SNR Spectra as a Quantitative Model for Image Quality in Polychromatic X-Ray Imaging

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1. Introduction

X-ray sources are relatively weak light sources, so a detection process in x-ray imaging is often noise-limited. This is especially true for computed tomography (CT), due to the need to acquire many images and the correspondingly longer measurement times. There are other detection problems in x-ray imaging that are not noise-limited, which we do not consider here.

It is therefore a natural assumption that, of the photons which pass through the sample, the detected fraction needs to be as high as possible to achieve a high image quality. Also, there should be as many photons generated as possible. We will call this concept “photon detection”, in which a good imaging device is equated with generating and detecting a high number of photons. Examples for work which (implicitly) use this concept to evaluate image quality are \[2,9,11,12,15\]

While photon detection is a sufficient model in some cases, it is an oversimplification in others: For example if polychromaticity or modulation transfer are important effects in x-ray imaging. Here polychromaticity means that different x-ray energies contribute to a single image. Generally, if different photons in one image have very different properties, photon detection is an oversimplification. To fully model these cases, we fundamentally consider the problem of signal detec-
**Table 1: Definitions for basic terms in the signal detection model.**

| term              | definition                                                                 |
|-------------------|-----------------------------------------------------------------------------|
| signal            | Sample properties and structure which are intended to be measured.          |
| data              | Result of a measurement.                                                   |
| noise             | Deviation between measured values and signal.                              |
| image quality     | Degree of reliability with which the sample structure of interest can be detected in a measured image. |
| modulation transfer | Image blur (signal deterioration) caused by the detection apparatus (source size, diffraction limit, ...). Quantified by the modulation transfer function (MTF). |
| poly-chromaticity | Photons have a broad energy distribution.                                   |

Detection is a measurement process in which a specific signal is reliably differentiated from noise.

To make this definition complete, the terms used are defined and explained in table 1.

The main difference to the concept of photon detection is that we consider the detection of a signal, not the detection of photons. Therefore we call this concept "signal detection". Photons are used to probe the signal, they are information carriers, not the signal itself.

In the case of x-ray imaging, the signal is the spatial distribution (structure) of the x-ray photon interaction strengths of the sample. The interaction can either be absorption or phase effects, which respectively correspond to the imaginary and real parts of the index of refraction. Usually, only parts of the sample structure is of interest. We define the term "image quality" to describe how well these can be detected.

This work is split into two parts. In the first part (section 2), we will take a closer look at what a quantitative model for image quality requires to be sufficiently accurate. We then consider if the common approaches satisfy these requirements and explain why SNR spectra are a good quantitative model. In the second part (the rest of this work), we develop a framework for understanding and evaluating polychromatic image quality based on SNR spectra.

2. Image Quality Measures

2.1. Effects to Include

We now want to find a physical model to describe image quality in signal detection quantitatively. One of the basic difficulties when designing a physical model is deciding which effects to include and which effects to ignore e.g. by an approximation. A careful consideration of appropriate approximations is required, otherwise the physical model will not describe reality correctly in the relevant cases.

The physical model considered in this work is that of a numerical value to describe image quality in a signal detection context. As a value to describe a quantity, we will call these different values "measures".

These measures are then used to optimize an imaging device or answering the question if one measurement technique is better than another. If the measure does not correctly describe the quantity, the wrong case might be considered superior.

In polychromatic x-ray imaging, effects from the following categories need to be included:

(A) Noisiness (intensity/poisson noise and other)

(B) Modulation transfer (MTF)

(C) Signal strength (absorption, phase or other)

(D) Superposition of contributions with different A—C

(E) Presence of artifacts (≈ noise)

Example images for how these effects influence image quality are given in figure 1. They consist of randomly placed balls of different size but identical maximal absorption length. These balls are then blurred.

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1. Note that these types of measures do not satisfy additivity – as would be expected from the use in mathematics.
Fig. 1: Simulated projection images of randomly placed balls of different sizes with the same number of balls for each size. Images b) and c) are d) plus another image with low image quality. In both cases, the resulting image has a lower image quality than d) alone. It can be seen that all effects have the same result: Small objects can or cannot be reliably detected depending on different properties of the imaging setup. For example noisiness (A) and MTF (B) have the same resulting effect and should therefore be quantified by a unified measure.

Note that from figure 1 d)–f) we can see that small differences in the image quality are not noticeable to a human observer. A difference by a factor of three can be differentiated well, much smaller differences would not be. This makes image quality optimizations by human estimate very imprecise.

The effects A-C scale the image quality. Ignoring one of these effects can give rise to severe misinterpretations. Effects D are important if one of the effects A, B or C differs greatly for different detected photons, even if the effect themselves would not need to be considered. In figure 2 an example for a strong energy dependency of the signal strength is shown. Different intensities (A) and varying signal strengths (C) have the same effect in a single image, but different effects in a superposition. Otherwise, both effects could be modeled as one effect. Effects E are included for completeness, but are not considered further in this work.

Fig. 2: Energy dependency of the signal strength for x-ray absorption for some materials. A variation by two orders of magnitude is realistic for broad polychromatic spectra. Approximating \( \mu \) as constant is generally only appropriate for monochromatic or very high energy imaging.
The two commonly used measures to describe and optimize image quality are DQE (= photon detection efficiency) and SNR (= ability/probability to detect signal). Both are used in temporal (scalar) and in spatial (frequency-dependent) forms.

2.2. Signal to Noise Ratio

The signal to noise ratio gives the strength of the signal in relation to the strength of the noise. It can be understood as an (inverse) relative measurement error. This concept is based on the fact that signal can only be reliably detected if it is stronger than the noise (relative error ≪ 1). While it is clear that higher SNR is better, defining a sufficiently high SNR depends on many factors and can not be done generally. The definitions of the temporal SNR_t and the SNR spectrum are:

\[ \text{SNR}_t = \frac{\text{pixel mean signal}}{\text{pixel standard deviation}} = \text{CNR}_t \]  
\[ \text{SNR}(u) = \frac{\text{image signal power spectrum}}{\text{image noise power spectrum}} \]

where \( u \) is the spatial frequency and \( \text{SNR}(u) \) is the SNR spectrum. “Spectrum” here means image spatial frequency spectrum, not light wavelength spectrum. Note that by “signal”, we always mean sample structure, not the light intensity. Our definition of \( \text{SNR}_t \) is identical to a contrast to noise ratio (CNR_t) – the contrast is the signal. This is consistent with how \( \text{SNR}(u) \) is defined.

