Efficacy of scalar leptoquark on $B_s \to (K^{(*)}, D_s^{(*)})\tau \bar{\nu}_\tau$ decay modes

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Abstract

We scrutinize the impact of various relevant scalar leptoquarks on the physical observables associated with the rare semileptonic decay processes of $B_s$ meson involving $b \to (u,c)\bar{l}\nu_l$ quark level transitions. We constrain the new parameter space consistent with the experimental limit on $\text{Br}(B_{u,c} \to \tau \nu_l)$, $\text{Br}(B \to \pi \tau \bar{\nu}_\tau)$, $R_{D^{(*)}}$, $R_{J/\psi}$ and $R_{\pi}^l$ observables. Using the allowed parameter space, we compute the branching ratios, forward-backward asymmetries, lepton and hardon polarization asymmetries of $B_s \to (K^{(*)}, D_s^{(*)})\tau \bar{\nu}_\tau$ decay modes. Forbye, we look at the possibility of existence of lepton non-universality in these processes.

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I. INTRODUCTION

Even though the Standard Model (SM) is currently the best theory of particle physics, it does not explain the complete picture of subatomic world. There are still some fundamental questions which are not answered within the domain of SM, so one has to search for new physics (NP) beyond it. In this aspect, the investigation of weak decays of $B$ mesons provide an excellent window. Recently various challenging anomalies in the sector of the violation of lepton flavor universality (LFU) in semileptonic B meson decays, especially in the measurements of $R_D^{(*)}, R_{J/\psi}$ have been observed. The measurements in the observables like $R_D = \frac{\text{Br}(B \to D \tau \bar{\nu}_\tau)}{\text{Br}(B \to D l \bar{\nu}_l)}$, $R_{D^*} = \frac{\text{Br}(B \to D^{*+} \tau \bar{\nu}_\tau)}{\text{Br}(B \to D^{*-} l \bar{\nu}_l)}$ by BaBar [1, 2], Belle [3–7] and LHCb [8–10] experiments deviate from their standard model predictions at 3.8$\sigma$ level [11]. Another observable, $R_{J/\psi} = \frac{\text{Br}(B_c \to J/\psi \tau \bar{\nu}_\tau)}{\text{Br}(B_c \to J/\psi l \bar{\nu}_l)}$ measured by LHCb shows a discrepancy at 1.7$\sigma$ [12] from the SM value. The combined data of $R_D^{(*)}$ by HFLAV Collaboration [11] are

$$R_D^{\text{Expt}} = 0.340 \pm 0.027 \pm 0.013,$$

$$R_{D^*}^{\text{Expt}} = 0.295 \pm 0.011 \pm 0.008,$$

respectively, whereas the value of $R_{J/\psi}$ by LHCb is [12]

$$R_{J/\psi}^{\text{Expt}} = 0.71 \pm 0.17 \pm 0.18,$$

with the first error as statistical and the second one as systematic. The SM predictions of all these observables $R_D$, $R_{D^*}$ [13–15], and $R_{J/\psi}$ [16–18] are $0.299 \pm 0.003$, $0.258 \pm 0.005$, and $0.289 \pm 0.01$ respectively. Since the uncertainties from the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the hadronic transition form factors are cancelled out to a large extent in all these ratios of branching fractions of these decay modes, any deviation in these observables from their SM values would definitely point out towards the signals of new physics. Besides these ratios, another discrepancy in $b \to u l \bar{\nu}_l$ processes is also noticed in the measured ratio [15]

$$R_l^l = \frac{\tau_{B^0}}{\tau_{B^-}} \frac{\text{Br}(B^- \to l^- \bar{\nu}_l)}{\text{Br}(B^0 \to \pi^+ l^- \bar{\nu}_l)}, \quad l = e, \mu,$$

where $\tau_{B^0}$ ($\tau_{B^-}$) is the life time of $B^0$ ($B^-$) meson. Taking the experimental results of the branching ratios of $B^-_u \to \tau^- \bar{\nu}_\tau$ and $B^0 \to \pi^+ l^- \bar{\nu}_l$ decay processes

$$\text{Br}(B^-_u \to \tau^- \bar{\nu}_\tau)^{\text{Expt}} = (1.09 \pm 0.24) \times 10^{-4},$$

$$\text{Br}(B^0 \to \pi^+ l^- \bar{\nu}_l)^{\text{Expt}} = (1.45 \pm 0.05) \times 10^{-4},$$

respectively.
with $\tau_{B^-}/\tau_{B^0} = 1.076 \pm 0.004$ from [19], we obtain

$$R_{\pi}^{l}|^{\text{Expt}} = 0.699 \pm 0.156,$$

which has also nearly 1σ discrepancy from its SM value $R_{\pi}^{l}|^{\text{SM}} = 0.583 \pm 0.055$.

Over the last few years, most of the research works have focused on the $B \to D^{(*)}\tau\bar{\nu}_\tau$ problems rather than the issues in the $B_s \to D_s^{(*)}l\bar{\nu}_l$ channels. Though both of these decays are driven by the $b \to c$ transition, the difference is in the spectator quark. The decay channels $B_s \to D_s^{(*)}l\bar{\nu}_l$ have the spectator strange quark whereas $B \to D^{(*)}l\bar{\nu}_l$ have either up or down quark. These decays have $SU(3)$ flavor symmetry with the dependence on CKM matrix element $V_{cb}$ and hence they should show similar properties in the limit of $SU(3)$ flavor symmetry. Further, studies of $B_{(s)} \to D_{(s)}^{(*)}\tau\bar{\nu}_\tau$ may help to determine the value of $|V_{cb}|$ with higher precision in both exclusive and inclusive measurements. Several authors have investigated the semileptonic $B_s \to D_s^{(*)}\tau\bar{\nu}_\tau$ decays within SM and the branching fractions have been computed using the constituent quark meson (CQM) model, QCD sum rules approach, the light cone sum rules (LCSR) approach [21], the covariant light-front quark model (CLFQM) [22], lattice QCD method [23–28] and in the perturbative QCD factorization approach [29–31]. Very recently the problem has been studied in Ref. [32] using effective theory formalism in presence of NP in a model independent way. Moreover the recent results of $R_{D^{(*)}}$ also motivates to analyse $b \to u\tau\bar{\nu}_\tau$ counterparts and look for the lepton non-universality (LNU) observables. With strange quark as spectator quark another possible decay channel of $B_s$ meson is $B_s \to K^{(*)}\tau\bar{\nu}_\tau$ decays with $b \to u\tau\bar{\nu}_\tau$ charged current interaction and which again, have been studied by a few authors [33–37] within SM. Recently $B_s \to K^{(*)}\tau\bar{\nu}_\tau$ decays have been worked out in the Ref. [38, 39] within SM and beyond. The transition $b \to u\tau\bar{\nu}_\tau$ process is also potentially sensitive due to the presence of a charged Higgs boson in new physics models like the two-Higgs-doublet model (2HDM) [40, 41] and in the minimal supersymmetric model (MSSM) [42, 43] and again, a systematic study of process both experimentally and theoretically can be helpful for the measurement of the smallest element of CKM matrix, $|V_{ub}|$.

In this concern, we would like to study these particular rare semileptonic decays of $B_s$ mesons with a lepton and a neutrino in the final state i.e., $B_s \to M\tau\bar{\nu}_\tau$ where $M$ is either $D_s^{(*)}$ mesons or $K^{(*)}$ mesons in the scalar leptoquark (SLQ) model. Leptoquark (LQ) is a unique hypothetical color triplet scalar (spin=0) or vector (spin=1) bosonic particle, which
acts as a bridge between the quark and lepton sectors, thus carries both baryon (B) and lepton (L) quantum numbers. The B and L numbers conserving LQs avoid rapid proton decays and are light enough to be seen in the present experiments. The existence of LQ is proposed in many new theoretical frameworks, such as the grand unified theories [44–47], Pati-Salam model [48–52], quark and lepton composite model [53] and the technicolor model [54, 55]. The phenomenology of SLQs in connection to only flavor anomalies, and both flavor and dark matter sectors has been studied extensively in the literature [56–83].

