The spectrum of screening masses near $T_c$: predictions from universality.

R. Fiore$^a$ †, A. Papa$^a$ † and P. Provero$^b$ ‡

$^a$ Dipartimento di Fisica, Universit`a della Calabria
Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza
I–87036 Arcavacata di Rende, Cosenza, Italy

$^c$ Dipartimento di Fisica Teorica dell’Universit`a di Torino
Istituto Nazionale di Fisica Nucleare, Sezione di Torino
via P. Giuria 1, I–10125 Torino, Italy

Abstract

We discuss the spectrum of screening masses in a pure gauge theory near the deconfinement temperature from the point of view of the dimensionally reduced model describing the spontaneous breaking of the center symmetry. Universality arguments can be used to predict the values of the mass ratios in the scaling region of the deconfined phase when the transition is of second order. One such prediction is that the scalar sector of the screening spectrum in $SU(2)$ pure gauge theory contains a bound state of the fundamental excitation, corresponding through universality to the bound state found in the 3D Ising model and $\phi^4$ theory in the broken symmetry phase. A Monte Carlo evaluation of the screening masses in the gauge theory confirms the validity of the prediction. We briefly discuss the possibility of using similar arguments for first order deconfinement transitions, and in particular for the physically relevant case of $SU(3)$.

† e-mail address: fiore.papa@cs.infn.it
‡ e-mail address: provero@to.infn.it
1 Universality prediction for the screening masses

QCD predicts the existence of a deconfinement temperature above which the fundamental degrees of freedom cease to be confined in color singlets and can propagate in isolation. This prediction is being experimentally tested in experiments where heavy nuclei are collided at high energies to produce such deconfined state of matter, the quark-gluon plasma. It is therefore imperative from the theoretical point of view to develop a better understanding of the behavior of gauge theories at temperatures above the deconfinement transition (for a recent review see e.g. Ref. [1]).

Since a non-zero temperature for a quantum field theory is equivalent to a finite size and periodic boundary conditions in Euclidean time, in many important respects gauge theories at high temperature can be treated as effectively living in one dimension less, a phenomenon generally referred to as dimensional reduction. More precisely, dimensional reduction can be defined through a specific pattern of pairwise degeneracies in the spectrum, characteristic of theories living in one less dimension than the original one, so that the occurrence of dimensional reduction can be quantitatively verified e.g. by Monte Carlo simulations. For SU(2) and SU(3) gauge theories in 3 + 1 dimensions, this comparison was successfully performed in [2], where convincing evidence was provided that dimensional reduction occurs for temperatures all the way down to the deconfinement one.

Much less is known about the actual form of the dimensionally reduced theory, especially for temperatures not too far from $T_c$, where perturbative methods are not reliable. The simplest hypothesis, that the effective theory is the pure gauge theory with the same gauge group in one less dimension and in the confined phase, is ruled out by numerical data on the spectrum of screening masses for SU(2) [3], while for SU(3) the agreement is better. A more sophisticated proposal was made in [4], where the effective 3D theory is taken to be a three-dimensional gauge theory coupled to an adjoint scalar. This effective theory has been shown to correctly reproduce the low-lying states of the spectrum for both SU(2) [5] and SU(3) [6] for temperatures down to $\sim 2T_c$.

On the other hand, in the temperature range just above $T_c$, we have the possibility of making a solid conjecture about the form of the dimensionally reduced theory, at least for the case of SU(2) or, in general, for gauge groups
such that the transition is of second order: this is the \textit{scaling region} above
the transition, where universality arguments hold and the effective theory
 can be conjectured to belong to the universality class of theories whose global
 symmetry group coincides with the center of the gauge group. For $SU(2)$, the
 relevant universality class is the one of the 3D Ising model. This application
 of universality arguments to the deconfinement transition was introduced
 by Svetitsky and Yaffe \cite{7} and many of its predictions have been accurately
 verified: the values of the critical indices (the most recent results can be
 found in Refs. \cite{8, 9, 10, 11}), universal amplitude ratios \cite{12}, and correlation
 functions at criticality \cite{13, 14}. In this approach, the fundamental degree
 of freedom of the effective theory is the Polyakov loop, which is the order
 parameter of the deconfinement transition.

