THE DEVELOPMENT OF MATHEMATICS MODULE BASED ON RELATIONAL THINKING

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ABSTRACT

Mathematical thinking and problem solving are important goal of learning mathematics. But in fact, mathematics learning in school was focused to compute the final answer. For example in solving algebraic equations, students have difficulty to solving these problems because students not understand about symbols of equivalencies. It's shows that student need a media that stimulates their relational thinking. Therefore, this research aims to develop mathematics module for secondary students based on relational thinking to increase ability in solving algebraic equation. This is a development research which refers to 3D model. The technique for data collection was taken from questionnaire and test. The results showed that the module included: validity of the contents 84.21%, validity for language 84.21% and validity of display 86.84%. The results of limited test to thirty students received a positive response 94.73% and score of relational included in good category, so that the module is valid as media for mathematical learning.

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INTRODUCTION

Mathematical thinking and solving mathematics problem are important goal of learning mathematics. Because students are expected to have the ability to think logically, analytically, systematically, critically, and creatively, and also they have ability to collaborate with other (Depdiknas, 2006). Mayer in Solso (1995) said one of the goals of thinking is problem solving. Therefore, to solve mathematics problems, students' thinking processes is very important used. But in fact, learning of mathematics in school focused to compute the final answer. Addition, subtraction, multiplication and
division are generally seen as a process of computational thinking only. Therefore the understanding of students was only procedural understanding, students according to the example procedure from their teacher.

Computational thinking is represent by twenty students on seventh grade in Kediri. The researcher find that sixteen students from twenty students who answered algebra equation correctly, can’t explain these reason of the problem solving process choosen. All students answered according to the procedure taught by their teachers. These facts indicate that the thinking of students to understand every step in solving mathematical problems less noticed by the teacher.

In other side, students have difficulty to solving algebraic equations because students are required to understand the symbol as an equal with the equivalence relation. In algebraic equations, students must focus on the concept of relationships. Despite the true solutions as an important part, but completing algebraic equation has a different character than applying the algorithm to get the final answer.

For example when students are given about $42 + x = 48 + 71$, then the student can do some strategies (1) by computing the sum on the right side next to subtract these results by $42$ to obtain $x = 77$ or with strategy (2) transform into a right-hand side $(42 + 6) + 71$ and use the associative of addition and seen the both sides of the relationship on the number $42$, the obtained $x = 6 + 71 = 77$. The thought process as a strategy (2) is to improve in this research because students thinking process already understood relationally not just procedurally.

According Doumas and Hummel (2005) relational thinking is the ability to understand the analogy between objects or events that seem different and apply abstract rules in new situations. Thus the relational thinking is a mental process that is characterized by building linkages between elements of the information provided by the prior knowledge or the knowledge about the properties or structure of mathematics for solving mathematical problems. Through the empowerment of relational thinking process, is expected to enhance the students' problem solving skills.

Based the importance of relational thinking and the important of algebra for secondary students as a new material for them, researcher would developed a media that stimulate relational thinking skills in learning algebra. For that initiated a development focused learning modules for teachers of SMP (Junior High School), which contains the solving algebraic equation with thought processes based relational thinking.

**LITERATURE REVIEW**

**A. Mathematics Modul**

The learning module is the unit of the smallest teaching and learning program learned by the students themselves individually or taught by the students to themselves (Winkel, 2009: 472). Meanwhile, according to Anwar (2010) learning module is a systematic and interesting teaching materials that cover the contents of materials, methods and evaluation that can be used independently to achieve the expected competencies. Based on some understanding of the module it can be concluded that the learning module is one form of teaching materials that are packaged in a systematic and interesting so easy to learn independently.
Anwar (2010) states that the characteristics of the learning module as follows:
1. **Self instructional**, students are able to these matters themselves, not dependent on other person.
2. **Self contained**, all learning materials from one unit of competency learned in one module intact.
3. **Stand alone**, develop module are not dependent on other media or should not be used in conjunction with other media.
4. **Adaptive**, the module should have a high adaptive to development of science and technology.
5. **User friendly**, the module should also seen the rules of friendship with the student.
6. **Consistency** in the use of fonts, spacing, and layout.

