STREAMING COLD COSMIC-RAY BACK-REACTION AND THERMAL INSTABILITIES ALONG THE BACKGROUND MAGNETIC FIELD

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ABSTRACT

Using a multi-fluid approach, we investigate the streaming and thermal instabilities of electron–ion–cosmic-ray astrophysical objects in which homogeneous cold cosmic rays have a drift velocity perpendicular to the background magnetic field. One-dimensional perturbations along the magnetic field are considered. The induced return current of the background plasma and back-reaction of cosmic rays are taken into account. It is shown that the cosmic-ray back-reaction results in a streaming instability with considerably higher growth rates than that due to the return current of the background plasma. This increase is by a factor of the square root of the ratio of the background plasma mass density to the cosmic-ray mass density. The maximal growth rate and the corresponding wavenumber are then found. Thermal instability is shown to be not subject to the action of cosmic rays in the model under consideration. The dispersion relation for thermal instability includes ion inertia. In the limit of a fast thermal energy exchange between electrons and ions, the isobaric and isochoric growth rates are obtained. The results can be useful for the investigation of electron–ion astrophysical objects such as galaxy clusters, including the dynamics of streaming cosmic rays.

Key words: cosmic rays – galaxies: clusters: general – instabilities – magnetic fields – plasmas – waves

1. INTRODUCTION

There is a growing interest in understanding the interactions of cosmic rays with plasma systems and their possible effects in astrophysics. Irrespective of the various mechanisms which are proposed for the generation of such high-energy particles, cosmic rays may interact with the existing turbulent motions in a plasma or may even excite them. In order to study cosmic rays, a particle description is needed—although the fluid approximation is also used for simplicity. Cosmic rays further induce ionization which may dramatically change the physical properties of a system. For example, ionization by cosmic rays plays a vital role in star formation near the Galactic center (e.g., Yusef-Zadeh et al. 2007) or in the dead zone of protoplanetary disks (Gammie 1996). Moreover, the heating rate is enhanced due to cosmic rays and this important effect has been studied in the context of structure formation in the interstellar medium (ISM) via thermal instability (e.g., Goldsmith et al. 1969; Field et al. 1969).

Another contribution of cosmic rays to the dynamical evolution of a system is their pressure. Many authors have studied the dynamical role of cosmic rays in structure formation on a large scale, through Parker instability (Parker 1966; Kuwabara & Ko 2006), magnetorotational instability (Khajenabi 2012), and even galactic winds and outflows (e.g., Everett et al. 2008). Recently, Wagner et al. (2005) and Shadmehri (2009) have extended the study of classical thermal instability (Field 1965) to include cosmic rays. Thermal instability has been used to explain the existence of structures not only in the ISM but also on very large scales such as in the intracluster medium (ICM). This instability is assumed as a possible mechanism for producing molecular filaments (Sharma et al. 2010) seen in galaxy clusters with short (<1 Gyr) cooling times (e.g., Conselice et al. 2001; Salomé et al. 2006; Cavagnolo et al. 2008; O’Dea et al. 2008).

Recent linear analysis of thermal instability with cosmic-ray pressure shows that instability is suppressed (Shadmehri 2009). Cosmic rays have been included by Shadmehri (2009; see also Sharma et al. 2010) in the framework of magnetohydrodynamic equations as a second fluid having the velocity of the thermal plasma. Numerical analysis shows that the cosmic-ray pressure can play an important role in the dynamics of cold filaments, making them much more elongated along the magnetic field lines than the Field length. This analysis is also consistent with observations (Sharma et al. 2010; see also Snodin et al. 2006). Also, the inclusion of cosmic rays is required to explain the atomic and molecular lines observed in filaments in clusters of galaxies (Ferland et al. 2009).

Moreover, there is another important effect of cosmic rays which has not been considered in the context of thermal instability, namely the presence of streaming cosmic rays. These particles are charged and their drift motion induces a current. This current results in the appearance of the return current provided by the background plasma (e.g., Zweibel 2003; Bell 2004, 2005; Riquelme & Spitkovsky 2009, 2010). The possible role of this effect in the generation of thermal instability needs to be considered. There is also another important issue—the amplification of magnetic fields. The classical cyclotron resonant instability has been proposed long ago to explain this process (Kulsrud & Pearce 1969). However, this mechanism has turned out to be unable to provide sufficient energy in the shock upstream plasma. In order to resolve this problem, a new non-resonant instability has been recently introduced that may provide a much higher energy (Bell 2004; see also Zweibel 2003). This instability, which is known as the Bell instability, has also been confirmed by nonlinear numerical simulations (e.g., Riquelme & Spitkovsky 2009). Subsequent works extended this instability into other directions by considering various physical factors (e.g., Reville et al. 2007; Reville & Bell 2012). However, previous work, except the paper by Bell (2005), has been restricted to the cosmic-ray drift speed parallel to the initial magnetic field. Bell (2005) has derived the general dispersion relation for arbitrary orientation of the background magnetic field, cosmic-ray current, and direction of perturbations. The particular case in which the cosmic-ray current is perpendicular to the initial magnetic field and where perturbations get excited along the latter has also been considered by Riquelme & Spitkovsky (2010).
In both of these previous papers, instabilities arose due to the return plasma current. Riquelme & Spitkovsky (2010) studied this perpendicular current-driven instability numerically and analytically in the linear regime. While their growth rate was similar to that of the cosmic-ray current-driven instability studied by Bell (2004), these authors did not include the cosmic-ray back-reaction in their analytical study.

The thermal instability in galaxy clusters in the multi-fluid approach has been considered by Nekrasov (2011, 2012). The related effects of cosmic rays were not included in these papers. Here, we take into account streaming cold cosmic rays. We consider a geometry in which homogeneous cosmic rays drift across the background magnetic field and perturbations arise along the latter. Such a geometry is analogous to that treated by Riquelme & Spitkovsky (2010). However, we include the cosmic-ray back-reaction. We also take into account the return plasma current. For simplicity, we here ignore the action of gravity (Sharma et al. 2010). The effects of the gravitational field in the multi-fluid approach have been investigated in detail in papers by Nekrasov & Shadmehri (2010, 2011). Thus, our present study extends previous analytical studies by considering not only the thermal effects but also the currents driven by cosmic rays and their back-reaction.

The paper is organized as follows. Section 2 presents the fundamental equations for plasma, cosmic rays, and electromagnetic fields used in this paper. The equilibrium state is discussed in Section 3. In Sections 4 and 5, the perturbed velocities of the ions and electrons, and the perturbed plasma current, are given, respectively. The corresponding results obtained for cosmic rays are provided in Sections 6 and 7. The total perturbed current is given in Section 8. Wave equations are found in Section 9. The dispersion relation including the plasma return current, cosmic-ray back-reaction, and terms describing the thermal instability is derived and its solutions are given in Section 10. In Section 11, a discussion of important results obtained is provided. Possible astrophysical implications are given in Section 12. Concluding remarks are summarized in Section 13.

