Some aspects of $B$ physics relevant for experiments in hadron colliders are discussed. These include the determination of parameters of the CKM Matrix and confirmation of its role in CP violation, studies of mixing of nonstrange and strange $B$ mesons, lifetimes of hadrons containing $b$ quarks, the use of “same-side” tagging of neutral $B$ mesons via correlations with charged pions through fragmentation or resonances, and the determination of CKM phases through the study of decays of $B$ mesons to pairs of light hadrons.

I. INTRODUCTION

While the first states containing the $b$ quark, the $\Upsilon$ resonances, were discovered in hadronic interactions at Fermilab (1), the study of $B$ mesons for many years has been largely the province of $e^+e^-$ colliders. With the reconstruction of large numbers of exclusive $B$ decays by the CDF Collaboration (2), that situation now has the potential to change. The present talk outlines some of the ways in which hadron colliders can exploit their innate advantage of large $B$ production rates to provide information complementary and in many cases superior to that available in electron-positron collisions.

We begin in Sec. II with a discussion of the Cabibbo-Kobayashi-Maskawa (CKM) matrix (3,4) and its role in CP violation as currently observed in the neutral kaon system. The role of $B$ mesons in checking this picture is outlined in Sec. III. We comment specifically on $B-\bar{B}$ mixing in Sec. IV and on systematics of lifetime differences in Sec. V. The identification of the flavor of a neutral $B$ meson by means of correlations with charged pions produced nearby in phase space, via fragmentation and/or resonances, is reviewed in Sec. VI. Decays of $B$ mesons to pairs of hadrons containing light ($u$, $d$, $s$) quarks can provide information on phases of CKM elements, as noted in Sec. VII. We mention some specific issues associated with studies in hadron colliders in Sec. VIII, and conclude in Sec. IX.

II. CKM MATRIX AND CP VIOLATION

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A. The Neutral Kaon System

The mass eigenstates of neutral kaons, \( K_S \) and \( K_L \), both of which decay to \( \pi \pi \), may be expressed in terms of a complex parameter \( \epsilon \):

\[
K_{S,L} = \frac{1}{\sqrt{2(1 + |\epsilon|^2)}} [(1 + \epsilon)K^0 \pm (1 - \epsilon)\bar{K}^0].
\] (1)

The leading candidate for the source of \( \epsilon \) at present is a CP-violating \( K^0 - \bar{K}^0 \) mixing term, dominated by the top quark in the loop diagram for \( s\bar{d} \to d\bar{s} \) and complex as a result of phases in the CKM matrix elements.

An analogue of the \( K_S - K_L \) system in the absence of CP (or time-reversal!) violation is provided by two resonant circuits with equal natural frequencies and damping terms, coupled through a resistor, so that the long-lived oscillations will be those in which the two systems are in phase with one another and no energy is dissipated in the resistor. Emulation of CP violation is harder. One may do so using asymmetric exchange of radiated energy between obliquely polarized antennas through a region in which Faraday rotation is occurring. Thus, for example, VLF (very-low-frequency) radio waves are subject to different attenuation when propagating in the ionosphere from east to west and from west to east (\( \text{[citation]} \)). A more “table-top” version of such a phenomenon could utilize the Hall effect.

B. The CKM Matrix

In a parametrization (\( \text{[citation]} \)) in which the rows of the CKM matrix are labelled by \( u, c, t \) and the columns by \( d, s, b \), we have

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \approx \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}. \] (2)

The third quark family ensures that there will be a non-trivial phase, taken here to be that of \( V_{ub} \). The elements on and below the diagonal are determined by unitarity. The phase of \( V_{td} \) is appreciable, while those of \( V_{cd} \) and \( V_{ts} \) are negligible for present purposes. These phases allow the standard \( V - A \) interaction to generate CP violation as a higher-order weak effect.

The four parameters are measured in the following manner (\( \text{[citation]} \)):

1. The parameter \( \lambda \) is measured by a comparison of strange particle decays with muon decay and nuclear beta decay, leading to \( \lambda \approx \sin \theta \approx 0.22 \), where \( \theta \) is the Cabibbo angle.

2. The dominant decays of \( b \)-flavored hadrons occur via the element \( V_{cb} = A\lambda^2 \). The lifetimes of these hadrons and their semileptonic branching ratios then lead to an estimate \( A = 0.79 \pm 0.06 \), or \( V_{cb} = 0.038 \pm 0.003 \).
(3) The decays of $b$-flavored hadrons to charmless final states allow one to measure the magnitude of the element $V_{ub}$ and thus to conclude that
$$\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09.$$ 

(4) The phase of $V_{ub}$, $\text{Arg}(V_{ub}^* V_{cb}) = \arctan(\eta/\rho)$, must be determined with the help of indirect information associated with contributions of higher-order diagrams involving the top quark.

The unitarity of $V$ and the fact that $V_{ud}$ and $V_{tb}$ are very close to 1 allow us to write $V_{ub}^* + V_{tb} \simeq A\lambda^3$, or, dividing by a common factor of $A\lambda^3$, $\rho + i\eta + (1 - \rho - i\eta) = 1$. The point $(\rho, \eta)$ thus describes in the complex plane one vertex of a triangle [with angle $\alpha$ in a conventional notation (8)] whose other two vertices are $(0, 0)$ (with angle $\gamma$) and $(0, 1)$ (with angle $\beta$).

C. Constraints on $\rho$ and $\eta$

We obtain a set of constraints on the parameters $\rho$ and $\eta$ in the CKM matrix in the following manner. More details may be found in Ref. (7); we have updated some values.

(1) Charmless $B$ decays lead to the estimate $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ and hence to the constraint $\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09$ mentioned above. The limits correspond to the dotted semicircles with center $(0,0)$ in Fig. 1.

(2) Recent averages (6) for the $B^0 - \bar{B}^0$ mixing parameter $\Delta m_d = 0.462 \pm 0.026$ ps$^{-1}$ and the $B^0$ lifetime $\tau(B^0) = 1.621 \pm 0.067$ ps can be combined
to yield $\Delta m/\Gamma = 0.75 \pm 0.05$. If interpreted in terms of a box diagram for $b\bar{d} \rightarrow d\bar{b}$ (dominated by the top quark), this value leads to $|V_{td}|$ reducing to $|1 - \rho - i\eta| = 1.03 \pm 0.22$. The corresponding limits are shown as the dashed lines in Fig. 1.

