The production of the doubly charmed baryon in deeply inelastic $ep$ scattering at the Large Hadron Electron Collider

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ABSTRACT: In this paper, we carry out a detailed study on the production of the doubly charmed baryon in deeply inelastic $ep$ scattering (DIS) for $Q^2 \in [2, 100]$ GeV$^2$, at the Large Hadron Electron Collider (LHeC) with $E_e = 60(140)$ GeV and $E_p = 7000$ GeV. To exclude the contributions from the diffractive productions and the $b$ hadron decays, we impose the kinematic cuts $0.3 < z < 0.9$, and $p^2_{t,\text{baryon}} > 1$ GeV$^2$ in the center-of-mass (CM) frame of $\gamma^* p$. Based on the designed LHeC luminosity, by detecting the decay channel $\Xi^+_{cc}(\Xi^{++}_{cc}) \rightarrow \Lambda^+_c$ with the subsequent decay $\Lambda^+_c \rightarrow pK^-\pi^+$, we predict that about 1880 (2700) $\Xi^+_{cc}$ events and 3750 (5400) $\Xi^{++}_{cc}$ events can be accumulated per year, which signifies the prospect of observing them via the DIS at the forthcoming LHeC. In addition, we also predict the distributions of a rich variety of physical observables in the laboratory frame and the $\gamma^* p$ CM frame, including $Q^2$, $p^2_t$, $Y$ (rapidity), $p^2_{t,\text{baryon}}$, $Y^*$, $W$, and $z$ distributions, respectively, which can provide helpful references for studying the doubly charmed baryon.

In conclusion, we think that in addition to the LHC, the LHeC is also a helpful platform for studying the properties of the doubly charmed baryon.

KEYWORDS: Doubly charmed baryon, DIS, NRQCD, LHeC
1 Introduction

The doubly charmed baryon, interpreted as a three-quark state with two heavy \( c \) quarks and a light quark \( q \) (\( q = u, d, s \)) based on the quark model [1–5], is very helpful for quantitatively testing the quantum chromodynamics (QCD). It has attracted a great deal of attention of the physicists. In year 2002 and 2005, the SELEX collaboration reported the observations of \( \Xi^{+}_{cc} \) via its decay channel \( \Xi^{+}_{cc} \rightarrow \Lambda^{+}_c K^- \pi^+ \) [6] and \( \Xi^{+}_{cc} \rightarrow pD^+ K^- \) [7]. However, the large production rates released by the SELEX Collaboration have not been confirmed by the BaBAR [8], the Belle [9, 10], and even the FOCUS [11] that is at the same collider of SELEX. In the past several years, the LHCb group has searched for the hadroproduced \( \Xi^{+}_{cc} (\Xi^{++}_{cc}) \) for many times [12–16], collecting fruitful results. While, the ratio \( R (= \frac{\sigma(\Xi^{+}_{cc})}{\sigma(\Lambda^+)} \frac{B(\Xi^{+}_{cc} \rightarrow \Lambda^+_c K^- \pi^+)}{B(\Lambda^+_c \rightarrow \pi^+ \pi^+ \pi^-)}} \) measured by LHCb [17] is still significantly below the value reported by the SELEX Collaboration. On theoretical aspect, the doubly charmed baryon productions at various high-energy colliders have been detailedly studied in the literatures [18–54], such as the \( e^+ e^- \) annihilation, the photoproduction in \( ep \) collision, the hadroproductions in \( pp \) (or \( pp \)) collision, and the indirect productions via the decays of \( Z \) boson, \( t \) quark, and the standard Higgs boson. Especially, a dedicated generator GENXICC [55–57] has been developed and has been frequently applied to study the production of doubly heavy baryon at the LHC.

In addition to the hadro- and photoproductions, the productions in deeply inelastic scattering (DIS) is another interesting process for the studies of the doubly charmed baryon, like the \( J/\psi \) case [58–75]. Analyzing the productions in DIS at finit \( Q^2 \) has theoretical and

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experimental advantages comparing to the inelastic photoproduction \((Q^2 \simeq 0)\). The large \(Q^2\), such as the kinematic region of \(Q^2 \in [2, 100] \text{GeV}^2\), corresponding to the inclusive \(J/\psi\) productions in DIS at the Hadron-Electron Ring Accelerator (HERA), may decrease the theoretical uncertainties [73] and make the perturbative calculations work better \(^1\); the contributions via the resolved photon are expected to be negligible since the probability of the resolved photon to emerge decrease rapidly with \(Q^2\). Furthermore, the background via the diffractive production is also expected to decrease faster with increasing \(Q^2\) than the case of the inelastic photoproduction process. On experimental aspects, the distinct signature of the scattered electron in the final state makes the process easier to be detected. Comparing to the hadro- and protoproduction, much more varieties of physical observables can be measured in the DIS process, such as \(p_t^2, Y\) (rapidity), \(p_t^s, Y^s, W, z, \text{and } Q^2\) distributions, where the left four variables are related to the doubly charmed baryon. Throughout the paper we employ the superscript “\(\ast\)” to denote the measured quantities in the center-of-mass (CM) frame of \(\gamma^*p\). In view of these advantages, the production in DIS can provide an ideal laboratory for studying the doubly charmed baryon, deserving a separate investigation.

