Colorless and colored gluon-clusters in nucleon?

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Abstract

It is suggested that virtual gluon-clusters exist in nucleon, and that such colorless and colored objects manifest themselves in the small $x_B$ region of inelastic lepton-nucleon scattering processes. The relationship between the space-time properties of such clusters and the striking features observed in these scattering processes is discussed. A phase-space model is used to show how quantitative results can be obtained in such an approach. The results of this model-calculation are in reasonable agreement with the existing data. Further experiments are suggested.
High precision data for deep-inelastic electron-proton scattering and photon-proton scattering in the small $x_B$ (down to $1.8 \times 10^{-4}$) region are now available\cite{1,2} from HERA. These data not only show that in this region, $F_2(x_B, Q^2)$ increases with decreasing $x_B$ at fixed $Q^2$, and increases like $\ell nQ^2$ at fixed $x_B$, but also show that a distinct class of events — the large rapidity gap events — exist. These striking features of the data, have already generated much interest\cite{3}. Are they related to one another? Can these observations shed light in understanding the structure of nucleon, and/or the reaction mechanism(s) of such collision processes?

It is known\cite{1,2,3} that the data for large rapidity gap events can be described\cite{4} in terms of the special Regge-pole, Pomeron, and/or other Pomeron-like objects, all of which are supposed to have parton-structures. It is also known\cite{5} since the 1970’s that Pomeron may be considered as a “gluon-system”. We are thus led to the following questions: Are these results suggesting that independent gluon-systems exist in proton? What properties, in particular space-time properties, should such systems — Pomeron and/or Pomeron-like objects — have? According to Regge Pole Models, the Regge-pole Pomeron, is associated with vacuum quantum-numbers; hence it is expected that these objects should be colorless and carry no isospin etc. But gluons are members of color-octet; hence a system of gluons should in general carry color. Can corresponding colored systems also exist in proton if colorless ones do? If no, why not? If yes, how do they manifest themselves experimentally?

In order to answer these questions, it seems useful to recall the following: The proton is a spatially extended object, and by performing deep-inelastic electron-proton scattering in the small $x_B$ and large $Q^2$ region, we wish to probe the structure of the proton by a beam of incoming virtual photons $\gamma^*(x_B, Q^2)$ of small transverse spatial dimension ($\sim 1/Q^2$) and of relatively short interaction-time (which is equal to $1/\nu \equiv 2Mx_B/Q^2$ when viewed from its rest frame, and it is $4P/(2M\nu - Q^2)$ when viewed in the electron-proton cms system where $P$ is the momentum the proton). In other words, in carrying out such experiments, we are asking ourselves: What kind of information about the proton structure can we
obtain by using virtual photons $\gamma^*(Q^2, x_B)$, with (transverse) resolution power $1/Q^2$, which interact with the constituents of the proton for a short time-interval $(1/\nu \equiv 2Mx_B/Q^2$ or $4P/(2M\nu - Q^2)$ in the reference frames mentioned above), provided that $\gamma^*(Q^2, x_B)$ is absorbed by the constituents of the proton, before it turns itself into lepton- or quark-pairs due to vacuum-fluctuation. Can such virtual photons tell us in particular whether (if yes how) the constituents of the proton interact with one another?

First of all, let us recall that, not only the spatial resolutions power, but also the interaction-time is important in studying this problem: In the limiting case, in which the interaction-time between the virtual photon and a (or a group of) constituent(s) is much longer than the average time for color-interaction to propagate between any two constituents of the proton, the information we get from “our messenger”, the virtual photon, will be: “Every constituent of the proton is directly or indirectly interacting with every other constituent of the same proton.”. Hence we are forced to say that, in this case, all constituents are interacting with one another. But, we are also forced to accept that not all constituents inside the proton are interacting with one another in a given time interval — especially when this time-interval is much shorter than the average time for color-interaction to propagate between any two constituents. This is because in the other limiting case, in which the interaction-time between the virtual photon and the struck constituent (of the proton) is extremely short — so short that it does not have time to communicate with any other constituent of the proton, every struck constituent (parton) can be considered as a free particle during the interaction-time. This is the well-known result of Parton Model and/or impulse-approximation, which explain(s) the existence of approximate Bjorken-scaling.

