Misallocation and Credit Market Constraints: the Role of Long-Term Financing

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Abstract

We measure aggregate productivity loss due to credit market constraints in a model with endogenous borrowing constraints, long-duration bonds, and costly equity payouts. Due to long-duration bonds, the model generates a realistic distribution of credit spreads. We structurally estimate our model using firm-level data on credit spreads from Thomson Reuters Bond Security Data and balance sheet data from Compustat. Credit market constraints increase aggregate productivity by 0.4% through their effect on the credit spread distribution. However, credit market constraints also interact with costly equity payouts, resulting in an overall productivity loss equal to 1.6%.

Keywords: misallocation, endogenous borrowing constraints, long-duration bonds

JEL Classification Numbers: E23, E44, G32, O47

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1 Introduction

The basic idea behind the large literature on misallocation is that distortions at the firm level show up as a reduction in total factor productivity (TFP) at the aggregate level. According to Restuccia and Rogerson (2013), one of the main approaches used in the literature to measure misallocation is to pick one or more frictions thought to be important and then build a dynamic model where these frictions are explicitly specified.\footnote{Restuccia and Rogerson (2013) call this the direct approach to measure misallocation. Examples of this approach include Restuccia and Rogerson (2008), Guner, Ventura, and Xu (2008), Caggese and Cuñat (2013), Khan and Thomas (2013), Midrigan and Xu (2014), Moll (2014), and Gourio and Roys (2014). On the other hand, the indirect approach is to infer reduced-form wedges directly from static first-order conditions. Following this approach, Hsieh and Klenow (2009) find very large TFP differences between countries. Correcting for additive measurement error, Bils, Klenow, and Ruane (2017) obtain smaller but still sizable TFP losses.} If the model can be disciplined with direct measures of the relevant frictions, this approach can yield convincing estimates of the magnitude and sources of misallocation.

Credit market constraints are perhaps the single most studied channel generating misallocation of capital.\footnote{For example, see Khan and Thomas (2013), Midrigan and Xu (2014), and Moll (2014).} The literature typically models credit market constraints using an exogenous collateral constraint. With such a constraint, the underlying distortion manifests itself through a shadow price (i.e., Lagrange multiplier), which has no direct empirical counterpart. Although it is possible to identify credit market constraints using broad statistics related to financing (e.g., leverage), the resulting multipliers cannot be externally validated in the data. Furthermore, the literature has typically abstracted from how credit market constraints interact with other features in the economy. Credit market constraints may affect misallocation not only by influencing borrowing costs, but also by altering the effect of other frictions or technological constraints. Therefore, modeling firms’ investment decisions within a realistic environment is crucial in order to identify the transmission mechanism of credit market constraints on misallocation.

In this paper, we measure the misallocation of capital using a model where credit market constraints can be directly disciplined using micro data and at the same time propagate or mitigate other existing frictions or technological constraints. There are three distinct features in our model. First, firms issue bonds balancing the tax advantage of debt with bankruptcy costs in case of default (Hennessy and Whited, 2007; Gourio, 2013). Lenders charge a credit spread to firms that have a higher probability of default. As a result, firms are endogenously constrained due to their inability to attract funds at a relatively low cost. Second, firms issue long-duration bonds (Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). With long-term financing, lenders take into account the whole sequence of default probabilities until the bond matures. The combination of endogenous borrowing constraints and long-duration bonds allows us to generate a realistic distribution of credit spreads. Third, firms
face capital adjustment costs and costly equity payouts (Hennessy and Whited, 2007; Jermann and Quadrini, 2012). The four model elements (credit market constraints, capital adjustment costs, equity payout costs, and taxes) introduce a wedge between the firm’s expected marginal product of capital and the user cost of capital.\(^3\)

The key advantage of our approach is that we can directly map the wedge to statistics we can measure in the data. For example, credit market constraints can be directly disciplined based on credit spreads. We structurally estimate our model using the Simulated Method of Moments (SMM). We obtain information on firm-level credit spreads from Thomson Reuters Bond Security Master Data and combine it with income and balance sheet data from Compustat. To our knowledge, this is the first paper that reproduces firm-level patterns regarding credit spreads, investment rates, and leverage using a structural estimation.\(^4\)

Credit market constraints have two effects in our model. First, they affect the distribution of credit spreads, and this generates variation in the wedge across firms. We call this the direct effect of credit market constraints on misallocation. Second, by affecting investment rates and dividend issuance, credit market constraints also influence the effect that capital adjustment costs, costly equity payouts, and taxes have on aggregate TFP. We call this the indirect effect of credit market constraints on misallocation.

We find a new and surprising result. The direct effect of credit market constraints increases aggregate TFP by 0.4%. Credit market constraints generate dispersion in credit spreads. If we did not have any other frictions or technological constraints, this would lead to TFP loss, similar to the findings in the literature (e.g., see Gilchrist, Sim, and Zakrajšek, 2013). However, consistent with the data, the model generates a negative covariance between credit spreads and firm-level productivity. In contrast, capital adjustment costs, equity payout costs, and taxes collectively have a relatively larger effect on high-productivity firms. As a result, credit market constraints (through credit spread variation) also tend to offset the productivity losses induced by the existing model environment.

Our second result is that the indirect effect of credit market constraints decreases aggregate TFP by 2.0%. Nevertheless, individual frictions or technological constraints respond differently to the introduction of credit market constraints. First, credit market constraints decrease the dispersion in investment rates across firms. As a result, there is less dispersion in

\(^3\)The reason we use expected and not realized marginal product of capital is that in our model firms choose capital one period in advance. As a result, the marginal product of capital could differ substantially across firms due to the ex-post realization of productivity (e.g., see Asker, Collard-Wexler, and De Loecker, 2014). We do not view such variation as interesting since it does not arise from an underlying distortion and it does not interact with credit market constraints.

\(^4\)Bai, Lu, and Tian (2018) estimate a model of firm financing using balance sheet data from Chinese firms. The authors calculate a firm-specific interest rate as the ratio of total interest payments to total debt. In general, interest expenses reflect the accumulation of multiple bond issuances over the previous years. In contrast, we use the credit spreads of current bond issuances that are more informative about financial distress.
the portion of the wedge that can be attributed to capital adjustment costs. The interaction of credit market constraints with capital adjustment costs increases aggregate TFP by 0.6%. Second, credit market constraints inhibit firms from smoothing dividend issuance across time. As a result, there is more dispersion in the portion of the wedge that can be attributed to equity payout costs. The interaction between credit market constraints and equity payout costs decreases aggregate TFP by 2.0%. Finally, the interaction of credit market constraints with taxes also decreases aggregate TFP by 0.6%.

The overall (direct and indirect) effect of credit market constraints is to decrease aggregate TFP by 1.6%. The effect on aggregate TFP is small because credit market constraints affect the wedge in multiple, and at times contradictory, ways. The transmission mechanism of credit market constraints on misallocation occurs through the indirect effect, a point missed by models where credit market constraints are analyzed in isolation.

Our exercise offers two lessons for policymakers. First, any policy that targets misallocation by affecting the distribution of credit spreads needs to take into account not only the dispersion of credit spreads, but also which types of firms have lower borrowing costs. Second, an effective way to insulate the economy from a financial shock is to create an environment with low taxes and where firms can adjust intertemporally dividend payouts without substantial cost. We do not explicitly evaluate such policies but simply highlight the mechanisms that are relevant for policy design.

Our paper differentiates itself from the large literature analyzing credit market constraints and misallocation. Midrigan and Xu (2014) and Moll (2014) analyze misallocation in a model with an exogenous collateral constraint. Gilchrist, Sim, and Zakrajšek (2013) discipline a static model to the observed distribution of credit spreads and find modest TFP losses from misallocation. We analyze misallocation in a dynamic model with endogenous borrowing constraints, long-duration bonds, capital adjustment costs, and costly equity payouts. As a result, our model has two contributions. First, credit market constraints show up through credit spreads that can be externally validated in the data. Second, credit market constraints affect misallocation through credit spread variation, but also through their interaction with other frictions or technological constraints. While the overall effect of credit market constraints is modest (as in the literature), we identify a new transmission mechanism through the indirect effect.

We are not the first to study the effect of credit market constraints within a rich model environment. Khan and Thomas (2013) model collateral constraints and capital adjustment costs jointly. In their paper, capital adjustment costs propagate financial shocks in the economy and reduce aggregate TFP. In contrast, we find that credit market constraints mitigate the TFP losses induced by capital adjustment costs. In Khan and Thomas (2013), the fi-
ancial shock generates misallocation by preventing young firms from quickly reaching their optimal scale. In our model, we abstract from firm entry and exit due to the nature of our data (large, publicly listed firms).

Our paper is also related to the growing literature on long-term financing. Our results regarding the shape of bond-price schedules are based on Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), who were the first papers to introduce long-term borrowing in a tractable model of sovereign default. We introduce long-term financing in a model with endogenous investment, and we structurally estimate the model using firm-level data. A paper closer to ours is Gordon and Guerron-Quintana (2018), who introduce capital in a sovereign default model with long-duration bonds and calibrate the model to cross-country moments. Gomes, Jermann, and Schmid (2016) focus on the role of monetary policy and inflation in a model with long-duration bonds and endogenous investment. Their model emphasizes the role of debt overhang due to long-duration bonds, while our model emphasizes the implications of long-duration bonds for credit spread variation, and thus misallocation. Finally, Crouzet (2017) and Sánchez, Sapriza, and Yurdagul (2018) use models with an endogenous maturity choice. In our paper, bonds mature probabilistically, which makes our model tractable enough to estimate using SMM.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes the model and analyzes how the wedge affects the firms’ investment decisions. Section 4 describes the empirical analysis, and Section 5 describes the structural estimation exercise. Section 6 reports the main results regarding misallocation. Section 7 explores how each of the key features of our model contribute to our main findings. Finally, Section 8 concludes.

2 Model

Our model combines the firm financing structure of Hennessy and Whited (2007) with long-duration bonds (Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). Firms borrow for two reasons. First, internal funds may be insufficient to reach the desired level of investment. Second, there is a tax benefit associated with debt that encourages all firms (even those that are unconstrained) to seek debt financing. On the other hand, firms face an endogenous limit on the amount they can borrow. The main friction that generates endogenous borrowing constraints is limited commitment. In particular, firms cannot commit to repaying their debt in the future, nor can they commit to future actions (such as the issuance of new debt), which also increase the probability of default. And since defaults incur deadweight losses, firms with less-healthy balance sheets face stricter borrowing limits.
2.1 Technology and Productivity

Firms are perfectly competitive and produce a single homogeneous good. The firm’s production function is \( f(z, k, n) = z [k^\alpha n^{1-\alpha}]^\gamma \), where \( z \) is total factor productivity, \( k \) is the capital input, and \( n \) is the labor input. The production function exhibits decreasing returns to scale (i.e., we assume \( \gamma \in (0, 1) \)). According to the “span of control” models of Lucas (1978) and Rosen (1982), diminishing returns to scale can be interpreted as a consequence of the diminishing returns to entrepreneurs in managing larger operations. Firms hire labor in a perfectly competitive labor market, so that profits are given by \( \pi(z, k) = \max_n \{ f(z, k, n) - wn \} \), where \( w \) is the wage.

Idiosyncratic productivity \( z \) follows an AR(1) process:

\[
\ln z' = \rho_z \ln z + \varepsilon
\]

where \( \varepsilon \) is an i.i.d. shock drawn from \( N(0, \sigma^2_\varepsilon) \). We denote by \( H(z'|z) \) and \( h(z'|z) \) the cumulative distribution and probability density functions, respectively, for next period’s productivity \( z' \), conditional on the current productivity \( z \).

The realization of next period’s productivity \( z' \) is not known when the investment decision takes place. Therefore, our model departs from Buera and Shin (2011), Moll (2014), and Midrigan and Xu (2014) where firms rent current capital after observing the realization of current productivity.

2.2 Long-Duration Bonds

In our model, a firm issues debt for two reasons. First, internal funds may be insufficient to reach the desired level of investment. Second, debt payments reduce a firm’s taxable income, which generates an additional incentive for firms to borrow, even if they have sufficient internal funds. As we discuss later, the firm has the option to default on its debt payments. In the case of default, there are deadweight losses that are reflected in the firm’s borrowing costs.

Following Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), we assume that a constant fraction \( \theta \) of all bonds will mature each period. This implies that tomorrow’s stock of bonds \( b' \) must satisfy

\[
b' = (1 - \theta)b + i_b
\]

where \( b \) is today’s stock of bonds and \( i_b \) is the number of bonds issued today. Bonds are assumed to be infinitesimally small. Therefore, while bonds mature probabilistically, there is no uncertainty about the number of bonds that will be outstanding next period.
Each bond will pay a coupon rate of $c$. For the $\theta b$ bonds that mature, the firm pays back the principal plus interest, so that the total payment today is $(1+c)\theta b$. For the $(1-\theta)b$ bonds that do not mature, the firm only pays the coupon, so that the total payment today is $c(1-\theta)b$. In total, the firm will make a payment of $(\theta+c)b$ on both maturing and non-maturing bonds.

If the firm issues $b' - (1-\theta)b$ bonds today, it will receive $q(z,k',b')[b' - (1-\theta)b]$, where $q(z,k',b')$ is the bond price (in Section 2.7 we explain how the bond price is determined). This price depends on today’s productivity $z$, capital $k'$, and the total number of bonds $b'$.\textsuperscript{5} A larger amount of bonds $b'$ increases the probability of default, which is reflected in a higher credit spread (i.e., a lower $q(z,k',b')$). When $\theta < 1$, firms have an incentive to dilute existing debt. More debt issuance today lowers $q(z,k',b')$ and reduces the value of outstanding bonds (i.e., $q(1-\theta)b$). In Section 3.1, we discuss in more detail the incentives of firms to issue debt.

Our model does not allow for an endogenous maturity choice. The maturity parameter, $\theta$, is fixed and firms do not have the ability to issue short-term debt. While this assumption is restrictive, it makes our model tractable enough to structurally estimate based on micro-level data. Nevertheless, long-duration bonds are actually preferable to short-duration bonds in Chatterjee and Eyigungor (2012) when their baseline model is extended to allow for a small probability of a rollover crisis.

For convenience, we assume that bonds issued in different periods are of equal seniority. We also assume that lenders impose no debt covenant restrictions. In principle, lenders could impose restrictions on firms regarding future debt issuance or the payment of dividends to shareholders.\textsuperscript{6} Our model does put restrictions on capital investment and equity payouts (see the next two sections) but these are not explicitly related to $b'$.

