Z boson pair production at LHC
in a stabilized Randall-Sundrum scenario

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Abstract

We study the Z boson pair production at LHC in the Randall-Sundrum scenario with the Goldberger-Wise stabilization mechanism. It is shown that comprehensive account of the Kaluza-Klein graviton and radion effects is crucial to probe the model: The KK graviton effects enhance the cross section of $gg \rightarrow ZZ$ on the whole so that the resonance peak of the radion becomes easy to detect, whereas the RS effects on the $q\bar{q} \rightarrow ZZ$ process are rather insignificant. The $p_T$ and invariant-mass distributions are presented to study the dependence of the RS model parameters. The production of longitudinally polarized Z bosons, to which the SM contributions are suppressed, is mainly due to KK gravitons and the radion, providing one of the most robust methods to signal the RS effects. The $1\sigma$ sensitivity bounds on $(\Lambda_\pi, m_\phi)$ with $k/M_{Pl} = 0.1$ are also obtained such that the effective weak scale $\Lambda_\pi$ of order 5 TeV can be experimentally probed.
I. INTRODUCTION

Recent advances in string theories have inspired particle physicists to approach the gauge hierarchy problem of the standard model (SM) in a very novel way, i.e., by introducing extra dimensions. Arkani-Hamed, Dimopoulos and Dvali (ADD) proposed that there exist $n$ large extra dimensions with factorizable geometry, whereas the SM fields are confined to our four-dimensional world $[1]$. The observed largeness of Planck scale $M_{\text{Pl}}$ is attributed to the large volume of the extra dimensions $V_n$, as can be seen from the relation $M_{\text{Pl}}^2 = M_S^{n+2}V_n$ with $M_S$ being the fundamental scale. Since the $M_S$ can be maintained around TeV scale, the hierarchy problem is answered. Criticism arose such that the ordinary gauge hierarchy is replaced by a new hierarchy between the $M_S$ and the compactification scale $\mu_c = V_n^{-1/n}$. Based on two branes and a single extra dimension with non-factorizable geometry, Randall and Sundrum (RS) have proposed another higher dimensional scenario where, without large volume of the extra dimension, the hierarchy problem is solved by a geometrical exponential factor $[2]$. Here the stabilization of the compactification radius is very crucial: Otherwise the study of the cosmological evolution in the RS scenario has shown that a nontrivial relationship between the matter densities on the two branes is required, which induces non-conventional cosmologies $[3]$. Goldberger and Wise (GW) have proposed a stabilization mechanism where a bulk scalar field with interactions localized on the two branes can generate for the modulus field a potential which allows a minimum appropriate to the hierarchy problem $[4]$.

Of great interest and significance is that the ADD and RS models could have chances to be detected at future collider experiments. Even more interestingly, they provide possible accounts for the recently reported deviation of the muon anomalous magnetic moment from the SM prediction $[5]$. In the ADD case, even though the coupling of each Kaluza-Klein (KK) graviton state with the SM fields is suppressed by Planck scale, summation over almost continuous KK spectrum compensates the suppression and leaves the effective coupling of $\sim 1/M_S$ $[6]$. In the RS scenario, the zero mode of the KK graviton states couples with
the usual Planck strength, whereas the masses and couplings of all the excited KK states are characterized by some electroweak scale $\Lambda_\pi$ \[7,8\]. This discrete spectrum is to yield the clean signal of graviton resonance production. Another key ingredient of the RS model is from the stabilization mechanism, the radion. Since the radion can be much lighter than $\Lambda_\pi$ \[9,10\], it is likely that the first signal of the RS effects come from the radion.

Various phenomenological aspects of the radion have been extensively studied in the literature: The decay modes of the radion are different from those of the Higgs boson (e.g., the radion with mass smaller $2m_Z$ dominantly decays into two gluons) \[9,10\]; without a curvature-scalar Higgs mixing, the radion effects on the oblique parameters of the electroweak precision observations are small \[11\]; the radion effects on the phenomenology at low energy colliders \[12\] and at high energy colliders \[13-15\] have been also discussed.

In the RS scenario, however, there is another key ingredient, the KK spectrum of gravitons. Especially at high energy collider, comprehensive effects of the radion and KK gravitons can be substantial. Moreover, in spite of its lighter mass than the KK gravitons, the coupling strength of the radion with the SM particles is weaker than that of the KK gravitons due to the following reasons: The characteristic scale of the radion coupling, inversely proportional to the vacuum expectation value (VEV) of the radion ($\Lambda_\phi$), is smaller than the coupling of the KK gravitons, since $\Lambda_\phi = \sqrt{6} \Lambda_\pi$ \[9\]; the degrees of freedom of the spin-0 radion are less than those of the spin-2 massive KK gravitons. And the $\Lambda_\phi$, usually treated as a free parameter, is also constrained through the relation with the $\Lambda_\pi$ which receives various constraints from the LEP II and Tevatron experiments \[8\]. It is to be shown that the comprehensive accommodation at high energy colliders leads to different phenomenologies from those only with the radion effects.

