Contributions of oscillator noises to the sensitivity of TAMA300

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Abstract. We present contributions of oscillator noises to the sensitivity of TAMA300. The oscillator is required for operating the interferometric detector. In order to estimate the contributions, phase and amplitude noises of the oscillator are measured, then transfer functions from these noises to the output of the interferometer are measured. The results show that neither phase nor amplitude noise limit the present sensitivity of TAMA300. Furthermore, models of the transfer functions are calculated to clarify how the noises pass through an interferometer. The frequency dependencies of the models agree with that of the measurement at the frequency above 200 Hz.

1. Introduction

We present contributions of oscillator noises to the sensitivity of TAMA300 [1], which is an interferometric gravitational wave detector in Japan. The oscillator is required for Schnupp modulation method, which is commonly employed by present large-scale interferometric detectors. An oscillator has two kinds of noises (oscillator noises): fluctuation of phase (phase noise) and fluctuation of amplitude (amplitude noise). Since the oscillator noises could be one of the noise sources in LCGT [2] (a future ground-based large-scale interferometer project in Japan) as well as LIGO [3], VIRGO [4] and GEO600 [5], it is important to make clear how the noises pass through an interferometer.

The configuration of TAMA300 is shown in Figure 1. The laser light is phase modulated with an electro-optic modulator (EOM) at a radio frequency (RF) before it is incident on a ring cavity (mode cleaner). This ring cavity is designed to transmit only the TEM₀₀ mode, which is determined by the geometry of the mode cleaner. Then the laser light is incident on the interferometer. The interferometer consists of two parts: a Michelson interferometer with 300-m-long Fabry-Perot arm cavities (hereafter “Michelson part”) and a power recycling cavity. The power recycling cavity, which is comprised of a power recycling mirror and the Michelson part, is resonant with the laser light to increase the effective power of the light inside the interferometer.

In this paper, the contributions from the noises of the master oscillator to the sensitivity of TAMA300, which is a prototype for LCGT, are experimentally estimated and analytically calculated. The noise contribution in the frequency domain is simply estimated as the product
of two terms, noise source and noise transfer function:

\[
\text{(Noise Contribution)} = \text{(Noise Source)} \times \text{(Noise Transfer Function)},
\]

where the noise source is phase or amplitude noise of the master oscillator.

2. Noise sources: oscillator noises
The output voltage of an oscillator with nominal amplitude \( V_0 \) and frequency \( \nu_0 \) can be written as

\[
v(t) = [V_0 + \delta V(t)] \cos [2\pi \nu_0 t + \delta \phi(t)],
\]

where \( \delta V(t) \) and \( \delta \phi(t) \) are time-dependent amplitude and phase variations, respectively. The oscillator phase noise is defined by \( \delta \phi(t) \), and oscillator amplitude noise is defined by

\[
\epsilon(t) \equiv \frac{\delta V(t)}{V_0}.
\]

Note that \( |\delta \phi(t)| \ll 1 \) and \( |\epsilon(t)| \ll 1 \).

The master oscillator, which is a VCXO (voltage controlled crystal oscillator) in this case, produces the RF voltage written as Eq.(2) with \( \nu_0 = 15.235 \) MHz. As shown in Figure 1, the RF voltage is applied to the EOM, thereby phase modulating the laser light. This produces RF sidebands around a carrier in the optical frequency domain. The mode cleaner and the power recycling cavity are resonant for both the carrier and the RF sidebands, while the arm cavity is only resonant for the carrier.

The sensitivity of the interferometric gravitational wave detector may depend on the accuracy of the master oscillator. Although the interferometer is controlled so that the carrier vanishes at the optical output port (or “dark port”), the AF sidebands induced by a gravitational wave also appear at the dark port. The RF sidebands also pass through the interferometer into the dark port due to an asymmetry of the Michelson part. The beat signal between the RF sidebands and the AF sidebands can be detected by a photodetector at the dark port, then the photocurrent is demodulated by the RF voltage of the local oscillator (LO), which is the master oscillator itself, to the original frequency band of the gravitational wave. The noises in the output voltage of the master oscillator could affect these processes.
Figure 2. Phase noise of the master oscillator in the frequency domain (measured value).

