A note on Gribov copies in 3D Chern-Simons theory

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Abstract

Using powerful tools of harmonic maps and integrable systems, all the Gribov copies in the Coulomb gauge in 3D Chern-Simons theory are constructed. Some issues about the Gribov and the modular regions are shortly discussed. The Gribov copies of the vacuum in 3D QCD in the Coulomb gauge are described. An interesting implication of the presence of Gribov copies is briefly pointed out.

Keywords: Chern-Simons theory, Gribov copies.
PACS: 11.15.-q; 11.10.Kk; 11.15.Bt; 02.30.Ik.
Preprint: CECS-PHY-08/02

1 Introduction

The Gribov ambiguity [1] is one of the most deep non perturbative phenomena in gauge theories: it is a global obstruction in achieving a global gauge fixing in linear derivative gauges (like Lorentz, Coulomb, Landau and so on) related to the non trivial topology of the space of non-Abelian gauge connections. Furthermore, as Gribov himself pointed out, it seems that such an ambiguity is closely related to the confinement in QCD once the path integral

\footnote{An interesting ambiguity, which in a sense is dual to the Gribov's one, was discovered in \cite{2}: the authors showed that there may exist gauge potentials which are not gauge equivalent but which generate the same curvature.}
is restricted to a ambiguity-free region (see, [3] and references therein; for two
detailed reviews see [4] [5]). Recently, it appeared an interesting paper [6]
in which an alternative argument has been presented which suggests a close
relation between Gribov ambiguity and confinement [2]. Such an argument is
intriguing in that it seems not to depend heavily on the explicit form of the
action so that it could be applied to other contexts.

This would provide some recent results in gravity (see, for instance, [7]
[8], [9], [10], [11] and references therein) with an interesting physical interpre-
tation. In such works, following an idea first pointed out by S. Weinberg in
[12] (see, for a related work on the same line, [13]), the authors found
strong evidence of an UV fixed point for gravity which could overcome the
perturbative non-renormalizability of gravity [14]. Such an UV fixed point
could correspond to a confinement phase transition in which only scalar fields
survive in the physical spectrum improving the UV behavior of gravity as
first pointed out in [15] [16] (see also [17] [18]).

However, many point about the relations between Gribov ambiguity and
confinement have to be clarified. In particular, the Gribov ambiguity is also
present in field theories with a vanishing beta function such as N=4 SUSY
Yang-Mills in four dimensions and Chern-Simons theory in three dimensions
(such a theory was introduced in the physical literature in the seminal paper
[19]). It is therefore interesting to analyze the issue of Gribov copies in
Chern-Simons case (which is simpler than the case of N=4 SUSY Yang-Mills
but highly non trivial) to see how such an ambiguity manifests itself in this
simpler case. Furthermore, there is no common agreement in the literature
about whether or not one has to restrict the path integral to an ambiguity
free region. In particular, in [20] it has been constructed a solvable model
(whose BRS analysis has been provided in [21]) in which one has to sum over
all the copies instead of restricting to an ambiguity free region.

The analysis of Gribov ambiguity in 3D Chern-Simons theory can help to
shed light on the above interesting questions in a context which is far simpler
than 4D QCD.

The paper is organized as follows: in the second section the Gribov ambi-
guity in the Chern-Simons case is shortly analyzed. In the third section, the
Gribov equation and its solutions are presented. In the fourth some possible

2At a first glance, one could simply use other gauge fixings free of ambiguities in order
to disproof such a relation but in four dimensions very often such gauge fixings have their
own problems (see [5] and references therein).
implications of the presence of Gribov copies in Chern-Simons theory are pointed out. Finally, conclusions and perspectives are presented.

