Research article

Study on \textit{Spirulina platensis} growth employing non-linear analysis of biomass kinetic models

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ABSTRACT

\textit{Spirulina platensis} has been considered a promising source of food supplement to combat malnutrition worldwide. Numerous investigations have stated its immune activity, ability to absorb CO\textsubscript{2} during the growth period, and antioxidant potential. Well-known theoretical biomass kinetic model sheds are capable of qualitative analysis of the fast microalgae growth. In this regard, we considered eight popular biomass models: Monod, Haldane, Andrews & Noack, Teissier, Hinshelwood, Yano & Koga, Webb and, Aiba model comprising analytical investigation within the numerical simulation. Besides, in this study, we establish a new mathematical biomass growth model by merging the well-known Hinshelwood and Yano & Koga models. We explored the most suitable \textit{Spirulina} growth model to minimize the overstated and understated growth trends in the assorted eight biomass kinetic models. Our findings show microalgae biomass growth and substrate diminishes along with time, and these results were compared with available experimental data. Results present a high value of R\textsuperscript{2} (0.9862), a low value of RSS (0.0813), AIC (-9.7277), and BIC (-8.2148) implied significantly fitted with the investigated data for the growth of \textit{Spirulina platensis} compared with popular eight studied models.

1. Introduction

Algae are photosynthetic organisms that can convert carbon dioxide (CO\textsubscript{2}) to foods and are used for fuel, potential drugs, high-value bio-actives, as well as numerous remarkable commercial goods (Aikawa et al., 2018; Bilal et al., 2017; Bleakley and Hayes 2017; Liu and Chen 2014; Wells et al., 2017; Yan et al., 2016). Microalgae culture has multiple promising advantages; for example, it uses abandoned land or damped canal for culturing, so it does not compete with fertile land, which means food security is not vulnerable. Algae cultivation contributes to CO\textsubscript{2} fixation and provides higher oil production compared to crop food per acre, and it helps to delete adverse materials from wastewater. Furthermore, some studies revealed that it boosts immunity and is active against heart disease, obesity, and other diseases (De Morais and Costa 2007; Metting and Pyne 1986; Metting 1996; Metzger and Largeau 2005; Ramanan et al., 2010; Singh et al., 2005; Walker et al., 2005). Recently, microalgae are getting biotechnological attention due to the nutraceutical compounds available in them (Miranda et al., 1998; Odjadjare et al., 2017). For example, algae and seaweed are excellent sources of protein comparable with that of soybean, egg, meat, and milk (Gouveia et al., 2008). Furthermore, \textit{Spirulina} has been widely used to produce biofuel, biogas, capsule, nutrition drinks and bars, noodles, fairness cream, powder, beauty soap, skincare products, lotion, shampoo, moisturizing gel, mask cream, eye zone repair gel, medicine, and other therapeutical products (Borges et al., 2013; Yan et al., 2016; Wells et al., 2017).

There is no need for fertile land or freshwater to cultivate marine algae/seaweed. But, the cultivation of microalgae and isolation process is not viable economically (Miranda et al., 1998). The filamentous cyanobacterium \textit{Spirulina platensis} grows in enormous volume in brackish water, freshwater, as well as the marine environment (Converti et al., 2006). The strains of \textit{Spirulina platensis}, initially isolated from Chad-an African lake, have seen extensive commercial production (Ciferri 1985). It has been grown all over the world owing to its highest...
concentration of protein (approximately 65–70%), fatty acid (linoleic acid, gamma-linolenic acid, oleic acid, etc.), some essential amino acids, vitamins, minerals, coloring agents, and enzymes (Vonshak and Richmond 1988).

Understanding the physiology, ecology, and biotechnology of biomass growth is crucially essential in which biomass kinetics theoretical framework is used as a popular tool to represent the structure of growth formation. In 1913, Michaelis and Menten proposed a mathematical model describing the kinetics of microbial growth and the lessening in substrate concentration in a specific period. Following, Monod, 1949; Tang et al., (1997); Roos et al., (2004) established a new non-linear model for substrate concentration and specific growth rate that is considered as a primitive model on biomass growth kinetics. Huppert et al. (2005) showed that Monod functional form enhances the kinetic growth model by substrate inhibition effects with dual-substrate model and biomass amounts, respectively. Notably, the Monod model has high accuracy for simple substrates and pure cultures (Contois 1959).

To explore the specific growth rate of the biomass as a function of substrate, Haldane (1930) introduced a model known as the Haldane kinetics model in the literature. Morozov (2016) stated that Haldane kinetics is the pioneer of the mathematical modeling of biological evolution. Many researchers widely accept this model for its inhibitive substance (Polizzi et al., 2017), and the specific growth rate is a non-linear function. According to Sathya et al. (2015) Haldane kinetic model was analyzed using the homotopy perturbation method for phenol biodegradation. Furthermore, Webb (1963) introduced the modified Haldane model, but it did not improve the situation significantly.

