Agreement in the presence of disagreeing rational players: The Huntsman Protocol

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Abstract

In this paper, a novel Byzantine consensus protocol among \( n \) players is proposed for the partially synchronous model. In particular, by assuming that standard cryptography is unbreakable, and that \( n > \max\left(\frac{3}{2}k + 3t, 2(k + t)\right) \), this protocol is an equilibrium where no coalition of \( k \) rational players can coordinate to increase their expected utility regardless of the arbitrary behavior of up to \( t \) Byzantine players.

We show that a baiting strategy is necessary and sufficient to solve this, so-called rational agreement problem. First, we show that it is impossible to solve this rational agreement problem without implementing a baiting strategy, a strategy that rewards rational players for betraying its coalition, by exposing undeniable proofs of fraud. Second, we propose the Huntsman protocol that solves the rational agreement problem by building recent advances in the context of accountable Byzantine agreement in partial synchrony. This protocol finds applications in distributed ledgers where players are incentivized to steal assets by leading other players to a disagreement on two distinct decisions where they “double spend”.

1 Introduction

Consider \( n \) players, each with some initial value. Solving the Byzantine consensus problem consists of designing a protocol guaranteeing that the \( n - t \) non-faulty of these players agree by outputting the same value and despite the presence of up to \( t \) arbitrarily behaving Byzantine players. Standard cryptography, which permits oblivious signed transfers and assumes computationally bounded players, has recently been used to build undeniable proofs of the identity of the players that lead the system to a disagreement [11]. Although this construction was not used in the game theoretical context, one can easily see the application to a baiting strategy that incentivizes rational players to solve
consensus: one can reward rational players for pretending to join a coalition in order to deceive the coalition by exposing undeniable proofs of fraud.

Placed in the game theory context, one can see a consensus protocol among rational players as a Nash equilibrium, however, a Nash equilibrium only prevents one rational player from increasing its utility by deviating solely, but it fails at preventing multiple rational players from increasing their utility by colluding and by all deviating together. The resilience to this type of coalition mentioned originally in the 50s [7] is needed to solve consensus despite a coalition of $k$ players. Ben-Porath [9] shows that one can simulate a Nash equilibrium with a central trusted mediator provided that there is a punishment strategy to threaten rational players in case they deviate and Heller [24] strengthens Ben-Porath’s result to allow coalitions. Abraham et al. [2] applied these results to the fully distributed setting, by showing that one can simulate a mediator with cheap talks by assuming the standard cryptography assumptions as mentioned above. A $(k, t)$-punishment strategy guarantees that if up to $k$ rational players deviate, then more than $t$ non-deviating players, by playing the punishment strategy, can lower the utility of these rational players.

Another challenge when making consensus Byzantine fault tolerant is for the equilibrium to be immune to $t$ Byzantine players that act arbitrarily or whose utility functions are unknown. Abraham et al. [2] were the first to formalize $k$-resilience, $t$-immunity and $\epsilon$-$(k, t)$-robustness. A protocol is a $k$-resilient equilibrium if no rational coalition of size $k$ can increase their utility by deviating in a coordinated way. A protocol is $t$-immune if the expected utility of the $n - t$ non-faulty players is not decreased regardless of the arbitrary behavior of up to $t$ Byzantine players. A protocol is $\epsilon$-$(k, t)$-robust if no coalition of $k$ rational players can coordinate to increase their expected utility by $\epsilon$ regardless of the arbitrary behavior of up to $t$ Byzantine players, even if the Byzantine players join their coalition.

1.1 Our result

In this paper, we show that a baiting strategy, that one can implement with these undeniable proofs-of-fraud, is necessary and sufficient to devise a consensus protocol that is robust to a coalition of up to $k$ rational players and $t$ Byzantine players, a problem we call the rational agreement problem. This result holds under the assumption of partial synchrony, where there is an unknown bound on the delay of messages [14].

First, we introduce the notion of baiting strategy as a particular case of punishment strategy and show that it is impossible to solve the rational agreement problem without a baiting strategy: A $(k, f, m)$-baiting strategy is a $(k - m, f)$-punishment strategy such that $0 < m \leq k$, and the $m$ rational players that do not collude would lower their utility by not playing the baiting strategy and deviating with the rest of the players of the coalition.

Second, we present a solution to the rational agreement problem that relies on a baiting strategy. To this end, we devise the Huntsman protocol for the partially synchronous model that extends an existing consensus protocol that is
t-accountable for $t < n/3$ to become $\epsilon$-$(k, t)$-robust when $n > \max(\frac{3k+3t}{2}, 2(k+t))$. The key idea is to reward a single player if it exposes the coalition to which it belongs. If the reward is larger than the individual return that a rational gains from causing a disagreement, then rational players in a coalition can find that the strategy to form a coalition and cause a disagreement is strictly dominated by the strategy to betray the coalition in the sequential game.

The solution thus finds relevance in the context of a distributed ledger where a disagreement can allow a player to steal digital assets but where requiring players to deposit some of their assets can be used to threaten them. In particular, our protocol pre-decides decisions from a $t$-accountable consensus protocol that it extends with the Byzantine Fault Tolerant Commit-Reveal protocol (BFTCR) that consists of two reliable broadcasts and one additional broadcast. BFTCR ensures the existence of a baiting strategy (baiting-dominance) and that the protocol still solves agreement even after playing the baiting strategy (baiting-agreement). We also add an additional property outside the problem definition, lossfree-reward, which states that the increase in utility for baiting rational players comes at no cost to non-deviating players. For this purpose, we introduce a deposit per player and reward to baiters, so that the system can always pay the reward by taking the deposits of the proven coalition at no cost for non-colluding players.

1.2 Related work

Considering fault tolerant distributed protocols as games requires to cope with a mixture of up to $k$ rational players and $t$ faulty players. The idea of mixing rational players with faulty players has already been extensively explored in the context of secret sharing and multi-party computation [27, 17, 12, 2]. In particular, the central third-party mediator that is typically relied upon was implemented with synchronous cheap talks [2], that are communications of negligible cost through private pairwise channels. This extension was indeed illustrated with an $\epsilon$-$(k, t)$-robust secret sharing protocol where $n > k + 2t$. It was later shown [1] that mediators could be implemented with asynchronous cheap talks in $\epsilon$-$(k, t)$-robust protocol when $n > 3(k + t)$. This adaptation makes it impossible to devise even a 1-immune protocol that would solve the consensus problem [20] as the communication model becomes asynchronous [16]. In this paper, we focus instead on the partially synchronous model, where the bound on the delay of messages is unknown [14], to design a protocol that solves consensus among $n$ players, where up to $t$ are Byzantine players and $k$ are rational players.

Consensus has been explored in the context of game theory. Some work focused on the conditions under which termination and validity is obtained for a non-negligible cost of communication and/or local computation [20, 6], without considering the incentives for rational players to cause a disagreement. This incentive is quite apparent in the blockchain context, where Bitcoin users hacked the network to double spend by simply leading sets of players to a disagreement.
(or fork) for long enough\footnote{https://www.cnet.com/news/hacker-swipes-83000-from-bitcoin-mining-pools/}. Some results consider the problem of consensus in the presence of rational players but do not consider failures\cite{19}. Leader election\cite{2}, which can be used to solve consensus indirectly, and consensus proposals\cite{21} focus on ensuring fairness defined as all players having an equal probability of their proposal being decided. Some proposals study consensus without focusing on fairness and mix faulty players with rational players\cite{4,8}, however, they consider the synchronous communication model.

Several research results focus more particularly on ensuring agreement, with some deriving from the BAR (Byzantine-Altruistic-Rational) model. However, these works considered either no Byzantine players\cite{20,15}, no coalitions of rational players\cite{5}, synchronous communication\cite{20,22,31,23} or solution preference\cite{22}. By assuming a larger payoff for agreeing than for disagreeing, solution preference requires that rational players never have an incentive to sabotage the consensus properties. To the best of our knowledge, we present the first work that considers bounds for the robustness of agreement against coalitions of Byzantine and rational players in partial synchrony.

