Sensitivity of $\beta$-decay rates to the radial dependence of the nucleon effective mass

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We analyze the sensitivity of $\beta$-decay rates in $^{78}$Ni and $^{100,132}$Sn to a correction term in Skyrme energy-density functionals (EDF) which modifies the radial shape of the nucleon effective mass. This correction is added on top of several Skyrme parametrizations which are selected from their effective mass properties and predictions about the stability properties of $^{132}$Sn. The impact of the correction on high-energy collective modes is shown to be moderate. From the comparison of the effects induced by the surface-peaked effective mass in the three doubly magic nuclei, it is found that $^{132}$Sn is largely impacted by the correction, while $^{78}$Ni and $^{100}$Sn are only moderately affected. We conclude that $\beta$-decay rates in these nuclei can be used as a test of different parts of the nuclear EDF: $^{78}$Ni and $^{100}$Sn are mostly sensitive to the particle-hole interaction through the B(GT) values, while $^{132}$Sn is sensitive to the radial shape of the effective mass. Possible improvements of these different parts could therefore be better constrained in the future.

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I. INTRODUCTION

Weak processes such as $\beta$-decay rates, electron capture, neutrino scattering and absorption play an important role during the late evolution of massive stars \cite{9}. They are largely responsible for the electron fraction in the core during the core-collapse phase and they continue to play a determinant role in the nuclear synthesis of the core during the late evolution of massive stars \cite{1}. They are largely responsible for the electron fraction in the core during the core-collapse phase and they continue to play a determinant role in the nuclear synthesis of the core during the late evolution of massive stars \cite{1}.

Since the pioneering work of Brown et al. \cite{9} it is known that the level density around the Fermi energy is stable in nuclei indicates that the in-medium nucleon effective mass is close to the bare mass. The description of giant resonances such as the giant dipole resonance requires, on the other hand, that the nucleon effective mass in the nuclear medium should be reduced as compared to its value in vacuum \cite{11}. Analysis of the momentum dependence of the nuclear optical potential also favors an in-medium effective mass lower than in vacuum \cite{11,12}.

These apparently diverging properties of the in-medium effective mass $m^*$ can be reconciled by considering the two contributions to $m^*$: the $k$-mass which is also called the non-locality mass, and the $\omega$-mass which is induced by dynamical correlations such as particle-phonon coupling \cite{13–17}. The coupling of the collective modes to the single-particle (s.p.) motion is, however, difficult to perform in a self-consistent approach. One of the main problems is coming from the fragmentation of s.p. strength which increases exponentially at each iteration of the self-consistent method. It has therefore been tried to include these correlation directly in the mean field, either at the level of the interaction with density-dependent gradient terms \cite{18}, or, loosing the relation with an interaction, at the level of the nuclear energy density functional (EDF) so as to produce a surface-peaked effective mass (SPEM) which, at the same time, does not strongly modify the mean-field \cite{8}. In this study, we will explore the second approach.

Predictions of $\beta$-decay rates throughout the nuclear chart within a consistent microscopic nuclear model are difficult. Tuning of models according to the system under study is usually performed, and the description of $\beta$-decay rates through a unique microscopic nuclear model does not exist. Since $\beta$-decay rates are known to depend strongly on the fine structure around the Fermi level, the difficulties to have a general description could be related to the common issue with mean-field models that the s.p. level density around the Fermi level is too low. The increase of the level density, by using for instance a model producing a SPEM could, in principle, lead to a better description of $\beta$-decay rates throughout the nuclear chart.

In microscopic approaches, calculations of nuclear $\beta$
-decay rates are rather complex. Due to phase-space amplification effects, the calculated $\beta$-decay rates are sensitive to both nuclear binding energies and $\beta$-strength functions. In an appropriate $\beta$-decay model, the correct amount of the integral $\beta$-strength should be placed within the properly calculated $Q_{\beta}$-window provided that the spectral distribution is also close to the “true” $\beta$-strength function. Furthermore, for consistency the model has to yield correct positions and strengths of the Gamow-Teller (GT) and first-forbidden resonances in the continuum. Another complication is related with the large-decay rates carried out consistently in the framework of Hartree-Fock-CERPA approach.

This paper is organized as follows. In Sec. II we briefly present the modifications to the nuclear EDF which produce a SPEM, and we describe the protocol used to adjust the strength of this correction. In Sec. III, we analyze the results of the calculations of $\beta$-decay rates in $^{78}$Ni and $^{100,132}$Sn and the properties of the giant quadrupole resonance (GQR) and Gamow-Teller resonance (GTR) of $^{208}$Pb. Conclusions are drawn in Sec. IV.

II. THE MEAN FIELD MODELS

Skyrme-type EDF are known to give an accurate description of masses and charge radii over the whole nuclear chart, from $Z = 8$ up to heavy elements. As most of the mean-field approaches, they however lead to a s.p. level density around the Fermi surface which is lower than the experimental one. Here, we introduce a correction term to the Skyrme EDF which leads to a SPEM and increases the average s.p. level density. We hereafter present this correction term and then briefly describe the calculations of $\beta$-decay rates carried out consistently in the framework of Hartree-Fock-CERPA approach.