Besides the models mentioned earlier, Aiba et al. (1968) investigated the kinetic growth model by substrate inhibition effects with pragmatic correlation and simulated data from experiments. Yano et al. (1966) stated a model that is based upon a theoretical analysis of the kinematic behavior of the growth inhibition at a high concentration of the substance. Schroder et al. (1997) used steady states continuous culture of Pseudomonas cepacia G4 and observed that Yano & Koga model fitted for various inhibition models better than any other known models. The model of Andrews (1968) described a model for continuous and batch cultures of biomass with substrate inhibition parameters (Beltrán-Prieto and Son Nguyen 2018). The main feature of this model is to establish a relation among substrate concentration to an inhibition parameter, and specific growth rate. Hinshelwood (1947) developed a model to illustrate the influence of alcohol in biomass cultivation. Ethanol inhibition affected cell growth, product yield, and specific growth rate, according to the study of Amenaghawon et al. (2012).

The objective of the research is to compare several popular kinetic models with existing experimental data in the literature and to propose a model blended from Yano & Koga and Hinshelwood models. The eight models are Monod, Teissier, Haldane, Andrews & Noack, Yano & Koga, Aiba, Webb, and Hinshelwood were used to explain the maximum specific growth rate, yield coefficient, substrate inhibition, and half-saturation parameters effect on biomass growth. Unlike previous works, this study mainly focuses on eight models with extensive analytical and numerical analysis with experimental values for four parametric effects. We studied the different growth parameters concerning biomass-growth concentration and substrate-diminish concentration for the Spirulina platensis. The proposed model illustrates the effect of substrate inhibition term on the Spirulina growth. We have simulated the validity of the model with experimental data using the statistical quantity, such as root mean square deviation (RMSD), correlation coefficient (R²), Akaike’s Information Criteria (AIC), and Bayesian Information Criteria (BIC).

2. Mathematical model and methods

2.1. Biomass growth model

The microbial biomass growth velocity is proportional to the present population,
\[ x'(t) = \mu(t) x(t) \]  (1)
where \( \mu(t) \) denotes the specific growth rate, biomass concentration indicated by \( x(t) \) and the time is expressed by \( t \), and the substrate concentration is significant since it allows the microorganism growth. The substrate concentration rate is transformed into biomass with some constant,
\[ s(t) = -\frac{1}{Y_{x/s}} x(t) \]  (2)
where \( Y_{x/s} \) is yield coefficient.

2.2. Specific growth rate

The specific growth rate \( \mu(t) \) was computed by the natural logarithm (ln) of biomass versus time, as in the following equation:
\[ \mu = \frac{\ln x_2 - \ln x_1}{t_2 - t_1} \]  (3)
where, \( x_1 \) and \( x_2 \) stands for the concentrations of biomass at times \( t_1 \) and \( t_2 \), respectively.

2.3. Yield coefficient

The yield coefficient (\( Y_{x/s} \)) computed according to the following equation,
\[ Y_{x/s} = \frac{x - x_0}{x_0 - s} \]  (4)
where \( x \) denotes the biomass concentration (g/l) at the final time, \( x_0 \) indicates the initial biomass concentration at the initial time, \( s \) (g/l) denotes the final concentration of substrate at the final time and, \( s_0 \) (g/l) represents the initial concentration of substrate at the initial time.

2.4. Half saturation parameter

The parameter, \( k_s \) called the half-saturation parameter, is the substrate concentration at half of the maximum specific growth rate. In the Monod model, if the substrate concentration value is less than \( k_s \), then the specific growth rate is linearly proportional to substrate concentration, and the growth rate is approximately a first-order kinetic equation. The value of the half-saturation parameter must be greater than zero, so the value of \( k_s^* < 1 \) and accordingly \( \mu_{\text{max}} \) is higher than the specific growth rate.

2.5. Maximum specific growth rate

The parameter \( \mu_{\text{max}} \) denotes the maximum specific growth rate. It is a function of environmental factors, such as temperature and light intensity (Goldman and Edward, 1974). In this study, we consider \( \mu_{\text{max}} \)
from 0.30 to 0.50. In this case, it indirectly depends on environmental factors.

In Figure 1, the specific growth rate depending on the following values of the parameter $μ_{\text{max}} = 0.39$ (h$^{-1}$), $k_i = 12.5$ (mg/l), $k_i = 158$ (mg/l), and $p = 0.001$ (curve fitting unitless parameter). Several models have been recommended intending to describe the method of growth kinetics. Some popular models are discussed in the following section.

### 2.6. Model description

Using (1) and (2) along with the following models, we get (5, 6, 7, 8, 9, 10, 11, 12) (detail description of the microbial growth rate and eight studied biomass kinetic models are provided in the Supplementary Material).

