Study of direct CP violation for $B_s^0 \rightarrow \pi^+\pi^0(K^0)$ decay in Perturbative QCD

Gang Lü1*, De-Sen Shi 1†, Na-Wang1‡.

1 College of Science, Henan University of Technology, Zhengzhou 450001, China

We investigate the direct CP violation for the charmless three-body decay process of $B_s^0 \rightarrow \rho(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$. The mechanisms of $\rho - \omega$, $\rho - \phi$ and $\omega - \phi$ mixing are considered in perturbative QCD approach. Isospin symmetry breaking from the decays of $\omega \rightarrow \pi^+\pi^-$ and $\phi \rightarrow \pi^+\pi^-$ are known to be tiny. However, we find the direct CP violation of the decay processes of $B^+_s \rightarrow \pi^+\pi^0\pi^0$ and $B^+_s \rightarrow \pi^+\pi^-K^0$ can reach 65% and 36% from isospin symmetry breaking, respectively. To better compare with experimental data in the future, the localized integrated direct CP violation of the decay processes are presented. The mechanism can provide a new way to explore CP violation from the three-body decay processes of $B_s$ meson for the LHC experiment.

I. INTRODUCTION

CP violation produces the differences between matter and antimatter. The sources of CP violation in particle physics have caused a great deal of attention since 1964 [1]. CP violation can occur from the hadron decay, the neutral hadron mixing and the interference between mixing and decay [2]. In Standard Model (SM), CP violation is related to the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes quark-level transition and changes sign under CP transforming. However, the strong phase is CP invariant for the decay amplitude. Direct CP violation in $b$ hadron decay occurs through interference with at least two amplitudes of different weak phase $\phi$ and strong phase $\delta$. The weak phase difference $\phi$ is determined by the CKM matrix elements, while the strong phase is generated by the rescattering or the effects of other hadron for two body decay process. In the three-body or multi-body decay processes, the strong phases arise from the intermediate resonance hadrons.

The hadronic components of the vacuum polarisation of the photon consist exclusively of the known vector mesons $\rho^0$, $\omega$ and $\phi$ in the vector meson dominance model (VMD) [3]. The photon couples to the neutral vector meson which is dominated by a two-pion state in the process $e^+e^- \rightarrow \pi^+\pi^-$. The $\omega \rightarrow \pi^+\pi^-$ and $\phi \rightarrow \pi^+\pi^-$ transitions from isospin symmetry breaking relate the $\omega - \rho$ and $\phi - \rho$ mixing. One can combine the intermediate state with the physical state by the unitary matrix. The dynamics mechanism can be obtained from the interference of $\rho^0$, $\omega$ and $\phi$ mesons [4]. The contribution of the intermediate resonance hadrons gives rise to the new strong phase, which may affect the CP violation of hadron decay.

In recent years, the LHCb collaboration has measured direct CP asymmetries in the decay channels of $B^\pm \rightarrow \pi^+\pi^+\pi^-$ and $B^\pm \rightarrow K^\pm\pi^+\pi^-$ [5][7]. CP violation has been observed in localized phase space region from the charmless

* Email: ganglv66@sina.com
† Email: sds163mail@163.com
‡ Email: wangna@haut.edu.cn
three-body $B$ meson decay. Especially, quasi-two-body $B \to PV$ resulting in three-body final states from vector meson decays are presented. In view of vector mesons resonances, the different types of resonant contributions are allowed to estimate the strong phase. CP violations are measured in charmless B decay which related to the $\rho(770) - \omega(782)$ mixing region. Including the $\omega(782)$ contribution, the CP violation related to the vector resonance is measured to be $A_{CP}(B^+ \to \rho(770)^0\pi^+ \to \pi^+\pi^+\pi^-) = -0.004 \pm 0.017$ and $A_{CP}(B^+ \to \rho(770)^0K^+ \to K^+\pi^+\pi^-) = 0.150 \pm 0.019$. The method comes from the approximation of a two-body interaction plus one spectator meson [8].

These processes improves our knowledge about the CP violation by precise measurements. The data sample collected by the upgraded LHCb detector will allow CP violation measurements for the $B_s$ decay in the near future. The three-body charmless decays of $B_s$ meson can provide us new possibilities to search CP violation. The three-body decay process is often dominated through the quasi-two-body decay channels by intermediate states. For two-body hadron decays, there are a number of factorization schemes to calculate the hadron matrix elements, such as QCD factorization (QCDF) [9–12], perturbative QCD (PQCD) [13–16] and soft co-linear effective theory (SCET) [17, 18]. Based on the QCD correction, the PQCD method can calculate the decay amplitudes by introducing a sudakov factor to avoid the endpoint divergence problem. The final state interaction is described by the two-particle distribution amplitude in the resonance region. The three-body decay process is treated as a quasi-two body decay process. At present, the perturbation QCD factorization method has been widely used in the theoretical calculation of B-meson three-body decay process [13–21].

In this paper, we consider the effect of the $\rho - \omega - \phi$ resonance for the CP violation, which is associated with new complex strong phase. The CP violation is presented from isospin symmetry breaking due to the new strong phase for the first order. We aim at the CP violation in the decay process of $B^0_s \to (\omega, \phi)\pi^0(K^0) \to \pi^+\pi^-\pi^0(K^0)$ in perturbative QCD approach. At the same time, the localized integrated CP violation can be obtained to compare with the results of experiments in near future.

The remainder of this paper is organized as follows. In Section II, we give theoretical framework including the form of the effective Hamiltonian, wave functions required in this work and theoretical derivation process for the $\rho - \omega - \phi$ resonance effect. Then, we present the decay diagrams and analytical formalization of the main decay processes in Section III. The detailed calculation of this work is in the Section IV and input parameters are presented in Section V. We present the numerical results in Section VI. Summary and conclusion are included in Section VII. The related functions defined in the text are given in Appendix.

## II. THEORETICAL FRAMEWORK

### A. EFFECTIVE HAMILTONIAN

With the operator product expansion, the effective weak Hamiltonian can be written as [22]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{eq}^* \left[ C_1(\mu) Q_1^u(\mu) + C_2(\mu) Q_2^u(\mu) \right] - V_{tb}V_{eq}^* \left[ \sum_{i=3}^{10} C_i(\mu) Q_i^u(\mu) \right] \right\} + H.c.,$$  \hspace{1cm} (1)
where $G_F$ is Fermi constant, $C_i$ ($i=1,...,10$) represent the Wilson coefficients, $V_{ub}, V_{uq}, V_{tb}$ and $V_{tq} (q = s, d)$ are the CKM matrix elements. The operators $O_i$ have the following forms:

$$
\begin{align*}
O_1 &= \bar{q}_a \gamma_\mu (1-\gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1-\gamma_5) b_\alpha, \\
O_2 &= \bar{q} \gamma_\mu (1-\gamma_5) u \bar{u} \gamma^\mu (1-\gamma_5) b, \\
O_3 &= \bar{q} \gamma_\mu (1-\gamma_5) b \sum_{q'} q'_\beta \gamma^\mu (1-\gamma_5) q'_\alpha, \\
O_4 &= \bar{q}_a \gamma_\mu (1-\gamma_5) b_\beta \sum_{q'} q'_\beta \gamma^\mu (1-\gamma_5) q'_\alpha, \\
O_5 &= \bar{q} \gamma_\mu (1-\gamma_5) b \sum_{q'} q'_\gamma \gamma^\mu (1+\gamma_5) q', \\
O_6 &= \bar{q}_a \gamma_\mu (1-\gamma_5) b_\beta \sum_{q'} q'_\gamma \gamma^\mu (1+\gamma_5) q'_\alpha, \\
O_7 &= \frac{3}{2} \bar{q} \gamma_\mu (1-\gamma_5) b \sum_{q'} e_q q' \bar{q}' \gamma^\mu (1+\gamma_5) q', \\
O_8 &= \frac{3}{2} \bar{q}_a \gamma_\mu (1-\gamma_5) b_\beta \sum_{q'} e_q q' \bar{q}'_\beta \gamma^\mu (1+\gamma_5) q'_\alpha, \\
O_9 &= \frac{3}{2} \bar{q} \gamma_\mu (1-\gamma_5) b \sum_{q'} e_q q' \bar{q}' \gamma^\mu (1-\gamma_5) q', \\
O_{10} &= \frac{3}{2} \bar{q}_a \gamma_\mu (1-\gamma_5) b_\beta \sum_{q'} e_q q' \bar{q}'_\beta \gamma^\mu (1-\gamma_5) q'_\alpha,
\end{align*}
$$

where $\alpha$ and $\beta$ are color indices, and $q' = u, d$ or $s$ quarks. In Eq. (2) $O_1^u$ and $O_2^u$ are tree operators, $O_3$–$O_6$ are QCD penguin operators and $O_7$–$O_{10}$ are the operators associated with electroweak penguin diagrams.