The Large Hadron Electron Collider (LHeC) is designed as a second generation of DIS \(ep\), and a first electron-ion collider [82, 83]. As reported in the Conceptual Design Report (CDR) of LHeC [82], it takes unique advantage of the intense, high energy hadron beams of the LHC, and a 60 GeV, to possibly 140 GeV, electron beam of high intensity based on a racetrack, energy recovery configuration using two 10 GeV electron linacs. Thus the LHeC will exceeds the luminosity of HERA by a factor of 100, reaching up to \(\mathcal{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1}\). Note that this value of \(\mathcal{L}\) is based on the original design for LHeC in 2012, from today’s experience with the LHC operation, the improved proton beam parameters of the HL-LHC (high luminosity) upgrade may lead to a significantly higher luminosity for the LHeC than the value of \(\mathcal{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1}\). As reported in Reference [84], by the parasitic operation in parallel to the HL-LHC \(pp\) collision, the up-to-date luminosity of LHeC could be improved to be \(\mathcal{L} = 0.8 \times 10^{34}\text{cm}^{-2}\text{s}^{-1}\), about one order of magnitude larger than the designed value in 2012. From these perspectives, the forthcoming LHeC allows a multitude of crucial DIS measurements to be performed. Thus, in this paper, we will for the first time carry out the studies on the doubly charmed baryon productions in DIS \((ep)\) at the LHeC.

The rest of the paper is organized as follows: In section 1, we give a description on the calculation formalism. In section 2, the phenomenological results and discussions are presented. Section 4 is reserved as a summary.

2 Calculation Formalism

2.1 General Formalism

It is a widely accepted view that the productions of the doubly charmed baryon can be factorized into three steps, c.f. Refs.[26, 46, 52]: the first step is to produce a \(c\) quark pair by the perturbative calculable hard processes; the second step is the \(c\) quark pair

\(^1\)Ref.[67] shows that the next-to-leading order QCD corrections to the inclusive \(J/\psi\) productions in DIS exhibits a rather good convergence, which is much better than the case of hadro- and photoproduction [76–81].
nonperturbatively forming a bounding \((cc)[n]\)-diquark in the spin-triplet \((n = [^3S_1]_3)\) or spin-singlet \((n = [^1S_0]_6)\) configurations that can be described by a matrix element, \(h_{3\bar{3}}\), accordingly in nonrelativistic Quantum Chromodynamics (NRQCD) [85]; the third step is the hadronization of the diquark into a physical colorless baryon \(\Xi_{ccq}\) \((q = u, d, s)\) by grabbing a light quark with possible soft gluons from the hadron. Here \(\bar{3}\) and \(6\) denote the color states. For convenience, hereinafter we label the doubly charmed baryon as \(\Xi_{cc}\) instead of \(\Xi_{ccq}\). During the hadronization procedure of the diquark into \(\Xi_{cc}\), one usually assumes the total evolving probability to be 100%, referring as the “direct evolution”. Among this total “100%” probability, the diquark fragmenting into \(\Xi_{cc}^+\)(ccd) and \(\Xi_{cc}^{++}\)(ccu) both accounts for 43%, respectively, and the ratio for \(\Omega_{cc}^+(ccs)\) is 14% [49, 86]. Chen et al. in Reference [35] pointed out that the direct evolution and the evolution via the heavy quark fragmentation model suggested by Reference [87] lead to almost the same numerical results, indicating that the direct evolution mechanism is precise enough to describe the production of the doubly charmed baryon.

![Figure 1](image.png)

**Figure 1**: The illustrative diagram for the \(\Xi_{cc}\) production in deeply inelastic \(ep\) scattering.

According to the above steps, we draw the illustrative diagram regarding the DIS production process \(e + p \rightarrow e + \Xi_{cc} + X\) in Figure 1, where \(e_i, e_f, \gamma^*, g,\) and \(p\) represent the initial and final electron, the virtual photon, the gluon parton, and the proton, respectively. To be specific, we shall focus on the contributions from the dominant gluon partonic process \(e + g \rightarrow e + \Xi_{cc} + \bar{c} + \bar{c}\); and the less-important contributions from the \(c\)-quark content in proton shall not be considered here. In addition, the observed value of \(Q\) is smaller than the mass of the \(Z\) boson, such as \(Q^2 \in [2, 100]\ \text{GeV}^2\), thus we neglect the contributions from the \(Z\) propagator, as well as the \(H^0\) propagator, taking only \(\gamma^*\) into consideration. By regarding the electron and proton as massless, we introduce some generally used invariants
Q^2 = -p_{7*}^2 = -(p_{e_i} - p_{e_f})^2,
W^2 = (p_{7*} + p_p)^2,
S = (p_p + p_{e_i})^2 = 2 p_p \cdot p_{e_i},
\hat{s} = (p_g + p_{7*})^2,
s = \hat{s} + Q^2 = 2 p_g \cdot p_{7*},
z = \frac{p_p \cdot p_{\Xi_{cc}}}{p_p \cdot p_{7*}}, \quad y = \frac{p_p \cdot p_{7*}}{p_p \cdot p_{e_i}}.
(2.1)

Based on the NRQCD and the collinear factorization, we factorize the $\Xi_{cc}$ production in DIS via $e + p \to e + \Xi_{cc} + \bar{c} + \bar{c}$ as

$$d\sigma(e + p \to e + \Xi_{cc} + \bar{c} + \bar{c}) = 
\int dx \sum_n f_{g/p}(x, \mu_f) d\hat{s}_{e+g\to e+(cc)[n]+\bar{c}+\bar{c}} \times \langle \mathcal{O}^{\Xi_{cc}}(n) \rangle;
(2.2)$$