A. The standard Skyrme functional

The standard Skyrme functional for the time-even energy density is expressed as

$$H_{\text{sky}}(r) = \frac{\hbar^2}{2m}r_0 + \sum_{t=0,1} C_t^0(\rho_0)\rho_t^2 + C_t^\Delta\rho_1 \Delta\rho_1 + C_t^\tau_1\rho_1 \tau_1 + \frac{1}{2} C_t^1 J_1^2 + C_t^\gamma J_1 \gamma \cdot J_1,$$

where the indices $t = 0, 1$ stand for the isoscalar and isovector part of the corresponding densities, respectively. For instance, the nucleonic densities $\rho_0$ and $\rho_1$ are defined as

$$\rho_0(r) = \rho_n(r) + \rho_p(r),$$
$$\rho_1(r) = \rho_n(r) - \rho_p(r), \quad (2)$$

where the densities $\rho_q$ ($q = n, p$) are expressed in terms of the s.p. wave functions $\phi^q_i$ as

$$\rho_q(r) = \sum_i |\phi^q_i(r)|^2. \quad (3)$$

The kinetic energy and spin-current densities, $\tau_1$ and $J_1$, are defined similarly. The coefficients $C_t^q$ in Eq. (1) are constants (see, e.g., Ref. [23]) except for the coefficient $C_t^0$ which depends on the isoscalar density $\rho_0$ as:

$$C_t^0(\rho_0) = C_t^0(0) + (C_t^0(\rho_{0,\text{sat}}) - C_t^0(0)) \left(\frac{\rho_0}{\rho_{0,\text{sat}}}\right)^\alpha, \quad (4)$$

where $\rho_{0,\text{sat}}$ is the saturation density in infinite nuclear matter.

The standard Skyrme functional can be separated into neutron and proton channels, and neutron and proton effective masses are introduced:

$$m \quad m^* = 1 + \frac{2m}{\hbar^2} \left[(C_0^0 + C_1^0)\rho_q + (C_0^0 - C_1^0)\rho_q\right], \quad (5)$$

Then, neutron and proton mean fields can be obtained (see appendix A).

Among the large number of Skyrme parametrizations, we have selected 6 of them based on the following requirements:

- First, the Skyrme EDF should predict $^{132}$Sn as a $\beta$-unstable nucleus at the mean field level. This is based on the common expectation that the Landau parameter $G_0^\tau$ in the spin-isospin channel is repulsive and will shift up the GT strength.

- Second, we wish to explore different values of effective mass in the bulk, and different isospin splittings of the effective mass.

In $^{132}$Sn the first condition can be related to the s.p. energy difference between the $\pi 2d_3^2$ and $\pi 2d_5^2$ states (which contribute mostly to the GT transition towards
the $1^+$ state of $^{132}$Sb). The lowest unperturbed transition energy is $\varepsilon_{2d_{5/2}} - \varepsilon_{2d_{3/2}} = \Delta M_{n-H}$, where the last term stands for the mass difference between the neutron and the hydrogen atom, $\Delta M_{n-H} = 0.782$ MeV. If this transition energy is positive at the mean field level - hereafter called the HF transition energy - the system is stable since the CERPA correlations could only push it up, while it is expected to be actually $\beta$-unstable. Anticipating the discussion of the results in Sec. III we observe that models having positive HF transition energies predict $\beta$-decay half-lives which are too large in $^{132}$Sn. We therefore consider only models having a HF energy difference $\varepsilon_{2d_{5/2}} - \varepsilon_{2d_{3/2}} < 0$. This condition is indeed quite drastic, and we found that an appreciable number of well established Skyrme models do not fulfill it. Among these are SIII [24], BSK14-17 [25], SKM$^*$ [26], SLy4-5 [27], SKO [28]. In addition, the models which predict that the HF transition energy is larger than 0.782 MeV are: RATP [24], SGII [27], LNS [28], LNS1, LNS5 [29], SKI1-5 [30], SAMI [31]. These models have therefore not been used here.

For the few remaining models, we restrict ourselves to the parametrizations SLyIII0.7, SLyIII0.8 and SLyIII0.9 [32] which predict in the bulk nuclear matter the effective mass values $m^*/m = 0.7, 0.8$ and 0.9, respectively. We have also considered the f–, f0 and f+ [33] models which predict an effective mass of 0.7 in symmetric matter, with either a positive, zero, or negative isospin splitting of the effective mass (ISEM) in neutron matter, defined as $m^*_n/m - m^*_p/m$. Notice that $m^*_n/m$ could be calculated in neutron matter without any ambiguity: the proton density shall simply be set to zero, see for instance Eq. 4.

