Dispersion management for atomic matter waves

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We demonstrate the control of the dispersion of matter wave packets utilizing periodic potentials. This is analogous to the technique of dispersion management known in photon optics. Matter wave packets are realized by Bose-Einstein condensates of \textsuperscript{87}Rb in an optical dipole potential acting as a one-dimensional waveguide. A weak optical lattice is used to control the dispersion relation of the matter waves during the propagation of the wave packets. The dynamics are observed in position space and interpreted using the concept of effective mass. By switching from positive to negative effective mass, the dynamics can be reversed. The breakdown of the approximation of constant, as well as experimental signatures of an infinite effective mass are studied.

The broadening of particle wave packets due to the free space dispersion is one of the most prominent quantum phenomena cited in almost every textbook of quantum mechanics. The realization of Bose-Einstein condensates of dilute gases allows for the direct observation of wave packet dynamics in real space on a macroscopic scale \cite{1}. Using periodic potentials it becomes feasible to experimentally study to what extent the matter wave dispersion relation can be engineered. This approach is similar to dispersion management for light pulses in spatially periodic refractive index structures \cite{2}.

First experiments in this direction have already been undertaken in the context of Bloch oscillations of thermal atoms \cite{2} and condensates \cite{3}. The modification of the dipole mode oscillation frequency of a condensate due to the changed dispersion relation in the presence of a periodic potential has been studied in detail \cite{2,3}. In contrast to these experiments where the center of mass motion was studied, we are investigating the evolution of the spatial distribution of the atomic cloud in a quasi one-dimensional situation. For the wave packet dynamics many new effects are expected due to the interplay between nonlinearity resulting from atom-atom interaction and dispersion. In particular, non-sweeping wave packets such as gap solitons \cite{3} and self trapped states \cite{4} are predicted. Our experiments show that even propagation in the linear regime with modified dispersion can change the shape of an initially Gaussian wave packet significantly.

For atomic matter waves inside a one-dimensional optical waveguide, we have achieved dispersion management by applying a weak periodic potential with adjustable velocity. Fig. 1 shows the results of an experiment in which the propagation of an atomic wave packet is studied in the normal (Fig. 1b) and anomalous (Fig. 1c) dispersion regime corresponding to positive and negative effective mass, respectively. A broadening of the wave packet is observed in both cases. The faster spreading in the case of anomalous dispersion is a consequence of the smaller absolute value of the negative effective mass. However, if we switch from one regime to the other during the propagation by changing the velocity of the periodic potential, the effects of normal and anomalous dispersion cancel. The wave packet, which has initially broadened under the influence of normal dispersion, reverses its expansion and compresses until it regains its initial shape (Fig. 1d). This is a direct proof of the realization of negative effective mass.

The concept of effective mass $m_{\text{eff}}$ allows to describe
the dynamics of matter wave packets inside a periodic potential in a simple way via a modified dispersion relation. The periodic potential in our experiments is well described by

$$V(x) = \frac{V_0}{2} \cos(Gx)$$

with a modulation depth $V_0$ on the order of the grating recoil energy $E_G = \hbar^2 G^2 / 8m$, with $G = 2\pi/d$ where $d = 417\text{nm}$ represents the spatial period. The energy spectrum of atoms inside the periodic potential exhibits a band structure $E_n(q)$ which is a periodic function of quasimomentum $q$ with periodicity $G$ corresponding to the width of the first Brillouin zone (Fig. 2a). In our experiment, we prepare condensates in the lowest energy band ($n=0$) with a quasimomentum distribution $w(q)$ centered at $q = q_c$ with an rms width $\Delta q \ll G$.

It has been shown by Steel et al. that in this case the condensate wave function in a quasi one-dimensional situation can be described by $\Psi(x,t) = A(x,t) \phi_{q_c}(x) \exp(-iE_0(q_c)t/\hbar)$, where $\phi_{q_c}$ represents the Bloch function in the lowest energy band corresponding to the central quasimomentum. The evolution of the envelope function $A(x,t)$, normalized to the total number of atoms $N_0$, is described by

$$i\hbar \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial x} \right) A = -\frac{\hbar^2}{2m_{\text{eff}}} \frac{\partial^2}{\partial x^2} A + \hat{g}|A|^2 A. \quad (1)$$

