Magnetic moments of negative parity heavy baryons in QCD

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Abstract

The magnetic moments of the negative parity, spin-1/2 baryons containing single heavy quark are calculated. The pollution that occur from the transitions between positive and negative parity baryons are removed by constructing the sum rules from different Lorentz structures.

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1 Introduction

Last decade was quite fruitful in the field of heavy baryon spectroscopy. At present, all baryons containing heavy charm and bottom quarks, except $\Omega_b$ baryon, as well as several heavy baryons with negative parity, have been observed by a number of collaborations (for a review see [1]). These progresses stimulated further theoretical studies, and experimental researches on heavy baryons at the existing facilities, especially at LHC.

Heavy baryons are well recognized to represent a rich “laboratory” for theoretical investigations. The properties of heavy baryons have been studied in framework of the various methods such as, relativistic quark model [2], variational approach [3], constituent quark model [4], lattice QCD model [5], QCD sum rules method [6]. Recent progress on this subject can be found in [7].

One of the most crucial quantities in discovering the internal structure of the baryons is their magnetic moments. Magnetic moments of the ground state $J^P = \frac{1}{2}^+$ heavy baryons have widely been studied in literature. Furthermore, magnetic moments of the heavy baryons have been investigated in the naive quark model in [8, 9], phenomenological quark model [10], relativistic three-quark model [11], variational approach [12], nonrelativistic quark model with screening and effective quark mass [13, 14], nonrelativistic hypercentral model [15], chiral constituent quark model [16], chiral bag model [17], chiral perturbation theory [18], traditional QCD sum rules [19], and light cone QCD sum rules (LCSR) [20–23], respectively.

In the present work, we calculate magnetic moments of the negative parity, spin-1/2 heavy baryons in framework of the light cone QCD sum rules method (for a review about this method, see for example [24]).

The body of the paper is organized as follows. In section 2, we construct the light cone QCD sum rules for the negative parity, heavy baryons. Section 3 is devoted to the numerical analysis of these sum rules for the magnetic moments. This section also contains discussion of the obtained results, and comparison with the prediction of the other approaches.

2 Construction of the sum rules for magnetic moments of the negative parity heavy baryons

In this section the LCSR for the negative parity baryons are derived. In order to formulate the relevant sum rules, we consider the following correlator,

$$\Pi(p,q) = i\int d^4 xe^{ipx} \langle 0| T\{\eta_Q(x)\bar{\eta}_Q(0)\}|0\rangle_\gamma ,$$

(1)

where $\gamma$ means the external magnetic field, $\eta(x)$ is the interpolating current of the corresponding heavy baryon with spin-1/2. In order to obtain the sum rules for magnetic moments of the heavy baryons the correlation function function is calculated in two different ways: a) In terms of the hadrons; b) performing the operator product expansion (OPE) over the twists of operators, and using the photon distribution amplitudes (DAs) which encode all nonperturbative effects. The sum rules for the magnetic moments of the heavy baryons are obtained by equating the coefficients of the appropriate Lorentz structures
that survive in parts (a) and (b). Finally, in order to enhance the contributions coming from the ground states, and suppress the contributions of the continuum and higher states, Borel transformation with respect to the momentum squared of the initial and final baryon states, and continuum subtraction procedure are performed, successively (for more about the LCSR, see for example [24]).

To be able to calculate the correlator function in terms of the hadrons, we insert the complete set of baryon states that carry the same quantum numbers as the interpolating current \( \eta(x) \). It should be noted here that the interpolating current can produce both positive and negative parity baryons from the vacuum state. Keeping this remark in mind, and isolating the contributions of the ground state baryons, we get

\[
\Pi(p, q) = \frac{\langle 0 | \eta | f_+ (p, s) \rangle}{p^2 - m_{f+}^2} \langle f_+ (p, s) | i_+ (p + q, s) \rangle \gamma_\nu \frac{\langle i_+ (p + q, s) | \bar{\eta}(0) \rangle}{(p + q)^2 - m_{i+}^2}
\]

\[
+ \frac{\langle 0 | \eta | f_- (p, s) \rangle}{p^2 - m_{f-}^2} \langle f_- (p, s) | i_+ (p + q, s) \rangle \gamma_\nu \frac{\langle i_+ (p + q, s) | \bar{\eta}(0) \rangle}{(p + q)^2 - m_{i+}^2}
\]

\[
+ \frac{\langle 0 | \eta | f_+ (p, s) \rangle}{p^2 - m_{f+}^2} \langle f_+ (p, s) | i_- (p + q, s) \rangle \gamma_\nu \frac{\langle i_- (p + q, s) | \bar{\eta}(0) \rangle}{(p + q)^2 - m_{i-}^2}
\]

\[
+ \frac{\langle 0 | \eta | f_- (p, s) \rangle}{p^2 - m_{f-}^2} \langle f_- (p, s) | i_- (p + q, s) \rangle \gamma_\nu \frac{\langle i_- (p + q, s) | \bar{\eta}(0) \rangle}{(p + q)^2 - m_{i-}^2} + \cdots ,
\]

(2)

where \( f_+(-) \) and \( i_+(-) \) correspond to the final and initial, positive (negative) parity baryonic states with spin \( s \); and \( m_{f+(-)} \) and \( m_{i+(-)} \) to their masses, respectively; \( q \) is the four-momentum of the photon; \( \cdots \) describe the contributions coming from higher states. In this expression, the first term describes positive to positive, the second term describes positive to negative, the third term describes negative to positive, and the fourth term describes negative to negative parity transitions, respectively.

The matrix elements in Eq. (2) are determined as,

\[
\langle 0 | \eta | + (p) \rangle = \lambda_+ u_+ (p), \\
\langle 0 | \eta | - (p) \rangle = \lambda_- \gamma_5 u_- (p), \\
\langle + (p) | \eta | + (p + q) \rangle = e \varepsilon^\mu \bar{u}(p) \left[ \gamma_\mu f_1 - \frac{i \sigma_\mu q^\nu}{2m_+} f_2 \right] u(p + q), \\
\langle - (p) | \eta | + (p + q) \rangle = e \varepsilon^\mu \bar{u}(p) \left[ \gamma_\mu f_1^* - \frac{i \sigma_\mu q^\nu}{2m_-} f_2^* \right] \gamma_5 u(p + q), \\
\langle - (p) | \eta | - (p + q) \rangle = e \varepsilon^\mu \bar{u}(p) \left[ \gamma_\mu f_1^* - \frac{i \sigma_\mu q^\nu}{2m_-} f_2^* \right] u(p + q),
\]

(3)

where \( \varepsilon^\mu \) and \( q^\mu \) are the four-momentum and -polarization vectors of a photon. Using the Gordon identity for the diagonal transitions, it can easily be shown that in the case a real photon is exchanged, i.e. \( q^2 = 0 \), the structure \( \gamma_\mu \) is proportional to \( f_1 + f_2 \) \( (f_1^* + f_2^*) \), and it describes the magnetic moment of the corresponding baryon. As has already been noted, the magnetic moments of the heavy \( J^P = \frac{1}{2}^+ \) baryons are calculated in [20, 23], and hence, in the present work we calculate the magnetic moments of the \( J^P = \frac{1}{2}^- \) baryons in framework of the LCSR.
Using the definition of the matrix elements given in Eq. (3), and performing summation over the spins of the heavy baryons, the correlation function can be written as,

\[
\Pi(p, q) = \frac{\lambda^2(f_1 + f_2)}{(m_+^2 - p_2^2)(m_+^2 - p_1^2)}(p_2 + m_+) \not\! \phi(p_1 + m_+)
\]

\[
+ \frac{\lambda^2(f^+_1 + f^+_2)}{(m_-^2 - p_2^2)(m_-^2 - p_1^2)}(p_2 - m_-) \not\! \phi(p_1 - m_-)
\]

\[
+ \frac{\lambda^+_+(f^+_2)}{(m_-^2 - p_2^2)(m_-^2 - p_1^2)} \left[ f^T_1 + \frac{m_- - m_+}{m_- + m_+} f^T_2 \right] (p_2 - m_-) \not\! \phi(p_1 + m_+)
\]

\[
+ \frac{\lambda^-_+(f^-_2)}{(m_-^2 - p_2^2)(m_-^2 - p_1^2)} \left[ f^T_1 + \frac{m_- - m_+}{m_- + m_+} f^T_2 \right] (p_2 + m_+) \not\! \phi(p_1 - m_-), \quad (4)
\]

