Nonlocal Spacetime-Matter

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We propose a nonlocal field theory for gravity in presence of matter consistent with perturbative unitarity, quantum finiteness, and other essential classical properties that we are going to list below. First, the theory exactly reproduces the same tree-level scattering amplitudes of Einstein’s gravity coupled to matter insuring no violation of macro-causality. Second, all the exact solutions of the Einstein’s theory are also exact solutions of the nonlocal theory. Finally, and most importantly, the linear and nonlinear stability analysis of the exact solutions in nonlocal gravity (with or without matter) is in one to one correspondence with the same analysis in General Relativity. Therefore, all the exact solutions stable in the Einstein’s theory are also stable in nonlocal gravity in presence of matter at any perturbative order.

I. INTRODUCTION

Nonlocal field theory aims to provide a simple, compact, and elegant solution to the quantum gravity issue. Indeed, gravity at quantum level is not special but exactly like all the other fundamental interactions. However, if classical gravity is described by the Einstein-Hilbert action, then the outcome of the quantization procedure shows divergences that drastically change the structure of the theory. In the technical jargon of quantum field theory, it is said that the Einstein’s theory of gravity is non-renormalizable. However, there is not any inconsistency between gravity and quantum mechanics: again, gravity is just non-renormalizable and need for a completion at high energy (a new action principle). This is something that does not happen in QED and QCD, but that physicists were called to face for the case of the Fermi theory of weak interactions. The renormalizability problem of the latter theory was overcome replacing the Fermi’s Lagrangian with a non abelian gauge theory. Therefore, exactly like for the weak interactions, also for gravity we need a new gravitational theory able to tame the infinities. Now we know that the only possible extension of the Einstein-Hilbert theory, in the field theory framework, that is consistent with unitarity and renormalizability (actually finiteness to all perturbative orders in the loop expansion) is the weakly nonlocal gravity. Another possibility is provided by the Lee-Wick quantum gravity that, however, needs further prescriptions at classical and quantum level, which are not specified by the action (references will be quoted shortly).

Records show that a nonlocal gravitational theory for gravity was proposed by Krasnikov [1] and studied two years later by Kuz’min [2]. However, only recently a multidimensional generalization of the theory [9], with particular attention to odd dimension [4], and an extension of the theory in even dimension [5], has been shown to be finite at quantum level. Furthermore, the Cutkosky rules [6] for a general nonlocal field theory has been derived in [7], where the perturbative unitarity was proven at any order in the loop expansion including the analysis of the anomalous thresholds. The macrocausality is also secured has proven in [10, 11] on the base of the Shapiro’s time delay. On the other hand, local Lee-Wick quantum gravity has been proposed in [12, 13], on the footprint of the seminal paper [14], in order to address the unitarity issue that plagues local higher derivative theories.

Despite the very encouraging, and we would like to say surprising results listed above, not much has been done for gravity in the presence of matter [15, 16]. A simple way to couple nonlocal gravity to matter is introducing supersymmetry. This is a relatively easy task when we have at our disposal a superspace formalism [18], but the construction of other theories is still incomplete, see for example the eleven dimensional supergravity [19].

In this paper we provide a recipe to construct a general nonlocal field theory for gravity coupled to matter (NLGM) on the base of the following four requirements (by Einstein’s theory we will mean Einstein’s gravity in presence of matter):

i all the solutions of Einstein’s gravity must be solutions of NLGM (this is an empirical requirement),

ii all the tree-level scattering amplitudes of NLGM theory must coincide with those of Einstein’s theory (this requirement guarantees macro-causality),

iii the stability analysis of the exact solutions in NLGM has to be in one to one correspondence with the same analysis in Einstein’s theory (namely if a solution is stable in Einstein’s gravity it is stable in NLGM too),

iv the theory has to be super-renormalizable or finite at quantum level and unitary at any perturbative order in the loop expansion.

As a final remark, we want to emphasize that the recipe provided in this article will allow us to construct the ultraviolet completion of any local two-derivatives theory, in any dimension, and regardless of the presence of gravity.
II. NONLOCAL GRAVITY-MATTER THEORY

In this section we first display the general theory accomplish for the requirements (i)-(iv) listed above in the previous section, and, afterwards, we will comments on such properties. Hence, let us start with the action,

\[ S[\Phi_i] = \int d^Dx \sqrt{-g} (\mathcal{L}_\ell + E_i F(\Delta)_{ij} E_j), \]  
(1)

\[ S_\ell = \int d^Dx \sqrt{-g} \mathcal{L}_\ell, \quad \mathcal{L}_\ell = \frac{2}{k^2} R + \mathcal{L}_m, \]  
(2)

\[ E_i(x) = \frac{\delta S_\ell}{\delta \Phi_i(x)}, \]  
(3)

where \( \Phi_i \) is a set of fields, placed in a vector of components labelled by the index \( i \), that include the metric and the matter’s fields. \( F(\Delta)_{ij}(x, y) \) is a symmetric (respect to the swap of the indexes \( i, j \) together with the spacetime points \( x, y \)) tensorial entire function whose argument is a tensorial differential operator \( \Delta \) that we are going to construct consistently with the stability of the exact solutions of the local theory (namely solutions of the equations of motion (EoM) \( E_i = 0 \)). Indeed, it straightforward to show that the requirement (i) is satisfied by explicitly computing the variation of the action \( 1 \) (up to total derivative terms and operators quadratic in the EoM \( E_i \)). The EoM for the nonlocal action \( 3 \) (see appendix \( A \) for more details) read:

\[ \mathcal{E}_k = E_k + 2\Delta_{ki} F_{ij} E_j + O(E^2) = 0. \]  
(4)

Since \( E_k \) are the Einstein’s EoM and the EoM for the matter, the following implication applies,

\[ E_k = 0 \implies \mathcal{E}_k = 0, \]  
(5)

where we introduced the Hessian operator of the local theory defined by (see appendix \( A \))

\[ \Delta_{ki} \equiv \frac{\delta E_i}{\delta \Phi_k} = \frac{\delta^2 S_\ell}{\delta \Phi_k \delta \Phi_i}. \]  
(6)

The same property, namely that the action consists on the Lagrangian in \( 2 \) plus a second operator quadratic in \( E_i \), secures that all the scattering amplitudes of the non-local theory in presence of matter are identical to those of Einstein’s gravity coupled to matter. The latter statement is based on a simple generalization of the theorem already used in \( 111 \) \( 111 \) \( 20 \). Therefore, also the requirement (ii) is satisfied. Notice that the reverse implication in \( 3 \) is in not true because the space of solutions of the nonlocal theory is generally larger then the one of the local Einstein’s theory coupled to matter.