The strength of the atom-atom interaction is given by $\hat{g} = \alpha_{\text{nl}} 2\hbar \omega_2 \alpha_l$, with the transverse trapping length $\alpha_l$, and a renormalization factor $\alpha_{\text{nl}} = 1/d \int_0^d dx |\phi_{q_c}|^4$. Besides the modification of the nonlinear term the periodic potential leads to a group velocity of the envelope $A(x,t)$ determined by the energy band via $v_g(q_c) = \hbar^{-1} \partial E_0(q_c)/\partial q_c$ (Fig. 2). In addition, the kinetic energy term describing the dispersion of the wave packet is modified by the effective mass (Fig. 2):

$$m_{\text{eff}}(q_c) = \hbar^2 \left( \frac{\partial^2 E_0(q)}{\partial q^2} \right)^{-1}_{q = q_c}.$$

It is important to note that group velocity and effective mass are assumed to be constant during the propagation. Since the approximation of constant effective mass corresponds to a parabolic approximation of the energy band, it is only valid for sufficiently small $\Delta q$ as can be seen in Fig. 2.

The general solution of Eq. (1) is a difficult task, but simple analytic expressions can be found in the special cases of negligible and dominating atom-atom interaction. Omitting the last term in Eq. (1) it is straightforward to see that $|m_{\text{eff}}|$ controls the magnitude of the dispersion term and thus the timescale of the wave packet broadening. A change in sign of $m_{\text{eff}}$ corresponds to time reversal of the dynamics in a frame moving with velocity $v_g$. In the regime where the atom-atom interaction is dominating, e.g. during the initial expansion of a condensate, the evolution of the envelope function can be found in standard nonlinear optics textbooks and in form of scaling solutions in the context of Bose-Einstein condensates. Note that in this regime the kinetic energy term is still relevant and thus a change of the sign of the effective mass will reverse the dynamics.

In the following we will analyze the obtained experimental results in more detail. The initial wave packet shown in Fig. 1(b) is characterized by $\Delta x_0 = 14.8(6)\mu m$ ($\Delta x$ is the r.m.s width of a Gaussian fit) and shows the density distribution of a Bose-Einstein condensate of $2 \cdot 10^4$ atoms in a three-dimensional dipole trap. Before releasing the atomic cloud into the one dimensional waveguide, a weak periodic potential along the waveguide is adiabatically ramped up to $V_0 = 2.8(2) E_G$ within 6 ms. This turns the initial Gaussian momentum distribution of the atoms into a Gaussian distribution of quasimomenta $w(q)$ centered at $q_c = 0$ in the lowest energy band of the lattice with a corresponding effective mass $m_{\text{eff}} = 1.25 m$. The density distribution shown in Fig. 1(c) is a result of propagation within the stationary periodic potential for $t = 26$ ms and exhibits a spread of $\delta x := \sqrt{\Delta x(t)^2 - \Delta x_0^2} = 18.4(12)\mu m$ in contrast to $\delta x_f = 20.2(14)\mu m$ for expansion without periodic potential. The resulting ratio $\delta x_f/\delta x = 1.10(15)$ indicates that the evolution is dominated by the nonlinearity, in which case one expects $\delta x_f/\delta x \approx \sqrt{m_{\text{eff}}/m} = 1.11(14)$. In the case of linear propagation one expects $\delta x_f/\delta x = m_{\text{eff}}/m = 1.25$.

The dynamics in the anomalous dispersion regime are investigated by initially accelerating the periodic potential within 3 ms to a velocity $v = v_q := \hbar G/2m$, thus preparing the atomic wave packet at the edge of the Brillouin zone, where $m_{\text{eff}} = -0.5 m$. The velocity is kept constant during the subsequent expansion. In the regime of negative mass a condensate exhibits collapse dynamics similar to condensates with attractive atom-atom interaction. Two-dimensional calculations for our experimental situation reveal that this collapse happens within the initial 3-6 ms of propagation. This leads to a fast reduction of the density and therefore of the nonlinearity due to excitation of transverse states. An indication of the population of transverse states is the observed increase of the transverse spatial extension of the wave packets for anomalous dispersion by almost a factor of two. The optical resolution of our setup does not allow for a quantitative analysis of the transverse broadening. The measured spread of $\delta x = 38.5(15)\mu m$ after 23 ms of expansion leads to a ratio $\delta x_f/\delta x = 0.46(5)$. This ratio represents an upper limit due to the initial collapse, indicating that a coherent nonlinear evolution in one dimension cannot describe the experimental results.