2 The Gribov ambiguity in Chern-Simons

The Gribov problem was discovered studying the Faddev-Popov procedure for quantizing Yang-Mills theory using path integral methods [1]. To carry on the Faddev-Popov procedure it is necessary to choose a gauge fixing. Usually, for practical computations, the more convenient gauge fixings are the Coulomb, the Landau, the Lorentz and so on. However, such gauge fixings do not fix the gauge in the non-Abelian case: there are gauge equivalent connections fulfilling the same (Coulomb, the Landau, the Lorentz and so on) gauge conditions. The Gribov ambiguity is not a special feature of path integral quantization: it also appears in the canonical formalism when applying the Dirac procedure (see, for instance, [22]). In gauge systems with first class constraints one needs to introduce suitable gauge fixing functions in such a way that these gauge fixing functions together with the first class constraints form a second class system of constraints to be analyzed with the Dirac method. In the presence of Gribov ambiguities such a procedure only works locally.

Let us see in detail the 3D Chern-Simons case: the action is

$$S = k_{\text{bare}} S_0 + S_{gf}(c, \overline{c}, A, \lambda), \quad S_0 = \frac{1}{4\pi} \int_M tr \left( AdA + \frac{2}{3} A^3 \right)$$

where the connection $A$ takes values in the algebra of $SU(N)$, the bare coupling constant $k_{\text{bare}}$ is an integer, the trace is in the fundamental representation, $M$ is a smooth three dimensional manifold. $S_{gf}$ represents the gauge fixing and the ghosts terms$^3$ which break the diffeomorphisms invariance of the first term due to the introduction of a background metric (which will be assumed to be flat and of Lorentzian signature). Here the Coulomb gauge (which allows to use powerful results in the theory of harmonic maps and integrable systems) will be considered

$$\chi = \partial_i A^i = 0$$

$^3$$\lambda$ is the Lagrange multiplier enforcing the gauge fixing, $c$ and $\overline{c}$ are the ghost and the anti-ghost and in the non-Abelian case the Faddev-Popov determinant also depends on $A$. 

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in which the splitting between time \((0 \sim t)\) and space \((i \sim x, y)\) indices will be introduced in a moment\(^4\). To the best of author’s knowledge, the first application of harmonic map theory to find Gribov copies in the four dimensional \(SU(2)\) Yang-Mills case can be found in [24]. The Singer theorem [25] tells that if Gribov ambiguities are present in the Coulomb gauge they are present in all the derivative gauges. Let us consider the case in which \(M = \Sigma \times R\) where \(R\) will be interpreted as the time direction so that one can decompose (using the notation of [26]) \(d\) and \(A\) as follows

\[
d = dt \otimes \frac{\partial}{\partial t} + \vec{d}, \quad A = A_0 + \vec{A}, \quad \vec{F} = \vec{d} \vec{A} + \vec{A}^2.
\]

\(S_0\) now reads

\[
S_0 = -\frac{1}{4\pi} \int_M tr \left( \vec{A} \frac{\partial}{\partial t} \vec{A} \right) + \frac{1}{2\pi} \int_M tr \left[ A_0 \vec{F} \right]
\]

in which \(A_0\) is a Lagrange multiplier enforcing \(\vec{F} = 0\) so that \(\vec{A}\) is locally flat. In the path integral formalism this implies the presence of a delta of \(\vec{F}\):

\[
\delta \left( \vec{F} \right)
\]

One can formally solve such a constraint as follows

\[
\vec{A} = U^{-1} \vec{d} U
\]

(where \(U\) is a single-valued map from \(\Sigma \times R\) to \(SU(N)\), we will be more precise on the choice of the space \(\Sigma\) in a moment) so that the Coulomb gauge condition (2) becomes

\[
\partial_i \left( U^{-1} \partial^j U \right) = 0.
\]

### 3 Gribov equation

Can one find different solutions of Eq. (3) giving rise to gauge equivalent gauge potentials? To answer to this question in a clear way it is useful to consider the case in which the space \(\Sigma\) is a two sphere

\[
\Sigma = S^2
\]

