Simple Scheme for Gauge Mediation

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We present a simple scheme for constructing models that achieve successful gauge mediation of supersymmetry breaking. In addition to our previous work [1] that proposed drastically simplified models using metastable vacua of supersymmetry breaking in vector-like theories, we show there are many other successful models using various types of supersymmetry breaking mechanisms that rely on enhanced low-energy $U(1)_R$ symmetries. In models where supersymmetry is broken by elementary singlets, one needs to assume $U(1)_R$ violating effects are accidentally small, while in models where composite fields break supersymmetry, emergence of approximate low-energy $U(1)_R$ symmetries can be understood simply on dimensional grounds. Even though the scheme still requires somewhat small parameters to sufficiently suppress gravity mediation, we discuss their possible origins due to dimensional transmutation. The scheme accommodates a wide range of the gravitino mass to avoid cosmological problems.

I. INTRODUCTION

Despite many new ideas, supersymmetry is still regarded as the prime candidate for physics beyond the standard model. If it exists at the TeV scale, it stabilizes the hierarchy between the electroweak and the Planck scales, allows for gauge coupling unification with the minimal particle content, has a natural candidate for the dark matter, and possibly connects coupling unification with the minimal particle content, has a phenomenologically successful manner.

Gauge mediation of supersymmetry breaking [2, 3] is an attractive solution to the phenomenological problems with supersymmetry. In particular, it naturally avoids excessive flavor-changing phenomena because gauge-mediated supersymmetry breaking effects are flavor universal. On the other hand, constructing explicit and realistic models of gauge mediation has been a rather nontrivial challenge that requires a fair amount of model-building efforts, and this aspect has been making the scenario appear a somewhat unlikely choice by nature.

In a previous paper [4], we have proposed a drastically simplified class of models for gauge mediation of supersymmetry breaking. The models have a supersymmetric $SU(N_c)$, $SO(N_c)$ or $Sp(N_c)$ gauge theory with massive quarks, massive vector-like messengers charged under the standard model gauge group, and a completely general superpotential among these fields. We have found it remarkable that this simple and general class of models can successfully break supersymmetry and generate a phenomenologically desired form of supersymmetry breaking masses, without any additional ingredients. This makes us conjecture that gauge mediation may be a rather generic phenomenon in the landscape of possible supersymmetric theories, which does not require any contrived or artificial structures that existed in many of the past models.

In this paper, we show that the success of the previous paper can extend more generally to even wider classes of theories. The low-energy structure of the models of Ref. [4] is such that, while the entire superpotential does not possess a $U(1)_R$ symmetry, terms relevant for supersymmetry breaking possess an accidental (and approximate) enhanced $U(1)_R$ symmetry. In the models of Ref. [4], this structure arises automatically at low energies, since $U(1)_R$ violating effects in the supersymmetry breaking sector arise from higher dimension operators and thus are suppressed by powers of the cutoff scale. In this paper we present many other models that are as simple as those in Ref. [4], and hence the simplicity of the scheme is not necessarily tied to the supersymmetry breaking mechanism of Ref. [4] on which the models of Ref. [4] were based.

In addition, in this paper we also consider the possibility that the $U(1)_R$ violating terms are suppressed (or absent) without obvious low-energy reasons. Such suppressions may arise through accidentally small parameters, as a property of string vacua, or for anthropic reasons. We also allow us to make a certain dynamical assumption on the sign of a Kähler potential term that is not calculable due to strong interactions. These relaxations of the requirements drastically enhance a variety of possible theoretical constructions leading to the structure described above. An important key to the success is the mass term for the messengers, which is simply one of the generic terms allowed by all symmetries.

In order for a model to be viable, several consistency conditions need to be met. Generic gravity-mediated supersymmetry breaking must be sufficiently small to avoid excessive flavor-changing and $CP$-violating processes. The impact of $U(1)_R$ violation, both at tree and loop levels, must be sufficiently small in the supersymmetry breaking sector to keep the essential dynamics intact. In addition, one should be concerned about cosmological constraints on the gravitino, moduli if any, the origin of the $\mu$ and $\mu B$ terms, and so on. Nonetheless, the framework we present here is sufficiently general and simple that we expect many models can be constructed to address these issues. In particular, the framework accommodates a wide range of the gravitino mass, $1 \text{ eV} \lesssim m_{3/2} \lesssim 10 \text{ GeV}$.

The simplicity and the variety of the models presented in this paper revitalize interest in the gauge mediation sce-
nario, and more generally in weak scale supersymmetry. They largely eliminate the concern about weak scale supersymmetry coming from the experimental non-observation of flavor-changing or CP-violating effects in addition to the ones in the standard model.

The organization of the paper is as follows. In the next section we describe the basic framework, and provide general discussions that apply to various explicit models presented in later sections. In Section II we present classes of models in which the supersymmetry breaking field is an elementary singlet. These models use accidental features to provide the approximate $U(1)_R$ symmetry. In Section IV we present models in which the supersymmetry breaking field arises as a composite field. In these models, $U(1)_R$ in the supersymmetry breaking sector arises automatically as an approximate low-energy symmetry. We present classes of models in which the sign of the relevant Kähler potential term can be reliably calculated in the low-energy effective theory, as well as those in which the sign is incalculable. In particular, a class of models presented in Section IV B enjoys the same level of success as the one in Ref. [3]. In Section V we discuss slightly different classes of models, which are nonetheless closely related to the ones presented in previous sections. Finally, in Section VI we discuss possible ways to naturally generate small parameters that are used in the models constructed in Sections II–IV. Conclusions are given in Section VII.

II. FRAMEWORK

In this section we present our basic framework. Explicit models within this framework will be given in later sections.

A. Basic idea

The basic idea is very simple. We consider the following superpotential

$$W = -\mu^2 S + \kappa S f \bar{f} + M f \bar{f},$$

where $S$ is a gauge singlet chiral superfield (elementary or composite), and $f, \bar{f}$ are messengers; $\mu$ is the scale of supersymmetry breaking, and $\kappa$ a coupling constant. The parameters $\mu^2, \kappa$ and $M$ can be taken real and positive without loss of generality. For concreteness, we take the messengers to be in $5 + 5^*$ representations of $SU(5)$ in which the standard model gauge group is embedded.

We assume that the Kähler potential for $S$ takes (approximately) the form

$$K = |S|^2 - \frac{|S|^4}{4\Lambda^2} + O\left(\frac{|S|^4}{\Lambda^4}\right),$$

expanded around the origin $S = 0$, where $\Lambda$ is a mass scale. (We assume a canonical Kähler potential for the messengers for simplicity.) This form of the Kähler potential is obtained, for instance, if there is an approximate low-energy $U(1)$ symmetry on $S$. This symmetry can be a $U(1)_R$ symmetry possessed by the first two terms of the superpotential, Eq. (1), under which $S, f$ and $\bar{f}$ carry the charges of 2, 0 and 0, respectively. In this case, the dynamics associated with (the generation of) these terms can be responsible for the Kähler potential of Eq. (2). (Explicit examples for such dynamics will be presented in later sections.) The $U(1)_R$ symmetry is violated by the last term of Eq. (1), but its effect on the Kähler potential can be suppressed as long as $M \gtrsim \kappa^2 \Lambda^2/4\pi$, as we will see below.

Let us first discuss the model at tree level specified by Eqs. (1–3). The potential is simply given by

$$V = |\mu^2 - \kappa f \bar{f}|^2 \left(1 + \frac{|S|^2}{\Lambda^2} + O\left(\frac{|S|^4}{\Lambda^4}\right)\right) + |\kappa S f + M f|^2 + |\kappa S \bar{f} + M \bar{f}|^2,$$

which has a global supersymmetric minimum at

$$S = -\frac{M}{\kappa}, \quad f = \bar{f} = \frac{\mu}{\kappa^{1/2}}.$$  (4)

This potential, however, also has a local supersymmetry breaking minimum at the origin of field space, $S = f = \bar{f} = 0$, as long as $M^2 > \kappa \mu^2$. The masses for the scalar components of $S$ and the messengers are given by $m^2_S = \mu^2/\Lambda^2$ and $m^2_f = M^2 \pm \kappa \mu^2$, respectively. Note that in order for this point to be a minimum, it is important that the sign of the second term in Eq. (1) is negative. This can be explicitly proven in some of the models presented in later sections, while in some others the sign should be simply assumed.

The tunneling rate from the local minimum to the true supersymmetric minimum can be easily suppressed. To estimate it, we can base our discussions on Ref. [3], calculating a semi-classical field theoretic tunneling rate for a toy triangular potential. While the expression worked out there cannot be literally applied to our case, we can still approximate our potential by a triangular form, obtaining the decay rate per unit volume

$$\Gamma/V \sim \mu^2 e^{-B}$$

with $B \sim 8\pi^2 M^2 \Lambda^2 / \kappa^3 \mu^4$. Since $\Lambda$ is expected to be $\gtrsim \mu$, the bounce action $B$ can be easily of $O(100)$ or larger for $M \gtrsim \kappa^{1/2} \mu$. To keep the lifetime of the local minimum much larger than the age of the universe, we need $\Gamma/V \ll H_0$, where $H_0 \simeq 1.6 \times 10^{-33}$ eV is the present Hubble constant. The bound is the strongest for $M^2 \approx \kappa \mu^2$ (the lowest messenger scale), but even then the constraints on the parameters are not very strong for $\Lambda \gtrsim M$.

