Research Article

Calculation of Stress Intensity Factors at Both Ends of the Indenter under Eccentric Loads

Xuefeng Yang and Yili Duo

1School of Mechanical Engineering, Liaoning Shihua University, Fushun 113001, China
2College of Pipeline and Civil Engineering, China University of Petroleum, Qingdao 266580, China

Correspondence should be addressed to Yili Duo; lnpuoyili@163.com
Received 10 December 2019; Accepted 13 March 2020; Published 10 April 2020

Copyright © 2020 Xuefeng Yang and Yili Duo. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

During rock breaking by drill, cone rolls on the rock surface and contact between gear teeth and surface is basically positive Mode-I plane indentation and bias Mode-I plane indentation. During bias action, there will be an asymmetrical Mode-I singular stress field at both ends of the indenter. The stress field singularity is of the same order as Mode-I crack tip and with the same function distribution. In this paper, based on the $J_2$ integral conservation law, a new method is established for solving stress intensity factor of bias Mode-I plane indentation, which not only provides respective stress intensity factor of both ends of the indenter but also compares relations between eccentricity, partial load, and stress intensity factor. The study not only provides a method for establishment of new cone drill engineering design but also improves rock layer boring efficiency.

1. Introduction

Rock-breaking technology is widely used in petroleum, coal mining and other energy industries, underground urban space construction and tunneling shield, and other military and civilian facilities, as well as environmental and architectural stone processing [1–3]. In 1952, the world’s first full-face rock tunnel boring machine was developed by American Robbins Company [4], opening a new era in lithostratigraphy full-face boring construction. With the rapid development of national economy, rock-breaking technology involves numerous engineering applications such as resource development, transportation, roads, and urban construction. As one of the major basic issues affecting economic and social development, it enjoys particularly prominent importance.

Rock breaking is directly related to drivage efficiency and reliability, whose concentrated expression is rock-breaking tool design and related theory. Scholars at home and abroad have undertaken a number of studies on the tool rock-breaking mechanism [5–10]. Tan Qing studied rock fracture characteristics of boring machine cutterhead under rock temperature variation [11]. The results showed that the higher the rock temperature, the lower the rock hardness, mechanical properties, and strength. Meanwhile, plus rapid increase in microcrack and cracks, rock-breaking efficiency is increased. But, rock is a poor conductor of heat, so there are certain difficulties in this method to improve rock-breaking efficiency. Choi and Lee [12] proposed disc cutter rock-breaking numerical research under joint characteristics. The results showed that when disc cutter rock-breaking direction was opposite to that of texture, rock-breaking input energy would be increased constantly with decrease in joint angle and spacing. Zhang et al., by comparing the mechanical model of interaction between forward and positively installed disc cutter and the rock [13, 14], proved with plasticity theory that rock-breaking specific energy of the forward installed disk cutter is low. Moreover, the greater the radius of the disc cutter, the greater the relative reduction amount of rock-breaking specific energy of the forward installed disc cutter. According to engineering statistics, disc cutter consumption is huge. Frequent replacement is caused due to tool life and fast wear, which not only delays construction period, causing staff fatigue, but also increases
project cost. Thus, cone-type tool becomes the major rock-breaking tool for hard rock heading machine, shield machine, and other equipment with wide application.

Stress-inducing mode, rock-breaking behavior, and application of the common drill cone, tunneling shield cone hob and hole drilling, and other rock-breaking tools in the rock matrix can be generally divided into Hertz concentrated induction method of spherical indentation stress [15, 16], concentrated induction method of cylindrical indentation stress, and singular induction method of plane indentation [17–19]. Wherein, both Hertz spherical indentation method and plane indentation method have applications in rock-breaking engineering. Their common feature is that gear teeth force the rock surface to induce local stress concentration. Hertz spherical indentation method is mainly characterized by maximum stress in the indenter or central indentation. Despite high peak stress, radius of the indenter will not be sufficiently small, so peak stress is after all finite. When rock strength is high, rock-breaking effect is limited, with high energy consumption and large damage area. Rock-breaking characteristic of cylindrical gear teeth is similar to that of the Hertz spherical indentation method, which will not be repeated here. However, even when the plane indentation method exerts a little load, high stress will be induced in the rock. Hence, the plane indentation method features prominent stress concentration, low load, and efficient rock breaking, which enjoy important engineering value. Therefore, in this paper, with two-dimensional plane indentation, for example, through the principle that the plane indentation method induces singular stress field on the rock surface, bias indentation stress intensity factor is given based on the J2 conservation integration method, and eccentricity effect on load size is analyzed to provide technical support for better results of the flat tooth cone drill in rock-breaking engineering applications.