In this connection, it is also useful to recall the following: Due to vacuum polarization in QED, the virtual photon may dissociate into a fermion-antifermion pair (lepton-pair or quark-pair) which exist for a certain time-interval; and the question whether the virtual photon reaches the struck constituent of the proton in form of a bare photon or in form of a fermion-pair, depends on the life-time of the virtual fermion-pair state. This life-time
can be calculated by standard methods with the help of the uncertainty principle. Viewed
from proton’s rest frame, it is of order $q_{||}/m_{\perp}^2$ where $q_{||}$ is the longitudinal momentum of
the photon, and $m_{\perp} \equiv (m^2 + p_{\perp}^2)^{1/2}$ is the so-called transverse mass of the fermion (with $m$
stands for the mass of the lepton or that of the quark, and $p_{\perp}$ its transverse momentum).
It means, viewed in this reference frame, the life-time of the virtual fermion-antifermion
pair state created by an energetic photon is long — much longer than the interaction-time
between a photon (with the same energy) and (the constituents of) the proton. Does this
mean that the “prober” is very likely not a bare photon? The difficulty is bypassed in Parton
Model by describing the electron-proton scattering in a “fast moving frame” in which the
proton (and thus all its constituents) are moving with light-velocity toward $\gamma^*$. We note
that, this method of bypassing the above-mentioned difficulty has also been used in all the
“QCD-corrected parton models” (See e.g. Refs 1-3 and the references given there). In fact,
all the parton (valence quark, sea quark and gluon) distributions extracted from the data
are based on theoretical interpretations and/or analyses made in reference-frames in which
the constituents are moving with (almost) light-velocity.

Viewed from such a fast moving reference frame, a gluon $g$ of the proton may dissociate
into a quark-antiquark pair, and the life-time of such a virtual quark-antiquark $q\bar{q}$ state is
$\tau_g \sim q_{g||}/m_{\perp}^2$ where $q_{g||}$ is the longitudinal momentum of the gluon $g$ and $m_{\perp}$ is the transverse
mass of the quark $q$ (antiquark $\bar{q}$). Hence, if $\tau_g$ is longer than the above-mentioned photon
($\gamma^*$) interaction-time $\tau_{int}$, this virtual $q\bar{q}$ state can be detected by the $\gamma^*$ with the given
interaction-time $\tau_{int}$. Now, $\gamma^*$ can only interact with charged constituents, a gluon can
dissociate into a virtual quark-antiquark pairs, and the life-time of such a virtual $q\bar{q}$-pair
is directly proportional to the longitudinal momentum of the gluon. These facts might
lead us to conclude that, for a photon $\gamma^*$ with given interaction-time $\tau_{int}$, only gluons with
sufficiently large longitudinal momenta could be (indirectly) detected by this $\gamma^*(\tau_{int})$. Can
gluons with lower longitudinal momenta also have a chance to be (indirectly) detected by
such a $\gamma^*(\tau_{int})$?
The answer is “Yes!” This is because according to QCD, quark-antiquark pairs can also be produced by gluon-gluon collisions, and the produced quark-antiquark pairs may again turn into gluons. One of the simplest examples is the process illustrated by the so-called “box-graph” (See Fig.1) in which two gluons turn into a quark-antiquark pair and then turn into two gluons. Now, imagine that, in a certain time-interval, two gluons interact with each other (via quark-exchange) and create a quark-antiquark pair which after a short moment turn into two gluons again; and during this short moment, a virtual photon $\gamma^*$ enters and interact with the quark or the antiquark for a time-interval $\tau_{int}$. Our messenger $\gamma^*(\tau_{int})$ would report: “I see charges distributed in space-time!”, and we know, because of energy-momentum conservation (and uncertainty principle), the life-time of such a virtual quark-antiquark state is proportional to the sum of the longitudinal momenta of these two gluons. This example explicitly demonstrates how two gluons may come together to form a quark-antiquark pair, which can be detected by virtual photons with a fixed interaction-time. If these two gluons, which form the quark-antiquark pair, for a given time-interval do not interact with other constituents of the proton (See the discussion at the beginning of the present paper!), we call this system ($c^*$) of gluons “a gluon-cluster”. We call the time-interval in which they do not interact with other constituents of the proton “the life-time of this gluon cluster”, and denote this quantity by $\tau_c$. Since three or more gluons can also form quark-antiquark pair(s), as illustrated in Fig.1, while the life-time of such quark-antiquark pair(s) state is directly proportional to the sum of all participating gluons. (This can be readily seen by straightforward generalization of the graphs shown in Fig. 1 to higher orders.) It should be mentioned in this connection, that the notion of gluon-cluster is not restricted to a system of two or more gluons which can form one and just one quark-antiquark pair. A gluon cluster $c^*$ is in general a system of gluons which, for a given time-interval $\tau_c$, interact with one another, and only with one another; they form quark-antiquark pairs, and the space-time distributions of the temporarily existing charged constituents can be detected by virtual photons in inelastic electron-proton scattering processes. The life-time of the gluon cluster $\tau_c$ can be calculated by using the uncertainty principle.
Kinematic considerations show that such a gluon-cluster, before it interacts (i.e. its charged constituents interact) with the incoming virtual photon $\gamma^*(x_B, Q^2)$, is itself virtual — in the sense that the total four-momentum $q_c$ of the system of gluons is such that the scalar-product $q_c^2$ is less than zero. This is why we denote such a cluster by $c^*$ (with an asterisk as superscript). It should also be mentioned in this connection that, in contrast to gluon-clusters, the four-momentum of a glueball is time-like. Perhaps, it is possible to produce glueballs and/or mesons by knocking-out colorless gluon-clusters under appropriate experimental conditions.