In order to sensitively probe the radion effects, we consider the process $gg \rightarrow ZZ$. Note that the radion interacts with the SM fields through the trace of the energy-momentum tensor: The coupling becomes stronger as the interacting SM particles are more massive; QCD trace anomaly enhances the coupling of the radion to a pair of gluons. This process has been extensively studied with special attention to the Higgs search at the LHC \[17\]. The double
leptonic decay of the Z-boson, \( gg \rightarrow ZZ \rightarrow l^+l^-l'^+l'^- \), generates a clean signal. Unfortunately the main background of the continuum production of the Z pair via \( q\bar{q} \) annihilation is known to be dominant except for a limited range of the Higgs resonance. For example, full one-loop calculations for the \( gg \rightarrow ZZ \) in the minimal supersymmetric standard model (MSSM) with the squark loop contributions have shown that the irreducible background of \( q\bar{q} \rightarrow ZZ \) surpasses even resonant peaks of the supersymmetric Higgs bosons \([19]\).

As shall be shown, comprehensive accommodation of both the KK graviton and radion effects is very crucial to explore the RS model. The RS effects on the gluon fusion process are much larger than those on the \( q\bar{q} \) process, which shall be apparently shown in the \( p_T \) and invariant-mass distributions. As a result, the radion effects on the \( gg \rightarrow ZZ \) have much more chance to be detected, while the radion effects on the \( q\bar{q} \rightarrow ZZ \) are negligible due to the smallness of \( m_q \). Moreover the measurement of the Z polarization shall provide another efficient method for probing the effects of the RS model.

This paper is organized as follows. In Sec. II, we briefly review the RS model with the GW mechanism, and summarize the effective Lagrangian between the KK graviton, radion and SM particles. Model parameters are carefully notified. In Sec. III, the parton level helicity amplitudes for \( gg \rightarrow ZZ \) and \( q\bar{q} \rightarrow ZZ \) to leading order are given. In Sec. IV, we present numerical results of the \( p_T \) and invariant-mass distributions. After presenting the RS model dependence on the distributions, we shall show the RS effects on various configurations of Z-boson polarization. The sensitivity bounds for parameter space of \( \Lambda_\pi \) and the radion mass are to be given. Finally, Sec. V deals with summary and conclusions.

**II. STABILIZED RANDALL-SUNDRUM SCENARIO**

For the hierarchy problem, Randall and Sundrum have proposed a five-dimensional non-factorizable geometry with the extra dimension compactified on a \( S_1/Z_2 \) orbifold of radius \( r_c \). Reconciled with four-dimensional Poincaré invariance, the RS configuration has the following solution to Einstein’s equations:
\[ ds^2 = e^{-2kr_c|\varphi|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\varphi^2, \]  

where \( 0 \leq |\varphi| \leq \pi \) and \( k \) is \( AdS_5 \) curvature. Two orbifold fixed points accommodate two three-branes, the hidden brane at \( \varphi = 0 \) and our visible brane at \( |\varphi| = \pi \). The arrangement of our brane at \( |\varphi| = \pi \) renders a fundamental scale \( m_0 \) to appear as the four-dimensional physical mass \( m = e^{-kr_c\pi}m_0 \). The hierarchy problem can be answered if \( kr_c \approx 12 \). From the four-dimensional effective action, the relation between the four-dimensional Planck scale \( M_{Pl} \) and the fundamental string scale \( M_S \) is obtained by

\[ M_{Pl}^2 = \frac{M_S^3}{k} (1 - e^{-2kr_c\pi}). \]  

The compactification of the fifth dimension leads to the following four-dimensional effective Lagrangian

\[ \mathcal{L} = -\frac{1}{M_{Pl}} T^{\mu\nu} h_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^\infty h_{\mu\nu}^{(n)}, \]  

where \( \Lambda_\pi \equiv e^{-kr_c\pi}M_{Pl} \) is at the electroweak scale. The coupling of the zero mode of the KK gravitons is suppressed by usual Planck scale, while those of the massive KK gravitons by the electroweak scale \( \Lambda_\pi \). The masses of the KK gravitons are also at the electroweak scale, given by

\[ m_n = k x_n e^{-kr_c\pi} = \frac{k}{M_{Pl}} \Lambda_\pi x_n, \]  

where the \( x_n \)'s are the \( n \)-th roots of the first order Bessel function. The condition \( k < M_{Pl} \) is to be imposed to maintain the reliability of the RS solution in Eq. (1). We take the value in the conservative range of \( 0.1 < k/M_{Pl} < 0.7 \). Then, the first excited KK graviton has mass slightly larger than 1 TeV for \( \Lambda_\pi \sim 3 \) TeV and so there might be a chance to see the effects of KK gravitons at future high energy colliders.

In the original RS scenario, the compactification radius \( r_c \) is assumed to be constant. According to the studies of cosmological evolution, however, two branes wants to blow apart, i.e., \( r_c \rightarrow \infty \), unless we impose a fine-tuning between the densities on two branes, which will lead to non-conventional cosmologies. A stabilization mechanism is required. Goldberger
and Wise have introduced a bulk scale field with the bulk mass somewhat smaller than
the $k$. The assumption of localizing the bulk scalar interactions on two branes determines
the four-dimensional effective potential for the radion, which can allow the minimum for
$kr_c \approx 12$ without an extreme fine-tuning. Furthermore, the radion mass is roughly an order
of magnitude below $\Lambda_\pi$ \[11\].