The baiting strategy that we introduced to reward traitors of a coalition is very similar to the betrayal used in the context of verifiable cloud-computing for counter-collusion contracts, assuming that the third party hosting the contracts is trusted\cite{13}. Our BFTCR protocol presents a novel implementation to select the winner of the baiting reward. There are a number of advantages of BFTCR compared to other state-of-the-art protocols. One may think that a solution similar to submarine commitments\cite{10} would work as well by, for example, hiding the proofs of fraud in a decision. However, such a solution does not prevent Byzantine players from always hiding in a submarine commitment their proofs of fraud, and revealing them only if a rational reveals their submarine commitment, which can act as a deterrent for rational players to not betray the coalition. Additionally, other protocols based on zero-knowledge proofs\cite{25} explicitly reveal the existence of an information to prove, which gives an additional advantage to other players in the coalition to also claim the same knowledge. To the best of our knowledge, BFTCR is the first protocol that tolerates coalitions of up to $k$ rationals and $t$ Byzantine players as long as $n > \max(\frac{3}{2}k+3t, 2(k+t))$.

1.3 Roadmap

The rest of the paper is structured as follows: Section\cite{2} presents our computational model and some preliminary definitions, Section\cite{3} introduces the definition of a baiting strategy and shows that it is impossible to solve the rational agreement problem without a baiting strategy, in Section\cite{4} we show bounds for a deposit and a reward to implement an effective baiting strategy, Section\cite{5} presents the Huntsman protocol, the first protocol that solves the rational agreement problem without synchrony and without solution preference.
2 Preliminaries

We consider a partially synchronous communication network, in which messages can be delayed by up to a bound that is unknown. For this purpose, we adapt the synchronous and asynchronous models of Abraham et al. \cite{2,1} to partial synchrony. We consider a game played by a set $N$ of players, where $|N| = n$, of type in $T = \{\text{Byzantine, rational, correct}\}$. In order to model partial synchrony, we introduce the scheduler as an additional player that will model the delay on messages derived from partial synchrony. The game is in extensive form, described by a game tree whose leaves are labeled by the utilities $u_i$ of each player $i$. We assume that players alternate making moves with the scheduler: first the scheduler moves, then a player moves, then the scheduler moves and so on. The scheduler’s move consists of choosing a player $i$ to move next and a set of messages in transit to $i$ that will be delivered just before $i$ moves (so that $i$’s move can depend on all the messages $i$ receives). Every non-leaf node is associated with either a player or the scheduler. The scheduler is bound to two constraints. First, the scheduler can choose to delay any message up to a bound, known only to the scheduler, before which he must have chosen all receivers of the message to move and provided them with this message, so that they deliver it before making a move. Second, the scheduler must eventually choose all players that are still playing. That is, if player $i$ is playing at time $e$, then the scheduler chooses him to play at time $e' \geq e$.

Each player $i$ has some local state at each node, which translates into the initial information known by $i$, the messages $i$ sent and received at the time that $i$ moves, and the moves that $i$ has made. An information set is the set of nodes in the game tree that contain the same local state for the same player, in that such player cannot distinguish them. The nodes where a player $i$ moves are further partitioned into information sets. We assume that the scheduler has complete information, so that the scheduler’s information sets just consist of the singletons.

Since faulty or rational players can decide not to move during their turn, we assume that players that decide not to play will at least play the default-move, which consists of notifying to the scheduler that this player will not move, so that the game continues with the scheduler choosing the next player to move. Thus, in every node where the scheduler is to move, the scheduler has as many moves as there are combinations of a player and of messages (including the empty set) that such player can deliver. Then, the selected player moves, after which the scheduler selects again the next player for the next node, and the messages it receives, and so on. The scheduler alternates thus with one player at each node down a path in the game tree up to a leaf. A run of the game is then a path in the tree from the root to a leaf.

Strategies. We denote the set of actions of a player $i$ (or the scheduler) as $A_i$, and a strategy $\sigma_i$ for that set of actions is denoted as a function from $i$’s information sets to a distribution over the actions.

We denote the set of all possible strategies of player $i$ as $\mathcal{S}_i$. Let $\mathcal{S}_I = \prod_{i \in I} \mathcal{S}_i$ and $A_I = \prod_{i \in I} A_i$ for a subset $I \subseteq N$. Let $\mathcal{S} = \mathcal{S}_N$ with $A_{-I} = \prod_{i \notin I} A_i$ and
$S_{-i} = \Pi_{i \notin I} S_i$. A joint strategy $\sigma = (\sigma_0, \sigma_1, ..., \sigma_{n-1})$ draws thus a distribution over paths in the game tree (given the scheduler’s strategy $\sigma_s$), being $u_i(\sigma, \sigma_s)$ player’s $i$ expected utility if $\sigma$ is played along with a strategy for the scheduler $\sigma_s$. A strategy $\theta_i$ strictly dominates $\tau_i$ for $i$ if for all $\phi_{-i} \in S_{-i}$ and all strategies $\sigma_s$ of the scheduler we have $u_i(\theta_i, \phi_{-i}, \sigma_s) > u_i(\tau_i, \phi_{-i}, \sigma_s)$.

Given some desired functionality $F$, a protocol is the recommended joint strategy $\sigma$ whose outcome satisfies $F$ for all strategies $\sigma_s$ of the scheduler, and an associated game $\Gamma$ for that protocol is defined as all possible deviations from the protocol [2]. In this case, we say that the protocol $\sigma$ implements the desired functionality. Note that both the scheduler and the players can use probabilistic strategies.

**Failure model.** $k$ players out of $n$ can be rational and up to $t$ of them can be Byzantine, while the rest are correct. Correct players follow the protocol: the expected utility of correct player $i$ is equal and positive for any run in which the outcome satisfies consensus, and 0 for any other run. Rational players can deviate to follow the strategy that yields them the highest expected utility at any time they are to move, while Byzantine players can deviate in any way, even not replying at all (apart from notifying the scheduler that they will not move). Rational players have greater utility for outcomes in which they caused a disagreement than from outcomes that satisfy consensus, but have no interest in deviating from consensus for anything else, in that they prefer to terminate and to guarantee validity. We will detail further the utilities of rational players after defining the consensus problem in this section.

We assume that if a coalition manages to cause a disagreement, then it obtains a payoff of at most $G$, which we call the total gain. This total gain may be, for example, the entire market value of the system. We assume that a coalition with $k$ rational players and $t$ Byzantine players split equally the total gain into $k$ parts, which we call the gain $g = G/k$, that is, Byzantine players are willing to give all the total gain from causing a disagreement to the rational players that collude (to incentivize the deviation for these rational players). Notice a disagreement of consensus can mean two or more disjoint groups of non-deviating players deciding two or more separate, conflicting decisions [29]. We speak of the disagreeing strategy as the strategy in which players collude to produce a disagreement, and of a coalition disagreeing to refer to a coalition that plays the disagreeing strategy. For ease of exposition, we consider in this work only disagreements into two values. Notice however that if the size of the coalition is less than half the total number of players $k + t < n/2$ (as is the case for the work that we present) then the coalition can only cause a disagreement into two values [29], whereas greater sizes of a coalition can cause disagreements into multiple values [28].

We let rational players in a coalition and Byzantine players (in or outside the coalition) know the types of all players, so that they know which players are the other Byzantine players, rational players and correct players, while the rest of the players only know the upper bounds on the number of rational and Byzantine players, i.e., $k$ and $t$ respectively, and their own individual type (that is, whether they are rational, Byzantine or correct).
Cheap talks. As we are in a fully distributed system, without a trusted central entity like a mediator, we assume **cheap-talks**, that is, private pairwise communication channels. We also assume negligible communication cost through these channels. Non-Byzantine players are also only interested in reaching consensus, and not in the number of messages exchanged. Similarly, we assume the cost of performing local computations (such as validating proposals, or verifying signatures) to be negligible.

Cryptography. We require the use of cryptography, for which we reuse the assumptions of Goldreich et al. [18]: polynomially bounded players and the enhanced trapdoor permutations. In practice, these two assumptions mean that players can sign unforgeable messages, and that they can perform oblivious transfer.

Robustness. Given that a Nash equilibrium only protects against single-player deviations, and our distributed system may be susceptible of a coalition of \( k \) rational and \( t \) Byzantine players, it is important to consider tolerating multiplayer deviations. We thus restate Abraham’s et al. [2] definitions of \( t \)-immunity, \( \epsilon \)-\((k, t)\)-robustness and the most recent definition of \( k \)-resilient equilibrium [1].

