Damping of sound waves by bulk viscosity in reacting gases

Miguel H. Ibáñez S.\textsuperscript{1}

Pedro L. Contreras E.

Centro de Física Fundamental, Universidad de los Andes, Mérida 5101, Venezuela.

pcontreras@ula.ve

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\textsuperscript{1}Now at Valle de Villa de Leyva, Colombia.
ABSTRACT

The very long standing problem of sound waves propagation in fluids is reexamined. In particular, from the analysis of the wave damping in reacting gases following the work of Einstein (20), it is found that the damping due to the chemical reactions occurs nonetheless the second (bulk) viscosity introduced by Landau & Lifshitz (1987) is zero. The simple but important case of a recombining Hydrogen plasma is examined.

Subject headings: Viscosity, sound waves, damping, Hydrogen plasma, reacting gas.
1. Introduction

Propagation of disturbances, in particular sound waves in hypothetical equilibrium fluids has been researched since the pioneer works \cite{25, 20, 17, 20, 19} and their main characteristics have been well established, i.e. waves propagate with certain velocity, and are damped by the irreversible processes say viscosities, thermal conduction and chemical reactions. Landau & Lifshitz \cite{1987} introduce a bulk (second) viscosity coefficient $\zeta$ in the equation of motion for accounting the dissipation of energy due to compression or expansion through transferring kinetic energy into internal degrees of freedom (such as chemical reactions, excitation of atomic/molecular levels, etc.). However, in the case of chemical reactions such approximation only holds if one neglects any others effects except the density change $\delta\rho$ due to the chemical reaction.

Henceforth, as it will be shown at the present note, the Landau approximation is rather restrictive. In fact, if $\xi$ is a parameter characterizing the degree of advance of chemical reaction in the fluid (say, the concentration of one chemical component) and $\xi_0$ its respective value at chemical equilibrium, which generally is a function of the equilibrium density and temperature, say $\xi_0(\rho_0, T_0)$ \cite{35}; henceforth, as it can be realized, in the Landau approximation \cite{1987, 14} the second viscosity coefficient is $\sim (\partial\xi_0/\partial\rho)_T$, therefore, when $(\partial\xi_0/\partial\rho)_T = 0$ the acoustic wave damping is also zero. However, when $(\partial\xi_0/\partial T)_\rho \neq 0$ the sound waves could be damped nonetheless the Landau bulk viscosity coefficient is zero, as it will be shown below.

The present analysis on the bulk viscosity is made for any reacting gas where the chemical reactions can be reduced to a net reaction that can be described by one parameter measuring the advance of the reaction \cite{37, 12}. However, for context, the results are applied to a Hydrogen plasma where the simple reaction $H^+ + e^- \rightleftharpoons H + (\chi)$ proceeds ($\chi$ being the ionization potential). The knowledge of the above plasma is of particular importance in
Astrophysics, say, the solar atmosphere (33; 34; 30; 2; 22), the interstellar gas (30; 31; 32) and more recently in the Intracluster gas (6; 26; 7; 8), in particular due to the fact that wave dissipation have been invoked as one of the mechanisms of heat input. However, a detailed study of the thermal behavior of the above plasmas is out the scope of the present study, which is particularly restricted to find an expression of the bulk viscosity coefficient in chemically active plasmas.

2. Basic Equations

In general, for a 1-D plane wave the wave number \( k \) and the frequency \( \omega \) are related by

\[
k = \frac{\omega}{c}.
\]  

(1)

The parameter \( c \) is defined by the relation

\[
c = \pm \sqrt{\frac{\partial p}{\partial \rho}},
\]  

(2)

where

\[
\frac{\partial p}{\partial \rho} = \frac{1}{\rho_0} \left[ p_0 - \left( \frac{\partial (pV)}{\partial V} \right) \right],
\]  

(3)

with \( V = 1/\rho \) and the equilibrium values denoted with the subindex \( 0 \). The relation (1) formally obtained by non-dispersive media also holds for dispersive media for which \( c \) is a complex quantity (as well as \( k \) (1987) and only for disturbances propagating in a non-reacting ideal fluid becomes the adiabatic sound speed

\[
c = c_s = \sqrt{\left( \frac{\partial p}{\partial \rho} \right)_s}.
\]  

(4)

