Distortion of Interference Fringes and the Resulting Vortex Production of Merging Bose-Einstein Condensates

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We investigate the distortion of interference fringes in colliding and merging Bose-Einstein Condensates due to inter-atomic interaction effects. We also consider the effect of the distortion on the spontaneous formation of vortices. The interference pattern depends crucially on two relevant parameters, the relative center-of-mass velocity and peak density of the initial state, which comprises a pair of initially well-separated, but coherent, condensate clouds. We identify three qualitatively distinct regimes of behavior during the subsequent evolution of the condensates: collision; expansion; merging. Using a comprehensive set of numerical simulations based on the Gross-Pitaevskii equation, we specify the cross-overs between these regimes and identify the system parameters required for dynamical instabilities and vortex creation. Interference fringes, which would be planar in the absence of interactions, are distorted by the combined effects of interaction and the non-uniform density profile of a harmonically trapped condensate. In appropriate parameter regimes, the distortion is sufficiently large to produce a net circulation and thereby a dynamical instability occurs by which vortices form with their cores on the interference minima.

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I. INTRODUCTION

Experiments in which a Bose condensed cloud of ultracold atoms is divided into two spatially separated components and then allowed to recombine have been widely reported [1–11]. These experiments are of fundamental interest, for example, in demonstrating the quantum nature of the condensate and in investigating decoherence. In addition, such processes are central to matter wave interferometry using ultra-cold atom condensates, which may have many technological applications. In general, there are two ways in which initially separated clouds can be made to evolve so that they subsequently overlap. Firstly, they can be allowed to expand by relaxing the confining potential that holds them apart [1, 3, 4]. Secondly, they can be subjected to external potential gradients that cause the clouds to move together and collide whilst maintaining their form [11]. In realistic scenarios, a merging process will, to some extent, involve both of these mechanisms. The resulting interference pattern will be more complex than in either of the idealized cases, particularly for systems in which inter-atomic interactions are involved. There has been some theoretical work on freely expanding condensate clouds in which inter-atomic interactions are shown to increase the expansion rate [12] and create nonuniform interference fringes [13, 14]. However, to date there has been little study of this general case or how it compares with either the purely expanding or purely colliding scenarios, in particular, the dynamics of curved fringes in the merging BECs.

It is now known that, while the behavior of very low density condensates can be well described using elementary single-particle quantum mechanics, systems in which the interactions are stronger show behavior which is quantitatively, and frequently qualitatively, different. In particular it was shown in Ref. [15] that the interference pattern formed when two counterpropagating interacting clouds overlap can give rise to the formation of persistent dark solitary waves and, subsequently, the nucleation of linear arrays of vortex rings. As well as having implications for real matter wave interferometers, these processes are of intrinsic interest as an experimentally controllable route to homogeneous quantum turbulence [16]. Recently, experiments have observed the spontaneous formation of vortices in a ring trap in which the condensate is split into three components and then allowed to recombine [12]. Related theoretical work focuses on the role of ramping down the trap potential and phase imprinting on the formation of these spontaneous vortices [18, 19]. Although the role of interference in the formation of vortices has been pointed out [18, 19], the detailed description of how the interference affects the formation of nonlinear excitations has not been shown.

In this paper, we model experiments in which a Bose-condensed cloud is divided adiabatically into two components, for example by using a tailored magnetic potential from an atom chip [4], the introduction of an optical barrier via a shaped blue-detuned laser [6] or by passing counterpropagating red detuned laser beams through an acousto-optic modulator driven at RF frequencies [8]. We examine how the process of both expansion and collision affects the interference patterns of two BECs by varying the initial peak densities of the clouds and their relative center-of-mass (c.m.) collision velocities (controlled by varying the accelerating potential in which they move). Our numerical simulations, based on the mean field Gross-Pitaevskii equation (GPE), show that in regimes where the interactions are significant, interference fringes first develop non-uniform spacings and then become curved. In extreme cases, the curvature can be-
come sufficient for there to be a net circulation around a localized core region leading to the formation of a vortex ring. Interactions are important at higher densities but at lower collision velocities. At higher collisional velocities the system cannot respond hydrodynamically to the high-density fringes before the clouds have passed through one another and the fringes have disappeared. Through the analysis of a set of simulations and known experiments, we argue that a connection between vortex formation and interference in merging BECs exists. Specifically, the interference of the condensates generates high-density peaks and extremely low-density valleys. The combined effects of inter-atomic repulsion and the non-uniform density profile of the condensates produce the additional degrees of freedom in the area of interference, with the result that the high-density areas from the interference peaks encircle the low-density areas between them.