A) Monod model (Monod, 1949)

$$\dot{x} = \frac{1}{(x_0 + s_Y Y_{s,i})μ_{\text{max}}} \left( (x_0 + s_Y Y_{s,i}) \ln \left( \frac{x}{x_0} \right) + k_i Y_{s,i} \ln \left( \frac{x - s_Y x_{s,i}}{x_0 - s_Y x_{s,i}} \right) \right)$$

B) Haldane model (Haldane, 1930)

$$\dot{x} = \frac{1}{(x_0 + s_Y Y_{s,i})μ_{\text{max}}} \left( (x_0 + s_Y Y_{s,i}) \ln \left( \frac{x}{x_0} \right) + k_i Y_{s,i} \ln \left( \frac{x - s_Y x_{s,i}}{x_0 - s_Y x_{s,i} - x} \right) \right) + \frac{1}{μ_{\text{max}} k_i Y_{s,i}} \left( (x_0 - x) + (x_0 + s_Y Y_{s,i}) \ln \left( \frac{x}{x_0} \right) \right)$$

where $c = s - px$

C) Andrews & Noack model (Andrews, 1968)

$$\dot{x} = \frac{1}{μ_{\text{max}}} \left( \frac{1 + \frac{1}{c} + \frac{k_i}{c^2}}{k_i p(x_0 - x) + \frac{k_i}{c}(px + c)} \ln \left( \frac{x}{x_0} \right) - \frac{1}{c} \frac{1}{c^2} \ln \left( \frac{px + c}{px_0 + c} \right) \right)$$

D) Hinshelwood model (Hinshelwood, 1947)

$$t = \frac{1}{μ_{\text{max}}(1 - k_0 q)} \left\{ (c + k_i) \ln \left( \frac{x}{x_0} \right) - \frac{k_i}{c} \ln \left( \frac{px + c}{px_0 + c} \right) \right\}$$

E) Teissier model (Yurt et al., 2002)

$$\Rightarrow t = \frac{1}{μ_{\text{max}}} \left\{ \frac{1}{1 - \frac{c}{c_i}} \ln \left( \frac{x}{x_0} \right) - \frac{k_i}{c} \ln \left( \frac{px + c}{px_0 + c} \right) \right\}$$

F) Webb model (Webb, 1963)

$$t = \frac{1}{μ_{\text{max}}} \left\{ \frac{c + k_i + \frac{c}{c_i}}{1 + \frac{c}{c_i}} \ln \left( \frac{x}{x_0} \right) - \frac{k_i}{c} \ln \left( \frac{px + c}{px_0 + c} \right) + \frac{k_i}{c} \ln \left( \frac{k_i + px + c}{k_i + px_0 + c} \right) \right\}$$

G) Yano & Koga model (Yano et al., 1966)

$$\Rightarrow t = \frac{1}{μ_{\text{max}}} \left\{ \left( 1 + \frac{k_i}{c_i} + \frac{c_i}{k_i} \right) \ln \left( \frac{x}{x_0} \right) - \frac{k_i}{c} \ln \left( \frac{px + c}{px_0 + c} \right) + \frac{p(x - x_0)}{k_i} \left( 1 + 2c + \frac{p^2(x_0^2 - x_0^2)}{2k_i} \right) \right\}$$

H) Alib model (Alib et al., 1968)

$$\Rightarrow t = \frac{k_i}{μ_{\text{max}}} \left\{ \frac{1}{c(k_c - c)} \ln \left( \frac{x}{x_0} \right) - \frac{1}{k_i} \frac{k_i}{c} \ln \left( \frac{px + c}{px_0 + c} \right) \right\} - \frac{k_i}{(k_c - c)^2} \ln \left( \frac{k_i - px - c}{k_i - px_0 - c} \right)$$

![Figure 1](image-url)

**Figure 1.** Biomass specific growth rate versus substrate according to the Monod, Haldane, Andrews & Noack, Aliba, Webb, Hinshelwood, Yano & Koga, Teissier model, modified proposed model, and maximum specific growth.
We also analyze the substrate concentration for all parameters. Substrate analysis is an essential substance for the optimized growth kinetics of biomass. Joshi et al. (2014) shows that *Spirulina platensis* when farmed on varying substrate concentrations, yield better progress. Organic substrates are capable of accelerating the growth of microalgae. In this study, we solve all models for the substrate concentration using Eqs. (1) and (2). The results are:

A) Monod model

\[
t = \frac{1}{\mu_{\text{max}}} \left\{-k_i \ln \left( \frac{s}{s_0} \right) + (c + k_i) \ln \left( \frac{s - c}{s_0 - c} \right) \right\} \tag{13}
\]

B) Haldane model

\[
t = \frac{1}{k_i \mu_{\text{max}}} \left\{(s - s_0) - \frac{k_i k_i}{c} \ln \left( \frac{s}{s_0} \right) + \frac{ck_i + k_i c + c^2}{c} \ln \left( \frac{s - c}{s_0 - c} \right) \right\} \tag{14}
\]

C) Andrews & Noack model

\[
t = \frac{1}{k_i \mu_{\text{max}}} \left\{ \frac{k_i}{c} \left( \frac{1}{s} - \frac{1}{s_0} \right) - \frac{k_i (c + k_i)}{c^2} \ln \left( \frac{s}{s_0} \right) + \frac{c k_i + k_i c + c^2}{c} \ln \left( \frac{s - c}{s_0 - c} \right) \right\} \tag{15}
\]

