Intersecting branes in pp-wave spacetime

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Abstract

We derive intersecting brane solutions in pp-wave spacetime by solving the supergravity field equations explicitly. The general intersection rules are presented. We also generalize the construction to the non-extremal solutions. Both the extremal and non-extremal solutions presented here are asymptotic to BFHP plane waves. We find that these solutions do not have regular horizons.

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1 Introduction

PP-wave spacetime provides an example of exact string theory background. It has been known for some time that pp-wave spacetime yields exact classical backgrounds for string theory, with all $\alpha'$ corrections vanishing [1, 2]. Also it has been shown recently that these backgrounds are exactly solvable in the light cone gauge [3]-[5]. Many of these backgrounds are obtained in the Penrose limit [6] of $AdS_p \times S^q$ type of geometry and preserve at least the same amount of supersymmetry as that of parent background [7, 8]. PP-wave backgrounds have also proven an interesting place to test the ideas of holography. It was conjectured in [9] that the type IIB string theory in $AdS_5 \times S^5$ background is dual to the $\mathcal{N} = 4$ super Yang-Mills theory on the boundary of $AdS_5$. Partly motivated by this, Berenstein, Maldacena and Nastase (BMN) [10] have argued that a particular sector of $\mathcal{N} = 4$ super Yang-Mills theory containing the operators with large $R$ charge $J$, is dual to Type IIB string theory on pp-wave background with RR flux. The fact that the string theory in pp-wave background is exactly solvable has opened up the window to understand the duality beyond the supergravity limit. The correspondence between the string states and black holes is also adapted to pp-wave backgrounds [11].

D-branes can probe the nonperturbative dynamics of the string theory and they have been used to study various duality aspects of string theory. Several aspects of D-branes in pp-wave spacetime e.g. the supergravity solutions, open string spectrum and the supersymmetric properties of branes and their bound states are studied extensively in the past [12]-[23]. Recently the focus has been on the most general pp-wave background with non-constant flux turned on [24]-[27]. D-brane solutions in these backgrounds and their supersymmetry properties are studied in refs. [28, 29]. In view of these recent developments in the study of D-branes and thermodynamics of strings in pp-wave background, it is interesting to give a systematic derivation of the general D-brane solutions in the pp-wave backgrounds.

In this paper we present a general class of intersecting brane solutions in the pp-wave background by using the method developed in ref. [30]. We start with a general ansatz for the metric and solve for the field equations of the supergravity. We also derive the intersection rules for the branes in this background. This is the pp-wave generalization of intersection rules for $p$-branes derived in [30] (see also [31]-[35] for discussion of brane intersections). The method used here also applies to other brane solutions [36] and is quite
useful.

The existence of black holes or so-called black branes in pp-wave spacetime is also discussed recently. In particular there are ‘no-go theorems’ for the existence of horizons in pp-wave spacetimes admitting a covariantly constant null isometry [37]-[39]. The covariant constancy condition is further relaxed and a similar ‘no-go theorem’ is proved in [39] for spacetimes which are asymptotic to plane wave spacetime. Though, a version of ‘no-go theorem’ is still lacking for the backgrounds with sources and admitting null isometry, some examples are studied in ref. [39]. It has been found that in pp-wave spacetime supported by non-zero 5-form flux, while some of the extremal solutions admit horizons, the corresponding non-extremal deformations result in naked singularity.\footnote{However, if we further relax the null isometry condition, it might be possible to find black branes with regular event horizon in pp-wave spacetime [39] (see also the recent paper by Gimon et. al. [40]).}

We further examine the tidal force in the parallel transported frame and find that these solutions all do not have regular horizons [41].

Here we look for the non-extremal deformations of brane solutions in pp-wave spacetime supported by NS-NS 3-form flux. We are interested in pp-wave background of the asymptotic form

$$ds_D^2 = -2dudv + K(y_\alpha, z_i) du^2 + \sum_{\alpha=2}^{d-1} dy_\alpha^2 + \sum_{i=1}^{\hat{d}+2} dz_i^2,$$

$$H^{(3)} = \partial_j b_k(z_i) \ du \wedge dz^j \wedge dz^k,$$

where $D = d + \hat{d} + 2$, the coordinates $u = (t + x)/\sqrt{2}$, $v = (t - x)/\sqrt{2}$ and $y_\alpha, (\alpha = 2, \ldots, d-1)$ parameterize the $d$-dimensional world-volume directions. The NS-NS 3-form, $H^{(3)}$ breaks the $SO(\hat{d} + 2)$ isometry of transverse $z_i \ (i = 1, \ldots, \hat{d} + 2)$ directions.

The plan of the paper is as follows. In sect. 2, we present classical solutions of intersecting branes in arbitrary dimension $D$ in pp-wave background in the presence of non-constant NS-NS 3-form flux, switched on along the transverse directions to the branes. We also present the intersection rules for branes in this background. In sect. 3, we generalize the above construction to non-extremal cases. The intersection rules along with the ‘blackening’ functions are derived by solving the Einstein equations. Sect. 4 is devoted to the discussion on the possible horizon and black hole solutions in the above pp-wave background. We conclude in sect. 5 with some remarks.
2 Extremal Solutions

The low-energy effective action for the supergravity system coupled to dilaton and $n_A$-form field strength is given by

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \sum_{A=1}^{m} \frac{1}{2n_A!} e^{a_A \phi} F_{n_A}^2 \right],$$

(2.1)

where $G_D$ is the Newton constant in $D$ dimensions and $g$ is the determinant of the metric. The last term includes both RR and NS-NS field strengths and $a_A = \frac{1}{2}(5 - n_A)$ for RR field strengths and $a_A = -1$ for NS-NS 3-form. We put fermions and other background fields to be zero.

From the action (2.1), one can derive the field equations/Bianchi identities

$$R_{\mu\nu} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \sum_A \frac{1}{2n_A!} e^{a_A \phi} \left[ n_A \left( F_{n_A}^2 \right)_{\mu\nu} - \frac{n_A - 1}{D - 2} F_{n_A}^2 g_{\mu\nu} \right],$$

(2.2)

$$\Box \phi = \sum_A \frac{a_A}{2n_A} e^{a_A \phi} F_{n_A}^2,$$

(2.3)

$$\partial_{\mu_1} \left( \sqrt{-g} e^{a_A \phi} F_{\mu_1 \cdots \mu_{n_A}} \right) = 0,$$

(2.4)

$$\partial_{[\mu} F_{\mu_1 \cdots \mu_{n_A}]} = 0.$$

(2.5)

We start with the most general ansatz for the metric in pp-wave spacetime consistent with the isometries of the background, and the NS-NS 3-form field strength:

$$ds_D^2 = e^{2u_0} [-2du dv + K(y_\alpha, z_i)du^2] + \sum_{\alpha=2}^{d-1} e^{2u_\alpha} dy_\alpha^2 + e^{2B} [dr^2 + r^2 d\Omega^2_{d+1}],$$

(2.6)

where $D = d + \tilde{d} + 2$, the coordinates $u$, $v$ and $y_\alpha, (\alpha = 2, \ldots, d - 1)$ parameterize the $d$-dimensional world-volume directions and the remaining $\tilde{d} + 2$ coordinates $z_i$ are transverse to the brane world-volume; these are interchangeably used here with the radial coordinate $r$ and $\tilde{d} + 1$ angles. $d\Omega^2_{d+1}$ is the line element of the $(\tilde{d} + 1)$-dimensional sphere. All the warp factors are assumed to depend on $r$ only. In this and next sections, $d$, $\tilde{d}$ and $D$ are general, but in sect. 4 we shall put $D = 10$. 

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The general ansatz for the background for an electrically charged \( q_A \)-brane is given by

\[
F_{uv2\ldots\alpha q r} = \epsilon_{uva2\ldots\alpha q} E', \quad (n_A = q_A + 2). \tag{2.7}
\]

Similarly, the magnetic case is given by

\[
F_{\alpha q_{A+1}\ldots\alpha_{d-1}a_{1}\ldots\alpha_{d+1}} = \frac{1}{\sqrt{-g}} e^{-q_A \phi} \epsilon_{\alpha q_{A+1}\ldots\alpha_{d-1}a_{1}\ldots\alpha_{d+1}r} \tilde{E}', \quad (n_A = D - q_A - 2) \tag{2.8}
\]

where \( a_1, \ldots, a_{d+1} \) denote the angular coordinates of the \((d + 1)\)-sphere. The functions \( E \) and \( \tilde{E} \) are also assumed to depend only on \( r \). The NS-NS 3-form responsible for the off-diagonal component of the metric is separately written as

\[
H_{aij} = \partial_i b_j, \tag{2.9}
\]

such that it satisfies the Bianchi identity. Here the indices \( i, j \) denote the directions transverse to the branes (\( z_i, z_j \) or \( r \) and angles).

