Research Article

The Wall Properties Effect on Peristaltic Transport of Micropolar Non-Newtonian Fluid with Heat and Mass Transfer

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The problem of the unsteady peristaltic mechanism with heat and mass transfer of an incompressible micropolar non-Newtonian fluid in a two-dimensional channel. The flow includes the viscoelastic wall properties and micropolar fluid parameters using the equations of the fluid as well as of the deformable boundaries. A perturbation solution is obtained, which satisfies the momentum, angular momentum, energy, and concentration equations for case of free pumping (original stationary fluid). Numerical results for the stream function, temperature, and concentration distributions are obtained. Several graphs of physical interest are displayed and discussed.

1. Introduction

The peristaltic transport is traveling contraction wave along a tube-like structure, and it results physiologically from neuromuscular properties of any tubular smooth muscle. Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a peristaltic mechanism may be involved in swallowing food through the esophagus, in urine transport from the kidney to the bladder through the ureter, movement of chyme in the gastrointestinal tract, in the transport of spermatozoa in the ducts efferentes of the male reproductive tracts and in the cervical canal, movement of ovum in the female fallopian tubes, the transport of lymph in the lymphatic vessels, and the vasomotion in small blood vessels such as arterioles, venues, and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomedical systems. It has now been accepted that most of the physiological fluids behave in general like suspensions of deformable or rigid particles in a Newtonian fluid. Blood, for example, is a suspension of red cells, white cells, and platelets in plasma. Another example is cervical mucus, which is a suspension of macromolecules in a water-like liquid [1]. However only a
few studies have considered this aspect since the initial investigation by Raju and Devanathan [2, 3]. The biviscosity fluid can represent the behavior of blood in vessels of small diameter as the mean shear rate is about 20–150 s⁻¹ [4].

Several studies have been made to analyze both theoretical and experimental aspects of the peristaltic motion of non-Newtonian fluids in different situations. Böhme and Friedrich [5] studied the mechanism of peristaltic transport of an incompressible viscoelastic fluid by means of an infinite train of sinusoidal waves traveling along the wall of the duct in the case of plane flow. The effects of an Oldroyd-B fluid on the peristaltic mechanism are examined by Hayat et al. [6], under the long wavelength assumption. They noticed that in the narrow part of the channel, the behavior of an Oldroyd-B fluid is much more different from that of a Newtonian fluid than in the wide part of the channel. El-Shehawey et al. [7] studied the peristaltic motion of an incompressible non-Newtonian fluid through a porous medium. They showed that the pressure rise increases as the permeability decreases and noted that both pressure rise and friction force do not depend on permeability parameter at a certain value of flow rate. Haroun [8] investigated the peristaltic flow of a third-order fluid in an asymmetric channel under the assumption of long wavelength approximation. He expanded the velocity components and pressure in a regular perturbation series in a small parameter Deborah number that contained the non-Newtonian coefficients appropriate to shear-thinning. The effects of slip boundary conditions on the dynamics of fluids in porous media is investigated by El-Shehawey et al. [9]. They studied the flow of non-Newtonian Maxwell fluids in an axisymmetric cylindrical tube (pore), in which the flow is induced by traveling transversal waves on the tube wall (peristaltic transport). Hakeem et al. [10] discussed the effect of an endoscope on the peristaltic mechanism of a generalized Newtonian fluid. Mekheimer [11] studied the effect of a uniform magnetic field on peristaltic transport of a blood in nonuniform two-dimensional channels, when blood is represented by a couple-stress fluid. Tsiklauri and Beresnev [12] analyzed the effect of viscoelasticity on the dynamics of fluids in porous media by studying the peristaltic flow of a Maxwell fluid in circular tube, in which the flow is induced by a wave traveling on the tube wall.

The micropolar fluid represents fluids, which consist of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. The theory of thermo-micropolar fluid was developed by Eringen [13]. Agrawal and Dhanapal [14] considered the micropolar fluid to be the model of blood flow in small arteries, and the calculation of theoretical velocity profiles is observed, in good agreement with experimental data [15]. They have many applications in physiological and chemical engineering [16]. Srinivasacharya et al. [17] studied the problem of peristaltic transport of a micropolar fluid in a circular tube. Girija Devi and Devanathan [18] studied the peristaltic flow of a micropolar fluid in a cylindrical tube with a sinusoidal deformation of small amplitude travelling down its flexible wall for the case of low Reynolds number flow devoid of wall properties like tension and damping. However, the wall properties are essential to be taken into consideration in various real situations. The peristaltic motion of a simple microfluid which accounts for microrotation and microstretching of the particles contained in a small volume element is studied by Philip and Chandra [19], using long wavelength approximation.

Combined heat and mass transfer problems are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occurs simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power
industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species [20]. Radhakrishnamacharya and Radhakrishna [21] investigated the peristaltic flow with heat transfer in a nonuniform channel. The effect of elasticity of the flexible walls on peristaltic transport of an incompressible viscous fluid, with heat transfer, in a two-dimensional uniform channel under long wavelength approximation is explained by Radhakrishnamacharya and Srinivasulu [22]. El Dabe et al. [23] studied heat and mass transfer of a steady slow motion of a Rivlin-Ericksen fluid in tube of varying cross-section with suction. Agrawal et al. [24] obtained numerical solutions of flow and heat transfer of a micropolar fluid at a stagnation point on a porous stationary wall. They observed that the heat sources increase the velocity and temperature in the pipe while the heat sinks decrease them.

Muthu et al. [25] carried out a study of the peristaltic motion of an incompressible micropolar fluid in two-dimensional channel. They investigated the effects of viscoelastic wall properties and micropolar fluid parameters on the flow using the equations of the fluid as well as of the deformable boundaries. The non-Newtonian property and equations of heat and mass transfer were not taken into their consideration. Because of the wide range of practical importance of the heat and mass transfer, the present study considered the heat and mass transfer of an unsteady peristaltic motion of a micropolar non-Newtonian biviscosity fluid [4]. The following analysis includes the dynamic boundary condition. Analytical approximate solutions for the stream function, microrotation velocity, temperature, and concentration equations are obtained as a power series in terms of the small amplitude ratio. We have shown the relation between the different parameters of motion in order to investigate how to control the motion of the fluid by changing these parameters.

2. Formulation of the Problem

Consider a two-dimensional symmetric unsteady flow of an incompressible micropolar biviscosity fluid in an infinite channel of uniform thickness $2d$, with heat and mass transfer. The walls of the channel are flexible membranes on which they are imposed traveling sinusoidal waves of moderate amplitude (see Figure 1).

We choose a rectangular coordinate system such that the axes $x$ and $y$ are in the directions of wave propagation and normal to the mean position of the membranes, respectively. The origin is located at the center line of the channel. It was shown by Wilson and Taylor [26] that the limiting process by which the biviscosity model is approached with the limiting process implicit in lubrication theory. Models of this type are much easier to handle mathematically than many models such as Oldroyd’s and represent the experimental facts just as well, at least in many cases.

The biviscosity model (which has been used in similar contexts by Nakayama and Sawada [4]) can be written as

\[
\tau_{ij} = \begin{cases} 
2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi \geq \pi_c, \\
2 \left( \mu_B + \frac{p_y}{\sqrt{2\pi c}} \right) e_{ij}, & \pi < \pi_c.
\end{cases}
\]  

(2.1)
The following quantity is introduced as a nondimensional parameter including $\pi_c$

$$
\gamma = \mu_B \sqrt{\frac{2\pi_c}{p_y}},
$$

(2.2)

where $\mu_B$ is the plastic viscosity, $p_y$ is the yielding stress, $\pi = e_{ij}e_{ij}$, where $e_{ij}$ is the $(i, j)$ component of the deformation rate and the value of $\gamma$ denotes the upper limit of apparent viscosity coefficient. The biviscosity model is approached in the limit $\gamma \to \infty$ (see Nakayama and Sawada [4] for a few further details).

The governing continuity, momentum, angular momentum, temperature, and concentration equations for this problem can be written as [13]

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
$$

(2.3)

$$
\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + \left( \frac{2\mu_B (1 + \gamma^{-1})}{2} + k \right) \nabla^2 u + k \frac{\partial \theta}{\partial y},
$$

(2.4)

$$
\rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} + \left( \frac{2\mu_B (1 + \gamma^{-1})}{2} + k \right) \nabla^2 v - k \frac{\partial \theta}{\partial x},
$$

(2.5)

$$
\rho J \left[ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right] = -2k \theta + \nabla^2 \theta + k \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right],
$$

(2.6)

$$
\rho C_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k_c \nabla^2 T + \left( \frac{2\mu_B (1 + \gamma^{-1})}{2} + k \right) \times \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]
$$

$$
+ 2k \left[ \theta^2 - \theta \left( \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right) + \nabla \left( \frac{\theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 \right]
$$

$$
+ \alpha_c \left[ \frac{\partial T}{\partial x} \frac{\partial \theta}{\partial y} - \frac{\partial T}{\partial y} \frac{\partial \theta}{\partial x} \right],
$$

(2.7)

$$
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{Dk_t}{T_m} \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} \right),
$$

(2.8)

where $u(x, y, t)$ and $v(x, y, t)$ are the velocity components in the $x$ and $y$ directions, respectively, $N_\theta(x, y, t)$ is the microrotation velocity component in the direction normal to both the $x$ and $y$ axes. Here $\rho$ is the density of the fluid, $P$ is the pressure, $J$ is the microinertia constant, $k$ is the vortex viscosity coefficient (also known as the coefficient of gyroviscosity), and $\nabla$ is the spin-gradient viscosity.

Further, the material constants $\mu_B$ and $k$ satisfy the following inequalities [13]:

$$
2\mu_B + k \geq 0, \quad k \geq 0, \quad \mu_B \geq 0.
$$

(2.9)
The temperature and concentration of fluid are \( T(x,y,t) \) and \( C(x,y,t) \), \( C_p \) is the specific heat at constant pressure, \( D \) is the coefficient of mass diffusivity, \( K_T \) is the thermal diffusion ratio, \( T_m \) is the mean fluid temperature, \( k_c \) is the thermal conductivity and \( \alpha_c \) is the heat conduction parameter for a micropolar fluid. The last term in (2.8) signifies the thermal-diffusion effect.

We consider a symmetric motion of the flexible walls in the boundary conditions. Let the vertical displacements of the upper and lower walls be \( \eta \) and \(-\eta\), respectively, where

\[
\eta(x,t) = a \cos \frac{2\pi}{\lambda} (x - ct),
\]

where \( a \) is the amplitude, \( \lambda \) is the wavelength, and \( c \) is the wave speed.

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero [25].

Thus the no-slip boundary conditions for the velocity and microrotation are

\[
\begin{align*}
    u &= 0, \\
    N_\theta &= 0 \quad \text{at} \quad y = \pm (d + \eta(x,t)).
\end{align*}
\]

The dynamic boundary conditions are imposed on the fluid by the symmetric motion of the flexible walls, which can be written as [18]

\[
\frac{\partial L(\eta)}{\partial x} = -\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] + \left( \frac{2\mu_B (1 + \gamma^{-1}) + k}{2} \right) \nabla^2 u + k \frac{\partial N_\theta}{\partial y},
\]

at \( y = \pm (d + \eta(x,t)) \),

where

\[
\frac{\partial L(\eta)}{\partial x} = -\ell \frac{\partial^3 \eta}{\partial x^3} + m \frac{\partial^3 \eta}{\partial t^2 \partial x^2} + n \frac{\partial^3 \eta}{\partial t \partial x^2}.
\]

Here \( L(\eta) \) is the pressure at the walls, \( \ell \) is the tension in the membrane, \( m \) is the mass per unit area, and \( n \) is the coefficient of viscous damping force.
The plate temperature starts oscillating about a nonzero mean temperature. Under these physical conditions, the temperature and concentration at the upper wall give

\[
T = T_w + \frac{a}{d} (T_w - T_m) \cos \left( \frac{2\pi (x - ct)}{\lambda} \right),
\]

\[
C = C_w + \frac{a}{d} (C_w - C_m) \cos \left( \frac{2\pi (x - ct)}{\lambda} \right), \quad \text{at } y = d + \eta(x,t),
\]

where \(C_m\) is the mean fluid concentration, \(T_w\) and \(C_w\) are the uniform temperature and concentration at the lower wall, that is,

\[
T = T_w, \quad C = C_w, \quad \text{at } y = -(d + \eta(x,t)).
\]

Equation (2.3) allows the use of the stream function \(\psi(x, y, t)\) such that

\[
u' = \frac{\partial \psi}{\partial y}, \quad \nu = -\frac{\partial \psi}{\partial x},
\]

