Spin chiral current induced by curvature of space-time

Sergey N. Andrianov and Sergey M. Kozyrev

Scientific center for gravity wave studies “Dulkyn”, Kazan, Russia

Rinat A. Daishev

1 Department of Mathematics and Department of Physics,
Kazan Federal University, Kremlevskaya str. 18, Kazan 420008, Russia

Klein-Gordon equation is derived for a particle in the brane model of Universe. It is compared with squared Dirac-Fock-Ivanenko equation and expression for a chiral current is obtained by this comparison. This expression defines chiral current through variation of spin connection gauge field that arises due to the symmetry in respect to local Lorenz transformations. So, the second derivative of gravitational gauge field determines variation of chiral current responsible for variation of mass. The role of these processes on the early stages of Universe evolution is discussed.

PAC numbers: 04.20, 03.50

I. INTRODUCTION

The evolution of particles in curved space can be described by generalized Klein-Gordon (KG) equation derived by Schrödinger [1]. KG equation in the same form was derived within brane model in paper [2] and thus it can be valid just after Big Bang and brane formation. But KG equation does not properly describe probability features of wave function. Besides, it properly describes position of particle in space but gives the solutions not only with positive time flaw but, also, with negative one. Therefore, motion of particles in our nowadays Universe with definite direction of local time flaw must be described by Dirac equation. Its generalization in curved space is named Dirac-Fock-Weyl (DFW) equation [3, 4] or in other notations Dirac-Fock-Ivanenko (DFI) equation [3, 5, 6].

Comparison of KG equation with squared DFW equation was performed in paper [7]. But spin connection in DFW equation was changed with that by some indefinite scalar matrix which quadrate was also included in KG equation therein. We perform here rigorous comparison of KG equation in its original form [1] with squared DFI equation and obtain formula for chiral current. Finally, we discuss generation of mass connected with chiral current variation on early stages of Universe evolution.

II. SQUARED DIRAC-FOCK-IVANENKO EQUATION

We have derived KG equation for a brane that has the following form [2]

\[ g^{ij} \nabla_i \nabla_j \psi = \left\{ \frac{1}{4} R - \left( \frac{mc}{\hbar} \right)^2 \right\} \psi \]  \hspace{1cm} (1)

coinciding with that of [1], where \( \nabla_i \) is covariant derivative, \( R \) is curvature, \( m \) is mass of a particle. Let’s consider, also, DFI equation

\[ i \gamma^i (\nabla_i + \Gamma_i) \psi = m \psi \] \hspace{1cm} (2)

with Dirac gamma matrixes \( \gamma^i \) and spin connection \( \Gamma_i \). Squaring of this equation yields

\[ (\gamma^i \gamma^j \nabla_i \nabla_j + (\gamma^i (\nabla_i \gamma^i) + \gamma^i \{ \Gamma_i, \gamma^i \}) \nabla_j + \gamma^i \nabla_i (\gamma^j \Gamma_j) + (\gamma^i \Gamma_i)(\gamma^j \Gamma_j)) \psi = -m^2 \psi. \] \hspace{1cm} (3)
If
\[ \frac{1}{2} \{ \gamma^i, \gamma^j \} = g^{ij}, \] (4)

\[ \gamma^i (\partial_i \gamma^j) + \{ (\gamma^i \Gamma_i), \gamma^j \} = 0, \] (5)

and
\[ \gamma^i (\partial_i \gamma^j) B_{ij} + \gamma^i \partial_i (\gamma^j \Gamma_j) + (\gamma^i \Gamma_i)(\gamma^j \Gamma_j) + (\gamma^i \Gamma_i)(\gamma^j B_{ij}) + (\gamma^i B_{ij})(\gamma^j \Gamma_j) = -\frac{1}{4} R, \] (6)

we come to Klein-Gordon equation (1). Relations (4, 5) coincide with that of [7]. Taking into account expression from [8]
\[ \partial_i \gamma^j + [\Gamma_i, \gamma^j] + \Gamma^j_{ik} \gamma^k = 0, \] (7)

we come as in [7] from (4) and (5) to the following relation between spin connection and Cristoffel symbol.
\[ \Gamma_i = \frac{1}{2} g^{ik} \Gamma^j_{ik}. \] (8)

Lets consider relation (6). Using (7) again we get
\[ \gamma^i \gamma^j \nabla_i \Gamma_j + \gamma^i \gamma^j \Gamma_i \Gamma_j = -\frac{1}{4} R, \] (9)

where \( \nabla_i \Gamma_j = \partial_i \Gamma_j - \Gamma^j_{ik} \Gamma_k \). Therefore, we obtain that
\[ \gamma^i \gamma^j D_i \Gamma_j = -\frac{1}{4} R. \] (10)

where \( D_i = \nabla_i + \Gamma_i \) is generalized covariant derivative.

