Abstract—Rate-Splitting multiple access (RSMA) has emerged as a key enabler in improving the performance of the beyond fifth-generation (5G) cellular networks. The existing literature has typically considered the sum rate of the users to evaluate the performance of RSMA. However, it has been shown in the existing works that maximizing the sum rate can result in asymmetric user performance. It significantly enhances one user’s rate at the cost of the rate of another RSMA user. Further, imperfections can reduce the performance of successive interference cancellation (SIC)-based RSMA. Therefore, in this letter, we consider the imperfection in SIC and derive suitable bounds on fractions of the power allocated for common and private messages and the fraction of common message intended for each user in an RSMA pair such that their individual RSMA rates are greater than their respective orthogonal multiple access (OMA) rates. Through simulations, we validate the derived bounds. We show that they can be used to appropriately select the RSMA parameters resulting in users’ RSMA rates being better than their respective OMA rates.

Index Terms—Beyond Fifth Generation (5G) cellular networks, Rate-Splitting Multiple Access (RSMA), Successive Interference Cancellation (SIC).

I. INTRODUCTION

RSMA splitting multiple access (RSMA) is gaining popularity due to its robustness, reliability, and high through-put in comparison to other multiple access schemes: orthogonal multiple access (OMA), non-orthogonal multiple access (NOMA) [1]. RSMA is an established multiple access scheme for which NOMA and space division multiple access (SDMA) act as special cases. For a downlink RSMA, the user’s message is split into common and private messages at the transmitter, and the successive interference cancellation (SIC) is executed at the receiver’s side [1].

In [2], the authors have worked on the sum-rate maximization for wireless networks that use RSMA in the downlink. It has been shown that RSMA can outperform orthogonal frequency division multiple access (OFDMA) and NOMA in terms of data rate with 23.5% and 19.6% gains, respectively.

II. RELATED WORK

The authors in [3] have developed a novel transmission scheme that utilizes RSMA with artificial noise and adaptive beam-forming to maximize the secrecy sum rate. In [4], the authors have applied RSMA for a multi-group and multi-cast scenario where each message is intended for a group of users. They have also designed the physical layer and performed a link-level simulation for RSMA and SDMA by assuming cellular and multi-beam satellite systems. The comparative analysis in [5] shows that RSMA is more robust and efficient than NOMA for multiple-input multiple-output (MIMO) settings. An extensive study on key multiple access technologies for aerial networks has been presented in [6]. The authors in [6] have modeled and analyzed the weighted sum-rate performance of two user networks served by an RSMA-based aerial base station (BS). The authors in [7] have derived an optimal power allocation strategy and demonstrated that RSMA with the proposed strategy is robust to degrading performance due to user mobility compared to the conventional MIMO strategies.

Most of the existing works on RSMA have concentrated on the sum-rate maximization and other aspects related to the sum rate of the users. However, a study on the individual user rates in RSMA is required. An asymmetric increase in user rates can result in unfairness between RSMA users. Motivated by this, we derive the bounds on the power allocation of common and private messages and the fraction of the individual common message for which the RSMA rates of strong and weak users will be greater than their respective OMA rates simultaneously. To the best of our knowledge, this is the first work that derives the bounds on power allocation and common message fraction by considering the imperfection in SIC for RSMA. Therefore, by fixing the power allocation fractions of common and their individual private messages and the fraction of common message within the derived bounds, individual rates of RSMA users will always be greater than their respective OMA rates. The main contributions of the letter are as follows.

- We derive bounds on power allocation coefficients of common and private messages and the fraction of individual common messages for RSMA for a two-user scenario with imperfect SIC.
- Through extensive simulations, we show that the bounds are close to the numerically obtained values.

The organization of the letter is as follows. In Section II, the system model is presented. The bounds on the fraction of common message for a user and power allocation coefficients for common and private messages are derived in Section III. The derived bounds are validated in Section IV.
Fig. 1: System Model.

conclusion and future work are presented in Section V.

