Effect of porosity and profile axial loading on elastic buckling and free vibration of functionally graded porous beam.

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Abstract. Elastic buckling and free vibration behaviour of porous functionally graded beam under profile axial mechanical loading is presented analytically. Symmetric and asymmetric porosity distribution with non linear material functional grading is presented. Buckling study is carried out first, then free vibration study under buckling loads is presented with profile loading. The effect of porosity, porosity pattern, slenderness ratio, boundary condition on critical buckling load and fundamental frequency is also studied. Ritz method with shear and normal deformable beam theory (SNDBT) is employed for formulation. It is observed that, with increase in porosity, buckling load decreases for both porosity distribution whereas, fundamental frequency increases for symmetric porosity distribution and is vice versa for asymmetric porosity distribution under all loading conditions.

1. Introduction

Functionally graded material (FGM), are inhomogeneous composite structures, whose properties are tailored in the desired directions based on the applications. Porosity induced structure with lower specific weight in combination with the FGMs have broad scope in defense, automobiles, aerospace and various other engineering fields. Buckling and free vibration study are both fundamental in structural design. Simsek et al. [1] studied static bending and buckling behaviour of functionally graded nano beams by nonlocal-Timoshenko theory analytically. Sunil et al. [2] experimentally studied vibration and buckling behaviour of synthetic foam core sandwich composite beam under compressive axial loading. Chen et al. [3] studied static bending and buckling of FG porous metal foam beam by shear deformable Timoshenko beam theory. Free vibration and buckling of FG-Carbon nano tubes reinforced composite cylindrical panel subject to thermal loading is presented by Vinod et al. [4] by finite element approach. Amol et al. [5] studied the effect of hybridization of glass and sisal fabric composite on thermal buckling characteristics experimentally. The hygro-thermal effect on the mechanical buckling of angle-ply and cross ply composite beams is studied by Mokhtar and Ashraf [6] analytically using Hyperbolic refined shear deformation theory. Nivesh et al. [7] studied vibration and buckling of FG-carbon nano-tube reinforced composite plate when subject to non uniform thermal loading by finite element approach. Vijay et al. [8] presented free vibration behaviour of FG-graphene reinforced composite plates, under non uniform edge loading analytically based on Reddy's theory. Mirmeyesam and Bekir [9] investigated free vibration and static buckling of porous FG reinforced...
plates, based on first order theory using spectral Chebyshev approach. Yang et al. [10] studied the dynamic response of Euler Bernoulli beam having open edge cracks, under axial compressive load and moving concentrated transverse load. Simsek and Kocaturk [11] studied the dynamic output of FG simply supported beam analytically, under the application of moving concentrated harmonic load. Arunkumar et al. [12] studied the bending and free vibration behaviour of foam filled truss panel by numerical approach. Vibration response of porous FG beam is presented by Chen et al. [13] using Ritz approach. Fang et al. [14] studied the free vibration and thermal buckling analysis of rotating axial non-local, FG nano beam based on the integration of Eringen's non-local elasticity theory with Euler-Bernoulli beam theory. Armagan and Metin [15] studied the elastic buckling of beams under axially varying in-plane mechanical loads for different boundary conditions. They employed Ritz method coupled with SNDBT theory. Thuc et al. [16] presented free vibration behaviour of composite beams under axial mechanical loading using SNDBT theory. The effect of axial loads on buckling and its mode shapes for FG beam is investigated by Melaibari et al. [17] using differential quadrature method(DQM). Ashish et al. [18] studied the vibration and acoustic response of porous graphene foam, by using refined plate theory. Vijay et al. [19] presented the vibro acoustic behaviour of isotropic plate under steady state excitation, with non uniform edge loading analytically by employing Reddy’s theory.

From the above literature survey, it can be said that the effect of profile axial mechanical loading on buckling and free vibration in the presence of porosity for a nonlinear FGM beams is a field still unexplored. Hence effort is made in presenting the behavior of different porosities under profile loading as a comparative study.

