Characteristic analysis of inductor in pulsed power supply changing with shape ratio

Shuangpeng Hao¹, Zhenxiao Li¹, Fuqiang Ma¹ and Baoming Li¹

¹ National Key Laboratory of Transient Physics, NJUST, Nanjing, Jiangsu, China 210094

E-mail: hspnjust@163.com

Abstract. Solenoid inductor is an important component of pulse power source, its volume, electrical performance, stress and magnetic field generated during discharge directly affect the relevant parameters and performance of pulse power source. This paper presents a calculation model for solenoid inductor in pulsed power supply, given the inductance value and the wire, the volume, stress, resistance and external magnetic intensity of the five inductance values inductor under different shape ratio are calculated by changing the number of turns, and discusses and analyzes the changes and extremum in the volume, resistance, stress, and magnetic intensity of inductor with shape ratio. This article has certain guiding significance for the design of solenoid inductors.

1. Introduction

As the key technology and energy source of electromagnetic emission, the pulse power supply is of high voltage, pulsed high current and high power [1,2]. Solenoid inductor is a key component of pulse power supply, which plays a role in limiting discharge current and adjusting pulse width [3,4]. With the development of pulsed power supply, increasingly higher requirements are placed on inductor, which require inductor to have lower resistance, occupy less space, and lower external magnetic field strength.

In the published literature, formulas for calculating inductance are “forward” formulas, meaning calculate the inductance value from the given dimensions such as size, shape, and number of turns [5]. “Backward” formulas for calculating inductance has no unique solution, a few large turns, or many small turns will deliver the same inductance value, a lot of research has done on their optimal forms. James Clerk Maxwell stated that when the total length and thickness of the wire being given, if the channel in which the coil is wound has a square transverse section, the mean diameter of the coil should be 3.7 times the side of the square-section of the channel, the form of a coil for which the coefficient of self-induction is a maximum [6]. Grover considers a range of practical examples including the “Most Economical Coil Shape” which is the “given piece of wire” optimum [5]. Takaaki Ibuchi gave the optimal design sizes of a single-layer solenoid to minimize copper loss for a given
wire diameter and inductance value [7]. In the design process of the solenoid inductor, given the inductance value, it is necessary to consider not only the resistance of the inductor, but also various factors such as volume, stress and external magnetic field.

In this paper, a solenoid inductor calculation model was established, and the formula for calculating the inductance value, volume, formula, resistance, and external magnetic field of the solenoid inductor was given. By changing the number of solenoid coil turns, that is, changing the shape parameter of the inductor, calculate the volume, resistance, stress and magnetic field strength of the inductor under different shape ratios, and Analyze and discuss its changes.

2. Calculation model

Three-dimensional perspective of the solenoid inductor is shown in figure 1. The solenoid coil, connection terminals and the insulation cylinder are the main components of the solenoid inductor. The solenoid coil are wound with copper wire of rectangular cross section. The connection terminal is used to connect the conductive copper plate and is welded on the end of the solenoid coil. The insulation cylinder made of epoxy resin reinforced with glass fibers, which has excellent insulation and mechanic properties. It is the skeleton of inductance and interlayer insulation material, is used to reinforce the structure and the spiral coil insulation.

![Solenoid Inductor Diagram](image)

**Figure 1.** Three-dimensional perspective of the solenoid inductor.

The assumptions are introduced here to reduces the computational difficulty;

1) Ignoring the effects of the import and export connection terminal;

2) Pulse current frequency is low, assuming current is evenly distributed along the cross section of the solenoid coil;

3) The solenoid coil can be simplified as a number of axisymmetric circular loops whose number is equal to the number of turns of the coil. Each circular loop is assumed to be separated from is neighboring loop by the pitch of the coil;

4) The magnetic conductivity of epoxy resin and glass fibers is the same as that of air, so insulation cylinder is not considered in the calculation of inductance and magnetic field.

The simplified solenoid inductor calculation model is shown in figure 2. The meanings of the parameters in the figure 2 are shown in the table 1.
Figure 2. Calculation model of the solenoid inductor.