The bulk properties of the selected interactions are given in Table I. It is observed that the saturation density $\rho_0$, the energy per particle at saturation $E_0$, and the symmetry energy $J_{sym}$ are very similar for these interactions. The slope of the symmetry energy $L_{sym}$ is varying between 24.75 and 43.79 MeV, which is a rather wide range, but these models are still considered as iso-soft ones. The incompressibility of SLyIII0.7, SLyIII0.8 and SLyIII0.9 is quite large. However, this does not affect the processes explored in this work. The main difference among these models comes from their effective masses and ISEM. The models SLyIII0.7, SLyIII0.8, SLyIII0.9 have a different effective mass in symmetric matter, and a positive ISEM. The models $f_-, f_0$ and $f_+$ have the same effective mass in symmetric matter and different signs for the ISEM. The models SLyIII0.7, SLyIII0.8, SLyIII0.9 and $f_+$ have a positive ISEM, as expected from microscopic BHF and DBHF calculations [34], while $f_0$ has no splitting and $f_-$ has a negative ISEM. Finally, the values of the Landau parameter for the models SLyIII0.7, SLyIII0.8 and SLyIII0.9 is given by

$$G'_0 = -N_0 \left[ \frac{1}{4} t_0 + \frac{1}{24} t_3 \rho \alpha^3 + \frac{1}{8} k_F^2 (t_1 - t_2) \right]$$

(6)

and for the models $f_-, f_0$ and $f_+$,

$$G'_0 = -N_0 \left[ \frac{1}{4} t_0 + \frac{1}{4} t_3 \rho \alpha^3 + \frac{1}{4} t_4 \rho \alpha^4 + \frac{1}{8} k_F^2 (t_1 - t_2) \right]$$

(7)

where $N_0 = 2k_F m^*/\pi^2 a^2$ is the level density, with $k_F$ being the Fermi momentum and $m^*$ the nucleon effective mass. In Eq. 7, the parameter $t_4$ is coming from an additional density-dependent term besides the usual density-dependent $t_3$ term, and the coefficient in front of the density-dependent term has been modified w.r.t. the standard notations [33]. The values of the Landau parameter $G'_0$ are given in the last column of Table I. At saturation density ($\rho = \rho_0$), the models SLyIII0.7, SLyIII0.8, SLyIII0.9 predict rather large values for $G'_0 \approx 0.3 - 0.35$, while the models $f_-, f_0$ and $f_+$ predict smaller value with $G'_0 \approx 0$. The forces $f_-, f_0$ and $f_+$ clearly predict not enough positive $G'_0$ values [19]. In addition to the different effective masses we therefore expect to observe substantial differences between these two sets of models in the charge-exchange channel.

### B. Surface-peaked effective mass correction

In Ref. [8], a surface-peaked effective mass correction to the Skyrme-type Hamiltonian was proposed with the following form,

$$\Delta \mathcal{H} = C_0 (\nabla \rho)^2 \tau (\nabla \rho)^2 + C_0^2 (\nabla \rho)^2 \rho^2 (\nabla \rho)^2,$$

(8)
and the new functional can be written as \( \mathcal{H} = \mathcal{H}_{sby} + \Delta \mathcal{H} \).

The first term of Eq. (8) is designed to modify the effective mass profile at the nuclear surface, while the second term is introduced in order to compensate the effects of the first term in the nuclear mean-field. Without the second term, the effects of the first term on the mean field are too large and drastically limit the possible values for the strength of the SPEM, as in Ref. [8].

The compensation was found to be optimal for intermediate mass and heavy nuclei if one uses the following constant relation between the two new parameters [8]:

\[
C_0^{\sigma^2(\nabla \rho)^2} = -10 \text{ fm } C_0^{(\nabla \rho)^2}. \tag{9}
\]

One can expect an impact of the SPEM on the properties of the lowest quadrupole excitation if the isoscalar terms [8] are taken into account. On the other hand, the energy-weighted sum rule (EWSR) is an integral characteristic and it is particularly sensitive to the giant-resonance properties which can be described by the EDF without the terms [8].

In the present work, the values of \( C_0^{(\nabla \rho)^2} = -210 \) and \(-420\) MeV fm\(^{10}\) are fixed so that the isoscalar quadrupole EWSR in \(^{208}\)Pb is modified by 1\% and 2\%, respectively. Consequently, a change of less than 0.04 of the neutron and proton effective masses at the nuclear surface of \(^{208}\)Pb is predicted, see Fig. 1. This procedure is slightly different from that used in Ref. [8], and it leads to a SPEM less strongly peaked at the surface. We have added the terms [8] without refitting the existing standard parameterizations.

Using this perturbative approach, we observe a small change of the binding energies which is larger than the tolerance of the protocol for the parameter fitting. In particular, in \(^{208}\)Pb the binding energy changes by 0.35\% for the SLyIII0.9 set, 0.37\% for the SLyIII0.8 set, 0.38\% for the SLyIII0.7 set and 0.45\% for the \( f_0, f_-, \) and \( f_+ \) sets. A fine tuning of other parameters in order to compensate for these energy changes has still to be done.

