Electromagnetic proton-neutron knockout off $^{16}$O: new achievements in theory

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Results for the cross sections of the exclusive $^{16}$O(e,e'pn)$^{14}$N and $^{16}$O(γ,pn)$^{14}$N knockout reactions are presented and discussed in different kinematics. In comparison with earlier work, a complete treatment of the center-of-mass (CM) effects in the nuclear one-body current is considered in connection with the problem of the lack of orthogonality between initial bound and final scattering states. The effects due to CM and orthogonalization are investigated in combination with different treatments of correlations in the two-nucleon overlap function and for different parametrizations of the two-body currents. The CM effects lead in super-parallel kinematics to a dramatic increase of the $^{16}$O(e,e'pn) cross section to the $1^+_2$ excited state (3.95 MeV) of $^{14}$N. In all the situations considered the results are very sensitive to the treatment of correlations. A crucial role is played by tensor correlations, but also the contribution of long-range correlations is important.

I. INTRODUCTION

It is well known that the independent particle shell model, in which nucleons move in a mean field, is not sufficient to describe all basic properties of atomic nuclei. This failure is a consequence of the strong short-range component of the NN-interaction, which induces into the nuclear wave function components beyond the mean field description. Thus, a careful evaluation of the corresponding short-range correlations (SRC) is inevitable for the understanding of nuclear structure in general [1].

Electromagnetically induced two-nucleon knockout is apparently the most promising tool to study SRC [2]. However, competing two-body mechanisms like meson-exchange currents (MEC), isobar-excitation or final state interactions (FSI) need to be taken into account as well in order to obtain a realistic description of the process. These “background” effects must be well under control in order to extract from experiment any information about the fundamental correlations. This requires a theoretical approach which should be as comprehensive as possible. Presently, different models are available (see [3–5] and references therein).

In the past years, the Pavia group has improved, step by step, its model for two-nucleon knockout. Recent improvements have been performed with respect to the treatment of FSI [6, 7], of the two-nucleon overlap function [8], of the ∆-current contribution [5], and, finally, of the center-of-mass (CM) effects in the electromagnetic current operator [9]. With respect to the last two issues, only proton-proton (pp) knockout has been considered, which is conceptually simpler than proton-neutron (pn) knockout due to the suppression of MEC contributions. However, pn-knockout is of specific interest for the study of tensor correlations (TC), which are due to the strong tensor component of the pion-exchange contribution to the NN-interaction.

In the present paper, we study thoroughly the impact of the above mentioned conceptual improvements on pn-knockout. The final aim – in the long term run – is to develop an approach which is as comprehensive as practically possible, so that hopefully important conclusions about the structure of correlations can be drawn by comparison with experiment.

The paper is organized as follows. In the next section, the main features of the theoretical approach are outlined. Special emphasis is devoted to the description of the various improvements performed in comparison with earlier work. Numerical results for the exclusive $^{16}$O(e,e'pn)$^{14}$N and $^{16}$O(γ,pn)$^{14}$N reactions in different kinematics are presented in sect. III. Some conclusions are drawn in sect. IV.

Finally, we want to stress the purely theoretical nature of the paper. A comparison with recent experimental data [10] is presently under investigation and will be discussed in forthcoming work [11].

II. THEORETICAL MODEL

The cross section of a reaction induced by a real or virtual photon, with momentum $q$, where two nucleons, with momenta $p'_1$, and $p'_2$, are ejected from a nucleus, can be written in terms of the transition matrix elements of the
charge-current density operator between initial and final nuclear states

\[ J^\mu(q) = \int < \Psi_f | J^\mu(r) | \Psi_i > e^{iq \cdot r} dr. \]  

(1)

Bilinear products of these integrals give the components of the hadron tensor and therefore the cross section \[2, 12\].

For an exclusive process, where the residual nucleus is left in a discrete eigenstate of its Hamiltonian, and under the assumption of a direct knock-out mechanism, the matrix elements of eq. (1) can be written as \[12, 13\]

\[ J^\mu(q) = \int \psi^*_i(r_1, r_2) J^\mu(r, r_1, r_2) \times \psi_i(r_1, r_2) e^{i q \cdot r} dr_1 dr_2. \]  

(2)

Three main ingredients appear in eq. (2): the two-nucleon overlap integral \( \psi_i \) between the ground state of the target and the final state of the residual nucleus, the nuclear current \( J^\mu \), and the two-nucleon scattering wave function \( \psi_f \).

In principle, the bound and scattering states, \( \psi_i \) and \( \psi_f \), are consistently derived from an energy-dependent non-Hermitian Feshbach-type Hamiltonian for the considered final state of the residual nucleus. They are eigenfunctions of this Hamiltonian at negative and positive energy values \[2, 12\]. In practice, it is not possible to achieve this consistency and the treatment of initial and final states proceeds separately with different approximations.