By introducing the following nondimensional quantities:

\[
x' = \frac{x}{d}, \quad y' = \frac{y}{d}, \quad t' = \frac{tc}{d}, \quad u' = \frac{u}{c}, \quad v' = \frac{v}{c}, \quad N_0' = \frac{dN_0}{c}, \quad P' = \frac{P}{\rho c^2}, \quad \eta' = \frac{\eta}{d},
\]

\[
q' = \frac{q}{cd}, \quad J' = \frac{J}{d^2}, \quad T' = \frac{T - T_m}{T_w - T_m}, \quad C' = \frac{C - C_m}{C_w - C_m}, \quad \varepsilon = \frac{a}{d}, \quad \alpha = \frac{2\pi d}{\lambda}, \quad \mu_1 = \frac{k}{\mu_B}, \quad \mu_B = \frac{\mu}{\mu_B},
\]

\[
\text{Re} = \frac{\rho c d}{\mu_B}, \quad N = \left( \frac{\mu_1}{2 + \mu_1} \right)^{1/2}, \quad \overline{N} = \left( \frac{\mu_1}{2(1 + \gamma^{-1}) + \mu_1} \right)^{1/2},
\]

\[
M = 2d \left( \frac{\mu_B}{Y} \right)^{1/2}, \quad \overline{M} = \left( \frac{M^2 (1 - \overline{N}^2)}{(1 - N^2)} \right)^{1/2},
\]

\[
\Gamma = \frac{M}{\overline{M}}, \quad \text{Re}_e = \frac{4\mu c d J}{Y (2\mu_B + k)}, \quad K_2 = \frac{n d}{\mu_B}, \quad K_3 = \frac{\ell^2 d}{\mu_B^2}, \quad m_1 = \frac{m}{\rho d}, \quad \text{Pr} = \frac{\mu_B C_p}{k_c},
\]

\[
\text{Ec} = \frac{c^2}{C_p(T_w - T_m)}, \quad \text{af} = \frac{\alpha c}{\mu_B C_p d}, \quad \text{Sc} = \frac{\mu_B}{D}, \quad \text{Sr} = \frac{k_T (T_w - T_m) \rho D}{T_m (C_w - C_m) \mu_B},
\]
substituting from (2.17) into (2.3)–(2.8), eliminating $P$ between (2.4) and (2.5), and dropping the accent mark for simplicity, the governing equations may be written as

$$
\frac{\partial}{\partial t} \nabla^2 \psi_x + \psi_y \nabla^2 \psi_y - \psi_x \nabla^2 \psi_y = \frac{2 (1 + \gamma^{-1}) + \mu_1}{2 \Re} \left[ \nabla^2 \nabla^2 \psi_x \right] + \frac{\mu_1}{\Re} \nabla^2 N_\theta,
$$

(2.18)

$$
Re \left( \frac{\partial N_\theta}{\partial t} + \psi_y \frac{\partial N_\theta}{\partial x} - \psi_x \frac{\partial N_\theta}{\partial y} \right) = 2 \left( 1 - N^2 \right) \nabla^2 N_\theta - N^2 M^2 \left( \nabla^2 \nabla^2 \psi \right),
$$

(2.19)

$$
\text{Re} \cdot \text{Pr} \left( \frac{\partial T}{\partial t} + \psi_y \frac{\partial T}{\partial x} - \psi_x \frac{\partial T}{\partial y} \right) = \nabla^2 T + \frac{2 (1 + \gamma^{-1}) + \mu_1}{2 \Pr \cdot \Ec} \left[ 4 \psi_x^2 + (\psi_{yy} - \psi_{xx})^2 \right]
+ 2 \mu_1 \cdot \Pr \cdot \Ec \left( N_\theta^2 + N_\theta \nabla^2 \psi \right) + \frac{4 \Pr \cdot \Ec}{M^2}
$$

$$
\times \left[ \left( \frac{\partial N_\theta}{\partial x} \right)^2 + \left( \frac{\partial N_\theta}{\partial y} \right)^2 \right] + \varepsilon \cdot \text{Pr} \left[ \frac{\partial T}{\partial x} \frac{\partial N_\theta}{\partial y} - \frac{\partial T}{\partial y} \frac{\partial N_\theta}{\partial x} \right],
$$

(2.20)

$$
Sc \cdot \text{Re} \left( \frac{\partial C}{\partial t} + \psi_y \frac{\partial C}{\partial x} - \psi_x \frac{\partial C}{\partial y} \right) = \nabla^2 C + Sc \cdot Sr \cdot \nabla^2 T.
$$

(2.21)

Also, the boundary conditions at $y = \pm(1 + \eta(x, t))$, when $\eta(x, t) = \varepsilon \cos(\alpha(x - t))$, are

$$
\psi_y = N_\theta = 0,
$$

(2.22)

$$
\frac{\partial \psi_y}{\partial x} = -[\psi_{yy} + \psi_y \psi_{yy} - \psi_x \psi_{yy}],
$$

(2.23)

while

$$
T = 1, \quad C = 1, \quad \text{at } y = -(1 + \eta(x, t)),
$$

$$
T = 1 + \varepsilon \cos(\alpha(x - t)), \quad C = 1 + \varepsilon \cos(\alpha(x - t)) \quad \text{at } y = 1 + \eta(x, t).
$$

(2.24)

The parameter $\varepsilon$ represents the amplitude ratio, $\alpha$ is wave number, $\Re$ is the Reynolds number, $\mu_1$ is the ratio between the viscosity coefficient for micropolar fluids and the classical viscosity coefficient, and $M$ is the micropolar fluid parameter characterizing spin-gradient viscosity. The parameter $M$ can be thought of as a fluid property depending upon the size of the microstructure [18]. We note that $Re$ is the modified Reynolds number and involves the quantity $J$. In this paper, we considered that the effect of microinertia is neglected and $Re$ is taken to be zero [24]. The non-dimensional quantities Pr are the Prandtl number, Ec is
Eckert number, $af$ is the dimensionless heat conduction coefficient of the micropolar fluid, $Sc$ is the Schmidt number, $Sr$ is the Soret number and represents the thermal-diffusion effect, $K_2$ and $K_3$ represent the dissipative and rigiditive feature of walls, and $m_1$ indicates the stiffness property of walls.

3. Method of Solution

Assume the amplitude ratio $\varepsilon$ of the wave to be small. As given by Muthu et al. [25], we seek for an approximate solution as a power series in terms of $\varepsilon$ in the form

$$
\psi(x, y, t) = \psi_0(y) + \frac{\varepsilon}{2} \left( \phi_1(y) e^{i\alpha(x-t)} + \phi_1^* e^{-i\alpha(x-t)} \right) + \frac{\varepsilon^2}{2} \left( \phi_{20}(y) + \phi_{22}(y) e^{2i\alpha(x-t)} + \phi_{22}^* e^{-2i\alpha(x-t)} \right) + \mathcal{O}(\varepsilon^3),
$$

$$
N_0(x, y, t) = N_0(y) + \frac{\varepsilon}{2} \left( \zeta_1(y) e^{i\alpha(x-t)} + \zeta_1^* e^{-i\alpha(x-t)} \right) + \frac{\varepsilon^2}{2} \left( \zeta_{20}(y) + \zeta_{22}(y) e^{2i\alpha(x-t)} + \zeta_{22}^* e^{-2i\alpha(x-t)} \right) + \mathcal{O}(\varepsilon^3),
$$

$$
T(x, y, t) = T_0(y) + \frac{\varepsilon}{2} \left( T_1(y) e^{i\alpha(x-t)} + T_1^* e^{-i\alpha(x-t)} \right) + \frac{\varepsilon^2}{2} \left( T_{20}(y) + T_{22}(y) e^{2i\alpha(x-t)} + T_{22}^* e^{-2i\alpha(x-t)} \right) + \mathcal{O}(\varepsilon^3),
$$

$$
C(x, y, t) = C_0(y) + \frac{\varepsilon}{2} \left( C_1(y) e^{i\alpha(x-t)} + C_1^* e^{-i\alpha(x-t)} \right) + \frac{\varepsilon^2}{2} \left( C_{20}(y) + C_{22}(y) e^{2i\alpha(x-t)} + C_{22}^* e^{-2i\alpha(x-t)} \right) + \mathcal{O}(\varepsilon^3).
$$

Here the asterisk denotes complex conjugate. The solutions for $\psi_0(y)$ and $N_0(y)$ are obtained as follows:

$$
\frac{d\psi_0}{dy} = K' (y^2 - 1) \left( 1 + \frac{N^2 T^2}{(1 - N^2)} \right) + 2K' \frac{N^2 T^2}{(1 - N^2)} \left( \frac{\cosh(NM) - \cosh(NMy)}{NM \sinh(NM)} \right),
$$

$$
N_0 = \frac{K' T^2}{(1 - N^2)} \left( \frac{\sinh(NMy) - y \sinh(NM)}{\sinh(NM)} \right),
$$

where $K' = (Re / (2(1+\gamma^{-1})+\mu_1))(dP/dx)_{y_0}$ is the poiseuille flow parameter for the micropolar fluid. For pure peristalsis, which means that the flow is generated by wall motion only. The
pressure gradient \( (dP/dx)_0 = 0 \); this implies that \( K' = 0 \), which gives \( \phi_0(y) = 0 \) and \( N_0(y) = 0 \). The expressions of \( \phi_1 \) and \( \xi_1 \) for the case of free pumping can be written as

\[
\phi_1(y) = A_3 \sinh a y + A_4 \sinh \beta y + a_1 A_1 \sinh r_1 y + a_2 A_2 \sinh r_2 y, \quad (3.6)
\]

\[
\xi_1(y) = A_1 \sinh r_1 y + A_2 \sinh r_2 y, \quad (3.7)
\]

where \( \beta^2 = \alpha^2 - 2i\alpha \text{Re} / (1 + (1 - \gamma^{-1}) + \mu_1) \), \( \lambda_1 = N^2 N^2 M^2 / (1 - N^2) \), \( \lambda_2 = N^2 M^2 / (1 - N^2) \),

\[
r_1 = \left[ \frac{1}{2} \left\{ \lambda_2 - \lambda_1 + (\alpha^2 + \beta^2) + \sqrt{(\lambda_2 - \lambda_1 - (\alpha^2 + \beta^2))^2 - 4(\alpha^2 \beta^2 - \lambda_1 \alpha^2 + \lambda_2 \beta^2)} \right\} \right]^{1/2},
\]

\[
r_2 = \left[ \frac{1}{2} \left\{ \lambda_2 - \lambda_1 + (\alpha^2 + \beta^2) - \sqrt{(\lambda_2 - \lambda_1 - (\alpha^2 + \beta^2))^2 - 4(\alpha^2 \beta^2 - \lambda_1 \alpha^2 + \lambda_2 \beta^2)} \right\} \right]^{1/2},
\]

\[
a_1 = \frac{-2 N^2}{r_1^2 - \beta^2}, \quad a_2 = \frac{-2 N^2}{r_2^2 - \beta^2}, \quad A_1 = \frac{\text{Re} N^2 M^2 \sinh r_2}{2 d_1}, \quad A_2 = \frac{-\text{Re} N^2 M^2 \sinh r_1}{2 d_1},
\]

\[
d_1 = \left( r_1^2 + (\lambda_1 - \lambda_2 - \alpha^2) \right) \sinh r_1 \cosh r_1 - \left( r_2^2 + (\lambda_1 - \lambda_2 - \alpha^2) \right) \sinh r_1 \cosh r_2,
\]

\[
A_3 = \frac{1}{\alpha (\beta^2 - \alpha^2) \cosh \alpha} \left( -\frac{2 \text{Re} \delta_1}{2 (1 + \gamma^{-1}) + \mu_1} + 2 N^2 (r_1 A_1 \cosh r_1 + r_2 A_2 \cosh r_2) \right)
\]

\[
+ a_1 A_1 r_1 \left( r_1^2 - \beta^2 \right) \cosh r_1 + a_2 A_2 r_2 \left( r_2^2 - \beta^2 \right) \cosh r_2,
\]

\[
A_4 = \frac{1}{\beta (\beta^2 - \alpha^2) \cosh \beta} \left( \frac{2 \text{Re} \delta_1}{2 (1 + \gamma^{-1}) + \mu_1} - 2 N^2 (r_1 A_1 \cosh r_1 + r_2 A_2 \cosh r_2) \right)
\]

\[
- a_1 A_1 r_1 \left( r_1^2 - \alpha^2 \right) \cosh r_1 + a_2 A_2 r_2 \left( r_2^2 - \alpha^2 \right) \cosh r_2,
\]

\[
\delta_1 = i \left( \frac{K_3 \alpha^3}{\text{Re}^3} - m_1 \alpha^3 \right) + K_2 \alpha^2 / \text{Re}.
\]

