Non-perturbative effect in the nucleon structure function and the Gottfried sum

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Abstract

We investigate the non-perturbative effect in the nucleon structure function and the Gottfried sum by using a non-perturbative quark propagator with lowest dimensional condensate contributions from the QCD vacuum. It is shown that the non-perturbative effect modifies the conventional quark-parton model formula of the nucleon structure function at finite $Q^2$ and suggests a non-trivial $Q^2$ dependence in the Gottfried sum.

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Within the framework of the quark parton model (QPM) [1], deep inelastic lepton-nucleon scattering is viewed simply as the sum of elastic scatterings of the lepton on “free” quarks in the nucleon. In the derivation of the inelastic structure function $F_{1,2}$, the incoherence assumption is made and the initial and final state interactions are neglected. The advantage of the simple QPM lies in the clear physical picture of the process and the simple form of the nucleon structure function which could be expressed as the sum of parton distributions weighted by the square of charge of corresponding partons (quarks plus antiquarks) for electron and muon probes. The QPM with QCD correction can give good descriptions of the large momentum transfer deep inelastic process. But recently, some experimental results are found to be beyond the predictions of the QPM. One of them is the violation of the Gottfried sum rule from the New Muon Collaboration (NMC) measurement of the ratio of the cross sections for unpolarized deep-inelastic scattering from deuterium and hydrogen targets [2]. The ratio of structure functions $F_d^2(x,Q^2)/F_p^2(x,Q^2)$ is extracted in the range $0.004 < x < 0.8$ at $Q^2 = 4.0 \text{ GeV}^2$. Assuming a smooth extrapolation of the data for $F_d^2/F_p^2$ from $x = 0.8$ to $x = 1$, adopting a Regge behavior $ax^b$ for $F_p^2 - F_n^2$ (a flavor nonsinglet quantity) in the region $x = 0.004 - 0.15$ and then extrapolating it to $x = 0$, the NMC reported the Gottfried sum, defined as

$$S_G = \int_0^1 \frac{dx}{x} [F_p^2(x) - F_n^2(x)],$$

(1)
with the result $S_G = 0.235 \pm 0.026$. This result is in striking contradiction with the Gottfried sum rule (GSR), which would have $S_G = \frac{1}{3}$ in the QPM \[3\].

Several different effects have been proposed as sources for this discrepancy between the NMC data and the GSR. They can be mainly referred to possibilities as follows: (1) a flavor distribution asymmetry in the sea of nucleons \[4, 5\], i.e., an excess of $d\bar{d}$ over $u\bar{u}$ pairs in the proton due to the pauli exclusive principle and the excess of a valence quark in the proton; (2) the unjustified $x \to 0$ extrapolation of the available data \[3\]; (3) the nuclear effects like mesonic exchanges in the deuteron \[7\] and nuclear bindings \[8\]; (4) isospin symmetry breaking between the proton and the neutrons \[9\].

We investigate in this letter the non-perturbative effect in the nucleon structure function and the GSR. We first study the $< \bar{q}q >$-corrected quark propagator by taking into account the lowest dimensional condensate contributions from the QCD vacuum. Using the obtained non-perturbative quark propagator, we describe the non-perturbative effect in the nucleon structure function based on the QPM and then discuss the consequence in the GSR. A measurable non-trivial $Q^2$ dependence in the Gottfried sum is suggested.

Let us start by writing the free quark propagator

$$i[S_F(x - y)]^{ab}_{\alpha\beta} = \langle 0 | T \bar{q}^a_\alpha (x) q^b_\beta (y) | 0 \rangle,$$  \hspace{1cm}(2)$$

where $a, b$ are color indices and $\alpha, \beta$ are spinor indices. As a simple model, we
consider, as shown in Fig. 1, the lowest-order Feynman diagrams generating the dimension-3 quark condensate and dimension-4 gluon condensate contributions from the QCD vacuum to the quark propagator. The free quark propagator without any modification of condensation can be expressed in momentum space as

\[ S^{-1}_F (p) = \not{p} - m_c \]  

with the perturbative (current-) quark mass \( m_c \) which can be neglected in large momentum transfer process. But in medium energy region, the quark propagator should be modified by taking into account non-perturbative effects [10, 11, 12]. In this letter we consider the non-perturbative effect from the Feynman diagrams as shown in Fig. 1 to the quark propagator. To derive the effect of dimension-3 quark condensate contribution to the quark propagator, we use the nonperturbative vacuum expectation value (VEV) of two quark fields

\[ <0 | \bar{q}^a(x) q^b(y) | 0 >_{NP} = \frac{1}{12} \delta_{ab} (1 + \frac{i m \gamma_{\mu} (x-y)^{\mu}}{4})_{\alpha \beta} < \bar{q} q > + \cdots. \]  

