Numerical study of proximity effect on critical field of rectangular type-II superconductor

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Abstract. Study of proximity effect on critical field of a rectangular type-II superconductor has been done. It was a numerical study using Time Dependent Ginzburg Landau (TDGL) equations as the basis. The superconductor conditioned in boundary with a non-superconducting material (i.e. metal). A uniform external field was applied to the superconductor in the direction of $z$-plane. Proximity effect or de Gennes boundary condition via extrapolation length ($b$) took account the properties caused by superconductor in contact with different material. As input to the simulation, we had three different dimensions each with $12 \times 12$, $32 \times 32$ and five varying value of $b$. We found that the lower critical field $H_{c1}$ will decrease as the increasing value of $b$ for all the rectangular superconductor, except for the size of $16 \times 64$. Any increment in the proximity effect also triggered a raise in the third higher critical field $H_{c3}$ for every grid size variations. We also concluded that the proximity effect is very effective to a superconductor with high number of circumference to area ratio.

1. Introduction

A lot of studies about magnetic properties of superconductor has done using Time Dependent Ginzburg-Landau equations as their foundation [1-8]. The Time Dependent Ginzburg-Landau equation, abbreviated and also widely known as TDGL equation, first developed by Gor'kov and Eliashberg now very successful in explaining properties and dynamics of superconductor [9]. Due to highly nonlinear nature of TDGL equations, it is common to solve the equation numerically [10]. One of several frequently used numerical solutions is $\psi U$ method. The method has been proven and worked great to solve the TDGL equation [1,3,4,5,6,7]. Previous study has succeeded acquiring the lower critical field and the third higher critical field using TDGL equations which were solved with $\psi U$ method [2].

In the practical usage of superconductor, it is very often to see that the superconductor is in contact with non-superconducting materials. This condition forces many studies about magnetic properties of superconductor to not abandoning the proximity effect [4,5,6].

The TDGL equations which govern the superconductivity order parameter $\psi$ and the vector potential $A$ in the zero-electric potential gauge are given by [1,6]:

$$\frac{\partial \psi(r,t)}{\partial t} = (\nabla - iA(r,t))^2 \psi(r,t) + \psi(r,t) - [\psi(r,t)]^2 \psi(r,t)$$

(1)


\[
\frac{\partial A(r,t)}{\partial t} - \frac{1}{2i} (\nabla \cdot \psi(r,t)) - \frac{1}{2} |\psi(r,t)\|^2 A(r,t) - \kappa^2 (R \cdot \nabla \times (\nabla \times A(r,t) - H_{\text{ext}}(r,t)))
\]

(2)

There are two equations that hold the boundary condition properties:

\[
\hat{n} \cdot [\nabla - iA]\psi = \frac{1}{b} \psi
\]

(3)

\[
\hat{n} \cdot [\nabla - iA]\psi = 0
\]

(4)

Equation (3) valid with the condition that superconductor is in contact with other materials. While equation (4) act if the superconductor is immersed in a vacuum. The proximity effect occurs when a superconductor is in contact with a normal metal. The proximity effect taken into account via de Gennes extrapolation length \( b \) in equation (3). All variables above are rescaled as: \( \psi \) in unit of \( \psi_0 = (|\alpha(T)|/\beta)^{1/2} \), \( A \) in unit of \( A_0 = \mu_0 H_{c2lin}(T) \xi(T) \), \( H_{\text{ext}} \) in unit of \( H_{c2lin}(T) \), \( \sigma \) in unit of \( \sigma_0 = 1/(\mu_0 \kappa(T)^2 D) \), \( r \) in unit of \( \xi(T) \), \( t \) in unit of \( \tau(T) = \xi(T)^2 / D \). Quantity \( \hat{n} \) is the normal vector of the superconductor boundary, \( \psi \) is order parameter, \( b \) is real number determining the boundary condition, \( \sigma \) is electrical conductivity, \( \alpha(T) \) and \( \beta \) are the Ginzburg-Landau free energy density coefficients, \( A \) is vector potential, \( H_{\text{ext}} \) is the external magnetic field, \( \xi(T) \) is coherence length, \( \kappa(T) \) is Ginzburg-Landau parameter and \( D \) is phenomenological diffusion constant.

The main topic discussed in the paper was proximity effect on the critical fields of the rectangular type-II superconductor. The proximity effect was observed through de Gennes extrapolation length. The critical fields in context are the lower critical field \( H_{c1} \) and the third higher critical field \( H_{c3} \).

2. Numerical Method

This study considered the superconductor as in figure 1. Rectangular shaped superconductor affected by spatially uniform, time-dependent external field pointing in \( z \)-plane direction. A normal metal situated in contact with the superconductor. In \( \psi U \) method, it is assumed that the superconductor contained \( N_x \times N_y \) cells of which have the size of \( \Delta x \times \Delta y \). There are three complex link variables held inside each cell [1,3]. They are introduced to help obtain a better numerical convergence at high magnetic field [10]. These set of conditions were studied with TDGL equations which solved numerically by \( \psi U \) method as have done in other previous studies [1,2,3,8]. A simulation program that has input and would yield desired output then generated based on this case.

