ON THE FATE OF CLOSE-IN EXTRASOLAR PLANETS

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ABSTRACT

It has been shown from the current observational data that there is a possible mass-period correlation for extrasolar planets, and this correlation is, in fact, related to the absence of massive close-in planets, which are strongly influenced by tidal interaction with the central star. We confirm that the model in Paetzold & Rauer is a good approximation for the explanation of the absence of massive close-in planets. We thus further determine the minimum possible semimajor axis for these planets to be detected during their lifetime and also study their migration timescale at different semimajor axes by the calculations of tidal interaction. We conclude that the mass-period correlation at the time when these planets were just formed was less tight than it is now observed if these orbital migrations are taken into account.

Subject headings: celestial mechanics — planetary systems — solar system: formation — solar system: general — stellar dynamics

1. INTRODUCTION

The number of discovered extrasolar planets has been increasing quickly during recent years. According to the Extrasolar Planets Catalog1 maintained by Jean Schneider, in 2002 May there were about 77 extrasolar planets around 69 main-sequence stars. These planets with a mass range from 0.16 to 17 Jupiter masses \((M_J)\) have semimajor axes from 0.04 to 4.5 AU and also a wide range of eccentricities. Interestingly, there is a planet moving on an extremely elongated orbit \((e = 0.927)\) around the solar-type star HD 80606 (Naef et al. 2001). These exciting discoveries provide great opportunities to understand the formation and evolution of planetary systems.

For example, Jiang & Ip (2001) showed that interaction with the disk is important to explain the original orbital elements during planetary formation. Yeh & Jiang (2001) analytically showed that the scattered planets should in general move on an eccentric orbit, and thus the orbital circularization must be important for scattered planets if they are now moving on nearly circular orbits (see Jiang & Yeh 2002 and I. G. Jiang & L. C. Yeh 2003, in preparation, for a follow-up).

In addition to the dynamic studies, Tabachnik & Tremaine (2002) used the maximum likelihood method to estimate the mass and period distributions of extrasolar planets and found there is a mass-period correlation, but they attributed their findings to the observational selection effect.

However, Zucker & Mazeh (2002) claimed that this mass-period correlation cannot be completely explained by the observational selection effect. They did some Monte Carlo simulations and show the real dependency between the mass and period of extrasolar planets. This mass-period correlation gives the paucity of massive close-in planets. Since they are supposed to be the easiest to detect, Zucker & Mazeh (2002) said this paucity was unlikely to be the result of any selection effect.

Paetzold & Rauer (2002) have reported tidal interaction as a possible explanation for the absence of massive close-in planets. They defined “critical mass” to be the maximum mass that the planet can have and survive under the tidal interaction with the central star for a particular semimajor axis. They determined the critical mass as a function of the semimajor axis for some assumed stellar dissipation factors and the ages of the planetary systems. Their results showed that most planetary systems are located at the permitted region of the ”critical mass-semimajor axis” plot (their Fig. 3), except the \(\tau\) Boo system, which needs more careful treatment for the assumed parameter values.

However, if these planets could be formed a bit farther from the central star initially, they should still survive under the tidal interaction and thus might be detected during the inward migration. One should keep in mind that the locations at which the planets are detected are not where they were formed. The planets from a more distant location could migrate inward to the region closer to the central star and probably have some chance of being detected by us.

To further investigate this problem, we carefully study the planetary migration due to tidal interaction. We try to include the effect of orbital eccentricity at the beginning, and we confirm that the model used in Paetzold & Rauer (2002) is a good approximation. We thus use a model similar to that of Paetzold & Rauer (2002) for the rest of the calculations. We describe our basic models for tidal interaction in § 2, and the results are in § 3. We provide concluding remarks in § 4.

2. THE MODELS FOR TIDAL INTERACTION

A tide is raised on the central star by the close-in planet because the force experienced by the side of the central star facing the planet is stronger than that experienced by the far side of the central star. We consider below the models for planets on both circular and eccentric orbits.