\(^4\)An interesting discussion of the possible phenomenological relevance of Gribov copies in three dimensional gauge theories can be found in [23].
since in this case the moduli space of flat connections is trivial. If Eq. (3) would only have the trivial solution $U = U_0$ a constant matrix in the group then there would be no Gribov copies. On the other hand, non-trivial Gribov copies correspond to different solutions of Eq. (3) giving rise to gauge equivalent potentials fulfilling the following boundary conditions:

$$\int_{\Sigma} tr (A_i)^2 d^2 \Sigma \approx \int_{\Sigma} tr \left( U^{-1} \partial_i U \right)^2 d^2 \Sigma < \infty. \quad (4)$$

Some beautiful results in the theory of harmonic maps (see [27]) and integrable systems (see for instance [28], [29]) provide one with the complete classification of the solutions of Eq. (3) fulfilling the boundary conditions (4) for a generic $SU(N)$. Thus, remarkably enough, one can construct all the Gribov copies in the Coulomb gauge in 3D Chern-Simons theory (in the case in which $\Sigma$ is a two sphere). In [27] it has been shown that all the solutions of Eq. (3) fulfilling the condition in Eq. (4) are of the form

$$U = U_0 \prod_{i=1}^{u} (1 - 2R_i), \quad R_i^2 = R_i \quad (5)$$

where the positive integer $u$ is the so-called unitons number, $U_0$ is a constant matrix in the group and $R_i$ are projectors which satisfy some first order differential equations. The integer $u$ is smaller than $N$: starting, for instance, with the trivial solution $U_0$ one can ”dress it” (see [28], [29] and references therein) with the factors $1 - 2R_i$ getting new solutions but this can only be done a finite number of times denoted by $u$. Another remarkable feature of the Gribov copies in the Coulomb gauge in 3D Chern-Simons case is that they are labelled by the discrete index.

This is quite different from the QCD case in four dimensions in which the Gribov equation admits continuous families of solutions: the Coulomb gauge condition in 3D Chern-Simons gauge theory together with the physical boundary condition (4) reduce to the integrable equation of the harmonic maps. QCD in three dimensions also has continuous families of Gribov copies due to the fact that, unlike the Chern-Simons case, the space of classical

\[5\]

\[6\]

\[5\] To the best of the authors knowledge there is no theorem telling that $u = N - 1$ but in many cases this is actually so.

\[6\] This is quite different from the standard dressing techniques in solitons theory and integrable systems which allow to construct solutions with an arbitrary large number of elementary solitons.
solutions is not made of pure gauge fields. However, the copies of the vacuum in 3D QCD on $S^2 \times R$ in the Coulomb gauge have exactly the same structure as the Gribov copies in 3D Chern-Simons on $S^2 \times R$ since the Gribov equation for the copies of the vacuum together with the physical boundary conditions reduce to Eqs. (3) and (4). Thus, harmonic map theory allows to construct all the Gribov copies of the vacuum in 3D QCD on $S^2 \times R$ in the Coulomb gauge which are also labelled by the unitons number. This is a peculiar property of Yang-Mills theory in 3 dimensions: Gribov copies of the vacuum can be avoided in 4D QCD by asking suitable boundary conditions (see for instance [4]). The complex structures present in a two dimensional space ($S^2$ in the present case) allow the construction of genuine Gribov copies of the vacuum in the Coulomb gauge in 3D QCD on $S^2 \times R$.

Thus, in the $SU(2)$ case the more general solutions are 1-uniton solutions. In the $SU(2)$ case the most general Gribov copy is

$$\begin{align*}
\overrightarrow{A} &= U^{-1} \overrightarrow{d} U \\
U &= U_0 (1 - 2R) \\
(1 - R) \partial_+ R &= 0, \quad R^2 = R
\end{align*}$$

where

$$\partial_+ = \frac{1}{2} (\partial_x + i \partial_y)$$

and $R$ is a holomorphic projector.

