A decimation method is applied to the tight binding model describing the two dimensional electron gas with next nearest neighbor interaction in the presence of an inverse golden mean magnetic flux. The critical phase with fractal spectrum and wave function exists in a finite window in two-dimensional parameter space. There are special points on the boundaries as well as inside the critical phase where the renormalization flow exhibits higher order limit cycles. Our numerical results suggest that most of the critical phase is characterized by a strange attractor of the renormalization equations.

75.30.Kz, 64.60.Ak, 64.60.Fr

I. INTRODUCTION

The two-dimensional electron gas with irrational magnetic flux is a well-known paradigm in the study of systems with two competing periodicities. The magnetic field results in reducing the problem to a one-dimensional tight binding model (TBM) known as the Harper equation. The Harper equation exhibits both extended (E) and localized (L) states. At the onset of transition corresponding to a periodic potential with square symmetry, the states are critical (C) with fractal spectra and wave functions. The scaling properties of the devil staircase spectra and the wave functions have been studied extensively using various renormalization group (RG) methods.

Recently, it was pointed out that the Harper equation also describes the isotropic XY quantum spin model in a modulating magnetic field of periodicity incommensurate with the periodicity of the lattice. It was shown that the presence of anisotropy in spin space fattened the critical point of the Harper equation resulting in a phase diagram where E, L, and C phases all existed in a finite measure parameter interval. The existence of a fat C phase provided a new scenario for the breakdown of analyticity in incommensurate systems. Furthermore, based on numerical results obtained using a new decimation method it was argued that the fat C phase was described by four distinct universality classes characterized by limit cycles of the RG flow.

The fat C phase was also reported recently in the TBM describing Bloch electrons moving on a tight binding square lattice where the coupling to next nearest neighbor (NNN) sites exceeded a certain threshold value compared to the nearest neighbor (NN) coupling. Using various analytical and numerical tools to study the scaling properties of the fractal eigenspectrum, the universality classes of the C phase were investigated. The E-C and C-L transition lines were conclusively shown to define a new universality class, different from the E-L transition line (which belonged to the Harper class). However, the analysis in the interior of the C phase was rather inconclusive.

II. TIGHT BINDING MODEL FOR BLOCH ELECTRONS IN A FIELD

The TBM describing Bloch electrons on a square lattice in a magnetic field with both NN hoppings $t_a$ and $t_b$
and NNN hoppings $t_{ab} = t_{ai}$ is [10]

\[
\{t_a + 2t_{ab}\cos[2\pi(\sigma(i + \frac{1}{2}) + \phi)]\} \psi_{i+1} \\
+ \{t_a + 2t_{ab}\cos[2\pi(\sigma(i - \frac{1}{2}) + \phi)]\} \psi_{i-1} \\
+ 2t_b\cos[2\pi(\sigma i + \phi)] \psi_i = E \psi_i
\] (1)

Here, $\sigma$ is the magnetic flux which we choose to be the inverse golden mean $\sigma = (\sqrt{5} - 1)/2$. This TBM was studied in detail in ref. [10]. Fig. 1 shows the phase diagram in the space of the parameters $\lambda = \frac{t_b}{t_a}$ and $\alpha = 2\frac{\tan}{\phi}$. The Harper equation corresponds to the limit $\alpha = 0$ where the NNN coupling term is zero. The phase diagram was obtained [10] using analytical methods to obtain the scaling behavior of the total band width (TBW) and the Lyapunov exponents and carrying out a numerical multifractal analysis. The lines AC (E-C transition) and CE (C-L transition) were found to be bicritical, i.e. the TBW scaled with the system size with the exponent $\delta = 2$. This was in contrast with the critical line BC separating the E and L phases where the exponent was known to be unity. Within the region bounded by the lines AC and CE and the $\alpha$-axis, where the NNN coupling dominated, the multifractal analysis did not lead to conclusive results on the universality. This was due to lack in convergence of the $f(\alpha)$ curve with the size of the system. Furthermore, their numerical calculation of the TBW was complicated by oscillatory terms superimposed on the power law. However, this regime was conjectured to be critical.