The two-nucleon overlap function (TOF) \( \psi_i \) contains information on nuclear structure and correlations. For the case of proton-neutron emission from \(^{16}\)O different approaches are used in \[8, 14, 15\] to calculate the TOF.

In the more sophisticated approach of \[8\], that we call SF-B, the TOF was computed partitioning the Hilbert space in order to determine the contribution of long-range correlations (LRC) and SRC separately. The LRC, describing the collective motion at low energy as well as the long range part of TC, were computed using the self-consistent Green’s function formalism \[16\] in an appropriate harmonic-oscillator (h.o.) basis. The effects of SRC due to the central and tensor part at high momenta were added by computing the appropriate defect functions with the Bonn-C NN-potential \[17\].

In the approach of \[14\], that we call SF-CC, the effects of SRC as well as TC were calculated within the framework of the coupled cluster method, using the so-called \( S_2 \) approximation and employing the Argonne V14 potential \[18\] for the NN-interaction. In this calculation the effects of LRC are accounted for in a simpler way and only knockout of nucleons from the (0p) shell is considered.

A much simpler approximation is used in \[15\], where the TOF is given by the product of a coupled and antisymmetrized shell model pair function and of a Jastrow-type central and state independent correlation function taken from \[19\]. In this approach, that we call SM-SRC, only SRC are considered and the final state of the residual nucleus is a pure two-hole state in \(^{16}\)O.

The final-state wave function \( \psi_f \) is written as the product of two uncoupled s.p. distorted wave functions, eigenfunctions of a complex phenomenological optical potential which contains a central, a Coulomb, and a spin-orbit term \[20\]. The effect of the mutual interaction between the two outgoing nucleons (NN-FSI) has been studied in \[6, 7, 21\] and is included in the present calculations only in some cases using the model applied in \[7\].

The nuclear current \( J^\mu \) is the sum of a one-body (OB) and a two-body (TB) contribution, \( i.e. \)

\[ J^\mu(r, r_1, r_2) = J^{\mu(1)}(r, r_1) + J^{\mu(2)}(r, r_1, r_2) \]  

(3)

The OB part includes the longitudinal charge term and the transverse convective and spin currents. The TB current is derived from the effective Lagrangian of \[22\], performing a non relativistic reduction of the lowest-order Feynman diagrams with one-pion exchange. We have thus currents corresponding to the seagull and pion-in-flight diagrams, and to the diagrams with intermediate \( \Delta \)-isobar configurations \[23\], \( i.e. \)

\[ J^{(2)}(r, r_1, r_2) = J^{\text{seagull}}(r, r_1, r_2) \]

\[ + J^\Delta(r, r_1, r_2). \]  

(4)

Details of the nuclear current components can be found in \[5, 23–25\]. In this paper, the parameters of the \( \Delta \)-current are fixed considering the NN-scattering in the \( \Delta \)-region, where a fairly good description of the scattering

\[1\] Spin/isospin indices are generally suppressed in the formulas of this paper for the sake of simplicity.
data can be achieved by choosing parameters similar to the ones of the full Bonn potential \cite{17,26}. This choice gives the parametrization \( \Delta(\text{NN}) \) discussed in \cite{5} where both \( \pi \)- and \( \rho \)-exchange are considered. It turns out that a comparable description of the NN-scattering data can be achieved in a conceptually simpler manner considering only \( \pi \)-exchange. In this choice, which is adopted in the present paper, the coupling constants \( f_{\pi NN}^2/(4\pi) = 0.078 \) and \( f_{\rho NN}^2/(4\pi) = 0.35 \) are used with a dipole form factor \( (n_{\pi NN}=n_{\rho NN}=1) \) using the cutoffs \( \Lambda_{\pi NN} = \Lambda_{\rho NN} = 700 \) MeV.\(^2\) With respect to the nonresonant pion-in-flight and seagull MEC, we use a dipole cutoff of 3 GeV in accordance with the Bonn-C potential \cite{17}. This large value leads to an upper estimate of the role of these contributions. In the following, we denote this choice of parameters for the TB currents as the “Bonn parametrization”.

In the previous calculations of \cite{8,14,15} the same coupling constants were used with a simpler regularization in coordinate space, both for the MEC and the \( \Delta \)-current, which in practice is similar to the unregularized prescription for the \( \Delta \)-current \( \Delta(\text{NoReg}) \) of \cite{5}. Although a bit simplistic, we denote in the following this previous prescription in shorthand as “unregularized parametrization”. The results obtained with the two parametrizations are compared in the next section.

