The characteristic black hole mass resulting from direct collapse in the early Universe

M. A. Latif,* D. R. G. Schleicher, W. Schmidt and J. C. Niemeyer

Institut für Astrophysik, Georg-August-Universität, Friedrich-Hund-Platz 1, D-37077 Göttingen, Germany

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ABSTRACT
Black holes of a billion solar masses are observed in the infant Universe a few hundred million years after the big bang. The direct collapse of protogalactic gas clouds in primordial haloes with $T_{\text{vir}} \geq 10^4 \text{ K}$ provides the most promising way to assemble massive black holes. In this study, we aim to determine the characteristic mass scale of seed black holes and the time evolution of the accretion rates resulting from the direct collapse model. We explore the formation of supermassive black holes via cosmological large eddy simulations (LES) by employing sink particles and following their evolution for 20,000 yr after the formation of the first sink. As the resulting protostars were shown to have cool atmospheres in the presence of strong accretion, we assume here that UV feedback is negligible during this calculation. We confirm this result in a comparison run without sinks. Our findings show that black hole seeds with characteristic mass of $10^3 \text{ M}_\odot$ are formed in the presence of strong Lyman–Werner flux which leads to an isothermal collapse. The characteristic mass is about two times higher in LES compared to the implicit large eddy simulations. The accretion rates increase with time and reach a maximum value of $10 \text{ M}_\odot \text{ yr}^{-1}$ after $10^4 \text{ yr}$. Our results show that the direct collapse model is clearly feasible as it provides the expected mass of the seed black holes.

Key words: methods: numerical – galaxies: formation – cosmology: theory – early Universe.

1 INTRODUCTION
Black holes of a few millions to billions solar masses dwell in the centre of present-day galaxies (Kormendy & Richstone 1995; Tremaine et al. 2002; McConnell et al. 2011; van den Bosch et al. 2012). These supermassive black holes (SMBHs) are not only present in the local Universe but have been observed at $z > 6$ (Fan et al. 2003, 2006; Mortlock et al. 2011). Their formation in the first billion years after the big bang is still an open question. Numerous models have been proposed to explain the origin and formation of SMBHs (Rees 1984; Djorgovski et al. 2008; Begelman & Shlosman 2009; Regan & Haehnelt 2009a; Tanaka & Haiman 2009; Volonteri 2010; Johnson et al. 2012, 2013; Haiman 2013). They include the collapse of dense stellar cluster due to relativistic instability (Portegies Zwart et al. 2004; Omukai, Schneider & Haiman 2008; Devecchi & Volonteri 2009), remnants of Population III stars (Haiman 2004; Latif et al. 2013c) and the direct collapse of protogalactic gas clouds (Oh & Haiman 2002; Bromm & Loeb 2003; Begelman, Volonteri & Rees 2006; Lodato & Natarajan 2011; Spaans & Silk 2006; Dijkstra et al. 2008; Djorgovski et al. 2008; Schleicher, Spaans & Glover 2010; Shang, Bryan & Haiman 2010; Johnson et al. 2011; Latif, Zaroubi & Spaans 2011a; Choi, Shlosman & Begelman 2013; Whalen et al. 2013b). Although the formation of SMBHs from stellar mass black holes may appear as the most natural way, the feedback from the stars creates a hindrance as they have to accrete at an Eddington rate all the time to reach the observed masses (Johnson & Bromm 2007; Alvarez, Wise & Abel 2009; Whalen & Fryer 2012). The masses of black holes resulting from the collapse of dense stellar clusters are relatively low with $\sim 10^3 \text{ M}_\odot$ (Devecchi & Volonteri 2009). On the other hand, the direct collapse seems the most plausible way to assemble SMBHs to provide higher mass black hole seeds.

The formation of SMBHs via direct collapse requires the suppression of fragmentation and efficient accretion of the gas on to the central object. Massive primordial haloes of $10^7–10^8 \text{ M}_\odot$ irradiated by strong Lyman–Werner background fluxes are the most plausible candidates (Omukai 2001; Johnson & Bromm 2007; Dijkstra et al. 2008; Schleicher et al. 2010; Shang et al. 2010; Johnson et al. 2011; Latif et al. 2011b, 2013a; Wolcott-Green, Haiman & Bryan 2011; Agarwal et al. 2012), see also Inayoshi & Omukai (2012) and Van Born & Spaans (2013) for alternatives. Numerical simulations support this scenario and show that in the presence of strong Lyman–Werner flux, an isothermal collapse yields massive objects (Bromm & Loeb 2003; Wise, Turk & Abel 2008; Regan & Haehnelt 2009b; Latif et al. 2011a, 2013b). The final fate of these objects is not yet very well understood and depends on the gas mass accretion rates.

