Propagation Parameters of Sea Surface Waves Inferred from Observations from Two Closely Spaced Vector Magnetometers

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The alternating magnetic dynamo field of sea surface waves, a consequence of their Lorentz electric field, has been observed with a pair of simultaneously operated, closely spaced tri-axial magnetometers. Measurements from a magnetometer located in the centre of a tiny, uninhabited island served to compensate measurements from a near-shore magnetometer for magnetic pulsations of ionospheric origin, leaving the water wave dynamo field, effective close to shore only, as the dominant residual magnetic field. Amplitude and frequency of waves and swell were recorded with a vertical accelerometer (wave rider buoy), floating on the sea surface. The wave rider data are in good agreement with those obtained from the magnetometers. Amplitude and phase relations between the three vector components of the magnetic oscillations yield a sea surface wave vector which is consistent with the swell propagation direction usually found in that area. The magnetic field data further demonstrate that the water mass motion close to shore was not confined to a vertical plane (as would be the case for freely propagating gravity waves in the open ocean). The motion rather took place in a plane inclined at about 40° from the horizontal, which is roughly twice the inclination of the island flanks. We conclude that the magnetometer measurements yield a reasonably accurate description of the surface wave water motion within about one wavelength from shore.

1. Introduction

ULF fluctuations of the geomagnetic field have routinely been observed with land-based magnetometers. The fluctuations result from the combined effects of primary electric currents flowing in the upper atmosphere and magnetosphere of the earth, and secondary (induced) electric currents flowing in the earth’s crust and upper mantle. When measurements of geomagnetic fluctuations are performed in the sea, on the sea bottom, or at low altitude above the sea, one may notice additional small oscillations of the magnetic field which are a signature of the sea surface wave dynamo. Sea water that moves with velocity v across the geomagnetic main field $B_0$, generates an electric $v \times B_0$ or Lorentz field which provides for an electromagnetic dynamo force. The high electrical conductivity of sea water (typically between 3 and 6 Sm$^{-1}$) permits the dynamo to drive an electric current which is via Ampère’s law associated with a magnetic field. Types of water mass motion relevant to this phenomenon include ocean currents (e.g. tidal currents), surface waves, and internal waves.

Fostered by improvements in the robustness and sensitivity of magnetic field sensors and by new developments in sensor concepts, the magnetic effect of ocean waves received considerable scientific attention during the 1960’s and 1970’s. A quantitative theoretical treatment of surface-wave associated magnetic variations observed above the sea surface was published by Crews and Futterman (1962). Their work was supplemented by Warburton and Caminiti (1964) who calculated the sub-surface magnetic field of sea surface waves. Weaver (1965) used a novel approach to calculate the magnetic field of sea surface waves above and below the surface of oceans with infinite and finite depth. Beal and Weaver (1970) followed with a calculation of the magnetic effects of internal waves in oceans of infinite and finite depth. Podney (1975) worked out a comprehensive theory of the magnetic field perturbations associated with
surface and internal waves in oceans of infinite and finite depth. Russian work on the subject was summarized by Sochel'nikov (1985).

In the wake of the theoretical work, several attempts were undertaken to observe the magnetic field of ocean waves. A number of successful measurements were made with total field sensors. They have the advantage of suffering minimally or not at all from rotational noise, i.e. from apparent magnetic fluctuations produced by mechanical oscillations of the sensor when it follows the oscillatory movement of the water mass elements. Maclure et al. (1964) measured simultaneously magnetic variations in deep water with a rubidium vapor magnetometer suspended from a spar buoy, and surface wave period and amplitude with an accelerometer floating on the surface. Fraser (1965) deployed a proton precession magnetometer and an echo sounder on the sea bottom at 40 m depth and showed that—under the assumption of a mean wave crest length of 300 metres—power spectra of ocean waves and magnetic field variations were consistent with theoretical predictions. Podney and Sager (1979a, b) exploited the high sensitivity of SQUID magnetometers to measure the vertical gradient of the east-west magnetic field variation over shallow water. They observed that the magnetic field gradient fluctuated in accordance with surface and internal waves which they recorded with pressure sensors, and current meter and thermistor chains, respectively.

