Chapter 1

Newtonian mechanics

1.1 Overview

Question: If a curling stone weighs 20 kg and is traveling at a speed of $0.5 \text{ m s}^{-1}$, with how much force did the curler throw it?

Question: What force is exerted when a 300 lb man falls 3 ft?

Question: I don’t completely understand Newton’s third law of motion. It says for every action, there is an equal and opposite reaction, but when we apply force to a book, why doesn’t the book apply the same force to us? And why are we able to push the book wherever we want, if, according to third law of motion, the book should also have an equal reaction force?

Question: If a bullet was traveling at $823 \text{ m s}^{-1}$ and hit an object that stopped it dead, how much force would be exerted on the target?

Every day I get questions like these, questions which say to me ‘I have a feeling for what force is, a push or a pull, but I have no idea how forces are related to the motion’. In the 18th century Isaac Newton (1642–1727) conceived three simple laws which tell us how we can understand how forces affect the physical world. In order to understand how the world works, not to mention be able to read this book, we must understand these three laws.

1.1.1 Newton’s first law

Imagine a book sitting on a table. It is at rest. There are two forces on it, its own weight (the force of the Earth pulling it down) and the force of the table keeping it from falling to the ground. Newton’s first law simply states that, because the book is at rest, the magnitude of the weight (pointing down) must be equal to the magnitude of the table force (but pointing up), or, to put it more elegantly, the net force is zero. If questioner #3 above were pushing with a force just right so that the book moved with constant speed across the table, there would be two new forces on the book, the
pusher pushing and the table resisting (called friction); but all forces (now four of them) on the book still add up to zero. An object which is at rest or moving with constant speed in a straight line is said to be in equilibrium. Newton’s first law can be expressed as follows. **The net force on an object in equilibrium is zero.** This law may seem obvious today, but it was revolutionary when Newton first stated it. Before Newton it was assumed that the natural state of an object was to be at rest and, in order to keep something moving, there had to be a force pulling or pushing it.

When we get to Newton’s second law, the first law will seem to be an unnecessary special case of the second. But the first law plays a much more important role than that. Suppose we ask the question ‘Is a law of physics always true?’ The answer, perhaps surprising, is no; there are usually conditions under which a law is true. Thinking about the first law, imagine that you are inside an accelerating car. You are at rest inside the automobile (in equilibrium according to the first law) but you feel the seat back pushing forward on you, an unbalanced force. Therefore Newton’s first law is untrue in this automobile. Whenever you find that Newton’s first law is true you are in what is called an **inertial frame of reference**. This is the more important role played by the first law, as a test whether Newton’s laws are true laws for you.

When you have found one inertial frame, you have found them all because any frame which moves with constant velocity relative to another is also an inertial frame. (This is proved in the appendix G.) Any frame which accelerates relative to an inertial frame is not one. Because the Earth rotates and revolves around the Sun, it is accelerating (see appendix D) and not an inertial frame. Fortunately, the accelerations involved are small enough to be negligible for many examples in everyday life. Later in the book, though, there will be examples of physics in non-inertial frames.

Incidentally, inertial frames get their name from an alternative name of the first law, the **law of inertia**. Inertia means unwillingness to change and the first law says that you need to push or pull on something at rest or moving with constant velocity to change its motion.

### 1.1.2 Newton’s second law

In order to discuss the second law, a brief detour to discuss units is imperative. Whole books have been written on this topic and I will be brief, assuming the reader already has a good sense of how we normally measure length, mass and time. I will usually use SI units, mass is a kilogram (kg), length is a meter (m), and time is a second (s). These are all operationally defined in rather complicated ways, but it is sufficient to simply think of a meter stick and a stopwatch for almost everything which will be discussed in this book. 1 kg = 1000 grams and 1 gram is the mass of 1 cm³ of water; or, you might like to think of a kilogram as having a weight of about 2.2 lb if you are in a country which uses imperial units. Note that I have not talked about how force is measured; that is part of what the second law is all about.

A brief discussion of acceleration is also in order. Everyone is comfortable with what velocity is, the distance traveled divided by the time to travel it; a scientist would call this the rate of change of position. Almost nobody, in my experience, is
really comfortable with what acceleration is. It is simply the rate of change of velocity. A dropped ball, for example, gains about $10 \text{ m s}^{-1}$ in velocity for each second it falls; after 1 s it is falling with a speed of $10 \text{ m s}^{-1}$, after 2 s with a speed of $20 \text{ m s}^{-1}$, 30 m s$^{-1}$ after 3 s, etc. The acceleration is $a = 10 \text{ (m s}^{-1}) \text{ s}^{-1} = 10 \text{ m s}^{-2}$.

To discover Newton’s second law you must interact with nature. We will never find physical laws by just sitting at a desk and thinking; we must make measurements which tell us how things happen. Newton’s second law is about how exerting a force on a mass changes its motion, i.e. how force, mass and acceleration are related. The experiment I propose is pretty simple: push or pull on a mass with a force and measure the acceleration. First vary the mass and hold the force constant, then vary the force and hold the mass constant. Hopefully, the resulting data will lead to some general law. But there is a problem. At this stage, force is a qualitative concept, a push or a pull, and if we cannot measure it then how can we vary it or even hold it constant? But, I can imagine having a machine which always exerts the same force. I could push with my hand using the muscles in my arm with very roughly the same force each time. Or, I could attach a spring to the mass and always pull with a force such that the spring was always stretched by the same amount, an improvement over my hand/arm machine. Oh, and I will call the force my machine exerts 1 baker (B). So, let’s do the first part of the experiment with one of my constant-force machines, pulling with a force of 1 B, on various masses. The data might look like the graph shown in the left panel of figure 1.1. Note that this makes sense because if the mass is large, then the acceleration is small and vice versa. The problem is that it is difficult to quantify the relationship between the two variables because the data lie on a curve, not a straight line. Suppose that, instead, we plot the acceleration as a function of the reciprocal of the mass (1/mass) as shown in the right panel of figure 1.1. This gives us a straight line which means the acceleration is proportional to the reciprocal of the mass, $a \propto 1/m$. Next, we hold the mass constant while we vary the force, first using one 1 B machine, then two, then three, etc. We would find that two gave twice the acceleration as one, three triple the acceleration, etc. In other words the acceleration is proportional to the force, $a \propto F$. Simple algebra says that therefore $a \propto F/m$. To me, this is Newton’s second law, a statement of experimental facts—acceleration is proportional to force and inversely proportional to mass.

![Figure 1.1](attachment:image1.png)
Since it is much more convenient to convert this to an equation, we introduce a proportionality constant $C$, $a = CF/m$. The choice of $C$ determines how we will measure force. The most clever choice, of course, would be $C = 1$ resulting in $F = ma$, the way we usually see the second law written. One unit of force, called a newton (N), is that force which, when applied to an object with a mass of 1 kg, results in an acceleration of 1 m $s^{-2}$, $1 \text{ N} = 1 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$ and is approximately 0.225 lb.

In conclusion, the second law is both a statement of an experimental fact and a definition of a unit of force.

1.1.3 Newton’s third law

The third law essentially says that forces in nature always appear in pairs and that if object $A$ exerts a force on object $B$, object $B$ exerts an equal and opposite force on object $A$. I will refer to those two forces as a Newton’s third-law pair and they always add up to zero. If you think about it, you will see that an alternative way of stating the third law is that the net force on an isolated system of interacting objects is zero, where an isolated system is one which has no forces acting on it other than the forces among its members.

There is often great confusion surrounding the third law. Carefully note from my first statement of the third law that the forces of a third-law pair are never on the same object. One of the questions cited at the beginning of this chapter wondered how we could move a book across a table since the action and reaction force always cancel out. But only one of those forces is on the book and only forces on the book determine how the book moves. For the same reason, we should not make the mistake of identifying equal and opposite forces automatically as third-law pairs. For example, the weight of a book sitting on a table points down and the force the table exerts on the book is equal but points up; these are equal and opposite because of the first law, they have nothing to do with the third law.

1.1.4 Linear momentum

Newton, in his landmark book *Philosophiæ Naturalis Principia Mathematica*, did not write the second law as $F = ma$, rather he said that the rate of change of motion is equal to the force. It will be important to understand what this means if we are to understand many of the Q&A examples in this book. Recall that the acceleration is the rate of change of velocity; it is customary in mathematics and physics to write this as $a = \Delta v/\Delta t$ where $\Delta v$ is the change in velocity and $\Delta t$ is the elapsed time. For example, if the velocity increases from 4 to 8 m $s^{-1}$ over a period of 2 s, the acceleration is $(8 - 4)/2 = 2 \text{ m} \cdot \text{s}^{-2}$. So now we can write $F = ma = m(\Delta v/\Delta t) = \Delta p/\Delta t$ where $p = mv$ is what Newton meant by the ‘motion’; $p$ is called the linear momentum today. (Note that since $m$ is constant in $F = ma$, $m\Delta v = \Delta (mv)$.) If the net force on a collection of objects is zero, the rate of change of linear momentum must be zero—linear momentum never changes! This is called conservation of linear momentum. Conservation principles are extremely useful in physics. Notice that conservation of linear momentum implicitly invokes the third law since the net force on an isolated system must be zero.
1.1.5 Energy

The mathematics behind the idea of energy is more complex and will be handled in appendix A of this book for the interested reader. The idea is that if a force is exerted on an object as it moves through some distance, work is done on the object which changes its energy. For many situations, the force is constant and along the path of the object so that the work can be written simply as \( W = Fs \) where \( W \) is the work and \( s \) is the distance traveled. Now, what changes if you do work on an object? Well, that is really an easy question to answer qualitatively if you understand the second law—if you push in the direction it is moving it speeds up and if you push opposite the direction it is moving it slows down. In other words, force causes acceleration which means either speed up or slow down in physics, so what changes is speed. Without any derivation (see appendix A), here is the way that speed changes:

\[
W = \Delta \left( \frac{1}{2}mv^2 \right) = \frac{1}{2}mv_{\text{final}}^2 - \frac{1}{2}mv_{\text{initial}}^2.
\]

This is often called the work–energy theorem. When you do work, you change the quantity \( K = \frac{1}{2}mv^2 \) which is called the kinetic energy of the object. Note that if there is no work done on a system its kinetic energy never changes. Again, we have discovered a conservation principle, conservation of energy, which states that a system on which no forces do work will have its total energy constant. Something called potential energy is useful, but for the most part it will not be needed for this book. I will briefly discuss and define potential energy in appendix A and write the potential energy for weight, \( mgy \).

