Performance Measurement in the Health Care Sector: a Leader-Follower Network DEA Model based on Double Frontier Analysis

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Abstract. The efficiency evaluation of laboratories as one of the most significant areas of healthcare plays a key role in the quality of laboratories management. In this paper, a three-stage structure with optimal desirable and undesirable inputs and outputs has been taken into consideration by us. This network comprises of a leader and two followers. The suggested model simulates the internal structure of a diagnostic lab (pre-test, test and post-test). The criteria for evaluation are achieved by using the Fuzzy Delphi technique. Due to the environmental, economic and social impacts of health care systems, the significance of sustainability criteria is obvious in the case study indicators. We use the non-cooperative method multiplicative DEA technique to evaluate the efficiency of the network from both the optimistic and pessimistic views. Moreover, a heuristic technique was used to convert non-linear models into linear ones. Finally, we suggest to use a k-means method to cluster DMUs into several groups with similar characteristics based on double-frontier Standpoint.

Keywords: Network DEA; Medical Diagnostic Laboratories; Sustainability; Non-Cooperative Game; Double-Frontier; Additional Inputs; Undesirable Outputs; K-means Algorithm.

1. Introduction

Currently, medical diagnostic laboratories (MDLs) are a significant part of the health organization in the real world. The MDL performs a vital role in the arenas of healthcare, diagnosis, maintenance and prevention of different illnesses. Based on global standards, the share of MDLs in the healthcare market is about 5.6 percent, which represents a significant contribution. Due to the economic, social and environmental situations of the MDLs, there is an increasing public demand for better-quality performance, reduced functional costs and improved quality of such organizations. Efficacy as the main pillar of growth, is one of the most commonly used mechanisms for evaluating the performance

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of a health care system including MDLs. Therefore, it is of special significance to survey the performance of MDLs by appraising efficiency and productivity. So we can judge the efficiency and inefficiency of the Decision Making Units (DMUs). Then, the lacks can be reduced and the strong points increased. Likewise, the optimal use of available resources will also happen. Throughout the past years, the role of the MDLs in the sectors of healthcare has made important progress and the jobs of Sohn et al. [1] and Fang et al. [2] are significant in this respect. It is obvious that, in order to evaluate the effectiveness of MDLs, we need to use an appropriate tool. Throughout the past years, numerous models suggested for evaluating performance according to two general parametric and non-parametric approaches. In this paper, we will utilize the network DEA as a nonparametric method. This technique chooses the efficient DMUs and creates the efficiency and inefficiency frontiers. This frontiers are two criteria for the measuring of other DMUs. In this research, we evaluate the efficiency by utilizing the DEA technique for the four aims. 1- DEA measures the efficiency of the system based on a logical model. 2- DEA discovers efficient and inefficient DMUs. 3- DEA specify the grade of inefficiency of the DMUs. 4- DEA specifies the patterns for the inefficient DMUs [3]. Since MDLs contain three levels (the pre-test, the test and the post-test) and have a multi-stage nature, in this study we apply network DEA (NDEA) models to evaluate efficiency. This paper considers a three-stage network with additional undesirable inputs and outputs. In fact, DEA has proposed as a theoretical framework for performance analysis, but its application in the field of health care has very low. This network is proposed to evaluate the performance and ranking of laboratory units with consideration of sustainability criteria (economic, social and environmental). So, we designed a three-stage network of three laboratory processes. In this regard, the pre-test process involves of the reception unit and the sample unit. The process of test involves of a test unit and the post-test process involves of the test results unit. The case study includes 25 MDLs in Tehran in the real world and subsequently, we shall analyze the results based on the double-frontier viewpoint. The structure of this paper has been rendered as follows: part (2) reviews the literature on the DEA method. Part (3) discusses the research methodology. Then, the mathematical modeling of the problem is described. In part (4), a heuristic approach is introduced to solve the nonlinear program. Part (5), comprises of the result of a case study of the paper. Lastly, part (6) is conclusion of the paper.
2. Literature Review

Currently, efficiency measurement is an extremely crucial issue towards a better understanding of problems in a system and planning for future improvement [4]. DEA is one of the most important and appropriate approaches for measuring the performance of DMUs [5]. DEA goals are to measure efficiency and present the efficient and inefficient units. Efficient units generate the maximal amount of outputs from the minimal amount of inputs and their efficiency equates to (1). These units form the efficiency frontier and the other units which have an efficiency ranging from (0 to 1) are identified as inefficient units. The DEA has such capacities, where each DMU, can be considered in segregation and specifies the efficiency appraisal on the basis of the distance of this unit till the efficiency frontier [6].

In his initial DEA task, Farrell [7], which was later, expensed by Charnes et al. [8], was reputedly known CCR or the (Charnes-Cooper-Rhodes DEA Model). Then, Banker et al. [9] developed the DEA and proposed BCC or the (Banker-Charnes-Cooper Model). DEA is a non-parametric linear programming model to evaluate the performance for a set of homogeneous DMUs with several inputs and outputs [10]. In classical DEA models, such as the CCR and BCC models, we do not consider the intermediate measures of DMUs or internal operations of DMUs and measure the efficiencies of DMUs as a “Black Box” [11]. In actual fact, the efficiency analysis in this method is carried out by the initial inputs and the final outputs. Ignoring the internal structure of systems leads to the classical DEA models where important information and segregation between the efficient units cannot be presented [12]. To overcome the problem, Fare and Grosskopf [13] suggested a network DEA (NDEA) model. In the NDEA models, the internal structure of systems and internal interactions of DMUs are taken under consideration until there is an increase in the accuracy of efficiencies. Kao [14] categorized the network models into three sets, namely, series, parallel and hybrid. Kao stated that, when activities in a system are protracted in respect to each other, the system is of a series structure; and whenever activities are in a parallel form alongside each other, the system has a parallel structure. Similarly, when there is a hybrid condition between the series and parallel aspects, a hybrid mode is engaged. The NDEA approach can simulate networks and then estimate the efficiencies [3,
15]. The internal structure of systems can simulate with the sub-DMUs either in series or in parallel. Thus, the systems with series and parallel structures are two very significant areas in NDEA [15]. For the parallel structure, the sum of the inputs or outputs of all stages are considered as the inputs or outputs of the whole structure, but in the series structure the inputs of first stage and the outputs of last stage are the inputs and outputs of the whole structure [16]. The general efficiency of the parallel and series structures are measured by the multiplicative and additive methods, respectively [17]. In recent years, the efficiency evaluation of the multi-stage is one of the most important topics in NDEA and the parallel and series structures are used by many researchers.

Kao [14] proposed a “closed system” with a series structure to be taken under consideration for intermediate measures, but without any additional input or output in each stage; whereas, Yu and Lin [18] used the NDEA to measure service effectiveness and technical efficiency. Kao [4] utilized the NDEA approach to evaluate the overall efficiency of the network with multi-stage and additional inputs. Cook et al. [19] used the network DEA for the evaluation of efficiency. In the network structure, the sub-DMUs have desirable or undesirable outputs.

The role of undesirable factors has been extremely crucial in NDEA, in the recent years, Liu et al. [20] used the clustering techniques and defined this part as one of the four Main parts of NDEA. For the first time, Fare et al. [21] took the undesirable factors under consideration to evaluate the efficiency in DEA models. Lu and Lo [22-25] categorized the methods for working with undesirable outputs in network DEA in the following three modes: 1-The first mode is to ignore the undesirable outputs which are done in order to simplify the models actually. 2-The second mode is to measure distances in such a manner, so as to limit the expansion of the undesired output or that the undesired output is modeled as a nonlinear network DEA model. 3-The third mode is to consider the undesired output as a desired input, or to employ the negative sign as a desirable output, or that a decrease in conversion is applied to them. Over the past few years, Wang et al. [26] and Wu et al. [27] contemplated on the role of undesired factors in manufacturing processes and utilized the NDEA to measure efficiency. In recent years, the evolution of unfavorable features has led to the use of undesirable factors for the generation of favorable aspects. For example, in a new approach, Wu et al. [28] considered an interactive network consisting of two stages, where the first stage inserts the
undesirable outputs to the second stage and ultimately, the second stage produces the desirable output and in actual fact, has utilized the undesirable outputs for production.

In recent years, NDEA models have undergone development and models combining this science, with the game theory branch have been rendered [29]. Li et al. [30] presented a network DEA model with two-stage, a part which holds a more significant viewpoint is named as “leader” and the other part is named as “follower”. The performance of the leader part is maximized to the optimum and thus, the performance of the follower part is secured by keeping a constant performance in the part of the leader. This exemplary, is designated as a Stackelberg Game. In another research, An et al. [31] considered a network in two stages in an interactive mode and compared the efficiency of this network in a cooperative and non-cooperative (leader-follower mode). In yet another research by Zhou et al. [32] who dealt with evaluating the efficiency performance of a multi-stage network in the black box and non-cooperative (leader-follower) mode, comparing their results with each other. A research performed by Du et al. [33] in the grounds of leader-follower, can also be designated, in which they studied a parallel structure in the cooperative and non-cooperative modes. On the basis of the abovementioned facts, the main difference between the black box and the network approach is summarized in the internal correlations of systems. Some of the previous researchers have studied on Bi-level programming under different situations as follows: Maiti and Roy [34] designed the model on bi-level programming for Stackelberg game under intuitionistic fuzzy environment. Maiti and Roy [35] offered and solved multi-choice stochastic bi-level programming problem in cooperative nature via fuzzy programming method; and Roy and Maiti [36, 37] studied on bi-level programming with multi-choice for Stackelberg game under fuzzy and stochastic environments.

