ANGEL CAPITALISTS EXIT DECISIONS UNDER INFORMATION ASYMMETRY: IPO OR ACQUISITIONS

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Abstract. Angel capital is an important source of fund for start-ups. Based on the characteristics of angel investment market and the emotional factors between angel investor and entrepreneur, we establish two different principal-agent models to study their impact on different exit mechanisms i.e., IPO and acquisition. We find that: 1) In the case of IPO: as the entrepreneur’s emotional factor increases, the optimal incentives decrease; but as the investor’s emotional factor increases, the optimal incentives and the efforts increase. 2) When it comes to acquisitions: with the rising of entrepreneur’s emotional factor, the optimal incentives decline; but the investor’s emotional factor does not affect the optimal incentives and efforts. 3) Under certain conditions, the exit decision is influenced only by the entrepreneur’s emotional factor. Moreover, IPO will be the best exit mechanism, only if the entrepreneur’s emotional factor is greater than a unique threshold.

1. Introduction. The shortage of funds has been a standing problem for start-ups [20], resulting in the crucial role of venture capitalists and angel investors play as start-up’s major financiers. Venture capitalists are distinct from angel investors in that they incline to invest in relatively mature enterprises with clear strategic objectives, and tend to obtain returns quickly through capital withdrawal. Thus, venture capitalists prefer investing in later stage of business, and barely invest in new technology industries [17]. On the contrary, there are some investors who incline to invest in high-tech enterprises inherent with higher risk and value the entrepreneur’s personal charm [16]. Such investors are angel investors in the financial market, and their investment is angel capital. Angel capital relieves the high-tech industry of the fund shortage problem in its initial stage, and is therefore supported by governments in many countries, such as China. According to the 2017 ZOL research report on the early investment industry released by zhongguancun angel investment alliance, the amount of funds in early investment industry increased by 15% in 2017 in China. This report also shows that angel investment funds are gradually concentrate on

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four areas: sharing economy, new retail, blockchain and artificial intelligence. In general, angel investment plays an important role in tertiary industry.

The existing literature has focused on the following.

First, the characteristics of angel investment. In 1995, Mason et al. define angel investors as individuals who have invested in high-tech enterprises in early stage [15]. In 2015, Wang et al. define angel investment as all of the first-round external investment in the early stage of the enterprise in pursuit of profitable equity [20]. The importance of investing companies in the early stage is emphasized in the literature, and this feature is used as a criterion for distinguishing angel investment from venture capital. Based on these definitions, we can see that early entrance is the essence of angel investment. As it is hard to obtain information about financial records at the beginning of the business, angel investors choose risky projects relying on experience or business network [2]. In recent years, many researchers focus on behavioral finance, and study how these factors affect investors or entrepreneurs. For example, in 2012 Mitteness et al. studied the relationship between entrepreneur’s passion and angel investor’s decision to invest [16]. In 2011, Fairchild assumed that there are emotional factors between angel and entrepreneur\(^1\), and studied how these factors influence entrepreneur in choosing between venture capitalists and angel investors [9]. Angel investors, unlike venture capitalists, are more emotional and attach more importance to the influences of psychological factors.

Secondly, angel investment in the context of information asymmetry. Due to information asymmetry, the problem of moral hazard exits that the agent may aim at maximizing personal interests, impairing the principal’s interests. The traditional principal-agent model believes that an agent will not be affected by psychological factors. However, this view has been questioned in recent years. Fairchild constrained the model under information asymmetry while taking into account the emotional factors to compare the optimal efforts and benefits among entrepreneurs cooperating with different types of investors [9]. In 2010, Engmaier et al. focused on fairness preference in the context of asymmetric information, and has proved that when the agent has fairness preference, the optimal contract curve is close to linear, though the research targets are not restricted to angel investment [7]. Similarly, in 2009 Wang et al. studied how fairness preference affects entrepreneurs’ free-riding behavior under information symmetry and information asymmetry respectively, and provides suggestions to angel investors on how to avoid free riding [19]. Under information asymmetry, Hellmann et al. studied the relationship between venture capitalists and angel capitalists, proving that angel capitalists and venture capitalists are friends in the early stage but enemies in later stage. He brought up corresponding methods to protect angel capitalists [11]. Likewise, in the context of asymmetric information, Elitzura et al. gave the equilibrium relationship among angel capitalists, venture capitalists, and entrepreneurs, assuming the coexistence of angel investors and venture capitalists [8].

Finally, the exit choices. On this topic, most literature does not distinguish between venture capital and angel. In terms of empirical research, lots of papers try to compare the ROI (return on investment) of different exit mechanisms for specific markets. In 2003, Through empirical analysis, Brau et al. found that IPO is preferred as exit route in the US, partially due to the sound legislation on company

\(^{1}\)The special emotional effects between angel investors and entrepreneurs, both of which will incorporate each other’s expected benefits into their own decisions, and increase the individual’s utility through the effects of these emotional factors
listing and relatively loose regulations in the US [3]. However, in 2000, Jeng et al. proved that in the more conservative European markets, investors prefer to exit through acquisition and merger [13]. Even in US, where the world’s most active IPO market lays, the number of firms going public is far less than the number of firms merged. Moreover, Qian et al. showed that: in China, the ROI from IPO is not higher than that from other exit alternatives, such as mergers and acquisitions [18]. The underlying causes include the imperfection of Chinese public offering market and the obstacle from the government’s scrutiny. Therefore, mergers and acquisitions have become the main exit choice in Chinese market, although it is not necessarily the best way. Therefore, a large number of Chinese companies, such as Baidu, Alibaba, choose to list abroad. Even if a company has successfully gone public in China, the efficiency of exit will be affected by various policy restrictions, so it is difficult to guarantee a higher rate of return.

Some other papers examine the effect of certain factors on capital exit by empirical methods. In 2008, Cumming found that the choice of exit route is closely related to the distribution of control power in a company [4]. When investors have more control, they incline to choose acquisitions as the exit mechanism. In 2011, Krishnan et al. focused on the relationship between investors’ reputations and the firm’s performance [14]. He found that investors’ reputation is positively correlated to four long-term performance indicators after IPO. So, it is reasonable to assume that investors who value the reputation more will be more likely to choose IPO as the ultimate way to exit. On the theoretical side, by building models, scholars have proved that the more efficient enterprises are, the more likely that they will exit through IPO [1, 10]. Interestingly, we got the same prediction in this paper.

From these studies, we find that: (1) Most research on capital exit mechanism focuses on the choice between IPO and acquisition. Another way of capital exit, bankruptcy liquidation, is not a subjective choice but a compulsory procedure. (2) Research on angel investment is often associated with the psychological factors of investors or entrepreneurs, but theoretical studies on exit choices are defective. For example, neither Bayar nor He has considered psychological influences on exit choices[1, 10]. Therefore, to make up for the imperfection in previous research, we introduce the emotional factors between angel and entreprenuers to the study of angel capitalists’ exit decision in this paper. On the basis of Fairchild’s theory[9], we studied the influences of emotional factors on optimal efforts, optimal incentives, and the ultimate payoff of investors in different exit modes.

We explore the exit decisions through modeling and analysis with the theory of behavioral finance. The main contributions of our analysis are as follows.

First of all, different exit mechanisms result in different returns for investors. Through IPO, the company compensates the investors directly with market capitalization. In the case of acquisition, the investors benefit from the one-off payment of acquisition. Therefore, we need to derive investors’ final utility under different exit mechanisms. At the same time, according to Fairchild’s assumption, there are often emotional factors between entrepreneurs and angel investors. We need to study whether the emotional factors have impacts on angel investors’ ultimate income, and how they work. In addition, we should give an explicit expression of angel investors’ utility in different exit modes.

Secondly, no matter how the investors exit later, they can never fully observe the degree of the entrepreneurs' efforts when they are in possession of the enterprise. So, the problem of moral hazard exists. According to Holmstrom’s theory [12], we
need to solve the optimal incentive problems in different exit decisions. Meanwhile, we need to study how the existence of emotional factors affects optimal incentives.

Thirdly after getting the optimal incentives, we need to solve for the entrepreneur’s optimal efforts under different exit decisions. Moreover, we should figure out the relationship between the emotional factors and these optimal efforts.

What’s more, the purpose of this paper is to explain how emotional factors affect the exit decision. We need to compare the ultimate utility of the investors in different exit mechanisms, and analyze how these emotional factors affect the investors’ choice of exit.