(3) The parameter $|\varepsilon_K| = (2.26 \pm 0.02) \times 10^{-3}$ can be reproduced with a CP-violating $K^0 - \bar{K}^0$ mixing term due to the box diagram for $s\bar{d} \rightarrow d\bar{s}$. The contribution of top quarks alone would be proportional to $\text{Im} (V_{td}^2) \sim \rho(1 - \eta)$; with a small additional charm contribution, one finds $\eta(1 + 0.35 - \rho) = 0.48 \pm 0.20$, whose limits are denoted by the solid hyperbolae in Fig. 1.

Here we have also used the recent average of CDF [11] and D0 [12] top quark masses, $m_t = 180 \pm 12$ GeV/$c^2$, vacuum saturation factors $B_K = 0.8 \pm 0.2$ and $B_B = 1$, a $B$ meson decay constant $f_B = 180 \pm 30$ MeV, and a QCD correction for the box diagram $\eta_B = 0.6 \pm 0.1$. A large region centered about $\rho \approx 0, \eta \approx 0.35$ is permitted. Nonetheless, it could be that the CP violation seen in kaons is due to an entirely different source, perhaps a superweak mixing of $K^0$ and $\bar{K}^0$ [12]. In that case one could probably still accommodate $\eta = 0$ (see Fig. 1), and hence a real CKM matrix. In order to confirm the predicted nonzero value of $\eta$, we turn to $B$ mesons for help.

III. ROLE OF $B$ MESONS

A. Direct $b$ Transitions

The error in the $b \rightarrow c$ CKM element is dominated by theoretical uncertainty in extracting information from the measured $B$ lifetime and semileptonic branching ratio. Even more theoretical uncertainty is associated in extraction of $|V_{ub}|$ from a small portion of the semileptonic decay spectrum involving leptons beyond the endpoint for $b \rightarrow c\ell\nu$. In the long run a measurement of the branching ratio for $B \rightarrow \tau\nu$ or $B \rightarrow \mu\nu$, though challenging, will be helpful in extracting the product $f_B|V_{ub}|$. The expected branching ratios are about $(1/2) \times 10^{-4}$ for $\tau\nu$ and $2 \times 10^{-7}$ for $\mu\nu$.

One should not forget the possibility of eventually getting information on $|V_{tb}|$ through the decay $t \rightarrow Wb$, which is expected to have a partial width of $1.8 \pm 0.4$ GeV for $m_t = 180 \pm 12$ GeV/$c^2$ and $|V_{tb}| \approx 1$.

B. $B - \bar{B}$ Mixing

The error on $|V_{td}|$ extracted from $\Delta m/\Gamma$ for nonstrange neutral $B$ mesons is dominated by uncertainty in $f_B$. In Sec. IV we shall mention some ways in which this uncertainty may be reduced, both by direct calculation and by comparison of mixing in nonstrange and strange $B$ mesons.
C. CP Violation

Two main avenues for detecting CP violating in systems involving $b$ quarks involve 1) decays to CP eigenstates [13], and 2) decays to CP non-eigenstates. In both cases, partial rates for particle and antiparticle decays are compared, but experimental aspects and interpretations differ.

1. In decays to CP eigenstates, one compares the partial rate for a decay of an initial $B^0$ with that for an initial $\bar{B}^0$. The interference of direct decays (such as $B^0 \rightarrow J/\psi K_S$) and those involving mixing (such as $B^0 \rightarrow \bar{B}^0 \rightarrow J/\psi K_S$) gives rise to rate asymmetries which can be easily interpreted in terms of the angles $\alpha$, $\beta$, $\gamma$. Thus, if we define

$$A(f) \equiv \frac{\Gamma(B_{t=0} \rightarrow f) - \Gamma(\bar{B}_{t=0} \rightarrow f)}{\Gamma(B_{t=0} \rightarrow f) + \Gamma(\bar{B}_{t=0} \rightarrow f)}$$

we have, in the limit of a single direct contribution to decay amplitudes,

$$A(J/\psi K_S, \pi^+ \pi^-) = -\frac{x_d}{1+x_d^2} \sin(2\beta, 2\alpha)$$

where $x \equiv \Delta m/\Gamma$. This limit is expected to be very good for $J/\psi K_S$, but some correction for penguin contributions is probably needed for $\pi^+ \pi^-$. The value $x_d = 0.75 \pm 0.05$ is nearly optimum to maximize the coefficient of $\sin(2\beta, 2\alpha)$.

For the range of parameters noted in Fig. 1, we expect $-0.4 \leq A(J/\psi K_S) \leq -0.1$, i.e., an asymmetry of a definite sign, and $-0.47 \leq A(\pi^+ \pi^-) \leq 0.3$, i.e., nearly any asymmetry within the possible limits imposed by the factor $x_d/(1+x_d^2)$. Thus, the $J/\psi K_S$ asymmetry is likely to provide a consistency check, while the $\pi^+ \pi^-$ asymmetry should be more useful in specifying the parameters $\rho, \eta$ (unless it lies outside the expected limits).

In order to employ this method it is necessary to know the flavor of the produced $B$ meson. We shall remark on one possible “tagging” method in Sec. VI.

2. In decays to CP non-eigenstates, the asymmetry is proportional to $\sin \Delta \phi_{wk} \sin \Delta \phi_{str}$, where $\Delta \phi_{wk}$ is a difference between weak phases of two interfering amplitudes, while $\Delta \phi_{str}$ is the corresponding strong phase shift difference in the two channels. Such channels might, for instance, be the $I = 1/2$ and $I = 3/2$ amplitudes for $B \rightarrow \pi K$. Here one is able to compare decays of charged $B$ mesons with those of their antiparticles, so the identification of the flavor of the decaying meson does not pose a problem. On the other hand, $\Delta \phi_{str}$ is generally expected to be small and quite uncertain for the energies characteristic of $B$ decays. We shall outline in Sec. VII some recent progress in using decays of charged $B$ mesons to provide information on CKM phases without necessarily having to observe a CP-violating decay rate asymmetry.
TABLE 1. Dependence of mixing parameter $x_s$ on top quark mass and $B_s$ decay constant.

| $m_t$ (GeV/c$^2$) | 168 | 180 | 192 |
|-------------------|-----|-----|-----|
| $f_{B_s}$ (MeV)   |     |     |     |
| 150               | 9   | 10  | 11  |
| 200               | 15  | 17  | 19  |
| 250               | 24  | 27  | 30  |

IV. $B - \bar{B}$ MIXING

A. Nonstrange $B$ mesons

As mentioned, the dominant error in extracting $|V_{td}|$ from $B^0 - \bar{B}^0$ mixing is associated with uncertainty in $f_B$. Lattice gauge theory calculations [14] indicate $f_B \approx 180 \pm 50$ MeV. A quark model calculation [15] uses isospin splittings in $D$ and $B$ mesons to estimate the heavy quark – light antiquark wave function at zero separation, obtaining $f_B = 180$ MeV with a negligible experimental error but a systematic error which is hard to estimate.