It is far from obvious \textit{a priori} that the higher masses in the spectrum
 should be universal, since only the lowest one contributes to the free energy.
 However, it has recently been shown that for the three dimensional Ising
 universality class, in the broken symmetry phase, there is strong evidence
 for such universality: the Ising model and lattice regularized $\phi^4$ theory both
 exhibit a rich spectrum of massive excitations, and the mass ratios coincide
 in the scaling region \cite{15}. Particularly interesting is the existence of an ex-
cited mass in the scalar sector at less than twice the mass of the fundamental
 excitation: being the interaction attractive in the broken symmetry phase, it
 is natural to interpret such state as a bound state of two fundamental par-
ticles, an interpretation that was confirmed by studying the Bethe-Salpeter
 equation for 3D $\phi^4$ theory in the continuum \cite{16}.

It is therefore natural to ask whether the same spectrum characterizes
 $SU(2)$ gauge theory in the scaling region above the deconfinement temper-
 ature. This paper is devoted to a numerical verification of this ansatz. In
 particular we will focus on the scalar sector: we will show that a bound
 state of the fundamental screening mass can indeed be extracted from the
 correlation functions of suitably defined operators, that are constructed from
 Polyakov loops in a manner similar to that employed in \cite{15} to study the
 spectrum of the Ising model. We also evaluated the lowest screening mass
 with angular momentum 2 in the scaling region and found a value compat-
 ible with the one of the Ising model. Preliminary results were reported in
 Ref. \cite{17}.

In the following section we describe in detail the method we used and the
 numerical results, while Section 3 is devoted to the discussion of the results
 and some speculation about the case of $SU(3)$. 

3
2 Numerical determination of the spectrum of scalar screening masses

As a first step we considered the connected wall-wall correlation function of the Polyakov loop on a lattice with \(N_x \times N_y \times N_z \times N_t\) sites and lattice spacing \(a\)

\[
G(|z_1 - z_2|) \equiv \langle \overline{P}(z_1) \overline{P}(z_2) \rangle - \langle \overline{P}(z_1) \rangle \langle \overline{P}(z_2) \rangle ,
\]

where

\[
\overline{P}(z) \equiv \frac{1}{N_x N_y} \sum_{n_1=1}^{N_x} \sum_{n_2=1}^{N_y} P(n_1a, n_2a, z)
\]

is the average of the Polyakov loop \(P(x, y, z) \equiv \text{Tr} \prod_{n_4=1}^{N_t} U_4(x, y, z, n_4a)\) over the \(xy\)-plane at a given \(z\). The wall average implies the projection at zero momentum in the \(xy\)-plane. The correlation function \(G(|z_1 - z_2|)\) takes contribution from all the screening masses in the \(0^+\) channel, i.e. from the lowest (fundamental) mass \(m_1\), from the first excited mass \(m_2\), and so on. For a periodic lattice, we have

\[
G(|z_1 - z_2|) = C_0 e^{-m_1 L_z} + C_1 \left[ e^{-m_1 |z_1 - z_2|} + e^{-m_1 (L_z - |z_1 - z_2|)} \right] + C_2 \left[ e^{-m_2 |z_1 - z_2|} + e^{-m_2 (L_z - |z_1 - z_2|)} \right] + \ldots ,
\]

with \(L_z = aN_z\). Here the dots represent the contribution from higher mass excitations and from multi-particle cuts. It should be noticed that, since \(\langle \overline{P}(z) \rangle \neq 0\) in the high temperature phase, a \(z\)-independent exponential term appears in the r.h.s. of Eq. (1) [19]. It is easy to verify that such term is sub-dominant with respect to the contribution of the fundamental mass and of the excited masses \(m_i, i > 1\) for which \(m_i < 2m_1\).

We discretized the theory on the lattice with the Wilson action and generated Monte Carlo configurations by the overheat-bath updating algorithm [20] with Kennedy-Pendleton improvement [21]. Measurements were taken every 10 upgradings and error analysis was performed by the jackknife method applied at different blocking levels.

We made simulations at two values of \(\beta\) slightly above the critical value \(\beta_c = 2.29895(10)\) determined in Ref. [12]: at \(\beta = 2.33\) (corresponding to \(T/T_c \simeq 1.10\)) we collected 1M measurements on a \(18^2 \times 36 \times 4\) lattice and at
\(\beta = 2.36\) (corresponding to \(T/T_c \simeq 1.21\)) we collected 1.5M measurements on a \(18^3 \times 4\) lattice.

