Observing gravitational waves from the post-merger phase of binary neutron star coalescence

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Abstract

We present an effective, low-dimensionality frequency-domain template for the gravitational wave (GW) signal from the stellar remnants from binary neutron star (BNS) coalescence. A principal component decomposition of a suite of numerical simulations of BNS mergers is used to construct orthogonal basis functions for the amplitude and phase spectra of the waveforms for a variety of neutron star (NS) equations of state and binary mass configurations. We review the phenomenology of late merger/post-merger GW emission in BNS coalescence and demonstrate how an understanding of the dynamics during and after the merger leads to the construction of a universal spectrum. We also provide a discussion of the prospects for detecting the post-merger signal in future GW detectors as a potential contribution to the science case for third generation instruments. The template derived in our analysis achieves >90\% match across a wide variety of merger waveforms and strain sensitivity spectra for current and potential GW detectors. Using a simple Monte Carlo simulation, we find a preliminary estimate of the typical uncertainty in the determination of the dominant post-merger oscillation frequency $f_{\text{peak}}$ of $\delta f_{\text{peak}} \sim 138$ Hz. Using recently derived correlations between $f_{\text{peak}}$ and the NS radii, this suggests potential constraints on the radius of a fiducial NS of $\sim 429$ m. Such measurements would only be possible for nearby ($\sim 30$ Mpc) sources with advanced LIGO but become more feasible for planned upgrades to advanced LIGO and other future instruments, leading to constraints on the

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high density NS equation of state which are independent and complementary to those inferred from the pre-merger inspiral GW signal. We study the ability of a selection of future GW instruments to provide constraints on the NS equation of state via the postmerger phase of BNS mergers.

Keywords: gravitational waves, data analysis, binary neutron star, post-merger, template

(Some figures may appear in colour only in the online journal)

1. Introduction

The first direct detection of gravitational waves (GW) [1] made by advanced LIGO (aLIGO) [2] in September 2015 has, at last, ushered in the age of gravitational wave astrophysics. Instruments such as advanced Virgo (advVirgo) [3] and KAGRA [4] will soon come online, eventually culminating in a world-wide network of GW observatories. The GW signal from the inspiral stage of binary neutron star (BNS) coalescence is amongst the most promising sources for this second generation of GW detectors. Observations of BNS GW inspiral signals from relatively nearby events (a few tens of Mpc) can lead to strong constraints on the supranuclear equation of state (EoS) via the impact on the phase evolution of the signal from tidal interactions during the latter stages of the merger [5–11].

For example, Read et al [6] have shown that neutron star (NS) radii could be constrained with an uncertainty of 10% for a single nearby (100 Mpc, assuming optimal orientation and sky-location) source. More recently, a number of full Bayesian analyses have been carried out which have used astrophysically motivated simulated populations of BNS merger events to develop and probe EoS constraints in the low signal-to-noise ratio (SNR) regime. In [17] it was found that only a few tens of inspiral events are required to measure NS tidal deformability to ~10%. Similar results are found in [18, 19], where the neutron star (NS) radius is determined to ±1 km and tidal deformability is determined to 10%–50% after a few tens of GW detections.

The focus of this work, however, is on the independent and complementary constraints on the EoS which may be obtained from the post-merger signal. Depending on the mass configuration of the system and the EoS, a BNS merger may result in prompt collapse to a black hole (high-mass, soft EoS) or the formation of a stable or quasi-stable NS remnant which again, may or may not collapse to a black hole depending on its mass and the EoS, while transient nonaxisymmetric deformations and quadrupolar oscillations in this remnant typically give rise to a richly structured, high-frequency (1–4 kHz) GW spectrum and a signal lasting ∼10–100 ms, [20–40]. Characterising the frequency content of GW signals from the post-merger system provides unique opportunities for GW asteroseismology: the dominant post-merger oscillation frequency \( f_{\text{peak}} \) exhibits a tight correlation with the radius of non-rotating NSs, with an overall uncertainty of a few hundred meters, depending on the total binary mass. For example, for a total binary mass of 2.7 \( M_\odot \), the uncertainty in the radius of a cold, nonrotating NS of mass 1.6 \( M_\odot \) (denoted as \( R_{1.6} \)) is about 175 m for symmetric mergers [35, 41]. Similar relationships between the dominant spectral features and stellar parameters have been confirmed elsewhere [37, 39]. A deeper understanding of the features of post-merger GW spectra has been provided in [30, 42, 43], where it was shown that the spectrum is

5 Reviews of the subject may also be found in [12–16].
dominated by a linear feature (quadrupolar oscillations), a quasi-linear feature (a coupling between quadrupolar and quasi-radial oscillations) and a fully nonlinear feature (a transient spiral deformation), leading to a classification scheme of the postmerger GW emission depending on the EoS and binary mass. More recently, efforts have been made to find correlations between the pre- and post-merger signals. In [44] the authors derive a relation between the tidal coupling constant $\kappa$ that determines the tidal interactions before and during the merger and the peak frequency $f_{\text{peak}}$ in the post-merger spectrum. Thus, measurements of the inspiral signal (which determine $\kappa$) could be used to constrain $f_{\text{peak}}$ by restricting its range of possible values and by combining measurements with those of the post-merger signal.

This connection between the tidal interactions and the post-merger oscillations highlights the complementarity of pre- and post-merger GW observations. The constraints arising from inspiral observations may be subject to systematic biases induced by errors in the phase due to missing high PN-order terms or insufficiently accurate descriptions of spin or tidal effects. These systematic errors can be as large as the statistical uncertainty in characterising the inspiral signal [19, 45]. While inspiral waveform models will continue to improve and incorporate such effects, we note that analyses of the post-merger signal are subject to a completely independent source of systematic error (e.g., the precise $f_{\text{peak}} - R_{1.6}$ relationship). Moreover, since the majority of pre-merger NSs are likely to have masses in the range $\sim 1.35 \pm 0.15 \, M_\odot$ (e.g. [46, 47]), the pre-merger waveforms are limited to probing the structure of NSs in that mass-range, while the post-merger signal allows us to probe the regime of higher masses (this is because, e.g. the central density of the remnant of a $1.35 + 1.35 M_\odot$ merger is close to the central density of a $1.6 M_\odot$ nonrotating star).

This high frequency component of the merger signal, however, will be somewhat challenging to observe in the upcoming generation of GW detectors. Typically, the most sensitive frequencies of ground-based GW instruments lie around 10–1000 Hz, with a rapidly diminishing sensitivity in the kHz regime. Additionally, the absence of a complete waveform model for the full pre- and post-merger signal, or even for the post-merger signal alone, currently prohibits the use of matched filtering and one must turn to more robust, but ultimately less sensitive unmodelled burst searches. For example, the study in [48] revealed that a typical realistic burst analysis yielded an effective range approximately 30%–40% of that which could be possible with an optimal matched filter.