Finally, the unit to measure energy and work is the joule (J), 1 J = 1 kg·m² s⁻². You are also probably familiar with the unit of power, the rate at which energy is used or created, the watt (W). 1 W = 1 J s⁻¹. A 100 watt light bulb consumes 100 J of energy each second.

1.1.6 This is all wrong!

As we shall learn in the second part of this book, Newtonian mechanics, as framed above, is only an excellent approximation to the true classical mechanics, special relativity. \( F = ma \) is wrong, \( K = \frac{1}{2}mv^2 \) is wrong, \( p = mv \) is wrong. But, this need not bother us here in chapter 1 because only when speeds become comparable to the speed of light, \( c = 671 000 000 \) mph, might you notice that Newtonian mechanics is not right. Interestingly, Newton’s expression for the second law, \( F = \Delta p/\Delta t \), is correct after linear momentum is slightly redefined.

1.2 Newton’s laws misunderstood

At the beginning of section 1.1 were listed several questions which demonstrate how Newton’s laws are often misconstrued. Let us now look at some of those questions along with the answers.

**Question:** Can you explain to me what exactly keeps molecules moving? With no energy being added, they should just eventually stop, shouldn’t they? Where does this energy that keeps them moving come from? In the end, does it all come down to radiation from the Sun?
**Answer:** You have fallen into one of the most common traps regarding misunderstanding how the Universe works. Newton’s first law states that an object which experiences no net force will continue to move with constant speed in a straight line. What this means is that if something is moving and nothing is pushing or pulling on it, then you do not have to do anything to keep it moving. In terms of energy, if something has a certain amount of energy, then it will retain that energy until some external agent changes it; this is called conservation of energy. I am not sure what you have in mind with your question, but probably the molecules moving around in a gas. As you probably know, the temperature of a gas is a measure of the average kinetic energy per molecule. If the gas is in thermal equilibrium with the walls, then when a molecule hits the wall it rebounds (on the average) with the same kinetic energy it had beforehand. You don’t have to do anything to keep it moving. Incidentally, if Newton’s first law were not true we would never have sent probes to the distant planets like Saturn and Jupiter or even the close ones like Mars and Venus. The reason is that if we had to keep the probe moving by burning an engine the whole way we could never carry enough fuel. What actually happens is that we burn up almost all the fuel escaping the Earth and acquiring a high speed and then we just turn off the engines and coast the rest of the way.

**Question:** If a curling stone weighs 20 kg and is traveling at a speed of 0.5 m s\(^{-1}\), with how much force did the curler throw it in N?

**Answer:** You cannot determine the force needed to give a particular mass a particular speed. Just to make that plausible, suppose you push on the 20 kg stone with a force of 2 N for 1 s; surely it will have a different result than if you push on the 20 kg stone with a force of 2 N for 2 s. There are two (in the end, equivalent) ways you can think about this problem:

- The impulse delivered by a force \(F\) in a time \(t\) is \(Ft\). The linear momentum of an object with mass \(m\) and speed \(v\) is \(mv\). The change in momentum is equal to the impulse and so, if the object starts at rest, \(Ft = mv\). For example, in your case \(Ft = 10 \text{ kg} \cdot \text{m s}^{-1}\) so you could push with a force of 10 N for 1 s.
- The work done by a force \(F\) pushing over a distance \(s\) is \(Fs\). The kinetic energy of an object with mass \(m\) and speed \(v\) is \(\frac{1}{2}mv^2\). The change in kinetic energy is equal to the work and so, if the object starts at rest, \(Fs = \frac{1}{2}mv^2\). For example, in your case \(Fs = 2.5 \text{ kg} \cdot \text{m}^2 \text{ s}^{-2}\) so you could push with a force of 10 N for a distance of 0.25 m.

In both cases, be sure to note that what the force is depends on how long or far it is applied.

Here the questioner thinks that the speed something acquires depends on how hard you push it, so that if you know its speed you know how hard it was pushed. But, it is acceleration, not speed, to which force is related and, as the answer shows, a small force over a large distance or time has the same effect as a large force over a small distance or time.

Sometimes the question can be answered if particular constraints are placed on it.
**Question:** We are working to produce a safety harness and the strap material we are using has a maximum Newton rating—we were trying to get an idea of what Newton rating would be needed to support a 300 lb man if he fell 3 ft. Being hunters (tree stand safety harness)—perhaps we are wording the question incorrectly.

**Answer:** What matters is how long it takes the falling guy to stop. The mass of a 300 lb guy is about 130 kg, the acceleration of gravity is about 10 m s\(^{-2}\), and so the weight of the guy is about 1300 N. You need that strong a strap just to hang him there at rest. If he falls 3 ft (about 1 m) he will be going about 4.5 m s\(^{-1}\). So, let’s call \(F\) the average force needed to stop him and \(t\) the time it takes him to stop; I reckon that \(F \approx 130(10 + (4.5/t))\). For example, if he takes \(\frac{1}{4}\) s to stop, \(F \approx 3600 \approx 809\) lb to stop him. The straps are probably pretty unstretchy, so your best bet would be to make the harness out of a stretchy material because, don’t forget, the bigger \(F\) is the more it is going to hurt during the stop.

Note that the average acceleration as he is stopping is \(\Delta v/\Delta t\) which is where the 4.5\(l/t\) came from; \(a = 4.5/l\). So the strap needs to do two things, hold up the weight \((mg)\) and provide the acceleration \((ma)\) where \(m = 130\) kg.

**Question:** If a bullet was traveling at 823 m s\(^{-1}\) and hit an object that stopped it dead how much force would be exerted on the target?

**Answer:** Here is the question which I get in one form or another which indicates how poorly understood the concept of force is! You cannot get the force because it depends on how quickly the bullet stops. If you mean by ‘stopped it dead’ that it stops instantaneously, then the force would be infinite. The average force is the change in momentum (mass times velocity) divided by the time to stop. So, you need also the mass of the bullet. Suppose the bullet had a mass 0.02 kg, then the change in momentum \((0.02 \times 823)\) is about 16 kg m s\(^{-1}\). If it stops in 0.01 s the average force is 1600 N = 360 lb, if it stops in 0.001 s the average force is 16 000 N = 3600 lb.

This next question comes up often. Newton’s third law says all forces have an equal and opposite mate. Why don’t they all add up to zero so that nothing ever moves?

**Question:** If action and reaction are always equal in magnitude and opposite in direction, why don’t they always cancel one another and leave no net force to accelerating a body?

**Answer:** Newton’s third law states that if one object exerts a force on a second, the second exerts an equal and opposite force on the first. Therefore, the ‘action/reaction’ forces are never exerted on one body. If you select a body to study, its motion is determined only by the forces exerted on it, not by forces exerted by it. Students often make mistakes with this ‘action/reaction’ thing because they tend to identify any pair of equal and opposite forces as being an ‘action/reaction’ pair. For example, a 1 lb book sitting on a horizontal table has two forces on it, its 1 lb weight...
pointing down and a force of 1 lb which the table exerts up on it (usually called the normal force); these have nothing to do with Newton’s third law but are equal and opposite because the book is in equilibrium and the force the table exerts is therefore required to be 1 lb up. If we now look at the table, the book exerts a 1 lb force down on it because of Newton’s third law; the ‘action/reaction’ pair is the force the table exerts on the book and the force which the book exerts on the table. Lots of novice physics students want to say that the weight of the book is the 1 lb force down on the table—this is totally false since this is a force on the book, not the table.

1.3 Air drag

If you have ever taken an introductory physics course, an often encountered phrase is ‘neglecting air drag’ or ‘neglecting air friction’. It is often a good idea to make approximations when you are just starting to learn something, deal with the relatively simple cases first. If you have a marble that you drop from 3 m, it is a very good approximation to neglect air drag, but what if you drop a cotton ball from 3 m or a marble from 1000 m? There are many examples of motion of objects where air drag is important and many questions I get are about cases where air drag is important.

Most problems of interest for objects moving through air can be well approximated as encountering a drag force \( F_d \) proportional to the square of the speed \( v \) of object, \( F_d \propto v^2 \). In fact, there is a fairly good expression for the proportionality constant necessary to make this an equation: \( F_d = C v^2 = (C_d A \rho/2) v^2 \) where \( C_d \) is the drag coefficient which depends on the shape of the object, \( A \) is its cross-sectional area, and \( \rho \) is the density of the air. A reasonable approximation if SI units are used is for \( C = \frac{1}{4} A \). An important thing to recognize if air drag is important is that there is what is known as a ‘terminal velocity’, \( v_t \), the speed which an object moving through the air tends toward. If dropped, it speeds up to \( v_t \), if projected at a higher speed it slows to \( v_t \). It is easy to calculate \( v_t \) because it is the speed where the drag force (up) becomes equal to the weight force (down), \( C v_t^2 = mg \) or \( v_t = \sqrt{(mg/C)} \). So, contrary to the simple Galileo-story result that all objects fall with the same acceleration, if two objects having identical shapes and sizes are dropped, the more massive one wins because the terminal velocity is larger because it is proportional to the square root of the mass.

One thing to keep in mind as we look at a few examples is that whenever you include air drag, your calculation is approximate. The details of air drag are very complicated and best done numerically with big computers if you are designing an airplane!

**Question:** How much does a lacrosse ball (2 inch diameter) slow down (horizontal velocity only) if thrown at 80 mph from the instant it is released until it reaches a point 10 m away, taking into account air resistance.