At the local supersymmetry breaking minimum, the messengers $f, \bar{f}$ both have supersymmetric and holomorphic supersymmetry breaking masses

$$M_{\text{mess}} = M + \kappa \langle S \rangle \approx M,$$

and

$$F_{\text{mess}} = \kappa \langle F_S \rangle = \kappa \mu^2.$$  (6)

Here, we have assumed that the expectation value of $S$, which is generated by $U(1)_R$ violating effects as we will see below, is small. The conditions that this requirement imposes on the parameters of the theory will be discussed shortly. The masses

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for the gauginos and scalars in the SSM sector are then generated by messenger loops [2,3] and of order

\[ m_{\text{SUSY}} \simeq \frac{g^2}{16\pi^2} \frac{\kappa \mu^2}{M} , \]  

(7)

where \( g \) represents generic standard model gauge coupling constants. Taking these masses to be of \( O(100 \text{ GeV} \sim 1 \text{ TeV}) \) corresponds to

\[ \frac{\kappa \mu^2}{M} \approx 100 \text{ TeV}. \]  

(8)

The gravitino mass, on the other hand, is given by

\[ m_{3/2} \approx \frac{\langle F_S \rangle}{M_{\text{Pl}}} \approx \frac{\mu^2}{M_{\text{Pl}}}, \]  

(9)

where \( M_{\text{Pl}} \approx 2.4 \times 10^{18} \text{ GeV} \) is the reduced Planck scale. Thus, requiring that gravity mediation gives only subdominant contributions to the scalar masses, \( m_{3/2} \lesssim 10 \text{ GeV} \), we find

\[ \mu \lesssim 10^{9.5} \text{ GeV}. \]  

(10)

B. Effects of \( U(1)_R \) violation

The existence of the supersymmetry breaking minimum at \( S = f = \bar{f} = 0 \) can be viewed as a result of the \( U(1)_R \) symmetry possessed by the first two terms of Eq. (6): \( R(S) = 2 \), \( R(f) = R(\bar{f}) = 0 \). This picture is corrected by \( U(1)_R \) violating effects coming from the other sectors and/or terms in the theory. One origin of \( U(1)_R \) violation arises from the superpotential terms \( S^2 \) and \( S^3 \), which are the (only) renormalizable terms, other than those in Eq. (6), allowed by the gauge symmetry.\(^1\) These terms can be automatically suppressed if \( S \) is a composite field generated at low energies (as in the models of Section IV) but in general must be suppressed for other reasons if \( S \) is elementary (as in the models of Section III). Denoting the extra terms as

\[ \Delta W = \frac{M_S}{2} S^2 + \frac{\kappa_S}{3} S^3, \]  

(11)

constraints on the parameters \( M_S \) and \( \kappa_S \) are obtained by requiring that the resulting shift of \( \langle S \rangle \) is smaller than \( \approx \Lambda \) (for the expansion of Eq. (6) to be valid) and that \( \approx M/\kappa \) (to avoid tachyonic messengers):

\[ |M_S| \lesssim \min \left\{ \frac{\mu^2}{\Lambda}, \frac{M \mu^2}{\kappa \Lambda^2} \right\}, \quad |\kappa_S| \lesssim \frac{\mu^2}{\Lambda^2}. \]  

(12)

Note that these conditions are not very restrictive. This is because we use field space with small \( S \), where there is a quadratic stabilizing potential for \( S \) arising from the second term in Eq. (6).

\(^1\) A linear term of \( S \) in the Kähler potential can be absorbed into the definition of the superpotential by the appropriate Kähler transformation.

Another source of \( U(1)_R \) violation comes from loops of the messengers, which do not respect \( U(1)_R \) because of the mass term. These loops generate the following Coleman–Weinberg effective potential for \( S \):

\[ \Delta V \approx \frac{\kappa^2 \mu^4}{16\pi^2} \mathcal{F} \left( \kappa S \right) \]  

\[ \approx \frac{5\mu^4}{16\pi^2} \left\{ \frac{\kappa^3}{M} (S + S^4) - \frac{\kappa^4}{2M^2} (S^2 + S^4 + \cdots) \right\}, \]  

(13)

where \( \mathcal{F}(x) \) is a real polynomial function with the coefficients of \( O(1) \) up to symmetry factors. In the second line, we have shown the coefficients explicitly, keeping only the leading terms in \( \kappa^2/M^2 \) (and dropping an irrelevant constant in \( \Delta V \)), which corresponds to the correction to the Kähler potential of the form \( \Delta K \approx (1/16\pi^2) \{ (\kappa^3/m) |S|^2 (S + S^4) + (\kappa^4/m^2) |S|^3 (|S| + (\kappa^4/m^2) |S|^2 (S^2 + S^4 + \cdots) \}. \) The effective potential of Eq. (13) pulls the minimum at \( S = 0 \) towards the negative direction, and reduces a mass-squared eigenvalue of \( S \) from \( \mu^4/\Lambda^2 \). Yet for

\[ M \gtrsim \frac{\kappa^2}{4\pi} \Lambda, \]  

(14)

we find that these effects are parametrically suppressed and the structure of the supersymmetry breaking sector is not significantly modified. In particular, the local minimum stays at small \( S \): \( |\langle S \rangle| \approx \kappa^3 \Lambda^2/16\pi^2 M \lesssim \min \{ M/\kappa, \Lambda \}. \) The condition for avoiding tachyonic messengers is

\[ M^2 \gtrsim \kappa^2 \mu^2. \]  

(15)

Note that the inequalities of Eqs. (14, 15) should be understood that order one coefficients are omitted.

In general, the 4 parameters of the theory \( \mu, \kappa, M \) and \( \Lambda \) are arbitrary, except that we expect \( \Lambda \gtrsim \mu \) if the higher dimension term in the Kähler potential of Eq. (6) is induced by the dynamics generating the first (two) term(s) of the superpotential of Eq. (6). By varying these parameters, a wide variety of physical pictures can arise. For \( \mu^2/\Lambda \gg M \), for example, we can first integrate out the \( S \) scalar, which is much heavier than the messengers, and then the low-energy theory below the \( S \) mass appears as the standard gauge mediation model, with the Lagrangian given by \( f d^2 \theta (M_{\text{mess}} + \theta^2 F_{\text{mess}}) f f + h.c. \). On the other hand, in the opposite limit of \( M \gg \mu^2/\Lambda \), we can first integrate out the messengers \( f \) and \( \bar{f} \). This generates “gaugino mass operators” \( f d^2 \theta S W^\alpha W_\alpha + h.c. \) as well as flavor universal “scalar mass operators” \( f d^2 \theta S^3 S \Phi^\dagger \Phi \), where \( W_\alpha \) represents the SSM gauge field strength superfields and \( \Phi \) the SSM matter and Higgs chiral superfields. The low-energy theory below the messenger mass, \( M \), is then a simple Polonyi-type model – Eqs. (6, 7) with \( f \) and \( \bar{f} \) set to zero – together with these operators, which are responsible for the masses of the gauginos and scalars in the SSM sector.

C. The origin of \( S \) and the scales of the theories

The framework described here represents a great simplification in building models of gauge mediation. The only re-
required aspect of model building is essentially to explain the origin of the quartic term in Eq. (3). There are many classes of explicit models that can be constructed in this framework, some of which will be presented in Sections III and IV. In the models where $S$ is an elementary singlet (the models in Section III), it must be assumed that the $U(1)_R$ violating terms of Eq. (2) are suppressed without obvious low-energy reasons. On the other hand, in the models where $S$ is a composite field (the models in Section IV), these terms are naturally suppressed. Suppose that the $S$ field consists of $n$ elementary fields, $S \sim Q^n/\Lambda_s^{n-1}$ ($n \geq 2$), where $Q$ and $\Lambda_s$ represent generic constituents of $S$ and the scale of compositeness, respectively. The parameters $M_S$ and $\kappa_S$ in Eq. (1) are then suppressed as

$$M_S \approx \frac{\Lambda_s^{2n-2}}{M_s^{2n-3}}, \quad \kappa_S \approx \frac{\Lambda_s^{3n-3}}{M_s^{4n-3}},$$

respectively. Here, $M_s$ is the cutoff scale of the theory.

The $S$ compositeness also suppresses the parameter $\kappa$ in Eq. (1), weakening the transmission of gauge mediation effects. Writing the fundamental superpotential, which replaces the first two terms of Eq. (1), schematically as

$$W \approx -\frac{\zeta}{M_s^{n-3}}Q^n + \frac{\eta}{M_s^{n-1}}Q^n f \bar{f},$$

we find

$$\mu^2 = \frac{\zeta \Lambda_s^{n-1}}{M_s^{n-3}}, \quad \kappa = \frac{\eta \Lambda_s^{n-1}}{M_s^{n-3}},$$

where we have defined the compositeness scale $\Lambda_s$ for $S = Q^n/\Lambda_s^{n-1}$. The requirement of Eq. (12) for preserving the approximate $U(1)_R$ symmetry was to have a metastable minimum around the origin to justify the analysis. It requires an unexplained suppression in $M_S$ for elementary $S$, while it is easy to satisfy for composite $S$.

