THE COLD PLUS HOT DARK MATTER MODEL FROM SUPERSYMMETRIC INFLATION

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The cold plus hot dark matter (CHDM) model is arguably the best theory we currently have for a consistent description of the observed large scale structure formation. This is especially true if the primordial density fluctuations are assumed to be essentially scale invariant, in which case a mixture with 20-25 % HDM, 5-10 % baryons, and the rest in CDM correctly predicted (in 1989) the quadrupole anisotropy measured a few years later by the COBE satellite. After a brief historical introduction, we present a model of supersymmetric inflation in which the CHDM model is neatly realized with a spectral index $n = 0.98$, while the dark matter consists of a few eV ‘tau’ neutrino and the LSP (essentially the ‘bino’). We also provide a comparison of this model against the observations.

1 Introduction

The idea that the dark matter in the universe may contain more than one non–baryonic component does not seem so ‘strange’ to this modern audience, but it was greeted with much skepticism when it was put forward in 1984 as the basis for a model of large scale structure formation. The inspiration for the CHDM model came from both particle physics and cosmology. As people began to search for models with axionic CDM, it quickly became clear that some of the best models also predicted non-zero neutrino masses. More importantly, it was shown that the presence of some amount of hot dark matter could go a long way in reconciling the (critical density) inflationary scenario with the observations.

Perhaps an analogy with particle physics can help highlight the situation. Even though the $SU(2) \times U(1)$ group may not appear as ‘attractive’ as say just $SU(2)$ or $SU(3)$, the fact is that a description of the electroweak interactions including both quarks and leptons could not be simultaneously achieved within the ‘nice’ looking $SU(2)$ or $SU(3)$, (even before the discovery of the weak neutral current). Similarly, you need the CHDM (and not the CDM) model to provide a consistent description of observations on scales varying between the galactic and the horizon size. This holds especially if the primordial density fluctuations are assumed to be scale invariant, as was the case in the original CHDM model. This model received a boost in 1992 when it became clear that a model with approximately 25 % HDM, 5-10% baryons, and the rest in CDM would beautifully match with the temperature fluctuation amplitude.
observed by the COBE satellite. It needs to be stressed that this was a prediction, NOT a ‘postdiction’!

Now to the rest of this talk. We first want to show how the CHDM model ‘fits’ in with the recent wave of interest in supersymmetry. In particular, we present a very simple framework for realizing an inflationary scenario with CHDM as the end product. Among other things, this model has a spectral index $n = 0.98$, negligible ‘gravity’ waves, and a density fluctuation amplitude which is proportional to $(M/M_{\text{Planck}})^2$, where $M \approx M_{\text{GUT}}$ denotes the gauge symmetry breaking scale. We conclude by presenting a comparison of this model with the current, ongoing, and planned observations.

## 2 Supersymmetric Inflation

As we will see in section 3 (also see talks by Liddle and Primack in these proceedings), the ‘standard critical density CHDM model’ with $n \approx 1$ provides a good fit to the present data on large scale structure. Several questions can now be asked: How can the CHDM model arise from an inflationary framework? Can inflation be associated with some gauge symmetry breaking in the early universe? What is the nature of the ‘cold’ and ‘hot’ components?

Since the standard $SU(3) \times SU(2) \times U(1)$ model has no obvious dark matter candidate, while its supersymmetric extension (also known as MSSM) only contains CDM, it seems clear that we should search for an inflationary model based on a larger gauge symmetry. One simple framework is offered by the supersymmetric extension of the left–right symmetric gauge models. The ‘light’ neutrinos in this scheme are necessarily massive, and with the LSP as ‘cold’ dark matter, we have a simple particle physics basis for the CHDM model. Remarkably, we find that the $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B−L}$ gauge model not only admits inflation, but that we can use it to ‘pin down’ the symmetry breaking scale of $SU(2)_R \times U(1)_{B−L}$. It turns out to be comparable to the SUSY GUT scale ($\sim 10^{16}$ GeV). The hope, of course, is that such models can be embedded in a supersymmetric grand unified framework.

