Perspectives on Quantum Gravity Phenomenology

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Abstract

The idea that quantum gravity manifestations would be associated with a violation of Lorentz invariance is very strongly bounded and faces serious theoretical challenges. Other related ideas seem to be drowning in interpretational quagmires. This leads us to consider alternative lines of thought for such phenomenological search. We discuss the underlying viewpoints and briefly mention their possible connections with other current theoretical ideas.

PACS numbers: 04.60.-m, 04.60.Pp, 04.80.-y 11.30.Cp.

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I. INTRODUCTION

The search for a reconciliation of the view of space-time as contemplated within the context of general relativity, and the principles of Quantum Theory has for most of its history been besieged by the seemingly inescapable conclusion that no information about the subject could, in practice, be expected to emerge from the empirical realm. Nevertheless the last few years we have witnessed a flare of interest on precisely this possibility. The change in outlook is due to the realization that in some simple scenarios, some hypothetical manifestations of the effects presumably associated with Quantum Gravity (Q.G.) could become observable. In those schemes the Q.G. effects would be associated with a distortion of the microscopic symmetry structure of space-time. These schemes can be divided into two subsets, in the first one assumes that the Lorentz symmetry is in fact broken and that Q.G. endows space-time with a preferential reference frame, a kind of resurrected “Ether”, while in a second class one introduces a modified Lorentz and /or Poincaré structure without invoking a preferential rest frame. Unfortunately these schemes suffer from some serious problems, which could in principle return us to the starting place with its bleak outlook on possible phenomenological guidance in the quest for a Quantum theory of Gravitation. However, now that the “taboo” about Q.G. Phenomenology has been broken, it seems appropriate to explore the notion in a larger context. In this spirit it should be mentioned that the study of some possibilities that involve manifestations of Q.G. in the extended Poincaré algebra in conjunction with the Heisenberg algebra are already the object of intensive research [1][2]. In this paper we explore some options along two different lines of thought; the first one, motivated to some degree by the ideas that where put forward in the context of the schemes that considered modification of the fundamental symmetries of space-time mentioned above, an that will be referred to as the “space-time micro-structure signature of Q.G.” and the second one inspired by the ideas of R. Penrose regarding the changes in standard Quantum Theory presumably tied with Gravitation, and an application to cosmology which we claim is already evidencing the need for some New Physics.

This article is organized as follows: In section 2 we discuss some of the bounds that have been placed in the models for Lorentz invariance violation associated with a preferential frame (Lorentz Invariance Violation or L.I.V. in short) followed by what we regard as a devastating argument against this possibility. In section 3 we briefly discuss a series problematic aspects of the ideas that have been considered in the context of modified fundamental symmetry structures of space-time without preferential frames. In section 4 we describe what seems to be the natural descendent of the schemes of L.I.V., and in section 5 we give short overview of R. Penrose’s proposals for a Quantum Gravity induced “collapse of the wave function” and follow it up with a recent analysis indicating that something of that sort is needed if one wants to justify the arguments (and their predictive success) leading from inflation to the birth of the cosmic structures.

II. RE-BIRTH AND RE-DEATH OF ETHER

One immediate result that emerges once one starts thinking about putting together General Relativity, and Quantum mechanics is that there is a natural scale with units of length that presumably signals the onset of the new physics: The Planck length $l_{Pl}$. The existence of such a fundamental length scale, when taken together with the well known special relativistic contraction of lengths, has motivated some researchers to consider the possibility
that at the fundamental level the space-time structure, which in the quantum context is most naturally thought to be granular, determines by itself a local preferential frame, where the granularity has indeed the isotropic scale \( l_{Pl} \) (in accordance with special relativity, in other frames the scale would be direction dependent). Such situation, so the argument goes, would become manifest, most conspicuously, through a modification in the dispersion relations for free particles, changing them into something like [3]:

\[
E^2 = (\vec{P})^2 + m^2 + \xi E^3 / M_{Pl}.
\] (1)

In such expression the preferential frame appears indirectly: the equation being clearly non Lorentz Invariant, would be valid in, at most, one reference frame: the preferential rest frame. If we denote by \( W^\mu \) the four velocity of the preferential frame such expression can be written in a Lorentz covariant language:

\[
P^\mu P_\mu = m^2 + \xi (W^\mu P_\mu)^3 / M_{Pl}.
\] (2)

As we will see, the existence of this four vector \( W^\mu \), quite often hidden from the discussions, will have dramatic consequences. In fact any modification of the dispersion relations is conceivable only in association with assumption of the existence of new fundamental objects such as \( W^\mu \).

It is worthwhile mentioning that, indeed, in the two most popular approaches to Quantum Gravity; String Theory (see [4]) and Loop Quantum Gravity (see [5]), it has been argued that there is room for precisely the types of effects just discussed.

Most of the work along these lines have centered in direct searches for evidence of such modifications in the dispersion relations, in photons and electrons and other elementary particles and interesting bounds have been obtained in the corresponding parameters \( \xi \) (which in principle are taken as different for the various types of particles). For a comprehensive discussion of these results see the review article [6]. The central conclusion is that the bounds, extracted directly from these analysis, on the value of the parameters \( \xi \), – which were in principle expected to be of order one – are in the range \( 10^{-4} \) – \( 10^{-9} \) for the most common particle species: photons, electrons, neutrons and protons (i.e. quarks).

The point we will focus on, and which represents the most devastating argument against this approach can be traced to the following observation: In the preceding discussion the underlying point of view has been that only very high energy particles are useful probes of these ideas, as seems to be indicated by the large suppression of order \( E/M_{Pl} \) of the last term in equations 1&2 above, relative to the dominant terms. However we must recall that Quantum Field Theory teaches us that particles of all energies contribute to the virtual process that underscores all real particle processes, and thus all processes (even low energy ones) are influenced by the high energy modifications. In fact we should keep in mind that the dispersion relations correspond in fact to the location of the poles in propagators of the quantum field corresponding to the particle type in question\(^1\). These issues where first considered in [7], where the modifications where assumed to be associated with dimension 5 operators, naturally suppressed by the Planck mass scale. In that work the

\[^1\] A note for the young reader: One should keep in mind that in working to combine special relativity and quantum mechanics one is taken quite generically into the realm of quantum field theory, and that particles cease to be fundamental entities and are viewed instead as certain type of exited states of the quantum fields.
authors found that generically quantum loops lead to unsuppressed Lorentz Violating corrections in the propagators, which would imply violations of Lorentz invariance of such magnitude as to be in blunt contradiction with observations. The dangerous expressions originated from quadratic diverging integrals – and to a lesser extent also the linearly diverging ones – that would appear in connection with the otherwise $M_{Pl}$ suppressed effective Lagrangian operators that loops expansion generate. An important detail that serves to illustrate a rather general point is the following: The L.I.V. operators in the original Lagrangian, all contained the triple product $W^\mu W^\nu W^\rho$. The dangerous integrals are of the form $\int (k_\mu k_\nu / k^2) d^4k \approx \eta_{\mu\nu}$. Within the effective field theory approach the dangerous integrals would need to be considered as having a cut-off at a the large mass scale $\Lambda$ and thus would be proportional to $\Lambda^2$. When taken together within the structure of the original L.I.V. operators one would end up with an effective term of order $\Lambda^2 / M_{Pl}$, and structure product $W^\nu W^\sigma W^\rho \eta_{\mu\nu} = -W^\rho$, which would thus lead clearly to a situation with an exceedingly large L.I.V.. The authors of this work then attempt to evade the disastrous conclusions, by considering an “add hoc” proposal to get rid of these dangerous terms: to replace in the structure of the dimension 5 operators every occurrence of the triple product of $W$’s by the tensor $C^{\mu\nu\rho} = W^\mu W^\nu W^\rho - (1/6)[\eta^{\mu\rho}W^\nu + \eta^{\nu\rho}W^\mu + \eta^{\nu\mu}W^\rho]$ which has the property that it vanishes upon contraction of any two induces with the metric tensor $\eta$. The odd thing about this is the following: The dangerous integrals are normally argued to give a result proportional to $\eta$ relying on Lorentz Symmetry arguments. Thus one’s position has effectively become the reliance on a symmetry – that is assumed to be broken – to ensure that the loop corrections do not generate the operators that would break it too badly. This sounds very dangerous. Indeed it has been shown in [8] that, upon the consideration of higher order diagrams, the scheme proposed in [7] fails, and one ends up with the large L.I.V. one was trying to avoid.