We can obtain numerical values of $C_i$ [23].

$$
\begin{align*}
C_1 &= -0.2703, & C_2 &= 1.1188, \\
C_3 &= 0.0126, & C_4 &= -0.0270, \\
C_5 &= 0.0085, & C_6 &= -0.0326, \\
C_7 &= 0.0011, & C_8 &= 0.0004, \\
C_9 &= -0.0090, & C_{10} &= 0.0022.
\end{align*}
$$

One can obtain numerical values of $a_i$. The combinations $a_i$ of Wilson coefficients are defined as [24]

$$
\begin{align*}
a_1 &= C_2 + C_1/3, & a_2 &= C_1 + C_2/3, \\
a_3 &= C_3 + C_4/3, & a_4 &= C_4 + C_3/3, \\
a_5 &= C_5 + C_6/3, & a_6 &= C_6 + C_5/3, \\
a_7 &= C_7 + C_8/3, & a_8 &= C_8 + C_7/3, \\
a_9 &= C_9 + C_{10}/3, & a_{10} &= C_{10} + C_9/3.
\end{align*}
$$
B. WAVE FUNCTION

The meson wave function is a function that describes how positive and negative quarks form hadrons, and how the momentum carried by the parton inside the meson is distributed. The wave function of the meson is non-perturbative and process independent. In perturbative QCD, transverse momentum is introduced due to the QCD corrections from the factorization approach [25 26]. The meson wave function is connected with the transverse momentum. The transverse momentum has a small effect on the light mesons, but a large effect on the heavy mesons from relevant results. Light-cone QCD sum rule is applied to calculate the non-perturbative contributions [27, 28]. The theoretical predictions are consistent with experimental results for most of the decay processes in $B/B_s \rightarrow M_2M_3$ by the wave function from Light-cone QCD sum rule [29 31].

For the B meson wave function, we adopt the model [32 33].

$$\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{M_B^2 x^2}{2 \omega_b} - \frac{1}{2} (\omega_b b)^2 \right], \quad (5)$$

Where $N_b$ is a normalization factor and $\omega_b$ is a shape parameter. For $B^0 (B^\pm)$, we use $\omega_b = 0.4 \pm 0.04 \, \text{GeV}$, which is determined by the calculation of form factor and other decay modes. Taking into account the small SU(3) breaking and the fact that the s quark is heavier than the u or d quark, we use the shape parameter $\omega_s = 0.5 \pm 0.05 \, \text{GeV}$ for the $B_s$ meson, indicating that the s quark momentum fraction is larger than that of the u or d quark in the $B^\pm$ or $B^0$ meson [34].

The distributed amplitudes in the wave function of the light pseudoscalar meson describe the distribution of the internal components of the hadron (quarks and antiquarks, parton) with momentum. At present, there is no reliable method to calculate, and the result obtained by light-cone QCD sum rule is usually adopted. For the distribution amplitude $\phi_{\pi(K)}^{A, T}$ and $\phi_{\pi(K)}^P$, which refer to the axial vector, tensor components and pseudoscalar of the wave function, respectively. We utilize the results for the $\pi$ meson obtained from the light cone sum rule including twist-3 contributions [29 30 35]:

$$\phi_{\pi(K)}^{A, T}(x) = \frac{f_{\pi(K)}}{2 \sqrt{2N_c}} 6x(1 - x) \left[ c1 + a_1^{(K)} C_1^{3/2} (2x - 1) + a_2^{(K)} C_2^{3/2} (2x - 1) + a_4^{(K)} C_4^{3/2} (2x - 1) \right], \quad (6)$$

$$\phi_{\pi(K)}^P(x) = \frac{f_{\pi(K)}}{2 \sqrt{2N_c}} \left[ 1 + \left( 30 \eta_3 - \frac{5}{2} \rho_3^{(K)} \right) C_2^{1/2} (2x - 1) - 3 \left( \eta_3 \omega_3 + \frac{9}{20} \rho_2^{(K)} (1 + 6 a_2^{(K)}) \right) C_4^{1/2} (2x - 1) \right], \quad (7)$$

$$\phi_{\pi(K)}^T(x) = \frac{f_{\pi(K)}}{2 \sqrt{2N_c}} (1 - 2x) \left[ 1 + 6 \left( 5 \eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho_2^{(K)} - \frac{3}{5} \rho_2^{(K)} a_2^{(K)} \right) (10x^2 - 10x + 1) \right], \quad (8)$$

Where $\rho_{\pi(K)} = \frac{m_{\pi(K)}}{m_{0(\pi(K))}}$. The Gegenbauer moments and the other parameters [30 31] are given as:

$$a_1^T = 0, a_2^{\pi, K} = 0.25 \pm 0.15, a_3^T = -0.015, a_4^{K} = 0.06, \quad (9)$$

$$\eta_3^{\pi, K} = 0.015, \omega_3^{\pi, K} = -3, m_0^{\pi} = 1.4 \pm 0.1 \, \text{GeV}, m_0^{K} = 1.6 \pm 0.1 \, \text{GeV}. \quad (10)$$
The momentum fraction \( x \) in the above distribution amplitude is the momentum fraction carried by the "quark". The relevant Gegenbauer polynomials are defined by [31]:

\[
C_1^{3/2}(t) = 3t, \quad C_2^{1/2}(t) = \frac{1}{2} (3t^2 - 1), \quad C_2^{3/2}(t) = \frac{3}{2} (5t^2 - 1), \quad (11)
\]

\[
C_4^{1/2}(t) = \frac{1}{8} (35t^4 - 30t^2 + 3), \quad C_4^{3/2}(t) = \frac{15}{8} (35t^4 - 14t^2 + 1). \quad (12)
\]

For vector mesons, the treatment is the same as for pseudoscalar mesons. We assume that the vector meson \( V \) moves in the "\( + \)" direction and its polarization vector is \( \epsilon \) and \( \epsilon \cdot P = 0 \). For a vector meson, which has longitudinal and transverse polarization, the corresponding wave function can be written as [30, 37]:

\[
\Phi_V = \frac{i}{\sqrt{2Nc}} \left\{ f_L \left[ M_V \phi_V^i (x) + \phi_V^i (x) \right] + M_V \phi_V^5 (x) \right\}, \quad (13)
\]

\[
\Phi_V (x) = \frac{i}{\sqrt{2Nc}} \left\{ f_T \left[ M_V \phi_V (x) + \phi_V^T (x) \right] + M_V i\epsilon_{\mu\nu\rho\sigma} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \phi_V^5 (x) \right\}. \quad (14)
\]

where \( \epsilon_{\mu\nu\rho\sigma} = -\epsilon^{\mu\nu\rho\sigma} = -1 \) and distributed amplitudes \( \phi_V (x) \) and \( \phi_V^T (x) \) indicate Twist-2 contributions.

For light vector mesons \( \rho, \omega \) and \( \phi \), the expressions for the distribution amplitudes \( \phi_V (x) \) and \( \phi_V^T (x) \) representing the Twist-2 contribution [38, 40] are:

\[
\begin{align*}
\phi_{\rho(x)} &= \frac{3f_\rho}{\sqrt{6}} x (1 - x) \left[ 1 + a_{2p}^\parallel C_2^{3/2} (t) \right], \\
\phi_{\omega(x)} &= \frac{3f_\omega}{\sqrt{6}} x (1 - x) \left[ 1 + a_{2p}^\parallel C_2^{3/2} (t) \right], \\
\phi_{\phi(x)} &= \frac{3f_\phi}{\sqrt{6}} x (1 - x) \left[ 1 + a_{2p}^\parallel C_2^{3/2} (t) \right], \\
\phi_{\rho^T(x)} &= \frac{3f_\rho}{\sqrt{6}} x (1 - x) \left[ 1 + a_{2p}^\parallel C_2^{3/2} (t) \right], \\
\phi_{\omega^T(x)} &= \frac{3f_\omega}{\sqrt{6}} x (1 - x) \left[ 1 + a_{2p}^\parallel C_2^{3/2} (t) \right], \\
\phi_{\phi^T(x)} &= \frac{3f_\phi}{\sqrt{6}} x (1 - x) \left[ 1 + a_{2p}^\parallel C_2^{3/2} (t) \right]. \quad (15)
\end{align*}
\]

where \( f_V \) denotes the decay constant of the longitudinal polarization, the newer Gegenbauer moments values taken are [31]:

\[
\begin{align*}
f_\rho &= 209 \pm 2 \text{MeV}, & f_\rho^T &= 165 \pm 9 \text{MeV}, \\
f_\omega &= 195 \pm 3 \text{MeV}, & f_\omega^T &= 145 \pm 10 \text{MeV}, \\
f_\phi &= 231 \pm 4 \text{MeV}, & f_\phi^T &= 200 \pm 10 \text{MeV}, \quad (16)
\end{align*}
\]

\[
\begin{align*}
a_{2p}^\parallel &= a_{2p}^\parallel = 0.15 \pm 0.07, & a_{2p}^\parallel &= 0.18 \pm 0.08, \\
a_{2p}^\parallel &= a_{2p}^\parallel = 0.14 \pm 0.06, & a_{2p}^\parallel &= 0.14 \pm 0.07.
\end{align*}
\]
There has been much theoretical discussion on the contribution distribution amplitudes \( \phi_V^0 \) and \( \phi_V^σ \) for the vector meson Twist-3. But there is still no good result [42, 43], the asymptotic expression currently used is

\[
\phi_V^0 (x) = \frac{3f_V^2}{2\sqrt{6}} t^2, \quad \phi_V^σ (x) = \frac{3f_V^2}{2\sqrt{6}} (-t), \\
\phi_V^3 (x) = \frac{3f_V}{8\sqrt{6}} (1 + t^2), \quad \phi_V^σ (x) = \frac{3f_V}{4\sqrt{6}} (-t).
\]

(17)

C. \( \rho - \omega - \phi \) RESONANCE EFFECT

According to the vector mesons dominance (VMD) model, \( e^+e^- \) annihilate into photons, which are polarized in a vacuum to form vector particles \( \rho^0(770), \omega(782), \phi(1020) \) and then decay into \( \pi^+\pi^- \) pairs. The momentum is also transmitted through VMD model. The intermediate state particle is a non-physical state, which can be converted to vacuum to form vector particles \( \rho \), \( \omega \), \( \phi \).

