Assembly Error Modeling and Calculating Method of Precision Mechanical System

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Abstract. Nowadays, there is no effective quantitative analysis method to describe the influence of design tolerance on the machining error of parts and the assembly accuracy of mechanical system at the design stage, which makes it difficult to analyze, predict and control the assembly accuracy in the actual assembly process. Aiming at this problem, a method for representation and modeling of geometric errors of parts is proposed. Furthermore, based on the above-mentioned geometric error modeling method of parts, a research method of geometric error transfer analysis and modeling for precision mechanical system oriented to actual assembly process is proposed.

1. Introduction
The assembly accuracy of precision mechanical system is affected by the processing accuracy of parts and assembly technology. The processing error of a single part is controlled by the processing accuracy, and the accumulated error in the assembly process is controlled by the assembly process. Su Chun [1] et al. studied the assembly error modeling of rigid parts and the reliability evaluation method of assembly accuracy. The part error is divided into position error and direction error, and described by position vector. Liu Weidong [2] et al. have studied the transmission mechanism of assembly errors, which are divided into geometric position deviation, shape deviation and assembly position deviation. Jiang Ke [3] studied the three-dimensional deviation analysis technology in the geometric accuracy design of mechanical products. Aiming at the problem of low accuracy in calculating the assembly success rate between parts, a model for calculating the assembly success rate of parts under the constraint of location priority based on matching contact state was proposed. Most of the existing research on assembly errors is aimed at geometric deviation, and the effect of geometric tolerance on assembly accuracy in design stage can not be quantitatively estimated. Aiming at the problem that there is no effective quantitative analysis method to describe the influence of geometric errors of parts and assembly process on the assembly accuracy of mechanical system, and it is difficult to analyze, predict and control the assembly accuracy in the actual assembly process. In this paper, a method of geometric error transfer analysis and modeling for precision mechanical system oriented to actual assembly process is presented.

2. Analysis of influencing factors on assembly accuracy of precision mechanical system
Precision mechanical system is composed of high precision parts. Assembly accuracy mainly refers to the actual accuracy of geometric parameters of mechanical products after assembly. The transmission and accumulation of geometric errors in the assembly process lead to the change of key geometric features, which will eventually be reflected in the assembly accuracy of the whole assembly system. The main influencing factors of assembly accuracy [4] are as follows:
(1) Machining accuracy of parts; (2) Assembly process adjustment; (3) Part stiffness; (4) Part thermal
deformation.

In order to simplify the complexity of geometric error transfer model, the following assumptions are
introduced into the analysis and modeling of geometric error transfer: (1) All parts are rigid bodies; (2)
No consideration of thermal deformation; (3) Small Error Hypothesis.

3. Key geometric feature description in assembly process

The assembly error is actually the deviation of the geometric features of the parts from their ideal
assembly position in the assembly body. Four coordinate systems are used to describe the key parts and
their geometric features in the assembly process, namely the reference coordinate system RCS, the part
coordinate system PCS, the feature coordinate system FCS and the matching system MCS, as shown in
Figure 1:

(1) Reference coordinate system: The reference coordinate system is generally fixed with the
assembly datum, base or worktable of the whole machine of the mechanical system. It is usually
established on the assembly datum level of the assembly datum part, which is used to determine the
position and direction of each part and its characteristics relative to the assembly datum in the assembly
process. It can also be used as the measurement benchmark in the assembly process of the whole
machine;

(2) Part coordinate system: Coordinate System for Parts Processing and Measuring. It is used to
represent the position and posture of each part in the assembly system relative to the reference coordinate
system;

(3) Feature coordinate system: Geometric centers of key features that affect assembly accuracy are
usually established to represent the position and orientation of key features relative to reference
coordinates or part coordinates;

(4) Matching coordinate system: Matching is the key link of assembly, and other features of parts
are carried by the matching features during assembly. Matching coordinate system is usually built on
the geometric center of the Parts’ matching position. In addition, in order to facilitate calculation, the
directions of each coordinate system are as consistent as possible with those of the reference coordinate
system.

![Assembly mounting surface](image)

Figure 1. Types of coordinate systems describing key geometric features in assembly process.