The existence of the fat C phase implies that the presence of the NNN coupling in the Bloch electron problem introduces new universality classes. The same does not happen if the cosine term in the Harper equation is just replaced by more generic periodic functions. [11]

Eq. (1) involving both diagonal and off-diagonal disorder bears some resemblance to the TBM describing quasiparticle fermion excitations in a quantum XY spin chain. [8] Unlike Harper, both the anisotropic XY spin chain and Eq. (1) exhibit C phase in a finite parameter interval. However, in the TBM (1) the fat C phase is observed beyond a critical value of the NNN hopping whereas in the spin model the fat C phase can be seen even with infinitesimal spin space anisotropy.

Motivated by the success of our decimation scheme to confirm the existence of a fat C phase and obtaining the universal behavior in the quantum spin chain [8], we now apply the method to Eq. (1). Our main focus is to obtain the universality classes of the fat C phase.

III. DECIMATION SCHEME

Our decimation approach describes the scaling properties of the wave functions for a specific value of energy. Although all quantum states are fractal, it is more useful to study states at the band edges. This is because the self-similar behavior is usually observed only for the minimum and maximum energy states and also for the band center if $E = 0$ is an eigenenergy. In order to get the overall picture, it is sufficient to consider only one of these states and we will focus on the quantum state corresponding to $E_{\text{min}}$.

In RG analysis, in addition to fixing the quantum state, one has to also fix the phase factor $\phi$ in Eq. (1). It has been pointed out in the previous studies [8, 7], the wave function $\psi_i$ obtained by iterating the TBM diverges unless the phase factor $\phi$ is tuned to some critical value. In the Harper model, the critical value of the phase factor is $\frac{\pi}{4}$ for the negative band edge. For this value, the main peak is centrally located and the wave function is symmetric about $i = 0$. In the study of the quantum spin model [6], the phase factor had to be varied continuously in the fat C phase so that the main peak could be centrally located and the resulting wave function became bounded. Determination of this critical phase factor was essential in order to find the RG limit cycles and to compute the universal scaling ratios. In general the phase factor $\phi$ for obtaining symmetric wave function need not be identical to the phase factor resulting in bounded wave functions. [6]

We consider an infinite lattice which extends in both positive and negative directions from the $i = 0$ site. In the decimation scheme, all sites except those labelled by positive as well as negative Fibonacci numbers are decimated. The resulting TBM connecting the wave function $\psi$ at two neighboring Fibonacci sites can be written as

\[
\psi(i + F_{n+1}) = c_n^+(i)\psi(i + F_n) + d_n^+(i)\psi(i) \\
\psi(i - F_{n+1}) = c_n^-(i)\psi(i - F_n) + d_n^-(i)\psi(i).
\] (2) (3)

The index $n$ above refers to the level of decimation.

For the Harper model it suffices to define only one set of the "decimation functions" $c_n(i)$ and $d_n(i)$ because of the symmetry of the wave function about $i = 0$. This implies that the scaling ratios are the same on the positive and negative side. This was also asymptotically true in the quantum spin model everywhere else except along the C-L transition line [6], where the wave functions were asymmetric about $i = 0$ resulting in vanishing scaling ratios on one side. [12] The reason why we now have to introduce separate decimation functions for the positive (+) and negative (−) side is that sometimes the asymptotic ($n \to \infty$) $c_n^0(0), d_n^+(0)$ appear to be shifted compared to $c_n^-0(0), d_n^-0(0)$. This type of shifted symmetry turns out to be helpful in locating higher order limit cycles and is discussed in detail in Section V.

