Supergravity Inflation on the Brane

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We study \( N = 1 \) Supergravity inflation in the context of the braneworld scenario. Particular attention is paid to the problem of the onset of inflation at sub-Planckian field values and the ensuing inflationary observables. We find that the so-called \( \eta \)-problem encountered in supergravity inspired inflationary models can be solved in the context of the braneworld scenario, for some range of the parameters involved. Furthermore, we obtain an upper bound on the scale of the fifth dimension, \( M_5 \lesssim 10^{-3} M_P \), in case the inflationary potential is quadratic in the inflaton field, \( \phi \). If the inflationary potential is cubic in \( \phi \), consistency with observational data requires that \( M_5 \simeq 9.2 \times 10^{-4} M_P \).

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I. INTRODUCTION

In supersymmetric theories with a single supersymmetry generator \( (N = 1) \), complex scalar fields are the lowest components, \( \phi^a \), of chiral superfields \( \Phi^a \). Masses for fields will be generated by spontaneous symmetry breaking so that the only fundamental mass scale is the reduced Planck mass, \( M = M_P/\sqrt{8\pi} \). The \( N = 1 \) supergravity theory describing the interaction of the chiral superfields, the scalar potential is given by

\[
V = e^{K/M^2} \left[ F^A \left( K_A^B \right)^{-1} F_B - 3 \frac{\left| W \right|^2}{M^2} \right] + D - \text{terms},
\]  

where

\[
F_A := \frac{\partial W}{\partial \Phi} + \left( \frac{\partial K}{\partial \Phi} \right) \frac{W}{M^2}, \quad K_A^B := \frac{\partial^2 K}{\partial \Phi^A \partial \Phi^B}.
\]  

The Kähler function, \( K \), sets the form of the kinetic energy terms of the theory while the superpotential, \( W \), determines the non-gauge interactions. For canonical kinetic energy terms, \( K = \sum_A \phi_A^* \phi_A \), the potential takes the relatively simple form

\[
V = \exp \left( \sum_A \frac{\phi_A^* \phi_A}{M^2} \right) \left[ \sum_B \frac{\partial W}{\partial \phi_B^*} \frac{\partial W}{\partial \phi_B} - 3 \frac{\left| W \right|^2}{M^2} \right] .
\]

Expanding the (slowly varying) inflaton potential about the value of the inflaton field at which the quantum fluctuations of interest are produced giving rise to structure formation, \( \phi \), by writing \( \varphi = \phi + \phi_0 \), we obtain a polynomial potential in \( \phi \)

\[
V = \Delta^4 \left[ 1 + c_1 \frac{\phi}{M} + c_2 \left( \frac{\phi}{M} \right)^2 + \cdots \right],
\]  

where we have factorized out the mass parameter \( \Delta \), which sets the scale of the inflationary phase and can be directly related with the amplitude of energy density perturbations. In this work, we shall consider a class of models in which the evolution of the inflaton dynamics is controlled by a single power at the point where the observed density fluctuations are produced; in this case, the potential assumes the following form

\[
V \simeq \Delta^4 \left[ 1 + c_n \left( \frac{\phi}{M} \right)^n \right].
\]  

The case where the second term is dominant and \( n = 2 \), a situation typical of chaotic inflationary models, has already been analysed in Refs. \([1, 2]\), where it is shown that specific features of the braneworld scenario allow for current observational constraints to be successfully accounted for with a single scale at the superpotential level; hence, difficulties with higher order non-renormalizable terms can be quite naturally avoided since it is possible to achieve successful inflation with sub-Planckian field values.

In this work, we consider the case where the first term is dominant and \( n = 2 \) or \( n = 3 \); regarding the former case, it is well known that a generic supergravity theory gives contributions of order \( \pm H^2 \) \([3]\) to the inflaton mass squared whereas inflation requires \( |m^2| \ll H^2 \), this is the so-called \( \eta \) problem. We will show that, in the braneworld scenario, this problem can be avoided, provided the \( D = 5 \) Planck mass satisfies the condition \( M_5 \lesssim 10^{16} \) GeV. This kind of analysis has recently been done for a particular supergravity F-term hybrid inflation model \([4]\), namely the one of Ref. \([5]\). It is relevant to point out that hybrid inflationary models \([4, 5]\) allow, as a result of the dynamics of two or more scalar fields, for successful realizations of the old inflationary type models, but have, in some instances, difficulties in what concerns initial conditions. However, these problems are shown to
be naturally solved in braneworld scenarios in the case
where the potential is dominated by the mass term \[8\].