2.1. Circular Orbits

If the close-in planet is moving on a circular, equatorial orbit, according to the tidal potential theory, this planet
would change its orbit following the formula below:

$$\frac{da}{dt} = \text{sign}(\Omega - n) \frac{3k m}{Q M} \left(\frac{R}{a}\right)^5 na,$$

where $a$ is the semimajor axis, $t$ is the time, $\Omega$ is the rotating angular speed of the central star, $k$ is the stellar Love number, $Q$ is the tidal dissipation function, $m$ is the planetary mass, $M$ is the mass of the central star, $R$ is the central star’s radius, and $n$ is the orbital mean motion, which is determined by

$$n = \sqrt{\frac{G(M + m)}{a^3}}. \quad (2)$$

We set $k = 0.2$ (Murray & Dermott 1999) and take $Q = 3.0 \times 10^5$ (the average value in Pätzold & Rauer 2002).

The above formula provides a good simple tool to study the tidal orbital decay for close-in planets. However, in fact, most discovered planets have a certain number of orbital eccentricities. Some of these eccentricities are even very big. We plan to include the effect of eccentricity in the calculations by the equations in the following subsection.

2.2. Eccentric Orbits

We know that the angular momentum is related to orbital eccentricity $e$. Thus, the evolution of semimajor axis $a$ due to tidal interaction should depend on eccentricity $e$ because the tidal torque changes the orbital angular momentum of planets.

The mechanical energy decreasing rate $dE/dt$ due to tidal interaction is

$$\frac{dE}{dt} = \Gamma \left(\frac{\Omega - d\theta}{dt}\right), \quad (3)$$

where $\Gamma$ is the magnitude of the torque, $\Omega$ is the spin angular speed of the central star, and $d\theta/dt$ is the orbital angular speed of the planet at a particular time.

Then $\Gamma$ can be approximated by

$$\Gamma = \frac{3}{2} \frac{k}{e^2} \frac{G m^2}{a^5} R^5 \frac{1}{Q}. \quad (4)$$

These parameters are defined in the last subsection.

The orbital angular speed of the planet can be expressed as

$$\frac{d\theta}{dt} = \frac{h}{r^2}, \quad (5)$$

where $h = [G(M + m)a(1 - e^2)]^{1/2}$ and $r$ is approximated as $r = a(1 - e \cos nt)$. Therefore,

$$\frac{dE}{dt} = \frac{3}{2} \frac{k}{e^2} \frac{G m^2}{a^5} R^5 \frac{1}{Q} \left[\Omega - \sqrt{G(M + m)a(1 - e^2)} \right] \left[\frac{\sqrt{G(M + m)a(1 - e^2)}}{a^2(1 - e \cos nt)}\right]. \quad (6)$$

On the other hand, the mechanical energy of the system can be expressed as

$$E = \frac{1}{2} I \Omega^2 - G \frac{M m}{2a}, \quad (7)$$

$$\frac{dE}{dt} = I \frac{d\Omega}{dt} + G \frac{M m \frac{da}{dt}}{2a^2}. \quad (8)$$

By Kepler’s third law,

$$G(M + m) = \frac{n^2 a^3}, \quad (9)$$

we have

$$\frac{dE}{dt} = I \frac{d\Omega}{dt} + \frac{M m}{2(M + m)} \frac{na^2}{a^4} \frac{da}{dt}. \quad (10)$$

Further, the angular momentum of the system is

$$L = I \Omega + \frac{M m}{M + m} a^2 n(1 - e^2)^{1/2}, \quad (11)$$

where $I$ is the moment of inertia of the central star and $e$ is the orbital eccentricity, and we have ignored the contribution from the spin of the planet.

By the conservation of angular momentum, $dL/dt = 0$, we have

$$I \frac{d\Omega}{dt} = -\frac{1}{2} \frac{M m}{M + m} \frac{na^2}{a^4} \frac{ed\Omega}{dt}.$$

In general, both terms on the right-hand side of equation (12) should be considered. The second term divided by the first term would be

$$e^2(1 - e^2) \left[\frac{63}{6} \frac{Q}{k \mu_p Q_p} \left(\frac{M}{m}\right)^2 \left(\frac{R_p}{R}\right)^5\right], \quad (13)$$

where equation (4.198) in Murray & Dermott (1999) has been used to estimate the value of $de/dt$, and we use $\mu_p$, $Q_p$, and $C_p$, etc., to replace the corresponding parameters $\mu$, $Q$, and $C$, etc., of equation (4.198) in Murray & Dermott (1999). If we use Jupiter as an example, this ratio would be about 1 when $e = 0.1$ and $Q/(k \mu_p Q_p) = 1$.