III. CHIRAL CURRENT.

Lets consider the first term in (10). It can be decomposed into commutator and anticommutator parts
\[ \gamma^i \gamma^j D_i \Gamma_j = \frac{1}{2} \{ \gamma^i, \gamma^j \} D_i \Gamma_j + \frac{1}{2} [\gamma^i, \gamma^j] \left( \frac{1}{2} (D_i \Gamma_j + D_j \Gamma_i) + \frac{1}{2} (D_i \Gamma_j - D_j \Gamma_i) \right). \] (11)

It can be rewritten as
\[ \gamma^i \gamma^j D_i \Gamma_j = g^{ij} D_i \Gamma_j + \frac{1}{4} [\gamma^i, \gamma^j] (\nabla_i \Gamma_j - \nabla_j \Gamma_i + \Gamma_i \Gamma_j - \Gamma_j \Gamma_i). \] (12)

Introducing spin curvature
\[ \Phi_{ij} = \nabla_i \Gamma_j - \nabla_j \Gamma_i + \Gamma_i \Gamma_j - \Gamma_j \Gamma_i = D_i \Gamma_j - D_j \Gamma_i \] (13)

we get from (12) the expression
\[ [\gamma^i, \gamma^j] \Phi_{ij} = -4 g^{ij} D_i \Gamma_j - R. \] (14)

that can be used for determination of \([\gamma^i, \gamma^j]\).

Another variant of this formula is derived as following. Spin curvature can be expressed as [8]
\[ \Phi_{ij} = -\frac{1}{8} [\gamma^i, \gamma^k] R_{ijk}. \] (15)
Substitution of (15) into (14) gives

$$
\left( \frac{1}{8} [\gamma^i, \gamma^j] [\gamma^\alpha, \gamma^\beta] - \frac{1}{4} \{\gamma^j, \gamma^\beta\} \{\gamma^i, \gamma^\alpha\} \right) R_{ij\alpha\beta} = 4g^{ij} D_i \Gamma_j,
$$

(16)

Using symmetry properties of metric tensor and Bianki identity we get

$$
\gamma^i \gamma^j \gamma^k \gamma^l R_{i\ell jk} = 4g^{ij} D_i \Gamma_j,
$$

(17)

The left hand side of (17) can be rewritten as

$$
\gamma^i \gamma^j \gamma^k \gamma^\ell R_{i\ell jk} = \delta^{ijps}_{klmn} \gamma^k \gamma^l \gamma^m \gamma^n R_{isjp},
$$

(18)

where tensor \( \delta^{ijps}_{klmn} \) is the generalized Kronecker symbol. Further, we use the identity

$$
\delta^{ijps}_{klmn} = \frac{1}{4!} \varepsilon^{ijps} \varepsilon_{klmn},
$$

(19)

substituting it in (18) to get

$$
\gamma^i \gamma^j \gamma^k \gamma^\ell R_{i\ell jk} = -i \gamma^5 \tilde{R},
$$

(20)

where

$$
\gamma^5 = \frac{i}{4!} \varepsilon_{klmn} \gamma^k \gamma^l \gamma^m \gamma^n,
$$

(21)

and

$$
\tilde{R} = \varepsilon^{ijps} R_{ijps}.
$$

(22)

Eventually, we get from (17) and (20) the following expression for \( \gamma^5 \):

$$
\gamma^5 = 4i \tilde{R}^{-1} g^{ij} D_i \Gamma_j
$$

(23)

or

$$
\gamma^5 = 4i \tilde{R}^{-1} D_i \Gamma^i.
$$

(24)