DEA with a double-frontier contemplates on two frontiers to compute the efficiency for each DMU. One is the efficient frontier and the other, the inefficient one. The efficiency calculated by the efficient frontier is called the optimistic efficiency, whereas, the efficiency computed by the inefficient frontier is known as the pessimistic efficiency [38]. In the optimistic view, each DMU, along with a set of efficient units, which form the efficient frontier, are compared. In the pessimistic view, each DMU, together with a set of inefficient units, forming the inefficient frontier are taken to comparison [30]. The value of the optimistic view is less than or equal to (1); whereas, the efficiency
of the pessimistic view is greater or equivalent to (1). The optimistic and pessimistic efficiency values are exactly equivalent to (1), if the DMU under evaluation, is placed respectively on the efficient or inefficient frontier [39, 40]. Though, when calculating the optimistic efficiency, the nearer the DMU proves to be to the efficiency frontier, the more desirable, whereas, in the case of computing the pessimistic efficiency, the further the distance of the DMU, the better and has an additional desirability. In fact, the double-frontier views each DMU from two outlooks and any conclusion which implies to only one of these perspectives, shall be one-sided and inadequate [41, 42]. For the first time, Doyle et al. [43] computed the efficiency of DMUs from the optimistic and pessimistic views. In recent years, many researchers have utilized the double-frontier for evaluating efficiency and stated various approaches towards calculating an overall performance; in this concern tasks in relative to Wang and Chin can be designated, who proposed a numerical measure for a general geometrical mean efficiency [44-47].

Producing a sustainable product as a practical way to minimize the environmental impacts of a product is the significant way for reaching sustainability [48]. In order to assess eco-efficient performance, Chen et al. [49] recently introduced the concept of "sustainable performance". This concept explains how achieve the desired output or lower undesired output in the process of production. Sustainability assessment is not limited to environmental criteria. Accordingly, three categories of sustainability factors (social, economic, and environmental) are presented in the literature. Determining sustainability goals requires some knowledge and comprehending of the current level of sustainability. This can be achieved through sustainability assessments taking into account all three factors of sustainability "economic, social, and environmental" [50]. Since the evaluation of a system involves a wide range of economic, social and environmental indicators, this leads to complex multi-criteria decision-making problems. A possible way to simplify the assessment is to define the concept of sustainability and to determine the importance of economic, social and environmental indicators [51]. Extensive research has conducted on methods and applications of DEA, but these efforts have focused mainly on assessing of DMUs in area of engineering. More recently, researchers have used the DEA to evaluate system performance by considering sustainability factors. However, many of these studies cover only environmental and economic aspects, but social
dimension has neglected as a dimension of sustainability [52, 53]. Table 1 reviews the studies which have applied the game theory methods in DEA. The last row of Table 1 presents characteristics of the current paper.

DEA has proposed as a theoretical framework for performance analysis, but its application in the field of health care has very low. As a summarization, contributions of this research are as below:

- A three-stage network is taken under consideration in regard to the additional desirable and undesirable inputs and outputs.
- We simulate a MDL with three level (pre-test, test and post-test) which allowed us to obtain important information about the causes of inefficiency and efficiency of laboratory units.
- We consider sustainability criteria (economic, social and environmental) to appraise the performance of MDLs, thus helping to improve the social, economic, and environmental problems of laboratories.
- The criteria for evaluation are gotten by utilizing the Fuzzy Delphi method
- A Double Frontier Approach is utilized to evaluate efficiency, in order to make results more realistic.
- k-means algorithm based on the double-frontier view is suggested to determine the efficient and inefficient points of the network.
- A heuristic technique is suggested to turn non-linear models into linear ones
- Implementation of the suggested model on an authentic example.

3. Methodology

In this study, the methodology is formed in four parts. In First part, the data and variables are gathered according to the library studies, interview and observation by Fuzzy Delphi technique. In Second part, a network DEA method is explained to evaluate the efficiency of units according to the double-frontier viewpoint. In Third part, a heuristic method is suggested to turn non-linear models into linear ones. Lastly, k-means algorithm based on the double-frontier view is suggested to determine the efficient and inefficient points of the network. In Fig. 1, the methodology is shown as below.

3.1. Identification of indicators
Variables are not identified and there is no structural for guidance. Therefore, the criteria for evaluation are achieved by analyzing organizational documents (articles, library studies) and observation (interview). Then, for screening the findings of this stage, experts’ opinions and the Fuzzy Delphi technique are used to achieve consensus about the influential criteria. In the Fuzzy Delphi technique, the information is established from the experts in the form of a written language and then evaluated by a Fuzzy technique. The questionnaire was considered with the goal of gaining expert opinions about the extent to which they agreed with the model’s criteria. Therefore, the experts have said their agreement by verbal variables (Very High, High, Medium, Low and Very Low). These variables are shown by Table 2 as follows:

A triangular fuzzy number denoted by \( M = (m, \alpha, \beta) \). \( m \) represents the mean value, \( \alpha \) is the left hand spreads of \( M \) and \( \beta \) is the right hand spreads of \( M \). The defuzzification value were calculated by Minkowski’s Formula in Table 2 as follows:

\[
x = m + \frac{\beta - \alpha}{4}
\]  

Firstly, we used two approaches of observation and documentation to achieve the most significant indicators in the MDL arena and to collected indicators. The proper indicators are showed in Table 3.

In the following, the effective criteria of the three MDL processes (pre-test, test, post-test) were gained from the Fuzzy Delphi technique. The conceptual model along with the descriptions of the criteria, is sent to the expert members. In the following, the amount of their agreement with criteria is taken. Given the offered options and the linguistic variables defined in the questionnaire, the results shown in Table 4 are shown. The Fuzzy average of each of the criterias are calculated by Formulas (3.2) and (3.3). In formulas (3.2) and (3.3), \( A_i \) and \( A_{ave} \) are represents the expert opinion \( i \) and the average of expert opinion, respectively. Also \( n \) represents the number of experts.

\[
A_i = (a_{1(i)}, a_{2(i)}, a_{3(i)}) \quad i = 1, 2, \ldots, n \tag{3.2}
\]

\[
A_{ave} = (m, \alpha, \beta) = \left( \frac{1}{n} \sum_{i=1}^{n} a_{1(i)}, \frac{1}{n} \sum_{i=1}^{n} a_{2(i)}, \frac{1}{n} \sum_{i=1}^{n} a_{3(i)} \right) \tag{3.3}
\]
In Table 4, the defuzzification operations and the triangular fuzzy average are calculated by Formulas (3.3) and (3.1), respectively. According to Cheng et al. [62], if the difference between the first and second rounds be lower than the threshold of very low (0.1), consensus has been made and the process will break. In the polls of the second round, the new questionnaire is designed and, with the prior viewpoint of opinion of each specialist and their degree of change with the viewpoint of other experts, is given to the expert members. Also, the members of the specialist group answered to the design questions again according to the opinions of other members of the group. The results of which are shown in Table 5.

In the last column of Table 5, expert members of the expert group have gotten agreement in totally criteria, except "available pace for service, number of false tests, waste weight", due to that the difference between the first and second round is lower than the threshold of very low (0.1). Therefore, we poll to continue for only three criteria. At third round, a third questionnaire Including three criteria was planned. According the prior point of opinion of each specialist and their degree of change with the view of other experts, the questionnaire was again given to the expert members. The fuzzy analysis of the results shown in Table 6.

Table 6 shows, the difference between the second and third rounds is lower than the threshold of very low (0.1). According to Cheng et al. [62], the consensus stops at this round. In the Table 5, the two criteria are in the range of Very low effect to Medium effect are removed. The criteria of available space for service, average sample transfer time, average waiting time for sampling, cost of consumables, garbage weight, income from admission, number of active experiments, number of false tests, number of kits, safety cost of sampling unit and safety cost of test unit, staff wage, sum of the scores of the laboratory standards and test response time are in the range of very high effect to high effect. Other criteria of correct number of tests, lab profit, number of patients' admitted and number of replies to the prepared tests are in the range of medium effect. Out of twenty-one effective criteria of MDLs, in two rounds of the polls, by removing three criteria: "cost of laboratory space and land value, and cost of staff welfare", lastly, eighteen executable criteria were specified in the area of
MDLs. Table 7 shows effective final indicators for measuring the efficiency of MDLs by Fuzzy Delphi technique.

3.2. Model Description

We study a set of n homogeneous DMUs that are shown by $\text{DMU}_j$ ($j=1,\ldots, n$), and each $\text{DMU}_j$ has three-stages with a complex internal structure as shown in Fig. 2. We define the desirable and undesirable inputs and outputs of the first stage by $x_{ij} \left( i_1 = 1, 2, \ldots, I_1 \right)$, $x_{ij} \left( i_2 = 1, 2, \ldots, I_2 \right)$, $y_{rj} \left( r_1 = 1, \ldots, R_1 \right)$ and $y_{rj} \left( r_2 = 1, \ldots, R_2 \right)$, respectively. We define the intermediate measures between stages by $z_{d_{ij}} \left( d_1 = 1, 2, \ldots, D_1 \right)$ and $z_{d_{ij}} \left( d_2 = 1, 2, \ldots, D_2 \right)$. The additional desirable inputs and undesirable outputs of the second stage are defined by $x_{ij} \left( i_3 = 1, \ldots, I_3 \right)$ and $y_{rj} \left( r_3 = 1, \ldots, R_3 \right)$ respectively. Lastly, we define, the additional desirable inputs and desirable outputs of the third stage by $x_{ij} \left( i_4 = 1, \ldots, I_4 \right)$ and $y_{rj} \left( r_4 = 1, \ldots, R_4 \right)$. Kao and Hwang [16] used uniform weights for the intermediate variables to evaluate the efficiency of a network. Therefore, we utilized similar weights for the intermediate variables in models. We show $v_{i_1}$, $v_{i_2}$, $v_{i_3}$ and $v_{i_4}$ as the weights of the inputs to the first, second and third stages, respectively. We show that the weights relative to the intermediate measures between stages by $\eta_{i_1}$ and $\eta_{i_2}$, respectively. The weights of the outputs for the first, second and third stages are defined by $u_{r_1}$, $u_{r_2}$, $u_{r_3}$ and $u_{r_4}$, respectively.