Theoretical models (Begelman, Rossi & Armitage 2008; Begelman 2010; Hosokawa, Omukai & Yorke 2012; Hosokawa et al. 2013; Schleicher et al. 2013; Whalen et al. 2013a) propose the
formation of supermassive stars (normal stars of higher masses) or quasi-stars (stars with black holes at the centre) as an intermediate stage to the formation of SMBHs. The work by Schleicher et al. (2013) suggests that for accretion rates $>0.14 \, M_\odot \, \text{yr}^{-1}$, the core of the star collapses into a black hole, forming a so-called quasi-star while lower accretion rates lead to the formation of a supermassive star. Hosokawa et al. (2012, 2013) show that for accretion rates $> \times 10^{-2} \, M_\odot \, \text{yr}^{-1}$, supergiant stars of $10^3 \, M_\odot$ form via rapid accretion while maintaining the cool atmospheres on the Hayashi track. Schleicher et al. (2013) have suggested that this may be true up to a mass scale of $3.6 \times 10^6 m_\odot M_\odot$, where $m$ is the mass accretion rate in units of $M_\odot \, \text{yr}^{-1}$. These models for the evolution of supermassive stars show that in the presence of accretion rates $> \times 10^{-2} \, M_\odot \, \text{yr}^{-1}$, such stars have a bloated envelope and lower surface temperatures, inhibiting the emission of ionizing flux. A recent study by Latif et al. (2013b) reported high accretion rates of $1 \, M_\odot \, \text{yr}^{-1}$ in these haloes and the possibility for the formation of massive objects in a short time.

In this paper, we explore the first time the characteristic mass scale of seed black holes and the time evolution of accretion rates in massive primordial haloes illuminated by a strong Lyman–Werner background UV flux. To accomplish this goal, we perform high-resolution cosmological large eddy simulations (LES) to ensure the collapse below parsec scales by exploiting the adaptive mesh refinement (AMR) method and employing sink particles to follow the accretion for longer time-scales. To verify our results, we also perform a comparison run without sink particles. We evolve the simulations for $20,000 \, \text{yr}$ after the formation of the first sink and employ a Jeans resolution of $32 \, \text{cells}$ to resolve turbulent eddies. The subgrid-scale (SGS) turbulence model of Schmidt, Niemeyer & Hillebrandt (2006) is used to take into account unresolved turbulence. This study will enable us to compute the masses of seed black holes formed in the massive primordial haloes and test the feasibility of a direct collapse model.

This paper is organized in the following way. In Section 2, we describe the numerical methods and simulations setup. We present our results in Section 3. In Section 4, we summarize the main findings of this study and discuss our conclusions.

## 2 Computational Methods

### 2.1 Simulation setup

The simulations presented here are performed using a modified version of the ENZO code (O’Shea et al. 2004; The Enzo Collaboration et al. 2013). ENZO is an Eulerian grid-based, massively parallel, cosmological AMR code. The hydrodynamical equations are solved using a third-order accurate piece-wise parabolic method. The dark matter dynamics is solved using the particle mesh technique.

The initial simulation setup is the same as in our previous study (Latif et al. 2013b). Here, we present a short summary of the initial conditions and the simulations setup. Our simulations are started with cosmological initial conditions at $z = 100$. We use the publicly available INIT package to generate nested grid initial conditions. These simulations were first run with a uniform grid resolution of $128^3$ and the dark matter haloes are selected at $z = 15$. The simulated volume has a comoving size of $1 \, \text{Mpc} \, h^{-1}$ and the most massive halo lies at the centre of the box. We further employ two initial nested refinement levels with a resolution of $128^3$ cells each and initialize $5767 \, 168$ particles to simulate the dark matter dynamics. In the central $62 \, \text{kpc}$ region of the halo, we allow $15$ dynamical refinement levels during the course of the simulations, which yields an effective resolution of subparsec scales ($10,000 \, \text{au}$ in comoving units). We resolve the Jeans length by $32$ cells during the entire course of the simulations. Once the maximum refinement level is reached, we employ sink particles to follow the evolution for $20,000 \, \text{yr}$ after the formation of the first sink.

