Finite element analysis of the mechanical behavior of a bi-hexagon grid structure

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Abstract
The space grid structure is widely used in large-span and large-scale buildings due to its excellent stiffness and lightweight. However, the joint connection is complicated since many struts emanate from one joint. In this paper, we proposed a novel joint connection with five struts and a space grid structure called the bi-hexagon grid structure (BHGS). The mechanical behavior of the proposed grids was investigated by comparing different double-layer and multi-layer grids by virtue of the Ansys parameter design language. The results show that the proposed structure can significantly reduce material consumption and improve installation efficiency. The comparison results also show the BHGS has a good stress distribution under uniform surface loads. However, the deformation is slightly large, which is acceptable considering the practical application. A configuration method and mechanical behaviors of a double-layer dome based on the bi-hexagon grid structure were presented as a particular application as well as the structural optimization. In addition, an assembly strategy was further given to realize the modular design.

Keywords
Space grid structure, bi-hexagon grid structure, modular design, mechanical behavior

Introduction
Over the past few decades, the construction projects, such as gymnasiums, convention centers, and transportation terminals, boosted the relative research progress of space grid structures characterized by lightweight, excellent stiffness, and economy. A typical space grid structure transferring forces and moments simultaneously comprises numerous joints and struts. Double-layer grids are the most common spatial structures. When the span becomes large, the double-layer grids become heavy and uneconomical, and then the multi-layer grids are introduced. Although current grid structures can meet various needs, the fabrication costs and the installation efficiency are getting more and more challenging, which need further improvement.

As is well known, there are several common topologies, including triangular pyramid,2 tetragonal pyramid,3 and other types.4 Pyramidal topology combinations create complicated joint connections, two of which are depicted in Figure 1. The marked red circle shows 10 or 12 struts emanating from an internal joint (not margin joints). By repeating those internal general joints horizontally and vertically, complicated space grid structures can be constructed. A mero connector with screw holes is usually utilized as a joint to connect struts as well as other prefabricated joint connectors.5

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Up to 18 struts can be connected at a single joint. Hence, the joint connections play a significant part in the cost of material consumption and installation efficiency. With the development of additive manufacturing technologies, challenging geometries like the multi-layer grids can be easily modeled through LSM, EBM, and NPJ, which are difficult for traditional manufacturers. The space grids modeled with periodic representative volume element by computer-aided design can meet the permeability and mechanical strength requirement in biomechanics and heat transfer in energetic fields.

On the other hand, the mechanical performance of a space grid structure is affected by many factors, such as materials, slenderness ratio, and the grid depth and support arrangements. Topology optimization is often used in the optimum design of space grid structures. Mashayekhi et al. performed the topology optimization of a double-layer grid by distinguishing the importance of struts and joints under loads. Due to the high degree of static indeterminacy, many struts do not bear much force. These struts are redundant and should be deleted or made slender. Moghadas et al. utilized neural networks to predict the optimal design of double-layer grids considering the design variables of the span and height of the grids and the cross-sectional area of the struts. In addition, a multiscale optimization approach was introduced by Montemurro et al. to simultaneously optimize the global and local geometric parameters at different scales without considering any simplifying hypotheses. The optimization strategy provides an optimum solution both in structure design and manufacture. However, little attention has been paid to the simplification of joint connections.

This paper aims to simplify joint connections of space grid structures. Meanwhile, in order to reduce the material consumption and improve the installation efficiency, a novel space grid structure called bi-hexagon grid structure is proposed, whose core joint unit is a five-strut joint. In Section 2, the concept design of the bi-hexagon grid structure is described. In Section 3, the mechanical properties of double-layer and multi-layer bi-hexagon grid structures are investigated by finite element analysis procedures. In section 4, a dome based on the bi-hexagon grid structure is configured, analyzed, and optimized. In section 5, the mechanical behaviors of multi-layer grids and an assembly strategy for realizing the modular design are discussed. Furthermore, the nonlinear buckling of the proposed structure was tested.