In order to evaluate the transition amplitude of eq. (2), for the three-body system consisting of the two nucleons, 1 and 2, and of the residual nucleus \( B \), it appears to be natural to work with CM coordinates \cite{9,12,27}

\[
 r_{1B} = r_1 - r_B, \quad r_{2B} = r_2 - r_B, \\
 r_B = \sum_{i=1}^{A} r_i/(A-2).
\]

The conjugated momenta are given by

\[
 p_{1B} = \frac{A-1}{A} p_1' - \frac{1}{A} p_2' - \frac{1}{A} p_B, \\
p_{2B} = \frac{1}{A} p_1' + \frac{A-1}{A} p_2' - \frac{1}{A} p_B, \\
P = p_1' + p_2' + p_B,
\]

where \( p_B = q - p_1' - p_2' \) is the momentum of the residual nucleus in the laboratory frame.

Applying this transformation to the matrix element of eq. (2) is essentially equivalent \cite{9} to substitute in the equation the operator \( \exp(iq \cdot r_1) \) with

\[
 \exp \left( i q \cdot \frac{A-1}{A} r_{1B} \right) \exp \left( -i q \cdot \frac{1}{A} r_{2B} \right).
\]

An analogous substitution applies to \( \exp(iq \cdot r_2) \) \cite{9}.

In spite of the fact that a OB operator cannot affect two particles if they are not correlated, it can be seen from eq. (9) that in the CM frame the transition operator becomes a two-body operator even in the case of a OB nuclear current. Only in the limit \( A \to \infty \) CM effects are neglected and the expression in eq. (2) vanishes for a pure OB current, when the matrix element is calculated using orthogonalized single particle (s.p.) wave functions. This means that, due to this CM effect, for finite nuclei the OB current can give a contribution to the cross section of two-particle emission independently of correlations. This effect is similar to the one of the effective charges in electromagnetic reactions \cite{28}.

The matrix element of eq. (2) contains a spurious contribution since it does not vanish when the transition operator is set equal to 1. This is due to the lack of orthogonality between the initial and the final state wave functions. The use of an effective nuclear current operator would remove the orthogonality defect besides taking into account space truncation effects \cite{2,29}. In the usual approach, however, the effective operator is replaced by the bare nuclear current operator. Thus, it is this replacement which may introduce a spurious contribution which is not specifically due to the different prescriptions adopted in practical calculations, but is already present in eq. (2), when \( \psi_i \) and \( \psi_f \) are eigenfunctions of an energy-dependent Feshbach-type Hamiltonian corresponding to different energies.

This spuriousity can be removed by subtracting from the transition amplitude the contribution of the OB current without correlations in the nuclear wave functions. This prescription was adopted in \cite{8,14,15}. In this way, however, not only the spuriousity is subtracted, but also the CM effect given by the two-body operator in eq. (9), which is present in the OB current independently of correlations and which is not spurious. The relevance of this effect was investigated in \cite{9} for the case of pp-knockout.

\footnote{The notation for the coupling constants and cutoffs is the same as in \cite{5}.}
A more accurate procedure to get rid of the spuriosity is to enforce orthogonality between the initial and final states by means of a Gram-Schmidt orthogonalization [30]. In this approach each one of the two s.p. distorted wave functions is orthogonalized to all the s.p. shell-model wave functions that are used to calculate the TOF, i.e., for the TOF of [8], to the h.o. states of the basis used in the calculation of the spectral function, which range from the 0s up to the 1p-0f shell. This procedure allows us to get rid of the spurious contribution to two-nucleon emission due to a OB operator acting on either nucleon of an uncorrelated pair, which is due to the lack of orthogonality between the s.p. bound and scattering states of the pair. In this approach, in consequence, no OB current contribution without correlations needs to be subtracted. Moreover, it allows us to include automatically all CM effects via eq. (9). This procedure was proposed and applied in [9] to electromagnetic pp-knockout and is applied in this paper to electromagnetic pn-knockout reactions.

III. RESULTS

In this section, numerical results are presented for the cross sections of the $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ and $^{16}\text{O}(\gamma,\text{pn})^{14}\text{N}$ reactions to discrete low-lying states in the residual nucleus. The main aim of this study is to investigate the effects of CM and orthogonalization in combination with different treatments of correlations in the TOF and for different parametrizations of the TB currents. The effects of the mutual interaction between the two outgoing nucleons is also considered with a few examples.

Calculations have been performed in different situations and kinematics. Of particular interest for our study is the case of the super-parallel kinematics, where for two-proton knockout the CM effects due to the OB current without correlations are particularly large [9].

In the so-called super-parallel kinematics the two nucleons are ejected parallel and anti-parallel to the momentum transfer and, for a fixed value of the energy transfer $\omega$ and of the momentum transfer $q$, it is possible to explore, for different values of the kinetic energies of the outgoing nucleons, all possible values of the recoil or missing momentum $p_B$. This kinematical setting, which is particularly favourable to emphasize short-range effects, has been widely investigated in our previous work, both for pp- and pn-knockout [5–8, 12–14, 31, 32], and is very interesting from the experimental point of view. In fact, the recent $^{16}\text{O}(e,e'\text{pp})^{14}\text{C}$ [33] and $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ [10] experiments carried out at MAMI were both centred on the same super-parallel kinematics. In [10] the data of the first measurements of the exclusive $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ reactions are not described by the theoretical model of [8]. Large discrepancies, both in shape and magnitude, are found between theoretical and experimental cross sections at low missing momenta. It is therefore of particular interest to investigate the relevance of the CM effects included in the present approach in comparison with the results of [8].