In Fig. 1 are shown the effective mass profiles in \(^{208}\)Pb for the \( f_0, f_- \) and \( f_+ \) models where we have considered different values of the parameter governing the strength of the SPEM, \( C_0^{(\nabla \rho)^2} = 0, -210 \) and \(-420\) MeV fm\(^{10}\). We remind that the differences between the models \( f_0, f_- \) and \( f_+ \) are mostly the ISEM in asymmetric matter: \( f_+ \) has \( m^*_n > m^*_p \) in neutron rich matter, while \( f_- \) has \( m^*_n < m^*_p \), and \( f_0 \) has \( m^*_n = m^*_p \) in the same conditions.
of isospin asymmetry. The effect of the sign difference of the effective mass splitting can also be observed on panels (a) and (d) without SPEM: Since $^{208}_{\text{Pb}}$ is a neutron rich nucleus, the neutron effective mass is larger than the proton one for the $f_+\sigma$ model, an opposite effect is found for $f_-$, and no effect is observed for $f_0$. Additionally, it is observed in panels (b), (c), (e) and (f) that the SPEM correction is almost unaffected by the effective mass splitting, since the correction is isoscalar.

C. Calculations of $\beta$-decay rates

We describe the collective modes in the charge-exchange random phase approximation (CERPA) using the same finite-rank interactions as above. Making use of the finite-rank separable approximation (FRSA) \[36–38\] for the p-h interaction enables us to perform CERPA calculations in very large configuration spaces. Although it is well known that the tensor interaction influences also the description of the $\beta^-$-decay half-lives \[39\], in the present study the tensor force is neglected in order to focus on the impact of the SPEM.

The experimentally known values of the half-lives put an indirect constraint on the calculated GT strength distributions within the $Q_\beta$-window. To calculate the half-lives an approximation worked out in Ref. \[40\] is used. It allows one to avoid an implicit calculation of the nuclear masses and $Q_\beta$-values. However, one should realize that the related uncertainty in constraining the parent nucleus ground state calculated with the chosen Skyrme interaction is transferred to the values of the neutron and proton chemical potentials. In the allowed GT approximation, the $\beta^\pm$-decay rate is expressed by summing the probabilities of the energetically allowed transitions (in units of $G_A^2/4\pi$) weighted with the integrated Fermi function. For the $\beta^-$-decay case we have:

$$T_{1/2}^{\beta^-} = \frac{D}{\left(\frac{G_A}{G_V}\right)^2 \sum_k f_{0}(Z+1, A, E_i - E_{1+}^k) B(GT)^k},$$

and for the $\beta^+$-decay case this becomes:

$$T_{1/2}^{\beta^+} = \frac{D}{\left(\frac{G_A}{G_V}\right)^2 \sum_k f_{0}(-Z+1, A, E_i - E_{1+}^k) B(GT)^k},$$

while $E_i - E_{1+}^k \approx \Delta M_{n-H} + \mu_n - \mu_p - E_k$. (10)

Expressions (10)-(14) will be used in the next section to calculate the $\beta$-decay rates and the collective modes. All the calculations are performed without any quenching factor.

III. RESULTS FOR COLLECTIVE MODES AND $\beta$-DECAY RATES

We now analyze first the results of the $\beta$-decay rates which are sensitive to the low-energy part of the CERPA strength, and then the GT collective modes. The effects of the SPEM will be discussed.

The p-h interaction in the spin-isospin channel is assumed of the following form:

$$V(r_1, r_2) = N_0^{-1} G_0(r_1) \sigma^{(1)}(\mathbf{r}) \cdot \sigma^{(2)}(\mathbf{r}) \cdot \tau^{(1)}(\mathbf{r}) \cdot \tau^{(2)}(\mathbf{r}) \delta(r_1 - r_2),$$

where $\sigma^{(i)}$ and $\tau^{(i)}$ are the spin and isospin operators. As expected, the largest contribution to the calculated

| Skyrme    | $C_0^{(\tau \rho)^2}$ | $^{132}_{\text{Sn}}$ | $^{190}_{\text{Sn}}$ | $^{78}_{\text{Ni}}$ |
|-----------|------------------------|----------------------|----------------------|---------------------|
| SLyIII0.7 | 0                      | 0.3                  | -7.2                 | -5.1                |
| SLyIII0.7 | -210                   | 0.2                  | -7.3                 | -5.2                |
| SLyIII0.7 | -420                   | 0.1                  | -7.3                 | -5.3                |
| SLyIII0.8 | 0                      | -0.6                 | -7.5                 | -6.2                |
| SLyIII0.8 | -210                   | -0.7                 | -7.5                 | -6.3                |
| SLyIII0.8 | -420                   | -0.9                 | -7.6                 | -6.4                |
| SLyIII0.9 | 0                      | -1.3                 | -7.7                 | -7.0                |
| SLyIII0.9 | -210                   | -1.4                 | -7.7                 | -7.2                |
| SLyIII0.9 | -420                   | -1.6                 | -7.8                 | -7.3                |

Table II: Energy differences between the dominant s.p. states in $^{132}_{\text{Sn}}$ and $^{190}_{\text{Sn}}$. For each Skyrme parameterization, the energy difference is calculated with the surface peaked term or without ($C_0^{\tau \rho \rho \rho} = 0$). See text for more details.