Solving for the dilaton in eq. (2.3) gives

\[
\phi'' + \frac{(d + 1)}{r} \phi' = -\frac{1}{2} \sum A \epsilon_A a_A S_A (E_A')^2, \tag{2.10}
\]

where \( S_A \) is defined as

\[
S_A = \exp \left( \epsilon_A a_A \phi - 2 \sum_{\alpha \in q_A} u_{\alpha} \right). \tag{2.11}
\]

The sum of \( \alpha \) runs over the world-volume of the branes (\( u, v \) and \((q_A - 1)\) \( y^\alpha \) coordinates, and so \( \sum_{\alpha \in q_A} u_{\alpha} = 2u_0 + \sum_{\alpha = 2}^{q_A} u_{\alpha} \) for example).

The Ricci tensors for the metric (2.6) are summarized in the appendix. The field eqs. (2.2) are simplified considerably by imposing the condition

\[
2u_0 + \sum_{\alpha = 2}^{d-1} u_{\alpha} + \tilde{d}B = 0. \tag{2.12}
\]

It is known that under this condition, all the known intersecting brane solutions can be derived \cite{30}-\cite{35}. We can see that this condition makes most of the field equations in (2.2) linear, allowing superpositions of obtained solutions, and it is closely related to BPS conditions. Thus it is also expected to be sufficient to impose this condition in deriving BPS brane solutions in the pp-wave background. We will relax this restriction in our search for non-extremal solutions in the next section.
Using the condition (2.12), we can write eqs. (2.2) in the following form:

\[ u''_0 + \frac{(\tilde{d} + 1)}{r} u'_0 = \sum_A \frac{D - q_A - 3}{2(D - 2)} S_A (E'_A)^2, \]  
\( (2.13) \)

\[ \left[ u''_0 + \frac{(\tilde{d} + 1)}{r} u'_0 + \frac{1}{2} K^{-1} \left( \Box^{(\tilde{d}+2)} + \sum_\alpha e^{2(B-u_0)} \partial_\alpha^2 \right) K \right] \]
\[ = \sum_A \frac{D - q_A - 3}{2(D - 2)} S_A (E'_A)^2 - \frac{1}{4} K^{-1} e^{-2u_0+2B+\phi} (\partial_i b_j)^2, \]  
\( (2.14) \)

\[ u''_\alpha + \frac{(\tilde{d} + 1)}{r} u'_\alpha = \sum_A \frac{\delta_A^{(\alpha)}}{2(D - 2)} S_A (E'_A)^2, \]  
\( (2.15) \)

\[ B'' + \tilde{d}(B')^2 + \frac{(\tilde{d} + 1)}{r} B' + 2(u'_0)^2 + \sum_{\alpha=2}^{d-1} (u'_\alpha)^2 \]
\[ = -\frac{1}{2} (\phi')^2 + \sum_A \frac{D - q_A - 3}{2(D - 2)} S_A (E'_A)^2, \]  
\( (2.16) \)

\[ B'' - \frac{1}{r^2} + \frac{(\tilde{d} + 1)}{r} (B' + \frac{1}{r}) - \frac{\tilde{d}}{r^2} = -\sum_A \frac{q_A + 1}{2(D - 2)} S_A (E'_A)^2, \]  
\( (2.17) \)

where \( \delta_A^{(\alpha)} \) is defined as

\[ \delta_A^{(\alpha)} = \begin{cases} 
D - q_A - 3 & \text{for } y_\alpha \text{ belonging to } q_A\text{-brane}, \\
-(q_A + 1) & \text{otherwise} 
\end{cases}, \]  
\( (2.18) \)

and \( \epsilon_A = +1(-1) \) is for electric (magnetic) backgrounds. The equations (2.13), (2.14), (2.15), (2.16) and (2.17) are the \( uv, uu, \alpha\beta, rr \) and \( ab \) components of the Einstein equations (2.2) respectively.

The field equation for the NS-NS 3-form (2.9) then leads to

\[ \partial_i b_j = e^{2u_0+2B+\phi} \mu_{ij}, \]  
\( (2.19) \)

where \( \mu_{ij} \) is constant.

From eqs. (2.13), (2.14) and (2.19), we get the following differential equation for \( K \):

\[ \Box^{(\tilde{d}+2)} K + \sum_\alpha e^{2(B-u_0)} \partial_\alpha^2 K = \frac{1}{2} e^{2u_0+2B+\phi} (\mu_{ij})^2, \]  
\( (2.20) \)

where \( \Box^{(\tilde{d}+2)} \) is the Laplacian in flat \( \tilde{d} + 2 \) dimensional spacetime.

Solving eq. (2.4) for the field strength gives

\[ S_A E'_A = c_A r^{-(\tilde{d}+1)}. \]  
\( (2.21) \)
Using this, one can integrate eqs. (2.10), (2.13), (2.15) and (2.17) to obtain a set of first order equations for \( \phi, u_0, u_\alpha \) and \( B \). They are

\[
\phi' = -\sum_A \frac{\epsilon_A a_A c_A}{2} \frac{E_A}{r^{d+1}} + \frac{c_\phi}{r^{d+1}},
\]

(2.22)

\[
u_0' = \sum_A \frac{D - q_A - 3}{2(D - 2)} c_A \frac{E_A}{r^{d+1}} + \frac{c_0}{r^{d+1}},
\]

(2.23)

\[
u_\alpha' = \sum_A \frac{\delta^{(\alpha)}_A}{2(D - 2)} c_A \frac{E_A}{r^{d+1}} + \frac{c_\alpha}{r^{d+1}},
\]

(2.24)

\[
B' = -\sum_A \frac{q_A + 1}{2(D - 2)} c_A \frac{E_A}{r^{d+1}} + \frac{c_b}{r^{d+1}}.
\]

(2.25)

Putting \( u_0', u_\alpha', B' \) into eq. (2.16) and equating \( E_A \)-independent part equal to zero, we get

\[c_\phi = c_0 = c_\alpha = c_b = 0,\]

(2.26)

and \( E_A \)-dependent part equal to zero, we get

\[
\sum_{A,B} \left[ M_{AB} \frac{c_A}{2} + r^\frac{d+1}{2} \left( \frac{1}{E_A} \right)' \delta_{AB} \right] \frac{c_B}{2} E_A E_B = 0,
\]

(2.27)

where

\[
M_{AB} = \frac{d (q_A + 1)(q_B + 1)}{(D - 2)^2} + \frac{d - 1}{2} \frac{\delta^{(\alpha)}_A \delta^{(\alpha)}_B}{(D - 2)^2} + \frac{1}{2} \epsilon_A a_A \epsilon_B a_B.
\]

(2.28)

\( M_{AB} \) being constant, eq. (2.27) cannot be satisfied for arbitrary function \( E_A \) unless the second term inside the square bracket is a constant. This gives

\[E_A = N_A H_A^{-1},\]

(2.29)

where the function \( H_A \) is defined as

\[H_A = 1 + \frac{Q_A}{r^d}.\]

(2.30)

Putting \( A = B \) in eq. (2.27), we get

\[
\frac{c_A}{2} = \frac{d Q_A}{N_A M_{AA}} \equiv \frac{d Q_A (D - 2)}{N_A \Delta_A},
\]

(2.31)
where \( \Delta_A \) is given by
\[
\Delta_A = (q_A + 1)(D - q_A - 3) + \frac{1}{2}a_A^2(D - 2). \tag{2.32}
\]

Integrating eqs. (2.22), (2.23), (2.24) and (2.25), we get
\[
\varphi = \sum_A \varepsilon_A q_A \frac{D - 2}{\Delta_A} \ln H_A,
\]
\[
u_0 = -\sum_A \frac{D - q_A - 3}{\Delta_A} \ln H_A,
\]
\[
u_\alpha = -\sum_A \delta_\alpha A \frac{\Delta_A}{\nu_0} \ln H_A,
\]
\[
B = \sum_A q_A + 1 \frac{\Delta_A}{\nu_0} \ln H_A, \tag{2.33}
\]

where the integration constants are put equal to zero by the requirement that asymptotically the warp factors approach to 1.