Denoting

\[
A = \frac{\lambda^2(f_1 + f_2)}{(m_+^2 - p_2^2)(m_+^2 - p_1^2)},
\]

\[
B = \frac{\lambda^2(f^+_1 + f^+_2)}{(m_-^2 - p_2^2)(m_-^2 - p_1^2)},
\]

\[
C = \frac{\lambda^+_+(f^+_2)}{(m_-^2 - p_2^2)(m_-^2 - p_1^2)} \left[ f^T_1 + \frac{m_- - m_+}{m_- + m_+} f^T_2 \right],
\]

\[
D = \frac{\lambda^-_+(f^-_2)}{(m_-^2 - p_2^2)(m_-^2 - p_1^2)} \left[ f^T_1 + \frac{m_- - m_+}{m_- + m_+} f^T_2 \right], \quad (5)
\]

the phenomenological part of the correlation function is given as,

\[
A(p_2 + m_+) \not\! \phi(p_1 + m_+) + B(p_2 - m_-) \not\! \phi(p_1 - m_-)
\]

\[
+ C(p_2 - m_-) \not\! \phi(p_1 + m_+) + D(p_2 + m_+) \not\! \phi(p_1 - m_-).
\]

This expression contains four type, positive–positive, negative–negative, positive–negative, negative–positive transitions. Among all transitions only the coefficient \( B \) contains negative–negative parity transition, whose solution at \( q^2 = 0 \) is \((f^+_1 + f^+_2)|_{q^2=0}\) corresponds to the magnetic moment of the negative parity heavy baryons. Having four unknowns, we choose the four structures \((\varepsilon \cdot p)I, (\varepsilon \cdot p)p, \not\! p\) and \( \not\! p\) which each leads to its own equation. Solving this system of four linear equations for the above-mentioned unknowns, we can easily find the unknown variable \( B \).

In order to obtain the sum rules we have to calculate the invariant functions \( \Pi_i \) from the theoretical side, for which the interpolating current of the corresponding hadrons must be known. According to the SU(3)\(_f\) classification, hadrons with single heavy quark belong to either sextet-symmetric or antitriplet-antisymmetric flavor representations. Therefore interpolating currents of the sextet (antitriplet) representation should be symmetric (antisymmetric) with respect to the light flavors. The heavy baryons \( \Sigma Q, \Xi'_Q \) and \( \Omega Q \) belong to the sextet, \( \Xi Q \) and \( \Lambda Q \) belong to the antitriplet representations of the SU(3)\(_f\) group.

Using this fact, the general form of the interpolating currents belonging to the sextet or antitriplet representation of the SU(3)\(_f\) group can be written in the following form (see
for example [25]),

\[
\eta^{(s)} = -\frac{1}{\sqrt{2}} \varepsilon^{abc}\left\{ (q_1^{aT}CQ^b)\gamma_5q_2^c + t(q_1^{aT}C\gamma_5Q^b)q_2^c + (q_2^{aT}CQ^b)\gamma_5q_1^c + (q_2^{aT}C\gamma_5Q^b)q_1^c \right\},
\]

\[
\eta^{(a)} = -\frac{1}{\sqrt{6}} \varepsilon^{abc}\left\{ 2(q_1^{aT}CQ^b)\gamma_5Q^c + 2t(q_1^{aT}C\gamma_5Q^b)Q^c + (q_1^{aT}CQ^b)\gamma_5q_2^c 
+ t(q_1^{aT}C\gamma_5Q^b)q_2^c - (q_2^{aT}CQ^b)\gamma_5q_1^c - t(q_2^{aT}C\gamma_5Q^b)q_1^c \right\},
\]

where \( t \) is an arbitrary parameter (\( t = -1 \) corresponds to the so-called Ioffe current); \( a, b, c \) are the color indices; and \( C \) is the charge conjugation operator. The light quark contents of the sextet and antitriplet representations are given in Table 1.

| \( a \)  | \( \Sigma^{c(++)}_{c(b)} \) | \( \Sigma^{c(+)0}_{c(b)} \) | \( \Sigma^{c(0-)}_{c(b)} \) | \( \Xi^{c(0-)}_{c(b)} \) | \( \Xi^{c(+)0}_{c(b)} \) | \( \Omega^{(0-)}_{c(b)} \) | \( \Omega^{(+)0}_{c(b)} \) |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( q_1 \) | \( u \) | \( u \) | \( d \) | \( s \) | \( s \) | \( u \) | \( d \) |
| \( q_2 \) | \( u \) | \( d \) | \( d \) | \( s \) | \( s \) | \( s \) | \( s \) |

Table 1: Quark contents of the heavy baryons belonging to the sextet and antitriplet representations.

Using the interpolating currents given in Eq. (6), one can easily calculate the theoretical part of the correlator function. As an example, we present the form of the correlation function for \( \Sigma^+_{\eta Q} \) in terms of the corresponding propagators,

\[
\Pi^{\Sigma^+_{\eta Q}} = -2\varepsilon^{abc}\varepsilon^{a'b'c'}\int d^4x\langle 0 | \sum_{t=1}^2 \sum_{k=1}^2 \left\{ A^k_u S^{a'b'}_{u} (x) A^k_a Tr S^{bb'}_{Q} (x) CA^k_{1} S^{aa'}_{u} (x) CA^k_{1} \right\} |0,\gamma \rangle ,
\]

\[
+ A^k_u S^{cc'}_{u} (x) (CA^k_{1})^T S^{bb'T}_{Q} (x) (CA^k_{1})^T S^{aa'}_{u} (x) A^k_{2} 
+ S^{aa'}_{u} (x) (CA^k_{1})^T S^{bb'T}_{Q} (x) (CA^k_{1})^T S^{cc'}_{u} (x) A^k_{2} 
+ S^{aa'}_{u} (x) A^k_{2} Tr S^{bb'T}_{Q} (x) CA^k_{1} S^{cc'T}_{u} (x) CA^k_{1} \right\} |0,\gamma \rangle ,
\]

where \( A^1_u = 1, A^2_u = t \gamma_5, A^2_u = \gamma_5, A^2_u = t, \) and \( S_{u} (x) \) and \( S_{Q} (x) \) are the full propagators of the light and heavy quarks.

The expressions of the correlator functions for the \( \Sigma^-_{\eta Q}, \Sigma^0_{\eta Q}, \Xi^0_{\eta Q}, \Xi^-_{\eta Q} \) and \( \Omega_{\eta Q} \) can be found by performing the following replacements,

\[
\Pi^{\Sigma^-_{\eta Q}} = \Pi^{\Sigma^+_{\eta Q}} (u \rightarrow d) ,
\]

\[
\Pi^{\Sigma^0_{\eta Q}} = \frac{1}{2} \left( \Pi^{\Sigma^+_{\eta Q}} + \Pi^{\Sigma^-_{\eta Q}} \right) ,
\]

\[
\Pi^{\Xi^-_{\eta Q}} = \Pi^{\Sigma^+_{\eta Q}} (u \rightarrow s) ,
\]

\[
\Pi^{\Xi^0_{\eta Q}} = \Pi^{\Sigma^+_{\eta Q}} (u \rightarrow d) .
\]

The light cone expression of the light quark propagator in external field is calculated in [26] in which it is found that The contributions of the nonlocal operators such as \( qGq, \)
\[ \bar{q}G^2 q, \bar{q}q\bar{q}q, \] are small. Neglecting these operators is justified in conformal spin expansion [27]. Note that in further analysis we retain only those terms that are linear in quark mass.