The module in this research is designing as instrument to improve relational thinking ability student in solving algebraic equation.

**B. Relational thinking**

Relational thinking is thinking use equivalence in another way. They are able to keep the two numbers either side of the equal sign as uncalculated pairs (Stepens, 2006). Relational thinkers seem to abstain from closing some operations because the possibilities of the variation depend on the numbers and operations involved. Baiduri (2011) point to the importance of enabling students to discern dimensions of variation in which some elements in mathematical sentences change while other elements remain unchanged.

For example when students are given about $42 + x = 48 + 71$, then the student can do strategies to transform into a right-hand side $(42 + 6) + 71$ and use the associative of addition and seen the both sides of the relationship on the number 42, the obtained $x = 6 + 71 = 77$. It marks relational thinking is the capacity to use possibilities of variation in number sentences.

**C. Solving Algebraic Equation Problem**

Equality and relational thinking are fundamental of algebraic learning. The meaning attributed the equal signs as deficiency of relational thinking could prevent the development of algebraic thinking and success in algebra (Stephens, 2006). As matter of fact, the studies on this issue in mathematics education literature have indicated that the students perceive “equal sign” as an operator sign rather than a relational sign (Carpenter, 2005). So in this research, definition of algebraic equation is an equation obtained by equating to zero a sum of a finite number of term each one of which is a product of positive integral powers (including the zero power) of the variables. For example these equation as follow:

Find the value of $x$ if $2 + \frac{1}{\frac{1}{1-x}} = \frac{5}{2}$ .......(1.1)

So student with relational thinking skill might answered

\[
2 + \frac{1}{\frac{1}{1-x}} = \frac{5}{2} \\
2 + \frac{1}{\frac{1}{1-x}} = 2 \cdot \frac{1}{2} \\
\frac{1}{\frac{1}{1-x}} = \frac{1}{2} \\
\frac{1}{1-x} = \frac{1}{2}
\]
Where:

\[ 1 + \frac{3}{1-x} = 2 \]
\[ 1 + \frac{3}{1-x} = 1 + 1 \]
\[ \frac{3}{1-x} = \frac{3}{3} \]

\[ 1 - x = 3 \text{ so } x = -2 \] .................(1.2)

To identify the aspects of weaknesses of students, the researcher prepared the assessment rubric for the mathematical relational thingking in solving algebraic equations. While for the ability of mathematics relational thingking there are this indicator are in table 1 as follows:

**Table 2.1 Scoring Rubric of Mathematics Relational Thingking In Solving algebraic Problem. Adapted from Scoring Rubric of Mathematics Relational Thingking (Stephens, 2006)**

| Scoring | Indicators |
|---------|------------|
| 0       | Relational thingking evident for all algebraic equations, for examples, through evidence of progressive calculations and use of algorithms to obtain relevant totals for addition and subtractions to solve the equivalence, even where these approach may have resulted in incorrect answers, and no evidence of any relational thingking; also where an answer only has been given without any working shown to indicate what method has been used. |
| 1       | A clear attempt to use relational thingking in at least one question, but not successfully executed. |
| 2       | Relationally thingking clearly shown in one question and successfully executed, even if the other problems are solved |
| 3       | Relational thingking clearly shown in at least two questions and executed, but where the remaining question or questions are not solved relationally or solved using incorrect relational thingking. |
| 4       | All questions solved clearly and successfully using relational thingking, computational solution of equation may also be provided in parallel. |