After simplification, we can get two expressions about \( R \), where \( \langle \rho_I | \rho \rangle \), \( \langle \omega_I | \omega \rangle \) and \( \langle \phi_I | \phi \rangle \) are equal to 1. Besides, \( \langle \rho_I | \omega \rangle \), \( \langle \rho_I | \phi \rangle \) and \( \langle \omega | \phi \rangle \) are equal to \( F_{\rho\omega} (s) \), \( F_{\rho\phi} (s) \) and \( F_{\omega\phi} (s) \) because of the isospin effect, where they are all order \( \mathcal{O}(\lambda) \), \( \lambda \leq 1 \) in the unitary matrix \( R(s) \) [4].

Based on isospin \( \rho_I^0 \), \( \omega_I \), \( \phi_I \) filed, we can construct the isospin basis vector \( |I, I_3> \), where \( I \) and \( I_3 \) refer to the isospin and the isospin third component, respectively. Hence, the physical particle state can be represented as a linear combination of the above basis vectors. We use \( M \) and \( N \) to represent the physical state and the isospin basis vector of the particle and then we can get the relationship equations \( \sum_M |M><M| = \sum_{M_I} |M_I><M_I| = I \) and \( \langle M | N \rangle = \langle M_I | N_I \rangle = \delta_{MN} \) according to the orthogonal normalization relation. We define \( M \) and \( D(s) \) as the mass squared operator and the propagator respectively. We can get the relationship between them according to the diagonalization for the physical states. We can get the physical states \( \rho^0 \), \( \omega \) and \( \phi \) as follows:

\[
\rho^0 = \rho_I^0 - F_{\rho\omega} (s) \omega_I - F_{\rho\phi} (s) \phi_I, \quad (18)
\]

\[
\omega = F_{\rho\omega} (s) \rho_I^0 + \omega_I - F_{\omega\phi} (s) \phi_I, \quad (19)
\]

\[
\phi = F_{\rho\phi} (s) \rho_I^0 + F_{\omega\phi} (s) \omega_I + \phi_I. \quad (20)
\]

Ignoring the contribution of higher order terms, we can diagonalize the equation \( W_I \) by the matrix \( R \) in the physical representation, where \( W_I \) is defined as the mass squared operator in an isospin field. Thus we can obtain the symmetry relationship of \( F \langle \rho_I | W | \omega_I \rangle = F \langle \omega_I | W | \rho_I \rangle \) and \( F \langle \rho_I | W | \phi_I \rangle = F \langle \phi_I | W | \rho_I \rangle \). We can describe the propagator of intermediate state particle from vector meson according to the representations of physics and isospin. \( D_{V_1V_2} \) and \( D_{V_1V_2}^I \) refer to the propagator \( D_{V_1V_2} = \langle 0|TV_{12}|0 \rangle \) and \( D_{V_1V_2}^I = \langle 0|TV_{12}^I|0 \rangle \) in the basis of physics and isospin, respectively. Hence we can get \( D_{\omega\rho}, D_{\rho\omega} \) and \( D_{\omega\phi} \), where \( D_{\omega\rho} = D_{\rho\omega}, D_{\rho\phi} = D_{\phi\rho} \) and \( D_{\omega\phi} = D_{\phi\omega} \).

In the state of physics, there are no \( \rho - \omega - \phi \) mixing so that \( D_{\rho\omega}, D_{\rho\phi} \) and \( D_{\omega\phi} \) are equal to zero. Besides, the
parameters of $\Pi_{\rho\omega}$, $\Pi_{\omega\phi}$, $\Pi_{\rho\phi}$, $F_{\rho\omega}$, $F_{\rho\phi}$ and $F_{\omega\phi}$ are order of $O(\lambda)$, ($\lambda \leq 1$). Any two or three terms multiplied together are of higher order and can be ignored. Hence, we can get

$$F_{\rho\omega} = \frac{\Pi_{\rho\omega}}{s_{\rho} - s_{\omega}}, \quad F_{\rho\phi} = \frac{\Pi_{\rho\phi}}{s_{\rho} - s_{\phi}}, \quad F_{\omega\phi} = \frac{\Pi_{\omega\phi}}{s_{\omega} - s_{\phi}}. \quad (21)$$

where $s_{\rho}$, $s_{\omega}$ and $s_{\phi}$ refer to the propagators of $\rho$, $\omega$ and $\phi$.

We can write

$$s_{V} = s - m_{V}^{2} + im_{V} \Gamma_{V}. \quad (22)$$

$s_{V}$, $m_{V}$, and $\Gamma_{V}$ ($V = \rho, \omega, \phi$) refer to the inverse propagator, mass and decay rate of the vector meson $V$, respectively.

From above equations, we can define ($\tilde{\Pi}_{\omega\phi}$ can be ignored in the next step of the calculation):

$$\tilde{\Pi}_{\rho\omega} = \frac{s_{\rho} \Pi_{\rho\omega}}{s_{\rho} - s_{\omega}}, \quad \tilde{\Pi}_{\rho\phi} = \frac{s_{\rho} \Pi_{\rho\phi}}{s_{\rho} - s_{\phi}}. \quad (23)$$

where the $\sqrt{s}$ denotes the invariant mass of the $\pi^{+}\pi^{-}$ pairs.

The $\rho - \omega$ mixing parameters were recently determined precisely by Wolfe and Maltman \[44, 45\]:

$$\Re \Pi_{\rho\omega}(m_{\rho}^{2}) = -4470 \pm 250_{\text{modl}} \pm 160_{\text{data}} \text{MeV}^{2},$$

$$\Im \Pi_{\rho\omega}(m_{\rho}^{2}) = -5800 \pm 2000_{\text{modl}} \pm 1100_{\text{data}} \text{MeV}^{2}, \quad (24)$$

The $\rho - \phi$ mixing parameters have been given near the $\phi$ meson \[46\]:

$$F_{\rho\phi} = (0.72 \pm 0.18) \times 10^{-3} - i (0.87 \pm 0.32) \times 10^{-3}. \quad (25)$$

The mixing parameter depends on the momentum including both the resonant and non-resonant contribution which absorbs the direct decay processes $\omega \rightarrow \pi^{+}\pi^{-}$ and $\phi \rightarrow \pi^{+}\pi^{-}$ from the isospin symmetry breaking effects. The mixing parameters $\tilde{\Pi}_{\rho\omega}(s)$ and $\tilde{\Pi}_{\rho\phi}(s)$ are the momentum dependence for $\rho - \omega$ mixing and $\rho - \phi$ mixing, respectively. We expect to search for the contribution of the mixing mechanism in the resonance region of $\omega$ and $\phi$ mass where two pions are also produced by isospin symmetry breaking.

One can express $\tilde{\Pi}_{\rho\omega}(s) = \Re \tilde{\Pi}_{\rho\omega}(m_{\omega}^{2}) + \Im \tilde{\Pi}_{\rho\omega}(m_{\omega}^{2})$ and $\tilde{\Pi}_{\rho\phi}(s) = \Re \tilde{\Pi}_{\rho\phi}(m_{\phi}^{2}) + \Im \tilde{\Pi}_{\rho\phi}(m_{\phi}^{2})$ and update the values\[47, 49\]:

| $\Re \tilde{\Pi}_{\rho\omega}(m_{\omega}^{2})$ | $\Im \tilde{\Pi}_{\rho\omega}(m_{\omega}^{2})$ |
|------------------------------------------|------------------------------------------|
| $-4760 \pm 440 \text{MeV}^{2}$         | $-6180 \pm 3300 \text{MeV}^{2}$         |

| $\Re \tilde{\Pi}_{\rho\phi}(m_{\phi}^{2})$ | $\Im \tilde{\Pi}_{\rho\phi}(m_{\phi}^{2})$ |
|------------------------------------------|------------------------------------------|
| $-796 \pm 312 \text{MeV}^{2}$          | $-101 \pm 67 \text{MeV}^{2}$            |
III. CP VIOLATION IN $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0$

A. Formalism

We take the decay process of $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0$ as an example to consider the CP violation. The decay rate of $\rho \rightarrow \pi^+ \pi^-$ is 100%. The decay diagram of $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0$ can be expressed in Fig.1. One can see that the quasi-two-body decay of $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0$ is associated the diagrams $(a) \sim (f)$ of Fig.1. For simplify, we only present a few of the major graphs.

