Structure functions in rotating Rayleigh–Bénard convection

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Abstract. A combined numerical–experimental investigation on the scaling of velocity structure functions in turbulent rotating Rayleigh–Bénard convection is carried out. Direct numerical simulations in a cylindrical domain and a horizontally periodic domain are compared with experiments using a cylindrical tank in which stereoscopic particle image velocimetry is employed. The turbulent length scales that govern the scaling of the structure functions are evaluated directly in the numerical simulations. They provide a framework for the interpretation of the structure functions. The composition of the domain (cylinder/periodic) has a quantitative effect on the length scales even in the fluid bulk. At lower rotation rates an additional scaling range due to rotation is found. At higher rotation rates a direct transition is observed from dissipation-range scaling at small separations to an uncorrelated state at larger separations.

1. Introduction

Rotation is an important ingredient of the large-scale geophysical flows found in the atmosphere and the oceans on our planet. These flows are generally driven by buoyancy. As a model for these flows we consider an extension to the classical Rayleigh–Bénard problem: a layer of fluid confined between horizontal plates, heated from below and cooled from above, is rotated about a vertical axis. Rotation causes a change in flow phenomenology: while non-rotating convection is typically ordered in mushroom-shaped plumes that tend to cluster into convection rolls in confined geometries (Krishnamurti & Howard, 1981; Ahlers et al., 2009), its rotating counterpart is made up of vortical columns that are aligned with the rotation vector (Boubnov & Golitsyn, 1986; Zhong et al., 1993; Sakai, 1997).

Here we focus on the effects of rotation on the small-scale structure of convective turbulence by calculating velocity structure functions (SFs) from measurements using stereoscopic particle image velocimetry (SPIV), as well as from direct numerical simulations (DNS). The SFs are defined as $S_p(r) = ⟨|\delta u(r)|^p⟩$, the moments of order $p$ of the velocity differences $\delta u(r) = u(x + r) - u(x)$ between points as a function of the separation $r$.

According to Kolmogorov’s classic theory of homogeneous and isotropic turbulence (Kolmogorov, 1941, hereafter cited as K41) the SFs should scale as $S_p(r) \sim r^{p/3}$ between the smallest length scale $r = η$ (the Kolmogorov length) and the integral length scale $L$. It was soon found,
however, that intermittent strong events occur in turbulence that cause deviations from pure K41 scaling, see, e.g., Frisch (1995).

The effect of buoyancy on the SFs has been discussed at length in the past (see Lohse & Xia, 2010, for a recent review). The theoretical picture based on dimensional analysis predicts the coexistence of two scaling ranges separated by the Bolgiano length $L_B$: for $η < r < L_B$ the K41 scaling is found, while for $L_B < r < L$ the steeper Bolgiano–Obukhov scaling $S_p(r) \sim r^{3p/5}$ is expected (Bolgiano, 1959; Obukhov, 1959, hereafter designated BO59). It is still inconclusive whether BO59 scaling exists in turbulent convection. However, effects of rotation are expected to be found in the SFs given the transitions in flow phenomenology and the related anisotropy (Kunnen et al., 2008b, 2010). The scaling properties of (isothermal) rotating turbulence have been investigated frequently, e.g. by Zhou (1995); Baroud et al. (2003); Müller & Thiele (2007); Mininni & Pouquet (2009); van Bokhoven et al. (2009). The limiting cases are the K41 result for small rotation rates, while for stronger rotation a scaling $S_p(r) \sim r^{p/2}$ is found above the rotational length scale $L_Ω$ for flows with zero mean helicity $u \cdot \omega = u \cdot (\nabla \times u)$ and a steeper scaling $S_p(r) \sim r^{3p/4}$ for flows with maximal mean helicity (Mininni & Pouquet, 2009).

We thus expect a transition from K41 scaling to rotation-dominated scaling with possibly some effects of buoyancy. The decisive factors in this process are the turbulent length scales $η$, $L_B$ and $L_Ω$. These length scales are calculated in the DNS and will be used in the interpretation of the SF results.

