A multi-temperature kinetic Ising model and its applications to partisanship dynamics in the US Senate

I. Mazilu, A. P. Lorson, S. Gibbs, W. Hanstedt, D. A. Mazilu

Washington and Lee University, 204 W. Washington Street, Lexington, VA 24450, USA

E-mail: mazilui@wlu.edu, lorsona21@mail.wlu.edu, gibbss21@mail.wlu.edu, hanstedt@grinnell.edu, mazilud@wlu.edu

Abstract. As the political landscape becomes increasingly complex, the classic paradigms used in political science have failed to remain relevant and other methods of study are needed. We introduce a population dynamics model and a multi-temperature kinetic Ising model to analyze the partisanship dynamics of the US Senate. We use Monte Carlo simulations, mean field theory and numerical analysis of the master equation of a system of 100 senators (agents) separated into various categories based on their political leanings and interactions with each other. Results show an interesting development of partisanship between the agents after a short time. The model can be extended to other cooperative stochastic systems in physics and social sciences.

1. Introduction

In recent years the areas of political and social sciences have become of interest to physicists and mathematicians as they provide a variety of interesting complex systems to study [1]. In politics, the majority of legislative voting analysis comes from third-party pollsters. Polling is an inherently biased process that more often than not involves a significant amount of human error [2]. On the other hand, voter models have been studied for a number of years in statistical physics. Analyzing voting behavior using statistical physics techniques may allow for more accurate models to be developed.

The United States Senate is one of two legislative voting bodies in The United States Federal Government. The Senate is comprised of 100 members, each with their own political leaning and agenda. Peer influence and external factors, such as the economy, public opinion, or international relations, have been proven to be powerful influences on the voting decisions of individual members. Using the US Senate as political background, we address two main questions. What are the partisanship dynamics of a group of individuals split into different categories depending
on their political affiliation? Is there a chance for consensus within a group of individuals with fixed political affiliations when it comes to a generic vote?

In order to address these questions, we offer two possible classes of models. The first class of models falls under the umbrella of population-dynamics models (section 2) and we present a simple three-state model, together with possible extensions. The second class of models are multi-temperature kinetic models that are governed by the Glauber dynamics [3] (section 3). We analyze these models by numerically solving the master equation for a one-dimensional multi-temperature kinetic Ising model. We finish our paper with some open questions and conclusions (section 4).

2. A three-state population dynamics model

Population dynamics is a well-established methodology that has been applied to a variety of systems. Most notably, a significant portion of population dynamics simulations have bases in the study of the spread of an infectious disease in a set population [4]. In population dynamics, the agents are split into different categories. The movement of these agents from one category to another is dictated by a system of ordinary differential equations. These equations often include correlation terms that create an interplay between the populations of each category. Depending on the system of equations, a steady-state can be achieved.

The overall idea of using population dynamics models to describe the dynamics of a group of politicians is to split the agents (in this case, US senators) into categories depending on their political affiliation and partisanship record [5]. Depending on the rules of a specific model, politicians are allowed to change categories with assigned probabilities that are derived from voting records of that particular individual, peer-to-peer interactions and external influence.

We illustrate here a simple three-category model, with agents divided into conservatives (population $P_1$), centrists (population $P_2$), and liberals (population $P_3$). We keep the total number $N = P_1 + P_2 + P_3$ constant and analyze the evolution of the percentage of conservatives, centrists and liberals $\frac{P_1}{N}, \frac{P_2}{N}, \frac{P_3}{N}$ with respect to time. $\alpha$ is defined as the probability of a liberal or conservative leaving their respective categories to become centrist and $\beta$ is defined as the probability of a centrist becoming a liberal or conservative.

The set of differential equations that dictates the time-evolution of these agents is:

$$\frac{dP_1}{dt} = -\alpha P_1 + \beta P_2$$
$$\frac{dP_2}{dt} = -2\beta P_2 + \alpha (P_1 + P_3)$$
$$\frac{dP_3}{dt} = -\alpha P_3 + \beta P_2$$

An important aspect of this model is the fact that a liberal has zero probability of becoming a conservative in one time step and vice versa. Each must first become centrists and then move into the respective opposite category based on the probabilities presented above. $\alpha$ and $\beta$ are varied and the results are compared. The initial values for $P_1$, $P_2$, and $P_3$ were based on approximations of the composition of the 114th United States Senate, a conservative leaning body with $P_1 = 40$, $P_2 = 30$, and $P_3 = 30$.