Figure 3. Amplitude noise of the master oscillator in the frequency domain (measured value).

Table 1. Interferometer imperfections in TAMA300 under consideration

| Description                                    | Symbol | Value                      |
|-----------------------------------------------|--------|----------------------------|
| Recycling-cavity differential length offset   | $dl_-$ | $6.5(\pm0.1) \times 10^{-10}$ m |
| Reflectivity mismatch between two arm-cavities for carrier | $\Delta r$ | $0.037(\pm0.017)$          |

Figure 2 and 3 show phase noise $\delta \phi$ and amplitude noise $\epsilon$ of the master oscillator in the frequency domain, respectively. The phase noise was measured by comparing the output phases of two independent oscillators. This technique ("two-oscillator technique" in [7]) is often used to measure the phase noise of an oscillator. The amplitude noise was measured by squaring the output voltage of the master oscillator to obtain the envelope of the amplitude variation. This technique is generally known as the squared detection method. Both noises are measured at the output port of RF power divider toward the EOM (see Figure 1).

3. Noise Transfer Functions
The transfer functions from the oscillator noises to the output of TAMA300 are experimentally measured. In addition, a model of noise transfer function is analytically calculated in order to clarify how the noises pass through an interferometer.

The output signal from an interferometer is regarded as displacement noise $\delta L_-$. Thus the transfer functions from the oscillator phase and amplitude noises can be written as $\delta L_- / \delta \phi$ and $\delta L_- / \epsilon$ in the frequency domain, respectively [see Eq.(1)].

The model of noise transfer function is based on [8], and summarized as follows: the oscillator noise is converted into laser-light noise at the EOM, couples with interferometer imperfections and contributes to the output noise of the detector.

The interferometer imperfections under consideration is shown in Table 1. The imperfections are zero if the detector is ideally constructed and operated. Although the interferometer is designed, controlled, and operated so that these imperfections are small, they are practically nonzero. Furthermore, it is difficult to measure the imperfections accurately. In this paper, the recycling-cavity differential length offset $dl_-$ was estimated by the residual voltage of the length control system, while the reflectivity mismatch between the two arm-cavities $\Delta r$ was estimated.
3.1. Phase Noise Coupling

The transfer function from the oscillator phase noise in TAMA300 is measured as follows: while the full interferometer is locked, the output of the master oscillator is phase modulated to induce a known amount of phase noise $\delta \phi$ at an audio frequency, and the detectors output at $\delta L_-$ in response to the induced $\delta \phi$ is measured. The measured transfer function is shown in Figure 4.

The calculated model of the transfer function shown in Figure 4 is

$$\frac{\delta L_-(f)}{\delta \phi} \simeq a \left(1 + i \frac{f}{f_{FP}} \right) \left(1 - \frac{1}{1 + i \frac{f}{f_{MC}}} \right),$$

where $f_{MC}$ and $f_{FP}$ are the cavity pole frequency of the mode-cleaner and the Fabry-Perot arm, respectively. Because the coupling constant $a$ is proportional to $d \times \Delta r$ (see Appendix A), the gain of transfer function depends on these imperfections. The second term comes from the fact that the sensitivity of this gravitational wave detector declines above $f_{FP}$. The third term gives the effect of the mode cleaner. The influence of the phase noise up to $f_{MC}$ is canceled out by the phase noise of the LO in the demodulation process. The influence above $f_{MC}$ is, however, filtered out by the mode cleaner, then the influence of the LO remains.

3.2. Amplitude Noise Coupling

The transfer function from the oscillator amplitude noise in TAMA300 is measured in the similar manner as the phase noise. The output of the master oscillator was amplitude modulated to induce a known amount of amplitude noise $\epsilon$ at an audio frequency, then the detector output, a $\delta L_-$ as a response to the induced $\epsilon$, was measured. The measured transfer function is shown in Figure 5.