To obtain the mass and heat transfer, we shall use the perturbation scheme (3.3), (3.4) in (2.20) and (2.21), and equating the zero-order terms on both sides, we obtain the following set of equations:

\[
\frac{d^2 T_0}{dy^2} = -\text{Ec} \cdot \text{Pr} \left[ \frac{d^2 \phi_0}{dy^2} \left( 2 \mu_1 N_0^2 + \frac{2 (1 + \gamma^{-1}) + \mu_1}{2} \frac{d^2 \phi_0}{dy^2} \right) + 2 \mu_1 N_0^2 + \frac{4}{M^2} \left( \frac{d N_0}{dy} \right)^2 \right], \quad (3.9)
\]

\[
\frac{d^2 C_0}{dy^2} = -\text{Sc} \cdot \text{Sr} \frac{d^2 T_0}{dy^2}, \quad (3.10)
\]

\[
T_0(\pm 1) = 1, \quad C_0(\pm 1) = 1. \quad (3.11)
\]
In case of plane Poiseuille flow of micropolar fluid \((\varepsilon = 0)\), the governing (3.9) and (3.10) may be solved in the light of the boundary conditions (3.11) with the constant pressure gradient \((\partial P/\partial x)\). At this stage, the temperature and concentration distributions \(T_0\) and \(C_0\), respectively, are given by

\[
T_0(y) = 1 - 2\text{Pr} \cdot \text{Ec} \cdot K^{1/2} \times \left[ \frac{1}{6} (y^4 - 1) + \frac{1}{M^2 \sinh^2 N M} \times \left\{ \frac{1}{4} (y^2 - 1) \left( 4\sinh^2 NM + M^2 \left( N^2 (2 - N^2) - \mu_1 (1 - N^2)^2 \right) \right) \right. \\
- 4 \sinh NM \left( y \sinh NM y - \sinh NM \right) \\
+ \frac{4 \sinh NM}{NM} \left( \cosh NM y - \cosh NM \right) \\
+ \frac{1}{8N^2} \left( N^2 (2 + N^2) + \mu_1 (1 - N^2)^2 \right) \right\} \right] ,
\]

\[
C_0(y) = 1 + \text{Sc} \cdot \text{Sr} \left( 1 - T_0(y) \right).
\]

Let us consider the case in which the pressure gradient \((\partial P/\partial x)\) vanishes. In this case there will be no flow if the wall motion stops. Hence \(q_0 = N_0 = 0\) and \(T_0 = C_0 = 1\), and equating the first-order terms on both sides of (2.20) and (2.21), we get the following set of equations:

\[
\frac{d^2 T_1}{dy^2} - \gamma_1^2 T_1 = 0,
\]

\[
\frac{d^2 C_1}{dy^2} - \gamma_2^2 C_1 = i \alpha \cdot \text{Sc} \cdot \text{Sr} \cdot \text{Pr} \cdot \text{Re} \cdot T_1,
\]

where

\[
\gamma_1^2 = \alpha^2 - i \alpha \cdot \text{Pr} \cdot \text{Re}, \quad \gamma_2^2 = \alpha^2 - i \alpha \cdot \text{Sc} \cdot \text{Re}.
\]

The boundary conditions are

\[
T_1(-1) = C_1(-1) = 0, \quad T_1(1) = C_1(1) = 1.
\]
The solutions of (3.13) with boundary conditions (3.15) are

\[ T_1(y) = \frac{\sinh(\gamma_1 + \gamma_3 y)}{\sinh 2\gamma_1}, \]  
\[ C_1(y) = \left(1 - \frac{Sc \cdot Sr \cdot Pr}{(Sc - Pr)}\right) \frac{\sinh(\gamma_2 + \gamma_3 y)}{\sinh 2\gamma_2} + \frac{Sc \cdot Sr \cdot Pr \sinh(\gamma_1 + \gamma_3 y)}{(Sc - Pr) \sinh 2\gamma_1}. \]  

(3.16)

(3.17)

In this case, the governing equations for \( T_{20} \) and \( C_{20} \) reduce to

\[ \frac{d^2 T_{20}}{dy^2} = \frac{i\alpha}{2} \Pr \cdot \text{Re} \left\{ \frac{d}{dy}(T_i \phi_i^* - \phi_i T_i^*) \right\} \]

\[- \Pr \cdot \text{Ec} \left\{ \frac{2(1 + \gamma^{-1}) + \mu_i}{2} \right\}

\times \left(4\alpha^2 \frac{d\phi_1}{dy} \frac{d\phi_1^*}{dy} + \frac{d^2 \phi_1}{dy^2} \frac{d^2 \phi_1^*}{dy^2} + \alpha^4 \phi_1 \phi_1^* + \alpha^2 \left(\phi_1 \frac{d^2 \phi_1^*}{dy^2} + \phi_1^* \frac{d^2 \phi_1}{dy^2}\right)\right)

\[+ 2\mu_i \left(\xi_1 \xi_1^* + \frac{1}{2} \left(\xi_1 \frac{d^2 \phi_1}{dy^2} + \xi_1^* \frac{d^2 \phi_1^*}{dy^2} - \alpha^2 (\xi_1 \phi_1^* + \xi_1^* \phi_1)\right)\right)\]

\[+ \frac{4}{M^2} \left(\frac{d\xi_1}{dy} \frac{d\xi_1^*}{dy} + \alpha^2 \xi_1 \xi_1^*\right)\right] - \frac{i\alpha}{2} \text{af} \cdot \Pr \frac{d}{dy}(T_i \phi_i^* - \phi_i T_i^*),\]

\[ \frac{d^2 C_{20}}{dy^2} = \frac{i\alpha}{2} \text{Sc} \cdot \text{Re} \left\{ \frac{d}{dy}(C_1 \phi_1^* - \phi_1 C_1^*) - \text{Sc} \cdot \text{Sr} \frac{d^2 T_{20}}{dy^2}\right\}, \]

(3.18)

with the boundary conditions

\[ T_{20}(\pm 1) = \mp \frac{1}{2} \left(T_{1y}(\pm 1) + T_{1y}^*(\pm 1)\right), \quad C_{20}(\pm 1) = \mp \frac{1}{2} \left(C_{1y}(\pm 1) + C_{1y}^*(\pm 1)\right). \]  

(3.19)

The solutions of (3.18) are

\[ T_{20}(y) = \frac{i\alpha}{4} \Pr \cdot \text{Re} \left[ \frac{1}{\sinh 2\gamma_1} \left(A_3^* g_1(\gamma_1, \alpha) + A_4^* g_1(\gamma_1, \beta) + A_5^* a_1^* g_1(\gamma_1, r_1^*) + A_6^* a_2^* g_1(\gamma_1, r_2^*) \right) \right. \]

\[- \frac{1}{\sinh 2\gamma_1} \left(A_3 g_1(\gamma_1^*, \alpha) + A_4 g_1(\gamma_1^*, \beta) + A_5 a_1 g_1(\gamma_1^*, r_1) + A_6 a_2 g_1(\gamma_1^*, r_2) \right) \right] \]
\[-E_c \cdot \text{Pr} \left[ \frac{2 \left( 1 + y^{-1} \right) + \mu_1}{4} \right. \]

\[\times \left( 2a^2 A_3 A_1^2 \cosh 2ay + 4a^2 (\alpha A_3 f_1 + r_1 A_1 a_1 f_2 + r_2 A_2 f_3 + \beta A_4 f_4) \right) \]

\[+ 2a^2 A_3 f_5 + \left( a^2 + r_1^2 \right) A_1 a_1 f_6 \]

\[+ \left( a^2 + r_2^2 \right) A_2 a_2 f_7 + \left( a^2 + \beta^2 \right) A_4 f_8 + \left( \frac{M^2 \mu_1 + 2a^2}{M^2} \right) \]

\[\times \left( A_1 (A_1^* g_3 (r_1, r_1^*) + A_2^* g_3 (r_1, r_1^*)) + A_2 (A_1^* g_3 (r_2, r_2^*) + A_2^* g_3 (r_2, r_2^*)) \right) \]

\[+ \frac{2}{M^2} (r_1 A_1 (r_1^* A_1^* g_2 (r_1, r_1^*) + r_2^* A_2^* g_2 (r_1, r_1^*)) \]

\[+ r_2 A_2 (r_1^* A_1^* g_2 (r_2, r_2^*) + r_2^* A_2^* g_2 (r_2, r_2^*)) \]

\[+ \frac{\mu_1}{2} \left( A_1 f_9 + A_2 f_{10} + A_4 \left( \beta^2 - a^2 \right) (A_1^* g_3 (\beta, r_1^*) + A_2^* g_3 (\beta, r_2^*)) \right) \right] \]

\[- \frac{i \alpha}{4} \text{af} \cdot \text{Pr} \left[ \frac{1}{\sinh 2\gamma_1} (A_1 g_1 (\gamma_1, r_1^*) + A_2^* g_1 (\gamma_1, r_2^*)) \right. \]

\[\left. - \frac{1}{\sinh 2\gamma_1} (A_1 g_1 (\gamma_1^*, r_1) + A_2 g_1 (\gamma_1^*, r_2)) \right] + A_3 y + A_6, \]

$$C_{20}(y) = A_8 + A_7 y - \text{Sc} \cdot \text{Sr} \cdot T_{20} + \frac{i \alpha}{4} \text{Sc} \cdot \text{Re}$$

$$\times \left\{ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \right) \frac{1}{\sinh 2\gamma_2} \right. \]

\[\times \left( A_3^* g_1 (\gamma_2, \alpha) + A_4^* g_1 (\gamma_2, \beta^*) + A_1 a_1^* g_1 (\gamma_2, r_1^*) + A_2 a_2^* g_1 (\gamma_2, r_2^*) \right) \]

\[+ \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \frac{1}{\sinh 2\gamma_2} \]

\[\times \left( A_1^* g_1 (\gamma_2^*, \alpha) + A_4 g_1 (\gamma_2^*, \beta) + A_1 a_1 g_1 (\gamma_2^*, r_1) + A_2 a_2 g_1 (\gamma_2^*, r_2) \right) \]

\[\left. + \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \right) \frac{1}{\sinh 2\gamma_2} \right. \]

\[\times \left( A_3 g_1 (\gamma_2^*, \alpha) + A_4^* g_1 (\gamma_2^*, \beta^*) + A_1 a_1 g_1 (\gamma_2^*, r_1^*) + A_2 a_2^* g_1 (\gamma_2^*, r_2^*) \right) \right\}. \]

where $g_1 - g_3$, $f_1 - f_{10}$, $K_3$ and $A_5 - A_8$ are given in the appendix.
Also, the equations governing for \( T_{22} \) and \( C_{22} \) are

\[
\frac{d^2 T_{22}}{dy^2} - \gamma_3^2 T_{22} = \frac{i\alpha}{2} \Pr \cdot \text{Re} \left( T_1 \frac{d\phi_1}{dy} - \phi_1 \frac{dT_1}{dy} \right) \\
- \Pr \cdot \text{Ec} \left[ \frac{2(1 + \gamma^{-1}) + \mu_1}{4} \left( \left( \frac{d^2 \phi_1}{dy^2} \right)^2 + \alpha^4 \phi_1^2 + 2\alpha^2 \left( \phi_1 \frac{d^2 \phi_1}{dy^2} - 2 \left( \frac{d\phi_1}{dy} \right)^2 \right) \right) \right. \\
+ \mu_1 \left( \mathcal{E}_1^2 + \mathcal{E}_1 \frac{d^2 \phi_1}{dy^2} - \alpha^2 \mathcal{E}_1 \phi_1 \right) + \frac{2}{M^2} \left( \left( \frac{d\mathcal{E}_1}{dy} \right)^2 - \alpha^2 \mathcal{E}_1^2 \right) \right] \\
- \frac{i\alpha}{2} \text{af} \cdot \Pr \left( T_1 \frac{d\xi_1}{dy} - \xi_1 \frac{dT_1}{dy} \right),
\]

\[
\frac{d^2 C_{22}}{dy^2} - \gamma_4^2 C_{22} = \frac{i\alpha}{2} \text{Sc} \cdot \text{Re} \left( C_1 \frac{d\phi_1}{dy} - \phi_1 \frac{dC_1}{dy} \right) - \text{Sc} \cdot \text{Sr} \left( \frac{d^2 T_{22}}{dy^2} - 4\alpha^2 T_{22} \right),
\]

(3.21)

where

\[
\gamma_3^2 = 4\alpha^2 - 2i\alpha \Pr \cdot \text{Re}, \quad \gamma_4^3 = 4\alpha^2 - 2i\alpha \text{Sc} \cdot \text{Re},
\]

(3.22)

with boundary conditions

\[
T_{22}(\pm 1) = \mp \frac{1}{2} T_1y(\pm 1), \quad C_{22}(\pm 1) = \mp \frac{1}{2} C_1y(\pm 1).
\]

(3.23)