![Diagram](image)

**FIG. 1:** The diagrams of $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0$ decay process.

In the diagram (a), $\bar{B}_s^0$ meson decays into $\pi^0$ and $\pi^+ \pi^-$ pair which is produced directly by $\rho^0$ resonance effect. Meanwhile, it is known that $\pi^+ \pi^-$ pair can also exist by the resonance effect of $\omega$ or $\phi$ meson, in which mixing parameters $\Pi_{\rho \omega}$ are involved. As shown in the diagram (b), $\rho^0$ meson can decay into $\pi^+ \pi^-$ by $\omega$ resonance. The mixing parameter $\Pi_{\rho \omega}$ is generated during the $\omega$ resonance, which as shown in the black dots of (b). Diagram of (c) is similar to the case of diagram (b), but the differences are the resonance effect is $\phi$ and the mixing parameter is $\Pi_{\rho \phi}$. In the diagram (d), we can know the decay process by $\omega \rightarrow \pi^+ \pi^-$ or $\phi \rightarrow \pi^+ \pi^-$. But compared to the process of $\rho \rightarrow \pi^+ \pi^-$, the decay rates of $\omega \rightarrow \pi^+ \pi^-$ and $\phi \rightarrow \pi^+ \pi^-$ are very little and we can ignore. It is easier to consider the contributions from the diagram (e) and diagram (f). Since $\omega$ and $\phi$ decay into $\pi^+ \pi^-$ through the resonance effect of $\omega - \phi$ mixing, its probabilities are tiny even to neglect. After considering above, we can see the decay process of $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0$ receives effectively diagrams contributions from $(a) \sim (c)$.

Then, the decay amplitude $A(\bar{A})$ of the process can be expressed as:

$$A = \langle \pi^+ \pi^- K^0 | H^T | \bar{B}_s^0 \rangle + \langle \pi^+ \pi^- K^0 | H^P | \bar{B}_s^0 \rangle,$$

where $\langle \pi^+ \pi^- K^0 | H^T | \bar{B}_s^0 \rangle$ and $\langle \pi^+ \pi^- K^0 | H^P | \bar{B}_s^0 \rangle$ refer to the contributions from the tree level and penguin level due to the operators, respectively.
The relative magnitude and phases between the tree and penguin operator contribution are defined as follows:

\[ A = \langle \pi^+\pi^- K^0| H^T |\bar{B}^0_s \rangle \left[ 1 + r e^{i(\delta + \phi)} \right]. \]  

(27)

From the diagrams (a), (b) and (c) in Fig.1, we can get:

\[ < \pi^+\pi^- K^0| H_T |\bar{B}^0_s > = \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho \omega} t_\omega + \frac{g_\rho}{s_\rho s_\phi} \tilde{\Pi}_{\rho \phi} t_\phi + \frac{g_\rho}{s_\rho} t_p, \]  

(28)

\[ < \pi^+\pi^- K^0| H^P |\bar{B}^0_s > = \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho \omega} p_\omega + \frac{g_\rho}{s_\rho s_\phi} \tilde{\Pi}_{\rho \phi} t_\phi + \frac{g_\rho}{s_\rho} p_p, \]  

(29)

where \( t_\rho(p_\rho), t_\phi(p_\phi) \), and \( t_\omega(p_\omega) \) are the tree (penguin) amplitudes, respectively. \( s_\rho, s_\omega \) and \( s_\phi \) refer to the propagations of \( \rho, \omega \) and \( \phi \), respectively. The coupling constant \( g_\rho \) is from decay process of \( \rho^0 \rightarrow \pi^+\pi^- \). The weak phase \( \phi \) is from the CKM matrix. The strong phase \( \delta \) and parameter \( r \) are dependent on the interference of the two level contribution and other mechanism. We can define

\[ r = \left| \frac{< \pi^+\pi^- K^0| H^P |\bar{B}^0_s >}{< \pi^+\pi^- K^0| H_T |\bar{B}^0_s >} \right|. \]  

(30)

We need give \( \sin \phi \) and \( \cos \phi \) to obtain the \( CP \) violating asymmetry. The weak phase \( \phi \) comes from the CKM matrix elements. In the Wolfenstein parametrization \[50\], one has

\[ \sin \phi = \frac{\eta}{\sqrt{\rho(1-\rho) - \eta^2} + \eta^2}, \]

\[ \cos \phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{\rho(1-\rho) - \eta^2} + \eta^2}. \]  

(31)

The differential \( CP \) asymmetry parameter can be defined as

\[ A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}. \]  

(32)

To better compare with experimental data in the future, we need consider the localized integrated direct \( CP \) violation of the decay processes. For the decay amplitude of the process of \( \bar{B}^0_s \rightarrow \rho^0 K^0 \rightarrow \pi^+\pi^- K^0 \), we consider the contributions of \( \bar{B}^0_s \rightarrow \rho^0 K^0 \) and \( \rho^0 \rightarrow \pi^+\pi^- \). The amplitude of the \( \bar{B}^0_s \rightarrow \rho^0 K^0 \) can be written as \( M_{\bar{B}^0_s \rightarrow \rho^0 K^0} = \alpha P_{\bar{B}^0_s} \cdot \epsilon^* (\lambda) \), where \( \epsilon \) is the polarization vector of \( \rho^0 \) and \( \lambda \) is its helicity. \( P_{\bar{B}^0_s} \) is the momenta of \( \bar{B}^0_s \) meson, and \( \alpha \) is the amplitude from the QCD correction and independent of \( \lambda \). The amplitude for \( \rho^0 \rightarrow \pi^+\pi^- \) is \( M_{\rho^0 \rightarrow \pi^+\pi^-} = g_\rho \epsilon (\lambda) \cdot (p_1 - p_2) \), where \( p_1 \) and \( p_2 \) are the momenta of \( \pi^+ \) and \( \pi^- \) produced by \( \rho^0 \), respectively. And \( g_\rho \) is the effective coupling constant for \( \rho^0 \rightarrow \pi^+\pi^- \). Hence, for the decay process \( \bar{B}^0_s \rightarrow \rho^0 K^0 \rightarrow \pi^+\pi^- K^0 \), the amplitude is \[51\] \[52\]:

\[ A = \frac{g_\rho \alpha}{s_\rho} P_{\bar{B}^0_s}^\mu \sum_{\lambda = \pm 1, 0} \epsilon^*_\mu (\lambda) \epsilon_\tau (\lambda) \cdot (p_1 - p_2)^\tau = -\frac{g_\rho \alpha}{s_\rho} P_{\bar{B}^0_s}^\mu \left[ g_{\mu \tau} - \frac{(p_1 + p_2)_\mu (p_1 + p_2)_\tau}{m^2_{\rho}} \right] (p_1 - p_2)^\tau, \]  

(33)

In the three body decay process, we obtain \( P_{\bar{B}^0_s} = p_1 + p_2 + p_3 \) and \( m_{ij}^2 = p_{ij}^2 \) since conservation of energy and
momentum. Thus, the amplitude can be written as:

\[ A = \frac{g_{\rho}}{s_p} \frac{M_{B^0_s \rightarrow \rho^0 K}}{P_{B^0_s} \epsilon^*} \left( \xi - s' \right) = \left( \xi - s' \right) \cdot M, \]

where \( \sqrt{s'} \) and \( \sqrt{s} \) is the high and low invariance mass of the \( \pi^+ \pi^- \) pair, respectively. \( \xi = \frac{1}{2} \left( s_{\text{max}}' + s_{\text{min}}' \right) \) with \( s_{\text{max}}' \) and \( s_{\text{min}}' \) being the maximum and minimum values of \( s' \) for a fixed \( s \), respectively.

By integrating the denominator and numerator of \( A_{\text{CP}} \) in this region, we get the localized integrated CP asymmetry, which can be measured by experiments and takes the following form:

\[ A_{\text{CP}}^O = \frac{\int_{s_1}^{s_2} ds \int_{s_1'}^{s_2'} ds' \left( \xi - s' \right)^2 \left( |A|^2 - |A'|^2 \right)}{\int_{s_1}^{s_2} ds \int_{s_1'}^{s_2'} ds' \left( \xi - s' \right)^2 \left( |A|^2 + |A'|^2 \right)}. \]  

We obtain \( \xi \left[ = \frac{1}{2} \left( s_{\text{min}}' + s_{\text{max}}' \right) \right] \) is related to \( s \) according to kinematics of the three body decay in the region \( \Omega \left( s_1 < s < s_2, s_1' < s < s_2' \right) \). In our calculations, we regard \( \xi \) as a constant since \( s \) varies in a small region. Hence, the terms \( \int_{s_1}^{s_2'} ds' \left( \xi - s' \right)^2 \) are cancelled and \( A_{\text{CP}}^O \) becomes independent of the high invariance mass of \( \pi^+ \pi^- \). In practice, to be more precise, we take into account the \( s \)-dependence of \( s_{\text{max}}' \) and \( s_{\text{min}}' \) in our calculations. We choose \( s_{\text{min}}' < s' < s_{\text{max}}' \) as the integration interval of the high invariance mass of \( \pi^+ \pi^- \) and regard \( \int_{s_{\text{min}}'}^{s_{\text{max}}'} ds' \left( \xi - s' \right)^2 \) as a factor which is dependent on \( s \) \cite{53, 54}.

