Entanglement in absorption processes

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Abstract

Entanglement can modify light–matter interaction effects and, conversely, these interactions can change the non-classical correlations present in the system. We present an example where these mutual connections can be discussed in a simple way at the graduate and advanced undergraduate levels. We consider the process of light absorption by multi-atom systems in non-product states, showing first that the absorption rates depend on entanglement. The reverse is also true, absorption processes can generate an hyperentangled atomic state involving in a non-product form both internal and spatial variables. This behavior differs from that of spontaneous emission, which disentangles atomic systems.

Keywords: entanglement, light–matter interaction, multi-atom absorption

(Some figures may appear in colour only in the online journal)

1. Introduction

Light–matter interaction effects can be modified by the presence of entanglement. These modifications have been studied in several contexts [1–4]. In particular, absorption and emission rates have been analyzed in detail [5]. In the case of spontaneous emission two experiments [6, 7] have tested these effects (see [8] for a discussion of the experiments interpretation).

Some examples of the reverse process, variations of the entanglement present in multi-atom systems due to light–matter interactions, have also been described in the literature. For instance, spontaneous emission disentangles initially correlated atomic systems [9, 10]. Another two well-known examples are the interaction of trapped ions with laser beams [11] and of atoms with the field in a cavity [12], where the light absorption generates entanglement in the ionic and atomic systems.

We consider an example, light absorption by two-atom systems, that presents the above ideas in a simple way. The calculations and concepts involved are not difficult and could be used at a graduate and advanced undergraduate level to introduce the subject. Moreover, our
presentation highlights a fundamental difference between absorption and spontaneous emission: the first can change atomic non-classical correlations whereas the second destroys them.

In the example analyzed here the atomic state is initially only entangled in the spatial variables, but after the absorption it becomes hyperentangled, involving the correlations both spatial and internal degrees of freedom. By evaluating the von Neumann entropy we show that the degree of entanglement does not change, but it is redistributed between the two degrees of freedom. The absorption process can generate hyperentanglement without changing the initial degree of entanglement. The evaluation of the von Neumann entropy involves non-orthogonal states. As these calculations are simple, this example could also be useful to teach at a graduate level how to evaluate entanglement when the states overlap, a situation frequently found in atomic and molecular physics.

2. Modification of absorption rates

We discuss in this section how entanglement changes the absorption rates in multi-atom systems. First, we describe the arrangement. A source prepares pairs of distinguishable atoms in a non-factorizable state traveling in opposite directions. The preparation of entangled states is not a simple task. In the case of photons there are several well-developed techniques as spontaneous parametric down-conversion, or those based on quantum dots in semiconductors and nanoscale impurities in diamonds. In the atomic case there is not a so vast literature on the generation issue. For our proposal we can invoke the experiments \[6, 7\], based on molecular photodissociation (although at variance with the two above references, the decaying atoms must be in their ground states instead of excited ones). Because of momentum conservation the two atoms travel in almost opposite directions. The state representing this preparation is

\[
|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\psi_L >_A |g >_A |\varphi_R >_B |g >_B + |\varphi_L >_B |g >_B |\psi_R >_A |g >_A)
\]

The labels \(A\) and \(B\) refer to the two atoms, and \(L\) and \(R\) denote opposite traveling directions. The symbol \(\varphi_L (\varphi_R)\) represents the center of mass (CM) wave function of atom \(A\) (\(B\)) moving towards \(L\) (\(R\)). On the other hand, \(|g >, i = A, B\) represents the electronic ground state of the atom \(i\).