Strictly speaking, the basic gas dynamic equations admit solutions in the form

\( \sim \exp(\beta t + i\mathbf{k}.\mathbf{r}) \), where \( \beta = \sigma - i\omega \) and \( \mathbf{k} = \mathbf{k}_r + i\mathbf{k}_i \), where \( \sigma \) and \( \omega \) are real quantities and \( \mathbf{k}_r \) and \( \mathbf{k}_i \) are real vectors. Therefore, one may write the sound disturbance as
\( \sim \exp(\sigma t - k_i x) \exp[i(k_r x - \omega t)] \) for the one-dimensional problem. The above can be interpreted as a wave of frequency \( \omega \), wavelength \( \lambda = 2\pi/k_r \), traveling along the \( x \)-axis with a phase velocity \( v = \omega/k_r \) and the amplitude \( \sim \exp(\sigma t) \exp(-k_i x) \), the first factor measures the attenuation (or growth if \( \sigma > 0 \)) in time, and the second factor measures the spatial absorption (or amplification if \( k_i < 0 \)) in ordinary progressive wave propagation studies (21; 18; 13). The present analysis is restricted to the spatial absorption of linear wave propagation in a chemically active fluids from where the bulk viscosity coefficient is calculated.

For reacting gases if the set of ”chemical reactions” which are in progress can be reduced to a single reaction \( \sum_j \nu_j A_j = 0 \) where \( A_j \) are the chemical symbols of the reagents and the coefficients \( \nu_j \) are positive or negative integers, there is at least on component \( j \) for which the concentration \( \xi_j = n_j/n \) goes to zero when the reaction proceeds to a sense indefinitely, here \( n \) denotes the total number density of atoms and \( n_j \) is the number density for gas particles of the \( j \)-th component. So, one may introduce the parameters \( \xi \), and \( a \), such that

\[
\xi_j = \frac{n_j}{n} = a \xi, \quad 0 \leq \xi \leq 1;
\]

(5)

where \( \xi \) denotes the degree of advance of the reaction and \( a \) the maximum number of abundance ratio of the \( j \)-th component to the total number of nuclei.

From the equation of continuity for the different components and the definition (5) one may obtain the rate equation (35; 37; 12)

\[
\frac{d\xi}{dt} + X(\rho, T, \xi(\rho, T)) = 0,
\]

(6)

where \( X(\rho, T, \xi(\rho, T)) \) is the net rate which at equilibrium \( X(\rho_0, T_0, \xi_0(\rho_0, T_0)) = 0 \).
Additionally, an ideal-like state equation will be assumed, i.e.

\[ p = \frac{R \rho T}{\mu(\xi)}, \]  

where \( R \) is the gas constant and \( \mu(\xi) \) is the mean molecular weight, \( \mu^{-1} = \sum_j \xi_j \).

On the other hand the internal energy per unit mass becomes

\[ u = A(\xi)RT + \chi N_0 a \xi, \]  

where \( \chi \) and \( N_0 \) denote the dissociation energy and the Avogadro’s number and

\[ A(\xi) = \sum_j \frac{\xi_j}{\gamma_j^{-1}}, \]  

\( \gamma_j \) being the specific heat-ratio for the \( j \)–th component.

For an adiabatic change, the energy equation can be written as

\[ RA(\xi) \frac{\delta T}{\delta \xi} - \frac{p}{\rho^2} \frac{\delta \rho}{\delta \xi} + RTB(\xi, T) \frac{\delta \xi}{\delta \xi} = 0, \]  

where

\[ B(\xi, T) = \frac{dA}{d\xi} + \frac{a\chi}{k_B T}, \]  

being \( k_B \) the Boltzmann constant. For linear disturbances close to the equilibrium

\[ RA_0 \delta T - \frac{p_0}{\rho_0^2} \delta \rho + RB_0 T_0 \delta \xi = 0, \]  

where \( A_0 = A(\xi_0) \) and \( B_0 = B(\xi_0, T_0) \) are the equilibrium values of the functions \( A(\xi) \) and \( B(\xi, T) \).

For fluctuations \( \sim \exp(-i\omega t) \) from Eq.(6) follows that the disturbances \( \delta \xi, \delta \rho, \) and \( \delta T \) are related by the equation

\[ \delta \xi = \frac{\xi^*_\rho}{1 - i\omega \tau} \delta \rho + \frac{\xi^*_T}{1 - i\omega \tau} \delta T, \]  

(13)
where $\tau = (\partial X/\partial \xi)^{-1}$ is the relaxation time which is a positive quantity for chemically stable gases; and where $\xi^*\rho = (\partial \xi_0/\partial \rho)_T$, and $\xi^*_T = (\partial \xi_0/\partial T)_\rho$ are the derivatives at equilibrium [37; 15; 12].