In this work, we consider three relevant SLQs, singlet $S_1(\bar{3}, 1, 1/3)$, doublet $R_2(3, 2, 7/6)$ and triplet $S_3(\bar{3}, 3, 1/3)$, which provide additional vector, scalar and tensor type couplings contributions to the SM. Constraining the new parameter space from $\text{Br}(B_{u,c} \to \tau \bar{\nu}_\tau)$, $\text{Br}(B \to \pi \tau \bar{\nu}_\tau)$, $R_{D(\ast)}$, $R_{J/\psi}$ and $R_{\ell \pi}$ parameters, we compute the branching ratios, forward-backward asymmetries, polarization asymmetries and lepton non-universality parameters of $B_s \to (D_s^{(*)}, K^{(*)})\tau \bar{\nu}_\tau$ processes.

The paper is organised as follows. We present the theoretical framework and the most general effective Hamiltonian associated with $b \to (u, c)\ell \bar{\nu}_l$ processes in section II. In section III, we provide the detailed discussion on the new scalar leptoquarks and the constrained on new couplings from the available experimetal results on feasible observables. The numerical analysis of all the physical observables of $B_s \to (D_s^{(*)}, K^{(*)})\tau \bar{\nu}_\tau$ decay modes in the presence of leptoquarks are given in section IV and section V summarize our estimated results.

II. THEORETICAL FRAMEWORK FOR THE ANALYSIS OF $b \to (u, c)\ell \bar{\nu}_l$ DECAY PROCESSES

Considering the neutrinos to be left-handed, the most general effective Lagrangian describing the $b \to q \tau \bar{\nu}_l$, $(q = u, c)$ transition is given by [84, 85]

$$
\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{qb} \left\{ (1 + V_L) \bar{L}_\gamma \gamma_\mu L \bar{q}_L \gamma^\mu b_L + V_R \bar{L}_\gamma \gamma_\mu L \bar{q}_R \gamma^\mu b_R + S_L \bar{L} R \gamma_\mu L \bar{q}_L b_R + S_R \bar{L} R \gamma_\mu L \bar{q}_R b_R + T_L \bar{L} R \sigma_{\mu\nu} \gamma_\mu \gamma_\nu L \bar{q}_R b_R \right\} + \text{h.c.}, \qquad (8)
$$

where $l = e, \mu, \tau$ are the flavor of neutrinos. Though the new Wilson coefficients $V_{L,R}, S_{L,R}, T_L$ are zero in the SM, they can have nonvanishing values in the presence of new physics.
Using the generalized effective Lagrangian \[8\], the branching ratios of $B \to Pl\bar{v}_l$ processes, where $P = D, D_s, \pi, K$ are the pseudoscalar mesons are given by \[86, 87\]

$$
\frac{d\text{Br}(B_s \to Pl\bar{v}_l)}{dq^2} = \tau_{B_s} \frac{G_F^2 |V_{q\bar{b}}|^2}{192\pi^3 M_{B_s}^3} q^2 \sqrt{\lambda_P(M_{B_s}, M_P, q^2)} \left(1 - \frac{m_l^2}{q^2}\right)^2 \times
$$

$$
\left\{ \begin{array}{l}
|1 + V_L + V_R|^2 \left(1 + \frac{m_l^2}{2q^2}\right) H_0^2 + \frac{3 m_l^2}{2 q^2} H_t^2 \\
+ 3 |S_L + S_R|^2 H_s^2 + 8 |T_L|^2 \left(1 + \frac{2 m_l^2}{q^2}\right) H_T \\
+ 3 \text{Re}[(1 + V_L + V_R)(S_L^* + S_R^*)] \frac{m_l}{\sqrt{q^2}} H_s H_t \\
- 12 \text{Re}[(1 + V_L + V_R)T_L^*] \frac{m_l}{\sqrt{q^2}} H_T H_0 \end{array} \right\},
$$

(9)

where the helicity amplitudes in terms of form factors ($F_{0,\pm}$) are expressed as

$$
H_0 = \sqrt{\frac{\lambda_P(M_{B_s}^2, M_P^2, q^2)}{q^2}} F_+(q^2), \quad H_t = \frac{M_{B_s}^2 - M_P^2}{\sqrt{q^2}} F_0(q^2),
$$

$$
H_s = \frac{M_{B_s}^2 - M_P^2}{m_b - m_q} F_0(q^2), \quad H_T = -\sqrt{\frac{\lambda_P(M_{B_s}^2, M_P^2, q^2)}{M_{B_s} + M_P}} F_T(q^2),
$$

(10)

with

$$
\lambda_P(a, b, c) = ((a - b)^2 - c)((a + b)^2 - c).
$$

(11)

The branching ratios of $B_s \to Vl\bar{v}_l$ with respect to $q^2$, where $V = K^*, D^*, D_s^*$ are the vector bosons are given by \[86, 87\]

$$
\frac{d\text{Br}(B_s \to Vl\bar{v}_l)}{dq^2} = \tau_{B_s} \frac{G_F^2 |V_{q\bar{b}}|^2}{192\pi^3 M_{B_s}^3} q^2 \sqrt{\lambda_V(M_{B_s}^2, M_V^2, q^2)} \left(1 - \frac{m_l^2}{q^2}\right)^2 \times
$$

$$
\left\{ \begin{array}{l}
(|1 + V_L|^2 + |V_R|^2) \left(1 + \frac{m_l^2}{2q^2}\right) \left(H_{V,+}^2 + H_{V,-}^2 + H_{V,0}^2\right) + \frac{3 m_l^2}{2 q^2} H_{V,t}^2 \\
- 2 \text{Re}[(1 + V_L)V_R^*] \left(1 + \frac{m_l^2}{2q^2}\right) \left(H_{V,0}^2 + 2 H_{V,+} H_{V,-}\right) + \frac{3 m_l^2}{2 q^2} H_{V,t}^2 \\
+ \frac{3}{2} |S_L - S_R|^2 H_s^2 + 8 |T_L|^2 \left(1 + \frac{2 m_l^2}{q^2}\right) \left(H_{T,+}^2 + H_{T,-}^2 + H_{T,0}^2\right) \\
+ 3 \text{Re}[(1 + V_L - V_R)(S_L^* - S_R^*)] \frac{m_l}{\sqrt{q^2}} H_s H_{V,t} \\
- 12 \text{Re}[(1 + V_L)T_L^*] \frac{m_l}{\sqrt{q^2}} (H_{T,0} H_{V,0} + H_{T,+} H_{V,+} - H_{T,-} H_{V,-}) \\
+ 12 \text{Re}[V_R T_L^*] \frac{m_l}{\sqrt{q^2}} (H_{T,0} H_{V,0} + H_{T,+} H_{V,+} - H_{T,-} H_{V,-}) \end{array} \right\},
$$

(12)
where the hadronic amplitudes $H_{V,0}$, $H_{V,t}$, $H_{V,0}$, $H_{V,0}$, $H_{V,t}$ and $H_{S}$ in terms of the form factors $V, A_{0,1,2}, T_{1,2,3}$ can be found in the Refs. [86, 87]. Besides the branching ratios, we also explore more physical observables in these decay modes in order to probe the structure of NP. The zero crossing of lepton forward-backward asymmetry is one of the interesting observable, which is defined as [86, 88]

$$A_{FB} = \frac{\int_{0}^{1} d\cos \theta d\cos \theta - \int_{-1}^{0} d\cos \theta d\cos \theta}{\int_{-1}^{1} d\cos \theta d\cos \theta}.$$  (13)

