Abstract.

Objective: Proton therapy is an emerging method in cancer therapy. One of the main developments is to increase the accuracy of the Bragg-peak position calculation, which requires more precise relative stopping power (RSP) measurements. An excellent choice is the application of proton computed tomography (pCT) systems which takes the images under similar conditions, as they use the same irradiation device and hadron beam for imaging and treatment. A key aim is to develop a precise image reconstruction algorithm for pCT systems to reach their maximal performance.

Approach: An image reconstruction algorithm was developed in this work, which is suitable to reconstruct pCT images from the deposited energy, position and direction measurement of individual protons. The flexibility of an iterative image reconstruction algorithm was utilised to appropriately model the trajectory of protons. Monte Carlo (MC) simulations of a Derenzo and a CTP404 phantom was used to test the accuracy of the image reconstruction.

Main results: The Richardson–Lucy algorithm was applied first and successfully for pCT image reconstruction. Authors used an averaged probability density based approach for the interaction (system) matrix generation, which is a relevant description to consider the uncertainty of the path of the protons in the patient. In case of an idealized setup, 1.43 lp/cm spatial resolution and 0.3% RSP accuracy was achieved.

Significance: This work is the first application of the Richardson–Lucy algorithm, for pCT image reconstruction.
1. Introduction

Hadron therapy is an emerging and efficient curing method against cancer. The increasing number of hadron therapy centers and the number of successful treatments demonstrate its success since the first proposal in the 50s (Lawrence, 1957). Today’s accelerator techniques led us to use protons or even heavier ions as bombarding particles. The application of massive hadron beams instead of massless X-ray results in more focused dose distribution (Durante et al., 2017). Indeed, using higher mass number ion beams than protons (He, C and O) can result in increased relative biological effectiveness (RBE) in the tumor volume, while keeping the RBE close to one in healthy tissues (Durante et al., 2021). The higher the dose gradient around the treated volume is required, the lower the uncertainty in the relative stopping power (RSP) distribution during dose planning is needed, to avoid insufficient dosage of the tumor or the overdose of organs at risk.

The developments of proton Computed Tomography (pCT) techniques are promising solutions for the above problems. Applying the same irradiation device, beam, and hadron for both the medical imaging and the treatments can significantly reduce the uncertainties of the imaging. To obtain this, two main imaging strategies exist:

(i) The first concept is to measure the average energy loss of the proton beam. This design is feasible from a technical point of view, but the achievable spatial resolution is poor with the clinically available proton beams (Krah et al., 2018).

(ii) The second concept is the so-called list mode imaging concept, which measures the energy loss and in parallel it estimates the path of each individual proton. Monte Carlo (MC) simulations and prototype measurements showed that this solution can meet the required spatial and density resolutions, so the focus is moved toward this direction (Johnson, 2017).

Nowadays, the pCT scanner R&Ds around the world are tending to reach the prototyping and clinical/pre-clinical testing phase, which requires the integration of the prototype scanners into clinical environment. Following the list mode strategy, the path estimation of individual protons is usually based on the measurements of the upstream and downstream tracker detector pairs, which concept is called the double-sided scanner design (see Figure 1). One important further step can be the abandonment of the upstream tracker detectors, and the application of a single-sided scanner design. The drawback of this latter concept is the less accurate proton path measurement, however the study by Sølie et al., 2020 concluded that the achievable spatial resolution meets the minimum requirement. Anyhow, the spatial resolution remains a point to improve of this solution, so even a small resolution increase would be important for the application in the clinical practice. The more precise modeling of the measurement inaccuracy in the system matrix is a possible way to reach this goal.

The realistic clinical applicability requires to complete the data taking and processing within minutes, which results in 1-10 million protons per second measurement rate. The ultimate goal would be to finish the image capturing within the minimal gantry rotation time. The LLU/UCSC Phase-II Scanner prototype detector showed data taking speed up to 1.2 million proton per second, which probably can be upgraded by 50% in the near future (Johnson et al., 2016). In order to increase the data taking rate to 10 million protons per second, two possible directions exist: the first is to apply faster readout frequency of 10 MHz at least, while the second is to measure multiple
proton tracks within one readout frame. The second solution fits mostly with the single-sided scanner design, because this solution avoids the pairing problem of the upstream and downstream measurements, which leads to track confusion in case of a double-sided scanner even with low number of protons in a frame.

