Optimal Beam Polarizations for New-Physics Search through $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$

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ABSTRACT

We perform an optimal-observable analysis of the final charged-lepton/b-quark momentum distributions in $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$ for various beam polarizations in order to study possible anomalous $t\bar{t}\gamma$, $tbW$ and $\gamma\gamma H$ couplings, which could be generated by $SU(2) \times U(1)$ gauge-invariant dimension-6 effective operators. We find optimal beam polarizations that will minimize the uncertainty in determination of those non-standard couplings. We also compare $e\bar{e}$ and $\gamma\gamma$ colliders from the viewpoint of the anomalous-top-quark-coupling determination.

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1. Introduction

Top-quark and Higgs-boson sectors are still not fully-tested regions of the electroweak physics. If there exists any new physics beyond the Standard Model (SM), it is plausible that its effects appear in those sectors. Therefore it is worth looking for experiments that are sensitive to top-quark and Higgs-boson properties, in particular to deviations from the SM predictions. Anomalous top-quark interactions will be tested by the various programs envisaged by the International Linear Collider (ILC) project [1]. In particular the photon-photon mode [2]–[4] of this collider will be able to probe efficiently the top-quark properties through $t\bar{t}$ production, as well as the Higgs-boson interactions. Therefore the $\gamma\gamma$ collider will prove to be a useful tool for searching for non-standard physics.

Indeed, compared to $e\bar{e}$ machines, $\gamma\gamma$ colliders present remarkable advantages, e.g., for the study of $CP$ violation in $\gamma\gamma H$ couplings [5]. In the case of $e\bar{e}$ collisions, the only relevant initial states are $CP$-even states $|e_L/R\bar{e}_R/L\rangle$ under the usual assumption that the electron mass can be neglected and that an $e\bar{e}$ pair annihilates dominantly into a single (virtual) vector-/axial-vector-boson. Therefore, all $CP$-violating observables must be constructed there from final-particle momenta/polarizations. In contrast, a $\gamma\gamma$ collider offers the unique possibility of preparing the polarization of the incident-photon beams which can be used to construct $CP$-violating asymmetries without relying on final-state information [5].

Because of this a number of authors have already considered top-quark production and decays in $\gamma\gamma$ collisions in order to study i) Higgs-boson couplings to the top quark and photon [6]–[11], or ii) anomalous top-quark couplings to the photon [12]–[14]. However, what will be observed in real experiments are combined signals that originate both from the process of top-quark production and, in addition, from its decays. Therefore, we have recently performed a model-independent analysis of $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$ [15], including all possible non-standard interactions together (production and decay) and applying the optimal-observable (OO) procedure to the final charged-lepton/b-quark momentum distributions.

In this work, we present a comprehensive analysis based on that framework, aiming to find optimal beam polarizations that minimize the uncertainty in determination of $t\bar{t}\gamma$-, $tbW$- and $\gamma\gamma H$-coupling parameters. Concerning the Higgs
couplings, we do not intend to go into its resonance region since our main interest is in \( t\bar{t} \) production/decay and also that region has already been studied in great details in existing literature [5]-[11]. Another goal here is to compare the \( e\bar{e} \) and \( \gamma\gamma \) colliders from the view point of the anomalous-top-quark-coupling determination.

The outline of this paper is as follows. After summarizing our fundamental framework in sec. 2, we give detailed numerical results of the analysis in sec. 3. In sec. 4 we perform a comparison of the \( e\bar{e} \) and \( \gamma\gamma \) colliders and the final section is devoted to conclusions and discussions. Some basic formulas and tools used in the analysis are described in detail in the appendix.

2. Framework

In this section, we summarize the basic elements of the framework used in the analyses, parts of which are described in more detail in appendices A1 and A2.

**Effective Lagrangian** We have used an effective low-energy Lagrangian parameterization [16] in order to describe possible new-physics effects, i.e., the SM Lagrangian is modified by the addition of a series of \( SU(3) \times SU(2) \times U(1) \) gauge-invariant operators, which are suppressed by inverse powers of a new-physics scale \( \Lambda \). Among those operators, the largest contribution comes from dimension-6 operators,\(^\text{21}\) denoted as \( O_i \), and we have the effective Lagrangian as

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i \left( \alpha_i O_i + \text{h.c.} \right) + O(\Lambda^{-3}).
\]

The operators relevant here lead to the following non-standard top-quark- and Higgs-boson-couplings: (1) \( CP \)-conserving and \( CP \)-violating \( t\bar{t}\gamma \) vertices, (2) \( CP \)-conserving and \( CP \)-violating \( \gamma\gamma H \) vertices, and (3) the anomalous \( tbW \) vertex. The corresponding coupling constants are denoted respectively by the five independent parameters \( \alpha_{\gamma}, \alpha_{\gamma2}, \alpha_{h1}, \alpha_{h2} \) and \( \alpha_d \). The explicit expressions for these couplings in terms of the coefficients of dimension-6 operators are to be found in appendices A1 and A2.

It is worth pointing out that the effective-Lagrangian parameterization is equally applicable to \( e\bar{e} \rightarrow t\bar{t} \) and \( \gamma\gamma \rightarrow t\bar{t} \). In the former case, no additional complications

\(^{21}\)Dimension-5 operators are not included since they violate lepton number [16] and are irrelevant for the processes considered here.
are encountered when replacing the effective-operator vertices by form factors (as given in appendix A3; see also [17, 18]) since all kinematic variables are fixed by the CM energy $\sqrt{s}$. This situation does not recur in $\gamma\gamma \rightarrow t\bar{t}$: the kinematic variables in the $t$-channel top exchange are not fixed by $s$, so, if we replace the effective couplings by form factors, the cross section will depend on the functional form of the latter. We will return to this point below.