We shall also consider the case where the quadratic
term gets cancelled and, therefore, the inflationary poten-
tial is cubic in \(\phi\), as is the case of the supergravity
natural inflation model of Ref. \[9\]. We show that
this model works in the braneworld scenario provided
\(M_5 \simeq 1.1 \times 10^{16}\) GeV.

II. REQUIREMENTS ON THE INFLATIONARY
POTENTIAL

We shall consider the five-dimensional brane scenario,
where one assumes that Einstein equations with a nega-
tive cosmological constant hold (an anti-De-Sitter space
is required) in D-dimensions and that matter fields are
confined to the 3-brane; then, the 4-dimensional Einstein
equation is given by \[10, 11\]:

\[
G_{\mu\nu} = -\Lambda g_{\mu\nu} + \frac{8\pi}{M_P^2} T_{\mu\nu} + \left(\frac{4\pi}{M_5^3}\right)^2 S_{\mu\nu} - E_{\mu\nu} ,
\]

where \(T_{\mu\nu}\) is the energy-momentum on the brane, \(S_{\mu\nu}\) is
a tensor that contains contributions that are quadratic
in \(T_{\mu\nu}\) and \(E_{\mu\nu}\) corresponds to the projection of the 5-
dimensional Weyl tensor on the 3-brane (physically, for a
perfect fluid, it is associated to non-local contributions to
the pressure and energy flux). In a cosmological frame-
work, where the 3-brane resembles our universe and the
metric projected onto the brane is an homogeneous and
isotropic flat Friedmann-Robertson-Walker (FRW) metric,
the Friedmann equation becomes \[10, 11\]:

\[
H^2 = \frac{\Lambda}{3} + \left(\frac{8\pi}{3M_P^2}\right) \rho + \left(\frac{4\pi}{M_5^3}\right)^2 \rho^2 + \frac{\epsilon}{a^4} ,
\]

where \(\epsilon\) is an integration constant. The four and five-
dimensional cosmological constants are related by

\[
\Lambda = \frac{4\pi}{M_5^3} \left(\Lambda_5 + \frac{4\pi}{3M_5^3}\right) \lambda^2 ,
\]

where \(\lambda\) is the 3-brane tension, and the four and five-
dimensional Planck scales through

\[
M_P = \sqrt{\frac{3}{4\pi \sqrt{\Lambda}}} .
\]

Assuming that, as required by observations, the cosmo-
logical constant is negligible in the early universe and
since the last term in Eq. \[8\] rapidly becomes unim-
portant after inflation sets in, the Friedmann equation
becomes

\[
H^2 = \frac{8\pi}{3M_P^2} \rho \left[1 + \frac{\rho}{2\lambda}\right]
\]

\[
= \left(\frac{4\pi}{3M_5^3}\right)^2 \rho^2 \left[1 + \frac{\rho_c}{\rho}\right] ,
\]

with

\[
\rho_c = \frac{3}{2\pi} \frac{M_5^6}{M_P^2} .
\]

Hence, the new term in \(\rho^2\) is dominant at high energies,
compared to \(\lambda^{1/4}\), i.e. \(\rho > \rho_c\), but quickly decays at lower
energies, and the usual four-dimensional FRW cosmology
is recovered.

Consistently, we shall assume that the scalar field is
confined to the brane, so that its field equation has the
standard form

\[
\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} .
\]

Consistency between the slow-roll approximation and
the full evolution equations requires that there are con-
straints on the slope and curvature of the potential. One
can define two slow-roll parameters \[1\]

\[
\epsilon = \frac{M_P^2}{16\pi} \left(\frac{V'}{V}\right)^2 \left[1 + \frac{V/\lambda}{(2 + V/\lambda)^2}\right] ,
\]

\[
\eta = \frac{M_P^2}{8\pi} \left(\frac{V''}{V}\right) \frac{1}{1 + \frac{V}{2\lambda}} .
\]

Notice that both parameters are suppressed by an ex-
tra factor \(\lambda/V\) at high energies and that, at low energies,
\(V < \lambda\), they reduce to the standard form. The value of \(\phi\)
at the end of inflation can be obtained from the condition

\[
\max\{\epsilon(\phi_F), |\eta(\phi_F)|\} = 1 .
\]

The number of e-folds during inflation is given by \(N = \int_{t_i}^{t_e} H dt\), which becomes \[8\]

\[
N \simeq -\frac{8\pi}{M_P^2} \int_{\phi_i}^{\phi_e} \frac{V}{V'} \left[1 + \frac{V}{2\lambda}\right] d\phi ,
\]

in the slow-roll approximation. We see that, as a result of
the modification in the Friedmann equation, the expan-
sion rate is increased, at high energies, by a factor \(V/2\lambda\).

