transforms.

The unitarity of the CKM matrix has been experimentally affirmed at the per mille level \[^{[20]}\], and usually parametrized in the Wolfenstein form \[^{[21]}\],

\[
V \simeq \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},
\]

where the CPV phase is placed in \(V_{ub}\) and \(V_{cd}\), \(\lambda \equiv V_{us} \simeq 0.22\) is the Cabibbo angle, and

\[
A\lambda^2 \simeq 0.041, \quad A\lambda^3\sqrt{\rho^2 + \eta^2} \sim 0.0036,
\]

are the strengths of \(V_{cb}\) and \(V_{ub}\), respectively. Agonizingly, the KM phase in the \(V\) matrix — confirmed by BaBar \[^{[22]}\] and Belle \[^{[23]}\] — is by far (roughly by \(10^{-10}\)) insufficient \[^{[24]}\] for the CPV condition of Sakharov to generate BAU. Furthermore, SM cannot provide the other condition, namely out of equilibrium, and electroweak phase transition in SM is a crossover. So, as the \(B\) factories were still even under construction, the experimenters had more than one reason to prefer ruling against the KM model. But Nature’s design was confirmed instead: all laboratory measurements of CPV so far can be explained by the KM phase.

### III. GELL-MANN PRINCIPLE AND G2HDM

G2HDM in the section title stands for General two Higgs doublet model. Having found empirically a Higgs boson weak doublet, even though three components appear as would-be Goldstone bosons, it seems totally reasonable to have a second doublet in Nature. But we wish to argue how this second doublet should “behave”, i.e. what its existence would bring forth.

**Three Types of 2HDM.** — Our first work (preceded slightly by Ref. \[^{[25]}\]) involving 2HDM was on the charged Higgs (\(H^+\)) effect on \(b \to s\gamma\) \[^{[26]}\], which adopted the “Natural Flavor Conservation” (NFC) condition \[^{[27]}\] of Glashow and Weinberg. Basically, in Model I, both \(u\)- and \(d\)-type quarks receive mass from just one scalar doublet, while in Model II, they receive mass from separate doublets. Because it automatically appears with supersymmetry (SUSY), 2HDM II is particularly popular. We found that in 2HDM II, the tan \(\beta\) factor associated with \(d\)-type quarks cancels out the cot \(\beta\) factor associated with \(u\)-type quarks, leaving an \(H^+\) correction term that is always constructive w.r.t. the SM effect. As much as \(b \to s\gamma\) developed into its own cottage industry, after it was observed by CLEO \[^{[28]}\], the reason that it provides the most stringent bound on \(m_{H^+}\) so far is rooted in this short-distance effect. The effect in 2HDM I, however, is destructive, and less studied.

Our second well-known 2HDM work is \[^{[29]}\] \(B \to \tau\nu\) decay. Having proposed \[^{[30]}\] two possible solutions to the then “\(B\) meson semileptonic branching ratio puzzle”, one of which is enhanced \(b \to c\tau\nu\) to \(O(10\%)\), only to see it ruled out by ALEPH \[^{[31]}\]. By curiosity, we checked \(B \to \tau\nu\), as it had not been touched upon. We found the \(H^+\) effect in 2HDM II to be destructive against SM, amounting to a multiplicative factor over the SM expectation that is independent of hadronic parameters. Once evidence emerged \[^{[32]}\] at Belle, the process became the second most powerful bound on the \(H^+\) boson.

Our third better known 2HDM work went beyond Models I & II. Although the NFC condition of Glashow and Weinberg sounded convincing, and going hand in hand with SUSY made 2HDM II quite popular, it was pointed out a decade later that, to demand each type of quarks receive mass from just one scalar doublet when two are present, may be overkill, i.e. maybe not needed. One motivation is the Fritzsch ansatz \[^{[33]}\],

\[
m_d = \left[ \begin{array}{c}
0 \\
\sqrt{m_dm_s}/m_s
\end{array} \right],
\]

which connects the Cabibbo angle and \(m_d/m_s\) mass ratio: \(\sin\theta_C \simeq \sqrt{m_d/m_s}\). Perhaps further stimulated by the ARGUS observation \[^{[34]}\] of finite \(B^0-\bar{B}^0\) mixing, Cheng and Sher extended \[^{[35]}\] the Fritzsch ansatz to 3 generations, with the trickling-off nature of \(\sqrt{m_im_j}\) as one goes off-diagonal, stressing that this provides some control of flavor changing neutral Higgs (FCNH) couplings, reducing the need for NFC.

A few years later, Sher proposed \[^{[36]}\] the search for \(b' \to bh\), where \(h\) is a Higgs boson of the 2HDM, and \(b'\) the 4th generation (4G) \(d\)-type quark. We learned of this paper because three out of its five references referred to \(b' \to bh\) decay \[^{[37]}\], which are loop-induced through the \(t'\) quark, an SM albeit 4G effect. We noticed that the extra assumption of 4G was not necessary, and immediately pointed out \[^{[38]}\] the process to look out for is \(t \to ch\), or \(h \to t\bar{c}\) if the generic exotic Higgs boson with FCNH \(tch\) coupling turns out to be heavier than the top quark. We further stressed that it is the fermion mass and mixing hierarchies, rather than the specific form of the Cheng-Sher ansatz, that allows one to drop the NFC condition. Since this is neither Model I nor II, the terminology of 2HDM III was suggested.

2HDM II is basically 2HDM III, where further developments would be elucidated in the next section.

**Gell-Mann Principle and a Second Higgs.** — Since 1992, we on-and-off worked on 2HDM III, e.g. elucidating \[^{[39]}\] the two-loop (Barr-Zee/Bjorken-Weinberg) mechanism for \(\mu \to e\gamma\), facilitated by a top loop, which also applies to \(\tau \to \mu\gamma\) \[^{[40]}\].

In revisiting \(\mu \to e\gamma\) and other muon Flavor Violation (\(\mu\)FV) processes recently \[^{[41]}\], perhaps influenced by our
living experience in Germany, one day the verse “what is not forbidden is allowed” came to mind. Searching the web, we found a Wikipedia entry “Everything which is not forbidden is allowed.” But, surprisingly, the second entry of the search was the Totalitarian principle:

“Everything not forbidden is compulsory.” (12)

which was traced to Gell-Mann [44], with the words “Anything that is not compulsory is forbidden.” used both ways. Gell-Mann referred to the state of affairs in “a perfect totalitarian state”, but in the context of strong interactions: “any process not forbidden by a conservation law does take place with appreciable probability.” Let us concern not with logical equivalence [45] here, but we shall call “Eq. (12)” the Gell-Mann Principle.

We now know firmly one Higgs doublet exists, and we know of no reason to forbid a second Higgs, the simple application of the Gell-Mann Principle dictates that a second scalar doublet, Φ′, must exist (G-M:1). As we will see in the next section, it is convenient to have Φ′ generating the v.e.v., hence ⟨ϕ0⟩ = 0 for Φ′, which does not contribute to v.e.v. Thus,

\[
\Phi' = \begin{bmatrix} \phi^0' \\ \phi^0 \end{bmatrix} \rightarrow \begin{bmatrix} H^+ \\ H + iA \end{bmatrix},
\]

after SSB is induced by ⟨ϕ0⟩ = v, where H and A are the exotic neutral scalar and pseudoscalar. We will discuss the Higgs potential in the next section.

One may immediately ask, “why not three scalar doublets?” While legitimate, let us say that this 2HDM, in particular the G2HDM that we will discuss, has enough new parameters, such that, short of model building, which we do not pursue, it is prudent to proceed with one extra doublet at a time (principle of simplicity).

The emergence of H, A and H+ exotic Higgs bosons in 2HDM is quite familiar, with the natural questions: “Where are they?”, and “What do they do?” We therefore return to Weinberg’s [15] original Model of Leptons, and the NFC work with Glashow [27], with an eye towards upholding the Gell-Mann Principle, Eq. (12).