Although no direct information on $f_B$ is available, we can check some of the predictions of the above models for $D$ mesons. Several experimental determinations [16] of the $D_s$ decay constant have appeared in the last two years, leading to $f_{D_s} \approx 300$ MeV. The estimate of Ref. [15] finds

$$f_D/f_{D_s} \simeq (m_d/m_s)^{1/2} \approx 0.8 \simeq f_B/f_{B_s},$$

or $f_D \approx 240$ MeV, not far below the present upper limit of 290 MeV [17]. Lattice estimates for the ratio in Eq. (5) range between 0.8 and 0.9.

B. Strange $B$ mesons

The mixing of neutral strange $B$ mesons is expected to involve a ratio $x_s \equiv \Delta m_s/\Gamma_s \approx 20$. Observation of this quantity poses a challenge but would provide very useful information on $f_{B_s}$, since the corresponding CKM elements are assumed well known from unitarity: $|V_{tb}| \approx 1$, $|V_{ts}| = 0.038 \pm 0.003$. In turn, one would estimate $f_B$ from $f_{B_s}$ using Eq. (5).

We show in Table 1 the dependence of $x_s$ on $f_{B_s}$ and $m_t$. We have updated a similar table in Ref. [8] on the basis of more recent information on the $B_s$ lifetime and the top quark mass. The dominant source of error on $x_s$ is primarily $f_{B_s}$. Our value of $f_{B_s}$ and Eq. (5) imply $f_B \approx 225$ MeV.

The estimate $f_B/f_{B_s} = 0.8 - 0.9$ and an experimental value for $x_s$ would allow us to tell whether the unitarity triangle had non-zero area by specifying $|1 - \rho - i\eta|$ [9]. Present bounds are not yet strong enough for this purpose.
Assuming that $|V_{ub}/V_{cb}| > 0.06$, one must show $0.73 < |1 - \rho - im| < 1.27$. Taking the $B_s$ and $B_d$ lifetimes to be equal, and assuming $0.7 < x_d < 0.8$, this will be so if $16 < x_s < 34$. An “ideal” measurement would thus be $x_s = 25 \pm 3$.

V. LIFETIMES

A. Charged vs. Neutral Nonstrange Mesons

The nonleptonic decay of a $B^0$ to a charmed final state involves the quark subprocess $b(\bar{d}) \to d\bar{u}c(\bar{d})$, where the parentheses denote the spectator quark. This is to be compared with the decay of a $B^-$, involving $b(\bar{u}) \to d\bar{u}c(\bar{u})$. Here we have assumed the weak current produces a $d\bar{u}$ pair.

The corresponding decays for charmed mesons are, for a $D^0$, $c(\bar{u}) \to u\bar{d}s(\bar{u})$, and, for a $D^+$, $c(d) \to ud\bar{s}(d)$. Now, $\Gamma_{tot}(D^0) \approx 2.5\Gamma_{tot}(D^+)$, as a result of different nonleptonic decay rates. This difference has been ascribed to many causes (19,20), but almost certainly is due in part to final-state interactions favoring the non-exotic final state in $D^0$ decays (21). An exotic state is one which cannot be formed from a quark-antiquark pair. In the case of $D^0$ decays, the $u$ and spectator $\bar{u}$ can annihilate one another, leaving a non-exotic $s\bar{d}$ final state. The $u\bar{d}s\bar{d}$ final state in $D^+$ decay is exotic. The study of amplitude triangles for such decays as $D \to K\pi$ and $D \to K^*\pi$ indicates that final-state phase differences between the non-exotic $I = 1/2$ and exotic $I = 3/2$ final states indeed can be significant (22).

It would be surprising if some remnant of this effect were not found in $B$ decays. The $d\bar{u}cd$ final state in $B^0$ decay can transform into a non-exotic $\bar{u}c$ state through $d\bar{d}$ annihilation, while the $d\bar{u}c\bar{u}$ final state in $B^+$ decay is exotic. So far, only upper limits on phase shift differences in channels like $\bar{B} \to (D\pi, D\rho, D^*\pi)$ have been obtained (23). Such phase differences should be observable if there really are lifetime differences between charged and neutral $B$’s and if they can be viewed in terms of final-state interactions.

Bigi et al. (20) estimate

$$\frac{\Gamma_{tot}(B^0)}{\Gamma_{tot}(B^+)} = 1 + 0.05 \frac{f_B^2}{(200 \text{ MeV})^2},$$

but their predictions also include a semileptonic $B$ decay branching ratio of at least 12%, whereas the experimental value seems to be below 11% (9). Thus there may be room for some additional enhancement of the hadronic decay rate. [The subprocess $b \to c\bar{c}s$, followed by $c\bar{c}$ annihilation, can lead to non-exotic states for both charged and neutral $B$ decays, accounting in part for the discrepancy between theory and experiment in the semileptonic branching ratio by enhancing both charged and neutral $B$ nonleptonic decays slightly.]

The experimental $B^0$ and $B^+$ lifetimes appear to be equal to within 7% (24). Greater accuracy is needed in order to test Eq. (5) incisively.
B. Baryons

The charmed baryon $\Lambda_c$ has a remarkably short lifetime: $\tau(\Lambda_c) \approx \frac{1}{2} \tau(D^0)$. It was predicted (24) that the $W$-exchange subprocess $cd \rightarrow su$, leading to $\Lambda_c(uccd) \rightarrow \Sigma^*(usu) \rightarrow$ hadrons, would significantly enhance the $\Lambda_c$ decay rate, as seems to be the case. A similar mechanism involving the subprocess $bu \rightarrow cd$ could enhance the $\Lambda_b$ decay rate. Indeed, $\tau(\Lambda_b) = 1.17 \pm 0.09$ ps (9), in contrast to the average meson lifetime $\bar{\tau}(B)$, which exceeds 1.5 ps (2,9). Bigi et al. (20) predict $\tau(\Lambda_c)/\bar{\tau}(B) \approx 0.9$, somewhat less of a difference than observed, but in the right direction.

C. Strange $B$ mesons

The decay of a $\bar{B}_s = b\bar{s}$ meson via the quark subprocess $b(\bar{s}) \rightarrow c\bar{c}s(\bar{s})$ gives rise to neutral final states which turn out to be predominantly CP-even (25). The mixing of $\bar{B}_s$ and $B_s$ leads to eigenstates $B_s^\pm$ of even and odd CP; the predominance of CP-even final states formed of $c\bar{c}s\bar{s}$ means that the CP-even eigenstate will have a shorter lifetime. With $\Delta \Gamma(B_s) \equiv \Gamma(B_s^+) - \Gamma(B_s^-)$ and $\bar{\Gamma}(B_s) \equiv [\Gamma(B_s^+) + \Gamma(B_s^-)]/2$, Bigi et al. (20) estimate

$$\frac{\Delta \Gamma}{\bar{\Gamma}(B_s)} \approx 0.18 \frac{f_{B_s}^2}{(200 \text{ MeV})^2},$$

possibly the largest lifetime difference in hadrons containing $b$ quarks.