While \( \text{SNR}_t \) is suitable to describe the lower noisiness of images with longer integration times, comparing cases with differences in the measurement setup (MTF differs e.g. due to a different/hardened x-ray spectrum, different screen thickness, ...) potentially leads to false comparisons. Optimizing \( \text{SNR}_t \) may thus lead to a lower image quality if this comes at the cost of a worse MTF.

Determining and evaluating both \( \text{SNR}_t \) and MTF can show this problem. It does not answer the question as to which case is optimal, which the \( \text{SNR}(u) \) does.

\( \text{SNR}_t \) is a linear quantity while \( \text{SNR}(u) \) is a squared quantity. The former is proportional to the signal amplitude while the latter is proportional to measurement integration time.

2.3. Detective Quantum Efficiency

A DQE is defined as a SNR transfer function to describe and optimize one part of the imaging device. Usually it is applied to describe the x-ray detector. It is generally calculated in the form

\[ \text{DQE} = \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} = \frac{\text{SNR}_{\text{detected}}}{\text{SNR}_{\text{ideal}}} \in [0, 1] \]  

This is intended to give an absolute optimum if \( \text{DQE} = 1 \). There exist many derived expressions for certain applications (simulation [2,10] or measurement [1,4]). The derivations assume monochromaticity but are applied to polychromatic imaging anyway. They assume that signal strengths cancel out and detector properties can be averaged over the x-ray spectrum. The common simplified expression for the DQE of an indirect detector is:

\[ \text{DQE}(u) = \frac{a}{1 + c - H_v(u)^{-2}} \]

where \( a, c \) and \( H_v(u) \) are the spectrally averaged x-ray absorption efficiency, x-ray photon conversion factor and optical (scattering) MTF. Effects from different samples are excluded by design in such a simplification, which means that effects (C) and (D) are neglected.

\( \text{SNR}_{\text{ideal}} \) and \( \text{SNR}_{\text{detected}} \) depend on the sample, which has a x-ray energy dependent transmission. This effective x-ray spectrum would need to be considered to correctly compute a polychromatic DQE and can be strongly sample-dependent.

Using a monochromatic DQE at different energies does not pose such problems. This is the standard approach for characterizing detectors for optical light (CMOS/CCD) [5], in which a similar quantity is called quantum efficiency (QE). It is the only application of the DQE concept in a polychromatic context which is accurate.

2.4. Indirect Detection

Generally, two types of x-ray imaging detectors can be differentiated:

- Direct detectors: These detectors count single x-ray photon detection events. They are principally able to determine the x-ray energy of a detected photon. They are also called counting detectors.

\[ \text{SNR}(u) \]

If a is included varies, but it must be included when considering the whole x-ray photon detection process.
○ Indirect detectors: These detectors first convert x-ray photons into visible light which is then detected. They usually integrate all events in a certain time frame, weighted by the energy of the x-ray photon. They are also called (energy) integrating detectors.

Due to the fact that indirect detectors are currently much cheaper to produce, most x-ray imaging setups use this kind of detector.

For indirect detection, the noise power spectrum \( N(u) \) is not a white spectrum, but is instead influenced by the detector MTF in the following way \[ N(u,E) = I(E) \left[ c(E)^2 H_v(u,E)^2 + c(E) \right] \]

Where \( E \) is the x-ray energy, \( H_v(u,E) \) is the optical detector MTF, \( I(E) \) is the x-ray intensity and \( c(E) \) is the conversion factor for x-rays to visible light. This equation is only valid for monochromatic x-rays and for a simplified imaging setup, but the general shape of \( N(u) \) is identical for polychromatic spectra on real setups. In a CT measurement, the reconstruction algorithm will also influence volume image noise.

The pixel noise \( \sigma_t \) (temporal standard deviation) for a noise power spectrum \( N(u) \) of a projection is given by Parseval’s relation:

\[
\sigma_t^2 = \frac{1}{A} \sum_{x,y} N(u_{x,y})
\]

where \( N(u_{x,y}) \) is one discrete Fourier coefficient and \( A \) is the number of these coefficients. Computing or measuring a \( \sigma_t \) for an indirect detector does not give useful results even for noisiness effects, because noise at all frequencies is averaged, although it may not decrease image quality at the structure size that is of interest.

If the image quality is reduced by decreasing \( H_v(u, E) \), both \( SNR_t \) and \( DQE_t \) will increase due to a decreasing \( \sigma_t \). Changing \( H_v(u, E) = 1 \) to \( H_v(u, E) = 0 \) decreases \( \sigma_t \) by a factor of \( \sqrt{c+1} \). If technically feasible, \( c > 100 \) is achieved in detectors built today. This effect can therefore be large. In other words, sharper indirect detectors are also temporally noisier. The \( SNR \) spectrum instead indicates that they have relatively weaker noise, which is correct.

Similar effects are produced by artifacts. Beam hardening artifacts or ring artifacts may e.g. decrease the \( SNR(u) \) at lower frequencies but fine detail can still be detected well. A temporal \( SNR \) cannot differentiate between low-frequency and high-frequency noise.

2.5. SNR Spectra

Which of the image quality measures discussed takes which physical effect into account is summarized in table 2.

Of those measures considered, only \( SNR \) spectra are left as a candidate for a generally useful image quality measure – they do not suffer from any of the problems discussed previously. While the other image quality measures do work in special cases, their restrictions have often been ignored and can lead to severe misinterpretations of physical effects on image quality.