**Weinberg Coupling and Gell-Mann Principle.** To spontaneously break the SU(2) L × U(1) symmetry 8 down to UQ(1), Weinberg introduced a weak scalar doublet field Φ, Eq. (2), and demanded ⟨ϕ0⟩ = v, as we have already seen. Charge symmetry, corresponding to QED, is not broken. However, the essence is that the left-handed leptons transform as a weak doublet

\[
\ell_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix},
\]

while the right-handed eR transforms as a singlet. Weinberg followed what one is taught in field theory: keep all terms allowed by symmetry. Hence, he wrote down

\[
\lambda_{\epsilon} \tilde{\ell}_L \Phi \epsilon_R,
\]

which is dimension-4 and gauge singlet, hence should be included in the Lagrangian. Upon SSB, the electron gains mass as we saw in Sec. II. So, λε is the Weinberg coupling for the electron, responsible for me generation. Subsequently, this formulation was generalized to three families for all types of charged fermions, namely

\[
\sum_{i,j} \left( \bar{\ell}_L \Lambda^e_{ij} e_{jR} + q_{iL} \Lambda^u_{ij} u_{jR} + q_{iL} \Lambda^d_{ij} d_{jR} \right) \Phi + h.c.,
\]

where q_{iL} ≡ [u_{iL}, d_{iL}], \Lambda^f are 3 × 3 Weinberg coupling matrices for f = e, d, u. A point to stress is that, because of the chiral nature of the weak interaction, the Λ^f matrices are complex. The usual program, then, is to diagonalize Λ^f by biunitary transforms, with real eigenvalues λ_f, where f now covers all charged fermions, and V is the difference between U_L and D_L, the left-handed transforms for u- and d-type quarks, respectively. The observation of Kobayashi and Maskawa is that, using all phase freedoms of quark fields, one irremovable phase remains [19] in V, as seen in Eq. (9).

Up to this point, on one hand one may find it difficult that the nine λ_f’s span 6 orders of magnitude in strength, and wonder what Nature has up her sleeves (which is part of the flavor problem). On the other hand, let us point out that the statement above,

“keep all terms allowed by symmetry”

of the Lagrangian, is nothing but the Gell-Mann Principle! As this preceded G-M:1, the application of Gell-Mann Principle to dictate the existence of a second scalar doublet, the mandatory existence of Weinberg interaction can be classified as G-M:0.

So, introducing the complex scalar doublet for sake of spontaneous breaking of SU(2)_L × U(1) to UQ(1) is necessarily accompanied by the Weinberg interaction. Thus, Eq. (1), or vector boson and fermion mass generation, are indeed achieved in one stroke.

**To NFC or Not To NFC.** We can now turn to the NFC condition [27] of Glashow and Weinberg.

Having invented the GIM mechanism [46] to explain the very strong suppression of e.g. K_L → μ⁺μ⁻ vs K⁺ → μ⁺ν, Glashow certainly had legitimate worries about FCNH when there is a second Higgs doublet. To wit, introducing the second doublet Φ′, there should be a second Weinberg interaction term,

\[
\sum_{i,j} \left( \bar{\ell}_L P^e_{ij} e_{jR} + \bar{q}_{iL} P^u_{ij} u_{jR} + \bar{q}_{iL} P^d_{ij} d_{jR} \right) \Phi' + h.c.,
\]

where P stands for the Capital Greek letter for ρ. Regardless of how we assign v.e.v. generation, the Λ^f and P^f matrices cannot be simultaneously diagonalized, and there would be FCNH couplings in general. This is the reason behind the Glashow-Weinberg NFC condition, which declares that for each fermion mass matrix, there can be only one Weinberg (usually called Yukawa) coupling matrix, i.e. absence of a second Weinberg matrix. Then, the fermion mass matrices are necessarily simultaneously diagonalized with the Weinberg matrices, and there are no FCNH couplings. *All fear is gone!*
But this is no less an Edict, as given by princes, hence authoritarian, and not in the scholarly tradition. Now, NFC is readily enforced by introducing a $Z_2$ symmetry, giving rise to 2HDM I & II, which correspond to linking the two Higgs doublets to $u$- and $d$-type quarks. Invoking a symmetry elevates the standing of NFC supericially in the context of the Gell-Mann Principle. It was also very convenient that it coincided with the outcome of SUSY, making 2HDM II popular. Alas, if SUSY had emerged quickly with the advent of the LHC, this picture would have gained credence. But we have not discovered anything beyond the $h(125)$, which by all counts resembles the Higgs boson of SM! Indeed, with the mindset of SUSY, it is easy to think of the extra doublet as decoupled $\mathcal{O}$, i.e. multi-TeV and out of reach.

But, should we give up $H, A$ and $H^*$ search? From the experimental point of view, the answer is clearly an emphatic “No!” Let LHC run its course, and finish what it is designed to do. We argue that NFC should be viewed the way it really is: ad hoc. It is NFC, and the familiar $Z_2$ symmetry, that should be dropped. Since the LHC has not revealed any new symmetry, we call the situation -Dyssymmetry-, playing on the word dissymmetry. After all, the Gell-Mann Principle should reign higher than the NFC dictum of Glashow and Weinberg. To put it differently, to remove $\Phi$ for fear of FCNH is itself unseemly.

If $m_t/m_b \gg 1$ was not anticipated, what were again purely empirical are the discoveries, circa 1983, of the long $b$ lifetime, and the absence of $b \to u$ vs $b \to c$ transitions. Sniffing these out, the experimental discoveries — divinations of Nature — inspired Wolfenstein [21] to write down his seminal parameterization, Eq. (9). Another thing unaware to Glashow and Weinberg — perhaps they would not care — at time of their writing Ref. [27], is the Fritzsch ansatz [23], Eq. (11), which curiously related the $m_d/m_s$ ratio to the Cabibbo angle. But we should stress that the quark mixing hierarchy

$$|V_{ub}|^2 \ll |V_{cb}|^2 \ll |V_{us}|^2 \ll |V_{cb}|^2 \approx 1, \quad (21)$$

is staggering, with $|V_{us}|^2 \simeq 0.05$, $|V_{cb}|^2 \sim 10^{-3}$, $|V_{ub}|^2 \sim 10^{-5}$. It is the combination of Eqs. (19)–(21), all emergent, that Nature seems to employ to hide the effects of the second scalar doublet from us, so far. This is curiously against the spirit of the Gell-Mann Principle, but we have to admit that Nature can have her ways.

**FCNH: an Experimental Question.** — A final point to make, before we move on to explore G2HDM and its consequences in the next section, is returning to the empirical nature of physics, yet again.

At the time it was proposed [10], be it $t \to ch^0$ or $h^0 \to t\bar{c}$, where $h^0$ is some neutral scalar boson, the top quark was not yet observed, and neither was any fundamental scalar boson. It was therefore a theoretical conjecture inspired by the Cheng-Sher ansatz [35], depending on whether $t$, or $h^0$, is the heavier one. We did extend beyond the Cheng-Sher ansatz and called it 2HDM III, i.e. neither 2HDM I nor II, but one where there is no $Z_2$ symmetry to enforce NFC. This general (not ad hoc) 2HDM that possesses extra Weinberg couplings, which includes FCNH, we now call G2HDM.

What happened after observation of $h(125)$ in 2012? If one looks up Particle Data Group [48], one would find entries for bounds on $t \to ch$ from ATLAS [49] and CMS [50], dating back to 2014. But, in particular ATLAS had already shown results for $t \to qh$ with $h \to \gamma\gamma$ at the EPS-HEP conference held 2013 in Stockholm. The point is, with $h(125)$ found lighter than top, there is no way to stop the experimentalists, young or not (e.g. Daniel Fournier of ATLAS) from $t \to ch$ search — because they can. Suppose someone resorts to authority and tells the valiant worker that Glashow and Weinberg forbade it since 1977, the first response would be shock: “What?!” But after a brief pause, the person would brighten up and say “Splendid, so much the better. Can Not Lose!” Because the bottom line would be: It is a PDG Entry. The current bound [53] at 95% C.L. is

$$B(t \to ch) < 7.3 \times 10^{-4}. \quad (22)$$

---

3 We continue to use $h$ for the 125 GeV boson. Experiments still use $H$, as nothing else has been discovered so far.
A second case is \( h \to \tau \mu \). Advocated by Ref. \[42\], in fact CMS saw a hint \[51\] at 2.4 \( \sigma \) with branching fraction \( \sim 1\% \) in 2015, but disappeared when more data was added. The current bound \[52\] at 95\% C.L. is
\[
\mathcal{B}(h \to \tau \mu) < 1.5 \times 10^{-3}. \quad \text{(CMS 2021) (23)}
\]
So, FCNH is an experimental question. And by generalization, it should be extended to all extra Weinberg couplings \( \rho'_{ij} \), for \( i,j = 1-3 \) and \( f = u, d, e \).

