Investigation of two-degree-of-freedom cylindrical electromagnetic energy harvesters

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Abstract. Harvesting energy from ambient excitations with broad spectrum of frequencies is still an open problem. We present in this paper three 2-degree-of-freedom (2-DOF) electromagnetic energy harvesters (EMEHs) to achieve broadband energy harvesting with adjustable bandwidth. The 2-DOF EMEHs are realized via suspending a 1-DOF cylindrical EMEH in a larger tube. The 2-DOF as well as the 1-DOF EMEHs are modeled, and the theoretical models have been validated by frequency sweeps performed by experiment. The parametric investigation indicates that the 2-DOF EMEHs have higher power output and wider working bandwidth than the 1-DOF EMEH. Among the three proposed 2-DOF EMEHs, employing two sets of coils to develop the movement of the two magnets affixed at the two ends of the 1-DOF EMEH’s cylindrical housing is more favored as it can provide the highest power level and adjustable closeness between the first and second power peaks.

1. Introduction
The continuous reduction in power consumption of small electronics has opened the possibility of achieving self-sustained systems using the energy harvested from ambient mechanical motions. The motion-to-electricity conversion can be performed through various transduction mechanisms, including piezoelectric [1, 2], inductive [3, 4], capacitive [5, 6], and triboelectric methods [7]. These mechanisms have been integrated with the spring-mass-damper oscillator to build energy harvesters that generate electricity from mechanical motions based on the exploitation of the vibration of the mechanical oscillators.

No matter what transduction mechanism is employed, the linear configuration of energy harvesters features simple structure, making it an object of a number of research efforts. However, the linear harvester usually suffers from narrow operating bandwidth and then poor adaptability. To overcome this limitation, different broadband strategies have been proposed, including tuning the resonance [8], integrating an array of linear oscillators in one harvester [9], up-converting the excitation frequency [10, 11], exploiting two-degree-of-freedom (2-DOF) structure [12, 13], and introducing nonlinear force into the harvesters [14, 15]. The improved performance achieved by the broadband strategies, particularly by the linear 2-DOF configuration, has been demonstrated by several studies [16, 17], but the cantilever beams used in the harvesters have raised concerns on the lifespan and durability of harvesters.

In recent years, electromagnetic energy harvesters (EMEHs) with suspended structure have attracted a great deal of attention due to its superior durability and nonlinear response to external
excitations [3, 18, 19]. Specifically, the EMEH with suspended structure, which is made up of a moveable center magnet, two endmost magnets, a set of coil and a tube, the coil is wound around the tube’s outer surface. The typical hardening response is exhibited in such suspended EMEH under harmonic excitations, enabling it to harvest energy from broadband excitations. Different from the bistable piezoelectric energy harvester (PEH) that exhibits typical hardening response [20, 21], there is no need for the suspended EMEH to overcome the high potential barrier that appears in the bistable PEH, removing the requirement for large excitations so as to drive the oscillator to jump between two stable equilibrium positions, which guarantees the high power output.

In order to expand the operating bandwidth of the suspended EMEH further, we upgrade the 1-DOF configuration to the 2-DOF configuration with three different layouts. Theoretical models for the three 2-DOF suspended EMEHs are established and verified by experiment. Parametric studies on the three 2-DOF EMEHs are implemented to exhibit their performance and seek after the optimal configuration.

2. 2-DOF EMEHs and models

Figure 1 shows the suspended EMEHs with various configurations. The 1-DOF EMEH consists of a tube, a moveable center magnet, two endmost magnets and a set of coil. The coil is wound around the tube’s outer surface. As shown in Figure 1(b), the 1-DOF EMEH is suspended magnetically within a large tube to constitute the first 2-DOF EMEH (layout-A). The second 2-DOF EMEH (layout-B) is transformed from layout-A by moving the coil from the inner tube to the outer tube, which still exploits the motion of the center magnet for power generation. The third 2-DOF EMEH (layout-C) converts the motion of magnets 1 and 1’ to electricity, which is realized via using two sets of coils that are wrapped around the outer tube and are aligned with the two magnets along the longitudinal direction of the tube. Three 2-DOF EMEH have the same mechanical structure, and the only difference is the position of the coil.