Researchers in efficiency analysis are likely to use input-oriented models, due to three major reasons. Firstly, because demand is on the growth and estimating demand is an intricate matter. Secondly, managers have more control over inputs than outputs. Thirdly, this model reflects the primary goals of policymakers, based on being responsible in responding to the requirements of people and units must reduce costs, or else, limit the use of resources. Thereby, in this research we utilize the input-oriented model. In accordance with Korhonen and Luptacik [63], we signify the undesirable output in the models with a negative mark as a desirable output. Also, we consider the undesirable input as a desirable input with a negative mark. Based on the opinions of managers, we will consider the first
stage (reception unit), the second stage (sampling and testing unit) and the third stage (test results unit) in the roles of the “leader”, “first follower” and “second follower”, respectively. Thence, we demonstrate the optimistic and pessimistic efficiencies of the leader’s stage with $\theta^L_o$ and $\varphi^L_o$ respectively; the optimistic and pessimistic efficiencies of the second and third stages as $\theta^{1F}_o$, $\theta^{2F}_o$ and $\varphi^{1F}_o$, $\varphi^{2F}_o$ respectively. In based on the ideas of managers, we depict the first stage as the leader and the second and third stages together, as a follower. On these bases, the optimistic and pessimistic efficiencies of the second and third stages together are defined as $\theta^{12F}_o$ and $\varphi^{12F}_o$ respectively.

We define the maximal efficiency of the leader stage on the bases of Li et al. [29] approach from the optimistic viewpoint as hereunder:

**Model 1**

$$\theta^L_o = \max \left\{ \frac{\sum_{d=1}^{D_1} \eta_{d}^1 z_{d,\eta} - \sum_{r=1}^{R_1} \mu_{r,\eta} y_{r,\eta} - \sum_{i=1}^{I_1} \nu_{i} x_{i,\eta}}{\sum_{i=1}^{I_1} v_{i} x_{i,\eta} - \sum_{i=2}^{I_2} v_{i} x_{i,\eta}} \right\}$$

s.t.

$$\sum_{d=1}^{D_1} \eta_{d}^1 z_{d,\eta} - \sum_{r=1}^{R_1} \mu_{r,\eta} y_{r,\eta} - \sum_{i=1}^{I_1} \nu_{i} x_{i,\eta} \leq 1, \quad j = 1, \ldots, n$$

$$\sum_{i=1}^{I_2} v_{i} x_{i,\eta} - \sum_{i=2}^{I_2} v_{i} x_{i,\eta} \leq 1, \quad j = 1, \ldots, n$$

$$\sum_{i=1}^{I_4} \nu_{i} x_{i,\eta} - \sum_{i=2}^{I_4} \nu_{i} x_{i,\eta} \leq 1, \quad j = 1, \ldots, n$$

The optimum efficiency has been demonstrated with the symbol (*) in the Model (1). Through model (1), the maximal efficiency of the leader’s stage ($\theta^L_o$) was attained on conditions that, none of the efficiencies of the other stages ($\theta^{1F}_o, \theta^{2F}_o, \theta^{12F}_o$) are more than (1) or...
\( \theta_{L}^{o} = \max \left\{ \theta_{o} | \theta_{1}^{j} \leq 1, \theta_{2}^{j} \leq 1, \theta_{3}^{j} \leq 1, j = 1, \ldots, n \right\} \). Model (1) is fractional and by using the Charnes-Cooper conversion [64], such as illustrated hereunder, they are converted to linear models:

Let \( T = \frac{1}{\sum_{i=1}^{\alpha_{1}} y_{i1}x_{i1}^{o} - \sum_{i=1}^{\alpha_{2}} y_{i2}x_{i2}^{o}} \), thus:

**Model 2**

\[
\theta_{o}^{L} = \max \left\{ \frac{\sum_{j=1}^{\varepsilon_{1}} \eta_{d_{1}}^{j}z_{d_{1}}^{j} + \sum_{j=1}^{\varepsilon_{2}} u_{r_{2}}^{j}y_{r_{2}}^{j} - \sum_{j=1}^{\varepsilon_{3}} u_{r_{3}}^{j}y_{r_{3}}^{j}}{\sum_{j=1}^{\varepsilon_{4}} y_{i1}^{j}x_{i1}^{j} - \sum_{j=1}^{\varepsilon_{5}} y_{i2}^{j}x_{i2}^{j}} \right\}, \quad j = 1, \ldots, n
\]

s.t. \( \sum_{j=1}^{\varepsilon_{1}} y_{i1}^{j}x_{i1}^{j} - \sum_{j=1}^{\varepsilon_{2}} y_{i2}^{j}x_{i2}^{j} = 1 \)

\[
\sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}}^{j}z_{d_{1}}^{j} + \sum_{r_{2}=1}^{R_{2}} u_{r_{2}}^{j}y_{r_{2}}^{j} - \sum_{r_{3}=1}^{R_{3}} u_{r_{3}}^{j}y_{r_{3}}^{j} - \left( \sum_{i_{1}=1}^{I_{1}} y_{i1}^{j}x_{i1}^{j} - \sum_{i_{2}=1}^{I_{2}} y_{i2}^{j}x_{i2}^{j} \right) \leq 0, \quad j = 1, \ldots, n
\]

\[
\sum_{d_{2}=1}^{D_{2}} \eta_{d_{2}}^{j}z_{d_{2}}^{j} - \sum_{r_{2}=1}^{R_{2}} u_{r_{2}}^{j}y_{r_{2}}^{j} - \left( \sum_{i_{1}=1}^{I_{1}} y_{i1}^{j}x_{i1}^{j} + \sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}}^{j}z_{d_{1}}^{j} \right) \leq 0, \quad j = 1, \ldots, n
\]

We, based on the tasks of Wang et al. [65], have modified Model (2) and have defined the efficiency of the leader stage from the pessimistic view according to the following:

**Model 3**

\[
\phi_{o}^{L} = \min \left\{ \frac{\sum_{j=1}^{\varepsilon_{1}} \eta_{d_{1}}^{j}z_{d_{1}}^{j} + \sum_{j=1}^{\varepsilon_{2}} u_{r_{2}}^{j}y_{r_{2}}^{j} - \sum_{j=1}^{\varepsilon_{3}} u_{r_{3}}^{j}y_{r_{3}}^{j}}{\sum_{j=1}^{\varepsilon_{4}} y_{i1}^{j}x_{i1}^{j} - \sum_{j=1}^{\varepsilon_{5}} y_{i2}^{j}x_{i2}^{j}} \right\}, \quad j = 1, \ldots, n
\]

s.t. \( \sum_{j=1}^{\varepsilon_{1}} y_{i1}^{j}x_{i1}^{j} - \sum_{j=1}^{\varepsilon_{2}} y_{i2}^{j}x_{i2}^{j} = 1 \)

\[
\sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}}^{j}z_{d_{1}}^{j} + \sum_{r_{2}=1}^{R_{2}} u_{r_{2}}^{j}y_{r_{2}}^{j} - \sum_{r_{3}=1}^{R_{3}} u_{r_{3}}^{j}y_{r_{3}}^{j} - \left( \sum_{i_{1}=1}^{I_{1}} y_{i1}^{j}x_{i1}^{j} - \sum_{i_{2}=1}^{I_{2}} y_{i2}^{j}x_{i2}^{j} \right) \geq 0, \quad j = 1, \ldots, n
\]

\[
\sum_{d_{2}=1}^{D_{2}} \eta_{d_{2}}^{j}z_{d_{2}}^{j} - \sum_{r_{2}=1}^{R_{2}} u_{r_{2}}^{j}y_{r_{2}}^{j} - \left( \sum_{i_{1}=1}^{I_{1}} y_{i1}^{j}x_{i1}^{j} + \sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}}^{j}z_{d_{1}}^{j} \right) \geq 0, \quad j = 1, \ldots, n
\]
\[ \sum_{r_j=1}^{R_j} u_{r_j} y_{r_j} - \left( \sum_{i_k=1}^{I_k} \sum_{i_k=1}^{I_k} x_{i_k j} + \sum_{d_j=1}^{D_j} \eta_{d_j} z_{d_j} \right) \geq 0, \quad j = 1, \ldots, n \]

\[ \eta_{d_j} d_{d_j}, u_{r_j}, u_{r_j}, v_{i_k}, v_{i_k}, v_{i_k} \geq \varepsilon; d_1 = 1, 2, \ldots, D_1; d_2 = 1, 2, \ldots, D_2; r_1 = 1, 2, \ldots, R_1; r_2 = 1, 2, \ldots, R_2; \]

\[ r_3 = 1, 2, \ldots, R_3; r_4 = 1, 2, \ldots, R_4; i_1 = 1, 2, \ldots, I_1; i_2 = 1, 2, \ldots, I_2; i_3 = 1, 2, \ldots, I_3; i_4 = 1, 2, \ldots, I_4. \]

The maximal optimistic and minimal pessimistic efficiencies of the leader stage (\( \theta_{o}^{L*} \) and \( \phi_{o}^{L*} \)) is gotten respectively, from models 2 and 3. To calculate the efficiency of the followers we will consider the second and third stages as one stage and gain the efficiency of the follower stage. Kao and Hwang [16] used the multiplicative approach to measure the overall efficiency of a series structure. We then define the efficiencies of the second and third stages of a system shown in Fig. 2 from the optimistic view as

\[ \theta_{o}^{12F} = \theta_{o}^{1F} \cdot \theta_{o}^{2F} \] Thus:

**Model 4**

\[ \theta_{o}^{12F} = \max \frac{\sum_{d_j=1}^{D_j} \eta_{d_j} z_{d_j}}{\sum_{i_k=1}^{I_k} y_{i_k j}} - \sum_{r_j=1}^{R_j} u_{r_j} y_{r_j} \]

s.t. \( \sum_{i_k=1}^{I_k} y_{i_k j} - \sum_{r_j=1}^{R_j} u_{r_j} y_{r_j} \leq 1, \quad j = 1, \ldots, n \)

\[ \sum_{d_j=1}^{D_j} \eta_{d_j} z_{d_j} \geq 0, \quad j = 1, \ldots, n \]

\[ \sum_{i_k=1}^{I_k} y_{i_k j} - \sum_{r_j=1}^{R_j} u_{r_j} y_{r_j} \leq 1, \quad j = 1, \ldots, n \]

\[ \sum_{d_j=1}^{D_j} \eta_{d_j} z_{d_j} = \theta_{o}^{L*} \]

