Springback and compensation in sheet metal forming reconsidered as an ill-posed problem

Christoph Hartmann, Lorenz Maier, Wolfram Volk*
Chair of Metal Forming and Casting, Technical University of Munich
Walther-Meissner-Strasse 4, 85748 Garching, Germany

*wolfram.volk@utg.de

Abstract. Springback is a fundamental challenge in sheet metal forming and a critical issue with respect to part quality and dimensional accuracy. To overcome the problem, different strategies for springback compensation have emerged in recent years. In this paper, we give a review on fundamental strategies of compensation for deviations in sheet metal forming and embed them, for the first time, in a general description framework rooted in the solution of ill-posed problems. On this basis, we conclude by briefly comparing different approaches and elaborate on future challenges and chances that naturally arise from the presented problem formulation.

1. Introduction
During the forming of sheet metal components, springback occurs on the component due to the elastic-plastic behavior of the material [1]. This springback is an important factor that has a decisive influence on the quality of the formed components. The size and type of springback is mainly determined by the stress distribution occurring in the part as a function of the forming process. The springback effect is influenced by multiple factors, such as the elastic-plastic properties, sheet thickness, lubrication, tool geometry, and others. Springback can occur both in a simple form in the sheet metal plane or perpendicular to the forming tool’s surface, and in a complex torsional effect on the component [2,3]. Thus, there is always a significant difference between tool and part geometry. In the sheet metal forming simulation, we can calculate straight-forward the amount of springback. Only considering a process-reliable compensation regime allows to find the proper tool geometries, which leads to the desired part geometry after springback. This is a great challenge especially for high-strength and ultra-high-strength steel grades [1,3]. In practice, this task is commonly met by iterative processes and tool modifications in order to achieve and maintain the required shape and dimensional accuracy.

2. Problem formulation
The problem of springback compensation is studied in many virtual and experimental investigations, not only because of its scientific complexity, but also because of its relevance for the industrial application. These works are based on a classical problem formulation, which we briefly condense in section 2.1. In this paper, we would like to introduce an alternative derivation of the compensation problem formulation from the viewpoint of Jacques Hadamard’s definition on well-posed problems [4], see section 2.2. This more fundamental perspective allows us to include process robustness and stochastic fluctuations in the discussion, in addition to the classification of current compensation approaches.
2.1. Classical problem formulation

The classical problem formulation originates from the objective that the deviation between the target geometry $t$ and the final component geometry after springback $g$ should be as small as possible [5,6]. This translates into a minimization problem with respect to the die geometry $d$ and process parameters, material, and adjustable variables consolidated in the parameter vector $p$. Since tool design and setting of parameters underlie certain restrictions, the variations during the minimization is subject to certain inequality and equality constraints $c_{in}(d,p) < l_{in}$ and $c_{eq}(d,p) = l_{eq}$, for example to prevent undercuts in the tool geometry. In addition, commonly a predetermined stopping criterion based on the geometrical deviation between target geometry $t$ and the final component geometry after springback $g$ is defined, because we always face uncertainties and scattering of the material and process parameters.

For a single stage sheet metal forming process, for example also conducting only the last relevant stage for compensating the whole process, we show the classical problem formulation in expression (1). Here, the overall objective is to identify a proper die geometry $d$.

$$
d = \arg\min_{d,p}(\|t - g(d,p)\|_2)$$

subject to $c_{in}(d,p) < l_{in} \land c_{eq}(d,p) = l_{eq}$

For multistage processes with $n$ stages, different of these problem formulations emerge. The minimization with respect to the final part deviations $\|t - g_n\|_2$ yields expression (2).

$$
d_i := \arg\min_{d_i,p_i}(\|t - g_n(d_n,p_n; g_1(d_1,p_1), ..., g_{n-1}(d_{n-1},p_{n-1}))\|_2)$$

\(\forall i = 1 \ldots n\) and subject to $c_{in}(d_i,p_i) < l_{in,i} \land c_{eq}(d_i,p_i) = l_{eq,i}$

An alternative for multistage processes with $n$ stages is to work with respect to each individual stage $i$. Hence, the geometrical deviation $\|t_i - g_i(d_i,p_i)\|_2$ should be minimized individually and we arrive at the global minimization expression (3), which may not coincide with results obtained by using expression (2).

$$
d_i := \arg\min_{d_i,p_i}(\|\sum_{i=1}^{n}t_i - g_i(d_i,p_i)\|_2)$$

\(\forall i = 1 \ldots n\) and subject to $c_{in}(d_i,p_i) < l_{in,i} \land c_{eq}(d_i,p_i) = l_{eq,i}$

2.2. Alternative derivation

Hadamard proposed three properties, well-posed problems should own: a solution that exists, is unique, and its behavior changes continuously with the initial boundary values [4]. In particular, inverse problems and complex hierarchical problems are not well-posed and are thus called ill-posed. In general, the springback compensation in sheet metal forming represents such an ill-posed problem.

