CP VIOLATION IN ANTINEUTRINO-ELECTRON ELASTIC SCATTERING

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In this paper we show that the elastic scattering of transversely polarized electron antineutrino beam off unpolarized electrons can be used to detect the CP-violating effects by measuring the azimuthal asymmetry of recoil electrons caused by the interference terms between the standard vector $c_L^V$, axial $c_A^A$ couplings of left-chirality antineutrinos and exotic scalar $c_R^S$ coupling of right-chirality ones in the differential cross section. It would be a positive evidence for the existence of the exotic antineutrino states. Moreover, we also show that the differential cross section for the $\overline{\nu}e^-$ scattering can be obtained from the one for the $\nu e$ scattering, if one makes the substitution $c_T^R \rightarrow -c_T^R$, $c_A^L \rightarrow -c_A^L$, $q \rightarrow -q$, $\hat{\eta}_\nu \rightarrow -\hat{\eta}_\nu$. Electron antineutrinos are assumed to be massive and to be polarized Dirac fermions coming from the polarized muon decay at rest. The results are presented in a limit of infinitesimally small antineutrino mass.

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1. Introduction

Neutrino-electron elastic scattering is a suitable process to test the CP violation, Lorentz structure, chiral structure and most of all possibility of
participation of the right-chirality neutrinos in the purely leptonic charged and neutral weak interactions. In addition, $\nu e$ scattering can also be used to probe if existing neutrinos are Dirac or Majorana fermions.

As is well known, the CP violation is observed only in the decays of neutral kaons and B-mesons [1] and is described by a single phase of the CKM matrix [2]. However, the baryon asymmetry of the universe can not be explained by the standard CKM phase only, and new sources of breaking CP symmetry is needed [3]. It is worth to notice that there is no proof of the CP violation in the leptonic processes, i.e. in (anti)neutrino-electron scattering or muon decay.

F. Wilczek et al. [4] considered the $\nu e$ scattering and searched for the effects of anomalous (nonstandard) Lorentz structure in the weak interactions. They admitted the most general local (derivative free) Lagrangian including the five types of Lorentz covariants; scalar, pseudoscalar, tensor, vector and axial-vector. In their case, the incoming neutrinos was always left-chirality and longitudinally polarized. In consequence, no interference terms between the standard and nonstandard couplings was present in the differential cross section. They also showed that the differential cross section for the $\overline{\nu} e^-$ scattering can be obtained from the one for the $\nu e$ scattering, if one simply substitutes $c_T \rightarrow -c_T$, $c_A \rightarrow -c_A$.

Cheng and Tung [5] proposed the measurement of polarization of the outgoing lepton in the framework of general local current-current interaction assuming only left-chirality incoming neutrinos. They pointed out that such measurement would allow to distinguish the V-A interactions from the other S, T, P admixtures. Since the direct tests would be extremely difficult at very high energies, they indicated the angular and spin correlation experiments for testing the Lorentz structure. This proposal involved the measurement of the outgoing lepton angular distribution in either the differential cross section or in certain asymmetry functions.

T.C. Yang [6] probed the $\nu e$ processes in which the right-chirality neutrinos, produced in the exotic S, T, P weak interactions, can take part and calculated the angular distribution of the scattered electrons for an unpolarized and a polarized electron target. He showed that if the right-chirality neutrinos are present in the incoming neutrino beam and $\nu_R e \rightarrow \nu_L e$ scattering occurs, their effect should be seen in near backward directions of the electron in the c. m. system.

In this paper, we study the elastic scattering of transversely polarized electron antineutrino beam on the unpolarized electron target and predict the new effects beyond the standard model of electroweak interactions [7]. The main goal is to show that the CP violation in the scattering of a mixture of the standard left-chirality electron antineutrinos and exotic right-chirality ones on the unpolarized electron target can be observed due to the
azimuthal asymmetry of recoil electrons. In addition, we show how to get the differential cross section for the $\nu_e$ scattering from the corresponding one for the case with incoming neutrinos.

We use the system of natural units with $\hbar = c = 1$, Dirac-Pauli representation of the $\gamma$-matrices and the $(+,−,−,−)$ metric [8].