Lastly, the remainder of this paper is structured as follows: Section 2 gives the hypotheses of the model. Section 3 introduces the basic models. In section 4, we make a comparative static analysis on angel investors’ choice of exit mechanisms. Section 5 gives the numerical experimental analysis. Section 6 gives conclusions and predictions for empirical testing.

2. Hypothesis of the model. Short of internal funding, start-ups need external finance to support their operation. However, as start-ups do not have financial records in the early stage, it is difficult to obtain funds from venture capitalists or banks. In this paper, we assume that venture entrepreneurs get investment from angel capitalists (ANs), and ANs need to exit in later stage of funding, either through IPO or acquisitions. However, in the context of information asymmetry, investors need to give incentives to entrepreneurs (ENs) so that ENs will make efforts. The whole process is illustrated in Figure 1. To facilitate the modeling, the assumptions of this paper is given first:

**Assumption 1.** EN’s efforts are divided into short-term efforts $e_m$, and long-term efforts $e_g$. Short-term efforts can increase the short-term cash flow and stimulate rapid growth in profits, but not lasting, such as overtime. Long-term efforts do not contribute to the company’s short-term profits, but are very beneficial to the company’s long-term development and can help build brand awareness. Therefore, long-term efforts increase the company’s future profits. The monetized costs of the short-term efforts $e_m$ is $c(e_m) = \frac{1}{2} e_m^2$, and the cost for long-term efforts $e_g$ is $c(e_g) = \frac{1}{2} e_g^2$, both functions satisfying $c'(\cdot) > 0$, and $c''(\cdot) > 0$.

**Assumption 2.** Short-term profit, denoted by $l_m$:

$$l_m = u + k_m e_m + \theta_m,$$

where $\theta_m$ describes short-term risks, and $\theta_m \sim N(0, \sigma_m^2)$. $k_m$ is the short-term productivity, and $u$ is a constant, portraying the foundation of the company’s short-term gains. Unlike [10], we believe that the intercept term of short-term gains function has nothing to do with production efficiency, same in long-term gains.
Assumption 3. Growth value, denoted by $l_g$:

$$l_g = v + k_g e_g + \theta_g,$$

where $\theta_g$ describes long-term risks, and $\theta_g \sim N(0, \sigma_g^2)$, $k_g$ is the long-term productivity, $v$ is a constant, portraying the foundation of the company’s long-term gains. Note that $\theta_m$ and $\theta_g$ are independent of each other.

Assumption 4. When AN chooses IPO as the final exit mechanism, the final long-term profit is $S^IPO_g = l_g - c$, and $c$ is the cost of IPO. When AN chooses acquisitions as the final exit mechanism, the final long-term profit is $S^{AC}_g = l_g + G + V_a$. $V_a$ is the value of acquirer, and $G$ is the synergy value through acquisitions. Different from [10] we think the fixed wage influence AN’s profit not through $S_g$. We can also follow [1], and interpret $V_a$ as the help from the acquirer.

Assumption 5. The emotional factor of EN is $\eta$ $(1 \geq \eta \geq 0)$. The emotional factor of AN is $\tau$ $(1 \geq \tau \geq 0)$. According to Fairchild’s theory[9], angel investors and entrepreneurs often have a good relationship, each one caring about the other’s payoff, so the increase in the payoff of EN can promote the utility of AN, and the increase in the payoff of AN can also promote the utility of EN. Emotional factors measure the degree of this relationship.

Assumption 6. AN is risk neutral. EN is risk aversion, and the risk aversion coefficient is $\rho$ ($\rho \geq 0$).

Assumption 7. AN’s investment is $I(I > 0)$ and the risk-free rate is $r(r > 0)$.

3. The basic model.

3.1. Angel capital IPO exit model under information asymmetry. According to Holmstrom’s theory, with information asymmetry, AN can’t observe the degree of EN’s effort, so the model needs to meet incentive compatibility constraints (IC) and participation constraint (IR).

Using a linear wage contract, EN’s payoff function under the IPO is:

$$\pi^IPO_{EN} = \alpha + \lambda S^IPO_g + \beta l_m - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2,$$

(3)

according to assumption 4, substituting (1),(2) into (3) we get:

$$\pi^IPO_{EN} = \alpha + \lambda (v + k_g e_g + \theta_g - c) + \beta (u + k_m e_m + \theta_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2,$$

(4)

where $\alpha$ is the fixed salary. Long-term and short-term incentives are $\lambda$ and $\beta$, respectively. So long-term and short-term compensations for EN are $\lambda(v + k_g e_g + \theta_g - c)$ and $\beta (u + k_m e_m + \theta_m)$. The monetized costs of EN are $\frac{1}{2} e_m^2 + \frac{1}{2} e_g^2$.

$$E(\pi^IPO_{EN}) = \alpha + \lambda (v + k_g e_g - c) + \beta (u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2, \quad \text{Var}(\pi^IPO_{EN}) = \lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2,$$

(5)

Fairchild’s theory assumes that this kind of emotional factors exist between AN and EN, and they are symmetrical. We think that this theory can be promoted and the emotional factors of both sides can be asymmetric. And symmetry is an special case of asymmetry. Also, we think it is rational to emphasize this asymmetry. For example, entrepreneurs tend to be more grateful to investors’ investment.
from the assumption 6 that EN is risk averse, EN’s certainty equivalent utility under IPO:

\[ CE^{IPO}_{EN} = E(\pi^{IPO}_{EN}) - \frac{\rho}{2} \text{Var}(\pi^{IPO}_{EN}), \]  

(6)

substituting (5) into (6), we have:

\[ CE^{IPO}_{EN} = \alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{\rho}{2}(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2). \]  

(7)

Meanwhile, according to the previous assumptions and model description, AN’s payoff is:

\[ \pi^{IPO}_{AN} = -\alpha + (1-\lambda)u + (1-\beta)\lambda m - I(1+r), \]  

(8)

according to assumption 4, substituting (1), (2) into (8) we have:

\[ \pi^{IPO}_{AN} = -\alpha + (1-\lambda)(v + k_g e_g + \theta_g - c) + (1-\beta)(u + k_m e_m + \theta_m) - I(1+r), \]  

(9)

where \((1-\lambda)(v + k_g e_g + \theta_g - c)\) and \((1-\beta)(u + k_m e_m + \theta_m)\) are the long-term and short-term gain of angel investor respectively, the cost of investment is \(I(1+r)\), and the fixed salary to an entrepreneur is \(\alpha\).

\[ E(\pi^{IPO}_{AN}) = -\alpha + (1-\lambda)(v + k_g e_g - c) + (1-\beta)(u + k_m e_m) - I(1+r), \]  

(10)

according assumption 5, angel investors and entrepreneurs often have a good relationship, each caring about the others’ payoff, each one take the others’ payoff into their own utility function, so we got the utility functions of EN and AN.

\[ U^{IPO}_{EN} = CE^{IPO}_{EN} + \eta E(\pi^{IPO}_{AN}), \]  

(11)

\[ U^{IPO}_{AN} = E(\pi^{IPO}_{AN}) + \tau E(\pi^{IPO}_{EN}), \]  

(12)

substituting (6),(8), (10) into equations (11) and (12), we got:

\[ U^{IPO}_{EN} = \alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{\rho}{2}(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) + \eta(-\alpha + (1-\lambda)(v + k_g e_g - c) + (1-\beta)(u + k_m e_m) - I(1+r)), \]  

(13)

\[ U^{IPO}_{AN} = -\alpha + (1-\lambda)(v + k_g e_g - c) + (1-\beta)(u + k_m e_m) - I(1+r) + \tau(\alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2), \]  

(14)

equation (13) satisfies the participation constraints:

\[ U^{IPO}_{EN} = \alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{\rho}{2}(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) + \eta(-\alpha + (1-\lambda)(v + k_g e_g - c) + (1-\beta)(u + k_m e_m) - I(1+r)) \geq \bar{U}. \]  

(15)

\(\bar{U}\) is reservation utility, the minimum level of utility that must be guaranteed by a contract to make it acceptable to an agent. However, AN will adjust the fixed salary so that equality holds. Namely, the participation constraint is tight, so (15) becomes:

\[ (IR) \ U^{IPO}_{EN} = \alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{\rho}{2}(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) + \eta(-\alpha + (1-\lambda)(v + k_g e_g - c) + (1-\beta)(u + k_m e_m) - I(1+r)) = \bar{U}, \]  

