Modeling the axially symmetric fracture evolution in deep-hole blasting

EN Sher
Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
E-mail: ensher@gmail.com

Abstract. When blasting deep-hole charges in a brittle rock mass, rock disintegration mainly proceeds due to development of radial fractures induced by dramatic tensile azimuthal stresses in the vicinity of a charge and along its surface. Coincidentally, appreciable tensile stresses are generated by blasting nearby ends of deep charges along their axes. Such stresses can initiate a rise and development of axially symmetric fractures. In the present paper the computational model is set forth for development of axially symmetric fractures nearby free surface in the vicinity of a charge end. Calculations of a fracture shape and size are reported.

1. Introduction
When blasting deep-hole charges in a brittle rock mass, an elastic compressive wave is generated and propagates from a hole in-depth of a rock mass; and the front of rock-breaking wave follows it in terms of the zone model of blasting [1 – 3]. As the elastic wave propagates, the stresses in it tend to expire and the front of rock-breaking wave also decelerates. When the rate of elastic wave development slows down to the maximum velocity of fracture propagation, and the azimuthal tensile stresses arise; all this can result in initiation and development of radial fractures [4, 5]. Thereto, the front of the rock-breaking wave comes to rest; the radial displacement of an elastic medium is fixed at boundary with broken rock. The elastic medium expansion gained at the first stage of blasting preserves henceforth thanks to resistance of broken rock to the radial compression, as broken ore yields under the dry friction law. Such expansion governs development of a system of radial fractures initiated by high tensile azimuthal stresses in the elastic zone of a rock mass.

Coincidently, in blasting the tensile stresses arise along charge axes in the vicinity of ends of buried elongated charges. This kind of stresses can initiate a rise and development of axially symmetric fractures. In the present paper the researchers propose the computational scheme for development of axially symmetric fractures in the vicinity of charge end nearby free surface. The calculations of shape and size of this-kind fractures are reported.

2. Methods and materials
Let assess the stress field, induced due to radial expansion of the boundary between an elastic zone and a rock-breaking zone under blasting of an elongated charge in a rock mass. Assume that such expansion proceeds uniformly along charge length and leads to lengthwise increase in a volume confined by the breaking zone boundary: \( \Delta V_b = 2\pi b u_b \), where \( b \), \( u_b \), is the radius of the breaking zone and radial displacement of the elastic zone boundary after blasting. The stress field in the environment of the breaking zone for an elongated charge can be approximately computed by...
considering the effect of expansion centers uniformly distributed along charge axis on the elastic space.

The field of stresses around the expansion center located at the origin of coordinates is described by expressions [6]:

\[
\begin{align*}
\sigma_{xx} &= C\mu \left( 1 - \frac{3x^2}{r^5} \right), \\
\sigma_{xz} &= C\mu \left( -\frac{3x^2}{r^5} \right), \\
\sigma_{zz} &= C\mu \left( 1 - \frac{3z^2}{r^5} \right), \\
r &= \sqrt{x^2 + y^2 + z^2}.
\end{align*}
\]

Here, \(4\pi C\) – increment of expansion center volume, \(\mu\) – modulus of medium shear. Given that a charge locates along axis \(z\) in interval \([-5, 5]\), then the stress field \(\sigma_{zz}\) under the total charge effect is determined at point \((x, 0, z)\) by integral:

\[
\sigma_{zz} = C_1\mu \int_{-5}^{5} \frac{1}{(x^2 + (s-z)^2)^{3/2}} \left[ 3(s-z)^2 \right] \left[ (x^2 + (s-z)^2)^{5/2} \right] ds,
\]

where \(C_1 = \Delta V_b / 4\pi\).

The measurement units suggested in calculations are: the breaking zone radius \(b\) taken as a distance unit and the shear modulus \(\mu\) as a stress unit. Assume as well that \(C=1\). Under this assumption the diagram \(\sigma_{zz}(1,0,z)\) for stress distribution \(\sigma_{zz}\) along boundary between the charge breaking zone and the elastic zone is shown in Figure 1.

![Figure 1. Tensile stress distribution \(\sigma_{zz}\) at the elastic zone boundary along charge axis.](image)

The narrow zones with higher tensile stresses along axis \(z\) are obvious nearby charge ends. Such stresses can cause rock breaking and formation of axially symmetric cracks.

The development of this-kind fractures was studied considering the influence of free surface on a character of breaking performance of deep-hole charge. Modeling the development of axially symmetric fractures was performed with software programs, designed earlier for computation of axially symmetric fracture development in blasting of a concentrated charge close to free surface and in hydrofracturing [7, 8].