In fact such problems can be seen to be rather generic, in the following sense: Let us take seriously the motivational arguments mentioned above, which lead to the suspicion that Quantum Gravity might be associated with a breakdown of Lorentz invariance, and let us consider that at the fundamental scale space-time has a discrete structure characterized by the Planck length, and take this to indicate an underlying granular structure in space-time, with such characteristic length scale, as seen from its proper reference frame which we will call the fundamental frame. In that case, the consistent treatment of the theory should include a provision indicating that there is a bound, with a fixed and specific value, on the physical wavelength of excitations as seen in the fundamental frame. Therefore, the quantum theory should not contain the corresponding excitations, either as real or as virtual particles. Similar considerations have in fact been made in proposing a saturation of the de-Broglie wave length at the Planck length as the momentum goes to infinity [9]. Thus every theory which we now consider as candidate to be a fundamental theory, should be regarded instead, as merely an effective theory and it should include a momentum cutoff, eliminating the unphysical excitations. That is, we must for instance take the standard model of particle physics and impose on it a cut-off on the particle’s 3-momentum as seen in the fundamental frame. This feature would in principle have to be combined with other features of the

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2 Such proper reference frame could be thought of as that in which the granular structure is maximally isotropic.

3 One could of course argue that the standard model is in reality also an effective theory, and that for some unspecified reasons such arguments should not apply to it directly but to some more fundamental and yet
effective theory, such as the change in the propagators of particles which would correspond to the modified dispersion relations. However we will see that, as shown first in [10], the effect of the frame dependent cut-off by itself is disastrous. In order to do this we consider the full propagator of a scalar particle in Yukawa theory. We focus on this theory, in order to illustrate the main point, because of its simplicity and because of the fact that is part of the standard model of particle physics, and thus we can rely on the wealth of knowledge about its phenomenology to confront with the consequences of the aforementioned ideas.

The theory is defined by the Lagrangian density:

\[ \mathcal{L} = \frac{1}{2} (\partial \phi)^2 - \frac{m_0^2}{2} \phi^2 + \bar{\psi} (i \gamma^\mu \partial_\mu - M_0) \psi + g_0 \phi \bar{\psi} \psi. \]  

(3)

We introduce next the cutoff on spatial momenta in the fundamental frame. Of course this is not a realistic model of Planck-scale granularity. However it does represent a field theory in which the basic Lagrangian gives Lorentz-invariant dispersion relations for low-energy classical modes, and in which there is a Planck-scale cutoff that is bound to a particular frame.

Therefore fermion bare propagators are modified according to

\[ \frac{i}{\gamma^\mu p_\mu - m_0 + i \epsilon} \rightarrow \frac{\gamma^\mu p_\mu - m_0 + \Delta (|\vec{p}|/\lambda) + i \epsilon}{i f (|\vec{p}|/\Lambda)}, \]  

(4)

and similarly the scalar bare propagators are modified according to

\[ \frac{i}{p^2 - M_0^2 + i \epsilon} \rightarrow \frac{i \tilde{f} (|\vec{p}|/\Lambda)}{p^2 - M_0^2 + \tilde{\Delta} (|\vec{p}|/\lambda) + i \epsilon}. \]  

(5)

The requirement on the functions \( f(x) \) and \( \tilde{f}(x) \) which specify the cut-off is that they go to 1 as \( x \to 0 \) to reproduce Lorentz Invariant low energy behavior and that they go to zero as \( x \to \infty \). The functions \( \Delta \) and \( \tilde{\Delta} \) would be specified by concrete proposals for the modified dispersion relations. We concentrate in examining the effect of the cutoff by itself, and thus the changes in the bare dispersion relations will be ignored. Let us consider the full propagator of the scalar field, and more specifically \( \Pi(p) \) its self-energy\(^4\). The parameter \( \Lambda \) is of order the Planck scale. Our choice that the cutoff function depends only on the size of the 3-momentum is for simplicity of calculation, and in fact simple changes in this choice do not change the main result [11].

The one-loop approximation to the self-energy \( \Pi(p) \) is given by the a standard “sausage” Feynman diagram. We wish to investigate its properties when the momentum \( p^\mu \) and the mass \( m \) are much less than the cutoff \( \Lambda \). Thus we make the customary Taylor expansion of \( \Pi \) about \( p = 0 \) and obtain

\[ \Pi(p) = A + p^2 B + p^\mu p^\nu W_\mu W_\nu \tilde{\xi} + \Pi^{(I)}(p^2) + O(p^4/\Lambda^2). \]  

(6)

Here \( W_\mu \) is the 4-velocity of the preferential frame, which appearance can be traced to eqs. 4 and 5 where \( |\vec{p}| = \sqrt{(\eta_{\mu\nu} + W_\mu W_\nu)p^\mu p^\nu} \). \( p^2 = p^\mu p^\nu \eta_{\mu\nu} \), with \( \eta_{\mu\nu} \) being the space-time

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\(^4\) In perturbation theory, \( \Pi(p) \) is the sum over one-particle-irreducible two point graphs for the scalar field.

unknown theory, but then, one would have to give up the argument that motivates the search for these quantum gravity effects using probes and interactions that are described in terms of this theory.
The coefficients $A$ and $B$ correspond to the usual Lorentz-invariant mass and wave function renormalization. The fourth term $\Pi^{(LI)}(p^2)$ is Lorentz-invariant. The third term however is clearly Lorentz violating. The coefficient $\tilde{\xi}$ is independent of $\Lambda$, and in fact explicit calculations give:

$$\tilde{\xi} = \frac{g^2}{6\pi^2} \left[ 1 + 2 \int_0^\infty dx x f'(x)^2 \right].$$

(7)

Although this term depends on the details of the function $f$ which models the microscopic quantum gravity effects, it is positive definite. Quantitatively the corresponding Lorentz violation is of order the square of the coupling, rather than being power-suppressed. One might want to treat this term as a renormalization of the space-time metric tensor however, there are many fields in the standard model that differ by the sizes of their couplings. Hence one way to describe the effect would be to say that each of these fields sees a different metric tensor and thus has a different limiting velocity. On the other hand, the limits on the differences in limiting velocities for different particle species are – even using only analysis that predate the latest round of studies – quite stringent [12] at the level $10^{-20}$ while the expected value of such differences is about $10^{-2}$ at the least given the values of the standard model coupling constants. Thus in the absence of a mechanism that would prevent this large Lorentz violations, while preserving some small ones, the ideas underlying the proposals discussed at the start of this section, would seem to be rather untenable. Recently Pospelov [13] has argued that supersymmetry might provide such mechanism. However he notes that the supersymmetry algebra contains the Lorentz Algebra, and thus it would seem problematic to argue that the latter is broken but the former is at work. He notes nevertheless that the protection from large Lorentz violations would work even if only the translation subgroup was unbroken. Here, we point out that Space-time granularity would break precisely such subgroup.

We should emphasize that while the idea that the Planck length might be some sort of “minimum measurable length”, seem to put into question the range of validity of Lorentz invariance, as shown in [14], the existence of a minimum measurable length does not of itself imply that local Lorentz invariance is violated any more than the discreteness of the eigenvalues of the angular momentum operators implies violation of rotational invariance in ordinary quantum mechanics. On a similar note, the work in [15] illustrates the point that a discrete structure of space-time does not by itself imply the existence of a preferential rest frame or the violation of Lorentz invariance.

We conclude that at present the theoretical ideas that pretend to connect a space-time granularity of quantum gravitational origin, with a violation of Lorentz invariance seem to be in serious trouble, to say the least.

