INSTANTON MODEL OF LIGHT HADRONS: FROM LOW TO HIGH ENERGIES *

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Instanton liquid model being effective model of the QCD vacuum describes well the hadrons at low as well as intermediate energies. Thus, contact with perturbative QCD results is possible providing the unique information about the quark-gluon distribution functions in the QCD vacuum and hadrons at low energy normalization point. As an illustrative example we consider the pion transition form factor for the process $\gamma^*\gamma^* \rightarrow \pi^0$ at space-like values of photon momenta. The leading and next-to-leading order power asymptotics of the form factor and the relation between the light-cone pion distribution amplitudes of twists 2 and 4 and the dynamically generated quark mass are found.

The interest to the pion transition form factor has recently revived due to its measuring by CLEO collaboration\(^1\) at large virtuality of one of the photons. The knowledge of the form factor at arbitrary photon virtualities would provide important information about the hadron contribution to the muon anomalous moment $g - 2$ and some other processes. Theoretically, the pion form factor $M_{\pi^0}(q_1^2, q_2^2)$ for the transition process $\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0(p)$, where $q_1$ and $q_2$ are photon momenta, is related to fundamental properties of QCD dynamics at low and high energies. At zero photon virtualities the value of the form factor and its slope (radius) is estimated within the chiral perturbative theory. In the opposite limit of large photon virtualities the leading momentum power dependence\(^2\) of the form factor supplemented by small radiative\(^3\) and power corrections\(^4\) is dictated by

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perturbative QCD (pQCD).

In the following, we discuss the approach that allows us to match these extremes and describe the intermediate energy region. This approach describes quark-meson dynamics within the effective model, where the quark-quark interaction induced by instanton exchange leads to spontaneous breaking of the chiral symmetry. It dynamically generates the momentum dependent quark mass \( M(k^2) \) that may be related to the nonlocal quark condensate. Specifically, we find the pion transition form factor in wide kinemtical region up to moderately large photon virtualities and extract from its asymptotics the pion distribution amplitudes (DAs) at normalization scale typical for hadrons.

The invariant amplitude for the process \( \gamma^* \gamma^* \rightarrow \pi^0 \) is given by

\[
A \left( \gamma^* (q_1, \epsilon_1) \gamma^* (q_2, \epsilon_2) \rightarrow \pi^0 (p) \right) = -ie^2 \varepsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu q_1^\rho q_2^\sigma M_{\pi^0} (q_1^2, q_2^2),
\]

where \( \epsilon_i^\mu (i = 1, 2) \) are the photon polarization vectors. Consider first the low energy region. With both photons real \( (q_i^2 = 0) \) one finds the result

\[
M_{\pi^0} (0, 0) = \frac{N_c}{6\pi^2 f_\pi} \int_0^\infty du \frac{uM(u) [M(u) - 2uM'(u)]}{D^3(u)} = \frac{1}{4\pi^2 f_\pi},
\]

where \( D(u) = u + M^2(u) \) and \( M'(u) = \frac{d}{du} M(u) \), consistent with the chiral anomaly and independent of the shape of \( M(k^2) \). Below, for the numerical analysis we choose the dynamical quark mass profile in the Gaussian form

\[
M_Q(k^2) = M_q \exp \left( -2k^2/\Lambda^2 \right),
\]

where we take \( M_q = 350 \) MeV and fix \( \Lambda = 1.29 \) GeV from the pion weak decay constant, \( f_\pi = 92.4 \) MeV. We also consider the shape given by the quark zero modes (z.m.) in the instanton field: \( M_I(k^2) = M_Q Z^2(k\rho) \), where \( Z(k\rho) = 2z [I_0(z)K_1(z) - I_1(z)K_0(z)] - I_1(z)K_1(z)/z \), with \( \rho = 1.7 \) GeV\(^{-1}\) being the inverse mean instanton radius and \( M_Q = 345 \) MeV. The mean square radius of the pion for the transition \( \gamma^* \pi^0 \rightarrow \gamma \) is found to be \( r_{\pi\gamma}^2 \approx 1/(2\pi^2 f_\pi^2) \) and is almost independent on the form of \( M(k^2) \).

At large photon virtualities \( Q^2 = -(q_1^2 + q_2^2) \) the model calculations reproduce the pQCD factorization result \((\omega = (q_1^2 - q_2^2)/(q_1^2 + q_2^2)) \)

\[
M_{\pi^0}(q_1^2, q_2^2)|_{Q^2 \rightarrow \infty} = J^{(2)} (\omega) \frac{1}{Q^2} + J^{(4)} (\omega) \frac{1}{Q^4} + O(\frac{1}{Q^6}) + O(\frac{1}{Q^6}).
\]

