Correlations among $T_c$, $A$, and $\rho_o$ within FL region of $T$-$p$ phase diagram of heavy-Fermion superconductors

M. ElMassalami$^1$, P. B. Castro$^1$, M. B. Silva Neto$^1$

$^1$ Instituto de Física, Universidade Federal do Rio de Janeiro, 21941-972 Rio de Janeiro RJ, Brazil
E-mail: massalam@if.ufrj.br, CASTRO.Pedro@nims.go.jp, mbsn@if.ufrj.br

Abstract. Extensively reported experimental observations indicate that on varying a control parameter, such as pressure $p$, within the phase diagram of most quantum critical heavy fermion HF superconductors, one identifies a cascade of distinct electronic states which may be magnetic, of Kondo-type, non-conventional superconducting, Fermi Liquid, FL, or non-FL character. Of particular interest is the part of the phase diagram wherein superconductivity emerges from a strongly renormalized FL state. This region resembles the overdoped region of the $T$-doping phase diagram of cuprate superconductors. Remarkably, within this highly nontrivial region, one identifies a universal correlation among $T_c$ and $A$: $\ln \frac{T_c}{\theta} \propto A^{\frac{1}{2}}$ ($\theta$ is a characteristic energy scale and $A$ is the coefficient of $T^2$ resistivity term). Commonly, these features are considered to be driven by a Spin-Fluctuation-mediated electron-electron scattering channel. On adopting such a channel and applying standard theories of Migdal-Eliashberg (superconductivity) and Boltzmann (transport), we derive analytic expressions that satisfactorily reproduce the aforementioned empirical correlations.

1 Introduction
Recently, much attention has been focused on the overdoped region of high-$T_c$ cuprate superconductors wherein only superconductivity and Fermi liquid, FL, state are involved \cite{1}. This region offers a welcome simplification since one avoids complications related to competing states such as antiferromagnetism, spin glass, stripes, or pseudogap. More importantly, the continuous and monotonic evolution of $T_c$ suggests that the same pairing channel is involved: Then, as that the trend of $T_c \rightarrow 0$ within this region is considered to be related to a decrease in the strength of the spin-fluctuation, SF, mediated pairing channel \cite{1} (a), it is tempting to correlate the shape of $T_c$ dome to the rise and fall of the strength of this channel.

In spite of this gain in simplification, analysis of this overdoped region is plagued with doping-induced disorder \cite{1}. But, there is a similar region within the $T$-$p$ phase diagrams of the closely related heavy fermion, HF, superconductors which does not manifest disorder effects \cite{2}. Indeed, the similarity is evidenced as: (i) Within this HF region, the pairing is considered to be mediated by SF modes \cite{3}; (ii) the superconductivity emerges from a strongly renormalized FL state; and (iii) $T_c \rightarrow 0$. Interestingly, in addition to avoiding the influence of competing states (antiferromagnetism, spin glass phase, non-FL state, etc.), here one also avoids disorder effects: a much better simplification, electing HFs as suitable candidates for such an investigation.

It is remarkable that, in spite of an absence of a deliberately introduced disorder, the residual resistivity, $\rho_o$, of HF superconductors is considerably high and manifest nonmonotonic baric evolution [see Fig. 1(d)]. We consider this resistivity excess to be due to a scattering from SF-related non-homogeneous spacial distribution within a HF superconducting sample: at each
instant, only some randomly-distributed micro-sized domains of a sample are assumed to sustain SF modes and that their average dimension is longer than the mean free path and coherent length. The concentration of these domains (a measure of the strength of the SF-mediated channel) is reflected in the excess of $\rho_s$. Moreover, we consider that the very same SF-mediated e-e coupling is responsible for the emergence of both superconductivity and FL character: This is reminiscent to that the electron-phonon interaction is responsible for both the BCS superconductivity and Bloch-Gruneisen $\beta T^3$ term. Accordingly, within the FL region, an increase in a control parameter (e.g. pressure) leads to a decrease in the size and concentration of the SF-bearing domains, till eventually one reaches, at very high pressure, the SF-free state wherein no modes are available for mediation and thus to a removal of SF-related features: quench of superconductivity ($T_c \rightarrow 0$), reduction of FL behavior ($A_{\text{tot}} \rightarrow A_{\text{c}}$), and reduction of residual resistivity ($\rho_{\text{tot}} \rightarrow \rho_{\text{c}}$). Below, two FL contributions are empirically identified: (i) a conventional FL contribution manifested as $A_{\text{c}}$; (ii) another, driven by SF-electron scattering channel, is manifested as $A_{\text{tot}} - A_{\text{c}}$.