3. Signal Detection Model

Understanding image quality in the context of signal detection requires that we model polychromatic effects. To do so, we will use \( SNR \) spectra. These are computed from the fraction of the signal power spectrum to the noise power spectrum, see eq. \[2\]. Signal detection image quality can therefore be reduced to the properties of these power spectra, which we will consider in the following.

3.1. Classes of Photons

To understand how the different detection properties (e.g. due to polychromaticity) affect \( SNR \) spectra, we will first consider the signal and noise spectra of photons with the same detection properties. This may be e.g. a monochromatic thin-screen model which is
then used as a building block for the polychromatic thick-screen model.

Following the use in set theory, we will call this a class of photons: The (detected) x-ray photons that have (approximately) identical detection properties. The two most important variables for detection properties are \( E \), the x-ray energy and \( z \), the position within the scintillation screen. The MTF depends on the latter and the depth intensity distribution is energy-dependent.

The actual image then consists of contributions from classes of photons – the superposition of the images of all individual classes.

3.2. Signal and Noise Superposition

To model \( SNR \) spectra of polychromatic images, the power spectra of signal and noise must be calculated for a superposition of images. In the strictest sense, this can be seen as a superposition of x-ray detection events (one image = one event). We thus need to derive the superposition equations for signal and noise power spectra.

If \( d_{\Sigma}(x) \) is the polychromatic superimposed intensity image at the pixel position \( x \) and \( d(x, E) \) the image contribution at energy \( E \) (= photon class), we get:

\[
d_{\Sigma}(x) = \sum_E d(x, E)
\]

(7)

Where the sum may also represent an energy integral, in which case \( d(x, E) \) is a density. In general, this sum may also be a multiple sum/integral over several variables \( (E, z, ...) \), where one combination of variables corresponds to a photon class.

The linear additive noise model can be used to split \( d \) into signal \( s \) and noise \( n \):

\[
d(x) = s(x) + n(x)
\]

(8)

The fact that there is a transformation between physical signal (volume distribution) and \( s(x) \) (projection image) is omitted here because it does not influence the results. In a volume \( SNR \) analysis, the noise is also transformed by the CT reconstruction. We can derive polychromatic expressions for signal and noise by adding up intensities in image space:

\[
s_{\Sigma}(x) = \sum_E s(x, E)
\]

(9)

(9)

\[
n_{\Sigma}(x) = \sum_E n(x, E)
\]

(10)

The image power spectrum is calculated as:

\[
D_{\Sigma}(u) = \left| \mathcal{F} \{ d_{\Sigma}(x) \} \right|^2 = S_{\Sigma}(u) + N_{\Sigma}(u)
\]

(11)

because noise is uncorrelated with any other signal/noise and the corresponding mixed terms vanish.

Similarly, for the noise power spectrum itself we get:

\[
N_{\Sigma}(u) = N_0(u) + \left| \mathcal{F} \{ \sum_E n(x, E) \} \right|^2
\]

(12)

\[
= N_0(u) + \sum_E N(u, E)
\]

(13)

Where \( N_0 \) are noise contributions in addition to photon noise, e.g. camera readout/dark noise. Scattered photons are counted in the noise sum (energy of the scattered photon), but usually do not contribute to the signal.

For the signal, we get:

\[
S_{\Sigma}(u) = \left| \mathcal{F} \{ \sum_E s(x, E) \} \right|^2,
\]

(14)

which is simplified in the next section.

3.3. Local Area Approximation

If we consider the image quality in a small area (locally), we can approximate the intensity as constant within that area. In the following, intensity thus stands for the number of x-ray photons detected in a local area (units of 1). A local area is much larger than a single pixel and contains weak sample structure. This also allows us to assume that for absorption, the signal strength is proportional to the absorption strength. The proportionality is given by a linear approximation of the absorption curve (lambert-beer law).

Because in the end we are interested in finding an optimum of the unit-free quantity \( SNR \), physical units are not considered. They can be restored if needed. We can thus write a simplified model for the signal and its power spectrum as:

\[
s(x, E) = I(E) [\kappa(E) \rho(x)] * h(x, E)
\]

(15)

\[
S(u, E) = P(u) \left( I(E) \kappa(E) H(u, E) \right)^2
\]

(16)

The signal is a product of the detected intensity \( I(E) \), an energy independent spatial matter density
\[ \rho(x) \leftrightarrow P(u), \] the MTF \( H(u, E) \) and an interaction strength \( \kappa(x, E) \). We have used \( \kappa(x, E) = \kappa(E) \) (single material sample) for simplicity—considering multi material samples would not give new effects. Note that we assume the imaging device to be a linear translationally invariant (LTI) system in the local area.

\( \kappa(E) \) can model almost any signal generating physical effect. This is usually x-ray absorption or phase contrast. In an cases like imaging with x-ray optics or grating interferometry \([7]\), \( \kappa(x, E) \) is also determined by the energy band for which the x-ray optics or gratings are optimized.

For a signal sum eq. \( (13) \) becomes:
\[ S_\Sigma(u) = P(u) \left[ \sum_{E} I(E)\kappa(E)H(u, E) \right]^2 \] (17)

The influence of the sample structure \( P(u) \) is a global scaling factor. The right hand side originally contains the square of the absolute value, but all sum contributions are real valued. It can be easily seen that signal sums are difficult to evaluate, because many physical effects need to be considered—even in a basic model.

Signal power spectra of large structures show higher amplitude at lower spatial frequencies and \( P(u) \) is then usually monotonously decreasing towards higher frequencies. The reason is that sample structures are usually monotonously decreasing towards higher frequencies. Larger structural details therefore have an intrinsically higher SNR spectrum value and are easier to detect. A lower SNR \( (u) \) usually has the effect that the detail resolution of a measurement gets worse.

