Meshing efficiency for sinusoidal movable tooth drive

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Abstract
Sine movable tooth drive has the smallest radial size. Working efficiency is one of the important factors of the performance evaluation of the drive. Here, based on analyzing entrainment speed and forces of sine movable tooth drive, the equations of the minimum oil film thickness and meshing efficiency for the drive is proposed. The minimum oil film thickness and meshing efficiency of the drive is studied, and the changes of the meshing efficiency along with main parameters are analyzed. Results show that the minimum oil film thickness between the movable teeth and central input shaft is higher than that between the movable teeth and shell, and the meshing efficiency of the drive is between 79.74% and 84.67%, and the average efficiency is 82.48% which is near the experimental result of 82.25%.

Keywords: Sine movable tooth drive, Meshing efficiency, Minimum oil film thickness

1. Introduction

Movable tooth drives have novel structure, high speed ratio and efficiency, simple process and large load-carrying ability. It can be widely used in aerospace, petroleum, precision instruments, robot, medical apparatus and other technical fields (Terada et al., 1988, 1990). In the 1960s, a sine ball drive mechanism was proposed (Нъезруков, 1963). In the 1980s and 1990s, the sine-ball speed reducer was studied, and the optimized design of the single-swing-tooth transmission was completed (Keith, 1982; Imase, 1997; Terada et al., 1995). Since the 21st century, a toothless gear reducer was developed and optimized (Sapsalev, 2004, 2012). A reciprocating motion type ball reducer was proposed which consists of an input rotor, a fixed rotor, a reduced motion rotor, several reciprocating motion sliders and two rows of steel balls. It can be preloaded to eliminate backlash. For the reducer, the motion principle was analyzed and the pressure angles were calculated (Terada, 2007). A two-stage cycloidal ball reducer was studied and the calculation method of the reducer tooth profile was put forward (Terada et al., 2009, 2010). A movable tooth thin plate reducer was designed, which can greatly improve the working efficiency (Nishibe et al., 2014). An eccentric movable tooth drive mechanism was proposed and the contact stress on the tooth surface was obviously decreased (Nam et al, 2013). A new type of eccentric oscillating movable tooth device was designed, which is engaged with the movable teeth by a toothed pin, and the weight of the device was reduced (Furuta, 2016). An electromagnetic harmonic movable tooth drive system was proposed and coupled vibrations about the system were investigated (Liang et al., 2015; Xu et al., 2016). A dynamic model of swing movable tooth drive was established and the modal analysis and investigation of the thermo-elastic coupling deformation of tooth surface were completed (Liang et al., 2017). For a movable tooth piezoelectric motor, the nonlinear dynamics model was established and chaotic vibration of drive system was analyzed (Li et al., 2017). A new two-stage sinusoidal movable tooth transmission was presented which has large speed ratio and small size (Xu et al., 2017). For the novel two-step movable tooth drive, forces and stress were analyzed (Xu et al., 2018).

In a word, a number of studies about movable tooth drive system were done. In above movable tooth drives, the sine movable tooth drive consists of a central input shaft, a fixed shell, an output guide frame and one row of steel balls. It has the smallest element number and radial size, and it is especially suitable for compact structure situations such as robot and oil drilling.
The drive mainly consists of a shell, a guide frame, a center shaft and several movable teeth (see Fig. 1). Sine track is machined on the center shaft. Several axial guide grooves are arranged on the guide frame, and the movable teeth are placed in each radial guide slot, respectively. The shell is installed outside the guide frame, and a sine track is machined inside the shell. The movable teeth must be installed in the sine tracks of the center shaft and the shell simultaneously. The shell is fixed, when the center shaft rotates, the center shaft track pushes the movable teeth, and the movable teeth drive the guide frame under the action of the shell raceway to output the power.

![3D diagram](image1.png) ![Assembly diagram](image2.png)

**Fig. 1 Sine movable tooth drive**

Working efficiency is one of the important factors of the performance evaluation of the drive. However, the research on working efficiency of the sinusoidal movable tooth drive system is rarely reported.

In this paper, based on the analysis of the entrainment speed and forces of meshing pair for the sine movable tooth drive, the equations of the minimum oil film thickness and meshing efficiency for the drive is proposed. The minimum oil film thickness and meshing efficiency of the drive is studied by using these equations, and changes of the minimum oil film thickness and meshing efficiency with the meshing position are revealed, and changes of the meshing efficiency along with main parameters are also given. In order to verify the correctness of the theoretical analysis, the efficiency experiment of this drive prototype is completed, and the test efficiency is very close to the calculated average efficiency. The research results are useful for performance design of the drive.