IV. CALCULATION

The perturbative QCD (PQCD) method is obtained by applying QCD correction to the decay amplitude for the two-body weak decay of B mesons based on the \( k_T \) factorization formalism. The basic idea is to take into account the transverse momentum \( k_T \) of the valence quarks in the hadrons. As a result, the end-point singularity can be avoided. On the other hand, the transverse momentum dependence introduces an additional energy scale which leads to double logarithms in QCD corrections. These terms could be resummed through the renormalization group approach, which results in the appearance of the Sudakov form factor. This form factor effectively suppresses the end-point contribution of the distribution amplitude of the mesons in the small transverse momentum region, making the calculation in the PQCD approach reliable. It is worth mentioning that in this framework, the so-called annihilation diagrams are also perturbatively calculable without introducing additional parameters. The PQCD approach has been successfully used to study a number of pure annihilation type decays, and these predictions were confirmed subsequently in experiments. Thus, in our view, this method is reliable in dealing with the non-factorisable process and finding the contribution to the annihilation diagram as well \cite{53, 54}.

For simplification, we take the decay process \( B_s^0 \rightarrow \rho^0 (\omega, \phi) K^0 \rightarrow \pi^+ \pi^- K^0 \) as an example to illustrate in detail. One need obtain the formalism of \( t_p, t_w, t_\phi \) and \( p_p, p_w, p_\phi \) to calculate the CP violation which are from the tree level and penguin level contributions, respectively. Based on CKM matrix elements of \( V_{ub} V_{ud}^* \) and \( V_{tb} V_{td}^* \), the decay
amplitude of $\bar{B}_s^0 \to \rho^0 K^0$ in perturbation QCD approach can be written as

$$\sqrt{2} M(\bar{B}_s^0 \to \rho^0 K^0) = V_{ub} V_{ud}^* t_\rho - V_{tb} V_{td}^* p_\rho,$$  \hspace{1cm} (35)$$

where $t_\rho$ and $p_\rho$ refer to the tree and penguin contributions respectively.

$$t_\rho = f_\rho F^{LL}_{B_s \to K} [a_2] + M^{LL}_{B_s \to K} [C_2],$$ \hspace{1cm} (36)$$

and

$$p_\rho = f_\rho F^{LL}_{B_s \to K} \left[ -a_4 + \frac{3}{2} a_7 + \frac{1}{2} a_{10} + \frac{3}{2} a_9 \right] + M^{LR}_{B_s \to K} \left[ -C_5 + \frac{1}{2} C_7 \right]$$

$$+ M^{LL}_{B_s \to K} \left[ -C_3 + \frac{1}{2} C_9 + \frac{3}{2} C_{10} \right] - M^{SP}_{B_s \to K} \left[ \frac{3}{2} C_8 \right] + f_{B_s} F^{LL}_{ann} \left[ -a_4 + \frac{1}{2} a_{10} \right]$$

$$+ f_{B_s} F^{SP}_{ann} \left[ a_6 + \frac{1}{2} a_8 \right] + M^{LL}_{ann} \left[ -C_3 + \frac{1}{2} C_9 \right] + M^{LR}_{ann} \left[ -C_5 + \frac{1}{2} C_7 \right].$$ \hspace{1cm} (37)$$

where $C_i$ are the Wilson coefficients. One can find the formalism of the function $F$ and $M$ by the appendix. $F_{B_s \to K}$ and $M_{B_s \to K}$ represent the contribution of factorable emission diagrams and annihilation-type diagrams, respectively. $F_{ann}$ and $M_{ann}$ represent the contribution of un-factorable emission diagrams and annihilation-type diagrams, respectively.

We will use $LL$ to denote the contribution from $(V - A)(V - A)$ operators, $LR$ to denote the contribution from $(V - A)(V + A)$ operators, and $SP$ to denote the contribution from $(S - P)(S + P)$ operators which result from the Fierz transformation of the $(V - A)(V - A)$ operators.

The $t_\rho$ and $p_\rho$ can be extracted by the amplitudes of $\bar{B}_s^0 \to \omega K^0$. The decay amplitudes can be written as

$$\sqrt{2} M(\bar{B}_s^0 \to \omega K^0) = V_{ub} V_{ud}^* t_\omega - V_{tb} V_{td}^* p_\omega,$$ \hspace{1cm} (38)$$

where

$$t_\omega = f_\omega F^{LL}_{B_s \to K} (a_2) + M^{LL}_{B_s \to K} (C_2),$$ \hspace{1cm} (39)$$

and

$$p_\omega = f_\omega F^{LL}_{B_s \to K} \left( 2a_3 + a_4 + 2a_5 + \frac{1}{2} a_7 + \frac{1}{2} a_{10} - \frac{1}{2} a_9 \right) + M^{LL}_{B_s \to K} \left( C_3 + 2C_4 - \frac{1}{2} C_9 + \frac{1}{2} C_{10} \right)$$

$$+ M^{LR}_{B_s \to K} \left( C_5 - \frac{1}{2} C_7 \right) - M^{SP}_{B_s \to K} \left( 2C_6 + \frac{1}{2} C_8 \right) + f_{B_s} F^{LL}_{ann} \left( a_4 - \frac{1}{2} a_{10} \right) + f_{B_s} F^{SP}_{ann} \left( a_6 - \frac{1}{2} a_8 \right)$$

$$+ M^{LL}_{ann} \left( C_3 - \frac{1}{2} C_9 \right) + M^{LR}_{ann} \left( C_5 - \frac{1}{2} C_7 \right).$$ \hspace{1cm} (40)$$

The amplitude for $\bar{B}_s^0 \to \phi K^0$ can be written as

$$\sqrt{2} M(\bar{B}_s^0 \to \phi K^0) = V_{ub} V_{ud}^* \phi - V_{tb} V_{td}^* \phi,$$ \hspace{1cm} (41)$$
where

\[ t_\phi = 0, \]  \hspace{1cm} (42) \]

and

\[
p_\phi = f_\phi F_{B_s \to K}^{LL} \left( a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 \right) + f_K F_{B_s \to \phi}^{LL} \left( a_4 - \frac{1}{2} a_{10} \right) - f_K F_{B_s \to \phi}^{SP} \left( a_6 - \frac{1}{2} a_8 \right) \\
+ M_{B_s \to K}^{LL} \left( C_{4} - \frac{1}{2} C_{10} \right) + M_{B_s \to \phi}^{LL} \left( C_{3} - \frac{1}{2} C_{9} \right) - M_{B_s \to \phi}^{SP} \left( C_{6} - \frac{1}{2} C_{8} \right) - M_{B_s \to K}^{LR} \left( C_{5} - \frac{1}{2} C_{7} \right) \\
+ f_{B_s} F_{\text{ann}}^{LL} \left( a_4 - \frac{1}{2} a_{10} \right) - f_{B_s} F_{\text{ann}}^{SP} \left( a_6 - \frac{1}{2} a_8 \right) + M_{\text{ann}}^{LL} \left( C_{3} - \frac{1}{2} C_{9} \right) - M_{\text{ann}}^{LR} \left( C_{5} - \frac{1}{2} C_{7} \right). \]  \hspace{1cm} (43) \]

V. INPUT PARAMETERS

The CKM matrix, which elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters \( A, \rho, \lambda \) and \( \eta \) [50]:

\[
\begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix}, \hspace{1cm} (44)\]

where \( O(\lambda^4) \) corrections are neglected. The latest values for the parameters in the CKM matrix are [57]:

\[
\begin{aligned}
\lambda &= 0.22506 \pm 0.00050, & A &= 0.811 \pm 0.026, \\
\tilde{\rho} &= 0.124^{+0.019}_{-0.018}, & \tilde{\eta} &= 0.356 \pm 0.011.
\end{aligned} \hspace{1cm} (45) \]

where

\[
\tilde{\rho} = \rho \left( 1 - \frac{\lambda^2}{2} \right), \hspace{0.5cm} \tilde{\eta} = \eta \left( 1 - \frac{\lambda^2}{2} \right). \hspace{1cm} (46) \]

From Eqs. (45) (46) we get

\[ 0.109 < \rho < 0.147, \hspace{0.5cm} 0.354 < \eta < 0.377. \]  \hspace{1cm} (47) \]

The other parameters are given as following [50, 57]:

| Parameter | Value              |
|-----------|--------------------|
| \( M_B \) | 5.2792 GeV         |
| \( M_W \) | 80.385 GeV         |
| \( M_\rho \) | 0.77526 GeV       |
| \( f_\pi \) | 0.13 GeV          |
| \( f_\phi \) | 0.1237 GeV        |
| \( f_\omega \) | 0.195 GeV        |
| \( M_\omega \) | 8.49 \times 10^{-3} GeV |
| \( \Gamma_\phi \) | 4.23 \times 10^{-3} GeV |
| \( \Gamma_\omega \) | 4/3                |
VI. NUMERICAL RESULTS

In this paper, we study the CP violation for the decay processes of $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$ and $\bar{B}_s^0 \to \pi^+\pi^-K^0$. One can find the CP violation is also affected by the weak phase difference, the strong phase difference and $r$ from Eq.(27) and Eq.(32). The weak phase depends on the CKM matrix elements which have little effect on our results. Hence, we present the results corresponding to central parameter values of CKM matrix elements. For the $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$ decay process, the results are presented in Fig.2, Fig.3 and Fig.4. One can easily find that the CP violation is changed for the decay process of $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$ when the invariant masses of the $\pi^+\pi^-$ pairs are in the area around the $\omega$ resonance range and the $\phi$ resonance range in Fig.2, which the maximum CP asymmetry can be achieved 65%.