Notice that the action \( 1 \) is very general and the recipe defined by the equations \( 1, 2, 3 \) and \( 4 \) applies to any system in any dimension, starting from a \( 1+0 \) system to a \( D \)-dimensional field theory. Therefore, \( 1 \) is actually an ultraviolet completion of any field theory, including a point-like action, with an arbitrary number of fields in presence or absence of the gravitational interaction.

In order to address the stability issue as stated in (iii) we have to pick out a form factor \( F_{ij} \) satisfying the following equation:

\[ 2\Delta_{ik} F(\Delta)_{kj} \equiv \left(e^{H(\Delta)} - 1\right)_{ij}. \]  
(7)

where \( H(\Delta) \) is an entire analytic function (see the appendix \( A \) for the explicit construction of \( F(\Delta) \)). Indeed, replacing \( 7 \) in \( 4 \) the EoM turn into

\[ \mathcal{E}_k = \left(e^{H(\Delta)}\right)_{kj} E_j + O(E^2) = 0. \]  
(8)

Since the function in front of the local EoM \( E_j \) is invertible, we can infer that the theory is ghost-free and only the perturbative degrees of freedom of Einstein’s gravity in presence of matter are allow to propagate. As marked by \( O(E^2) \) in the EoM \( 8 \) and consistently with the previous results published in \( 21, 22 \), the EoM for the perturbations of the metric and all the other fields of the theory are the same as in Einstein’s local gravity with matter. Hence, the stability is guarantee at any perturbative order. Let us expand on this statement. We can invert the exponential factor and rewrite \( 8 \) as

\[ \tilde{\mathcal{E}}_i \equiv E_i + \left(e^{-H(\Delta)}\right)_{ik} \left[O(E^2)\right]_k = 0. \]  
(9)

Now, given an exact background solution of the NLGM theory compatible with \( E_k = 0 \), we can derive the EoM for the perturbations defined through an expansion of the fields, and then of the EoM, in a small dimensionless parameter \( \epsilon \), namely

\[ \Phi_i = \sum_{n=0}^{\infty} \epsilon^n \Phi_i^{(n)}, \]  
(10)

\[ E_k(\Phi_i) = \sum_{n=0}^{\infty} \epsilon^n E_k^{(n)}, \quad \tilde{\mathcal{E}}_k(\Phi_i) = \sum_{n=0}^{\infty} \epsilon^n \tilde{\mathcal{E}}_k^{(n)}. \]  
(11)

Assuming that the fields \( \Phi_i^{(0)} \) satisfy the local background EoM, namely

\[ E_k^{(0)}(\Phi_i^{(0)}) = 0, \]  
(12)

2 For dimensional reasons, the operator \( \Delta \), being the argument of the entire function \( H \), has to be divided by a proper power of the mass scale \( \Lambda \). For example, in \( D = 4 \), if we are differentiating the Einstein’s EoM respect to the metric we should divide by \( \Lambda^4 \), if we differentiate the Einstein’s EoM respect to a scalar field or the matter EoM respect to the metric in both cases we should divide by \( \Lambda^3 \), finally, if we differentiate the matter EoM respect to the scalar field we have to divide by \( \Lambda^2 \).
it is extremely simple to prove the following theorem, which is a slight generalization of the theorems proved in \[21, 22\].

**Theorem.** In the NLGM theory, all perturbations (for gravity and matter) satisfy the same EOM of the perturbations in Einstein’s gravity coupled to matter, namely

\[ \delta^{(n)}(\Phi_i^{(n)}) = 0 \quad \Longrightarrow \quad E^{(n)}(\Phi_i^{(n)}) = 0 \quad \text{for } n > 0, \quad (13) \]

where the label “n” stays for the perturbative expansion of the tensors \( \delta \) and the EoM \( E_k \) at the order “n” in all the perturbations \( \Phi_i^{(n)} \).

The proof is a straightforward consequence of the EoM \[9\], which coincides with the Einstein’s \( E_k = 0 \) EoM in presence of matter up to operators \( O(E^2) \), and of the invertibility of the exponential form factor. The perturbative expansion that provides the details of the proof is identical to the one for pure gravity. Therefore, we remind the reader to \[22\], but a short proof is given in the appendix [D]. In particular, it deserves to be notice that in the perturbative expansion in \( \epsilon \) of equation (8) the exponential form factor always contributes at the zero order, namely \( e^0 \). Moreover, for the standard model of particle physics the Hessian resulting from the local action is diagonal (or constant) at the order \( e^0 \) (see [23]). Hence, the inversion (9) is actually trivial at any perturbative order.

### III. THE QUANTUM THEORY

Making use of the previous results, we show perturbative unitarity and finiteness of the theory \[1\] with form factor \[7\], and Hessian operator given in \[6\] and \[A1\].

