Surrogate Search As a Way to Combat Harmful Effects of Ill-behaved Evaluation Functions

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November 4, 2014

Abstract

Recently, several researchers have found that cost-based satisficing search with A* often runs into problems. Although some “work arounds” have been proposed to ameliorate the problem, there has been little concerted effort to pinpoint its origin. In this paper, we argue that the origins of this problem can be traced back to the fact that most planners that try to optimize cost also use cost-based evaluation functions (i.e., $f(n)$ is a cost estimate). We show that cost-based evaluation functions become ill-behaved whenever there is a wide variance in action costs; something that is all too common in planning domains. The general solution to this malady is what we call a surrogate search, where a surrogate evaluation function that doesn’t directly track the cost objective, and is resistant to cost-variance, is used. We will discuss some compelling choices for surrogate evaluation functions that are based on size rather than cost. Of particular practical interest is a cost-sensitive version of size-based evaluation function where the heuristic estimates the size of cheap paths, as it provides attractive quality vs. speed tradeoffs.

1 Introduction

Much of the scale-up and research focus in the automated planning community in the recent years has been on satisficing planning. Unfortunately, there has not been

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* An extended abstract of this paper appeared in the proceedings of SOCS 2010. This research is supported in part by ONR grants N00014-09-1-0017 and N00014-07-1-1049, the NSF grant IIS-0905672, and by DARPA and the U.S. Army Research Laboratory under contract W911NF-11-C-0037.
a concomitant increase in our understanding of satisficing search. Too often, the “theory” of satisficing search in automated planning defaults to doing (W)A∗ with evaluation functions that optimize over the problem objective. While this approach has shown itself to be empirically better than some other techniques, there is a disconnect between the behavior of the evaluation functions used in the search and the computational efficiency we seek when performing satisficing search. Finding a satisficing solution entails removing the requirement of optimality in favor of better performance, but the approach only goes half way; it removes the optimality requirement without attention to efficiency. This turns out not to be too much of a problem in domains with uniform cost values on each edge of the search graph, but it can lead to a significant performance hit in problems with non-uniform values. The issue is that the evaluation function can be slow-rising with respect to the search depth, trapping the search into performing an inordinate number of expansions.

As we shall see, these ill-behaved evaluation functions can cause arbitrarily poor performance and crop up over a spectrum of optimization problems in automated planning; where we often want to consider non-uniform edge costs in the search space. Problems that involve minimizing the costs of actions in a plan (i.e., when performing cost-based planning) offer a quintessential example of the issue, but many other planning search spaces have this quality as well. Indeed, any search space where the evaluation function fails to increase in concordance with the search depth can suffer. As such, the common objective in temporal planning of makespan minimization suffers an equally dissatisfying performance drop when using makespan itself as the evaluation function [1]. This is because many search decisions that involve adding actions in parallel to the current plan will give zero increase in the evaluation function. Because of this, most satisficing temporal planners already more or less ignore the objective function during search, except to prune plans with respect to incumbent solutions.

To make our discussion more concrete, let us consider a cost-based planning problem for which the cost-optimal and second-best solution to a problem consist of 10 and 1000 unspecified actions respectively. The optimal solution may be the longer one. Generally, a combinatorial search can be seen to consist of two parts: a “discovery” part where the (optimal) solution is found and a “proof” part where the optimality of the solution is verified. An A∗ search conflates these phases together and terminates only when it picks the optimal path for expansion. So, how long should it take just to discover the 10 action plan? How long should it take to prove (or disprove) its optimality? In general (presuming PSPACE/EXPSPACE ≠ P):
1. Discovery should require time exponential in, at most, 10.

2. Proof should require time exponential in, at least, 1000.

That is, in principle, the only way to (domain-independently) prove that the 10 action plan is better or worse than the 1000 action one is to in fact go and discover the 1000 action plan. Thus, A* search with cost-based evaluation function will take time proportional to \( b^{1000} \) for either discovery or proof. Simple breadth-first search discovers a solution in time proportional to \( b^{10} \) (and proof in \( O(b^{1000}) \)).

We use both abstract and benchmark problems and show that this is a systematic weakness of any search that uses an ill-behaved evaluation function that grows arbitrarily slowly with respect to search depth. Using planning with action costs as the guiding example (without loss of generality), we shall see that if \( \varepsilon \) is the smallest cost of an action (after all costs are normalized so the most expensive action costs 1 unit), then the time taken to discover a depth \( d \) optimal solution will be \( b^{d\varepsilon} \). If all actions have same cost, then \( \varepsilon \approx 1 \), whereas if the actions have significant cost variance, then \( \varepsilon \ll 1 \). For a variety of reasons, most real-world planning domains do exhibit high cost variance, thus presenting an “\( \varepsilon \)-cost trap” that forces any cost-based satisficing search to dig its own (\( \frac{1}{\varepsilon} \) deep) grave.

We thus argue that the decision to use ill-behaved evaluation functions for satisficing search should be reevaluated given their susceptibility to \( \varepsilon \)-cost traps, even when interested in that function’s evaluation of the resulting plan. But what exactly should take their place? We suggest using a better-behaved surrogate evaluation function in lieu of the ill-behaved one. There are two desiderata for such surrogate evaluation:

1. they should be immune to the \( \varepsilon \)-cost traps by increasing at a reasonable rate with respect to search depth and

2. they should track the (true) objective well.

In the case of action costs, we will consider two size-based branch-and-bound alternatives: the straightforward one which completely ignores costs and sticks to a purely size-based evaluation function, and a more subtle one that uses a cost-sensitive size-based evaluation function (specifically, the heuristic estimates the size of the cheapest cost path; see Section 2). We show that both of these out-perform cost-based evaluation functions in the presence of \( \varepsilon \)-cost traps, with the second one providing better quality plans (for the same run time limits) than the first in our empirical studies.
The issue takes a slightly different flavor when it comes to finding satisficing solutions in temporal planning, where the usual stated objective is makespan minimization. Balancing this with the avoidance of $\varepsilon$-cost traps takes great care, though temporal planners that use heuristic search tend to limit use of the objective only for pruning without directly using makespan for the heuristic value (c.f., Temporal Fast Downward [8] and POPF [2]). We seek to enhance such surrogate search approaches to pay more attention to the makespan minimization objective. To these ends, we suggest the use of a lookahead technique based on the usefulness of actions with respect to the objective function.