**Conceptual design of the bi-hexagon grid structure**

Honeycomb structures are widely used in the industry domain with the advantages of lightweight and high stiffness. The hexagonal structure inspired a novel spatial design by rotating and translating hexagonal planes. As summarized schematically in Figure 2, three hexagonal planes are placed with identical spacing \( H = 2L \cos 30^\circ \). The plane S1 can be obtained by rotating the top plane T1 90° clockwise about the edge on the left-hand side. In a similar method, planes S3 and S5 can be configured. A spatial structure with honeycomb planes in two perpendicular directions is constructed through simple space operation. However, it is not structurally stable because half of the connecting nodes are independent of adjacent nodes, which can be solved by adding additional planes to strengthen the connection. Therefore, planes S2 and S4 are added by duplicating and shifting plane S1. Planes T2 and T4 can also be inserted by translating plane T1.

A complete model of the bi-hexagon grid structure is generated with five layers, as shown in Figure 2(d). It can be seen that each layer is a regular hexagonal plane from the top view and side view. In this context, this new space structure is called a bi-hexagon grid structure (BHGS). Moreover, it can be noted that the distance between adjacent layers is fixed. Each internal joint of the BHGS has five struts of equal length, which can be observed in Figure 3, where a local detail is marked in color for better observation. The struts numbered 1, 2, and 3 are in the same plane, while the struts numbered 3, 4, and 5 are in the other perpendicular plane. The BHGS is arrays of five-strut joint units regularly. The joint unit is sufficiently simplified from more than 10 struts to 5 struts. The mechanical behavior of the simplified grid structure will be investigated in the following sections.

**Mechanical behavior of double-layer grids**

This section involves analyzing and comparing three double-layer grids to obtain their mechanical behavior differences, including the deformation and stress.
Three double-layer grid structures

The double-layer grid structures are classified by three regular polygonal patterns (polygons with identical sides), that is, quadrilateral (two-way), triangle (three-way), and hexagon (three-way) where the way number is the number of arrangement directions of grid members. Two- and three-way grids are widely used due to their high strength and uniform stress, especially in extreme conditions. By creating basic internal shapes like triangular and tetragonal pyramids between the upper and lower layer, the structural stability can be significantly improved.

In this paper, three different grids were selected, as shown in Figure 4. Each structure has five views: the plan view, the view of the top-layer struts, the view of the bottom-layer struts, the view of middle members, and the side view. Figure 4(b) and (c) shows two typical double-layer grids: the square-on-square grids (SSG) and the square-on-square offset grids (SSOG). The proposed BHGS is depicted in Figure 4(d). In order to keep consistent with the naming of the former two grids, we name it the hexagon-on-hexagon offset grids (HHOG).

We took the distance between adjacent parallel struts equal to keep different grid sizes the same. As shown in Figure 5, the top view of the SSG, the SSOG, and the HHOG are put together. The HHOG has only one type of strut length. The parallel distance of 1.385 m was confirmed when the strut length was...
supposed to be 0.8 m. The grid of 11.080 m length and 11.080 m width was selected to ensure the grid integrity. Meanwhile, the layer depth was determined to be 0.693 m \((0.8 \times \cos 30^\circ)\). Hence, the span ratios of the three grids have the same value of 0.0625.

All the struts used have the same round hollow cross-sections with the identical steel material, whose properties are listed in Table 1. After modeling these three grid structures, some essential features can be summarized as listed in Table 2.

From the table above, the advantages of HHOG can be concluded. Firstly, the HHOG has only one type of struts with a length of 0.8 m, while the SSG has three types, and the SSOG has two types. It is effortless to fabricate a single type of struts and is convenient to complete the joint connections without considering

| Property | Outer diameter (mm) | Thickness (mm) | Elasticity modulus (GPa) | Poisson’s ratio | Density (kg/m\(^3\)) |
|----------|---------------------|----------------|-------------------------|----------------|---------------------|
| Value    | 43                  | 3              | 200                     | 0.3            | 7580                |
confusion. Secondly, the HHOOG has the least strut number 4 emanating from each internal node while the number of the other two grids is 6/7/8. The joint structure can be effectively simplified because only four screw holes are needed for assembly. Thirdly, the materials consumption can be easily obtained by summing all the strut lengths together. The total strut length of the HHOOG is 514.4 m. Considering the diameter and wall thickness of the strut, the grid we proposed can save lots of materials.