**2. Entrainment speed of meshing pair**

Letting $\sigma^{(1)}(\alpha, \vec{i}_1, \vec{j}_1, \vec{k})$, $\sigma^{(2)}(\alpha, \vec{i}_2, \vec{j}_2, \vec{k})$, $\sigma^{(3)}(\alpha, \vec{i}_3, \vec{j}_3, \vec{k})$ and $\sigma^{(0)}(\alpha, \vec{i}_0, \vec{j}_0, \vec{k})$ be the coordinate systems fixed on the center shaft, guide frame, shell and movable teeth, respectively. Common origin of the coordinate system $\sigma^{(1)}(\alpha, \vec{i}_1, \vec{j}_1, \vec{k})$, $\sigma^{(2)}(\alpha, \vec{i}_2, \vec{j}_2, \vec{k})$ and $\sigma^{(3)}(\alpha, \vec{i}_3, \vec{j}_3, \vec{k})$ is point $O$. The shell is fixed, so $\sigma^{(3)}(\alpha, \vec{i}_3, \vec{j}_3, \vec{k})$ is a fixed coordinate system. $\varphi$ is the rotation angle between fixed coordinate system and central shaft coordinate system. Take the frame coordinate system $\sigma^{(2)}(\alpha, \vec{i}_2, \vec{j}_2, \vec{k})$ as the reference coordinate system. $\varphi_1$ and $\varphi_3$ are the rotation angles between central shaft or shell and the reference coordinate system (see Fig. 2).

![Coordinate systems](image3.png)  

**Fig. 2 Coordinate systems of the drive system**
As shown in Fig. 2, the equation of the tooth surface of the movable tooth is

\[ \vec{r}_0 = r \cos v \vec{v}_0 + r \cos u \sin v \vec{j}_0 + r \sin u \sin v \vec{k}_0 \]  

(1)

where \( v, u \) are spherical parameters; \( r \) is radius of the movable tooth.

The velocity of the central axis relative to the guide frame at the meshing point between them (contact point between them) is (Litvin, 1994)

\[ \vec{v}_{12} = \vec{h}_{12} \times \vec{r}_0 + \vec{h}_{12} \times \vec{\xi} \]  

(2)

where \( \vec{\xi} = R\vec{h}_2 + C\vec{k}_2; \) \( C \) is axial displacement of the movable tooth center, \( C = A \sin(Z_1 \phi_1); \) \( \vec{h}_{12} \) is angular velocity of the central axis relative to the guide frame, \( \vec{h}_{12} = \left( 1 - \frac{1}{i_{12}} \right) \omega \vec{k}_2 = \frac{Z_3}{Z_1 + Z_3} \omega \vec{k}_2; \) \( \omega \) is absolute angular velocity of central axis; \( Z_1 \) is the period number of the sinusoidal track on center shaft; \( Z_3 \) is the period number of the sinusoidal track on shell.

Eq. (1) for calculating radius \( r_0 \) and related parameters are substituted into Eq. (2), yields

\[ \vec{v}_{12} = -Z_3 \frac{\omega r \cos u \sin v \vec{v}_2}{Z_1 + Z_3} + \frac{Z_3}{Z_1 + Z_3} \omega (R + r \cos v) \vec{j}_2 \]  

(3)

The absolute velocity of the center axis at the meshing point is

\[ \vec{v}_1 = \vec{v}_{12} \cdot \frac{i_{12}}{i_{12} - 1} = \vec{v}_{12} \cdot \frac{Z_1 + Z_3}{Z_3} \]  

(4)

The velocity of the movable tooth in the guide frame coordinate system is

\[ \vec{v}_{02} = \frac{d\vec{\xi}}{dt} + \vec{h}_{02} \times \vec{r}_0 \]  

(5)

where \( \vec{h}_{02} \) is rotational angular velocity of movable teeth relative to guide frame, \( \vec{h}_{02} = 0 \).