Multiple proton measurements fit best for silicon pixel trackers and silicon pixel sensor based range counters as presented by the Bergen pCT Collaboration (Pettersen et al., 2017; Pettersen et al., 2019; Alme et al., 2020). However, multiple proton measurements can be done by applying three silicon strip detectors rotated relative to each other as presented by the PRaVDA collaboration (Esposito et al., 2018). Another layout has been designed by the iMPACT group. The ProXY detector combines the two acceleration possibilities with monolithic active pixel detectors. Applying this layout, about 50 MHz readout frequency can be reached, and it is planned to measure multiple-proton events (Mattiazzo et al., 2015).

In this paper a novel imaging method is proposed for these pCT detector concepts. The Richardson–Lucy algorithm (Richardson, 1972; Lucy, 1974) is used for the first time for pCT imaging. This method originates from astronomy and was found to be the maximum-likelihood expectation-maximization (ML-EM) solution of the image reconstruction of emission tomography (Shepp et al., 1982), which was later applied for the image reconstruction of a single-sided pCT scanner. The paper is organized as follows: section 2 begins with the general approach to the image reconstruction problem itself, followed by the presentation of the details of the Richardson–Lucy algorithm and the proton-phantom interaction model. Section 3 compares detector designs and presents the image reconstruction algorithm. Section 3 also contains the evaluation of the spatial- and density resolution of phantoms. Results are summarized and discussed in section 4 and 5, respectively.

2. The Image Reconstruction Algorithm

The role of the image reconstruction is to determine the relative stopping power (RSP) distribution from the measured data. Two families of image reconstruction techniques exist: the first family contains the filtered backprojections, in contrast with the second family, which includes the iterative reconstructions.

The backprojections usually use integrals along straight lines, which is naturally an inaccurate approximation for the scattered proton trajectory. The so-called distance-driven backprojection belongs to this family, which can take into account the curvature of the most likely path (MLP)
of the protons during the filtered backprojection (Rit et al., 2013). The method provides a fairly
good spatial resolution, however, it requires very high statistics, which means that a higher dose
must be applied, and therefore may not be accepted for clinical use.

The second category of image reconstruction models represents the imaging process as the
interaction between proton tracks and volumetric pixels (voxels) in the reconstruction space. This
approach is particularly well-suited for handling curved proton trajectories and achieves reasonably
good spatial and density resolution with acceptable statistical performance. However, it requires
higher computational power. The method models the imaging as a large linear system of equations,
which is described by the following general algebraic form:

$$\mathbf{y} = \mathbf{A} \cdot \mathbf{x},$$  \hspace{1cm} (1)

where \(\mathbf{y}\) is a one-dimensional vector, which has typically \(m = 10^8-10^9\) elements. The \(\mathbf{y}\) contains
the water equivalent path length (WEPL) reduction of the protons in the reconstruction area.
Variable \(\mathbf{x}\) is also a one-dimensional vector (typically \(n = 10^5-10^7\) elements) containing the relative
stopping power (RSP) of the voxels. Finally, \(\mathbf{A}\) is the so-called system matrix, which has \(m \times n\)
elements of \(10^{13-10^{16}}\). The system matrix contains the interaction coefficients between protons
and voxels. The information in matrix \(\mathbf{A}\) can be described as the (expected) length of the proton’s
path in the voxel. In practice, \(m\) is usually a larger integer than \(n\), so the linear equation system
is over-determined. The goal of the image reconstruction in general, is to determine the values of
vector, \(\mathbf{x}\) with the knowledge of vector, \(\mathbf{y}\) and system matrix, \(\mathbf{A}\).

