The general Two-Higgs doublet eXtensions of the SM: a saucerful of secrets

J.L. Díaz-Cruz, A. Díaz-Furlong, and J.H. Montes de Oca Y.

Cuerpo Académico de Partículas, Campos y Relatividad Facultad de Ciencias Físico-Matemáticas,
BUAP. Apdo. Postal 1364, C.P. 72000 Puebla, Pue., México

We discuss the most general formulation of the Two-Higgs doublet model, which incorporates flavor changing neutral scalar interactions (FCNSI) and CP violation (CPV) from several sources. CP violation can arise either from Yukawa terms or from the Higgs potential, be it explicit or spontaneous. We show how the model, which is denoted as 2HDM-X, reduces to some versions known in the literature (2HDM-I,II,III), as well as some of their variants (top, lepton, dark) denoted here as 2HDM-IV. We also discuss another limit that includes CPV and Yukawa four textures to control FCNSI, which we denote as 2HDM-V. We evaluate the CPV asymmetry for the decay $h \to bcW$, which may allow to test the patterns of FCNSI and CPV, that arises in these models.

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I. INTRODUCTION

Despite the success of the Standard Model (SM) in the gauge and fermion sectors, the Higgs sector remains the least tested aspect of the model [1], which leaves the puzzles associated with the mechanism of electroweak symmetry breaking (EWSB) still unsolved. On one hand, the analysis of radiative corrections within the SM [2–5], points towards the existence of a Higgs boson, with a mass of the order of the EW scale, which in turn could be detected at the LHC [6, 7]. On the other hand, the SM is often considered as an effective theory, valid up to an energy scale of $O(TeV)$, that eventually will be replaced by a more fundamental theory [8], which will explain, among other things, the physics behind EWSB and perhaps even the origin of flavor. Many examples of candidate theories, which range from supersymmetry [9–11] to strongly interacting models [12, 13] as well as some extra dimensional scenarios [14–16], include a multi-scalar Higgs sector. In particular, models with two scalar doublets have been studied extensively [17–19], as they include a rich structure with interesting phenomenology [20–22].

Several versions of the 2HDM have been studied in the literature [23]. Some models (known as 2HDM-I and 2HDM-II) involve natural flavor conservation [24], while other models (known as 2HDM-III) [23], allow for the presence of flavor changing scalar interactions (FCNSI) at a level consistent with low-energy constraints [25]. There are also some variants (known as top, lepton, neutrino), where one Higgs doublet couples predominantly to one type of fermion [27], while in other models it is even possible to identify a candidate for dark matter [26]. The definition of all these models, depends on the Yukawa structure and symmetries of the Higgs sector [28–32], whose origin is still not known. The possible appearance of new sources of CP violation is another characteristic of these models [33].

In this paper we aim to discuss the most general version of the Two-Higgs doublet model (2HDM), which incorporates flavor or CP violation from all possible sources [34–36]. We also discuss how the general model, denoted here as 2HDM-X, reduces in certain limits to the versions known as 2HDM-I,II,III, as well as some of their variants which we shall name as 2HDM-IV and 2HDM-V. The logic of the naming scheme that we adopt here, consists in identifying distinctive physical characteristics that can be associated with the models and have sufficient merit to single out them.

Within model I (2HDM-I) where only one Higgs doublet generates all gauge and fermion masses [1], while the second doublet only knows about this through mixing, and thus the Higgs phenomenology will share some similarities with the SM, although the SM Higgs couplings will now be shared among the neutral scalar spectrum. The presence of a charged Higgs boson is clearly the signal beyond the SM. Within 2DHM-II one also has natural flavor conservation [24], and its phenomenology will be similar to the 2HDM-I, although in this case the SM couplings are shared not only because of mixing, but also because of the Yukawa structure. On the other hand, the distinctive characteristic of 2HDM-III is the presence of FCNSI, which require a certain mechanism in order to suppress them, for instance one can imposes a certain texture for the Yukawa couplings [37], which will then predict a pattern of FCNSI Higgs couplings [38]. Within all those models (2HDM I,II,III) [39–41], the Higgs doublets couple, in principle, with all fermion families, with a strength proportional to the fermion masses, modulo other parameters.

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*Electronic address: ldiaz@sirio.ifuap.buap.mx
†Electronic address: adiazfurlong@yahoo.com
‡Electronic address: halim@esfm.ipn.mx
II. A GENERAL FORMULATION OF THE 2HDM AND ITS LIMITING CASES.

The Two-Higgs doublet extension of the SM includes two scalar doublets of equal hypercharge, denoted by: \( \Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)^T \). Depending on the Yukawa matrices \( Y_{1,2}^q (q = u, d) \) that are allowed, one defines the particular versions of the 2HDM. FCNSI appear at tree level when more than one Higgs doublet couples to both types of quarks (\( u \) and \( d \)) \cite{10}, and a certain mechanism should be invoked in order to bring them under control. The CP properties of the Higgs boson depend on the symmetries of the potential \cite{44}. In order to clarify the discussion of the many models that have been presented in the literature, we shall present in the next subsections, a classification scheme for these models, which have different patterns of FCNSI and CP properties. We shall discuss in this paper first the most general formulation of the 2HDM-X, and will consider the 2HDM versions usually discussed in the literature, which are known as 2HDM-I,II,III, as well as some variants (2HDM-IV). We shall discuss then in detail, some cases that have not been considered before, which we denote as 2HDM-V. Although the 2HDM-X suffers from the FCNSI problem, we shall discuss it first in general terms, without referring the specific mechanism that is used to address the problem, which will be done later in this section.

A. A classification of models

We shall define here the different types of models according to their Yukawa structure, the Hermiticity of the Yukawa matrices and the CP properties of the bosonic Higgs sector. Thus, the most general version of the 2HDM is defined through the following assumptions:

i) In principle we allow each Higgs doublet to couple to both type of fermions, expecting that some particular structure of the Yukawa matrices is responsible for the suppression of flavor changing neutral scalar interactions (FCNSI).

ii) The Yukawa matrices are allowed in general to be non-Hermitian, i.e., \( Y_{fi} \neq Y_{fi}^\dagger \) (\( f = u, d, l, i = 1, 2 \)). The limit when the Yukawa matrices are hermitic defines a particular version of the models.

iii) The Higgs potential admits in principle both spontaneous or explicit CPV.

Then the known limiting models (2HDM I,II,III), are obtained by relaxing some of those assumptions, namely:

1. The 2HDM-I \cite{17,46,47} is defined by considering that only one Higgs doublet generates the masses of all types of fermions, as it happens in the SM. This type of model can be obtained by assuming an additional \( Z_2 \) discrete symmetry. Under a variant of this model, where the second doublet does not mix with the first doublet, it is possible to identify a neutral scalar as a dark matter candidate \cite{23}, which makes it very attractive.

2. For the so called 2HDM-II \cite{17,47,48}, each Higgs doublet couples only to one type of quark, and then FCNSI do not appear at tree level. A variant of the \( Z_2 \) discrete symmetry is considered here, similarly to the case of the 2HDM-I. Two limiting cases can be considered, namely: the 2HDM-IIa, with CP conserving Higgs sector, and THDM-IIb where the Higgs sector is CP violating \cite{49}. This model is also attractive because it corresponds to the Higgs sector of the Minimal supersymmetric standard model (MSSM), at tree-level \cite{6}.
3. Within the 2HDM-III\textsuperscript{[38]}, one considers all possible couplings among the Higgs doublets and fermions in the Yukawa sector; thus, it is possible to have FCNSI in this case. According to the extended classification that we try to motivate here, we shall also assume that within 2HDM-III the Yukawa matrices are Hermitic, whereas the Higgs potential is CP conserving. Thus, CP violation only arise from the CKM phase. A particular version of this model, widely studied in the literature, assumes Hermitic Yukawa matrices with 4-textures\textsuperscript{[37]}, which has under control the FCNSI problem\textsuperscript{[38]}. It also happens that when one considers loop effects within the MSSM, its Higgs sector also becomes of type III\textsuperscript{[50]}, which again makes attractive this version of the 2HDM. Although it is not often explicitly stated, we shall consider that within 2HDM-III the Higgs doublets couple in principle with all three families of quarks and leptons.

