Field interpretation of General Relativity

M L Fil’chenkov and Yu P Laptev
Peoples’ Friendship University of Russia, Moscow, Russia
E-mail: fmichael@mail.ru

Abstract. A field interpretation of General Relativity, without resorting to Einstein-Hilbert’s equations, has been presented for three cases: a static centrally symmetric gravitational field, radiation of gravitational waves and a homogeneous isotropic cosmological model. The possibility of an effective Riemannian space being used in General relativity should not been ruled out.

1. Introduction
According to R. Feynman, gravity has both a field and geometric interpretation [1]. A. Eddington wrote: “… it is likely that some of the phenomena will be determined by comparatively simple equations in which the components of curvature of the world do not appear; such equations will be the same for a curved region as for a flat one…” [2]. Below the field interpretation of General Relativity will be exemplified by three well-known cases: a) a static centrally symmetric gravitational field, b) radiation of gravitational waves, c) a homogeneous isotropic cosmological model, without resorting to Einstein-Hilbert’s equations.

2. Static centrally symmetric gravitational field
The action has the form:

\[ S = -mc \int ds , \]  

where

\[ ds = c d\tau \sqrt{1 - \frac{v^2}{c^2}} \]  

is the interval,

\[ d\tau = dt \sqrt{g_{00}} \]  

is the proper time in a static gravitational field defined from the gravitational frequency shift [3]

\[ \frac{\Delta \omega}{\omega} = \sqrt{8 g_{00}} - 1 , \]  

where

\[ g_{00} = 1 + \frac{2\phi}{c^2} . \]  

\( \phi \) is the gravitational potential satisfying Poisson's equation

\[ \Delta \phi = 4\pi G \rho_{\text{eff}} , \]  

\( \rho_{\text{eff}} \) is the effective mass density.
The energy is defined as
\[ E = v \frac{\partial L}{\partial v} - L, \]  
where
\[ L = -mc^2 \sqrt{g_{00}} \sqrt{1 - \frac{v^2}{c^2}}, \]  
is the Lagrangian. From formulae (7), (8) we obtain [4]
\[ E = \frac{mc^2 \sqrt{g_{00}}}{\sqrt{1 - \frac{v^2}{c^2}}}. \]  

In fact, we are constructing an effective Riemannian space with \( g_{00}g_{11} = -1 \), using the static gravitational field satisfying Poisson's equation in a flat space. The curvature of such a Riemannian space is simulated by a mutual effect of gravity with the potential \( \phi \) and motion with the velocity \( v \) on the proper time of a test particle in Minkowski space. Taking into account that in the static centrally symmetric field, apart from the energy \( E \), the angular momentum
\[ J = \frac{\partial L}{\partial \phi} - \frac{mr^2 \phi}{\sqrt{g_{00}}} \sqrt{1 - \frac{v^2}{c^2}} \]
at \( \theta = \frac{\pi}{2}, \) \( (\phi \text{ is the polar angle}) \) is conserved, and we obtain the law of motion
\[ t = \frac{Jc}{E} \int \frac{dr}{g_{00}\sqrt{1 - \left( m^2 c^2 + \frac{J^2}{r^2} \right) g_{00}}} \frac{c^2}{E^2}, \]  
and the trajectory
\[ \phi = \frac{Jc}{E} \int \frac{dr}{r^2 \sqrt{1 - \left( m^2 c^2 + \frac{J^2}{r^2} \right) g_{00}}} \frac{c^2}{E^2}. \]  
of a test particle [5].

The gravitational potential \( \phi = -\frac{GM}{r} \) for the Newtonian field created by a point source with the mass \( M \) is given by Poisson's equation (6). From formula (5) we have the expression
\[ g_{00} = 1 - \frac{2GM}{c^2 r}, \]  
that completely coincides with the temporal component of Schwarzschild's metric derived from General Relativity. Thus, formula (5) proves to be valid in the general case, but not only in the nonrelativistic case, as often mentioned. Its left side is the temporal component of the metric satisfying Einstein-Hilbert's equations, and the right one contains the gravitational potential satisfying Poisson's equation (6).

Substituting (13) and \( E = mc^2 \) into (12), we have
\[ \frac{mv^2}{2} = \frac{GMm}{r}, \]  
wherefrom P. Laplace obtained a formula for the gravitational radius [6]
\[ r = \frac{2GM}{c^2}. \]
for \( v = c \).

The classical gravitational effects [2,5,7] can be found from the formula for the trajectory (12):

\[
\Delta \varphi = 2 \left| \varphi(R_o) - \varphi(\infty) \right| - \pi = \frac{2r_s}{R_o},
\]

(16)

where \( R_o \) is the Sun radius;

b) Mercury’s perihelion shift

\[
\Delta \varphi_p = 2 \left| \varphi(r_a) - \varphi(r_p) \right| - 2\pi = \frac{3\pi r_a^2}{p},
\]

(17)

where \( r_a \) is the aphelion, \( r_p \) the perihelion, \( p \) Mercury’s orbit parameter, \( r_s \) the gravitational radius of the Sun.