Orthogonal projection based iterative algorithms are widely used for pCT image reconstruction
as presented by Gordon et al., 1970; Censor et al., 2008; Herman, 2009; Penfold et al., 2010; Plautz
et al., 2016; Johnson, 2017. In this work we applied the Richardson – Lucy deconvolution, which
reached reasonable quality increase in the field of emission tomography (Shepp et al., 1982), and
have not been used earlier for proton computed tomography.

2.1. The Richardson – Lucy Algorithm

The Richardson – Lucy deconvolution iteration cycle (Richardson, 1972; Lucy, 1974) originates
from the field of optics, and known as a fixed point iteration. The iterative solution is based on
the formula,

$$x_i^{k+1} = x_i^k \frac{1}{\sum_j A_{ij}} \sum_j \sum_l y_j A_{lj} x_l^k A_{ij},$$  \hspace{1cm} (2)

for every \(i = 1, \ldots, n\), where \(n\) is the length of vector \(x\), which contains the RSP of the voxels,
\(k\) is the number of iterations, matrix elements \(A_{ij}\) contain the interaction coefficients between the
proton trajectories and the voxels, \(j = 1, \ldots, m\) is the index of the trajectories, where \(m\) is the
number of the trajectories, \(y_j\) contain the integrated RSP along the trajectories, which is equivalent
to the WEPL reduction of the protons traveling along the trajectories. The \(y_j/\sum_l A_{lj} x_l^k\) term is
usually called Hadamard ratio, and it represents the ratio of the integrated RSP along the proton
path and its estimate based on the voxel values, calculated in the previous iteration.

‡ Specifies for this study is explained in section 3.3 in details.
2.2. The Proton-Phantom Interaction

Instead of the most simple straight line approximation, the literature uses the estimated (most likely) path of the protons, based on the upstream and downstream measurements of proton track position and angle in case of a double-sided scanner design. Novel formulae are available in (Schneider, 1994; Williams, 2004; Schulte et al., 2008; Krah et al., 2018) to calculate the MLP of the protons.

In case of a single-sided scanner design, where upstream measurements are not available, the beam information can be used. Certainly, this contains much more uncertainty than a precise measurement, which must be included in the calculations. The formalism of Schulte et al., 2008 was extended by Krah et al., 2018 to deal with the uncertainty of the measurements and the beam as well, so their formulae were applied in this work.

In order to incorporate the increased uncertainty that is originating from the missing upstream measurement and the scattering of the proton inside the patient, we have implemented a simplified probability density based approach. This was suggested by Williams, 2004 and applied earlier for a double-sided setup by Wang et al., 2010.

The determination of the proton-phantom interaction was divided into the MLP calculation and the approximation of the probability density around the MLP. The calculation was organized into the following steps:

(i) The outgoing direction on the phantom hull is determined by the measured direction of the proton, neglecting any interactions between the first detector layer and the phantom hull. Similarly, the entry directions are determined by the known beam parameters.

(ii) The perpendicular entry and outgoing coordinates are calculated based on the MLP formulae of Schulte et al., 2008; Krah et al., 2018 at the directional incoming and outgoing positions, respectively.

(iii) From computational point of view, the MLP calculation in the whole phantom would not be feasible, therefore inside the phantom a third order spline approximation is applied, which is found to be accurate and was suggested by Williams, 2004.

(iv) The probability density around the MLP is approximated by a Gaussian (Williams, 2004). To simplify the calculations, the standard deviation ($\sigma$) of this Gaussian was considered constant along the proton path in the phantom. This approach does not deal with the uncertainty characteristics along the proton path, but it includes the uncertainty that is originating from the scattering of the proton in the phantom.

The proposed approach strikes a balance between physical accuracy and computational efficiency in determining the most likely paths of the protons. It offers a well-founded, physically motivated calculation, while maintaining the computational efficiency necessary for practical application.

3. Simulations with the Algorithm

The ultimate goal of the pCT imaging for proton therapy is to provide a solid basis for accurate and reliable dose planning. It is traditionally a challenge to define a proper measure, which correctly characterizes the goodness of a reconstructed RSP distribution, therefore general image properties
(spatial and density resolution, image noise) are usually used to quantify image quality. To study this, the imaging of dedicated spatial and density resolution phantoms were simulated with Monte Carlo techniques, reconstructed by the formerly described method and evaluated following the instructions later on this section.