2. BASIC EQUATIONS FOR PLASMA AND COSMIC RAYS

The fundamental equations for a plasma considered here are the following:

\[ \frac{\partial v_j}{\partial t} + v_j \cdot \nabla v_j = - \nabla p_j + \frac{q_i}{m_j} E + \frac{q_i}{m_j} v_j \times B, \]  

(1)

the equation of motion,

\[ \frac{\partial n_j}{\partial t} + \nabla \cdot n_j v_j = 0, \]  

(2)

the continuity equation, and

\[ \frac{\partial T_j}{\partial t} + v_j \cdot \nabla T_j + (\gamma - 1) T_j \frac{\nabla \cdot v_j}{\nabla \cdot v_j} \]

\[ \quad = - (\gamma - 1) \frac{1}{n_j} \mathcal{L}_i (n_i, T_i) + \nu_{ie} (n_e, T_e) (T_e - T_i) \]

(3)

and

\[ \frac{\partial T_e}{\partial t} + v_e \cdot \nabla T_e + (\gamma - 1) T_e \frac{\nabla \cdot v_e}{\nabla \cdot v_e} \]

\[ \quad = - (\gamma - 1) \frac{1}{n_e} \nabla \cdot q_e - (\gamma - 1) \frac{1}{n_e} \mathcal{L}_e (n_e, T_e) \]

\[ \quad - \nu_{ei} (n_i, T_e) (T_e - T_i), \]  

(4)

the temperature equations for the ions and electrons. In Equations (1) and (2), the subscript \( j = i, e \) denotes ions and electrons, respectively. The notations in Equations (1)–(4) represent the following: \( n_j \) and \( m_j \) are the charge and mass of species \( j \), respectively; \( v_j \) is the hydrodynamic velocity; \( n_i \) is the number density; \( p_j = n_j T_j \) is the thermal pressure; \( T_j \) is the temperature; \( \nu_{ie} \) is the frequency of thermal energy exchange between ions (electrons) and electrons (ions) being \( \nu_{ie} (n_e, T_e) = 2 \nu_{ie} \), where \( \nu_{ie} \) is the collision frequency of ions with electrons (Braginskii 1965); \( n_i \nu_{ei} (n_i, T_e) = n_i \nu_{ei} (n_i, T_e) \), \( \gamma \) is the ratio of the specific heats; \( E \) and \( B \) are the electric and magnetic fields; and \( c \) is the speed of light in a vacuum. Here, for simplicity, we do not take into account collisions between the ions and electrons in the momentum equation. Their effect upon thermal instability has been treated by Nekrasov (2011, 2012), where particular conditions that allow one to ignore collisions have been found. However, the thermal exchange should be included because its timescale is comparable with the dynamical time. The value \( q_e \) in Equation (4) represents the electron heat flux (Braginskii 1965). In a weakly collisional plasma, which is considered here, the electron Larmor radius is much smaller than the electron collisional mean free path. In this case, the electron thermal flux is mainly directed along the magnetic field,

\[ q_e = - \chi_e b (b \cdot \nabla) T_e, \]  

(5)

where \( \chi_e \) is the electron thermal conductivity coefficient and \( b = B/B \) is the unit vector along the magnetic field. We only take into account the electron thermal flux given by Equation (5) because the longitudinal ion thermal conductivity is considerably smaller (Braginskii 1965). We also assume that the thermal flux in the equilibrium is absent. The cooling and heating of plasma species in Equations (3) and (4) are described by the function \( \mathcal{L}_j (n_j, T_j) = n_j^2 \Gamma_j (T_j) - n_j \Gamma_j \) where \( \Gamma_j \) and \( \Gamma_j \) are the cooling and heating functions, respectively. The form of this function is somewhat different from the commonly used cooling–heating function \( \chi (Fiel’d 1965) \). Both functions are connected to each other via the equality \( \mathcal{L}_j (n_j, T_j) = m_j n_j \chi_e \). Our choice is analogous to those of Begelman & Zweibel (1994), Bogdanović et al. (2009), and Parrish et al. (2009). The function \( \Lambda_j (T_j) \) can be found, for example, in Tozzi & Norman (2001).

For relativistic cosmic rays, we use equations in the following form (e.g., Lontano et al. 2002):

\[ \frac{\partial (p_{re} \rho_{re})}{\partial t} + v_{cr} \cdot \nabla (p_{re} \rho_{re}) = - \frac{\nabla \rho_{cr}}{\rho_{cr}} + q_{cr} \left( \frac{1}{c} v_{cr} \times B \right), \]  

(6)

\[ \left( \frac{\partial}{\partial t} + v_{cr} \cdot \nabla \right) \left( \frac{p_{cr} \gamma_{cr}^2}{\rho_{cr}} \right) = 0, \]  

(7)

where

\[ R_{cr} = 1 + \frac{\Gamma_{cr} T_{cr}}{\Gamma_{cr} - 1 m_{cr} c^2}, \]  

(8)

In these equations, \( p_{cr} = \gamma_{cr} m_{cr} c^2 \) is the momentum of a cosmic-ray particle having rest mass \( m_{cr} \) and velocity \( v_{cr} \), \( q_{cr} \) is the charge, \( p_{cr} = \gamma_{cr} n_{cr} c^2 \) is the kinetic pressure, \( n_{cr} \) is the number density in the laboratory frame, \( \Gamma_{cr} \) is the adiabatic index, and \( \gamma_{cr} = (1 - v_{cr}^2/c^2)^{-1/2} \) is the relativistic factor. The continuity equation is the same as Equation (2) for \( j = cr \). Equation (8) can be used for both cold nonrelativistic, \( T_{cr} \ll m_{cr} c^2 \), and hot relativistic, \( T_{cr} \gg m_{cr} c^2 \), cosmic rays. In the first (second) case, we have \( \Gamma_{cr} = 5/3 \) (4/3) (Lontano et al.
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The general form of the value \( R_a \) applied to any relations between \( T_e \) and \( m_e c^2 \), can be found, e.g., in Toepfer (1971) and Dzhavakhishvili & Tsintadze (1973).

Equations (1)–(4), (6), and (7) are solved together with Maxwell’s equations

\[
\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}
\]

and

\[
\nabla \times B = \frac{4\pi}{c} j \text{,}
\]

where the current density \( j = j_\parallel + j_\perp = \sum q_i n_i v_i + j_\perp \).

Below, we first consider an equilibrium state in which there is a stationary cosmic-ray current. This can be done due to the condition of quasi-neutrality

\[
q_{ni} n_i = q_{ne} n_e + q_{cn} n_{cn} = 0
\]

where we have taken into account the condition of quasi-neutrality \( q_{ni} n_i + q_{ne} n_e + q_{cn} n_{cn} = 0 \) (the number density \( n_{cn} \) is that in the laboratory frame). This condition is satisfied in astrophysical plasmas due to cosmic-ray charge neutralization from the background environment (Alfvén 1939). We note that Equation (14) describes the total equilibrium current of electrons, ions, and cosmic rays. Due to quasi-neutrality, the current produced by the electric drift velocity \( v_0 \) (see Equation (12)), which is common for all species, is equal to zero. Substituting Equation (14) into Equation (11), we obtain

\[
\frac{\partial E_0}{\partial t} = -4\pi q_e n_{e0} u_{cr} \frac{c^2}{c_A^2 + c_e^2},
\]

where \( c_A = (B_0^2/4\pi m_e n_{e0})^{1/2} \) is the ion Alfvén velocity. We note that for the \( c_e^2 \gg c_A^2 \) case, this equation has been derived by Riquelme & Spitkovsky (2010). Substituting Equation (15) into Equation (12) for ions, we find the return plasma current assuming \( c_e^2 \gg c_A^2 \):

\[
j_{ret} = q_i n_i (v_i - v_0) = -q_e n_{e0} u_{cr},
\]

which is equal to the cosmic-ray current and has the opposite direction. The induced plasma current drift velocity \( u_{pl} = v_i - v_0 \) is equal to \( -q_i n_{i0}/q_e n_{e0} u_{cr} \). Using Equation (15), we see that the polarizational cosmic-ray drift velocity \( R_{cr} c_e / \omega_{cr} B_0 \partial E_0 / \partial t \) can be ignored in comparison to \( u_{cr} \) under the condition \( m_i n_{i0} \gg R_{cr} c_e m_e n_{e0} \). If cosmic rays are not too relativistic, this condition is satisfied. The plasma drift velocity \( u_{pl} \) will be taken into account in the consideration of both streaming and thermal instability which is provided below.

