Low-scale minimal linear seesaw model
for neutrino mass and flavor mixing

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We consider an extension of the Standard Model with three right-handed (RH) neutrinos and a Dirac pair of extra sterile neutrinos, odd under a discrete $Z_2$ symmetry, in order to have left–right symmetry in the neutrino content and obtain tiny neutrino masses from the latter ones only. Our working hypothesis is that the heavy RH neutrinos do not influence phenomenology at low energies. We use the usual high-scale seesaw to suppress all of the mass terms involving RH neutrinos and a low-scale minimal variant of the linear seesaw led by the Dirac mass of the extra sterile neutrinos to provide the small mass of active neutrinos. One of the active neutrinos is massless, which fixes the mass of the other two on the basis of a soft breaking of the $Z_2$ symmetry. The mixing between the extra neutrinos makes for a particle that effectively behaves like a Dirac sterile neutrino with mass around the GeV level.

Keywords: Beyond Standard Model; neutrino mass; minimal seesaw; linear seesaw; heavy right-handed neutrino.

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1. Introduction

The finding of neutrino masses and their flavor mixing together with the existence of dark matter (DM) are major experimental facts that ask for extensions of the Standard Model (SM). The addition of right-handed (RH) neutrinos, singlets under the gauge symmetry of the SM, seems to be the simplest input to bring about significant results. They can be combined with the left-handed (LH) neutrinos to generate Dirac mass terms ($m_D$) via Yukawa couplings with the Higgs doublet, just like for charged leptons and quarks.

A minimalist approach is to introduce one RH neutrino per generation. However, this requires extremely tiny Yukawa couplings to explain the observed masses, which suggests the existence of a new fundamental mass scale. This appears naturally in the high-scale Type-I seesaw mechanism for neutrino mass generation $^{1-5}$ where the RH neutrinos are assumed to possess heavy Majorana masses ($m_R$). In this
scenario, the neutrino mass matrix in the basis $(\nu_L, \nu_R)$ is given by

$$
\mathbf{M}_\nu = \begin{pmatrix}
0 & m_D \\
m_D^T & m_R
\end{pmatrix}.
$$

(1)

Here, $\nu_R = C \nu_R^T$, using the notation of Ref. 6. The physical states after diagonalization of the mass matrix include light ($\nu$) and heavy ($\nu_H$) sectors of Majorana neutrinos with masses

$$
m_\nu \simeq -m_D m_R^{-1} m_D^T, \quad m_{\nu_H} \simeq m_R \gg m_\nu.
$$

(2)

From the point of view of model building, a natural expectation is a Dirac neutrino mass $m_D$ similar in size to the Dirac mass of the charged lepton, i.e.

$$
m_D \approx m_\tau \sim 1 \text{ GeV}
$$

(3)

for the third generation of neutrinos, which leads to $m_\nu \sim 10^{-2}$ eV for $m_R \sim 10^{12}$ GeV. In this scheme, the mixing between the active neutrinos and the heavy RH neutrinos is of order $m_D/m_R \simeq \sqrt{m_\nu/m_{\nu_H}} \ll 1$, so that any experimental observation of flavor violation involving charged leptons would imply the existence of new physics at a much lower scale. Models with low Majorana masses for the RH neutrinos were then proposed, in which the lightest Majorana RH neutrino plays the role of a keV DM candidate. 7,8

Well-motivated variants of these schemes include the inverse seesaw,9–11 the linear seesaw,12–14 and the extended seesaw,15,16 which are usually known as low-scale seesaw mechanisms. These scenarios introduce an additional singlet neutrino per generation, such that in the new basis $(\nu_L, \nu_R, S_L)$ the full mass matrix becomes

$$
\mathbf{M}_\nu = \begin{pmatrix}
0 & m_D & \mu'_L \\
m_D^T & 0 & M_D \\
\mu'^T_L & M_D^T & \mu_L
\end{pmatrix},
$$

(4)

where $\mu'_L$ and $M_D$ are the mass terms induced by the couplings of the new fermion to the LH and RH neutrinos, respectively, while $\mu_L$ is the new Majorana mass.

After block matrix diagonalization, the active neutrino mass has the form

$$
m_\nu \simeq m_D M_D^{-1} \mu_L M_D^{-1} m_D^T
$$

(5)

in the inverse seesaw case, with $\mu'_L = 0$. Compared to Eq. (2), there is an extra suppression with respect to the classical seesaw given by $m_\nu \simeq (m_D^2/M_D)(\mu_L/M_D)$, which leads to $M_D \sim 1$ TeV for $m_\nu \sim 10^{-2}$ eV, $m_D$ as in Eq. (3), and $\mu_L \sim 10$ keV, attributed to the smallness of lepton number violation.

