Tests of gravity theories with Galactic Center observations

Alexander F. Zakharov\textsuperscript{1,2,3,4}

\textsuperscript{1}Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117218 Moscow, Russia
zakharov@itep.ru

\textsuperscript{2}Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia

\textsuperscript{3}National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 115409, Moscow, Russia

\textsuperscript{4}North Carolina Central University, Durham, NC 27707, USA

Received Day Month Year
Revised Day Month Year

An active stage of relativistic astrophysics started in 1963 since in this year, quasars were discovered, Kerr solution has been found and the first Texas Symposium on Relativistic Astrophysics was organized in Dallas. Five years later, in 1967–1968 pulsars were discovered and their model as rotating neutron stars has been proposed, meanwhile J. A. Wheeler claimed that Kerr and Schwarzschild vacuum solutions of Einstein equations provide an efficient approach for astronomical objects with different masses. Wheeler suggested to call these objects black holes. Neutron stars were observed in different spectral band of electromagnetic radiation. In addition, a neutrino signal has been found for SN1987A. Therefore, multi-messenger astronomy demonstrated its efficiency for decades even before observations of the first gravitational radiation sources. However, usually, one has only manifestations of black holes in a weak gravitational field limit and sometimes a model with a black hole could be substituted with an alternative approach which very often looks much less natural, however, it is necessary to find observational evidences to reject such an alternative model. At the moment only a few astronomical signatures for strong gravitational field are found, including a shape of relativistic iron $K_{\alpha}$ line, size and shape of shadows near black holes at the Galactic Centers and M87, trajectories of bright stars near the Galactic Center. After two observational runs the LIGO–Virgo collaboration provided a confirmation for an presence of mergers for ten binary black holes and one binary neutron star system where gravitational wave signals were found. In addition, in last years a remarkable progress has been reached in a development of observational facilities to investigate a gravitational potential, for instance, a number of telescopes operating in the Event Horizon Telescope network is increasing and accuracy of a shadow reconstruction near the Galactic Center is improving, meanwhile largest VLT, Keck telescopes with adaptive optics and especially, GRAVITY facilities observe bright IR stars at the Galactic Center with a perfecting accuracy. More options for precision observations of bright stars will be available with creating extremely large telescopes TMT and E-ELT. It is clear that the Galactic Center (Sgr A* ) is a specific objects for observations. Our Solar system is located at a distance around 8 kpc from...
the Galactic Center (GC). Earlier, theorists proposed a number of different models including exotic ones for GC such as boson star, fermion ball, neutrino ball, a cluster of neutron stars. Later, some of these models are ruled out or essentially constrained with consequent observations and theoretical considerations. Currently, a supermassive black hole with mass around \(4 \times 10^6 M_\odot\) is the most natural model for GC. Using results of observations for trajectories of bright stars in paper \[1\] the authors got a graviton mass constraint which is comparable and consistent with constraints obtained recently by the LIGO-Virgo collaboration. Later, we consider opportunities to improve current graviton mass constraints with future observations of bright stars \[2\]. Similarly, from an analysis of bright star trajectories one could constrain a tidal charge which was predicted by a gravity theory with an additional dimension \[3\].

**Keywords:** Black holes; supermassive black holes; gravitational lensing; the Galactic Center; Large telescopes; VLBI interferometry.

**PACS numbers:** 04.20.-q, 04.30.Tv, 04.70.-s, 04.70.Bw, 04.80.Cc, 98.35.Jk

1. **A history of black hole concept**

At the moment there is no consensus concerning a starting point for black hole concept, some people say that it was introduced at the end of XVIII century in studies of the Reverend John Michel and Pierre Simon Laplace \[4, 5, 6\], some people say that it was started since the Schwarzschild paper \[7\]. It is well-known that one of the first solutions of general relativity was spherically symmetric vacuum solution found by Schwarzschild \[7\], however, properties of the solution are rather unusual and it was a natural question about an opportunity to use such a solution for real physical objects. In 1939 A. Einstein investigated the issue and arrived at a negative conclusion that the Schwarzschild singularities (Einstein called the event horizon in such a way) do not exist in physical reality \[11\]. Later, Einstein’s assistant Peter Bergmann supported the Einstein opinion about a unreality of Schwarzschild solution in one of the first books on general relativity \[11\]. As it was found in thirties of the XX century the final fate of star depends on its mass for white dwarfs and neutron stars. Soon after the discovery of Fermi–Dirac statistics in 1926 \[12, 13\], Fowler supposed that degenerate electron gas pressure is counterbalancing an action of gravity \[14\], while soon after that Soviet theorist Ya. Frenkel evaluated typical mass range for white dwarfs in \[15\]. A detailed analysis of the Frenkel’s paper and its consequences for an existence of mass limits of white dwarfs are presented in \[16\]. British physicist E. Stoner was the first researcher who found in the simple model of mass density distribution that white dwarfs should have upper mass limit \[17\] and later this statement was generalized by S. Chandrasekhar and L.D. Landau \[18, 19, 20\]. In paper \[21\], the authors considered Landau’s paper \[19\] and related historical content and the authors concluded that Landau contributed a lot in a development of the neutron star concept in spite of the fact that words ”neutrons” or ”neutron stars” were not used in \[19\], moreover, Landau noted that the conclusion about an upper mass limit for white dwarfs looks so extraordinary that there are

\[\text{An interesting discussions of historical aspects of the issue are presented in}\ [8, 9].\]
regions in stars where laws of quantum mechanics and quantum (Fermi – Dirac) statistics should be violated. Soon after the neutron discovery Baade and Zwicky outlined a scenario for formations of neutron stars could be observed as supernova explosions [24]. Later G. Gamow concluded that similarly to white dwarfs neutron stars should have an upper limit for their masses [22]. In his consideration Gamow used Newtonian approximation for a gravitational field of neutron stars and the analysis has a numerical mistake which was corrected in [23]. After a few years, J. R. Oppenheimer and G. M. Volkoff generalized the Gamow’s result considering gravity in a general relativity approach [25], meanwhile J. R. Oppenheimer and H. Snyder analyzed a spherically symmetric collapse of a dust ball which is forming a Schwarzschild vacuum solution [26], however at this time people thought that such theoretical model is too simple and artificial to be correct because in their solution dust ball formed space-time singularity in a finite interval of a proper time.