Static analysis under loads

Three double-layer grids described in Section 3.2 were analyzed by virtue of the APDL (Ansys Parametric Design Language). Based on the Timoshenko beam theory, BEAM188 was selected for its broad application from slender to moderately stubby beam structures. In addition, the loads exerted on the grids were the same. A vertical uniform surface load of 100 N/m² was applied on the top layer. The surf154 element was utilized to deal with the surface load. The six degrees of freedom of all the peripheral joints in the upper layer for all grids were restricted.

The SSG has a total of 130 nodes, 32 of which are supported, giving 98 degrees of freedom corresponding to the vertical motions of the 98 unsupported nodes. And the SSOG has a total number of 145 nodes, 32 of which are supported, giving a total of 113 degrees of freedom corresponding to the vertical motions of the 113 unsupported nodes. For the HHOOG, 50 nodes are supported, and a total of 290 degrees of freedom corresponds to the vertical movements of the 290 unsupported nodes. The results are based on linear analysis. A mesh convergence was conducted to determine the final accurate results as shown in Table 3. The element size used in each grid varies from 0.8 to 0.05 m. The comparison results indicate that the mesh is converged.

The final deformation for the three grids is presented in Figure 6. The maximum displacement of the SSG is 0.723 mm, which is better than the displacements of the other two grids. Although the SSOG has eight struts at most of its joints, the maximum displacement of the SSOG is still more significant than that of the SSG due to the increased self-weight from increased struts. The strut number of the SSG and SSOG at a joint mentioned above is two times that of the HHOOG. Accordingly, the deformation of HHOOG is more considerable. Furthermore, the peak displacement of the

| Table 2. Essential characteristics of different grids. |
|------------------------------------------------------|
| Grid type | Square-on-square grids (SSG) | Square-on-square offset grids (SSOG) | Hexagon-on-hexagon offset grids (HHOG) |
| Length types of struts | 3 (1.549, 1.385, 0.693 m) | 2 (1.385, 0.693 m) | 1 (0.8 m) |
| Strut number at an internal joint | 6/7/8 | 7/8 | 4 |
| Sum of all strut lengths (m) | 523.191 | 661.760 | 514.400 |

| Table 3. Mesh sensitivity analysis of different grids. |
|------------------------------------------------------|
| Grid type | Element size (m) | Total element number | Max. displacement (mm) |
|------------|----------------|----------------------|-----------------------|
| SSG | 0.05 | 10,542 | 0.751 |
| | 0.1 | 5327 | 0.751 |
| | 0.2 | 2688 | 0.750 |
| | 0.4 | 1458 | 0.749 |
| | 0.8 | 389 | 0.723 |
| | 0.05 | 13,312 | 0.917 |
| | 0.1 | 6656 | 0.917 |
| | 0.2 | 3328 | 0.917 |
| | 0.4 | 1792 | 0.916 |
| | 0.8 | 512 | 0.878 |
| | 0.05 | 10,288 | 1.293 |
| SSOG | 0.1 | 5144 | 1.293 |
| | 0.2 | 2572 | 1.292 |
| | 0.4 | 1286 | 1.289 |
| | 0.8 | 643 | 1.234 |
| HHOG | 0.1 | 5144 | 1.293 |
| | 0.2 | 2572 | 1.292 |
| | 0.4 | 1286 | 1.289 |
| | 0.8 | 643 | 1.234 |

Figure 6. The displacements distribution: (a) the SSG, (b) the SSOG, and (c) the HHOOG.
HHOG is 1.4 times the value of the SSOG. Considering the displacement ratio to the strut length or the total length is small, the final deformation will be acceptable for practical application.

