Analysis of Modal Characteristics of Long-Span Suspension Bridges with Ruled Surface Spatial Cables

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Abstract. In this paper, new concepts of single-leaf hyperboloid spatial cable suspension bridges and hyperbolic parabolic spatial cable suspension bridges are proposed. Combined with the 4000 m long-span Messina Strait Bridge, dynamic modal analyses of the spatial cable suspension bridges are carried out. The results show that the spatial suspension bridges have a torsional vibration mode appearing much later, a torsional-bending frequency ratio greatly improved, a critical wind speed for flutter greatly improved, and better wind stability, as compared with the parallel suspension bridge.

1. Introduction
With the increase of the span of suspension bridges, the critical wind speed for flutter decreases greatly, and the 2000m-span seems to be an insurmountable limit for the traditional suspension bridges with parallel cable systems. How to ensure sufficient wind stability of long-span suspension bridges is a difficult problem for long-span suspension bridges today [1-5].

The structural stiffness of long-span suspension bridges is mainly derived from the main cables, so the focus of improving the overall stiffness of the suspension bridges should be on the main cables. There are mainly 3 ways to improve the wind resistance of the structure: improving the overall stiffness of the structure, controlling the vibration characteristics of the structure and improving the aerodynamic characteristics of the section [6-10].

The single-leaf hyperboloid and hyperbolic paraboloid, as shown in figures 1 and 2, are typical quadric ruled surfaces, which may be composed of two families of straight lines, and have important application values in buildings to form building frames. Buildings with ruled surfaces possess excellent mechanical properties. A spatial cable network is constructed with characteristics of the ruled surface.

![Figure 1. Univalent hyperboloid cable.](image1)

![Figure 2. Hyperbolic paraboloid cable.](image2)
With the Messina Strait Bridge as the background, analyses of the dynamic modal characteristics of the 4000m long-span spatial cable suspension bridge were carried out, to explore the wind stability of the single-leaf hyperboloid and hyperbolic paraboloid spatial cable suspension bridges, for opening up a new way of thinking, for the convenience of future construction of long-span suspension bridges.

2. Finite Element Model
Finite element models are shown in figure 3. According to the design of the 4000 m Messina Bridge, the span ratio is 1/10, the sling spacing is 40m, the full bridge deck width is 60.0 m, the sub-span suspension bridge is 1360 m+4000 m+1360 m, each side of the bridge deck is a one-way 3-lane highway part and the middle part of the bridge deck is a double-track railway, and two spatial cable systems of single-leaf hyperboloid and hyperbolic paraboloid respectively are adopted in order to meet the wind stability of the bridge.

The suspension bridge with the single-leaf hyperboloid spatial cable system adopts the triangular arrangement method, with a total of 24 cables, each having a diameter of 0.8 m; the suspension bridge with the hyperbolic parabolic spatial cable system has a total of 18 cables, each having a diameter of 1.0 m; and the suspension bridge with parallel cables has 2 cables, each having a diameter of 2.7 m according to the equal area principle. The main cable is made of steel, with a standard strength of 2000 Mpa, an elastic modulus of 1.95*10^5 Mpa, a weight of 78.5 kN/m, and a Poisson's ratio of 0.3.

The top ring beam of the spatial cable suspension bridge adopts an elliptical form, with a long axis dimension of 100 m, and a short shaft dimension of 50 m. The middle ring beam consists of a series of elliptical steel ring beams of different sizes, where the waist ellipse has a long axis of 50 m and a short axis of 25 m, and the single-leaf hyperboloid spatial cable and the middle ring beam are firmly connected through a special clamp to form a structure system of the spatial cable suspension bridge.

3. Dynamic Modal Analysis
Considering that the super-long-span suspension bridge across the strait is huge, MIDAS was used to carry out the initial analysis of the suspension bridge to determine the shape of the bridge and the initial stress of the cable, and then the fine analysis was performed by ANSYS, where the modal analysis was carried out by the iterative method. Some typical vibration modes of the spatial suspension bridges and the parallel suspension bridge are given in figures 4, 5 and 6 respectively. The typical frequencies and modal characteristics of the three bridges are shown in tables 1, 2 and 3.
Figure 4. Vibration modes of single-leaf hyperboloid spatial cable suspension bridge.

