Effect of simulating parity-odd observables in high energy heavy ion collisions on 
Balance Functions of charged particles and elliptic flow of pions 

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At the early stage of heavy ion collisions, non-trivial topologies of the gauge fields can be created resulting in an imbalance of axial charge density and eventually separation of electric charges along the direction of the magnetic field produced in such collisions. This process is called the chiral magnetic effect (CME). In this work we implement such a charge separation at the partonic level in AMPT for Au+Au collisions at \( \sqrt{s} \approx 200 \) GeV to study its consequence on experimental observables. We present the effects on the pion elliptic flow (\( v_2 \)) and the charged particle Balance Function (BF) for varying strengths of initial charge separation. We find that the shape of the balance function is sensitive to the increasing charge separation. The \( v_2 \) of pion shows a strong decreasing trend at higher transverse momentum (\( p_T \)) with increasing charge separation.

I. INTRODUCTION

In high energy heavy ion collisions, gluonic configuration such as sphalerons and instantons [1–5] can change the left-handed quarks to right-handed ones and vice versa through axial anomaly [6–9]. The interactions between these gluonic configurations and the quarks break the parity (P) and charge conjugation parity (CP) symmetry due to this axial anomaly. This results in a net chirality due to P and CP breaking and generates an asymmetry between the number of left and right-handed fermions. The P and CP violation result in a separation between the positive and negative charges along the direction of magnetic field that is produced in heavy ion collisions [10]. This Phenomenon of QCD anomaly-driven charge separation is referred to as the Chiral Magnetic Effect (CME). Several experimental measurements have been dedicated towards the search for CME at RHIC and LHC[11–16].

It has been estimated that in high energy heavy-ion collisions spectator protons produce a strong magnetic fields \( eB_y \approx m_c^2 \) or \( \sim 3.14 \times 10^{14} \) T [4]. A P- and CP-odd domain in the presence of a large magnetic field can generate chirality by inducing \( up \) – \( down \) asymmetry in the production of quarks and antiquarks. In this work we implement this charge separation in an event generator known as A Multi-Phase Transport(AMPT) model.

This paper is organized as follows, in section II we have described the AMPT model and in section III, we describe the method of charge separation that is implemented at quark level. In section IV, we have discussed the observables and present results and summary in section V and VI respectively.

II. A MULTI-PHASE TRANSPORT MODEL

In this work, we have implemented charge separation at the partonic level in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) using the AMPT model. The AMPT model consists of different components with the heavy ion jet interaction generator (HIJING) to implement the initial conditions, Zhang’s parton cascade (ZPC) for modelling the partonic scatterings, the Lund string fragmentation model or a quark coalescence model for hadronization, and a relativistic transport (ART) model for hadronic re-scattering[17–19]. HIJING provides spatial and momentum distributions of the minijet partons and of the soft string excitations [20, 21]. The cascading of partons are carried out using the ZPC model [22]. The parton interaction cross section is set at 10 mb in this work. AMPT model has two versions , one is the Default AMPT and the other is the string melting(SM) version. In the default AMPT model, partons are combined with their parent strings when they stop interacting and the resulting strings are converted to hadrons using the Lund string fragmentation model [23–24]. In the AMPT with string melting [26–28], a quark coalescence model is used instead to combine partons into hadrons. In the string melting mechanism, all excited strings that are not from the projectile and the target nucleons and without any interactions are converted to partons according to the flavor and spin structures of their valence quarks. Subsequently the dynamics of the hadronic matter is described by a hadronic cascade, which is based on the ART model [29, 30].

III. PROCEDURE OF GENERATING CHARGE SEPARATION

The manifestation of the Chiral magnetic effect is seen by a charge separation along the direction of the magnetic field. The charge separation is a result of P and CP odd domains. In our implementation of the CME effect in heavy ion collisions, we introduce a charge separation and
make sure that the current thus introduced is along the direction of the magnetic field. In semi-peripheral collisions, as has been argued earlier, spectator protons create a magnetic field perpendicular to the reaction plane. We thus introduce charge separation perpendicular to the reaction plane.

For having such a direction of the charge separation, we first choose regions that are perpendicular to the reaction plane. The momenta of the quarks in those selected regions are then modified in such a way that it results in a net positive charge in the upward direction and a net negative charge in the downward direction.

To achieve this, we replace a fraction of total number of upward going negatively charged quarks with downward going positively charged quarks and vice versa. In practice, \( -p_y \) of a positively charged \( u \) quark and \( +p_y \) of a negatively charged \( \bar{u} \) quark are flipped to each other making positively charged quark upgoing and negatively charged quark down going. Similarly flipping takes place between the \( +p_y \) of a negatively charged \( d \) quark with the \( -p_y \) of a positively charged \( \bar{d} \) quark. Fig. 1 shows schematically the method of charge separation mechanism we have implemented in AMPT-SM model. As shown in the figure, before flipping, each of the regions marked \( a \) and \( b \), lying perpendicular to the reaction plane are with net-charges of \( \pm \frac{1}{2}e \). Now, after flipping the corresponding regions are with charges of \( \pm \frac{1}{2}e \) respectively thereby generating a charge separation perpendicular to the reaction plane.