Clearly then, there is great motivation and opportunity to develop more sensitive, more targeted analysis techniques and effective models which will bring us closer to the sensitivity offered by a matched filtering analysis. It is the goal of this work to explore a principal component analysis (PCA) based approach to constructing precisely such an effective model for the high-frequency component of the BNS merger signal. We construct a catalogue of 50 numerical waveforms from the merger and post-merger evolution of a variety of BNS systems with various EoSs and mass configurations. The magnitude and phase spectra of the waveforms in the catalogue are then decomposed into orthogonal bases using PCA. These basis functions can then be used to construct a frequency-domain waveform template which provides, on average, a 93% match for the waveforms in the catalogue for both aLIGO and a variety of potential upgrades and new GW instruments.

The structure of this paper is as follows: in section 2 we provide a detailed review of BNS merger and post-merger phenomenology, focussing on the resulting features in the GW spectrum and hence how one may constrain the NS EoS. Section 3 summarises the expected detectability of the high-frequency BNS waveforms used in this study, assuming a matched-filtering approach and a variety of current and potential future GW instruments. In section 4 we describe and characterise our PCA-based frequency-domain waveform template in terms
of waveform match and provide estimates of the uncertainties in $f_{\text{peak}}$ and $R_{1.6}$ based on a simple Monte Carlo study as a preliminary guide to its potential. Finally, section 5 provides a summary and some concluding remarks relating to the planned applications of this model and the potential for similar approaches to enhance unmodelled burst analyses.

2. Properties of postmerger GW spectra and constraining the NS EOS

2.1. Types of merger dynamics and GW spectra

For symmetric (i.e., equal component masses) and mildly asymmetric binaries the GW postmerger spectra of NS mergers (see e.g. right panel of figure 1 or figure 1 in [42]) show a generic behaviour in the sense that certain features of the spectrum depend in a particular way on the total binary mass and the high-density EoS [42]. Specifically, distinct peaks in the spectrum can be associated with distinct mechanisms generating those features, and the frequency and strength of the different GW peaks are determined by the total binary mass and EoS. The presence or absence of certain secondary peaks in the spectrum, together with their relative strengths is determined by the quasi-linear coupling between the quasi-radial and quadrupolar oscillation modes and by the orbital motion of antipodal bulges of a spiral deformation in the remnant. The characteristics of these distinct spectral features can be used to classify the post-merger dynamics of the system [42].

The most striking feature of the postmerger spectrum is a major peak generated by the dominant quadrupolar oscillation of the remnant, which is present in all models that form a NS merger remnant. The determination of the frequency of this peak in a GW measurement is the focus of this work because the peak frequency scales tightly with the radii of nonrotating NSs (see figure 4 below and discussion in [34, 49]) and thus provides strong constraints on the only incompletely known EoS of NS matter. Apart from this main peak, there can be up to two pronounced secondary peaks at frequencies below $f_{\text{peak}}$.

One of the secondary features is a peak generated by the quasi-linear interaction between the dominant quadrupolar oscillation and the quasi-radial mode of the remnant (the latter does not appear strongly in the GW spectrum on its own) [30]. The corresponding peak of the quasi-linear mode coupling has an amplitude proportional to the product of the amplitudes of the quadrupolar mode and of the quasi-radial mode, while its frequency, which we denote as $f_{2-0}$, is equal to the difference of the frequencies of these two modes, i.e. $f_{2-0} = f_{\text{peak}} - f_0$, where $f_0$ is the frequency of the quasi-radial mode. The $f_{2-0}$ feature is particularly pronounced for relatively high total binary masses and soft EoSs. Another secondary spectral peak is produced by the orbital motion of antipodal bulges, which form during the merging as a spiral deformation and then orbit around the inner remnant for a few milliseconds [42] (see figure 2). This dynamical feature is present in addition to the main emission at $f_{\text{peak}}$ for the first few milliseconds after merging. Bulges moving with an orbital frequency $f_{\text{bulges}}$ result in a peak in the GW spectrum at $f_{\text{spiral}} = 2f_{\text{bulges}}$. This finding receives further support by the time-frequency map of the GW signal shown in figure 1 for the 1.35–1.35 $M_\odot$ merger with the TM1 EoS [50, 51]. The time-frequency map has been produced using a continuous wavelet transform with a Morlet basis. We provide further details of this time-frequency

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6 We employ the quadrupole formula for the GW extraction. Reference [23] has found a very good agreement between the GW emission computed via the quadrupole formula and an extraction based on gauge-invariant Moncrief variables. In particular, the frequencies are reproduced with high accuracy. Clearly, the association of a particular GW peak with some dynamical feature should be treated with caution. However, the coincidence of GW frequencies and frequencies of the dynamical features is striking (see also figures 3–8 in [30]).
decomposition in the appendix. One can clearly recognise that in the early postmerger phase there are two distinct frequencies simultaneously contributing to the GW signal. The frequency of the dominant remnant oscillation is present for many milliseconds. The secondary peak at $f_{\text{spiral}}$ is generated within the first few milliseconds, when the antipodal bulges are pronounced (see figure 2). There is no evidence for a strong time variation of the frequencies, especially of the dominant frequency, which was suggested as an explanation for the structure of the GW spectrum in \cite{52, 53}. A roughly constant dominant frequency has also been seen in \cite{54}.

The information in the time-frequency map of the GW signal can be related to the dynamical behaviour of the remnant, which we illustrate by the evolution of the rest-mass density in the equatorial plane for the same simulation (see figure 2). The time step of the different snapshots are marked in the time-frequency map (figure 1) by vertical lines. Evidently, the presence of antipodal bulges at the outer remnant coincides with the presence of power at $f_{\text{spiral}}$ in the time-frequency map. It is apparent that the $f_{\text{spiral}}$ feature is initially particularly strong exceeding even the emission at $f_{\text{peak}}$; the antipodal bulges are strongest during and immediately after merging and the spiral deformation forming the bulges initially comprises large parts of the remnant (see upper right panel in figure 2). In figure 2, the antipodal bulges complete approximately one orbit from the top right to the bottom left panel in about 1.2 ms. Thus, the orbital frequency $f_{\text{bulges}} = 1/1.2 \text{ ms} = 0.833 \text{ kHz}$ is expected to produce a peak at $f_{\text{spiral}} = 2f_{\text{bulges}} = 1.67 \text{ kHz}$, where a peak is found in the spectrum (see figure 1). For comparison we also show the time-frequency analysis for the SFHO EoS and component masses $1.35 - 1.35 \epsilon M_\odot$ in figure 3. Here the secondary peak at 2.2 kHz likely arises from the $f_{2\cdot 0}$ feature. An examination of the hydrodynamical data for this model reveals an $f_{\text{bulges}}$ of about 1.25 kHz (resulting in $f_{\text{spiral}} \approx 2.5 \text{ kHz}$), whereas the frequency of the quasi-radial mode is $f_0 = 1.0 \text{ kHz}$, and thus the $f_{2\cdot 0}$ peak is expected to occur at about 2.2 kHz.