Klein et al. (1975) attempted to measure the magnetic field of sea surface waves with a pair of search coils, one horizontally and the other vertically mounted in a non-magnetic frame. The device was deployed on the sea bottom in various shallow water areas of the Mediterranean Sea. The authors suggested that the spectral maxima found between 0.1 and 0.2 Hz were a manifestation of the surface wave dynamo field.

Fig. 1. The Formiche di Grosseto sea area, with part of Tuscany visible in the upper right corner. Inlay map of Italy with dot indicating the location of this area. Bathymetry lines in metres.
However, no oceanographic data were presented (e.g. measurements from a pressure sensor or a wave rider) which could have provided evidence that frequency and amplitude of the maxima in the magnetic field spectra coincided with maxima in the surface wave spectrum. The authors did also not discuss the possibility of a mechanical sensor oscillation, synchronous with the water mass oscillation, which can become a problem for a vector magnetometer. Estimation of the cross spectrum between the horizontal and the vertical magnetic field could have helped to resolve this problem.

The most recent analysis of experimental work on the subject we are aware of was published by Ochadlik (1989) who analysed simultaneously recorded magnetic and oceanographic data. In one experiment, a helium vapor magnetometer was mounted on a research tower placed in shallow water, and a pressure transducer was installed on the sea bottom. In another experiment, a dual-sensor helium vapor magnetometer was towed by a plane flying at low altitude over large-amplitude ocean swell. Ochadlik found his data to be consistent with Weaver’s theory.

2. Measurements

Between 12 and 28 September 1995, magnetic and oceanographic measurements were carried out simultaneously on and near the tiny, uninhabited island of Formica Grande (370 m long, 230 m wide, located at 43.6°N, 10.9°E, 14 km off the Italian west coast). It is the largest of a group of rocky islands called Formiche di Grosseto, which are aligned on a northwest-southeast striking rock bank rising rather sharply from an otherwise flat and gently sloping sea floor (Fig. 1). The transition from the inclined island flanks to the almost level sea floor occurs about 300 m northeast of the island at 100 m depth and 450 m southwest at 115 m depth. An autonomously operating lighthouse constitutes the only significant civilisation landmark on Formica Grande. It is powered by solar energy collected with an array of silicon panels and stored in truck batteries. The lighthouse building served as our field laboratory and housed our technical equipment, including a small Diesel generator. Except for the generator, which was operated during daytime hours only, no source of anthropogenic electromagnetic noise exists on the island. For this paper we selected only data which were collected during the night when the generator was shut off and the instruments were powered by batteries.

Two tri-axial fluxgate magnetometers were operated on the island at various locations. A continuously recording floating buoy housing a vertical accelerometer ("wave rider") was moored some 600 m southwest of the island. The magnetometer and wave rider measurements were recorded digitally with 48.25 Hz and 2.56 Hz sampling rate, respectively. In this paper we discuss data from a six-hour nighttime

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Fig. 2. Formica Grande, the northwestmost island of the Formiche di Grosseto group, with lighthouse and magnetometer sites.
interval in which we encountered the highest swell of the campaign period. This interval coincided with a setup in which one sensor (Mag2) was located in the centre of Formica Grande (site M2) and the other (Mag1) at its southeastern tip (site M1), 160 m away from M2 (Fig. 2). The intensity of waves and swell was considerably smaller during the four other nights in which the sensors were operated at the same locations, with the result that the signal-to-noise ratio of the magnetic dynamo field was much lower and yielded statistically less certain results.

The magnetic fluctuations recorded in the centre of the island were used to compensate the recordings from the shore site for magnetic variations of ionospheric origin. The compensation is necessary because the amplitude of magnetic fluctuations induced by swell was of the same order of magnitude as the average amplitude of magnetic fluctuations of ionospheric origin. A compensation is possible because the horizontal scale length of the ionospheric fluctuations is several orders of magnitudes larger than that of the swell, in other words, the spatial gradient of the ionospheric fluctuations is much smaller than that of the swell-associated magnetic dynamo field.