2. Methodology
A beam having length $L$, width $b$ and height $h$, is taken under consideration with three boundary conditions [Pinned-Pinned(P-P), Fixed-Fixed (F-F) and Fixed-Pinned (F-P)]. Symmetric (Porosity pattern-I) and Asymmetric (Porosity Pattern-II) porosity distribution are studied. The reader is referred to Chen et al. [13] for details regarding material distribution. Four different profile loadings($N(x)$) are studied as shown in figure 1. The external axial in-plane profile mechanical loading is given by equation (1). The critical buckling load is determined from equation (2) and natural frequencies are obtained from equation (3).

\[
N(x) = N_0 \left[ \alpha_1 \left( x + \frac{L}{2} \right)^2 + \alpha_2 \left( x + \frac{L}{2} \right) + \alpha_3 \right]
\]

\[
([K] - N_0[M])\{U\} = 0
\]

\[
([K - G] - \omega^2[M])\{U\} = 0
\]

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**Figure 1.** Fixed-Fixed beam with profile load description.
Where, $\alpha_1$, $\alpha_2$ and $\alpha_3$ are constants which depend on the type of profile loading, $\omega$ is fundamental natural frequency and $N_0$ is critical buckling load, $[M]$ mass matrix, $[K]$ stiffness matrix and $[G]$ the geometric stiffness matrix.

3. Results

In this study, a beam of $L=1m$, $b=h$, is used and studied with two slenderness ratios ($L/h$) of 20 and 100. Critical buckling load and critical fundamental frequencies are calculated for different porosity distribution and porosity intensities ($\zeta$) under various profile loading. The buckling load ($P_0$) and fundamental natural frequencies ($\omega_0$) are expressed in non-dimensional form which are given in equation (4) and equation (5) respectively, the symbols $E$, $I$, $\vartheta$, $\rho$ meaning young's modulus, moment of inertia, poission's ratio and density respectively.

$$P_{cr} = \frac{P_0 L^2}{EI}$$  \hspace{1cm} (4)

$$\bar{\omega} = \omega_0 L \left( \frac{\rho (1 - \vartheta^2)}{E} \right)^{1/2}$$  \hspace{1cm} (5)

3.1. Mechanical Buckling Study

Effect of different profile loading, slenderness ratio ($L/h$) and porosity co-efficient ($\zeta$), on the dimensionless buckling co-efficient ($P_{cr}$), for a beam of $L=1m$ and F-F boundary condition (BC), for two porosity patterns is presented in table 1. Results show that, as porosity increases, the stiffness of the structure decreases, hence a decreasing trend of $P_{cr}$ is observed for all the investigated cases. With increase in slenderness ratio from 20 to 100, the critical buckling load decreases by a large margin for all loading cases, this is because of decrease in structural stiffness with increasing ($L/h$) ratio. It is observed that, NLR provides the highest critical buckling load where as NLL profile loading provides the lowest critical buckling load considering both PP-I (porosity pattern-I) and PP-II (porosity pattern-II). It can be said that, PP-I with symmetric porosity distribution has higher critical buckling load compared to PP-II, for all the loading cases studied.

Referring to figure 2 (a) and (b), a decreasing trend of $P_{cr}$ for both PP-I and PP-II is observed with increase in porosity, with PP-II showing a higher slope. It can be observed that, $P_{cr}$ for Neon and NPS profile load is comparable for both the porosity distributions. Referring to figure 2 (c) and (d), it can be said that, as the boundary conditions change from F-F, F-P to P-P, the stiffness of the structure decreases, hence a proportionate decrease in $P_{cr}$ is observed for both porosity distributions, under all the loading cases. The effect of ($L/h$) ratio on critical buckling load is studied in figure 2 (e) and (f) for F-F BC, it can be observed that, there is proportionate decrease in critical buckling load as ($L/h$) ratio increases from 20 to 100, for all the loading conditions.

Hence from the buckling study, it can be concluded that, symmetric porosity distribution provides better buckling strength compared to asymmetric porosity distribution.
Table 1. Effect of porosity intensity (ζ), porosity pattern, slenderness ratio (L/h) and type of profile loading on dimensionless buckling co-efficient of beams with PP-I and PP-II (L=1m, F-F BC).

| L/h | ζ   | Porosity Pattern-I (PP-I) | Porosity Pattern-II (PP-II) |
|-----|-----|---------------------------|-----------------------------|
|     |     | Ncon | NLL  | NLR  | NPS | Neon | NLL  | NLR  | NPS |
| 20  | 0.2 | 70.2940 | 52.9290 | 101.8571 | 68.5309 | 50.0137 | 96.2537 | 64.7580 |
| 20  | 0.4 | 64.6961 | 48.7163 | 93.7353 | 63.0726 | 56.2689 | 42.3691 | 81.5326 | 54.8575 |
| 100 | 0.2 | 69.6640 | 52.4830 | 100.8105 | 67.8968 | 65.8206 | 49.5865 | 95.2524 | 64.1508 |
| 100 | 0.4 | 64.1360 | 48.3200 | 92.8041 | 62.5091 | 55.7700 | 42.0157 | 80.7040 | 54.3552 |

