THE FEASIBILITY OF OBTAINING FINITE SOURCE SIZES FROM MACHO-TYPE MICROLENSESING LIGHT CURVES

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ABSTRACT

Recent discussion of the effects of finite source size on high-magnification microlensing events due to MACHOs motivates a study of the feasibility of observing such effects and extracting the source radius. Simulated observations are generated by adding Gaussian error to points sampled on theoretical microlensing light curves for a limb-darkened, extended source. These simulated data sets are fitted in an attempt to see how well the fits extract the radius of the source. The source size can be fitted with reasonable accuracy only if the impact parameter of the event, \( p \), is less than the stellar radius, \( R_* \). It is possible to distinguish “crossing” events, ones where \( p < R_* \), from “noncrossing” events if the light curve is well sampled around the peak and photometric error is small—i.e., \( \geq 3 \) observations while the lens transits the disk of the source, and \( \sigma_{\text{phot}} < 0.08 \) mag. These requirements are just within the reach of current observational programs; the use of an early-warning system and multiple observing sites should increase the likelihood that \( R_* \) can be fitted. The programs used to simulate and fit data can be obtained via anonymous ftp.

Subject headings: gravitational lensing — stars: low-mass, brown dwarfs

1. INTRODUCTION

Paczyński (1986) noted that if the dark matter constituents of the halo are massive enough, then the optical depth to microlensing is on the order of \( 10^{-6} \). In the past few years, searches for gravitational microlensing events in the LMC and the Galactic bulge have discovered many plausible candidates through long-term monitoring of millions of stars (OGLE: Optical Gravitational Lensing Experiment, Udalski et al. 1993; EROS: Expérience de Recherche d’Objets Sombres, Aubourg et al. 1993; MACHO: Massive Compact Halo Objects collaboration, Alcock et al. 1993; DUO: Disk Unseen Objects program, Alard et al. 1995). All these projects share the goal of trying to measure the amount of dark matter in the Galactic halo that is in the form of massive compact halo objects (MACHOs).

This study is motivated by recent papers that show the shape of the light curve around the peak to be considerably different from the one predicted for a point source in events where the impact parameter is less than or equal to the source radius (Witt & Mao 1994; Nemiroff & Wickramasinghe 1994; Gould 1994). The ability to use the light curve of an event to determine the source size, \( R_* \), where \( R_E \) is the Einstein radius, is valuable because we already have an independent estimate of \( R_* \) using spectral type. This allows us to use the shape of the light curve to obtain a direct estimate of the value of \( R_E \), which helps to obtain a more accurate estimate of the mass of the lens. The traditional way of estimating \( R_E \) requires assumptions of the statistical measures of the relative velocities. Also, knowing the values of \( R_E \) and \( t_0 \), the time it takes for the lens to traverse a distance \( R_E \) relative to the source in the deflection plane, gives us the transverse velocity.

The approach taken in this paper is limited to single-lens microlensing. Witt (1995) predicts that 3% of microlensing events caused by lenses in the Galactic bulge should show noticeable effect due to the finite source size. Deviations from the point-source light curve due to effects such as parallax and binary lenses were ignored for the purposes of this study. Parallax effects have been observed, but occur on timescales much larger than those on which finite source size effects occur (Alcock et al. 1995). Binary lens events, while not insignificant, should only make up \( \sim 10\% \) of microlensing candidates toward the Galactic bulge (Mao & Paczyński 1991). However, in the future the technique used in this study may also be used to investigate these types of deviations.

While it is important to identify the possible effects of finite source size on the light curve, it is equally important to test the feasibility of observing such effects. After a systematic study of generated light curves for single-lens events, this paper determines a lower limit on the sampling rate and photometric accuracy of observations needed to reliably extract \( R_* \) from an event. The outline of the paper is as follows: in § 2, the general method for generating light curves and the approach to feasibility testing are discussed; in § 3, the results are presented and recommendations for observing programs are made.