The comparison is shown in fig. 1 for the cross section of the $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ reaction to the $1^+_2$ excited state of $^{14}\text{N}$ at 3.95 MeV. This is the state that is mostly populated in pn-knockout [10, 34, 35]. Results for this transition are therefore of particular interest. Calculations have been performed in the same super-parallel kinematics already considered in [8] and realized in the experiment [10] at MAMI, i.e. the incident electron energy is $E_0 = 855$ MeV, $\omega = 215$ MeV, and $q = 316$ MeV/c. The CM contribution included in the orthogonalized approach produces a huge enhancement of the cross section calculated with the OB current. The results are shown in the right panel of the figure. The enhancement is larger than one order of magnitude at low missing momenta and is only slightly reduced at higher momenta. The difference between the cross section obtained in the orthogonalized approach and the one of [8] is reduced when also the TB currents are included in the calculations. The results are shown in the left panel of the figure. In the approach of [8] the cross section is dominated by the TB currents, in particular by the $\Delta$-current. In contrast, the OB current dominates the cross section in the orthogonalized approach and only a small enhancement is given by the TB currents. The differences obtained in the orthogonalized approach with the two prescriptions for the TB currents are appreciable although not very important. The Bonn parametrization reduces the cross section by at most 30-40%. In the final cross section the difference between the results of the two approaches remains large, a bit less than one order of magnitude at low momenta, in the maximum region, and it is still sizable, although considerably reduced, at higher momenta.

The results given by the two parametrizations on the separate contributions of the different components of the TB current are displayed in fig. 2 and compared with the contribution of the OB current and with the final cross section. The calculations have been performed with the orthogonalized approach for the $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ reaction to the $1^+_2$ state in the super-parallel kinematics and with the SF-B overlap function. The CM contribution included in the orthogonalized approach gives large effects on the OB current and, as it has been shown in fig. 1, on the final cross section. In contrast, the use of orthogonalized s.p. bound and scattering wave functions produces only negligible effects on all the terms of the TB current. Therefore the results for the separate TB components obtained with the unregularized parametrization used in [8], and displayed in the right panel of fig. 2, are practically the same as in [8]. The differences obtained with the Bonn parametrization, which are displayed in the left panel of the figure, are

\[ \text{TB current} \]
FIG. 1: The differential cross section of the $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ reaction to the $1^+_2$ excited state (3.95 MeV) of $^{14}\text{N}$ as a function of $p_B$ in a super-parallel kinematics with $E_0 = 855$ MeV, electron scattering angle $\theta_e = 18^\circ$, $\omega = 215$ MeV, and $q = 316$ MeV/c. The proton is emitted parallel and the neutron antiparallel to the momentum transfer. Different values of $p_B$ are obtained changing the kinetic energies of the outgoing nucleons. Positive (negative) values of $p_B$ refer to situations where $p_B$ is parallel (anti-parallel) to $q$. The final results given by the sum of the OB and the TB currents are displayed in the left panel, the separate contribution of the OB current is shown in the right panel. The TOF from the two-nucleon spectral function of [8] (SF-B) is used in the calculations. The dotted lines give the results of [8], the dashed and solid lines are obtained with the present approach, where the orthogonalization of s.p. bound and scattering states is enforced and all CM effects are taken into account. The solid and dashed lines in the left panel are obtained with different parametrizations of the TB currents, i.e., our previous unregularized parametrization, as in [8], (dashed) and the Bonn parametrization (solid).

FIG. 2: The differential cross section of the $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ reaction to the $1^+_2$ state as a function of $p_B$ in the same super-parallel kinematics as in fig. 1. Calculations are performed in the orthogonalized approach. TOF as in fig. 1. Separate contributions of the OB, seagull, pion-in-flight and $\Delta$-current are shown by the dotted, short-dashed, dotted-dashed, and long-dashed lines, respectively. The solid lines give the total cross section. The Bonn parametrization is used for the TB currents in the left panel, the unregularized parametrization in the right panel.

appreciable for the seagull and pion-in-flight MEC and huge for the $\Delta$-current, whose contribution is dramatically reduced with the Bonn parametrization. In both cases, however, the cross section is dominated by the OB current. With the Bonn parametrization the role of the TB currents is negligible and the final cross section is in practice entirely due to the OB current. With the parametrization used in the right panel of the figure the contribution of the $\Delta$-current enhances the OB cross section by 30-40% and is responsible for the difference between the two final results.