In order to show the perturbative unitarity of the NLGM theory we do not need to compute the propagator because the EoM already tell us that we have only one pole in \( k^2 = -m_i^2 \) for each field, exactly like in the local theory (see appendix [D]). Therefore, the singularities of the amplitudes are obtained from the Landau’s equations as in the local case (this is due to the analytic structure of the form factor \( F_{ij} \) and the Cutkosky rules are the same of the local theory, too. Hence, we can export the outcome of \[7\] to the general theory presented in this paper, and the unitarity is guaranteed at any perturbative order in the loop expansion. We remind that the theory has to be defined in Euclidean space and the physical amplitudes are afterwards obtained employing the analytic continuation from complex to real external energies. Last comment is about the non-diagonal elements of the operator \( \Delta \). Indeed, one can easily see that such components are at least linear in the fields [23] and, therefore, can not affect the propagators around the Minkowski spacetime.

We can finally address the issue of the quantum divergences. The differential operator \( \Delta \) in \[A1\] is a second order differential operator, hence, it is always possible to choose a form factor exp \( H(\Delta \lambda) \), asymptotically polynomial [3, 5], to cancel all the divergences from two loops onwards. However, in even dimension we still have one loop divergences that can likely be removed by adding other operators to the action [1] provided they are at least cubic in the Einstein’s equations of motion \( E_k \). For the sake of simplicity we can consider the NLGM theory in odd dimension where we do not have one-loop divergences in dimensional regularization scheme. Therefore, the theory proposed in this paper is surely finite in odd dimensions and super-renormalizable in even dimensions.

### IV. CONCLUSIONS

We have explicitly constructed a nonlocal theory for all fundamental interactions, including gravity, that at classical level has the same solutions and the same stability properties of the local Einstein’s theory in presence of matter. Moreover, the theory reproduces all and only the same tree-level scattering amplitudes of the local standard model of particle physics in presence of gravity, securing that there is no causality violation [10, 25]. At quantum level, the theory is unitary and surely finite in odd dimension, while in even dimension there are only one-loop divergences that can be removed adding few more local operators on the footprint of what has been done for pure gravity without and with the cosmological constant [5, 20].

Thus, this paper lays strong foundations for an ultraviolet completion of the standard model of particle physics and gravity.

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**Appendix A: Equations of Motion**

In this section we derive the EoM for a general nonlocal theory providing all the details of the calculation. We will make use of the following definition,

\[
\Delta_{ki}(y, x) = \frac{\delta E_i(x)}{\delta \Phi_k(y)} = \frac{\delta^2 S}{\delta \Phi_k(y) \delta \Phi_i(x)} = \Delta_{ki}(x) \frac{\delta^{D}(x - y)}{\sqrt{-g(y)}},
\]

(A1)
and we will use the functional derivative consistent with the Dirac delta distribution in curved spacetime,
\[
\frac{\delta \Phi_i(x)}{\delta \Phi_j(y)} = \frac{\delta^D(x - y)}{\sqrt{-g(y)}} \delta_{ij}.
\]

(A2)

The variation of the action, introducing the short notation for the integral measure \( \int d\mu(x) \equiv \int d^D x \sqrt{-g(x)} \) or simply \( \int_x \), reads:
\[
\delta S = \int d\mu(x) \left[ \delta \Phi_i E_i + \delta E_i F_{ij} E_j + E_i F_{ij} \delta E_j + O(E^2) \right],
\]
\[
= \int d\mu(x) \left[ \delta \Phi_i(x) E_i(x) + \int d\mu(y) \left( \frac{\delta E_i(x)}{\delta \Phi_k(y)} F_{ij}(x) E_j(x) + E_i(x) F_{ij}(x) \frac{\delta E_j(x)}{\delta \Phi_k(y)} \delta \Phi_k(y) + O(E^2) \right) \right]
\]
\[
= \int d\mu(x) \left[ \delta \Phi_k(x) E_k(x) + \int d\mu(y) \left( \frac{\delta E_i(x)}{\delta \Phi_k(y)} F_{ij}(x) E_j(x) + E_i(x) F_{ij}(x) \frac{\delta E_j(x)}{\delta \Phi_k(y)} \delta \Phi_k(y) + O(E^2) \right) \right],
\]

(A3)

where in the last term we just changed name to the indexes in order to have the same \( \Delta \) operator (see (A1)) of the last by one term. Let us now introduce the following definition,
\[
v_i(x) = \int d^D y \sqrt{-g(y)} \Delta_{kl}(y, x) \delta \Phi_k(y) \equiv \int d\mu(y) \Delta_{kl}(y, x) \delta \Phi_k(y).
\]

(A4)

Therefore, replacing the definitions (A1) and (A4) in (A3) we get:
\[
\delta S = \int d\mu(x) \left[ \delta \Phi_k(x) E_k(x) + \int d\mu(y) \left( \Delta_{kl}(y, x) \delta \Phi_k(y) F_{ij}(x) E_j(x) + E_i(x) F_{ij}(x) \Delta_{kl}(y, x) \delta \Phi_k(y) + O(E^2) \right) \right]
\]
\[
= \int d\mu(x) \left[ \delta \Phi_k(x) E_k(x) + \int d\mu(y) \left( v_i(x) F_{ij}(x, y) E_j(y) + E_j(x) F_{ij}(x, y) v_i(y) \right) + O(E^2) \right].
\]

(A5)

In the last term we now change name to the integration variables, namely \( x \to y \) and \( y \to x \), hence
\[
\delta S = \int d\mu(x) \delta \Phi_k(x) E_k(x) + \int d\mu(x) \int d\mu(y) \left( v_i(x) F_{ij}(x, y) E_j(y) + E_j(x) F_{ij}(x, y) v_i(x) \right) + O(E^2).
\]

(A6)