$\gamma\gamma$ colliders Following the standard approach [2], each photon beam originates as a laser beam back-scattered off an electron ($e$) or positron ($\bar{e}$) beam. The polarizations of the initial-state are characterized by the electron and positron longitudinal polarizations $P_e$ and $P_{\bar{e}}$, the maximum average linear polarizations $P_t$ and $P_{\bar{t}}$ of the laser photons with the azimuthal angles $\varphi$ and $\bar{\varphi}$ (defined in the same way as in [2]), and their average helicities $P_\gamma$ and $P_{\bar{\gamma}}$. The photon polarizations $P_{t,\gamma}$ and $P_{\bar{t},\bar{\gamma}}$ satisfy

$$0 \leq P_t^2 + P_{\gamma}^2 \leq 1, \quad 0 \leq P_{\bar{t}}^2 + P_{\bar{\gamma}}^2 \leq 1,$$

and combine with the azimuthal angles to form the following polarization density matrices:

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + P_\gamma & -P_t e^{-2i\varphi} \\ -P_t e^{2i\varphi} & 1 - P_\gamma \end{pmatrix}, \quad \bar{\rho} = \frac{1}{2} \begin{pmatrix} 1 + P_{\bar{\gamma}} & -P_{\bar{t}} e^{2i\bar{\varphi}} \\ -P_{\bar{t}} e^{-2i\bar{\varphi}} & 1 - P_{\bar{\gamma}} \end{pmatrix}. \quad (3)$$

For linear polarization, we denote the relative azimuthal angle by $\chi \equiv \varphi - \bar{\varphi}$, which we fixed to be $\pi/4$ by the following procedure: we calculated the cross section $\sigma(\gamma\gamma \rightarrow t\bar{t})$ to first order in $\alpha_{\gamma_1,\gamma_2,h_1,h_2}$, we found that the terms proportional to $\alpha_{\gamma_2}$ and those to $\alpha_{h_2}$ were the most sensitive to $\chi$ and that these were maximized when $\chi = \pi/4$ (this was previously noticed in [12] concerning the $\alpha_{\gamma_2}$ term).

Cross sections When calculating the cross section $d\sigma(\gamma\gamma \rightarrow t\bar{t})$, the photon beams do not have definite spins and momenta as in $e\bar{e}/p\bar{p}$ colliders; for back scattered photon the spin information is given in terms of the Stokes parameters and the momentum distribution by the photon spectrum function. The cross section is calculated similarly to parton-model calculations (see, for example, [15] for more details). Taking this into account the calculation is straightforward but the final
expressions are very lengthy and will not be displayed here; to simplify the algebraic manipulations we used FORM [19].

In deriving the distributions of secondary fermions (= \ell/b) produced by the above cross section and the decay widths \( d\Gamma(t \rightarrow \ell X/bX) \), we use the narrow-width approximation thus treating the decaying \( t \) and \( W \) as on-shell particles; this enables us to use the Kawasaki-Shirafuji-Tsai formalism [20]. We have also neglected all contributions quadratic in \( \alpha_i \) (\( i = \gamma_1, \gamma_2, h_1, h_2, d \)), so that the angular-energy distributions of the secondary fermions \( \ell/b \) in the \( e\bar{e} \) (the initial electron-positron beams) CM frame can be expressed as

\[
\frac{d\sigma}{dE_{\ell/b}d\cos\theta_{\ell/b}} = f_{SM}(E_{\ell/b}, \cos\theta_{\ell/b}) + \sum_i \alpha_i f_i(E_{\ell/b}, \cos\theta_{\ell/b}),
\]

(4)

where \( f_{SM} \) and \( f_i \) are calculable functions: \( f_{SM} \) denotes the SM contribution, \( f_{\gamma_1,\gamma_2} \) describe the anomalous \( CP \)-conserving and \( CP \)-violating \( tt\gamma \)-vertices contributions respectively, \( f_{h_1,h_2} \) those generated by the anomalous \( CP \)-conserving and \( CP \)-violating \( \gamma\gamma H \)-vertices, and \( f_d \) that by the anomalous \( tbW \)-vertex.

**Optimal-observable technique** The optimal-observable technique [21] is a useful tool for estimating expected statistical uncertainties in various coupling measurements. Suppose we have a cross section

\[
\frac{d\sigma}{d\phi}(\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi),
\]

(5)

where \( f_i(\phi) \) are known functions of the location in final-state phase space variables \( \phi \) and \( c_i \)'s are model-dependent coefficients. The goal is to determine the \( c_i \)'s. This can be done by using appropriate weighting functions \( w_i(\phi) \) such that

\[
\int w_i(\phi) \Sigma(\phi) d\phi = c_i.
\]

In general different choices for \( w_i(\phi) \) are possible, but there is a unique choice for which the resultant statistical error is minimized. Such functions are given by

\[
w_i(\phi) = \sum_j X_{ij} f_j(\phi) / \Sigma(\phi),
\]

(6)

where \( X_{ij} \) is the inverse matrix of \( \mathcal{M}_{ij} \) which is defined as

\[
\mathcal{M}_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} d\phi.
\]

(7)
When we use these weighting functions, the statistical uncertainty of $c_i$ becomes

$$\Delta c_i = \sqrt{X_{ii} \sigma_T/N} ,$$

(8)

where $\sigma_T \equiv \int (d\sigma/d\phi)d\phi$ and $N$ is the total number of events.

