Degenerate neutrino mass models revisited

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Abstract

A parametrisation of the degenerate neutrino mass matrix obeying $\mu - \tau$ symmetry, is introduced for detailed numerical analysis. The present parametrisation for degenerate models has the ability to lower the solar mixing angle below the tri-bimaximal value $\tan^2 \theta_{12} = 0.5$, while maintaining the condition of maximal atmospheric mixing angle and zero reactor angle. The combined data on the mass-squared differences derived from various oscillation experiments, and also from the bounds on absolute neutrino masses in $0\nu\beta\beta$ decay and cosmology, gives certain constraints on the validity of the degenerate models.

Keywords. Degenerate neutrino mass model, absolute neutrino masses, solar mixing angle below tribimaximal mixings.

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1 Introduction

Discrimination of neutrino mass patterns among three possible cases, viz., degenerate, normal and inverted hierarchical models, has drawn considerable attentions in the last one decade. This has renewed interest with the presently available precise observational data from neutrino oscillation experiments as well as bounds on absolute neutrino masses. The present neutrino oscillation data gives the following information for the parameters at $3\sigma$ from global $3\nu$ oscillation analysis.\[1\]

\[
\begin{align*}
\Delta m^2_{21}(10^{-5}eV^2) &= (7.4 - 7.9) \text{ (best-fit at } 7.7), \\
| \Delta m^2_{32}(10^{-3}eV^2) | &= (2.1 - 2.8) \text{ (best-fit at } 2.4), \\
\tan^2 \theta_{12} &= (0.41 - 0.58) \text{ (best-fit at } 0.45), \\
\tan^2 \theta_{23} &= (0.56 - 2.03) \text{ (best-fit at } 1.0), \\
\sin^2 \theta_{13} &= 0.016 \pm 0.010.
\end{align*}
\]

At present there is no precise information about the absolute neutrino masses. One could expect only the upper bounds on the absolute neutrino masses from various experiments like the Tritium $\beta$ decay, Neutrinoless double beta decay and the cosmological observations. The bounds on the absolute neutrino masses are \[2, 3\]

\[
\begin{align*}
m_{\nu_e} &= (\sum_i m_i^2 |U_{ei}|^2)^{1/2} < 2.3eV \text{ (Tritium } \beta \text{ decay)} \\
m_{ee} &= |\sum_i m_i U_{ei}^2| < 0.3 - 1.0eV \text{ (0$\nu$}\beta\beta \text{ decay)} \\
m_{\text{cosmo}} &= \sum_i |m_i| < 0.61eV \text{ (cosmological bounds)}
\end{align*}
\]

While both normal and inverted hierarchical models are well within the above observational bounds, and are therefore far from ruling out at the moment, the degenerate models are considered almost disfavoured in the literature, following the present bounds on absolute neutrino masses, particularly from cosmological and neutrinoless double beta decay bounds. However a closer analysis reveals that such assertion is only correct, specifically for larger absolute neutrino masses $m_0 \sim 0.4eV$. There are still possibilities for certain degenerate models which allow lower values of neutrino masses $m_0 \sim 0.1eV$ valid for lower values of solar mixings. This is one of the objectives for further investigations in the present work.

Degenerate models are generally classified according to their CP-parity pattern in their mass eigenvalues $m_i = (m_1, m_2, m_3)$, viz., Type IA: $(++)$, Type IB:$(+++)$, Type IC: $(++-)$ respectively. It will be shown in the present
work that both Type IB and IC are almost disfavoured by the presently available data on absolute neutrino masses. This is due to the fact that the predicted absolute neutrino masses are in the range \( m_0 \sim 0.4eV \) in these two cases. On the other hand, Type IA has many interesting properties which are yet to be explored, particularly variation of \( m_0 \) with solar mixing \( \tan^2 \theta_{12} \), and also a partial cancellation of even and odd CP parity in the first two mass eigenvalues appeared in the expressions of \( m_{\nu_e} \) and \( m_{ee} \). This makes Type IA degenerate model far from ruling out. It provides enough scope for future experiments to go the sensitivity down to \( |m_{ee}| = 0.03eV \).

