PIG STRUCTURE FUNCTION $F^π_2$ In the Valon Model

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Partonic structure of constituent quark (or valon) in the Next-to-Leading Order is used to calculate pion structure function. This is a further demonstration of the finding that the constituent quark structure is universal, and once it is calculated, the structure of any hadron can be predicted thereafter, using a convolution method, without introducing any new free parameter. The results are compared with the pion structure function from ZEUS Coll. Leading Neutron Production in $e^+p$ collisions at HERA. We found good agreement with the experiment. A resolution for the issue of normalization of the experimental data is suggested. In addition, the proportionality of $F^π_2$ and $F^p_2$, which have caused confusion in the normalization of ZEUS data is discussed and resolved.

I. INTRODUCTION

ZEUS Collaboration at HERA has recently published [1] data on pion structure function, $F^π_2$, using the leading neutron production in $e^+p$ collision. The data do suggest that there is a simple relation between proton structure function, $F^p_2$, and the pion structure function $F^π_2$. This assertion is believable and points to the direction that there exists a more basic and universal structure inside all hadrons. In Ref.[1] the normalization of $σ_{γπ}$ and hence, the pion structure function is fixed by two different methods: (a) dominance of one pion exchange, and (b) use of the additive quark model. The two normalizations differ by a factor of two. The additive quark model makes no statement about the leading baryon production while the extraction of $F^π_2$ is completely based on meson exchange dynamics. A criticism of this procedure is given in [2]. It is evident that the issue of normalization of the data is more uncertain than it was thought before. ZEUS Coll. now believes that most likely the final result will lie between the two options [3]. The issue is what to use for $F(t)$ that parameterizes the shape of the pion cloud in the proton. In this paper we will offer an alternative, which does not rely on those assumptions and renders support to a normalization, which lies between the two options used by the ZEUS Coll. In what follows we have used the structure function of a Constituent Quark (CQ), (in Ref. [4] it is called valon and hence the valon model) which is universal to all hadrons and from there, with a convolution method, the structure function of pion is obtained. The motivation for such an approach is based on the fact that such a model for soft production has found phenomenological success at $\sqrt{s} < 100$ GeV and low $p_T$. It should, however, be understood that when $s$ is high enough to generate a significant component of hard subprocesses ($\sqrt{s} > 200 GeV$), $σ_{tot}$ and average $p_T$ will both increase and inclusive distributions will lose their scaling behavior. Nevertheless, the soft component is unchanged, and the model remains valid.

Our knowledge of hadronic structure is based on the hadron spectroscopy and the Deep Inelastic Scattering (DIS) data. In the former picture quarks are massive particles and their bound states describe the static properties of hadrons; while the interpretation of DIS data relies upon the QCD Lagrangian, where the hadronic structure is intimately connected with the presence of a large number of partons. It has been shown [5] that it is possible to perturbatively dress a valence QCD Lagrangian field to all order and construct a constituent quark in conformity with the color confinement. Therefore, the assumption of constituent quark as a valence quark with its cloud of partons is reasonable. The Cloud is generated by QCD dynamics. A complete description and calculational procedure for obtaining the constituent quark structure, and hence, the hadronic structure functions are detailed in [6].

II. FORMALISM

By definition, a valon is a universal building block for every hadron. In a DIS experiment at high enough $Q^2$ it is the structure of valon that is being probed, while at low enough $Q^2$ this structure cannot be resolved and it behaves

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as the valence quark and the hadron is viewed as the bound state of its valons. For a U-type valon one can write its structure as,

\[ F^U_2(z, Q^2) = \frac{4}{9} z(q_\pi + q_\pi) + \frac{1}{9} z(q_\pi + q_\pi + q_\pi + q_\pi) + \ldots \]  

(1)

where all the functions on the right-hand side are the probability functions for quarks having momentum fraction \( z \) of a U-type valon at \( Q^2 \). These functions are calculated in Ref. [6] in the next-to-leading order and we will not go into the details here. Suffices to note that the functional forms of the parton distributions in a constituent quark, (or valon), is as follows [6]:

\[ zq_{\text{valon}}(z, Q^2) = a z^b(1 - z)^c \]  