Thikonov proposed a mathematical framework to treat ill-posed problems by regularization [7,8]. The basic form of the framework is shown in expression (4), where $D$ is the so-called data term and $R$ is the so-called regularization term. The data term $D$ contains the considered measure for the primary optimization objective. The regularization term $R$ introduces additional information to condition the problem formulation.

$$
x := \arg\min_{x}(D(A,x,B) + \alpha R(x))$$

Here, $\alpha$ is a scalar weighting factor balancing between the data term $D$ and the regularization term $R$, $x$ are the free variables and parameters to be identified, $A$ represents the underlying process or model, and $B$ contains information and data on the process at hand. From a physical and engineering perspective, in our opinion, the regularization $R$ provides possibilities to enforce further desirable features in the derived solution $x$, in addition to the properties contained in the data term $D$ [9,10]. This consideration extends the purely mathematically motivated perspective of Thikonov's approach [9].

Multistage processes with $n$ stages may be seen, for example, as underdetermined problems with infinite solutions (many adjustable and interacting parameters), hence, fall into one category of ill-posed
problems.\(^1\) To determine and select a desired solution additional information and adjustable parameters contained in the vector \(\varphi\) may be used in the regularization [9,10]. The formalization of the underdetermined problem yields expression (5), where the deviation is conducted with respect to last stage, as already introduced in expression (2). The tolerance \(\varepsilon\) on the geometrical deviations are strongly related to the classical stopping criteria.

\[
d_i := \arg \min_{d_i; p_i} \left( R(d_i; p_i; \varphi_i) \right) \quad \forall \, i = 1, \ldots, n
\]

subject to \(\| t - g_n(d_n; p_n; g_1(d_1; p_1), \ldots, g_{n-1}(d_{n-1}; p_{n-1})) \|_2 < \varepsilon\)

subject to \(c_{\text{in}}(d_i; p_i; \varphi_i) < l_{\text{in},i} \land c_{\text{eq}}(d_i; p_i; \varphi_i) = l_{\text{eq},i}\)

A redundant reformulation by augmentation using a Lagrangian multiplier \(\alpha\) yields expression (6a) that except from the regularization term \(R\) highly corresponds to the classical problem formulation, see expressions (1-3).

\[
d_i := \min_{d_i; p_i} \left( \| t - g_n(d_n; p_n; g_1(d_1; p_1), \ldots, g_{n-1}(d_{n-1}; p_{n-1})) \|_2 + \alpha R(d_i; p_i; \varphi_i) \right)
\]

\[
\forall \, i = 1 \ldots n \text{ and subject to } c_{\text{in}}(d_i; p_i; \varphi_i) < l_{\text{in},i} \land c_{\text{eq}}(d_i; p_i; \varphi_i) = l_{\text{eq},i}\]

We base our further elaboration on the topic of springback compensation on this alternative derivation and show how incorporation of additional important aspects emerge naturally from it.

3. Problem key elements

The key elements of the springback compensation problem can all be identified in expression (6a), as indicated in expression (6b). The innermost key element is the prediction of springback, here represented through the component geometry \(g_n\). The term called measurement represents the data term \(D\) in the original expression (4), and hence defines how the actual data is compared to the target. In case of a pure geometric analysis, the data term consists of the deviations between target \(t\) and predicted component geometry \(g_n\). The compensation represents the outermost key element. We understood the compensation as the updating scheme to solve for the non-linear optimization still subject to certain inequality and equality design restrictions \(c_{\text{in}}(d_i; p_i; \varphi_i) < l_{\text{in},i} \land c_{\text{eq}}(d_i; p_i; \varphi_i) = l_{\text{eq},i}\).

\[
d_i := \min_{d_i; p_i} \left( \| \frac{t - g_n(d_n; p_n; g_1(d_1; p_1), \ldots, g_{n-1}(d_{n-1}; p_{n-1}))}{\text{measurement}} \|_2 + \alpha R(d_i; p_i; \varphi_i) \right)
\]

\[
\forall \, i = 1 \ldots n \text{ and subject to } c_{\text{in}}(d_i; p_i; \varphi_i) < l_{\text{in},i} \land c_{\text{eq}}(d_i; p_i; \varphi_i) = l_{\text{eq},i}\]

3.1. Determination of the stress state and springback prediction

The springback prediction in expression (6b) presents as a two-step process, where the stress state \(S\) determined in the first step serves as the input for calculating the springback geometry of the component \(g\) and the internal stresses \(S_{\text{int}}\) remaining in the component after removal of the tools.