III. IF IT AINT BROKEN WHY NOT TRY BENDING IT?.

This title has perhaps an exceedingly negative connotation, and fairness requires that the reader be warned that at this point, it can not be argued that these approaches are unviable. However, given the tremendous difficulties that such schemes seem to face, in particular as regards to the physical interpretation that one is to give to the mathematical structures, and which will be briefly discussed below, I can only give my own personal pessimistic outlook for this line of thought. Nevertheless, I shall point to a particular deviation that seems to me more promising not only because it is simpler, but rather because, not only is it grounded
in a rigorous mathematical foundation, but is based on a method that is successful in what

can be considered as similar instances.

This second point of view towards Quantum Gravity Phenomenology is based on the

idea that Lorentz Invariance might not be broken and that there would be therefore no

preferential frame at all, but that instead, the local geometry of space-time would exhibit
departures from that described by special relativity. The options that have been considered

can be classified as tied to the notions: 1) that the Lorentz algebra might be replaced by

some sort of nonlinear mathematical structure, 2) that the Lorentz algebra, in fact the full

Poincaré algebra might be unified with the Heisenberg algebra (i.e. including space-time

coordinates) and then modified, and 3) that the space-time structure in itself might become

non-commutative.

To go into the detailed way in which problems appear in each one of these alternatives

would be far beyond the scope of this paper. However the main problems will be mentioned

for the benefit of the reader.

Consider first the scenarios where the Lorentz algebra is supposed to appear in a nonlinear

form, i.e. where the commutators are not longer linear functions of the generators. It has

been shown that in the cases that have been studied \cite{16} one can perform a nonlinear

redefinition of the generators in which the algebra takes again a linear Lie algebra form

(if one adds the central generator and then proceeds as in \cite{17}). One then takes the view

that those are not the variables that one measures. Then the issue becomes what are the

variables that we measure?, and this takes us to the assumptions underlying the way we built

our detectors and other devices that measure energy momentum etc.. Here we note that

the conservation of energy-momentum is one of the basic principles in which we base our

measurements of these quantities, our design of the apparatuses to measure them, and that

this fact is a particularly important aspect of such measurements for high energy particles.

Thus, the issue becomes what are the quantities that are conserved?, and this takes us to

the issue of how one obtains total energy and total momentum for composite objects, which

is essentially linked to the selection of the co-product for the algebra generators.

If one wants something different from the standard situation one would need a nontrivial

(non-primitive) co-product \cite{18}. The cases that have been analyzed are based on the selection

of asymmetric (or non-commutative) co-products, where, say, the total energy of a pair of

particles depends on the way we order them calling them first or second. Here one faces a

very serious problem because there does not seem to exist a canonical recipe for deciding

in each specific situation, which order to take, i.e. given two particles in a collider, which

one is called the first and which is second affects their total energy and momentum, a

clearly disastrous situation, as one would not know how to proceed. Thus one would lack

an interpretation scheme for the formalism, a fact that makes it impossible to use, at least,

for phenomenology.

In fact even if one chooses a symmetric but nontrivial co-product one faces the so called

“spectator problem” where the system of two particles would transform differently if con-

sidered as a subsystem of, say a three particle system, that when considered by itself. In

that case one would not know in principle how to proceed, as even a particle in a remote

region of the universe, which happened to be in the regime where the non-linearity becomes

important, would affect the physics of, say, a scattering process at a Fermilab. It thus seems

that any recourse to a nontrivial co-product puts us in an essentially untenable situation.

Let us note that some of the problems that these proposals face have been pointed out before

\cite{19}.
The second option would start from the requirement that one maintains a Lie Algebra structure and a trivial co-product, and modify only the Lie Algebra structure constants. Within this set of ideas, the most promising ones seem to be those that arise from considerations of algebraic stability applied to the Poincaré-Heisenberg algebra, \([1], [2]\). It is noteworthy that such considerations would take one directly from the Galilean algebra, to the Lorentz algebra, and from the commutative algebra of functions over phase space, to the Heisenberg algebra \([20]\). Unfortunately, and as clearly noted by the authors of \([2]\) this approach also suffers from interpretational difficulties, connected to the fact that for composite systems the position operators can not be reasonably expected to be additive. In other words, in such schemes one is asked to consider nonstandard commutators involving the 4-position and 4-momentum operators and a new central operator, while the Lorentz sector remains untouched. In devising an experiment we need to have an unambiguous identification of these objects with the quantities one measures. It seems rather clear that the objects we would call the position operators in these schemes can not be identified with the actual position of objects that are obtained during measurements\(^5\), and then one is at a loss as to what can be an actual test of the scheme. It is a fact that the momentum operators do not seem, at first sight, to suffer from the same problems that afflict the position operators but the issue of what objects can they in fact be associated with, seems a bit confusing as the momentum operators are intimately connected to the position operators as they both trace their origin to conjugated pair of variables. Thus it seems that one could not associate such momentum operators to objects to which one can not associate the corresponding position operators. These points are not made to suggest that the scheme is unviable, as I do not think it is, but rather to stress the fact that, the hope for its applicability lies in a profound interpretational analysis that would clarify the status of “4-position observables” and connect them to the objects found in the real world.

The third set of ideas, the so called non-commutative geometry program starts usually with the postulate that the Minkowski coordinates do not commute but the commutators are functions of these coordinates themselves and not of any other generators. Thus one assumes that they satisfy a fundamental commutation relation such as

\[
[x^\mu, x^\nu] = i\theta^{\mu\nu},
\]

where \(\theta^{\mu\nu}\) is a fundamental antisymmetric c-number tensor. Here we note that, if taken at face value and using it without further modifications\(^6\), the problem is that there is no Lorentz invariant antisymmetric tensor of rank 2. This point should not confused with the covariance of antisymmetric tensors such as the electromagnetic field strength \(F^{\mu\nu}\). The issue is of course, that once we write the specific numerical proposal for the matrix \(\theta^{\mu\nu}\),

\(^5\) The issue is the following: The position operator is not expected to de additive (to find the position of an hydrogen atom one does not add the position of the proton wit that of the electron), while the symmetry of the construction would require similar coproducts for position and momentum operators (the momentum of composite systems as usual being additive). On the other hand if the coproduct is nontrivial, the issue would be how to deal with the position operators for composite objects.

\(^6\) It is possible of course, to bring in a more elaborated structure to remove the problems of a naive interpretation. In fact in the more methodical formulations many other objects become non-commutative, including for instance entries of the matrices representing the generators of the Lorentz transformations, resulting in new notions of invariance and new types of invariant tensors \([21]\)
such specific value can be associated at most with one specific reference frame, and then the issue is: which one? In other words, while in the case of $F^{\mu\nu}$ its specific values in a given frame and a given physical situation are determined by the field equations of motion and the boundary conditions (the latter of course have different specific values in the various frames), in the case of a fixed fundamental object associated with the space-time structure such as $\theta^{\mu\nu}$ those elements are not available. Therefore any specific recipe for $\theta^{\mu\nu}$ could only be done in association with one specific frame, and furthermore it would imply the existence of a preferential frame where the corresponding matrix with specific values has a particularly simple form. For instance if at all possible it will be only only in one particular frame that we could say that the $\theta^{0i}$ components are all zero. Thus this scheme incorporates the selection of preferential frames, and is thus, as pointed out in [10], susceptible to the same problems we encountered in section II. Of course one should stress that it is conceivable that a scheme can be constructed that would overcome these difficulties and in this regard its worthwhile to mention the proposals in [22].

