Micromechanics Analysis of Elastic Properties of Flexible Matrix TWF Composites

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Abstract. Using carbon fibers triaxial woven fabric (TWF) as a reinforcing material and compounding with a suitable flexible matrix material to form a composite shell film structure. It is a new type of high precision deployable antenna implementation. In this paper, the elastic properties of TWF composites using silicone rubber are studied. Using the micromechanics method, the three-dimensional unit cell characterizing the structural properties of the braided composite material is carried out. Homogenization finite element analysis will ultimately yield the elastic modulus of the entire composite. Firstly, considering the fiber bundle interlacing, the unit cell is finely geometrically modelled. Next, the unit cell is analyzed by finite element and periodic boundary conditions are applied. The method of homogenization is used to obtain the volume average stress and strain through six independent loading analysis, and the stiffness matrix is obtained, and the engineering constant is finally obtained. Finally, the effect of fiber volume fraction on elastic properties is analyzed. At present, there are few researches on flexible matrix TWF composites at home and abroad. The research on the elastic properties of the material has laid a solid foundation for further research on the performance of the material in the future.

1. Introduction
For a long time now and in the future, the demand for large space deployable reflectors (LDRs) is growing and growing. For scientific, communications and Earth observation missions, reflectors of medium (4 to 8 m), large (8 to 15 m) and even very large (up to 25 m) sizes will be required [1]. The accuracy of the reflection surface of large-scale space antenna reflectors is also required to meet the needs of Ka-band satellite communication and higher-frequency Earth observation [2]. Existing large expandable reflectors use a metal mesh as the reflecting surface (RS), and the mesh expandable reflector is more mature and easier to implement than other large expandable reflectors. However, this type of antenna often encounters a series of problems in achieving high precision, and it requires a high price in material selection, structural design, thermal control design, manufacturing debugging time, cost, and the like.

The use of a single-layer TWF composite in association with a large expandable spatial reflector is to use a flexible matrix, its stiffness falls between the metal mesh and most of the deformable shell. This new type of shell membrane reflector [4, 5] uses a carbon TWF reinforced flexible matrix composite as the reflective surface (Fig. 1). Due to its small bending stiffness determined by the silicone rubber matrix, the need for tensioning is eliminated. The bending stiffness helps to obtain a double-bend parabolic shape, providing a highly accurate reflective surface while maintaining easy folding performance to meet high-precision expandable requirements. Therefore, the research on the composite material has very important theoretical significance and engineering value.
2. Structure and parameters of TWF composites
The structure of the triaxial woven fabric (TWF) and the structural parameters of the selected unit cell, the material properties of the fiber and matrix of the TWF composite are introduced. The calculation process of the fiber bundle material properties is carried out.

2.1. Unit cell structure parameters and material properties
Triaxial Woven Fabric (TWF) is a planar triaxial fabric consisting of three different directions of yarn or tow system in a plane, Interwovening with each other at a certain angle (0° and ±60°) according to a specific interlacing law[3]. The material studied in this paper is carbon fiber reinforced silicone rubber, in which carbon fiber is T300 type (each fiber bundle contains 1000 T300 fibers), and the matrix is made of silicone rubber S690. The material has a fiber volume fraction of 42% and a thickness of about 0.15 mm.

The selected unit cell and geometric parameters are as shown in the figure below. The direction of the 0° fiber bundle is defined as the $x$ direction, and the $y$ direction is perpendicular to it.

The material properties of the fiber and matrix are shown in the following table:

| Material properties | T300 fiber | S690 matrix |
|---------------------|------------|-------------|
| Density $[kg/m^3]$  | 1760       | 1060        |
2.2. Calculation of fiber bundle material properties

Along the x-axis of the tow, the tensile modulus $E_i$ and Poisson's ratio $\nu_{12}$ are obtained from the mixing rule using known fiber and matrix properties:

$$E_i = \varphi_f E_{i,f} + (1 - \varphi_f)E_m$$

(1)

$$\nu_{12} = \varphi_f \nu_{12,f} + (1 - \varphi_f)\nu_m$$

(2)

Determine the transverse tensile modulus $E_2$ using the Halpin-Tsai semi-empirical equation:

$$E_2 = E_3 = E_m \frac{1 + \xi \eta \varphi_f}{1 - \eta \varphi_f}$$

(3)

