Stress-strain state of elastomeric vibroseismic insulators

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Abstract. A method for solving problems of viscoelastic deformation of structures made of weakly-compressible elastomers, based on the application of the hereditary Boltzmann-Volterra theory, is considered. To consider the weak compressibility and eliminate the "false shift" effect, the moment scheme of the finite element is used.

1. Introduction

Every year over a hundred earthquakes occur on the planet, the victims of which are people who die under the rubble of buildings. In addition, the development of technological equipment and transport systems lead to an increase in the vibration impact on buildings and structures. Therefore, the creation and improvement of various structures that reduce the risk of destruction during vibration and seismic impacts [1,2], is extremely important. One of the most effective devices are vibration and seismic insulators based on rubber-metal structures (Figure 1). The use of such vibration-seismic protection systems allows: to ensure protection of objects during earthquakes and man-made impacts, while the estimated cost of construction is reduced by 3–6%, the material consumption of buildings and structures is reduced by 5–10%; the scope of application of standard structures is expanding by building up areas with increased seismicity, the permitted building height buildings is also increasing [3].

Vibroseismic insulators protect machines and buildings against seismic impacts not only in horizontal and vertical planes, but also from torsion. It is believed that it is torsion in combination with unfavorable factors, in particular with the vertical component of shocks, that is the main cause of catastrophic destruction during earthquakes. In addition, the use of rubber-metal layered vibration insulators allows protecting buildings and people in them from the effects of the subway, road and rail transport [4].

Figure 1. General view of rubber-metal high-damping structures of rubber seismic blocks on natural rubber.

The main damper in such structures is an elastomeric element, the dynamic properties of which depend on the initial stresses arising under static loading.

2. Result of research

Due to the specificity of the mechanical properties of rubber, the determination of the parameters of the stress-strain state of rubber vibration insulators is a difficult task.

The finite element method (FEM) [5-8] is a universal numerical method for calculating rubber vibration seismic insulators, which allows one to considering the asymmetry of loads and fixings. The use of finite element models makes it possible to obtain a complete picture of the stress-strain state. At the same time, the traditional FEM does not allow considering such a property of rubber as the weak
compressibility of the material; therefore, we will use a specially developed moment finite element scheme for weakly compressible materials [7]. The peculiarity of the moment scheme lies in the triple approximation of displacement vector components, components of deformation tensor and volume change function. When expanding in a Maclaurin series the functions of displacements, functions of deformations and functions of volume change in the vicinity of the origin of the local coordinate system of an element, only the associated components are retained in the expansion of these functions. In addition, the use of MSFE avoids the "false shift" effect.

In the study of compressible and weakly compressible elastomers, a special place is occupied by the integral law of the state, which connects small increases in stresses and strains based on the ordinary Hooke’s law in metric of deformed volume. It is widely used in solving static and quasi-static problems in the field of large deformations for nonlinear-elastic and thermo-viscoelastic bodies that require integration by parameter [7]. The increment in stresses and strains is related by the ratio

\[
\sigma_{ij} = 2\mu \left( G^{mi} G^{nj} d\varepsilon_{mn} - \frac{1}{3} G^{ij} d\theta \right) + B G^{ij} d\theta ,
\]

(1)

For a linear material, the volume compression function is determined by the formula:

\[
\theta = \varepsilon_{ii} = I_1 (\hat{\varepsilon}),
\]

(2)

where \( I_1 (\hat{\varepsilon}) = I_1 \) is the first invariant of the Cauchy-Green’s strain tensor.

Applying a triple approximation of displacements, deformations and volume change functions using power functions of a special form

\[
\psi_{(pqr)}^{(pqr)} = \frac{(x_1)^p (x_2)^q (x_3)^r}{p! q! r!},
\]

(3)

and establishing a connection between the corresponding components of the expansions, one can write down the equilibrium equations

\[
\begin{bmatrix} A^T & F_{\mu}^T & C_{\mu}^{ij} & F_{\mu}^T \end{bmatrix} \begin{bmatrix} F_{\mu} \end{bmatrix} A\{u_i\} + \begin{bmatrix} A^T & F_{\phi}^T & C_{\phi}^{ij} & F_{\phi}^T \end{bmatrix} \begin{bmatrix} F_{\phi} \end{bmatrix} A\{u_i\} = \{R_i\},
\]

\[
\begin{bmatrix} G^{\alpha} + G_{\phi}^{\alpha} \end{bmatrix} \{u_i\} = \{R_i\},
\]