Model (4) demonstrates the maximal overall efficiency of the second and third stages in Fig. 2; and measures it from the optimistic view, on condition that the efficiency of all the stages is less than (1)
and to the method of Li et al. [30], the efficiency of the leader’s stage must remain constant. We, based on the tasks of Wang et al. [65], have modified Model (4) and have defined the minimal efficiency of the overall follower stages from the pessimistic viewpoint according to the following:

Model 5

\[
\varphi_o^{12F} = \min \frac{\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1} \rho - \sum_{i_1=1}^{I_1} \eta_{i_1} y_{i_1} \rho - \sum_{i_2=1}^{I_2} \eta_{i_2} y_{i_2} \rho}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1} \rho + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1} \rho}
\]

s.t. \[
\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1} \rho = \sum_{i_1=1}^{I_1} \eta_{i_1} y_{i_1} \rho - \sum_{i_2=1}^{I_2} \eta_{i_2} y_{i_2} \rho \geq 1, \quad j = 1, \ldots, n
\]

\[
\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1} \rho \geq 1, \quad j = 1, \ldots, n
\]

\[
\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1} \rho + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1} \rho \geq 1, \quad j = 1, \ldots, n
\]

Models (4 and 5) are nonlinear and in the fourth part of this research, an innovative method in resolving it is utilized. In assuming that, the models (4 and 5) are resolved, we define the maximal optimistic efficiency and the minimal pessimistic efficiency of a network shown in Fig.2 respectively in formula (3.4) as below:

\[
\theta^{overall*}_o = \theta^{L*}_o \cdot \theta^{12F*}_o, \quad \varphi^{overall*}_o = \varphi^{L*}_o \cdot \varphi^{12F*}_o
\]

(3.4)

We, based on the tasks of Wang and Chin [39], define the overall efficiency according to the double-frontier in formula (3.5) according to the following:

\[
\varphi^{*}_o = \sqrt{\theta^{overall*}_o \cdot \varphi^{overall*}_o}
\]

(3.5)
So we performed the modeling of the network shown in Fig 2, from three approaches (the optimistic, pessimistic and double-frontier). In the next, we utilize the k-means algorithm to cluster the DMUs into several groups based on the double-frontier view and these results are shown in the case study section.

4. Model Solution

Two exploratory approaches are proposed for the optimistic and pessimistic views relatively. It is for the first time that we have developed an exploratory approach, in relevance with the pessimistic perspective or condition, which, with the best of our findings, we had not been able to perform the modeling, as to this conceptual approach under pessimistic conditions, till date. Similarly, we have also implemented an exploratory approach from the optimistic standpoint, in relative to the leader-follower concept. Thereby, the exploratory approach from the pessimistic angle is proposed in this paper. Models (4) and (5) cannot be converted into linear models because of the additional inputs and outputs in the stages. Thus, we propose the heuristic approach given hereunder for solving models (4) and (5).

4.1. A heuristic technique to solve the optimistic view

The objective function of model (4) is the product of multiplicative efficiency of the two-stages. We consider $\theta_o^{1F}$ as a variable in the objective function, which is between the $[0, \theta_o^{1F_{max}}]$ interval and change. We define $\theta_o^{1F}$ in the figure below, so that we can move it within the interval.

$$\theta_o^{1F} = \theta_o^{1F_{max}} \cdot k_i \Delta \varepsilon, \quad k_i = 0, 1, \ldots, \left\lceil \frac{\theta_o^{1F_{max}}}{\Delta \varepsilon} \right\rceil + 1$$

(4.1)

In the formula (4.1), we consider $\Delta \varepsilon$ as a step size and of a very small value; and define $\theta_o^{1F_{max}}$ as the maximum optimistic efficiency of the first follower stage and from the following model its value is capable of being computed.

Model 6

$$\theta_o^{1F_{max}} = \max \left\{ \theta_o^{1F}, \theta_j^{1F} \leq 1, \theta_j^{2F} \leq 1, \quad j=1, \ldots, n \right\}$$

All the variables are non-negative in model (6). The said models have attained a maximum efficiency of the first follower stage, on condition that, the efficiency of the stages is less than (1). This model is
fractions and by using the Charnes-Cooper conversion [64], such as, given below, it arise modified into linear models:

Model 7

$$\theta_{o}^{F_{\max}} = \max_{\theta} \left\{ \frac{1}{2} \sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}} z_{d}^{o} \right\}$$

s.t.

$$\sum_{d_{1}=1}^{D_{1}} y_{i} x_{i} + \sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}} z_{d}^{o} = 1$$

$$\sum_{d_{1}=1}^{D_{1}} \eta_{d_{1}} z_{d}^{o} + \sum_{r_{2}=1}^{R_{2}} u_{r_{2}} y_{r_{2}} - \sum_{r_{2}=1}^{R_{2}} u_{r_{2}} y_{r_{2}} - \left( \sum_{i_{1}=1}^{I_{1}} v_{i_{1}} x_{i_{1}} - \sum_{i_{2}=1}^{I_{2}} v_{i_{2}} x_{i_{2}} \right) \leq 0, \quad j = 1, \ldots, n$$

$$\sum_{d_{2}=1}^{D_{2}} \eta_{d_{2}} z_{d}^{o} - \sum_{r_{3}=1}^{R_{3}} u_{r_{3}} y_{r_{3}} - \left( \sum_{i_{3}=1}^{I_{3}} v_{i_{3}} x_{i_{3}} + \sum_{d_{2}=1}^{D_{2}} \eta_{d_{2}} z_{d}^{o} \right) \leq 0, \quad j = 1, \ldots, n$$

$$\sum_{r_{4}=1}^{R_{4}} u_{r_{4}} y_{r_{4}} - \left( \sum_{i_{4}=1}^{I_{4}} v_{i_{4}} x_{i_{4}} + \sum_{d_{2}=1}^{D_{2}} \eta_{d_{2}} z_{d}^{o} \right) \leq 0, \quad j = 1, \ldots, n$$

$$\eta_{d_{1}}, \eta_{d_{2}}, u_{r_{3}}, u_{r_{2}}, u_{r_{1}}, u_{r_{1}} v_{i_{1}}, v_{i_{1}} \geq 0; d_{1} = 1, 2, \ldots, D_{1}; d_{2} = 1, 2, \ldots, D_{2}; r_{1} = 1, 2, \ldots, R_{1}; r_{2} = 1, 2, \ldots, R_{2};$$

$$r_{3} = 1, 2, \ldots, R_{3}; r_{4} = 1, 2, \ldots, R_{4}; i_{1} = 1, 2, \ldots, I_{1}; i_{2} = 1, 2, \ldots, I_{2}; i_{3} = 1, 2, \ldots, I_{3}; i_{4} = 1, 2, \ldots, I_{4}.$$
s.t. \[
I_k \sum_{i_1=1}^{I_k} y_{i_1} x_{i_1 \rho} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, \rho} = 1
\]
\[
\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, j} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 j} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j} - \left( \sum_{i_1=1}^{I_k} y_{i_1} x_{i_1 j} - \sum_{i_2=1}^{I_2} y_{i_2} x_{i_2 j} \right) \leq 0, \quad j = 1, \ldots, n
\]
\[
\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d, j} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j} - \left( \sum_{i_1=1}^{I_k} y_{i_1} x_{i_1 j} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, j} \right) \leq 0, \quad j = 1, \ldots, n
\]
\[
\sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j} - \left( \sum_{i_1=1}^{I_k} y_{i_1} x_{i_1 j} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, j} \right) \leq 0, \quad j = 1, \ldots, n
\]
\[
\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, \rho} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 \rho} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 \rho} - \theta_0^{1F} \left( \sum_{i_1=1}^{I_k} y_{i_1} x_{i_1 \rho} - \sum_{i_2=1}^{I_2} y_{i_2} x_{i_2 \rho} \right) = 0.
\]
\[
\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, \rho} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 \rho} - \theta_0^{1F} \left( \sum_{i_1=1}^{I_k} y_{i_1} x_{i_1 \rho} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d, \rho} \right) = 0
\]
\[
\theta_0^{1F} \in \left[ 0, \theta_0^{1F\text{-max}} \right]
\]
\[
\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_2}, u_{r_3}, v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4} \geq \varepsilon; d_1 = 1, 2, \ldots, D_1; d_2 = 1, 2, \ldots, D_2; r_1 = 1, 2, \ldots, R_1; r_2 = 1, 2, \ldots, R_2; r_3 = 1, 2, \ldots, R_3; r_4 = 1, 2, \ldots, R_4; i_1 = 1, 2, \ldots, I_1; i_2 = 1, 2, \ldots, I_2; i_3 = 1, 2, \ldots, I_3; i_4 = 1, 2, \ldots, I_4.
\]

In model (9), by utilizing formula (4.1), we increase the value of \( k_1 \) from (0) to a high level, so that each time the model can be solved with the new \( \theta_0^{1F} \). We resolve all the returns of the conditions of the \( k_1 \) model and illustrate the responses with \( \theta_0^{12F} (k_1) \). By comparing the overall values of \( \theta_0^{12F} (k_1) \), we describe \( \theta_0^{12F} \) as the maximal efficiency of the total sum of the follower stages. It should be noted, we have tested our proposed method in two modes and each time have taken one stage into consideration as variables. Given that at this point, the efficiency of a stage is unique, hence, the results of these two methods are outstandingly in approximation to each other and in order to explain, we have broached one of these two conditions to describe our above approach.