D) Teissier model

\[
t = \frac{k_i}{\mu_{\text{max}}} \left\{ \frac{1}{c} \ln \left( \frac{s - c}{s_0 - c} \right) - \ln \left( \frac{s}{s_0} \right) \right\} \tag{16}
\]

E) Hinshelwood model

\[
t = \frac{k_i}{\mu_{\text{max}} (1 - p k_i)} \left\{-k_i \ln \left( \frac{s}{s_0} \right) + (c + k_i) \ln \left( \frac{s - c}{s_0 - c} \right) \right\} \tag{17}
\]

F) Yano & Koga model

\[
t = \frac{1}{k_i \mu_{\text{max}}} \left\{ \frac{c^2}{2} \frac{s_0^2}{s} + (c + k_i) \left( s - s_0 \right) - \frac{k_i k_i}{c} \ln \left( \frac{s}{s_0} \right) + \frac{c^2 + c k_i + k_i k_i}{c} \ln \left( \frac{s - c}{s_0 - c} \right) \right\} \tag{18}
\]

G) Webb model

\[
t = \frac{1}{\mu_{\text{max}}} \left\{ \frac{k_i}{c} \ln \left( \frac{s}{s_0} \right) + \frac{c^2 + c k_i + k_i k_i}{c (c + k_i)} \ln \left( \frac{s - c}{s_0 - c} \right) + \frac{k_i k_i}{k_i (k_i + c)} \ln \left( \frac{s + k_i}{k_i (k_i + c)} \right) \right\} \tag{19}
\]

\[
t = \frac{1}{(x_0 + x_0 y_{x/x}) \mu_{\text{max}} (1 - k_i q)} \left\{ (x_0 + x_0 y_{x/x}) \ln \left( \frac{x}{x_0} \right) + k_i y_{x/x} \ln \left( \frac{x}{x_0} \right) + s_0 y_{x/x} - x \right\} + \frac{1}{y_{x/x} \mu_{\text{max}} k_i (1 - k_i q)} \left\{ x_0 (x_0) + (x_0 + s_0 y_{x/x}) \ln \left( \frac{x}{x_0} \right) \right\} \tag{21}
\]

H) Aiba model

\[
t = \frac{k_i}{\mu_{\text{max}}} \left\{ -\frac{k_i}{c} \ln \left( \frac{s}{s_0} \right) + \frac{c + k_i}{c (k_i - c)} \ln \left( \frac{s - c}{s_0 - c} \right) + \frac{k_i + k_i}{k_i (k_i - c)} \ln \left( \frac{k_i - s}{k_i - s_0} \right) \right\} \tag{20}
\]

From this solution, we can easily describe the substrate concentration for different parameters.

2.7. Proposed modified model

In this study, it is observed that Hinshelwood and Yano & Koga model fits very well with the experimental results. Regarding these pertinent issues, these two models were analyzed rigorously with the half-saturation parameter, $k_i$, yield coefficient, $Y_{x/x}$, maximum specific growth rate, $\mu_{\text{max}}$, and substrate inhibition parameter, $k_i$. It is noticeable that the above-discussed models have a significant effect when changing the values of the three parameters $k_i$, $Y_{x/x}$, and $\mu_{\text{max}}$. Surprisingly, the parameter $k_i$ has an impact only on Hinshelwood and Yano & Koga models but no effect on the other six models. In literature, there is a lack of discussions about the impact of the substrate inhibition parameter analysis for growth kinetic models. Based on this observation, this study proposed a modified model for *Spirulina* biomass growth, which is furnished in Table 1.

The analytic solution of the proposed modified model by using Eqs. (1) and (2)

### Table 1. Specific growth rate equation for biomass kinetic models.

| Model                        | Growth rate of biomass |
|------------------------------|------------------------|
| Monod (1949)                 | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i}$ |
| Haldane (1968)               | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i}$ |
| Andrews & Noack (1968)       | $\mu = \mu_{\text{max}} \frac{s(t)}{1 + \frac{s(t)}{k_i}}$ |
| Aiba (1968)                  | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i}$ |
| Webb (Edward, 1970)          | $\mu = \mu_{\text{max}} \frac{s(t) + k_i}{s(t) + k_i}$ |
| Yano and Koga (1969)         | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i + c}$ |
| Teissier (1970)              | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i + c}$ |
| Hinshelwood                  | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i + c}$ |
| Proposed model               | $\mu = \mu_{\text{max}} \frac{s(t)}{s(t) + k_i + c}$ |

3. Statistical analysis

In this study, all models are verified by using root mean square deviation error (RMSD) and coefficient of determination ($R^2$) statistics. RMSD error can be calculated by using the equation below:

$$RMSD = \sqrt{\frac{\sum_{j=1}^{n} (x_{j,m} - x_{j,p})^2}{n}}$$

Where $x_{j,p}$ comes from the model data and $x_{j,m}$ is the experimental value, $j$ is the experimented time step, and $n$ is the experimented time step in the total period. The coefficient of determination ($R^2$) lies
between 0 and 1. It is a statistical quantity that represents the ratio for an independent variable and a dependent variable in a regression model. Independent variables explain the dependent variables. R-squared describes the intensity of the relationship between dependent and independent variables. R-squared estimates the capability of a model to envisage experimental data. In model selection criteria, AIC and BIC are commonly operated. When we compare the AIC and BIC for different models, it is better to have low AIC and low BIC values. Japanese statistician Hirotugu Akaike (1969) proposed a model selection criterion based on the relation between the relative Kullback-Leibler (K-L) distance and the maximum log-likelihood. The mathematical form of AIC’s is as follows:

$$AIC = 2q - 2l$$  \hspace{1cm} (23)

where \(2l = -n(\log 2\pi + \log(error) - \log n + 1)\), \(n\) is the number of observations, and \(q\) is the parameter numbers of the model.