Our general protocol involves preparing each of a pair of clouds in the lowest (mean field) energy state of a harmonic trap. The prepared clouds are then displaced in opposite directions and allowed to evolve when subject to a harmonic trap potential, which need not be the same as that used to prepare the initial clouds. In general, two dynamical processes will occur before the two clouds overlap. Firstly the clouds can undergo (ballistic) c.m. motion in response to the trap potential. Secondly the clouds can change shape as they evolve (most notably expanding or contracting). In general, of course, both processes will occur but we can identify two extreme limits. If two copies of a non-interacting Bose gas are prepared in the lowest energy state of a given harmonic trap and are then placed at symmetrical points away from the center of that trap, the clouds will move ballistically without changing volume until they overlap spatially. We refer to this as a “colliding process”. Alternatively, two (interacting) clouds prepared as above can be initially held a fixed distance apart and then allowed to evolve after switching off the trap potential. In this case, the clouds will expand until they overlap. We refer to such a process as an “expanding” process. Intermediate cases where neither ballistic c.m. motion nor expansion dominate will be referred to as “merging” processes.

This paper is organized as follows. Section II formally describes the construction of the initial state of two colliding, expanding, and merging condensates, and the numerical procedure for simulating the dynamics of the two condensates. In Section III, we derive the interference formulae which quantitatively describe the interference pattern of two condensates coming together. We also identify three distinct dynamical regimes in the cloud dynamics. Section IV shows the typical interference patterns of the three different processes and the formation of spontaneous vortices in merging condensates. Also the correlation between vortex formation and interference in merging process is shown. In section V, we summarize general properties in the parameter space. Section VI contains our conclusions.

FIG. 1: Solid curves: schematic representations of the effective potential for trapping two condensates (a) and for the merging them (b). Shaded areas in (a), (b) represent the atom density profile $|\Phi(z, 0, 0)|^2$.

II. SIMULATIONS

In the most general case, our simulations have the following features. We begin with a trap potential of the form

$$V_0(r, z) = \frac{1}{2} m \omega^2_{\perp} r^2 + \frac{1}{2} m \omega^2_{\parallel} z^2,$$

where $(r, \theta, z)$ are polar coordinates of positions within the trap. We choose the form for the function $\phi_0(r, z)$ which minimizes the energy functional

$$E[\phi_0] = 2\pi \int_{-\infty}^{\infty} dz \int_0^\infty dr \left\{ \frac{\hbar^2}{2m} |\nabla \phi_0|^2 + V_0(r, z) |\phi_0|^2 + \frac{1}{2} g |\phi_0|^4 \right\},$$

subject to the normalization constraint

$$N = 2\pi \int_{-\infty}^{\infty} dz \int_0^\infty dr |\phi_0|^2 r dr dz$$

where $g = 4\pi \hbar^2 a_s / m$ is the inter-atomic coupling constant and $a_s$ is the s-wave scattering length. We use the value $a_s = 2.9$nm and $m = 3.82 \times 10^{-26}$kg for a Sodium-23 condensate throughout this paper.