In 1963 the first star-like object with large redshift has been discovered [28]. This object was observed earlier with Cambridge radio survey and it was called 3C273. It was observed in a survey with Parkes Radio Telescope by Lunar occultation method [27]. M. Schmidt evaluated the redshift of this object and it was rather high $z \approx 0.158$, while J. B. Oke estimated a flux [29]. It turned out that its luminosity in around 100 times higher than our Galaxy and immediately astronomers and astrophysicists started to discuss possible physical processes to provide so huge energy release. Analyzing old photo plates where was 3C273 image, Soviet astronomers and A. S. Sharov and Yu. N. Efremov found variabilities with time scale $T_{\text{var}}$ around a few days [30]. It means that a size 3C273 is less than $cT_{\text{var}}$ (where $c$ is a speed of light) or less than the Solar system size. It was a very challenging task to propose a theoretical model for a huge energy release from a very small space region.

In December 1963 the first Texas Symposium on Relativistic Astrophysics has been organized in Dallas, quasars were discovered a few months before it and Roy Patrick Kerr found his solution which describes rotating black holes. In 1964 the term ”quasar” was introduced by Hong-Yee Chiu as a short version of ”quasi-stellar radio source” in his detailed review about the First Texas Symposium on Relativistic Astrophysics [31]. Astronomers and astrophysicists analyzed different possibilities to explain typical quasar features and excluding very exotic points of view there is an opinion that this high luminosity is connected with a conversion of binding energy of accreting matter into radiation or with an opportunity to transform rotational energy of black hole in radiation. In both cases a rotating black hole solution found by R. P. Kerr in 1963 is the important component to provide a huge energy release from a very small space region. A creation of quasar energy release is a bright example of efficiency of interaction scientists working in astrophysics and general relativity. Kip Thorne reminded [32] that initially astronomers and astrophysicists were rather skeptical in respect to exact solutions of Einstein equations, but fortunately, Achilles Papapetrou paid a special attention to Kerr solution. In his concluding remarks T. Gold expressed his hope that a fruitful interaction between
astrophysicists and relativists will continue.

An introduction of term "black hole" is associated with J. A. Wheeler, but such a concept has been used earlier at the first Texas Symposium on Relativistic astrophysics in December 1963 as it was explained by A. Rosenfield in an article for "Life" magazine published on January 24, where he discussed the Hoyle – Fowler model, namely "as Hoyle theorized, gravitational collapse gravitational collapse would be "catastrophe implosion" on a cosmic scale”, but criticizing the Hoyle’s model and demonstrating its weaknesses, Rosenfield wrote "... instead of intensely radiating object sending out lavish quantities of light and radio energy, gravitational collapse would result it in invisible "Black hole" in the universe” [33].

Before a publication of the "Life" issue with the Rosenfield’s article, on January 18, 1964 Ann Ewing reported about the meeting of the American Association for the Advancement of Science in Cleveland in January 1964 with a short article "Space may be peppered with "black holes"" as a title [34], therefore, clearly that in December 1963 at the First Texas Symposium and in January 1964 at the AAAS meeting astronomers and relativists discussed black holes in sky, however it is still unknown exactly who were these persons, see, also discussions in book [35] and in article "50 years later, its hard to say who named black holes" by T. Siegfried in internet journal "Science News" where the author wrote "but it didn’t catch on until Wheeler began using it a few years later. Perhaps Wheeler still gets credit, Bartusiak said. He never said he originated the term. What was important is that he had the authority to give the scientific community permission to use the term black hole.” [35] Bartusiak had conversations with Hong-Yee Chiu who was a member of the Institute of Advanced Studies on Princeton from 1959 until 1961 and based on his reminding, it was noted that R. Dicke discussed a collapse of massive stars with a formation of something similar to "black hole of Calcutta" [35]. A description of this historical event could find see for instance, in records by John Zephaniah Holwell [36] where the author reminded the historical case when after the fall of Fort Williams Siraj ud-Daulah, the Nawab of Bengal, ordered to place many British prisoners of war at a very small room in June 1756 (see also a description of the vent in a popular book [37]). Really, it is a nice illustration of a huge concentration of mass in a very limited space region.

Therefore, according to Bartusiak’s opinion probably Dicke introduced the term for the scientific community [35], J.A. Wheeler was reminding circumstances which were stimuli to introduce the black hole concept [38]. When information about the discovery reached US scientific community, Wheeler started to think about possible theoretical model for pulsars discovered in UK in 1967 (the corresponding paper was published with a delay only in 1968 [39] since initially these observational data

bThese grains of pepper for nearby supermassive black holes in the Galactic Center and in the center of M87 could be observed as shadows (small dark spots in the sky) with the Event Horizon Telescope.

*chttps://www.sciencenews.org/blog/context/50-years-later-it%E2%80%99s-hard-say-who-named-black-holes.
were classified) and Wheeler understood that a pulsar model had to include gravitationally completely collapsed object. In the autumn 1967, he was invited to attend a conference where possible interpretations of pulsar phenomenon were discussed and Wheeler expressed his concern about unsuccessful searches for suitable terminology for "completely collapsed object" and somebody in audience suggested "How about a black hole". According to Wheeler’s opinion it was extremely useful and fruitful suggestion and Wheeler used this term in his Sigma XiPhi Beta Kappa lecture in the West Ballroom of the New York Hilton a few weeks later, on December 29, 1967 and it was included in his corresponding paper in the spring of 1968 [40].

Now it is known that a pulsar phenomenon is powered by a neutron star as it was discussed before the pulsar discovery by F. Pacini [41] and soon after the discovery by T. Gold [42]. F. Pacini and T. Gold worked in Cornell University and clearly it was not a pure incident that two principal authors of the pulsar model were from the same institution. From this piece of history of black hole concept and pulsar phenomenon, one could conclude if a great mind (J.A. Wheeler in our case) would started to think about an important challenging problem, he (she) often could find a scientific brilliant or in other words something which is very important even in the case of a failure to reach an initial goal.