3.2. Free Vibration Study

Effect of different profile loading, slenderness ratio (L/h), and porosity co-efficient (ζ), on the dimensionless fundamental frequency (ω̅), for a beam of L=1m and F-F boundary condition (BC), for two porosity patterns is presented in table 2. Fundamental frequencies are extracted under the buckling load of 0.5 times the lowest critical buckling load considering all the loading scenario for L/h of 20 and 100 respectively. The lowest critical buckling load for (L/h) of 20 is 36,98,783 N and 5,886 N for slenderness ratio of 100. It can be observed that, with increase in porosity, ω̅ increases for PP-I and decreases for PP-II, under all loading conditions and slenderness ratios. As slenderness ratio increases from 20 to 100, ω̅ decreases for all loading cases. This can be explained as, with increase in length of structure, the stiffness of the structure decreases hence, fundamental frequency decreases.

Referring to figure 3 (a),(b),(c) and (d), it can be said that, ω̅ increases with increase in porosity for PP-I and it decreases for PP-II. It is also observed that, as buckling load increases from 0.25Pcr to 0.75Pcr, the fundamental frequencies shift towards higher frequency domain, due to increase in stiffness of the structure with increasing loading. This trend is same for all four loading cases. It is observed that, NLL profile loading offers higher fundamental frequencies whereas NLR loading offers the lowest of the four cases for both PP-I and PP-II FGM beams.

Referring to figure 4 (a) and (b), the effect of boundary condition has been presented here for PP-I and PP-II FGM beams respectively. As BC changes from F-F, F-P to P-P, the stiffness of the structure decreases, hence ω̅ decreases proportionately for all four loading cases. The decrease in ω̅, due to increase in slenderness ratio from 20 to 100, because of decrease in stiffness is observed in figure 4 (c) and (d) for PP-I and PP-II FGM beams respectively.
Figure 2. Effect of porosity coefficient ($\zeta$), slenderness ratio ($L/h$) and boundary conditions (BC) on buckling characteristics of FGM-beams with PP-I and PP-II.
Table 2. Effect of porosity intensity($\zeta$), porosity pattern, slenderness ratio(L/h) and type of profile loading on dimensionless fundamental frequency of FGM-beams with PP-I and PP-II. (L=1m, F-F BC, buckling load $\Delta P=0.5P_{cr}$, $P_{cr}=36,98,783N$ for (L/h)=20 and $P_{cr}=5,886N$ for (L/h)=100).

| Loading Pattern | Porosity Pattern-1(PP-I) | Porosity Pattern-2(PP-II) |
|-----------------|--------------------------|---------------------------|
|                 | Neon | NLL | NLR | NPS | Neon | NLL | NLR | NPS |
| 0.2             | 0.33724 | 0.34731 | 0.32672 | 0.33708 | 0.33001 | 0.34032 | 0.31923 | 0.32985 |
| 20              | 0.34010 | 0.35094 | 0.32875 | 0.33993 | 0.32278 | 0.33422 | 0.31073 | 0.32256 |
| 0.6             | 0.34650 | 0.35834 | 0.33406 | 0.34629 | 0.31368 | 0.32678 | 0.29974 | 0.31338 |
| 100             | 0.2   | 0.06739 | 0.06941 | 0.06529 | 0.06736 | 0.06584 | 0.06379 | 0.06591 |
|                 | 0.4   | 0.06798 | 0.07015 | 0.06571 | 0.06794 | 0.06450 | 0.06679 | 0.06209 | 0.06446 |
|                 | 0.6   | 0.06928 | 0.07165 | 0.06679 | 0.06924 | 0.06269 | 0.06531 | 0.05991 | 0.06263 |

Figure 3. Effect of porosity coefficient($\zeta$) and porosity intensity on dimensionless fundamental frequency of FGM-beams with PP-I and PP-II under varying buckling load $\Delta P$, for various profile loadings. ($P_{cr}=36,98,783N$).
4. Conclusions
Effect of profile in-plane axial mechanical loading, porosity pattern, porosity intensity, \((L/h)\) ratio and BC on buckling and free vibration of FGM beam is presented in this study. Ritz method is used in conjunction of SNDBT theory to carry out the study in a comparative manner. The following fundamental conclusions can be derived from this study.

- Profile axial loading has significant effect on the buckling and vibration characteristics.
- Porosity effect decreases the critical buckling load.
- Porosity pattern has significant contribution on the free vibration characteristics of the P-FGM beam.

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