The expression of the light quark operator in the presence of external field is given as,

\[ S_q(x) = \frac{i\gamma_\mu}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} \left( 1 - i \frac{m_q}{4} \right) - \frac{x}{192} m_0^2 \langle \bar{q}q \rangle \left( 1 - i \frac{m_q}{6} \right) \]

\[ - ig_s \int_0^1 du \left[ \frac{x}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma_{\mu\nu} - \frac{i}{4\pi^2 x^2} u x^\mu G_{\mu\nu}(ux) \gamma^\nu \right. \]

\[ - \frac{i m_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left( \ln \frac{-x^2 \Lambda^2}{4} + 2 \gamma_E \right) ] + \cdots, \tag{9} \]

where \( \Lambda \) is the cut-off energy separating perturbative and nonperturbative domains, and \( \gamma_E \) is the Euler constant.

In calculating the correlation function from the QCD side we also need the expression for the heavy quarks, whose explicit form in the coordinate space can be expressed as,

\[ S_Q(x) = \frac{m_Q^2}{4\pi^2} \left\{ \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{\gamma_\mu}{-x^2} K_2(m_Q \sqrt{-x^2}) \right\} \]

\[ - \frac{g_s}{16\pi^2} \int_0^1 du G_{\mu\nu}(ux) \left[ (\sigma^{\mu\nu} \gamma_\mu + \gamma_\mu \sigma^{\mu\nu}) \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + 2 \sigma^{\mu\nu} K_0(m_Q \sqrt{-x^2}) \right] + \cdots, \tag{10} \]

where \( K_i(m_Q \sqrt{-x^2}) \) are the modified Bessel functions. Taking into account the expressions of the light and heavy propagators, the correlation function given in Eq. (7) can be calculated from the QCD side, from which we observe three different type of contributions:

a) Perturbative contributions, i.e., photon interacts with the quark propagators perturbatively. Technically this contribution can be calculated by replacing the one of the free quark operators (the first two terms in Eqs. (9) and (10)) by,

\[ S_{\text{free}}(x) \rightarrow \int d^4 y S_{\text{free}}(x - y) \mathcal{A}(y) S_{\text{free}}(x - y), \tag{11} \]

and the remaining two propagators are the free ones. b) In the case when photon interacts with the heavy quark perturbatively, the free part must be removed at least in one of the light quark propagators. c) Nonperturbative contributions, i.e., photon interacts with the light quark fields at large distance. This contribution can be calculated by replacing one of the light quark operators by,

\[ S^{ab}_{\alpha\beta} \rightarrow -\frac{1}{4} \left( q^a \Gamma_i q^b \right) (\Gamma_i)_{\alpha\beta}, \tag{12} \]

and the remaining quarks constitute the full quark propagators. Here, \( \Gamma_j \) are the full set of Dirac matrices \( \gamma_j = \{ I, \gamma_5, \gamma_\mu, i\gamma_\mu \gamma_5, \sigma_{\mu\nu}/\sqrt{2} \} \). When Eq. (12) is used in calculation of the nonperturbative contributions, we see that matrix elements of the form \( \langle \gamma(q)|\bar{q}\Gamma_i q|0 \rangle \) are needed. These matrix elements are defined in terms of the photon distribution amplitudes in the following way (see [28]),
\[ \langle q | \bar{q}(x) \sigma_{\mu} q(0) | 0 \rangle = -i e_q \bar{q} q (\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu}) \int_0^1 du e^{i q x u} \left( \chi \varphi_\gamma(u) + \frac{x^2}{16} A_\lambda(u) \right) \]

\[ - \frac{i}{2 q x} e_q \bar{q} q \left[ x_{\mu} \left( \varepsilon_{\mu} - \frac{\varepsilon x}{q x} \right) - x_{\nu} \left( \varepsilon_{\nu} - \frac{\varepsilon x}{q x} \right) \right] \int_0^1 du e^{i q x u} h_\gamma(u) \]

\[ \langle q | \bar{q}(x) \gamma_5 q(0) | 0 \rangle = e_q f_3 \gamma_5 \left( \varepsilon_{\mu} - q_{\mu} \frac{\varepsilon x}{q x} \right) \int_0^1 du e^{i q x u} \psi^\mu(u) \]

\[ \langle q | \bar{q}(x) \gamma_5 q(0) | 0 \rangle = -\frac{1}{4} e_q f_3 \gamma_5 \varepsilon_{\mu \alpha \beta \varepsilon} q^{\alpha} x^{\beta} \int_0^1 du e^{i q x u} \psi^\mu(u) \]

\[ \langle q | \bar{q}(x) g_s G_{\mu \nu} (v x) q(0) | 0 \rangle = -i e_q \bar{q} q (\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu}) \int D \alpha_i e^{i (\alpha q + \nu a) x} S(\alpha_i) \]

\[ \langle q | \bar{q}(x) g_s G_{\mu \nu} i \gamma_5 (v x) q(0) | 0 \rangle = -i e_q \bar{q} q (\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu}) \int D \alpha_i e^{i (\alpha q + \nu a) x} \tilde{S}(\alpha_i) \]

\[ \langle q | \bar{q}(x) g_s i \gamma_5 (v x) \gamma_5 q(0) | 0 \rangle = e_q f_3 \gamma_5 \alpha (\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu}) \int D \alpha_i e^{i (\alpha q + \nu a) x} A(\alpha_i) \]

\[ \langle q | \bar{q}(x) g_s G_{\mu \nu} (v x) i \gamma_5 q(0) | 0 \rangle = e_q f_3 \gamma_5 \alpha (\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu}) \int D \alpha_i e^{i (\alpha q + \nu a) x} V(\alpha_i) \]

\[ \langle q | \bar{q}(x) \sigma_{\alpha \beta} g_s G_{\mu \nu} (v x) q(0) | 0 \rangle = e_q \bar{q} q \left\{ \left[ (\varepsilon_{\mu} - q_{\mu} \frac{\varepsilon x}{q x}) g_{\alpha \nu} - \frac{1}{q x} (q_{\alpha} x_{\nu} + q_{\nu} x_{\alpha}) \right] \bar{q} \alpha \right. \]

\[ - \left( \varepsilon_{\mu} - q_{\mu} \frac{\varepsilon x}{q x} \right) \left( g_{\beta \nu} - \frac{1}{q x} (q_{\beta} x_{\nu} + q_{\nu} x_{\beta}) \right) q_{\alpha} \]

\[ - \left( \varepsilon_{\nu} - q_{\nu} \frac{\varepsilon x}{q x} \right) \left( g_{\alpha \mu} - \frac{1}{q x} (q_{\alpha} x_{\mu} + q_{\mu} x_{\alpha}) \right) q_{\beta} \]

\[ + \left( \varepsilon_{\nu} - q_{\nu} \frac{\varepsilon x}{q x} \right) \left( g_{\beta \mu} - \frac{1}{q x} (q_{\beta} x_{\mu} + q_{\mu} x_{\beta}) q_{\alpha} \right) \left[ D \alpha_i e^{i (\alpha q + \nu a) x} T_1(\alpha_i) \right] \]

\[ + \left[ \varepsilon_{\alpha} - q_{\alpha} \frac{\varepsilon x}{q x} \right] \left( g_{\mu \beta} - \frac{1}{q x} (q_{\mu} x_{\beta} + q_{\beta} x_{\mu}) \right) q_{\nu} \]

\[ - \left( \varepsilon_{\alpha} - q_{\alpha} \frac{\varepsilon x}{q x} \right) \left( g_{\nu \beta} - \frac{1}{q x} (q_{\nu} x_{\beta} + q_{\beta} x_{\nu}) \right) q_{\mu} \]

\[ - \left( \varepsilon_{\beta} - q_{\beta} \frac{\varepsilon x}{q x} \right) \left( g_{\mu \alpha} - \frac{1}{q x} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\nu} \]

\[ + \left( \varepsilon_{\beta} - q_{\beta} \frac{\varepsilon x}{q x} \right) \left( g_{\nu \alpha} - \frac{1}{q x} (q_{\nu} x_{\alpha} + q_{\alpha} x_{\nu}) \right) q_{\mu} \left[ D \alpha_i e^{i (\alpha q + \nu a) x} T_2(\alpha_i) \right] \]