In order to apply this technique to eq.(4), we first have to calculate $M_{ij}$ using $f_{SM}$ and $f_i$

$$M_{ij} = \int dE_{\ell/b} d\theta_{\ell/b} f_i(E_{\ell/b}, \cos \theta_{\ell/b}) f_j(E_{\ell/b}, \cos \theta_{\ell/b}) / f_{SM}(E_{\ell/b}, \cos \theta_{\ell/b})$$

(9)

and its inverse matrix $X_{ij}$, where $i, j = 1, \cdots, 6$ correspond to SM, $\gamma 1, \gamma 2, h1, h2$ and $d$ respectively. Then, according to eq.(8), the expected statistical uncertainty for the measurements of $\alpha_i$ is given by

$$\Delta \alpha_i = \sqrt{X_{ii} \sigma_T/N_{\ell/b}} ,$$

(10)

where

$$\sigma_T = \int dE_{\ell/b} d\theta_{\ell/b} f_{SM}(E_{\ell/b}, \cos \theta_{\ell/b}).$$

(11)

In this calculation we will not probe the Higgs-resonance region which has been extensively studied previously (see, for example, [7]). Therefore, since we work to lowest order in the $\alpha_i$, we compute the number of secondary fermions, $N_{\ell/b}$, from the SM total cross section multiplied by the lepton/b-quark detection efficiency $\epsilon_{\ell/b}$ and the integrated $e\bar{e}$ luminosity $L_{e\bar{e}}$; this leads to $N_{\ell/b}$ independent of $m_H$.

3. Anomalous couplings and optimal polarizations

In our previous work [15], where our main concern was to construct a fundamental framework for practical analyses, we used (1) $P_e = P_{\bar{e}} = 1$ and $P_t = P_{\bar{t}} = P_{\gamma} = P_{\bar{\gamma}} = 1/\sqrt{2}$, and (2) $P_e = P_{\bar{e}} = P_{\gamma} = P_{\bar{\gamma}} = 1$ as typical polarization examples and performed an OO-analysis. Inverting the matrix $M_{ij}$, we noticed that the numerical results for $X_{ij}$ are often unstable [15]: even a tiny variation of $M_{ij}$ changes $X_{ij}$ significantly. This indicates that some of $f_i$ have similar shapes and therefore their coefficients cannot be disentangled easily. The presence of such instability forced us to forgo our initial goal of determining all the couplings at once through this process alone. That is, we assume that some of $\alpha_i$’s have been
measured in other processes (e.g., in $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell^{\pm} X$), and we performed an analysis with smaller number of independent parameters.

For example, when estimating the statistical uncertainty in simultaneous measurements of $\alpha_{\gamma 1}$ and $\alpha_{h 1}$ (assuming all other coefficients are known), we need only the matrix components with indices 1, 2 and 4. In such a “reduced analysis”, we allowed only “stable solutions” according to the following criterion: we calculate the selected $\Delta \alpha_i$ rounding $M_{ij}$ first to three and then to two decimal places, obtaining $\Delta \alpha_i^{[3]}$ and $\Delta \alpha_i^{[2]}$ respectively. We then accept the result as a stable solution if $|\Delta \alpha_i^{[3]} - \Delta \alpha_i^{[2]}| / \Delta \alpha_i^{[3]} \leq 0.1$ for $i = \gamma 1$, $h 1$.

In this work, we took $\sqrt{s_{e\bar{e}}} = 500$ GeV and $\Lambda = 1$ TeV and minimized the statistical uncertainties $\Delta \alpha_i$ by choosing the polarization parameters from the set $\{P_e, e = 0, \pm 1; P_{t, \tilde{t}} = 0, 1/\sqrt{2}, 1; P_{\gamma, \tilde{\gamma}} = 0, \pm 1/\sqrt{2}, \pm 1\}$. We also considered 3 values of the Higgs mass, $m_H = 100$, 300 and 500 GeV, which correspond to the widths $\Gamma_H = 1.08 \times 10^{-2}$, 8.38 and 73.4 GeV respectively according to the standard-model formula.

Although we again did not find any stable solution in the four- and five-parameter analysis, we did find some solutions not only in the two- but also in the three-parameter analysis. This is in marked contrast to the results in [15], where we found no stable solution for the three-parameter analysis. In order to insure acceptable statistical precision we required the solutions to satisfy the following conditions:

- **Three-parameter analysis**: the resulting uncertainties must obey $\Delta \alpha_i \leq 0.1$ for at least two of the three unknown couplings, for an integrated luminosity of $L_{e\bar{e}} = 500$ fb$^{-1}$ (without detection-efficiency suppression).

- **Two-parameter analysis**: after selecting the Higgs-boson mass and the secondary fermion ($b$ or $\ell$) that will be observed, we found many stable solutions. We then selected those pairs $\{\Delta \alpha_i, \Delta \alpha_j\}$ that satisfy $\Delta \alpha_{i,j} \leq 0.1$ for a luminosity of $L_{e\bar{e}} = 500$ fb$^{-1}$, and which minimize $(\Delta \alpha_i)^2 + (\Delta \alpha_j)^2$.

The results are presented below. We did not fix the detection efficiencies $\epsilon_{\ell/b}$ since they depend on detector parameters and will improve with the development of detection technology.
1) Three parameter analysis

⊕ Final charged-lepton detection

\[ m_H = 500 \text{ GeV} \]

- \( P_e = P_{\bar{e}} = 0, \quad P_t = P_{\bar{t}} = 1 / \sqrt{2}, \quad P_\gamma = -P_{\bar{\gamma}} = 1 / \sqrt{2}, \quad N_\ell \simeq 6.1 \times 10^3 \epsilon_\ell \)

\[ \Delta \alpha_{\gamma_2} = 0.94 / \sqrt{\epsilon_\ell}, \quad \Delta \alpha_{h_2} = 0.11 / \sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.042 / \sqrt{\epsilon_\ell}. \quad (12) \]

Strictly speaking, this result does not satisfy our condition for the three-parameter analysis, but we show it since \( \Delta \alpha_{h_2} \) exceeds the limit by only 0.01.