Where

$$\eta = \frac{E_{2,f} - E_m}{E_{2,f} + \xi E_m}$$

(4)

The parameters $\xi = 2$ are a measure of composite reinforcement. Similarly, for shear modulus $G_{12}$, the Halpin-Tsai semi-empirical relationship is used.

$$G_{12} = G_{13} = G_m \frac{(G_{12,f} + G_m) + \varphi_f (G_{12,f} - G_m)}{(G_{12,f} + G_m) - \varphi_f (G_{12,f} - G_m)}$$

(5)

Obtaining shear modulus $G_{23}$ by solving the following quadratic equation.

$$\left( \frac{G_{23}}{G_m} \right)^2 A + \left( \frac{G_{23}}{G_m} \right) B + C = 0$$

(6)

The calculation formula for $A$, $B$, and $C$ is referred to in Ref. 9.

With the shear modulus $G_{23}$, the Poisson's ratio $\nu_{23}$ can be calculated.

$$G_{23} = \frac{E_2}{2(1 + \nu_{23})}$$

(7)

The engineering constant of the fiber bundle is finally obtained, as shown in the following table:

| Material properties | Fiber bundle material properties. |
|---------------------|----------------------------------|
| $E_1/MPa$           | 230000                           |
| $E_2/MPa$           | 15000                            |
| $G_{12}/MPa$        | 6000                             |
| $\nu_{12}$          | 0.2                              |
| $E_3/MPa$           | 96601                            |
| $E_3/MPa$           | 3.8058                           |
| $G_{13}/MPa$        | 0.9825                           |
| $G_{23}/MPa$        | 1.3608                           |
| $\nu_{12} = \nu_{13}$| 0.3711                           |
| $\nu_{23}$          | 0.3984                           |
3. Micromechanics Analysis
Homogenization finite element analysis of the three-dimensional unit cell characterizing the structural properties of the braided composite is applied, and periodic boundary conditions are applied, and finally the elastic modulus of the entire composite is obtained.

3.1. Establishment of a unit cell model
Figure 4 shows the TWF unit cell geometry model that is established by the software ProE. The solid is a staggered fiber bundle, and the substrate is transparent (the actual material hexagonal hole and the rectangular boundary should have no matrix at the four corners).

The geometric model established in the ProE is imported into the finite element analysis software ABAQUS to establish a finite element model. Due to the complex structure of the TWF composite, the tetrahedral elements are discrete for the unit cell, and the unit cell grid is freely divided, as shown in Figure 6:

3.2. Periodic boundary condition
The periodic boundary conditions required to analyse a unit cell are based on a translational symmetry transformation. Under the translational symmetry transformation, for 3-D composites of size 2a × 2b × 2c, the relative displacement between the opposite faces of the unit cells are as follows:

\[
\begin{align*}
(u_{x=a} - u_{x=-a})_{y,z} &= 2ae_{11}^0 \quad (v_{x=a} - v_{x=-a})_{y,z} = 0 \quad (w_{x=a} - w_{x=-a})_{y,z} = 0 \\
(u_{y=b} - u_{y=-b})_{x,z} &= 2be_{12}^0 \quad (v_{y=b} - v_{y=-b})_{x,z} = 2be_{22}^0 \quad (w_{y=b} - w_{y=-b})_{x,z} = 0 \\
(u_{z=c} - u_{z=-c})_{x,y} &= 2ce_{13}^0 \quad (v_{z=c} - v_{z=-c})_{x,y} = 2be_{23}^0 \quad (w_{z=c} - w_{z=-c})_{x,y} = 2ce_{33}^0
\end{align*}
\]

(8)

The relative displacement for the edges (except the vertices) and vertex are referred to in Ref. 11.