While some issues that arise with ill-behaved evaluation functions have also been observed in the past (e.g., [1, 17]), and some work-arounds have been suggested, our main contribution is to bring to the fore their fundamental nature; that the general phenomenon can be observed in any planning problem where an evaluation function does not grow fast enough with the search depth (e.g. measured in terms of node expansion operations).

The rest of the paper is organized as follows. In the next section, we present some preliminary notation, and formally introduce cost-based, size-based as well as cost-sensitive size-based search alternatives for searching with action costs. Next, we present two abstract and fundamental search spaces, which demonstrate that ill-behaved evaluation functions are “always” needlessly prone to $\varepsilon$-cost traps (Section 3). Section 4 strengthens the intuition behind this analysis by viewing best-first search as flooding topological surfaces set up by evaluation functions. We will argue that of all possible topological surfaces (i.e., evaluation functions) to choose for search, non-uniform ones based on action costs are of the worst. In Section 5 we propose a solution to this malady in terms of surrogate evaluation functions. We describe two candidate surrogates for cost-based search and a kindred approach for makespan minimization in temporal planning. In Section 6 we put all this analysis to empirical validation by experimenting with LAMA [17] and SapaReplan [19] for searching with action costs, and Temporal Fast Downward for searching for minimum makespan in a temporal setting. The experiments do show that surrogate search alternatives can out-perform previous, ill-behaved evaluation functions.

2 Setup and Notation

We gear the problem set up to be in line with the prevalent view of state-space search in modern, state-of-the-art satisficing planners. First, we assume the current
popular approach of reducing planning to graph search. That is, planners typically model the state-space in a causal direction, so the problem becomes one of extracting paths, meaning whole plans do not need to be stored in each search node. More important is that the structure of the graph is given implicitly by a procedure $\Gamma$, the child generator, with $\Gamma(v)$ returning the local subgraph leaving $v$; i.e., $\Gamma(v)$ computes the subgraph $(N^+[v], E(\{v\}, V - v)) = (\{u \mid (v, u) \in E\} + v, \{(v, u) \mid (v, u) \in E\})$ along with all associated labels, weights, and so forth. That is, our analysis depends on the assumption that an implicit representation of the graph is the only computationally feasible representation, a common requirement for analyzing the A* family of algorithms [9, 5].

The search problem is to find a path from an initial state, $i$, to some goal state in $\mathcal{G}$. We use costs as a proxy for any single evaluation objective (e.g., action costs or makespan), and let them be represented as edge weights, where $c(e)$ is the cost of the edge $e$. Let $g^*_c(v)$ be the (optimal) cost-to-reach $v$ (from $i$), and $h^*_c(v)$ be the (optimal) cost-to-go from $v$ (to the goal). Then $f^*_c(v) := g^*_c(v) + h^*_c(v)$, the cost-through $v$, is the cost of the cheapest $i$-$\mathcal{G}$ path passing through $v$. For discussing smallest solutions, let $f^*_a(v)$ denote the smallest $i$-$\mathcal{G}$ path passing through $v$. We can also consider the size of the cheapest $i$-$\mathcal{G}$ path passing through $v$, which we call $f^*_a(v)$.

We define a search node $n$ as equivalent to a path represented as a linked list (of edges). In particular, we distinguish this from the state of $n$ (its last vertex), $n. v$. We say $n.a$ (for action) is the last edge of the path and $n.p$ (for parent) is the subpath excluding $n.a$. Let $n' = na$ denote extending $n$ by the edge $a$ (so $a = (n. v, n'. v)$). The function $g_c(n)$ ($g$-cost) is just the recursive formulation of path cost: $g_c(n) := g_c(n.p) + c(n.a)$ ($g_c(n) := 0$ if $n$ is the trivial path). So $g^*_c(v) \leq g_c(n)$ for all $i$-$v$ paths $n$, with equality for at least one of them. Similarly let $g_a(n) := g_a(n.p) + 1$ (initialized at 0), so that $g_a(n)$ is an upper bound on the shortest path reaching the same state $(n. v)$.

A goal state is a target vertex where a plan may stop and be a valid solution. We fix a computed predicate $G(v)$ (a blackbox), the goal, encoding the set of goal states. Let $h_c(v)$, the heuristic, be a procedure to estimate $h^*_c(v)$. (Sometimes $h_c$ is considered a function of the search node, i.e., the whole path, rather than just the last vertex.) The heuristic $h_c$ is called admissible if it is a guaranteed lower bound. An inadmissible heuristic lacks the guarantee, but might anyways be coincidentally admissible. Let $h_s(v)$ estimate the remaining depth to the nearest goal, and let $\hat{h}_s(v)$ estimate the remaining depth to the cheapest reachable goal. Importantly, anything goes with such heuristics. Later we note that taking the size of a heuristic that optimizes on cost is an acceptable (and practical) interpretation of $\hat{h}_s(v)$.
We focus on two different definitions of $f$ (the evaluation function). Since we study cost-based planning, we consider

$$f_c(n) := g_c(n) + h_c(n, v)$$

(1)

This is the (standard, cost-based) ill-behaved evaluation function of $A^*$: cheapest-completion-first. We compare this to

$$f_s(n) := g_s(n) + h_s(n, v)$$

(2)

This is the canonical size-based (or search distance) surrogate evaluation function, equivalent to $f_c$ under uniform weights. Any combination of $g_c$ and $h_c$ is cost-based; any combination of $g_s$ and $h_s$ is size-based (e.g., breadth-first search is size-based). The evaluation function $\hat{f}_s(n) := g_s(n) + \hat{h}_s(n, v)$ is also size-based, but cost-sensitive.