Presented below are the results from our three-state system with no peer-to-peer interactions built into the system of equations.
Figure 1. The evolution of the three population categories ($P_1$ - conservatives, $P_2$ - centrists, and $P_3$ - liberals) for probabilities $\alpha = 0.05$ and $\beta = 0.04$. The time scale is arbitrary.

Figure 2. The evolution of the three population categories ($P_1$ - conservatives, $P_2$ - centrists, and $P_3$ - liberals) for probabilities $\alpha = 0.05$ and $\beta = 0.075$. The time scale is arbitrary.

Figure 1 presents the evolution of each category for the case when the probabilities $\alpha$ and $\beta$ are very close to one another. We found that the closer these two probabilities were to one another, the more the centrist politicians grew in number. The steady-state is achieved in approximately the same number of time steps. On the other hand, if the probabilities are starkly different from one another, as in Figure 2, where $\beta$ is almost twice the value of $\alpha$, the conservatives and liberals make up larger portions of the population. This interesting difference is easily interpreted in this simple model. In order for the Congress to be more polarized than how it started out, the liberals and conservative must be twice as stubborn as the centrists.

3. Multi-temperature kinetic models with Glauber dynamics

We now discuss a different approach to the problem using a generalization of a benchmark spin system model [3] solved by R. Glauber and known in literature as the Glauber model. Glauber solved the one-dimensional Ising model exactly and calculated the magnetization and the two-point spin correlation functions. An interesting generalization of Glauber’s model is the two-temperature kinetic Ising model that was first introduced by Racz and Zia [6] who calculated exactly the two-point correlation functions for the steady state.

The one-dimensional multi-temperature kinetic model is defined as a chain of sites that can be occupied with a spin with two possible values: $-1$ (“spin down”) and $+1$ (“spin up”). Each cell interacts with its two nearest neighbors and is in contact with a heat bath of temperature $T_n$.

The transition between any two configurations $\sigma$ and $\sigma'$ is described by the generalized Glauber transition rates $W[\sigma \rightarrow \sigma']$. The rate $W$ is non-zero only if the two configurations differ in the spin of a single particle. The rate at which site $n$ has its spin flipped is given by:
\[ W_n = \frac{1}{2} - \frac{\gamma_n}{4} s_n (s_{n-1} + s_{n+1}) \]  

(2)

where \( k_B \) is Boltzmann’s constant. The factor \( \gamma_n (0 \leq \gamma_n \leq 1) \) is related to the temperature of cell \( n \) by \( \gamma_n = \tanh(\frac{2k_B T_n}{s_0}) \), and \( s_n \) is the state (+1 or -1) of the \( n \)-th cell. This rate equation prescribes a spin flip rate of \( 1/2 \) for a cell if cells to the left and right have opposite spins, \( (1 - \gamma_n)/2 \) if adjacent spins are the same and the same as that of cell \( n \), and \( (1 + \gamma_n)/2 \) if adjacent spins are the same and opposite that of cell \( n \). The time scale is arbitrary.

The magnetization of the system is defined as:

\[ m_i(t) = < s_i(t) > = \sum_s s_i P(s_1,..s_N, t). \]  

(3)

For the case of periodic boundary conditions, the equations of motion for the magnetization, derived from the master equation are:

\[ \frac{d m_n}{d t} = -m_n + \frac{\gamma_n}{2} (m_{n+1} + m_{n-1}) \]  

(4)

For non-periodic boundary conditions (hard walls), the equations for the spin magnetizations are (the bulk of the spins is represented by \( n = 2...N - 1 \)):

\[ \frac{d m_1}{d t} = -m_1 + \frac{\gamma_1}{2} m_2 \]

\[ \frac{d m_n}{d t} = -m_n + \frac{\gamma_n}{2} (m_{n+1} + m_{n-1}) \]

\[ \frac{d m_N}{d t} = -m_N + \frac{\gamma_N}{2} m_{N-1} \]  

(5)

The master equations are solved using standard numerical Python packages (ODEINT from SCIPY). The initial states and temperatures are assigned using NUMPY arrays, and subsequent states are collectively stored in a matrix. Each particle’s change is tracked over time and specific particles of interest can be singled out for plotting in addition to the overall magnetization.