Using the defining property of the Fibonacci numbers, $F_{n+1} = F_n + F_{n-1}$, the following recursion relations are obtained for $c_n$ and $d_n$ (we will omit the +, − indices if the equations do not depend upon them) [11, 7]:

\[
\psi(i + F_{n+1}) = c_n^+(i)\psi(i + F_n) + d_n^+(i)\psi(i) \\
\psi(i - F_{n+1}) = c_n^-(i)\psi(i - F_n) + d_n^-(i)\psi(i).
\] (2) (3)
\[c_{n+1}(i) = c_n(i + F_n)c_{n-1}(i + F_n) - d_n^{-1}(i)d_{n+1}(i) \quad (4)\]
\[d_{n+1}(i) = -d_n(i)[d_n(i + F_n) + c_n(i + F_n)c_{n-1}(i + F_n)]c_{n-1}(i). \quad (5)\]

For a fixed \(i\), the above coupled equations for the decimation functions define a RG flow which asymptotically \((n \to \infty)\) converge on an attractor. In our earlier studies, the E, C, and L phases were distinguished by the distinctions in the attractors of the RG flow. In the Harper as well as in the quantum spin case, the C phase was characterized by a nontrivial asymptotic p-cycle at the band edges with \(p\) equal to 3 or 6.

The existence of a nontrivial \(p\)-cycle for the decimation functions often implies that the wave function is neither extended nor localized and exhibits the self-similarity described by
\[
\psi(i) \approx \psi([\sigma^p i + 1/2]) \quad (6)
\]
where \([\ ]\) denotes the integer part. The \(p\)-cycle of the self-similar bounded wave function can be used to define the universal scaling ratios
\[
\zeta_j = \lim_{n \to \infty} |\psi(F_{pn+j})/\psi(0)|; \quad j = 0, ..., p - 1. \quad (7)
\]

This equation describes the decay of the wave function with respect to the central peak. A well-defined limit \(\zeta_j\) exists for an integer \(p\) for which asymptotically \(\psi(F_{n+p}) \approx \psi(F_n)\). For an even \(p\), it often happens that \(|\psi(F_{n+p/2})| \approx |\psi(F_n)|\) so that the above equation defines actually only \(p/2\) different scaling ratios. Whenever any scaling ratio \(\zeta_j\) takes a finite value between zero and unity, the wave function over the infinite system is neither localized nor extended. In order to fully characterize the self-similarity of a wave function, an infinite number of scaling ratios have to be defined. However, using the ones defined above one can already separate different universality classes from each other.

The numerics to demonstrate a \(p\)-cycle was rather challenging for the case where \(E = 0\) was not an eigenenergy. This was because at many points in the phase diagram, the energy was required with 16 digits precision (machine double precision) in order to see the asymptotic cycle with two or more digits of precision. Even for tridiagonal matrices, we were able to determine the energy only up to 12 digits. With this precision, the decimation equations could be iterated only about 16 times. The conjecture for the existence of a limit cycle provided a very efficient Newton method where the energy and the limit cycle were determined self-consistently. Diagonalization routines provided a good starting value of the energy which in principal could be improved to an arbitrary precision. At many points where the transients were rather long, the quadruple precision was used to confirm the existence of a limit cycle.

### IV. HIGHER ORDER SYMMETRIC LIMIT CYCLES

Fig. 1 shows the E, C, and L phases of the model in the \(\lambda - \alpha\) space. The iteration of decimation equations shows that the BC critical line defining the boundary between the E and L phases is described by a 3-cycle of the RG flow. On this line (with the exception of the point C) the decimation functions flow to the same limit cycle as for the critical Harper equation (point B). However, the point C is described by a different 3-cycle defining a new universality class. Table I compares the universal scaling ratios in these two universality classes. Fig. 2(a-b) show the wave function in these two cases. In both cases, a bounded wave function was obtained for \(\phi = 1/2\) and the wave function as well as decimation functions were symmetric about the center of the lattice: i.e. \(c^+ = c^-\) and \(d^+ = d^-\).

