Evidence for three dimensional superconductivity in the cuprates oxides

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We review the empirical scenario emerging from the measured doping dependence of the transition temperature and the anisotropy parameter $\gamma = \xi_{ab}/\xi_c$, defined as the ratio of the correlation lengths parallel and perpendicular to the ab-planes. It suggests that two dimensional models cannot explain the occurrence of superconductivity in the cuprates. This conclusion is confirmed and extended in terms of a novel scaling relation. It involves the transition temperature, the aerial superfluid density in the ground state, $\gamma$ and the dynamic critical exponent of the quantum superconductor to insulator transition. The important implication there is that a non vanishing superfluid density in the ground state of the cuprates is unalterably linked to an anisotropic but 3D condensation mechanism.

It has been a long-standing question whether two dimensional (2D) models alone can describe and explain the essential experimental observations of superconductivity in cuprates oxides. On the one hand models of the CuO$_2$ plane alone, such as the t-J or Hubbard model, have been successful in explaining properties of the undoped insulator, including the renormalization of charge dynamics and the occurrence of a spin gap. Moreover, given the crystal structures of the superconducting cuprates oxides, it is to be expected that the physical properties of single crystals will show a high degree of anisotropy. This is indeed the case; by way of example the ratio $\rho_c/\rho_{ab}$ between the out-of-plane $\rho_c$ and in-plane $\rho_{ab}$ resistivity is very large. On the other hand, serious questions can be raised. The first is the observation that when the ground state is a superconductor both components of resistivity drop to zero at the same temperature $T_c$, so that the phase transition has genuine 3D character. The second emerges from the empirical scenario emerging from the measured doping dependence of the transition temperature and the anisotropy parameter $\gamma$, depicted in Fig.1. It shows that after passing the so called underdoped limit, at $x_o \approx 0.05$, where the quantum insulator to superconductor (QSI) transition occurs, $T_c$ rises and reaches a maximum value $T^m_c$ at $x_m \approx 0.16$. With further increase of $x$, $T_c$ decreases and finally vanishes in the overdoped limit $x_o \approx 0.27$. Here the system undergoes a quantum superconductor to normal state (QSN) transition. From Fig.1 it is also seen that decreasing dopant concentration is also accompanied by a raise of anisotropy. In tetragonal cuprates it is defined as the ratio $\gamma = \xi_{ab}/\xi_c$, of the correlation lengths parallel and perpendicular to the ab-planes. Although the fundamental reason for the anisotropy increase is not clear, it reveals a 3D-2D crossover with reduced dopant concentration. Note that the limit $\gamma \rightarrow \infty$ implies 2D critical behavior. The observation that in the underdoped regime $T_c$ falls with increasing anisotropy $\gamma$ (see Fig.2) suggests that superconductivity in the cuprates is inevitable a 3D phenomenon.

This letter addresses these issues by providing experimental and theoretical evidence that a non vanishing superfluid density in the ground state of the cuprates oxides is unalterably linked to an anisotropic but 3D condensation mechanism. We review the scenario emerging from the experimental data for the doping dependence of $T_c$ and anisotropy $\gamma$. It suggests that two dimensional models cannot explain the occurrence of superconductivity in the cuprates. This conclusion is confirmed and extended in terms of a new scaling relation. It involves $T_c$, the aerial superfluid density in the ground state, $\gamma$ and the dynamic critical exponent of the 2D-QSI transition. The important implication there is that a non vanishing superfluid density in the ground state of the cuprates is unalterably linked to an anisotropic but 3D condensation mechanism. This finding clearly reveals that two dimensional models cannot explain the occurrence of su-
perconductivity in these materials.

A generic property of cuprate superconductors is the existence of a phase transition, \( T_c(x) \), separating the superconducting from the normal conducting phase, as well as the so-called underdoped and overdoped regimes. This behavior is thought to be universal for cuprate superconductors [21]. A glance to Fig. 1 reveals that it is very well described by the empirical relation

\[
T_c(x) = T_c^m(1 + 82.6(x - 0.16)^2), \tag{1}
\]
due to Presland et al. [22]. In practice, there are only a few compounds, including \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \), \( \text{HgBa}_2\text{CuO}_{4+x} \) [23] and \( \text{Bi}_2\text{Sr}_2\text{CuO}_{6+x} \) [24], for which the dopant concentration \( x \) can be varied continuously throughout the entire doping range. In other cuprates, including \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{6+x} \) [25], \( \text{YBa}_2\text{Cu}_3\text{O}_{7-x} \) [26] and \( \text{Y}_1-x\text{Pr}_x\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta} \) [27], only the underdoped and optimally doped regimes appear to be accessible.