It was shown that the radion couples to ordinary matter through the trace of the sym-
metric and conserved energy-momentum tensor with TeV scale suppressed coupling;

$$\mathcal{L} = \frac{1}{\Lambda_\phi} \phi T^\mu_\mu,$$

\hspace{1cm} (5)

where $\Lambda_\phi$, the VEV of the radion field, is related such that $\Lambda_\phi = \sqrt{6} \Lambda_\pi$ \[9\]. The coupling
of the radion with a fermion or massive gauge boson pair is the same as that of the Higgs,
except for a factor of $(v/\Lambda_\phi)$, with $v$ being the VEV of the SM Higgs boson. The massless
gluons and photons also contribute to the $T^\mu_\mu$, due to the trace anomaly which appears as
the scale invariance of massless fields is broken by the running of gauge couplings \[20\]. Thus
the interaction Lagrangian between two gluons and the radion or Higgs boson is

$$\mathcal{L}_{h(h)g-g} = \left[ \left( \frac{v}{\Lambda_\phi} \right) \left\{ b_3 + I_{1/2}(z^h_t) \right\} \phi + I_{1/2}(z^h_t)h \right] \frac{\alpha_s}{8\pi v} \text{Tr}(G^C_{\mu\nu}G^{C\mu\nu}),$$

\hspace{1cm} (6)

where $z^\phi(h)_t = 4 m_t^2 / m^2_\phi(h)$, $m_t$ is the top quark mass, and the QCD beta function coefficient
is $b_3 = 11 - 2n_f / 3$ with the number of dynamical quarks $n_f$. The loop function $I_{1/2}(z)$ is
defined by

$$I_{1/2}(z) = z [1 + (1 - z) f(z)],$$

\hspace{1cm} (7)

where the $f(z)$ is

$$f(z) = \begin{cases} \arcsin^2(1/\sqrt{z}), & z \geq 1, \\ -\frac{1}{4} \left[ \ln \left( \frac{1+\sqrt{1-z}}{1-\sqrt{1-z}} \right) - i\pi \right]^2, & z < 1. \end{cases}$$

\hspace{1cm} (8)

It is to be noted that the phenomenology of radions and KK gravitons in the RS model can
be determined by three parameters, $m_\phi$, $\Lambda_\pi$, and $k/M_{Pl}$.
III. Z BOSON PAIR PRODUCTION AT LHC

A. The $gg \to ZZ$ helicity amplitudes

For the process $g(\lambda_1)g(\lambda_2) \to Z(\lambda_3)Z(\lambda_4)$ there are, in general, 36 helicity amplitudes according to two and three polarization states of initial gluons and final $Z$-bosons, respectively. Various symmetry arguments reduce these 36 amplitudes into eight independent ones [17]. Parity invariance implies

$$M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = M^*_{-\lambda_1 -\lambda_2 -\lambda_3 -\lambda_4}. \quad (9)$$

Bose statistics and the standard form of the $Z$-boson polarization vectors demand

$$M_{++++}(\beta) = M_{++++}(-\beta), \quad (10)$$
$$M_{+++\beta}(\beta) = M_{+-+}(\beta), \quad (11)$$
$$M_{+--}(\beta) = M_{+-+}(\beta), \quad (12)$$
$$M_{+-+}(\beta) = M_{+-+}(-\beta), \quad (13)$$
$$M_{+++0}(\beta) = M_{+++0}(\beta) = M_{++0\beta}(-\beta) = M_{++0\beta}(-\beta), \quad (14)$$
$$M_{+--0}(\beta) = -M_{+-0+}(-\beta) = M_{+-0+}(-\beta) = -M_{+-0+}(-\beta). \quad (15)$$

For the explicit calculations, we consider the following four momenta for the initial gluons and final $Z$ bosons in the gluon-gluon center-of-momentum (c.m.) frame:

$$p_1 = \frac{\sqrt{s}}{2} (1, 0, 0, 1), \quad p_2 = \frac{\sqrt{s}}{2} (1, 0, 0, -1), \quad (16)$$
$$p_3 = \frac{\sqrt{s}}{2} (1, \beta \sin \theta, 0, \beta \cos \theta), \quad p_4 = \frac{\sqrt{s}}{2} (1, -\beta \sin \theta, 0, -\beta \cos \theta),$$

where $\beta = \sqrt{1 - 4m_Z^2/s}$ and $\hat{t} = (p_1 - p_3)^2$. The polarization vectors for the spin-1 particles with momentum $p^\mu = (p^0, \vec{p})$ are

$$\epsilon^\mu(p, \lambda) = \frac{e^{i\lambda \phi_\rho}}{\sqrt{2}} (0, -\lambda \cos \theta \cos \phi_\rho + i \sin \phi_\rho, -i \cos \phi_\rho - \lambda \cos \theta \sin \phi_\rho, \lambda \sin \theta), \quad (17)$$
$$\epsilon^\mu(p, 0) = \frac{|\vec{p}|/m, p^0 \hat{\rho}/m}, \quad (18)$$
where the angles $\theta$ and $\phi^\hat{p}$ specify the direction of $\vec{p}$.