2. Contact Pressure Problem

2.1. Plane Indentation. In recent years, plane indentation boundary cracking damage has been a major concern. As shown in Figure 1, plane indentation-induced progressive stress field demonstrates \( r^{-1/2} \) singularity, with the same order as crack-tip singular stress field [20–23] and essentially the same mechanics. Existence of singular stress field will inevitably lead to rock damage and cracking. And theoretically, its peak stress can be infinite even under small load.

It is worth noting that, unlike research objects of conventional fracture mechanics, indentation boundary is general without initial cracks. Under the act of plane indentation singular stress field, the cracking problem of the indentation boundary is actually the cracking problem without the initial crack surface or the crack initiation problem, which carries certain challenges, with difficulty much higher than the cracking problem. However, this kind of indentation-induced singular stress field is near surface stress field, with stress singular point and K-control area on the surface of the rock mass. The field intensity of the field stress is under sole control of indentation stress intensity factor kind. Even if ambient boundary conditions change, singularity and distribution of stress field will not be undermined, with only impact on the size of indentation stress intensity factor. This characteristic provides an ideal fracture mechanics model for research on flat tooth drilling tool fracture-type rock-breaking technology and applications, which is especially suitable for development of the new flat tooth cone drilling tool and shield flat hob and can form new and more perfect rock fragmentation theories.

2.2. Rectangular Indenter Contact Problem. Two typical rock-cutting saws commonly used in engineering are shown in Figure 2. In drilling engineering, the interface of the rock and bit interface process is very complex. In order to describe the rock failure and mechanism, many scholars usually select a simplified indentation test using the flat-end indenter [24, 25], and the indentation hardness is also usually used to assess the drillability of rocks [26–28]. Figure 2(a) shows a typical flat-tipped indenter, and the saw tooth of the drill is composed of a cylindrical shell periodic array flat-end diamond indenter. The mechanism of this type of drilling tool for rock breaking is that the rock is ground by diamond abrasive with the control of metal bond. The rock-breaking mechanism of the circular saw is shear failure, as shown in Figure 2(b). It is worth noting that both rock-breaking tools have a rectangular indenter, this is also the focus of this paper. Zhao et al. [29] took the brittle rock granite as the research object to test the performance of different tools. Guha et al. simulated flat punch indentation from the perspective of a moulding technique using a modified Fleck–Hutchinson plasticity model [30]. Campbell and Gill investigated the flat punch ISE across many orders of magnitude, and an analytic expression for the loading and unloading response is derived [31]. Nevertheless, there were almost no reports regarding the eccentric load problem.

The common and yet unique character is that these rock-cutting tools can be modeled by periodic rigid and flat-tipped indenters, shown in Figure 2. It has been pointed out from the previous work that a singular stress field exists in an incompressible substrate at the sliding contact edge of a rigid flat-ended indenter pressing down onto the substrate. It should be emphasized that such a singular indentation stress field is sufficient to generate microcracks on the contact surface even if it is free of any microcracks. This surface cracking mechanism can potentially be utilized to assist rock cutting, in additional to the diamond grits embedded on the cutter surface.

3. Plane Indentation Configuration and Singular Stress Field

As is shown in Figure 1(a) of the plane indentation contact surface, width of the rectangular rigid indenter is \( 2l \), load on the rigid indenter is \( P \), acting on the smooth half-plane, Poisson’s ratio is \( \nu \), and elastic modulus is \( E \). Indentation interfacial stress distribution according to classical analysis is as follows [32, 33]:
When coordinate transformation is given, make \( x = r - l \) and expand with the binomial distribution method; then, pressure distribution between the indentation interface near the indenter is as follows:

\[
p(x_1) = -\frac{P}{\pi \sqrt{r^2 - x^2}}. \tag{1}
\]

Based on flat-end indenter (two-dimensional) induced Mode-I singular stress field, cracking behavior of the indentation boundary is studied. As shown in Figure 1(a), when the rigid flat-end indenter acts on the half-plane frictionless surface, there is the following stress field in the indentation boundary [34]:

\[
\begin{pmatrix}
\sigma_{rr} \\
\sigma_{r\theta} \\
\sigma_{\theta\theta}
\end{pmatrix}
= \frac{K_{1\text{-ind}}}{\sqrt{2\pi r}} 
\begin{pmatrix}
\cos \frac{\theta}{2} \left(1 + \sin^2 \frac{\theta}{2}\right) \\
\cos^3 \frac{\theta}{2} \\
\sin \frac{\theta}{2} \cos^3 \frac{\theta}{2}
\end{pmatrix}
. \tag{3}
\]

As can be known from the above equation, for the crack shown in Figure 1(b), stress distribution at \( x_2 = 0 \) is consistent with the indentation problem in Figure 1(a). Thus, the indentation problem in Figure 1(a) is equivalent to the crack problem in Figure 1(b).