In the linear seesaw mechanism, with $\mu_L = 0$, one has instead

$$
m_\nu \simeq -m_D (\mu'_L M_D^{-1})^T - \mu'_L M_D^{-1} m_D^T.
$$

(6)

Note here that there is an extra suppression with respect to the classical seesaw according to $m_\nu \simeq (m_D^2/M_D)(2\mu'_L/m_D)$. Now, for a Dirac neutrino mass in accord with Eq. (3), and $\mu'_L \sim 10$ keV also attributed to the smallness of lepton number
violation, we obtain $m_\nu \sim 10^{-2}$ eV for $M_D \sim 10^3$ TeV. The $M_D$ scale can be brought down to the TeV range as done for the inverse seesaw, making the model more appealing, by using the similarity condition with the first generation of charged leptons instead of Eq. (3), i.e. $m_D \approx m_e \sim 1$ MeV, but maintaining $m_\nu \sim 10^{-2}$ eV and $\mu_L$ at the keV scale.

Alternatively, in the extended seesaw scheme, with $\mu'_L = 0$ and $\mu_L = 0$, $m_\nu = 0$ at the tree level. An extension of all of these scenarios with new singlet neutrinos can lead to the appearance of DM candidates.

No definitive evidence, however, has been observed of effects from extensions of the SM with RH neutrinos. It is then natural to assume that these particles are just elements of a large-scale physics, preventing their production at current high-energy accelerators. Our working hypothesis in this letter is that the standard RH neutrinos are superheavy, exactly as in the original classic seesaw, so that they do not influence phenomenology at low energies and therefore they are neither the relevant particles to generate the tiny neutrino mass of active neutrinos nor the particles able to provide a DM candidate, in contrast with the former seesaw schemes.

If neutrino masses have no relation to RH neutrinos, then Eqs. (2), (5), and (6) should be modified according to couplings of new sterile neutrinos incorporated to the spectrum, suppressing the mass terms associated with RH neutrinos ($m_D$, $M_D$, and $m_R$), that is, a seesaw with sterile neutrinos having a scale which is much lower than the scales of the above mechanisms, so relaxing the restrictions of having to deal with heavy RH neutrinos to explain the smallness of masses. Yet, as long as the new neutrinos are simply singlets under the SM gauge group, we should end up with one of the above-mentioned low-scale seesaw models.

Motivated by these expectations, we consider a new simple low-energy model which gives explanations of the origin and nature of neutrino mass and flavor mixing. We assume a seesaw mechanism where three RH neutrinos are added to the SM with high-scale Majorana masses so that they become decoupled generating highly suppressed terms for active neutrinos, just as in the canonical high-scale seesaw mechanism. We think of RH neutrinos as part of a large scale physics, featuring, for example, left–right symmetry, with very weak, practically unobservable effects at low energies. Thus, the origin of the smallness of neutrino masses would not be in the heavy RH neutrinos. It is an alternative to seesaw mechanisms where neutrino masses depend on suppressed Dirac mass terms which involve RH neutrinos, such as the canonical one and its inverse and linear extended variants, among others published in the literature.

To realize a low-scale seesaw mechanism for the generation of neutrino mass, without invoking the ordinary RH neutrinos, we simply introduce a Dirac pair of sterile fermions, $N_R$ and $N_L$. Besides, in order to succeed in completing this scheme, we consider a $Z_2$ symmetry under which the RH neutrinos and all of the fermions in the SM have charge +1 (labeling the even state), whereas the new sterile neutrinos
have charge −1 (labeling the odd state). Thus, while the usual low-scale seesaw mechanisms operate with singlet fermions which are in the $Z_2$ even state, our model works with extra neutrinos in the $Z_2$ odd state. A soft breaking of this symmetry would generate the tiny mass of neutrinos.\textsuperscript{17}

The three RH neutrinos, partners of the three LH neutrinos, are introduced to restore left–right symmetry in the neutrino content of the $Z_2$ even sector. The extra sterile neutrinos do not break this symmetry because they are in the $Z_2$ odd state.

In order to understand the origin and soft breaking for the $Z_2$ symmetry considered here, we invoke the so-called presymmetry described in Ref.\textsuperscript{18}. Presymmetry characterizes an underlying electroweak theory of quarks and leptons based on the requirements of a left–right symmetry in fermionic content and a charge symmetry between initially postulated fractional electroweak charges. It is built upon the $U(1)_{B-L}$ global symmetry, which forbids Majorana mass terms. Presymmetry is broken at the level of quarks and leptons, making possible symmetry breaking terms such as the Majorana mass terms for RH neutrinos and those related to the $Z_2$ symmetry. In a sense, presymmetry reflects the high affinity of LH neutrinos with their RH partners before the breaking that separates or dissociates them. It is the root of the $Z_2$ symmetry through which we introduce the new leveling of singlet fermions. Presymmetry also explains why the three RH neutrinos are required, despite them assumed to be so heavy that they do not influence phenomenology at low energies.

