SPIN STRUCTURE FUNCTIONS*

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I. INTRODUCTION

The study of spin-dependent effects in deep inelastic scattering originated with the work of the SLAC–Yale Collaboration headed by Vernon Hughes in the early 1970’s.\(^1\)\(^,\)\(^2\) This subject has been reinvigorated by the surprising results published in 1987 by the European Muon Collaboration\(^3\) which also had a large representation from Yale. It is highly appropriate, therefore, to dedicate this talk here at Yale to the fascinating subject of the spin structure of the nucleon as probed at large momentum transfer in lepton scattering and related processes.

Deep inelastic processes have long been recognized for their dual roles, on the one hand as precise quantitative tests of QCD — largely through the logarithmic \(Q^2\)-variation generated by radiative corrections — and on the other hand as precise and well-understood probes of hadron structure — an approach summarized at the most elementary level by the well-known parton model interpretation of deep inelastic structure functions. Often, when parton model phenomena have clear interpretations, they have led to surprises and major revisions of our concept of hadrons as bound states of quarks and gluons. Three examples come to mind:

1. The “Momentum Sum Rule,” recognized in the early 1970’s\(^4\) and checked experimentally soon after, showed that about 50% of the nucleon’s “momentum” is carried by gluons.

A surprise to quark modelists, this result is actually predicted by QCD at asymptotic \(\ln Q^2 (\ln Q^2/\Lambda^2 \gg 1)\).\(^5\) The fact that it seems to work even at moderate \(Q^2\) suggests either that asymptopia is precocious or that glue is an intrinsic component of the nucleon bound state.

2. The “Gottfried Sum Rule,”\(^6\) which is not a sum rule at all but instead a measurement of the isospin asymmetry of the antiquarks in the proton: \(\int dx (\bar{u}_p(x) - \bar{d}_p(x))\). The

\[ \int dx (\bar{u}_p(x) - \bar{d}_p(x)) \]
recent NMC precision measurement of this quantity\textsuperscript{7} firmly established the existence of an asymmetry, and requires quark modelists to entertain effects which deform the sea on account of the valence quarks in a hadron.

3. Finally, and perhaps most significant, the EMC measurement of $g_1^{p}(x)$ can be interpreted\textsuperscript{8,9} as a direct measure of the fraction of the nucleon’s spin to be found on the spin of the quarks. The result: $12.0 \pm 9.4 \pm 13.8\%$, caught everyone by surprise. A reasonably sophisticated quark model estimate was $60 \pm 12\%$.\textsuperscript{8,10}

Naive quark models are apparently too naive. Even rather sophisticated models with relativistic quarks, ambient gluon fields and distant meson clouds do not easily accommodate these unusual results.

The EMC result, which originates in an unexpectedly small spin asymmetry at low-$x$ has stimulated a reconsideration of spin effects in deep inelastic processes. Much to the surprise of those of us involved in this undertaking, the reconsideration has led to a much richer and more rational picture of the role of spin in deep inelastic processes than existed before. The interpretation of transverse spin and of chirality in hard processes\textsuperscript{*} has had to be rewritten. Along the way, the importance of polarization in deep inelastic processes with hadronic initial states (e.g. polarized Drell-Yan) has come to the fore. The principal object of my talk is to outline a unified description of spin-dependent structure functions which has emerged in the previous few years. Much of my work on this subject was performed in collaboration with Xiangdong Ji and Aneesh Manohar and my debt to them is substantial.

\textsuperscript{*} I use the term \textit{hard processes} in the same sense as the fine book of the same title by B. L. Ioffe, V. A. Khoze and L. N. Lipatov.\textsuperscript{11} The reader should bear in mind that “hard” is to be construed as the opposite of “soft,” not “easy.” Indeed, “hard processes” are among the few \textit{easy} aspects of QCD!
My talk will be organized as follows:

I. Introduction

II. How to count: enumerating and interpreting the independent structure functions of hadrons.

III. The $g_1$ saga: some remarks on the longitudinal spin asymmetry measurement and interpretation.

IV. $h_1(x, Q^2)$: a new structure function and its relation to chirality, transversity and polarized Drell–Yan.

V. Twist-3: measuring quark-gluon correlations with $g_T(x, Q^2)$ and $h_L(x, Q^2)$.

II. HOW TO COUNT:

Enumerating and Interpreting the Structure Functions of Hadrons

Merely counting the independent “structure functions” of hadrons turns out to be a non-trivial problem. Its complexity increases with the degree of departure from scaling at large-$Q^2$. Effects which scale (modulo logarithms of $Q^2$ from radiative corrections) at large-$Q^2$ are “twist-2.” Those which vanish as $1/Q$ (again, mod logarithms) are “twist-3,” and so on. The discussion here will be complete through twist-3. Twist-4 effects which vanish like $1/Q^2$ require a considerable investment in formalism and are much harder to extract from experiment.