In fig. 3 the cross sections of the $^{16}\text{O}(e,e'\text{pn})^{14}\text{N}$ reaction are displayed for transitions to different final states of the residual nucleus: the $1^+_2$ ground state, the $0^+$ (2.31 MeV), and the $2^+$ (7.03 MeV) excited states of $^{14}\text{N}$. The calculations are performed in the super-parallel kinematics and with the SF-B overlap functions. The shape of the recoil-momentum distribution for each state is determined by the CM orbital angular momentum $L$ of the knocked-out pair. Different partial waves of relative and CM motion contribute to the TOF. Each transition is characterized by different components, with specific values of $L$. The relative weights of these components, which are given by the two-nucleon removal amplitudes included in the TOF, determine the shape of the recoil-momentum distribution [8].
The CM contribution included in the orthogonalized approach gives in fig. 3 an enhancement of the OB cross section that depends on the final state considered. The results are shown in the right panels of the figure. For the ground state the enhancement is of about the same size as the one found in fig. 1 for the \(1^+_1\) state. A smaller effect is obtained for the \(0^+\) and \(2^+\) states. The enhancement is large for all the states considered in the figure at low values of \(p_B\) and is strongly reduced beyond 200 MeV/\(c\), where for the \(0^+\) and \(2^+\) states the difference between the results of the two approaches becomes very small. The difference is reduced in the left panels, where also the contribution of the TB currents is included. For the \(1^+_1\) ground state a significant enhancement of the final cross section is obtained in the orthogonalized model at low values of \(p_B\). The sensitivity of the results to the parametrization used for the TB currents is for this state similar to the one found for the \(1^+_2\) state in fig. 1. For the \(0^+\) and \(2^+\) states, where the TB currents play a more important role, the sensitivity of the calculated cross sections to the parameters used in the TB currents (dashed vs. solid) is somewhat larger and generally larger than the difference produced by the use of orthogonal s.p. wave functions (dotted vs. dashed). The effects due to the orthogonalization are very small and even negligible at high values of \(p_B\). It can be noted that when also the TB currents are included, the final cross section to the \(0^+\) state calculated in the orthogonalized approach is lower than the one calculated with our previous approach [8]. This result is due to the different and combined effect of the different components of the TB current. The cross section calculated with the Bonn parametrization are generally lower than the ones calculated with the unregularized parametrization.

In the \(^{16}\text{O}(e,e'pn)^{14}\text{N}\) measurements reported in [10] the energy resolution was sufficient to distinguish groups of states in the residual nucleus but not good enough to separate individual states. Therefore, the cross sections were measured for the group of states in the excitation energy range between 2 and 9 MeV and include the \(0^+, 1^+_2, \) and \(2^+\)
FIG. 4: The differential cross section of the $^{16}$O($e,e'pn$)$^{14}$N reaction to the $1^+_2$ state as a function of $p_B$ in the same super-parallel kinematics as in fig. 1. The final results given by sum of the OB and the TB currents are displayed in the left panel, the separate contribution of the OB current is shown in the right panel. The calculations are performed in the orthogonalized approach and with the Bonn parametrization for the TB currents. Results obtained with different TOF’s are compared: SF-B (solid), SF-CC (dashed), and SM-SRC (dotted).

states. The present results in figs. 1 and 3 indicate that the $1^+_2$ state dominates the cross section, the contribution of the $2^+$ state is competitive only for recoil-momentum values above 250 MeV/$c$, and the contribution of the $0^+$ state is always negligible. The strong enhancement of the cross section to the $1^+_2$ state at low values of $p_B$, that is due to the CM effects in the OB current included in the orthogonalized approach, is able to resolve the discrepancies found in [10] in comparison with the experimental data and give a much better agreement [11]. A careful comparison with the data requires calculations for a number of kinematical settings covering the energy and angular ranges subtended by the experimental set-up and will be presented in a forthcoming paper [11].

At next, we discuss the central point of these studies, namely the sensitivity of the cross sections to the treatment of correlations in the TOF. The cross sections calculated with the TOF’s of [8] (SF-B), [14] (SF-CC), and [15] (SM-SRC) are compared in fig. 4. The calculations are performed with the orthogonalized approach for the reaction $^{16}$O($e,e'pn$)$^{14}$N to the $1^+_2$ state in the super-parallel kinematics. The three overlap functions give large differences, both on the shape and the magnitude of the calculated cross sections. With the simpler SM-SRC overlap function, where only SRC are taken into account, the contribution of the OB current is negligible and is up to about three orders of magnitude lower than the one obtained with the more complete and sophisticated approach SF-B. When the TB currents are added, the SM-SRC cross section is enhanced by about one order of magnitude, the difference between the SM-SRC and SF-B cross sections is reduced but remains very large, up to about two orders of magnitude in the maximum region and somewhat smaller at high values of $p_B$. The results in fig. 4 have been obtained with the Bonn parametrization for the TB currents. A calculation with the parametrization used in our previous calculations enhances the contribution of the $\Delta$-current, but does not change significantly the final cross sections and therefore the main features of the results shown in the figure.