We shall see that the extra sterile neutrinos generate a massless neutrino within a linear seesaw scheme, providing thus a simple solution to the neutrino mass and mixing problems of the SM. To summarize, in this letter, we propose a low-scale (at or below GeV) minimal variant of the linear seesaw mechanism in a scenario with left–right symmetry in the neutrino content. We organize the work as follows. In Sec. 2, we refer to the SM extended with RH neutrinos, which generate neutrino masses via the usual high-scale seesaw mechanism. The extra sterile neutrinos, which are protected by the discrete $Z_2$ symmetry, become passive spectators. In Sec. 3, we examine the low-scale minimal variant of the linear seesaw mechanism now involving the new sterile neutrinos, which, keeping the heavy RH neutrinos decoupled and assuming a soft breaking of the $Z_2$ symmetry, induce the actual neutrino masses. Phenomenological aspects of the model are discussed in Sec. 4. Conclusions are given in Sec. 5.

2. Seesaw with RH Neutrinos Keeping Extra Sterile Neutrinos Decoupled

We start with the SM extended with three RH neutrinos ($\nu_{R\alpha}$, $\alpha = e, \mu, \tau$) in the $Z_2$ even sector plus a Dirac pair of sterile neutrinos ($N_R, N_L$) in the $Z_2$ odd sector, then having a left–right symmetry in the neutrino content. The terms of the gauge-invariant Lagrangian relevant to the neutrino masses are

$$-\mathcal{L} \supset y_\nu \bar{\ell}_L \phi \nu_R + \frac{m_{R\alpha}}{2} \bar{\nu}_R^\alpha \nu_R + M_D \bar{N}_L N_R + h.c.,$$

(7)
where, omitting flavor and isospin indexes, $\ell_L$ denotes the LH lepton doublet and $\tilde{\phi} = i\sigma_2\phi^*$ represents the charge conjugated field of the scalar Higgs doublet. We consider a Dirac mass term for the extra sterile neutrinos and invoke the underlying presymmetry to forbid their Majorana mass terms, allowing these only for RH neutrinos. Although they can be generated via the vacuum expectation value of gauge singlet scalars that couple to the gauge singlet fermions, in this paper we proceed with the effective Lagrangian given in Eq. (7). To make the point, we here neglect the mixing of the extra sterile neutrinos with the LH and RH neutrinos. This Lagrangian is invariant under the $Z_2$ symmetry and also consistent with conservation of lepton parity $(-1)^L$, but, being established at the level of leptons, it is not under presymmetry. Even though presymmetry breaking allows the LH and RH neutrino couplings to change, Majorana mass terms for the extra sterile neutrinos remain excluded.

After the electroweak symmetry breaking, Dirac mass terms with $m_D = y_\nu \langle \phi^\circ \rangle$ are induced. In the ($\nu_L, \nu_R^c, N_R^c, N_L$) basis, the full 8x8 neutrino mass matrix takes the form

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D^T & m_R & 0 & 0 \\ 0 & 0 & 0 & M_D \\ 0 & 0 & M_D & 0 \end{pmatrix}.$$  \hfill (8)

In the limit $m_R \gg m_D$, the diagonalization of the mass matrix leads to light Majorana neutrinos ($\nu$) and a very heavy Majorana neutrino ($\nu_H$), just as in the classical high-scale seesaw scenario, together with a Dirac neutrino ($N$) that appears as an inactive spectator. We have

$$m_\nu \simeq -m_D m_R^{-1} m_D^T, \quad m_{\nu_H} \simeq m_R, \quad M_N^\pm = \pm M_D,$$  \hfill (9)

where $m_\nu$ and $m_{\nu_H}$ are 3x3 matrices.

The tiny neutrino mass would be the only trace at low energies left by the decoupled heavy RH neutrinos. Moreover, an increase of the $m_R$ would mean an increase of the $m_D$, breaking even more the expected similarity of the Dirac mass terms of neutrinos and their charged leptonic weak partners.