In the case of dispersion management Fig. 1(d)
wave packet was first subjected to normal dispersion for 17 ms at \( q_c = 0 \). The time of subsequent propagation with anomalous dispersion at \( q_c = G/2 \) was adjusted to achieve the minimal wave packet size of \( \Delta x = 15.4(2) \mu m \). The minimum was achieved for times ranging from 7 ms to 9 ms which is in rough agreement with the expected time resulting from effective mass considerations \( \sqrt{0.5/1.25 \times 17 \text{ ms}} = 10.7 \text{ ms} \). The experimental results exhibit a rapid increase of the wave packet width for overall propagation times longer than 27 ms also indicating collapse dynamics.

Before presenting the detailed experimental studies of the applicability of the constant effective mass approximation and evidence of infinite masses, we turn to a brief description of our setup.

The wave packets have been realized with a \(^{87}\)Rb Bose-Einstein condensate. The atoms are collected in a magneto-optical trap and subsequently loaded into a magnetic time-orbiting potential trap. By evaporative cooling we produce a cold atomic cloud which is then transferred into an optical dipole trap realized by two focused Nd:YAG laser beams with 60 \( \mu m \) waist crossing at the center of the magnetic trap. Further evaporative cooling is achieved by lowering the optical potential leading to pure Bose-Einstein condensates with up to \( 3 \cdot 10^4 \) atoms in the \( |F = 2, m_F = +2 \rangle \) state. By switching off one dipole trap beam the atomic matter wave is released into a trap acting as a one-dimensional waveguide with radial trapping frequency \( \omega_{\perp} = 2\pi \cdot 80 \text{ Hz} \) and longitudinal trapping frequency \( \omega_l = 2\pi \cdot 1.5 \text{ Hz} \).

The periodic potential is realized by a far off-resonant standing light wave with a single beam peak intensity of up to 5 W/cm\(^2\). The chosen detuning of 2 nm to the blue off the D2 line leads to a spontaneous emission rate below 1 Hz. The frequency and phase of the individual laser beams are controlled by acousto-optic modulators driven by a two channel arbitrary waveform generator allowing for full control of the velocity and amplitude of the periodic potential. The light intensity and thus the absolute value of the potential depth was calibrated independently by analyzing results on Bragg scattering [13] and Landau Zener tunneling [4].

The wave packet evolution inside the combined potential of the waveguide and the lattice is studied by taking absorption images of the atomic density distribution after a variable time delay. The density profiles along the waveguide, \( n(x, t) \), are obtained by integrating the absorption images over the transverse dimension \( z \).

Since the assumption of a constant effective mass used so far is only an approximation, it is important to check its applicability in experiments. Therefore we investigate the dynamics of wave packets prepared at the Brillouin zone edge \( (q_c = G/2) \) for different potential depths. The observed density profiles after 60 ms of propagation are shown in Fig. 2d-f). While both the initial wave packet shape \( n(x, 0) \) and the quasimomentum distribution \( w(q) \) are measured to be approximately Gaussian, the wave packet changes its shape during evolution. We attribute this distortion to the invalidity of the constant effective mass approximation, which assumes that the populated quasimomenta experience the same negative curvature of \( E_0(q) \). Since the range of quasimomenta fulfilling this criterion becomes smaller with decreasing modulation depth, a more pronounced distortion of the wave packet shape for weak potentials is expected (see Fig. 2d).

This explanation is confirmed more quantitatively by comparing the observed wave packets with the results of an integration of Eq. 1 neglecting the nonlinear term. Since the initial collapse of the condensate cannot be described by a linear theory, we take a Gaussian function fitted to the density distribution measured at 20 ms as the initial wave packet for the numerical propagation. Due to the fact that this is not a minimum uncertainty wave packet we add a quadratic phase in real space such that the Fourier transform of the wave packet is consistent with the measured momentum distribution. In first approximation this takes into account the initial expansion including the repulsive atom-atom interaction. For

![Image](image_url)
the existence of another very interesting feature of the band structure: we currently investigate these features in more detail. This could be an indication of entering the tight binding regime where the nonlinear effect of self trapping, i.e. stopping and non-spreading wave packets, has been predicted by Trombettoni et al. We are currently investigating the transport properties in this regime in more detail.

In conclusion, we have demonstrated experimentally that the dispersion of atomic matter waves in a waveguide can be controlled using a weak periodic potential. Matter wave packets with positive, negative and infinite effective masses are studied in the regime of weak and intermediate potential heights. The preparation of matter waves with engineered dispersion \( m_{\text{eff}} < 0 \) is an important prerequisite for the experimental investigation of atomic gap solitons and other effects arising from the coherent interplay of nonlinearity and dispersion in periodic potentials.

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