Having in mind that the gluons are subjected to confining color forces, the magnitude of which increases with increasing distance within a hadron, and that the clusters are formed in a random manner, it is not difficult to imagine that the typical spatial extension of a gluon-cluster is comparable with that of a hadron, and that there are in general spatial overlaps between different clusters of the same proton. But, as we have explained in connection with the definition of the clusters, during the given interaction-time $\tau_{int}$, the cluster struck by $\gamma^*(Q^2, x_B)$ can be considered as a free (that is not interacting with other cluster or hadrons) object, provided that $\tau_{int}$ is shorter than the life-time of the cluster. (As we have already mentioned at the beginning of this paper, the question “who is interacting with whom” depends on the comparison between two time-intervals: the time-interval the “prober” does his measurement and the time-interval the probed objects need to communicate with one another!) Furthermore, since gluons are members of a color-octet, gluon-clusters are expected to be either in the color-singlet (colorless) or in one of the possible color-multiplet (colored) states. Hence, there should be two kinds of gluon-clusters, which we denote by $c^*_0$ and $c^*_m$ respectively. This means, once we accept the widely accepted relationship between color and confinement, we also have to accept that the spatial distribution of $c^*_0$ should be very much different from that of $c^*_m$. In particular, the former may exist beyond the “average border” for the colored constituents of the proton. Taken together the fact that there is no color-connection between the colorless clusters $c^*_0$ and the rest of the proton,
this immediately lead us to the following conclusion: The interactions between the virtual photon $\gamma^*$ with $c_0^*$'s are peripheral $\gamma^*$-nucleon collisions; and it is this kind of collisions which is responsible for the large rapidity gap events. The interactions between the virtual photon $\gamma^*$ with the colored clusters $c_m^*$'s do not contribute to the large rapidity gap events, but they do contribute to the inclusive cross-section for deep-inelastic lepton-nucleon collisions, and hence to the structure functions of the nucleon.