### 2.3 Capital Adjustment Costs

According to Cooper and Haltiwanger (2006), there are large periods of investment inaction at the plant level, which are complemented by periods of intensive adjustment of the capital stock. Moreover, during periods of adjustment, investment does not appear to be excessively volatile but relatively smooth. There are two standard assumptions that together can generate such investment patterns. First, there is a fixed cost to undertake investment, and second, investment is associated with quadratic adjustment costs.

Intuitively, investors are more reluctant to buy long-duration bonds if firms’ investment is too volatile. As a result, we focus only on the second type of costs, which seem more relevant

\textsuperscript{5}Since the default decision depends on the total number of outstanding bonds tomorrow, $b'$, $q(z,k',b')$ depends on $b'$ rather than the amount of debt issued today (i.e., $b' - (1-\theta)b$).

\textsuperscript{6}Debt covenant restrictions are relatively uncommon in the investment grade bond market. For example, in Reisel (2014), only 5.92% of investment-grade bonds include restrictions on the additional payouts and additional debt (see Table 4 in their paper).
for our model. In particular, we assume that capital investment is subject to adjustment costs $g(k, k')$, which take the following form:

$$g(k, k') = \phi_k \left( \frac{k' - (1 - \delta)k}{k} \right)^2 k.$$  

The parameter $\phi_k$ controls the cost of adjusting investment each period. Adjustment costs are symmetric to positive or negative investment adjustments.

### 2.4 Equity Payout Costs

Each period, firms choose the level of dividends $d$. To control the substitution between debt and equity, we assume that shareholders incur a cost whenever the firm issues dividends (i.e., $d > 0$). Our modeling assumption for equity payout costs hinders an investment strategy whereby firms sell all their capital and subsequently distribute large payouts to shareholders shortly before defaulting. We view such sudden dividend payouts as unrealistic.

In principle, we could also assume that shareholders pay a cost when the firm is raising external equity (i.e., $d < 0$). However, costs to distribute funds to shareholders in the future are implicitly a cost to raise funds from shareholders today. Therefore, since equity payout costs also affect equity issuance, we economize on parameters by modeling costs only when $d > 0$.

We summarize the costs paid by shareholders with the following function:

$$\Lambda(d) = \begin{cases} 
0 & \text{if } d < 0 \\
C_d(d) & \text{if } d \geq 0 
\end{cases}$$

where $C_d(d) = \int_0^d c_d(x)dx$ is the total equity payout cost and $c_d(d) = 1 - \exp(-\phi_d d)$ is the marginal equity payout cost. The parameter $\phi_d$ controls the speed to which marginal payout costs increase with the level of dividends.

Equity payout costs are modeled, in a reduced-form way, to capture both pecuniary and non-pecuniary costs. For example, they can be interpreted to include dividend taxes or costs associated with share repurchases. But they can also capture the preferences of managers to smooth dividends. Lintner (1956) documents the tendency of firms to smooth dividends. While such smoothing behavior could arise from an agency friction, we do not model these frictions directly. With a similar motivation, Hennessy and Whited (2007) assume a progressive dividend tax, while Jermann and Quadrini (2012) assume a quadratic equity payout cost.

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7 Indeed, when we explicitly added equity issuance costs, our results regarding leverage, credit spreads, and misallocation did not change.
2.5 Taxes

We incorporate a realistic tax structure. Investors must pay a constant tax rate, $\tau_i$, on interest income. Meanwhile, firms pay a tax on their corporate income. The firm’s taxable income, $x$, is assumed to be profits minus economic depreciation and interest expense:

$$x = \pi(z, k) - \delta k - cb$$

where $\delta$ is the depreciation rate of capital and $cb$ is the interest expense. As in Hennessy and Whited (2007), the corporate tax rate is assumed to be $\tau_c$ when taxable income is positive, and $\tau_c - c$ when taxable income is negative. Therefore, the firm’s total corporate tax bill is given by

$$T_c(x) = \begin{cases} 
\tau^+ x & \text{if } x \geq 0 \\
\tau^- x & \text{if } x < 0 
\end{cases}.$$ 

In order to capture loss limitations in the U.S. corporate tax code, we will assume that $\tau^- < \tau^+$.

2.6 Value Functions and Default Decision

A firm that does not default today chooses dividends $d$, capital $k'$, and debt $b'$ according to the following program:

$$V(z, k, b) = \max_{d,k',b'} \left\{ d - \Lambda(d) + \beta E \left[ \max(V(z', k', b'), 0) \right| z \right\} \right.$$ (1)

subject to

$$d + k' = e(z, k, b) + q(z, k', b') [b' - (1 - \theta)b] - g(k, k') \right.$$ (2)

$$e(z, k, b) = \pi(z, k) - T_c \left[ \pi(z, k) - \delta k - cb \right] + (1 - \delta)k - (\theta + c)b.$$ (3)

Today the firm’s shareholders receive dividends, $d$, minus the equity payout cost, $\Lambda(d)$. Future dividends are discounted with $\beta = \eta/(1 + r(1 - \tau))$, where the parameter $\eta \in (0, 1]$ reflects the extra impatience of the firm. This parameter will help generate default in equilibrium. Nevertheless, due to long-duration bonds, we can generate a realistic level of default without resorting to unrealistically low values of $\eta$. The risk-free interest rate $r$ is exogenous in our analysis. We define internal equity, $e(z, k, b)$, as the sum of after-tax profits and undepreciated capital minus current payments to bondholders. According to the firm’s budget constraint,
the firm has access to three sources of funding for capital. First, the firm can issue new debt (e.g., \( q(b' - (1 - \theta)b) \)). Second, the firm can utilize internal equity (e.g., \( e \)). Third, the firm can resort to external equity (e.g., by choosing \( d < 0 \)).

Next period, after the realization of \( z' \), the firm chooses whether to default. If the firm defaults, its continuation value is zero. However, defaulting firms go through bankruptcy reorganization. In particular, lenders recover (a fraction of) the firm’s value as a compensation for default. The firm continues operating with the same productivity \( z' \) and capital level \( k' \) as if there was no default. The only difference is that firms do not have any debt obligations anymore (\( b' = 0 \)). Since \( z' \) and \( k' \) retain their values, there is no misallocation in our model due to the extensive margin (entry and exit of firms).

We can define a threshold, \( z'_d(k', b') \), such that the firm will default next period only for realizations of productivity \( z' < z'_d(k', b') \). This threshold is defined to be the value of idiosyncratic productivity, \( z'_d \), such that the firm is just indifferent between defaulting and not defaulting:

\[
V(z'_d(k', b'), k', b') = 0. 
\]

The value of not defaulting (i.e., \( V(z', k', b') \)) may be strictly positive for any value of \( z' \). In this case, the firm will not default next period and the default threshold is \( z'_d = 0 \).

### 2.7 Determination of Bond Price

Bonds are purchased by risk-neutral lenders. In the case of default, the firm continues operating but it goes through bankruptcy reorganization. This assumption reflects common practices under bankruptcy laws in which the owners’ rights and interests are ended and the company’s creditors are given ownership of the newly reorganized company.\(^8\)

In default, lenders receive equity in the reorganized firm but pay a bankruptcy cost. If the firm defaults tomorrow, lenders will recover \((1 - \xi)V(z', k', 0)\), where \( \xi \) is the bankruptcy cost parameter and \( V(z', k', 0) \) is the value of the firm with no debt. As a result, the price of the bond, \( q(z, k', b') \), is set to guarantee lenders an expected pre-tax return equal to the risk-free rate:

\[
q(z, k', b') = \frac{1}{1 + r} \left[ R^d(z, k', b') + R^{nd}(z, k', b') \right] 
\]

---

\(^8\)An alternative would be to follow Hennessy and Whited (2007) and assume that debt is renegotiated to the point where the firm is just indifferent between defaulting and not defaulting. However, in our model with long-duration bonds, this assumption would imply that the defaulting firm would renegotiate to a relatively high level of debt, which it would immediately dilute by issuing even higher debt next period and then defaulting again.
where

\[ R^d(z, k', b') \equiv \frac{1}{b} \int_{0}^{z_d(k', b')} (1 - \xi) V(z', k', 0) h(z'|z)dz' \]

\[ R^{nd}(z, k', b') \equiv \int_{z'_d(k', b')}^{\infty} \left[ \theta + c + (1 - \theta)q(z', k'(z', k', b'), b'(z', k', b')) \right] h(z'|z)dz'. \]

There are two components that determine the bond price. The first component, \( R^d(z, k', b') \), reflects payments lenders will receive in default states (i.e., when \( z' < z'_d(k', b') \)). In this case, lenders receive equity in the reorganized firm and pay the bankruptcy cost. The second component, \( R^{nd}(z, k', b') \), reflects payments lenders receive in non-default states (i.e., when \( z' \geq z'_d(k', b') \)). In this case, lenders receive a payment \( (\theta + c) \) on all maturing and non-maturing bonds today. For the outstanding bonds that do not mature, the lender can expect to receive more payments in the future. The value of these claims is \( (1 - \theta)q(z', k'', b'') \), where \( q(z', k'', b'') \) is the price of the unmatured bonds tomorrow. Notice that the price tomorrow depends not only on the realized productivity tomorrow, \( z' \), but also on \( k'' = k'(z', k', b') \) and \( b'' = b'(z', k', b') \), which are the firm’s choices for capital and debt tomorrow.

In one-period bond models (i.e., \( \theta = 1 \)), the bond price depends only on the probability the firm will default next period. The probability of defaulting in two periods or more has no effect on today’s price. In contrast, in models with long-duration bonds, today’s price is affected by the possibility of default in each future state of the world until the bond matures. For example, if the firm is expected to issue a high level of debt next period (i.e., high \( b'' \)), then the probability of default after two periods increases and next period’s bond price \( q(z', k'', b'') \) decreases. This will be reflected in the price of bonds today, \( q(z, k', b') \).

To make the above discussion more transparent, we plot in Figure 1 the cumulative probability of default over various horizons and the bond price. In our framework, we can distinguish between the probability of default in the short-run (the next couple of periods, for example) from the probability of default over a longer horizon (10 years ahead). We make this distinction by showing in the left panel of Figure 1 how the cumulative probability of default (after one period, two periods, etc.) depends on today’s choice of \( b' \). The right panel of Figure 1 plots the bond price \( q \) as a function of \( b' \). Both figures have been plotted for a given \( z \) and \( k' \).

In all horizons the default probability increases with a higher debt issuance \( b' \). A highly indebted firm will have smaller value and hence will find default more attractive. What’s noteworthy is how, as \( b' \) increases, default over longer horizons becomes likely before default over shorter horizons does. In Figure 1, if the firm issues very little \( b' \), then it is likely that the firm will neither default over a long horizon (i.e., within 10 years) nor over a short horizon (i.e., within three years). As \( b' \) increases, it may then become likely that the firm will default.
Figure 1: Probability of Default by Horizon (Left Panel) and Bond Price Schedule (Right Panel)

Note: The left panel plots how the cumulative probability of default (for selected time horizons) depends on today’s choice of \( b' \). The right panel plots the bond price, \( q \), as a function of today’s \( b' \). Both figures have been plotted for a given \( z \) and \( k' \). Note that the model has been calibrated so that the risk-free price of the bond is 1.

over a 10-year horizon, but is still unlikely to default within three years. As \( b' \) increases even more, the firm is certain to default within three years as well. The firm has to issue a large amount of debt to make it likely that the firm will default next period.

The link between the cumulative probability of default and \( b' \) explains the shape of the bond price schedule. Even as \( b' \to 0 \), the bond price is strictly less than the risk-free bond price (normalized to 1). Lenders fear that, over a long horizon, the firm might be in a position to default on this debt. As \( b' \) gradually increases – and as default on shorter horizons becomes even more relevant – the bond price gradually decreases. The probability of default over longer horizons (due to long-term financing) is key to generating a smoothly decreasing bond price schedule. This will help us match the credit spread distribution. With one-period bonds, the only relevant default probability is the default probability after one period. As a result, firms can issue a relatively large amount of debt at the risk-free rate and the cross-sectional distribution of credit spreads will be degenerate (at zero). We show this in Section 7.2.

Therefore, the main friction that limits the ability of firms to borrow is limited commitment. The firm cannot commit to repaying its debt tomorrow nor at any time in the future. Furthermore, the firm cannot commit to not taking any actions regarding future choices of \((d, k', b')\), which may result in losses for current debtholders. Due to lack of commitment (and also bankruptcy costs), bondholders demand a higher credit spread to supply funds to the
firm. As a result, the overall amount that firms are able to borrow is reduced.

3 Model Analysis

In this section, first, we analyze the optimal debt issuance policy with a particular focus on the effect of long-duration bonds. Second, we analyze the optimal capital policy. In the presence of frictions or technological constraints, the firm does not equate the expected marginal product of capital with the user cost of capital. We show how each friction and technological constraint contributes to a wedge in the firm’s optimality condition. Third, we explain how we measure aggregate productivity and misallocation of capital across firms.

3.1 Optimal Financing Policy

We describe how the optimal choice for next period’s debt \( b' \) is determined for a given choice of next period’s capital \( k' \). Optimizing Equation (1) with respect to \( b' \) gives

\[
[1 - \Lambda'(d)] \left( q + \frac{\partial q}{\partial b'} [b' - (1 - \theta)b] \right) = E \left[ (1 - \chi')(1 - \Lambda'(d')) \{ \theta + c + (1 - \theta)q' - c\tau' \} \right], \tag{5}
\]

where \( \chi' = 1 \{ z' < z'_d(k', b') \} \) is an indicator for whether the firm defaults tomorrow, \( q' \) is tomorrow’s bond price, and \( \tau'_c \) is tomorrow’s marginal corporate income tax rate. The left-hand side of Equation (5) represents the marginal benefit of an additional unit of debt issuance, holding fixed \( k' \). If the firm issues one more unit of debt, it is able to increase dividends by \( q + (\partial q/\partial b') [b' - (1 - \theta)b] \). The marginal value of these additional dividends depends on whether the firm is issuing equity \((d < 0)\) or issuing dividends \((d > 0)\). If the firm is issuing equity, then term \( \Lambda'(d) \) is equal to zero. If the firm is issuing dividends, then term \( 1 - \Lambda'(d) \) is equal to \( 1 - c_d(d) \). Therefore, equity payout costs reduce the marginal benefit of additional debt issuance.

Looking at the left-hand side of Equation (5), we can see that existing debt \( b \) increases the marginal benefit of debt issuance (since \( \partial q/\partial b' < 0 \)). The firm benefits from issuing more debt since additional debt issuance reduces the price the firm receives on debt issued today, but not debt issued in previous periods. The more indebted a firm is (i.e., higher \( b \)), the stronger is its incentive to issue more debt to dilute existing claims. This incentive to overborrow is a notable feature of models with long-duration bonds. If \( \theta = 1 \), then the current stock of outstanding bonds does not affect the incentives to issue new debt.