In his paper [40], where Wheeler used the term "black hole" in the written form at the first time, he considered the Crab Nebula which is associated with a supernovae remnant SN 1054 as a clear confirmation of Baade – Zwicky scenario for neutron star formation where it was suggested that neutrons stars could be formed in supernova explosions [24]. Really the object has been observed by Chinese astronomers for 21 months since July 4, 1054 (according to the our current chronology). Clearly, that one thousand years ago Chinese astronomers and astrollogists used their own chronology and their own names for constellations. Dutchman sinologist J. J. L. Duyvendak identified the Guest Star described in Chinese records with Crab Nebula [44], see also consequent astronomical discussions given by N. U. Mayall and J. H. Oort [45]. At this time when these two papers were prepared Duyvendak and Oort worked in University of Leiden (Oort worked in the Observatory of the University), while Mayall worked in the Lick Observatory (California, USA) and had intensive communications with Baade. Soon after a publication of these two papers [44, 45] W. Baade and R. Minkowski studied properties of Crab Nebula and concluded that Chinese Nova 1054 A. D. was a supernova of type I [46, 47]. Later, the Crab nebula was identified with a radio source in 1963 and as a X-ray source in 1964 and as a pulsar in 1968 (see also discussion of the source in [? ]). There are a couple of conclusions from Wheeler’s considerations in [40]. First, sometimes, people could recognize a significance and justify an importance of some activity (observations of ancient Chinese astronomers in our case) for fundamental science only after centuries (or even Millennium) and at this period one could

\[d\]The standard model in particle physics has been proposed in 1967, see an overview of foundations of the model and its development in [43], therefore, 1967–1968 were very for physics.
think that the activity was useless for fundamental study but it was not. Second, a scientific knowledge is a result of activity of enthusiastic people working in different areas, therefore, multi-disciplinary investigations are often very fruitful. In addition, one could note that rather routine observations almost 1000 years ago, their careful storage, translations and interpretations of old historical records give crucial contribution in studies of nuclear matter which are still very important and interesting for a scientific community. Third, a scientific process is very fragile and it may be terminated due to many different reasons, for instance, old historical records may be destroyed, forgotten or ignored and in this case a confirmation of the Baade–Zwicky scenario for NS formation will be incomplete. In a more general case action or inaction could lead to a complete solution (or failure) of a puzzle for a phenomenon of Nature or to a case when pieces of the puzzle will be blank, fuzzy or be in disorder.

Properties of nuclear matter, quark–gluon plasma are still very important for theoretical studies and experimental science and a rapid development of Mega Science facilities for studies of nuclear matter such FAIR (Facility for Antiproton and Ion Research) (Darmstadt, Germany) and Nuclotron Based Facility (NICA, Dubna, Russia) is this claim. Therefore, as we discussed, even ancient observations of stars have an important value for fundamental physics and nuclear physics, in particular, but a justification of astronomical observations for a fundamental science could come many years later.

It is clear that a suitable terminology is extremely important and the term "black hole" is used in different context. For instance, Google found more than $10^9$ documents where people used this term, therefore it is very popular not only in science. However, Wheeler’s suggestion to use the black hole term consists of two important issues, first, in spite of infinite time of gravitating object for complete gravitational collapse, a distant observer could use a limiting metric which is result of the collapse, since after a finite time interval differences between a dynamical metric of collapsing matter and a static or stationary metric which describes a result of gravitational collapse are very small, therefore, one could use a simpler static (stationary) model. Second, Wheeler believed that black holes have to exist as results of stellar collapses and as engines in AGNs and quasars. At the end of this section I would like to point out an interesting paper [49] where historical aspects of a black hole concept development were also discussed. Recently a relativistic astrophysics development before 1940 was discussed in [50] in a more wider context.

2. Astrophysical black holes

It is clear that classical black holes should not emit particles and photons, but in 1974 S. Hawking found that black holes should emit particles due to an existence of quantum tunneling [51] (many years before the Hawking’s paper publication famous Soviet theorist V. N. Gribov argued in his conversations with Ya. B. Zeldovich that black holes have to radiate [52, 53]). The Hawking radiation could be important for
Tests of gravity theories with Galactic Center observations

black holes with masses significantly smaller than stellar ones. Such black holes may be formed at the early stage of Universe evolution and these black holes are called primordial ones. The Hawking radiation for black holes with stellar masses and heavier is negligible in comparison with other astrophysical processes. An opportunity to discover massive black holes with observations of electromagnetic radiation from accreting matter has been discussed by E. Salpeter [54] and Ya. B. Zeldovich [55]. Evaluating the energy for inner most stable circular orbit in the Schwarzschild metric one could conclude that an energy release could be a few percent of accreting matter mass (or which is comparable with nuclear fusion energy release), while for extreme Kerr metric such an energy release could be almost 50% of mass or comparable with energy release in annihilation.

2.1. Black holes with stellar masses

Almost independently with the first X-ray observations of astronomical sources theoretical models of disk accretion have been developed in [56, 57, 58, 59] and in these papers it was predicted that black holes and neutron stars with stellar masses could be found in X-ray stellar binary systems. Several BH systems have been found in X-ray binaries with masses ($M_{BH} = 4 - 16 M_\odot$) and around one hundred neutron stars (NSs) as X-ray binaries, see for instance, papers [62, 63, 64].

It we take a look at a mass distribution of compact objects which are black holes and neutron stars in binary stellar systems, we could see that there is a clear mass gap between NSs and BHs with stellar masses and at the first glance, such a property does not look natural. Sometimes even alternative theories of gravity have been used to explain such a phenomenon [65], however a fine tuning procedure for parameters of population synthesis gives an opportunity to solve such a puzzle [66, 67].

2.2. Supermassive black holes in galactic centers

Supermassive black holes with masses in range ($10^6 - 10^{10}) M_\odot$ exist in quasars, active galactic nuclei and in centers of spiral galaxies and in this case they could evolve in interactions with their galaxies [68]. In active galactic nuclei black hole masses are evaluated with the reverberation method or a spectroscopy of absorption and emission lines. Namely, variations in the strengths of the central source in a quasar will generate variations in the strengths and profiles of the emission lines. These reverberations (or "echos") in the emission lines will delay in respect to continuum variations. The reverberation method was suggested in [69], however, similar ideas were proposed earlier in [70], see also papers [63, 64, 71] for more recent reviews.

