Study Pure Annihilation Decays $B^0_s(\bar{B}^0_s) \to D^{\pm}\pi^\mp$ in PQCD Approach

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Abstract The rare decays $B^0 \to D^{\pm}\pi^\mp$ and $\bar{B}^0 \to D^{+}\pi^\mp$ can occur only via annihilation-type diagrams in the standard model. In this paper, we calculate branching ratios of these decays in perturbative QCD approach ignoring soft final state interaction. From our calculation, we find that their branching ratios are at $O(10^{-6})$ with large CP asymmetry, which may be measured in LHC-b experiment in the future.

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1 Introduction

The rich data from two $B$ factories make the study of $B$ physics a very hot topic. A lot of study has been made, especially for the CP violation problem. The Cabibbo–Kobayashi–Maskawa (CKM) angle $\beta = \phi_1$ has already been measured. However the other two angles are difficult to measure in $B$ factories. The study of $B_s$ meson decay is needed for this purpose. Some work on $B_s$ decays has already been done.

In this work, we will explore four decay channels, namely $B^0 \to D^{\pm}\pi^\mp$ and $\bar{B}^0 \to D^{\mp}\pi^{\pm}$. There is only one kind of contribution for each of the decay modes, thus there is no direct CP violation for them. However the remaining two angles are still CP violation induced by mixing, although they are decays with charged final states (non-CP-eigenstates). They are quite complicated since altogether four are involved simultaneously.

From these decays, we find that the four quarks in final states are different from the ones in $B^0$ meson. We call this mode pure annihilation-type decay. In the factorization approach, this decay is described as $B^0_s$ annihilating into vacuum and final states mesons produced from vacuum afterwards. They are rare decays. Up to now, only PQCD approach can calculate this kind of modes effectively. Using PQCD approach, we have calculated many of this kind of decays and some decays have been measured in $B$ factory. Some information about PQCD in detail can be found in Ref. [7].

In standard model language, for decay $B_s \to D\pi$, a $W$ boson exchange causes $b\bar{s} \to u\bar{c}$, and $d\bar{d}$ in final state are produced from a gluon. This is also called $W$ exchange diagram. This gluon can be emitted by any one of the quarks participating in the four-quark interaction. This is shown in Fig. 1. We consider the $B^0_s$ meson at rest for simplicity. In this frame, this gluon has $O(M_B/2)$ momenta, that is to say, this is a hard gluon. We can perturbatively treat the process by six-quark interaction.

![Fig. 1 The diagram of PQCD approach.](image)

In this work, we will give the PQCD calculation of these two decays in the next section, and discuss the numerical results in Sec. 3. At last we conclude this study in Sec. 4.

2 Calculation

The non-leptonic $B^0_s$ decays $B^0_s \to D^{\mp}\pi^{\pm}$ and $\bar{B}^0 \to D^{\pm}\pi^\mp$ are rare decays. For decay $B^0_s \to D^{\mp}\pi^{\pm}$, the effective Hamiltonian at the scale lower than $M_W$ is given as

$$\mathcal{H}_1 = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs} \left[ C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu) \right],$$

where the four-quark operators are

$$O_1 = (\bar{b}s)_{V-A}(\bar{c}u)_{V-A}, \quad O_2 = (\bar{b}u)_{V-A}(\bar{c}s)_{V-A},$$

with the definition $(\bar{q}_1q_2)_{V-A} \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. $C_{1,2}$ are Wilson coefficients at renormalization scale $\mu$. For decay $B^0_s \to D^{\mp}\pi^{\pm}$, the effective Hamiltonian reads

$$\mathcal{H}_2 = \frac{G_F}{\sqrt{2}} V_{cs} V_{ub} \left[ C'_1(\mu)O'_1(\mu) + C'_2(\mu)O'_2(\mu) \right],$$

Footnotes:

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where the four-quark operators are
\[ O'_1 = (\bar{b}s)_{V-A}(\bar{u}c)_{V-A}, \quad O'_2 = (\bar{b}c)_{V-A}(\bar{u}s)_{V-A}. \quad (4) \]

In PQCD, the decay amplitude is expressed as\[^7\]
\[
\text{Amplitude} \sim \int \frac{d^4 k_1 d^4 k_2 d^4 k_3}{2^4 \pi^2} \text{Tr} \left[ C(t) \Phi_B^2(k_1) \Phi_D(k_2) \right]
\times \Phi_{\pi}(k_3) H(k_1, k_2, k_3, t) e^{-S(t)}. \quad (5)
\]