Here, we will consider the case in which the background temperatures of the electrons and ions are equal, \( T_e = T_i = T_0 \). However, for convenience in following the symmetric contribution of the ions and electrons, we keep the notations \( T_0 \) and \( T_{i0} \) in general calculations. In this case, the thermal equations in equilibrium are given by

\[
L_e (n_{e0} , T_{i0}) = L_e (n_{e0} , T_{e0}) = 0.
\]

4. PERTURBED VELOCITIES OF IONS AND ELECTRONS

We will investigate one-dimensional perturbations depending on the \( z \)-coordinate. Equations for perturbed velocities of ions and electrons are given in Appendix A. We consider Equation (A3) under the condition \( \omega_{crj} \gg \partial^2 / \partial t^2 \). Then, the transverse velocities are given by

\[
v_{j1x} = \frac{q_j}{m_j \omega_{crj}} E_{1x} + \frac{q_j}{m_j \omega_{crj}^2} \frac{\partial E_{1x}}{\partial t} - \frac{q_j}{m_j \omega_{crj}^3} \frac{\partial^2 E_{1x}}{\partial t^2},
\]

\[
v_{j1y} = -\frac{q_j}{m_j \omega_{crj}} E_{1y} + \frac{q_j}{m_j \omega_{crj}^2} \frac{\partial E_{1y}}{\partial t} + \frac{q_j}{m_j \omega_{crj}^3} \frac{\partial^2 E_{1y}}{\partial t^2},
\]

where Equation (A2) has been used. We also have taken into account that \( B_{1z} = 0 \) (see Equation (9)). The longitudinal velocities \( v_{1z1} \) and \( v_{e1z} \) are given by Equation (A13):

\[
L v_{1z1} = H_{1z}, L v_{e1z} = H_{e1},
\]

where the values \( L \), \( H_{1z} \), and \( H_{e1} \) are defined by Equations (A9), (A12), and (A14)–(A20).

5. PERTURBED PLASMA CURRENT

It is known that the contribution of streaming flows to the dispersion relation is due to the difference of their background velocities (e.g., Nekrasov 2008, 2009a, 2009b, 2009c). Therefore, for simplicity, we do not need to take into account the contribution of the electric drift, \( v_0 \), which is common for all species (see Section 3) and does not contribute to the dispersion relation. We can also choose the appropriate frame of reference
7. PERTURBED COSMIC-RAY CURRENT

The linear perturbations of the components of the cosmic-ray current \( j_{\text{cr1}} = q_{\text{cr}n_{i0}} v_{\text{cr}1} + q_{\text{cr}n_{e0}} u_{\text{cr}1} \) are equal to

\[
4\pi j_{\text{cr1}x} = \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} E_{1x} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \frac{\partial E_{1x}}{\partial t} - \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \frac{\partial^2 E_{1y}}{\partial t^2},
\]

\[
4\pi j_{\text{cr1}y} = -\frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} E_{1y} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \frac{\partial E_{1y}}{\partial t} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \frac{\partial^2 E_{1x}}{\partial t^2} - 4\pi q_{\text{cr}n_{i0}} n_{i0} c H_{1z} \left( \frac{\partial}{\partial t} \right)^{-1} \frac{\partial H_{1z}}{\partial z},
\]

\[
4\pi j_{\text{cr1}z} = 4\pi q_{\text{cr}n_{e0}} H_{1z},
\]

where \( \omega_{\text{pcr}} = \left( 4\pi n_{i0} q_{\text{cr}}^2 / m_{\text{cr}} \right)^{1/2} \) is the cosmic-ray plasma frequency. To obtain Equation (24), we have used Equations (21) and (22) and the continuity equation for cosmic rays. The value of \( H_{1z} \) is defined by Equation (23).

8. PERTURBED TOTAL CURRENT

We now calculate the components of the perturbed total current \( j_1 = j_{\text{pl1}} + j_{\text{cr1}} \). Adding Equations (20) and (24), we obtain

\[
4\pi j_{1x} = \left( \frac{\omega_{\text{pi}}^2}{\omega_{\text{ci}}} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \right) \frac{\partial E_{1x}}{\partial t} - \left( \frac{\omega_{\text{pi}}^2}{\omega_{\text{ci}}} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \right) \frac{\partial^2 E_{1y}}{\partial t^2},
\]

\[
4\pi j_{1y} = \left( \frac{\omega_{\text{pi}}^2}{\omega_{\text{ci}}} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \right) \frac{\partial E_{1y}}{\partial t} + \left( \frac{\omega_{\text{pi}}^2}{\omega_{\text{ci}}} + \frac{\omega_{\text{pcr}}^2}{\omega_{\text{ccr}}} \right) \frac{\partial^2 E_{1x}}{\partial t^2} - 4\pi q_{\text{pl}n_{i0}} n_{i0} c H_{1z} \left( \frac{\partial}{\partial t} \right)^{-1} \frac{\partial H_{1z}}{\partial z},
\]

\[
4\pi j_{1z} = 4\pi q_{\text{pl}n_{e0}} H_{1z} + 4\pi q_{\text{cr}n_{i0}} n_{i0} c H_{1z} + 4\pi q_{\text{cr}n_{e0}} H_{1z},
\]

where we have used the condition of quasi-neutrality \( q_{\text{pl}n_{i0}} + q_{\text{cr}n_{e0}} + q_{\text{cr}n_{i0}} = 0 \).

9. WAVE EQUATIONS

To obtain the wave equations, we have substituted the current (Equation (25)) into Maxwell’s Equation (10). Omitting small terms because of the condition \( \partial^3 / \partial \omega_{\text{ci}} \partial t \ll 1 \) and also assuming \( \gamma_{\text{cr0}}^3 / \partial / \partial t / \partial \ll 1 \), we find in the one-dimensional case

\[
\frac{c^2}{\omega_{\text{ccr}}} \left( \frac{\partial}{\partial t} \right)^{2} \frac{\partial^2 E_x}{\partial z^2} - E_x = \varepsilon_{xx} E_{1x},
\]

\[
\frac{c^2}{\omega_{\text{ccr}}} \left( \frac{\partial}{\partial t} \right)^{2} \frac{\partial^2 E_y}{\partial z^2} - E_y = \varepsilon_{yy} E_{1y} - \varepsilon_{yz} E_{1z},
\]

\[
0 = -\varepsilon_{yz} E_{1y} + \varepsilon_{zz} E_{1z}.
\]
Here, the following notations are introduced:

\[ \varepsilon_x = \frac{\omega_{pi}^2}{\omega_c^2} + \frac{\omega_{pc}^2 \nu_{c0}^3}{\omega_{cc}^2}, \]

\[ \varepsilon_y = \frac{\omega_{pi}^2}{\omega_c^2} + \frac{\omega_{pc}^2 \nu_{c0}^3}{\omega_{cc}^2}, \]

\[ \varepsilon_z = \left( \frac{\omega_{pi}^2 u_{pl}^2 D G_2}{L} + \frac{\omega_{pc}^2 u_{cr}^2}{L_{ct}} \right) \frac{\partial}{\partial t} \frac{\partial^2}{\partial z^2} \]

\[ + \left( \frac{\omega_{pi}^2 u_{pl}^2 D G_1}{L} + \frac{\omega_{pc}^2 u_{cr}^2}{L_{ct}} \right) \frac{\partial}{\partial t} \frac{\partial}{\partial z}, \quad (29) \]

\[ \varepsilon_{zz} = \left[ \frac{\omega_{pi}^2 D}{L} u_{pl} \left( G_2 + q_e n_{e0} q_i n_{i0} G_4 \right) + \omega_{pc}^2 u_{cr}^2 \right] \left( \frac{\partial}{\partial t} \frac{\partial}{\partial z} \right)^{-1} \frac{\partial}{\partial z}. \]

\[ \varepsilon_{zz} = \left( \omega_{pe}^2 G_3 + \omega_{pi}^2 G_1 \right) \frac{D}{L} + \omega_{pc}^2 u_{cr}^2 \frac{1}{L_{ct}}. \]

When deriving Equations (26)–(28), we have used Equations (9), (23), and (A18) without the contribution of terms proportional to \( \nu_{c0} \).

Equation (26) describes magnetosonic waves including the contribution of cosmic rays at the conditions under consideration. Equations (27) and (28) describe streaming instability due to cosmic-ray flow and thermal instability when subjected to cosmic rays. We have when \( u_{cr} = 0 \), thermal instability is defined by Equation (28), \( \varepsilon_{zz} = 0 \), and \( E_{1z} \neq 0 \) (Nekrasov 2011).