4. Family non-universal assignments are also possible, for instance we can have models where one doublet couples to all types of quarks, but a second doublet only couples to the 3rd family\textsuperscript{[27]}. Several possibilities have been considered in the literature, which we denote as 2HDM-IV, depending on whether the second doublet couples to the whole third family or only to the top (2HDM-IV-t)\textsuperscript{[42,43]}. We shall also include within this category, those models where one doublet couples only to charged leptons or to neutrinos\textsuperscript{[42,43]}.

The properties of these models are summarized in tables\textsuperscript{I, II}. Table\textsuperscript{II} shows the assumptions for the different types of the 2HDM which are considered in this work. Besides the cases, that have been discussed in the literature, we can define still another class of models, which we denote as 2HDM-V, where besides having FCNSI at tree level, also include extra sources of CP violation, either from the Yukawa or the Higgs sectors. Within this class, we shall consider the following sub-cases:

(a) The 2HDM-Va has Hermitian Yukawa matrices, but the Higgs sector is CP violating. To work out a concrete example we shall also consider a four-texture for the Yukawa matrices.

(b) For the 2HDM-Vb we will not assume Hermiticity in the Yukawa matrices, while the Higgs sector is CP conserving. Again, to work out an specific example we shall consider the four-texture case for the Yukawa matrices.

### B. Solutions to the FCNC problem

When both Higgs doublets couple to up- and down-type fermions, FCNSI are allowed\textsuperscript{[10]}. An acceptable suppression for FCNSI can be achieved with the following mechanisms:

| Model type | Up quarks | Down quarks | Charged leptons | Neutral leptons |
|------------|-----------|-------------|----------------|----------------|
| 2HDM-I     | $H_1$     | $H_1$       | $H_1$          | $H_1$          |
| 2HDM-II    | $H_2$     | $H_1$       | $H_1$          | $H_2$          |
| 2HDM-III   | $H_{1,2}$ | $H_{1,2}$   | $H_{1,2}$      | $H_{1,2}$      |
| 2HDM-IV    | $H_1$     | $H_1$       | $H_1$          | $H_2$          |

**TABLE I:** Higgs interaction with fermions for 2HDM types.

| Model type | FCNC | Hermiticity | Higgs sector | CP |
|------------|------|-------------|--------------|----|
| I          | $\times$ | -           | -            | -  |
| II         | $\times$ | -           | -            | -  |
| III        | $\checkmark$ | $\checkmark$ | $\checkmark$ (CKM) | -  |
| Va         | $\checkmark$ | $\checkmark$ | x            | -  |
| Vb         | $\checkmark$ | x           | $\checkmark$ | -  |

**TABLE II:** Symmetries under the different types of 2HDM's
After getting a correct SSB \([59–62]\), the Higgs doublets are decomposed as follows:

\[
\tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3
\]

The most general structure of the Yukawa lagrangian for the quark fields, can be written as follows:

\[
\mathcal{L}_Y^{\text{quarks}} = \overline{q}_L Y_1^D \phi_1 d_R^0 + \overline{q}_L Y_2^D \phi_2 d_R^0 + \overline{q}_L Y_1^U \phi_1 u_R^0 + \overline{q}_L Y_2^U \phi_2 u_R^0 + h.c.,
\]

where \(Y_{1,2}^{u,d}\) are the 3 \(
\times 3\) Yukawa matrices, \(q_L\) denotes the left handed quark doublets and \(u_R, d_R\) correspond to the right handed singlets. Here \(\tilde{\phi}_{1,2} = i\sigma_2 \phi_{1,2}\). The superscript zero means that the quarks are weak eigenstates. After getting a correct SSB \([51, 62]\), the Higgs doublets are decomposed as follows:

\[
\phi_1 = \begin{pmatrix}
\frac{\varphi_1^+}{\sqrt{2}}, \\
v_1 + \varphi_1^+ + i\chi_1
\end{pmatrix}
\]
\[ \phi_2 = \left( \frac{\varphi^+_2}{\sqrt{2}} \right) . \]

where the v.e.v.'s \( v_1 \) and \( v_2 \) are real and positive, while the phase \( \xi \) introduces spontaneous CP violation. Now, we transform the quarks to the mass eigenstate basis through the rotations: \( u_{L,R} = U_{L,R} u^0_{L,R} \), \( d_{L,R} = D_{L,R} d^0_{L,R} \), to obtain:

\[
\mathcal{L}^{\text{quarks}}_Y = \bar{u}_L Y^D_1 \varphi^+_1 D^\dagger_R d_R + \bar{d}_L D_L Y^D_1 \varphi^+_1 + i \chi_1 \varphi^+_2 D^\dagger_R d_R
\]

\[+ \bar{u}_L Y^D_2 \varphi^+_2 D^\dagger_R d_R + \bar{d}_L D_L Y^D_2 \varphi^+_2 + i \chi_2 \varphi^+_1 D^\dagger_R d_R
\]

\[+ \bar{u}_L Y^D_1 \varphi^+_1 - i \chi_1 U^\dagger_R u_R - \bar{d}_L D^\dagger_L Y^D_1 \varphi^+_1 - \bar{u}_R^\dagger D^\dagger_L Y^D_2 \varphi^+_2 U^\dagger_R u_R
\]

\[+ \bar{u}_L M^U u_R + \bar{d}_L M^D d_R + h.c., \]

Then, the (diagonal) mass matrices are given as follows:

\[
M^U = \frac{v_1}{\sqrt{2}} \tilde{Y}^U_1 + e^{-i \xi} \frac{v_2}{\sqrt{2}} \tilde{Y}^U_2
\]

and

\[
M^D = \frac{v_1}{\sqrt{2}} \tilde{Y}^D_1 + e^{i \xi} \frac{v_2}{\sqrt{2}} \tilde{Y}^D_2,
\]

where \( \tilde{Y}^U_{1,2} = U_L Y^U_{1,2} U^\dagger_R \) and \( \tilde{Y}^D_{1,2} = D_L Y^D_{1,2} D^\dagger_R \). Then, one can split the Yukawa couplings into the neutral and charged terms, both for the up and down sector. The neutral couplings for the up sector are given in terms of (four-components) Dirac spinors as follows:

\[
\mathcal{L}^{\text{neutral}}_{\text{up}} = \bar{u}_L Y^U_1 \varphi^+_1 - i \chi_1 P_R u + \bar{u}_L Y^U_2 \varphi^+_2 - i \chi_2 P_R u
\]

\[+ \bar{u}_L Y^U_1 \varphi^+_1 + i \chi_1 P_L u + \bar{u}_L Y^U_2 \varphi^+_2 + i \chi_2 P_L u
\]

\[+ \bar{u}_L M^U u, \]

where \( P_{L,R} = \frac{1 \mp \gamma_5}{2} \) are the chiral operators. In order to arrive to the final form of the Yukawa lagrangian we need to include the Higgs mass eigenstates. When one allows for the possibility of having CP violation in the Higgs potential, the CP even and CP-odd components get mixed [63]. This CPV Higgs mixing is included as follows,

\[
\begin{pmatrix}
\varphi_1 \\
\varphi_2 \\
\chi_1 \\
\chi_2
\end{pmatrix} = R
\begin{pmatrix}
H_1 \\
H_2 \\
H_3 \\
H_4
\end{pmatrix}
\]

with \( H_4 \) is a Goldstone boson. The matrix \( R \) can be obtained when by relating equations [3] and [4] with the physical Higgs mass eigenstate

\[
\Phi_a = \left( \frac{1}{\sqrt{2}} v_a + \frac{1}{\sqrt{2}} \sum_{r=1}^{4} (q_{r1} v_a + q_{r2} e^{-i \theta_{r3}} w_a) H_r \right).
\]
TABLE IV: Mixing angles for Higgs bosons which consider spontaneous and explicit CPV [63].

| \( q_r \) | \( q_{r1} \) | \( q_{r2} \) |
|---|---|---|
| 1 | \( \cos \theta_{12} \cos \theta_{13} \) | \( -\sin \theta_{12} - i \cos \theta_{12} \sin \theta_{13} \) |
| 2 | \( \sin \theta_{12} \cos \theta_{13} \) | \( \cos \theta_{12} - i \sin \theta_{12} \sin \theta_{13} \) |
| 3 | \( \sin \theta_{13} \) | \( i \cos \theta_{13} \) |
| 4 | \( i \) | 0 |

where \( a = 1, 2, r = 1, ..., 4 \) and \( \hat{v}_a, \hat{w}_a \) are the components of the orthogonal eigenvectors of unit norm

\[
\hat{v} = \left( \hat{v}_1, \hat{v}_2 \right) = \left( \cos \beta, e^{i\xi} \sin \beta \right)
\]

and

\[
\hat{w} = \left( \hat{w}_1, \hat{w}_2 \right) = \left( -e^{-i\xi} \sin \beta, \cos \beta \right).
\]

The values of \( q_{ra} \) are written as combination of the \( \theta_{ij} \), which are the mixing angles appearing in the rotation matrix that diagonalize the mass matrix for neutral Higgs; table IV shows the different values for the \( q_r \)’s.