The bicritical line AC was found to exhibit two new universality classes: At the point A, the decimation functions were found to asymptotically converge to a period-6 limit cycle for \(\phi = 1/2\). The regime bounded by the points A and C on the line AC (excluding the points A and C) appears to be described by a unique symmetric limit cycle of the period 12 for \(\phi = 1/2\). The 12-cycle is particularly clear for the middle point M (\(\lambda = .5, \alpha = 1\)). However, for other points on this line, the RG flow may exhibit long transients before settling on the limit cycle of the point M. This is illustrated in Fig. 3 for the point MM (\(\lambda = .25, \alpha = 1\)). The RG iterations begin close to the 6-cycle of the point A and then follow the 3-cycle of the point C for a while before approaching the 12-cycle of the point M. There is additional complexity involved because in order to observe the approach we had to shift the data for MM by 6 decimation levels. We used the quadruple precision to confirm that the MM limit cycle asymptotically converged to the limit cycle of M.

Symmetric period-12 cycles were also observed at the points F, G, and H which fall on the line \(t_a = t_{ab}\). However, the limit cycle at each point was different implying different scaling properties and hence different universality classes.

It should be noted that with the exception of the points B, C, and H, all the limit cycles observed for \(\phi = 1/2\) resulted in asymptotically diverging wave functions (see Fig. 2). The decimation functions corresponding to diverging wave functions cannot be used to obtain universal scaling ratios for the new universality classes. In section V, we determine the bounded wave functions by tuning the phase factor.

We also studied systematically other points on the lines CE and FH and points inside the C phase, but we did not see any evidence of limit cycles for \(\phi = 1/2\). Since the numerics should be sufficient to show at least cycles of length 12 (unless there are very very long transients),
we can conjecture that there are no cycles of the order 12 or shorter on these lines with the phase 1/2. In section VI, we present some evidence that the rest of the C phase is described by a strange attractor of the RG flow.

V. SHIFTED SYMMETRY AND PERIOD DOUBLING

Fig. 4 shows the bounded wave functions obtained by tuning the phase factor \( \phi \) to a certain critical value. Unlike the symmetric wave functions of section IV, these wave functions are not symmetric about \( i = 0 \). It turns out that this asymmetry is due to a constant shift between the wave functions on the positive and negative sides. Numerical iteration of the TBM (1) shows that in addition to Eq. (7), the wave functions on the positive and negative sides are asymptotically related by

\[
\psi(F_n) \approx \psi(-F_{n+s}) \quad (8)
\]

Instead of the symmetric solution for the decimation functions corresponding to \( c_n^+(0) = c_n^-(0), d_n^+(0) = d_n^-(0) \) for all decimation levels \( n \), we found the asymptotic equations

\[
\begin{align*}
c_n^+(0) &\approx c_{n+s}^-(0) \\
d_n^+(0) &\approx d_{n+s}^-(0),
\end{align*}
\]

i.e. asymptotically there was a shift of \( s \) levels between the positive and negative decimation functions. Moreover, in above + and − could be interchanged with the same shift \( s \), which implied that \( c_n^+(0) \approx c_{n+s}^-(0) \approx c_{n+2s}^+(0) \), i.e. the asymptotic period \( p = 2s \). The shift \( s \) was found to be equal to the period of the symmetric limit cycle discussed in section IV. Therefore, the phenomenon of shifted symmetry resulted in doubling the period of a limit cycle for the decimation functions.

The points F and G, which respectively fall on the intersection of the line \( t_a = t_{ab} \) and the lines \( t_b = 0 \) and the self-dual line BD, exhibit the phenomenon of shifted symmetry with \( s = 12 \) (see Table II). This value implies the asymptotic cycle-length 24.

The phenomenon of shifted symmetry is very crucial in locating the limit cycles of period 24. This is because with double precision arithmetics, the RG equations can be iterated only about 24 times. Without having the shifted symmetry we could not have deduced that the asymptotic period is 24 as we could not go to big enough decimation levels to see the full cycle on one side only. This is shown explicitly in Table II. The same shift and period is observed for the critical phase at the point M in the middle of the line AC. At the point A the shift is 6 and the asymptotic period therefore 12.