(16)
at the same time, due to information asymmetry, EN also needs to meet the incentive constraint. EN can decide on optimal efforts to maximize the utility:

\[ (IC) \quad e^*_m, e^*_g \in \arg \max_{e_m, e_g} \alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} \sigma^2_g - \frac{1}{2} \sigma^2_m \]

\[-\frac{\rho}{2} (\lambda^2 \sigma^2_g + \beta^2 \sigma^2_m) + \eta (-\alpha + (1 - \lambda)(v + k_g e_g - c) + (1 - \beta)(u + k_m e_m) - I(1 + r)). \]

Lastly, because of the information asymmetry, AN can’t control the degree of EN’s efforts. Therefore, AN’s decision should maximize his own utility by controlling the incentives. The objective function is:

\[ \max_{\alpha, \lambda, \beta} U^{IPO}_{AN} = -\alpha + (1 - \lambda)(v + k_g e_g - c) + (1 - \beta)(u + k_m e_m) - I(1 + r) \]

\[ + \tau(\alpha + \lambda(v + k_g e_g - c) + \beta(u + k_m e_m) - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g). \]

Summing up the (16), (17) and (18), we have the principal-agent model (I) with IPO as the angel capital’s investment exit decision under information asymmetry:

\[ \max_{\alpha, \lambda, \beta} U^{IPO}_{AN} \]

\[ \text{s.t.} \quad \{ (IR) \quad U^{IPO}_{EN} = U \}
\]

\[ (IC) \quad e^*_m, e^*_g \in \arg \max_{e_m, e_g} U^{IPO}_{EN} \]

solving the model (I) leads to the following conclusions:

**Proposition 1 (incentives under IPO).** If the angel investor adopts the IPO exit mechanism, the optimal incentives which the AN offers to the EN are:

\[ \beta^*_{IPO} = \frac{k^2_m (1 - \tau \eta)(1 - \eta)^2}{\rho(1 - \tau) \sigma^2_g + (1 - \tau \eta) k^2_m (1 - \eta)^2}, \]

\[ \lambda^*_{IPO} = \frac{k^2_m (1 - \eta)^2}{\rho(1 - \tau) \sigma^2_g + (1 - \tau \eta) k^2_g (1 - \eta)^2}, \]

**Proof.** In the case of using IPO as the exit method, derivation from equation (13)

\[ \left\{ \begin{array}{l}
\frac{\partial U^{IPO}_{AN}}{\partial e_m} = k_m(\beta + \eta(1 - \beta)) - e_m = 0 \\
\frac{\partial U^{IPO}_{AN}}{\partial e_g} = k_g(\lambda + \eta(1 - \lambda)) - e_g = 0
\end{array} \right. \]

we have:

\[ e^*_{m,IPO} = k_m(\eta + \beta(1 - \eta)), e^*_{g,IPO} = k_g(\eta + \lambda(1 - \eta)). \]

And easy to verify \( \frac{\partial^2 U^{IPO}_{AN}}{\partial e_m^2} = \frac{\partial^2 U^{IPO}_{AN}}{\partial e_g^2} = -1, \frac{\partial^2 U^{IPO}_{AN}}{\partial e_m \partial e_g} = 0 \). Therefore, the extreme value is the maximum value. Substituting (19) into the participation constraint (16) to get:

\[ \alpha = -\frac{1}{(1 - \eta)}(-U + (\eta + \lambda(1 - \eta))(v + k_g e^*_{g,IPO} - c) + (\eta + \beta(1 - \eta)). \]

\[ (u + k_m e^*_{m,IPO}) - \frac{1}{2}(e^*_{m,IPO})^2 - \frac{1}{2}(e^*_{g,IPO})^2 - \frac{\rho}{2}(\lambda^2 \sigma^2_g + \beta^2 \sigma^2_m) - \eta I(1 + r)), \]

(20)
then we have:

\[ U_{AN}^{IPO} = -\frac{1 - \tau}{1 - \eta} (1 + \eta) v + k_g^2 (\eta + \lambda (1 - \eta) - c) + (1 + \eta) \frac{1 - \tau}{1 - \eta} \cdot (u + k_m^2 (\eta + \beta (1 - \eta))) - \frac{\rho}{2} \frac{1 - \tau}{1 - \eta} \left( \lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2 \right) - (1 + \eta) \frac{1 - \tau}{1 - \eta} I (1 + r) \]

\[
- \frac{1}{2} (\tau + \eta) (k_m^2 (\eta + \beta (1 - \eta))^2 + k_g^2 (\eta + \lambda (1 - \eta))^2).
\]

Deriving first-order conditions:

\[
\begin{align*}
\frac{\partial U_{AN}^{IPO}}{\partial \lambda} &= k_g^2 (1 + \eta) \frac{1 - J}{1 - \eta} (1 - \eta) - \rho \lambda \frac{1 - \tau}{1 - \eta} \sigma_g^2 \\
&\quad - (\tau + \frac{1 - \tau}{1 - \eta}) k_g^2 \eta (1 - \eta) - \frac{1 - \tau}{1 - \eta} k_g^2 (1 - \eta)^2 \lambda = 0 \\
\frac{\partial U_{AN}^{IPO}}{\partial \beta} &= k_m^2 (1 + \eta) \frac{1 - \tau}{1 - \eta} (1 - \eta) - \rho \beta \frac{1 - \tau}{1 - \eta} \sigma_m^2 \\
&\quad - (\tau + \frac{1 - \tau}{1 - \eta}) k_m^2 \eta (1 - \eta) - \frac{1 - \tau}{1 - \eta} k_m^2 (1 - \eta)^2 \beta = 0
\end{align*}
\]

then we have:

\[
\lambda_{IPO}^* = \frac{k_g^2 (1 - \tau \eta)(1 - \eta)^2}{\rho (1 - \tau) \sigma_g^2 + (1 - \tau \eta) k_g^2 (1 - \eta)^2},
\]

\[
\beta_{IPO}^* = \frac{k_m^2 (1 - \tau \eta)(1 - \eta)^2}{\rho (1 - \tau) \sigma_m^2 + (1 - \tau \eta) k_m^2 (1 - \eta)^2},
\]

And easy to verify: \( \frac{\partial U_{AN}^{IPO}}{\partial \lambda} = \frac{\partial U_{AN}^{IPO}}{\partial \beta} = - (\tau + \frac{1 - \tau}{1 - \eta}) k_g^2 (1 - \eta)^2 < 0 \), \( \frac{\partial^2 U_{AN}^{IPO}}{\partial \lambda \partial \beta} = 0 \). Satisfying the optimization conditions, and the proposition 1 QED.

Proposition 1 gives the expressions of optimal long-term and short-term incentives in the case of IPO. For the analysis of the optimal incentive given by Proposition 1, we get the following corollaries:

**Corollary 1.** If the angel investor adopts the IPO as the exit mechanism, the optimal incentives increase in the investor’s emotional factor \( \tau \) but decreases in the entrepreneur’s emotional factor \( \eta \).

**Proof.** Derivation from equation (21),(22), we have:

\[
\begin{align*}
\frac{\partial \lambda_{IPO}^*}{\partial \tau} &= \frac{\rho k_g^2 (1 - \eta)^3 \sigma_g^2}{\left[ \rho (1 - \tau) \sigma_g^2 + (1 - \tau \eta) k_g^2 (1 - \eta)^2 \right]^2} > 0, \\
\frac{\partial \lambda_{IPO}^*}{\partial \eta} &= \frac{[- \tau k_g^2 (1 - \eta)^2 - 2 k_g^2 (1 - \tau \eta)(1 - \eta)] \rho (1 - \tau) \sigma_g^2}{\left[ \rho (1 - \tau) \sigma_g^2 + (1 - \tau \eta) k_g^2 (1 - \eta)^2 \right]^2} < 0,
\end{align*}
\]

Similarly, the above conclusion is also true for \( \beta_{IPO}^* \), the corollary 1 QED.

Proposition 1 and corollary 1 reveal that the incentives are influenced by emotional factors. When AN’s emotional factor \( \tau \) increase, AN is more concerned about the benefits of EN, so the AN will give more incentives to EN. Conversely, when EN’s emotional factor \( \eta \) increases, EN is more concerned about the benefits of AN, so AN will take advantage of EN’s emotional factor to increase personal gain by reducing incentives for EN.
Corollary 2. When EN’s emotional factor $\eta$ approaches 1, we have $\lambda^*_{IPO} = \beta^*_{IPO} = 0$.

