An Improved Particle Swarm Optimization algorithm for Optimal Control of ASP Flooding

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Abstract. An optimization for injection of alkali/surfactant/polymer (ASP) flooding in oil recovery is considered in this paper. This optimal control problem (OCP) is formulated as a parameter identification system, where the objective function is revenue maximization and the governing equation is multiphase flow in porous media. We use the ASP concentrations and slug size as the control and give the pointwise constraint for the control. An improved particle swarm optimization (IPSO) which is a particle swarm optimization (PSO) with second-order oscillatory in velocity, is applied to solve the OCP. Finally, an example of the OCP for ASP flooding is exposed and the results show that the IPSO method is effective and feasible.

1. Introduction
With the arrival of high water cut period, the production of oil is increasingly difficult to meet the demands for economic development. A great number of oil recovery methods are proposed. Among them, the alkali/surfactant/polymer (ASP) flooding which is an important tertiary oil recovery technology can enhance oil production obviously. The advantage here is that the basic idea is to utilize three displacing agents could change the physicochemical property to enhanced oil recovery [1]. Because of the high price of displacing agents, it is important to optimize the injection strategies and maximize the profit as much as possible. The optimal control method has been researched in ASP flooding in recent years. Lei et al. [2] presented a mixed integer iterative programming to optimize the polymer flooding. Zerpa et al. [3] used UTCHEM to simulate the multiple surrogates optimization of ASP flooding process.

The optimal control problem of an ASP flooding process is studied in this paper. The objective function of the OCP is given by maximizing the economic benefit which could be non-convex. The governing equations are a set of partial differential equations (PDEs), which are a continuity equation, three diffusion equations for alkali/surfactant/polymer respectively. The ASP injection process could be regarded as a multi-stages process. Each stage is the slug. The injection concentration of every slug should be constant in the scheme. The ASP injection strategies which should be optimized includes ASP concentrations and slug size. The ASP concentration should be given the constraint for upper and lower.

For solving the OCP of ASP flooding proposed, the maximum principle and gradient based methods are not appropriate due to the non-convex. The new approach introduced here is based on PSO. Many optimal algorithms are developed and suitably updated for OCP in the last one decade. Particle swarm optimization algorithm is a evolutionary computation method proposed by Kennedy and Eberhart [4] in 1995, inspired from the motion of bird groups. It has been successfully applied in many research fields due to the advantages of simple operation, fast convergence speed, few
parameters and easy implementation. However, there still exits some disadvantages such as lose population diversity, fall into local extremal and low precision in general PSO. This paper presents a IPSO method which makes use of the oscillatory in the parameter value to regulate the global and local search capabilities. It improve the convergence speed and the accuracy of the PSO.

The rest of this paper is organized as follows. In section 2 the optimal control model of ASP flooding is built. In section 3 the steps of solving OCP based on a IPSO method are introduced. In section 4 an example of OCP for two dimensional (2-D) ASP flooding accompanied with the optimal results is given. And in section 5 some conclusions are derived.

2. Optimal Control Model

In this section, we would give the optimal control model for the ASP flooding. This is a distributed parameter control problem which aim is to get the maximum benfit by the optimal injection scheme. In the optimal control model, the injection volume of the alkali, surfactant, and polymer in the injection wells is the control variable, and the state variables are the pressure, grid concentration, and water saturation.

2.1. Object function

For ASP flooding, we might wish to increase the profit and reduce the producing cost. The object function is given mathematically by,

$$\max_{u} J = \int_0^t \int_\Omega \left[ \chi(1 - f_w) q_{out} - \chi_0 q_{in} u_0 \right] dx \, dt,$$

(1)

where $\chi$ is the discount rate, $q_{in}$ and $q_{out}$ are the volume flow rate of injection and production, $\chi_0$ is the price of three displacing agents, and other parameters would be given in next subsection.

2.2. Governing equations

The governing equations for the optimal control model are the model about interaction of alkali, surfactant, and polymer. Let $\Omega \subseteq \mathbb{R}^2$ denote the domain of reservoir with boundary $\partial \Omega$, and $\bar{n}$ denotes the unit outward normal on $\partial \Omega$. The governing equations are following the model in [5]. Given a point $(x, y) \in \Omega$ and an instant $t \in (0, t_f)$, the governing equations are as following.