The above findings on the time-frequency characteristics of the $f_{\text{spiral}}$ peak are consistent with the explanations of its origin presented in \cite{42}. The $f_{\text{spiral}}$ feature is particularly strong for
mergers with relatively low binary masses and stiff EoSs because less compact NSs favour the spiral deformation and the formation of the antipodal bulges during merging. In contrast, binaries with more massive components, i.e. very compact stars, merge with a higher impact velocity, which favours a strong excitation of the quasi-radial mode of the remnant, leading to a strong $f_{2-0}$ feature, while the spiral deformation becomes less pronounced. For intermediate cases, i.e. moderately high binary masses, both secondary peaks are clearly present with comparable strength and distinguishable in frequency. Overall, this implies that for a given EoS the binary mass determines the presence and strength of the different secondary features. According to the classification scheme introduced in [42], one can identify three different types of spectra: high-mass/soft EOS binaries produce spectra where the dominant secondary peak is $f_{2-0}$ (type I mergers). For intermediate binary masses and EOS stiffness, both the $f_{\text{spiral}}$ and $f_{2-0}$ features are present with roughly comparable amplitude (type II mergers). Low-mass/stiff EOS binaries produce spectra with a strong $f_{\text{spiral}}$ peak and an absent $f_{2-0}$ feature (type III mergers). See [42, 43] for further discussion.

Figure 2. Evolution of the rest-mass density in the equatorial plane for a $1.35–1.35 \, M_\odot$ merger with the TM1 EoS [50, 51]. Black and white dots indicate the positions of selected fluid elements constituting the antipodal bulges, which generate a distinct peak in the GW spectrum. (A low number of iso-density contours is chosen for a better identification of the different remnant components. This choice leads to an artificially coarse visualisation of the simulation data.)
2.2. Universal post-merger spectra and measuring the NS radius

For a fixed total binary mass the frequencies of the three different peaks depend in a particular way on the EoS, which can be characterized by the radius or compactness of nonrotating NSs [34, 42, 49] (see also [53, 55] for the dependence of the strongest secondary feature on compactness, without distinguishing the different nature of secondary peaks). The two secondary frequencies show a tight correlation with the dominant postmerger frequency \( f_{\text{peak}} \). This is shown in figure 4 for 1.35–1.35 \( M_\odot \) mergers. The least-square fits in figure 4 are given by \( f_{\text{spiral}} = 0.806 \cdot f_{\text{peak}} - 0.190 \) kHz and \( f_{2\pi} = 1.002 \cdot f_{\text{peak}} - 1.080 \) kHz (with frequencies in kHz). Similar relations hold for other binary masses.
The existence of generic spectral features with predictable behaviour suggests that the construction of a universal spectrum should be feasible through the appropriate alignment of the main peaks from spectra for various EoSs. Figure 5 shows the GW spectra for equal mass binaries \(1.35 - 1.35 \times M_\odot\) with different EoSs. To accentuate the morphological similarities between the spectra without introducing artificial differences from different overall amplitude scales, the waveforms have been normalised such that the root-sum-squared amplitude is unity

\[
h_{\text{rss}} = \sqrt{\int_{-\infty}^{\infty} |h(t)|^2 dt} = 1. \tag{1}
\]

In the right panel, we rescale the frequency axis such that the dominant quadrupolar oscillation peak feature is located at a common reference value (2.6 kHz) for all models. Apart from small variations of the secondary features a remarkable universality of the spectra is found, which can be explained as follows: we choose a reference peak frequency of \(f_{\text{ref}} = 2.6 \text{ kHz}\). Thus, for a spectrum with the main peak at \(f_{\text{peak}}\) the factor for rescaling the frequency is \(a = f_{\text{ref}} / f_{\text{peak}}\). This factor \(a\) is also applied to the frequencies of the secondary peaks. Therefore, a rescaled secondary peak \(f_{\text{sec}}\) (i.e. \(f_{\text{spiral}}\) or \(f_{\text{2-0}}\)) is located at 

\[
a f_{\text{sec}} = f_{\text{ref}} f_{\text{sec}} / f_{\text{peak}}.
\]

Since the fraction \(f_{\text{sec}} / f_{\text{peak}} \equiv c\) is approximately constant and similar for both secondary features (see figure 4), a rescaled secondary feature occurs at approximately the same frequency \(c \cdot f_{\text{ref}}\) for all models.

This universality of the scaled spectra suggests that it should be possible to produce a model from the mean spectrum, computed over a number of numerical simulations, plus some small deviations. In section 4, we demonstrate that PCA provides an approach to solve exactly this problem by producing an orthonormal basis constructed from a superposition of the mean-centered spectra. Furthermore, we find that the perturbations from the mean spectrum are generally well described by a small number of basis functions.

It is important to stress here that, for distances which allow for the detection of post-merger GW emission, the individual masses of the binary components can be determined with an accuracy of a few per cent \([56-58]\). Furthermore, current observations suggest that BNS mass configurations will not be dramatically asymmetric (see e.g. the compilation of NS masses in \([47]\)). Peak frequencies which are recovered within this data analysis study, are
converted to NS radii via
\[ R_{1.6} = a f_{\text{peak}}^2 + b f_{\text{peak}} + c, \tag{2} \]
describing the empirical relation between NS radii and the dominant postmerger oscillation frequency for fixed total binary masses. Depending on the total binary mass the coefficients of equation (2) take different values because tight empirical relations between \( R_{1.6} \) and \( f_{\text{peak}} \) can only be found for fixed binary masses. The coefficients are given in Table 1 together with the maximum deviation between the fit (equation (2)) and the numerical data underlying the fit. These deviations are included as systematic uncertainties in our error analysis in section 4, which, in addition, considers statistical errors introduced by the measurement of \( f_{\text{peak}} \) in noisy GW data. Note that for total binary masses of 2.4 \( M_\odot \) and 3.0 \( M_\odot \) smaller deviations are found if a fiducial NS mass different from 1.6 \( M_\odot \) is chosen [49]. For asymmetric binary systems we employ the relation of the same total binary mass because asymmetric mergers do not yield peak frequencies much different from the frequencies resulting from mergers with a symmetric setup. For total binary masses different from the ones reported in Table 1 we linearly interpolate/extrapolate in mass between the listed relations.

### Table 1.

| \( M_{\text{tot}} (M_\odot) \) | \( a \) | \( b \) | \( c \) | \( \Delta R \) (m) |
|-----------------------------|------|------|------|--------------|
| 2.4                         | 1.143 | -8.79 | 27.7 | 236          |
| 2.7                         | 1.099 | -8.57 | 28.1 | 175          |
| 3.0                         | -0.463 | 0.580 | 18.5 | 254          |

2.3. BNS merger waveforms used in this study

The numerical waveforms used in this study rely mostly on the calculations discussed in [34, 41, 42, 48, 49], where further information can be found. Additional waveform models employed here are obtained within the same physical and numerical model, for which further details are provided in [24, 35, 59, 60]. The calculations are performed with a relativistic smooth particle hydrodynamics scheme, which imposes the conformal flatness condition for the solution of the Einstein equations [61, 62]. For determining the frequencies in the GW spectra this numerical approach has shown a quantitatively very good agreement (below a few per cent) with grid-based hydrodynamics schemes in full general relativity (comparisons are discussed e.g. in [34, 49, 54, 55, 63]). The code can employ temperature-dependent microphysical EoSs. The model neglects the effects of neutrinos and magnetic fields, which have a practically negligible impact on the GW frequencies, which are mostly determined by stellar structure (e.g. [33, 64]). This enables a very high computational efficiency of the code to produce many numerical waveforms, which is crucial for the present study. The construction of the empirical frequencies relations is not affected by the numerical resolution (see [34, 49]), and for the determination of the empirical relations we employ solely temperature-dependent EoSs, which is indispensable for an accurate determination of the GW peak frequencies [60]. We note that the numerical model may underestimate the GW amplitude