**Answer:** I prefer to work in metric units so 80 mph is about \( v_0 = 35 \text{ m s}^{-1} \) and the diameter is about \( D = 6 \text{ cm} = 0.06 \text{ m} \). I will also need the mass of a lacrosse ball which I looked up to be about \( m = 0.15 \text{ kg} \). Now, for a ball of this size traveling
through air with this velocity, the air resistance force is proportional to the square of the velocity. Therefore Newton’s second law is of the form \(-Cv^2 = ma = m(dv/dt)\) where \(C\) is a constant which can be calculated approximately as \(C = 0.22D^2\) for a sphere in air. Therefore we must solve the differential equation \((dv/dt) + 0.00079v^2 = 0\). (I completely ignore gravity because the ball starts with zero velocity in the vertical direction and flies for only a very short time.) If you know differential equations, then this is not particularly difficult to solve. I will do that later. For starters, however, it is instructive to make a reasonable approximation and see what we get. I am going to say that I expect, over so short a distance as 10 m and starting with such a large initial velocity, that the acceleration will not change much. So I will say that the acceleration at the beginning, \(a_0 = -0.00079 \times 35^2 = -0.97 \text{ m s}^{-2}\), does not change much over the flight. So we have a uniform acceleration problem and we can say \(x = v_0t + \frac{1}{2}a_0t^2 = 10\) and solve for \(t\); I find that \(t = 0.29\text{ s}\). Finally, we can obtain the estimated final velocity, \(v = v_0 + a_0t = 35 - 0.97 \times 0.29 = 34.7 \text{ m s}^{-1}\). So the ball loses about 0.9% of its initial velocity.

For anyone interested in the exact solution of the differential equation, here it is. The solutions to the equation are \(v = v_0/(1 + kt)\) And, \(x = (v_0/k)\ln(1 + kt)\) where \(k = Cv_0/m\). Solving these I find that \(t = 0.29\text{ s}\) and \(v = 33.2 \text{ m s}^{-1}\). So, only about 5% of the velocity is lost.

The previous question was done in two ways. One very important thing to know in science is how to make approximations to make a problem more manageable without getting incorrect results. One of the things which makes air drag problems tricky is that the force depends on the speed and so the acceleration does also. In this problem I suggested that, since the time it takes a fast lacrosse ball to go a short distance must be really small, the velocity, and therefore the acceleration, does not change very much. Then we can use the equations for uniform acceleration to solve the problem, much more familiar to many of you than the more difficult solution to the differential equation. And the exact and approximate solutions give you pretty much the same result.

This next question is about baseball. It is well known that a curve ball happens because of air drag but I had not realized how much a ball slows down in the brief time it takes a fastball to reach the plate. Here I use the ‘exact’ solution (in quotes because all air drag calculations are approximate).

**Question:** Based on physics, is a 90 mph fastball slower or faster than a 95 mph fastball? At work we are trying to determine if the 95 mph fastball loses energy faster than a 90 mph fastball. Your answer is greatly appreciated.

**Answer:** You are asking two questions; if a 95 mph ball loses energy faster than a 90 mph fastball (it does) and if the one which starts out faster ends up slower (it does not). For the details of the following, see the earlier lacrosse ball answer. Following the (exact) solution in that earlier answer, I find that the 95 mph ball reaches the plate in 0.47 s and arrives at the plate with a speed of about 80.8 mph. The 90 mph ball reaches the plate in 0.50 s and arrives at the plate with a speed of
about 76.3 mph. So, each loses about 14 mph with the faster ball losing a bit more. This surprised me but I found another reference saying that something like 10 mph is what is lost, so my calculations are reasonable. So they do not lose energy significantly differently (the faster pitch lost more speed in a shorter time so its average rate of change of speed was indeed bigger). (I used 3 inches for the diameter, 0.145 kg for the mass, and 60 ft 6 inches for the distance to the plate.) There is certainly no way that one could characterize a 95 mph fastball as slower than a 90 mph fastball.

Often I am called on to settle arguments. Here is an example involving air drag and the sports of tennis and badminton.

**Question:** A friend of mine and I have an argument over which is the faster sport, tennis or badminton. The criterion is how long it would take to serve a tennis ball/shuttlecock from one side of an Olympic sized tennis/badminton court to the player waiting on the other side assuming that both are standing on the out of bounds line. We are assuming ideal conditions and that the players in both cases are equally strong and fast.

**Answer:** You may not realize it, but your question is mostly about air drag on projectiles. I seem to get more questions about air drag than just about anything else except maybe variations of the twin paradox. Maybe that is because it is perhaps the most important phenomenon mostly swept under the rug in most elementary physics courses. There are several instances of earlier questions involving baseballs and lacrosse balls which are very similar to this one. For high speed projectiles, air drag is very important; e.g. a 100 mph baseball loses about 10 mph by the time it crosses the plate. Approximations have to be made to quantify the situation you are interested in, but I feel the results I will present are pretty close to what happens on the court. The approximations are:

- I neglect gravity because the times involved are sufficiently short that the ball/shuttlecock will not fall far or very much change its vertical speed.
- I assume that the drag is proportional to the square of the speed—twice the speed, four times the force of drag. This is an excellent approximation for these speeds, these objects.
- The form of the force I use is \( F \approx \frac{1}{4} A v^2 \) where \( A \) is the cross-sectional area presented to the wind. Here \( A = \pi R^2 \) where \( R \) is the radius of the ball or the outer circle of the feathers. This probably slightly overestimates the force for the tennis ball (whose ‘hairs’ have the function of decreasing the drag) and underestimates it for the shuttlecock (whose ‘feathers’ are designed to increase drag).
- Data for tennis:
  \[ v_0 = 73 \text{ m s}^{-1} = 163 \text{ mph} \]
  \[ R = 0.032 \text{ m} = 1.26 \text{ in} \]
  \[ m = 0.057 \text{ kg} = 2 \text{ oz} \]
  back line to back line distance: 24 m
- Data for badminton:
\[ v_0 = 92 \text{ m s}^{-1} = 206 \text{ mph} \]
\[ R = 0.025 \text{ m} = 1 \text{ in} \]
\[ m = 0.005 \text{ kg} = 0.18 \text{ oz} \]

back line to back line distance: 13.4 m

I used the fastest recorded serves for the velocity off the racquets, \( v_0 \). If you integrate \( F = ma \), you get the following solutions:
\[ v = \frac{v_0}{1 + kt} \text{ and } x = \left( \frac{v_0}{k} \right) \ln(1 + kt) \] where \( k = \frac{1}{4} Av_0/m \).

Here are the results:

- The tennis ball takes 0.39 s to travel the distance, arrives with a speed of 52 m s\(^{-1}\) (116 mph), a loss of 21 m s\(^{-1}\) (47 mph) or 29%.
- The badminton shuttlecock takes 0.30 s to travel the distance, arrives with a speed of 25 m s\(^{-1}\) (56 mph), a loss 67 m s\(^{-1}\) (150 mph) or 73%.

I will leave it to you to argue about what these numbers tell you about which ‘is the fastest sport’. According to your criterion, the shuttlecock arrives earlier but with a much lower speed. The shuttlecock starts off the fastest because it has a smaller mass and can therefore have a larger acceleration from the force from the racquet. But it slows down very rapidly mainly because of its small mass. Figure 1.2 shows the speeds as functions of time over the flight time of each.

Finally, in our exploration of air drag, here are a few questions in which terminal velocity is the focus.

**Question:** If I were to drop an empty wine bottle out of an airplane flying at say 35 000 ft above the ocean at 300 mph, would the bottle hit the surface of the water hard enough to break the bottle? I read somewhere something about terminal velocity.
velocity being 120 mph, so would the resistance of the atmosphere slow the wine bottle to 120 mph by the time it made impact with the ocean? And would 120 mph be enough to shatter the wine bottle, or would it depend on how choppy the seas were versus a flat water surface?

**Answer:** When I answer questions involving air drag and terminal velocity, I usually use the approximation that (in SI units) the force $F$ of air drag is $F \approx \frac{1}{4} A v^2$ where $A$ is the area presented to the wind and $v$ is the speed. So, as something falls, the faster it goes the greater the drag force on it so that, eventually, when the drag equals the weight, the object will be in equilibrium and fall with constant speed. Since the weight $W$ is $mg$ where $m$ is the mass and $g = 9.8 \text{ m s}^{-2}$, the terminal velocity $v_t$ can be calculated: $\frac{1}{4} A v_t^2 \approx mg$ or $v_t \approx 2 \sqrt{(mg/A)}$. So the terminal velocity depends on the mass and size of the falling object and your 120 mph is most likely not correct. Also, how it falls determines the terminal velocity since it has a much bigger area falling broadside than with the top or bottom pointing down. I figure that if it falls broadside there will be a bigger force on the fat side than the neck which will cause a net torque which will make it want to turn with its neck pointing down; so I will assume that is how it falls. I happened to have an empty wine bottle in my recycle bin which has a mass of about 0.5 kg and a diameter of about 8 cm. When I calculate the terminal velocity I get $v_t \approx 63 \text{ m s}^{-1} = 140 \text{ mph}$. The 120 mph number you heard was probably a typical terminal velocity of a human, and it is just coincidence that the wine bottle has a terminal velocity close to that.

It is hard to say whether it would break or not. I think probably not. Suppose that it took 1 s to stop. Then the average force on the bottle would be $F = ma = (0.5 \text{ kg} \times 63 \text{ m s}^{-1})/(1 \text{ s}) = 31.5 \text{ N} \approx 7 \text{ lb}$ which the bottle should be able to withstand easily. I know that they say that at high speeds hitting the water is like hitting a brick wall, but if the stopping time were 0.1 s the force would still only be about 70 lb.

**Question:** If you shoot a bullet straight up into the air, its velocity at the very top of the trajectory is zero, even if only for an instant, as its upward velocity slows to nothing before becoming downward velocity. Downward vertical velocity then increases in the earthward direction. Would the velocity ever become dangerous if it landed on a living person? Is the weight of the bullet important? Does the atmosphere restrict the downward velocity?