The seepage continuity equation is

$$\frac{\partial S}{\partial t} = -\frac{Q}{A\phi} \nabla f_w.$$

(2)

The adsorption diffusion equation of surfactant is

$$\phi S_w \frac{\partial C_s}{\partial t} = -v_w \nabla C_s + \nabla \left( D_s \phi \nabla C_s \right) - \rho_s \frac{\partial \Gamma_s}{\partial t}.$$

(3)

The adsorption diffusion equation of polymer is

$$\phi S_w \frac{\partial C_p}{\partial t} = -v_w \nabla C_p + \nabla \left( D_p \phi \nabla C_p \right) - \rho_p \frac{\partial \Gamma_p}{\partial t}.$$

(4)

The adsorption diffusion equation of alkali is

$$\phi S_w \frac{\partial C_a}{\partial t} = -v_w \nabla C_a + \nabla \left( D_a \phi \nabla C_a \right) - \rho_a \frac{\partial \Gamma_a}{\partial t} - R_a,$$

(5)

where $a$ is the expression of alkali, $s$ is the expression of surfactant, $p$ is the expression of polymer, $C_a, C_s, C_p$ are the concentration of alkali, surfactant and polymer, respectively. $D_a, D_s, D_p$ are the diffusion coefficient of the three variable, $S_w$ is the water saturation, $A$ denote the core cross section area, $f_w$ denote $s$ the moisture content, $v_w$ is the seepage speed of water phase. $\rho_s$ is the core density, $\Gamma_s, \Gamma_p, \Gamma_a$ are the adsorbing capacity of core for different displacing agents, and $R_a$ denotes the alkali consumption.
The initial conditions are
\[ S_w(x, y, t)|_{t=0} = 1 - S_o, \quad C_o(x, y, t)|_{t=0} = 0, \quad \theta = \{a, s, p\} . \quad (6) \]
For the boundary, we assume the injection point is \((0,0)\) and \(\Gamma = \delta \Omega \setminus (0,0)\). So the boundary conditions are
\[ f_w(0,0, t) = 1.0; \quad S_w(0,0, t) = 1.0 - S_o; \quad \frac{\partial C_o(0,0, t)}{\partial n} = \frac{v_w}{D_o \phi}(C_o - u_\theta) , \quad (7a) \]
\[ \frac{\partial S_w}{\partial n}|_n = 0, \quad \frac{\partial f_w}{\partial n}|_n = 0, \quad \frac{\partial S_o}{\partial n}|_n = 0 , \quad (7b) \]
where \(u_\theta\) is the injection concentration of displacing agents. And we could control the concentration at different strategys as
\[ u_\theta = \begin{cases} u_{\theta i} & t_{i-1} < t \leq t_i, \ i = 1,2,\ldots, P, \\ 0 & t_p < t \leq t_f , \end{cases} \quad (8) \]
where \(P\) is the number of slugs, \([t_{i-1}, t_i]\) and \(u_{\theta i}\) denote the slug size for the i-th slug and the injection concentration of the alkali, surfactant and polymer in the i-th slug.

2.3. Constraint condition
The objective function (1) should be given the constraint condition of ASP concentration
\[ u_{\theta \text{min}} \leq u_\theta \leq u_{\theta \text{max}}, \quad (9) \]
where \(u_{\theta \text{min}}\) and \(u_{\theta \text{max}}\) denote the minimal and maximum injection concentrations for alkali, surfactant and polymer, respectively.

3. Optimal control problem solution based on MPSO
PSO (Particle swarm optimization) is a stochastic search method inspired by the coordinated motion of animals living in groups. This technique works by all group members is to find the most favorable location within a specified search space through the change in direction and velocity of each individual particle is the effect of cognitive, social and stochastic influences. IPSO method is a special PSO method which accelerate convergence by increasing oscillation during parameter selection. This method would be proposed for solving IOP and we would show the details in subsections.

3.1. IPSO optimization
The difference between IPSO and PSO is in the updating of the velocity by the habit of bug. In this part we use the velocity update method which is proposed by Jiang L. [6] as an operator to modify the PSO. The conventional PSO algorithm is implemented as following: suppose that the current position \(x_i\) of the particle \(x = (x_1, x_2, \ldots, x_n)\) is to be given \(,\) the new position \(x_{i+1}\) is calculated using the following equation:
\[ v_{i+1} = \phi v_i + \alpha_i \left[ \gamma_{1,i} (P_i - x_i) \right] + \alpha_i \left[ \gamma_{2,i} (G - x_i) \right] \quad (10) \]
\[ x_{i+1} = x_i + v_{i+1} \quad (11) \]
where the vectors \(x^k\) and \(v^k\) are current position and velocity of the i-th particle in the k-th generation. \(P_i\) is the personal best position of each individual and \(G\) is the global best position observed among all particles up to the current generation. The parameters \(\gamma_{1,i}, \gamma_{2,i} \in [0,1]\) are uniformly distributed random values and \(\phi\) is the inertial weight. The algorithm is easy to fall into the local optimum. So the oscillatory convergence needs to be added into the algorithm to jump out of the local optimum solution and improve the search performance and accuracy. The modified PSO algorithm is as following:
The modified velocity of the i-th particle in the k-th generation and \( \beta, \beta_1, \beta_2, \xi_1, \xi_2 \) are the second-order oscillations parameter.