4.2 A heuristic technique to solve the pessimistic view
Our analogous optimistic approach, $\Phi_o^{1F}$ is considered as a variable in the objective function model (8), which change between interval $[\Phi_o^{1F-min}, M]$. We describe $\Phi_o^{1F}$ in the following manner, so that we can move them within the interval.

$$\Phi_o^{1F} = \Phi_o^{1F-min} + k_i \Delta \epsilon, \quad k_i = 0, 1, \ldots, \left[\frac{M - \Phi_o^{1F-min}}{\Delta \epsilon}\right] + 1$$

(4.2)

We take “M” as a larger value and $\Delta \epsilon$ as a similar optimistic approach, as a step size and of very small value. Moreover, $\Phi_o^{1F-min}$ is the minimal pessimistic efficiency of the first follower stage, and from the following model its value is capable of being computed.

**Model 10**

$$\Phi_o^{1F-min} = \min \left\{ \Phi_o^{1F} \mid \Phi_j^1 \geq 1, \Phi_j^{1F} \geq 1, \Phi_j^{2F} \geq 1, \quad j=1, \ldots, n \right\}$$

The entire variables are non-negative in the model (10). The said models have attained a minimum efficiency of the first follower stage, on condition that, the efficiency of the stages is more than (1). This model is fractions and by using the Charnes-Cooper conversion [64], such as, given below, it is modified into linear model:

**Model 11**

$$\Phi_o^{1F-min*} = \min \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 \rho} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 \rho}$$

s.t. $$\sum_{i_3=1}^{I_3} y_{i_3} x_{i_3 \rho} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 \rho} = 1$$

$$\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 j} + \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j} - \sum_{i_3=1}^{I_3} u_{i_3} y_{i_3 j} - \left( \sum_{i_3=1}^{I_3} y_{i_3} x_{i_3 j} - \sum_{i_2=1}^{I_2} y_{i_2} x_{i_2 j} \right) \geq 0, \quad j=1, \ldots, n$$

$$\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j} - \left( \sum_{i_3=1}^{I_3} y_{i_3} x_{i_3 j} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 j} \right) \geq 0, \quad j=1, \ldots, n$$

$$\sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j} - \left( \sum_{i_3=1}^{I_3} y_{i_3} x_{i_3 j} + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j} \right) \geq 0, \quad j=1, \ldots, n$$

$$\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_1} \mid r_1, j=1, 2, \ldots, D_1; d_2 = 1, 2, \ldots, D_2; r_1 = 1, 2, \ldots, R_1; r_2 = 1, 2, \ldots, R_2;$$
\( r_1 = 1, 2, \ldots, R_1; r_2 = 1, 2, \ldots, R_2; i_1 = 1, 2, \ldots, I_1; i_2 = 1, 2, \ldots, I_2; i_3 = 1, 2, \ldots, I_3; i_4 = 1, 2, \ldots, I_4. \)

By determining the value of \( \Phi_o^{1F_{\text{min}}} \) with the help of model (11), we convert model (5) to model (12) as follows:

**Model 12**

\[
\Phi_o^{12F^o} = \min \left\{ \Phi_o^{1F}, \Phi_o^{2F} \mid \Phi_j^{1F} \geq 1, \Phi_j^{2F} \geq 1, \Phi_o^{1E} = \frac{O_o}{I_o}, \Phi_o^{1F} \in \left[ \Phi_o^{1F_{\text{min}}}, M \right] \right\}
\]

It should be noted that similar to the optimistic approach in the model (12), we consider \( \Phi_o^{1F} \) in the objective function as a variable and a constraint which specify this variable and together with its interval modification, it was supplemented to the model. The model (12) is a fractional one and by using the Charnes-Cooper conversion [64], such as, given below, is modified into a linear model.

**Model 13**

\[
\Phi_o^{12F^o} = \min \Phi_o^{1F} \sum_{i_{\rho}=1}^{R_1} \sum_{j=1}^{D_1} y_{r_{\rho} j} x_{i_{\rho} j} \\
\text{s.t.} \sum_{i_{\rho}=1}^{R_1} y_{r_{\rho} j} x_{i_{\rho} j} + \sum_{d_{\rho}=1}^{D_1} \eta_{d_{\rho}} z_{d_{\rho}} = 1 \\
\sum_{d_{\rho}=1}^{D_1} \eta_{d_{\rho}} z_{d_{\rho}} = \min \left\{ \Phi_o^{1F}, \Phi_o^{2F} \mid \sum_{i_{\rho}=1}^{R_1} y_{r_{\rho} j} x_{i_{\rho} j} - \sum_{i_{\rho}=1}^{R_1} y_{r_{\rho} j} x_{i_{\rho} j} \right\} \geq 0, \ j = 1, \ldots, n \\
\sum_{d_{\rho}=1}^{D_1} \eta_{d_{\rho}} z_{d_{\rho}} = \min \left\{ \Phi_o^{1F}, \Phi_o^{2F} \mid \sum_{i_{\rho}=1}^{R_1} y_{r_{\rho} j} x_{i_{\rho} j} + \sum_{d_{\rho}=1}^{D_1} \eta_{d_{\rho}} z_{d_{\rho}} \right\} \geq 0, \ j = 1, \ldots, n \\
\sum_{d_{\rho}=1}^{D_1} \eta_{d_{\rho}} z_{d_{\rho}} = \min \left\{ \Phi_o^{1F}, \Phi_o^{2F} \mid \sum_{i_{\rho}=1}^{R_1} y_{r_{\rho} j} x_{i_{\rho} j} \right\} \geq 0, \ j = 1, \ldots, n \\
\sum_{d_{\rho}=1}^{D_1} \eta_{d_{\rho}} z_{d_{\rho}} = \min \left\{ \Phi_o^{1F}, \Phi_o^{2F} \mid \sum_{i_{\rho}=1}^{R_1} y_{r_{\rho} j} x_{i_{\rho} j} \right\} = 0 \\
\Phi_o^{1F} \in \left[ \Phi_o^{1F_{\text{min}}}, M \right] \\
\eta_{d_{1}}, \eta_{d_{2}}, u_{r_{1}}, u_{r_{2}}, u_{r_{3}}, u_{r_{4}}, v_{i_{1}}, v_{i_{2}}, v_{i_{3}}, v_{i_{4}} \geq 0; d_{1} = 1, 2, \ldots, D_1; d_{2} = 1, 2, \ldots, D_2; r_{1} = 1, 2, \ldots, R_1; r_{2} = 1, 2, \ldots, R_2; \}
\]
In model (13) by utilizing formula (4.2), the value of $k_1$ is increased from (0) to a high level, so that, we can resolve the model with the new $\Phi^{12F}_o$. We solve all the returns of the conditions of the $k_1$ models and show the responses with $\Phi^{12F}_o(k_1)$. By comparing all the values of $\Phi^{12F}_o(k_1)$, we define $\Phi^{12F}_o$ the minimal efficiency of $\Phi^{12F}_o(k_1)$ in Fig. 1 from the pessimistic view. It should be observed that, we have tested our proposed method in two modes and each time have taken one stage into consideration as variables. Given that at this point, the efficiency of a stage is unique, hence, the results of these two methods are outstandingly in approximation to each other and in order to explain, we have broached one of these two conditions to describe our above approach.

5. Case Study Description

According to the statistics released by the Iranian Health Reference Laboratory, 5611 MDLs (public and private sectors) are operating in the country. The shares of the public and private sectors are 57 percent and 43 percent, respectively. There are 933 active MDLs in Tehran, which include 16.7 percent of the total share of the country. Of these, 71 percent and 29 percent are managed by the private and public sectors, respectively. The statistics show that, unlike the number of MDLs in the country, most of which are available to the public sector, most of the MDLs are under private sector management in Tehran. Due to the importance of the private sector in Tehran, our case study is related to the private MDLs of Tehran. In this regard, the sample size in this study includes 25 MDLs selected by cluster sampling from private MDLs in Tehran. In this segment we will study the performance of private MDLs in Tehran. MDLs contain three main processes (pre-test, test, and post-test) as shown in Fig. 3.

To evaluate the efficiency of the MDL, environmental, social and economic criteria are considered. Cost of consumables, income from admission, MDL profit and staff wage are introduced as economic criteria. Staff safety as social criteria is the costs that each service center payments for dangers that exist and accidents which happen in workplace. This criterion is considered to protect the health of the
staff, patients and other clients, as well as protecting the environment. The variables are shown in Table 8 as follows:

We have many variables of inputs and outputs. Therefore, Tables 9 and 10 provide the data in 2017. The inputs are shown in Table 9 and the outputs are rendered in Table 10. Thereby, we measure the efficiency of the structure as shown in Fig. 3 from both, the optimistic and pessimistic views.

According the opinions of the experts, we consider the step size in the models $\Delta \varepsilon=0.01$, $M=3$ and $\varepsilon=0.001$. We implemented our heuristic approach, for the two models (4 and 5). The values gained for the first follower are shown in the Table 11 as follows:

By studying the values of $k_1$, we found the optimal values of network shown in Fig. 2 to occur when the first follower is at their maximum and minimum values. For this reason, all values of $k_1$ vanish.

Table 12 shows the efficiencies based on the optimistic, pessimistic and double-frontier views.

In Table 12, the maximum and minimum mean efficiency of the course is shown in gray and green respectively. The second column of Table 12 show that the efficiency scores of period 22 is the highest with a total efficiency of 1 and the efficiency scores of period 12 is the lowest with a total efficiency of 0.228176, from the optimistic view. From the third, fourth and fifth columns of Table 12, we note that the average optimistic efficiency values of the leader, first follower and second follower are of the following values respectively, (0.73, 0.99, 0.82). It should be noted that, the closer the efficiency value is closer to (1), from the optimistic standpoint and the farther the efficiency value is from (1), from the pessimistic standpoint, the DMU proves to have a better condition. The sixth column of Table 12, show that the efficiency scores of period 22 is the highest with a total efficiency of 3.80496; whereas, units 2, 12, 14, 15, 20, 21, 24 and 25 display a common and overall efficiency of 1 are the worst units, from the pessimistic view. Similarly, Results demonstrate that the average pessimistic efficiency values of the leader and the first and second followers are (1.88, 1.04, 1.32), respectively. By checking the all results, we note that the difference in optimistic and pessimistic views. Given, that this paper proposes a double-frontier approach for evaluate the overall efficiency, thereby, the best and poorest DMUs are units 22 and 25 with overall efficiencies of 2.804719 and 0.541537 respectively. (Column (10) of Table 12). In order to the final ranking, we use the double-
frontier view that we have illuminated in the part (3). So, the efficiency values of 25 DMUs are rated in the Table 13 as bellow:

The third and sixth columns of Table 13 demonstrate the clustering of the DMUs by k-means method based on double-frontier view that we have explained in the Section (3). Table 13 reports that units 7, 18, 19, 22 are located in the first cluster. Units 1, 2, 3, 5, 6, 8, 9, 11, 13, 16, 17 are placed in the second cluster. Also, units 4, 10, 12, 14, 15, 20, 21, 23, 24, 25 are placed in the third cluster.