II. SYSTEM MODEL

We consider a downlink RSMA system for a BS as depicted in Fig. 1. Let \( \mathcal{U} = \{1, 2, 3, \ldots, N\} \) be the set of users associated with the BS \( b \) under consideration for a given user association scheme and \( N \) be the total number of users connected with BS \( b \). In downlink transmission, the signal-to-interference-plus-noise ratio (SINR) from BS \( b \) to user \( u \) \((u \in \mathcal{U})\) on a subchannel \( m \) in case of OMA is given as

\[
\gamma_u = \frac{P_t \|h_u\|^2}{\sigma^2 + I_u},
\]  
(1)

where \( P_t \) is the maximum transmit power of BS \( b \), \( \|h_u\|^2 \) is the channel gain between user \( u \) and BS \( b \), and \( \sigma^2 \) is the noise variance. \( I_u = \sum_{b \neq B, B \in \mathcal{B}} P_t^b \|h_{u}^b\|^2 \) is the total interference from neighboring BSs, where \( P_t^b \) and \( B \) is the transmit power of neighboring BSs and set of BSs in a given area, respectively. The downlink rate for user \( u \) based on the log-rate model is given as

\[
R_{\text{oma}}^u = \frac{1}{2} \log_2 (1 + \gamma_u),
\]  
(2)

We consider a 1-layer RSMA system for a pair of RSMA users, i.e., \( s \) and \( w \) \((s \text{ and } w \in \mathcal{U})\). The proposed bounds can be implemented for all the RSMA pairs. Without loss of generality, we assume that \( \|h_s\|^2 > \|h_w\|^2 \). Therefore, we denote \( s \) as the strong user and \( w \) as the weak user. The users’ messages, \( s \) and \( w \) are divided into their respective common and private messages. Using RSMA, different power levels are assigned to common and private messages of strong and weak users. Let \( P_c \), \( P_ps \), and \( P_pw \) be the power of common stream of \( s \) and \( w \), private stream of \( s \), and private stream of \( w \), respectively, such that \( P_c + P_ps + P_pw \leq P_t \).

The receiver of the respective users performs SIC to decode their messages. At the receiver side, the user \( s \) decodes the common message by treating the interference from the private messages as noise. Then, the user \( s \) subtracts the common message from the received signal to extract its own private message by treating other users’ private messages as noise. Therefore, the SINR of the common message of user \( s \) is given as follows.

\[
\gamma_{cs} = \frac{P_c \|h_s\|^2}{(P_ps + P_pw) \|h_s\|^2 + \sigma^2 + I_u},
\]  
(3)

Dividing numerator and denominator of (3) by \((\sigma^2 + I_u)\) and replacing \( \|h_s\|^2/({\sigma^2 + I_u}) \) with \( \gamma_s/P_t \) from (1), we get

\[
\gamma_{cs} = \frac{P_c \gamma_s}{(P_c - P_t) \gamma_s + P_t}.
\]  
(4)

Additionally, the SINR of the private message of user \( s \) is

\[
\gamma_{ps} = \frac{P_ps \gamma_s}{(P_t - P_c) \gamma_s + P_t},
\]  
(5)

where \( P_c = \alpha_c P_t \), \( P_ps = \lambda(P_t - P_c) \), \( P_pw = (1 - \lambda)(P_t - P_c) \), \( \alpha_c \in (0, 1) \) is the power fraction of common message, \( \lambda \in (0, 1) \) is the power fraction of the private message of \( s \), and \( \beta \in [0, 1] \) represents the coefficient of SIC imperfection. Similarly, for user \( w \), the SINRs of common and private messages are, respectively, \( \gamma_{cw} \) and \( \gamma_{pw} \).

\[
\gamma_{cw} = \frac{P_c \gamma_w}{(P_t - P_c) \gamma_w + P_t},
\]  
(6)

\[
\gamma_{pw} = \frac{P_pw \gamma_w}{(P_t - P_c) \gamma_w + P_t}.
\]  
(7)

Therefore, the RSMA rates for user \( s \) and \( w \) based on the SINRs obtained in (4), (5), (6), and (7) are

\[
R_{rsma}^s = \tau R_{\text{comm}}^s + R_{priv}^s,
\]  
(8)

\[
R_{rsma}^w = (1 - \tau) R_{\text{comm}}^w + R_{priv}^w.
\]  
(9)

Using (2), the OMA sum-rate for the same pair is given as

\[
SR_{\text{oma}} = R_{\text{oma}}^s + R_{\text{oma}}^w.
\]  
(10)

III. BOUNDS ON COMMON MESSAGE FRACTION OF S (\( \tau \)), POWER FRACTION OF COMMON MESSAGE (\( \alpha_c \)), AND POWER FRACTION OF PRIVATE MESSAGE OF S (\( \lambda \))

In this section, we derive bounds on \( \tau \), \( \alpha_c \), and \( \lambda \) for which the individual RSMA rates of user \( s \) and \( w \) \((R_{rsma}^s \text{ and } R_{rsma}^w)\) should be greater than their respective OMA rates \((R_{oma}^s \text{ and } R_{oma}^w)\).