Additionally, from Eqs.(7), (6), and (12) the Eq.(3) becomes

$$\frac{\partial p}{\partial \rho} = \frac{p_0}{\rho_0} [1 + Q], \quad (14)$$

the $Q$ factor is given by

$$Q = \frac{(1 - i\omega \tau) - (\mu_0 B_0 + \mu_\xi A) \rho \xi^*_\rho - T \xi^*_T \mu_\xi / \mu}{\mu_0 [A (1 - i\omega \tau) + T \xi^*_T B_0]}, \quad (15)$$

$\mu_\xi$ being the derivative of the molecular weight with respect to the chemical parameter. It is important to mention that the above relation (14) for a particular simply chemical reaction was obtained in an early paper by [20].

In the limiting when $\omega \tau \to \infty$ (frozen chemistry), $Q \to 1/\mu_0$ and in the opposite limiting $\omega \tau \to 0$ (the chemical equilibrium follows the fluctuation)

$$Q = \frac{1 - (\mu_0 B_0 + \mu_\xi A) \rho \xi^*_\rho - T \xi^*_T \mu_\xi / \mu}{\mu_0 [A + T \xi^*_T B_0]}, \quad (16)$$

In the limiting case when the fluctuation $\delta \xi$ is only due to the change of density $\xi^*_T = 0$, the Eq.(15) reduces to

$$Q = \frac{(1 - i\omega \tau) - (\mu_0 B_0 + \mu_\xi A) \rho \xi^*_\rho}{\mu_0 A (1 - i\omega \tau)}. \quad (17)$$

On the opposite limit when $\xi^*_\rho = 0$,

$$Q = \frac{(1 - i\omega \tau) - T \xi^*_T \mu_\xi / \mu}{\mu_0 [A (1 - i\omega \tau) + T \xi^*_T B_0]}, \quad (18)$$

If in the limiting $\xi^*_T = 0$ additionally $\xi^*_\rho = 0$, henceforth $Q = 1/\mu_0 A = \gamma - 1$ and therefore from Eq.(14) $\sqrt{\partial p/\partial \rho} = \sqrt{\gamma p_0/\rho_0}$ (being $\gamma$ the specific heat ratio) becomes the isentropic sound speed $c^2_s$ in a non-reacting ideal gas, as it should be.
It is interesting to point out that in the Landau approximation Landau & Lifshitz (1987) (pp. 308–312), where the fluctuation $\delta \xi$ is assumed to occur at constant entropy $S$, i.e. the change of pressure $p$ is due only to the change of density $\delta \rho$ produced by the fluctuation in the chemical parameter $\delta \xi$,

$$\frac{\partial p}{\partial \rho} = \frac{1}{1 - i\omega \tau} \left[ c_0^2 - i\omega \tau c_\infty^2 \right], \quad (19)$$

and $c_0$ is given by

$$c_0^2 = \left( \frac{\partial p}{\partial \rho} \right)_{eq} = \left( \frac{\partial p}{\partial \rho} \right)_\xi + \left( \frac{\partial p}{\partial \xi} \right)_\rho \left( \frac{\partial \xi_0}{\partial \rho} \right), \quad c_\infty^2 = \left( \frac{\partial p}{\partial \rho} \right)_\xi. \quad (20)$$

From Eq. (15) one obtain the corresponding parameter $Q_L$ in the Landau approximation, i.e.

$$Q_L = -\frac{1}{1 - i\omega \tau} \frac{\mu_\xi}{\mu_0} \rho_\xi^* \rho. \quad (21)$$

Finally, in the limiting case when $\xi_\rho^* = 0$, it follows that $\sqrt{\partial p/\partial \rho} = \sqrt{p_0/\rho_0}$, i.e. the sound propagation would occur with the isothermal sound speed as it is expected. Additionally, at the Landau’s approximation the effects of the chemical reaction may be accounted for introducing a second viscosity coefficient in the motion equation given by the following expression

$$\zeta = \frac{\rho_0 \tau}{1 - i\omega \tau} \left[ c_\infty^2 - c_0^2 \right] = \frac{\rho_0 \tau}{1 - i\omega \tau} \frac{p_0 \mu_\xi}{\mu_0} \xi_\rho^*, \quad (22)$$

i.e. the Landau bulk viscosity coefficient (in g cm$^{-1}$s$^{-1}$), as it can be readily verified from Eq.(20).