We employ a strong Lyman–Werner flux of strength $10^3$ in units of $J_{13}$ produced by the first generation of stars with radiative spectra of $10^7 \, \text{K}$ (Omukai 2001; Johnson & Bromm 2007; Dijkstra et al. 2008; Schleicher et al. 2010; Latif et al. 2011b; Wolfson-Green et al. 2011; Agarwal et al. 2012; Safranek-Shrader et al. 2012). To follow the thermal evolution, we self consistently solve the rate equations of $H$, $H^+$, $\text{He}$, $\text{He}^+$, $\text{He}^{++}$, $e^-$, $H^-, \text{H}_2$, $\text{H}_3^+$ in the cosmological simulations. We ignore the effect of self-shielding as our main focus is on $\text{H}_2$-free haloes.

To take into account the unresolved turbulence, we use the subgrid-scale turbulence model proposed by Schmidt et al. (2006). The adaptively refined LES technique is used to apply the SGS turbulence model in cosmological AMR simulations (Maier et al. 2009). We perform LES and compare our results with implicit large eddy simulations (ILES). The approach of LES is based on separating the resolved and unresolved scales, and connect them through an eddy-viscosity closure for the transfer of energy between the grid scales. The turbulent viscosity is computed from the grid scale and the SGS turbulence energy, i.e. the kinetic energy associated with numerically unresolved turbulent velocity fluctuations. On the other hand, ILES uses only the numerical dissipation produced from the discretization errors of fluid dynamics equations.

In all, we have performed six simulations (three LES, three ILES) with sink particles for three distinct haloes A, B and C and a comparison run without sinks particles for halo A. The halo properties are given in the table 1 of Latif et al. (2013b). Their collapse redshifts are $12.6$, $10.8$ and $13.6$, respectively. For a comparison run without sinks, simulations are evolved adiabatically after they reach the maximum refinement level. This method allows us to follow the collapse for a longer times.

### 2.2 Sink particles

The need to resolve the Jeans length to stellar densities and Courant constraints on the calculation of the time step make it impossible to follow the collapse to the smallest scales while evolving the simulations for a long time. Therefore, sink particles are introduced to represent the gravitationally bound objects undergoing a free-fall collapse. This approach has been successfully employed in both smoothed particle hydrodynamics as well as in AMR codes (Bate, Bonnell & Price 1995; Krumholz, McKee & Klein 2004; Federrath et al. 2010). Here, we employ the sink particle algorithm by Wang et al. (2010) to represent the protostars. Sinks are created when a grid cell is at the maximum refinement level, the density exceeds the Jeans density (i.e. it violates the Truelove criterion) and the overall flow is convergent, which is implicitly covered by a density threshold check. Furthermore, particles are merged if they are created within the accretion radius. The initial mass of the sinks is calculated such that the cell is Jeans stable after the subtraction of the sink mass. The initial velocity of the sinks is computed based on momentum conservation. Further details of the sinks algorithm can be found in the reference paper (Wang et al. 2010).

The sink particles in our case do not accrete gas directly from the grid as typically computed using the Bondi–Hoyle accretion. This effect is compensated by allowing the formation of additional sinks and, subsequently, they are merged if they are formed within the
accretion radius which we choose as the Jeans length. Using sink particles, we follow the collapse for longer dynamical time-scales. This approach enables us to compute the masses of seed black holes and the time evolution of mass accretions.

3 MAIN RESULTS

3.1 Simulations with sink particles

We have performed six cosmological simulations with sink particles (three LES and three ILES) for three distinct haloes A, B and C and employing a constant Lyman–Werner background UV flux of $J_{21} = 10^3$ for stellar spectra of $10^5$ K. We followed the evolution for 20,000 yr after the simulations reached the maximum refinement level and determined the characteristic mass scale of the most massive objects. The results obtained from the cosmological LES are presented here. After the simulations are started at $z = 100$, massive haloes are formed around redshift 18, and gas falls into the dark matter potentials and gets shock heated.