Pseudo-code for best-first branch-and-bound search of implicit graphs is shown below. It continues searching after a solution is encountered and uses the current best solution value to prune the search space (line 4). The search is performed on a graph implicitly represented by $\Gamma$, with the assumption being that the explicit graph is so large that it is better to invoke expensive heuristics (inside of EVALUATE) during the search than it is to just compute the graph up front. The question considered by this paper is how to implement EVALUATE.

```plaintext
BEST-FIRST-SEARCH(i, G, \Gamma, h_c)
1 initialize search
2 while open not empty
3 n = open.remove() // Expand s
4 if BOUND-TEST(n, h_c) then continue
5 if GOAL-TEST(n, G) then continue
6 if DUPLICATE-TEST(n) then continue
7 s = n.v
8 star = \Gamma(s)
9 for each edge a = (s, s') from s to a child s' in star
10 n' = na
11 f = EVALUATE(n')
12 open.add(n', f)
13 return best-known-plan // Optimality is proven.
```

With respect to normalizing costs, we can let $\varepsilon := \frac{\min_a c(a)}{\max_a c(a)}$, that is, $\varepsilon$ is the least cost edge after normalizing costs by the maximum cost (to bring costs into the range $[0, 1]$). We use the symbol $\varepsilon$ for this ratio as we anticipate actions with
high cost variance in real world planning problems. For example: boarding versus flying (ZenoTravel), mode-switching versus machine operation (Job-Shop), and (unskilled) labor versus (precious) material cost.

![Diagram](image)

Figure 1: An $\varepsilon$-cost trap for cost-based search.

3 Two Canonical Case of $\varepsilon$-cost Trap

In this section we argue that the mere presence of $\varepsilon$-cost edge weights misleads search, and that this is not an accidental phenomenon, but a systemic weakness of the very concept of “(ill-behaved) cost-based evaluation functions + systematic search + combinatorial graphs”. We base this analysis in two abstract search spaces, in order to demonstrate the fundamental nature of such traps.

For our analysis, the first abstract space we consider is a simple, non-trivial, non-uniform cost, intractably large, search space: the search space of an enormous cycle with one expensive edge. The second abstract space we consider is a more natural model of search (in planning): a uniform branching tree. Traps in these
Figure 2: A trap for cost-based search involving exponentially large subtrees on $\varepsilon$-cost edges.
spaces are exponentially sized and connected sets of $\varepsilon$-cost edges: not the common result of, for example, a typical random model of search. We briefly consider why planning benchmarks naturally give rise to such structure.

### 3.1 Cycle Trap

In this section we consider the simplest abstract example of the $\varepsilon$-cost ‘trap’. The notion is that applying increasingly powerful heuristics, domain analysis, learning techniques, . . . , to one’s search problem transforms it into a simpler ‘effective graph’ — the graph for which Dijkstra’s algorithm [6] produces isomorphic behavior. For example, let $c'$ be a new edge-cost function obtained by setting edge costs to the difference in $f$ values of the edge’s endpoints: Dijkstra’s algorithm on $c'$ is A* on $f$. Similarly take $\Gamma'$ to be the result of applying one’s favorite incompleteness-inducing pruning rules to $\Gamma$ (the child generator), say, helpful actions [12]; then Dijkstra’s algorithm on $\Gamma'$ is $A^*$ with helpful action pruning.

We presume the effective search graph remains very complex despite clever inference (or there is nothing to discuss). If there is a problem with search behavior in an exceedingly simple graph then we can suppose that no amount of domain analysis, learning, heuristics, and so forth, will incidentally address the problem: such inference must specifically address the issue of non-uniform costs. When none of the bells and whistles consider non-uniform costs to be a serious issue, the search permits wildly varying edge “costs” even in the effective search graph: $\varepsilon \approx \varepsilon' = \frac{\min c'(e)}{\max c'(e)}$. We demonstrate that this by itself is enough to produce very troubling search behavior: $\varepsilon$-cost is a fundamental challenge to be overcome in planning.

There are several candidates for simple non-trivial state-spaces (e.g., cliques), but clearly the cycle is fundamental (what kind of ‘state-space’ is acyclic?). So, the state-space we consider is the cycle, with associated exceedingly simple metric consisting of all uniform weights but for a single expensive edge. Its search space is certainly a simple non-trivial search space: the rooted tree on two leaves. So the single unforced decision to be made is in which direction to traverse the cycle: clockwise or counter-clockwise.

$\varepsilon$-cost Trap: Consider a counting problem of making some variable, $x$, encoded in $k$ bits represent $2^k - 2 \equiv -2 \pmod{2^k}$, starting from 0, using only the operations of increment and decrement. We illustrate the search in Figure 1. There are 2 minimal solutions: incrementing $2^k - 2$ times, or decrementing twice. Set the cost

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1 Systematic inconsistency of a heuristic translates to analyzing the behavior of Dijkstra’s algorithm with many negative ‘cost’ edges, a typical reason to assume consistency in analysis.
of incrementing and decrementing to 1, except for transitioning between \( x \equiv 0 \) and \( x \equiv -1 \) costs, say, \( 2^{k-1} \) (in either direction). Then the 2 minimal solutions cost \( 2^k - 2 \) and \( 2^{k-1} + 1 \), or, normalized, \( 2(1 - \varepsilon) \) and \( 1 + \varepsilon \). Cost-based search loses:

While both approaches prove optimality in exponential time \( (O(2^k)) \), size-based search discovered that optimal plan in constant time.

The heuristic perceives all movement on the cycle to be irrelevant to achieving high quality plans. The state with label -2 is one interesting way to leave the cycle, there may be (many) others. \( C \) denotes the cost of one such continuation from -2, and \( d \) its depth. Edge weights nominally denote changes in \( f_c \): as given, locally, these are the same as changes in \( g_c \). But increasing \( f_s \) by 1 at -1 (and descendants) would, for example, model instead the special edge as having cost \( \frac{1}{2} \) and being perceived as worst-possible in an undirected graph.

**Performance Comparison: All Goals.** The goal \( x \equiv -2 \) is chosen to best illustrate the trap. Consider the discovery problem for other goals. With the goal in the interval \( 2^k \cdot [0, \frac{1}{2}] \) cost-based search is twice as fast. With the goal in the interval \( 2^k \cdot [\frac{1}{2}, \frac{3}{2}] \) the performance gap narrows to break-even. For the last interval, \( 2^k \cdot [\frac{2}{3}, 1] \), the size-based approach takes the lead — by an enormous margin.