3.1. The Proton CT Scanner Model

A single-sided detector design (figure 2) with a 230 MeV/u pencil beam was investigated. Following the realistic beam model of Sølie et al., 2020, the full width at half maximum (FWHM) of the Gaussian beam was set to 7 mm (with about 3 mm standard deviation), the spot divergence was chosen to be 2.8 mrad and the spot emittance was 3.0 mrad×mm.

![Figure 2: Single-sided, list mode detector design.](image)

Three different detector layer setups were compared in this study: the first is an idealized detector with no measurement errors, the second is a silicon pixel tracker modeled after the design of the Bergen pCT Collaboration (Pettersen et al., 2019; Alme et al., 2020). Finally, the third is a silicon strip detector based tracker layer, followed the LLU/UCSC Phase-II Scanner design of the Loma Linda University (LLU) and the University of California at Santa Cruz (UCSC) (Johnson et al., 2016). We note, that the results of this work is valid for an envisioned single-sided scanner, built of the LLU/UCSC Phase-II Scanner tracker layers, in comparison with the existing LLU/UCSC Phase-II Scanner which is a state-of-the-art double-sided setup. The properties of the three detector layer setups, the idealized, the silicon pixel, and the silicon strip, are summarized in table 1.

As presented in Figure 2, the idealized setup is a single sensitive plane, but in the latter two realistic cases each tracker layer contains two sensitive planes similarly as in the existing technological solutions. If the detection is based on a silicon pixel detector, a double structure of two equivalent sensitive planes is needed to be applied to fully cover the alternating sensitive and readout electronics panels. While applying silicon strip detectors, two separate planes are required for the perpendicular geometrical $x$ and $y$ directions. The schematic structures of the detector layers are shown in figure 3. Table 1 contains the joint material budget of these double layers. The WEPL resolution of both realistic setups was chosen to be 3 water equal mm (standard deviation of a normal distribution), which was added to the simulated range straggling in the phantom. This
Table 1: Comparison of tracker detector pair model parameters: Ideal setup, with no measurement errors, Silicon pixel detector based on the design of the Bergen pCT Collaboration (Pettersen et al., 2019; Alme et al., 2020), and Silicon strip detector model following the structure of the LLU/UCSC Phase-II Scanner (Johnson et al., 2016).

is a realistic uncertainty—however, this is a very rudimentary model, as the measurement error is likely to depend on the remaining range of the protons behind the patient. The distance of the first detector pair to the rotation axis (isocenter) was chosen for 400 mm in all cases, which results in 300 mm and 325 mm detector-phantom distance for the Derenzo and the CTP404 phantoms, respectively. Similar distances were used for portal detectors applied in photon therapy gantries, so they can be considered to be realistic for the future pCT devices as well (Krah et al., 2018).

**Figure 3:** The structure of the investigated layers.

### 3.2. The Applied Phantoms

In order to quantify the effectiveness of the Richardson–Lucy algorithm, a standardized evaluation method is required. In our study, two widely-applied phantoms have been used to test the goodness of the reconstruction. The RSP distribution was reconstructed in one plane of the phantoms, so the phantoms were considered to be offset invariant in the direction of the rotation axis. To ensure the offset invariance, 400 mm thick phantoms were simulated in the axial direction.

The spatial resolution of the reconstruction was measured with the Monte Carlo imaging of the Derenzo phantom: a 200 mm diameter water cylinder, which contains six sectors of 1.5-6 mm diameter aluminum rods, specially chosen for the current analysis. The original idea of this phantom comes by Derenzo et al., 1977.

The RSP reconstruction accuracy (also referred to as density resolution) has been evaluated
with the CTP404 phantom The Phantom Laboratory, 2022. It is designed to measure how accurately a material property is reconstructed in a homogeneous region of the phantom. The CTP404 phantom is a 150 mm diameter epoxy cylinder, which contains 8 different material inserts with a diameter of 12.2 mm.