*Recently one of the pioneers of these theoretical studies N. I. Shakura published interesting historical reminding of an initial period of X-ray astronomy [60, 61].
2.3. Primordial black holes

As it was proposed many years ago in \cite{72, 73}, at the early stage of the Universe evolution black holes could be formed with rather wide mass spectrum and in particular, their masses could be rather small and Hawking radiation should be significant for such objects \cite{74}. These black holes are called primordial or PBHs \cite{75}. Currently, PBHs are not discovered yet, however, they are very attractive as preferable models for many astrophysical puzzles such as a formation of supermassive black holes with high redshifts and discoveries of relatively heavy binary black holes with LIGO – Virgo gravitational interferometers \cite{76}. If such binary black holes are results of stellar evolution then their progenitors should have very strong stellar winds and binary black holes are placed not in vacuum but in matter and at the moment of a final merger of black holes an gravitational wave energy around $3M_\odot c^2$ (as for the first GW event GW150914) was released as strong gravitational wave in a fraction of second, then it is naturally to expect to detect an electromagnetic counterpart. It was proposed PBHs with intermediate masses as objects forming dark matter \cite{77}. Astrophysical applications of PBHs with different masses are considered in a number of reviews \cite{78, 79, 80} (see also references therein). Observational consequences of PBH clusters were considered in \cite{81}.

3. Observations of bright stars near the Galactic Center

The supermassive black hole in our Galactic Center is the closest one, therefore, this object is very attractive and astronomers observe the Galactic Center in different spectral band including $\gamma$, X-ray, IR, optical, radio and mm-band \cite{82, 83, 84, 85, 86, 87, 88, 89}. Moreover, such an object is a natural laboratory to test general relativity and its alternatives in a strong gravitational field limit \cite{90, 91, 92, 93, 94}. The GRAVITY collaboration observed S2 star pericenter passage in May 2018 reported about the discovery of general relativity effects for this star \cite{95}. Currently there are two groups observing bright stars near the Galactic Center with largest telescopes equipped with adaptive optics facilities. One (US) group uses the twin Keck telescopes with 10 m diameters at Hawaii, another ESO-MPE group uses four VLT telescopes with 8 m diameters and now with GRAVITY facilities. Roughly speaking, results of these two groups are consistent and complimentary. Results of observations showed that in the first approximation stars are moving along elliptical orbits and therefore, one could conclude that motions of these stars are fitting rather well with a potential of point like mass around $M_{SBH} = 4 \times 10^6 M_\odot$ in the framework of Newtonian gravity law. One of the most interesting probes for a gravitational potential at the Galactic Center is S2 star. It has eccentricity $e = 0.88$, period $T = 16$ yr and an expected visible relativistic

\footnote{Recently with data analysis of two observational runs the LIGO–Virgo collaboration reported about discoveries of ten gravitational wave signals from binary black hole mergers and one signal from binary neutron star merger, see \cite{82}.}
precession of its orbit is around $\Delta_s \approx 0.83$ mas\cite{96,97} in assumption that extended mass distributions such as a stellar cluster or dark matter near the Galactic Center do not have a significant impact on relativistic precession of its orbit\cite{91,92}. The Keck uncertainties in the S2 star orbit reconstruction are around $\sigma_{\text{Keck}} \approx 0.16$ mas\cite{98}, while for Thirty Meter Telescope (TMT) which will be constructed within a several years uncertainties are $\sigma_{\text{TMT}} \approx 0.015$ mas.

4. GRAVITY: first discoveries

4.1. Gravitational redshift for S2 star

Both teams (Keck and VLT ones) continuously improve a precision for S2 star orbit reconstruction for both teams, for example, for MPE–ESO team operating with VLTs, since in 1990s a precision of SHARP facilities was around 4 mas, in 2000s NACO had a precision around 0.5 mas, but in 2018 GRAVITY reached a precision around 30 $\mu$as\cite{95}. In May 2018 the GRAVITY collaboration evaluated a gravitational redshift in the orbit of S2 star near its pericenter passage. In addition, the GRAVITY collaboration estimated relativistic precession of its orbit and showed that observational data are much better fitted with GR model in the first PN approach than in comparison with Newtonian one. The authors introduced a parameter $f$ and $f = 0$ corresponds to the Newtonian approach for gravitational redshift while $f = 1$ corresponds to the first post-Newtonian approximation of general relativity. Therefore, general relativity successfully passed an additional test. It means that almost after 100 years after the confirmation of the GR prediction about a deflection of light during Solar eclipse in 1919\cite{99} when astronomers tested three different options, namely, a) a deflection of light describes with the GR law; b) a deflection of light describes with the Newtonian law, and c) there is no a deflection of light in gravitational field. Now astronomers checked GR prediction in much stronger gravitational field at high distances from our Solar system and Einstein’s theory of relativity successfully passed one important test more\cite{95}. A theoretical approach for gravitational redshift evaluation if sources are moving in binary system was developed in\cite{100,101,102}. In May 2018 S2 star passed its pericenter and the GRAVITY collaboration established that relativistic corrections have to be taken at the period near this passage. Near its pericenter S2 star had a total space velocity $V_{\text{peri}} \approx 7650$ km/s or $\beta_{\text{peri}} = V_{\text{peri}}/c = 2.55 \times 10^{-2}$\cite{95}. Computing PPN(1) correction for a total gravitational redshift following\cite{95,100,101,102}

$$z_{\text{GR}} = \frac{\Delta \lambda}{\lambda} = B_0 + B_0.5\beta + B_1\beta^2 + O(\beta^3),$$

(1)

where $B_1 = B_{1,tD} + B_{1,\text{grav}}$, $B_{1,tD} = B_{1,\text{grav}} = 0.5$, $B_{1,tD}$ is the special relativistic transverse Doppler effect, $B_{1,\text{grav}}$ is the general relativistic gravitational redshift,\footnote{Recently, it was reported precise measurements of gravitational redshifts with Galileo satellites\cite{103}, an opportunity to test GR predictions with accuracy around $10^{-5}$ for gravitational redshifts using Radioastron observational data discussed in\cite{104}.}
$B_{0.5} = \cos \theta$, where $\theta$ is the angle between the velocity vector and line of sight \cite{101}, while the total redshift $B_0$ which is independent on a star velocity $\beta$

$$B_0 = z_\odot + z_{\text{gal}} + z_{\text{star}} + \frac{1}{2} \Upsilon_0,$$

therefore, the redshift $B_0$ consists of four terms, namely, $z_\odot$ is a redshift due a total motion of the Sun and the Earth in respect to Galactic Center and blue shift due to potential of the Sun and the Earth, $z_{\text{gal}}$ is a redshift due to Galaxy potential, $z_{\text{star}}$ is redshift due to the star’s potential, the term $\frac{1}{2} \Upsilon_0 = \frac{GM}{2a}$ is due to the location of star in the SMBH potential \cite{101}. The GRAVITY collaboration estimated the total redshift from spectroscopical observations and concluded that it corresponds to $z \approx 200 \text{ km/s} / c$ \cite{95}. One could represent the total redshift obtained from spectroscopical observations in the form \cite{95} (as it was noted earlier $f$ reflects a weight of post-Newtonian term) or more precisely,

