A Study of the Relationship between Fractal Dimension of Boundary Trace and Stability of the Loess-bedrock Landslide

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Abstract. Through the analysis and summary of the characteristics of the boundary trace of the Loess bedrock landslides in the Baota district of Yan'an, it is found that the characteristics are clear and obvious and have some similarity. This paper based on the fractal dimension theory, the fractal dimension of boundary trace of 8 typical loess bedrock landslide in Baota district are calculated by box counting method. A fractal dimension evaluation of landslide steady state model is established by the fractal dimension of the landslide boundary track correlation and stability of landslide. The correctness of the model is Verifified through the example. The study shows that the characteristics of the Loess bedrock landslide boundary trajectory can be used to evaluate the stability of the landslides.

1. Preface
The Baota district of Yan'an City is one of the typical mountain cities in the hinterland of the Loess Plateau. The special geographical environment has led to frequent geological disasters in this area. According to statistics of the 1:50,000 Geological Disaster Survey in Baota District, Yan'an City, Shaanxi Province in 2006, there were 396 geological disasters in Baota District, including 293 landslides, 52 collapses, and 51 unstable slopes. In many landslides, the loess-bedrock contact surface landslide is the most common type, accounting for 76.8% of the total number of landslides [1]. In recent years, with the development of "new area construction, old city reconstruction, the relocation of mountain residents, channel management, municipal facilities supporting, and urban landscape upgrading", major achievements have been made in the construction of urban system in Yan'an city. A lot of infrastructure construction must increase the burden of the geological environment, leading to the revival of the stable ancient landslide and the old landslide, among which the landslide is most common in the loess-bedrock contact surface.

In this paper, by summarizing the characteristics of the boundary trace of the loose-bedrock landslide in the Baota district, based on the fractal dimension theory, the fractal dimension of the typical landslide boundary trajectory is calculated, and the correlation between the fractal dimension and the stability is analyzed and applied. It is verified by engineering examples. The study are able to provide reference for stability evaluation and emergency management of the same type landslide.
2. Characteristics of typical loess-bedrock landslide boundary trace

Baota district is located in the central Ordos massif, because did not suffer from major tectonic erosion, keeping the basic characteristics of the sedimentary basin, and the Quaternary loess cover large area, formed the special geological environment at present: the loess deposits are thick and extensive; the loess sedimentary base is a continental clastic sedimentary rock group, which is different from loess engineering geology; the internal and external dynamic geological processes form the landform and geomorphological features of ravines and crisscross. These geological environments provide good material and space conditions for the development of geological disasters.

Most of the landslide of loess - bedrock slid from the contact surface of the loess and rock, which is mainly developed in the area of valley landscape and human activities. The boundary trajectories of them are clearly visible and have certain similarities. Including the following aspects:

1) Characteristics of appearance

The landslide of loess-bedrock in the Baota district is affected by gully or underlying bedrock surface, and the whole mostly likes "dumpling" (Fig 1). Most the rear walls of the landslides are in the shape of a circle chair or an arc, usually in the upper part, and the slope of them is 60° to 80°.

Under the effect of rainfall, the water in the depression is drained to both sides and gradually forms deep gully. The slope surface erosion is serious, causing a number of small gullies and ponors.

![Figure 1. The whole map of Er Zhuang ke landslide](image)

(2) Characteristics of the boundary

1) The trailing edge of the landslide: as the most obvious feature of the boundary, it is mostly steep-walled, with significantly grade which is larger than the original slope, tending to be basically consistent with the original slope direction, ranging in height from a few meters to a dozen meters, and tending from integrity to breakage by suffered natural weathering and erosion. 2) The side circles of landslide: divided into two parts: the side wall and the sliding body boundary. The upper part of the landslide side is the side wall, and its characteristics are similar to the trailing edge. The sliding body boundary is the boundary of landslide sliding mass, most of which is bounded by ravine and ridge line. After the landslide occurred, the slope was piled up and extended to both sides. Due to the different historical time of the landslide, the degree of clarity was different. 3) Front of landslides: mainly exposed in the toe of slope or river valleys. Because of the effects of water erosion, artificial excavation, etc., the front edge characteristics are not obvious. But the shear outlet is exposed clearly in the loess bedrock interface (Fig.2, Fig.3).
3. Study on the fractal dimension characteristics of landslide boundary trace

