Temperature Effect and Parameter Optimization of Silicon Resonant Accelerometer Based on Electrostatic Rigidity

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Abstract. Temperature stability, one of the most important parameters of highly accurate MEMS accelerometers, is always calibrated by the temperature coefficients of performance, thus reducing the temperature coefficients has significance to acquire a higher stability of micro devices. This paper proposed an in-depth analytical study on the temperature coefficients of silicon resonant micro-accelerometer based on electrostatic rigidity. And an corresponding optimization method was introduced to reduce the influence of the temperature on the scale factor and improve the device stability over temperature. The results showed the temperature coefficient and the temperature sensitivity of the scale factor were 9Hz/g/°C and 1280ppm/°C, respectively, which are equal to 1.6% and 49% of the original values respectively.

1. Introduction
Resonant principle has been widely used for measuring physical parameters such as mass, acceleration, force, flow or pressure due to its good resistance to noise or electrical interference and simple interface to a digital system[1]. Micro accelerometer based on the resonance exhibited many advantages over other type ones because they converted the input acceleration into frequency, a quasi-digital signal with a higher resistance to disturbance[2-7]. As a mechanical sensor, however, the resonant accelerometers were inevitably subject to the effects of temperature change, which is characterized by the temperature coefficients of performance usually including scale factor and zero bias. In some applications, such as the inertial navigation system(INS), the acceleration measurement error caused by the temperature stability of scale factor after the second integral would seriously affect the positioning precision of the navigation system. Therefore, scale factor stability needed to be carefully considered in the design of accelerometers.

This paper focused on the frequency thermal stability of the silicon resonant accelerometer based on electrostatic rigidity. Current investigations indicated that the performance drift of micro devices could result from the temperature effects of residual stress, structure, materials and circuitry[8-10]. The temperature change could cause the change of Young's modulus and structural dimensions of resonator[11]. The residual stress induced by the mismatch of thermal expansion coefficient between silicon and substrate would experience a redistribution during the temperature change[12-14]. The stability issue could be solved by constant temperature control[15], temperature compensation[16] and isolated encapsulation stress, etc[17-18]. However, those techniques always came with a more complicated system and larger volume of micro devices. In this paper, a novel measure was proposed...
to improve the temperature stability of silicon resonant accelerometer based on parameter optimization. It was found that the temperature stability of scale factor could be improved dramatically by adjusting the stiffness ratio.

2. Modeling and Theory Analysis

2.1. Principle of the Silicon Resonant Accelerometer

The simplified structure of silicon resonant accelerometer based on electrostatic rigidity is shown in figure 1[1]. It consists of the double-ended tuning fork (DETF), driving capacitors, sensing capacitors, proof mass and supporting springs.

![Figure 1. A schematic diagram of the resonant accelerometer.](image)

The DETF resonant state was sustained by the oscillating circuit connecting to driving and sensing electrodes. The sensitive proof mass supported elastically by folded beams and was bonded to a glass substrate, which moves near or away from the electrode of DETF under inertial force and changed the gap distance of sensing capacitor. Then, the stiffness of the DETF were reduced or increased by the electrostatic force form between mass and DETF and resulted in the change of the output frequency. The dynamic equation of the vibrating beam of DETF can be expressed as[1] :

$$m\ddot{Y} + c\dot{Y} + \kappa Y = F_{elec}(x,Y,t)$$  \hspace{1cm} (1)

where $x,Y$ are the vertical displacement of the vibrating beam and proof mass, respectively, $m,c$ and $\kappa$ are effective mass of the vibrating beam, damping coefficient, effective mechanical stiffness of vibrating beam, respectively. The electrostatic force $F_{elec}(x,Y,t)$ can be expressed as:

$$F_{elec}(x,Y,t) = \frac{\varepsilon AV_d^2}{2(g_0-x-Y)^2} - \frac{N_e h (V_{d1} + V_c \sin(\Omega t))^2}{2g_0}$$  \hspace{1cm} (2)

where, the first part on the right is sensing electrostatic forces, while the second part is driving electrostatic forces. $g_0$ is the gap distance of driving capacitor. The $V_{d2}$ is DC voltage of the proof mass, the DETF is connected to ground. The driving voltage is $V_c \sin(\Omega t)$ with DC bias voltage $V_{d1}$, which is added in the fixed comb electrodes, $\Omega$ is frequency of AC voltage. The other parameters shown in equation (2) were described in [1].