![Diagram of charge separation mechanism](image)

**FIG. 1.** (Color online) Charge Separation mechanism shown schematically

After the implementation of flipping at the partonic level, the evolution of the system follows. The fraction \( f \) of the total number of quarks that have been flipped is taken a parameter. We calculate \( f \times M \) for every event, where ‘M’ is the multiplicity of \( u \) or \( \bar{u} \) quarks and the smaller of the two resultant numbers has been chosen as the number of quarks to be flipped. Similar procedure has been followed for \( d \) and \( \bar{d} \).

In this work, \( f=0.1,0.2,0.3,0.4,0.5 \) & 0.6 have been used. The AMPT-SM has been used in which the quarks are hadronized by coalescence method as discussed earlier. The observables discussed in section IV have been studied for the finally produced hadrons.

**IV. OBSERVABLES**

In this section, we discuss two observables to investigate the sensitivity of charge separation effects generated by different flipping fractions. The Observables are (a) charge balance function and (b) elliptic flow coefficient \( (v_2) \).

**A. Balance Function**

Charge dependent correlation is a powerful tool to study the properties of a system created in high energy heavy-ion collisions. These correlations are studied with a correlation function called balance function (BF). Balance function has been shown to be sensitive to the mechanisms of charge formation and the subsequent relative diffusion of the balancing charges\[33\]. The BF gives the normalized net correlation between oppositely charged particles. The Balance Function that can be studied as a function of rapidity \( (y) \), pseudo-rapidity \( (\eta) \) or azimuthal angle \( (\phi) \) is defined as \[32, 33\]

\[
B = \frac{1}{2} \left[ \frac{\langle N_{(a,b)} \rangle - \langle N_{(a,a)} \rangle}{\langle N_{a} \rangle} + \frac{\langle N_{(b,a)} \rangle - \langle N_{(b,b)} \rangle}{\langle N_{b} \rangle} \right],
\]

where \( a \) and \( b \) could be different kinds of particles. For example \( a \) could refer to all negative particles and \( b \) to all positive particles. Here, \( N_{a,b} \) counts pairs of opposite charges satisfying a criterion that their relative rapidity/pseudo-rapidity or \( \phi \) is within a specified range, whereas \( N_{a} \) is the number of positive or negative particles in the chosen phase space. Here the angular brackets represent averaging over the events.

The BF is measured by making combinations of two types of particles called trigger and associated particles of different ranges of the transverse momenta \( (p_{T,tr} \& p_{T,as}) \) such that \( p_{T,as} < p_{T,tr} \). Since the combination of \( p_{T,as} < p_{T,tr} \) is required, double counting of pairs is suppressed and the factor \( \frac{1}{2} \) can be removed. The balance function then reads as \[33\]

\[
B(\Delta \eta, \Delta \phi, p_{T,tr}, p_{T,as}) = C_{+,+}(\Delta \eta, \Delta \phi, p_{T,tr}, p_{T,as}) - C_{+,+}(\Delta \eta, \Delta \phi, p_{T,tr}, p_{T,as}) - C_{+,+}(\Delta \eta, \Delta \phi, p_{T,tr}, p_{T,as}) - C_{+,+}(\Delta \eta, \Delta \phi, p_{T,tr}, p_{T,as}),
\]

(2)
with

\[ C_{a,b}(\Delta \eta, \Delta \varphi, p_{T, tr}, p_{T, as}) = \frac{\langle N_{a_{corr}}^{\phi}(\Delta \eta, \Delta \varphi, p_{T, tr}, p_{T, as}) \rangle}{\langle N_{a_{corr}}^{\phi} p_{T, tr} \rangle}. \]  

Here we have used pseudorapidity($\eta$) between -0.8 to 0.8 and $p_{T, tr}$ and $p_{T, as}$ between 0.2 to 2 GeV/c.

First, a two-dimensional correlation function is generated in ($\Delta \eta, \Delta \phi$) which is then projected on $\Delta \phi$ for further investigations. The Balance function correlation follows a characteristic structure with a peak at near side ($\Delta \eta = 0, \Delta \phi = 0$). In this paper we have studied the sensitivity of the BF structure with the varying fraction of charge flipping.

**B. Elliptic Flow of Pion**

Flow is an observable providing experimental information about the equation of state and the transport properties of the created medium \[35\]. In non-central collisions, geometrical overlap region and the initial matter distribution is anisotropic (almond-shaped). For an interacting matter, the initial spatial asymmetry is converted through multiple collisions into momentum anisotropy\[35\]. The azimuthal dependence of the particle yield can be decomposed in a Fourier series as

\[ E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_x dp_y dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_R)] \right) \]  

where $E$ is the energy of the particle, $p$ is the momentum, $p_t$ is the transverse momentum, $\phi$ is the azimuthal angle, $y$ is the rapidity, and $\Psi_R$ is the reaction plane angle (plane defined by the beam axis and the impact parameter). The sine terms in the above Fourier expansion vanish because of the reflection symmetry with respect to the reaction plane. The $n$-th harmonic co-efficient defined as $v_n$ is $\langle \cos[n(\phi - \Psi)] \rangle$ where $\langle .. \rangle$ denotes average over particles\[35\]. The second Fourier coefficient $v_2$ called elliptic flow ($\cos(2\phi)$) is the quantity of our interest. In our simulation, $\Psi_R = 0$ as per the implementation of AMPT model.