One could measure $\bar{\Gamma}(B_s)$ using semileptonic decays, while the decays to CP eigenstates could be measured by studying the correlations between the polarization states of the vector mesons in $B_s^\pm \rightarrow J/\psi K_S$. [For a similar method applied to decays of other pseudoscalar mesons see, e.g., Ref. (26).]

The ratio of the mass splitting to the width difference between CP eigenstates of strange $B$'s is predicted to be large and independent of CKM matrix elements (27,28) (to lowest order, neglecting QCD corrections which may be appreciable):

$$\left| \frac{\Delta m}{\Delta \Gamma} \right| \approx \frac{2}{3\pi} \frac{m_b^2 h(m_b^2/M_W^2)}{m_b^2} \left( 1 - \frac{8}{3} \frac{m_c^2}{m_b^2} \right)^{-1} \approx 200!$$

Here $h(x)$ is a function which decreases monotonically from 1 at $x = 0$ to 1/4 as $x \rightarrow \infty$; it is about 0.53 for the present value of $m_t$. If the mass difference $\Delta m/\bar{\Gamma} \approx 20$ turns out to be too large to measure, the width difference $\Delta \Gamma/\bar{\Gamma} \approx 1/10$ may be large enough to detect.

VI. FLAVOR TAGGING AND SPECTROSCOPY

In the decays of neutral $B$ mesons to CP eigenstates, it is necessary to know the flavor of the meson at time of production. A conventional means for
“tagging” the flavor of a $B$ is to identify the flavor of the meson produced in association with it. At a hadron collider or in high energy $e^+e^-$ collisions as at LEP, this method suffers only from the possible dilution of the “tagging” signal by $B^0 - \bar{B}^0$ mixing, and from the difficulty of finding the “tagging” hadron. However, in the reaction $e^+e^- \to B^0\bar{B}^0$ at threshold, the odd $C$ of the pair implies that any CP-odd asymmetry will be an odd function of the time difference between decays of the two mesons, leading to the need for asymmetric collisions [or, possibly, ingenious schemes (29) for enhancing vertex resolution in symmetric machines].

In this section I would like to discuss recent progress in tagging neutral $B$ mesons by means of the hadrons produced nearby in phase space. This method, also proposed (30) for tagging strange $B$'s via associated kaons, has been the subject of recent papers devoted to correlations of nonstrange $B$'s with charged pions (31).

A. Fragmentation vs. Resonances

The existence of correlations between $B$ mesons and pions can be visualized either in terms of a fragmentation picture or in terms of explicit resonances.

In a fragmentation picture, if a $b$ quark picks up a $\bar{d}$ quark from the vacuum to become a $\bar{B}^0$ meson, and a charged pion containing the corresponding $d$ quark is generated, that pion will be a $\pi^-$. It is likely to lie “near” the $\bar{B}^0$ in phase space, in the sense that its transverse momentum with respect to the $B$ is low, its rapidity is correlated with that of the $B$, or the effective mass of the $\pi B$ system is low. Similarly, if a $\bar{b}$ quark picks up a $d$ quark to become a $B^0$, the charged pion containing the corresponding $\bar{d}$ will be a $\pi^+$.

The signs of the pions in the above correlations are those which would have resulted from the decays $B^{**+} \to B^{(*)0}\pi^+$ or $B^{**-} \to B^{(*)0}\pi^-$. We utilize the double-asterisk superscript to distinguish $B^{**}$'s from the hyperfine partners of the $B$'s, the $B^{**}$'s, which are only about 46 MeV heavier than the $B$'s and cannot decay to them via pions.

The importance of explicit narrow $B^{**}$ resonances is that they permit reduction of combinatorial backgrounds. Thus, we turn to what is expected (and, more recently, observed) about such resonances.

B. Spectroscopic predictions

We shall briefly recapitulate material which has been presented in more detail elsewhere (32,33). In a hadron containing a single heavy quark, that quark ($Q = c$ or $b$) plays the role of an atomic nucleus, with the light degrees of freedom (quarks, antiquarks, gluons) analogous to the electron cloud. The properties of hadrons containing $b$ quarks then can calculated from the corresponding properties of charmed particles by taking account (34) of a few
simple “isotope effects.” If \( q \) denotes a light antiquark, the mass of a \( Q\bar{q} \) meson can be expressed as

\[
M(Q\bar{q}) = m_Q + \text{const.}[n, L] + \frac{\langle p^2 \rangle}{2m_Q} + a \frac{\langle \sigma_q \cdot \sigma_Q \rangle}{m_q m_Q} + \mathcal{O}(m_Q^{-2}) \quad .
\] (9)

Here the constant depends only on the radial and orbital quantum numbers \( n \) and \( L \). The \( \langle p^2 \rangle / 2m_Q \) term expresses the dependence of the heavy quark’s kinetic energy on \( m_Q \), while the last term is a hyperfine interaction. The expectation value of \( \langle \sigma_q \cdot \sigma_Q \rangle \) is \((+1, -3)\) for \( J^P = (1^-, 0^-) \) mesons. If we define \( M \equiv [3M(1^-) + M(0^-)]/4 \), we find

\[
m_b - m_c + \frac{\langle p^2 \rangle}{2m_b} - \frac{\langle p^2 \rangle}{2m_c} = M(B\bar{q}) - M(c\bar{q}) \simeq 3.34 \text{ GeV} \quad .
\] (10)

so \( m_b - m_c > 3.34 \text{ GeV}, \) since \( \langle p^2 \rangle > 0 \). Details of interest include (1) the effects of replacing nonstrange quarks with strange ones, (2) the energies associated with orbital excitations, (3) the size of the \( \langle p^2 \rangle \) term, and (4) the magnitude of hyperfine effects. In all cases there exist ways of using information about charmed hadrons to predict the properties of the corresponding \( B \) hadrons. A recent comparison of the charmed and beauty spectra may be found in Ref. (33). For S-wave states the predictions of the heavy-quark symmetry approach work rather well.