3.1. Fractal dimensions of landslide

Fractal theory is first proposed by Manddelbrot, an American mathematician, in his study of irregular but self-similar system in nature. It has been applied in the fields of physics, geology, chemistry and so on.

In 1989, Manddelbrot used fractals to quantitatively describe the geometric distribution and characteristics of landslides. The fractal theory was introduced into the study of landslide disaster, and then a series of related studies were carried out by scholars. Kubota, Yokoi, Y., Goltz, et al. made a lot of in-depth studies on the fractal characteristics of landslide distribution, and concluded that the fractal dimension of landslides can be used to assess landslide risk. Wang Zhiwang introduced the fractal structural features of the landslide, including material particles, the slide track, the boundary trace, the deformation displacement and evolution process of the landslide. Based on the fractal theory, Wu Shuren calculated the fractal dimension of the boundary trace of the HuangLashi and Huanghai landslides in the Three Gorges, the ChangJiang River, the results showed that the landslide geometry has obvious fractal features. Zheng Mingxin took the Xintan landslide in the ChangJiang River and the Huangci landslide in Gansu Province as examples, used the fractal theory to quantitatively describe the landslide displacement and velocity fractal characteristics, and analyzed the dynamic correlation with the landslide. Wu Shuren analyzed the fractal dimensions of the boundary trace of the 11 major landslides in Badong County of the Three Gorges Reservoir area using box dimension method, The results showed the more complex the structure of the landslide trajectory, the higher the value of fractal dimension.

The fractal dimension has various calculation forms. According to different objects, the commonly used methods are Hausdorff dimension, information dimension, correlation dimension, similar dimension, capacity dimension, etc. For the landslide boundary trace, it belongs to the statistical measurement of a certain small area. It is simple and objective to use the capacity dimension calculation. The capacity dimension, also known as the box dimension method, is one of the most widely used dimensions. Its calculation method is: using a square side length of r to measure the geometric boundary map of the landslide, counting the number of the square lattice occupied by the landslide boundary trace as N(r), then changing the length of side, counting the number of squares with different sizes in the boundary trace, finally making use of the follow formula to calculate:

$$D = \lim_{r \to 0} \frac{\lg N(r)}{\lg (r)}$$  \hspace{1cm} (1)

In actual calculation, the slope of the straight line segment is fitted by the least squares method on the double logarithmic graph with r as the abscissa and N(r) as the ordinate, which is the fractal dimension D sought.
According the measuring scale of landslide, the number of the square lattice with 1,5,10,20,40,80mm occupied by landslide boundary trace are counted separately(Table 1). These data are projected onto a double logarithmic graph, and calculated using least square method. The slope of the fitting curve is obtained, which is the fractal dimension.

Table 1 Fractal dimension of typical landslide boundary trajectory

| Square size /mm | Trace length /m | ErZhuang ke | Xiao Bian gou1 | Xiao Bian gou2 | Wang Jia gou1 | Wang Jia gou2 | Qiao Gou | Ding Jia gou | Zhao Zhuang |
|-----------------|-----------------|-------------|----------------|----------------|---------------|---------------|----------|-------------|------------|
| 1               | 2               | 1150        | 914            | 1478           | 131           | 113           | 125      | 1013        | 270        |
| 5               | 10              | 300         | 184            | 312            | 26            | 24            | 27       | 195         | 52         |
| 10              | 20              | 134         | 88             | 153            | 14            | 12            | 13       | 90          | 26         |
| 20              | 40              | 66          | 46             | 75             | 8             | 6             | 8        | 42          | 13         |
| 40              | 80              | 34          | 22             | 38             | 5             | 3             | 3        | 18          | 5          |
| 80              | 160             | 18          | 10             | 15             | 2             | 1             | 1        | 8           | 2          |