A. Bounds on \( \tau \)

We consider that \( R_{rsma}^s > R_{oma}^s \) and \( R_{rsma}^w > R_{oma}^w \) for user \( s \) and \( w \), respectively. Using these constraints and (5), (6), (8), and (9), we derive the upper and lower bounds on \( \tau \). The constraints are expressed as follows

\[
\tau R_{\text{comm}}^s + R_{\text{priv}}^s > R_{rsma}^s.
\]  
(11)

Substituting \( R_{\text{comm}}^s \), \( R_{\text{priv}}^s \), and \( R_{rsma}^s \) with their respective expressions from (6), (5), and (8) in (12), we obtain

\[
\tau > \frac{1}{\log_2(1 + \gamma_{cw})} \left[ \frac{1}{2} \log_2(1 + \gamma_s) - \log_2(1 + \gamma_{ps}) \right].
\]  
(13)
Substituting $\gamma_{cw}$ and $\gamma_{ps}$ from (6) and (5) in (13) and further solving (13), we get
\[
\log_2 \left( \frac{\sqrt{(1+\gamma_s)(\lambda \gamma_s(w-1) + (1-\lambda - \gamma_s))}}{\alpha_s (\lambda \gamma_s + \gamma_s w/(1-\alpha_s) + 1)} \right) > 0.
\] (14)

Similarly, the constraint for user $w$ is given as follows
\[
(1 - \tau) P_w^{\text{comm}} + P_w^{\text{priv}} > P_w^{\text{oma}}.
\] (15)

Using $\gamma_{cw}$ and $\gamma_{pw}$ from (6) and (7) in (15), we get
\[
\log_2 \left( \frac{\sqrt{1+\gamma_s(\lambda \gamma_s - \gamma_s - 1) + \gamma_s + 1}}{\alpha_s (\lambda \gamma_s + \gamma_s w/(1-\alpha_s) + 1)} \right) > 0 \implies \gamma_{ps} < \sqrt{1+\gamma_s - 1}.
\] (16)

The bounds derived in (13) and (16), respectively, are considered as lower and upper bound for $\tau$ and referred to as $\tau_{\text{lower}}$ and $\tau_{\text{upper}}$, respectively. Note that the upper and lower bounds on $\tau$ depend on $\alpha_c$, $\beta$, and $\lambda$.

### B. Bounds on $\alpha_c$

The range of $\tau$ is $(0,1)$. Rearranging (12), we get
\[
\tau > \frac{R_w^{\text{oma}} - R_w^{\text{priv}}}{R_w^{\text{comm}}}. \quad (17)
\]

Considering $\tau > 0$ in (17), we obtain,
\[
\frac{R_w^{\text{oma}} - R_w^{\text{priv}}}{R_w^{\text{comm}}} > 0 \implies R_w^{\text{oma}} > R_w^{\text{priv}}, \quad (18)
\]

Substituting the expression of $\gamma_{ps}$ from (5) in (18), and solving further, we get
\[
\alpha_c > \sqrt{1+\gamma_s(\lambda \gamma_s - \gamma_s - 1) + \gamma_s + 1} / \gamma_s (\lambda \gamma_s + \gamma_s w/(1-\alpha_s) + 1 - \beta) \equiv \alpha_{\text{LB}}. \quad (19)
\]

The $\alpha_c$ should satisfy (19) for a given $\lambda$ and $\beta$ for $\tau_{\text{lower}}$ to be greater than 0. Similarly, considering and rearranging (15), we get,
\[
\tau < 1 - \left( \frac{R_w^{\text{oma}} - R_w^{\text{priv}}}{R_w^{\text{comm}}} \right). \quad (20)
\]

Solving (20) by imposing the constraint that the right-hand side (RHS) of the expression should be less than 1, we get,
\[
\frac{R_w^{\text{oma}} - R_w^{\text{priv}}}{R_w^{\text{comm}}} > 0 \implies R_w^{\text{oma}} > R_w^{\text{priv}}, \quad (21)
\]

Replacing $\gamma_{pw}$ with (7) in (21) and further solving, we get
\[
\alpha_c < \sqrt{1+\gamma_s(\lambda \gamma_s + 1) - (1 + \gamma_s w)} / \gamma_s (\lambda \gamma_s + \gamma_s w/(1-\alpha_s) + 1 - \beta). \quad (22)
\]

Along with (19), the $\alpha_c$ should also satisfy (22) for a given $\lambda$ and $\beta$ for $\tau_{\text{upper}}$ to be less than 1. We also need to select an $\alpha_c$ such that $\tau_{\text{lower}} < \tau_{\text{upper}}$. Hence, solving the inequality (14) < (16), we get the following cubic equation in $\alpha_c$.