The SM-SRC cross section is dominated by the TB currents and, as such, it is practically unaffected by the use of orthogonalized s.p. bound and scattering wave function. Incidentally, we note that for the separate contribution of the OB current the CM effects included in the orthogonalized approach give with SM-SRC a much smaller effect than with SF-B and SF-CC. This result confirms what was already obtained in [9], i.e. that the relevance of these CM effects depends on the TOF used in the calculation. The very large enhancement of the OB current contribution due to these CM effects makes the OB current dominant in the cross sections calculated with the SF-B and SF-CC overlap functions. We note that in the calculations of [8] and [14] the cross section to the $1^+_2$ state obtained with SF-B and SF-CC was in both cases dominated by the TB $\Delta$-current.

The much larger contribution of the OB current with the SF-B and SF-CC overlap functions emphasizes the crucial role played by TC, that are very important in proton-neutron emission and are neglected in the simpler SM-SRC calculation. In the SF-B and SF-CC overlap functions SRC and TC are accounted for consistently in the defect functions, which are calculated in the two TOF’s within different methods.

In fig. 4 the SF-B cross section is generally larger than the SF-CC one. The SF-B result overshoots the SF-CC one up to a factor of 6 in the maximum region. The differences are strongly reduced for values of $p_B$ greater than 100 MeV/$c$.

The differences between the cross sections calculated with SF-B and SF-CC are due to the different treatments of all the contributions to the TOF. A more complete calculation of LRC in an extended shell-model basis is performed
in SF-B [8]. Moreover, the normalization of the two-nucleon overlap amplitudes is higher in the SF-B calculation. The difference in the shape of the cross section is due to the different mixing of configurations in the two cases. The defect functions are not only mixed differently by the different two-nucleon removal amplitudes, but different models are used to generate them in the two calculations, as well as different NN-interactions: Bonn-C in [8] Argonne V14 in [14].

The effect of the mutual interaction between the two outgoing nucleons (NN-FSI) has been neglected in the calculations presented till now. NN-FSI has been studied within a perturbative treatment in [6, 7], where it is found that the effect depends on the kinematics, on the type of reaction, and on the final state of the residual nucleus. NN-FSI effects are in general larger in pp- than in pn-knockout. For the $^{16}$O(e,e$'pn$)$^{14}$N reaction in the super-parallel kinematics the effects of NN-FSI were found small but non negligible [7]. The calculations in [7] were performed with SF-CC [14]. Since NN-FSI is sensitive to the various ingredients of the calculations, it can be interesting, also in view of the comparison with the (e,e$'pn$) data of [10], to investigate its effects in the present orthogonalized approach.

The contribution of NN-FSI to the cross section of the reaction $^{16}$O(e,e$'pn$)$^{14}$N to the $1^+$ and $2^+$ excited states of $^{14}$N in the super-parallel kinematics is shown in fig. 5. The results obtained in the approach considered till now (DW), where only the interaction of each one of the outgoing nucleons with the residual nucleus is considered, are compared with the results of the more complete treatment (DW-NN) where also the mutual interaction between the two outgoing nucleons is included within the same perturbative approach as in [7]. The calculations have been performed in the orthogonalized approach with the more refined SF-B overlap function and with the Bonn parametrization for the TB currents. For the $1^+_2$ state the contribution of NN-FSI is quite moderate. It is practically negligible at low values of $p_B$, where the cross section has its maximum, and it is appreciable at high missing momenta, where the cross section is much lower. In this region the slight enhancement and change of shape produced by NN-FSI might be appreciated in the comparison with the data [11]. For the $2^+$ state NN-FSI enhances the cross section up to about one order of magnitude at low values of $p_B$, where, however, the cross section remains completely dominated by the $1^+_2$ state, and is negligible above 100 MeV/c, and, therefore, also for the higher recoil momenta where the contribution of the $2^+$ state can be comparable to the one of the $1^+_2$ state.

The comparison between the cross sections obtained in the present approach, where orthogonality is enforced between s.p. bound and scattering states, and in the previous approach, where the contributions of the OB current without correlations is subtracted in the transition amplitude, is presented in fig. 6 for the photoinduced reaction $^{16}$O($\gamma$,$pn$)$^{14}$N to the $1^+_2$ excited state of $^{14}$N. Calculations have been performed in a super-parallel kinematics and for an incident photon energy which has the same value, $E_\gamma = 215$ MeV, as the the energy transfer in the (e,e$'pn$) calculations of figs. 1-5. Although the super-parallel kinematics is not well suited for ($\gamma$,$pn$) experiments, this case can be interesting for a theoretical comparison with the corresponding results of the electron induced reaction.