There is one additional region of interest. Taking the goal in the interval \( 2^k \cdot [\frac{2}{3}, \frac{3}{4}] \) there is a trade-off: size-based search finds a solution before cost-based search, but cost-based search finds the optimal solution first. Concerning time till optimality is proven, the cost-based approach is monotonically faster (of course). Specifically, the cost-based approach is faster by a factor of 2 for goals in the region \( 2^k \cdot [0, \frac{1}{2}] \), not faster for goals in the region \( 2^k \cdot [\frac{2}{3}, 1] \), and by a factor of \( (\frac{1}{2} + 2\alpha)^{-1} \) (bounded by 1 and 2) for goals of the form \( x \equiv 2^k (\frac{1}{2} + \alpha) \), with \( 0 < \alpha < \frac{1}{4} \).

**Performance Comparison: Feasible Goals.** Considering all goals is inappropriate in the satisficing context; to illustrate, consider \( k = 1000 \) as an example of a large value of \( k \). Fractions of exponentials are still exponentials — even the most patient reader working out this example will have forcibly terminated either search long before receiving any useful output. Except if the goal is of the form \( x \equiv 0 \pm f(k) \) for some sub-exponential \( f(k) \). Both approaches discover (and prove) the optimal solution in the positive case in time \( O(f(k)) \) (with size-based performing twice as much work). In the negative case, only the size-based approach manages to discover a solution (the optimal one, in time \( O(f(k)) \)) before being killed. Moreover, while it will fail to produce a proof of such before termination, based on our understanding of the domain, we can show it to be posthumously correct. \( (2^k - f(k)) > 2^k \cdot \frac{3}{4} \) for any sub-exponential \( f(k) \) with large enough \( k \).

**How Good is Almost Perfect Search Control?** Keep in mind that the represen-
tation of the space as a simple \( k \) bit counter is not available. In particular what ‘increment’ actually stands for is an inference-motivated choice of a single operator out of a large number of executable and promising operators at each state — in the language of Markov Decision Processes, we are allowing inference to be so close to perfect that the optimal policy is known at all but 1 state. Only one decision remains ... but no methods cleverer than search remain. Still the graph is intractably large. Cost-based search only explores in one direction: left, say. In the satisficing context such behavior is entirely inappropriate. What is appropriate? We want the search to explore the space so that a solution that exists only one step to the right can still be found, even if it is not optimal.

3.2 Branching Trap

In the counter problem the trap is not even combinatorial; the search problem consists of a single decision at the root, and the trap is just an exponentially deep path. For example, appending a large enough Towers of Hanoi problem to a planning benchmark, setting its actions at \( \epsilon \)-cost, will hurt cost-based search — even given the perfect heuristic for the puzzle! Besides Hanoi, though, exponentially deep paths are not typical of planning benchmarks. So in this section we demonstrate that exponentially large subtrees on \( \epsilon \)-cost edges are also traps.

Consider \( x > 1 \) high cost actions and \( y > 1 \) low cost actions in a uniform branching tree model of search space. The model is appropriate up to the point where duplicate state checking becomes significant. See the example illustrated in Figure 2.

We have two rather distinct kinds of physical objects that exist in the domain with primitive operators at rather distinct orders of magnitude; supposing uniformity and normalizing, one type involves \( \epsilon \)-cost and the other involves cost 1. In this case, there is a low-cost subspace, a high-cost subspace, and the full space, where each is a uniform branching tree. As trees are acyclic, it is probably best to think of these as search (rather than state) spaces. As depicted, planning for an individual object is trivial as there is no choice besides going forward. Other than that no significant amount of inference is being assumed, and in particular the effects of a heuristic are not depicted. For cost-based search to avoid “dying” due to memory or time computational resources, the heuristic would need to forecast every necessary cost 1 edge, so as to reduce its weight closer to 0. (Note that the aim of a heuristic is to drive all the weights to 0 along optimal/good paths, and to infinity for not-good/terrible/dead-end choices.) If any cut of the space across such edges (separating good solutions) is not foreseen, then backtracking into all of the
low-cost subspaces so far encountered commences, to multiples of depth $\varepsilon^{-1}$ — one such multiple for every unforeseen cost 1 cut.

Observe that in the object-specific subspaces (the paths), a single edge ends up being multiplied into such a cut of the global space. Suppose the solution of interest costs $C$, in normalized units, so the solution lies at depth $C$ or greater. Then cost-based search faces a grave situation: $O((x + y^{\frac{1}{\varepsilon}})^C)$ possibilities will be explored before considering all potential solutions of cost $C$.

A size-based search only ever considers at most $O((x + y)^d) = O(b^d)$ possibilities before consideration of all potential solutions of size $d$. But the more interesting question is how long it takes to find solutions of fixed cost rather than fixed depth. Note that $\frac{C}{\varepsilon} \geq d \geq C$. Assuming the high cost actions are relevant, that is, some number of them are needed by solutions, we have that solutions are not actually hidden as deep as $\frac{C}{\varepsilon}$. To help see this, suppose that solutions tend to be a mix of high and low cost actions in equal proportion. Then the depth of those solutions with cost $C$ is $d = 2 \left( \frac{C}{1+\varepsilon} \right)$ (i.e., $\frac{d}{2} \cdot 1 + \frac{d}{2} \cdot \varepsilon = C$). At such depths the size-based approach is the clear winner: $O((x + y)^{2C}) \ll O((x + y^{\frac{1}{\varepsilon}})^C)$ (normally).

Consider the case where $x = y = \frac{b}{2}$, then:

\[
\text{size effort/cost effort} \approx \frac{b^{2C}/(x + y^{\frac{1}{\varepsilon}})^C}{b^{1+\varepsilon}/(x + y^{\frac{1}{\varepsilon}})^C} = \frac{b^{2C}/y^{\varepsilon}}{b^{1+\varepsilon}/y^{\varepsilon}} < \frac{2^{C}}{b^{1+\varepsilon}},
\]

and, provided $\varepsilon < \frac{1 - \log_2 b}{1 + \log_2 b}$ (for $b = 4$, $\varepsilon < \frac{1}{3}$), the last is always less than 1 and, for that matter, goes quickly to 0 as $C$ increases and/or $b$ increases and/or $\varepsilon$ decreases.

