Abstract

We present a family of spherically symmetric spacetimes, specified by the density profile of a vacuum dark energy, which have the same global structure as the de Sitter spacetime but the reduced symmetry which leads to a time-evolving and spatially inhomogeneous cosmological term. It connects smoothly two de Sitter vacua with different values of cosmological constant and corresponds to anisotropic vacuum dark fluid defined by symmetry of its stress-energy tensor which is invariant under the radial boosts. This family contains a special class distinguished by dynamics of evaporation of a cosmological horizon which evolves to the triple horizon with the finite entropy, zero temperature, zero curvature, infinite positive specific heat, and infinite scrambling time. Non-zero value of the cosmological constant in the triple-horizon spacetime is tightly fixed by quantum dynamics of evaporation of the cosmological horizon.
The holographic principle was first formulated [1] as the requirement that the number of independent quantum degrees of freedom \( N_d \) contained in a given spatial volume \( V \) is bounded from above by the surface area of the region, \( N_d(V) \leq A/4 \). It led to a conjecture that a physical system can be completely specified by data stored on its boundary, which obtained the name of holographic principle [2, 3]. This view was strongly supported by the AdS/CFT duality between quantum gravity in AdS space and a certain conformal field theory on its boundary [4] which defines quantum gravity non-perturbatively and involves only physical degrees of freedom admitted by the holographic principle [5].

Astronomical observations testify for an early inflationary stage in our universe evolution and provide convincing evidence that now it is dominated at above 70 % of its density by a dark energy responsible for its accelerated expansion due to negative pressure, \( p = w \rho, \ w < -1/3 \) [6], with the best fit \( w = -1 \) [7] corresponding to a positive cosmological constant \( \lambda \), so that we are living in the universe asymptotically de Sitter in both past and future.

The question of existence of a holographic duality between quantum gravity in de Sitter space and an Euclidean conformal field theory on its boundary, DS/CFT correspondence [8], is one of the most intriguing questions discussed in the literature. A holographic description of de Sitter space requires the symmetries of de Sitter space to act on its asymptotic boundary as conformal mappings [9]. It encounters the de Sitter complementarity principle [9] referred to also as observer complementarity [10] since it represents the symmetry relating observations of observers passing out of each other’s static patches [11]. The key point is that the de Sitter horizon is observer-dependent [12]. Each observer sees the surrounding spacetime as a finite closed cavity bounded by his horizon with the finite temperature and entropy. The de Sitter complementarity precludes the existence of the appropriate limits for the boundary correlators in the DS/CFT correspondence if the entropy is finite [9].

An additional trouble comes from the fact that de Sitter horizon is a fast scrambler [11]. The scrambling time for a quantum system at temperature \( T \) is the time needed to thermalize information, given that the interactions are between bounded clusters of degrees of freedom [13]. A universal bound on scrambling time \( t_* \) for a given entropy \( S \) can be estimated as [11]

\[
t_* \geq A T^{-1} \hbar S^{1/2}
\]

where \( A \) is a certain constant. For de Sitter horizon \( t_* = T^{-1} l \ln(l/l_P) \) [11] where \( T \) is the temperature of the horizon \( l \). This qualifies it as a fast scrambler and leads to the conclusion that it cannot be described by any kind of two-dimensional field theory [11], although there could exist a dual description based on the matrix quantum mechanics [14]. However, the question of consistence with observer complementarity remains open - different static patches are related by the non-compact \( O(4, 1) \) symmetry of de Sitter space, and the problem is how to implement such a symmetry on the matrix degrees of freedom which describe only a single static patch [11]. In [15] the no-go theorem was proved that the non-compact \( O(4, 1) \) symmetry of de Sitter space cannot be realized if the entropy is finite [11] with the conclusion that if the entropy is finite then the symmetry of different causal patches must be broken [15].

In this note we present a family of spherically symmetric spacetimes, specified by the density profile of a vacuum dark energy, which has the same global structure as the de Sitter spacetime but the reduced symmetry which leads to a time-evolving and spatially inhomogeneous cosmological term. This family contains a special class distinguished by dynamics of evaporation of a horizon which evolves to thermodynamically stable state with an infinite scrambling time.
The Einstein cosmological term $\Lambda g_{ik}$ corresponds to the maximally symmetric stress-energy tensor

$$T_k^i = \rho_{\text{vac}} \delta_k^i; \quad \rho_{\text{vac}} = (8\pi G)^{-1} \Lambda$$

which is responsible for the high symmetry of de Sitter spacetime. The tensor (2) was interpreted as a Lorentz-invariant vacuum fluid by Lemaître as early as in 1934 [16], and in 1965 Gliner supplemented this interpretation with the elegant argument: the medium specified by (2) has an infinite set of co-moving reference frames, so that an observer cannot in principle measure his/her velocity with respect to it [17], which is the intrinsic property of a vacuum [18].