It is convenient to write the following relation, for \( a = 1, 2 \)

\[
\varphi_1 + i\chi_1 = \sum_r \left( q_{r1} \cos \beta - q_{r2} e^{-i(\theta_{23} + \xi)} \sin \beta \right) H_r
\]

and

\[
\varphi_2 + i\chi_2 = \sum_r \left( q_{r1} e^{i\xi} \sin \beta + q_{r2} e^{-i\theta_{23}} \cos \beta \right) H_r.
\]

Then, we arrive to the final general form of the neutral Higgs boson couplings for the up-type quarks:

\[
\mathcal{L}_{\text{neutral}}^{\text{up}} = \overline{u}_i \left( S_{ijr}^{u} + \gamma^5 P_{ijr}^{u} \right) u_j H_r + \overline{u}_i M_{ij}^{U} u_j,
\]

with

\[
S_{ijr}^{u} = \frac{1}{2} \sqrt{2} \sin \theta_{23} \left( q_{k1}^* + q_{k1} - \tan \beta \left( q_{k2}^* e^{i(\theta_{23} + \xi)} + q_{k2} e^{-i(\theta_{23} + \xi)} \right) \right)
\]

\[
+ \frac{1}{\sqrt{2} \cos \beta} \left( q_{k2} e^{i\theta_{23}} \overline{Y}_2 U^{*}_{ij} + q_{k2} e^{-i\theta_{23}} Y_2^{*} U^{*}_{ij} \right)
\]

and

\[
P_{ijr}^{u} = \frac{1}{2} \sqrt{2} \sin \theta_{23} \left( q_{k1} - q_{k1} - \tan \beta \left( q_{k2}^* e^{i(\theta_{23} + \xi)} - q_{k2} e^{-i(\theta_{23} + \xi)} \right) \right)
\]

\[
+ \frac{1}{\sqrt{2} \cos \beta} \left( q_{k2} e^{i\theta_{23}} \overline{Y}_2 U^{*}_{ij} - q_{k2} e^{-i\theta_{23}} Y_2^{*} U^{*}_{ij} \right)
\]

Similarly, for the down-type quarks we find:

\[
\mathcal{L}_{\text{neutral}}^{\text{down}} = \overline{d}_i \left( S_{ijr}^{d} + \gamma^5 P_{ijr}^{d} \right) d_j H_r + \overline{d}_i M_{ij}^{D} d_j,
\]

1 Here we shall follow closely the notation of Haber and O’Neil [63].
with

\[ S_{ijr}^d = \frac{1}{2v} M_{ij}^D \left[ q_{k1} + q_{k1} - \tan \beta \left( q_{k2} e^{i(\theta_{23} + \xi)} + q_{k2} e^{-i(\theta_{23} + \xi)} \right) \right] \]

\[ + \frac{1}{2\sqrt{2} \cos \beta} \left( q_{k2} e^{-i\theta_{23} Y^D_2} + q_{k2} e^{i\theta_{23} Y^D_2} \right) \]  \hspace{1cm} (19)

and

\[ P_{ijr}^d = \frac{1}{2v} M_{ij}^D \left[ q_{k1} - q_{k1} + \tan \beta \left( q_{k2} e^{i(\theta_{23} + \xi)} - q_{k2} e^{-i(\theta_{23} + \xi)} \right) \right] \]

\[ + \frac{1}{2\sqrt{2} \cos \beta} \left( q_{k2} e^{-i\theta_{23} Y^D_2} - q_{k2} e^{i\theta_{23} Y^D_2} \right) \]  \hspace{1cm} (20)

On the other hand, the Yukawa couplings for charged states are given by:

\[ \mathcal{L}_{Y}^U = \pi \left[ \varphi^+_V \left( \sqrt{2} M^D_1 - e^{i\xi} \tan \beta Y^D_2 \right) \frac{I + \gamma^5}{2} + \varphi^+_V \tilde{Y}^D_2 \frac{I + \gamma^5}{2} \right. \]

\[ - \frac{\varphi^+_V}{2} \left( \sqrt{2} M^U_1 - e^{i\xi} \tan \beta \tilde{Y}^U_2 \right) V - \frac{\varphi^+_V}{2} \tilde{Y}^U_2 \]  \hspace{1cm} \[ \left. + h.c. \right) \]  \hspace{1cm} (21)

where \( V \) denotes the CKM matrix. The physical eigenstates for the charged Higgs boson \( (H^+) \) can be obtain through the following rotation:

\[ \begin{pmatrix} \varphi^+_V \\ \varphi^+_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -e^{i\xi} \sin \beta \\ e^{i\xi} \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \]  \hspace{1cm} (22)

Therefore, the Yukawa couplings for charged Higgs are

\[ \mathcal{L}_{Y}^U = \pi \left[ H^+ e^{-i\xi} M^U_1 V \frac{I - \gamma^5}{\sqrt{2}} - H^+ e^{-i\xi} V M^D \frac{I + \gamma^5}{\sqrt{2}} \right. \]

\[ + \frac{1}{\cos \beta} H^+ \left( V \frac{I + \gamma^5}{2} - \tilde{Y}^U_2 \right) \]  \hspace{1cm} \[ \left. + h.c. \right) \]  \hspace{1cm} (23)

**IV. SOME LIMITING CASES**

**A. The THDM-V with explicit CP violation (2HDM-Va)**

In this case we assume the hermiticity condition for the Yukawa matrices, but the Higgs sector could be CP violating. For simplicity we shall consider that the Yukawa matrices obey a four-texture form, and CP is violated explicitly in the Higgs sector.

As it is discussed in the appendix A, the assumption of universal 4-textures for the Yukawa matrices, allows to express one Yukawa matrix in terms of the quark masses, and parametrize the FCNSI in terms of the unknown coefficients \( \chi_{ij} \), namely \( \tilde{Y}_{2ij}^U = \sqrt{m_i m_j} \chi_{ij} \), where the hermiticity condition reads \( \chi_{ij} = \chi_{ji}^\dagger \). These parameters can be constrained by considering all types of low energy FCNC transitions. Although these constraints are quite strong for transitions involving the first and second families, as well as for the b-quark, it turns out that they are rather mild for the top quark.

Then, from (10) and (17), one obtains within 2HDM-Va, the following expressions for the couplings of the neutral Higgs bosons with up-type quarks, namely:

\[ S_{ijr}^u = \frac{1}{2v} M_{ij}^U \left[ q_{r1}^* + q_{r1} - \tan \beta (q_{r2}^* + q_{r2}) \right] + \frac{\sqrt{m_i m_j}}{2\sqrt{2} v \cos \beta} \chi_{ij} \left( q_{r2}^* + q_{r2} \right) \]  \hspace{1cm} (24)
eigenstates can be written now in terms of the angles $\alpha$ and $\beta$, the neutral Higgs mixing, corresponds to the standard notation. The expressions (10) for the neutral Higgs masses assumes a 4-texture for the Yukawa matrices, the Higgs-fermion couplings further simplify as in the 2HDM-III, one has that the Yukawa matrices obey a 4-texture form, and also

\[ H_{ij} = \begin{pmatrix} \cos \theta & -\sin \theta \cos \phi \\ \sin \theta & \cos \theta \cos \phi \end{pmatrix}, \]

\[ H_{ij} = \begin{pmatrix} \cos \theta & -\sin \theta \cos \phi \\ \sin \theta & \cos \theta \cos \phi \end{pmatrix}, \]

then, without loss of generality, we can assume that

\[ \hat{H}_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \]

for the CP-conserving limit $\hat{w}_a$ and $\hat{v}_a$ have a vanishing phase $\xi = 0$. Additionally, when one assumes a 4-texture for the Yukawa matrices, the Higgs-fermion couplings further simplify as $\hat{Y}^U_{2ij} = \chi_{ij} \frac{\sqrt{m_i m_j}}{v}$. Then, the corresponding coefficient equation for up sector and $h^0 (r = 1)$ are

\[ S^u_{ij1} = \frac{1}{v} M^U_{ij} \left[ \sin (\beta - \alpha) - \tan \beta \cos (\beta - \alpha) \right] + \frac{\sqrt{m_i m_j}}{2\sqrt{2v}} \frac{\cos (\beta - \alpha)}{\cos \beta} \left( \chi_{ij} + \chi_{ij}^\dagger \right), \]

\[ P^u_{ij1} = \frac{\sqrt{m_i m_j}}{2\sqrt{2v}} \frac{\cos (\beta - \alpha)}{\cos \beta} \left( \chi_{ij} - \chi_{ij}^\dagger \right), \]