Making use of the following integrated symmetric property of the Hessian \( \Delta \) (see the appendix [C]), namely
\[
\int d^D x \sqrt{-g(x)} \int d^D y \sqrt{-g(y)} A_i(x) \Delta_{ij}(x, y) B_j(y) = \int d^D y \sqrt{-g(y)} B_j(y) \Delta_{ij}(x, y) A_i(x),
\]
(A7)

the variation turns into:
\[
\delta S = \int d\mu(x) \delta \Phi_k(x) E_k(x) + \int d\mu(x) \int d\mu(y) \left( v_i(x) F_{ij}(x, y) E_j(y) + v_i(x) F_{ij}(x, y) E_j(y) \right) + O(E^2)
\]
\[
= \int d\mu(x) \delta \Phi_k(x) E_k(x) + \int d\mu(x) \int d\mu(y) 2 v_i(x) F_{ij}(x, y) E_j(y) + O(E^2)
\]
\[
= \int d\mu(x) \left[ \delta \Phi_k(x) E_k(x) + \int d^D y \sqrt{-g(y)} \delta \Phi_k(x) \Delta_{kl}(y, x) \right] \int d^D z \sqrt{-g(z)} 2 F_{ij}(x, z) E_j(z) + O(E^2).
\]

(A8)

Notice that the operators
\[
v_i(x), \quad \Delta_{ij}(x, y), \quad F_{ij}(x, y), \quad E_j(y),
\]
(A9)
can be freely interchanged because each of them is in a closed integral form, namely they are not differential operators acting on their right or left side but they are actually integrated quantities in which differential operators, if any, act on Dirac’s delta distributions. Therefore, in a more compact and implicit notation:
\[
\delta S = \int d^D x \sqrt{-g(x)} \left[ \delta \Phi_k(x) E_k(x) + \delta \Phi_k(x) 2 \Delta_{kl}(x) F_{ij}(x) E_j(x) + O(E^2) \right]
\]
\[
= \int d^D x \sqrt{-g(x)} \delta \Phi_k(x) \left[ E_k(x) + 2 \Delta_{kl}(x) F_{ij}(x) E_j(x) + O(E^2) \right],
\]

(A10)
and:

\[ E_i(y) = \frac{\delta S}{\delta \Phi_i(y)} = \int d^D x \sqrt{-g(x)} \frac{\delta \Phi_k(x)}{\delta \Phi_i(y)} \left[ E_k(x) + 2 \Delta_{ki}(x) F_{ij}(x) E_j(x) + O(E^2) \right] \]

\[ = E_l(x) + 2 \Delta_{li}(x) F_{ij}(x) E_j(x) + O(E^2) \],

(A11)

where we used (A2). Finally, the EoM for the nonlocal theory read:

\[ E_k = E_k + 2 \Delta_{ki} F_{ij} E_j + O(E^2) = 0, \]

(A12)

and, making again explicit the dependence on spacetime points, the EoM (A12) should be written

\[ E_k(x) = E_k(x) + \int d\mu(y) \int d\mu(z) 2 \Delta_{ki}(x,y) F_{ij}(y,z) E_j(z) + O(E^2) = 0. \]

(A13)

In order to further simplify the above equation of motion (A13) and rid out of the instabilities, we now expand on the operator \( F_{ij} \) defined in (7). The analytic form factor \( F_{ij} \) is given as a solution of the following equation,

\[ 2 \Delta_{ik} F_{kj}(\Delta) = 2 F_{ik}(\Delta) \Delta_{kj} = (e^H)_{ij} - 1_{ij}. \]

(A14)

Indeed, if we define

\[ F_{ij}(\Delta) = \sum_{n=0}^{+\infty} a_n(\Delta^n)_{ij} \quad \text{and} \quad (e^H(\Delta))_{ij} = \sum_{n=0}^{+\infty} b_n(\Delta^n)_{ij}, \]

(A15)

by replacing (A15) in (A14) we get:

\[ 2 \Delta \sum_{n=0}^{+\infty} a_n\Delta^n = \sum_{n=0}^{+\infty} b_n\Delta^n - 1 \]

\[ 2 \Delta \sum_{n=0}^{+\infty} a_n\Delta^n = b_0 + \sum_{n=1}^{+\infty} b_n\Delta^n - 1 \quad \text{and assuming} \quad b_0 = 1 \quad \text{or} \quad H(0) = 0, \]

\[ 2 \Delta \sum_{n=0}^{+\infty} a_n\Delta^n = \sum_{n=1}^{+\infty} b_n\Delta^n \]

\[ 2 \Delta \sum_{n=0}^{+\infty} a_n\Delta^n = \Delta \sum_{n=1}^{+\infty} b_n\Delta^{n-1} \quad (n - 1 = k) \]

\[ 2 \Delta \sum_{n=0}^{+\infty} a_n\Delta^n = \Delta \sum_{k=0}^{+\infty} b_{k+1}\Delta^k. \]

(A16)

Between the third last and the last but one step (A16) we do not need to define the inverse of \( \Delta \) because in the sum on the right side we have \( n > 0 \). Therefore, comparing the left and right side of the last equality in (A16) we figure out the relation between the coefficients \( a + n \) and \( b_n \), namely

\[ a_n = \frac{b_{n+1}}{2}. \]

(A17)

Replacing (A14) in (A13),

\[ E_k(x) = E_k(x) + \int d\mu(z) (e^H(\Delta) - 1)_{kj}(x,z) E_j(z) + O(E^2) = 0. \]

(A18)

where the functional identity in (A18) is defined by

\[ 1_{kj}(x,z) = \delta_{kj} \frac{\delta(x-z)}{\sqrt{-g(z)}}, \]

(A19)
Therefore, (A18) turns into:

\[ E_k(x) = E_k(x) + \int d\mu(z) \left( e^{H(\Delta)} \right)_{kj}(x, z) \delta_k \frac{\delta(x - z)}{\sqrt{-g(z)}} E_j(z) + O(E^2) = 0 \]

\[ E_k(x) = E_k(x) + \int d\mu(z) \left( e^{H(\Delta)} \right)_{kj}(x, z) E_j(z) - \int d\mu(z) \delta_k \frac{\delta(x - z)}{\sqrt{-g(z)}} E_j(z) + O(E^2) = 0 \]

\[ E_k(x) = E_k(x) + \int d\mu(z) \left( e^{H(\Delta)} \right)_{kj}(x, z) E_j(z) - E_k(x) + O(E^2) = 0 \]

\[ E_k(x) = \int d\mu(z) \left( e^{H(\Delta)} \right)_{kj}(x, z) E_j(z) + O(E^2) = 0 \]  \hspace{1cm} (A20)