$$z_{\text{tot}} = z_K + f(z_{\text{GR}} - z_K),$$

where $z_K = B_0 + B_{0.5} \beta$ is the Keplerian redshift, $f = 0$ corresponds to Keplerian (Newtonian) approach, while $f = 1$ corresponds to PPN(1) approach. The GRAVITY collaboration found that $f = 0.90 \pm 0.09|_{\text{stat}} \pm 0.15|_{\text{sys}}$ and the authors also concluded that S2 data are inconsistent with a pure Newtonian dynamics. Since $f$-value is slightly less than its expected value estimated with pure PPN(1) fit, perhaps an extended mass distribution of stellar cluster or dark matter should be taken into account in this model and in this case future observations of relativistic redshifts and astrometric monitoring the bright stars will help to evaluate parameters of an extended mass distribution and estimate a contribution of time dependent terms in a gravitational potential near the supermassive black hole. Comparing fits for Schwarzschild and Newtonian precessions for a point like mass potential and observational data for S2 star, the GRAVITY collaboration evaluated $f$-value from observational data (without any precession) and they concluded that the $f$-value is much closer to GR quantity $f = 1$ or more precisely, the GRAVITY collaboration found $f = 0.94 \pm 0.09$.

4.2. Observations of motions of hot spots near ISCO

The GRAVITY collaboration observed trajectories of two bright flares on July 22 and July 28, 2018, as well as an orbit of fainter flare on May 27, 2018 \cite{105}. The authors claimed that the position centroids exhibited clockwise looped motion on the sky, on scales of typically 150 $\mu$as over a time interval around a few tens of minutes, corresponding to about 30% the speed of light. Meanwhile, the flares exhibited continuous rotation of the polarization angle, with about the same 45($\pm$15) min period as that of the centroid motions. Typical radius of spot orbits are around 7 $M_{SBH}$ (in mass units), while the innermost stable circular orbit (ISCO) radius is 6 $M_{SBH}$ for a Schwarzschild black hole. Since there is a strong demand to explain these
observational data quantitatively, it is expected that a detailed theoretical model
describing polarization variations of hot spots would be created shortly.

4.3. **Spacially resolved rotation of broad line region for 3C273**

Recently GRAVITY collaboration observed the first quasar 3C273 and found a
spatial offset (with a spatial resolution of $10^{-5}$ arcseconds, or about 0.03 parsecs
for a distance of 550 million parsecs) between the red and blue photo-centres of
the broad Paschen-$\alpha$ line of the quasar 3C 273 perpendicular to the direction of its
radio jet [106]. The authors fitted with a broad-line-region model of a thick disk
of gravitationally bound material orbiting a black hole of $3 \times 10^8$ solar masses and
they concluded that disk radius is around 150 light days and earlier a disk radius
in the range 100–400 light days was found previously using reverberation mapping,
therefore, new estimates are consistent with previous ones.

5. Constraints on alternative theories of gravity with observations
   of bright stars

5.1. **Massive graviton constraints**

A few years ago F. Dyson considered opportunities to detect a graviton [107] and
his answer was rather pessimistic since in the future it will be very hard to de-
tect graviton in physical experiments (or observations), independently on a version
of gravity theory where graviton could be massive or massless. However, if graviti-
on is massive there a number of different ways to constrain a graviton mass from
astronomical observations could be used [108, 109]. Initially theories of massive
gravity have pathologies including discontinuities and a presence of ghosts, how-
ever, recently theorists found ways to create ghost-free theories of massive gravity
[109]. In 2016 the LIGO–Virgo collaboration reported about the first detection of
gravitational waves from a merger of a binary black hole system (the event was
detected on September 14, 2015 and it is called GW150914) [110]. Moreover, the
LIGO–Virgo collaboration considered a theory of massive gravity as an alternative
for conventional general relativity and found a constraint for the graviton Compton
wavelength $\lambda_g > 10^{13}$ km which could be interpreted as a constraint for a
graviton mass $m_g < 1.2 \times 10^{-22}$ eV [110]. Later, the LIGO–Virgo collaboration
LIGO reported about the discovery of the third GW event from merging the BHs
with 31 and 19 solar masses at redshift $z = 0.19$ observed on January 4, 2017,
named GW170104 and the authors significantly improved an upper graviton mass
constraint $m_g < 7.7 \times 10^{-23}$ eV [111]. In August 2017 a gravitational wave signal
has been detected from binary neutron star merger (GW170817). This source was
observed by space spacecrafts Fermi and INTEGRAL and many ground based facili-
ties, including global robotic network Master [112]. The LIGO – Virgo collaboration
together with its partner groups observing the same astronomical sources with dif-
ferent facilities found that constraints on speed of gravitational waves from binary
neutron star merger (GW170817) are $-3 \times 10^{-15} < (v_g - c)/c < 7 \times 10^{-16}$ [113]. Since graviton energy is $E = h f$, therefore, assuming a typical LIGO frequency range $f \in (10, 100)$, from the dispersion relation one could obtain a graviton mass estimate $m_g < 3 \times (10^{-21} - 10^{-20})$ eV which is a weaker estimates than previous ones obtained from binary black hole signals detected by the LIGO team [114]. In the case of massive graviton, one could use Yukawa gravitational potential in a form $\propto r^{-1} \exp(-r/\lambda_g)$, and in this case a lower bound for Compton wavelength $\lambda_g$ of the graviton is connected with an upper bound of its mass

$$m_g(upper) = h c/\lambda_g.$$ (4)

Some time ago, in paper [115] we obtained constraints on Yukawa gravity from observational data for the S2 star orbit. Later, we found constraints on graviton mass $m_g < 2.9 \times 10^{-21}$ eV from available observational data [1] (see also [116, 117, 118] for more details). In these considerations we used available and relatively old observational data to constrain a graviton mass. Later, Keck group followed our ideas to improve our estimates with new observational data $m_g < 1.6 \times 10^{-21}$ eV [98]. In paper [2] we considered perspectives to improve current graviton mass estimates found with future observational data for S2 and other bright stars observed with VLT and Keck telescopes, in particular, we evaluated orbital precession for Yukawa potential and obtained an upper limit for a graviton mass assuming that GR prediction about orbital precession will be confirmed with future observations. In this case as it was shown in [2] the longest Compton wavelength could be expressed as

$$\Lambda \approx \sqrt{\frac{(a\sqrt{1 - e^2})^3}{3GM}} \approx \sqrt{\frac{(a\sqrt{1 - e^2})^3}{6R_S}},$$ (5)

where $a$ is a semi-major axis and $e$ eccentricity for a selected orbit. Therefore, after observations of bright stars for several decades an upper bound for a graviton mass could reach around $5 \times 10^{-23}$ eV.