The extra Weinberg couplings in 2HDM are \[53, 54\]
\[
- \bar{u}_i \bar{d}_j R \ell_j H^+ - \bar{u}_i \left[ (V^d_{ij}) R - (\rho^u V_{ij}) L \right] d_j H^+ - \frac{1}{\sqrt{2}} \sum_{f=t,c} \bar{f}_i \left( \lambda^f_{ij} c_{\gamma} + \rho^f_{ij} s_{\gamma} \right) H - i \text{sgn}(Q_f) \rho^f_{ij} \bar{A}_i \tau_{\mu} - \left( \lambda^f_{ij} s_{\gamma} - \rho^f_{ij} c_{\gamma} \right) \mu_f + h.c., \quad \text{(24)}
\]
where \( i, j \) are summed over generations, \( L, R \) are projection operators, and \( V \) is the CKM matrix, with lepton matrix taken as unity due to vanishing neutrino masses. With \( h \) identified as the observed SM-like Higgs boson, one can read off Eq. \( (24) \) and see that \( c_{\gamma} \) is the \( h-H \) mixing angle between the two \( CP \)-even scalars. As we shall see in the next section, the nonobservation of \( t \to ch \) and \( h \to \tau \mu \) so far may have another surprise of Nature behind it: small \( c_{\gamma} \), that \( h \), the SM sector, and \( H \), the exotic sector, do not seem to mix much.

\section{BIG ISSUES & NEW HIGGSS/FLAVOR ERA}

\subsection{Heaven and Earth}

By Heaven we refer to baryogenesis. In particular, electroweak baryogenesis (EWBG) is preferred by down to Earth physicists, such as at the LHC, or those working on exquisite low energy precision experiments (LEPEs). Take any pair of the three, however, there is tension.

Baryogenesis calls for additional CPV that is beyond SM (BSM), i.e. beyond the Kobayashi-Maskawa phase. EWBG, which further requires a strongly first order electroweak phase transition (1st EWPT), is attractive because it is more testable. But hereby lies the tension: EWBG calls for large BSM CPV, but we have not seen any BSM physics at the LHC so far!

EWBG also easily runs into tension with LEPEs, where a recent flagship example is the ACME experiment aimed at electron electric dipole moment (eEDM). The current ACME bound \[50\] on eEDM is,
\[
|d_e| < 1.1 \times 10^{-29} \text{ e cm}, \quad \text{(ACME 2018) (25)}
\]
at 90\% C.L. As EWBG demands rather large BSM CPV, as well as new particles with sufficient dynamics to instigate 1st EWPT, it is usually not trivial to evade the probing eye of LEPEs such as ACME. As we shall see, EWBG can be achieved \[57\] in 2HDM, but its initial prediction for eEDM was ruled out by ACME 2018, Eq. \( (25) \), within a year! Fortunately, Nature seems to provide a mechanism \[55\] to render \( d_e \) small.

Finally, the tension between LHC experiments and LEPEs is, plainly, asymmetric competition. While we have not observed any BSM physics at the LHC, the “table-top” LEPEs (which ACME truly is), with its claim to be able to probe higher scales, causes further angst for the behemoth LHC experiments.

\textbf{EWBG in 2HDM. —} The CMS hint \[51\] for \( h \to \tau \mu \) caused some stir and hope in the HEP circle. In Fall 2016, there was a talk on EWBG that combined \( h \to \tau \mu \) with \( \tau \to \mu \gamma \), employing large \( \rho_{\tau \mu} \) or \( \rho_{\tau \gamma} \) (vs \( \lambda_7 \simeq 0.01 \)) as CPV source \[59\] for baryogenesis. After the seminar, we spoke with the speaker and commented: “This is too intricate for it to work in Nature.”, remarking further, “However, \( \rho_{tt} \) is naturally \( \mathcal{O}(1) \) and complex. I would bet that it can provide EWBG.”

The context of the talk was basically 2HDM (i.e. 2HDM III \[40\]), and we learned from the talk that the extra Higgs bosons in 2HDM could in principle provide 1st EWPT. And finally, we started to pay attention to the CPV phases that 2HDM provide, and set off to work. The results turned out better than anticipated.

As seen from Fig.\[1\] with the Weinberg coupling \( \lambda_7 \simeq 1 \) in SM, it is plausible that \( \rho_{tt} \) and \( \rho_{tc} \) are \( \mathcal{O}(\lambda_7) \) hence \( \mathcal{O}(1) \) order 1. This is the reason behind our comment on the leading effect for EWBG. We thereby showed \[57\] that
\[
\rho_{tt} \mathcal{O}(1) \quad \text{and} \quad \rho_{tc} \mathcal{O}(1) \quad \text{provide a robust driver for BAU.}
\]
This is illustrated in Fig.\[2\]. The baryon over entropy, \( Y_B = n_B/s \), is plotted \[57\] in units of the observed \[61\]
\[
Y_B^{\text{obs}} \simeq 8.59 \times 10^{-11}, \quad \text{(Planck 2013) (27)}
\]
vs the strength of \( \rho_{tt} \), where we scan over the phases of \( \rho_{tt} \) and \( \rho_{tc} \). To further discern the cause and effect, we plot the scatter points for \( 0.1 \leq |\rho_{tc}| \leq 0.5 \) (0.5 \( \leq |\rho_{tt}| \leq 1.0 \)) in purple bullets (green crosses). For the bulk of the plot, the two sets can be barely distinguished, so \( \rho_{tt} \) is the driver for BAU and appears quite robust. But for \( |\rho_{tt}| \lesssim 0.07 \) or so, the main effect seems to peter out, and we find that \( 1 \approx Y_B/Y_B^{\text{obs}} \leq 2-3 \) to be populated more by \( 0.5 \leq |\rho_{tt}| \leq 1.0 \). This indicates that, when \( |\rho_{tt}| \lesssim 0.05 \) and becomes ineffective, \( \rho_{tt} \) at \( \mathcal{O}(1) \) provides a second, backup \[57\] ineffective for BAU. This was also illustrated heuristically with numerical 2 \times 2 examples for the 2-3 sector of the \( \rho^4 \) matrix.

\footnote{The other element, \( \rho_{tt} \), has to be relatively small \[60\] as it enters e.g. \( b \to s \gamma \) with chiral enhancement.}
ACME had advanced the eEDM bound only a few years before our work, setting the impressive limit (compared with e.g. neutron EDM, i.e. nEDM),

\[ |d_e| < 8.7 \times 10^{-29} \text{ e cm, \ (ACME 2014)} \] (28)

with the claim that the experiment knew how to improve by an order of magnitude. Since a sizable \( \rho_{tt} \) with CPV phase can affect eEDM via the two-loop mechanism \[63\], i.e. the Barr-Zee diagrams (see Fig. 3), they ought to be checked. For simplicity, we set \( \rho_{ee} = 0 \), i.e. we made estimates only with effects from \( \lambda_\text{t} \text{Im} \rho_{tt} \). We were aware that this kind of assumption normally would require a symmetry. However, assuming \( \rho_{ee} = 0 \) in our numerics, we gave a “predicted” range of \( Y_B/Y_{B^{\text{obs}}} \) > 1 that falls between Eq. (28) and the projected improvement, hence can be probed by ACME — with chance for discovery.

To our surprise, ACME reached their advertised improvement \[56\], Eq. (25), in less than a year and half after our paper was posted \[57\], and it ruled out the entire range we had “predicted”.

Any mechanism to render eEDM small?

**Facing ThO EDM: Cancellation Mechanism.**

Fortunately, the answer is yes \[58\], and turned out simpler than anticipated\[5\]. Restoring \( \rho_{ee} \), many diagrams have to be accounted for. The leading Barr-Zee diagram is basically inserting \( \phi \to \gamma \gamma^* \) with top loop into the \( ee \gamma \) effective coupling, which we call the \( d^e_{ee} \gamma \) term, with \( \phi \) summed over \( h, H, A \). Besides the \( \lambda_\text{t} \text{Im} \rho_{tt} \) term with respective couplings at \( t \) end, one now also has \( \text{Im} \rho_{ee} \lambda_t \) effects. Interestingly, we find \[58\] a cancellation mechanism that boils down to a simplified “ansatz” of

\[
\frac{\text{Im} \rho_{tt}}{\text{Im} \rho_{ff}} = r \frac{\lambda_f}{\lambda_t}, \quad \text{Re} \rho_{ff} = - r \frac{\lambda_f}{\lambda_t},
\] (29)

i.e. the extra Weinberg couplings follow the mass-mixing hierarchy pattern seen in SM couplings. In Eq. (29), \( r \) depends on details of the loop functions involved.