For $H^0 (r = 2)$ one finds:

\[ S^u_{ij2} = \frac{1}{v} M^U_{ij} \left[ \cos (\beta - \alpha) - \tan \beta \sin (\beta - \alpha) \right] + \frac{\sqrt{m_i m_j}}{2\sqrt{2v}} \frac{\sin (\beta - \alpha)}{\cos \beta} \left( \chi_{ij} + \chi_{ij}^\dagger \right), \]

\[ P^u_{ij2} = \frac{\sqrt{m_i m_j}}{2\sqrt{2v}} \frac{\sin (\beta - \alpha)}{\cos \beta} \left( \chi_{ij} - \chi_{ij}^\dagger \right), \]

Finally, for $A^0 (r = 3)$ one obtains:

\[ S^u_{ij3} = i \frac{\sqrt{m_i m_j}}{2\sqrt{2v}} \frac{\sin (\beta - \alpha)}{\cos \beta} \left( \chi_{ij} - \chi_{ij}^\dagger \right), \]

\[ P^u_{ij3} = \frac{i}{2v} M^U_{ij} \tan \beta - i \frac{\sqrt{m_i m_j}}{2\sqrt{2v}} \frac{\sin (\beta - \alpha)}{\cos \beta} \left( \chi_{ij} + \chi_{ij}^\dagger \right). \]

C. The 2HDM of type I, II and III

It is interesting, and illustrative, to consider the limit when the general model becomes the 2HDM-III, within 2HDM-III, one has that the Yukawa matrices obey a 4-texture form, and also $Y_f = Y_f^\dagger$, namely:

\[ \hat{Y}^U_{2ij} = \chi_{ij} \frac{\sqrt{m_i m_j}}{v}. \]
The condition of Hermiticity means then $\chi_{ij} = \chi_{ij}^{\dagger}$. Within 2HDM III, we shall consider that the Higgs sector is CP conserving. Therefore the Yukawa couplings take the following form. For $h^0$ one gets,

$$S_{ij1}^u = \frac{1}{v} M_{ij}^U \left( \sin(\beta - \alpha) + \tan \beta \cos(\beta - \alpha) \right) - \chi_{ij} \sqrt{m_i m_j} \frac{\cos(\beta - \alpha)}{\cos \beta},$$

while

$$P_{ij1}^u = 0.$$

Then for $H^0$,

$$S_{ij2}^u = -\frac{1}{v} M_{ij}^U \left( \frac{\sin \alpha}{\cos \beta} + \chi_{ij} \sqrt{m_i m_j} \frac{\sin(\beta - \alpha)}{\sqrt{2v}} \right),$$

and

$$P_{ij2}^u = 0.$$

Then for $A^0$,

$$S_{ij3}^u = 0,$$

and

$$P_{ij3}^u = -i \frac{\chi_{ij} \sqrt{m_i m_j}}{\sqrt{2v \cos \beta}}.$$

One can also reduce the general model to the 2HDM types I and II, by eliminating some of the Yukawa matrices $Y_{ij}^{U,D} = 0$ and $Y_{ij}^U = Y_{ij}^D = 0$, accordingly. The tables V and VI summarize the corresponding results, they include the expressions for the neutral Higgs couplings with up and down type quarks, and similar results hold for the leptons.
TABLE VII: Explicit values of the Yukawa couplings for neutral Higgs in 2HDM-II.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$i$ & $S_{ij}^u$ & $P_{ij}^u$ & $S_{ij}^d$ & $P_{ij}^d$ \\
\hline
1 & $-\cos \alpha M_{ij}^D$ & $0$ & $\sin \alpha M_{ij}^D$ & $0$ \\
2 & $-\sin \alpha M_{ij}^D$ & $0$ & $\cos \alpha M_{ij}^D$ & $0$ \\
3 & $0$ & $\pm \cos \beta M_{ij}^D$ & $0$ & $\pm \sin \beta M_{ij}^D$ \\
\hline
\end{tabular}
\end{table}

FIG. 1: Tree level Feynman diagrams for the decay. Right diagram is for $h \rightarrow W^- \bar{b}c$ while left diagram is for $h \rightarrow W^- \bar{t}c$.

V. PROBING THE CP VIOLATING HIGGS COUPLINGS THROUGH THE DECAY $h \rightarrow c\bar{b}W$

In this section we shall evaluate the asymmetry coefficient for the decay $h \rightarrow c\bar{b}W$ in order to analyze presence of both FCNSI and CPV within the 2HDM-X. In the SM the FCNC are suppressed, but in the 2HDM extensions these processes are found even at tree level. We consider the neutral Higgs boson decay both FCNSI and CPV within the 2HDM-X. In the SM the FCNC are suppressed, but in the 2HDM extensions these processes are found even at tree level. We consider the neutral Higgs boson decay $h \rightarrow W^- \bar{b}c$ at tree level. Two diagrams contribute to this decay, the first one is through the FCNC coupling diagram is shown on left figure 1. The other one is through $h \rightarrow W^+W^- \rightarrow W^- \bar{b}c$, also shown in figure 1.

The couplings of the neutral Higgs with the quarks and the charged boson W with the neutral Higgs are written as $i (S_{231}^u + g^5 P_{231}^u)$ and $igM_W q_1 g_{\mu\nu}$, respectively. The other vertices are the usual SM contribution. The average amplitude for these diagrams is thus

$$\langle |\mathcal{M}|^2 \rangle = \langle |\mathcal{M}_1|^2 \rangle + \langle |\mathcal{M}_2|^2 \rangle + \mathcal{M}_1 \mathcal{M}_2 + \mathcal{M}_2 \mathcal{M}_1$$

(39)

We can obtain an approximation when the terms proportional to the charm and bottom masses are neglected. Then, the expressions for the squared amplitudes are

$$\langle |\mathcal{M}_1|^2 \rangle = \frac{g^2}{4M_W^2} |P_i(q)|^2 [4 |S_{231}^u|^2 P_{231}^u|^2 p_1 \cdot p_2 p_1 \cdot q p_3 \cdot q + 2 |S_{231}^u|^2 P_{231}^u|^2 M_{Wp_2}^2 p_2 \cdot q p_3 \cdot q$$

$$+ \left( |S_{231}^u| + P_{231}^u |2 m_i^2 - |S_{231}^u|^2 - P_{231}^u|^2 q^2 \right) (2 p_1 \cdot p_2 p_1 \cdot p_3 + M_{Wp_2}^2 p_3) \right],$$

(40)

$$\langle |\mathcal{M}_2|^2 \rangle = g^4 (q_{11})^2 |V_{cb}|^2 |P_W(k)|^2 \left( M_{Wp_2}^2 p_3 + 2 p_1 \cdot p_2 p_1 \cdot p_3 \right),$$

(41)

$$\mathcal{M}_1 \mathcal{M}_2 = g^4 m_{W} (S_{231}^u + P_{231}^u) q_{11} V_{cb} P_i^\nu(q) P_W(k) \left( M_{Wp_2}^2 p_3 + 2 p_1 \cdot p_2 p_1 \cdot p_3 \right) + \mathcal{M}_2 \mathcal{M}_1$$

(42)

and

$$\mathcal{M}_2 \mathcal{M}_1 = g^4 m_{W} (S_{231}^u + P_{231}^u) q_{11} V_{cb} P_W(k) P_i(q) \left( M_{Wp_2}^2 p_3 + 2 p_1 \cdot p_2 p_1 \cdot p_3 \right).$$

(43)
where the W boson propagator is written in the Feynman-t’Hooft gauge \( P_W(k) = (k^2 - M_W^2 + iM_W\Gamma_W)^{-1} \) and \( \rho_t(q) = \frac{(q^2 - m_t^2 + im_t\Gamma_t)}{-1} \). In order to find the asymmetry coefficient we also need to calculate the conjugate decay, that is, \( h \to W^+ \bar{c} \). We denote the average amplitude as

\[
\overline{|M|^2} = |M_1|^2 + |M_2|^2 + M_1^\dagger M_2 + M_2^\dagger M_1.
\]