In this work, we are interested in analyzing the involved SF-mediated interaction channel and how this channel influences the evolution of $A$, and $T_c$ within the FL region of a typical archetype HF superconductor (analysis of further HF examples are given in Ref. [4]). Our analysis identified universal correlations among $\rho_s$, $A$, and $T_c$. As mentioned above, we argue that the SF-electron scattering channel is responsible for the emergence of the superconductivity, the FL character, and the correlations among them. On adopting such a mechanism and applying standard Migdal-Eliashberg theory for description of superconductivity and Boltzmann theory for description of transport properties, we derive analytic expressions that compare favorably with the empirically-obtained correlations.

**Figure 1.** (a) $T$-$p$ phase diagram of CeCoIn$_5$. Also shown are the baric evolution of $T_c(p)$ in (b), $A_{\text{tot}}(p)$ in (c), and $\rho_{\text{tot}}(p)$ in (d). All curves were taken from the continuous line (the visual guide) of Ref. [3]. The red symbols and hatched area identify the FL region while the dashed lines are visual guides. For $p \rightarrow 4.4$ GPa, $T_c \rightarrow 0$, $A \rightarrow A_0 = 0.082(4) \mu \Omega \text{cm}$ and $\rho_0 \rightarrow \rho_0^c = 0.130(5) \mu \Omega \text{cm}$ (all shown as green large filled circle). (c) A plot of $A_{\text{tot}} - A_0$ versus $\rho_{\text{tot}}^c - \rho_0^c$. The solid curve is the best fit to Eq. (3) ($A_1 = 6.0(2) \times 10^4 \Omega^{-1} \text{cm}^{-1}/K^2$). (f) $T_c$ versus $(\rho_0 - \rho_0^c)$. The solid calculated line is based on Eq. (5). (g) $\ln(T_c)$ versus $(A - A_0)^{-1}$. The best fit to the linearization of Eq. (4) (Fit parameters $\theta = 3.2(1)K$, $F = 1.8(1) \times 10^{-4}(\Omega \text{cm}/K^2)^{1/2}$). (h) The generalized plot of $T_c(p)$ versus $\lambda = \frac{(A - A_0)^{1/2}}{F}$ of the tabulated HF superconductors [2, (c)], [07, 3, 8, 9, 10, 11, 12, 13, 14, 15]. Each pair of sample-dependent $\theta$ and $F$ was evaluated within the pressure range wherein Eq. (4) is valid [see Figs. 3(b)]. As evident, the distribution of ($T_c/\theta$, $\lambda$) extends up to $\approx (0.6, 2.1)$. This is similar to the limit reported for strong superconductors, see Ref. [16] and Fig. 4 of Ref. [17].
2 Results and Analysis

2.1 Empirical analysis: Identification of correlations among \( \rho_o, A, \) and \( T_c \)

The \( T-p \) phase diagram of CeCoIn$_5$ [2] is shown in Fig. 1(a) while the baric evolution of \( T_c(p), A_{tot}(p) \) and \( \rho_{o}^{tot}(p) \) are shown in Figs. 1(b), 1(c) and 1(d), respectively. All data of Figs. 1 were taken from Ref. [5]. As shown in the figures, the FL character is manifested at sufficiently higher pressure and is evidenced as a quadratic-in-\( T \) resistivity contribution, marked by the hatched area and the \( n=2 \) notation. We are interested in the FL region which starts at the event of switching from NFL into FL character and ends when \( T_c \to 0 \). Specifically, the \( T_c \to 0 \) event at 4.4 GPa is taken as signaling the pressure-induced quench of the SF-related FL contributions and that all other parameters extrapolate to non-SF-bearing limits: namely, \( A_{tot} \to A_o \) and \( \rho_{o}^{tot} \to \rho_{o}^{c} \). Then, based on Matthiessen’s rule, we identify two residual resistivities: \( \rho_{o}^{c} \) and \( (\rho_{o}^{tot} - \rho_{o}^{c}) \). In fact, as mentioned above, we identify two contributions: (i) The non-SF-bearing contribution leading to \( \rho_{o}^{c} \), \( A_o \), a weak coupling constant \( \lambda_o \), and a mean free path \( \ell_o \) (\( \ell_o \ll \ell_o \)).

(ii) The SF-bearing contribution leading to \( \rho_{o} = \rho_{o}^{tot} - \rho_{o}^{c} \), \( A = A_{tot} - A_o \), \( \lambda = \lambda_{tot} - \lambda_o \), and \( \ell = 1/(\ell_{tot} - \ell_o) \). Based on such a clear distinction, we were able to identify three correlations among \( T_c, A, \) and \( \rho_o \). As shown in Fig. 1(e,f,g), these are, respectively, \( |A_{tot} - A_o| \propto |\rho_{o}^{tot} - \rho_{o}^{c}|^2 \), \( \ln \frac{T_c}{\bar{T}} \propto |\rho_{o}^{tot} - \rho_{o}^{c}|^{-1} \), and \( \ln \frac{T_c}{\bar{T}} \propto (A_{tot} - A_o)^{-2} \).

