Effect of quantum correction and black body radiation on Jeans instability of porous medium

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Abstract. The Jeans instability of self-gravitating gaseous plasma is re-discussed in the framework of viscous, self-gravitating porous medium under the influence of quantum correction and black body radiation. The mathematical solution of the problem has been obtained through the normal mode analysis and the dispersion relation has been derived with the help of linearized perturbation equations. It is found that quantum corrections and black body radiation modify the fundamental Jeans criterion of gravitational instability and a new instability quantum corrected radiative instability is found. It is pointed here that the role of quantum correction is to stabilize the considered system by decreasing the value of critical Jeans wave-length. The stability of the system is discussed by applying Rought-Hurwitz criteria.

1. Introduction

There has been an accrued interest on astrophysics, motivated by applications in understanding the various collective process in gaseous plasma, which is ubiquitous in space, including diffuse and dense interstellar shell, circumstellar shell, ionosphere, nova ejecta, star envelopes dark and dark interiors molecular clouds. The starting point for modern star cosmogony is that stars are formed and reach a state similar to that of the sun owing to the gravitational condensation of rarefied clouds of gas. It has been established as a fact that stars are formed as a result of the gravitational construction of fragments of molecular clouds, which are the sorts of collective behavior. The gravitational instability of an infinite homogeneous self-gravitating gaseous cloud was first discovered by Jeans [1]. The problem of instability of self-gravitating plasma in a magnetic field is of considerable astrophysical significance. A detailed analysis of the effect of magnetic field and rotation on the self-gravitational instability of an infinite homogeneous medium has been discussed by Chandrasekhar [2]. Chieze [3] has studied the formation of these clouds and suggested that the fragmentations occur near the gravitational instability threshold. These clouds are unstable against fragmentation when they approach gravitational instability in a fixed pressure environmental. The problem of an isothermal gas sphere under the action of external pressure has been investigated by Ebert [4], who pointed out that the disturbances of length scale approximately equal to the $\lambda_J$ Jeans length based on the central density were unstable to gravitational collapse. Since $M \propto \rho \lambda_J^3 \propto \rho^{-2/3}$, this considerably reduced the minimum unstable mass and demonstrated that an ‘O’ star...
cloud form in the center of an interstellar cloud. Hunter [5] has studied the growth of perturbations in a gravitationally contracting isothermal gas cloud and he points out that perturbations with the initial length scale of order less than \( \lambda_j \), grew less rapidly relative to the background density than did perturbations of substantially large dimension. Recently Prajapati et al. [6] have discussed the problem of self-gravitational instability of rotating viscous hall plasma with arbitrary radiative heat-loss functions and electron inertia. In addition to this, there has been interesting in understanding the role of Black Body radiation in star formation and molecular cloud condensation process. An important point to be noted in this case, that the heat-loss process is a major cause for the condensation of large astrophysical compact objects. In this problem we have considered that for the case where the perturbations under heat-loss by black body radiation under the plasma correction, since radiative pressure in the inertial area of the star is usually small in comparison with the gaseous pressure, in interior of hot and large plasma cloud like H regions etc., where temperature is rather high while the density is low, it should be important to take into account the radiative processes. The radiative heat-loss functions considered earlier by Chandrasekhar [7]. Many researchers [8-15] is studies of the self-gravitational instability of gaseous plasma, the role of quantum corrections and black body radiation has not been analyzed. It would, therefore, be of interest to examine the effect of black body radiation and quantum corrections. On the Jeans instability of self-gravitating viscous infinite homogeneous porous medium. We hope that the presence result will help to understand the structure formation in interstellar medium.