Now using the second identity in (A1) we end up with:

\[ E_k(x) = \int d\mu(z) \left( e^{H(\Delta)} \right)_{kj} \frac{\delta(x, z)}{\sqrt{-g(z)}} E_j(z) + O(E^2) = 0 \]

\[ E_k(x) = \left( e^{H(\Delta_x)} \right)_{kj} E_j(x) + O(E^2) = 0 . \]  \hspace{1cm} (A21)

The equations (7), (A15), and (A17) allow us to avoid to invert the \( \Delta \) operator. Indeed, it deserves to be notice that the definition of \( \Delta^{-1} \) is extremely delicate. The Hessian \( \Delta \) is usually not invertible because of gauge invariance\(^3\) and one usually adds a gauge fixing term to the (local) action in order to get the inverse. If we define

\[ F_{ij} \equiv \left( \frac{e^{H(\Delta_x)} - 1}{2\Delta} \right)_{ij} , \]  \hspace{1cm} (A22)

instead of (7) or (A14), then \( \Delta^{-1} \) will be part of the definition of the theory and one might worry about an explicit break of the gauge invariance of the theory because of the gauge fixing term. However, it will not be the case because \( \Delta^{-1} \) would appear in the intermediate steps of our derivation, but not in the final action and in the EoM. Indeed, the function \( F_{ij} \) is analytic in \( \Delta \) and the presence of \( \Delta^{-1} \) in (A22) is just formal. If we wanted to be mathematically rigorous we should add a gauge fixing term to (6), namely

\[ \Delta \rightarrow \Delta + H_{GF} , \]  \hspace{1cm} (A23)

which is invertible. However, in the EoM and in the action \( \Delta^{-1} \) would disappear because it is always multiplied by \( \Delta \). Therefore, we can safely take the limit of zero gauge fixing parameters to finally recover gauge invariance for the action and the EoM.

### Appendix B: Propagators

In order to implement the recipe developed in the main text, we here consider two explicit examples: pure gravity and a general scalar theory. In particular, we will derive the tree-level propagator in both cases.

1. **Graviton propagator**

As a first example we consider the purely gravitational theory for which the only non zero component of the \( \Delta \) operator is \( \Delta_{11} \), but in the flat spacetime background \( g_{\mu\nu} = \eta_{\mu\nu} \) and \( E_{\mu\nu} = 0 \) (Einstein’s EoM), hence

\[^3\] Also the operator \( \Box \) is not invertible in flat space because of the zero mode/s, but not because of gauge invariance.
\[
\Delta_{\mu\nu,\alpha\beta}(y, x) = \frac{\delta^2 S_\ell}{\delta g^{\mu\nu}(y) \delta g^{\alpha\beta}(x)} = \frac{2}{\kappa^2} \left[ \frac{\delta^2 \left( \int d^D z \sqrt{-g(z)} L_g(z) \right)}{\delta g^{\mu\nu}(y) \delta g^{\alpha\beta}(x)} \bigg|_{g=\eta} \right] = 2 \frac{\delta}{\delta g^{\mu\nu}(y)} \left( \frac{\delta \left( \int d^D z \sqrt{-g(z)} L_g(z) \right)}{\delta g^{\alpha\beta}(x)} \right) \bigg|_{g=\eta} = 2 \frac{\delta}{\delta g^{\mu\nu}(y)} \left( \frac{\delta R_{\alpha\beta}(x)}{\delta g^{\mu\nu}(y)} - \frac{1}{2} g_{\alpha\beta}(x) g^{\gamma\delta}(x) \frac{\delta R_{\gamma\delta}(x)}{\delta g^{\mu\nu}(y)} \right) = \frac{2}{\kappa^2} \left[ \frac{1}{2} \left( \eta_{\alpha\mu} \eta_{\beta\nu} + \eta_{\beta\mu} \eta_{\alpha\nu} \right) \Box x - \eta_{\mu\nu} \eta_{\alpha\beta} \Box x + \ldots \right] \delta^D(x-y) = \frac{2}{\kappa^2} \left[ (P^{(2)}(x) - (D-2)P^{(0)}(x))_{\mu\nu,\alpha\beta} \Box x \delta^D(x-y) \right]
\]

where the “dots” stay for other second order derivative terms. The first three steps in (B1) are general for any background, while from the fourth step we restricted to the case of the Minkowski spacetime. Furthermore, \(P^{(2)}, P^{(0)}\) are the spin-projectors in \(D\)-dimensions whose definitions read [15] [24]:

\[
P^{(2)}_{\mu\nu,\rho\sigma}(x) = \frac{1}{2} (\theta_{\mu\sigma} \theta_{\nu\rho} + \theta_{\mu\rho} \theta_{\nu\sigma}) - \frac{1}{D-1} \theta_{\mu\nu} \theta_{\rho\sigma},
\]

\[
P^{(0)}_{\mu\nu,\rho\sigma}(x) = \frac{1}{D-1} \theta_{\mu\nu} \theta_{\rho\sigma},
\]

\[
\theta_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu \partial_\nu, \quad \omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu - \eta_{\mu\nu}}{\Box}.
\]

Notice that the last by one equality in (B1) is exact because the projectors reconstruct also the terms shortly indicated with dots.

Finally, the \(\Delta\)-operator for the purely gravitational theory reads as follows,

\[
\Delta(x, y) = \frac{1}{\kappa^2} \left[ P^{(2)}(x) - (D-2)P^{(0)}(x) \right] \Box x \delta^D(y-x),
\]

where we did not displayed the four spacetime indexes.