2. METHOD

The shape of the light curve for a gravitationally lensed extended source can be fully described by five parameters: \( p \), the impact parameter, which is the angular separation between the lens and the unlensed position of the source at the time of maximum magnification (from now on given in units of \( R_E \)); \( R_* \), the source radius (also given in units of \( R_E \)); \( t_0 \), the time it takes for the lens to traverse a distance \( R_E \) relative to the source in the deflection plane; \( t_{\text{max}} \), the time of maximum magnification; and \( m_{\text{obs}} \), the unlensed or baseline magnitude of the source.

The Einstein ring radius, the distance an image would appear from the line of sight if the lens and source were exactly aligned, is given by

\[
R_E^2 = \frac{4 G M D}{c^2} , \quad D \equiv \frac{D_d(D_s - D_d)}{D_s} , \quad (1)
\]

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In this paper, however, all sources are considered to be the trajectory of the lens is derived by Mao & Witt (1994).

The expression for the observed magnitude of a disk of constant surface brightness as a function of position along the trajectory of the lens is derived by Witt & Mao (1994).

$$\frac{p}{R_{E}} = 0.0, 0.055, 0.2, 0.5.$$ The timescale $t_\ast$ is defined as the time it takes the lens to move a distance $R_E$ with respect to the source, where $R_E$ is the Einstein ring radius. The other input values for these curves are $M_{lon} = 0.1 M_\odot, D_L = 8$ kpc, $D_S = 9$ kpc, $R_\ast = 10 R_\odot$, and $V = 100$ km s$^{-1}$. This gives $R_\ast/R_E = 0.055$. The $p = 0$ event is a “crossing” event; the $p = 0.055$ event is one where the lens grazes the edge of the source; the other two are “noncrossing” events.

where $D_L$ and $D_S$ are the distances to the lens of mass $M_\ast$ and to the source, respectively.

The ability to simulate observations not only gives one a large number of sample data sets on which to test potential observational programs, but it also facilitates error analysis by allowing one to compare the values extracted by a fitting routine with the “true” input values. In this model, input parameters were chosen and the theoretical light curve was evenly sampled within each daily observing period, with sampling frequency $n$. In theory, the length of this daily observing period can range from 1 to 24 hr. The photometric errors were simulated by adding a random Gaussian error in magnitude.

$$\sigma_{\text{phot}}(m) = \sigma_0 10^{(m - m_0)/3.5}, \quad \sigma_0 = \sigma_{\text{phot}}(m_0)$$

(Udalski et al. 1994a). Values for $\sigma_0$ range from 0.01 to 0.25 mag. This dependence was derived from an empirical fit to median OGLE error at different magnitudes. In the future, with better statistics, we will be able to use error histograms to empirically fit the error distribution, and thereby avoid making assumptions of the Gaussian nature of the error.

To test for fitting accuracy, a simulated data set is generated with specific, known parameters. Then, a best-fit microlensing light curve is obtained with a program that uses the Levenberg-Marquardt method for nonlinear fits in multiple dimensions to minimize $\chi^2$ (Press et al. 1992). Figure 2 shows a set of simulated observations and the best-fit light curve. The rms photometric error at the baseline magnitude, $\sigma_0$, was taken to be 0.1 mag.

Many different data sets were created and fitted with the goal of investigating error in the extracted source radius as a function of $p, \sigma_0, n$, and the length of the daily observing period. There are a large number of plausible lensing configurations, and it would be extremely time-consuming and computer-intensive to examine them all. It was the goal of this study to examine a few extreme cases and a few pertinent ones so that trends in error would be evident, and so that rough, conservative guidelines could be suggested for observing programs. Those interested in testing different cases can access the original C code used to simulate and fit observations at astro.princeton.edu via anonymous ftp. After login, change directory to bp/finite. The read.me file contains the information about all other files, their names and sizes.