**Answer:** A falling bullet experiences a downward force of its own weight and an upward force of air drag. The result of the air drag, which increases with speed, is to have the falling object eventually reach a maximum velocity called the terminal velocity which is determined by its weight and its geometry (which is why you can jump out of an airplane with a parachute). A .30 caliber bullet weighing about 10 grams has a terminal velocity of about 90 m s$^{-1}$ (about 200 mph) and a .50 caliber bullet weighing about 42 grams has a terminal velocity of about 150 m s$^{-1}$ (about 335 mph). A bullet traveling 60 m s$^{-1}$ (about 130 mph) can penetrate the skull so, yes, a falling bullet is dangerous. Dozens of people are killed every year by celebratory gunfire.
**Question:** Why is it difficult to calculate the terminal velocity for a cat falling from a high rooftop?

**Answer:** I do not know what you mean ‘difficult to calculate’. We can estimate it pretty easily, but certainly not do it precisely. First of all, any calculation having to do with air friction is going to have approximations and assumptions. For something like a cat, roughly 2 kg (4.4 lb), falling, it is a very good approximation to say that the drag force is proportional to the square of the velocity. It turns out that a fairly good approximation for the force is \( F = \frac{1}{4} A v^2 \) where \( A \) is the area the falling object presents to the onrushing wind and \( v \) is the velocity (this is only for SI units). Since it depends on \( A \), it depends on how the cat orients itself: if in a ball he will fall much faster than if all spread out. Suppose we take the area of a falling cat to be about \( 20 \text{ cm} \times 40 \text{ cm} = 0.08 \text{ m}^2 \). Then the force will be about \( 0.02v^2 \).
Now, the cat’s weight is about \( mg = 2 \times 9.8 \approx 20 \text{ N} \). When the force of air friction is equal to the weight force down, the cat will fall with a constant velocity called the terminal velocity: \( 0.02v_t^2 = 20 \), so \( v_t = \sqrt{20/0.02} \approx 30 \text{ m s}^{-1} = 67 \text{ mph} \). If you google ‘terminal velocity of a cat’ you will find the number 60 mph, so my approximations were evidently reasonable. There, now, that wasn’t so difficult, was it?

This last question is interesting in that, since cats have a relatively low terminal velocity, they usually survive falls from high buildings. In fact, they are more likely to survive falls from higher than seven stories because, at the time they reach terminal velocity, they instinctively relax and spread out. The next question concerns an animal with a much smaller terminal velocity.

**Question:** Why is it that if you blow a spider suspended by her web she floats out but then when this pendulum swings back it stops when the web is vertical and doesn’t swing back and forth? Is it due to the air friction as it comes back to equilibrium or perhaps the dynamic structure of the web strand that absorbs energy that would have made the web swing back and forth?

**Answer:** It is caused by air drag. This is called a damped oscillator. If there were no air, the spider would swing back and forth with constant amplitude, just like a clock pendulum (apart from the little friction from bending the thread she hangs from). A spider has so little mass that her terminal velocity is very small—drop her off the roof and she will not get hurt because she quickly comes to a constant velocity because the air drag, which can be approximated as being proportional to her speed, quickly becomes equal to her weight. If the air drag is not too big, the pendulum will swing back and forth with ever decreasing amplitude; this is called underdamped. For larger drag, as in the case of your spider, she never crosses over the equilibrium and just slowly approaches the bottom of her swing; this is called overdamping. There is a third possibility called critically damped, but it is qualitatively just like overdamping, so let’s not go there.

### 1.4 Gravity and Kepler’s laws

Gravity is the force which you are most aware of. This is strange because, gravity is the weakest force in nature. How can that be? The only reason that it is so pervasive in your life is that there happen to be very huge accumulations of mass scattered throughout the Universe (like the Earth, the Sun, etc) and mass is the source of gravitational force. Newton discovered that two objects of mass \( M_1 \) and \( M_2 \), separated by a distance \( r \), exert a force \( F \) on each other given by \( F = GM_1M_2/r^2 \) where \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \text{kg}^{-2} \) is the universal constant of gravitation. Now you can appreciate what a weak force gravity is: two 1 kg masses separated by 1 m exert a force on each other of \( 6.67 \times 10^{-11} \text{ N} \); this is about 100 times smaller than the weight of one speck of dust.

How did Newton figure this out? Astronomy was a science which was around long before physics. Amazingly accurate measurements of the positions of planets were made by the Danish astronomer Tycho Brahe and his assistant German
astronomer Johannes Kepler in the 16th century, nearly a century before Newton’s birth. Using these data, Kepler was able to describe how the planets move in their orbits using his now-famous three laws; the laws, however, were purely empirical, meaning that they resulted from just describing data, not any physical principle. Newton’s triumph was that his law of gravitation was able, along with his three laws of mechanics, to explain Kepler’s laws.

Finally, note that near the Earth’s surface the force $W$ on a mass $m$ is approximately $W = m(GM/R^2) \equiv mg$ where $M = 6 \times 10^{24}$ kg and $R = 6.4 \times 10^6$ m are the mass and radius, respectively, of the Earth. So $g$, the acceleration due to gravity, is $g = 9.8 \text{ m s}^{-2}$.

A reasonable question was how Newton could know $G$. He never did.

**Question:** How was the value of $G = 6.67 \times 10^{-11}$ derived? How did Newton get the value of the constant $G$?

**Answer:** $G$ is a fundamental constant of nature, it cannot be derived. Newton did not get the value of $G$, the best he could do was get the product $GM$ where $M$ is the mass of the Sun. He shows that $GM = 4\pi^2a^3/T^2$ where $a$ is the semimajor axis of the orbit of a planet and $T$ is the period. This is a derivation of Kepler’s third law and is the real triumph of Newton. It was 70 years after Newton’s death that the first measurement of $G$ was made by Cavendish. You can also get $M_{\text{Earth}} G = R^2 g$ where $M_{\text{Earth}}$ is the mass of the Earth, $R$ is the radius of the Earth, and $g = 9.8 \text{ m s}^{-2}$; so Newton could have found the ratio $M_{\text{Earth}}/M$ without knowing $G$.

You have probably heard the legendary tale of Galileo dropping balls from the Tower of Pisa, finding them all falling in the same time. It is a question frequently submitted to *Ask The Physicist*.

**Question:** Why do two objects of different masses reach the ground at the same time and what are the factors that affect their motion?

**Answer:** The motion of a mass $m$ is determined by Newton’s second law, $F = ma$, where $F$ is the net force on $m$ and $a$ is its acceleration. A mass in free fall (no air friction) has only one force on it, its own weight $W$ which is the force with which the Earth pulls on it. It turns out that the weight is proportional to the mass, that is $W = mg$ where $g$ is a proportionality constant called the acceleration due to gravity. So, if you have two masses, $m$ and $M$, you can calculate their accelerations, $a$ and $A$, respectively: $A = W/M = g$ and $a = W/m = g$. Since both have the same acceleration, they fall identically. (You can see why $g$ is called the acceleration due to gravity.) The factors which must be satisfied for this to be true are that air friction is negligibly small and the masses must be small compared to the mass of the Earth.

It turns out that there is a profound physical truth here. There are really two kinds of mass we have discovered now. One is inertial mass, the property which resists acceleration when you push on it; the other is gravitational mass, the property which
allows objects which have it to create and feel gravity. The fact that different masses have the same acceleration implies that the two masses are identical. Experiments bear this out to remarkable accuracy. This is important in general relativity, the modern theory of gravity, and will be revisited in chapter 3.

I have had many variations of the next question. Above, the important role in physics history played by Kepler’s laws was emphasized. Here is a question the answer to which states these laws and uses two of them.

**Question:** The Earth orbits around the Sun. If we stopped the Earth in orbit and then let it fall straight towards the Sun, how long would it take to reach the Sun?

**Answer:** The questioner sent me a bunch of data about the masses of the Sun and Earth, the radius of the Earth’s orbit, and Newton’s universal constant of gravitation. But, you do not need any of that stuff—all you need to know is Kepler’s first and third laws and the fact that the period of Earth’s (approximately circular) orbit is one year. Kepler’s laws are as follows.

- **First law:** The orbit of a planet is an ellipse with a semimajor axis $a$ and with the Sun at one focus of the ellipse.
- **Second law:** A planet in its orbit sweeps out equal areas in equal times, so it moves faster as it gets closer to the Sun.
- **Third law:** The square of the period $T$ of an orbit is proportional to the cube of its semimajor axis, $T^2 \propto a^3$.

The Earth’s orbit is very nearly circular and a circular orbit has a semimajor axis equal to the radius of the circle, so $a_1 = R_O$ where $R_O$ is the radius of the Earth’s orbit; the eccentricity of a circle is 0. The other extreme is an ellipse with eccentricity 1 which is a straight line from the Sun to the Earth and so the semi-major axis for a ‘dropped Earth’ is $a_2 = R_O/2$. (To help you visualize this, figure 1.3 shows an elliptical orbit very close to the straight line orbit; just squeeze it a little bit flatter.) If we can cleverly deduce the period of this orbit, one-half that period will be the answer to your question. Using the third law,

\[
\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} = \frac{R_O^3}{(R_O/2)^3} = 8
\]

\[
T_2 = \frac{T_1}{\sqrt{8}} = 0.354 \text{ years.}
\]

So, the time to go half a period is 0.177 years = 64.6 days.