Table 1. List of principal symbols.

| Symbols | Meaning |
|---------|---------|
| $a$     | The inner radii of circular loops |
| $p$     | The distance between two adjacent loops (pitch of spiral coil) |
| $t$     | The cross-sectional dimension of copper wire in axial direction |
| $w$     | The cross-sectional dimension of copper wire in radial direction |
| $N$     | The number the loops |
| $b$     | Outside insulation thickness in radial direction |
| $s$     | End insulation thickness in axial direction |

3. Calculation equations

3.1. Calculation equations of inductance value

The inductance of the designed inductor should conform to the requirement of inductor electrical parameters. According to literature [5] and [8], the inductance of the solenoid inductor is given by

$$L = \frac{\mu_0 H_0}{W^2} \sum_{n_2=1}^{N} \sum_{n_1=1}^{N} \int_{0}^{\pi} \int_{r_1}^{r_2} \int_{z_1}^{z_2} \cos \theta d\theta \, d\phi \, dz$$

$$= \frac{\mu_0 H_0}{W^2} \sum_{n_2=1}^{N} \sum_{n_1=1}^{N} \left[ r_2^2 \int_{r_1}^{r_2} \phi_1^{(n_2-1)+1} \, dr_1 \right] \int_{z_1}^{z_2} \frac{dz_2}{R}$$

(1)

where:

$$R = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \theta + (z_2 - z_1)^2}$$

(2)

Where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space (vacuum), $r_1$, $r_2$, $z_1$ and $z_2$ are the cylindrical coordinates corresponding to the radii and azimuths of coil $n_1$ and coil $n_2$, respectively, and $\theta$ corresponds to the angular coordinate in a cylindrical coordinate system.
3.2. Calculation equation of the inductor volume
As the requirements for miniaturization of pulse power source modules become higher and higher, the inductance needs to occupy as little space as possible. The volume of inductance is:

\[ V = 2\pi \left( (N-1)p + t + 2s \right)(a + w + b) \quad (3) \]

3.3. Calculation equations of the inductor maximum stress
The resultant force of radial magnetic force of each ring of the solenoid coil is zero, so only the axial magnetic force is considered. The axial magnetic force of the first turn of the solenoid circle with the highest axial magnetic force is:

\[ F_{z_{\text{max}}} = \sum_{n=2}^{N} \frac{\mu_0 I^2}{w^2 t^2} \int_{0}^{\pi} \cos \theta d\theta \int_{a}^{r_{1}} r_{1} dr_{1} \int_{a}^{w} r_{1} dr_{1} \int_{(k-1)p}^{(k-1)p+t} \int_{(n-1)p}^{(n-1)p+t} \left( \frac{z_{2} - z_{1}}{R^2} \right) dz_{2} \quad (4) \]

Where the symbols \( I \) is the maximum electric currents imposed coils. The maximum stress on the insulation cylinder is:

\[ \sigma_{z_{\text{max}}} = \frac{F_{z_{\text{max}}}}{\pi (w^2 + 2aw)} \quad (5) \]

The maximum stress calculated by (5) should be less than the ultimate compressive strength of epoxy resin, will not cause the inductor material failure.

3.4. Calculation equation of the inductor magnetic intensity
The strong magnetic field generated by the pulse inductor during the discharge process has greatly restricted the development requirements for the miniaturization of pulse power supply. According to literature [9], The magnetic strength at point \((r, z)\) is:

\[ B(r, z) = \frac{\mu_0 I}{4\pi wt} \sum_{n=1}^{N} \int_{a}^{r_{1}} r_{1} dr_{1} \int_{0}^{\pi} d\theta \left[ \int_{(k-1)p}^{(k-1)p+t} \frac{(z_{2} - z_{1}) \cos \theta}{R^3} dz_{2} + \frac{(a)_{p+t} r_{1} - r_{2} \cos \theta}{R^3} dz_{2} \cdot e_{z} \right] \quad (6) \]

3.5. Calculation equation of the inductor resistance
The low-resistance inductor can improve the emission efficiency of pulse power supply and reduce the absorbed heat. It makes sense to minimize the resistance of the inductor. The resistance of inductance is:

\[ R_{z} = \rho \frac{l}{S} = \rho \frac{\sqrt{2\pi (2a + w)}}{wt} \quad (7) \]

Where the length and cross-sectional area of solenoid coil are expressed by \( l \) and \( S \). the resistivity of the solenoid coil material copper are expressed by \( \rho \).
4. Solenoid inductor characteristics as a function of shape ratio

In this section, by changing the number of turns of the coil, that is, changing the shape ratio of the spiral coil, we discuss the characteristics dependence on the shape ratio for 10µH, 20µH, 30µH, 40µH and 50µH solenoid inductor. The fix parameters are \(w=12\text{mm}, b=s=t=10\text{mm}, p=12\text{mm}, l=100\text{kA}\).