We plot \( |d_e| \) as the long-dashed line vs \( r \) in Fig. 4, together with the ACME 2014 and 2018 bounds as the two shaded regions, where the result for setting \( \rho_{ee} = 0 \) corresponds to \( r \to 0 \) and ruled out already by ACME 2018. Near exact cancellation occurs at \( r \approx 0.74 \). Then there are the subleading \( d^e_{ee}Z \) and \( d^e_{ee}W \) effects (see Ref. \[58\] for details), given as dot-dashed and dotted curves in Fig. 4 shifting the cancellation point slightly upward to the (red) solid curve for \( d_e \). But that is not all. The ACME experiment actually measures the precession of eEDM in the strong internal electric field of the ThO molecule. Therefore, one also has to account for the \( T \)-violating \( eN \) scattering effect, \( \alpha_{T\text{ThO}}C_S \), which involves all types of quarks. Though a weaker effect than \( d^e_{ee}Z \) and \( d^e_{ee}W \), it finally shifts \( d_e \) to the (black) solid curve corresponding to the measured \( d_{T\text{ThO}} \), which turns out to lie close to the original \( d^e_{ee} \).

The moral: take no shortcuts.

\[\text{We do not give any details here. Suffice it to say that our results are consistent with Ref. \[64\].}\]
The result \( [58] \) allows a possible reduction by two orders of magnitude, to be probed by eEDM experiments. Interestingly, the cancellation mechanism of Eq. (29) rhymes with the mass-mixing hierarchy that emerged after the Glashow-Weinberg NFC proposal. Ref. \([58]\) illustrated further the enlarged parameter space for EWBG allowed by this “natural” cancellation mechanism.

In particle physics, one is more familiar with nEDM, which already posed a challenge by putting strong constraints on BSM phases such as in SUSY. Heavily investing on the ultracold neutron (UCN) approach, even just a few years back, nEDM seemed poised for a major reworking on the ultracold neutron (UCN) approach, even just a few years back, nEDM seemed poised for a major reworking under \( Z_2 \), the usual notation \( [63] \) is \( m_{ij}^2 \) and \( \lambda_i \), with \( \lambda_6 = \lambda_7 = 0 \). We assume \( V(\Phi, \Phi') \) to be \( CP \) conserving for simplicity, hence all parameters in Eq. (31) are real.

**Upper Bound, 1st EWPT, and Landau Pole.**—Without \( Z_2 \) symmetry, \( \Phi \) and \( \Phi' \) cannot be distinguished (hence the familiar \( \tan \beta \) is unphysical), and we choose the Higgs basis where \( \Phi \) generates \( v \), while \( \mu_{22}^2 > 0 \). The two minimization conditions \( [63, 60, 65] \) are

\[
\mu_{11}^2 = -\frac{1}{2} \eta_1 v^2, \quad \mu_{12}^2 = \frac{1}{2} \eta_6 v^2, \tag{32}
\]

where the first is the same as in SM, except the factor of 1/2 due to convention. The second minimization condition eliminates \( \mu_{12}^2 \) altogether, and \( \eta_6 \) is the sole parameter for \( \hbar \)-\( H \) mixing. This is in contrast with usual \( Z_2 \) models, where \( \lambda_6 \) (and \( \lambda_7 \)) is absent, but requires both \( m_{11}^2 \) and \( m_{22}^2 \) to be negative, while the “soft breaking” \( m_{ij}^2 \) parameter serves both roles of \( \hbar \)-\( H \) mixing and the decoupling mass. Thus, G2HDM is more intuitive than 2HDM I and II, where \( \mu_{22}^2 > 0 \) is the decoupling mass for the exotic doublet. The exotic Higgs masses are

\[
m_{H^+}^2 = m_{22}^2 + \frac{1}{2} \eta_3 v^2, \tag{33}
m_{A}^2 = m_{H^+}^2 + \frac{1}{2} (\eta_4 - \eta_6) v^2, \tag{34}
M_{\text{even}}^2 = \begin{bmatrix} \eta_1 v^2 \\ \eta_6 v^2 \end{bmatrix}, \tag{35}
\]

where the last equation is for the \( CP \)-even Higgs sector, which we will return to shortly.

With \( v \simeq 246 \) GeV from the first minimization condition of Eq. (32), the dimensionless Higgs quartic couplings, \( \eta_i \) for \( i = 1 \) and 3–6 contribute to \( m_S^2 \) \((S = H, A, H^+)\) modulo \( v^2 \). If we demand that \( v \) sets the scale for \( V(\Phi, \Phi') \), then all \( \eta_i \) plus the dimensionless ratio \( \mu_{22}^2/v^2 \) being \( O(1) \) is the common sense “naturalness”\(^7\). Imposing this “naturalness”, we find

\[
m_S \lesssim 600 \text{ GeV}. \quad (S = H, A, H^+) \tag{36}
\]

\(^7\) Of course, \( \mu_{22}^2 \) is an independent scale parameter. However, before going to the decoupling limit, one should certainly probe \( \mu_{22}^2 = \mathcal{O}(v^2) \). Otherwise, the decoupling limit of \( \mu_{22}^2 \gg v^2 \) would render all dynamical effects mute.
We remark with interest that, with $v$ as the scale parameter, to have all $\eta_i$ at $O(1)$ is just what is needed for generating 1st-EWPT [69], one of the prerequisites for EWBG. As already stated, $\mu_{22}/v^2 < O(1)$ is also needed, otherwise decoupling would damp all scattering amplitudes, making EWBG infeasible. Thus, we find a natural argument for having 1st-EWPT in 2HDM.

Having the heuristic upper bound of Eq. (36), we will later argue a lower bound (see Fig. 5) from the apparent smallness of $h-H$ mixing. Here we note that, having $\eta_i = O(1)$ for $i = 1-7$, the “common sense” naturalness [8] has its implications, namely the Landau pole associated with scalar fields. Even with $|\eta_i| = 1$ for some $i$ implies the Landau pole is not far above 10 TeV [60]. Thus, $O(1)$ quartics imply a strong coupling scale at several 10 TeV and would be a boon to a 100 TeV $pp$ collider [9]. Since none of the $\eta_i$s have yet been measured and some may be negative, the Landau pole scale is highly uncertain, which we put at 10–20 GeV.

To understand the 1st-EWPT and probe the Landau pole scale of the 2HDM Higgs potential, numerical studies with lattice field theory are needed, which could go hand in hand with experimental search.

Leading Search Modes at the LHC. — Shortly after the EWBG work [57] and Ref. [60], we proposed [70] the search modes via $tS$ associated production,

$$\begin{align*}
    cg &\rightarrow tA/H \rightarrow tt\bar{c}, \quad \text{(Same-Sign Top)} \quad (37) \\
    cg &\rightarrow tA/H \rightarrow t\bar{t}t, \quad \text{(Triple-Top)} \quad (38)
\end{align*}$$

where production goes through $\rho_{tc}$ (see Fig. 6), while $A/H \rightarrow t\bar{c}, t\bar{t}$ decays go through $\rho_{tc}$ and $\rho_{tt}$, respectively, likely the two largest extra Weinberg couplings that can also serve as drivers [57] for EWBG. The Same-Sign Top signature, $SS2t+j$, where the extra jet is useful as a discriminant, is promising already [70] with full Run 2 data, while adding Run 3 data can only help.

The triple-top signature is more exquisite, with SM cross section only at a few fb$^{-1}$ [71], and can allow more scrutiny. But high statistics is needed, hence more suited [70] for the High Lumi LHC (HL-LHC). A source of frustration is the dedicated search for $tt\bar{t}t$ by both ATLAS and CMS, targeting the SM cross section [71] at ~12 fb$^{-1}$, which is considerably larger than tri-top in SM. So, we had to use $4t$ search results, both Control Regions (CR) and Signal Regions (SR), to extract [72] some constraint on tri-top parameter space. Understandably, $4t$ search has a “measurable” SM target, but it amazes us that, with $tt\bar{t}t$ cross sections that could be over a hundred times larger than $4t$ in SM, no dedicated search exists so far. Philosophically, one could say that, shooting for the small but accessible SM $4t$ process, one should be more or less cover large enhancement of tri-top, which is the basis of Ref. [72]. The latest $4t$ search of CMS [73] and ATLAS [74], both based on full Run 2 data, are consistent with SM, with ATLAS a little on the high side. Adding future Run 3 data should seal it.