For the dimension-4 gluon condensate contribution to the quark propagator, we take the nonperturbative VEV of two gluon fields

\[ <0 | A^a_\mu(x) A^b_\nu(y) | 0 >_{NP} = \frac{1}{4} x^{\lambda} y^{\rho} < 0 | G^a_{\lambda \mu} G^b_{\rho \nu} | 0 > + \cdots \]  

in the fixed point gauge \( x^{\mu} A_\mu(x) = 0 \), where

\[ <GG> = <0 | G^a_{\lambda \mu}(0) G^a_{\lambda \mu}(0) | 0 >. \]
Under the chain approximation, one can obtain the complete quark propagator in momentum space \[12\]

\[
S^{-1}_F(p) = \not{p}[1 + \frac{g^2<\bar{q}q>(1-\xi)m}{9p^2} + \frac{g^2<GG>m^2_{\xi}}{12(p^2-m^2_{\xi})^3}] \\
- [m_c + \frac{g^2<\bar{q}q>(4-\xi)}{9p^2} + \frac{g^2<GG>m_p^2}{12(p^2-m^2_{\xi})^3}],
\]

where \(m\) in \(S^{-1}_F(p)\) arises from incorporating the QCD equation of motion [12]

\[
(i\not{D} - m)\psi = 0. \quad (8)
\]

It is necessary to emphasize that \(m\), which includes the effect of the condensates of non-perturbative QCD, is different from the purely perturbative (current-) quark mass \(m_c\). Note also that \(S^{-1}_F(p)\) is gauge parameter \(\xi\) dependent due to the internal gluon line appearing in Fig. 1(b). In common sense, the current quark mass \(m_c\) is small, and it can be neglected in large momentum transfer process, which is equivalent to neglecting the gluon condensate term in \(S^{-1}_F(p)\). Therefore \(S^{-1}_F(p)\) can be rewritten as

\[
S^{-1}_F(p) = \not{p} - M(p) \quad (9)
\]

with

\[
M(p) = \frac{g^2}{9p^2} <\bar{q}q> [(4 - \xi) - \frac{(1 - \xi)\not{p}m}{p^2}] \quad (10)
\]

We require the pole of the \(<\bar{q}q>\)-corrected propagator corresponding with \(m\) in equation of motion (8), i.e.,

\[
M(p)|_{\not{p}=m} = \frac{g^2}{3m^2} <\bar{q}q> = m. \quad (11)
\]
From this equation, one can obtain the solution of \( m \) which is independent of gauge parameter \( \xi \)

\[
m = M(p)|_{p=m} = \left( \frac{4\pi \alpha_s(Q^2) \langle \bar{q}q \rangle}{3} \right)^{1/3}.
\]  

(12)

Thus the \( \langle \bar{q}q \rangle \)-corrected quark propagator can be written as

\[
S_F^{-1}(p) = \hat{p} - \left( \frac{4\pi \alpha_s(Q^2) \langle \bar{q}q \rangle}{3} \right)^{1/3},
\]  

(13)

where the strong coupling constant \( \alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln \Lambda^2 / Q^2} \) with \( \beta_0 = 11 - \frac{2}{3} N_F \)

and the dimensional parameter \( \Lambda = 0.250 \text{ GeV} \) for \( N_F = 3 \).

We now discuss the effect of the \( \langle \bar{q}q \rangle \) condensate in the nucleon structure function by means of the non-perturbative quark propagator Eq. (13).