Additionally, the axial stress distribution was taken into account. The sizable axial stress should be decreased for the possibility of strut crack. The maximum axial stresses in three grids can be observed in Figure 7. The positive value is tensile stress, while the negative value is compressive stress. The peak stress of the HHOG is 7.05 MPa and is the least of the three grids. In addition, due to the uneven stress distribution, the SSG and SSOG can be divided into two parts: the first part is stress concentrating struts located at the center of the lower layer; the other part is the rest of the struts under normal axial stress. However, the stress distribution of the HHOG is prone to a uniform condition. That means the HHOG can bear higher loads. In brief, the HHOG has a good stress state.

In order to obtain the true behaviors of different grids, the non-linear static analysis was performed. As can be seen from Figure 8, the relationship between applied force and deformation is not linear. The maximum displacement increases with increasing applied force acting on the top layer. The SSG has excellent performance compared to the HHOG, which is relatively prone to large deformation.

However, contrary to the deformation, the maximum axial stress of the HHOG is the least of three grids, as reported in Figure 9. No significant differences in the SSG and SSOG for peak stress. The large deformation in the SSOG can be explained by the strut number at an internal joint in the SSOG being far less than the other two grids. The simple joint design of the SSOG weakens the overall stiffness but relieves the stress concentration due to numerous strut elements.

**Application of the HHOG**

In this section, the construction way of a dome structure based on the HHOG is described as well as the mechanical behaviors.

**Formation of a double-layer dome**

The HHOG studied above is a flat configuration. This section utilizes the HHOG to model a double-layer dome, a shell structure, the formation method of which
is inspired by a geodesic spherical reticulated shell. The geodesic dome is a spherical structure with a regular polygonal arrangement of struts. The Climatron greenhouse built in the USA is a typical specimen of the geodesic dome.

The dome with inner radius \( R_1 \) and outer radius \( R_2 \) is sketched in Figure 10. The entire dome surface can be divided into six identical regions. Six arc lines sharing the sphere surface of radius \( R_1 \) converge at the top point. The included angle of two adjacent lines is 60°. The arc line is evenly divided into \( N \) segments from the top point to the edge point. The corresponding central angle \( \alpha \) can be obtained. \( L \) is the span and \( h \) is the rise height, which can be found in equations (1) and (2).

\[
\alpha = \arctan \frac{L/2 \sqrt{R_1^2 - (L/2)^2}}{N} \quad (1)
\]
\[
R_1 = \frac{h^2 + (L/2)^2}{2h} \quad (2)
\]

Orthodromic lines connecting points 1-1, 2-2, etc. are evenly divided into \( N \) segments from the left point to the right point in each region. Triangles can be generated by connecting these segment points. The final triangular configuration of the lower layer grid is depicted in Figure 10(b). Similarly, the triangles can be constructed by increasing the radius from \( R_1 \) to \( R_2 \). Because the HHOG is based on hexagons, 24 triangles are utilized to form a hexagon whose edge is straight lines connecting points 2-2, as the yellow color shows. The HHOG dome consists of the upper dome in yellow, the lower dome in green, and the middle members in blue.

Actually, the shell structure is totally different from the flat one. It does not strictly follow the rules of the bi-hexagon structure on a curved surface, as described in Section 2. Most of the struts are of different lengths except the symmetrical ones. Moreover, the six struts of a hexagon are on the same sphere surface instead of a plane. In order to reduce the length difference, geodesic spherical triangulated grids are used. The length of the three sides of each triangle keeps close to each other as possible. Meanwhile, a more considerable segment number \( N \) can generate denser triangulated grids in order to make struts approximate in length.

With the help of the APDL, the double-layer dome model can be automatically created by command flow. Figure 11 depicts a double-layer dome with a span of 100 m, a rise of 20 m, and a shell thickness of 2 m. All the outmost joints are not on the same circle due to the particularity of the geodesic dome. The material properties of struts were the same as listed in Table 1 of Section 3 except for the diameter and wall thickness. For the large span situation, we adopted the struts with an external diameter of 0.22 m and a wall thickness of 0.01 m.
The total number of members making up the dome is 805. The dome has a total of 2523 nodes, 42 of which are supported, giving 2481 degrees of freedom corresponding to the vertical motions of the 2481 unsupported nodes. The sensitivity analysis of the model outputs to the element size was also performed, as shown in Table 4. The peak displacement of the HHOG dome converges at 14.806 mm.