The initial state for the simulations is then obtained by setting the order parameter to have the form

$$\Phi(r, z, t = 0) = \sqrt{\frac{1}{2(1 + Q)}} (\phi_0(r, z - \Delta) + \phi_0(r, z + \Delta)),$$

where

$$Q = \frac{1}{N} 2\pi \int_{-\infty}^{\infty} dz \int_0^\infty dr r r \phi_0(r, z - \Delta) \phi_0(r, z + \Delta),$$

representing a pair of clouds displaced in opposite directions along the z-axis, as shown in Fig.1(a). The parameter $Q$ ensures correct normalization of $\Phi$ in situations in which two atomic clouds are not initially fully separated. It is worth pointing out that the representation in terms of
a single order parameter, \( \Phi \), implies that the two clouds are fully coherent and it is not the case that there are \( N/2 \) atoms in each cloud.

The subsequent evolution of the system is determined by numerically solving the Gross-Pitaevskii equation

\[
i \hbar \frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Phi + V(r,z)\Phi + g|\Phi|^2\Phi, \quad (6)
\]

assuming that the initial rotational symmetry about the \( z \)-axis is preserved, using a 2D Crank-Nicholson algorithm adapted to correctly treat the polar variable \( r \). The potential is now

\[
V(r,z) = \frac{1}{2}m\omega_r^2 r^2 + \frac{1}{2}m\omega_z^2 z^2, \quad (7)
\]

where in general \( \omega_r \leq \omega_{r,0} \) and \( \omega_z \leq \omega_{z,0} \).

III. GENERAL DESCRIPTION OF TWO MERGING CONDENSATE CLOUDS

In this section, we analyze the interference pattern generated by the merger of two condensates and identify three types of behavior and their corresponding parameter regimes. We refer to the two extreme cases as collision and expansion, and to the generalized intermediate case as merging. While the interference fringes described by the (linear) Schrödinger equation are planar, the interactions included in the GPE via the non-linear term, \( g|\Phi|^2\Phi \), give rise to phenomena such as curved fringes [1]. The repulsive interaction depends on the density distribution in the interference region of the two condensate clouds. Consequently, there is a position-dependent rate of expansion, resulting in different fringe spacings at different positions.

Our purpose is to explore the relationship between nonlinear interactions and the expansion rate of condensate peaks, which we quantify using \( \eta = \int_0^\infty z|\Phi(z,t)|^2dz/\int_0^\infty |\Phi(z,0)|^2dz \). The system of interest is a series of condensate fringes with similar shapes but different densities. The presence of interactions should cause the fringes to expand at different rates, and hence their spacings to change. In order to approximately simulate the expansion of individual fringes, we consider the 1D model of a freely expanding single condensate initially held in a harmonic trap. To consider different fringe densities, the numbers of atoms in the model are changed, and then the trap frequency adjusted to create a similar density profile. Inspired by a 3D model of a classical gas [21], we introduce a scaling factor \( \eta(t) \) for our 1D case, where the condensate width \( R(t) = \eta(t)R(0) \). The analytic solution of the expansion rate can be derived by adapting the method used in [20] to the 1D case. In the Thomas-Fermi approximation, \( \eta(t) \) satisfies the dynamic equation

\[
\dot{\eta} = \frac{\omega_r^2}{\eta^2}, \quad (8)
\]

where \( \omega \) is the trap frequency. By integrating Eq. (8), we find

\[
\frac{1}{2} \left( \sqrt{\eta(\eta - 1)} + \log \left| \sqrt{\eta + \sqrt{\eta - 1}} \right| \right) = \frac{\omega t}{\sqrt{2}} \quad (9)
\]

For small \( t \), the solution for Eq. (9) is

\[
\eta = 1 + \frac{1}{2} \omega^2 t^2. \quad (10)
\]

If \( t \) is large, \( \eta >> 1 \), then we obtain

\[
\eta = \sqrt{2} \omega t. \quad (11)
\]

Eqs. (10) and (11) indicate that a higher peak density, corresponding to larger \( \omega \), causes a higher rate of expansion in 1D freely expanding clouds. Fig. 2 shows that our simulations are in basic agreement with analytic prediction that a higher density leads to a larger expansion rate. Although we fit the numerical data well at small and large \( t \) respectively using generic parabola and linear formulae, the fitting formulae are not the same to Eq. (11) and Eq. (11). This originates that at short time, the expansion of the cloud is speeded up due to the interatomic interaction, and then the Thomas-Fermi shape of the density distribution is broken down.