Extended models with additional singlet neutrinos in the $Z_2$ even state can lower the scale of the $m_R$, as shown in Sec. 1. However, this produces a proliferation of particles in a scenario where no signs of RH neutrinos have been established yet. Other possibilities, where the $m_R$ are naturally held up at a high scale, are then worth considering, giving up the idea that the small neutrino mass has to do with the suppressed Dirac mass term $m_D$, as in the seesaw mechanisms we have referred to previously.
3. Seesaw with Extra Sterile Neutrinos Having RH Neutrinos Decoupled

We now consider the low-scale seesaw mechanism for neutrino mass generation led by the extra sterile neutrinos. New terms are introduced in the Lagrangian switching on the tiny mixing of these neutrinos with the active neutrinos and the RH neutrinos. The $Z_2$ symmetry that differentiates the new sterile neutrinos from the others is broken softly. The reading of the Lagrangian in Eq. (7) extended with the new mixing terms is

$$-\mathcal{L} \supset y_\nu \overline{\ell_L} \phi \nu_R + y'_\nu \overline{\ell_L} \phi N_R + y'_L \overline{\ell_L} \phi N^c_L + \frac{m_R}{2} \overline{\nu_R} \nu_R$$

$$+ M_D N_L N_R + \mu'_{N_L} N_R \nu_R + \mu'_{N_R} N_L \nu_R + \text{h.c.}. \quad (10)$$

As explained above, all of the terms in this Lagrangian that produce soft breaking of the $Z_2$ and $U(1)_{B-L}$ symmetries are consistent with the breaking of presymmetry.

When the Higgs field gets its vacuum expectation value, the mass matrix that follows from Eq. (10) in the $(\nu_L, \nu^c_R, N^c_R, N_L)$ basis can be written in blocks as

$$M_\nu = \begin{pmatrix} 0 & m_D & m'_D & \mu'_L \\ m'^T_D & m_R & \mu'_R & \mu'_L \\ m'^T_R & \mu'_R & 0 & M_D \\ \mu'^T_L M_D & 0 & M_D & 0 \end{pmatrix}, \quad (11)$$

where $m_D = y_\nu \langle \phi^0 \rangle$, $m'_D = y'_\nu \langle \phi^0 \rangle$, and $\mu'_L = y'_L \langle \phi^0 \rangle$. Note that $m_D$ and $m_R$ are $3 \times 3$ matrices, while $m'_D$ and $\mu'_L$ together with $\mu'_R$ and $\mu'_D$ are $3 \times 1$ matrices. Comparing Eqs. (8) and (11), we see that the smallness of the added masses does arise from the $Z_2$ symmetry, which becomes a good symmetry as the new parameters go to zero. We use the high-scale seesaw mechanism under the assumption that the Majorana masses are larger than all of the other mass parameters.

The block diagonalization of $M_\nu$ leads to a mass matrix, which after neglecting all the suppressed seesaw contributions (e.g. $m^2_D/m_R \ll m'_D, \mu'_L$), is reduced to the following form in the $(\nu_L, N^c_R, N_L)$ basis

$$M'_\nu = \begin{pmatrix} 0 & m'_D & \mu'_L \\ m'^T_D & 0 & M_D \\ \mu'^T_L M_D & 0 & M_D \end{pmatrix}. \quad (12)$$

This is the mass matrix left when the line and the column of $m_R$ in the mass matrix in Eq. (11) are removed, which should be contrasted with that in Eq. (4) relative to the linear seesaw. It is equivalent to having in the neutrino sector the effective Lagrangian

$$-\mathcal{L}_{\text{eff}} \supset y_\nu \overline{\nu_L} \phi \nu_R + y'_\nu \overline{\nu_L} \phi N^c_R + M_D \overline{N_L} N_R + \text{h.c.}, \quad (13)$$

i.e. the Yukawa sector of the SM augmented with terms involving the extra sterile neutrinos. Thus, no effects of RH neutrinos in the $Z_2$ even state remain at low energies, in agreement with the null experimental evidence so far proving the existence of RH neutrinos. The mass hierarchy is now $m'_D, \mu'_L \ll M_D$, with $m'_D \sim \mu'_L$. 

The block diagonalization of $\mathcal{M}_\nu'$ leads to the $3 \times 3$ light neutrino mass matrix