The absolute velocity of the movable tooth is

\[ \vec{v}_{0g} = \frac{1}{i_{12}} \cdot \vec{v}_1 + \vec{v}_{02} \cdot \frac{i_{12}}{i_{12} - 1} \]  

(6)

At the meshing point, the resultant velocity of the movable teeth relative to the central axis is

\[ \vec{v}_{h1} = \vec{v}_{0g} + \vec{v}_1 \]

\[ = -\omega r \cos u \sin v \left( \frac{Z_i}{Z_1 + Z_3} + 1 \right) \vec{i}_2 + \left[ \omega r \cos v_1 \left( 1 + \frac{Z_1}{Z_1 + Z_3} \right) + \omega R \right] \vec{j}_2 + A\omega \cos (Z_1 \phi_1) \vec{k}_2 \]  

(7)

In a same manner, the resultant velocity of the movable teeth relative to the shell at the meshing point can be given by

\[ \vec{v}_{h3} = -\omega r \cos u \sin v_3 \left( \frac{Z_3}{Z_1 + Z_3} \right) \vec{i}_2 + \left[ \omega r \cos v \left( \frac{Z_1}{Z_1 + Z_3} \right) \right] \vec{j}_2 + A\omega \cos (Z_3 \phi_3) \vec{k}_2 \]  

(8)

Projection of the resultant velocity in normal direction of contact line is

\[ \vec{v}_{h0} = \frac{\vec{v}_{h1} \cdot \vec{b}}{||\vec{b}||} \]  

(9)

where \( \vec{b} \) is vector in normal direction of contact line, \( \vec{b} = \lambda_1 \vec{r}_{uu} + \lambda_2 \vec{r}_{0u}; \) \( \lambda_i = \frac{1}{D^2} (-\Phi_i G + \Phi_i F) \);
\[
\lambda_2 = \frac{1}{D^2}
\left(-\Phi_x E + \Phi_y F\right); \quad E = \left(v_{0u}\right)^2 = r^2 \sin^2 \nu; \quad F = \left(v_{0v}\right)^2 = r^2; \quad G = \left(v_{0w}\right)^2 = r^2; \quad D^2 = EG - F^2;
\]

\[
\Phi(u, v, \varphi_i) = \frac{Z_3}{Z_1 + Z_3} \alpha \sin \nu \left[R \cos u + A Z_i \sin u \cos (Z_i \varphi_i)\right].
\]

Then the entrainment velocity of the meshing pair is the sum of the absolute velocity of the central axis or shell and the absolute velocity of the movable teeth at meshing point, that is,

\[
U_{0u} = \frac{1}{2} \left(\ddot{v}_{bot} + v_{0z}\right) = \frac{1}{2} \left(\frac{\ddot{v}_{bot} \cdot \ddot{b}}{|\ddot{b}|} + v_{0z}\right)
\]

where \(v_{0z}\) is the absolute velocity of the movable teeth at meshing point,

\[
v_{0z} = \frac{Z \omega}{(Z_1 + Z_3)} \sqrt{R^2 + A^2 Z^2} \cos^2 (Z_i \varphi_i); \text{ for the movable tooth and the center axis, } i=1; \text{ for the movable tooth and the shell, } i=3.
\]

### 3. Forces in meshing pair

Without considering the contact friction and gravity in the movable tooth drive, the spherical center \(o\) of the movable tooth is assumed to be the coordinate origin, and the x-axis, y-axis and z-axis represent the radial, circumferential and axial directions of the sine-tooth drive, respectively (see Fig. 3). From the force relationship, it is known that

\[
\begin{align*}
F_{1i} \cos \alpha_{n1i} + F_{3i} \cos \alpha_{n3i} &= 0 \\
F_{1i} \sin \alpha_{n1i} \cos u_{ti} + F_{3i} \sin \alpha_{n3i} \cos u_{ti} + F_{2i} &= 0 \\
F_{1i} \sin \alpha_{n1i} \sin u_{ti} + F_{3i} \sin \alpha_{n3i} \sin u_{ti} &= 0
\end{align*}
\]

Where \(F_{1i}\) is the force applied to movable tooth by central input shaft; \(F_{2i}\) is the force applied to movable tooth by guide frame; \(F_{3i}\) is the force applied to movable tooth by shell; \(\alpha_{n1i}\) is the contact angle between movable tooth and central input shaft; \(\alpha_{n3i}\) is the contact angle between movable tooth and shell; \(u_{ti}\) and \(\dot{u}_{ti}\) are the direction angles of the contact lines for two pairs of meshing teeth, respectively.