\( \gamma^5 \) can be used for the determination of chiral spin current according to the formula

$$
\gamma^{k5} = \overline{\psi} \gamma^k \gamma^5 \psi,
$$

(25)

where \( \psi \) is spinor wave function. Substitution of (24) into (25) yields

$$
j^{k5} = 4i \overline{\psi} \gamma^k \tilde{R}^{-1} D_i \Gamma^i \psi.
$$

(26)

Spin connection in equation (26) can be regarded as gauge gravitational field related to the symmetry of local Lorenz transformations. \( D_i \Gamma^i \) in (26) is gravitational force. When the brane radius exceeds critical value gravitational force supporting the quark-gluon \([9, 10]\) brane becomes weaker than the influence of strong interaction and quarks combine into separate elementary particles. Variation of chiral current describes generation of mass when quarks combine producing the particles. So, variation of our chiral current is connected with creation and decay of elementary particles on the background of strongly varying gravitational field in the early Universe.

IV. CONCLUSION

Thus, we have considered Klein-Gordon equation for a particle on brane derived by us earlier using variation principle. Solutions of Klein-Gordon equation have time reversal symmetry and thus brane after its emergence due Big Bang supported this symmetry. But then this symmetry was spontaneously broken and we must deal now with Dirac equation. However, time reversing symmetry breaking still does not yield definite time arrow. The result of this symmetry breaking is the emergence of particl’s spin that can be considered as rotation of particle in extra
dimension. Particle can not move already in additional dimension because of space compactification but it can rotate in it. Massive particle rotating in extra dimension produces gravitational field on brane and thus spin connectivity through this field.

Spin projection on momentum direction defines helicity of the particle that coincides with chirality for massless particles. No coordinate transformation can change the helicity of massless particle since no physical body can move faster than light. So, particle can exist simultaneously in local times $\tau$ and $-\tau$ for any given value of brane radius that can be regarded as global time since these states do not mix. In the case of massive particles, helicity coincides with chirality only when the symmetry to parity transformations is valid for a particle. Here, coordinate transformation can change the sign of spin projection on particle’s momentum direction but parity transformation (inversion of coordinates) returns it back. So, particle can again be simultaneously characterized by two contra-directional local times. Only chiral symmetry breaking leads eventually to collision of local times $\tau$ and $-\tau$ and formation of time arrow with definite time $t$. With that, inertial mass of the particle becomes larger because now the particle does not experience free evolution with times $\tau$ and $-\tau$ but evolution in definite time $t$ with a process of time arrow formation.

Squaring Dirac-Fock-Ivanenko equation gives Klein-Gordon equation. We have derived the expression for gamma five matrix through the derivative of spin connection by the comparison of squared DFI equation with KG equation and used it for determination of chiral spin current. This chiral spin current is anomalous spin current corresponding to spontaneous chiral symmetry breaking of mass particle in the space of DFI equation solutions. It describes creation and decay of elementary particles and is connected with variation of gravitational gauge field gradient that could be rather intensive in early Universe. Generation of mass is one of crucial steps in the evolution of early Universe. 99 percents of nucleon mass comes out of light quarks combination in more heavy hadrons as a result of this chiral symmetry breaking.

[1] E. Schrödinger, Sitz. Preuss. Akad. Wiss. Berlin 105 (1932).
[2] Global Journal of Science Frontier Research: A, Physics and Space Science. 13, 1 (2013).
[3] V. A. Fock. Z. Phys. 57, 261 (1929).
[4] H. Weyl. Z. Phys. 56, 330 (1929).
[5] V. A. Fock, D., Ivanenko. Z. Phys. 54, 798 (1928).
[6] V. A. Fock, D., Ivanenko. Z. Phys. 60, 648 (1929).
[7] A. D. Alhaidari and A. Jellal. Dirac and Klein-Gordon Equations in Curved Space, arXiv: 1106, (2011).
[8] H. A. Weldon. Fermions without vierbeins in curved space-time. Phys. Rev., D V.63, No.10, P.104010 (2001).
[9] Giuseppe De Risi, Tiberiu Harko, Francisco S. N. Lobo and Chun Shing Jason Pun. Nucl. Phys. B 805, 190 (2008).
[10] Kh.Saaidi, A. Mohammadi, T. Golanbari, H. Sheikhhahmadi, and B. Ratra. Phys. Rev. D 86, 045007 (2012).