In the theoretical front there are several attempts to find out the most viable neutrino mass models, and among them the neutrino mass models obeying \( \mu - \tau \) symmetry \([4, 5, 6, 7, 8]\), have drawn considerable attention. Neutrino mass matrix having \( \mu - \tau \) symmetry, leads to maximal atmospheric mixings \( \tan^2 \theta_{23} = 1 \) and zero reactor angle \( \theta_{13} = 0 \), whereas the solar angle is purely arbitrary. This has to be fixed by the input values of the parameters in the mass matrix. The tribimaximal mixings (TBM) \([9, 10]\) with \( \tan^2 \theta_{12} = 0.5 \) is a special case of this symmetry. There are four unknown elements present in a general \( \mu - \tau \) symmetric mass matrix and it is difficult to solve these four unknown elements from only three equations involving observational data on \( \tan 2\theta_{12}, \Delta m_{21}^2 \) and \( \Delta m_{32}^2 \). Thus

\[
\begin{pmatrix}
m_{11} & m_{12} & m_{12} \\
m_{12} & m_{22} & m_{23} \\
m_{12} & m_{23} & m_{22}
\end{pmatrix}
\]

(1)

where the eigenvalues and solar mixings are

\[
m_1 = m_{11} - \sqrt{2} \tan \theta_{12} m_{12},
\]

\[
m_2 = m_{11} + \sqrt{2} \cot \theta_{12} m_{12},
\]

\[
m_3 = m_{22} - m_{23},
\]

\[
\tan 2\theta_{12} = \frac{2\sqrt{2}m_{12}}{m_{11} - m_{22} - m_{23}}.
\]

In our earlier works we simply considered possible parametrisation with lesser numbers of free parameters consistent with available data for practical solution of the mass matrix. We have already reported our method for parametrisation with only two parameters \( \eta \) and \( \epsilon \) and the ratio of the these two parameters named as flavour twister \([11]\), is responsible for lowering the solar
angle below tri-bimaximal solar mixing. We have already parametrised both normal and inverted hierarchical neutrino mass matrices\cite{12}. In TBM mixing we have the value of the solar mixing angle, $(\tan^2 \theta_{12} = 0.5)$. But, the recent global $3\nu$ oscillation analysis has shown a mild departure from the tri-bimaximal neutrino mixings, $\tan^2 \theta_{12} = 0.45$ and the present work has its own relevance here. Such parametrisation thus reduces to two unknown free parameters $\eta$ and $\epsilon$ in addition to an overall neutrino mass scale $m_0$, making the three equations solvable in practice.

In section 2 we first extend our earlier method of parametrisation of $\mu - \tau$ symmetric mass matrices, to a particular degenerate neutrino mass model Type 1A($m_1, -m_2, m_3$) denoted by CP-parity pattern $(+-+)$, and also identify the flavour twister term to obtain tribimaximal mixings and then modify it for deviations from tribimaximal mixings. Similar analysis is also carried out for Type IB and IC degenerate models. Section 3 concludes with a brief summary.

2 Parametrisation of degenerate models

2.1 Degenerate Type 1A ($m_1, -m_2, m_3$)

The zeroth order left-handed Majorana mass matrix \cite{13} for Type IA degenerate model, is given by

$$m_{LL}^o = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} m_0$$ \hspace{1cm} (2)

where $m_0$ is the overall scale factor and mass eigenvalues are $\text{Diag}(1, -1, 1)m_0$. A complete light neutrino mass matrix $m_{LL}$ for the degenerate neutrino mass model (Type 1A) with $\mu - \tau$ symmetry in eq.(2) is now modified as

$$m_{LL} = \begin{pmatrix} \epsilon - 2\eta & -c\epsilon & -c\epsilon \\ -c\epsilon & 1/2 - d\eta & -1/2 - \eta \\ -c\epsilon & -1/2 - \eta & 1/2 - d\eta \end{pmatrix} m_0$$ \hspace{1cm} (3)
where $c$ and $d$ are just real constants, and $\eta$ and $\epsilon$ are the unknown parameters. The mass matrix still maintains the $\mu - \tau$ symmetry but the arbitrary solar angle can be fixed at the particular value by choosing a set of values for $c, d, \eta$ and $\epsilon$.

The mass matrix in eq.(3) predicts an expression for solar mixing angle,

$$\tan 2\theta_{12} = \frac{2\sqrt{2}(-c\epsilon)}{\epsilon - 2\eta - 1/2 + d\eta + 1/2 + \eta} = -\frac{2\sqrt{2}c}{1 + (d - 1)\eta/\epsilon}$$  \hspace{1cm} (4)$$

In general, any mass matrix obeying $\mu - \tau$ symmetry, can be diagonalized by the following unitary matrix,

$$U_{MNS} = \begin{pmatrix}
\cos \theta_{12} & -\sin \theta_{12} & 0 \\
\sin \theta_{12} & \cos \theta_{12} & 0 \\
\frac{\sqrt{2}}{\sqrt{2}} & \frac{\sqrt{2}}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}$$  \hspace{1cm} (5)$$