The amplitude of scalar perturbations is given by \[8\]

\[
A_s^2 \simeq \left(\frac{512\pi}{75M_P^6}\right) \frac{V^3}{V''} \left[1 + \frac{V}{2\lambda}\right]^3 \bigg|_{k = aH} ,
\]
where the right-hand side should be evaluated as the moving scale equals the Hubble radius during inflation, so that the amplitude of scalar perturbations is increased relative to the standard result at a fixed value of $\phi$ for a given potential.

The scale-dependence of the perturbations is described by the spectral tilt \[ n_s - 1 \equiv \frac{d \ln A^2}{d \ln k} \simeq -6 \epsilon + 2 \eta \quad , \] where the slow-roll parameters are given in Eqs. (14) and (15). Notice that, as $V/\lambda \to \infty$, the spectral index of scalar perturbations is driven towards the Harrison-Zel’dovich spectrum, $n_s \to 1$.

Finally, the ratio between the amplitude of tensor and scalar perturbations is given by \[ (A_s^T/A_s^s)^2 \simeq \frac{3 M_p^2}{16 \pi} \left( \frac{V''}{V} \right)^2 \frac{2 \lambda}{V} \quad . \] In what follows we shall use the inflationary observables defined to study two generic supergravity inflationary models of the type described by Eq. (5). We should stress that we shall be interested in achieving a successful inflationary scenario with sub-Planckian field values so as to avoid the abovementioned problems with higher order non-renormalizable terms.

### III. QUADRATIC POTENTIAL

We first consider the case where the potential is quadratic in $\phi$ and we rewrite it as

\[ V = V_0 + \frac{1}{2} m^2 \phi^2 \quad , \]

and assume that the first term is dominant.

In supergravity, effective mass squared contributions of fields are given by

\[ \frac{1}{2} m^2 = 8 \pi \frac{V_0}{M_p} \approx 3 H^2 \quad \] since the horizon of the inflationary De Sitter phase has a Hawking temperature given by $T_H = H/2 \pi$.

Contributions like the ones of Eq. (22) lead to $\eta \equiv M_p^2 V''/8 \pi V \approx 2$; however, the onset of inflation requires $\eta < 1$. Within the braneworld scenario, however, $\eta$ is modified, at high energies, by a factor $\Lambda/V$ (see Eq. (19)). Hence, if the quantity, $\alpha \equiv V_0/\lambda$, is sufficiently large, this problem is automatically solved by the brane corrections, as in this case, $\eta \simeq 4/\alpha$ for large $\alpha$.

We shall now see that a constraint on $M_5$ can be obtained from the requirement that the magnitude of the energy density perturbations ensuing from our inflationary setup explains the anisotropies in the CMB radiation observed by COBE.

The number of $e$-foldings, $N$, in terms of $\chi$ is given by

\[ N = \alpha \left[ \frac{1}{4} \log \left( \frac{\phi_I}{M_p} \right) + \frac{2 \pi}{M_p^2} (\phi_I^2 - \phi_F^2) + \frac{16 \pi^2}{M_p^4} (\phi_I^4 - \phi_F^4) \right] \quad \] Using the high energy approximation and $V \simeq V_0$ in Eq. (18), we obtain for $A_s^2$

\[ A_s^2 \simeq \frac{64 \pi V_0^6}{75 M_p} \lambda^3 m^4 \chi^2 \quad , \]

where $\chi$ is the value of $\phi$ when scales corresponding to large-angle CMB anisotropies, as observed by COBE, left the Hubble radius during inflation. For $N_\chi \approx 55$, $\phi_I = \phi_*$ and $\phi_F = \beta M$ in Eq. (23), we get

\[ \phi_* \simeq 0.2 \beta \exp \left( \frac{220}{\alpha} \right) M_p \quad , \]