Using boundary conditions (3.23), the expressions of \( T_{22} \) and \( C_{22} \) are

\[
T_{22}(y) = A_9 \cosh \gamma_3 y + A_{10} \sinh \gamma_3 y + \frac{i\alpha}{4} \Pr \frac{1}{\sinh 2\gamma_1} \\
\times \left[ \text{Re} \left( \alpha A_3 g_4(\gamma_1, \alpha) + \beta A_4 g_4(\gamma_1, \beta) - \gamma_1 A_9 g_5(\gamma_1, \alpha) - \gamma_1 A_4 g_5(\gamma_1, \beta) \right) \right. \\
+ (a_1 \text{Re} - \text{af}) \left( r_1 A_1 g_4(\gamma_1, r_1) - \gamma_1 A_1 g_5(\gamma_1, r_1) \right) \\
+ (a_2 \text{Re} - \text{af}) \left( r_2 A_2 g_4(\gamma_1, r_2) - \gamma_1 A_2 g_5(\gamma_1, r_2) \right) \right]
\]
\[- \text{Pr} \cdot \text{Ec} \left\{ \frac{2}{8} \left( 1 + \gamma^{-1} \right) + \mu_1 \left( 4a^4A_2^2g_8(a) + \left( r_1^2 + a^2 \right)^2 A_1^2a_1^2g_8(r_1) \right) \right. \]
\[+ \left( r_2^2 + a^2 \right)^2 A_2^2a_2^2g_8(r_2) + (\beta^2 + a^2)^2 A_4^2g_8(\beta) \]
\[+ 4a^2A_3f_{11} + 2A_1a_1 \left( a^2 + r_1^2 \right) f_{12} + 2A_2a_2 \left( a^2 + r_2^2 \right) f_{13} \]
\[- 4a^2 \left( a^2 A_3g_9(a) + r_1^2 A_1^2a_1^2g_9(r_1) \right) \]
\[+ r_2^2 A_2^2a_2^2g_9(r_2) + \beta^2 A_4^2g_9(\beta) + 2aA_3f_{14} \]
\[+ 2r_1A_1a_1f_{15} + 2r_2A_2a_2f_{16} \right) \]
\[+ \frac{\mu_1}{2} \left( \left( 1 + \left( r_1^2 - a^2 \right) a_1 \right) A_2^2g_8(r_1) + \left( 1 + \left( r_2^2 - a^2 \right) a_2 \right) A_2^2g_8(r_2) \right) \]
\[+ A_1f_{17} + A_2A_4 \left( \beta^2 - a^2 \right) g_8(r_2, \beta) \]
\[+ \frac{1}{M^2} \left( r_1^2 A_1^2g_9(r_1) + r_2^2 A_2^2g_9(r_2) + 2r_1r_2A_1A_2g_7(r_1, r_2) \right) \]
\[- a^2 \left( A_1^2g_8(r_1) + A_2^2g_8(r_2) + 2A_1A_2g_6(r_1, r_2) \right) \right) \right\}, \]

(3.24)

\[C_{22}(y) = A_{11} \cosh \gamma_4 y + A_{12} \sinh \gamma_4 y \]
\[+ \frac{i\alpha}{4} \text{Sc} \cdot \text{Re} \left\{ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\left( \text{Sc} - \text{Pr} \right)} \right) \frac{f_{18}}{\sinh 2\gamma_2} + \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\left( \text{Sc} - \text{Pr} \right)} \frac{f_{19}}{\sinh 2\gamma_1} \right\} \]
\[+ \frac{\text{Pr} \cdot \text{Sc} \cdot \text{Sr}}{\left( \text{Sc} - \text{Pr} \right)} \left( A_9 \cosh \gamma_3 y + A_{10} \sinh \gamma_3 y \right) - \frac{i\alpha}{4} \text{Sc} \cdot \text{Sr} \cdot \text{Pr} \cdot \text{Re} \frac{f_{20}}{\sinh 2\gamma_1} + \text{Pr} \cdot \text{Ec} \cdot \text{Sc} \cdot \text{Sr} \]
\[\times \left[ \frac{2}{8} \left( 1 + \gamma^{-1} \right) + \mu_1 \left( \frac{32\alpha^6A_2^2}{(\gamma_3\gamma_4)^2} + \left( r_1^2 + a^2 \right)^2 A_1^2a_1^2g_{14}(r_1) + \left( r_2^2 + a^2 \right)^2 A_2^2a_2^2g_{14}(r_2) \right) \right. \]
\[+ (\beta^2 + a^2)^2 A_4^2g_{14}(\beta) + 4a^2A_5f_{21} \]
\[+ 2 \left( a^2 + r_1^2 \right) A_1a_1f_{22} + 2 \left( a^2 + r_2^2 \right) A_2a_2f_{23} \]
\[- 4a^2 \left( r_1^2 A_1^2a_1^2g_{15}(r_1) + r_2^2 A_2^2a_2^2g_{15}(r_2) + \beta^2 A_4^2g_{15}(\beta) \right) \]
\[+ 2aA_3f_{24} + 2r_1A_1a_1f_{25} + 2r_2A_2a_2f_{26} \right) \right\} \]
\[
+ \frac{\mu_1}{2} \left( (1 + \left( r_1^2 - a^2 \right) a_1) A_1^2 g_{14}(r_1) + (1 + \left( r_2^2 - a^2 \right) a_2) A_2^2 g_{14}(r_2) \right) \\
+ A_1 f_{27} + A_2 A_4 \left( \beta^2 - \alpha^2 \right) g_{16}(r_2, \beta) \\
+ \frac{1}{M^2} \left( r_1^2 A_1^2 g_{15}(r_1) + r_2^2 A_2^2 g_{15}(r_2) + 2r_1 r_2 A_1 A_2 g_{17}(r_1, r_2) \right) \\
- \alpha^2 \left( A_1^2 g_{14}(r_1) + A_2^2 g_{14}(r_2) + 2 A_1 A_2 g_{16}(r_1, r_2) \right) \right] \\
+ \frac{i \alpha}{4} a f \cdot \Pr \cdot Sc \cdot Sr \cdot \frac{f_{28}}{\sinh 2 \gamma_1}.
\]
(3.25)

Hence \( g_{14} - g_{17}, f_{11} - f_{28} \) and \( A_9 - A_{12} \) are given in the appendix.

Using the expressions for \( \Psi_1, T_1, \), \( C_1, T_{20}, C_{20}, T_{22}, \) and \( C_{22} \) in (3.1), (3.3), and (3.4), we can determine the stream function \( \Psi(x, y, t) \), temperature \( T(x, y, t) \), and concentration \( C(x, y, t) \) of micropolar fluid for the case of free pumping as

\[
\Psi(x, y, t) = \frac{\varepsilon}{2} \left( \phi_1(y) e^{i \alpha(x-t)} + \phi_1^*(y) e^{-i \alpha(x-t)} \right) + o(\varepsilon^2),
\]
(3.26)

\[
T(x, y, t) = 1 + \frac{\varepsilon^2}{2} \left( T_1(y) e^{i \alpha(x-t)} + T_1^*(y) e^{-i \alpha(x-t)} \right) \\
+ \frac{\varepsilon^2}{2} \left( T_{20}(y) + T_{22} e^{2i \alpha(x-t)} + T_{22}^*(y) e^{-2i \alpha(x-t)} \right),
\]
(3.27)

\[
C(x, y, t) = 1 + \frac{\varepsilon^2}{2} \left( C_1(y) e^{i \alpha(x-t)} + C_1^*(y) e^{-i \alpha(x-t)} \right) \\
+ \frac{\varepsilon^2}{2} \left( C_{20}(y) + C_{22} e^{2i \alpha(x-t)} + C_{22}^*(y) e^{-2i \alpha(x-t)} \right),
\]
(3.28)

where \( \phi_1^*, T_1^*(y), C_1^*(y), T_{22}^*(y), \) and \( C_{22}^*(y) \) are conjugate to (3.6), (3.16), (3.17), (3.24), and (3.25), respectively.

4. Results and Discussion

To discuss the effect of the parameters of micropolar, non-Newtonian fluid, and viscoelastic membrane properties on the solutions of the considered problem. A numerical results are calculated from formula (3.6), (3.27), and (3.28) for the temperature \( T(x, y, t) \), concentration \( C(x, y, t) \) and stream function \( \Psi(x, y, t) \) and shown by Figures 2–24. Take into account the parameters \( \varepsilon \) and \( \alpha \) have the numerical value 2.0, while parameter \( \alpha \) and amplitude ratio \( \varepsilon \) take the value 0.5 and 0.01, respectively, but \( \alpha \) takes the value 2 for the stream function.

Figures 2–4 illustrate the effects of \( \mu_1 = (k/\mu_B) \), \( M = (2d (\mu_B/Y)^{0.5}) \), and \( af \), in order, on the temperature distribution with taking into account that the elastic wall \( (K_2 = 0) \). The
The temperature $T$ is equal to $\mu_1/\mu_1^4/\mu_1^{10}$.

Figure 2: The variation of temperature distribution versus $y$, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $Re = 10$, $\gamma = 0.8$, $Pr = 5$, $Ec = 0.5$, $M = 10$, $af = 1$, $K_2 = m_1 = 0$, and $K_3 = 10$ for various values of $\mu_1$.

Figure 3: The variation of temperature distribution versus $y$, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $Re = 10$, $\gamma = 0.8$, $Pr = 5$, $Ec = 0.5$, $\mu_1 = 10$, $af = 1$, $K_2 = m_1 = 0$, and $K_3 = 10$ for various values of $M$.

ratios $k/\mu_B$ and $\gamma/\mu_B$ measure the relative strengths of the vortex viscosity coefficient to the viscosity coefficient and the spin gradient viscosity coefficient to the viscosity coefficient, respectively. These coefficients $\mu_B$, $k$, and $\gamma$ may be greater or equal to zero. For example, the blood 50% hematocrit has the values $\mu_B = 0.0029$, $k = 0.000232$, and $\gamma = 0.000001$ [27]. When the viscous effects are much larger than the spin gradient viscosity effects, $(\gamma/\mu_B)$ may tend to be zero, and the parameter $M$ will tend to infinity. Ahmadi [28] has stated that the parameter $\mu_1$ depends on the shape and concentration of the microelements while the parameter $M$ signifies the size of microstructure. In this context, small (large) value of $\mu_1$ means low (high) concentration of the microelements. Similarly, a given small (large) value of $M$ is related to small (large) size of the particles. It is noted from Figure 2 that the temperature distribution
The temperature $T$ decreases with the increasing of $\mu_1$ near the lower wall, while it increases as $\mu_1$ increases when $y \geq -0.7$. In Figure 3, it can be seen that the temperature distribution increases as $M$ increases. Also, Figure 4 shows that the temperature increases as $af$ increases but at $y > 0.8$, an opposite effect is noticed near the upper wall. The results in our study are consistent with those which are obtained by Agarwal et al. [24] in case of isothermal wall.

The choice $K_2 = 0$ implies that the walls move up and down with no damping force on them and hence indicates that the membrane is treated as an elastic wall. The parameter $K_2$ depends upon the wall tension and represents the rigid nature of the walls. Figures 5, 6, and 7 illustrate the effect of the parameters of the viscoelastic wall properties $K_2$, $K_3$, and $m_1$ on the temperature distribution, respectively. It is found that the temperature distribution
The temperature $T$ increases as $K_2$ increases as shown in Figure 5, but near the lower wall, it decreases with the increase of $K_2$. The temperature distribution decreases with the increase of both $K_3$ and $m_1$, but when $y \geq -0.7$, it starts increasing as both $K_3$ and $m_1$ increase for the case of elastic walls ($K_2 = 0$), as shown in Figures 6 and 7. This phenomenon reflects the fact that the increasing of the values of tension or dissipative for the membrane leads to loss of heat from the walls and gives it to the fluid. Therefore, the temperature distribution of fluid will increase.

In Figure 8, we see the effect of Reynolds number $Re$ on the temperature distribution. It is observed that the temperature distribution behaves a dual role with the variation of $Re$. The variations of temperature distribution for various values of Prandtl number $Pr$ and
Eckert number Ec are displayed in Figures 9 and 10, respectively. The graphical results of Figures 9 and 10 indicate that the temperature of fluid increases as Pr and Ec increase. While for $|y| > 0.7$, near the lower and upper wall, the temperature decreases with the increase of Pr and Ec. In the case when the temperature of the plate is periodic, Das et al. [29] showed that the temperature decreases with the increase of the Prandtl number of Newtonian fluid. In Figure 11, the temperature is plotted against $y$ for various values of the upper limit apparent viscosity coefficient $\gamma$; it is observed that an increase of $\gamma$ increases the temperature.

Figures 12–18 reveal the influence of physical parameters entering in the problem on the concentration distribution of fluid. The effect of the coupling parameter $\mu_1$ on concentration is indicated in Figure 12; we find that the concentration distribution of fluid
The temperature $T_{\gamma}$ decreases with the increase of $\mu_1$. It has been observed that the concentration distribution decreases with the increase of $af$, but an opposite effect is seen at $y > 0.8$. This is noticed in Figure 13. Also, in Figures 12 and 13, for small values of $\mu_1$ and $af$, its effects are rather insignificant. In Figures 14, 15, and 16, it is found that the increase of the dissipative, rigid nature of the walls and stiffness in the walls causes a decrease of the concentration distribution. The behavior of the concentration distribution with the nondimensional distance $y$ for various values of Soret number Sr is depicted in Figure 17. It is clear, from Figure 17, that the effect of Sr on the concentration can be neglected near the lower wall. When $-0.8 < y < 0.5$, the concentration decreases with the increase of Sr, but at $y > 0.5$, an increase in Sr leads to
The concentration $C_\mu$ is equal to $1$, $4$, and $10$.

**Figure 12:** The variation of concentration distribution versus $y$, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $Re = 10$, $\gamma = 0.8$, $Pr = 5$, $Ec = 0.5$, $M = 10$, $af = 1$, $K_2 = m_1 = 0$, $K_3 = 10$, $Sc = 1$ and $Sr = 1$ for various values of $\mu_1$.

The concentration increases near lower and upper walls, but it increases with the increase of $\gamma$ at the middle of the channel.

**Figure 13:** The variation of concentration distribution versus $y$, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $Re = 10$, $\gamma = 0.8$, $Pr = 5$, $Ec = 0.5$, $\mu_1 = 10$, $M = 10$, $K_2 = m_1 = 0$, $K_3 = 10$, $Sc = 1$ and $Sr = 1$ for various values of $af$.