In this equation, \( C(t) \) is Wilson coefficient at scale \( t \) with leading order QCD correction. \( \Phi_i \) are light-cone wave functions, which describe the non-perturbative contributions. They cannot be theoretically calculated directly. Fortunately, they are process-independent. \( e^{-S(t)} \) is called Sudakov factor, which comes from the resummation of soft and collinear divergence. This Sudakov factor suppresses the soft contributions, which makes the perturbative calculation of hard part reliable. By including the \( k_r \) dependence of the wave functions and Sudakov form factor, this approach is free of endpoint singularity. Thus, the work left is to calculate the perturbative hard part \( H(t) \).

The structures of the meson wave functions are
\[
B^0_s(P): \ [P + m_u] \gamma_5 \phi_b(x), \quad (6)
\]
\[
D(P): \ \gamma_5 [P + m_d] \phi_d(x), \quad (7)
\]
\[
\pi(P): \ \gamma_5 [P \phi_A(x) + m_0 \phi_T(x)] + m_0 (\phi_T - \phi_T(x)) \quad (8)
\]

with \( m_0 = m_s^2/(m_u + m_d) = 1.4 \text{ GeV} \) characterizing the chiral breaking scale. And the light-like vectors are defined as \( n = (1, 0, 0) \) and \( v = (0, 1, 0) \). In the above functions, \( \phi_i \) are distribution amplitude wave functions.

According to the effective Hamiltonian (1), the diagrams contributing to \( B^0_s \to D^+ \pi^- \) are drawn in Fig. 2. Just as stated in Sec. 1, this decay has only annihilation diagrams. With the meson wave functions and Sudakov factors, the hard amplitude for factorizable annihilation diagrams in Figs. 2(a) and 2(b) is

![Fig. 2](image_url)

**Fig. 2** Leading order Feynman diagrams contributing to decay \( B^0_s \to D^+ \pi^- \).

The total decay amplitude for \( B^0_s \to D^+ \pi^- \) is given as
\[ A = f_A F_a + M_a. \quad (11) \]

The decay width is expressed as
\[ \Gamma(B^0_s \to D^+ \pi^-) = \frac{G^2_F M_B^3}{128 \pi} (1 - r^2) |V_{cb} V_{us} A|^2. \quad (12) \]
As the case $B_s^0 \to D^-\pi^+$, we also draw diagrams Fig. 3 using Eq. (3). The amplitude for factorizable annihilation diagrams (a) and (b) results in $-F_a$. The amplitude for the non-factorizable annihilation diagram results in 

$$M'_a = \frac{256\pi}{3\sqrt{2}N_c}M_B^2\int_0^1 \, dx_1 \, dx_2 \, dx_3 \, \int_0^\infty \, b_1 \, dB_1 \, b_2 \, dB_2 \, \phi_B(x_1, b_1) \phi_D(x_2, b_2) \left[ \left( (1 - x_3) \phi_\tau^\pi(x_3, b_2) + r(x_2 + 1 - x_3) \phi_\tau^\pi(x_3, b_2) + r(x_2 - 1 + x_3) \phi_\tau^\pi(x_3, b_2) \right) E_m(t_m) \right] f_a(x, b_2) \left[ \left( (1 - x_3) \phi_\tau^\pi(x_3, b_2) + r(3 + x_2 - x_3) \phi_\tau^\pi(x_3, b_2) + r(1 - x_2 - x_3) \phi_\tau^\pi(x_3, b_2) \right) E_m(t_m) \right] f_a(x, b_1) \right] . \tag{13}$$

Thus, the total decay amplitude $A'$ and decay width $\Gamma$ for $B_s^0 \to D^-\pi^+$ decay is given as

$$A' = -f_a F_a + M'_a, \tag{14}$$

$$\Gamma(B_s^0 \to D^-\pi^+) = \frac{G_F^2 \, M_B^2}{128\pi} (1 - r^2) |V_{ub} V_{cs} A'|^2 . \tag{15}$$

The decay widths for CP conjugated mode, $\bar{B}_s^0 \to D^+\pi^-$, are the same expressions as $B_s^0 \to D^-\pi^+$ with the conjugate of CKM matrix elements.