\[
-\alpha_c^3 \gamma_w^2 B + \alpha_c^2 \left[ \gamma_w \left( \gamma_w (\gamma_w + 1) - C \right) \right] - \gamma_w (\beta - 1)^2 (\gamma_w + 1) + \alpha_c \frac{\gamma_w (C - D) + C - (\beta - 1)(\gamma_w + 1)(\gamma_s - \gamma_w)}{A} < 0, \quad (23)
\]

where
\[
A = \sqrt{1+\gamma_s} \sqrt{1+\gamma_w}, \quad B = \beta - 1 + \lambda (1-\lambda),
\]
\[
C = \beta (\gamma_s + \gamma_w) + \gamma_w (\beta - \lambda (1-\lambda)) - \lambda (\gamma_s \gamma_w (1-\lambda) + \gamma_w - \gamma_s - \gamma_s),
\]
\[
D = \lambda (\gamma_s \gamma_w (1-\lambda) + \gamma_w - \gamma_s) + \gamma_s + 1.
\]

The $\alpha_c$ should satisfy (19), (22), and (23) for a given $\lambda$ and $\beta$. The expressions in (19), (22), and (23) are functions of $\lambda$ and $\beta$. Thus, $\lambda$ and $\beta$ have a significant impact on the value of $\alpha_c$. Therefore, we need to choose a $\lambda$ such that the bounds of $\alpha_c$ always lie between 0 and 1. Considering this condition, we next derive bounds on $\lambda$.

### C. Bounds on $\lambda$

We consider the constraint that $\alpha_c$ lies between $(0, 1)$. Applying this constraint on the bounds of $\alpha_c$ derived in (19) and (22), we derive the bounds on $\lambda$. Considering numerator of (19) to be greater than 0, and further solving, we get
\[
\lambda > (\sqrt{1+\gamma_s} - 1) / \gamma_s. \quad (24)
\]

Now, applying the positivity constraint on the denominator of (19), we obtain,
\[
\lambda > (\sqrt{1+\gamma_s} - 1)(1-\beta) / \sqrt{1+\gamma_s}. \quad (25)
\]

By applying the constraint on (19) that $\alpha_c < 1$, we get
\[
\frac{\sqrt{1+\gamma_s(\lambda \gamma_s - \gamma_s - 1) + \gamma_s + 1}}{\gamma_s (\lambda \gamma_s + \gamma_s w/(1-\alpha_s) + 1 - \beta)} < 1. \quad (26)
\]

Considering that (25) is true, we obtain,
\[
\beta > -1 / \gamma_s. \quad (27)
\]

We already know that $\beta \in [0, 1]$. Hence, $\beta$ always satisfies (27). This implies that the lower bound on $\alpha_c$ in (19) is always less than 1. Similarly, applying the positivity constraint on the numerator and denominator of (22), we get
\[
\lambda > \sqrt{1+\gamma_w - 1} / \gamma_w, \quad (28)
\]
\[
\lambda > \beta (\sqrt{1+\gamma_s} - 1 + 1) / \sqrt{1+\gamma_s}. \quad (29)
\]
Hence, the upper bound on $\alpha$. The condition obtained in (30) cannot be true as $\beta$ get of (23) for various $\lambda$.

The condition obtained in (30) cannot be true as $\beta$ get of (23) for various $\lambda$. Therefore, it is not a strict upper bound on $\alpha$. By setting (22) less than 0. However, we found that those bounds are insignificant. Due to space constraints, we are not presenting them in this letter.

D. Strict lower bound on $\lambda$ and Strict upper bound on $\alpha_c$

By setting (22) less than 1 and assuming (29) to be true, we get

$$\beta < -\frac{1}{\gamma_w}.$$ (30)

The condition obtained in (30) cannot be true as $\beta \in [0, 1]$. Hence, the upper bound on $\alpha_c$ in (22) cannot be less than 1. Therefore, it is not a strict upper bound on $\alpha_c$. Further, we have also derived a few other bounds on $\lambda$ by applying the constraint that the numerator and denominator of (19) and (23) are less than 0. However, we found that those bounds are insignificant. Due to space constraints, we are not presenting them in this letter.

