Behavior of a Double-Layer Pipeline Under Static Load

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Abstract. The authors investigated the features in the behavior of various components of "pipe in pipe" system when changing the internal pressure in the annular space. During the research, the dependence of plastic pipe creep modulus on tensile stress was received. Using a method of least squares, the authors calculated the empirical coefficients for dependences of plastic pipe creep modulus on pipeline service period and stresses in the pipe material caused by static load. They revealed an unequivocal relationship between the parameters of a double-layer pipeline with the maximum use of plastic or steel pipelines resistance.

1. Introduction
As a rule, the use of "pipe in pipe" system in construction practice means pulling a plastic pipe inside a steel pipeline [2]. Hereby we get a new double-layer pipeline consisting of pipes made from different materials. Modeling the behavior of such a pipeline under a load is complicated by non-linear deformation properties of a plastic pipe interacting with a steel pipeline [6], which in its turn has linear deformation properties.

Moreover, using this design, the repair technology allows us to get a pipeline, various layers of which seal tightly to each other [7], wherein clearance between them is virtually eliminated. Nevertheless, in practice there may be loose fit pipelines [4] and the presence of any size of the annular space between the internal and external pipelines.

The present article solves the problem of defining the rational parameters of a pipeline section, that is the ratio of diameters for given values of the outside diameter and wall thickness of a steel pipeline. We assume the existence of a gap between pipelines in "pipe in pipe" system.

2. Materials and methods
Strain properties of plastics are characterized by a non-linear relationship between strain and stress that causes it [1]. It is known that creep modulus $E$ of the pipe material is a value that depends on stress $\sigma$. This dependence is manifested differently for various materials. Because of the insignificance it can be ignored for steel materials up to a certain limit - the elastic limit.

However, it is noticeable even at relatively low (compared to steel) stresses for plastic materials [9], and if we ignore this dependence, it could cause serious errors in mechanical calculations. As can be seen from the graph (figure 1) built according to table 4 of the Code SP 42-101-96, creep modulus $E$ varies linearly with changes in stress. Tabulated function $E(\sigma)$ can be approximated by a linear dependence of the form:

$$E = E_0 - K\sigma,$$  \hspace{1cm} (1)

where $E_0$, $K$ – model parameters determined from the condition of the minimum sum of squared deviations.
The sum of squared deviations:

\[ \Delta = \sum_{i=1}^{n} (E_i - E_0 + K\sigma_i)^2. \]  

(2)

Resolving equations of a method of least squares look like:

\[
\begin{aligned}
ne_0 - K \sum_{i=1}^{n} \sigma_i &= \sum_{i=1}^{n} E_i; \\
E_0 \sum_{i=1}^{n} \sigma_i - K \sum_{i=1}^{n} \sigma_i^2 &= \sum_{i=1}^{n} E_i \sigma_i.
\end{aligned}
\]  

(3)

where \( n \) – the number of data points.

The system of equations (3) has the following solution:

\[
E_0 = \frac{\sum_{i=1}^{n} E_i \sigma_i - \sum_{i=1}^{n} E_i \sum_{i=1}^{n} \sigma_i^2}{\left(\sum_{i=1}^{n} \sigma_i\right)^2 - n \sum_{i=1}^{n} \sigma_i^2};
\]  

\[
K = \frac{n \sum_{i=1}^{n} E_i \sigma_i - \sum_{i=1}^{n} E_i \sum_{i=1}^{n} \sigma_i}{\left(\sum_{i=1}^{n} \sigma_i\right)^2 - n \sum_{i=1}^{n} \sigma_i^2}.
\]  

(4)

Calculations were carried out for the data shown in table 4 of SP 42-101-96. And the following results were obtained for the dependence (1).

For the 10-year period of pipeline service:

\[ E = 245.17 - 26.67\sigma. \]  

(5)

For the 25-year period of pipeline service:

\[ E = 224.60 - 24.36\sigma. \]  

(6)

And for the 50-year period of pipeline service:

\[ E = 217.67 - 25.93\sigma. \]  

(7)

Parameter \( E_0 \) values are expressed in MegaPascals.

The nonlinearity of the strain properties of plastic pipes \([5]\) may be accounted by successive approximations with a quasi-static formulation of the problem. In this case, creep modulus values
should be calculated step by step according to (1) using the previous step stress value obtained by formulas for a linear dependence of stress and strain. Wherein the process of successive approximations terminates automatically when sufficient clarification of the required value is reached.