Proof. From equation (21), (22), we have:

$$
\lim_{\eta \to 1} \lambda^*_{IPO} = \lim_{\eta \to 1} \frac{k^2_g(1-\tau \eta)(1-\eta)^2}{\rho(1-\tau)\sigma^2_g + (1-\tau \eta)k^2_g(1-\eta)^2} = 0,
$$

$$
\lim_{\eta \to 1} \beta^*_{IPO} = \lim_{\eta \to 1} \frac{k^2_m(1-\tau \eta)(1-\eta)^2}{\rho(1-\tau)\sigma^2_m + (1-\tau \eta)k^2_m(1-\eta)^2} = 0,
$$
corollary 2 QED. 

The intuition behind corollary 2 is that when EN’s emotional factor $\eta$ approaches 1, which means that EN’s emotional dependence is very strong, AN will give no incentive to EN, it will be the same as the situation under full information or symmetric information.

Why does this corollary not maintain when AN’s emotional factors increase? The reason is that the foundation of moral hazard problem lies in the fact that the principal cannot observe the degree of efforts made by agent. When EN’s emotional factor is too strong, the increase in AN’s earning is equivalent to the increase of EN’s own income, so AN does not need to give any incentive to EN. However, if the emotional factor of AN is aggravated, EN still has a motivation to laze. Unless given certain incentives, EN will not work hard. Just like the description in corollary 1, AN’s emotional factor increases the incentives. Unfortunately, as we will prove in the next section, this corollary is not satisfied in the case of acquisitions.

Corollary 3. In the case of IPO, EN’s sentiment factor $\eta$ has a stronger impact than AN’s emotional factor $\tau$.

Proof. let $\eta = \tau = a$, from equation (21) and (22) we have:

$$
\frac{\partial \lambda^*_{IPO}}{\partial \tau} = \frac{\rho k^2_g(1-a)^3 \sigma^2_g}{[\rho(1-a)\sigma^2_g + (1-a^2)k^2_g(1-a)^2]^2},
$$

$$
\frac{\partial \lambda^*_{IPO}}{\partial \eta} = \frac{\rho k^2_g(1-a)^3 \sigma^2_g(a-2)}{[\rho(1-a)\sigma^2_g + (1-a^2)k^2_g(1-a)^2]^2},
$$
since $0 \leq a \leq 1$, we have $|\frac{\partial \lambda^*_{IPO}}{\partial \tau}| < |\frac{\partial \lambda^*_{IPO}}{\partial \eta}|$. Therefore, EN’s emotional factor has a greater impact on motivation than AN’s emotional factor. Corollary 3 QED.

From corollary 1 we know that both the emotional factors $\eta$ and $\tau$ affect the optimal incentives, but in opposing effects. We can’t help asking: whose emotional factor has stronger impact on the optimal incentives? We find that under the same conditions, if $\eta = \tau$, then the incentive will still decline in the rise of this symmetrical emotional factor, that is, the influence of EN’s emotional factors prevail.

Proposition 2 (optimal efforts under IPO). If angel investor choose IPO exit mechanism, EN’s best efforts are:

---

3In the classic principal-agent model, under the symmetry of information, the principal can fully observe and control the agent, so there is no need to give any incentives.
Proof. Substituting equations (21), (22) into (19), then we have:

\[ e^*_{m, IPO} = k_m \frac{\rho(1 - \tau) \eta \sigma^2_m + (1 - \tau \eta) k^2_m (1 - \eta)^2}{\rho(1 - \tau) \sigma^2_m + (1 - \tau \eta) k^2_m (1 - \eta)^2}, \]

\[ e^*_{g, IPO} = k_g \frac{\rho(1 - \tau) \eta \sigma^2_g + (1 - \tau \eta) k^2_g (1 - \eta)^2}{\rho(1 - \tau) \sigma^2_g + (1 - \tau \eta) k^2_g (1 - \eta)^2}. \]

Proposition 2 QED.

From proposition 2, we have:

**Corollary 4.** In the IPO, optimal efforts increase as AN emotional factor \( \tau \) increases.

Proof. Let \( k_m^2 (1 - \eta)^2 = A, \rho \sigma^2_m = B \), according equations (23), (24) we have:

\[ e^*_{m, IPO} = k_m \frac{\rho(1 - \tau) \eta \sigma^2_m + (1 - \tau \eta) k^2_m (1 - \eta)^2}{(1 - \tau \eta) A + (1 - \tau) B}, \]

\[ e^*_{g, IPO} = k_g \frac{\rho(1 - \tau) \eta \sigma^2_g + (1 - \tau \eta) k^2_g (1 - \eta)^2}{(1 - \tau \eta) A + (1 - \tau) B}. \]

Then:

\[ \frac{\partial e^*_{m, IPO}}{\partial \tau} = k_m \frac{(1 - \eta)^2 AB}{[(1 - \tau \eta) A + (1 - \tau) B]^2} > 0. \] (25)

The same is true for \( e^*_{g, IPO} \). Therefore, the long-term (short-term) optimal effort level rises as the emotional factors of AN rise.

From corollary 1, we know that when AN’s emotional factor \( \tau \) strengthens, AN will give EN more incentives to stimulate EN for more efforts. However, when EN’s emotional factor increases, the situation is more complicated. Intuitively, we think that the increase will stimulate EN’s efforts, but it is not the case. On the one hand, the rise of EN’s emotional factor decreases the incentives, depressing the efforts; on the other hand, the rise of emotional factor makes EN more concerned about AN’s benefits, therefore imposing more efforts. We will give illustrative example on this in the numerical part.

3.2. Angel capital acquisitions exit model under information asymmetry.

In the case of acquisitions (AC), AN cannot observe EN’s efforts, so moral hazard exists just as in IPO. The first thing we need to solve is how much the acquirer pays in the case of mergers and acquisitions. According to the hypothesis of He, under perfect competition, when a large company acquires a venture company, the value of the acquirer remains unchanged before and after the acquisition [10]. Moreover, we have reasons to believe that there will be no emotional factors between the acquirer and EN. Therefore, we use the conclusion of He.\(^4\)

**Proposition 3 (the purchase price).**

\[ P = k_m e_m + u + k_g e_g + v + G - U - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g - \frac{1}{2} \rho (\lambda^2 \sigma^2_g + \beta^2 \sigma^2_m). \] (26)

\(^4\)readers can refer Human Capital, Management Quality, and the Exit Decisions of Entrepreneurial Firms (He et al., 2016), and check the equation (17) in their article.
Under the assumption of perfectly competitive market, the purchase price paid to AN includes synergy value $G$ and the expected output of the entire company, i.e. $k_m e_m + u + k_g e_g + v$. At the same time, since there is no emotional factor between the acquirer and EN, the amount of the price deducted only includes the reservation utility of EN, i.e. $\bar{U}$, the monetized costs of EN i.e. $\frac{1}{2}e^2_m + \frac{1}{2}e^2_g$, and the risk premium $\frac{1}{2} \rho(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)$.

Since AN is the direct beneficiary of acquisitions, we have the expected return of AN:

$$E(\pi^{AC}_{AN}) = P - I(1 + r),$$

where $I(1 + r)$ is the investment cost, substituting the equation (26) into (27), we have:

$$E(\pi^{AC}_{AN}) = k_m e_m + u + k_g e_g + v + G - \bar{U} - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g - \frac{1}{2} \rho(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) - I(1 + r).$$

(28)

Considering the benefits of entrepreneurs in the context of acquisitions:

$$\pi^{AC}_{EN} = \alpha + \lambda S^AC_g + \beta I_m - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g,$$

(29)

according to assumption 4, substituting (1), (2), similar to the previous deduction, we have:

$$E(\pi^{AC}_{EN}) = \alpha + \lambda(v + k_g e_g + G) + \beta(u + k_m e_m) - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g,$$

$$Var(\pi^{AC}_{EN}) = \lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2.$$

(30)

then we have certainty equivalent of EN income in the case of acquisitions:

$$CE^{AC}_{EN} = E(\pi^{AC}_{EN}) - \frac{\rho}{2} Var(\pi^{AC}_{EN}),$$

(31)

substituting (30) into (31), we have:

$$CE^{AC}_{EN} = \alpha + \lambda(v + k_g e_g + G + \theta_g) + \beta(u + k_m e_m + \theta_m) - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g - \frac{\rho}{2} (\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2).$$