Schwarz G. (1978) proposed the Bayesian information criterion, which uses posterior probability and Bayesian information. Mathematically, BIC can be evaluated by the following equation-

$$BIC = q \log(n) - 2l$$  \hspace{1cm} (24)

4. Results and discussion

This study illustrates *Spirulina platensis* growth for several existing biomass models and compares them with experimental data available in the literature (Çelekli and Yavuzatmaca 2009). We began by considering the influence of the biomass concentration growth rate, as shown in Figures 2, 3 and 4 (and Supplementary Figures S1-S3). Figures 5 and 6 represent the outcomes of substrate concentration.

Figure 2 displays the set of line graphs for six biomass models: (A) Haldane, (B) Andrews (C) Hinshelwood (D) Yano & Koga (E) Webb, and (F) Aiba for inhibition parameter values 120, 140, 160, 180, and 200 respectively. It illustrates that the Haldane, Andrews & Noack, Webb, and Aiba models are independent of the inhibition parameter, while the Hinshelwood and Yano & Koga models are dependent on it. In Yano & Koga model, biomass concentration increases with the increase of inhibition parameter. However, the complete opposite trend is observed for the Hinshelwood model; biomass concentration increases as the...
inhibition parameter decreases. Besides, it can be said that the biomass concentration increases when the yield coefficient rises for all considered models (Supplementary Figure S1).

After observing Supplementary Figure S1, we can demonstrate the following remarks: Hinshelwood and Yano & Koga model biomass yield rates are slower than other models. Monod, Haldane, Andrews & Noack, Teissier, Webb, and Aiba model attains 3.29 g/l at approximately 300 days, but Hinshelwood and Yano & Koga model need 352 days and 332 days, respectively. After 250 h, biomass growth exhibits a stationary phase for all models. The specific growth rate of biomass concentration increases with the increase of specific growth rates for kinetic models. Subsequently, biomass concentration remains constant for all models except Hinshelwood and Yano & Koga model. The exponential phase was observed within 50–200 h and then attained the maximum biomass growth rate. It is observed from Supplementary Figure S2, biomass escalations with increases for specific growth rates. Consequently, it is found from Supplementary Figure S3, Yano & Koga, and Hinshelwood model exhibits the higher growth rate of biomass from the other models for the half-saturation parameter. In 150 h, biomass concentration distribution shows more variance for these two models. After 250 h, the biomass growth curve appears more stable for all models.

To show the holistic tendency, we present Figure 3, which represents the biomass concentration rate for all considered models. After 50–200 h, experimental results are close to the Yano & Koga and Hinshelwood model. Monod, Haldane, Andrews & Noack, Teissier, Webb, and Aiba model predicts the higher growth than experimental data as well as the Hinshelwood model. Hinshelwood model exhibits over expected results than empirical data, but Yano & Koga shows under predicted data. The experimental data exhibited an S-shaped curve (Narang 2006), which is like the logistics growth model (Baccaër 2011; Islam et al., 2017). In this study, Logistics, Richards (1959), Schenute (1981), Gompertz (1825), and Stannard models (Kyurkchiev and Iliev 2016) were not considered because these models do not contain any substrate or nutrient related parameters. When we considered the delay time for the logistics equation, it can be transformable to the Monod equation using an approximation on Taylor expansion (Lobry et al., 1992). In this study, from Figure 5, it is apparent that for increasing the inhibition parameter, there is no effect of substrate concentration on the Haldane, Andrews & Noack, Webb, and Aiba model. But Hinshelwood and Yano & Koga’s model has significant outcomes. If the inhibition parameter is increasing, substrate concentration decreases slowly for the Hinshelwood model. But for Yano & Koga model, substrate concentration increases when the inhibition parameter increases. These two models exhibit the opposite characteristics for the inhibition parameter. Supplementary Figures S4, S5, S6, and S7 correspond to the concentration of substrate for various values of the particular parameters, maximum specific growth rate (\( \mu_{max} \)), observed yield coefficient (\( Y_{x/s} \)), and half-saturation concentration (\( k_s \)). From Supplementary Figure S4, it is estimated that the substrate concentration lessens when the specific growth rate escalates. It is also observed that from 50 h to 200 h, substrate concentration decreased rapidly. After 300 h, substrate concentration flattens for Monod, Haldane & Noack, Teissier, Webb, and Aiba model except for Hinshelwood and Yano & Koga model. From Supplementary Figures S5 and S6, it is stated that the substrate concentration diminishes when the yield coefficient and half-saturation constant increase. It is evident from Figure 6 that the substrate concentration decreases faster for the Teissier model and then Monod, Haldane, and others. After 100 h, the substrate concentration is approximately 0.76 mg/l, 0.68 mg/l, and 0.56 mg/l, corresponding to Yano & Koga model, Hinshelwood model, and Monod model. Similarly, we observed that after 150 h, the pattern of substrate concentration follows the same pattern. For Yano & Koga and Hinshelwood model, substrate concentration decreases slowly than other models.