Table 1. Natural frequencies and modes of single-leaf hyperboloid spatial cable suspension bridge.

| Order | Frequency (Hz) | Modal shape          | Order | Frequency (Hz) | Modal shape          |
|-------|----------------|----------------------|-------|----------------|----------------------|
| 1     | 0.027465       | Primary transverse bending | 11    | 0.134836       | Transverse bending   |
| 2     | 0.04191        | Transverse bending    | 12    | 0.140725       | Stiffening beam      |
| 3     | 0.051054       | Stiffening beam       | 13    | 0.148272       | Vertical displacement|
| 4     | 0.058772       | Transverse bending    | 14    | 0.164108       | Vertical bending     |
| 5     | 0.076872       | Vertical bending      | 15    | 0.172719       | Transverse bending   |
| 6     | 0.079975       | Transverse bending    | 16    | 0.188972       | Vertical bending     |
| 7     | 0.095256       | Vertical bending      | 17    | 0.205847       | Transverse bending   |
| 8     | 0.096099       | Transverse bending    | 18    | 0.211389       | Primary torsion      |
| 9     | 0.106183       | Primary torsion       | 33    | 0.336883       | Torsional            |
| 10    | 0.111906       | Primary transverse bending | 34    | 0.339503       | Torsional            |

Figure 5. Vibration modes of hyperbolic paraboloid spatial cable suspension bridge.
Table 2. Natural frequencies and vibration modes of hyperbolic paraboloid spatial cable suspension bridge.

| Order | Frequency (Hz) | Modal shape                      | Order | Frequency (Hz) | Modal shape                      |
|-------|----------------|----------------------------------|-------|----------------|----------------------------------|
| 1     | 0.026228       | Primary transverse bending       | 11    | 0.088938       | Transverse bending               |
| 2     | 0.040039       | Transverse bending               | 12    | 0.093262       | Stiffening beam Vertical displacement |
| 3     | 0.042648       | Stiffening beam Vertical displacement | 13    | 0.097241       | Transverse bending               |
| 4     | 0.042712       | Transverse bending               | 14    | 0.101470       | Vertical bending                 |
| 5     | 0.053685       | Vertical bending                 | 15    | 0.107000       | Transverse bending               |
| 6     | 0.060571       | Transverse bending               | 16    | 0.109820       | Vertical bending                 |
| 7     | 0.062198       | Vertical bending                 | 17    | 0.111880       | Transverse bending               |
| 8     | 0.070245       | Transverse bending               | 18    | 0.113970       | Primary torsion                  |
| 9     | 0.082209       | Primary torsion                  | 27    | 0.202940       | Torsional                        |
| 10    | 0.083308       | Primary transverse bending       | 28    | 0.211810       | Torsional                        |

Figure 6. Vibration modes of parallel suspension bridge.

Table 3. Natural frequencies and vibration modes of parallel suspension bridge.

| Order | Frequency (Hz) | Modal shape                      | Order | Frequency (Hz) | Modal shape                      |
|-------|----------------|----------------------------------|-------|----------------|----------------------------------|
| 1     | 0.026539       | Primary transverse bending       | 11    | 0.095823       | Transverse bending               |
| 2     | 0.041035       | Transverse bending               | 12    | 0.10567        | Stiffening beam Vertical displacement |
| 3     | 0.049206       | Stiffening beam Vertical displacement | 13    | 0.109286       | Transverse bending               |
| 4     | 0.057457       | Transverse bending               | 14    | 0.126261       | Vertical bending                 |
| 5     | 0.064962       | Vertical bending                 | 15    | 0.12894        | Transverse bending               |
| 6     | 0.075142       | Transverse bending               | 16    | 0.13773        | Vertical bending                 |
| 7     | 0.075322       | Vertical bending                 | 17    | 0.146811       | Transverse bending               |
| 8     | 0.088758       | Transverse bending               | 18    | 0.160321       | Primary torsion                  |
| 9     | 0.091135       | Primary torsion                  | 19    | 0.163044       | Primary torsion                  |
| 10    | 0.091512       | Primary transverse bending       | 20    | 0.165603       | Torsional                        |
By comparing the results of the natural vibration characteristics of the above three types of bridges, we can come to the following conclusions.