The effects of various physical parameters on the stream function $\psi$ are indicated in Figures 19–24. In these figures the stream function is plotted versus the dimensionless distance $y$, and it is clear that there is an inversely non-linear relation between $y$ and $\psi$. 

The effects of the thermal-diffusion parameter $Sr$ on the concentration distribution, in case of non-Newtonian fluid, has been studied by El Dabe et al. [23]. The result in Figure 17, at the upper wall, is in agreement with those obtained by El Dabe et al. [23]. Figure 9 illustrates the effects of the upper limit apparent viscosity coefficient $\gamma$ on the concentration distribution. It is found that the concentration decreases near lower and upper walls, but it increases with the increase of $\gamma$ at the middle of the channel.
and that the obtained curves of all figures will intersect at the origin where \( y = 0, \psi = 0 \).

Figure 19 shows the variation of \( \psi \) with \( y \) for different values of the coupling parameter \( \mu_1 \); it is observed that, in the region \(-1 \leq y < 0\), the stream function decreases by increasing \( \mu_1 \), while in the region \( 0 < y \leq 1 \), \( \psi \) increases by increasing \( \mu_1 \). Note that, when \( \mu_1 \geq 1 \), the peristaltic pumping region becomes slightly wider as \( \mu_1 \) increases, and if \( \mu_1 < 1 \), it is slightly narrow as \( \mu_1 \) decreases. Figure 20 illustrates the stream function \( \psi \) for different values of \( M \). From this figure, it is observed that the effect of \( M \) on \( \psi \) is similar to the effect of \( \mu_1 \) on \( \psi \) illustrated in Figure 19. In Figures 21 and 22, the effects of Reynolds number \( Re \) and the parameter of dissipates of walls \( K_2 \) on the stream function \( \psi \), respectively, are reverse to the
The concentration $C_m$ increases by increasing both of Re and $K_2$, while in the region $0 < y < 1$, $\psi$ decreases by increasing both of Re and $K_2$. Note that the peristaltic pumping region becomes slightly wider as Re and $K_2$ increase. Figures 23 and 24 show that the behaviors of the stream function $\psi$ with the dimensionless distance $y$ for different values of the parameter of rigidities feature of walls $K_3$ and upper limit apparent viscosity coefficient $\gamma$, respectively, are similar to the behaviors of $\psi$ for different values of $\mu_1$ and $M$ illustrated in Figures 19 and 20, with the only difference that the peristaltic pumping will disappear in these cases.
Figure 18: The variation of concentration distribution versus $y$, with fixed $\epsilon = 0.01$, $\alpha = 2$, $Re = 10$, $Pr = 5$, $Ec = 0.5$, $\mu_1 = 10$, $M = 10$, $af = 1$, $K_2 = m_1 = 0$, $K_3 = 10$, $Sc = 1$ and $Sr = 1$ for various values of $\gamma$.

Figure 19: The variation of streamlines versus $y$, with fixed $\epsilon = 0.01$, $\alpha = 0.5$, $Re = 10$, $\gamma = 0.8$, $M = 10$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of $\mu_1$.

5. Conclusion

In this paper, we study the peristaltic flow of a micropolar biviscosity fluid in channel with dynamic boundary condition. Also, we take into considerations that the temperature and concentration of lower wall are constant while they are periodic for the upper wall. This study is an extension of the work of Muthu et al. [25]. The analytical expressions are constructed for the stream function, microrotation velocity, temperature, and concentration distributions as a power series in terms of small amplitude ratio in case of free pumping. The effects of various physical parameters acting on the problem are discussed by a set of graphs. The micropolar
parameters $\mu_1, M, \text{ and } \alpha \text{ and wall parameters } K_2, K_3, \text{ and } m_1 \text{ which describe the viscoelastic behaviors of the flexible wall are discussed through numerical computations. The obtained results can be outlined and summarized as follows.}

(1) The temperature increases with the increase each of $K_2, K_3, \text{ and } m_1$, but near the lower wall it decreases.

(2) The temperature for different values of $K_2, K_3, \text{ and } m_1$ becomes greater with increasing the dimensionless distance $y$ and reaches maximum at $y = 1$.

(3) The concentration decreases with the increase each of $K_2, K_3, \text{ and } m_1$. 

Figure 20: The variation of streamlines versus $y$, with fixed $\varepsilon = 0.01, \alpha = 0.5, \text{Re} = 10, \gamma = 0.8, \mu_1 = 1, K_2 = m_1 = 0 \text{ and } K_3 = 10$ for various values of $M$.

Figure 21: The variation of streamlines versus $y$, with fixed $\varepsilon = 0.01, \alpha = 0.5, \gamma = 0.8, \mu_1 = 1, \text{Re} = 10, K_2 = m_1 = 0 \text{ and } K_3 = 10$ for various values of Re.
(4) The temperature and concentration increase as \( \gamma \) increases, but the concentration decreases near the lower and upper walls.

(5) The streamlines intersect at a critical value of \( y (=0) \) after which all the obtained curves will have an opposite behavior.

(6) For any value \( y < 0.8 \), the temperature increases with the increase of \( af \), while for any value \( y \geq 0.8 \), all the obtained lines will behave in an opposite manner to these behavior when \( y < 0.8 \).
The functions $A_1 - A_{12}$ and the constants $A_5 - A_12$ are given by

\begin{align*}
 g_1(a,b) &= \frac{\sinh(a + (a+b)y) - \sinh(a + (a-b)y)}{a+b}, \\
 g_2(a,b) &= \frac{\cosh((a+b)y)}{(a+b)^2} \frac{\sinh((a-b)y)}{(a-b)^2}, \\
 g_3(a,b) &= \frac{\cosh((a+b)y)}{(a+b)^2} - \frac{\cosh((a-b)y)}{(a-b)^2}, \\
 f_1 &= r_1^* A_1^* a_1^* g_2(a, r_1^*) + r_2^* A_2^* a_2^* g_2(a, r_2^*) + \beta^* A_4^* g_2(a, \beta^*), \\
 f_2 &= \alpha_1^* A_3^* g_2(r_2, \alpha) + r_1^* A_1^* a_1^* g_2(r_1, r_1^*) + r_2^* A_2^* a_2^* g_2(r_1, r_2^*) + \beta^* A_4^* g_2(r_1, \beta^*), \\
 f_3 &= \alpha_2^* A_3^* g_2(r_2, \alpha) + r_1^* A_1^* a_1^* g_2(r_2, r_1^*) + r_2^* A_2^* a_2^* g_2(r_2, r_2^*) + \beta^* A_4^* g_2(r_2, \beta^*), \\
 f_4 &= \alpha_2^* A_3^* g_2(\beta, \alpha) + r_1^* A_1^* a_1^* g_2(\beta, r_1^*) + r_2^* A_2^* a_2^* g_2(\beta, r_2^*) + \beta^* A_4^* g_2(\beta, \beta^*), \\
 f_5 &= (\alpha^2 + r_1^2) A_1^* a_1^* g_3(a, r_1^*) + (\alpha^2 + r_1^2) A_1^* a_1^* g_3(a, r_2^*) + (\alpha^2, \beta^2) + A_4^* g_2(a, \beta^*),
\end{align*}

Figure 24: The variation of streamlines versus $y$, with fixed $\varepsilon = 0.01, \alpha = 0.5, \mu_1 = 1, M = 10, K_2 = m_2 = 0$ and $K_3 = 10$ for various values of $\gamma$.

(7) The concentration has an opposite behavior compared to the temperature with $\alpha_f$ (i.e., for any value $y < 0.8$, the concentration decreases with the increase of $\alpha_f$, while for any value $y \geq 0.8$, it increases).

**Appendix**

The functions $g_1 - g_{17}, f_1 - f_{28}$ and the constants $A_5 - A_{12}$ are given by
\[ f_6 = 2a^2 A_2^* g_3(r_1, \alpha) + \left( a^2 + r_1^2 \right) A_1^* a_1^* g_3(r_1, r_1^*) \]
\[ + \left( a^2, r_2^2 \right) A_2^* a_3^* g_3(r_1, r_2^*) + \left( a^2 + \beta^2 \right) A_4^* g_3(r_1, \beta^*), \]
\[ f_7 = 2a^2 A_3^* g_3(r_2, \alpha) + \left( a^2 + r_1^2 \right) A_1^* a_1^* g_3(r_1, r_1^*) \]
\[ + \left( a^2 + r_2^2 \right) A_2^* a_3^* g_3(r_1, r_2^*) + \left( a^2 + \beta^2 \right) A_4^* g_3(r_2, \beta^*), \]
\[ f_8 = 2a^2 A_3^* g_3(\beta, \alpha) + \left( a^2 + r_1^2 \right) A_1^* a_1^* g_3(\beta, r_1^*) \]
\[ + \left( a^2 + r_2^2 \right) A_2^* a_3^* g_3(\beta, r_2^*) + \left( a^2 + \beta^2 \right) A_4^* g_3(\beta, \beta^*), \]
\[ f_9 = A_1^* \left( a_1^* \left( r_1^2 - a^2 \right) + a_1 \left( r_1^2 - \alpha^2 \right) \right) g_3(r_1, r_1^*) \]
\[ + A_2^* \left( a_2^* \left( r_2^2 - a^2 \right) + a_2 \left( r_2^2 - \alpha^2 \right) \right) g_3(r_1, r_2^*) + A_4^* \left( \beta^2 - a^2 \right) g_3(r_1, \beta^*), \]
\[ f_{10} = A_1^* \left( a_1^* \left( r_1^2 - a^2 \right) + a_2 \left( r_2^2 - a^2 \right) \right) g_3(r_2, r_1^*) \]
\[ + A_2^* \left( a_2^* \left( r_2^2 - a^2 \right) + a_2 \left( r_2^2 - \alpha^2 \right) \right) g_3(r_2, r_2^*) + A_4^* \left( \beta^2 - a^2 \right) g_3(r_2, \beta^*), \]
\[ \delta_2(a, b) = \frac{\sinh(2a + b)}{a + b} - \frac{\sinh(2a - b)}{a - b} - \frac{2a \sinh b}{a^2 - b^2}, \]
\[ \delta_3(a, b) = \frac{\cosh(a + b)}{(a + b)^2} + \frac{\cosh(a - b)}{(a - b)^2}, \]
\[ \delta_4(a, b) = \frac{\cosh(a + b)}{(a + b)^2} - \frac{\cosh(a - b)}{(a - b)^2}, \]
\[ \delta_5(a, b) = \frac{\sinh(2a + b)}{a + b} - \frac{\sinh(2a - b)}{a - b} + \frac{2a \sinh b}{a^2 - b^2}, \]
\[ \lambda_3 = r_1^* A_1^* \delta_3(\alpha, r_1^*) + r_2^* A_2^* a_2^* \delta_3(\alpha, r_2^*) + \beta^* A_4^* \delta_3(\alpha, \beta^*), \]
\[ \lambda_4 = a A_3^* \delta_3(r_1, \alpha) + r_1^* A_1^* a_1^* \delta_3(r_1, r_1^*) + r_2^* A_2^* a_2^* \delta_3(r_1, r_2^*) + \beta^* A_4^* \delta_3(r_1, \beta^*), \]
\[ \lambda_5 = a A_3^* \delta_3(r_2, \alpha) + r_1^* A_1^* a_1^* \delta_3(r_2, r_1^*) + r_2^* A_2^* a_2^* \delta_3(r_2, r_2^*) + \beta^* A_4^* \delta_3(r_2, \beta^*), \]
\[ \lambda_6 = a A_3^* \delta_3(\beta, \alpha) + r_1^* A_1^* a_1^* \delta_3(\beta, r_1^*) + r_2^* A_2^* a_2^* \delta_3(\beta, r_2^*) + \beta^* A_4^* \delta_3(\beta, \beta^*), \]
\[ \lambda_7 = \left( a^2 + r_1^2 \right) A_1^* a_1^* \delta_4(\alpha, r_1^*) + \left( a^2 + r_2^2 \right) A_2^* a_2^* \delta_4(\alpha, r_2^*) + \left( a^2 + \beta^2 \right) + A_4^* \delta_4(\alpha, \beta^*), \]
\[ A_5 = -\frac{1}{4} \left( \frac{\gamma_1 (1 + \cosh 2\gamma_1)}{\sinh 2\gamma_1} + \frac{\gamma_1^* (1 + \cosh 2\gamma_1^*)}{\sinh 2\gamma_1^*} \right) \]

\[ - \frac{i\alpha}{8} Pr \cdot \text{Re} \left( \frac{1}{\sinh 2\gamma_1} \left( \frac{\sinh (2\gamma_1 + \alpha)}{\gamma_1 + \alpha} - \frac{\sinh (2\gamma_1 - \alpha)}{\gamma_1 - \alpha} + \frac{2\gamma_1 \sinh \alpha}{\gamma_1^2 - \alpha^2} \right) A_3 \right. \]

\[ + \left. \left\{ \frac{\sinh (2\gamma_1 + \beta^*)}{\gamma_1 + \beta^*} - \frac{\sinh (2\gamma_1 - \beta^*)}{\gamma_1 - \beta^*} + \frac{2\gamma_1 \sinh \beta^*}{\gamma_1^2 - \alpha^2} \right\} A_4 \right) \]