$$m_\nu \simeq -\frac{m'_D \mu'_L}{M_D} - \frac{\mu'_L m'_D}{M_D}$$

and the heavier neutrino masses

$$M_N^+ \simeq M_D + \frac{(m'_D + \mu'_L)^T (m'_D + \mu'_L)}{2M_D},$$

$$M_N^- \simeq -M_D - \frac{(m'_D - \mu'_L)^T (m'_D - \mu'_L)}{2M_D}. \tag{15}$$

Equation (14) implies that just one of the three active neutrinos is massless at the tree level, without constraining the values of the active–sterile mixing couplings, as it can be seen if the determinant of the matrix $m_\nu$ is calculated in general (it is nonzero in the case of two generations). Quantum corrections to this null mass eigenvalue turn out to be vanishingly small.\(^2\) Note that as expected from the null trace of the mass matrix, $\mathcal{M}_\nu'$, $tr(m_\nu) + M_N^+ + M_N^- = 0$. Also, the mass splitting of the pseudo-Dirac fermion goes with the neutrino masses. In other words, the neutrino masses remove the degeneracy of the pair composing what otherwise would be a Dirac particle. As already mentioned, the smallness of $m'_D$ and $\mu'_L$, with $m'_D \sim \mu'_L$, is protected by the $Z_2$ symmetry, and when they are small, $M_D$ is small too, lowering the scale of the seesaw mechanism. The flavor states will be combinations of the three SM-like neutrino mass eigenstates ($\nu$) and the heavier mostly sterile pseudo-Dirac neutrino ($N$).

The resulting mass matrices in Eqs. (12) and (14), as well as the eigenvalues in Eq. (15), are very similar in form to the ones found in the standard linear seesaw mechanism, with the difference that the texture of our matrix is consequence of a different set up, which ends up with mass terms at a much lower scale, without including the Dirac mass of RH neutrinos. Here, it is important to remark the differences with such a linear seesaw given the close resemblance between Eqs. (6) and (14). One might argue that the whole procedure of adding extra sterile neutrinos and decoupling the RH neutrinos is a mere relabeling of the linear seesaw (albeit with additional decoupled particles), i.e. $\nu_R \to N_R$, then obtaining the neutrino mass matrix simply by replacing the notation $m_D \to m'_D$.

However, it is not just a matter of labeling differently. In fact, the usual linear seesaw operates with the singlet fermions $\nu_R, S_L$, which are in the $Z_2$ even state, while our low-scale minimal linear seesaw model works with the extra neutrinos $N_R, N_L$, which are in the $Z_2$ odd state. Thus, when we compare the mass terms $m_D \nu_L \nu_R$ (in the standard linear seesaw) and $m'_D \nu_L N_R$ (in our model) to each other, for instance, we cannot claim that all of this is just a change of notation: there is invariance under $Z_2$ symmetry transformations ($\nu_{L,R} \to \nu_{L,R}; N_{R,L} \to -N_{R,L}$) in the former case, but not in the latter one. Moreover, under a soft breaking of $Z_2$ we have $m'_D \ll m_D$. Clearly, the two models we are referring to are not the same.
Furthermore, the seesaw mass matrix given in Eq. (12) contains elements 
\((m'_D, \mu'_L)\) associated with the sterile neutrinos \(N_R, N_L\) in the \(Z_2\) odd state, while the usual linear seesaw mass matrix contains those \((m_D, \mu'_L)\) related to the singlet fermions \(\nu_R, S_L\) in the \(Z_2\) even state. Being \(m'_D \sim \mu'_L \ll m_D\), the mass of the heavy Dirac neutrino \((M_D)\) in the new variant is much lower than the mass of the heavy neutrino predicted by the standard linear seesaw.

Note in particular that in the known linear seesaw, the linearity is because of \(m_D\), which in our context is \(m'_D\). Besides, the usual linear seesaw does introduce three pairs of additional neutrinos, in contrast to the realization of the low-scale minimal seesaw presented in this paper, where a single pair of extra neutrinos is sufficient to explain the light neutrino masses. And even though this last statement is well established in minimal seesaw models\(^{21-23}\) also with singlet neutrinos that can have masses below the TeV scale, the light neutrino masses still depend on the Dirac mass term \(m_D\) with RH neutrinos in the \(Z_2\) even state, which in our case goes away with the decoupling of RH neutrinos via the high-scale seesaw mechanism.

On the other hand, now regarding the neutrino mass in Eq. (14) from the phenomenological point of view, there is an additional suppression with respect to the classical seesaw and the usual linear seesaw (see Eqs. (2) and (6)) given by

\[
m_\nu \simeq \frac{m_D^2}{M_D} \frac{2\mu'_L}{m_D} \frac{m'_D}{m_D},
\]