2. Equations of the problem

Let us consider an infinite homogeneous, self-gravitating viscous gaseous molecular clouds quantum plasma incorporating black body radiation. We introduced the quantum effects through the Bohm potential term in the momentum transfer equation. For simplicity of the problem, we ignore the additional convective instability, effects which may occur in such a system if the temperature \( T \) is a decreasing function of a coordinate, we shall assume \( T \) to be constant. The medium is taken optically thick and the black body radiation is assumed. The momentum transfer equation for magnetized quantum plasma is

\[
\rho \frac{dv}{dt} = -\nabla p + \rho \nabla \phi + \rho \left( \frac{v^2}{k_1} - 1 \right) v + \frac{h^2 \rho}{2m_i m_e} \nabla \left( \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right) + \rho \frac{\nabla^2 \phi}{\nabla} \quad (1)
\]

\[
\varepsilon \frac{d\rho}{dt} + \nabla \cdot (\rho v) = 0 \quad (2),
\]

\[
\nabla^2 \phi + 4\pi G \rho = 0 \quad (3),
\]

\[
\frac{dT}{T} = (\gamma - 1) \frac{dp}{\rho} \quad (4)
\]

where \( p = p_g + p_r \) is the total pressure of the medium, \( p_g = \rho RT \) is the gas pressure and \( p_r = K_B T^{4/3} \) is the radiation pressure, \( K_B \) is the Boltzmann’s constant. Also, the symbols \( \rho, \gamma, v, \phi, T, G, T \) and \( \Gamma \) denotes the number density, adiabatic index, fluid velocity, gravitational potential, kinematic viscosity, universal gravitational constant, temperature and radiative heat function, respectively. \( \hbar \) Planck’s constant divided by \( 2\pi m_e m_i \) are the electron and ion mass, respectively. Adiabatic changes in an enclosure containing matter and radiation are given by equation (4), (Chandrasekhar, 1957)

\[
\Gamma = \left( 1 + \frac{\Gamma_1 - b}{4 - 3b} \right), \Gamma_1 = \left( b + \frac{(4 - 3b)^2(\gamma - 1)}{b + 12(1 - b)(\gamma - 1)} \right), b = \left( \frac{p_g}{p_g + p_r} \right) \quad (5)
\]

If the radiation is negligible then, \( \Gamma = \Gamma_1 = \gamma \) while in the case of \( p_g \ll p_r \) is \( \Gamma = \Gamma_1 = 4/3 \).

3. Linearized perturbation equations

In the linearization, we write the space and time dependent physical quantities \( \rho, P, v \) and \( \phi \), in the form of the sum of the equilibrium and perturbed part as

\[
\rho = \rho_0 + \delta \rho, \quad p = p_0 + \delta p, \quad v = v_0 + \delta v, \quad \phi = \phi_0 + \delta \phi \quad (6)
\]

The terms with subscript ‘0’ denote the equilibrium part of the physical quantities. Perturbation in fluid velocity, fluid pressure, fluid density and gravitational potential are given as, \( \delta v = (0,0,v_z) \delta p \delta \rho \) and \( \delta \phi \),
respectively. Using equation (6) in equations, (1) to (4) we write the linearized perturbation equations of infinite conducting quantum plasma, by removing ‘0’ from subscript in the equilibrium quantities, for simplicity. Thus, we obtain linearized perturbation equations of the considered system as,

\[
\frac{\partial \delta \mathbf{v}}{\partial t} + \frac{RT}{\rho} \frac{\partial \delta \rho}{\partial z} + R(1 + 4R_p) \frac{\partial \delta T}{\partial z} - \rho \theta \left( \frac{1}{k_1} \right) \mathbf{v} - \frac{\hbar^2}{4m_e m_i} \frac{\partial}{\partial z} \left( \nabla^2 \delta \rho \right) = 0
\]

\[
\frac{\partial \delta \rho}{\partial t} + \rho \frac{\partial \delta \mathbf{v}}{\partial z} = 0
\]

\[
\frac{\partial^2 \delta \phi}{\partial z^2} + 4\pi G \delta \rho = 0
\]

\[
\frac{\partial \delta T}{\partial t} - (\Gamma - 1) \frac{T}{\rho} \frac{\partial \delta \rho}{\partial t} = 0
\]

Where, \( v \equiv v_z R_p = p_r/ p_g \) denote the ratio of the radiation pressure and gas pressure, and \( \Gamma \) is assumed to be constant.