### 5.2. Tidal charge constraints

Reissner – Nordström – de-Sitter black hole could arise in theories with an extra dimension [114] or in Horndeski scalar-tensor theories [120]. The line element of the spherically symmetric Reissner – Nordström – de-Sitter metric is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$ (6)

where function $f(r)$ is defined as

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{1}{3} \Lambda r^2,$$ (7)

$M$ is a black hole mass, $Q$ is its charge and $\Lambda$ is cosmological constant. In the case of a tidal charge [119] or Horndeski scalar-tensor theories [120], $Q^2$ could be negative. For simplicity, we call $Q^2$ a tidal charge. In paper [3] it was shown that a total
relativistic advance for metric (6) in PPN(1) approximation could be expressed in an analytical form

$$\Delta \theta^{\text{total}} = \frac{6\pi M}{L} - \frac{\pi Q^2}{ML} + \frac{\pi \Lambda a^3 \sqrt{1-e^2}}{M}. \quad (8)$$

As one can see apocenter shift dependences on eccentricity and semi-major axis are the same for Schwarzschild and Reissner–Nordström cases while corresponding factors \(6\pi M\) and \(-\frac{\pi Q^2}{M}\) are different, therefore, it is very hard to distinguish a presence of a tidal charge and black hole mass evaluation uncertainties since black hole mass uncertainty could mimic a existence of a tidal charge. For \(Q^2 > 0\), there is an apocenter shift in the opposite direction in respect to GR advance, while for \(Q^2 > 0\) an apocenter shift direction coincides with relativistic advance direction. If we apply these relations for the supermassive black hole at the Galactic Center, in paper \([3]\) bounds in \(Q^2\) and \(\Lambda\) are presented for current and future accuracies for Keck and Thirty Meter telescopes which were discussed in \([98]\). Similarly to papers \([3, 121]\) if we use uncertainty \(\sigma_{\text{GRAVITY}} = 0.030\) mas for the GRAVITY facilities as it was found in \([93]\) \((\delta_{\text{GRAVITY}} = 2\sigma_{\text{GRAVITY}})\) or \(\Delta \theta^{(GR)}_{S2} = 13.84 \delta_{\text{GRAVITY}}\) for S2 star and assuming again that GR predictions about orbital precession of S2 star will be confirmed with \(\delta_{\text{GRAVITY}}\) accuracy \((or \ |\frac{\pi Q^2}{ML}| \lesssim \delta_{\text{GRAVITY}})\), one could conclude that

$$|Q^2| \lesssim 0.432 M^2, \quad (9)$$

or based on results of forthcoming observations with GRAVITY facilities one could expect to reduce significantly a possible range of \(Q^2\) parameter in comparison with a possible hypothetical range of \(Q^2\) parameter in comparison with current and future Keck data.

6. Conclusions

As it was noted earlier, precise observations of bright stars near the Galactic Center is very efficient tool to check alternative theories of gravity and to investigate a presence of an extended mass distribution near the Galactic Center. In our simple approach we compared theoretical estimate for pericenter (apocenter) shifts with their estimates obtained from observations. Using this approach we discussed an opportunities to evaluate parameters of supermassive black hole, stellar cluster and dark matter cloud near the Galactic Center or evaluate parameters of alternative gravity model analyzing apocenter (pericenter) advance after at least one star revolution, however, in the future we will have a possibility to evaluate a static gravitational potential at the Galactic Center analyzing only very small parts of stellar orbits similarly to \([122]\), where it was shown even around 40% of stellar phase coverage is enough for an orbit reconstruction. However, if a contribution of time-dependent component of gravitational potential caused by stellar encounters...
or moving invisible mass concentrations is significant, then an orbit reconstruction problem may be more complicated. One could obtain the graviton mass constraint from an analysis of S2 star trajectory and the bound is consistent and comparable with the constraint presented recently by the LIGO collaboration, in particular, future observations of bright stars will give an opportunity to obtain a graviton mass estimate which is slightly better than the best current graviton mass estimate found by the LIGO–Virgo collaboration.

If we assume that the supermassive black hole at the Galactic Center is described with Reissner – Nordström – de-Sitter metric, a tidal charge $Q^2$ (or corresponding parameter in Horndeski gravity theories) could be constrained in an efficient way with monitoring the bright IR stars.

Acknowledgements

The author thanks D. Borka, V. Borka Jovanović, F. De Paolis, G. Ingrosso, P. Jovanović, S. M. Kopeikin, A. A. Nucita, S.G. Rubin, B. Vlahovic for useful discussions. A. F. Z. thanks also NSF (HRD-0833184) and NASA (NNX09AV07A) at NASA CADRE and NSF CREST Centers (NCCU, Durham, NC, USA) for a partial support and the organizers of the Fourth International Conference on Particle Physics and Astrophysics (ICPPA-2018) for their attention to our contribution.