### 3. Collisionally ionized Hydrogen plasma

For context, at the present section the above results will be applied to the simple but important examples of an ionized Hydrogen gas when it is collisionally ionized. As it will
shown the damping of sound waves becomes zero at the Landau approximation, but different from zero at Einstein approximation.

A collisionally ionized Hydrogen plasma can be considered as a reacting plasma where the reaction
\[ H^+ + e^- \rightleftharpoons H^0 + \chi, \]  
proceeds with the following expressions
\[ A = \frac{1}{(\gamma - 1)\mu}, \quad B_0 = \frac{1}{\gamma - 1} + \frac{\chi}{k_B T}, \quad \mu = \frac{1}{1 + \xi}, \]  
\( \xi \) being the degree of the ionization, \( \chi \) the Hydrogen ionization potential and \( k_B \) the Boltzmann constant, the sub-index 0 indicating equilibrium values has been omitted. Additionally, the generalized ionization recombination rate function \((37; 12)\) becomes equal to
\[ X = N_0 \rho \alpha(T) \xi^2 - N_0 \rho q(T) \xi(1 - \xi) = 0, \]  
therefore at equilibrium
\[ \xi^*(T) = \frac{q(T)}{\alpha(T) + q(T)}, \]  
the total recombination coefficient \( \alpha(T) \) is given by
\[ \alpha(T) = \frac{2.06 \times 10^{-11}}{T} \left( 0.5 \ln \Theta + \frac{0.47}{\Theta^{1/3}} - 0.32 \right) \frac{cm^3}{s}, \]  
and the collisional ionization rate follows the expression
\[ q(T) = 5.85 \times 10^{-11} \sqrt{T} \exp(-\Theta) \frac{cm^3}{s}, \]  
where \( \Theta = 1.579 \times 10^5/T \) \((27; 10; 11)\). The above approximation holds \((24; 5)\) in the range of \( 3.5 \times 10^3 \lesssim T(K) \lesssim 1.58 \times 10^5 \).

For a collisionally ionized Hydrogen plasma from Eq.\((26)\) follows that \( \partial \xi^*/\partial \rho = 0 \), and therefore the second viscosity in the Landau approximation, Eq.\((22)\), is also zero. However
from Eq. (14) the speed of sound $c$ becomes

$$c = \sqrt{\frac{p}{\rho}} (1 + Q), \quad (29)$$

with

$$Q = \frac{\gamma - 1}{1 + (\gamma - 1) B \mu T \xi_T/(1 - i \omega \tau)} \left[1 - \mu T \xi_T/(1 - i \omega \tau)\right], \quad (30)$$

i.e. damping effect occurs due to the irreversible process inherent to the chemical reaction, as it follows from the fact that $c$ becomes a complex quantity as well as the wave number $k$ Eq.(1), and which can be written as $k = k_r + i k_i$, where $k_r$ and $k_i$ are real quantities, $k_i$ being the damping coefficient which has to be a positive quantity (15).

On the other hand, from Eqs. (25)-(28) the relaxation time becomes

$$\tau = \frac{1}{N_0 \rho q(T)}. \quad (31)$$

i.e. the relaxation time $\tau > 0$ and therefore the Hydrogen plasma is chemically stable.

The damping per unit wave length $2 \pi k_i/k_r$ and the phase velocity $v_{ph}/c_T$ normalized to the isothermal sound speed $c_T = \sqrt{p_0/\rho_0}$ have been plotted in Figs. (1a) and (1b) respectively, as functions of temperature for three different values of $\omega \tau$ (10\(^{-1}\) dash line, 1 thick line, and 10 point line). Regardless the value of $\omega \tau$ the damping shows maxima, and the phase speed shows minima at a temperature close to $\log T = 4.16$, temperature at which the function $\xi_T^*$ becomes a maximum and the effect of the recombination-ionization process become important. At very low (neutral Hydrogen) as well as at very high temperatures (ionized Hydrogen), the damping tends to be zero Fig.(1a), and the sound velocity tends to be the isentropic sound speed Fig.(1b), as it is expected from simple physical considerations.