3. RESULTS AND DISCUSSION

In the following discussion, the quantity $R_{\ast,\text{fit}}$ is used to describe the fitted value for $R_\ast$; the value $p_{\text{fit}}$ describes the fitted value for $p$. Figure 3a plots the percent error in $R_{\ast,\text{fit}}$ against $p_{\text{fit}}$. Every point represents the fit to simulated observations of an event with random $p$. The fitted value of the impact parameter is plotted on the $x$-axis (as opposed to $p$) because that is what is measured in real data sets. Each extracted value of $R_\ast$ has an error $(R_{\ast,\text{fit}} - R_\ast)$. With the exception of $p$, all the input parameters are identical for limb-darkened disks with limb-darkening parameter 0.6, using the formulation of Aller (1953).

When the source and lens move relative to each other in the deflection plane, the magnification varies with time. An example of such a light curve is depicted in Figure 1. Each light curve is labeled with the impact parameter of the source trajectory. Time is given in units of $t_\ast$, and $R_\ast$ for all curves is 0.055 $R_E$. Parts of the computer code used to generate these light curves were obtained from B. Paczynski.
Fig. 3.—(a) Percent error in \( R_{*, \text{fit}} \) against \( \rho_{\text{fit}} \). Error in the fitted source size is small if \( p < R_* \). When \( p > R_* \), errors rapidly increase. Every point represents a simulated observation for an event with a random \( p \) and all other parameters held constant. Points represented by crosses are “crossing” event simulations. Circles signify “noncrossing” event simulations with the radius of the circle proportional to \( p - R_* \). In addition to the input parameters used in Fig. 1, \( m_0 = 17.0 \text{ mag}, \sigma_0 = 0.0222 \text{ mag}, \) and 24 hour coverage is assumed. The dashed line is the line on which \( \rho_{\text{fit}} = \rho_{*, \text{fit}} \). All points to the left of this line represent simulations in which \( \rho_{\text{fit}} < \rho_{*, \text{fit}} \), i.e., apparent crossing events. All points to the right of this line are simulations in which \( \rho_{\text{fit}} > \rho_{*, \text{fit}} \); apparent noncrossing events. If all apparent crossings correspond to true crossings, then we are able, from the fitted parameters alone, to distinguish true crossings from true noncrossings. This figure exhibits a scenario in which this is true. All “crossing” events were fitted with relatively high accuracy. (b) Percent error in \( R_{*, \text{fit}} \) against the error in \( \rho_{*, \text{fit}} \). The relationship between \( \Delta R_*, R_{*, \text{fit}} - R_*, \) and \( \Delta \rho \) is linear for \( \Delta R_* > 0 \). The slope of this relationship is a function of the photometric errors and the sampling. Larger errors and fewer observations cause the slope to increase. If this “error slope,” \( \Delta R_*/\Delta \rho \), has a value less than 1, i.e., less than that of the dashed line in Fig. 3a, then apparent crossings will correspond to true crossings.

Fig. 3a also illustrates how photometric error quickly blurs any information on the source radius when \( p > R_* \). On the other hand, for “crossing” events, fitting is fairly reliable. The question that follows is: for real observations, where we do not know nature’s input values, can we distinguish “crossing” data sets from “noncrossing” ones? The answer to this question is yes, if our observations are accurate enough.

The dashed line in Fig. 3a is the line on which \( \rho_{\text{fit}} = R_{*, \text{fit}} \). All points to the left of this line represent simulations in which \( \rho_{\text{fit}} < R_{*, \text{fit}} \), i.e., apparent crossing events. All points to the right of this line are simulations in which \( \rho_{\text{fit}} > R_{*, \text{fit}} \), apparent noncrossing events. If all apparent crossings correspond to true crossings, i.e., all points to the left of the line are crosses, then we are able, from the fitted parameters alone, to distinguish true crossings from true noncrossings. Figure 3a shows a scenario in which this is true. Likewise, all apparent noncrossing events (right of line) correspond to true noncrossing events (circles).

