Damage assessment in PRC and RC beams by dynamic tests

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Abstract. The present paper reports on damaged prestressed reinforced concrete (PRC) beams and reinforced concrete (RC) beams experimentally investigated through dynamic testing in order to verify damage degree due to reinforcement corrosion or cracking correlated to loading. The experimental program foresaw that PRC beams were subjected to artificial reinforcement corrosion and static loading while RC beams were damaged by increasing applied loads to produce bending cracking. Dynamic investigation was developed both on undamaged and damaged PRC and RC beams measuring natural frequencies and evaluating vibration mode shapes. Dynamic testing allowed the recording of frequency response variations at different vibration modes. The experimental results are compared with theoretical results and discussed.

1. Introduction

The damage of concrete structures reinforced both with prestressed wires or tendons and normal steel bars is a topic of interest for researchers and practitioners. Damage of prestressed reinforced concrete (PRC) and reinforced concrete (RC) beams may be the result of insufficient reinforcement, large deflections, poor concrete quality, steel corrosion linked to environmental conditions [1]. Although a number of interesting experimental studies have been developed in recent years, the structural behaviour of damaged reinforced concrete elements still needs to be fully investigated. Fruitful algorithms based on dynamic testing have been proposed to address the problem of locating and quantifying structural damage in both beams with homogeneous material and reinforced concrete beams using changes in the structure’s vibration characteristics [2; 3-5].

Frequency response data is directly measured through test data and can provide much more damage information in a desired frequency range, even if the experimental data obtained could be contaminated by measurement errors [6-8]. This paper reports on damaged PRC beams and RC beams experimentally investigated through dynamic testing in order to assess damage degree due to reinforcement corrosion or cracking correlated to loading. In terms of PRC beams, the experimental program foresaw the analysis of the dynamic response of beams subjected to artificial reinforcement corrosion and static bending with beams subjected only to static loading. As for RC beams the comparison of response between three beams was developed considering the effects of increasing loading. Dynamic investigation was carried out on both damaged and undamaged PRC and RC beams measuring natural frequencies and evaluating vibration mode shapes. The dynamic tests allowed recording frequency response variations at different modes of vibration as well as verifying that frequency values of natural vibration were reduced by damage due to cracking.
2. Flexural vibration of uniform beams

In the case of natural vibration of a uniform beam with homogeneous material, the well known equation is obtained [9]:

\[
\frac{\partial}{\partial x^2} \left( EI \frac{\partial^2 v}{\partial x^2} \right) + \rho A \frac{\partial^2 v}{\partial t^2} = 0
\]  

(1)

where: \( E \) is Young’s modulus; \( I \) is the second moment of area of the cross-section; \( \rho A \) is the mass per unit length of beam. The solution of Eq. (1) is given in the following form:

\[ v(x,t) = V(x) \cos(\omega t - \alpha) \]  

(2)

where \( \omega \) is the circular frequency and \( \alpha \) is a phase angle. Introducing Eq. (2) in Eq. (1), the following is obtained:

\[ \frac{\partial}{\partial x^2} \left( EI \frac{\partial^2 V}{\partial x^2} \right) + \rho A \omega^2 V = 0 \]  

(3)

Eq. (3) is simplified in the following way:

\[ \frac{d^4 V}{dx^4} - \lambda^4 V = 0 \]  

(4)

with \( \lambda^2 = \frac{\rho A}{EI} \omega^2 \). The general solution is shown below:

\[ V(x) = B_1 \sinh \lambda x + B_2 \cosh \lambda x + B_3 \sin \lambda x + B_4 \cos \lambda x \]  

(5)

Eq. (5) defines the natural dynamic behaviour of a homogeneous beam without any consideration on the constraints at the ends. Five unknown parameters have to be determined: the constants \( B_i \) per \( i=1,2,3,4 \) and the eigenvalue \( \lambda \). The problem may be solved considering the equilibrium and compatibility equations for the boundary conditions. In the case of both ends free, the following conditions are obtained [2]:

\[ B_2 = B_4 \quad B_1 = B_3 \]  

(6)

\[ V''(L)=0 \Rightarrow (\sinh(\lambda L) - \sin(\lambda L))B_3 + (\cosh(\lambda L) - \cos(\lambda L))B_4 = 0 \]  