To make sure that our Monte Carlo ensembles were not contaminated by configurations with mixed phase, we plotted histograms of the Polyakov loop averaged over the lattice configuration. In all cases, histograms showed two non-overlapping peaks located on opposite sides with respect to the origin, thus indicating that no configurations with mixed phase were present in the thermal equilibrium ensemble.

We determined the effective mass through

\[
G(z) = C_0 e^{-m_{\text{eff}} L_z} + C \left[ e^{-m_{\text{eff}} z} + e^{-m_{\text{eff}} (L_z - z)} \right]
\]

(2)

and found for both \(\beta\) values that data for \(m_{\text{eff}}\) as a function of \(z\) reach a plateau value, corresponding to the fundamental mass \(m_1\), as \(z\) is increased towards \(N_z/2\). The deviations from the plateau value at small \(z\) can be attributed to lattice artifacts and to the possible effect of other physical states. In order to single out the latter contribution, we rescaled the values of \(m_{\text{eff}}\) and \(z\) by \(m_1 \equiv 1/\xi\) and \(\xi\), respectively, and put together on the same plot data from \(\beta = 2.33\) and \(\beta = 2.36\). As shown in Fig. 1, data from the two different \(\beta\)'s fall on the same curve before reaching the plateau, indicating that the pre-asymptotic behavior of the effective mass is not a lattice artifact, but it is instead a physical effect, due to higher mass excitations.

Universality arguments hold in the scaling region of the deconfinement transition: in order to check whether our choice of \(\beta\) is in fact within this region, one can compare the values of the lowest screening mass \(m_1\) at \(\beta_1 = 2.33\) and \(\beta_2 = 2.36\), whose ratio is predicted by universality to scale as:

\[
\frac{am_1(\beta_1)}{am_1(\beta_2)} \sim \left( \frac{\beta_1 - \beta_c}{\beta_2 - \beta_c} \right) ^ \nu
\]

(3)

where \(\nu \sim 0.63\) is the correlation length critical index for the three dimensional Ising model. The values we measured\(^1\) give

\[
\frac{am_1(\beta_1)}{am_1(\beta_2)} = \frac{0.3654(31)}{0.5431(61)} = 0.673(13)
\]

(4)

\(^1\)The value quoted for \(\beta_1\), which is the point where the analysis of the whole spectrum was performed, was obtained with the variational method described below. The value at \(\beta_2\) was obtained simply from the long-distance behavior of \(m_{\text{eff}}\) as defined in Eq. (2). The value obtained with the latter method at \(\beta_1\) is compatible with the one quoted.
Figure 1: Effective screening masses in physical units vs the separation between walls in physical units at $\beta = 2.33$ on a $18^2 \times 36 \times 4$ lattice and $\beta = 2.36$ on a $18^3 \times 4$ lattice.

We conclude that the scaling region includes, within the accuracy of our data, these two values of $\beta$, and hence that such region extends at least up to temperatures $\sim 1.2 T_c$. On the other hand, the value of the mass at $T = 2T_c$ reported in Ref. [3] is rather far from the scaling prediction, therefore the scaling region does not extend up to $2T_c$.

In order to detect possible excited states, we adopted the so-called “variational” method [22, 23]. The method consists of the following steps: (i) defining a basis of (wall-averaged) operators $\{O_\alpha\}$, (ii) computing the con-
nected cross-correlation matrix among these operators,
\[ C_{\alpha\beta}(z) = \langle O_{\alpha}(z)O_{\beta}(0) \rangle - \langle O_{\alpha}(z) \rangle \langle O_{\beta}(0) \rangle , \]
(6)

(iii) diagonalizing \( C_{\alpha\beta}(z) \) to obtain the eigenvalues \( \lambda_i(z) \) and finally (iv) extracting the masses \( m_i \) through the relation
\[ \lambda_i(z) = c_0 + c \left[ e^{-m_i z} + e^{-m_i (L_z - z)} \right] . \]
(7)

The fundamental mass \( m_1 \) corresponds to the leading eigenvalue \( \lambda_1 \), the first excited mass \( m_2 \) corresponds to the next-to-leading eigenvalue \( \lambda_2 \), and so on. In practical simulations, for each value of \( z \) we have a numerical determination of an “effective” mass \( m_i(z) \) through Eq. (7). As \( z \) increases, \( m_i(z) \) reaches a plateau value which we identify with the screening mass \( m_i \).

