Viscoelastic repetitive creep and recovery in bituminous materials

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Abstract

Repetitive creep and recovery tests in bituminous materials— asphalt and asphalt paving mix are studied at high service temperatures. The non-exponential-stretched exponential type of continuous retardation spectrum is defined and used for the calculation of creep compliance function in repetitive shear and tensile creep and recovery experiments and also in the wheel tracking test on asphalt paving mix. It is shown that the used model, with only five adjustable parameters, can describe the test quite well. The model is also adequate for the description of the composite compliance function constructed from the linear viscoelastic data and the apparently nonlinear compliance function obtained in the wheel tracking test, for asphalt paving mix.
1. Creep and recovery in viscoelastic material

When material is subjected to the tensile or shear stress of sufficiently high amplitude it attains a state of irreversible deformation; and, finally, ruptures. These changes in materials (preceding these last two stages) are associated with structural inhomogeneties on a microscopic or molecular level (Ferry, 1980). The formation of inhomogeneties at temperatures above the glass transition temperature can be influenced by the presence of filler particles, viscoelastic relaxation, and formation of a crystalline phase and the history of repetitive stressing. Repetitive stress is frequently used as the primary testing method, when the strength and durability of a material is the main engineering concern.

In this study, we investigated three methods of repetitive loading and unloading used for the testing of asphalt binders and paving mixes: the first was the repeated creep and recovery shearing test used for binders; the second was the uniaxial repetitive loading and unloading of cylindrical paving mix samples and the third one was the wheel-tracking test where moving loads are applied through wheels with solid rubber tires, which travelled with a reciprocating motion on specimens of paving mixes.

Starting with the linear viscoelastic description of repetitive loading and unloading we tried to find a unified approach to such tests and extend the description of the tested materials into a domain of their nonlinear behavior.

The linear viscoelastic constitutive equation for shear strain has the form, [1, 2]

\[ \gamma(t) = \int_{-\infty}^{t} J(t-t') \dot{\tau}(t') dt' \]  

where \( J \) represents the shear creep compliance function, \( \gamma \) is the shear strain, \( \tau \) is the shear stress, and \( \dot{\tau} = d\tau / dt' \).

Repeated creep and recovery test, in shear, uses as an input a train of shear stress pulses of height \( \tau_0 \) and duration \( t \) seconds. Each pulse starts at \( nt_1 \) where, \( n = 0, 1, 2, \ldots, N \), and the duration of each creep and recovery cycle is \( t_1 \) seconds.

In typical shear creep experiment

\[ \tau(t') = \tau_0 H(t'), \ \tau_0 = \text{const.} \]  

where \( H(t') \) is the unit step function.

Thus, the shear stress in repeated creep and recovery test is given as

\[ \tau(t') = \tau_0 \sum_{n=0}^{N} \left[ H(t'-nt_1) - H(t'-(nt_1 + t)) \right] \]  

and the accumulated shear compliance is

\[ J(t) = \sum_{n=0}^{N} \left[ H(t-nt_1) J(t-nt_1) - H(t-(nt_1 + t)) J(t-(nt_1 + t)) \right] \]
If the studied creep test consisting of $N$ cycles is in the linear viscoelastic domain (this domain is generally a three-dimensional element of the time-temperature-shear stress space) then only the model of the shear compliance function is needed for the description of the test.

With the help of memory function, $m(s) = -dJ(s)/ds$, the linear viscoelastic constitutive equation for a material capable to flow leads to [3,1,2]

$$\gamma(t) = J_g \tau_0 + J_D \Psi(t) \tau_0 + \tau_0 / \eta_0$$

(5)

Where $\psi$ is the creep function, with properties: $\psi(0)=0$, $\psi(\infty)=1$, then,

$$J_g + J_D = J^c_0$$

(6)

where $J^c_0$ is the steady state compliance [1].

The question of the linear viscoelastic domain for dynamic creep has to be addressed. Even if this domain is covered by a certain number of repeated creep and recovery cycles the boundary of this domain will be eventually crossed when the number of cycles is increased. Of course the boundary is also determined by the temperature and by the magnitude of the applied shear stress. After leaving the linear viscoelastic domain the test will be governed by nonlinear viscoelasticity and finally the domain of ultimate properties and the rupture will be reached. Even if the ultimate properties of asphalt (above the glass transition temperature) are not known the rate at which this “final stage of the life of material” is reached is of utmost importance, for engineering applications. This rate is closely related to the viscoelastic properties of the material [1], and thus the proper description of the dynamic creep test in the linear viscoelastic domain is important.