Using eqs. (2.33), one can write down the expression (2.11) for \( S_A \) as
\[
S_A = H_A^2. \tag{2.34}
\]

By use of eqs. (2.33), eq. (2.20) for \( K \) becomes
\[
\left( \Box^{(d+2)} + \sum_{\alpha=2}^{d-1} \prod_A H_A^2 \frac{\gamma_\alpha^{(\alpha)}}{\Delta_A} \partial_\alpha^2 \right) K = -\frac{1}{2}(\mu_{ij})^2 \prod_A H_A^l_A, \tag{2.35}
\]

where we have defined
\[
\gamma_\alpha^{(\alpha)} = \begin{cases} D - 2 & \text{for } y_\alpha \text{ belonging to } q_A\text{-brane} \\ 0 & \text{otherwise} \end{cases}, \tag{2.36}
\]

and
\[
l_A = \frac{4(q_A + 1) + \varepsilon_A a_A(D - 2) - 2(D - 2)}{\Delta_A}. \tag{2.37}
\]

We note that for D-branes in \( D = 10 \), \( l = 0 \) and \( \frac{2(D-2)}{\Delta_A} = 1 \). For a single D\(_{q_A}\)-brane, eq. (2.35) admits a solution of the form
\[
K = c + \frac{Q_{ij}}{r^d} - \frac{(\mu_{ij})^2}{32} \left( r^2 + \sum_\alpha y_\alpha^2 + \frac{(q_A - 1)}{(d - 2)} \frac{Q_A}{r^{d-2}} \right), \quad (\text{for } \tilde{d} \neq 2) \tag{2.38}
\]
and

\[ K = c + \frac{Q}{r^\tilde{d}} - \frac{(\mu_{ij})^2}{32} \left( r^2 + \sum_\alpha g_\alpha^2 - (q_A - 1) Q A \ln r \right), \quad \text{for } \tilde{d} = 2 \] (2.39)

Now, using eqs. (2.21) and (2.31), we can determine the normalization constant \( N_A \) as

\[ N_A = \sqrt{\frac{2(D-2)}{\Delta A}}. \] (2.40)

Our metric and background fields are thus finally given by, after putting all the warp factors etc. we obtained by solving the Einstein equations,

\[
    ds_D^2 = \prod_A H_A^{2q_A+1} \left[ \prod_A H_A^{-2q_A^2} \left\{ -2dudv + K du^2 \right\} \right. \\
    \left. + \sum_{\alpha=2}^{d-1} \prod_A H_A^{-2q_A^{(\alpha)}} dy_\alpha^2 + dr^2 + r^2 d\Omega_{d+1}^2 \right], \\
    E_A = \sqrt{\frac{2(D-2)}{\Delta A}} H_A^{-1}. \tag{2.41}
\]

where \( \gamma_A^{(\alpha)} \) is defined in (2.36) and the function \( K \) is in eq. (2.35).

For \( A \neq B \), we have \( M_{AB} = 0 \) from eq. (2.27). This gives the intersection rules for the branes. If \( q_A \)-brane and \( q_B \)-brane intersect over \( \bar{q}(\leq q_A, q_B) \) dimensions, this gives

\[ \bar{q} = \frac{(q_A + 1)(q_B + 1)}{D - 2} - 1 - \frac{1}{2} \epsilon_A \epsilon_B q_A q_B. \tag{2.42} \]

For D-branes

\[ \epsilon_A \epsilon_B = \frac{3 - q_A}{2}, \tag{2.43} \]

and we get

\[ \bar{q} = \frac{q_A + q_B}{2} - 2. \tag{2.44} \]

The results presented here are the pp-wave generalization of the intersection rules already discussed in the literature [30]-[35]. The amount of supersymmetry preserved by these brane configurations can be obtained by solving the Killing spinor equations explicitly. In the present case, the lightcone directions are lying along the brane, whereas the other pp-wave directions are transverse to the brane world-volume. The supersymmetric properties
of such D-brane configurations and their bound states are already discussed in [19, 29] for $D = 10$. For the D-branes in the background under consideration, there always exist 16 ‘standard’ Killing spinors ($\epsilon_{\pm}$) satisfying $\Gamma^u \epsilon_{\pm} = 0$ [42, 20]. The rest of the supersymmetry preservation depends on the number of solutions to the condition $(\partial_i b_j) \Gamma^{ij} \epsilon_{\pm} = 0$ and the standard D-brane supersymmetry conditions. For the special case of $H_{u12} = H_{u34}$, the condition $(\partial_i b_j) \Gamma^{ij} \epsilon_{\pm} = 0$ breaks half of the supersymmetry and the D-brane configurations in this background preserve 1/8 supersymmetries [19, 29].

3 Non-Extremal Solutions

In this section, we present the non-extremal generalization of the solutions analyzed in the previous section. This could be done by directly starting with a metric ansatz with blackening functions along with the arbitrary warp factors and then solving the field equations to fix each of them accordingly. We follow a slightly different approach. Instead of putting the blackening functions directly into the metric ansatz, without any loss of generality, we shall deform the condition (2.12) as

$$2u_0 + \sum_{\alpha=2}^{d-1} u_\alpha + \tilde{d}B = \ln g,$$

where $g$ being a function of $r$ only.

In our derivation of extremal solutions in the preceding section, we have set the above combination to zero. This is sufficient to find extremal solutions. Here we relax this restriction and search for general solutions.

Using (3.1), eqs. (2.2) can be rewritten as

$$\left[ u''_0 + \left( \frac{g'}{g} + \frac{(\tilde{d} + 1)}{r} \right) u'_0 \right] = \sum_A \frac{D - q_A - 3}{2(D - 2)} S_A (E'_A)^2,$$

$$\left[ u''_\alpha + \left( \frac{g'}{g} + \frac{(\tilde{d} + 1)}{r} \right) u'_\alpha + \frac{1}{2} \frac{g' K'}{g K} + \frac{1}{2} K^{-1} \left( \Box^{(\tilde{d}+2)} + \sum_{\alpha} e^{2(B-u_\alpha)} - 4 \partial_\alpha^2 \right) \right]$$

$$\delta_{\alpha\beta} = \sum_A \frac{\delta^{(a)}_A}{2(D - 2)} S_A (E'_A)^2,$$
\[
\begin{align*}
B'' + \tilde{d}(B')^2 + \left( -\frac{g'}{g} + \frac{(\tilde{d} + 1)}{r} \right) B' + \left( \frac{g'}{g} \right)' + 2(u_0')^2 + \sum_{\alpha=2}^{d-1}(u'_\alpha)^2 \\
&= -\frac{1}{2}(\phi')^2 + \sum_A \frac{D - q_A - 3}{2(D - 2)} S_A(E'_A)^2, \\
\left[ B'' - \frac{1}{r^2} + \left( \frac{g'}{g} + \frac{(\tilde{d} + 1)}{r} \right) \left( B' + \frac{1}{r} \right) - \tilde{d} \right] &= -\sum_A \frac{q_A + 1}{2(D - 2)} S_A(E'_A)^2,
\end{align*}
\] (3.5)

where \( S_A \) is defined as
\[
S_A = \exp \left( \epsilon_A a_A \phi - 2 \sum_{\alpha \in q_A} u_\alpha \right). 
\] (3.7)

Similarly the dilaton eq. (2.3) can be written as
\[
\phi'' + \left( \frac{g'}{g} + \frac{(\tilde{d} + 1)}{r} \right) \phi' = -\frac{1}{2} \sum_A \epsilon_a a_A S_A(E'_A)^2, 
\] (3.8)

Solving the equation for the field strengths gives
\[
S_A E'_A = c_A g^{-1} r^{-(\tilde{d}+1)}. 
\] (3.9)

To solve for \( g(r) \), we multiply eqs. (3.2) and (3.6) by 2 and \( \tilde{d} \) respectively and add to eq. (3.4). Using the condition (3.1), we get
\[
g(r) = 1 - \left( \frac{r_0}{r} \right)^{2\tilde{d}}. 
\] (3.10)

Solving the dilaton eq. (3.8) and eqs. (3.2), (3.4) and (3.6) by using the solution (3.9), we get the following set of first order eqs.
\[
\phi' = -\sum_A \frac{\epsilon_A a_A c_A}{2} E_A h^{-1} + c_\phi h^{-1}, 
\] (3.11)
\[
u_0' = \sum_A \frac{D - q_A - 3}{2(D - 2)} c_A E_A h^{-1} + c_0 h^{-1}, 
\] (3.12)
\[
u'_\alpha = \sum_A \frac{\delta^{(\alpha)}_A}{2(D - 2)} c_A E_A h^{-1} + c_\alpha h^{-1}, 
\] (3.13)
\[
B' = -\sum_A \frac{q_A + 1}{2(D - 2)} c_A E_A h^{-1} + \frac{1}{d} g' + c_B h^{-1}, 
\] (3.14)

where \( h \) is defined as
\[
h = gr^{\tilde{d}+1}. 
\] (3.15)
Putting values of $u'_0, u'_\alpha, B'$ into eq. (3.5) and equating the $E_A$-independent part to zero, we get the condition

$$2c_0^2 + \sum_{\alpha=2}^{d-1} c_\alpha^2 + \bar{d}c_\phi^2 + \frac{1}{2}c_\phi^2 = 4\bar{d}(\bar{d} + 1)r_0^{2\bar{d}}, \quad (3.16)$$

and similarly, $E_A$-dependent part equal to zero gives

$$\sum_{A,B} \left[ M_{AB} \frac{c_A}{2} + \left( h\left( \frac{1}{E_A} \right)' + \tilde{c}_A \right) \delta_{AB} \right] c_B = 0, \quad (3.17)$$

where

$$M_{AB} = \bar{d}(q_A + 1)(q_B + 1) \frac{(D - 2)^2}{(D - 2)^2} + \sum_{\alpha=0}^{\bar{d}} \frac{\delta^{(\alpha)}_A \delta^{(\alpha)}_B}{(D - 2)^2} + \frac{1}{2} \epsilon_{AaA} \epsilon_{BbB}, \quad (3.18)$$

and

$$\tilde{c}_A = -2\bar{d}c_0 q_A + 1 \frac{q_A}{D - 2} + 2 \sum_{\alpha=0}^{d-1} \frac{\delta^{(\alpha)}_A}{D - 2} c_\alpha - \epsilon_{AaA} c_\phi. \quad (3.19)$$