\[ + \frac{1}{q x} (q_{\mu} x_{\nu} - q_{\nu} x_{\mu}) (\varepsilon_{\alpha} q_{\beta} - \varepsilon_{\beta} q_{\alpha}) \left[ D \alpha_i e^{i (\alpha q + \nu a) x} T_3(\alpha_i) \right] \]

\[ + \frac{1}{q x} (q_{\alpha} x_{\beta} - q_{\beta} x_{\alpha}) (\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu}) \left[ D \alpha_i e^{i (\alpha q + \nu a) x} T_4(\alpha_i) \right] \}

where \( \varphi_\gamma(u) \) is the leading twist-2, \( \psi^\mu(u) \), \( \psi^\alpha(u) \), \( A \) and \( V \) are the twist-3, and \( h_\gamma(u) \), \( A \), \( T_i \) \((i = 1, 2, 3, 4) \) are the twist-4 photon DAs, and \( \chi \) is the magnetic susceptibility. The
The measure $D\alpha_i$ is defined as

$$\int D\alpha_i = \int_0^1 d\alpha_q \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_q - \alpha_q - \alpha_g).$$

As has already been noted in determining the magnetic moments of heavy baryons, we need four equations. Separating the coefficients of the structures $(\varepsilon \cdot p)I$, $(\varepsilon \cdot p)\not p$, $\not p$ and $\not q$ from the theoretical parts, and equating them to the corresponding structures in the phenomenological parts, one can obtain from Eq. (5) the sum rules which describe “positive (negative) parity $\rightarrow$ positive (negative) parity” transitions; as well as “positive (negative) parity $\rightarrow$ negative (positive) parity” transitions. Solving these equations for the “negative parity $\rightarrow$ negative parity” transitions, the following sum rules are obtained,

$$\lambda^2 \mu \left[2(m_+ + m_-)^2\right] = -(m_- + m_+) \left(P_1^{th} - m_+ P_2^{th}\right) - 2m_+ P_3^{th} + 2P_4^{th}, \quad (14)$$

where $\mu = (f_1^* + f_2^*)|q^2=0$ is the magnetic moment of the corresponding negative parity baryons.

Performing Borel transformation over the variables $-p^2$ and $-(p + q)^2$ in order to enhance contributions of the ground states, and suppress the continuum and higher state contributions; and subtracting the continuum contributions, we finally obtain the following sum rules for the negative parity baryons

$$\mu = \frac{e_m^2/M^2}{\lambda^2 \left[2(m_+ + m_-)^2\right]} \left\{- (m_- + m_+) \left(P_1^{th} - m_+ P_2^{th}\right) - 2m_+ P_3^{th} + 2P_4^{th}\right\}, \quad (15)$$

where we set $M_1^2 = M_2^2 = 2M^2$. The expressions of $P_i^{th}$ are quite lengthy, so we do not present their expressions here.

Our final attempt in this section is the calculation of residues of the negative parity heavy baryons, which are needed in determination of the magnetic moments. These residues are determined from an analysis of the two-point correlator function,

$$\Pi^M(q^2) = i \int d^4xe^{iqx} \langle 0 | T \{\eta_Q(x)\bar{\eta}_Q(0)\} | 0 \rangle, \quad (16)$$

where the superscript $M$ means mass sum rule. This correlation function have two structures, namely, $\not p$ and unit matrix $I$, and can be written as,

$$\Pi^M(q^2) = \Pi_1^M \not p + \Pi_2^M I. \quad (17)$$

Saturating (16) with positive and negative parity baryons, we get

$$\Pi^M(q^2) = \frac{\lambda_-^2 (\not p - m_-)}{m_-^2 - p^2} + \frac{\lambda_+^2 (\not p + m_+)}{m_+^2 - p^2}. \quad (18)$$

Eliminating the positive parity baryons, and performing Borel transformation over the variable $-p^2$, we get the following sum rules for the masses and residues of the negative
parity baryons.

\[ |\lambda_-|^2 = \frac{1}{\pi} \frac{c^{m_2/M^2}}{m_+ + m_-} \int_{m_b^2}^{s_0} dse^{-s/M^2} \left[ m_+ \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s) \right] , \]

\[ m_2^- = \frac{\int_{m_b^2}^{s_0} ds dse^{-s/M^2} \left[ m_+ \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s) \right]}{\int_{m_b^2}^{s_0} dse^{-s/M^2} \left[ m_+ \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s) \right]} . \]

The expressions of \( \text{Im}\Pi_1^M(s) \) and \( \text{Im}\Pi_2^M(s) \) for the \( \Sigma_0^b \) baryon are presented in Appendix B.

### 3 Numerical analysis

This section is devoted to the numerical analysis of the sum rules for the \( J^P = \frac{1}{2}^- \) heavy baryons obtained in the previous section. The main input parameters of the light cone QCD sum rules are the photon distribution amplitudes (DAs). The photon DAs are obtained in [28], and for completeness we present their expressions in Appendix C. Sum rules for the magnetic moment, together with the photon DAs, also contain the following input parameters: quark condensate \( \langle \bar{q}q \rangle \), \( m_0^2 \) that appears in determination of the vacuum expectation value of the dimension-5 operator \( \langle \bar{q}Gq \rangle = m_0^2 \langle \bar{q}q \rangle \), magnetic susceptibility \( \chi \) of quarks, etc. In the present analysis we use \( \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \rangle_{\mu=1} \text{GeV} = -(0.243)^3 \text{GeV}^3 \) [29], \( \langle \bar{s}s \rangle |_{\mu=1} \text{GeV} = 0.8 \langle \bar{u}u \rangle |_{\mu=1} \text{GeV} \), \( m_0^2 = (0.8 \pm 0.2) \text{GeV}^2 \) [30]. The values of the magnetic susceptibilities can be found in numerous works (see for example [31–33]). In our numerical analysis we use \( \chi(1 \text{ GeV}) = -2.85 \text{ GeV}^2 \) obtained in [33].

Having determined input parameters, in this section we shall proceed with the analysis of the sum rules for the magnetic moments of the \( J^P = \frac{1}{2}^- \) heavy baryons. The sum rules contain three auxiliary parameters: continuum threshold \( s_0 \), Borel mass square \( M^2 \), and \( t \) appearing in the expression of the interpolating current. Accordingly to the QCD sum rules philosophy, any measurable quantity must be independent of these parameters. For this reason we try to find such regions of these parameters where magnetic moments are insensitive to their variations. This issue can be accomplished by the following three-step procedure. First, at fixed values of \( s_0 \) and \( t \), we try to find region of \( M^2 \) where magnetic moment is independent of its variation. The upper bound of \( M^2 \) is determined by requiring that the contributions of higher states and continuum constitute, say, less than 40% of the contribution coming from the perturbative part. The lower bound of \( M^2 \) is obtained by demanding that higher twist contributions are less than the leading twist contributions. Analysis of our sum rules leads to the following regions of \( M^2 \) where magnetic moments are independent on its variation.

\[ 2.5 \text{ GeV}^2 \leq M^2 \leq 4.0 \text{ GeV}^2 , \text{ for } \Sigma_c, \Xi_c^0, \Lambda_c, \Xi_c^+ , \]

\[ 4.5 \text{ GeV}^2 \leq M^2 \leq 7.0 \text{ GeV}^2 , \text{ for } \Sigma_b, \Xi_b^+, \Lambda_b, \Xi_b^- . \]

Next, we try to find the domain of variation of the continuum threshold \( s_0 \), which is the energy square where the continuum starts. The difference \( \sqrt{s_0} - m \), \( m \) being the ground state mass, is the energy needed for the excitation of the particle to its first excited state,
and usually this difference is varies in the range between 0.3 GeV and 0.8 GeV. In our analysis we use the average value $\sqrt{s_0} - m = 0.5$ GeV.

