Influence of nonuniform critical current density profile on magnetic field behavior of AC susceptibility in 2D Josephson Junction Arrays

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Employing mutual-inductance measurements we study the magnetic field dependence of complex AC susceptibility of artificially prepared highly ordered (periodic) two-dimensional Josephson junction arrays of unshunted Nb–AlO\(_x\)–Nb junctions. The observed behavior can be explained assuming single-plaquette approximation of the overdamped model with an inhomogeneous critical current distribution within a single junction.

**Keywords:** D. Josephson Junction Arrays; D. AC susceptibility; D. Inhomogeneous critical current distribution

I. INTRODUCTION

Despite the fact that Josephson Junction Arrays (JJA) have been actively studied for decades, they continue to contribute to the variety of intriguing and peculiar phenomena. To name just a few recent examples, it suffice to mention the so-called paramagnetic Meissner effect and related reentrant temperature behavior of AC susceptibility, observed both in artificially prepared JJA and granular superconductors (for recent reviews on the subject matter, see, e.g. \cite{1-4} and further references therein). So far, most of the investigations have been done assuming an ideal (uniform) type of array. However, it is quite clear that, depending on the particular technology used for preparation of the array, any real array will inevitably possess some kind of non-uniformity, either global (related to a random distribution of junctions within array) or local (related to inhomogeneous distribution of critical current densities within junctions). For instance, recently a comparative study of the magnetic remanence exhibited by disordered (globally nonuniform) 3D-JJA in response to an excitation with an AC magnetic field was presented \cite{5}. The observed temperature behavior of the remanence curves for arrays fabricated from three different materials (\(Nb, YBa_2Cu_3O_7\) and \(La_{1.85}Sr_{0.15}CuO_4\)) was found to follow the same universal law regardless of the origin of the superconducting electrodes of the junctions which form the array.

In the present paper, through an experimental study of complex AC magnetic susceptibility \(\chi(T, h_{AC})\) of the periodic (globally uniform) 2D-JJA of unshunted Nb–AlO\(_x\)–Nb junctions, we present evidence for existence of the local type non-uniformity in our arrays. Specifically, we found that in the mixed state region \(\chi(T, h_{AC})\) can be rather well fitted by a single-plaquette approximation of the overdamped 2D-JJA model assuming a nonuniform (Lorentz-like) distribution of the critical current density within a single junction.

II. EXPERIMENT

Our samples consisted of 100 × 150 unshunted tunnel junctions. The unit cell had square geometry with lattice spacing \(a = 46\mu m\) and a junction area of \(5 \times 5\mu m^2\). The critical current density for the junctions forming the arrays was about 600\(A/cm^2\) at 4.2\(K\), giving thus \(I_C = 150\mu A\) for each junction.

We used the screening method \cite{6} in the reflection configuration to measure the complex AC susceptibility \(\chi = \chi' + i\chi''\) of our 2D-JJA (for more details on the experimental technique and set-ups see \cite{7-9}). Fig.1 shows the obtained experimental data for the complex AC susceptibility \(\chi(T, h_{AC})\) as a function of the excitation field \(h_{AC}\) for a fixed
temperature below $T_C$. As is seen, below 50mOe (which corresponds to a Meissner-like regime with no regular flux present in the array) the susceptibility, as expected, practically does not depend on the applied magnetic field, while in the mixed state (above 50mOe) both $\chi'(T, h_{AC})$ and $\chi''(T, h_{AC})$ follow a quasi-exponential field behavior of the single junction Josephson supercurrent (see below).

### III. DISCUSSION

To understand the observed behavior of the AC susceptibility, in principle one would need to analyze the flux dynamics in our overdamped, unshunted JJA. However, given a well-defined (globally uniform) periodic structure of the array, to achieve our goal it is sufficient to study just a single unit cell (plaquette) of the array. (It is worth noting that the single-plaquette approximation proved successful in treating the temperature reentrance phenomena of AC susceptibility in ordered 2D-JJA [4,7,8] as well as magnetic remanence in disordered 3D-JJA [5].) The unit cell is a loop containing four identical Josephson junctions. Since the inductance of each loop is $L = \mu_0 a \simeq 64pH$ and the critical current of each junction is $I_C = 150\mu A$, for the mixed-state region (above 50mOe) we can safely neglect the self-field effects because in this region the inductance related flux $\Phi_L(t) = LI(t)$ (here $I(t)$ is the total current circulating in a single loop [10]) is always smaller than the external field induced flux $\Phi_{ext}(t) = B_{ac}(t)S$ (here $S \simeq a^2$ is the projected area of a single loop, and $B_{ac}(t) = \mu_0 h_{AC} \cos \omega t$ is an applied AC magnetic field). Besides, since the length $L$ and the width $w$ of each junction in our array is smaller than the Josephson penetration depth $\lambda_J = \sqrt{\Phi_{0}/2\pi\mu_0 d_j\phi_0}$ (where $d_j\phi_0$ is the critical current density of the junction, $\Phi_0$ is the magnetic flux quantum, and $d = 2\lambda_L + \xi$ is the size of the contact area with $\lambda_L(T)$ being the London penetration depth of the junction and $\xi$ an insulator thickness), namely $L \simeq w \simeq 5\mu m$ and $\lambda_J \simeq 20\mu m$ (using $j_{c0} = 600A/cm^2$ and $\lambda_L = 39nm$ for Nb at $T = 4.2K$), we can adopt the small-junction approximation [10] for the gauge-invariant superconducting phase difference across the ith junction (by symmetry we assume that $\phi_1 = \phi_2 = \phi_3 = \phi_4 \equiv \phi_i$)