### 4. Energy Release Rate for Boundary Cracking

This section describes a method for calculating the energy release rate associated with the initiation of a crack, starting from the free boundary. Consider a three-dimensional elastostatic boundary problem for a material contained within the boundary \( S + s \) illustrated in Figure 3, where the portion \( s \) of the boundary is traction-free, and external loading only imposes tractions on \( S \). Without changing the boundary conditions on \( S \), impose a continuously varying sequence of static solutions, related to the displacement \( u \), given by a time-like parameter "\( t \)." The result of energy release rate per unit time, \( \frac{\partial \Pi}{\partial t} \), is given, i.e.,

\[
\frac{\partial \Pi}{\partial t} = \int_s \nu_i m_i \, ds,
\tag{5}
\]

where \( \nu_i \) denotes the "velocity" of the points on \( s \) and \( m_i \) is the current outward normal to \( s \). In the case of two-dimensional deformation fields, relevant to the present problem, the energy release rate remains in the same form as equation (5).

Let \( \nu = e_1 = \Delta s/\Delta t \), which corresponds to two components of unit boundary shift, so that \( e_1 = \cos \alpha \), \( e_2 = \sin \alpha \) for the two-dimensional solids, where \( \alpha \) is the angle between shift \( \Delta \) and \( e_1 \), and represents the direction of the boundary shift. Let \( n_i = -m_i \) be the unit inward normal on boundary \( s \), which means that the boundary \( s \) moves inward, e.g., the case of crack growth. Thus, the energy release rate of boundary shift is given by
Now, let all points on the boundary $s$ move in the same direction, and combine this result with the conservation law for two-dimensional problem $I_j$. The energy release rate expressed by equation (6) can then be rearranged along the $v$-direction, i.e.,

$$G = J_1 \cos \alpha + J_2 \sin \alpha.$$  

(7)

From the geometrical point of view, boundary cracking or crack initiation regardless whether it occurs at a crack tip, or a notch, or from a smooth boundary, can be considered as a boundary shift, or a change in the boundary condition, with the limit $s \rightarrow 0$ taken, as shown in Figure 4. Then, the energy release rate of boundary cracking can be defined as

$$G = (J_1)|_{s \rightarrow 0} \cos \alpha + (J_2)|_{s \rightarrow 0} \sin \alpha,$$  

(8)

where $(J_1)|_{s \rightarrow 0}$ denotes the energy release rate of the boundary $s$ shifting in $x_1$-direction or the driving force of boundary cracking in $x_1$-direction and $(J_2)|_{s \rightarrow 0}$ denotes the energy release rate of the boundary $s$ shifting in $x_2$-direction.
or the driving force of boundary cracking in $x_2$-direction when the limits taken exist.

The geometrical model for crack-tip shattering initiated from a free boundary in $n$ directions with $s = s_1 + s_2 + \cdots + s_n \to 0$ is illustrated in Figure 5 for the case of $n = 3$ and $s = s_1 + s_2 + s_3 \to 0$. By following the same procedure from equations (5)–(8), the energy release rate for crack-tip shattering is obtained, i.e.,

$$ G \overset{\text{s} \to 0}{=} \sum_{j=1}^{n} \left[ (J_1) \bigg|_{s \to 0} + (J_2) \bigg|_{s \to 0} \right] \cos a_i. $$

As expected, the boundary shift or cracking should be moving inwards within the solid, and $(J_2)_{s \to 0}$ must be positive indicating an energy release.

5. Conservation Law

5.1. $J$ Integral. For closed curve, free of cracks or holes, as shown in Figure 6, based on the $J$ integral conservation law, the following zero integral expression can be obtained [35, 36]:

$$ J_j = \int_{s} (wn_j - T \cdot n_i) \, ds, \quad j = 1, 2. \quad (10) $$

Equation (10) contains two integral components ($J_1$ and $J_2$). When the crack plane is parallel to axis $x_1$, $J_1$ means crack growth energy release rate and $J_2$ means crack opening energy release rate. Both can be used to calculate stress intensity factor of elastic body cracks, including calculation of plane indentation stress intensity factor. This paper will provide the main calculation process for calculating bias Mode-I plane indentation stress intensity factor based on integral $J_2$ and will then compare eccentricity size.