The \( B^* - B \) hyperfine splitting scales as the inverse of the heavy-quark mass: \( B^* - B = (m_c / m_b)(D^* - D) \). Consequently, while \( D^{*+} \rightarrow D^0 \pi^+ \) and \( D^{*+} \rightarrow D^+ \pi^0 \) are both allowed, leading to a useful method (33) for identifying charmed mesons via the soft pions often accompanying them, the only allowed decay of a \( B^* \) is to \( B\gamma \). No soft pions are expected to accompany \( B \) mesons. One must look to the next-higher set of levels, the \( B^{**} \) resonances consisting of a \( \bar{b} \) quark and a light quark in a P-wave, or the fragmentation process mentioned above, for the source of pions correlated with the flavor of \( B \) mesons.

One can use heavy-quark symmetry or explicit quark models to extrapolate from the properties of known \( D^{**} \) resonances to those of \( B^{**} \) states. Two classes of such resonances are expected (36), depending on whether the total angular momentum \( j = s_q + L \) of the light quark system is 1/2 or 3/2. Here \( s_q \) is the light quark’s spin and \( L = 1 \) is its orbital angular momentum with respect to the heavy antiquark. The light quark’s \( j = 1/2, 3/2 \) can couple with the heavy antiquark’s spin \( S_Q = 1/2 \) to form states with total angular momentum and parity \( J^P = 0^{+}_{1/2}, 1^{+}_{1/2}, 1^{+}_{3/2}, 2^{+}_{3/2} \).

The \( 0^{+}_{1/2} \) and \( 1^{+}_{1/2} \) states are expected to decay to a ground-state heavy meson with \( J^P = 0^- \) or \( 1^- \) and a pion via an S-wave, and hence to be quite broad. No evidence for these states exists in the \( c\bar{q} \) or the \( b\bar{q} \) system. By contrast, the \( 1^{+}_{3/2} \) and \( 2^{+}_{3/2} \) states are expected to decay to a ground-state heavy meson and a pion mainly via a D-wave, and hence to be narrow. Candidates
TABLE 2. P-wave resonances of a heavy antiquark and a light quark \( q = u, d \). In final states \( P, V \) denote a heavy 0\(^-\), 1\(^-\) meson. For strange states, add about 0.1 GeV/c\(^2\) to the masses.

| \( J^P \) | \( \bar{c}q \) mass (GeV/c\(^2\)) | \( bq \) Mass (GeV/c\(^2\)) | Allowed final state(s) |
|---------|-----------------|-----------------|-----------------|
| \( 2^+ \) | 2.46\(^{+}\)\(^a\) | \( \sim 5.77^{+}\)\(^b\) | \( P\pi, V\pi \) |
| \( 1^+ \) | 2.42\(^c\) | \( \sim 5.77^{+}\)\(^b\) | \( V\pi \) |
| \( 1^\frac{1}{2} \) | \(< 2.42^{+}\)\(^a\) | \(< 5.77^{+}\)\(^b\) | \( V\pi \) |
| \( 0^\frac{1}{2} \) | \(< 2.42^{+}\)\(^c\) | \(< 5.77^{+}\)\(^b\) | \( P\pi \) |

\( \text{a) Observed value. See Ref. (37).} \)

\( \text{b) Predicted by extrapolation from corresponding } D^{\ast\ast} \text{ using heavy-quark symmetry.} \)

\( \text{c) Predicted in most quark models.} \)

for all the nonstrange and strange \( D^{\ast\ast} \) states of this variety have been identified (37). The known nonstrange \( D^{\ast\ast} \) resonances (identified in both charged and neutral states) are a 2\(^+\) state around 2.46 GeV/c\(^2\), decaying to \( D\pi \) and \( D^\ast\pi \), and a 1\(^+\) state around 2.42 GeV/c\(^2\), decaying to \( D^\ast\pi \). In addition, strange \( D^{s^{\ast\ast}} \) resonances have been seen, at 2.535 GeV/c\(^2\) (a candidate for \( 1^\frac{3}{2}\)\(^+\)) and 2.573 GeV/c\(^2\) (a candidate for \( 2^\frac{3}{2}\)\(^+\)). Thus, adding a strange quark adds about 0.1 GeV/c\(^2\) to the mass.

Once the masses of \( D^{\ast\ast} \) resonances are known, one can estimate those of the corresponding \( B^{\ast\ast} \) states by adding about 3.32 GeV (the quark mass difference minus a small binding correction). The results of this exercise are shown in Table 2. The reader should consult Ref. (38) for more detailed predictions based on potential models and for relations between decay widths of \( D^{\ast\ast} \) states and those of the \( B^{\ast\ast} \)'s. Thus, the study of excited charmed states can play a crucial role in determining the feasibility of methods for identifying the flavor of neutral \( B \) mesons.

C. Spectroscopic observations

The OPAL (39), DELPHI (40), and ALEPH (41) Collaborations have now observed \( B\pi \) correlations which can be interpreted in terms of the predicted \( J^P = 1^\frac{3}{2}, 2^\frac{3}{2} \) states. The OPAL data are shown in Fig. 2. Similar plots have been presented by DELPHI and ALEPH. OPAL also sees a \( BK \) correlation.

In all experiments one is able to measure only the effective mass of a \( B\pi \) system. If a \( B^{\ast\ast} \) decays to \( B^\ast\pi \), the photon in the \( B^\ast \rightarrow \gamma B \) decay is lost, leading to an underestimate of the \( B^{\ast\ast} \) mass by about 46 MeV but negligible energy smearing (31). Thus, the contributions to the \( B\pi \) mass distribution
FIG. 2. Evidence for B\(^{\ast \ast}\) resonances obtained by the OPAL Collaboration (39). 
(a) \(B\pi\) background-subtracted mass distribution; (b) \(BK\) background-subtracted mass distribution. The solid and cross-hatched histograms correspond to the contributions of \(2^+\) and \(1^+\) resonances in two-resonance fits based on the mass splittings and branching ratios predicted in Ref. (38).

The fitted masses of the nonstrange and strange resonances are respectively 30 MeV lower and 40 MeV higher than the predictions of Ref. (38). The difference could well be due to additional contributions to the nonstrange channel from the lower-lying \(0^+_1/2\) and \(1^+_1/2\) states or from nonresonant fragmentation. The corresponding strange states might lie below \(BK\) threshold.

The OPAL results imply that \((18 \pm 4)\%\) of the observed \(B^+\) mesons are accompanied by a “tagging \(\pi^-\)” arising from \(B^{*0}\) decay. By isospin reflection, one should then expect \((18 \pm 4)\%\) of \(B^0\) to be accompanied by a “tagging \(\pi^+\)” arising from \(B^{*+}\) decay. This is good news for the possibility of “same-side tagging” of neutral \(B\) mesons. [Another \((2.6 \pm 0.8)\%\) of the observed \(B^+\) mesons are accompanied by a “tagging \(K^-\).” The isospin-reflected kaon is...
neutral, and unsuitable for tagging.]