In the SM, there are two types of Feynman diagrams for the $gg \rightarrow ZZ$ process, through the box and triangle quark loops shown in Fig. 1. Despite loop suppression by a factor of $\alpha_s^2$, high luminosity of the gluon in a proton at the LHC yields substantial cross section for the process. The SM helicity amplitudes have been studied in detail. We refer the reader to Ref. [17].

In the RS model, KK gravitons mediate the $s$-channel Feynman diagram at tree level (see Fig. 2). The helicity amplitudes are cast into

$$
\mathcal{M}^{G\lambda_1\lambda_2\lambda_3\lambda_4}_{\lambda_1\lambda_2\lambda_3\lambda_4}(gg \rightarrow ZZ) = -\frac{1}{8\Lambda_n^2} \sum_n \delta^{ab} \hat{s} - m_n^2 A_{\lambda_1\lambda_2\lambda_3\lambda_4},
$$

where $\delta^{ab}$ denotes the color factor. There are six non-vanishing independent helicity amplitudes:

$$
A_{++++} = -\frac{1}{2}(\beta^4 - 1)(\hat{t} - \hat{u})^2 + \frac{1}{2}(\beta^2 + 1)\hat{s}^2 + (\hat{t} + \hat{u})\hat{s},
$$

$$
A_{+-+-} = \frac{1}{2}\{\beta(\hat{t} - \hat{u}) + \hat{s}\}^2,
$$

$$
A_{+-+-} = \frac{1}{2}(\beta^2 - 1)\{\beta^2(\hat{t} - \hat{u})^2 - \hat{s}^2\},
$$

$$
A_{-+00} = -\frac{\sqrt{\hat{s}}}{2\sqrt{2m_Z}}(1 - 1/\beta^2)(\hat{t} - \hat{u} + \beta\hat{s})\sqrt{(\beta\hat{s})^2 - (\hat{t} - \hat{u})^2},
$$

$$
A_{++00} = \frac{(1 + \beta^2)((1 + \beta^2)\hat{s} + 2(\hat{t} + \hat{u})\hat{s}}{8m_Z^2},
$$

$$
A_{+-00} = \frac{(1 - 1/\beta^2)(\beta^2 - 2)((\hat{t} - \hat{u})^2 - \beta^2\hat{s}^2)\hat{s}}{8m_Z^2}.
$$

The second ingredient of the RS model, the radion, couples to two gluons through its Yukawa coupling to a quark inside a triangle loop diagram, as well as through QCD trace anomaly. Since the radion interaction with quarks is proportional to the mass, only top quark loop is to be considered. Two of the eight independent helicity amplitudes receive non-vanishing contributions from the radion:

$$
\mathcal{M}^{\phi}_{++++} = -\frac{\alpha_s\delta^{ab}}{\pi}[b_{QCD} + I(z_t)] \left(\frac{m_Z}{\Lambda_\phi}\right)^2 \frac{1}{\hat{s} - m_\phi^2 + i\Gamma_\phi m_\phi} \hat{s}^2 (\beta^2 + 1),
$$

$$
\mathcal{M}^{\phi}_{++00} = -\frac{\alpha_s\delta^{ab}}{\pi}[b_{QCD} + I(z_t)] \left(\frac{m_Z}{\Lambda_\phi}\right)^2 \frac{1}{\hat{s} - m_\phi^2 + i\Gamma_\phi m_\phi} \hat{s}^2 (\beta^2 + 1).
$$
B. The $q\bar{q} \to ZZ$ helicity amplitudes

In the SM the process $q\bar{q} \to ZZ$ surpass the gluon fusion process due to the presence of tree level Feynman diagrams. Accurate structure functions at low $x$ and higher order QCD corrections are necessary for the precise ratio of the $q\bar{q}$ annihilation and gluon fusion processes. Even though the $K$-factor of $\mathcal{O}(30\%)$ of the $q\bar{q} \to ZZ$ is known [21], the absence of the corresponding calculation for the gluon fusion process leads us not to include any higher order corrections. We also assume that the uncertainties with leading contributions from soft gluon emission in both processes cancel to some extent in the ratio of cross sections, hardly affecting our main interest, the distribution shapes.

In the SM, there are $t$- and $u$-channel Feynman diagrams. Since the radion coupling to fermions is proportional to the fermion mass, we can safely neglect the radion effects here. The $s$-channel diagram mediated by KK gravitons, which is similar to the diagram (a) in Fig. 2, still influence the production. For the process $q(\lambda_1)\bar{q}(\lambda_2) \to Z(\lambda_3)Z(\lambda_4)$, the helicity amplitudes due to KK gravitons are written by

$$
\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{G}(q\bar{q} \to ZZ) = -\frac{1}{4\Lambda^2} \sum_n \frac{\delta^{\alpha\beta}}{\hat{s} - m_n^2} B_{\lambda_1\lambda_2\lambda_3\lambda_4},
$$