This paper is organised as follows. First, in section 2 we present the experimental and numerical methods employed in this work. A discussion of the turbulent length scales from DNS is given in section 3. Section 4 contains the presentation of the structure function results, which will be interpreted with the aid of the turbulent length scales. We summarise our findings in section 5.

2. Experimental and numerical methods

2.1. Experimental setup
A cylindrical tank of diameter and height $D = H = 23$ cm is filled with water containing PIV seeding particles and is placed on a rotating table, so that the rotation axis and cylinder axis coincide. A heated copper plate seals the cylinder from below. A transparent cooling chamber seals the cylinder from above. The temperature difference $ΔT$ between top and bottom is controlled; an effective Rayleigh number (dimensionless temperature difference) $Ra = gαΔTH^3/(νκ) = 6 \times 10^8$ is maintained (Kunnen et al., 2010). Here $g$ is the gravitational acceleration, $α$ is the thermal expansion coefficient, and $ν$ and $κ$ are the diffusion coefficients for momentum and temperature, respectively. The Prandtl number is under these conditions $σ = ν/κ = 6.4$. The rotation rate, expressed in dimensionless form by the Taylor number $Ta = (2ΩH^2/ν)^2$, is varied between experiments: $0 \leq Ta \leq 2.2 \times 10^{10}$.

A corotating laser is used to generate a light sheet that traverses the cylinder horizontally, illuminating the seeding particles. Two cameras mounted above the tank at a mutual angle record the tracer patterns. The stereoscopic view allows for the simultaneous resolution of the three components of velocity in the light sheet plane using an SPIV algorithm. Two measurement heights are considered. The effective measurement area at height $z = 0.8H$ is a rectangle of $15 \times 12$ cm$^2$ centered at the rotation axis; at height $z = 0.5H$ the covered area is $12 \times 9$ cm$^2$. The mutual field of view of the cameras is reduced in size as $z$ is lowered due to the off-axis placement and the nonperpendicular viewing angle of the cameras.
2.2. Numerical methods

In the DNS we consider an incompressible Boussinesq fluid, the flow of which is governed by the following set of equations:

\begin{align}
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \left( \frac{\sigma T a}{Ra} \right)^{1/2} \hat{z} \times \mathbf{u} &= -\nabla p + \left( \frac{\sigma}{Ra} \right)^{1/2} \nabla^2 \mathbf{u} + T \hat{z}, \\
\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T &= \frac{1}{(\sigma Ra)^{1/2}} \nabla^2 T, \\
\nabla \cdot \mathbf{u} &= 0.
\end{align}

(1a)

(1b)

(1c)

Here \( \mathbf{u} \) is the dimensionless velocity scaled with the convective velocity \( U = (g\alpha \Delta T H)^{1/2} \), \( t \) is the dimensionless time scaled with \( \tau = H/U \), \( \hat{z} \) is the vertical unit vector pointing upward, parallel to the rotation vector and antiparallel to gravity, \( p \) is the reduced pressure modified by the centrifugal term, and \( T \) is the temperature normalised with \( \Delta T \) such that \( 0 \leq T \leq 1 \).

We simulate the flow governed by these equations in two configurations.

The first uses a cylindrical flow domain where the cylinder axis is aligned with the rotation axis. All the bounding walls are no-slip. The sidewall is adiabatic: \( \partial T / \partial r = 0 \), where \( r \) is the radial coordinate. The bottom and top plates are kept at constant temperatures \( T = 1 \) and \( 0 \), respectively, prescribing a destabilising temperature gradient. The grid is finer near the bottom and top plates, as well as close to the sidewall, so that the boundary layers are represented with enough accuracy (Stevens et al., 2010; Shishkina et al., 2010). Due to the cylinder geometry a reformulation in cylindrical coordinates \((r, \theta, z)\) is natural. Central second-order finite-difference discretisations are used. To avoid the occurrence of singularities in the discretised equations the flow is solved in terms of the vector \((ru_r, u_{\theta}, u_z)\) and by using a staggered grid. Due to the staggered grid, only terms containing the radial velocity component need to be evaluated at the cylinder axis \( r = 0 \), which conveniently reduce to zero as \( ru_r = 0 \) there. For more details on the discretisation we refer to Verzicco & Orlandi (1996). The discretised system is solved by a fractional-step procedure with the elliptic equation inverted using trigonometric expansions in the azimuthal direction and a direct solver for the other two directions. For time-stepping a third-order Runge-Kutta scheme is employed.