The plot of the $\sin\delta$ and $r$ as a function of $\sqrt{s}$ is presented in Fig.3 and Fig.4, respectively. One can find the $\sin\delta$ and $r$ varies sharply when the invariant masses of the $\pi^+\pi^-$ pairs are in the area around the $\omega$ resonance range and changes slightly around the $\phi$ resonance range comparing with the $\omega$ resonance range. For the decay of $\bar{B}_s^0 \to \rho^0 (\omega, \phi) \pi^0 \rightarrow \pi^+\pi^-\pi^0$, we obtain the CP asymmetries vary from 65% to -23% and from 0% to -0.05% at the $\rho - \omega$ resonance range and the $\rho - \phi$ resonance range, respectively.

One can see that the CP violation is changed sharply for the decay process of $\bar{B}_s^0 \to \pi^+\pi^-K^0$ when the invariant masses of the $\pi^+\pi^-$ pairs are around the $\omega$ resonance range and the $\phi$ resonance range in Fig.5, which the maximum CP asymmetry can be achieved 36%. For the decay process of $\bar{B}_s^0 \to \pi^+\pi^-K^0$ process, CP violation reach 62% and
2\% when $\pi^+\pi^-$ pairs are around regions of $\rho - \omega$ mixing and $\rho - \phi$ mixing. Similarly, the values of $\sin \delta$ and $r$ can also change at the resonance regions in Fig.6 and Fig.7.

Further more, we obtain the the localized integrated direct CP violation of the decay processes of $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$ and $\bar{B}_s^0 \to \pi^+\pi^-K^0$ are $-0.008 \pm 0.002$ and $-0.017 \pm 0.003$ at the resonance regions, respectively.

VII. SUMMARY AND CONCLUSION

This paper presents $\rho - \omega - \phi$ interference effect caused by isospin breaking. The new strong phase can be generated by the resonance contribution of $\rho - \omega$, $\rho - \phi$ and $\omega - \phi$. Meanwhile, the mechanism is applied to the decay process of $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$ and $\bar{B}_s^0 \to \pi^+\pi^-K^0$. It is found that a large CP violation can be generated in the resonance range. In the decay process of $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$, the CP violation can reach a maximum of 65\%. In the decay process of $\bar{B}_s^0 \to \pi^+\pi^-K^0$, the CP violation can reach a maximum of 36\%.

After integration of the decay processes, we obtain the local CP violation for the decay process of $\bar{B}_s^0 \to \pi^+\pi^-\pi^0$ and $\bar{B}_s^0 \to \pi^+\pi^-K^0$ are:

$$A_{CP}^Q(\bar{B}_s^0 \to \pi^+\pi^-\pi^0) = -0.008 \pm 0.002, \quad (48)$$

$$A_{CP}^Q(\bar{B}_s^0 \to \pi^+\pi^-K^0) = -0.017 \pm 0.003. \quad (49)$$

There are a lot of studies on B meson decays, including two-body decays as well as multi-body decays. Among such decays, quasi-two-body decays form an important class in the study of bottom and charm meson physics. We take B meson decay $B \to RP_3$ as an example, where R and $P_3$ are respectively an intermediate resonant state and a hadron and R further decays to two hadrons $P_{1,2}$. We apply the factorization relation which also known as the narrow width approximation (NWA) to factorize the process as two sequential two-body decays: $B(B \to RP_3 \to P_1P_2P_3) = B(B \to RP_3)B(B \to P_1P_2)$. It is known that the width $\omega$ and $\phi$ are relatively small, which one can neglect the effects safely as quasi-two-body decay processes. The decay rate of $\rho(770)$ is large. Therefore, the correction should be considered. The correction factor is at level 7\% for the quasi-two-body decays process of $B^- \to \rho(770)\pi^- \to \pi^+\pi^-\pi^-$ from the QCD factorization approach. The parameter $\eta_R$ is introduced to identify the degree of approximation for $\Gamma(B \to RP_3)B(B \to P_1P_2) = \eta_R \Gamma(B \to RP_3 \to P_1P_2P_3)$ [58, 59]. One can ignore the effect of NMA for the
calculation of CP violation since the $\eta_R$ can be divided out as a constant. Therefore, we neglect the effects of this correction in this work.

In recent years, the large data collected by the LHC have allowed accurate measurements of direct CP violation in B meson decays. Theoretical developments using different methods have already led to many predictions of CP violation. The Atlas and CMS experiments focus on the B physics program and the search of new physics. The LHC have made a major upgrade and increased its luminosity by a factor of five beyond its design value recently. The search for direct CP violation in charmless $B_s$ decay may be measured in the near future. The CP violation can be presented in the regions of $\rho - \omega$ and $\rho - \phi$ regions by reconstructing the $\rho$, $\omega$ and $\phi$ mesons when the invariant masses of $\pi^+\pi^-$ are at the resonant regions. We hope that our predictions will provide useful guidance for future experiments.

VIII. ACKNOWLEDGEMENTS

One of the authors (Gang L) thanks Professor Zhen-Hua Zhang, Jing-Juan Qi and Chao-Wang for helpful discussions.

IX. APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

The functions related with the tree and penguin contributions are presented with PQCD approach [60, 61]. The hard scales $t$ are chosen as

\begin{align*}
  t_a &= \max\{\sqrt{x_3}m_{B_s}, 1/b_1, 1/b_3\}, \quad (50) \\
  t'_a &= \max\{\sqrt{x_1}m_{B_s}, 1/b_1, 1/b_3\}, \quad (51) \\
  t_c &= \max\{\sqrt{1-x_3}m_{B_s}, 1/b_2, 1/b_3\}, \quad (52) \\
  t'_c &= \max\{\sqrt{x_2}m_{B_s}, 1/b_2, 1/b_3\}, \quad (53) \\
  t_b &= \max\{\sqrt{x_1x_3}m_{B_s}, \sqrt{(x_1-x_2)x_3}m_{B_s}, 1/b_1, 1/b_2\}, \quad (54) \\
  t_d &= \max\{\sqrt{x_2(1-x_3)}m_{B_s}, \sqrt{1-(1-x_1-x_2)x_3}m_{B_s}, 1/b_1, 1/b_2\}, \quad (55) \\
  t'_d &= \max\{\sqrt{x_2(1-x_3)}m_{B_s}, \sqrt{(x_1-x_2)(1-x_3)}m_{B_s}, 1/b_1, 1/b_2\}, \quad (56) \\
  t'_b &= \max\{\sqrt{x_2x_3}m_{B_s}, \sqrt{(x_1-x_2)x_3}m_{B_s}, 1/b_1, 1/b_2\}. \quad (57)
\end{align*}
The function $h_c(x_1, x_3, b_1, b_3) = K_0(\sqrt{x_1}x_3 b_1) \left[ \theta(b_1 - b_3) K_0(\sqrt{x_3}b_1) I_0(\sqrt{x_3}b_3) \right] + \theta(b_3 - b_1) K_0(\sqrt{x_3}b_3) I_0(\sqrt{x_3}b_1) S_t(x_3), \quad \text{(58)}$

$h^1_c(x_1, x_2, b_1, b_2) = K_0(\sqrt{x_1}x_2 b_1) \left[ \theta(b_1 - b_2) K_0(\sqrt{x_2}b_1) I_0(\sqrt{x_2}b_2) \right] + \theta(b_2 - b_1) K_0(\sqrt{x_2}b_2) I_0(\sqrt{x_2}b_1), \quad \text{(59)}$

$h_a(x_1, x_2, x_3, b_1, b_2) = \left[ \theta(b_2 - b_1) I_0(M_B \sqrt{x_1} x_3 b_1) K_0(M_B \sqrt{x_1} x_3 b_2) \right] + \theta(b_2 - b_1) K_0(-i \sqrt{x_2} x_3 b_1) I_0(\sqrt{x_2} x_3 b_2), \quad \text{(60)}$

$h^y_1(x_1, x_2, x_3, b_1, b_2) = K_0(-i \sqrt{x_2} x_3 b_1) \left[ \theta(b_1 - b_2) K_0(-i \sqrt{x_2} x_3 b_1) I_0(\sqrt{x_2} x_3 b_2) \right] + \theta(b_2 - b_1) K_0(-i \sqrt{x_2} x_3 b_1) I_0(\sqrt{x_2} x_3 b_2), \quad \text{(61)}$

$h^y_2(x_1, x_2, x_3, b_1, b_2) = K_0(i \sqrt{x_2} x_3 b_1) \left[ \theta(b_1 - b_2) K_0(-i \sqrt{x_2} x_3 b_1) I_0(\sqrt{x_2} x_3 b_2) \right] + \theta(b_2 - b_1) K_0(-i \sqrt{x_2} x_3 b_1) I_0(\sqrt{x_2} x_3 b_2), \quad \text{(62)}$