Consider the inclusive lepton-nucleon scattering

\[
l + N \rightarrow l + X
\]  

(14)

where the hadronic structure is entirely contained in the tensor \( W_{\mu\nu} \)

\[
W_{\mu\nu} = (2\pi)^3 \sum_X \langle P|J_\mu|X\rangle \langle X|J_\nu|P\rangle \delta^4(P_X - P - q)
\]

\[
= (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})W_1 + \frac{1}{M^2}(P_\mu - \frac{P_\nu q_\mu}{q^2}q_\nu)(P_\nu - \frac{P_\mu q_\nu}{q^2}q_\mu)W_2;
\]  

(15)

here \( M \) is the mass of the nucleon. If \( W_{\mu\nu} \) is given, one can extract \( W_1 \) and \( W_2 \) through the following formulas:

\[
W_1 = \frac{1}{2}[c_2 - (1 - \frac{\nu^2}{q^2})c_1](1 - \frac{\nu^2}{q^2})^{-1};
\]  

(16)

\[
W_2 = \frac{1}{2}[3c_2 - (1 - \frac{\nu^2}{q^2})c_1](1 - \frac{\nu^2}{q^2})^{-2};
\]  

(17)
with \( c_1 \equiv W_\mu^\mu \) and \( c_2 \equiv \frac{P_\mu P_\nu}{2M^2} W_{\mu\nu} \) \cite{[13]}. All non-perturbative effects are entirely contained in \( W_{\mu\nu} \). In this letter, we try to study the non-perturbative effect in the nucleon structure function from the quarks in the QCD physical vacuum. We suppose that the proton is made up of bound partons that appear as “free” Dirac particles but with the non-perturbative propagator because of being in the QCD vacuum. With the incoherence assumption, one parton contribution to \( W_{\mu\nu} \) is

\[
\begin{align*}
\begin{array}{l}
w_{\mu\nu} = (2\pi)^3 \frac{1}{2} \sum_{s,s'} \sum_{p'} < \vec{p}, s | J_{\mu} | \vec{p}', s' > < \vec{p}', s' | J_{\nu} | \vec{p}, s > \delta^4(p' - p - q) \\
= e_i^2 \int \frac{d^3p'}{2p_0'} \delta^4(p' - p - q) \frac{1}{2} \text{Tr} [\gamma_{\mu}(p' + m)\gamma_{\nu}(p + m)].
\end{array}
\end{align*}
\] (18)

According to the trace theorem that the trace of an odd number of \( \gamma \)'s vanishes, \( w_{\mu\nu} \) can also be equivalently expressed as

\[
\begin{align*}
\begin{array}{l}
w_{\mu\nu} = e_i^2 \int \frac{d^3p'}{2p_0'} \delta^4(p' - p - q) \frac{1}{2} \text{Tr} [\gamma_{\mu}(p' - m)\gamma_{\nu}(p - m)];
\end{array}
\end{align*}
\] (19)

i.e.,

\[
\begin{align*}
\begin{array}{l}
w_{\mu\nu} = e_i^2 \int \frac{d^3p'}{2p_0'} \delta^4(p' - p - q) \frac{1}{2} \text{Tr} [\gamma_{\mu}S_{\nu}^{-1}(p')\gamma_{\nu}S_{\mu}^{-1}(p)],
\end{array}
\end{align*}
\] (20)

where \( iS_{\nu}(p) \) is the quark propagator and \( e_i \) is the charge of quark in unit of \( e \). Generally, one should take the complete non-perturbative quark propagator including the correction due to \( < \bar{q}q >, < GG > \) and higher dimensional condensate. However, as a simple qualitative analysis, we take only the \( < \bar{q}q > \)-corrected quark propagator given by Eq. \([13]\). For the sake of simplicity, we adopt the parton picture in which the parton 4-momentum is
expressed as $p^\mu = yP^\mu$ ($0 \leq y \leq 1$) with the nucleon 4-momentum $P^\mu$; i.e., we assume that all transverse momenta are negligible and that no parton moves oppositely to the nucleon. Using Eqs. (16) and (17), we can extract one quark contribution of type $i$ quark to $W_2$, and the corresponding contribution to the nucleon structure function $F_2^{(i)} = \nu w_2$ is

$$F_2^{(i)}(y) = 2Mx^2e_i^2\delta(y - x)R_{NP}(Q^2), \quad (21)$$

with

$$x = \frac{Q^2}{2M\nu}, \quad (22)$$

and

$$R_{NP}(Q^2) = 1 - \frac{4}{Q^2}(\frac{\alpha_s}{3} <\bar{q}q>)^{2/3}. \quad (23)$$