Before we proceed to discuss in more detail the relationship between the proposed gluon-clusters and the existing inelastic electron-proton scattering data, it seems useful to ask: How much do we know about the dynamics of cluster-formation and cluster-decay? How much details do we need to know about the dynamics of such formation- and decay-processes in order to understand the results obtained the experiments performed at HERA? The answer to the first question is: “Very little.” It is the case, although we adopt QCD for the description of the elementary interactions between the constituents (quark and gluons). As we can see from the examples shown in Fig.1, the color-interactions between the constituents in a cluster are in general very complicated, and we are not sure whether it is useful and/or meaningful to calculate the lowest order or certain sets of graphs by using perturbative QCD. To attack such random formation process using non-perturbative methods in which the interaction-time with the “prober” is also taken into account, lattice QCD calculations seems to be an attractive possibility. Discussions on the possibility of writing a Monte-Carlo program based on such ideas — in particular the idea of also taking the interaction-time between the “prober” and the “probed object” is taken into account in such calculations — are underway; but we are still rather far from our goals. Fortunately enough, the answer to the second question is also “Very little.”! In fact, in the present paper, we discuss the problem by using a statistical approach; and we show that phase space considerations without any dynamical input are sufficient to give a reasonable description of the striking characteristic features of the HERA-data.

To be more precise, let us consider such gluon-clusters and their interactions with virtual
photons $\gamma^*$ (with given $Q^2$ and $x_B$) in a fast moving frame in which the proton is moving practically with the velocity of light. We denote the four-momentum of the proton by $P = (E_p, 0, 0, |\vec{P}|)$, that of the cluster by $q_c = (q^0_c, q^\perp_c \cos \varphi_c, q^\perp_c \sin \varphi_c, q^\parallel_c)$ and that of the remnant of the proton by $P'$. Here, we consider only the case in which the remnant of the proton remains a proton (the generalization is straightforward) and obtain that, for $E_p^2 \gg Q^2 \gg M^2$ ($M$ is the proton mass),

$$\tau_c = \frac{1}{\Delta E} \approx \frac{\xi_c E_p}{Q^2_c},$$

(1)

where $\Delta E = P^0 + |\vec{q}_c| - E_p = |\vec{q}_c| - q^0_c$ is the energy difference between the initial nucleon and the virtual state (which consists of the cluster $c^*_i$ and the proton remnant). In the above-mentioned reference frame, the variable $\xi_c \equiv (q_c \cdot q)/(q \cdot P)$ is simply the fractional energy of the cluster.

Colored clusters can only exist inside the nucleon, that is in a restricted spatial region, the length of which is of order $1/M$ (the Compton wavelength of the nucleon). This means, due to confinement, the $c^*_m$'s are expected to stay in a volume $V_m$ of the order of $M^{-3}$ independent of the four-momenta $q_c$ of the clusters. In contrast to this, the $c^*_0$'s can leave the nucleon and exist in a spatial region which depends on the spatial distance that a $c^*_0$ can propagate. In the infinite momentum frame, the volume of this spatial region is given by $V_0 = \pi r_0^2 l$. Here $r_0 \propto \tau_c$ is a measure of the transverse distance which $c^*_0$ can travel during its lifetime $\tau_c$. Since everything practically moves with the same (light) velocity in the longitudinal direction, $l$ depends, if at all, only weakly on $q_c$ and is taken as a constant, $l \sim M^{-1}$, independent of $q_c$. Hence, we obtain, for the four dimensional phase-space $\Omega_i$ for $c^*_i$ with $i = 0$ and $m$, the following:

$$\Omega_i(\xi_c, Q^2_c) \equiv \tau_c V_i \propto \begin{cases} \tau^3_c M^{-1}, & \text{for } i = 0 \\ \tau_c M^{-3}, & \text{for } i = m \end{cases}$$

(2)

Now, we consider the number-density $N_i(q_c)$ of $c^*_i$ ($i = m, 0$) with fixed $q_c$. It is clear that, unless there are special dynamical reasons which forbid such considerations, the simplest
ansatz for this quantity is to assume that it is proportional to the corresponding allowed
(4-dimensional) phase-space $\Omega_i$. That is:

$$N_i(q_c) = \kappa_i M \delta(q_c^0 - \xi_c E_p) \Omega_i(\xi_c, Q_c^2)$$

Here, the $\delta$-function takes care of to energy-momentum conservation which requires $q_c^0 \approx \xi_c E_p$ for $E_p \gg Q^2 \gg M^2$; and $\kappa_i$ with $i = 0, m$ are two constants.