where $d\hat{s}_{e+g\to e+(cc)[n]+\bar{c}+\bar{c}}$ is the perturbative calculable short distance coefficient (SDC), representing the production of a configuration of the $(cc)[n]$ intermediate quark pair with $n = [^3S_1]^3$ or $[^1S_0]^6$. The universal nonperturbative long distance matrix element $\langle \mathcal{O}^{\Xi_{cc}}(n) \rangle$ stands for the probability of $(cc)[n]$ pair into the corresponding diquark, subsequently fragmenting into $\Xi_{cc}$, i.e., $h_{^{3}S_{1}} \times 100\%$ based on the direct evolution mechanism. $f_{g/p}(x, \mu_f)$ is the gluon parton distribution function evaluated at the factorization scale $\mu_f$. The hard partonic SDC can be written as

$$d\hat{s}_{e+g\to e+(cc)[n]+\bar{c}+\bar{c}} = \frac{1}{2xS N_c N_s} |\mathcal{M}|^2 d\Phi.
(2.3)$$

where $1/(N_c N_s)$ is the colour and spin average factor; $|\mathcal{M}|^2$ and $d\Phi$ are squared matrix element and the 4-body phase space, respectively. In the following subsections, we depict the method of how to calculate $|\mathcal{M}|^2$ and $d\Phi$.

2.2 $|\mathcal{M}|^2$

For the subprocess $e + g \to e + (cc)[n] + \bar{c} + \bar{c}$, there are in total 48 Feynman diagrams, half of which are illustrated in Figure 2 ($p_4 \leftrightarrow p_5$). The other 24 ones can be obtained by exchanging the two identical $c-$quark lines inside the diquark. Seeing that we have set the relative velocity between the two constituent $c$ quarks in the diquark to be zero, the two parts of 24 diagrams contribute identically. Thus at the cross section level we only need to calculate the 24 diagrams in Figure 2, and multiply a factor of $2^2$. Simultaneously we should introduce an additional factor of $1/(2!)^2$ to deal with the identity of the two constituent $c$ quarks inside the diquark, and that of the two $\bar{c}$ quarks in the final states. To calculate $|\mathcal{M}|^2$, we decompose it as

$$|\mathcal{M}|^2 = L^{\mu\nu} H_{\mu\nu},
(2.4)$$

where $L^{\mu\nu}$ denote the leptonic tensor regarding $e_i \to e_f + \gamma^*$; $H_{\mu\nu}$ represents the hadronic tensor corresponding to $\gamma^* + g \to (cc)[n] + \bar{c} + \bar{c}$.
2.2.1 Leptonic tensor

By a direct calculation, one can obtain

\[ L^{\mu\nu} = \frac{1}{(Q^2)^2} 4\pi\alpha' \text{Tr} \left[ p_{\epsilon_i} \gamma_\mu \bar{p}_{\epsilon_f} \gamma_\nu \right] \]
\[ = \frac{1}{(Q^2)^2} 8\pi\alpha Q^2 \left( -g^{\mu\nu} + \frac{4p_{\epsilon_i}^\mu p_{\epsilon_i}^\nu - 2p_{\epsilon_i}^\mu p_{\gamma^*}^\nu - 2p_{\gamma^*}^\mu p_{\epsilon_i}^\nu}{Q^2} \right) \]
\[ = \frac{8\pi\alpha Q^2}{Q^2} L^{\mu\nu}. \]  

(2.5)

If only one final-state hadron (e.g. \( \Xi_{cc} \)) other than the scattered electron is observed, \( L^{\mu\nu} \) can be decomposed into the linear combination of four independent Lorentz invariant structures as \[65–67]\]

\[ l^{\mu\nu} = A_g \left( -g^{\mu\nu} - \frac{p_{\gamma^*}^\mu p_{\gamma^*}^\nu}{Q^2} \right) + A_L \epsilon_L^\mu \epsilon_L^\nu + A_L T \epsilon_L^\mu \epsilon_T^\nu + A_T \epsilon_T^\mu \epsilon_T^\nu \]  

(2.6)

where

\[ \epsilon_L = \frac{1}{Q} \left( p_{\gamma^*} + \frac{2Q^2}{s} p_g \right), \]
\[ \epsilon_T = \frac{1}{p_t^2} \left( p_{\Xi_{cc}} - p p_g - z p_{\gamma^*} \right), \]  

(2.7)
and

\begin{align*}
A_g &= 1 + \frac{2(1-y)}{y^2} - \frac{2(1-y)}{y^2} \cos(2\psi^*), \\
A_L &= 1 + \frac{6(1-y)}{y^2} - \frac{2(1-y)}{y^2} \cos(2\psi^*), \\
A_{LT} &= \frac{2(2-y)}{y^2} \sqrt{1-y} \cos(\psi^*), \\
A_T &= \frac{4(1-y)}{y^2} \cos(2\psi^*),
\end{align*}

(2.8)
in which \( \rho \) is defined as

\[
\rho = \left( \frac{p_t^\ast s^2}{2} + M_{\Xi cc}^2 \right) / z + zQ^2.
\]

(2.9)

Here, \( p_t^2 \) is the square of the transverse momentum of \( \Xi_{cc} \); \( \psi^* \) denotes the azimuthal angle of the hadron (\( \Xi_{cc} \)) production plane around the z axis relative to the lepton plane expanded by the incoming and the outgoing electrons. Substituting Equation (2.7) into Equation (2.6), with the help of the current conservation equation \( p^\mu H_{\mu\nu} = 0 \), one can obtain a more convenient form of \( l\mu\nu \)

\[
l\mu\nu = C_1 (-g\mu\nu) + C_2 p_{g\nu}^\mu + C_3 \frac{p_{g\nu}^{\mu} p_{\Xi cc}^{\nu} + p_{\Xi cc}^{\mu} p_{g\nu}^{\nu}}{2} + C_4 p_{\Xi cc}^{\mu} p_{\Xi cc}^{\nu},
\]