where non-zero and independent $B$'s are, in the parton c.m. frame,

$$
B_{++--} = \frac{\hat{s}(\hat{t} - \hat{u})}{\beta} (\beta^2 - 1) \sin \theta, \\
B_{+-+} = \frac{\hat{s}(\hat{t} - \hat{u} + \beta\hat{s})}{\beta} \sin \theta, \\
B_{+-0} = -\frac{\beta^2 - 1}{2^{3/2} m_Z} \frac{\hat{s}^{3/2}(1 + \cos \theta)\{ \beta\hat{s} - 2(\hat{t} - \hat{u}) \}}{2^{3/2} m_Z \beta}, \\
B_{++-} = \hat{s}^2 \frac{\sin \theta(\cos \theta - 1)}{\beta}, \\
B_{+-0} = -\frac{(\beta^2 - 1)\hat{s}^{3/2}(1 - \cos \theta)(\beta\hat{s} + 2(\hat{t} - \hat{u}))}{2^{3/2} m_Z \beta}, \\
B_{+-00} = \frac{\hat{s}^2(\beta^2 - 1)(\beta^2 - 2)(\hat{t} - \hat{u})}{4 m_Z^2 \beta} \sin \theta,
$$

where $\cos \theta = (\hat{t} - \hat{u})/\beta\hat{s}$. 

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IV. CONTINUUM Z BOSON PAIR PRODUCTION AT THE LHC

The physical production rate of $Z$ boson pair at $pp$ colliders is obtained by convoluting over the parton structure functions $^{22}$:

$$\sigma(pp \rightarrow ij \rightarrow ZZ) = \int dx_1 dx_2 \sum_{i,j} f_i(x_1, Q^2) f_j(x_2, Q^2) \hat{\sigma}_{ij}(x_1 x_2 s),$$

(28)

where $i$ and $j$ denote the parton such as the gluon or quark, and the $x_1$ and $x_2$ denote the momentum fraction of the parton from the parent proton beam. For the numerical analysis, we use the leading order MRST parton distribution functions (PDF) $^{23}$. The QCD factorization and renormalization scales $Q$ are set to be the $m_Z$. The $Q^2$-dependence is expected to be small on the distribution shapes. The c.m. energy at the LHC is $\sqrt{s} = 14$ TeV. And we have employed the kinematic cuts of $p_T \geq 25$ GeV and $|\eta| \leq 2.5$ throughout the paper. As discussed before, the RS effects are determined by three parameters, $(\Lambda_{\pi}, k/M_{\text{Pl}}, m_\phi)$. From the above arguments, we consider $\Lambda_{\pi} = 2, 3, 5$ TeV, $k/M_{\text{Pl}} = 0.1, 0.3, 0.7$, and $m_\phi = 300, 500, 700$ GeV.

The SM Higgs boson mass has not been experimentally confirmed yet. Recently, the ALEPH group has reported the observation of an excess of $3 \sigma$ in the search of the SM Higgs boson, which corresponds to the Higgs mass about 114 GeV $^{24}$. As the operation of LEP II has been completed, the decision whether the observations are only the results of statistical fluctuations or the first signal of the Higgs boson production is suspended until the Tevatron II and/or LHC running $^{25}$. In the following, the Higgs boson mass is set to be 114 GeV except for the comparison of the contributions from the Higgs boson and radion with the same mass in Fig. 3.

Before presenting numerical results, some discussions on the unitarity violation of the RS model are in order here. As can be seen in the effective Lagrangian of Eqs. (3) and (5), the RS model generically undergoes the unitarity violation at high energies $\sqrt{s} \gg \Lambda_{\pi}$. In Ref. $^{26}$, the elastic process $\gamma\gamma \rightarrow \gamma\gamma$ has been examined to obtain the bound from partial wave unitarity on the ratio $\sqrt{s}/\Lambda_{\pi}$ in the RS model. This process can yield very sensitive
bounds since the RS effects mediated by KK gravitons are dominant due to the absence of the SM contributions. The $J$-partial wave amplitude is defined \[27\] by

\[
a_{J}^{\mu\mu'} = \frac{1}{64\pi} \int_{-1}^{1} d\cos\theta \, d_{J}^{\mu\mu'}(\cos\theta) \, [-iM_{\lambda_{1}\lambda_{2}\lambda_{3}\lambda_{4}}] ,
\]

where the $M_{\lambda_{1}\lambda_{2}\lambda_{3}\lambda_{4}}$ is the helicity amplitudes, $\mu = \lambda_{1} - \lambda_{2}$, $\mu' = \lambda_{3} - \lambda_{4}$, and the $d_{J}^{\mu\mu'}$ is the Wigner functions \[28\]. Unitarity implies that the largest eigenvalue ($\chi$) of $a_{J}^{\mu\mu'}$ is to be $|\chi| \leq 1$. The reliability of perturbative calculations is approximately guaranteed by the conditions $|\chi| = 1$ and $|\text{Re}(\chi)| = 1/2$. The helicity amplitudes, of which the dominant contribution at high energies comes from the KK gravitons, are

\[
M_{++++} = M_{----} = -i \frac{s^{2}}{\Lambda_{\pi}^{2}} \sum_{n} [D_{n}(t) + D_{n}(u)] ,
\]