Here $\theta$ is the angle between the three-momenta of $\tau$ and $B_s$ in the lepton rest frame. As like the $R_D^{(\ast)}$ LNU parameters, we also define the lepton universality violating parameters $R_{P(V)}^\ell$ as

$$R_{P}^\ell = \frac{\text{Br}(B_s \to P\tau \bar{\nu}_\tau)}{\text{Br}(B_s \to Pl \bar{\nu}_l)},$$  (14)
$$R_{V}^\ell = \frac{\text{Br}(B_s \to V\tau \bar{\nu}_\tau)}{\text{Br}(B_s \to Vl \bar{\nu}_l)}, \quad l = e, \mu.$$  (15)

Other amazing observables are the polarization asymmetry parameters. The $\tau$ polarization asymmetry of $B_s \to P\tau \bar{\nu}_\tau$ decay modes are given as [86],

$$P_{\tau} = \frac{\Gamma(B \to D^{(\ast)}\tau \bar{\nu}_\tau)|_{\lambda_\tau=1/2} - \Gamma(B \to D^{(\ast)}\tau \bar{\nu}_\tau)|_{\lambda_\tau=-1/2}}{\Gamma(B \to D^{(\ast)}\tau \bar{\nu}_\tau)|_{\lambda_\tau=1/2} + \Gamma(B \to D^{(\ast)}\tau \bar{\nu}_\tau)|_{\lambda_\tau=-1/2}},$$  (16)

and the $V( = K^*, D_s^*)$ longitudinal polarization parameters are defined as [88],

$$P_{V} = \frac{\Gamma(B_s \to V\tau \bar{\nu}_\tau)|_{\lambda_V=0} + \Gamma(B_s \to V\tau \bar{\nu}_\tau)|_{\lambda_V=1} + \Gamma(B_s \to V\tau \bar{\nu}_\tau)|_{\lambda_V=-1}}{\Gamma(B_s \to V\tau \bar{\nu}_\tau)|_{\lambda_V=0} - \Gamma(B_s \to V\tau \bar{\nu}_\tau)|_{\lambda_V=1} - \Gamma(B_s \to V\tau \bar{\nu}_\tau)|_{\lambda_V=-1}}.$$  (17)

The detailed expressions for the $q^2$ distributions for various $\tau$ and $V$ polarization states can be found in the Ref. [86].

### III. MODEL WITH SCALAR LEPTOQUARKS

In the presence of scalar LQ, the interaction Lagrangian responsible for the $b \to q\ell\bar{\nu}$ processes are given by [86],

$$\mathcal{L}_{\text{LQ}} = (y_{3L}^{ij} \bar{u}_{iR} \ell_{jL} + y_{2R}^{ij} \bar{Q}_{iL} i\sigma_2 \ell_{jR}) R_2 + (y_{1L}^{ij} \bar{Q}_{iL} i\sigma_2 L_{jL} + y_{1R}^{ij} \bar{u}_{iR} \ell_{jR}) S_1 + y_{3L}^{ij} \bar{Q}_{iL} i\sigma_2 \ell_{jR} S_3,$$  (18)

where $i$ and $j$ are the generation indices, $Q_{iL}$ ($u_{iR}, \, d_{iR}$) and $L_{jL}$ ($\ell_{jR}$) are the left (right) handed quark and lepton $SU(2)_L$ doublets (singlets) respectively. Here $Q^{c}_{iL}$ and $u^{c}_{iR}$ are the
charge-conjugated fermion fields. After performing the Fierz transformations, we obtain the additional Wilson coefficients contributions to the $b \to q \tau \bar{\nu}_l$ processes as \cite{86}

\[
V_L = \frac{1}{2\sqrt{2}G_F V_{mb}} \sum_{k=1}^{3} V_{k3} \left[ \frac{y_{1L}^{kL} y_{2L}^{m3*}}{2M^2_{S_1^{1/3}}} - \frac{y_{3L}^{kL} y_{3L}^{m3*}}{2M^2_{S_2^{1/3}}} \right], \tag{19a}
\]

\[
V_R = 0, \tag{19b}
\]

\[
S_L = 0, \tag{19c}
\]

\[
S_R = \frac{1}{2\sqrt{2}G_F V_{mb}} \sum_{k=1}^{3} V_{k3} \left[ \frac{y_{1L}^{kL} y_{1L}^{m3*}}{2M^2_{S_1^{1/3}}} - \frac{y_{2L}^{mL} y_{2L}^{k3*}}{2M^2_{R_2^{1/3}}} \right], \tag{19d}
\]

\[
T_L = \frac{1}{2\sqrt{2}G_F V_{mb}} \sum_{k=1}^{3} V_{k3} \left[ \frac{y_{1L}^{kL} y_{1L}^{m3*}}{8M^2_{S_1^{1/3}}} - \frac{y_{2L}^{mL} y_{2L}^{k3*}}{8M^2_{R_2^{1/3}}} \right], \tag{19e}
\]

where $V_{k3}$ denotes the elements of CKM matrix. Here $y_{xL}^{ij}$ and $y_{xR}^{ij}$ ($x = 1, 2, 3$) are the leptoquark couplings in the mass basis of the down type quarks and charged leptons. The upper index in the LQ mass denotes the electric charge of LQ.

A. Constraint on leptoquark couplings

With the idea on new Wilson coefficients in mind, we now move on to constrain the new parameters from the available experimentally feasible flavor observables like $\text{Br}(B_{u,c} \to \tau \bar{\nu}_\tau)$, $\text{Br}(B \to \pi \tau \bar{\nu}_\tau)$, $R_\pi^d$, $R_{D(\ast)}$ and $R_{J/\psi}$. We assume that only the third generation lepton receives the additional new physics contributions arising due to the scalar leptoquarks exchange and the couplings with light leptons are considered to be SM like. The SM branching ratios of $B_q \to \pi l \nu_l$ processes obtained by using the masses of all the particles, lifetime of $B_q$ meson, CKM matrix elements from \cite{19} and the $B \to \pi$ form factors from \cite{89,92}, are given by

\[
\text{Br}(B^0 \to \pi^+ \mu^- \bar{\nu}_\mu)^{\text{SM}} = (1.35 \pm 0.10) \times 10^{-4}, \tag{20}
\]

\[
\text{Br}(B^0 \to \pi^+ \tau^- \bar{\nu}_\tau)^{\text{SM}} = (9.40 \pm 0.75) \times 10^{-5}. \tag{21}
\]

Although, the branching ratio of the muonic channel agrees reasonably well with the experimental value\cite{6}, the tau-channel is within its current experimental limit \cite{19}

\[
\text{Br}(B^0 \to \pi^+ \tau^- \bar{\nu}_\tau)^{\text{Expt}} < 2.5 \times 10^{-4}. \tag{22}
\]
Including the new physics contribution, the branching ratios of $B_q \to l \bar{\nu}_l$ processes are given by [88]

$$\text{Br}(B_q \to l \bar{\nu}_l) = \frac{G_F^2 |V_{ql}|^2}{8 \pi} \tau_{B_q} f_{B_q}^2 m_l^2 M_{B_q} \left(1 - \frac{m_l^2}{M_{B_q}^2}\right)^2 \times \left|(1 + V_L - V_R) - \frac{M_{B_q}^2}{m_l (m_b + m_q)} (S_L - S_R)\right|^2.$$ \hspace{1cm} (23)