The DELPHI data can be fit with a single peak having a mass of $M = 5732 \pm 5 \pm 20$ MeV and width $\Gamma = 145 \pm 28$ MeV. The number of $B_{u,d}^{*\ast}$ per $b$ jet is quoted as $0.27 \pm 0.02 \pm 0.06$. The ALEPH peak occurs at $M(B\pi) - M(B) = (424 \pm 4 \pm 10)$ MeV/$c^2$, with Gaussian width $\sigma = (53 \pm 3 \pm 9)$ MeV/$c^2$. The ALEPH $B^{\ast\ast}$ signal is characterized by a production rate

$$\frac{B(Z \rightarrow b \rightarrow B_{u,d}^{*\ast})}{B(Z \rightarrow b \rightarrow B_{u,d})} = (27.9 \pm 1.6 \pm 5.9 \pm 3.8)\%,$$

where the first error is statistical, the second is systematic, and the third is associated with uncertainty in ascribing the peak to the contribution of various resonances. Multiplying the DELPHI and ALEPH $B^{\ast\ast}$ rates by the isospin factor of $2/3$ to compare with the OPAL result, we find complete agreement among the three.

**D. Isospin and Correlations**

In principle it should be possible to calibrate the correlations between charged pions and neutral $B$'s by comparing them with the isospin-reflected correlations between charged pions and charged $B$'s (whose flavor may be easier to identify). Thus, the enhancement of the non-exotic $\pi^+ B^0$ channel with respect to the exotic $\pi^- B^0$ channel should be the same as that of the non-exotic $\pi^- B^+$ channel with respect to the exotic $\pi^+ B^+$ channel. What can spoil this relation? I. Dunietz and I (42) have explored several instances in which care is warranted in making this comparison. Some of the differences could be real, but there are many sources of potential instrumental error against which one has concrete remedies.

1. **Interaction with the producing system** can lead to final states which need not be invariant under isospin reflection. For example, although a pair of gluons would produce a $b\bar{b}$ pair with isospin $I = 0$, the fragmentation process could involve picking up quarks from the producing system (e.g., proton or antiproton fragments) in a manner not invariant with respect to $u \leftrightarrow d$ substitution. Similarly, the production of a $B$ meson through diffractive dissociation of a proton (which has more valence $u$ quarks than $d$ quarks) need not be invariant under isospin reflection.

2. **Misidentification of associated charged kaons as pions** can lead one to overestimate the charged $B$ -- charged pion correlations. One expects $B^+ K^-$ correlations, as seen by OPAL, but not $B^0 K^+$ correlations. As mentioned, the isospin reflection of a charged kaon is neutral, and would not contribute to a correlation between charged particles and neutral $B$'s.

3. **Pions in the decay of the associated $B$** will not be produced in an isospin-reflection-symmetric manner. One must be careful not to confuse them with primary pions.
4. Different time-dependent selection criteria for charged and neutral $B$'s can lead one to mis-estimate the mixing of neutral $B$'s with their antiparticles. (I thank P. Derwent for pointing this out.) It is possible to make an unfortunate cut on $B^0$ lifetime which enhances the mixing considerably with respect to the value obtained by integrating over all times.

5. Overestimates of particle identification efficiencies can lead to confusion in identification of the flavor of a neutral $B$ through the decay $B^0 \rightarrow J/\psi K^{*-0} \rightarrow J/\psi K^+ \pi^-$. It is possible, especially for $K^+$ and $\pi^-$ with equal laboratory momenta, to confuse them with $\pi^+$ and $K^-$, while still keeping them in a $K^*$ peak.

The CDF Collaboration at Fermilab has been studying charged pion - $B$ correlations ever since the reports of Ref. (31) appeared, but no public announcement of these results has yet appeared. The intent of the present subsection is not to provide excuses for this failure to report results, but rather to provide ideas for ways to view the data in which such correlations are likely to be robust. They should certainly be present in hadron collider data.

VII. DECAYS

In this section we turn to decays of $B$ mesons to CP non-eigenstates. We have suggested (43) that relations between decays of charged $B$'s to pairs of light pseudoscalar mesons based on flavor SU(3) (44) could provide information on weak phases by means of rate measurements alone. The latest chapter in this story has been written since the Workshop.

A. $\pi\pi$ and $\pi K$ final states

Two years ago the CLEO Collaboration (45) presented evidence for a combination of $B^0 \rightarrow K^+ \pi^-$ and $\pi^+\pi^-$ decays, generically known as $B^0 \rightarrow h^+\pi^-$. On the basis of 2.4 fb$^{-1}$ of data, the most recent result (46) is $B(B^0 \rightarrow h^+\pi^-) = (1.81^{+0.6}_{-0.5} +0.2 \pm 0.2) \times 10^{-5}$. Although one still cannot conclude that either decay mode is nonzero at the $3\sigma$ level, the most likely solution is roughly equal branching ratios (i.e., about $10^{-5}$) for each mode.

Other results (47) of the CLEO Collaboration on related modes include the upper bounds $B(B^0 \rightarrow \pi^0\pi^0) < 1.0 \times 10^{-5}$, $B(B^+ \rightarrow \pi^+\pi^0) < 2.3 \times 10^{-5}$, and $B(B^+ \rightarrow K^+\pi^0) < 3.2 \times 10^{-5}$. Interesting levels for the last two modes (48) are probably around $(1/2) \times 10^{-5}$, and probably $10^{-6}$ or less for $\pi^0\pi^0$. With good particle identification and a factor of several times more data, it appears that CLEO will be able to make a systematic study of decay modes with two light pseudoscalars. What can it teach us?
B. SU(3) relations and the phase $\gamma$

We mentioned earlier that rate asymmetries in the decays $B^0 \rightarrow \pi^+\pi^-$ and $\bar{B}^0 \rightarrow \pi^+\pi^-$ could provide information on the weak angle $\alpha$, as long as a single quark subprocess dominated the decay. Additional contributions from penguin diagrams (48) can be taken into account by means of an isospin triangle construction (49) involving the relation

$$A(B^0 \rightarrow \pi^+\pi^-) = \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) + \sqrt{2}A(B^0 \rightarrow \pi^+\pi^0),$$

and the corresponding relation for the charge-conjugate processes. Here we define amplitudes such that a partial width is always proportional to the square of an amplitude with the same coefficient.

A similar amplitude quadrangle applies to the decays $B \rightarrow \pi K$ (50):

$$A(B^+ \rightarrow \pi^+ K^0) + \sqrt{2}A(B^+ \rightarrow \pi^0 K^+) = A(B^0 \rightarrow \pi^- K^+) + \sqrt{2}A(B^0 \rightarrow \pi^0 K^0).$$

When combined with the corresponding relation for $\bar{B}^0$ and $B^-$ decays, and used in conjunction with the time-dependence of the decays ($B^0$ or $\bar{B}^0$) $\rightarrow \pi^0 K_S$, these quadrangles are useful in extracting the weak phase $\alpha$.