As an example, in Fig. (1) we present the dependence of the magnetic moments of $\Sigma_c^0$ baryon on $M^2$, at $s_0 = 12$ GeV$^2$, and at several fixed values of $t$. It follows from these figures that, indeed, we have very good stability of the magnetic moment $\mu$ as $M^2$ varies in its above-mentioned working region. We also analyze these dependencies at $s_0 = 11$ GeV$^2$ and $s_0 = 13$ GeV$^2$; and find out that the discrepancy in the values of the magnetic moment is about 10%. In other words, the magnetic moments of $\Sigma_c^0$ baryon exhibits the expected insensitivity to the variations in $s_0$ and $M^2$.

Having decided the working regions of $M^2$ and $s_0$, the third and last step is to find the working region of the parameter $t$ in which the predictions for the values magnetic moments of heavy baryons show good stability. For this aim, we study the dependence of the magnetic moment of the $\Sigma_c^0$ baryon on $\cos \theta$, where $t = \tan \theta$. We observe from our numerical analysis that, the magnetic moments of all baryons are independent of the variation in $\cos \theta$ when it varies in the region $-0.7 \leq \cos \theta \leq -0.4$. Our numerical analysis predicts that the magnetic moment of the $\Sigma_c^0$ baryon is $\mu = (-2.0 \pm 0.1)\mu_N$, where $\mu_N$ is the nuclear magneton. Performing similar analysis we have calculated the magnetic moments of the other $J^P = \frac{1}{2}^-$ heavy baryons whose results are presented in Table 2. We note here that, in many cases, the naive expectation that the relation between the negative and positive parity baryons, i.e.,

$$|\mu_-| = \frac{m_+}{m_-} |\mu_+|,$$

is violated considerably. This violation can be attributed to the fact that in our analysis we take into account contributions coming from positive-to-positive and nondiagonal transitions.

|       | $\mu$ | $\mu$ |
|-------|-------|-------|
| $\Sigma_c^+$ | 1.3 ± 0.3 | $\Sigma_c^{++}$ | 2.2 ± 0.2 |
| $\Sigma_c^0$ | 0.5 ± 0.05 | $\Sigma_c^+$ | 0.15 ± 0.02 |
| $\Sigma_c^-$ | -0.3 ± 0.1 | $\Sigma_c^0$ | -2.0 ± 0.1 |
| $\Xi_c^-$ | -0.4 ± 0.1 | $\Xi_c^0$ | -2.0 ± 0.2 |
| $\Xi_c^0$ | 0.4 ± 0.1 | $\Xi_c^{++}$ | 0.15 ± 0.02 |
| $\Omega_c^-$ | -0.3 ± 0.1 | $\Omega_c^0$ | -2.0 ± 0.2 |
| $\Lambda_c^0$ | -0.11 ± 0.02 | $\Lambda_c^+$ | 1.3 ± 0.2 |
| $\Xi_c^-$ | -0.7 ± 0.1 | $\Xi_c^0$ | 1.6 ± 0.2 |
| $\Xi_c^0$ | -0.12 ± 0.02 | $\Xi_c^+$ | 1.2 ± 0.2 |

Table 2: Magnetic moments of the negative parity, spin-1/2 baryons containing single heavy quark belonging to the sextet and antitriplet representations, in units of nuclear magneton $\mu_N$

In conclusion, we have employed the light cone QCD rules to calculate the magnetic moments of negative parity baryons containing single heavy quark. In determination of
the magnetic moments of these baryons the contamination coming from the positive parity baryons, as well as from the transitions between opposite parity baryons are eliminated by considering different Lorentz structures. A comparison our results on the magnetic moments of the negative parity baryons with the predictions of other approaches, such as quark model, bag model, chiral perturbation theory, lattice QCD, etc., would be interesting.
Appendix A: Photon distribution amplitudes

Explicit forms of the photon DAs [28]:

\[ \varphi_\gamma(u) = 6w\bar{u} \left[ 1 + \varphi_2(\mu)C^2_2(u - \bar{u}) \right], \]
\[ \psi^\gamma(u) = 3[3(2u - 1)^2 - 1] + \frac{3}{64}(15w^\gamma - 5w^A)[3 - 30(2u - 1)^2 + 35(2u - 1)^4], \]
\[ \psi^\alpha(u) = [1 - (2u - 1)^2][5(2u - 1)^2 - 1] \left( 1 + \frac{9}{16}w^\gamma - \frac{3}{16}w^A \right), \]
\[ \mathcal{A}(\alpha_i) = 360\alpha_\gamma\alpha_\gamma\alpha_\gamma^2 \right[ 1 + w^A_\gamma \frac{1}{2}(7\alpha_g - 3) \right], \]
\[ \mathcal{V}(\alpha_i) = 540w^\gamma_\gamma(\alpha_\gamma - \alpha_\gamma)\alpha_\gamma\alpha_\gamma\alpha_\gamma^2, \]
\[ h_\gamma(u) = -10(1 + 2\kappa^+)C^2_2(u - \bar{u}), \]
\[ A(u) = 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^2(10 - 15u + w^2) \ln(u) \]
\[ + 2\bar{u}^3(10 - 15\bar{u} + w^2) \ln(\bar{u})], \]
\[ T_1(\alpha_i) = \frac{1}{120}(3\zeta_2 + \zeta_2^+)(\alpha_\gamma - \alpha_\gamma)\alpha_\gamma\alpha_\gamma\alpha_\gamma, \]
\[ T_2(\alpha_i) = 30\alpha_\gamma^3(\alpha_\gamma - \alpha_\gamma)[(\kappa + \kappa^+)(1 + 2\alpha_\gamma) + \zeta_2(3 - 4\alpha_\gamma)], \]
\[ T_3(\alpha_i) = \frac{1}{120}(3\zeta_2 - \zeta_2^+)(\alpha_\gamma - \alpha_\gamma)\alpha_\gamma\alpha_\gamma\alpha_\gamma, \]
\[ T_4(\alpha_i) = 30\alpha_\gamma^3(\alpha_\gamma - \alpha_\gamma)[(\kappa + \kappa^+)(1 + 2\alpha_\gamma) + \zeta_2(3 - 4\alpha_\gamma)], \]
\[ S(\alpha_i) = 30\alpha_\gamma^3[(\kappa + \kappa^+)(1 - \alpha_\gamma) + (\zeta_1 + \zeta_1^+)(1 - \alpha_\gamma)(1 - 2\alpha_\gamma) \]
\[ + \zeta_2[3(\alpha_\gamma - \alpha_\gamma)^2 - \alpha_\gamma(1 - \alpha_\gamma)]], \]
\[ \tilde{S}(\alpha_i) = -30\alpha_\gamma^3((\kappa + \kappa^+)(1 - \alpha_\gamma) + (\zeta_1 - \zeta_1^+)(1 - \alpha_\gamma)(1 - 2\alpha_\gamma) \]
\[ + \zeta_2[3(\alpha_\gamma - \alpha_\gamma)^2 - \alpha_\gamma(1 - \alpha_\gamma)]]. \]

The parameters entering the above DA’s are borrowed from [28] whose values are \( \varphi_2(1 GeV) = 0, \)
\( w^\gamma_\gamma = 3.8 \pm 1.8, \)
\( w^A_\gamma = -2.1 \pm 1.0, \)
\( \kappa = 0.2, \)
\( \kappa^+ = 0, \)
\( \zeta_1 = 0.4, \)
\( \zeta_2 = 0.3, \)
\( \zeta_1^+ = 0, \) and
\( \zeta_2^+ = 0. \)
Appendix B

In this appendix we give the expressions of the invariant amplitudes $\Pi_1^M$ and $\Pi_2^M$ entering into the mass sum rule for the $\Sigma_{b}^0$ baryon. Here in this appendix, and in appendix C the masses of the light quarks are neglected.