**Problem statement.** It was suggested that a vertical blasthole charge of length \(H\) loaded at depth \(h\) relative to free surface, Figure 2. It was also assumed that in the charge blasting process at the moment when the development of the breaking zone came to the end, the external elastic zone of the rock mass was subjected to loading specified with the following parameters: breaking zone radius \(b\) and radial compression stress at the boundary between breaking zone and elastic zone \(q\).
The free surface was modeled at \( z = 0 \) for a large-radius disk fracture \( R \) with stress-free edges. A shape of boundary between the breaking zone and an elastic space was taken as a cylinder of radius \( b \) and length \( H \).

It was suggested that the elastic space at boundary with the breaking zone was uniformly loaded with pressure \( \mu \) over cylinder surface. The problem to solve was reduced to a dimensionless form. Radius of the breaking zone is taken as a length unit; a value of shear modulus \( \mu \) is taken as a stress unit.

The calculation data on stress distribution at the boundary between the elastic zone and rock-breaking zone for blasthole charge of length 10 and loading depth 1 is shown in Figure 3, where curve 1 corresponds to distribution of axial tension \( \sigma_{zz}(1,0,z) \), curve 2 is for azimuthal tension \( \sigma_{yy}(1,0,z) \).

It is explicit from the curves that azimuthal tension dominating along the charge leads to development of radial fractures, but at the same time the axial tension nearby charge end closest to the free surface is even higher than that of azimuthal tension. Hence it follows that in this case the development of axially symmetric fractures is quite possible along with radial fractures.

Figure 2. Scheme of charge loading relative to free surface.

Figure 3. Distribution of axial \( \sigma_{zz}(z) \) and azimuthal \( \sigma_{yy}(z) \) stresses at elastic zone boundary along charge axis (curves 1, 2).
The problem on development of an axially symmetric fracture in the zone of axial tensile stress concentration was studied at the first stage to consider the study problem on failure of a brittle rock mass nearby the blasthole charge end. For this purpose, a horizontal initial circular crack (Figure 3) of small width was provided at depth $z_0$ in the axial tensile stress zone in the scheme of the charge loaded relative to the free surface. Development of this initial axially symmetric fracture advanced step-by-step. At every step the field of stresses was evaluated in the vicinity of fracture tip; direction of its further propagation was estimated relative to the maximum tension, and amount of released energy was evaluated. The computation recycles made it possible to identify a trajectory of fracture evolution up to its approach or even its outlet to the free surface and variations in the stress intensity factor (SIF) with fracture growth.

3. Results

Figure 4 demonstrates dimensionless calculated trajectories of the axially symmetric fracture development at different burial lengths $z_0$ of initial fractures in a blasthole charge of 10 in length and 1 in depth: fracture shapes are shown in Figure 4a, and graphs of variations in SIF vs. radial coordinate growth of fracture tip $x$ are presented in Figure 4b. It is explicit in Figure 4 that at the first stage the initial fracture under the tensile stress nearby the top charge face develops along the main axis of the stress field, moreover, its SIF sharply jumps, thus indicating its accelerated development. At the second stage the fracture front is directed to the free surface at angle $\approx 30^\circ$, thereto, SIF growth slows down.

![Figure 4](image)

Figure 4. Calculation parameters of axially symmetric fractures: (a) trajectories of their development; (b) SIF versus radial coordinate of fracture tip $x$. Curves 1–4 correspond to respective initial depth $z_0 = 1.2, 1.3, 1.4, 1.5$.

At the third stage the fracture propagates in parallel to the free surface, therewith its SIF is lowering and fracture growth is slowing down. The fact that SIF of a fracture is monotonically growing, thus indicating: if a fracture starts its propagation, then velocity of its development used to increase until its outlet to the free surface. Excess of axial tension $\sigma_{zz}$ over tension strength $\sigma_t$ of a rock to be disintegrated can be assumed as the condition to initiate fracture development.

4. Discussion

Consider a possibility of axially symmetric fracture development in the vicinity of the top end of a buried blasthole charge in a brittle rock like granite. According to reference data on granite Young’s modulus is $E \approx 5 \times 10^{10}$ Pa, Poisson ratio is $\nu = 0.3$, compression strength is $\sigma_c \approx 2 \times 10^8$ Pa, tension
strength is $\sigma_c \approx 1 - 2 \times 10^7$ Pa. Let us apply the quasistatic approach [4] to evaluation of a rock-breaking zone size and compressive stress at its boundary with the elastic zone under blasting of a blasthole charge, loaded with tetryl in granite. In the case of blasting a hole charge of 45 diameter in granite the computation in terms of the procedure described in [9] gives expansion of explosion cavity up to $30$ mm and radius of the rock-breaking zone formed of $b = 0.5$ m. Moreover the expanded breaking zone exerts pressure $q = 3.35 \times 10^7$ Pa on the elastic space. Using these data we compute the axial tension stress for fractures 1-4 in Figure 4a just at the start of their development at the boundary between the breaking zone and the elastic zone. The stress values computed for fractures 1-4 in Figure 4a, and recalculated considering magnitude $q$ are as follows $16.7, 13.7, 11.4$ and $8.7$ MPa thereto coordinated of their initial location in a dimensional form are $-0.6, -0.65, -0.7, -0.75$ m. The charge loading depth is $0.5$ m, its length is $5$m.