5.2. $J_1$ and $J_2$ Integral Calculation of the Mode-I Crack. The integral path is a circle with a radius of $r$, and the integral is shown in Figure 6. Mode-I crack stress field expression is

$$ \begin{align*}
\sigma_{11} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\
\sigma_{22} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\
\sigma_{12} &= \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}.
\end{align*} \quad (11) $$

For homogeneous, isotropia material elastomers, the elastic strain energy density is

$$ w = \frac{1}{2} \sigma_{ij} \varepsilon_{ij}. \quad (12) $$

In equation (11), $\theta$ is the angle with the $x_1$ axis, rad. Then, substituting equation (11) to equation (12), we find

$$ w = \frac{(1 + \nu)K_I^2}{4E\pi r} \left[ \frac{1}{3} \left( 5 - \cos 2\theta + 4 \cos \theta \right) - 2\mu(1 + \cos \theta) \right]. \quad (13) $$

where $\nu$ is Poisson’s ratio and $E$ is the modulus of elasticity, GPa. The expression of surface stress $T$ is

$$ T_1 = \sigma_{11}n_1 + \sigma_{12}n_2, \quad (14) $$

$$ T_2 = \sigma_{21}n_1 + \sigma_{22}n_2. \quad (15) $$

Substituting equation (11) to equations (14) and (15), the following result can be found:

$$ T_1 = \frac{K_I}{\sqrt{2\pi r}} \left( \frac{3}{4} \cos \frac{3\theta}{2} + \frac{1}{4} \cos \frac{\theta}{2} \right), \quad (16) $$

$$ T_2 = \frac{K_I}{\sqrt{2\pi r}} \frac{3}{2} \cos \frac{\theta}{2} \sin \theta. \quad (17) $$

The expression of Mode-I crack tip displacement field is...
From equations (16), (17), and (21)–(24), it will yield
\[ T_1 \frac{\partial u_1}{\partial x_1} + T_2 \frac{\partial u_2}{\partial x_2} = \frac{(1 + \nu)K_i^2}{64\pi E} \left[ -2 \cos 3\theta \right. \\
\left. + (40 - 48\nu) \cos 2\theta + (18 - 32\nu) \cos \theta + (16\nu - 24) \right] \]
\[ + (40 - 48\nu) \sin 2\theta + (22 - 32\nu) \sin \theta \].

From equations (13), (25), and (26), according to equation (10), \( J_1 \) and \( J \) can be expressed as follows:
\[ J_1 = \frac{(1 - \nu^2)K_i^2}{2\pi E} \int (1 - \cos 2\theta) d\theta, \]
\[ J_2 = \frac{(1 - \nu^2)K_i^2}{2\pi E} \int (-\sin 2\theta) d\theta. \]

6. Influencing Factors of Flat Tooth Cone Bias

6.1. Bias Indentation Stress Intensity Factor. Positive stress intensity factor problem of the ideal smooth surface has been discussed in literature [37]. Bias Mode-I plane indentation is shown in Figure 7, and unbalance loading indentation stress intensity factor can be expressed as superposition of positive \( P \) and bending moment \( M \) stress intensity factor.
In this paper, finite width rigid indenter with width at W and thickness at B (wherein $B = 1$) is taken as an example to study stress intensity factor under bias, as shown in Figure 8. In the case of concentrated force, choose a closed path of integration $S_{abcdefghij}$ as shown in Figure 8(a). And the following results can be obtained according to equation (10) based on $J_2$ [38–40]:

$$J_{2-P} = \oint_{S_{abcdefghij}} (wn_{2} - T_{i}u_{1,2}) ds = 0. \quad (29)$$

On path $S_{k}$ and $S_{d}$, there exist $T_{i} = 0$ and $n_{2} = 0$. And the following calculation results can be obtained:

$$J_{2-P} = \int_{S_{k}} (wn_{2} - T_{i}u_{1,2}) ds = \int_{S_{d}} (wn_{2} - T_{i}u_{1,2}) ds = 0. \quad (30)$$

Path $S_{gh}$ and $S_{ij}$ are straight lines, while path $S_{k}$ and $S_{d}$ are quadrants. When $S_{ij}$ and $S_{gh}$ are in $K$-control area at the corner of the indenter, it is not difficult to obtain the following calculation results:

$$J_{2-P} = \int_{S_{gh}} (wn_{2} - T_{i}u_{1,2}) ds = \int_{S_{ij}} (wn_{2} - T_{i}u_{1,2}) ds = 0, \quad (31)$$

$$J_{2-P} = \int_{S_{gh}} (wn_{2} - T_{i}u_{1,2}) ds = \int_{S_{ij}} (wn_{2} - T_{i}u_{1,2}) ds = \frac{(1 - \nu^2)K_{1}^{2-\text{ind}}}{2\pi E} \quad \text{(plane strain)}. \quad (32)$$