$M_{AB}$ being constant, eq. (2.27) cannot be satisfied for arbitrary function $E_A$ unless the second term inside the square bracket is a constant. This gives

$$E_A = \frac{N_A}{1 - \beta_A(1 - f^{-\alpha_A})}, \quad (3.20)$$

where

$$\alpha_A = \frac{\tilde{c}_A}{2dr_0^{\bar{d}}}, \quad (3.21)$$

$\beta_A$ and $N_A$ are constants and the function $f(r)$ is defined as

$$f(r) = \frac{1 - \left( \frac{r_0}{r} \right)^{\bar{d}}}{1 + \left( \frac{r_0}{r} \right)^{\bar{d}}}, \quad (3.22)$$

Putting $A = B$ in eq. (3.17), we get

$$\frac{c_A}{2} = \frac{\tilde{c}_A(\beta_A - 1)}{N_A M_{AA}} \equiv \frac{\tilde{c}_A(\beta_A - 1) D - 2}{N_A \Delta_A}, \quad (3.23)$$

where $\Delta_A$ is given in (2.32).
Integrating eqs. (3.11), (3.12), (3.13) and (3.14), we get

\[
\phi = \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A} \ln \tilde{H}_A + \frac{c_\phi}{2dr_0^d} \ln f,
\]

\[
u_0 = - \sum_A \frac{D - q_A - 3}{\Delta_A} \ln \tilde{H}_A + \frac{c_0}{2dr_0^d} \ln f,
\]

\[
u_\alpha = - \sum_A \frac{\delta_\alpha^{(\alpha)}}{\Delta_A} \ln \tilde{H}_A + \frac{c_\alpha}{2dr_0^d} \ln f,
\]

\[
B = \sum_A \frac{q_A + 1}{\Delta_A} \ln \tilde{H}_A + \frac{1}{d} \ln g + \frac{c_b}{2dr_0^d} \ln f,
\]

(3.24)

where \( \tilde{H}_A \) is given by

\[
\tilde{H}_A = N_A E_A^{-1} f^{\alpha_A} = \left\{ 1 - \beta_A (1 - f^{-\alpha_A}) \right\} f^{\alpha_A},
\]

(3.25)

and the integration constants are fixed by the requirement that asymptotically the warp factors approach to 1.

Using eqs. (3.24), one can write down the expression (3.7) for \( S_A \) as

\[
S_A = N_A^2 E_A^{-2} f^{\alpha_A}.
\]

(3.26)

Now, using eqs. (3.9) and (3.23), we can determine the normalization constants \( N_A \) as

\[
N_A = \sqrt{\frac{2(\beta_A - 1) (D - 2)}{\beta_A}} \frac{1}{\Delta_A}.
\]

(3.27)

We also have

\[
2c_0 + \sum_{\alpha=2}^{d-1} c_\alpha + \tilde{c_0} = 0,
\]

(3.28)

from the relation (3.1). By use of this relation, \( \tilde{c}_A \) in eq. (3.19) can also be written as

\[
\tilde{c}_A = 2 \sum_{\alpha \in q_A} c_\alpha - \epsilon_A a_A c_\phi.
\]

(3.29)

Our metric and background fields are thus finally given by, after putting all the warp factors etc. that we get by solving the Einstein equations,

\[
ds_D^2 = \prod_A \tilde{H}_A^{2 \frac{q_A + 1}{\Delta_A}} \left[ \prod_A \tilde{H}_A^{-2 \frac{D-2}{\Delta_A}} f^{c_0/\tilde{d}_0^d} \left\{ - 2dudv + Kdu^2 \right\} \right.

+ \sum_{\alpha=2}^{d-1} \prod_A \tilde{H}_A^{-2 \frac{\delta_\alpha^{(\alpha)}}{\Delta_A}} f^{c_\alpha/\tilde{d}_0^d} \left( d^2 + r^2 d\Omega_{d+1}^2 \right) \right],

\]

\[
E_A = \sqrt{\frac{2(\beta_A - 1) (D - 2)}{\beta_A}} \frac{1}{\Delta_A} \tilde{H}_A^{-1} f^{\alpha_A}.
\]

(3.30)
where the function $K$ is given from eq. (3.2) and (3.3) by

$$
\left[\Box^{(d+2)} + g^{-1} \partial_r g \partial_r + \sum_{\alpha=2}^{d-1} g^{2/\tilde{d}} f^{(c_\alpha - c_\omega)}/\tilde{d}_0 \prod_A \tilde{H}^{\tilde{d}^{(\alpha)}}_A \partial_{\tilde{\alpha}}^2 \right] K
$$

$$
= -\frac{(\mu_{ij})^2}{2} g^{2/\tilde{d}} f^{(c_\alpha + c_\phi)/2}/\tilde{d}_0 \prod_A \tilde{H}^{\tilde{d}^{(\alpha)}}_A. \quad (3.31)
$$

The extremal limit corresponds to $r_0 \to 0$ and $\beta_A \to \infty$, keeping $\beta_A r_0^{\tilde{d}^{(\alpha)}}$ finite. In this limit, noting that $\alpha_A$ is finite from (3.16), (3.19) and (3.21), we find

$$
\tilde{H}_A \to 1 + 2\alpha_A \beta_A \left( \frac{r_0}{r} \right)^{\tilde{d}^{(\alpha)}}, \quad (3.32)
$$

so that $2\alpha_A \beta_A r_0^{\tilde{d}^{(\alpha)}}$ is a parameter corresponding to the charge $Q_A$ in the extremal solution.

For $A \neq B$, we have $M_{AB} = 0$ from eq. (3.17). This gives the intersection rules for the branes. If $q_A$-brane and $q_B$-brane intersect over $\bar{q}(\leq q_A, q_B)$ dimensions, this gives

$$
\bar{q} = \frac{(q_A + 1)(q_B + 1)}{D - 2} - 1 - \frac{1}{2} \epsilon_A \epsilon_B a_A a_B. \quad (3.33)
$$

We also get the rule (2.44) for D-branes. We, once again, would like to point out that the above analysis is consistent with the flat space analysis performed in [30]-[35]. These branes are nonsupersymmetric for arbitrary values of non-extremal parameter $r_0$. One can check that in the limit $r_0 \to 0$, both $f$ and $g$ goes to one and we get back to the supersymmetric solutions presented in the previous section.

### 4 Black branes and Horizons

Non-extremal D-brane solutions usually admit horizons and are known as black branes. Though this is certainly true in flat spacetime, not all non-extremal solutions in pp-wave admit regular horizon [39, 41]. In particular, if the background admits a null Killing isometry, the corresponding brane solutions are found to have naked singularities. While the authors of [39] have analyzed pp-wave background supported by 5-form RR fields, our backgrounds have different isometries and it would be interesting to examine if one can find some black branes in this background. Also since the construction of the general solutions in the preceding sections is somewhat abstract, it would be instructive to present explicit examples of D-branes in the background (1.1) and discuss properties of their horizons.
First let us review the criteria developed in ref. [39] for the existence of horizon in pp-wave spacetimes. For the black holes in spacetimes which are not asymptotically flat e.g. pp-waves in our context, no rigorous definition of event horizon exists, partly because of difficulty in identifying the future null infinity. So one has to content with the working hypothesis that a black hole is the region of the spacetime causally disconnected from the asymptotic infinity. Thus the coordinate time along timelike or null geodesics to reach the asymptotic infinity is arbitrarily large. PP-wave spacetime admits two Killing vectors \( \frac{\partial}{\partial u} \) and \( \frac{\partial}{\partial v} \). For \( \frac{\partial}{\partial u} \) timelike, coordinate time can be identified with \( u \) and the following criteria for the existence of the horizon will apply.

Suppose the warp factors behave in the vicinity of the horizon at \( r_0 \) as

\[
e^{2u_0} \sim (r - r_0)^{2a}, \quad e^{2B} \sim (r - r_0)^{2b}, \quad K \sim (r - r_0)^{2h} \text{ sgn } K. \tag{4.1}
\]

Then the conditions for the existence of the horizon are:

\[
a + b > -1, \quad a - b \geq 1 \, \text{ if } h \geq 0
\]
\[
a + b + |h| > -1, \quad a - b \geq 1 + |h| \, \text{ if } h < 0 \quad \text{and} \quad \text{ sgn } K = +
\]
\[
a + b > -1, \quad a - b \geq 1 \, \text{ if } K \sim |\ln(r - r_0)|. \tag{4.2}
\]

For \( \frac{\partial}{\partial u} \) spacelike or null, one has to choose the coordinate time as some linear combination of \( u \) and \( v \) and second condition is replaced by

\[
a + b + |h| > -1, \quad a - b \geq 1 - |h| \, \text{ if } h < 0 \quad \text{and} \quad \text{ sgn } K = +. \tag{4.3}
\]

For the pp-wave spacetime which are asymptotic to BFHP plane wave [7] (as is the case with our solutions), \( \frac{\partial}{\partial u} \) is always timelike and one can use the first set of conditions given above. In this case, it should be noted that condition with \( h < 0 \) is stronger than the condition with \( h \geq 0 \) in the sense that if there exists no horizon for \( h \geq 0 \), and then it cannot exist for \( h < 0 \) either. Since our solutions are asymptotic to plane wave spacetime, we do not have to worry about the second set of conditions.

