Experimental test of the relation between coherence and path information

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Quantum coherence stemming from the superposition behaviour of a particle beyond the classical realm, serves as one of the most fundamental features in quantum mechanics. The wave-particle duality phenomenon, which shares the same origin, has a strong relationship with quantum coherence. Recently, an elegant relation between quantum coherence and path information has been theoretically derived. Here, we experimentally test such new duality by $l_1$-norm measure and the minimum-error state discrimination. We prepare three classes of two-photon states encoded in polarisation degree of freedom, with one photon serving as the target and the other photon as the detector. We observe that wave-particle-like complementarity and Bagan’s equality, defined by the duality relation between coherence and path information, is well satisfied. Our results may shed new light on the original nature of wave-particle duality and on the applications of quantum coherence as a fundamental resource in quantum technologies.
coherence was recognised early as a superposition of optical fields in the theory of electromagnetic waves. Together with the energy quantisation, a quantum version of coherence has become one of the most fundamental features that can mark the departure of quantum mechanics from the classical realm.\(^1\)\(^-\)\(^5\) Carrying out general quantum operations remotely under local operations and classical communication requires quantum states that contain consumable resources. Quantum entanglement is found to have strong connection with coherence\(^6\)\(^-\)\(^8\) and may even originate from it.\(^9\) The development of quantum technologies demands a reassessment of fundamental resources such as quantum coherence, including relations with other quantum physical phenomena and rigorous quantitative description.\(^10\)\(^-\)\(^17\) Among these quantum physical phenomena, wave-particle duality has been a unified picture to fully describe the behaviour of quantum-scale objects. Quantitative characterisations of such wave-particle duality relations have been extensively investigated, aiming to set an upper bound on the sum of the wave behaviour and the particle behaviour for a given interferometer.\(^3\)\(^,\)\(^4\) Based on the fringe visibility \(V\) of the interference pattern and the path distinguishability \(D\), a wave-particle duality relation is given by \(V^2 + D^2 \leq 1\), which means that full wave-behaviour \((V = 1)\) implies no particle behaviour \((D = 0)\) and vice-versa.

Recently, Bagan et al.\(^18\) proposed another elegant relation to characterise the new wave-particle-like duality based on the coherence \(C\)\(^i\) which quantifies the wave nature of a state and the path information which is given by the minimum-error state discrimination between the detector state and the target state. In this paper, we experimentally test such new duality relation by \(l\)\(_i\)norm measure and the minimum-error state discrimination. By preparing three classes of two-photon states that have different upper bounds of quantum coherence, we are able to investigate the new wave-particle-like complementarity in different regions. In every class of state, we continuously tune the detector state and observe clear duality trade-off as well as the upper bound on the quantum coherence of the target photon. The Bagan’s equality defined by the duality relation between coherence and path information can be well satisfied.

**Results**

**Theoretical description of the duality relation.** Consider a particle (target state) entering an \(N\)-port interferometer via a generalised beam splitter. Once the particle interacts with the detector (detector state), the state of the entire system is described by

\[
|\psi\rangle = \sum_{i=1}^{N} \sqrt{p_i} |i\rangle |\eta_i\rangle,
\]

which is the superposition of the initial target particle state \(|i\rangle\) and the detector state \(|\eta_i\rangle\). The coherence of the target state is given by \(C = C(i)\langle p/N\rangle\), where \(\rho = Tr_{\text{det}}(|\psi\rangle\langle \psi|) = \sum_{i=1}^{N} \sqrt{p_i} |\eta_i\rangle\langle \eta_i| |i\rangle\langle i|\), \(C(i)\langle p/N\rangle = \sum_{i=1}^{N} |p_i|\), where \(|p_i|\) is the absolute value of entry \(\rho_{ij}\) of the density matrix \(\rho\) under the reference basis.\(^10\) Here, \(C\) quantifies the wave nature of the target particle. The detector states are introduced to quantify the path information of the target particle by tracing out the target particle state, \(\rho_{\text{det}} = Tr_{\text{tar}}(|\psi\rangle\langle \psi|) = \sum_{i=1}^{N} p_i |\eta_i\rangle\langle \eta_i|\). If the detector states \(\rho_i\) are orthogonal, no coherence property of the target particle will be obtained. To discriminate among the detector states \(|\eta_i\rangle\), one employs the minimum-error strategy by using an \(N\)-element positive operator valued measure (POVM) with elements \(\{\Pi_j\}\). Then the average probability of successfully identifying the state is \(P = \sum_{i=1}^{N} p_i (\Pi_i |\eta_i\rangle |\eta_i\rangle\). It is proved that \(C\) and \(P\) satisfy the following new relation raised by ref.\(^18\),

\[
\left( P - \frac{1}{N} \right)^2 + C^2 \leq \left( 1 - \frac{1}{N} \right)^2.
\] (2)

This upper bound represents a trade-off between the path information and the coherence of the target particle. In our experiment, we consider the case of \(N = 2\). It is worth mentioning that when \(N = 2\), the inequality becomes an equality which we denote as Bagan’s equality. In this case, \(P\) can be maximised either by an optimised POVM or calculated by an analytic solution.\(^19\) It should be noticed that this new definition of path information is different from ref.\(^3\). Here, we use the success probability in minimum-error state discrimination, while ref.\(^3\) uses the difference of two probabilities for taking either one of the two paths.