It took us a while to consider $H^+$ effects, limited in part by intuition arising from the familiar 2HDM II. Motivated by Belle progress [75] and Belle II prospects, in studying the $B \rightarrow \mu\nu$ decay, we finally noticed [76] the $H^+$ coupling in Eq. (24) has its own features. A $b \rightarrow u$ process on the quark side, it involves two possibilities: $\sum_i \rho_{ib} V_{ui}$, summing over $d$-type quarks, one can show [76] it is highly suppressed: $\sum_i \rho_{iu}^* V_{ib} = \rho_{tu}^* V_{tb} + \rho_{cu}^* V_{cb} + \rho_{uu} V_{ub} \cong \rho_{tu} V_{tb}$, where $\rho_{cu}$ is highly constrained by $D^0$ mixing, and $\rho_{uu}$ is suppressed by mass-mixing hierarchy, and each receive further CKM suppression. We noticed that, on one hand there is no $\rho_{tu}$ coupling in 2HDM II, on the other hand, the associated $V_{tb}$ stands against the (2HDM II) intuition that $V_{ub}$ governs the process. On lepton side, the $\nu$ escapes Belle undetected, so it could be $\bar{\nu}_\tau$, hence bring in $\rho_{\mu\tau}$. The upshot is that $B \rightarrow \mu\nu$ decay probes the FCNH product,

$$\rho_{tu} \rho_{\mu\tau}, \quad (39)$$

and receives an astonishing $|V_{tb}/V_{ub}|$ enhancement!

Flavor considerations matter, and one should put away old intuitions, which are nothing but prejudice.

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8 For Weinberg couplings, only $\lambda_1 \approx 1$ seems “natural”, while the emergent mass hierarchy of Eq. (17) is part of the flavor riddle.

9 In principle, SUSY may get a second lease of life.
The production is through \( \rho \) via \( bH \) at amplitude level. Because \( b \) is so much lighter than \( t \), the \( cg \rightarrow bH^+ \) process is more efficient. The signature of single top plus two \( b \)-jets may look a little dubious, but our study [77] did not find any obvious issue.

Our proposed production and decay processes are a bit unconventional in the context of 2HDM, where most intuition is from 2HDM II, which has SUSY as support. But in these \( Z_2 \) models, the “attractive” thing is that there is practically one parameter, \( \tan \beta \), i.e. the ratio of v.e.v.s between the two doublets, aside from \( H, A, H^+ \) masses.

In contrast, the extra Weinberg couplings in \( G2HDM \) open up several new parameters, hence production and decay possibilities. One expects these would reflect the observed mass-mixing pattern seen in the Weinberg couplings of SM, as we learned from the Cheng-Sher ansatz, as well as from surviving electron EDM constraint. That is the reason we have focused more on \( \rho_{tt} \) and \( \rho_{tc} \).

As a precursor to our lower mass bound on exotic scalars, let us consider an extra pseudoscalar Higgs boson \( A \). We had worked on the \( cg \rightarrow tA \) process [78] for the LHC long before its advent. Well into the LHC era, it is curious whether a light \( A \), e.g. \( m_A \lesssim 300 \text{ GeV} \), can still be viable. Though below \( t\bar{t} \) threshold, one still has to turn off \( \rho_{tt} \), as it would induce \( gg \rightarrow A \), although one can also suppress \( A \rightarrow hZ \) by small \( h-H \) mixing (called “alignment”, as discussed later). The upshot [79] is that an \( A \) with mass between \( tc \) and \( t\bar{t} \) thresholds could still survive so far, and \( cg \rightarrow tA \rightarrow t\bar{t}c \) of Eq. (37) can probe sizable \( \rho_{tc} \) at the LHC, accessing the second mechanism of EWBG in \( G2HDM \). The \( \rho_{tc} \) mechanism does not generate eEDM, hence is not constrained by ACME.

There are many other search possibilities, depending on thresholds and other parameters. We have worked out e.g. \( gg \rightarrow A, H \rightarrow t\bar{t} \) [60], \( \tau\mu \) [80]; \( cg \rightarrow tA \rightarrow tZH \) [54], where \( A \rightarrow ZH \) is a weak decay; \( cg \rightarrow tH \rightarrow thh \) [81] and \( cg \rightarrow bH^+ \rightarrow bW^+ \) [82], which depend on \( h-H \) mixing; or \( cg \rightarrow bH^+ \rightarrow bAW^+ \) [83]. For these and other works, we refer to our brief review [84] for discussion.

As far as we know, programs along some of these lines have started in both ATLAS and CMS.

**Remark: \( A \rightarrow t\bar{t} \) at 400 GeV?**—With finite \( \rho_{tt} \), the \( H \) and \( A \) bosons can be produced via gluon-gluon fusion (ggF), \( gg \rightarrow H, A \), and it would likely decay back to \( t\bar{t} \), as \( \rho_{tt} = O(1) \) is plausible. This “resonance” in the \( t\bar{t} \) channel, however, is known to run into trouble: QCD production of \( t\bar{t} \). The enormous cross section of the latter means the loop-produced ggF process would be like foam on the waves of the wide ocean. The interference turns the resonance peak into a peak-dip structure [55], making the analysis rather difficult.\(^\text{10}\) But ATLAS pioneered the way with 8 TeV data [57], finding no hint.

For some time, CMS gave no results. But this changed in 2019, first with a somewhat “oblique” CMS PAS [88]. “Oblique” because no sensational words appeared in the Abstract, Introduction, nor Summary, although in the text, the word “excess” was used more than a few times,\(^\text{11}\) so nobody noticed, nor cared, even by summer conferences. Words finally broke through in the paper [89]: in essence “a signal-like excess for the pseudoscalar hypothesis (largest) at 400 GeV, 4% of \( \Gamma_{tot} \), with 3.5\( \sigma \) (1.9\( \sigma \)) local significance (with look-elsewhere-effect).” However, though plots were given, no extracted values for coupling modifiers \( g_{H\bar{t}t} \) and \( g_{A\bar{t}t} \)—corresponding to our \( |\rho_{tt}|/\lambda_t \) (real values were assumed)—were provided. The result was based on \( \sim 36 \text{ fb}^{-1} \) data at 13 TeV. CMS has not yet updated with full Run 2 data, while ATLAS has not revealed any 13 TeV result on this so far.

How seriously should one take this hint of excess, in a difficult process for analysis? One could throw a host of problems at the result, the chief one being the proximity to \( t\bar{t} \) threshold. The earlier ATLAS analysis [87] took \( m_{\bar{t}t} > 500 \text{ GeV} \) in part to avoid such issues. We will leave experimental issues to the experiments.

Incorporating a discussion on \( A \rightarrow t\bar{t} \) in our “At 3t” paper, Ref. [72] argued that having both \( \rho_{tt} \) and \( \rho_{tc} \) at \( \approx 1 \) strength can in principle explain the excess. We also noted that complex \( \rho_{tt} \) values “rotate” in \( H-A \) space [90], e.g. \( H \) could feign as \( A \) for purely imaginary \( \rho_{tt} \) (hence the analysis should be done with complex coupling modifiers). The need for \( \rho_{tt}, \rho_{tc} \approx 1 \) is in part to reduce the decay branching ratios, but such large strength for the relatively low \( m_A \) causes other tensions. The proposed \( cg \rightarrow bH^+ \) production [77] brings in new worries, e.g. sizable cross section for relatively high \( H^+ \) masses. If \( H^+ \rightarrow AW^+ \) is allowed [83], followed by \( A \rightarrow t\bar{t}, t\bar{t} \), one would face 4\( t \) bounds. So, we find the hint for a 400 GeV pseudoscalar by CMS dubious. However, such \( t\bar{t} \) resonance searches should be watched, while the experiments should provide the coupling modifier values.

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\(^{11}\) We found out the hint of “excess” only when trying to incorporate the result into Ref. [72].
C. Nature’s Flavor Design and Lower Mass Bound

When we used the Gell-Mann Principle to demand the existence of a second Higgs boson doublet $\Phi'$, implying the existence of exotic Higgs bosons $H$, $A$ and $H^+$, it is not particularly new, but we raised two related questions:

“Where are they?”

“What do they do?”