In Figs. (1c) and (1d) the damping per unit wave length $(2 \pi k_i/k_r)$ and the normalized phase velocity $(v_{ph}/c_T)$ are respectively plotted but as functions of $\omega \tau$ for temperatures slight lower ($\log T = 4.04$, (dash line) and higher $\log T = 4.28$, (point line) than $\log T = 4.16$.
(thick line). The damping per unit wave length becomes a maximum very close to the $\omega \tau$ value Fig. (1c) where the inflexion point of $v_{ph}/c_T$ occurs as in Fig. (1d). Regardless of the temperature value, waves with $\omega \tau \ll 1$ propagate as adiabatic disturbances in a gas at chemical equilibrium, and those with $\omega \tau \gg 1$ as adiabatic disturbances in a gas where the chemical reaction is frozen. In the above limiting cases the disturbances tend to be undamped waves Fig. (1c) as it should be.

3.1. Photo-ionized Hydrogen plasma

In this section the results of Section 1 will be applied to a photo-ionized Hydrogen plasma model i.e. an optically thin Hydrogen plasma ionized by a background radiation field of averaged photon energy $E$ and photo-ionization rate $\varsigma$. The net rate function $X(\rho, T, \xi)$ is given by (5) as

$$X(\rho, T, \xi) = \frac{N_0}{\rho^2} [\xi^2 [1 - \xi] - \xi q] - \xi (1 + \phi) \varsigma,$$

(32)

$\alpha$ is the total recombination coefficient ($cm^3 s^{-1}$) which is given be Eq. (27), $q$ is the collisional ionization rate ($cm^3 s^{-1}$) according to (11), $\phi(E, \xi)$ is the number of secondary electrons which in general is a function of the energy mean photon energy $E$ and the ionization $\xi$, (28), and $\varsigma$ is the photo-ionization rate in $s^{-1}$. The last term of the right hand side of (32) just accounted for this effect. Therefore, the corresponding terms in the energy equation Eq. (10), have to be be added for consistency. For accounted the heat input and output of energy by radiation. So, instead of Equation (10) one obtains

$$RA(\xi) \delta T - \frac{\rho}{\rho^2} \delta \rho + RB(\xi, T)T \delta \xi + \delta L(\rho, T, \xi) = 0,$$

(33)

Where the net heat/cooling function becomes

$$L(\rho, T, \xi) = N_0 \{ \rho^2 [(1 - \xi) \xi \Lambda_{eH} + \xi^2 \Lambda_{eH^+}] \} - N_0 \{ (1 - \xi)[E_h + (1 + \phi)\chi] \}$$
where \( \Lambda_{eH} \) and \( \Lambda_{eH^+} \) are the cooling losses by \( e-H \) and \( e-H^+ \) collisions \(^{15}\) neglecting secondary electrons \( \phi = 0 \).

According to \(^{28}\), \( 0.002 \lesssim \phi \lesssim 0.366 \), for \( 0.95 \gtrsim \xi \gtrsim 10^{-4} \) the exact value depending on \( E \) (which depends on the particular optical depth in the gas) strictly speaking a self-consistent radiative transfer problem should be worked out, and which is out the scope of the present paper, whose aim is restricted to obtain an indicative value of the bulk viscosity for a photo-ionized Hydrogen plasma. Therefore, if in a first approximation the production of secondary electrons is neglected \( (\phi = 0) \), from Eq. \(^{32}\) an explicit form the ionization \( \xi^*(\rho, T) \) at equilibrium can be obtained, i.e.

\[
\xi^*(\rho, T) = \frac{N_0 \rho q - \varsigma + \sqrt{B_p}}{2N_0 \rho (\alpha + q)}, \tag{34}
\]

with

\[
B_p = (N_0 \rho q + \varsigma)^2 + 4N_0 \rho \alpha \varsigma, \tag{35}
\]

otherwise the solution for \( \xi \) at equilibrium becomes an implicit function of \( T \) and \( \rho \), and for its calculation one must proceed numerically. The correction introduced by the secondary electrons is equivalent to an increase of the value of the photo-ionization rate, as it can be verified from Eq. \(^{32}\).