which leads to \(M_D \sim 10\ \text{GeV}\) for \(m_\nu \sim 10^{-2}\ \text{eV}\) and \(\mu'_L \sim m'_D \sim 10\ \text{keV}\), now attributed to the soft breaking of lepton number conservation and \(Z_2\) symmetry. This means \(m'_D \sim 10^{-5} m_D\) for \(m_D\) in accord with the similarity condition in Eq. (3), and thus a seesaw scale \((M_D)\) which is about 5 orders of magnitude smaller than the scale of the usual linear seesaw. This can be shrunk to \(m'_D \sim 10^{-2} m_D\) if Eq. (3) is replaced by the similarity condition involving the first generation of charged leptons. But we keep the \(Z_2\) symmetry breaking terms at the keV scale, same as done for the inverse and linear seesaw mechanisms, making the model more appealing and stressing its differences. As argued in Sec. 1, the soft breaking of the \(Z_2\) symmetry would have its origin in the presymmetry breaking, allowing then the coupling of LH and RH neutrinos to the extra sterile neutrinos. Interesting enough, the new variant has a seesaw scale at the GeV range independently of the similarity condition used between the size of the Dirac neutrino mass \((m_D)\) and the Dirac mass of the charged lepton, making thus a difference with the others. Whatever view is taken on this, we can state unequivocally that we here have a low-scale (at or below the GeV) minimal variant of the linear seesaw mechanism within a scenario with left–right symmetry in the neutrino content that has not been dealt with yet.

4. Phenomenological Aspects of the Model

The neutrino phenomenology at low energies relies drastically upon the extra sterile neutrinos in the \(Z_2\) odd state, whose couplings to the active neutrinos in the even
state determine their decay channels and lifetimes.

The prediction that the lightest active neutrino is massless excludes the possibility of having a quasi-degenerate mass spectrum and fixes the masses of the other two active neutrinos to be \((8.6 \times 10^{-3}, 0.05)\) eV in the normal hierarchy \((m_1 = 0, m_2 < m_3)\) or \((4.92 \times 10^{-2}, 0.05)\) eV in the inverted hierarchy \((m_3 = 0, m_1 < m_2)\),\(^24\) almost doubling the total sum of neutrino masses of the normal ordering.

Based on these results, we now revise the parameter space of our model starting at the keV scale to see if the heavy sterile neutrino in the \(Z_2\) odd state classifies as a DM candidate. Its decay into the lighter SM particles in the even state is a fact that has to be pondered as the \(Z_2\) symmetry is broken softly. We need to make certain that its lifetime is longer than the age of the Universe. We also have to corroborate that this heavy neutrino that drives the seesaw complies with the constraint on DM relic abundance. Since the mass scale of \(N\) is much higher than the mass splitting, the Dirac pair of sterile neutrinos becomes approximately degenerate and it can be treated as a Dirac neutrino.

The dominant decay channel of the heavy neutrino would be \(N \rightarrow 3\nu\) through active–sterile neutrino mixing and weak interaction of \(\nu\). Neglecting the tiny difference between the two Majorana sterile neutrinos and so the neutrino masses, the decay width of this decay mode is given by\(^{25,26}\)

\[
\Gamma_{N \rightarrow 3\nu} \approx \frac{G_F^2 M_N^5}{96\pi^3} |\Theta_{\nu N}|^2, 
\]

where \(|\Theta_{\nu N}|\) is the small mixing parameter between the active and sterile neutrinos defined in our model as

\[
|\Theta_{\nu N}|^2 = \sum_{\alpha=e,\mu,\tau} \left( |\Theta_{\nu_{\alpha}N_L}|^2 + |\Theta_{\nu_{\alpha}N_R}|^2 \right),
\]

in which

\[
\Theta_{\nu_{\alpha}N_L} \approx \frac{m_{D\alpha}^\prime}{M_D}, \quad \Theta_{\nu_{\alpha}N_R} \approx \frac{\mu_{L\alpha}^\prime}{M_D}.
\]

With \(M_N\) in the 10 keV range, it leads to

\[
\Gamma_{N \rightarrow 3\nu} \simeq \frac{|\Theta_{\nu N}|^2}{1.4 \times 10^{14}} \left( \frac{M_N}{10 \text{ keV}} \right)^5.
\]

Thus, from the lifetime of \(N\) defined as \(\tau_N = 1/\Gamma_N\), and an age for the Universe of about \(4.4 \times 10^{17}\) s,\(^27\) we obtain the following bound on the active–sterile mixing:

\[
|\Theta_{\nu N}|^2 \ll 3.3 \times 10^{-4} \left( \frac{10 \text{ keV}}{M_N} \right)^5.
\]

An additional well-known decay channel is the radiative decay \(N \rightarrow \nu\gamma\), with a width given by\(^26\)

\[
\Gamma_{N \rightarrow \nu\gamma} \approx \frac{27\alpha}{8\pi} \Gamma_{N \rightarrow 3\nu} = \frac{1}{128} \Gamma_{N \rightarrow 3\nu},
\]
which predicts an outgoing photon energy of around half the mass of the sterile neutrino, thus suggesting a keV line in the spectra of galaxies to be searched by X-ray telescopes. In the early Universe, the production of our sterile neutrinos would have been dominated by the so-called non-resonant Dodelson–Widrow mechanism, which occurs at high temperatures due to the active–sterile neutrino mixing. Since the sterile neutrinos do not experience the SM forces and their mixing angles are so small, they would not have been in thermal equilibrium with the known particles and produced then in non-equilibrium processes. The abundance of DM is fixed by the DM density \( \Omega_{\text{DM}} h^2 = 0.12 \), where \( h \) is the reduced Hubble constant.