**Figure 1.3.**

To actually do this problem by brute force, integrating Newton’s second law for the gravitational force which changes as the Earth falls into the Sun, is very tricky and certainly beyond the understanding of the average interested layperson. This problem emphasizes, once again, an important thing to learn in science—it is often most illuminating to make reasonable approximations to difficult problems. The solution above is not actually perfectly accurate, it treats the Sun and the Earth
as point masses which is certainly not true. Also, the Earth would have infinite velocity when it turned around in its straight line orbit and its acceleration would be infinite at that time, obvious nonsense. But, if we realize that the real end of the trip is at the surface of the Sun and that distance is tiny compared to the total distance to the Earth’s orbit, most of the time must have been spent in the fall to the surface, the tiny amount farther being a negligible contribution. I show in appendix B that the time neglected is less than about 9 min of the 64.6 day total. As we shall see in chapter 2, for Newtonian mechanics to be applicable the speed must be much smaller than the speed of light and in this case the Earth has about 0.4% the speed of light when it gets to the surface, again shown in appendix B.

The following question is also a frequent one. Variations on ‘what happens when I drop a stone in a hole drilled through the Earth?’

**Question:** A sky-diver is falling toward Earth. A tunnel has been previously excavated completely through the Earth at exactly the location of the skydiver’s landing. He continues his dive through the tunnel without touching the sides of the tunnel. I believe that Newton would have had him stop at the Earth’s core. Where would Einstein have him stop?

**Answer:** First, since this is clearly an idealized problem, let us neglect air friction (which is, of course, not negligible because the sky-diver has a terminal speed before he hits the ground). Until he enters the tunnel he is accelerating with a constant acceleration down. When he enters the tunnel, he experiences less and less force as he goes deeper because there is less and less of the Earth pulling on him (all of the Earth outside him exerts no force) until finally at the center he has zero force on him but he has his highest velocity of the whole trip since he has been speeding up the whole time. Now as he moves away from the center he slows down. When he re-enters the other end of the tunnel he has exactly the same speed as he had when he entered it. He continues until he reaches the altitude from which he originally jumped at which point he turns around and begins the process all over again. Newton and Einstein would both agree on this. If air friction were included, he would not go as far and if he happened to stop at the center of the Earth, he would stay there forever. If the air friction were included, the general solution to the problem would be that he would oscillate back and forth going less far each time until he finally stopped in the center. There is an interesting aspect of this problem: if the Earth had its mass uniformly distributed through its volume (it does not), when the sky-diver is inside the tunnel, he moves exactly like he were a mass on an ideal spring.

The next question has to do with golfing on the Moon.

**Question:** My son has a question that I can’t seem to find an answer to researching on the web. What force is behind a golf ball when hit on the Moon? I appreciate your time on answering this for my son.

**Answer:** I think there is some confusion about what force is here. Here are all the forces a golf ball experiences here on Earth:

- The club, traveling with some speed, hits the ball and exerts a contact force on it for a very short time but it is a very big force and it results in the ball
acquiring a very large velocity. As soon as the ball leaves the club, there is no force ‘keeping it moving’. If there were no other forces, it would keep going forever with the speed with which it left the club.

- Once it is started, gravity pulls down on it which is the force which eventually does bring the ball back to the ground.
- As it flies through the air, it experiences air drag which can be very complicated. Essentially, it is a force trying to slow it down and the bigger the speed is the bigger this force is.
- If it happens to be spinning, the air drag can act asymmetrically so that the ball curves. This is what is called a hook or a slice in golf (depending whether it curves left or right, respectively, for a right-handed golfer).
- Of course, when it hits the ground, it experiences forces from the ground which ultimately bring it to rest.

What is different on the Moon?

- If the club is the same club with the same speed, there is no difference for this force. Therefore, the ball launches just the same as on Earth.
- The Moon is much smaller than the Earth and the result is that the gravity on the Moon is much weaker. Therefore, this force (trying to pull the ball back down) is much smaller and the ball will go a lot farther.
- Since there is no air on the Moon, there is no drag and this also results in the ball going much farther.
- The ball will not curve on the Moon, regardless of how much spin it has.
- When it hits the ground, things are about the same as on Earth except that all the forces are smaller, again because of gravity being smaller, so the ball rolls farther before it stops (also because it is going much faster when it hits the ground than it would have been on Earth).
1.5 Physics of everyday life

Physics applies to many every-day situations. Even analogies between societal issues and physics can be found, as the next question illustrates.

**Question:** It’s a question the answer to which I wish to use as an analogy when I make talks to citizen groups regarding homelessness; and specifically in response to the complaint by some in the audience that the homeless need to just pick themselves up by their own bootstraps and stop being a burden on society. I keep trying to explain to them that once one has fallen all the way down (as opposed to just tipping over a little, or even falling to one’s knees; and especially once they’ve slipped so through certain kinds of society’s cracks), it actually takes more effort to get back up again than it took to knock the person down. (And, trust me, it does.) *The Physicist: The questioner wishes to compare the energy necessary to tip over a cylinder of radius $R$ whose center of gravity is a distance $h$ above the floor to the energy required to lift it back up.*) What is the amount of energy needed to tip it over from vertical to horizontal compared to the amount of energy needed to tip it back up and make it vertical again? I’m looking for a ratio.

**Answer:** Figure 1.4 illustrates the situation described below. (See appendix C.) To tip it over, you have to move the center of gravity (COG) so it is above the point on the floor where the cylinder touches the floor; to do this you must raise the COG a distance $d = h\sqrt{(1 + (R/h)^2)} - 1$. The work necessary to do this is $W_{\text{fall}} = mgh\sqrt{(1 + (R/h)^2)} - 1$. If $R$ is much smaller than $h$, this may be approximated as $W_{\text{fall}} \approx \frac{1}{2}mgh(R/h)^2$. The work necessary to lift it back up is $W_{\text{lift}} = mg(h - R)$. Again, if $R$ is much smaller than $h$, $W_{\text{lift}} \approx mgh$. So, the ratio is $W_{\text{lift}}/W_{\text{fall}} \approx 2(h/R)^2$. For example, if $h = 5R$, $W_{\text{lift}}/W_{\text{fall}} \approx 50$; it takes 50 times the work to lift as to push over!

![Figure 1.4. A cylinder upright, about to tip over, and fallen.](image)

**Question:** Could you explain why the driver of a car must keep her foot on the accelerator to maintain a constant speed and therefore why energy is needed to maintain the car’s speed?

**Answer:** Wouldn’t it be great if we could have a car which had no energy loss? Unfortunately, the world has forces which we cannot avoid which take energy away from something moving along. These fall into the category of frictional forces: a spinning wheel has friction in its bearings which will eventually cause it to stop; an object moving thought the air has air resistance which will eventually
stop it as it moves along; the tires are not perfectly elastic and as they roll they are being continually deformed and undeformed and energy is lost. Without all these forces, we could accelerate up to speed and disengage the engine from the wheels and turn it off and just cruise. However, one can work hard to minimize these forces in the design of cars; making the cars aerodynamic, reducing the weight, and other tricks can minimize the energy we lose.

Bicycle stability involves some pretty heavy physics, but some aspects of it can be understood fairly easily as in the following question. Look for this question to come up again in section 1.6.

**Question:** Why, when a cyclist is turning round a bend, why does he lean inward?

**Answer:** Figure 1.5 is a simplified picture of the bike and cyclist, the circle representing the center of mass. Now, the sum of all the vertically directed forces must add to zero, \(-mg + N = 0\) which tells you that \(N = mg\). And, the sum of all the horizontally directed forces must equal mass times acceleration, \(F_f = \frac{mv^2}{r}\). So, given \(m\), \(v\) and \(r\), you now know all the forces. But you still need to know the angle of lean for the cyclist to not topple over. This is achieved by summing all torques (about the center of mass) and setting it equal to zero (so that it does not start to rotate in the plane of the page), \(\Sigma \tau = 0 = NL \sin \theta - F_fL \cos \theta = mgL \sin \theta - (\frac{mv^2}{r})L \cos \theta\). And so, solving for \(\theta\), \(\theta = \tan^{-1}(\frac{v^2}{rg})\). (See appendices C and D.)

![Figure 1.5. A bicycle turning a curve.](image)

The physics of towing something is illustrated in the next two questions.

**Question:** If I am towing a vehicle from a standing start is there an equation for calculating the amount of force I would be using for example if I tow a vehicle that weighs twelve tonnes because it has wheels and is therefore not a ‘dead weight’ how do I work out how much force I would be exerting on the tow rope/towing vehicle and also how would I factor in different gradients as it would
obviously require greater force on an upslope. This came up in my workplace where our towropes are rated to three tonnes and I was trying to explain that it does not mean you could not tow a vehicle over that weight.

**Answer:** This is a good question to illuminate elementary Newtonian physics. Your referring to ‘dead weight’ really has no meaning in physics, but you apparently mean that the object can move with little friction. So, let’s assume there is no friction; this is, of course, never true, but it puts an upper limit to anything I do. On level ground, any force will move the vehicle if there is no friction. What matters is how quickly you start it moving, in other words what the acceleration is. For example, suppose you have a 4 lb fish hanging on a 5 lb test line; if you pull it up slowly you will land it, but if you try to jerk it up really fast the line will break.

The physical principle in play here is Newton’s second law, \( F = ma \) where \( F \) is the force, \( m \) is the mass, and \( a \) is the acceleration of \( m \) due to \( F \). So, in your case, \( m = 12 \ t = 12 \ 000 \ kg \); the maximum force you can apply is \( 3 \ t = 29,420 \ N \) because the 3 t rating means that it can hold up a 3 t mass which has a weight of \( 3000 \times 9.8 \ N \). So the maximum acceleration is \( a_{\text{max}} = 29.420/12 \ 000 \ = 2.45 \ m \ s^{-2} = 5.5 \ mph \ s^{-1} \). This means that if you speed up to 5.5 mph in 1 s, the rope will almost break. Of course, there will be friction and so to be safe I would recommend a factor of roughly two, an acceleration of about 3 mph s\(^{-1}\) would probably be safe. Here is an equation you can use (which does not include any safety factor): \( a_{\text{max}} = 22(M_T/M_V) \ mph \ s^{-1} \) where \( M_V \) is the mass of the vehicle, and \( M_T \) is the mass rating of the towrope. If you are trying to tow up a hill which makes an angle \( \theta \) with the horizontal, you need to apply a factor of \( \sin \theta \) to the equation above, \( a_{\text{max}} = 22(M_T/M_V) \sin \theta \), because some of the vehicle’s weight is now directed down the hill instead of straight down. If the grade is \( 30^0 \), for example, \( \sin \theta = \frac{1}{2} \).