The right-hand side of Equation (5) represents the marginal cost of servicing the additional debt, holding fixed \( k' \). Tomorrow, the firm will make an additional payment of \((\theta + c)\) on the
maturing and non-maturing debt. The firm will be obligated to make additional payments in the future on the non-maturing debt. The marginal increase in these future payments is captured by the \((1 - \theta)q'\) term. Moreover, since debt is tax deductible, debt issuance reduces the corporate income tax paid by the firm. This provides an incentive for the firm to issue more debt.

### 3.2 Optimal Investment Policy

Using the optimality condition for capital \(k'\), we describe how various frictions/technological constraints affect the first-best level of investment. Holding fixed the firm’s choice for \(b'\), the first-order condition for \(k'\) is

\[
E \left[ \frac{\partial \pi(z', k')}{\partial k'} \right] + 1 - \delta = \text{Wedge},
\]

where

\[
\text{Wedge} \equiv \left( \frac{1 - \Lambda'(d')}{\beta} \right) \left[ 1 - \frac{\partial q'}{\partial k'} (b' - (1 - \theta)b) + \frac{\partial g(k, k')}{\partial k'} \right] + E \left[ \{\Lambda'(d') + \chi'(1 - \Lambda'(d'))\} \left( \frac{\partial \pi(z', k')}{\partial k'} + 1 - \delta \right) \right] + E \left[ (1 - \chi')(1 - \Lambda'(d')) \left( \frac{\partial T_c'}{\partial k'} + \frac{\partial g(k', k'')}{\partial k'} \right) \right].
\]

According to Equation (6), firms invest until the expected marginal product of capital (EMPK) plus \(1 - \delta\) is equal to the capital wedge, denoted by \(\text{Wedge}\). Our model economy features four elements that affect aggregate productivity through the wedge: credit market constraints (CMC), capital adjustment costs (CAC), equity payout costs (EPC), and taxes (T). Without these features, \(\text{Wedge} = 1 + r\) and thus EMPK equals the user cost of capital \((r + \delta)\). In this case, the EMPK is equated across firms. In our benchmark economy, however, each of the four features generates variation in \(\text{Wedge}\) across firms and thus generates dispersion in the cross-sectional distribution of EMPK.

To understand the transmission mechanism of credit market constraints, we now define four individual wedges that correspond to the four model elements in our economy. Each individual wedge, \(\text{Wedge}_j\), isolates the effect of feature \(j \in \{\text{CMC, CAC, EPC, T}\}\) in Equation (7).

---

9In our model, credit market constraints generate a wedge in the first-order condition for capital. However, in other classes of models, it is possible for borrowing constraints to generate a wedge in the first order condition for labor (e.g., see Jermann and Quadrini, 2012).
Wedge due to Credit Market Constraints  To isolate the effect of credit market constraints, we re-compute the wedge assuming that only credit market constraints are present. As a result, we set \( g(k, k') = 0 \), \( \Lambda(d) = 0 \) and \( T_c(x) = 0 \) in Equation (7). This yields the credit market constraints wedge:

\[
Wedge_{CMC} \equiv \frac{1}{\beta} \left[ 1 - \frac{\partial q(z, k', b')}{\partial k'}(b' - (1 - \theta)b) \right] + E \left[ \chi' \left\{ \frac{\partial \pi'}{\partial k'} + 1 - \delta \right\} \right].
\] (8)

The larger the variation in credit spreads and default probabilities across firms, the larger the variation in the wedge due to credit market constraints. In our quantitative analysis, we discipline our model based on the empirical distribution of credit spreads and default probabilities. This mapping of the wedge to direct empirical counterparts is a key part of our contribution.

Wedge due to Capital Adjustment Costs  To isolate the effect of capital adjustment costs, we assume \( \partial q/\partial k' = 0 \), \( \chi' = 0 \), \( \Lambda(d) = 0 \) and \( T_c(x) = 0 \) in Equation (7). This yields the capital adjustment cost wedge:

\[
Wedge_{CAC} = \frac{1}{\beta} + \frac{2\phi_k}{\beta} \left( \frac{k' - (1 - \delta)k}{k} \right)
- 2\phi_kE \left[ \frac{1}{2} \left( \frac{k'' - (1 - \delta)k'}{k'} \right)^2 + (1 - \delta) \left( \frac{k'' - (1 - \delta)k'}{k'} \right) \right].
\] (9)

The wedge depends on the investment rate \( [k' - (1 - \delta)k]/k \) as well as the correlation between investment rates across periods. The wedge is largest when high (low) investment rates in the current period tend to be associated with low (high) investment rates in the future. In the case of capital adjustment costs, the relevant empirical moments are the standard deviation of investment rates and the autocorrelation of investment rates.

Although \( Wedge_{CAC} \) is directly affected by capital adjustment costs (through the presence of \( \phi_k \)) it is also indirectly affected by credit market constraints. Credit market constraints change the firms’ investment rates and hence the cross-sectional distribution of \( [k' - (1 - \delta)k]/k \). As a result, credit market constraints affect misallocation of capital through the presence of capital adjustment costs.

Wedge due to Equity Payout Costs  To isolate the effect of equity payout costs, we assume that \( \partial q/\partial k' = 0 \), \( \chi' = 0 \), \( g(k, k') = 0 \) and \( T_c(x) = 0 \) in Equation (7). This yields the equity payout cost wedge:

\[
Wedge_{EPC} \equiv \frac{1}{\beta} + E \left[ \Lambda'(d') \left\{ \frac{\partial \pi'}{\partial k'} + 1 - \delta \right\} \right] - \frac{\Lambda'(d)}{\beta}.
\] (10)
Equity payout costs affect the wedge through variation in dividend issuance across time. Notice that the wedge is largest when dividend issuance today is low relative to tomorrow. In contrast, if marginal equity payout costs are constant over time (i.e., $\Lambda'(d) = \Lambda'(d') = \text{constant}$), then the wedge disappears and equity payout costs would have no effect on misallocation.

Similar to the case of capital adjustment costs, $\text{Wedge}_{EPC}$ is also affected by the presence of credit market constraints. In particular, credit market constraints affect the incentive of firms to smooth dividends across time. A borrowing constrained firm would like to reduce dividends $d$ today and increase future dividends $d'$. Without credit market constraints, firms would be free to use debt to smooth dividend issuance. As a result, credit market constraints have an effect on the firm’s EMPK through the presence of equity payout costs.

**Wedge due to Taxes** Finally, to isolate the effect of corporate income taxes, we assume that $\partial q/\partial k' = 0, \chi' = 0, g(k, k') = 0$ and $\Lambda(d) = 0$ in Equation (7). This yields the tax wedge:

$$Wedge_T \equiv \frac{1}{\beta} + E \left[ \frac{\partial T'}{\partial k'} \right].$$

(11)

Variation in expected marginal tax rates across firms generates misallocation. Once more, credit market constraints have an effect on $Wedge_T$ by affecting firms’ decisions and ultimately the distribution of marginal tax rates.

The distinction between the direct and indirect effects of credit market constraints is a key part of our analysis. Credit market constraints have a direct effect on investment through the credit market constraints wedge (i.e., $\text{Wedge}_{CMC}$). However, credit market constraints also interact with the other three wedges ($\text{Wedge}_{CAC}, \text{Wedge}_{EPC}, \text{Wedge}_T$). Therefore, to study the overall effect of credit market constraints, it is crucial to consider their effect jointly in a realistic model environment.

### 3.3 Aggregate Productivity and the Wedge

We now show how the wedge is related to aggregate TFP. To do so, we first derive an expression for TFP in our benchmark economy. Namely, we can derive the following aggregate relationship (see Appendix A.3 for details):

$$Y = A \left[ K^\alpha N^{1-\alpha} \right]^\gamma$$
where $Y$ is aggregate output, $K$ is aggregate capital, $N$ is aggregate labor, and $A$ is TFP. Credit market constraints can reduce output in two ways. First, they reduce the overall level of $K$. Second, they increase misallocation of capital across firms, which results in a lower $A$.

To understand the second channel, we derive the following expression for TFP (again, see Appendix A.3 for details):

$$A = \left[ \frac{\sum_i \left( \frac{E[(z_i')^{1/(1-(1-\alpha)\gamma)}|z_i]}{(w_k^i)^{\alpha\gamma/(1-(1-\alpha)\gamma)}} \right)^{1-(1-\alpha)\gamma} \left( 1 - \frac{1-(1-\alpha)\gamma}{1-\gamma} \right)^{1-(1-\alpha)\gamma}}{\sum_i \left( \frac{E[(z_i')^{1/(1-(1-\alpha)\gamma)}|z_i]}{w_k^i} \right)^{\alpha\gamma/(1-(1-\alpha)\gamma)}} \right]^{1-\gamma} \quad (12)$$

where $z_i$ is the productivity of firm $i$ and $w_k^i$ is the gross deviation of firm $i$’s expected marginal product of capital (EMPK) from the economy-wide average. The firm-specific wedges (derived in the previous section) thus affect TFP. To gauge the extent to which TFP is affected, we first define a benchmark level of TFP. Specifically, we re-allocate capital and labor across firms to maximize total output subject to the constraints that aggregate capital is equal to $K$ and aggregate labor is equal to $N$. In this case, EMPK is equated across all firms and $w_k^i = 1$ for all firms. Therefore, the benchmark level of TFP is

$$\bar{A} = \left[ \sum_i E[(z_i')^{1/(1-(1-\alpha)\gamma)}|z_i] \right]^{1-(1-\alpha)\gamma} \left( 1 - \frac{1-(1-\alpha)\gamma}{1-\gamma} \right)^{1-(1-\alpha)\gamma}.$$

TFP loss is the percentage difference between $\bar{A}$ and $A$:

$$\text{TFP loss} = \frac{\bar{A}}{A} - 1. \quad (13)$$

Two important statistics that determine the TFP loss are the variance of the wedge $\text{Var}(w_k^i)$ and the covariance of the wedge with productivity $\text{Cov}(w_k^i, \ln z_i)$. Namely, higher cross-sectional dispersion in EMPK (i.e., higher $\text{Var}(w_k^i)$) is associated with higher TFP losses. While $\text{Var}(w_k^i)$ is informative about the magnitude of TFP loss, $\text{Cov}(w_k^i, \ln z_i)$ is informative about the direction of TFP losses. When $\text{Cov}(w_k^i, \ln z_i) > 0$, high-productivity firms are under-investing and have too little capital. TFP could be increased by transferring resources to more productive units. In contrast, high-productivity firms have too much capital if $\text{Cov}(w_k^i, \ln z_i) < 0$, and TFP would increase by allocating more resources to less-productive

\footnote{If $w_k^i$ is log-normally distributed, TFP loss is directly proportional to $\text{Var}(w_k^i)$.}
units.

Finally, since in our model firms choose next period’s capital stock before they learn next period’s productivity, it is possible for aggregate productivity to decrease due to firms under- or over-investing prior to their productivity being realized. We do not view these losses to be interesting, as they reflect underlying uncertainty over productivity (e.g., see Asker, Collard-Wexler, and De Loecker, 2014) and do not interact in any way with credit market constraints. As a result, we net out such effects by considering TFP loss to be zero when the expected MPK is equated across firms.

4 Empirical Analysis

In this section, we conduct our empirical analysis. First, we start with a description of our data. Second, we report some summary statistics. Third, we analyze the relationship between firm-level TFP and firm characteristics (such as credit spreads).

4.1 Description of Datasets

Our analysis is based on two firm-level datasets: Standard and Poor’s Compustat industrial files and Thomson Reuters Bond Security Master Data (henceforth, TR). Compustat includes detailed income statement, balance-sheet, and cash flow data of publicly listed companies. We use annual fundamental data from 1984-2015. We impose selection criteria common in the literature. First, we exclude financial firms (SIC 6000–6999) and utilities (SIC 4900–4999). Second, we drop any firm-year observations without information on assets, capital stock, debt, or equity. Third, we drop observations that violate the accounting identity of assets equal to equity plus debt (normalized by assets) by more than 10%. Fourth, we drop firms affected by the 1988 accounting change (GM, GE, Ford, Chrysler) and include only firms reporting in USD. And finally, we keep companies that appear more than 10 consecutive years in the sample. This leaves us with a total of 7,632 firms and 113,389 observations.

Our second dataset, Thomson Reuters Bond Security Master Data, provides information on primary issuances of corporate bonds between 1980-2015. There are a total of 18,480 bond deals in our data. Available information includes the name of the company issuing the bond, the market value of the issue, the issue date, the type and purpose of the bond issuance, the maturity of the bond, and the credit spread paid by the issuer (defined as the interest rate

11 These criteria are common in studies employing Compustat data. See Bernanke, Campbell, and Whited (1990) for details.
12 We find that around 70% of bond deals took place after the year 2000.
Table 1: Compustat vs. NIPA

|                         | Compustat/NIPA | Bond-issuing firms/NIPA |
|-------------------------|----------------|-------------------------|
| Sales Share             | 0.49           | 0.19                    |
| Employment Share        | 0.58           | 0.20                    |
| Investment Share        | 0.30           | 0.10                    |
| Correlation of Sales Growth | 0.80        | 0.88                    |
| Correlation of Employment Growth | 0.61     | 0.57                    |
| Correlation of Investment Growth | 0.91   | 0.94                    |

Note: This table uses time series from the national income accounts (NIPA), Compustat firms, and a subset of Compustat firms that have issued at least one bond in our sample (denoted “bond-issuing firms”). We focus on two set of statistics: (a) the share of sales, employment, and investment in Compustat or bond-issuing firms in Compustat relative to the aggregate and (b) the correlation between sales, employment, and investment growth between NIPA, Compustat, and bond-issuing firms in Compustat.

paid over a Treasury bill of similar maturity).\textsuperscript{13} We drop bond issuances (1) below 1 million dollars, (2) with a maturity larger than 40 years, and (3) with a credit spread less than 5 basis points. These restrictions leave us with 18,369 deals. When we merge Compustat with TR data, we are left with 4,372 bond deals.

In Table 1, we compare statistics regarding sales, employment and investment between aggregate national income accounts (NIPA), Compustat firms, and Compustat firms that have issued bonds in our data (merged sample). Compustat firms represent a fairly large share of the aggregate economy: 49% of aggregate sales, 58% of aggregate employment, and 30% of aggregate investment. Bond-issuing Compustat firms represent a smaller but still sizable share of the aggregate economy: 19% of aggregate sales, 20% of aggregate employment, and 10% of aggregate investment. The correlation between sales and investment growth across time is high when comparing both Compustat and NIPA and bond-issuing Compustat firms and NIPA. On the other hand, the correlation of employment growth in NIPA and Compustat is low, around 0.6, as most of the employment growth usually comes from small firms. In Table 2 we describe the construction of variables in our empirical analysis. For most of the variables, there is a direct model counterpart.