The notion of \( k \)-resilience is motivated in distributed computing by the need to tolerate a coalition of \( k \) rational players that can all coordinate actions. A joint strategy is \( k \)-resilient if no coalition of size \( k \) can gain greater utility by deviating in a coordinated way.

**Definition 2.1** \((k\text{-resilient equilibrium})\). A joint strategy \( \sigma \in S \) is a \( k \)-resilient equilibrium (resp. strongly \( k \)-resilient equilibrium) if, for all \( K \subseteq N \) with \( |K| \leq k \), all \( \sigma_K \in S_K \), all strategies \( \sigma_s \) of the scheduler, and for some (resp. all) \( i \in K \) we have \( u_i(\sigma_K, \sigma_{-K}, \sigma_s) \geq u_i(\sigma_K, \sigma_{-K}, \sigma_s) \).

The notion of \( t \)-immunity is motivated in distributed algorithms by the need to tolerate \( t \) Byzantine players. An equilibrium is \( t \)-immune if the utilities of the non-Byzantine players is not decreased by the deviations of up to \( t \) other players.

**Definition 2.2** \((t\text{-immunity})\). A joint strategy \( \sigma \in S \) is \( t \)-immune if, for all \( T \subseteq N \) with \( |T| \leq t \), all \( \sigma \in S_T \), all \( i \notin T \) and all strategies of the scheduler \( \sigma_s \), we have \( u_i(\sigma_{-T}, \sigma_T, \sigma_s) \geq u_i(\sigma, \sigma_s) \).

A joint strategy is an \( \epsilon \)-(\( k \),\( t \))-robust equilibrium if no coalition of \( k \) rational players can coordinate to increase their expected utility by \( \epsilon \) regardless of the arbitrary behavior of up to \( t \) Byzantine players, even if the Byzantine players join their coalition. We illustrate it however with \( \epsilon \) because of the use of cryptography, that is, in order to account for the (small) probability of the coalition breaking cryptography, as previously done by state of the art [2]:

**Definition 2.3** \((\epsilon \text{-}(k, t)\text{-robust equilibrium})\). A joint strategy \( \sigma \in S \) is an \( \epsilon \)-(\( k \),\( t \))-robust (resp. strongly \( \epsilon \)-(\( k \),\( t \))-robust) equilibrium if for all \( K, T \subseteq N \) such that \( K \cap T = \emptyset \), \( |K| \leq k \), and \( |T| \leq t \), for all \( \sigma_T \in S_T \), for all \( \phi_K \in S_K \), for some (resp. all) \( i \in K \), and all strategies of the scheduler \( \sigma_s \), we have
Notice we use the most recent definition of $k$-resilient equilibrium [1], which in turn slightly varies the definition of an $\epsilon$-$(k, t)$-robust equilibrium. We define here strong resilience and strong robustness to refer to the stronger versions of these properties [2]. The notion of $t$-immunity in game theory as per defined here-above is equivalent to the notion of Byzantine fault tolerance in distributed computing.

Given some game $\Gamma$ and desired functionality $F$, we say that a protocol $\sigma$ is a $k$-resilient protocol for $F$ if $\sigma$ implements $F$ and is a $k$-resilient equilibrium. For example, if $\sigma$ is a $k$-resilient protocol for the consensus problem, then in all runs of $\sigma$, every non-deviating player terminates and agrees on the same valid value. We extend this notation to $t$-immunity and $\epsilon$-$(k, t)$-robustness. The required functionality of this paper is thus reaching agreement.

**Punishment strategy.** We also restate the definition of a punishment strategy [2] as a threat that correct and rational players can play in order to prevent other rational players from deviating. For example, in society, the judicial system is an effective punishment strategy against committing a crime. Not terminating a protocol if just one player deviates can also be a punishment strategy against deviating from the protocol.

In the following we need to consider a set of players that will act against the coalition by bidding. The punishment strategy guarantees that if $k$ rational players deviate, then $t + 1$ players can lower the utility of these rational players by playing the punishment strategy.

**Definition 2.4** ($(k, t)$-punishment strategy). A joint strategy $\rho$ is a $(k, t)$-punishment strategy with respect to $\sigma$ if for all $K, T, P \subseteq N$ such that $K, T, P$ are disjoint, $|K| \leq k$, $|T| \leq t$, $|P| > t$, for all $\tau \in S_T$, for all $\phi \in S_K$, for all $i \in K$, and all strategies of the scheduler $\sigma_s$, we have:

$$u_i(\sigma_{\neq T}, \tau_T, \sigma_s) > u_i(\sigma_{N - (K \cup T \cup P)}, \phi_K, \tau_T, \rho_P, \sigma_s).$$

Intuitively, a punishment strategy represents a threat to prevent rational players from deviating, in that if they deviate, then players in $P$ can play the punishment strategy $\rho$ and the deviating rational players decrease their utility with respect to following $\sigma$.

**Accountability.** Previous work introduced signatures in consensus protocol messages, guaranteeing that for a disagreement to occur, at least $t + 1$ players must sign conflicting messages, and once these messages are discovered by a correct player, such player can prove the fraudsters to the rest of correct players through *Proofs-of-Fraud (PoFs)* [11, 28].

We also adapt to this model the property of accountability, recently defined for consensus [11]:

**Definition 2.5** ($t$-accountability). Let $\sigma$ be a protocol that implements agreement. Suppose that a disagreement takes place, then $\sigma$ is $t$-accountable if all
correct players will eventually gather enough proof that at least \( t + 1 \) players deviated to cause the disagreement.

In order to distinguish the value \( t \) from \( t \)-immunity and from \((k, t)\)-robustness, we set \( t = \left\lceil \frac{n}{4} \right\rceil - 1 \) in the rest of this paper. We will make use of \( f \) instead to refer to more restrictive bounds for Byzantine players, i.e., \( f \leq t \).

**Rational agreement.** In the remainder we are interested in proposing a consensus protocol that is immune to up to \( t \) Byzantine failures and robust to a coalition of up to \( k \) rational and \( f \) Byzantine players, so we restate the Byzantine consensus problem \([20]\) in the presence of rational players: The \textit{Byzantine consensus problem} is, given \( n \) players, each with an initial value, to ensure (i) \textit{agreement} in that no two non-deviating players decide different values; (ii) \textit{validity} in that the decided value has to be proposed; and (iii) \textit{termination} in that eventually every non-deviating player decides.

**Definition 2.6** (rational agreement). Consider a system with \( n \) players, a protocol \( \sigma \) solves the rational agreement problem if it implements consensus, and is \( t \)-immune and \(-\varepsilon\)-(\( k, f \))-robust for some \( k, f > 0 \) such that \( n \leq 3(k + f) \).

### 3 Impossibility Result

In this section, we introduce a baiting strategy as a particular case of punishment strategy and show that it is necessary to devise a consensus protocol resilient to a coalition of \( k \) rational players and immune to \( t \) Byzantine players.

Our solution to agreement in the presence of rational and Byzantine players, presented in Section 4, consists of rewarding rational players for betraying the coalition. One may wonder whether rewarding rational players in a coalition is the only way to obtain \(-\varepsilon\)-(\( k, f \))-robustness that tolerates coalitions of size \( n \leq 3(k + f)n \) in partial synchrony. To demonstrate the need for a reward, we first formalize a type of \(-\varepsilon\)-(\( k, f \))-punishment strategy, which we call a \((k, f, m)\)-baiting strategy. A \((k, f, m)\)-baiting strategy is a \((k-m, f)\)-punishment strategy such that \( k \geq m > 0 \), and these \( m \) rational players prefer to actually play the baiting strategy than to deviate with the rest of the players in the coalition. That is, \( m \) players of the coalition need to play the baiting strategy for it to succeed, and at least \( m \) rational players in the coalition prefer to play the baiting strategy than to deviate with the coalition.