Stability coefficient k 1.138 1.055 1.094 1.023 1.084 1.042 1.034 1.116
fractal dimension D 1.021 1.058 1.033 1.121 1.050 1.084 1.107 1.029

3.2. The relationship between the fractal dimension and the stable of landslide

Based on the above methods, eight typical landslides in the Baota district were selected, and the fractal dimension and stability coefficient were calculated respectively (Tab.1). The fractal dimension of the landslides are between 1.021 and 1.121, the stability coefficients of the landslides are between 1.138 and 1.023. The stability and fractal dimension showed a significant negative correlation, that is, the worse the stability, the higher the fractal dimension. Then the fractal dimension and the stability coefficient are projected onto a double logarithmic graph, and the least squares method is used for linear regression fitting (Fig.4). The fitting relationship is:

\[ \log k = 0.0007(\log D)^{-0.967} \]  

(2)

Figure 4. Correlation between Fractal Dimensions and Stability Coefficients of Boundary Trace

According to the division of landslide stability in the Code for Geological Investigation of Landslide Prevention (GB/T32864-2016) and the formula (2) in this paper, the stability of the landslide is divided by the fractal dimension of the landslide boundary trace. This method can quickly assess the stability of the landslide by measuring the landslide boundary, but the traditional evaluation
method needs to do a lot of sampling, experiment and numerical calculation. So the method can provide reference for the hazard and emergency management of such landslide.

Table 2 Classification of Stable of landslides

| stability coefficient $k$ | $k < 1.00$ | $1.00 \leq k < 1.05$ | $1.05 \leq k < 1.15$ | $K \geq 1.15$ |
|--------------------------|------------|----------------------|----------------------|--------------|
| fractal dimension of landslide boundary trace $D$ | $D > 1.425$ | $1.425 \leq D < 1.075$ | $1.075 \leq D < 1.025$ | $1.025 \leq D < 1.0$ |
| Stable of landslides     | Instability | Less stability       | Basic stability       | Stability    |

4. Application instance

The QingHuasi landslide was selected in Baota district of Yan'an to evaluate its the stability by the fractal dimension of landslide boundary trace, and the results were verified with the traditional limit equilibrium method.

4.1. The characteristics of the landslide boundary trace

The QingHuasi landslide is located in the loess girder area on the west side of the DaTang power plant of Yan'an. Material composition of landslides consists mainly of Quaternary landslide deposition, the Pleistocene loess and the paleosol of the Middle Pleistocene. The sliding zone is divided into two parts: the middle and back of the sliding zone are between the loess layers, and the lower and front part of the sliding zone are the top surface of the Triassic sand-mudstone. It is a typical loess-bedrock interface landslide (Fig 5).

![Figure 5. The whole map of the QingHuasi Landslide](image)

The apparent characteristics of the landslide are as follows: The overall terrain is east low west high, south low north high, and tilted from west to east. The average angle of the slope is relatively low, about 20°. But the undulations are large, and there are some scarps. A total of number of ponors are 27 which appeared on the landslide platform where have the growth of gullies.

The characteristics of the boundary: ①the northwest-western-western side is bounded by the trailing edge of a landslide as round-backed armchair, which is 10 to 20 meters high. There are obvious signs of sliding scratches. The trailing edge extend north and south to form the side wall of the landslide. ②Both sides of the landslide are bounded by the edge of the landslide deposit. The apparent colour difference is obvious. The colour of the landslide deposit is dominated by yellow-brown, the Malan loess is mainly pale yellow, the ancient soil is mainly red-brown, so the boundary is
clear. ③ The front edge of the landslide is the northwest-southeast extension of the I-grade terraces and floodplain of the PanLong River. Because of the sliding of landslide, the leading edge of spreads in a meandering pattern. At the same time, the south bank of the river is formed some different heights of steepness due to the lateral erosion of the foot of the slope.