The general properties of the haloes, 20,000 yr after the formation of the first sink, are shown in Fig. 1. The formation of molecular hydrogen remains suppressed in the presence of strong H$_2$ photodissociating background and, consequently, an isothermal collapse occurs. The central temperature of these haloes is about 7000 K. The maximum density in the haloes is a few times $10^{-18}$ g cm$^{-3}$. There is an initial rise in the density at larger radii and then it becomes almost constant. This trend is observed for all haloes. The deviation in the density radial profiles from an isothermal behaviour may result both from the removal of the gas by sink particles from the grid as well as the formation of a disc due to the non-zero angular momentum. The typical radial velocities are a few km s$^{-1}$ which indicate the relatively high gas infall rates at this stage of the collapse. The radial profile of the mass accretion rates shows that the accretion rate in the surroundings of the halo is about $1 M_\odot$ yr$^{-1}$ and decreases down to $10^{-3} M_\odot$ yr$^{-1}$ in the core of halo, the region which corresponds to the Jeans length. This behaviour seems to be consistent for all haloes. The ratio of rotational to circular velocities is about 1 in the surroundings of the halo and declines towards the centre of the scale which corresponds to the Jeans length. It further shows that there is high degree of rotational support in the halo. The mass profiles increase with $r^2$, the deviations from this behaviour come from the differences in the underlying density structure for various runs.

The state of the simulations at their collapse redshift is shown in Fig. 2 and is presented by the density projections. The fragmentation in these haloes remains suppressed and a single massive sink is formed in all haloes except one ILES run as discussed below. We note that the underlying density distribution is different from halo to halo for both LES and ILES. We attribute these differences to the properties of the haloes and the occurrence of various processes such as the removal of gas by the sinks and turbulent stresses. We further show the time evolution of the density structure for the ILES run (halo C) in Fig. 3 where the formation of multiple sinks is observed. The formation of a second sink takes place about 7000 yr after the formation of the first sink. The third sink particle in this case is formed about 10,000 yr after the formation of the first sink. The mass of the most massive sink is about $10^5 M_\odot$. The masses of two particles are almost comparable and may lead to a binary or even multiple system. Fragmentation in this halo is due to the local compression of the gas by the turbulence. Similarly, fragmentation was observed in our previous study (Latif et al. 2013b) with a different approach employed for the evolution of simulations. Here, no fragmentation is found in LES runs for all simulated haloes.

The time evolution of the mass accretion rates for the most massive sinks is shown in Fig. 4 for both LES and ILES. The accretion rate increases with time and reaches a peak value of about 10 solar masses per year in about a time of a few thousand years. Such an accretion rate is in accordance with Bondi–Hoyle accretion,

$$M = \frac{4\pi\rho G^2 M^2}{c^3}.$$
Figure 2. Density projections are shown for the central 50 pc 20,000 yr after the formation of the first sink. The white spheres represent sink particles and are overplotted on the density projections. They have typical masses of $\sim 10^5 \, M_\odot$.

Figure 3. Time evolution of the density projections is shown in this figure for an ILES run (halo C). The time in years after the formation of the first sink is shown in each projection. The white spheres represent sink particles and are overplotted on the density projections. They have typical masses of $\geq 10^4 \, M_\odot$.

Considering $M = 10^5 \, M_\odot, c_s = 12 \, \text{km s}^{-1}$ and $\rho = 5 \times 10^{-19} \, \text{g cm}^{-3}$, the typical values in our case, the expected accretion rate from the above equation is 9.7 $M_\odot \, \text{yr}^{-1}$ comparable to the values in our simulations. After about 10,000 yr, the accretion starts to become constant for LES. This time-scale can be understood from the infall of a point mass $m$ at distance $R$ from a point source of mass $M$. The expression for the infall time can be derived from Kepler’s third law of motion and is given as

$$T_{\text{inf}} = \frac{\pi}{2} \frac{R^{3/2}}{\sqrt{2 G (M + m)}}.$$  

(2)
For a point source of mass $10^5 \, M_{\odot}$ and a distance of $3 \times 10^4$ au, the infall time is about 9400 yr and is in good agreement with our results. This characteristic behaviour is noticed for all haloes and for both LES and ILES runs. The decline in accretion rates after 10 000 yr in one of the ILES runs is due to the formation of multiple sinks in halo C and a decrease in the density in the surroundings of the sink particle. The constant accretion rates after 10 000 yr are a consequence of enhanced rotational support in the halo with time.