**Definition 3.1** \((k, f, m)\)-baiting strategy. A joint strategy \( \eta \) is a \((k, f, m)\)-baiting strategy with respect to a strategy \( \sigma \) if \( \eta \) is a \((k - m, f)\)-punishment strategy with respect to \( \sigma \), with \( 0 < m \leq k \) and for all \( K, T, P \subseteq N \) such that \( K \cap T = \emptyset, |P \cap K| \geq m, P \cap T = \emptyset, |K| \leq k - m, |T| \leq f, |P| > t \), for all \( \sigma \in S_T \), all \( \phi_K \in S_K \setminus \{\sigma_K\} \), all \( \phi_P \in S_P \), all \( i \in P \), and all strategies of the scheduler \( \sigma_s \), we have:

\[
\forall \sigma, \phi_K, \phi_P, \tau_T, \tau_P, \sigma_s \in S_T, \forall \sigma \in S_T, \forall \phi_K \in S_K \setminus \{\sigma_K\}, \forall \phi_P \in S_P, \forall i \in P, \forall \sigma_s \in S_T,
\]

\[
u_i(\sigma, \phi_K, \phi_P, \tau_T, \tau_P, \sigma_s) \geq u_i(\sigma, \phi_K, \phi_P, \tau_T, \tau_P, \sigma_s).
\]
Additionally, we call this strategy a strong \((k, f, m)\)-baiting strategy in the particular case where for all rational coalitions \(K \subseteq N\) such that \(|K| \leq k + f\), \(|K \cap P| \geq m\) and all \(\phi_{K \setminus P} \in \mathcal{S}_{K \setminus P}\) we have:

\[
\sum_{i \in K} u_i(\mathcal{G}_{N-\{K \cup P\}}, \phi_{K \setminus P}, \eta_P, \sigma_s) \leq \sum_{i \in K} u_i(\mathcal{G}, \sigma_s).
\] (1)

A baiting strategy illustrates a situation where at least \(m\) rational players in the coalition may be interested in baiting other \(k + f - m\) rational and Byzantine players into a trap: the \(k + f\) of them collude to deviate initially, just so that these \(m\) players can prove such deviation by playing the baiting strategy, and get a reward for exposing this deviation. Such a strategy has a significant impact in a protocol to implement the agreement functionality. A strong baiting strategy defines a baiting strategy in which the fact that \(m\) deviating players play the baiting strategy does not yield higher payoff to the entire coalition as a whole (if such coalition was made only by rationals), compared to following the protocol. This prevents a coalition of rationals from colluding together so as to play the baiting strategy on themselves only with the purpose of splitting the baiting reward among the colluding members. Notwithstanding, neither a baiting strategy nor a strong baiting strategy show that if these \(m\) players play the baiting strategy, then the protocol implements the desired functionality.

The reason why a \((k, f, m)\)-baiting strategy is relevant to the consensus problem is that without such a strategy it is not possible to obtain a consensus protocol that is \((k, t)\)-robust where \(k > 0\). We show this result in Theorem 3.1. The proof is similar to that of the impossibility of \(t\)-immune consensus under partial synchrony for \(t \geq n/3\) [14], since a partition of rational and Byzantine players can exploit two disjoint partitions of correct players to lead them to different decisions. Let us recall that we do not assume solution preference, and thus the payoffs from a disagreement can be significantly greater than those of agreeing for rational players. We defer the proof to Appendix A.

**Theorem 3.1.** It is impossible to obtain a protocol \(\mathcal{G}\) that implements rational agreement unless there is a \((k, f, m)\)-baiting strategy with respect to \(\mathcal{G}\), with \(m > \frac{k + f - n}{2} + t\).

Theorem 3.1 shows the need for a baiting strategy to solve rational agreement for \(n \leq 3(k + f)\). In Section 4 we illustrate the values of a reward and deposit per player to make a strong baiting strategy. In Section 5 we solve rational agreement with cheap-talk to provide bounds for robustness in a fully distributed protocol.

### 4 A strong baiting strategy for the rational agreement problem

In this section, we focus on the key idea of this paper: what are the values required for a deposit per player and a reward to players for baiting the coalition
that make a baiting strategy. Thus, we show how to implement a strong baiting strategy by introducing a deposit and a reward. Note that we already proved in Section 3 that we need a baiting strategy for a protocol to solve the rational agreement problem. We focus here on the properties that we aim at for such a baiting strategy. Given a protocol \( \overrightarrow{\pi} \) that implements consensus and is \( t \)-immune, we will extend it to implement the rational agreement problem, in that we will prove all the three following properties:

- Baiting-dominance: There is a \((k, f, m)\)-baiting strategy \( \overrightarrow{\eta} \) with respect to \( \overrightarrow{\pi} \), for \( m > \frac{k + f - n}{2} + t \).
- Baiting-agreement: \( \overrightarrow{\eta} \) implements agreement.
- Lossfree-reward: \( \overrightarrow{\eta} \) is a strong baiting strategy and no non-deviating player gets its utility decreased by the outcome of \( m \) deviating players playing \( \overrightarrow{\eta} \), compared to the outcome of following \( \overrightarrow{\pi} \).

Baiting-dominance states the necessary condition that a baiting strategy exists, while baiting-agreement guarantees that playing such a baiting strategy still leads to agreement. Lossfree-reward guarantees that such a baiting strategy is a strong baiting strategy.

We introduce a deposit per player, and also require such deposit to be big enough so that the deposit taken from the exposed coalition is enough to pay the reward, satisfying lossfree-reward. We thus analyze in Theorem 4.1 the required values for such deposit and reward necessary to incentivize at least \( m \) rational players in a coalition to follow a baiting strategy, depending on the size of the coalition and on the maximum total gain from disagreeing \( G \). In Section 5 we show the first protocol that solves the rational agreement problem, the required reward and deposit depending on the maximum total gain of a disagreement \( G \) derived from the analysis that we do in this section.

In partial synchrony, it is trivial to obtain a \( k \)-resilient protocol that implements agreement for \( k < n \) and without Byzantine players, i.e., \( f = 0 \): players do not decide until they hear from all other players with the same decision. Similarly, it has been showed that it is possible to obtain \( t \)-immune protocols that implement agreement [14]. Before deepening into better results, we introduce the deposit per player \( L \) and a baiting reward \( R \). For this purpose, we assume a \( t \)-accountable consensus protocol, such as Polygraph [11].

**Dominating disagreements.** Since the protocol is \( t \)-accountable, we add a baiting reward \( R \) for player \( i \) if \( i \) can prove to the rest of the players that a coalition of at least \( t + 1 \) players are trying to cause a disagreement. If multiple players are eligible for the baiting reward, then only one is chosen at random to win the reward, and the rest are treated as fraudsters that did not bait. Players can prove that a coalition is trying to cause a disagreement through proofs-of-fraud (PoFs) which undeniably show two conflicting messages signed by the same set of players. The reward is only given to \( i \) if \( i \) exposes this coalition before the coalition causes the disagreement (i.e., before both partitions of correct players decide different decisions). In order to reduce the expected utility from playing
the baiting strategy, we require all players to place a minimum deposit $L$. If these PoFs expose at least $t+1$ players including the baiter (i.e., the player that follows the baiting strategy), then $t$ non-baiting players lose the deposit amount $L$, while the baiter is rewarded with a baiting reward $R$. Our goal is to set $R$ and $L$ so that we implement a baiting strategy for a set $M$ of rational players in the coalition, such that if others in the coalition bait, then for all $i \in M$, player $i$ is better off also trying to bait and getting the reward, while if the rest of the players in the coalition do not bait, then if $i$ baits then $i$ obtains a larger utility than if $i$ was not baiting too.

We explore here the possible runs, assuming that we already have such a baiting strategy, and what each of these runs means for the payoffs of a rational player $i$:

1. Rational players including $i$ contribute to reaching agreement and follow the protocol $\bar{\sigma}$, getting some utility $u_i(\bar{\sigma}_{-T}, \bar{\tau}_T) \geq \epsilon$ where $\epsilon > 0$.

2. Some rational players collude with $i$ and deviate to disagree, playing strategy $\bar{\phi}$ with some Byzantine players $T$ and other rational players $K$ such that $|K \cup T| \geq n/3$, $K \cap T = \emptyset$, obtaining utility $u_i(\bar{\sigma}_{N-K-T}, \bar{\phi}_{K \cup T}) = g$.