**Experimental test of the duality relation.** We experimentally generate the two-photon polarisation entangled state via type-II spontaneous parametric down-conversion.\(^20\) As shown in Fig. 1a, a 405 nm UV laser is first coupled into a single mode fiber to acquire a high quality spatial beam profile. After adjusting the polarisation of the pump laser with a combination of a half-wave plate (HWP) and a quarter-wave plate, the pump laser enters the Sagnac interferometer and is focused on a periodically-poled Potassium Titanyl Phosphate (PPKTP) crystal. The clockwise and the anti-clockwise components then interfere at the central polarisation beam splitter (PBS) and generate a superposition state of these two components. The generated singlet state can be written as \(|\psi^-\rangle = \frac{1}{\sqrt{2}}(|H_2 V_2\rangle - |V_2 H_2\rangle\). Two band pass filters centred at the desired wavelength of the down-converted photons are employed to block the UV laser. With careful alignment of the Sagnac interferometer, we are able to observe the polarisation visibility as high as 97.7% in \(H/V\) basis and 98.3% in \(D/A\) basis. Then we further characterise the entanglement quality by conducting state tomography.\(^21\) The measured purity and concurrence of the entangled state is 0.963 ± 0.0016 and 0.964 ± 0.0016 respectively, which suggests that we have obtained a very pure state with high entanglement quality.

In order to observe the coherence properties of the target states, the detector states \(|\eta_i\rangle\) should be non-orthogonal. In our experiment, we first use a HWP to generate the following two-qubit state with four components,

\[
|\psi\rangle = a|H_1 H_2\rangle + b|H_1 V_2\rangle + \gamma|V_1 H_2\rangle + \delta|V_1 V_2\rangle,
\] (3)

where \(\alpha\), \(\beta\), \(\gamma\) and \(\delta\) are parameters based on the rotation in Hilbert Space. Note that these parameters are highly correlated and can be determined by the rotation angle of HWP and the normalisation condition. Since \(|\psi^-\rangle\) state is invariant under arbitrary unitary rotation in Hilbert space, polarisation-dependent loss should be additionally introduced to generate non-orthogonal basis of the detector states. Here, we choose fused silica as Brewster window to introduce this polarisation-dependent loss into the above four components.

We experimentally characterise the transmission rate of fused silica with different incidence angles and the results agree well with the theoretical prediction (see Fig. 1b). We choose 60° as the incidence angle where the transmission rate is 99.7 and 71.9% for horizontal and vertical polarisation respectively (denoted as \(\epsilon_f\) and \(\epsilon_v\)). After passing through this state preparation section
We can see that the detector qubit state is a result of a polarisation analysis measurement setup composed of a half-wave plate (HWP), a quarter-wave plate (QWP) and a PBS.

For an asymptotic limit case, we use a PBS to replace the Brewster window to generate a separable state \( \{ \psi_{\text{II}}^{\text{I}} \} \). With \( \{ \psi_{\text{II}}^{\text{I}} \} \), we can observe full complementarity between quantum coherence and path information. It should be noticed that the three classes of states are not the entire states of the system but part of states retrieved in a smaller subspace with postselection. Such solution has been proven to be exactly equivalent to the ideal one by Sandu Popescu, Lucien Hardy, and Marek Zukowski.

These generated states are then analysed by quantum state tomography with maximum likelihood reconstruction. From the tomographic details, we are able to calculate both entanglement and purity. The entanglement is reestimated by the off-diagonal term of the two-qubit state density matrix. In practice, the entanglement is prepared to approach the ideal value of 0.5. We achieve 0.49 in our experiment. After tracing out the detector qubit, the quantum coherence quantity of the target qubit will reveal the imperfection with a small deviation. Another thing we should mention is that the entanglement quality do affect the degree of complementarity between quantum coherence and path information. From this new duality relation. As we change the number of Brewster window, the state we generated is also in the opposite case.

We can see that the measured C and P follow the theoretical curve with some small deviations. From the analytical view, the entanglement quality is reflected by the off-diagonal term of the two-qubit state density matrix. In practice, the entanglement is prepared to approach the ideal value of 0.5. We achieve 0.49 in our experiment. After tracing out the detector qubit, the quantum coherence quantity of the target qubit will reveal the imperfection with a small deviation. Another thing we should mention is that the entanglement quality do affect the degree of complementarity between quantum coherence and path information. From this new duality relation. As we change the number of Brewster window, the state we generated is also in the opposite case.