The calculated model of the transfer function shown in Figure 5 is

$$\frac{\delta L_-}{\epsilon}(f) \simeq b \left(1 + i \frac{f}{f_{FP}} \right) \left(-1 \frac{1}{1 + i \frac{f}{f_{MC}}} \right).$$

Figure 4. Transfer function from oscillator phase noise to the output of TAMA300: full circle for measured values, red solid line for calculated model assuming coupling constant $a < 0$, and green dashed line for calculated model assuming $a > 0$.

Figure 5. Transfer function from oscillator amplitude noise to the output of TAMA300: full circle for measured values, red solid line for calculated model assuming coupling constant $b < 0$, and green dashed line for calculated model assuming $b > 0$. 

from each arm-cavity finesse.
3.3. Comparisons between the measurements and the calculations

Figure 4 shows that the measurement of the phase noise transfer function agrees with the estimated model above 200 Hz, whereas the sign of the coupling constant $a$ fluctuates. Figure 5 shows that the measurement of the amplitude noise transfer function follows the estimated model above 200 Hz, whereas the coupling constant $b$ is overestimated. Since the coupling constant $b$ is proportional to $dl_-$, the disagreement should be caused by inaccurate estimate of this imperfection. If it is true, the coupling constant $a$ should be underestimated by a factor of 10. The imperfection $\Delta r$ should be increased ten times to meet the measurement, however the actual value of the imperfection must not be such a large one. In addition, the coupling constant $a$ fluctuate in sign. The imperfection $\Delta r$ should be caused by the difference between the qualities of mirrors, which should not fluctuate in short time. Therefore the discrepancy suggests that additional parameters dominate the coupling constant $a$. For example, a strange and large sensitivity to the phase noise was measured in LIGO, during thermal compensation in the optics of power recycling cavity [9].

Below 200 Hz, the models do not agree with the measurements. The disagreement suggests that these models should not be considered valid in this frequency range. Since many feedback systems, such as alignment control, are operated in this frequency band, the noises could pass through these systems into the output of the detector.

4. Estimations of oscillator noise contributions

The contributions of oscillator noises to the sensitivity of TAMA300 are shown in Figure 6. These noise contributions are estimated from the measured noise levels measured transfer functions. Figure 6 shows that neither phase (green line) nor amplitude noise (blue line) limits the present sensitivity level of TAMA300 at any frequency.

Moreover, contributions of the oscillator noises seem lower than the theoretical sensitivity limit of the detector for almost all frequencies. This means that the quality of the present master oscillator is sufficient for the sensitivity of TAMA300 to reach the goal with the oscillator. If, in fact, a bit lower amplitude noise is required around 100 Hz, it will be provided with amplitude stabilization.
5. Conclusions
We present the contributions from oscillator noises to the sensitivity of TAMA300. The noise contributions are well suppressed below the present sensitivity level of the detector. The output noises of the master oscillator used to operate the interferometer are experimentally measured. Furthermore, the noise transfer functions are not only measured, but also calculated based on a theoretical model. Then the contributions from the output noise of the master oscillator to the sensitivity of TAMA300 are estimated.

The frequency dependency of the models agree with the measurements at frequencies above 200Hz. Below 200 Hz the models are, however, not in agreement with the measurements. The discrepancy might result from some feedback system operating in this frequency region, such as the alignment control system, which may transfer these noises into the output of the detector. The magnitude of the calculated transfer function is not in agreement with the measurement for oscillator amplitude noise. This could be caused by the uncertainty of the coupling constant, which is a function of the interferometer imperfections. Therefore it is important to know how to measure accurately the real interferometer imperfections, so that we can predict the contributions of oscillator noises to a future large-scale detector.

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Appendix A.
The coupling constants $a, b$ are described as follows:

$$a = -dl - \frac{\Delta r}{s_0 \tan \alpha}, \quad (A.1)$$

$$b = -dl - \frac{\bar{r}}{s_0}, \quad (A.2)$$

where $s_0 \sim 320$, $\alpha \sim 0.27$, and $\bar{r} \sim 0.93$ correspond to the sensitivity for phase deviation of an arm-cavity, Schnupp asymmetry in the Michelson part, and mean of amplitude reflectivity of two arm-cavities for carrier, respectively.

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