3. Player $i$ deviates to bait other rational players into colluding with some Byzantine players such that $|K \cup T| \geq n/3$, $K \cap T = \emptyset$, and this deviation consists of playing strategy $\bar{\eta}$ to expose the colluding players via PoFs and obtain the baiting reward. As a result, player $i$ obtains utility $u_i(\bar{\sigma}_{N-K-T}, \bar{\phi}_{K \cup T-M}, \bar{\eta}_M) = \frac{R}{m} - \frac{m-1}{m}L$, where $M$ is the set of players of the coalition that bait, i.e. $i \in M$, with $|M| = m$. $1/m$ represents the probability of winning the bait, while $(m - 1)/m$ the probability of not winning it.

4. Player $i$ deviates to disagree only to suffer a trap baited by another rational (or group of rational players), obtaining utility $u_i(\bar{\sigma}_{N-K-T}, \bar{\phi}_{K \cup T-M}, \bar{\eta}_M) \leq -L$.

5. In any run where the protocol does not terminate, player $i$ obtains utility $u_i(\bar{\sigma}_{N-T-{i}}, \tau_T, \theta_i) \leq 0$.

6. Player $i$ contributes to reaching agreement but a coalition causes a disagreement so $i$ obtains utility $-\infty$.

Notice that the runs 4, 5 and 6 are strictly dominated by following the protocol. Our goal is to make runs represented by 3 runs that also implement agreement and that strictly dominate runs represented by 2.

**Playing the reward.** We will obtain $\epsilon$-$(k, f)$-robustness by setting the reward $R$ and deposit $L$ such that at most $k + f - m \leq t$ players in a coalition will play the disagreeing strategy (i.e. not enough to actually cause a disagreement). For this, one might think that implementing a baiting strategy with the reward and deposit might not be enough: we need to discourage coalitions from actually playing the baiting strategy, since the system would have to pay the reward $R$, for
and thus the coalition can effectively steal some funds from the system. However, if the system can use the deposited amount $L$ from at least $t$ certified fraudsters in the coalition to pay for the baiting reward $R$, then the system does not lose any funds, while obtaining agreement. That is, as long as $R < \sum_{i=1}^{t} L$ then the coalition as a whole loses more funds than it gains from playing the baiting strategy, while the system obtains agreement, i.e., loss-free-reward. Instead, another approach could be to actually discourage playing the reward by setting $pR - (1 - p)L < 0$, but it is clear that this requires higher values of the deposit per player to discourage this, and so the former inequation will allow a lower deposit $L$.

Nevertheless, notice that if the coalition consists entirely of rational players then they do not actually play this strategy: they all individually lose more than they can gain from deviating. Notwithstanding, agreement will still be guaranteed at no cost to non-deviating players, although it is unclear whether Byzantine players with unexpected utilities and the goal to break the system would be interested in giving their funds to rational players, if it does not cause some damage on non-deviating players.

**Making enough rational players follow the baiting strategy.** Before we describe our implementation for a baiting strategy, we show in Theorem 4.1 which values of $L$ and $R$ make the disagreeing strategy a strictly dominated strategy by the baiting strategy for at least $m$ rational players (i.e., a dominated strategy even if player $i$ already knows that $m - 1$ other players are baiting too).

**Theorem 4.1.** Let $\bar{\sigma}$ be a protocol that implements rational agreement if at least $m$ rational players in a coalition play the baiting strategy $\bar{n}$, and suppose players cannot play $\bar{n}$ after seeing $m$ other players playing $\bar{n}$ (because of delivering messages that expose the coalition from other players). Then such protocol is $\epsilon$-$(k - f, f)$-robust for some $f$, with $f \leq t$, if the following three properties are satisfied:

1. Each player is required to deposit $L = d \cdot G$, being $d > \frac{m}{k(1-m+1)}$,
2. If multiple players bait, then the system chooses one at random to reward him with $R$ such that $dGt > R > \frac{G}{m}$ and
3. The protocol slashes the deposits of the provably fraudulent players that do not win the reward.

**Proof.** Recall that the gain is split equally among all $k$ rational players in the coalition $g = G/k$. To guarantee lossfree-reward, the sum of losses from the coalition must always be more than the reward given for the coalition to always lose funds while failing to disagree, that is $\sum_{i} L > R \iff L > \frac{R}{t}$.

As such, the baiting strategy must strictly dominate the strategy to disagree for rational players, even after a rational player sees another $m - 1$ other rational players also playing the same baiting strategy. Since the probability of winning the bait among $m$ players is uniform $p(m) = \frac{1}{m}$ we have that the utility for a player to play the baiting strategy knowing that another $m - 1$ players are playing the same strategy is $p(m)R - q(m)L$ where $q(m) = 1 - p(m)$. If instead the
player disagrees then the player’s utility is $G_k$. As such, we obtain that the baiting strategy strictly dominates the disagreeing strategy if $p(m)\mathcal{R} - q(m)\mathcal{L} > \frac{G_k}{k}$ and replacing $\mathcal{R}$ by $t\mathcal{L}$, and $\mathcal{L}$ by $d\mathcal{G}$ we obtain:

$$d > \left(k(tp(m) - q(m))\right)^{-1} \iff d > \left(k\left(\frac{t-m+1}{m}\right)\right)^{-1} \iff d > \frac{m}{k(t-m+1)}.$$ 

While for the reward, we already know $\sum_i \mathcal{L} > \mathcal{R}$ while also $\mathcal{R} > G$ and thus:

$$d\mathcal{G}t > \mathcal{R} > \frac{G}{k}$$

\[ \square \]

It is possible to derive from Theorem 4.1 results for the number of Byzantine players for a given deposit. That is, suppose that the baiting strategy only requires 1 rational player to play the baiting strategy, and let $\mathcal{L} = d \cdot \mathcal{G}$, then every coalition of size at least $t+1$ players has at least $k = t+1 - f$ rational players, and thus the maximum amount of Byzantine players tolerated for $\epsilon$-$(k-f,f)$-robustness is $f < t+1 - \frac{1}{n}$. For example, let us set the deposit $\mathcal{L} = d\mathcal{G}$ to $d = \frac{1}{n}$, i.e., the total deposit is $D = \mathcal{L} \cdot n = \mathcal{G}$, and $n = 100$, it follows that the protocol is $\epsilon$-$(k-f,f)$-robust and $f \leq 30$. If instead $d = \frac{1}{3n}$, then $f \leq 24$.

5 The Huntsman Protocol: Implementing Rational Agreement with Cheap-talk

In the previous sections, we designed a deposit mechanism so that $m$ rational players will play a baiting strategy instead of the disagreeing strategy, while guaranteeing lossfree-reward, provided that there is a protocol that already provides such baiting strategy. For this purpose, we introduced a deposit per player and a reward dependant on an estimated maximum gain of attackers, and analyzed the values to ensure that $m$ rational players follow that baiting strategy.

In this section, we provide a protocol that can make any $t$-accountable protocol solve rational agreement and be $\epsilon$-$(k,t)$-robust for $n > \frac{3}{2}k + 3f$ and $n > 2(k+f)$ in what we call the Huntsman Protocol.

First, we implement fifo-channels by requiring players to add a sequence number to their messages, so that a correct player $i$ does not deliver a message from player $j$ with a sequence number $s$ until all previous messages with sequence number $s' < s$ have been delivered. This way, if a coalition splits correct players into different partitions to achieve a disagreement, the messages from one partition cannot be delivered in the other, unless they are all delivered by order of increasing sequence number. This means that non-deviating players
will be able to tell that there is a disagreement before considering later messages from non-deviating players from the other partition.

We also assume that all players have deposited the required amount $L$ before starting the protocol. This deposit is either released back to their players after at least $n - t$ players decide a decision (whether this is the same or different decisions), or slashed to some players and released to others if a fraud is exposed, as we will show later.

