Comment on “Are the spectra of geometrical operators in Loop Quantum Gravity really discrete?” by B. Dittrich and T. Thiemann

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I argue that the prediction of physical discreteness at the Planck scale in loop gravity is a reasonable conclusion that derives from a sensible ensemble of hypotheses, in spite of some contrary arguments considered in an interesting recent paper by Dittrich and Thiemann. The counter-example presented by Dittrich and Thiemann illustrates a pathology which does not seem to be present in gravity. I also point out a common confusion between two distinct frameworks for the interpretation of general-covariant quantum theory, and observe that within one of these, the derivation of physical discreteness is immediate, and not in contradiction with gauge invariance.

I. INTRODUCTION

In loop quantum gravity (LQG) [1, 2, 3, 4], some operators representing geometrical quantities have discrete spectra [5]. Does it follow that geometrical quantities have discrete spectra? That is, does LQG predicts that a Plank-scale measurement of these quantities would yield quantized outcomes? That is, does LQG truly predicts discreteness of physical space? These operators are not themselves gauge-invariant; but it has been argued that under plausible hypotheses their spectra provide evidence for physical discreteness [5]. The plausibility of these additional hypotheses has raised a certain debate in the past.

In a recent paper [6] Dittrich and Thiemann reconsider the debate and argue for a negative answer. They present some toy models as counter-examples to the expectation that a discrete spectrum implies physical quantization.

Here, I argue that the existence of these counter-examples makes an interesting point, but does not change the fact that physical discreteness in LQG is a reasonable conclusion deriving from a sensible ensemble of hypotheses. There are three distinct arguments to this effect.

(i) As discussed below, the key counter-example presented in [6] is based on a pathology which does not appear to be present in gravity, at least in this form.

(ii) Physical evidence is distinct from mathematical proof. Physics requires tentative generalizations and implicit assumptions. In loop gravity, evidence is towards discreteness, and in support of the familiar relation between discrete spectra and quantization. Lack of a rigorous mathematical proof does not imply lack of evidence. ¹

(iii) Most importantly, the analysis in [6] is based on a specific framework for interpreting general-covariant quantum theory. Dittrich and Thiemann refer to my own work on the subject, in particular to the notion of “partial observable” and the paper [7]. However, ideas discussed in [7] are related to an interpretation which is different from the one they use. Under this second interpretation physical discreteness follows immediately, without contradicting gauge invariance.

The interest of the Dittrich-Thiemann paper is that it shows how distinct ways of extending the interpretation might yield different physical predictions. This is an important contribution, which deserves to be better understood. But in either case, the evidence remains strong towards the conclusion that LQG implies fundamental discreteness at Planck scale.

I point out the difference between two frameworks for interpreting general-covariant quantum theory in Sec. II. The counter-example in [6] is discussed in Sec. III. Some general arguments about discreteness of space in LQG are recalled in Sec. IV. In the conclusion, I summarize the evidence for discreteness in LQG.

II. TWO INTERPRETATIONS OF GENERAL-COVARIANT QUANTUM THEORY

There is a common confusion between two distinct approaches to the issue of observability in general-covariant quantum systems. (In my own work I have studied both of them, beginning with the first, and then shifting to the second.)

¹ In this regard, a stronger wording in the first online version of [6], (“fundamental discreteness in LQG is an empty statement”) has been removed in the second version.
I. The formulation based on the notion of “evolving constant” or “complete observables” discussed in Refs. 9, 11, 12. This is the interpretation that Dittrich and Thiemann utilize.

II. The formulation derived from the paper 8 on partial observables, and summarized for instance in Ref. 2.

Let me briefly summarize the two, in order to clarify their difference. I give here only the basic structure of the two interpretations. Both have been discussed in depth in the literature. See the references cited for motivation and for the numerous details needed to make these pictures physically and mathematically precise. I assume here for simplicity that we are dealing with systems defined by a single constraint.

I. A general-covariant system is defined by a phase space \( \Gamma \), which is equipped with Poisson brackets. Its dynamics is determined by a (“Hamiltonian”) constraint \( H \), which is a real function on \( \Gamma \). The quantities that represent physical measurements are real functions \( F \) on \( \Gamma \) that have vanishing Poisson brackets with \( H \) on the surface \( H = 0 \). These are called Dirac observables. A special class of these functions is given by the “evolving constants”. An evolving constant \( F_\tau \) is a one-parameter family of Dirac observables determined by two functions on \( \Gamma \): a “clock” function \( t \) and a function \( f \). \( F_\tau \) is defined by two properties: (i) it is a Dirac observable and (ii)

\[
(F_\tau = f)|_{t=\tau},
\]

that is, \( F_\tau = f \) on the surface \( t = \tau \) in \( \Gamma \). The physical interpretation of the evolving constant \( F_\tau \) is that it represents “the value of \( f \) when the ‘clock’ variable \( t \) has the value \( \tau \)”. However, notice that in this framework one assumes that there is no meaning to measuring \( t \) by itself, or \( f \) by itself.