The familiar quark and gluon distribution of the parton model are actually special cases of general light-cone correlation functions. A couple of generic examples are

\[ f(x) \equiv \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle P | \phi(0) \phi(\lambda n) | P \rangle \]  
(2.1)

\[ E(x, x') \equiv \int \frac{d\lambda d\lambda'}{4\pi^2} e^{-i\lambda x - i\lambda' x'} \langle P | \phi(0) \phi(\lambda n) \phi(\lambda' n) | P \rangle \]  
(2.2)
where $p^\mu = (p, 0, 0, p)$, $n^\mu = \left(\frac{1}{2p}, 0, 0, -\frac{1}{2p}\right)$, $P^\mu = p^\mu + \frac{M^2}{2} n^\mu$, $\phi(x)$ is a generic field, and $p$, which is arbitrary, labels the frame. $p = \frac{M}{2}$ corresponds to the target rest frame and $p \to \infty$ is the “infinite momentum frame.” Both $f(x)$ and $E(x, x')$ depend non-trivially on a renormalization scale, $\mu^2$ [For a careful discussion see Ref. [12].] In an asymptotically free theory like QCD in which $\beta(g) \sim g^3$ for small $g$, the $\mu$-dependence of $f(x)$ and $E(x, x')$ is at most a power of $\ln \mu$. These logarithms do not change the structure of our results and they can be considered separately, hence I will often suppress the renormalization scale.

Both $f(x)$ and $E(x, x')$ are ground state correlation functions with the correlation taken along a tangent to the light-cone. $F(x)$ has a simple, probabilistic interpretation obtained by inserting a complete set of states:

$$f(x) = \sum_X \left| \langle P | \phi(0) | X \rangle \right|^2 \delta \left( P_X^+ - (1 - x) P^+ \right)$$

so $f(x)$ is the probability to find a quantum of $\phi$ with $k^+ = xP^+$ in the target state (see Fig. 1a). $E(x, x')$ does not have a probabilistic interpretation (see Fig. 1b). Some general properties of distributions such as $E(x, x')$ are developed in Ref. [13].
There are relatively simple rules for enumerating those quark and gluon distribution functions which contribute effects at twist-2 (i.e. $O(1)$ at large $Q^2$) and twist-3 (i.e. $O(1/Q)$ at large-$Q^2$):

1. Identify independent degrees of freedom when QCD is canonically quantized on the light-cone.

The solution to this problem is well-known: The quark field is decomposed with light-cone projection operators, $P_{\pm} = \frac{1}{2} \gamma^{\pm} \gamma^\mp$, where $\gamma^{\pm} \equiv \frac{1}{\sqrt{2}} (\gamma^0 \pm \gamma^3)$ —

$$\psi_{\pm} \equiv P_{\pm} \psi . \quad (2.4)$$

The gluon field, $A^\mu$, is decomposed with respect to light cone coordinates — $A^\mu \rightarrow (A^+, A^1, A^2, A^-)$. In the gauge $A^+ = 0$, then $\psi_+$ and $\vec{A}_\perp = (A^1, A^2)$ are independent variables, whereas $\psi_-$ and $A_-$ are dependent, constrained variables

$$A_- = A_- [\psi_+, \vec{A}_\perp]$$

$$\psi_- = \psi_- [\psi_+, \vec{A}_\perp] . \quad (2.5)$$

The independent degrees of freedom, $\psi_+$ and $\vec{A}_\perp$, carry helicity $\pm \frac{1}{2}$ and $\pm 1$, respectively. The dependent degrees of freedom $\psi_-$ and $A_-$ carry helicity $\pm \frac{1}{2}$ and $0$, respectively.

2. To enumerate all twist-2 distributions and exhibit their target spin dependence, count independent helicity amplitudes for forward parton-hadron scattering using only independent degrees of freedom.