\[
M_{++--} = M_{----} = -i \frac{u^{2}}{\Lambda_{\pi}^{2}} \sum_{n} [D_{n}(s) + D_{n}(t)] ,
\]

\[
M_{++--} = M_{----} = -i \frac{t^{2}}{\Lambda_{\pi}^{2}} \sum_{n} [D_{n}(s) + D_{n}(u)] ,
\]

where $D_{n}(s) = 1/(s - m_{n}^{2} + im_{n}\Gamma_{n})$. The odd $J$-partial wave amplitudes vanish due to Bose-Einstein statistics in the elastic $\gamma\gamma$ scattering. And we have $a_{22}^{2} = a_{-2-2}^{2}$ and $a_{2-2}^{2} = a_{2-2}^{2}$ from the parity arguments. The non-vanishing eigenvalues $\chi_{i}$ are $a_{00}^{2}$ and $2a_{22}^{2}$. Numerical estimation leads to $\sqrt{s} \lesssim 3.1\Lambda_{\pi}$ for $k/M_{\text{Pl}} = 0.1$, $\sqrt{s} \lesssim 5.7\Lambda_{\pi}$ for $k/M_{\text{Pl}} = 0.3$, and $\sqrt{s} \lesssim 9.8\Lambda_{\pi}$ for $k/M_{\text{Pl}} = 0.7$. In what follows, a very conservative bound is to be employed such as $\sqrt{s} \leq 0.9\Lambda_{\pi}$. In order to eliminate the concern for unitarity violation, it is reasonable that we restrict our region to the region where our perturbative calculations are trustworthy. This can be achieved by excluding data with high invariant-mass. In the following, therefore, we constrain the invariant mass to be less than 1.8 TeV, to be consistent with our parameterizations of $\Lambda_{\pi}$.

First we present the $p_{T}$ and invariant-mass distributions for unpolarized $Z$-bosons. Figure 3 shows the radion and KK graviton effects for the process $gg \rightarrow ZZ$ on the distributions. For comparison, both the Higgs boson and radion masses are set to be 300 GeV with $\Lambda_{\pi} = 2$ TeV and $k/M_{\text{Pl}} = 0.1$. The long dashed line denotes the SM results with the Higgs mass
of 300 GeV. The short dashed line includes only the radion effects, whereas the dotted line incorporates both the KK graviton and radion effects. The SM result of $q\bar{q} \rightarrow ZZ$ denoted by the dash-dotted line is also plotted for comparison. The $p_T$ distributions apparently show that the KK graviton effects enhance the cross section on the whole, which increases the chance to probe the presence of the radion. Any other models for new physics beyond the SM, including supersymmetric models, hardly generate such elevated resonance behavior.

Even in the large $\Lambda_\pi$ cases where the KK graviton effects are negligible, the distinction between the Higgs boson and radion is, in principle, possible: The resonance peak of the radion becomes narrower with increasing $\Lambda_\pi$ because the radion total decay width ($\Gamma_\phi$) is inversely proportional to $\Lambda_\pi^2$. Figure 4 shows the resonance shapes of the Higgs boson and radion with the same mass, through the invariant mass spectrum of the $Z$ pair. It can be seen that if both have the same mass, the radion shows sharper resonance peak than the Higgs boson. This is generic since the total decay width of the radion is smaller than that of the Higgs boson due to the radion’s larger VEV.

In Fig. 5, we present the RS effects on the process $q\bar{q} \rightarrow ZZ$, which are determined solely by the $\Lambda_\pi$ due to the ignorance of the radion influence. The KK gravitons can be recognized by broad peaks. The RS effects are less important than those on the $gg \rightarrow ZZ$ process: The effects appear beyond 300 GeV of $p_T$ and 700 GeV of $M_{ZZ}$, generating at most $10^{-3}$ pb/GeV of the differential cross section with respect to $p_T$ or $M_{ZZ}$; the effects on the gluon fusion shall be shown to appear in the low $p_T$ and $M_{ZZ}$ region where the cross sections are sizable.

Now we illustrate each parameter dependence of the RS model to the gluon fusion process. In Fig. 6, we plot the distributions for the $m_\phi = 300, 500, 700$ GeV, with $\Lambda_\pi = 2$ TeV and $k/M_{Pl} = 0.1$. It can be seen that the resonance peak of lighter radions is sharper. If the radion is quite heavy (around 500 GeV in this parameter space), the contribution of the KK gravitons to the $p_T$ distributions overwhelms the radion resonance; the invariant-mass distribution is more appropriate to probe. If the radion is too heavy (more than 700 GeV in this case), large contributions of the KK gravitons obscure the radion effects. Figure
presents the \( \Lambda_\pi \)-dependence on the \( gg \rightarrow ZZ \) process for \( \Lambda_\pi = 2, 3, \) and \( 5 \) TeV with \( m_\phi = 300 \) GeV and \( k/M_{Pl} = 0.1 \). The \( M_{ZZ} \) distributions show a sharp resonance peak of the radion and the successive broad peaks of the KK gravitons. In the \( p_T \)-distributions, the KK graviton effects yield a plateau region. The case of \( \Lambda_\pi \gtrsim 5 \) TeV would be difficult to probe. The \( k/M_{Pl} \) dependence is presented in Fig. 8 for \( k/M_{Pl} = 0.1, 0.3, \) and \( 0.7 \) with \( m_\phi = 300 \) GeV and \( \Lambda_\pi = 2 \) TeV. Since the \( k/M_{Pl} \) is proportional to the masses of the KK gravitons (see Eq. (4)), the KK graviton effects in the large \( k/M_{Pl} \) cases are hardly detected.

