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An analytical model of transducer array arrangement for guided wave excitation and propagation on cylindrical structures

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Abstract. Ultrasonic guided wave (GW) inspection is one of the non-destructive testing (NDT) techniques available for the engineering structures. Compared with other NDT techniques, guided waves can propagate a long distance with a relatively high sensitivity to defects in the structure. In order to increase the performance for pipe inspections to meet higher requirements under different conditions, the optimisation of piezoelectric transducer array design is still a need, as the technique is currently subject to a complex analysis due to wide number of guided wave modes generated. This can be done by optimising the transducer array design. In this paper, it is described an analytical mode of a set of piezoelectric transducer arrays upon torsional wave mode T(0,1) excitation in a tubular structure. The proposed analytical model for predicting signal propagation is validated by using finite element analysis in ABAQUS and three-dimensional laser vibrometer experiments for transducer array characterisations. The proposed analytical model works well and very fast for simulating transducer excitation and wave propagation along cylindrical structures. This will significantly reduce the complexity of guided wave analysis, enhancing effectively the structural health of structures and subsequently reducing the industry maintenance cost.

1. Introduction

Ultrasonic guided waves can propagate a long distance with a lower operation frequency range for defect detection when compared with other non-destructive testing techniques. Currently, a 3%-9% [1] of cross-sectional area reduction of a pipe wall-thickness can be detected by using guided waves for pipeline inspections. However, the higher level of sensitivity in pipe inspection is still an area of interest for industry. To develop guided wave technique for defect detection, the method for predicting signal is proposed to simulate guided wave excitation and propagation by a set of transducers in a tubular structure. This will significantly reduce the complexity of guided wave analysis, enhancing effectively the structural health of structures and subsequently reducing the industry maintenance cost. It exists many wave modes at a frequency in cylindrical structures, including axisymmetric and non-axisymmetric wave modes. Gazis [2] has investigated the propagation of fundamental axisymmetric wave modes and higher-order non-axisymmetric flexural type wave modes in pipes. Silk and Bainton [3] have introduced a nomenclature of $X(m,n)$ to describe these wave modes, in which $X$ represents
wave mode characteristics, namely L, T and F for longitudinal, torsional and flexural, respectively, \( m \) indicates the circumferential order, and \( n \) indicates the mode order of occurrence. The core objective of guided wave testing is to only generate axisymmetric guided waves for defect detection, such as the longitudinal wave modes L(0,1) and L(0,2) or torsional wave mode T(0,1). Generally, a set of circumferential arrays or “rings” are clamped around the surface area of a pipe in guided wave testing. To suppress non-axisymmetric wave modes, the transducers are equally spaced around the cylindrical structure for the generation of a pure axisymmetric wave mode [4]. Then, the transmitted signal can form unidirectional propagation by using multiple rings [5]. It needs each ring with 360 degrees uniform loading in the circumferential direction. However, many commercial tools have gaps between their transducers. As a result, the axisymmetric wave mode is generated with some other non-axisymmetric wave modes by the excitation of transducers with non-uniform transducer spacings. This problem can be solved by developing transducer array design. In this paper, an analytical model is proposed for predicting the transmitted signal propagates along an 8-inch schedule 40 steel pipe. The number of transducers used to generate different wave modes are investigated. The excitation and propagation of guided waves are modelled and evaluated by using the analytical method in Matlab. Angular profiles are used to describe the normalised amplitude distributions of the circumferential displacement on the pipe model. Furthermore, the related finite element (FE) models in ABAQUS [6-8] are simulated to compare the predictive simulations and experimental measurements by using a three-dimensional laser vibrometer [9-10] which are undertaken to validate the analytical model.

2. Analytical model of transducer array design

2.1. Dispersion curves

The characteristics of guided wave modes are described through dispersion curve diagrams as shown in Figures 1(a)-(b). It can be described as an example set of phase and group velocity dispersion curves for an 8-inch (219.1 mm outer diameter) and schedule 40 (8.18 mm wall thickness) steel pipe at the frequency range, from 0 to 80 kHz. These are calculated numerically using a semi-analytical finite element (SAFE) method [11-13].