4. Mechanism of geometric error transfer and accumulation in assembly process

In the process of mechanical assembly as shown in Figure 2, with the continuous assembly and
combination of different parts in the assembly process, a number of assembly mounting surfaces are
formed. The geometric errors and assembly errors of parts are continuously transmitted and
accumulated through the assembly mounting surfaces, forming a geometric error transmission chain
with the assembly mounting surfaces as the basic transmission unit of errors. The geometric errors of
all the features in the error transfer chain affect the assembly accuracy of the whole machine.
5. A method for calculating geometric errors of parts

Small displacement vectors are vectors composed of small displacements generated by rigid bodies with six motion components, it was introduced of the tolerance research field by Bourdet [5] in 1996, and it is suitable for representing the deviation of ideal shape features (or geometric elements) [6]. The geometric error mathematical method of SDT is used to express the position and direction of geometric errors of parts, and the small displacement vector can be expressed as \( D = (dx, dy, dz, \delta x, \delta y, \delta z) \).

Among them, \( \delta x \) denotes the slight variation of rotation around X axis, \( \delta y \) denotes the slight variation of rotation around Y axis, \( \delta z \) denotes the slight variation of rotation around Z axis, \( dx \) denotes the slight variation of translation along X axis, \( dy \) denotes the slight variation of translation along Y axis, and \( dz \) denotes the slight variation of translation along Z axis. The small changes mentioned above are the components of geometric element errors caused by machining errors.

The concept of constancy is clearly defined in the ISO 17450-1 standards [7]: when a geometric element moves along the X, Y and Z axes or rotates around the X, Y and Z axes, its characteristics remain unchanged. Then the characteristic invariance of the corresponding direction is called the invariance degree, which is the concept corresponding to the degree of freedom in kinematics. Constancy denotes that the pose change of geometric elements in the corresponding direction has no effect on the shape characteristics of geometric elements, and the corresponding SDT component is zero.

For example, the cylindricity error component is expressed as:

\[
\begin{pmatrix}
    \delta x_y, \\
    \delta y_x, \\
    \delta z_y, \\
    \delta x_y, \\
    \delta y_x, \\
    0
\end{pmatrix} = 
\begin{pmatrix}
    dx, \\
    dy, \\
0
\end{pmatrix},
\]

the methods that can be used to determine the error components of shape and position are maximum value method, 0.618 method, and random value method, etc. For the geometric errors calculated by the maximum method, it will exceed the range of error. In order to make the calculation method converge, the golden section point method, i.e. 0.618 method, is introduced in mathematical calculation.

6. Mathematical model of geometric error transfer in assembly process

Part surface machining error is one of the important factors that lead to assembly error, and it is also an inevitable error in manufacturing process. The tolerance of geometric elements of parts is the allowable error range bandwidth of geometric elements given according to the functional requirements of parts and considering the processing cost, processing technology and other factors comprehensively. In practical engineering, the engineer satisfies the precision requirement of product assembly by giving the tolerance of each part, and the actual error of the geometric elements of the part is the important basis for solving the mounting surface error and the assembly error. Therefore, it is an important prerequisite for solving assembly accuracy to establish the calculation method of geometric elements of parts, obtain the expression of actual geometric elements, and establish the calculation model of geometric error transfer. Homogeneous transformation matrix of \( 4 \times 4 \) matrix, which is used to express the spatial posture relationship between rigid bodies in robot kinematics, is used to describe the spatial relationship between the actual posture and the ideal posture of each part in assembly system. As shown in Figure 3, the geometric error transfer in the assembly process of precision mechanical system can be equivalent to the error transfer in coordinate transformation.
As shown in Figure 4, Part 1 is a fixed basic part, on which the global coordinate system $O_1X_1Y_1Z_1$ is established, the local coordinate system $O_2X_2Y_2Z_2$ is established on part 2, and the local coordinate system $O_3X_3Y_3Z_3$ is established on part 3. Definition of $O'_2X'_2Y'_2Z'_2$ is actual coordinate system of Part 2 with error, and $O'_3X'_3Y'_3Z'_3$ is actual coordinate system of Part 3 with error. Definition: $(x_{i1}, y_{i1}, z_{i1})$ expresses the ideal coordinate value of the $i$ local coordinate system in the $i-1$ coordinate system, for example, with the first transfer of error, $(x_{i2}, y_{i2}, z_{i2})$ represents the ideal coordinate value of $O_2$ in the $O_1$ coordinate system, and the second transfer, $(x_{i3}, y_{i3}, z_{i3})$ represents the ideal coordinate value of $O_3$ in the $O_2$ coordinate system, and so on.