![FIG. 2: The time-dependent expansion rate of the 1D freely expanding condensate, with \( g_{\Sigma D} = 1.3a_s \), from our simulation (curves from top to bottom have \( N = 100, 400, 600, 800 \)); Insert: short-time behaviors.](image-url)

Furthermore, we take into account the effect of the repulsive interaction on interference fringes by introducing a variable \( \alpha \), which is a modified coefficient depending monotonically on the expansion rate of interference fringes. Qualitative insight into the properties of the interference of two condensate waves can be gained by using a Gaussian ansatz,

\[
\phi(z, t) = C \sqrt{\frac{1}{1 + i\chi(t)}} e^{-(1 - i\chi(t))z^2/2\sigma^2(t)}, \quad (12)
\]

for the wave function, where \( C = (\pi\ell_0^2)^{-1/4}, \chi(t) = \hbar t/m\ell_0^2 + \alpha t/\hbar \ell_0^2 \) and \( \sigma^2(t) = \ell_0^2 (1 + \chi^2(t)) \).

Now, we arrange a pair of wavepackets initially at positions \( \pm L/2 \), \( \Phi(z, 0) = \frac{1}{\sqrt{2}} (\phi(z - \frac{L}{2}, 0) + \phi(z + \frac{L}{2}, 0)) \).
Then they move toward each other with the same speed \( v = \hbar k^\prime / m \), described by
\[
\Phi(z, t) = \frac{1}{\sqrt{2}}(\phi(z - L/2, t)e^{-ik^\prime z} + \phi(z + L/2, t)e^{ik^\prime z}),
\]
where the wavevector \( k^\prime \) is related to the c.m. speed of atom cloud, depending on the variation of trap potential. Inserting Eq. (12) into Eq. (13), we deduce the expression of \(|\Phi(z, t)|^2\), and the cosine term in \(|\Phi(z, t)|^2\) shows the effective wavelength for the interference fringes,
\[
\lambda = \frac{2\pi|l_0^2 + (\frac{m}{\hbar^2} + \frac{\hbar^2}{m})|}{L(\frac{h^2}{m} + \frac{\hbar^2}{m}) + |l_0^2 + (\frac{h^2}{m} + \frac{\hbar^2}{m})|^22k^\prime}.
\]
Assuming that \( g_{1D}|\Phi|^2 \) is zero (i.e., single-particle picture) and the shape of the clouds is frozen, the time-dependent terms in Eq. (13) vanish and thus \( \lambda = \frac{\pi}{m\hbar} \), analogous to classical interference.

Setting, \( k^\prime = 0 \) so that the centers of the initial states don’t move and \( g_{1D}|\Phi|^2 = 0 \), means that the two wave packets expand freely and the interference fringe spacing then is
\[
\lambda = \frac{2\pi(l_0^2 + \frac{\hbar^2z^2}{m})}{\hbar L}. \tag{15}
\]
We refer to this regime as expanding. In general, the first term in the bracket in Eq. (15) can be neglected with respect to the second term at large \( t \), resulting in the fringe spacing \( \lambda = \hbar t/mL \), which has been used in experiment \( 1 \) to explain the interference of two expanding condensates.

Except for these limiting cases, all other processes of two condensates coming together are in the merging regime. Obviously the inter-atomic interaction affects the interference pattern in merging process. For example, if the role of the nonlinear interaction is involved in the collision, Eq. (14) becomes
\[
\lambda = \frac{\pi(l_0^2 + \frac{\hbar^2z^2}{m})}{\frac{\alpha^2}{\hbar^2} + (l_0^2 + \frac{\hbar^2z^2}{m})k^\prime}, \tag{16}
\]
where the larger inter-atomic interaction, the larger \( \alpha \), leading to the larger \( \lambda \).