The quantum theory is defined by a kinematical Hilbert space \( K \), which quantizes \( \Gamma \), and a constraint operator \( \hat{H} \). The space of the (possibly generalized) states that satisfy \( \hat{H} \psi = 0 \) is the physical state space \( \mathcal{H} \) and inherits a scalar product from \( K \). Dirac observables, and in particular evolving constants, become operators on \( \mathcal{H} \). Transition amplitudes are given by the scalar product in \( \mathcal{H} \) between their eigenstates. Physical discreteness is determined by the spectra of the Dirac observables. In particular, the “measurement of the value of \( f \) when the ‘clock’ variable \( t \) has the value \( \tau \)” is predicted to have quantized outcomes if and only if the operator \( F_\tau \) has discrete spectrum.

II. A general-covariant system is defined by a set of kinematical quantities \( q_n \) called “partial observables”. These are physical quantities that can be measured, but are not necessarily predictable 8. (An example of a quantity considered “measurable but not predictable” is the usual time variable \( t \)). The space of the partial observables is called the extended configuration space \( C \). Dynamics is given by a (Hamiltonian) constraint \( H \) on \( \Gamma = T^* C \).

The quantum theory is defined by the kinematical Hilbert space \( K \) that quantizes \( \Gamma \) and by the (possibly generalized) “projection” operator

\[
P : K \to \mathcal{H}
\]

to the space \( \mathcal{H} \) of the solutions of \( \hat{H} \psi = 0 \). The probability amplitude for observing the values \( q_n \) if the values \( q_n' \) have been observed is given by

\[
\langle q_n | P | q_n' \rangle,
\]

where \( | q_n \rangle \) is the eigenvector of the partial-observable \( \hat{q}_n \) in \( K \). If the operator \( \hat{q}_n \) on \( K \) has discrete spectrum, then the measurement of the corresponding partial observable is predicted to have quantized outcomes.

Both frameworks reduce to the standard interpretation of quantum mechanics when applied to a conventional quantum system. In this case \( q_n = (q_0, q_n) = (t, q_n) \) where \( q_n \) are coordinates of the usual configuration space, and \( H = p_0 + H_0(p_0, q_n) \), where \( p_n \) are the momenta conjugate to \( q_n \) and \( H_0(p_0, q_n) \) is the conventional Hamiltonian. The quantum operators \( \hat{q}_n \) and \( \hat{p}_n \) are the usual Schrödinger operators, while the evolving-constant operator \( (\hat{Q}_n)_\tau \) determined by \( q_n \) and \( t \) is the Heisenberg operator

\[
(\hat{Q}_n)_\tau = e^{-iH_\tau} \hat{q}_n e^{iH_\tau}.
\]

Notice that the operators \((\hat{Q}_n)_\tau \) and \( \hat{q}_n \) have the same spectrum, because \( e^{-iH_\tau} \) is unitary. Hence \((\hat{Q}_n)_\tau \) has discrete spectrum if and only if \( \hat{q}_n \) does.

In the general case, the two interpretations are often compatible, but not always. The paper 2 interestingly points out a number of cases where the two appear to lead to different predictions concerning discreteness. Which one is then the correct generalization of quantum theory? I shall not enter this discussion here (a few comments are below, in Sec. III). I only mention that my own opinion has somehow shifted over the years from I to II, and I have given arguments for this in Ref. 2.

What is relevant for the present discussion is the following. The geometrical operators that have discrete spectra in LQG can be considered partial observables. If one follows interpretation II, the conclusion that a physical quantity has somehow shifted over the years from I to II, and I have given arguments for this in Ref. 2.
For instance, it is in principle possible that the partial observables \( f \) giving the area of a coordinate surface has discrete spectrum, while the corresponding complete observable \( F_x \) representing the area of a physically defined surface, has continuous spectrum. This is the possibility that Dittrich and Thiemann point out. It is a possibility. Is it a plausible one, for the geometrical quantities in LQG?