The next question is a little different because rather than worrying about the tow line breaking you need to worry about traction of the wheels of the towing vehicle as well as its power, ability to deliver the energy.

**A similar question:** Watching a TV commercial showing how mighty a pickup truck is—it’s **towing the space shuttle**, which weighs (according to the announcer) 292 000 lb (146 tons). Now I know that it’s not as if the pickup is lifting 146 tons—I figure the load on the little pin hooking the shuttle to the pickup will be (initially) 146 tons times the coefficient of friction for the tarmac upon which both vehicles are riding—am I right?
Answer: Assuming there is negligible friction in the bearings of the carriage for the shuttle, it is not hard to get the shuttle moving with a small acceleration. In the previous question, though, the numbers were much smaller than in your case where there is what appears to be a steel towing bar which would far exceed the strength of a towrope to tow things with weights of several tons rather than several hundred. So, with such a strong ‘towrope’ you might think that you could have as big an acceleration as you like. For example, if the breaking strength of the pin (probably the weakest link) were 100 tons, my little formula above would say that you could have an acceleration up to about \(22 \times \frac{100}{146} \approx 15 \text{ mph s}^{-1}\) (0–60 in 4 s)! This will obviously not happen. There are two considerations you need to think about. First, the force which provides the acceleration is actually the static friction between the truck wheels and the road; the biggest this force can be is \(f = \mu W\) where \(\mu\) is the coefficient of static friction and \(W\) is the weight of the truck. For rubber on dry concrete, \(\mu \approx 0.7\) and the weight of a Toyota Tundra pickup is about 3 tons, so \(f = 3 \times 0.7 \approx 2 \text{ tons};\) so, the maximum acceleration is only about 0.3 mph s\(^{-1}\). The second consideration is how rapidly the truck can deliver the energy needed to move the load, in other words its power rating of about 300 hp. I calculate that the maximum acceleration with a 146 ton load would be about 4 mph s\(^{-1}\). So, it appears that the main limiting factor on the acceleration is the possibility of the tires spinning. Keep in mind that these are all just rough estimates, but they give the general picture. (See appendix E.)

Many questions I have received ask things like if a box is full of birds which are flying, does it weigh less than if they were all roosting. These questions can get a bit convoluted although the question after the next one will be such a question. A simpler question is about a juggler.

Question: Is a juggler, while juggling three weights or any number really, lighter at all times than she would be if she merely carried the weight about her person? If so then by how much, when and why? If not then what does happen to their weight while they juggle at the various times they are and are not in contact with the juggled objects?

Answer: First, I am a stickler for the use of the word weight. The juggler’s weight is the force which the Earth exerts on her and so it is always the same unless she overeats or goes on a diet. But, the apparent weight (what would be read by a scale she is standing on) depends what is going on with the balls. If all the balls are in the air at some time, her apparent weight will be her actual weight. If she is simply holding one ball, the scale will read her weight plus the ball’s weight. If she is in the process of juggling one of the balls, she is exerting an upward force which will be larger than the weight of the ball (Newton’s second law); but, because of Newton’s third law, we can conclude that the ball exerts an equal and opposite force on her; and so the force read by the scale will be larger than the weight of the ball plus juggler. Only if she throws a ball downward would her apparent weight be smaller.

Now the bird-in-a-box problem.
**Question:** If there is a trailer full of birds and the birds are sitting on the bottom, does it weigh the same as if all the birds are flying?

**Answer:** (In this answer, when I say ‘weigh’ it means what a scale would read.) There is more than one answer to this question. Let us assume that the birds are hovering or moving with constant velocities. In that case, each bird stays in flight because the air exerts a force up on it equal to the bird’s weight; but Newton’s third law requires that the bird therefore exerts an equal downward force on the air. The air is part of the trailer, so the net weight of the whole truck is unchanged. Another possibility would be if the birds have an acceleration with a vertical component; the simplest example is that all the birds are in freefall inside (probably not what you had in mind by ‘flying’) in which case the birds would not contribute to the weight (neglecting any air friction or buoyancy) and the overall weight would be smaller. Or, if all the birds were at some instant accelerating upward, the air would be exerting an upward force on them greater than their weight so the trailer would measure heavier.

To finish off this section, here is a clever way to exert a force much bigger than the maximum force with which you can push.

**Question:** Today on NPR’s *Cartalk*, someone called in a physics question. I would like to have a definite answer (very easy for you I’m sure). Here it is: A lady’s car is stuck in the mud. She of course is alone with no phone and is a physicist. She ties a rope to her car bumper and a nearby tree. She then finds the mid-point of the rope and pushes with max effort which she estimates to be 300 N. The car just begins to budge with the rope at about a 5° angle. With what force is the rope pulling on the car? Ray, co-host of *Cartalk*, said to find the sine of 5 degrees and then multiply by 300. Then he changed it to cosine of 5 degrees and multiply by 300. If any of these is right, I don’t understand why.

**Answer:** One of my favorite shows! Neither of the answers is right which is surprising since Tom and Ray are both are MIT grads. Here is how you do the problem (see figure 1.6). The point where she is pulling is in equilibrium, so the vector sum of the three shown vectors (her 300 N pull and the tensions in the two halves of the rope) must equal zero. The components perpendicular to her pull must add to zero, so the tension \( T \) in each side of the rope is the same. This comes from \( T_1 \cos(5^\circ) - T_2 \cos(5^\circ) = 0 \), so \( T_1 = T_2 = T \). Similarly, the components parallel to her pull must sum to zero, so \( 300 - T \sin(5^\circ) - T \sin(5^\circ) = 0 \). So, \( T = 300/(2 \sin(5^\circ)) = 1721 \text{ N} \).

![Figure 1.6.](image-url)
1.6 Accelerated frames and fictitious forces

In section 1.1 we discussed noninertial frames, frames where Newton’s first law is false and Newton’s second law cannot be applied. A noninertial frame is, essentially, any frame which accelerates relative to any inertial frame. Amazingly, it is possible to force Newtonian mechanics to be valid in an accelerating frame if you judiciously add forces which do not exist, called fictitious forces. It is easiest to start the discussion of fictitious forces with a question involving linear acceleration.

**Question:** What is the force that causes you to fall over when a moving bus comes to an immediate stop? I’m having an argument with my teacher over what the answer is, it would be great if you could explain!

**Answer:** When the bus is stopping, it is accelerating and so it is a noninertial frame. That means that Newton’s laws are not valid if you are riding inside the bus. But, if we watch you from the bus stop, Newton’s laws do apply and we conclude that if you move with the bus, there must be a force which is causing you to accelerate also. Friction provides a force which, except under extreme circumstances, accelerates your feet along with the bus; but, unless you are holding on to something, there is nothing to provide a force on your upper body which therefore tends to keep going forward as the bus stops. All this says that the reason you fall forward is not due to any force, rather it is due to the lack of a force. There is, though, another way to look at this problem. If you are in an accelerating frame, like the bus, you can force Newton’s laws to be true by adding fictitious forces. In the bus which has an acceleration \( a \) you can invent a fictitious force \( F_{\text{fictitious}} \) on any mass \( m \) in the bus, \( F_{\text{fictitious}} = -ma \); the negative sign means that the fictitious force points in the direction opposite the acceleration. If you do that, Newton’s laws become true inside the bus and the force \( F_{\text{fictitious}} \) may be thought of as being the force which provides your acceleration. Note that the acceleration is opposite the direction the bus is moving when it is stopping, and so the fictitious force is forward as you know if you have fallen over in a stopping bus. When the bus is speeding up you tend to fall backwards. Since there are two answers here, depending on how you choose to view the problem, maybe you and your teacher are both right!

From this you learn that the secret to making Newtonian mechanics work in noninertial frames is to add fictitious forces to masses \( m \) whose direction is opposite the direction of the acceleration \( a \) of the frame and of magnitude \( ma \). Let us now re-examine the bicycle going around a curve which we looked at earlier. Here we encounter that best-known of nonexistent forces, the centrifugal force.

**Question:** Why when a cyclist is turning round a bend, does he lean inward?

**Answer:** Figure 1.7 shows the forces (real and fictitious) on the cyclist. The circle represents the center of mass of the system. Since he is moving in a circle of radius \( r \) and with speed \( v \), he experiences a centripetal acceleration \( a_c = v^2/r \) to the left. The forces on him are his own weight \( mg \), the normal force \( N \) up from the road, and the frictional force \( F_f \) which is the force providing the acceleration. If you want to apply Newton’s second law in the frame of reference of the cyclist, which is not an
inertial frame, you must add the fictitious centrifugal force $ma_c$ as shown in figure 1.7. Note that if he were not leaning, there would be an unbalanced torque about the point where the tire touches the ground, $\tau = mLv^2/r$ where $L$ is the distance to the center of mass, which would cause him to rotate clockwise, that is to fall over. When he leans, though, the weight also exerts a torque, so the two torques can balance if the angle is just right: $mgL \sin \theta = mLv^2 \cos \theta / r$, or $\theta = \tan^{-1}(v^2/rg)$.

The centrifugal force is often suggested in sci-fi movies or books as a source of artificial gravity. Imagine that you are inside a very large hollow cylinder with radius $R$ which is rotating around its axis such that the speed of the outer surface of the cylinder is $v$. You and the cylinder, your home, are in outer space with no gravity around. If you are at the inside surface and rotating with the cylinder, you are in a noninertial frame with acceleration, pointing toward the axis, of $v^2/R$. Then, if $v$ and $R$ have been chosen such that $v^2/R = g$, you will experience an apparent force $mg$, just as if you were standing on the surface of the Earth. Of course, $R$ must be much larger than your height or else your head and feet will experience different accelerations. Suppose that $R = 200$ m; taking $g \approx 10$ m s$^{-2}$, $v = \sqrt{2000} = 45$ m s$^{-1}$.