The square terms are the same as the above, \( \overline{|M_{1,2}|^2} = |M_{1,2}|^2 \), while for the interference terms we have

\[
\overline{M_1^\dagger M_2} = \frac{g^4 m_t}{M_W} (S_{231}^u + P_{231}^u) q_{11} V_{cb} P_t(q) P_W^* (k) \left( M_W^2 p_2 \cdot p_3 + 2 p_1 \cdot p_2 p_1 \cdot p_3 \right)
\]

and

\[
\overline{M_2^\dagger M_1} = \frac{g^4 m_t}{M_W} (S_{231}^u + P_{231}^u) q_{11} V_{cb} P_t^* (q) P_W (k) \left( M_W^2 p_2 \cdot p_3 + 2 p_1 \cdot p_2 p_1 \cdot p_3 \right).
\]

Then, the width for the decay is

\[
\Gamma_h \to W^+ \bar{c} = \frac{m_h}{256\pi} \int \int_{R_{xy}} \left( |M_1|^2 + |M_2|^2 + M_1^\dagger M_2 + M_2^\dagger M_1 \right) dx dy,
\]

where the dimensionless variables are defined as \( x = \frac{2E_1}{m_h} \) and \( y = \frac{2E_2}{m_h} \). All details about the decay kinematics were included in the appendix B. The definition for the asymmetry coefficient is

\[
A_{CPV} = \frac{\Gamma_h \to W^+ \bar{c} - \Gamma_h \to W^- \bar{c}}{\Gamma_h \to W^+ \bar{c} + \Gamma_h \to W^- \bar{c}}.
\]

The final result, for the decay asymmetry is given by:

\[
A_{CPV} (S_{ijk}^u, P_{ijk}^u, \text{Re} (q_{k1}), m_h) = \frac{2V_{cb} \text{Re} (q_{k1}) \text{Im} \left( S_{ijk}^u + P_{ijk}^u \right) (J_{10} + J_{12})}{f \left( S_{ijk}^u, P_{ijk}^u, \text{Re} (q_{k1}), m_h \right)},
\]

where

\[
f \left( S_{ijk}^u, P_{ijk}^u, \text{Re} (q_{k1}), m_h \right) = \frac{1}{4g} \left[ |S_{ijk}^u - P_{ijk}^u|^2 (J_1 + J_2 - J_4 - J_6) + |S_{ijk}^u + P_{ijk}^u|^2 (J_5 + J_7) \right] + g \text{Re} (q_{k1}) |V_{cb}|^2 (J_7 + J_8) + 2 \text{Re} (q_{k1}) \text{Re} \left( S_{ijk}^u + P_{ijk}^u \right) V_{cb} (J_9 + J_{11}).
\]

The \( J \)'s are integrals obtained from the decay kinematics, which are shown in appendix B as well as the others parameters defined in previous sections.

**A. Asymmetry in 2HDM-Va**

Let us discuss now the resulting expression for \( A_{CPV} \) for two subcases within 2HDM-V. We fix \( i = 2 \) and \( j = 3 \) in equations (24) and (25) in order to obtain the appropriate parameters within 2HDM-Va, then we find:

\[
S_{231}^u = \frac{\sqrt{m_c m_t}}{2\sqrt{2} v \cos \beta} \chi_{23} (q_{12}^* + q_{12})
\]

and

\[
P_{231}^u = \frac{\sqrt{m_c m_t}}{2\sqrt{2} v \cos \beta} \chi_{23} (q_{12}^* - q_{12}).
\]

Then, the asymmetry coefficient is
\[ A_{2HDM-Va} = \frac{\sqrt{m_c m_t V_{cb}} \chi_{23} (J_{10} + J_{12}) \cos^2 \theta_{12} \cos \theta_{13} \sin \theta_{13}}{\sqrt{2M_W f(\beta, \theta_{12}, \theta_{13}, \chi_{23}, m_h) \cos \beta}}, \]  

(52)

where

\[ f(\beta, \theta_{12}, \theta_{13}, \chi_{23}, m_h) = \frac{m_c m_t \chi_{23}^2}{32M_W^2 \cos^2 \beta} (\sin^2 \theta_{12} + \cos^2 \theta_{12} \sin^2 \theta_{13}) (J_1 + J_2 + J_3 - J_4 + J_5 - J_6) \]

\[ + |V_{cb}|^2 \cos^2 \theta_{12} \cos^2 \theta_{13} (J_7 + J_8) - \frac{\sqrt{m_c m_t V_{cb}} \chi_{23}}{\sqrt{2M_W \cos \beta}} \cos \theta_{12} \cos \theta_{13} \sin \theta_{12} (J_9 + J_{11}). \]  

(53)

### B. Asymmetry in 2HDM-Vb

The appendix [A] shows the four-texture structure for Yukawa matrices, nevertheless for practical evaluation of the asymmetry we write the texture parameter in Euler complex form as \( \chi_{23} = |\chi_{23}|e^{i\theta_{23}} \). Then, we fix the equations \((24)\) and \((28)\) for \( i = 2 \) and \( j = 3 \) in order to obtain the required element for 2HDM-Vb,

\[ S^u_{231} = \frac{\cos(\beta - \alpha)\sqrt{m_c m_t}}{2\sqrt{2}v \cos \beta} (\chi_{23} + \chi_{23}^\dagger) \]  

(54)

and

\[ P^u_{231} = \frac{\cos(\beta - \alpha)\sqrt{m_c m_t}}{2\sqrt{2}v \cos \beta} (\chi_{23} - \chi_{23}^\dagger). \]  

(55)

Then, for this case the asymmetry coefficient is given by:

\[ A_{2HDM-Vb} = \frac{gV_{cb}\sqrt{m_c m_t} |\chi_{23}| \sin \nu_{23} \cos(\beta - \alpha) \sin(\beta - \alpha)}{\sqrt{2M_W \cos \beta} f(\alpha, \beta, \chi_{23}, m_h)} (J_{10} + J_{12}), \]  

(56)

where

\[ f(\alpha, \beta, \chi_{23}, m_h) = \frac{g m_c m_t \cos^2(\beta - \alpha)}{32M_W^2 \cos^2 \beta} |\chi_{23}|^2 (J_1 + J_2 - J_4 - J_6 + J_4 + J_5) \]

\[ + \frac{g \sqrt{m_c m_t} V_{cb}}{\sqrt{2M_W}} |\chi_{23}| \cos \nu_{23} \sin(\beta - \alpha) \cos(\beta - \alpha) \cos \beta \]  

\[ + g \sin^2(\beta - \alpha) |V_{cb}|^2 (J_7 + J_8). \]  

(57)

### C. Numerical results

We shall discuss in detail the result for 2HDM-Vb. The asymmetry depends of the five free parameters. One of them is the Higgs boson mass which appears in the \( J \) integrals, the other ones are the mixing angles \( \alpha, \beta \) and the complex parameter \( \chi_{23} \). The mixing angle \( \beta \) is taken within the values \( 1 < \tan \beta < 50 \) [64]. For the mixing angle \( \alpha \) we study three scenarios, \( \alpha < \beta, \alpha \approx \beta \) and \( \alpha > \beta \). The phase \( \nu_{23} \) is fixed to the value 0.1 in order to analyzed a similar value to the phase from the CKM matrix. For each scenario we take two possible values for \( |\chi_{23}| \). Therefore, the scenarios studied here are:

i) \( \alpha < \beta \) for \( |\chi_{23}| = 0.9 \) and \( |\chi_{23}| = 0.1 \), figure 2

ii) \( \alpha \approx \beta \) for \( |\chi_{23}| = 0.9 \) and \( |\chi_{23}| = 0.1 \), figure 3

iii) \( \alpha > \beta \) for \( |\chi_{23}| = 0.9 \) and \( |\chi_{23}| = 0.1 \), figure 4

We use the reported values \( m_t = 171.2 \) GeV, \( m_b = 4.2 \) GeV, \( m_c = 1.27 \) GeV, \( M_W = 80.39 \) GeV and \( \sin \theta_W = 0.231 \) [64].