### III. THE DITTRICH-THEIMANN COUNTER-EXAMPLE

In [7], various examples are presented where the quantity \( f \) is represented by an operator \( \hat{f} \) in \( K \) which has discrete spectrum, while a corresponding evolving constant operator \( F_x \) does not. As stated in [7], most of these examples can be taken as irrelevant for the situation in LQG because they differ substantially from the LQG case. First, the configuration space of the models is non compact, while for the geometry part of LQG the configuration space is compact. Second, \( f \) and \( t \) do not commute, which is a different situation than the one expected in LQG. But in [7] there is also an example presented as a “baby version” of LQG, meant to show what could effectively go wrong in LQG in expecting a discrete spectrum of a kinematical operator to naturally imply a discrete spectrum of a corresponding complete observable.

The model is defined by the phase space \( T^*(S^1 \times R) \), namely the cotangent bundle of a cylinder. The angular variable of the cylinder can be called \( \alpha \), by analogy with the gravitational connection \( A \). Since it varies on the compact space \( S^1 \), its conjugate variable \( p_\alpha \) is given in the kinematical state space \( K \) by an operator with discrete spectrum. The longitudinal variable of the cylinder can be called \( x \), and its conjugate momentum \( p_x \). Hence: \( \alpha \in S^1, x \in R \). Dittrich and Thiemann choose a dynamics defined by a hamiltonian constraint that can be written in the form

\[
H = \cos(\alpha - x) - 1 = 0 \tag{5}
\]

and show that the spectrum of the evolving constant \( (P_\alpha)_t \), determined by \( p_\alpha \) and by the clock variable \( t := p_x \) is continuous. More precisely, we can avoid the ill-defined angular variable \( \alpha \) by defining \( h = e^{ixi} \)

\[
h = e^{ix} \tag{6}
\]

in \( S^1 \), and the constraint reads then

\[
H = h - e^{ix} = 0. \tag{7}
\]

(this should be multiplied by \( i \) to generate a real evolution.) As Dittrich and Thiemann nicely explain, \( T^*(S^1 \times R) \) constrains the cylinder down to a spiral in it (see Fig. 1), and the dynamical disappearance of the quantization of \( p_\alpha \) is due to the following fact. While its conjugate variable \( \alpha \) varies over a compact space \( S^1 \) in the unconstrained phase space, the variation of \( \alpha \) gives a flow along the non-compact spiral, when we follow it along the constraint surface.

More precisely. Recall that a characteristic indication that an operator will have discrete spectrum is that the hamiltonian flow generated by the corresponding classical quantity is compact. The flow generated by \( p_\alpha \) over \( T^*(S^1 \times R) \) is compact: it is an angular rotation of the cylinder. But there can be no non-trivial compact flow over the spiral.

Notice that this happens because the constraint \( \equiv \) is a periodic function of \( x \). In this way, the same value of \( h \) determines an infinite set of disconnected values of \( x \), and therefore \( \equiv \) determines a one-to-many relation from \( S^1 \) to the spiral. The compact flow along \( S^1 \) is then necessarily “opened-up” to a non-compact flow along the spiral. In particular, there is no embedding of \( S^1 \) into the constraint surface \( \equiv \). Any circle \( S^1 \) is necessarily “opened-up” by the constraint \( \equiv \). This is what destroys the quantum discreteness.

As far as quantum gravity is concerned the relevant question is then whether the same can happen in general relativity. Could there be a similar mechanism that forbids the compact action of a flow? The gravitational variables used in LQG are holonomies of the gravitational \textit{SU}(2) Ashtekar-Barbero connection \( A \). Consider one such holonomy, and call it \( \hat{h} \). It lives in the compact space \( SU(2) \). The gravitational constraints can be expressed in terms of the holonomies \[1\]. Can it be that varying \( \hat{h} \) in \( SU(2) \) along the constraint surface gives necessarily a non-compact flow, as in the “baby version” above? For this to happen, there should be no embedding of \( SU(2) \) in the constraint surface of general relativity, possibly coupled with matter.

But say we consider GR plus a set of scalar fields. The constraints are then given by the standard gravitational constraints plus the matter energy, or momentum. Thus, we can always solve the constraints by simply choosing
an initial configuration of matter with given energy and momentum. But the dependence of these on the matter degrees of freedom is linear or quadratic, and not a periodic, or more complicated function. Hence there is no obvious way of forcing the SU(2) flow obtained by varying $h$ to open-up to a non-compact one on the constraint surface, as in the counter-example. It is thus reasonable to expect discreteness to survive for some clock choice.