Fig. 3b displays the linear relationship between \( \Delta \rho \), the error in \( \rho_{*, \text{fit}} \) and \( \Delta R_* \), the error in \( R_{*, \text{fit}} \). As in Fig. 3a, every point represents simulated observations of an event with a random \( p \) and all other input parameters held constant. We would expect errors in \( R_{*, \text{fit}} \) and errors in \( \rho_{*, \text{fit}} \) to be coupled because the light curve is most sensitive to both of these parameters around the time of maximum magnification. Hence, the greater the error in the fitted impact parameter, the greater the error in the fitted stellar radius needs to be in order to fit the points on the light curve. The slope of this relationship is a function of the photometric errors and the sampling. Larger errors and fewer observations cause the slope to increase. If this “error slope,” \( \Delta R_*/\Delta \rho \), has a value less than 1, i.e., less than that of the dashed line in Fig. 3a, then the relation between \( p \) and \( R_* \) will be the same for both input and extracted values, meaning that

\[
p < R_* \Leftrightarrow \rho_{\text{fit}} < R_{*, \text{fit}}.
\]

As a result, we should be able to distinguish between “crossing” events and “noncrossing” events using just the fitted values.

The error slope depends on many factors. For any given lensing geometry, the most important ones are the baseline photometric error, \( \sigma_0 \), and the observational coverage, determined by both the sampling frequency, \( n \), and the length of the daily observing period.

3.1. Photometric Error

Assuming that there are an adequate number of evenly spaced observations on a light curve, the photometric error is the quantity that defines the error slope. The \( \chi^2 \) surface as a function of both \( p \) and \( R_* \) possesses a valley in which there is a global minimum. The location of this minimum in the valley is very sensitive to errors in photometry; this can cause it to move significantly. Error slopes were determined using least-squares fitting in plots like Fig. 3b. Only points for which \( \Delta R_{*, \text{fit}} > 0 \) were fitted. This is because (1) the fact that \( R_{*, \text{fit}} \) cannot be negative skews the slope for points where \( \Delta R_{*, \text{fit}} < 0 \), and (2) these points will never cross into the \( \rho_{\text{fit}} < R_{*, \text{fit}} \) regime, as exhibited by the lower half of Figure 3a. The trend derived from this rough analysis shows that for \( \sigma_0 > 0.08 \text{ mag}, \) the error slope is greater than 1 and grows slowly with increasing \( \sigma_0 \); for \( \sigma_0 < 0.08 \text{ mag}, \) the source radius from a light curve if the lens does not cross the disk of the source.
the error slope decreases rapidly for decreasing $\sigma_0$. The value $\sigma_0 = 0.08$ mag can be used as a rough upper limit on the photometric error.

However, each lensing configuration is different and some may be more tolerant of photometric errors than others. Since $\sigma_0$ is the baseline photometric error, and the part of the light curve with which we are concerned is the peak, the error slope will also be a function of lensing amplitude—photometric error decreases as the source brightens. In events where the source radius is large ($R_\ast > 0.5R_E$), the total magnification is significantly lower than for the point-source case, and the resulting photometric errors at the peak are relatively large. However, in these events the effect of the finite disk is so pronounced that, overall, it is still possible to obtain a reasonable fit to the source radius provided there are an adequate number of observations during the crossing of the disk by the lens.

3.2. Sampling Frequency

Even with dense sampling, errors in photometry still prevent fitting from being perfect. Sparse coverage of an event can cause a fitting routine to wander aimlessly, not possessing enough information to determine the global minimum. Frequent and evenly spaced measurements around the time of the peak magnification are essential for good convergence of the fitting routine.

Experiments with the fitting of simulated data sets have shown that some lensing configurations will be more forgiving toward sparse sampling than others. The smaller the impact parameter and the larger the source radius in units of $R_E$, the easier it is to “resolve” the disk. The most important result obtained from these simulations is that regardless of how high the sampling frequency is on other parts of the light curve, there is very little hope of fitting the source radius without at least one observation while the lens is transiting the disk of the source.

A conservative estimate requires at least three observations during the disk crossing in order to have an error slope less than 1. The timing of the observations is most important for a good fit. A single observation exactly at the time of maximum magnification will often result in a good extraction of the source radius. In a sense, it is in an effort to obtain this one point that we must take as many observations as possible around the time of maximum. Multiple observations also reduce the effect of photometric errors and increase the certainty of the fit.