The gauge-mediated contribution to the SSM superparticles is given by (see Eq. (9))

$$m_{\text{SUSY}} \approx \frac{g^2}{16\pi^2} \frac{\zeta \eta \Lambda_s^{2n-2}}{M_s^{2n-4} M_{\text{mess}}},$$

where we have denoted the messenger mass explicitly as $M_{\text{mess}}$, leaving the possibility that the term with the $S$ expectation value contributes significantly in Eq. (9). We find that $m_{\text{SUSY}}$ is suppressed by $(\Lambda_s/M_s)^{2n-2}$. The contribution from gravity mediation is (see Eq. (9))

$$m_{3/2} \approx \frac{\zeta \Lambda_s^{n-1}}{M_s^{n-3} M_{\text{Pl}}},$$

Dividing Eq. (20) by Eq. (19), and using the stability condition $M_{\text{mess}}^2 \gtrsim \kappa \mu^2$ (see Eq. (15)), we obtain

$$m_{3/2}/m_{\text{SUSY}} \approx \frac{16\pi^2}{g^2} \frac{M_s^{n-1} M_{\text{mess}}}{\eta \Lambda_s^{2n-2} M_{\text{Pl}}} \gtrsim 100 \left(\frac{\zeta}{\eta}\right) M_s/M_{\text{Pl}}.$$  

In order for the gauge-mediated contribution to dominate over the gravity-mediated one, we must have $m_{3/2}/m_{\text{SUSY}} \lesssim O(0.01 \sim 0.1)$. This requires either small $\zeta$, large $\eta$, small $M_s$, or a combination of these:

$$\sqrt{\frac{\zeta}{\eta}} \frac{M_s}{M_{\text{Pl}}} \lesssim O(10^{-4} \sim 10^{-3}).$$

(A large value for $\eta$ is obtained by generating the nonrenormalizable coupling $W \sim Q^n f \bar{f}$ by integrating out heavy fields below $M_s$ in the theory.) In the case that $S$ is a two-body composite, i.e. $n = 2$, this condition is satisfied simply by having small mass parameters for elementary fields: $W \sim mQQ$ with $m \ll M_s$, which corresponds to having small $\zeta$.

A large variety of theoretical constructions allowed in this framework can lead to a wide range of the parameters $\mu$, $\kappa$, $M$ and $\Lambda$. This implies in particular that the framework accommodates a wide range of the gravitino mass, $1 \text{eV} \lesssim m_{3/2} \lesssim 10 \text{GeV}$. The smallest gravitino mass is obtained when $M^2 \approx \kappa \mu^2 \approx (100 \text{TeV})^2$ and $\kappa \approx O(1 \sim 4\pi)$. Such a light gravitino is useful to avoid cosmological problems associated with the gravitino.

III. THEORIES WITH ELEMENTARY SINGLETS

In this section we present classes of models in which the supersymmetry breaking field $S$ is an elementary singlet. As discussed in the previous section, this case requires accidental suppressions in $U(1)_R$ violating terms. Nonetheless it is quite nontrivial that successful gauge mediation is obtained in very simple models once such suppressions are assumed.

A. Tree-level supersymmetry breaking

An obvious candidate for producing the required Kähler potential of Eq. (2) is the good-old O’Raifeartaigh model [3]. We replace the first term of Eq. (1) (and the second term of Eq. (2)) by

$$W = -\mu^2 S + \lambda S X^2 + m XY,$$

where $S$, $X$ and $Y$ are singlet fields having the canonical Kähler potential (up to terms suppressed by the cutoff scale). Here, we simply assume that possible terms $S^2$, $S^3$, $X^2$, $SYX$, $Y^2$ and $SY^2$ are somehow suppressed. (The other terms can be forbidden by a discrete $Z_2$ symmetry under which $X$ and $Y$ are odd.) The parameters $\mu^2$, $\lambda$ and $m$ are taken real and positive without loss of generality.

The superpotential of Eq. (23) breaks supersymmetry due to the incompatibility between $F_S = 0$ and $F_Y = 0$. For $m^2 > 2\lambda \mu^2$, the minimum is at $X = Y = 0$. The field $S$ is a flat direction at tree level, but is stabilized at the origin due to radiative corrections to the Kähler potential.

These corrections can be calculated most easily by computing the Coleman-Weinberg effective potential for $S$, arising from loops of $X$ and $Y$. The mass matrix of the $X$ and $Y$
fermions in the basis $(\psi_X, \psi_Y)$ is
\[
\begin{pmatrix}
2S & m \\
m & 0
\end{pmatrix},
\]
while that of the scalars in the basis $(X, X^\dagger, Y, Y^\dagger)$ is
\[
\begin{pmatrix}
2\lambda m_S & -2\lambda \mu^2 & 2\lambda m S^\dagger & 0 \\
-2\lambda \mu^2 & m^2 + 4\lambda^2 |S|^2 & 2\lambda m S^\dagger & 0 \\
2\lambda m S & 0 & m^2 & 0 \\
0 & 2\lambda m S^\dagger & 0 & m^2
\end{pmatrix}.
\]

The resulting Coleman-Weinberg potential can be expanded around the origin of $S$ as
\[
\Delta V = \frac{\lambda^4 \mu^4}{3\pi^2 m^2} |S|^2 - \frac{3\lambda^6 \mu^4}{10\pi^2 m^4} |S|^4 + \cdots,
\]
where we have dropped an unimportant constant and kept only the leading terms in $\lambda \mu^2/m^2$. Note that since the superpotential of Eq. (23) possesses a $U(1)_R$ symmetry under which $S$, $X$ and $Y$ carry the charges of 2, 0 and 2, respectively, the potential of Eq. (24) is a function only of $|S|^2$. This, therefore, corresponds to the Kähler potential corrections of the form of Eq. (3), with $\lambda^2 = 3\pi^2 m^2/\lambda^4$.

To summarize, the complete superpotential of the model presented here is given by the combination of Eqs. (6) and (23):
\[
W = -\mu^2 S + \lambda S X^2 + m X Y + \kappa S f \bar{f} + M f \bar{f}.
\]
The other possible renormalizable terms must be suppressed as discussed in Section 11B. The Kähler potential can be canonical.

### B. Dynamical models

Another class of models that reproduces the super- and Kähler potentials of Eqs. (4, 5) uses supersymmetry breaking theories of Ref. [3], based on quantum modified moduli space. Consider an $SU(2)$ gauge theory with four doublets $Q_i$ and six singlets $S^{ij}$ $(i, j = 1, \ldots, 4)$. It is convenient to exploit the local equivalence of $SU(4)$ and $SO(6)$ groups for the flavor symmetry, and regard both singlets $S^{ij}$ and mesons $M_{ij} = Q_i \bar{Q}_j$ to be in the vector representation of $SO(6)$. For the sake of presentation, we assume that flavor $SO(6)$ is explicitly broken to $SO(5)$ by superpotential interactions, and refer to $SO(5)$ vectors $S_a$, $M_a$ $(a = 1, \ldots, 5)$ and singlets $S_6$, $M_6$. The superpotential, which replaces the first term of Eq. (1) and the second term of Eq. (2), is then given by
\[
W = -\lambda_5 S_6 M_6 - \lambda S_6 M_6.
\]
The couplings $\lambda_5$ and $\lambda$ can be taken real and positive without loss of generality. At quantum level, the theory confines with the following quantum modified moduli space [3]:
\[
Pf(Q_i Q_j) = M_a M_a + M_6 M_6 = \Lambda^4,
\]
where $\Lambda_4$ is the dynamical scale of $SU(2)$ gauge interactions. Because this constraint contradicts with the conditions for a supersymmetric vacuum $\partial W/\partial S_a = \partial W/\partial S = 0$, the theory breaks supersymmetry. Assuming $\lambda_5 > \lambda$, the minimum is at $M_a = S_6 = 0$ and $M_6 = \Lambda^2$. We can thus eliminate $M_6$ using the constraint as $M_6 = (\Lambda^4 - M_a M_a)^{1/2}$, and the superpotential of Eq. (28) becomes
\[
W = -\lambda_5 S_6 M_a - \lambda S_6 (\Lambda^4 - M_a M_a)^{1/2},
\]
where we have denoted $S_6$ simply as $S$. The field $S$ is a flat direction at tree level. We thus need to consider quantum effects to find where the minimum is for $S$. For $S \gg \Lambda_4$, the potential grows logarithmically with $S$ [1]. This can be shown explicitly because in this regime a weakly coupled description in terms of the fundamental quarks $Q_i$ is valid, so that the wavefunction renormalization factor $Z_S$ can be reliably calculated. The potential is $V_{\text{eff}} = Z_S^{-1}(S) |F_S|^2$, which grows for large $S$ because of the Yukawa coupling $\lambda$.

The behavior of the potential for small $S$ is more subtle. It was shown, however, in Ref. [11] that the behavior of the Kähler potential around the origin of $S$ is indeed of the type in Eq. (6). The quartic correction due to strong coupling of $Q_i$ is not calculable. Yet noting that only the combination $\lambda S$ couples to the strong sector, the contribution to the effective Kähler potential of $S$ coming from strong coupling physics at the scale $\Lambda'_s = 4\pi \Lambda_s$ is given by
\[
K = \frac{\Lambda^2}{(4\pi)^2} G \left( |S|^2 / \Lambda^2 \right),
\]
where $G(x)$ is a polynomial function with the coefficients of $O(1)$ up to symmetry factors, and the factor of $4\pi$ is inserted using naive dimensional analysis [13]. The quartic correction to the Kähler potential for $S$ is therefore of $O(\lambda^4/16\pi^2 \Lambda^2_s)$ from the strong sector.