These lead to a combined question:

“What hides $H, A, H^+$ effects from our view?” (41)

For 2HDM II, there are specific predictions for exotic scalar couplings, governed by $\tan \beta = v_1/v_2$, the ratio of v.e.v.s of the two doublets. One could enhance $d$-type quark and charged lepton $\ell$ couplings, or enhance $u$-type quark couplings, including the dominant top, or treat them on more equal footing, i.e. $\tan \beta \sim 1$. For a long time, $H^+$ search was for $m_{H^+} < m_t$, and only more recently do we have search bounds for $pp \to tH^+(b)$ from CMS [91] and ATLAS [92]. The parton level process is

$$\bar{b}_g \to \bar{t}H^+, \quad (42)$$

which can be compared with our $cg \to bH^+$ process, Eq. (40) or Fig. [7]. In G2HDM, Eq. (42) is governed by $\rho_u V_{tb}$, while Eq. (40) is governed by $\rho_c V_{tb}$, and are quite independent, the latter receiving $|V_{tb}/V_{cb}|^2$ enhancement [77] in rate compared with 2HDM II, while phase space favored. It also implies that $H^+ \to c\bar{b}$ proceeds with $\rho_c V_{tb}$ coupling and can compete strongly with $H^+ \to \bar{t}b$, which is quite unconventional. The measurement of $tH^+(b)$ production, taking the $H^+ \to c\bar{b}$ dilution into account, has been used to constrain the extra Weinberg couplings. However, CMS has yet to update with full Run 2 data. Both experiments have [91, 92] a more stringent than expected bound at 600–700 GeV, putting some pressure on heavy $H^+$.

**Nature’s Flavor Design.** So, we have illustrated that, similar to $H, A \to c\bar{t}$ and $t\bar{t}$ can mutually dilute each other, $H^+ \to c\bar{b}$ and $tb$ decays can also mutually dilute each other, and should be taken into account. This is part of the broader “flavor design” of Nature that addresses the question in Eq. (41).

Let us articulate once more:

1. (Fermion) Mass-Mixing Hierarchy

The fermion mass hierarchies, Eqs. (19) and (20), and mixing hierarchy Eq. (21), all emergent, seem to have worked in concert such that we have been oblivious to extra Weinberg couplings, $\rho^f_{ij}$ trickle off going off-diagonal, (43)

as originally noted by Cheng and Sher [35]. These hierarchies form the flavor enigma, which we do not understand, but seem to be Nature’s choice, as emphasized by the word emergent.

2. (Emergent) Alignment

Another phenomenon, not quite related to fermion flavor, emerged after the discovery of $h(125)$:

The $h$ boson resembles $H_{SM}$ rather closely. (44)

So far we have not seen deviations in its properties from $H_{SM}$, the Higgs boson of SM. If there is a second Higgs doublet — by Gell-Mann Principle there must be! — then $h-H$ mixing is small, i.e. the mixing angle $c_{\gamma} \equiv \cos \gamma$ (called $\cos(\beta-\alpha)$ in 2HDM II convention) between the two CP-even scalars is small. This alignment phenomenon can readily account for the absence of $t \to c\bar{h}$ and $h \to \tau\mu$ so far. That is, Eqs. (22) and (23) can be satisfied without requiring e.g. $\rho_c$ to be small.

Historically, whether in the Cheng-Sher ansatz [35] or our proposal [40] of $t \to c\bar{h}$ (alternatively, $h \to t\bar{c}$ in “2HDM III”, Higgs sector mixing was glossed over, reflecting the expectation that the 5 Higgs bosons would share some common spectrum, and would be well-mixed. As it now stands, and as we have illustrated in Sec. IV.B, there seems some gap between $h$ at 125 GeV and the exotic $H, A, H^+$ bosons, and Nature threw in further a small $c_{\gamma}$ to protect $h$ from FCNH couplings.

NFC of Glashow-Weinberg should be retired.

We elucidate alignment further below to extract some lower bound on exotic Higgs boson masses, but turn now to state a curiosity.

3. Near diagonal $\rho^d$: Nature’s mysterious ways

In the SM context, $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, and $B^0_s - \bar{B}_s^0$ systems are our most sensitive probes of flavor and CP violation. But absence of New Physics so far in these sectors (bounds are still suitably accommodating), as well as in e.g. $B_q \to \mu^+ \mu^-$, means that the $\rho^d$ matrix is near-diagonal.

Numerically, it is not that serious, e.g. $\rho_{sb}$ only needs to be an order of magnitude smaller than $\rho_{tb}$, which is already constrained by $[61] b \to s\gamma$ — another sensitive probe — to be small, $\rho_{sb} \lesssim \lambda_b \sim 0.02$. However, were it not the case, that Nature put in just a tad larger off-diagonal elements in $\rho^d_{ij}$, we would have detected the effects via these exquisite systems, and discovered BSM much earlier. Instead, all along we just confirmed SM, from the fantastic working of GIM, to the present day lack of firm BSM effects.

**Alignment & Lower Mass Bound.** Let us turn to some subtlety of alignment in G2HDM. From
Eqs. (33)–(35), we had inferred an upper bound of Eq. (36) by argument of $O(1)$ Higgs quartics and $\mu_{Z^0}/v^2$, the decoupling parameter, also needed for achieving 1st-EWPT. By the same token, we argue that one has a rough lower bound on $m_H$, or the generic $m_A$.

Diagonalizing Eq. (35) by

$$R_T^2 M^2_{\text{even}} R_\gamma = \begin{bmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{bmatrix}, \quad R_\gamma = \begin{bmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{bmatrix},$$

(45)

where $s_\gamma \equiv \sin \gamma$. In 2HDM II convention, $\gamma = \beta - \alpha$ is the relative angle between the Higgs basis and the neutral Higgs mass basis, hence basis-independent. Fixing $m_h \approx 125$ GeV, without solving explicitly for $m_H$, the mixing angle $c_\gamma$ satisfies two relations

$$c_\gamma^2 = \frac{\eta_1 v^2 - m_h^2}{m_H^2 - m_h^2}, \quad \sin 2\gamma = \frac{2\eta_0 v^2}{m_H^2 - m_h^2}.$$  

(46)

Since we know that $c_\gamma$ is small, i.e. approximate alignment [15], $s_\gamma \to -1$ much faster than $c_\gamma \to 0$, hence the second relation simplifies to

$$c_\gamma \simeq \frac{-\eta_0 v^2}{m_H^2 - m_h^2}, \quad \text{(approx. alignment)}$$

(47)

which is the familiar mixing formula for a two-level system. Since $m_h^2$ is small compared with $v^2$ (roughly $v^2/4$ in the present convention), we see that for $m_H^2 \gtrsim v^2$, small $c_\gamma$ can only be achieved by having $\eta_0$ small.

In fact, Ref. [66] was motivated by the statement that $O(1)$ Higgs quartics are needed for 1st-EWPT [69]. But from the SUSY mindset, $\eta_0$ appeared to be small [71][95]. Concerned with logical consistency (one quartic kept small defeats reasoning of generic $O(1)$ quartics), the thought was to investigate how large $\eta_0$ could be, and we found that $\eta_0$ at $O(1)$ was truly allowed.

Let us illustrate [66] some simple but subtle aspects of alignment in G2HDM, free from any SUSY distinction. For $m_A = m_{H^+} = 350, 475, 600$ GeV, i.e. custodial symmetry to control $T$-parameter and implemented by $\eta_1 = \eta_5 \in (0.5, 2)$, hence $m_H$ is heavier than $m_A = m_{H^+}$, we plot in Fig. 8 the two controlling quartics for $h - H$ mixing, $\eta_1$ vs $\eta_0$. In Fig. 8, bullets are for $-\cos \gamma = 0.1, 0.2, 0.3$, where two sets are connected by (red) dashed lines. The unmarked line for $c_\gamma = -0.3$ roughly coincides with the 95% C.L. limit from $S-T$ data [66], which cuts off the parameter space of each “horn”, turning into dotted lines; small $c_\gamma$ and $\Delta T$ constraint seem correlated.

To illustrate $O(1)$ quartics, let us arbitrarily take the middle (pink) “horn”, and $(\eta_0, \eta_1) \simeq (1.25, 0.6)$. Both values are clearly $O(1)$, with $c_\gamma \sim -0.25$. One can approach $|c_\gamma| \simeq 0.2$ by picking a larger value of $\eta_0 \simeq 1.5$ in the $m_A = m_{H^+} = 600$ GeV (grey) “horn”. So how is this achieved? Note that the $\eta_1$ value of $\sim 0.6$ is 2.3 times the value of $\sim 0.26$ that corresponds to SM value. This enhanced $hhh$ coupling itself would have implications for the electroweak phase transition. But now we see that the eigenvalue of $m_H \simeq 125$ GeV is actually maintained by the familiar level repulsion: the lower state is pushed down to 125 GeV, while the large $\eta_0$ mixing quartic pushes the second state up to a higher $m_H$ value, and helps suppress $c_\gamma$ through the denominator of Eq. (17).