\[ + \frac{i\alpha}{8} \cdot \text{Pr} \left( \frac{1}{\sinh 2\gamma_1} \left( \frac{\sinh (2\gamma_1^* + \alpha)}{\gamma_1^* + \alpha} - \frac{\sinh (2\gamma_1^* - \alpha)}{\gamma_1^* - \alpha} + \frac{2\gamma_1^* \sinh \alpha}{\gamma_1^*^2 - \alpha^2} \right) A_3 \right. \]

\[ + \left. \left( \frac{\sinh (2\gamma_1^* + \beta)}{\gamma_1^* + \beta} - \frac{\sinh (2\gamma_1^* - \beta)}{\gamma_1^* - \beta} + \frac{2\gamma_1^* \sinh \beta}{\gamma_1^*^2 - \alpha^2} \right) A_4 \right) \]

\[ + \frac{1}{\sinh 2\gamma_1} \left( (a_1 \text{Re} - a_1^*) A_1 \right. \]

\[ + \left. \left( \frac{\sinh (2\gamma_1^* + \alpha)}{\gamma_1^* + \alpha} - \frac{\sinh (2\gamma_1^* - \alpha)}{\gamma_1^* - \alpha} + \frac{2\gamma_1^* \sinh \alpha}{\gamma_1^*^2 - \alpha^2} \right) A_3 \right. \]

\[ + \left. \left( \frac{\sinh (2\gamma_1^* + \beta)}{\gamma_1^* + \beta} - \frac{\sinh (2\gamma_1^* - \beta)}{\gamma_1^* - \beta} + \frac{2\gamma_1^* \sinh \beta}{\gamma_1^*^2 - \alpha^2} \right) A_4 \right) \]

\[ \lambda_8 = 2a^2 A_3^2 \delta_4 (r_1, \alpha) + \left( a^2 + r_1^2 \right) A_1^* A_1^* \delta_4 (r_1, \alpha) \]

\[ + \left( a^2 + r_2^2 \right) A_2^* A_2^* \delta_4 (r_1, r_2^*) + \left( a^2 + \beta^2 \right) A_2^* \delta_4 (r_1, \beta^*) , \]

\[ \lambda_9 = 2a^2 A_3^2 \delta_4 (r_2, \alpha) + \left( a^2 + r_1^2 \right) A_1^* A_1^* \delta_4 (r_2, r_1^*) \]

\[ + \left( a^2 + r_2^2 \right) A_2^* A_2^* \delta_4 (r_2, r_2^*) + \left( a^2 + \beta^2 \right) A_2^* \delta_4 (r_2, \beta^*) , \]
\[\lambda_{10} = 2a^{2}A_{1}^{*}b_{4}(\beta, a) + \left(a^{2} + r_{1}^{*}\right)A_{1}^{*}a_{1}^{*}b_{4}(\beta, r_{1}^{*}) + \left(a^{2} + r_{2}^{*}\right)A_{2}^{*}a_{2}^{*}b_{4}(\beta, r_{2}^{*})
+ \left(a^{2} + b^{*}\right)A_{4}^{*}b_{4}(\beta, b^{*}),\]

\[\lambda_{11} = A_{1}^{*}(a_{1}^{*}(r_{1}^{*} - a^{2}) + a_{1}(r_{1}^{*} - a^{2}))b_{4}(r_{1}, r_{1}^{*})
+ A_{2}^{*}(a_{2}^{*}(r_{2}^{*} - a^{2}) + a_{1}(r_{2}^{*} - a^{2}))b_{4}(r_{1}, r_{2}^{*}) + A_{4}^{*}(b^{*} - a^{2})b_{4}(r_{1}, b^{*}),\]

\[\lambda_{12} = A_{1}^{*}(a_{1}^{*}(r_{1}^{*} - a^{2}) + a_{2}(r_{2}^{*} - a^{2}))b_{4}(r_{2}, r_{1}^{*})
+ A_{2}^{*}(a_{2}^{*}(r_{2}^{*} - a^{2}) + a_{2}(r_{2}^{*} - a^{2}))b_{4}(r_{2}, r_{2}^{*}) + A_{4}^{*}(b^{*} - a^{2})b_{4}(r_{2}, b^{*}),\]

\[f_{11} = A_{1}a_{1}(r_{1}^{*} + a^{2})g_{6}(\alpha, r_{1}) + A_{2}a_{2}(r_{2}^{*} + a^{2})g_{6}(\alpha, r_{2}) + A_{4}(\beta^{2} + a^{2})g_{6}(\alpha, \beta),\]

\[f_{12} = A_{2}a_{2}(r_{2}^{*} + a^{2})g_{6}(r_{1}, r_{2}) + A_{4}(\beta^{2} + a^{2})g_{6}(r_{1}, \beta),\]

\[f_{13} = A_{4}(\beta^{2} + a^{2})g_{6}(r_{2}, \beta),\]

\[f_{14} = r_{1}A_{1}a_{1}g_{7}(\alpha, r_{1}) + r_{2}A_{2}a_{2}g_{7}(\alpha, r_{2}) + \beta A_{4}g_{7}(\alpha, \beta),\]

\[f_{15} = r_{2}A_{2}a_{2}g_{7}(r_{1}, r_{2}) + \beta A_{4}g_{7}(r_{1}, \beta),\]

\[f_{16} = \beta A_{4}g_{7}(r_{2}, \beta),\]

\[f_{17} = \left(2 + (r_{2}^{*} - a^{2})a_{1} + (r_{2}^{*} - a^{2})a_{2}\right)A_{2}g_{6}(r_{1}, r_{2}) + (\beta^{2} - a^{2})A_{4}g_{6}(r_{1}, \beta),\]

\[f_{18} = aA_{3}g_{10}(\gamma_{2}, a) + \beta A_{4}g_{10}(\gamma_{2}, \beta) + r_{1}A_{1}a_{1}g_{10}(\gamma_{2}, r_{1}) + r_{2}A_{2}a_{2}g_{10}(\gamma_{2}, r_{2})
- \gamma_{2}(A_{3}g_{11}(\gamma_{2}, a) + A_{4}g_{11}(\gamma_{2}, \beta) + A_{1}a_{1}g_{11}(\gamma_{2}, r_{1}) + A_{2}a_{2}g_{11}(\gamma_{2}, r_{2})),\]

\[f_{19} = aA_{3}g_{10}(\gamma_{1}, a) + \beta A_{4}g_{10}(\gamma_{1}, \beta) + r_{1}A_{1}a_{1}g_{10}(\gamma_{1}, r_{1}) + r_{2}A_{2}a_{2}g_{10}(\gamma_{1}, r_{2})
- \gamma_{1}(A_{3}g_{11}(\gamma_{1}, a) + A_{4}g_{11}(\gamma_{1}, \beta) + A_{1}a_{1}g_{11}(\gamma_{1}, r_{1}) + A_{2}a_{2}g_{11}(\gamma_{1}, r_{2})),\]

\[f_{20} = aA_{3}g_{12}(\gamma_{1}, a) + \beta A_{4}g_{12}(\gamma_{1}, \beta) + r_{1}A_{1}a_{1}g_{12}(\gamma_{1}, r_{1}) + r_{2}A_{2}a_{2}g_{12}(\gamma_{1}, r_{2})
- \gamma_{1}(A_{3}g_{12}(\gamma_{1}, a) + A_{4}g_{12}(\gamma_{1}, \beta) + A_{1}a_{1}g_{12}(\gamma_{1}, r_{1}) + A_{2}a_{2}g_{12}(\gamma_{1}, r_{2})),\]

\[f_{21} = \left(r_{1}^{*} + a^{2}\right)A_{1}a_{1}g_{16}(\alpha, r_{1}) + \left(r_{2}^{*} + a^{2}\right)A_{2}a_{2}g_{16}(\alpha, r_{2}) + (\beta^{2} + a^{2})A_{4}g_{16}(\alpha, \beta),\]
\[
\begin{align*}
f_{22} &= \left( r_2^2 + a^2 \right) A_2 a_2 g_{16}(r_2) + \left( \beta^2 + a^2 \right) A_4 g_{16}(r_1, \beta), \\
f_{23} &= \left( \beta^2 + a^2 \right) A_4 g_{16}(r_2, \beta), \\
f_{24} &= r_1 A_1 A_1 g_{17}(\alpha, r_1) + r_2 A_2 a_2 g_{17}(a, r_2) + \beta A_4 g_{17}(\alpha, \beta), \\
f_{25} &= r_2 A_2 a_2 g_{17}(r_1, r_2) + \beta A_4 g_{17}(r_1, \beta), \\
A_6 &= \frac{1}{4} \left( \frac{\gamma_1 (1 - \cosh 2\gamma_1)}{\sinh 2\gamma_1} + \frac{\gamma_1^* (1 - \cosh 2\gamma_1^*)}{\sinh 2\gamma_1^*} \right) \\
&+ \frac{ia}{8} \Pr \left[ -\Re(A_3 \delta_2(\gamma_1, \alpha) + A_1^* \delta_2(\gamma_1^*, \beta^*)) + A_1^* (af - \Re a_1^*) \delta_2(\gamma_1, r_1^*) \\
&+ A_2^* (af - \Re a_2^*) \delta_2(\gamma_1, r_2^*) \right] + \frac{ia}{8} \Pr \left[ 2(1+\gamma)^{-1} + \mu_1 \right] \\
&\times \left[ \Re(A_3 \delta_2(\gamma_1^*, \alpha) + A_4 \delta_2(\gamma_1^*, \beta^*)) + A_1 (a_1 \Re -af) \delta_2(\gamma_1^*, r_1) A_2^*(a_2 \Re -af) \delta_2(\gamma_1^*, r_2) \right] \\
&+ Ec \cdot Pr \left[ \frac{2(1+\gamma^{-1}) + \mu_1}{4} \right] \\
&\times \left( 2a^2 A_3 A_3^* \cosh 2\alpha + 4a^2 (\alpha A_3 \lambda_3 + r_1 A_1 a_1 \lambda_4 + r_2 A_2 a_2 \lambda_5 + \beta A_4 \lambda_6) + 2a^2 A_3 \lambda_7 \\
&+ (a^2 + r_1^2) A_1 a_1 \lambda_8 + (a^2 + r_2^2) A_2 a_2 \lambda_9 + (a^2 + \beta^2) A_4 \lambda_{10} \right] + \frac{2}{M^2} \\
&\times (r_1 A_1 (r_1^* A_1^* \delta_3(r_1, r_1^*) + r_2^* A_2^* \delta_3(r_1, r_2^*))) \\
&+ r_2 A_2 (r_2^* A_1^* \delta_3(r_2, r_1^*) + r_2^* A_2^* \delta_3(r_2, r_2^*)) + \left( \mu_1 + \frac{2a^2}{M^2} \right) \\
&\times (A_1 (A_1^* \delta_4(r_1, r_1^*) + A_2^* \delta_4(r_1, r_2^*)) + A_2 (A_1^* \delta_4(r_2, r_1^*) + A_2^* \delta_4(r_2, r_2^*))) \\
&+ \frac{\mu_1}{2} \left( A_1 \lambda_{11} + A_2 \lambda_{12} + A_4 (\beta^2 - a^2) (A_1^* \delta_4(\beta, r_1^*) + A_2^* \delta_4(\beta, r_2^*)) \right),
\end{align*}
\]
\[
A_7 = -\frac{1}{4} \left\{ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \right) \left[ \gamma_2 \left( \frac{1 + \cosh 2\gamma_2}{\sinh 2\gamma_2} \right) + \gamma_2^* \left( \frac{1 + \cosh 2\gamma_2^*}{\sinh 2\gamma_2^*} \right) \right] + \left( \frac{\text{Pr}}{\text{Sc} - \text{Pr}} + 1 \right) \text{Sc} \cdot \text{Sr} \left[ \gamma_1 \left( \frac{1 + \cosh 2\gamma_1}{\sinh 2\gamma_1} \right) + \gamma_1^* \left( \frac{1 + \cosh 2\gamma_1^*}{\sinh 2\gamma_1^*} \right) \right]\right\} \\
- \frac{ia}{8} \text{Sc} \cdot \text{Re} \left\{ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \right) \times \left[ \frac{1}{\sinh 2\gamma_2} \left( A_3^* \delta_3(\gamma_2, \alpha) + A_4^* \delta_3(\gamma_2, \beta^*) + A_1^* a_1^* \delta_3(\gamma_2, r_1^*) \right) + A_1 a_1 \delta_3(\gamma_2^*, r_1) + A_2 a_2 \delta_3(\gamma_2^*, r_2) \right] \right\},
\]