### 3.1 Modular vs Gribov regions in 3D Chern-Simons

An important issue in the analysis of Gribov ambiguity is the relation between the modular and the Gribov region (see [30], [31], [32], [33]). In the present case, the Gribov region $\Omega$ is defined as the region of local minima of the functional $S_g[U]$ of $U$ which is defined on the gauge orbit of $A_i$

$$S_{A_i}[U] = \int_{S^2} dS^2 tr \left(A_i^U\right)^2$$

where $dS^2$ is the volume element on the sphere and $A_i^U$ is the gauge transformed of $A_i$ with the gauge transformation generated by $U$ (which, in general, may also be homotopically non-trivial). At the Gribov horizon $\partial \Omega$ the lowest eigenvalue of the Faddev-Popov operator vanishes. The modular region $\Lambda$ is defined as the global minima of $S_g[U]$. It can be shown that at
the global minima of the above action, the vector potential is transverse and
the Faddev-Popov operator is positive. The restriction to the interior of the
Gribov region $\Omega$ is not enough to avoid copies so that, if one wants to restrict
the path integral to an ambiguity free region, then it is necessary to consider
the interior of the modular region $\Lambda$ (since, unless suitable identifications are
performed, copies may appear on the boundary $\partial \Lambda$). It may also happen
that portions of the boundaries of the modular region $\partial \Lambda$ and of the Gribov
region $\partial \Omega$ coincide (see [30], [31], [32], [33]). At a practical level, it is much
more difficult to work with the restriction to the modular region than it is
with the restriction to the Gribov region (a nice review on the implementa-
tion of such a restriction can be found in [34]). Being the moduli space of
flat connection of $S^2$ trivial, the above action is proportional to the positive
definite action which defines the harmonic maps

$$ S[U] = \int_{S^2} dS^2 \text{tr} \left( U^{-1} \partial_i U \right)^2 $$

(6)

It is quite obvious that the global minima are the constant solutions $U = U_0$
which give rise to a vanishing gauge potential so that the interior of the
modular region in this case reduces to the trivial gauge potential. Thus, one
can answer the question of whether or not in this case $\Lambda$ and $\Omega$ coincide by
investigating if the non-trivial unitons solutions are local minima or simply
saddle points of the action (6). In the $SU(2)$ case the above action is equiv-
alent to the two dimensional $\mathbb{C}P^1$ model (the generic $\mathbb{C}P^{n-1}$ models were
introduced in the physical literature in [35] [36], see also [37]) and the more
general 1-uniton solutions are nothing but instanton solutions of the $\mathbb{C}P^1$
model (to the best of author’s knowledge, instantons in the generic $\mathbb{C}P^{n-1}$
models were firstly constructed in [38]). If one neglects the zero modes which
correspond to translations of the position of the instanton and to constant
rescalings of its size, well known topological arguments would suggest that
instanton solutions are local minima.

This is a crucial point, to define functional determinants in quantum
field theory it is a standard procedure to ”quotient out” trivial zero modes
but in the present case one is asking ”are the instanton solutions of the
two dimensional $\mathbb{C}P^1$ model local minima?” Strictly speaking, the answer
is ”no” because of the flat directions: if one change the position (or the
size) of the instanton the value of the action does not change. Consequently,
the Faddev-Popov determinant evaluated at the instanton solutions vanishes
and they are neither in the interior of $\Lambda$ nor in the interior of $\Omega$. On the
other hand, if one takes the opposite view that instanton solutions which are related by translations of their positions and/or by constant rescalings of their size represent the same solution then one has to factor out the zero modes obtaining a local minimum. In any case, it should be noted that the instanton solutions of the $CP^1$ model are homotopically non-trivial so that the $SU(2)$ Gribov copy of the vacuum $A_i = (U_I)^{-1} \partial_i (U_I)$ (where $U_I$ is the instanton solution of the $CP^1$ model) is obtained by acting on the vacuum with a homotopically non-trivial gauge transformation. Thus, one can argue that the Faddev-Popov determinant vanishes using the elegant argument in [39].