**METHODOLOGY**

This study is kind of the development research. The stage of designing module follow the procedure of Thiagarajan model and Semmel & Semmel (1974). Development model that used is refer to 4-D model modified that called 3-D model. Where consist of 3 steps namely define, design and develop. This model without disseminate stage because limited of time and cost. The research instrument used in questionnaire validation for expert, observation sheets, algebraic quation testing, assessment rubrics, and the questionnaire responses of students.
The try out subject in this research were: (1) a group experts consisting of expert design development of instructional media, material expert of algebra, expert of linguistic, (2) user group for limited trial consisting the seventh grade students of SMP Ar Risalah Kediri and MTs. Al Ikhlas Kediri, East Java, Indonesia. The data collected was in the for of qualitative and quantitative data. Quantitative data was obtained through questionairs assessment development products prepared with Likert scale by material expert, expert design, expert linguistic and unlimited users as well as the assessment of students respons, limited test and rubric of relational thingking skill for limited users of module, while qualitative data was obtained through the comment and suggestion, both from expert and limited users. Data collecting instrument used consist of questionaires validation, observation sheet during learning, algebraic quation testing, assessment rubrics, and the questionnaire responses of students.

The data analysis technique used in this research is descriptive analysis techniques. Descriptive data analysis was used to analyze the data in the form of record suggestions and respons from the validations questionnaire. To analyze validity of module, guidlines for the categorization of validities of module are in table 3.1 as follows

**Table 3.1 Capabilities of Relational Thingking Categorization, Adapted from Anwar (2010)**

| Total Score Achieved (X) | Category   |
|-------------------------|------------|
| 4 < RTV ≤ 5             | Very valid |
| 3 < RTV ≤ 4             | Valid      |
| 2 < RTV ≤ 3             | Less valid |
| 1 < RTV ≤ 2             | Not valid  |

RTV = Average of each aspect module criteria  
(Submitted: author)

Descriptive statistical analysis used to analyze the data in the form of scores relational thingking when student solving algebraic equation problem. To analyze relational thingking score, guidlines for the categorization of relational thingking skill are in table 3.2 as follows

**Table 3.2 Capabilities of Relational Thingking Categorization, Adapted from Sugiyono (2008)**

| Total Score Achieved (X) | Category   |
|-------------------------|------------|
| X > Xi + 1.5 Si         | Very High  |
| Xi + 0.5Si < X ≤ Xi + 1.5Si | High    |
| Xi - 0.5Si < X ≤ Xi + 0.5Si | Medium  |
| Xi -1.5Si < X ≤ Xi – 0.5Si | Low      |
| X ≤ Xi - 1.5 Si         | Very Low   |

Xi = ½ (the possible maximum score from rubric + possible minimum score from rubric) and Si = 1/6 (the possible maximum score from rubric - possible minimum score from rubric)  
(Submitted: author)
So the diagram that illustrates the development process of module and achievement targets presented in figure 3.1 as follows:

**Stage 1: Define**
- Analysis of relational thinking theory, problem solving ability, algebraic equation, characteristics of good module and 2013 Curricula

**Stage 2: Design**
- The activity of this stage consists of identifying indicators of learning objective in algebraic equation, identifying basic competence, initial design of mathematics module in 7th grade in Junior High School.

**Stage 3: Develop**
- Validation of draft 1, revise, trials to 7th student, revise, drafting these module manuscript (draft 2) and

If draft 1 is not valid, the next step is **redesign**

![Diagram](image)

**Figure 3.1** The Development Process of Module And Achievement Targets on Built-up Model March-May 2017
(Source: author)

**RESULTS and DISCUSSION**

Result of development are described as follows:

**Stage 1: Define**

This stage set and defining relational thinking in learning activity by conducting analysis purpose and material limitations. In stage of define, five activity that must be done namely: front-end analysis, student analysis, concept and material analysis, assignment and formulation of learning purpose. Stage of define in this research are described as follows:

a. **Front-End Analysis**

Equality and relational thinking are fundamental of algebraic learning. The meaning attributed the equal signs as deficiency of relational thinking could prevent the development of algebraic thinking. Relational thinking is think about perceive “equal sign” as relational sign. In curricula 2013, algebra is new material for seventh grade student, so they used media to help them solved algebra problems.

b. **Student Analysis**

Information gathering and analysis from the limited test showed that student still rely solving the problem likes the teacher processed. The limited test result in 7th students SMP Ar-Risalah Lirboyo Kediri, from 18 students, only 5 students has relational thinking skill, and from 20 students in MTs Al Ikhlas Kediri only 2 students has relational thinking skill. Based this analysis, clearly shows that the students are required relational thinking in solving algebraic equation.