The generic helicity amplitude shown in Fig. 2 gives a visual depiction of the generic light-cone correlation function of Eq. (2.1). Since there is no momentum transfer to the target, the amplitude corresponds to forward scattering and takes place along the $\hat{e}_3$-axis in coordinate
Therefore helicity (≡ angular momentum about $\hat{e}_3$) is conserved: $h + H = h' + H'$ and furthermore $A(h, H \rightarrow h', H') = A(-h, -H \rightarrow -h', -H')$ and $A(h, H \rightarrow h', H') = A(h', H' \rightarrow h, H)$ are consequences of parity and time reversal invariance, respectively. With these relations, we enumerate the independent helicity amplitudes for quarks and gluons scattering from targets with spin-1/2 and spin-1 in Table 1.

$$\begin{align*}
A(hH \rightarrow h'H')
\end{align*}$$

| Target Spin-1/2 | Target Spin-1 |
|-----------------|---------------|
| Quarks          |               |
| $A\left( \frac{1}{2} \; \frac{1}{2} \rightarrow \frac{1}{2} \; \frac{1}{2} \right)$ | $A\left( \frac{1}{2} \; 1 \rightarrow \frac{1}{2} \; 1 \right)$ |
| $A\left( \frac{1}{2} \; -\frac{1}{2} \rightarrow \frac{1}{2} \; -\frac{1}{2} \right)$ | $A\left( \frac{1}{2} \; -1 \rightarrow \frac{1}{2} \; -1 \right)$ |
| $A\left( -\frac{1}{2} \; \frac{1}{2} \rightarrow \frac{1}{2} \; -\frac{1}{2} \right)$ | $A\left( -\frac{1}{2} \; 1 \rightarrow \frac{1}{2} \; 0 \right)$ |
| $A\left( -\frac{1}{2} \; -\frac{1}{2} \rightarrow \frac{1}{2} \; \frac{1}{2} \right)$ | $A\left( \frac{1}{2} \; 0 \rightarrow \frac{1}{2} \; 0 \right)$ |
| Gluons          |               |
| $A\left( 1 \; \frac{1}{2} \rightarrow 1 \; \frac{1}{2} \right)$ | $A\left( 1 \; 1 \rightarrow 1 \; 1 \right)$ |
| $A\left( 1 \; -\frac{1}{2} \rightarrow 1 \; -\frac{1}{2} \right)$ | $A\left( 1 \; -1 \rightarrow 1 \; -1 \right)$ |

Table 1: Twist-2 helicity amplitudes.
Each amplitude in Table 1 corresponds to a different quark or gluon distribution function (a function of $x$ and $\ln q^2$). There are three quark and two gluon distributions for a spin-1/2 target:

$$f_1(x) \propto A \left( \frac{1}{2} \frac{1}{2} \rightarrow \frac{1}{2} \frac{1}{2} \right) + A \left( \frac{1}{2} -\frac{1}{2} \rightarrow \frac{1}{2} -\frac{1}{2} \right)$$

$$g_1(x) \propto A \left( \frac{1}{2} \frac{1}{2} \rightarrow \frac{1}{2} \frac{1}{2} \right) - A \left( \frac{1}{2} -\frac{1}{2} \rightarrow \frac{1}{2} -\frac{1}{2} \right)$$

$$h_1(x) \propto A \left( -\frac{1}{2} \frac{1}{2} \rightarrow \frac{1}{2} -\frac{1}{2} \right)$$

$$G(x) \propto \left( 1 \frac{1}{2} \rightarrow 1 \frac{1}{2} \right) + A \left( 1 -\frac{1}{2} \rightarrow 1 -\frac{1}{2} \right)$$

$$\Delta G(x) \propto A \left( 1 \frac{1}{2} \rightarrow 1 \frac{1}{2} \right) - A \left( 1 -\frac{1}{2} \rightarrow 1 -\frac{1}{2} \right)$$

The spin-average and helicity difference structure functions for quarks ($f_1(x)$ and $g_1(x)$) and gluons ($G(x)$ and $\Delta G(x)$) are well-known. There is experimental information on $f_1$, $g_1$ and $G$. The helicity-flip distribution function $h_1(x)$ is little-known because it does not figure prominently in electron scattering. Its interpretation is the subject of Section IV below. Note that there is no helicity-flip gluon distribution at leading twist.