The contributions to the nucleon structure function $F_2(x_B, Q^2)$, $F_2(x_B, Q^2|c_m^* + c_0^*) = F_2(x_B, Q^2|c_m^*) + F_2(x_B, Q^2|c_0^*)$, can be calculated. They are given by,

$$F_2(x_B, Q^2|c_i^*) = \int_D d^4q_c N_i(q_c) F_2^i(q, q_c),$$

where $F_2^i$ stands for “the structure function of the cluster $c_i^*$”.

What do we mean by “the structure function of the cluster $c_i^*$ (i=0,m)”? Why do we simply add the contribution from $c_0^*$ and that from $c_m^*$? These questions are not difficult to answer. This is because, having the proposed picture in mind, what we need to know are the following: For a given $Q^2$ and a given $x_B$, how large is the chance for $\gamma^*(Q^2, x_B)$ to hit a charged member of a gluon-cluster of life-time $\tau_c$? Now, it is clear that probability for such a $\gamma^*(Q^2, x_B)$ to hit a charged constituent of a gluon-cluster $c_i^*$ (i=m,0) is different from zero if such a cluster exists. It is also not difficult to imagine that we can express this probability, in analogy to that for a hadron under such circumstances, as “the structure function of the cluster $c_i^*$” which is in general a function of the kinematic variables $Q^2, x_B, q_c^2 \equiv -q_c^2, x_Bc \equiv Q_c^2/(2qq_c)$. Here, $q$ and $q_c$ are the four momenta of $\gamma^*$ and $c_i^*$ respectively. In particular, since a $\gamma^*(Q^2, x_B)$ can hit either a charged constituent of a $c_m^*(Q_c^2, x_Bc)$ or that of a $c_0^*(Q_c^2, x_Bc)$, the probability for $\gamma^*(Q^2, x_B)$ to hit a charged constituent of either $c_m^*(Q_c^2, x_Bc)$ or $c_0^*(Q_c^2, x_Bc)$ should be the sum of the corresponding possibilities. This is why we add the structure functions of $c_m^*$ and $c_0^*$. In this connection, it is also interesting to compare the structure function of a virtual gluon-cluster with that of a virtual meson — in particular a virtual pion. The latter is similar to the former in the sense that it also describes the probability
for the virtual photon in an inelastic electron-proton scattering to interact with a charged constituent of a virtual (that is space-like) subunit of the proton. But, in contrast to the former, it is formed by quark-antiquark collisions, and thus manifest itself outside the small-$x_B$ region where the quark/antiquark-distributions dominate. This is why, we do not consider them in the present paper.

The physical region $D$ of the above-mentioned gluon-clusters is characterized by: $q_{cL}^2 \geq 0$, $q_{c}^0 + q^0 > 0$ and $(q + q_c)^2 \geq m_\pi^2$. Hence, the integration limits, and therefore the integrals, are functions of $x_B$ and $Q^2$ — even when $F_{c_i}^2$ are taken as constants (see the discussions below). For the explicit evaluation of $F_2(x_B, Q^2|c_i^*)$ given by Eq.(4), it is convenient to use, instead of $(q_{c}^0, \vec{q}_c)$, the following set of variables $(q_{c}^0, \xi_c, Q_c^2, \varphi_c)$. The integration limits $\xi_{cmin}(x_B, Q^2) \leq \xi_c \leq \xi_{cmax}(x_B, Q^2)$ and $Q_{cmin}^2(\xi; x_B, Q^2) \leq Q_c^2 \leq Q_{cmax}^2(\xi; x_B, Q^2)$ are determined by the above-mentioned kinematic constraints. In order to see the qualitative features of the relevant quantities, it is useful to know that, for $Q^2 \gg M^2$, we have:

$$Q_{cmax}^2 \approx Q^2(\xi_c/x_B - 1),$$

$$Q_{cmin}^2 \approx M^2 \xi_c^2/(1 - \xi_c),$$

and $\xi_{cmax} \approx 1$, $\xi_{cmin} \approx x_B$.

