Supplementary Information for

The protective benefits of tsunami mitigation parks and ramifications for their strategic design

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This PDF file includes:
- Figs. S1 to S19
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- References for SI reference citations

Other supplementary materials for this manuscript include the following:
- Movies S1 to S4
Cost Comparison Between Tsunami Walls and Mitigation Parks

Needless to say, many factors contribute to a cost-benefit analysis aimed at comparing different approaches to mitigating tsunami risk. One of these is the construction cost of a tsunami mitigation park as compared to a seawall. According to Reuters (1), the cost of 395 km of a 12.5 m tsunami wall built in Japan was 12.74 billion USD in 2018, which is equivalent to 32.3 million USD per km of wall. If we select parameters such that one line of hills most closely resembles a solid wall like the one built in Japan, i.e., there is no space between 12.5m high ellipsoidal hills with a 25 m diameter base, we find that we need 40 hills per km. These hills are equivalent to an approximate volume of 330,000 m$^3$ per km. We use an order of magnitude cost of 10 USD per m$^3$ for excavation and backfill used in the United States for estimating roadway construction costs (2).

The rough cost estimate we get is 3.3 million USD per km, which is an order of magnitude less than the wall. Obviously, the cost of earthwork is very dependent on whether the material is sourced locally or hauled in, how much compaction is needed, and project design. We did not include permitting, project management costs, and other similar project costs, since they are location-specific.

...more importantly, there are other contributors to the cost of a tsunami mitigation park such as the value of coastal land. In some countries like the U.S., coastal land is very valuable and the additional space required for a hillscape could outweigh potential savings in construction costs, highlighting the need to weigh the two options carefully on a site-by-site basis. In other countries like Indonesia, where almost 55,000 km of coastline are exposed to an intermediate to high tsunami risk, sea walls are problematic not only for cost reasons but also due to their negative impact on coastal ecosystems, potential moral-hazard implications (3), and possible adverse consequences for livelihoods that are tied to coastal appearance and access such as tourism and fisheries. Another potential co-benefit of tsunami-mitigation parks is that they could provide higher ground, which could serve as an evacuation service as considered in the GeoHazards International project in Padang, Indonesia. It is interesting to note that while the negative externalities of large tsunami walls, or co-benefits of a hybrid risk solution like mitigation hills, are rarely considered directly in a cost-benefit analysis, they are important considerations for local communities and decision-makers.

Effect of Hill Spacing on the Protective Benefit

An indirect test for the validity of our argument that reflection provides the main protective benefit is to quantify the dependence of the protective benefit on hill spacing. Increasing the spacing between hills should decrease the amount of reflected flux, because the frontal surface area of the hills is reduced relative to the surface area of the wave front. To test this hypothesis we run an experiment in which we fix the incoming wave amplitude, wave length, and the height of the ellipsoidal hills and vary only the spacing between the hills. In this experiment we use an initial condition with wavelength $r_{obs}/\lambda = 0.015$ and wave amplitude $A/H = 1$. We fix $r_{obs}$ and vary $\Delta_{obs}$ to produce $\Delta_{obs}/2r_{obs} = [1, 2]$. The lower limit of this range has the hills touching at their edges, while in the upper limit the spacing is equal to the diameter of the hills.

We compute the reflected flux by comparing the onshore flux from the no hill case to the onshore flux for each of the different hill spacings. To compare between the cases we compute the total flux, $F$, as the time integral of the flux during the period of net onshore flow through the alongshore transect, which we define as $T_{u>0}$.

$F = \int_{Tu>0} F dt$

Figure S1 shows how much of the total possible onshore flux is reflected in the presence of the obstacles. As expected, increasing the hill spacing decreases the amount of reflected flux.

Contrasting Mitigation Hills to a Uniform Wall

In the limit of hill spacing trending to zero, the hills blend together into an alongshore uniform levee. We ran a set of 9 simulations similar to those described in Table S1, except we now consider hill spacings of 30m, 60m, and 120m for N-waves with amplitudes of 6m, 15m, and 30m. We keep the hill dimension, height and radii, constant at 15m and 30m respectively.