## IV. DISTORTION OF FRINGES IN TWO INTERFERING CONDENSATES

In this section, we illustrate numerically “collision”, “merging” and “expanding” behaviors of two BECs. In particular, we focus on the role of the repulsive inter-atomic interactions in the distortion of fringes and the resulting spontaneous vortex formation.

### A. System I: Free Expansion of Well Separated Clouds

We set the initial separation of the two condensates, \( \Delta = 4.8l_{\parallel} \) with \( l_{\parallel} = \sqrt{\hbar/\mu \omega_{\parallel,0}}, \) where each condensate wavefunction \( \phi \) is the groundstate solution of Eq. (6) with trap frequencies \( \omega_{\parallel,0} = 2\pi \times 180\text{Hz} \) and \( \omega_{\perp,0} = 2\pi \times 120\text{Hz} \). Then, the system is allowed to evolve freely by setting \( \omega_{\parallel} = \omega_{\perp} = 0 \). In the absence of interactions, the order parameter has the form \( \Phi(r, z - \Delta, t) + \Phi(r, z + \Delta, t) \) where \( \Phi(r, z, t) = (m\omega_{\parallel}^2)^{3/4} \alpha_\perp(t) \sqrt{\alpha_\parallel(t)} \times e^{-(\alpha_1(t) - i\beta_1(t))z^2/2\omega_{\perp}^2} e^{-\alpha_\perp(t)z^2/2\omega_{\parallel}^2} \times (\cosh(\frac{2\alpha_\parallel(t)\Delta z}{l_{\parallel}^2}) - \cos(\frac{2\beta_\parallel(t)\Delta z}{l_{\parallel}^2}))) \tag{17} \)

where \( \alpha_j = 1/(1 + \omega_j^2 t^2) \) and \( \beta_j = \omega_j t/(1 + \omega_j^2 t^2) \).

Hence at arbitrary time \( t \) we have
\[
n(r, z, t) = |\Phi(r, z, t)|^2 = 2\alpha_1^2(t)\alpha_\parallel(t) \times e^{-\alpha_\perp(t)z^2/\omega_{\perp}^2} e^{-\alpha_\perp(z^2 + \Delta^2)/\omega_{\parallel}^2} \times (\cosh(\frac{2\alpha_\parallel(t)\Delta z}{l_{\parallel}^2}) - \cos(\frac{2\beta_\parallel(t)\Delta z}{l_{\parallel}^2}))) \tag{18} \)

From the cosine term in Eq. (18), we deduce that the length scale characterizing the interference fringes is
\[
\lambda(t) = \frac{\pi l_{\parallel}^2 1 + \omega_{\parallel}^2 t^2}{\Delta \omega_{\parallel} t} \tag{19} \]

which, for \( t > 2\pi/\omega_{\parallel} \), behaves as \( \lambda(t) \sim (\pi l_{\parallel}^2/\Delta)\omega_{\parallel} t = \pi\hbar t/\mu \Delta \).

FIG. 3: Gray-scale plots of atom density (black=high) in the \( z-r \) plane (axes inset) for double condensates, evolving with the process of expanding at \( t = 5.81\text{ms} \) (a) and 10.21ms (b). 0 and 1 label respectively zeroth- and first-order fringes.

In Fig (a), our simulations show that, at very low densities (\( N = 3 \times 10^3 \)), the growth of the fringe spacing is the same for the zeroth- and first-order fringes (depicted in Fig (b)) and linear in \( t \), which matches the non-interacting expression well \( 1,12 \). At a slightly higher density (\( N = 4 \times 10^5 \)), the interference dynamics exhibits three regimes, as indicated in Fig (b). In regime I (\( t < 2.25\text{ms} \)), the growth of the fringe spacings is identical for the zeroth- and first-order fringes due to the low densities in the initial merging region. Note that this is distinct from the non-interacting behavior because the nonlinear interaction increases the expansion rate of the clouds \( 12 \). In regime II (\( 2.25\text{ms} < t < 9.75\text{ms} \)), the
spacing of the zeroth-order fringe grows faster than that of the first-order fringe, indicating that the ideal fringe pattern is distorted in the $z$-direction. Specifically, owing to its higher density, the central fringe is thickened relative to its neighbors. In regime III ($t > 9.75\text{ms}$), the growth rate of the zeroth- and first-order fringe spacings reduces gradually, approaching the non-interacting limit. For enough long times, the difference in the zeroth- and first-order fringe spacings becomes negligible and the fringes are uniform.