References

1. A. F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, J. Cosm. Astropart. Phys. (JCAP) 05 (2016) 045.
2. A. F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, J. Cosm. Astropart. Phys. (JCAP) 04 (2018) 050.
3. A. F. Zakharov, Eur. Phys. J. C 78 (2018) 689.
4. J. Michell, Phil. Trans. R. Soc. London 74 (1784) 35.
5. P. S. Laplace, Exposition du Systeme du Monde (Imprimerie du Cerde-Social, Paris, 1796).
6. P. S. Laplace, Allgemeine Geographische Ephemeriden 4 (1799) 1.
7. K. Schwarzschild, Sitzungsberichte der Koniglich Preussischen Akademie der Wissenschaften (1916) 189; see English translation in Gen. Rel. Grav. 35 (2003) 951.
8. J. Eisenstaedt, Arch. Hist. Exact Sci. 27 (1982) 157.
9. C. Montgomery, W. Orchiston and I. Whittingham, J. Astron. Hist. and Heritage 12 (2009) 90.
10. A. Einstein, Ann. Math. 40 (1939) 922.
11. P. G. Bergmann, Introduction to the theory of relativity (New York, Prentice-Hall, 1942).
12. E. Fermi, Rend. Lincei 3 (1926) 145; see also English translation in arXiv:cond-mat/9912229.
13. P. A. M. Dirac, Proc. Roy. Soc. London. Ser. A 112 (1926) 661.
14. R. H. Fowler, *Mon. Not. Roy. Astron. Soc.* **87** (1926) 114.
15. Ya. Frenkel, *Zeitschrift Phys.* **50** (1928) 234.
16. D. G. Yakovlev, *Phys. Usp.* **37** (1994) 609.
17. E.C. Stoner, *Philos. Mag.* **9**, 944 (1930).
18. S. Chandrasekhar, *Astrophys. J.* **34**, 81 (1931).
19. L. D. Landau, *Phys. Z. Sowjetunion* **1**, 285 (1932).
20. S. Chandrasekhar, *Observatory* **57**, 373 (1934).
21. D. G. Yakovlev, P. Haensel, G. Baym and C. J. Pethick, *Phys. – Usp.* **56** (2013) 289.
22. G. Gamow, *Structure of atomic nuclei and nuclear transformations* (Oxford Press, 1937).
23. H. Ludwig and R. Ruffini, *J. Korean Phys. Soc.* **65** (2014) 892.
24. W. Baade and F. Zwicky, *Phys. Rev.* **45** (1934) 138.
25. J. R. Oppenheimer and G. M. Volkoff, *Phys. Rev.* **55** (1939) 374.
26. J. R. Oppenheimer, H. Snyder, *Phys. Rev.* **56** (1939) 455.
27. C. Hazard, M. B. Mackey and A. J. Shimming, *Nature* **197** (1963) 1037.
28. M. Schmidt, *Nature* **197** (1963) 1040.
29. J. B. Oke, *Nature* **197** (1963) 1040.
30. A. S. Sharov and Yu. N. Efremov, *Information Bulletin on Variable Stars on 18 April 1963*, N 23.
31. H. Y. Chiu, *Physics Today* **17** (1964) 21.
32. K. S. Thorne, *Black Holes and Time Warps* (New York, W. W. Norton & Company, 1994)
33. A. Rosenfield, *Life on 24 January* (1964) 11.
34. A. Ewing, *Science Newsletter for January 18* **85** (1964) 39.
35. M. Bartusiak, *Black hole: how an idea abandoned by Newtonians, hated by Einstein, and gambled on by Hawking became loved* (New Haven & London, Yale University Press, 2015)
36. J. Z. Holwell and friends, *India tracks* (Second edition, revised and corrected) (London, Printed for T. Becket and P.A. De Hondt, 1774).
37. J. Narlikar, *The lighter side of gravity* (San Francisco, W. H. Freeman and Company, 1982).
38. J. A. Wheeler and K. Ford, *Geons, Black Holes, and Quantum Foam: A Life in Physics* (New York, W W Norton & Company, Inc., 2000).
39. A. Hewish, S. J. Bell, J. D. H. Pilkington, P. F. Scott and R. A. Collins, *Nature* **217** (1968) 567.
40. J. A. Wheeler, *American Scientist* **56** (1968) 1.
41. F. Pacini, *Nature* **216** (1968) 567.
42. T. Gold, *Nature* **218** 731.
43. S. Weinberg, *Phys. Rev. Lett.* **121** (2018) 2200011.
44. J. J. L. Duyvendak, *Publ. Astron. Soc. Pacific* **54** (1942) 91.
45. N. U. Mayall and J. H. Oort, *Publ. Astron. Soc. Pacific* **54** (1942) 95.
16 REFERENCES

46. W. Baade, Astrophys. J. 96 (1942) 188.
47. R. Minkowski, Astrophys. J. 96 (1942) 199.
48. I. S. Shklovsky, Supernovae (New York, John Wiley & Sons, 1968).
49. C. A. R. Herdeiro and J. P. S. Lemos, arXiv:1811.06587.
50. L. Bonolis, Eur. Phys. J. H 42 (2017) 311.
51. S. W. Hawking, Nature 248 (1974) 30.
52. A. A. Anselm et al., Physics – Uspekhi 41 (1998) 407.
53. I. B. Khriplovich, General Relativity (New York, Springer, 2005).
54. E. E. Salpeter, Astrophys. J. 140 (1964) 796.
55. Ya. B. Zeldovich, Sov. Phys. Dokl. 9 (1964) 195.
56. J. E. Pringle and M J Rees, Astron. Astrophys. 21 (1972) 1.
57. N. I. Shakura, Astron. Zhurn. 49 (1972) 921.
58. N. I. Shakura and R. A. Sunyaev, Astron. Astrophys. 24 (1973) 337.
59. I. D. Novikov and K. S. Thorne, in Black Holes, eds. C de Witt and B S de Witt (New York, Gordon and Breach, 1973) p. 343.
60. N. I. Shakura, 2014 Phys. Usp. 57 (2014) 407.
61. N. I. Shakura, Preprint arXiv:1809.11137[physics.hist-phys].
62. J. M. Corral-Santana, J. Casares, T. Munoz-Darias et al., Astron. Astrophys. 587 (2016) A61.
63. A. M. Cherepashchuk, Phys. Usp. 59 (2016) 702.
64. A. M. Cherepashchuk, Astron. Rep. 61 (2017) 265.
65. V. V. Sokolov, in Particle and Astroparticle Physics, Gravitation and Cosmology: Predictions, Observations and New Projects - Proc. of the XXX-th International Workshop on High Energy Physics, eds. R. A. Ryutin and V. A. Petrov (Singapore, World Scientific, 2015), p. 320.
66. K. Belczynski, G. Wiktorowicz, C. L. Fryer et al., Astrophys. J. 757 (2012) 91.
67. G. Wiktorowicz, G. Belczynski and T. J. Maccarone, Preprint arXiv:1312.5924[astro-ph.HE].
68. J. Kormendy, L. C. Ho, Ann. Rev. Astron. Astrophys. 51 (2013) 511.
69. R. D. Blanford and C. F. McKee, Astrophys. J. 255 (1982) 419.
70. A. M. Cherepashchuk and V. M. Lyutyi, Astrophys. Lett. 13 (1973) 165.
71. M. C. Gaskell and L. C. Sparke, Astrophys. J. 305 (1986) 175.
72. Ya. B. Zeldovich and I. D. Novikov, Sov. Astron. 10 (1967) 602.
73. S. W. Hawking, Mon. Not. R. Astron. Soc. 152 (1971) 75.
74. B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. 168 (1974) 399.
75. A. G. Polnarev and M. Yu. Khlopov, Sov. Phys. Usp. 28 (1985) 213.
76. A. D. Dolgov, Physics – Uspekhi 61 (2018) 115.
77. G. F. Chapline and P. H. Frampton, JCAP 11 (2016) 042.
78. M. Yu. Khlopov, Res. Astron. Astrophys. 10 (2010) 495.
79. K. M. Belotsky, A. E. Dmitriev, E. A. Esipova et al., Mod. Phys. Lett. A 29 (2014) 1440005.
REFERENCES