From figs.2, 3 and 4 we obtain asymmetry values of the order \( 10^{-3} \) to \( 10^{-2} \) (\( 10^{-4} \) to \( 10^{-3} \) and \( 10^{-3} \)) for scenario i) (ii) and iii) within 2HDM-Vb. We have also analyzed the numerical results for the CP asymmetry for case 2HDM-Va. We also find that the size of this asymmetry depends strongly on the phases.
FIG. 2: The asymmetry as function of $\tan \beta$ for scenario 1, on left for $|\chi_{23}| = 0.1$ and on right for $|\chi_{23}| = 0.9$

FIG. 3: The asymmetry as function of $\tan \beta$ for scenario 2, on left for $|\chi_{23}| = 0.1$ and on right for $|\chi_{23}| = 0.9$

FIG. 4: The asymmetry as function of $\tan \beta$ for scenario 3, on left for $|\chi_{23}| = 0.1$ and on right for $|\chi_{23}| = 0.9$
VI. CONCLUSIONS

In this paper we have presented a broad discussion of the most general formulation of the Two-Higgs doublet extension of the SM, which we name as 2HDM-X. Then, we have defined in a model named 2HDM-V, which has the possibility of including both FCNC and CPV, and have presented the corresponding Lagrangian for both the neutral and charged Higgs sectors.

The limits when 2HDM-X reduces to one of the known versions (2HDM-I, II, III) has also been discussed; in these cases each pattern of Higgs-Yukawa couplings holds for all families. To identify the class of family non-universal models, we have used the label 2HDM-IV, where we include models where one Higgs doublet couples only to a certain type of fermion, for instance to the top quark or the third family, or to neutrinos only.

Finally, we have also evaluated the CPV asymmetry for the decay $h \rightarrow c\overline{b}W$, which allows to test the presence of both FCNC and CPV that associated with model V. We found that for certain optimal range of parameters the decay asymmetry could be of $O(10^{-2})$ to $O(10^{-4})$. These asymmetry values for three scenarios were obtained in the case of the 2HDM-Vb. Similar results arise within 2HDM-Va. The asymmetry behavior has a dependency proportional to the mixing complex parameter $\chi_{23}$. The mixing angles $\alpha$ and $\beta$ control the shape of the graphs. The asymmetry keeps same shape for the Higgs boson mass range between 115 GeV and 160 GeV.

In order to detect this asymmetry we could have to resort to a linear collider, since the final state seems difficult to reconstruct at a hadron collider. Although a final conclusion would require a detailed simulation study, which we plan to address in a future publication [65].

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Appendix A: 2HDM-III with four-Textures

Here we shall summarize the result for 2HDM-III, namely we assume that both Yukawa matrices $Y^q_1$ and $Y^q_2$ have the four-texture form and are Hermitic; following the conventions of [25], the quark mass matrix is then written as:

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C^*_q & \tilde{B}_q & B_q \\ 0 & B^*_q & A_q \end{pmatrix}.$$ 

when $\tilde{B}_q \rightarrow 0$ one recovers the six-texture form. We also consider the hierarchy:

$$|A_q| \gg |\tilde{B}_q|, |B_q|, |C_q|,$$ 

which is supported by the observed fermion masses.

Because of the hermicity condition, both $\tilde{B}_q$ and $A_q$ are real parameters, while the phases of $C_q$ and $B_q$, $\Phi_{B_q,C_q}$, can be removed from the mass matrix $M_q$ by defining: $M_q = P^T_q \tilde{M}_q P_q$, where $P_q = diag[1, e^{i\Phi_{C_q}}, e^{i(\Phi_{B_q}+\Phi_{C_q})}]$, and the mass matrix $\tilde{M}_q$ includes only the real parts of $M_q$. The diagonalization of $\tilde{M}_q$ is then obtained by an orthogonal matrix $O_q$, such that the diagonal mass matrix is: $\bar{M}_q = O^T_q \tilde{M}_q O_q$.

The lagrangian (2) can be expanded in terms of the mass-eigenstates for the neutral ($h^0, H^0, A^0$) and charged Higgs bosons ($H^\pm$). The interactions of the neutral Higgs bosons with the d-type and u-type are given by ($u, u' = u, c, t$ and $d, d' = d, s, b$),
\[ \mathcal{L}_Y^q = \frac{g}{2} \left( \frac{m_d}{m_W} \right) \bar{d} \left[ \cos \alpha \cos \beta \delta_{dd} + \sqrt{2} \sin(\alpha - \beta) \left( \frac{m_W}{m_d} \right) \left( \tilde{Y}_d^d \right) \right] d' H^0 \\
+ \frac{g}{2} \left( \frac{m_d}{m_W} \right) \bar{d} \left[ -\sin \alpha \cos \beta \delta_{dd} + \sqrt{2} \cos(\alpha - \beta) \left( \frac{m_W}{m_d} \right) \left( \tilde{Y}_d^d \right) \right] d' h^0 \\
+ \frac{ig}{2} \left( \frac{m_u}{m_W} \right) \bar{u} \left[ \sin \alpha \sin \beta \delta_{uu'} - \sqrt{2} \sin(\alpha - \beta) \left( \frac{m_W}{m_u} \right) \left( \tilde{Y}_u^u \right) \right] u' H^0 \\
+ \frac{g}{2} \left( \frac{m_u}{m_W} \right) \bar{u} \left[ \cos \alpha \sin \beta \delta_{uu'} - \sqrt{2} \cos(\alpha - \beta) \left( \frac{m_W}{m_u} \right) \left( \tilde{Y}_u^u \right) \right] u' h^0 \\
+ \frac{ig}{2} \left( \frac{m_u}{m_W} \right) \bar{u} \left[ -\cot \beta \delta_{uu} + \sqrt{2} \sin(\alpha - \beta) \left( \frac{m_W}{m_u} \right) \left( \tilde{Y}_u^u \right) \right] \gamma^5 u' A^0. \] (A1)

The first term, proportional to \( \delta_{qq'} \), corresponds to the modification of the 2HDM-II over the SM result, while the term proportional to \( \tilde{Y}_q^q \) denotes the new contribution from 2HDM-III. Thus, the \( f^q \phi^q \) couplings respect CP-invariance, despite the fact that the Yukawa matrices include complex phases; this follows because of the Hermiticity conditions imposed on both \( Y_q^q \) and \( Y_q^q \).

The corrections to the quark flavor conserving (FC) and flavor violating (FV) couplings, depend on the rotated matrix: \( \tilde{Y}_q^q = O_q^T P_q Y_q^q P_q^T O_q \). We will evaluate \( \tilde{Y}_q^q \) assuming that \( Y_q^q \) has a four-texture form, namely:

\[ Y_q^q = \begin{pmatrix} 0 & C_2^q & 0 \\ C_2^q & 0 & B_2^q \\ 0 & B_2^q & A_2^q \end{pmatrix}, \quad |A_2^q| \gg |B_2^q|, |B_2^q|, |C_2^q|. \] (A2)

The matrix that diagonalizes the real matrix \( \hat{M}_q \) with the four-texture form, is given by:

\[ O_q = \begin{pmatrix} \sqrt{\lambda_1 (4 \lambda_1 - A_q)} & \eta_1 \sqrt{\lambda_1 (4 \lambda_1 - A_q)} & \eta_1 \sqrt{\lambda_1 \lambda_2 (4 \lambda_1 - A_q)} \\ -\eta_1 \sqrt{\lambda_1 (4 \lambda_1 - A_q)} & \sqrt{\lambda_1 (4 \lambda_1 - A_q)} & \sqrt{\lambda_2 (4 \lambda_1 - A_q)} \\ \eta_1 \sqrt{\lambda_1 (4 \lambda_1 - A_q)} & -\eta_1 \sqrt{\lambda_1 (4 \lambda_1 - A_q)} & \sqrt{\lambda_2 (4 \lambda_1 - A_q)} \end{pmatrix}, \]

where \( m_q^q = \lambda_1^q \), \( m_q^q = \lambda_2^q \), \( m_q^q = \lambda_3^q \), \( m_q^q = \lambda_4^q \), and \( \eta_q = \lambda_3^q / m_2^q \) ( \( q = u, d \) ). With \( m_u = m_u^q \), \( m_c = m_2^q \), and \( m_t = m_3^q \);

\[ m_d = m_1^q, \quad m_s = m_2^q, \quad \text{and} \quad m_b = m_3^q. \]