In order to resolve disagreements before deciding them, we add a Byzantine Fault Tolerant (BFT) commit-reveal (BFTCR) phase at the end of a \(t\)-accountable consensus protocol shown in Algorithm 1 that consists of two reliable broadcasts per player, plus an additional broadcast per player. The purpose of the first group of reliable broadcasts is to reliably broadcast the encrypted PoFs, should a player own them, or an encrypted hash of the decision otherwise. We say that the commitment is the encrypted content that each player decides to broadcast in this first reliable broadcast. In line 14 players then start the second reliable broadcast by broadcasting a list of the first \(n - t\) delivered commitments. We define a valid candidate to win the reward $R$ as a member of a deviating coalition that committed to bait the coalition independently of what other $m$ players of the coalition did. The objective of the BFTCR protocol is to distinguish valid candidates from deviating players who try to become valid candidates only after they see that the disagreement will not succeed. Finally, the purpose of the calls to broadcast on lines 17 and 19 is to deliver the keys to decrypt the encrypted messages. A player $i$ thus reveals his commitment by broadcasting the key. Thus, the Huntsman protocol consists of an accountable consensus protocol (i.e., Polygraph [11]) with the added BFTCR phase before deciding, and with the requirements on a deposit per player and a reward for baiting as depicted in Section 4.

As such, we speak of a predecision for a decision of the \(t\)-accountable consensus protocol, whereas a decision is now the outcome of the BFTCR protocol. The protocol terminates when at least $n - t$ messages are decrypted with the same hash of the predecisions in line 24 or when at least $t + 1$ messages are decrypted (without counting players that are proven to be false through a PoF) with a reward to a chosen baiter and a punishment to the remaining players that are listed in the list of PoFs in line 34. Note that \(t\)-accountability does not guarantee that a baiter will gather enough PoFs before a disagreement takes place. We prove that baiters will gather enough PoFs before a disagreement as part of Theorem 5.1.

The BFTCR protocol selects the winner of the bait among the list of PoFs by running an additional consensus, in the call to select_winner in line 32, which executes an additional consensus where all participating players share the PoFs they know about and the valid candidates, along with the proofs that such players are valid candidates, to then select a winner from among the valid candidates. We refer to this consensus as the winner consensus. A correct player $i$ considers a baiter $j$ as a valid candidate if $i$ can see $j$'s commitment to bait in at least $t + 1$ messages from the second reliable broadcast. We prove in Theorem 5.3 that no deviating player can become a valid candidate after
seeing more than \( m - 1 \) other players baiting. In lines 33 and 34 fraudsters are punished and baiter is rewarded, respectively. Finally, the call to resolve(...) resolves the two disagreeing predecisions by deterministically choosing one of them (i.e., lexicographical order, or the union of both if the application allows it).

Algorithm 1 BFT commit-reveal protocol for player \( i \)

1: State:
2: \( \text{encrypted}_\text{msgs} \), list of delivered encrypted messages from the first group reliable broadcasts, initially \( \emptyset \)
3: \( \text{list}_\text{encrypted}_\text{msgs} \), list of delivered encrypted messages by other players from the first group of reliable broadcasts, initially \( \emptyset \)
4: \( \text{decrypted}_\text{msgs} \), list of delivered decrypted messages from the first group of reliable broadcasts, initially \( \emptyset \)
5: \( \{\text{RB}_1^j\}_{j=0}^m \), the first group of reliable broadcasts where \( j \) is the source
6: \( \{\text{RB}_2^j\}_{j=0}^m \), the second group of reliable broadcasts where \( j \) is the source
7: hashes, a dictionary where keys are hashes and value are integers, initially it does not contain any key or value
8: local_hash, local hash of the predecided value, according to this player
9: POF_received, boolean, initially False
10: \( i, \text{i_msg}, i_{\text{key}}, i_{\text{encrypted_msg}} \), player’s id, message, key, and encrypted message

11: \( \text{RB}_1^1.\text{start}(i_{\text{encrypted_msg}}) \)
12: Upon RB-delivering \( \text{encrypted_msg} \) from reliable broadcast \( \text{RB}_1^j \)
13: \( \text{encrypted}_\text{msgs}[j] \leftarrow \text{encrypted}_\text{msgs} \)
14: if (size(encrypted_msg) \( \geq n - t \)) then \( \text{RB}_1^2.\text{start}(\text{encrypted}_\text{msgs}) \)
15: Upon RB-delivering \( \text{encrypted}_\text{msgs} \), from reliable broadcast \( \text{RB}_2^j \)
16: \( \text{list}_\text{encrypted}_\text{msgs}[j] \leftarrow \text{encrypted}_\text{msgs}_j \)
17: if (size(list_\text{encrypted}_\text{msgs}) \( \geq n - t \) and size(encrypted_msg) \( \geq n - t \)) then broadcast(i_{\text{key}}, i)
18: Upon delivering key from \( j \) and RB-delivering from reliable broadcasts \( \text{RB}_1^1 \) and \( \text{RB}_2^j : \)
19: broadcast(key, j)
20: \( \text{decrypted}_\text{msgs}[j] \leftarrow \text{decrypt(encrypted}_\text{msgs}, \text{key}) \)
21: if (\( \text{decrypted}_\text{msgs}[j] \)\.type = HASH) then
22: \( \text{hash} \leftarrow \text{decrypted}_\text{msgs}[j]\.\text{get_hash}() \)
23: \( \text{hashes}[\text{hash}] \leftarrow 1 \)
24: if (\( \text{hashes}[\text{hash}] \geq n - t \) and local_hash = hashes[\text{hash}]) then deliver(hash)
25: else if (\( \text{decrypted}_\text{msgs}[j] \)\.type = POF) then
26: \( \text{POFs} \leftarrow \text{decrypted}_\text{msgs}[j]\.\text{get_POFs}() \)
27: if (verify(POFs)) then list_POFs[j] \( \leftarrow \text{POFs} \)
28: POF_received \( \leftarrow \text{True} \)
29: if (POF_received) then
30: \( \text{msgs_filtered} \leftarrow \text{keys(encrypted}_\text{msgs}) \setminus \text{keys(POFs)} \)
31: if (size(msgs_filtered) \( \geq t + 1 \)) then
32: baiter, fraudsters, predec_1, predec_2 \( \leftarrow \text{select_winner(list}_\text{encrypted}_\text{msgs}, \text{list}_\text{POFs}) \)
33: punish(fraudsters)
34: reward(baiter)
35: resolve(predec_1, predec_2)

Solving rational agreement with Cheap-talk.

We prove in this Section that the proposed Huntsman protocol implements
rational agreement and is an $\epsilon$-$(k, f)$-robust for $n > \frac{3}{2}k + 3f$ and $n > 2(k + f)$. For this purpose, we prove first that we implement a baiting strategy that solves baiting-agreement, baiting-dominance and lossfree-reward. We will prove them in this order. We will refer to the baiting strategy as the strategy that reveals the coalition with PoFs. We prove in Theorem 5.3 that this is a strong baiting strategy (i.e., baiting-dominance and lossfree-reward). (As $m$ depends on $k$ and $f$ we introduce the function $m(k, f)$ to refer to this value in the remainder of the paper). However, we first show in Theorem 5.1 that if $m(k, f) = \lfloor \frac{k + f - n}{2} + 1 \rfloor$ rational players play the baiting strategy, then the protocol implements agreement. That is, we prove baiting-agreement. Additionally, note that the winner consensus solves consensus for $n > 9/5(k + f)$ because at least $t + 1$ players of the coalition will not participate in it, as has already been shown [28], and we consider $n > 2(k + f)$. That is, because at least $t + 1$ players do not participate in the winner consensus (since they are provably fraudulent), at most $n' = n - (t + 1) < 2n/3$ players participate in the winner consensus. Since the maximum coalition size, i.e. $k + f$, is $k + f < n/2$, then the remaining players of the coalition that could participate in the winner consensus are $t' = n/2 - (t + 1) < n/6$, and thus $t' < n'/3$ and the winner consensus solves consensus.

**Theorem 5.1.** Let $n$ players execute the Huntsman protocol, out of which there can be a coalition of $k$ rational players and $f$ Byzantine players such that $n > 2(k + f)$ and $n > \frac{3}{2}k + 3f$. Suppose $m(k, f) = \lfloor \frac{k + f - n}{2} + 1 \rfloor$ rational players in the coalition play the baiting strategy. Then, the only possible outcome is to pay the reward and resolve the disagreement on the predecisions.