2. Stretched retardation spectrum in asphalt

The linear viscoelastic creep function is related to the continuous retardation spectrum, $L(\Lambda)$, by the relation [Gross, Ferry]

$$\Psi(t) = \int_0^\Lambda L(\Lambda) \left[1 - \exp\left(-t / \Lambda\right)\right] d\Lambda$$

(7)

By studying various conventional and polymer modified asphalts we have found that their behavior in the repeated creep and recovery test can be better described when one assumes that $L(\Lambda)$ has the form of a “stretched” gamma distribution function (normalized)

$$L(\Lambda) = \frac{\alpha a (\alpha \Lambda)^a}{\Gamma(1+1/a)} \exp\left[-(\alpha \Lambda)^a\right]$$

(8)

and the creep function is also given in terms of the stretched time, i.e.

$$\Psi(t) = \int_0^\Lambda L(\Lambda) \left(1 - \exp\left[-(t / \Lambda)^a\right]\right) d\Lambda$$

(9)

In this case the accumulated shear compliance function has the form
\[ J(t) = J_g + J_D \left[ 1 - \frac{2\sqrt{\alpha t^{1+\alpha}}}{\Gamma\left(1+\frac{1}{\alpha}\right)} K_{1+\frac{1}{\alpha}}(2\sqrt{\alpha t^{1+\alpha}}) \right] + \frac{t}{\eta_0} \]  

(10)

where \( K_{1+1/\alpha} \) is Macdonald’s function of order \( (1+1/\alpha) \), and \( \Gamma \) represents the gamma function.

The fit of (10), to the accumulated compliance for conventional asphalt modified by 4\% (by weight) of styrene-butadiene-styrene copolymer, \( T=60^\circ\text{C} \), \( \tau_0=300\text{Pa} \), is shown in Figure 1. Here, only the first cycle (creep for 1s and recovery for 9s) was fitted and then propagated to one hundred cycles.

3. Repeated creep and recovery in asphalt paving mix

When paving mix samples are tested, tensile loading is usually preferred to the shear loading. In these tests, the mix samples are much larger than the samples of binders, which are tested in conventional rheometers. The testing of paving mixes is generally accompanied by a change in volume, which complicates the form of the linear constitutive equation e.g. [1,2]. One can assume that the tensile creep function \( D(t) \) is equal to \( J(t)/3 \), only if Poisson’s ratio is close to 0.5. This situation can arise when the change in volume is negligible in comparison with the change in shape. In this case, the applied tensile stress \( \sigma \), produces a time-dependent strain, \( \varepsilon(t) = \sigma D(t) = \sigma J(t)/3 \). Let us assume that these conditions are satisfied in the studied repetitive tensile loading and unloading of the mix samples. Since in the test suggested for paving mixes, the stress pulses of 0.1s duration and waiting (“recovery”) time of 0.9s are recommended, the axial loading stress (haversine) then have the form:
\[ \sigma(t) = A \sin^2 \left( 10 \pi (t - \beta) \right) \sum_{n=0}^{N-1} \left[ H(t - n - \beta) - H(t - n - 0.1 - \beta) \right] \] (11)

Because of the haversine loading (amplitude A, and the time shift, \( \beta \)), the accumulated strain in this repetitive test is

\[ \varepsilon_{\text{acc}}(t) = \frac{10 \pi A}{3} \int J(t - t') \sin 20 \pi (t' - \beta) \left[ H(t' - n - \beta) - H(t' - n - \beta - 0.1) \right] dt' \] (12)

The fit of the strain in one typical loading cycle (with \( J \) given by (10)) is shown in Figure 2.

![Figure 2](image)

**Figure 2.** Fit of strain response in one cycle. Nominal stress 138kPa, \( T = 40^\circ\text{C} \). Mix prepared with PMA binder portrayed in Fig.1.

It was observed that the testing machine produced haversine loading that is far from the ideal one. However, even from imperfect loading, one can determine the nominal values of parameters, \( A \) and \( \beta \).

For \( N \) on the order of 10,000 and higher, the attempts to fit the data of \( \varepsilon_{\text{acc}} \) to (12) are not practical and one can only proceed with the approximate modelling of the complete test (10,000 cycles). Moreover in typical test, the testing machine does not capture the complete shape of accumulated strain. Usually, only the values of \( \varepsilon(t) \) at the ends of cycles are reported. In these cases, a great deal of information is lost, and one has to implement the relation (12) at \( t = N + \beta \):

\[ \varepsilon_{\text{acc}}(N + \beta) = \frac{AN}{60 \eta_0} - \frac{AJ_d}{3 \Gamma \left( 1 + \frac{1}{a} \right)} \sum_{n=0}^{N-1} \int_0^{2\pi} \sin z \left[ \alpha (N - n - \frac{z}{20 \pi}) \right] \frac{a+1}{z} dz \]

\[ K \left( 1 + \frac{1}{a} \right) \left[ 2 \alpha (N - n - \frac{z}{20 \pi}) \right] dz \] (13)
The integral in (13) can be approximated by the mean value theorem, and the collected data of accumulated strain can be fitted to (13). The result of this fitting, for paving mix portrayed in Fig.2, is shown in Fig.3.