However this criterion (hereafter referred to as the first criterion), even when it is satisfied, might not be sufficient to guarantee the existence of a horizon [41]. It is not difficult to see that all scalar curvature invariants in the background (2.6) behave regularly in the near horizon limit. However, there may be divergence in the Riemann tensors...
Now the geodesic deviation equation for a test particle moving in the spacetime (2.6) is given by

\[
\frac{D^2 x^\mu}{d\tau^2} = -R^\mu_{\nu \rho \sigma} u^\nu x^\rho u^\sigma \tag{4.4}
\]

where \(x^\mu(\tau)\) and \(u^\mu\) are the displacement vector and four velocity of the test particle respectively. Hence the geodesic deviation equation may become singular and an observer traveling along a causal timelike or null geodesics will feel infinite tidal forces. The relative motion has an invariant meaning only in an orthonormal frame (with basis vectors \(e_a\) obeying \(e_a \cdot e_b = \eta_{ab}\)). The geodesic equation in this frame becomes

\[
\dddot{x}^i = -R^i_{\ 0j0} x^j, \tag{4.5}
\]

where \(x^i = e^i_{\mu} x^\mu\) etc. The most natural choice for the orthonormal frame is the parallel transported frame and we shall see, by some examples, that the Riemann tensors as measured in the parallel transported frame diverges for all these cases, in contrast to the first criterion mentioned above. This suggests that the first criterion is only a necessary condition but not sufficient.

For simplicity, we set the parameters \(\alpha, \beta, \gamma\) and \(\delta\) as

\[
\alpha = \frac{c_0}{2dr_0^2}, \quad \beta = \frac{c_2}{2dr_0^2} = \frac{c_3}{2dr_0^2} = \cdots, \quad \gamma = \frac{c_b}{2dr_b^2}, \quad \delta = \frac{c_0}{2dr_0^2}. \tag{4.6}
\]

Also since we are interested in the near-horizon limit of warp factors, it is useful to note the behaviour of functions \(\tilde{H}_A, f\) and \(g\) in this limit:

\[
\tilde{H}_A \sim (r - r_0)^{-|A| + \alpha A}, \quad f \sim (r - r_0), \quad g \sim (r - r_0). \tag{4.7}
\]

4.1 D3-brane

First we consider the metric for the extremal D3-brane solution

\[
ds^2 = H_3^{-\frac{1}{2}} \left[ -2dudv + K du^2 + dy_2^2 + dy_3^2 \right] + H_3^\frac{1}{2} \left[ dz_1^2 + \cdots + dz_6^2 \right], \quad H_{uij} = \mu_{ij}. \tag{4.8}
\]

The corresponding eq. (2.35) for \(K\) becomes

\[
\left( \square^{(6)} + H_3 (\partial_{y_2}^2 + \partial_{y_3}^2) \right) K = -\frac{(\mu_{ij})^2}{2}, \quad \tag{4.9}
\]
where $\Box^{(6)}$ is the Laplacian in the six-dimensional transverse space spanned by $z_1, \ldots, z_6$ and $H_A$ is given by eq. (2.30) with $d = 4$.

Equation (4.9) admits a solution of the form

$$K = c + \frac{Q}{r^4} - \frac{1}{32} (\mu_{ij})^2 \left( r^2 + y_2^2 + y_3^2 + \frac{Q_3}{r^2} \right). \tag{4.10}$$

So for the extremal D3-brane solution $r_0 = 0$, $a = -b = 1$ and $h = -2$ and the criteria for the existence of the horizon is not satisfied.

Next we consider the non-extremal deformation of the above solution. The metric of the non-extremal solution is given by

$$ds^2 = \tilde{H}_3^{-\frac{1}{2}} f^{2\alpha} (-2dudv + Kdu^2) + \tilde{H}_3^{-\frac{1}{2}} f^{2\beta} (dy_2^2 + dy_3^2)$$

$$+ \tilde{H}_3^{\frac{1}{2}} f^{2\gamma} (dz_1^2 + \cdots + dz_6^2), \tag{4.11}$$

where $\tilde{H}_3$ is given by eq. (3.25) with $\tilde{d} = 4$. The corresponding eq. (3.31) for $K$ then becomes

$$\left( \Box^{(6)} + g^{-1} \partial_r g \partial_r + \frac{1}{2} f^{2(\gamma-\beta)} \tilde{H}_3 (\partial_{y_2}^2 + \partial_{y_3}^2) \right) K = -\frac{(\mu_{ij})^2}{2} g^{\frac{1}{2}} f^{2\alpha + 2\gamma + \delta}. \tag{4.12}$$

The parameters $\alpha, \beta$ and $\gamma$ satisfy the relation

$$\alpha + \beta + 2\gamma = 0, \tag{4.13}$$

and the relation (3.16) becomes

$$8\alpha^2 + 24\gamma^2 + 16\alpha\gamma + \delta^2 = \frac{5}{2}. \tag{4.14}$$

Using these relations, we find from (4.11)

$$a = \frac{|\alpha_{D3}|}{8} - \alpha_{D3} + \alpha, \quad b = -\frac{|\alpha_{D3}|}{8} - \alpha_{D3} + \gamma + \frac{1}{4}, \quad \alpha_{D3} = -8\gamma. \tag{4.15}$$

It is not difficult to see that the relations (4.2), (4.13)-(4.15) cannot be satisfied for real values of parameters $\alpha, \gamma,$ and $\delta$. So the non-extremal deformation of the D3-brane solution does not admit horizon.

Now we would like to check how the curvature tensors behave in the near horizon limit. As already mentioned, the possible divergent quantity is the Riemann tensor measured
in an orthonormal frame. A natural choice for it is the parallel transported frame as emphasized in [41]. The Riemann tensors in the parallel transported frame are given by

\[
R_{\mu
u\rho\sigma}^{\text{tp}t\rho} \equiv R_{\mu
u\rho\sigma}^{\text{tp}t\rho t\rho p^\sigma},
\]

\[
R_{\mu
u\rho\sigma}^{\text{tp}t\rho} \equiv R_{\mu
u\rho\sigma}^{\text{tp}t\rho n^\sigma_i},
\]

\[
R_{\mu
u\rho\sigma}^{\text{tp}t\rho} \equiv R_{\mu
u\rho\sigma}^{\text{tp}t\rho n^\sigma_j},
\]

where \( t^\mu, p^\mu, n^\mu_i, n^\mu_j \) are the unit vectors in the parallel transported frame.

Explicit form of these vectors for D3-brane can be read off from ref. [41]. Using the expression for Riemann tensors given in the appendix, one can see that

\[
R_{\mu\nu\rho\sigma}^{\text{tp}t\rho} \sim \left( K_{,rr} + \frac{3K_{,r} \cos^2(\tau)}{r} \right),
\]

\[
R_{\mu\nu\rho\sigma}^{\text{tp}t\rho} \sim \frac{1}{r} K_{,rr} + \delta_{ij} r^2 K_{,r},
\]

\[
R_{\mu\nu\rho\sigma}^{\text{tp}t\rho} \sim \frac{1}{r^2} \left( K_{,y_i y_j} + 3 \delta_{ij} K_{,r} \right)
\]

for the extremal solution, and they diverge as \( \frac{1}{r^6} \), \( \frac{1}{r^4} \) for \( i \neq j \) and \( \frac{1}{r^3} \) for \( i = j \), respectively. Similarly we can see that some components of the Riemann tensors behave as \( \frac{1}{(r - r_0)^a} \) with \( a > 0 \) for non-extremal solutions and hence are divergent in near horizon limit. This means that there is no regular horizon in these extremal and non-extremal solutions, in agreement with the above conclusion.

### 4.2 D3-D3 system

Next let us consider the extremal D3-D3 system. The metric is given by

\[
ds^2 = H_3^{-\frac{1}{2}} H_3'^{-\frac{1}{2}} (-2 du dv + K du^2) + H_3^{-\frac{1}{2}} H_3'^{\frac{1}{2}} (dy_2^2 + dy_3^2) + H_3^{\frac{1}{2}} H_3'^{-\frac{1}{2}} (dy_4^2 + dy_5^2) + H_3^{-\frac{1}{2}} H_3'^{\frac{1}{2}} (dz_1^2 + \cdots + dz_4^2),
\]

\[
H_{\mu ij} = \mu_{ij}.
\]

The corresponding eq. (3.31) for \( K \) then becomes

\[
[\Box^{(4)} + H_3 (\partial^2_{y_2} + \partial^2_{y_3}) + H_3' (\partial^2_{y_4} + \partial^2_{y_5})] K = -\frac{1}{2} (\mu_{ij})^2,
\]

where \( \Box^{(4)} \) is again the Laplacian in the four-dimensional transverse space spanned by \( z_1, \cdots, z_4 \) and \( H_3 \) and \( H_3' \) are given by eq. (2.30) with \( \tilde{d} = 2 \).