Figure 3. Geometric error transfer path in assembly process.

Figure 4. Assembly parts schematic diagram.

Figure 5. Drawing of ideal posture and actual posture of parts.

Defined as follows: the ideal coordinate of the parts are $\tilde{O}_1(x_{i1}, y_{i1}, z_{i1}, \hat{\beta}_{i1}, \hat{\gamma}_{i1})$, $\tilde{O}_2(x_{i2}, y_{i2}, z_{i2}, \hat{\beta}_{i2}, \hat{\gamma}_{i2})$, and $\tilde{O}_3(x_{i3}, y_{i3}, z_{i3}, \hat{\beta}_{i3}, \hat{\gamma}_{i3})$. The global coordinate system $O_1X_1Y_1Z_1$ has no angle deviation on its surface. The angle deviation of each part only affects the coordinate system position of the next level part, but has no effect on the coordinate system position of its own part. When the coordinate axes are all ideal, $(\hat{\beta}, \hat{\gamma}) = \{0\}$. If $O_1$, $O_2$, and $O_3$ have the following errors respectively:

$$
\Delta \tilde{O}_1 = (\Delta x_{i1}, \Delta y_{i1}, \Delta z_{i1}, 0, 0, 0)
$$

$$
\Delta \tilde{O}_2 = (\Delta x_{i2}, \Delta y_{i2}, \Delta z_{i2}, \Delta \hat{\beta}_{i2}, \Delta \hat{\gamma}_{i2})
$$

$$
\Delta \tilde{O}_3 = (\Delta x_{i3}, \Delta y_{i3}, \Delta z_{i3}, \Delta \hat{\beta}_{i3}, \Delta \hat{\gamma}_{i3})
$$
the coordinate values of each coordinate system after the occurrence of minor translation and minor angle deviation are \( \bar{O}'_1 \), \( \bar{O}'_2 \), and \( \bar{O}'_3 \) calculated by the following formula respectively:

\[
\bar{O}'_1(x_{01}', y_{01}', z_{01}'; 1) = \begin{bmatrix}
1 & 0 & 0 & \Delta x_{01} & 1 & 0 & 0 & x_{01}' \\
0 & 1 & 0 & \Delta y_{01} & 0 & 1 & 0 & y_{01}' \\
0 & 0 & 1 & \Delta z_{01} & 0 & 0 & 1 & z_{01}' \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\bar{O}'_1(x_{01}', y_{01}', z_{01}'; 1) = \begin{bmatrix}
1 & 0 & 0 & \Delta x_{02} & 1 & 0 & 0 & x_{02}' \\
0 & 1 & 0 & \Delta y_{02} & 0 & 1 & 0 & y_{02}' \\
0 & 0 & 1 & \Delta z_{02} & 0 & 0 & 1 & z_{02}' \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\bar{O}'_3(x_{03}', y_{03}', z_{03}'; 1) = \begin{bmatrix}
1 & -\Delta y_2 & \Delta \beta_2 & \Delta \alpha_3 & 1 & 0 & 0 & x_{03}' \\
\Delta y_2 & 1 & -\Delta \beta_2 & \Delta \alpha_3 & 0 & 1 & 0 & y_{03}' \\
-\Delta \beta_2 & \Delta \alpha_3 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

From the above formula, it can be concluded that with translation and rotation errors, the actual coordinate system of \( \bar{O}'_1 \) is \((\Delta x_{01} + x_{01}, \Delta y_{01} + y_{01}, \Delta z_{01} + z_{01}, 0,0,0)\), the actual coordinate system of \( \bar{O}'_2 \) is \((\Delta x_{02} + x_{02} + x_{02}', \Delta y_{02} + y_{02} + y_{02}', \Delta z_{02} + z_{02} + z_{02}', 0,0,0)\), and the actual coordinate system of \( \bar{O}'_3 \) is \((\Delta x_{03} + x_{03} + x_{03}', \Delta y_{03} + y_{03} + y_{03}', \Delta z_{03} + z_{03} + z_{03}', 0,0,0)\). and the actual coordinate system of \( \bar{O}'_3 \) is