2. Basic assumptions

We consider the $\nu_e$ scattering, when the incoming electron antineutrino beam is a mixture the left-chirality electron antineutrinos produced in the standard vector charged weak interaction and the right-chirality ones produced in the exotic scalar charged weak interaction. This beam comes from the polarized muon decay at rest and has a nonzero value of the transverse antineutrino spin polarization $\eta^\perp_\mu$. A direction of this transversal polarization is assigned with respect to the production plane spanned by the direction of initial muon polarization $\hat{\eta}_{\mu}$ and of the outgoing antineutrino momentum $\hat{q}$. The reaction plane is spanned by the direction of antineutrino momentum $\hat{q}$ and of the outgoing electron momentum $\hat{p}_e$, see Fig.1. As is known, the polarization vector $\hat{\eta}_{\mu}$ can be expressed, with respect to the $\hat{q}$, as a sum of the longitudinal component of the muon polarization $(\hat{\eta}_{\mu} \cdot \hat{q})\hat{q}$ and transverse component of the muon polarization $\eta^\perp_{\mu}$, that is defined as $\eta^\perp_{\mu} = \hat{\eta}_{\mu} - (\hat{\eta}_{\mu} \cdot \hat{q})\hat{q}$.

We assume that the left-chirality antineutrinos are detected in the standard $V−A$ charged interaction with the unpolarized electrons, while the
right-chirality ones are detected in the exotic scalar one. In the limit of vanishing antineutrino mass, the left-chirality antineutrino has a positive helicity, while the right-chirality one has a negative helicity. We want to show how the differential cross section for the $\bar{\nu}e^-$ scattering depends on the CP-violating relative phase between the standard vector and exotic scalar couplings. We also assume that a detector is able to measure both the recoil electron scattering angle and the azimuthal angle of outgoing electron momentum with a high angular resolution. Because we allow for the nonconservation of the combined symmetry CP, the transition amplitude includes the complex coupling constants denoted as $c_L^V$, $c_L^A$ and $c_R^S$ respectively to the initial electron antineutrino of left- and right-chirality:

$$M_{\bar{\nu}e} = \frac{G_F}{\sqrt{2}} \{(\bar{\nu}_{e^c}^\alpha \gamma^\alpha (c_L^V - c_L^A\gamma_5) u_e)(\bar{\nu}_{\nu_e} \gamma_\alpha (1 - \gamma_5)\nu_{\nu^c})$$

$$+ \frac{1}{2} c_R^S (\bar{\nu}_{e^c} u_e)(\bar{\nu}_{\nu_e} (1 - \gamma_5)\nu_{\nu^c}),$$

(1)

where $u_e$ and $\bar{\nu}_{e^c}$ ($\bar{\nu}_{\nu_e}$ and $\nu_{\nu^c}$) are the Dirac bispinors of the initial and final electron (electron antineutrino) respectively. $G_F = 1.16639(1) \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant. $c_L^V = -0.040 + 1$, $c_L^A = -0.507 + 1$.

An admittance of tensor and pseudoscalar couplings of the right-chirality antineutrinos does not change qualitatively the conclusions from the studies. In addition, if the incoming antineutrino beam consists only of the left-chirality antineutrinos produced in the standard and exotic weak interactions, there is no interference between the $c_L^{V,A}$ and $c_R^S$ couplings in the differential cross section, when $m_\nu \rightarrow 0$. We do not consider this scenario.

3. Azimuthal asymmetry of recoil electrons

The laboratory differential cross section for the $\bar{\nu}e^-$ scattering, in the limit of vanishing antineutrino mass, has the form:

$$\frac{d^2\sigma}{dy_e d\phi_e} = \left(\frac{d^2\sigma}{dy_e d\phi_e}\right)_{(V,A)} + \left(\frac{d^2\sigma}{dy_e d\phi_e}\right)_{(S)} + \left(\frac{d^2\sigma}{dy_e d\phi_e}\right)_{(V,S)},$$

(2)

$$\left(\frac{d^2\sigma}{dy_e d\phi_e}\right)_{(V,A)} = \frac{E_\nu m_e G_F^2}{4\pi^2} \left\{ (1 + \hat{\nu}_e \cdot \hat{q}) \left[ (c_L^V - c_L^A)^2 + (c_L^V + c_L^A)^2 (1 - y_e)^2 - \frac{m_e y_e}{E_\nu} ((c_L^V)^2 - (c_L^A)^2) \right] \right\},$$