![Dispersion Curve Diagram](attachment:image.png)
In this paper, the torsional type wave mode T(0,1) is excited to study guided waves excitation and propagation along cylindrical structures because the phase and group velocities for torsional wave mode T(0,1) have an invariable value in frequencies. Therefore, the fundamental wave mode T(0,1) will propagate without dispersion effect in time domain. However, some torsional-flexural guided wave modes \( F(m,2) \) (\( m = 1, 2, 3, \ldots \) as its ‘family’) will be generated simultaneously when the higher-order wave modes are not completely cancelled. As a result, the wave modes are superposed over time with a similar velocity. The wave speeds for all generated wave modes depend on the properties of structural materials. In the analytical models, characteristic velocities are compressional (longitudinal) bulk wave velocity \( c_L = 5960 \) m/s and shear (torsional) bulk wave velocity \( c_T = 3260 \) m/s of the waveguide as shown in Figure 1.

2.2. Incident signal and source influence

In order to inspect reflections from features on structures for ultrasonic guided wave testing, the excitation of the incident signal is often a pulse with several cycles. With an increased number of cycles, it can reduce the frequency bandwidth of the pulse so that the incident wave is less dispersive for propagating over a distance. In general, wave modes are created by a loading source in any direction and they can be undertaken by a Hanning windowed signal, given by [14]

\[
u(t) = \frac{1}{2} [1 - \cos \left( \frac{2\pi f_c t}{n_c} \right)] \sin (2\pi f_c t) \tag{1}\]

where \( t \) is the time, \( f_c \) is the centre frequency of the incident signal, and \( n_c \) is the number of cycles for the Hanning windowed signal. Generally, narrow band signals are used in a 5-cycle or 10-cycle Hanning window [15]. Therefore, a 10-cycle, 60 kHz Hanning windowed signal as the incident pulse can be selected to excite torsional wave mode T(0,1). Thus, its waveform is calculated by using equation (1) and the normalised displacement amplitude in time domain is presented in Figure 2.
Figure 2. The incident signal pulse, a 10-cycle 60 kHz Hanning windowed signal in time domain.

Figure 3. Transducer arrays design with a set of piezoelectric transducers clamped the surface of a pipe model.

Loading area is one of the excitation conditions to determine a specific wave group. If a shear loading is uniformly distributed with the excitation area on the pipe surface in Figure 3. The loading region is $2\alpha$ (circumferential length) \times $2L$ (axial length). The related functions using a normal mode expansion (NME) method are explained in Ditri and Rose [16]. Applying the loading conditions give the stress amplitude of any wave mode in the positive propagation direction. Rose [17] describes that the weighting function $A^{mn}$ of non-axisymmetric wave modes can be calculated by using a normalised weighting function, given by:

$$A_{m}^{nn} = A_{0m}^{nn} \cdot \frac{\sin(m\alpha)}{ma} \cdot \frac{\sin(k^{nn} - k_0^{nn})}{(k^{nn} - k_0^{nn})L} \cdot m, n = 1, 2, 3,...$$  \hspace{1cm} (2)
where \( A_0^{mn} = 1 \) (the amplitude factor of the axisymmetric wave mode T(0,1)), and \( k_0^{on}, k_0^{mn} \), the angular wavenumber, which is based on the corresponding frequency and phase velocity of the wave mode at that frequency. Thus, the amplitude factors for all generated torsional-type flexural wave modes can be determined in equation (2).