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7 We note that, even in the absence of a measurement of mass ratio, the \( f_{\text{peak}} \sim R_{1.6} \) relation is quite robust for a constant chirp mass, which is generally recovered to high precision. See e.g. [43].
because of the numerical damping (see also [48]). This aspect is irrelevant for the present study because effectively the amplitude is set by the uncertain source distance. Consequently, the derived detection rates may conservatively be taken as lower limits. Overall, the very good agreement between the present physical and numerical model and other calculations is highly encouraging given the significant differences particularly in the numerical approach (e.g. Lagrangian scheme versus Eulerian hydrodynamics) [34, 49, 54, 55, 63]. This points to a certain reliability of the empirical frequency-radius relations, which are the basis for NS radius measurements discussed here. The insensitivity may not be surprising considering that the main peak is generated by the fundamental quadrupolar fluid mode, which is a rather robust feature of a stellar configuration since the frequency is essentially determined by the merger remnant’s size [34, 49]. The EoS models for the hydrodynamical simulations are chosen to cover a large variety including very stiff and very soft EoSs (see table 2), and a variety of binary mass configurations are used. All EoSs are compatible with a maximum NS mass of \( \sim 2 M_\odot \) [65, 66]. Figure 6 illustrates the selection of waveforms used in this study in terms of their EoSs and mass configurations.

### 3. Detectability

We now discuss the expected detectability of the post-merger GW signal in current and planned GW instruments. A natural, preliminary measure of detectability is the matched-filter SNR one would obtain given a perfect model, or template, for the signal waveform \( h \) in GW detector data \( s \). The matched-filter SNR is defined as

\[
\rho = \frac{(s|h)}{\sqrt{(h|h)}}. \tag{3}
\]
The optimal SNR, where the template $h$ exactly matches detector output is then simply

$$\rho_{\text{opt}} = \sqrt{(h|h)},$$

where $(.,.)$ is the usual inner product [78]:

$$ (a|b) = 4 \Re \int_{f_{\text{low}}}^{f_{\text{ny}}} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_0(f)} \, df, $$

and $S_0(f)$ is the noise spectrum of a given GW detector and the asterisk indicates complex conjugation. Note that we impose a lower bound $f_{\text{low}}$ on the frequency over which the inner product is evaluated in order to target the detectability of the high-frequency part of the signal. In this study we use $f_{\text{low}} = 1$ kHz. The inner product is evaluated up to the Nyquist frequency of the spectrum, 8192 Hz in this study. We also characterise detectability in terms of horizon distance $D_{\text{hor}}$: the distance at which an optimally oriented source yields an SNR at least as large as some nominal threshold, $\rho_k$. For GW searches in which the time of arrival of the signal and the source sky-location are unknown, it is typical to evaluate horizon distances with $\rho_k = 8$. In our application, however, we envisage a hierarchical ‘triggered’ analysis, similar to that described in [48], wherein the earlier, lower-frequency inspiral portion of the coalescence signal has already been detected at high confidence. It is likely then that the time of coalescence has been determined to an accuracy of a few or a few tens of milliseconds and we can significantly reduce the threshold used to define the horizon distance. Following [48], we choose $\rho_k = 5$. Finally, we can determine the rate $\dot{N}_{\text{det}}$ with which we will obtain signals with $\rho_u \geq 5$ from the expected number of BNS mergers which are accessible to a search with a given horizon distance [79]. For the purposes of this study, we assume the ‘realistic’ rate of BNS coalescence from [79]: $R_{\text{re}} = 100 \text{ MWEG}^{-1}\text{Myr}^{-1}$, where MWEG stands for Milky Way Equivalent Galaxies. See [79] for the conversion to a rate per volume.

We now compute each of the figures of merit (the SNR for an optimally oriented source at 50 Mpc; the horizon distance assuming an SNR threshold $\rho_k = 5$ and the expected detection rate $\dot{N}_{\text{det}}$) for aLIGO [2, 80], as well as the following selection of proposed
upgrades to aLIGO and new facilities. The following descriptions emphasise the expected increases in sensitivity relative to aLIGO only over 1–4 kHz; the band of interest for the post-merger signal. Comparisons with the increased range and sensitivity to the earlier inspiral part of the signal are left to future studies. Note also that we take aLIGO to be the most sensitive of the second generation GW detectors; instruments such as advVirgo and Kagra offer comparable or reduced sensitivity in the frequency regime of interest to this study. It should be noted, however, that a network of X detectors with comparable sensitivity could improve the range of a search by a factor of up to $\sqrt{X}$ with respect to the single detector expectation, assuming stationary Gaussian noise and an optimal analysis. We restrict our estimates to single detector ranges and rates in the interests of conservatism and simplicity.

LIGO A+ \cite{81, 82} a set of upgrades to the existing LIGO facilities, including frequency-dependent squeezed light, improved mirror coatings and potentially increased laser beam sizes. Noise amplitude spectral sensitivity would be improved by a factor of $\sim 2.5–3$ over 1–4 kHz. A+ could begin operation as early as 2017–18.

LIGO voyager (LV) \cite{82} a major upgrade to the existing LIGO facilities, including higher laser power, changes to materials used for suspensions and mirror substrates and, possibly, low temperature operation. LV would become operational around 2027–28 and offer noise amplitude spectral sensitivity improvements of $\sim 4.5–5$ over 1–4 kHz.

LIGO cosmic explorer (CE) \cite{82} a new LIGO facility rather than an upgrade, with operation envisioned to commence after 2035, probably as part of a network with LV. In its simplest incarnation, CE would be a straightforward extrapolation of A+ technology to a much longer arm length of 40 km, referred to as CE1 which would be $\sim 14\times$ more sensitive than aLIGO over 1–4 kHz. An alternative extrapolation is that of Voyager technology to the 40 km arm length, referred to as CE2. CE2 is only $\sim 8\times$ more sensitive than aLIGO for the frequency range of interest in this study. For simplicity, we consider only CE1.

Einstein telescope (ET-D) \cite{83, 84} the European third-generation GW detector. In this work, we consider the ET-D configuration which is comprised of two individual interferometers where one targets low frequency sensitivity and the other high frequency sensitivity. Both interferometers will be of 10 km arm length and housed in an underground facility. Furthermore, the full observatory will consist of three such detectors in a triangle.
arrangement. ET-D is \approx 8 \times more sensitive than aLIGO over 1–4 kHz. Due to the network configuration (i.e., the alignment of the component instruments) the effective sensitivity of ET-D is \approx 18\% higher than that for a single ET-D detector.