The model-independent way to make $\Lambda$ variable without introducing additional entities is to reduce symmetry of (2) (which enforces $\Lambda = \text{const}$ by $T_k^i = 0$) while keeping its vacuum identity, i.e. Lorentz-invariance in one or two spatial direction and thus impossibility to fix some of velocity components. A stress-energy tensor with a reduced symmetry such that only one or two of its spatial eigenvalues coincides with the temporal eigenvalue, represents a vacuum dark fluid with the equation of state $p = -\rho$ in the distinguished direction(s) [19], which makes it intrinsically anisotropic. This immediately promotes the cosmological constant $\Lambda$ to a dynamical component $\Lambda^0$ of a variable (by $T_k^i = 0$) cosmological term $\Lambda_k^i = 8\pi G T_k^i$ [20].

The spherically symmetric vacuum is specified by [21]

$$T_0^0 = T_1^1 \quad (p_r = -\rho)$$

Transversal pressure is given by

$$p_\perp = -\rho - \frac{r}{2} \rho'$$

and the metric has the form

$$ds^2 = g(r) dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2$$

with the metric function

$$g(r) = 1 - \frac{2GM(r)}{r}; \quad M(r) = 4\pi \int_0^r \rho(x)x^2 dx$$

(6)

For the source terms satisfying the weak energy condition, regular solutions have obligatory de Sitter center and monotonically decreasing density $\rho(r)$ [22]. Therefore we can separate in (6) the minimal vacuum density $\rho_\lambda$ as the background density. If we introduce $\rho(r) = \rho_d(r) + \rho_\lambda$ where $\rho_d(r)$ is a dynamical density decreasing from the value at the center $\rho_0 = (8\pi G)^{-1}\Lambda$ to zero at infinity, ”cosmological constant” $\Lambda_0^0$ evolves monotonically from $\Lambda + \lambda$ to $\lambda$ [22]. In the case of two vacuum scales, $\Lambda = 8\pi G \rho_0$ at the center, and $\lambda = 8\pi G \rho_\lambda$ at infinity, geometries are defined by three parameters: two length scales related to limiting vacuum densities, $r_0 = \sqrt{3/\Lambda}$ and $l = \sqrt{3/\lambda}$, and the mass parameter $M = 4\pi \int_0^\infty \rho_d(r)r^2 dr$.

The curvature scalar

$$R = 8\pi G (4\rho + r \rho') = -32\pi G p_\perp$$

(7)

can have two zero points since $p_\perp$ is negative for $r \to 0$ and for $r \to \infty$.

The behavior of the metric function $g(r)$ is dictated by the number of vacuum scales through behavior of the transversal pressure $p_\perp$ in a source term. By the Einstein equation [18]

$$8\pi G p_\perp = g''/2 + g'/r$$

(8)

it determines the maximal number and character of $g(r)$ extrema which in turn determines the maximal number of spacetime horizons [23]. In the case of two vacuum scales, $p_\perp = -\rho$ as $r \to 0$.
and as \( r \to \infty \), hence \( g(r) \) can have two maxima (one obligatory in de Sitter center \( r = 0 \)), one minimum and not more than three zeros, and spacetime can thus have not more than three horizons [24]. In the 3-horizon case the metric (5) describes a regular black hole with de Sitter center in the asymptotically de Sitter space [25] shown in Fig.1.

![Figure 1: Typical behavior of a metric function \( g(r) \) for the case of three horizons: a black hole horizon \( r_b \), a cosmological horizon \( r_c \), and an internal horizon \( r_a \) related to the de Sitter center.](image1)

For a black hole the mass parameter is confined within \( M_{cr1} < M < M_{cr2} \). Values \( M_{cr1} \) and \( M_{cr2} \) depend on the parameter \( q = \sqrt{\Lambda/\lambda} \) [25]. Double-horizon configurations \( M_{cr1} \) and \( M_{cr2} \) represent, respectively, an extreme black hole and a regular modification of the Nariai solution. Configurations \( M < M_{cr1} \) and \( M > M_{cr2} \) correspond to one-horizon spacetimes (see Fig.2).