Deformation and stress

All the outmost joints of the lower layer were fixed. A vertical load of 100 N was applied vertically on each joint of the upper layer. The deformation and stress results are plotted in Figure 12.

The peak displacement of 0.01 m appears at the center of the dome and takes a proportion of 1/10,000 of the whole span. Compared with an 800 m Kiewitt type mega-latticed dome with a proportion of 1/608, the double-layer dome has an excellent stiffness. Among six types of stress, axial stress is the most important one, a combination of stress generated by axial forces and bending moments. The positive stress represents the tensile stress, while the negative stress represents the compression stress. From Figure 12(b), it can be seen that the maximum axial stress is 15 MPa, and the corresponding strut is in the compression status. The maximum axial tensile stress is 7.5 MPa, half of the peak compressive stress. Meanwhile, most struts of upper and lower layers and most of the middle members are in compression except the struts close to the dome edge. The forces of the middle members are so small that the dome can be designed to be lighter. Because the loads are transferred from the top layer to the lower layer, then to the supporting struts, the maximum axial force occurs near the fixed points.

In practical situations, the material costs play a crucial role in the entire construction costs. For the purpose of reducing steel consumption, structural optimization was carried on. According to equation (3), the total mass of the dome is defined as the objective function. The strut diameters of the upper and lower layers as well as middle members are selected as design variables. The axial stress and the central joint displacement are used as state variables.

\[
\begin{align*}
\min f(l_i, A_i) &= \sum_{i=1}^{n} \rho l_i A_i \\
\text{s.t.} \quad &0.1 \leq D_{\text{upper}}, \ D_{\text{middle}}, \ D_{\text{lower}} \leq 0.3 \text{ m} \quad (3) \\
&0 \leq U_{\text{max}} \leq 0.020 \text{ m} \\
&0 \text{ Pa} \leq |\sigma_{x\text{-max}}| \leq 1.5 \times 10^7 \text{ Pa}
\end{align*}
\]

where \(f\) is the objective function, \(l_i\) is the strut length, \(A_i\) is the strut section area, \(D\) is the strut diameter, \(U\) is the central joint displacement, and \(\sigma\) is the axial stress. All the wall thickness of struts is 0.005 m.

We are able to obtain the minimum weight double-layer dome subjected to stress and displacement constraints by developing the command flow in the APDL. The random design optimization method was first used to find the initial feasible solution. Then First-order optimization method was utilized to obtain the best

| Grid type | Element size (m) | Total element number | Max. displacement (mm) |
|-----------|-----------------|----------------------|-----------------------|
| SSG       | 0.25            | 16,767               | 14.806                |
|           | 0.5             | 8577                 | 14.806                |
|           | 1               | 4452                 | 14.698                |
|           | 2               | 2240                 | 14.326                |
|           | 4               | 1407                 | 10.842                |

Figure 12. (a) Contour plot of sum displacement and (b) contour plot of internal axial stress.
solution. The final optimal weight is converged as presented in Figure 13.

Since the first-order optimization method is based on the sensitivity of the objective function to the variables involved, it is suitable for accurate optimization analysis. First-order optimization method was used in this paper to carry out the solution search. The penalty function approach, as the constraint-handling technique, was applied to transform the constrained problem into an unconstrained one by adding penalty functions. The forward difference of 0.2% of the design variable range was used to compute the gradient of the dependent variables with respect to the design variables. When the search direction was determined, the line search step size of 10% was applied to the maximum range of design space at each iteration to minimize the unconstrained problem. Meanwhile, a maximum number of iterations 45 was specified to end each loop. The optimized diameters of $D_{\text{upper}}$, $D_{\text{lower}}$, and $D_{\text{middle}}$ are 0.101, 0.300, and 0.101 m, respectively. The maximum stress in the dome is 9.558 MPa which does not exceed the allowable stress. Moreover, the maximum displacement is 0.008 m. The weight of the dome can be minimized to $3.629 \times 10^5$ kg. In addition, the optimization results show that the struts of the lower layer have a bigger diameter because they bear greater forces.