When the separation of the atoms is large, with no spatial overlap between them, they interact with the light. We consider classical light in the linear (without multiple absorptions) regime. In order to the atomic correlations be able to modify absorption rates it is not necessary to consider more sophisticated types of light (quantum or entangled) than the classical one, or to move to the nonlinear regime. The beams must contain the absorption frequencies of the two atoms; we can use light beams with different frequencies or a single broad band beam. As we assume a low intensity of the beams (linear regime), after the interaction the atomic states evolve as

\[
|\phi_j >_A |g >_A \rightarrow \alpha |\phi_j >_A |e >_A + \beta |\phi_j >_A |g >_A
\]

and

\[
|\varphi_j >_B |g >_B \rightarrow \gamma |\varphi_j >_B |e >_B + \delta |\varphi_j >_B |g >_B
\]
with \( j = L, R \). The coefficients obey the relations \( |\alpha|^2 + |\beta|^2 = 1 \) and \( |\gamma|^2 + |\delta|^2 = 1 \). The wave functions \( \tilde{\phi}_j \) and \( \tilde{\varphi}_j \) include the recoil after the absorption, and \( |e> \) denotes the excited internal state.

Note that the equations (2) and (3) are only valid in the low-intensity beam approximation. This approximation is quantitatively expressed by the relations \( |\alpha|^2 \ll 1 \) and \( |\gamma|^2 \ll 1 \), indicating that only single absorptions are relevant in the problem. The probability of multi-absorption processes is very low and can be neglected. In the presence of multi-photon absorptions we enter in a nonlinear regime and new terms should be added to these equations.

The final state after the interaction is

\[
|\psi_f> = \frac{1}{\sqrt{2}} (\alpha\gamma|\tilde{\phi}_L>_A |\tilde{\varphi}_R>_B + \gamma\alpha|\tilde{\phi}_R>_A |\tilde{\varphi}_L>_B |e>_A |e>_B + |\cdots>.
\]

where \( |\cdots> \) contains the rest of terms, which do not lead to double absorptions.

We represent the above arrangement in figure 1.

From the expression for \( \psi_f \) we can calculate the probabilities of double absorptions. There are two alternatives that contribute to the probability of double absorption, (i) absorption by an atom of type \( A \) in \( L \) and by one of type \( B \) in \( R \), and (ii) absorption by an atom of type \( B \) in \( L \) and by one of type \( A \) in \( R \). These alternatives are not distinguishable (both atoms can absorb at both sides of the arrangement) and consequently, according to the rules of quantum theory, we must add probability amplitudes instead of probabilities. Finally, the probability of double absorption is

\[
P_{\text{dou}} = \left| \frac{1}{\sqrt{2}} \alpha\gamma + \frac{1}{\sqrt{2}} \gamma\alpha \right|^2 = 2|\alpha\gamma|^2.
\]

The above probability differs from that of atoms in product states. In effect, when the initial atomic state is, instead of the pure one \( \psi_0 \), a mixture of \( |\tilde{\phi}_L>_A |g>_A |\tilde{\varphi}_R>_B |g>_B \) and \( |\tilde{\phi}_R>_A |g>_A |\tilde{\varphi}_L>_B |g>_B \) with equal weights \( 1/2 \), the double absorption probability changes to

\[
P_{\text{dou}}^{\text{mix}} = \frac{1}{2} |\alpha\gamma|^2 + \frac{1}{2} |\gamma\alpha|^2 = |\alpha\gamma|^2.
\]

The probability of double absorption in the entangled state doubles that in product ones. We conclude that the absorption probabilities in multi-atom systems depend on entanglement.
The experimental verification of the above ideas relies on the quantum theory of detection. In our case its implementation is simple. We do not need to measure the atoms and their internal states. As an excited atom shortly emits a photon because of spontaneous emission we only need standard optical detectors. The presence or absence of photon detections at every side of the arrangement tell us if the atoms were excited or not.

3. Modification of entanglement

In this section we describe the reverse of the above behavior, showing that the absorption process can also modify the entanglement distribution of the atomic system. The explicit expression for $\psi_f$ is

$$|\psi_f\rangle = \alpha \delta (|\varphi_L >_A |\varphi_R >_B + |\varphi_R >_A |\varphi_L >_B) |e >_A |g >_B + \beta \gamma (|\varphi_L >_A |\varphi_R >_B + |\varphi_R >_A |\varphi_L >_B) |g >_A |l >_B + \beta \delta (|\varphi_L >_A |\varphi_R >_B + |\varphi_R >_A |\varphi_L >_B) |g >_A |g >_B).$$

From this expression it is immediate to see that $\psi_f$ is entangled, but in a very different way from $\psi_0$.