(2.10)

where

\[
C_1 = A_g, \\
C_2 = \frac{4Q^2}{s^2} (A_L - 2\beta A_{LT} + \beta^2 A_T), \\
C_3 = \frac{4Q}{p_t^2 s} (A_{LT} - \beta A_T), \\
C_4 = \frac{1}{p_t^2} A_T,
\]

(2.11)

with

\[
\beta = \left( \frac{p_t^2 + M_{\Xi cc}^2}{2p_t^2 Q} \right) / z + zQ^2.
\]

(2.12)

Comparing the two leptonic tensor forms in Equations (2.5) and (2.10), one can find that the four basic tensors in Equation (2.10) are only correlated to the hadronic process momentum \( (p_g, p_{\Xi cc}) \), which can greatly reduce the complication of the subsequent contraction with \( H_{\mu\nu} \). In Refs.[66, 67], the leptonic tensor in Equation (2.10) have been utilized to calculate the \( J/\psi \) productions in DIS at HERA.

### 2.2.2 Hadronic tensor

To begin with, taking that the virtual photon is attached to the fermion line including \( p_4 \) as an example, we write the hadronic amplitude \( \mathcal{M}_H^\mu \) in a general form

\[
\mathcal{M}_H^\mu = C_{\bar{g}s} e e \left\{ \bar{u} \left( \frac{P_{\Xi cc}}{2}, s_1 \right) \Gamma_{\xi s f (-k_{\xi-1}, m_c)} \cdots \Gamma_{\mu} \cdots s_f (-k_1, m_c) \Gamma_{\nu} v(p_4, s_4) \right\} \times \left\{ \bar{u} \left( \frac{P_{\Xi cc}}{2}, s_2 \right) \Gamma'_{\xi' s f (q_1, m_c)} \cdots s_f (q_{n-1}, m_c) \Gamma'_{\nu} v(p_5, s_5) \right\},
\]

(2.13)
where \( s_f(k, m) \) (\( k = k_i \) or \( q_i \)) is the fermion propagator, \( \Gamma_1, \cdots, \Gamma_{\bar{c}}, \Gamma_1', \cdots, \Gamma_{\bar{c}}' \) are the interaction vertices, \( C \) is the color factor, and \( e_c \) is the electric charge of \( c \) quark. For the first fermion line in Equation (2.13),

\[
A = \bar{u} \left( \frac{p_{\xi, c}}{2}, s_1 \right) \Gamma_{\xi} s_f(-k_{\xi-1}, m_c) \cdots \Gamma_{\mu} \cdots s_f(-k_1, m_c) \Gamma_{\bar{c}} v(p_4, s_4),
\]  

(2.14)

we have

\[
A = A^T = v^T(p_4, s_4) \Gamma_{\bar{c}}^T \Gamma_{\bar{c}}^T(-k_1, m_c) \cdots \Gamma_{\mu}^T \cdots s_f^T(-k_{\xi-1}, m_c) \Gamma_{\xi}^T \bar{u} \left( \frac{p_{\xi, c}}{2}, s_1 \right)^T.
\]  

(2.15)

By inserting the charge conjugate matrix \( C = -i\gamma^2\gamma^0 \), which satisfies the following equations

\[
CC^{-1} = 1, \quad v^T(p_4, s_4)C = -\bar{u}(p_4, s_4), \quad C^{-1} \bar{u} \left( \frac{p_{\xi, c}}{2}, s_1 \right)^T = v \left( \frac{p_{\xi, c}}{2}, s_1 \right),
\]

\[
C^{-1} \Gamma_{\xi} C = -\Gamma_1, \quad C^{-1} s_f(-k_1, m_c) C = s_f(k_1, m_c),
\]

(2.16)

we obtain

\[
A = v^T(p_4, s_4) C C^{-1} \Gamma_{\xi}^T \bar{u} \left( \frac{p_{\xi, c}}{2}, s_1 \right)^T \Gamma_{\bar{c}}^T \cdots \Gamma_{\mu}^T \cdots s_f^T(-k_{\xi-1}, m_c)
\times \quad C C^{-1} \Gamma_{\xi}^T \bar{u} \left( \frac{p_{\xi, c}}{2}, s_1 \right)^T \Gamma_{\bar{c}}^T \cdots \Gamma_{\mu}^T \cdots s_f^T(-k_{\xi-1}, m_c)
\]

\[
= (-1)^{\xi+1} \bar{u}(p_4, s_4) \Gamma_1 s_f(k_1, m_c) \cdots \Gamma_{\mu} \cdots s_f(k_{\xi-1}, m_c) \Gamma_{\xi} v \left( \frac{p_{\xi, c}}{2}, s_1 \right).
\]

(2.17)

Combining Equations (2.13), (2.14), and (2.17), the hadronic amplitude \( \mathcal{M}^H_{\mu} \) can be transformed as

\[
\mathcal{M}^H_{\mu} = (-1)^{\xi+1} \bar{u}(p_4, s_4) \Gamma_1 s_f(k_1, m_c) \cdots \Gamma_{\mu} \cdots s_f(k_{\xi-1}, m_c) \Gamma_{\xi} v \left( \frac{p_{\xi, c}}{2}, s_1 \right)
\times \quad \bar{u} \left( \frac{p_{\xi, c}}{2}, s_2 \right) \Gamma_1 s_f(q_1, m_c) \cdots \Gamma_{\mu} \cdots s_f(q_{\xi-1}, m_c) \Gamma_{\xi} v(p_5, s_5),
\]

\[
= (-1)^{\xi+1} \bar{u}(p_4, s_4) \Gamma_1 s_f(k_1, m_c) \cdots \Gamma_{\mu} \cdots s_f(k_{\xi-1}, m_c) \Gamma_{\xi}
\times \quad \Pi^0_{p_{\xi, c}} \times \Gamma_1 s_f(q_1, m_c) \cdots \gamma_5 \cdot s_f(q_{\xi-1}, m_c) \Gamma_{\xi} v(p_5, s_5),
\]