Using the decay constants $f_{B_u} = 190.5 \pm 4.2$ MeV, $f_{B_c} = 489 \pm 4 \pm 3$ MeV from [93, 94] and rest input parameters from [19], the branching ratios of $B_{u,c}^+ \to \tau^+ \nu_\tau$ processes in the SM are found to be

$$\text{Br}(B_u^+ \to \tau^+ \nu_\tau)^{\text{SM}} = (8.48 \pm 0.5) \times 10^{-5},$$ \hspace{1cm} (24)

$$\text{Br}(B_c^+ \to \tau^+ \nu_\tau)^{\text{SM}} = (3.6 \pm 0.14) \times 10^{-2}.$$ \hspace{1cm} (25)

Considering the current world average of the $B_c$ lifetime, the upper limit on the branching ratio of $B_c^+ \to \tau^+ \nu_\tau$ process is [95]

$$\text{Br}(B_c^+ \to \tau^+ \nu_\tau) \lesssim 10\%.$$ \hspace{1cm} (26)

To compute the allowed regions of new parameters associated with $b \to u \tau \bar{\nu}_\tau$ processes, we compare the theoretically predicted values of $\text{Br}(B_u^+ \to \tau^+ \nu_\tau)$, $R_{ul}$ with their corresponding $3\sigma$ range of observed experimental results and for $b \to c \tau \bar{\nu}_l$ transitions, we use the experimental limits on $R_{D^{(*)}}$, $R_{J/\psi}$ parameters and the branching ratio of $B_c \to \tau \nu_\tau$ channel. The upper limit on the branching ratio of $B^0 \to \pi^+ \tau^- \bar{\nu}_\tau$ process is also used to constrain the new couplings of $b \to u \tau \bar{\nu}_l$. By reason of zero contribution of tensor coupling to the branching ratios of $B_{u,c}^+ \to \tau^+ \nu_\tau$ processes and the lack of precise determination of the form factors associated with tensorial operators for $B_c \to J/\psi \tau \bar{\nu}_\tau$ decay mode, the tensor operator part contribution from $\text{Br}(B^0 \to \pi^+ \tau^- \bar{\nu}_\tau)$ and $R_{D^{(*)}}$ observables are only included in this analysis. We consider two cases of couplings, (a) couplings as real and (b) couplings as imaginary.

1. Real couplings

Considering the leptoquarks couplings as real, the constrained plots of the $S_1$ (top-left panel) and $S_3$ (top-right panel) SLQ masses and their corresponding new couplings related
with \( b \to u\tau\bar{\nu}_\tau \) process are presented in Fig. [1]. In the bottom-left (bottom-right) panel of Fig. [1], we show the allowed space of scalar type couplings and \( S_1 \) (\( R_2 \)) SLQ masses. For the case of \( b \to c\tau\bar{\nu}_\tau \), the constraints on \( y_{1L}^{23} y_{1L}^{23*} \) (\( y_{3L}^{33} y_{3L}^{33*} \)) product of couplings of the \( S_1(\mathcal{S}_3) \) leptoquarks and their corresponding masses are depicted in the top-left (top-right) panel of Fig. [2]. The bottom-left panel of Fig. [2] represents the allowed region of \( y_{1L}^{23} y_{1R}^{23*} \) couplings and masses of \( S_1 \) leptoquarks and the corresponding plot for \( R_2 \) leptoquark is shown in the bottom-right panel. In Table [1], we give the allowed ranges of leptoquark masses and real couplings obtained by imposing the extrema conditions.

2. **Complex couplings**

Considering the leptoquarks couplings as complex, the constraint on the real and imaginary parts of \( y_{1L}^{33} y_{1L}^{13*} \) (\( y_{1L}^{33} y_{1R}^{13*} \)) couplings of \( S_1 \) leptoquark is depicted in the top-left (bottom-left) panel of Fig. [3]. The allowed space of \( y_{3L}^{33} y_{3L}^{13*} \) and \( y_{2L}^{33} y_{2L}^{13*} \) couplings obtained from the \( \text{Br}(B_u \to \tau\nu_1) \), \( \text{Br}(B \to \pi\tau\nu_1) \) and \( R^L_\pi \) experimental data are shown in the top-right and...
FIG. 2: Constraints on scalar leptoquark masses and couplings (real) associated with $b \to c\tau\bar{\nu}_\tau$ decay processes.

bottom-right panels of Fig. 3. Imposing the experimental limit on $\text{Br}(B_c \to \tau\nu_\tau)$, $R_{D^{(*)}}$ and $R_{J/\psi}$ observables, the constraints on real and imaginary parts of the vectorial type couplings of $S_1$ ($S_3$) leptoquark are presented in the top-left (top-right) panel of Fig. 4. The corresponding constrained plots for the scalar couplings of $S_1$ and $R_2$ leptoquarks are given in the bottom-left and bottom-right panel of this figure. The obtained allowed new parameter space is presented in Table II.

IV. NUMERICAL ANALYSIS

In this section, we perform the numerical analysis of the branching ratios and physical observables of $B_s \to (K^{(*)}, D_s^{(*)})\tau\bar{\nu}_\tau$ processes. For numerical estimation, we use the particle masses, life time of $B_s$ meson and the CKM matrix elements from [19]. The predictions for the various observables require sufficient knowledge of the associated hadronic form factors. For $\bar{B}_s \to K^+l^-\bar{\nu}_l$ decay processes, we consider the perturbative QCD (PQCD) calculation [33, 34] based on the $k_T$ factorization [98, 99] at next-to-leading order (NLO) in $\alpha_s$ [100],
TABLE I: Minimum and maximum values of the leptoquark couplings (real) and masses.

| Decay processes | Leptoquarks | Couplings | Real part (Min, Max) | Mass of leptoquark (Min, Max) |
|-----------------|-------------|-----------|----------------------|-------------------------------|
| $b \to u \tau \nu_\tau$ | $S_1$ | $y^{23}_{1L}y^{13*}_{1L}$ | $(-3.5, 3.0)$ | $(1100, 2000)$ |
|                  | $S_3$ | $y^{23}_{3L}y^{13*}_{3L}$ | $(-3.0, 3.5)$ | $(1100, 2000)$ |
|                  | $R_2$ | $y^{13}_{2L}y^{33*}_{2R}$ | $(0.3, 3.5)$ | $(1100, 2000)$ |
| $b \to c \tau \bar{\nu}_\tau$ | $S_1$ | $y^{23}_{1L}y^{13*}_{1L}$ | $(0.0, 2.2)$ | $(1100, 2000)$ |
|                  | $S_3$ | $y^{23}_{3L}y^{13*}_{3L}$ | $(-0.32, 0.0)$ | $(1100, 2000)$ |
|                  | $R_2$ | $y^{23}_{2L}y^{33*}_{2R}$ | $(0.0, 0.32)$ | $(1100, 2000)$ |

TABLE II: Minimum and maximum values of the real and imaginary parts of the leptoquarks couplings.