VI. STRANGE SET OF THE RENORMALIZATION FLOW

We explored the idea of describing the region bounded by the lines AC and CE by a strange set of the RG flow. Fig. 5 shows the two-dimensional projection of the attractor obtained by plotting an inverse decimation function for two subsequent decimation levels in various parts of the phase diagram. Having \( E_{min} \) up to 12 digits, the RG equations are estimated to give correct decimation functions up to about 16 levels. Transients were taken into account by excluding the first six decimation levels from the data. It is interesting to note that the iteration of the decimation functions in three different parameter regimes namely the line FH, the region CHG (excluding the line CH), and the region ACGF appear to asymptotically converge on roughly the same set. Similar figures were obtained also on the line CG, GD and the lines FG and GH. The fact that different parts of the phase diagram are described by similar invariant sets makes us to exclude the possibility that the observed behavior is due to long transients. However, although the possibility of a very long limit cycle cannot be completely ruled out, we believe that the observed behavior suggests that the interior of the fat C phase (excluding the special points which exhibit limit cycle) is attracted to a unique invariant set of the RG equations. We conjecture that the set is a strange attractor.

The iterates of the RG flow on the CE line seem to lie on the inner boundary of the invariant set corresponding to the interior of the C phase (see Fig. 5). In the previous studies [10], the CE line defining the boundary of C and L phases was found to be bicritical. In analogy with AC line, we would expect that the decimation functions on the CE line converge to a limit cycle of the order 24 for \( \phi = 1/2 \). We did not see any evidence of this cycle. However, its existence can not be ruled out specially in view of the possibility of long transients and the fact that even the symmetric limit cycle could be of order 24. Therefore, the problem of determining the universality class along the CE line describing the C-L transition remains open.

VII. CONCLUSIONS

In this paper, we demonstrate that our decimation scheme is an extremely useful tool to study general quasiperiodic TBMs. The Bloch electron with NNN interaction and the anisotropic quantum XY chain in a transverse field are two known examples where the C phase exists in a finite parameter range. In the spin problem, the C phase was characterized by four different limit cycles of the RG flow. \[ ] The present study shows that in the Bloch electron case the situation is lot more complex: in addition to six new universal limit cycles,
which correspond to self-similar wave functions, there is strong numerical evidence of a RG strange set. To best of our knowledge, this is the only example of a quasiperiodic model where the golden mean incommensurability does not result in self-similar wave functions at the band edges. Even in the regime where no limit cycles exist, the RG scheme provides a clear distinction between the E, C, and L phases: in the E and L phases, the decimation functions are trivial, and in the C phase they assume finite non-trivial values. It should be noted that in this regime where the C phase is not described by the limit cycles of the RG equations, the previous studies showed lack of convergence in $f(\alpha)$ curve.

The general case of TBM (1) where the NNN interactions $t_{ab}$ and $t_{ab}$ are not equal provides an interesting limit of the Bloch electron on a triangular lattice. Our studies have shown that the universality class of this model is related to the subconformal universality class of the Ising model. This establishes an interesting relationship between the anisotropic Bloch electron with NNN interaction and anisotropic quantum spin chains.

In the quantum spin problem, the fattening of the C phase is due to the broken O(2) symmetry in spin space which translates to the broken U(1) symmetry in the quasiparticle fermion Hamiltonian. For the Bloch electron, the existence of C phase is due to a NNN interaction. Although at this point we are unable to pin point the commonality between the symmetric breaking in these two problems, we believe that the origin of the fat C phase and new universality classes may be tied to certain broken symmetries of the models.

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FIG. 1. The phase diagram of the anisotropic electron gas with $t_{ab} = t_{ab}$. The solid lines BC, AC, and CE are respectively the E-L, E-C, and C-L transition lines. With the exception of the point C, the BC line is described by the Harper universality class. The bicritical line AC is described by three different universal limit cycles corresponding to the points A, C, and the regime in between A and C. In addition, the points F, G, and H are also described by limit cycles of the RG flow. The period of the limit cycle (see section IV and V) is indicated in a bracket close to the point. The two entries inside the bracket describe the symmetric and the shifted-symmetry periods. For example, (12) near the point H shows that it exhibits only the symmetric limit cycle (bounded wave function) with period $p = 12$. The (12,24) near the point G shows that it exhibits both the symmetric as well as the shifted-symmetry limit cycles of periods 12 and 24, respectively.