c. **Concept and Material Analysis**
Based of observation, it is also known that algebraic equation materials that are used is still general and some parts are not accordance with the characteristics of students residence. In this case, it is recognized the need of algebraic equations, so the purpose of this research was explained concept of algebraic equation as an equation obtained by equating to zero a sum of a finite number of term each one of which is a product of positive integral powers.

d. Assignment Analysis

To analyze relational thingking skill and problem solving ability used some instrumens likes algebraic quation testing, assessment rubrics and quessionaire. And to analyze validity of module, te module must have good criteria such as: self instructional, self contained, stand alone, adaptive, used friendly and consistent.

e. Formulation of Learning Purpose

This analysis intended to convert the purpose of assignment analysis and concept analysis into specific learning objectives that expressed by the behaviour of student in learning. In this study, the purpose of learning module are 1) students can explaine concept of algebraic equation as relational equivalence , 2) students has relational thingking skill in problem solving.

Stage 2: Design

Product is developed in form a module which is based on relational thingking. This module for 7th grade in Junior High School. The development of this module consist of three criteria: visual design, content component and text or language design. Result of design is called **draft-1**. The activity of this stage begun from identify indicator of learning objective in algebraic equation, identify basic competence, initial design of mathematics module in 7th grade in Junior High School, and drafting these module manuscript.

Physical design the module consist of outers cover and the size of the yard. The page size is A4 with a weight of 80 grams as standar ISO (International Organization for Standardization). Text design with Times New Roman type 12pt and the visual design dominate for green colour and equivalance picture.

Therefore, the component of the contents of the module that was developed in modules include an introduction, table of contents, instructions for use, the parts of the module, algebraic equation testing, and self assessment.

Stage 3: Develop

At this stage there are four activities, namely;

a. Validation

Stage of develop, started with validation by expert then validation are conducted to the mathematics module is designed in the stage 2 (draft-1). Subject matter experts as three person who have competence in the field of mathematics, especially algebra (one lecturer of mathematics education), the person competent in the field of graphics (one lecturer of educational technology) and the person competent in language (one lecturer of English language) . Validation are conducted content material, format and language that is used. Aspect that are observed namely: whether subject matter, whether sentences in module is easy to understand, whether sentences in module is not ambiguous, whether display and content module is interesting, whether the questions varied, and whether sentences in questions is not ambiguous. Data Revision for module presented in table 4.1 as follow:
Table 4.1 The Result Validity for Module (Draft-1) By Experts

| Aspect            | Experts | The Average Score For Each Aspect | Category |
|-------------------|---------|-----------------------------------|----------|
| self instructional| 4 3     | 3,3                               | Valid    |
| self contained    | 2 3     | 2,6                               | Less Valid |
| stand alone       | 3 3     | 2,6                               | Less Valid |
| adaptive          | 3 2     | 2,6                               | Less Valid |
| used friendly     | 3 3     | 3                                 | Less Valid |
| consistent        | 3 4     | 3,6                               | Valid    |

(Source: validation sheet)

b. Revise

Suggestion of experts is used to completion the module. Refinement have been done formatting, grammar (so contains no more sense) and graph or pictures (so has a meaning that can be understood by student). Result of revised module is called draft-2. Data Revision for module presented in table 4.2 as follow:

Table 4.2 The Result Revision for Module By Experts

| Recommendation From Expert | Second Revision |
|----------------------------|-----------------|
| Content: It is advisable use more proper words | Problem of Algebraic Equations has been revised. |
| Display : Reproduce more images | Relevant images has been added |
| Language : There are several misspelled | Writting revision |

(Source: validation sheet)

c. Trials

After get draft-2, the next activity is doing trials on limited group. The trials as called as limited because of its dependence on the scedule learning materials in school where tested. Trial was conducted in two school in Kediri namely: SMP Ar-Risalah Lirboyo Kediri City with 18 students for a limited trial and MTs Al Ikhlas Kediri Distric with 20 students for a limited trial, because according Sadiman (2012) that the small group trial awarded to 10-20 students who can represent the target population. After students learning mathematics module, they get algebraic test and quotionare. Aspect of observed consist of: whether sentences in module is easy to understand, whether sentences in module is not ambigous, whether display and content module is interesting, whether the questions varied, and whether sentences in questions is not ambigous.