\[
A_8 = \frac{1}{4} \left\{ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \right) \left[ \gamma_2 \left( \frac{1 - \cosh 2\gamma_2}{\sinh 2\gamma_2} \right) + \gamma_2^* \left( \frac{1 - \cosh 2\gamma_2^*}{\sinh 2\gamma_2^*} \right) \right] + \left( \frac{\text{Pr}}{\text{Sc} - \text{Pr}} + 1 \right) \text{Sc} \cdot \text{Sr} \left[ \gamma_1 \left( \frac{1 - \cosh 2\gamma_1}{\sinh 2\gamma_1} \right) + \gamma_1^* \left( \frac{1 - \cosh 2\gamma_1^*}{\sinh 2\gamma_1^*} \right) \right]\right\} \\
- \frac{ia}{8} \text{Sc} \cdot \text{Re} \left\{ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{Sc} - \text{Pr}} \right) \times \left[ \frac{1}{\sinh 2\gamma_2} \left( A_3^* \delta_2(\gamma_2, \alpha) + A_4^* \delta_2(\gamma_2, \beta^*) + A_1^* a_1^* \delta_2(\gamma_2, r_1^*) \right) + A_1 a_1 \delta_2(\gamma_2^*, \alpha) + A_4 \delta_2(\gamma_2^*, \beta) + A_1 a_1 \delta_2(\gamma_2^*, r_1) + A_2 a_2 \delta_2(\gamma_2^*, r_2) \right] \right\}
\]
\[
\frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{(\text{Sc} - \text{Pr})} \left[ \frac{1}{\sinh 2y_1} (A_3^* \delta_2(y_1, a) + A_4^* \delta_2(y_1, b^*) + A_5^* a_i \delta_2(y_1, r_i^*) + A_6^* a_i \delta_2(y_1, r_i^*))
\right.
\[
- \frac{1}{\sinh 2y_1^*} (A_3 \delta_2(y_1^*, a) + A_4^* \delta_2(y_1^*, b) + A_5^* a_i \delta_2(y_1^*, r_i) + A_6^* a_i \delta_2(y_1^*, r_i)) \right] \Bigg],
\]

\[
g_4(a, b) = \frac{\sinh(a + (a + b)y)}{(a + b)^2 - y_3^2} + \frac{\sinh(a + (a - b)y)}{(a - b)^2 - y_3^2},
\]

\[
g_5(a, b) = \frac{\sinh(a + (a + b)y)}{(a + b)^2 - y_3^2} - \frac{\sinh(a + (a - b)y)}{(a - b)^2 - y_3^2},
\]

\[
g_6(a, b) = \frac{\cosh((a + b)y)}{(a + b)^2 - y_3^2} - \frac{\cosh((a - b)y)}{(a - b)^2 - y_3^2},
\]

\[
g_7(a, b) = \frac{\cosh((a + b)y)}{(a + b)^2 - y_3^2} + \frac{\cosh((a - b)y)}{(a - b)^2 - y_3^2},
\]

\[
g_8(a) = \frac{\cosh(2ay)}{4a^2 - y_3^2} + \frac{1}{y_3^2},
\]

\[
g_9(a) = \frac{\cosh(2ay)}{4a^2 - y_3^2} - \frac{1}{y_3^2},
\]

\[
g_{10}(a, b) = \frac{\sinh(a + (a + b)y)}{(a + b)^2 - y_3^2} + \frac{\sinh(a + (a - b)y)}{(a + b)^2 - y_3^2},
\]

\[
g_{11}(a, b) = \frac{\sinh(a + (a + b)y)}{(a + b)^2 - y_3^2} - \frac{\sinh(a + (a - b)y)}{(a + b)^2 - y_3^2},
\]

\[
g_{12}(a, b) = \frac{(a + b)^2 - 4a^2 \sinh(a + (a + b)y)}{(a + b)^2 - y_3^2} + \frac{(a - b)^2 - 4a^2 \sinh(a + (a - b)y)}{(a - b)^2 - y_3^2},
\]

\[
g_{13}(a, b) = \frac{(a + b)^2 - 4a^2 \sinh(a + (a + b)y)}{(a + b)^2 - y_3^2} - \frac{(a - b)^2 - 4a^2 \sinh(a + (a - b)y)}{(a - b)^2 - y_3^2},
\]

\[
g_{14}(a) = \frac{4(a^2 - a^2) \cosh(2ay)}{(4a^2 - y_3^2) (4a^2 - y_3^2)} + \frac{4a^2}{(y_3 y_4)^2},
\]

\[
g_{15}(a) = \frac{4(a^2 - a^2) \cosh(2ay)}{(4a^2 - y_3^2) (4a^2 - y_3^2)} - \frac{4a^2}{(y_3 y_4)^2},
\]

\[
g_{16}(a, b) = \frac{(a + b)^2 - 4a^2 \cosh(a + (a + b)y)}{(a + b)^2 - y_3^2} - \frac{(a - b)^2 - 4a^2 \cosh((a - b)y)}{(a - b)^2 - y_3^2},
\]
\[
\begin{align*}
g_{17}(a, b) &= \frac{(a + b)^2 - 4a^2 \cosh(a + (a + b)y) + (a - b)^2 - 4a^2 \cosh((a - b)y)}{(a + b)^2 - \gamma_4^2} + \frac{(a - b)^2 - 4a^2 \cosh((a - b)y)}{(a - b)^2 - \gamma_3^2}, \\
f_{26} &= \beta A_4 g_{17}(r_2, \beta), \\
f_{27} &= \left(2 + \left(r_1 - \alpha^2 \right)a_1 + \left(r_2 - \alpha^2 \right)a_2 \right)A_2 g_{16}(r_1, r_2) + A_4 \left(\beta^2 - \alpha^2 \right)g_{16}(r_1, \beta), \\
f_{28} &= r_1 A_1 g_{12}(\gamma_1, r_1) + r_2 A_2 g_{12}(\gamma_1, r_2) - \gamma_1 A_1 g_{13}(\gamma_1, r_1) - \gamma_1 A_2 g_{13}(\gamma_1, r_2), \\
\delta_6(a, b) &= \frac{\sinh(2a + b) - \sinh b}{(a + b)^2 - \gamma_3^2} + \frac{\sinh(2a - b) - \sinh b}{(a - b)^2 - \gamma_3^2}, \\
\delta_7(a, b) &= \frac{\sinh(2a + b) - \sinh b}{(a + b)^2 - \gamma_3^2} - \frac{\sinh(2a - b) + \sinh b}{(a - b)^2 - \gamma_3^2}, \\
\delta_8(a, b) &= \frac{\sinh(2a + b) + \sinh b}{(a + b)^2 - \gamma_3^2} + \frac{\sinh(2a - b) - \sinh b}{(a - b)^2 - \gamma_3^2}, \\
\delta_9(a, b) &= \frac{\sinh(2a + b) + \sinh b}{(a + b)^2 - \gamma_3^2} - \frac{\sinh(2a - b) - \sinh b}{(a - b)^2 - \gamma_3^2}, \\
\delta_{10}(a) &= \cosh(2a) + \frac{1}{4a^2 - \gamma_3^2}, \\
\delta_{11}(a) &= \cosh(2a) - \frac{1}{4a^2 - \gamma_3^2}, \\
\delta_{12}(a, b) &= \frac{\cosh(a + b)}{(a + b)^2 - \gamma_3^2} + \frac{\cosh(a - b)}{(a - b)^2 - \gamma_3^2}, \\
\delta_{13}(a, b) &= \frac{\cosh(a + b)}{(a + b)^2 - \gamma_3^2} - \frac{\cosh(a - b)}{(a - b)^2 - \gamma_3^2}, \\
\delta_{14}(a, b) &= \frac{\sinh(2a + b) - \sinh b}{(a + b)^2 - \gamma_4^2} + \frac{\sinh(2a - b) + \sinh b}{(a - b)^2 - \gamma_4^2}, \\
\delta_{15}(a, b) &= \frac{\sinh(2a + b) - \sinh b}{(a + b)^2 - \gamma_4^2} - \frac{\sinh(2a - b) + \sinh b}{(a - b)^2 - \gamma_4^2}, \\
\delta_{16}(a, b) &= \frac{\sinh(2a + b) + \sinh b}{(a + b)^2 - \gamma_4^2} + \frac{\sinh(2a - b) - \sinh b}{(a - b)^2 - \gamma_4^2}, \\
\delta_{17}(a, b) &= \frac{\sinh(2a + b) + \sinh b}{(a + b)^2 - \gamma_4^2} - \frac{\sinh(2a - b) - \sinh b}{(a - b)^2 - \gamma_4^2}, \end{align*}
\]
\[
\begin{align*}
\delta_{16}(a, b) &= \frac{(a + b)^2 - 4a^2 \sinh(2a + b) - \sinh b}{(a + b)^2 - \frac{1}{14}} + \frac{(a - b)^2 - 4a^2 \sinh(2a - b) + \sinh b}{(a - b)^2 - \frac{1}{14}}, \\
\delta_{19}(a, b) &= \frac{(a + b)^2 - 4a^2 \sinh(2a + b) - \sinh b}{(a + b)^2 - \frac{1}{14}} + \frac{(a - b)^2 - 4a^2 \sinh(2a - b) + \sinh b}{(a - b)^2 - \frac{1}{14}}, \\
\delta_{20}(a, b) &= \frac{(a + b)^2 - 4a^2 \sinh(2a + b) + \sinh b}{(a + b)^2 - \frac{1}{14}} + \frac{(a - b)^2 - 4a^2 \sinh(2a - b) - \sinh b}{(a - b)^2 - \frac{1}{14}}, \\
\delta_{21}(a, b) &= \frac{(a + b)^2 - 4a^2 \sinh(2a + b) - \sinh b}{(a + b)^2 - \frac{1}{14}} + \frac{(a - b)^2 - 4a^2 \sinh(2a - b) + \sinh b}{(a - b)^2 - \frac{1}{14}}, \\
\delta_{22}(a) &= \frac{4(a^2 - \alpha^2)}{4a^2 - \frac{1}{14}} \cosh(2a) + \frac{4a^2}{(\frac{1}{14})^2}, \\
\delta_{23}(a) &= \frac{4(a^2 - \alpha^2)}{4a^2 - \frac{1}{14}} \cosh(2a) - \frac{4a^2}{(\frac{1}{14})^2}, \\
\delta_{24}(a, b) &= \frac{(a + b)^2 - 4a^2 \cosh(2a + b) + \cosh(a - b)}{(a + b)^2 - \frac{1}{14}} + \frac{(a - b)^2 - 4a^2 \cosh(2a - b) - \cosh(a - b)}{(a - b)^2 - \frac{1}{14}}, \\
\delta_{25}(a, b) &= \frac{(a + b)^2 - 4a^2 \cosh(2a + b) + \cosh(a - b)}{(a + b)^2 - \frac{1}{14}} + \frac{(a - b)^2 - 4a^2 \cosh(2a - b) - \cosh(a - b)}{(a - b)^2 - \frac{1}{14}}, \\
\lambda_{13} &= A_1a_1(\gamma_1^2 + \alpha^2) \delta_{13}(\alpha, r_1) + A_2a_2(\gamma_2^2 + \alpha^2) \delta_{13}(\alpha, r_2) + A_4(\beta^2 + \alpha^2) \delta_{13}(\alpha, \beta), \\
\lambda_{14} &= A_2a_2(\gamma_2^2 + \alpha^2) \delta_{13}(r_1, r_2) + A_4(\beta^2 + \alpha^2) \delta_{13}(r_1, \beta), \\
\lambda_{15} &= A_4(\beta^2 + \alpha^2) \delta_{13}(r_2, \beta), \\
\lambda_{16} &= r_1A_1a_1 \delta_{12}(\alpha, r_1) + r_2A_2a_2 \delta_{12}(\alpha, r_2) + \beta A_4 \delta_{12}(\alpha, \beta), \\
\lambda_{17} &= r_2A_2a_2 \delta_{12}(r_1, r_2) + \beta A_4 \delta_{12}(r_1, \beta), \\
\lambda_{18} &= \beta A_4 \delta_{12}(r_2, \beta), \\
\lambda_{19} &= \left(2 + (\gamma_1^2 - \alpha^2)\right) a_4 + \left(\gamma_2^2 - \alpha^2\right) a_2 A_2 \delta_{13}(r_1, r_2) + \left(\beta^2 - \alpha^2\right) A_4 \delta_{13}(r_1, \beta), \\
\lambda_{20} &= \alpha A_3 \delta_{14}(\gamma_2, \alpha) + \beta A_4 \delta_{14}(\gamma_2, \beta) + r_1A_1a_1 \delta_{14}(\gamma_2, r_1) + r_2A_2a_2 \delta_{14}(\gamma_2, r_2) \\
&- \gamma_3( \lambda_3 \delta_{15}(\gamma_2, \alpha) + A_4 \delta_{14}(\gamma_2, \beta) + A_1a_1 \delta_{15}(\gamma_2, r_1) + A_2a_2 \delta_{15}(\gamma_2, r_2)),
\end{align*}
\]
\[
\begin{align*}
\lambda_{21} &= a A_3 \delta_{14}(y_1, a) + \beta A_4 \delta_{14}(y_1, \beta) + r_1 A_1 a_1 \delta_{14}(y_1, r_1) + r_2 A_2 a_2 \delta_{14}(y_1, r_2) \\
&\quad - \gamma_1 A_3 \delta_{15}(y_1, a) + A_4 \delta_{14}(y_1, \beta) + A_1 a_1 \delta_{15}(y_1, r_1) + A_2 a_2 \delta_{15}(y_1, r_2)), \\
\lambda_{22} &= a A_3 \delta_{18}(y_1, a) + \beta A_4 \delta_{18}(y_1, \beta) + r_1 A_1 a_1 \delta_{18}(y_1, r_1) + r_2 A_2 a_2 \delta_{18}(y_1, r_2) \\
&\quad - \gamma_1 A_3 \delta_{19}(y_1, a) + A_4 \delta_{19}(y_1, \beta) + A_1 a_1 \delta_{19}(y_1, r_1) + A_2 a_2 \delta_{19}(y_1, r_2)), \\
\lambda_{23} &= (r_1^2 + \alpha^2) A_1 a_1 \delta_{25}(a, r_1) + (r_2^2 + \alpha^2) A_2 a_2 \delta_{25}(a, r_2) + (\beta^2 + \alpha^2) A_4 \delta_{25}(a, \beta), \\
\lambda_{24} &= (r_2^2 + \alpha^2) A_2 a_2 \delta_{25}(r_1, r_2) + (\beta^2 + \alpha^2) A_4 \delta_{25}(r_1, \beta), \\
\lambda_{25} &= (\beta^2 + \alpha^2) A_4 \delta_{25}(r_2, \beta), \\
\lambda_{26} &= r_1 A_1 a_1 \delta_{24}(a, r_1) + r_2 A_2 a_2 \delta_{24}(a, r_2) + \beta A_4 \delta_{24}(a, \beta), \\
\lambda_{27} &= r_2 A_2 a_2 \delta_{24}(r_1, r_2) + \beta A_4 \delta_{24}(r_1, \beta), \\
\lambda_{28} &= \beta A_4 \delta_{24}(r_2, \beta), \\
\lambda_{29} &= (2 + (r_1^2 - \alpha^2) a_1 + (r_2^2 - \alpha^2) a_2) A_2 \delta_{25}(r_1, r_2) + A_4 (\beta^2 - \alpha^2) \delta_{25}(r_1, \beta), \\
\lambda_{30} &= r_1 A_1 \delta_{18}(y_1, r_1) + r_2 A_2 \delta_{18}(y_1, r_2) - \gamma_1 A_1 \delta_{19}(y_1, r_1) - \gamma_1 A_2 \delta_{19}(y_1, r_2), \\
\lambda_{31} &= a A_3 \delta_{16}(y_2, a) + \beta A_4 \delta_{16}(y_2, \beta) + r_1 A_1 a_1 \delta_{16}(y_2, r_1) + r_2 A_2 a_2 \delta_{16}(y_2, r_2) \\
&\quad - \gamma_2 (A_3 \delta_{17}(y_2, a) + A_4 \delta_{17}(y_2, \beta) + A_1 a_1 \delta_{17}(y_2, r_1) + A_2 a_2 \delta_{17}(y_2, r_2)), \\
\lambda_{32} &= a A_3 \delta_{16}(y_1, a) + \beta A_4 \delta_{16}(y_1, \beta) + r_1 A_1 a_1 \delta_{16}(y_1, r_1) + r_2 A_2 a_2 \delta_{16}(y_1, r_2) \\
&\quad - \gamma_1 (A_3 \delta_{17}(y_1, a) + A_4 \delta_{17}(y_1, \beta) + A_1 a_1 \delta_{17}(y_1, r_1) + A_2 a_2 \delta_{17}(y_1, r_2)), \\
\lambda_{33} &= a A_3 \delta_{20}(y_1, a) + \beta A_4 \delta_{20}(y_1, \beta) + r_1 A_1 a_1 \delta_{20}(y_1, r_1) + r_2 A_2 a_2 \delta_{20}(y_1, r_2) \\
&\quad - \gamma_1 (A_3 \delta_{21}(y_1, a) + A_4 \delta_{21}(y_1, \beta) + A_1 a_1 \delta_{21}(y_1, r_1) + A_2 a_2 \delta_{21}(y_1, r_2)), \\
\lambda_{34} &= r_1 A_1 \delta_{20}(y_1, r_1) + r_2 A_2 \delta_{21}(y_1, r_2) - \gamma_1 A_1 \delta_{21}(y_1, r_1) - \gamma_1 A_2 \delta_{21}(y_1, r_2),
\end{align*}
\]
\[ A_9 = \frac{1}{2 \cosh \gamma_3} \times \left\{ \frac{1}{2} \gamma_1 \left( \frac{1 - \cosh 2\gamma_1}{\sinh 2\gamma_1} \right) - \frac{i\alpha}{4} \frac{\Pr}{\sinh 2\gamma_1} \right\} \times \left[ \text{Re} \left( \alpha A_3 \delta_6 (\gamma_1, \alpha) + \beta A_4 \delta_6 (\gamma_1, \beta) - \gamma_1 A_3 \delta_7 (\gamma_1, \alpha) - \gamma_1 A_4 \delta_7 (\gamma_1, \beta) \right) + (a_1 \text{ Re} - \text{ af}) \right\} \times (r_1 A_1 \delta_6 (\gamma_1, r_1) - \gamma_1 A_1 \delta_7 (\gamma_1, r_1)) + (a_2 \text{ Re} - \text{ af}) (r_2 A_2 \delta_6 (\gamma_1, r_2) - \gamma_1 A_2 \delta_7 (\gamma_1, r_2)) + \left. \frac{\text{Pr} \cdot \text{ Ec}}{4} \times \left( \frac{2}{1 + \gamma^{-1}} + \mu_1 \right) \times \left( 4\alpha^4 A_3 \delta_{10} (\alpha) + (r_1^2 + \alpha^2) A_1^2 a_1^2 \delta_{10} (r_1) + (r_2^2 + \alpha^2) A_2^2 a_2^2 \delta_{10} (r_2) \right) \right. \]