As pointed out previously, our main aim here is to compute the characteristic mass scale of seed black holes resulting from the direct collapse model. The mass distribution of the sinks for individual runs is illustrated in Fig. 6. It is clearly visible from the figure that LES runs produce higher mass sinks compared to the ILES runs. It can be noted that a single sink particle of $10^5 \, M_{\odot}$ is formed per halo except for halo C which has two additional particles of about $5 \times 10^4$ and $8 \times 10^4 \, M_{\odot}$.

3.2 Comparison run without sinks

In addition to our six cosmological runs with sink particles, we have performed one cosmological simulation where we evolve the simulation adiabatically at densities above a few times $10^{-18}$ g cm$^{-3}$ after reaching the maximum refinement level. The properties of the halo are shown in Fig. 7 after 20 000 yr of evolution. The density radial profile shows an isothermal behaviour at larger radii and becomes almost flat in the centre due to the adiabatic evolution. The temperature is about 8000 K and starts to increase with density to
Figure 7. Radially binned spherically averaged radial profiles are shown for the run without sinks (for halo A). The radial profiles of the enclosed mass and the density are shown in the top-left and -right panels. The middle-left panel shows the rotational velocity profile while the middle-right panel depicts the temperature radial profiles. The bottom panels show the mass accretion rate and radial velocity profiles.

make the collapse stable at the smallest scales. The radial infall velocity is about $10 \text{ km s}^{-1}$ and becomes almost constant in the centre. The mass accretion rate is about $1 \text{ M}_\odot \text{ yr}^{-1}$ as collapse proceeds on the larger scales and decreases down to the $10^{-4} \text{ M}_\odot \text{ yr}^{-1}$ within the Jeans length. The rotational velocity is low in the centre, peaks around $10^5 \text{ au}$ and declines down as it follows the Keplerian velocity. The mass radial profiles increase with $r^2$ in the centre and becomes flat at larger radii. This indicates that most of the mass lies in the central clump.

The state of the simulations after 20000 yr of the evolution is shown in Fig. 8. It is clearly visible from the density projections that a disc is formed in the centre of the halo. The mass of the disc is $\sim 10^5 \text{ M}_\odot$ equivalent to the sink mass in the corresponding run. The formation of a parsec size disc is according to the expectation of theoretical models for the black hole formation (Lodato & Natarajan 2006). Both approaches provide high masses in relatively short time-scales. This verification of results without sinks puts our estimates on even sounder footing.

We further show the time evolution of the mass radial profile for this run as depicted in the left-hand panel of Fig. 9. It can be noticed that initially the mass follows an $r^2$ behaviour within the Jeans length and then increases linearly with radius. This trend is consistent with an isothermal density profile. Over the passage of time, the mass increases due to the infall of gas in the centre of

Figure 8. Density projections are shown for the central 10 pc for the run without sinks. The left- and right-hand panels show the projections along the $y$- and $z$-axis.
the halo and profile gets flattened. A disc of $\sim 10^5 M_\odot$ is formed and most of the mass lies in the central clump as indicated by the flat mass profile. The time evolution of the accretion rates radial profile is shown in the right-hand panel of Fig. 9. Similar to sinks’ simulation, accretion rate increases with time and reaches a few solar masses per year in about 10 000 yr. We note that the accretion rate measured here effectively probes a larger scale, and the profiles indicate an increase towards smaller scales. We therefore consider them to be consistent with sink particle runs.

4 DISCUSSION

We present here the results from the first cosmological LES employing sink particles and following the collapse for 20 000 yr after their formation. These simulations are performed for three distinct haloes and the results are compared with ILES. The main objective of this study is to compute the characteristic mass scale of seed black holes resulting from the direct collapse model. We also computed the time evolution of mass accretion rates in massive primordial haloes irradiated by a strong Lyman–Werner UV background flux.