$h^y_3(x_1, x_2, x_3, b_1, b_2) = \left[ \theta(b_1 - b_2) K_0(i \sqrt{x_2} x_3 b_1) I_0(i \sqrt{x_2} x_3 b_2 M_B) \right] + \theta(b_2 - b_1) K_0(-i \sqrt{x_2} x_3 b_1) I_0(-i \sqrt{x_2} x_3 b_1 M_B), \quad \text{(63)}$

$h^y_4(x_1, x_2, x_3, b_1, b_2) = \left[ \theta(b_1 - b_2) K_0(i \sqrt{x_2} x_3 b_1) I_0(i \sqrt{x_2} x_3 b_2 M_B) \right] + \theta(b_2 - b_1) K_0(-i \sqrt{x_2} x_3 b_1) I_0(-i \sqrt{x_2} x_3 b_1 M_B), \quad \text{(64)}$

where $J_0$ is the Bessel function and $K_0$, $I_0$ are the modified Bessel functions $K_0(-ix) = -\frac{x}{2} J_0(x) + i \frac{x}{2} J_0(x)$, and $F_{(1)}$’s are defined by

$$F_{(1)}^2 = (x_1 - x_2) x_3, \quad F_{(2)}^2 = (x_1 - x_2) x_3.$$ \quad \text{(66)}$$

The $S_t$ re-sums the threshold logarithms $\ln^2 x$ appearing in the hard kernels to all orders and it has been parameterized as

$$S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,$$ \quad \text{(67)}$$

with $c = 0.3$. In the nonfactorizable contributions, $S_t(x)$ gives a very small numerical effect on the amplitude \textsuperscript{63}. The Sudakov exponents are defined as

$$S_{ab}(t) = s \left( x_1 \frac{m_B}{\sqrt{2}} b_1 \right) + s \left( x_3 \frac{m_B}{\sqrt{2}} b_3 \right) + s \left( (1 - x_3) \frac{m_B}{\sqrt{2}} b_3 \right) - \frac{1}{\beta_t} \left[ \ln \ln(t/\Lambda) - \ln(b_3 \Lambda) + \ln \ln(t/\Lambda) - \ln(b_3 \Lambda) \right],$$ \quad \text{(68)}$$

$$S_{cd}(t) = s \left( x_2 \frac{m_B}{\sqrt{2}} b_1 \right) + s \left( x_2 \frac{m_B}{\sqrt{2}} b_2 \right) + s \left( (1 - x_2) \frac{m_B}{\sqrt{2}} b_2 \right) + s \left( x_3 \frac{m_B}{\sqrt{2}} b_1 \right) + s \left( (1 - x_3) \frac{m_B}{\sqrt{2}} b_1 \right) - \frac{1}{\beta_t} \left[ \ln \ln(t/\Lambda) - \ln(b_3 \Lambda) + \ln \ln(t/\Lambda) - \ln(b_2 \Lambda) \right],$$ \quad \text{(69)}$$
\[ S_{ef}(t) = s \left( x_1 \frac{m_B}{\sqrt{2}}, b_1 \right) + s \left( x_2 \frac{m_B}{\sqrt{2}}, b_2 \right) + s \left( 1 - x_2 \frac{m_B}{\sqrt{2}}, b_2 \right) + s \left( x_3 \frac{m_B}{\sqrt{2}}, b_2 \right) + s \left( 1 - x_3 \frac{m_B}{\sqrt{2}}, b_2 \right) \]
\[ - \frac{1}{\beta_1} \left[ \ln \frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + 2 \ln \frac{-\ln(t/\Lambda)}{-\ln(b_2\Lambda)} \right]. \tag{70} \]

The explicit form for the function \( s(k, b) \) is \([34]\):
\[ s(k, b) = \frac{2}{3\beta_1} \left[ \hat{q} \ln \left( \frac{\hat{q}}{b} - \hat{q} + \hat{b} \right) + A^{(2)} \left( \frac{\hat{q}}{b} - 1 \right) \right. \]
\[ - \left. \left[ A^{(2)} - \frac{1}{3\beta_1} (2\gamma_E - 1 - \ln 2) \right] \ln \left( \frac{\hat{q}}{b} \right), \tag{71} \right. \]

where the variables are defined by
\[ \hat{q} \equiv \ln[k/(\sqrt{\Lambda})], \quad \hat{b} \equiv \ln[1/(b\Lambda)], \tag{72} \]
and the coefficients \( A^{(i)} \) and \( \beta_i \) are
\[ \beta_1 = \frac{33 - 2n_f}{12}, \tag{73} \]
\[ A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln \left( \frac{1}{2} e^{\gamma_E} \right), \tag{74} \]

\( n_f \) is the number of the quark flavors and \( \gamma_E \) is the Euler constant.

The decay amplitudes \( F^{LL}_{B_s \to M_3} \), \( M^{LL}_{B_s \to M_3} \), \( F^{LL}_{ann} \) and \( M^{LL}_{ann} \) induced by inserting the \((V - A)(V - A)\) operators are
\[ f_p F^{LL}_{B_s \to M_3} = 8\pi G_F C_F f_p M^{LL}_{B_s} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 b_3 b_3 \phi_{B_s} (x_1, b_1) \cdot \left\{ a_i (t_a) E_c (t_a) \left[ (1 + x_3) \phi^A_3 (x_3, b_3) ight. \right. \]
\[ + r_3 (1 - 2x_3) \left( \phi^P_3 (x_3, b_3) + \phi^T_3 (x_3, b_3) \right) \right\} h_c (x_1, x_2, b_3, b_3) + 2r_3 \phi^P_3 (x_3, b_3) a_i \left( t'_a \right) E_c \left( t'_a \right) h_c (x_3, b_3, b_3), \tag{75} \]

\[ f_B F^{LL}_{ann} = 8\pi G_F C_F f_B M^{LL}_{ann} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_3 \phi_{B_s} (x_1, b_1) \cdot \left\{ a_i (t_c) E_c (t_c) \left[ (x_3 - 1) \phi^A_3 (x_2, b_3) \phi^A_3 (x_3) \right. \right. \]
\[ - 4r_3 \phi^P_3 (x_2) \phi^P_3 (x_3) + 2r_3 \phi^P_3 (x_2) \phi^P_3 (x_3) + 2r_3 \phi^P_3 (x_2) \phi^T_3 (x_3) \phi^P_3 (x_3) + 2r_3 \phi^P_3 (x_2) \phi^T_3 (x_3) \phi^P_3 (x_3) \phi^A_3 (x_3) \]
\[ - \phi^T_3 (x_2) \phi^T_3 (x_3) + 2r_2 \phi^T_3 (x_2) + 2r_2 \phi^T_3 (x_2) + \phi^T_3 (x_3) \phi^A_3 (x_3) \phi^A_3 (x_3) \] \]
\[ a_i \left( t'_c \right) E_c \left( t'_c \right) h_a (1 - x_3, b_3, b_2), \tag{76} \]

\[ f_p M^{LL}_{B_s \to M_3} = 32\pi C_F M^{LL}_{B_s} \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_2 \phi_{B_s} (x_1, b_1) \phi^A_2 (x_2) \left\{ \left[ (1 - x_2) \phi^A_2 (x_3) - r_3 x_3 \phi^P_3 (x_3) \right. \right. \]
\[ - \phi^T_3 (x_3) \right\} a_i (t_b) E_c (t_b) h_n (x_1, 1 - x_2, b_3) + h_n (x_1, x_2, b_3, b_2) \left[ (x_2 + x_3) \phi^A_3 (x_3) \right. \]
\[ + r_3 x_3 \phi^P_3 (x_3) \phi^A_3 (x_3) \] \]
\[ a_i \left( t'_b \right) E_c \left( t'_b \right), \tag{77} \]
\[ M^{LL}_{ann} = 32\pi C_F M^4_B / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \{ h_{na}(x_1, x_2, x_3, b_1, b_2) \} [x_2 \phi^P_{\omega}(x_2) \phi^A_{\omega}(x_3) ]