10. Dispersion Relation

Equations (27) and (28) together with Equations (29) are given in their general form. Making use of the Fourier analysis for perturbations proportional to \( \exp(ikz - i\omega t) \), we obtain

\[ \varepsilon_{zz} \left( \frac{k^2 c^2}{\omega^2} - 1 \right) = \varepsilon_{yy} - \varepsilon_{yzz} \varepsilon_{yy}, \quad (30) \]

Substituting into Equation (30) expressions for \( \varepsilon_{yy}, \varepsilon_{yzz}, \varepsilon_{yzz}, \) and \( \varepsilon_{zz} \), which are defined by Equation (29), we find

\[ \varepsilon_{zz} \left( \frac{k^2 c^2}{\omega^2} - 1 - \frac{c_A^2}{c_A^2} \right) = \varepsilon_{zz} \alpha_1 - \varepsilon_{zz} \alpha_3, \quad (31) \]

where \( c_A \) is the Alfvén velocity, including the contribution of cosmic rays:

\[ \frac{c^2}{c_A^2} = \frac{\omega_{pi}^2}{\omega_c^2} + \frac{\omega_{pc}^2 \nu_{c0}^3}{\omega_{cc}^2}. \]

In Equation (31), we have introduced the notations

\[ \alpha_1 = \omega_{pi}^2 u_{pl}^2 D G_2 \frac{L}{L_{ct}}, \]

\[ \alpha_2 = \omega_{pi}^2 u_{pl}^2 D G_1 \frac{L}{L_{ct}}, \]

\[ \alpha_3 = \omega_{pi}^2 D \frac{L}{L_{ct}} u_{pl} \left( G_2 + q_e n_{e0} q_i n_{i0} G_4 \right), \quad (32) \]

\[ + \omega_{pc}^2 u_{cr}^2 \frac{1}{L_{ct}}. \]

To calculate the right-hand side of Equation (31) with values \( \alpha_{1,2,3} \) given by Equation (32) it is convenient to consider the expression \( L^2 (\varepsilon_{zz} \alpha_1 - \varepsilon_{zz} \alpha_3) \). Carrying out the calculations and taking into account that \( q_i n_{i0} + q_e n_{e0} \approx 0 \) and \( u_{cr} \gg \nu_{ioy} \), we obtain

\[ L^2 (\varepsilon_{zz} \alpha_1 - \varepsilon_{zz} \alpha_3) = \omega_{pe}^2 u_{pl}^2 \left( \omega_{pe}^2 G_2 G_3 + \omega_{pi}^2 G_1 G_4 \right) \frac{D^2}{L_{ct}^2}, \]

\[ + \omega_{pc}^2 u_{cr}^2 \left( \frac{\omega_{pe}^2 G_2 + \omega_{pi}^2 G_1}{L_{ct}} \right) \frac{D}{L_{ct}}. \quad (33) \]

It can be shown that the value \( \omega_{pe}^2 G_2 G_3 + \omega_{pi}^2 G_1 G_4 \) acquires the simple form

\[ \omega_{pe}^2 G_2 G_3 + \omega_{pi}^2 G_1 G_4 = \omega_{pe}^2 L, \quad (34) \]

where we have taken \( q_i = -q_e \) (see Equation (A19)). The value \( \omega_{pe}^2 G_2 + \omega_{pi}^2 G_1 \) can be given in the following form:

\[ \omega_{pe}^2 G_2 + \omega_{pi}^2 G_1 = -\omega_{pe}^2 D \left( \omega^2 - k^2 C_i^2 \right), \quad (35) \]

where \( C_i^2 \) is defined by

\[ m_i D^2 C_i^2 = T_{io} \left( W_i V_i + 2W_j V_j + V_i \Omega_e - V_i \Omega_i \right), \quad (36) \]

In the value \( \varepsilon_{zz} \) on the left-hand side of Equation (31), we ignore the contribution of unity which has arisen due to the displacement current. It is easy to see that this can be done if \( \omega_{pe}^2 \gg \omega^2 \) for the cold plasma and \( \omega_{pe}^2 \gg k^2 C_i^2 \) for the warm plasma, when the wavelength of perturbations is much larger than the Debye length. We also omit from \( \varepsilon_{zz} \) the negligible contribution of cosmic rays. Thus, we have

\[ \varepsilon_{zz} = \left( \omega_{pe}^2 G_3 + \omega_{pi}^2 G_1 \right) \frac{D}{L}. \quad (37) \]

Substituting Equations (33)–(35) and (37) into Equation (31), we derive the following dispersion relation:

\[ \omega^2 \frac{c^2}{c_A^2} - k^2 c^2 = \frac{\omega_{pe}^2 u_{pl}^2 k^2}{\left( \omega^2 - k^2 C_i^2 \right)} + \frac{\omega_{pc}^2 u_{cr}^2 k^2}{\left( \gamma_{cr0} \omega_0^2 - k^2 C_{vec}^2 \right)}, \quad (38) \]

where we have also neglected the contribution of the displacement current in the round brackets of Equation (31). Equation (38) in its general form describes the streaming and thermal instabilities. This equation includes the plasma return current and cosmic-ray back-reaction. Below, we consider some particular cases.

10.1. Streaming Instability Without Cosmic-ray Back-reaction

In this case, we ignore the contribution of the cosmic-ray term in the dispersion relation defined by Equation (38). We also set all the frequencies \( \Omega = 0 \). Then Equation (38) can be written in the form

\[ \omega^2 \frac{c^2}{c_A^2} - k^2 c^2 = \frac{4\pi j_{vec}^2 m_i^2}{ho_0 \left( \omega^2 - k^2 C_i^2 \right)}, \quad (39) \]

where \( c_s = \sqrt{\gamma \left( T_{io} + T_{io} \right) / m_i} \) is the plasma sound velocity, \( c_A = \omega_A / \omega_{pe} \), \( \rho_0 = m_i n_{i0} \), and \( j_{vec} \) is defined by Equation (16). Equation (39) coincides with Equation (9) in Riquelme & Spitkovsky (2010; see also Bell 2005).
10.2. Streaming Instability with Cosmic-ray Back-reaction