Generalizing from this example, the size-based approach is faster at finding solutions of any given cost, as long as (1) high-cost actions constitute at least some constant fraction of the solutions considered (high-cost actions are relevant), (2) the ratio between high-cost and low-cost is sufficiently large, (3) the effective search graph (post inference) is reasonably well modeled by an infinite uniform branching tree (i.e., huge enough to render duplicate checking negligible, or at least not especially favorable to cost-based search), and most importantly, (4) the cost function in the effective search graph still demonstrates a sufficiently large ratio between high-cost and low-cost edges (no inference has attempted to compensate).
4 Search Topology and Satisficing Solutions

We view evaluation functions \((f)\) as topological surfaces over search nodes, so that generated nodes are visited in, roughly, the order of \(f\)-altitude. With non-monotone evaluation functions, the set of nodes visited before a given node is all those contained within some basin of the appropriate depth — picture water flowing from the initial state: if there are dams then such a flood could temporarily visit high altitude nodes before low altitude nodes. (With very inconsistent heuristics — large heuristic weights — the metaphor loses explanatory power, as there is nowhere to go but downhill.)

All reasonable choices of search topology will eventually lead to identifying and proving the optimal solution (e.g., assume finiteness, or divergence of \(f\) along infinite paths). Some search topologies will produce a whole slew of suboptimal solutions along the way, eventually reaching a point where one begins to wonder if the most recently reported solution is optimal. Others report nothing until finishing. The former are interruptible \([23]\), which is one way to more formally define satisficing\(^2\). Admissible cost-based topology is the least interruptible choice: the only reported solution is also the last path considered. Define the cost-optimal footprint as the set of plans considered. Gaining interruptibility is a matter of raising the altitude of large portions of the cost-optimal footprint in exchange for lowering the altitude of a smaller set of non-footprint search nodes — allowing sub-optimal solutions to be considered. Note that interruptibility comes at the expense of total work.

So, along the lines of confirming the intuition that interruptibility is a reasonable notion of satisficing: the cost-optimal approach is a poor choice for satisficing solutions. Said another way, proving optimality is about increasing the lower bound on true value, while solution discovery is about decreasing the upper bound on true value. It seems appropriate to assume that the fastest way to decrease the upper bound is more or less the opposite of the fastest way to increase the lower bound — with the notable exception of the very last computation one will ever do for the problem: making the two bounds meet (proving optimality).

Intuitively the search should not be completely blind to costs: just defensive about possible \(\varepsilon\)-traps. For size-based topology, with respect to any cost-based variant, the ‘large’ set is the set of longer yet cheaper plans, while the ‘small’ set is the shorter yet costlier plans. In general one expects there to be many more longer plans than shorter plans in combinatorial problems, so that the increase in

\(^2\)Another way that Zilberstein suggests is to specify a contract; the 2008 and 2011 planning competitions have such a format \([10]\).
total work is small, relative to the work that had to be done eventually (exhaust the many long, cheap, plans). The additional work is considering exactly plans that are costlier than necessary (potentially suboptimal solutions). The idea of the trade-off is good, but even the best version of a purely size-based topology will not be the best trade-off possible.

Not finding the cheapest path first comes with the price of re-expansion, so the satisficing intent comes hand in hand with re-expansion of states. Indeed, duplicate detection and re-expansion are, in practice, important issues. Besides the obvious kind of re-expansion that IDA* [13] performs between iterations, it is also true that it considers paths which A* never would (even subsequent to arming IDA* with a transposition table) — it is not really true that one can reorder consideration of paths however one pleases. In particular at least some kind of breadth-first bias is appropriate, so as to avoid finding woefully suboptimal plans to states early on, triggering giant cascades of re-expansion later on.

5 Handling $\varepsilon$-Cost Traps with Surrogate Search

We propose that a principled way to combat the effects of $\varepsilon$-cost traps is to swap the objective-sensitive evaluation function, which is ill-behaved, with a surrogate evaluation function that is well behaved, but does not directly track the objective. While surrogate functions avoid the $\varepsilon$-cost traps, they do so at the expense of solution quality. To make up for this, we will consider branch-and-bound search regimes, where the search can continuously improve the quality of the solution. Obviously, for a given objective, there can be multiple surrogate evaluation functions that differ in terms of how closely/loosely they track the objective. They offer a spectrum of computation vs. quality tradeoffs—with the functions that track the objective function more closely converging on higher quality solutions, but taking considerably longer time to find even one solution. The art here is to strike a good balance in this tradeoff. Specifically, the challenge is in finding effective surrogate evaluations that increase in accordance with search depth while navigating the tradeoff between defending against $\varepsilon$-cost traps and focusing search on the objective.

To concretely illustrate the art of picking good surrogate evaluation functions, we consider two distinct planning problems—a cost-based planning planning problem, as well as a temporal planning problem involving makespan minimization. In the former case, we first show that a purely sized-based surrogate evaluation function is able to avoid the search cost imposed by $\varepsilon$-cost trap. We then describe a more
sophisticated surrogate evaluation function that estimates the size of a cost-optimal plan, and show that it has better performance in terms of solution quality. The empirical support for these claims is provided by experiments run on LAMA, a state of the art planner. For the makespan optimizing temporal planning, most practical methods, such as Temporal Fast Downward, already abandon objective functions directly tracking makespan. By viewing this decision in terms of the surrogacy of the evaluation function, we propose a method for further improving cost-vs-quality tradeoff that involves performing a lookahead based on the usefulness of actions according to the objective function.

5.1 Surrogate Evaluation Functions for Cost-based Planning

For cost-based planning we consider two kinds of surrogate evaluation functions: the first is purely based on size, while the second is also sensitive to costs.