2.2 Theoretical analysis: interpretation of the correlations among \( \rho_o, A, \) and \( T_c \)

It is taken that within a HF superconducting sample, the SF-related quasiparticles can be created or annihilated and that their mediation of the e-e interaction, \( V_{ee}^s \), is the driving mechanism behind the surge of the features understudy. As usual, the \( I^2 \chi(\omega) \) [\( I \) is the exchange coupling while \( \chi(\omega) \) is the dynamic susceptibility] plays the same role as that of Eliashberg’s electron-phonon spectral function \( \alpha^2 F(\omega) \). \( V_{ee}^sf \) can then be obtained from the integral over frequencies of such an Eliashberg’s type spectral function. Therefore, any modification in the interaction strength \( V_{ee} \) or redistribution of the spectral weight \( V_{ee} \) would modify both \( T_c \) and \( A \) as well as their correlation.

On adopting the traditional strategy for calculating \( T_c(\ell) \), we solved the set of coupled linearized Eliashberg’s equations within the BCS limit. In this limit, \( T_c(\ell) \) is determined by the superconducting coupling constant \( \lambda(\ell) = N(e_F)V_{ee}^sf(\ell) = 2\int d\omega \omega^2 \chi(\omega) \omega^{opt} \omega^2 \), where \( \omega^{opt} \) is an optimal frequency at which \( \lambda \) is maximal, and can thus be written as [19]:

\[
T_c(\ell) = \theta \exp \left( \frac{-1 - \lambda(\ell)}{\lambda(\ell) - \mu^*} \right),
\]

where \( \mu^* \) is the Coulomb pseudopotential and \( \theta \) is an energy scale that depends on the cut-off bandwidth \( \omega^s \). On applying a variational approach to the linearized version of Boltzmann’s transport equations within the relaxation time approximation (wherein the inverse scattering time is calculated by the use of Fermi’s golden rule) we obtain [19] \( \Delta \rho = A(\ell)T^2 \) and

\[
A(\ell) = F^2_\ell \left| \frac{\lambda(\ell) - \mu^*}{1 + \lambda(\ell)} \right|^2.
\]

Here, \( F_\ell \) represents the efficiency of momentum relaxation and the availability of phase space for scattering [19]. Within the FL phase (where \( \ell \) is long, \( 1 \ll k_F \ell < \infty \), \( \rho_o \) is small, and \( k_F \) is the Fermi wave number), Eq. (2) can be expanded to give the following relation [19]:

\[
A(\ell) \approx A_o + A_1(\rho_o) + A_2(\rho_o)^2,
\]
where $A_i(i = 0, 1, 2)$ are functions of $F_\ell$, $\mu^*$, and $\lambda_0 = \lambda(\ell \to \infty)$. The finding of Fig. 4 indicates that Eq. 2 is reduced to $(A - A_0) \propto (\rho_0)^2$: a contribution with a dominant $A_2$ but a negligible $A_1$ (no Koshino-Taylor contribution).

Finally, on combining the expressions for $T_c(\ell)$ in Eq. 1 and $A(\ell)$ in Eq. 2 we arrive at

$$T_c(\ell) = \theta e^{-F}/\sqrt{A(\ell)}.$$  \hspace{1cm} (4)

This universal and exact kinematic scaling relation (valid for long $\ell$, low $\rho_0$, within the Fermi-liquid region) is the essence of the the fits in Figs. 4g,h.

Finally, a simplified but approximate relation between $T_c$ and $(\rho_0^{\text{tot}} - \rho_0^2)$ can be obtained in the specific case wherein the quadratic-in-$\rho_0$ term in Eq. 3 is dominant leading to

$$T_c(\ell) \approx \theta \exp\left(\frac{-F}{\sqrt{A_2(\rho_0^{\text{tot}}(\ell) - \rho_0^2)}\right}.$$  \hspace{1cm} (5)

On substituting $\theta$, $F$, $A_2$, and $\rho_0^2$, the solid curve of Fig. 4f is obtained with no adjustment.

3 Discussion and Conclusion

Analysis of Fig. 4 indicates that on moving out of the FL region towards the quantum critical point, one observes a monotonic and continuous enhancement in $\rho_0$, $A$, and $T_c$: an indication that the same interaction channel is in operation and that its strength is enhanced on reducing the pressure. However, Fig. 4 also indicates that below the FL region our analytical expressions do not reproduce the evolution of $A(p)$ or $T_c(p)$. This shortcoming is related to the breakdown of the long-$\ell$ condition and as such does not invalidate our analysis within the FL region.

In summary, we identified three empirical correlations among $T_c$, $A$, and $\rho_0$ within the FL region of the investigated $T$-$p$ phase diagrams. These correlations are attributed to a SF-mediated electron-electron interaction. Theoretically, on adopting many-body techniques, we derive analytic expressions for $T_c$ and $A$ and their correlations that reproduce satisfactorily the aforementioned empirical correlations. It is worth adding that our internally-consistent empirical and theoretical analyses are much helped by the clarity in identifying the difference between the normal (non-SF-related) and the SF-related contributions.

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