Consider the following globally supersymmetric renormalizable superpotential $W$

$$W = \kappa S\tilde{\phi}\phi - \mu^2 S \quad (\kappa > 0, \mu > 0),$$

where $\phi$, $\tilde{\phi}$ denote the standard model singlet components of a conjugate pair of $SU(2)_R \times U(1)_{B−L}$ doublet left handed superfields, and $S$ is a gauge singlet left handed superfield. An R-symmetry, under which $S \to e^{i\alpha}S$, $\tilde{\phi}\phi \to \tilde{\phi}\phi$, and $W \to e^{i\alpha}W$, can ensure that the rest of the renormalizable terms are either absent or irrelevant. Note that the gauge quantum numbers of $\phi$ are precisely those of the ‘matter’ right handed neutrinos. But they are distinct (!)
superfields and, in particular, the latter do not have the conjugate partners. From $W$, one writes down the potential $V$ as a function of the scalar fields $\phi, \bar{\phi}, S$:

$$V(\phi, \bar{\phi}, S) = \kappa^2 |S|^2 \left[ |\phi|^2 + |\bar{\phi}|^2 \right] + |\kappa \phi \bar{\phi} - \mu^2|^2 + D - \text{terms.} \quad (2)$$

The D-terms vanish along the D-flat direction $\phi = \bar{\phi}^*$ which contains the supersymmetric minimum

$$\langle S \rangle = 0, \quad \langle |\phi| \rangle = \langle |\bar{\phi}| \rangle = \mu/\sqrt{\kappa} \equiv M. \quad (3)$$

Using an appropriate R-transformation, $S$ can be brought to the real axis, i.e., $S = \sigma/\sqrt{2}$, where $\sigma$ is a normalized real scalar field.

The important point now is that in the early universe the scalar fields are displaced from the above minimum. In particular, for $S > S_c = M$, the potential $V$ is minimized by $\phi = \bar{\phi} = 0$. The energy density is dominated by $\mu^4$ which therefore leads to an exponentially expanding inflationary phase (hybrid inflation). As emphasized in [6], there are important radiative corrections under these conditions. At one loop, and for $S$ sufficiently larger than $S_c$, the inflationary potential is given by

$$V_{\text{eff}}(S) = \mu^4 \left[ 1 + \frac{\kappa^2}{16\pi^2} \left( \ln \left( \frac{\kappa^2 S^2}{\Lambda^2} \right) + \frac{3}{2} - \frac{S_c^4}{12S^4} + \ldots \right) \right]. \quad (4)$$

Using equation (4), one readily finds [8] the fundamental quantity:

$$(\Delta T/T)_Q \approx 8\pi(N_Q/45)^{1/2}(M/M_P)^2, \quad (5)$$

where $(\Delta T/T)_Q$ is the cosmic microwave quadrupole anisotropy amplitude. Here $N_Q \approx 50 - 60$ denotes the relevant number of e-foldings experienced by the universe between the time the quadrupole scale exited the horizon and the end of inflation. We also find the primordial density fluctuation spectral index $n \approx 0.98$. From equation (4), one finds $\kappa \approx \frac{8\pi^{3/2}}{\sqrt{N_Q}} y_Q \left( \frac{M}{M_P} \right)$, where $y_Q = x_Q(1 - 7/(12x_Q^2)) + \ldots$ with $x_Q = S_Q/M$, and $S_Q$ is the value of the scalar field $S$ when the scale which evolved to the present horizon size crossed outside the de Sitter horizon during inflation.

The inflationary phase ends as $S$ approaches $S_c$ from above. Write $S = xS_c$, where $x = 1$ corresponds to the phase transition from $G \to H$ which,
it turns out, more or less coincides with the end of the inflationary phase (this is checked by noting the amplitude of the quantities $\epsilon = \frac{M^2}{8\pi}(V'/V)^2$ and $\eta = \frac{M^2}{8\pi}(V''/V)$, where the prime refers to derivatives with respect to the field $\sigma$). Indeed, the $50 - 60$ e-foldings needed for the inflationary scenario can be realized even with $x \approx 2$. An important consequence of this is that with $S \sim 10^{16}$ GeV, the supergravity corrections are negligible.