For relatively far away indentation contact area and indentation contact area near the surface, the following approximate expression can be obtained:

$$J_{2-P} = \int_{S_{u}} (wn_{2} - T_{i}u_{1,2}) ds = \left[\bar{w} + P\bar{u}_{2,2}\right], \quad (33)$$

$$J_{2-P} = \left(\int_{S_{u}} (wn_{2} - T_{i}u_{1,2}) ds = -\left[\bar{w} + P\bar{u}_{2,2}\right]\right), \quad (34)$$

wherein $\bar{u}_{i}$ represents the strain of the neutral axis, which can be obtained by bending theory in material mechanics. The symbol “−” indicates far-field cross section, and symbol “+” indicates near-field cross section. $\bar{w}$ represents strain energy density per unit length. Then, substitute equations (30)–(34) into equation (29) and obtain that

$$J_{2-P} = \left(1 - \nu^2\right)K_{1}^{2-\text{ind}} + 2\int_{S_{u}} w_{p} ds = \left[\bar{w} + P\bar{u}_{2,2}\right] - \left[\bar{w} + P\bar{u}_{2,2}\right] = \frac{P}{2} (\bar{u}_{2,2} - \bar{u}_{2,2}), \quad (35)$$

wherein $\bar{u}_{2,2}$ and $\bar{u}_{2,2}$ may be expressed as follows [41, 42]:

$$\bar{u}_{2,2} = \frac{-P}{EA}, \quad (36)$$

$$u_{2,2} = \frac{P}{EA} \int_{0}^{1} \frac{d\xi}{1 - (2a/W)\sqrt{1 - \xi^2}} \quad (37)$$

In the formula, $a$ represents the crack length, $A = WB$ shows the cross-sectional area at the far field $S_{u}$. Due to
movement and cracking of the indentation border and that $S_{ob}$ and $S_{ef}$ are not in $K$-control area, its integral value is very small which can be ignored. Then, substitute equations (36) and (37) into equation (35) and obtain

$$J_2 = \frac{(1 - \nu^2)K_{1\text{-ind}}^2}{\pi E} = \frac{P}{2}(\bar{u}_{2,2} - \bar{u}_{2,2}^+)$$

$$= \frac{P}{2} \left( \frac{P}{E W} \int_0^1 \frac{d\xi}{1 - (2a/W)\sqrt{1 - \xi^2}} - \frac{P}{E W} \right).$$

According to the above formula, stress intensity factor under concentrated force can be expressed as

$$K_{1\text{-ind}} = \frac{\sqrt{\pi P}}{\sqrt{2W(1 - \nu^2)}} \left( \int_0^1 \frac{d\xi}{1 - (2a/W)\sqrt{1 - \xi^2}} - 1 \right)^{1/2}.$$  \hspace{1cm} (39)

Under concentrated moment, closed integration path $S_{abcdefgija}$ is still chosen. As shown in Figure 8(a), it can be obtained according to the integral conservation law and by referring to the above calculation method that

$$J_{2-M} = \frac{(1 - \nu^2)K_{1\text{-ind-M}}^2}{\pi E} + 2 \int_{S_{ob}} w_{M} ds = \left( \bar{w}^+ - M^{-\bar{\phi}^-} \right)$$

$$- \left( \bar{w}^- - M^{-\bar{\phi}^+} \right) = \frac{M}{2} \left( \bar{\phi}^+ - \bar{\phi}^- \right).$$  \hspace{1cm} (40)

In the formula, $\bar{\phi}^+$ and $\bar{\phi}^-$ are the curvature of cracked cross section and crack-free cross section after deformation [43]. $I$ represents the inertia moment of the neutral axis of cross section (wherein, $B = 1$):

$$\bar{\phi}^- = \frac{M}{EI},$$

$$I = \frac{BW^3}{12},$$

$$\bar{\phi}^+ = \frac{M}{EI} \int_0^1 \frac{d\xi}{(1 - (2a/W)\sqrt{1 - \xi^2})}. \hspace{1cm} (41)$$

Then, substitute equations (41) and (42) into equation (40). In equation (40), $M = nP$, where $P$ is equal to the eccentric loads $P$ and $n$ is the distance from the center line (Figure 7(a)).

Figure 8: Contact model for elastic solids with finite boundaries and the crack model.
Due to two nonsymmetrical Mode-I singular stress fields at the indenter corner under eccentric load, as shown in Figure 9, two different stress intensity factors, in this paper, singular stress field at both ends of the indenter is defined as

\[
K_{I\text{-ind-M}} = \frac{P \sqrt{\pi}}{2W(1-\nu^2)} \left( \int_0^1 \frac{12 \times (n/W)^2}{1 - 2a/W \sqrt{1 - \xi^2}} \, d\xi - 12 \times \left( \frac{n}{W} \right)^2 \right)^{1/2}, \tag{43}
\]

L-end Mode-I singular stress field and R-end Mode-I singular stress field. Stress intensity factor at both ends of the indenter obtained via equations (39) and (43) is

\[
K_{I\text{-ind-R}} = \frac{P \sqrt{\pi}}{2W(1-\nu^2)} \left[ \int_0^1 \frac{1}{1 - 2a/W \sqrt{1 - \xi^2}} + \frac{12 \times (n/W)^2}{1 - 2a/W \sqrt{1 - \xi^2}} \, d\xi - 12 \times \left( \frac{n}{W} \right)^2 \right]^{1/2}, \tag{44}
\]

\[
K_{I\text{-ind-L}} = \frac{P \sqrt{\pi}}{2W(1-\nu^2)} \left[ \int_0^1 \frac{1}{1 - 2a/W \sqrt{1 - \xi^2}} - \frac{12 \times (n/W)^2}{1 - 2a/W \sqrt{1 - \xi^2}} \, d\xi + 12 \times \left( \frac{n}{W} \right)^2 \right]^{1/2}. \tag{45}
\]

As can be known from equations (44) and (45), this method innovatively provides different stress intensity factors at both ends of the indenter under bias and takes into account the impact of eccentricity on stress intensity factor.