Diagonalizing the mass matrix (3) we get the three mass eigenvalues as,

$$m_1 = \frac{1}{2}(\epsilon - 3\eta - d\eta + \chi)m_0, \hspace{1cm} (6)$$

$$m_2 = \frac{1}{2}(\epsilon - 3\eta - d\eta - \chi)m_0, \hspace{1cm} (7)$$

$$m_3 = (1 + \eta - d\eta)m_0, \hspace{1cm} (8)$$

$$\chi^2 = \epsilon^2 + 8c^2\epsilon^2 - 2\eta\epsilon + 2d\eta\epsilon + \eta^2 - 2d\eta^2 + d^2\eta^2 \hspace{1cm} (9)$$

The choice of $c = d = 1$ in eq.(4) leads to tribimaximal mixings (TBM), $\tan^2 \theta_{12} = 0.5$ ($\tan 2\theta_{12} = 2\sqrt{2}$). With the input value $m_0 = 0.4\text{eV}$, the values of the free parameters are extracted from the data of neutrino oscillation mass parameters, as $\epsilon = 0.661145$ and $\eta = 0.165348$ respectively. As seen in (4) there is no flavour twister term in this case.

In the next step the flavour twister $\eta/\epsilon$ responsible for lowering solar angle, is then introduced by taking $c < 1$ and $d > 1$. Using the known values of $\eta$ and $\epsilon$ already fixed in TBM, along with those of $c$ and $d$ for a particular choice of solar angle, the value of $m_0$ is extracted. Table 1 gives the numerical results for a few selected cases. A scattered plot in Fig.1 using the points
generated within the allowed ranges of neutrino oscillation mass parameters, depicts the dependence of \( \tan^2 \theta_{12} \) on \( m_0 \). This leads to \( m_{\text{cosmos}} \) within the upper bound from cosmology. For \( \tan^2 \theta_{12} = 0.45 \), we get \( \sum |m_i| = 0.275 \text{eV} \) and \( |m_{ee}| = 0.033 \text{eV} \). It can be emphasised that the predictions on other oscillation parameters are consistent with latest data.

### 2.2 Degenerate Types 1B and 1C

We now briefly perform similar analysis for other two degenerate models - Type IB with CP-Parity \((+++\)) and Type IC with \((+--\)). The structures of their mass matrices are given below:

**Degenerate Type 1B \((m_1, m_2, m_3)\)**

\[
\begin{pmatrix}
1 - \epsilon - 2\eta & -\epsilon \epsilon & -\epsilon \\
-\epsilon \epsilon & 1 - d\eta & -\eta \\
-\epsilon \epsilon & -\eta & 1 - d\eta
\end{pmatrix}
\begin{pmatrix}
1.0 \\
0.931 \\
0.868
\end{pmatrix}
\]

**Degenerate Type 1C \((m_1, m_2, m_3)\)**

- Parameters:
  - \( c \)  
  - \( d \)  
  - \( m_0 (\text{eV}) \)  
  - \( m_1 (\text{eV}) \)  
  - \( m_2 (\text{eV}) \)  
  - \( m_3 (\text{eV}) \)  
  - \( \Delta m^2_{21} \times 10^{-5} \text{eV}^2 \)  
  - \( \Delta m^2_{23} \times 10^{-3} \text{eV}^2 \)  
  - \( \tan^2 \theta_{12} \)  
  - \( |m_{ee}| (\text{eV}) \)  
  - \( \sum |m_i| (\text{eV}) \)  

\[
\begin{array}{ccc}
1.0 & 0.931 & 0.868 \\
1.0 & 1.011 & 1.025 \\
0.4 & 0.141 & 0.100 \\
0.39664 & 0.13119 & 0.08793 \\
-0.39674 & -0.13148 & -0.08793 \\
0.40000 & 0.14074 & 0.09959 \\
7.838 & 7.651 & 7.685 \\
2.600 & 2.521 & 2.186 \\
0.5 & 0.475 & 0.450 \\
0.132 & 0.047 & 0.033 \\
1.193 & 0.403 & 0.275
\end{array}
\]

Table 1: Numerical calculation for Type IA degenerate model. Different values of parameters \( c, d, \epsilon \) and \( \eta \) along with the corresponding ranges of \( \Delta m^2_{21} \) and \( \Delta m^2_{23} \), \( m_{ee}, \sum |m_i| \) and \( \tan^2 \theta_{12} \) for fixed value of \( \tan^2 \theta_{23} = 1.0 \) and \( \sin \theta_{13} = 0 \).
Figure 1: Scattered plot showing the variation of solar mixing $\tan^2 \theta_{12}$ with the scale of absolute neutrino masses $m_0 (eV)$ in Type IA degenerate model with CP parity pattern (+-+).