4.2 Summary Statistics

Table 3 reports summary statistics for Compustat and TR data, respectively. In Compustat, the mean leverage ratio is 0.29 while the mean assets/sales ratio is 2.56. These estimates are close to estimates using aggregate data. We define the investment rate as investment-to-
Table 2: Definition of Variables: Model vs. Data

| Variable       | Notation | Definition                                                                                                                                 |
|----------------|----------|-------------------------------------------------------------------------------------------------------------------------------------------|
| Capital        | k        | Gross value of property, plant, and equipment (PPENT, data item #8)                                                                     |
| Investment     | i        | Capital expenditures on property, plant, and equipment (CAPXV, #30) minus sales of capital stock (SPPE, #107).                             |
| Depreciation   | δ        | Current depreciation (DP, #14)                                                                                                          |
| Dividends      | d        | Common dividends (DVC, #268) plus preferred dividends (DVP, #270) minus stock repurchases (PRSTKC, #115)                                  |
| Employees      | n        | Stock of employees (EMP, #29)                                                                                                           |
| Leverage       | qb/k'    | Debt in current liabilities (DLC, #34) plus long-term debt (DLTT, #9) divided by book value of assets (AT, #6)                             |
| Sales          | y        | Total Sales (SALE, #12)                                                                                                                 |
| Value added    | –        | Sales minus materials                                                                                                                    |
| Materials      | –        | Total Expenses [(SALE, #12) - (OIBDP, #13)] - Labor Expenses [(EMP, #29) × Wages]                                                        |
| Credit spread  | (θ + c)/q - θ - r | Interest rate paid over a similar maturity Treasury bill                                                                                   |
| Productivity   | z        | See main text                                                                                                                           |

Note: We describe the empirical counterpart of model variables. For variables computed using Compustat data we include the definition and data item in parentheses.

capital ratio. The mean investment rate in Compustat is 0.16, and the standard deviation is 0.26. Cooper and Haltiwanger (2006) analyze data from the Longitudinal Research Database and find an investment rate around 0.12 and a standard deviation of 0.33. Due to their size and access to equity markets, Compustat firms are naturally less volatile in their investment decisions. Around 6% of firms do not adjust their investment and 21% of firms change their investment by more than 20%. These results are in accord with the large literature on investment dynamics suggesting the presence of costs to adjust capital. The corresponding numbers in Cooper and Haltiwanger (2006) are 8% and 18%, respectively.

In our data, 9.8% of Compustat firms issue a bond. The median number of bond issuances by a single firm in all years is four while the maximum is 88. In some cases, firms issue multiple bonds of different amounts and maturities during the same time period. Moreover, firms raise funds through the bond market at a relatively high frequency. A firm issuing a bond in year t re-issues a bond with probability 45% between years [t, t + 2] and with probability 57% between years [t, t + 4]. The average amount raised in a deal is around $365 million. The distribution is highly skewed: the maximum amount issued is $2 billion. Most of the issues in our data involve long-term financing. The average bond matures in 11.6 years.
| Variable | #Obs. | Mean | SD   | Min  | p(10) | p(25) | p(50) | p(75) | p(90) | Max  |
|----------|-------|------|------|------|-------|-------|-------|-------|-------|------|
| **Compustat (Firms)** |       |      |      |      |       |       |       |       |       |      |
| Leverage | 133,786 | 0.29 | 0.42 | 0.00 | 0.00  | 0.03  | 0.20  | 0.38  | 0.60  | 3.23 |
| Assets/Sales | 129,730 | 2.55 | 7.29 | 0.20 | 0.43  | 0.63  | 0.96  | 1.68  | 3.53  | 61.6 |
| Investment rate | 119,991 | 0.16 | 0.22 | -0.05 | 0.01 | 0.04 | 0.08 | 0.17 | 0.36 | 1.82 |
| Inaction rate (|< 1%|) | 119,991 | 0.06 |       |      |       |       |       |       |      |
| Spike rate (> 20%) | 119,991 | 0.21 |       |      |       |       |       |       |       |      |
| Autocorrelation | 119,991 | 0.41 |       |      |       |       |       |       |       |      |
| Dividends/profits | 94,861 | 0.21 | 0.43 | 0.0 | 0.0 | 0.05 | 0.23 | 0.55 | 3.06 |
| Autocorrelation | 94,861 | 0.17 |       |      |       |       |       |       |       |      |
| **Thomson Reuters (Bond deals)** |       |      |      |      |       |       |       |       |       |      |
| Mkt. val. of issue ($mil.) | 18,369 | 365.5 | 364.6 | 3.5 | 28.3 | 126.6 | 268.6 | 475.1 | 795.8 | 2000.0 |
| Maturity (years) | 18,369 | 11.6 | 8.59 | 1 | 5 | 6 | 10 | 11 | 30 | 40 |
| Credit spread (p.p.) | 18,369 | 2.32 | 2.16 | 0.05 | 0.60 | 0.90 | 1.55 | 3.00 | 5.37 | 9.59 |

Note: We report statistics from the cross-sectional distribution over the entire period sample. All variables are winsorised at the 1%. The inaction rate and the spike rate are the fraction of firms with investment rates less than 1% (in absolute value) and more than 20%, respectively. For Compustat, the different number of observations across variables is due to missing values.

The main focus of the analysis is on the distribution of credit spreads. On average, firms pay a spread of 2.3% over a T-Bill of similar maturity. The standard deviation is 2.0%. Figure 2 shows the cross-sectional distribution of credit spreads in the data. The sizable dispersion reflects a wide variety of bond qualities in our data. Our findings are very close to Gilchrist and Zakrjšek (2012). The authors use secondary market prices of outstanding securities between 1973-2010 to construct bond yields. They find a credit spread mean of 2.0% with a slightly larger standard deviation of 2.8%.14

In Table 3 we used all bond deals. When we examine bond deals issued only by Compustat firms (around 4,129 deals) the results remain largely intact. The average maturity is 11 years, the average credit spread is 2.10 p.p., and the standard deviation decreases slightly to 2.0 p.p. (see Appendix Section A.2). Hence, credit quality can vary substantially in our data even if we restrict the sample to bond-issuing Compustat firms.

14When we look at time variation in credit spreads, we document a countercyclical pattern similar to the findings of Gilchrist and Zakrjšek (2012). In addition, during the Great Recession, bond issuance increased. Issuance of new bonds totaled around $80 billion in 2007 and increased to $130 billion during the crisis. This was the result of more firms choosing bond issuance as a means of financing. The annual number of bond deals increased from around 200 to 400 per year. In contrast, given bond issuance, the average amount of issuance decreased. The average amount per bond deal decreased from around $350 million to around $300 million. The increase in the number of issuances occurred because firms substituted bank loan financing with corporate debt issuance. For a more detailed analysis of the time-series variation in bank loans and bond issuance see Adrian, Colla, and Shin (2012) and Karabarbounis (2015).
4.3 Productivity and Firm Characteristics

A key part of our analysis is the relationship between firm-level total factor productivity (TFP) and firm characteristics. To estimate firm-level TFP we use the following specification:

\[ y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + z_{it} + \varepsilon_{it}, \]

where all variables are in logs. \( y_{it} \) is value added by firm \( i \) in year \( t \), and \( k_{it} \) and \( l_{it} \) are the capital and labor inputs, respectively, by firm \( i \) in year \( t \). \( z_{it} \) is the TFP of firm \( i \) in year \( t \), which is observed by the firm but unobserved by the econometrician. Finally, \( \varepsilon_{it} \) represents either shocks unobserved to both the econometrician and the firm (and hence, not influencing labor inputs) or measurement error.

We translate nominal variables such as value added or total value of capital into physical units using the GDP price deflator and the price index for private fixed investment, both available from the Bureau of Economic Analysis. To transform the capital stock into physical units, we take into account that capital stock is shaped based on investment that occurred in different time periods. Following the method by Brynjolfsson and Hitt (2003), we calculate the average capital age as accumulated depreciation divided by current depreciation. As a result, we deflate capital by the investment deflator of the respective year.

\[^{15}\text{As suggested by İmrohoroğlu and Tüzel (2014), we smooth further capital age by taking a three-year moving average.}\]
Table 4: Relationship between TFP and Firm Characteristics

| TFP Decile  | 1st  | 2nd  | 3rd  | 4th  | 5th  | 6th  | 7th  | 8th  | 9th  | 10th | 10th-1st |
|-------------|------|------|------|------|------|------|------|------|------|------|----------|
| Leverage    | 1.00 | 1.06 | 1.04 | 0.97 | 0.92 | 0.90 | 0.85 | 0.82 | 0.73 | 0.71 | -0.29    |
| Assets/Sales| 1.00 | 0.66 | 0.58 | 0.56 | 0.55 | 0.60 | 0.64 | 0.68 | 0.77 | 1.03 | +0.03    |
| Investment Rate | 1.00 | 0.76 | 0.86 | 0.99 | 1.15 | 1.29 | 1.52 | 1.73 | 2.15 | 2.93 | +1.93    |
| Dividends/Profits | 1.00 | 0.68 | 0.53 | 0.51 | 0.50 | 0.51 | 0.52 | 0.57 | 0.61 | 0.59 | -0.64    |

Note: We divide firms based on their TFP into 10 bins. For each bin, we calculate the average leverage, assets-sales ratio, investment rate, and dividends-profits ratio. For each variable we normalize the series based on the value in the first bin. We summarize the relationships using the difference between the 10th to first decile.

There are several challenges to estimate firm-level total factor productivity. First, there is a simultaneity problem between the firm’s choices and the firm’s productivity. For example, if a firm hires more workers because it is more productive, then coefficient $\beta_l$ might be upward biased and productivity $z_{it}$ might be downward biased. Moreover, there is a selection problem. The most productive firms are more likely to issue equity for a longer time and thus stay in the sample longer. To deal with these shortcomings, we employ the approach developed by Olley and Pakes (1996). We leave a description of this technique to Section A.1 in the appendix.

Second, our measure of productivity is revenue-based (revenue total factor productivity or TFPR), so it is jointly affected by true, physical productivity (TFPQ) as well as changes in prices (Foster, Haltiwanger, and Syverson, 2008). Moreover, firm value added is likely to be measured with error. We address – to some extent – these concerns using a fixed effects estimator when analyzing the effect of TFP on credit spreads (Equation (14) below). It is less likely that measurement error would change over time for the same firm (see for example, Bils, Klenow, and Ruane, 2017). Moreover, as Kaplan and Menzio (2015) show, variation in good prices is very prevalent across stores but small for the same store across time. Therefore, although we do recognize that our productivity measure partially captures price variation, we mitigate this concern by using a fixed effects estimator.\textsuperscript{16}

Table 4 explores the relationship between firm-level TFP and firm-level leverage, assets-to-sales ratio, investment rate, and the dividends-profits ratio. In particular, we sort firms into 10 bins based on their TFP level. For each bin, we report the average leverage ratio, assets-to-sales ratio, investment rate, and dividends-profits ratio. We have around 9,000 firm-year observations per bin. For each statistic, we normalize bins by the value of the first decile.

\textsuperscript{16}Note that our approach is also susceptible to an input price bias as we deflate capital using aggregate deflators. For this case, we explicitly check the magnitude of the bias by using not aggregate but industry-specific deflators available by the NBER-CES database. The deflators are available at the six-digit NAICS level but only for a subset of industries that reduces substantially the number of our observations. We compare our estimates for productivity and its relation to credit spreads (to be discussed in Table 5 below) when we deflate by the mean investment deflator and by industry-specific deflators. We present results in Table 13 in Appendix A.2. Our estimates are not substantially affected by using the industry-specific deflators.

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Table 5: Determinants of Credit Spreads

| Dependent Variable | $S_{ijt}$ (Credit Spreads) |
|--------------------|-----------------------------|
| Specification      | (1)            | (2)            | (3)            | (4)            | (5)            | (6)            |
| $z_{it}$ (Productivity) | -0.06*** | -0.37*** | -0.01 | -0.33*** | (0.02) | (0.05) | (0.01) | (0.05) |
| $[b'/k']_{it}$ (Leverage) |                          | 0.27*** | 0.27*** | 0.28*** | 0.26*** | (0.03) | (0.05) | (0.03) | (0.04) |
| St. Deviation of Equity Returns | 1.04*** | 0.41*** | 1.04*** | 0.39*** | (0.05) | (0.05) | (0.05) | (0.05) |

# Observations 4,151 4,151 3,780 3,780 3,644 3,644
R-squared 0.38 0.47 0.57 0.48 0.57 0.54
# Firms 724 724 625 625 608 608
Year/Bond F.E. Yes Yes Yes Yes Yes Yes
Firm F.E. No Yes No Yes No Yes

Note: This table reports estimates from the regression in Equation (14). $S_{ijt}$ is the log credit spread paid by firm $i$ associated with bond $j$ issued in year $t$, $z_{it}$ is TFP (in logs) of firm $i$ in period $t$, $[b'/k']_{it}$ is log-leverage, and we also include the logged standard deviation of yearly equity returns for firm $i$ as calculated from CRSP. One, two, and three stars denote significance at the 10%, 5%, and 1%, respectively.

Firms at the 10th decile have around 70% of the leverage and almost twice the investment rate of firms in the first decile. The relationship between TFP and the assets-to-sales ratio is non-monotonic. Our results are consistent with the findings in İmrohoroğlu and Tüzel (2014).

In Section 3.2, we have shown that the credit market constraints wedge depends on the credit spread. Moreover, the sign and magnitude of the covariance between the wedge and productivity is crucial for our results regarding the direct effect of misallocation (see Section 6). We analyze this covariance by exploring the empirical relationship between firm-level productivity and credit spreads. In particular, we run the following regression:

$$S_{ijt} = \beta_0 + \beta_1 z_{it} + \beta_2 [b'/k']_{it} + \beta_3 [\text{St. Dev. Equity Returns}]_{it} + \alpha_t + \eta_i + X_j + \varepsilon_{ijt}. \quad (14)$$

The dependent variable $S_{ijt}$ is the log credit spread paid by firm $i$ associated with bond $j$ issued in year $t$. We regress the spread to firm-level TFP $z_{it}$ and log leverage $[b'/k']_{it}$. We also include the standard deviation of yearly equity returns for firm $i$. Equity volatility is estimated using data from the CRSP daily stock files during the period of our analysis. We capture time-varying common factors using a time dummy $\alpha_t$. Moreover, we include a firm fixed effect $\eta_i$. Finally, we include bond characteristics $X_j$ such as the value of the issue (in logs) and the maturity of the bond.
Table 5 shows the results of our regression. All specifications use time fixed effects and bond characteristics. We first regress spreads on firm-level TFP without and with fixed effects (specifications (1) and (2)). The fixed effect differences out time-invariant cross-sectional variation in firm characteristics and turns out to be very important. Without fixed effects, a 1% increase in TFP decreases credit spreads by 0.06%. When we include firm fixed effects, a 1% increase in TFP decreases credit spreads by 0.37%. We view the discrepancy to be related with our discussion on revenue-based productivity versus physical productivity. If productive firms charge lower prices, then a revenue-based measure of productivity will be biased toward zero. Since price variation is likely to be more prevalent in the cross-section (Kaplan and Menzio, 2015), the fixed effects estimator increases the coefficient.