(2.18)

where \( \Pi^0 \) and \( \Pi^\rho \) are the spin projector operators \([88]\)

\[
\Pi^0 = \frac{1}{\sqrt{8m_c^2}} \left( \frac{1}{2} - m_c \right) \gamma^5 \left( \frac{1}{2} + m_c \right),
\]

\[
\Pi^\rho = \frac{1}{\sqrt{8m_c^2}} \left( \frac{1}{2} - m_c \right) \gamma^\rho \left( \frac{1}{2} + m_c \right),
\]

(2.19)

which are for the spin singlet \((1S_0)\) and the spin triplet \((3S_1)\), respectively. From Equation (2.18), it is not difficult to find the calculation for the doubly charmed baryon is similar to the charmonium case, except for the factor of \((-1)^{\xi+1}\) and the color factor.

For the color-singlet charmonium, since the constituent \( c \) and \( \bar{c} \) must have the same color so as to guarantee the colorless state of the quarkonium, we use \( \delta_{ij}/\sqrt{2} \) to characterize
its color with \((i, j = 3)\) denoting the color indices of \(c/\bar{c}\). \(\sqrt{2}\) is the normalized factor. However, in the case of diquark, by the fact that \(3 \otimes 3 = 3 \oplus 6\) in \(SU_c(3)\) group, it can be either in anti-triplet 3 or in sextuplet 6 color state. Based on this, we introduce the function \(G_{ijk} / \sqrt{2}\) to describe the diquark color, where \(i, j, k = 3\) represents the color indices of the two constituent \(c\) quarks and that of the diquark, respectively. \(G_{ijk}\) is identical to the antisymmetric \(\varepsilon_{ijk}\) (3) and the symmetric \(f_{ijk}\) (6), which satisfy the following equations

\[
\varepsilon_{ijk}\varepsilon_{i'j'k'} = \delta_{ii'}\delta_{jj'} - \delta_{jj'}\delta_{ii'},
\]

\[
f_{ijk}f_{i'j'k'} = \delta_{ii'}\delta_{jj'} + \delta_{jj'}\delta_{ii'},
\]

(2.20)

### 2.3 Phase space

To deal with the 4-body phase space \(d\Phi\), we first decompose it into two components

\[
d\Phi = d\Phi_L d\Phi_H,
\]

(2.21)

where

\[
d\Phi_L \equiv \frac{d^3 p_{t_1}}{(2\pi)^3 2p_{t_1}^0},
\]

\[
d\Phi_H \equiv (2\pi)^4 \delta^4(p_{\gamma^*} + p_g - p_{\Xi_{cc}} - p_4 - p_5) \frac{d^3 p_{\Xi_{cc}}}{(2\pi)^3 2p_{\Xi_{cc}}^0} \frac{d^3 p_4}{(2\pi)^3 2p_4^0} \frac{d^3 p_5}{(2\pi)^3 2p_5^0}.
\]

(2.22)

Integrating over the azimuthal angle of the outgoing electron, one can obtain

\[
d\Phi_L = \frac{1}{(4\pi)^2} S dQ^2 dW^2.
\]

(2.23)

By introducing a new momentum \(p_{45} = p_4 + p_5\) with \(p_{45}^2 = s_1\), \(d\Phi_H\) can be rewritten as

\[
d\Phi_H = \left[ (2\pi)^4 \delta^4(p_{\gamma^*} + p_g - p_{\Xi_{cc}} - p_{45}) \frac{d^3 p_{\Xi_{cc}}}{(2\pi)^3 2p_{\Xi_{cc}}^0} \frac{d^3 p_4}{(2\pi)^3 2p_4^0} \right] \frac{d^3 p_5}{(2\pi)^3 2p_5^0} \times \left[ \frac{(2\pi)^4 \delta^4(p_{45} - p_4 - p_5) ds_1}{2\pi (2\pi)^3 2p_4^0 (2\pi)^3 2p_5^0} \right] = d\Phi_{H_1} \times d\Phi_{H_2}.
\]

(2.24)

Integrating over the three-momentum of \(p_{45}\), we have (see e.g. Reference [66])

\[
dx d\Phi_{H_1} = \frac{1}{32\pi^2 p_{45}^0} \delta(p_{\gamma^*}^0 + p_g^0 - p_{\Xi_{cc}}^0 - p_{45}^0) dp_{t_1}^2 dz d\psi^* dx \]

\[
= \frac{1}{(4\pi)^2(W^2 + Q^2) z(1 - z)} dp_{t_1}^2 dz d\psi^*,
\]

(2.25)

In the second step of Equation (2.25), we have utilized \(\delta(p_{\gamma^*}^0 + p_g^0 - p_{\Xi_{cc}}^0 - p_{45}^0)\) to integrate over \(dx\), thus the value of \(x\) has been fixed at

\[
x = \frac{a + Q^2}{W^2 + Q^2}, \text{ with } a = \frac{p_{t_1}^2 + M_{\Xi_{cc}}^2}{z} + \frac{p_{t_1}^2 + s_1}{1 - z},
\]