Assuming that the sterile neutrino is the sole DM particle, it is given by

\[
\Omega_{\text{DM}} h^2 \sim 0.12 \left( \frac{|\Theta_{\nu N}|^2}{10^{-9}} \right) \left( \frac{M_N}{10 \text{ keV}} \right)^2.
\]  

(23)

On the other hand, we can make (see Ref. 8)

\[
|\Theta_{\nu N}|^2 = \frac{1}{M_D^2} \sum_{\alpha=e,\mu,\tau} \left( |m'_{D\alpha}|^2 + |\mu'_{L\alpha}|^2 \right) = \frac{1}{M_N} \text{tr}(m_\nu) = \frac{1}{M_N} \sum_{i=1}^3 m_i.
\]

(24)

If we choose \( M_N \simeq 7 \text{ keV} \) as a possible DM sterile neutrino mass and adopt the normal hierarchy for the active neutrino masses, we get \( |\Theta_{\nu N}|^2 \simeq 0.7 \times 10^{-5} \) and so \( |m'_{D\alpha}|, |\mu'_{L\alpha}| \) of a few eV, which complies with the bound in Eq. (21) but it is too high to do it with the DM condition. Thus, the possibility that the Dirac sterile neutrino in the \( Z_2 \) odd state be a DM candidate at the keV scale is ruled out, as expected. The sterile neutrinos cannot be at the eV range either because of the cosmological bound on the number of relativistic neutrino species. We therefore assume \( |m'_{D\alpha}| \) and \( |\mu'_{L\alpha}| \) at the keV level as in the usual low-scale inverse and linear seesaw schemes. For illustrative purposes, we establish benchmark points choosing \( |m'_D| \sim |\mu'_L| \sim 4 \text{ keV} \) and the normal hierarchy for neutrino masses. This leads to

\[
M_N = \frac{\sum_{\alpha} \left( |m'_{D\alpha}|^2 + |\mu'_{L\alpha}|^2 \right)}{\sum_i m_i} \sim 1 \text{ GeV},
\]

(25)

that is, a sterile neutrino at the GeV scale, consistent with the value obtained in Sec. 3. In contrast, the inverse and linear seesaw schemes expect a rich phenomenology just at or above the TeV level.

Note in particular that from Eq. (14) we have \( m'_D/M_D \sim m_\nu/\mu_L \) which, when compared with \( m_D/M_D \sim m_\nu/\mu_L \) and \( (m_D/M_D)^2 \sim m_\nu/\mu_L \) obtained from Eqs. (6) and (5), respectively, indicates a much lower scale for the seesaw mechanism \( (M_D) \) if \( \mu'_L \) and \( \mu_L \) are taken at the keV range, with \( m'_D \sim \mu'_L \ll m_D \).

It is also worth mentioning that the almost Dirac nature of the pair of heavy neutrinos means no significant impact on the lepton number violating processes such as the neutrinoless double beta decays. It can, however, mediate lepton flavor violating processes like the \( \mu \rightarrow e\gamma \) transition. But, the active–sterile neutrino
mixing angles are not large enough to be at the range of the current experimental sensitivity.\textsuperscript{31} Yet the GeV sterile neutrino is potentially accessible directly by collider experiments.\textsuperscript{32}

Finally, we should stress that the superheavy RH neutrinos are decoupled from active neutrinos and therefore do not contribute to the low seesaw matrix in Eq. (14). Nevertheless, they can give rise to the baryon asymmetry of the Universe through high-scale leptogenesis.\textsuperscript{33} We here note that a low-scale resonant leptogenesis with the two extra sterile neutrinos, nearly degenerate in their masses, is also viable.\textsuperscript{20}

5. Conclusions

We have extended the SM with three RH neutrinos, $\nu_R$, plus a Dirac pair of sterile neutrinos, $N_R, N_L$, as a way to have left–right symmetry in neutrino content and produce small neutrino masses just from the last ones. A discrete $Z_2$ symmetry was considered, under which all particles have charge $+1$, except the extra sterile neutrinos which have charge $-1$. It is a minimal extension in the sense that additional Dirac pairs of sterile neutrinos in the odd state can be included. We have argued that this $Z_2$ symmetry would have its origin in the so-called presymmetry, which typifies an underlying electroweak theory of quarks and leptons with left–right symmetry in fermionic content and initially postulated symmetric fractional charges, broken at the level of quarks and leptons. This presymmetry, built on the global $U(1)_{B-L}$ symmetry, also guarantees that the extra sterile neutrinos cannot have Majorana mass terms.