**3 Numerical Evaluation**

Considering SU(3) symmetry, we use the distribution amplitude of the $B_s^0$ meson similar to $B$ meson:

$$\phi_B(x, b) = N x^2(1 - x)^2 \exp \left[ - \frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2 \right] . \tag{16}$$

which is adopted in Refs. [9] ~ [11]. $N$ is a normalization factor, which can be get from the normalized relation

$$\int_0^1 dx \phi_\pi(x, b = 0) = \frac{f_\pi}{2\sqrt{2}N_c} . \tag{17}$$

For $D$ meson, the distribution amplitude is

$$\phi_D(x) = \frac{3}{\sqrt{2}N_c} f_D x(1 - x) \{ 1 + a_D (1 - 2x) \} , \tag{18}$$

which is fitted from experiments.[12] The wave functions of the $\pi$ meson have been derived in Refs. [13] and [14],

$$\phi_\pi^A(x) = \frac{3 f_\pi}{\sqrt{2}N_c} x(1 - x) \{ 1 + 0.44 C_2^{3/2} (2x - 1) + 0.25 C_4^{3/2} (2x - 1) \} , \tag{19}$$

$$\phi_\pi^B(x) = \frac{f_\pi}{\sqrt{2}N_c} \{ 1 + 0.43 C_2^{1/2} (2x - 1) + 0.09 C_4^{1/2} (2x - 1) \} , \tag{20}$$

$$\phi_\pi^C(x) = \frac{f_\pi}{\sqrt{2}N_c} (1 - 2x) \times \left[ 1 + 0.55 (10x^2 - 10x + 1) \right] \tag{21}$$

with the Gegenbauer polynomials

$$C_2^{1/2}(t) = \frac{1}{2} (3t^2 - 1) ,$$

$$C_4^{1/2}(t) = \frac{1}{8} (35t^4 - 30t^2 + 3) ,$$

$$C_2^{3/2}(t) = \frac{3}{2} (5t^2 - 1) ,$$

$$C_4^{3/2}(t) = \frac{15}{8} (21t^4 - 14t^2 + 1) . \tag{22}$$

The other input parameters are listed below,[15]

$$f_{B_s} = 230 \text{ MeV}, \quad \omega_b = 0.5 \text{ GeV}, \quad f_D = 240 \text{ MeV},$$

$$C_D = 0.8 \pm 0.2, \quad f_s = 132 \text{ MeV}, \quad M_{B^0_s} = 5.37 \text{ GeV},$$

$$m_d = 1.87 \text{ GeV}, \quad m_0 = 1.4 \text{ GeV},$$

$$\tau_{B_s^0} = 1.46 \times 10^{-12} \text{ s}, \quad |V_{ub}| = 0.043, \quad |V_{us}| = 0.22 ,$$

$$|V_{ub}| = 0.0036 , \quad |V_{cs}| = 0.974 . \tag{23}$$

With the above parameters, we show the decay amplitudes calculated in Table 3. The predicted branching ratios are

$$\text{Br}(B_s^0 \to D^+\pi^-) = 8.3 \times 10^{-7} , \tag{24}$$

$$\text{Br}(B_s^0 \to D^-\pi^+) = 2.9 \times 10^{-6} . \tag{25}$$
Table 1 Decays amplitudes \(10^{-3} \text{ GeV}\) with parameters equations (9) \(\sim\) (13).

| \(B_s^0 \to D^+\pi^-\) | \(B_s^0 \to D^-\pi^+\) |
|----------------|----------------|
| \(f_B F_o\) | 0.51 \(-\) 1.3i | \(-f_B F_o\) | 0.51 \(+\) 1.3i |
| \(M_o\) | \(-16.1\) \(-\) 19.1i | \(M'_o\) | \(-1.8\) \(-\) 19.1i |
| \(A\) | \(-15.6\) \(-\) 20.4i | \(A'\) | \(-2.3\) \(-\) 17.8i |
| \(\text{Br}\) | \(8.3 \times 10^{-7}\) | \(\text{Br}\) | \(3.0 \times 10^{-6}\) |

From the above results, we find that the branching ratio of decay \(B_s^0 \to D^+\pi^-\) is smaller than that of decay \(B_s^0 \to D^-\pi^+\). The CKM element in decay \(B_s^0 \to D^+\pi^-\) is \(V_{cb}^* V_{us}\), but in \(B_s^0 \to D^-\pi^+\) the CKM element is \(V_{ub}^* V_{cs}\). Although \(|V_{cb}^* V_{us}|\) and \(|V_{ub}^* V_{cs}|\) are both \(O(\lambda^3)\) in Wolfenstein parametrization, the value of \(|V_{ub}^* V_{cs}|/|V_{cb}^* V_{us}|\) is equal to 0.37. The branching ratio of \(B_s^0 \to D^+\pi^-\) is 3 times larger than that of \(B_s^0 \to D^-\pi^+\), which is mainly due to the CKM factor.