Today, medical sciences depend on the laboratory diagnosis to discover the cause of the disease and timely and rapid treatment. Accordingly, the number of diagnostic laboratories is increasing day by day in different cities of the world. Therefore, trying to emulate leading laboratories can be an important step for continuous improvement of their performance. The results of this study indicate that most private labs in Tehran are not efficient. The reasons of inefficiency of laboratories can be identified as follow: 1) one of the most important sources of municipal wastes production is hospitals, health centers, physicians, clinics and MDLs. Among them, the laboratories produce the large amount of infectious waste that are of great importance to health and the environment. Releasing this waste into the environment can cause and transmit a variety of diseases including hepatitis B, C, and AIDS. Proper management of waste performs a significant role in the performance of laboratories. 2) Analyzing the standards and criteria that any laboratory system needs to be upgraded. Therefore, the quality management achievements in the lab are advantages such as: enhancing the assurance of the accuracy of the results provided by the labs, ensuring the continuous calibration of lab equipment, standardizing the procedures for the management of laboratories, and improves the level of customer-oriented laboratories. 3) The ability to differentiate and excel a laboratory is the large coverage of services. Therefore, increasing the geographic coverage of services each laboratory is important. 4) Factors such as price growths of kits shows the MDLs require to expense management. Averagely, 45 percent of the total expense required is related to the consumables in every MDL. So, the costs management has an important role in increasing efficiency. Considering the above reasons for growing the performance of MDL, the four methods are suggested as follows: 1) Separation of laboratory wastes at the place of production, collection and labeling, transportation to the location of safe place, packing, temporary storage, transportation from the place of production and loading and
also the final disposal stage. All steps are designed according to the performance and breadth of each laboratory. All staff members should be educated and notified of the procedures in writing. 2) Investigating the factors and determining the status of the laboratory and recording the results in the form of weaknesses and strengths, and determining the gap between the existing and the desired situation may provide appropriate and effective strategies for standardizing the laboratories. 3) Provision of services to smaller laboratories in view of the increasing diversity and capacity of the experiments is one of the ways in which successful labs operate. The use of sampling units and the use of information and communication knowledge is also one of the requirements for the coverage of services. 4) The operation management method with removing unnecessary points lead to reduces additional laboratory costs and increases productivity.

6. Conclusions

The services of laboratory centers are covered an important part of the activities of many health centers and research organizations. Since the performance of clinical and the researches laboratories shows a vital role in the quality and efficiency of health care and research activities, the need for solutions for evaluating and improving their performance has attracted the attention of the world's scientific and professional communities for many years. The performance measurement in laboratory centers is also important for managers and authorities in health centers and research organizations. By doing so, they can provide areas for improvement and increase productivity in the organization through identifying their strengths and weaknesses.

The purpose of any performance appraisal program is to increase efficiency and improve effectiveness. This goal is achieved through helping the laboratories to do their best by developing their skills and knowledge to meet the future needs of the work units. It is important that the tasks are done properly in laboratories which will improve the quality of the test results and increase the effectiveness of the services and research achievements. Effectiveness of services in clinical laboratories leads to a quick diagnosis of illness and to save the lives of patients. Also, the effectiveness of their research achievements and their commercialization will lead to the growth and self-sufficiency of research organizations. In this study, we measured the efficiency of selected private
MDLs in Tehran through NDEA approach. In this regard, the efficiency evaluation of 25 private MDLs in Tehran was examined. To evaluate the efficiency of MDLs, there are several indicators that may be used for different approaches, but in most studies, researchers, regardless of the approach they use to evaluate, try to find better or more appropriate indicators. Therefore, it is important to investigate and identify the most effective factors for evaluating the performance of MDLs. Therefore, in order to facilitate the correct and well-informed decision-making in this area, given the lack of literature, the identification of effective factors was done using the Fuzzy Delphi method. The Delphi team was composed of 11 members consisting of professors, administrators, technical officials and experts in the field of private MDLs. The final indicators consisted of 7 input indicators, 3 intermediate indicators and 9 output indicators. It is noteworthy that there were three criteria of sustainability (social, economic and environmental) in some of the selected indicators.

The efficiency evaluation of a network opens the “black box” and deliberates on the internal structures and innermost interactions of the system. The black box method ignores the internal structure of systems and this task lead to inaccuracy in the results. This paper presents the modeling method and solution for evaluating the efficiency of a complex system with additional inputs and undesirable outputs. We tried to pay attention to the intra-system activities using the proposed model. In this study, we simulated a three-stage series structure of a MDL in a real-world. This network model of three processes (The pre-test process, The test process, The post-test process). The pre-testing process contain of the reception unit. Also, the testing process consists of the sampling and testing unit. Finally, the post-test process includes of the test results unit. The model presented in this article is an innovative model, and similar research was not found in the field of MDLs as a network analysis. According to managers, we used a non-cooperative model with a double-frontier method to measure the performance of the lab units and then to convert nonlinear programs into linear programs by a heuristic method. This approach provides more information about inefficient units through penetration to the depth of the system. In fact, the double-frontier method, views each unit from two outlooks (the optimistic and pessimistic views) and any result which implies to only one of these perspectives, shall be one-sided and inadequate. The evaluating efficiency with the double-frontier method, shall lead to an increase in accuracy. The heuristic approach proposed in this research can be
used to solve a three-stage system. The model becomes complex for higher-stage systems, because of the additional inputs and outputs; and thereby, increases the solution time significantly. To overcome the problem we can change the movement step (Δε).

The ranking results show that the units 22 and 21 are the best and poorest units in terms of efficiency, respectively. Also, the results of clustering techniques are displayed as 4, 11 and 10 units, which are placed in the first, second and third clusters, separately. We found out that variant performance indicators such as monitoring and control of wastes, geographic coverage, review of the pre-test process (the reception unit), providing appropriate and effective strategies for standardization of laboratories and cost of consumables (kits) due to currency fluctuations are significant elements to determine efficiency. Other results of this research are providing application indicators that can be used in future research by other researchers. We provided the results of this research to the managers which might lead to improved MDL services with adopting suitable methods. For researches in the future, Due to the problems in gathering data, modeling should be done with imprecise data. Also, since the activities of an enterprise such as MDLs is not sectional over a period of time, but is continuous activity, therefore, the cross-sectional efficiency assessment cannot provide a realistic answer to the performance of laboratories. Therefore, network analysis in dynamic mode is also recommended.

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Table 1. Classification of Studies on DEA-Game Theory method

| Reference | Type of game       | Structure Of Network | Additional inputs | Undesirable output | Type of modelling          | Type of frontier          | Sustainability |
|-----------|--------------------|----------------------|-------------------|--------------------|-----------------------------|---------------------------|----------------|
| Hwang et al. [54] | Cooperative | One-stage | - | ✓ | Linear programming | Optimistic view | - |
| Kao and Hwang [16] | Cooperative | Two-stage | - | - | Linear programming | Optimistic view | - |
| Wang et al. [55] | Cooperative | Two-stage | - | - | Linear programming | Optimistic view | - |
| Kou et al [14] | Cooperative | Two-stage | ✓ | - | Linear programming | Optimistic view | - |
| Li et al. [30] | Non-cooperative | Two-stage | ✓ | - | Linear programming | Optimistic view | - |
| Liang et al. [29] | Cooperative and Non-cooperative | Two-stage | - | - | Linear programming | Optimistic view | - |
| Wu et al. [27] | Cooperative | Two-stage | ✓ | ✓ | Linear programming | Optimistic view | - |
| Zhou et al. [32] | Non-cooperative | Two-stage | - | - | Non-linear programming | Optimistic view | - |
| An et al. [31] | Cooperative and Non-cooperative | Two-stage | ✓ | - | Non-linear programming | Optimistic view | - |
| Wu et al. [28] | Cooperative and Non-cooperative | Two-stage | ✓ | ✓ | Non-linear programming | Optimistic view | - |
| Du et al. [31] | Cooperative and Non-cooperative | Three-stage | - | - | Linear programming | Optimistic view | - |
| Youseti et al. [56] | Cooperative | Three-stage | ✓ | - | Non-linear programming | Optimistic view | - |
| Shabanpour et al. [57] | Cooperative | One-stage | - | - | Linear programming | Double-frontier | - |
| Badiezadeh et al. [42] | Cooperative | Three-stage | ✓ | ✓ | Linear programming | Double-frontier | - |
| current paper | Non-cooperative | Three-stage | ✓ | ✓ | Non-linear programming | Double-frontier | ✓ |