Figure 3 shows a ten-minute interval of the northward and eastward components of the magnetic fluctuations observed with the island shore magnetometer, the magnetic field difference between island shore and island centre, and the same difference enlarged by a factor of ten. The curves are offset along the ordinate for better graphical distinction. Obviously, the magnetic field measured with Mag2 does not completely compensate the measurements from Mag1. Besides long-period residuals, more or less regular fluctuations of about 7 s period remain as the most spectacular residuals. They are associated with the dynamo field of swell, as we will see below.

3. Spectral Analysis

We performed a spectral analysis on a six-hour time interval starting September 14, 1995, at 01:11 UTC. The interval was selected because it fell into the period of the highest swell of the entire campaign.
Before 01:11 UTC, radio transmission from the wave rider buoy suffered from failures, and after 07:20 UTC, magnetic noise caused by technicians working on the island impaired the correlation between measurements from the two magnetometers considerably. Prior to the spectral analysis, the magnetometer and the wave rider data were low-pass filtered (with 0.4 and 0.5 Hz cutoff, respectively) and decimated to sampling rates of 1 Hz and 1.28 Hz, respectively.

Power density spectra of magnetic variations in the north, east and vertical components were estimated in two different ways. Periodogram estimates were obtained by averaging 336 FFT spectra from 128-point time segments with 50% overlap (following a method proposed by Welch (1967)). Each individual time segment was centred and tapered with a 4-point Blackman-Harris window. Under the assumption that the individual time segments are different realisations of the same stationary stochastic process, the tapered segments can be considered to be virtually independent of each other despite overlapping (Harris, 1978). The equivalent degrees of freedom of the spectrum reach 99% of 672, which translates into a 99% confidence interval ranging from -1.2 dB to +1.3 dB around the estimated power at each point of the spectrum.

The maximum entropy (ME) method, on the other hand, was applied to the full-length time series. Instead of using the Burg (1975) algorithm (which minimizes the residual sum of squares with respect to the highest coefficient), we employed an algorithm described by Barrodale and Erickson (1980) (which minimizes the residual sum of squares with respect to all coefficients but requires more computer time and memory). Various criteria to determine the number of ME coefficients (i.e. the order of the stochastic process) have been proposed (e.g. Akaike, 1974, 1979; Parzen, 1974; Schwarz, 1978). We chose a different way and recalculated the ME spectra for process orders between 4 and 28 and for various time series lengths. We noticed that for orders exceeding 15 (magnetic field) and 20 (wave height) the spectra varied very little with increasing order. We therefore decided to estimate the ME spectra from 15-order and 20-order process representations, respectively, using a combined forward and backward prediction scheme.

Figure 4 shows ME auto spectra of the north and east components of the magnetic field fluctuations from Mag1 (upper panel), Mag2 (centre panel) and of the residual field (the difference, computed in the time domain, between Mag1 and Mag2) (lower panel). To be able to assess the significance of the magnetic field residual, we have included in each panel the upper limit of the magnetometer self noise, based on the manufacturer-supplied sensor specifications (dotted line). Also plotted is the effective surface wave spectrum, which we obtained by correcting the measured wave height spectrum for the wave rider response function and multiplying it with a wave efficiency factor $\omega_0/\omega$ ($\omega_0$ is a reference frequency which we chose to be 2\pi*0.14 Hz, and $\omega$ the wave angular frequency). This factor accounts for the frequency-dependent efficiency of the wave dynamo: For a bottom depth $D > \lambda/3$ ($\lambda$ denotes the wavelength), the surface wave dispersion relation can be approximated by $\omega^2 = g \cdot k$ ($g$ means the gravity acceleration and $k$ the wave number), and the magnetic dynamo field scales with the factor $(g/k)^{1/2} = g/\omega$ (Podney, 1975).

