Evolutionary Game Analysis of Government Regulation and Enterprise Emission from the Perspective of Environmental Tax

Yazong Mai
State Grid Energy Research Institute, Beijing, China
maiyyazzong@163.com

Abstract: In the context of the upcoming implementation of the environmental tax policy, there is a need for a focus on the relationship between government regulation and corporate emissions. To achieve the real effect of environmental tax policy, government need to regulate the illegal emissions of enterprises. Based on the hypothesis of bounded rationality, this paper analyses the strategic set of government regulators and polluting enterprises in the implementation of environmental tax policy. By using the evolutionary game model, the utility function and payoff matrix of the both sides are constructed, and the evolutionary analysis and strategy adjustment of the environmental governance target and the actual profit of the stakeholders are carried out. Thus, the wrong behaviours could be corrected so that the equilibrium of the evolutionary system can be achieved gradually, which could also get the evolutionary stable strategies of the government and the polluting enterprises in the implementation of environmental tax policy.

1. Introduction
Polluting enterprises are the main source of China's pollution emissions, and also the main object of environmental tax. With the increase of collection intensity and collection standards, environmental tax levy the biggest resistance from the polluting enterprises, to be its focus. Therefore, in the environmental tax perspective, the government regulation and corporate emissions between the existence of the scientific relationship between the scientific demonstration and reasonable analysis, help to protect the effective implementation of environmental tax policy.

2. Model Construct
The evolutionary game model of government regulation involves two types of participant: 1) polluting enterprises; 2) Government regulation department. Each game is the enterprise and the regulatory department of random matching, assuming that the government regulatory action set to (strong supervision, weak supervision), the action set for the enterprise is (standard emission, illegal emission). Both sides of the game are bounded rational, enterprises in the strict environmental control of the government situation may still choose illegal emissions, and the government in the case of corporate illegal emissions may also choose to sit idly by, that is, to allow both sides of the game mistakes.

In the process of environmental tax levy, it is assumed that the government chooses "strong supervision" as x (0 ≤ x ≤ 1), the probability of choosing "weak supervision" is 1-x; enterprises choose " standard emission" probability y (0 ≤ y ≤ 1), the probability of "illegal emission" is 1-y. The
production cost when the enterprise is discharged is $C_1$, and the production cost when the illegal discharge is $C_2$ ($C_2 < C_1$). The profit of the market for the products produced by the enterprise is $P$.

In the case of the government's choice of "strong supervision", it is assumed that the illegal enterprises will be found and will be punished by $M_1$. The government departments that choose "strong supervision" can receive positive incentives $S$, including the higher government departments' incentives for their environmental work assessment and public recognition of the environmental protection work of government departments. "Strong supervision" need to invest more human, material and financial resources, set up government departments, "strong supervision" law enforcement costs $G_1$.

In the case of government choice of "weak supervision", assuming that corporate illegal emissions will not be found by government regulators, but the illegal emissions of enterprises have $z$ ($0 \leq z \leq 1$) the possibility of being found by the public or media exposure. At this point, the company will be the reputation of the loss of $R$ ($R > 0$) and the government regulators punish $M_1$. At the same time, the government regulators, because there is no timely detection of illegal polluting business, will be the loss of $M_2$, including the relevant person in charge of administrative accountability, loss of local people's trust. In the case of "weak supervision", the cost of law enforcement for government departments is $G_2$ ($G_2 < G_1$).

Based on the above assumptions, we can derive the income matrix of government regulators and polluting enterprises under different behavior choices (Table 1).

| Both sides of the game | Government regulators |
|------------------------|-----------------------|
| Polluting enterprises  | strong supervision $(x)$ | weak supervision $(1-x)$ |
| Standard emission $(y)$ | $P-C_1, S-G_1$ | $P-C_1, -G_2$ |
| Illegal emission $(1-y)$ | $P-C_2-R-M_1, S-G_1$ | $P-C_2-z(R+M_1), -G_2-zM_2$ |

Assuming that government departments choose $E_{11}, E_{12}$ and $E_{1}$ for "strong supervision" and "weak supervision", respectively, according to the income matrix, we can see:

$$E_{11} = y(S-G_1) + (1-y)(S-G_1) = S-G_1$$  
(1)

$$E_{12} = -yG_2 + (1-y)(-G_2 - zM_2) = zM_2(y-1) - G_2$$  
(2)

$$\bar{E}_1 = xE_{11} + (1-x)E_{12}$$  
(3)