The schematic diagrams of various EMEHs are shown in Figure 2, in which the magnetic interaction between magnets is represented as the nonlinear spring and the origin of the Cartesian coordinate system is set to the midpoint of the tube’s axis. Let \( u_0, u_1, \) and \( u_2 \) represent the absolute coordinates of the base, \( M_1, \) and \( M_2, \) respectively, the electromechanical model of the 1-DOF EMEH is written as

\[
\begin{align*}
M_1 \ddot{z}_1 + \eta_1 \dot{z}_1 + \lambda_1 M_1 g \cos \theta + \gamma_1 I_1 &= F_{m1} - M_1 \dot{u}_2 \\
L_1 \ddot{I}_1 + (R_1 + R_{11}) I_1 &= \gamma_1 \dot{z}_1
\end{align*}
\]

where \( z_1 = u_1 - u_2; \lambda_1 \) is the mechanical friction coefficient between the center magnet and the tube; \( M_1 \) is the mass of the center magnet; \( \eta_1 \) is the mechanical damping of mass \( M_1; \theta \) denotes the angle of the tube to the ground; \( g \) is the gravitational acceleration; \( \gamma_1 \) denotes the electromechanical coupling coefficient; \( F_{m1} \) is the magnetic force between the endmost magnets (1 and 1’) and the center magnet; \( R_1 \) is the load; \( I_1 \) is the current flowing through the resistor \( R_1; L_1 \) and \( R_{11} \) are, respectively, the inductance and resistance of the coil.

![Figure 1](image-url)

**Figure 1.** Suspended EMEHs with a variety of structures: (a) 1-DOF, (b) 2-DOF layout-A, (c) 2-DOF layout-B, and (d) 2-DOF layout-C.
Figure 2. Schematic diagrams of various EMEHs: (a) 1-DOF, (b) layout-A, (c) layout-B, and (d) layout-C.

Let $z_2 = u_2 - u_0$, the mathematical model of the 2-DOF EMEH of layout-A come in the following form

$$\begin{align*}
(M_1 + M_2)\ddot{z}_2 + \eta_2\dot{z}_2 + \lambda_2g \cos \theta + \gamma_A I_A &= F_{m2} - (M_1 + M_2)\ddot{u}_0 - M_1\dddot{z}_1 \\
M_2\dddot{z}_2 + \eta_2\ddot{z}_2 + \lambda_2M_2g \cos \theta + \gamma_A I_A &= F_{m1} - M_1(\dddot{u}_0 + \dddot{z}_1) \\
L_2\dot{I}_A + (R_0 + R_A)I_A &= \gamma_A (\dddot{z}_1 + \dddot{z}_2)
\end{align*}$$

(2)

where $M_2$ is the mass of the 1-DOF EMEH excepting the center magnet; $F_{m2}$ is the magnetic force from the interaction of the inner tube and the endmost magnets (2 and 2'); $\lambda_2$ is the mechanical friction coefficient between the inner tube and outer tube; $\eta_2$ is mechanical damping of mass $M_2$. Similarly, the electromechanical model of the 2-DOF EMEHs of layout-B and layout-C can be described by the equation (3) and (4), respectively.

$$\begin{align*}
(M_1 + M_2)\ddot{z}_2 + \eta_2\dot{z}_2 + \lambda_2(M_1 + M_2)g \cos \theta + \gamma_{ib} I_B &= F_{m2} - (M_1 + M_2)\dddot{u}_0 - M_1\dddot{z}_1 \\
M_2\dddot{z}_2 + \eta_2\ddot{z}_2 + \lambda_2M_2g \cos \theta + \gamma_{ib} I_B &= F_{m1} - M_1(\dddot{u}_0 + \dddot{z}_1) \\
L_B\dot{I}_B + (R_0 + R_{ib})I_B &= \gamma_{ib} (\dddot{z}_1 + \dddot{z}_2) \\
(M_1 + M_2)\ddot{z}_2 + \eta_2\dot{z}_2 + \lambda_2(M_1 + M_2)g \cos \theta + \gamma_{ic} I_C &= F_{m2} - (M_1 + M_2)\dddot{u}_0 - M_1\dddot{z}_1 \\
M_2\dddot{z}_2 + \eta_2\ddot{z}_2 + \lambda_2M_2g \cos \theta + \gamma_{ic} I_C &= F_{m1} - M_1(\dddot{u}_0 + \dddot{z}_1) \\
L_C\dot{I}_C + (R_0 + R_{ic})I_C &= \gamma_{ic} (\dddot{z}_1 + \dddot{z}_2)
\end{align*}$$