(1) The first few modes of the spatial cable suspension bridge and the parallel cable suspension bridge are similar. The spatial suspension bridge has a slightly higher frequency at the first few orders than the parallel suspension bridge.

(2) From the graphs of vibration modes, the cable vibration of the parallel suspension bridge appears at the fifth order. However, the 24 cables of the hyperboloid suspension bridge are connected as a whole. The coordinated vibration of the cable beam system strengthens the stability of the whole structure of the suspension bridge.

(3) The torsional vibration mode appears in the parallel suspension bridge at the eleventh order and the twentieth order with frequencies of 0.095823 Hz and 0.165603 Hz respectively, in the hyperbolic paraboloid spatial suspension bridge at the twenty-seventh order and the twenty-eighth order with frequencies of 0.20294 Hz and 0.21181 Hz respectively, and in the single-leaf hyperboloid spatial suspension bridge at the thirty-third order and the thirty-fourth order with frequencies of 0.336883 Hz and 0.339503 Hz respectively. The spatial suspension bridges have the torsional vibration mode appearing much later than the parallel suspension bridge.

(4) The torsional-bending frequency ratio of the single-leaf hyperboloid spatial suspension bridge, the hyperbolic paraboloid spatial suspension bridge, and the parallel suspension bridge is 4.38, 3.78, and 1.28, respectively. The greater the torsional-bending frequency ratio is, the better the static wind stability and flutter stability of the suspension bridges are, indicating that the spatial suspension bridges have good wind resistance.

4. Analysis of Flutter Stability
Flutter is self-excited divergent instability of structures under the action of wind. In this paper, the flutter stability of suspension bridges is analyzed with the Selberg equation [4], which is used to calculate the critical wind speed for flutter of the separated flow.
Selberg equation:

\[ V_{cr} = \eta_s \eta_\alpha \omega_t b \sqrt{\frac{r}{b_1}} \mu \left[ 1 - \left( \frac{\omega_t}{\omega_c} \right)^2 \right] \]

In the equation (1), \( \eta_s \) is the main beam section shape coefficient, \( \eta_\alpha \) is the attack angle effect coefficient of wind, and \( \eta_s \) and \( \eta_\alpha \) are taken as 1 for the flat section at 0° wind attack angle. \( r \) is the radius of gyration of the bridge section (the stiffening beam and the main cable), \( b_1 \) is the half bridge width of the section of the stiffening beam, \( \mu \) is the ratio of bridge density to air density, and \( \omega_c \) and \( \omega_t \) stand for the lowest order torsional frequency and vertical circle frequency, respectively.

\( V_{cr} \) and \( V_{cr}^p \) denote the critical wind speed for flutter of the suspension bridge with a hyperboloid spatial cable system and the suspension bridge with a parallel cable system, respectively; \( \omega_t^d \) and \( \omega_t^p \) denote the lowest order torsional frequency and vertical bending frequency respectively, of the suspension bridge with a single-leaf hyperboloid spatial cable system; and \( \omega_t^p \) and \( \omega_t^p \) denote the lowest order torsional frequency and vertical bending frequency respectively, of the suspension bridge with a parallel cable system. The same coefficients are omitted to obtain equations (2) and (3):