In the generic $SU(N)$ case the situation is less clear: strictly speaking, the unitons number is not well defined since if one adds, for instance, a uniton factor to a two-unitons solution the resulting expression may be equivalent to a 1-uniton solution. For this reason, in [27] the minimal unitons number was defined as the minimal number of unitons that are required to construct a given solution of Eq. (3) fulfilling the boundary conditions (4). In the generic case one may expect that a solution of Eq. (3) is simply a saddle point of the action (6). Solutions representing local minima should be characterized by some notion of ”minimality” or ”indecomposability” otherwise intuitive physical arguments based on the action (6) would suggest the possibility that such a solution may ”decay” into some more fundamental solutions but already in the $SU(3)$ case the computations are quite involved and it is not possible to obtain explicit expressions for the solutions in the general case. The $SU(2)$ example suggests that in the generic $SU(N)$ case also there could be instanton-like solutions but, being homotopically non-trivial, the argument in [39] ensures the vanishing of the Faddev-Popov determinant (unless zero modes are quotiented out): this interesting point deserves further investigation.

4 Chern-Simons shift

In Chern-Simons theory there is a well known debate in the literature: the seminal paper of Witten [40] shed light on the close relations between knot theory, conformal field theory and Chern-Simons theory. It allowed the computations of the Jones polynomials in knot theory using Chern-Simons perturbations theory and some inputs from conformal field theory. Chern-Simons theory is renormalizable by power counting; the beta function and
the anomalous dimensions of the elementary fields are vanishing to all orders in perturbation theory (see [11]). The only free parameter of the model is the renormalized coupling constant $k_{\text{ren}}$ which is fixed by the normalization conditions. Therefore, on the Chern-Simons side, such polynomials turn out to depend on a parameter $q$ which is related to the renormalized Chern-Simons coupling constant $k_{\text{ren}}$:

$$q = \exp \left[ -i \frac{2\pi}{k_{\text{ren}}} \right]$$  

and this conclusion should not depend on the regularization scheme. The debate in the literature is about the relation between $k_{\text{ren}}$ and the bare coupling constant $k_{\text{bare}}$ appearing in the action (1). It has been shown in [26] that the essential point behind the shift

$$k \rightarrow k + N$$  

is that in the evaluation of expectation values one must integrate out the gauge degrees of freedom and in so doing non-trivial Jacobians appear. The source of the shift is the anomaly in these Jacobians\footnote{Indeed, there are regularization schemes which give rise to similar results in covariant gauges (see, for instance, [12]). However such kind of results depend on the regularization scheme and with different regularization schemes one can also obtain shift different from Eq. (8). In any case, it is fair to say that the regularization schemes which appear to be "more natural" always give rise to integer shifts.}. While the essential point stressed, for instance, in [43]\footnote{The point of view followed in [43] can be provided with mathematically sound basis: see [44] and references therein. Similar results have been obtained in [45].} is that in order to agree with the so-called exact skein relations (see for instance [44]) known in knot theory the parameter $q$ has to depend on $k$ as in Eq. (7) so that the shift in Eq. (8) should refer to the bare unphysical coupling constant:

$$k_{\text{bare}} \rightarrow k_{\text{bare}} + N.$$  

The presence of Gribov copies can shed some light on the relations between these two points of view. A first consideration is that in the Abelian 3D Chern-Simons theory (which has no Gribov ambiguity) there is no such a shift. Non-trivial Jacobians (which are the source of the shift [26]) may arise because of the fact that $(A/G) \times G$ is not diffeomorphic\footnote{Note that the decomposition $(A/G) \times G$ should be a global realization of the local Faddev-Popov procedure to fix the gauge in order to take care of the path integral over the gauge degrees of freedom. In the presence of Gribov ambiguity this local procedure fails to extend globally.} to $A$ (where $A$ is the
Chern-Simons configuration space and $G$ is the non-Abelian gauge group): this phenomenon generates the Gribov ambiguity as well \cite{25}. To have a "pictorial" idea of how Gribov copies manifest themselves one can observe that in the path integral

$$Z(\kappa_{\text{bare}}) = \int DA\lambda DC\sigma \exp i (\kappa_{\text{bare}}S_0 (A) + S_{gf} (A, c, \tau)),$$