We have used a seesaw with superheavy RH neutrinos without raising the Dirac masses connected with electroweak symmetry breaking, avoiding thus to alter the natural similarity between neutrinos and charged leptons. This similitude has been maintained and the RH neutrino mass terms suppressed via the standard high-scale seesaw, so that the contribution of RH neutrinos becomes too small to be worth consideration. In contrast to most of known type-I seesaw mechanisms, our working hypothesis is that the RH neutrinos in the $Z_2$ even state do not influence phenomenology at low energies. This effectively reduces the content of the seesaw mechanism to three active neutrinos in the even state plus two sterile neutrinos in the odd state. Yet, the three RH neutrinos are required as presymmetry is demanded.

The main prediction of the minimal seesaw is that one of the active neutrinos is massless, which fixes the small mass of the other two; in a more indicative sense, three fermion generations are required to generate a massless neutrino. It has been shown that this smallness of neutrino masses can be produced through a low-scale seesaw with the Dirac pair of sterile neutrinos in the odd state. This couple becomes a pseudo-Dirac fermion with a mass splitting given by the sum of light neutrino masses. To explain the weakness of its coupling to the active neutrinos, we have introduced a soft breaking for the $Z_2$ symmetry. We have explained that this would have its origin in the presymmetry breaking, which allows the coupling of LH and
RH neutrinos to change, but keeping forbidden the Majorana mass terms for the extra sterile neutrinos.

We have made clear that the almost Dirac sterile neutrino leading to the tiny neutrino masses cannot be at the keV scale because it does not give the correct DM relic abundance. In addition, as a low-scale alternative with no RH neutrinos to the standard linear and inverse seesaw mechanisms, the scale of the sterile neutrino should lie well below the TeV scale, that is, about the GeV range if we take into account that the small $Z_2$ symmetry breaking terms should be at the keV scale, same as done for them, making the model more appealing and stressing its differences. As a matter of fact, this new variant has the seesaw scale at the GeV level independently of the similarity condition used between the Dirac neutrino mass and the Dirac mass of the charged lepton, making therefore a difference with those ones.

The resulting light neutrino mass matrix is very similar in form to the mass matrix of the linear seesaw mechanism, albeit with a single pair of sterile neutrinos instead of three and a different parameter regime, implying the lightest active neutrino to be massless at tree level. Because of the similarity, one can assert that the seesaw under consideration is a low-scale minimal variant of the linear seesaw. It is worth emphasizing, however, that they are not the same. The usual linear seesaw operates with singlet fermions which are in the $Z_2$ even state, while our low-scale minimal linear seesaw model works with the extra neutrinos which are in the $Z_2$ odd state. Moreover, the mass of the heavy Dirac neutrino, $M_D$, in the new variant turns out to be much lower than the mass of the heavy neutrino predicted by the usual linear seesaw. Phenomenological aspects of the minimal approach, related to lepton number violation and lepton flavor violation processes, have been discussed in the literature. 31

On the other hand, although the superheavy RH neutrinos are irrelevant to the generation of neutrino masses, they can give rise to the baryon asymmetry of the Universe through high-scale leptogenesis. A low-scale resonant leptogenesis with the two extra sterile neutrinos is also viable as they are nearly degenerate in their masses.

Experimental tests of the model would be the finding of active neutrino masses at the predicted scale and the appearance of a Dirac sterile neutrino at the GeV scale in the particle spectrum. Our minimal seesaw also implies no phenomenology associated with ordinary RH neutrinos with mass in the keV to TeV range, as proposed by the alternative low-scale seesaw models that require RH neutrinos. Here, we remark that these RH neutrinos are the ones that would be charged under a $U(1)_{B-L}$ gauge symmetry and an eventual $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge extension of the SM, 34–36 in contrast to our sterile neutrinos which would remain singlets in such a scenario. The whole scheme proposed in this work would be ruled out if signals of RH weak currents are found at low energies.

The puzzle of the observed mixing between leptons was not addressed by this paper. The simple structure of the mass matrix that comes from the model, however, allows to accommodate straightforwardly the so-called tri-bimaximal mixing
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pattern as well as its deviations according to experimental data without invoking non-abelian discrete flavor symmetries and symmetry-breaking scalar flavon fields, in the first place. Analysis of these possibilities will be presented elsewhere.

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