### 3.2. Solution of OCP for ASP flooding based on IPSO

In this part we would show how to solve the optimal control problem (1)-(9) by the IPSO method. Since there are \( P \) slugs in the ASP flooding process, we switch time for every slug together and define the vector \( \mathbf{q} = [u_{10}, u_{20}, \cdots, u_{p0}, t_1, t_2, \cdots, t_P] \) as the optimization variables and the optimal control problem could be parameterized as

\[
\max_{\mathbf{q}} J = \sum_{j=1}^{P} \int_{t_{j-1}}^{t_j} \int_{\Omega} \left( f_w(1 - f_w)q_{\text{out}} - \xi_\rho q_{\text{in}} u_{\rho} \right) dx \, dt + \int_{t_P}^{t_f} \int_{\Omega} \left( 1 + \chi \right)^{-\chi} \left[ \xi_\rho (1 - f_w)q_{\text{out}} \right] dx \, dt
\]

where \( t_0 = 0 \).

As we know, the optimal control problem need to be solved coupled the PDE equations with the right initial and boundary conditions. The finite volume method is used to solve the PDE equations of the ASP flooding system (2)-(7b). By the parameterization, the original optimal control problem could be changed into the MOCP as the following form:

\[
\begin{align}
\max J(q_1, q_2) \\
\text{s.t.} \\
q_{1\max} \leq q_1 \leq q_{1\max} \\
q_{2\max} \leq q_2 \leq q_{2\max} \\
q_1 = [u_{10}, u_{20}, \cdots, u_{p0}] \\
q_2 = [t_1, t_2, \cdots, t_P].
\end{align}
\]

(15)

The given IPSO algorithm is used to solve the resulted MOCP (15).

### 4. Numerical example of optimal control for ASP flooding

In order to show the efficiency of the IPSO given in Section 3, a numerical example is given in this section. We also use the PSO algorithm to compute the example as a comparison.

Considering the 2-D optimal control problem for ASP flooding process, the domain \( \Omega = [0,1]^2 \), and there is a ASP injection well at (0,0) and an oil production well at (1,1). The optimal control problem is formulated by (15), the governing equation (2)-(7b) and the constraint conditions (9). We consider four slugs for the polymer concentration. The time domain is (0,1). The polymer value constraint condition is \( 0 \leq u_{\rho} \leq 1, i = 1, 2, 3 \). The data for the model is given in Table 1.

| sign | value | sign | value |
|------|-------|------|-------|
| \( x \) | 7 | \( \zeta_\rho \) | 40 |
| \( \zeta_\rho \) | 4 | \( Q \) | 0.5 |
| \( A \) | 18 | \( \phi \) | 0.4728 |
| \( v_w \) | 0.0045 | \( D_s \) | 0.2 |
| \( \rho_r \) | 2.0 | \( D_p \) | 0.6 |
The ASP concentrations solved by PSO is $u_a = [0.7730, 0.5407, 0.5041]$, $u_s = [0.2494, 0.1744, 0.1626]$, $u_p = [0.5411, 0.3785, 0.3529]$ and the switching time $t = [0.0436, 0.1323, 0.1645]$. The performance is $J = 1.0196$. The optimal performance obtained by IPSO is $J_{opt} = 1.0342$. The optimal injection concentrations $u_{a,opt} = [0.8071, 0.4873, 0.5811]$, $u_{s,opt} = [0.2522, 0.1523, 0.1816]$, $u_{p,opt} = [0.5650, 0.3411, 0.4067]$. The optimal switching time $t_{opt} = [0.0464, 0.1509, 0.1886]$. From the results we could obtain that the IPSO is more accurate than PSO for this problem.

5. Conclusion
The optimal control problem of ASP flooding is successfully solved by IPSO in this paper. The proposed IPSO method, which which accelerate convergence by increasing oscillation during parameter selection, is an effective method for solving OCP. The non-convex OCPs which are difficult to solve by conventional methods such as maximum principle and gradient based algorithm can be solved easily by the new approach. An example for the optimal injection strategies of ASP flooding is studied. Compared with the solution based on PSO, the results show the effectiveness of IPSO for solving the optimal control problem of ASP flooding.

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