In our simulations, the signal-to-noise ratio allowed to extract a screening mass only from the two leading eigenvalues.

The effectiveness of this method relies on the choice of a “good” set of operators; it is convenient to define operators living on different length scales (for instance, by use of recursive definitions). Moreover, by defining operators with non-trivial transformation under spatial rotation, it is possible to look for states with non-zero angular momentum.

As a first set of operators with \( 0^+ \) quantum numbers, we considered those which can be built by adapting to the present case the recursive “smoothing” procedure used in Ref. \[ 15 \] for the 3D Ising model and the 3D lattice regularized \( \phi^4 \) theory:
\[ P^{(0)}(x, y, z) = P(x, y, z) , \quad P^{(n+1)}(x, y, z) = \text{sign}(u) \left[ (1 - w)|u| + wy \right] , \]
(8)

where \( u \) is the average of four nearest values of the Polyakov loop on the \( xy \)-plane,
\[ u = \frac{1}{4} \left( P^{(n)}(x-a, y, z) + P^{(n)}(x, y-a, z) + P^{(n)}(x+a, y, z) + P^{(n)}(x, y+a, z) \right) , \]
and we chose \( w = 0.1 \) and \( y = \langle P(x, y, z) \rangle \). We considered a subset of six such (wall-averaged) operators corresponding to \( n = 0, 3, 6, 9, 12 \) and 15 smoothing steps. At \( \beta = 2.33 \) \( (T/T_c \simeq 1.10) \) on a \( 18^2 \times 36 \times 4 \) lattice, we collected \( \sim 1.6 \text{M} \) measurements.
The relevant observable to check our ansatz is the ratio between the first excited and the fundamental screening masses in the scalar channel, which in the 3D Ising and $\phi^4$ theories assumes the value

$$\frac{m_2}{m_1} = 1.83(3).$$

The effective values of this ratio for $SU(2)$ gauge theory are shown in Fig. 2: while the determination of this ratio is much more difficult than in the Ising model, the compatibility of the result with the expectations from universality is satisfactory. We have also observed that the excited scalar state cannot be detected when only the first four operators of our set (0, 3, 6 and 9 smoothing levels) are included in the analysis. This fact suggests that the first excited state in the $0^+$ channel couples with operators living at rather large length scales.

![Figure 2: Ratio of the first excited effective screening mass in the $0^+$ channel to the fundamental one vs $z$ at $\beta = 2.33$ on a $18^2 \times 36 \times 4$ lattice. The statistics of the simulation is $\sim 1.6M$.](image)

Finally, in order to look for states with non-zero angular momentum, we
considered a second set of $2^+$ operators, inspired by Ref. 18.

$$P^{(n)}(z) = \frac{1}{N_x N_y} \sum_{n_1=1}^{N_x} \sum_{n_2=1}^{N_y} P(n_1 a, n_2 a, z) \times \left[ P(n_1 a + n a, n_2 a, z) - P(n_1 a, n_2 a + n a, z) \right], \quad (10)$$

with $n = 1, \ldots, 5$. By the same procedure adopted in the case of the screening mass in the $0^+$ channel, we got for the fundamental mass in the $2^+$ channel the value $m_1^{(2^+)} = 0.86(21) a^{-1}$. Again, we studied the observable $m_1^{(2^+)/m_1^{(0^+)}}$, i.e. the ratio between the fundamental mass in the $2^+$ channel, determined from this second basis of operators, and the fundamental mass in the $0^+$ channel, determined from the first basis of operators. The result of our determination, as a function of $z$, is shown in Fig. 3. Despite the noisiness, one can again see that the result is compatible with the value found \textsuperscript{18} in the 3D Ising universality class\textsuperscript{2}

$$\frac{m_1^{(2^+)}}{m_1^{(0^+)}} = 2.56(4). \quad (11)$$

3 Discussion

The numerical results we have obtained are in agreement with our starting hypothesis: the spectrum of screening masses for $SU(2)$ gauge theory near $T_c$ is indeed the one predicted by universality arguments, i.e. it coincides with the spectrum of the transfer matrix of the 3D Ising model in the scaling region of the broken-symmetry phase. In particular a bound state of two elementary screening masses can be detected. An unambiguous verification of the coincidence of the two spectra will require both the analysis of more states and a substantial increase in the accuracy in the masses we have determined.