Pure Size-Based Evaluation Function: Here we replace cost-based evaluation function with a pure size-based one. That is, we search with \( f \) value being the size (number of actions) in the solution, even though we are interested in optimizing the cost of the solution. In particular, the heuristic will estimate the size of the shortest length path from the current state. By using size-based evaluation function, we effectively force \( \varepsilon \) to be 1. Since the search is not cost-focused, the first solution it finds may not necessarily have high quality, however the search will avoid \( \varepsilon \)-cost traps. We handle quality of solutions by employing a branch-and-bound search.

Cost-Sensitive Size-Based Evaluation Function: This is an improvement of the pure-size based evaluation function that improves its quality focus while still retaining \( \varepsilon \) being 1. Specifically, the evaluation function here is still measured in terms of size or number of actions, but the heuristic aims to estimate the size of the cheapest cost path from the current state. By tracking the cheapest cost path, the evaluation function becomes more quality focused, and by measuring that path in terms of the number of actions (rather than the cumulative cost), it avoids the \( \varepsilon \)-cost trap. Once again, the quality of the solution can be improved with a branch-and-bound search regime. To elaborate further, the cost-sensitive size-based evaluation functions can use the standard relaxed-plan heuristics that select the relaxed plan itself in terms of the cost [7]. However, rather than take the cost of the relaxed plan, we take the size of the relaxed plan, computing \( \hat{h}_s \), and use \( \hat{f}_s = g_s + \hat{h}_s \) for the evaluation function (see Section 2).
5.2 Surrogate Evaluation Functions for Temporal Planning

In the case of temporal planning, we look toward enhancing search methodologies that already do not directly search over the objective of makespan minimization. To do this, we perform a lookahead over useful search operators. We define useful search operators as those whose absence would lead to a worse quality solution. They relate to the idea of useless operators defined by Wehrle, et al. [21]; indeed, we can find a useful operator by using the same method for finding useless operators. We say an operator \( o \) is useful in a state \( s \) if \( d^\text{opt}(s) > d(o(s)) \), where \( d^\text{opt}(s) \) is the optimal distance to a goal state without an operator \( o \) (i.e., \( d(s) \) and \( d^\text{opt}(s) \) are perfect heuristics). We say the state \( o(s) = s' \) is useful if the operator generating it is useful and we can use this notion to enhance satisficing planners that use best-first search. We define the heuristic degree of usefulness of an operator \( o \) as \( \upsilon_o(s) = h^\text{opt}(s) - h^o_m(o(s)) \). With this information, we create a new search approach that interleaves local search decisions using degree of usefulness on makespan when we detect a \( g \)-value plateau (i.e., no change in makespan). In other words, we supplement the surrogate evaluation function with a one-step lookahead procedure. This procedure generates a set of child states for the best node on the queue and calculates their “makespan-usefulness” using a heuristic on makespan. The procedure expands the most makespan-useful node (unless no nodes are makespan-useful), then puts all expanded nodes back into the best-first search queue, ordered by the surrogate function.

6 Empirical Evaluation

In this section, we verify that the effect of \( \varepsilon \)-cost traps in ill-behaved evaluation functions is disruptive to planner performance, both in cost-based planning and temporal planning. We shall see that the use of size-based surrogate search does better in the case of cost-based planning. We will also see the effect in temporal planning in the planning framework of Temporal Fast Downward, discuss Temporal Fast Downward’s surrogate search and see how our lookahead technique improves plan quality over it.

We demonstrate existence of the problematic planner behavior when using ill-behaved evaluation functions in both LAMA-2008 for cost-based planning and Temporal Fast Downward for temporal planning. In LAMA-2008 we use problems in the travel domain (simplified ZenoTravel, zoom and fuel removed), as well as two other IPC domains. In Temporal Fast Downward we use domains from the International Planning Competition of 2008 (IPC-2008). Analysis of LAMA is com-
Figure 3: Rendezvous problems. Diagonal edges cost 7,000, exterior edges cost 10,000. Board/Debark cost 1.

plicated by many factors, so we also test the behavior of SapaReplan on simpler instances (but in all of ZenoTravel). The first set of problems concern a rendezvous at the center city in the location graph depicted in Figure 3; the optimal plan arranges a rendezvous at the center city. The second set of problems is to swap the positions of passengers located at the endpoints of a chain of cities.

6.1 Cost-Based Planning

Evaluation of Cost-based planning on LAMA

In this section we demonstrate the performance problem wrought by $\varepsilon$-cost in a state-of-the-art (2008) planner — LAMA [17], the leader of the cost-sensitive (satisficing) track of IPC’08 [10]. With a completely trivial recompilation (set a flag) one can make it ignore the given cost function, effectively searching by $f_s$, i.e., pure-size. This methodology was evaluated by Richter and Westphal ([17]). With slightly more work one can do better and have it use $\hat{f}_s$ as its evaluation function, i.e., use a cost-sensitive heuristic estimate of $\hat{d}$ and allow the search be size-based, but still compute costs correctly for branch-and-bound. We call this latter modification LAMA-size. Ultimately, the observation is that LAMA-size outperforms LAMA — an astonishing feat for such a trivial change in implementation.

LAMA defies analysis in a number of ways: landmarks, preferred operators, dynamic evaluation functions, multiple open lists, and delayed evaluation, all of which effect potential search plateaus in complex ways. Nonetheless, it is essentially a cost-based approach.

\(^3\)Options: ‘FILi’.
Table 1: Percentage IPC score improvement on LAMA variants.

| Domain       | LAMA | LAMA-size |
|--------------|------|-----------|
| Rendezvous   | 70.8%| 83.0%     |
| Elevators    | 79.2%| 93.6%     |
| Woodworking  | 76.6%| 64.1%     |

Results. With more than about 8 total passengers, LAMA is unable to complete any search stage except the first (the greedy search). For the same problems, LAMA-size finds the same first plan (the heuristic values differ, but not the structure), but is then subsequently able to complete further stages of search. In so doing it sees marked improvement in cost; on the larger problems this is due only to finding better variants on the greedy plan. Other domains are included for broader perspective, woodworking in particular was chosen as a likely counter-example, as all the actions concern just one type of physical object and the costs are not wildly different. For the same reasons we would expect LAMA to out-perform LAMA-size in some cost-enhanced version of Blocksworld.