⊕ Final bottom-quark detection

\[ m_H = 100 \text{ GeV} \]

- \( P_e = P_{\bar{e}} = 1, \quad P_t = P_{\bar{t}} = 1 / \sqrt{2}, \quad P_\gamma = -P_{\bar{\gamma}} = 1 / \sqrt{2}, \quad N_b \simeq 4.2 \times 10^4 \epsilon_b \)

\[ \Delta \alpha_{h_1} = 0.086 / \sqrt{\epsilon_b}, \quad \Delta \alpha_{h_2} = 0.21 / \sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.037 / \sqrt{\epsilon_b}. \quad (13) \]

\[ m_H = 500 \text{ GeV} \]

- \( P_e = P_{\bar{e}} = 0, \quad P_t = P_{\bar{t}} = 1 / \sqrt{2}, \quad P_\gamma = -P_{\bar{\gamma}} = 1 / \sqrt{2}, \quad N_b \simeq 2.8 \times 10^4 \epsilon_b \)

\[ \Delta \alpha_{\gamma_2} = 0.61 / \sqrt{\epsilon_b}, \quad \Delta \alpha_{h_2} = 0.054 / \sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.052 / \sqrt{\epsilon_b}. \quad (14) \]

2) Two parameter analysis

⊕ Final charged-lepton detection

Independent of \( m_H \)

- \( P_e = P_{\bar{e}} = -1, \quad P_t = P_{\bar{t}} = 1, \quad P_\gamma = P_{\bar{\gamma}} = 0, \quad N_\ell \simeq 1.0 \times 10^4 \epsilon_\ell \)

\[ \Delta \alpha_{\gamma_1} = 0.051 / \sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.022 / \sqrt{\epsilon_\ell}. \quad (15) \]

This result is independent of \( m_H \) since the Higgs-exchange diagram does not contribute to the determination of \( \alpha_{\gamma_1} \) and \( \alpha_d \) within our approximation.

\[ m_H = 100 \text{ GeV} \]

- \( P_e = P_{\bar{e}} = -1, \quad P_t = P_{\bar{t}} = 1 / \sqrt{2}, \quad P_\gamma = P_{\bar{\gamma}} = 1 / \sqrt{2}, \quad N_\ell \simeq 1.9 \times 10^4 \epsilon_\ell \)

\[ \Delta \alpha_{h_1} = 0.034 / \sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.017 / \sqrt{\epsilon_\ell}. \quad (16) \]

\[ m_H = 300 \text{ GeV} \]
• \( P_e = P_w = -1, P_t = P_{\ell} = 0, P_{\gamma} = P_{\bar{\gamma}} = 1, \quad N_\ell \simeq 2.4 \times 10^4 \epsilon_\ell \)
\[
\Delta \alpha_{h1} = 0.013/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.015/\sqrt{\epsilon_\ell}.
\] (17)

\( m_H = 500 \text{ GeV} \)

• \( P_e = P_w = -1, P_t = P_{\ell} = 0, P_{\gamma} = P_{\bar{\gamma}} = 1, \quad N_\ell \simeq 2.4 \times 10^4 \epsilon_\ell \)
\[
\Delta \alpha_{h1} = 0.023/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.015/\sqrt{\epsilon_\ell}.
\] (18)

• \( P_e = P_w = -1, P_t = P_{\ell} = 0, P_{\gamma} = P_{\bar{\gamma}} = 1, \quad N_\ell \simeq 2.4 \times 10^4 \epsilon_\ell \)
\[
\Delta \alpha_{h2} = 0.030/\sqrt{\epsilon_\ell}, \quad \Delta \alpha_d = 0.015/\sqrt{\epsilon_\ell}.
\] (19)

⊕ Final bottom-quark detection

\( m_H = 100 \text{ GeV} \)

• \( P_e = P_w = -1, P_t = P_{\ell} = 1/\sqrt{2}, P_{\gamma} = -P_{\bar{\gamma}} = -1/\sqrt{2}, \quad N_b \simeq 4.2 \times 10^4 \epsilon_b \)
\[
\Delta \alpha_{h1} = 0.058/\sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.026/\sqrt{\epsilon_b}.
\] (20)

\( m_H = 300 \text{ GeV} \)

• \( P_e = P_w = -1, P_t = P_{\ell} = 1/\sqrt{2}, P_{\gamma} = -P_{\bar{\gamma}} = -1/\sqrt{2}, \quad N_b \simeq 4.2 \times 10^4 \epsilon_b \)
\[
\Delta \alpha_{h1} = 0.009/\sqrt{\epsilon_b}, \quad \Delta \alpha_{h2} = 0.074/\sqrt{\epsilon_b}.
\] (21)

• \( P_e = P_w = 1, P_t = P_{\ell} = 1/\sqrt{2}, P_{\gamma} = -P_{\bar{\gamma}} = -1/\sqrt{2}, \quad N_b \simeq 4.2 \times 10^4 \epsilon_b \)
\[
\Delta \alpha_{h1} = 0.025/\sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.019/\sqrt{\epsilon_b}.
\] (22)

• \( P_e = P_w = 1, P_t = P_{\ell} = 1/\sqrt{2}, P_{\gamma} = -P_{\bar{\gamma}} = 1/\sqrt{2}, \quad N_b \simeq 4.2 \times 10^4 \epsilon_b \)
\[
\Delta \alpha_{h2} = 0.065/\sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.010/\sqrt{\epsilon_b}.
\] (23)