(4)

where \( A \) is a matrix of correspondence between functions of the form of a finite element and basis power functions;

\( F_{\mu}, F_{\phi} \) are transformation matrices of the expansion of displacements and expansions of deformations and the function of changing the volume, respectively;

\( G^{\alpha} + G_{\phi}^{\alpha} \) are the stiffness matrix of a weakly compressible solid.

To considering the nonlinear behavior of the material, the volume deformation function can be calculated by the formula

\[
\theta = \left( \frac{dV - dv}{dv} \right) = \sqrt{\frac{G}{g}} - 1 = \sqrt{I_3} \left( \hat{G}^{\alpha} \right) - 1.
\]

(5)

Then for nonlinear weakly compressible material the modified Hooke's law (1) has the form:

\[
\sigma_{ij} = \int_0^1 \left( G^{mi} G^{nj} - \frac{1}{3} G^{ij} G^{mn} \right) d\varepsilon_{mn} - \int_0^{\hat{\varepsilon}} B \left( \sqrt{I_3} - 1 \right) dG^{ij}.
\]

(6)

The strain tensor can be represented as the sum of linear and nonlinear terms.
\[ \varepsilon_{mn} = \varepsilon_{mn}^L + \varepsilon_{mn}^N, \]  
(7)

where \( \varepsilon_{mn}^L = \frac{1}{2} \left( C^j_{m} u_{j,m} + C^j_{n} u_{j,n} \right) \) - linear terms of strain tensor,

\[ \varepsilon_{mn}^N = \frac{1}{2} u_{j,m} u_{j,n} \] - nonlinear terms of strain tensor, \( u_i \) - displacement vector components,

\( C^j_{n} \) - transformation tensor components.

The variation of the strain energy in this case has the form

\[ \delta W = \iiint \left( \sigma_{ij}^L + \sigma_{ij}^N \right) \delta \left( \varepsilon_{ij}^L + \varepsilon_{ij}^N \right) dv, \]  
(8)

In this case, the solution can be obtained using the modified Newton-Kantorovich method.

To describe the viscoelastic properties of rubber, the hereditary Boltzmann-Volterra theory and the stress tensor are used in the form of an integral equation using the instantaneous and equilibrium elastic characteristics of the material.

\[ \sigma^H(t) = E^{ij}(t) \left( \varepsilon_{ij}(t) - \int_0^t K(t - \tau) \varepsilon_{ij}(\tau) d\tau \right). \]  
(9)

The Rabotnov’s fractional-exponential nucleus is used as a Volterra integral equation nucleus

\[ K(t - \tau) = \lambda(t - \tau)^{\alpha} \sum_{n=0}^{\infty} \left( -\beta - \lambda \right)^n \frac{(t - \tau)^{n(1+\alpha)}}{\Gamma [(n+1)(1+\alpha)]}, \]  
(10)

where \(-1 < \alpha < 0; \beta > 0; \lambda > 0\) - material constants.

Most often, structures made of highly elastic materials work in conditions of large deformations, when linear dependences no longer work. When calculating thin-layer elastomeric elements of structures operating in conditions of limited deformation, it is necessary to teach the weak stylistics of the elastomer. To describe the behavior of highly elastic materials, considering compressibility and nonlinearity, special laws of state are required, which considering the properties of material at large deformations.