In (B1) we used the following functional derivatives [11],

\[
\frac{\delta R_{\alpha\beta}(x)}{\delta g^{\mu\nu}(y)} = \left[ \frac{1}{4} \left( g_{\alpha\mu} g_{\beta\nu} + g_{\beta\mu} g_{\alpha\nu} \right) \Box x + \frac{1}{2} g_{\mu\nu} \nabla_\alpha \nabla_\beta - \frac{1}{2} \left( g_{\alpha\mu} \nabla_\beta \nabla_\nu + g_{\alpha\nu} \nabla_\beta \nabla_\mu \right) \right] \frac{\delta^D(x-y)}{\sqrt{-g(y)}}; \quad g^{\alpha\beta}(x) \frac{\delta R_{\alpha\beta}(x)}{\delta g^{\mu\nu}(y)} = \left[ g_{\mu\nu} - \frac{1}{2} \left( \nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu \right) \right] \frac{\delta^D(x-y)}{\sqrt{-g(y)}}.
\]

We can now expand the purely gravitational theory at the second order in the perturbation \(h_{\mu\nu}\) in the Minkowski background, which satisfy the EoM \(E_{\mu\nu} = 0\), namely

\[
S[g] = S[\bar{g} + h] = S[\bar{g}] + \frac{1}{2} \int d\mu(x_1) d\mu(x_2) h^{\mu\nu}(x_1) \frac{\delta^2 S[\bar{g}]}{\delta h^{\rho\sigma}(x_1) \delta h^{\rho\sigma}(x_2)} h^{\rho\sigma}(x_2) + O(h^3), \quad h^{\alpha\beta} \equiv \delta g^{\alpha\beta}.
\]
where the second order term in the above expansion of the action reads:

\[ S_h^{(2)} = \int d^Dx \left[ -\frac{2}{\kappa^2} \frac{1}{\sqrt{-g}} R + \sqrt{-g} E_{\alpha\beta} F(\Delta_{\lambda})^{\alpha\beta,\mu\nu} E_{\mu\nu} \right] \]

\[ = \frac{1}{2} \int d\mu_y \int d\mu_x h^{\alpha\beta}(y) \left( \frac{\delta^2 S}{\delta h^{\alpha\beta}(y) \delta h^{\mu\nu}(x)} \right) h^{\mu\nu}(x) \]

\[ = \frac{1}{2} \int d\mu_y \int d\mu_x h^{\alpha\beta}(y) \left( \frac{\delta^2 S_{\ell}}{\delta h^{\alpha\beta}(y) \delta h^{\mu\nu}(x)} \right) h^{\mu\nu}(x) \]

\[ + \frac{1}{2} \int d\mu_y \int d\mu_x h^{\gamma\delta}(y) \left[ \frac{\delta}{\delta h^{\gamma\delta}(y)} \frac{\delta}{\delta h^{\rho\sigma}(x)} \left( \int d\mu_{\alpha\beta} E_{\alpha\beta}(z) F(\Delta_{\lambda}(z))^{\alpha\beta,\mu\nu} E_{\mu\nu}(z) \right) \right] h^{\rho\sigma}(x) \]

\[ = \frac{1}{2} \int \int h^{\alpha\beta}(y) \left( \frac{\delta^2 S_{\ell}}{\delta h^{\alpha\beta}(y) \delta h^{\mu\nu}(x)} \right) h^{\mu\nu}(x) + 2 h^{\gamma\delta}(y) \left( \int d\mu_{\alpha\beta} E_{\alpha\beta}(z) F(\Delta_{\lambda}(z))^{\alpha\beta,\mu\nu} \frac{\delta E_{\mu\nu}(x)}{\delta h^{\rho\sigma}(y)} h^{\rho\sigma}(x) \right) \]

The overall 1/2 factor is due to the functional expansion at the second order, while the multiplicative factor 2 in the second term at the forth step comes from symmetric property of \( \Delta \), namely \( \Delta(x, z)_{\rho\sigma,\mu\nu} = \Delta(z, x)_{\mu\nu,\rho\sigma} \). Using again the latter property,

\[ S_h^{(2)} = \frac{1}{2} \int d\mu_x \int d\mu_y \left[ h^{\alpha\beta}(y) \frac{\delta E_{\alpha\beta}(x)}{\delta h^{\alpha\beta}(y)} h^{\mu\nu}(x) + h^{\gamma\delta}(y) \int d\mu_{w} \int d\mu_{z} \frac{\delta E_{\alpha\beta}(w)}{\delta h^{\gamma\delta}(y)} 2F(w, z)^{\alpha\beta,\mu\nu} \Delta(x, z)_{\mu\nu,\rho\sigma} h^{\rho\sigma}(x) \right] \]

Changing the name of the indexes in the second quadratic term in \( h_{\mu\nu} \), we get:

\[ S_h^{(2)} = \frac{1}{2} \int d\mu_y \int d\mu_x \left[ h^{\alpha\beta}(y) \frac{\delta E_{\alpha\beta}(x)}{\delta h^{\alpha\beta}(y)} h^{\mu\nu}(x) + h^{\gamma\delta}(y) \left( \int d\mu_{w} \int d\mu_{z} \frac{\delta E_{\alpha\beta}(w)}{\delta h^{\gamma\delta}(y)} 2F(w, z)^{\alpha\beta,\mu\nu} \Delta(x, z)_{\mu\nu,\rho\sigma} h^{\rho\sigma}(x) \right) \right] \]