The leading and next-to-leading order asymptotic coefficients

\[
J^{(2)} (\omega) = \frac{1}{3} f_\pi \int_0^1 dx \frac{\phi_\pi^{(2)}(x)}{1 - \omega^2 (2x - 1)^2},
\]

\[
J^{(4)} (\omega) = \frac{1}{3} f_\pi \Delta^2 \int_0^1 dx \frac{1 + \omega^2 (2x - 1)^2}{[1 - \omega^2 (2x - 1)^2]^2} \phi_\pi^{(4)}(x)
\]
are expressed in terms of the light-cone pion distribution amplitudes (DA), \( \varphi_\pi(x) \), that are predicted by the model at the low normalization scale \( \mu^2 \sim \Lambda^2 \sim \rho^{-2} \) (Fig. 1, for Gaussian \( M(k^2) \))

\[
\varphi_\pi^{(2)}(x) = \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty du \cdot \\
\frac{F(u + i\lambda \pi, u - i\lambda \pi)}{D(u - i\lambda \pi) D(u + i\lambda \pi)} [xM(u + i\lambda \pi) + (x \leftrightarrow \pi)], \quad (4)
\]

\[
\varphi_\pi^{(4)}(x) = \frac{1}{\Lambda^2} \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty du \cdot \\
uF(u + i\lambda \pi, u - i\lambda \pi) \frac{2}{D(u - i\lambda \pi) D(u + i\lambda \pi)} [xM(u + i\lambda \pi) + (x \leftrightarrow \pi)], \quad (5)
\]

where \( F(u, v) = \sqrt{\frac{M(u)}{M(v)}} \). The parameter \( \Delta^2 \) characterizing the scale of the power corrections in the hard exclusive processes is

\[
\Delta^2 = \frac{N_c}{4\pi^2 f_\pi^2} \int_0^\infty du \cdot \frac{u^2 M(u)(M(u) + \frac{1}{2}uM'(u))}{D^2(u)}, \quad (6)
\]

Its value is predicted \( \Delta^2 = 2.41(2.74)\pi^2 f_\pi^2 \) for Gaussian (zero mode) shape of \( M(k^2) \), correspondingly. As it is clear from Fig. 1, the leading order pion DA, \( \varphi_\pi^{(2)}(x) \), is close to the asymptotic form that is in agreement with the results obtained previously in\(^8\,9\). In the leading order the similar results within the instanton model have been derived earlier in\(^10\).

The asymptotic coefficients \( J(\omega) \) may be written in the form

\[
J^{(2)}(\omega) = -\frac{1}{\pi^2 f_\pi} \int_0^\infty du \int_0^\infty dv \cdot \\
\left\{ \frac{M^{1/2}(z_-)}{D(z_-)} \frac{\partial}{\partial z_+} \left[ \frac{M^{3/2}(z_+)}{D(z_+)} \right] + (z_- \leftrightarrow z_+) \right\}, \quad (7)
\]

\[
J^{(4)}(\omega) = \frac{2}{\pi^2 f_\pi} \int_0^\infty du \int_0^\infty dv \cdot \\
\left\{ \frac{M^{1/2}(z_-)}{D(z_-)} \frac{M^{3/2}(z_+)}{D(z_+)} + u \frac{\partial}{\partial z_+} \left[ \frac{M^{3/2}(z_+)}{D(z_+)} \right] + (z_- \leftrightarrow z_+) \right\}, \quad (8)
\]

where \( z_\pm = u \pm v(1 \pm \omega) \). With the model parameters given above we find for the process \( \gamma^* \gamma^* \to \pi^0 \) the values \( J^{(2)}(\omega = 1) = 1.83(2.13)f_\pi \) consistent with the CLEO fit \( J^{(2)}_{\text{CLEO}}(1) = (1.74 \pm 0.32)f_\pi \) and the power correction \( J^{(4)}(1) / J^{(2)}(1) = 2.97(3.62)\pi^2 f_\pi^2 \).

The model form factors presented in Fig. 2 (for Gaussian \( M(k^2) \)) take into account the perturbative \( \alpha_s(Q^2) - \) corrections\(^3\) to the leading twist-2
term with the running coupling that has zero at zero momentum. The perturbative corrections to the twist-4 contribution and the power corrections generated by the twist-3 pion DAs are expected to be inessential.

In conclusion, within the covariant nonlocal model describing the quark-pion dynamics we obtain the $\pi\gamma^*\gamma^*$ transition form factor in the region up to moderately high momentum transfer squared, where the rapid power-like asymptotics takes place. At larger virtualities the pQCD evolution of the DA slowly goes to the asymptotic limits. From the comparison of the kinematical dependence of the asymptotic coefficients of the transition pion form factor, as it is given by pQCD and the nonperturbative model, the relations between the pion DAs and the dynamical quark mass and quark-pion vertex are derived.

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Figure 1. The pion distribution amplitudes (normalized by unity): the model predictions for twist-2 (solid line) and twist-4 (dashed line) components and the asymptotic limits of twist-2 (dotted line) and twist-4 (dash-dotted line) amplitudes.

Figure 2. The pion-photon transition form factors $Q^2 F_{\pi \gamma^*} (Q^2)$ (solid line) and $F_{\pi \gamma^* \gamma^*} (Q^2) = M_{\phi^0} (-Q^2/2, -Q^2/2)$ (dashed line) and their perturbative limits $2f_\pi$ (dotted line) and $4f_\pi/3$ (short dashed line). The experimental points ($Q^2 F_{\pi \gamma^*}$) are taken from [1].