Equation (4.18) admits a solution of the form

\[
K = c + \frac{Q}{r^2} - \frac{1}{32} (\mu_{ij})^2 \left( r^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2 - 2Q_3 \ln r - 2Q_3' \ln r \right).
\]
So for the extremal D3-D3 system \( a = -b = 1 \) and \( h = -1 \) and the criteria for the existence of the horizon is satisfied. However, we shall see that this is not sufficient for the existence of the regular horizon.

Now let us consider the non-extremal deformation of the D3-D3 system. The metric is given by

\[
\begin{align*}
\text{ds}^2 &= H_3^{-\frac{1}{2}} H_{y^3}^{\frac{1}{2}} f^{2\alpha} (-2dudv + Kd_2) + H_3^{-\frac{1}{2}} H_{y^3}^{\frac{1}{2}} f^{2\beta} (dy_2^2 + dy_y^2) \\
&\quad + H_3^{\frac{1}{2}} H_{y^3}^{\frac{1}{2}} f^{2\alpha} (dy_4^2 + dy_5^2) + H_3^{\frac{1}{2}} H_{y^3}^{\frac{1}{2}} f^{2\gamma} g(dz_1 + \cdots + dz_4), \\
H_{uij} &= \mu_{ij}.
\end{align*}
\]

The corresponding eq. (3.31) for \( K \) then becomes

\[
\begin{align*}
[\square^{(4)} + g^{-1} \partial_r g \partial_r + H_3 f^{(\gamma - \beta)} g(\partial_{y_2}^2 + \partial_{y_1}^2) \\
+ H_{y^3} f^{(\gamma - \beta)} (\partial_{y_4}^2 + \partial_{y_5}^2)] K &= -\frac{(\mu_{ij})^2}{2} f^{2\alpha + 2\gamma + \delta} g.
\end{align*}
\]

The parameters \( \alpha, \beta \) and \( \gamma \) satisfy the relation

\[
\alpha + 2\beta + \gamma = 0.
\]

The relation (3.16) becomes

\[
6\alpha^2 + 6\gamma^2 + 4\alpha\gamma + \delta^2 = 3.
\]

Using these relations and choosing same value for \( \alpha_{D3} \) for both D3-branes, we find from (4.20)

\[
a = \frac{|\alpha_{D3}|}{4} - \frac{\alpha_{D3}}{4} + \alpha, \quad b = -\frac{|\alpha_{D3}|}{4} - \frac{\alpha_{D3}}{4} + \gamma + \frac{1}{2}, \quad \alpha_{D3} = 2(\alpha - \gamma) = \alpha_{D3'}.
\]

We again find that the conditions (4.2) for the existence of the horizon cannot be satisfied for real values of parameters \( \alpha, \gamma, \) and \( \delta \) obeying the above conditions. So the non-extremal deformation of the D3-D3 brane solution also does not admit horizon.

On the other hand, we have examined the Riemann tensors in the parallel transported frame and found that they again diverge. Thus, both the extremal and non-extremal solutions do not have regular horizon. This is contrary to the above result due to the first criterion that the extremal solution admits a regular horizon.
4.3 D3-D5 system

Next let us consider the extremal D3-D5 system. The metric is given by

\[
\begin{align*}
\text{ds}^2 &= H_3^{-1/2}H_5^{-1/4}(-2dudv + Kd^2 + dy_5^2) + H_3^{-1/2}H_5^{3/4}dy_3^2 \\
&\quad + H_3^{1/2}H_5^{-1/2}(dy_4^2 + dy_5^2 + dy_6^2) + H_3^{1/2}H_5^{3/4}(dz_1^2 + dz_2^2 + dz_3^2),
\end{align*}
\]

\[H_{uij} = \mu_{ij}. \tag{4.25}\]

The corresponding eq. (3.31) for \(K\) then becomes

\[
\begin{align*}
&\left[\Box^{(3)} + H_3H_5\partial_{y_2}^2 + H_3\partial_{y_3}^2 + H_5(\partial_{y_4}^2 + \partial_{y_5}^2 + \partial_{y_6}^2)\right]K = -\frac{1}{2}(\mu_{ij})^2. \tag{4.26}
\end{align*}
\]

where \(\Box^{(3)}\) is once again the Laplacian in the three-dimensional transverse space spanned by \(z_1, z_2, z_3\) and \(H_3\) and \(H_5\) are given by eq. (2.30) with \(\tilde{d} = 1\).

Equation (4.26) admits a solution of the form

\[
K = c + \frac{Q}{r} - \frac{1}{32}(\mu_{ij})^2\left(r^2 - 2(Q_3 + 2Q_5)r - 2Q_3Q_5\ln r + y_2^2 + y_3^2 + y_4^2 + y_5^2 + y_6^2\right). \tag{4.27}
\]

So for the extremal D3-D5 solution \(a = \frac{3}{8}, b = -\frac{5}{8}\) and \(h = -\frac{1}{2}\) and the criterion for the existence is not satisfied.

Now let us consider the non-extremal deformation of D3-D5 system. The metric is given by

\[
\begin{align*}
\text{ds}^2 &= H_3^{-1/2}H_5^{1/4}f^{2\alpha}(-2dudv + Kd^2) + H_3^{-1/2}H_5^{1/4}f^{2\beta}dy_3^2 + H_3^{-1/2}H_5^{3/4}f^{2\beta}dy_3^2 \\
&\quad + H_3^{1/2}H_5^{-1/2}(dy_4^2 + dy_5^2 + dy_6^2) + H_3^{1/2}H_5^{3/4}f^{2\gamma}g^2(dz_1^2 + dz_2^2 + dz_3^2),
\end{align*}
\]

\[H_{uij} = \mu_{ij}, \quad e^\phi = \tilde{H}_5^{-1/2}f^\delta. \tag{4.28}\]

where \(\tilde{H}_3\) and \(\tilde{H}_5\) are given by eq. (3.25) with \(\tilde{d} = 1\). The corresponding eq. (3.31) for \(K\) then gives

\[
\left[\Box^{(3)} + g^{-1}\partial_r g\partial_r + \tilde{H}_3\tilde{H}_5f^{2(\gamma-\beta)}g^2\partial_{y_4}^2 + \tilde{H}_3f^{2(\gamma-\beta)}g^2\partial_{y_3}^2 \\
+ \tilde{H}_5f^{2(\gamma-\beta)}g^2(\partial_{y_4}^2 + \partial_{y_5}^2 + \partial_{y_6}^2)\right]K = -\frac{(\mu_{ij})^2}{2}f^{2\alpha+2\gamma+\delta}g^2. \tag{4.29}\]

The parameters \(\alpha, \beta, \gamma\) satisfy the relation (3.28)

\[
2\alpha + 5\beta + \gamma = 0. \tag{4.30}\]
The relation (3.16) becomes

\[ 28\alpha^2 + 8\alpha\gamma + 12\gamma^2 + 5\delta^2 = 20. \tag{4.31} \]

Using these relations, we find from (4.28)

\begin{align*}
a &= \frac{|\alpha_{D3}| - \alpha_{D3}}{8} + \frac{|\alpha_{D5}| - \alpha_{D5}}{16} + \alpha, \\
b &= -\frac{|\alpha_{D3}| - \alpha_{D5}}{8} - \frac{3(|\alpha_{D5}| - \alpha_{D5})}{16} + \gamma + 1, \\
\alpha_{D3} &= \frac{12}{5}\alpha - \frac{4}{5}\gamma, \quad \alpha_{D5} = \frac{4}{5}\alpha - \frac{8}{5}\gamma + \delta \tag{4.32}
\end{align*}

We find that again the conditions (4.2) for the existence of the horizon cannot be satisfied for real values of parameters \(\alpha, \gamma,\) and \(\delta\) obeying the above conditions. So the non-extremal deformation of the D3-D5 brane solution also does not admit horizon.

One also finds that the Riemann tensor in the parallel transported frame for this solution diverges for both the extremal and non-extremal solutions, in agreement with the above conclusion.