In above sections, based on the conservation law, a direct and simple method is proposed to estimate the indentation stress intensity factor for singular stress field induced by eccentric loads. For various crack configurations, as shown in Figure 8(b), the stress intensity factors have been found and collected in the handbooks [39]:

\[
K_I = \left[ F_P + F_M \frac{3M}{l} \right] \frac{\sqrt{\pi a}}{2\sqrt{W/l}}, \tag{46}
\]

\[
F_P = 1.1221 - \left( \frac{a}{W} \right)^2 - 0.06 \left( \frac{a}{W} \right)^3 + 0.728 \left( \frac{a}{W} \right)^3, \tag{47}
\]

\[
F_M = \frac{4}{3\pi} \left[ 1 + \frac{l}{W} + 1.5 \left( \frac{l}{W} \right)^2 + 2.5 \left( \frac{l}{W} \right)^3 \right] - 7.52 \left( \frac{l}{W} \right)^4 + 21.216 \left( \frac{l}{W} \right)^5. \tag{48}
\]

Here, a hypothesis is given by \( n/l = 0.3 \), so the following is obtained:

\[
\frac{n}{W} = 0.3 \left( 0.5 - \frac{a}{W} \right), \tag{49}
\]

and then equation (44) is taken as an example to compare with equation (46), as displayed in Figure 10.

The results are in good agreement with the superposition of stress intensity factors for the bending moment and the normal load, and Figure 10 shows that equation (44) is correct. Traditionally, “Handbook of Stress Intensity Factors" is needed when stress intensity factors are calculated.

However, through the study of this paper, equations (46)–(48) can be replaced by equation (44) to obtain stress intensity factor. This method not only has more accurate results but also saves calculation costs. What are especially worth noticing are equations (44) and (45) also indicate that the boundary cracking of Mode-I plane indentation with positive loads and eccentric load has different critical cracking, and they are different from the crack growth problem, but they all have the same asymptotic singular stress field, the same mechanical essence, and the homologous cracking mechanism.

### 6.2 Impact Analysis of Eccentricity

In this paper, through equations (44) and (45), effect of different eccentricity values on eccentric load is analyzed. Figure 11(a) shows that, in the case of increasing crack expansion, with increase in eccentricity, eccentric load will gradually become smaller under constant stress intensity factor; stress intensity factor will gradually become larger under constant load. The difference is that, although Figure 11(b) shows that when eccentricity changes from 0.1 to 0.3, the situation is opposite to the case of Figure 11(a), and value change from 0.1 to 0.3 causes little effect which can be ignored. At the same time, research shows that when eccentricity is >0.3, imaginary number will appear in stress intensity factor of the left indenter, and with indefinitely growing eccentricity, the left indenter will leave the base. This is not conducive to improving drilling efficiency, which will wear the right indenter and reduce service life of the indenter. This finding not only enlightens our drilling efficiency improvement but also means great significance for dimensional design of drill and solution of the engineering application problem.
Figure 9: Nonsymmetrical Mode-I singular stress field at the indenter corner under eccentric load.

Figure 10: Scheme of eccentric load stress intensity factor ($n/l = 0.3$ and $\mu = 0$).

Figure 11: Effect of eccentricity on $K/P$. (a) R-end of the indenter. (b) L-end of the indenter.
7. Conclusion

(1) Different stress intensity factors at both ends of the indenter under bias are given based on the $J_2$ integral conservation law, no longer equates in positive stress intensity factor. The calculation of the stress intensity factor can be simplified from 3 formulas into one formula based on this methodology. Therefore, the calculation process is not only simple with good accuracy but also can provide a reference for engineering application.

(2) When stress intensity factor remains constant, eccentricity growth can reduce input load, but when eccentricity is $\geq 0.3$, imaginary number will appear in stress intensity factor of the left indenter; when stress intensity factor is constant, indenter width reduction can also lower input load.

(3) Study on the relationship between stress intensity factor, eccentricity, and indenter width can enable us to optimize flat tooth cone design, reduce energy consumption, and improve rock-breaking efficiency.

Data Availability

The data of numerical results are generated during the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Scientific Research Project of the Department of Education of Liaoning Province (No. L2019044) and the Natural Science Foundation of Liaoning Province (No. 2019-ZD-0056).