(3)

$$\left(\frac{d^2\sigma}{dy_e d\phi_e}\right)_{(S)} = \frac{E_\nu m_e G_F^2}{4\pi^2} (1 - \hat{\nu}_e \cdot \hat{q}) \left\{ \frac{1}{8} y_e \left( y_e + \frac{m_e}{E_\nu} \right) |c_R^S|^2 \right\},$$

(4)
\[
\left( \frac{d^2 \sigma}{dy_e d\phi_e} \right)_{(V S)} = \frac{E_{\nu} m_e G_F^2}{4\pi^2} \left\{ \sqrt{y_e(y_e + 2 \frac{m_e}{E_{\nu}})} \left[ \eta_{\vec{T}} \cdot \hat{\mathbf{p}}_e \right] \text{Re}(c_Y^L c_{S}^{R*}) + \eta_{\vec{T}} \cdot (\hat{\mathbf{p}}_e \times \hat{\mathbf{q}}) \text{Im}(c_Y^L c_{S}^{R*}) \right\},
\]

is the ratio of the kinetic energy of the recoil electron \( T_e \) to the incoming (anti)neutrino energy \( E_{\nu} \); \( \theta_e \) is the angle between the direction of the outgoing electron momentum \( \hat{\mathbf{p}}_e \) and the direction of the incoming (anti)neutrino momentum \( \hat{\mathbf{q}} \) (recoil electron scattering angle); \( m_e \) is the electron mass; \( \phi_e \) is the angle between the production plane and the reaction plane spanned by \( \hat{\mathbf{p}}_e \) and \( \hat{\mathbf{q}} \).

We see that the term with interference between the standard \( c_Y^L \) and exotic \( c_Y^S \) couplings does not depend on the antineutrino mass, so does not vanish in the limit of vanishing antineutrino mass. It is proportional to the transverse components of the initial antineutrino spin polarization, both \( CP \)-even and \( CP \)-odd. This interference can be rewritten as follows:

\[
\frac{y_e}{E_{\nu}} = \frac{T_e}{E_{\nu}} = \frac{m_e}{E_{\nu}} \frac{2\cos^2\theta_e}{(1 + \frac{m_e}{E_{\nu}})^2 - \cos^2\theta_e},
\]

where \( \alpha_{SV} \equiv \alpha_Y^S - \alpha_Y^L \), - the relative phase between the \( c_Y^S \), \( c_Y^L \) couplings. The interference contribution is linear in the \( c_Y^S \) coupling and contains the relative phase \( \alpha_{SV} \) which could generate the CP violation, when \( \alpha_{SV} \neq 0 \) or \( \pi \). The appearance of above interference in the cross section should manifest the observation of azimuthal asymmetry of the scattered electrons. This asymmetry does vanish even for \( \alpha_{SV} = 0 \), however there is different azimuthal dependence in the case of CP conservation and CP nonconservation. It is necessary to point out that with the proper choice of angle \( \phi \), a measurement of the maximal asymmetry of the cross section could detect the CP-violating phase. The Fig.2 shows the possible effect of the CP violation connected with the interference term \( c_Y^L c_{S}^{R*} \) proportional to the \( |\eta_{\vec{T}}| \).

To give a numerical example of expected event number, we assume that in our analysis an antineutrino source is located in the center of the ring detector and is polarized perpendicularly to the ring. Moreover, we assume that \( T_{th}^{th} = 100keV \) - a detector threshold; \( N_e = 2.097 \cdot 10^{34} \) - number of electrons in fiducial volume of the detector; \( \epsilon = 1 \) - an efficiency of the
Fig. 2. Plot of the $d\sigma/d\phi_e$ as a function of the $\phi_e$ for $\hat{\eta}_\nu \cdot \hat{q} = 0.996, \vert \eta_\nu^\perp \vert = 0.088, \eta_e = 1/2, \vert c_S^R \vert = 0.088, \vert c_V^L \vert = 0.96, \vert c_A^L \vert = 0.493$. The solid line is for the SM case, the long-dashed line corresponds to the CP violation for $\alpha_{VS} = \pi/2$, while the short-dashed line represents the CP symmetric case for the $\alpha_{VS} = 0$.