Taking into account the back-reaction of cosmic rays, Equation (38) becomes
\[ \omega^2 c_s^2 - k^2 c_A^2 = \frac{4\pi j_{tot} k^2}{\rho_{tot} (\omega^2 - k^2 c_s^2)} + \frac{4\pi j_{tot} k^2}{\rho_{tot} (\gamma c_{tot} \omega - k^2 c_{scr}^2)}. \]
(40)
where \( \rho_{tot} = n_{tot} T_{tot} \) and \( j_{tot} = q_{tot} c_{tot} T_{tot} \). Since \( j_{tot} \neq j_{tot} \), the second term on the right-hand side of Equation (40) is considerably larger than the first one—roughly by a factor of \( n_{tot} / n_{tot} \) ≫ 1. Thus, the back-reaction of the streaming cosmic rays results in much more powerful instability than seen in the induced background plasma streaming. It should be noted that this conclusion is satisfied for the conditions considered in this paper. From Equation (40), omitting the first term on the right-hand side, we can find the wavenumber \( k_m \) and growth rate \( \delta_m = (-i \omega) \) of the fastest growing mode:
\[ k_m^2 = \frac{8\pi j_{tot}^2 Y_{c0} c_A^2}{\rho_{tot} c_s^2} \left( \frac{c_s^2}{c_{scr}^2 - c_s^2} \right) \left\{ 1 + \frac{(\gamma c_{tot} - c_s^2 - c_A^2)^2}{4(\gamma c_{tot} - c_s^2 - c_{scr}^2)^2} \right\}^{-1/2} \]
and
\[ \delta_m^2 = \frac{2\pi j_{tot}^2 c_A}{\rho_{tot} c_s^2} \left( \frac{c_s^2}{c_{scr}^2 - c_s^2} \right) \left\{ 1 + \frac{(\gamma c_{tot} - c_s^2 - c_A^2)^2}{4(\gamma c_{tot} - c_s^2 - c_{scr}^2)^2} \right\}^{1/2} \]
(41)
Let us find asymptotical expressions for \( k_m \) and \( \delta_m \). In the case \( \gamma c_{tot} c_{scr}^2 \gg c_s^2 \), we have
\[ k_m^2 = \frac{4\pi j_{tot}^2 Y_{c0} c_s^2}{\rho_{tot} c_s^2} \frac{Y_{c0} c_s^2}{c_{scr}^2}, \quad \delta_m^2 = \frac{4\pi j_{tot}^2 c_s^2}{\rho_{tot} c_s^2} \frac{Y_{c0} c_s^2}{c_{scr}^2}. \]
Thus, \( \delta_m = \sqrt{\frac{\gamma c_{tot} c_{scr}^2}{c_s^2}} \), \( k_m = \sqrt{\frac{\gamma c_{tot} c_{scr}^2}{c_s^2}} \). In the opposite case, \( \gamma c_{tot} c_{scr}^2 \ll c_s^2 \), we obtain
\[ k_m^2 = \frac{4\pi j_{tot}^2 Y_{c0} c_s^2}{\rho_{tot} c_s^2} \frac{Y_{c0} c_s^2}{c_{scr}^2}, \quad \delta_m^2 = \frac{4\pi j_{tot}^2 c_s^2}{\rho_{tot} c_s^2} \frac{Y_{c0} c_s^2}{c_{scr}^2}. \]
(44)
The relation between \( \delta_m \) and \( k_m \) is the same as for solutions (43). From Equations (43) and (44), we can write expressions for \( k_m^2 \) and \( \delta_m^2 \), which unite both limiting cases:
\[ k_m^2 = \frac{4\pi j_{tot}^2 Y_{c0} c_s^2}{\rho_{tot} c_s^2} \frac{Y_{c0} c_s^2}{c_{scr}^2} (Y_{c0} c_s^2 + c_{scr}^2), \quad \delta_m^2 = \frac{4\pi j_{tot}^2 Y_{c0} c_s^2}{\rho_{tot} c_s^2} \frac{Y_{c0} c_s^2}{c_{scr}^2} (Y_{c0} c_s^2 + c_{scr}^2). \]
(45)
In the resonance case, \( \gamma c_{tot} c_{scr}^2 \approx c_s^2 \), we find from Equations (41) and (42)
\[ k_m^2 = \frac{\pi j_{tot}^2}{\rho_{tot} c_s^2} \frac{1}{c_{scr}^2}, \quad \delta_m^2 = \frac{\pi j_{tot}^2}{\rho_{tot} c_s^2} \frac{1}{Y_{c0} c_s^2}. \]
(46)
As we see, the magnitudes given by Equation (45) in the resonance case are only twice as large as those in Equation (46). Thus, Equation (45) can be applied to any relation between \( \gamma c_{tot} c_{scr}^2 \) and \( c_s^2 \) with good accuracy.

10.3. Thermal Instability with Cosmic-ray Back-reaction

From Equation (38), it is clear that in this case thermal instability is described by the equation
\[ \omega^2 - k^2 c_s^2 = 0, \]
(47)
where \( c_s^2 \) is given by Equation (36). If we set \( u_{cr} = 0 \), then Equation (47) is satisfied. In the case \( u_{cr} \neq 0 \), the value \( (\omega^2 - k^2 c_s^2)^{-1} \) is multiplied by a small coefficient in Equation (38) in comparison with the second term on the right-hand side of this equation. Therefore, Equation (47) is kept. Thus, cosmic rays do not influence thermal instability under the conditions considered in this paper.

If we set in Equation (47) \( T_{i0} = T_{e0} = T_0 \), then this equation coincides with Equation (47) (Nekrasov 2011) without the inertia term. We note that in that paper, the perturbation of the thermal energy exchange frequency has been taken into account. We further take \( \Omega_{te} = \Omega_{ei} = \Omega_e \). Then, Equation (47) is given by
\[ \delta^2 (\delta^2 + \beta_2 \delta + \beta_4) + \frac{1}{2} k^2 c_s^2 (2 \gamma \delta^2 + \beta_1 \delta + \beta_2) = 0, \]
(48)
where, as above, \( \delta = -i \omega \) and \( c_s = (2 \gamma T_0 / m_i)^{1/2} \). The following notations are introduced:
\[ \beta_1 = (\gamma + 1) (\Omega_x + \Omega_{Te} + \Omega_{Ti}) - \Omega_{ne} - \Omega_{ni} + 4 \gamma \Omega_x, \]
\[ \beta_2 = (\gamma - 1) (\Omega_x + \Omega_{Te} - \Omega_{ne} + \Omega_{ni} + \Omega_x) + (\Omega_x + \Omega_{Te} - \Omega_{ne} + \Omega_{ni} - \Omega_x) \Omega_x, \]
\[ \beta_3 = (\gamma - 1) (\Omega_x + \Omega_{Te} - \Omega_{ne} + \Omega_{ni} - \Omega_x) \Omega_x, \]
\[ \beta_4 = (\gamma - 1) (\Omega_x + \Omega_{Te} - \Omega_{ne} + \Omega_{ni} - \Omega_x) \Omega_x. \]
The frequencies \( \Omega_{Te,i}, \Omega_{ne,i}, \Omega_{ei,i}, \) and \( \Omega_x \) are given by Equation (A7). We note that \( \Omega_x = (\gamma - 1) Y_{c0} k^2 / n_0 \). In the general form, Equation (48) can be solved numerically.

We now treat Equations (48) and (49) in the limit \( \Omega_x \gg \Omega_{Te,i}, \Omega_{ne,i}, \Omega_{ei,i} \). In the short-wavelength limit, \( k^2 c_s^2 \ll \delta^2 \), the dispersion relation has the form
\[ \delta^2 + 2 \Omega_x \delta + \frac{1}{2} \Omega_{Te} \Omega_x = 0, \]
(50)
where \( \Omega_{Te,n} = \Omega_x + \Omega_{Te} - \Omega_{ne} + \Omega_{Ti} - \Omega_{ni} \). The solution of Equation (50) is the following:
\[ \delta = - \frac{1}{2} \Omega_{Te,n}. \]
(51)
This solution corresponds to Field’s isobaric solution (Field 1965). In the long-wavelength limit, \( k^2 c_s^2 \ll \delta^2 \), we have the equation
\[ \delta^2 + 2 \Omega_x \delta + \Omega_{Te} \Omega_x = 0, \]
(52)
where \( \Omega_T = \Omega_x + \Omega_{Te} + \Omega_{Ti} \). The solution of Equation (52) is
\[ \delta = - \frac{1}{2} \Omega_T, \]
(53)
which corresponds to Parker’s isochoric solution (Parker 1953).
10.4. Thermal Instability without Cosmic-ray Back-reaction

For ultrarelativistic cosmic rays, \( \gamma_{\text{crit}} \to \infty \), their back-reaction is absent (see Appendix B). In this case, Equation (38) takes the form

\[
\delta^2 + k^2 C_r = \frac{\omega_p^2 n_{\text{cr}0}^2}{n_{i0}^2} \left( \frac{k^2 c_r^2 A_i}{\delta^2 + k^2 c_r^2 A_i} \right),
\]

where

\[
C_r = \frac{1}{2 \gamma^2 c_s} \frac{2 \gamma \delta^2 + \beta_1 \delta + \beta_2}{\delta^2 + \beta_3 \delta + \beta_4}.
\]

In Equation (54), we have assumed that \( \eta_{\text{cr}} = q_i \) and \( n_{\text{crit}} \approx c \). We also consider that \( n_{i0} = n_{\text{cr}0} \). In the low-frequency regime, \( \delta^2 \ll k^2 c_r^2 A_i \), Equation (54) together with Equation (55) is given by

\[
2 \gamma \delta^2 + k^2 c_r^2 \frac{2 \gamma \delta^2 + \beta_1 \delta + \beta_2}{\delta^2 + \beta_3 \delta + \beta_4} = 2 \gamma \omega_p^2 n_{\text{cr}0}^2 n_{i0}^2.
\]

If we assume that \( \delta^2 \gg \omega_p^2 n_{\text{cr}0}^2 / n_{i0}^2 \), then we return to the case considered in the previous section. In this case, the plasma return current plays no role. When the opposite condition, \( \delta^2 \ll \omega_p^2 n_{\text{cr}0}^2 / n_{i0}^2 \), is satisfied, then Equation (56) takes the form

\[
2 \gamma \delta^2 + k^2 c_r^2 \frac{2 \gamma \delta^2 + \beta_1 \delta + \beta_2}{\delta^2 + \beta_3 \delta + \beta_4} \frac{n_{\text{cr}0}^2}{n_{i0}^2} = a.
\]