\[
\Pi_1^M = \frac{3}{256\pi^4}\left\{ -m_b^3 M^6[5 + t(2 + 5t)] [m_b^4 \mathcal{I}_5 - 2m_b^2 \mathcal{I}_4 + \mathcal{I}_3] \right\} \\
+ \frac{1}{192\pi^4} m_b^4 M^2 \left[ \langle g_s^2 G^2 \rangle (1 + t + t^2) - 18m_b \pi^2(-1 + t^2) (\langle dd \rangle + \langle \bar{u}u \rangle) \right] \mathcal{I}_3 \\
+ \frac{1}{3072\pi^4} m_b^2 M^2 \left[ -\langle g_s^2 G^2 \rangle (13 + 10t + 13t^2) + 288m_b \pi^2(-1 + t^2) (\langle dd \rangle + \langle \bar{u}u \rangle) \right] \mathcal{I}_2 \\
+ \frac{e^{-m_b^2/M^2}}{73728m_b^2 \pi^4} \left\{ -\langle g_s^2 G^2 \rangle m_b(1 + t)^2 + 768m_b m_0^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^4(-1 + t)^2 \\
- 56\langle g_s^2 G^2 \rangle m_b^2 \pi^2(-1 + t^2) (\langle dd \rangle + \langle \bar{u}u \rangle) \right\} \\
+ \frac{1}{168 M^2 \pi^2} \langle g_s^2 G^2 \rangle m_b(-1 + t^2)(\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) \mathcal{I}_1 \\
+ \frac{e^{-m_b^2/M^2}}{18432 M^4 \pi^2} m_b^3 \left[ \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) + 384m_b \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^2(-1 + t^2) \right] \\
+ \frac{e^{-m_b^2/M^2}}{1728 M^6} m_b^2 \langle g_s^2 G^2 \rangle(-1 + t)^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \\
- \frac{e^{-m_b^2/M^2}}{1728 M^8} m_b^2 m_0^2 \langle g_s^2 G^2 \rangle(-1 + t)^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \\
- \frac{e^{-m_b^2/M^2}}{3456 M^{10}} m_b^4 m_0^2 \langle g_s^2 G^2 \rangle(-1 + t)^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \\
- \frac{e^{-m_b^2/M^2}}{768 m_b \pi^2} \left[ \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) + 32m_b \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^2(-1 + t^2) \right] \\
+ \frac{1}{256\pi^2} m_b \langle \bar{u}u \rangle \langle \bar{d}d \rangle (-1 + t^2) \left[ \langle g_s^2 G^2 \rangle - 13m_b^2 m_0^2 \right] \mathcal{I}_2 + 6m_b^2 \mathcal{I}_1, \right.
\]

\[
\Pi_2^M = -\frac{3}{256\pi^4}\left\{ -m_b^3 M^6(-1 + t)^2 [m_b^4 \mathcal{I}_4 - 2m_b^2 \mathcal{I}_3 + \mathcal{I}_2] \right\} \\
+ \frac{1}{3072\pi^4} m_b^4 M^2 \left\{ 4m_b^2 \left[ \langle g_s^2 G^2 \rangle(-1 + t)^2 + 72m_b (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \pi^2(-1 + t^2) \right] \mathcal{I}_3 - 3\langle g_s^2 G^2 \rangle(-1 + t)^2 \mathcal{I}_2 \right\} \\
- \frac{7e^{-m_b^2/M^2}}{256\pi^2} m_b^2 m_0^2 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) \\
+ \frac{1}{1024\pi^4} m_b^4 M^2 \left\{ m_b \left[ 3m_b \langle g_s^2 G^2 \rangle(-1 + t)^2 + 4m_b^2 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \pi^2(-1 + t^2) \right] \mathcal{I}_2 - 2\langle g_s^2 G^2 \rangle(-1 + t)^2 \mathcal{I}_1 \right\} \\
- \frac{e^{-m_b^2/M^2}}{73728 M^2 \pi^4} m_b \left[ \langle g_s^2 G^2 \rangle^2(-1 + t)^2 + 1536m_b^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^4(3 + 2t - 3t^2) \right] \\
+ \frac{e^{-m_b^2/M^2}}{18432 M^4 \pi^2} m_b \left[ -11m_b^2 \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) \right].
\]
\[-32 \left( \langle g_s^2 G^2 \rangle - 12 m_0^2 m_b^2 \right) \langle \bar{d}d \rangle \langle \bar{u}u \rangle \langle 5 + 2t + 5t^2 \rangle \]
\[+ \frac{e^{-m_b^2/M^2}}{1728 M^6} m_b \left( m_b^2 - 3m_0^2 \right) \langle g_s^2 G^2 \rangle \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2) \]
\[+ \frac{e^{-m_b^2/M^2}}{576 M^8} m_b^5 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2) \]
\[- \frac{e^{-m_b^2/M^2}}{3456 M^6} m_b^5 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2) \]
\[+ \frac{e^{-m_b^2/M^2}}{36864 m_b \pi^4} \left[ \langle g_s^2 G^2 \rangle^2 (-1 + t)^2 - 1536 m_b^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^4 (5 + 2t + 5t^2) \right] + 96 m_b \langle g_s^2 G^2 \rangle \left( \langle \bar{u}u \rangle + \langle \bar{d}d \rangle \right) \pi^2 (-1 + t^2) \),

(1)

where

\[\mathcal{I}_n = \int_{m_b^2}^{\infty} ds \frac{e^{-s/M^2}}{s^n}.\]
Appendix C

In this Appendix we present the expressions of the invariant functions $\Pi_i$ appearing in the sum rules for the magnetic moment of $\Sigma_0$ baryon.

1) Coefficient of the $\langle \varepsilon \cdot p \rangle I$ structure

$$
\Pi_1 = -\frac{3}{128 \pi^4} (1 + t)^2 (\varepsilon_b + \varepsilon_d + \varepsilon_u) m_b^3 M^3 (I_2 - 2m_b^2 I_3 + m_b^4 I_4)
$$

$$
+ \frac{1}{1536 \pi} (-1 + t) m_b^4 M^4 \left\{ 3 \left[ (\varepsilon_d + \varepsilon_u)(1 - t) \langle g_s^2 G^2 \rangle + 48(1 + t)e_b m_b \pi^2 (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) \right] I_2
$$

$$
+ 4m_b^2 \left[ (\varepsilon_d + \varepsilon_u)(-1 + t) \langle g_s^2 G^2 \rangle + 72m_b (1 + t) \pi^2 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) \right] I_3 \right\}
$$

$$
+ \frac{3}{16 \pi^2} (-1 + t^2) m_b^4 M^4 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_u \langle \bar{u}u \rangle) \tilde{j} (r) I_3
$$

$$
+ \frac{1}{16 \pi^2} (-1 + t^2) (\varepsilon_d + \varepsilon_u) f_{3\gamma} m_b^2 M^2 (I_2 - m_b^2 I_3) \psi^\nu (u_0)
$$

$$
+ \frac{3}{32 \pi^2} (-1 + t^2) e_b m_b^2 M^2 (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) I_1
$$

$$
+ \frac{1}{768 \pi^2} m_b M^2 I_2 \left\{ -3(-1 + t^2) m_b \left[ \langle \bar{d}d \rangle (7\varepsilon_b - 2\varepsilon_u) m_b^2 + 24(\bar{d}d) e_b m_b^2
$$

$$
+ (7\varepsilon_b - 2\varepsilon_u) m_b^2 \langle \bar{u}u \rangle + 24e_b m_b^2 \langle \bar{u}u \rangle \right) + 2(-1 + t^2) \langle \varepsilon_d + \varepsilon_u \rangle f_{3\gamma} \langle g_s^2 G^2 \rangle \psi^\nu (u_0) \right\}
$$

$$
- \frac{e^{-m_b^2/M^2}}{2304 m_b \pi^2} (-1 + t^2) M^2 \left\{ 9(1 + t^2) m_b^2 m_b \langle \bar{d}d \rangle e_b + 12e_u \langle \bar{d}d \rangle + 7e_b \langle \bar{u}u \rangle + 12e_d \langle \bar{u}u \rangle
$$

$$
+ 2 f_{3\gamma} \left[ (\varepsilon_d + \varepsilon_u)(-1 + t) \langle g_s^2 G^2 \rangle + 96(1 + t) m_b \pi^2 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_u \langle \bar{d}d \rangle) \right] \psi^\nu (u_0) \right\}
$$