In addition to the perturbative annihilation contributions, there are other pictures existing such as soft final states interaction.\[^{16}\] In Ref. [5], the results from the PQCD approach for \(B^0 \to D^- K^+\) is consistent with experiment well, which tells us that the soft final states interaction may not be important. So we think their effects are small and ignore them in our calculation.

Unfortunately, there are no data for these two decays in experimental side up to now. We think that the LHC-b experiment can measure these decays in the future. The results can test this PQCD approach and show some information about new physics.

Table 2 The sensitivity of the branching ratios to change of \(\omega_b\) and \(a_D\).

| \(\omega_b\) | \(\text{Br}(B_s^0 \to D^+\pi^-)\) \(10^{-7}\) | \(\text{Br}(B_s^0 \to D^-\pi^+)\) \(10^{-6}\) |
|---|---|---|
| 0.45 | 9.5 | 3.5 |
| 0.50 | 8.3 | 2.9 |
| 0.55 | 7.5 | 2.5 |
| \(a_D\) | \(\text{Br}(B_s^0 \to D^+\pi^-)\) \(10^{-7}\) | \(\text{Br}(B_s^0 \to D^-\pi^+)\) \(10^{-6}\) |
| 0.6 | 7.6 | 2.6 |
| 0.8 | 8.3 | 2.9 |
| 1.0 | 9.1 | 3.4 |

The calculated branching ratios in PQCD approach are sensitive to various parameters such as the parameters in Eqs. (16) \(\sim\) (23). The uncertainty taken by \(m_h\) has been argued in many papers,\[^{10,11}\] and it is strictly constrained by \(B \to \pi\) form factor. In Table 2, we show the sensitivity of the branching ratio to change of \(B_s^0\) and \(D\) distribution amplitude functions. It is found that the uncertainty of the branching ratio in PQCD is mainly due to \(\omega_b\), which characterizes the shape of \(B_s^0\) meson wave function.

Considering most of the uncertainty,\[^{4}\] we give the branching ratios of these two decays with suitable range of \(\omega_b\) and \(a_D\). Thus we can give our results

\[
\text{Br}(B_s^0 \to D^+\pi^-) = (8.3 \pm 1.2) \times 10^{-7} \left( \frac{f_B f_D}{230 \text{ MeV} \times 240 \text{ MeV}} \right)^2 \left( \frac{|V_{ub}^* V_{cs}|}{0.0036 \times 0.974} \right)^2, \tag{26}
\]

\[
\text{Br}(B_s^0 \to D^-\pi^+) = (2.9 \pm 0.5) \times 10^{-6} \left( \frac{f_B f_D}{230 \text{ MeV} \times 240 \text{ MeV}} \right)^2 \left( \frac{|V_{ub}^* V_{cs}|}{0.0412 \times 0.224} \right)^2. \tag{27}
\]

The \(CP\) violation information in decay \(B_s(\bar{B}_s) \to D^\pm \pi^\mp\) is very complicated. There are four kinds of decays,

\[
g = \langle D^+ \pi^- | H | B_s^0 \rangle \propto V_{ub}^* V_{cs}, \quad h = \langle D^+ \pi^- | H | \bar{B}_s^0 \rangle \propto V_{ub} V_{cs},
\]

\[
\bar{g} = \langle D^- \pi^+ | H | \bar{B}_s^0 \rangle \propto V_{ub}^* V_{cs}, \quad \bar{h} = \langle D^- \pi^+ | H | B_s^0 \rangle \propto V_{ub} V_{cs}, \tag{28}
\]

which determine the decay matrix element of \(B_s^0 \to D^+\pi^-\) and \(D^-\pi^+\), and of \(\bar{B}_s \to D^-\pi^+\) and \(D^+\pi^-\). They are already shown in the previous section. There is only one kind of contribution for each of the decay modes, thus there is no direct \(CP\) violation for them. However there is still \(CP\) violation induced by mixing, although they are decays with charged final states.\[^{4}\]