In order to estimate the ‘reheat’ temperature we take account of the fact that the inflaton consists of the two complex scalar fields $S$ and $\theta = (\delta \phi + \delta \tilde{\phi})/\sqrt{2}$, where $\delta \phi = \phi - M$, $\delta \tilde{\phi} = \tilde{\phi} - M$, with mass $m_{infl} = \sqrt{2}\kappa M$. We mainly concentrate on the decay of $\theta$. Its relevant coupling to ‘matter’ is provided by the non-renormalizable superpotential coupling (in symbolic form):

$$\frac{1}{2} \left( \frac{M_{\nu c}}{M^2} \right) \bar{\phi} \nu^c \nu^c,$$

where $M_{\nu c}$ denotes the Majorana mass of the relevant right handed neutrino $\nu^c$. Without loss of generality we assume that the Majorana mass matrix of the right handed neutrinos has been brought to diagonal form with positive entries. Clearly, $\theta$ decays predominantly into the heaviest right handed neutrino permitted by phase space. (The field $S$ can rapidly decay into higgsinos through the renormalizable superpotential term $\xi S h^{(1)} h^{(2)}$ allowed by the gauge symmetry, where $h^{(1)}$, $h^{(2)}$ denote the electroweak higgs doublets which couple to the up and down type quarks respectively, and $\xi$ is a suitable coupling constant. Note that after supersymmetry breaking, $\langle S \rangle \sim M_S$, where $M_S \sim$ TeV denotes the magnitude of the breaking.)

Following standard procedures (we will soon comment on the issue of parametric resonance), and assuming the MSSM spectrum, the ‘reheat’ temperature $T_R$ is given by

$$T_R \approx \frac{1}{7} (\Gamma_\theta M_P)^{1/2},$$

where $\Gamma_\theta \approx (1/16\pi)(\sqrt{2} M_{\nu c}/M)^2 \sqrt{2}\kappa M$ is the decay rate of $\theta$. Substituting $\kappa$ as a function of $N_Q$, $y_Q$, and $M$, we find

$$T_R \approx \frac{1}{12} \left( \frac{56}{N_Q} \right)^{1/4} \sqrt{y_Q} M_{\nu c}.$$  

Several comments are in order:

i. For $x_Q$ on the order of unity the ‘reheat’ temperature is essentially determined by the mass of the heaviest right handed neutrino the inflaton can decay into;
ii. The well known gravitino problem requires that $T_R$ lie below $10^8 - 10^{10}$ GeV, unless a source of late stage entropy production is available. Given the uncertainties, we will interpret the gravitino constraint as the requirement that $T_R \lesssim 10^9$ GeV.

iii. In deriving equation (8) we have ignored the phenomenon of parametric resonance. This is justified because the oscillation amplitude is of order $M$ (not $M_P$!), such that the induced scalar mass ($\sim M_{\nu}$) is smaller than the inflaton mass $\sqrt{2}kM$. Note that here $M_{\nu}$ denotes the mass of the heaviest right handed neutrino super-multiplet the inflaton can decay into.

To proceed further we will need some details from the see-saw mechanism for the generation of light neutrino masses. For simplicity, we will ignore the first family of quarks and leptons. The Majorana mass matrix of the right handed neutrinos can then be brought (by an appropriate unitary transformation on the right handed neutrinos) to the diagonal form with real positive entries

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \quad (M_1, M_2 > 0). \quad (9)$$

An appropriate unitary rotation can then be further performed on the left handed neutrinos so that the (approximate) see-saw light neutrino mass matrix $m_D \mathcal{M}^{-1} \tilde{m}_D$, $m_D$ being the neutrino Dirac matrix, takes the diagonal form

$$m_D \frac{1}{\mathcal{M}} \tilde{m}_D = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}. \quad (10)$$

$(m_1, m_2$ are, in general, complex). In this basis of right and left handed neutrinos, the elements of

$$m_D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad (11)$$

are not all independent. They can be expressed in terms of only three complex parameters $a$, $d$, and $\eta$, where $\eta = \frac{M_1/M_2}{(b/a)} = \frac{M_2/M_1}{(c/d)}$.

We will now assume that $m_D$ coincides asymptotically (at the SUSY GUT scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV) with the up type quark mass matrix as is the case in many GUT models. Restricting ourselves, from now on, to the case where $|\eta| \sim 1$ and $M_1/M_2 \gg 1$, we have $|a| \gg |b|$ and $|c| \gg |d|$. Without much loss of generality we can further take $|c| \ll |a|$ so that $a$ is the dominant element in $m_D$. Under these assumptions the asymptotic top and charm masses are
\[ |m_i| \approx |a| \text{ and } |m_c| \approx |d| (1 + \eta^2). \] Since \(|m_2| \ll |m_1|\), we can make the following identification of the light neutrino mass eigenstates

\[ m_{\nu_\tau} = |m_1| = \frac{|a|^2}{M_1} (1 + \eta^2), \quad m_{\nu_\mu} = |m_2| = \frac{|d|^2}{M_2} (1 + \eta^2). \] (12)