We see from Equation (57) that in the limiting case \( a \ll 1 \) (\( a \gg 1 \)) the numerator (denominator) tends to zero. In the case \( a \sim 1 \), the dispersion relation is modified. In general, the solution is a mixture of the isobaric and isochoric perturbations. However, the qualitative character of thermal instability does not change. Thus, the plasma return current does not influence thermal instability in the low-frequency regime. In the high-frequency regime, \( \delta^2 \gg k^2 c_r^2 A_i \), Equation (54) becomes

\[
2 \gamma \delta^2 + \beta_1 \delta + \beta_2 = 2 \gamma \omega_p^2 n_{\text{cr}0}^2 n_{i0}^2 \frac{c_r^2 A_i}{\delta^2} = b,
\]

where we have assumed that \( \delta^2 \ll \omega_p^2 k c_A n_{\text{cr}0} / n_{i0} \). Again, if \( b \ll 1 \), then the numerator (denominator) on the left-hand side of this equation tends to zero. Thus, the plasma return current has no effect on thermal instability in these limiting cases. From Equation (58), we can also obtain the growth rate

\[
\delta = \omega_p n_{\text{cr}0} c_A / c_s
\]

which must exceed the growth rate of thermal instability.

11. DISCUSSION

In this paper, no conditions have been used for the background plasma except for \( \omega_p^2 \gg \delta^2 / \delta t^2 \), which is usually satisfied in astrophysical settings. For cosmic rays, we have assumed that \( \gamma_{\text{crit}} \delta / \omega_{\text{crit}} \delta t \ll 1 \) (see Section 9). This condition can be satisfied for moderately relativistic cosmic rays. However, it can be violated for ultrarelativistic cosmic rays. The last condition, for example, for the growth rate (45) in the case \( \gamma_{\text{crit}} \delta^2 \gg c_r^2 A_i^2 \), can be written in the form

\[
\gamma_{\text{crit}}^3 \left( \frac{n_{\text{cr}}}{n_{i0}} \right)^{1/2} \frac{n_{\text{crit}}}{c_{\text{crit}}} \ll 1,
\]

where we have assumed that \( c_A \sim c_{\text{crit}} \) and \( \omega_{\text{crit}} \sim \omega_c \). In the limit \( n_{\text{crit}} \to c \), Equation (59), taking into account that \( T_{\text{cr}} \ll m_c c^2 \), can be violated for sufficiently dense cosmic rays. In the opposite case, \( \gamma_{\text{crit}}^{-1} c_{\text{crit}}^2 \ll c_A^2 \), the corresponding condition is given by

\[
\gamma_{\text{crit}}^{5/2} \left( \frac{n_{\text{crit}}}{n_{i0}} \right)^{1/2} \frac{u_{\text{cr}}}{c} \ll 1.
\]

We see from Equation (38) that at the conditions under consideration and for one-dimensional perturbations along the background magnetic field, cosmic rays do not influence thermal instability (see Sections 10.3 and 10.4). At the same time, the back-reaction of cosmic rays results in aperiodic streaming instability much more powerful than that caused by the return current of the background plasma. The maximal growth rate is achieved for sufficiently cold cosmic rays and large magnetic fields such that \( c_A^2 \gg \gamma_{\text{crit}} c_{\text{crit}}^2 \). In this case, the growth rate is equal to

\[
\delta_{\text{max}} \sim \omega_{\text{pcr}} \frac{u_{\text{cr}}}{c} \gamma_{\text{crit}}^{-1/2}
\]

and the wavenumber is

\[
k_{\text{m}} = \frac{\omega_p^2}{c} \frac{n_{\text{crit}}}{c_{\text{crit}}} \frac{\gamma_{\text{crit}}^{-1/4}}{c (c_{\text{crit}} c_A)^{1/2}}.
\]

Thus, in particular cases such as those considered in this paper, the cosmic-ray back-reaction must be invoked to study cosmic-ray streaming instabilities.

If we take into account the cosmic-ray back-reaction we see that thermal instability (Equation (47)) is not influenced by the action of cosmic rays in the model under consideration. The multi-fluid dispersion relation includes ion inertia and has the general form except for \( T_{i0} = T_{\text{cr}} \). In the limit of fast thermal energy exchange in which \( \Omega_k \) is much larger than all other frequencies, the isobaric and isochoric growth rates have been obtained (Equations (51) and (53)). Ultrarelativistic cosmic rays do not experience the back-reaction. In this case, the plasma term with the return current is contained in the dispersion relation (Equation (54)). However, as we have shown, this term also does not influence thermal instability.

We note that all instabilities considered in this paper are connected with the particle dynamics along the background magnetic field.

We have explored the situation in which cosmic rays drift across the background magnetic field. This model has been considered by Riquelme & Spitkovsky (2010) for the problem of magnetic field amplification in the upstream region of supernova remnant shocks. However, such a model can also be applied to the ICM where cosmic rays are an important ingredient (Loewenstein et al. 1991; Guo & Oh 2008; Sharma et al. 2009, 2010). In another model, cosmic rays drift along the magnetic field. This case has been investigated by Bell (2004; see also Riquelme & Spitkovsky 2009). In both cases the growth rates are the same (Bell 2004, 2005; Riquelme & Spitkovsky 2010). Such a situation can also be encountered in the ICM. In the papers by Bell (2004, 2005) and Riquelme & Spitkovsky (2010), the return current of the background plasma has only been addressed in the analytical treatments. The cosmic-ray back-reaction has been included in a numerical analysis by Riquelme & Spitkovsky (2009, 2010) where they have found that this effect results in saturation of instability. However, the influence of the cosmic-ray back-reaction on the growth rate remained unknown.
As we have found in this paper, the cosmic-ray back-reaction drives the instability whose growth rate is proportional to $n_{e0}^{-1/2}$, but not $n_{i0}^{-1/2}$ (as it is for the growth rate due to the return plasma current). Therefore the system enters into a nonlinear regime considerably faster in the presence of cosmic-ray back-reaction, though it is unclear if nonlinear perturbation amplitude also attains a much larger magnitude. Therefore, this instability can produce enhanced magnetic field amplification in both the upstream medium of shocks and the ICM at the linear stage. However, we think further numerical simulations will clarify the importance of the back-reaction effect in a nonlinear regime, which is beyond the scope of the present study.

12. ASTROPHYSICAL IMPLICATIONS

Our linear analysis of the instabilities related to current-driven instability by cosmic rays is applicable to a variety of environments. Although such a type of instability was originally suggested for magnetic field amplification in shocks by supernovae, we think this instability may play a significant role wherever there is a strong cosmic-ray streaming. For example, if the supernova-driven shock propagates through a hot and low-density medium (i.e., superbubbles), then current-driven instability may exist. Even at larger scales, such as shocks in the ICM, we may expect to find this type of instability under some conditions. Most of the previous analytical studies have been restricted to cosmic rays drifting along the ambient magnetic field without consideration of their back-reaction and possible thermal effects. Interestingly, our analysis shows that the inclusion of the back-reaction will lead to a much stronger instability in comparison to the previous studies where this effect is ignored. So, we expect that the magnetic field is amplified to a larger value in the presence of the back-reaction of cosmic rays. This implies more confinement of cosmic rays with excited turbulent motions in the nonlinear regime and accordingly the acceleration of cosmic rays to higher energies.

In some supernova remnants such as IC 443, SN 1006, Kepler, Tycho, etc., the supernova-driven shocks are propagating in their partially ionized ambient medium. This was a good motivation to extend the cosmic-ray current-driven instability theory by Bell (2004) from the MHD approach to a two-fluid system consisting of ions, electrons, cosmic rays, and neutrals. This deserves further study, but we may expect that the stabilizing effect of collisions will be compensated by the back-reaction of cosmic rays.