Then the rotated form \( \tilde{Y}_q^q \) has the general form,

\[ \tilde{Y}_q^q = O_q^T P_q Y_q^q P_q^T O_q \]

\[ = \begin{pmatrix} (\tilde{Y}_q^q)_{11} & (\tilde{Y}_q^q)_{12} & (\tilde{Y}_q^q)_{13} \\ (\tilde{Y}_q^q)_{21} & (\tilde{Y}_q^q)_{22} & (\tilde{Y}_q^q)_{23} \\ (\tilde{Y}_q^q)_{31} & (\tilde{Y}_q^q)_{32} & (\tilde{Y}_q^q)_{33} \end{pmatrix}. \] (A3)

However, the full expressions for the resulting elements have a complicated form, as it can be appreciated, for instance, by looking at the element \( (\tilde{Y}_q^q)_{22} \), which is displayed here:

\[ (\tilde{Y}_q^q)_{22} = \eta_1 [C_2^q e^{i \phi_{c_q}} + C_2^q e^{-i \phi_{c_q}}] \frac{(A_q - \lambda_2^q)}{m_3^q - \lambda_2^q} \sqrt{\frac{m_2^q m_3^q}{A_q m_2^q} + \frac{B_2^q A_q - \lambda_2^q}{m_3^q - \lambda_2^q}} \]

\[ + \frac{A_2^d A_q - \lambda_2^q}{m_3^q - \lambda_2^q} \left[ B_2^d e^{i \phi_{b_q}} + B_2^d e^{-i \phi_{b_q}} \right] \frac{(A_q - \lambda_2^q)(m_3^q - A_q)}{m_3^q - \lambda_2^q} \] (A4)
where we have taken the limits: $|A_q|, m^q_3, m^q_2 \gg m^q_1$. The free-parameters are: $B^q_2, B^q_3, A^q_2, A_q$.

To derive a better suited approximation, we will consider the elements of the Yukawa matrix $Y^q_2$ as having the same hierarchy as the full mass matrix, namely:

$$C^q_2 = c^q_2 \sqrt{m^q_1 m^q_2 m^q_3} \frac{A_q}{\mu}$$ \hspace{1cm} (A5)

$$B^q_2 = b^q_2 \sqrt{(A_q - \lambda^q_2)(m^q_3 - A_q)}$$ \hspace{1cm} (A6)

$$B^q_3 = b^q_3 (m^q_3 - A_q + \lambda^q_3)$$ \hspace{1cm} (A7)

$$A^q_2 = a^q_2 A_q.$$ \hspace{1cm} (A8)

Then, in order to keep the same hierarchy for the elements of the mass matrix, we find that $A_q$ must fall within the interval $(m^q_3 - m^q_2) \leq A_q \leq m^q_3$. Thus, we propose the following relation for $A_q$:

$$A_q = m^q_3 (1 - \beta_q z_q),$$ \hspace{1cm} (A9)

where $z_q = m^q_2/m^q_3 \ll 1$ and $0 \leq \beta_q \leq 1$.

Then, we introduce the matrix $\tilde{\chi}^q$ as follows:

$$\tilde{(Y^q_2)}_{ij} = \begin{pmatrix} m^q_1 m^q_2 m^q_3 \chi^q_{ij} \\ \sqrt{m^q_1 m^q_2 m^q_3} \chi^q_{ij} \end{pmatrix} \hspace{1cm} (A10)$$

which differs from the usual Cheng-Sher ansatz not only because of the appearance of the complex phases, but also in the form of the real parts $\chi^q_{ij} = |\tilde{\chi}^q_{ij}|$.

Expanding in powers of $z_q$, one finds that the elements of the matrix $\tilde{\chi}^q$ have the following general expressions:

$$\tilde{\chi}^q_{11} = \left[ \tilde{b}^q_2 - (c^q_2 e^{i\Phi_{Cq}} + c^q_2 e^{-i\Phi_{Cq}}) \right] \eta_q + \left[ a^q_2 + \tilde{b}^q_2 - (b^q_2 e^{i\Phi_{Bq}} + b^q_2 e^{-i\Phi_{Bq}}) \right] \beta_q$$

$$\tilde{\chi}^q_{12} = \left[ (c^q_2 e^{-i\Phi_{Cq}} - \tilde{b}^q_2) - \eta_q \left[ a^q_2 + \tilde{b}^q_2 - (b^q_2 e^{i\Phi_{Bq}} + b^q_2 e^{-i\Phi_{Bq}}) \right] \right] \beta_q$$

$$\tilde{\chi}^q_{13} = \left[ (a^q_2 - b^q_2 e^{-i\Phi_{Bq}}) \right] \eta_q \sqrt{\beta_q}$$

$$\tilde{\chi}^q_{22} = \tilde{b}^q_2 \eta_q + \left[ a^q_2 + \tilde{b}^q_2 - (b^q_2 e^{i\Phi_{Bq}} + b^q_2 e^{-i\Phi_{Bq}}) \right] \beta_q$$

$$\tilde{\chi}^q_{23} = \left[ b^q_2 e^{-i\Phi_{Bq}} - a^q_2 \right] \sqrt{\beta_q}$$

$$\tilde{\chi}^q_{33} = a^q_2.$$ \hspace{1cm} (A11)

While the diagonal elements $\tilde{\chi}^q_{ii}$ are real, we notice (Eqs. 14) the appearance of the phases in the off-diagonal elements, which are essentially unconstrained by present low-energy phenomena. As we will see next, these phases modify the pattern of flavor violation in the Higgs sector. For instance, while the Cheng-Sher ansatz predicts that the FCNC couplings $(Y^q_2)_{13}$ and $(Y^q_2)_{23}$ vanish when $a^q_2 = b^q_2$, in our case this is no longer valid for $\cos \Phi_{Bq} \neq 1$. Furthermore the FCNC couplings satisfy several relations, such as: $|\tilde{\chi}^q_{23}| = |\tilde{\chi}^q_{13}|$, which simplifies the parameter analysis. Similar expressions can be obtained for the lepton sector.

**Appendix B: Decay kinematics for $h \rightarrow b\bar{b}W$**

For sake of simplicity we introduce the dimensionless scaled variables

$$\mu_t = \frac{m_t^2}{m_h^2}$$ \hspace{1cm} (B1)

and
\[ (x, \ y, \ z) = \left( \frac{2F_1}{m_h}, \frac{2F_2}{m_h}, \frac{2F_3}{m_h} \right). \]

(B2)

With this notation we can write the energy conservation as

\[ x + y + z = 2. \]

(B3)

In the Higgs rest frame, we just consider the contribution of the \( \mu_1 \), because \( \mu_1 \gg \mu_2, \mu_3 \), and find the momentum expressions

\[ p_1 \cdot p_2 = \frac{m_2^2}{2} (x + y + \mu_1 - 1), \quad \text{(B4)} \]
\[ p_1 \cdot p_3 = \frac{m_2^2}{2} (1 - y - \mu_1), \quad \text{(B5)} \]
\[ p_2 \cdot p_3 = \frac{m_2^2}{2} (1 - x + \mu_1), \quad \text{(B6)} \]
\[ p_1 \cdot q = \frac{m_2^2}{2} (x + y + \mu_1 - 1), \quad \text{(B7)} \]
\[ p_2 \cdot q = \frac{m_2^2}{2} (x - y - \mu_1), \quad \text{(B8)} \]
\[ p_3 \cdot q = \frac{m_2^2}{2} (2 - x - y), \quad \text{(B9)} \]
\[ k^2 = m_2^2 (1 + \mu_1 - x), \quad \text{(B10)} \]
\[ q^2 = m_2^2 (x + y - 1). \quad \text{(B11)} \]

Now, the functions \(|P_t(q)|^2\), \(|P_W(q)|^2\), \(P_t(q)P_W(k)\) and \(P_t(q)P_W^*(k)\) with the dimensionless variables can be written as

\[ |P_t(q)|^2 = \frac{1}{m_h^2} \frac{1}{(x + y - 1 - \mu)^2 + \mu \Gamma^2}, \quad \text{(B12)} \]
\[ |P_W(k)|^2 = \frac{1}{m_h^4} \frac{1}{(1 - x)^2 + \mu \Gamma^2}, \quad \text{(B13)} \]
\[ P_t(q)P_W^*(k) = \frac{(x + y - 1 - \mu)(1 - x) + \sqrt{\mu \Gamma} \Gamma + i \left[ \sqrt{\mu \Gamma} (1 - x) - \sqrt{-\mu \Gamma} (x + y - 1 - \mu) \right]}{m_h^2 \left[ (x + y - 1 - \mu)^2 + \mu \Gamma^2 \right] \left[ (1 - x)^2 + \mu \Gamma^2 \right]}, \quad \text{(B14)} \]

and

\[ P_t(q)P_W^*(q) = \frac{(x + y - 1 - \mu)(1 - x) + \sqrt{\mu \Gamma} \Gamma - i \left[ \sqrt{\mu \Gamma} (1 - x) - \sqrt{-\mu \Gamma} (x + y - 1 - \mu) \right]}{m_h^2 \left[ (x + y - 1 - \mu)^2 + \mu \Gamma^2 \right] \left[ (1 - x)^2 + \mu \Gamma^2 \right]}, \quad \text{(B15)} \]

Here \( \mu = \frac{m_2^2}{m_h^2} \), \( \Gamma^2 = \frac{1}{m_h^2} \) and \( \gamma^2 = \frac{\Gamma^2}{m_h^2} \), with \( \Gamma_t \approx 1.28 \) GeV and \( \Gamma_W \approx 2.14 \) GeV are the SM full width decay for top quark and W boson, respectively. 