The final state is hyperentangled. Hyperentanglement refers to entanglement involving more than one degree of freedom. In our case we clearly have hyperentanglement as the CM and internal degrees are involved. In general, the hyperentanglement studied in the literature has the form of entangled states in each one of the degrees of freedom, and all of these states in product form (in our arrangement would be $(|\varphi_L >_A |\varphi_R >_B + |\varphi_R >_A |\varphi_L >_B) (|e >_A |g >_B + |g >_A |l >_B)$).

For $\psi_f$ is no longer possible to express the state as a product of the CM and internal parts (as it was the case for $\psi_0$).

Initially we only had correlations in the CM variables. After the interaction we have entanglement between the two variables, a form clearly different from the initial one. We conclude that the process of absorption has modified the entanglement of the system, generating hyperentanglement.

We can associate the generation of hyperentanglement with the recoil of the atom. If we neglect the effect of the recoil we can make the approximation $\delta \approx \varphi$ and $\varphi \approx \varphi$. Then the final state can be approximated as (using equations (2) and (3) without recoil)

$$|\psi_f^{approx}\rangle = \frac{1}{\sqrt{2}} (|\varphi_L >_A |\varphi_R >_B + |\varphi_R >_A |\varphi_L >_B)$$

$$\times (|e >_A + \beta |g >_A)(|e >_B + \delta |g >_B).$$

where, as in the initial state, there is only entanglement between the two particles in the spatial variables.

If the initial state in not entangled the situation will change drastically. Imagine that initially we have the product state $|\varphi_L >_A |g >_A |\varphi_R >_B |g >_B$. It changes to $(\alpha |e >_A + \beta |g >_A)(|e >_B + \delta |g >_B)$ after the interaction with the light. This is also in a product form and, consequently, the absorption does not generate entanglement between $A$ and $B$. Thus, the initial presence of some degree of entanglement in the atomic system (in addition to the process of recoil) is a necessary condition for the generation of hyperentanglement in our arrangement.

In the above paragraphs we have only given a qualitative description of the problem. In the next section we address the subject from a more quantitative point of view. We evaluate the initial and final degrees of freedom showing that there is not generation of entanglement but only a redistribution between different degrees of freedom.
4. Evaluation of the entanglement degree

We evaluate the entanglement degree after the absorption of the photons. We use the von Neumann entropy as measure of the entanglement degree. The evaluation is a little bit involved because after the recoil the states $\hat{\phi}$ and $\hat{\varphi}$ overlap with $\phi$ and $\varphi$. In order to correctly deal with that overlap we introduce states orthogonal to the last ones, $\phi^\perp$ and $\varphi^\perp$, such that the first ones can be expressed as

$$|\hat{\phi}_i> = a|\phi_i> + b|\phi^\perp_i>$$

and

$$|\hat{\varphi}_i> = c|\varphi_i> + d|\varphi^\perp_i>$$

with $i = L, R$. We assume by the matter of simplicity $a, b, c, d$ to be real, obeying the relations $a^2 + b^2 = 1$ and $c^2 + d^2 = 1$. In [13] a similar calculation, in a different context, has been carried out to determine the entanglement degree of overlapping states.