\[ + \left( \beta^2 + \alpha^2 \right) A_4 \delta_{10} (\beta) + 4\alpha^2 A_3 \lambda_{13} + 2A_1 a_1 \left( \alpha^2 + r_1^2 \right) \lambda_{14} \]

\[ + 2A_2 a_2 \left( \alpha^2 + r_2^2 \right) \lambda_{15} \]

\[ - 4\alpha^2 \left( \alpha^2 A_3^2 \delta_{11} (\alpha) + (r_1^2 + \alpha^2) A_1^2 a_1^2 \delta_{11} (r_1) + (r_2^2 + \alpha^2) A_2^2 a_2^2 \delta_{11} (r_2) + \beta \delta_{12} (\beta) \right) \] + 2\alpha A_3 \lambda_{16} + 2r_1 A_1 a_1 \lambda_{17} + 2r_2 A_2 a_2 \lambda_{18}\]

\[ + \mu_1 \left( \left( 1 + (r_1^2 - \alpha^2) a_1 \right) A_1^2 \delta_{10} (r_1) + \left( 1 + (r_2^2 - \alpha^2) a_2 \right) A_2^2 \delta_{10} (r_2) \right) + A_1 \lambda_{19} + A_2 A_4 \left( \beta^2 - \alpha^2 \right) \delta_{13} (r_2, \beta) \] + \frac{2}{M^2} \left( r_1^2 A_1^2 \delta_{11} (r_1) + r_2^2 A_2^2 \delta_{11} (r_2) + 2r_1 r_2 A_1 A_2 \delta_{12} (r_1, r_2) \right) \]

\[ - \left( A_1^2 \delta_{10} (r_1) + A_2^2 \delta_{10} (r_2) + 2A_1 A_2 \delta_{13} (r_1, r_2) \right) \right), \]

\[ A_{10} = \frac{1}{2 \cosh \gamma_3} \left\{ \frac{1}{2} \gamma_1 \left( \frac{1 + \cosh 2\gamma_1}{\sinh 2\gamma_1} \right) + \frac{i\alpha}{4} \frac{\Pr}{\sinh 2\gamma_1} \right\} \times \left[ \text{Re} \left( \alpha A_3 \delta_6 (\gamma_1, \alpha) + \beta A_4 \delta_6 (\gamma_1, \beta) - \gamma_1 A_3 \delta_9 (\gamma_1, \alpha) - \gamma_1 A_4 \delta_9 (\gamma_1, \beta) \right) + (a_1 \text{ Re} - \text{ af}) (r_1 A_1 \delta_6 (\gamma_1, r_1) - \gamma_1 A_1 \delta_9 (\gamma_1, r_1)) \right. \]

\[ + (a_2 \text{ Re} - \text{ af}) (r_2 A_2 \delta_6 (\gamma_1, r_2) - \gamma_1 A_2 \delta_9 (\gamma_1, r_2)) \right), \]
\[ A_{11} = \frac{1}{2 \cosh \gamma_4} \left\{ \frac{1}{2} \left[ \left( 1 - \frac{Sc \cdot Sr \cdot Pr}{(Sc - Pr)} \right) \gamma_2 \left( \frac{1 - \cosh 2\gamma_2}{\sinh 2\gamma_2} \right) + \frac{\gamma_1 \cdot Sc \cdot Sr \cdot Pr}{Sc - Pr} \left( \frac{1 - \cosh 2\gamma_1}{\sinh 2\gamma_1} \right) \right] \right. \\
- \frac{i \alpha}{4} Sc \cdot \Re \left\{ \left( 1 - \frac{Sc \cdot Sr \cdot Pr}{(Sc - Pr)} \right) \frac{\lambda_{20}}{\sinh 2\gamma_2} + \frac{Sc \cdot Sr \cdot Pr \lambda_{21}}{(Sc - Pr) \sinh 2\gamma_1} \right\} \\
- \frac{Pr \cdot Sc \cdot Sr}{Sc - Pr} 2A_9 \cosh \gamma_3 + \frac{i \alpha}{4} Sc \cdot Sr \cdot Pr \cdot \Re \frac{\lambda_{32}}{\sinh 2\gamma_1} \\
- Pr \cdot Ec \cdot Sc \cdot Sr \left[ \frac{(1 + \gamma^{-1}) + \mu_1}{2} \right] \times \left( \frac{32\alpha^2 A_3^2}{(\gamma_3 \gamma_4)^2} + (r_1^2 + \alpha^2)^2 A_1^2 a_1^2 \delta_{22}(r_1) \right. \\
+ \left( r_2^2 + \alpha^2 \right)^2 A_2^2 a_2^2 \delta_{22}(r_2) \\
+ (\beta^2 + \alpha^2)^2 A_3^2 \delta_{22}(\beta) + 4\alpha^2 A_3 \lambda_{23} \\
+ 2(\alpha^2 + r_1^2) A_1 a_1 \lambda_{24} \\
+ \left. 2(\alpha^2 + r_2^2) A_2 a_2 \lambda_{25} \\
- 4\alpha^2 \left( r_1^2 A_1^2 a_1^2 \delta_{23}(r_1) + r_2^2 A_2^2 a_2^2 \delta_{23}(r_2) + \beta^2 A_3^2 \delta_{23}(\beta) \right) \\
+ 2\alpha A_3 \lambda_{26} + 2r_1 A_1 a_1 \lambda_{27} + 2r_2 A_2 a_2 \lambda_{28} \right) \right\} \\
+ \mu_1 \left( \left( 1 - \frac{r_1^2 - \alpha^2}{a_1} \right) A_3^2 \delta_{22}(r_1) + \left( 1 - \frac{r_2^2 - \alpha^2}{a_2} \right) \right. \\
\times A_3^2 \delta_{22}(r_2) \\
+ \left. A_1 a_1 + A_2 A_4 (\beta^2 - \alpha^2) \delta_{25}(r_2, \beta) \right) \\
+ \frac{2}{\lambda^2} \left( r_1^2 A_1^2 \delta_{23}(r_1) + r_2^2 A_2^2 \delta_{23}(r_2) + 2r_1 r_2 A_1 A_2 \delta_{24}(r_1, r_2) \\
- \alpha^2 \left( A_1^2 \delta_{22}(r_1) + A_2^2 \delta_{22}(r_2) + 2A_1 A_2 \delta_{25}(r_1, r_2) \right) \right) \\
\left. - \frac{i \alpha}{4} af \cdot Pr \cdot Sc \cdot Sr \cdot \frac{\lambda_{30}}{\sinh 2\gamma_1} \right\}. \]
\[ A_{12} = \frac{-1}{2 \cosh \gamma_4} \left\{ \frac{1}{2} \left[ \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{(Sc} - \text{Pr})} \right) \gamma_2 \left( 1 + \cosh 2 \gamma_2 \right) \right] \right. \\
+ \left. \left( \frac{1}{	ext{Sc} \cdot \text{Pr}} \right) \left( 1 - \frac{\text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{(Sc} - \text{Pr})} \right) \frac{\lambda_{31} \text{Sc} \cdot \text{Sr} \cdot \text{Pr}}{\text{(Sc} - \text{Pr})} \right\} \\
+ \frac{\text{Pr} \cdot \text{Sc} \cdot \text{Sr}}{\text{(Sc} - \text{Pr})} \cdot 2 A_{10} \sinh \gamma_3 - \frac{i \alpha}{4 \text{Sc} \cdot \text{Sr} \cdot \text{Pr} \cdot \text{Re} \cdot \lambda_{33} \sinh 2 \gamma_1} \\
+ \frac{i \alpha}{4 \text{af} \cdot \text{Pr} \cdot \text{Sc} \cdot \text{Sr} \cdot \lambda_{34} \sinh 2 \gamma_1} \right\}. \quad (A.1) \]

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