We can then get the useful relations

\[ M_2 \approx \frac{m_{\nu_\mu}^2 m_{\nu_\tau}^2}{m_{\nu_\tau} m_{\nu_\mu}} \frac{1}{M_1}, \quad |1 + \eta^2| \approx \frac{m_{\nu_\mu}}{m_1} M_1. \] (13)

We are now ready to draw some important conclusions concerning neutrino masses that are more or less model independent. Assuming that the inflaton predominantly decays to the heaviest right handed neutrino (i.e. \(M_{\nu_\tau} = M_1\) in equation (8)) and employing condition (ii), we obtain \(M_1 \approx 9.3 \times 10^9\) GeV for \(N_Q \approx 56\) and \(x_Q \approx 2\). Equation (14) then implies an unacceptably large \(m_{\nu_\tau}\) for \(|\eta| \approx 1\). Thus, we are led to our first important conclusion: the inflaton should decay to the second heaviest right handed neutrino and consequently \(M_{\nu_\tau} = M_2\) in equation (8). Combining this equation with equation (13) we obtain

\[ T_R \approx \frac{1}{12.1} \left( \frac{56}{N_Q} \right)^{1/4} \frac{m_{\nu_\mu}^2 m_{\nu_\tau}^2}{m_{\nu_\tau} m_{\nu_\mu}} \frac{y_Q}{M_1} \approx 1.2 \times 10^{22} \frac{y_Q}{M_1} \text{ GeV}. \] (14)

Here we put \(N_Q = 56\) which is easily justifiable by standard methods at the end of the calculation after having fixed the values of all relevant parameters. Also, we took \(m_t = 120\) GeV, \(m_c = 0.25\) GeV, which are consistent with the assumption that below \(M_{\text{GUT}}\) the theory reduces to MSSM with large \(\tan \beta\). Moreover, we took \(m_{\nu_\mu} \approx 10^{-2.8}\) eV which lies at the center of the region consistent with the resolution of the neutrino solar puzzle via the small angle MSW mechanism. The value \(m_{\nu_\tau} \approx 4\) eV is consistent with the light tau neutrino playing an essential role in the formation of large scale structure in the universe.

The value of \(M_1\) is restricted by the fact that the inflaton should not decay to the corresponding right handed ‘tau’ neutrino

\[ M_1 \geq \frac{m_{\text{inf}}}{2} = \frac{\kappa M}{\sqrt{2}} \approx \left( \frac{45\pi}{2} \right)^{1/2} \frac{y_Q}{N_Q} M_P \left( \frac{\Delta T}{T} \right)_Q \approx y_Q 1.2 \times 10^{13} \text{ GeV}. \] (15)

It is interesting to note that since the right handed neutrinos acquire their masses from superpotential terms \(\lambda \bar{\phi} \phi \nu^c\), where \(M_c = M_P/\sqrt{8\pi} \approx 2.4 \times 10^{18}\)
GeV and \( \lambda \approx 1 \), \( M_1 = 2\lambda M^2/M_c \approx 2.9 \times 10^{13} \) GeV \( (M \approx 5.9 \times 10^{15} \) GeV for \( N_Q = 56 \), \( (\Delta T/T)_Q = 6.6 \times 10^{-6} \)). Thus, from equation (15), \( y_Q \approx 2.4 \) which implies \( x_Q \approx 2.6 \), and restricts the relevant part of inflation at values of \( S \sim 10^{16} \) GeV.

To maximize the primordial lepton asymmetry (see below) we choose the bound in equation (15) to be saturated. Equation (14) then gives

\[
T_R \approx y_Q^{-1/2} 9.7 \times 10^8 \left( \frac{\Delta T/T}{6.6 \times 10^{-6}} \right)^{-1} \left( \frac{N_Q}{56} \right)^{3/4} \left( \frac{m_c}{0.25 \text{GeV}} \right) \left( \frac{m_1}{120 \text{GeV}} \right) \left( \frac{m_{\nu_{\mu}}}{10^{-2.8} \text{eV}} \right) \left( \frac{m_{\nu_{\tau}}}{4 \text{eV}} \right)^{-1} \text{GeV},
\]

(16)

which satisfies condition (ii) for all allowed values of \( y_Q \). Eq. (13) implies

\[
M_2 \approx y_Q^{-1} 1.2 \times 10^{10} \text{ GeV}, \quad |1 + \eta|^2 \approx 3.4 y_Q.
\]

(17)

This implies that the errors in the asymptotic formulas for the top and charm masses are < 1%.