The second configuration is a rectangular domain of size \( L_x \times L_y \times L_z = 2 \times 2 \times 1 \), with periodic boundary conditions in the horizontal directions. In the vertical directions isothermal no-slip boundaries border the fluid. Again a grid refinement is applied close to the plates. The finite-difference formulations are based on the symmetry-preserving discretisation proposed by Verstappen & Veldman (2003). Mass and momentum conservation is ensured by conservation of the symmetries of the differential operators found in the Navier-Stokes equations in their discrete representations. By use of a Richardson extrapolation the scheme is fourth-order accurate in space. Time integration uses a so-called one-leg scheme (one evaluation of the fluxes per time step) similar to the well-known Adams-Bashforth scheme. In the periodic domain it implicitly assumed that the length scales governing the flow are much smaller than the horizontal dimension of the domain, i.e. the periodicity length does not affect the length scales of the turbulence. \textit{A posteriori} this is validated; the turbulent length scales are much smaller than two.

The DNS runs are carried out at \( Ra = 1 \times 10^9 \), \( \sigma = 6.4 \). Due to its relevance for comparison with the experiments we consider the flow in the cylindrical domain at several Taylor numbers \( Ta = 0, \ldots, 4.8 \times 10^9 \). We calculate one reference case in the periodic domain at \( Ta = 2 \times 10^7 \), where rotational effects are expected in the SFs, but rotation is not yet dominant.
3. Turbulent length scales

The turbulent length scales that bound the SF scaling ranges are formulated in terms of the dissipation rates of turbulent kinetic energy $\epsilon$ and the temperature variance dissipation rate $N$, which are given in the current dimensionless units by:

$$
\epsilon = \frac{1}{2} \left( \frac{\sigma}{Ra} \right)^{1/2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \quad N = \frac{1}{(Ra \sigma)^{1/2}} \left( \frac{\partial T}{\partial x_i} \right)^2,
$$

where summation over repeated indices is implied. Although difficult to measure in experiments, the dissipation rates are readily available in DNS. The angular brackets denote averaging in time and in the homogeneous directions, viz. the periodic directions in the rectilinear domain and the azimuthal direction in the cylinder. There is a strong dependence of the mean dissipation rates on the vertical coordinate, and, in the cylinder, on the radial coordinate.

For the velocity SFs the smallest length scale in the inertial range is the Kolmogorov length $\eta$, which in the current dimensionless form is given by

$$
\eta = \left( \frac{\sigma}{Ra} \right)^{3/8} \epsilon^{-1/4}.
$$

The transition between the K41 and BO59 scalings is expected at the Bolgiano length scale $L_B$ (Bolgiano, 1959; Obukhov, 1959; Lohse & Xia, 2010), which is in this case

$$
L_B = \frac{\epsilon^{5/4}}{N^{3/4}}.
$$

Additionally, the rotation adds a length scale $L_\Omega$ which is based on a dimensional estimate including the rotation rate $\Omega$ and $\epsilon$ (Zhou, 1995), here given by

$$
L_\Omega = 2^{3/2} \left( \frac{Ra}{\sigma T a} \right)^{3/4} \epsilon^{1/2},
$$

where the numerical prefactor is due to the factor 2 in the definition of the Taylor number $Ta$. However, since all three length scales $\eta$, $L_B$ and $L_\Omega$ are based on dimensional arguments only, numerical prefactors may be needed to match with the actual results.