The circumference of the cylinder is $2\pi R = 1257$ m, so the time to make one revolution is $1257/45 = 280$ s = 0.47 h; the rotation is at a rather lazy rate of about two rotations per hour, a reasonable model for a space habitat. (See appendices C and D.)

The work–energy theorem, $W = \frac{1}{2}mv_{\text{final}}^2 - \frac{1}{2}mv_{\text{initial}}^2$ is derived (see appendix A) starting with Newton’s laws. The following question asks whether this is true in noninertial frames.

**Question:** Is work–energy theorem valid in noninertial frames?

**Answer:** The work–energy theorem says that the change in kinetic energy of an object is equal to the work all forces do on it. Imagine that you are in an accelerating rocket ship in empty space, a noninertial frame. You have a ball in
your hand and you let go of it. You observe this ball to accelerate opposite the
direction in which the ship is accelerating and therefore see its kinetic energy
change. But, there are no forces acting on it so no work is done. Another way you
could come to this conclusion is that the work–energy theorem is a result of
Newton’s laws and Newton’s laws are not valid in noninertial frames. You can,
though, force the work–energy theorem to be valid if you introduce fictitious
forces, a way to force Newton’s laws to work in noninertial frames. If you invent
a force on the objects of mass $m$ in the accelerating ($a$) rocket ship above of
$F_{\text{fictitious}} = -ma$, this force will appear to do the work equal to the change in
kinetic energy. (See appendix C.)

1.7 Wagers, arguments and disagreements

The Physicist is often called on to settle disputes. There are already a couple
examples in earlier sections, the accelerated bus question and the badminton/tennis
question. Here are a couple of others.

Question: My friend and I had a drunken argument. I would like independent
council to weigh in (there’s $300 on the line). I was given a unique bottle opener
by a friend who is a brewer for a craft brewery in the northeast. It is a flat piece of
wood with a smooth screw embedded near one end. The argument is as follows.

- **Person A:** There is less force required to open the bottle pulling up with the
screw positioned between the cap and the user (top panel in figure 1.8).

![Figure 1.8. A unique bottle opener, person A above, person B below.](image)
• Person B: There is less force required to open the bottle pressing down with the cap positioned between the screw and the user, (bottom panel in figure 1.8).

Can you prove either argument successfully?

**Answer:** The questioner also provided the information that $R = 94/16''$ and $d = 17/16''$. To answer the question I will compute the force which the nail exerts on the bottle top for equal forces by the user. Whichever of these is the biggest is the winner. Doing this is a simple first-semester physics statics problem, most easily done by summing the torques in each case about the point on the bottle cap just opposite the nail; that point is a distance $R$ from the end where $F$ is applied for person A and a distance $R - 2d$ for person B. I find that the nail exerts a force of $F_B = F[(R/d) - 2]$ for person B and a force of $F_A = F(R/d)$ for person A; person A is the winner of the bet. For your numbers, $R/d = 5.53$ and the ratio of the forces is $F_A/F_B = 5.53/3.53 = 1.57$, making option A 57% bigger, quite definitive. (If a $300 bet is really on the line, don’t forget to reward The Physicist!) (See appendix A.)

I am pleased to report that these barroom physics enthusiasts did indeed send a generous contribution to Ask The Physicist! Here is another question, this one about friction.

**Question:** I am writing in the interest of hopefully resolving a question which had arisen in my workplace. One gentleman poses the hypothetical situation of a motionless tank sitting on solid ice which he describes as ‘very slick and smooth—so much so that if one were to toss a penny across the surface then it would glide on endlessly’. He posits that the tank is then started and attempts to move forward. His position is that the tank will not be able to move as the treads would simply spin on the ice. His detractor posits that the treads are moved by the wheels inside the treads and that this would be able to propel the tank forward. So, would this tank be able to move forward or not? If so, what properties of physics would make it be able to move and, if not, why would this tank not be able to move forward? The gentleman’s scenario also posits that there is no friction between the tank treads and the ice. Is it realistic, physically speaking, to posit these two surfaces touching and no friction existing between them?

**Answer:** How genteel you are! The gentleman who says that the tank will not move forward if the ice is perfectly frictionless is correct. It is the force of friction which accelerates the tank forward, not the force which the wheels exert on the treads; if the wheels exert a force on the treads, then Newton’s third law says the treads exert an equal and opposite force on the wheels so the two cancel each other out if you are looking at the tank as a whole. No it is not possible to have a perfectly frictionless surface; it is possible to get a good enough approximation, however, to do an experiment which should convince the second gentleman.

There is an important lesson here. The force which propels a wheeled vehicle is the force of static friction between the wheels and the surface they are rolling on (or the force of kinetic friction if they are spinning).

Here is a dispute regarding torques and center of gravity.
**Question:** I have a question related to weight/mass placement on a bar. My friend and I are weight lifters. We got into a discussion about the center of gravity on the bar. Here is the question. We are using a 45 lb plate on each side and also have a 5 and 10 on each side, each taking up the same space and the end of the bar is the same distance from the last weight and will not change. Does it change anything if the weights are not in the same order, from one side to the other? My friend says the side with the 45 lb plate close to the end is slightly heavier because the ratio has changed. I say nothing has changed because the weights on the bar are still taking up the same space. I believe it would only change if the distance to the end of the bar is changed, which it is not. I hope I explained this well enough.

**Answer:** Assuming that the bar itself is uniform (has its center of gravity (COG) at its geometrical center), the COG of the total barbell depends on the location of the weights. Relative to the center of the bar, the position of the center of gravity may be written as 

$$
\text{COG} = \left(45x_1 + 10x_2 + 5x_3 - 45x_4 - 10x_5 - 5x_6\right)/120
$$

where the $x_i$ are the distances of weights from the center. Suppose that the weights are placed symmetrically ($x_1 = x_4$, $x_2 = x_5$, $x_3 = x_6$); then COG = 0, the center of the bar. Now, suppose we interchange two of the weights, exchange the 45 lb with the 10 lb on one side:

$$
\text{COG} = \left(45x_2 + 10x_1 + 5x_3 - 45x_4 - 10x_5 - 5x_6\right)/120 \\
= (45x_1 + 10x_2 - 45x_2 - 10x_1)/120 \\
= (35/120)(x_1 - x_2);
$$

since $x_1 \neq x_2$, COG $\neq 0$, the barbell is no longer balanced. If that explanation is too mathematical for you, try a more qualitative argument. Each weight $W$ a distance $D$ from the center exerts a torque about the center and the magnitude of that torque is $WD$. The net torque due to all weights must be zero if the bar is to balance at its center. This means that the sum of all the $WD$s on one side must be precisely equal to those on the other if the barbell is to be balanced about its center. If you change the $D$ on only one side, the bar will not be balanced at its center. (This qualitative argument is just the mathematical argument in words.) What certainly does not change is the total weight. (See appendix C.)

### 1.8 These are a few of my favorite things

This final section in chapter 1 collects some of my favorite questions and answers, questions which I found very interesting or questions from which I learned or just plain cool questions.

This first question surprised me greatly. I would never have guessed that gravity would pull two dice separated by a few centimeters together in a matter of hours.

**Question:** Two dice are suspended in outer space with no visible forces acting on them. Their centers of mass are 10 cm apart, and they each have an identical mass of .0033 kg. How long would it take for the force of gravity between them to cause them to touch? (We will assume they are volumeless for ease in calculation).
Answer: This seems a very difficult problem because the gravitational force between them changes as they get closer and so it is not a case of uniform acceleration. However, this is really just a special case of the Kepler problem (the paths of particles experiencing $1/r^2$ forces) which I have done in detail before. You can go over that in detail. For your case, $K = Gm_1m_2 = 6.67 \times 10^{-11} \times (3.3 \times 10^{-3})^2 = 7.26 \times 10^{-16}$ N·m²·kg⁻², the reduced mass is $\mu = m_1m_2/(m_1+m_2) = 0.0033/2 = 1.65 \times 10^{-3}$ kg, and the semimajor axis $a = 2.5$ cm $= 2.5 \times 10^{-2}$ m. Now, from the earlier answer, $T = \sqrt{(4\pi\mu a^3/K)} = 5.98 \times 10^4$ s. The time you want is $T/2 = 2.99 \times 10^4$ s. This is only 8.3 h and seemed too short to me. To check if the time is reasonable, I calculated the starting acceleration and assumed that the acceleration was constant and each die had to go 5 cm; this time should be longer than the correct time because the acceleration increases as the masses get closer. The force on each die at the beginning is $K/r^2 = 7.26 \times 10^{-16}/0.052 = 3.04 \times 10^{-13}$ N; so, the resulting initial acceleration is $F/m = 3.04 \times 10^{-13}/3.3 \times 10^{-3} = 9.21 \times 10^{-11}$ m·s⁻². So, assuming uniform acceleration, $0.05 = \frac{1}{2}at^2 = 4.61 \times 10^{-11}t^2$. Solving, $t = 3.3 \times 10^4$ s. So, the answer above is, indeed, reasonable.

The following question is one of my very favorites because it is so deceptively simple and yet so subtle to understand. I pondered this on and off for several days and finally needed to talk it over with some other physicists. I would like to acknowledge a very useful discussion over pizza with friends and colleagues A K Edwards, W G Love, R S Meltzer and R L Anderson.

Question: My question has to do with traction and the movement of a wheel (a wheel alone). Traction is essential for its movement both linear and circular. But if we throw a wheel forward it rolls some meters and then it stops (and falls). Which force is responsible for the decrease in its velocity? Because if traction is parallel to the ground facing backwards, then linear movement’s negative acceleration is explained but not angular negative acceleration. If traction is parallel to the ground facing forward then angular negative acceleration is explained but not linear. If traction is zero then which force decreases both velocities linear and angular?

