STABILITY FOR A CUBIC FUNCTIONAL EQUATIONS
IN NON-ARCHIMEDEAN NORMED SPACES

CHANG IL KIM* AND CHANG HYEOB SHIN**

Abstract. In this paper, we investigate the functional equation
\[ f(3x + y) + f(3x - y) = f(x + 2y) + 2f(x - y) + 6f(2x) + 3f(x) - 6f(y) \]
and prove the generalized Hyers-Ulam stability for it in non-Archimedean normed spaces.

1. Introduction and preliminaries

S. M. Ulam [15] raised a question concerning the stability of functional equations in 1940: Let \( G_1 \) be a group and let \( G_2 \) be a metric group with the metric \( d(\cdot, \cdot) \). Given \( \epsilon > 0 \), does there exist a \( \delta > 0 \) such that if a mapping \( h : G_1 \rightarrow G_2 \) satisfies the inequality \( d(h(xy), h(x)h(y)) < \delta \) for all \( x, y \in G_1 \), then there exists a homomorphism \( H : G_1 \rightarrow G_2 \) with \( d(h(x), H(x)) < \epsilon \) for all \( x \in G_1 \)?

In 1941, Hyers [6] solved the Ulam problem for the case of approximately additive functions in Banach spaces. Since then, the stability of several functional equations have been extensively investigated by several mathematicians [2, 5, 7, 8]. Rassias [13], Jun and Kim [9] and Park and Jung [12] introduced the following functional equations

\begin{align*}
(1.1) \quad & f(x + 2y) + 3f(x) = 3f(x + y) + f(x - y) + 6f(y) \\
(1.2) \quad & f(2x + y) + f(2x - y) = 2f(x + y) + 2f(x - y) + 12f(x) \\
(1.3) \quad & f(3x + y) + f(3x - y) = 3f(x + y) + 3f(x - y) + 48f(x)
\end{align*}

Received November 13, 2014; Accepted July 22, 2015.

2010 Mathematics Subject Classification: Primary 39B82, 39B52.

Key words and phrases: generalized Hyers-Ulam stability, cubic functional equation, non-Archimedean space.

Correspondence should be addressed to Chang Hyeob Shin, seashin@hanmail.net.
and they established general solutions and the generalized Hyers-Ulam-Rassias stability problem for this functional equations, respectively. It is easy to see that the function \( f(x) = cx^3 \) is a solution of the functional equations (1.1), (1.2) and (1.3). Thus, it is natural that (1.1), (1.2) and (1.3) are called a cubic functional equations and every solution of the cubic functional equation is said to be a cubic mapping.

In this paper, we consider the following functional equation

\[
  f(3x + y) + f(3x - y)
  = f(x + 2y) + 2f(x - y) + 6f(2x) + 3f(x) - 6f(y).
\]

(1.4)

We prove the generalized Hyers-Ulam stability of (1.4) in complete non-Archimedean normed spaces.

A valuation is a function \( | \cdot | \) from a field \( K \) into \([0, \infty)\) such that, for any \( r, s \in K \), the following conditions hold: (i) \( |r| = 0 \) if and only if \( r = 0 \), (ii) \(|rs| = |r||s|\), (iii) \(|r + s| \leq |r| + |s|\).

A field \( K \) is called a valued field if \( K \) carries a valuation. The usual absolute values of \( \mathbb{R} \) and \( \mathbb{C} \) are examples of valuations. If the triangle inequality is replaced by \(|r + s| \leq \max\{|r|, |s|\}\) for all \( r, s \in K \), then the valuation \(| \cdot |\) is called a non-Archimedean valuation and a field with a non-Archimedean valuation is called non-Archimedean field. If \(| \cdot |\) is a non-Archimedean valuation on \( K \), then clearly, \(|1| = |−1|\) and \(|n| \leq 1\) for all \( n \in \mathbb{N} \).