The three body decay rate is given by the formula

\[ d\Gamma_{h \rightarrow WZ} = \frac{|M|^2}{2m_h} \frac{d^3 \vec{p}_1}{(2\pi)^4 2E_1} \left[ \frac{d^3 \vec{p}_2}{(2\pi)^4 2E_2} \right] \left[ \frac{d^3 \vec{p}_3}{(2\pi)^4 2E_3} \right] (2\pi)^4 \delta^4(p - p_1 - p_2 - p_3). \quad \text{(B16)} \]

Using the delta function to perform the \( d\vec{p}_3 \) integral and setting the polar axis along \( \vec{p}_1 \), we have

\[ \Gamma_{h \rightarrow WZ} = \Gamma_{11} + \Gamma_{22} + \Gamma_{12}, \quad \text{(B17)} \]
where we define

\[ \Gamma_{11} = \frac{m_h}{256\pi^3} \int \int_{R_{xy}} |M_1|^2 \, dx \, dy, \]

\[ \Gamma_{22} = \frac{m_h}{256\pi^3} \int \int_{R_{xy}} |M_2|^2 \, dx \, dy, \]  \( \text{(B18)} \)

and

\[ \Gamma_{12} = \frac{m_h}{256\pi^3} \int \int_{R_{xy}} (M_1^* M_2 + M_2^* M_1) \, dx \, dy, \]

\[ \text{(B19)} \]

with the \( R_{xy} \) region is defined by

\[ \frac{1}{2} \left( 2 - x - \sqrt{x^2 - 4\mu_1} \right) \leq y \leq \frac{1}{2} \left( 2 - x + \sqrt{x^2 - 4\mu_1} \right) \]

\[ \text{(B21)} \]

and

\[ 2\sqrt{\mu_1} \leq x \leq 1 + \mu_1. \]

\[ \text{(B22)} \]

We can obtain the next results whether we write equations (B10), (B11), (B12) and (B13) with dimensionless parameters and substitute in equations (B18), (B19) and (B20),

\[ \Gamma_{11} = \frac{2g^2m_h}{(16\pi)^3} \left[ |S_{231}^u - P_{231}^u|^2 (J_1 + J_2 - J_4 - J_6) + |S_{231}^u + P_{231}^u|^2 (J_3 + J_5) \right], \]

\[ \text{(B23)} \]

\[ \Gamma_{22} = \frac{g^4q_{11}^2 |V_{cb}|^2 m_h}{512\pi^3} [J_7 + J_8], \]

\[ \text{(B24)} \]

\[ \Gamma_{12} = \frac{|S_{231}^u + P_{231}^u| g^3q_{11} V_{cb} m_h}{256\pi^3} (J_0 \sin \theta + J_{10} \cos \theta + J_{11} \sin \theta + J_{12} \cos \theta), \]

\[ \text{(B25)} \]

and

\[ \tilde{\Gamma}_{12} = \frac{|S_{231}^u + P_{231}^u| g^3q_{11} V_{cb} m_h}{256\pi^3} (J_0 \sin \theta - J_{10} \cos \theta + J_{11} \sin \theta - J_{12} \cos \theta). \]

\[ \text{(B26)} \]

The \( J_i \) integrals, for \( i = 1, \ldots, 12 \), are given by

\[ J_1 = \frac{1}{\mu_1} \int \int_{R_{xy}} \frac{(x + y - \mu_1 - 1) (x + y + \mu_1 - 1) (2 - x - y)}{(x + y - 1 - \mu)^2 + \mu \Gamma^2} \, dx \, dy, \]

\[ \text{(B27)} \]

\[ J_2 = \int \int_{R_{xy}} \frac{(x + y - \mu_1 - 1) (2 - x - y)}{(x + y - 1 - \mu)^2 + \mu \Gamma^2} \, dx \, dy \]

\[ \text{(B28)} \]

\[ J_3 = \frac{\mu}{\mu_1} \int \int_{R_{xy}} \frac{(x + y - \mu_1 - 1) (1 - y - \mu_1)}{(x + y - 1 - \mu)^2 + \mu \Gamma^2} \, dx \, dy, \]

\[ \text{(B29)} \]

\[ J_4 = \frac{1}{\mu_1} \int \int_{R_{xy}} \frac{(x + y - 1) (x + y - \mu_1 - 1) (1 - y - \mu_1)}{(x + y - 1 - \mu)^2 + \mu \Gamma^2} \, dx \, dy, \]

\[ \text{(B30)} \]

\[ J_5 = \mu \int \int_{R_{xy}} \frac{1 - x + \mu_1}{(x + y - 1 - \mu)^2 + \mu \Gamma^2} \, dx \, dy, \]

\[ \text{(B31)} \]
FIG. 5: Graphics for the integrals of the $\Gamma_{11}$.

FIG. 6: Graphics for the integrals of the $\Gamma_{22}$ and $\Gamma_{12}$.

\begin{align*}
J_6 &= \int \int_{R_{xy}} \frac{(x+y-1)(1-x+\mu_1)}{(x+y-1-\mu)^2 + \mu \Gamma^2} dxdy, \quad (B32) \\
J_7 &= \mu_1 \int \int_{R_{xy}} \frac{(1-x+\mu_1)}{(1-x)^2 + \mu_1 \gamma^2} dxdy, \quad (B33) \\
J_8 &= \int \int_{R_{xy}} \frac{(x+y+\mu_1-1)(1-y-\mu_1)}{(1-x)^2 + \mu_1 \gamma^2} dxdy, \quad (B34) \\
J_9 &= \sqrt{\mu \mu_1} \int \int_{R_{xy}} \frac{(1-x+\mu_1) \left[ (x+y-1-\mu) (1-x) + \sqrt{\mu \mu_1 \gamma} \Gamma \right]}{(x+y-1-\mu)^2 + \mu \Gamma^2} \left[ (1-x)^2 + \mu_1 \gamma^2 \right] dxdy, \quad (B35) \\
J_{10} &= \sqrt{\mu \mu_1} \int \int_{R_{xy}} \frac{(1-x+\mu_1) \left[ \sqrt{\mu \Gamma} (1-x) - \sqrt{\mu_1 \gamma} (x+y-1-\mu) \right]}{(x+y-1-\mu)^2 + \mu \Gamma^2} \left[ (1-x)^2 + \mu_1 \gamma^2 \right] dxdy, \quad (B36) \\
J_{11} &= \sqrt{\frac{\mu}{\mu_1}} \int \int_{R_{xy}} \frac{(x+y+\mu_1-1)(1-y-\mu_1) \left[ (x+y-1-\mu) (1-x) + \sqrt{\mu \mu_1 \gamma} \Gamma \right]}{(x+y-1-\mu)^2 + \mu \Gamma^2} \left[ (1-x)^2 + \mu_1 \gamma^2 \right] dxdy, \quad (B37)
\end{align*}
\[ J_{12} = \sqrt{\frac{\mu}{\mu_1}} \int \int_{R_{xy}} \frac{(x + y + \mu_1 - 1)(1 - y - \mu_1)(\sqrt{\mu} (1 - x) - \sqrt{\mu_1} \gamma (x + y - 1 - \mu))}{(x + y - 1 - \mu)^2 + \mu \gamma^2} \ dx \ dy. \]

(B38)

The graphics for the \( J_i, i = 1, ..., 12, \) are shown in the figures 5 and 6.
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