\( m_H = 500 \text{ GeV} \)

• \( P_e = P_w = -1, P_t = P_{\ell} = 1, P_{\gamma} = P_{\bar{\gamma}} = 0, \quad N_b \simeq 4.6 \times 10^4 \epsilon_b \)
\[
\Delta \alpha_{h1} = 0.030/\sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.018/\sqrt{\epsilon_b}.
\] (24)

• \( P_e = P_w = -1, P_t = P_{\ell} = 1, P_{\gamma} = P_{\bar{\gamma}} = 0, \quad N_b \simeq 4.6 \times 10^4 \epsilon_b \)
\[
\Delta \alpha_{h2} = 0.028/\sqrt{\epsilon_b}, \quad \Delta \alpha_d = 0.014/\sqrt{\epsilon_b}.
\] (25)

Using these results one can find (given \( m_H \)) the most suitable polarization for a determination of a given pair of coefficients.

Note that it is difficult to simultaneously determine \( \alpha_{\gamma 1} \) and \( \alpha_{\gamma 2} \) in either the two- or three-parameter analyses. Also, although we did find some new stable solutions that would allow for a determination of \( \alpha_{\gamma 1} \) in the lepton analysis, the
expected precision is rather low. Nevertheless this demonstrates that the use of purely linear laser polarization is crucial for measuring $\alpha_{\gamma 1}$. Unfortunately, the statistical uncertainty for $\alpha_{\gamma 2}$ is still large, even in this improved analysis, so we did not include it among our examples. Other processes must be used to determine this parameter; for a review see [22].

We found that there are many combinations of polarization parameters that make uncertainties of $\alpha_{h1, h2}$ and $\alpha_d$ relatively small. For instance, analyzing the $b$-quark final state with the choices $P_e = P_\bar{e} = -1$, $P_t = P_{\bar{t}} = 1/\sqrt{2}$, $P_\gamma = -P_{\bar{\gamma}} = -1/\sqrt{2}$ enables us to probe the Higgs-photon couplings $\alpha_{h1}$ and $\alpha_{h2}$ of a 300 GeV Higgs-boson.

As already mentioned, the results are obtained for $\Lambda = 1$ TeV. If the scale of new physics equals $\Lambda = \lambda$ TeV, then all the above results ($\Delta \alpha_i$) are replaced with $\Delta \alpha_i/\lambda^2$, which means that the right-hand sides of eqs.(12)–(25) giving $\Delta \alpha_i$ are all multiplied by $\lambda^2$.

Some additional comments are in order here.

- If we are only interested in measuring the decay coefficient $\alpha_d$, then the optimal polarization should be adjusted to maximize the top-production with no significant Higgs exchange contribution (this is because we keep only linear terms in the anomalous couplings). However, if $\alpha_d$ and $\alpha_{h1}$ or $\alpha_{h2}$ are to be determined, then certain compromise of the SM $t\bar{t}$-production rate is necessary as one also needs a significant contribution from the Higgs-boson exchange.

- If, on the other hand, only Higgs couplings are to be measured, then the optimal polarization would make the Higgs-exchange diagram as large as possible. It is obvious that for the most precise determination of the $\gamma\gamma H$ couplings, one should go to the resonance region\footnote{That would require adjustments of polarizations of the initial electron and laser beams, tuning initial electron energies and choosing large conversion distance, for details see [4]. Then the $\gamma\gamma$ spectrum would peak at $\sqrt{s_{\gamma\gamma}} \simeq 0.8\sqrt{s_{e\bar{e}}}$. Here, since we do not consider $m_H = 400$ GeV, we are never in the resonance region, as mentioned in Introduction.} in order to increase the Higgs production rate. A detailed study of $CP$-violating effects in $\gamma\gamma \rightarrow H$ has been performed, e.g., in [7]. There, for the luminosity $L_{e\bar{e}} = 20$ fb$^{-1}$, the
authors estimate 3-σ limits for $\alpha_{h2}$ ($d_{\gamma\gamma} = (v/\Lambda)^2\alpha_{h2} + \cdots$ in the notation of [7]) at the level of $10^{-3} - 10^{-4}$ depending on the Higgs-boson mass. Correcting for the luminosity adopted here ($L_{ee} = 500 \text{ fb}^{-1}$) it corresponds to our 1-σ uncertainty for $\alpha_{h2}$ also of the order of $10^{-3} - 10^{-4}$, so smaller by about two orders of magnitude than the precision obtained here for the off-resonance region.

4. Comparing $e\bar{e}$ and $\gamma\gamma$ colliders

Since both $t\bar{t}\gamma$ and $tbW$ couplings contribute to $\gamma\gamma \to t\bar{t}$ and $e\bar{e} \to t\bar{t}$, it is pertinent to compare the sensitivity to those anomalous couplings in these two types of colliders.