**Definition 1.1.** Let \( X \) be a vector space over a scalar field \( K \) with a non-Archimedean nontrivial valuation \(| \cdot |\). A function \(| \cdot | : X \rightarrow \mathbb{R}\) is called a non-Archimedean norm (valuation) if it satisfies the following conditions:

(a) \(|x| = 0\) if and only if \( x = 0 \),
(b) \(|rx| = |r||x|\),
(c) the strong triangle inequality (ultrametric), that is,

\[
|x + y| \leq \max\{|x|, |y|\}
\]

for all \( x, y \in X \) and all \( r \in K \).

If \(| \cdot |\) is a non-Archimedean norm, then \((X, | \cdot |)\) is called a non-Archimedean normed space.

Let \((X, | \cdot |)\) be a non-Archimedean normed space. Let \( \{x_n\} \) be a sequence in \( X \). Then \( \{x_n\} \) is said to be convergent if there exists an \( x \in X \) such that \( \lim_{n \to \infty} |x_n - x| = 0 \). In that case, \( x \) is called the limit of the sequence \( \{x_n\} \) and one denotes it by \( \lim_{n \to \infty} x_n = x \). A sequence \( \{x_n\} \) is said to be Cauchy in \((X, | \cdot |)\) if \( \lim_{n \to \infty} \|x_{n+p} - x_n\| = 0 \) for all
p ∈ \mathbb{N}. By (c) in Definition 1.1,
\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| \mid m \leq j \leq n - 1\} \quad (n > m)
and hence a sequence \{x_n\} is Cauchy in \((X, \| \cdot \|)\) if and only if sequence \{x_{n+1} - x_n\} converges to zero in \((X, \| \cdot \|)\). By a complete non-Archimedean normed space we mean one in which every Cauchy sequence is convergent.

Throughout this paper, \(X\) is a non-Archimedean normed space and \(Y\) a complete non-Archimedean normed space.

2. The Generalized Hyers-Ulam stability for \((1.4)\)

In 2003, Jun and Kim [10] introduced the following cubic functional equation
\(f(x + 2y) + f(x - 2y) + 6f(x) = 4f(x + y) + 4f(x - y)\)
and proved the generalized Hyers-Ulam stability for it in Banach spaces. In this section, we prove the generalized Hyers-Ulam stability of functional equation \((1.4)\) in complete non-Archimedean normed spaces. We start the following theorem.

**Theorem 2.1.** Let \(f : X \rightarrow Y\) be a mapping. Then \(f\) satisfies \((1.4)\) if and only if \(f\) is cubic.

**Proof.** Suppose that \(f\) satisfies \((1.4)\). Letting \(x = y = 0\) in \((1.4)\), we have \(f(0) = 0\). Letting \(y = 0\) in \((1.4)\), we have
\(f(3x) - 3f(2x) - 3f(x) = 0\)
for all \(x \in X\) and letting \(x = 0\) in \((1.4)\) and rel pacing \(y\) by \(x\), we have
\(7f(x) - f(-x) - f(2x) = 0\)
for all \(x \in X\). Letting \(y = x\) in \((1.4)\), we have
\(f(4x) - f(3x) - 5f(2x) + 3f(x) = 0\)
for all \(x \in X\). By \((2.2)\) and \((2.4)\), we get
\(f(4x) = 2^3f(2x)\)
for all \(x \in X\). Re palcing \(2x\) by \(x\) in \((2.5)\), we get
\(f(2x) = 2^3f(x)\)
for all \(x \in X\). By \((2.2)\) and \((2.6)\), we get
\(f(3x) = 3^3f(x)\)
for all \( x \in X \). By (2.3) and (2.6), we get
\[
(2.8) \quad f(-x) = -f(x)
\]
for all \( x \in X \). Replacing \( y \) by \( 3y \) in (1.4), by (2.6) and (2.7), we have
\[
(2.9) \quad 27f(x+y)+27f(x-y) = f(x+6y)+2f(x-3y)+51f(x)-6f(3y)
\]
for all \( x, y \in X \). Interchanging \( x \) and \( y \) in (2.9), by (2.8), we have
\[
(2.10) \quad 27f(x+y)-27f(x-y) = f(6x+y)-2f(3x-y)+51f(x)-6f(3x)
\]
for all \( x, y \in X \). Replacing \( y \) by \( 2y \) in (2.10), by (2.6), we have
\[
(2.11) \quad 27f(x+2y)-27f(x-2y) = f(3x+y)-2f(3x-2y)+408f(y)-6f(3x)
\]
for all \( x, y \in X \). Letting \( y = -y \) in (2.11), by (2.8), we have
\[
(2.12) \quad 27f(x-2y)-27f(x+2y) = f(3x-y)-2f(3x+2y)-408f(y)-6f(3x)
\]
for all \( x, y \in X \). By (2.11) and (2.12), we get
\[
(2.13) \quad 8[f(x+y)+f(x-y)]-2[f(3x+2y)+f(3x-2y)]-12f(3x) = 0
\]
for all \( x, y \in X \). Letting \( x = \frac{x}{3} \) in (2.13), we have
\[
f(x+2y)+f(x-2y)+6f(x) = 4f(x+y)+4f(x-y)
\]
for all \( x, y \in X \) and so \( f \) is additive-quadratic-cubic [10]. By (2.6), \( f \) is cubic. The converse is trivial.