2.2. Residual stress of the resonant beam

The residual stress was mainly caused by the thermal expansion difference between silicon chip structure and glass substrate layer during processing. When the temperature of silicon accelerometer dropped to room temperature from processing condition, the thermal stresses including normal stresses and shearing and transverse normal (peeling) stresses, arose in the interface of the attachment material. In this paper, the residual stress(or thermal stress) was predicted by the theory of Tri-material assembly by E.Suhir [19-21]. The assembly structure could be expressed by figure 2(a). Note that it's not a typical adhesively assembled structure. Actually the upper component #1 and #2 are the silicon
micro beam structure, component #1 represents the resonant beam and component #2 represents the anchor points. They have the same material, so the interface between them is fictitious for ease of analysis. The lower component #3 is the glass substrate. In this picture, \( l \) is a half-length of the resonant beam, \( L \) is the length of one anchor point.

![Image of micro beam structure](image)

**Figure 2**[20]. (a) The assembly structure model; (b) Forces and moments at the x cross section.

The coefficients of linear thermal expansion of the assembly components are related as follows: 

\[
\alpha_1 = \alpha_2 < \alpha_3, \quad \text{Shearing stresses in the interfaces, when the assembly is cooled down by the temperature } \Delta T, \text{ are schematically shown in figure 2(b). Section I-II is the half structure of resonant beam, There is no direct contact between #1 and #3 layer structure, so it is only subject to the force from the dividing plane I, including axial forces (} T_{e1} \text{ and } T_{e3} \text{) and he moments (} M_I \text{ and } M_2 \text{) at the } x \text{ direction. And the section II-III is a complete three-layer structure, and the first and third layers are subjected to the opposite forces in the structure I (see figure 2(b)).}
\]

According to [20], the axial force \( T_{e1} \) and stress \( \sigma \) acting on the resonant beam can be expressed as:

\[
T_{e1} = \frac{(\alpha_1 - \alpha_2) T}{(h_1 + 2h_2 + h_3)^2} + \frac{(\alpha_2 + \alpha_3) \tau_1(0) - (\alpha_1 + \alpha_3) \tau_1(0)}{4(D_1 + D_2)} + \frac{(h_2 + 2h_3)^2}{4(D_1 + D_2)} (\lambda_3 + \lambda_1) l
\]

\[
\sigma = \frac{T_{e1}}{h_3} + \frac{h\theta_0 E_{\text{eff}}}{2l}
\]

In the equation (3), the second term on the right is a smaller amplitude than the first term, this term is omitted. In the equation (4), the first term on the right represents the axial stress introduced by the shearing force, and the second term represents the axial stress introduced by the moment. Rewrite equation (4):

\[
\sigma = \frac{n_2 h_2 (n_2 h_2^3 + h_1^3 (4h_1 + 6h_2 + 3h_3))}{h_1^4 + n_2^2 h_2^4 + 2n_2 h_1 h_2 (2h_1^2 + 6h_2^2 + 2h_3^2 + 6h_1 h_3 + 3h_2 h_3 + 6h_2 h_3)} E_i (\alpha_2 - \alpha_{\text{glass}}) (T_0 - T)
\]

where, \( n_2 = \frac{E_2}{E_1} \), define \( E_{\text{eff}} \) as the equivalent Young’s modulus for component 1##. \( E_{\text{eff}} = \frac{n_2 h_1 (n_2 h_1^3 + h_1^3 (4h_1 + 6h_2 + 3h_3))}{h_1^4 + n_2^2 h_1^4 + 2n_2 h_1 h_2 (2h_1^2 + 6h_2^2 + 2h_3^2 + 6h_1 h_3 + 3h_2 h_3 + 6h_2 h_3)} E_i = \eta E_i \), where, the parameter \( \eta \) is only related to the thickness and the ratio of the Young’s modulus between the structural layers. Then, the residual axial load of the resonant beam can be expressed as:

\[
N = \sigma A_i = (\alpha_2 - \alpha_{\text{glass}}) (T_0 - T) E_{\text{eff}} A_i
\]

where, \( A_i \) is the cross-sectional area of the resonant beam, \( T_0 \) is the initial temperature on the die packing process, and \( T \) is the working ambient temperature.