The proposed model consists of two parts: the first part is related to substrate consumption, and the last part is associated with substrate inhibition. The proposed model shows better performance than previously
studied models. The RMSD of the Hinshelwood and Yano & Koga model is 0.1086 and 0.1207, respectively. But other models have higher values than these two models, which is shown in the following Table 2. The R-squared values of the Hinshelwood model and Yano & Koga model are 0.973 and 0.9582, respectively. The R-squared values of the Hinshelwood and Yano & Koga models are higher than that of the other models.

The RMSD of the modified proposed model is 0.081393, the R-squared value is 0.9862, AIC is -9.7277, and BIC is -8.2148, whereas the study

Figure 5. Substrate concentration profiles of (A) Haldane, (B) Andrews & Noack, (C) Hinshelwood, (D) Yano & Koga, (E) Webb, and (F) Aiba model versus time t for fixed values of $\mu_{\text{max}} = 0.39$ time $^{-1}$, $Y_{c/s} = 3.09$, $k_{i} = 12.5$ mg/l, and different values of $k_{i} = 120, 140, 160, 180,$ and 200 mg/l respectively.

Figure 6. Substrate concentration profiles of the *Spirulina platensis* cultures measured for Monod, Haldane, Teissier, Hinshelwood, Yano & Koga, Webb, Andrews & Noack, and Aiba model at different times.

Table 2. RMSD, $R^2$, AIC, and BIC values of the kinetic models.

| Model               | RMSD      | $R^2$   | AIC      | BIC      |
|---------------------|-----------|---------|----------|----------|
| Monod               | 0.373007  | 0.9606  | 1.4954   | 2.4031   |
| Haldane             | 0.373262  | 0.9606  | 3.5021   | 4.7125   |
| Andrews & Noack     | 0.380012  | 0.9531  | 3.6814   | 4.8918   |
| Teissier            | 0.40717   | 0.9567  | 2.3717   | 3.2795   |
| Hinshelwood         | 0.108673  | 0.9727  | -6.8372  | -5.3243  |
| Yano & Koga         | 0.155987  | 0.9618  | -5.2229  | -4.0125  |
| Webb                | 0.381455  | 0.9600  | 3.7193   | 4.9297   |
| Aiba                | 0.376514  | 0.9612  | 1.5889   | 2.4967   |
| Proposed model      | 0.081393  | 0.9862  | -9.7277  | -8.2148  |
found the best RMSD of 0.1086, the R-squared value is 0.973, AIC is -6.8372, and BIC is -5.3243 from the studied models.

Though Haldane, Webb, Aiba, and Andrews and Noack’s models include the inhibition parameter, there is no significant effect on these models when we change the inhibition parameter value. Hinshelwood, and Yano, and Koga models, as well as our proposed hybrid model, have a significant effect when we change the inhibition parameter values (ranges from 120 to 200). The present study showed better fitting of experimental values of algae cultivation with the three mathematical models. Future studies are required to further understand the effect of inhibition parameters on the algae growth model.

5. Conclusions

The present study demonstrates insight into the fast growth of *Spirulina platensis* that incorporates popular eight biomass growth models and experimental data. One of our primary concerns while studying theoretical comparisons is to illustrate the more well-suited model compared with experimental data. We found that Hinshelwood and Yano & Koga model results are more compatible than others. Further, it is observed that the substrate inhibition parameter has significant effects on Hinshelwood and Yano & Koga model. Our proposed models had a high $R^2$ (0.9862), a low RSS (0.0813), AIC ($-9.7277$), and BIC ($-8.2148$) in this investigation, implying that this recommended modified model was considerably matched with the experimental data for *Spirulina platensis* growth compared to the popular eight studied models. Thus, the authors anticipate that this study provides a suitable guide for algae growth kinetics and a profound understanding of various parameters with realistic circumstances.