The orthogonalized approach enhances the cross section calculated with the OB current. This effect, that is shown in the right panel of fig. 6, is large although a bit lower than the one found in the corresponding situation for the (e,e$'pn$) reaction in fig. 1. The difference between the two results is significantly reduced when also the contribution of the TB currents is included. The final cross sections calculated in the two approaches differ at most by a factor of about 2 (dotted vs. dashed). As becomes obvious from the left panel, a larger difference is given in the case of the ($\gamma$,$pn$) reaction by the treatment of the TB currents. The cross section calculated with the orthogonalized
FIG. 6: The differential cross section of the $^{16}\text{O(}\gamma,\text{pn)}^{14}\text{N}$ reaction to the $1^+_2$ state as a function of $p_B$ in a super-parallel kinematics with $E_\gamma = 215$ MeV. Calculations are performed in the DW approach and with the same TOF as in fig. 1. Line convention for left and right panels as in fig. 1.

FIG. 7: The differential cross section of the $^{16}\text{O(}\gamma,\text{pn)}^{14}\text{N}$ reaction to the $1^+_2$ state as a function of $p_B$ in the same super-parallel kinematics as in fig. 6. Line convention for left and right panels as in fig. 4.

approach and with the Bonn parametrization (solid) is lower than the one obtained with the previous approach and the unregularized parametrization (dotted).

The cross sections displayed in fig. 6 have been calculated with the SF-B overlap function. The results obtained, with the orthogonalized approach and the Bonn parametrization, for the different TOF’s are compared in fig. 7. Also in this case large differences are found in the shape and in the magnitude of the calculated cross sections. The contribution of the OB current calculated with SF-B and SF-CC is much larger than with SM-SRC. The difference is, however, less dramatic than in the corresponding situation of fig. 4 for the $(e,e'\text{pn})$ reaction. In the final cross sections calculated with SF-B and SF-CC the TB currents produce a moderate enhancement of the OB contribution. In contrast, with SM-SRC the cross section is dominated by the TB currents which enhance the OB cross section by more than one order of magnitude. As a consequence of this enhancement, the final result with SM-SRC turns out to be closer to the SF-B one.

The effect of NN-FSI on the cross section of the reaction $^{16}\text{O(}\gamma,\text{pn)}^{14}\text{N}$ to the $1^+_2$ state in the super-parallel kinematics is shown in fig. 8. NN-FSI enhances the cross section. This effect is very small for low values of $p_B$ and increases at higher values, where the cross section decreases.

A different kinematical situation is considered in fig. 9, where the cross section of the reaction $^{16}\text{O(}\gamma,\text{pn)}^{14}\text{N}$ to the $1^+_2$ state has been calculated with an incident photon energy $E_\gamma = 400$ MeV in a coplanar symmetrical kinematics, where the two nucleons are ejected at equal energies and equal but opposite angles with respect to the momentum transfer. In this kinematical setting different values of $p_B$ are obtained changing the scattering angles of the two outgoing nucleons. In this case the cross sections are practically unaffected by the use of orthogonalized s.p. bound and scattering wave functions and also the effect of NN-FSI is small. The results are sensitive to the TB currents and to their treatment. The cross sections calculated with the SF-B overlap function for the two parametrizations are compared in the left and right panels. With the Bonn parametrization the enhancement produced by the TB currents
FIG. 8: The differential cross section of the $^{16}\text{O}(\gamma,\text{pn})^{14}\text{N}$ reaction to the $1^+_2$ state as a function of $p_B$ in the same super-parallel kinematics as in fig. 6. The calculations are performed in the orthogonalized approach and with the Bonn parametrization for the TB currents. TOF as in fig. 1. Line convention: DW-NN (solid), DW (dashed).

FIG. 9: The differential cross section of the $^{16}\text{O}(\gamma,\text{pn})^{14}\text{N}$ reaction to the $1^+_2$ state in a coplanar symmetrical kinematics with $E_\gamma = 400$ MeV as a function of the scattering angle of the outgoing nucleons. Calculations are performed in the orthogonalized approach and with the same TOF as in fig. 1, i.e. SF-B. The dotted lines give the separate contribution of the OB current, the dotted-dashed lines the sum of the OB and seagull currents, the dashed lines the sum of the OB, seagull, and pion-in-flight currents, and the solid lines the final result, where also the contribution of the $\Delta$-current is included. The Bonn parametrization is used for the TB currents in the left panel, the unregularized parametrization in the right panel. FSI are treated within the DW approach.

in the cross section is within a factor of 2 and the main contribution to this enhancement is given by the seagull current. A strong enhancement of the $\Delta$-current contribution is produced by the unregularized parametrization. In this case the contribution of the $\Delta$-current is dominant and increases the cross section by more than one order of magnitude.