(32)

Similar to the previous analysis, according to Fairchild emotional theory, we get the utility function of AN and EN in the case of acquisitions:

$$U^{AC}_{EN} = CE^{AC}_{EN} + \eta E(\pi^{AC}_{AN}),$$

(33)

$$U^{AC}_{AN} = E(\pi^{AC}_{AN}) + \tau E(\pi^{AC}_{EN}),$$

(34)

next, put (28), (30), (32) into (33), (34), and we get:

$$U^{AC}_{EN} = \alpha + \lambda(v + k_g e_g + G) + \beta(u + k_m e_m) - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g - \frac{1}{2} \rho(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)$$

$$+ \eta[k_m e_m + u + k_g e_g + v + G - \bar{U} - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g - \frac{1}{2} \rho(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)$$

$$- I(1 + r)],$$

(35)

$$U^{AC}_{AN} = k_m e_m + u + k_g e_g + v + G - \bar{U} - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g - \frac{1}{2} \rho(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) - I(1 + r)$$

$$+ \tau(\alpha + \lambda(v + k_g e_g + G) + \beta(u + k_m e_m) - \frac{1}{2} e^2_m - \frac{1}{2} e^2_g),$$

(36)
It is required to satisfy the participation constraint (IR) of EN. Similar to the preceding content, the participation constraint is tightened.

\[(IR) \quad \alpha + \lambda (v + k_g e_g + G) + \beta (u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{\rho}{2} (\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) + \eta [k_m e_m + u + k_g e_g + v + G - \bar{U} - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{1}{2} \rho (\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)] - I(1 + r) = \bar{U}.
\]

Also, due to information asymmetry, EN can determine his own efforts to maximize his personal utility, so we need incentive compatibility constraint:

\[(IC) \quad e_g, e_m \in \arg \max_{e_g, e_m} \alpha + \lambda (v + k_g e_g + G) + \beta (u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{\rho}{2} (\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) + \eta [k_m e_m + u + k_g e_g + v + G - \bar{U} - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2] - I(1 + r),\]

because of the information asymmetry, AN can only maximize the objective function (29) by controlling the incentives:

\[
\max_{\alpha, \lambda, \beta} k_m e_m + u + k_g e_g + v + G - \bar{U} - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2 - \frac{1}{2} \rho (\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) - I(1 + r) \\
+ \tau (\alpha + \lambda (v + k_g e_g + G) + \beta (u + k_m e_m) - \frac{1}{2} e_m^2 - \frac{1}{2} e_g^2).\]

Therefore, in the case of acquisitions, there is moral hazard. We have the principal-agent model (II):

\[
\max_{\alpha, \lambda, \beta} U^{AC}_{AN} \\
\text{s.t.} \quad (IR) \quad U^{AC}_{EN} = \bar{U} \\
(IC) \quad e^*_{m}, e^*_{g} \in \arg \max_{e_m, e_g} U^{AC}_{EN}
\]

Solving the principal-agent model (II), we get the following conclusions:

**Proposition 4 (incentives under acquisitions).** If the angel investor adopts acquisitions as the exit route, the optimal incentives that AN offers to EN are:

\[
\beta^{*}_{AC} = \frac{k_m^2}{k_m^2 + \rho (1 + \eta)^2 \sigma_m^2}, \quad \lambda^{*}_{AC} = \frac{k_g^2}{k_g^2 + \rho (1 + \eta)^2 \sigma_g^2},
\]

**Proof.** Derived from (35), we have:

\[
\begin{align*}
\frac{\partial U^{AC}_{EN}}{\partial e_g} &= (\eta + \lambda) k_g - (1 + \eta) e_g = 0 \\
\frac{\partial U^{AC}_{EN}}{\partial e_m} &= (\eta + \lambda) k_m - (1 + \eta) e_m = 0
\end{align*}
\]

then we have:

\[
e_{g,AC} = \frac{(\eta + \lambda) k_g}{(1 + \eta)}, \quad (40)
\]
\[ e_{m, AC}^* = \frac{(\eta + \beta)k_m}{(1 + \eta)}. \] (41)

And the second-order condition that is easy to verify. According to equation (37), the fixed salary of AN to EN is obtained in the case of acquisition:

\[ \alpha_{AC} = -(\nu + k_g e_{g, AC}^* + G)(\eta + \lambda) - (\eta + \beta)(k_m e_{m, AC}^* + u) + \frac{1}{2}(1 + \eta)((e_{m, AC}^*)^2 + (e_{g, AC}^*)^2) + \frac{\rho}{2}(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)(1 + \eta) + (\eta + 1)\bar{U} + \eta(1+r). \] (42)

Substituting (40), (41), (42) into equation (34):

\[
U_{AN}^{AC} = E(\pi_{AN}^{AC}) + \tau E(\pi_{EN}^{AC})
\]

\[
= E(l_m) + v + k_g e_{g, AC}^* + G - \bar{U} - \frac{1}{2}(e_{m, AC}^*)^2 - \frac{1}{2}(e_{g, AC}^*)^2
\]

\[
- \frac{1}{2} \rho(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) + \tau[\alpha + \lambda E(S_{AC}^{EN}) + \beta E(l_m)] - \frac{1}{2}(e_{m, AC}^*)^2 - \frac{1}{2}(e_{g, AC}^*)^2
\]

\[
- \frac{\rho}{2}(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)] - I(1+r)
\]

\[
= (1 - \tau \eta)(k_m e_{m, AC}^* + u) + (1 - \tau \eta)\nu k_g e_{g, AC}^* + G) - (1 - \tau - \tau \eta)\bar{U}
\]

\[
- \frac{1}{2}(1 - \tau \eta)((e_{m, AC}^*)^2 + (e_{g, AC}^*)^2) - \frac{\rho}{2}(1 - \tau \eta)(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2)
\]

\[
- (1 - \tau \eta)I(1+r)
\]

\[
= \frac{1 - \tau \eta}{1 + \eta}((\beta k_m^2 + \eta k_m^2) + (1 - \tau \eta)\nu + (1 - \tau \eta)(\nu + G) + \frac{1 - \tau \eta}{1 + \eta}(k_m^2 + \eta k_m^2)
\]

\[
- (1 - \tau - \tau \eta)\bar{U} - \frac{1}{2}(1 + \eta)^2(\beta k_m + \eta k_m)^2 - \frac{1}{2}(1 + \eta)^2(\lambda k_g + \eta k_g)^2
\]

\[
- \frac{\rho}{2}(1 - \tau \eta)(\lambda^2 \sigma_g^2 + \beta^2 \sigma_m^2) - (1 - \tau \eta)I(1+r)
\]

\[
= \frac{1 - \tau \eta}{1 + \eta}k_m^2 + (1 - \tau \eta)\nu + (1 - \tau \eta)(\nu + G) + \frac{1 - \tau \eta}{1 + \eta}\eta k_g^2 - (1 - \tau - \tau \eta)\bar{U}
\]

\[
- \frac{1}{2}(1 + \eta)^2(\lambda^2 k_g^2 + \beta^2 k_g^2) - \frac{1 - \tau \eta}{1 + \eta}\eta k_g^2 - (1 - \tau - \tau \eta)I(1+r)
\]

\[
+ \beta \frac{1 - \tau \eta}{1 + \eta}k_g^2
\]

\[
- \beta k_m^2 + \frac{1 - \tau \eta}{1 + \eta}\eta k_g^2 + \frac{1 - \tau \eta}{1 + \eta}k_g^2
\]

\[
- \beta^2 k_m^2 - \frac{1 - \tau \eta}{1 + \eta}\eta k_g^2 + \frac{1 - \tau \eta}{1 + \eta}k_g^2
\]

\[
- \beta^2(1 - \tau \eta)k_g^2 - \frac{\rho}{2}(1 - \tau \eta)\sigma_g^2 - \lambda^2(1 - \tau \eta)k_g^2 + \frac{\rho}{2}(1 - \tau \eta)\sigma_g^2
\]

Let \( \frac{\partial U_{AC}^{AC}}{\partial \beta} = \frac{\partial U_{AC}^{AC}}{\partial \lambda} = 0 \), we have:

\[
\beta_{AC}^* = \frac{k_m^2}{k_m^2 + \rho(1 + \eta)^2 \sigma_m^2},
\] (43)

\[
\lambda_{AC}^* = \frac{k_g^2}{k_g^2 + \rho(1 + \eta)^2 \sigma_g^2},
\] (44)

also, it is easy to verify the second-order condition: \( \frac{\partial^2 U_{AC}^{AC}}{\partial e_m^2} = \frac{\partial^2 U_{AC}^{AC}}{\partial e_y^2} = 0; \frac{\partial^2 U_{AC}^{AC}}{\partial e_m \partial e_y} = 0 \). Proposition 4 QED.