It is possible to define a quantity that is not dependent on the amount of structure in the sample or on the shape of the signal power spectrum. We will call this the detection effectiveness spectrum, which is derived from eq. \( (13) \) and eq. \( (17) \) and defined as:
\[ DE(u) = \frac{SNR_\Sigma(u)}{P(u)\eta} = \frac{S_\Sigma(u)}{N_\Sigma(u)P(u)\eta} = \frac{\left[ \sum_{E} I(E)\kappa(E)H(u, E) \right]^2}{\left[ N_0(u) + \sum_{E} N(u, E) \right] \eta} \] (18)

Introducing the cost function \( \eta \) further allows to compare measurements with e.g. different measurement times (\( \eta = \text{time} \)). In medical imaging, using the dose as a cost function can be appropriate. Optimizing the SNR \((u) \) will give the same result as optimizing the \( DE(u) \) if \( \eta \) is constant. In the following considerations, the SNR \((u) \) could usually be replaced by the \( DE(u) \).

The \( DE(u) \) does not depend on the specific sample structure and thus describes only the imaging setup itself. Its main advantage compared to the SNR \((u) \) is that the \( DE(u) \) allows comparable measurements. In contrast to e.g. a DQE, this quantity models polychromatic effects. Sample-dependent properties of an imaging setup are included by the \( DE(u) \).

### 3.4. SNR Spectra of Superpositions

One important aspect of SNR spectra are the effects that come from superimposing images with different properties. From eq. \( (13) \) and eq. \( (17) \) we get:
\[ SNR_\Sigma(u) = P(u) \left[ \sum_{k} I_k\kappa_kH_k(u) \right]^2 \] (19)

where \( k \) is the index for different contributions to the resulting image – a contribution may consist of a single photon class or can itself be a sum of photon classes. While both sums are linear superpositions, the square in the numerator makes many simplifications impossible. An important property of eq. \( (19) \) is that the signal contributions are weighted with the signal strength \( \kappa \) and MTF \( H \), while the noise contributions are not.

We can see from eq. \( (19) \) that adding two images with greatly differing SNR (due to relative differences in signal strength or MTF) will result in an image with a lower SNR than that of the one image with the higher SNR. The following triangle inequality holds:
\[ SNR_{1+2}(u) \leq SNR_1(u) + SNR_2(u) \] (20)

Detecting more photons (image 2) in addition to the reference situation (image 1) increases the overall SNR if:
\[ SNR_{1+2}(u) > SNR_1(u) \] (21)
\[ \iff \frac{I_1\kappa_1H_1 + I_2\kappa_2H_2}{N_0 + N_1 + N_2} > \frac{I_1\kappa_1H_1}{N_0 + N_1} \] (22)
\[ \iff \frac{I_1\kappa_1H_1 + I_2\kappa_2H_2}{I_1\kappa_1H_1} > \sqrt{\frac{N_0 + N_1 + N_2}{N_0 + N_1}} \] (23)
\[ \iff \frac{I_2\kappa_2H_2}{I_1\kappa_1H_1} > \sqrt{1 + \frac{N_2}{N_0 + N_1}} - 1 \] (24)
If this condition is violated, the additional photons (image 2) would decrease SNR if they are detected. In this equation, the absolute signal strengths are not important, but the relative strengths are.

For an infinitely small addition \( N_2 \ll N_1 \) we can approximate the square root above and get:

\[
\frac{I_2 \kappa_2 H_2}{N_2} \geq \frac{1}{2} \frac{I_1 \kappa_1 H_1}{N_1 + N_0}
\]

While this is an interesting limit case, it does not have any practical application. We define these naturally arising quantities as the "signal detection strength":

\[
SDS(u, E) = \frac{I(E) \kappa(E) H(u, E)}{N(u, E)}
\]

A SDS can be calculated either for a photon class or for a set of photon classes \( \{E\} \):

\[
SDS(u, \{E\}) = \frac{\sum(E) I(E) \kappa(E) H(u, E)}{\sum(E) N(u, E)}
\]

For \( N_0 \ll N_1 \) we can rewrite eq. (25) to:

\[
SDS_2(u, E) > \frac{1}{2} SDS_1(u, E)
\]

For an ideal detector \( (I = N) \), the SDS simplifies to:

\[
SDS(u, E) = \kappa(E) H(u, E)
\]

Note that for absorption imaging, \( H(u, E) \leq 1 \) and \( SDS(u, E) \leq \kappa(E) \). For phase contrast imaging, \( H(u, E) \) may be \( > 1 \) (or even \( \gg 1 \)).

From eq. (28) we can see that a small intensity addition requires at least half the SDS of the existing image to contribute positively. On the other hand, for a large intensity addition \( (I_2 \gg I_1) \), \( SNR_{1+2} > SNR_1 \) is always true. As an example for an intermediate case, if \( N \), \( H \) and \( I \) are equal for both images, the condition becomes \( \kappa_2 > 0.41 \kappa_1 \).

### 3.5. Weighted Superposition

As discussed in [13], applying an energy-dependent weighting factor before the superposition of the image contributions can increase the SNR. This is called "energy weighting" and there are two variants:

- Multiplication with an energy-dependent weighting factor after the detection of the x-ray photons. This is what is usually discussed as energy weighting. Because sufficiently good energy-resolving 2D imaging detectors are currently not technically feasible, this is mostly a theoretical concept.
- We will call this effect "computational energy weighting" (CEW).

A CEW weight \( w \) scales SDS with \( 1/w \).

- Implicit weighting by the different detected intensities \( I(E) \) (effective x-ray spectrum). This is influenced e.g. by the source spectrum or the detector absorption efficiency spectrum. This weighting can be changed by purposefully decreasing the detected intensity e.g. in specific energy ranges.
- We will call this effect "detection energy weighting" (DEW).

In both cases, optimizing the weighting can lead to an increase in SNR. The increases vary for different experimental conditions between a few percent and factors \( > 10 \). The larger effect sizes are for very broad spectra that are not useful without energy weighting.