(2)

\[ z\bar{q}_{\text{valon}}(z, Q^2) = a z^b(1 - z)^{\gamma}[1 + \eta z + \xi z^{0.5}] \]  

(3)

The parameters \( a, b, c, \alpha, \) etc. are functions of \( Q^2 \) and are given in the appendix of Ref.[6]. The above parameterization of the parton distribution in a valon is for light quarks, \( u \) and \( d \). For heavy quarks additional phenomenological assumptions are needed to be made. It is known that in proton the strange quark distribution is smaller than up quark by a factor of 2 at some regions of \( x \) and by the time \( x \) reaches down to \( 10^{-3} \), we have \( x\bar{s} = x\bar{u} \). As for the charm quark content of proton, the picture is less clear. Early treatments of heavy parton distributions assumed that for \( Q^2 > m^2_Q \), the heavy quark, \( Q \), should be considered as massless. At the opposite extreme, the heavy quark has never been regarded as part of the nucleon sea but produced perturbatively through photon-gluon fusion. The number of flavors remain fixed, regardless of \( Q^2 \). This treatment is used in GRV 94 parton distribution [7]. Both schemes have their own deficiency. For example, in the former scheme, the heavy quark should not be treated as massless for \( Q^2 \geq m^2_Q \) and in the latter scheme one cannot incorporate large logarithms at \( Q^2 >> m^2_Q \). CTEQ [8] on the other hand, have used an interpolation, which produces relative features of both schemes. In this treatment, the heavy quarks are essentially produced by photon-gluon fusion when \( Q^2 \approx m^2_Q \) and considered as massless quark when \( Q^2 >> m^2_Q \). In our treatment, however, since we are dealing with low \( x \)-values, we will assume \( z\bar{s} = z\bar{u} \), and \( z\bar{c} = \alpha z\bar{s} \), where \( \alpha \) is a factor taken to be equal to the ratio of the strange and charm quark masses when \( Q^2 < m^2_{\text{charm}} \); and \( \alpha = 1 \) for \( Q^2 > m^2_{\text{charm}} \). Although such an undertaking is not free of ambiguity, however, that does not change our qualitative arguments regarding the normalization of the data.

Equations (1-3) completely determine the structure of a valon. The structure function of any hadron can be written as the convolution of the structure function of a valon with the valon distribution in the hadron:

\[ F^h_2(x, Q^2) = \sum_{\text{CQ}} \int_{-1}^{1} dyG_{\text{CQ/valon}}(y)F^\text{CQ}_2\left(\frac{x}{y}, Q^2\right) \]  

(4)

where summation runs over the number of CQ’s in a particular hadron. \( F^\text{CQ}_2(z, Q^2) \) denotes the structure function of a CQ (\( U, D, \bar{U}, \bar{D} \), etc.), as given in equation (1), and \( G_{\text{CQ/valon}}(y) \) is the probability of finding a valon carrying momentum fraction \( y \) of the hadron. It is independent of the nature of probe and its \( Q^2 \) value. Following [4], [6], and [9], for the case of pion we have:

\[ G_{\text{CQ/valon}}(y) = \frac{1}{B(\mu + 1, \nu + 1)}y^\mu(1 - y)^\nu \]  

(5)

where \( G_{\text{CQ/valon}} \) is the U-valon distribution of in \( \pi^+ \) as well as the D-valon distribution in \( \pi^- \). Similar expression for \( G_{\text{CQ/valon}} \), anti-valon distribution in a pion, is obtained by interchanging \( \mu \leftrightarrow \nu \). In the above equation \( B(i, j) \) is Euler \( \beta \) function and \( G_{\text{CQ/valon}}(y) \) are non-invariant distributions satisfying the following number and momentum sum rules:

\[ \int_0^1 G_{\text{CQ}}(y)dy = 1 \quad \sum_{\text{CQ}} \int_0^1 y G_{\text{CQ}}(y)dy = 1 \]  

(6)