| Decay processes | Leptoquarks | Couplings | Real part (Min, Max) | Imaginary part (Min, Max) |
|-----------------|-------------|-----------|----------------------|---------------------------|
| $b \to u \tau \nu_\tau$ | $S_1$ | $y^{23}_{1L}y^{13*}_{1L}$ | $(-1.4, 0.6)$ | $(-3.5, 3.5)$ |
|                  | $S_3$ | $y^{23}_{3L}y^{13*}_{3L}$ | $(-0.6, 1.4)$ | $(-3.5, 3.5)$ |
|                  | $R_2$ | $y^{13}_{2L}y^{33*}_{2R}$ | $(0.3, 1.6)$ | $(-2.0, 2.0)$ |
| $b \to c \tau \bar{\nu}_\tau$ | $S_1$ | $y^{23}_{1L}y^{13*}_{1L}$ | $(-0.2, 0.6)$ | $(-3.5, 3.5)$ |
|                  | $S_3$ | $y^{23}_{3L}y^{13*}_{3L}$ | $(-0.35, 0.1)$ | $(-1.4, 1.4)$ |
|                  | $R_2$ | $y^{23}_{2L}y^{33*}_{2R}$ | $(-0.6, 0.2)$ | $(-3.5, 3.5)$ |

which gives

$$F_{1}^{B_s \to K}(q^2) = F_{1}^{B_s \to K}(0) \left( \frac{1}{1-q^2/M_{B_s}^2} + \frac{a_1q^2/M_{B_s}^2}{(1-q^2/M_{B_s}^2)(1-b_1q^2/M_{B_s}^2)} \right),$$

$$F_{0}^{B_s \to K}(q^2) = \frac{F_{0}^{B_s \to K}(0)}{(1-a_0q^2/M_{B_s}^2 + b_0q^4/M_{B_s}^4)}.$$  \hspace{1cm} (27)
The values of the parameters $a_{0,1}$, $b_{0,1}$ and $F_{0,1}^{B_s \to K}$ can be found in the Ref. [33]. The $q^2$ dependence of the form factors $V, A_{0,1,2}, T_{1,2,3}$ associated with $B_s \to K^* \tau \bar{\nu}_\tau$ decay modes can be parametrized as [101]

\[
\begin{align*}
    f_1^{B_s \to K^*}(q^2) &= \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{fit}^2}, & f_1 &= V, A_0, T_1, \\
    f_2^{B_s \to K^*}(q^2) &= \frac{r_2}{1 - q^2/m_{fit}^2}, & f_2 &= A_1, T_2, \\
    f_3^{B_s \to K^*}(q^2) &= \frac{r_1}{1 - q^2/m_{fit}^2} + \frac{r_2}{(1 - q^2/m_{fit}^2)^2}, & f_3 &= A_2, \bar{T}_3,
\end{align*}
\]

where $T_3$ is related to $\bar{T}_3$ and the values of the parameters involved in the calculation of form factors are taken from Ref. [101]. The $B_s \to D^{(*)}_s \tau \bar{\nu}_\tau$ form factors, computed by using the PQCD approach [30] are used in this analysis, which can be parametrized as

\[
F^{B_s \to D^{(*)}_s}(q^2) = \frac{F_{B_s \to D^{(*)}_s}(0)}{(1 - a q^2 / M_{B_s}^2 + b q^4 / M_{B_s}^4)},
\]

where $F^{B_s \to D^{(*)}_s}$ stand for the form factors $F_{0,+, V, A_{0,1,2}}$. The fitting values of the $a$ and $b$ parameters can be found in [30]. Since there is no PQCD results on the form factors...
FIG. 4: Constraints on real and imaginary part of the leptoquark couplings associated with $b \rightarrow c\tau \bar{\nu}_\tau$ decay processes.

associated with tensor operator, we show our results for $B_s \rightarrow D_s^{(*)}l\bar{\nu}_l$ process with vanishing tensor form factors.

Now the stage is ready with detailed expressions of physical observables, required input parameters and the constrained new Wilson coefficients for complete numerical anlaysis. Using these input values, the predicted branching ratios $B_s \rightarrow (K^{(*)}, D_s^{(*)})\mu \bar{\nu}_l$ processes in the SM are given by

\begin{align*}
\text{Br}(B_s \rightarrow K\mu \bar{\nu}_\mu)^{\text{SM}} &= (1.044 \pm 0.084) \times 10^{-4}, \\
\text{Br}(B_s \rightarrow K^{*}\mu \bar{\nu}_\mu)^{\text{SM}} &= (3.43 \pm 0.275) \times 10^{-4}, \\
\text{Br}(B_s \rightarrow D_s\mu \bar{\nu}_\mu)^{\text{SM}} &= (2.17 \pm 0.174) \times 10^{-2}, \\
\text{Br}(B_s \rightarrow D_s^{*}\mu \bar{\nu}_\mu)^{\text{SM}} &= (4.82 \pm 0.386) \times 10^{-2}.
\end{align*}

In the following subsection, we discuss the impact of individual scalar leptoquarks on the branchings ratios and the optimized physical observables of semileptonic $B_s$ decay modes.
A. $S_1$ scalar leptoquark

This subsection is dedicated to the analysis of $B_s \rightarrow (K^{(*)}, D_s^{(*)})\tau\bar{\nu}_\tau$ physical observables by using the singlet $S_1(3,1,1/3)$ scalar LQ. This LQ will contribute additional $V_L$, $S_R$ and $T_L$ coefficients to the SM. The constraint on the the couplings (real and complex) and masses associated with this leptoquark are obtained by using the $\text{Br}(B_{u,c} \rightarrow \tau\nu_\tau)$, $\text{Br}(B \rightarrow \pi\tau\tau)$, $R_\pi$, $R_{D^{(*)}}$ and $R_{J/\psi}$ observables, as discussed in section III. Using the allowed space of real and complex couplings and the leptoquark mass from Table I and II, we show the variation of the branching ratios of $B_s \rightarrow K\tau\bar{\nu}_\tau$ (top-left panel), $B_s \rightarrow K^{*}\tau\bar{\nu}_\tau$ (top-right panel), $B_s \rightarrow D_s\tau\bar{\nu}_\tau$ (bottom-left panel) and $B_s \rightarrow D_s^{*}\tau\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in Fig. 5. Here the cyan bands represent the contributions coming due to the $S_1$ SLQ exchange, where coupling is complex. The orange bands are due to the SLQ contributions with real couplings. The red dashed lines present the central values of standard model and their corresponding theoretical uncertainties, arising due to the uncertainties associated with hadronic form factors and CKM matrix elements, are shown in gray color.

We found that the branching ratios of all these processes deviate significantly from their corresponding SM predictions due to the additional contribution from the complex coupling of $S_1$ scalar leptoquark. Though the region of real coupling case provide deviation from the SM results, these are comparatively less than the case of complex coupling. The numerical values of the branchings of these decay modes for the SM and for all the cases of $S_1$ leptoquark couplings are presented in the Table III.

Beyond the branching ratios, another interesting observable is the zero crossing of forward-backward asymmetry. The $q^2$ variation of the forward-backward asymmetries of $B_s \rightarrow K\tau\bar{\nu}_\tau$ (top-left panel), $B_s \rightarrow K^{*}\tau\bar{\nu}_\tau$ (top-right panel), $B_s \rightarrow D_s\tau\bar{\nu}_\tau$ (bottom-left panel) and $B_s \rightarrow D_s^{*}\tau\bar{\nu}_\tau$ (bottom-right panel) decay modes in the $S_1$ scalar leptoquark model are presented in Fig. 6. The case of real leptoquark coupling provides significant deviation in the forward-backward asymmetries of $B_s \rightarrow K^{(*)}\tau\bar{\nu}_\tau$ processes from their SM values, whereas the case of complex leptoquark coupling provides comparatively less deviation. The constrained couplings(real and complex) have almost negligible impact on the forward-backward asymmetries linked to the $B_s \rightarrow D_s^{(*)}\tau\bar{\nu}_\tau$ channels. The numerical values of the forward-backward asymmetry are given in Table III.