Modal analysis

Block Lanczos method with 10 extraction modes and 10 extended modes was used for modal analysis. The first 10-step natural frequencies are listed in Table 5.

![Figure 13. The iteration progress of the total mass of the dome.](image)

![Figure 14. The first fourth modal shape: (a) the first mode shape, (b) the second mode shape, (c) the third mode shape, and (d) the fourth mode shape.](image)

The first-order modal frequency is 2.666 Hz indicating that the dome’s stiffness is significant. The spectrum is a little dense, especially for high frequency. The first few modal shapes are principal shapes, as plotted in Figure 14. The first shape is similar to the second one after 180 degrees of rotation.

Mechanical behavior of multi-layer grids

In this section, two types of grid structures are modeled in 3, 5, 7, and 9 layers, respectively, to distinguish the performance difference. Meanwhile, an assembly method was proposed for a multi-layer bi-hexagon grid structure.

Differences in three perpendicular directions

The BHGS is anisotropic like most grid structures. Nevertheless, there exist two directions where geometric characteristics are the same. As can be seen from Figure 15, three directions are perpendicular to each other, consistent with the view directions mentioned in the configuration progress. The first and second direction

| Mode | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10    |
|------|------|------|------|------|------|------|------|------|------|-------|
| Frequency (Hz) | 2.666 | 2.939 | 3.341 | 3.546 | 4.197 | 4.299 | 4.437 | 4.870 | 5.208 | 5.460 |
are full of hexagonal planes while the third direction has a pyramid-shaped frame instead of planes. It is essential to determine which direction is the most appropriate direction to apply loads. We modeled a minimal and complete BHGS in Figure 15 to compare three situations under loads. The material properties of struts were the same as listed in Table 1. The bottom joints were fixed, and a surface load of 200 N/m² was applied on top joints in three directions, respectively. The results indicate that the first and second directions have the same mechanical behavior as listed in Table 6. The situation of the third direction is worse than that of the other two directions. The peak displacement of the third direction is $7.26 \times 10^{-2}$ m, which is almost 23.8 times that of the former two. The axial stress differences can also manifest the superiority of the first and second directions. Hence, the first and second directions are the right direction for load-bearing. Considering the assembly progress, the first direction is recommended to apply loads.

**Multi-layer grids**

When the span is large, the structure deflection of double-layer grids will not meet the requirements. Multi-layer grids can significantly increase the stiffness and bearing capacity. They can also be used as core structures of sandwich structures and as supporting systems for vibration isolation.

The SSOG can be extended to multi-layer grids by creating mirror bodies about the symmetric plane, which is the plane of upper layer grids. The extended grid is actually the octet truss structure described by Fleck. In this paper, the octet truss structure was selected to be compared with the BHGS. The overall length and width of these two grids are the same, and the depth of layers are identical, as represented in Figure 5 in Section 3. The layer numbers can be adjusted to increase and decrease the height of the whole grid. Figure 16 illustrates two nine-layer grid structures and corresponding joint units. It will undoubtedly take more time to complete the installation of a 12-strut joint. The space left by the previously installed struts for the non-installed struts is quite limited.