(7)

\[ V'''(L)=0 \Rightarrow (\cosh(\lambda L) - \cos(\lambda L))B_3 + (\sinh(\lambda L) + \sin(\lambda L))B_4 = 0 \]  

(8)

In the matrix form, considering an unknown vector \( Z \):

\[ Z = \begin{pmatrix} B_3 \\ B_4 \end{pmatrix} \Rightarrow [H(\lambda)]Z = 0 \quad \text{with} \quad \lambda = \lambda(\omega) \]  

(9)

being the matrix of coefficient equal to:

\[ [H(\lambda)] = \begin{bmatrix} \sinh(\lambda L) - \sin(\lambda L) & \cosh(\lambda L) - \cos(\lambda L) \\ \cosh(\lambda L) - \cos(\lambda L) & \sinh(\lambda L) + \sin(\lambda L) \end{bmatrix} \]  

(10)

Since the condition that the determinant of coefficient matrix is null, the expression of circular natural frequency \( \omega_r^f \) for generic mode \( r \) of vibration in the case of both ends free, is obtained:

\[ \omega_r^f = \left( a_r \frac{r\pi}{L} \right)^2 \cdot \sqrt{\frac{EI}{\rho A}} \]  

(11)

where \( a_r \) depends on the different modes of vibration: 1.506; 1.25; 1.167; 1.125 and 1.1 for the first five modes of vibration. For a prestressed beam the Eq. (1) is modified [9] as follows taking into account an external constant axial compressive force \( N \) equivalent to tensile force applied to wires:

\[ \frac{\partial}{\partial x^2} \left( EI \frac{\partial^2 v}{\partial x^2} \right) + N \frac{\partial^2 v}{\partial t^2} + \rho A \frac{\partial^2 v}{\partial t^2} = 0 \]  

(12)

The circular natural frequency \( \omega_r \) has been evaluated as follows:
\[ \overline{\omega} = \omega^f \cdot \sqrt{1 - \frac{N \cdot L^2}{EI \cdot \xi^2 \cdot r^2 \pi^2}} \]  

(13)

where \( \xi \) a coefficient linked to different modes of vibration.

3. Experimental research on PRC beams

Response comparisons of the results of the dynamic tests on damaged and undamaged PRC beams were carried out in laboratory. Damage was due to steel corrosion obtained in the beam by an artificial electrochemical process [3, 4]. During the experimental research three beams were tested: B0-B1-B2: B0 was subjected to bending test to evaluate the development of damage for PRC beam model up to failure; B1 and B2 under bending and dynamic tests. The dimensions of double T section were: 100x130mm, respectively, for base and height; 30mm for width. The length of the beams was 2.45m.

![Figure 1](image1.png)

Figure 1. (a) Set-up of bending tests of PRC beam; set-up for dynamic tests

After its’ construction, beam B2 was subjected to a corrosion process through three artificial corrosion cycles for a period of 3 months. Beams B1 and B2 were successively tested by bending for different loads value (figure 1b). At each degree of damage, after removing the load from the beams, dynamic vibration tests were carried out in the condition of free-free ends.

The maximum moment value for the two first cycles was equal for both beams: 1\(^{st}\) cycle of load up to a value of moment \( M_1 = 1.20 \text{kNm} \) less than the moment of cracking - this step is shown as damage \( D_1 \); 2\(^{nd}\) cycle of load up to a value of moment \( M_2 = 1.81 \text{kNm} \) bigger than the value of cracking - damage \( D_2 \). Finally, the 3\(^{rd}\) cycle of load was obtained increasing the value of vertical load up to a moment \( M_3 = 2.26 \text{kNm} \) for B1 and \( M_3 = 1.95 \text{kNm} \) for B2 model - damage \( D_3 \).

| Beam | Degree of Damage | Ratio of Moment \( \text{M/M}_{\text{M, max}} \) |
|------|------------------|---------------------------------|
| B1   | \( D_0 \)        | 0.0                             |
|      | \( D_1 \)        | 0.34                            |
|      | \( D_2 \)        | 0.51                            |
|      | \( D_3 \)        | 0.63                            |
| B2   | \( D_0 \)        | 0.0                             |
|      | \( D_1 \)        | 0.34                            |
|      | \( D_2 \)        | 0.51                            |
|      | \( D_3 \)        | 0.55                            |

Table 1 shows the ratio of experimental bending moment with the maximum value of moment for each damage degree \( D_i \). The experimental dynamic analysis of beams B1 and B2 was developed to detect damage and its effects on the structural behaviour for both the initial phase of the corrosion process
when cracks are not present on the concrete’s surface and after the bending loading at different degrees of damage.