detector for antineutrino energy above threshold; $N_\mu = 10^{20}$ - number of muons decaying per year. This number gives the antineutrino flux, i.e. the number of antineutrinos passing through $S_D = 2\pi R \cdot D = 305490cm^2$ (where $R = L = 2205cm$ is the inner radius of the ring that is equal to the distance from the antineutrino source, $D = 22.05cm$ is the thickness of the ring detector) in the direction perpendicular to the $\hat{n}_\mu$ according to the SM: $\Phi^\perp_{\bar{\nu}} = 1.497 \cdot 10^{18}cm^{-2}s^{-1}$. For the SM, the event number does not depend on the $\phi_e$ and one expects $dN_e/d\phi_e \simeq 1.52 \cdot 10^8$ events (recoil electrons) per year. To calculate the event number we used the antineutrino spectral function, see Appendix B. If the exotic $c_S^R$ coupling is present in $\bar{\nu}e^-$ scattering, the azimuthal asymmetry of the event number should occur.

It is worth to notice that a knowledge of the differential cross section for $\nu e^-$ scattering allows to get the correct formula for $\bar{\nu}e^-$ scattering, making the simple substitutions. The calculated formula for the laboratory differential cross section in the case of $\nu e^-$ scattering, in the limit of vanishing neutrino
mass, is as follows:

\[
\frac{d^2\sigma}{dy_e d\phi_e} = \left( \frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V,A)} + \left( \frac{d^2\sigma}{dy_e d\phi_e} \right)_{(S)} + \left( \frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V,S)}, \quad (8)
\]

\[
\frac{d^2\sigma}{dy_e d\phi_e} \bigg|_{(V,A)} = \frac{E_\nu m_e G_F^2}{4\pi^2} \left\{ (1 - \hat{\eta}_\nu \cdot \hat{q}) \left[ (c_V^L + c_A^L)^2 
\right.
\right.
\right.
\left. \left. + (c_V^L - c_A^L)^2 (1 - y_e)^2 - \frac{m_e y_e}{E_\nu} \left( \left( c_V^L \right)^2 - \left( c_A^L \right)^2 \right) \right] \right\}, \quad (9)
\]

\[
\frac{d^2\sigma}{dy_e d\phi_e} \bigg|_{(S)} = \frac{E_\nu m_e G_F^2}{4\pi^2} \left\{ \frac{1}{8} y_e \left( y_e + 2 \frac{m_e}{E_\nu} \right) \left| c_S^R \right|^2 \right\}, \quad (10)
\]

\[
\frac{d^2\sigma}{dy_e d\phi_e} \bigg|_{(V,S)} = \frac{E_\nu m_e G_F^2}{4\pi^2} \left\{ \sqrt{y_e (y_e + 2 \frac{m_e}{E_\nu})} \left[ - \eta_{\nu}^\perp \cdot (\hat{p}_e \times \hat{q}) \right] Im(c_V^L c_S^{R^*}) 
\right.
\right.
\right.
\left. \left. + (\eta_{\nu}^\perp \cdot \hat{p}_e) Re(c_V^L c_S^{R^*}) \right\}. \quad (11)
\]

We see that the Eq. (2) can be obtained from the Eq. (8) by substituting \( c_V^R \rightarrow -c_V^R, \ c_A^L \rightarrow -c_A^L, \ q \rightarrow -q, \ \hat{\eta}_\nu \rightarrow -\hat{\eta}_\nu, \ \eta_{\nu}^\perp \rightarrow -\eta_{\nu}^\perp. \) In addition, \( \hat{\eta}_\nu \cdot \hat{q} \) changes the sign respectively to the definition of the density operator for the polarized neutrino and antineutrino, see Appendix A.

4. Conclusion

We have shown that the scattering of the electron antineutrino beam, produced in the decays of polarized muons at rest, on the unpolarized electron target can be used to measure the CP violation in leptonic weak interactions. An appropriate observable to unambiguous test would be the observation of azimuthal asymmetry of recoil electrons generated by nonzero interference terms between the standard \( c_V^L \) and exotic \( c_S^R \) couplings, proportional to the magnitude of \( \eta_{\nu}^\perp. \)

According to the standard model, the angular distribution of scattered electrons should be symmetric. The detection of the azimuthal asymmetry would indicate the possible existence of the right-chirality antineutrino states (it means that in this case right-chirality antineutrinos have negative helicity when for \( m_\nu \rightarrow 0 \)).