Apart from the labor-consuming work, it needs to choose the right struts because there are two types of struts. Therefore, simplifying the joint connections is a

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**Table 6. Mechanical behaviors in three directions.**

|                      | Max. displacement (m) | Max. axial compressive stress (MPa) | Max. axial tensile stress (MPa) |
|----------------------|-----------------------|------------------------------------|---------------------------------|
| The first/second direction | $2.89 \times 10^{-3}$ | 1.01                               | 0.532                           |
| The third direction   | $6.88 \times 10^{-2}$ | 4.59                               | 4.39                            |

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![Figure 15. Three perpendicular directions.](image1)

![Figure 16. Multi-layer grid structures: (a) the nine-layer octet truss structure, (b) the nine-layer bi-hexagon grid structure, (c) the joint unit of 12 struts, and (d) the joint unit of five struts.](image2)
A direct way to improve assembly efficiency. The applied forces and the imposed displacement constraints for the two grids are similar. The rightmost and leftmost joints of each even layer are fixed, and a uniform surface load of 100 N/m² is applied on the top layer. The comparison results of two grids with 5-, 7-, and 9-layer are graphed in Figure 17. The material consumption depends on the sum of strut length when struts utilize the same material and have identical cross-sections. As the layer quantity increases, the total length of struts used increases proportionally. The material consumption of the nine-layer octet truss structure is 4.03 km which is 1.44 times that of the nine-layer bi-hexagon grid structure. Therefore, it can really save material costs.

The maximum axial stresses of the two grids are both compress stress. The peak axial stress of the BHGS is always lower than that of the octet truss structure. Meanwhile, the stress distribution of BHGS is more uniform, which decreases the risk of strut fracture. As a result of the sharp drop of strut quantities at a single joint, the maximum displacement of the bi-hexagon grid structure becomes large. Moreover, the displacements show a declining trend when the layer quantities increase. However, the deformation is relatively small and is acceptable compared with the strut length.

In summary, the multi-layer BHGS is a periodic array of five-strut joint units. Through the static analysis, its stress state is be found to be prior to the octet truss structure. In order to reduce the maximum deformation, it is necessary to choose proper strut diameters, strut lengths, reasonable layer numbers and consider diversified constraints. In the actual application, the deformation is inevitable in any grid structure. The buckling and fracture are also a critical factor affecting the utilization.

The mechanical performances of multi-layer space grids on nonlinear analysis are worth exploring. The overall trends in the multi-layer grids were observed to be similar to those of the bi-layer grids. For all grids, the greater the total force, the greater the peak displacement and stress. To facilitate the comparison between different grid configurations, the curves were plotted in solid and dotted lines, as illustrated in Figure 18. The octet truss structure (OTS) behaves better than the BHGS in the maximum displacement. For 3-layer grids, the BHGS and OTS are both susceptible to applied vertical force and have sizable deformations. The steep curves obtained for the 5, 7, and 9-layer grids indicate that large deformations still exist for the BHGS. The influence of layer number is significant that the effect of varying applied force from

Figure 17. Comparisons of multi-layer grids: (a) the sum of strut length, (b) the maximum stress, and (c) the maximum displacement.
7-layer OTS is not prominent since the stiffness of grids is large enough.

However, unlike results presented in bi-layer grids, the curves of the maximum stress for the OTS and BHGS show weak nonlinear trends, as shown in Figure 19. For all layers, the BHGS has minor stress, especially for 3-layer grids. For 9-layer grids, the peak stress of the BHGS and OTS are close when subjected to an initial external force. It stands out that when continuously increasing the external force, the BHGS began to behave better than the OTS. Furthermore, the OTS and BHGS come to the same peak stress state when the external force is large enough. This was caused by the configuration method that the BHGS has more struts than the OTS to share the stress. Each strut of the BHGS is fully involved in decreasing the maximum stress occurring in a single strut. With the increase of layer numbers, the stress difference decreases because of growing stiffness.

**Way to construct multi-layer bi-hexagon grids**

When the multi-layer grid structure is employed, the assembly process can be greatly simplified by modular design. The mero system is a good invention that is often used to connect up to 18 struts. As mentioned in Section 1, it is complicated to assemble such joint structures due to not only the strut quantities at a joint but also the strut types. There are at least two types of struts for a common joint where 12 struts converged. However, the proposed BHGS has one type of struts. The manufacturing and installation of struts are simplified.