\[ \Phi(r, z, t) = \frac{1}{\sqrt{2}}Ce^{-r^2/2l^2_\perp}\left( e^{-iK(t)z}e^{-(z-Z(t))^2/2l^2_\parallel} + e^{iK(t)z}e^{-(z+Z(t))^2/2l^2_\parallel} \right) \]  

(20)

where $Z(t) = \Delta \cos(\omega_\parallel t)$ and $K(t) = \frac{\Delta}{l_\parallel} \sin(\omega_\parallel t)$. When the clouds are maximally overlapping ($t = \pi/2\omega_\parallel$), the fringe spacing is given by,

\[ \lambda = \frac{2\pi}{K(t)} = \frac{2\pi l^2_\parallel}{\Delta}. \]  

(21)

B. System II: Collision of Well Separated Clouds

In this set of simulations, the initial state is prepared as before for a range of $N$. However, the trap remains present during the dynamical process, as depicted in Fig.1(b), so that the two components are accelerated towards one another. For low-density condensates, the system behaves, at least over one period of the longitudinal trap, like an ideal Bose gas. Since $\omega_\perp = \omega_\perp,0$ and $\omega_\parallel = \omega_\parallel,0$, the initial state is a superposition of two coherent states of the oscillator potential, so that

\[ \Phi(r, z, t) = \frac{1}{\sqrt{2}}Ce^{-r^2/2l^2_\perp}\left( e^{-iK(t)z}e^{-(z-Z(t))^2/2l^2_\parallel} + e^{iK(t)z}e^{-(z+Z(t))^2/2l^2_\parallel} \right) \]  

(20)

where $Z(t) = \Delta \cos(\omega_\parallel t)$ and $K(t) = \frac{\Delta}{l_\parallel} \sin(\omega_\parallel t)$. When the clouds are maximally overlapping ($t = \pi/2\omega_\parallel$), the fringe spacing is given by,

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\[ \lambda = \frac{2\pi}{K(t)} = \frac{2\pi l^2_\parallel}{\Delta}. \]  

(21)
In our simulations, the initial density distribution is shown in Fig.5(a). Subsequently the two clouds approach and collide at speed $\mathbf{v}$ given approximately by

$$\frac{1}{2} m \mathbf{v}^2 = \frac{1}{2} m \omega^2 || \Delta^2,$$

which gives $\mathbf{v} = \omega || \Delta$ and $\lambda = h/m \mathbf{v} = 2 \pi \mathbf{r}^2 / \Delta$. We find that the collision velocity is $\mathbf{v} = 6.08 \pi || \omega || = 1.07 \times 10^{-2} \text{ms}^{-1}$ and $\lambda = 1.036 \pi || = 1.62 \times 10^{-13} \text{m}$.

FIG. 6: The peak density of the zeroth- (dashed curve) and first-order fringes (solid curve), and the zeroth- (solid curve with crosses) and first-order fringe width (dashed curve with circles) versus the initial peak density of the condensate, $n_0$, in “colliding” process with fixed $g$.

Fig.6 shows the effect of the nonlinear term on the fringe pattern of the colliding condensates with different atom numbers. For an initial peak density $n_0 < 1.52 \times 10^{14} \text{cm}^{-3}$, the peak density of the zeroth- and first-order fringes increases linearly with atom number, as expected for interference in a system governed by a linear wave equation. In this regime, for given initial trap frequencies and separation of the clouds, the fringe spacings are independent of the initial density distribution. However, when $n_0 > 1.52 \times 10^{14} \text{cm}^{-3}$, the peak densities of the two fringes depend sublinearly on $n_0$. The gap between the two fringes increases with $n_0$, with the result that the differences between fringe spacings increase as well. When this difference in the high-density region becomes sufficiently large, the fringes become significantly curved leading, ultimately, to vortex formation.