Other proposals one finds in the literature, start by writing:

$$[x^0, x^i] = i\lambda x^i, \quad \text{or}, \quad [x^\mu, x^\nu] = f^{\mu\nu}_{\rho} x^\rho, \quad (9)$$

These schemes in my view suffer from another serious problem: Lets focus on the first proposal: If we pretend to view this within the interpretational framework of quantum mechanics, the corresponding uncertainty relations indicate that one could not find a simultaneous eigenvector of $x^0$ and $x^i$, except when the corresponding eigenvalue of $x^i$ happens to be zero. Thus one can not localize an event both in space and in time (taking as we said the interpretational framework directly from QM) unless that event is at the origin of the coordinates. The issue is then: Where is the origin of coordinates? It would be clearly unphysical to state that certain points in space become physically differentiated just because we chose them as the origin of coordinates. It is clear that the second scheme suffers from similar problems: If the objects $x^\mu$ have any relation whatsoever with the coordinates we measure it is clear that the non-commutativity becomes larger for larger values of the coordinates (the same can be said for eigenvalues, expectation values, for the corresponding operators, and in fact for any adjudication of some real values to these objects). In other words the effect decreases as we approach the origin of the coordinates. But where on the universe is this point or region? The only option seem to be to change the interpretational scheme, but then we find ourselves again in a similar conundrum as that of the first direction we explored in this section.

It is of course possible that these problems might be overcome but at this point it is fair to say that the situation if far from clear. For further reading on Quantum Field theory in non-commutative space-times see [23].

IV. WHAT MIGHT BE THERE?

In view of the hardships one encounters in trying to reconcile the naive idea of a granular structure of space-time – which would naturally be associated with a preferential reference frame where the granularity takes say the most symmetric form – with the tight phenomenological bounds that have been obtained and with the clear expectations from Quantum Field Theory, that the effects of such granular structure would be only lightly suppressed, one is lead to consider more subtle possibilities. In this section we will discuss an alternative way
in which a granular structure of space-time might appear, and which would be immune from
the previous considerations while still, in principle, susceptible to a phenomenological study.
The idea will be discussed in a rather heuristic way and it is fair to say that there is at
this point no concrete realization of the proposal. However one can as usual employ the
symmetry principles to restrict the possible phenomenological manifestations. These ideas
have been studied first in [24].

We have at this point no real good geometrical picture of how a granularity might be
associated to space-time while strictly preserving the Lorentz, and Poincaré symmetries.
One can point however to the Poset program [15] as one that seems to embody such scheme,
however we will not at this point commit to any such specific proposal. Instead we seek
guidance in analogies with some simple ideas from solid state physics. Thus, we consider
the case of a crystal, and note that when a large crystal has the same symmetry (say cubic)
of the fundamental crystal, one could expect no deviations from fully cubic symmetry, as
a result of the discrete nature of the fundamental building blocks. In fact one would not
expect in such situation that the discrete structure of the crystal could be revealed at the
macroscopic level by any deviation from precise cubic symmetry. The discrete structure
might be studied, of course, but NOT by looking at deviations from such symmetry. How-
ever if one considers a macroscopic crystal whose global form is not compatible with the
structure of the fundamental crystals, say hexagonal, the surface will necessarily include
some roughness, and thus a manifestation of the granular structure, would occur through a
breakdown of the exact hexagonal symmetry.

Our ideas will be guided by the simple picture above, which will be transported mutatis
mutandis from the crystal and the cubic symmetry to the space-time and the Lorentz sym-
metry. Thus, we will start by assuming that the underlying symmetry of the fundamental
structure of space-time is itself the Lorentz Symmetry, which would naturally leads us to
expect no violation of the symmetry at the macroscopic level when the space-time is macro-
scopically Lorentz invariant. Thus, the large scale Lorentz Symmetry is protected by the
symmetry of the fundamental granular structure.

Thus in a region of space-time normally considered as well approximated by Minkowski
metric, the granular structure of the quantum space-time would not become manifest
through the breakdown of its symmetry. However, and following with our solid state anal-
ogy, we are lead to consider the situation in which the macroscopic space-time is not fully
compatible with the symmetry of its basic constituents.

The main point is then, that in the event of a failure of the space-time to be exactly
Minkowski in an open domain, the underlying granular structure of quantum gravity origin,
could become manifest, affecting the propagation of the various matter fields. Such situation
should thus involve the Riemann tensor, which is known to precisely describe the failure of
a space-time to be Minkowski over an open region. Thus the non-vanishing of Riemann
would correspond to the macroscopic description of the situation where the microscopic
structure of space-time might become manifest. Moreover, we can expect, due to the implicit
correspondence of the macroscopic description with the more fundamental one, that the
Riemann tensor would also indicate the space-time directions with which the sought effects
would be associated.

This selection of special space-time directions, embodies a certain analogy within the
current approach, to the global selection of a preferential reference frame that was implicit
in the schemes towards Quantum Gravity Phenomenology described in section II.

With this ideas in mind we turn now to proposing the corresponding phenomenology.
That would imply the consideration of an effective description in the way that the Riemannian curvature could affect, in a nontrivial manner, the propagation of matter fields. Thus we need to consider the Lagrangian terms representing such couplings.

Before we do so, we recall that the Ricci tensor represents that part of the Riemann tensor which, at least on shell, is locally determined by the energy momentum of matter at the events of interest. Thus the coupling of matter to the Ricci tensor part of the Riemann tensor would, at the phenomenological level, reflect a sort of pointwise self interaction of matter that would amount to a locally defined renormalization of the usual phenomenological terms such as a the mass or the kinetic terms in the Lagrangian.

However we are interested in the underlying structure of space-time rather that the self interaction of matter. Thus we would need to ignore the aspects that encode the latter, which in our case would corresponds to all Lagrangian terms containing the Ricci tensor, coupled to matter fields. The remainder of the Riemann tensor, i.e. the Weyl tensor, can thus be thought, to reflect the aspects of the local structure of space-time associated solely with the gravitational degrees of freedom.

Therefore we are lead to consider the coupling of the Weyl tensor with the matter fields. We note that in the absence of gravitational waves, the Weyl tensor is also connected with the nearby “matter sources” but such connection involves the propagation of their influence through the space-time and thus the structure of the latter would be playing a central role in the way the influences become manifest. In this sense the Weyl tensor reflects the “non-local effects” of the matter in contrast with the Ricci tensor or curvature scalar that are determinable from the latter in a completely local way.

We further assume observer covariance and the absence of globally defined non-dynamical tensor fields. We are interested in the minimally suppressed terms, those that are only suppressed by the first power of $M_{\text{Planck}}$, which would naturally correspond to the dimension 5 operators, while considering the coupling of the fundamental fields of the standard model, bosons and fermions, to the Weyl tensor. Using the fact that the Weyl tensor, as the Riemann tensor, has mass dimension 2, while the fermions have mass dimension 3/2 and the bosons have mass dimension 1, one can show that there are no non-vanishing dimension five operators coupling the Weyl tensor to the fields of the standard model [24]. Thus one can either take this as an indication that the effects one is looking for are more strongly suppressed or search for somehow more indirect approaches. In [24] we take the latter approach and consider the following schema.

One considers the Weyl tensor viewed as a tensor of type $\left(2,2\right)$ as a mapping from the space of antisymmetric tensors of type $\left(0,2\right)$, $\mathcal{S}$ into itself. As is well known the space-time metric endows the six dimensional vector space $\mathcal{S}$ with a pseudo-Riemannian metric of signature $(+++---)$. Then the Weyl tensor is a symmetric operator on this space $\mathcal{S}$, which can therefore be diagonalized, and thus has a complete set of eigenvectors (which are however not necessarily orthogonal). We will assume for simplicity, that all eigenvalues are different, and consider only the eigenvectors $\Xi^{(i)}$ corresponding to non-vanishing eigenvalues $\lambda^{(i)}$, by fixing the normalization of these eigenvectors to be $\pm 1$ (also drop the null eigenvectors). Next we use the antisymmetric tensors $\Xi_{\mu\nu}^{(i)}$ and their associated eigenvalues $\lambda^{(i)}$ to construct the types of Lagrangian terms we are interested in. Finally we look for terms linear in these objects, and recalling that the eigenvalues $\lambda^{(i)}$ have the dimension of the Riemann tensor, we have the least possible suppressions in each sector as follows: In the scalar sector there is in fact no candidate of dimension 5 or 6 for such term. In the vector boson sector, taking into account the requirements of gauge invariance, we are lead to a dimension 6 term
\[ \mathcal{L}_m = \frac{\xi}{M_{Pl}^2} \sum_i \lambda^i \Xi^{(i)}_{\mu\nu} \text{Tr} \left( F^\mu_\rho F^{\rho\nu}_\nu \right). \]  

(10)

It is worthwhile pointing out that in a purely \( U(1) \) sector one can write a dimension 4 term,

\[ \mathcal{L}_m = \xi \sum_i \lambda^i \Xi^{(i)}_{\mu\nu} F^{\mu\nu}. \]

(11)

This is an unsuppressed term, which is rather surprising, however given that such term can not be written in the case of non-abelian gauge fields, together with fact that in the standard model the \( U(1) \) sector mixes with the \( SU(2) \) sector suggest that such terms should be absent. We have no tighter argument regarding this possibility at this point, but we will not consider it any further in view of the last observation.