The time-dependent decay rates for \(B_s \to D^\pm \pi^\mp\) are given by

\[
\Gamma_{D^\pm \pi^\mp}(t) = (1 \pm A_{CP}) e^{-t/\tau_{B_s}} \left\{ 1 + (S_{D\pi} \pm \Delta S_{D\pi}) \sin(\Delta m t) + (C_{D\pi} \pm \Delta C_{D\pi}) \cos(\Delta m t) \right\}, \tag{29}
\]

and \(\bar{B}_s \to D^\pm \pi^\mp\) by

\[
\Gamma_{D^\pm \pi^\mp}(t) = (1 \pm A_{CP}) e^{-t/\tau_{\bar{B}_s}} \left\{ 1 - [(S_{D\pi} \pm \Delta S_{D\pi}) \sin(\Delta m t) + (C_{D\pi} \pm \Delta C_{D\pi}) \cos(\Delta m t)] \right\}. \tag{30}
\]

\[^{1}\] Although the uncertainty taken by CKM matrix elements is large, we will not discuss them in this work, since they are only an overall factor here for branching ratios.
Utilizing Eq. (28), we can get

\[
C_{D\pi} = A_{CP} = 0, \quad \Delta C_{D\pi} = \frac{1 - |h/g|^2}{1 + |h/g|^2}, \quad S_{D\pi} = \frac{2|h/g| \sin \gamma \cos \delta}{1 + |h/g|^2}, \quad \Delta S_{D\pi} = \frac{-2|h/g| \sin \delta \cos \gamma}{1 + |h/g|^2}.
\]

In deriving the above formulas we have neglected the small weak phase \(\arg(q/p) = V_{tb}^*V_{ts}/V_{tb}V_{ts} = 2\lambda^2\eta < 2\pi\), in Wolfenstein parametrization.\(^{[17]}\)

We can calculate these parameters related to decays \(B^0_s(\bar{B}^0_s) \to D^{\pm} \pi^\mp\) in our PQCD approach. Through calculation, we get

\[
\Delta C_{D\pi} = -0.56.
\]

The parameters \(S_{D\pi}\) and \(\Delta S_{D\pi}\) are related to \(\gamma\). The results are shown in Fig. 4. If we can measure the time-dependent spectrum of the decay rates of \(B^0_s\) and \(\bar{B}^0_s\), we can extract the CKM angle \(\gamma\) and strong phase \(\delta\) in Eq. (31) by Fig. 4. The parameter \(C_{D\pi} = A_{CP} = 0\) is from the fact that there is only one kind of contribution to each of the decays. Any deviation from zero will be a signal of new physics contribution.

![Fig. 4 CP violation parameters of \(B^0_s(\bar{B}^0_s) \to D^{\pm} \pi^\mp\); \(\Delta S_{D\pi}\) (dash-dotted line) and \(S_{D\pi}\) (solid line) as functions of CKM angle \(\gamma\).](image)

### 4 Summary

Recent study shows that PQCD approach works well for charmless \(B\) decays\(^{[10,11]}\) as well as for channels with one charmed meson in the final states.\(^{[5,12]}\) Because the final state mesons move very fast, each of them carries more than 2 GeV energy, there is not enough time for them to exchange soft gluons. So we can ignore the soft final states interaction. Due to disadvantages in other approach such as general factorization approach\(^{[18]}\) and BBNS approach\(^{[19]}\) pure annihilation decay can be calculated reliably only in PQCD approach.

\[
s(Q, b) = \int_{1/b}^{Q} \frac{d\mu'}{\mu'} \left[ \frac{2}{3} (2\gamma_{\mu} - 1 - \log 2) + C_F \log \frac{Q}{\mu'} \right] \frac{\alpha_s(\mu')}{\pi} + \left( \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3} \beta_0 \log \frac{\gamma_q}{2} \right) \left( \frac{\alpha_s(\mu')}{\pi} \right)^2 \log \frac{Q}{\mu'},
\]

\(\gamma_{\mu} = 0.57722 \cdots\) is Euler constant, and \(\gamma_q = \alpha_s/\pi\) is the quark anomalous dimension.

In this paper, we calculate \(B^0_s \to D\pi\) decays, which occur purely via annihilation-type diagrams. The branching ratios are still sizable at the order of \(10^{-6}\). There will also be sizable CP violation in these decays. They will be measured in future LHC-b experiment, which may bring some information about new physics to us.

### Appendix: Some Functions

The definitions of some functions used in the text are presented in this appendix. In the numerical analysis we use one loop expression for strong coupling constant,

\[
\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)},
\]

where \(\beta_0 = (33 - 2n_f)/3\) and \(n_f\) is the number of active flavors at appropriate scale. \(\Lambda\) is QCD scale, which we use as 250 MeV at \(n_f = 4\). We also use leading logarithms expressions for Wilson coefficients \(C_{1,2}\) presented in Ref. [8]. Then, we put \(m_t = 170\) GeV, \(m_w = 80.2\) GeV, and \(m_b = 4.8\) GeV.