We find that using a hill spacing of 30m with hill radii of 30m results in overlapping hills that effectively behave like a seawall. In Figure S18, we show the free surface shortly after impact ($t = 140 s$) and during run-down ($t = 400 s$) for relatively small, medium, and large tsunamis, as defined in the manuscript by amplitude-to-hill-height ratios of 2/5, 1, and 2 respectively.

We find that for all the tsunami amplitudes tested, the biggest reflection and smallest runup occur with the smallest spacing between the hills, i.e., when the hills are effectively a levee, as shown in Figures S18A, S18C, S18E. One of the challenges then becomes to balance the added protective benefit of the wall with the potential drawbacks associated with the presence of a wall, including negative impacts to coastal ecosystems, potential moral-hazard implications, and possible adverse consequences for local livelihood from fisheries to tourism. The relative benefits of a wall are clearest for small tsunamis, where a wall has the potential to prevent flooding entirely, as shown in Figure S18B. For intermediate to large tsunamis, however, the tsunami will partially overspill or entirely overrun the wall and the differences in protective benefit become less clear.

Another important difference between a continuous wall and a hillscape occurs during the run-down stage of the tsunami. We find that for configurations without gaps for water to drain, the water that overtopped the hills during tsunami impact is now trapped behind the walls and may have to be pumped out. This is especially clear in cases with relatively large tsunamis, such as in Figures S18D and S18F, where water has been permanently trapped for hill spacings of 30m and 60m, causing a longer flooding duration. The construction of the wall hence requires the joint design of a drainage solution, while a hillscape...
facilitates drainage by design. Summarizing, we find that both approaches have their pros and cons, suggesting that site-specific modeling is necessary to weigh these options and determine how much protection can be achieved with the given budget.

Ellipsoidal as Compared to Conical Hills

The manuscript gives an overview over some of the tsunami-mitigation-hill projects that exist or are currently under construction (see Figure 1 in the main manuscript). The form of the hills varies for the different projects. While the tsunami-mitigation park in Constitución Chile entails conical hills, several mitigation parks in Indonesia have a rectangular or ellipsoidal shape such as in the case of the project in South Java. One motivation for choosing an ellipsoidal hill shape is to provide a flat evacuation space at the top of the hill. Here, we test whether the specific shape of the hills significantly reduces the protective benefit of the tsunami-mitigation park.

Two Hill Shapes. Most of the simulations presented in our main manuscript assume an ellipsoidal hill shape, inspired by the Morino project. The ellipsoidal hills chosen to conduct the numerical experiments entail a vertical and hence wall-like orthogonal break in slope at their base which is conducive to reflection. To verify that reflection is not unique to this particular hill shape, we test the effect of replacing ellipsoidal hills with conical hills as featured in the designs for the tsunami mitigation park at Constitución, Chile. Contrary to the ellipsoidal hill shape, conical hills have a constant slope.

We create the topography for \(i = 1, 2, \ldots, N\) hills, with centers defined by a given spacing \(\Delta\) at \((x_i, y_i)\), such that the elevation at a given point on a conical hill, \(z_c(x, y)\), or ellipsoidal hill, \(z_e(x, y)\), is given by equations (1,2):

\[
z_c(x, y) = \begin{cases} \frac{H}{r} (r - l), & \text{if } l \leq r \\ 0, & \text{if } l > r \end{cases}
\]

\[
z_e(x, y) = \begin{cases} \frac{H}{r} \tan(\arccos(l/r)), & \text{if } l \leq r \\ 0, & \text{if } l > r \end{cases}
\]

where \(l = \sqrt{(x - x_i)^2 + (y - y_i)^2}\).

Ellipsoidal Hills Provide More Reflective Protective Benefit. We repeat our simulations testing the effect of wave amplitude while replacing ellipsoidal hills with conical hills with the same height and circular footprint. By changing hill shape while keeping other model parameters constant we investigate how much of the reflected flux is a consequence of hill shape. Figure S2 compares the time evolution of reflected flux between simulations with ellipsoidal and conical hills. The conical hill case has a smaller reflected flux than the ellipsoidal hill case, but is still associated with significant reflected flux. Figure S3 shows the proportion of total flux reflected as a function of wave amplitude for the long wavelength case. The flux reflected is less for the conical hills than for the ellipsoidal hills at all wave amplitudes. While not shown here, these results also hold for the shorter wavelengths. These results demonstrate that the ellipsoidal hill shape is more reflective than the conical hill shape, but that reflection is still a significant mechanism through which conical hills remove kinetic energy from the onshore flow. We emphasize that we do not attempt to quantify erosion here, which is likely also affected by hill shape.