The spectra shown in Fig. 4 are very smooth, and the spectral power rises above system noise only below 0.2 Hz. The spectra from the shore site magnetometer (Mag1) show a minor peak at 0.14 Hz which coincides with the peak of the effective wave spectrum. The difference spectrum straddles the system noise level over virtually the entire frequency band, except around 0.14 Hz where the spectral power exceeds the noise by 15 dB (north component) and 20 dB (east component). If the vector time series from the two magnetometers were not identical, their difference would not vanish. Consequently, the power spectra of the difference time series would exhibit a certain amount of signal power in addition to the system noise power. This is not the case with our data, except for frequencies around 0.14 Hz and for the lowest frequencies. We therefore conclude that the time series from Mag1 and Mag2 are indeed almost identical, except for a quasi-DC component (which gives rise to the residual power at the low end of the spectrum) and oscillations with a mean period of some seven seconds (which produce the spectral peak at 0.14 Hz). This peak is statistically significant at a confidence level of more than 99.9% (a number obtained from the corresponding periodogram estimates). We suggest that it results from the swell dynamo field. The swell dynamo is effective only at the shore site and negligible in the centre of the island.
The dynamo field is therefore not cancelled out in the process of differencing the time series from the two magnetometers. We will return to this point further below in our discussion.

Figure 5 displays a stack of spectral parameters obtained from the same data set. The estimates for this figure were made using the periodogram technique in the way described above. The upper panel provides, in principle, the same information as the lower panel of Fig. 4. It shows the power spectra of the magnetic field differences in the north and east components, supplemented by the vertical component. The spectrum of the vertical component, too, shows a peak near 0.14 Hz, but the larger residual power at low frequencies indicates that the correlation between the two sites is weaker in the vertical component. We suggest that this is, at least in part, related to man-made magnetic noise, possibly from an underwater power cable. A cable running at 10° bearing from north, i.e. almost perpendicular to the line connecting Mag1 and Mag2, passes the island at some 1.7 km distance in 120 m water depth. An electric current flowing in the cable would produce a magnetic perturbation the power of which would be lower by a factor of $(1700/120)^2 = 200$ (i.e. $-46$ dB) in the horizontal than in the vertical magnetic field component. A change of the cable current intensity by 10 A would be observed as a 0.1 nT change of the vertical magnetic field difference between Mag1 and Mag2. Consequently, low-frequency stochastic fluctuations of the cable current with some 10 A peak-to-peak can explain the higher residual power in the vertical magnetic field. While being observed in the vertical component (at a level of $-30$ dB), they would be completely hidden in the horizontal components (where they would reach only $-76$ dB).

The magnetic north and east components reach peaks of 45 pT/Hz$^{1/2}$ and 63 pT/Hz$^{1/2}$, respectively, around 0.14 Hz. The peak value of the vertical magnetic field cannot be determined with equal accuracy.
because the vertical magnetic field power is substantially higher than the system noise level in the entire frequency band below 0.2 Hz, and it is difficult to distinguish between what can be attributed to the wave excitation and what is simply incoherent background noise. We can only estimate that the peak likely reaches up to between 55 and 60 pT/Hz$^{1/2}$, and we use 57 pT/Hz$^{1/2}$ in the following discussion. The wave height spectrum (suppressed in this figure) yields a peak amplitude of 1.4 m/Hz$^{1/2}$ for the 0.14 Hz swell.

The next lower panel shows the cross power of the north/east and east/vertical difference fields, together with the instrument noise level. The cross power is almost everywhere lower than the noise except for the remarkable peak around 0.14 Hz. The information from the two top panels is combined in the third panel to yield the spectral coherency. The larger residual noise in the vertical component results in a lower coherency between the east and vertical components than is found between the north and east components. Only the very low frequencies which may be dominated by man-made noise, seem to indicate a higher degree of coherency between the east and vertical difference fields. This is consistent with our suggestion that the measurements were affected by electric current fluctuations in an underwater power cable. The north/east coherency exceeds 0.5 within the shaded area and reaches a maximum of 0.81, and the east/vertical coherency exceeds 0.45 in its leftmost section (from the left edge to the white delimiter line) and reaches a maximum of 0.48. The coherency distribution demonstrates that there is a rather high degree of correlation in the swell frequency band, in particular between magnetic north and east fluctuations, and very little correlation between signals outside this band.
The bottom panel shows the phase difference between the two pairs of magnetic field differences, north/east and east/vertical. Because of the low coherency, the phase information is highly uncertain outside the shaded area and is most certain inside. Within the shaded areas, the north component lags the east component by some 60° and the east component the vertical component by about 75° on the average. For 672 degrees of freedom and a coherency exceeding 0.45, the statistical 99% confidence limit of the phase is bound by ±14° around each estimate.