The beam for the dynamic tests was divided into 27 points. The distance between each point was 87.5 mm. The condition of the beam having free ends was simulated by hanging the beam with elastic springs. The dynamic measures were obtained using an impact hammer and working in a 0 and 800 Hz range of frequencies with a resolution of 0.5 Hz. For every set of dynamic analysis the response to the impulsive load by hammer at each of the 27 points was recorded by an accelerometer linked to a multi-analysis system Pulse Labshop 5.1 and using Me’scope Modal software. The frequencies were extracted by transformed signals in the frequency domain by the Fast Fourier Transform (FFT) technique.

3.1. Dynamic response of PRC beam B2 subjected to damage by corrosion

The experimental data recorded for beam B2 before the corrosion process are compared with the theoretical data obtained by dynamic analysis of the beam and the FEM analysis (table 1). Figure 2 shows the natural modal deformation for the beam’s first four modes of vibration obtained by FEM analysis.

![Figure 2. Modes of vibration of free-free PRC beam by FEM analysis (STRAUS CODE).](image)

The theoretical frequency values for free-free beam are evaluated neglecting the effect of compressive force on the dynamic response by the Eq. (13) assuming the following values: area of section $A = 8.845 \times 10^3 \text{ mm}^2$; moment of inertia $I = 1.7 \times 10^7 \text{ mm}^4$; Young’s modulus $E = 3.096 \times 10^3 \text{ N/mm}^2$; density $\rho = 2.4 \times 10^{-9} \text{ N\cdot s}^2/\text{mm}^2$.

| Table 2. Theoretical and experimental frequency data for B2 model | $f_1$ (Hz) | $f_2$ (Hz) | $f_3$ (Hz) | $f_4$ (Hz) |
|---------------------------------------------------------------|---------|---------|---------|---------|
| Eulero-Bernoulli beam model                                   | 93.476  | 257.591 | 505.167 | 834.596 |
| Experimental B2 beam model                                   | 93.628  | 249.870 | 461.107 | 706.271 |
| FEM                                                           | 94.060  | 251.050 | 507.000 | 838.000 |

The dynamic measures of frequencies $f_1$, $f_2$, $f_3$ and $f_4$ recorded for B2 beam from the beginning of the corrosion process are shown in table 2. Figure 3 shows the envelope of Frequency Response Functions (FRFs) obtained by the vibration analysis of B2 due to impulse load by impact hammer. This test was carried out at time $t=0$ when the electrochemical process of corrosion was at the beginning.
Figure 3. Envelope of FRFs by dynamic tests of B2 at the beginnings of corrosion process (t=0).

Table 3. Experimental frequency data for B2 beam during the corrosion phase

| Time    | $f_1$ (Hz) | $\Delta f_1/f_1$ (%) | $f_2$ (Hz) | $\Delta f_2/f_2$ (%) | $f_3$ (Hz) | $\Delta f_3/f_3$ (%) | $f_4$ (Hz) | $\Delta f_4/f_4$ (%) |
|---------|------------|----------------------|------------|----------------------|------------|----------------------|------------|----------------------|
| t=0     | 93.628     | -                    | 249.870    | -                    | 461.107    | -                    | 706.271    | -                    |
| 42 days | 94.174     | -0.583               | 249.520    | 0.140                | 459.556    | 0.336                | 706.975    | 0.100                |
| 56 days | 93.870     | -0.258               | 248.803    | 0.427                | 457.838    | 0.709                | 705.046    | -0.173               |
| 111 days| 94.330     | -1.407               | 248.978    | 0.173                | 459.150    | -1.108               | 705.638    | -0.015               |
| 128 days| 93.628     | 0.000                | 247.170    | 1.080                | 453.600    | 0.327                | 701.860    | 0.624                |

The B2 beam was tested by vibration as free-free beam during the process of artificial corrosion damage. The dynamic response as frequency values after the beginning of corrosion process (t=0) for about three months are shown in table 2 with the variations of values (in percent) as $(f_{i,0} - f_i) / f_{i,0}$.