Finite element analysis of springback-induced shape deviations requires appropriate modeling of the sheet metal component’s stress state \(S\) after forming, since the elastic energy stored causes final springback of the sheet metal component [1,11]. Multiple influence factor affect the stress state prediction, including the elastic-plastic behavior during forming, such as the original anisotropy of the sheet and its development, as well as process-related influences, such as resulting deformation paths.

\(^1\) Depending on the actual problem formulation, the problem formulations of the springback compensation may fall into different categories, for example also overdeterminacy. However, in general, these always remain in the class of ill-posed problems.
lubrication, or blank holder force. In equation (7), these influencing factors are summarized and described using the parameter vector \( \mathbf{p} \) that for example may contain the loading elastic modulus \( E_l \), flow curve \( k_f \), Coulomb friction coefficient \( \mu \), as well as other necessary process and material characteristics [12]. The mapping \( f_{st} \) represents the determination of the component’s stress state \( \mathbf{S} \) after forming with respect to the parameter vector \( \mathbf{p} \) and the die geometry \( \mathbf{d} \).

\[
\mathbf{S} = f_{st}(\mathbf{d}, \mathbf{p} = E_l, k_f, \mu \ldots)
\]  

Equation (8) represents the second step using the mapping \( f_{sp} \). In addition to the stress state, the unloading behavior of the sheet material plays a decisive role for the springback estimation. Therefore, the parameter vector \( \mathbf{p} \) may for example contain an unloading elastic modulus \( E_u \).

\[
(\mathbf{g}, \mathbf{S}_{int}) = f_{sp}(\mathbf{S}, \mathbf{p} = E_u \ldots)
\]

3.2. Measurement and derivation of compensation

The measurement term in expression (6b) defines how the data and the target are compared. We used the term measurement here, since in almost any case today, the data term consist of a geometric distance calculation between the predicted component geometry \( \mathbf{g}_n \) and the target geometry \( \mathbf{t} \).

To perform a geometric measurement, a measurement direction \( \mathbf{n} \) and a distance measure need to be defined. The commonly used distance measure is the Euclidean norm \( || \cdot ||_2 \). An important challenge is the identification of consistent material points in the target \( \mathbf{t} \) and the part geometry \( \mathbf{g} \). To overcome this problem, multiple methods has been proposed [13–16]. As a result, each geometric point in the target can be associated with its actual counterpart in the predicted geometry. The data term reads accordingly.

\[
D = ||\mathbf{t} - \mathbf{g}_n(d_n, \mathbf{p}_n; \mathbf{g}_1, \ldots, \mathbf{g}_{n-1})||_2
\]

The compensation in expression (6b) represents the updating \( u_{cl} \) of the variables, \( \mathbf{d}^{(j+1)} \) in the presented case, to be solved for in the non-linear minimization problem between the iterations \( j \) and \( j + 1 \). In the classical problem setup, see section 2.1 , the compensation is formally restricted to minimize for the data term \( D \), hence geometrical deviations only and without the possibility for regularization \((\alpha = 0)\).

\[
\mathbf{d}^{(j+1)} = u_{cl}(\mathbf{d}_i^{(j)}, \mathbf{p}_i^{(j)}) \forall i = 1 \ldots n
\]

The alternative derivation, see section 2.2 , enables to incorporate regularization \( R^{(j)} \), additional information, and properties \( \mathbf{q}_i \) of the updated variables \( \mathbf{d}^{(j+1)} \) leading to the updating \( u_{al} \).

\[
\mathbf{d}^{(j+1)} = u_{al}(\mathbf{d}_i^{(j)}, \mathbf{p}_i^{(j)}, \mathbf{q}_i^{(j)}, R^{(j)}(\mathbf{d}_i, \mathbf{p}_i; \mathbf{q}_i)) \forall i = 1 \ldots n
\]

4. Problem characteristics

The key elements of springback compensation are characterized by different elements of the nested problem, for example numerics, modeling, or stochastics. In this paper, we exemplify the material description and the separation of deterministic and stochastic influences.

4.1. Material testing and modelling

Material modeling and identification of material parameters play an important role, especially for the prediction of component’s springback geometry \( \mathbf{g}_n \) and internal stress state \( \mathbf{S}_{int} \). For example, one thrust is the careful description of the transition from the elastic to the plastic domain, which has a decisive effect on the prediction of the stress state \( \mathbf{S} \) before springback [17]. Another thrust is to address the different behavior under loading and unloading conditions, together with non-linear elastic material properties that generally vary with the deformation history [1,18].