Suppose that the nucleon state contains $f_i(y)dy$ parton states of the type $i$ in the interval $dy$, then

$$F_2 = \sum_i \int_0^1 dy f_i(y)F_2^{(i)}. \quad (24)$$

We adopt the convention of Ref. [13] in which a parton state has $2p_0$ partons per unit volume, while a nucleon state has $P_0/M$ nucleons per unit volume. Therefore, in one nucleon, the number of partons of type $i$, in the interval $dy$ is $f_i(y)$ multiplied by $\frac{2p_0}{(P_0/M)} = 2My$, i.e., $q_i(y)dy = 2Myf_i(y)dy$, where $q_i(y)$ is the quark parton distribution with constraint of parton flavor number conservation. Summing all contributions of all quarks in the nucleon, we
obtain the structure function of the nucleon

\[ F_2(x) = \sum_i q_i(x) x e_i^2 R_{NP}(Q^2) = \sum_i \tilde{q}_i(x) x e_i^2, \tag{25} \]

where

\[ \tilde{q}_i(x) = q_i(x) R_{NP}(Q^2) \tag{26} \]

which is different from \( q_i(x) \) since \( q_i(x) \) represents the probability distribution of quarks of type \( i \) and satisfies the parton flavor number sum rule, but \( \tilde{q}_i(x) \) does not. From Eq. (25), we see that the structure function, with a reducing factor of \( R_{NP}(Q^2) \), is no longer simply the sum of the parton distributions multiplied by the square charge of corresponding partons (quarks plus antiquarks) in the nucleon at finite \( Q^2 \). In fact, there are some results in recent experimental data which show explicit violations of conventional parton sum rules. In this letter we take the violation of GSR as an example, although the non-perturbative effect should also have consequences in other parton sums. To show explicitly the non-perturbative effect in the structure function, we give the values of \( R_{NP}(Q^2) \) for corresponding \( Q^2 \) in Tab. 1. In the estimate of \( R_{NP}(Q^2) \), we have taken the standard phenomenological value of the quark condensate \( \langle \bar{q}q \rangle = -(0.25\text{GeV})^3 \) obtained from QCD sum rule [10].

The available observed Gottfried sum is \( S_G = 0.235 \) at \( Q^2 = 4 \text{ GeV}^2 \) by the NMC experiment and we need \( R_{NP} = 0.705 \) to explain the data if there is no other sources for the violation of the GSR. Our calculated
value is $R_{NP} = 0.922$ at $Q^2 = 4$ GeV$^2$ and is too small to explain the data. However, we see from Tab. 1 that there is non-trivial $Q^2$ dependence in the calculated $R_{NP}(Q^2)$. Thus we would expect a measurable $Q^2$ dependence in the Gottfried sum. From the observation of $Q^2$ dependences in several other parton sums, we know that this expectation is not unreasonable and the magnitude of the $Q^2$ dependence is of the order $0.06 \text{GeV}^2/Q^2$ which is consistent with the $Q^2$ dependence as observed in experiments and as estimated from the perturbative QCD (pQCD) and higher twist effects in other parton sums [14]. The pQCD corrections to flavor number conservation are expected to be small and to have little consequence on the GSR [15]. The effect considered in this letter is from the non-perturbative QCD vacuum and thus should include both of some pQCD and higher twist effects.

In summary, we investigate the non-perturbative effect in the nucleon structure function and the Gottfried sum by taking into account the lowest dimensional condensate contributions from the QCD vacuum in the quark propagator. We find a non-trivial modification of the conventional quark parton model formula of the nucleon structure function at finite $Q^2$ and suggest a measurable $Q^2$ dependence in the Gottfried sum.
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Table 1. $R_{NP}(Q^2)$ for corresponding $Q^2$

| $Q^2$ | $R_{NP}(Q^2)$ |
|-------|---------------|
| 2.0   | 0.823         |
| 4.0   | 0.922         |
| 10    | 0.973         |
| 20    | 0.987         |
| 100   | 0.998         |
Figure Captions

Fig. 1. The non-perturbative quark propagator including:

(a). The perturbative free quark propagator;

(b). Lowest-order correction due to the nonvanishing value of $<\bar{q}q>$;

(c). Lowest-order correction due to the nonvanishing value of $<GG>$. 
Fig. 1