By analyzing proposition 4 we get the following corollaries:
Corollary 5. When angel capitalist adopts mergers and acquisitions as the exit route, the optimal incentives have nothing to do with the investor’s emotions but decline as the entrepreneur’s emotional factor rises.

Proof. According equations (43), (44), we have:

\[
\frac{\partial \beta_{AC}^*}{\partial \tau} = 0, \quad \frac{\partial \lambda_{AC}^*}{\partial \tau} = 0; \quad \frac{\partial \beta_{AC}^*}{\partial \eta} < 0, \quad \frac{\partial \lambda_{AC}^*}{\partial \eta} < 0,
\]

corollary 5 QED.

From the conclusions above, we can see that in the case of mergers and acquisitions, the incentive has nothing to do with AN’s emotional factor \(\tau\). The reason behind this is that the income of AN comes from the purchase price given by the acquirer, and the acquirer has no emotional factor with EN. However, similar to Corollary 1, when EN’s emotional dependence is aggravated, then AN will use this emotional factor to reduce the incentives for EN.

Corollary 6. When EN’s emotional factors approach 1, we have \(\lambda_{AC}^* = \beta_{AC}^* > 0\).

Proof. According equations (43), (44), we have:

\[
\lim_{\eta \to 1} \beta_{AC}^* = \frac{k_m^2}{k_m^2 + \rho(1 + \eta)^2 \sigma_m^2} \frac{k_m^2}{k_m^2 + 4 \rho \sigma_m^2} > 0,
\]
\[
\lim_{\eta \to 1} \lambda_{AC}^* = \frac{k_g^2}{k_g^2 + 4 \rho \sigma_g^2} > 0,
\]
corollary 6 QED.

In the corollary 2 we have stated that in the case of IPO, when EN’s emotional factor approaches 1, \(\lambda_{IPO}^* = \beta_{IPO}^* = 0\). Therefore, the investor does not need to give the entrepreneur any incentive, but this condition does not hold in the case of mergers and acquisitions. The reason is that in the case of IPO, even if the company is listed, it is still a whole entity, and the income it generates is distributed between EN and AN. When EN’s emotional factor approaches 1, EN considers the benefits of AN in his own decisions, so AN does not need to give EN any incentives. However, in the case of mergers and acquisitions, AN’s income is the purchase price given by the acquirer instead of the residual value after the removal of EN’s payoff from the whole profit. Therefore, if the emotional factor of EN increases, the optimal incentive will decrease, but limited to above 0.

Proposition 5 (optimal efforts under acquisitions). If angel capital withdraws with the acquisitions, EN pays the optimal effort:

\[
e_m^{*\text{AC}} = \frac{k_m^2 + \eta \rho(1 + \eta) \sigma_m^2}{k_m^2 + \rho(1 + \eta)^2 \sigma_m^2} k_m, \quad e_g^{*\text{AC}} = \frac{k_g^2 + \eta \rho(1 + \eta) \sigma_g^2}{k_g^2 + \rho(1 + \eta)^2 \sigma_g^2} k_g
\]

Proof. Substituting equations (43), (44) into equations (40), (41), we have:

\[
e_m^{*\text{AC}} = \frac{k_m^2 + \eta \rho(1 + \eta) \sigma_m^2}{k_m^2 + \rho(1 + \eta)^2 \sigma_m^2} k_m, \quad (45)
\]
\[
e_g^{*\text{AC}} = \frac{k_g^2 + \eta \rho(1 + \eta) \sigma_g^2}{k_g^2 + \rho(1 + \eta)^2 \sigma_g^2} k_g, \quad (46)
\]

proposition 5 QED.
By analyzing proposition 5 we get the following corollaries:

**Corollary 7.** The best efforts are not related to the investor’s emotional factor.

*Proof.* According to equations (45), (46), we have:

\[ \frac{\partial e^*_{m, AC}}{\partial \eta} = \frac{k_m \rho \sigma^2_m}{(k^2_m + \rho (1 + \eta)^2 \sigma^2_m)^3} (-k^2_m + \rho (1 + \eta)^2 \sigma^2_m); \quad \frac{\partial e^*_{m, AC}}{\partial \tau} = 0, \]

Corollary 7 QED.

Similar to the essential reason described in Corollary 5: in the case of acquisitions, AN’s payoff comes directly from the purchase price given by the acquirer, and the acquiring party has no emotional relationship with EN. Similarly, EN’s optimal efforts have nothing to do with AN’s emotional factors. We will explore the relationship between optimal efforts and emotional factors \( \eta \) in the section of numerical simulation.

4. **The choice of angel capital exit mechanism.** In the preceding section, we have obtained the optimal incentives and the optimal effort function of EN. Now we need to compare AN’s utilities in different exit routes, explore a correct way to choose between different exit methods, and give out the conditions and thresholds of these choices.

We have already obtained the optimal incentives given by AN and the best efforts made by EN in the case of IPO and acquisition. Substituting equations (20), (21), (22), (23), (24) into equation (14), we have:

\[ U_{AN}^{IPO} = -\frac{1 - \tau}{1 - \eta} \bar{U} + (1 + \eta) \frac{1 - \tau}{1 - \eta} (v + k_g \eta - c) + (1 + \eta) \frac{1 - \tau}{1 - \eta} (u + k_m \eta) - (1 + \eta) \frac{1 - \tau}{1 - \eta} I(1 + r) - \frac{1}{2} \tau \left( \frac{1 - \tau}{1 - \eta} \right) (k^2_m \eta^2 + k^2_g \eta^2) + \frac{1 - \tau}{2} \rho (1 - \tau) \sigma^2_m + 2(1 - \tau) k^2_m (1 - \eta)^2. \]

Substituting equations (42), (43), (44), (45), and (46) into equation (refeq36), we have:

\[ U_{AN}^{AC} = \frac{1 - \tau \eta}{1 + \eta} \eta k^2_m + (1 - \tau \eta) (u + v + G) + \frac{1 - \tau \eta}{1 + \eta} \eta k^2_g - (1 - \tau - \tau \eta) \bar{U} - \frac{1}{2} \frac{1 - \tau \eta}{(1 + \eta)^2} \eta^2 k^2_m - \frac{1}{2} \frac{1 - \tau \eta}{(1 + \eta)^2} \eta^2 k^2_g - (1 - \tau \eta) I(1 + r) + \frac{k^2_m (1 - \tau \eta)}{2(k^2_m + \rho (1 + \eta)^2 \sigma^2_m)(1 + \eta)^2} + \frac{k^2_g (1 - \tau \eta)}{2(k^2_g + \rho (1 + \eta)^2 \sigma^2_g)(1 + \eta)^2} \]

To facilitate our analysis, let \( \sigma^2 = \sigma^2_m = \sigma^2, k^2_m = k^2_g = k^2, u = v = \bar{U} = 0. \) By analyzing the investors’ utility differences under different exit mechanism, i.e. \( \Delta(\Delta = U_{AN}^{IPO} - U_{AN}^{AC}) \), we get the following conclusions:

**Proposition 6 (the choose between two exit methods).** There is a unique threshold \( c^* \) for the IPO listing cost of Angel Capital. When \( c < c^* \), IPO is chosen as the capital exit route. Threshold \( c^* \) is negatively correlated with synergistic utility \( G \), but when the entrepreneur’s emotional factor increases, it will weaken the impact of synergy utility.
Proof.  