The upshot is that there is large parameter space, the body of the “horns”, rather than the “tip of the horn” where all “horns” converge, as $\eta_0$ necessarily shrinks with $c_\gamma$, with $\eta_1 v^2 \to m_h^2$ as required by the first part of Eq. (46), in the approach to the alignment limit.

If having Higgs quartics $O(1)$ demand $\eta_0 > 0.3$, then the bulk of the horn roughly demands

$$m_H \gtrsim v \approx 250 \text{ GeV}.$$  

(48)

There is a catch, though. Eq. (48) is not fool proof, as Nature can impose a small $\eta_0$ when the rough bound of Eq. (48) is not kept, at her will. What may be unclear is whether 1st-EWPT can be maintained. But that can arise from other means, at Nature’s will.

D. Glimpse of the New Flavor Era

We have three new $3 \times 3$ extra Weinberg matrices, each element in principle complex, so altogether 54 new flavor parameters. It may seem horrible to have so many new parameters in G2HDM (but don’t forget the many flavor parameters of SM). On the other hand, it is all the more remarkable that we have not seen hints of their existence so far — the Flavor Design of Sec. IV.C!

Actually, there have been a host of suitably strong hints from flavor sector for BSM physics, the so-called “$B$-anomalies”. In order of their first appearance (none are viewed as established by itself): $R_D, R_{D^*}$ anomaly; $P_L^s$ anomaly; $R_K, R_{K^*}$ anomaly. Although people tried G2HDM with $R_D, R_{D^*}$ anomaly, but it was insufficient.

The first $R_{D^*}$ anomaly started with BaBar [97], which found deviations of $B \to D^{(*)} \tau \nu$ from $B \to D^{(*)} \mu \nu$. It became a sensation when it was confirmed by LHCb [98] in 2015. The $P_L^s$ anomaly is from LHCb, in the named variable in some $q^2$ bins in the angular analysis of $B^0 \to K^{*0} \mu^+ \mu^-$ [99]. The $R_K$ [100], then $R_{K^*}$ [101] anomalies are also from LHCb, in the ratio of $B \to K^{(*)} \mu \nu$ with $B \to K^{(*)} e e$ in some choice $q^2$ range. These “$B$-anomalies” are quite well known, so we do not describe
them further, but refer to our time-capsule snapshot of 2018 [102], where a brief discussion of each “anomaly”, together with a critique from experimental perspective, is given. Nothing has drastically changed since then.

Two BSM possibilities emerge from the $B$-anomalies, one is some $Z'$, another is some leptoquark (LQ), each accounting for a subset of the $B$-anomalies. We will use the LQ from “PS$^{3n}$” [105], i.e. having three copies of the Pati-Salam symmetry [106], as a contrast example.

One should not forget the recent result announced by the FNAL Muon g-2 experiment [103], confirming the Pati-Salam symmetry [106], as a contrast example.

A second experiment, Mu3e, searches for $\mu \to e\gamma$. With $\rho_{ee} = \lambda_\epsilon$, however, the tree-level effect is too suppressed, and $\mu \to e\gamma$ is dominated by the $\mu e\gamma$ dipole, marked by the (red) $\Downarrow$. Though within reach of Mu3e, measurement would come late.

A third type of experiment, $\mu \to e$ conversion on Nuclei, or $\mu N \to e N$, may become the champion. This is because of ambitious plans to improve on existing bound [48] by six orders or more, starting with COMET at KEK and Mu2e at FNAL. Our (red) $\Downarrow$ in Fig. 9 may not look too impressive. But unlike $\mu \to 3e$, at the nuclei side, the $(\bar{e} \mu)(\bar{q} q)$ four-Fermi operators probe all $\rho_{qq}$s, and the ability to use different nuclei increases the versatility. What is important, however, is to realize the many orders of magnitude improvement. Thing to watch!

Turning to $\tau$ decays, the analogous modes are $\tau \to \mu\gamma$ and $\tau \to \mu\mu$. Abiding by Eq. (19) and with $\rho_{tt}$ fixed by $\mu \to e\gamma$, it would take [107] close to full Belle II data to discover $\tau \to \mu\gamma$, while the expected dipole dominance indicates that $\tau \to \mu\mu$ lies out of reach.

So, there are some prospects for discoveries in $\mu$ decays and $\tau$ decays processes. But they are not guaranteed, as it depends on $\rho_{tt}$ and $\rho_{ee}$ values together saturate the current MEG bound [48]. That is the reason we have the (red) $\Downarrow$, that the G2HDM “prediction” lies below.

**Various Rare $B$ Decays and LQ Contrast.**—With $\tau \to \mu\gamma$, we already noticed the grey shaded band...
that more or less fills the space from current bound\cite{111} to
the projection for Belle II. This is from a phenomenologi-
cal study\cite{109} of the PS$^3$ model due to the $\bar{b}\tau$-flavored
leptoquark, which carry also lower generation flavors,
with the experimental result taken as constraint.

Having three copies of PS symmetry to avoid fla-
vor constraints would violate lepton universality, lead-
ing to LFV such as $B \to K\tau\mu$ and $B_s \to \tau\mu$ by gen-
eral arguments\cite{110}. In the PS$^3$ study of Ref.\cite{109},
$B_s \to \tau\mu$ as large as $\sim 5 \times 10^{-4}$ was predicted. Within
a year, however, LHCb pushed the bound down\cite{111} by
more than an order of magnitude, ruling the bulk of the
predicted region\cite{109}, and the PS$^3$ practitio-
ers had to tweak right-handed couplings, giving this new
"grey band"\cite{112} in Fig. 9, with current experimental
bound\cite{111} biting right in. This interaction between the-
ory and experiment in $B_s \to \tau\mu$ enters\cite{13} is as it should
be and exciting. The reach of LHCb with given data
remains to be seen, but likely can improve with data.

From Ref.\cite{109} to Ref.\cite{112}, the experimental con-
straint on $\tau \to \mu\gamma$ has barely moved, awaiting Belle II
data. For $B^+ \to K^-\mu^+\tau^+$\cite{110}, the current bound\cite{48}
is held by BaBar\cite{114}, which used full hadronic tag of
the other $B$, and look for events in the $m_\tau$ bin on sig-
nal side, given that $K^+$ and $\mu^+$ are well-identified tracks.
We note with interest that LHCb has found a method for
generating "grey band"\cite{112} in Fig. 9, with current experimental
bounds\cite{111} only slightly worse than BaBar. Thus, LHCb would soon take over in
pushing down the bound to access the grey band from PS$^3$.
So this could be the smoking gun, or the way to
further rule out PS$^3$. But Belle has not yet followed up even
with BaBar\cite{114} in this mode! We assume this
would happen soon, and that Belle II would come head
to head with LHCb in the coming years. The two other
"grey band" modes to probe LQ effects in PS$^3$, $B_s \to \tau\tau$
and $B \to K\tau\tau$ is less crisp than $B_s \to \tau\mu$ and $B \to K\mu\tau$.

The discussion above is rather exciting for the discover-
ability potential, and we would be more than happy if it hap-
sens. But otherwise we are no fans of LQs (Why LQ
Now?!\cite{102})\cite{102} looking around to at projections in
G2HDM, we note that $B_s \to \tau\tau$ and $B \to K\tau\tau$ have SM
expectations, the (red) $\star$’s in Fig. 9. The interference
effect with G2HDM would be similar to $B_s \to \mu\mu$ discussed
below. For $B_s \to \tau\mu$ and $B \to K\mu\tau$ have 4-5 orders of magnitude above experimental sensitivity,
while final states with $\mu\tau$ are even farther away. This
is because we adhere to Eq. (49), and illustrates our an-
twer to the question in Eq. (41): the mass-mixing hier-
archy, alignment, and near diagonal $\rho_{ij}^d$ continue to hide
G2HDM from our view in LFV $B$ decays.

\textbf{G2HDM–SM Interference Modes.} Finally, we
come to several very interesting modes that can enjoy
interference with SM process, and are elevated probes of
G2HDM, namely $B \to \mu\mu$, $B_d \to \mu\mu$ and $B^- \to \mu\nu\tau\nu$.
The decays $B_s \to \mu\mu$ and $B_d \to \mu\mu$ have been of
great interest since CDF times, largely due to a very
well known $(\tan\beta)^6$ enhancement in SUSY-like settings.
Alas, as Tevatron passed the torch on to the LHC, this
potentially large enhancement was eventually excluded
by experiment. We now have good indication\cite{48} that
$B_s \to \mu\mu$ is even slightly smaller than SM expectation.