Finally, in the fermion sector we have a term,

\[ \mathcal{L}_\psi = \frac{\xi}{M_{Pl}} \sum_i \lambda^i \Xi^{(i)}_{\mu\nu} \bar{\Psi}_a \gamma^\mu \gamma^\nu \Psi_a. \]

(12)

Thus the fermions seem to provide the most promising probes, which seems a fortunate situation, in this scheme.

One could also consider coupling directly a scalar made out of the standard model fields to an appropriate power of a scalar constructed out of the Weyl tensor such as \( (W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma})^{1/2} \). This proposal, departs slightly from the spirit of the suggestion the space-time structure would naturally and locally select preferential space-time directions. The lack this feature would tend to make the effects, in principle much harder to detect experimentally. On the other hand this line opens the way to consider, effects that would not be suppressed by \( M_{\text{Planck}} \) at all, such as

\[ \mathcal{L}_\psi = (W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma})^{1/4} \bar{\Psi}\Psi. \]  

(13)

These type of terms in which the space-time structure appears only as a scalar coupled to the matter fields would correspond to a space-time dependence of mass or coupling constants, controlled by local curvature. As we mentioned, the fact that they exhibit no particular signature, would tend to make the related effects, very difficult to probe.

Regarding phenomenology one should thus, concentrate clearly in the fermion sector as leading to the most promisingly observable effects.

Before continuing we write again the corresponding Lagrangian term, taking now into account, a possible flavor dependence, which could be thought to arise from the detailed way the different fields interact with the virtual excitations that intimately probe the underlying space-time structure. Thus we consider:

\[ \mathcal{L}_f^{(2)} = \sum_a \frac{\xi_a}{M_{Pl}} \sum_i \lambda^i \Xi^{(i)}_{\mu\nu} \bar{\Psi}_a \gamma^\mu \gamma^\nu \Psi_a. \]  

(14)

where \( a \) denotes flavor. Next we note that we have in principle the same types of effects that have been considered in the Standard Model Extension (SME) \([25]\) but only with terms of the form \(-1/2H_{\mu\nu} \bar{\Psi} \sigma^{\mu\nu} \Psi \). Moreover, here the tensor \( H_{\mu\nu} \) must be identified with \(-\frac{2\xi}{M_{Pl}} \sum_i \lambda^i \Xi^{(i)}_{\mu\nu} \), and thus has a predetermined space-time dependence dictated by the surrounding gravitational environment. Therefore, special care has to be taken when comparing
different experiments at different sites\textsuperscript{7}, by taking into account the differences in the surrounding environment that leads to variable values of the relevant curvature related tensors.

Finally we briefly comment on the related phenomenology: The relevant experiments must be associated with both, relative large gravitational tidal effects (indicating large curvature) in the local environment together with probes involving polarized matter as the explicit appearance of the Dirac matrix $[\gamma^\mu, \gamma^\nu]$ indicates. Both conditions seem from the onset difficult to achieve, and to control. Polarized matter is usually highly magnetic and thus electromagnetic disturbance would need to be controlled to a very high degree as they would tend to obscure any possible effects. Gravitational field gradients are usually exceedingly small on Earth and even in the solar system.

Thus, neutrinos crossing regions of large curvature, seem like very good candidates to be studied in this context. We note in particular that a term of the sort we are considering could lead to neutrino oscillations even if they are massless, in close analogy with the ideas exposed in \cite{27}.

Next we note that the terms in question do not violate CPT so that that particular phenomenological avenue is closed. On the other hand other discrete symmetries, particularly CP could, depending on the environment and state of motion of the probes seem to be open channels for investigation.

In this light it would be very interesting to consider the Neutral Kaon system where, as in the fifth force scenario, one would look for energy dependence of the system’s parameters \cite{28}. On the other hand, as we mentioned before one expects the that the useful probes would involve polarized matter which would seem to rule out the usefulness of Neutral Kaons. However one should consider other particles, such as neutrinos that might combine, some sort of flavor oscillation with a nontrivial polarization structure.

All these ideas are of course in need of a much more detailed study.

We end this section by pointing out a quite different proposal regarding possible manifestations of Quantum Gravity: the possibility that a underlying discrete structure of time, would lead to a fundamental decoherence in quantum mechanics considered in \cite{29}. This idea is quite intriguing and in fact is closer in spirit to the ideas and proposals that we address in the next section.

\section{What Seems to Be There}

This might sound like a strange title, as it indicates that there is in fact some sort of evidence for a manifestation of Quantum Gravity. We will argue that indeed there is something out there that requires new physics for its understanding. It is of course not at all clear that the problem we will discuss should be related to Quantum Gravity, but since that is the only sphere of fundamental physics for which we have so far failed to find a satisfactory conceptual understanding\textsuperscript{8} we find quite natural to associate the two. In fact

\textsuperscript{7} This is reminiscent of the Situation encountered with the studies of the “Fifth Force” proposals\cite{26}.

\textsuperscript{8} There are of course many open issues in fundamental understanding of physics that are not in principle connected with the issue of quantum gravity, however it is only in this latter field that the problems seem to be connected with deep conceptual issues and where one can envision the possibility that their resolution might require a fundamental change of paradigm, as the would be the case if we find we must modify the laws of quantum mechanics.
the ideas of Penrose regarding the fundamental changes, that he argues[30], are needed in Quantum Mechanics and their connection to quantum Gravity, are a inspirational precedent for the analysis first reported in [31].

There are for instance lingering interpretational problems in quantum mechanics, in particular in connection with the measurement problem. For instance, and as it is often emphasized by R. Penrose, we have in the Copenhagen interpretation, and in fact in any practical application of the theory, two quite different evolution processes: the U process or unitary evolution applied when systems are not subjected to a measurement, and the R process or state reduction process which makes its appearance whenever a measurement is invoked. The point is that without recourse to the R process the theory can make no predictions. But, when exactly should we in principle call upon the R process becomes a question that is not addressed within the theory. Other interpretations have similar problems, for instance in the many worlds interpretation one has the universe splitting with every measurement. However the issue of how in principle do we determine what constitutes a measurement, is no resolved. These issues have prompted R. Penrose to propose that Quantum gravity might play a role in triggering a real dynamical collapse of the wave function of systems [30]. His proposals would have a system collapsing whenever the gravitational interaction energy between two alternative realizations that appear as superposed in a wave function of a system reaches a certain threshold which is identified with $M_{\text{Planck}}$. The ideas can in principle lead to observable effects and in fact experiments to test them are currently being contemplated [32] (although it seems that the available technology can not yet be pushed to the level where actual tests might be expected to become a reality soon). We have considered in [31] a situation for which there exist already a wealth of empirical information and which we have argued can not be fully understood without involving some New Physics, whose required features would seem to be quite close to Penrose’s proposals: The quantum origin of the seeds of cosmic structure.