Corroborating Key Finding with another Tsunami Model – GeoClaw

While the usage of numerical models for probing complex, nonlinear flow problems is common, and sometimes the only way of modeling the flow dynamics at its actual spatial scale, there typically are differences between models that arise from the specific numerical methodology used. To reduce potential model bias, we verify that our results are robust to a change in the numerical methodology from NUMA used in the main manuscript to the open-source community model GeoClaw (v 5.4.1). We emphasize that we do not expect the two models to match exactly in every aspect, because they are based on different numerical representations of the governing equation.

GeoClaw is a finite-volume, 2D shallow water equation solver popular in the tsunami modeling community that was adopted in 2012 by the US National Tsunami Hazard Mitigation Program (4–9). It relies on a low order (up to 2nd) finite volume discretization of the shallow-water equations. One characteristic of finite-volumes in the context of hyperbolic conservation laws is their inherent numerical stability as a consequence of numerical dissipation that is naturally embedded into the discretization. From the point of view of a user, this numerical diffusion is advantageous because it helps preserve numerical stability of the solution.

In contrast to GeoClaw, NUMA contains an explicit stabilization term to suppress high-frequency numerical errors. Its main advantage lies in leveraging high-order discontinuous/continuous Galerkin (DG/CG) methods, which naturally translate into zero-dissipation and zero dispersion errors and exponential error decay when the order of the interpolation polynomials is increased. In this study, we run NUMA2D in its discontinuous Galerkin modality. We note that the DG-based NUMA has been developed since 2002 (although not explicitly called NUMA until 2010) and has been benchmarked extensively for ocean modeling including tsunami simulations in (10–18).

Comparison Cases. We select three cases to directly compare results between NUMA2D and GeoClaw, shown in Table S1. In all cases, tsunami wavelength \(A\) is 2000m, hill height \(H\) is 15m, and hill radius \(r\) is 30m. We vary tsunami wave amplitude \(A\) and hill spacing \(\Delta\) to investigate how the two models perform with various hill geometries and initial conditions.
Model Setup. The GeoClaw model setup, shown in Figure S4, is similar to the NUMA2D model setup with some minor differences. Similar to the NUMA2D model, we use a free-slip reflective boundary condition for the longitudinal edges, but use open extrapolation boundary conditions at the far edges to reduce domain size. To reduce the computational cost while still capturing the complex hydrodynamics around the hills, Adaptive Mesh Refinement (AMR) was adopted. In NUMA2D the grid refinement is handled by using coarse grid spacing far from the shore where velocity is nearly zero while the spacing is gradually refined in the shore proximity where water depths are shallow and gradients become important. The NUMA2D grid is always high-resolution near the hills in contrast to the GeoClaw grid which adaptively refines in response to the hydrodynamics. An example of the mesh used by GeoClaw is shown in Figure S5 whereas a schematic of the NUMA2D grid refinement is shown in Figure S6; a detailed visualization of the NUMA2D high-order grid elements is shown in Figure S7.

Since we expect GeoClaw’s second-order accurate Lax-Wendroff scheme to be more numerically diffusive than NUMA2D, we conduct convergence tests shown in Figure S8. We find that an 8m grid size is adequate for the coarsest level of the AMR mesh, which we refine down to 1m in the vicinity of the hills. We handle friction by using a uniform Manning’s value of $n = 0.025$, which is physically representative of a ‘clean and straight channel’, or a ‘floodplain with short grass’ (19). Finally, we assume the parameter in our experiments. In Figure S19 we compare the simulation results at times $t = [160, 175, 220]$ s to the tabulated data available in (20). These validation results show that our numerical model reasonably reproduces the long-wave runup effect of evenly spaced hills on a shore-normal tsunami is alongshore uniform, and find that a 320m-wide domain is sufficiently large to distinguish reflections at the boundary from the primary reflection effect of the hills. All code needed to reproduce our GeoClaw simulations is available on the Sigma Research Group Gitlab: http://zapad.stanford.edu/sigma/tsunami-mitigation-parks. A detailed README file provides instructions.