Answer: One of the reasons I love doing Ask the Physicist is because I often learn things I did not know or had never thought about. You would think that a guy who has been teaching introductory physics courses for nearly 50 years would find this question simple. But, indeed I was puzzled by it because, as I have found by thinking about it and talking to some friends, I wasn’t thinking beyond the friction force (which questioner calls traction) being simply the only force in the horizontal direction and obviously stopping the forward motion after some distance. I never addressed the angular acceleration of the wheel before. What frictional forces are important to understand the rolling of a wheel? Most introductory physics classes talk only about the contact forces of static friction and kinetic friction. Kinetic friction is not applicable to this problem because the wheel is not slipping on the ground, and static friction might be important, but not necessarily. If we have a round wheel rolling on a flat horizontal surface (don’t look at figure 1.9 yet!), there
are three possible forces—the weight which must be vertical, pass through the center of mass, and (assuming it is a uniform wheel) pass through the point of contact; the friction, which must be parallel to the surface and pass through the point of contact; and the normal force which must be perpendicular to the surface and pass through the point of contact. If you now sum torques about the point of contact (as noted by the questioner), there are none! So, there can be no angular acceleration; if we have stipulated that the wheel does not slip, then there can be no linear acceleration either and the wheel will roll forever and no friction is required. But we all know better! A real wheel will eventually slow down. The key is that there is no such thing as a perfectly round wheel or a perfectly flat surface, one or both must be deformed. In that case, we have to think about a new kind of friction called rolling friction, the friction the wheel has because of the rolling. This is different from the static friction, and static friction may still be present to keep the wheel from slipping. A perfectly round wheel cannot have rolling friction as I showed above, it must deform which means that there is no longer a 'point (or line) of contact' but now an area of contact. Since the normal force is only constrained to act somewhere where the two are in contact, it is now possible (in fact inevitable) that this force will not act through the center of mass of the wheel. That is the whole key to answering this question. So, finally, the answer: refer to figure 1.9 where I have drawn the forces \( mg, N, \) and \( f \). The weight is still constrained to be vertically down and pass through the center of mass (blue cross). The normal force is constrained to be vertical and act somewhere where the wheel and ground are in contact, drawn a distance \( d \) to the left. The frictional force (which now includes both static and rolling friction) is constrained to act at the surface and parallel to it. I choose a coordinate system with \( x \) to the left and \( y \) up; the axis (red cross) about which I will sum torques is at the ground directly under the center of mass and positive torque results in an angular acceleration which is positive when acceleration of the center of mass is positive (counterclockwise around the axis). All is now straightforward: \( \Sigma F_x = -f = ma, \Sigma F_y = N - mg = 0, \Sigma \tau_x = Nd = Ia = Ia/L \) where \( I \) is the moment of inertia about the red cross and \( L \) is the distance from the red cross to the blue cross. Finally, \( N = mg, a = -f/m, \) and \( d = fl/(Lm^2g) \). (See appendix C.)
Sometimes I find myself in a debate which, like in the following case, lets me view a question I had dismissed with an open mind and coming around to common ground with the questioner. Here is the disturbing thing, though: after finally having given an answer to his question which explained how a blown-out tire at high speed could retain its roundness (but definitely not without the presence of the sidewalls), the questioner wrote thanking me for proving that sidewalls were not needed at high speeds! Guess he was less able to listen to the other side of the argument than I was.

**Question:** In 1973 a physics instructor explained that the sidewalls of a regulation tire need not be present if the velocity of the vehicle was above a speed of 65+ mph. I tried to explain this to family members at Christmas and was scoffed at and then ridiculed. The physics instructor had previously worked at a GMC/Chevrolet plant. His job had been to change out instruments on GM cars running around a track and in excess of 100 mph. In one of the test runs his driver advised him that they had had a blowout and he needed to get out from under the dash quickly and get safety belted in. Then the driver slowed down and at some critical speed he almost, but not quite, lost control and they did not crash but came close to it. The instructor was a good instructor in that he made the physics relevant to the real world. Also this is why tires need sidewalls as they won’t hold up in gravity and below a specific velocity.

**Answer:** This is nonsense. If there is no air pressure to connect the tire to the axle, which would be the case if there were no sidewalls, what is going to hold up the weight of the car?

**Follow-up question:** No not really. If you get the tire up to speed, as well as providing forward momentum, the circumference and the center point about which the tire is rotating will hold the tire up even if there is a blow out as the forward speed or acceleration is sufficient to hold it up and will prevent collapse of the tire above a critical speed. Once the speed drops below this critical speed, the tire will start to collapse and, according to the physics instructor, all hell broke loose on the track and only the driver’s expertise insured that they were able to stop safely. If you are above the critical speed, the outer rim of the tire need not have anything to hold it up. The key elements are:

1. Tires are inflated to the recommended PSI.
2. The vehicle was an experimental test GMC product running in excess of 100 mph.
3. When the driver announced that there had been a blowout, the car was under control and my physics teacher was not aware anything was amiss.
4. He was alive to prove it to the class, using physics concepts that escape me.

**Answer:** Well, maybe I misunderstand something here, but let’s boil this problem down to the simplest equivalent I can think of: imagine a tire with sidewalls and just an axle which is supported by the sidewall, shown on the left in figure 1.10. Now, we would agree, I believe, that if the sidewall suddenly disappeared, the axle would fall because there would be nothing holding up that weight. How is that
situation any different if the car is moving? So, let’s agree that ‘the sidewalls… need not be present’ is wrong because there has to be some physical contact of the outer surface of the tire and the axle. So, my first answer was a knee-jerk response to the notion that the sidewalls were not needed.

The answer you will like: However, there is still a way that you might have a point. When the blowout occurs, the pressure inside the tire is lost; this pressure is typically 30 PSI = 21 000 N m\(^{-2}\) above atmospheric pressure (which is about 100 000 N m\(^{-2}\)). If the car is sitting still, this loss of pressure results in the wheel collapsing because the sidewalls alone are insufficient to hold up the weight of the car unless the force due to the pressure pushing on the outer part of the tire holds the sidewalls taut. Now, imagine that you are driving with some speed \(V\) and viewing a spinning tire from its axis, you see every point on the outer surface of the wheel accelerating with an acceleration \(V^2/R\) where \(R\) is the radius of the tire. Therefore, every little piece of the tire with mass \(m\) experiences a (fictitious) force (called the centrifugal force) of \(mV^2/R\). That would be equivalent to there being a pressure \(P\) exerted on that little piece of tire of \(P = mV^2/(aR)\) where \(a\) is the area of that little piece. But, every little piece behaves like this, so it is equivalent to a pressure of \(P = MV^2/(AR)\) acting on the outer surface where \(M\) is the mass of the tire (assuming the sidewalls are a small fraction) and \(A = 2\pi RW\) is the area of the outer surface and \(W\) is the tread width. So, if that pressure is equal to 21 000 N m\(^{-2}\), it will be like the blowout never happened! I took \(R \approx 16\) in \(\approx 0.4\) m, \(W \approx 12\) in \(\approx 0.3\) m, and \(M \approx 20\) lb \(\approx 9\) kg and solved 21 000 = \(MV^2/(AR) = MV^2/(2\pi R^2 W)\) and found \(V = 27\) m s\(^{-1}\) = 60 mph. (Incidentally, the ‘forward momentum’ has nothing to do with it.)

The following question was fun because it got me thinking about how strong a rotating structure has to be. I am not an engineer and only claim to do an order-of-magnitude calculation here, but it seems to jive pretty well with estimates given by the questioner.

**Question:** I’ve read about space habitat concepts for a while and I’ve run into an interesting concept. The concept I’ve run into is the McKendree cylinder which is basically an O’Neill cylinder made of carbon nanotubes. The O’Neill cylinder made of steel would be 32 km long and 6 km in diameter. The McKendree
cylinder would be 4600 km long and 460 km in diameter. And the maximum length for the McKendree cylinder is 10 000 km and diameter of 1000 km. So the McKendree could be built a lot bigger than an O’Neill one because the carbon nanotubes have greater endurance. But a habitat of thousands of kilometers seems to be really big when compared to what we can build from other materials. And as I recall we don’t have any ways to produce carbon nanotubes in large quantities. Is it theoretically possible to build a habitat 10 000 km long and 1000 km wide out of carbon nanotubes? And is the McKendree cylinder more of a theoretical design than a practical design that actually could be built?

Answer: I presume that the issue is more a strength issue than anything else. To illustrate how the strength of the material and its mass determine the size the habitat can be, consider a rotating string of beads, each of mass \( m \). The rotation rate must be such that \( a = \frac{v^2}{R} = g \) where \( v \) is the tangential speed of each bead. Therefore each bead must experience a force \( F = mg \). This force can only come from the two strings attaching each bead to its nearest neighbors and, from figure 1.11, \( F = mg = 2T \sin \theta \). But, we will imagine many, many beads on this string and we will call the distance between them \( d \); so we can make the small angle approximation that \( \sin \theta \approx \theta = \frac{d}{R} \). Solving for \( T \), \( T = \frac{mgR}{2d} \). Now imagine that the beads are atoms; \( d \) will be about the same for steel or carbon, \( g \) is just a constant, \( m_{\text{steel}} \approx 5m_{\text{carbon}} \), and the Young’s modulus of carbon nanotubes is about five times bigger than steel, \( T_{\text{steel}} \approx \frac{T_{\text{carbon}}}{5} \). So, \( R_{\text{carbon}}/R_{\text{steel}} \approx \frac{T_{\text{carbon}}}{T_{\text{steel}}}/\left(\frac{m_{\text{carbon}}}{m_{\text{steel}}}\right) \approx 25 \). Your numbers are \( R_{\text{carbon}}/R_{\text{steel}} \approx 460/6 = 77 \); I would have to say that my calculation is pretty good given that I have made very rough estimates and I am not an engineer! I do not know what considerations would limit the length of the habitat. (Of course, neither of these models is currently practical to actually build, so call them theoretical if you like. However, there would certainly be no problem building them if resources and manufacturing capabilities were available.) (See appendix D.)

Figure 1.11. Forces on the edge of a rotating space station.