The stress-strain state of a plastic pipeline located inside a sealed steel pipeline can be schematically represented as follows (figure 2).

![Figure 2. The cross section of “pipe in pipe” system: 1 - plastic pipe; 2 - steel pipe.](image)

As shown in figure 2, there is a plastic pipeline inside a steel pipeline. The internal volume of the plastic pipe is designated as $V_1$ and the volume of the annulus is designated as $V_2$. The inner pressure $P_1$ causes the accordant radial stress $\sigma_1$ and radial displacement in the wall of the plastic pipe, which in its turn causes a change in the volume of the annulus $V_2$ and pressure $P_2$ according to Clapeyron equation ($PV = \text{const}$) [9].

The total radial stress in the wall of the plastic pipeline may be expressed as:

$$\sigma_{r1} = \frac{(P_1 - P_2)D_{1in}}{2\delta_1}, \quad (8)$$

where $D_{1in}$ – the inside diameter of the plastic pipe; $\delta_1$ – the wall thickness of the plastic pipe.

The corresponding radial displacement of the plastic pipe wall would be:

$$\Delta r_1 = \frac{\sigma_{r1}D_1}{2E_1} = \frac{(P_1 - P_2)D_{1in}D_1}{4\delta_1E_1}, \quad (9)$$

where $E_1$ – the elastic modulus of plastics determined by (1); $D_1$ – the median diameter of the plastic pipe cross section area.

The radial stress in the wall of the steel pipeline may then be calculated as:

$$\sigma_{r2} = \frac{P_2D_{2in}}{2\delta_2}, \quad (10)$$

where $D_{2in}$ – the inside diameter of the steel pipe; $\delta_2$ – the wall thickness of the steel pipe.

And the corresponding radial displacement of the steel pipe wall would be:
\[ \Delta r_2 = \frac{\sigma_1 D_2}{2E_2} = \frac{P_2 D_{\text{in}} D_2}{4\delta_2 E_2} , \]  
where \( D_2 \) – the median diameter of the steel pipe cross section area; 
\( E_2 \) – the elastic modulus of steel, which is equal to 2,000,000 MPa.

After pipes are deformed, the volume of the annulus became equal to:
\[ V_2' = \pi \left( \frac{D_{\text{in}}}{2} + \Delta r_2 - \frac{D_{\text{out}}}{2} - \Delta r_1 \right)^2 , \]  
where \( D_{\text{out}} \) – the outside diameter of the plastic pipe;

According to Clapeyron equation, the gas volume change will cause a corresponding change in gas pressure. The equation of gas state for the isothermal process looks like:
\[ P_2 V_2 = P_2' V_2' , \]  
where \( P_2 \) and \( V_2 \) – the pressure and volume of gas in the annular space prior to pipe deformation; 
\( P_2' \) and \( V_2' \) – the pressure and volume of gas in the annular space after pipe deformation.

In consideration of (12) and (13), we find a new gas pressure value:
\[ P_2' = \frac{P_2 V_2}{V_2'} . \]  

Then, we repeat the calculation of stresses and the corresponding radial displacements of the pipe wall using the new value of gas pressure in the annular space. Continuing the calculations (4) - (10), we obtain a new increment of gas pressure in the annular space, and each approaching will reduce these increments. We will continue the calculations as long as the increments are notable.

However, the actual conditions of pipeline construction demand the determination of rational parameters of the “pipe in pipe” system cross section.

Pipeline cross section parameters \( D_{\text{out}} \) and \( \delta_1 \) must satisfy the following conditions for given values of the outside diameter \( D_{\text{out}} \) and wall thickness \( \delta_1 \) of a steel pipe (figure 1):

At first, the radial stress in the plastic pipe \( \sigma_1 \) should not exceed the rated plastic resistance \( R_{1p} \):
\[ \sigma_1 \leq R_{1p} . \]  
Secondly, the radial stress in the steel pipe \( \sigma_2 \) should not exceed the rated steel resistance \( R_{1s} \):
\[ \sigma_2 \leq R_{1s} . \]  
Substituting (16) into (10) we obtain:
\[ R_{1s} \geq \frac{P_2 D_{\text{in}}}{2\delta_2} . \]  
Consequently:
\[ P_2 \leq \frac{R_{1s} 2\delta_2}{D_{\text{in}}} . \]  

The pressure inside the plastic pipeline is limited by the strength of the plastic pipeline and may be calculated as:
\[ P_1 \leq P_2 + \frac{R_{1p} 2\delta_1}{D_{\text{out}} - 2\delta_1} . \]  

Taking the atmospheric pressure as the initial pressure in the annulus and observing Clapeyron equation, we find the initial volume of the annulus from the expression:
\[ P_2' V_2' = V_2 , \]  
where \( V_2 \) and \( V_2' \) – the volume of the annular space before and after deformation, respectively; 
\( P_2' \) – the pressure of gas in the annular space after pipe deformation.