3.3. Observational Programs

Since the time it takes for a lens to cross the disk of a source is usually on the order of one day (and that is for $p = 0$), multisite coverage, while not an absolute necessity, would greatly increase the chances of resolving a stellar disk. If we assume that there are only 8 hr a night during which a group can observe at one site, then the disk crossing of an event with $t_0 \sim 4$ days and $R_\ast = 0.05R_E$ has a window of only 5 hr, and can be missed entirely if timing is less than fortuitous. Moreover, the light curves of short $t_0$ events are more prone to exhibit effects due to finite source size because they are likely to involve small lensing masses, which implies a small $R_E$ and a large $R_\ast/R_E$. Early-warning systems, like the ones currently used by the MACHO and OGLE groups (Udalski et al. 1994b), allow resources to be concentrated on a single event and will increase the density of photometry around the time of maximum magnification. Multisite monitoring of microlensing events, with telescopes in Australia, South Africa, and Chile, has already been initiated by the PLANET collaboration (Probing Lensing Anomalies Network; Albrow et al. 1995).

To a limited extent, good photometry can compensate for less than ideal observational coverage and vice versa. However, good quality in both respects is needed to increase the chances that observations of the light curve of a “crossing” event will still contain information on the source size. Nemiroff & Wickramasinghe (1994) emphasize the importance of having enough photometry in order to resolve inflection points in the light curve. However, this demanding criterion need not be met since the source size can be fitted directly as a free parameter, a technique that requires a much lower sampling rate.

The error in $R_{\ast, \text{fit}}$ shows no discernible trend as a function of $t_0$, $f_{\text{max}}$, or $m_0$.

Time resolution and coverage are currently the limiting factors in microlensing observations. Photometric errors in OGLE observations are already within the prescribed limits. Present observational programs that are fortunate enough to detect an extremely high magnification event in the early stages will most likely be able to determine whether or not it is a “crossing” event. For shorter $t_0$ events, the ability to take observations from more than one site coupled with an early-warning system greatly increases the chances of resolving stellar disks.

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REFERENCES

Alard, C. 1995, in IAU Symp. 173, Astrophysical Applications of Gravitational Lensing, ed. C. S. Kochanek & J. N. Hewitt (Dordrecht: Kluwer), 215

Albrow, M., et al. 1995, in IAU Symp. 173, Astrophysical Applications of Gravitational Lensing, ed. C. S. Kochanek & J. N. Hewitt (Dordrecht: Kluwer), 227

Alcock, C. A., et al. 1993, Nature 365, 621

1995, ApJ, 454, L125

Aller, L. H. 1953, Astrophysics: The Atmospheres of the Sun and Stars (New York: Ronald), 207

Aubourg, E., et al. 1993, Nature, 365, 623

1995, ApJ, 421, L71

Mao, S., & Paczynski, B. 1991, ApJ, 374, L37

Nemiroff, R. J., & Wickramasinghe, W. A. D. T. 1994, ApJ, 424, L21

Paczynski, B. 1986, ApJ, 304, L1

Press, W., Teukolsky, S., Vetterling, W., & Flannery, B. 1992, Numerical Recipes in C, 2nd ed. (New York: Cambridge Univ. Press), 683

Udalski, A., Szymański, M., Kalużyński, J., Kubiaκ, M., Krzemieński, W., Mateo, M., Preston, G. W., & Paczynski, B. 1993, Acta Astron., 43, 289

Udalski, A., Szymański, M., Kalużyński, J., Kubiaκ, M., Mateo, M., Krzemieński, W., & Paczynski, B. 1994b, Acta Astron., 44, 227

Udalski, A., Szymański, M., Stanek, K. Z., Kalużyński, J., Kubiaκ, M., Mateo, M., Krzemieński, W., Paczynski, B., & Venkat, R. 1994a, Acta Astron., 44, 165

Witt, H. J. 1995, ApJ, 449, 42

Witt, H. J., & Mao, S. 1994, ApJ, 430, 505