Evaluation of Cost-Based Planning on SapaReplan

We also consider the behavior of SapaReplan on the simpler set of problems. This planner is much less sophisticated in terms of its search than LAMA, in the sense that it does not use dual queues or lazy evaluation. The problem is just to swap the locations of passengers located on either side of a chain of cities. A plane starts on each side, but there is no actual advantage to using more than one (for optimizing either of size or cost): the second plane exists to confuse the planner. Observe that smallest and cheapest plans are the same. So in some sense the concepts have become only superficially different; but this is just what makes the problem interesting, as despite this similarity, still the behavior of search is strongly affected by the nature of the evaluation function. We test the performance of $\hat{f}_s$ and $f_c$, as well as a hybrid evaluation function similar to $\hat{f}_s + f_c$ (with costs normalized). We also test hybridizing via tie-breaking conditions, which ought to have little effect given the rest of the search framework.

Results. The size-based evaluation functions find better cost plans faster (within...
the deadline) than cost-based evaluation functions. The hybrid evaluation function also does relatively well, but not as well as could be hoped. Tie-breaking has little effect, sometimes negative.

We note that Richter and Westphal (2010) also report that replacing cost-based evaluation function with a pure size-based one improves performance over LAMA in multiple other domains. Our version of LAMA-size uses a cost-sensitive size-based search \(\hat{h}_s\), and our results, in the domains we investigated, seem to show bigger improvements over the size-based variation on LAMA obtained by completely ignoring costs \(h_s\), i.e., setting the compilation flag. Also observe that one need not accept a tradeoff: calculating \(\log_{10} \varepsilon^{-1} \leq 2\) and choosing between LAMA and LAMA-size appropriately would be an easy way to improve performance simultaneously in ZenoTravel (4 orders of magnitude) and Woodworking \((< 2\) orders of magnitude).

Finally, while LAMA-size outperforms LAMA, our theory of \(\varepsilon\)-cost traps suggests that cost-based search should fail even more spectacularly. In an earlier technical report describing this work [4], we took a much closer look at the travel domain and present a detailed study of which extensions of LAMA help it temporarily mask the pernicious effects of cost-based search. Our conclusion is that both LAMA and SapaReplan manage to find solutions to problems in the travel domain despite the use of a cost-based evaluation function by using various tricks to induce a limited amount of depth-first behavior in an A*-framework. This has the potential effect of delaying exploration of the \(\varepsilon\)-cost plateaus slightly, past the discovery of a solution, but still each planner is ultimately trapped by such plateaus before being able to find really good solutions. In other words, such tricks are mostly serving to mask the problems of cost-based search (and \(\varepsilon\)-cost), as they merely delay failure by just enough that one can imagine that the planner is now effective (because it returns a solution where before it returned none). Using a size-based evaluation function more directly addresses the existence of cost plateaus, and not surprisingly leads to improvement over the equivalent cost-based approach — even with LAMA.

### 6.2 Evaluation of Temporal Planning on Temporal Fast Downward

The problem of \(\varepsilon\)-cost also exists when attempting minimize makespan using an makespan-based, ill-behaved evaluation function. In previous work on this subject
Table 2: IPC metric on SapaReplan variants in ZenoTravel.

| Mode                      | 2 Cities Score | 2 Cities Rank | 3 Cities Score | 3 Cities Rank |
|---------------------------|----------------|---------------|----------------|---------------|
| Hybrid                    | 88.8%          | 1             | 43.1%          | 2             |
| Size                      | 83.4%          | 2             | 43.7%          | 1             |
| Size, tie-break on cost   | 82.1%          | 3             | 43.1%          | 2             |
| Cost, tie-break on size   | 77.8%          | 4             | 33.3%          | 3             |
| Cost                      | 77.8%          | 4             | 33.3%          | 3             |

Table 3: From useful actions paper.

| Domain                  | cov | qual |
|-------------------------|-----|------|
| crewplanning-strips     | 4   | 4.00 |
| elevators-numeric       | 2   | 2.00 |
| elevators-strips        | 3   | 2.98 |
| openstacks-adl          | 8   | 7.58 |
| openstacks-strips       | 27  | 20.93|
| parcprinter-strips      | 6   | 5.31 |
| pegsol-strips           | 22  | 21.15|
| sokoban-strips          | 10  | 10.00|
| transport-numeric       | 3   | 3.00 |
| woodworking-numeric     | 18  | 16.91|
| overall                 | 103 | 93.86|

Benton et al. ([1]) tested the planner Temporal Fast Downward to see the effect of this; their results over IPC-2008 domains are shown in Table 3.

We present some results of using useful search operators as a way to complement surrogate evaluation function. The best-first search in TFD uses a modified version of the context-enhanced additive heuristic [11] that sums the durations as costs, $h_{dur}^{cea}$, meaning the heuristic captures a sequential view of actions. To avoid noise from other search enhancement techniques, we disabled deferred evaluation for our experiments. To detect heuristic-useful operators, we used the heuristic of Benton et al. [1] that utilizes a Simple Temporal Network to reschedule heuristic plans. We implemented the useful operator lookahead discussed earlier into TFD for a planner we call $TFD^{gusef}$. We run both TFD and $TFD^{gusef}$ on the temporal domains from the IPC 2008 (except modeltrain, as in our tests TFD in general is unable to solve more than
2 problems in this domain). This benchmark set is known to contain plateaus on $g_m$. The experiments were run on a 2.7 GHz AMD Opteron processor, with a timeout of 30 minutes and a memory limit of 2 GB.

The IPC scores of our experiment are found in Table 5:

| Domain            | TFD | TFD$^{g_m,u}$ |
|-------------------|-----|---------------|
| Crewplanning      | 22.44 | 22.57 |
| Elevators         | 13.88 | 15.40 |
| Openstacks        | 25.79 | 27.81 |
| Parcprinter       | 8.73  | 8.47 |
| Pegsol            | 28.73 | 28.81 |
| Sokoban           | 10.93 | 10.79 |
| Transport         | 5.06  | 4.68 |
| Woodworking       | 19.72 | 19.93 |
| Overall           | 135.28 | 138.48 |

We used the elevators-numeric and openstacks-adl variants for our results on the respective domains following the IPC-2008 rules by using the domain variant that all our planner versions did best in.