β\(^{-}\)-decay half-life comes from the 1\(^{+}\) state, the structure of which is dominated by one unperturbed configuration. They are the 1p-1h configurations \{π2d\(^{2}\), ν2d\(^{2}\}\}, \{ν1g\(^{2}\), π1g\(^{2}\}\} and \{π2p\(^{2}\), ν2p\(^{2}\}\} of 132\(^{\text{Sn}}\), 100\(^{\text{Sn}}\) and 78\(^{\text{Ni}}\), respectively. In other words, the 1\(^{+}\) state is non-collective and, therefore, the β-decay is related to the lowest unperturbed \(^{\text{I}}\)\(^{+}\) energy. We first examine the s.p. energy differences given in Table III for the selected Skyrme models and for various strength of the SPEM parameter \(C_{0}^{\text{(νν)}}\). They are small (about 1 MeV) in 132\(^{\text{Sn}}\) but rather large in 78\(^{\text{Ni}}\) and 100\(^{\text{Sn}}\) (5 to 8 MeV). In 100\(^{\text{Sn}}\), the energy differences without SPEM are mostly sensitive to the Coulomb component of the EDF, with a small additional effect due to the effective mass (the larger effective mass, the smaller the energy difference). In 132\(^{\text{Sn}}\), the energy difference is related mostly to the symmetry energy: the larger the symmetry energy, going from SLyIII0.9 to SLyIII0.7 for instance, the larger the energy difference. In addition, the increase of the effective mass also contributes, with a smaller impact, to the decrease the energy difference, as it can be deduced from the comparison of the energy difference for the forces \(f_{-}\), \(f_{0}\), and \(f_{+}\) which has increasing effective mass in neutron rich matter, see Fig. I. It can be seen (cf. Table III) that the shifts in the energy differences between the cases without, and with maximal SPEM \((C_{0}^{\text{(νν)}}=-420\text{ MeV fm}^{10})\) are almost constant and independent of the models considered. It varies by about 0.3 MeV in 132\(^{\text{Sn}}\) and in 78\(^{\text{Ni}}\). From Table II, we can anticipate that the SPEM will have a larger impact on the calculation of the β half-life of 132\(^{\text{Sn}}\) and a weaker one in the case of 78\(^{\text{Ni}}\) and 100\(^{\text{Sn}}\). In 132\(^{\text{Sn}}\), the experimental value is -1.305 MeV [42]. No data exist for 78\(^{\text{Ni}}\) and 100\(^{\text{Sn}}\).

The \(E_{i} - E_{i+}\) energies, the \(B(GT)\)\(^{-}\) values and β\(^{-}\)-decay half-lives of 132\(^{\text{Sn}}\) and 78\(^{\text{Ni}}\) are given in Tables III and IV the β\(^{+}\)-decay properties of 100\(^{\text{Sn}}\) in Table V. The evolution of the transition energies and the \(B(GT)\)\(^{-}\) values is reflected in the half-life behaviour, see Eqs.(10) and (12). As in Table III the results shown in Tables III, IV and V correspond to the selected interactions with and without the SPEM represented by the value of the parameter \(C_{0}^{\text{(νν)}}\). In the case of 132\(^{\text{Sn}}\), the model SLyIII0.7 predicts positive energy differences for the dominant transition of the β-decay half-lives (see Table III), and it leads to half-lives which are much larger than the experimental value, as anticipated.

One can see from Tables III, IV and V that the β-decay half-lives are much more sensitive to the effective mass distribution in the case of the low-Q\(_{β}\) nucleus 132\(^{\text{Sn}}\) than in 100\(^{\text{Sn}}\) and 78\(^{\text{Ni}}\). For 132\(^{\text{Sn}}\), a strong decrease of the half-life can be directly correlated to either the increase of the effective mass in symmetric matter, or to the increase of the SPEM, while in 78\(^{\text{Ni}}\) and 100\(^{\text{Sn}}\), the correlation, while still being present, is much less pronounced. This can be easily understood from the energy difference of the most important transition given in Table III. The energy differences are much smaller in the case of 132\(^{\text{Sn}}\) than

### Table III: SPEM effects on β\(^{-}\)-decay properties of 132\(^{\text{Sn}}\).

| Skyrme | \(C_{0}^{\text{(νν)}}\) (MeV fm\(^{10}\)) | \(E_{i} - E_{i+}\) (MeV) | \(B(GT)\)\(^{-}\) | \(T_{1/2}\) (s) |
|--------|---------------------------------|----------------|-----------------|----------------|
| SLyIII0.7 | 0                         | 0.07             | 2.6             | 389400         |
| SLyIII0.7 -210 | 0.21                      | 2.7              | 9840            |
| SLyIII0.7 -420 | 0.34                      | 2.7              | 1930            |
| SLyIII0.8 | 0                         | 0.97             | 2.5             | 57             |
| SLyIII0.8 -210 | 1.11                      | 2.5              | 33              |
| SLyIII0.8 -420 | 1.26                      | 2.6              | 21              |
| SLyIII0.9 | 0                         | 1.70             | 2.4             | 6.7            |
| SLyIII0.9 -210 | 1.84                      | 2.5              | 4.7             |
| SLyIII0.9 -420 | 2.01                      | 2.6              | 3.3             |