As we have shown before for the non-rotating case (Kunnen et al., 2008c), $\epsilon$ tends to attain higher values close to the no-slip walls where viscous boundary layers are formed, while $N$ has its highest values near the isothermal plates in the thermal boundary layers. In the fluid bulk there is a smaller dependence of $\epsilon$ and $N$ on the actual position under consideration. For horizontal cross-sections of the bulk the length scales $\eta$ and $L_B$ are almost constant.

The rotation, while not explicitly present in the definitions of $\epsilon$ and $N$, changes the magnitude and spatial distribution of the dissipation rates. The domain composition is also an important factor, as is shown in figure 1. Here the vertical distributions of $\epsilon$ and $N$ are shown at $Ta = 2 \times 10^7$ in a comparison between cylinder (close to the cylinder axis) and periodic domain. In the central part of the domain there is a quantitative difference between the two domains: in the periodic domain $\epsilon$ is roughly 75% higher than in the cylinder, while at some heights $N$ is about two times as large as in the periodic domain than in the cylinder. A note on the boundary-layer behaviour of $\epsilon$ in the cylinder: in practice we approximate $\epsilon$ in the cylinder by the so-called pseudo-dissipation $\tilde{\epsilon} = (\sigma/Ra)^{1/2} \langle |\nabla u|^2 \rangle$ (Pope, 2000, p. 132). The difference between the two formulations is negligibly small, except for deep within the viscous boundary layer, where $\tilde{\epsilon}$ goes to 0 while $\epsilon$ attains its maximal value (periodic domain).
The effects of rotation on the turbulent length scales are evaluated at the two measurement heights $z = 0.5$ and 0.8. We know that at $Ta_c = 2.5 \times 10^7$ there is a transition between turbulent states in the flow, evidenced by a ‘kink’ in the heat transfer graph as a function of rotation (Kunnen et al., 2008a; Stevens et al., 2009). It is related with a change in flow phenomenology from a large-scale convection roll to vortical plumes.

The dependence on $Ta$ of the Kolmogorov length $\eta$ is depicted in figure 2. There is only

**Figure 1.** Comparison of the vertical distribution of the dissipation rates between the cylinder and the periodic domain at $Ta = 2 \times 10^7$: (a) kinetic energy dissipation rate $\epsilon$, (b) thermal variance dissipation rate $N$.

**Figure 2.** Rotational dependence of the Kolmogorov length $\eta$ at the two measurement heights $z = 0.5$ and 0.8. The symbols on the left vertical axis represent the non-rotating case $Ta = 0$. The vertical dashed line indicates $Ta_c = 2.5 \times 10^7$.

**Figure 3.** Rotational dependence of the Bolgiano length $L_B$ at the two measurement heights $z = 0.5$ and 0.8. The symbols on the left vertical axis represent the non-rotating case $Ta = 0$. The vertical dashed line indicates $Ta_c = 2.5 \times 10^7$. 
a minor dependence on rotation. Furthermore, the small difference between the results at the two measurement heights vanishes at higher $Ta$, indicating a homogeneous bulk distribution of $\epsilon$ and $\eta$ under strong rotation. The large quantitative difference in $\epsilon$ between cylinder and periodic domain is also reflected in $\eta$, although a smaller and opposite difference is found due to the small negative exponent $\epsilon^{-1/4}$ of equation (3).

The same evaluation is carried out for the Bolgiano length $L_B$ in figure 3. Under slow rotation $L_B$ is not so different from its non-rotating value; at $z = 0.5$ a slight increase is found. For $Ta > Ta_c$, however, at both measurement positions $L_B$ is reduced when $Ta$ increases. The Bolgiano length is larger in the periodic domain than in the cylinder.