2.3. Dispersive wave propagation and linear superposition analysis

The received distance-trace can be evaluated at any distance of wave propagation by considering the phase shift of frequency components. Wilcox [18] has described dispersive propagation of guided wave along structures in equation (3), as given by:

\[
g(z) = \int_{-\pi}^{\pi} U(\omega)e^{-ik(\omega)z} d\omega
\]  

(3)

where \( U(\omega) \) is the fast Fourier transform (FFT) of the received time-trace \( u(t) \) from equation (1), \( k(\omega) \) is the angular wavenumber as a function of angular frequency, \( \omega \) and \( z \) is the propagation distance of the guided wave which is measured from the source. The function of particle displacement amplitude at any position of cylindrical structures in the circumferential direction, is given by:

\[
A_d = \cos(m\theta)
\]  

(4)

where \( A_d \) is particle displacement amplitude and the function of \( \cos(m\theta) \) is the angular distribution function of the particle displacement created by wave modes in the circumferential direction. For \( m = 0 \), the mode is axisymmetric torsional, according to the waveform from equation (1). For \( m > 0 \), the mode is torsional-type flexural. The propagation of all generated wave modes along the 8-inch schedule 40 steel pipe can be determined by dispersion curves in Figure 1. For a linear superposition analysis of guided waves propagation along cylindrical structures, the angular profiles are obtained by summing up the angular profile of each mode in a family, wave modes T(0,1) and F(m,2) with their weighting amplitude factors. Therefore, when a set of transducers are excited synchronously as shown in Figure 3, a polar plot of the circumferential displacements at any distance from the transducer array can be calculated by linearly superposing all generated wave modes. Moreover, the received distance-trace can also be converted to time-trace depending on its inverse fast Fourier transform (IFFT).

3. Results

The purpose of transducer array arrangement for guided wave testing is to generate axisymmetric guided waves only on cylindrical structures. That is an ideal condition if a transducer can produce 360 degrees uniform loading in the circumferential direction. However, many commercial pipe inspection tools consist of a set of independent piezoelectric transducers. In this paper, a thickness-shear d15 element (PZT-5H) is used for empirical work. The excitation region of the transducer is 13 mm x 3 mm and it is clamped around the pipe surface. Therefore, the analytical simulations for a group of transducers excitation are calculated for predicting guided waves propagation along a pipe. A Hanning windowed pulse of 10 cycles is used so that a transducer array is excited at a bandwidth of 12 kHz with the centre frequency 60 kHz synchronously to generate guided waves in an 8-inch schedule 40 steel pipe. There are 10 transducers in a ring excited simultaneously. The circumferential length of each source is 6.8 degrees and the spacing between two transducers is approximately 32.4 degrees. Figure 4 illustrates that the angular profile simulations of torsional type wave modes T(0,1) and its family F(m,2) superposition are plotted at four axial distances, including \( z = 0 \) m, 0.1 m, 0.25 m and 0.5 m. The distribution of normalised displacement amplitudes in the circumferential direction is
described by using a polar plot. Results show that torsional-type flexural wave modes have not been completely cancelled by using the 10 transducers with the transducer spacing (approximately 32.4 degrees) in the circumferential direction. The models are used to excite a group of non-axisymmetric mode orders from \( m = 1 \) to 12 at the centre frequency 60 kHz as shown in Figure 1(a). To reduce the undesired wave modes, one can increase the excitation region of the transducer or the number of transducers [19] in order to make an axisymmetric wave for pipeline inspections.

![Angular profile simulations of 60 kHz wave modes T(0,1) and F(m,2) in an 8-inch schedule 40 steel pipe at several axial distances from 10 transducers in a ring, including (a) \( z = 0 \) m, (b) \( z = 0.1 \) m; (c) \( z = 0.25 \) m; and (d) \( z = 0.5 \) m.](image)

**Figure 4.** Angular profile simulations of 60 kHz wave modes T(0,1) and F(m,2) in an 8-inch schedule 40 steel pipe at several axial distances from 10 transducers in a ring, including (a) \( z = 0 \) m, (b) \( z = 0.1 \) m; (c) \( z = 0.25 \) m; and (d) \( z = 0.5 \) m.