The observed baryon asymmetry of the universe can be generated by first producing a primordial lepton asymmetry via the out-of-equilibrium decay of the right handed neutrinos, which emerge as decay products of the inflaton field at ‘reheating’ \(^{14}\). It is important though to ensure that the lepton asymmetry is not erased by lepton number violating 2-2 scatterings at all temperatures between \( T_R \) and 100 GeV\(^{15}\). In our case this requirement is automatically satisfied since at temperatures above \( 10^7 \) GeV the lepton asymmetry is protected\(^{14}\) by supersymmetry, whereas at temperatures between \( 10^7 \) and 100 GeV, as one can easily show, these 2-2 scatterings are well out of equilibrium. The out-of-equilibrium condition for the decay of the right handed neutrinos is also satisfied since \( M_2 \gg T_R \) for all relevant values of \( x_Q \). The primordial lepton asymmetry is estimated to be

\[
\frac{n_L}{s} \approx \frac{9}{8\pi m_{infl}} \frac{T_R}{M_1} \frac{M_2}{M_1} \frac{\text{Im}(m_D^4 m_D^4/|\langle h^{(1)} \rangle|^2)}{21}.
\]

Equation (11) combined with the fact that \( |c||d| \ll |a||b| \) then gives

\[
\frac{n_L}{s} \lesssim \frac{9}{8\pi m_{infl}} \frac{T_R}{M_1} \frac{M_2}{M_1} \frac{m_2^2}{|\langle h^{(1)} \rangle|^2},
\]

(19)

which, using equations (13) - (17) and the fact that \( |\langle h^{(1)} \rangle| \approx 174 \) GeV for large \( \tan \beta \), becomes

\[
\frac{n_L}{s} \lesssim y_Q^{-7/2} 6.6 \times 10^{-9} \left( \frac{\Delta T/T}{6.6 \times 10^{-6}} \right)^{-4} \left( \frac{N_Q}{56} \right)^{15/4}
\]
For $x_Q \approx 2 \ (y_Q \approx 1.7)$, this gives $n_L/s \lesssim 10^{-9}$ which is large enough to account for the observed baryon asymmetry. Also $T_R \approx 7 \times 10^8$ GeV, $M_1 \approx 2 \times 10^{13}$ GeV, $M_2 \approx 7 \times 10^9$ GeV, and $m_{infl} \approx 4 \times 10^{13}$ GeV for the same value of $x_Q$.

In supersymmetric models the lightest supersymmetric particle (LSP) is expected to be stable and is a leading cold dark matter candidate. If we couple this with a tau neutrino of mass $\sim 2-6$ eV we are led to the well tested (CHDM) model.

To summarize, among the key features of the inflationary models we have discussed one could list the role played by radiative corrections in the early universe, the realization of inflation at scales well below $M_P$ so that the gravitational corrections can be adequately suppressed, and the constraints on the two heaviest right handed neutrino masses. The cold plus hot dark matter combination which results is an important consequence.

### 3 Comparison of predictions to Large Scale Structure Observations

Inflationary critical density cold plus hot dark matter models were recently tested against measurements of large scale structure (see ref. [18] and references therein). The model described here was shown to be quite compatible with observations as long as the heaviest neutrino mass $m_{\nu} \sim 2-7$ eV and the Hubble constant turns out to be $h \lesssim 0.55$, where $h$ is the value of the Hubble constant in units of $100$ km s$^{-1}$ Mpc$^{-1}$. It is interesting to note that the restriction on the Hubble constant from the age of the universe (which was not used in ref. [18]) yields a nearly identical constraint. Observational determinations of the Hubble constant have not yet settled down to a precise value. Although some current determinations have found $h \simeq 0.7 - 0.8$, lower values are still being seen. In fact, a recent survey of determinations using observations made by the Hubble Space Telescope finds $h = 0.55 \pm 0.10$. However, the greatest difficulty with Hubble constant determinations is overcoming systematic errors, and we await with interest more precise results.