Therefore, from Eq. \(^{34}\) one obtains

\[
\xi^*_\rho = \frac{\rho B_{pp} + 2\sqrt{B_p} \varsigma - 2B_p}{4\sqrt{B_p}N_0 \rho^2 (\alpha + q)}, \tag{36}
\]

and

\[
\xi^*_T = \frac{B_{pT} (\alpha + q) - 2B_p (\alpha_T + q_T)}{4\sqrt{B_p}N_0 \rho (\alpha + q)^2} + \frac{2N_0 \rho \sqrt{B_p} [\alpha q_T - q\alpha_T + \varsigma (\alpha_T + q_T)]}{4 \sqrt{B_p}N_0 \rho (\alpha + q)^2}, \tag{37}
\]

here \( B_{pT} = \partial B_p / \partial T \), \( B_{pp} = \partial B_p / \partial \rho \), \( \alpha_T = \partial \alpha / \partial T \) and \( q_T = \partial q / \partial T \). Similarly to the previous sub-section, from Eqs. \(^{1}\), \(^{14}\), and \(^{15}\) one may calculate both, the real and imaginary parts of \( c \) and \( k \). However, for this particular plasma \( \xi^* \) is a function of both \( \rho \), and \( T \) instead of \( T \) only as given by Eq. \(^{26}\).
Fig. (2a) is a 3D plot of the ionization rate $\xi^*$ as a function of $T(K)$ and density $n(cm^{-3})$ (in Fig. (2a) the red color refers to temperatures close to 5.000 K; on the other hand, the magenta color gives the highest temperatures, which are of the order of 30.000 K, for a fixed value of the photo-ionization $\zeta = 5 \times 10^{-13}s^{-1}$). Fig. (2b) shows a 3D plot of the ionization $\xi^*$ as a function of the density $n(cm^{-3})$ and the photo-ionization $\zeta(s^{-1})$, for a fixed value of temperature ($logT = 4.16K$), spanning in the range of values for the galactic interstellar medium (16). In Fig. (2b) the color indicates the values of the density, red color refers to densities $n$ near zero values, and magenta color indicates values of the density $n$ close to 100 ($cm^{-3}$). From both Figs. (2a), and (2b) follows that the effect of the ionizing radiation is to increase the ionization at any temperature, respect to that resulting by collisions only, however the strong ionization occurring at temperature $\approx 2 \times 10^4 K$ is determined by collisions, for galactic values of the photo-ionization rate $\zeta(s^{-1})$.

As it can be verify the presence of the ionizing radiation field shifts the value of $\xi^*_T$ towards higher temperatures ($log T = 4.21$, for $\zeta = 5 \times 10^{-13}s^{-1}$) and smooth the change of the damping per unit wave length with the temperature for any wave frequency, as it can be shown comparing Fig. (1a) with Fig. (3a) in which the damping $2 \pi k_i/k_r$ is plotted as a function of $T$, for $\zeta = 5 \times 10^{-13}s^{-1}$, and the same three values of $\omega \tau$ shown in Fig. (1a) but for a rate given by (32) instead of (25). The change of value of the maxima of the damping per unit wave depends of the value of $\omega \tau$, in particular increases for $\omega \tau = 1$, additionally they are shifted towards higher values of $T$ following the shift of the maximum of $\xi^*_T$ as follows from physical considerations.

Accordingly, the change of the phase velocity produced (taking into account the photo-ionization) can be seen comparing Fig. 3b with Fig. 1b. The minimum is shifted towards higher temperatures but they are smoothed at high frequencies ($\omega \tau$) as shown by comparing the point lines ($\omega \tau = 10$) in the the above two figures.
At a particular temperature, the changes of the damping per unit wave length and the corresponding to the phase velocity are small (for galactic vales of the photo-ionization $\varsigma$) as it can be shown comparing Figs. (3c) with (1c) and Figs. (3d) with (1d), respectively. Generally, the qualitative and quantitative effects of the photo-ionization are small respect to those produced by collisions only in an atomic Hydrogen gas, as far as sound wave propagation is concerned, and in the range of values of the parameters above considered.