### Table IV: SPEM effects on β\(^{-}\)-decay properties of 78\(^{\text{Ni}}\).

| Skyrme | \(C_{0}^{\text{(νν)}}\) (MeV fm\(^{10}\)) | \(E_{i} - E_{i+}\) (MeV) | \(B(GT)\)\(^{-}\) | \(T_{1/2}\) (s) |
|--------|---------------------------------|----------------|-----------------|----------------|
| SLyIII0.7 | 0                         | 5.49             | 1.0             | 0.157          |
| SLyIII0.7 -210 | 5.61                      | 1.1              | 0.140           |
| SLyIII0.7 -420 | 5.74                      | 1.1              | 0.121           |
| SLyIII0.8 | 0                         | 6.60             | 1.0             | 0.057          |
| SLyIII0.8 -210 | 6.73                      | 1.0              | 0.051           |
| SLyIII0.8 -420 | 6.87                      | 1.0              | 0.045           |
| SLyIII0.9 | 0                         | 7.48             | 1.0             | 0.025          |
| SLyIII0.9 -210 | 7.61                      | 1.0              | 0.023           |
| SLyIII0.9 -420 | 7.79                      | 1.0              | 0.020           |

Expt. 1.794±0.009  39.7±0.8
in the case of $^{100}$Sn and $^{78}$Ni, which makes the $\beta^-$-decay half-lives more sensitive to a small modification of the s.p. energies induced by the SPEM.

Let us examine whether the SPEM could improve the agreement between the model predictions and the experimental values. As one can see from Tables III, IV and V, the inclusion of the terms \( \nabla \) leads to minor effects on the \( B(GT)_{1/2}^+ \) values. We find that the SPEM induces an increase of the transition energies and it results in a decrease of the half-lives. We first concentrate on the models SLyIII0.7, SLyIII0.8 and SLyIII0.9, which correspond to different values of the effective mass in symmetric matter. From the comparison of the theoretical predictions with the experimental half-lives shown in Table I, it is difficult to conclude which model is better: For $^{132}$Sn, the model SLyIII0.8 is preferred, for $^{78}$Ni and $^{100}$Sn, it is SLyIII0.7. Now, if we concentrate on the models \( f_+ \), \( f_0 \) and \( f_- \), it is \( f_+ \) which always comes the closest to the experimental value. This indicates that, in addition to the effective mass, the residual interaction is very important. It was already anticipated that the value of the Landau parameter \( G' \), for the selected models (cf. Table I), could have an impact on charge-exchange related observables. For the models \( f_+ \), \( f_0 \) and \( f_- \), the values of \( G' \) are too small. Since the impact of the SPEM on the $\beta^-$-decay rates in $^{78}$Ni and $^{100}$Sn is quite small, these nuclei could be, in the future, used to calibrate the residual interaction almost independently from the profile of the

| Skyrme     | \( C_0^{s(p)} \) (MeV fm\(^{10}\)) | \( E_\beta - E_{1/2}^{GT} \) (MeV) | \( B(E^2; 0^+ \rightarrow 2^+_{1/2}) \) (e\(^2\) fm\(^4\)) | \( T_{1/2} \) (s) |
|------------|----------------------------------|----------------------------------|---------------------------------|-----------------|
| SLyIII0.7  | 0                                | 4.26                             | 15.2                            | 0.232           |
| SLyIII0.7  | -210                             | 4.38                             | 15.2                            | 0.221           |
| SLyIII0.7  | -420                             | 4.42                             | 15.2                            | 0.213           |
| SLyIII0.8  | 0                                | 4.60                             | 15.1                            | 0.178           |
| SLyIII0.8  | -210                             | 4.64                             | 15.1                            | 0.172           |
| SLyIII0.8  | -420                             | 4.67                             | 15.0                            | 0.167           |
| SLyIII0.9  | 0                                | 4.86                             | 15.1                            | 0.138           |
| SLyIII0.9  | -210                             | 4.90                             | 15.0                            | 0.134           |
| SLyIII0.9  | -420                             | 4.92                             | 15.0                            | 0.131           |

Expt. 3.08±0.34 1.16±0.20

| Skyrme     | \( C_0^{s(p)} \) (MeV fm\(^{10}\)) | Energy (MeV) | \( B(E^2; 0^+_{1/2} \rightarrow 2^+_{1/2}) \) (e\(^2\) fm\(^4\)) |
|------------|----------------------------------|--------------|---------------------------------|
| \( f_+ \)  | 0                                | 5.12         | 3130                            |
| \( f_+ \)  | -210                             | 5.09         | 2530                            |
| \( f_+ \)  | -420                             | 5.09         | 2180                            |
| \( f_0 \)  | 0                                | 5.13         | 3250                            |
| \( f_0 \)  | -210                             | 5.09         | 2650                            |
| \( f_0 \)  | -420                             | 5.09         | 2310                            |
| \( f_- \)  | 0                                | 5.09         | 3440                            |
| \( f_- \)  | -210                             | 5.06         | 2850                            |
| \( f_- \)  | -420                             | 5.06         | 2500                            |

Expt. 4.09 3000±300

FIG. 2: (Color online) The quadrupole strength distribution of $^{208}$Pb. Solid, dashed and dotted lines correspond to RPA calculations with \( f_0 \), \( f_- \) and \( f_+ \) models, respectively. The experimental centroid of the GQR is at 10.89±0.30 MeV [46].