Now we replace the product in the box, i.e. \( 2F(\Delta)\Delta \), with \( [7] \),

\[ S_h^{(2)} = \frac{1}{2} \int dDx \left[ h^{\alpha\beta}(y) \frac{\delta E_{\alpha\beta}(x)}{\delta h^{\alpha\beta}(y)} e^{H(\Delta)} h^{\mu\nu}(x) + h^{\gamma\delta}(y) \left( \int d\mu_{w} \int d\mu_{z} \frac{\delta E_{\alpha\beta}(w)}{\delta h^{\gamma\delta}(y)} e^{H(\Delta)}(w, x)^{\alpha\beta,\rho\sigma} - \frac{\delta (w, x)^{\alpha\beta,\rho\sigma}}{\sqrt{-g}(x)} \right) h^{\rho\sigma}(x) \right] \]

In the last three steps we replaced \( \sqrt{-g} = 1 \) because we are expanding around the Minkowski background. In the last step of \( [B6] \) we used the result \( [B7] \). Finally, replacing the argument of the form factor \( \exp H(\Delta_{\lambda}) \) in \( [B6] \) with
we get

\[ S^{(2)} = \frac{1}{2\kappa^2} \int d^D x \left\{ h^{\alpha\beta} \left[ \left( P^{(2)} - (D - 2) P^{(0)} \right) \Box \right] \gamma_{\mu\nu} \right\} \]

\[ = \frac{1}{2\kappa^2} \int d^D x \left\{ h^{\alpha\beta} \left[ \left( P^{(2)} e^{H \left( \frac{2}{\kappa^2} \right)} - (D - 2) P^{(0)} e^{H \left( \frac{-(D - 2)}{\kappa^2} \right)} \right) \Box \right] \gamma_{\mu\nu} \right\} \equiv \frac{1}{2} \int d^D x h^{\alpha\beta} \mathcal{O}_{\alpha\beta,\mu\nu} h^{\mu\nu}, \tag{B7} \]

from which after introducing the gauge fixing it is obtained the following gauge independent part of the graviton propagator \[ \mathcal{O}^{-1} = \kappa^2 \left( \frac{P^{(2)}}{\Box e^{H \left( \frac{2}{\kappa^2} \right)}} - \frac{P^{(0)}}{\Box (D - 2) e^{H \left( \frac{-(D - 2)}{\kappa^2} \right)}} \right). \tag{B8} \]

The tree-level unitarity is guaranteed whether the asymptotically polynomial entire function \( H(z) \) is such that \( H(z) = 0 \). Moreover, \( H(z) = H(-z) \) in order to ensure the super-renormalizability of the theory.

2. Free and Interacting Scalar Fields

For a free scalar field, the nonlocal Lagrangian, the local EoM \( E_m \), and the \( \Delta_{22} \) operator are:

\[ \mathcal{L}_m = \frac{1}{2} \phi \Box \phi + E_m F(\Delta)_{22} E_m, \]

\[ E_m = \Box \phi, \tag{B9} \]

\[ \Delta_{22} = \delta E_m / \delta \phi = \Box. \tag{B10} \]

Replacing \( E_m \) and \( \Delta_{22} \) in \( \mathcal{L}_m \) we end up with the following nonlocal Lagrangian,

\[ \mathcal{L}_m = \frac{1}{2} \phi \Box \phi + \left( \Box \phi \right) e^{H \left( \frac{2}{\kappa^2} \right)} - \frac{1}{2 \Box} \left( \Box \phi \right) = \frac{1}{2} \phi \Box e^{H \left( \frac{2}{\kappa^2} \right)} \phi. \tag{B11} \]

Therefore, the propagator is proportional to:

\[ e^{-H \left( \frac{2}{\kappa^2} \right)} / \Box. \tag{B12} \]

For an interacting scalar field, whose local Lagrangian and EoM read

\[ \mathcal{L}_m^{(loc)} = \frac{1}{2} \phi \Box \phi - V(\phi), \tag{B13} \]

\[ E_m = \Box \phi - V'(\phi), \tag{B14} \]

the nonlocal theory is:

\[ \mathcal{L}_m = \frac{1}{2} \phi \Box \phi - V(\phi) + \left( \Box \phi - V'(\phi) \right) F(\Delta)_{22} \left( \Box \phi - V'(\phi) \right), \]

\[ \Delta_{22} = \Box - V''(\phi), \]

\[ F(\Delta^{(2)}_{\lambda})_{22} = \frac{e^{H \left( \frac{2}{\kappa^2} \right) - V''(\phi)} - 1}{2 \left( \Box - V''(\phi) \right)}. \tag{B15} \]

If we switch off the interaction, the Lagrangian \[ \mathcal{B}15 \] turns into \[ \mathcal{B}11 \].
Appendix C: Symmetry properties of the Hessian

In this section we explicitly prove that the Hessian operator $\Delta$ is symmetric, namely
\[
\int dx \int dy \, A_i(x) \Delta_{ij}(x,y) B_j(y) = \int dx \int dy \, B_j(y) \Delta_{ij}(y,x) A_i(x) .
\] (C1)

In order to prove the above statement is sufficient to consider the following quite general action operator,
\[
S = \int dx \, \Phi_1(x) (\partial^2 \Phi_2(x)) \Phi_3(x) ,
\] (C2)

where $\Phi_1(x), \Phi_2(x), \Phi_3(x)$ are three general fields. For the sake of simplicity we assume $\Phi_3(x)$ to be an external classical field and we compute the components of the Hessian’s operator, which is a $2 \times 2$ matrix in the space of the fields $\Phi_1(x)$ and $\Phi_2(x)$,
\[
\frac{\delta S}{\delta \Phi_1(y)} = \int dx \, \delta(x-y) (\partial^n_2 \Phi_2(x)) \Phi_3(x) ,
\] (C3)
\[
\Delta_{21}(z,y) = \frac{\delta^2 S}{\delta \Phi_2(z) \delta \Phi_1(y)} = (-1)^n \int dx \, \delta(x-z) \partial^n_2 \left( \delta(x-y) \Phi_3(x) \right) = (-1)^n \partial^n_2 \left( \delta(z-y) \Phi_3(z) \right) ,
\] (C4)
\[
\frac{\delta S}{\delta \Phi_2(y)} = (-1)^n \int dx \, \delta(x-y) \partial^n_2 \left( \Phi_1(x) \Phi_3(x) \right) ,
\] (C5)
\[
\Delta_{12}(z,y) = \frac{\delta^2 S}{\delta \Phi_1(z) \delta \Phi_2(y)} = (-1)^n (-1)^n \int dx \, \delta(x-z) \Phi_3(x) \partial^n_2 \left( \delta(x-y) \right) = \Phi_3(z) \partial^n_2 \left( \delta(z-y) \right) .
\] (C6)

In deriving the components of the Hessian we integrating by parts several times. Moreover, the Hessian operator has zero diagonal elements because the action (C2) is linear in all fields.