On the other hand, the Coleman–Weinberg potential for $S$ due to loops of $M_a$ gives the quartic term of $S$ in the effective Kähler potential at $O(\lambda^2 / \Lambda^2_s)$. Here, we have assumed that $\lambda_5$ and $\lambda$ are of the same order of magnitude, and $\Lambda^2 / \Lambda^2_s$ arises from the product of the one-loop factor, $1/16\pi^2$, four couplings of $S, \lambda^4$, and the inverse square of the $M_a$ masses, $1/(\Lambda_s)^2$. We thus find that for a perturbative value of $\lambda$, i.e. $\lambda \lesssim 4\pi$, the calculable correction dominates over the in calculable one in Eq. (31). Indeed, one can show that for $\lambda_5 > \lambda$, the potential has a minimum at $S = 0$ with positive curvature. With the renormalized $\lambda_5$, $\lambda$ and $\Lambda_s$ in Eq. (31), the

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2 The model works equally well if $SO(6)$ is completely broken by superpotential interactions analogous to Eq. (23).

3 The real parts of $M_a$ become Nambu–Goldstone bosons of a spontaneously broken $SO(6)$ symmetry and hence massless in the limit $\lambda_5 \rightarrow \lambda_5 + \eta$. Their mass squared goes negative for $\lambda_5 < \lambda$, and the new minimum gives a nonvanishing $F$ component for $S_6$, instead of $S$. 
bosons have a mass matrix

\[
m_B^2 = \begin{pmatrix} \lambda_S^2 \Lambda_s^2 & -\lambda_S \lambda_a \Lambda_a S & 0 & 0 \\ -\lambda_S \lambda_a \Lambda_a S^\dagger & \lambda_a^2 \Lambda_s^2 + \lambda^2 |S|^2 & 0 & -\lambda^2 \Lambda_s^2 \\ 0 & 0 & -\lambda^2 \Lambda_s^2 & \lambda_S \lambda_a \Lambda_a S \lambda_a^2 \Lambda_s^2 + \lambda^2 |S|^2 \end{pmatrix},
\]

while the fermions

\[
m_F^2 = \begin{pmatrix} \lambda_S^2 \Lambda_s^2 & -\lambda_S \lambda_a \Lambda_a S^\dagger & \lambda_S \lambda_a \Lambda_a S \lambda_a^2 \Lambda_s^2 + \lambda^2 |S|^2 \end{pmatrix}.
\]

Here, we have used the fact that the kinetic terms for \( M_a \) are given by \( K = M_a^2 M_a / \lambda_a^2 \). The curvature \( m_B^2 \) at the origin, defined as \( V = V_0 + m_B^2 |S|^2 + O(|S|^4) \), can then be calculated as

\[
m_B^2 = \frac{5 \Lambda_s^2}{32 \pi^2} (\lambda_a^2 - \lambda^2)^2 \ln \frac{\lambda_a^2 + \lambda^2}{\lambda_a^2 - \lambda^2} + 2 \lambda^2 \Lambda_s^2 \ln \left( \frac{\lambda_a^2 + \lambda^2}{\pi e \Lambda_s^2} \right),
\]

and we find that \( m_B^2 \geq 0 \) for all \( \lambda_a \geq \lambda \). This explicit calculation confirms our power counting, \( m_B^2 \approx \lambda^2 |F_S|^2 / 16 \pi^2 \Lambda_s^2 \approx \lambda^4 \Lambda_s^2 / 16 \pi^2 \).

The theory is not calculable for \( S \sim \Lambda_s / \lambda \), and hence it in principle allows for a local minimum there [13]. If there is indeed a local minimum at \( S \sim \Lambda_s / \lambda \), it also provides a phenomenologically acceptable minimum. In this paper we have picked the minimum close to the origin \( S \approx 0 \), since we know it exists and thus is on a firmer theoretical footing.

To summarize, the complete superpotential of the model is given by the combination of Eqs. (1) and (28):

\[
W = -\lambda_S S_a M_a - \lambda S_a M_0 + \kappa S_a f \bar{f} + M f \bar{f}.
\]

The other possible gauge-invariant, renormalizable terms must be suppressed. Their coefficients must be smaller than of \( O(\min\{\Lambda_s, M_f\}) \) for dimensionful ones and of \( O(1/16 \pi^2) \) for dimensionless ones. The model reduces to the one of Eqs. (1) at low energies. The correspondence of the scales is given by \( \mu^2 = \lambda_a^2 \Lambda_s^2 \) and \( \Lambda \approx 4 \pi \Lambda_s / \lambda \). Extensions of the model to other gauge groups, \( Sp(N_c) \) \((N_c > 1)\) and \( SU(N_c) \) \((N_c > 2)\), are straightforward.

IV. THEORIES WITHOUT ELEMENTARY SINGLETS

Models in the previous section contain elementary singlets \( S \), so that the superpotential terms \( S^2 \) and \( S^3 \) must be suppressed “by hand” to obtain the approximate \( U(1)_B \) symmetry in the supersymmetry breaking sector. In this section, we present models that do not contain any fundamental singlets. The effective singlet \( S \) arises as a composite field at low energies, which allows for natural suppressions of the \( S^2 \) and \( S^3 \) terms in the low-energy effective superpotentials. One class is our previous work [1] and its straightforward generalizations based on the supersymmetry breaking mechanism of Ref. [4], where the negative quartic term in the Kähler potential originates from loops of light fields. However, the success of our scheme is not limited to this class of models. We also show other classes of models which enjoy comparable success, with tree-level or dynamical origin of the negative quartic term.

A. Models of Ref. [1] and their straightforward variations

We begin by reviewing a class of models constructed in our previous work Ref. [1]. Strictly speaking, these models do not reduce to the one given by Eqs. (1), since there are several “\( S \)” fields that carry nonvanishing \( F \)-component expectation values, \( F_S \). This slightly changes the situation. For example, turning on expectation values of the messengers cannot absorb all the \( F_S \)’s, so it does not lead to a supersymmetric minimum. Nonetheless, the basic structure of the models is still that of Section I, and many of the analyses there remain without any essential changes. At the qualitative level, even the constraint from tunneling can persist. We simply have to reinterpret the tunneling to the supersymmetric minimum as that to a lower, phenomenologically unacceptable minimum, which may arise by turning on messenger expectation values.

The models employ \( SU(N_c) \), \( SO(N_c) \) or \( Sp(N_c) \) gauge theories with massive vector-like quarks. Here we consider an \( SU(N_c) \) gauge theory for definiteness, and denote quark and antiquark chiral superfields by \( Q^i \) and \( \bar{Q}^i \) \((i = 1, \ldots, N_f)\). We take the number of quark flavors to be in the range \( N_c + 1 \leq N_f < \frac{3}{2} N_c \). The tree-level superpotential in this sector is given by

\[
W = m_{ij} \bar{Q}^i Q^j.
\]

We adopt the basis in which the quark mass matrix is diagonal, \( m_{ij} = -m_i \delta_{ij} \) with \( m_i \) real and positive. We consider that all the masses are different to avoid (potentially) unwanted Nambu–Goldstone bosons, and assume that they are ordered as \( m_1 > m_2 > \cdots > m_{N_c} > 0 \) without loss of generality.

For \( m_i \ll \Lambda_s \), the theory breaks supersymmetry on a local minimum, where \( \Lambda_s \) is the dynamical scale of \( SU(N_c) \) [4]. After integrating out the excitations of masses of order \( (m_\Lambda_s)^{1/2} \), the relevant degrees of freedom are \( S^{(i,j)}(i,j = N_f - N_c + 1, \ldots, N_f) \) with the superpotential

\[
W = -m_i \Lambda_s S^{ij},
\]

where we have assumed \( m_i \sim m \) for simplicity. These degrees of freedom obtain masses of order \( (m_\Lambda_s)^{1/2} / 4 \pi \) due to the corrections to the Kähler potential. This, therefore, reproduces the essential structure of Eqs. (1), (2).

The complete superpotential in the electric theory is given by the combination of the quark mass terms, Eq. (36), and general interactions of the quarks with the messengers [1]:

\[
W = m_{ij} \bar{Q}^i Q^j + \frac{\lambda_{ij}}{M_s} \bar{Q}^i Q^j f \bar{f} + M f \bar{f},
\]
where \( M_s \) is the cutoff scale of the theory, and \( \lambda_{ij} \) are dimensionless constants. The correspondence between the scales of the present model and those in Section II is given by \( \mu^2 \simeq m M_s, \kappa \simeq \lambda A_s/M_s \) and \( \Lambda \simeq 4\pi/(m M_s)^{1/2} \), where we have assumed \( m \sim m \) and \( \lambda_{ij} \sim \lambda \).