Another essential property in this context is the anisotropy parameter \( \gamma \). In tetragonal cuprates it is defined by \( \gamma = \xi_{ab}/\xi_c \), because \( \xi_a = \xi_b = \xi_{ab} \). In the normal state, where \( \gamma = \xi_{ab}/\xi_c = \sqrt{\rho_{ab}/\rho_c} \), it can be inferred from resistivity [24] and magnetic torque [15] measurements. In the superconducting state, where \( \gamma = \xi_{ab}/\xi_c = \lambda_{ab}/\lambda_c \), it follows from magnetic torque [13,23] and penetration depth data [23,28]. \( \lambda_{ab} \) and \( \lambda_c \) are the London penetration depth due to supercurrents flowing parallel and perpendicular to the ab-planes, respectively. In a mean-field treatment of the Ginzburg-Landau Hamiltonian \( \gamma \) can also be expressed in terms of the effective masses \( M_a \) and \( M_c \) for the motion of the pairs parallel and perpendicular to the ab-planes as \( \gamma = \sqrt{M_c/M_{ab}} \). As shown in Fig. 2 for \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \), the anisotropy parameter \( \gamma_{T_c} \), evaluated close to \( T_c \), increases as the dopant concentration is reduced and appears to diverge in the underdoped limit. This doping dependence of \( \gamma_{T_c} \), also observed in \( \text{HgBa}_2\text{CuO}_{4+\delta} \) [24] and \( \text{YBa}_2\text{Cu}_3\text{O}_{7-x} \) [28], is thought to be generic and uncovers the 3D-2D crossover, tuned by reducing the dopant concentration \( x \). Consequently, the doping does not control \( T_c \) only, but the dimensional crossover, not included in 2D models, as well. Since \( T_c \) tends to zero while \( \gamma_{T_c} \) appears to diverge in the underdoped limit, it is instructive to consider the doping dependence of \( T_c \gamma_{T_c} \), and \( T_c \gamma_{T=0} \). A glance to Figs. 2 and 3 makes it clear that these quantities scale nearly linearly with the relative distance from the overdoped limit, \( x_0 - x \). The straight lines correspond to

\[
T_c \gamma_{T_c} = a_{T_c,\gamma}(x_0 - x) \tag{2}
\]
and

\[
T_c \gamma_{T=0} = a_{T=0,\gamma}(x_0 - x), \tag{3}
\]
respectively. To check the consistency of the empirical relations (1), (2) and (3) we included in Fig. 2 the \( T_c \) values, resulting from Eqs. (2) and (3), and the experimental data for \( \gamma \). Moreover, we included \( \gamma_{T_c}(x) \), expressed in terms of Eqs. (1) and (3).

Close to both, the QSI and the QSN transition, the empirical relation (1) for \( T_c \) implies the power law behavior

\[
T_c = 27c^m\sqrt{82.6|x - x_{u,o}|}, \tag{4}
\]
and with the empirical doping dependence of \( T_c \gamma_{T_c} \) (Eq. (2)) and \( T_c \gamma_{T=0} \) (Eq. (3)) the expressions:
Their divergence in the underdoped limit \((x = x_u)\) implies a 2D-QSI transition and for any \(x_u < x \leq x_0\), where \(\gamma_{T_c}\) and \(\gamma_{T=0}\) are finite, a superconducting state with anisotropic but genuine 3D character. Consequently, the available experimental data for the doping dependence of \(T_c\), \(\gamma_{T=0}\) and \(\gamma_{T_c}\) strongly suggest that two dimensional models cannot explain the occurrence of superconductivity in the cuprates. Despite this, the vast majority of theoretical models focus on single Cu-O planes, i.e., on the limit of zero intracell and intercell c-axis coupling.