Main Finding Holds. The purpose of our GeoClaw simulations is to reproduce our key findings with another model to reduce model bias. We observe a similar reflection of the tsunami wave from the hills with the community-endorsed tsunami model GeoClaw. In order to provide a side by side comparison of the results from both models, we show a time-series for each comparison case at a given point in space. We select a point 1,000m offshore in order to capture the N-wave before breaking to ensure both models are forced in a similar way. We find that this distance offshore is far enough outside the noise of the wave-hill interaction but not so far that the slightly higher numerical diffusion in the GeoClaw solution reduces the height of the reflected wave notably. In Figures S9, S10, and S11 we show that GeoClaw and NUMA2D appear to be in agreement with respect to the height of the reflected wave.

Additional Verification Case for NUMA2D. Here we present an additional validation of 1D tsunami run up on a sloping beach (20) to show that NUMA2D is suitable for capturing the impact of a tsunami on the coast as many previous application focused on open-ocean propagation of tsunamis (10) or flow problems on the sphere more generally (13, 17, 18). In this runup validation test the length of the computational domain is $x = [-500, 50, 000]$ m, where the dry beach is 500m long. The initial waveform follows the N-wave definition given by Eq. 3 (21):

$$
\eta = a_1 \exp\{-\hat{k}_1(x - \hat{x}_1)^2\} - a_2 \exp\{\hat{k}_2(x - \hat{x}_2)^2\},
$$

where the constants $(a_1, a_2, \hat{k}_1, \hat{k}_2, \hat{x}_1, \hat{x}_2) = (0.006, 0.018, 0.4444, 4.0, 4.1209, 1.6384)$ are scaled for an 8m domain. To scale the N-wave to the $L = 50000$ m long domain, we introduce the scaling factor $\xi = L/8$ and re-express Eq. 3 with respect to $x_{1,2} = \hat{x}_{1,2} \xi$ and $k_{1,2} = k_{1,2}/\xi^2$; we also utilize larger amplitudes $(a_1, a_2) = (3.0, -8.8)$. We define the parameters of Eq. 3 as $\hat{x}_1 = 1000 + 0.5151125\lambda$, $\hat{x}_2 = 1000 + 0.2048\lambda$, $k_1 = 28.416/\lambda^2$, $k_2 = 256/\lambda^2$, $a_1 = A$, and $a_2 = A/3$, where $\lambda$ and $A$ are parameters of our simulations. The parameter $A$ is roughly the height of the offshore propagating bore. The parameter $\lambda$ is the wavelength of the initial condition. The horizontal spacing between the center of two hills, $\Delta$, is the spatial configuration parameter in our experiments. In Figure S19 we compare the simulation results at times $t = [160, 175, 220]$ s to the tabulated data available in (20). These validation results show that our numerical model reasonably reproduces the long-wave runup benchmark results.

Potential and Kinetic Energies

Potential energy is calculated as

$$
E_p(t) = \rho g \int \int h(t, x, y) dz dx dy
= \frac{1}{2} \rho g \int \int h(t, x, y)^2 dx dy
$$

The analytical solutions are calculated in an $x - z$ domain so that the potential energy per unit width becomes

| Case Number | $A$ (m) | $\Delta$ (m) | $A/H$ | $\Delta/2r$ |
|-------------|---------|--------------|-------|-------------|
| 1           | 15      | 120          | 1     | 2           |
| 2           | 6       | 60           | 0.4   | 1           |
| 3           | 30      | 60           | 2     | 1           |

Table S1. Selected cases for model validation
Kinetic energy is calculated as

\[
E_k(t) = \frac{1}{2} \rho \int \int h(t,x,y) |U(t,x,y)|^2 dx dy = \frac{1}{2} \rho \int h(t,x,y)|U(t,x,y)|^2 dx dy
\]

which is evaluated per unit width as

\[
E_{k,width}(t) = \frac{1}{2} \rho \int h(t,x)|U(t,x)|^2 dx
\]