![Fig. 3 Force analysis](image-url)
\[
\frac{F_{1yz}}{F_{1ycz,\text{max}}} = \frac{\Delta s \cdot \cos u_{iy}}{\Delta s \cdot \cos u_{i\text{max}}}
\]

(12)

where \( u_{i\text{max}} = \arctan \frac{R}{AZ_i} \), \( R \) is the radial radius of the sinusoidal raceway on the central input shaft.

Torque balance equation of center input shaft is

\[
\sum_{i=1}^{Z_2} F_{1yz} \cdot \cos u_{iy} \cdot R_i = T_i
\]

(13)

where \( T_i \) is input torque; \( Z_2 \) is the number of the movable teeth.

From the geometric relationship, the following equation can be given

\[
(r' - r) \left( \cos \alpha_{ni} - \cos \alpha_{ni,3} \right) = R_1 - R_3
\]

(14)

where \( r' \) is the radius of the sinusoidal raceway.

Combined with equations (11)-(14), the forces in meshing pair of the sine movable tooth drive can be determined.

Equation of working load per unit length is

\[
W = \frac{F_{m}}{l}
\]

(15)

where \( F_{m} \) is the normal load on movable tooth; \( l \) is the length of the contact line, \( l = r \sin \left( \frac{\pi}{2} - u_i \right) \), \((i = 1, 3)\); For the normal load between movable tooth and central input shaft, \( F_{m}=F_{1i} \); For the normal load between movable tooth and shell, \( F_{m}=F_{3i} \).

4. Minimum oil film thickness and efficiency

According to the theory of elasto-hydrodynamic lubrication, the minimum oil film thickness of the sinusoidal movable tooth drive can be calculated as

\[
H_{\text{min}} = 2.65 \alpha^{0.54} \left( \frac{\eta_0 U_{bi}}{E' W} \right)^{0.7} R_s^{0.43}
\]

(16)

where \( H_{\text{min}} \) is the minimum oil film thickness; \( \alpha \) is pressure viscosity coefficient of the lubricant oil; \( \eta_0 \) is dynamic viscosity of lubricating oil under atmospheric pressure; \( E' \) is the combined elastic modulus of material; \( R_s \) is the combined curvature radius of the meshing pair, \( \frac{1}{R_s} = \frac{1}{r} - \frac{1}{r'} \).

In the sinusoidal movable tooth drive, both sliding and rolling occur, so the meshing power loss can be divided into sliding friction loss and rolling friction loss, that is,

\[
P = P_H + P_R
\]

(17)

where \( P \) is friction power loss; \( P_H \) is sliding friction power loss; \( P_R \) is rolling friction power loss.

The sliding friction power loss can be calculated as

\[
P_H = f \cdot F_n \cdot v \times 10^{-3}
\]

(18)

where \( f \) is sliding friction coefficient; \( v \) is relative sliding velocity at meshing point.

The rolling friction force is linearly related to the minimum oil film thickness. The empirical formulas summarized by experiments are as follows
\[ T_R = 9H_{\text{min}} \cdot 10^{-4} \]  

(19)

where \( T_R \) is rolling friction force.

The rolling friction power loss can be calculated as

\[ P_R = T_R \cdot l_i \cdot U_i \cdot 10^{-3}, (i = 1, 3) \]  

(20)

where \( P_R \) is rolling friction power loss.

Then, the meshing efficiency of the sine movable tooth drive can be calculated as follows

\[ \eta = \frac{P_m - (\Sigma P_S + \Sigma P_R)}{P_m} \times 100\% \]  

(21)

where \( P_m \) is the input power; \( \Sigma P_S \) is total sliding friction power loss; \( \Sigma P_R \) is total rolling friction power loss.