We finally comment on an example of straightforward variations of the models reviewed above. In the above models, the effective supersymmetry breaking fields are two-body composite states, \( S_i^j \sim \bar{Q}^i Q^j \), so that the supersymmetry breaking superpotential of Eq. (37) [\( \beta \)] comes from dimension-two operators in the ultraviolet, Eq. (36). We can, however, also consider models in which the supersymmetry breaking fields are n-body composite states with \( n > 2 \). Consider, for example, an \( SU(N_c) \) gauge theory with \( N_f \) massless vector-like quarks, \( Q_i \) and \( \bar{Q}^i \) (\( i = 1, \cdots, N_f \)), and a massless adjoint chiral superfield \( X \). The superpotential of the theory is then

\[
W = \lambda \text{Tr} X^3 - \zeta Q^i X Q^i.
\]

For \( \frac{1}{2}N_c + 1 \leq N_f < \frac{2}{3}N_c \), this theory has a dual magnetic description which is infrared free [14]. The dual theory is an \( SU(2N_f - N_c) \) gauge theory with \( N_f \) vector-like quarks, \( q_i \) and \( \bar{q}_i \), an adjoint, \( Y \), and elementary singlets, \( M^{ij} = Q^i Q^j / \Lambda_s \) and \( S^{ij} = Q^i X Q^j / \Lambda_s^2 \). The magnetic theory has the superpotential

\[
W = -\frac{\lambda}{3} \text{Tr} Y^3 + \frac{\lambda}{\Lambda_s} \text{M}^{ij} \bar{q}_i Y q_j + \lambda S^{ij} \bar{q}_i q_j - \zeta_{ij} \Lambda_s^2 S^{ij}.
\]

(The first term is absent for \( N_f = \frac{1}{2}N_c + 1 \).) Here, we have normalized the fields \( q_i, \bar{q}_i, Y, \) and \( S^{ij} \) to have canonical mass dimensions in the infrared, and we have taken \( \Lambda_{\text{mag}} \equiv \Lambda_s \) for simplicity.\(^5\)

We find that the last two terms of Eq. (40) have the identical structure with the corresponding terms in the previous model.\(^6\) The \( S^{ij} \) fields can thus serve the role of the supersymmetry breaking fields. The stability of \( S^{ij} \) is ensured by loops of the dual quarks \( q_i \) and \( \bar{q}_i \), and potentially unwanted light fields obtain masses from higher dimension operators omitted in Eq. (39). (Under the existence of higher dimension operators, an appropriate vacuum must be chosen in the dual magnetic theory.) Together with the couplings to the messengers

\[
W = \eta_{ij} \frac{M_s^2}{\Lambda_s} \bar{Q}^i X Q^j f \bar{f},
\]

this provides gauge mediation models in which the effective supersymmetry breaking fields are three-body composite states \((n = 3 \text{ in the language of Section II.D)}\).

According to the general discussions in Section II.C, the models require small couplings \( \zeta_{ij} \) or an enhancement of the operators of Eq. (31).

### B. SO(10) model with \( \psi(16) \) and \( H(10) \)

A general philosophy advocated in Ref. [1] is to discard a \( U(1)_R \) symmetry altogether at the level of a fundamental theory. An approximate \( U(1)_R \) symmetry should then arise in the low-energy effective theory as an accidental property of the supersymmetry breaking sector. Presumably the earliest calculable model of supersymmetry breaking without a \( U(1)_R \) symmetry is an \( SO(10) \) gauge theory with two chiral superfields, \( \psi(16) \) and \( H(10) \) [7]. This theory breaks supersymmetry under the existence of an \( H \) mass term, and can be regarded as a continuous deformation of an incalculable model of supersymmetry breaking, \( SO(10) \) with a single \( 16 \) [8], since they hold the same Witten index [9]. We can thus use this theory to construct a model of gauge mediation by coupling it to the messengers, along the lines of Ref. [1].

In the absence of a superpotential, the theory has global symmetries listed in Table I. These symmetries are explicitly broken under the existence of the most general renormalizable superpotential consistent with the gauge symmetry:

\[
W = \lambda \psi \psi H - \frac{m}{2} H^2.
\]

The general \( D \)-flat directions are parameterized by gauge-invariant polynomials \( X = \psi \psi H / Y = H^2 \). At a generic point in \( X \)-\( Y \) space, the gauge group is broken to \( SO(7) \), whose gaugino condensation generates a nonperturbative superpotential

\[
W_{\text{np}} = \frac{\Lambda_s^{21/5}}{X^{2/5}},
\]

where \( c \) is a calculable \( O(1) \) numerical coefficient, and \( \Lambda_s \) the dynamical scale of \( SO(10) \). Since the value of \( c \) is not important in the rest of the discussions, we set \( c = 1 \) by suitably changing the normalization of \( \Lambda_s \).

The model is calculable when \( \lambda \ll 1 \) and \( m \ll \Lambda_s \). In this limit, we can first ignore the mass term \(-mH^2/2 \) and find a numerical coefficient, and the general discussions in Section II.C, the models require small couplings \( \zeta_{ij} \) or an enhancement of the operators of Eq. (31).

| \( U(1)_R \) | \( U(1)_M \) |
|---|---|
| \( -3 \) | \( 1 \) |
| \( -5 \) | \( 2 \) |
| \( 0 \) | \( 4 \) |

**TABLE I:** Global symmetries of the \( SO(10) \) model in the absence of a superpotential.

\(^4\) The absence of the masses is not crucial. They just have to be suppressed sufficiently so that they do not alter the essential dynamics.

\(^5\) A similar superpotential to Eq. (40) is obtained for \( N_f = \frac{1}{2}(N_c + 1) \) if \( N_c \) is odd. In this case the relevant infrared degrees of freedom are \( q_i, \bar{q}_i \) and \( S^{ij} \), so that the first two terms of Eq. (40) are absent. The model also works in this case, since a possible nonperturbative superpotential term \( \text{det} S^{ij} S^{lk} M^{ik} \) is irrelevant.

\(^6\) A more complicated case without an accidental low-energy \( U(1)_R \) symmetry was considered in Ref. [10].

\(^7\) This is a non-standard embedding, where \( SO(7) \) is embedded into the \( SO(8) \) subgroup of \( SO(10) \) after we use the triality that switches the vector representation and one of the Majorana–Weyl spinor representations.
moduli space of supersymmetric vacua

\[ \langle X \rangle = \left( \frac{2}{3} \right)^{\frac{5}{7}} \Lambda_0^\lambda, \quad Y : \text{arbitrary}. \] (44)

One can verify that the \( U(1)_M \) anomalies are saturated by the composite \( Y \) alone. As long as \( \lambda \ll 1 \) and hence \( \langle X \rangle \gg \Lambda_0^\lambda \) the theory is weakly coupled, and the \( \text{Kähler} \) potential for \( X \) and \( Y \) can be worked out with the tree-level approximation. We use a similar technique to that in [57]. The result is

\[ K = x^2 + \frac{1}{\sqrt{2}} |X| + \frac{1}{4x^2} |Y|^2. \] (45)

Here, \( x \) is the real positive solution to the equation \( \partial K/\partial x = 0 \):

\[ 4x^4 - \sqrt{2} |X| |x - |Y|^2 = 0, \] (46)

which can be solved analytically using Ferrari’s method. With \( X \) integrated out along the moduli space of Eq. (44), the low-energy theory is one with \( X \) alone, whose \( \text{Kähler} \) potential can be expanded around the origin as

\[ K = \frac{3}{2} \langle X \rangle^{2/3} + \frac{|Y|^2}{2 \langle X \rangle^{2/3}} - \frac{|Y|^4}{6 \langle X \rangle^2} + O(|Y|^6). \] (47)

It is guaranteed that this \( \text{Kähler} \) potential depends only on the combination \( |Y|^2 \) because of the \( U(1)_M \) invariance of the theory in the absence of the mass term \( -m Y/2 \). We thus find that the low-energy theory, characterized by the \( \text{Kähler} \) potential of Eq. (47) and the linear superpotential term \( W = -m Y/2 \), has an accidental \( U(1)_R \) symmetry, under which \( Y \) carries a charge of \( +2 \).

The rest of the discussion reduces to the general one in Section 4. Note that the negative coefficient for the quartic term in the \( \text{Kähler} \) potential originates not from one-loop effects as in the models in Section [45] but rather from the tree-level \( \text{Kähler} \) potential along the \( D \)-flat directions. Correspondingly, there are no other light fields in the theory to generate the quartic term, which makes the model more easily compatible with cosmology. The coupling to the messengers is given by

\[ W = \frac{\eta}{2 M_*} H^2 f \tilde{f}. \] (48)

The correspondence of the scales can be worked out easily by canonically normalizing the \( Y \) field in Eq. (17): \( S \equiv Y/\sqrt{2} \langle X \rangle^{1/3} \). It is given by \( \mu^2 \simeq m \Lambda_0 / \lambda^{5/21} \), \( \kappa \simeq \eta \Lambda_0 / \lambda^{5/21} M_* \), and \( \Lambda \simeq \Lambda_0 / \lambda^{5/21} \). For an appropriate range of the parameters, the superpotential of the model can be a generic one compatible with the gauge symmetry, as in the models of Ref. 45.

In fact, the basic dynamics of the model just described is more general. Consider a model of dynamical supersymmetry breaking in which some of the classical flat directions are lifted by superpotential interactions. By choosing these interactions appropriately, one can make expectation values of fields larger than the dynamical scale, and thus make the model calculable. Now, if the model allows for making only one field \( S \) significantly lighter than the rest of the excitations (such as the \( Y \) field above), then one can write a low-energy effective theory that contains only a single composite field \( S \). By shifting the origin of \( S \) such that \( \langle S \rangle = 0 \) at the minimum, the superpotential contains a linear term, and the \( \text{Kähler} \) potential takes generically the form of Eq. (46). We can then construct a model of gauge mediation simply by coupling the gauge invariant operator \( S \) to the messenger bilinear \( f \bar{f} \) in the superpotential.