d. 2nd Revise

After trials has been done, researcher revise the module (draft-2). The revisons are based on invention of trial result. Data after 2nd revision for module presented in table 4.2 as follow:
### Table 4.2 The Result Validity of Module For Increase Relational Thingking Skill

| Aspect                     | Content and Category                                                                 |
|----------------------------|---------------------------------------------------------------------------------------|
| validity of the contents   | 84.21% (valid) → 32 respondens (students) from 38 respondens said that the algebraic equation problem in module is vary |
| validity for language      | 86.84% (valid) → 33 respondens (students) from 38 respondens said that the sentence of module is easy to understand, and the sentences does not have double meanings. |
| validity of display        | 84.21% (valid) → 32 respondens (students) from 38 respondens said that display of module is interest. |
| students response          | 94.73% (positive response) → 36 respondens (students) from 38 respondens interesting for applying module in mathematics learning. |
| score of relational rubric | Good Average → total score archive in high category.                                    |

(Source: validation sheet)

After 2nd revision for module, researcher get the final product as the valid module as media for learning. The final module presented in figure 4.1 as follow:

![Figure 4.1 The Presentation of Final project (Source: author)](image)

The final module can easy used and it can increase student ability in solving algebra problems.

**CONCLUSION**

This module based relational thingking may provide an oppotunity for student to learn and have to skill to observe and understand equivalence concept and they get flexibility in algebraic equation problem. Students also practice solving algebraic equation problem with relational thingking. This module also have advantages it is use as a companion teaching materials that can developed problem solving ability in learning mathematics.

Researcher hope taht next time, this module can develope until disseminate stage.
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REFERENCES
[1] Anwar, Ilham. 2010. *Pengembangan Bahan Ajar. Bahan Kuliah Online*. Direktori UPI, Bandung.
[2] Baiduri. 2011. *Profil Berpikir Relasional Siswa SD Menyelesaikan Masalah Matematika ditinjau dari Kemampuan Matematika dan Gender*. Disertasi tidak dipublikasikan. UNESA Surabaya.
[3] Carpenter, T. P., Levi, L., Frankie, M. L., & Zeringue, J. K. (2005) *Algebra In Elementary School: Developing Relational Thingking*, ZDM, 37(1), pp 53-59.
[4] Depdiknas. 2006. *Kurikulum Tingkat satuan Pendidikan*. Jakarta: Pusat Kurikulum, Balitbangdiknas, Jakarta.
[5] Depdiknas, 2013. *Kurikulum 2013*. Jakarta: Pusat Kurikulum, Balitbangdiknas, Jakarta.
[6] Doumas. L. A. A. and Hummel. J. E. 2005. *Approaches To Modeling Human Mental Representations: What Works, What Doesn’t, And Why*. In Holyoak. K.J and Morisson. R.G (Ed). The Cambridge Handbook of Thinking and Reasoning. Cambridge. University Press
[7] Sadiman, dkk. 2012. *Media Pendidikan*. Jakarta: Rajagrafindo Persada
[8] Solso, Robert L. 1995. *Cognitive Psychology*. Boston. Allyn and Bacon
[9] Stephens, A. C. 2006. *Equivalence and Relational Thingking: Preservice Elementary Teacher Awareness Of Opportunities and Misconceptions*. Journal of mathematics teacher education, 9, pp 249-278
[10]Sugiyono. 2008. *Memahami Penelitian Kualitatif*. Bandung: CV.ALFABETA
[11]Thiagarajan and Semmel & Semmel. 1974. *Instructional Development for Training Teachers of Exceptional Children*. Bloomington: Indiana University.
[12]Winkel. 2009. *Psikologi Pengajaran*. Yogyakarta : Media Abadi