\]

\[ -4r_2 r_3 \phi^P_{\omega}(x_2) (1 - x_2) (\phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3)) + r_2 r_3 x_3 (\phi^P_{\omega}(x_2) - \phi^T_{\omega}(x_3)) + (1 - x_3) r_2 r_3 (\phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3)) + r_2 r_3 x_2 (\phi^P_{\omega}(x_2) - \phi^T_{\omega}(x_3)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3)) \]

\[ + \phi^0_{\omega}(x_3)] a_1(t_4) E_a(t_4) + \{ 1 - x_3 \} \phi_{B_s}^A(x_3) \]

\[ + (1 - x_3) r_2 r_3 \phi_{B_s}^P(x_3) \}

From the \((S + P)(S - P)\) operators, we can get

\[ f_{M_2 B_s \rightarrow M_3} = 16\pi r_2 C_F \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \{ a_i(t_a) E_c(t_a) [\phi_{B_s}^A(x_3) + r_3 (2 + x_3) \phi^P_{\omega}(x_3) ] \}

\[ - [x_2 \phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3)) + r_2 r_3 x_2 (\phi^P_{\omega}(x_2) - \phi^T_{\omega}(x_3)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3)) \]

\[ + \phi^0_{\omega}(x_3)] a_1(t_4) E_a(t_4) + \{ 1 - x_3 \} \phi_{B_s}^A(x_3) \]

\[ + (1 - x_3) r_2 r_3 \phi_{B_s}^P(x_3) \}

The decay amplitude for \((V - A)(V + A)\) operators can be written as follows

\[ M^{LR}_{B_s \rightarrow M_3} = 32\pi C_F m^4_B / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \{ h_{na}(x_1, 1 - x_2, x_3, b_1, b_2) \}

\[ \times [(1 - x_2) \phi^A_{\omega}(x_2) (\phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) + r_3 x_3 (\phi^0_{\omega}(x_2) - \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) + \phi^T_{\omega}(x_3)) \]

\[ + (1 - x_2) r_3 (\phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3))] a_1(t_a) E_a(t_a) - h_{na}(x_1, 1 - x_2, x_3, b_1, b_2) \]

\[ \times [x_2 \phi^A_{\omega}(x_2) (\phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) + r_3 x_3 (\phi^0_{\omega}(x_2) - \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3)) + r_2 r_3 x_3 \]

\[ (\phi^P_{\omega}(x_2) + \phi^T_{\omega}(x_2)) (\phi^0_{\omega}(x_3) - \phi^T_{\omega}(x_3))\]
\[ M_{\text{ann}}^{LR} (M_2, M_3, a_i) = 32\pi G_F m_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s} (x_1, b_1) \{ h_{n_a} (x_1, x_2, x_3, b_1, b_2) [r_2 (2 - x_2) 
abla \phi_2^T (x_2) + \phi_2^T (x_2)] \phi_3^A (x_3) - r_3 (1 + x_3) \phi_2^A (x_2) (\phi_3^P (x_3) - \phi_3^T (x_3)) \} a_i (t_d) E'_a (t_d) + h_{n_a} (x_1, x_2, x_3, b_1, b_2) \]

\[ r_2 x_2 (\phi_2^P (x_2) + \phi_2^T (x_2)) \phi_3^A (x_3) + r_3 (x_3 - 1) \phi_2^A (x_2) \]

\[ (\phi_3^P (x_3) - \phi_3^T (x_3)) a_i (t_d) E'_a (t_d) \}

(83)

[1] J. H. Christenson, J. W. Cronin, Phys. Rev. Lett. 13, 138-140 (1964).
[2] T. Gershon and V.V. Gligorov, Rept. Prog. Phys. 80, 046201 (2017).
[3] N.M. Kroll, T.D. Lee, B. Zumino, Phys. Rev. 157, 1376 (1967).
[4] Gang Lü and Yan-Lin Zhao, Chinese. Physics. C11, 113101 (2022).
[5] R. Aaij et al. (LHCb), Phys. Rev. Lett. 112, 011801 (2014).
[6] R. Aaij et al. (LHCb), Phys. Rev. D90, 112004 (2014).
[7] I. Bediaga and C. G"obel, Prog. Part. Nucl. Phys. 114, 103808 (2020).
[8] LHCb collaboration, arXiv:2206.02038v1 [hep-ex] (2022).
[9] M. Beneke, G. Buchalla, M. Neubert, Phys. Rev. Lett. 83, 1914 (1999).
[10] M. Beneke, G. Buchalla, Nucl. Phys. B591, 313-418 (2000).
[11] Martin Beneke, Matthias Neubert, Nucl. Phys. B675, 333-415 (2003).
[12] Xin-Qiang Li, Ya-Dong Yang, Xing-Bo Yuan, Phys. Rev. D89 054024(2014).
[13] Hsiang-nan Li, George F. Sterman, Nucl. Phys. B381, 129-140 (1992).
[14] Wen-Fei Wang, Hsiang-nan Li, Phys. Lett. B763, 29-39 (2016).
[15] Y. Y. Keum, Hsiang-nan Li, A. I. Sanda, Phys. Rev. D63, 054008 (2001).
[16] C.D. L"u, K.Ukai and M.Z. Yang, Phys. Rev. D63, 074009 (2001).
[17] Christian W. Bauer, Dan Pirjol, Phys. Rev. Lett. 87, 201806 (2001).
[18] C. W. Bauer, D. Pirjol, I. W. Stewart, Phys. Rev. D65, 054022 (2002).
[19] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. 45, 952 (1980).
[20] A. B. Carter and A. I. Sanda, Phys. Rev. D23, 1567 (1981).
[21] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. B193, 85-108 (1981).
[22] G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. 68,1125 (1996).
[23] C.-D. L"u, K. Ukai, M.-Z. Yang, Physical Review D63, 074009 (2001).
[24] A. Ali, G. Kramer, C.-D. L"u, Phys. Rev. D58, 094009(1998); 59, 014005 (1998); Y. H. Chen, H. Y. Cheng, B. Tseng, K. C. Yang, Phys. Rev. D60, 094014 (1999).
[25] Tao Huang, Xing-Gang Wu, Phys. Rev. D70, 093013 (2004).
[26] Xing-Gang Wu, Tao Huang, Phys. Rev. D84, 074011 (2011).
[27] T. Kurimoto, H.-n. Li, A.I. Sanda, Phys.Rev. D65, 014007 (2002).
[28] C.D. L"u, M.Z. Yang, Eur. Phys. J. C28, 515 (2003).
[29] C.-D. L"u, K. Ukai, M.-Z. Yang, Phys. Rev. D63, 074009 (2001).
[30] Zhen-Jun Xiao, Zhi-Qing Zhang, Xin Liu, Li-Bo Guo, Phys. Rev. D78, 114001 (2008).
[31] Gang Lü, Qin-Qin Zhi, High. Energy. Phys. 6927130 (2019).
[32] A. Ali, G. Kramer, C. D. Lü, Y. L. Shen, Phys. Rev. D76, 074018 (2007).
[33] C.D. Lü, M.Z. Yang, Eur. Phys. J. C28, 515 (2003).
[34] The physics of B meson, Science Press, Zhen-Jun Xiao (2022).
[35] P. Ball, J. High Energy Phys. 09005 (1998); 01010(1999).
[36] P. Ball, V. M. Braun, Y. Koike, K. Tanaka, Nucl. Phys. B529, 323 (1998).
[37] P. Ball, V. M. Braun, Nucl. Phys. B543, 201 (1999).
[38] P. Ball, V. M. Braun, Phys. Rev. D54, 2182 (1996).
[39] P. Ball, R. Zwicky, J. High Energy Phys. 02034 (2006).
[40] P. Ball, M. Boglione, Phys. Rev. D68, 094006 (2003).
[41] Y. Y. Charng, T. Kurimoto, H.-n. Li, Phys. Rev. D74, 074024 (2006).
[42] H.-n. Li, Phys. Lett. B622, 63 (2005).
[43] H.-n Li, S. Mishima, Phys. Rev. D71, 054025 (2005).
[44] C.E. Wolfe, K. Maltman, Phys. Rev. D80, 114024 (2009).
[45] C.E. Wolfe, K. Maltman, Phys. Rev. D83, 077301 (2011).
[46] M.N. Achasov, et.al., Nucl. Phys. B569, 158 (2000).
[47] H.B. O'Connell, A.W. Thomas, A.G. Williams, Nucl. Phys. A623, 559 (1997).
[48] K. Maltman, H.B. O'Connell, A.G. Williams, Phys. Lett. B376, 19 (1996).
[49] S. Gardner and H.B. O'Connell, Phys. Rev. D57, 2716 (1998).
[50] L. Wolfenstein, Phys. Rev. Lett. 51,1945 (1983); Phys. Rev. Lett. 13, 562 (1964).
[51] Z.-H. Zhang, X.-H. Guo, Y.-D. Yang, Phys. Rev. D87, 076007 (2013).
[52] X.-H. Guo, O. Leitner, A.W. Thomas, Phys. Rev. D63, 056012 (2001).
[53] Chao Wang, Zhen-Hua Zhang, Eur. Phys. J. C75, 11536 (2015).
[54] R. Aaij et al. (LHCb), Phys. Rev. D90, 112004 (2014).
[55] H.-n. Li, K. Ukai, Phys. Lett. B813, 136058 (2021).
[56] Ahmed Ali, Gustav Kramer, Ying Li, Cai-Dian Lü, Yue-Long Shen, Wei Wang, Phys. Rev. D76, 074018 (2007).
[57] Ahmed Ali, Gustav Kramer, Ying Li, Cai-Dian Lü, Yue-Long Shen, Wei Wang, Phys. Rev. D76, 074018 (2007).
[58] H.-Y. Cheng, C.-W. Chiang, C.-K. Chua, Phys. Rev. D103, 036017 (2021).
[59] H.-Y. Cheng, C.-W. Chiang, C.-K. Chua, Phys. Lett. B813, 136058 (2021).
[60] Ahmed Ali, Gustav Kramer, Ying Li, Cai-Dian Lü, Yue-Long Shen, Wei Wang, Phys. Rev. D76, 074018 (2007).
[61] Y.Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B 504, 6 (2001).
[62] Zhen-Jun Xiao, Dong-qin Guo, and Xin-fen Chen, Phys. Rev. D75, 014018 (2007).
[63] H.-n. Li, K. Ukai, Phys. Lett. B555, 197 (2003).
[64] C.-D. Lü, K. Ukai, and M.-Z. Yang, Phys. Rev. D63, 074009 (2001).