![Figure 1. Rectangular shaped superconductor and the three important variables](image)

We set the simulation input with time step of 0.010, mesh spacing 0.5×0.5, \( \kappa = 2.0 \), initial applied external field 0 with increment of \( 1 \times 10^{-6} \) and various grid sizes with a basis of 12×12 and
32 × 32. The two basis represent a small and large size superconductor. More details of the grid size input shown in table 1.

| Input Name | \( N_x \times N_y \) | Area | Circumference | \( b \) |
|------------|----------------------|------|---------------|------|
| SA1        | 9 × 16               | 144  | 50            | 3    |
| SA2        | 9 × 16               | 144  | 50            | 5    |
| SA3        | 9 × 16               | 144  | 50            | 10   |
| SA4        | 9 × 16               | 144  | 50            | 20   |
| SA5        | 9 × 16               | 144  | 50            | 30   |
| SB1        | 8 × 18               | 144  | 52            | 3    |
| SB2        | 8 × 18               | 144  | 52            | 5    |
| SB3        | 8 × 18               | 144  | 52            | 10   |
| SB4        | 8 × 18               | 144  | 52            | 20   |
| SB5        | 8 × 18               | 144  | 52            | 30   |
| SR1        | 12 × 12              | 144  | 48            | 3    |
| SR2        | 12 × 12              | 144  | 48            | 5    |
| SR3        | 12 × 12              | 144  | 48            | 10   |
| SR4        | 12 × 12              | 144  | 48            | 20   |
| SR5        | 12 × 12              | 144  | 48            | 30   |
| LA1        | 16 × 64              | 1024 | 160           | 3    |
| LA2        | 16 × 64              | 1024 | 160           | 5    |
| LA3        | 16 × 64              | 1024 | 160           | 10   |
| LA4        | 16 × 64              | 1024 | 160           | 20   |
| LA5        | 16 × 64              | 1024 | 160           | 30   |
| LB1        | 8 × 128              | 1024 | 272           | 3    |
| LB2        | 8 × 128              | 1024 | 272           | 5    |
| LB3        | 8 × 128              | 1024 | 272           | 10   |
| LB4        | 8 × 128              | 1024 | 272           | 20   |
| LB5        | 8 × 128              | 1024 | 272           | 30   |
| LR1        | 32 × 32              | 1024 | 128           | 3    |
| LR2        | 32 × 32              | 1024 | 128           | 5    |
| LR3        | 32 × 32              | 1024 | 128           | 10   |
| LR4        | 32 × 32              | 1024 | 128           | 20   |
| LR5        | 32 × 32              | 1024 | 128           | 30   |

3. Result and Discussion

The TDGL equations has been solved numerically and we obtained the magnetization, external field, order parameter and other contained quantities. Since our main discussion was the critical fields, we concerned only to \(|\psi|^2\) and \(H_{\text{ext}}\). From plotting those two variables we obtained the critical fields of the superconductor. The first local maximum determined the lower critical field \(H_{\text{C1}}\) and the zero-point determined the third upper critical field \(H_{\text{C3}}\) [2]. After we obtained the critical fields with the corresponding de Gennes extrapolation length \(b\) value, we plotted it to obtain two graphs shown in figure 2 and 3.

From the graphs obtained, we can see several interesting phenomena. In the group of small size superconductor, the lower critical field \(H_{\text{C1}}\) decreases when de Gennes extrapolation length \(b\) is raised. It is valid for either rectangular or square superconductor. However, in the large size group,
$H_{C1}$ increases when $b$ is raised only for the square $(32 \times 32)$ and $16 \times 64$. The proximity effect seems to give opposite response to $8 \times 128$ in term of $H_{C1}$ value within the group. When it comes to the third higher critical field, all of the grid size variation gives the same response. The value of $H_{C3}$ is increases when the value of proximity is raised. It is true for both square and rectangular superconductor. Regarding the geometry, one can discuss about the circumference to area ratio. The small group superconductor obviously has the bigger ratio over the large group. Here at figure 4 & 5 we shall see that varying value of $b$ has less impact to the $H_{C1}$ value of the large group. It means that the proximity effect is less effective to superconductor with an area much more bigger than its circumference. After all, the simulation is programmed to measure the order parameter $\psi$ with the TDGL equations except for those near the interface which it used the boundary condition equation instead.

**Figure 2.** Graph of $H_{C1}$ function $b$

**Figure 3.** Graph of $H_{C3}$ function $b$

**Figure 4.** Value of $H_{C1}$ for the small size group

**Figure 5.** Value of $H_{C1}$ for the large size group

### 4. Conclusion
The effect of de Gennes boundary condition (proximity effect) on the critical fields of rectangular shaped type-II superconductor has been examined by numerical simulation. We found that the lower critical field $H_{C1}$ will decrease as the increasing value of $b$ for all the rectangular superconductor, except for the size of $16 \times 64$. Any increment in the proximity effect also triggered a raise in the third higher critical field $H_{C3}$ for every grid size variations. We also concluded that the proximity effect is very effective to a superconductor with high number of circumference to area ratio.
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