To calculate the von Neumann entropy we need to determine the eigenvalues of the reduced density matrix or, equivalently, the coefficients of the Schmidt form of the state $\psi_f$ [3]. In the last approach we express the state in the matrix form

$$\Lambda \equiv \begin{pmatrix} 0 & \hat{\Lambda} \\ \hat{\Lambda} & 0 \end{pmatrix}$$

with

$$\hat{\Lambda} \equiv \begin{pmatrix} \beta \delta & \alpha \delta a & \alpha \delta b \\ \beta \gamma a & \alpha \gamma a c & \alpha \gamma a b \\ \beta \gamma d & \alpha \gamma d a & \alpha \gamma d b \end{pmatrix}$$

where the matrix $\Lambda$ is written in the basis $|\phi_L, g>\lambda_I, |\phi_L, e>\lambda_{I}, |\phi_R, g>\lambda_I, |\phi_R, e>\lambda_{I}, |\phi^\perp_L, g>\lambda_I, |\phi^\perp_L, e>\lambda_{I}, |\phi^\perp_R, g>\lambda_I, |\phi^\perp_R, e>\lambda_{I}$ for the particle $A$ (rows) and $|\varphi_L, g>\lambda_{II}, |\varphi_L, e>\lambda_{II}, |\varphi_R, g>\lambda_{II}, |\varphi_R, e>\lambda_{II}, |\varphi^\perp_L, g>\lambda_{II}, |\varphi^\perp_L, e>\lambda_{II}, |\varphi^\perp_R, g>\lambda_{II}, |\varphi^\perp_R, e>\lambda_{II}$ for $B$ (columns). We have not included the normalization factor $1/\sqrt{2}$ in these expressions because it will disappear in the final normalization of the diagonalized state.

Now, the Schmidt form can be obtained from the diagonalization of $\Lambda$. The coefficients of the Schmidt form correspond to the eigenvalues of the matrix, given by the solutions of

$$\det(\Lambda - \lambda I) = 0,$$

where $I$ is the $6 \times 6$ identity matrix. The explicit form of this equation is

$$\lambda^6 - \lambda^4(\alpha \gamma (ac + bd) + \beta \delta)^2 = 0$$

with quadruple null solution $\lambda^4 = 0$ and

$$\lambda_{\pm} = \pm (\alpha \gamma (ac + bd) + \beta \delta).$$

Finally, the normalized diagonalized state reads

$$|\psi_f> = \frac{\lambda_+}{\sqrt{\lambda_+^2 + \lambda_-^2}}|\chi_+> + \frac{\lambda_-}{\sqrt{\lambda_+^2 + \lambda_-^2}}|\chi_-> = \frac{1}{\sqrt{2}}|\chi_+> + \frac{1}{\sqrt{2}}|\chi_->$$

with $\chi$ and $\bar{\chi}$ denoting the eigenkets associated with these eigenvalues.
Now, we are in position to evaluate the von Neumann entropy. We have $S = -(1/2)\log_2(1/2) - (1/2)\log_2(1/2) = 1$. The entropy of the final state is 1, as it can be also directly derived from equation (16), corresponding to the maximally entangled form of a qubit.

With respect to the initial atomic state, the form of the spatial variables is also that of a maximally entangled qubit. On the other hand, the state of the internal variables is a product one and does not contribute to the entanglement degree. Consequently, the initial entanglement is also $S = 1$. The degree of entanglement does not change because of the light absorption. The entanglement has been only redistributed between more degrees of freedom.

5. Discussion

We have analyzed in a simple example the mutual dependence between entanglement and absorption. This is a two-sided relation. The modifications associated with the presence of entanglement on the absorption/emission rates of multi-atom systems have been extensively described in the literature. We have emphasized the much more asymmetric behavior of the reverse effects. Light–matter interactions can change (in the case of absorption) or destroy (for spontaneous emission) non-classical correlations.

It is also instructive to present the process of entanglement redistribution in terms of the local operations and classical communication (LOCC) paradigm. This is a central element in the characterization of entanglement measures [14]. According to it, physical operations that only affect to one of the components of a multi-particle system, and information transmitted between different components via classical means cannot modify the entanglement degree of the full system. This is our case. As the photon absorptions occur at well-separated places they are local operations and the process is within the LOCC paradigm. The total degree of entanglement does not change. However, the paradigm says nothing about how this entanglement is shared among the different variables, allowing for the redistribution.