the parts affected by the Gribov copies are the gauge fixing and ghosts parts while, being $\kappa_{\text{bare}}$ an integer, the factor $\exp i (\kappa_{\text{bare}}S_0 (A))$ is constant on the gauge orbit of $A$ (it is worth to recall here that in the present case there is only the gauge orbit of the vacuum). Due to the "multi-unitons" solutions (see Eq. (5)) of Eq. (3) one can imagine the following formal splitting inside the path integral

$$\int DA\delta (\partial_i (A^i)) \approx \int DA \prod_{n=0}^{\mu} \delta (A - A_n)$$

where the $\delta$ of the gauge fixing condition appears after the integral over $\lambda$.

The above product is over the unitons solutions\footnote{10} $U_n$ of Eq. (3) fulfilling the boundary conditions in Eq. (4)

$$A_n = U_n^{-1}dU_n.$$

The above splitting gives rise to $u + 1$ factors

$$\int DA \prod_{n=0}^{u} \delta (A - A_n) \exp i (\kappa_{\text{bare}}S_0 (A)) \int DC\sigma \exp i (S_{gf} (A, c, \tau)).$$

This suggests the shift of the coupling constant

$$\kappa_{\text{bare}} \rightarrow \kappa_{\text{bare}} + u + 1 = \kappa_{\text{bare}} + N = \kappa_{\text{ren}}$$

since, as it has been argued in \cite{40}, the shift arises from the ghosts and gauge fixing terms while $\exp i (\kappa_{\text{bare}}S_0 (A))$ is constant on the gauge orbit. Of course, the above formal manipulations are far from being conclusive\footnote{11}.
since the issues of regularization are rather subtle in Chern-Simons theory. Nevertheless, they show that the presence of Gribov copies affects the final result and that if one would restrict the measure to an ambiguity-free region the final result would be different. Further obstacles to provide the above arguments with a rigorous basis come from the lacking of a complete understanding of harmonic maps. Firstly, to the best of the author’s knowledge, there is no general theorem stating that the unitons number $u$ is equal to $N - 1$. Furthermore, there are no complete results in the theory of harmonic map in the cases in which the spatial topology is more complicated. It is also not perfectly known the relation between the unitons number and the Casimirs of generic non-Abelian gauge groups. A mathematical refinement of the above argument could clarify the non-perturbative origin of the shift in a regularization independent way. A more "physical" approach to investigate the consequences of the presence of Gribov copies could be the computation of the Chern-Simons path integral with the restriction to the modular region.

5 Conclusions and perspectives

In the present paper, using powerful tools of harmonic maps and integrable systems, all the Gribov copies in the Coulomb gauge in 3D Chern-Simons theory have been constructed. This is the first example of a simple yet non-trivial gauge theory in which all the Gribov copies in the Coulomb gauge can be determined. The same construction also works in the case of all the Gribov copies of the vacuum in 3D QCD in the Coulomb gauge. This result gives rise to the possibility to relate the presence of Gribov copies and the famous shift of the Chern-Simons coupling constant. It would be very interesting to provide this relation with more sound basis. It is would be also interesting to compute the Chern-Simons path integral implementing the restriction to a copies-free region: this analysis could shed new light (in a case which is much simpler than 4D QCD) both on the differences between the Gribov and the modular regions and on whether or not one has to restrict the path integral to an ambiguity free region.
Acknowledgements

I would like to thank A. Anabalon and J. Zanelli: without their questions, interesting discussions and suggestions this work would not have been done. I would also like to thank S. Sorella for encouraging comments and important suggestions and G. Vilasi for discussions and suggestions about integrable systems. This work was supported by Fondecyt grant 3070055. The Centro de Estudios Científicos (CECS) is funded by the Chilean Government through the Millennium Science Initiative and the Centers of Excellence Base Financing Program of Conicyt. CECS is also supported by a group of private companies which at present includes Antofagasta Minerals, Arauco, Empresas CMPC, Indura, Naviera Ultragas and Telefo’nica del Sur.

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