In fact one of the major claimed successes of Inflationary cosmology is its reported ability to predict the correct spectrum for the primordial density fluctuations that seed the growth of structure in our Universe. However when one thinks about it one immediately notes that there is something truly remarkable about it, namely that out of an initial situation which is taken to be perfectly isotropic and homogeneous and based on a dynamics that preserves those symmetries one ends with a non-homogeneous and non isotropic situation. Most of our colleagues who have been working in this field for a long time would reassure us, that there is no problem at all by invoking a variety of arguments. It is noteworthy that these arguments would tend to differ in general from one inflationary cosmologist to another [33]. Other cosmologists do acknowledge that there seems to be something unclear at this point [34]. In a recent paper [31] a critical analysis of such proposals has been carried out indicating that all the existing justifications fail to be fully satisfactory. In particular, the cosmological situation can be seen to be quite different from any other situation usually treated using quantum mechanics when one notes the fact that while in analyzing ordinary situations quantum mechanics offers us, at least one self consistent assignment at all times of a state of the Hilbert space to our physical system (we are of course thinking of the Schroedinger picture). It is well known, that in certain instances there might be several mutually incompatible assignments of that sort, as for instance when contemplating the two descriptions offered by two different inertial observers who consider a given a specific EPR experiment. However, as we said, in all known cases, one has at least one description available. The reader might want to attempt to conceive of such assignment – of a state at
each time – when presented with any of the proposed justifications offered to deal with the issue of the transition from a symmetric universe to a non-symmetric one. The reader will find that each instance he/she will be asked to accept one of the following: i) our universe was not really in that symmetric state (corresponding to the vacuum of the quantum field), ii) our universe is still described by a symmetric state, iii) at least at some points in the past the description of the state of our universe could not be done within quantum mechanics, iv) quantum mechanics does not correspond to the full description of a system at all times, or v) our own observations of the universe mark the transition from a symmetric to an asymmetric state. It should be clear that none of these represent a satisfactory alternative, in particular if we want to claim that we understand the evolution of our universe, its structure – including ourselves –, as the result of the fluctuations of quantum origin in the very early stages of our cosmology. Needless is to say that none of these options will be explicitly called upon in the arguments one is presented with, however one or more would be hidden, perhaps in a subtle way, underneath some of the aspects of the explanation. For a more thorough discussion we refer the reader to [31].

The interesting part of these situation is that one is forced to call upon to some novel physical process to fill in the missing or unacceptable part of the justification of the steps that are used to take us from that early and symmetric state, to the asymmetric state of our universe today, or the state of the universe we photograph when we look at the surface of last scattering in the pictures of the CMB. In [31] we have considered in this cosmological context a proposal calling for a self induced collapse of the wave function along the general lines conceived by Penrose, and have shown that the requirement that one should obtain results compatible with current observations is already sufficient to restrict in important ways some specific aspects of these novel physics. Thus, when we consider, that the origin of such new physics can be traced to some aspects of quantum gravity, one is already in a position of setting phenomenological constraints at least on this aspect of the quantum theory of gravitation.

In the following we give a short description of this analysis for the benefit of the reader. The staring point is as usual the action of a scalar field coupled to gravity.

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R[g] - 1/2 \nabla_a \phi \nabla_b \phi g^{ab} - V(\phi) \right],
\]

(15)

where \( \phi \) stands for the inflaton or scalar field responsible for inflation and \( V \) for the inflaton’s potential. One then splits both, metric and scalar field into a spatially homogeneous (‘background’) part and an inhomogeneous part (‘fluctuation’), i.e. \( g = g_0 + \delta g, \phi = \phi_0 + \delta \phi \).

The unperturbed solution correspond to the standard inflationary cosmology which written using a conformal time, has a scale factor

\[
a(\eta) = -\frac{1}{H\eta},
\]

(16)

and with the scalar \( \phi_0 \) field in the slow roll regime. The perturbed metric can be written

\[
d\sigma^2 = a(\eta)^2 \left[ -(1 + 2\Psi) d\eta^2 + (1 - 2\Psi) \delta_{ij} dx^i dx^j \right],
\]

(17)

where \( \Psi \) stands for the relevant perturbation and is called the Newtonian potential.

The perturbation of the scalar field leads to a perturbation of the energy momentum tensor, and thus Einstein’s equations at lowest order lead to

\[
\nabla^2 \Psi = 4\pi G \dot{\phi}_0 \dot{\phi}.
\]

(18)
Now, write the quantum theory of the field \( \delta \phi \). It is convenient to consider instead the field \( y = a \delta \phi \). We consider the field in a box of side \( L \), and decompose the real field \( y \) into plane waves

\[
y(\eta, \vec{x}) = \frac{1}{L^3} \sum_k \left( \hat{a}_k y_k(\eta) e^{i\vec{k} \cdot \vec{x}} + \hat{a}_k^\dagger \bar{y}_k(\eta) e^{-i\vec{k} \cdot \vec{x}} \right),
\]

where the sum is over the wave vectors \( \vec{k} \) satisfying \( k_i L = 2\pi n_i \) for \( i = 1, 2, 3 \) with \( n_i \) integers.

It is convenient to rewrite the field and momentum operators as

\[
\hat{y}(\eta, \vec{x}) = \frac{1}{L^3} \sum_k e^{i\vec{k} \cdot \vec{x}} \hat{y}_k(\eta), \quad \hat{\pi}(y, \vec{x}) = \frac{1}{L^3} \sum_k e^{i\vec{k} \cdot \vec{x}} \hat{\pi}_k(\eta),
\]

where \( \hat{y}_k(\eta) \equiv y_k(\eta) \hat{a}_k + \bar{y}_k(\eta) \hat{a}_k^\dagger \) and \( \hat{\pi}_k(\eta) \equiv g_k(\eta) \hat{a}_k + \bar{g}_k(\eta) \hat{a}_k^\dagger \) with

\[
y_k^{(\pm)}(\eta) = \frac{1}{\sqrt{2k}} \left( 1 \pm \frac{i}{\eta k} \right) \exp(\pm i k \eta),
\]

and

\[
g_k^{\pm}(\eta) = \pm i \sqrt{\frac{k}{2}} \exp(\pm i k \eta).
\]

As we will be interested in considering a kind of self induced collapse which operates in close analogy with a “measurement”, we proceed to work with Hermitian operators, which in ordinary quantum mechanics are the ones susceptible of direct measurement. Thus we decompose both \( \hat{y}_k(\eta) \) and \( \hat{\pi}_k(\eta) \) into their real and imaginary parts \( \hat{y}_k(\eta) = \hat{y}_k^R(\eta) + i \hat{y}_k^I(\eta) \) and \( \hat{\pi}_k(\eta) = \hat{\pi}_k^R(\eta) + i \hat{\pi}_k^I(\eta) \) where

\[
\hat{y}_k^{R,I}(\eta) = \frac{1}{\sqrt{2}} \left( y_k(\eta) \hat{a}_k^{R,I} + \bar{y}_k(\eta) \hat{a}_k^{\dagger R,I} \right), \quad \hat{\pi}_k^{R,I}(\eta) = \frac{1}{\sqrt{2}} \left( g_k(\eta) \hat{a}_k^{R,I} + \bar{g}_k(\eta) \hat{a}_k^{\dagger R,I} \right).
\]

We note that the operators \( \hat{y}_k^{R,I}(\eta) \) and \( \hat{\pi}_k^{R,I}(\eta) \) are therefore hermitian operators. Note that the operators corresponding to \( k \) and \( -k \) are identical in the real case (and identical up to a sign in the imaginary case).

Next we specify our model of collapse, and follow the field evolution through collapse to the end of inflation. We will assume that the collapse is somehow analogous to an imprecise measurement of the operators \( \hat{y}_k^{R,I}(\eta) \) and \( \hat{\pi}_k^{R,I}(\eta) \) which, as we pointed out are hermitian operators and thus reasonable observables. These field operators contain complete information about the field (we ignore here for simplicity the relations between the modes \( k \) and \( -k \)).