**Derivation of the Energy Flux**

To derive the energy flux from the shallow water equations we follow the approach used in (22) for the Navier-Stokes equation. We start with the governing equations for momentum and continuity, excluding the stabilization terms included for numerical purposes:

\[
\frac{\partial h \mathbf{u}}{\partial t} + \nabla \cdot (h \mathbf{u} + \frac{g}{2} (h^2 - H^2_b) \mathbf{I}) + g \eta \nabla \cdot (H_b \mathbf{I}) = -n^2 g \frac{|\mathbf{u}|}{H^{1/3}}, \quad [4]
\]

\[
\frac{\partial h}{\partial t} + \nabla \cdot (h \mathbf{u}) = 0, \quad [5]
\]

The 3rd and 4th terms are a rewrite of the free surface gradient term, \(gh \nabla \cdot \eta \mathbf{I} = \nabla \cdot (\frac{g}{2} (h^2 - H^2_b) \mathbf{I}) + g \eta \nabla \cdot H_b \mathbf{I}\), representing the force exerted onto the water volume by the tilt of the evolving water surface, yielding

\[
\frac{\partial h \mathbf{u}}{\partial t} + \nabla \cdot (h \mathbf{u}) = -gh \nabla \cdot \eta \mathbf{I} - n^2 g \frac{|\mathbf{u}|}{H^{1/3}}. \quad [6]
\]

Since the water surface is a free surface, it cannot do work and thereby alters the energy balance only through dissipation, similarly to the frictional loss of energy at the lower interface represented by the second term on the right hand side. From an energetic point of view, both terms on the right hand contribute only to the dissipation occurring at the upper and lower surface respectively.

To derive the energy flux, we take \(\mathbf{u} \cdot (6), gh \ (5)\), sum the results and divide by two:

\[
\frac{\partial}{\partial t} \left( hq + \frac{gh^2}{2} \right) + \nabla \cdot \left[ \frac{gh^2}{2} \mathbf{u} + hq \mathbf{u} \right] = \phi \quad [7]
\]

\[
\frac{\partial}{\partial t} (hq + \frac{gh^2}{2}) + \nabla \cdot \mathbf{f} = \phi \quad [8]
\]

\[
\mathbf{f} = \mathbf{u} \left[ \frac{gh^2}{2} + hq \right], \quad [9]
\]

where \(q = \frac{1}{2} \mathbf{u} \cdot \mathbf{u}\) and \(\phi\) is the total dissipation due to the Manning model of bottom shear. By construction, the dynamic numerical dissipation that is added for numerical stability only contributes to the dissipation of the high-wave number energy build-up triggered by numerical instabilities in the proximity of sharp fronts. Equation 9 is the local energy flux, which can be computed at each location and at each timestep.

**Effect of Bathymetry Slope on Protective Benefit of Hills**

Tsunami runup is highly dependent on bathymetry (23, 24). In this study, we use a constant slope \(\alpha = 1/50\) to narrow our focus on hill geometry. We now consider additional slopes to check whether the primary reflection of onshore flux is significantly altered for other slopes. To inform the selection of these slopes, we process GEBCO (25) raster files to extract bathymetry transects normal to the shore for convergent plate boundaries in regions where parks are being built: Chile, Japan, and Indonesia. We plot these transects from the shoreline to the plate boundary in Figure S12. Since we initialize our tsunami wave 2km from shore, we use these figures to guide our choice of slope. We decide to study two additional slopes, \(\alpha = 3/40\) and \(\alpha = 1/100\). For these simulations, we modify the bathymetry for elevations below the still water level. We do not modify the topography because doing so would also alter the elevation of the hills.
We compare three slopes and find that the height of the reflected wave is not significantly affected as shown in Figure S13. Since GeoClaw is a 2D depth-averaged model, it does not fully capture the nonlinear dynamics associated with wave breaking. It does, however, simulate quite different wave breaking behavior for the N-waves caused by the different slopes, as shown in Movie S1. In Figure S14a, we observe the wave with $\alpha = 3/40$ approaching shore faster than the other waves, and shoaling in such a manner that a higher tsunami is created. Most notably, the run-down processes appear to differ significantly for each slope, as shown in Figure S14b. In general, scenarios with steeper bathymetries appear to exhibit a faster tsunami run-down. Due to the highly nonlinear behaviors involved with wave breaking for different slopes, tsunami properties, and hill geometries, it is not wise to draw general conclusions. We do feel confident in stating that the primary reflective benefit of mitigation hills, the reflection of onshore flux, is not sensitive to the slope of the bathymetry.