### 4.4 D5-NS5-D3 system

Next let us consider the extremal D5-NS5-D3 system. The metric is given by

\begin{align*}
ds^2 &= H_3^{-\frac{1}{2}}H_5^{-\frac{1}{2}}H_5'^{-\frac{1}{2}}(-2dudv + Kdu^2) + H_3^{-\frac{1}{2}}H_5^{-\frac{1}{2}}H_5'^{-\frac{1}{2}}(dy_2^2 + dy_3^2 + dy_4^2) \\
&\quad + H_3^{-\frac{1}{2}}H_5'^{-\frac{1}{2}}dy_5^2 + H_3^{-\frac{1}{2}}H_5'\partial y_5^2 + H_3^{-\frac{1}{2}}H_5' \partial y_6^2 + H_3^{-\frac{1}{2}}H_5' \partial y_7^2 (dz_1^2 + dz_2^2 + dz_3^2), \\
H_{\mu ij} &= \mu_{ij}. \tag{4.33}
\end{align*}

The corresponding eq. (3.31) for \(K\) then becomes

\[ [\Box^{(3)} + H_5H_5'\partial_{y_2}^2 + \partial_{y_3}^2 + \partial_{y_4}^2 + H_3H_5\partial_{y_5}^2 + H_3H_5'\partial_{y_6}^2]K = -\frac{(\mu_{ij})^2}{2}H_5', \tag{4.34} \]

where \(\Box^{(3)}\) is once again the Laplacian in the three-dimensional transverse space spanned by \(z_1, z_2, z_3\) and \(H_3, H_5\) and \(H'_5\) are given by eq. (2.30) with \(\tilde{d} = 1.\)

Equation (4.34) admits a solution of the form

\begin{align*}
K &= c + \frac{Q}{r} - \frac{1}{32}(\mu_{ij})^2 \left( r^2 - 2(2Q_5 - 2Q_5' + Q_3)r - 6Q_5Q_5' \ln r - 2Q_3Q_5 \ln r \\
&\quad - 2Q_3Q_5' \ln r + y_2^2 + y_3^2 + y_4^2 + y_5^2 + y_6^2 \right). \tag{4.35}
\end{align*}
So for the extremal D5-NS5-D3 solution $a = \frac{1}{2}, b = -1$ and $h = -\frac{1}{2}$ and the first criterion for the existence is satisfied. However, we shall see that this is not sufficient for the existence of the regular horizon.

Now let us consider the non-extremal deformation of D5-NS5-D3 system. The metric is given by

$$ds^2 = H_3^{-\frac{1}{2}} H_5^{-\frac{1}{2}} H_5'^{-\frac{1}{2}} f^{2\alpha} (-2dudv + Kd\alpha^2) + H_3^\frac{1}{2} H_5^{-\frac{1}{2}} H_5'^{-\frac{1}{2}} d^2(y_2 + dy_3 + dy_4)$$

$$+ H_3^{-\frac{1}{2}} H_5^{-\frac{1}{2}} H_5'^{-\frac{1}{2}} f^{2\beta} dy_5^2 + H_3^{-\frac{1}{2}} H_5^{-\frac{1}{2}} H_5'^{-\frac{1}{2}} f^{2\gamma} dz_1^2 + dz_2^2 + dz_3^2,$$

$$H_{\mu ij} = \mu_{ij}, \quad e^\phi = \tilde{H}_5^{-\frac{1}{2}} H_5'^{\frac{1}{2}} f^\phi. \quad (4.36)$$

where $\tilde{H}_3, \tilde{H}_5$ and $\tilde{H}_5'$ are given by eq. (3.25) with $\tilde{d} = 1$. The corresponding eq. (3.31) for $K$ then gives

$$[\Box^{(3)} + g^{-1} \partial_r g \partial_r + \tilde{H}_5 H_5' f^{2(\gamma-\beta)} g^2 (\partial^2_{y_2} + \partial^2_{y_3} + \partial^2_{y_4}) + \tilde{H}_3 H_5 f^{2(\gamma-\beta)} g^2 \partial^2_{y_5}$$

$$+ \tilde{H}_3 H_5' f^{2(\gamma-\beta)} g^2 \partial^2_{y_6}] K = -\frac{(\mu_{ij})^2}{2} f^{2\alpha+2\beta+\delta} g^2 \tilde{H}_5'. \quad (4.37)$$

The parameters $\alpha, \beta, \gamma,$ satisfy the relation (3.28):

$$2\alpha + 5\beta + \gamma = 0. \quad (4.38)$$

The relation (3.16) becomes

$$28\alpha^2 + 8\alpha\gamma + 12\gamma^2 + 5\delta^2 = 20. \quad (4.39)$$

Using these relations, we find from (4.36)

$$a = \frac{1}{8} |\alpha_D| - \frac{1}{4} |\alpha_D| + \frac{1}{16} |\alpha_{NSS'}| - \frac{1}{16} |\alpha_{NSS'}| + \alpha,$$

$$b = -\frac{1}{8} |\alpha_D| - \frac{3}{16} (|\alpha_D| - |\alpha_{DS}|) - \frac{3}{16} (|\alpha_{NSS'}| - |\alpha_{NSS'}|) + \gamma + 1,$$

$$\alpha_{D3} = \frac{12}{5} \alpha - \frac{4}{5} \gamma, \quad \alpha_{DS} = \frac{4}{5} \alpha - \frac{8}{5} \gamma + \delta, \quad \alpha_{NSS'} = \frac{4}{5} \alpha - \frac{8}{5} \gamma - \delta \quad (4.40)$$

We find that the conditions (4.2) for the existence of the horizon cannot be satisfied for real values of parameters $\alpha, \gamma,$ and $\delta$ obeying the above conditions. So the non-extremal deformation of the D5-NS5-D3 system does not admit horizon.

We have examined the Riemann tensors as measured in the parallel transported frame for this solution and find that they again diverge for both the extremal and non-extremal solutions. Thus the above result according to the first criterion is not sufficient to guarantee the existence of the regular horizon for the extremal solution.
5 Summary and Discussion

In this paper we have constructed a general class of intersecting brane solutions in pp-wave spacetime with nonconstant NS-NS flux. The intersection rules for the branes in this background are derived by solving Einstein equations in $D$ dimensional space-time. Though the asymptotic form of the solutions is quite different from flat space, the intersection rules are found to be the same as that of flat space-time. The supersymmetric properties of the brane configuration are also outlined. Moreover, we generalize our construction to non-extremal solutions as well. The corresponding set of intersection rules are also derived.

We have considered the possibility of the existence of horizon with some examples of intersecting branes in the pp-wave background with three form flux. Here we have first used the criteria developed in ref. [39], which indicates that there can be a regular horizon for some extremal solutions, but the non-extremal deformations do not admit any horizon. However, the criterion is not sufficient to guarantee the existence of the horizon [41]. We have explicitly shown that the Riemann tensors diverge in the parallel propagated frame for the brane solutions in pp-wave background for which the first criterion suggests that there can exist a regular horizon.

One would conclude that the brane solutions do not admit the regular horizon in pp-wave spacetime in general and the corresponding solutions are singular. However, as emphasized in [41], the presence of these 'pp-singularities' close off the spacetime near the horizon and analytic extension beyond the horizon is not possible. In this sense these singularities may be regarded as the physical boundary of the spacetime. It is also possible that stringy effects can resolve these singularities.

D-brane systems have been quite useful in studying nonperturbative properties of field theories realized on them. The pp-wave background is one of the rare examples on which string theories can be solved exactly. It would be interesting to examine our solutions further and see how these nice properties expected to be reflected in our brane solutions can give further insight into the field theories as well as string theories.
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A Curvature tensors

In this appendix, we summarize the curvature tensors necessary for our derivation of brane solutions.

The non-zero Christoffel symbols for the metric (2.6) are

\[
\begin{align*}
\Gamma^u_{ur} &= u'_0, \\
\Gamma^v_{ur} &= -\frac{1}{2}K', \\
\Gamma^r_{uv} &= u'_0 e^{2u_0 - 2B}, \\
\Gamma^r_{uu} &= -K (u'_0 + \frac{1}{2}K') e^{2u_0 - 2B}, \\
\Gamma^r_{rr} &= B', \\
\Gamma^\alpha_{\beta r} &= u'_\alpha \delta_{\alpha \beta}, \\
\Gamma^r_{\alpha \beta} &= -u'_\alpha e^{2u_\alpha - 2B} \delta_{\alpha \beta}, \\
\Gamma^a_{br} &= (B' + \frac{1}{r}) \delta_{ab}, \\
\Gamma^r_{ab} &= -r^2 (B' + \frac{1}{r}) g_{ab}, \\
\Gamma^a_{bc} &= \frac{1}{2} g^{ad} (g_{bd,c} + g_{cd,b} - g_{bc,d}),
\end{align*}
\]

where \( g_{ab} \) is the metric for the sphere of radius \( r \) and the prime denotes a derivative with respect to \( r \).