Assuming that the expected return and average expected return of "standard emission" and "illegal emission" are $E_{21}, E_{22}$ and $\bar{E}_2$, respectively:

$$E_{21} = x(P-C_1) + (1-x)(P-C_1) = P-C_1$$  
(4)

$$E_{22} = x(P-C_2-R-M_1) + (1-x)[P-C_2-z(R+M_1)]$$  
$$= P-C_2 - [1 - (1-x)(1-z)](R+M_1)$$  
(5)

$$\bar{E}_2 = yE_{21} + (1-y)E_{22}$$  
(6)

Constructing the dynamic equation of the replicator between government and enterprise strategy is:

$$F(x) = \frac{dx}{dt} = x(E_{11} - \bar{E}_1) = x(1-x)[S + G_2 - G_1 - zM_2(y-1)]$$  
(7)
\[ F(y) = \frac{dy}{dt} = y(E_2 - \bar{E}_2) = y(1 - y)[C_2 - C_1 + (1 - (1 - x)(1 - z))(R + M_1)] \]  

(8)

### 3. Stability Analysis

Let \( F(x) = 0 \) and \( F(y) = 0 \) and solve the dynamic equation of the replicator of the government and the enterprise strategy. The five local equilibrium points of the system are \((0, 0), (0, 1), (1, 0), (1, 1)\) and \((p, q)\).

\[
p = \frac{C_1 - C_2 - z(R + M_1)}{(1 - z)(R + M_1)}, \quad q = \frac{S + G_2 - G_1 - zM_2}{zM_2} \tag{9}
\]

The equilibrium point solved by the replica dynamics equation is not necessarily the ESS (Evolutionarily Stable Strategy) of the system. According to the authentication method proposed by Friedman, the ESS of the differential equation system can be derived from the local stability analysis of the Jacobian matrix (abbreviated as \( J \)) of the system.

\[
J = \begin{bmatrix}
\frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\
\frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y}
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \tag{10}
\]

When the Jacobian matrix \( J \) satisfies the following two conditions at the same time, the local equilibrium point of the system becomes the evolution stabilization strategy:

1. \( \det J = |a_{11} a_{12}| = a_{11}a_{22} - a_{12}a_{21} > 0 \)
2. \( \text{tr} J = a_{11} + a_{22} < 0 \)

In the formula,

\[
a_{11} = [S + G_2 - G_1 - zM_2(y - 1)](1 - 2x) \]
\[
a_{12} = -zM_2x(1 - x) \]
\[
a_{21} = (R + M_1)(1 - z)y(1 - y) \]
\[
a_{22} = \{C_2 - C_1 + [1 - (1 - x)(1 - z)](R + M_1)\}(1 - 2y)
\]

\((0, 0), (0, 1), (1, 0), (1, 1)\) and \((p, q)\) of the system are substituted into \( a_{11}, a_{12}, a_{21}, a_{22} \) respectively. Specific values are shown in Table 2.

| Equilibrium point | \( a_{11} \) | \( a_{12} \) | \( a_{21} \) | \( a_{22} \) |
|-------------------|------------|------------|------------|------------|
| \((0, 0)\)        | \( S + G_2 - G_1 + zM_2 \) | 0           | 0           | \( C_2 - C_1 + z(R + M_1) \) |
| \((0, 1)\)        | \( S + G_2 - G_1 \) | 0           | 0           | \( -[C_2 - C_1 + z(R + M_1)] \) |
| \((1, 0)\)        | \( -(S + G_2 - G_1 + zM_2) \) | 0           | 0           | \( C_2 - C_1 + R + M_1 \) |
| \((1, 1)\)        | \( -(S + G_2 - G_1) \) | 0           | 0           | \( -(C_2 - C_1 + R + M_1) \) |
| \((p, q)\)        | 0           | \( a \)    | \( b \)    | 0           |

Among them,

\[
a = \frac{zM_2(C_1 - C_2 - R - M_1)[C_1 - C_2 - z(R + M_1)]}{(1 - z)^2(R + M_1)^2} \tag{11}
\]

\[
b = \frac{(R + M_1)(1 - z)(S + G_2 - G_1 - zM_2)(2zM_2 - S - G_2 + G_1)}{(zM_2)^2} \tag{12}
\]

Since \((p, q)\) at the point \( a_{11} + a_{22} = 0 \), does not meet the conditions\(^\circ\), the local equilibrium point \((p, q)\) is not an evolutionary stabilization strategy. Thus, the evolution strategy of government and corporate strategy can only be the remaining four local equilibrium.