Figure 7 shows the design sensitivity spectra for each of these instruments, again focussing on the 1–4 kHz range of interest for the post-merger BNS GW signal. For comparison, we also show the amplitude spectrum of a typical BNS waveform (the TM1 1.35 +1.35 example discussed in section 2) for an optimally oriented source at 50 Mpc. Finally, the figures of merit describing the detectability of the post-merger signal for each instrument are summarised in table 3. Since there are five instruments, 50 waveforms and 4 figures of merit, we choose to summarise the results for each instrument in terms of the 10th, 50th and 90th percentiles, evaluated over the 50 waveforms used in the study.

| Instrument | SNR_{full} | D_{hor} (Mpc) | \dot{N}_{\text{det}} (year^{-1}) |
|-----------|------------|---------------|---------------------------------|
| aLIGO     | 2.99$^{+2.37}_{-2.16}$ | 29.89$^{+34.57}_{-37.56}$ | 0.01$^{+0.03}_{-0.01}$ |
| A+        | 7.89$^{+0.16}_{-0.23}$ | 78.89$^{+10.67}_{-13.52}$ | 0.13$^{+0.20}_{-0.10}$ |
| LV        | 14.06$^{+0.16}_{-0.35}$ | 140.56$^{+181.29}_{-181.60}$ | 0.41$^{+0.58}_{-0.21}$ |
| ET-D      | 26.65$^{+0.32}_{-0.81}$ | 266.52$^{+342.80}_{-208.06}$ | 2.85$^{+5.98}_{-2.71}$ |
| CE        | 41.50$^{+0.52}_{-0.95}$ | 414.62$^{+535.221}_{-329.68}$ | 10.59$^{+27.78}_{-3.35}$ |

4. A waveform model using PCA

The optimal data analysis method for the identification and characterisation of a GW signal in noisy data is matched-filtering, wherein an exact analytic model, or template, for the waveform is convolved with data stream from a network of GW detectors. Unfortunately, the physical complexity of the merging BNS system is such that detailed numerical simulations are required to produce even an approximate waveform. Furthermore, since the physical parameters of the system are essentially unknown, many such simulations would be required in order to build a template bank to maximise the likelihood of signal detection.

We are, therefore, confronted with a similar data analysis problem to that in the analysis of GWs from core collapse supernovae: the absence of an accurate analytic waveform template, a limited number of computationally expensive and approximate simulations and a requirement to significantly reduce the complexity of the modelling problem to facilitate the use of an approximate matched-filter. Motivated by the work in [85, 86], we find that we can construct an effective waveform model from a basis constructed using PCA of a suite of merger simulations comprised of systems with different equations of state, masses and mass ratios.

Our goal is to reduce the complexity of the modelling problem from a high-dimensional physical parameter space, where the waveforms are modelled directly through numerical
simulation, to a lower-dimensional problem to model the dominant features of the waveform. PCA of a catalogue of simulated waveforms provides a solution to precisely this problem. Denoting the time-domain merger waveform as \( h(t) \), its complex Fourier spectrum is given by

\[
\tilde{h}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h(t) e^{i\omega t} dt = A(\omega) \exp[i\phi(\omega)],
\]

where \( A(\omega) = |\tilde{h}(\omega)| \) and \( \phi(\omega) = \arg[\tilde{h}(\omega)] \) are the magnitude and phase spectra of signal \( h(t) \), respectively. In a similar spirit to the approach described in [87] we construct orthonormal bases for the amplitude \( A(\omega) \) and phase spectra \( \phi(\omega) \) separately, using similar a PCA decomposition to that described in [85, 86]. PCA forms a basis from the eigenvectors of the covariance matrix of some set of data. The procedure is as follows:

(i) Collate a representative sample of \( m \) binary merger GW waveforms, sampled at 16384 Hz. This sample of waveforms is hereafter referred to as our training catalogue. Each waveform is normalised to unit root-sum-squared amplitude \( h_{\text{rss}} \) (equation (1)) to reduce catalogue variance from different amplitude scales and emphasise morphological differences.

(ii) Compute the complex Fourier spectra of the time-domain waveforms in our catalogue. A Tukey window is initially applied to the time-domain signals to minimise spectral ringing and the waveforms are zero-padded to a uniform 16384 samples. The complex spectra \( \tilde{h}(\omega) \) are computed using the fast Fourier transform. The amplitude and phase spectra are computed from the absolute values and arguments of the complex frequency series and the phase spectra are unwrapped to yield smooth functions, each of \( n = 8192 \) samples.

(iii) The unique feature to the analysis presented in this work is our choice of feature alignment, an absolutely key component to PCA. In [85], for example, the GW waveforms are aligned such that the peak amplitudes lie at a common reference time removing the need for the PCA to account for trivial variance in the catalogue. The analogous procedure in our application is to align features in the frequency domain. Each amplitude spectrum is rescaled such that the dominant post-merger peak, labelled \( \omega_{\text{peak}} \), is aligned to a common reference value \( \omega_{\text{align}} \). This alignment is achieved by computing a set of frequencies \( \omega' = \frac{\omega_{\text{align}}}{\omega} \times \omega \), where \( \omega \) are the angular frequencies of the original spectrum. We then interpolate the original spectrum to the new frequencies where the dominant spectral feature (the post-merger oscillation peak) is aligned. Although it is not
perfect, this geometric scaling (as opposed to a simple linear shift) also helps to align the sub-dominant $f_{2,0}$ and $f_{\text{spiral}}$ features. Three examples of original and aligned amplitude spectra are shown in figure 8.

(iv) Next, we construct an $m \times n$ matrix $D$ where each row corresponds to the $n$-sample feature-aligned amplitude spectrum of each waveform after subtracting the mean spectrum (averaged over the $m$ waveforms). The mean amplitude spectrum, evaluated over our $m = 50$ waveforms is shown in the left panel of figure 9.

(v) Finally, we perform the PCA decomposition in which we compute the eigenvectors of the empirical covariance matrix $DD^\top$. Following [86] and noting the change in row/column convention for the data matrix, the centered data matrix $D$, of dimension $m \times n$, can be factorised using singular value decomposition

$$D = USV^\top,$$

where $U$ and $V$ are orthonormal matrices with dimensions $m \times k$ and $n \times k$, respectively; $S$ is a diagonal matrix of singular values of $D$ in descending order and $k = \text{rank}(D) \leq \min(m, n)$. The columns of $V$, $v_1 \ldots v_k$, contain the eigenvectors of the covariance matrix $DD^\top$, our principal components, and the singular values in $S$ are the square roots of the eigenvalues $\lambda_i$ of the covariance matrix $DD^\top$. Finally, the columns of $U$ contain the eigenvectors of $D^\top D$. The principal components $v_1 \ldots v_k$, comprise an orthonormal basis of the rows (i.e., the aligned and centered waveforms) in $D$ so that each of the aligned waveform amplitude (or phase) spectra can be represented as a linear sum of principal components and the mean. For example, the aligned amplitude spectrum of the first waveform can be constructed as:

$$A_1^\prime (\omega) = \langle A^\prime (\omega) \rangle + \sum_{i=1}^{m} \beta_i v_i (\omega),$$

where $\langle A^\prime (\omega) \rangle$ is the mean amplitude spectrum over the aligned waveform catalogue and $\beta_1 \ldots \beta_m$ are weighting coefficients given by the projection of the centered $A_1 (\omega)$ onto the principal component basis

$$B = [A_1 (\omega) - \langle A^\prime (\omega) \rangle]. V,$$

---

8 Note that this matrix is transposed relative to the descriptions in [85, 86] for more straightforward comparison with other PCA literature and use with software packages such as that offered in [88].
and $\beta_1, \ldots, \beta_k$ are the elements of $B$. The original waveform magnitude spectrum $A_i(\omega)$ is, at last, obtained by applying the inverse of the alignment procedure in step (iii). Figure 9 shows the mean aligned magnitude spectrum $\langle A'(\omega) \rangle$ and the first principal component as computed for the 50 waveforms described in section 2.3. Note that we have chosen to align the dominant post-merger peak to a value of 2710 Hz; this choice is essentially arbitrary and simply corresponds to the mean of the peak frequencies in the catalogue. It is important to note here that the value of $f_{\text{peak}}$ is a free parameter in the spectral model; in practice, its value must be inferred from GW observations.