**Proof.** Suppose two predecisions that two partitions of players not in the coalition $A$ and $B$ predecided, such that $A \cap B = \emptyset$, and $|A| + |B| + k + f \leq n$. For any of the predecisions of the disagreement to be decided, players not in the coalition $A$ and $B$ must be able to decide at least one of the decisions either with $A$ or with $B$. Therefore, $|A| + k + f \geq n - t$ and also $|B| + k + f \geq n - t$. We consider now how many $m$ rational players out of $k$ must bait (i.e., must not contribute to disagree) for a disagreement to necessarily fail. This value must be such that $|A| + (k - m) + f < n - t$ and same for $B$’s partition, which solves to $m > \frac{k + f - n}{2} + t$. Therefore, as long as at least $m(k, f) = \lfloor \frac{k + f - n}{2} + 1 \rfloor$ rational players play the baiting strategy, the only possible outcome is for one of them to get the reward. and for the disagreement on predecisions to be resolved before deciding any of the two.

Notice also that if this reward is paid, then it is not possible to decide any of the hashes of the BFTCR protocol. The winner consensus starts for player $i$ when at least $t + 1$ players have been proven fraudulent to $i$. Since the players that are provably fraudulent do not count towards this winner consensus, that means that there are $n' = n - (t + 1)$ participants in the winner consensus. For the winner consensus to guarantee termination, it must guarantee that it requires at most $n - 2t'$ players replying to terminate, where $t'$ are the remaining players from the coalition after removing the $t + 1$ proven to be fraudulent. Let us define a function $g(k, f) = \min(f + k - (t + 1), f)$, then the winner consensus
must be robust against $t' = g(k, f)$ colluding players, meaning it requires $n - g(k, f)$ replies from different players, for a correct player to terminate. From this $n - g(k, f)$, at most $n - g(k, f) - (k + f)$ are replies from correct players. Since there are $n - k - f$ correct players, by the time the winner consensus terminates there might be $c = n - k - f - (n - g(k, f) - (k + f))$ correct players that have not participated in the winner consensus. As such, if $c + k + f \geq n - t$ then the coalition can still decide a decision after receiving the reward from the winner consensus. We thus require $c + k + f < n - t \iff g(k, f) + k + f < n - t \iff \min(f + k - t, f) + k + f < n - t$ which means that $n > \frac{3}{2}k + 3f$ and $n > 2(k + f)$ must be true for this to occur.

Next, we prove in Theorem 5.2 that the addition of a BFTCR protocol still guarantees consensus.

**Theorem 5.2.** Let $n$ players execute the Huntsman protocol, out of which there can be a coalition of $k$ rational players and $f$ Byzantine players such that $n > 2(k + f)$ and $n > \frac{3}{2}k + 3f$. Suppose $m(k, f) = \left\lceil \frac{k + f - n}{t - m(k, f) + 1} \right\rceil + 1$ rational players in the coalition play the baiting strategy if they participate in a disagreement on the predecisions. Then the Huntsman protocol satisfies consensus and is $t$-immune.

We show in Theorems 5.1 and 5.2 that, provided $m(k, f)$ rational players play the baiting strategy if there is a disagreement in the predecisions, the Huntsman protocol solves the rational agreement problem. We only have left to prove that baiting the coalition is in fact a strong $(k, f, m(k, f))$-baiting strategy that $m(k, f)$ players will play, to guarantee baiting-dominance and loss-free-reward. We show this in Theorem 5.3.

**Theorem 5.3.** Let $n$ players execute the Huntsman protocol, out of which there can be a coalition of $k$ rational players and $f$ Byzantine players such that $n > 2(k + f)$ and $n > \frac{3}{2}k + 3f$. Suppose each of the following predicates holds:

1. Each player is required to deposit $L = d \cdot G$, where $d > \max(k, f) \left\lceil \frac{m(k, f)}{k(t - m(k, f) + 1)} \right\rceil$,
2. If multiple players bait, then the system chooses one of these players at random to reward it with $R$ such that $dG > R > \max(k, f) \left( \frac{d}{m(k, f)} \right)$, and
3. The protocol slashes the deposits of the provably fraudulent players that do not win the reward.

Then the strategy to bait on the coalition is a strong $(k, f, m(k, f))$-baiting strategy.

Notice that the greater the size of the coalition, the greater $d$ must be in order for the protocol to be $c$-$(k, f)$-robust. However, for $n > \frac{3}{2}k + 3f$ with $n > 2(k + f)$, since for every two rational players that join the coalition one Byzantine must leave, the coalition that maximizes the total deposit $D = Ln = dGn$ is a coalition of $k = 1$ rational player and $f = t$ Byzantine players, and that means $d > \frac{1}{t + 1}$. Corollary proves such robustness.
**Corollary.** Let $\mathcal{P}$ be the Huntsman protocol. Then $\mathcal{P}$ is a $\epsilon$-$(k, f)$-robust for the rational agreement problem for $n > \frac{3}{2}k + 3f$ and $n > 2(k + f)$ if the following three properties are satisfied:

1. Each player is required to deposit $L = d \cdot G + \delta$, where $d = \left\lceil \frac{1}{n - 1} \right\rceil + \delta$ and $\delta > 0$,

2. If multiple players bait, then the system chooses one at random to reward him with $R = G + \frac{2}{7}$, and

3. The protocol slashes the deposits of the provably fraudulent players that do not win the reward.

Thus, there are only two possible outcomes of the Huntsman protocol:

- If the coalition is made by so many rational players that the utilities of deviating do not compensate the risk of losing the deposit, then the Huntsman protocol will provide agreement at predecision level, and no reward $R$ will be paid to any player,

- If the coalition has enough Byzantine players to make the deviation into two predecisions profitable, then enough $m(k, f)$ rational players in the coalition will bait so that the disagreement on predecisions can safely be resolved and decided, and one rational player among the baiters will receive a reward $R$, paid entirely by the deposits of the rest of the provably fraudulent players.

In both scenarios, the Huntsman protocol implements rational agreement, it is thus $\epsilon$-$(k, f)$-robust for $n > \frac{3}{2}k + 3f$ and $n > 2(k + f)$, and $t$-immune.

### 6 Conclusion

We have presented a necessary and sufficient baiting strategy to solve the rational agreement problem with partial synchrony. Based on this strategy, we also proposed a novel Byzantine consensus protocol among $n > \max(\frac{3}{2}k + 3f, 2(k + f))$ players, where $k$ players are rationals and $f$ are Byzantine. By building upon standard cryptography and a recent accountable Byzantine agreement protocol, this protocol tolerates the coordinated changes of strategy of $k$ rational players and $f$ Byzantine players. As future work, it would be interesting to explore whether our bound $n > \max(\frac{3}{2}k + 3f, 2(k + f))$ is tight, and considering the impact of non-negligible costs of computation and communication.

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A Missing Proofs

**Lemma A.1.** It is impossible to obtain a protocol \( \varphi \) that implements the agreement functionality, is \( t \)-immune and \( (k, f) \)-robust, \( k \geq 0 \) and \( f = \max(t - k + 1, 0) \) unless there is a \( (k, f, m) \)-baiting strategy with respect to \( \varphi \), for \( m > \frac{k + f - n}{2} + t \).

**Proof.** We refer to Lamport et al.’s [26] work for the impossibility of increasing \( t \geq n/3 \) and obtaining agreement (i.e., for \( k = 0 \)).