Note that in the RS model the magnitude of the five-dimensional curvature \( (R_5 = -20 k^2) \) is required to be smaller than \( M_S^2 (\simeq M_{Pl}^2) \) for the reliability of the classical RS solution derived from the leading order term in the curvature. The value of \( k/M_{Pl} \) less than about 0.1 is theoretically favored [18].

We present the influence of the Z polarization measurement on the distributions. Figures 9 and 10 are for the polarization states \( Z_T Z_T \) and \( Z_L Z_L \), respectively. We set \( \Lambda_\pi = 2 \) TeV, \( k/M_{Pl} = 0.1 \) and \( m_\phi = 300 \) GeV for illustration. As expected from the scalar nature of the radion and the spin-2 nature of massive KK gravitons, longitudinally polarized Z bosons are produced more in the RS model, while the SM production of \( Z_L Z_L \) through both gluon fusion and \( q\bar{q} \) annihilation is suppressed. Thus the measurement of the longitudinally polarized Z bosons would provide one of the most robust methods to single out, in particular, the radion effects of the RS model.

Unfortunately, an event cannot determine the polarization of the Z boson. The angular distributions for the Z boson decay, \( Z \rightarrow f \bar{f} \), provide some information on the Z polarizations. To leading order, the RS effects can be ignored in the Z decay. Neglecting the mass of the final state fermions, the angular distributions of the Z decay rate are given by, in the rest frame of the decaying Z, [16]

\[
\frac{1}{\Gamma_f} \frac{d\Gamma^\pm}{d \cos \chi} = \frac{3}{8} \left[ \alpha_f (1 \mp \cos \chi)^2 + (1 - \alpha_f)(1 \pm \cos \chi)^2 \right]
\]  

(31)

for the transversely polarized Z bosons, and

\[
\frac{1}{\Gamma_f} \frac{d\Gamma^0}{d \cos \chi} = \frac{3}{4} \sin^2 \chi
\]

(32)

\[13\]
for the longitudinally polarized $Z$ bosons. For the charged leptonic decay, we have 
\[ \alpha_f = \frac{(1 - 2x_W^2)/(1 - 4x_W + 8x_W^2)}{\Gamma_f} \]
with $x_W \equiv \sin^2 \theta_W$. The partial width $\Gamma_f$ is for the normalization, 
and the $\chi$ is the angle between the fermion momentum direction and the spin axis as seen 
in the $Z$ rest frame (the $Z$ boost direction in the helicity basis). Therefore, an appropriate 
cut on the $\chi$ would select more data of longitudinally polarized $Z$ bosons. We define the 
ratio $\mathcal{R}$ as follows, which is proportional to the observable RS effects:

\[
\mathcal{R} \equiv \frac{\sigma(pp \rightarrow ZZ \rightarrow l^+l^-l'^+l'^-)_{SM+RS}}{\sigma(pp \rightarrow ZZ \rightarrow l^+l^-l'^+l'^-)_{SM}},
\]

where $l, l' = e, \mu, \tau$. In order to focus on the detection of the radion effects, we employ the 
kinematic cut, e.g., $p_T < 125$ GeV only for the estimates of the $\mathcal{R}$. Numerical analysis shows 
that with the kinematic cut of $|\cos \chi| < 0.3$ the $\mathcal{R}$ increases by 49% with respect to the $\mathcal{R}$ 
without any cut on $\chi$. Thus appropriate kinematic cuts on the transverse momentum of $Z$ 
 boson and the angle $\chi$ can enhance the signal for the radion effects by, e.g., about 50%.

Finally, we estimate the $1\sigma$ search bounds on the $\Lambda_\pi$ and the radion mass, which is 
obtained by comparing the total cross sections with and without the RS effects: Even if the 
cross section of longitudinally polarized $Z$ bosons ($\sigma_{LL}$) and/or the radion resonance peak 
are powerful methods to probe the RS effects, higher sensitivity bounds, which is usually 
relevant in the case of no signal, are obtained from the observable with larger event number. 
As discussed before, the RS model has unitarity violation at high energies, which may induce 
unphysically large contributions to the total cross section. One possible way is to exclude 
data with high invariant-mass, i.e.,, to employ an upper cut on the $M_{ZZ}$ such as $\sqrt{s} \leq 0.9 \Lambda_\pi$. 
Thus we require that

\[ \frac{\sigma_{SM+RS}^\leq - \sigma_{SM}^\leq}{\sqrt{\sigma_{SM}^\leq}} \sqrt{\mathcal{L}} \epsilon \geq 1, \]

where the $\sigma^\leq$ denotes the total cross section with an additional kinematic cut of $M_{ZZ} < 900$ GeV. The $\epsilon$ is the reconstruction efficiency for the $Z$-boson pair, which is the squared 
branching ratio of the $Z$ boson into $e^+e^-$ or $\mu^+\mu^-$. The LHC luminosity $\mathcal{L}$ is 100 pb$^{-1}$. In 
Fig. [II], we show the $1\sigma$ attainable bounds on $\Lambda_\pi$ and $m_\phi$ with $k/M_{Pl} = 0.1$. Even with
restrictive efficiency, the Λπ of about 5 TeV can be experimentally examined. Since the radion has relatively small influence upon the total cross section, the pT or invariant-mass distribution would be more appropriate to signal the radion effects.