Ricci Tensors:

The non-zero Ricci tensors of the metric are

\[
\begin{align*}
R_{uv} &= e^{2(u_0 - B)} \left[ u''_0 + u'_0 \left\{ 2u'_0 + \sum_{\alpha=2}^{d-1} u'_\alpha + \tilde{d} B' + \frac{\tilde{d} + 1}{r} \right\} \right], \\
R_{uu} &= -e^{2(u_0 - B)} K \left[ u''_0 + \frac{\tilde{d} + 1}{r} u'_0 + \frac{1}{2} K^{-1} \Box (\tilde{d} + 2) K \\
& \quad + \partial_i \left( u_0 + \frac{1}{2} \ln K \right) \partial^i \left( 2u_0 + \sum_{\alpha=2}^{d-1} u_\alpha + \tilde{d} B \right) \right] - \frac{1}{2} \sum_{\alpha=2}^{d-1} e^{2(u_0 - u_\alpha)} \partial^2_{\alpha} K, \\
R_{\alpha \beta} &= -e^{2(u_\alpha - B)} \left[ u''_\alpha + u'_\alpha \left\{ 2u'_\alpha + \sum_{\gamma=2}^{d-1} u'_\gamma + \tilde{d} B' + \frac{\tilde{d} + 1}{r} \right\} \right] \delta_{\alpha \beta}, \\
R_{rr} &= -2u''_0 - \sum_{\alpha=2}^{d-1} u''_\alpha - (\tilde{d} + 1) \left( B'' - \frac{1}{r^2} \right) + B' \left\{ 2u'_0 + \sum_{\alpha=2}^{d-1} u'_\alpha + (\tilde{d} + 1) \left( B' + \frac{1}{r} \right) \right\} \\
& \quad -2(u'_0)^2 - \sum_{\alpha=2}^{d-1} (u'_\alpha)^2 - (\tilde{d} + 1) \left( B' + \frac{1}{r} \right)^2,
\end{align*}
\]
where $\hat{R}_{abc}{}^d$ is the Riemann tensor for the sphere part with metric $g_{ab}$.

**References**

[1] D. Amati and C. Klimcik, “Nonperturbative computation of the Weyl anomaly for a class of nontrivial backgrounds”, Phys. Lett. B 219 (1989) 443.

[2] G. T. Horowitz and A. R. Steif, “Space-time singularities in string theory”, Phys. Rev. Lett. 64 (1990) 260.

[3] R. R. Metsaev, “Light cone gauge formulation of IIB supergravity in AdS$_5 \times S^5$ background and AdS/CFT correspondence”, Phys. Lett. B 468 (1999) 65, hep-th/9908114.

[4] R. R. Metsaev, “Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background”, Nucl. Phys. B 625 (2002) 70, hep-th/0112044.
[5] R. R. Metsaev and A. A. Tseytlin, “Exactly solvable model of superstring in Ramond-Ramond plane wave background”, Phys. Rev. D **D65** (2002) 126004, hep-th/0202109.

[6] R. Penrose, “Any space-time has a plane wave as a limit”, in Differential geometry and relativity, pp.271-275, Reidel, Dordrecht, (1976).

[7] M. Blau, J. Figuero-O’Farrill, C. Hull and G. Papadopoulos, “A new maximally supersymmetric background of IIB superstring theory”, JHEP **0201**, 047 (2000), hep-th/0110242;

[8] M. Blau, J. Figuero-O’Farrill and G. Papadopoulos, “Penrose limits and maximal supersymmetry”, Class. Quant. Grav. **19** (2000) L87, hep-th/0201081; M. Blau, J. Figuero-O’Farrill and G. Papadopoulos, “Penrose limits, supergravity and brane dynamics”, Class. Quant. Grav. **19** (2002) 4753, hep-th/0202111.

[9] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity”, Adv. Theor. Math. Phys. **2** (1998) 231; Int. J. Theor. Phys. **38** (1999) 1113, hep-th/9711200.

[10] D. Berenstein, J. Maldacena and H. Nastase, “Strings in flat space and pp waves from $N = 4$ super Yang-Mills”, JHEP **0204** (2002) 013, hep-th/0202021.

[11] M. Li, “PP-wave black holes and the matrix model”, JHEP **0305** (2003) 031, hep-th/0212345.

[12] A. Dabholkar and S. Parvizi, “Dp Branes in pp-wave background”, Nucl. Phys. B **641** (2002) 223, hep-th/0203231.

[13] A. Kumar, R. R. Nayak and Sanjay, “D-brane solutions in pp-wave background”, Phys. Lett. B **541** (2002) 183, hep-th/0204025.

[14] K. Skenderis and M. Taylor, “Branes in AdS and pp-wave spacetimes”, JHEP **0206** (2002) 025, hep-th/0204054.

[15] P. Bain, P. Meessen and M. Zamaklar, “Supergravity solutions for D-branes in Hpp-wave backgrounds”, Class. Quant. Grav. **20** (2003) 913, hep-th/0205106.
[16] M. Alishahiha and A. Kumar, “D-brane solutions from new isometries of pp-waves”, Phys. Lett. B 542 (2002) 130, hep-th/0205134.

[17] Y. Michishita, “D-branes in NSNS and RR pp-wave backgrounds and S-duality”, JHEP 0210 (2002) 048, hep-th/0206131.

[18] P. Bain, K. Peeters and M. Zamaklar, “D-branes in a plane wave from covariant open strings”, Phys. Rev. D 67 (2003) 066001, hep-th/0208038.

[19] A. Biswas. A. Kumar and K. L. Panigrahi, “p-p’ branes in pp-wave background”, Phys. Rev. D 66 (2002) 126002, hep-th/0208042.

[20] M. Cvetic, H. Lu, C. N. Pope and K. S. Stelle, “Linearly-realised worldsheet supersymmetry in pp-wave background”, hep-th/0209193.

[21] R. R. Nayak, “D-branes at angle in pp-wave background”, Phys. Rev. D 67 (2003) 086006, hep-th/0210230.

[22] L. F. Alday and M. Cirafici, “An example of localized D-branes solution on pp-wave backgrounds”, JHEP 05 (2003) 006, hep-th/0301253.

[23] K. Skenderis and M. Taylor, “Open strings in the plane wave background I: Quantization and symmetries”, hep-th/0211011. K. Skenderis and M. Taylor, “Open strings in the plane wave background II: Superalgebras and spectra”, hep-th/0212184.

[24] J. Maldacena and L. Maoz, “Strings on pp-waves and massive two dimensional field theories”, JHEP 0212 (2002) 046, hep-th/0207284.

[25] J. G. Russo and A. A. Tseytlin, “A class of exact pp-wave string models with interacting light-cone gauge actions”, JHEP 0209 (2002) 035, hep-th/0208114.

[26] N. Kim, “Remarks on type IIB pp waves with Ramond-Ramond fluxes and massive two dimensional nonlinear sigma models”, Phys. Rev. D 67 (2003) 046005, hep-th/0212017.

[27] G. Bonelli, “On Type II strings in exact superconformal non-constant RR backgrounds”, JHEP 0301 (2003) 065, hep-th/0301089.
[28] Y. Hikida and S. Yamaguchi, “D-branes in pp-waves and massive theories on worldsheet with boundary”, JHEP 0301 (2003) 072, hep-th/0210262.

[29] K. L. Panigrahi and Sanjay, “D-branes in pp-wave spacetime with nonconstant NS-NS flux”, Phys. Lett. B 561 (2003) 284, hep-th/0303182.

[30] N. Ohta, “Intersection rules for non-extreme p-branes”, Phys. Lett. B 403 (1997) 218, hep-th/9702164.

[31] G. Papadopoulos and P. K. Townsend, “Intersecting M-branes”, Phys. Lett. B 380 (1996) 273, hep-th/9603087.

[32] A. A. Tseytlin, “Harmonic superpositions of M-branes”, Nucl. Phys. B 475 (1996) 149, hep-th/9604035.

[33] K. Behrndt, E. Bergshoeff and B. Janssen, “Intersecting D-Branes in ten and six dimensions”, Phys. Rev. D 55 (1997) 3785, hep-th/9604168.

[34] J. P. Gauntlett, D. A. Kastor and J. Traschen, “Overlapping branes in M theory”, Nucl. Phys. B 478 (1996) 544, hep-th/9604179.

[35] R. Argurio, F. Englert and L. Houart, “Intersection rules for p-branes”, Phys. Lett. B 398 (1997) 61, hep-th/9704190;

[36] N. Ohta, “Intersection rules for S-branes”, Phys. Lett. B 558 (2003) 213, hep-th/0301095; N. Ohta, “Null-brane solutions in supergravities”, Phys. Lett. B 559 (2003) 270, hep-th/0302140.

[37] V. E. Hubeny and M. Rangamani, “No horizons in pp-waves”, JHEP 0211 (2002) 021, hep-th/0210234.

[38] V. E. Hubeny and M. Rangamani, “Causal structures of pp-waves”, JHEP 0212 (2002) 043, hep-th/0211195. V. E. Hubeny and M. Rangamani, “Generating asymptotically plane wave spacetimes”, JHEP 0301 (2003) 031, hep-th/0211206.

[39] J. T. Liu, L. A. Pando Zayas and D. Vaman, “On horizons and plane waves”, hep-th/0301187.
[40] E. G. Gimon, A. Hashimoto, V. E. Hubeny, O. Lunin and M. Rangamani, "Black strings in asymptotically plane wave geometries", hep-th/0306131.

[41] G. T. Horowitz and H. Yang, Black strings and classical hair, Phys. Rev. D 55 (1997) 7618, hep-th/9701077;
D. Brecher, A. Chamblin and H.S. Reall, “AdS/CFT in the infinite momentum frame”, Nucl. Phys. B 607 (2001) 155, hep-th/0012076.

[42] M. Cvetic, H. Lu and C. N. Pope, “M-theory pp-waves, Penrose limits and supernumerary supersymmetries”, Nucl. Phys. B 644 (2002) 65, hep-th/0203229.