(2.26)
As for \( d\Phi_{H_2} \), in the CM frame of the two final-state \( \bar{c} \), we have

\[
p_{45} = (\sqrt{s_1}, 0, 0, 0),
\]

\[
p_{4} = (p_{4}^{0}, |\vec{p}_{4}| \sin \theta_{\bar{c}} \cos \phi_{\bar{c}}, |\vec{p}_{4}| \sin \theta_{\bar{c}} \sin \phi_{\bar{c}}, |\vec{p}_{4}| \cos \theta_{\bar{c}}),
\]

\[
p_{5} = (p_{5}^{0}, -|\vec{p}_{4}| \sin \theta_{\bar{c}} \cos \phi_{\bar{c}}, -|\vec{p}_{4}| \sin \theta_{\bar{c}} \sin \phi_{\bar{c}}, -|\vec{p}_{4}| \cos \theta_{\bar{c}}).
\]

(2.27)

Then

\[
d\Phi_{H_2} = (2\pi)^4 \delta^4(p_{45} - p_4 - p_5) \frac{ds_1}{2\pi} \frac{d^3p_4}{(2\pi)^3 2p_4^0} \frac{d^3p_5}{(2\pi)^3 2p_5^0}
\]

\[
= \frac{ds_1}{2\pi} \frac{1}{(2\pi)^2} \frac{|\vec{p}_4|}{2} d\phi d\Omega d^4p_4 \delta(p_4^2 - m_{\bar{c}}^2) \delta^4(p_{45} - p_4 - p_5)
\]

\[
= \frac{1}{(4\pi)^3} \sqrt{1 - \frac{4m_{\bar{c}}^2}{s_1}} ds_1 d\cos \theta_{\bar{c}} d\phi_{\bar{c}}.
\]

(2.28)

Combining Equations (2.23), (2.25), and (2.28), we finally obtain\(^2\)

\[
dx d\Phi = \frac{1}{(4\pi)^7 S(W^2 + Q^2) z(1 - z)} \sqrt{1 - \frac{4m_{\bar{c}}^2}{s_1}} dQ^2 dW^2 d\phi d\psi \frac{dz}{ds_1} d\cos \theta_{\bar{c}} d\phi_{\bar{c}}.
\]

(2.29)

3 Phenomenological results

3.1 Kinematic cuts and input parameters

According to the CDR of LHeC \cite{82, 83}, the designed beam energy of the incident electron and proton are: \( E_e = 60 \text{ GeV} \), to possibly 140 GeV, and \( E_P = 7000 \text{ GeV} \). Reviewing the inclusive \( J/\psi \) productions in DIS at HERA \cite{72–75}, the measurements mainly cover the range of \( p_{t,J/\psi}^2 > 1 \text{ GeV}^2, 0.3 < z < 0.9, 2 < Q^2 < 100 \text{ GeV}^2 \), and \( 50 < W < 225 \text{ GeV} \). The cuts regarding \( p_{t,J/\psi}^2 \) and \( z \) are applied to suppress the effects via the diffractive productions, which cannot yet be reliably described by purely perturbative QCD calculations, and that from the \( b \) hadrons decay. As for the \( \Xi_{cc} \) case, the situation is similar, so we shall also carry out the cut operation to avoid kinematic overlaps with the diffractive productions and the decay of \( b \) hadrons. Considering the HERA is the unique available \( ep \) collider by now, it is natural and reasonable to take its experimental conditions for reference to assume the kinematic cuts for the \( \Xi_{cc} \) productions in DIS at LHeC. To be specific, in our calculations, we adopt the following cut conditions

\[
\begin{align*}
p_{t,\Xi_{cc}}^2 &> 1 \text{ GeV}^2, \quad W > 50 \text{ GeV}, \\
2 &< Q^2 < 100 \text{ GeV}^2, \\
0.3 &< z < 0.9.
\end{align*}
\]

(3.1)

The other input parameters are listed below

\(^2\)Note that, the derivation of \( d\Phi_{H_2} \) is based on the CM frame of \( p_ap_b \), thus in the calculations one should transform to the \( \gamma p \) CM frame, or the laboratory frame.
1) The charm quark mass is taken as $m_c = M_{\Xi_{cc}}/2 = 1.75$ GeV [46, 49]; the fine structure constant is $\alpha = 1/128$.

2) The default factorization and renormalization scales are chosen to be $\mu_f = \mu_r = \mu_0 = \sqrt{Q^2 + M_{\Xi_{cc}}^2}$.

3) According to the velocity scaling rule of NRQCD, we take the usual assumption that $h_3$ and $h_6$ have the equal values [26, 45, 46], which are related to the wave function at the origin [20, 45]

$$h_3 = h_6 = |\Psi_{(cc)}(0)|^2 = 0.039 \text{ GeV}^3.$$  \hspace{1cm} (3.2)

### 3.2 Integrated cross sections

The integrated cross sections of $\Xi_{cc}$ via the two configurations of $(cc)[^3S_1]_3$ and $(cc)[^1S_0]_6$, under the kinematic cuts listed in Equation (3.1), are predicted to be:

for $E_e = 60$ GeV,

$$\sigma_{(cc)[^3S_1]_3} = 19.5^{+7.20}_{-5.20} \text{ pb},$$

$$\sigma_{(cc)[^1S_0]_6} = 2.42^{+0.90}_{-0.65} \text{ pb},$$

and for $E_e = 140$ GeV,

$$\sigma_{(cc)[^3S_1]_3} = 28.0^{+7.90}_{-6.60} \text{ pb},$$

$$\sigma_{(cc)[^1S_0]_6} = 3.48^{+1.01}_{-0.82} \text{ pb},$$

where the uncertainties are induced by varying $\mu_r (= \mu_f)$ from 0.5$\mu_0$ to 2$\mu_0$. From Equations (3.3) and (3.4), one can find: halving or doubling $\mu_r$ and $\mu_f$ simultaneously around the default value of $\mu_0$ will arouse a 30% ~ 40% variation of the integrated cross section; the integrated cross sections corresponding to $E_e = 140$ GeV is about 40% larger in magnitude than that of $E_e = 60$ GeV; the contribution via the configuration of $(cc)[^3S_1]_3$ dominates over that of $(cc)[^1S_0]_6$, accounting for about 90% in the total predictions.