80. B. Carr, F. Kühnel and M. Sandstad, Phys. Rev. D 94 (2016) 083504.
81. K. M. Belotsky, V. I. Dokuchaev, Yu. N. Eroshenko et al., Preprint arXiv:1807.06590v1 [astro-ph.CO].
82. B. P. Abbott, R. Abbott, T. D. Abbott et al., Preprint arXiv:1811.12907v2 [astro-ph.HE].
83. A. Eckart, R. Schödel and C. Straubmeier, The Black Hole at the Center of the Milky Way (London, Imperial College Press, 2005).
84. V. I. Dokuchaev and Yu. N. Eroshenko Phys. Usp. 58 (2015) 772.
85. V. I. Dokuchaev and Yu. N. Eroshenko JETP Letters 101 (2015) 777.
86. A. F. Zakharov, Intern. J. Mod. Phys. D 27 (2018) 1841009.
87. A. F. Zakharov, J. Phys.: Conf. Ser. 934 (2017) 012037.
88. V.I. Dokuchaev and N.O. Nazarova, Preprint arXiv:1804.08030v2 [astro-ph.HE].
89. V.I. Dokuchaev, arXiv: arXiv:1812.06787v1 [astro-ph.HE].
90. A. F. Zakharov, A. A. Nucita, F. De Paolis and G. Ingrosso, New Astron. 10 (2005) 479.
91. A. F. Zakharov, A. A. Nucita, F. De Paolis and G. Ingrosso, Phys. Rev. D 76 (2007) 062001.
92. A. A. Nucita, F. De Paolis, G. Ingrosso et al., Proc. Astron. Soc. Pac. 119 (2007) 349.
93. D. Borka, P. Jovanović, V. Borka Jovanović, A. F. Zakharov, Phys. Rev. D 85 (2012) 124004.
94. A. F. Zakharov, Phys. Rev. D, 90 (2014) 062007.
95. GRAVITY Collaboration: R. Abuter, A. Amorim, N. Amugu et al., Astron. & Astrophys. Lett. 615 (2018) L15.
96. S. Gillessen, P. M. Plewa, F. Eisenhauer et al., Astrophys. J. 837 (2017) 30 (19pp).
97. D. S. Chu, T. Do, A. Hees et al., Astrophys. J. 854 (2018) 12 (10pp).
98. A. Hees, T. Do, A. M. Ghez et al. Phys. Rev. Lett. 118 (2017) 211101.
99. F. W. Dyson, A. S. Eddington and C. Davidson, Phil. Trans. R. Soc. London. Series A, 220 (1920) 291.
100. S. M. Kopeikin and L. M. Ozernoy, Astrophys. J. 523 (1999) 771.
101. T. Alexander, Phys. Rep. 419 (2005) 65.
102. S. Zucker, T. Alexander, S. Gillessen et al., Astrophys. J. 639 (2006) L21.
103. S. Herrmann, F. Finke, M. Lülf et al., Phys. Rev. Lett. 121 (2018) 231102.
104. D. A. Litvinov, V. N. Rudenko, A. V. Alakoz et al., Phys. Lett. A 382 (2018) 2192.
105. GRAVITY Collaboration: R. Abuter, A. Amorim, M. Bauböck et al., Astron. & Astrophys. Lett. 618 (2018) L10.
106. GRAVITY Collaboration: E. Sturm, J. Dexter, O. Pfuhl et al., Nature 563 (2018) 657.
107. F. Dyson, Intern. J. Mod. Phys. A 28 (2013) 1330041 (14 pages).
REFERENCES

108. A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. 82 (2010) 939.
109. C. de Rham, J. T. Deskins, A. J. Tolley et al., Rev. Mod. Phys. 89 (2017) 025004.
110. B. P. Abbott et al., Phys. Rev. Lett. 116 (2016) 061102.
111. B. P. Abbott et al., Phys. Rev. Lett. 118 (2017) 221101.
112. V. M. Lipunov et al., Astrophys. J. Lett. 850 (2017) L1.
113. B. P. Abbott et al., Astrophys. J. Lett. 848 (2017) L13.
114. A. F. Zakharov, P. Jovanović, D. Borka et al., Intern. J. Mod. Phys.: Conf. Ser. 47 (2018) 1860096.
115. B. Borka, P. Jovanović P, V Borka Jovanović and A.F. Zakharov, J. Cosm. Astropart. Phys. JCAP 11 (2013) 050.
116. A. F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, Eur. Phys. J. Web Conf. 125 (2016) 01011.
117. A. F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, J. Phys.: Conf. Series 798 (2017) 012081.
118. A. F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, Eur. Phys. J. Web of Conf. 138 (2017) 010010.
119. D. Dadhich, R. Maartens, Ph. Papadopoulos and V. Rezania, Phys. Lett. B 487 (2001) 1.
120. E. Babichev, C. Charmousis and A. Lehébel, J. Cosm. Astropart. Phys. (JCAP) 04 (2017) 027.
121. A. Zakharov, Eur. Phys. J. Web Conf. 125 (2018) 01010.
122. K. K. O’Neil, G. D. Martinez, A. Hees et al., Preprint [arXiv:1809.05400] [astro-ph.EP].