**Algorithm 1** Steps for calculating RSMA rates of strong and weak users using the derived bounds.

1: **INPUTS:** $\gamma_s$ and $\gamma_w$.
2: **OUTPUTS:** $\tau$, $\alpha_c$, $\lambda$, $R_{s}^{\text{rsma}}$, and $R_{w}^{\text{rsma}}$.
3: For a given $\gamma_s$ and $\gamma_w$, compute $\lambda_{\text{lower}}$ using (23).
4: Select $\lambda > \lambda_{\text{lower}}$.
5: Using the selected $\lambda$, compute $\alpha_{\text{LB}}$ and $\alpha_{\text{UB}}$ using (19) and (23), respectively.
6: Select an $\alpha_c$ such that $\alpha_{\text{LB}} < \alpha_c < \alpha_{\text{UB}}$.
7: Using the selected $\alpha_c$ and $\lambda$, compute $\tau_{\text{lower}}$ and $\tau_{\text{upper}}$ using (14) and (16), respectively.
8: Select a $\tau$ such that $\tau_{\text{lower}} < \tau < \tau_{\text{upper}}$.
9: Using the selected $\tau$, $\alpha_c$, and $\lambda$, compute $R_{s}^{\text{rsma}}$ and $R_{w}^{\text{rsma}}$ as in (8) and (9), respectively.

**Fig. 4:** Variation of the rates of $s$ and $w$ w.r.t. $\lambda$.

**Fig. 3:** Variation of the suitable upper bound (23) and lower bounds of $\alpha_c$ (19) w.r.t. $\lambda$.

**Fig. 2:** Variation of the cubic equation and region of operation of $\alpha_c$ with respect to (w.r.t.) $\lambda$. 

mentioned in (23). From Fig. 2, it is observed that for the lower bound on $\lambda$ computed using (24), (25), (28), and (29), no $\alpha_c$ satisfies (23). As we increase $\lambda$, we observe that for $\lambda \geq 0.865$, there exists $\alpha_c$ which satisfies $\alpha_c > \alpha_{\text{LB}}$ and (23). Therefore, the lower bound of $\lambda$ obtained using Section III-C is not a strict lower bound. The suitable lower bound of $\lambda$ is the lowest value of $\lambda$ for which there exists a solution of (23), which is denoted as $\lambda_{\text{lower}}$. In addition, (23) provides the suitable upper bound of $\alpha_c$ denoted by $\alpha_{\text{UB}}$ which is the point where the curve cuts the $x$-axis after the vertical line. Similarly, we have plotted the $\alpha_{\text{LB}}$ and $\alpha_{\text{UB}}$ for $\lambda \geq 0.865$ (obtained from Fig. 2). Since the bounds of $\alpha_c$ depend on $\lambda$ as shown in (19) and (23), we have plotted the LHS of (23) for various $\lambda$ starting from 0.7. The values of $\alpha_{\text{LB}}$ for different $\lambda$ are evaluated using (19). The vertical lines in Fig. 2 represent the lower bounds of $\alpha_c$ for different $\lambda$. The inequality shown in (23) states that the cubic expression in $\alpha_c$ on the LHS should be less than 0. Therefore, for a particular $\lambda$, we need to select $\alpha_c$ such that it satisfies $\alpha_c > \alpha_{\text{LB}}$ and the inequality

In this section, we validate the aforementioned bounds through simulation. We consider a two users scenario with interference as in (2). Fig. 4 illustrates the variation of RSMA and OMA rates (in bps/Hz) of $s$ and $w$ w.r.t. $\lambda$. For $\gamma_s = 6$ dB, $\gamma_w = 2$ dB, $\beta = 0$ (i.e., for perfect SIC), $P_t = 1$ W, we have observed that with increase in $\lambda$, the $\text{SR}^{\text{rsma}}$ increases
the bounds w.r.t upper bound of $\alpha$ are $0$ are functions of the same set of we cannot present the plots. Furthermore, similar results are obtained by varying $\tau$ are approximately equal to the $\alpha$ lower and upper bound of $\alpha$ are greater than their respective OMA rates. Moreover, the bounds are almost equal to the theoretical bounds computed using $\alpha$ bounds using simulation. We have numerically computed $\alpha$, $\lambda$, and $\tau$, which are validated using simulation. In our future work, we aim to compute the closed-form expressions of the lower bound of $\lambda$ and $\alpha_{UB}$. We also aim to derive bounds on $\beta$ in our future work.

V. CONCLUSION

In this letter, we have derived the bounds on the power allocation coefficients of the common and private messages and the fraction of the individual common message of the users for which the RSMA rates of users will be greater than their respective OMA rates. We have validated the derived bounds using simulation. We have numerically computed $\alpha_c$, $\lambda$, and $\tau$, which are validated using simulation. In our future work, we aim to compute the closed-form expressions of the lower bound of $\lambda$ and $\alpha_{UB}$. We also aim to derive bounds on $\beta$ in our future work.

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