A small variation of this picture is obtained, for example, in the \( SU(5) \) model with \( A(10) \), \( F(2) \), and two \( F_i(5') \) \((i = 1, 2)[57] \). This model can be viewed as a continuous deformation of the incalculable model with only \( A \) and one \( F \)[57], once a mass term is given to a pair of \( F \) and \( F' \). The most general superpotential of the model is

\[ W = \lambda A \bar{F}_1 \bar{F}_2 + \lambda' A A F - m \bar{F}_1 F, \] (49)

while the nonperturbative superpotential is

\[ W_{np} = \frac{\Lambda^6}{(|A \bar{F}_1 \bar{F}_2 |(A A F))^{1/2}}, \] (50)

where \( \Lambda \) is the dynamical scale of \( SU(5) \). There is no \( U(1)_R \) symmetry, but there is a global \( SU(2) \times U(1) \) symmetry in this model. In terms of the gauge-invariant polynomials \( X = (1/2)(A \bar{F}_1 \bar{F}_2), Y = (1/\sqrt{2})(A A F) \) and \( S_i = \bar{F}_i F \), the tree-level \( \text{Kähler} \) potential can be worked out and expanded in \( S_i \) as

\[ K = 6(|X| + |Y|)^2/3 + \sum_i S_i S_i \frac{2(|X| + |Y|)^{2/3}}{2(|X| + |Y|)^{2/3}} - \frac{\sum_i S_i^2 S_i^2}{24(|X| + |Y|)^2} + O(S_i^6). \] (51)

The global \( SU(2) \times U(1) \) invariance of the theory guarantees that it depends on \( S_i \) only through the combination \( \sum_i S_i^2 S_i \). After minimizing the superpotential without the mass term \( (m = 0) \), both \( X \) and \( Y \) are fixed and can be integrated out. The low-energy theory consists of \( S_i \) alone, which saturates the \( SU(2) \times U(1) \) anomalies. Given the negative quartic term in the \( \text{Kähler} \) potential, the mass term breaks supersymmetry with a stable minimum at the origin \( S_i = 0 \). One can verify that both \( S_1 \) and \( S_2 \) acquire positive squared masses. In fact, this model generalizes to \( SU(2k + 1) \) with an antisymmetric tensor \( A \), one fundamental \( F \) and \( (2k - 2) \) antifundamentals \( F_i \). With \( A F_i \) and \( A F_i \bar{F}_j \) terms in the superpotential and the nonperturbative superpotential \( W_{np} \propto |(A F_i) \bar{F}_j|^{-1/2} \), the low-energy theory is given in terms of \( S_i = \bar{F}_i F \) that match the anomalies of the global \( Sp(k-1) \times U(1) \) symmetry. A mass term \( m \bar{F}_i F \) would break supersymmetry with a stable minimum at the origin. Then the coupling to the messengers \( \bar{F}_i F \bar{f} \bar{f} / M_* \) makes gauge mediation possible.

### C. Models with incalculable \text{Kähler} potentials

The first model we present here uses the supersymmetry breaking theory of Ref. [29], based on the phenomenon of
quantum smooth-out of classical singularities in moduli space. Consider an $SU(2)$ gauge theory with a single chiral superfield $Q$ in the $I = 3/2$ representation. The gauge invariant chiral operator in this theory is $u = QQQQ$, and we introduce the following tree-level superpotential:

$$W = -\frac{\zeta}{M_s}u,$$

where $M_s$ is the cutoff scale of the theory, presumably of order $M_{Pl}$, and $\zeta$ a dimensionless constant. Since $u$ saturates nontrivial 't Hooft anomaly matching conditions [23], we expect that $u$ is the only low-energy degree of freedom. The Kähler potential for $u$ is then given by

$$K = \Lambda_s^2 G \left( \frac{|u|^2}{\Lambda_s^2} \right),$$

where $\Lambda_s$ is the dynamical scale of $SU(2)$ gauge interactions. For $|u| \ll \Lambda_s$, $G(x)$ is expected to be a polynomial function with the coefficients of $O(1)$ up to symmetry factors — the classical singularity at the origin of $u$ is smoothed out by quantum effects.

Denoting the field with the canonical dimension by $S = u/\Lambda_s^4$, the low-energy super- and Kähler potentials for $|S| \ll \Lambda_s$ take the form given by the first term of Eq. (1) and Eq. (2), respectively. In the present theory, however, the sign of the quartic term in Eq. (3) is incalculable, while it must be negative in order for the model to work. We thus make a dynamical assumption that the sign of this term is negative.

The complete superpotential of the model is given by

$$W = -\frac{\zeta}{M_s}u + \eta M_s^3 f\bar{f} + M f\bar{f},$$

where $\eta$ is a dimensionless constant. The model reduces at low energies to that of Eqs. (4), (8), with $\mu^2 = \zeta\Lambda_s^4/M_s$, $\kappa = \eta\Lambda_s^4/M_s^3$ and $\Lambda \simeq \Lambda_s$. In addition to the terms in Eq. (4), the most general terms consistent with the gauge symmetry may present. The coefficients of these terms need not be suppressed, since the constraints of Eq. (12) are almost automatically satisfied because of the compositeness of $S$.

As discussed in Section II.4, the dimensionless coupling $\zeta$ must be small in order for the model to work (for $\eta \sim 1$ and $M_s \sim M_{Pl}$; see Eq. (22)). Possible parameter regions include, for example, $\Lambda_s \simeq 10^{15}$ GeV, $M_s \simeq 10^{18}$ GeV, $\zeta \simeq 10^{-8}$, $\eta \simeq 1$, and $M \simeq 10^6$ GeV.

A nearly identical analysis can be made on an $SU(6)$ gauge theory with a rank-three antisymmetric tensor $A^{ijk}$. For general $D$-flat configurations, the gauge group is broken to $SU(3) \times SU(3)$, each of which develops a gaugino condensate. Depending on the relative phase between the two condensates, the nonperturbative superpotential

$$W_{np} = \left(1 + (-1)^{1/3}\right) \frac{\Lambda_s^5}{(A^4)^{1/2}},$$

can identically vanish. The composite field $A^4$ saturates the $U(1)_R$ and $U(1)_R^3$ anomalies, so we expect it to have a non-singular Kähler potential at the origin. An introduction of a linear term in $A^4$ would then break supersymmetry. As in the $SU(2)$ model, however, the quartic term in the Kähler potential is not calculable. We thus have to assume that its coefficient is negative in order to use this theory.

Yet another example is $SO(N)$ theories with $N - 4$ vectors. They have two inequivalent branches, one with and the other without a dynamical superpotential [25]. All anomalies are saturated by the mesons $M^{ij} = Q^iQ^j$. Adding a mass term to just one of the flavors, the theory breaks supersymmetry. Again the quartic term in the Kähler potential is not calculable and we have to simply assume that its coefficient is negative to use this theory.

V. RELATED MODELS

In this section we present models that do not exactly fall in the category discussed in Section II. We first present models in which the low-energy effective theories contain more than one field, $S$. In general, these theories have multiple composite fields $F_i$ ($i = 1, 2, \cdots$) at low energies, which are stabilized due to complicated Kähler potentials. Models of gauge mediation are then obtained by coupling the degree of freedom responsible for supersymmetry breaking to the messengers in the superpotential. In the case that the Kähler potentials are complicated, the low-energy fields $F_i$ cannot be regarded simply as multiple copies of an $S$ field, in contrast with the case in some of the previous models such as the ones in Section IV A.

We then consider models that do not contain any degree of freedom which is significantly lighter than the dynamical scale. While these models are generally incalculable, models of gauge mediation can be obtained by coupling the messengers to appropriate composite operators.

While the models discussed in this section do not have an identical low-energy structure to those of Section II, they share many features. In particular, the basic constructions of the models are quite similar — we simply prepare a supersymmetry breaking model that has a stable supersymmetry breaking minimum (either global or local), and then couple the operator responsible for supersymmetry breaking to the bilinear of (generically massive) messengers. Many of the general analyses in Section II also persist. In particular, a general constraint on parameters in Eq. (23) persists, despite the fact that the powers of $\zeta$ appearing in the gauge-mediated and gravity-mediated contributions in Eqs. (13) (20) can now be different. Here, $\zeta$ represents the coefficient in front of the operator responsible for supersymmetry breaking in the superpotential. We will prove this fact in Section IV A.

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8 For an alternative derivation of inequivalent branches, see [24].

9 Discrete anomalies are not matched because of a condensate $\langle Q^N - 4\bar{W}^a\bar{V}_a \rangle \neq 0$; see [23].
A. Models with multiple low-energy fields

A class of theories that breaks supersymmetry dynamically is chiral gauge theories which do not possess classical flat directions, and in which global symmetries are spontaneously broken [21, 27, 28]. These theories have stable supersymmetry breaking vacua, and one can construct models of gauge mediation by coupling these theories to (generically massive) messengers $f, f$. Here we present one such theory explicitly, and analyze its relations to the class of models discussed in Section II.

