Dimension two gluon condensates in a variety of gauges and a gauge invariant Yang-Mills action with a mass

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We give a short overview of our work concerning the dimension two operator $A_2$ in the Landau gauge and its generalizations to other gauges. We conclude by discussing recent work that leads to a renormalizable gauge invariant action containing a mass parameter, based on the operator $F_{\mu\nu}F^{\mu\nu}$.

1. Introduction

Recent years have witnessed a great deal of interest in the possible existence of mass dimension two condensates in gauge theories, see for example [1,2,3,4,5,6,7,8,9] and references therein for approaches based on phenomenology, operator product expansion, lattice simulations, an effective potential and the string perspective. There is special interest in the operator

$$A_{2\min} = \langle V T \rangle^{-1} \min_{U \in SU(N)} \int d^4x \; (A^U)^2, \quad (1)$$

since it is gauge invariant due to the minimization along the gauge orbit. It should be mentioned that obtaining the global minimum is delicate due to the problem of gauge (Gribov) ambiguities [10,11]. As is well known, local gauge invariant dimension two operators do not exist in Yang-Mills gauge theories. The nonlocality of (1) is best seen when it is expressed as [12]

$$A_{2\min} = \frac{1}{VT} \int d^4x \left[ A^a_{\mu} \left( \delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A^a_{\nu} \right] \quad + \ldots. \quad (2)$$

The relevance of the condensate $\langle A_2 \rangle_{\min}$ was discussed in [12], where it was shown that it can serve as a measure for the monopole condensation in the case of compact QED.

2. Measurement of $\langle A_2 \rangle_{\min}$

All efforts so far concentrated on the Landau gauge $\partial A = 0$. The preference for this particular gauge fixing is obvious since the nonlocal expression (2) reduces to a local operator, more precisely

$$\partial A = 0 \Rightarrow A_{2\min} = A^2. \quad (3)$$

In the case of a local operator, the Operator Product Expansion (OPE) becomes applicable, and consequently a measurement of the soft (infrared) part $\langle A_2 \rangle_{\text{OPE}}$ becomes possible. Such an approach was followed in e.g. [5] by analyzing the appearance of $\frac{1}{q^2}$ power corrections in (gauge variant) quantities like the gluon propagator or strong coupling constant, defined in a particular way, from lattice simulations. Let us mention that already two decades ago attention was paid to the condensate $\langle A^2 \rangle$ in the OPE context [13].
The condensate \( \langle A^2 \rangle_{\text{OPE}} \) can be related to an effective gluon mass, see e.g. [11]. Effective gluon masses have found application in some phenomenological studies. Also lattice simulations of the gluon propagator revealed the need for massive parameters [14,15,16].

A more direct approach for a determination of \( \langle A^2 \rangle \) in the Landau gauge was presented in [3]. A meaningful effective potential for the condensation of the Local Composite Operator (LCO) \( A^2 \) was constructed by means of the LCO method. This is a nontrivial task due to the composite nature of the considered operator. We consider pure Euclidean SU(N) Yang-Mills theories with action

\[
S_{YM} = \int d^4 x \left( \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + S_{gf} \right),
\]

\[
S_{gf} = \int d^4 x \left( b^a \partial_\mu A_\mu^a + \epsilon^{abc} \partial_\mu D_\mu^b e_c \right). \tag{4}
\]

We couple the operator \( A^2 \) to the Yang-Mills action by means of a source \( J \),

\[
S_J = S_{YM} + \int d^4 x \left( \frac{1}{2} J A_\mu^a A^\mu_a - \frac{1}{2} \zeta J^2 \right). \tag{5}
\]

The last term, quadratic in the source \( J \), is necessary to kill the divergences in vacuum correlators like \( \langle A^2(x)A^2(y) \rangle \) for \( x \to y \), or equivalently in the generating functional \( W(J) \), defined as

\[
e^{-W(J)} = \int [\text{fields}] e^{-\int d^4 x S_J}. \tag{6}
\]

The presence of the LCO parameter \( \zeta \) ensures a homogenous renormalization group equation for \( W(J) \). Its arbitrariness can be overcome by making it a function \( \zeta(g^2) \) of the strong coupling constant \( g^2 \), allowing one to fix \( \zeta(g^2) \) order by order in perturbation theory in accordance with the renormalization group equation.