5. Results and discussion

Using Eq. (16), the minimum oil film thickness distribution of the sine movable tooth drive can be determined (see Fig. 4). The parameters of the sine movable tooth drive are shown in Table 1. Besides it, the input speed of the drive is 225 r/min and the input torque is 0.06 Nm. Here, \( \alpha = 1.5 \times 10^{-8} \text{m}^2/\text{N}, \eta_0 = 80 \times 10^{-3} \text{Pa} \cdot \text{s} \) and \( E' = 2.19 \times 10^{11} \text{Pa} \). In Fig. 4, \( H_{\text{min}} \) is minimum oil film thickness between movable tooth and central input shaft at meshing contact point. \( H_{\text{min}} \) is minimum oil film thickness between movable tooth and shell at meshing contact point. Results show:

| Parameter                                      | Value |
|-----------------------------------------------|-------|
| Speed ratio \( i_{12} \)                     | 6     |
| Sine raceway amplitude \( A \) (mm)           | 1.5   |
| Central axial rotary radius of movable teeth \( R \) (mm) | 10.5  |
| movable tooth radius \( r \) (mm)             | 2.0   |
| Sinusoidal raceway radius \( r' \) (mm)       | 2.2   |
| sinusoidal raceway period (center input axis) \( Z_1 \) | 1     |
| sinusoidal raceway period (shell) \( Z_3 \)  | 5     |

![Fig. 4 Minimum oil film thickness distribution of sine movable tooth drive](image)

(1) The minimum oil film thickness \( H_{\text{min}} \) between the movable tooth and central input axis is about 0.1432 \( \mu \text{m} \) ~ 0.1479 \( \mu \text{m} \), and the minimum oil film thickness \( H_{\text{min}} \) between the movable tooth and the shell is about 0.0414 \( \mu \text{m} \) ~ 0.0589 \( \mu \text{m} \) when the input shaft speed \( n = 225 \text{r/min} \). The minimum oil film thickness between the movable teeth and
central input shaft is generally higher than those between the movable teeth and shell because the entrainment speed between the movable teeth and central input shaft is higher than that between the movable teeth and shell.

(2) The distribution of the minimum oil film thickness varies periodically, and the fluctuation range of the minimum oil film thickness between the movable tooth and center input shaft is smaller than that between the movable tooth and shell. This is because the change rate of entrainment velocity between the movable tooth and shell is higher than that between the movable tooth and central input axis.

Let the root-mean-square values of two surface roughness be $\sigma_1$ and $\sigma_2$. The effective roughness of two surfaces can be calculated as $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$. Thus, film thickness ratio $\lambda$ can be determined as below

$$\lambda = \frac{H_{\text{min}}}{\sigma} = \frac{H_{\text{min}}}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

When $\lambda > 3$, good lubrication performance can be obtained, which is called full film elasto-hydrodynamic lubrication. When $1 \leq \lambda \leq 3$, the lubrication film is thinner and even contact occurs between some micro-convex bodies of two mesh surfaces, which is referred to as the partial membrane elasto-hydrodynamic lubrication. When $\lambda < 1$, the load is mainly carried by the micro-convex body, it is called as the state of thin film lubrication.

The minimum oil film thickness and the corresponding film thickness ratio for the sine movable tooth drive vary with the surface roughness of the sine raceway and the speed of input shaft (see Tables 2-4). It is known:

| Table 2 Minimum oil film thickness distribution at different speeds |
|---------------------------------------------------------------|
| input speed (rpm) | 225 | 500 | 1000 | 1500 | 2000 |
| $H_{\text{min}}$ (μm) | 0.143–0.147 | 0.250–0.258 | 0.407–0.420 | 0.540–0.558 | 0.661–0.682 |
| $H_{\text{min}}$ (μm) | 0.041–0.058 | 0.072–0.103 | 0.118–0.167 | 0.156–0.222 | 0.191–0.272 |

| Table 3 lubrication states of meshing pairs for Ra=0.1 μm |
|-----------------------------------------------------------|
| input speed (rpm) | 200 | 500 | 1000 | 1500 | 2000 |
| movable teeth and central shaft | partial film | partial film | partial film | full film | full film |
| movable teeth and shell | thin film | thin film | partial film | partial film | partial film |

| Table 4 lubrication states of meshing pairs for Ra=0.2 μm |
|-----------------------------------------------------------|
| input speed (rpm) | 200 | 500 | 1000 | 1500 | 2000 |
| movable teeth and central shaft | thin film | thin film | partial film | partial film | partial film |
| movable teeth and shell | thin film | thin film | thin film | thin film | thin film |

(1) As the surface roughness is 0.1 μm, the state of partial film elasto-hydrodynamic lubrication is formed between the movable teeth and central shaft, and the state of thin film lubrication is formed between the movable teeth and shell under condition that input speed is below 1000 rpm.