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Monte Carlo simulation with the Poissonian statistics of the detection process taken into account complementarity also increases. The solid curves are the theoretical results based on the three different classes of states. Error bars are calculated with

Figs. 2a–c, the three classes of states show apparent difference in degree of complementarity.

Apart from direct calculation of $P$ value from the tomographic data, we also use optimised POVM to directly measure the maximum probability of successfully identifying the state with \{ $\psi_i^{\text{III}}$ \}. We theoretically constructed the projective operators according to the formula in ref. 24. The results are given in Fig. 3a and the measured results agree with the theoretical prediction very well. Error bars in Fig. 3a are directly calculated by using the propagation of error formulas. We have also compared the mismatch between the $P$ values in Figs. 2c and 3a, as shown in Fig. 3b. Except for the result of $\psi_4^{\text{III}}$, the distances between the two different results are nearly invisible, which implies that the two methods are equivalent in principle.

Finally, we test the upper bound of this new duality relation by summing the square of both $C$ and $P$ with our experimental results. As shown in Fig. 4, all the measured results meet the Bagan’s equality with acceptable errors. Each $C$ and $P$ error is estimated via 1000 rounds of Monte Carlo simulation based on
Discussion

We have experimentally tested the recently derived duality relation between quantum coherence and path information with polarization encoded entangled two-qubit state. We propose and demonstrate a new way to generate different classes of partial entangled states and therefore can explore Bagan’s equality for the case of $N = 2$ under different conditions. Note that it is impossible to obtain the same results by using classical light in our scheme and our setup. We become aware that Yuan et al. also test Bagan’s equality with heralded single photon state. A classical light field (no matter whether a weak laser or thermal light) applied on Yuan et al.’s setup will generate exactly the same results. In our experiment, we design and prepare an exotic two-photon entangled state specifically for rigorously testing Bagan’s theory in the quantum regime.

Furthermore, our results show a potential connection between the two quantum resources of coherence and entanglement. Although Bagan’s theory doesn’t require entanglement, with different class of partial entangled quantum states, we are able to test Bagan’s duality trade-off with different degree of complementarity. Since the relation between coherence and entanglement remains an important and open topic, our scheme and setup may serve as a new platform to simultaneously investigate the roles and their relations to coherence, path information and entanglement.

Last but not least, our scheme can be extended to larger $N$ for testing multi-path Bagan’s theory and for other quantum tasks. The key operation in generating our exotic two-photon entangled states is to introduce polarization-dependent differential transmission rates on each arm of a Bell state. Such operation can be also applied onto other multi-photon states, such as GHZ states, cluster states and even multi-photon mixed states. Especially applying our operation onto GHZ states will enable a genuine test of multi-path Bagan’s theory, which will go beyond equality and test the duality between coherence and path information in an inequality fashion. There have been demonstrations of high fidelity multi-photon entanglement state, thus it is experimentally feasible to generate a multi-path exotic state of our type to test Bagan’s theory and to explore the relation between coherence, path information and multi-partite entanglement, even though much technical effort still remains to be made.

The wave-particle duality relations are of significance in quantum mechanics, while the quantum coherence plays a central role in quantum information processing. Our results may inspire further theoretical and experimental investigations on such fundamental researches and strengthen the prominent role of coherence in quantum physics and quantum technologies.

Methods

Experiments. Entangled photon pairs are generated by using type-II spontaneous parametric down-conversion. A 5 mW 405 nm laser diode pumps a 25 mm periodically-poled KTiOPO4 (PPKTP) crystal in a Sagnac interferometer. The beam spot size at the center of the crystal is 220 μm. The degenerate down-conversion photon pairs at 810 nm are filtered by 3 nm bandpass filter, then coupled into single mode fibers and guided to the state preparation stage. The coincidence rate is about 30 kHz. By either inserting Brewster windows or placing a periodically-poled KTiOPO4 (PPKTP) crystal in a Sagnac interferometer. The wave-particle duality relations are of significance in quantum mechanics, while the quantum coherence plays a central role in quantum information processing. Our results may inspire further theoretical and experimental investigations on such fundamental researches and strengthen the prominent role of coherence in quantum physics and quantum technologies.

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Data availability

The data that support the findings of this study are available from the corresponding author on reasonable request.

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Author contributions
X.M.J. conceived the project and designed the experiment. Z.H.M., S.M.F. and V.V. developed the theory. J.G., Z.Q.J., C.H., L.F.Q., R.J.R., H.T. and X.M.J. performed the experiment. J.G., X.M.J., Z.H.M. and S.M.F. analysed the data and wrote the paper.

Additional information
Competing interests: The authors declare no competing interests.

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