Supplemental Material : Geometry and Topology Tango in Ordered and Amorphous Chiral Matter

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(Dated: 23 novembre 2021)
**Video 1 : Increasing spatial disorder for a domain wall between two phases.** Video based on figure 6 of the main text. First row : Sketch of the frame geometry for increasing positional disorder quantified by the maximal amplitude of the random displacements $|\delta|/a$. All panels show the vicinity of a boundary between two different insulators defined on the same frame but with different positions of the stronger couplings. The lines’ width indicates the magnitude of the coupling strength. In all panels $t'/t = 20$. In the lefmost panel, we indicate the choice of the unit cell and of the crystallographic axes. Second row : Corresponding chiral polarization fields. The color indicates the orientation of $\Pi(x)$. Third row : Magnitude of the zero-mode wave function. The zero mode is located at the boundary between topologically inequivalent states even on amorphous frames. Fourth row : Probability density function of the $\theta$, the local orientation of the chiral polarization field. The distributions are peaked on the same two directions (vertical lines) regardless of the magnitude of disorder. This reveals the coexistence of two distinct topological phases robust to positional disorder.

**Video 2 : Increasing correlation length of the interaction disorder of a single phase.** Video based on figure 7 of the main text. The magnitude of the interaction disorder is fixed to $\epsilon_D = 15$. First row : Sketch of the honeycomb frame and of the coupling strengths for increasing spectral disorder. The strengths of the couplings are represented by the width of the dark lines. Their randomness is quantified by the variance of the Gaussian couplings $\epsilon_D$ and the spatial correlation length $\xi$. Second row : Corresponding chiral polarization fields. The color indicates the orientation $\theta$ of $\Pi(x)$. Third Row : Magnitude of the zero-energy modes on the $A$ (red) and $B$ (blue) sites. Fourth row : Probability density function of the orientation $\theta$. Remarkably, even in the disordered cases, the distribution peaks only at values characteristic of the three phases of the homogeneous chiral Hamiltonian. As the correlation length increases, different phases get bigger and better delimited.