\[
\bar{O}'_3(x_{03}', y_{03}', z_{03}'; 1) = \begin{bmatrix}
1 & -\Delta y_2 & \Delta \beta_2 & \Delta \alpha_3 & 1 & 0 & 0 & x_{03}' \\
\Delta y_2 & 1 & -\Delta \beta_2 & \Delta \alpha_3 & 0 & 1 & 0 & y_{03}' \\
-\Delta \beta_2 & \Delta \alpha_3 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Figure 5 shows the ideal and actual position of the part after error transfer and accumulation. If there are \( n \) parts to be assembled, the actual pose of the end of the system is \( \bar{O}_n(x_{0n}', y_{0n}', z_{0n}', \beta_{0n}', \beta_{0n}', \gamma_{0n}') \) after error transfer and accumulation, which is obtained from the following formula.

\[
\bar{O}_n(x_{0n}', y_{0n}', z_{0n}', 1) = \prod_{i=0}^{n} H_i T_i
\]
Where $H_i$ is the error matrix, $H_i = \begin{bmatrix} 1 & -\Delta \gamma_{i-1} & \Delta \beta_{i-1} & \Delta x_0 \\ \Delta \gamma_{i-1} & 1 & -\Delta \delta_{i-1} & \Delta y_0 \\ -\Delta \beta_{i-1} & \Delta \delta_{i-1} & 1 & \Delta z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $T_i$ is the ideal coordinate value of the coordinate system, $T_i = \begin{bmatrix} 1 & 0 & 0 & x_{0i} \\ 0 & 1 & 0 & y_{0i} \\ 0 & 0 & 1 & z_{0i} \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Formula (7) is a mathematical model for calculating the transmission and accumulation of geometric errors. According to this formula, the influence of geometric errors of the former part on the position and posture of the next part can be calculated after the transmission and accumulation of errors.

7. Case study

Taking the assembly of parts in a gear system as an example, according to the previous modeling method, as shown in the figure 6, the assembly is carried out according to the sequence of parts 1-2-3-4. Part 2 is a bearing. Bearings are regarded as ideal rigid parts, and their errors are not included. The geometric tolerances of other parts are shown in Table 1.

![Figure 6. Part assembly diagram of a gear system.](image)

| Matching parts | Design tolerance /mm |
|----------------|----------------------|
| Part 1         | Cylindricity of inner hole is 0.004; Coaxiality is 0.01; End face runout of inner hole is 0.006. |
| Part 3         | Coaxiality is 0.005; End face runout is 0.015. |
| Part 4         | End face runout is 0.01. |

Part 1 is the base fixed part, on which the reference coordinate system $O_0X_0Y_0Z_0$ is established. Establishment of local coordinate system $O_1X_1Y_1Z_1$ on End Face of Inner Hole of Part 1, $O_2X_2Y_2Z_2$ is the local coordinate system of part 2, and $O_3X_3Y_3Z_3$ is the local coordinate system of part 3. The ideal coordinate values of each coordinate system are shown in Table 2. Each part forms error transmission and accumulation through matching surfaces, uses 0.618 method to get the value of the tolerance.
involved, and then uses formula (7) to carry out geometric error transmission and accumulation calculation. The coordinate system of the actual axis of Part 3 is obtained to be (0.0088,-0.0041,12.0014,0.0003,0.0003,0).

Table 2. Ideal coordinate values of reference and local coordinate systems.

| Coordinate systems | X    | Y    | Z    |
|--------------------|------|------|------|
| O₀                 | 0    | 0    | 0    |
| O₁                 | 0    | 0    | 2    |
| O₂                 | 0    | 0    | 10   |
| O₃                 | 0    | 0    | -25.3|

8. Conclusion
The existing tolerance analysis is based on tolerance bandwidth to describe the processing error of parts. In actual assembly, if the actual measurement is not carried out, the assembly accuracy of the system after assembly can not be predicted. In this paper, a method for calculating geometric errors of parts is presented. The value of geometric errors is calculated by 0.618 method. Based on the above method, a research method of geometric error transfer analysis and modeling for precision mechanical system oriented to actual assembly process is proposed. According to the design tolerance of parts in the design stage, the possible errors in actual machining and their effects on assembly accuracy are estimated, and the prediction and evaluation of assembly accuracy in the design stage are realized.

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