Given two general fields $A(z)$ and $B(y)$, we now prove the following identity,
\[
\int dz \int dy \, A(z) \Delta_{21}(z,y) B(y) = \int dz \int dy \, B(y) \Delta_{12}(y,z) A(z) .
\] (C7)

Replacing (C4) in the left hand side of (C7) we find:
\[
\int dz \int dy \, A(z) \Delta_{21}(z,y) B(y) = \int dz \int dy \, A(z) \left[ (-1)^n \partial^n_2 \left( \delta(z-y) \Phi_3(z) \right) \right] B(y)
\]
\[
= (-1)^n \int dz \int dy \, ( -1)^n \left( \partial^n_2 A(z) \right) \delta(z-y) \Phi_3(z) B(y)
\]
\[
= \int dy \, \left( \partial^n_2 A(y) \right) \Phi_3(y) B(y)
\]
\[
= \int dy \, ( -1)^n A(y) \partial^n_2 \left( \Phi_3(y) B(y) \right) .
\] (C8)

Similarly, we replace (C6) in the right hand side of (C7),
\[
\int dz \int dy \, B(y) \Delta_{12}(y,z) A(z) = \int dz \int dy \, B(y) \left[ \Phi_3(y) \partial^n_2 \left( \delta(y-z) \right) \right] A(z)
\]
\[
= \int dz \int dy \, ( -1)^n \partial^n_2 \left( B(y) \Phi_3(y) \right) \delta(y-z) A(z)
\]
\[
= \int dy \, ( -1)^n A(y) \partial^n_2 \left( \Phi_3(y) B(y) \right) = (C8) .
\] (C9)

Hence, we have proved (C7).

We can also swap the fields $A(z)$ and $B(y)$ in (C9) and the result does not change,
\[
\int dz \int dy \, A(z) \Delta_{12}(y,z) B(y) = \int dz \int dy \, A(z) \left[ \Phi_3(y) \partial^n_2 \left( \delta(y-z) \right) \right] B(y)
\]
\[
= \int dz \int dy \, ( -1)^n \partial^n_2 \left( \Phi_3(y) B(y) \right) \delta(y-z) A(z)
\]
\[
= \int dy \, ( -1)^n A(y) \partial^n_2 \left( \Phi_3(y) B(y) \right) = (C8) .
\] (C10)

which also proves the statement in the text below formula (A9).
Appendix D: Proof of the theorem in the main text

In order to prove the theorem about the linear and nonlinear stability of the nonlocal theory \([11]\), we have to expand perturbatively in \(\epsilon\) (see \([11]\)) the EoM \([8]\),

\[
\mathcal{E} = e^{H(\Delta_\Lambda)} E + O(E^2) = 0 .
\]  

(D1)

Since we want to study the stability of exact solutions of the Einstein’s theory coupled to matter, we assume to expand around a metric consistent with \(E^{(0)} = 0\) \([12]\) (for the sake of simplicity we use the bold notation in place of the latin indexes for the fields).

Hence, at the zero order in \(\epsilon\), i.e. \(\epsilon^0\), we have:

\[
e^{H(0)(\Delta_\Lambda)} E^{(0)} + O(E^{(0)2}) = 0 ,
\]  

(D2)

which is satisfied because by hypothesis \(E^{(0)} = 0\).

At the first order \(\epsilon^1\), we get:

\[
e^{H(1)(\Delta_\Lambda)} E^{(0)} + e^{H(0)(\Delta_\Lambda)} E^{(1)} + O(E^{(0)}E^{(1)}) = 0 \implies E^{(1)} = 0 ,
\]  

(D3)

where we used \(E^{(0)} = 0\).

At the second order \(\epsilon^2\), we get:

\[
e^{H(2)(\Delta_\Lambda)} E^{(0)} + e^{H(1)(\Delta_\Lambda)} E^{(1)} + e^{H(0)(\Delta_\Lambda)} E^{(2)} + O(E^{(1)}E^{(1)}) + O(E^{(2)}E^{(0)}) = 0 \implies E^{(2)} = 0 ,
\]  

(D4)

where we used \(E^{(0)} = 0\) and \(E^{(1)} = 0\).

Finally, at the order \(\epsilon^n\),

\[
e^{H(n)(\Delta_\Lambda)} E^{(0)} + e^{H(n-1)(\Delta_\Lambda)} E^{(1)} + e^{H(n-2)(\Delta_\Lambda)} E^{(2)} + \ldots + e^{H(0)(\Delta_\Lambda)} E^{(n)} +
\]

\[+ O(E^{(n)}E^{(0)}) + O(E^{(n-1)}E^{(1)}) + \ldots + O(E^{(1)}E^{(n-1)}) + O(E^{(0)}E^{(n)}) = 0 \implies E^{(n)} = 0 ,
\]  

(D5)

where we used: \(E^{(0)} = 0, E^{(1)} = 0, \ldots, E^{(n-1)} = 0\).

Therefore,

\[
\mathcal{E}^{(n)} = 0 \implies E^{(n)} = 0 .
\]  

(D6)

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