In order to further simplify the assembly process, we proposed a way to construct the multi-layer BHGS. As graphed in Figure 20, the joint with five struts is separated from the bi-hexagon grids. Each strut is divided into two parts. By this way a joint unit is obtained consisting of a joint and five shortened struts whose length is half the original strut. That means each joint of the
grids is a part of the joint unit. There are two pairing ways between two adjacent joint units. The first pairing way is to pair struts marked “1/2/4/5.” For the convenience of showing the grid structure, struts were marked with different colors. Actually, the first pairing way is unique without considering colors due to the symmetric characteristics. The second pairing way is to pair struts marked “3.” These struts are paired by flange connections or other methods.

The BHGS can be obtained by pairing the joint units. In order to shorten the assembly time, two basic modules were designed. The first module is M-shaped comprising of three joint units. The second module is of hexagon arrangement consisting of six joint units. As presented in Figure 21, these two modules contain the aforementioned pairing methods. Two beneficial configuration schemes are recommended, like the grid layer incorporating five module-1 and two module-2 in Figure 21(b) and the other layer encompassing four module-1 and four module-2 in Figure 21(c). Such layers can be easily constructed horizontally and vertically using simple rules. Therefore, the workload and efficiency of field assembly can be significantly improved.

The construction method proposed above is significantly valid for the application of the BHGS in offshore engineering. The joints and struts should be made to be hollow ones to provide the total buoyancy. Figure 22 sketches one layer grid for the application.

**Nonlinear buckling**

In order to investigate the buckling behavior of the BHGS, a simple compression test was carried out. A miniature model with struts of the slenderness ratio 16 was manufactured by fused deposition modeling as graphed in Figure 23. The solid strut is made of a mixture of nylon and glass fibers. The specimen has a dimension of $80.0 \times 55.4 \times 55.4$ mm. And it was tested on a microprocessor-controlled universal material testing machine named CTM8050. A load was applied at the speed of 1 mm per minute to keep the quasi-static condition.

The load-displacement curve is obtained as plotted in Figure 24. In the beginning, the linear relationship shows a high elastic modulus and a high bearing capacity. The strut is nearly vertical. As the load increases, the peripheral struts have a slight deformation. When the displacement reaches 6.41 mm occupying 11.57% of the total height, the specimen becomes unstable. The corresponding maximum load is 271.73 N.

**Conclusion**

This paper presents the mechanical performance of double-layer and multi-layer proposed bi-hexagon grid structures through static and modal analysis. The Ansys parametric design language based on the Timoshenko beam theory was used to create and
analyze structures under the same dimensions, materials, cross-section, constraints, and surface loads. The results show that the BHGS has distinct superiority in material consumption, installation efficiency, and stress distribution. The following conclusions can be drawn:

- The five-strut joint units separated from the BHGS can significantly simplify the complexity of the joint structures. Meanwhile, the installation efficiency can be improved considering strut types and the joint complexity without distinguishing different struts, especially when the recommended assembly strategy for multi-layer BHGS was performed.

- The material consumption is decreased both in double-layer HHOG or multi-layer BHGS. The three-layer BHGS has the least material consumption (840 m in terms of length) of three double-layer grids. When multi-layer grids are utilized, the BHGS is in a superior position for reducing costs.

- The stress distribution is improved when the structure is subject to a vertical uniform surface load. The maximum axial stress of the three-layer BHGS is 61% of the peak stress of the three-layer extended SSOG. However, the deformation of the BHGS is not ideal compared with other grid structures because of the fewer struts at the joint.

- The double-layer dome based on the HHOG combines the advantages of the BHGS and the geodesic dome. The minimum dome weight of 362.9 tons was obtained by structural optimization.

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**Appendix**

**Abbreviations**

The following abbreviations are used in this manuscript:

- APDL: Ansys parametric design language
- BHGS: Bi-hexagon grid structure
- EBM: Electron Beam Melting
- HHOG: Hexagon-on-hexagon offset grids
- LSM: Selective Laser Melting
- NPJ: Nano Particle Jetting
- OTS: Octet truss structure
- SSG: Square-on-square grids
- SSOG: Square-on-square offset grids