For the CEW, if we multiply with a weight \( w(u, E) \), we get:

\[
SNR_{CEW}(u, w) = P(u) \frac{[\sum w(u, E) I(E) \kappa(E) H(u, E)]^2}{\sum w(u, E)^2 N(u, E)}
\]

To compute the optimal energy weighting function \( w_{opt} \) for CEW, we determine the maximum of the SNR spectrum depending on \( w(u, E) \):

\[
\frac{dSNR_{CEW}(u, w)}{dw} \bigg|_{w_{opt}} = 0
\]

which is solved by any CEW weight proportional to

\[
w_{opt}(u, E) \propto \frac{I(E) \kappa(E) H(u, E)}{N(u, E)} = SDS(u, E)
\]

This is an extension of the known result \( w_{opt}(u, E) \propto \kappa(E) \) [13]. For a detector with a limited energy resolution, eq. (27) can be used. To avoid artificial image blurring, it is sufficient to only apply a relative blurring factor in this way:

\[
w_{opt}(u, E) = \frac{I(E) \kappa(E) H(u, E)}{N(u, E) H_0(u)}
\]
Where \( H_0(u) \) can e.g. be the energy-averaged MTF. Note that the SNR spectrum is not changed when a Fourier filter is applied, but the image may be easier or more difficult to interpret. After application of the optimal CEW, the signal detection strength \( SDS \) is constant with respect to \( E \):

\[
SDS_{\text{opt. CEW}}(u, E) = \text{const}(E)
\]  

(34)

The special case of a task-independent (t.i.) measurement case is defined as a case with energy independent signal detection strength (without weighting):

\[
SDS_{t.i.}(u, E) = \text{const}(E)
\]  

(35)

\[
\Rightarrow w_{\text{opt},t.i.}(u, E) = \text{const}(E)
\]  

(36)

\[
\Rightarrow SNR_{\Sigma,t.i.}(u) = \sum_E SNR_{t.i.}(u, E)
\]  

(37)

In this case, the optimal DQE also gives the optimal SNR. This is the simplest possible case which is often implicitly used to understand detection efficiency. To do so, eq. (35) must be assumed to hold true approximately. This is only appropriate in some special cases, e.g. monochromatic imaging.

If we compare eq. (34) and eq. (35), we can see that applying an optimal CEW has the effect that the result of the CEW appears to be task-independent. The CEW itself is of course task-dependent. This also means that task-independency is the case in which there is an optimal superposition of the SNR contributions.

For the optimal DEW, there appears to be no analytical expression. The optimal DEW is thus the case with the maximal SNR. Eq. (24) can be used to find approaches that may increase SNR. The general principle is that if the x-ray photons at specific energy ranges do not contribute positively to SNR, a reduction of their intensity increases SNR.

4. Examples

4.1. Two Images Superposition

As the simplest possible example for SNR spectra of an image superposition, we will consider adding two monochromatic images:

\[
d_{1+2}(E) = d_1 \delta(E - E_1) + d_2 \delta(E - E_2)
\]  

(38)

Table 3: Examples for the \( SNR(u_a) \) of image sums for different physical effects represented by \( \kappa \) with \( \kappa_1 = 1 \) and \( SNR_1 = 1 \). Cases a) and b) are e.g. iron for 30 keV and 60 keV. Note that for a) to d), the fraction of detected photons (\( \kappa \) polychromatic DQE) is twice as high for the sum image, but for a) and b) the image quality is lower. See section 3.3 for cases e)+f).

We assume the following conditions:

\[
P(u_a)H_1(u_a) = P(u_a)H_2(u_a) = 1; \quad I_1 = I_2 = 1; \quad E_1/E_2 = 0.5
\]  

(39)  

(40)

for a structure size of \( u_a \). This effectively excludes modulation transfer effects from our examples—this can be done in a theoretic model without loss of generality but would be almost useless for a real application.

This is a realistically polychromatic case whose effect sizes approximate real cases. Inserting all these values into equations (13) and (17) yields:

\[
N_{1+2}(u_a) = I_1 + I_2
\]  

(41)

\[
S_{1+2}(u_a) = (I_1 \kappa_1 + I_2 \kappa_2)^2
\]  

(42)

\[
N_1(u_a) = I_1; \quad N_2(u_a) = I_2
\]  

(43)

\[
S_1(u_a) = (I_1 \kappa_1)^2; \quad S_2(u_a) = (I_2 \kappa_2)^2
\]  

(44)

Where we assume an ideal photon counting detector. Using the specific values listed above gives:

\[
SNR_1 = SNR_1(u_a) = 1
\]  

(45)

\[
SNR_2 = \kappa_2^2
\]  

(46)

\[
SNR_{1+2} = \frac{(1 + \kappa_2)^2}{2}
\]  

(47)

For some examples of \( \kappa_2 \) the SNR values for image sums are computed in table 3. From these examples \( ^4 \) Including \( H(u_a) \) as a factor in \( \kappa \) restores the full properties.

---

\(^4\) Including \( H(u_a) \) as a factor in \( \kappa \) restores the full properties.
we can see that the additionally detected intensity \( I_2 \) may reduce the overall image quality significantly if the \( \text{SNR}_2 \) of this additional intensity is low compared to the \( \text{SNR}_1 \) of \( I_1 \). A difference in a MTF may also produce this effect.

The reason for this behavior is easily explained by the fact that if \( \kappa_2^2 \gg \kappa_1^2 \), \( I_2 \) contributes little signal but all of its noise to the image sum. A lower detected intensity at \( E_2 \) would decrease the noise.

For this example, a high enough intensity can compensate for a low signal strength in the following way:

\[
\text{SNR}_{1+2} \geq \text{SNR}_1 \iff I_2 \geq I_1 \frac{1 - 2\kappa_2}{\kappa_2^2} \quad (48)
\]

This simple example allows us to evaluate the effect (non-optimal) energy weighting has on the \( \text{SNR} \), which is shown in figure 3 for CEW and DEW. The examples only qualitatively represent realistic cases, but demonstrate the effects which different energy weightings have on \( \text{SNR} \).