(2) As the surface roughness is 0.1 μm, the state of full film elasto-hydrodynamic lubrication is formed between the movable teeth and central shaft, and the state of partial film lubrication is formed between the movable teeth and shell under condition that input speed is above 1000 rpm.

(3) As the surface roughness is 0.2 μm, the state of partial film elasto-hydrodynamic lubrication is formed between the
movable teeth and central shaft for input speed above 500rpm, and the state of thin film lubrication is formed between the movable teeth and central shaft for input speed below 500rpm.

(4) As the surface roughness is 0.2 μm, the state of thin film lubrication is formed between the movable teeth and shell for input speed below 2000 rpm.

Substituting the parameters of sine movable tooth drive in Table 1 into Eq. (21), the distribution of meshing efficiency and its changes with main parameters can be obtained, as shown in Fig. 5 and Fig. 6. As can be seen from Figs. 5 and 6:

(1) The meshing efficiency of the sinusoidal movable tooth drive is between 79.74% and 84.67%, the variation period of efficiency is 6π/5, and the average efficiency is 82.48%.

(2) With the increase of sine raceway amplitude A, rotary radius R of the movable tooth center, movable tooth radius r and input speed n, the meshing efficiency of the sinusoidal movable tooth drive increases. Among above parameters, the radius r of the movable tooth has the greatest influence. When the input torque and speed of the drive are fixed, the input power will not change. With the increase of the amplitude of the sinusoidal raceway A, the center radius R of the movable teeth and the radius r of the movable teeth, the sliding friction power loss of the meshing pair between the single movable tooth and the central shaft will decrease, and the efficiency of the drive increases. With the increase of input speed n, the oil film thickness between movable teeth and meshing parts increases and the sliding friction power loss decreases, so the efficiency of the drive increases.

Fig. 5 Mesh efficiency distribution of sinusoidal movable tooth drive

(3) When the speed ratio i = 5, the average meshing efficiency of the drive is 83.87%; When the speed ratio i = 7, the average meshing efficiency of the drive is 78.58%. With the increase of the speed ratio, the meshing efficiency of the drive decreases, and the changing period of meshing efficiency decreases as well. With the increase of the speed ratio, the sliding friction power loss of the meshing pair for single movable tooth and central shaft will decrease, but the total friction power loss will increase due to the increase of meshing tooth number, so the efficiency of the drive decreases.
To verify the correctness of the theoretical analysis, the prototype with the parameters shown in Table 1 is produced (see Fig. 7a). The efficiency of the experimental prototype is measured, and the test platform is shown in Fig. 7b. The transmission efficiency of the drive prototype is measured at 180 rpm and 225 rpm of input shaft speed. The test results are shown in Fig. 8. The results show that:

As the load torque increases, the efficiency of the drive system first increases to a peak value and then gradually decreases. With the increase of input shaft speed, the transmission efficiency increases gradually.

When the input shaft speed is 225 rpm, the meshing efficiency of the drive is between 79.74% and 84.67%, and the average efficiency is 82.48%, which is near to the experimental efficiency of 82.25%. The results illustrate correctness of the theoretical analysis in this paper.
6. Conclusion

In this paper, based on the analysis of the entrainment speed and forces for the sine movable tooth drive, the equations of the minimum oil film thickness and meshing efficiency for the drive is proposed. The minimum oil film thickness and meshing efficiency of the drive are studied, and the changes of the meshing efficiency along with main parameters are analyzed. Results show:

1. The minimum oil film thickness between the movable teeth and central input shaft is higher than that between the movable teeth and shell. As the surface roughness is \(0.1 \mu\text{m}\) and input speed is above 1000 rpm, the full film lubrication is formed between the movable teeth and the central shaft, and the partial film lubrication is formed between the movable teeth and the shell. As input speed is below 1000 rpm, the partial film lubrication is formed between the movable teeth and the central shaft, and the thin film lubrication is formed between the movable teeth and shell.

2. With the increase of sine raceway amplitude, rotary radius of movable tooth center, movable tooth radius and input speed, the meshing efficiency of sinusoidal movable tooth drive increases. Among above parameters, the radius of the movable tooth has the greatest influence.

3. The meshing efficiency of the sinusoidal movable tooth drive is between 79.74\% and 84.67\%, and the average efficiency is 82.48\% which is near to the experimental efficiency of 82.25\%.
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