In order to recover an energy interpretation, the term \( \propto J^2 \) can be removed by employing a Hubbard-Stratonovich transformation

\[
1 = \int \sigma e^{-\frac{1}{g^2}(\frac{\sigma}{\zeta} + \frac{1}{2} A^2 - \zeta J)^2}, \tag{7}
\]

leading to the action

\[
S = S_{YM} + S_\sigma,
\]

\[
S_\sigma = \int d^4 x \left( \frac{\sigma^2}{2g^2\zeta} + \frac{1}{2g^2\zeta} \sigma A^2 + \frac{1}{8\zeta} (A^2)^2 \right). \tag{8}
\]

A key ingredient for the LCO method is the renormalizability of the operator \( A^2 \). It was proven in [6] that \( A^2 \) is renormalizable to all orders of perturbation theory, making use of the Ward identities.

Starting from (8) it is possible to calculate the effective potential \( V(\sigma) \). The correspondence \( \langle \sigma \rangle = -g \langle A^2 \rangle \) consequently provides evidence for a nonvanishing dimension two gluon condensate using an effective potential approach, if \( \langle \sigma \rangle \neq 0 \). It is clear from (8) that \( \langle \sigma \rangle \neq 0 \) induces an effective gluon mass. \( V(\sigma) \) was calculated to two loop order in [3,17], and a nonvanishing condensate is favoured as it lowers the vacuum energy. The ensuing effective gluon mass was a few hundred MeV.

Before ending this section, we want to stress that the value \( \langle A^2 \rangle_{\text{LCO}} \) has no clear connection with \( \langle A^2 \rangle_{\text{OPE}} \). The former one is derived from an effective potential calculated in perturbation theory, thus a priori only reliable in the UV regime, while the latter one finds it origin in the IR sector. Furthermore, the notion of a dynamical gluon mass does not imply the existence of (physical) massive gluons. Our results should rather be taken as giving evidence for the appearance of nonperturbative mass parameters in the gluon propagator, as also found by lattice simulations.

3. \( \langle A^2 \rangle_{\text{min}} \) beyond the Landau gauge?

The question arises what can be said about the dimension two condensate in a gauge other than the Landau gauge? As the operator \( A^2_{\text{min}} \) is then clearly nonlocal, it falls beyond the applicability of the OPE. It is also unclear how e.g. renormalizability or an effective potential approach could be established for nonlocal operators.

Nevertheless, in several other gauges, we have shown that other dimension two, renormalizable, local operators exist. We generalized the LCO method and showed that these operators condense and give rise to a dynamical gluon mass, see Table 1 and [7,19,20,21]. In the maximal Abelian gauge, it was found that only the off-diagonal gluons \( A_\mu^a \) acquire a dynamical mass, a fact qualitatively consistent with the lattice results from
At the cost of introducing a set of bosonic ($B_{\mu\nu}^a$, $B_{\mu\nu}^c$) and a set of fermionic ghost fields ($G_{\mu\nu}^a$, $G_{\mu\nu}^c$), antisymmetric in their Lorentz indices and belonging to the adjoint representation. The local gauge invariance is respected with respect to

\[ \delta A_\mu^a = -D_{\mu}^a \omega^b, \]

\[ \delta B_{\mu\nu}^a = gf^{abc} \omega^b B_{\mu\nu}^c, \]

\[ \delta B_{\mu\nu}^c = gf^{abc} \omega^b B_{\mu\nu}^b, \]

\[ \delta G_{\mu\nu}^a = gf^{abc} \omega^b G_{\mu\nu}^c, \]

\[ \delta G_{\mu\nu}^c = gf^{abc} \omega^b G_{\mu\nu}^b. \]

Having found a reasonable classical action, we need to take a look at the quantum properties of the action (11).