References

[1] H. Mroueh and I. Shahrour, "A simplified 3D model for tunnel construction using tunnel boring machines," Tunneling and Underground Space Technology, vol. 23, no. 1, pp. 38–45, 2008.

[2] Y. Koyama, "Present status and technology of shield tunneling method in Japan," Tunneling and Underground Space Technology, vol. 18, no. 2-3, pp. 145–159, 2003.

[3] Y. Lu, J.-M. Ma, Q.-J. Xu, and M.-N. Han, "TBM in the future of China," Marine Georesources & Geotechnology, vol. 22, no. 3, pp. 185–193, 2004.

[4] M. R. Wang, D. H. Li, and J. J. Zhang, Rock Tunnel Boring Machine (TBM) Construction and Engineering Examples, China Railway Press, Beijing, China, 2004.

[5] M. J. Jackson and M. P. Hitchiner, High Performance Grinding and Advanced Cutting Tools, Springer Briefs in Applied Sciences and Technology, New York, NY, USA, 2013.

[6] A. M. Adaskin and V. N. Butrim, "Study of wear rate of cutting tools made from Cr-based high-temperature alloy during operation depending on the material properties of the cutting tool and the rate of cutting," Journal of Friction and Wear, vol. 35, no. 5, pp. 407–413, 2014.

[7] R. E. Gertsch, Rock Toughness and Disc Cutting, University of Missouri Rolla, Rolla, MO, USA, 2000.

[8] Y. V. Rubtsov, G. V. Konnova, V. S. Shchetinin, and S. V. Zolotoreva, "Improving the cutting mechanism of a disk-type wood chipper," Russian Engineering Research, vol. 31, no. 1, pp. 28–30, 2011.

[9] Z.-H. Zhang and F. Sun, "The three-dimension model for the rock-breaking mechanism of disc cutter and analysis of rock-breaking forces," Acta Mechanica Sinica, vol. 28, no. 3, pp. 675–682, 2012.

[10] Y. Chen, "Experimental study of rock-breaking with an offset single cone bit," Petroleum Science, vol. 5, no. 2, pp. 179–182, 2008.

[11] Q. Tan, G.-j. Zhang, Y.-m. Xia, and J.-f. Li, "Differentiation and analysis on rock breaking characteristics of TBM disc cutter at different rock temperatures," Journal of Central South University, vol. 22, no. 12, pp. 4807–4818, 2015.

[12] S.-O. Choi and S.-J. Lee, "Numerical study to estimate the cutting power on a disc cutter in jointed rock mass," KSCE Journal of Civil Engineering, vol. 20, no. 1, pp. 440–451, 2016.

[13] Z. H. Zhang, X. M. Hu, L. Meng, and F. Sun, "Theoretical analysis of efficiency of rock breaking by disc cutters," Journal of Basic Science and Engineering, vol. 20, no. 1, pp. 199–206, 2012.

[14] Z.-H. Zhang, S. Fei, and M. Liang, "The comparative analysis of rocks' resistance to forward-slanting disc cutters and traditionally installed disc cutters," Acta Mechanica Sinica, vol. 32, no. 4, pp. 690–695, 2016.

[15] V. Magnenet, C. Auvray, S. Djordem, and F. Homand, "On the estimation of elastoplastic properties of rocks by indentation tests," International Journal of Rock Mechanics and Mining Sciences, vol. 46, no. 3, pp. 635–642, 2009.

[16] A. Carpinteri, B. Chiaia, and S. Invernizzi, "Numerical analysis of indentation fracture in quasi-brittle materials," Engineering Fracture Mechanics, vol. 71, no. 4–6, pp. 567–577, 2004.

[17] R. Mouginot and D. Maugis, "Fracture indentation beneath flat and spherical punches," Journal of Materials Science, vol. 20, no. 12, pp. 4354–4376, 1985.

[18] L. Nicola, A. F. Bower, K.-S. Kim, A. Needleman, and E. Van der Giessen, "Surface versus bulk nucleation of dislocations during contact," Journal of the Mechanics and Physics of Solids, vol. 55, no. 6, pp. 1120–1144, 2007.

[19] M. I. Porter and D. A. Hills, "Note on the complete contact between a flat rigid punch and an elastic layer attached to a dissimilar substrate," International Journal of Mechanical Sciences, vol. 44, no. 3, pp. 509–520, 2002.

[20] A. E. Giannakopoulos, T. C. Lindley, and S. Suresh, "Aspects of equivalence between contact mechanics and fracture mechanics: theoretical connections and a life-prediction methodology for fretting-fatigue," Acta Materialia, vol. 46, no. 9, pp. 2955–2968, 1998.

[21] A. E. Giannakopoulos and S. Suresh, "Theory of indentation of piezoelectric materials," Acta Materialia, vol. 47, no. 7, pp. 2153–2164, 1999.