Fig. 7 depicts the variation of the lepton non-universality parameters $R_K^{\tau\mu}$ (top-left
FIG. 5: The variation of branching ratios of $\bar{B}_s \rightarrow K^+l^−\bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \rightarrow K^{+}\bar{l}^−\bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \rightarrow D^+s l^−\bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \rightarrow D^{*+}s l^−\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $S_1$ scalar leptoquark model. Here cyan bands are due to the contribution from $S_1$ leptoquark with coupling as complex. The orange bands stand for the allowed regions of real leptoquark couplings. The red dashed lines represent the standard model contributions with their corresponding theoretical uncertainties are shown in gray bands.

panel), $R_{K^*(s)}^\tau$ (top-right panel), $R_{D_s}^\tau$ (bottom-left panel) and $R_{D_{s}^{*}}^\tau$ (bottom-right panel) with respect to $q^2$. It is found that, the allowed real coupling region has more effect on $R_{K^*(s)}^\tau$ LNU parameters and the complex leptoquark couplings affect the $R_{D_s^{*}}^\tau$ parameters. In Table III, the numerical values of the lepton non-universality parameters are shown.

The plots in the Fig. 5, show the effect of new parameter space on the $\tau$-polarization asymmetries of $B_s \rightarrow K\tau\bar{\nu}_\tau$ (top-left panel), $B_s \rightarrow K^{*}\tau\bar{\nu}_\tau$ (top-right panel), $B_s \rightarrow D_s\tau\bar{\nu}_\tau$ (bottom-left panel) and $B_s \rightarrow D_{s}^{*}\tau\bar{\nu}_\tau$ (bottom-right panel) processes. We observe profound deviation in the polarization asymmetry of $B_s \rightarrow K^{(s)}\tau\bar{\nu}_\tau$ decay modes due to the $S_1$ SLQ contribution with the coupling as real. The contribution from the complex LQ couplings also deviate the $\tau$-polarization asymmetry parameters from their SM predictions. The complex couplings region affect the $\tau$-polarization asymmetry $B_s \rightarrow D_s\tau\bar{\nu}_l$ significantly. There is no much impact of new physics on the $B_s \rightarrow D_{s}^{*}\tau\bar{\nu}_l$ decay process. The $K^*$ ($D_{s}^{*}$) polarization asymmetry plot for the $B_s \rightarrow K^*\tau\bar{\nu}_\tau$ ($B_s \rightarrow D_{s}^{*}\tau\bar{\nu}_\tau$) mode is presented in the left panel.
FIG. 6: The variation of forward-backward asymmetry of $\bar{B}_s \rightarrow K^{+}\tau^{-}\bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \rightarrow K^{*+}\tau^{-}\bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \rightarrow D^{+}_s\tau^{-}\bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \rightarrow D^{*+}_s\tau^{-}\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $S_1$ scalar leptoquark model.

(right panel) of Fig. 9. The predicted numerical values of all these observables are given in III.

B. $S_3$ scalar leptoquark

The triplet $S_3(\bar{3}, 1, 1/3)$ scalar leptoquark contributes only additional $V_L$ Wilson coefficient to the SM. The allowed new parameter space obtained from the available experimental data on relevant observables, for both real and complex coupling cases are already provided in section III. Now using the constrained parameters, the branching ratios of $B_s \rightarrow K\tau\bar{\nu}_\tau$ (top-left panel), $B_s \rightarrow K^{*}\tau\bar{\nu}_\tau$ (top-right panel), $B_s \rightarrow D^+_s\tau\bar{\nu}_\tau$ (bottom-left panel) and $B_s \rightarrow D^{*+}_s\tau\bar{\nu}_\tau$ (bottom-right panel) decay modes with $q^2$ in the $S_3$ scalar leptoquark model are shown in Fig. 10. Here the cyan color bands are arising due to the constrained complex couplings and the magneta bands represent the new physics contributions to the branching ratios predicted from the allowed region of $S_3$ leptoquark with real couplings. We found that, the branching ratios of all these decay modes deviate significantly from SM for the case of both complex and real couplings. The predicted branching ratios for all the cases of new couplings are given in Table IV.
FIG. 7: The variation of lepton non-universality parameters of $\bar{B}_s \to K^+\tau^-\bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \to K^{*+}\tau^-\bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \to D_s^+\tau^-\bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \to D_s^{*+}\tau^-\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $S_1$ scalar leptoquark model.

In Fig. 11, the lepton non-universality parameters for $B_s \to D_s\tau\bar{\nu}_\tau$ and $B_s \to D_s^{*}\tau\bar{\nu}_\tau$ are shown in the left and right panels, respectively. For this observables, the complex leptoquark coupling are found to be more effective in comparison to the case of real coupling. However, the impact of $S_3$ leptoquark on the $R_{\tau\mu}^T$ parameter of $B_s \to K^{(*)}\tau\bar{\nu}_\tau$ processes are found to be negligible. The numerical values of the LNU parameters of all these decay modes are presented in Table IV. We don’t find any deviation in the forward-backward asymmetry, lepton and hadron polarization asymmetry parameters of semileptonic $B_s$ decay processes due to the additional NP contributions from $S_3$ scalar LQ.

C. $R_2$ scalar leptoquark

After discussing the impact of $S_{1,3}$ scalar leptoquarks on the flavor observables of $B_s$ decay modes mediated by the $b \to (u,c)\tau\bar{\nu}_\tau$ transitions, we now proceed to check the same physical observables in the $R_2(3,2,7/6)$ SLQ model. $R_2$ LQ is doublet under $SU(2)$ and contributes only scalar and tensor type couplings to the SM. The variation of branching ratios of $B_s \to K\tau\bar{\nu}_\tau$ (top-left panel), $B_s \to K^{*}\tau\bar{\nu}_\tau$ (top-right panel), $B_s \to D_s\tau\bar{\nu}_\tau$ (bottom-
FIG. 8: The variation of lepton polarization asymmetry parameters of $\bar{B}_s \to K^{+}\tau^{-}\bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \to K^{*+}\tau^{-}\bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \to D_s^{+}\tau^{-}\bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \to D_s^{*+}\tau^{-}\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $S_1$ scalar leptoquark model.

FIG. 9: The variation of hardon polarization asymmetry parameters of $\bar{B}_s \to K^{*+}\tau^{-}\bar{\nu}_\tau$ (left panel) and $\bar{B}_s \to D_s^{*+}\tau^{-}\bar{\nu}_\tau$ (right panel) processes with respect to $q^2$ in the $S_1$ scalar leptoquark model.

left panel) and $B_s \to D_s^*\tau\bar{\nu}_\tau$ (bottom-right panel) processes with $q^2$, obtained by using the allowed parameter space of $R_2$ leptoquark [I, II] are given in Fig. 12. Here the blue bands are due to the constrained real couplings and the cyan bands are for the complex $R_2$ leptoquark couplings. We observe that, the branching ratios of $B_s \to (K,D_s)$ modes show profound deviation from their corresponding SM results due to the additional complex coupling contributions. The real coupling has more effect on $\text{Br}(B_s \to K^*\tau\bar{\nu}_\tau)$ in comparison to the case of complex parameters. Where as there is no deviation in the branching ratio of $B_s \to D_s^*\tau\bar{\nu}_\tau$ due to the presence of $R_2$ leptoquark. The predicted values of branching ratios
TABLE III: The predicted values of the branching ratios and other physical observables of $B_s \to (K^{(*)}, D_s^{(*)})\tau\bar{\nu}_\tau$ processes in the SM and in the $S_1$ scalar leptoquark model. Here RC represents the real coupling region and CC stands for complex coupling.