Finally, we consider the rotational length $L_\Omega$. This is depicted in figure 4. The rotational length scale is only defined for $|\Omega| > 0$. Again there is only a minor difference between $z = 0.5$ and 0.8. However, a rather strong dependence on $Ta$ is found, with a slope change around the transitional $Ta$ value. For $Ta > Ta_c$ the power-law scaling $L_\Omega \sim Ta^{-3/4}$ expected by the definition (5) is observed. For $Ta < Ta_c$ rotation has a less pronounced effect.

From the evaluations of the length scales we can conclude that there are two regimes with the boundary corresponding to the previously reported heat transfer transition at $Ta_c = 2.5 \times 10^7$ (Kunnen et al., 2008a; Stevens et al., 2009). Above the transition point a more pronounced dependence of the length scales on $Ta$ is observed.

4. Structure functions

With the information gained on the turbulent length scales we can now consider the SFs and interpret the rotational effects. We will confine ourselves to the scaling of the second-order SFs of the vertical velocity component, since the vertical component of velocity is directly affected by buoyancy.

The SFs $S_2^{u_z}$ as calculated from the cylinder DNS are displayed in figure 5 as a function of the dimensionless separation $r$. The two measurement heights $z = 0.5$ and 0.8 are considered. Several Taylor numbers are included to probe the effects of rotation. These plots are shifted vertically for clarity. The lowest curve is at $Ta = 0$. $Ta$ increases from bottom to top, the top curve corresponding to $Ta = 1.2 \times 10^9$. On each curve the corresponding length scales $L_B$
Figure 5. Structure functions $S_u^2$ calculated in the radial direction from the cylinder DNS at various Taylor numbers. From bottom to top: $Ta = 0$, $4.7 \times 10^6$, $1.9 \times 10^7$, $7.5 \times 10^7$, $3.0 \times 10^8$ and $1.2 \times 10^9$. Two measurement heights are included: (a) $z = 0.5$, (b) $z = 0.8$. The corresponding turbulence length scales $L_B$ (circles) and $L_\Omega$ (crosses) are included, as are reference slopes (thin solid lines). The curves are shifted apart vertically for clarity.

and $L_\Omega$ are indicated. The Kolmogorov scale $\eta$ is not indicated; it attains a value $r \approx 6 \times 10^{-3}$ in all cases which is on the far left-hand side of each curve.

On the low-$r$ end the SFs describe the expected $\sim r^2$ scaling characteristic of the dissipation range (Pope, 2000, p. 195). The transition to the inertial range, formally at $r = \eta$, is actually found at eight to ten times larger distances. In the three lowest curves, at both measurement heights, a K41 scaling range can be recognised. BO59 scaling is absent, even though $L_B$ is well within the measurement range. Effects of rotation are found for $r > L_\Omega$ in the cases when $L_\Omega > L_B$. A tendency to rotation-dominated scaling $\sim r^{\rho/2}$ is found for $r > L_\Omega$. At both heights it is found that rotation reduces the integral length scale $L$, the scale at which the SF reaches a plateau with constant value for $r > L$. The plateau indicates that velocities at these separations are uncorrelated. At the two highest $Ta$ values shown there is a nearly direct transition from dissipation-range scaling to the plateau.

The same analysis has been carried out for the SFs calculated from SPIV measurements. These results are shown in figure 6. The SFs at various $Ta$ are depicted along with the length scales based on the results in figures 2, 3 and 4 interpolated to the corresponding $Ta$ values. It must be noted that experiment and DNS do not match exactly in terms of $Ra$: the DNS were run at $Ra = 1 \times 10^9$, while the experiments had an effective Rayleigh number $Ra = 6 \times 10^8$ (Kunnen et al., 2010).