**Effect of N-Wave Shape on Protective Benefit of Hills**

An N-wave is a more appropriate representation of a tsunami caused by seismic fault dislocation than a soliton (26). Depending on location relative to a fault, N-wave shapes can have a leading depression or a leading elevation. Leading depression N-waves may lead to higher runups and could be more destructive (27). We test both N-wave types to determine if the protective benefit of mitigation hills depends on wave shape. To model a leading elevation N-wave, we modify one parameter such that $\hat{x}_2 = 1000 + 0.825425\lambda$, which essentially shifts the center of the smaller, negative Gaussian symmetrically about the center of the main Gaussian. We run simulations for both N-wave shapes using the parameters in comparison case 1 and find that the height of the reflected wave is not affected, as shown in Figures S15, S16c. We initialize the depression of the N-wave to be symmetric about the main Gaussian, meaning that the centroids of the two N-waves were not aligned and the leading depression was closer to the hills than the leading elevation, as shown in Figure S16a. By the time the two N-waves shoal and impact the hills, their shape is quite similar as shown in Figure S16b, although the velocity fields are different. Interestingly, we find that the reflection from the leading elevation N-wave occurs slightly earlier due to differences in the velocity field. We note that run-down processes appear to be more significantly impacted by wave shape than the initial reflection as shown in Movie S2.