Ultimately, our goal is to construct a reduced basis from which any post-merger waveform can be reconstructed to some accuracy. It is, therefore, helpful to understand the relative importance of each principal component. A measure of the total variance in the centered catalogue is given by the trace of the covariance matrix $\text{tr}(DD^T) = \sum_{i=1}^{k} \lambda_i$. The variance explained by $p$ principal components is, then, the sum of the first $p$ eigenvalues:

$$\sigma_{\text{PCA}}^2 = \sum_{i=1}^{p} \sqrt{s_i},$$

where $s_i$ are the singular values from equation (7). The post-merger waveforms can then be approximated by using a reduced basis with $p < m$, with the choice of $p$ based on capturing a reasonable degree of variance in the catalogue, and equation (8). Figure 10 shows the cumulative explained variance for both the magnitude and phase spectra of the waveforms in our catalogue (i.e., equation (10)) as a function of the number of principal components. One can immediately see that the variation between the waveforms is dominated by the rich and varied structure in the magnitude spectra; only $\sim 60\%$ of the total variance is explained by the first principal component of the magnitude spectra, while $\sim 99\%$ of the variance in the phase spectra is described by the first component.

### 4.1. PCA templates: characterisation and expected performance

Remembering that our goal is to build an approximate waveform template for matched filtering, a useful figure of merit to characterise PCA-based model is the waveform match $M$,
which describes the fraction of the optimal SNR for a given signal \( s(t) \) which is captured by the waveform template \( h(t) \):

\[
\mathcal{M} = \max_{t_0, \phi_0} \frac{(s|h)}{\sqrt{(h|h)(s|s)}},
\]

(11)

where \((\cdot|\cdot)\) is the inner-product, defined by equation (5), maximised over the start time \( t_0 \) and initial phase offset \( \phi_0 \) of the signal. The match is normalised such that \( \mathcal{M} = 1 \) for a perfect template and zero for an template which is orthogonal to the target signal. In the following examples, the match is computed assuming the aLIGO noise curve. Figure 11 shows an example of the reconstructed time series and magnitude spectrum for the TM1 1.35 + 1.35 system considered earlier in section 2. The time series is given by the inverse Fourier transform of the complex spectrum constructed from the separate amplitude and phase PCA.

As expected, when we use the full PCA basis with \( p = m \) and include the waveform in the data matrix \( D \), we obtain a complete basis which allows a perfect reconstruction such that \( \mathcal{M} = 1 \).

It is unlikely that nature will provide us with a signal which exactly matches one of those contained in the set of training data \( D \). The right panel of figure 11 again shows the original and reconstructed TM1 1.35 + 1.35 waveform, except now this waveform has been excluded from the training data. In addition, we use only the first principal component for both the amplitude and phase spectra. With this more realistic example and a much smaller parameter space, we are still able to reconstruct the target signal with a match \( \mathcal{M} = 0.96 \).

We now compute similar matches for all of the waveforms in our catalogue and for the different instruments described in section 3. To begin, we compute the match using the aLIGO noise curve as a function of the number of principal components used and compare the results of including and excluding each waveform from the data used to compute the PCA. These results are summarised in figure 12 with the mean, minimum, maximum and the tenth and ninetieth percentiles over the matches computed for each of the fifty waveforms. The left panel shows the results when all of the waveforms are used while the right panel summarises the matches when each waveform is removed from the catalogue prior to

---

\(^9\) Again, the value of \( f_{\text{peak}} \) is assumed known here; the match here represents the best case scenario.
computing the PCA. We see that, as before, perfect reconstruction fidelity is attained using the full basis when all waveforms are used. In contrast, the match remains approximately constant with respect to the number of principal components used when each waveform being matched is excluded from the training data. This is a reflection of the fact that the lower-order principal components represent the most common generic features in the catalogue, while the higher-order components are essentially minor corrections to the mean which may not be present in the waveform which is excluded. Given the quite respectable matches obtained with just the first principal component and its apparent robustness, we propose modelling the high-frequency GW spectrum for BNS mergers using equation (8) with $m = 1$.

We now repeat the match calculation for each of the instrument noise curves described in section 3, using just the first principal component. The 10th, 50th (i.e., the median) and 90th

**Figure 12.** Reconstructed waveform matches (see equation (11)) as a function of the number of principal components used in the reconstruction. Left: match when the test waveform is included in the training data. As expected using the full principal component basis allows for perfect reconstruction fidelity ($\mathcal{M} = 1$). Right: here, matches are computed from a principal component basis wherein the waveform whose match is calculated is withheld from the training data.

**Figure 13.** Template matches for each instrument discussed in section 3. Bar height indicates the median match evaluated across all waveforms and error bars indicate the 10th and 90th percentiles in the match. Templates consist of the mean waveform and a single principal component. When evaluating the match for each waveform, that waveform is removed from the catalogue used for the PCA.
percentiles, computed over the matches for different waveforms, are summarised in figure 13 and listed explicitly in table 4. We find that the PCA templates yield a match of $\sim 0.93$ across all of the instruments considered. Variations in the match arise from differences in the shapes of the noise curve, i.e., the denominator in equation (5); in the kHz regime, where sensitivity is limited by photon shot-noise, the noise curves mostly only differ in their overall amplitude scale and we do not expect significant variations in match quality.

### 4.2. Implications for parameter estimation

Given this approximate waveform template it is useful to determine its effectiveness in parameter estimation and, ultimately, the extraction of astrophysics. Recall from section 2 that the single most robust feature of the GW spectrum for these signals is the presence of a dominant spectral peak due to excitation of the post-merger remnant’s quadrupolar $f$-mode oscillation. Recall also that the peak frequency of this excitation in systems with total binary masses of $2.7 M_\odot$ correlates strongly with the radius $R_{1.6}$ of a fiducial nonspinning $1.6 M_\odot$ NS across a wide variety of equations of state. Our goal then, is to determine how accurately we might expect to measure $f_{\text{peak}}$ using our PCA-based waveform model. In this section, we review the PCA waveform template and derive preliminary estimates for the accuracy with which we may measure $f_{\text{peak}}$ and hence $R_{1.6}$ given current and planned GW observatories.