$$
+ \frac{e^{-m_b^2/M^2}}{48 M^2} (-1 + t^2) f_{3\gamma} m_b^2 m_b^2 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) \psi^\nu (u_0)
$$

$$
+ \frac{e^{-m_b^2/M^2}}{13824 M^4 \pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle m_b^2 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) \left[ 9m_b^2 + 16 f_{3\gamma} \pi^2 \psi^\nu (u_0) \right]
$$

$$
+ \frac{e^{-m_b^2/M^2}}{1728 M^6} (-1 + t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_b^2 m_b^2 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) \psi^\nu (u_0)
$$

$$
- \frac{e^{-m_b^2/M^2}}{3456 M^8} (-1 + t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_b^2 m_b^2 (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) \psi^\nu (u_0)
$$

$$
+ \frac{1}{768 \pi^2} (1 + t^2) \left[ -2 \langle g_s^2 G^2 \rangle (\varepsilon_u \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) - 21 e_b m_b^2 m_b^2 \langle \bar{d}d \rangle + \langle \bar{u}u \rangle \right] I_1
$$

$$
+ \frac{384 \pi^2}{96} (-1 + t^2) \langle g_s^2 G^2 \rangle (\varepsilon_d \langle \bar{d}d \rangle + \varepsilon_u \langle \bar{u}u \rangle) \tilde{j} (h_7)
$$

$$
+ \frac{384 \pi^2}{96} (-1 + t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle (\varepsilon_u \langle \bar{d}d \rangle + \varepsilon_d \langle \bar{u}u \rangle) \psi^\nu (u_0) .
$$
2) Coefficient of the $(\varepsilon \cdot p) \not{\phi}$ structure

\[
\Pi_2 = -\frac{1}{128\pi^4} \frac{m_b^2 M^6}{4} \left\{ 3(5 + 2t + 5t^2)(e_d + e_u)m_b (\mathcal{I}_3 - 2m_b^2 \mathcal{I}_4 + m_b^4 \mathcal{I}_5) \\
+ e_b \left[ (3 + 2t + 2t^2)\mathcal{I}_2 - 3(1 + t)^2m_b^2 \mathcal{I}_3 - 3(-1 + t)^2m_b^2 \mathcal{I}_4 + (3 - 2t + 3t^2)m_b^2 \mathcal{I}_5 \right] \right\} \\
+ \frac{1}{128\pi^4} (3 + 2t + 3t^2) e_b m_b^2 M^4 \left\{ -\mathcal{I}_1 + 3m_b^2 \mathcal{I}_2 - 3m_b^4 \mathcal{I}_3 + m_b^6 \mathcal{I}_4 \right\} \\
+ \frac{1}{1536\pi^4} m_b^2 M^2 \left\{ (5 + 2t + 5t^2)(e_d + e_u)\langle g_s^2 G^2 \rangle \\
+ 288(-1 + t^2)m_b \pi^2 \left[ e_u \langle \bar{u}d \rangle + e_d \langle \bar{u}u \rangle + e_b \langle \langle \bar{d}d \rangle + \langle \bar{u}u \rangle \rangle \right] \right\} (\mathcal{I}_2 - m_b^2 \mathcal{I}_3) \\
- \frac{e^{-m_b^2/M^2}}{768m_b^2 M^2 \pi^2} (-1 + t^2)\langle g_s^2 G^2 \rangle m_b^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
+ \frac{e^{-m_b^2/M^2}}{1536M^4 \pi^2} (-1 + t^2)\langle g_s^2 G^2 \rangle m_b (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
- \frac{e^{-m_b^2/M^2}}{384m_b \pi^2} (-1 + t^2)\langle g_s^2 G^2 \rangle \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \\
+ \frac{1}{64\pi^2} 3(-1 + t^2) m_b \pi^2 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \mathcal{I}_1 \\
+ \frac{1}{128\pi^2} (3 + 2t + 3t^2) m_b \pi^2 \left\{ \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \langle g_s^2 G^2 \rangle \\
- m_b^2 m_b^2 \left[ 7e_b \langle \langle \bar{d}d \rangle + \langle \bar{u}u \rangle \rangle + 12 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \right] \right\} \mathcal{I}_2 .
\]

3) Coefficient of the $\not{\phi}\not{\phi}$ structure

\[
\Pi_3 = \frac{1}{128\pi^4} m_b^2 M^4 \left\{ (-1 + t^2)(e_d + e_u) \left[ \mathcal{I}_2 - 2m_b^2 \mathcal{I}_3 \right](i_1(S) + i_1(S, 1) \\
+ i_1(T_2, 1) - i_1(T_4, 1) - 2i_1(T_2, v) + 2i_1(T_4, v) \right] + 12m_b^2 \mathcal{I}_3 \bar{\Psi}(h_\gamma) \right\} \\
+ (-1 + t^2)(e_d + e_u) f_{3\gamma} m_b \left( -\mathcal{I}_2 + m_b^2 \mathcal{I}_3 \right) \psi_{\alpha\alpha}(u_0) \right\} \\
+ \frac{e^{-m_b^2/M^2}}{9216m_b^2 \pi^2} (-1 + t^2)(e_d + e_u) f_{3\gamma} \langle g_s^2 G^2 \rangle M^2 \psi_{\alpha\alpha}(u_0) \\
+ \frac{e^{-m_b^2/M^2}}{96} (-1 + t^2) f_{3\gamma} M^2 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \psi_{\alpha\alpha}(u_0) \\
- \frac{1}{3072\pi^2} (-1 + t^2)(e_d + e_u) f_{3\gamma} \langle g_s^2 G^2 \rangle m_b M^2 \mathcal{I}_2 \psi_{\alpha\alpha}(u_0) \\
- \frac{e^{-m_b^2/M^2}}{384M^2} (-1 + t^2) f_{3\gamma} m_b^2 m_b^2 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \psi_{\alpha\alpha}(u_0) \\
- \frac{e^{-m_b^2/M^2}}{6912M^4} (-1 + t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_b^2 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \psi_{\alpha\alpha}(u_0)
\]
\[- \frac{e^{-m_b^2/M^2}}{13824 M^6} (-1 + t^2) f_3 g_s^2 G^2 m_b^2 m_0^2 m_1^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \psi^{\alpha'}(u_0) \]

\[+ \frac{e^{-m_b^2/M^2}}{27648 M^8} (-1 + t^2) f_3 g_s^2 G^2 m_b^2 m_1^4 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \psi^{\alpha'}(u_0) \]

\[+ \frac{e^{-m_b^2/M^2}}{9216 \pi^2} (-1 + t^2) \left\{ (g_s^2 G^2) (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \left[ i_1(S) + i_1(\bar{S}, 1) + i_1(T_2, 1) - i_1(T_4, 1) \right] - 2i_1(T_2, v) + 2i_1(T_4, v) - 12j(h, \gamma) \right\} - 4f_3 \gamma m_0^2 \pi^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \psi^{\alpha'}(u_0) \right\} . \tag{1}