We applied this model to the US Senate by considering the situation where all the senators are voting. Each site represents a senator who, given a specific issue, votes ‘yes’ (spin +1) or ‘no’ (spin -1). We considered hard boundaries to reflect the lack of influence between the most liberal and most conservative senators. We split the lattice into categories based on their political leanings, each distinct category receiving its own temperature bath. We assign initial average vote values based on their category, and allow their interactions over time to influence both the individual and collective vote magnetization.

We find that regardless of initial conditions, given enough time, all of the individual magnetizations (average individual vote) will move towards a steady-state of zero (lack of consensus for a ‘yes’ or ‘no’ vote.) The collective magnetization (total average vote) moves in the direction of ‘lower temperatures’, where the specific temperature can be interpreted as a quantified ‘stubbornness’. Higher temperatures are equivalent to less stubbornness and lead to a quicker change in the average vote.
We also find that the senators at the edge of the political spectrum move inwards more quickly than their neighbors of the same political category. This is a result of having one less connection, and in the master equation we see that it is these connections which are resisting the inevitable movement towards the steady-state. As a result, the outside edges encourage the change of the outermost categories, speeding up the process of the entire system reaching the steady-state.

Figure 3. The evolution of individual magnetizations for a system with a population $N = 100$ split into 5 categories (A through E) with respective generic ‘temperatures’ of 2.5; 4.5; 5.5; 3.5; and 2.0. The time scale is arbitrary.

Figure 4. The evolution of the overall magnetization of the five-temperature system with a population $N = 100$ presented in Figure 3. The time scale is arbitrary.

Used in the context of a political system, this model leads to a dire conclusion: no matter the initial conditions (vote), given enough time all categories are moving toward zero magnetization, which means no consensus (more apathy) for the whole system, but also no definitive vote for each category. This is not a very realistic conclusion, since legislation does get passed once in a while (for the 114th US Congress, approximately 9% of the legislation introduced was passed.) A very interesting feature for the overall magnetization of the system is the extremum of the magnetization curve. That means that the short-term behavior of the system is very different. If the final vote is taken early enough in the process, there is a chance of a non-zero magnetization, and a vote majority. This is much more realistic for how things operate in the US Senate. The vote happens a limited number of times (more likely one or two times, at most). So the decision tends to fall under ‘yes’ or ‘no’.

To complement the numerical solutions of the master equation, we also used Monte Carlo simulations that implemented the Glauber transition rate presented in equation (2). We present below a sample comparison for the overall magnetization (overall vote) for a one-dimensional system with 100 sites, divided into fifths with temperatures of 1, 0.75, 1.25, 1, 0.75. Initially, the first half of the sites were spin 1 and the second spin -1. As shown in Figure 5, we achieved a very good agreement between the methods. For clarity, we focused on the short-term behavior of the system before it reaches steady state, which happens on a different time scale than for the example presented in Figure 4.
4. Conclusions

We presented in this paper two possible classes of models that may be used to study the dynamics of a group of individuals with different political leanings in terms of the evolution of its categories, or the evolution of a vote for a repeated voting process within the group. Analyzing any voting system in a purely quantitative manner presents unique challenges that are often hard to overcome. Specifically, with our model, the United States Senate is a small non-representative sample with many factors that cannot be accounted for quantitatively. For example, a conservative politician who votes nearly always along party lines may vote to fund a specific ‘liberal’ issue because it would positively affect the state that they represent. Personal biases and the influence of lobbying are also very hard to quantify as there has been little reputable research published on either of these topics. However, our population model did succeed in depicting the slow and inevitable polarization that has been observed in our modern-day political system.

We used the multi-temperature kinetic Ising model as a toy model to depict the voting dynamics for a group of senators split into five categories: far left, moderate left, centrist, moderate right, far right. This is an interesting modeling exercise that can be also used in the classroom as an example of the versatility of statistical physics models. This model also has the potential to be extended by introducing, for example, the equivalent of an external magnetic field, which, in this case, can account for external factors such as public opinion, lobbying, economic conditions or international relations.

However, studying multi-temperature kinetic systems goes beyond modeling US political problems. These models are extremely versatile for a variety of fields, from biochemistry to polymer science, to social systems. We plan to work on a theoretical framework for these systems to identify patterns in connection with special matrices in matrix theory.
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