In examining SU(3) relations among $B \rightarrow PP$ amplitudes, where $P$ is a pseudoscalar meson, we found that one of the diagonals of the amplitude quadrangle for $B \rightarrow \pi K$ (corresponding to an amplitude with isospin $I = 3/2$) could be related to the purely $I = 2$ amplitude for $B^+ \rightarrow \pi^+\pi^0$. We obtained the relation

$$A(B^+ \rightarrow \pi^+ K^0) + \sqrt{2}A(B^+ \rightarrow \pi^0 K^+) = \hat{r}_u \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0),$$

where $\hat{r}_u \equiv (f_K/f_\pi)|V_{us}/V_{ud}|$. The $B^+ \rightarrow \pi^+ K^0$ amplitude is expected to be dominated by a (gluonic) penguin contribution, involving mainly the combination of CKM elements $V_{ub}^* V_{ts}$, whose electroweak phase is $\pi$. The electroweak phase of the $B^+ \rightarrow \pi^+\pi^0$ amplitude is just $\gamma = \text{Arg}(V_{ub}^*)$. Thus, in the absence of strong-interaction phase shift differences, the shape of the amplitude triangle would give $\gamma$. One could account for strong-interaction phases by comparing the amplitude triangle for $B^+ \rightarrow \pi^+\pi^0$ with that for $B^- \rightarrow \pi^+\pi^0$ (43). We estimated (51) that in order to measure $\gamma$ to $10^\circ$ one would need a sample including about 100 events in the channels $\pi^0 K^\pm$.

C. Electroweak penguins

The analyses of Ref. (13) assumed that the only penguin contributions to $B$ decays were gluonic in nature. Consequently, one could treat the flavor-dependence in terms of an effective $\bar{b} \rightarrow \bar{d}$ or $\bar{b} \rightarrow \bar{s}$ transition since the gluon couples to light quarks in a flavor-symmetric manner. Thus, the $I = 3/2$ amplitude in $B \rightarrow \pi K$ (the diagonal of the $\pi K$ amplitude quadrangle mentioned above) was due entirely to the Cabibbo-suppressed “tree-diagram” process $\bar{b} \rightarrow \bar{u}s\bar{s}$, whose weak phase was well-specified.

It was pointed out (12,53) that in certain penguin-dominated $B$ decays such as $B \rightarrow \pi K^*$ and $\pi K$, electroweak penguin amplitudes were large enough to
compete favorably with the tree amplitude in the $I = 3/2$ channel. In contrast
to gluonic penguins, the virtual photon or $Z$ emitted in an electroweak penguin
diagram does not couple to light quarks in a flavor-symmetric manner, and
possesses an $I = 1$ component. Specifically, if one decomposes amplitudes
into isospin channels,

$$A(B^+ \to \pi^+ K^0) = (1/3)^{1/2}A_{3/2} - (2/3)^{1/2}A_{1/2},$$
$$A(B^+ \to \pi^0 K^+) = (2/3)^{1/2}A_{3/2} + (1/3)^{1/2}A_{1/2},$$

(13)

Deshpande and He (53) find, in a specific calculation, that

$$A_{1/2} \sim -0.75e^{i\gamma}e^{i\delta_{T,1/2}} + 7.3e^{i\delta_{P,1/2}},$$
$$A_{3/2} \sim -1.06e^{i\gamma}e^{i\delta_{T,3/2}} + 0.84e^{i\delta_{P,3/2}},$$

(14)

where the first term in each equation is the “tree” contribution (of lowest order
in electroweak interactions), while the second term is the penguin contribution.
Only the electroweak penguin contributes to the $I = 3/2$ amplitude,
but with magnitude comparable to the tree contribution. The electroweak
penguin spoils the relation of Eq. (12).

Recently several of us re-examined the effects of SU(3) breaking (54) and
electroweak penguins (55), to see if one could extract electroweak penguin
effects directly from the data. Since our previous SU(3) decomposition gave a
complete set of reduced amplitudes, electroweak penguins only changed the
interpretation of these amplitudes, so that a separation of electroweak penguin
effects was not possible merely on the basis of SU(3).

As pointed out by Deshpande and He (56), certain amplitudes (notably
those for $B_s \to (\pi^0 \text{ or } \rho^0) + (\eta \text{ or } \phi)$) are expected to be dominated by elec-
troweak penguins. We noted that the $\pi K$ amplitude quadrangle could be writ-
ten in such a manner that one if its diagonals was equal to $\sqrt{3}A(B_s \to \pi^0 \eta_8),$ where $\eta_8$ denotes an unmixed octet member. The shape of the quadrangle,
shown in Fig. 3, is uniquely determined, up to possible discrete ambiguities.
The case of octet-singlet mixtures in the $\eta$ requires us to replace the $\sqrt{3}$
by the appropriate coefficient; one can show that the SU(3) singlet contribution
of the $\eta$ is unimportant in this case.

The quadrangle has been written in such a way as to illustrate the fact

(43) that the $B^+ \to \pi^+ K^0$ amplitude receives only penguin contributions in
the absence of $O(f_B/m_B)$ corrections. The weak phases of $b \to s$ penguins,
which are dominated by a top quark in the loop, are expected to be $\pi$. We
have oriented the quadrangle to subtract out the corresponding strong phase,
and define corresponding strong phase shift differences $\tilde{\delta}$ with respect to the
strong phase of the $B^+ \to \pi^+ K^0$ amplitude.

The $I = 3/2$ $\pi K$ amplitude is composed of two parts, as noted above. We
can rewrite it slightly as

$$A_{3/2} = |A_T|e^{i\gamma}e^{i\tilde{\delta}_{T,3/2}} - |A_P|$$

(15)
FIG. 3. Amplitude quadrangle for $B \to \pi K$ decays. (a) $A(B^+ \to \pi^+ K^0)$; (b) $\sqrt{2}A(B^+ \to \pi^0 K^+)$; (c) $\sqrt{2}A(B^0 \to \pi^0 K^0)$; (d) $A(B^0 \to \pi^- K^+)$; (e) the diagonal $D_2 = \sqrt{2}A(B_s \to \pi^0 \eta_8)$; (f) the diagonal $D_1 = A_{3/2}$ corresponding to the $I = 3/2$ amplitude.