**V. RESULTS**

Fig. 2 shows the net electric charge distributions on the transverse plane before flipping of quarks in Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{GeV}$. The contours indicate the net charge density profile. One can see that the distribution is symmetric. Fig. 3 shows the net electric charge distributions after flipping with 20% flipping fraction.

It is clearly observed that an out of plane charge separation is generated after introduction of charge separation effects in the AMPT model.

![FIG. 2. (Color online) Net electric charge distributions on the transverse plane before flipping of quarks.](image1)

We have studied the effect of flipping on the average energy of the partons at freeze out i.e energy of parton after ZPC. Fig. 4 shows the variation of average parton energy with freeze out time. It is seen that, in general, the energy is higher for partons having smaller freeze out time and reduces for larger freeze out time. Interestingly, the average energy of partons increase after flipping presumably due to an electric field acting on them thereby accelerating the partons. These accelerated partons are likely to enhance the average momenta of the produced particles as shown in Fig. 5 where the $\phi$ distribution of pions shows a clear increasing $\langle p_T \rangle$ with increasing flipping fractions. $\langle p_T \rangle$ of pions increases at bins $\phi=1.57$ and $\phi = -1.57^\circ$ where flipping takes place.

![FIG. 3. (Color online) Net electric charge distributions on the transverse plane after flipping with a 20% flipping fraction.](image2)
leading to a peak at $\pi^e$.

![Figure 4](image-url)  
**FIG. 4.** (Color online) Average energy of partons as a function of freeze out time from string melting AMPT model.

![Figure 5](image-url)  
**FIG. 5.** (Color online) $\langle p_T \rangle$ with azimuthal angle of pions for different flipping fractions.

In this work, we propose the balance function of charged particles and the elliptic flow of pions as the observables. In high energy heavy ion collisions, these observables might be studied with centrality as the magnitude of the magnetic field create in such collisions depends on centrality. In the present study, changing flipping fractions represents different magnitude of the produced magnetic field and can be compared with collisions of various centralities.

Fig. 6 and Fig. 7 show the charged particle balance function at different flipping fractions. It is clearly seen that the shape of the balance function evolves with the flipping fraction. While for no-flipping it shows a peak~0°, the peak slowly shifts towards $\pi^e$ at 60% flipping. This is clearly the effect of net charge produced perpendicular to the reaction plane after flipping. The net charge results in dilution of the near side peak(0°) and starts producing back to back charge separation

![Figure 6](image-url)  
**FIG. 6.** (Color online) $BF(\Delta \phi)$ for Au+Au minimum bias collisions which at $\sqrt{s_{NN}} = 200$GeV with no flipping.

![Figure 7](image-url)  
**FIG. 7.** (Color online) BF for Au+Au minimum bias at $\sqrt{s_{NN}} = 200$GeV with different flipping fractions.

Fig. 8 shows the elliptic flow ($v_2$) of pions as a function of $p_T$ for different flipping fractions in the initial partonic state of the collisions. It is observed that the elliptic flow of pion increases up to $p_T \sim 1.1$ GeV/$c$ and then decreases at higher $p_T$. In general as there is an increase in out of plane particle production at higher $p_T$, $v_2$ starts decreasing reaching a negative value at flipping fraction > 50%.

VI. SUMMARY

In this study, momenta of initial partons of AMPT generator has been flipped to generate an out-of-plane charge separation in Au+Au collisions at $\sqrt{s} = 200$GeV. The fraction of a type of quark($u, \bar{u}, d, \bar{d}$) has been taken as a variable. This charge separation represents the effect of
odd-parity observable in heavy ion collisions where magnetic fields are generated. We have studied the effect of their charge separation on two widely used observables i.e charge particle BF & elliptic flow of pions. The observables are chosen in such a way that these represent to characterize the effect of net-charge & their distribution on azimuthal plane. Different fraction must be considered to represent varying centrality in such collisions. In this study, with varying fraction of flipping, both the BF & $v_2$ show significant sensitivity with the peak of the BF shifting from $\Delta \phi = 0$ towards $\Delta \phi = \pi$ with increasing flipping fraction, the $v_2$ of pions decreases at higher $p_T$. The reduction in $v_2$ with respect to no-flipping scenario depends on the flipping fraction. It is possible to further study the shape evolution of the balance function by calculating the shape variables like width or asymmetry as have been discussed in \[39\]. For the study of $v_2$ alone, however, the decreasing trend with $p_T$ could be explained using energy loss. It is therefore suggested to look at both the observables together for making an unambiguous conclusion on the generation of parity-odd effects in high energy heavy ion collisions.

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