For \( k > 0 \) with \( f < n/3 \), assume the contrary: let \( \varphi \) be a protocol such that there is no \( (k, f, m) \)-baiting strategy with respect to \( \varphi \) and \( \varphi \) is \( (k, f) \)-robust, for \( f = \max(t - k + 1, 0) \), \( k > 0 \). Since the protocol is \( t \)-immune and it works under partial synchrony, the protocol must not require more than \( n - t \) players participating in it in order to take a decision, or else the Byzantine players could prevent termination. Consider a partition of the network between 4 disjoint subsets \( N = K \cup A \cup B \cup F \), where \( K \) are the rational players (there is at least one), \( F \) are the Byzantine players, i.e., \( |F| + |K| = f + k \geq t + 1 \), and \( A \) and \( B \) are the rest of the players such that \( |A| + |B| \leq n - t - 1 \) and both \( |A| + |F| + |K| \geq n - t \) and \( |B| + |F| + |K| \geq n - t \) hold (recall \( t = \lceil \frac{n}{3} \rceil - 1 \)). Let \( \theta \) be the strategy in which the rational players in \( K \) deviate with Byzantine players in \( F \) and achieves a disagreement between players in \( A \) and players in \( B \). If the players in \( F \) and \( K \) are all Byzantine and rational players, then such a disagreement is always possible and the utility for each rational player is, by definition of the model, greater than that of reaching agreement. Notice also that since \( f = \max(t - k + 1, 0) \), if \( m > \frac{k + f - n}{2} + t \) rational players do not deviate to cause such disagreement, we have that at least one of \( |A| + |F| + |K| - m < n - t \) and \( |B| + |F| + |K| - m < n - t \) holds, or both: for this value of \( m \) the deviants cannot cause a disagreement, where as this does not hold for \( m \leq \frac{k + f - n}{2} + t \). It follows that it is necessary to encourage at least \( m > \frac{k + f - n}{2} + t \) rational players not to deviate into causing a disagreement, i.e. a \( (k, f, m) \)-baiting strategy is necessary.

**Theorem A.2 (Theorem 3.1).** It is impossible to obtain a protocol \( \varphi \) that implements rational agreement unless there is a \( (k, f, m) \)-baiting strategy with respect to \( \varphi \), for \( m > \frac{k + f - n}{2} + t \).

**Proof.** By definition, every \( (k, f) \)-robust protocol for \( n \leq 3(k + f) \) must also be \( (k, f) \)-robust, for some \( k \geq 0 \) and \( f = \max(t - k + 1, 0) \). Therefore it derives from Lemma [A.1]
Theorem A.3 (Theorem 5.2). Let \( n \) players execute the Huntsman protocol, out of which there can be a coalition of \( k \) rational players and \( f \) Byzantine players such that \( n > 2(k + f) \) and \( n > \frac{3}{2}k + 3f \). Suppose \( m(k, f) = \left\lfloor \frac{k + f - n}{t} \right\rfloor + 1 \) rational players in the coalition play the baiting strategy if they participate in a disagreement on the predecisions. Then the Huntsman protocol satisfies consensus and is \( t \)-immune.

Proof. Since \( f \leq t \), it is clear that if there is no disagreement on the predecisions, then rational and correct players are more than \( n - t \) and thus the protocol terminates and guarantees validity and agreement. If there is instead a disagreement on the predecision, then as long as \( m(k, f) \) players play the baiting strategy, by Theorem 5.1 the only outcome is to pay the reward and resolve the disagreement.

Theorem A.4 (Theorem 5.3). Let \( n \) players execute the Huntsman protocol, out of which there can be a coalition of \( k \) rational players and \( f \) Byzantine players such that \( n > 2(k + f) \) and \( n > \frac{3}{2}k + 3f \). Suppose each of the following predicates holds:

1. Each player is required to deposit \( \mathcal{L} = d \cdot G \), where \( d > \max(k, f) \left( \frac{m(k, f)}{k-m(k,f)+1} \right) \),
2. If multiple players bait, then the system chooses one of these players at random to reward it with \( R \) such that \( dGt > R > \max(k, f) \left( \frac{c}{m(k,f)} \right) \), and
3. The protocol slashes the deposits of the provably fraudulent players that do not win the reward.

Then the strategy to bait on the coalition is a strong \((k, f, m(k,f))\)-baiting strategy.

Proof. We show first that if \( m(k, f) \) rational players in the coalition play the baiting strategy, becoming valid candidates to win the reward, then the remaining \( k + f - m(k, f) \) cannot become valid candidates of the winner consensus after seeing \( m(k, f) \) players become valid candidates. Given that the non-baiting members of the coalition are trying to terminate a disagreement, they will still split non-deviating players into two partitions \( A \) and \( B \) for the BFTCR protocol. Hence, we look at how many rational players must take part in both partitions of the BFTCR protocol. Notice that \( |A| + |B| + f + k \geq n \), \( |A| + k + f > n - t \) and \( |B| + k + f > n - t \), and thus \( c = (n - t) - \frac{n + f - k}{2} \) is the number of members of the coalition that must participate in a partition for it to terminate, with \( A \cap B = \emptyset \) thanks to using fifo-channels. We are interested in calculating the minimum number of rational players out of these \( c \), this is why we include as many Byzantine players as possible, i.e., \( c - f \). Notice also that we want to see how many rational players must take part in both partitions, meaning that we are interested in \( c - f - \frac{k}{2} = (n - t) - \frac{n + f}{2} \geq m(k, f) \) for \( n > \frac{3}{2}k + 3f \). Both partitions will include at least \( m(k, f) \) repeated rational players, which will bait. What is left to prove is that by the time they reveal their commitment, the remaining players cannot collude to try and become valid.
candidates of the winner consensus too. That is, each of the $m(k, f)$ players can wait for $n - t$ deliveries of the second reliable broadcast before revealing their commitment by broadcasting their key, and they also can wait to at least deliver $\min(n - k - f, n - t)$ from each partition. This means that $|A| + k + f \geq n - t$ and same for $|B|$, i.e. $|B| + k + f \geq n - t$, which translates into the existence of $|A| + |B| \geq 2(n - t) - 2k - 2f$ correct players that delivered at least $m(k, f)$ commitments by baiters. Notice that $|A| + |B| \geq t + 1$ for $n > 2(k + f)$. Thus, we must calculate for which values of $k$ and $f$ the remaining players cannot become valid candidates, that is, for which values of $k$ and $f$ other players that did not bait yet cannot include the new commitment to bait in $t + 1$ valid second reliable broadcasts. Since the remaining set of correct players $C$ such that $|C| = n - f - k - |A| - |B|$ are $|C| = n - k - f - (2(n - t) - 2k - 2f)$, we calculate for which values of $k$ and $f$ we have $|C| + k + f - t \leq t$, which results in $n > 2(k + f)$. As a result, no deviating player can decide to bait and become a valid candidate for the winner consensus only after seeing another $m(k, f)$ players baiting, where becoming a valid candidate means having their commitment to bait included in at least $t + 1$ messages from the second reliable broadcast.

Finally, since $m(k, f)$ players are interested in playing the baiting strategy even after they see $m(k, f) - 1$ rational players playing the baiting strategy, then at least $m(k, f)$ play such strategy. If $m(k, f)$ play the baiting strategy, then from Theorem 5.1 the only outcome is to pay the reward once and resolve the disagreement. We have shown here that the reward must in turn be paid to one of the members in $m(k, f)$, that is, players cannot wait to see $m(k, f)$ playing the baiting strategy in order to play the baiting strategy. Thus, analogously to Theorem 4.1 we calculate $R$ and $L = dG$ so that the strategy to bait the coalition strictly dominates the strategy to finish the disagreement, obtaining a strong baiting strategy for $d > \max(k, f)\left(\frac{m(k, f)}{m(k, f) + 1}\right)$ and $dG > R > \max(k, f)\left(\frac{G}{m(k, f)}\right)$.

**Corollary** (Corollary 5). Let $\tilde{\sigma}$ be the Huntsman protocol. Then $\tilde{\sigma}$ is an $\epsilon$-(k, f)-robust for the rational agreement problem for $n > \frac{3}{2}k + 3f$ and $n > 2(k + f)$ if the following three properties are satisfied:

1. Each player is required to deposit $L = d \cdot G + \delta$, where $d = \frac{1}{\left\lceil \frac{1}{2}\right\rceil - 1} + \delta$ and $\delta > 0$,
2. If multiple players bait, then the system chooses one at random to reward him with $R = G + \frac{\delta}{2}$, and
3. The protocol slashes the deposits of the provably fraudulent players that do not win the reward.

**Proof.** Theorem 5.3 proves baiting-dominance and lossfree-reward, Theorem 5.1 proves baiting-agreement and Theorem 5.2 proves consensus. Finally, Theorem 4.1 proves rational agreement and $\epsilon$-(k, f)-robustness when $n > \frac{3}{2}k + 3f$ and $n > 2(k + f)$ for the aforementioned values of the deposit per player and the reward. \qed

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