Declarations

Author contribution statement

Mir Shariful Islam: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.
K M Ariful Kabir: Conceived and designed the experiments; Performed the experiments.
Jun Tanimoto: Contributed reagents, materials, analysis tools or data; Wrote the paper.
Bidyut Baran Saha: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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Data availability statement

Data will be made available on request.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

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References

Aiba, S., Shoda, M., Nagatani, M., 1968. Kinetics of product inhibition in alcohol fermentation. Biotechnol. Bioeng. 10 (6), 845–864.
Aikawa, S., Inokuma, K., Wakai, S., Sasaki, K., Ogino, C., Chang, J.S., Hasunuma, T., Kondo, A., 2018. Direct and highly productive conversion of cyanobacteria *Anabaena platensis* to ethanol with CaCl$_2$ addition. Biotechnol. Biofuels 11 (50).
Aminahavavan, N.A., Okeimen, C.O., Obide, S.E., 2012. Kinetic modelling of ethanol inhibition during alcohol fermentation of corn stover using Saccharomyces cerevisiae. Int. J. Eng. Res. Afr. 2 (4), 798–803.
Andrews, J.F., 1968. A mathematical model for the continuous culture of microorganisms utilizing inhibitory substrates. Biotechnol. Bioeng. 10 (6), 707–723.
Bacar, N., 2011. A Short History of Mathematical Population Dynamics. Springer nature, 978-3-642-77297-1141.
Beltran-Prieto, J.C., Son Nguyen, L.H.B., 2018. Numerical analysis of initial amount of substrate and biomass in substrate inhibition process. WSEAS Trans. Syst. Control 13 (3), 491–496.
Beyenal, H., Chen, S.N., Lewandowski, Z., 2003. The double substrate growth kinetics of *Pseudomonas aeruginosa*. Enzym. Microb. Technol. 32 (1), 92–98.
Bilal, M., Rashheed, T., Ahmed, I., Iqbal, H.M.N., 2017. High-value compounds from microalgae with industrial exploitability - a review. Front. Biotechnol. 9, 319–342.
Blakeney, S., Hayes, M., 2017. Algal protein extraction, application, and challenges concerning production. Foods 6 (3), 33.
Borges, J.A., Rosa, G.M., Meza, L.H.R., Hennard, A.A., Souza, M.R.A.Z., Costa, J.A.V., 2013. *Spirulina* sp. LEB-18 culture using effluent from the anaerobic digestion. Braz. J. Chem. Eng. 30 (2), 277–288.
Celekli, A., Yavuzatmaca, M., 2009. Predictive modeling of biomass production by *Spirulina platensis* as function of nitrate and NaCl concentrations. Bioreour. Techn. 100 (5), 1847–1851.
Ciferni, O., 1983. *Spirulina*, the edible microorganism. Microb. Rev. 47 (4), 551–578.
Contois, D.E., 1959. Kinetics of bacterial growth: relationship between population density and specific growth rate of continuous cultures. J. Gen. Microbiol. 21 (1), 40–50.
Converti, A., Lodl, A., Del Reghetto, M., Solcini, C., 2006. Cultivation of *Spirulina platensis* in a combined air-filtr-turbulnt reactor system. Biochem. Eng. J. 32 (1), 13–18.
De Morais, M.G., Costa, J.A.V., 2007. Biofixation of carbon dioxide by *Spirulina* sp. and *Scenedesmus* obliquus cultivated in a three-stage serial tubular photobioreactor. Biotechnol. Bioeng. 129 (3), 439–445.
Deng, F., Neuweiler, I., Nogueira, R., et al., 2021. Modeling of symbiotic bacterial biofilm growth with an example of the Streptococcus–Veillonella sp. system. Bull. Math. Biol. 83 (48), 1–36.
Godman, J.C., Edwards, J.C., 1974. A kinetic approach to the type of temperature effect on algal growth. Limnol. Oceanogr. 19 (5), 756–766.
Gompertz, B., 1825. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. Phil. Trans. Roy. Soc. Lond. 115, 513–585.
Gouveia, L., Batista, A.P., Sousa, I., Raymundo, A., Bandarra, N.M., 2008. Microalgae in novel food products. In: Food Chemistry Research Developments. Nova Science Publishers, Inc.
Haldane, J.B.S., 1930. In: Haldane, J.B.S., Monographs on Biochemistry, M.A. (Eds.), Enzymes, 7. Longmans Green & Co., London, p. 235.
Hinshelwood, C.N., 1947. The Chemical Kinetics of the Bacterial Cell. Clarendon Press, Oxford.
Huppert, A., Blasius, S., Olinky, R., Stone, L., 2005. A model for seasonal phytoplankton blooms. J. Theor. Biol. 236 (3), 276–290.
Isaeva, M.D., Shariful, Islam, Shafiur, Mir, Khan, A.F.K.M., Banglalee, Z.I., 2017. A procedure to fit an interpolating curve to a set of logistic data. Dhaka Univ. J. Sci. 65, 103–105.
Joshi, M., Kaur, K., Mishra, T., Singh, S., 2014. To evaluate Lab scale cultivation of *Spirulina platensis* using different substrates and to evaluate its chlorophyll and protein content. Int. Res. J. Biol. Sci. 3 (1), 22–30.
Kurych, A., Sil, I., 2016. On some growth curve modelling: approximation theory and applications. Int. J. Trend Res. Develop. 3 (3), 2943–9333.
Kolyer, J.R., Flandrois, J.P., Carret, G., et al., 2019. Monod's bacterial growth model revisited. Bta. Math. Biol. 54, 117–122.
Liu, J., Chen, F., 2014. Biology and industrial applications of chlorella: advances and prospects. In: Posten, C., Feng Chen, S. (Eds.), Microalgae Biotechnology. Advances in Biochemical Engineering/Biotechnology 153. Springer, Cham.
McHenry, J.L., Werkner, A.G., 2002. Characterization of bioactivity in treatment wetlands utilising an enzymatic assay. CSCE/ASCE Int. Conf. Environ. Eng. Int. Perspect. Environ. Eng. 38 (41).
Meyting, B., Pyne, J.W., 1986. Biologically active compounds from microalgae. Enzym. Microb. Technol. 8, 386–394.
Meyting, F.B., 1996. Biodiversity and application of microalgae. J. Ind. Microbiol. 17, 477–489.
Metzger, P., Largeau, C., 2005. Bovycoccus brasilian: a rich source for hydrocarbons and related ether lipids. Appl. Microbiol. Biotechnol. 66, 486–496.
Michaelis, L., Menten, M.L., Goody, R.S., Johnson, K.A., 1913. Die Kinetik der Invertinwirkung/The kinetics of inverte action. Miranda, M.S., Cimino, R.G., Barros, S.B.M., Mancini-Filho, J., 1998. Antioxidant activity of the microalga *Spirulina platensis*. Braz. J. Med. Biol. Res. 31 (8), 1075–1079.
Monod, J., 1949. The growth of bacterial cultures. Annu. Rev. Microbiol. 3, 371–394.
Morozov, A., 2016. Modelling biological evolution: linking mathematical theories with empirical realities. J. Theor. Biol. 405, 1–4.
Narang, A., 2006. Comparative analysis of some models of gene regulation in mixed-substrate microbial growth. J. Theor. Biol. 242 (2), 489–501.
Ojdajdare, E.C., Mutanda, T., Olaniran, A.O., 2017. Potential biotechnological application of *Microalgae*: a critical review. Crit. Rev. Biotechnol. 37 (1), 37–52.
Polizzi, B., Bernard, O., Ribot, M., 2017. A time-space model for the growth of microalgae biofilms for biofuel production. J. Theor. Biol. 432, 55–79.
Ramanan, R., Kannan, K., Deshkar, A., Yadav, R., Chakrabarti, T., 2010. Enhanced algal CO2 sequestration through calcite deposition by Chlorella sp. and Spirulina platensis in a mini-raceway pond. Bioreour. Technol. 101 (8), 2616–2622.