Summing up the contributions of $(cc)[^3S_1]_3$ and $(cc)[^1S_0]_6$, we can collect about $1.8(2.5) \times 10^6 \Xi_{cc}$ events corresponding to $E_e = 60(140)$ GeV in one year (assuming 10$^7$ seconds running time), based on the up-to-date designed luminosity of LHeC $\mathcal{L} = 0.8 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$. By further considering the decay chain of $\Xi_{cc}^{++} \rightarrow \Lambda_{cc}^{+} K^- \pi^+$ with the cascade decay $\Lambda_{cc}^+ \rightarrow pK^-\pi^+$ [12], or $\Xi_{cc}^{++} \rightarrow \Lambda_{cc}^{+} K^-\pi^+\pi^+$ with $\Lambda_{cc}^+ \rightarrow pK^-\pi^+$ [13, 89], we can accumulate about 1880 reconstructed $\Xi_{cc}^{++}$ events, and 3750 reconstructed $\Xi_{cc}^{++}$ events, corresponding to $E_e = 60$ GeV. In the case of $E_e = 140$ GeV, the numbers change to be about 2700 and 5400, respectively. The thousands of reconstructed events per year strongly indicate the probability to hunt for the $\Xi_{cc}^{++}$ and $\Xi_{cc}^{++}$ events via the DIS at the LHeC.

For comparison, we now stop to test the prospect of observing the $\Xi_{cc}$ productions in DIS at the HERA. By adopting the same kinematic cuts as applied for the $J/\psi$ production [73], we have

$$\sigma_{\Xi_{cc}} = 3.61(3.69) \text{ pb},$$

(3.5)
corresponding to \( E_e = 27.5 \) GeV and \( E_P = 820(920) \) GeV. According to the integrated luminosity of HERA, about 70 pb\(^{-1}\) in the years 1997-2000 [73], only less than 300 \( \Xi_{cc} \) events can be generated there. Thus, it could be extremely hard to search for either \( \Xi_{cc} \) or \( \Xi_{cc}^{++} \) at the HERA by detecting their decaying into \( \Lambda_c^+ \) with subsequent \( \Lambda_c^+ \rightarrow pK^-\pi^+ \).

**Table 1:** The integrated cross sections (unit: pb) of \( \Xi_{cc} \) corresponding to different \( Q^2 \) bins (unit: GeV\(^2\)) under the kinematic cuts in Equation (3.1). \( N_{\Xi_{cc}}^{++} \) and \( N_{\Xi_{cc}}^{+} \) denote the number of the \( \Xi_{cc}^{++} \) and \( \Xi_{cc}^{+} \) events per year, reconstructed by the decay chains of \( \Xi_{cc}^{+} \rightarrow \Lambda_c^+K^-\pi^+ \) with \( \Lambda_c^+ \rightarrow pK^-\pi^+ \), and \( \Xi_{cc}^{++} \rightarrow \Lambda_c^+K^-\pi^+ \) with \( \Lambda_c^+ \rightarrow pK^-\pi^+ \).

| \( Q^2_{\text{bin}} \) | \( \sigma_{\Xi_{cc}} \) | \( N_{\Xi_{cc}}^{++} \) | \( N_{\Xi_{cc}}^{+} \) | \( \sigma_{\Xi_{cc}} \) | \( N_{\Xi_{cc}}^{++} \) | \( N_{\Xi_{cc}}^{+} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 2 – 3.5         | 6.14 ± 2.07     | 528 ± 141       | 1056 ± 356      | 6.67 ± 2.15     | 746 ± 175       | 1491 ± 472      |
| 3.5 – 6.5       | 5.73 ± 1.53     | 493 ± 132       | 986 ± 354       | 8.16 ± 1.90     | 702 ± 163       | 1404 ± 327      |
| 6.5 – 12        | 4.32 ± 1.14     | 372 ± 98        | 743 ± 287       | 6.21 ± 1.36     | 534 ± 126       | 1068 ± 320      |
| 12 – 20         | 2.52 ± 1.00     | 217 ± 58        | 433 ± 172       | 3.65 ± 0.88     | 314 ± 64        | 628 ± 151       |
| 20 – 40         | 2.05 ± 1.02     | 176 ± 47        | 353 ± 141       | 3.03 ± 1.00     | 261 ± 86        | 527 ± 172       |
| 40 – 100        | 1.13 ± 0.31     | 97 ± 27         | 194 ± 79        | 1.72 ± 0.59     | 148 ± 37        | 296 ± 74        |

**Figure 3:** The differential cross sections as a function of \( Q^2 \) under the kinematic cuts in Equation (3.1), corresponding to \( E_e = 60 \) GeV and \( E_e = 140 \) GeV, respectively. “Total” denotes the sum of the contributions via \((cc)^3S_1]/3\) and \((cc)^1S_0]/6\). The band is caused by the variation of \( \mu_r (= \mu_f) \) from 0.5\( \mu_0 \) to 2\( \mu_0 \).