![Figure 2: Metric function for spacetime with de Sitter center asymptotically de Sitter as \( r \to \infty \).](image2)

Cosmological evolution of an anisotropic fluid is described by the Lemaître class models. The static spherically symmetric metric (5) can always be transformed to the Lemaître form [24, 26, 23] with the line element [18]

\[
ds^2 = d\tau^2 - e^{\nu(R,\tau)}dR^2 - r^2(R,\tau)d\Omega^2,
\]

where \( e^{\nu(R,\tau)} = r'^2 = (\partial r/\partial R)^2 \) for the case when each 3-hypersurface \( \tau = \text{const} \) is flat [18], which guarantees fulfilment of the spatial flatness condition \( \Omega = 1 \) required by observations.
The function $r(R, \tau)$ is found by integration of the Einstein equations which gives [27, 24]

$$\tau - \tau_0(R) = \int \frac{dr}{\sqrt{2GM(r)/r}}; \quad \mathcal{M}(r) = 4\pi \int_0^r \rho(x)x^2dx$$

and the function $e^{\nu(R, \tau)}$ is found from (10) as $e^{\nu(R, \tau)} = 2GM(r(R, \tau))/r(R, \tau)(\tau'_0(R))^2$ [24, 26].

For the case of the source terms specified by (3), regular cosmological solutions are asymptotically de Sitter at the early and late time, and can describe evolution from an inflationary beginning to the late accelerated expansion, guided by dynamical vacuum dark energy closely related to spacetime symmetry [24, 23].

Each one-horizon spacetime has the global structure of de Sitter spacetime [26], but spacetime symmetry is essentially different. The horizon $r_h$ is not observer-dependent as in the de Sitter case, but real horizon due to reduced symmetry (3) which breaks the $O(4,1)$ symmetries by involving the distinguished center $r = 0$.

An observer in the R-region $0 \leq r < r_h$ (a static causal patch) can detect the Hawking radiation from the cosmological horizon $r_h$. Dynamics of quantum evaporation of horizon can tell us which of one-horizon spacetimes is the most appropriate to represent evolution from the de Sitter beginning to the de Sitter end.

The Gibbons-Hawking temperature on a horizon $r_h$ with a surface gravity $\kappa_h$ is given by [12]

$$kT_H = \frac{\hbar}{4\pi c} |g'(r_h)|$$

A general approach for defining thermodynamical variables in a multi-horizon spacetime with non-zero pressure was developed by Padmanabhan [28]. He considered a canonical ensemble of spacetime metrics from the class (5) at the constant temperature of the horizon determined by the periodicity of the Euclidean time in the Euclidean continuation of the Einstein action. The partition function calculated as the path integral sum [28], can be written as

$$Z(kT_h) = Z_0 \exp \left[ \frac{1}{4} \left( 4\pi r_h^2 \right) - \frac{1}{kT_h} \left( \frac{|g'| r_h}{g'} \right)^2 \right] \propto \exp \left[ S(r_h) - \frac{E(r_h)}{kT_h} \right]$$

which gives [28] (in the geometrical units $c = G = \hbar = 1$)

$$S = \pi r_h^2; \quad E = \frac{|g'|}{g} \left( \frac{A_h}{16\pi} \right)^{1/2} = \frac{|g'| r_h}{g} 2$$

where $S$ is the entropy, $E$ is the thermodynamical energy, and $A_h$ is the horizon area.

A specific heat, $C_h = dE_h/dT_h$ is given by [29]

$$C_h = \frac{2\pi r_h}{g'(r_h)+g''(r_h)r_h}$$

The cosmological horizon $r_h$ of an observer in the R-region $0 \leq r < r_h$, is the boundary of his manifold, and the second law of horizons thermodynamics reads $dS_h \geq 0$ ([30] and references therein). It requires $dt_h \geq 0$, so that the horizon moves outwards. Increasing of a cosmological horizon leads to decreasing of the mass parameter $M$ [31, 29, 30].

One-horizon spacetime with $M > M_{\text{cr}}$ and three extrema, evolves during evaporation towards the double-horizon spacetime with $M = M_{\text{cr}}$ with zero temperature by virtue of (11). It is thermodynamically unstable since in a maximum $g' = 0$, $g'' < 0$ and hence $C_h < 0$. 