The shape ratio of solenoid inductor as the following equation:

\[
\beta = \frac{NP}{2a + w}
\]  

\(\text{(8)}\)

Figure 3. Volume as a function of shape ratio.

Figure 3 shows the volume dependence on the shape ratio for solenoid inductor. The results shown in figure 3 indicate that the volume of solenoid inductor reaches a minimum and remains constant when \(\beta\) is greater than 2.4. When \(\beta\) is less than 2.4, the smaller the value of \(\beta\), the larger the volume of the inductor. When \(\beta\) is 0.13, the volume of solenoid inductor is more than double that when \(\beta\) is greater than 2.4. It can also be seen in figure 3 that the minimum volume of the inductor is in a multiple relationship with the inductance value of the inductor.

![Figure 3](image)

The calculated resistance is shown in figure 4. Resistance is minimum at a solenoid shape ratio is 0.4. When \(\beta\) is greater than 0.4, the resistance value of the inductor increases as shape ratio \(\beta\) increases. The resistances of the five inductors are more than half larger when \(\beta\) is 7 than when \(\beta\) is 0.4. It can be seen that the shape ratio has a greater impact on the resistance of the inductor. It can also be seen in

![Figure 4](image)
figure 4 that the larger the inductance value, the greater the resistance value of the inductor.

![Figure 4](image)

Figure 4. Stress as a function of shape ratio.

Figure 5 shows the maximum stress dependence on the shape ratio for solenoid inductor. It can be seen from figure 5 that when $\beta$ is about 0.65, the maximum stress on the insulation cylinder is the largest. The maximum stress value of the five inductors is more than half when the shape ratio $\beta$ is 0.65 than when $\beta$ is 7. Therefore, when the mechanical properties of the insulation material of the inductor cannot meet the design requirements of the inductor, the shape parameters ratio $\beta$ of the inductor can be adjusted to meet the design requirements.

![Figure 5](image)

Figure 5. Stress as a function of shape ratio.

![Figure 6](image)

Figure 6. External magnetic intensity as a function of shape ratio.
Figure 6 (a), (b), (c), and (d) are the magnetic intensity on the inductance axis at the inductor end, 2cm, 4cm and 6cm away from the inductor end respectively (points of A, B, C and D in figure 2. As shown in figure 6, with the same shape ratio $\beta$, the larger the inductance value, the greater the external magnetic intensity of the inductor. It can be seen from figure 6(a) that the magnetic intensity on the inductance axis at the inductor end (point A) is maximum when the shape ratio $\beta$ is 1.7. When the shape ratio $\beta$ is greater than 1.7, the magnetic intensity at point A gradually decreases as the shape ratio $\beta$ increases. At the same position, the larger the inductance value, the larger the shape ratio $\beta$ at the maximum magnetic intensity. When the inductance value is 10$\mu$H, the shape ratio $\beta$ at the maximum magnetic intensity at points A, B, C and D are 1.7, 0.9, 0.6, and 0.45 respectively. Therefore, the farther away from the inductor, the smaller the shape ratio $\beta$ at the maximum magnetic intensity.

5. Conclusion
In this paper a calculation model of solenoid inductor is established, and gives the calculation formulas for the inductance, volume, resistance, maximum stress, and external magnetic intensity of solenoid inductor.

The volume, resistance, maximum axial stress and magnetic intensity of five kinds of inductors under different shape ratios are calculated, and their variation laws are analyzed.

It is obtained that when the shape ratio of the inductor is greater than 2.4, the volume of the inductor reaches a minimum and remains unchanged, the inductor has the smallest resistance when the shape ratio is 0.4. When the shape ratio of the inductor is about 0.65, the maximum stress of the inductor is the largest, when the shape ratio of the inductor is about 1.7, the magnetic intensity on the inductance axis at the inductor end.

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