Figure 4. Exp. frequency values vs. time of corrosion process for B2 beam.

From the inspection of the experimental results (figure 4) it appears that the corrosion process in absence of cracking of concrete does not significantly influence the frequency data and the dynamic response of the beam.
3.2. Dynamic response of PRC beams B1- B2 subjected to damage by bending

Beams B1 and B2 were successively tested by bending load at different levels of loading (table 2). The experimental frequency values obtained by the dynamic tests are indicated in the following tables.

Table 4. Experimental frequency data for B1

| M/M\text{u,th} t=2 years | f_1 (Hz) | Δf_1/f_1 (%) | f_2 (Hz) | Δf_2/f_2 (%) | f_3 (Hz) | Δf_3/f_3 (%) | f_4 (Hz) | Δf_4/f_4 (%) |
|--------------------------|----------|--------------|----------|--------------|----------|--------------|----------|--------------|
| 0                        | 96.931   | -            | 255.990  | -            | 476.340  | -            | 724.400  | -            |
| 0.34                     | 96.616   | 0.32         | 255.830  | 0.06         | 475.700  | 0.13         | 723.560  | 0.12         |
| 0.51                     | 94.164   | 2.85         | 254.110  | 0.73         | 469.910  | 1.35         | 717.370  | 0.97         |
| 0.63                     | 92.468   | 4.60         | 251.480  | 1.76         | 466.970  | 1.97         | 709.700  | 2.03         |

Table 5. Experimental frequency data for B2

| M/M\text{u,th} t=2 years | f_1 (Hz) | Δf_1/f_1 (%) | f_2 (Hz) | Δf_2/f_2 (%) | f_3 (Hz) | Δf_3/f_3 (%) | f_4 (Hz) | Δf_4/f_4 (%) |
|--------------------------|----------|--------------|----------|--------------|----------|--------------|----------|--------------|
| 0                        | 95.622   | -            | 251.410  | -            | 464.480  | -            | 713.540  | -            |
| 0.34                     | 94.426   | 1.25         | 248.960  | 0.97         | 460.770  | 0.80         | 706.110  | 1.04         |
| 0.51                     | 91.824   | 3.97         | 247.360  | 1.61         | 454.270  | 2.2          | 696.420  | 2.4          |
| 0.55                     | 90.464   | 5.39         | 244.020  | 2.94         | 452.050  | 2.68         | 688.960  | 3.45         |

In tables 4 and 5 variations of frequency values are evaluated considering the difference of frequency value of i-mode for the initial condition undamaged by cracking due to bending and the frequency value of i-mode at the j-degree of damage. It can be seen that although evident phenomena of damage such as cracks on the compressive concrete’s surface were not present, the PRC beam, B2, subjected to artificial corrosion process, suffered from a reduction of frequency values greater than the values recorded in beam B1. In figure 5 a comparison of the experimental variations of the frequency values for the first mode of the two beams B1 and B2 are shown at different degrees of damage expressed by the ratio M/M\text{u,th}. The comparison allows us to conclude that corrosion leads to a decrease in frequency values. The effect of damage may be actually evaluated when the beam is subjected to service loads.

Figure 5. Comparison of exp. variation frequency values vs. M/M\text{u,th} for B1 and B2 (1\text{st} mode).
4. Experimental research on RC beams

Dynamic tests developed with the same aforementioned procedure were carried out on three RC beams (B1, B2, B3) measuring 3.75m in length and with a rectangular section measuring $150 \times 250 \text{mm}^2$ [5]. B1 and B2 beams were reinforced with four longitudinal steel bars with a diameter of 10mm and 14mm, respectively; B3 with four steel bars having a diameter of 10mm and 16mm. Eight millimetre diameter stirrups were disposed at step 150mm. The RC beams were subjected to monotonic load, P, at two points in the middle of the span. The bending test foresaw that the mid zone section of beam was to be subjected only to bending moment M (figure 6a). Four damage degrees D1...D4 (figure 6b) are correlated to M1...M4 bending values. The beams were analysed by free vibration due to impulse load at free-free ends (figure 7) for each damage degree.