\[ \Delta = U_{IPO}^{AN} - U_{AC}^{AN} \]

\[ = 2(1 + \eta) \frac{1 - \tau}{1 - \eta} k^2 \eta^2 - (\tau + \frac{1 - \tau}{1 - \eta})k^2 \eta^2 + \frac{(1 - \tau \eta)^2 k^4 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \]

\[ - (1 + \frac{1 - \tau}{1 - \eta})I(1 + r) - c(1 + \eta) \frac{1 - \tau}{1 - \eta} - \frac{2(1 - \tau \eta) k^2 \eta^2}{1 + \eta} \]

\[ + \frac{k^4 (1 - \tau \eta)}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \]

\[ + (1 - \tau \eta)G - (1 - \tau \eta) I(1 + r) \}

\[ = 2k^2 \eta^2 (\frac{1 - \tau \eta}{1 - \eta} - \frac{1 - \tau \eta}{1 + \eta}) + k^2 \eta^2 (\frac{1 - \tau \eta}{1 + \eta}) - \frac{1 - \tau \eta}{1 - \eta} I(1 + r) \]

\[ - \frac{1 - \eta \tau c + k^4 \left\{ \frac{(1 - \tau \eta)^2 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \]

\[ - \frac{(1 - \tau \eta)}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \}

\[ \left[ \frac{(1 - \tau \eta)^2 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \right] \}

let \( \Delta > 0 \), we have:

\[ c < - \eta I(1 + r) - (1 - \eta) G + \frac{1 - \eta}{1 - \eta \tau} \left\{ 2k^2 \eta^2 (\frac{1 - \tau \eta}{1 - \eta} - \frac{1 - \tau \eta}{1 + \eta}) \right\}

\[ + k^2 \eta^2 (\frac{1 - \tau \eta}{1 + \eta}) - \frac{1 - \tau \eta}{1 - \eta} I(1 + r) \]

\[ - \frac{1 - \eta \tau c + k^4 \left\{ \frac{(1 - \tau \eta)^2 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \]

\[ - \frac{(1 - \tau \eta)}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \}

\[ \left[ \frac{(1 - \tau \eta)^2 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \right] \}

then we have a threshold:

\[ c^* = - \eta I(1 + r) - (1 - \eta) G + \frac{1 - \eta}{1 - \eta \tau} \left\{ 2k^2 \eta^2 (\frac{1 - \tau \eta}{1 - \eta} - \frac{1 - \tau \eta}{1 + \eta}) \right\}

\[ + k^2 \eta^2 (\frac{1 - \tau \eta}{1 + \eta}) - \frac{1 - \tau \eta}{1 - \eta} I(1 + r) \]

\[ - \frac{1 - \eta \tau c + k^4 \left\{ \frac{(1 - \tau \eta)^2 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \]

\[ - \frac{(1 - \tau \eta)}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \}

\[ \left[ \frac{(1 - \tau \eta)^2 (1 - \eta)^3}{\rho(1 - \tau) \sigma^2 + (1 - \tau \eta) k^2 (1 - \eta)^2} \right] \}

So when \( c < c^* \) chooses an IPO as the capital exit method. The smaller the threshold, the greater the resistance to the listing. Derivation from \( c^* \), we have: \( \frac{\partial c^*}{\partial \sigma} < 0, \frac{\partial^2 c^*}{\partial \sigma^2} > 0 \). We see that an increase in entrepreneur's emotional factor reduces the impact of synergy. Proposition 6 QED.

When angel investors choose exit mechanisms between IPO and acquisitions, the first thing to consider is the IPO listing cost. If the IPO cost is too high, it will not choose the IPO as exit route. With this rationale, we calculate the unique threshold for the listing cost of \( c^* \). When \( c < c^* \), IPO is chosen as the capital exit route. Obviously, the smaller the threshold value, the greater the resistance to listing.

When angel investors compare IPO and acquisitions, they also need to consider the synergy effect brought by acquisitions. In the case of acquisitions, according to equation (19), AN's benefits include all synergies. When the synergy effect is too large, investors are more willing to choose acquisitions as exit route, so the resistance
to listing increases, we have $\frac{\partial c}{\partial \tau} < 0$. But, when the entrepreneur’s emotional factor is too high, it will weaken the synergy effect, and we have $\frac{\partial^2 c}{\partial \eta \partial \tau} > 0$. The reason for this effect is that EN’s emotional factor is too high, and his incentives decline, so the utility of EN is reduced, and then AN’s utility drops indirectly through emotional factor $\tau$, which weakens the synergy effect.

Next, we will consider a more specific case where entrepreneurs are risk neutral. In reality, many entrepreneurs, especially start-up entrepreneurs, are often the initiators of the project. We believe that they are not risk averse but risk-neutral, like the investors.

**Proposition 7.** If EN is risk neutral:
1. When $k^2 - I(1 + r) - c < 0$, AN’s optimal choice is acquisitions;
2. When $k^2 - I(1 + r) - c > 0$, there is a unique threshold $\eta^*$, AN chooses IPO as the exit route, otherwise choose acquisitions;
3. Threshold has the following relationship with other variables: $\frac{\partial \eta^*}{\partial c} > 0, \frac{\partial \eta^*}{\partial \tau} > 0, \frac{\partial \eta^*}{\partial G} > 0, \frac{\partial \eta^*}{\partial I(1 + r)} < 0$.

**Proof.** Considering the risk neutrality of entrepreneurs, we have:

$$
\Delta = U_{AN}^{IPO} - U_{AN}^{AC} \\
= 2k^2(1 - \tau \eta) \frac{\eta(1 + \eta)^2}{2(1 - \eta)(1 + \eta)^2} + k^2(1 - \tau \eta) \frac{\eta(1 + \eta)^2}{2(1 - \eta)(1 + \eta)^2} \\
- c \frac{1 - \tau \eta}{1 - \eta} - G(1 - \tau \eta) - \frac{1}{1 - \eta} I(1 + r) - \tau \eta I(1 + r) \\
= \frac{1}{2(1 - \eta)} [2k^2 \eta - 2c - 2G(1 - \eta) - 2\eta I(1 + r)] \\
- \tau \eta (2k^2 \eta - 2c - 2G(1 - \eta) - 2\eta I(1 + r)) \\
= \frac{1}{2(1 - \eta)} (1 - \tau \eta)(2k^2 \eta - 2c - 2G(1 - \eta) - 2\eta I(1 + r))
$$

let $2k^2 \eta - 2c - 2G(1 - \eta) - 2\eta I(1 + r) \leq 0$, then we have $\Delta = \frac{1}{2(1 - \eta)} (1 - \tau \eta) \Omega$. So, when $\Omega > 0$, AN choose IPO as the exit route, when $\Omega < 0$, AN choose acquisition as the exit route. We know $\Omega = 2k^2 \eta - 2c - 2G(1 - \eta) - 2\eta I(1 + r) = \eta(2k^2 + 2G - 2I(1 + r)) - 2c - 2G$, it is a linear function on $\eta$.

If $\eta = 0$, then $\Omega_{\eta=0} = -2c - 2G < 0$. If $\eta = 1$, then $\Omega_{\eta=1} = 2(k^2 - I(1 + r) - c)$, so:
when $k^2 - I(1 + r) - c < 0$, AN can only choose acquisition; when $k^2 - I(1 + r) - c > 0$, there is a threshold $\eta^* = \frac{c + G}{k^2 + 2G - I(1 + r)}$, and only if $\eta > \eta^*$, AN choose IPO, or otherwise, AN chooses acquisition.

Derive from $\eta^*$, we have $\frac{\partial \eta^*}{\partial c} > 0, \frac{\partial \eta^*}{\partial G} > 0, \frac{\partial \eta^*}{\partial I(1 + r)} > 0, \frac{\partial \eta^*}{\partial \tau} > 0$. Proposition 7 QED.

We have already explained in the corollary 2, when the emotional factor of EN is too strong, the situation of the IPO will be the same in the case of complete information. In this case, $c^*_{m,IPO} = k_m, c^*_{g,IPO} = k_g$. Therefore, the income generated by the entire venture company is $k_m^2 + k_g^2$. removing the effort cost of EN $\frac{1}{2} k_m^2 + \frac{1}{2} k_g^2$, the IPO listing cost $c$, and the investment $I(1 + r)$, it is exactly the first threshold given in the conclusion $\frac{1}{2} k_g^2 + \frac{1}{2} k_m^2 - I(1 + r) - c$ (recall that $\frac{1}{2} k_g^2 + \frac{1}{2} k_m^2 = k^2$). If this value is less than zero, the return from IPO will be negative.
anyway, so the choice of IPO is excluded. If this first threshold is greater than zero, there is another threshold $\eta^*$. When the emotional factor $\eta$ is greater than this threshold, AN will choose IPO as the optimal exit route. The intuition of this conclusion is that if AN hopes to go public, AN would like EN to have a stronger emotional factor, because in the case of acquisition, the income of AN comes from the purchase price given by the acquirer, and there is no emotional factors between the acquirer and EN. Therefore, the stronger the emotional factors of EN, the more favorable the IPO, but the more unfavorable the acquisition. At the same time, this conclusion illustrates that the choice between IPO and acquisition is only related to EN’s emotional factor. Finally, the first part of the third article is easier to understand, that is, if the listing cost is too high, the IPO gets more difficult, and the threshold of EN’s emotional factor $\eta^*$ will become higher. Similar intuition hold for synergy value $G$. For investment, if the value is too high, going through IPO may lead to the problem that the investment cannot be recovered, so the requirement for EN’s emotional factor will be higher. Finally, for $k$, which reflects the productivity of venture companies, and the intuition behind $\frac{\partial \eta^*}{\partial k} < 0$ is the same way as He’s paper: high-quality companies are more likely to choose IPOs, while low-quality companies will choose to be acquired.