A first problem is the renormalizability. The action (11) as it stands is not renormalizable. Fortunately, we were able to prove to all orders of perturbation theory the renormalizability of the following slightly more general action

\[ S_{\text{phys}} = S_{cl} + S_{gf}, \quad (13) \]

\[ S_{cl} = \int d^4x \left( \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{im}{4} (B - \overline{B})_{\mu\nu} F_{\mu\nu}^a \right) + \frac{1}{4} (\overline{B}_{\mu\nu} D^a_{\rho\sigma} D^b_{\rho\sigma} B_{\mu\nu}^c - \overline{G}_{\mu\nu}^a D^a_{\rho\sigma} D^b_{\rho\sigma} G_{\mu\nu}^c) - \frac{3}{8} m^2 \lambda_1 \left( \overline{B}_{\mu\nu} B_{\mu\nu}^a - \overline{G}_{\mu\nu}^a G_{\mu\nu}^c \right) + m^2 \lambda_3 \frac{32}{16} (\overline{B}_{\mu\nu} B_{\mu\nu}^a - \overline{G}_{\mu\nu}^a G_{\mu\nu}^c) + \frac{\lambda_{abcd}}{16} \left( \overline{B}_{\mu\nu} B_{\mu\nu}^a - \overline{G}_{\mu\nu}^a G_{\mu\nu}^c \right) \times \left( \overline{B}_{\rho\sigma} \rho_{\rho\sigma} - \overline{G}_{\rho\sigma} G_{\rho\sigma} \right) \right), \quad (14) \]

\[ S_{gf} = \int d^4x \left( \frac{\alpha}{2} b^a b^b + c^a \partial_{\mu} A_\mu^a + \overline{c}^a \partial_{\mu} D_{\mu\nu}^a b^b \right), \]

in the class of linear covariant gauges. $\lambda_{abcd}$ is an invariant rank 4 tensor coupling while $\lambda_1$ and $\lambda_3$ are mass couplings. The classical action $S_{cl}$ is still invariant with respect to the gauge transformations (12). The gauge fixed action itself enjoys a generalized BRST symmetry, generated by the nilpotent transformation

\[ s A_{\mu}^a = -D_{\mu}^a \overline{b}^b, \]

\[ s B_{\mu\nu}^a = gf^{abc} b^c B_{\mu\nu}^b, \]

\[ s G_{\mu\nu}^a = gf^{abc} b^c G_{\mu\nu}^b, \]

\[ s \overline{c}^a = b^a, \overline{b}^a = 0, \overline{s}^2 = 0. \]

| GAUGE                | OPERATOR             |
|----------------------|----------------------|
| linear covariant     | $\frac{1}{2} A_{\mu}^a A_{\mu}^b$ |
| Curci-Ferrari        | $\frac{1}{2} A_{\mu}^a A_{\mu}^b + \alpha_{\mu}^c \epsilon^a$ |
| maximal Abelian      | $\frac{1}{2} A_{\mu}^a A_{\mu}^b + \alpha_{\mu}^c \epsilon^a$ |

Table 1

Gauges and their renormalizable dimension two operator
In [23,24], we also presented various renormalization group equations to two loop order, confirming the renormalizability at the practical level. Various consistency checks are at our disposal in order to establish the reliability of these results, e.g. the gauge parameter independence of the anomalous dimension of gauge invariant quantities or the equality of others, in accordance with the output of the Ward identities in [23]. Furthermore, we proved in [24] the equivalence of the model (13) with the ordinary Yang-Mills theory in the case that $m \equiv 0$. An open question is what the physical excitations are of the model in the case that $m \neq 0$. A useful tool in discussing this token will be the nilpotent BRST charge associated to (15).

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