**V. SUMMARY AND CONCLUSION**

We have studied the gg → ZZ process at LHC as a probe of the Randall-Sundrum scenario with the Goldberger-Wise stabilization mechanism. Even though the process has been regarded significant in various aspects (e.g., in examining the Higgs sector), the main background of the continuum production of q̅q → ZZ is known to be dominant in the SM and MSSM. It has been shown that the comprehensive effects of Kaluza-Klein gravitons and the radion enhance the chance to probe the model: The RS effects on the q̅q → ZZ process, which are generated only by KK gravitons, are much smaller than those on the gg → ZZ one; the KK graviton effects increase the cross section of gg → ZZ throughout the pT and invariant-mass distributions; the resonance shape of the radion is distinguishable from that of the Higgs boson. Numerical results of the pT and invariant-mass distributions have been obtained to show the dependence of the RS model parameters, (Λπ, k/MPl, mφ). The distinction between the Higgs boson and radion even with the same mass is feasible since the resonance peak of the radion is narrower than that of the Higgs boson in most of the parameter space. The pT and invariant-mass distributions for the polarized Z boson pair have been also presented. We have shown that especially the production of longitudinally polarized Z bosons, to which the SM contributions are suppressed, receives substantial corrections due to KK gravitons and the radion. Polarization measurements would provide one of the most robust methods to signal the RS effects. The 1σ sensitivity bounds on (Λπ, mφ) with k/MPl = 0.1 have been also obtained. The Λπ of about 5 TeV can be experimentally searched even with restrictive experimental efficiency. In conclusion, the channel of Z-boson pair production at LHC with the measurement of the Z-polarizations and the kinematic distributions can provide an efficient method to probe the effects of the
RS scenario with the modules fields being stabilized by the Goldberger-Wise mechanism.

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FIG. 1. Feynman diagrams for the process $gg \rightarrow ZZ$ in the SM.

FIG. 2. Feynman diagrams for the process $gg \rightarrow ZZ$ mediated (a) KK gravitons and (b) the radion in the RS model.
FIG. 3. The \( p_T \) and invariant-mass distributions of the \( gg \to ZZ \) process when both the SM Higgs boson and radion masses are 300 GeV with \( \Lambda_\pi = 2 \) TeV and \( k/M_{Pl} = 0.1 \). The SM results for \( q\bar{q} \to ZZ \) are plotted for comparison.
FIG. 4. The resonance peak shapes in the invariant-mass distributions of the $gg \rightarrow ZZ$ process when both the SM Higgs boson and radion masses are 300 GeV with $\Lambda_\pi = 2$ TeV and $k/M_{Pl} = 0.1$. 
FIG. 5. The $p_T$ and invariant-mass distributions of the $q\bar{q} \to ZZ$ process for $\Lambda_\pi = 2$, 3, and 5 TeV. The SM results for $gg \to ZZ$ are plotted for comparison.
FIG. 6. The $p_T$ and invariant-mass distributions of the $gg \to ZZ$ process for $m_{\phi} = 300, 500,$ and 700 GeV with $\Lambda = 2$ TeV and $k/M_{Pl} = 0.1$. The SM results for $q\bar{q} \to ZZ$ are plotted for comparison.
FIG. 7. The $\Lambda_\pi$-dependence on the $p_T$ and invariant-mass distributions of the $gg \to ZZ$ process for $\Lambda_\pi = 2, 3, \text{ and } 5 \text{ TeV with } m_\phi = 300 \text{ GeV and } k/M_{Pl} = 0.1.$
FIG. 8. The $k/M_{Pl}$-dependence on the $p_T$ and invariant-mass distributions of the $gg \rightarrow ZZ$ process for $k/M_{Pl} = 0.1, 0.3, \text{ and } 0.7$ with $m_\phi = 300 \text{ GeV}$ and $\Lambda_\pi = 2 \text{ TeV}$.
FIG. 9. The $p_T$ and invariant-mass distributions of the $gg \to Z_TZ_T$ process. We set $k/M_{Pl} = 0.1$, $m_\phi = 300$ GeV and $\Lambda_\pi = 2$ TeV.
FIG. 10. The $p_T$ and invariant-mass distributions of the $gg \to Z_L Z_L$ process. We set $k/M_{Pl} = 0.1$, $m_\phi = 300$ GeV and $\Lambda_\pi = 2$ TeV.
FIG. 11. The $1\sigma$ sensitivity bounds on $(\Lambda_\pi, m_\phi)$ with $k/M_{Pl} = 0.1$. 