The following qualitative features can be read off from Eqs.(1-6): Both $N_i(q_c)$’s ($i = 0, m$) depend on $Q_c^2$ and $\xi_c$, and both decrease with increasing $Q_c^2$ for fixed $\xi_c$. For a given $\xi_c$, the average number $\langle N_i \rangle$ of clusters $c_i$ which can be detected by the photon $\gamma^*(Q^2, x_B)$ can be readily obtained by integrating $N_i(q_{c}^0, \xi_c, Q_c^2, \varphi_c)$ in the physical domain $D$ over $q_{c}^0$, $Q_c^2$ and $\varphi_c$. The results are the following: $\langle N_0(x_B, Q^2; \xi_c) \rangle$ is proportional to $\xi_c^3/Q_{cmin}^4$, and $\langle N_m(x_B, Q^2; \xi_c) \rangle$ is directly proportional to $\ell n(Q_{cmax}^2/Q_{cmin}^2)$. Thus we have,

$$\langle N_0(x_B, Q^2; \xi_c) \rangle \sim \frac{1}{\xi_c^3},$$

$$\langle N_m(x_B, Q^2; \xi_c) \rangle \sim \ell n\frac{Q^2}{M^2 \xi_c^2}(\xi_c/x_B - 1),$$

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for $x_B < \xi_c \ll 1$. It is interesting to note in particular that the latter has a term proportional to $\ln Q^2/M^2$, and this dominates the $Q^2$-dependence of $F_2(x_B, Q^2|c^*_m)$. The former is expected to have significant influence in particular on $\xi_c$-dependence of the “diffractive structure function” $F_2^{D(3)}(x_{Bc}, Q^2, \xi_c)$ which will be discussed below.

Let us now examine the second factor in the integrand of Eq.(4), the structure function of the cluster $c^*_i$, $F_{2i}^c$, which is a priori not known. As a first step, we ask: What kind of $Q^2$- and $x_B$-dependence do we expect to see for $F_2(x_B, Q^2|c^*_i)$, if the dynamical details of such a process do not play a role at all? To see this, we took $F_{2i}^c$ as a constant. It is interesting to see that such a simple ansatz is consistent with the experimental finding in $e^-p$-scatterings/1,2,3/ and that in $e^+e^-$-processes/9/.

In terms of the variables $(q^0_c, \xi_c, Q^2_c, \varphi_c)$, we obtain from Eqs.(2) and (4), with $F_{2i}^c =$const, the following:

$$F_2(x_B, Q^2|c^*_i) = \lambda \kappa_i \int_{\xi_{c\min}}^{\xi_{c\max}} d\xi_c \int_{Q^2_{c\min}}^{Q^2_{c\max}} dQ^2_c \Omega_i(\xi_c, Q^2_c),$$

(9)

where $\lambda \approx 4\pi M/(1 + x_B E_p/E_e)$ ($E_e$ is the electron energy) coming from the Jacobi one obtains from the variable-transformation from the four-component of $q_c$ to $(q^0_c, \xi_c, Q^2_c, \varphi_c)$ and the integration over $\varphi_c$. The $\Omega_i(\xi_c, Q^2_c)$’s are given by Eq.(3). This means, under the assumption that gluon-clusters indeed exists in the small $x_B$ region, and that the dynamical details about such cluster can be neglected in a statistical approach, their contributions to $F_2(x_B, Q^2)$ namely the sum of $F_2(x_B, Q^2|c^*_i)$’s can now be calculated and be compared with data/1,2,3,6,7/. Such calculations have been carried out by using the exact expressions for the kinematic boundaries. The two unknown parameters $\kappa_i$’s ($i = m, 0$) times the overall normalization constant are determined by adjusting them to the normalization of the data points. The results are shown in Figs.2a and 2b. In order to see whether/how they contribute to the large rapidity gap events, we have also calculated the rapidity distribution of the produced hadrons. To do this, we wrote a Monte-Carlo program to generate events according to Eq.(4) and took the following into account: While a $\gamma^*-c^*_0$ system can fragment
independently, a $\gamma^* - c_m^*$ system (because of the color lines) has to fragment together with the rest of the proton. The fragmentation was performed by using the Lund model/10/ as implemented in JETSET/11/. (The details of this Monte-Carlo calculation will be published elsewhere/12/). The result is shown in Fig. 3.