TABLE V: SPEM effects on $\beta^+$-decay properties of $^{100}$Sn. Data are from Ref. [13].

TABLE VI: SPEM effects on the energy and $B(E2)$-value for the up-transition to the first $2^+$ state in $^{208}$Pb. Data are from Ref. [13].
effective mass. The modification of $G_0^i$ could be obtained from a refitting of the Skyrme functional with a different value of the strength of the SPEM. One could increase $G_0^i$ by about 0.1-0.2 by introducing the spin-density dependent extension of the Skyrme model. This will be left for future investigations.

Up to this point, we have mostly focused on the relation between the SPEM and the low energy part of the strength, since it represents the main contribution to the $\beta$-decay rates. We now turn to the higher energy part and show in Figs. 2 and 3 the effects of the SPEM on the properties of the GQR and GTR in $^{208}$Pb. In the figures, the calculated strength distributions are folded out with a Lorentzian distribution of 1 MeV width. The excitation energies refer to the ground state of the parent nucleus $^{208}$Pb. The arrows indicate the maxima of the strength distributions corresponding to the case of the $f_+$ model and $C_0^{\nu} = 0$ MeV fm$^{10}$. Since the isoscalar quadrupole EWSR is changed by only about 1% by the SPEM, we expect the collective modes at higher energy to be only marginally impacted.

The GQR strength distribution consists mostly of a main peak. Comparing the cases without SPEM and with maximal SPEM, we find that the peak is shifted down by about 500 keV. One can notice that the GQR strength distribution is almost identical for the three models $f_0$, $f_+$ and $f_-$. As can be seen from Table VI, while the $2^+_1$ energy is practically unaffected by the SPEM, the $B(E2)$ value is decreasing as the SPEM increases. Some overestimate of the experimental energy indicates that there is room for the two-phonon effects.

The GTR is much more fragmented than the GQR, as seen in Fig. 3. The strength distribution is globally shifted up as the isospin splitting is going from positive ($f_+$) to negative values ($f_-$). As in the case of the GQR, the high energy peaks of the strength distribution are shifted to lower energies (by about 500 keV) as the SPEM gets larger. This is an effect of the slight increase of the level density induced by the SPEM.

IV. CONCLUSIONS

Starting from different Skyrme EDF which predict $^{132}$Sn $\beta$-unstable, we have studied the effects of introducing a surface-peaked effective mass on top of existing Skyrme models. The main effect of this additional term is a compression of the s.p. level spacing around the Fermi level, or equivalently, an increase of the level density. This systematically increases the $\beta$-decay rates (i.e., decreases the half-lives). The collective modes at higher energy are only slightly impacted by the SPEM.

This work is a first step towards improving Skyrme functionals by adding extra terms to the energy functional. Our motivation is based on both having a better agreement with nuclear data, and also predicting weak transition rates for astrophysical applications. The results of our analysis allow for a better understanding of the effects at play. The $\beta$-decay rates in doubly-magic unstable nuclei ($^{100,132}$Sn, $^{78}$Ni) are indeed very sensitive both to the s.p. energies and residual interactions, and none of the Skyrme models selected in this work are fully satisfactory in this respect. From our analysis, we have however identified two nuclei ($^{100,132}$Sn, $^{78}$Ni) where the $\beta$-decay half-lives are only weakly impacted by the SPEM. They can be considered as good benchmark nuclei since they potentially offer the possibility to calibrate the residual interaction, with a weak influence of the effective mass. In a complementary approach, $^{132}$Sn could be used to test different strengths of the SPEM, for a fixed residual interaction.

The tensor force has not been considered in this work, although it can affect the neutron-proton s.p. energies in some cases. We have aimed at understanding just the contribution of the SPEM to the $\beta$-decay and GT mode in order to disentangle the respective roles of the effective mass and the residual interaction. An additional
modification of the Skyrme functional was proposed earlier in order to stabilize the nuclear matter equation of state \cite{40,51}. It has been recently used in nuclei and, since it brings an additional repulsive term to the $G_0$ Landau parameter, it was shown to shift the centroids of the GT collective mode to higher energies by a few hundreds keV up to 1 MeV \cite{52}. In the future, we plan to explore the predictions of a general mean field model including all these ingredients, and to compare them to known experimental data, as done in this work. These calibration processes are important to set-up boundaries for the additional parameters before making predictions for astrophysical cases.