4. Validations

To verify the analytical method, the FE modelling setup is illustrated by the commercial software ABAQUS Standard/Explicit module for simulating guided wave propagation along an 8-inch schedule 40 steel pipe. The related numerical results are to compare the predictive results from the analytical model. In each case, the pipe model is 4.45 m long and the excitation is applied over 24 points in a ring with a 33 degrees gap between the start and end transducers at the top of the pipe model. The spacing between the other transducers is approximately 14.2 degrees. The transducer array consists of three rings with a 30 mm ring spacing which is clamped at one end. Moreover, the 24 receiving points
spaced equally around the pipe are located at 2 m away from the pipe end. The excitation is in the tangential direction at each point and the three rings are designed to excite wave modes by using the inverted signal and time-delay method for cancelling backward signal [4]. In the finite element models, the Explicit module is used to produce a wave field of transient elastic wave. The property of the steel pipe is defined as a linear isotropic material with Young's modulus $E = 216.9$ GPa, mass density $\rho = 7932$ kg/m$^3$ and Poisson's ratio $\nu = 0.2865$. A total of 829,830 hexahedral elements are generated for the finite element mesh, it takes around 60 mins to complete for each case. The approximate global size of 3.2 mm is defined to satisfy at least 8 elements per wavelength for accurate results and the elements were 8-node linear bricks with reduced integration (element type C3D8R). Furthermore, a test rig is designed for empirical work and the experimental measurements are carried out by using a three-dimensional laser vibrometer system as shown in Figure 5. The experimental rig is composed of 24 PZT-5H patches in each ring, which are excited in the thickness-shear direction. The excitation region 13 mm × 3 mm is used and the loading in the circumferential direction is to generate the torsional wave mode T(0,1) for the experimental validation.

![Figure 5. Set up of laser vibrometer experiment](image)

![Figure 6. Normalised power amplitudes of the transducer arrays with a 30 mm ring spacing in frequency domain, analytical prediction, experiment.](image)
The power amplitude of the transducer arrays with the 30 mm ring spacing is evaluated by using analytical simulations. Figure 6 shows the predictive results are validated by experimental measurements at a range of frequencies, is increased from 35 kHz to 65 kHz in a series of steps. The two sets of results for normalised power amplitudes agree well with each other in different frequencies.

![Figure 6](image)

Figure 6. Angular profile results for normalised power amplitudes of transducers in 3 rings of transducer arrays with 32 transducers in each ring with a 33-degree gap, including (a) 30 kHz, (b) 40 kHz, (c) 50 kHz, and (d) 60 kHz.

Figure 7. Angular profile results of torsional type wave modes in an 8-inch schedule 40 steel pipe at different frequencies from transducer arrays with 3 rings, 24 transducers in each ring with a 33-degree gap, including (a) 30 kHz, (b) 40 kHz, (c) 50 kHz, and (d) 60 kHz. Analytical simulations, experimental measurements.

Figure 7 illustrates the distribution of circumferential displacement amplitudes of torsional-type wave modes T(0,1) and F(m,2) at the axial distance 2 m away from the pipe end by using a polar plot. The related results are produced by using analytical simulations, finite element models and experimental measurements. A 10-cycle Hanning windowed signal pulse is transmitted along the pipe length at the centre frequencies 30 kHz, 40 kHz, 50 kHz, and 60 kHz. Results show that the quasi-axisymmetric waves are generated because of the non-axisymmetric transducer spacing, but there is excellent agreement between these results.
5. Conclusions
In this paper, an analytical model is proposed, and it works well and very fast for simulating one single transducer or a set of transducer array excitation at different frequencies and the generated wave propagation along cylindrical structures. The purpose of the work is to develop the analytical model for the prediction of guided wave propagation. This will effectively help to develop transducer performance and subsequently reducing the manufacturing cost. The proposed analytical model provides faster solutions to three-dimensional guided wave problems for a long cylindrical structure when compared to computational analysis of finite element models. The analytical method is compared with finite element modelling and then validated by experimental data. The models in all cases agree well with each other.

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