Furthermore, our theoretical formula Eq. (10) shows a proper explanation for the nonuniform fringes. The higher density in the zeroth-order fringe, characterized by larger $\alpha$, leads to a larger fringe spacing than that of the first-order fringe.

C. System III: Merging of Well Separated Clouds

Here, we choose the initial state to have $n_0 = 4.6 \times 10^{14} \text{m}^{-3}$ and $\Delta = 4.33 \times 10^{-11} \text{m}$. The initial wave function $\phi_0$ is obtained by numerically minimizing the mean field energy Eq. (2) with the trap frequencies, $\omega_{||,0} = 2 \pi \times 800 \text{Hz}$ and $\omega_{\perp,0} = 2 \pi \times 533 \text{Hz}$. Then at time $t = 0$, we change the trap frequencies $\omega_0 = 2 \pi \times 180 \text{Hz}$ and $\omega_\perp = 2 \pi \times 120 \text{Hz}$. Since the trap used for the evolution of the two clouds is much less confining than the trap used in preparing them, each cloud expands due to internal pressure as well as undergoing bulk motion. $\Delta$ here is smaller than that in system II and hence the relative velocity of the two clouds at maximal overlap, $\mathbf{v} = 2.8 \pi || \omega ||$, is slightly smaller than in system II.

Fig.5(c)-(h) show the time evolution of the density profile. The process of both expansion and collision leads to a distorted fringe pattern with the characteristic configuration that there are “thicker” fringes in the center of the cloud and a larger fringe spacing, towards the edge of the cloud where interaction effects are negligible and the fringe spacing is closer to its non-interacting value (Fig.5(c)). This curved fringe pattern is similar to that observed in experiment [1] where condensates with short separation tend to form large variations in density in the merging region. After the clouds have completely merged, the fringe spacings become smaller, and the distinction between the center and edge part as well as the zeroth-order and first-order peak, reduces correspondingly (Fig.5(d)).

The complex interference pattern can also be interpreted by Eq.(14). At the beginning of the merging ($t < 2.1 \text{ms}$), the expanding speed of an atom cloud $v_c$ is larger than its c.m. speed, $v_n$, due to very large $n_0$, and thus the fringe spacings are dominated by the term $\alpha t/\hbar$ in Eq. (14). It implies that the larger peak density of the fringes, the larger $\alpha$, resulting in larger $\Delta$. Therefore, the fringe spacings approaching $z = 0$, are slightly larger than those at the edge, originated from the peak density of the zeroth-order fringe larger than those of high-order fringes. With time evolution in the merging process, the increasing $k$ and $\hbar t/2m$ in Eq. (14) gradually dominate, resulting in smaller fringe spacings and less variation of the fringe spacings between the middle and edge of the clouds.

As shown in Fig.5(d) and (e), the strong interactions in the center of the clouds lead to a net radial flow of atoms in the high density fringes, triggering a sound wave in the central peak to propagate radially from position 1 to 2. Our calculation shows that $v_c \approx L_{1,2}/t_{1,2} = 11.1 \text{mm} \text{s}^{-1}$, larger than $2v_n \approx 9.8 \text{mm} \text{s}^{-1}$. This rapid outflow leads to complete depletion of the center of trap after the two clouds have passed through one another, resulting in the formation of a dark soliton (e). When the two clouds recollide at $2\mathcal{F}$, both are recollapsing radially leading to an enhanced density in the bright fringes causing an even greater longitudinal expansion and greater density gradients in the radial direction (f). The curvature of the fringes is thus even greater than in the first collision, (g), and is sufficient to generate a net circulation around localized regions that are fully depleted: vortices are generated, as seen in (h).
the competition between $v_m$ and $v_r$, additional degrees of freedom are generated, with the result that the high-density areas from the interference peaks encircle the low-density areas between them (Fig.5(g)). The positions of the vortex rings correlate naturally with the previous positions of interference valleys. Naturally, the formation of vortices from quantum reflection of high-density and low-velocity BEC [15] is analogous with our results.