(3)

(4)

where $R_{ib}$ is the load; $I_i$, $L_i$ and $R_i$ are the output current, inductance, and resistance, respectively; $\gamma_i$ is the electromechanical coupling coefficient; and the subscript $i$ denotes A (layout-A), B (layout-B), or C (layout-C). The magnetic forces $F_{m1}$ and $F_{m2}$ in the models can be calculated by a cuboidal magnet model, in which the cylindrical magnets are equivalent to cuboidal magnets which have the same thickness and volume as the former [4].
It is worth mentioning that the magnetic forces between the endmost magnets (2 and 2’) and the center magnet are ignored in the electromechanical model. In order to evaluate the influence of this magnetic force on the model, a kinetic analysis is carried out under the acceleration amplitude of 4 g. The result shows that the minimum gap between the endmost magnet 2 (2’) and the center magnet is 17.5 mm where the repulsive force is about 0.59 N. For the magnetic force $F_{m1}$ and $F_{m2}$, the values at the minimum gap are 6.12 N and 20.46 N, respectively, which are much larger than the former. The comparison shows that the magnetic force between the endmost magnet 2 (2’) and the center magnet has inessential effect on the electromechanical model of the 2-DOF EMEH.

![Figure 3. Prototypes and experimental setup.](image)

### 3. Model validation

The experimental setup for examining the theoretical models is shown in Figure 3 along with the 1-DOF and 2-DOF (layout-A) prototypes. Since the 2-DOF EMEHs has a similar configuration (except the difference in the position of the coil), we only test the model for the layout-A. Following a procedure given by Liu et al. [22], the experiment parameters for the 1-DOF and 2-DOF prototypes are determined, as shown in Table 1. The 1-DOF EMEH is tested by the frequency sweep operation from 15 Hz to 30 Hz. The experimental excitation is sinusoidal and the amplitude of acceleration is 0.8g, whereas for the 2-DOF EMEH, the amplitude of acceleration is also 0.8g and the frequency is varied from 10 Hz to 30 Hz.

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| $M_1$ (g) | 11.3  | $M_2$ (g) | 29.2  |
| $\eta_1$ (Nsm$^{-1}$) | 0.07  | $\eta_2$ (Nsm$^{-1}$) | 0.20  |
| $\lambda_1$ | 0.05  | $\lambda_2$ | 0.05  |
| $\gamma_A$ (Wb m$^{-1}$) | 0.6   | $R_{LA}$ (Ω) | 12   |
| $\gamma_B$ (Wb m$^{-1}$) | 0.45  | $R_{LB}$ (Ω) | 15   |
| $\gamma_C$ (Wb m$^{-1}$) | 1.1   | $R_{LC}$ (Ω) | 15   |
| $L_A$ (H) | 0.0275 | $D_1$ (mm) | 29   |
| $L_B$ (H) | 0.060  | $D_2$ (mm) | 25   |
| $L_C$ (H) | 0.047  | $L$ (mm) | 66   |

For the 1-DOF EMEH, the voltage response exhibits the typical hardening phenomenon, which is revealed by both simulation results and experimental measurements, as shown in Figure 4. Although small differences between model and experiment in terms of peak voltage and corresponding frequency can be observed, which is probably due to the presence of friction between the center magnet and the inner tube but is not taken in account in the simulations, the simulations predict the same trends of nonlinear behavior as the experiments. For the 2-DOF EMEH, not only the hardening response but also two voltage peaks can be observed from the voltage output, and the experimental results agree well with the theoretical results, as shown in Figure 5.
4. Performance evaluation

This section presents theoretical power outputs of EMEHs with various configurations, where the power is calculated by \( P = \frac{V_{\text{RMS}}^2}{R_L} \) with \( V_{\text{RMS}} \) denoting the root-mean-square (RMS) voltage through the load resistor \( R_L \). The experimentally measured electromechanical coupling coefficients for the 2-DOF layout-B and layout-C are 0.45 and 1.1, respectively. Throughout the simulations, the excitation is a sinusoidal excitation and the acceleration amplitude is 0.8g.