2.3. Improvement of Stability of Scale Factor

The resonant frequency of DETFs can be expressed as:

3
where, $\kappa_i$, $\kappa_j$ represent the electrostatic stiffness of two resonant beams in DETFS, respectively, and their values are obtained based on the mechanical dynamics equation (1). $\kappa$ is the effective mechanical stiffness of vibrating beams. The value of $\kappa$ with residual stress can be expressed as:

$$\kappa = \frac{E_i I}{I_i'} - a_i - \frac{N}{I_a} a_i$$

(8)

where, $I$ is the moment of inertia of the beam cross-section. $a_i$ and $a_2$ are constants determined by vibration modes of vibrating beam. $\kappa_a$ is the effective mechanical stiffness of vibrating beams regardless of the residual stress effect. Combining equation (6) and equation (8), the temperature coefficient of mechanical stiffness is:

$$\frac{\partial \kappa}{\partial T} = \frac{a_i E_{eff} A_i}{I} \cdot \left[\frac{\partial \alpha_i}{\partial T} - \frac{\partial \alpha_i}{\partial T} (T_0 - T)\right]$$

(9)

Here, the thermal expansion coefficient of the glass ($\alpha_{glass}$) also has good thermal stability as a constant.

The scale factor of the resonant accelerometer can be expressed as:

$$S_f = \Delta_f \left|_{avg} = f_0 \cdot \frac{6\gamma a(M \kappa - mk_i) [g_o k k_i - 2\gamma (\kappa + k_i)]^3}{g_o \kappa [g_o k k_i - \gamma (3\kappa + 2k_i)]^4}\right|_{avg} \cdot f_0 = \frac{1}{2\pi} \sqrt{\frac{\kappa}{m}}$$

(11)

where, $f_0$ is the mechanical natural frequency of DETFs. $\gamma = \frac{\epsilon A V_i^2}{2 \rho_0^2} = \frac{N_i \epsilon h V_i^2}{2 \rho_0^2}$ Remaining parameters shown in above are described in table 1. As the working ambient temperature $T$ changed, the residual thermal stress would change, causing a drift in the scale factor.

$$CT_{S_f} = \frac{\partial S_f}{\partial T} = f_0 \frac{\partial \kappa}{\partial T} \cdot \frac{3g_o k k_i [g_o k k_i - 2\gamma (\kappa + k_i)]^3}{g_o \kappa [\gamma (3\kappa + 2k_i) - g_o k k_i]} \cdot \left[(g_o k k_i)^2 (\kappa M - 3k_i m)ight] + \left[2\gamma^2 [3\kappa^2 M + \kappa^2 k_i (7M - 9m) - \kappa k_i^2 (17m + 2M) - 2k_i^3 m]\right]$$

(12)

is the temperature coefficient of scale factor.

$$CET_{S_f} = \frac{CT_{S_f}}{S_f}$$

(13)

is defined as the temperature sensitivity of the scale factor. The larger the value is, the more significant the temperature effect and the worse the thermal stability of the scale factor of the resonant accelerometer would be. equation (11) and equation (12) are substituted into equation (13) so as to obtain the expression of $CET_{S_f}$. 
Previous studies have shown that the thermal stability of the resonant accelerometer was improved in virtue of designing the isolating structures[17-18], temperature compensation[16] or other methods[15], which accordingly reduced the residual thermal force. Equation (14) showed that by optimizing the configuration of the structural parameters (such as: $M, m, k_s, g_o, ...$), the absolute values in curly braces \{\} were infinitesimal, and the thermal stability of the resonant accelerometer could be effectively improved even if the value of the residual stress is not changed.