3.3. The Workflow

A simulation code was developed to use and test the Richardson–Lucy algorithm, which was divided into the following steps (illustrated in Figure 4):

(i) The data taking was simulated with Monte Carlo method. Beside the pencil beam model, the phantoms were modeled appropriately in the simulations. We used the Geant4 (version 11.0.0) (Agostinelli et al., 2003; Allison et al., 2006) with GATE (version 9.2) (Jan et al., 2004; Jan et al., 2011) to simulate the phantom-beam interactions. In the reference physics list settings QGSP\_BIC\_EMY was activated for these calculations. Data taking of one slice was simulated from 180 directions in 2° and 2 mm steps. In total, \( \sim 3.7 \) million and \( \sim 5.4 \) million primary protons were used for the CTP404 and Derenzo phantoms respectively, that didn’t suffer nuclear collisions (see the next step). We note, that instead of a full-fledged detector simulation, the measurement uncertainties for the various detector setups were assigned in a later step to the ideal, exact position, direction and energy of the protons at the first detector layer.

(ii) The position and direction uncertainties for the realistic detector setups were simulated using correlated Gaussian distributions, and added to the exact positions and directions of simulated protons. The measurement uncertainty was calculated based on the guideline of Krah et al., 2018. The WEPL measurement error also was randomly assigned from a Gaussian distribution to the WEPL of the protons calculated from their energy losses, simulated in the previous step, according to the parameters listed in table 1.

(iii) In order to filter out the protons which undergo nuclear collisions in the patient, a 3-sigma filtering has been applied for the direction and WEPL of the protons originating from the same beam spot, as it was suggested and used by Schulte et al., 2008.

(iv) Calculation of the most probable incoming and outgoing position of the protons on a cylinder around the phantom hull was performed, based on the formulation of Schulte et al., 2008;
Krah et al., 2018. The diameter of this cylinder was chosen to be 10 mm wider than that of the phantom hull in order to avoid lace-like artefact, which appeared, when the diameter of the phantom hull and the cylinder were the same.

(v) The Richardson–Lucy algorithm was used to reconstruct the RSP distribution from the individual proton histories. On-the-fly system matrix calculation was applied based on simplified Gaussian probability density around a third order spline approximation of the MLP, where the probability density is characterized by the $\sigma$ variance of the Gaussian (given in mm units).

(vi) In the final step, the spatial resolution was evaluated based on the reconstruction of the Derenzo phantom. The density resolution was calculated from the reconstructed CTP404 phantom.

Calculations were done on the machines of the Wigner Scientific Computing Laboratory’s hardware. The computationally demanding part of the algorithm was running on Nvidia 1080 Ti Graphical Processing Unit (GPU) cards. The algorithm has been designed so that the list of all available proton tracks are grouped and processed in small chunks (typically consisting $2 \times 10^5$ protons) that fit in the memory of the used GPU device. Within a chunk, the weights of the iterations defined by Eq. (2) should be calculated once and only once (Erdogan et al., 1999). The iteration on each chunk is processed until a given number of $k$ cycles is reached, or a certain, adaptively set loss value is achieved (typically after 20-30 cycles). This loss is defined as the mean squared error between two consecutive cycles. This ensures that the processing of the given chunk is terminated, and therefore the computation time is optimal.

3.4. Evaluation of the Derenzo Phantom

Spatial resolution is a critical parameter in evaluating the performance of an image reconstruction algorithm. It refers to the ability of the imaging system to distinguish between small structures in the object being imaged. In essence, it measures the contrast between adjacent features—higher contrast implies that smaller structures can be differentiated effectively. Typically, spatial resolution is quantified using metrics such as line pairs per millimeter (lp/mm) or line pairs per centimeter (lp/cm), where a higher number of line pairs indicates a better resolution (Smith, 2007; Flay et al., 2012).

In any imaging system, the inherent effect is similar to a blurring of the image, where fine details become smeared out. This blurring can be mathematically characterized by the Point Spread Function (PSF), which describes how a single point in the object space is distributed in the image space. The PSF essentially encapsulates the extent of the blurring effect introduced by the imaging system.