- $e\bar{e}$ colliders

The assumption that the on-mass-shell $t\bar{t}$ are produced through $s$-channel (axial-)vector-boson-exchange fixes all the kinematics in $t\bar{t}Z$ and $t\bar{t}\gamma$ vertices as a function of $\sqrt{s}$ and $m_t$ only. For the subsequent two-body on-shell $t$ decay, the kinematics is also fixed (just by masses). Therefore, in this framework, we can perform a very general analysis without worrying about the momentum dependence of all the effective vertices, i.e., not referring to the effective Lagrangian but treating the anomalous couplings as form factors (which could be momentum dependent). As we have shown earlier [17, 18], momentum distributions of the secondary lepton and the $b$-quark can serve as a mean to measure of real parts of the anomalous form factors. There, we used the anomalous magnetic- and electric-dipole-type couplings $\delta C_\gamma$ and $\delta D_\gamma$ for $t\bar{t}\gamma$ vertex and $f_2^R$ for $tbW$ vertex given; see appendices A2 and A3. The correspondence to $\alpha_{\gamma 1}, \alpha_{\gamma 2}$ and $\alpha_d$ is

$$\delta C_\gamma = -\frac{4\sqrt{2}m_t v}{gA^2}\alpha_{\gamma 1}, \quad \delta D_\gamma = i\frac{4\sqrt{2}m_t v}{gA^2}\alpha_{\gamma 2}, \quad \text{Re}(f_2^R) = \alpha_d,$$

where $g$ is the $SU(2)$ coupling and $v$ is the electroweak vacuum expectation value ($\simeq 250 \text{ GeV}$). Within the effective-Lagrangian framework $\alpha_{\gamma 1, \gamma 2}$ are real numbers, so $\delta D_\gamma$ is purely imaginary. Since only the real parts of the form factors can be measured through the distributions of the final fermions, $e\bar{e}$
colliders are sensitive only to $\delta C_\gamma (\alpha_{\gamma 1})$ and $f^R_2 (\alpha_d)$.\footnote{Note that we are discussing only the couplings which contribute to both $e\bar{e}$ and $\gamma\gamma$ processes. Of course, at $e\bar{e}$ colliders it is possible to determine the $t\bar{t}Z$ anomalous couplings to which $\gamma\gamma$ machines are completely blind; see \cite{17,18}.}

\subsection*{\gamma\gamma colliders}
Due to the presence of the $t$-channel diagram, in the case of the $\gamma\gamma$ collider the kinematics of the $t\bar{t}\gamma$ vertex is not fixed by $\sqrt{s}$ and the masses. In order to calculate distributions of secondary particles, one would need to integrate over momenta on which the $t\bar{t}\gamma$ form factor may depend. Therefore for $\gamma\gamma$ colliders we will not go beyond the effective-Lagrangian framework in which all the anomalous couplings are just given by constant coefficients. In \cite{15} we have shown that for the $\gamma\gamma$ scattering we could in general determine both real and imaginary parts of the anomalous $\gamma$ couplings.\footnote{Indeed, $\alpha_{\gamma 1}$ and $\alpha_{\gamma 2}$ are respectively the real part and the imaginary part of one parameter as shown in \cite{33} and \cite{34} in appendix A1. Calculating cross sections within our approximation, the imaginary part of any coupling cannot contribute unless the Levi-Civita tensor terms appearing in $\gamma$-matrix calculations survive. In case of $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell X$ process, we do not have enough number of independent vectors to keep those terms non-vanishing, while we do have for $\gamma\gamma$ process. In order to keep the Levi-Civita tensor terms non-zero in the final result in $e\bar{e}$-process analyses, we would need to introduce some additional independent vectors by, e.g., defining an angular asymmetry, see for instance \cite{23}.}

Because of the above remarks, in order to compare $e\bar{e}$ and $\gamma\gamma$ colliders we will adopt the framework of the effective Lagrangian. Then it is clear that there are only two couplings which can be measured at both machines: $\alpha_{\gamma 1}$ and $\alpha_{d}$. Therefore, we present results of one-parameter OO analysis (i.e., assuming that only one coupling is to be determined at a time) for them using the final-lepton distributions. We show the highest expected precision of each parameter obtained while varying the polarization parameters.

\begin{itemize}
  \item $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell X$
    \begin{itemize}
      \item e-1) $\Delta\alpha_{\gamma 1} = 0.02/\sqrt{\epsilon_{\ell}}$ for $P_e = -1$ and $P_{\bar{e}} = +1$, $N_\ell \simeq 1.5 \times 10^5 \epsilon_{\ell}$
      \item e-2) $\Delta\text{Re}(f^R_2) = 0.003/\sqrt{\epsilon_{\ell}}$ for $P_e = -1$ and $P_{\bar{e}} = +1$, $N_\ell \simeq 1.5 \times 10^5 \epsilon_{\ell}$
    \end{itemize}
  \item $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X$
    \begin{itemize}
      \item \gamma-1) $\Delta\alpha_{\gamma 1} = 0.03/\sqrt{\epsilon_{\ell}}$ for $P_e = P_{\bar{e}} = \pm 1$ and $P_\ell = P_\gamma = 1$, $N_\ell \simeq 1.0 \times 10^4 \epsilon_{\ell}$
      \item \gamma-2) $\Delta\alpha_{\gamma d} = 0.01/\sqrt{\epsilon_{\ell}}$ for $P_e = P_{\bar{e}} = -1$ and $P_\gamma = 1$, $N_\ell \simeq 2.4 \times 10^4 \epsilon_{\ell}$
    \end{itemize}
\end{itemize}
As one can see the precision obtained for $\delta C_\gamma (\alpha_{\gamma 1})$ is of the same order although the $e\bar{e}$ machine seems to be slightly favored. On the other hand the precision for $f_{t2}^R (\alpha_d)$ is much better in $e\bar{e}$ than in $\gamma\gamma$. This simply comes from the difference in the expected event numbers $N_\ell$ obtained for each optimal polarizations for the same $e\bar{e}$ luminosity. So, the $e\bar{e}$ machine is superior as far as the determination of the top-quark decay parameters is concerned.