In accordance with the condition of strength, the maximum permissible strain of the radial displacement of the plastic pipe wall will be:
\[
\Delta r_i = \frac{R_i p D_i}{2E_i}.
\]

Having in mind that the value of the initial gap between the walls of the plastic and steel pipes cannot be less than the value determined by (21), we find:

\[
D_{\text{out}} = D_{2\text{in}} - 2\Delta r_i
\]

Substituting (21) into (22) we obtain:

\[
D_{\text{out}} = D_{2\text{in}} - \frac{R_i p}{E_i} (D_{\text{out}} - \Delta r_i)
\]

Therefore:

\[
D_{\text{out}} = \frac{E_i}{E_i + R_i p} \left( D_{2\text{in}} + \frac{R_i p}{E_i} \Delta r_i \right)
\]

On the other hand, a decrease in \(V_2\) as a result of radial displacement of the plastic pipe wall should provide an increase in the annulus pressure from one atmosphere to a value determined by the expression (18). That is, according to Clapeyron equation, the volume of the annulus should be reduced \(P_2'\) times (see (20)).

\[
V_2' = \pi \left( \frac{D_{2\text{in}}}{2} + \Delta r_2 - \frac{D_{\text{out}}}{2} - \Delta r_1 \right)^2 = \frac{V_2}{P_2'}.\]

From (25) we obtain:

\[
\left( \frac{D_{2\text{in}}}{2} + \Delta r_2 - \frac{D_{\text{out}}}{2} - \Delta r_1 \right)^2 = \frac{1}{P_2'} \left( \frac{D_{2\text{in}}}{2} - \frac{D_{\text{out}}}{2} \right)^2.
\]

Consequently:

\[
\frac{D_{2\text{in}}}{2} + \Delta r_2 - \frac{D_{\text{out}}}{2} - \Delta r_1 - \frac{1}{\sqrt{P_2'}} \left( \frac{D_{2\text{in}}}{2} - \frac{D_{\text{out}}}{2} \right) = 0.
\]

That is:

\[
\frac{D_{2\text{in}}}{2\sqrt{P_2'}} \left( \sqrt{P_2'} - 1 \right) + \Delta r_2 - \Delta r_1 = \frac{D_{\text{out}}}{2\sqrt{P_2'}} \left( \sqrt{P_2'} - 1 \right).
\]

Expressing \(D_{\text{out}}\) from the equation (28), we get:

\[
D_{\text{out}} = \frac{2\sqrt{P_2'}}{\sqrt{P_2'} - 1} (\Delta r_2 - \Delta r_1) + D_{2\text{in}}.
\]

Next, we combine (29) and (24):

\[
\frac{E_i}{E_i + R_i p} \left( D_{2\text{in}} + \frac{R_i p}{E_i} \Delta r_i \right) = \frac{2\sqrt{P_2'}}{\sqrt{P_2'} - 1} (\Delta r_2 - \Delta r_1) + D_{2\text{in}}.
\]

3. Results and Conclusion

Thus, we obtain the plastic pipe wall thickness corresponding to the conditions (15) and (16):

\[
\delta_1 = \frac{E_i + R_i p}{R_i p} \left( \frac{2\sqrt{P_2'}}{\sqrt{P_2'} - 1} (\Delta r_2 - \Delta r_1) + D_{2\text{in}} \right) - \frac{E_i}{R_i p} D_{2\text{in}}.
\]

Equations (24) - (31) give us unambiguous double-layer pipeline parameters, taking into account the maximum resistance of both plastic and steel pipelines.

The results of this work can be used not only in the area of constructing urban infrastructure facilities, water supply and sewerage. Pipelines consisting of parts having different physical properties
may find application in the construction of underwater crossings by the "pipe in pipe" method, and it already concerns directly oil and gas pipelines, including pipes of average diameters. The given work can also be used for the calculations of forecasting the service life of double-layer pipeline sections [3], selection of rational parameters for their design, and ensuring the maximum long service period of underwater crossings [8].

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