| Observables | Values for SM | Values for RC | Values for CC |
|-------------|---------------|---------------|---------------|
| $B_s$ Br    | $(6.65 \pm 0.532) \times 10^{-5}$ | $(5.35 - 9.36) \times 10^{-5}$ | $(2.43 - 3.056) \times 10^{-4}$ |
| ↓ $\langle A_{FB}^r \rangle$ | 0.278 ± 0.022 | 0.25 - 0.42 | 0.348 - 0.369 |
| $K$ $\langle P_\tau \rangle$ | $-0.1567 \pm 0.013$ | $-0.283 \to 0.657$ | 0.22 - 0.368 |
| $\langle R_{K}^{\tau\mu} \rangle$ | 0.6365 | 0.512 - 0.896 | 2.33 - 2.93 |
| $B_s$ Br    | $(1.85 \pm 0.148) \times 10^{-4}$ | $(0.68 - 2.95) \times 10^{-4}$ | $(4.38 - 6.09) \times 10^{-4}$ |
| ↓ $\langle A_{FB}^r \rangle$ | $-0.182 \pm 0.015$ | $-0.196 \to -0.015$ | $-0.131 \to -0.111$ |
| $K^*$ $\langle P_\tau \rangle$ | $-0.585 \pm 0.045$ | $-0.61 \to 0.122$ | $-0.405 \to -0.27$ |
| $\langle P_{K^*} \rangle$ | 0.191 ± 0.015 | 0.185 - 0.313 | 0.213 - 0.225 |
| $\langle R_{K^*}^{\tau\mu} \rangle$ | 0.54 | 0.198 - 0.858 | 1.275 - 1.772 |
| $B_s$ Br    | $(1.39 \pm 0.111) \times 10^{-2}$ | $(1.39 - 2.26) \times 10^{-2}$ | $(3.93 - 4.67) \times 10^{-2}$ |
| ↓ $\langle A_{FB}^r \rangle$ | 0.358 ± 0.029 | 0.353 - 0.358 | 0.329 - 0.333 |
| $D_s$ $\langle P_\tau \rangle$ | 0.2 ± 0.016 | 0.2 - 0.284 | 0.418 - 0.422 |
| $\langle R_{D_s}^{\tau\mu} \rangle$ | 0.6415 | 0.6415 - 1.04 | 1.808 - 2.2867 |
| $B_s$ Br    | $(2.23 \pm 0.178) \times 10^{-2}$ | $(2.23 - 3.215) \times 10^{-2}$ | $(4.5 - 5.62) \times 10^{-2}$ |
| ↓ $\langle A_{FB}^r \rangle$ | $-0.0996 \pm 0.008$ | $-0.107 \to -0.0996$ | $-0.12 \to -0.118$ |
| $D_s^*$ $\langle P_\tau \rangle$ | $-0.514 \to 0.0411$ | $-0.514 \to -0.5$ | $-0.4766 \to -0.4746$ |
| $\langle P_{D_s^*} \rangle$ | 0.0113 ± 0.0009 | 0.0111 - 0.0113 | 0.0107 - 0.0108 |
| $\langle R_{D_s^*}^{\tau\mu} \rangle$ | 0.463 | 0.463 - 0.6673 | 0.9313 - 1.167 |

for both real and complex couplings cases are presented in Table [V].

Fig. [13] describes the variation of the forward-backward asymmetry of $B_s \to K\tau\bar{\nu}_\tau$ (top-left panel), $B_s \to K^*\tau\bar{\nu}_\tau$ (top-right panel), $B_s \to D_s\tau\bar{\nu}_\tau$ (bottom-left panel) and $B_s \to D_s^*\tau\bar{\nu}_\tau$ (bottom-right panel) with respect to $q^2$. The $A_{FB}$ of $B_s \to K^{(*)}$ deviate significantly due to additional $R_2$ leptoquark whereas very minor effect of new parameters are observed for $B_s \to D_s^{(*)}$ modes. The numerical values of forward-backward asymmetries for both real and complex couplings cases are given in Table [V].

In Fig. [14], we depict the variation of $R_{K}^{\tau\mu}$ (top-left panel), $R_{K^*}^{\tau\mu}$ (top-right panel), $R_{D_s}^{\tau\mu}$
FIG. 10: The variation of branching ratios of $\bar{B}_s \to K^{+}\tau^{-}\bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \to K^{*+}\tau^{-}\bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \to D_s^{+}\tau^{-}\bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \to D_s^{*+}\tau^{-}\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $S_3$ scalar leptoquark model. Here cyan bands represent the case of complex $S_3$ leptoquark coupling and the magenta bands stand for real coupling.

FIG. 11: The variation of lepton nonuniversality parameters of $\bar{B}_s \to D_s^{+}\tau^{-}\bar{\nu}_\tau$ (left panel) and $\bar{B}_s \to D_s^{*+}\tau^{-}\bar{\nu}_\tau$ (right panel) processes with respect to $q^2$ in the $S_3$ scalar leptoquark model. (bottom-left panel) and $R_{D_s^{*}}^{\tau\mu}$ (bottom-right panel) LNU parameters. The impact of $R_2$ leptoquark on the $R_{D_s^{*}}^{\tau\mu}$ LNU parameters are found to be negligible. Table V contains the numerical values of all the LNU parameters.

The $\tau$ polarization asymmetries of all these decay modes are given in Fig. 15. The left and right panel of Fig. 16 presents the $K^*$ and $D_s^*$ polarization asymmetry parameters respectively. In Table V, the predicted numerical values of lepton and hardon polarization asymmetries are listed.
TABLE IV: The predicted values of the branching ratios and other physical observables of $B_s \rightarrow (K, D_s)\tau\bar{\nu}_\tau$ processes in the SM and in the $S_3$ scalar leptoquark model. Here RC represents the real coupling and CC stands for complex coupling.

| Observables | Values for RC | Values for CC |
|-------------|---------------|---------------|
| $\text{Br}(B_s \rightarrow K\tau\bar{\nu}_\tau)$ | $(0.26 - 2.47) \times 10^{-4}$ | $(0.993 - 1.71) \times 10^{-4}$ |
| $\text{Br}(B_s \rightarrow K^*\tau\bar{\nu}_\tau)$ | $(0.73 - 6.9) \times 10^{-4}$ | $(2.77 - 4.78) \times 10^{-4}$ |
| $\langle R^\mu_{D_s} \rangle$ | $0.6415 - 1.812$ | $1.317 - 1.654$ |
| $\text{Br}(B_s \rightarrow D_s^*\tau\bar{\nu}_\tau)$ | $(2.26 - 3.94) \times 10^{-2}$ | $(2.86 - 3.59) \times 10^{-2}$ |
| $\langle R^\mu_{D_s^*} \rangle$ | $0.463 - 1.307$ | $0.95 - 1.193$ |

FIG. 12: The variation of branching ratios of $\bar{B}_s \rightarrow K^+\tau\bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \rightarrow K^{*+}\tau\bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \rightarrow D_s^+\tau\bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \rightarrow D_{s}^{*+}\tau\bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $R_2$ scalar leptoquark model. Here cyan bands represent the case of complex $S_3$ leptoquark coupling and the blue bands stand for real coupling.

V. SUMMARY AND CONCLUSION

To summarize the article, we have studied the rare semileptonic decay processes of $B_s$ meson i.e, $B_s \rightarrow (K, D_s)\tau\bar{\nu}_\tau$ mediated by the $b \rightarrow (u, c)\tau\bar{\nu}_\tau$ quark level transitions in
FIG. 13: The variation of forward-backward asymmetry of $\bar{B}_s \rightarrow K^+ \tau^- \bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \rightarrow K^{*+} \tau^- \bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \rightarrow D_s^+ \tau^- \bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \rightarrow D_s^{*+} \tau^- \bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $R_2$ scalar leptoquark model.