For pion, the numerical values are: \( \mu = 0.01, \nu = 0.06 \). We have, however, tried a range of values for \( \mu \) and \( \nu \) and did not find much sensitivity on \( F^\pi_2 \) data against this variations. In [4] [10] it is estimated that \( \mu = \nu = 0., \) which is very close to the values that are used in this paper. The flatness or almost flatness of the valon distribution in pion is attributed to the fact that the valons are more massive than the pion, so they are tightly bound. From \( SU(2) \)
symmetry one should expect that $\mu = \nu$. In our calculation, $\mu$ is slightly different from $\nu$, indicating a small violation of $SU(2)$ symmetry. This violation is very small and the data on $F_2^\pi$ is not sensitive enough to make a large difference. Significant asymmetry is observed in proton sea and its implications are discussed in Ref. [6] in the context of the valon model.

The pion has two valons (or constituent quarks), for example, $\pi^+$ has a U and a $\bar{D}$, therefore, the sum in equation (4) has only two terms. Parton distributions in a pion, say $\pi^+$, is obtained as:

$$u_{\text{val.}}^{\pi^+}(x, Q^2) = \int_x^1 \frac{dy}{y} G_{\pi^+}^{\nu}(y) u_{\text{val.}}^\nu \left( \frac{x}{y} Q^2 \right)$$

(7)

$$d_{\text{val.}}^{\pi^+}(x, Q^2) = \int_x^1 \frac{dy}{y} \bar{G}_{\pi^+}^{\bar{\nu}}(y) d_{\text{val.}}^{\bar{\nu}} \left( \frac{x}{y} Q^2 \right) dy$$

(8)

$$q_{\text{val.}}^{\pi^+}(x, Q^2) = \int_x^1 \frac{dy}{y} G_{\pi^+}^{\bar{\nu}}(y) q_{\text{val.}}^{\bar{\nu}} \left( \frac{x}{y} Q^2 \right) + \int_x^1 \frac{dy}{y} \bar{G}_{\pi^+}^{\nu}(y) q_{\text{val.}}^\nu \left( \frac{x}{y} Q^2 \right)$$

(9)

Similar relations can be written for $\pi^-$ and $\pi^0$. In the above equations the subscripts $U$ and $\bar{D}$ are the two valon types in $\pi^+$. Equations (7, 8, 9) along with equation (4) completes the evaluation of the pion structure function. In figures (1) we present $F_2^\pi(x, Q^2)$, as calculated above, for the fixed $Q^2$'s corresponding to the ZEUS data of Ref.[1].

III. DISCUSSION AND THE DATA

In the previous section we outlined the procedure for calculating $F_2^\pi(x, Q^2)$. The results are shown in figures (1) by the square points. From the figures, it is evident that for smaller $x$ and lower $Q^2$ values, the results of the model calculations are closer to the additive quark model normalization of the ZEUS Coll. data (see figure 19 of Ref.[1]). As we move towards the large $x$ values the calculated structure function decreases and gets closer to the effective flux normalization of the data. If we are to trust in the valon model results, which provides a very good description for the wealth of proton structure function data as well as other hadronic processes, we can conclude that the two normalizations used by ZEUS may be relevant to different kinematical regions and lends support to the assessment made by the ZEUS collaboration that the final results will lie between the two options[3]. It is true that the valon model resembles the additive quark model in that the contributions from each valon are added up. But here we are mainly dealing with the parton content of each valon, which is derived from the perturbative QCD. The valon distribution in pion serves only as a phenomenological mimic of the pion wave function. We further note that in the above calculation none of the ZEUS data for pion structure function (see figure 18 of Ref.[1]); namely

$$F_2^{\pi(\text{EF})}(x, Q^2) \approx kF_2^p(x, Q^2)$$

(10)

with the proportionality constant, $k = 0.361$. We have calculate the right-hand side of Eq. (10) in the valon model and compared it with the effective flux normalization, $F_2^{\pi(\text{EF})}(x, Q^2)$, in the left hand side. The copmarison is presented in figures (1). As one can see from the figures, the relationship holds rather well at all $Q^2$ values. To avoid any misleadings, we emphasize that the direct calculation of $F_2^\pi(x, Q^2)$ in the valon model (Square points in figures (1)) is different from $F_2^{\pi(\text{EF})}(x, Q^2)$ and hence, does not support the effective flux normalization of $F_2^\pi$. In other words, our finding merely states that if we scale $F_2^\pi$ by a factor of $k = 0.37$ we arrive at equation (10). Figures (1) also indicates that the above relationship holds rather well at lower $Q^2$ values. As we move to higher $Q^2$ this relationship at the lowest $x$-value gets blurred, but at large $x$ and high $Q^2$ it continues to hold.