The spatial resolution achieved in the experiments was not sufficient to accurately resolve the dissipation-range scaling at small $r$: the Kolmogorov scale $\eta$ was smaller than the minimal $r$ value. This is a trade-off due to the current interest in the larger separations, where the changes due to rotation are expected. At $Ta = 0$ an indication of BO59 scaling is found for $r > L_B$ at $z = 0.5$, while at $z = 0.8$ another scaling exponent $r^{4/3}$ appears. We discussed this topic before in detail (Kunnen et al., 2008c): the presence of the large-scale circulation causes a shear-dominated scaling range at $z = 0.8$ with characteristic scaling $S_p \sim r^{2\beta/3}$ (Lohse, 1994). At $Ta = 2.1 \times 10^7$ (second curve from below) the scalings are inconclusive, although some effect of rotation may be recognised at the large-$r$ end. The differences between DNS and experiment
Figure 6. Structure functions $S_{u^2}$ calculated in the radial direction form the experiments at various Taylor numbers. From bottom to top: $Ta = 0, 2.1 \times 10^7, 8.4 \times 10^7, 3.4 \times 10^8,$ and $5.3 \times 10^9$. Two measurement heights are included: (a) $z = 0.5$, (b) $z = 0.8$. The corresponding turbulence length scales $L_B$ (circles) and $L_\Omega$ (crosses) are included, as are reference slopes (thin solid lines). The curves are shifted apart vertically for clarity.

(Ra is different, and the top boundary in the experiment is not a perfect heat conductor as it is in the DNS) are likely causes of the discrepancy. Nevertheless, at higher $Ta$ a very similar result to the DNS is found: a direct transition from dissipation-range scaling to the plateau.

The SFs are also calculated from the periodic DNS. These results are compiled in figure 7. Although the magnitude of the length scales is different than in the cylindrical geometry, the general shape of the curves is quite similar. The dissipation-range scaling is found on the small-$r$ end. For $r \gtrsim 0.3$ the SFs do not rise further, in contrast with the cylinder case. It is plausible that the large-scale circulation in the cylinder enhances the correlation length, while in the periodic geometry no such large flow structure is present (or at least not as strong).

5. Conclusions

The effects of rotation on the velocity structure functions of turbulent convection have been investigated with SPIV measurements and DNS. The DNS also provided estimates for the turbulent length scales that govern the scaling of the SFs. The composition of the domain (cylinder or periodic) has a direct quantitative effect on these length scales even in the flow far away from the boundaries. Furthermore, it is found that the rotational length scale is a good indicator of the length scale above which rotation can affect the SFs. The existence of a BO59 scaling range is inconclusive, but it is found that at the considered nonzero rotation rates it is absent.

An interesting observation is that in the high-$Ta$ cases, for which $L_\Omega < L_B$, the transition to the plateau in the SFs is found at $r = L_B$. In the original formulation of the theory by Bolgiano (1959) and Obukhov (1959), which was actually derived for turbulence in a stably stratified fluid, the scale $L_B$ was regarded as the scale of energy input. There may be an intimate connection of $L_B$ with the vortical plumes that govern the flow. The vortices may act as the source of energy injection, so that $L_B$ may be an indicator of their characteristic size. We are currently investigating this connection in more detail.
Figure 7. Structure functions $S^{u_z}_2$ calculated in the horizontal direction from a DNS in a horizontally periodic domain at $Ta = 2 \times 10^7$. The curves obtained at the two measurement heights $z = 0.5$ and $0.8$ are shifted apart vertically for clarity.