We discuss the remaining four local equilibrium points in order to test the possibility of becoming an evolutionary stabilization strategy. According to the government regulators received by the positive
incentive $S$ and the sewage enterprises due to be reported or exposure to the reputation of the loss of $R$ and government regulators punishments $M_i$ different range of values, divided into the following six cases to discuss:

① when $S < G_1 - G_2 - zM_2$ and $R + M_1 < \frac{c_1 - c_2}{z}$

The stability analysis of the four local equilibrium points is shown in table 3. It can be seen that only the point $(0, 0)$ is a progressive stable fixed point in the dynamic evolution process, which corresponds to the evolution stabilization strategy.

**Table 3. Stability Analysis of Local Equilibrium Point in Case ①**

| Equilibrium point | detJ | trJ | Stability     |
|-------------------|------|-----|---------------|
| $(0, 0)$          | +    | -   | ESS           |
| $(0, 1)$          | -    | Uncertain | Saddle point |
| $(1, 0)$          | -    | Uncertain | Saddle point |
| $(1, 1)$          | +    | +   | Unstable point|

② when $S < G_1 - G_2 - zM_2$ and $R + M_1 > \frac{c_1 - c_2}{z}$

The stability analysis of four local equilibrium points is shown in table 4. It can be seen that only the point $(0, 1)$ is a progressive stable fixed point in the dynamic evolution, which corresponds to the evolutionary stability strategy.

**Table 4. Stability Analysis of Local Equilibrium Point in Case ②**

| Equilibrium point | detJ | trJ | Stability     |
|-------------------|------|-----|---------------|
| $(0, 0)$          | -    | Uncertain | Saddle point |
| $(0, 1)$          | +    | -   | ESS           |
| $(1, 0)$          | +    | +   | Unstable point|
| $(1, 1)$          | -    | Uncertain | Saddle point |

③ when $G_1 - G_2 - zM_2 < S < G_1 - G_2$ and $R + M_1 < \frac{c_1 - c_2}{z}$

The stability analysis of four local equilibrium points is shown in table 5. It can be seen that only the point $(1, 0)$ is a progressive stable fixed point in the dynamic evolution process, which corresponds to the evolutionary stability strategy.

**Table 5. Stability Analysis of Local Equilibrium Point in Case ③**

| Equilibrium point | detJ | trJ | Stability     |
|-------------------|------|-----|---------------|
| $(0, 0)$          | -    | Uncertain | Saddle point |
| $(0, 1)$          | +    | -   | ESS           |
| $(1, 0)$          | +    | +   | Unstable point|
| $(1, 1)$          | -    | Uncertain | Saddle point |

④ when $G_1 - G_2 - zM_2 < S < G_1 - G_2$ and $R + M_1 > \frac{c_1 - c_2}{z}$

The stability analysis of four local equilibrium points is shown in table 6. It can be seen that only the point $(1, 0)$ is the asymptotic stability fixed point in the dynamic evolution process, which corresponds to the evolution stabilization strategy.

**Table 6. Stability Analysis of Local Equilibrium Point in Case ④**

| Equilibrium point | detJ | trJ | Stability     |
|-------------------|------|-----|---------------|
| $(0, 0)$          | +    | +   | Unstable point|
| $(0, 1)$          | +    | -   | ESS           |
| $(1, 0)$          | -    | Uncertain | Saddle point |
| $(1, 1)$          | -    | Uncertain | Saddle point |

⑤ when $S > G_1 - G_2$ and $R + M_1 < \frac{c_1 - c_2}{z}$

The stability analysis of four local equilibrium points is shown in table 7. It can be seen that only the point $(1, 0)$ is a progressive stable fixed point in the dynamic evolution process, which corresponds to
the evolutionary stability strategy.

Table 7. Stability Analysis of Local Equilibrium Point in Case®

| Equilibrium point | detJ | trJ | Stability       |
|-------------------|------|-----|-----------------|
| (0, 0)            | -    | -   | Uncertain       |
| (0, 1)            | +    | +   | Stable point    |
| (1, 0)            | -    | -   | ESS             |
| (1, 1)            |      |     | Saddle point    |

® when \( S > G_1 - G_2 \) and
\[ R + M_1 > \frac{c_1-c_2}{z} \]

The stability analysis of the four local equilibrium points is shown in table 8. It can be seen that only the point (1, 1) is a progressive stable fixed point in the dynamic evolution, which corresponds to the evolution stabilization strategy.

Table 8. Stability Analysis of Local Equilibrium Point in Case®

| Equilibrium point | detJ | trJ | Stability       |
|-------------------|------|-----|-----------------|
| (0, 0)            | +    | +   | Unstable point  |
| (0, 1)            | -    |     | Saddle point    |
| (1, 0)            | -    | -   | Saddle point    |
| (1, 1)            | +    | -   | ESS             |

4. Result Analysis

Based on the analysis of the above six cases, the evolution stabilization strategy (ESS) under different conditions can be summarized as shown in table 9.