Consider an $SU(3) \times SU(2)$ gauge theory with the matter content $Q(3,2), U(3^*,1), D(3^*,1)$ and $L(1,2)$, with the tree-level superpotential $W = \zeta QDL$. This theory breaks supersymmetry at the vacuum with expectation values for the fields $v \sim \Lambda_\zeta/\zeta^{1/7}$ [28]. Here, $\Lambda_\zeta$ is the dynamical scale of $SU(3)$, which we assume to be larger than that of $SU(2)$. The vacuum energy is given by $V \sim \zeta^{10/7} \Lambda_\zeta^2$.

It is useful to analyze the theory in terms of the gauge-invariant composite fields: $X_1 = QDL$, $X_2 = QUL$ and $X_3 = \text{det}(Q_i Q^j)$, where $Q_i \equiv (D,U)$ and $j = 1, 2$ is the $SU(2)$ index. Including nonperturbative effects, the low-energy effective superpotential is

$$W = \zeta X_1 + 2 \frac{A^2}{X_3}$$

(56)

For $\zeta \ll 1$, expectation values for the fields are much larger than the dynamical scale, $v \gg \Lambda_\zeta$, so that the Kähler potential is well approximated by the tree-level one. In terms of the composite fields, it is given by [24]

$$K = 24 \frac{A + B x}{x^2},$$

(57)

where $A = (X_1^4 X_2 + X_2^4 X_2)/2, B = (X_3^4 X_3)^{1/2}/3,$ and

$$x = 4 \sqrt{B \cos \left(\frac{1}{3} \arccos \left(\frac{A}{B^{3/2}}\right)\right)}.$$  

(58)

By minimizing the resulting scalar potential, we find that the minimum is at

$$\langle X_1 \rangle \approx 0.5 \frac{\Lambda_\zeta^3}{\zeta^{3/7}}, \quad \langle X_2 \rangle = 0, \quad \langle X_3 \rangle \approx 2.5 \frac{\Lambda_\zeta^4}{\zeta^{4/7}},$$

(59)

with

$$F_{X_1} \approx -2.57 \zeta^{3/7} \Lambda_\zeta^4, \quad F_{X_2} = 0, \quad F_{X_3} \approx 3.42 \zeta^{2/7} \Lambda_\zeta^5,$$

(60)

where $F_{X_i}$ represent the vacuum expectation values for the auxiliary components of chiral superfields $X_i$. We can thus construct a model of gauge mediation by coupling $X_1$ to the messengers in the superpotential.

The relevant superpotential for the messengers is

$$W = \frac{\eta}{M_*} QDL f f + M f f,$$

(61)

where $M_*$ is the cutoff scale of the theory. The supersymmetric and holomorphic supersymmetry breaking masses for the messengers are given by

$$M_{\text{mess}} = M + \frac{\eta}{M_*^2} \langle X_1 \rangle \approx M + \frac{\eta \Lambda_\zeta^4}{\zeta^{3/7} M_*^2},$$

(62)

and

$$F_{\text{mess}} = \frac{\eta}{M_*^2} \langle F_{X_1} \rangle \approx \frac{\eta \zeta^{3/7} \Lambda_\zeta^4}{M_*^2},$$

(63)

where we have omitted $O(1)$ coefficients. The condition for avoiding tachyonic messengers is

$$M_{\text{mess}} \gtrsim \frac{\eta^{1/2} \zeta^{3/14} \Lambda_\zeta^2}{M_*}.$$  

(64)

As long as this condition is satisfied, the minimum in the original theory stays as a local supersymmetry-breaking minimum in the theory with the messengers.

The resulting gauge-mediated contribution to the scalar and gaugino masses in the SSM sector is of order

$$m_{\text{SUSY}} \approx \frac{g^2}{16 \pi^2 M_*^2 M_{\text{mess}}},$$

(65)

while generic gravity-mediated contributions to the SSM-sector scalars, arising from Kähler potential terms of the form $Q^1 Q \Phi_i \Phi_i / M_{Pl}^2$, $U^1 U \Phi_i \Phi_i / M_{Pl}^2$ and so on, are of order

$$m_{3/2} \approx \frac{\eta^{5/2} \Lambda_\zeta^2}{M_{Pl}^2},$$

(66)

where $\Phi$ represents matter and Higgs superfields in the SSM sector. To obtain $m_{\text{SUSY}} = O(100 \text{ GeV} \sim 1 \text{ TeV})$, we need to take

$$\frac{\zeta^{3/7} \eta \Lambda_\zeta^4}{M_*^2 M_{\text{mess}}} \approx 100 \text{ TeV}. $$

(67)

The tunneling rate from the supersymmetry breaking minimum to the true supersymmetric (runaway) minimum does not give a very strong constraint on the parameters.

From the expressions in Eqs. (65)–(66), we find that the ratio of gravity- to gauge-mediated contributions is given by

$$\frac{m_{3/2}}{m_{\text{SUSY}}} \approx \frac{16 \pi^2 \zeta^{2/7} M_{\text{mess}}^2}{g^2 \eta \Lambda_\zeta^2 M_{Pl}} \gtrsim \frac{100 \zeta^{1/2} M_*}{\eta^{1/2} M_{Pl}}.$$  

(68)

Note that this inequality is identical to the corresponding one in Section II C, Eq. (21), despite the fact that the powers of $\zeta$ in Eqs. (43)–(46) are different from the corresponding ones in Section II C. Therefore, the general bound in Eq. (23) also applies to the present case. In particular, the coupling $\zeta$ must be suppressed for $\eta \sim 1$ and $M_* \sim M_{Pl}$. An example of phenomenologically successful parameter regions in the present model is $\zeta \sim 10^{-6}, \eta \sim 1, \Lambda_\zeta \sim 10^{12-15} \text{ GeV}$, $M_* \sim 10^{13} \text{ GeV}$ and $M \sim 10^{5-6} \text{ GeV}$.
One can show that the general bound of Eq. (22) applies quite generally in the present class of theories. Suppose that the gauge-invariant chiral superfield operator $O$ that couples to $f\bar{f}$ and gives $F_{\text{mess}}$ and (a part of) $M_{\text{mess}}$ consists of $n$ elementary fields, $O \sim Q^n$, and the superpotential term generating the auxiliary component expectation value for $O$ is given by $W = \zeta O/M_n^{n-3}$. (The SU(3) × SU(2) model discussed above corresponds to the case with $n = 3$.) Characteristic field expectation values, $\langle Q \rangle$, depend on the non-perturbatively generated superpotential, but can in general be parameterized by $\langle Q \rangle \sim \Lambda_{s}/\zeta_{n}$, where $\Lambda_{s}$ is the dynamical scale for the relevant gauge interactions and $\alpha > 0$. (The SU(3) × SU(2) model has $\alpha = 1/7$). The $F$-term expectation value for $Q$ is then given by $F_{Q} \sim \zeta \langle Q \rangle^{n-1}/M_n^{n-3} \sim \zeta^{1-(n-1)\alpha}M_{s}^{n-1}/M_n^{n-3}$ (which implies $\alpha < 1/(n - 1)$ since $F_{Q}$ should vanish for $\zeta \rightarrow 0$). The coupling to the messengers takes the form $W = (\eta/M_n^{n-1})O f\bar{f}$, so the messenger masses are given by

$$M_{\text{mess}} \approx M + \frac{\eta\Lambda_{s}^{n}}{\zeta M_{n}^{n-1}},$$

(69) and

$$F_{\text{mess}} \approx \frac{\zeta^{1-2(n-1)\alpha}\eta\Lambda_{s}^{2n-2}}{M_{s}^{2n-4}},$$

(70)

where we have used the fact that the Kähler potential is approximately canonical in terms of the ultraviolet fields $Q$ for $\zeta \lesssim 1$. The gauge-mediated and gravity-mediated contributions are given by

$$m_{\text{SUSY}} \approx \frac{g^{2}/16\pi^{2}}{M_{n}^{n-1}}\frac{\zeta^{1-2(n-1)\alpha}\eta\Lambda_{s}^{2n-2}}{M_{n}^{n-1}},$$

(71)

$$m_{3/2} \approx \frac{\zeta^{1-(n-1)\alpha}}{M_{s}^{n-3}},$$

(72)

respectively. Using the stability condition for the messengers $M_{\text{mess}} \gtrsim F_{\text{mess}}$, we then find the same inequality as Eq. (68):

$$\frac{m_{3/2}}{m_{\text{SUSY}}} \approx \frac{16\pi^{2}/g^{2}}{M_{s}^{n-1}}\frac{\zeta^{1/2}}{\eta^{1/2}},$$

(73)

which implies the general bound of Eq. (22).

B. Incalculable models

Consider an SU(5) gauge theory with $\psi(10)$ and $\phi(5^*)$. This theory does not have a classical flat direction, and has an exact global $U(1)_{R}$ symmetry with the charge assignment $R(\psi) = 1$ and $R(\phi) = -9$. The difficulty of satisfying the 't Hooft anomaly matching condition suggests that the $U(1)_{R}$ symmetry is dynamically broken. The exact global symmetry of the theory, $U(1)_{A}$ with the charge assignment $A(\psi) = 1$ and $A(\phi) = -3$, may or may not be broken.\textsuperscript{10} The broken $U(1)_{R}$ symmetry, together with the absence of a classical flat direction, then suggests that supersymmetry is dynamically broken at the vacuum.

