CHANGING AND UNCHANGING OF THE DOMINATION NUMBER OF A GRAPH: PATH ADDITION NUMBERS

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Abstract

Given a graph $G = (V, E)$ and two its distinct vertices $u$ and $v$, the $(u,v)$-$P_k$-addition graph of $G$ is the graph $G_{u,v,k}$ obtained from disjoint union of $G$ and a path $P_k : x_0, x_1, \ldots, x_{k-1}$, $k \geq 2$, by identifying the vertices $u$ and $x_0$, and identifying the vertices $v$ and $x_{k-1}$. We prove that $\gamma(G) - 1 \leq \gamma(G_{u,v,k})$ for all $k \geq 1$, and $\gamma(G_{u,v,k}) > \gamma(G)$ when $k \geq 5$. We also provide necessary and sufficient conditions for the equality $\gamma(G_{u,v,k}) = \gamma(G)$ to be valid for each pair $u, v \in V(G)$. In addition, we establish sharp upper and lower bounds for the minimum, respectively maximum, $k$ in a graph $G$ over all pairs of vertices $u$ and $v$ in $G$ such that the $(u,v)$-$P_k$-addition graph of $G$ has a larger domination number than $G$, which we consider separately for adjacent and non-adjacent pairs of vertices.

Keywords: domination number, path addition.

2010 Mathematics Subject Classification: 05C69.

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Received 18 June 2018
Revised 2 November 2018
Accepted 3 November 2018