5. Conclusions and discussions

We have performed a detailed analysis of the process $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$ in order to find optimal beam polarizations that minimize uncertainties in the determination of $t\bar{t}\gamma$-, $tbW$- and $\gamma\gamma H$-coupling parameters. To estimate the uncertainties we have applied the optimal-observable procedure to the final lepton/$b$-quark momentum distribution in $\gamma\gamma \rightarrow t\bar{t} \rightarrow \ell X/bX$. We have also compared the $e\bar{e}$ and $\gamma\gamma$ colliders from the point of view of the anomalous-top-quark-coupling determination.

Applying the optimal observable technique, we have again encountered the problem of “unstable-solutions” (see also [15]) and concluded that there is no stable solution when trying to determine more than three anomalous couplings simultaneously. However, in contrast to [15], allowing for more polarization choices, we have obtained stable solutions with three couplings. We also found a number of two-parameter solutions, most of which allow for the determination of the $\gamma\gamma H$ and $tbW$ couplings. The expected precision of the measurement of the Higgs coupling is of the order of $10^{-2}$ (for the scale of new physics $\Lambda = 1$ TeV). This shows that the $\gamma\gamma$ collider will be useful for testing the Higgs sector of the SM.

We also found that $e\bar{e}$ colliders will do slightly better than $\gamma\gamma$ colliders for the determination of $CP$-conserving $t\bar{t}\gamma$ and $tbW$ couplings (assuming the validity of the effective-Lagrangian framework). One should not forget, however, that $e\bar{e}$ colliders can only measure the real part of the $t\bar{t}\gamma$ and $tbW$ couplings as long as we perform full integration over the final-particle momenta.

Apart from the $t\bar{t}\gamma$- and $tbW$-coupling determinations, the $\gamma\gamma \rightarrow t\bar{t}$ and $e\bar{e} \rightarrow t\bar{t}$ processes are sensitive to different types of couplings. The former provides information on $\gamma\gamma H$ couplings, while the $t\bar{t}Z$ couplings can be tested only via the latter. Therefore it is fair to conclude that the measurements from both colliders will com-
plement one another. In this respect it should be noted that $\gamma\gamma H$ coupling could also be measured at $e\bar{e}$ colliders using final states such as $e\bar{e} \rightarrow \gamma H$. However the expected uncertainty is two orders of magnitude larger than at $\gamma\gamma$ colliders, see [7, 24]. Therefore, the $\gamma\gamma$ collider is definitely superior as far as the determination of $\gamma\gamma H$ couplings is concerned.

Let us consider the top-quark-coupling determination in an ideal case such that the beam parameters could be easily tuned and that the energy is sufficient for the on-shell Higgs-boson production, assuming that the Higgs-boson mass is known from the Large Hadron Collider. Then the best strategy would be to adjust polarizations and tune the initial electron energies to construct semi-monochromatic $\gamma\gamma$ beams such that $\sqrt{s_{\gamma\gamma}} \approx m_H$ and on-shell Higgs bosons are produced. This would allow for precise $\alpha_{h1, h2}$ measurement, so the virtual Higgs effects in $\gamma\gamma \rightarrow t\bar{t}$ would be calculable. Unfortunately, as we have shown earlier, it is difficult to measure $\alpha_{\gamma 2}$ by looking just at $\ell X/bX$ final states from $\gamma\gamma \rightarrow t\bar{t}$. Therefore to fix $\alpha_{\gamma 2}$, one should, for example, measure the asymmetries described in [12] to determine the top-quark electric-dipole moment, which is proportional to $\alpha_{\gamma 2}$. Then, following the analysis presented here, one can determine $\alpha_{\gamma 1}$ and $\alpha_d$.

Finally, one must not forget that it is necessary to take into account carefully the Standard Model contribution with radiative corrections when trying to determine the anomalous couplings in a fully realistic analyses. In particular this is significant when we are interested in $CP$-conserving couplings since the SM contributions there are not suppressed unlike the $CP$-violating terms. On this subject, see for instance [25].

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A1. Dimension-6 operators inducing $\gamma \gamma \to t \bar{t}$

Following the Buchm¨uller and Wyler scenario [16], operators of dim.6 that could contribute to the continuum top-quark-production process $\gamma \gamma \to t \bar{t}$ read:

$$O'_{uB} = (\bar{q}\sigma^{\mu\nu}u)\tilde{\varphi}B_{\mu\nu}, \quad O_{uW} = (\bar{q}\sigma^{\mu\nu}\tau^i u)\tilde{\varphi}W_{i\mu\nu}. \tag{27}$$

Each of the above operators contains both CP-violating and CP-conserving parts.

On the other hand, the following operators contribute to $\gamma \gamma \to t \bar{t}$ through the resonant s-channel Higgs-boson exchange:

$$O_{\varphi W} = (\varphi^\dagger \varphi)\tilde{W}^{i\mu\nu} W^i_{\mu\nu}, \quad O_{\varphi B} = (\varphi^\dagger \varphi)B_{\mu\nu}B^{\mu\nu}/2, \quad O_{\varphi' B} = (\varphi'^\dagger \varphi')\tilde{B}_{\mu\nu}B^{\mu\nu}, \quad O_{\varphi' B} = (\varphi'^\dagger \varphi')B_{\mu\nu}B^{\mu\nu}/2. \tag{28}$$

The operators that contain the dual tensors (e.g., $\tilde{B}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}/2$ with $\epsilon_{0123} = +1$) are CP odd while the remaining are CP even.