To analyze the frequency behavior of the system, the Fourier transform of the PSF is computed, resulting in the Modulation Transfer Function (MTF). The MTF provides a comprehensive description of how different spatial frequencies (i.e., the rate of change of intensity in the image) are transferred by the imaging system. High spatial frequencies correspond to finer details in the image, and a higher MTF value at these frequencies indicates that the system can accurately reproduce those details.
The spatial resolution is commonly defined by the point on the MTF curve where the MTF value drops to 10% of its maximum. This MTF(10%) value represents the spatial frequency beyond which the imaging system’s ability to differentiate between structures diminishes significantly, serving as a standard benchmark for comparing the resolution of different imaging systems or reconstruction algorithms. This method to acquire the MTF(10%) values was motivated by Sølie et al., 2020, however we note that various evaluation methods may provide slightly different spatial resolution values.

For the evaluation of the reconstructed image of the Derenzo phantom, an automatized strategy has been developed, that is illustrated on Figure 5. The main steps of the evaluation follows as:

(i) The background intensity (defined by the mean intensity around the middle of the phantom) is subtracted from the image.

(ii) For each rod size, an attempt is made to extract each individual rod from the given sector using classical image processing techniques, using the OpenCV library (Bradski, 2000). If the contrast is not high enough and the image of the rods cannot be separated, the sector is excluded from the evaluation.

(iii) The image segments of the individual rods are averaged, resulting in the point spread function for the given sector.

(iv) The 2-dimensional Fourier transform of the PSF images results in the modulation transfer functions, from which the MTF(10%) is determined after taking the radial profile and performing a sigmoid fit.

(v) If the MTF(10%) for more than one sector has been extracted successfully, then their average represents the spatial resolution of the given image.

Figure 5: The main steps of the determination of the spatial resolution on a Derenzo phantom.

3.5. Evaluation of the CTP404 Phantom

In order to robustly determine the average RSP of the inserts on the reconstructed images, yet another automatized algorithm has been implemented as the following:
(i) At given reconstruction resolution (determined by the mm/pixel values), the exact center position of each insert is determined on the ground truth image (which has perfect contrast, zero noise and blurring).

(ii) On the reconstructed CTP404 images, the mean RSP around the previously determined center positions was calculated.

The extracted RSP values can be compared to the ground truth RSP values, as it was presented in details by Alme et al., 2020.

4. Results

In this study the proof-of-concept performance of the Richardson – Lucy-algorithm was investigated using a single-sided detector setup, simulated by a simplified Monte Carlo model. The center line of all beams coincided in one plane, perpendicular to the rotation axis. Moreover, every proton was assigned into this layer, without taking into account the deviation of their path in the direction of the rotation axis. This 256×256 image slice was reconstructed using 1 mm/pixel reconstruction resolution. For this resolution, the variance of the probability density around the MLP was chosen to be $\sigma = 1$ mm.

Figure 6: Richardson – Lucy algorithm based reconstruction of the CTP404 (top row) and Derenzo (bottom row) phantoms. The ideal, the pixel detector, and the strip detector layers are shown, respectively from left to right.
The result of the reconstructed images of the CTP404 and Derenzo phantoms are shown in Figure 6, on the top and bottom rows respectively. The colored sections on the CTP404 images indicate the reconstructed RSP values of the inserts—see the details below. The left-hand side column of Figure 6 presents the idealized case, while the middle and right-hand side columns show the more realistic silicon pixel and silicon strip detector models, respectively.

To quantify the quality of the reconstructed images, the density resolution and its relative difference is shown in Figure 7a, while Figure 7b shows the evolution of the spatial resolution (as defined in Subsection 3.4). The reconstructed RSP values and their errors are summarized in Table 2.

![Figure 7: Left panel: The reconstructed RSP values from the CTP404 phantom and their relative differences compared to the ground truth values, for the various material inserts. Right panel: the average spatial resolution in the function of the processed proton tracks.](image)

The average relative RSP difference for the listed materials was found to be 0.28% for the ideal, 0.85% for the silicon pixel, and 1.31% for the silicon strip setups after processing 3.7 million protons. This density resolution exceeds the required 1% accuracy for the ideal and silicon pixel setups (Poludniowski et al., 2015). In all investigated setups, the highest RSP difference (-2.06%, -2.91% and -4.52%) was measured at the Teflon insert, which has the highest RSP value.