We have been only concerned with the entanglement behavior in the atomic system. The light beams have been considered as classical auxiliary tools and we have not take care about their properties (we have only demanded them not to be very intense and to contain the adequate frequencies). A potential extension of the work would be to study similar processes with quantum and entangled light. As it is well-known the extension can offer advantages over the classical framework. For instance, in [15] the authors described an increase of exciton oscillator strengths in absorption processes by semiconductor quantum wells. Similarly, the Fourier limitations on spectral resolution can be circumvented [16] and the antibunching effects improved [17].

Although the main aim of the paper is pedagogical we must briefly consider potential applications of the scheme. In general, the entanglement generated this way is short lived because it quickly disappears with the subsequent spontaneous emission. Nevertheless, there is a scenario where it is possible, in principle, to exploit it. This potentially interesting scenario considers transitions to metastable excited states. In this case, we have the possibility of manipulating the system during an interval of time long enough (similar to those of some processes in ion trapping) to try of exploiting the resource. For instance, we could study if the non-product form of hyperentanglement described here differs from the standard product one. It could be also on the basis of schemes to distribute pre-existing entanglement between various degrees of freedom initially not correlated.
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References

[1] Ficek Z and Tanaś R 2002 Entangled states and collective nonclassical effects in two-atom systems Phys. Rep. 372 369
[2] Dowling J P 2008 Quantum optical metrology— the lowdown on high-N00N states Contemp. Phys. 49 125
[3] Tichy M C, Mintert F and Butchleitner A 2011 Essential entanglement for atomic and molecular physics J. Phys. B 44 192001
[4] Fedorov M V, Efremov M A, Kazakov A E, Chan K W, Law C K and Eberly J H 2005 Spontaneous emission of a photon: wave-packet structures and atom-photon entanglement Phys. Rev. A 72 032110
[5] Sancho P 2016 Atomic absorption and emission in non-product states Eur. Phys. J. D 70 188
[6] Tanabe T, Odagiri T, Nakano M, Kumagai Y, Suzuki I H, Kitajima M and Kouchi N 2010 Effect of entanglement on the decay dynamics of a pair of $H(2p)$ atoms due to spontaneous emission Phys. Rev. A 82 040101(R)
[7] Urbain X, Dochain A, Lauzin C and Fabret B 2015 Absence of entanglement effect on the decay dynamics of $H(2p)$ pairs produced by VUV photodissociation of $H_2$ J. Phys.: Conf. Ser. 635 112085
[8] Sancho P 2017 Entanglement, identity, and disentanglement in two-atom spontaneous emission Phys. Rev. A 95 032116
[9] Yu T and Eberly J H 2004 Finite-time disentanglement via spontaneous emission Phys. Rev. Lett. 93 140404
[10] Yu T and Eberly J H 2009 Sudden death of entanglement Science 323 598
[11] Zoller P, Cirac J I, Duan L and García-Ripoll J J 2004 Implementing quantum information processing with atoms, ions and photons Proc. Les Houches Summer School 2003
[12] Walther H, Varcoe B, Englert B-G and Becker T 2006 Cavity quantum electrodynamics Rep. Prog. Phys. 69 1325
[13] Lo Franco R and Compagno G 2016 Quantum entanglement of identical particles by standard information-theoretic notions Sci. Rep. 6 20603
[14] Plenio M B andVirmani S 2007 An introduction to entanglement measures Quant. Inf. Comput. 7 1
[15] Salazar L J, Guzman D A, Rodríguez F J and Quiroga L 2012 Quantum-correlated two-photon transitions to excitons in semiconductor quantum wells Opt. Express 20 4770
[16] Dorfman K E, Schlawin F and Mukamel S 2016 Nonlinear optical signals and spectroscopy with quantum light Rev. Mod. Phys. 88 045008
[17] López Carreño J C, Sánchez Muñoz C, del Valle E and Laussy F P 2016 Excitation with quantum light II. Exciting a two-level system Phys. Rev. A 94 063826