\[ \frac{V_{cr}^d}{V_{cr}^p} = \frac{\omega_t^d \sqrt{1 - \left( \frac{\omega_t^d}{\omega_t^p} \right)^2}}{\omega_t^p \sqrt{1 - \left( \frac{\omega_t^p}{\omega_t^p} \right)^2}} = 0.336883 \times \sqrt{\frac{1 - \left( \frac{0.075142}{0.095823} \right)^2}{1 - \left( \frac{0.076872}{0.336883} \right)^2}} = 5.51 \]

\[ \frac{V_{cr}^p}{V_{cr}^p} = \frac{\omega_t^p \sqrt{1 - \left( \frac{\omega_t^p}{\omega_t^p} \right)^2}}{\omega_t^p \sqrt{1 - \left( \frac{\omega_t^p}{\omega_t^p} \right)^2}} = 0.095823 \times \sqrt{\frac{1 - \left( \frac{0.076872}{0.095823} \right)^2}{1 - \left( \frac{0.076872}{0.336883} \right)^2}} = 5.51 \]
\[ V_{cr}^{s} = \sqrt{\frac{1 - (\frac{\omega_0^s}{\omega_t})^2}{1 - (\frac{\omega_0^p}{\omega_t})^2}} \times \frac{0.20294 \times 0.20294}{0.075142 \times 0.095823} = 3.29 \]

Therefore, compared with the suspension bridge with a parallel cable system, the critical wind speed for flutter of the single-leaf cable hyperboloid spatial cable suspension bridge and the hyperbolic paraboloid spatial cable suspension bridge has greatly improved by 451% and 229%, respectively.

Referring to the design of Messina Bridge, according to results of the wind tunnel test, \( r=25.65 \text{ m} \) and \( \mu=45.87 \) can be obtained for the suspension bridge.

Therefore, the critical wind speed for flutter of the single-leaf hyperboloid spatial cable suspension bridge, the hyperbolic parabolic spatial cable suspension bridge and the parallel cable suspension bridge is: \( V_{cr}^{s}=388.54 \text{ m/s}, V_{cr}^{p}=231.98 \text{ m/s} \) and \( V_{cr}^{p}=70.51 \text{ m/s} \), respectively.

5. Conclusions

In this paper, ANSYS finite element models of 4000 m spatial cable suspension bridges and traditional parallel cable suspension bridges are established. By analyzing the dynamic modal characteristics and flutter stability of the three types of bridges, we come to main conclusions as follows.

(1) The spatial cable suspension bridges have good integrity. The parallel suspension bridge has a vibration mode dominated by the main cable vibration at the fifth order, while the suspension bridges with the single-leaf hyperboloid and hyperbolic parabolic spatial cable systems always maintain coordinated vibration.

(2) The spatial cable suspension bridges have greater torsion resistance. The torsional vibration mode appears in the parallel suspension bridge at the eleventh order and the twentieth order, in the single-leaf hyperboloid spatial suspension bridge at the thirty-third order and the thirty-fourth order, and in the hyperbolic paraboloid spatial suspension bridge at the twenty-seventh order and the twenty-eighth order. The corresponding torsional frequency of the spatial suspension bridges is greatly improved.

(3) The torsional-bending frequency ratio of the spatial suspension bridge, the hyperbolic paraboloid spatial suspension bridge, and the parallel suspension bridge is 4.38, 3.78, and 1.28, respectively. The greater the torsional-bending frequency ratio is, the better the wind resistance is.

(4) The critical wind speed for flutter of the single-leaf hyperboloid spatial cable suspension bridge, the hyperbolic parabolic spatial cable suspension bridge and the parallel cable suspension bridge is: \( V_{cr}^{s}=388.54 \text{ m/s}, V_{cr}^{p}=231.98 \text{ m/s} \) and \( V_{cr}^{p}=70.51 \text{ m/s} \), respectively. The flutter stability of the spatial cable suspension bridges has been greatly improved.

(5) The single-leaf hyperboloid spatial cable suspension bridge and the hyperbolic parabolic spatial cable suspension bridge have good wind stability. These novel spatial cable suspension bridges are expected to be useful in the construction of 3000-5000 m super-long-span suspension bridges.

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