Table 9. Evolutionary Stabilization Strategy Summarization under Different Conditions

| Condition            | Corresponding situation | ESS      |
|----------------------|--------------------------|----------|
| \( S < G_1 - G_2 - z M_2, \ R + M_1 < \frac{c_1-c_2}{z} \) | ©                       | (0, 0)   |
| \( S < G_1 - G_2, \ R + M_1 > \frac{c_1-c_2}{z} \)   | ©, ©                    | (0, 1)   |
| \( S > G_1 - G_2 - z M_2, \ R + M_1 < \frac{c_1-c_2}{z} \) | ©, ©                    | (1, 0)   |
| \( S > G_1 - G_2, \ R + M_1 > \frac{c_1-c_2}{z} \)   | ©                       | (1, 1)   |

In case ©, the government regulators are less motivated to implement strict supervision. Assuming that the law enforcement costs of "strong supervision" and "weak supervision" are stable. The greater the positive incentive for government departments, the lower the likelihood of image loss and administrative accountability. For enterprises, if the circumstances of its illegal acts of deportation by the public or the media exposed, the loss of its reputation and by the government departments punished less than the proceeds of its illegal emissions, enterprises have sufficient economic incentives to carry out illegal emissions. The game analysis result shows that the evolution of this situation is strategy of "weak government supervision, corporate illegal emissions", which will make the environmental tax repeat the mistakes of pollute charge system, which means it can’t effectively play the function of environmental taxes and the establishment of environmental tax targets.

In case © and ©, we can see that the evolutionary strategy of the situation is "weak government supervision, corporate discharge", although the supervision of government regulators to maintain a low level. But because of the public and the media by the high degree of supervision and high chase, the higher the cost of illegal, companies still have to choose to discharge standards. In this case, the public and the media are required to have sufficient capacity and authority to supervise, which requires government departments to provide the public, media and other social supervision forces to provide smooth participation and reporting channels. At the same time, it is necessary to disclose the information of the production and emission data of the pollutant discharge enterprises, especially the
data on pollution emission and control so that the social supervision forces such as the public can truly realize effective third party supervision.

In situation③ and⑤, the government regulators have enough incentives to strengthen supervision, but because of its low penalties, not enough to make the company's illegal income below the law-abiding income, which led to the enterprise is still illegal production and emissions, which is the actual situation of the majority of polluting enterprises in China. Through the evolutionary game analysis we can see that this situation under the evolution of stability strategy is "strong government supervision, corporate illegal emissions".

In case⑥, we can see that the evolutionary stabilization strategy in this situation is "government strong supervision, enterprise standard discharge". In the face of the dual pressures of government and social supervision, the illegal risks and costs of sewage enterprises will be high, so that enterprises have to carry out clean production and discharge standards. In this case, it avoids the obstruction of poor supervision of government regulators and the low cost of illegal enterprises, which will help the environmental tax policy to play its function of pollution control and fund raising, and finally realize its policy objectives to improve the environment.

5. Conclusion

Through the game analysis we can see that there are two ways to achieve the pollutant to the "standard discharge" target strategy evolution: The first way is the case of government "weak supervision", the sewage enterprises due to the public and the media highly supervised, as well as the face of pollution behaviour was exposed after the high government punishments and corporate reputation loss costs, making the enterprise illegal costs higher than the illegal income, in this case, even if the government supervision is not high, but companies still have to choose to discharge standards. The second way is the case of government "strong supervision", in addition to strong third-party supervision, the government regulators due to the large enough positive incentives will be a strong supervision of the sewage business. And accordingly improve the punishment, making the enterprise is facing a huge risk of illegal emissions. In this case, the stability of the emission strategy of enterprises have to target the goal of "standard emission."

To achieve strong supervision and management, the first need for government officials is to handle the relationship between economic development and environmental protection correctly. The central government have to face up to the important position of environmental protection, given the subordinate government sufficient positive incentives. For example, in the performance assessment to increase the proportion of environmental protection work, no longer GDP on the success or failure, for environmental protection work is not in place, to increase administrative accountability. Second, increase the illegal sewage enterprises supervision and punishment, strict implementation of the relevant provisions of the environmental tax policy, to steal the row, tax evasion, power hunting and other illegal acts severely punished. Third, improve the public participation system, the implementation of "Environmental Protection Public Participation" in the provisions of timely and accurate disclosure of various types of environmental information. And expand the scope of the public, and effectively protect the public and other third-party power to obtain environmental information, participation and supervision of environmental protection rights, to ensure that the public's right to know and participate in channels to safeguard public environmental rights and interests.

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