Richards, F.J., 1959. A flexible growth function for empirical use. J. Exp. Bot. 10 (29), 290–300.

Roos, K.F., Martin Jr., J.B., Moser, M.A., 2004. A Manual for Developing Biogas Systems at Commercial Farms in the United States. AgSTAR Handbook 70. U.S. Environmental Protection Agency.

Sathya, R., Rasi, M., Rajendran, L., 2015. Non-linear analysis of Haldane kinetic model in phenol degradation in batch operations. Kinet. Catal. 56, 141–146.

Schenute, J., 1981. A versatile growth model with statistically stable parameters. Can. J. Fish. Aquat. Sci. 38, 1128–1140.

Schroeder, M., Miller, C., Posten, C., Deckwer, W.D., Hecht, V., 1997. Inhibition kinetics of phenol degradation from unstable steady-state data. Biotechnol. Bioeng. 54 (6), 567–576.

Singh, S., Kate, B.N., Banecjee, U.C., 2005. Bioactive compounds from cyanobacteria and microalgae: an overview. Crit. Rev. Biotechnol. 25 (3), 73–95.

Tang, B., Sitomer, A., Jackson, T., 1997. Population dynamics and competition in chemostat models with adaptive nutrient uptake. J. Math. Biol. 35 (4), 453–479.

Vonshak, A., Richmond, A., 1988. Mass production of the blue-green alga Spirulina: an overview. Biomass 15 (4), 233–247.

Walker, T.L., Purton, S., Becker, D.K., Collet, C., 2005. Microalgae as bioreactors. Plant Cell Rep. 24 (11), 629–641.

Webb, J.L., 1963. Enzyme and Metabolic Inhibitors. I. Academic Press.

Wells, M.L., Potin, P., Craigie, J.S., Raven, J.A., Merchant, S.S., Hellwell, K.E., Smith, A.G., Camire, M.E., Brawley, S.H., 2017. Algae as nutritional and functional food sources: revisiting our understanding. J. Appl. Phycol. 29, 949–982.

Yan, N., Fan, C., Chen, Y., Hu, Z., 2016. The potential for microalgae as bioreactors to produce pharmaceuticals. Int. J. Mol. Sci. 17 (6), 962.

Yano, T., Nakahara, T., Kamiyama, S., Yamada, K., 1966. Kinetic studies on microbial activities in concentrated solution: Part. I effect of excess sugars on oxygen uptake rate of a cell free respiratory system. Agric. Biol. Chem. 30 (1), 42–48.

Yurt, N., Sears, J., Lewandowski, Z., 2002. Multiple substrate growth kinetics of Leptothrix discophora SP-6. Biotechnol. Prog. 18 (5), 994–1002.