$$\phi_i(x, t) = \phi_0 + \frac{2\pi B_{ac}(t)d}{\Phi_0} x \tag{1}$$

where $\phi_0$ is the initial phase difference.

The net magnetization of the plaquette is $M(t) = SI_s(t)$ where the maximum supercurrent (corresponding to $\phi_0 = \pi/2$) through an inhomogeneous Josephson contact reads

$$I_s(t) = \int_0^L dx \int_0^w dy j_{i\omega}(x, y) \cos \phi_i(x, t) \tag{2}$$

For the explicit temperature dependence of the Josephson critical current density

$$j_{i\omega}(T) = j_{i\omega}(0) \left[ \frac{\Delta(T)}{\Delta(0)} \right] \tanh \left[ \frac{\Delta(T)}{2k_B T} \right] \tag{3}$$

we used the well-known [11] analytical approximation for the BCS gap parameter (valid for all temperatures), $\Delta(T) = \Delta(0) \tanh \left( 2.2\sqrt{\frac{T_{c0}}{T}} \right)$ with $\Delta(0) = 1.76k_B T_C$.

In general, the values of $\chi'(T, h_{AC})$ and $\chi''(T, h_{AC})$ of the complex harmonic susceptibility are defined via the time-dependent magnetization of the plaquette as follows:

$$\chi'(T, h_{AC}) = \frac{1}{\pi h_{AC}} \int_0^{2\pi} d(\omega t) \cos(\omega t) M(t) \tag{4}$$

and

$$\chi''(T, h_{AC}) = \frac{1}{\pi h_{AC}} \int_0^{2\pi} d(\omega t) \sin(\omega t) M(t) \tag{5}$$

Using Eqs. (1)-(5) to simulate the magnetic field behavior of the observed AC susceptibility of the array, we found that the best fit through all the data points and for all temperatures is produced assuming the following nonuniform distribution of the critical current density within a single junction [10].
It is worthwhile to mention that in view of Eq.(2), in the mixed-state region the above distribution leads to approximately exponential field dependence of the maximum supercurrent $I_s(T, h_{AC}) \approx I_s(T, 0)e^{-h_{AC}/h_0}$ which is often used to describe critical-state behavior in type-II superconductors [12]. Given the temperature dependencies of the London penetration depth $\lambda_L(T)$ and the Josephson critical current density $j_c(0)$, we find $h_0(T) = \Phi_0/2\pi\mu_0\lambda_L(T)L \approx h_0(0)(1-T/T_C)^{1/4}$ for the temperature dependence of the characteristic field near $T_C$. This explains the improvement of our fits (shown by solid lines in Fig.1) for high temperatures because with increasing the temperature the total flux distribution within a single junction becomes more regular which in turn validates the use of the small-junction approximation.

In summary, we found clear experimental evidence for the influence of the junction nonuniformity on magnetic field penetration into the periodic 2D array of unshunted Josephson junctions. By using the well-known AC magnetic susceptibility technique, we have shown that in the mixed-state regime the AC field behavior of the artificially prepared array is reasonably well fitted by the single-plaquette approximation of the overdamped model of 2D-JJA assuming inhomogeneous (Lorentz-like) critical current distribution within a single junction.

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FIG. 1. The dependence of the magnetic susceptibilities, \( \chi'(T, h_{AC}) \) and \( \chi''(T, h_{AC}) \), on AC magnetic field amplitude \( h_{AC} \) for different temperatures: \( T = 4.2K \) (a,b), \( T = 6K \) (c,d), and \( T = 8K \) (e,f). Solid lines correspond to the fitting of the 2D-JJA model with nonuniform critical current profile for a single junction (see the text).