After processing 5.4 million protons, the spatial resolution was found to be 1.43 lp/cm for the ideal setup, 1.17 lp/cm and 0.94 lp/cm for the silicon pixel and the silicon strip detector based setups, respectively. As it is visible also on the lower panels of Figure 6, the detector uncertainties result in significant blurring, while on the other hand with an ideal setup, even the fine structures of the smallest rods can be recognized.
Table 2: The difference between the real and the reconstructed RSP of the eight inserts. The epoxy data was measured in a 4 mm radius circle in the middle of the phantom.

5. Discussion and Comparison

Sølie et al., 2020 reached a 3.8 lp/cm spatial resolution with the ideal and 3.2 lp/cm spatial resolution with a realistic single-sided detector setup, which was found to be 75% and 82% of their simulation results for an ideal and realistic double-sided setup, respectively. In Collins-Fekete et al., 2016, a spatial resolution of 5.76 lp/cm has been reported without detector effects using $10^7$ primary protons. The current, proof-of-concept version of the developed Richardson–Lucy algorithm achieved 37% and 24% spatial resolution of the aforementioned studies.

The obtained resolution was reached by optimizing the number of the primary protons and spatial coverage to be calculated within the order of 10 minutes running on a common, commercial GPU card. Certainly, using a stronger GPU can immediately lead us to increase the batch number of protons and therefore the resolution. Furthermore, in the current proof-of-concept development phase only a handful of computational optimization techniques have been applied, therefore with future developments the algorithm will provide excellent potential to become a competitive and clinically viable method.

The achieved density resolution was already found to be around the required 1% RSP accuracy, which indicates the relevance of the proposed algorithm as well. However, there is room for improvement in this regard as well: the currently applied simplified probability density model could be superseded with a more detailed probability map (Wang et al., 2010), while a true 3-dimensional reconstruction would further improve the result.

6. Summary

In this work the proof-of-concept application of the Richardson–Lucy algorithm with probability density based proton-phantom interaction calculation for proton CT image reconstruction has been presented. We applied clinically realistic setups and parameters in the Monte Carlo simulations: 400 mm detector-isocenter distance, 3-5 million primary protons per image slice, and realistic beam and detector characteristics. For testing and for the evaluation of the resolutions two widely used phantoms were applied: the Derenzo and the CTP404 ones.
We have concluded that the presented reconstruction method already meets the required density resolution, with similar values as the state-of-art prototypes. Regarding spatial resolution, as of today, quantitatively more accurate methods are available in e.g. Solie et al., 2020 and Collins-Fekete et al., 2016. However, the proposed method has the potential to achieve qualitatively competitive results and medical applicability.

We plan to continue the algorithm development, with a focus on the spatial resolution and reconstruction time, which limited the investigations of the current work to only one layer. Indeed, the possibility of the speedup of the algorithm has been also directed by parallel booster methods and machine learning techniques.