![Graph showing accumulated strain of mix portrayed in Fig.3. Fit to the mean value approximation of (13). Nominal stress = 138 kPa, T = 40°C.](image)

4. Wheel tracking test in asphalt paving mix

The rutting of asphalt pavements is one of the major distresses observed on the roads in regions where the pavement temperature can be relatively high during the summer. Rutting deformations are commonly found in the wheel paths of the slow lanes of major roads, and the approaches to traffic-light junctions. Due to the inconvenience and high costs of frequent repairs in these areas, road agencies and contractors are experimenting with different asphalt mixtures and also with the use of asphalt binders produced by different technologies (oxidized, modified, etc.) to combat the problem of rutting. Full-scale test tracks are costly to construct and maintain. They are beyond the reach of most highway agencies and researchers, although this test method is most desirable. Compared with full-scale track tests, laboratory-size wheel-tracking tests are cheaper, with the advantage that specimen preparation and tracking tests can usually be completed within a couple of days. The test offers a good simulation of stress conditions generated in the actual pavement; therefore, in principle, it provides a meaningful tool for the prediction of the rutting potential of asphalt pavements. This has been confirmed by the analysis of the performance of asphalt mixes in the wheel-tracking rutting test and their behavior on site [4, 5].

The used wheel-tracking apparatus consisted of two parallel sets of loading wheels. This allowed for the testing of two specimens simultaneously. Rubber tire 47mm wide with diameter of 203.2mm was used in this study. The nominal contact stress of 636.363kPa approximates the contact stress produced by the wheel and obtained from the load of 700N and the contact area of the wheel (approximated from the imprint of the wheel on a tested plate). Each wheel traveled 230 mm before reversing direction, and the device operated at 52 wheel passes per minute. The rut depth of the test specimens was determined from
the measurement of the vertical distance of the axis of the wheel of interest from a predetermined reference level.

Under the assumption that the tested paving mix is a viscoelastic material (with Poisson’s number not too different from 0.5) and the loaded material element (in the middle of the wheel path) undergoes only tensile deformation the tensile compliance function can be defined and the values of the accumulated tensile compliance $D(t)$ can be calculated from the measured accumulated tensile deformation (rut depth) of the material element. The wheel tracking tests were performed at high service temperature of 58°C and the observed deformations were well outside of the deformations observed in standard dynamic testing of paving mixes, at this reference temperature. The dynamic testing provided at several temperatures and the frequency window (0.1Hz, 25Hz) allowed to construct the master curves of the components of the tensile complex modulus $E^*(\omega)$ at the reference temperature of 58°C and calculate the discrete relaxation and retardation spectra for the tested materials. From the retardation spectrum the linear viscoelastic tensile compliance function $D(t)$ was calculated and appended to the (nonlinear) $D(t)$ obtained from the wheel tracking test. Interestingly enough it was found that when the form of the tensile compliance is given as in (10) and the viscous flow term is written with the stretched time i.e. $\eta_0^\phi$, the compounded data of the tensile compliance can be fitted to such modified model (10) quite well, see Fig.4.

![Figure 4. Compounded tensile compliance- linear viscoelastic and wheel tracking. Paving mix with the same binder as in Fig. 1. $T = 58^\circ$C.](image)

5. Conclusion

The use of the stretched time in the retardation spectrum and it the calculation of the creep function was beneficial for the description of repeated creep and recovery tests in both asphalt binder and asphalt paving mix. The derived model of the creep compliance (for the shear and also for the tensile type of loading) contains only five adjustable parameters in contrast with the classical linear viscoelasticity where, just the discrete retardation spectrum must contain about eleven modes, i.e. 22 adjustable parameters, in
order to describe the linear accumulated compliance, in the tested materials. Certainly and especially in the paving mixes the discussed tests will “cross” the boundary of linear viscoelasticity very soon when time of the test is increased or the applied stress is large and/or when the temperature is higher than 50°C. In such cases the discussed model of creep compliance can be adjusted by considering that the “flow” term of the compliance depends on the stress. These modifications of the presented model are currently investigated.

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