We plan to consider the simple case when $e^2 Q/(k \mu_p Q_p)$ is small enough and the second term can be ignored. We will leave the more general case in which both orbital migration and circularization need to be included to future work.

Thus,

$$\frac{dE}{dt} = \frac{1}{2} \left(n - \Omega \sqrt{1 - e^2}\right) \frac{M m}{M + m} \frac{na^2}{a^4} \frac{da}{dt}. \quad (14)$$

From equations (14) and (6), we have

$$\frac{da}{dt} = 3k \frac{G m^2}{a^7} \frac{R^5}{Q} \frac{M + m}{M m} \left(n - \Omega \sqrt{1 - e^2}\right)^{-1} \left[\frac{\Omega}{\sqrt{G(M + m)a(1 - e^2)}}\right] \left[\frac{\sqrt{G(M + m)a(1 - e^2)}}{a^2(1 - e \cos nt)}\right]. \quad (15)$$

where $\Omega$ is related to $a$ by equation (11). Given an assumed initial angular momentum $L$, etc., $a$ can be solved numerically by equation (15).

3. RESULTS

By applying the equations in the preceding section, we can study the inward migration of planets due to tidal interaction. As different cases, we place the planet at five different initial semimajor axes: 0.02, 0.03, 0.04, 0.05, and also 0.06 AU. Each panel in Figure 1 shows the plots of these semimajor axes as a function of time for a mass of a particular
value. Thus, the five curves on each panel of Figure 1 result when the planetary masses for Figures 1a–1d are assumed to be 5, 2, 1, and 0.5 \( M_J \), respectively. Since 2 Gyr is about the age of the \( \tau \) Boo system, we thus regard 2 Gyr as the typical age of extrasolar planetary systems. Those planets that can survive for 2 Gyr under the tidal interaction can possibly be detected.

All the curves are the results when we assume the planets move on circular orbits, and the triangles are the results when the planets move on eccentric orbits [assume \( e = 0.5 \) and \( \frac{e^2 Q}{(k\mu_0)_{\text{orb}}} \) is small enough]. In general, the results of eccentric orbits are quite similar to the results of circular orbits and the ignorance of eccentricity will not affect the determination of the planet survival timescale, etc. This confirms that the equations used in Pätzold & Rauer (2002) are good approximations, and we thus use the model of circular orbits for all the rest of the calculations.

Figures 1a–1d show that when the initial semimajor axis \( a_i = 0.06 \) AU the planet would have only a tiny migration during 2 Gyr. The planet can easily survive under the tidal interaction. If the initial semimajor axis \( a_i = 0.05 \) AU, the orbital semimajor axis decays a bit more. If the initial semimajor axis \( a_i = 0.04 \) AU, the planet falls into the central star when \( t \) is about 1 Gyr for the case of 5 \( M_J \) but still survives for all other cases. When the initial semimajor axis \( a_i = 0.03 \) AU, the planet falls into the central star within 1.5 Gyr. If the initial semimajor axis \( a_i = 0.02 \) AU, the planet approaches the central star almost immediately.

The detection probability for a particular range of semimajor axes depends on how much time the planet can survive around that range. We plot the time the planet should spend from one semimajor axis \( a_j \) to another semimajor axis \( a_{j+1} \) (we assume \( a_j > a_{j+1} \)) during the orbital decay in Figure 2. There are two sets of \( a_j \); one makes \( \delta a = a_j - a_{j+1} = 0.005 \)
AU (dotted lines), another set has \( \delta a \equiv a_j - a_{j+1} = 0.0025 \) AU (solid lines). Figures 2a–2d are the results when we set the planetary mass to be 5, 2, 1, and 0.5 \( M_J \), respectively.

In Figures 2a and 2b, the planet spends more than 1 Gyr at around 0.05 AU, and thus the planet is likely to survive during 2 Gyr. However, the planet only stays around 0.04 AU for about 0.5 Gyr and around 0.03 AU for about 0.2 Gyr only. These timescales are considerably smaller than the age of the planetary system, and thus the planets initially formed around these locations are very unlikely to be observed.

When the planetary mass is smaller as in Figure 2c and 2d, the planet can survive for much longer (more than 1.5 Gyr) around 0.04 AU and still only stays around 0.03 AU for on the order of 0.5 Gyr. This implies that the probability the planet will be detected at around 0.03 AU is very small.