V. GENERAL FEATURES IN THE PARAMETER SPACE

To explore the interference properties and characterize the spontaneous formation of vortices over the parameter space, we have performed a comprehensive set of numerical simulations to obtain Fig.7. Each data point in Fig.7(a) and (b) is derived from a set of at least 12 dynamic calculations. We identify the three types of process through their interference characteristics. In an expansion process, the interference fringes are time-dependent and uniform. In the colliding process, the fringes are static and uniform while the clouds are fully overlapping. The interference pattern in a merging process is time-dependent and nonuniform.

Fig.7(a) demonstrates that starting in the freely expanding regime, moving to higher initial densities or smaller initial separations will produce the more general merging behavior. This is exhibited by a loss of homogeneity in the fringe spacing and, in turn, fringe curvature. The upwards curve of the boundary between expanding and merging behavior is similar to that for the boundary between colliding and merging behavior shown in (Fig.7(b)). The rigorous explanation of this trend might be derived from the quantitative analysis of the effect of nonlinear interaction on the fringe spacing, which is beyond the range of this paper.

Based on the analysis of the dynamics over a large portion of the parameter space, Fig.7(c) summarizes the formation of vortices in merging BECs. Above a critical initial density, $n_c$, vortices will form during the return overlap of the two clouds within a definite range of initial c.m. velocities. If the velocity is too low then damping of the motion during the first overlap effectively arrests the dynamics. In addition, the low velocity leads to a large fringe spacing so that, even towards the edges of the cloud, the curvature is never sufficient to cause a net circulation. If the collision velocity is too high, the hydrodynamic response of the cloud is too slow and the clouds have separated before any significant distortion occurs [22] (inserted plot in Fig.7(c)). Our simulations show that the curvature of the interference fringes is of importance in the formation of vortices and that the number of fringes within the cloud determine the number of vortices. This is shown in the inserted plots of Fig.7(c) where the five fringes from the short initial separation result in the formation of two vortex rings while the nine fringes from the large initial separation cause six vortex rings. The connection between the distortion of fringes and the formation of vortices can explain the experiments in which more vortices are generated in the faster merging, interfering region [6]. The faster merging produces more interference fringes, creating the possibility for the formation of more vortices in the region. The diagram in Fig.7 is based on the specific material parameters described in the previous sections. Simulations using different parameters give results that are similar but with the cross-over lines shifted. In general, for the smaller values of the coupling constant, $g$, and with the larger numbers of atoms, it is easier to observe the crossover from colliding process to merging process.

Finally, we address the effect of the halo of scattered atoms, produced by the counter-propagating condensates, on the interference pattern. The phenomena that two BECs collide at a sufficiently high velocity, a halo of elastically scattered atoms is produced, have been
demonstrated by both experiments and theory. In the expanding and merging processes of our system, the scattered atoms might not have significant effect on the interference patterns because the rapid decrease of the densities corresponding to the fast expansion of the clouds would reduce the rate of atom-atom collision in merging areas. For the colliding process, the visibility of the interference can be reduced and the fringes can be distorted due to the scattered atoms. However, these effects can be suppressed or avoided if the density and velocity of clouds are low, or the colliding process is controlled by properly adjusting the barriers.

VI. SUMMARY

In summary, we have investigated vortex formation and interference instability of two merging condensates and identified three distinct regimes in situations relevant to atom interferometry. Our simulations show an explanation about recent experimental work, where the faster merging three BECs creates more vortices. The regularity of vortex formation and interference might allow for the design of experiments to study vortex creation and the dynamics of regular, linear, vortex arrays. Our latest calculations show clearly that one could control the density of vortices generated by tuning the fringe spacing.

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