Of course, this does not prove that complete observables corresponding to physically determined areas have discrete spectrum in LQG. But it shows that the counter-example presented depends on a peculiar pathology that does not appear to be reproduced in general relativity.

IV. ARGUMENTS FOR DISCRETENESS

I review some general physical arguments in favor of the conclusion that LQG predicts physical discreteness, beginning with those in favor of interpretation II.

(i) Kinematical character of quantum discreteness. In conventional quantum theory, the discreteness of a physical variable is a kinematical property. It is independent from the dynamics.

For instance, the momentum $p_\varphi$ of a particle that moves on a circle, namely whose position is given by an angular variable $\varphi \in S_1$, is quantized \textit{whatever} is the Hamiltonian $H_0(\varphi, p_\varphi)$. I find it hard to believe that this kinematical character of quantum discreteness does not survive in a general-covariant theory.

The distinction between kinematics and dynamics becomes subtle in a general-relativistic theory, but it still exists. For instance, consider the general-relativistic dynamics of the solar system. Kinematics determines the relevant variables (distances of the planets, angles in the sky, various proper times...), while the expected relations that the equations of motion of the theory impose among these variables constitute the dynamics.

It is reasonable to expect that this distinction and kinematical character of the discreteness survives in the quantum theory. Interpretation II is consistent with usual quantum theory, and with this idea. It is therefore a sensible hypothesis for extending quantum theory to the general-relativistic regime.

To be more specific, consider the particle on a circle, in a system where angular momentum is not conserved:

$$\frac{dp_\varphi(t)}{dt} = -\frac{\partial H_0(\varphi, p_\varphi)}{\partial \varphi} \neq 0. \quad (8)$$

In the quantum theory, different operators are related to the measurement of the momentum of the particle: the Schrödinger operator

$$\hat{p}_\varphi = -i\hbar \frac{d}{d\varphi} \quad (9)$$

and the one-parameter family of Heisenberg operators

$$(\hat{P}_\varphi)_t = e^{i\hbar H_0 t} \hat{p}_\varphi e^{-i\hbar H_0 t}. \quad (10)$$

If we describe this system using a general-covariant formalism (see for instance [2]) then (9) can be identified with a kinematical operator in $\mathcal{K}$, while (10) can be identified with an evolving-constant operator in $\mathcal{H}$.\(^3\)

Now, a measurement of the momentum of this particle yields quantized outcomes. This physical fact is described by a mathematical aspect of the theory; which one? There are two possible answers: (a) the momentum is quantized because the operator $\hat{p}_\varphi$ has discrete spectrum, or (b) because the operator $(\hat{P}_\varphi)_t$ has discrete spectrum. These two answers are equivalent for this system, because the two operators have the same spectrum. But they are not equivalent for a generic general-covariant system, where a kinematical operator like $\hat{p}_\varphi$ and an evolving-constant like $(\hat{P}_\varphi)_t$ may have different spectra. If quantum discreteness is a kinematical effect that has nothing to do with the dynamics, it seems to me that the correct physical answer that we have to take over to general-relativistic physics is the first: momentum is quantized because the kinematical operator $\hat{p}_\varphi$ has discrete spectrum. This leads to interpretation II.

(ii) Gauge invariance. The common argument presented in favor of interpretation I is gauge invariance. The argument goes as follows. Only gauge-invariant quantities have physical meaning, and kinematical quantities in $\mathcal{K}$ are not gauge invariant. Therefore the spectrum of a partial observables, which is an operators on $\mathcal{K}$, cannot be physically meaningful.

As clarified in [12], the reason we must assume that only the gauge-invariant quantities can be measured and predicted is that the equations of motion do not determine the evolution of the non-gauge-invariant ones. The only quantities whose evolution is well determined are the Dirac observables. This fact is taken into account within interpretation II, where transition amplitudes describe gauge-invariant correlations, and all predictions are indeed gauge-invariant.

But in physics we utilize quantities that we measure but cannot predict: these are the independent variables with respect to which we express evolution. In non relativistic physics, the prototype of these quantities is the time variable $t$. In general-relativistic physics there is no preferred time variable and any physical quantity can play the role of independent variable. Therefore there is nothing wrong in referring to quantities that are not themselves Dirac observables. (This is done also within interpretation I: an example is the parameter $\tau$ that parametrizes an evolving constants). We just have to do so properly, respecting the overall gauge-invariance of the theory and its predictions.