### 3.2. Physical Implications

The aim of the present section is to compare the value of the three absorption coefficients corresponding to: (1) the bulk viscosity $\tilde{k}_b = c_T k_i / \omega$, (2) the dynamical viscosity $\tilde{k}_\nu$, and (3) the thermal conduction $\tilde{k}_\kappa$ which are given by \[ (18; 1987) \], i.e.

$$\tilde{k}_\nu = \frac{2 \omega \nu}{3 c_T^2 \gamma^{3/2}},$$

where $\nu$ is the kinematic viscosity, and

$$\tilde{k}_\kappa = \frac{\omega (\gamma - 1) \chi}{2 c_T^2 \gamma^{3/2}},$$

in which $\chi$ corresponds to the thermometric conductivity, \[ (24; 29; 3; 18) \]. The problem of sound wave propagation in a self-consistent model of the atomic gas in the galaxy and other plasmas of interest in Astrophysics, for which $H_e$ and ions of $H_e$, and ions of heavy elements included will be published elsewhere.

Incidentally, another irreversible process in plasmas, is due to the frictional force between ions of mass $m_i$ (and velocity $v_i$), and neutral particles of mass $m_n$ (and velocity $v_n$) \[ (3) \]. The time scale for equalizing the velocities can be easily calculated from the respective Braginskii relations, from which one obtains the equation

$$\tau_{ni} \approx \frac{(m_i + m_n)}{(\sigma v)(\rho_i + \rho_n)},$$
where \( \langle \sigma v \rangle \) is a mean value of the product of the cross-section and the relative velocity averaged over all velocities. As it can be easily verified, generally \( \tau_{ni} \ll \tau \), additionally the frictional damping becomes independent on the wave-length \( \lambda \), and it is only important for oscillations with very high frequencies, and in plasmas with very low ionization \( \text{\textsuperscript{3, 23, 36}} \). Such effect will not be considered at the present discussion.

Figs. (4a), (4b), and (4c), are plots of the absorption coefficients \( k_b \) (thin line), \( k_\nu \) (dash line), \( k_\kappa \) (point line), and the total absorption \( k_{tot} = k_b + k_\nu + k_\kappa \) (thick line) in units of \( \text{cm}^{-1} \), as functions of the temperature \( T \) for \( n = 1 \) \( (\text{cm}^{-3}) \), a photo-ionization rate value of \( \varsigma = 5 \times 10^{-13} \text{s}^{-1} \), and three different values of the frequency \( \omega \tau = 10^{-1} \), 1 and 10, respectively. The relaxation time is plotted in Fig. (4d) for \( n = 1 \) and three values of the photo-ionization rate \( \varsigma = 5 \times 10^{-14} \) (dash line), \( 5 \times 10^{-13} \) (thick line) and \( 10^{-12} \) (point line), \( \text{s}^{-1} \). Due to the fact that the effect of damping of sound waves is linear, it is worthy to calculate the total absorption coefficients due to the above three effects. The absorption by bulk viscosity becomes the dominant one in the range of temperatures where recombination-ionization takes place \( 4.2 \times 10^3 \lesssim T \lesssim T_M(\omega \tau) \), where \( T_M(\omega \tau) \) is a function of the wave frequency, increasing when \( \omega \tau \) decreases as can be seen in the above Figs. (4a), (4b), and (4c). At high temperatures \( (T > T_M) \) and high ionization, the thermal conduction (by electrons) dominates, instead, at low temperatures \( T \lesssim 4.2 \times 10^3 \text{ K} \), the thermal conduction by neutral atoms becomes the dominating one. At frequencies \( \omega \tau \gtrsim 1 \) the bulk viscosity coefficient shows a conspicuous (relative) maximum. On the other hand dynamical viscosity is much more lower (more than one order of magnitude) than both, bulk viscosity, and thermal conduction in the range of temperature under consideration. In conclusion, in a photo-ionized Hydrogen plasma the bulk viscosity is the most important damping mechanism in the range of temperature \( 4.2 \times 10^3 \lesssim T \lesssim T_M(\omega \tau) \). Fig.(4d) is a plot of the relaxation time \( \tau (= |X_\xi|^{-1} \text{ s}) \) as a function of temperature for three different values of the photo-ionization rate \( \varsigma = 5 \times 10^{-14} \) (dash line), \( \varsigma = 5 \times 10^{-13} \) (thick line), \( \varsigma = 10^{-12} \) (point
line). As it is expected the relaxation time $\tau$ sharply decreases at $T \sim 10^4 \sim 2 \times 10^4 K$ and its value is close to $\tau \sim 10^5 y$. Therefore, for a typical number density $n \sim 1$ (for instance) the space scale for damping ranges between $\sim 0.03 pc$ at high temperatures ($\sim 5 \times 10^4 K$), and $\sim 30 pc$ at low temperatures ($\sim 10^4 K$), for frequencies $\omega \sim 10^{-5}y^{-1}$.