4.2. Fractal dimension of the landslide boundary trace
The boundary trace of the QingHuasi landslide is complex because there are two secondary landslides on both the north and south sides. Therefore, in the calculation of the fractal dimension, the boundary trace of the secondary landslide is taken into. The values are calculated separately by the calculation method described in section 2.1 (Tab.3, Fig.6).

Table 3 Measurement data of the boundary trace of QingHuasi landslide

| Square size/mm | 1  | 5  | 10 | 20 | 40 | 80 |
|----------------|----|----|----|----|----|----|
| Trace length/m |    |    |    |    |    |    |
| No secondary landslides measured value | 2  | 10 | 20 | 40 | 80 | 160 |
| Secondary landslides | 1515 | 299 | 138 | 68 | 33 | 16 |

Figure 6. Calculation of fractal dimension of landslide boundary trace

According to the division of the stable state in Table 2, the boundary trace of the old landslide of QingHuasi has a fractal dimension of 1.041, which in basic stability. When the secondary landslide is formed or partially slipped, the boundary trace has a fractal dimension of 1.076, which in less stability. The stability coefficient is 1.084 and 1.046 when bring the fractal dimension into formula 1. But the stability coefficients calculated according to the traditional limit equilibrium method is 1.074 and 1.025. The relative error is 0.9% and 2%, and the evaluation result is the same. It can be seen that the fractal dimension of the boundary trace of the loess-bedrock landslide can be used to evaluate the stability of such landslides.
5. Conclusion
In this paper, based on the summary of the characteristics of boundary trace of the loess-bedrock landslides, the relationship between the fractal dimension of the boundary trace and the stability coefficient is fitted through the eight typical landslides. The fractal dimension value is used instead of the landslide stability coefficients to classify the stability of landslide, and the accuracy and robustness of the model are verified by the actual landslide. The main conclusions are as follows:

(1) The loess-bedrock landslide has obvious characteristics of boundary trace which has certain similarities. Through the research on the correlation between the fractal dimension and stability, the fractal dimension of the boundary trace can be used to evaluate the stability of landslides. According to the negative correlation between the two, it can be seen that the process of the boundary trace increasing is consistent with the process of landslide deformation instability. The more obvious increase of the fractal dimension of landslide boundary trace, the more obvious the decrease of stability.

(2) When there are Secondary landslides in the stage of extended deformation in the old landslide, a new trace appear, and the fractal dimension increases. At the same time, because the boundary trace of the secondary landslides are not connected, the new boundary trace is highly independent from the fractal dimension of the original boundary trace. With the expanding of secondary landslides, the traces are interconnected, so that the fractal dimension increases sharply, the landslide will slide.

(3) From the fractal geometry analysis, when the fractal dimension of the landslide boundary trace is less than 1, the fractal structure is stable and the landslide is stillness. When the fractal dimension is more than 1, proliferation phenomenon of the fractal structure begins to appear, and the landslide starts peristaltic deformation. There are some cracks on the surface of the landslide.

6. Expectation
In this study, the following points are still unclear:

(1) The model established in this paper using the fractal dimension of the landslide boundary trace to evaluate its stability state is only applicable to the loess-bedrock landslide in the Baota district of Yan’ an, where the boundary trace is clear and has obvious characteristics. There is still needed to verify other types of landslides in other areas.

(2) Is there a critical fractal dimension on the fractal dimension of the boundary trace? Due to the process of landslide is a time course, the relationship can be theoretically established between the fractal dimension and time, and the critical fractal dimension and its occurrence time can be accurately calculated. The result can provide a reference for landslide early warning.

(3) This study focuses on the natural evolution of landslides. When human engineering activities such as excavating foothill and filling hillsides, cause strong damage to the landslide boundary trace, the variation trend and rule of fractal dimension need to be further studied.

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