In addition to the classic material description, we see an increasing importance of being able to properly model and estimate the property fluctuations of a certain sheet metal material within a batch, but also across batches. These fluctuations are currently included in the compensation, for example, via sampling and metamodelling on the basis of the classic sample formulation [19,20], see section 2.1 .
4.2. Deterministic and stochastic influences

In general, not only the material description is affected by stochastic influences, but also, other parameters of the vector $p_t$ or additional information in the vector $q_t$. Therefore, the data term and the dimensional deviations, respectively, are composed of a deterministic part $\Delta_{\text{det}}$, including systematic errors, and a stochastic part $\Delta_{\text{sto}}$, including non-systematic errors.

$$\|t - g_n(d_n, p_n; g_1(d_1, p_1), \ldots, g_{n-1}(d_{n-1}, p_{n-1}))\|_2 = \Delta_{\text{sto}}(d_i, p_i; \varphi_i) + \Delta_{\text{det}}(d_i, p_i; \varphi_i) \quad (12)$$

The classical problem formulation, see section 2.1., acts solely on the deterministic portion $\Delta_{\text{det}}$ of the deviations, normally assuming a Gaussian distribution on fluctuating variables [21]. The alternative derivation, see section 3.2., enables to adapt the minimization goal and introduce also target on process robustness directly into the minimization expression through regularization. In this way the stochastic properties of the different fluctuating variables may be included directly into the compensation regime.

5. Compensation approaches and problem solving

Systematic springback compensation is almost exclusively driven by finite element simulations [22]. Currently, there is hardly any feedback of information and improvements from the real tryout of the forming tools or the press shop. Experimental data is used for the validation of the numerical analysis and rarely for compensation purposes in sheet metal forming due to the expensive tools, contrary to other manufacturing technologies [21,23].

Hence, forming and springback simulation form the backbone of compensation in sheet metal forming right now [3,24]. The different compensation methodologies rely on the numerical results and estimate new design variable sets iteratively solving for the non-linear minimization problem [25]. In essence, despite proposed sampling and metamodeling techniques, the compensation methods do not differ between deterministic deviations $\Delta_{\text{det}}$ and stochastic deviations $\Delta_{\text{sto}}$. This is also reflected in the present design variable updating schemes, the actual compensation steps, that entirely rely on combined deviation values, either in the spatial domain, see section 5.1., or in a corresponding stress domain, see section 5.2. and section 5.3.

5.1. Displacement adjustment method.

The displacement adjustment method directly uses the measurement between the target geometry $t$ and the formed part after springback $g_n^{(\cdot)}$ information, see section 3.2., to compute the die geometry update $d^{(j+1)}$ [14] using the linear mapping $u_{\text{da}}$.

$$d^{(j+1)} = d^{(j)} + u_{\text{da}}(t - g_n^{(j)}) = d^{(j)} + \beta(t - g_n^{(j)}) \quad (13)$$

Here, $\beta$ represents a scalar compensation factor that controls for the amount of die geometry adjustment. $\beta$ may also play an important role for the convergence behavior of the displacement adjustment compensation procedure, which bears certain similarities to gradient descent methods.

Different enhancements of the original displacement adjustment method has been proposed focusing on measurement direction [15,26] and material point correspondence [13]. The displacement adjustment method has been also proposed in bulk forming [21,27] or additive manufacturing [23] to compensate for geometrical deviations.

5.2. Spring forward method.

The spring forward method compensates for deviations based on internal forces [28]. The update of the tool geometry is thus calculated from the target geometry $t$ and the current stress state $S^{(j)}$ before springback based on the current tool geometry $d^{(j)}$. The inverted internal forces are imposed on the target geometry $t$ to identify the die geometry update $d^{(j+1)}$. This loading step is called spring forward. The resulting geometry serves as the updated tool geometry $d^{(j+1)}$. Equation (14) formalizes the die geometry update using the spring forward mapping $u_{\text{sf}}$. 

5
\[ d^{(j+1)} = t + u_{sf}(S^{(j)}(d^{(j)})) \]

Since springback generally occurs under high tensile stresses, the inverted stresses are compressive stresses. Accordingly, buckling in sheet metal components may occur in the spring forward step, which may result in restrictions and convergence problems.