Reference [18] assumed a baryon fraction of $\Omega_{baryon} = 0.064 \ (0.5/h)^2$, which is smaller than the currently allowed upper limits from big bang nucleosynthesis. The range consistent with galactic chemical evolution and some recent high redshift deuterium abundances, is

$$0.08 \leq \Omega_b(h/0.5)^2 \leq 0.12,$$

(21)

which is somewhat higher. This increase in the allowed baryon fraction helps reconcile the high baryon fractions observed in clusters (see, e.g. ref. [21]) with
Figure 1: We compare the predicted filtered density contrast (mass fluctuation amplitude \(\delta M/M\)) in this inflationary CHDM model against observations. We compare two \(n = 0.98\) models with \(\Omega_\nu = 0.25\) and \(h = 0.55\) (solid line) and \(\Omega_\nu = 0.20\) and \(h = 0.50\) against a variety of observations (see text.) We also show a CDM model \((\Omega_\nu = 0, h = 0.5, n = 1.00)\) for comparison. The error bars are 2 \(\sigma\) (95 % confidence).

a critical density CHDM universe. Therefore we will assume that our critical density universe has a baryon fraction \(\sim 10\%\).

We now compare the supersymmetric inflationary model against observations. First of all, our model suggests that we have only one neutrino flavor with a mass in the eV range, while another popular version suggests that there are two nearly degenerate (in mass) flavors. The reason is that with two flavors one can decrease the amplitude of cluster scale fluctuations while still getting a reasonable epoch of galaxy formation. However, as we have shown elsewhere, a larger baryon fraction mimics the effect of increasing the number of neutrino flavors in a lower baryon fraction model. Thus we get comparable results to the two flavor, lower baryon model.

The model has \(\Omega = 1, h < 0.6, n = 0.98, \Omega_b = 0.10\), and 1 flavor of neutrino with mass in the few eV range. To see how such a specific prescription compares with observations, we plot in figure 1 the COBE normalized rms filtered density contrast as a function of the filtering length. The two models have \(\Omega_\nu = 0.25, h = 0.55\) (solid) and \(\Omega_\nu = 0.20, h = 0.50\) (dashed). The observations are shown with 95% confidence limits, so a model with \(\Omega_\nu = 0\),
Figure 2: We compare the predictions of temperature anisotropies in this inflationary CHDM model against observations [29] (1σ errors). The solid line is our inflationary model (supplied by A. A. de Laix).

$h = 0.5$ (the CDM model - dotted line) strongly violates observational limits. The observational limits in figure 2 are the large scale streaming velocities (POTENT, ref. [24]), the galactic counts-in-cells measurements from the APM - Mt. Stromlo survey which have been corrected for a linear bias factor of 1.4, the x-ray cluster abundance constraint from ref. [18], and the 95% confidence lower limit required to make early galaxies (damped Ly-α systems - ref. [26]).

The cluster abundance is a synthesized linear theory constraint from x-ray measurements of clusters. The best way to test these data are with detailed hydrodynamic numerical simulations. These are computer intensive and time consuming, but allow for much more detailed comparison to observations. We note that simulations with similar parameters $n = 1.00$, $\Omega_c = 0.2$, $h = 0.5$, $\Omega_b = 7.5\%$ and two massive neutrino flavors show remarkably good agreement with observations.

The early galaxy formation constraint could be revised if many more high redshift (z > 4) galaxies are observed. However, there are indications that the galactic number density drops off beyond $z \sim 3$, indicating that most galaxies formed relatively recently as we expect in a cold plus hot dark matter universe (see, e.g. ref. [28] for a recent reference).
Lastly we compare the temperature anisotropy predictions of our models with observations. In figure 2 we plot results from a variety of CMB experiments (1 $\sigma$ errors) along with predictions for our inflation model $\Omega_\nu = 0.25$, $n = 0.98$, $h = 0.55$ and $\Omega_b = 0.10$ (solid line) We see that the present CMB data is consistent with the inflationary CHDM model although the data are not yet very discriminating. Data from planned and ongoing experiments should be able to test this model much more precisely.

We see that starting from a very simple superpotential we have arrived not only at a successful model of inflation but also a beautiful picture of large scale structure formation which is quite consistent with present large scale structure observations. For particle physics the most important predictions include a massive ‘tau’ neutrino in the 2-7 eV range, (a “smoking gun” of the CHDM model), as well as an LSP which is more or less pure bino. For cosmology, we predict an essentially scale invariant spectrum (index $n \approx 0.98$) and an absence of gravity waves.

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