Our findings show that black hole seeds with characteristic masses of $10^5 M_\odot$ are formed in a short time-scale of 20 000 yr after their formation. It is further found that the characteristic masses are two times higher in LES. The time evolution of the accretion rates shows a characteristic behaviour, it increases with time and reaches a peak value of $10 M_\odot$ yr$^{-1}$. The accretion rate becomes almost constant as the rotational support is increased in the halo at later times. We further noticed that multiple sinks are formed in one halo with masses between $5 \times 10^4$ and $10^5 M_\odot$. It is worth mentioning that we confirmed our estimates for the masses of sinks by evolving simulations adiabatically soon after they reached the maximum refinement level (an alternative approach to sinks) and found similar results. We further stress that our estimates for the characteristic mass are robust as they are confirmed from two independent approaches.

The results from this study suggest that the formation of supermassive stars of $10^5 M_\odot$ seems the most plausible outcome as an intermediate stage to the formation of SMBHs. This is in accordance with the prediction of theoretical studies (Begelman 2010; Hosokawa et al. 2012, 2013; Schleicher et al. 2013). We have further shown that higher accretion rates of $\geq 0.1 M_\odot$ yr$^{-1}$ can be maintained for longer time-scales. Our simulations show that SGS turbulence favours higher accretion rates compared to the ILES and the resulting seed black holes are about two times more massive.

It is expected that supermassive protostars produce accretion luminosity feedback during their early stage. Our calculations did not take into account this effect. We expect that the accretion luminosity feedback will have only minor impact on the masses of seed black holes as similar study exploring this impact in minihaloes reported no significant impact (Smith et al. 2011, 2012). The UV feedback is expected to occur when the star mass exceeds $10^5 M_\odot$ (Hosokawa et al. 2013) which may further influence the growth of such stars (Hosokawa et al. 2012; Johnson et al. 2012). Numerical simulations exploring the impact of UV feedback should be performed in the future.