5.3. Displacement compatible spring forward method.

The displacement compatible spring forward compensation approach consists of two elastic finite element analyses [2,29]. First, a compatible stress state is estimated using the current deviations \( t - g_n^{(j)} \) to load the target geometry \( t \) elastically. The resulting compatible stress state is inverted, may be combined with the internal stress state of the component \( S_{int}^{(j)} \), and then applied to the component geometry associated with the actual die geometry \( d^{(j)} \). After stress relaxation, an updated die geometry \( \bar{d}^{(j+1)} \) is obtained using the displacement compatible spring forward mapping \( u_{dc-sf} \).

\[ d^{(j+1)} = d^{(j)} + u_{dc-sf}(S_{int}^{(j)}, t - g_n^{(j)}) \]

As with other stress-based methods, the inversion of the stress state in the displacement compatible spring forward method, in particular the conversion from tension to compression, involves difficulties. However, it should be noted that due to the compatible stress state, not only the three translational degrees of freedom are available for the compensation, but also the generally independent three rotational degrees of freedom, we definitely observe in springback of complex components [2,3].

5.4. Inline monitoring and control

Currently, the focus is increasingly on inline inspection and control, since the required component accuracies have now reached the range of stochastic deviations \( \Delta_{sto} \) and remaining deterministic deviations \( \Delta_{det} \), such as model errors [9,30]. Up to now, concepts for inline monitoring and control have been developed detached from the initial process design for a variety of reasons, which is particularly consistent with the classic problem formulation, see section 2.1. If the possibilities of inline monitoring or control are already included in the initial design of the process in the optimization routine, sensor positions or possibilities for additional actuators can be integrated, for example. The alternative problem formulation, see section 2.2, offers a basis for this and shows its feasibility.

6. Conclusion

With the alternative approach to the problem of springback compensation in sheet metal forming, we have shown a systematic way not only to classify current approaches, but also to integrate current challenges, such as stochastic influences or the integration of sensors and actuators, into the process design optimization problem.

6.1. Limitations and challenges

Approaches that originate in the classical problem formulation, see section 2.1, define the problem of springback compensation very rigidly. Up to now, established updating procedures in the iterative process are very strongly oriented towards the use of finite element analyses and the direct integration of measurement data is hardly provided for. Stochastic deviations, robustness of the overall process, or even the integration of process sensors and actuators are difficult to integrate systematically starting from the basic expressions (1-3).

However, the fundamental challenge must and will be to ensure and actively shape component dimensional stability in multi-stage processes, but also other component properties.

6.2. New opportunities and industrial relevance

The presented derivation and extended reformulation of the problem, see section 2.2, opens a systematic way to include additional aspects in the process design. Thus, the following modification of expression (6) shows the possibilities to incorporate robustness criteria, stochastic deviations as well as
process monitoring and control by identifying a suitable set of variables \( \{d_i, p_i; \varphi_i\} \) that describe the die geometry, adjustable (process) parameters and additional degrees of freedom, like sensors or actors.

\[
d_i, p_i; \varphi_i := \min_{d_i, p_i; \varphi_i} \left( D(\Delta_{\text{det}}(d_i, p_i; \varphi_i)) + \alpha_{\text{rob}} R_{\text{rob}}(\Delta_{\text{sto}}(d_i, p_i; \varphi_i)) + \alpha_{\text{con}} R_{\text{con}}(d_i, p_i; \varphi_i) \right)
\]

\[
\forall \ i = 1 \ldots n \text{ and subject to } c_{\text{in}}(d_i, p_i; \varphi_i) < l_{\text{in},i} \land c_{\text{eq}}(d_i, p_i; \varphi_i) = l_{\text{eq},i} \quad (16)
\]

The three terms in the minimization expression (16) represent three potential interests in today’s process design, component quality represented by \( D \), which must not be limited to geometry but may also include for example mechanical or electrical properties, overall process robustness represented by \( R_{\text{rob}} \), and process controllability represented by \( R_{\text{con}} \). Each of the three terms can be considered as an independent model in a hierarchical structure, so that different data-driven and theory-driven approaches can be combined. The weighting factors \( \alpha_{\text{rob}} \) and \( \alpha_{\text{con}} \) allow to balance between the three objectives. This generic framework enables to incorporate additional objectives.

In industrial application, sampling and metamodeling-based tools for a priori estimation of deterministic and stochastic deviations are already available, like AutoForm-Sigma or Dynamore LS-OPT, and may be directly incorporated in the presented framework. Furthermore, the presented approach opens the possibility to evaluate all information and sensor data accumulating in the design phase, tryout and production with the objective to include them in a holistic process design.

Based on these considerations, new updating rules can be developed which, for instance, also include measurement data and simulation data in equal measure. Process robustness also has a temporal dimension, for example, which represents a new facet and the modern possibilities in sensor and control technology may be taken into account.

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