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Table 4: From g-value plateaus paper.

| Domain                | $f_c = g_c + h_c$ | $f_m = g_m + h_m$ | $f_{mw} = g_m + h_w$ |
|-----------------------|------------------|-------------------|---------------------|
|                       | $r_g$ $r_f$ cov  | $r_g$ $r_f$ cov  | $r_g$ $r_f$ cov  |
| crewplanning-strips   | 0.03 0.55 11     | 0.98 0.83 4      | 0.95 0.09 12      |
| elevators-numeric     | 0.06 0.03 4      | 0.57 0.27 2      | 0.48 0.05 4       |
| elevators-strips      | 0.07 0.05 3      | 0.53 0.25 3      | 0.44 0.04 4       |
| openstacks-adl        | 0.15 0.89 30     | 1.00 0.88 8     | 0.92 0.02 30      |
| openstacks-strips     | 0.14 0.88 30     | 0.67 0.29 27    | 0.71 0.06 30      |
| parcprinter-strips    | 0.16 0.08 12     | 0.90 0.37 6      | 0.76 0.09 6       |
| pegsol-strips         | 0.17 0.09 25     | 0.85 0.25 22     | 0.82 0.08 26      |
| sokoban-strips        | 0.28 0.16 11     | 0.78 0.32 10     | 0.77 0.10 10      |
| transport-numeric     | 0.23 0.06 2      | 0.74 0.48 3      | 0.58 0.09 3       |
| woodworking-numeric  | 0.08 0.12 18     | 0.72 0.26 18     | 0.55 0.04 19      |
| overall               | 0.09 0.21 146    | 0.84 0.50 103    | 0.68 0.07 144     |

Table 5: IPC scores when using surrogate useful operator in TFD.

| Domain | TFD | TFD$^{g_m,u}$ |
|--------|-----|---------------|
| Crewplanning | 22.44 | 22.57 |
| Elevators | 13.88 | 15.40 |
| Openstacks | 25.79 | 27.81 |
| Parcprinter | 8.73  | 8.47 |
| Pegsol | 28.73 | 28.81 |
| Sokoban | 10.93 | 10.79 |
| Transport | 5.06  | 4.68 |
| Woodworking | 19.72 | 19.93 |
| Overall  | 135.28 | 138.48 |
Figure 4: Anytime behavior showing the change in IPC score as time progresses.

than TFD. However, at about 10 minutes $T_{FD}^{{{g}_{useful}}}$ dominates TFD.

7 Related Work

Others have noted issues with using cost-based search and suggested surrogate approaches. Perhaps most relevant to our own analysis is the work by Wilt and Ruml ([22]). They perform an empirical analysis similar to ours that shows typical heuristic search suffers as the result of high ratios between the cost of search operators. They then prove heuristic error bounds in specific cases of search (i.e., using a consistent heuristic and invertible operators) that may be the beginnings of an explanation of the problem. Further, using our earlier investigations ([3], [1]) as a guide, they empirically show that search approaches using “search distance” (i.e., size) as a component have better performance.

Work done by Thayer and Ruml ([20]) has considered the issues of blending size and cost in the design of evaluation functions and search algorithm modification. Their work combines a type of surrogate evaluation over solution size with cost estimates using different search queues. The yet earlier, deeply insightful, work that Thayer, Ruml, and Westphal build on is that of Pohl into the typical poor behaviour of $A^*$ when given a plethora of equally good choices. That work led first to the development of $WA^*$ (in a phrase, use $f = g + wh$ as surrogate), and then on to depth-dependent weighting variants of $A^*$ (let $w$ vary somewhat intelligently with depth, i.e., penalize size) [15, 16]. To quote Pohl quoting Kowalski [14]: “...have a critical defect. They [$A^*$] investigate *all* equally meritorious alternative paths. *In difficult problem spaces where any solution path is desired, *
this procedure is inappropriately breadth first.” (emphasis added).

Dechter and Pearl ([5]) give a highly technical account of the properties of generalized best-first search strategies, focusing on issues of computational optimality, but mostly from the perspective of search constrained to proving optimality in the path metric. Additionally, Richter and Westphal ([17]) have a thorough empirical analysis of cost issues in standard planning benchmarks in their planner LAMA. Subsequently LAMA was modified to include a surrogate, size-based greedy best-first search search to find a first solution before beginning the same search approach as its previous version [18].

8 Conclusion

Several researchers have noted the troublesome behavior of cost-based search in many planning benchmarks, suggesting several ad hoc work arounds for it. In this paper, we tried to provide a general explanation for the malady. Specifically, we argued that the origins of this problem can be traced back to the fact that most planners that try to optimize cost also use cost-based evaluation functions (i.e., $f(n)$ is a cost estimate). We showed that cost-based evaluation functions become ill-behaved whenever there is a wide variance in action costs; something that is all too common in planning domains. The general solution to this malady is what we call a surrogate search, where a surrogate evaluation function that does not directly track the cost objective, and is resistant to cost-variance, is used. To shed light on the art of devising good surrogate evaluation functions, we proposed cost-surrogates in cost-based planning and makespan-surrogates in temporal planning. For the former, we discussed some compelling choices for surrogate evaluation functions that are based on size rather than cost, and showed that they are immune to the difficulties faced by cost-based search. Of particular practical interest is a cost-sensitive version of size-based evaluation function where the heuristic estimates the size of cheap paths, as it provides attractive quality vs. speed tradeoffs.

For temporal planning, we proposed a method that significantly improves upon the surrogate evaluation function by tracking operator usefulness. Our empirical evaluations with state-of-the-art planners demonstrate the effectiveness of surrogate search. A worthwhile direction for future work is finding other surrogate evaluation functions that strike an even better balance between resistance to $\varepsilon$-cost traps, and focus on the objective.
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