For a given mapping \( f : X \to Y \), we define the difference operator
\[
Df : X^2 \to Y
\]
by
\[
Df(x,y) = f(3x+y)+f(3x-y) - f(x+2y) - 2f(x-y) - 6f(2x) - 3f(x) + 6f(y)
\]
for all \( x, y \in X \).

**Theorem 2.2.** Let \( \phi : X^2 \to [0, \infty) \) be a mapping such that
\[
(2.14) \quad \lim_{n \to \infty} \frac{\phi(2^n x, 2^n y)}{2^{jn}} = 0
\]
for all \( x, y \in X \) and let for each \( x \in X \), the following limit
\[
(2.15) \quad \lim_{n \to \infty} \max \left\{ \{ \frac{\phi(2^j x, 0)}{2^j} : 0 \leq j < n \} \cup \left\{ \phi(2^j x, 2^j x) \bigg| 2^j x \right\} : 0 \leq j < n \right\}
\]
denoted by $\tilde{\phi}(x)$, exist. Suppose that $f : X \rightarrow Y$ is a mapping satisfying
\begin{equation}
\|Df(x,y)\| \leq \phi(x,y)
\end{equation}
for all $x, y \in X$. Then there exists a cubic mapping $C : X \rightarrow Y$ such that
\begin{equation}
\|C(x) - f(x)\| \leq \frac{1}{2^n} \tilde{\phi}(x)
\end{equation}
for all $x \in X$. In addition, if the limit
\begin{equation}
\lim_{i \to \infty} \lim_{n \to \infty} \max \left\{ \frac{1}{2^n} \phi(2^{j-1}x, 0) : i \leq j < n + i \right\} \cup \left\{ \frac{\phi(2^{j-1}x, 2^{j-1}x)}{2^{3(j-1)}} : i \leq j < n + i \right\} = 0
\end{equation}
exists for all $x \in X$, then $C$ is the unique cubic mapping satisfying (2.17).

Proof. Putting $x = y = 0$ in (2.16), we have
\begin{equation}
\|f(0)\| \leq \frac{1}{2^n} \phi(0,0)
\end{equation}
and since $1 \leq \frac{1}{2^n}$, we get
\begin{equation}
\|f(0)\| \leq \frac{1}{2 \cdot 2^n} \phi(0,0) \leq \frac{1}{2 \cdot 2^n} \phi(0,0)
\end{equation}
for all $n \in \mathbb{N}$. By (2.14), $f(0) = 0$.