[22] A. C. Fischer-Cripps, "Predicting hertzian fracture," Journal of Materials Science, vol. 32, no. 5, pp. 1277–1285, 1997.

[23] B. Yang and S. Mall, "On crack initiation mechanisms in fretting fatigue," Journal of Applied Mechanics, vol. 68, no. 1, pp. 76–80, 2001.

[24] D. Li, S. J. Li, Y. Shen, and L. J. Cao, "Fractal characteristics of rock fragmentation process induced by indenters," Chinese Journal of Geotechnical Engineering, vol. 35, no. 2, pp. 314–319, 2013.

[25] L. X. Li, "Mechanism on equivalent press-pole brokeed for press-head to insert brittle rock in equilibrium," Journal of Xiangtan Mining Institute, vol. 18, no. 1, pp. 13–16, 2003.
[26] X. C. Shi, Z. W. Tao, Y. F. Meng et al., “The mechanism of rock breakdown during bit-tooth penetration: a review,” Geotechnical Science and Technology Information, vol. 33, no. 4, pp. 225–230, 2014.

[27] G. Wijk, “The stamp test for rock drillability classification,” International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, vol. 26, no. 1, pp. 37–44, 1989.

[28] L. Zhang, X. F. Fu, G. R. Liu, S. B. Li, W. Li, and S. Qu, “Models for evaluating craters morphology, relation of indentation hardness and uniaxial compressive strength via a flat-end indenter,” Open Geosciences, vol. 10, no. 1, pp. 289–296, 2018.

[29] F. Zhao, H. Wang, Z. Ye, Y. Liu, and Y. Li, “Study on energy consumption characteristics of different tools under impact load,” Advances in Civil Engineering, vol. 2019, Article ID 3104102, 7 pages, 2019.

[30] S. Guha, S. Sangal, and S. Basu, “Numerical investigations of flat punch molding using a higher order strain gradient plasticity theory,” International Journal of Material Forming, vol. 7, no. 4, pp. 459–467, 2014.

[31] C. J. Campbell and S. P. A. Gill, “An analytical model for the flat punch indentation size effect,” International Journal of Solids and Structures, vol. 171, no. 15, pp. 81–91, 2019.

[32] M. Entacher, E. Schuller, and R. Galler, “Rock failure and crack propagation beneath disc cutters,” Rock Mechanics and Rock Engineering, vol. 48, no. 4, pp. 1559–1572, 2015.

[33] V. Buljak, G. Cocchetti, and G. Maier, “Calibration of brittle fracture models by sharp indenters and inverse analysis,” Fracture Phenomena in Nature and Technology, vol. 184, pp. 123–136, 2014.

[34] A. Nadai and P. G. Hodge, Theory of Flow and Fracture of Solids, Harvard University, Cambridge, MA, USA, 1963.

[35] Y. J. Xie and D. A. Hills, “Quasibrittle fracture beneath a flat bearing surface,” Engineering Fracture Mechanics, vol. 75, no. 5, pp. 1223–1230, 2008.

[36] Y. J. Xie, X. H. Wang, X. Z. Hu, and X. Z. Zhu, “Fracture-based model of periodic-arrayed indentation for rock cutting,” International Journal of Rock Mechanics and Mining Sciences, vol. 76, pp. 217–221, 2015.

[37] Y. J. Xie, K. Y. Lee, X. Z. Hu, and Y. M. Cai, “Applications of conservation integral to indentation with a rigid punch,” Engineering Fracture Mechanics, vol. 76, no. 7, pp. 949–957, 2009.

[38] B. Budiansky and J. R. Rice, “Conservation laws and energy-release rates,” Journal of Applied Mechanics, vol. 40, no. 1, pp. 201–203, 1973.

[39] H. Tada, P. C. Paris, and G. R. Irwin, The Stress Analysis of Cracks Handbook, ASME Press, New York, NY, USA, 2000.

[40] J. R. Rice, “A path independent integral and the approximate analysis of strain concentration by notches and cracks,” Journal of Applied Mechanics, vol. 35, no. 2, pp. 379–386, 1968.

[41] Y. J. Xie, P. N. Li, and H. Xu, “On KI estimates of cracked pipes using an elliptical hole model and elementary beam strength theory of cracked beams,” Engineering Fracture Mechanics, vol. 59, no. 3, pp. 399–402, 1998.

[42] Y. J. Xie, X. Zhang, and X. H. Wang, “An exact method on penny-shaped cracked homogeneous and composite cylinders,” International Journal of Solids and Structures, vol. 38, no. 38-39, pp. 6953–6963, 2001.

[43] Y. J. Xie, “An analytical method on circumferential periodic cracked pipes and shells,” International Journal of Solids and Structures, vol. 37, no. 37, pp. 5189–5201, 2000.