13. CONCLUSION

Using the multi-fluid approach, we have investigated streaming and thermal instabilities of electron–ion astrophysical plasma with homogeneous cold cosmic rays drifting across the background magnetic field. We have taken into account the return current of the background plasma and the back-reaction of cosmic rays for one-dimensional perturbations along the magnetic field. It has been shown that the cosmic-ray back-reaction results in the streaming instability having a considerably larger growth rate than that due to the usually treated return current of the background plasma. The maximal growth rates and corresponding wavenumbers have been found.

Thermal instability has been shown to be not subject to the action of cosmic rays in the model under consideration. The dispersion relation for thermal instability in the multi-fluid approach has been derived and includes ion inertia. In the limit of fast thermal energy exchange between electrons and ions, the isobaric and isochoric growth rates have been obtained.

The results of this paper will be useful for the investigation of electron–ion astrophysical objects, such as galaxy clusters, including the dynamics of streaming cosmic rays.

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APPENDIX A

A.1. Perturbed Velocities of Ions and Electrons

We set in Equation (1) $v_j = v_{j0} + v_{j1}$, $p_j = p_{j0} + p_{j1}$, $E = E_0 + E_1$, and $B = B_0 + B_1$. For perturbations depending only on the $z$-coordinate, we have $v_{j0} \cdot \nabla = 0$. Then the linearized Equation (1) takes the form

$$\frac{\partial v_{j1}}{\partial t} = -\frac{\nabla T_{j1}}{m_j} - \frac{T_{j0}}{m_j n_{j0}} \nabla n_{j1} + \frac{F_{j1}}{m_j} + \frac{q_j}{m_j} v_{j1} \times B_0,$$  \hspace{1cm} (A1)

where we have used $p_{j1} = n_{j0} T_{j1} + n_{j1} T_{j0}$ ($n_j = n_{j0} + n_{j1}$, $T_j = T_{j0} + T_{j1}$) and we introduced the notation

$$F_{j1} = \frac{q_j}{m_j} E_1 + \frac{q_j}{m_j} v_{j0} \times B_1.$$  \hspace{1cm} (A2)

We find from Equation (A1) the following equations for $v_{j1x,y}$:

$$\frac{\partial^2 v_{j1x}}{\partial t^2} + \omega_{cj}^2 v_{j1y} = \omega_{cj} F_{j1y} + \frac{\partial F_{j1x}}{\partial t}, \hspace{1cm} \frac{\partial^2 v_{j1y}}{\partial t^2} + \omega_{cj}^2 v_{j1x} = -\omega_{cj} F_{j1x} + \frac{\partial F_{j1y}}{\partial t}.$$  \hspace{1cm} (A3)

Applying $\partial/\partial t$ to the $z$-component of Equation (A1) and using the linearized continuity Equation (2), we obtain

$$\frac{\partial^2 v_{j1z}}{\partial t^2} - \frac{T_{j0}}{m_j} \frac{\partial^2 v_{j1z}}{\partial z^2} = -\frac{1}{m_j} \frac{\partial^2 T_{j1}}{\partial z \partial t} + \frac{\partial F_{j1z}}{\partial t}.$$  \hspace{1cm} (A4)

A.2. Perturbed Temperatures of Ions and Electrons

Let us now find equations for the temperature perturbations. Linearized versions of Equations (3) and (4) for one-dimensional perturbations are given by

$$D_{i1} T_{i1} = -C_i \frac{n_{i1}}{n_{i0}} + \Omega_{ie} T_{e1},$$  \hspace{1cm} (A5)

$$D_{e1} T_{e1} = -C_e \frac{n_{e1}}{n_{e0}} + \Omega_{ie} T_{i1}.$$
where the following notations are introduced:

\[
\begin{align*}
D_{i1} &= \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{ie} C_{i1} = T_{i0} \left[ -\left( \gamma - 1 \right) \frac{\partial}{\partial t} + \Omega_{ni} \right], \\
D_{i1e} &= \frac{\partial}{\partial t} + \Omega_{\chi} + \Omega_{Te} + \Omega_{ie}, C_{i1e} = T_{e0} \left[ -\left( \gamma - 1 \right) \frac{\partial}{\partial t} + \Omega_{ne} \right].
\end{align*}
\] (A6)

The frequencies \( \Omega \) in Equations (A5) and (A6) are the following:

\[
\begin{align*}
\Omega_{Ti} &= (\gamma - 1) \frac{\partial L_{i} (n_{i0}, T_{i0})}{n_{i0} \partial T_{i0}}, \quad \Omega_{ni} = (\gamma - 1) \frac{\partial L_{i} (n_{i0}, T_{i0})}{T_{i0} \partial n_{i0}}, \\
\Omega_{Te} &= (\gamma - 1) \frac{\partial L_{e} (n_{e0}, T_{e0})}{n_{e0} \partial T_{e0}}, \quad \Omega_{ne} = (\gamma - 1) \frac{\partial L_{e} (n_{e0}, T_{e0})}{T_{e0} \partial n_{e0}}, \\
\Omega_{\chi} &= -\left( \gamma - 1 \right) \frac{\partial}{\partial t} - \frac{\chi_{0}}{\Omega_{e0}} \frac{\partial^2}{\partial z^2}, \\
\Omega_{ie} &= \nu_{ie} (n_{i0}, T_{e0}), \quad \Omega_{ei} = \nu_{ei} (n_{i0}, T_{e0}).
\end{align*}
\] (A7)

In deriving Equation (A5), we have used Equation (17) and Equations (2) and (5) in their linearized form. From Equation (A5), we can express temperature perturbations through the number density perturbations

\[
\begin{align*}
DT_{i1} &= -D_{i1e} C_{i1} \frac{n_{i1}}{n_{i0}} - \Omega_{e0} C_{i1} \frac{n_{i1}}{n_{i0}}, \\
DT_{e1} &= -D_{i1e} C_{i1} \frac{n_{e1}}{n_{e0}} - \Omega_{e0} C_{i1} \frac{n_{e1}}{n_{e0}}.
\end{align*}
\] (A8)

where

\[
D = D_{i1} D_{i1e} - \Omega_{e0} \Omega_{ei}.
\] (A9)

To proceed further, we apply the operator \( \partial / \partial t \) to Equation (A8) and use the continuity equation. As a result, we obtain

\[
\begin{align*}
D \frac{\partial T_{i1}}{\partial t} &= D_{i1e} C_{i1} \frac{\partial v_{i1z}}{\partial z} + \Omega_{e0} C_{i1} \frac{\partial v_{i1z}}{\partial z}, \\
D \frac{\partial T_{e1}}{\partial t} &= D_{i1e} C_{i1} \frac{\partial v_{e1z}}{\partial z} + \Omega_{e0} C_{i1} \frac{\partial v_{e1z}}{\partial z}.
\end{align*}
\] (A10)

We have to introduce these equations into Equation (A4).

A.3. Equations for Longitudinal Velocities \( v_{i1z} \) and \( v_{e1z} \)

Let us rewrite Equation (A4) for each component of the species and use Equation (A10). Then we obtain

\[
\begin{align*}
L_{i1} v_{i1z} + L_{2i} v_{i1z} &= D \frac{\partial F_{i1z}}{\partial t}, \\
L_{i1e} v_{e1z} + L_{2e} v_{i1z} &= D \frac{\partial F_{e1z}}{\partial t}.
\end{align*}
\] (A11)

Here, the following notations are introduced:

\[
\begin{align*}
L_{i1} &= D \frac{\partial^2}{\partial t^2} + \frac{1}{m_i} (D_{i1e} C_{i1} - T_{i0} D) \frac{\partial^2}{\partial z^2}, \ L_{2i} = \frac{1}{m_i} \Omega_{e0} C_{i1} \frac{\partial^2}{\partial z^2}, \\
L_{i1e} &= D \frac{\partial^2}{\partial t^2} + \frac{1}{m_e} (D_{i1e} C_{i1e} - T_{e0} D) \frac{\partial^2}{\partial z^2}, \ L_{2e} = \frac{1}{m_e} \Omega_{ei} C_{i1e} \frac{\partial^2}{\partial z^2}.
\end{align*}
\] (A12)