Putting $y = 0$ in (2.16), we have
\begin{equation}
\|f(3x) - 3f(2x) - 3f(x)\| \leq \frac{1}{2^n} \phi(x,0)
\end{equation}
for all $x \in X$. Putting $y = x$ in (2.16), we have
\begin{equation}
\|f(4x) - f(3x) - 5f(2x) + 3f(x)\| \leq \phi(x,x)
\end{equation}
for all $x \in X$. By (2.19) and (2.20), we get
\begin{equation}
\|f(4x) - 8f(2x)\| \leq \max \left\{ \frac{1}{2^n} \phi(x,0), \phi(x,x) \right\}
\end{equation}
for all $x \in X$. Replacing $x$ by $2^{n-1}x$ and dividing by $2^{3(n+1)}$ in (2.21), we get
\[ \left\| \frac{f(2^{n+1}x) - f(2^nx)}{2^{3(n+1)}} \right\| \leq \frac{1}{|2|^6} \max \left\{ \frac{1}{|2|^{3(n-1)}} \phi(2^{n-1}x, 0), \frac{\phi(2^{n-1}x, 2^{n-1}x)}{|2|^{3(n-1)}} \right\} \]

for all \( x \in X \). By (2.14) and (2.22), we get \( \left\{ \frac{f(2^nx)}{2^m} \right\} \) is Cauchy sequence. Since \( Y \) is complete, we conclude that \( \left\{ \frac{f(2^nx)}{2^m} \right\} \) is convergent.

Set \( C(x) := \lim_{n \to \infty} \frac{f(2^nx)}{2^m} \).

Using induction one can show that

\[ \left\| \frac{f(2^nx) - f(x)}{2^{3n}} \right\| \leq \frac{1}{|2|^6} \max \left\{ \frac{1}{|2|} \phi(2^{j-1}x, 0), \frac{\phi(2^{j-1}x, 2^{j-1}x)}{|2|^{3(j-1)}} : 0 \leq j < n \right\} \]

for all \( n \in \mathbb{N} \) and all \( x \in X \). By taking \( n \) to infinity in (2.23) and by (2.15), we obtain (2.17). Replacing \( x \) and \( y \) by \( 2^nx \) and \( 2^ny \), respectively, and dividing by \( |2|^3n \) in (2.16) and taking the limit as \( n \to \infty \), by (2.14), we get

\[ C(3x+y) + C(3x-y) = C(x+2y) + 2C(x-y) + 6C(2x) + 3C(x) - 6C(y) \]

for all \( x, y \in X \). Therefore the mapping \( C : X \to Y \) satisfies (1.4) and so by Theorem 2.1, \( C \) is cubic.

Suppose that (2.18) holds. If \( C' \) is another cubic mapping satisfying (2.17), then by (2.18),

\[ \left\| C(x) - C'(x) \right\| = \lim_{i \to \infty} \frac{1}{|2|^{3i}} \left\| C(2^ix) - C'(2^ix) \right\| \]

\[ \leq \lim_{i \to \infty} \frac{1}{|2|^{3i}} \max \left\{ \left\| C(2^ix) - f(2^ix) \right\|, \left\| f(2^ix) - C'(2^ix) \right\| \right\} \]

\[ \leq \frac{1}{|2|^6} \lim_{i \to \infty} \lim_{n \to \infty} \max \left\{ \frac{1}{|2|^{3i}} \phi(2^{j-1}x, 0), \frac{\phi(2^{j-1}x, 2^{j-1}x)}{|2|^{3(j-1)}} : |2|^{3(j-1)} : 0 \leq j < n \right\} \]

for all \( x \in X \) and so \( C = C' \). \( \square \)
Stability for a cubic functional equations in non-Archimedean normed spaces

From Theorem 2.2, we obtain the following corollary concerning the stability of (1.4).

**Corollary 2.3.** Let $\alpha_i : [0, \infty) \to [0, \infty)$ ($i = 1, 2, 3$) be mappings satisfying

(i) $\alpha_i([2]) \neq 0$,

(ii) $\alpha_i([t]) \leq \alpha_i([2])\alpha_i(t)$ for all $t \geq 0$, and

(iii) $\alpha_1([2]) < |2|^{\frac{3}{2}}, \alpha_2([2]) < |2|^3$, and $\alpha_3([2]) < |2|^3$.