**Vegetation Complements Protective Benefit of Hills**

Tsunami mitigation parks are a hybrid nature-based solution involving an engineered element, the hills, and a green element, vegetation. The hills provide an opportunity for strategic design, as shown in this study, and the vegetation provides complementary benefits in two main ways: first, drag can slow down the flow and thus reduce runup and second, roots can hold sediment in place and prevent erosion. We do not study the latter, but briefly evaluate the former by repeating the simulations in comparison case 1 where the area surrounding the hills has a modified Manning’s $n$ value. We consider values of $n = 0.050$, representative of a "floodplain with high grass"; and $n = 0.200$, representative of a "floodplain with dense willows (19)." Since the hills are set back from the shore by 60m, we consider a vegetated area surrounding the hills where $0 \leq x \leq 120$. We find that adding a large area with higher drag surrounding the hills serves to reduce tsunami runup, as shown in Figure S17b, c and in Movie S4. Vegetation modeled with $n = 0.050$, or "high grass", provides a minimal reduction in runup, whereas vegetation modeled with $n = 0.200$, or "dense trees", has a much higher impact in terms of reducing runup and also increasing the size of the reflected wave. Planting a dense forest on the hills could reduce runup and also enhance the protective benefit of the hills in terms of reflection. It is worth noting that Manning’s $n$ values are empirically derived under experimental conditions that are not as extreme as tsunamis, and we may be overestimating the amount of drag that vegetation exerts on flow due to the fact that it can bend, be damaged, or even uprooted. These results are shown mostly to highlight that further work on hybrid solutions is needed to fully understand their value.
Fig. S1. Reflected mass flux as a percentage of total onshore mass flux in the absence of obstacles with a fixed hill wave amplitude and hill height.
Fig. S2. Timeseries of mass flux during net onshore flow through alongshore transect at alongshore line through hill centers. Comparison between mass flux with no hill, with ellipsoidal and conical hills, and the differences between no hill and hill (reflected flux).
Fig. S3. Percentage of reflected mass flux relative to net mass flux with no hill as a function of $A/H$ for simulations with ellipsoidal hills and with conical hills.
Fig. S4. Model setup not to scale.
Fig. S5. AMR example grid with 8m, 4m, and 1m cells, i.e. a factor of 2 and 4 refinement, from left to right. We refine the grid in specified regions to accurately capture dynamics in the vicinity of the hills. We determine adequate AMR boundaries in the x-direction by trial and error. For purposes of convenient model setup and result visualization we use grid sizes of 1, 4, and 8 meters.
Fig. S6. Illustrative representation of the NUMA2D grid refinement scheme. The alongshore grid spacing is fixed at 2 m. A) In deep water far from shore the grid is long and rectangular, the alongshore velocity component is zero. The maximum cross-shore grid spacing at the far offshore boundary of the domain is 4.5 m. B) The grid compresses in the x-dimension as the water depth grows shallower. C) In the vicinity of the shore and hills the grid spacing is roughly square. The minimum cross-shore grid spacing is 1.7 m.
Fig. S7. Visualization of grid elements in the proximity of the hills.
Fig. S8. Time-series of convergence tests for the tsunami described in comparison case 1 with no hills at a fixed point 1000m offshore (x=1000, y=0). We note that convergence is achieved around 8m, with almost no difference in tsunami shoaling, breaking, runup and run-down. The purpose of this convergence test was to determine the size of the coarsest mesh needed. No hills were used in model setup since mesh refinement up to 1m takes place in the vicinity of the hills. See model setup.
Fig. S9. Time-series for comparison case 1 at a fixed point 1000m offshore (x=1000, y=0)
Fig. S10. Time-series for comparison case 2 at a fixed point 1000m offshore (x=1000, y=0)
Fig. S11. Time-series for comparison case 3 at a fixed point 1000m offshore (x=1000, y=0)
Fig. S12. Bathymetry transects for a) Japan at 37.5° N, b) Chile at 24.5° S, and c) Indonesia at 110° E from shoreline to convergent plate boundary. Slopes of $\alpha = 1/100, 1/50, 3/40$ shown in dashed yellow, orange, and brown lines respectively.
Fig. S13. Time-series for case 1 showing slope comparison at a fixed point 1000m offshore (x=1000, y=0)
Fig. S14. Snapshots in time for case 1 showing slope comparison at a) $t = 60 \text{s}$: we observe the waves shoaling at different speeds, slower for shallow bathymetries probably due to bottom friction b) $t = 160 \text{s}$: we observe the reflected wave in all slope scenarios, and run-down processes occurring for slope $\alpha = 3/40$ while runup is still occurring in the other two cases.
Fig. S15. Time-series for N-wave comparison at a fixed point 1000m offshore (x=1000, y=0)
Fig. S16. Snapshots in time for case 1 showing N-wave comparison at a) $t = 0\,\text{s}$: we show how the different N-wave shapes are initialized symmetrically about the center of the primary Gaussian, meaning the leading elevation N-wave is actually closer to shore b) $t = 80\,\text{s}$: we observe that despite the leading elevation N-wave being initialized closer to shore, the two shoal in such a way that they appear very similar before impact; although not shown or considered in detail, this implies the velocity fields must be significantly different c) $t = 250\,\text{s}$: we observe a similar reflected wave for both N-wave shapes, and note that the run-down process appears to be more interesting in terms of N-wave effects.
Fig. S17. Snapshots in time for case 1 showing vegetation comparison at a) $t = 85s$: we show the N-wave is identical for all vegetation cases before impact b) $t = 160s$: the runup for a bare slope and "high grass" cases is very similar, with the biggest difference noticed in the region immediately behind the hills, while the runup for the "dense trees" case is significantly lower and the reflected wave is bigger and noticed sooner c) $t = 290s$: here we show the run-down process, which is not the focus of this study but we note that a higher drag leads to more water held behind the hills and probably a longer run-down period.
Fig. S18. Free surface transects of tsunami waves with amplitudes of 6m (A,B), 15m (C,D), and 30m (E,F) shortly after impact with hills at $t = 140$ s (A,C,E) and later during run-down at $t = 400$ s (B,D,F).
Fig. S19. 1D tsunami runup as computed with DG compared to benchmark solution. Near field plot of the water surface at $t = [160, 175, 220]$ s.
Movie S1. slopesMovieS1.mp4 shows the effect of different bathymetry slopes

Movie S2. nWaveMovieS2.mp4 shows the effect of different N-wave shapes

Movie S3. vegMovieS3.mp4 shows the effect of vegetation

Movie S4. case1SurfaceS4.mp4 shows an animation of the free surface to better visualize complex hydrodynamics

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