Degenerate Type 1C($m_1, m_2, -m_3$)

\[
m_{LL}^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0
\]

\[
(12)
\]

\[
m_{LL} = \begin{pmatrix} 1 - \epsilon - 2\eta & -\epsilon \eta & -\epsilon \\ -\epsilon \eta & -\eta & 1 - d\eta \\ -\epsilon & 1 - d\eta & -\eta \end{pmatrix} m_0
\]

\[
(13)
\]

where the zeroth-order mass matrix with mass eigenvalues $\text{Diag}(1, 1, -1)m_0$, is given by
Figure 2: Scattered plot showing a relation of solar mixing $\tan^2 \theta_{12}$ with the scale of absolute neutrino masses $m_0 (eV)$ in Type IB degenerate model with CP parity pattern $(+++)$

$$m^0_{LL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$$  \hspace{1cm} (14)

The above two models (Type IB and IC) are similar except the interchange of (22) and (23) elements, which imparts a negative sign before the third mass eigenvalue $m_3$. Both give the same expression for the solar mixing angle as

$$\tan 2\theta_{12} = \frac{2\sqrt{2}c}{1 + (1 - d)\eta/\epsilon}$$  \hspace{1cm} (15)

Using the condition $c = d = 1$ for tribimaximal mixings and input value $m_0 = 0.4eV$, the values of $\eta$ and $\epsilon$ are solved as $\eta = 8.3138 \times 10^{-5}$ and $\epsilon = 0.00395$. These two values are again used for lowering the solar angle corresponding to the condition $c < 1$ and $d < 1$. However the neutrino mass scale is always obtained at $m_0 = 0.4eV$ leading to $\sum |m_i| = 1.194eV$ and $m_{ee} = 0.397eV$ respectively. Table 2 gives some representative numerical
| Parameters | I   | II  | III  |
|-----------|-----|-----|------|
| c         | 1.0 | 0.945 | 0.868 |
| d         | 1.0 | 0.998 | 0.998 |
| $m_0$(eV) | 0.4 | 0.4 | 0.4 |
| $m_1$(eV) | 0.39677 | 0.39678 | 0.39678 |
| $m_2$(eV) | 0.39687 | 0.39687 | 0.39687 |
| $m_3$(eV) | 0.4000 | 0.40000 | 0.40000 |
| $\Delta m^2_{21}(10^{-5}eV^2)$ | 7.936 | 7.619 | 7.143 |
| $\Delta m^2_{23}(10^{-3}eV^2)$ | 2.492 | 2.494 | 2.494 |
| $\tan^2 \theta_{12}$ | 0.5 | 0.45 | 0.421 |
| $|m_{ee}|$(eV) | 0.397 | 0.397 | 0.397 |
| $\sum |m_i|$(eV) | 1.194 | 1.194 | 1.194 |

Table 2: Numerical calculation for Type IB degenerate model. Different values of parameters $c,d,c$ and $\eta$ along with the corresponding ranges of $\Delta m^2_{21}$ and $\Delta m^2_{23}$, $m_{ee}, \sum |m_i|$ and $\tan^2 \theta_{12}$ for fixed value of $\tan^2 \theta_{23} = 1.0$ and $\sin \theta_{13} = 0$.

There is no variation of solar angle with neutrino mass as shown in Fig.2. Such degenerate models are generally disfavoured by the bounds with absolute neutrino masses.

3 Summary and conclusion

To summarise, we have presented a method of parametrisation of the degenerate neutrino mass models obeying $\mu - \tau$ symmetry, and this enables to lower the solar mixing angle below the tri-bimaximal value $\tan^2 \theta_{12} = 0.5$, while maintaining the conditions of maximal atmospheric mixing angle and zero reactor angle. The predictions from these mass models are consistent with the data on the mass-squared differences derived from various oscillation experiments, and also from the bounds on absolute neutrino masses from $0\nu\beta\beta$ decay and cosmology. Type IA degenerate model with CP-parity pattern $(+-+)$ predicts a variation of absolute neutrino mass scale with the solar mixing angle, and the best-fit value $\tan^2 \theta_{12} = 0.45$ corresponds to the absolute neutrino mass scale $0.1eV$. The model is not yet ruled out and therefore far from discrimination. The other two types IB and IC having CP parity patterns $(+++)$ and $(+-)$ respectively, do not possess such variation and the neutrino mass scale is almost fixed at about $0.4eV$ for a wide range.
of solar mixing angle. These models are found to be almost disfavoured by the available data from absolute neutrino masses. The present result has profound implications for future experiments on the discrimination of neutrino mass hierarchy.

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