Let $f : X \to Y$ be a mapping such that

$$\|Df(x, y)\| \leq \delta [\alpha_1(\|x\|)\alpha_1(\|y\|) + \alpha_2(\|x\|) + \alpha_3(\|y\|)]$$

for all $x, y \in X$ and some $\delta > 0$. Suppose that $|2| < 1$. Then there exists a unique cubic mapping $C : X \to Y$ such that

$$\|C(x) - f(x)\| \leq \frac{1}{|2|^6} \tilde{\phi}(x)$$

for all $x \in X$, where

$$\tilde{\phi}(x) = \delta |2|^2 \max \left\{ \frac{\alpha_2(\|x\|)}{\alpha_2([2])}, |2| \left[ \frac{\alpha_1(\|x\|)}{\alpha_1([2])} \right]^2 + \frac{\alpha_2(\|x\|)}{\alpha_2([2])} + \frac{\alpha_3(\|x\|)}{\alpha_3([2])} \right\}.$$

**Proof.** Let $\phi(x, y) = \delta [\alpha_1(\|x\|)\alpha_1(\|y\|) + \alpha_2(\|x\|) + \alpha_3(\|y\|)]$. Then for any $n \in \mathbb{N}$

$$\phi(2^n x, 2^n y) = \delta \frac{|2|^{3n}}{|2|^{3n}} \left[ \alpha_1([2^n]\|x\|)\alpha_1([2^n]\|y\|) + \alpha_2([2^n]\|x\|) + \alpha_3([2^n]\|y\|) \right]$$

$$\leq \delta \left[ \left( \frac{\alpha_1([2])}{|2|^3} \right)^n \alpha_1(\|x\|)\alpha_1(\|y\|) + \left( \frac{\alpha_2([2])}{|2|^3} \right)^n \alpha_2(\|x\|) + \left( \frac{\alpha_3([2])}{|2|^3} \right)^n \alpha_3(\|y\|) \right]$$

for all $x, y \in X$. By (iii), we have

$$\lim_{n \to \infty} \phi(2^n x, 2^n y) = 0$$

for all $x, y \in X$. Hence $\phi$ satisfies (2.14) in Theorem 2.2.

Let $x \in X$ and $j \in \mathbb{N} \cup \{0\}$. Then

$$\frac{1}{|2|} \phi(2^{j-1} x, 0) \leq \delta \frac{|2|}{|2|} \left( \frac{\alpha_2([2])}{|2|^3} \right)^{j-1} \alpha_2(\|x\|)$$

and
\[ \frac{\phi(2^j x, 2^{j-1} x)}{2^{3(j-1)}} \leq \delta \left[ \left( \frac{(\alpha_1|2|)^2}{|2|^3} \right)^{j-1} (\alpha_1|\|x\||)^2 \right. \\
+ \left( \frac{\alpha_3|2|}{|2|^3} \right)^{j-1} \alpha_2|\|x\|| + \left( \frac{\alpha_3|2|}{|2|^3} \right)^{j-1} \alpha_3|\|x\|| \right] \]

for all \( x \in X \). By (iii), we obtain

\[
\lim_{i \to \infty} \lim_{n \to \infty} \max \left\{ \frac{1}{|2|} \phi(2^j x, 0) : i \leq j < n + i \right\} \cup \\
\left\{ \frac{\phi(2^j x, 2^{j-1} x)}{2^{3(j-1)}} : i \leq j < n + i \right\} = 0
\]

for all \( x \in X \) and so \( \phi \) satisfies (2.18) in Theorem 2.2. Hence by Theorem 2.2, we have the result. \( \square \)

**Example 2.4.** Let \( \delta > 0 \) and \( p \) be a real number with \( p > \frac{3}{2} \). Suppose that \( |2| < 1 \). Let \( f : X \to Y \) be a mapping satisfying

\[ \|Df(x, y)\| \leq \delta(\|x\|^p||y||^p + \|x\|^{2p} + \|y\|^{2p}) \]

for all \( x, y \in X \). Then there exists a unique cubic mapping \( C : X \to Y \) satisfying (1.4) such that

\[ \|C(x) - f(x)\| \leq \delta |2|^{-2(p+2)} \max\{1, 3|2|\} \|x\|^{2p} \]

for all \( x \in X \).