The function \(E'_f, E_m,\) and \(E'_m\) including Wilson coefficients are defined as

\[
E'_f(t) = [C_1(t) + C_2(t)] \alpha_s(t) e^{-S_D(t) - S_{\pi}(t)}, \quad (A2)
\]

\[
E_m(t) = C_2(t) \alpha_s(t) e^{-S_B(t) - S_D(t) - S_{\pi}(t)}, \quad (A3)
\]

\[
E'_m(t) = C_1(t) \alpha_s(t) e^{-S_B(t) - S_D(t) - S_{\pi}(t)}, \quad (A4)
\]

where \(S_B, S_D,\) and \(S_{\pi}\) result from summing both double logarithms caused by soft gluon corrections and single ones due to the renormalization of ultra-violet divergence. The above \(S_{B,D,\pi}\) are defined as

\[
S_B(t) = s(x_1 p_1^+, b_1) + 2 \int_{1/b_1}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'), \quad (A5)
\]

\[
S_D(t) = s(x_2 p_2^+, b_1) + 2 \int_{1/b_2}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'), \quad (A6)
\]

\[
S_{\pi}(t) = s(x_3 p_3^+, b_3) + s(1 - x_3) p_3^+, b_3) + 2 \int_{1/b_3}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'), \quad (A7)
\]

where \(s(Q, b)\), so-called Sudakov factor, is given as\(^{[20]}\)
The functions $h_a$, $h_a^{(1)}$, and $h_a^{(2)}$ in the decay amplitudes consist of two parts: one is the jet function $S_t(x_i)$ derived by the threshold resummation\cite{21} and the other is the propagator of virtual quark and gluon. They are defined by

$$h_a(x_2, x_3, b_2) = S_t(1 - x_3) \left( \frac{\pi i}{2} \right)^2 H_0^{(1)} (MB \sqrt{(1 - r^2)} x_2 (1 - x_3) b_2)$$

$$\times \left\{ H_0^{(1)} (MB \sqrt{(1 - r^2)} (1 - x_3) b_2) J_0(MB \sqrt{(1 - r^2)} (1 - x_3) b_2) \theta(b_2 - b_3) + (b_2 \leftrightarrow b_3) \right\},$$

(A9)

$$h_a^{(j)}(x_1, x_2, x_3, b_1, b_2) = \left\{ \frac{\pi i}{2} H_0^{(1)} (MB \sqrt{(1 - r^2)} x_2 (1 - x_3) b_1) J_0(MB \sqrt{(1 - r^2)} x_2 (1 - x_3) b_2) \theta(b_1 - b_2) + (b_1 \leftrightarrow b_2) \right\}$$

$$\times \left( \frac{K_0(MbF_{(j)})}{b_1} \right), \quad \text{for } F_{(j)}^2 > 0,$$

(A10)

where $H_0^{(1)}(z) = J_0(z) + iY_0(z)$, and $F_{(j)}$'s are defined by

$$F_{(1)}^2 = (1 - r^2)(x_1 - x_2)(1 - x_3), \quad F_{(2)}^2 = x_1 + x_2 + (1 - r^2)(1 - x_1 - x_2)(1 - x_3).$$

(A11)

We adopt the parametrization for $S_t(x)$ of the factorizable contributions,

$$S_t(x) = \frac{2^{1+2\epsilon} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c, \quad c = 0.3,$$

(A12)

which is proposed in Ref. [22]. In the non-factorizable annihilation contributions, $S_t(x)$ gives a very small numerical effect to the amplitude. Therefore, we drop $S_t(x)$ in $h_a^{(1)}$ and $h_a^{(2)}$. The hard scale $t$'s in the amplitudes are taken as the largest energy scale in the $H$ to kill the large logarithmic radiative corrections,

$$t_1^a = \max(MB, \sqrt{(1 - r^2)(1 - x_3)}),$$

(A13)

$$t_2^a = \max(MB, \sqrt{(1 - r^2)}x_2, 1/b_1, 1/b_3),$$

(A14)

$$t_m^j = \max(MB \sqrt{|F_{(j)}^2|}, MB \sqrt{(1 - r^2)x_2(1 - x_3)}, 1/b_1, 1/b_2).$$

(A15)

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