The aligned magnitude spectrum at frequency $f$ of our waveform model is given by

$$A'(f) = \langle A'(f) \rangle + \beta_1 v_1(f),$$

(12)

where the spectrum is aligned such that the dominant peak lies at frequency $f_{\text{align}}$. $\langle A'(f) \rangle$ is the mean magnitude spectrum of the catalogue, $v_1(f)$ is the first principal component and $\beta_1$ is the coefficient of an arbitrary waveform’s projection onto $v_1(f)$. The final magnitude spectrum is given by interpolating the aligned spectrum $A'(f)$ to a set of new frequencies

$$A'(f) \rightarrow A(f): f \rightarrow \frac{f_{\text{peak}}}{f_{\text{align}}} \times f$$

(13)

and an identical procedure is applied to the phase spectrum $\phi(f)$. In this prescription then, the peak frequency $f_{\text{peak}}$ is a direct parameter of the model.

We adopt a fiducial detection threshold which demands that a signal is observed with SNR $\geq 5$. At low SNRs, and given the large uncertainty in the peak frequency, it is unclear

### Table 4. Expected template performance using the PCA methodology. $\mathcal{M}$ is the match given by a frequency-domain template composed of the mean and a single principal component, evaluated for magnitude and phase separately. $\delta f_{\text{peak}}$ and $\delta R_{1.6}^{\text{stat}}$ are the expected statistical uncertainties in the peak frequency and NS radius, respectively. $\delta R_{1.6}$ is the combined systematic and statistical error in the NS radius, assuming the systematic errors given in table 1.

| Instrument | $\mathcal{M}$ | $\delta f_{\text{peak}}$ (Hz) | $\delta R_{1.6}^{\text{stat}}$ (m) | $\delta R_{1.6}$ (m) |
|------------|--------------|-------------------------------|----------------------------------|-------------------|
| aLIGO      | 0.93^10.9^10^10 | 135.718^18.9^18.9 | 363.4^478^478.7 | 429.1^537.0^537.0 |
| A+         | 0.93^10.9^10^10 | 136.4^10^8.8 | 359.7^227.5 | 425.5^316.0^316.0 |
| LV         | 0.94^10.9^10^10 | 139.0^10^6.6 | 375.3^278.5 | 420.9^318.2^318.2 |
| CE         | 0.94^10.9^10^10 | 138.1^8.5 | 363.9^234.7 | 424.9^341.8^341.8 |
| ET-D       | 0.94^10.9^10^10 | 138.8^15^5.3 | 401.8^506.7 | 443.1^536.1^536.1 |
that Fisher matrix calculations such as those in [49] are sufficient to provide a realistic estimate of the expected accuracy in the determination of $f_{\text{peak}}$, although they do provide a lower bound on the possible uncertainty. We have, therefore, performed a small scale Monte-Carlo study wherein each of the simulated post-merger signals is scaled to an optimal $\text{SNR} = 5$ and added to simulated Gaussian noise. The Gaussian noise is coloured in such a way that its noise spectrum follows that of each of the instruments described in section 3. The peak frequency of the signal in this simulated observation is then determined by maximising the likelihood of the data $d$, given the PCA template $h$, over the peak frequency $f_{\text{peak}}$, start time $t_0$ and phase offset $\phi_0$,

$$f_{\text{peak}} = \arg \max \{ \exp[(d - h(f_{\text{peak}}))d - h(f_{\text{peak}}))]\}. \quad (14)$$

To estimate the distribution of the $f_{\text{peak}}$ measurement thus obtained, the measurement is repeated in 500 different noise realisations for each post-merger waveform. The uncertainty in the determination of $f_{\text{peak}}$ is then taken to be the width of that distribution. We note that this procedure also allows us to quantify any bias in the recovery of $f_{\text{peak}}$, which would manifest as a systematic offset in the mode of the distribution. In fact, we find that this bias is typically an order of magnitude smaller than the statistical uncertainty given by the width of the distribution. We therefore neglect this second order error in the determination of the peak frequency using the PCA templates.

Figure 14 and table 4 summarise the expected frequency uncertainties obtained across waveforms and instruments from the Monte-Carlo simulations. Uncertainties are evaluated at $\text{SNR} = 5$, corresponding to a source at the horizon distance. As with the match summary from earlier the results for each instrument are summarised with the 10th, 50th and 90th percentiles computed over the different waveforms. The expected frequency uncertainty is quite consistent between the different instruments and we find $\delta f_{\text{peak}} \approx 138 \text{ Hz}$.

Finally, we use equation (2) to explore the corresponding accuracy with which NS radii may be inferred. Here, we simply propagate the expected error in the $f_{\text{peak}}$ determination to that in the NS radius using equation (2):

![Figure 14](image.png)

**Figure 14.** Expected uncertainties in parameter estimation based on Monte Carlo simulations recovered with the single-PC waveform template. Bar height indicates the median value evaluated across waveforms and error bars indicate the 10th and 90th percentiles. Left: expected uncertainties in the determination of $f_{\text{peak}}$. Right: expected uncertainties in the determination of the NS radius $R_{1.6}$. 


\[
\frac{\delta R_{1.6}^{\text{stat}}}{\delta f_{\text{peak}}} \approx \frac{\partial R_{1.6}}{\partial f_{\text{peak}}},
\]
(15)

\begin{align*}
\delta R_{1.6} &= (2a f_{\text{peak}} + b) \cdot \delta f, \\
&= (2a f_{\text{peak}} + b) \cdot \delta f,
\end{align*}
(16)

where the fitting coefficients \(a\) and \(b\) for a given total mass are given in table 1. The radius errors are only computed for systems with the masses quoted in table 1. The errors thus obtained represent the statistical uncertainty in the radius, arising from the measurement of a signal in noisy data. The fits given by equation (2) are also subject to systematic errors which, as described in section 2.2, we take to be the maximum deviation in the \(f_{\text{peak}} - R_{1.6}\) relationship across a variety of EoSs, at each total mass. To arrive at a total expected error in the determination of the radius \(\delta R_{1.6}\) then, we quote the quadrature sum of the statistical and systematic errors:

\[
\delta R_{1.6} = \sqrt{(\delta R_{1.6}^{\text{stat}})^2 + (\delta R_{1.6}^{\text{sys}})^2}.
\]
(17)

The expected radius errors yielded by this procedure are summarised in the right panel of figure 14 and in table 4 with the usual breakdown by instrument and percentile summary statistics.

5. Summary and outlook

The wealth of information contained in the high-frequency spectrum of BNS mergers means that there is a strong motivation to develop effective models for the merger and post-merger phase of the coalescence GW signal. Through consideration of the general morphology of the post-merger spectrum and the phenomenology during and after the merger, we have determined that the high-frequency complex spectrum is remarkably well modelled by an orthogonal basis constructed from a catalogue of numerical simulations using a PCA decomposition.