Figure 3 shows the \( \ln(a/AU) - \ln(M/M_J) \) plots for all discovered extrasolar planets, where \( M \) is the planetary mass and \( a \) is the semimajor axis. The data for these planets are from Extrasolar Planets Catalog, 2002 May. In Figure 3a, we take \( a \) to be the values of current semimajor axes of these discovered extrasolar planets. However, in Figures 3b–3d, we take \( a \) to be the planetary semimajor axes backward in time for 2, 6, and 12 Gyr, respectively. The values of \( a \) backward in time can be obtained by equation (1).

In Figures 3b–3d, we find that most planets do not move on the \( \ln(a/AU) - \ln(M/M_J) \) plane, but some of them do move a lot when they are plotted backward in time.
It is quite obvious that the planets line up on the left side of the plots in Figures 3b–3d, and the position of this line hardly moves from Figure 3b to 3d. This line can be approximated by

\[
\ln\left(\frac{M}{M_J}\right) = \frac{1}{2} \left[ 28 \ln\left(\frac{a}{\text{AU}}\right) + 62 \right].
\]

We can also see that those planets that do not move much are all on the right side of this line. This line can thus be regarded as the “critical line”: all planets on the right side of this line would not migrate much during their lifetime, but all the planets standing on this critical line of Figures 3b–3d would move to the upper left corner of Figure 3a after 2, 6, or 12 Gyr, and finally all the planets on the left side of this line would migrate inward quickly to approach the central star and thus cannot be detected.

4. CONCLUDING REMARKS

As dynamic friction successfully explained the orbit of the Sagittarius dwarf galaxy (Jiang & Binney 2000), tidal interaction can indeed explain the current observed mass-period correlation reported by Zucker & Mazeh (2002). The results in Figure 1 give us the full picture of inward migration due to tidal interaction. We found that 0.03 AU seems to be the critical semimajor axis for a planet with mass of the order of the \( \tau \) Boo system to survive for 2 Gyr. This is consistent with the current observational results that the smallest semimajor axis of a discovered planet is about 0.04 AU.

On the other hand, we can also check this minimum possible semimajor axis from another point of view. In Figure 2 the timescale for a planet’s survival is smaller if the planet is closer to the central star initially and the time a planet can stay around 0.03 AU is considerably less than 2 Gyr, which was regarded as the typical age of these planetary systems. Because the timescale is too short, the probability of detecting the planet is very small.

Moreover, we discover the interesting observational “critical line” on the \( \ln(a/\text{AU}) - \ln(M/M_J) \) plane. All the planets on the left side of this line would migrate inward quickly to approach the central star and thus cannot be detected.

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Fig. 3.—Plot of \( \ln(a/\text{AU}) - \ln(M/M_J) \) for all discovered extrasolar planets in the Extrasolar Planets Catalog, 2002 May. (a) Current discovered configuration, (b) backward in time for 2 Gyr, (c) backward in time for 6 Gyr, and (d) backward in time for 12 Gyr.
Therefore, the initial configuration on the \( \ln(a/\text{AU}) - \ln(M/M_J) \) plane might be composed of all the points on Figure 3b plus those points which might have been on the left side of the “critical line” about 2 Gyr ago but have disappeared in Figure 3a because these planets have already fallen into the central star. From this point of view, even though there is a correlation between mass and period for current discovered planets as claimed by Zucker & Mazeh (2002), this correlation could be weaker or less obvious at the time when these planets were just formed since we can add an arbitrary number of “possible” planets on the left side of our observational “critical line” if there is no difficulty in theory for planets to form there. This tells us that we should be careful when we try to link the mass-period correlation to the theory of planetary formation.

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REFERENCES

Jiang, I.-G., & Binney, J. 2000, MNRAS, 314, 468
Jiang, I.-G., & Ip, W.-H. 2001, A&A, 367, 943
Jiang, I.-G., & Yeh, L.-C. 2002, Int. J. Bifurcation and Chaos, in press
Murray, C. D., & Dermott, S. F. 1999, Solar System Dynamics (Cambridge: Cambridge Univ. Press)
Naef, D., et al. 2001, A&A, 375, L27
Pätzold, M., & Rauer, H. 2002, ApJ, 568, L117
Tabachnik, S., & Tremaine, S. 2002, MNRAS, 335, 151
Yeh, L.-C., & Jiang, J.-G. 2001, ApJ, 561, 364
Zucker, S., & Mazeh, T. 2002, ApJ, 568, L113