as, for sub-Planckian field values, the logarithmic term in Eq. (23) dominates. We prefer to leave $\phi_*$ as a free parameter since in hybrid models, which are of particular interest because they can give rise to quadratic potentials of the type we are studying once some other field is held at the origin by its interaction with $\phi$, inflation may end due to instabilities triggered by the dynamics of the other field. It then follows that the amount of inflation

\[ m^2 \simeq \frac{220}{\alpha} \frac{\pi^2}{6} \frac{1}{\lambda} \quad , \]
critically depends on the value of the inflaton field, \( \phi_I \), and hence on \( \beta \), at the time the instabilities arise. Notice that these instabilities are quite necessary in order to end inflation as \( \epsilon = 64\pi\phi^2/\alpha M_P^2 \ll 1 \) for \( \alpha > > 1 \) and sub-Planckian field values. Moreover, it is clear that the condition \( \epsilon \approx 1 \) cannot be met subsequently as the field is decreasing.

We also mention that the problem of initial conditions for hybrid models, discussed in Ref. 8, can also be solved in this model due to the brane corrections.

Inserting Eq. (23) into Eq. (24) and using the fact that the observed value from COBE is \( A_s = 2 \times 10^{-5} \), we obtain for \( M_5^2 \):

\[
M_5^2 \approx \frac{8\pi}{\alpha^2} \times 10^{-5} \beta \exp(220/\alpha) M_P^3 .
\]  (26)

Therefore, \( M_5 \) depends on the importance of the brane corrections via the \( \alpha \) parameter and the value of the field at the end of inflation through \( \beta \). In Figure 1 we show contours of the inflationary observables \( n_s \) and \( r \equiv 4\pi \left( \frac{\lambda}{\alpha^2} \right)^2 \), as given by Eqs. (13) and (23), in terms of parameters \( \alpha \) and \( \beta \). The shaded regions in Figure 1 are obtained by requiring that the bounds resulting from latest CMB data from BOOMERANG [13] and MAXIMA [14] and DASI [15] are satisfied. Also shown is the contour corresponding to \( N = 70 \) (we have chosen the initial value of the inflaton field to be sub-Planckian, \( \phi_I = 0.2 \ M_P \)) and, after applying the condition that there is sufficient inflation, \( N \geq 70 \), only the darker region remains allowed. This analysis clearly leads to a constraint on parameter \( \alpha \), \( \alpha \geq 140 \). Finally, also in Figure 1 we have superposed contours of the scale \( M_5 \), from which we can derive an upper bound on this quantity

\[
M_5 \lesssim 10^{-3} M_P .
\]  (28)

Finally, we mention that \( \Delta \) (or instead \( V_0 \)) in Eq. (4) can be estimated from the bound on the reheating temperature so as to avoid the gravitino problem (see Ref. 16 and references therein)

\[
T_{RH} \lesssim 2 \times 10^9 \text{ GeV}, \quad 6 \times 10^9 \text{ GeV} ,
\]

for \( m_{3/2} = 1 \text{ TeV}, \quad 10 \text{ TeV} \) ,

leading to \( \Delta \lesssim 6.71 \times 10^{-4} M_P \), as the reheating temperature is given by [5]

\[
T_{RH} \approx 5.5 \times 10^{-1} \frac{\Delta^3}{M_P^2} . \tag{30}
\]

### IV. CUBIC POTENTIAL

We shall now consider the case where, due to some cancellation mechanism, the quadratic term is absent and the potential is cubic in \( \phi \):

\[
V = \Delta^4 \left[ 1 + c_3 \left( \frac{\phi}{M} \right)^3 \right] . \tag{31}
\]

As mentioned before, we shall assume that the first term is dominant. For instance, the model of Ref. 9 corresponds precisely to this case, with \( c_3 = -4 \). We shall consider a generic \( c_3 \) and adapt our results for this particular example.

We start by computing the slow-roll parameters \( \epsilon \) and \( \eta \):

\[
\epsilon \approx \frac{18c_3^2}{\alpha} \left( \frac{\phi}{M} \right)^4 , \tag{32}
\]

\[
\eta \approx -\frac{12c_3}{\alpha} \left( \frac{\phi}{M} \right) , \tag{33}
\]

where \( \alpha \equiv \Delta^4 \).