The virtuality of \( \gamma^* \), representing by \( Q^2 \), is a key variable for the DIS process, thus it is interesting to investigate the dependence on it. For the integrated cross section, we choose some \( Q^2 \) bins between \( 2 < Q^2 < 100 \) GeV\(^2\) to present their values in Table 1. From the data in this table, we find that the dominant contributions are localized in the relatively small \( Q^2 \) scope, for instance, the proportion taken by the contribution of \( 2 < Q^2 < 20 \) GeV\(^2\) to the total result (\( 2 < Q^2 < 100 \) GeV\(^2\)) is about 85\%. Although the integrated cross sections of \( 20 < Q^2 < 40 \) GeV\(^2\) and \( 40 < Q^2 < 100 \) GeV\(^2\) are not as large as that of
2 ~ 20 GeV², accounting for about 10% and 5%, respectively, the corresponding hundreds of events also allow us to perform the measurements in those two Q² regions. Regarding the differential cross sections, we draw the Q² distributions in Figure 3. Throughout this paper, we adopt the binning pattern, instead of continuous curves, to present the predictions on the differential cross sections, as the J/ψ case at HERA [73]. One can find that, as Q² increases, dσ/dQ² decreases rapidly; the contribution of the (cc)[1S₀]₆ configuration becomes increasingly important.

3.3 Differential cross sections

| Variable     | Distribution |
|--------------|--------------|
| p²ₓ (GeV²)   | Eₑ = 60 GeV  |
| Y (Ξcc)      |              |
| W (GeV)      |              |
| z            |              |

Figure 4: The distributions of p²ₓ(Ξcc), Y(Ξcc), p²ᵧ(Ξcc), Y*(Ξcc), W, and z under the kinematic cuts in Equation (3.1), corresponding to Eₑ = 60 GeV. “Total” denotes the sum of the contributions via (cc)[3S₁]₃ and (cc)[1S₀]₆. The band is caused by the variation of μᵣ (= μₓ) from 0.5μ₀ to 2μ₀.
Figure 5: The distributions of $p_t^2(\Xi_{cc})$, $Y(\Xi_{cc})$, $p_t^2(\Xi_{cc})$, $Y^*(\Xi_{cc})$, $W$, and $z$ under the kinematic cuts of Equation (3.1), corresponding to $E_e = 140$ GeV. “Total” denotes the sum of the contributions via $(cc)[3S1]_8$ and $(cc)[1S0]_6$. The band is caused by the variation of $\mu_r(=\mu_f)$ from $0.5\mu_0$ to $2\mu_0$.

As stated in section 1, comparing to the hadro- and photoproduction, the production in DIS allow us to measure more varieties of physical observables. Therefore, to serve as a preliminary, useful reference awaiting for the future measurements by LHeC, in addition to $d\sigma/dQ^2$, we further provide the differential cross sections of $\Xi_{cc}$ with respect to $p_t^2$, $Y$, $p_t^2$, and $Y^*$, as well as the $W$, $z$ distributions, in Figures 4 and 5, which correspond to $E_e = 60$ GeV and $E_e = 140$ GeV, respectively. Following the conventions of HERA, the forward direction of $Y^*$ is defined as that of the incident virtual photo; $Y^*$ is taken to be positive in the direction of the incoming proton. As can be seen in the two figures,

1) As $p_t^2$ goes up, the differential cross sections continuously decrease, and the relative significance of the $(cc)[1S0]_6$ configuration increases gradually. With regard to the $p_t^2$
distribution, the situation is similar to the $p_t^2$ case, except that there is a small peak around $p_t^2 \sim 2.5 \text{ GeV}^2$.

2) For $Y^*$ (rapidity) distribution, the differential cross section is seriously asymmetric. To be specific, the value of $d\sigma/dY^*$ at $Y^* = 4$ is about five orders of magnitudes larger than that at $Y^* = 0$, which indicates, in the $\gamma^* p$ CM frame, $\Xi_{cc}$ is much more likely to be generated in the direction of the virtual photon rather than that of the incoming proton. The $Y$ distribution is also asymmetric, for example, $\left. \frac{d\sigma}{dY} \right|_{Y = -1.5}$ is about two times larger in magnitude than $\left. \frac{d\sigma}{dY} \right|_{Y = 1.5}$, from which we can learn that, in the laboratory frame, more $\Xi_{cc}$ events will be generated along the direction of the incident electron.

3) For $W$ distribution, the available $W$ values are scattered from tens to thousands, and the peak of the differential cross section is localized at the $W$ range of $100 \sim 150 \text{ GeV}$. Regarding the $z$ distribution, $d\sigma/dz$ corresponding to small $z$ is several times bigger than that for large $z$. This inequality implies that the small (mid) $z$ region is the main source of the $\Xi_{cc}$ events.

4 Summary

In this manuscript, we have carried out the studies on the productions of the doubly charmed baryons, $\Xi_{cc}^+$ and $\Xi_{cc}^{++}$, in deeply inelastic $ep$ scattering at the LHeC with $E_e = 60, 140 \text{ GeV}$ and $E_p = 7000 \text{ GeV}$. Imposing the kinematic cuts as adopted in observing the inclusive $J/\psi$ productions in DIS at the HERA, about 1880 ($2700$) $\Xi_{cc}^+$ events and 3750($5400$) $\Xi_{cc}^{++}$ events can be accumulated in one year by hunting for the decay chains of $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$ with the cascade decay $\Lambda_c^+ \rightarrow p K^- \pi^+$, and $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ with $\Lambda_c^+ \rightarrow p K^- \pi^+$.

We also provide the predictions on the distributions of $Q^2$, $p_t^2$, $Y$, $p_t^2$, $Y^*$, $W$, and $z$, which may serve as a useful reference for the future measurements at LHeC. Thousands of reconstructed events manifest the possibility of observing $\Xi_{cc}^+$ and $\Xi_{cc}^{++}$ baryons via the DIS production process at the LHeC. Thus we think, in addition to LHC, the LHeC can also be a good laboratory for investigating the properties of the doubly heavy baryons.

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