3.3. Homogenization analysis
The stiffness of the composite changes rapidly with position, a simpler solution is sought by introducing an average modulus. One way to achieve this average modulus is to use homogenization theory. The component of the average modulus is determined by applying six independent load cases.

\[
\begin{align*}
\begin{bmatrix}
\varepsilon_{11}^0 \\
\varepsilon_{22}^0 \\
\varepsilon_{33}^0 \\
\varepsilon_{23}^0 \\
\varepsilon_{31}^0 \\
\varepsilon_{12}^0
\end{bmatrix}
= &
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{23} \\
\varepsilon_{31} \\
\varepsilon_{12}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11}^0 \\
\varepsilon_{22}^0 \\
\varepsilon_{33}^0 \\
\varepsilon_{23}^0 \\
\varepsilon_{31}^0 \\
\varepsilon_{12}^0
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11}^0 \\
\varepsilon_{22}^0 \\
\varepsilon_{33}^0 \\
\varepsilon_{23}^0 \\
\varepsilon_{31}^0 \\
\varepsilon_{12}^0
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{23} \\
\varepsilon_{31} \\
\varepsilon_{12}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11}^0 \\
\varepsilon_{22}^0 \\
\varepsilon_{33}^0 \\
\varepsilon_{23}^0 \\
\varepsilon_{31}^0 \\
\varepsilon_{12}^0
\end{bmatrix}
\end{align*}
\]

(9)
The mechanical properties of composites can be represented by three-dimensional constitutive equations, where the matrix form is as follows:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
e_{23} \\
e_{31} \\
e_{12}
\end{bmatrix}
\]  

(10)

\(C_{ij}\) is the elastic modulus, and the flexibility matrix \(S_{ij}\) can be obtained by inversion. The coefficients of the homogenized stiffness matrix of the composite are calculated by the solution of six load conditions (as described in Equation 9) on the unit cell.

From the flexibility matrix coefficients, the engineering constants can be obtained as follows:

\[
E_{11} = 1/S_{11}, E_{22} = 1/S_{22}, E_{33} = 1/S_{33},
\]

\[
G_{12} = 1/S_{66}, G_{23} = 1/S_{44}, G_{31} = 1/S_{55},
\]

\[
v_{12} = -S_{12}/S_{11}, v_{23} = -S_{23}/S_{22}, v_{31} = -S_{31}/S_{33}.
\]

(11)

4. Results

4.1. TWF composite mechanical properties

In order to determine the effective modulus, stress analysis is performed by applying 6 independent loads to the unit cells, from which volume average stress and strain are obtained. Periodic boundary conditions, written in Python, run in ABAQUS.

Stiffness matrix \(C_{ij}\):

\[
\begin{bmatrix}
9047.90 & 2806.15 & 9.08598 & 0 & 0 & 0 \\
2806.15 & 8398.37 & 9.43017 & 0 & 0 & 0 \\
9.08598 & 9.43017 & 10.0653 & 0 & 0 & 0 \\
0 & 0 & 0 & 2796.62 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.72782 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.80043
\end{bmatrix}
\]

The final engineering constants are shown in the following table:

| \(E_{11}/\text{MPa} \) | \(E_{22}/\text{MPa} \) | \(E_{33}/\text{MPa} \) | \(G_{12}/\text{MPa} \) | \(G_{23}/\text{MPa} \) | \(G_{31}/\text{MPa} \) | \(v_{12} \) | \(v_{23} \) | \(v_{31} \) |
|---|---|---|---|---|---|---|---|---|
| 8106.78 | 7523.72 | 10.05 | 2796.62 | 0.8 | 0.73 | 0.33 | 0.35 | 0.39 |

4.2. Effect of fiber volume fraction on elastic properties of materials

\(V_f\) =40%, 42%, and 45% were selected to discuss the influence of the change of fiber volume fraction on the elastic properties of the material. The graph shows the variation of the elastic modulus with the volume fraction of the fiber. It can be seen that as \(V_f\) increases, \(E_{11}, E_{22}\) and \(G_{12}\) both increase.
5. Conclusion
In this paper, the elastic properties of flexible matrix TWF composites are analysed by meso-method. Fine-grained modelling is performed by selecting the appropriate volume repeating unit-unit cell, and the material properties of the fiber bundle are obtained by mixing rules. Next, the finite element analysis is carried out, periodic boundary conditions are applied to the unit cell, and the volume average stress and strain are obtained by six independent loading analysis by using the homogenization method to obtain the stiffness matrix, and finally the engineering constant is obtained. The effect of fiber volume fraction on the elastic properties is analysed. It can be seen that the fiber volume fraction has a great influence on the elastic properties of the material, directly affecting and controlling the macroscopic mechanical properties of the material.

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