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REFERENCES

Agarwal B., Khochfar S., Johnson J. L., Neistein E., Dalla Vecchia C., Livio M., 2012, MNRAS, 425, 2854
Alvarez M. A., Wise J. H., Abel T., 2009, ApJ, 701, L133
Bate M. R., Bonnell I. A., Price N. M., 1995, MNRAS, 277, 362
Begelman M. C., 2010, MNRAS, 402, 673
Begelman M. C., Shlosman I., 2009, ApJ, 702, L5
Begelman M. C., Volonteri M., Rees M. J., 2006, MNRAS, 370, 289
Begelman M. C., Rossi E. M., Armitage P. J., 2008, MNRAS, 387, 1649
Bromm V., Loeb A., 2003, ApJ, 596, 34
Choi J.-H., Shlosman I., Begelman M. C., 2013, ApJ, 774, 149
Devecchi B., Volonteri M., 2009, ApJ, 694, 302
Dijkstra M., Haiman Z., Mesinger A., Wyithe J. S. B., 2008, MNRAS, 391, 1961
Djorgovski S. G., Volonteri M., Springel V., Bromm V., Meylan G., 2008, Proceedings of the MG11 Meeting, Part 1, The Eleventh Marcel Grossmann Meeting: On Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories. World Scientific Press, Singapore
Fan X., Strauss M. A., Schneider D. P., Becker R. H., White R. L., Haiman Z., Gregg M., 2003, AJ, 125, 1649
Fan X., Strauss M. A., Richards G. T., Hennawi J. F., Becker R. H., White R. L., Diamond-Stanic A. M., 2006, AJ, 131, 1203
Federrath C., Banerjee R., Clark P. C., Klessen R. S., 2010, ApJ, 713, 269
Haiman Z., 2004, ApJ, 613, 36
Haiman Z., 2013, in Wiklind T., Mobasher B., Bromm V., eds, Astrophysics and Space Science Library Vol. 396, The First Galaxies. Springer-Verlag, Berlin, p. 293
Hosokawa T., Omukai K., Yorke H. W., 2012, ApJ, 756, 93
Hosokawa T., Yorke H. W., Inayoshi K., Omukai K., Yoshida N., 2013, preprint (arXiv:1308.4457)
Inayoshi K., Omukai K., 2012, MNRAS, 422, 2539
Johnson J. L., Bromm V., 2007, MNRAS, 374, 1557
Johnson J. L., Khochar S., Greif T. H., Durier F., 2011, MNRAS, 410, 919
Johnson J. L., Whalen D. J., Fryer C. L., Li H., 2012, ApJ, 750, 66
Johnson J. L., Whalen D. J., Li H., Holz D. E., 2013, ApJ, 771, 116
Kormendy J., Richstone D., 1995, ARA&A, 33, 581
Krumholz M. R., McKee C. F., Klein R. I., 2004, ApJ, 611, 399
Latif M. A., Zaroubi S., Spaans M., 2011a, MNRAS, 411, 1659
Latif M. A., Schleicher D. R. G., Spaans M., Zaroubi S., 2011b, A&A, 532, A66
Latif M. A., Schleicher D. R. G., Schmidt W., Niemeyer J., 2013a, MNRAS, 430, L588
Latif M. A., Schleicher D. R. G., Schmidt W., Niemeyer J., 2013b, MNRAS, 433, 1607
Latif M. A., Schleicher D. R. G., Schmidt W., Niemeyer J., 2013c, ApJ, 772, L3
Lodato G., Narajna P., 2006, MNRAS, 371, 1813
Maier A., Iapichino L., Schmidt W., Niemeyer J. C., 2009, ApJ, 707, 40
McConnell N. J., Ma C.-P., Gebhardt K., Wright S. A., Murphy J. D., Lau R. T., Graham J. R., Richstone D. O., 2011, Nat, 480, 215
Mortlock D. J. et al., 2011, Nat, 474, 616
Oh S. P., Haiman Z., 2002, ApJ, 569, 558
Omukai K., 2001, ApJ, 546, 635
Omukai K., Schneider R., Haiman Z., 2008, ApJ, 686, 801
O’Shea B. W., Bryan G., Bower J., Norman M. L., Abel T., Harkness R., Kritsuk A., 2004, preprint (arXiv:astro-ph/0403044)
Portegies Zwart S. F., Baumgardt H., Hut P., Makino J., McMillan S. L. W., 2004, Nat, 428, 724
Rees M. J., 1984, ARA&A, 22, 471
Regan J. A., Haehnelt M. G., 2009a, MNRAS, 396, 343
Regan J. A., Haehnelt M. G., 2009b, MNRAS, 393, 858
Safranek-Shrader C., Agarwal M., Federrath C., Dubey A., Milosavljević M., Bromm V., 2012, MNRAS, 426, 1159
Schleicher D. R. G., Spaans M., Glover S. C. O., 2010, ApJ, 712, L69
Schleicher D. R. G., Palla F., Ferrara A., Galli D., Latif M., 2013, A&A, 558, A59
Schmidt W., Niemeyer J. C., Hillebrandt W., 2006, A&A, 450, 265
Shang C., Bryan G. L., Haiman Z., 2010, MNRAS, 402, 129
Smith R. J., Glover S. C. O., Clark P. C., Greif T., Klessen R. S., 2011, MNRAS, 414, 3633
Smith R. J., Hosokawa T., Omukai K., Glover S. C. O., Klessen R. S., 2012, MNRAS, 424, 457
Spaans M., Silk J., 2006, ApJ, 652, 902
Tanaka T., Haiman Z., 2009, ApJ, 696, 1798
The Enzo Collaboration et al., 2013, preprint (arXiv:e-prints)
Tremaine S. et al., 2002, ApJ, 574, 740
Turk M. J., Smith B. D., Oishi J. S., Skory S., Skillman S. W., Abel T., Norman M. L., 2011, ApJS, 192, 9
Van Borm C., Spaans M., 2013, A&A, 553, L9
van den Bosch R. C. E., Gebhardt K., Gültekin K., van de Ven G., van der Wel A., Walsh J. L., 2012, Nat, 491, 729
Volonteri M., 2010, A&AR, 18, 279
Wang P., Li Z.-Y., Abel T., Nakamura F., 2010, ApJ, 709, 27
Whalen D. J., Fryer C. L., 2012, ApJ, 756, L19
Whalen D. J. et al., 2013a, ApJ, 768, 195
Whalen D. J., Johnson J. L., Smith J., Meiksin A., Heger A., Even W., Fryer C. L., 2013b, ApJ, 774, 64
Wise J. H., Turk M. J., Abel T., 2008, ApJ, 682, 745
Wolkolt-Green J., Haiman Z., Bryan G. L., 2011, MNRAS, 418, 838

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