Since our model produces very good fit to the proton structure function data in a wide range of both $x = [10^{-5}, 1]$ and $Q^2 = [0.45, 10000]$ GeV$^2$, we have also attempted to investigate the relation:

$$F_2^\pi(x, Q^2) \approx \frac{2}{3} F_2^p \left( \frac{2}{3} x, Q^2 \right)$$

(11)

which is based on color-dipole BFKL-Regee expansion and corresponds to the ZEUS's additive quark model normalization. We have calculated both sides of the equation (11) in the valon model. The results are shown by the solid
lines and square points in Figures (1). Although We get similar results as in Fig.19 of Ref.[1], but this does not say much about the pion structure function data, because the additive quark model normalization of the data is based on the above equation. It only restates that our model, indeed, correctly produces proton structure function. The main ingredient of our model is the partonic content of the constituent quark which is calculated based on QCD dynamics. Convolution of this structure with the constituent quark distribution in the pion appears, to some extent, to give support for the additive quark model normalization of the pion structure function data. In fact, we agree with Ref. [3] that the final results should be somewhere between the two normalizations used by ZEUS, being much closer to the additive quark model scheme than the pion flux scheme.

It is worth to note that the following relationship also holds very well between ZEUS’s pion flux normalization data and proton structure function:

\[ F_2^{π(EF)}(x,Q^2) = \frac{1}{3} F_2^p \left( \frac{2}{3} x, Q^2 \right) \] (12)

This relationship is essentially the same as Eq. (10), except for the factor \( \frac{2}{3} \) in front of \( x \) that makes a small correction to the factor \( k \) of Eq. (10). In Figure (2) we have presented the right-hand sides of both equations (10) and (12) along with the effective flux normalization data from Ref.[1] at a typical value of \( Q^2 = 15 \text{GeV}^2 \). The same feature is also prevalent for the other values of \( Q^2 \).

Both Eqs. (11) and (12) indicate that, regardless of the choice of normalization scheme, the valence structure of proton, as compared to pion, is shifted to the lower \( x \) by a factor of \( \frac{2}{3} \). So that valence \( x \) in proton corresponds to \( \frac{2}{3} x \) in pion.

ZEUS has observed that the rate of neutron production in photo-production process, in comparison to that of \( pp \) collision, drops to half and from this observation ZEUS Coll. has concluded that \( \sigma_{γπ} \approx \frac{1}{4} \sigma_{γp} \) whereas one expects to get a rather \( \frac{2}{3} \), both from Regge factorization and the additive quark model[2]. This poses a problem on the understanding of the dynamics of the interaction. A tentative resolution is that the discrepancy can be resolved if we suppose that in the process each valon interacts independently. That is the impulse approximation. Suppose that \( δ \) denotes the number of valons in the target proton that suffers a collision and \( δ_i \) denotes the number of collisions that \( i^{th} \) valon of the projectile encounters. If we define the integer \( \sigma = \sum_i δ_i \), then we will have \( δ \leq σ \leq 3δ \).

Furthermore, let \( P_δ(σ) \) represents the probability that out of \( δ \) independent collisions that target valons encounters, \( σ \) valonic collisions occur. If \( p \) is the probability that either of the other valons in the projectile also interact, then the probability for \( δ \) collisions will have a binomial distribution having \( σ = δ - δ \) valonic collision by \( i = 2 \) and 3 valons out of a maximum \( 2δ \) possible such collisions [11]. Now, for a real photon, we can assume that it may fluctuate into mesons with two valons. If we denote the number of possible valonic collisions in \( γ - p \) interaction by \( δ' \) then the mean number of such collisions will be \( 2δ' p \) whereas in \( pp \) collisions it will be \( 3δp \). The observed reduction in the rate of neutron production in two processes and the conclusion that \( σ_{γπ} \approx \frac{1}{4} σ_{pp} \) implies that \( \frac{δ'}{δ} = \frac{1}{2} \). That is, there are twice as many collisions in \( pp \) interactions than in photo-production.