All these operators lead to the following Feynman rules for on-shell photons, which are necessary for our calculations:

1. CP-conserving $t \bar{t}\gamma$ vertex
   $$\frac{\sqrt{2}}{\Lambda^2} v_{\alpha_1} k_1 \gamma_\mu, \tag{29}$$
2. CP-violating $t \bar{t}\gamma$ vertex
   $$i\frac{\sqrt{2}}{\Lambda^2} v_{\alpha_2} k_2 \gamma_\mu \gamma_5, \tag{30}$$
3. CP-conserving $\gamma\gamma H$ vertex
   $$-\frac{4}{\Lambda^2} v_{\alpha_{h1}} [(k_1 k_2)g_{\mu\nu} - k_1\mu k_2\nu], \tag{31}$$
4. CP-violating $\gamma\gamma H$ vertex
   $$\frac{8}{\Lambda^2} v_{\alpha_{h2}} k_1^\rho k_2^\sigma \epsilon_{\rho\sigma\mu\nu}, \tag{32}$$

where $k$ and $k_{1,2}$ are incoming photon momenta, $v$ is the EW vacuum expectation value ($\simeq 250$ GeV) and $\alpha_{\gamma_1,\gamma_2,h_{1,2}}$ are defined as

$$\alpha_{\gamma_1} \equiv \sin \theta_W \text{Re}(\alpha_{uW}) + \cos \theta_W \text{Re}(\alpha'_{uB}), \tag{33}$$
\[ \alpha_{\gamma 2} \equiv \sin \theta_W \text{Im}(\alpha_{uW}) + \cos \theta_W \text{Im}(\alpha'_{uB}), \quad (34) \]

\[ \alpha_{h1} \equiv \sin^2 \theta_W \text{Re}(\alpha_{\varphi W}) + \cos^2 \theta_W \text{Re}(\alpha_{\varphi B}) - 2 \sin \theta_W \cos \theta_W \text{Re}(\alpha_{W B}), \quad (35) \]

\[ \alpha_{h2} \equiv \sin^2 \theta_W \text{Re}(\alpha_{\varphi W}) + \cos^2 \theta_W \text{Re}(\alpha_{\varphi B}) - \sin \theta_W \cos \theta_W \text{Re}(\alpha_{W B}). \quad (36) \]

In our notation, the SM \( f \bar{f} \gamma \) coupling is given by

\[ eQ_f \gamma \mu, \]

where \( e \) is the proton charge and \( Q_f \) is \( f \)'s electric charge in unit of \( e \) (e.g., \( Q_u = 2/3 \)).

### A2. Dimension-6 operators inducing \( t \to bW \)

The top-quark decay vertex is also affected by some dim.6 operators. For the on-shell \( W \) boson it will be sufficient to consider just the following contributions to the \( tbW \) amplitude since other possible terms do not interfere with the SM tree-level vertex when \( m_b \) is neglected:

\[ \Gamma^\mu_{tbW} = - \frac{g}{\sqrt{2}} \bar{u}(p_b) \left[ \gamma^\mu f_1^L P_L - \frac{i \sigma^\mu \nu k_\nu}{M_W} f_2^R P_R \right] u(p_t), \quad (37) \]

where \( g \) denotes the \( SU(2) \) gauge coupling constant, \( P_{L,R} \equiv (1 \mp \gamma_5)/2 \), and \( f_1^L \) and \( f_2^R \) are given by

\[ f_1^L = 1 + \frac{v}{A^2} \left[ \frac{m_t}{2} (\alpha_{D u} - \alpha_{\bar{D} u}) - 2v \alpha_{(3)\varphi q} \right], \quad (38) \]

\[ f_2^R = - \frac{v}{A^2} M_W \left[ \frac{4}{g} \alpha_{uW} + \frac{1}{2} (\alpha_{D u} - \alpha_{\bar{D} u}) \right], \quad (39) \]

with \( \alpha_{D u}, \alpha_{\bar{D} u} \) and \( \alpha_{(3)\varphi q} \) being correspondingly the coefficients of the following operators:

\[ \mathcal{O}_{D u} = (\bar{q} D_\mu u) D^\mu \bar{\varphi}, \quad \mathcal{O}_{\bar{D} u} = (\bar{D}_\mu \bar{q}) u D^\mu \bar{\varphi}, \quad \mathcal{O}_{(3)\varphi q} = i (\varphi^i D_\mu \tau^i \varphi) (\bar{q} \gamma^\mu \tau^i q). \quad (40) \]

In the main text, we express \( \text{Re}(f_2^R) \) as \( \alpha_d \).

### A3. General invariant amplitude of \( e\bar{e} \to t\bar{t} \)

We assume that all non-standard effects in the production process \( e\bar{e} \to t\bar{t} \) can be represented by the following corrections to the photon and \( Z \)-boson vertices contributing to the \( s \)-channel diagrams:

\[ \Gamma_{vtt}^\mu = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \left\{ A_v + \delta A_v - (B_v + \delta B_v) \gamma_5 \right\} + \frac{(p_t - p_\ell)^\mu}{2 M_t} (\delta C_v - \delta D_v \gamma_5) \right] v(p_\ell), \quad (41) \]
where \( v = \gamma, Z \) and
\[
A_\gamma = \frac{4}{3} \sin \theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{v_t}{2 \cos \theta_W}, \quad B_Z = \frac{1}{2 \cos \theta_W}
\]
with \( v_t = 1 - 8 \sin^2 \theta_W / 3 \). In addition, contributions to the vertex proportional to \((p_t + p_\overline{t})^\mu\) are also allowed, but their effects vanish in the limit of zero electron mass. Therefore, we can say that this form is practically the most general invariant one. Among the above new form factors, \( \delta A_{\gamma,Z}, \delta B_{\gamma,Z} \) and \( \delta C_{\gamma,Z} \) are parameterizing \( CP \)-conserving, while \( \delta D_{\gamma,Z} \) describes \( CP \)-violating non-standard interactions. A complete list of these non-standard couplings expressed through coefficients of \( \text{dim.6} \) effective operators is to be found, e.g., in the second paper in [17].

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