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References

AHLERS, G., GROSSMANN, S. & LOHSE, D. 2009 Heat transfer and large scale dynamics in turbulent Rayleigh–Bénard convection. Rev. Mod. Phys. 81, 503.
BARoud, C. N., PLaPP, B. B., SWINNEY, H. L. & SHE, Z.-S. 2003 Scaling in three-dimensional and quasi-two-dimensional rotating turbulent flows. Phys. Fluids 15, 2091–2104.
VAN BOKHOVEN, L. J. A., CLERCX, H. J. H., VAN HELIST, G. J. F. & TrielING, R. R. 2009 Experiments on rapidly rotating turbulent flows. Phys. Fluids 21, 096601.
BOLGIANO, R. 1959 Turbulent spectra in a stably stratified atmosphere. J. Geophys. Res. 64, 2226–2229.
BOUBNOV, B. M. & GOLITSYN, G. S. 1986 Experimental study of convective structures in rotating fluids. J. Fluid Mech. 167, 503–531.
FRISCH, U. 1995 Turbulence: the legacy of A. N. Kolmogorov. Cambridge: Cambridge University Press.
KOLMOGOROV, A. N. 1941 Dissipation of energy in isotropic turbulence. Dokl. Akad. Nauk SSSR 32, 19–21.
KRISHNAMURTI, R. & HOWARD, L. N. 1981 Large-scale flow generation in turbulent convection. Proc. Natl. Acad. Sci. USA 78, 1981–1985.
KUNNEN, R. P. J., CLERCX, H. J. H. & GEURTS, B. J. 2008α Breakdown of large-scale circulation in turbulent rotating convection. EPL 84, 24001.
Kunnen, R. P. J., Clercx, H. J. H. & Geurts, B. J. 2008b Enhanced vertical inhomogeneity in turbulent rotating convection. *Phys. Rev. Lett.* **101**, 174501.

Kunnen, R. P. J., Clercx, H. J. H., Geurts, B. J., van Bokhoven, L. J. A., Akkermans, R. A. D. & Verzicco, R. 2008c Numerical and experimental investigation of structure function scaling in turbulent Rayleigh–Bénard convection. *Phys. Rev. E* **77**, 016302.

Kunnen, R. P. J., Geurts, B. J. & Clercx, H. J. H. 2010 Experimental and numerical investigation of turbulent convection in a rotating cylinder. *J. Fluid Mech.* **642**, 445–476.

Lohse, D. 1994 Temperature spectra in shear flow and thermal convection. *Phys. Lett. A* **196**, 70–75.

Lohse, D. & Xia, K.-Q. 2010 Small-scale properties of turbulent Rayleigh–Bénard convection. *Annu. Rev. Fluid Mech.* **42**, 335–364.

Mininni, P. D. & Pouquet, A. 2009 Helicity cascades in rotating turbulence. *Phys. Rev. E* **79**, 026304.

Müller, W.-C. & Thiele, M. 2007 Scaling and energy transfer in rotating turbulence. *EPL* **77**, 34003.

Obukhov, A. M. 1959 The influence of hydrostatic forces on the structure of the temperature field in turbulent flow. *Dokl. Akad. Nauk SSSR* **125**, 1246–1248.

Pope, S. B. 2000 *Turbulent Flows*. Cambridge: Cambridge University Press.

Sakai, S. 1997 The horizontal scale of rotating convection in the geostrophic regime. *J. Fluid Mech.* **333**, 85–95.

Shishkina, O., Stevens, R. J. A. M., Grossmann, S. & Lohse, D. 2010 Boundary layer structure in turbulent thermal convection and its consequences for the required numerical resolution. *New J. Phys.* **12**, 075022.

Stevens, R. J. A. M., Verzicco, R. & Lohse, D. 2010 Radial boundary layer structure and Nusselt number in Rayleigh–Bénard convection. *J. Fluid Mech.* **643**, 495–507.

Stevens, R. J. A. M., Zhong, J.-Q., Clercx, H. J. H., Ahlers, G. & Lohse, D. 2009 Transitions between turbulent states in rotating Rayleigh–Bénard convection. *Phys. Rev. Lett.* **103**, 024503.

Verstappen, R. W. C. P. & Veldman, A. E. P. 2003 Symmetry-preserving discretization of turbulent flow. *J. Comput. Phys.* **187**, 343–368.

Verzicco, R. & Orlandi, P. 1996 A finite-difference scheme for three-dimensional incompressible flow in cylindrical coordinates. *J. Comput. Phys.* **123**, 402–413.

Zhong, F., Ecke, R. E. & Steinberg, V. 1993 Rotating Rayleigh–Bénard convection: asymmetric modes and vortex states. *J. Fluid Mech.* **249**, 135–159.

Zhou, Y. 1995 A phenomenological treatment of rotating turbulence. *Phys. Fluids* **7**, 2092–2094.