The context of scalar leptoquark model. Our main motivation was to see how the singlet $S_1(\bar{3},1,1/3)$, doublet $R_2(\bar{3},2,7/6)$ and the triplet $S_3(\bar{3},3,1/3)$ leptoquarks affect the branching ratios and other physical observables like forward-backward asymmetry, lepton non-universality parameter, lepton and hadron polarization asymmetry associated with these decay modes. The $S_1$ leptoquark contributes additional vector, scalar and tensor couplings to the standard model, where as the $S_3$ leptoquark contributes only $V_L$ coefficient and the $R_2$ leptoquark provides scalar and tensor couplings contributions. We have considered that the new physics contributes only to the tau lepton and the contribution to the first and second generation leptons are assumed to be SM like. We then considered two valid cases of leptoquark couplings, real and complex. We have used the experimental limit on the branching ratios of $B_u \rightarrow \tau \nu_\tau$ and $B \rightarrow \pi \tau \tau$ and the $R_\pi$ parameters to constrain the new leptoquark couplings related to the $b \rightarrow u \tau \bar{\nu}_\tau$ transition. For the case of $b \rightarrow c \tau \bar{\nu}_\tau$ processes, we have computed the allowed parameter space by using the branching ratio of $B_c \rightarrow \tau \nu_\tau$ obtained by using the life time of $B_c$ meson and the $R_{D^{(*)}}$, $R_{J/\psi}$ parameters. Using the allowed parameter space, we have estimated the branching ratios and other observables of $B_s \rightarrow (K^{(*)}, D_s^{(*)}) \tau \bar{\nu}_\tau$ modes.
We have observed that the branching ratios of all these decay processes deviate significantly from their corresponding SM results due to the new contributions from the complex couplings of $S_1$ leptoquark. However the region of the constrained real couplings show negligible effect on the branching ratios of these decay modes. The impact of $S_1$ leptoquark on the forward-backward asymmetry of $B_s \to K^{(*)}\tau^{-}\bar{\nu}_\tau$ are found to be more sizable in comparison to the $B_s \to D_s^{(*)}\tau^{-}\bar{\nu}_\tau$ decay processes, where as opposite results are observed in the case of lepton non-universality parameters. The constrained couplings of $S_1$ leptoquark affect the $R_{\tau\mu}^{K^{(*)}}$ parameters more comparatively than the $R_{K^{(*)}}^{\tau\mu}$ observables. The lepton and hadron polarization asymmetries of all the decay modes have also shifted profoundly except the $B_s \to D_s^{(*)}\tau^{-}\bar{\nu}_\tau$ process.

The branching ratios of all the discussed semileptonic decay modes of $B_s$ meson have shown significant deviation in the $S_3$ leptoquark model. Both the real coupling and the complex coupling have sizable impact on the branching ratios. Though the $R_{K^{(*)}}^{\tau\mu}$ LNU parameters have shown negligible deviation, the $R_{D_s^{(*)}}^{\tau\mu}$ parameters have shifted from their SM results. The new couplings of $S_3$ leptoquark have not shown any effect on the forward-backward asymmetry, lepton and hardon polarization asymmetry.
FIG. 15: The variation of lepton polarization asymmetry parameters of $\bar{B}_s \to K^+ \tau^- \bar{\nu}_\tau$ (top-left panel), $\bar{B}_s \to K^{*+} \tau^- \bar{\nu}_\tau$ (top-right panel), $\bar{B}_s \to D_s^+ \tau^- \bar{\nu}_\tau$ (bottom-left panel) and $\bar{B}_s \to D_s^{*+} \tau^- \bar{\nu}_\tau$ (bottom-right panel) processes with respect to $q^2$ in the $R_2$ scalar leptoquark model.

FIG. 16: The variation of hardon polarization asymmetry parameters of $\bar{B}_s \to K^{*+} \tau^- \bar{\nu}_\tau$ (left panel) and $\bar{B}_s \to D_s^{*+} \tau^- \bar{\nu}_\tau$ (right panel) processes with respect to $q^2$ in the $R_2$ scalar leptoquark model.

The $R_2$ leptoquark have provided significant impact on the branching ratios and other observables of $B_s \to K^{(*)} \tau \bar{\nu}_\tau$. However it has observed that, this leptoquark is very less sensitive to the observables of $B_s \to D_s^{(*)} \tau \bar{\nu}_\tau$ processes.

To conclude, we have observed that the physical observables of rare semileptonic decay modes of $B_s$ meson are too sensitive to the new physics contribution to the SM arising due to the scalar leptoquark exchange. As like $R_{D^{(*)}}$, $R_{J/\psi}$ parameters associated with the $B \to D^{(*)} \tau \bar{\nu}_\tau$ processes, the $B$-factories as well as the LHCb should check the violation of
TABLE V: The predicted values of the branching ratios and other physical observables of $B_s \rightarrow (K^{(*)}, D_s^{(*)} \tau \bar{\nu}_{\tau})$ processes in the SM and in the $R_2$ scalar leptoquark model. Here RC represents the real coupling region and CC stands for complex coupling.

| Observables | Values for RC | Values for CC |
|-------------|---------------|---------------|
| $B_s$ Br    | $(6.2 - 7.0) \times 10^{-5}$ | $(1.25 - 1.278) \times 10^{-4}$ |
| $\langle A_{FB} \rangle$ | $-0.029 \rightarrow 0.27$ | $-0.204 \rightarrow -0.029$ |
| $K$ $\langle P_{\tau} \rangle$ | $-0.516 \rightarrow -0.322$ | $0.054 - 0.347$ |
| $\langle R_{K}^{\mu} \rangle$ | $0.593 - 0.671$ | $1.198 - 1.224$ |
| $B_s$ Br    | $(2.0 - 3.15) \times 10^{-4}$ | $(2.47 - 3.93) \times 10^{-4}$ |
| $\langle A_{FB} \rangle$ | $-0.134 \rightarrow 0.047$ | $-0.041 \rightarrow 0.1143$ |
| $K^*$ $\langle P_{\tau} \rangle$ | $-0.6 \rightarrow -0.5$ | $-0.31 \rightarrow -0.278$ |
| $\langle P_{K^*} \rangle$ | $0.21 - 0.281$ | $0.19 - 0.285$ |
| $\langle R_{K}^{\mu} \rangle$ | $0.591 - 0.917$ | $0.719 - 1.146$ |
| $B_s$ Br    | $(1.34 - 1.39) \times 10^{-2}$ | $(2.0 - 3.74) \times 10^{-2}$ |
| $\langle A_{FB} \rangle$ | $0.358$ | $0.255 - 0.3$ |
| $D_s$ $\langle P_{\tau} \rangle$ | $0.1674 - 0.2$ | $0.45 - 0.7$ |
| $\langle R_{D_s}^{\mu} \rangle$ | $0.16 - 0.6415$ | $0.934 - 1.72$ |
| $B_s$ Br    | $(2.23 - 2.24) \times 10^{-2}$ | $(2.1 - 2.2) \times 10^{-2}$ |
| $\langle A_{FB} \rangle$ | $-0.0996 \rightarrow -0.097$ | $-0.164 \rightarrow -0.115$ |
| $D_s^*$ $\langle P_{\tau} \rangle$ | $-0.519 \rightarrow -0.514$ | $-0.475 \rightarrow -0.374$ |
| $\langle P_{D_s^*} \rangle$ | $0.0113 - 0.0114$ | $0.0099 - 0.011$ |
| $\langle R_{D_s^*}^{\mu} \rangle$ | $0.43 - 0.464$ | $0.438 - 0.457$ |

lepton universality in their corresponding $B_s$ decay modes i.e, $B_s \rightarrow D_s^{(*)} \tau \bar{\nu}_{\tau}$, the observation of which would provide the indirect hints for possible existence of leptoquarks.

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