The corresponding charge-conjugate quadrangle has one diagonal equal to

$$\tilde{A}_{3/2} = |A_T| e^{-i\gamma} e^{i\delta_{T,3/2}} - |A_P|,$$

so that one can take the difference to eliminate the electroweak penguin contribution:

$$A_{3/2} - \tilde{A}_{3/2} = 2i|A_T| \sin \gamma e^{i\delta_{T,3/2}}.$$

The quantity $|A_T|$ can be related to the $I = 2 \pi\pi$ amplitude in order to obtain $\sin \gamma$. Specifically, if we neglect electroweak penguin effects in $B^+ \to \pi^+ \pi^0$ (a good approximation), we find that

$$|A_T| = \lambda (f_K/f_\pi) \sqrt{2} |A(B^+ \to \pi^+ \pi^0)|.$$

Thus, we can extract not only $\sin \gamma$, but also a strong phase shift difference $\delta_{T,3/2}$, by comparing Eqs. (16) and (18). If such a strong phase shift difference exists, the $B$ and $\bar{B}$ quadrangles will have different shapes, and CP violation in the $B$ system will already have been demonstrated.

The challenge in utilizing the amplitude quadrangle in Fig. 3 is to measure $B(B_s \to \pi^0 \eta)$, which has been estimated to be only $2 \times 10^{-7}$! Very recently (since the Workshop) Deshpande and He have pointed out that the amplitude triangle

$$2A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \sqrt{6}A(B^+ \to \eta K^+)$$

implied by the SU(3) relations of Refs. (43,55), where $\eta_8$ is the octet component of the $\eta$, permits one to specify the quantity $A_{3/2} - \tilde{A}_{3/2}$ in Eq. (17) and extract $\gamma$ as above. Here there is some delicacy associated with the SU(3) singlet component of the physical $\eta$. 
D. Other final states

1. **PV final states** \( (V = \rho, \omega, K^*, \phi) \) are characterized by more graphs (and hence more reduced SU(3) amplitudes), since one no longer has the benefit of Bose statistics as in \( B \to PP \) decays. There still exist quadrangle relations in \( \rho K \) and \( K^*\pi \) decays, however. Remarkably, if \( \Delta S = 0 \) gluonic penguin diagrams (small in any case) are approximately equal for the cases in which the spectator quark ends up in a vector meson and in a pseudoscalar [see Ref. (55) for details], the previous quadrangle construction still holds if we replace \( \sqrt{3}A(B_s \to \pi^0\eta_s) \) with \( \sqrt{2}A(B_s \to \pi^0\phi) \), \( A(B \to \pi K) \) with \( A(B \to \pi K^*) \), and \( A(B^+ \to \pi^+\pi^0) \) with \( A(B^+ \to \pi^0\rho^+) \).

Deshpande and He (56) predict \( B(B_s \to \pi^0\phi) \approx 2 \times 10^{-8} \), which probably means that the quadrangle reduces to two nearly overlapping triangles (whose shapes will consequently be difficult to specify). On the other hand, in \( PV \) decays, the effects of electroweak penguins then may not be so important if the dominant processes are characterized by branching ratios of order \( 10^{-5} \) as in \( B \to PP \) decays.

Hints of some signals have been seen in some \( PV \) channels in the latest CLEO data (46), but only upper limits are being quoted. These are fairly close to theoretical expectations in the case of some \( \pi\rho \) channels.

2. **VV final states** satisfy Bose statistics. Since the total angular momentum of the decaying particle is zero, the \( (\text{space}) \times (\text{spin}) \) part of the \( VV \) wave function will be symmetric, as in \( PP \) final states (58). Thus, there should exist amplitude relations for each relative orbital angular momentum \( \ell \). If one \( \ell \) value dominates the decays, such relations might be tested using triangles constructed of square roots of decay rates, as in the \( PP \) case.

VIII. SPECIFICS FOR HADRON COLLIDERS

In this section we comment on miscellaneous issues which deserve more attention if hadron collider experiments are to make a dent in the problem of CP violation in the \( B \) meson system.

A. Vertex resolution

As we mentioned earlier, if different minimum decay lengths are adopted for selection of different \( B^0 \) decay modes, the data sample will contain different admixtures of \( B^0 \) and \( \bar{B}^0 \) in different decay modes. Apparent inconsistencies can show up in charged pion – neutral \( B \) correlations.

A trigger based on the presence of secondary vertices may be the only way to pick out elusive decays such as \( B \to \pi\pi \) in sufficient numbers for CP studies.
B. Particle Identification

In the comparison of $B \rightarrow \pi \pi$ and $B \rightarrow \pi K$ decay modes, we have seen that good discrimination between pions and kaons is essential. Some distinctive processes of $B$ production and decay involve large number of kaons. Consider, for example the production of a $B_s = \bar{b}s$ with a kaon “tag.” If the subsequent decay is $B_s \rightarrow D^- \pi^+ \rightarrow \phi \pi^- K^+ \rightarrow K^+K^-\pi^-K^+$, a total of four kaons in the final state (counting the “tagging” kaon) must be identified.

C. Particle-Antiparticle Correlations

In the production of a $b\bar{b}$ pair in high-energy $e^+e^-$ collisions, as at LEP, the $b$ and $\bar{b}$ are separated from one another by many units of rapidity, with many hadrons filling the gap in between. In hadronic $b\bar{b}$ production, there may be a continuum of rapidity separations between $b$ and $\bar{b}$, ranging from the very small (as in fragmentation of a hard gluon to $b\bar{b}$) to the very large (as in back-to-back production of $b$ and $\bar{b}$ jets). The “same-side” tagging method is likely to be more relevant for widely separated $b$ and $\bar{b}$. One must learn more about the hadrons which fill the gap between a $b$ and a $\bar{b}$ in hadronic collisions, and tailor one’s tagging methods accordingly.

IX. CONCLUSIONS

The most promising advantage for the study of $B$ physics at the Tevatron (and ultimately, at the LHC) consists of the high rate of $b$ production. The relatively high $b\bar{b}$ effective mass expected in such processes implies that “same-side” tagging of neutral $B$ mesons in order to identify CP-violating rate asymmetries deserves serious study. A study of lifetime differences of hadrons containing $b$ quarks at an accuracy of several percent will permit the checking of detailed predictions of these differences.

The innate advantage (based on rate) of the Fermilab detectors for identification of tagged neutral $B$ mesons became clear several years ago when the first reconstructed $B$ mesons were presented by the CDF Collaboration. However, “same-side” tagging results from CDF have been nearly two years in gestation. Meanwhile, the recent identification of “$B^*$” mesons at LEP has brought to mind the last of Satchel Paige’s six rules for staying young

\[\text{Don’t look back. Something may be gaining on you.}\]

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58. We thank H. J. Lipkin for a discussion on this point.

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