We have the following result which is analogous Theorem 2.2 for the functional equation (1.4).

**Theorem 2.5.** Let \( \phi : X^2 \to [0, \infty) \) be a mapping such that

\[ \lim_{n \to \infty} 2^{3n} \phi\left( \frac{x}{2^n}, \frac{y}{2^n} \right) = 0 \]

for all \( x, y \in X \) and for each \( x \in X \), and let for each \( x \in X \), the following limit

\[ \lim_{n \to \infty} \max \left\{ \left\{ \frac{2^{3(j+2)}}{|2|} \phi\left( \frac{x}{2^{j+2}}, \frac{x}{2^{j+2}} \right) : 0 \leq j < n \right\} \cup \\
\left\{ \frac{2^{3(j+2)}}{|2|} \phi\left( \frac{x}{2^{j+2}}, \frac{x}{2^{j+2}} \right) : 0 \leq j < n \right\} \right\} \]

denoted by \( \phi_1(x) \), exist. Suppose that \( f : X \to Y \) is a mapping satisfying \( f(0) = 0 \) and

\[ \|Df(x, y)\| \leq \phi(x, y) \]
for all \(x, y \in X\). Then there exists a cubic mapping \(C : X \rightarrow Y\) satisfying (1.4) such that
\[
\|C(x) - f(x)\| \leq \frac{1}{|2|^{6}} \phi_{1}(x)
\]
for all \(x \in X\). In addition, if the limit
\[
\lim_{i \to \infty} \lim_{n \to \infty} \max \left\{ \frac{|2|^{3(j+2)}}{|2|} \phi \left( \frac{x}{2^{j+2}} \right) : i \leq j < n + i \right\} \cup \left\{ \frac{|2|^{3(j+2)}}{|2|} \phi \left( \frac{x}{2^{j+2}}, \frac{x}{2^{j+2}} \right) : i \leq j < n + i \right\} = 0,
\]
then \(C\) is the unique cubic mapping satisfying (2.24).

The following corollary is an immediate consequence of Theorem 2.5.

**Corollary 2.6.** Let \(\alpha_{i} : [0, \infty) \rightarrow [0, \infty)\) \((i = 1, 2, 3)\) be mappings satisfying
(i) \(\alpha_{i}(\frac{1}{|2|}) \neq 0\),
(ii) \(\alpha_{i}(\frac{t}{|2|}) \leq \alpha_{i}(\frac{1}{|2|}) \alpha_{i}(t)\) for all \(t \geq 0\), and
(iii) \(\alpha_{1}(\frac{1}{|2|}) < \frac{1}{|2|^{rac{p+1}{2}}}\), \(\alpha_{2}(\frac{1}{|2|}) < \frac{1}{|2|^{rac{p+1}{2}}}\), and \(\alpha_{3}(\frac{1}{|2|}) < \frac{1}{|2|^{rac{p+1}{2}}}\).