We can see that for CEW, there is always a unique maximum at \( w_2 = \kappa_2 \). For DEW, there is a unique minimum, while the maximum is obtained for \( I_2 \to \infty \), and for \( \kappa_2 < \kappa_1 / 2 \) an additional local maximum exists at \( I_2 = 0 \). If we assume that a maximal value for \( I_2 \) is given by the physical circumstances (e.g. source spectrum), the optimal weighting is either to use this maximal value or use \( I_2 = 0 \) (depending on \( \kappa_2 \)). The optimal DEW weight can thus be interpreted as a mask function that is either 0 or 1. This is usually a step function that is 1 at low energies and 0 above some threshold.

### 4.2. Simple Polychromatic Simulations

\( \text{SNR} \) simulations for simple x-ray spectra are given in figure 4. They are generated by analytically computing \( \text{SNR}(u) \), using eq. (19). Both examples use the same source spectra, detector and sample, but different energy-dependencies of the signal strength. These examples demonstrate the magnitude of the effect different energy weighting effects have on \( \text{SNR} \), they are not meant to describe an actual setup. See fig. 2 for the absorption coefficient for aluminium (Al), which gives the energy-dependent sample transparency (0.1 mm thickness) and the signal strength for the absorption signal (approximately \( \kappa(E) \propto E^{-3} \)). The x-ray inline phase contrast signal strength is assumed to be \( \kappa(E) \propto E^{-2} \).

An ideally absorbing detector is used and the source is assumed to have constant target power (product of tube voltage and current is constant). Its spectrum is a bremspectrum given by Kramers’ law, which is sufficient as a rough approximation. Higher tube voltages correspond to a higher degree of polychromaticity.

To produce plots that are easy to interpret, we again evaluate the \( \text{SNR} \) spectrum at a specific spatial frequency \( u_0 \) and assume an energy-independent MTF. Applications in real world examples must avoid this kind of simplification.

The monochromatic \( \text{SNR} \) curve is from the optimal monochromatic x-ray spectrum (at 6.1 keV for absorption and 7.0 keV for phase contrast) with the intensity given by the sum over the corresponding
Fig. 4: Simulated (photo-)absorption $SNR(u_a)$ (top) and x-ray inline phase contrast $SNR(u_a)$ (bottom) curves for a bremspectrum source with constant source power. The corresponding detected x-ray spectra are shown in the middle, where the highest energy in a curve is the tube voltage.

The optimal CEW curve uses the signal strength as the weight. The optimal DEW curve only includes photons below the optimal threshold, shown as vertical dashed lines in fig. 4 (middle). The "$\propto E$ CEW" is the energy integrating indirect detector case ($w \propto E$) and assumes efficient conversion (see eq. [5], $c^2H^2 \gg c$).

Both for photoabsorption and phase contrast signals, only the lower energy x-ray photons have a sufficiently high $SDS$ to contribute positively to the $SNR$. This effect is weaker for phase contrast due to a weaker energy dependency. In this example, the $SNR$ curves are always ordered: $\propto E$ CEW $< \text{no weight} \leq \text{optimal DEW} < \text{optimal CEW} < \text{monochromatic}$. The differences get larger for broader x-ray spectra.

Direct detectors ("no weight") here have an intrinsically higher image quality than indirect detectors ("$\propto E$ CEW") and this benefit is larger for higher degrees of polychromaticity. This difference is caused solely by the different energy weighting, as both detectors are otherwise assumed to be perfect absorbers and without additional noise.

It can be seen that weighting down (CEW) or not detecting (DEW) specific photons can increase $SNR$ by large factors. Note that the monochromatic $SNR$ is directly proportional to the cumulative intensity of the source spectrum.

The following simple rules for energy weighting can be seen in fig. 4. For optimal CEW, every additional photon increases $SNR$. For optimal DEW no additional photon decreases $SNR$. For all other weightings, additional photons can decrease or increase $SNR$.

If we use this example for a $SNR$ optimization, we can see that very low tube voltages would be optimal if CEW or DEW cannot be implemented.

We could use the optimal CEW case as $SNR_{\text{ideal}}$ in eq. [3] and the $\propto E$ CEW as $SNR_{\text{detected}}$ to compute an accurate polychromatic $DQE$ for an indirect detector with ideal absorption and efficient conversion. It has $DQE \ll 1$ for broad spectra due to its energy weighting. Computing a polychromatically averaged $DQE$ using eq. [4] without consideration of the signal strengths would give $DQE = 1$. This is where sample-independent $DQE$ models fail.

In real applications, samples usually have varying thicknesses and a setup must be optimized to give high $SNR$ at a combination of thicknesses.
4.3. Optimal Screen Thickness

$SNR(u_a)$ curves for different thicknesses $t$ of a x-ray absorbing screen (e.g. scintillator) are shown in figure 5. The screen is approximated as an idealized photoabsorber with an absorption constant $\propto E^{-3}$ (no absorption edges). All other properties and the sample are the same as above and a source spectrum with a tube voltage of 38 kV is used in the three upper plots. The average absorption values are computed as the weighted detected spectral average of the x-ray absorption efficiency of the detection screen. In real cases, the detector MTF depends on the x-ray detection position within the screen and the intensity distribution within the screen is energy-dependent.

We can see that for the ”no weight” and the ”$\propto E$ CEW” cases, there is an optimal screen thickness. Using a thicker screen reduces the $SNR$ due to the lower $SDS$ of the additionally detected higher energy photons. The ”optimal CEW” and ”optimal DEW” methods however prevent this effect. In addition, a thinner screen may have a better MTF (this effect is not simulated here).

In fig. 5 (bottom) the values of the average x-ray absorption of the detector for the $t$ with the maximal $SNR$ are shown. It can be seen that in the absence of optimal CEW or DEW, lower average absorption values are optimal for higher degrees of polychromaticity. This is an application of the DEW, as discussed before.