Next we confirm and extend this empirical scenario by invoking the scaling theory of quantum critical phenomena. This theory predicts that close to the QSI and QSN transition \(T_c(x)\) scales as

\[
T_c(x) = a_{QSI} T_c \left( x - x_u \right)^{z_{QSI}} \nu_{QSI},
\]

(6)

and

\[
T_c(x) = a_{QSN} T_c \left( x - x_u \right)^{z_{QSN}} \nu_{QSN},
\]

(7)

respectively. \(z\) is the dynamic critical exponent, \(\nu\) the exponent of the correlation lengths \(\xi(T=0) \propto \delta^{-\nu}\), \(a_{QSI}\) and \(a_{QSN}\) denote nonuniversal critical amplitudes. Moreover, supposing that the QSI transition occurs in 2D, \(T_c\) and the zero temperature in-plane penetration depth \(\lambda_{ab}(T=0)\) are not independent but in the limit \(x - x_u \to 0\) related by the universal relation

\[
\frac{T_c}{\lambda_{ab}^2(T=0)} = \frac{1}{\Omega_2} \left( \frac{\Phi_0^2}{16\pi^2 k_B} \right),
\]

(8)

where \(\Omega_2\) is a universal number. Here the bulk superconductor corresponds to a stack of independent superconducting slabs of thickness \(\delta_s\). Experimentally, this relation is well confirmed for various families of underdoped cuprates in terms of the empirical Uemura plot [11, 13, 21].

On the other hand, at finite temperature there is along the 3D phase transition line \(T_c(x)\) the universal relation [12, 22]

\[
k_B T_c = \frac{\Phi_0^2}{16\pi^2 \gamma_{T_c} \lambda_{ab,0}}.
\]

(9)

\(\xi_{ab,0}\) and \(\lambda_{ab,0}\) are the finite temperature critical amplitudes of in-plane correlation length for \(T \leq T_c\) and penetration depth, respectively. Matching with the quantum behavior [11, 24] and [25] requires that the finite temperature critical amplitudes scale as \(\xi_{ab,0} \propto \lambda_{ab}(T=0) \propto (x - x_u)^{-\nu_{QSI}}\) and \(\lambda_{ab}^2 \propto \lambda_{ab}^2(0) \propto (x - x_u)^{z_{QSI}}\nu_{QSI}\). Substitution into Eq. (9) leads to the remarkable result that close to the 2D-QSI transition \(\gamma_{T_c}\) diverges as

\[
\gamma_{T_c} \propto (x - x_u)^{-\nu_{QSI}},
\]

(10)

On the other hand, approaching the 2D-QSI transition at \(T = 0\) along the doping axis, the singular part of the ground state energy density scales in 3D as

\[
\hat{E}_{c}^{3D} = \lambda_{3} \left( \frac{\sqrt{\xi_{ab}^2} \xi_c}{\lambda_{ab}^2} \right)^{-1} \propto \lambda_{3} \gamma_{T=0} \left( \xi_{ab}^2 \xi_c^{-1} \right)^{-1},
\]

in 2D,

\[
\hat{E}_{c}^{2D} = \lambda_{3} \left( \frac{\sqrt{\xi_{ab}^2} \xi_c}{\lambda_{ab}^2} \right)^{-1} \propto \lambda_{3} \gamma_{T=0} \left( \xi_{ab}^2 \xi_c^{-1} \right)^{-1} \propto \lambda_{3} \gamma_{T=0} \left( \xi_{ab}^2 \xi_c^{-1} \right)^{-1},
\]

where \(\lambda_{3}\) and \(\lambda_{2}\) are universal numbers and \(\xi_c \propto (x - x_u)^{-\nu}\) is the temporal correlation length having units of energy. The 3D-2D crossover requires that both expressions give close to the QSI transition the same doping dependence. This yields

\[
\gamma_{T=0} = \frac{\lambda_{3} \xi_{ab}^2}{\Omega_{2} \lambda_{ab}^2} \propto (x - x_u)^{-\nu_{QSI}},
\]