Specifications (3) and (4) run separately credit spreads on leverage and the volatility of firm equity returns. This is motivated by the extensive literature trying to identify the determinants of credit spreads (Duffee, 1998; Collin-Dufresne, Goldstein, and Martin, 2001; Driessen, 2005; Gilchrist, Sim, and Zakrajšek, 2014). Leverage and equity volatility are identified as important determinants of the probability of default on debt obligations. A 1% increase in leverage increases credit spreads by 0.27%. This is unaffected by the inclusion of fixed effects. Moreover, a 1% increase in the volatility of equity increases credit spreads by 1.04% without fixed effects and 0.41% with fixed effects. Specifications (5) and (6) include all regressors – namely, TFP, leverage and equity volatility. Compared to specifications (1)-(2), the coefficient of TFP becomes less negative, which can be explained by the inclusion of the other regressors. Once more, the inclusion of fixed effects turns out to be very important. With fixed effects, we find that a 1% increase in TFP decreases credit spreads by 0.33%.

Although not reported in Table 5, credit spreads increase with the amount issued from a bond. Moreover, conditional on the amount issued and year, longer maturity is associated with higher credit spreads. The relationship between maturity and spreads is affected by the fact that better firms issue bonds of longer duration (Helwege and Turner, 1999). In our specification, we can overcome this challenge by looking at the term structure paid by the same firm.

Our regression explains around 50% of the variation in credit spreads. The challenge to capture the determinants of credit spreads is well known in the literature as the credit spread puzzle. Driessen (2005) analyzes the determinants of credit spreads, taking into account most of the variables proposed by the literature. He finds that only two-thirds of the variation is explained by his specification. Our explanatory power is somewhat lower but within the ranges found in the literature.
5 Model Estimation

In this section, we discuss our structural estimation technique, the estimated parameters, and assess the model’s fit with the data.

5.1 Estimation Strategy

One set of parameters is set outside of the model (literature or data), and the remaining parameters are estimated using Simulated Method of Moments (SMM). With SMM, we solve for the unknown parameters by minimizing the weighted sum of squared errors between model and data moments.\(^\text{17}\) We select moments in the data that are particularly informative (based on our theory) for the wedge and misallocation.

Table 6 lists the calibrated parameters. The model is computed at an annual frequency. We normalize the wage rate to 1 and set the annual risk-free rate \(r\) equal to 4%. The returns to scale parameter, \(\gamma\), is set to 0.85, which is in the middle of the estimates of Burnside, Eichenbaum, and Rebelo (1995). We set the capital share parameter equal to \(\alpha = 0.35\) and the tax parameters to be \(\tau_i = 0.296\), \(\tau_c^- = 0.2\), and \(\tau_c^+ = 0.35\), which are typical values in the literature (e.g., Katagiri (2014)). We set the coupon rate, \(c\), to be equal to the risk-free rate. This is just a normalization that ensures the risk-free bond price is equal to 1. The expected maturity of a bond in our model is \(\sum_{t=1}^{\infty} t \theta (1 - \theta)^{t-1} = 1/\theta\). Since the average maturity in our data is 11.8 years, we set \(\theta = 1/11.8 \approx 0.085\). The depreciation rate is set to \(\delta = 0.08\) to match the median investment rate in our data.

| Parameter                  | Notation | Value | Target / Source               |
|----------------------------|----------|-------|-------------------------------|
| Wage                       | \(w\)    | 1     | Normalization                 |
| Risk-free rate             | \(r\)    | 0.04  | Typical in literature         |
| Coupon                     | \(c\)    | 0.04  | Normalization                 |
| Capital share              | \(\alpha\) | 0.35  | Capital-income share          |
| Returns to scale           | \(\gamma\) | 0.85  | Burnside, Eichenbaum, and Rebelo (1995) |
| Min. corporate tax rate    | \(\tau_c^-\) | 0.2   | Katagiri (2014)               |
| Max. corporate tax rate    | \(\tau_c^+\) | 0.35  | Katagiri (2014)               |
| Interest income tax rate   | \(\tau_i\) | 0.296 | Katagiri (2014)               |
| Bond maturity              | \(\theta\) | 0.085 | TR Bond Security Data         |
| Depreciation rate          | \(\delta\) | 0.08  | Median investment rate        |

The remaining parameters are estimated via SMM to match moments computed from our data. We estimate six parameters: the productivity parameters \((\rho_z, \sigma_{\varepsilon_z})\), the discount factor

\(^{17}\)For a review of SMM, see Streublæv and Whited (2012).
η, the bankruptcy cost ξ, the equity payout cost parameter φ_d, and the capital adjustment cost parameter φ_k.

The six parameters are estimated using a total of nine moments. The moments are reported in Panel A of Table 7. We choose moments that are informative about the parameters we seek to estimate. To pin down the parameters of the productivity process (ρ_z, σ_{εz}), we estimate a first-order autoregressive process in log-sales, controlling for firm fixed effects, and we target the persistence and the standard deviation of the residual.

The average firm-level assets-to-sales ratio is informative for the discount factor (η). If firms discount the future more, then a higher fraction of profits will be distributed to shareholders and a lower fraction will be re-invested. Hence the assets-to-sales ratio will be lower. The median leverage ratio is informative for several parameters, including the discount factor η, the bankruptcy cost ξ, and the equity payout cost parameter φ_d. With a lower discount factor, firms choose higher leverage ratios, making default more likely. With a higher bankruptcy cost ξ, lenders charge higher credit premiums for a given leverage ratio, which discourages firms from borrowing. The equity payout cost parameter φ_d affects the firm’s incentive to distribute dividends and ultimately how much debt to issue.

To inform our estimation, we also use the median credit spread and the mean default rate. The expected default rate on corporate bonds is constructed using Moody’s estimates on default probabilities on different classes of bonds. In our sample, for example, 20% of bond deals are rated as A1, 15% as Baa1, 12% as Baa2, and 10% as A3. The average default rate is calculated by combining the distribution of bonds over categories and the default probability for each class. The average default rate of bonds turns out to be 1.56%. The median credit spread and the average default rate are informative about the bankruptcy cost ξ and the discount factor η. Lenders charge higher credit spreads if bankruptcy costs are higher. Moreover, when the discount factor is lower, firms default at a higher frequency, generating higher credit spreads.

We also target the elasticity of credit spread with respect to productivity. In the data, there is a negative relationship between credit spreads and firm-level TFP. As mentioned before, the bankruptcy cost ξ and the discount factor η affect the credit spread distribution and hence the elasticity with respect to productivity. Moreover, the elasticity depends on the equity payout cost parameter φ_d. The equity payout cost hinders an investment strategy whereby firms sell all their capital and subsequently distribute large payouts to shareholders shortly before defaulting. When we allowed such strategies to take place (by setting φ_d = 0), the

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18 In the model, the firm’s credit spread is r^* - r, where r is the risk-free rate and r^* = (θ + c)/q − θ is the yield-to-maturity on the promised sequence of payments.

19 More than half of our firms in the sample are rated by Moody’s. Since every maturity has a different default probability, we choose the probability of defaulting after 12 years. This is the average maturity we observe in our sample.
Table 7: Model Fit and Estimated Parameters

| Panel A: Moments                      | Model  | Data  |
|--------------------------------------|--------|-------|
| Autocorrelation of log sales         | 0.78   | 0.75  |
| Standard deviation of innovation to log sales | 0.48   | 0.50  |
| Mean capital/sales ratio             | 2.48   | 2.55  |
| Median credit spread                 | 1.22%  | 1.55% |
| Mean default rate                    | 1.63%  | 1.56% |
| Std dev. of investment rates         | 0.24   | 0.22  |
| Median leverage ratio                | 0.34   | 0.20  |
| Credit spread-TFP elasticity         | -0.53  | -0.33 |
| Autocorrelation of dividends/profits | 0.18   | 0.17  |

| Panel B: Parameters                  | Notation | Value |
|--------------------------------------|----------|-------|
| Productivity persistence             | \( \rho_z \) | 0.670 (0.006) |
| Std dev. of productivity innovation  | \( \sigma_{\epsilon z} \) | 0.210 (0.002) |
| Discount factor                      | \( \eta \) | 0.972 (0.001) |
| Bankruptcy cost                      | \( \xi \) | 0.100 (0.005) |
| Capital adjustment cost             | \( \phi_k \) | 0.045 (0.001) |
| Equity payout cost                  | \( \phi_d \) | 0.500 (0.018) |

Note: Panel A reports the targeted moments in the model and the data. Panel B reports the resulting parameters, estimated via Simulated Method of Moments (SMM). Standard errors are reported in parentheses.

The elasticity turned from negative to positive as it was the most productive firms that defaulted.

The standard deviation of investment rates is informative for the capital adjustment cost wedge. In particular, the standard deviation of investment rates is lower when \( \phi_k \) is higher. And finally, the autocorrelation of dividends/profits is informative for the equity cost parameter, \( \phi_d \). The equity cost parameter affects the incentives of firms to smooth dividends over time.

5.2 Estimation Results and Model Fit

Although we use six parameters, our model overall does a good job matching our nine empirical moments (see Panel A of Table 7). In particular, the model matches very closely the median credit spread, the standard deviation of investment rates, the mean assets/sales
Figure 3: Credit Spreads and Productivity: Model vs. Data

Note: This figure plots the relationship between credit spreads and firm-level TFP in the model and the data. The data source is Compustat and Thomson Reuters Bond Security Master Data. Each point in the scatter plot represents a bin of firms (total of 80 bins).

ratio, the average default ratio, and the autocorrelation of dividends/profits.

The model also matches well the negative cross-sectional relationship between credit spreads and productivity. This is shown in Figure 3. Highly productive firms are able to attract financing at a lower cost since (a) their incentive to default is lower and (b) the lenders can recover a highly valued firm in the case of default. The negative covariance between credit spreads and productivity turns out to be critical for our results regarding the effect of credit market constraints on misallocation (see Section 6.2).

We now evaluate our model’s fit with respect to untargeted moments (see Table 8). For example, in our estimation, we targeted the median credit spread but not other moments of the credit spread distribution. Figure 4 shows the distribution of credit spreads in the data (left panel) and the distribution generated in the model (right panel). Dispersion of credit spreads in the data is substantial. The bottom 10%, median, and top 10% pay less than 0.5, 1.5 and more than 5 percentage points, respectively. The model generates a dispersed distribution of credit spreads close to what we observe in the data. The standard deviation of credit spreads in the model is 0.9 percentage points versus 2.0 in the data.

Our model also replicates the positive relationship between leverage and credit spreads. In the model, highly leveraged firms have a larger incentive to default and so they are charged
Figure 4: Credit Spread Distribution: Data (Left Panel) and Model (Right Panel)

Note: Left panel shows the credit spread distribution in percentage points from Thomson Reuters Bond Security Master Data (authors’ calculations). The right panel shows the credit spread distribution from the simulated data.

Table 8: Untargeted Moments: Model vs. Data

| Moments                                    | Model  | Data  |
|--------------------------------------------|--------|-------|
| St. dev. of credit spreads                 | 0.92%  | 2.03% |
| Investment rate | $|i/k| |
| < 1% (Inaction)                             | 7.19%  | 6.97% |
| > 20% (Spike)                               | 34.3%  | 17.7% |
| Autocorrelation                             | 0.16   | 0.41  |
| 10th - 1st TFP decile                      | -4.44  | 0.03  |
| Assets/Sales                               | 0.44   | 1.93  |
| Investment/Capital                          | -0.20  | -0.29 |
| Leverage                                   | -2.02  | -0.64 |
| Elasticity of credit spreads wrt Leverage   | 0.63   | 0.26  |
| Mean dividends/profits                      | 0.61   | 0.21  |
| Std. dev. of dividends/profits              | 0.73   | 0.43  |

Note: This table reports untargeted moments from our benchmark model and in the data.

a higher credit spread. The leverage coefficient that comes out of the regression (see Equation (14)) when we use the model-generated data is 0.63. This is somewhat higher than the coefficient of 0.26 in the data.

In addition, the model matches well the relationship between firm-level TFP and leverage. The model reproduces qualitatively the relationship between investment rate and firm-level TFP, but investment rates are less responsive than in the data. Similarly, the model reproduces
qualitatively the relationship between the dividends/profits ratio and firm-level TFP, but in
this case, the model is more responsive than what the data suggest. The model is not successful
capturing the relationship between TFP and assets/sales ratio. In the data, the relationship
is non-monotonic, while in the model it is monotonically decreasing.

Table 8 also reports untargeted moments regarding the investment rates. The inaction
rate (investment rate in absolute terms less than 1%) is 7.1% which is close to the empirical
counterpart of 6.9%. However, the model over-predicts the fraction of firms undertaking a
large investment relative to their capital (34% versus 17% in the data). Moreover, the model
underpredicts the autocorrelation of investment rates (0.16 versus 0.41 in the data). Finally,
the model cannot match well untargeted statistics regarding dividend issuance. In the model,
firms issue three times more dividends (relative to their profits) than in the data. Moreover,
the dispersion in the model is also higher than in the data (0.73 versus 0.43, respectively).

6 Quantitative Results

In this section, we present our main findings. We show how credit market constraints
affect TFP directly, but also indirectly through their interaction with other frictions and
technological constraints.

6.1 Overall Effect of Credit Market Constraints

Table 9 analyzes the overall effect of credit market constraints on misallocation. We
report the results for our benchmark economy as well as an economy without credit market
constraints. In the economy without credit market constraints, we eliminate bankruptcy costs
and limited commitment.\footnote{We model commitment by requiring that firms buy back existing debt at the risk-free price before issuing
new debt or before dis-investing.} For each model, we compute various aggregate statistics, including
TFP loss. Note that for the case without credit market constraints, we compute the wage at
which the aggregate level of labor is the same as our benchmark economy.