Members of the Bergen pCT Collaboration

Max Aehle\textsuperscript{a}, Johan Alme\textsuperscript{b}, Gergely Gábor Barnaföldi\textsuperscript{c}, Gábor Böröcz\textsuperscript{d}, Tea Bodova\textsuperscript{b}, Vyacheslav Borshchov\textsuperscript{d}, Anthony van den Brink\textsuperscript{e}, Mamdouh Chaar\textsuperscript{b}, Bence Dudás\textsuperscript{f}, Viljar Eikeland\textsuperscript{g}, Gregory Feofilov\textsuperscript{f}, Christoph Garth\textsuperscript{h}, Nicolas R. Gauger\textsuperscript{a}, Ola Grottvik\textsuperscript{b}, Havard Helstrup\textsuperscript{h}, Sergey Igolkin\textsuperscript{f}, Szófia Jólesz\textsuperscript{c}, Ralf Keidel\textsuperscript{i}, Chinarat Kobdaj\textsuperscript{j}, Tobias Kortus\textsuperscript{a}, Lisa Kusch\textsuperscript{a}, Viktor Leonhardt\textsuperscript{h}, Shruti Mehendale\textsuperscript{b}, Raju Ningappa Mulawade\textsuperscript{b, Odd Harald Odland}\textsuperscript{k}, George O’Neill\textsuperscript{b}, Gábor Papp\textsuperscript{b}, Thomas Peitzmann\textsuperscript{a}, Helge Egil Seime Pettersen\textsuperscript{k}, Pierluigi Piersimoni\textsuperscript{b,m}, Maksym Protsenko\textsuperscript{d}, Max Rauch\textsuperscript{b}, Attiq Ur Rehman\textsuperscript{b}, Matthias Richter\textsuperscript{a}, Dieter Röhrich\textsuperscript{b}, Joshua Santana\textsuperscript{a}, Alexander Schilling\textsuperscript{a}, Joao Seco\textsuperscript{p}, Arnon Songmoolnak\textsuperscript{b}, Jarle Ramso Solie\textsuperscript{b}, Ákos Sudár\textsuperscript{c,r}, Ganesh Tambave\textsuperscript{b}, Ihor Tynychuk\textsuperscript{k}, Kjetil Ullaland\textsuperscript{b}, Mónika Varga-Kőfaragó\textsuperscript{e}, Lennart Volz\textsuperscript{a}, Boris Wagner\textsuperscript{b}, Steffen Wendzel\textsuperscript{b}, Alexander Wiebel\textsuperscript{b}, RenZheng Xiao\textsuperscript{b}, t, Shining Yang\textsuperscript{b}, Hiroki Yokoyama\textsuperscript{a}, Sebastian Zillien\textsuperscript{i}

 a) Chair for Scientific Computing, University of Kaiserslautern- Landau, 67663 Kaiserslautern, Germany; b) Department of Physics and Technology, University of Bergen, 5007 Bergen, Norway; c) HUN-REN Wigner Research Centre for Physics, 29–33 Konkoly–Thege Miklós út, H-1121 Budapest, Hungary; d) Research and Production Enterprise "LTU" (RPELTU), Kharkiv, Ukraine; e) Institute for Subatomic Physics, Utrecht University/Nikhef, Utrecht, Netherlands; f) St. Petersburg University, St. Petersburg, Russia; g) Scientific Visualization Lab, University of Kaiserslautern- Landau, 67663 Kaiserslautern, Germany; h) Department of Computer Science, Electrical Engineering and Mathematical Sciences, Western Norway University of Applied Sciences, 5020 Bergen, Norway; i) Center for Technology and Transfer (ZTT), University of Applied Sciences Worms, Worms, Germany; j) Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, Thailand; k) Department of Oncology and Medical Physics, Haukeland University Hospital, 5021 Bergen, Norway; l) Institute for Physics, Eötvös Loránd University, 1/A Pázmány P. Sétány, H-1117 Budapest, Hungary; m) UniCamillus - Saint Camillus International University of Health Sciences, Rome, Italy; n) Department of Physics, University of Oslo, 0371 Oslo, Norway; o) Department of Biomedical Physics in Radiation Oncology, DKFZ—German Cancer Research Center, Heidelberg, Germany; p) Department of Physics and Astronomy, Heidelberg University, Heidelberg, Germany; q) Department of Diagnostic Physics, Division of Radiology and Nuclear Medicine, Oslo University Hospital, Oslo, Norway; r) Budapest University of Technology and Economics, Budapest, Hungary; s) Biophysics, GSI Helmholtz Center for Heavy Ion Research GmbH, Darmstadt, Germany; t) College of Mechanical & Power Engineering, China Three Gorges University, Yichang, People’s Republic of China; u) Department of Radiology, Faculty of Medicine, Chulalongkorn University, 1873 Rama IV Rd, Pathum Wan, Bangkok, 10330, Thailand; v) Eindhoven University of Technology, Eindhoven, Netherlands;

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