Let \( |\Xi\rangle \) be any state in the Fock space of \( \hat{y} \). Let us introduce the following quantity:

\[
\hat{d}_k^{R,I} = \langle \hat{a}_k^{R,I} \rangle_\Xi.
\]

Thus the expectation values of the modes are expressible as

\[
\langle \hat{y}_k^{R,I} \rangle_\Xi = \sqrt{2} \Re (y_k g_k^{R,I}), \quad \langle \hat{\pi}_k^{(y)R,I} \rangle_\Xi = \sqrt{2} \Im (g_k d_k^{R,I}).
\]

(24)

For the vacuum state \( |0\rangle \) we have of course:

\[
\langle \hat{y}_k^{R,I} \rangle_0 = 0, \langle \hat{\pi}_k^{(y)R,I} \rangle_0 = 0,
\]

while their corresponding uncertainties are

\[
(\Delta \hat{y}_k^{R,I})_0^2 = (1/2)|y_k|^2(hL^3), \quad (\Delta \hat{\pi}_k^{R,I})_0^2 = (1/2)|g_k|^2(hL^3).
\]

(25)

The collapse
Now we will specify the rules according to which collapse happens. Again, at this point our criteria will be simplicity and naturalness. Other possibilities do exist, and may lead to different predictions.

What we have to describe is the state $|\Theta\rangle$ after the collapse. We need to specify $d_{k}^{R,I} = \langle \Theta|\hat{a}_{k}^{R,I}|\Theta\rangle$ In the vacuum state, $\hat{y}_{k}$ and $\hat{\pi}_{k}^{(y)}$ individually are distributed according to Gaussian distributions centered at 0 with spread $(\Delta \hat{y}_{k})^{2}$ and $(\Delta \hat{\pi}_{k}^{(y)})^{2}$ respectively. However, since they are mutually non-commuting, their distributions are certainly not independent. In our collapse model, we do not want to distinguish one over the other, so we will ignore the non-commutativity and make the following assumption about the (distribution of) state(s) $|\Theta\rangle$ after collapse:

$$\langle \hat{y}_{k}^{R,I}(\eta_{c}^{k})\rangle_{\Theta} = x_{k,1}\sqrt{(\Delta \hat{y}_{k}^{R,I})^{2}} = x_{k,1}|y_{k}(\eta_{c}^{k})|\sqrt{\hbar L^{3}/2},$$  

$$\langle \hat{\pi}_{k}^{(y)}(\eta_{c}^{k})\rangle_{\Theta} = x_{k,2}\sqrt{(\Delta \hat{\pi}_{k}^{(y)})^{2}} = x_{k,2}|g_{k}(\eta_{c}^{k})|\sqrt{\hbar L^{3}/2},$$

where $x_{k,1}, x_{k,2}$ are distributed according to a Gaussian distribution centered at zero with spread one. From these equations we solve for $d_{k}^{R,I}$. Here we must recognize that our universe, corresponds to a single realization of the random variables, and thus each of the quantities $x_{k,1,2}$ has a single specific value. Latter we will see how to make relatively specific predictions despite of these features.

Next we focus on the expectation value of the quantum operator which appears in our basic formula

$$\nabla^{2}\Psi = s\Gamma$$  

(28)

(where we introduced the abbreviation $s = 4\pi G\dot{\phi}_{0}$) and the quantity $\Gamma$ as the aspect of the field that acts as a source of the Newtonian Potential. In the slow roll approximation we have $\Gamma = \delta \dot{\phi} = a^{-1}\pi^{y}$. We want to say that, upon quantization, the above equation turns into

$$\nabla^{2}\Psi = s\langle \hat{\Gamma}\rangle.$$  

(29)

Before the collapse occurs, the expectation value on the right hand side is zero. Let us now determine what happens after the collapse: To this end, take the Fourier transform of (29) and rewrite it as

$$\Psi_{k}(\eta) = \frac{s}{k^{2}}\langle \hat{\Gamma}_{k}\rangle_{\Theta}.$$  

(30)

Let us focus now on the slow roll approximation and compute the right hand side, we note that $\delta \dot{\phi} = a^{-1}\pi^{y}$ and hence we find

$$\langle \hat{\Gamma}_{k}\rangle_{\Theta} = \sqrt{\hbar L^{3}k^{2}}\frac{1}{2a}F(k),$$

where

$$F(k) = (1/2)[A_{k}(x_{k,1} + ix_{k,2}^{I}) + B_{k}(x_{k,2}^{R} + ix_{k,1}^{I})],$$  

(31)

with

$$A_{k} = \frac{\sqrt{1 + z_{k}^{2}}}{z_{k}}\sin(\Delta_{k}); \quad B_{k} = \cos(\Delta_{k}) + (1/z_{k})\sin(\Delta_{k})$$  

(32)

and where $\Delta_{k} = k\eta - z_{k}$ with $z_{k} = \eta_{c}^{k}k$.

Next we turn to the experimental results. We will for the most part, disregard the changes to dynamics that happen after re-heating and due to the transition to standard (radiation
dominated) evolution. The quantity that is measured is $\Delta T(\theta, \varphi)$ which is a function of the coordinates on the celestial two-sphere which is expressed as $\sum_{lm} \alpha_{lm} Y_{lm}(\theta, \varphi)$. The angular variations of the temperature are then identified with the corresponding variations in the “Newtonian Potential” $\Psi$, by the understanding that they are the result of gravitational red-shift in the CMB photon frequency $\nu$ so $\delta T / T = \delta \nu / \nu = \delta (\sqrt{g_{00}}) / \sqrt{g_{00}} \approx \delta \Psi$.

The quantity that is presented as the result of observations is $OB_l = l(l+1)C_l$ where $C_l = (2l+1)^{-1} \sum_m |\alpha_{lm}|^2$. The observations indicate that (ignoring the acoustic oscillations, which is anyway an aspect that is not being considered in this work) the quantity $OB_l$ is essentially independent of $l$ and this is interpreted as a reflection of the “scale invariance” of the primordial spectrum of fluctuations.

Then, as we noted the measured quantity is the “Newtonian potential” on the surface of last scattering: $\Psi(\eta_D, \vec{x}_D)$, from where one extracts

$$\alpha_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y_{lm}^* d^2\Omega.$$  

(33)

To evaluate the expected value for the quantity of interest we use (30) and (31) to write

$$\Psi(\eta, \vec{x}) = \sum_k sU(k) \frac{\hbar k}{k^2} \sqrt{k^2} \frac{1}{L^3} \frac{F(\vec{k})e^{ik\cdot\vec{x}}}{2a},$$  

(34)

where we have added the factor $U(k)$ to represent the aspects of the evolution of the quantity of interest associated with the physics of period from re-heating to de-coupling, which includes among others the acoustic oscillations of the plasma.

After some algebra we obtain

$$\alpha_{lm} = s \sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} \sum_k \frac{U(k)\sqrt{k}}{k^2} F(\vec{k})4\pi i^l j_l((|\vec{k}|R_D)Y_{lm}(\hat{k}),$$  

(35)

where $\hat{k}$ indicates the direction of the vector $\vec{k}$. It is in this expression that the justification for the use of statistics becomes clear. The quantity we are in fact considering is the result of the combined contributions of an ensemble of harmonic oscillators each one contributing with a complex number to the sum, leading to what is in effect a 2 dimensional random walk whose total displacement corresponds to the observational quantity. To proceed further we must evaluate the most likely value for such total displacement. This we do with the help of the imaginary ensemble of universes, and the identification of the most likely value with the ensemble mean value. Now we compute the expected magnitude of this quantity. After taking the continuum limit we find,

$$|\alpha_{lm}|^2_{M.L.} = \frac{s^2 \hbar}{2\pi a^2} \int \frac{U(k)^2 C(k)}{k^4} j_l^2((|\vec{k}|R_D)k^3dk,$$  

(36)

where

$$C(k) = 1 + (2/z_k^2) \sin(\Delta_k)^2 + (1/z_k) \sin(2\Delta_k).$$  

(37)

The last expression can be made more useful by changing the variables of integration to $x = kR_D$ leading to

$$|\alpha_{lm}|^2_{M.L.} = \frac{s^2 \hbar}{2\pi a^2} \int \frac{U(x/R_D)^2 C(x/R_D)}{x^4} j_l^2(x)x^3dx,$$  

(38)
which in the exponential expansion regime where $\mu$ vanishes and in the limit $z_k \to -\infty$ where $C = 1$, and taking for simplicity $U(k) = U_0$ to be independent of $k$, (neglecting for instance the physics that gives rise to the acoustic peaks), we find:

$$|\alpha_{lm}|^2_{M.L.} = \frac{s^2U_0^2\hbar}{2a^2} \frac{1}{l(l+1)}. \quad (39)$$

Now, since this does not depend on $m$ it is clear that the expectation of $C_l = (2l + 1)^{-1} \sum_m |\alpha_{lm}|^2$ is just $|\alpha_{lm}|^2$ and thus the observational quantity $OB_l = l(l+1)C_l = \frac{s^2U_0^2\hbar}{2a^2}$ independent of $l$ and in agreement with the scale invariant spectrum obtained in ordinary treatments and in the observational studies.

