Fluctuation superconductivity in uniform and nonuniform rings

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Abstract. Due to fluxoid quantization, the average current around a superconducting ring close to its critical temperature is an oscillatory function of the magnetic field. For the case of uniform one-dimensional rings, there is an exact theory that enables us to evaluate this current. Using a scanning SQUID device, it has been possible to isolate magnetic signals that are seven orders of magnitude smaller than the applied field. This technique was applied to several samples, intended to be uniform. A numerical simulation method which we have recently developed enables us to evaluate the average fluctuating current also in the case that the ring is not uniform. We find that the behavior of most rings is indeed in agreement with the assumption that they are uniform, but for some of the rings nonuniformity must be invoked. The difference between the behavior of a uniform and a nonuniform ring is most pronounced near half-integer number of fluxoids, and close to the temperature at which a critical point has been found.

1. Introduction
Superconducting rings have attracted the interest of physicists during many decades, because they are an easily accessible system in which measurable quantities depend on the enclosed flux, rather than on local fields only. More precisely, these quantities depend on the magnetic flux modulo $\Phi_0 = \hbar c/2e$, as expected from quantum behavior. One of the milestones in this endeavour was the Little–Parks experiment [1], in which the transition temperature is an oscillatory function of the applied flux.

A closer look at the Little–Parks results shows that, as expected from a one-dimensional system, there is actually no phase transition. What experimentalists call transition temperature is in most cases the temperature at which the resistance becomes some given fraction of the normal resistance. Since the Little–Parks effect is most pronounced for small samples, fluctuations are especially important. Superconducting rings are a compelling system for studying fluctuations: theoretically, they are a nontrivial system which is not invariant under time reversal and has a suitable phase diagram; experimentally, the influence of fluctuations is quite directly detectable.

A theory for the evaluation of resistivity above the critical temperature was developed by Aslamazov and Larkin [2]. An exact theory for the evaluation of the average current around a uniform loop, as a function the temperature and enclosed flux, was developed by von Oppen and Riedel [3]. An independent theory, which discusses the influence of the shape of the loop and uses a two-level approximation is due to Daumens et al. [4].

The von Oppen–Riedel theory was tested by Zhang and Price [5]. They measured the susceptibility and found values larger by an order of magnitude than predicted. Very recently,
Koshnick et al. [6] repeated this experiment for several samples with widths of the order of 100 nm, using a scanning SQUID, and found good agreement with the theory. Rosario et al. [7] measured conductivity in ultrathin cylinders at half-integer number of fluxoids, and found exponential dependence on the temperature above the transition, in contrast with the power dependence predicted by Aslamazov–Larkin.

In this article we study the influence of the width and nonuniformity of the loop on the average current. We focus on the Little–Parks temperature range, i.e., the range between the temperatures at which the transition would occur, in the absence of fluctuations, for integer and for half-integer flux. For higher temperatures, currents are small and experimental and numeric values are noisy; for lower temperatures, there is hysteresis and the statistical average becomes irrelevant.

The motivation for this study is our previous finding that nonuniformity can qualitatively modify the Little–Parks phase diagram [8, 9]. Even in the absence of fluctuations, there is a range of temperatures for which there is a continuous passage between consecutive fluxoid states. At the lower end of this range, there is a critical point which we have called \( P_2 \). Below \( P_2 \), the passage between fluxoid states is hysteretic. At \( P_2 \), the derivative of the current with respect to the flux diverges. We therefore expected that a remnant of these features should persist in the presence of thermal fluctuations.

2. Experiment
The susceptibilities of many rings were measured [6], one at a time, using a scanning SQUID that can isolate magnetic signals that are seven orders of magnitude smaller than the applied flux. The samples were aluminum rings with radii of the order of a micron and linewidth of the order of a hundred nanometers. One field coil applies up to 50 Gauss of field to the sample, whose response couples a magnetic flux into the pickup loop. A second counterwound loop cancels the SQUID response to the applied field to within one part in \( 10^4 \). Additional modulation coils maintain the optimum working point.

3. A Langevin approach
We have developed a numeric code that can take into account the width of the rings and variations of the cross section with the angle. Our approach is based on the time-dependent Ginzburg-Landau equations, and takes thermal fluctuations into account by adding Langevin terms. The size of these terms is determined by the fluctuation-dissipation theorem [10].

For 1D loops it is possible to gauge out the electromagnetic potential and deal with the evolution of the gauge-invariant order parameter only. However, even in this gauge in which the electromagnetic potential appears to be excluded from the description of the system, its thermal fluctuations are still essential.

Our method remains applicable for situations in which the von Oppen-Riedel procedure becomes untractable. We also developed a 1D model that approximates the behavior of wide rings.

3.1. Why nonuniform rings
In the absence of thermal fluctuations, the phase diagram of a nonuniform loop differs qualitatively from that of a uniform loop. For nonuniform loops there is a critical point at \( \Phi \approx (n + 0.5)\Phi_0 \) and at a temperature that depends on the eccentricity. For temperatures above this critical point, there is a smooth passage between consecutive fluxoid states. The transition between states is mediated by the migration of a vortex between the inner and the outer boundary. In some cases the vortex migrates across the thinnest part of the loop and in other cases across the widest part. Recent experiments have detected the path of this vortex. An additional source of interest in nonuniform loops is the claim that they could rectify thermal
Figure 1. Current as a function of flux. The lines are experimental data and the points were calculated. The blue line is what would be obtained if the samples were perfectly uniform.

Figure 2. Fluctuation region. The black lines at the left and right show the current that would be obtained if there were no thermal fluctuations. The sample considered in this figure has a 0.35 width/ radius ratio. All the adjustable parameters of our model were fixed outside the region shown in this figure.

noise and sustain a dc voltage. We have found support for this claim in the case of a model that involves two Josephson junctions with nonsinusoidal current-phase relationship.

3.2. Why wide rings
We are especially interested in the fluctuation region, where the average current in the ring is particularly small. Wider rings carry larger currents and therefore provide us with better signal to noise ratios. It is therefore important to have at hand a theoretical tool that makes the analysis of this system possible.
4. Results
If we expand the cross section of the ring as a function of the angle in a Fourier series, we find that the first harmonic has the largest influence on the shape of the current as a function of the flux. Although the samples were intended to be uniform, we have found that some of them deviate noticeably from uniform cross section (Fig. 1). The deviation is felt most strongly in the vicinity of half-integer flux and at the lowest temperatures for which the passage between consecutive fluxoid states is smooth. For wide samples, we find good agreement between our model and experiment, even in the fluctuation region (Fig. 2). Outside the fluctuation region, the width yields a reduction of the average current, which is proportional to the square of the width/radius ratio.

5. Plans for the future
We intend to use our code to determine the distribution of the vortex paths during fluxoid transitions and the distribution of the lifetime of metastable states.

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