The value of \( \phi \) at the end of inflation can be obtained from Eq. (16); we get, from \( |\epsilon| \approx 1 \)

\[
\phi_F \approx \left( \frac{\alpha}{18c_3^2} \right)^{1/4} M \tag{34}
\]

while, from \( |\eta| \approx 1 \), we obtain

\[
\phi_F \approx \left( \frac{\alpha}{12|c_3|} \right) M . \tag{35}
\]

Hence, the prescription to be used depends on the value of \( \alpha \). For \( c_3 = -4 \), we see that the two prescriptions coincide for \( \alpha \approx 26 \).

The number of \( e \)-foldings, \( N \), is given by:

\[
N = \frac{\alpha M}{6|c_3|} \left[ \frac{1}{\phi_I} - \frac{1}{\phi_F} \right] = \frac{\alpha M}{6|c_3|\phi_I} - 2 , \tag{36}
\]

using Eq. (19) (see below). Therefore, sufficient inflation to solve the cosmological horizon/flatness problems, that is \( N > 70 \), is achieved for

\[
\phi_I < 2.3 \times 10^{-3} \left( \frac{\alpha}{|c_3|} \right) M , \tag{37}
\]

yielding, for \( \alpha = 26 \) and \( c_3 = -4 \):

\[
\phi_I < 7.5 \times 10^{-2} M_P . \tag{38}
\]
In order to match CMB anisotropies as observed by COBE, we compute the value of $\phi$, $\phi_*$, that corresponds to $N = 55$:

$$\phi_* = \frac{\alpha M}{342 |c_3|} \ .$$  \hfill (39)

Inserting this result into Eq. (33), we obtain, in the high energy limit:

$$A_*^2 \simeq \frac{64\pi}{75M_P^2} \alpha^3 \left( \frac{V}{V'} \right)^2 V$$

$$= \frac{\alpha^4}{5400 \pi^2 c_3^2 \phi_*^4} \ ,$$ \hfill (40)

which determines $M_5$, using Eqs. (38) and (39):

$$M_5 = 7.3 \times 10^{-2} \left( \frac{A_*}{|c_3|} \right)^{1/3} M_P = 9.2 \times 10^{-4} M_P \ ,$$ \hfill (41)

where we have set $|c_3| = 4$. Naturally, Eq. (40) can also be used to extract the inflationary scale, $\Delta$:

$$\Delta = 9.9 \times 10^{-4} \left( \frac{\alpha}{c_3} \right)^{1/4} M_P \ .$$ \hfill (42)

Requiring that the reheating temperature, computed using Eq. (31) and the above result, obeys the bounds necessary to avoid the gravitino problem, Eq. (29), leads, in turn, to the following bounds on $\alpha$ (for $|c_3| = 4$)

$$\alpha \lesssim 3.4 \ , \ 14.7 \ ,$$

for $m_{3/2} = 1$ TeV, 10 TeV . \hfill (43)

Notice that for such low values of $\alpha$, the relevant prescription for the end of inflation is the one given by Eq. (33). Moreover, we find that, within our approximations, $n_s$ does not depend on $\alpha$ and we get, for $|c_3| = 4$, $n_s \simeq 0.93 \ ,$ \hfill (44)

and

$$r = 2.5 \times 10^{-8} \frac{\alpha^2}{|c_3|^2} \simeq 5 \times 10^{-7} \ ,$$ \hfill (45)

where we have set $\alpha \simeq 14.7$; these values are clearly consistent with the observations. Notice that the observational limit on $r$ implies a relatively weak bound on $\alpha$, namely $\alpha < 577$.

V. CONCLUSIONS

In this work, we have analysed a class of $N = 1$ supergravity inflationary models in which the evolution of the inflaton dynamics is controlled by a single power (quadratic or cubic in the inflaton field) at the point where the observed density fluctuations are produced, in the context of the braneworld scenario. We find that the so-called $\eta$-problem encountered in such models can be naturally solved in the context of braneworld scenario for some range of $\alpha$, the ratio of the dominant term in the inflationary potential and the brane tension, and the value of the inflaton field at the end of inflation, $\phi_*$. For an inflationary potential with a quadratic term, we find that consistency with data requires $\alpha \gtrsim 140$. The implied mass scale of the fifth dimension is $M_5 \lesssim 10^{-3} M_P$. For an inflationary potential that is cubic we find that, for consistency with observational data, $M_5 \simeq 9.2 \times 10^{-4} M_P$. Finally, we have checked that the gravitino problem can be avoided by a proper choice of the scale of the inflationary potential, $\Delta$, in both models.

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