From Equation (A11), we find equations for \( v_{i1z} \) and \( v_{e1z} \):

\[
L v_{i1z} = H_{i1}, \quad L v_{e1z} = H_{e1},
\] (A13)

where

\[
\begin{align*}
L &= L_{i1} L_{1e} - L_{2i} L_{2e}, \\
H_{i1} &= D \frac{\partial}{\partial t} (L_{i1} F_{i1z} - L_{2i} F_{e1z}), \\
H_{e1} &= D \frac{\partial}{\partial t} (L_{i1} F_{e1z} - L_{2e} F_{i1z}).
\end{align*}
\] (A14)

A.4. Simplification of Operators Defining \( v_{i1z} \) and \( v_{e1z} \)

Let us introduce the following notations:

\[
\begin{align*}
W_i &= \gamma \frac{\partial}{\partial t} + \Omega_{Ti} - \Omega_{ni}, \quad V_i = \frac{\partial}{\partial t} + \Omega_{Ti}, \\
W_e &= \gamma \frac{\partial}{\partial t} + \Omega_{Te} - \Omega_{ne}, \quad V_e = \frac{\partial}{\partial t} + \Omega_{\chi} + \Omega_{Te}.
\end{align*}
\] (A15)

Then, the following operators take the form

\[
\begin{align*}
D &= V_i V_e + \Omega_{i} V_i + \Omega_{e} V_e, \\
&= \frac{1}{T_{i0}} (D_{i1e} C_{i1e} - T_{i0} D) = W_i (V_e + \Omega_{ei}) + \Omega_{e0} V_e, \\
&= \frac{1}{T_{e0}} (D_{i1e} C_{i1e} - T_{e0} D) = W_e (V_i + \Omega_{ie}) + \Omega_{e0} V_e, \\
L &= D \frac{\partial^4}{\partial t^4} + L_1 \frac{\partial^4}{\partial z^4} + \frac{T_{i0} T_{e0}}{m_i m_e} L_2 \frac{\partial^4}{\partial z^4},
\end{align*}
\] (A16)

where

\[
\begin{align*}
L_1 &= -\frac{T_{e0}}{m_e} [W_e (V_i + \Omega_{ei}) + V_i \Omega_{ei}], \\
&= -\frac{T_{i0}}{m_i} [W_i (V_e + \Omega_{ie}) + V_e \Omega_{ie}], \\
L_2 &= W_i W_e V_i + W_i V_i (W_e + V_e) \Omega_{ie} + W_e V_e (W_i + V_i) \Omega_{ei} + W_i V_i \Omega_{ie} + W_e V_e \Omega_{ei} + (W_i + V_i) \Omega_{ie} \Omega_{ei}.
\end{align*}
\] (A17)

We further have

\[
\begin{align*}
\left( \frac{\partial}{\partial t} \right)^{-1} H_{i1} &= \frac{q_i}{m_i} G_1 \left( E_{1z} + \frac{v_{0z}}{c} B_{1y} \right) - \frac{v_{0y}}{c} \frac{q_i}{m_i} G_1 B_{1x}, \\
\left( \frac{\partial}{\partial t} \right)^{-1} H_{e1} &= \frac{q_e}{m_e} G_3 \left( E_{1z} + \frac{v_{0z}}{c} B_{1y} \right) - \frac{v_{0y}}{c} \frac{q_i}{m_i} G_3 B_{1x},
\end{align*}
\] (A18)

where the following notations are introduced

\[
\begin{align*}
G_1 &= D \frac{\partial^2}{\partial t^2} - \frac{T_{e0}}{m_e} \\
&\times \left[ W_i V_i + W_i \Omega_{ei} + V_e \Omega_{ie} - \frac{q_i}{q_i} (W_i - V_i) \Omega_{ei} \right] \frac{\partial^2}{\partial z^2}, \\
G_2 &= D \frac{\partial^2}{\partial t^2} - \frac{T_{i0}}{m_i} \left[ W_i V_i + W_i \Omega_{ei} + V_e \Omega_{ie} \right] \frac{\partial^2}{\partial z^2}, \\
G_3 &= D \frac{\partial^2}{\partial t^2} - \frac{T_{i0}}{m_i} \left[ W_i V_i + W_i \Omega_{ei} + V_e \Omega_{ie} \right] \frac{\partial^2}{\partial z^2}, \\
G_4 &= \frac{T_{i0}}{m_e} (W_i - V_i) \Omega_{ei} \frac{\partial^2}{\partial z^2}.
\end{align*}
\] (A19)
In Equations (A18), we have used the expressions (see Equation (A2))
\[
\begin{align*}
F_{t1z} &= \frac{q_i}{m_i} \left( E_{1z} c + v_{0z} B_{1y} - \frac{v_{0y}}{c} B_{1z} \right), \\
F_{c1z} &= \frac{q_e}{m_e} \left( E_{c1z} c + v_{0z} B_{c1y} \right),
\end{align*}
\]
(A20)
where \(v_{0z} = c E_{0y} / B_0\).

APPENDIX B

B.1. Perturbed Velocity of Cosmic Rays

For the cold, nonrelativistic, \(T_{cr} \ll m_{cr} c^2\), cosmic rays, the linearized Equation (6) takes the form
\[
\gamma_{cr0} \frac{\partial v_{cr1}}{\partial t} + \gamma_{cr0} \frac{v_{cr1}}{c^2} \frac{\partial v_{cr1}}{\partial t} = - \nabla p_{cr1} + \frac{q_{cr}}{m_{cr} c} v_{cr1} \times B_0,
\]
where
\[
F_{cr1} = \frac{q_{cr}}{m_{cr}} \left( E_1 + \frac{1}{c} \mathbf{u}_{cr} \times \mathbf{B}_1 \right)
\]
and \(u_{cr}\) is directed along the \(z\)-axis. We have used \(\gamma_{cr} = \gamma_{cr0} \mathbf{u}_{cr} \cdot \mathbf{v}_{cr}/c^2\), where \(\gamma_{cr0} = (1 - u_{cr}^2/c^2)^{-1/2}\). Equations (B1) and (B2) do not include \(v_0\) in Equation (13). From Equation (B1), we find the following equations for \(v_{cr1x, y}\):
\[
\begin{align*}
\left( \gamma_{cr0} \frac{\partial^2}{\partial t^2} + \omega_{cr0}^2 \right) v_{cr1x} &= \omega_{cr} F_{cr1x} + \gamma_{cr0} \frac{\partial F_{cr1x}}{\partial t}, \\
\left( \gamma_{cr0} \frac{\partial^2}{\partial t^2} + \omega_{cr0}^2 \right) v_{cr1y} &= -\omega_{cr} F_{cr1y} + \gamma_{cr0} \frac{\partial F_{cr1y}}{\partial t}.
\end{align*}
\]
The \(z\)-component of Equation (B1) is given by
\[
\gamma_{cr0} \frac{\partial v_{cr1z}}{\partial t} = - \frac{1}{m_{cr} n_{cr0}} \frac{\partial p_{cr1}}{\partial z} + F_{cr1z}.
\]
From Equation (7) in the linear approximation, we find
\[
p_{cr1} = \rho_{cr0} \Gamma \left( \frac{n_{cr1}}{n_{cr0}} - 2 \frac{u_{cr1} v_{cr1z}}{c^2} \right).
\]
Applying the operator \(\partial/\partial t\), substituting Equation (B5), and using the continuity equation for cosmic rays in Equation (B4), we obtain
\[
L_{cr} v_{cr1z} = H_{cr1}.
\]
Here,
\[
\begin{align*}
L_{cr} &= \gamma_{cr0} \frac{\partial^2}{\partial t^2} - c_{sc}^2 \frac{\partial^2}{\partial z^2}, \\
H_{cr1} &= c_{sc}^2 \gamma_{cr0}^2 \frac{u_{cr1} v_{cr1z}}{c^2} \frac{\partial^2 v_{cr1z}}{\partial t^2} + \frac{\partial F_{cr1z}}{\partial t},
\end{align*}
\]
where \(c_{sc} = (p_{cr0} \Gamma / m_{cr} n_{cr0})^{1/2}\) is the sound speed of cosmic rays. We note that the first term on the right-hand side in the definition of \(H_{cr1}\) in Equation (B7) is connected with the perturbation of the Lorentz factor.

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