Focusing on the frequency band covered by the shaded area, we combine the amplitudes of the magnetic field vector differences with their phase relations, and obtain spectral amplitude, spatial orientation, and rotation sense of the mean magnetic field polarization vector. Eventually we infer the mean motion of the water mass elements and the sea surface wave propagation vector from the magnetic field oscillation. That is even more information about the swell than our non-directional wave rider provides.

4. Water Motion Inference

In this section we reconstruct the swell-associated water mass motion close to the island, using the information obtained from the spectral analysis of magnetic field fluctuations. We begin with a recollection of the relevant physical parameters pertaining to the spectral peaks. They are: (1) dominant frequency of swell and magnetic field difference, 0.14 Hz; (2) swell peak intensity, 1.4 m/Hz1/2; (3) peak intensities of north, east, and vertical magnetic field differences, 45, 63, and 57 pT/Hz1/2, respectively; (4) mean phase lag magnetic north versus east, 60°, and east versus vertical, 75°. From the swell frequency of 0.14 Hz we infer a wavelength of 75 m using the approximate dispersion relation \( \omega^2 = g \cdot k \) (see above).

The electrical conductivity of the sea water was measured to be 4.5 Sm\(^{-1}\). The elements of the geomagnetic main field at Formica were: total field strength \( B_0 = 45 \mu T \), inclination \( I = 60° \), declination \( D = 0° \).

From the numerical values obtained for the peak power densities and phase differences between the magnetic field components we can construct a magnetic field polarization ellipse, under the assumption that the magnetic field and the sea water oscillate harmonically with amplitudes equivalent to the root of the peak power densities. In parametric representation, the ellipse is determined by the three functions \( x_M(t) \), \( y_M(t) \), and \( z_M(t) \), which denote the north, east, and vertical magnetic components, respectively:

\[
\begin{align*}
   x_M(t) & = A_X \sin(\omega t - \pi/3) \\
   y_M(t) & = A_Y \sin(\omega t) \\
   z_M(t) & = A_Z \sin(\omega t + 5\pi/12)
\end{align*}
\]

with \( \omega = 2\pi \cdot 0.14 \) Hz, \( A_X = 45 \) pT, \( A_Y = 63 \) pT, and \( A_Z = 57 \) pT. The corresponding mean water motion can similarly be modeled by a parameterized ellipse

\[
\begin{align*}
   x_W(t) & = \Sigma^{-1} \cdot T^{-1} \cdot A_X \sin(q_0 - \omega t + \pi/3) \\
   y_W(t) & = \Sigma^{-1} \cdot T^{-1} \cdot A_Y \sin(q_0 - \omega t) \\
   z_W(t) & = \Sigma^{-1} \cdot T^{-1} \cdot A_Z \sin(q_0 - \omega t - 5\pi/12)
\end{align*}
\]

with \( T \) denoting a distance-independent scale factor which depends on the electrical conductivity of the sea water, the wavelength, the strength of the geomagnetic main field, and the orientation of the motion ellipse, and \( \Sigma \) denoting a scale factor which depends on the ratio between wavelength and minimum distance between sensor and water waves. An arbitrary phase factor, \( q_0 \), has been added to indicate that we do not know the phase relationship between water mass motion and magnetic field oscillation, because
the wave rider was too far from the island (about eight wavelengths) to guarantee phase coherency between wave rider and magnetometer measurements. The actual phase lag, though, is irrelevant for our objective, the construction of the polarization ellipse. The sign of the angular frequency, $\omega$, is reversed because the vectors of the polarization ellipses of magnetic field variation and water motion share the same plane but circulate in opposite sense. The water motion in a wave propagating in eastern direction is right-hand polarized with respect to a north-pointing vector, while the associated magnetic variation is left-hand polarized above the sea surface, and vice versa. Therefore, the polarization sense of the magnetic field yields an estimate of the wave propagation direction (i.e. the orientation of its $k$-vector).