The average absorption for ”no weight” is identical with a polychromatically averaged $DQE$ (eq. 3). Thus the ”no weight” and the ”$\propto E$ CEW” cases have an optimal $SNR$ at a specific value of the polychromatic $DQE$ which is different from the maximal $DQE$. Raising the $DQE$ usually has other costs (e.g. worse MTF), so that in this case, increasing the $DQE$ beyond its optimum can have direct and indirect disadvantages. The ”optimal CEW” and ”optimal DEW” cases do not benefit significantly if the detector thickness is increased beyond this point.

5. Discussion

Different Use Cases

The technical capabilities of different x-ray imaging setups vary greatly. The samples are also very different: Imaging e.g. a 100$\mu$m thick metallic sample...
at 100 nm resolution or a 500 nm organic sample at 500 µm resolution requires very different setups.

High resolution x-ray imaging (typical sampling < 10 µm) generally uses lower energies (e.g. < 10 keV) due to the higher x-ray transparency of thinner samples. Detecting smaller structures also intrinsically requires a higher SNR, so its optimization is generally more important for reducing measurement time than for coarser structures. The signal in high resolution x-ray imaging mainly consists of photoabsorption and phase contrast. In low resolution x-ray imaging, larger samples are investigated. They require higher x-ray energies. For the latter, Compton scattering produces a significant part of the absorption signal.

Within the range of x-ray imaging applications, medical imaging represents a very narrow range. Models for image quality which may be appropriate for these cases can be inappropriate in a different context, e.g. material science. Imaging scientists may therefore need to use models different from those used in medical imaging (e.g. DQE).

SNR spectra can be used to optimize image quality including phase contrast effects [7,14]. The latter can be interpreted as a physical highpass filter [6]. For x-ray imaging setups with (sub-)micrometer resolution, phase contrast is often the strongest contribution to the detected signal.

If one imaging device is used for imaging very different samples, some compromise must be made on the setup components. Also, if one sample has very different attenuation lengths along the beam direction, a compromise must be made to optimize image quality for the corresponding different x-ray spectra.

It is not possible to derive general rules that apply to every case. For any specific use case, optimizing the SNR spectrum by measurements is required. For a set of similar use cases, one can derive general rules. Such a set can e.g. be imaging with a photoabsorption signal and a bremsstrahlung source. A general rule in this example is the fact that only lower x-ray energies contribute to image quality while higher energies deteriorate it. Additionally, simulations are an important method to discover ways in which an imaging setup may be optimized.

**Structure Size**

Using SNR spectra implies that sample structure (of a specific size) is of interest. Because $SNR(u)$ usually strongly decreases to higher $u$ (smaller structures), larger structures are always detected well in a specific imaging measurement. Smaller structures can only be detected down to a minimal size (“spatial resolution”) which is given by the properties of the setup and the measurement configuration. For example, the measurement time can be increased to resolve smaller details.

Depending on the structure size of interest, different imaging setups may be optimal. This fact is modeled well by SNR spectra. Optimal detection depends on what is intendend to be detected, which in itself is a decision which needs to be made by the person doing the investigation.

**Temporal Measures**

Using temporal quantities ($SNR_t$ or $DQE_t$) in an imaging context is fundamentally wrong. Temporal quantities are defined as properties within a single pixel. Due to the fact that the signal in imaging almost always spans multiple pixels, temporal quantities have no reliable quantitative relation to image quality. Additionally, the absolute value of a temporal quantity depends on the size of one pixel, which is an arbitrary choice. For indirect detectors, temporal measures also neglect the noise correlations that exist (see section 2.4).

The measurement error in a single pixel does not solely determine image quality. Instead, image quality depends on the image and what we want to detect within. Temporal quantities must be used only in specific cases and in imaging only if no absolute scale is necessary.

**Optimizing Imaging Setups**

Designing a good imaging setup or operating it well is conceptually different in a polychromatic use case than in a monochromatic use case. For monochromatic imaging, an optimal setup generates the highest possible intensity and detects all photons. In the polychromatic case one can apply a CEW and detecting or generating less photons may increase the SNR.

If optimal CEW is not possible, then the optimal polychromatic setup needs to detect as many photons as possible from some photon classes and no photons from the other photon classes, see eq. (24). Depending on the differentiating criterion, this may be very
difficult to achieve. While a detector can be designed to less efficiently detect x-ray photons of a specific energy range, this is much more difficult for x-ray photons that were absorbed at a disadvantageous position in the screen.

Counting detectors which can set an upper limit to the x-ray energy of the counted detection events are ideally suited for DEW. Setting the upper limit in such a way that photons with comparatively low SDS are not counted may potentially result in a large increase in SNR (see fig. 4).

For an indirect detector, the SNR can be increased by choosing a thinner screen for lower energy imaging – even if the absorption efficiency at the relevant energy range may then be slightly lower (see fig. 5). This effect is separate from the better MTF of the thinner screen. In many imaging setups, the screen is already thin enough for its use cases due to technical limitations or MTF considerations.

**Testable Predictions**

An image quality optimization based on SNR spectra or temporal SNR makes predictions which can be tested directly. This is done with a direct measurement, see [10] or [14] for a method. A SNR optimization should ideally be based on measurements for the specific use case. Using simulations instead requires that the simulation method was thoroughly tested with direct measurements on the same or a sufficiently similar device. This makes it possible to notice errors in the theory or in the assumptions made, which is a basic requirement if one wants to rely on such a model [9].

Concerning DQE, note that optimizing SNR may lead to a different optimal setting than optimizing polychromatically averaged DQE. This difference stems from the fact that in practice DQE measurements must use assumptions that may be wrong.

A model can both be tested by physical measurements and also by checking if it is without internal contradictions. Here, internal contradictions mainly mean that the model does not actually describe what it is supposed to describe. This is for example the case when temporal SNR is used in imaging.

**6. Conclusions**

The concept of signal detection allows a systematic investigation of physical effects that result from varying signal or detection properties for the different detected photons. Image quality measures like the $DQE(u)$ falsely assume these differences to be unimportant and are therefore unable to model cases where they are.