Typically, the waveform templates thus constructed yield a match of \(M \sim 0.93\), over the frequency range 1–4 kHz, with the majority of the waveforms used in this study. While the typical desideratum in most matched-filtering analyses is \(M \gtrsim 0.97\), it is worth noting that the only other systematic and well-quantified estimate of GW search effectiveness to date has been the burst analysis reported in [48]. While the burst analysis is robust to uncertainties in the waveform it was found that its effective range was only \(\sim 40\%\) that of an optimal matched filter analysis. The PCA model presented in this work therefore holds the potential to double or even triple the sensitivity offered by existing analyses. Furthermore, a preliminary Monte Carlo analysis reveals that the uncertainty in the determination of the peak post-merger oscillation frequency is \(\delta f_{\text{peak}} \approx 138\) Hz for sources at the detection horizon. This results in a statistical uncertainty on the radius of a 1.6 \(M_\odot\) NS of \(\delta R_{1.6}^{\text{stat}} \approx 373\) m for sources with sufficient power at 1–4 kHz or proximity to Earth to produce \(\text{SNR} = 5\). Assuming conservative estimates for the systematic error in the \(f_{\text{peak}} - R_{1.6}\) relation when the binary masses are known, we find the total error in the radius is \(\delta R_{1.6} \approx 429\) m. For comparison, the analysis in [48] found a maximum uncertainty of 250 m within a smaller sample of EoSs (see figure 11(b) in [48]).

There are, however, some important differences between these analyses. The estimates presented in [48] computed the error in the recovery of the radius by directly comparing the recovered value of \(R_{1.6}\), measured via \(f_{\text{peak}}\) and the radius relation in that paper, whereas here we simply propagate the expected uncertainty in the \(f_{\text{peak}}\) measurement through to that in \(R_{1.6}\). We then include a very conservative estimate of the systematic error in the fit used.
Additionally, the burst analysis requires a relatively large SNR before sufficient GW signal power is acquired to generate a detection candidate; by this time, the peak frequency itself can be quite easily resolved. In contrast, the results presented in this work have been produced at a fixed, and relatively low, SNR since our intention here is simply to demonstrate the feasibility of the approach; a systematic study using a realistic analysis pipeline and astrophysically motivated population of signals will be explored more fully in a forthcoming study. It is also worth noting that this study uses an expanded set of merger simulations spanning a wider selection of total masses with appropriate \( f_{\text{peak}} = R_{0.6} \) relations chosen for each total mass; the previous study only reported radius measurements for \( M_{\text{tot}} = 2.7 \).

For aLIGO the horizon distance with an optimal template for SNR = 5 is generally \( D_{\text{hor}} \approx 30 \text{ Mpc} \) with a plausible signal rate of approximately 1 event per 100 years, comparable to the rate of Galactic supernovae (see table 3). Our template, however, will lose \( \sim 10\% \) of this SNR, resulting in a proportional decrease in the horizon distance. In fact, thanks to the local over-density of galaxies, the impact on signal rate from this mismatch is rather negligible (\( \leq 10\% \)) until we consider the ET-D or CE sensitivities.

In addition to the obvious benefit of potentially yielding a greater detection horizon than an unmodelled search, it is worth highlighting a number of other advantages of using even a rather ad hoc waveform template such as ours. The strain sensitivity spectrum of GW detectors and the short duration of post-merger GW signals suggest that the pre-merger inspiral signal will always be observed at high SNR for any source which is sufficiently close to observe the post-merger signal. This will lead to a quite precise determination of the time of coalescence, potentially a constraint on the sky-location and constraints on the binary masses. If we also assume that the merger does indeed result in the formation of a stable or quasi-stable NS remnant then the analysis need only consist of inferring the parameters of our waveform model and does not necessarily require a signal to produce an SNR above some detection threshold. Instead, our estimate of the parameters (e.g., \( f_{\text{peak}} \)) simply have greater uncertainty for low SNR signals. This is in contrast to typical burst analyses which require signals to be sufficiently loud that they produce statistically significant loud pixels in the time-frequency plane. We note, however, that a time-frequency PCA could very easily be used to ‘inform’ burst clustering algorithms to better target signals such as these where there is a rich time-frequency structure.

For the purposes of this study, we adopted an SNR threshold \( \rho_{\text{th}} = 5 \) as a fiducial point of reference; in practice, however, it may be possible to determine \( f_{\text{peak}} \) at larger distances (although with correspondingly greater uncertainty) than suggested in this work. Since the Fisher matrix approximation is not valid in the low SNR regime, this point will be investigated via a full Bayesian analysis using our PCA templates in a future study.

Finally, it is worth mentioning that the construction of PCA templates leads to an intriguing and natural way to feed GW observations back to the numerical modelling community to produce a feedback loop for the estimation of the NS EoS and refinement of our waveform models. For now we simply sketch the basic algorithm as follows, leaving an example implementation to future work:

(i) Construct a PCA template from a coarsely sampled catalogue of merger waveforms, whose \( f_{\text{peak}} \) span the full frequency space permitted by allowed EoSs and masses.
(ii) Following a nearby BNS detection, determine the probable component masses from the inspiral signal and the best estimate of \( f_{\text{peak}} \) from the PCA template constructed in (i).
(iii) Produce a refined, more finely sampled catalogue of merger waveforms (with new simulations if necessary) which correspond only to those EoSs and mass configurations
which are compatible with the observations in (ii). Construct a new PCA template from
this catalogue.

This process could then be iterated until some desirable stopping criterion, such as
reaching some critical value of the minimum match between the PCA templates and wave-
forms used, is reached. This approach may provide an avenue to go beyond simply deter-
mining \( f_{\text{peak}} \) and allow an accurate reconstruction of the full spectrum of the underlying
signal. We see then that the application of PCA to construct robust and simple phenomen-
ological templates for the characterisation of post-merger BNS signals holds great promise on
its own, and may provide a useful tool to augment other approaches such as those in
[43, 44, 48].

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Appendix. Continuous wavelet transform of the post-merger signal

In section 2, we used a continuous wavelet transform to explore the time-frequency properties
of the post-merger signal. Here, we expand on some details of this decomposition. The
continuous wavelet transform of a signal \( s(t) \) is defined by

\[
W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t) \psi \left( \frac{t - b}{a} \right) dt,
\]

where \( a > 0 \) represents a set of ‘scales’ to which a mother wavelet function \( \psi \) may be dilated
to produce a set of self-similar basis functions. The parameter \( b \) results in a translation of the
wavelet in time and provides time localisation of signal power. Choosing a mother wavelet
which is well-localised in the time and frequency domains thus results in a time-frequency
decomposition of signal \( s(t) \) which can resolve finite temporal and frequency features. A
complete description of wavelet analysis may be found in [89]. The time-frequency maps
shown in figures 1 and 3 were produced using the Morlet wavelet basis

\[
\psi(x) = \frac{1}{\pi} \exp(2\pi i f_0 x) - \exp(-2\pi^2 f_0^2) \exp(x^2/2),
\]

where \( x = \frac{t - b}{a} \) and \( f_0 \) is the center frequency of the mother wavelet. By considering the
response to a sinusoid of frequency \( f \), it may be shown that \( W(a, b) \) is peaked at
\( a = f_0/f \), so that the foci of the time-frequency response are centered at frequencies
\( tf_0/f, f_0/f \). The center frequency of the mother wavelet \( f_0 \) thus determines the set of frequency resolutions available
to the transform. We find, via visual inspection, that a center frequency \( f_0 = 3 \) yields time-
frequency maps which best resolve both the temporal and frequency content of the post-

\[10 \text{ For discretised time series data, } f = \frac{f_0}{f_s}, \text{ where } f_s \text{ is the sample frequency of the data.}\]
merger signals investigated in this work. Note that we have used the pyCWT software library available at [90] to perform these decompositions.

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