At first sight, application of these methods to the case of $SU(3)$ does not seem possible, given that the deconfinement transition is first order. However, the transition is rather weak, both in the gauge theory and in the 3-state Potts model that would be in the same universality class if universality

\footnotetext[2]{This mass ratio was actually evaluated in Ref. 18 for the 3D Ising gauge model: however duality implies that in the broken symmetry phase the spectrum coincides with the one of the 3D Ising model.}
Figure 3: Ratio of the fundamental effective screening mass in the $2^+$ channel to the fundamental one in the $0^+$ channel vs $z$ at $\beta = 2.33$ on a $18^2 \times 36 \times 4$ lattice. The statistics of the simulation is $\sim 1.8M$. The horizontal lines represent the same mass ratio for the 3D Ising model [18], with its uncertainty.

arguments applied. Therefore the three-dimensional correlation length at criticality, while not diverging, assumes rather high values: hence, there exists a range of temperatures in which the correlation length is large in terms of lattice spacing, and it is not unreasonable to expect the spectra of the two theories to be similar in such region. We plan to investigate this issue studying the spectrum of the three-dimensional 3-state Potts model in the broken symmetry phase, and comparing it to the spectrum of $SU(3)$ pure gauge theory near $T_c$.

Acknowledgement. We are grateful to M. Caselle for many useful discussions.
References

[1] J. B. Kogut, D. Toublan and D. K. Sinclair, arXiv:hep-lat/0208076.

[2] S. Datta and S. Gupta, Nucl. Phys. B 534 (1998) 392 [arXiv:hep-lat/98-06034].

[3] S. Datta and S. Gupta, Phys. Lett. B 471 (2000) 382 [arXiv:hep-lat/99-06023].

[4] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B 503 (1997) 357 [arXiv:hep-ph/9704416].

[5] A. Hart and O. Philipsen, Nucl. Phys. B 572 (2000) 243 [arXiv:hep-lat/9908041].

[6] A. Hart, M. Laine and O. Philipsen, Nucl. Phys. B 586 (2000) 443 [arXiv:hep-ph/0004060].

[7] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B 210 (1982) 423.

[8] S. Fortunato, F. Karsch, P. Petreczky and H. Satz, Nucl. Phys. Proc. Suppl. 94 (2001) 398 [arXiv:hep-lat/0010026].

[9] R. Fiore, A. Papa and P. Provero, Nucl. Phys. Proc. Suppl. 106 (2002) 486 [arXiv:hep-lat/0110017].

[10] R. Fiore, A. Papa and P. Provero, Phys. Rev. D 63 (2001) 117503 [arXiv:hep-lat/0102004].

[11] A. Papa and C. Vena, arXiv:hep-lat/0203007.

[12] J. Engels and T. Scheideler, Nucl. Phys. B 539 (1999) 557 [arXiv:hep-lat/9808057].

[13] F. Gliozzi and P. Provero, Phys. Rev. D 56 (1997) 1131 [arXiv:hep-lat/9701014].

[14] R. Fiore, F. Gliozzi and P. Provero, Phys. Rev. D 58 (1998) 114502 [arXiv:hep-lat/9806017].

[15] M. Caselle, M. Hasenbusch and P. Provero, Nucl. Phys. B 556 (1999) 575 [arXiv:hep-lat/9903011].
[16] M. Caselle, M. Hasenbusch, P. Provero and K. Zarembo, Nucl. Phys. B 623 (2002) 474 [arXiv:hep-th/0103130].

[17] R. Fiore, A. Papa and P. Provero, arXiv:hep-lat/0208020.

[18] V. Agostini, G. Carlino, M. Caselle and M. Hasenbusch, Nucl. Phys. B 484 (1997) 331 [arXiv:hep-lat/9607029].

[19] I. Montvay and P. Weisz, Nucl. Phys. B 290 (1987) 327.

[20] R. Petronzio and E. Vicari, Phys. Lett. B 254 (1991) 444.

[21] A. D. Kennedy and B. J. Pendleton, Phys. Lett. B 156 (1985) 393.

[22] A. S. Kronfeld, Nucl. Phys. Proc. Suppl. 17 (1990) 313.

[23] M. Lüscher and U. Wolff, Nucl. Phys. B 339 (1990) 222.