We have commented that the smallness of $B_q \to \mu\mu$
(lack of BSM indications in $K^0$, $B_q$ and $B_s$ meson
mixings) demands a near diagonal $\rho_{ij}^q$. But can the SM
dominance of these modes, by interference, help us probe
G2HDM? It could happen. But there can be both tree
or loop effects in G2HDM, and possibilities abound
(too many parameters). How to approach it?

The G2HDM effects at tree level in $B \to \mu\nu$ and $\tau\nu$
have also been touched upon, that subtleties involving
charged Higgs effects and the undetected neutrino go be-
ond the intuition from 2HDM II. In fact, careful analy-
sis\cite{76} suggest $B \to \tau\nu$ to be SM-like, but $B \to \mu\nu$
probes the unique FCNH product $\rho_{\mu\nu}\rho_{\tau\nu}$ (Eq. (39)), with
$\rho_{\mu\nu}$ entering in $V_{tb}/V_{ub}$ enhancement, and $\rho_{\tau\nu}$
entering through the unobserved neutrino. The decay can be
enhanced or suppressed. As seen from the current al-
lowed (green) range in Fig. 9 the variation is not huge
around SM expectation. But measured precisely, any de-
viation\cite{20} of $B(B \to \mu\nu)/B(B \to \tau\nu)$ from 0.0045 would
not only be SM, but beyond\cite{76} 2HDM II.
The four modes of $B_s, d \to \mu^+\mu^-$ and $B^- \to \mu^-\nu$, $\tau^-\nu$
have good chance to become legacy modes for
LHCb/CMS and Belle II, while $\mu\nu F$ and $\tau\nu$ pursuits
should also be watched. Not to be forgotten are $K^0$, $B_q$
and $B_s^0$ (and also $D^0$) meson mixings, as well as $b \to s\gamma$.
For $B_s$ mixing, the CPV phase would become a probe of
CPV in G2HDM, while $b \to s\gamma$ has quite a few param-
eters entering, making phenomenological analysis chal-
 lenging. We clearly see a new flavor era emerging.

\textbf{Anecdote: Muon $g−2$ as Harbinger.}— The new
FNAL muon $g−2$ result\cite{103} shows that the experi-
mental side is now confirmed, and would unfold further as
data accumulates. Although a new lattice study\cite{114}
casts some doubt on the “theory consensus” SM predic-
tion\cite{117}, hence on the 4.2$\sigma$ significance\cite{103}, one should
take the discrepancy seriously at present.

There is a known one-loop mechanism\cite{118,119} to
account for muon $g−2$ in 2HDM III, i.e. G2HDM. With
either $H$ or $A$ at 300 GeV in mass, we employ\cite{120} large
LFV Weinberg coupling $\rho_{\mu\nu} \sim 0.2$ ($\sim 2\alpha$) to
account for the muon $g−2$ anomaly. But one needs to
be close to the alignment limit of very small $h−H$ mixing
to evade $h \to \tau\tau$ bound, Eq. (23). This motivates the
check with $gg \to H$, $A \to \tau\mu$ search\cite{80}. We find\cite{120}
that a recent result from CMS\cite{121}, using 36 fb$^{-1}$ data at

\footnotesize
13 The current bound\cite{48} from BaBar has just been improved in-
crementally by Belle\cite{108} with full dataset, which means Belle II
would continue to push the bound as data accumulates.

14 A fresh result on this mode just came out from Belle\cite{113}, show-
ing that Belle II can compete with LHCb in this mode.
13 TeV, already puts more stringent bound on the extra top Weinberg coupling $\rho_{tt}$ than from $\tau \to \mu\gamma$ through the two-loop mechanism, which is quite interesting.

The stringent constraint on $\rho_{tt}$ can be eased by allowing the second extra top Weinberg coupling, $\rho_{tc}$, to dilute the $H/A \to \tau\mu$ decay branching ratio, as the mass is below $t\bar{t}$ threshold. As $H^+$ is taken as heavier, this then motivates \cite{120} the search for the novel signature of $c\bar{g} \to bH^+ \to \tau\mu bW^+$, as well as $c\bar{g} \to bH^+ \to t\bar{c}bW^+$ (same-sign dilepton plus two $b$-jets and additional jet, with missing $p_T$) at the LHC, on top of possible same-sign top plus jet signatures from $c\bar{g} \to tH/A \to t\bar{c}c$.

The one-loop mechanism for solving muon $g-2$ with large $\rho_{\tau\mu}$ has strong repercussions on flavor physics, in particular muon-related physics, and may harbinger \cite{122} a new $\mu/\tau$ era. As discussed earlier, a similar one-loop diagram for $\mu \to e\gamma$ with $\tau$ in the loop, the enhanced $\rho_{\tau\mu}$ could allow \cite{122} even $\rho_{ee} \sim \lambda_e$ to bring the rate right into the sensitivity of MEG II! It also further elevates $\mu N \to e N$, making it a potentially superb probe of $\rho_{qq}$. By same token, $\tau \to \mu\gamma$ can probe $\rho_{\tau\tau}$ to below $\lambda_\tau$ strength, and $\tau \to 3\mu$ can probe $\rho_{\tau\mu}$ to $O(\lambda_\mu)$.

Thus, if large $\rho_{\tau\mu}$ is behind muon $g-2$ anomaly, there can be strong implications for collider and flavor physics. For example, we might soon see indications for $gg \to H, A \to \tau\mu$ at the LHC with full Run 2 data, or MEG II could discover $\mu \to e\gamma$ soon. Although $\rho_{\tau\mu} \simeq 20\lambda_\tau$ grossly violates our conservative Eq. (19), is it up to Nature’s choice, and maybe that is what she’s been trying to tell us through the muon $g-2$ anomaly for two decades. But we think that Nature cannot whimsically move $\rho_{ij}$ elements around at random, for our many sensitive flavor probes would have picked them up.

V. CONCLUSION: DECADAL MISSION

The discovery of the Higgs boson $h(125)$ is certainly the landmark event at the Large Hadron Collider. But, after a dozen years running, a second “discovery” might be called No New Physics (NNP).

LHC certainly has made tremendous progress, in detailed measurements and in pushing down on many search bounds. But with NNP in sight, HEP seems rudderless. Putting it differently, there may now be too many little rudders and engines. For instance, with the Weakly Interacting Massive Particle paradigm strongly cornered, the bandwidth for Dark Matter (DM) search has turned practically infinite, spanning from infinitesimal “wave” DM to massive black holes. Thus, there are many out-of-the-box ideas and searches for Long-Lived Particles (LLPs), and one strong candidate, the Axion, even spins off Axion-Like Particle (ALP) searches. But given the “infinite” bandwidth, like shooting in the dark, any given approach hitting target has very low likelihood. The spreading-thin of manpower is not in the “targeted” tradition of HEP.

Another trend is to accept that there are no new particles below some scale $\Lambda$, then make an expansion in $1/\Lambda$. This effective field theory (EFT) approach is not unreasonable, and has gained in popularity by noting NNP. But, have we exhausted all possibilities of dimension-4 (Lagrangian) operators?

In this Invited Review, we have outlined what is in our mind “a most-likely New Physics in plain sight”, but most in HEP See Not: a 2nd Higgs doublet — with two sets of dimension-4/Lagrangian couplings — i.e. Extra Weinberg couplings $\rho'_{ij}$ ($f = u,d,\ell$), and Higgs quartic couplings $\eta_i$ ($i = 1–7$). Impact:

- **Heavens:** $\eta_8$ at $O(1)$ drive first order EWPT; $\rho_{tt}$ (or $\rho_{tc}$) drives Baryogenesis;
- **Earth:** $\rho_{ee}$ helps evade ACME’18 $d_e$ bound \cite{19}
- **Sub-TeV** $H, A, H^+$, effects hidden so far by: Flavor structure (mass-mixing hierarchies); & emergent Alignment (small $h-H$ mixing).
- **Proposed:**
  - Direct Search Modes at LHC;
  - & Myriad Indirect Flavor Probes.
- **Bonus:** Landau pole at $10–20$ TeV $\Rightarrow$ New Scale.

In summary, we have before us the

**Decadal Mission**

Find the $H, A, H^+$ bosons and crack the Flavor code!

The Mission would take several decades to unfold.

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