When we add credit market constraints, the variance of the overall wedge increases and
TFP loss increases by 1.6%. Small TFP losses are a relatively common finding in the literature
analyzing credit market constraints. For example, Midrigan and Xu (2014) and Moll (2014)
find small effects of financial frictions due to persistent productivity shocks and self-financing.\footnote{Midrigan and Xu (2014) do find substantial losses from misallocation due to entry and exit of firms. We
do not consider the extensive margin in our model since our data includes only publicly listed firms and not
newly formed establishments.} Gilchrist, Sim, and Zakrajšek (2013) find that dispersion in credit spreads is not large enough
to generate a significant amount of misallocation. In our case, the effect of credit market
constraints on misallocation is small because they affect the wedge in multiple, and at times contradictory, ways. We illustrate this point in Section 6.2 by decomposing the overall effect into a direct and an indirect effect.

Table 9: Overall Effect of Credit Market Constraints

|                     | Model w/o CMC | Benchmark Model |
|---------------------|---------------|-----------------|
| Wage                | 1.12          | 1.00            |
| N                   | 0.40          | 0.40            |
| K/N                 | 4.32          | 3.10            |
| Y/N                 | 2.03          | 1.81            |
| Median EMPK         | 0.12          | 0.15            |
| TFP loss            | 2.6%          | 4.2%            |

Note: The model without credit market constraints (CMC) eliminates bankruptcy costs and limited commitment. TFP loss is computed according to Equation (13).

In addition, when we add credit market constraints, the capital-labor ratio decreases by 28%, and the median EMPK increases by about 0.03. Therefore, while credit market constraints have a modest effect on misallocation and the dispersion of EMPK, they have a large effect on the average level of EMPK and aggregate capital. To the extent that all firms are affected by credit market constraints, this shows up in the level of EMPK. To the extent that firms are affected in different ways (e.g., low- versus high-productivity firms), this shows up in the dispersion of EMPK.

6.2 Decomposing the Transmission of Credit Market Constraints

We have shown that the overall effect of credit market constraints is to generate more misallocation. However, the overall effect is the combination of the direct effect, which originates from cross-sectional variation in credit spreads, and the indirect effect between credit market constraints and other frictions and technological constraints.

When we move from a model with no credit market constraints to our benchmark economy, aggregate TFP decreases because the variance of the overall wedge increases. As discussed in Section 3.2, the overall wedge can be decomposed into four individual wedges: (1) the credit market constraints wedge ($Wedge_{CMC}$), (2) the capital adjustment cost wedge ($Wedge_{CAC}$), (3) the equity payout cost wedge ($Wedge_{EPC}$), and (4) the tax wedge ($Wedge_{T}$). To distinguish between the direct and indirect effects of credit market constraints, we investigate how the introduction of credit market constraints affects the variance of the overall wedge through each individual wedge. Specifically, for both the model with and without credit market constraints, we compute the TFP loss attributable to feature $i \in \{CMC, CAC, EPC, T\}$
Table 10: The Direct and Indirect Effects of Credit Market Constraints on TFP

| Transmission Mechanism | Model w/o CMC | Benchmark Model | Change |
|------------------------|---------------|-----------------|--------|
| Credit Market Const.   | Direct Effect | 0.0%            | -0.4%  | -0.4% |
| Capital Adj. Costs     | Indirect Effect | 1.5%           | 0.9%   | -0.6% |
| Equity Payout Costs    | Indirect Effect | 0.4%           | 2.4%   | 2.0%  |
| Taxes                  | Indirect Effect | 0.7%           | 1.3%   | 0.6%  |
| All                    | Direct + Indirect Effect | 2.6%         | 4.2%   | 1.6%  |

Note: To decompose total TFP loss into direct and indirect effects, we decompose the wedge into four individual wedges that separately isolate the effect of credit market constraints, capital adjustment costs, equity payout costs, and taxes. Then we construct the marginal contribution of each wedge to the overall TFP loss by decomposing the variance of the overall wedge using Equation (15).

as follows:

\[
TFP \text{ loss}_i = TFP \text{ loss} \left( \frac{\text{Var}(\text{Wedge}_i) + \sum_{j \neq i} \text{Cov}(\text{Wedge}_i, \text{Wedge}_j)}{\text{Var} \left( \sum_j \text{Wedge}_j \right)} \right). \tag{15}
\]

With this approach, the marginal contribution of feature \( i \) to TFP loss depends not only on the variance of \( \text{Wedge}_i \), but also on the covariance of \( \text{Wedge}_i \) with the other wedges. \( \text{Wedge}_i \) has a larger effect on the overall variance when it covaries positively with the other wedges.

The direct effect of credit market constraints is the \textit{change} in TFP loss\(_{CMC}\) when we move from a model without credit market constraints to a model with credit market constraints. Similarly, the indirect effect is the change in TFP loss\(_i\) for \( i \in \{CAC, EPC, T\} \) between the model with and without credit market constraints. For example, for the case of capital adjustment costs, the indirect effect of credit market constraints is captured by the change in TFP loss\(_{CAC}\).

In Table 10, we report TFP loss for each wedge and for each model (with and without credit market constraints). TFP loss due to the credit market constraints wedge (\( \text{Wedge}_{CMC} \)) is naturally zero. When we add credit market constraints, TFP loss due to the credit market constraints wedge decreases by 0.4%. Therefore, our first finding is that the direct effect of credit market constraints reduces misallocation. TFP loss due to all the other features in the economy increases by a total of 2.0%. Therefore, the overall increase in TFP loss by 1.6% arises due to the indirect effects. In the following, we explain in more detail what is driving the direct and indirect effects of credit market constraints.

**Direct Effect of Credit Market Constraints**

When we introduce credit market con-
straints, TFP loss\textsubscript{\textit{CMC}} decreases. Looking at Equation (15), we can see that TFP loss\textsubscript{\textit{CMC}} depends on \text{Var}(\text{Wedge}_{\text{CMC}}) and \textstyle \sum_{j \neq \text{CMC}} \text{Cov}(\text{Wedge}_{\text{CMC}}, \text{Wedge}_j). As we highlighted in Figure 4, credit market constraints generate dispersion in credit spreads and this increases \text{Var}(\text{Wedge}_{\text{CMC}}) and thus generates misallocation. If our model had a single distortion, then the direct effect would generate TFP loss. This is exactly what Gilchrist, Sim, and Zakrajšek (2013) find since they omit any other frictions or technological constraints from the analysis. However, in our model, TFP loss\textsubscript{\textit{CMC}} also depends on the covariance between the credit market constraints wedge with the other individual wedges (\textstyle \sum_{j \neq \text{CMC}} \text{Cov}(\text{Wedge}_{\text{CMC}}, \text{Wedge}_j)). The direct effect can decrease TFP loss\textsubscript{\textit{CMC}} as long as the sum of the covariances between the individual wedges is negative and also larger in magnitude than the variance of \text{Wedge}_{\text{CMC}}.

Figure 5 shows the covariance between each wedge and productivity. The middle left panel of Figure 5 shows that when we eliminate credit market constraints, there is no dispersion in the credit market constraints wedge across firms (\text{Wedge}_{\text{CMC}}). In this economy, there is no variation in credit spreads and the probability of default is zero for all firms. When we add credit market constraints, as mentioned above, \text{Var}(\text{Wedge}_{\text{CMC}}) increases due to the higher dispersion in credit spreads and default probabilities.

However, \text{Cov}(\text{Wedge}_{\text{CMC}}, \ln z) turns negative. The negative covariance between the credit market constraints wedge and productivity stems from the negative relationship between credit spreads and productivity. In the model, more productive firms can borrow at a lower cost for two reasons. First, due to higher productivity, the firm has less incentive to default and lose part of its future value. As a result, lenders are willing to supply more funds to the firm at a low cost. Second, even if default takes place, the high recovery value of a high-productivity firm allows lenders to charge a smaller credit spread. As shown in Section 5.2, our model replicates the empirical relationship between credit spreads and productivity very closely.

Although \text{Cov}(\text{Wedge}_{\text{CMC}}, \ln z) turns negative, the covariance of all other wedges with productivity turns positive (middle right panel and lower panels of Figure 5). As a result, \textstyle \sum_{j \neq \text{CMC}} \text{Cov}(\text{Wedge}_{\text{CMC}}, \text{Wedge}_j) < 0, which explains why the direct effect reduces misallocation. Credit market constraints affect mostly low-productivity firms. Other frictions or technological constraints affect mostly high-productivity firms. Therefore, credit market constraints reduce misallocation directly through credit spreads, because they offset the productivity losses induced by the existing model environment.

Our results indicate that variation in credit spreads might be desirable as long as it affects mostly low-productivity firms. While such variation increases the dispersion in the credit market constraints wedge, it also decreases the covariance between the credit market constraints wedge and the other wedges. Therefore, any policy that targets misallocation by affecting the distribution of credit spreads needs to take into account not only the dispersion of credit
Figure 5: Effect of Credit Market Constraints on Individual Wedges

Note: Bottom four panels show a scatter plot of four individual wedges (credit market constraints, capital adjustment costs, equity payout costs, and taxes) and productivity. We plot the wedges for our benchmark model with credit market constraints and a model without credit market constraints. Top panel shows the scatter plot between the overall wedge (all frictions and technological constraints) and productivity.
spreads but also which type of firms have lower borrowing costs.

**Indirect Effect of Credit Market Constraints** Since the overall effect of credit market constraints is to decrease aggregate TFP, the main transmission mechanism occurs through the indirect effect. However, the individual frictions and technological constraints respond very differently when we eliminate credit market constraints.

When we add credit market constraints, TFP loss due to capital adjustment costs decreases (Table 10). This is because dispersion in the capital adjustment cost wedge decreases (see the middle-right panel of Figure 5). To gain intuition for this result, we repeat the expression for the capital adjustment cost wedge (see Equation (9)):

\[
Wedge_{CAC} = \frac{1}{\beta} + \frac{2\phi_k}{\beta} \left( \frac{k' - (1 - \delta)k}{k} \right) - 2\phi_k E \left[ \frac{1}{2} \left( \frac{k'' - (1 - \delta)k'}{k'} \right)^2 + (1 - \delta) \left( \frac{k'' - (1 - \delta)k'}{k'} \right) \right].
\]

The variance of this wedge is largest when the dispersion in investment rates is high and when investment rates across periods are volatile. When we introduce credit market constraints, dispersion and volatility of investment rates decrease and thus aggregate productivity losses associated with capital adjustment costs decrease as well.

Similar to our exercise, Khan and Thomas (2013) jointly model credit market constraints and capital adjustment costs. In their paper, capital adjustment costs interact with collateral constraints, propagate shocks in the economy, and decrease aggregate TFP. In contrast, we find that credit market constraints mitigate the productivity losses induced by capital adjustment costs. In Khan and Thomas (2013), the financial shock generates misallocation by preventing young firms from quickly reaching their optimal scale. In our model, we abstract from firm entry and exit due to the nature of our data (large, publicly listed firms).

While credit market constraints decrease TFP loss directly and indirectly through capital adjustment costs, they increase TFP loss through equity payout costs and taxes. Quantitatively, the strongest indirect effect comes from the equity payout cost wedge. To understand the mechanism, we repeat the definition of the equity cost wedge (Equation 10):

\[
Wedge_{EPC} \equiv \frac{1}{\beta} + E \left[ \Lambda'(d') \left( \frac{\partial \pi'}{\partial k'} + 1 - \delta \right) \right] - \frac{\Lambda'(d)}{\beta}.
\]

Equity payout costs create an incentive for firms to smooth dividend issuance across time. Notice that the wedge is largest when dividend issuance today is low relative to tomorrow. Credit market constraints affect the incentives of firms to smooth dividends. Consider the
firm’s budget constraint (see Equation (2)), which we repeat here:

\[ d + k' = e(z, k; b) + q(z, k', b') [b' - (1 - \theta)b] - g(k, k'). \]

A firm with low equity \( e \) is more likely to be borrowing constrained. Such a firm, in order to finance capital \( k' \), would like to reduce dividends \( d \) today (maybe even making \( d \) negative) and increase future dividends \( d' \). Without credit market constraints, this firm would be free to use debt to smooth dividend issuance. With credit market constraints, the ability of the firm to issue debt is limited and misallocation arises because of the presence of equity payout costs.

Our results suggest that an effective way to mitigate credit market constraints is to design policies that target equity payout costs. For example, by making it easier for firms to adjust dividends, misallocation from credit market constraints may be reduced. We do not explicitly evaluate such policies, but our exercise highlights the mechanisms that are relevant for policy design.

7 Inspecting the Mechanism

In this section, we explore which model element is responsible for our findings. First, we shut down endogenous default and consider an economy with an exogenous collateral constraint. Second, we discuss the role of long-term financing. Third, we consider the role of productivity persistence for TFP loss.

7.1 The Role of Endogenous Default

In our benchmark model, firms are endogenously constrained due to limited commitment. We now study a version of our model with an exogenous collateral constraint. Instead of assuming long-duration bonds with limited commitment, we assume firms issue one-period bonds subject to the following constraint:

\[ b' \leq \psi k'. \]  \hspace{1cm} (16)

As there is no default in this version of the model, firms are able to issue bonds at the price \((1 + c)/(1 + r)\). Parameter \( \psi \) governs the magnitude of borrowing constraints. We set \( \psi = 0.34 \) to match the leverage ratio in our benchmark model. All other parameters are the same as in our benchmark parameterization.\(^{22}\) We compare this economy to one in which the

\(^{22}\)One exception is that there are no bankruptcy costs in our exogenous collateral constraint model (as
Table 11: Credit Market Constraints and TFP: Exogenous Collateral Constraint

|                          | Transmission Mechanism | Model w/o CMC | Benchmark Model | Change |
|--------------------------|------------------------|---------------|-----------------|--------|
| Credit Market Const.     | Direct Effect          | 0.4%          | -0.7%           | -1.1%  |
| Capital Adj. Costs       | Indirect Effect        | 1.5%          | 1.0%            | -0.5%  |
| Equity Payout Costs      | Indirect Effect        | 0.1%          | 2.5%            | 2.4%   |
| Taxes                    | Indirect Effect        | 0.7%          | 1.3%            | 0.6%   |
| All                      | Direct + Indirect Effect | 2.7%      | 4.1%            | 1.4%   |

Note: TFP loss decomposition for the model with an exogenous collateral constraint. To decompose total TFP loss into direct and indirect effects, we decompose the wedge into four individual wedges that separately isolate the effect of credit market constraints, capital adjustment costs, equity payout costs, and taxes. Then we construct the marginal contribution of each wedge to the overall TFP loss by decomposing the variance of the overall wedge using Equation (15).

The exogenous collateral constraint is relaxed to $\psi = 1$.23

Table 11 reports the result of this exercise. The introduction of an exogenous credit market friction increases overall TFP loss by 1.4%. In our benchmark economy, the effect of credit market constraints on TFP loss is 1.6%. Therefore, the exogenous collateral constraint has a similar effect on misallocation as an endogenous borrowing constraint.