Consider one more static field being generated by a charged massive point source. The effective mass of this source is of the form

\[
M_{\text{eff}} = M - \frac{Q}{c^2 r},
\]

(18)

where the first term corresponds to an attractive gravitational mass \( M \) and the second one to an effective repulsive mass corresponding to the proper electrostatic energy of the charge \( Q \). Poisson's equation (6) for the effective mass density, which is related to the effective mass by the formula

\[
\rho_{\text{eff}} = 4\pi \int \rho_{\text{eff}} r^2 dr,
\]

(19)

has the solution as follows:

\[
\phi_{\text{eff}} = -\frac{GM}{r} + \frac{GQ^2}{2c^2 r^2}.
\]

(20)

From formula (5) we obtain the expression

\[
\phi_{00} = 1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}
\]

(21)

known as Reissner-Nordström’s metric.

3. Gravitational Radiation

Einstein's formula for the intensity of gravitational radiation [8]

\[
I = \frac{G}{5c^5} \bar{D}_{\alpha\beta} \bar{D}^{\alpha\beta},
\]

(22)

where

\[
\bar{D}_{\alpha\beta} = \int \rho_m \left( x_\alpha x_\beta - \frac{1}{3} \delta_{\alpha\beta} \right) dV
\]

(23)

is the quadrupole moment and \( \rho_m \) the mass density, is derivable from the intensity of the electric quadrupole radiation by substituting the photon by the graviton, the charge density by the mass density and introducing the gravitational constant [5]. Based on an analogy between Coulomb's law for charges and Newton's law for masses, we assume that accelerated masses radiate gravitational waves in the same way as accelerated charges radiate electromagnetic ones. The gravitational radiation is a quadrupole one due to the absence of a dipole moment, since the gravitational charge-to-mass ratio is constant. The spin of the graviton \( s = 2 \) is being equal to the minimum change of the orbital moment \( \Delta l = 2 \) in the quadrupole transitions. It in no way follows that a gravitational symmetric tensor field is related to a curved space-time. The astronomical data [9-10] validating a reality of gravitational waves support only the quadrupole formula (22) but are, in general, unrelated to a space-time curvature.

From General Relativity we have a zero divergence of the energy momentum tensor:
\( T^{ik} = \frac{\partial \left( T^{ik} + t^i k \right)}{\partial x^k} = 0 \), \hspace{1cm} (24)

where \( t^i k \) is the pseudotensor describing a free gravitational field, which is not invariant under coordinate transformations [11].

**4. Homogeneous Isotropic Cosmological Model**

From Einstein-Hilbert's equations follow Friedmann's equations for a homogeneous isotropic cosmological model [12]

\[
\frac{a^2}{2} - \frac{4\pi G \varepsilon a^2}{3c^2} = -\frac{kc^2}{2},
\]

\[
\ddot{a} = -\frac{4\pi G}{3} \rho_{\text{eff}},
\]

where \( a \) is the scale factor, \( \rho_{\text{eff}} c^2 = \varepsilon + 3p \), \( \varepsilon \) the energy density, \( p \) the pressure, \( k = 0, \pm 1 \) the model parameter. Equation (25) is the first integral of equation (26), providing

\[
a\ddot{\varepsilon} + 3(p + \varepsilon) \dot{a} = 0,
\]

which is the first law of thermodynamics for an adiabatic expansion of the Universe. From equation (27) it follows that

\[
\varepsilon \sim a^{-3(w+1)},
\]

where \( p = w \varepsilon \) is the barotropic equation of state for \( w = \text{const} \).

Friedmann's first equation was obtained by W. McCrea and E. Milne [13] as well in the framework of Newtonian theory of gravity from the energy conservation law of a unit mass on an expanding ball in Euclidean space using the relations:

\[
\phi(a) = -\frac{GM}{a}, \quad M = \frac{4}{3} \pi a^3 \rho,
\]

i.e. substituting \( r \) by \( a \) in the Newtonian gravitational potential.

Consider the case of de Sitter's vacuum:

\[
w = -1, p = -\varepsilon, \varepsilon = \varepsilon_0, \rho_{\text{eff}} = -\frac{2\varepsilon_0}{c^2}.
\]

Substituting \( \rho_{\text{eff}} \) from (30) into Poisson's equation (6), we have

\[
\phi_{\text{ds}} = \frac{c^2 r^2}{2r_0^3},
\]

where

\[
\frac{1}{r_0} = \frac{8\pi G \varepsilon_0}{3c^2},
\]

\( r_0 \) is de Sitter's horizon defining the law of de Sitter's Universe expansion \( a(t) \sim \exp \left[ \frac{ct}{r_0} \right] \), which follows from (25) for \( k = 0 \), \( \varepsilon = \varepsilon_0 \). Substituting \( \phi = \phi_{\text{ds}} \) into (5), we obtain the expression

\[
\varepsilon_{00} = 1 - \frac{r^2}{r_0^2}
\]

known as de Sitter's static metric [14].

Thus, we regain Newton's gravity for Friedmann's cosmological models and Poisson's equation for de Sitter's static model, without using Einstein-Hilbert's equations.
5. Conclusion
The results of importance for a static field, gravitational waves and cosmology may be obtained without resorting to Einstein-Hilbert's equations in a curved space-time. It is unlikely accidental, there are too many coincidences. More likely, this indicates that General Relativity is overdetermined, i.e. its mathematical formalism is unnecessarily complicated to obtain simple or even trivial results. The field interpretation of gravity, i.e. degeomerization of General Relativity, does not change its results; hence it is quite admissible alongside the geometric one. The possibility of an effective Riemannian space being used in General Relativity should not be ruled out.

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