Temporal measures like the $SNR_t$ should principally not be used in imaging due to the fact that an image consists of more than one pixel.

We use SNR spectra as a quantitative model for image quality within the framework of signal detection. This allows a robust optimization of image quality in polychromatic x-ray imaging. The predictions of this model can and should be tested with direct measurements. An image quality optimization on an existing imaging device can be done with direct measurements without the need to correctly model its physical properties.

The main disadvantage SNR spectra have is that they are not an absolute quantitative measure. The actual value depends on the object. While this is necessary for an accurate model, a quantity that depends only on the imaging setup is more convenient. This is solved by simplifying the SNR spectra to a quantity like the detection effectiveness $DE(u)$, eq. (18), at the cost of a more complex measurement process. The $DE(u)$ is not as convenient as an absolutely normalized quantity like the $DQE(u)$, but more robust and accurate.

For polychromatic imaging, we have shown that in a case with varying signal strengths, reducing the detected intensity or weight of photons with a relatively low signal detection strength ($\approx SDS$, eq. (26)) can increase the SNR and therefore the image quality. While the absolute signal strength does not influence an image quality optimization, relative strength does. Of course, increasing the detected intensity of photons with a high $SDS$ does increases image quality. In principle, this result can be applied to any imaging setup.

The model developed here is especially useful for high resolution x-ray imaging and phase contrast imaging. We found that depending on the application, direct detectors can have an intrinsically higher SNR than indirect detectors, due to the different energy weightings.

It is likely that much work analyzing image quality using the $DQE$ or temporal SNR will transfer to the concept of signal detection without large differences in the conclusion. In these cases, the benefit will mainly be that the underlying assumptions be-
come clear, which in turn may prevent false generalizations. In other cases, using the signal detection model is required to accurately model and optimize the image quality of a polychromatic x-ray imaging device.

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Supplementary Material

- The three Jupyter notebooks (Python) that were used to generate figures, including the simulations.
- Python code for a SNR spectra evaluation from measurements as described in the A. Includes examples in two Jupyter notebooks.

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A. Measuring SNR Spectra

An important aspect of the SNR Spectra is that they can directly be measured. The method described in
SNR can be used for this purpose—also in applications other than phase contrast. In the following, we will give a short description of how to apply this method. The measurement consists of using an appropriate test object of specific material composition and thickness as sample and acquiring a series of \( K \) images \( \{d_k\}_k \) with this sample. The images can either be projections or CT reconstructions. Individual images must differ only in their noise realizations. The individual exposure times can be shorter than those used in practice; the images should be visibly noisy. The averaged image is defined as:

\[
d_{\text{avg}}(x) = \frac{1}{K} \sum_{k=1}^{K} d_k(x) \quad (49)
\]

If \( D(u) \) is the average power spectrum of the \( d_k \), \( D_{\text{avg}}(u) \) the power spectrum of \( d_{\text{avg}}(x) \), \( \tau \) the integration time of the x-ray detector, then the SNR spectrum for this setup can be calculated as:

\[
\text{SNR}_r(u) = \frac{D_{\text{avg}}(u) - K D(u) - (1 - \frac{1}{K})A(u)}{D(u) - D_{\text{avg}}(u)} \quad (50)
\]

Where \( A(u) \) is the power spectrum of the known image artifacts (e.g. reference image noise). This formula works for \( K \geq 2 \) but gives better results for \( K > 20 \). Note that for computing the power spectra from real images with a FFT, using an appropriate window function is necessary to get an accurate result.

If a reference image is used to normalize the intensity of the projection image, the noise power spectrum of a single reference image can be measured similarly as above with:

\[
N_{\text{ref}}(u) = \frac{D_{\text{ref}}(u) - D_{\text{avg,ref}}(u)}{1 - \frac{1}{K_{\text{ref}}}} \quad (51)
\]

This is similar to the denominator of eq. (50) except for the factor which canceled out. If the average of the ref images is used for the normalization, it gives a contribution to \( A(u) \) of the form:

\[
A(u) = \ldots + \frac{N_{\text{ref}}(u)}{K_{\text{ref}}} \quad (52)
\]

Detector imperfections or other image artifacts not modeled by \( A(u) \) can falsely increase the calculated signal and SNR. These signal artifacts usually appear as a lower limit to SNR at the higher spatial frequencies where the actual SNR becomes very small. For a good measurement, this lower limit can be as low as \( 10^{-3} \), which is not a practical problem.

SNR spectra measurements from different samples ("test phantoms") can generally not be compared and their absolute values have limited usefulness. The reason is that different amounts of sample structure lead to different SNR spectra. In many cases, comparability is not needed or the results can be stated as a relative difference between different conditions for the same sample.

Otherwise, the detection effectiveness spectrum defined in eq. (15) can be used. This quantity can be determined for an experimental setup if the SNR spectrum is measured and the object spectrum \( P(u) \) is computed for the object used. For full comparability, the sample thicknesses must be approximately the same for all measurements that need to be compared. It is possible to choose a test phantom for which \( P(u) \) can easily be computed, e.g. one consisting of balls. This implies that the SNR measurement is done for the CT reconstruction image.

Test phantoms may need to be combined with x-ray filters of different thickness to simulate different sample thicknesses. The cumulative thickness of all objects in the beam is an integral part of the definition of the test phantom. Any test phantom should satisfy the following properties:

- Its power spectrum should not change with small differences (\(< 1^\circ \) or \(< \) cone angle) in the orientation of the sample to the beam. Round shapes are ideal, e.g. balls with a diameter of 20-50 voxels.
- It should represent a type of sample or a category of samples which will realistically be used.

Another possibility of obtaining comparable measurements is to define standardized test phantoms. Examples of such definitions are: (1) A geometrically defined object of a specific material. (2) A random object with a defined statistic, e.g. a size distribution of balls and a specific container geometry.

Note that robust measurements of the temporal SNR require the same type of measured data but have a slightly different evaluation. Estimating temporal SNR from a region with constant gray values is prone to errors.