As in the benchmark economy, we decompose the overall wedge into individual wedges. The only difference is that the credit market constraints wedge is now given by

$$
Wedge_{CMC} = \frac{1}{\beta} (1 - \psi \mu),
$$

(17)

where $\mu$ is the Lagrange multiplier on the collateral constraint. Because an increase in capital $k'$ relaxes the borrowing constraint, the direct effect of a more binding collateral constraint is to reduce the (exogenous) credit market constraints wedge.24 The direct effect of credit market constraints reduces misallocation as in the benchmark model. Moreover, overall, credit market constraints generate more misallocation, which implies that the transmission mechanism occurs through the indirect effect.

Therefore, there is no qualitative difference between our model with endogenous default and a model with an exogenous collateral constraint. Endogenous default is not necessary.

23 When $\eta < 1$, a value of $\psi = 1$ guarantees that $E[MPK] = r + \delta$ when there are no other distortions. We get similar results when we set $\eta = 1$ and $\psi$ to a very large number.

24 In our model, if $g(k, k') = 0$ and $d = 0$, then the budget constraint is $k' = e + b'$ (using $(1+c)/(1+r) = 1$). Using the firm’s budget constraint, we can re-write the exogenous borrowing constraint ($b' \leq \psi k'$) as $k' \leq e/(1-\psi)$. In this case, an increase in $k'$ pushes the firm closer to the constraint. However, this transformation depends crucially on the restriction that $d = 0$. 

for our results. But then what is the added value of incorporating an endogenous borrowing constraint? With exogenous collateral constraints, the underlying distortion manifests itself through a shadow price (i.e., Lagrange multiplier), which has no direct empirical counterpart. In contrast, in our benchmark model with endogenous borrowing constraints, the distortion shows up through credit spreads that can be externally validated in the data. This allows us to verify empirically the covariance between the credit market constraint wedge and productivity. In both models, the covariance is negative. However, with endogenous default, this covariance takes the form of a relationship between credit spreads and firm productivity that we can empirically verify.

7.2 The Role of Long-Term Financing

In this section, we analyze how long-duration bonds affect our results. In particular, we consider an economy with endogenous default but where bonds mature after one year ($\theta = 1$). As mentioned in Section 2.7, long-term financing allows us to generate a realistic distribution of credit spreads. What matters with long-term financing is the whole sequence of default probabilities until the bond matures. As a result, lenders charge a premium based on the long-run probability of default.

With one-period bonds, what matters for firms is the one-period default probability. It turns out that with one-period bonds, larger issuances increase the credit spreads very steeply. As a result, all firms prefer to pay the risk-free rate. In contrast, with long-term financing, credit spreads increase more gradually since they reflect the whole path of future default probabilities, not just next period’s default probability. Figure 6 (right panel) shows that the credit spread distribution is degenerate at the risk-free rate when $\theta = 1$. In contrast, in our benchmark model ($\theta = 0.085$), the credit spread distribution has sizable dispersion, closer to what we observe in the data.

In Table 12, we compare a model without credit market constraints to a model with credit market constraints. Since with a one-period bond there is no effect on credit spreads, the direct and the indirect effects of credit market constraints are zero. Therefore, long-term financing is necessary for our findings.

\footnote{The shape of bond-price schedules with long-term financing was first analyzed by Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) in the context of a sovereign default model without capital. Our paper uses the insights developed in these papers to analyze how long-term financing affects misallocation of capital.}
Figure 6: Credit Spread Distribution: Benchmark and One-Period Bond Model

Note: The left panel shows the credit spread distribution in the benchmark model ($\theta = 0.085$), and the right panel shows credit spread distribution in a one-period bond model ($\theta = 1$). Credit spreads are reported in percentage points.

Table 12: Credit Market Constraints and TFP: One-Period Bond Model

| TFP loss | Transmission Mechanism | Model w/o Benchmark CMC | Model Benchmark | Change |
|----------|------------------------|-------------------------|-----------------|--------|
| Credit Market Const. | Direct Effect | 0.0% | 0.0% | 0.0% |
| Capital Adj. Costs | Indirect Effect | 1.3% | 1.3% | 0.0% |
| Equity Payout Costs | Indirect Effect | 0.7% | 0.7% | 0.0% |
| Taxes | Indirect Effect | 0.6% | 0.6% | 0.0% |
| All | Direct + Indirect Effect | 2.6% | 2.6% | 0.0% |

Note: TFP loss decomposition for the model with a one-period bond. To decompose total TFP loss into direct and indirect effects, we decompose the wedge into four individual wedges that separately isolate the effect of credit market constraints, capital adjustment costs, equity payout costs, and taxes. Then we construct the marginal contribution of each wedge to the overall TFP loss by decomposing the variance of the overall wedge using Equation (15).
Note: The left panel plots the relationship between the persistence of productivity ($\rho_z$) and TFP loss. We show the relationship in a model without credit market constraints and our benchmark model with credit market constraints. The right panel shows the effect of credit market constraints on TFP loss. In each case, the standard deviation of the innovation to productivity ($\sigma_{e_z}$) is adjusted so that the unconditional variance ($\sigma_{e_z}^2/(1-\rho_z^2)$) is the same as our benchmark economy. All other parameters are the same.

7.3 The Role of Persistence

A standard result in the literature is that steady-state productivity losses due to financial frictions are smaller when productivity is more persistent (Buera and Shin, 2011; Moll, 2014; Midrigan and Xu, 2014). With a higher persistence, high-productivity firms accumulate internal funds quickly and thus they can reach their optimal scale faster. We now evaluate how the persistence in productivity affects misallocation by varying the parameter, $\rho_z$, in our benchmark economy. For each value of $\rho_z$, we adjust the conditional variance $\sigma_{e_z}^2$ so that the unconditional variance $\sigma_{e_z}^2/(1-\rho_z^2)$ remains the same as in our benchmark parameterization. All other parameters are held fixed at their benchmark values.

Figure 7 shows the relationship between productivity persistence and TFP loss in our model. The left panel plots TFP loss in our benchmark economy and in our model without credit market constraints. The right panel plots the difference in TFP loss between the two economies. The effect of persistence in our model is non-monotonic. Misallocation increases up to $\rho_z = 0.7$ and then decreases. The decreasing part of the schedule is based on the standard intuition provided in the literature. With higher persistence, high-productivity firms accumulate much more internal equity and can reach their optimal scale through self-financing.

However, we find that for $\rho_z < 0.7$, persistence results in a smaller effect of credit market constraints on misallocation. The reason is that we have a dynamic investment decision that is different from the aforementioned literature. In particular, we assume firms make investment decisions one period in advance, before they learn tomorrow’s productivity. In contrast, the
literature often models a static problem whereby firms choose the current level of capital given current productivity. In our economy, when shocks are i.i.d. ($\rho_z = 0$), all firms choose the same capital stock $k'$. As a result, there is no dispersion in the expected marginal product of capital and there is no misallocation. This explains the non-monotonic relationship. If the model was static, and firms rented capital based on their current productivity, misallocation would be positive even with i.i.d. shocks.

In sum, our model confirms the intuition mentioned in the literature that if shocks become very persistent, misallocation would be eliminated. But at the same time, our model highlights a different channel that is based on the timing of investment and the realization of productivity.

8 Conclusion

Our paper evaluates the contribution of credit market constraints to misallocation in a model with endogenous borrowing constraints, long-duration bonds, capital adjustment costs, costly equity payouts, and taxes. Due to long-duration bonds, the model generates a realistic distribution of credit spreads. We distinguish between the direct and indirect effects of credit market constraints on misallocation of capital. The direct effect captures the effect of credit market constraints on the cross-sectional variation in credit spreads. The indirect effect corresponds to how credit market constraints propagate or mitigate the effect of other frictions or technological constraints on aggregate productivity.

The direct effect of credit market constraints increases aggregate TFP by 0.4%. This happens because more productive firms face lower credit spreads, a relationship we confirm in the data. In contrast, other frictions and technological constraints collectively affect high-productivity firms. As a result, credit spread variation offsets the productivity losses induced by the existing model environment. We find that the overall effect of credit market constraints is to decrease aggregate TFP by 1.6%, which means that the main transmission mechanism occurs through the indirect effect.

Our exercise provides guidance for policymakers regarding the appropriate mix of policies to reduce misallocation. First, policies that directly target credit spread variation, without taking into account which firms have lower borrowing costs, may actually be ineffective at reducing misallocation. Second, the bulk of misallocation occurs through the indirect effect of credit market constraints. As a result, an effective way to reduce misallocation is to design policies that target other frictions or technological constraints. One example is to make it easier for firms to adjust dividends. Our paper does not explicitly evaluate such policies, but it does highlight the transmission mechanisms that are relevant for policy design. We leave a complete quantitative evaluation of such policies for future research.
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A Appendix

A.1 Estimation of TFP

We estimate TFP of firm $i$ at time $t$ using the following specification

$$ y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + z_{it} + \varepsilon_{it} \quad (A.1) $$

To deal with issues of endogeneity and selection we employ the approach of Olley and Pakes (1996). Their method relies on the following assumptions.

1. Labor $l_{it}$ is chosen at time $t$ based on a static decision problem.
2. Capital at $t+1$ depends on investment at period $t$: $k_{it+1} = (1 - \delta)k_{it} + i_t$.
3. Firm $i$ makes the investment decision at time $t$ based on the observed productivity $z_{it}$ and the current stock of capital $k_{it}$. Hence the investment policy rule is given by $i_{it} = f(z_{it}, k_{it})$.
4. $f$ is strictly monotonic in $z_{it}$ so that the policy rule can be inverted to obtain $z_{it} = f^{-1}(i_{it}, k_{it})$.

We assume that $f^{-1}$ is a third order polynomial in $i_{it}$ and $k_{it}$ and their interaction $i_{it}k_{it}$. Substituting into Equation (A.1) we get

$$ y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \gamma_1 i_{it} + \gamma_2 i_{it}^2 + \gamma_3 i_{it}^3 + \gamma_4 k_{it} + \gamma_5 k_{it}^2 + \gamma_6 k_{it}^3 + \gamma_7 i_{it} k_{it} + \gamma_8 i_{it}^2 k_{it} + \gamma_9 i_{it}^3 k_{it} + \varepsilon_{it} \quad (A.2) $$

From this regression we cannot separately identify $\beta_k$ from the $\gamma$’s. But given that we have eliminated the unobserved component we can consistently estimate coefficient $\beta_l$ as well as $\hat{\Phi}_{it}$ defined as

$$ \hat{\Phi}_{it} = \hat{\beta}_0 + \hat{\beta}_k k_{it} + \hat{\gamma}_1 i_{it} + \hat{\gamma}_2 i_{it}^2 + \hat{\gamma}_3 i_{it}^3 + \hat{\gamma}_4 k_{it} + \hat{\gamma}_5 k_{it}^2 + \hat{\gamma}_6 k_{it}^3 + \hat{\gamma}_7 i_{it} k_{it} + \hat{\gamma}_8 i_{it}^2 k_{it} + \hat{\gamma}_9 i_{it}^3 k_{it} $$

Given our estimates we can take the expectation of Equation (A.1)

$$ E[y_{it} - \beta_l l_{it} \mid \chi_{it} = 1] = \beta_0 + \beta_k k_{it} + E[z_{it} \mid z_{i,t-1} \chi_{it} = 1] $$

where $\chi_{it}$ is an indicator that the firm survived. Let $E[z_{it} \mid z_{i,t-1} \chi_{it} = 1]$ be a function of lagged productivity and the survival probability: $g(z_{t-1}, \hat{P}_t)$ where $\hat{P}_t$ is the probability of
Table 13: Robustness

|                      | Benchmark | Merged Compustat/TR | Industry-Level Deflators |
|----------------------|-----------|---------------------|--------------------------|
|                      | # Obs.    | Mean                | # Obs.                   | Mean                | # Obs.    | Mean                |
| Leverage             | 133,786   | 0.29                | 16,526                   | 0.31                | 55,580    | 0.26                |
| Assets/Sales         | 129,730   | 2.55                | 16,504                   | 1.39                | 54,100    | 2.98                |
| Investment rate      | 119,991   | 0.16                | 15,641                   | 0.11                | 50,789    | 0.16                |
| Inaction rate (| < 1%|) | 119,991   | 0.06                | 15,641                   | 0.02                | 50,789    | 0.06                |
| Spike rate (> 20%)   | 119,991   | 0.21                | 15,641                   | 0.12                | 50,789    | 0.20                |
| Bond amount ($million) | 18,369   | 365.5               | 4,326                    | 421.0               | 18,369    | 365.5               |
| Maturity (years)     | 18,369    | 11.6                | 4,326                    | 11.0                | 18,369    | 11.6                |
| Mean credit spread (p.p.) | 18,369   | 2.32                | 4,326                    | 2.12                | 18,369    | 2.32                |
| St. dev. of credit spread (p.p.) | 18,369   | 2.16                | 4,326                    | 2.05                | 18,369    | 2.16                |
| Elasticity of credit spreads wrt TFP | 4,129     | -0.06****           | 4,129                    | -0.06***            | 1,399     | -0.42****           |
|                      | W/o controls, no FE | 4,129 | -0.37***           | 4,129                    | -0.37***            | 1,399     | -0.60***           |
|                      | W/o controls, FE   | 4,127     | -0.01              | 4,127                    | -0.01              | 1,399     | -0.40***           |
|                      | W/ controls, no FE | 4,127     | -0.33***           | 4,127                    | -0.33***            | 1,399     | -0.54***           |

Note: This table reports a collection of statistics for three specifications: “Benchmark,” bond issuing firms in Compustat (denoted “Merged Compustat/TR”) and a specification using industry-level deflators. One, two, and three stars denote significance at the 10%, 5%, and 1%, respectively.

survival. Olley and Pakes (1996) suggest modeling the first using $\Phi_{it-1} - \beta_k k_{it-1}$ and for the second using the predicted probability from a probit on survival indicator on a polynomial including capital and investment. As a result we run the following regression

$$y_{it} - \hat{\beta}_t i_{it} = \beta_0 + \beta_k k_{it} + \delta_1[\Phi_{it-1} - \beta_k k_{it-1}] + \delta_2[\Phi_{it-1} - \beta_k k_{it-1}]^2$$

We estimate Equation (A.1) using a non-linear OLS estimator and derive estimates for $\beta_k$.

### A.2 Robustness

We explore how our main findings vary for different sample or for different specifications. In Table 13 we report the benchmark values which is a combination of statistics reported in Tables 3 and 5. Column “Merged Compustat/TR” considers firms that appear in both samples (i.e., Compustat firms that have issued at least one bond). Naturally, the number of observations drops significantly. The most notable discrepancy occurs for the assets-to-sales ratio which drops from 2.55 to 1.39.