One of the distinctive properties of elastomers is that when deformed in a highly elastic state, the balance between forces and displacements is established over a period of time. The high viscosity, the pronounced relaxation nature of the stresses, the geometrically and physically nonlinear nature of the deformation require the application of the mathematical apparatus of the nonlinear three-dimensional theory of viscoelasticity. The relationship between the components of stress and strain tensors for nonlinear viscoelastic weakly compressible material can be taken as Hooke’s law, replacing the elastic constants - the compression modulus and the shear modulus - by Volterra integral operators:

\[ \tilde{\mu} \varphi = \mu \varphi(t) - \int_{-\infty}^{t} K_{\mu}(t - \tau) \varphi(\tau) d\tau, \]  
\[ \tilde{B} \varphi = B \varphi(t) - \int_{-\infty}^{t} K_{b}(t - \tau) \varphi(\tau) d\tau. \]  
(11)

In this way, all the terms due to nonlinear and viscoelastic properties can be selected.
To solve the problem of viscoelastic deformation, we use the spatiotemporal approximation of displacements. Moving at any point of the element is approximated as:

\[ u = \sum_{i=1}^{M} u_i(t) N_i(x_k), \quad (k = 1, 2, 3) \]  

(12)

where \( u_m \) is time-dependent approximation of the displacement of the \( i \)-th node; \( N_i(x_k) \) is the basic functions of spatial variables.

The time domain can also be divided into linear elements, then enter the basic functions:

\[ u(t) = \sum_{m=1}^{n} u_m N(t) \]  

(13)

where \( u_m \) is displacement in the \( m \)-th node, ie at time \( t_m \).

The solving equations of viscoelastic deformation can be represented as

\[
\int_{t_n}^{t_{n+1}} \left[ G \left( u(t) - \int_{0}^{t} R(t - \tau) u(\tau) d\tau \right) \right] dt = \int_{t_n}^{t_{n+1}} N_n P dt
\]

(14)

Integration is performed only on the \( n \)-th period of time. Therefore, we can use the approximation of displacements (13) for \( u(t) \).

\[
K \int_{t_n}^{t_{n+1}} N_n \left[ u_n(1 - t) + u_{n+1} t - \int_{0}^{t} R(t - \tau) \left( u_n \left( 1 - \frac{\tau - t_n}{\Delta t_n} \right) + u_{n+1} \left( \frac{\tau - t \lambda n}{\Delta t_n} \right) \right) d\tau - \int_{t_n}^{t_{n+1}} N_n P dt \]

(15)

Then, assuming that the matrix \( G \) is constant on the element, equation (14) after integration takes the form:

\[ Gu_{n+1} \left[ \frac{1}{3} - \frac{R_2}{\Delta t} + \frac{R_1 t_n}{\Delta t} \right] = -Gu_{n} \left[ \frac{2}{3} - R_1 \left( 1 + \frac{t_n}{\Delta t} \right) + \frac{R_2}{\Delta t} \right] + P \]

(16)

where

\[ R_1 = \int_{0}^{t} K(t - \tau) d\tau \]

\[ R_2 = \int_{0}^{t} K(t - \tau) \tau d\tau \]

Thus, a system of equations of hereditary viscoelasticity is obtained

On the basis of the considered approach, the characteristics of the stress-strain state of a seismic support with a diameter of \( d = 500 \) mm and a height of a rubber layer of \( h = 50 \) mm are determined. Rubber grade 2959, elastic modulus \( G = 0.7 \) MPa. Poisson’s ratio \( \nu = 0.499 \). Parameters of the Rabotnov’s nucleus \( \alpha = 0.60, \beta = 1.72, \lambda = 0.38 \). Loading \( P \approx 30 \) kN.

Figures 2-5 show the results of solving the problem of the stress-strain state of the structure.
Figure 2. Dependence of axial (1) and radial (2) displacements on time.

Figure 3. Distribution of radial stresses.

Figure 4. Dependence of axial (1) and (2) radial stresses.
3. Conclusion
The damping elements of the considered vibration seismic insulators operate under conditions of preliminary stresses caused by the initial and installation loads, which affect the dissipative properties of such structures. To calculate the initial stress state of an elastomeric element, its viscoelastic properties and weak compressibility of the material are taken into account. The analysis of the distribution of compressive stresses makes it possible to evaluate the stiffness properties of the vibration isolator.

Dependences of axial and radial displacements on time and on load have been determined; dependences of axial and radial stresses; the distribution of radial stresses is constructed. The analysis of the distribution of compressive stresses makes it possible to evaluate the stiffness properties of the vibration isolator.

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