FIG. 2. (a-g) shows the wave function at the points B, C, A, M, F, G, and H corresponding to the phase factor $\phi = 1/2$ such that the reflection symmetry about $i = 0$ is preserved. This causes the wave function to diverge for the points A, M, F, and G while the wave function at the point B, C, and H is bounded. In these plots, the maximum value of the wave function is scaled to unity.
Fig. 3. (a) The inverse decimation function $1/c_n(0)$ vs. $n$ ($\phi = 1/2$) along the bicritical line AC: The data for the point MM ($\lambda = .25, \alpha = 1$; shown by crosses), shifted by 6 decimation levels, eventually follows the 12-cycle of the point M ($\lambda = .5, \alpha = 1$; solid line with small crosses showing the locations of the periodic orbit).

Fig. 4. (a-d) show the absolute value of the wave function at the points A ($\phi = 1/4$), M ($\phi = .3202185$), F ($\phi = .2777...$), and G ($\phi = 1/3$). The phase factor $\phi$ is chosen so that the main peak is centrally located resulting in a bounded wave function.

Fig. 5. A two-dimensional projections of the inverse decimation functions inside the fat C phase. The data has been obtained by sampling on the line FH (a), in the region CHG (b), and (c) of the whole C phase which includes (a) and (b) and also the square ACGF and the points above it. The dark dots correspond to the data obtained along to the line CE (excluding the points C and H).

Table I. The universal scaling ratios $\zeta_j (j = 1, 3)$ at the point B (Harper) and at the point C (bicritical).

| $j$ | $\zeta_j(B)$ | $\zeta_j(C)$ |
|-----|--------------|--------------|
| 0 (0, 2, 8, 34, 144, 610,...) | 0.2107 | 0.1712 |
| 1 (1, 3, 13, 55, 233,...) | 0.2107 | 0.2353 |
| 2 (1, 5, 21, 89, 377,...) | 0.2107 | 0.2387 |

Table II. The decimation functions for the point G ($\phi = 1/3$) at site $i = 0$ showing the shifted symmetry with $s = 12$ and thus indirectly implying the limit cycle of length 24. We see that $c_n^\dagger(0) \approx c_{n+12}(0)$ and also $d_n^\dagger(0) \approx d_{n+12}(0)$.

| $n$ | $c_n^\dagger(0)$ | $c_n(0)$ | $d_n^\dagger(0)$ | $d_n(0)$ |
|-----|----------------|---------|-----------------|---------|
| 6   | -0.817         | -2.477  | -3.499E-02      | 0.109   |
| 7   | 2.852          | -6.304  | -1.257          | -3.952  |
| 8   | -4.149         | 5.509   | 1.439           | -0.501  |
| 9   | 1.990          | 1.176   | 0.191           | -4.571E-02 |
| 10  | -4.463         | -1.166  | -1.685          | 0.302   |
| 11  | 2.733          | -4.122  | -1.108          | -1.808  |
| 12  | -3.255         | 42.235  | 0.498           | -9.109  |
| 13  | 1.517          | 1.559   | 0.140           | -1.328E-02 |
| 14  | 4.982          | -6.744  | 3.607           | 0.250   |
| 15  | -3.738         | -0.831  | -0.427          | -1.720E-02 |
| 16  | -1.961         | -1.957  | 0.467           | 8.547E-02 |
| 17  | -12.536        | 9.281   | -6.087          | 0.944   |
| 18  | -2.507         | -0.809  | 0.109           | -3.454E-02 |
| 19  | -6.266         | 2.874   | -3.942          | -1.26   |
| 20  | 5.532          | -4.132  | -0.502          | 1.436   |
| 21  | 1.174          | 1.996   | -4.567E-02      | 0.191   |
| 22  | -1.168         | -4.457  | 0.303           | -1.684  |
| 23  | -4.119         | 2.738   | -1.807          | -1.168  |
| 24  | 42.288         | -3.256  | -9.106          | 0.498   |