Table 2. Triangular Fuzzy Numbers of variables

| Linguistic variables | Triangular fuzzy number | Defuzzification value |
|----------------------|-------------------------|-----------------------|
| Very low             | (0, 0, 0.25)            | 0.0625                |
| Low                  | (0.25, 0.15, 0.15)      | 0.25                  |
| Medium               | (0.5, 0.25, 0.25)       | 0.5                   |
| High                 | (0.75, 0.15, 0.15)      | 0.75                  |
Table 3. Indicators effective in evaluating the efficiency of MDLs

| Row | Indicator                                           | Observation | Leleu et al. [58] | Asandului et al. [59] | Hamid Abu Bakar et al. [60] | Yousefi et al. [56] | Patra and Ray [61] |
|-----|-----------------------------------------------------|-------------|-------------------|------------------------|-----------------------------|---------------------|-------------------|
| 1   | Available space for service                         | ✓           |                   |                        |                             |                     |                   |
| 2   | Average sample transfer time                        | ✓           |                   |                        |                             |                     |                   |
| 3   | Average waiting time for sampling                   | ✓           |                   |                        |                             |                     |                   |
| 4   | Correct number of tests                             | ✓           |                   |                        |                             |                     |                   |
| 5   | Cost of consumables                                 | ✓           |                   |                        |                             |                     |                   |
| 6   | Cost of laboratory space and land value              | ✓           | ✓                  |                        |                             |                     |                   |
| 7   | Cost of staff welfare                               | ✓           |                   |                        |                             |                     |                   |
| 8   | Garbage weight                                      | ✓           |                   |                        |                             |                     |                   |
| 9   | Income from admission                               | ✓           |                   |                        |                             |                     |                   |
| 10  | Lab profit                                          | ✓           |                   |                        |                             |                     |                   |
| 11  | Number of active experiments                        | ✓           |                   |                        |                             |                     |                   |
| 12  | Number of false tests                               | ✓           |                   |                        |                             |                     |                   |
| 13  | Number of kits                                      | ✓           |                   |                        |                             |                     |                   |
| 14  | Number of patients' admitted                        | ✓           | ✓                  | ✓                      |                             |                     |                   |
| 15  | Number of responses of the prepared tests           | ✓           |                   |                        |                             |                     |                   |
| 16  | Number of samples                                   | ✓           |                   |                        |                             |                     |                   |
| 17  | Safety cost of sampling unit                        | ✓           |                   |                        |                             |                     |                   |
| 18  | Safety cost of test unit                            | ✓           |                   |                        |                             |                     |                   |
| 19  | Staff wage                                          | ✓           |                   |                        |                             |                     |                   |
| 20  | Sum of the scores of the laboratory standards       | ✓           |                   |                        |                             |                     |                   |
| 21  | Test response time                                  | ✓           |                   |                        |                             |                     |                   |

Table 4. The average expert opinions in the first Round

| Row | Indicator                                           | Triangular Fuzzy average | Defuzzification value average (x) |
|-----|-----------------------------------------------------|--------------------------|-----------------------------------|
| 1   | Available space for service                         | (0.54 0.79 0.96)         | 0.58                              |
| 2   | Average sample transfer time                        | (0.54 0.77 0.97)         | 0.59                              |
| 3   | Average waiting time for sampling                   | (0.56 0.79 0.97)         | 0.60                              |
| 4   | Correct number of tests                             | (0.49 0.74 0.89)         | 0.52                              |
| 5   | Cost of consumables                                 | (0.59 0.84 0.96)         | 0.62                              |
| 6   | Cost of laboratory space and land value              | (0.06 0.29 0.54)         | 0.12                              |
| 7   | Cost of staff welfare                               | (0.09 0.24 0.49)         | 0.15                              |
| 8   | Garbage weight                                      | (0.61 0.86 0.96)         | 0.64                              |
| 9   | Income from admission                               | (0.57 0.84 0.96)         | 0.61                              |
| 10  | Lab profit                                          | (0.37 0.64 0.89)         | 0.43                              |
| 11  | Number of active experiments                        | (0.46 0.71 0.91)         | 0.51                              |
| 12  | Number of false tests                               | (0.51 0.74 0.89)         | 0.55                              |
| 13  | Number of kits                                      | (0.59 0.84 0.96)         | 0.62                              |
| 14  | Number of patients' admitted                        | (0.39 0.64 0.82)         | 0.43                              |
| 15  | Number of samples                                   | (0.69 0.86 0.99)         | 0.72                              |
| 16  | responses of the prepared tests                     | (0.31 0.54 0.77)         | 0.37                              |
| 17  | Safety cost of sampling unit                        | (0.69 0.86 0.99)         | 0.72                              |
| 18  | Safety cost of test unit                            | (0.46 0.39 0.64)         | 0.52                              |
| 19  | Staff wage                                          | (0.64 0.92 0.97)         | 0.7                               |
| 20  | Sum of the scores of the laboratory standards       | (0.64 0.89 0.99)         | 0.66                              |
### Table 5. The average expert opinions in the second Round

| Row | Indicator                                      | Triangular Fuzzy average | Defuzzification value average | Difference of the first and second Rounds |
|-----|-----------------------------------------------|--------------------------|-------------------------------|------------------------------------------|
| 1   | Correct number of tests                       | (0.49 0.59 0.77)         | 0.53                          | 0.01                                     |
| 2   | Average sample transfer time                  | (0.57 0.72 0.94)         | 0.62                          | 0.03                                     |
| 3   | Test response time                            | (0.67 0.74 0.89)         | 0.7                           | 0.04                                     |
| 4   | Cost of laboratory space and land value       | (0.11 0.17 0.42)         | 0.17                          | 0.05                                     |
| 5   | Number of samples                             | (0.74 0.79 0.92)         | 0.77                          | 0.05                                     |
| 6   | Average waiting time for sampling             | (0.62 0.72 0.89)         | 0.66                          | 0.06                                     |
| 7   | Cost of staff welfare                         | (0.15 0.19 0.44)         | 0.21                          | 0.06                                     |
| 8   | Safety cost of test unit                      | (0.53 0.66 0.86)         | 0.58                          | 0.06                                     |
| 9   | Safety cost of sampling unit                  | (0.75 0.81 0.94)         | 0.78                          | 0.06                                     |
| 10  | responses of the prepared tests               | (0.39 0.54 0.76)         | 0.44                          | 0.07                                     |
| 11  | Cost of consumables                           | (0.65 0.73 0.88)         | 0.69                          | 0.07                                     |
| 12  | Lab profit                                    | (0.44 0.61 0.86)         | 0.5                           | 0.07                                     |
| 13  | Number of kits                                | (0.66 0.76 0.94)         | 0.7                           | 0.08                                     |
| 14  | Sum of the scores of the laboratory standards | (0.71 0.79 0.94)         | 0.75                          | 0.09                                     |
| 15  | Number of patients' admitted                  | (0.47 0.62 0.84)         | 0.52                          | 0.09                                     |
| 16  | Number of active experiments                  | (0.54 0.71 0.96)         | 0.6                           | 0.09                                     |
| 17  | Staff wage                                    | (0.76 0.79 0.89)         | 0.79                          | 0.09                                     |
| 18  | Income from admission                         | (0.66 0.74 0.89)         | 0.7                           | 0.09                                     |
| 19  | Available space for service                   | (0.66 0.74 0.96)         | 0.72                          | 0.14                                     |
| 20  | Garbage weight                                | (0.78 0.84 0.96)         | 0.81                          | 0.17                                     |
| 21  | Number of false tests                         | (0.70 0.79 0.94)         | 0.74                          | 0.19                                     |

### Table 6. The average expert opinions in the third Round

| Row | Indicator            | Triangular Fuzzy average | Defuzzification value average (x) | Difference of the second and third Rounds |
|-----|----------------------|--------------------------|-----------------------------------|------------------------------------------|
| 1   | Number of false tests| (0.64 0.89 1.09)         | 0.69                              | 0.05                                     |
| 2   | Garbage weight       | (0.69 0.96 1.11)         | 0.73                              | 0.08                                     |
| 3   | Available space for service | (0.56 0.79 1.07) | 0.63                              | 0.09                                     |

### Table 7. Effective Indicators for the efficiency evaluation of MDLs

| Row | Indicator                                      | Row | Indicator                                      |
|-----|-----------------------------------------------|-----|-----------------------------------------------|
| 1   | Available space for service                   | 11  | Number of kits                                |
| 2   | Average sample transfer time                  | 12  | Number of patients' admitted                  |
| 3   | Average waiting time for sampling             | 13  | Number of samples                             |
| 4   | Correct number of tests                       | 14  | responses of the prepared tests               |
| 5   | Cost of consumables                           | 15  | Safety cost of sampling unit                  |
| 6   | Garbage weight                                | 16  | Safety cost of test unit                      |
| 7   | Income from admission                         | 17  | Staff wage                                    |
| 8   | Lab profit                                    | 18  | Sum of the scores of the laboratory standards |
| 9   | Number of active experiments                  | 19  | Test response time                            |
| 10  | Number of false tests                         |     |                                               |
Table 8. Variables of input, Intermediary and output

| Input variables       | Intermediary variables | Output variables         |
|-----------------------|------------------------|--------------------------|
| Number of active experiments ($X_1$) | Number of samples ($Z_1$) | Average waiting time for sampling ($Y_1$) |
| Available pace for service ($X_2$) | Correct number of tests ($Z_2$) | Number of patients' admission ($Y_2$) |
| Cost of consumables ($X_3$) |                        | Income from admission ($Y_3$) |
| Safety cost of sampling unit ($X_4$) |                        | Average sample transfer time ($Y_4$) |
| Safety cost of test unit ($X_5$) |                        | Number of false tests ($Y_5$) |
| Number of kits ($X_6$) |                        | Test response time ($Y_6$) |
| Staff wage ($X_7$) |                        | Garbage weight ($Y_7$) |
|                       |                        | Number of responses of the prepared tests ($Y_8$) |
|                       |                        | Sum of the scores of the laboratory standards ($Y_9$) |
|                       |                        | Lab profit ($Y_{10}$) |