It is worth to point out that if the incoming antineutrino beam consists only of the left-chirality and longitudinally polarized antineutrinos, there is no interference \( Re(c_V^L c_S^{R^*}) \) or \( Im(c_V^L c_S^{R^*}) \) connected with CP violation, and the electron angular distribution is symmetric. This is in agreement with the Wilczek results.

We also have noticed a general regularity that a knowledge of the differential cross section for \( \nu e^- \) scattering allows to write the corresponding formula
for $\bar{\nu}e^-$ scattering, if one substitutes $c^R_T \rightarrow -c^R_T$, $c^L_A \rightarrow -c^L_A$, $q \rightarrow -q$, $\hat{\eta}_\nu \rightarrow -\hat{\eta}_\nu$, so $\eta^\perp_\nu \rightarrow -\eta^\perp_\nu$, and one uses the correct definitions of the density operators for polarized antineutrino (neutrino).

The high-resolution neutrino-electron experiments require very large detectors and intense polarized neutrino sources which are very well understood (shape and normalization) to accumulate enough statistics because the cross section for $\nu e$ scattering is tiny. In addition, such experiments should run long (one year) and the detectors must distinguish the electrons from various potential background sources. New detectors should also measure both the polar angle and the azimuthal angle of the outgoing electrons with high resolution.

Appendix A

Spin polarization 4-vector of massive (anti)neutrino and density operator of the polarized (anti)neutrino

The formula for the spin polarization 4-vector of massive antineutrino $S'_\nu$ moving with the momentum $q$ is as follows:

$$S'_\nu = (S'^0_\nu, S'_\nu),$$  \hspace{1cm} (A.1)

$$S'^0_\nu = \frac{|q|}{m_\nu} (\hat{n}_\nu \cdot \hat{q}),$$  \hspace{1cm} (A.2)

$$S'_\nu = - \left( \frac{E_\nu}{m_\nu} (\hat{n}_\nu \cdot \hat{q}) \hat{q} + \hat{n}_\nu - (\hat{n}_\nu \cdot \hat{q}) \hat{q} \right),$$  \hspace{1cm} (A.3)

where $\hat{n}_\nu$ is the unit 3-vector of the antineutrino polarization in its rest frame. The formula for the density operator of the polarized antineutrino in the limit of vanishing antineutrino mass $m_\nu$ is given by:

$$\lim_{m_\nu \rightarrow 0} \Lambda^{(\nu)} = \lim_{m_\nu \rightarrow 0} \frac{1}{2} \left\{ (q^\mu \gamma_\mu) \left[ 1 + \gamma_5 (S'^0_\nu \gamma_\mu) \right] \right\},$$  \hspace{1cm} (A.4)

$$= \frac{1}{2} \left\{ (q^\mu \gamma_\mu) \left[ 1 - \gamma_5 (\hat{n}_\nu \cdot \hat{q}) - \gamma_5 S'^\perp_\nu \cdot \gamma \right] \right\},$$  \hspace{1cm} (A.5)

where $S'^\perp_\nu = \left( 0, \eta^\perp_\nu \right) = \hat{n}_\nu - (\hat{n}_\nu \cdot \hat{q}) \hat{q}$. We see that in spite of the singularities $m_\nu^{-1}$ in the polarization four-vector $S'_\nu$, the density operator $\Lambda^{(\nu)}$ remains finite including the transverse component of the antineutrino spin polarization \[10\].

The corresponding formula for the spin polarization 4-vector of massive neutrino $S'_\nu$ moving with the momentum $q$ is as follows:

$$S'_\nu = (S'^0_\nu, S'_\nu),$$  \hspace{1cm} (A.6)
\[ S^0_\nu = \frac{|q|}{m_\nu} (\hat{\eta}_\nu \cdot \hat{q}), \]  
(A.7)

\[ S'_\nu = \frac{E_\nu}{m_\nu} (\hat{\eta}_\nu \cdot \hat{q}) \hat{q} + \hat{\eta}_\nu - (\hat{\eta}_\nu \cdot \hat{q}) \hat{q}. \]  
(A.8)