In Summary, following the Einstein (1920) (20) work based on propagation of sound waves in reacting gases, the bulk viscosity coefficient introduced by Landau & Lifshitz (1987) (Eq. 22) has been generalized to chemically active gases. The bulk viscosity coefficient $\sim k_i$ becomes the imaginary part of the wave vector $k$ calculated from Eqs.(1, 29 and 30). In particular, for a collisionally ionized Hydrogen gas, the bulk viscosity in the Landau approximation becomes zero, but it is different from zero at the present approximation, see results in section 3. For context, additionally the bulk viscosity is also calculated for a photo-ionized Hydrogen gas for values of parameters characteristic of the high latitude atomic gas in the Galaxy.
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Fig. 1.— 1a. The damping per unit wave length $2\pi k_i/k_r$ as a function of temperature for three different values of the dimensionless frequency $\omega \tau = 10^{-1}$ (dash line), 1 (thick line) and 10 (point line). 1b. The phase velocity $v_{ph}/c_T$ normalized to the isothermal sound speed $c_T (= \sqrt{p_0/\rho_0})$ as a function of temperature for three different values of the dimensionless frequency $\omega \tau = 10^{-1}$ (dash line), 1 (thick line) and 10 (point line). 1c. The damping per unit wave length $2\pi k_i/k_r$ as a function of the dimensionless frequency for three different values of the temperature $\log T = 4.04$ (dash line), $\log T = 4.16$ (thick line) and $\log T = 4.28$, (point lines). 1d. The phase velocity $v_{ph}/c_T$ normalized to the isothermal sound speed as a function of the dimensionless frequency for three different values of the temperature $\log T = 4.04$ (dash line), $\log T = 4.16$ (thick line) and $\log T = 4.28$, (point lines).
Fig. 2.—2a. The equilibrium ionization $\xi^*$ as a function of temperature $T(\text{K})$ and density $n$ (cm$^{-3}$) for a photo-ionization rate $\varsigma = 5 \times 10^{-13}$s$^{-1}$ (In Fig. 2a, the red color refers to temperatures close to 5.000 K, and the magenta color refers to the highest temperatures of the order of the 30.000 K). 2b. The equilibrium ionization $\xi^*$ in a 3D plot as a function of the density $n$(cm$^{-3}$) and the ionization rate $\varsigma$(s$^{-1}$) for a fixed value of temperature ($\log T = 4.16K$), and where the color palette indicates the values of the density, i.e. red color refers to $n$ around zero, and magenta color indicates values of the density $n$ close to 100 (cm$^{-3}$).
Fig. 3.— 3a. As Fig. (1a) for a photo-ionized gas with a rate given by the expression (32) and $\zeta = 5 \times 10^{-13}\text{s}^{-1}$. 3b. As Fig. (1b) for a photo-ionized gas with a rate given by the expression (32) and $\zeta = 5 \times 10^{-13}\text{s}^{-1}$. 3c. As Fig. (1c) for $\zeta = 5 \times 10^{-13}\text{s}^{-1}$. 3d. As Fig. (1d) for $\zeta = 5 \times 10^{-13}\text{s}^{-1}$. 
Fig. 4.— 4a. The absorption coefficients $k_b$ (thin line), $\nu$ (dash line), $k_\kappa$ (point line) and the total absorption (thick line) $k_{tot} = k_b + k_\nu + k_\kappa$ as functions of temperature $T$ for $n = 1 \text{ cm}^{-3}$, a photo-ionization rate $\zeta = 5 \times 10^{-13} \text{s}^{-1}$ and a dimensionless frequency $\omega\tau = 10^{-1}$.

4b. As Fig. (4a) for $\omega\tau = 1$.

4c. as Fig. (4a) for $\omega\tau = 10$.

4d. is a plot of the relaxation time $\tau = (|X_\zeta|^{-1} \text{ (s)})$ as a function of temperature $T$ for three different values of the photo-ionization rate $\zeta = 5 \times 10^{-14}$ (dash line), $\zeta = 5 \times 10^{-13}$ (thick line), and $\zeta = 10^{-12}$ (point line)