Let \(f : X \rightarrow Y\) be a mapping such that \(f(0) = 0\) and
\[
\|Df(x, y)\| \leq \delta [\alpha_{1}(\|x\|) \alpha_{1}(\|y\|) + \alpha_{2}(\|x\|) + \alpha_{3}(\|y\|)]
\]
for all \(x, y \in X\) and some \(\delta > 0\). Then there exists a unique cubic mapping \(C : X \rightarrow Y\) such that
\[
\|C(x) - f(x)\| \leq \frac{1}{|2|^{6}} \phi_{1}(x)
\]
for all \(x \in X\), where
\[
\phi_{1}(x) = \delta |2|^{6} \max \left\{ \frac{1}{|2|} \left( \alpha_{2}(\frac{1}{|2|}) \right)^{2} \alpha_{2}(\|x\|), \left( \alpha_{1}(\frac{1}{|2|}) \right)^{4} \alpha_{1}(\|x\|)^{2} + \left( \alpha_{2}(\frac{1}{|2|}) \right)^{2} \alpha_{2}(\|x\|) + \left( \alpha_{3}(\frac{1}{|2|}) \right)^{2} \alpha_{3}(\|x\|) \right\}.
\]

**Example 2.7.** Let \(\delta > 0\) and \(p\) be a real number with \(p < \frac{3}{2}\). Suppose that \(|2| < 1\). Let \(f : X \rightarrow Y\) is a mapping satisfying \(f(0) = 0\) and
\[
\|Df(x, y)\| \leq \delta (\|x\|^{p} \|y\|^{p} + \|x\|^{2p} + \|y\|^{2p})
\]
for all $x, y \in X$. Then there exists a unique cubic mapping $C : X \rightarrow Y$ satisfying (1.4) such that

$$
\|C(x) - f(x)\| \leq \delta|2|^{-(4p+1)}\max\left\{1, 3|2|\right\}\|x\|^{2p}
$$

for all $x \in X$.

References

[1] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan 2 (1950), 64-66.
[2] S. Czerwik, *Functional equations and Inequalities in several variables*, World Scientific, New Jersey, London, 2002.
[3] M. E. Gordji and M. B. Savadkouhi *Stability of cubic and quartic functional equations in non-Archimedean spaces*, Acta Appl. Math. (2010), 1321-1329.
[4] M. E. Gordji and M. B. Savadkouhi *Stability of a mixed type cubic-quartic functional equation in non-Archimedean spaces*, Applied Mathematics Letter 23 (2010), 1198-1202.
[5] G. L. Forti, *Hyers-Ulam stability of functional equations in several variables*, Aequationes Math. 50 (1995), 143-190.
[6] D. H. Hyers, *On the stability of linear functional equation*, Proc. Natl. Acad. Sci. USA 27 (1941), 222-224.
[7] D. H. Hyers, G. Isac, and T. M. Rassias, *Stability of functional equations in several variables*, Birkhäuser, Boston, 1998.
[8] D. H. Hyers and T. M. Rassias, *Approximate homomorphisms*, Aequationes Math. 44 (1992), 125-153.
[9] K.-W. Jun and H.-M. Kim *The generalized Hyers-Ulam-Rassias stability of a cubic functional equation*, J. Math. Anal. Appl. 274 (2002), 867-878.
[10] K.-W. Jun and H.-M. Kim *On the Hyers-Ulam-Rassias stability of a general cubic functional equation*, Math. Inequal. Appl. 6 (2003), 289-302.
[11] M. Sal Moslehian and G. Sadeghi, *Stability of two types of cubic functional equations in non-Archimedean spaces*, Real Analysis Exchange 33 (2007), no. 2, 375-384.
[12] K. H. Park and Y. S. Jung, *Stability of a cubic functional equation on groups*, Bull. Korean Math. Soc. 41 (2004), 347-357.
[13] J. M. Rassias, *Solution of the Ulam stability problem for cubic mappings*, Glasnik Matematički 36 (2001), 63-72.
[14] F. Skof, *Approssimazione di funzioni δ-quadratic su dominio restretto*, Atti. Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. 118 (1984), 58-70.
[15] S. M. Ulam, *A collection of mathematical problems*, Interscience Publ., New York, 1960.
Stability for a cubic functional equations in non-Archimedean normed spaces.

* Department of Mathematics Education
Dankook University
Yongin 448-701, Republic of Korea
E-mail: kci206@hanmail.net

** Department of Mathematics
Soongsil University
Seoul 156-743, Republic of Korea
E-mail: seashin@hanmail.net