![Figure 4. Voltage output of 1-DOF EMEH: (a) simulation and (b) experiment.](image)

4.1. Effects of varying the length of inner tube

For the 1-DOF EMEH, varying the length of the (inner) tube has ignorable effect on the power output, but a longer tube can shift the working frequency band to the left (lower frequency), as shown in Figure 6. Encapsulating the 1-DOF EMEH in a larger tube (112 mm long) enables the harvester (layout-A) to generate two power peaks with adjustable separation in terms of frequency. Although the power output can also be maximized through changing the length of the inner tube, the closeness between two power peaks is affected by the optimization of the power output. An inner tube with a length of around 71 mm enables the 2-DOF EMEH of layout-A to generate significant power output with two close power peaks. For the layout-B, although the peak power and operating frequency band can be regulated by changing the length of the inner tube, the second peak power is insignificant, making it even inferior to the 1-DOF EMEH. For the layout-C, decreasing the length of the inner tube can enhance the value of the first peak power remarkable but at the cost of second peak power output, which can improve the harvester’s performance of energy scavenging under excitations with ultra-low frequency. On the contrary, when the length of the inner tube is increasing, the second peak power will increase rapidly but the first peak power shows a significant reduction.

![Figure 5. Voltage output of 2-DOF EMEH of layout-A: (a) simulation and (b) experiment.](image)

4.2. Effects of varying the magnetic mass
Power outputs of various EMEHs with different magnetic masses are shown in Figure 7, in which \( r \) indicates the ratio between \( M_1 \) and \( M_2 \). It should be noted that the total mass \((M_1 + M_2)\) remains constant and the \( L \) is set to be 71 mm throughout the simulations in this section. About the 1-DOF EMEH, we find that increasing the mass of the center magnet (in the meanwhile decreasing the value of \( M_2 \) to keep \( M_1 + M_2 \) unchanged) can not only shift the working frequency band to the left but also improve the peak power output, a phenomenon similar to the linear harvesters. For the 2-DOF EMEH of layout-A, increasing the value of \( r \) can lower the first peak frequency but push the second peak frequency toward the right, resulting in enlarged interval between the two peaks. Significant power outputs from the two peak frequencies can be achieved when \( r \) ranges from 0.54 to 1.85. For the layout-B, the first peak frequency is smaller than the peak frequency of the 1-DOF EMEH for the same value of \( r \), but the second peak power is insignificant for all values of \( r \) considered in this study. For the layout-C, the power output at the first peak frequency decreases with increasing \( r \) from 0.18 to 0.82, and then maximizes with increasing \( r \) further to 1.22, after which the first peak power attenuates again. Moreover, increasing the value of \( r \) lowers the first peak frequency but enhances the second peak frequency, leading to enlarged separation between the two peaks.

Overall, the 2-DOF EMEHs can work under lower frequency and provide larger power than the 1-DOF EMEH. Among the three 2-DOF EMEHs, the layout-C has the best performance based on the power output, followed by the layout-A. For the layout-B, the power output at the second peak frequency is scarcely discernible, making it inferior to the other two 2-DOF EMEHs.

\[ \text{Figure 6. Power outputs of various EMEHs with different lengths of inner tube: (a) 1-DOF, (b) layout-A, (c) layout-B, and (d) layout-C.} \]

5. Conclusions
Three 2-DOF EMEHs are proposed in this paper to achieve broadband energy harvesting. For comparison purpose, theoretical models for the 2-DOF EMEHs and 1-DOF are established and validated by experiment. The theoretical results indicate that improving the 1-DOF EMEH to 2-DOF EMEHs can not only expand the operating bandwidth but also enhance power output. However, for
the 2-DOF EMEHs, certain tradeoff is required between the peak power output and the enlarged bandwidth (or the closeness between the first and second peak frequencies). Among the three proposed 2-DOF EMEHs, the layout-C is more favored as it can provide the highest power level and adjustable bandwidth.

Figure 7. Power outputs of various EMEHs with different magnetic masses: (a) 1-DOF, (b) layout-A, (c) layout-B, and (d) layout-C.

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