For ocean depths $D > \lambda/3$, the scale factor $T$ can be approximated after Podney (1975) with less than 10% error by the expression

$$T = \frac{1}{2} \mu_0 \sigma \omega^{-1} B_p$$

with $\mu_0$ denoting the vacuum permeability, $\sigma$ the electrical conductivity of sea water, and $B_p$ the intensity of the geomagnetic main field in the plane of the polarization ellipse. In case of a uniform wave field only $B_p$ is relevant; the geomagnetic field perpendicular to the polarization ellipse plays no role in the dynamo process. Although the condition of uniformity is certainly violated by the presence of the island, we use it as a first approximation in our description of the water motion. We note, though, that the ellipses are not confined to a vertical plane, as is the case for freely propagating waves in an unbound ocean.

Fig. 6. Ellipse (not to scale): projection into horizontal plane of mean swell-associated water mass motion close to the southern shore of Formica Grande. Shade of gray indicates vertical displacement of water mass elements (darker = lower, lighter = higher). Polarization sense indicated by curved arrows. Long arrow: propagation direction of swell coming from the Strait of Bonifacio.
Figure 6 shows the projection of the mean water motion ellipse onto the horizontal plane, positioned below a map of Formica Grande (taken from Fig. 2). North points toward the top and east toward the right. The vertical component of the water motion is represented by different shades of gray, darkest tone means point of lowest, lightest tone means point of highest displacement of the water mass elements. The polarization sense of the water motion is marked by small curved arrows. Note that island and ellipse are not drawn to scale. The ellipse semi-axes are of the same order of magnitude as the mean wave amplitude, that is of the order of 1 metre. A more precise scaling can not be given because we do not know exactly how much the amplitude of the water mass displacement close to shore differs from that recorded in open waters.

The direction from the Strait of Bonifacio (located between Corsica and Sardinia) to the Formiche islands is indicated in Fig. 6 by an arrow across the ellipse. It nearly coincides with the orientation of the major axis of the ellipse projection. Orientation of the major ellipse axis and polarization sense of the water motion suggest that the surface wave vector, $k$, was pointing approximately northeastward, nearly aligned with the arrow. This is in agreement with long-term observations of ship operators which indicate that swell encountered in the Formiche area very often originates from the Strait of Bonifacio.

The inclination of the ellipse is qualitatively in agreement with the expected water motion. The inclined island flanks allow the upward moving water to move toward the island and force the downward moving water to move away from the island. For waves not propagating perpendicular to the island flanks, the originally circular motion, which was confined to a vertical plane, is converted into a motion in a plane inclined in the same sense as the island flank. In the case of Formica Grande, the flank gradient at the southeastern edge of the island is mainly south-north oriented. The topography thus forces the water to deviate from the original, southwest-northeast oriented motion, and to assume an additional north-south motion up and down the island flank, which is approximately in phase with the vertical motion but not with the original horizontal motion. We conclude that the three-dimensional orientation of the ellipse represents qualitatively the true motion of the water mass elements on the island flank. Note that only the water motion very close to the island, within about one wavelength, contributes efficiently to the observed magnetic field oscillation. The discussion further below makes clear that the scale factor $S$ becomes very small for distances exceeding one wavelength.

Theory (Podney, 1975) states that in the case of a uniform surface wave field the magnetic polarization ellipse above the sea surface degenerates into a circle. Our polarization ellipse is indeed almost circular, the major and minor axes of the fully three-dimensional magnetic field polarization ellipse, for instance, are found to be 64 pT and 61 pT, respectively. In order to facilitate quantitative comparison with theory, we approximate the polarization ellipse by a circle of the same area (which has a radius of 62 pT), and assume a circular water motion with an amplitude equivalent to the wave peak power observed away from the island, i.e. 1.4 m. The intensity of the geomagnetic main field in the plane of the polarization ellipse, $B_p$, amounts to 22.4 µT. We can then calculate scale factors $T$ and $S$, with $S$ being an equivalent to $\Sigma$ and being expressed after Podney (1975) as