The formula for the density operator of the polarized neutrino in the limit of vanishing neutrino mass \( m_\nu \) is given by:

\[
\lim_{m_\nu \to 0} \Lambda^{(s)}_\nu = \lim_{m_\nu \to 0} \frac{1}{2} \left\{ (q^\mu \gamma_\mu) + m_\nu \left[ 1 + \gamma_5 (S'^{\mu}_{\nu} \gamma_\mu) \right] \right\} 
\]  
(A.9)

\[
= \frac{1}{2} \left\{ (q^\mu \gamma_\mu) \left[ 1 + \gamma_5 (\hat{\eta}_\nu \cdot \hat{q}) + \gamma_5 S'_\nu \cdot \gamma \right] \right\}. \quad \text{(A.10)}
\]

**Appendix B**

**Antineutrino spectral function**

The formula for the electron antineutrino spectral function in case of the polarized muon decay at rest, when the exotic \( g^{S}_{LR} \) coupling of the right-chirality antineutrinos in addition to the standard \( g^{V}_{LL} \) coupling of the left-chirality antineutrinos is admitted, takes the form:

\[ \frac{d^2 \Gamma}{dyd\Omega_\nu} = \left( \frac{d^2 \Gamma}{dyd\Omega_\nu} \right)_{(V)} + \left( \frac{d^2 \Gamma}{dyd\Omega_\nu} \right)_{(S)} + \left( \frac{d^2 \Gamma}{dyd\Omega_\nu} \right)_{(V S)}, \]

\[ \left( \frac{d^2 \Gamma}{dyd\Omega_\nu} \right)_{(V)} = \frac{G_F^2 m^5_\mu}{128 \pi^4} \left\{ |g^{V}_{LL}|^2 (1 + \hat{\eta}_\sigma \cdot \hat{q}) (1 + \hat{\eta}_\mu \cdot \hat{q}) y^2 (1 - y) \right\}, \]  
(B.2)

\[ \left( \frac{d^2 \Gamma}{dyd\Omega_\nu} \right)_{(S)} = \frac{G_F^2 m^5_\mu}{3072 \pi} |g^{S}_{LR}|^2 (1 - \hat{\eta}_\sigma \cdot \hat{q}) \]  
\[ \cdot y^2 \left\{ (3 - 2y) - (1 - 2y) \hat{\eta}_\mu \cdot \hat{q} \right\}, \]  
(B.3)

\[ \left( \frac{d^2 \Gamma}{dyd\Omega_\nu} \right)_{(V S)} = \frac{G_F^2 m^5_\mu}{256 \pi^4} \left\{ |\eta^{\perp}_\sigma| |\eta^{\perp}_\mu| |g^{V}_{LL}| |g^{S}_{LR}| \cos (\phi - \alpha_{VS}) \right\} \]  
\[ \cdot y^2 (1 - y), \]  
(B.4)

where \( \hat{\eta}_\sigma, (\hat{\eta}_\sigma \cdot \hat{q}) \hat{q}, \) and \( \hat{\eta}_\sigma^{\perp} \) denote the unit polarization vector, its longitudinal component, and transverse component of the outgoing \( \nu_e \) in its rest system, respectively. \( \hat{\eta}_\mu \) is the unit polarization vector of the initial muon.
in its rest frame. \( y = \frac{2E_{\nu}}{m_\mu} \) is the reduced antineutrino energy for the muon mass \( m_\mu \), it varies from 0 to 1, and \( d\Omega_\nu \) is the solid angle differential for \( \nu_e \) momentum \( \hat{q} \).

The interference term is presented for the case when \( \hat{\eta}_\mu \cdot \hat{q} = 0 \). \( \phi \) is the angle between the \( \eta^\perp_{\nu} \) and the \( \eta^\perp_\mu \), see Fig. 1. \( \alpha_{VS} \equiv \alpha^L_V - \alpha^R_S \) is the relative phase between the \( g^V_{LL} \) and \( g^S_{LR} \).

With the use of the current data \[9\], the upper limit on the magnitude of the transverse antineutrino polarization and lower bound on the longitudinal antineutrino polarization have been calculated, see \[11\]:

\[
|\eta^\perp_{\nu}| = 2\sqrt{Q^T_L(1 - Q^T_L)} \leq 0.088, \quad \hat{\eta}_{\nu} \cdot \hat{q} = 2Q^T_L - 1 \geq 0.996, \quad (B.5)
\]

\[
Q^T_L = 1 - \frac{1}{4}|g^S_{LR}|^2 \geq 0.998, \quad (B.6)
\]

where \( Q^T_L \) is the probability of the \( \bar{\nu}_e \) to be left-chirality.

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