Comparison of tensile stress values for initial fractures 1-4 with tension strength of granite $\sigma_c \approx 1 - 2 \times 10^7$ Pa indicates that axially symmetric fractures 1-3 can really arise and develop under blasting of hole charge of $5$ m length and $0.5$ m depth in granite. By the calculations this-type blasting can produce explosion crater of $5$ m in diameter and $0.5$ m in depth.

The present research computation results indicate that blasting of a buried blasthole charge in a brittle rock can initiate axially symmetric fracturing nearby the top end of the charge with further development of new-born fractures and their outlet to the free surface. A characteristic feature of these fractures is the resultant ratio of blasting crater radius to crater depth: $R_V / h \approx 2.5 - 3$. It is experimentally proved that such shallow craters are actually observed. Abundant experimental data on evaluation of crater radius resulted from buried charge blasting are reported in publications [10, 11]. The experiments were carried out at mines of Severouralsky bauxite producer. According to the experimental procedure blastholes of different lengths were made on perpendicularly exposed surface; the new-made holes were loaded with different hole underloading levels. After blasting the radius of the crater formed in the free surface $R_V$ was measured. It is appeared that it is limited with the radius of radial fracture zone $R_c$ for a blasthole charge. Herewith, given that $H \geq 5b R_V / h \approx \sqrt{(R_c / h)^2 - 1}$, if $h/b = 1$, as it was assumed in the present computations, then $R_V / h = \sqrt{(R_c / b)^2 - 1}$. By the data cited in [11] the ratio between radii of a fracturing zone and the rock-breaking zone is $R_c / b \approx 3.5 - 5$. Therefore, in this case, $R_V / h \approx 2.8 - 4.9$, so it is rather close to our calculations.

5. Conclusions
In the present research a probability of axially symmetric fracture formation in blasting of a deep-buried blasthole charge in a brittle rock mass is under consideration.

1. Computation of the stress field at boundary of the elastic zone and rock breaking zone at the moment of its stoppage in blasting of blasthole charge is made. Considerable tensile stresses are detected along axis of the elastic zone.

2. Modeling of axially symmetric fractures development in environment of the top end of a deep blasthole charge nearby the free surface. The shape and size of this-type fractures are identified. Calculated parameters are compared to available experimental data. It is justified that the new-developed theory is in satisfactory compliance with the experimental evidence.

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References

[1] Grigoryan SS 1967 Some issues of mathematical theory of deformation and failure of hard rocks *Applied Mathematics and Mechanics* Vol 31 Issue 4

[2] Rodionov VN, Adushkin VV, Romashev AN, et al 1971 *Mechanical Effect of Underground Blasting* Moscow: Nedra (in Russian)

[3] Chedvik P, Koks A, and Hopkinson G 1966 *Mechanics of Deep Underground Blasting* Moscow: Mir (in Russian)

[4] Sher EN 1997 Dynamics consideration in description of brittle media failure in string charge blasting *Applied Mechanics and Technical Physics* No 3 pp 484–493

[5] Sher EN and Chernikov AG 2015 Computing the radial fracture system initiated under blasting of elongated charge in brittle rock *J. Fundament. Appl. Min. Sci.* No 2 pp 299–303

[6] Rabotnov YuN 1988 *Mechanics of Solid Body Deformation* Moscow: Nauka (in Russian)

[7] Sher EN and Mikhailov AM 2008 Modeling axially symmetric crack growth under blasting and fracturing *J. Min. Sci.* Vol 44 No 5 pp 473–481

[8] Mikhailov AM 2000 Calculation of stresses around a crack *J. Min. Sci.* Vol 36 No 5 pp 445–451

[9] Aleksandrova NI and Sher EN 1999 Effect of dilation on rock breaking by explosion of a cylindrical charge *J. Min. Sci.* Vol 35 No 4 pp 400–408

[10] Kutuzov BN and Andrievsky AP 2002 *New Theory and Novel Rock Disintegration Processes by Blasting of Elongated Charges* Novosibirsk: Nauka (in Russian)

[11] Andrievsky AP, Kutuzov BN and Matveev PF 1996 Formation of the blast crater in a rock mass blast-loaded by column charges *J. Min. Sci.* Vol 32 No 5 pp 390–394