$$S = \exp(-k \cdot h)$$

where $h$ denotes the height of a hypothetical observation point above the sea surface. Essentially, Eq. (4), in combination with Eqs. (3) and (2), tells us how far a magnetic sensor should have been placed above the surface of an open ocean in order to observe a circularly polarized magnetic oscillation with the same amplitude as what we observed at the M1 site, always under the condition that the same swell amplitude and frequency prevailed. For the parameter $h$, which we call the “equivalent sensor altitude”, we obtain 41 m using Podney’s theory. Incidentally, this is also roughly the length of the shortest line-of-sight between Mag1 and the shore where the swell hit the island (approximately southwest of M1; in Fig. 6, it is the point where the arrowhead almost touches the island contour). Mag1 was much closer to the shore northwest of M1, but waters at that side of the island were protected from the swell, and the waves were not high enough to generate a measurable magnetic dynamo field. It is also clear that $S$ is reduced by a
factor of the order of 500 if \( h \) is replaced by \( h + \lambda \), i.e. if the motion of a water mass element at a point one wavelength further away is considered. For the same reason, it was sufficient to place our local reference sensor, Mag2, about two wavelengths inland to ensure that it remained practically unaffected by the ocean wave dynamo.

5. Conclusion

We have demonstrated that magnetic field oscillations observed close to an island shore can be used to infer the water motion associated with surface waves and swell. The measurement of the magnetic fluctuations yields the direction of the wave propagation, and orientation and inclination of the plane in which the water oscillates. However, magnetic measurements on an island are not yet suitable to estimate the wave height accurately. The good agreement between hypothetical sensor altitude and measured distance of the magnetometer from shore is probably accidental and not backed by published theory.

Podney and predecessors developed their theory for the geometrically simple case of an unbound ocean with a plane, strictly horizontal bottom and perfectly two-dimensional, freely propagating gravity waves in which the water mass motion is confined to a vertical plane containing the wave propagation vector. This is different from our situation in which we consider the water motion close to an island with sloped flanks. To our knowledge, the complex topography of an island has not yet been treated analytically. Examples of various topographic structures have, however, been studied by Miles and Dosso (1979, 1980), using reduced-size analogue models of some representative topographic features, with consistent scaling of the physical parameters. A numerical simulation of wave propagation and magnetic field excitation in specific topographic settings may provide an alternative approach to dealing with complex topographies. To our knowledge, such kind of simulation studies has not yet been published.

Miles and Dosso performed measurements of wave-induced magnetic fields on laboratory models of various coastal shapes, shelf profiles, dykes and sea mounts. They found that in very shallow water, and close to topographic ocean bottom irregularities, the magnetic field of surface waves can deviate substantially from the field over a uniform ocean. Among the models they investigated, the sea mount model meets best, though not completely, the conditions prevailing at Formica Grande. Their scale model is equivalent to a setting in which a 106 m deep ocean with 4.5 Sm\(^{-1}\) electrical conductivity is threaded by a vertical magnetic field of 50 µT intensity. A nonconducting sea mount with flanks of 60° inclination, topped by a platform which extends over 200 m horizontally, reaches up to 7.6 m below the surface. Surface waves of 0.064 Hz frequency propagate in the ocean and across the sea mount. The magnetic field probe is positioned 24 m above the sea surface. The scale model measurements indicated that at the edge of the sea mount (and continuing toward its centre), the magnetic field amplitude can be reduced by some 50% compared to a reference field measured over a deep ocean with a plane bottom.

We conclude that our island-based, bistatic vector-magnetometer measurements provide for an accurate description of sea surface wave frequency and propagation direction. Even our results on wave height and details of water motion close to shore appear to agree qualitatively with what we expect to observe. However, because of the complexity of the island topography and the subsequent breakdown of various assumptions made in the theoretical papers (such as the infinite clepth approximation, which requires \( D > \lambda/3 \) and becomes valid in our setting only at a distance of more than one wavelength from shore), a quantitative description of the wave height and water mass motion remains uncertain. Presently available theory does not necessarily yield reliable numbers for observation points close to shore, it yields only numbers for points farther away where the water motion is not affected by the obstacle of an island such as Formica Grande.

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