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Appendix A: Decomposition of the Skyrme functional into neutron and proton channels

Here, the Skyrme functional is expressed in terms of the neutron and proton densities instead of the isoscalar and isovector densities,

$$H_{\text{sky}}(r) = \sum_{q=n,p} h_q^p + h_q^\n + h_q^J,$$  \hspace{1cm} (A1)

where the different terms of the energy density are

$$h_q^p = \frac{\hbar^2}{2m} f_q^{\text{Sky}} \tau_q + (C_0^p + C_1^p) \rho_q^2 + (C_0^p - C_1^p) \rho_q \bar{q},$$ \hspace{1cm} (A2)

$$h_q^\n = - (C_0^{\Delta n} + C_1^{\Delta n}) (\nabla \rho_q)^2 - (C_0^{\Delta n} - C_1^{\Delta n}) \nabla \rho_q \cdot \nabla \bar{q},$$ \hspace{1cm} (A3)

$$h_q^J = \frac{1}{2} (C_0^J + C_1^J) J_q^2 + \frac{1}{2} (C_0^J - C_1^J) J_q J_{\bar{q}} - [(C_0^{\nabla J} + C_1^{\nabla J}) \nabla \rho_q + (C_0^{\nabla J} - C_1^{\nabla J}) \nabla \bar{q}] \cdot J_q,$$ \hspace{1cm} (A4)

and the effective mass factor $f_q^{\text{Sky}} = m/m_q^*$. is defined as

$$f_q^{\text{Sky}} = 1 \pm \frac{2m}{\hbar^2} \left[ (C_0^\tau + C_1^\tau) \rho_q + (C_0^\tau - C_1^\tau) \bar{q} \right].$$ \hspace{1cm} (A5)

By functional derivation the one-body Hamiltonian $H_q$ is obtained as,

$$H_q = - \frac{\hbar^2}{2m} \nabla \cdot f_q^{\text{Sky}}(r) \nabla + V_q(r) \pm \frac{i}{2} \sum_{\sigma'} [W_q \cdot (\nabla \times \langle \sigma | \sigma' \sigma' \rangle) + (\nabla \times \langle \sigma | \sigma' \sigma' \rangle) \cdot W_q]$$ \hspace{1cm} (A6)

where the central potential is given by

$$V_q^{\text{Sky}}(r) = V_q^p(r) + V_q^\n(r) + V_q^J(r).$$ \hspace{1cm} (A7)

Here, the central-density potential is given by:

$$V_q^p(r) = \tau_q + (C_0^\tau - C_1^\tau) \rho_q$$

$$+ 2 [ (C_0^\tau + C_1^\tau) \rho_q + (C_0^\tau - C_1^\tau) \bar{q} ]$$

$$+ \frac{\partial}{\partial \rho_0} (C_0^\tau + C_1^\tau) \rho_q^2 + \frac{\partial}{\partial \rho_0} (C_0^\tau - C_1^\tau) \rho_q \bar{q}],$$ \hspace{1cm} (A8)

the central-gradient potential by:

$$V_q^{\n}(r) = 2 (C_0^{\Delta n} + C_1^{\Delta n}) \nabla \rho_q^2 + 2 (C_0^{\Delta n} - C_1^{\Delta n}) \nabla \bar{q}^2,$$ \hspace{1cm} (A9)

and the central-$J$ potential by:

$$V_q^J(r) = (C_0^{\nabla J} + C_1^{\nabla J}) \nabla \cdot J_q + (C_0^{\nabla J} - C_1^{\nabla J}) \nabla \cdot \bar{J}_q.$$ \hspace{1cm} (A10)

The spin-orbit potential is:

$$V_q^{\text{SO}}(r) = - (C_0^{\nabla J} + C_1^{\nabla J}) \nabla \rho_q - (C_0^{\nabla J} - C_1^{\nabla J}) \nabla \bar{q}$$

$$+ (C_0^J + C_1^J) J_q - (C_0^J - C_1^J) J_{\bar{q}}.$$ \hspace{1cm} (A11)

Appendix B: Modification of the mean-field equations induced by the SPEM

The kinetic energy correction induced by the effective mass in Eq. (A2) is now given by $f_q = f_q^{\text{Sky}} + f_q^{\text{cor}}$ where

$$f_q^{\text{cor}} = \frac{2m}{\hbar^2} C_0^{\tau(\nabla \rho)^2} (\nabla \rho(r))^2,$$ \hspace{1cm} (B1)

and the mean field central potential \cite{A7} reads:

$$V_q(r) = V_q^{\text{Sky}}(r) + V_q^{\text{cor}}(r),$$ \hspace{1cm} (B2)

where $V_q^{\text{Sky}}(r)$ is the mean field deduced from the Skyrme interaction, e.g. Eq. \cite{A7}, and $V_q^{\text{cor}}(r)$ is the correction term induced by Eq. \cite{A8}:

$$V_q^{\text{cor}}(r) = - 2 C_0^{\tau(\nabla \rho)^2} \left( \tau(r) \nabla \rho(r) + \nabla \tau(r) \nabla \rho(r) \right)$$

$$- 2 C_0^{\rho^2(\nabla \rho)^2} \left( \rho(r)(\nabla \rho(r))^2 + \rho(r)^2 \nabla^2 \rho(r) \right).$$ \hspace{1cm} (B3)
[52] P. Wen, L.-G. Cao, J. Margueron and H. Sagawa, Phys. Rev. C 89, 044311 (2014)