4) Coefficient of the $\not{g}$ structure

\[\Pi_4 = \frac{1}{256 \pi^4} m_b^2 M^8 \left\{ 3(e_d + e_u) m_b^2 \left[ (1 + t)^2 \mathcal{I}_3 + 4(1 + t^2) m_b^2 \mathcal{I}_4 - (5 + 2t + 5t^2) m_b^4 \mathcal{I}_5 \right] \right. \]

\[- e_b \left[ (3 + 2t + 3t^2) \mathcal{I}_2 + 3(1 + t^2) m_b^2 \mathcal{I}_3 + (3 + 2t + 3t^2) m_b^4 \mathcal{I}_4 + (3 - 2t + 3t^2) m_b^6 \mathcal{I}_5 \right] \right\} \]

\[+ \frac{1}{256 \pi^4} e_b m_b^2 M^6 \left[ 4t \mathcal{I}_1 + 3m_b^2 \mathcal{I}_2 + 6tm_b^2 \mathcal{I}_3 + 3t^2m_b^2 \mathcal{I}_2 - 6m_b^4 \mathcal{I}_3 - 6t^2m_b^4 \mathcal{I}_3 + (3 + 2t + 3t^2) m_b^6 \mathcal{I}_4 \right] \]

\[+ \frac{1}{128 \pi^2} (e_d + e_u) f_3 m_b^2 M^6 \left[ (1 + t)^2 \mathcal{I}_2 - 2(1 + 4t + t^2) m_b^2 \mathcal{I}_3 \right] i_2(A, v) \]

\[+ \frac{1}{128 \pi^2} (e_d + e_u) f_3 m_b^2 M^6 \left[ (1 + t)^2 \mathcal{I}_2 - 4(1 + t + t^2) m_b^2 \mathcal{I}_3 \right] i_2(V, v) \]

\[+ \frac{1}{32 \pi^2} (3 + 2t + 3t^2)(e_d + e_u) f_3, m_b^4 M^6 \mathcal{I}_3 \psi^\nu(u_0) \]

\[+ \frac{1}{64 \pi^2} (-1 + t^2) m_b^3 M^6 (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \chi(- \mathcal{I}_2 + m_b^2 \mathcal{I}_3) \varphi'_{\gamma}(u_0) \]

\[- \frac{1}{128 \pi^2} (3 + 2t + 3t^2)(e_d + e_u) f_3, m_b^4 M^6 \mathcal{I}_3 \psi^\alpha(u_0) \]

\[+ \frac{e^{-m_b^2/M^2}}{4608 m_b^2 \pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle M^4 \left( e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle \right) \chi \varphi'_{\gamma}(u_0) \]

\[+ \frac{1}{3072 \pi^4} m_b^4 M^4 \left\{ - (5 + 2t + 5t^2)(e_d + e_u) \langle g_s^2 G^2 \rangle \right. \]

\[- 288(-1 + t^2) m_b^2 \pi^2 \left[ \langle \bar{d}d \rangle (e_b + e_u) + (e_b + e_d) \langle \bar{u}u \rangle \right] \mathcal{I}_3 \]

\[+ \frac{1}{128 \pi^2} (-1 + t^2) m_b M^4 (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \left[ i_1(S, 1) - i_1(\bar{S}, 1) + i_1(T_1, 1) \right] \]

\[- i_1(T_2, 1) + i_1(T_3, 1) - i_1(T_4, 1) \right\} \mathcal{I}_1 \]

\[+ \frac{1}{1536 \pi^4} m_b m_b M^4 \left\{ m_b \left[ (1 + t^2)(e_d + e_u) \langle g_s^2 G^2 \rangle + 114(-1 + t^2)e_b m_b \pi^2 \langle \bar{d}d \rangle + \langle \bar{u}u \rangle \right] \right. \]

\[- (1 + t^2) \pi^2 \left( e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle \right) \left[ 6m_b^2 \left( 2i_1(S, 1) - 2i_1(\bar{S}, 1) - 2i_1(T_1, 1) + i_1(T_2, 1) + i_1(T_4, 1) \right. \\ - 6i_1(S, v) - 2i_1(T_3, v) + 2i_1(T_4, v) - A'(u_0) \right) + \langle g_s^2 G^2 \rangle \chi \varphi'_{\gamma}(u_0) \]
\[- \frac{e^{-m_b^2/M^2}}{4608 m_b \pi^2} (1 + t^2) \langle g_s^2 G^2 \rangle M^2 \left( (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \right) \left[ i_1(\mathcal{T}_1, 1) - i_1(\mathcal{T}_3, 1) \right] + 6i_1(\mathcal{S}, v) + 2i_1(\mathcal{T}_3, v) - 2i_1(\mathcal{T}_4, v) \]

\[+ \frac{e^{-m_b^2/M^2}}{9216 \pi^2} \left( (e_d + e_u) f_{3\gamma} \langle g_s^2 G^2 \rangle M^2 \left[ (1 + 6t + t^2) i_2(\mathcal{A}, v) + (3 + 2t + 3t^2) i_2(\mathcal{V}, v) \right] \right) \]

\[+ \frac{e^{-m_b^2/M^2}}{2304 \pi^2} f_{3\gamma} M^2 \left[ - (3 + 2t + 3t^2) (e_d + e_u) \langle g_s^2 G^2 \rangle - 288 (-1 + t^2) m_b \pi^2 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \right] \psi^{\nu}(u_0) \]

\[+ \frac{1}{256 \pi^2} \left( -1 + t^2 \right) m_b \langle \bar{u}u \rangle \left( (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \right) \left[ \langle g_s^2 G^2 \rangle m_0^2 + \pi^2 f_{3\gamma} \left( \langle g_s^2 G^2 \rangle - 6m_0^2 m_b^2 \right) \right] \times \left( -4\psi^{\nu}(u_0) + \psi^{\alpha}(u_0) \right) \]

\[+ \frac{e^{-m_b^2/M^2}}{9216 M^4 \pi^2} \left( -1 + t^2 \right) \langle g_s^2 G^2 \rangle m_b \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \left[ - 3m_0^2 m_b^2 - 2\pi^2 f_{3\gamma} (3m_0^2 - 2m_b^2) \left( 4\psi^{\nu}(u_0) - \psi^{\alpha}(u_0) \right) \right] \]

\[+ \frac{1536 M^6}{e^{-m_b^2/M^2}} \left( -1 + t^2 \right) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^3 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \left( 4\psi^{\nu}(u_0) - \psi^{\alpha}(u_0) \right) \]

\[- \frac{e^{-m_b^2/M^2}}{9216 M^8} \left( -1 + t^2 \right) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^5 \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \left( 4\psi^{\nu}(u_0) - \psi^{\alpha}(u_0) \right) \]

\[- \frac{e^{-m_b^2/M^2}}{18432 M^2} \left( -1 + t^2 \right) \langle g_s^2 G^2 \rangle m_b \left( \langle \bar{d}d \rangle e_d + e_u \langle \bar{u}u \rangle \right) \left[ 2i_1(\mathcal{T}_1, 1) - 2i_1(\mathcal{T}_3, 1) + 12i_1(\mathcal{S}, v) \right] \]

\[+ 4i_1(\mathcal{T}_3, v) - 4i_1(\mathcal{T}_4, v) - \mathcal{A}'(u_0) \]

\[- \frac{e^{-m_b^2/M^2}}{1536 m_b \pi^2} \left( -1 + t^2 \right) \left( e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle \right) \left[ \langle g_s^2 G^2 \rangle (m_0^2 - 2m_b^2) + 12 f_{3\gamma} m_0^2 m_b^2 \pi^2 \left( 4\psi^{\nu}(u_0) - \psi^{\alpha}(u_0) \right) \right] \].

The functions $i_n$ ($n = 1, 2$), $\tilde{j}(f(u))$, and $\mathcal{I}_n$ are defined as:

\[i_1(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_q, \alpha_q, \alpha_g) f(v) \delta'(k - u_0) , \]

\[i_2(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_q, \alpha_q, \alpha_g) f(v) \delta''(k - u_0) , \]

\[\tilde{j}(f(u)) = \int_{u_0}^1 du f(u) , \]
\[ I_n = \int_{m_n^2}^{\infty} ds \frac{e^{-s/M^2}}{s^n}, \]

where

\[ k = \alpha_q + \alpha_g \bar{v}, \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}, \] and \( \bar{v} = 1 - v. \]
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Figure captions

**Fig. (1)** The dependence of the magnetic moment of the Σ^0_c baryon on M^2, at several fixed values of t, and at s_0 = 12.0 GeV^2.

**Fig. (2)** The dependence of the magnetic moment of the Σ^0_c baryon on cos θ, at several fixed values of M^2, and at s_0 = 12.0 GeV^2.
Figure 1:

$s_0 = 12.0 \text{ GeV}^2$

Figure 2:

$s_0 = 12.0 \text{ GeV}^2$

$M^2 = 3.0 \text{ GeV}^2$  
$M^2 = 4.0 \text{ GeV}^2$