It is natural to consider that the $U(1)_{R}$ symmetry and supersymmetry are broken, respectively, by the vacuum expectation values of the lowest and highest components of the operator $S = W^{\alpha}W_{\alpha}$, where $W_{\alpha}$ is the gauge field-strength superfield for SU(5).\textsuperscript{11} We assume here that this is indeed the case: $\langle S \rangle \sim \Lambda_{s}^{3}$ and $F_{S} \sim \Lambda_{s}^{4}$, where $\Lambda_{s}$ is the dynamical scale of SU(5). A model of gauge mediation is then obtained by introducing the following Lagrangian terms for the messengers:

$$\mathcal{L} = \int d^{2} \theta \left( \frac{1}{m^{2}} f\bar{f}W^{\alpha}W_{\alpha} + M f\bar{f} \right) + \text{h.c.},$$

(74)

where $m$ is a scale associated with the generation of the $f\bar{f}W^{\alpha}W_{\alpha}$ term (assuming $O(1)$ dimensionless couplings). The supersymmetric and holomorphic supersymmetry breaking masses for the messengers are then given by

$$M_{\text{mess}} \approx M + \frac{\Lambda_{s}^{3}}{m^{2}},$$

(75)

$$F_{\text{mess}} \approx \frac{\Lambda_{s}^{4}}{m^{2}}.$$

(76)

The gauge-mediated and gravity-mediated contributions are given by

$$m_{\text{SUSY}} \approx \frac{g^{2}}{16\pi^{2}}\frac{\Lambda_{s}^{4}}{m^{2}M_{\text{mess}}},$$

(77)

$$m_{3/2} \approx \frac{\Lambda_{s}^{4}}{m^{2}M_{\text{Pl}}}. $$

(78)

There are several constraints for the model to work. First, the messenger stability requires that $M_{\text{mess}}^{2} > F_{\text{mess}}$. Assuming $m \gg \Lambda_{s}$, this implies that $M_{\text{mess}}$ is dominated by the first term in Eq. (75) and that

$$M \gtrsim \frac{\Lambda_{s}^{2}}{m}. $$

(79)

We then require the gauge-mediated contribution to dominate the gravity-dominated one, leading to

$$m^{2}M \lesssim O(10^{-4} \sim 10^{-3}).$$

(80)

Equations (79) (80) imply that the parameter $m$ should be at least 3 or 4 orders of magnitude smaller than $M_{\text{Pl}}$. The first

\textsuperscript{10} The $U(1)_{A}$ symmetry is most likely not broken as its anomalies can be matched by a simple three-fermion composite $(\psi^{ij}\psi_{ij})\phi_{j}$, where $i, j$ are SU(5) fundamental indices and the parenthesis represents the contraction of spinor indices.

\textsuperscript{11} There are other natural candidate chiral superfields for nonvanishing highest-component expectation values: $\mathcal{D}^{2}(\text{Tr} e^{V^2} \psi^{T} e^{V} \psi)$ and $\mathcal{D}^{2}(\phi^{(e^{V^{T}} \phi)} e^{V} \phi)$. They are, however, identical to $W^{\alpha}W_{\alpha}$ because of the Konishi anomaly [3].
term of Eq. (74) with such a small $m$ can be easily generated, for example, by introducing a pair of vector-like fields $F(r) + \bar{F}(r^*)$ under $SU(5)$, as well as the superpotential
\[ W = \frac{\lambda}{M_s} F\bar{F} f \bar{f} + M_F F \bar{F}, \]
where $M_s$ is the cutoff scale of the theory, presumably of order $M_{Pl}$, and $M_F$ takes a value in the range
\[ \Lambda_s \lesssim M_F \lesssim M_s. \]
After integrating out the $F, \bar{F}$ fields, we obtain the first term of Eq. (74) with
\[ m^2 \approx 8\pi^2 \frac{M_F M_s}{\lambda}. \]
Finally, to obtain $m_{\text{SUSY}} = O(100 \, \text{GeV} \sim 1 \, \text{TeV})$, we need
\[ \frac{\Lambda^4}{m^4 M_s} \approx 100 \, \text{TeV}. \]

To summarize, the model has an $SU(5)$ gauge symmetry with the matter content $\psi(10), \phi(5^*), F(r)$ and $\bar{F}(r^*)$. The superpotential is given by
\[ W = M_F F \bar{F} + \frac{\lambda}{M_s} F\bar{F} f \bar{f} + M_f f \bar{f}, \]

VI. GENERATING SMALL PARAMETERS

The models discussed so far have used small parameters, e.g. small dimensionless couplings and/or mass parameters that are hierarchically smaller than the cutoff scale. It is rather easy to generate these small parameters dynamically, using supersymmetric dynamics.

As an example, let us consider an $SU(2)$ gauge theory with 4 quark chiral superfields $Q_i$ ($i = 1, \cdots, 4$). There are six gauge-invariant meson operators constructed out of $Q_i$, which can be decomposed into a 5-plet, $(QQ)_m$ ($m = 1, \cdots, 5$), and a singlet, $(Qg)$, under the $SP(4) \simeq SO(5)$ subgroup of the flavor $SU(4) \simeq SO(6)$ symmetry. Nonperturbative $SU(2)$ dynamics induce vacuum expectation values for these operators $(QQ)_m^2 + (Qg)^2 = \Lambda'^4$, where $\Lambda'$ is the dynamical scale of $SU(2)$ [53]. We now introduce the superpotential term $W = k \tilde{Z}_m (QQ)_m$, where $\tilde{Z}_m$ is an $SU(2)$-singlet chiral superfield and $k$ a coupling constant. This leads to $\langle (QQ)_m \rangle = 0$ and $\langle (Qg) \rangle = \Lambda'^2$, which can be used as a general scale generation mechanism through the $(QQ)$ operator [50]. Specifically, we can generate small mass and/or dimensionless parameters, simply by replacing them by (some powers of) the operator $(QQ)$ suppressed by appropriate powers of the cutoff scale. For a sufficiently large value of $k$, this does not disturb the original dynamics of the models.

Another way of generating small parameters is to use the gaugino condensation of a supersymmetric pure Yang-Mills theory. Suppose we replace small parameters by (some powers of) the bilinear of the gauge field-strength Yang-Mills superfield, $W^\alpha W_\alpha$ suppressed by appropriate powers of the cutoff scale. The low-energy effective Lagrangian is then obtained essentially by setting $W^\alpha W_\alpha = \Lambda'^3$, where $\Lambda'$ is the dynamical scale of the Yang-Mills theory. This, therefore, dynamically generates small parameters [51]. While this process also generates other (small) terms in the superpotential, minima of the potential in the original models are maintained in general, with only small shifts in expectation values of the fields.

It is model dependent if these scale generation mechanisms lead to a theory in which the interactions are the most general ones consistent with symmetries. For the models in our previous work Ref. [1], this question has been discussed in Ref. [32]. In the $SO(10)$ model of Section IV B we can consider a discrete $R$ symmetry under which the mass term of the $H$ field is replaced by (some powers of) the gaugino condensation of a pure Yang-Mills theory. For example, we can consider a $Z_{r,R}$ symmetry with the charge assignment $R(\psi) = -2$ and $R(H) = -1$. This leads to an unsuppressed coupling $\lambda$, with the $H$ mass term given by $m \approx (W^\alpha W_\alpha)^2/M_s^2 = \Lambda'^6/M_s^8$.

VII. CONCLUSIONS

In this paper, we have presented a simple scheme for constructing models that achieve successful gauge mediation of supersymmetry breaking. It uses the essence of the success of the models in our previous work [1], which relies on an approximate $U(1)_R$ symmetry for the field that breaks supersymmetry. We have clarified essential ingredients for the scheme: (i) a negative quartic term for the supersymmetry breaking field in the Kähler potential, (ii) an adequate suppression of explicit breaking of the $U(1)_R$ symmetry, and (iii) an explicit mass term for the messengers.

We have shown various possible origins for (i). The negative quartic term in the Kähler potential may arise at the one-loop level due to light fields in the low-energy theory, at tree level in the calculable Kähler potential along $D$-flat directions, or at the nonperturbative level for composite fields.

On general grounds, we need an unexplained suppression of explicit $U(1)_R$ breaking terms if the supersymmetry breaking field is an elementary singlet. It could arise accidentally in certain string compactifications or due to anthropic reasons on the landscape of theories. On the other hand, models in which a composite field breaks supersymmetry naturally suppress the $U(1)_R$ breaking effects and are very attractive.
Finally, the explicit mass term for the messengers may well arise from dimensional transmutation at a scale much lower than the cutoff scale, due to quantum modified moduli space or gaugino condensations.

We also pointed out that the successful models do rely on small parameters to sufficiently suppress gravity-mediated supersymmetry breaking which may be flavor non-universal and/or $CP$-violating. An example is the small quark masses in Ref. [1]. Such small parameters again can well arise from dimensional transmutation.

Given a wide variety of classes of models that achieve successful gauge mediation, it is also clear that the scheme accommodates a wide range of the gravitino mass to alleviate cosmological problems concerning the gravitino and moduli if any.

We conclude that gauge mediation of supersymmetry breaking is a rather generic phenomenon on the landscape of supersymmetric theories. This observation largely eliminates the concern about low-energy supersymmetry due to the absence of anomalous flavor-changing and $CP$-violating effects. This revitalizes interest in supersymmetry below the TeV scale, which will be probed by the forthcoming LHC experiments.

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