5. Numerical experimental analysis. In the previous sections, we have given the intuition behind the conclusions. In the lack of data, numerical simulation is an effective way to test the previous conclusions. The purpose of this section is to use MATLAB for numerical analysis by taking parameters, observe the conclusions drawn before more intuitively, and further explain the conclusions and corollaries from the previous sections.

5.1. Incentives under different exit modes. In Figure 2, the relationship between the optimal incentive $\beta^*\text{IPO}$ and the emotional factors $\tau$, $\eta$ is given. When the emotional factor $\tau$ of AN increases, the optimal incentive $\beta^*\text{IPO}$ increases, and when the emotional factor $\eta$ of EN increases, the optimal incentive $\beta^*\text{IPO}$ decreases. Meanwhile, for the fact that when the emotional factor $\eta$ is large, we can observe from the picture that the optimal incentive $\beta^*\text{IPO}$ is close to zero, and this proves that when the emotional factor of EN is large, the incentive plan is the same as that under the symmetric information. These verify the corollary 1 and corollary 2. Then, we observe the marked points in figure 2. When the emotional factors satisfy the condition $\eta = \tau$, AN gives EN the incentive $\beta^*\text{IPO}$, which is decrementing. It corroborates corollary 3 in that EN’s emotional factor $\eta$ has a stronger impact than AN’s emotional factor $\tau$. Finally, we can also observe that when the emotional factor of AN is high, the incentive is close to $1^5$. This results from the fact that the emotional factor $\tau$ is very high, and AN increases the proportion of EN’s income into its own utility. Therefore, AN will augment the incentives to EN as much as possible, and the degree increases until it reaches $1$.

In Figure 3, the relationship between optimal incentives $\beta^*_\text{AC}$ and emotional factors $\tau$, $\eta$ is given. Similarly, we can see that: first, the optimal incentive is not related to $\tau$, and it decreases in the emotional factor $\eta$. However, unlike IPOs, even if the emotional factor $\eta$ increases to 1, it does not make $\beta^*_\text{AC}$ drop to 0. These verifies the corollary 5 and corollary 6.

\footnote{From proposition 2 we have $\beta^*\text{IPO} = \frac{k^2 m (1-\tau \eta)(1-\eta)^2}{\mu(1-\tau) \sigma^2_m + (1-\tau) \kappa^2_m (1-\eta)^2}$, then $\lim_{\tau \to 1} \beta^*\text{IPO} = 1$}
5.2. Optimal efforts under different exit choice. Figure 4 depicts the relationship between $e^*_{m,IPO}$ and the emotional factors, we observed that the optimal effort $e^*_{m,IPO}$ increases with the increase of $\tau$, thus supporting corollary 4. However, $e^*_{m,IPO}$ is not significantly correlated with emotional factors $\eta$. On the one hand, we can see from Figure 2 that the increase in $\eta$ reduces the incentive of EN, depressing EN’s efforts. On the other hand, the increase in $\eta$ gives EN satisfaction from the payoff of AN, which may actually stimulate EN’s efforts.
Now we turn to Figure 5, focusing on the relationship between $e_{m,AC}^*$ and the emotional factor $\eta$. $e_{m,AC}^*$ is uncorrelated with emotional factors $\tau$, supporting corollary 7. But with a distinct U shape\(^6\), and a shreshold $\tilde{\eta} = \sqrt{\frac{k_m}{\rho\sigma_m^2}} - 1$, $e_{m,AC}^*$ increases with $\eta$ when $\eta > \tilde{\eta}$.

5.3. Comparison of AN utility under different exit modes. In order to verify proposition 7. We show a set of three-dimensional figures depicting the relationship between $\Delta (\Delta = U_{AN}^{IPO} - U_{AN}^{AC})$ and various factors, and draw the contour at the bottom. In Figure 6, the contour line of $\Delta = 0$ is a straight line parallel to the $\tau$ axis, indicating that the emotional factor $\tau$ does not affect the choice of exit mode. In figure 7, we can see with the contour line of $\Delta = 0$ that as the cost of IPO, $c$ increases, $\eta$ is also on the rise. In Figure 8, the contour line of $\Delta = 0$ shows that as the investment cost $I$ increases, $\eta$ is increasing. Lastly, in Figure 9 the contour of $\Delta = 0$ implies that as the $k_m$ increases, $\eta$ is falling. Therefore, most of the conclusions in Proposition 7 have been verified.

6. Conclusions and predictions. Angel capitalists generally have two options for investment exit: IPO and acquisitions. When making exit decisions, they compare the personal utilities under different exit mechanisms. Entrepreneurs and angel investors are influenced by emotional factors. In other words, they incorporate each other’s benefits in their own utility functions. This paper focuses on these emotional factors in the study of exit model. By doing comparative static analysis, we have the following conclusions:

Firstly, whichever exit method chosen, as EN’s emotional factor strengthens, the incentives to EN decline. Secondly, when AN chooses IPO as the exit route, the incentives will increase with AN’s emotional factor rising, but in the case of acquisition, AN’s emotion will not affect the incentives. Thirdly, if AN chooses IPO, EN will make more efforts with an increase in AN’s emotional factor, but AN’s emotional factor will not affect the optimal efforts of EN in acquisition. Moreover, synergy gains through acquisition can make IPO uneconomic, but EN’s emotional factor weakens this effect. Last but not least, assuming that EN is risk neutral, only EN’s emotional factor will affect the final choice of exit route. The higher EN’s emotional factor, the more likely AN chooses IPO as the exit mechanism. There is a threshold of EN’s emotional factor. When EN’s emotional factor rises above it, the firm will go public. As the firm’s efficiency increases, the threshold value goes down. Therefore, high-efficient firms are willing to go public.

According to Fairchild’s theory, the source of emotional factors between investors and entrepreneurs may be their long-term relationship or friendship. Therefore, we propose a few falsifiable points for empirical tests:

H1. In a risky project, if the investors and entrepreneurs are relatives or long-term business partners, IPO is more likely to be chosen as the final exit mechanism (according to Proposition 6 and Proposition 7).

H2. Assume that the emotional factors are equal. If an investor has a long-term partnership with the entrepreneur, then the incentives to the entrepreneur are relatively small, regardless of the exit method (according to Corollary 3 and Corollary 5).

\[^6\frac{\partial e_{m,AC}^*}{\partial \eta} = \frac{k_m\rho \sigma_m^2}{(k_m + \rho(1 + \eta)^2\sigma_m^2)} \left(-k_m^2 + \rho(1 + \eta)^2\sigma_m^2 \right), \text{if } \rho \sigma_m^2 < k_m^2 < 4\rho \sigma_m^2, \text{there is threshold } \tilde{\eta} = \sqrt{\frac{k_m^2}{\rho\sigma_m^2}} - 1]
H3. Highly-efficient firms are more likely to choose IPO as the exit route (according to Proposition 7).

H4. Since we think it’s hard for pioneers to achieve synergy. The industry’s leading companies are generally listed publicly, and follow-up of small enterprises will generally be acquired by these leading large enterprises. (according to Proposition 7).
\[ k_m = k_g = 5, \]
\[ c = G = I = 10, \quad U = \]
\[ v = u = r = 0 \]

\[ \bar{U} = v = u = \]
\[ r = 0, \quad \tau = 0.5, k_g = \]
\[ k_m = 5, \quad G = I = 10 \]

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Figure 8. \( \bar{U} = v = u = \\
= 0, \tau = 0.5, k_g = \\
k_m = 5, G = c = 10 \)

Figure 9. \( \bar{U} = v = u = \\
= 0, \tau = 0.5, k_g = 5, \\
G = c = I = 10 \)

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