Table 9. Set of inputs and intermediate measures for the 25 diagnostic laboratories in 2017

| DM | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $Z_1$ | $Z_2$ |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1  | 0.6   | 0.652 | 0.841 | 0.873 | 0.693 | 0.873 | 0.69  | 0.812 | 0.874 |
| 2  | 1     | 0.739 | 0.865 | 0.927 | 1     | 0.927 | 0.739 | 0.857 | 0.925 |
| 3  | 0.583 | 0.822 | 0.631 | 0.677 | 0.663 | 0.677 | 0.559 | 0.629 | 0.669 |
| 4  | 0.592 | 0.77  | 0.502 | 0.538 | 0.667 | 0.538 | 0.941 | 0.5   | 0.535 |
| 5  | 0.85  | 0.87  | 0.932 | 1     | 0.933 | 1     | 0.919 | 0.929 | 1     |
| 6  | 0.817 | 0.713 | 0.817 | 0.877 | 0.837 | 0.877 | 0.31  | 0.814 | 0.872 |
| 7  | 0.3   | 0.665 | 0.617 | 0.662 | 0.6   | 0.662 | 0.592 | 0.614 | 0.66  |
| 8  | 0.35  | 0.722 | 0.574 | 0.615 | 0.733 | 0.158 | 1     | 0.571 | 0.616 |
| 9  | 0.867 | 0.865 | 0.789 | 0.835 | 1     | 0.835 | 0.362 | 0.786 | 0.834 |
| 10 | 0.75  | 0.8   | 0.645 | 0.69  | 0.77  | 0.69  | 0.017 | 0.643 | 0.689 |
| 11 | 0.5   | 0.778 | 0.717 | 0.765 | 0.673 | 0.765 | 0.931 | 0.714 | 0.764 |
| 12 | 0.717 | 0.565 | 0.445 | 0.474 | 0.7   | 0.474 | 0.655 | 0.443 | 0.474 |
| 13 | 0.233 | 1     | 0.288 | 0.311 | 0.333 | 0.311 | 0.097 | 0.107 | 0.285 |
| 14 | 0.85  | 0.804 | 0.857 | 0.873 | 0.96  | 0.873 | 0.626 | 0.32  | 0.855 |
Table 10. Set of outputs for the 25 diagnostic laboratories in 2017

| DMU | y_1  | y_2  | y_3  | y_4  | y_5  | y_6  | y_7  | y_8  | y_9  | y_{10} |
|-----|------|------|------|------|------|------|------|------|------|-------|
| 1   | 0.423| 0.351| 0.876| 0.125| 0.075| 0.036| 0.451| 0.351| 0.98  | 1.0    |
| 2   | 0.462| 0.899| 0.916| 0.25  | 0.051| 0.012| 0.522| 0.899| 0.82  | 0.74   |
| 3   | 0.423| 0.505| 0.673| 0.083| 0.061| 0.03 | 0.273| 0.505| 0.64  | 0.95   |
| 4   | 0.538| 0.496| 0.535| 0.167| 0.047| 0.143| 0.32 | 0.496| 0.55  | 0.50   |
| 5   | 0.5  | 0.829| 0.978| 0.375| 0.033| 0.06 | 0.53 | 0.829| 0.44  | 0.89   |
| 6   | 0.615| 0.789| 0.865| 0.167| 0.047| 0.286| 0.261| 0.789| 0.725 | 0.86   |
| 7   | 0.615| 0.428| 0.647| 0.75  | 0.042| 1   | 0.266| 0.428| 0.675 | 0.82   |
| 8   | 0.692| 0.416| 0.591| 0.063| 0.047| 0.036| 0.387| 0.416| 0.49  | 0.25   |
| 9   | 0.692| 0.865| 0.838| 0.25  | 0.037| 0.286| 0.542| 0.865| 0.85  | 0.52   |
| 10  | 0.692| 0.468| 0.692| 0.271| 0.037| 0.048| 0.564| 0.468| 0.425 | 0.75   |
| 11  | 0.615| 0.522| 0.767| 0.313| 0.047| 0.018| 0.266| 0.522| 0.525 | 0.96   |
| 12  | 0.654| 0.427| 0.476| 0.104| 0.051| 0.143| 0.325| 0.427| 0.575 | 0.28   |
| 13  | 0.269| 0.107| 0.312| 0.125| 0.173| 0.143| 0.278| 0.107| 0.5   | 0.41   |
| 14  | 0.5  | 0.323| 0.875| 0.25  | 0.252| 0.429| 0.608| 0.323| 0.5   | 0.93   |
| 15  | 0.5  | 0.134| 0.295| 0.125| 0.262| 0.429| 0.581| 0.115| 0.975 | 0.35   |
| 16  | 0.538| 0.279| 0.503| 0.167| 0.374| 0.143| 0.601| 0.277| 1     | 0.49   |
| 17  | 0.692| 0.465| 0.71 | 0.063| 0.294| 0.036| 0.648| 0.465| 0.94  | 0.69   |
| 18  | 0.615| 0.489| 0.308| 0.208| 0.336| 0.054| 0.591| 0.489| 0.69  | 0.24   |
| 19  | 0.577| 0.567| 0.429| 0.375| 0.318| 0.03 | 0.665| 0.567| 0.725 | 0.34   |
| 20  | 0.615| 0.477| 0.293| 0.75  | 0.341| 1   | 0.606| 0.477| 0.69  | 0.19   |
| 21  | 0.385| 0.038| 0.076| 1     | 0.196| 0.143| 0.347| 0.038| 0.84  | 0.07   |
| 22  | 0.917| 0.835| 1     | 0.997| 0.767| 0.997| 0.776| 1     | 0.993  |
Table 11. Values of the maximum and minimum efficiencies of the first follower

| DM  | Optimistic View | Pessimistic View |
|-----|----------------|-----------------|
|     | \( k_1 \) | \( \theta_{1F}^{\text{max}} \) | \( \varphi_{1F}^{\text{min}} \) | \( k_1 \) | \( \theta_{1F}^{\text{max}} \) | \( \varphi_{1F}^{\text{min}} \) |
| 1   | 0 | 1 | 0 | 1.06362 | 14 | 0 | 1 | 0 | 1.02517 |
| 2   | 0 | 1 | 0 | 1.08218 | 15 | 0 | 1 | 0 | 1.02923 |
| 3   | 0 | 1 | 0 | 1.07317 | 16 | 0 | 1 | 0 | 1.02001 |
| 4   | 0 | 0.99185 | 0 | 1.07551 | 17 | 0 | 0.96213 | 0 | 1.0248 |
| 5   | 0 | 1 | 0 | 1.08612 | 18 | 0 | 1 | 0 | 1.01567 |
| 6   | 0 | 1 | 0 | 1.0782 | 19 | 0 | 1 | 0 | 1.01635 |
| 7   | 0 | 1 | 0 | 1.06064 | 20 | 0 | 0.9905 | 0 | 1 |
| 8   | 0 | 1 | 0 | 1.08898 | 21 | 0 | 0.98689 | 0 | 1 |
| 9   | 0 | 0.99718 | 0 | 1.07487 | 22 | 0 | 1 | 0 | 1.01478 |
| 10  | 0 | 1 | 0 | 1.08286 | 23 | 0 | 1 | 0 | 1.00491 |
| 11  | 0 | 1 | 0 | 1.08152 | 24 | 0 | 0.99346 | 0 | 1.02735 |
| 12  | 0 | 0.99589 | 0 | 1.07942 | 25 | 0 | 1 | 0 | 1.0314 |
| 13  | 0 | 0.99706 | 0 | 1 | | | | | |

Table 12. Results based on the optimistic, pessimistic and double-frontier views

| DM  | Optimistic View | Pessimistic View | Double Frontier |
|-----|----------------|-----------------|----------------|
|     | \( \theta_{1F}^{\text{overall*}} \) | \( \theta_{1F}^{L*} \) | \( \theta_{1F}^{2F*} \) | \( \varphi_{1F}^{\text{overall*}} \) | \( \varphi_{1F}^{L*} \) | \( \varphi_{1F}^{2F*} \) | \( \varphi_{1F}^{\text{overall*}} \) |
| 1   | 0.79044 | 1 | 1 | 0.79044 | 1.725818 | 1.52304 | 1.06362 | 1.065362 |
| 2   | 0.74755 | 1 | 1 | 0.74755 | 1.45366 | 1 | 1 | 1.08218 |
| 3   | 0.88415 | 0.88415 | 1 | 1 | 3.190746 | 1.92187 | 1.07317 | 1.167971 |
| 4   | 0.428705 | 0.53989 | 0.99185 | 0.800585 | 2.229599 | 1.73271 | 1.07551 | 1.167971 |
| 5   | 0.74256 | 1 | 1 | 0.74256 | 2.04121 | 1.87936 | 1.08612 | 1.065362 |
| 6   | 0.719244 | 0.82592 | 1 | 0.87084 | 2.44752 | 1.68241 | 1.0782 | 1.065362 |
| 7   | 0.88457 | 1 | 1 | 0.88457 | 5.331549 | 3.55631 | 1.06064 | 1.065362 |
| 8   | 0.358562 | 0.80945 | 1 | 0.44297 | 3.152826 | 2.89521 | 1.08898 | 1.063246 |
| 9   | 0.510268 | 0.75612 | 0.99718 | 0.676758 | 2.60279 | 1.78698 | 1.07487 | 1.355152 |

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| unit | rank | cluster | unit | rank | cluster |
|------|------|---------|------|------|---------|
| DMU_{22} | 1 | 1 | DMU_{8} | 14 | 2 |
| DMU_{18} | 2 | 1 | DMU_{2} | 15 | 2 |
| DMU_{19} | 3 | 1 | DMU_{4} | 16 | 3 |
| DMU_{7} | 4 | 1 | DMU_{10} | 17 | 3 |
| DMU_{16} | 5 | 2 | DMU_{20} | 18 | 3 |
| DMU_{3} | 6 | 2 | DMU_{15} | 19 | 3 |
| DMU_{11} | 7 | 2 | DMU_{14} | 20 | 3 |
| DMU_{6} | 8 | 2 | DMU_{23} | 21 | 3 |
| DMU_{13} | 9 | 2 | DMU_{24} | 22 | 3 |
| DMU_{5} | 10 | 2 | DMU_{21} | 23 | 3 |
| DMU_{1} | 11 | 2 | DMU_{12} | 24 | 3 |
| DMU_{9} | 12 | 2 | DMU_{25} | 25 | 3 |

Table 13. Rank and cluster of DMUs based on the double-frontier view
Figures with their numbers

Fig. 1 The methodology of research

Fig. 2 The structure of three-stage network system with additional inputs and undesirable outputs
Fig. 3 Three-stage network of a MDL

**Biographies**

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