Thus, the predicted value for the $OB_l$ is $[31]$,

$$OB_l = \left(\pi/6\right)G\hbar\frac{(V')^2}{V}U_0^2 = \left(\pi/3\right)\varepsilon(V/M_{Pl}^2)U_0^2, \quad (40)$$

where we have used the standard definition of the slow roll parameter $\varepsilon = (1/2)M_{Pl}^2(V'/V)^2$ which is normally expected to be rather small. We note that if one could avoid $U$ from becoming too large during re-heating, the quantity of interest would be proportional to $\varepsilon$ a possibility that was not uncovered in the standard treatments, so one could get rid of the “fine tuning problem” for the inflationary potential, i.e. even if $V \sim M_{Pl}^4$, the temperature fluctuations in the CMB would be expected to be small.

Now let us focus on the effect of the finite value of times of collapse $\eta_c k$, that is, we consider the general functional form of $C(k)$. The first thing we note is that in order to get a reasonable spectrum there seems to be only one simple option: That $z_k$ be essentially independent of $k$ that is the time of collapse of the different modes should depend on the mode’s frequency according to $\eta_c k = z/k$. This is a remarkable conclusion which would provide relevant information about whatever the mechanism of collapse is.

Let’s turn next to one simple proposal about the collapse mechanism which following Penrose’s ideas is assumed to be tied to Quantum Gravity, and examine it with the above results in mind.

### A. A version of ‘Penrose’s mechanism’ for collapse in the cosmological setting

Penrose has for a long time advocated that the collapse of quantum mechanical wave functions might be a dynamical process independent of observation, and that the underlying mechanism might be related to gravitational interaction. More precisely, according to this suggestion, collapse into one of two quantum mechanical alternatives would take place when the gravitational interaction energy between the alternatives exceeds a certain threshold. In fact, much of the initial motivation for the present work came from Penrose’s ideas and his questions regarding the quantum history of the universe.

A very naive realization of Penrose’s ideas in the present setting could be obtained as follows: Each mode would collapse by the action of the gravitational interaction between it’s own possible realizations. In our case one could estimate the interaction energy $E_l(k, \eta)$ by considering two representatives of the possible collapsed states on opposite sides of the Gaussian associated with the vacuum. Let us interpret $\Psi$ literally as the Newtonian potential and consequently the right hand side of equation (18) as the associated matter density $\rho$. 

19
Therefore, $\rho = \dot{\phi}_0 \Gamma$, with $\Gamma = \pi^y/a$. Then we would have:

$$E_I(\eta) = \int \Psi^{(1)}(x, \eta) \rho^{(2)}(x, \eta) dV = a^3 \int \Psi^{(1)}(x, \eta) \rho^{(2)}(x, \eta) d^3x,$$

(41)

which when applied to a single mode becomes:

$$E(\eta) = (a^3/L^6) \Psi^{(1)}(\eta) \rho^{(2)}(\eta) \int d^3x = (a^3/L^3) \Psi^{(1)}(\eta) \rho^{(2)}(\eta),$$

(42)

where $(1), (2)$ refer to the two different realizations chosen. Recalling that $\Psi_{\kappa} = (s/k^2) \Gamma_{\kappa}$, with $s = 4\pi G \dot{\phi}_0$, and using equation (25), we get $|<\Gamma_{\kappa}>|^2 = \hbar^2 L^3(1/2a)^2$. Then

$$E_I(k, \eta) = (\pi/4)(a/k)\hbar G(\dot{\phi}_0)^2.$$

(43)

In accordance to Penrose’s ideas the collapse would take place when this energy reaches the ‘one-graviton’ level, namely when $E_I(k, \eta) = M_p$, where $M_p$ is the Planck mass, thus one gets $z_k = \pi \hbar G(\dot{\phi}_0)^2/MA_p$. So $z_k$ is independent of $k$ which leads to a roughly scale invariant spectrum of fluctuations in accordance with observations. Thus a naive realization of Penrose’s ideas seems to be a good candidate to supply the element that we argued is missing in the standard accounts of the emergence of the seeds of cosmic structure from quantum fluctuations during the inflationary regime in the early universe.

VI. CONCLUSIONS

The dramatic change in outlook that has taken place in the last few years regarding the possibility – despite early pessimistic assessments – that some aspects of quantum gravity might be after all experimentally accessible is a very healthy development for the quantum gravity community. Bringing back to the realm of empirical falsificability of ideas, a discipline that seemed to wander ever deeper into the abyss of unchecked lucubrations, can not but reassure us, that the discipline still lies within the boundaries of scientific research. The early proposals that there might be a breakdown in Lorentz Violation associated with a discrete structure that quantum gravity is supposed to endow space-time with, lead in fact to a vigorous program, which at this time, and due not only to the direct bounds obtained but more importantly to the severe restrictions that QFT puts on these ideas, have to be regarded with a strong dose of skepticism, as the lesson from that stage seem to be in the direction of requiring Quantum Gravity to be free of such effects. The lasting legacy of this episode, on the other hand, is, I believe, the lesson that we should not give up so easily in our quest for phenomenological manifestations of quantum gravity. In this regard the successors of the program should be divided in three groups, first those ideas that suffer from a lack of clear interpretational status including some for which the existence of a sensible interpretational scheme is highly dubious, and which are briefly discussed in section 2, then there are some ideas that seem to be well defined and have a rather clear interpretational

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9 There seems to be at this time a single situation where some indications that a breakdown of Lorentz invariance could be at play: The absence of a GZK cut-off in the cosmic ray spectrum [35]. The evidence is still rather controversial [36] and, on the other hand it is not clear whether some simpler explanations, perhaps including new physics, but unconnected with the issues at hand, do exist.
status, and which could, in principle be subjected to experimental investigations, such as the specific search for a gravitationally induced collapse of the wave function proposed by Penrose [30], the proposals of Pullin and Gambini about a gravitationally induced fundamental decoherence [29], the ideas about a possible non-standard manifestations of curvature in extended quantum systems, first proposed in [24] and reviewed in section 3. Finally, as was described in section 4, and first reported in [31] the recognition that there are very intriguing aspects of our understanding of the origin of the seeds of cosmic structure, which seem to “account” for the observations, in the sense that the predictions and observations are in agreement, but that on the other hand suffer from unjustified identifications, problematic interpretations, and do not pass a careful and profound examination. In other words the recognition that something else seems to be needed for the whole picture to work, could be pointing us towards an actual manifestation quantum gravity. We have shown that not only the issues are susceptible of scientific investigation based on observations, but that a simple account of what is needed seem to be provided by the extrapolation of Penrose’s ideas to the cosmological setting.

We end by stressing that it might well be that we are at the dawn of a new era regarding Quantum Gravity; but we would do well by keeping an open mind, as it is quite likely that such new era, as any region which is truly virgin to exploration, will look rather different that what was expected on arrival.

Acknowledgments

It is a pleasure to acknowledge very helpful conversations with Chryssomalakos. This work was supported in part by DGAPA-UNAM IN108103 and CONACyT 43914-F grants.

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