A specific heat, $C_h = dE_h/dT_h$ is given by [29]
Now let us consider a one-horizon spacetime with the metric function shown in Fig.3 which does not have extrema in the T-region \( r > r_h \) but an inflection point distinguished by two conditions: 
\[
g'(r_i) = 0; g''(r_i) = 0.
\]

For one-horizon spacetime with the inflection point, specific heat of the horizon is always positive. Its sign is the same as the sign of its denominator \( g' + g'' r_h \). Taking into account (4) written in the units where \( G = 1 \), we get 
\[
g' + g'' r_h = 16 \pi p_\perp r_h - g'.
\]

In a region where \( p_\perp \geq 0 \), denominator is positive since \( g' \leq 0 \) everywhere; in a region where \( p_\perp \leq 0 \), eq.(4) gives 
\[
g'' r_h \leq -2g' \quad \text{and thus} \quad g' + g'' r_h \leq -g' \quad \text{with} \quad -g' \geq 0, \quad \text{hence} \quad C_h \geq 2\pi r_h/|g'|.
\]

Evaporation of the cosmological horizon in this case goes towards the point where decreasing mass achieves a certain critical value \( M = M_{cr} \) at which the metric function vanishes, which corresponds to the triple horizon \( r_h = r_t \) where the metric function, its first and second derivatives vanish (Fig.4) [29, 30].

The specific heat tends to infinity at the triple horizon which testifies that the triple-horizon spacetime is thermodynamically stable final product of evaporation of the cosmological horizon in the course of evolution of a one-horizon spacetime with an inflection point.

All above analysis was done for an arbitrary density profile with the only condition imposed on the stress-energy tensor: the weak energy condition needed to have de Sitter center which in the cosmological context corresponds to the first inflation. The basic generic features of the triple horizon \( r_h = r_t \) are the finite entropy, zero temperature, zero transversal pressure, zero curvature, and infinite positive specific heat. Applying the universal bound (1), we can qualify the triple horizon as extremely long scrambler.
Let us emphasize that the triple-horizon spacetime is not just one of possibilities to describe vacuum dark energy by some one-horizon spacetime. It is distinguished by the quantum dynamics of the horizon as the only thermodynamically stable final product of its evaporation. Moreover, evaporation stops completely \( T_h = 0; \ C_h \to \infty \) at a finite non-zero value of a vacuum density. The value of the cosmological constant is tightly fixed by the final point in the evaporation of the cosmological horizon. The triple horizon satisfies three algebraic equations: 
\[ g(r_t) = 0; \ g'(r_t) = 0; \ g''(r_t) = 0, \]
which define uniquely \( M_{cr}, \ r_t, \) and \( q_{cr} \). We have to choose only the initial vacuum scale \( \rho_0 \), and then \( g_{cr}^2 = \rho_0/\rho_\lambda \) will give us the value of \( \rho_\lambda \).

Now we can adopt some density profile to check whether the preferred by horizon dynamics value of the cosmological constant is close to its observed value.

We choose the density profile [21] which was used to produce the above pictures illustrating generic behavior of spherically symmetric spacetimes with vacuum dark fluid,

\[ \rho = \rho_0 \exp \left( -r^3/2GMr_0^2 \right) \] (15)

We adopt \( \rho_0 = \rho_{GUT} \) and \( E_{GUT} = 10^{15} \) GeV (then \( \rho_{GUT} = 5 \times 10^{77} g \ cm^{-3} \) and \( r_0 = 2.4 \times 10^{-25} \) cm), and calculate \( r_t, \ M_{cr}, \) and \( q_{cr} \), which gives \( r_t = 5.4 \times 10^{28} \) cm, \( M_{cr} = 2.33 \times 10^{56} \) g, and \( q_{cr}^2 = 1.37 \times 10^{107} \). We obtain remarkable coincidence of the calculated parameter \( q_{cr}^2 \) with that corresponding to our universe with \( \rho_\lambda \ (\text{obs}) \simeq 3.6 \times 10^{-30} g \ cm^{-3} \) [7] which gives \( q^2 = \rho_{GUT}/\rho_\lambda \ (\text{obs}) = 1.39 \times 10^{107} \). (The de Sitter horizon corresponding to \( \rho_\lambda \) is \( l = 9 \times 10^{28} \) cm.)

We found that all essential information is stored on the cosmological horizon and determines its quantum evolution in accordance with the spirit of the holographic principle. We would like to hope that the reduced symmetry of spacetime with the triple horizon with an infinite scrambling time could be also more friendly to a relevant CFT correspondence than the full symmetry of the de Sitter spacetime and fast-scrambling character of its horizon.

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