3. Result and Discussion

Example was calculated by MTALAB-based theory, and the simulation parameters were expressed in table 1. According to equation (14), the stiffness of the folded beam $k_s$ was adjusted depending on the stiffness ratio $K(=\kappa_0 / k_s)$. The optimal stiffness ratio $K=60.5$ was obtained after finding the minimum absolute value of braces \{\} in equation (14) at a room temperature $T=25^\circ C$. After optimization, both the temperature coefficient and the temperature sensitivity of the scale factor were significantly improved, as shown in figure 3. Under initial conditions, of $K=55$, the temperature coefficient and the temperature sensitivity of the scale factor were 9Hz/g/$^\circ C$ and 1280ppm/$^\circ C$, respectively, and values of the two parameters were reduced to 1.6% and 49% of the original values respectively after optimization.

![Figure 3](image1.png)

**Figure 3.** Compare the values of $CT_{sf}$ and $CET_{sf}$ under the initial conditions($K=55$) and after optimization($K=60.5$). (a) Simulated the temperature coefficient of scale factor $CT_{sf}$ versus temperature $T(^\circ C)$; (b) Simulated the temperature sensitivity of the scale factor $CET_{sf}$ versus temperature $T(^\circ C)$. 

\[
CET_{sf} = \frac{1}{2} + \frac{4\gamma^2k_s^2m + 2\gamma mk_s^2(11\gamma - 3g_o k_s)\kappa}{(M\kappa - mk_s)(g_o\kappa k_s - \gamma(3\kappa + 2k_s))[g_o\kappa k_s - 2\gamma(\kappa + k_s)]} + \frac{2k_s[g_o^2k_s^2m + 6\gamma^2(m - M) + g_o\gamma k_s(M - 5m)]\kappa^2}{(M\kappa - mk_s)(g_o\kappa k_s - \gamma(3\kappa + 2k_s))[g_o\kappa k_s - 2\gamma(\kappa + k_s)]} - \frac{M[6\gamma^2 - 5g_o\gamma k_s + g_o k_s^2]\kappa^3}{(M\kappa - mk_s)(g_o\kappa k_s - \gamma(3\kappa + 2k_s))[g_o\kappa k_s - 2\gamma(\kappa + k_s)]} \frac{\partial \kappa}{\kappa T} 
\]
4. Conclusions
This paper mainly studies the temperature stability of the silicon resonant accelerometer based on electrostatic rigidity under residual thermal stress. In the study, the analytical formula of scale factor about residual thermal stress has been deduced, based on which the relevant parameters have been optimized. Simulation results show that the temperature stability of the scale factor has been improved significantly.

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| Table 1. The simulation parameters. |
|------------------------------------|
| Parameter | $E_1$ (Gpa) | $E_2$ (Gpa) | $h_1$ (um) | $h_2$ (um) | $h_3$ (um) | $l$ (um) |
| Value     | 160         | 62.7        | 40         | 4          | 50          | 400       |
| Parameter | $v_1$, $v_2$ | $v_3$ | $\alpha$ (F/m) | $g_0$ (um) | $b$ (um) | $L$ (um) |
| Value     | 0.22        | 0.2        | 8.5e-12    | 2.6e-9     | 5.6e-8    | 5.12e-8   |
| Parameter | $V_{d1}$, $V_{d2}$ (V) | $V_c$ (V) | $\alpha_{glue}$ (°C) | $g_0$ (um) | $b$ (um) | $L$ (um) |
| Value     | 8           | 0.1        | 3.25e-6    | 2          | 8          | 200       |
| Parameter | $T_f$ (°C) | $T_0$ (°C) | $\alpha_1$ | $\alpha_2$ | $\eta$ | $g(m/s^2)$ |
| Value     | 25          | 355        | 500        | -12.3      | 0.39       | 9.8       |
| Parameter | $C_1$, $C_2$, $C_3$, $C_4$ = -0.03,-9.37,0.42,89 ; $C_5$, $C_6$, $C_7$, $C_8$ = 0.04,12.41,0,10.76 |

**Table 1.** The simulation parameters.
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