(11)

in agreement with Eq.(10). Accordingly, given a 2D-QSI transition we confirmed the divergence of \(\gamma_{T_c}\) and \(\gamma_{T=0}\), emerging from the empirical scenario (Eq. (6)). Moreover, comparing Eqs. (6), (8) and (10) this scenario also suggests a 2D-QSI transition with critical exponents \(z_{QSI} = 1\) and \(\nu_{QSI} = 1\). Although the experimental data are rather sparse close to this transition, there is mounting evidence for \(z_{QSI} \approx 1\) and \(\nu_{QSI} \approx 1\) [10, 27]. These estimates are close to theoretical predictions [33, 34], from which \(z_{QSI} = 1\) is expected for a disordered bosonic system with long-range Coulomb interactions independent of dimensionality and \(\nu_{QSI} \geq 1 \approx 1.03\) in D=2. They are also consistent with Monte Carlo calculations on the dirty-boson Hamiltonian, yielding \(\nu_{QSI} \approx 0.9 \pm 0.15\) [35], as well as with experiments on the 2-QSI transition in thin (InO, Bi, MoGe) films, where \(\tau_{QSI}\) clusters around \(\tau_{QSI} = 1.2 \pm 0.2\) [36, 37]. In this transition, the loss of phase coherence is due to the localization of the pairs which is ultimately responsible for the transition. As the 3D-QSN transition is concerned, the estimate for \(z_{QSN}\) is 1. This value is consistent with the disordered d-wave superconductor to normal state transition considered by Herbut [38], with \(z_{QSN} \approx 1\), \(\nu_{QSN} \approx 1\), and \(\tau_{QSN} \approx 1/2\). Upon using Eqs. (6), (10) and (11) one obtains for \(T_c(x)\), \(\gamma_{T_c}(x)\) and \(T_c(x)\) the expression

\[
T_c \gamma_{T_c} \propto (x - x_u)^{-\nu_{QSI}(z_{QSI} - 1)},
\]

(12)

For \(z_{QSI} = 1\) these quantities remain finite, as suggested by the experimental data shown in Figs. [2] and [3]. More importantly, eliminating \(x - x_u\) from the scaling forms [12], [24], [25] and [26] we obtain close to the 2D-QSI transition the novel scaling form

\[
T_c \propto n_s(T = 0) \propto \gamma_{T_c}(x)^{-\nu_{QSI}} \propto \gamma_{T=0}^{-\nu_{QSI}},
\]

(13)

where \(n_s(T = 0, x) \propto d_s/\lambda_{ab}^2(T = 0, x)\) is the aerial superfluid number density. It relates the superconducting properties to the anisotropy parameters, fixing the dimensionality of the system, for any \(z_{QSI}\). Since \(z_{QSI} > 0\), this leads to the conclusion that superconductivity in the
cuprates oxides is unalterably linked to a finite $\gamma_{T=0}$, implying an anisotropic but 3D condensation mechanism.

Finally we note that in the regime where thermal fluctuations dominate, $T_c^{-\gamma}$, is proportional to the number density $\langle |\Psi|^2 \rangle$ of the pairs at $T_c$. The complex order parameter field $\Psi$ represents the wavefunction of the pairs of charge 2e. In the mean-field approximation, where $\langle |\Psi|^2 \rangle = \langle |\Psi| \rangle^2$, this quantity vanishes for $T \geq T_c$. Thus, the plot $T_c^{-\gamma}$ versus $x$, shown in Fig.3, also reveals the existence of pairs at and above $T_c$, as well as a raise of their number density with decreasing dopant concentration.

In conclusion, we have shown that the scenario emerging from the empirical doping dependence of $T_c$, $\gamma$, and $\gamma_{T=0}$ is remarkably consistent with the scaling properties resulting from a 3D critical line $T_c(x)$, ending in the underdoped and overdoped limit. Here the doping tuned QSI and 3D-QSN transitions occur. Given the evidence for a 2D-QSI transition and the associated 3D-2D crossover, tuned by decreasing dopant concentration, we derived the scaling relation (13), relating the essential characteristics of the superconducting phase to the anisotropy parameters, fixing the dimensionality of the system. It implies that for any finite value of the dynamic critical exponent ($z_{QSI}$) of the quantum superconductor to insulator transition, a non vanishing transition temperature and superfluid aerial superfluid density ($n_s(T=0,x)$) in the ground state, require a finite anisotropy. The important conclusion there is that a non vanishing superfluid density in the ground state of the cuprates oxides is unalterably linked to an anisotropic but 3D condensation mechanism. This finding clearly reveals that two dimensional models cannot explain the occurrence of superconductivity in the cuprates. They single out, however, theories that ascribe the phenomenon to interlayer coupling [40–43].

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