The corresponding cross sections calculated with the SF-CC and SM-SRC overlap functions for the reaction $^{16}\text{O}(\gamma,\text{pn})^{14}\text{N}$ to the $1^+_2$ state in the coplanar symmetrical kinematics with $E_\gamma = 400$ MeV are displayed in fig. 10. With both SF-CC and SM-SRC, as well as with SF-B in fig. 9, the cross section is sensitive to the parametrization used for the TB currents. When the unregularized parametrization is used in the calculations, the $\Delta$-current is dominant both with SM-SRC and SF-CC. With SM-SRC the $\Delta$-current gives the main contribution also when the Bonn parametrization is used. For every TOF here considered the cross sections obtained with the two parametrizations differ by about one order of magnitude, while the shape is not particularly affected by the treatment of the TB currents. In contrast, large differences, both in the shape and in the magnitude of the calculated cross sections, are obtained with different TOF’s. This confirms that the treatment of correlations in the two-nucleon wave function affects all the ingredients of the calculations, and not only the OB current but also the TB currents.
Electromagnetically induced two-nucleon knockout is an ideal tool to study the role of correlations in the nuclear wave function going beyond the shell-model approach. In this paper, we have presented recent improvements in the theoretical model of proton-neutron knockout, which is of specific interest for the study of tensor correlations, which are suppressed in proton-proton knockout.

In comparison with earlier studies, a more complete treatment of CM effects has been included in the model. In the CM frame the transition operator becomes a two-body operator even in the case of a one-body nuclear current. As a consequence, the one-body current can give a contribution to the cross section of two-particle emission independently of correlations. These CM effects were not properly taken into account in our previous calculations for pn-knockout. They have been included in this work enforcing orthogonality between s.p. initial and final states by means of a Gram-Schmidt orthogonalization. This procedure, that was applied in [9] to pp-knockout, allows us to naturally include all the CM effects as well as to get rid of possible spurious contributions to two-nucleon emission, due to the lack of orthogonality between bound and scattering states obtained by the use of an energy-dependent optical model potential.

The treatment of the two body seagull, pion-in-flight and ∆-currents has been improved using a more realistic regularized approach which is consistent with the description of elastic NN-scattering data. The role of the mutual interaction between the two outgoing nucleons, that is usually ignored, has been reconsidered in combination with the present improvements. Last but not least, the sensitivity of the cross sections to NN-correlations has been studied comparing results obtained with different two-nucleon overlap functions, where correlations are included with more refined or simpler approaches.

It turns out that the effect of these different aspects strongly depends on the chosen kinematics. In the super-parallel kinematics, which is of particular interest with respect to experiment, the CM effects included with the enforced orthogonalization lead to a dramatic increase, up to an order of magnitude, of the contribution of the one-body current, which becomes dominant in the $^{16}O(e,e'pn)^{14}N$ reaction. In the final cross section the enhancement is particularly large for the transition to the $1^+_2$ (3.95 MeV) excited state of $^{14}N$ and for low values of the missing momentum. For different final states or in different kinematics the influence of these effects is small or even negligible. In particular, it is negligible when the cross section is dominated by the two-body currents, that are not sensibly
affected by the orthogonalization procedure.

The regularized treatment of the two-body currents leads in general to a dramatic reduction of the $\Delta$-current contribution with respect to the results obtained with the previous treatment. In contrast, the corresponding effects on the nonresonant seagull and pion-in-flight meson-exchange currents are of minor importance. The difference due to the regularized and unregularized parametrizations on the final cross section depends on the role played by the $\Delta$-current. In the super-parallel kinematics for the $(e,e'p)n$ reaction, the latter is only of minor importance so that the resulting difference between the different parametrizations is within a factor of about 2. Larger differences are obtained in the $(\gamma,p)n$ reaction, where in the symmetrical kinematics the regularized parametrization reduces the cross section by more than one order of magnitude.

The effect of the mutual interaction between the two outgoing nucleons is in general moderate although not negligible. This contribution does not change the qualitative features of the cross sections but it should be evaluated for a more careful comparison with the experimental data.

Dramatic differences, both in the shape and magnitude of the calculated cross sections are given, in all the considered situations and kinematics, by different treatments of correlations in the two-nucleon wave function. Correlations affect both one-body and two-body current contributions. A crucial role is played by tensor correlations. When tensor correlations are neglected in the overlap function the contribution of the one-body current to the cross section is strongly underestimated and becomes always negligible. The cross sections of the $^{16}\text{O}(e,e'p)^{14}\text{N}$ reaction in super-parallel kinematics differ up to about two orders of magnitude depending on whether tensor correlations are taken into account or not.

In addition, also a careful treatment of long-range correlations appears to be essential. In the calculations long-range correlations affect the two-nucleon removal amplitudes. These amplitudes determine the weights of the partial waves of relative and CM motion in the overlap function and may therefore affect the shape and also the magnitude of the cross section.

In conclusion, we may expect that the comparison with presently available data may lead to important conclusions about the structure of correlations and therefore to a test of our present understanding of nuclear structure in general. This topic will be outlined in future work [11].

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