![Figure 6](image1.png)

**Figure 6.** (a) Mid zone of beam under exp. bending test (b) D4 damage degree for B2

![Figure 7](image2.png)

**Figure 7.** Set-up for dynamic test of RC beam: points 1, ... 21 of position of accelerometer; (1) RC beam; (2) hammer; (3) accelerometer, CH1, CH2: channels to analyzer FFT.

Response was measured at different points 1, ... 21, using an accelerometer. The frequencies were extracted by transformed signals in the frequency domain by the FFT technique. Considering Eq. (11) with Young’s modulus $E=3.673 \times 10^3 \text{N/mm}^2$; $\rho=2.5 \times 10^{-6} \text{Ns}^2/\text{mm}^4$ and moment of inertia $I=20.1 \times 10^7 \text{mm}^4$, the theoretical frequency values of the undamaged beams were evaluated (table 6).

| Table 6. Theoretical and experimental frequency data for undamaged RC beams |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|
| Eulero-Bernoulli beam model | $f_1$ | $f_2$ | $f_3$ | $f_4$ (Hz) |
| Exp. undamaged B1 beam      | 71.180 | 196.160 | 384.690 | 635.540 |
| Exp. undamaged B2 beam      | 71.847 | 194.245 | 367.693 | 580.446 |
| Exp. undamaged B3 beam      | 70.300 | 192.676 | 362.495 | 577.280 |

In Figure 8 the envelope of FRFs are shown in the 0-800Hz range obtained by dynamic testing. A translation on the left of the higher points of the diagrams linked to an increase of damage degree from D0 to D4 and a decrease of frequency values may be noted for different modes of vibration for all beams B1, B2 and B3.
Figure 8. Envelope of FRFs for B1, B2, B3 for damage degrees from D0 to D4.
The assessment of the RC beams at different degrees of damage due to cracking as a consequence of load increasing may be carried out considering the following experimental diagrams in figures 9 and 10 where $\Delta f_i = f_i - f_{i0}$, $f_{i0}$ the frequency value at undamaged state.

In figure 9 the frequency ratio in percent decreases versus the ratio in percent between the experimental value of bending moment $M$ on bending capacity $M_{\text{max}}$ considering four modes of vibration. In figures 10 (a) and (b) the frequency ratio is referred to the first mode of vibration.

![Figure 9](image9.png)

**Figure 9.** Frequency ratio $\Delta f_i / f_{i0}$ vs. moment ratio $M / M_{\text{max}}$ for B1.

![Figure 10](image10.png)

**Figure 10.** Frequency ratio $\Delta f_i / f_{i0}$ vs. moment ratio $M / M_{\text{max}}$ for B2 and B3 (1st mode).
5. Conclusions
Dynamic tests on PRC and RC beams were carried out in an experimental research investigating the behaviour of beams subjected to both environmental cause of damage producing corrosion of reinforcement and to beams subjected to load increase. The damage phenomenon was represented by the softening of the compressive concrete in PRC beam subjected to corrosion and visible cracks are not present on the concrete’s surface. In the case of cracking due to bending, damage was located at the intrados zone of the beams in the mid span area where the loading was applied. Dynamic tests were carried out considering the condition of free-free ends for the beams analyzed and subjected to vibration for impulsive force applied on a point by impact hammer. The main parameters providing us with useful information regarding damage degree were the frequency values. The following conclusions were obtained on the basis of the aforementioned frequency measures:

1. Frequency values recorded in dynamic testing of PRC beams, undamaged or subjected to an increasing corrosion process with softening of concrete by micro cracking, were almost constant;
2. Damage due to corrosion may be not conveniently studied with dynamic tests;
3. Damage due to bending by increasing of applied loads produced a reduction of frequency values that may be evaluated experimentally;
4. In the case of PRC beam subjected to corrosion, the variations of values are greater than in PRC beam undamaged by corrosion;
5. The variations of frequency values in damaged RC beams may reach high values as 15-25 per cent undervalue of moment for service conditions of loading.

Although the dynamic response of PRC and RC beams are influenced by a number of errors, such as the condition of restraints, the results obtained in the experimental research can lead us to consider the technique adequate for assessing damage to beams with non homogeneous material.

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Acknowledgement
This research was supported by research funds of University Politecnica delle Marche for two consecutive years. The author would like to express his gratitude to the Administration of the University Politecnica delle Marche.