4. Dispersion relation

Let us assume the perturbation of all the quantities vary as, \( \exp(-i\sigma t + ikz) \) (11)

Where, \( \sigma \) is the frequency of harmonic perturbations and \( k \) is the wave number in \( z \)-direction. Combining equations (7) to (10), we get the dispersion relation

\[
\omega^2 - \theta_k \omega + \frac{k^2 RT}{\epsilon} \left[ 1 + (1 + 4R_p)(\Gamma - 1) \right] - \frac{4\pi G \rho \theta_k}{\epsilon} + \frac{\hbar^2 k^4}{4m_e m_i \epsilon} = 0
\]

Where, \( \omega = i\sigma \), the growth rate of instability, \( \theta_k = \theta \left( k^2 + \frac{1}{k_1} \right) \) is viscosity, \( k_1 \) is permeability. This dispersion relation (12) shows the combined influence of viscosity, black body radiation, porosity and quantum correction on the Jeans instability of optically thick quantum plasma. If we ignore the quantum correction and viscosity then this dispersion relation reduces to Vranjes and Cadez [10]. Thus, the dispersion relation is modified by the presence of viscosity and quantum correction. The stability of the system, represented by preceding equation, since all the coefficient of equation (12) is positive that the necessary condition for instability of the system is satisfied. To obtain the sufficient condition, the principal diagonal minors of Hurwitz matrix must be positive, which are shown below.

\[
\Delta_1 = \theta_k > 0, \Delta_2 = \theta_k \frac{k^2 RT}{\epsilon} \left[ 1 + (1 + 4R_p)(\Gamma - 1) \right] - \frac{4\pi G \rho \theta_k}{\epsilon} + \frac{\hbar^2 k^4 \theta_k}{4m_e m_i \epsilon} > 0
\]

\[
\Delta_3 = \left( \frac{k^2 RT}{\epsilon} \left[ 1 + (1 + 4R_p)(\Gamma - 1) \right] + \frac{4\pi G \rho}{\epsilon} + \frac{\hbar^2 k^4}{4m_e m_i \epsilon} \right) \Delta_2 > 0
\]

We see that all \( \Delta's \) are positive so we find that a Quantum Correction, Black body radiation, the viscous porous medium is stable the system. The condition of instability is obtained from the constant term of equation (12) and the condition is instability is given as,

\[
\frac{k^2 RT}{\epsilon} \left[ 1 + (1 + 4R_p)(\Gamma - 1) \right] + \frac{\hbar^2 k^4}{4m_e m_i \epsilon} - \frac{4\pi G \rho}{\epsilon} < 0
\]

Thus the system is unstable if

\[
\frac{\lambda_f}{\lambda_c} \geq \left( 1 + (1 + 4R_p)(\Gamma - 1) \right) + \frac{\hbar^2}{4m_e m_i \epsilon} z^2 \right]^{-1/2}
\]

where \( \lambda_f = \sqrt{\pi \gamma^2 / \rho G} \) is critical Jean’s wavelength and \( \lambda_c = 2\pi / k \), is the critical wavelength associated with black body radiations. The above equation (14) shows the quantum and black body corrected condition of relative instability. It is clear that this relative instability is affected by the presence of viscosity. In the absence of quantum correction, the above condition of relative instability is identical to Vranjes and Cadez [10]. Thus we have found the new condition of instability which is modified by quantum corrections term. It is also noted from relative instability condition (14) that the role of quantum corrections terms is to stabilize the considered system by increasing the value of critical Jean’s wavelength.
Now we plot the curves between normalized growth rates of Jeans instability versus normalized wave number. In figure 1 and 2 we have plotted the real root of growth rate against wave number for various values of quantum correction and porosity. It is clear from this figure that for any wave number value the growth rate of instability decreases as increasing the value of quantum correction and porosity. Hence the quantum correction and porosity have a stabilizing influence on the system.

5. Conclusions
In the present paper, it is found that the viscosity has a dissipative effect but do not affect the Jeans expression. We have obtained a new modified relative instability which is affected by black body radiations, quantum corrections, and Porosity terms. It is pointed out that the black body radiation has a stabilizing influence on the considered problem which is further increased by quantum corrections terms.

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