There would be a contradiction if we had different

\(^3\) More precisely, here $\mathcal{K} = L_2[\mathbb{R}^2, d\varphi dt]$ and $\mathcal{H}$ is the generalized subspace of $\mathcal{K}$ given by the solutions of the Schrödinger equation. The evolving constant operator $(\hat{P}_\varphi)_t$ is well defined in $\mathcal{H}$ because it commutes with $H = i\hbar \frac{d}{d\varphi} - H_0$.\)
defined the surfaces. Mute. Their algebra does not depend on how we have... physical versions of these geometrical quantities. For instance, area elements of intersecting surfaces do not commute. Their algebra does not depend on how we have defined the surfaces.

(iii) Gauge fixing. In classical general relativity, the evolution of quantities that depend on the spacetime coordinates $x^\mu$ is under-determined by the equations of motion. This fact can be interpreted in two distinct but physically equivalent ways. (See for instance [11].) Accordingly, the spacetime coordinates $x^\mu$ can be given two different meanings, both consistent and viable.

According to the first, the coordinates $x^\mu$ are irrelevant mathematical labels that can be changed at will. The only quantities that have a physical interpretation are those that are independent on the choice of these coordinates. This is the interpretation which is is most commonly considered in quantum gravity.

According to the second, the coordinates $x^\mu$ describe physical position with respect to a physical reference system whose dynamics we do not care describing. Then the under-determinacy of the evolution simply reflects the fact that we are neglecting the evolution equations of the matter forming the physical reference system. In this case coordinate-dependent quantities represent gauge-invariant observables of a larger system where we have gauge-fixed the coordinates to some physical value. In other words, the gauge-invariant gravitational degrees of freedom on a physical reference system are described by the same variables as the gauge-dependent gravitational field variables.

In simpler words, we can gauge-fix the coordinates by choosing them to be determined by a chosen physical rods and clocks. Then non-diff-invariant observables in the pure gravity theory correspond precisely to diff-invariant observables in the matter+gravity theory. This is the analog of the fact that the Maxwell potential $A_\mu$ describes a physically observable quantity, if we work in a formalism in which we have entirely fixed the gauge.

The fact that this is possible in the classical theory suggests that the same could happen in the quantum theory. That is, it is reasonable to expect that the gauge-invariant geometrical operators have the same mathematical form as the gauge dependent ones in pure gravity. And therefore the same spectrum. This expectation is reinforced by the following consideration.

(iv) Commutator algebra. Discreteness depends on the commutation structure of the relevant geometrical quantities. Such structure does not change among different physical versions of these geometrical quantities. For instance, area elements of intersecting surfaces do not commute. Their algebra does not depend on how we have defined the surfaces.

Compare this with the angular momentum in non relativistic quantum theory: angular momentum is always quantized, with the same eigenvalues, irrespectively on whether it is the angular momentum of an atom, a proton, or a molecule, in spite of the fact that the various angular momentum operators are different in the different cases. The reason is of course that the angular momentum functions may be different, but their $SO(3)$ commutator algebra is the same. Similarly, the commutation structure of the components of, say, the area of any physical object must be dictated by the geometry of the gravitational field, not by specific features of the object whose area is considered.

(iv) Clock-dependence of the discreteness. Finally, let’s reconsider interpretation I. Notice that in this framework the discreteness of the spectrum may well depend on the choice of the clock. One should therefore distinguish between two possibilities: that discreteness of a quantity $f$ is lost with any clock, or that it is lost with some clock.

The first case appears unlikely in a realistic situation, in the light of the discussion of the example in Sec. III. If some compact flow exist at all on the physical phase space, then some complete observable will give it and will likely have discrete spectrum.

What is then the interpretation of the second case? Suppose a partial observable $f$ has discrete spectrum if measured at the time determined by a clock $t_1$, but continuous spectrum if measured at the time determined by a clock $t_2$. What is the physical interpretation of this situation? Isn’t this an indication that the procedure for determining $t_2$ disturbs quantum mechanically the determination of $f$?

V. CONCLUSION

The paper [8] is interesting because it points out cases where two interpretations of general-covariant quantum theory can yield different results. These cases should be better understood and the relative merits and open questions in the two interpretations deserve to be better investigated.

If interpretation I is physically correct, then we do not have a hard theorem to the effect that LQG predicts physical discreteness, but we have a number of compelling plausibility arguments. Counter examples appear to depend on pathologies that do not have an obvious correspondence in the theory.

If interpretation II is correct, then discreteness of LQG is immediate and compatible with gauge invariance. The interpretation II appears to me not only more manageable to extract physics from the theory [13], but also more plausible on physical ground.

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