However, statistics associated with bond issuance remain largely intact (with the exception
of average bond issuance). In particular, bonds for both publicly listed and non-publicly listed firms have similar average maturity and average credit spread. Note that to compute the elasticity of credit spreads to TFP we used the merged sample so by construction the first and the second column are identical.

Finally, we check if the elasticity of credit spreads changes significantly when we use industry-specific deflators to deflate capital. The industry-specific deflators available by the NBER-CES database and are available at the 6-digit NAICS level but only for a subset of industries which reduces substantially the number of our observations. There are two differences relative to our Benchmark estimates. First, even without fixed effects the elasticity of credit spreads with respect to productivity is negative and statistically significant. Second, the estimates with fixed effects also increase to a range closer to what our model predicts.

A.3 TFP Loss

To compute TFP losses, we take as given the aggregate stock of capital $K = \sum_i k'_i$ and labor $N = \sum_i E[n'_i(z'_i)|z_i]$ from the original economy and solve the following problem:

$$\max_{k'_i, n'_i(z'_i)} \sum_i E \left[ z'_i(k'_i)^\alpha (n'_i(z'_i))^{(1-\alpha)\gamma} | z_i \right]$$

subject to

$$\sum_i k'_i = K$$
$$\sum_i E[n'_i(z'_i)|z_i] = N$$

We are using the fact that aggregate capital and labor are constant in a stationary steady state (i.e., $K' = K$ and $N' = N$). The solution to this problem takes into account the fact that firms choose capital one period in advance, before they learn tomorrow’s productivity. To maximize total expected output, we choose $k'_i$ for each firm $i$ and a rule for tomorrow’s labor $n'_i(z'_i)$ as a function of tomorrow’s realized productivity, subject to the constraint that the total capital stock tomorrow is $K$ and the total labor stock tomorrow is $N$.

Let $\hat{\alpha} = \alpha \gamma$ and $\hat{\beta} = (1-\alpha) \gamma$. The Lagrangian for this problem is given by:

$$\mathcal{L} = \sum_i \sum_{z'_i} \pi(z'_i|z_i) \left[ z'_i(k'_i)^{\hat{\alpha}} (n'_i(z'_i))^{\hat{\beta}} \right] + \lambda_k \left[ K - \sum_i k'_i \right] + \lambda_n \left[ N - \sum_i \sum_{z'_i} \pi(z'_i|z_i)n'_i(z'_i) \right]$$

where $\pi(z'_i|z_i)$ is the probability of transitioning to $z'_i$, given that current productivity is $z_i$. 

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The first order conditions are then

\[ \lambda_k = E \left[ \hat{\alpha} z_i'(k_i')^{\hat{\alpha}-1} n_i'(z_i')^{\hat{\beta}} \Big| z_i \right] \] (A.3)

\[ \lambda_n = \hat{\beta} z_i'(k_i')^{\hat{\alpha}} n_i'(z_i')^{\hat{\beta}-1} \] (A.4)

where \( \lambda_k \) and \( \lambda_n \) are the Lagrange multipliers. The solution to this problem requires that the expected marginal product of capital (EMPK) and the marginal product of labor (MPL) are both equated across firms.

Solving Equations A.3 and A.4 for \( k_i' \) and \( n_i'(z_i') \), we obtain:

\[ k_i' = \left( \frac{\hat{\alpha}}{\lambda_k} \right)^{\frac{1-\hat{\beta}}{1-\gamma}} \left( \frac{\hat{\beta}}{\lambda_n} \right)^{\frac{\hat{\beta}}{1-\gamma}} E \left[ (z_i')^{1/(1-\hat{\beta})} \Big| z_i \right]^{\frac{1-\hat{\beta}}{1-\gamma}} \] (A.5)

\[ n_i'(z_i') = \left( \frac{\hat{\alpha}}{\lambda_k} \right)^{\frac{1-\hat{\alpha}}{1-\gamma}} \left( \frac{\hat{\beta}}{\lambda_n} \right)^{\frac{1-\hat{\alpha}}{1-\gamma}} (z_i')^{1/(1-\hat{\beta})} E \left[ (z_i')^{1/(1-\hat{\beta})} \Big| z_i \right]^{\frac{\hat{\alpha}}{1-\gamma}} \] (A.6)

Take expectations to get the expected value of \( n_i'(z_i') \) tomorrow:

\[ E \left[ n_i'(z_i') | z_i \right] = \left( \frac{\hat{\alpha}}{\lambda_k} \right)^{\frac{1}{1-\gamma}} \left( \frac{\hat{\beta}}{\lambda_n} \right)^{\frac{\hat{\beta}}{1-\gamma}} E \left[ (z_i')^{1/(1-\hat{\beta})} \Big| z_i \right]^{\frac{1-\hat{\beta}}{1-\gamma}} \] (A.7)

Substituting Equations A.5 and A.7 into \( \sum_i k_i' = K \) and \( \sum_i E[n_i'(z_i')] = N \), respectively, we obtain

\[ \left( \frac{\hat{\alpha}}{\lambda_k} \right)^{\frac{1-\hat{\beta}}{1-\gamma}} \left( \frac{\hat{\beta}}{\lambda_n} \right)^{\frac{\hat{\beta}}{1-\gamma}} \Gamma = K \] (A.8)

\[ \left( \frac{\hat{\alpha}}{\lambda_k} \right)^{\frac{1-\hat{\alpha}}{1-\gamma}} \left( \frac{\hat{\beta}}{\lambda_n} \right)^{\frac{\hat{\beta}}{1-\gamma}} \Gamma = N \] (A.9)

where

\[ \Gamma \equiv \sum_i E \left[ (z_i')^{1/(1-\hat{\beta})} \Big| z_i \right]^{\frac{1-\hat{\beta}}{1-\gamma}} \]

Solving Equations A.8 and A.9 for \( \lambda_k \) and \( \lambda_n \), we obtain

\[ \lambda_k = \hat{\alpha} K^{\hat{\alpha}-1} N^{\hat{\beta}} \Gamma^{1-\gamma} \] (A.10)

\[ \lambda_n = \hat{\beta} K^{\hat{\alpha}} N^{\hat{\beta}-1} \Gamma^{1-\gamma} \] (A.11)
Substituting for \( \lambda_k \) and \( \lambda_n \) in Equation (A.5), we obtain

\[
k'_i = \left( \frac{K'}{\Gamma} \right) E \left[ \left( z'_i \right)^{1/(1-\beta)} \bigg| z_i \right]^{\frac{1-\beta}{1-\gamma}} \quad (A.12)
\]

This is the capital choice which maximizes output. Similarly, we can use Equation (A.9) to substitute for \( \lambda_k \) and \( \lambda_n \) in Equation (A.6). We then obtain an expression for labor \( n'_i(z'_i) \):

\[
n'_i(z'_i) = \frac{N}{\Gamma} \left( z'_i \right)^{1/(1-\beta)} E \left[ \left( z'_i \right)^{1/(1-\beta)} \bigg| z_i \right]^{\frac{1-\beta}{1-\gamma}} \quad (A.13)
\]

This is the rule for labor tomorrow which maximizes output.

Given the new level of capital \( k'_i \) in Equation (A.12) and the rule for labor tomorrow \( n'_i(z'_i) \) in Equation (A.13), we can then compute TFP losses as the percentage difference between output (with inputs allocated according to Equations A.12 and A.13) and the original level of output.

We now derive an expression for TFP. In our benchmark economy, the optimal labor, \( n'_i(z'_i) \), and capital policies, \( k'_i \), for firm \( i \) are chosen to satisfy the following first order conditions:

\[
E \left[ \hat{\alpha} z'_i (k'_i)^{\hat{\alpha}-1} (n'_i)^{\hat{\beta}} \right] = \bar{r} w_i^k \quad (A.14)
\]

\[
\hat{\beta} z'_i (k'_i)^{\hat{\beta}-1} = \bar{w} \quad (A.15)
\]

where \( \bar{w} \) is the average MPL and \( \bar{r} \) is the average EMPK across all firms. We impose that the MPL will be equated across all firms, which is true in our benchmark economy. However, the EMPK, in general, will not be equated across firms. This is represented by the firm-specific wedge, \( w_i^k \), which is now the gross deviation of the firm’s EMPK from the economy-wide average. When the EMPK is equated across all firms, \( w_i^k = 1 \) for all \( i \).

Equations A.14 and A.15 imply the following optimal demands for capital and labor:

\[
k'_i = \left( \frac{\hat{\alpha}}{\bar{r} w_i^k} \right)^{\frac{1-\beta}{1-\gamma}} \left( \frac{\hat{\beta}}{\bar{w}} \right)^{\frac{1-\gamma}{1-\gamma}} E \left[ \left( z'_i \right)^{1/(1-\beta)} \bigg| z_i \right]^{\frac{1-\beta}{1-\gamma}} \quad (A.16)
\]

\[
n'_i(z'_i) = \left( \frac{\hat{\alpha}}{\bar{r} w_i^k} \right)^{\frac{1-\alpha}{1-\gamma}} \left( \frac{\hat{\beta}}{\bar{w}} \right)^{\frac{1-\gamma}{1-\gamma}} (z'_i)^{1/(1-\beta)} E \left[ \left( z'_i \right)^{1/(1-\beta)} \bigg| z_i \right]^{\frac{1-\beta}{1-\gamma}} \quad (A.17)
\]

Taking expectations of the optimal labor demand in Equation (A.17) yields

\[
E[n'_i(z'_i)|z_i] = \left( \frac{\hat{\alpha}}{\bar{r} w_i^k} \right)^{\frac{1-\alpha}{1-\gamma}} \left( \frac{\hat{\beta}}{\bar{w}} \right)^{\frac{1-\gamma}{1-\gamma}} E \left[ \left( z'_i \right)^{1/(1-\beta)} \bigg| z_i \right]^{\frac{1-\beta}{1-\gamma}} \quad (A.18)
\]
Let $K = \sum_i k'_i$ denote the aggregate capital stock. Substituting for the optimal capital demand from Equation (A.16) yields

$$K = \left( \frac{\hat{\alpha}}{\bar{r}} \right)^{1-\beta} \left( \frac{\hat{\beta}}{\bar{w}} \right)^{1-\gamma} \Gamma_K \tag{A.19}$$

where

$$\Gamma_K = \sum_i \left( \frac{E \left[ (z'_i)^{1/(1-\hat{\beta})} | z_i \right]}{w'_i^k} \right)^{1-\hat{\beta}}^{1-\gamma} \tag{A.20}$$

Similarly, let $N = \sum_i E[n'_i(z'_i)|z_i]$ denote the aggregate labor stock. Substituting for the optimal expected labor demand from Equation (A.18) yields

$$N = \left( \frac{\hat{\alpha}}{\bar{r}} \right) \frac{\hat{\alpha}}{\hat{\beta}} \left( \frac{\hat{\beta}}{\bar{w}} \right)^{1-\hat{\alpha}} \Gamma_N \tag{A.21}$$

where

$$\Gamma_N = \sum_i \left( \frac{E \left[ (z'_i)^{1/(1-\hat{\beta})} | z_i \right]}{w'_i^k} \right)^{1-\hat{\beta}}^{1-\gamma} \tag{A.22}$$

Using Equations A.19 and A.21 to substitute for $\bar{w}$ and $\bar{r}$ in Equations A.16 and A.17, we get

$$k'_i = \frac{K}{\Gamma_K} \left( \frac{E \left[ (z'_i)^{1/(1-\hat{\beta})} | z_i \right]}{w'_i^k} \right)^{(1-\hat{\beta})/(1-\gamma)}$$

$$n'_i(z'_i) = \frac{N}{\Gamma_N} \left( z'_i \right)^{1/(1-\hat{\beta})} \left( \frac{E \left[ (z'_i)^{1/(1-\hat{\beta})} | z_i \right]}{w'_i^k} \right)^{\hat{\alpha}/(1-\gamma)}$$

Taking expectations of $n'_i(z'_i)$:

$$E[n'_i(z'_i)|z_i] = \frac{N}{\Gamma_N} \left( \frac{E \left[ (z'_i)^{1/(1-\hat{\beta})} | z_i \right]}{(w'_i^k)^{\hat{\alpha}/(1-\beta)}} \right)^{(1-\hat{\beta})/(1-\gamma)}$$

Substituting for the optimal $k'_i$ and $n'_i(z'_i)$ in the production function, $y'_i = z'_i(k'_i)^{\hat{\alpha}}(n'_i)^{\hat{\beta}}$, we
obtain output for each firm tomorrow:

\[ y_i' = \left( z_i' \right)^{1/(1-\hat{\beta})} \left( \frac{E \left[ (z_i')^{1/(1-\hat{\beta})} \mid z_i \right]}{w_i^k} \right)^{\hat{\alpha} / \hat{\gamma}} \left( \frac{K}{\Gamma_K} \right)^{\hat{\alpha}} \left( \frac{N}{\Gamma_N} \right)^{\hat{\beta}} \]

Taking expectations, we obtain:

\[ E[y_i' \mid z_i] = \left( \frac{E \left[ (z_i')^{1/(1-\hat{\beta})} \mid z_i \right]}{(w_i^k)^{\hat{\alpha} / (1-\hat{\beta})}} \right)^{1-\hat{\beta}} \left( \frac{K}{\Gamma_K} \right)^{\hat{\alpha}} \left( \frac{N}{\Gamma_N} \right)^{\hat{\beta}} \]

Aggregating across all firms, we get

\[ Y = AK^{\hat{\alpha}}N^{\hat{\beta}} \]

where \( A \) is TFP:

\[ A = \frac{\Gamma_N^{1-\hat{\beta}}}{\Gamma_K^{\hat{\alpha}}} = \left[ \sum_i \left( \frac{E \left[ (z_i')^{1/(1-\hat{\beta})} \mid z_i \right]}{(w_i^k)^{\hat{\alpha} / (1-\hat{\beta})}} \right)^{1-\hat{\beta}} \right]^{1-\gamma} \left[ \sum_i \left( \frac{E \left[ (z_i')^{1/(1-\hat{\beta})} \mid z_i \right]}{(w_i^k)^{\hat{\alpha} / (1-\hat{\beta})}} \right)^{1-\hat{\beta}} \right]^{\hat{\alpha}} \]

We can compare this level of TFP to a benchmark level of TFP, in which we remove all distortions and technological constraints are removed. In this case, \( w_i^k = 1 \), and TFP becomes:

\[ \bar{A} = \left[ \sum_i E \left[ (z_i')^{1/(1-\hat{\beta})} \mid z_i \right] \right]^{1-\gamma} \]

We can then define the TFP loss as the percentage difference between \( \bar{A} \) and \( A \).

\[
\text{TFP loss} = \frac{\bar{A}}{A} - 1 = \left[ \sum_i E \left[ (z_i')^{1/(1-\hat{\beta})} \mid z_i \right] \right]^{1-\gamma} - 1
\]