IV. CONCLUSION

We have demonstrated that the assumption of the existence of a basic structure in hadrons is a reasonable model to investigate the hadronic structure. Pion structure function measurement by ZEUS Coll. provides additional tests and further validation of the Model. We have presented a resolution to the issue of the normalization of the data. Our results are based on QCD dynamics and suggest that the correct normalization of \( F_2^π \) is closer to the additive quark model normalization for low \( x \) region. The observed reduction in the rate of neutron production in photo-production as compared to \( pp \) collision is accounted for and concluded that there are twice less valonic collisions in photo-production than in \( pp \) collision. Furthermore, it appears that there are simple relations among the structure functions of hadrons; namely they are proportional and the proportionality ratio seems to depend on the normalization scheme chosen.

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Figure 1(a): Pion structure function at $Q^2 = 7 \text{ GeV}^2$. The circles and the triangles are pion flux and additive quark model normalizations of the data [1], respectively. Squares are the calculated results from the valon model (see the text for details.) The solid line represents $2/3 F_2^\pi (2/3 x, Q^2)$ and the dotted line is $0.371 * F_2^\pi (x, Q^2)$, both are calculated using the model.
Figure 1(d): Pion structure function at $Q^2 = 60$ GeV$^2$. The circles and the triangles are pion flux and additive quark normalizations of the data [1], respectively. Squares are the calculated results from the valon model (see the text for details.) The solid line represents $2/3 F_2^\pi (2/3 x, Q^2)$ and the dotted line is $0.371^* F_2^p (x, Q^2)$, both are calculated using the model.
$F_{2}^{\pi}(x, Q^{2})$ at $Q^{2} = 120$ GeV$^2$. The circles and the triangles are pion flux and additive quark normalizations of the data [1], respectively. Squares are the calculated results from the valon model (see the text for details.) The solid line represents $2/3 F_{2}^{p}(2/3 x, Q^{2})$ and the dotted line is $0.371 \times F_{2}^{p}(x, Q^{2})$, both are calculated using the model.
Figure 1(b): Pion structure function at $Q^2 = 15 \text{ GeV}^2$. The circles and the triangles pion flux and additive quark model normalizations of the data [1], respectively. Squares are the calculated results from the valon model (see the text for details.) The solid line represents $\frac{2}{3} F_2^\pi (2/3 x, Q^2)$ and the dotted line is $0.371 \times F_2^p (x, Q^2)$, both are calculated using the model.
Figure 1(c): Pion structure function at $Q^2 = 30 \text{ GeV}^2$. The circles and the triangles are pion flux and additive quark model normalizations of the data [1], respectively. Squares are the calculated results from the valon model (see the text for details.) The solid line represents $2/3 F_{2P}^\pi (2/3 x, Q^2)$ and the dotted line is $0.371 \times F_{2P}^\pi (x, Q^2)$, both are calculated using the model.
Figure 1(f): Pion structure function at $Q^2 = 240 \text{ GeV}^2$. The circles and the triangles are from ZEUS Coll. [1] for two different normalizations. The squares are calculated from the valon model (see the text for details.) The solid line represents $\frac{2}{3} F_2^\pi (2/3 \ x, \ Q^2)$ and the dotted line represents $0.371 \ F_2^\pi (x, Q^2)$, both are calculated using the model.
Figure 2. The effective flux normalization of Pion structure function data as compared with the scaled proton structure function of Eqs. (10) and (12). The data points are from Ref. [1]. The solid line is $0.371 \times F^p_2(x, Q^2)$ pertinent to Eq. (10) while the dashed line is $1/3 F^p_2(2/3 x, Q^2)$ pertinent to Eq. (12), both are calculated from the valon model at $Q^2 = 15 GeV^2$. 