Furthermore, we note that the measured/1,2,3/ “proton’s diffractive structure functions” $F_2^{D(4)}(\beta, Q^2, x_p, t)$ and $F_2^{D(3)}(\beta, Q^2, x_p) \equiv \int F_2^{D(4)}(\beta, Q^2, x_p, t) dt$ can also be calculated and that the variables used in Refs.1 and 2 are closely related to those we used in this paper. In fact, we have: $x \equiv x_B$, $\beta \equiv x_B c$, $x_p \equiv \xi_c$, $t \equiv q_c^2$; and in the case $F_2^{ci} = \text{const}$, we have,

$$F_2^{D(4)}(x_B c, Q^2, \xi_c, Q_c^2) = \lambda \kappa_0 \Omega_0(\xi_c, Q_c^2), \quad (10)$$

$$F_2^{D(3)}(x_B c, Q^2, \xi_c) = \lambda \kappa_0 \int_{Q_c^{2\text{min}}}^{Q_c^{2\text{max}}} dQ_c^2 \Omega_0(\xi_c, Q_c^2). \quad (11)$$

They can now be calculated without any adjustable parameters and the comparison of the obtained results with the experimental findings should be considered as further tests of the proposed picture. In fact, Eq.(10) shows that $F_2^{D(4)}(x_B c, Q^2, \xi_c, Q_c^2)$ should in particular be proportional to $\xi_c^3$ for fixed $Q_c^2$ and proportional to $Q_c^{-6}$ for fixed $\xi_c$. Eq.(11) implies that $F_2^{D(3)}(x_B c, Q^2, \xi_c)$ should behave approximately in the same way as the $\langle N_0(x_B, Q^2; \xi_c) \rangle$ does, namely it should be approximately proportional to $1/\xi_c$ [see Eq.(7)]. While the latter is consistent with the existing data/1,2,3/ (see Fig.4), the former can be checked by future experiments.

In conclusion, it is suggested that virtual gluon-clusters exists, and they manifest themselves in the small $x_B$ region of inelastic lepton-nucleon scattering processes. It is shown that the space-time properties of the gluon-clusters play a very special role in understanding the striking properties observed in such scattering processes. To be more precise, it is shown that the persisting $\ln Q^2$-dependence of $F_2(x_B, Q^2)$ and the existence of large rapidity events are closely related to each other; in fact they directly reflect the space-time properties of the colored and the colorless clusters. In the framework of a statistical approach
discussed in this paper, the gluon-clusters are randomly formed, and the dynamical details of
cluster-formation and cluster-decay are completely neglected. In order to make quantitative
comparisons with the data\textsuperscript{1−3}, a phase-space model is used to carry out the calculations.
It is seen that the results of this model-calculation are in reasonable agreement with the
existing data. Further studies along this line will show whether/how the dynamical aspects
of such clusters can significantly effect the obtained results.

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\[ \gamma^* \quad c^* \quad c^* = + \quad + \ldots \]

Fig. 1. An illustrative example: a gluon-cluster \( c^* \) that absorbs a photon \( \gamma^* \).
Fig. 2. $F_2(x_B,Q^2)$, proton’s structure function, (a) as function of $Q^2$ for different $x_B$-values, and (b) as a function of $x_B$ for different $Q^2$-values. The data points are taken from Refs.[1,2,3,7]. The curves are the calculated results. Here, as well as in Fig.3 and 4, the lines show the calculated result with the $F_2^{ci}(x_{Bc}) = \text{const.}$
Fig. 3. The distribution of $\eta_{\text{max}}$ compared with the data (Refs. 1 and 2). Note that the kinematic cuts used by H1 and ZEUS are different.
Fig. 4. $F_{2}^{D(3)}(x_{Bc}, Q^2, \xi_c)$, the diffractive structure function, as function of $\xi_c$ for different values of $Q^2$ and $x_{Bc}$. The data are taken from Refs.1 and 3 (Note that $\beta \equiv x_{Bc}$ and $x_{P} \equiv \xi_c$.) The lines are the calculated results.