For a spin-1 target there are several new distribution functions beyond the obvious generalizations of $f_1$, $g_1$, $h_1$, $G$ and $\Delta G$. Two are quadrupole distributions for quarks and gluons,

$$b_1(x) \propto A \left( \frac{1}{2} 1 \rightarrow \frac{1}{2} 1 \right) + A \left( \frac{1}{2} -1 \rightarrow \frac{1}{2} -1 \right) - 2A \left( \frac{1}{2} 0 \rightarrow \frac{1}{2} 0 \right)$$

$$B_1(x) \propto A \left( 1 1 \rightarrow 1 1 \right) + A \left( 1 -1 \rightarrow 1 1 \right) - 2A \left( 1 0 \rightarrow 1 0 \right)$$

and one is a gluon helicity-flip structure function unique to targets with $J \geq 1$:

$$\Delta(x) \propto A \left( -1 1 \rightarrow 1 -1 \right)$$

$b_1(x)$ and $\Delta(x)$ have novel properties and interesting implications for nuclear physics.16
3. To enumerate twist-3 distributions which enter inelastic hard processes, count independent helicity amplitudes for forward parton-hadron scattering with one-independent and one-dependent degree of freedom.*

The dependent gluon mode is $A^-$ with helicity 0. The dependent quark mode is $\psi_-$ with helicity $\pm 1/2$. To distinguish $\psi_-$ from $\psi_+$ in helicity amplitudes we denote it $\psi_{-1/2}$ in helicity amplitudes. Table 2 summarizes the independent twist-3 helicity amplitudes for spin-1/2 and spin-1 targets.

|                   | Target Spin-1/2 | Target Spin-1 |
|-------------------|-----------------|---------------|
| **Quarks**        |                 |               |
| $A\left(\frac{1}{2}, \frac{1}{2} \rightarrow \frac{1}{2}, \frac{1}{2}\right)$ | $A\left(\frac{1}{2}, 1 \rightarrow \frac{1}{2}, 1\right)$ | $A\left(\frac{1}{2}, -1 \rightarrow \frac{1}{2}, -1\right)$ |
| $A\left(\frac{1}{2}, -\frac{1}{2} \rightarrow \frac{1}{2}, -\frac{1}{2}\right)$ | $A\left(-\frac{1}{2}, 1 \rightarrow \frac{1}{2}, 0\right)$ | $A\left(-\frac{1}{2}, 1 \rightarrow \frac{1}{2}, 0\right)$ |
| $A\left(-\frac{1}{2}, \frac{1}{2} \rightarrow \frac{1}{2}, -\frac{1}{2}\right)$ | $A\left(\frac{1}{2}, 0 \rightarrow \frac{1}{2}, 0\right)$ | $A\left(\frac{1}{2}, 0 \rightarrow \frac{1}{2}, 0\right)$ |
| **Gluons** | | |
| $A\left(0, \frac{1}{2} \rightarrow 1, -\frac{1}{2}\right)$ | $A\left(0, 1 \rightarrow 1, 0\right)$ | $A\left(0, 0 \rightarrow 1, -1\right)$ |

**Table 2: Twist-3 Helicity Amplitudes**

From Table 2 we see that a description of the nucleon requires three twist-3 quark distributions, one spin averaged,

$$e(x) \propto A\left(\frac{1}{2}, \frac{1}{2} \rightarrow \frac{1}{2}, \frac{1}{2}\right) + A\left(\frac{1}{2}, -\frac{1}{2} \rightarrow \frac{1}{2}, -\frac{1}{2}\right), \quad (2.10)$$

* This rule does not give rise to all twist-3 distributions. The complete description of twist-3 requires the introduction of distributions which depend on two variables like Eq. (2.2). However, the only ones which appear to arise at tree level (i.e. ignoring QCD radiative corrections) in hard processes through order $1/Q$ are the ones captured by this rule.
one helicity difference,

$$h_L(x) \propto A\left(\frac{1}{2}, \frac{1}{2} \rightarrow \frac{1}{2}, \frac{1}{2}\right) - A\left(\frac{1}{2}, -\frac{1}{2} \rightarrow \frac{1}{2}, -\frac{1}{2}\right) ,$$  \hspace{1cm} (2.11)

and one helicity flip,

$$g_T(x) \propto A\left(-\frac{1}{2}, \frac{1}{2} \rightarrow \frac{1}{2}, -\frac{1}{2}\right) .$$  \hspace{1cm} (2.12)

There is only one twist-3 gluon distribution for the nucleon and it involves helicity flip

$$G_T(x) \propto A\left(0, \frac{1}{2} \rightarrow 1, -\frac{1}{2}\right) .$$  \hspace{1cm} (2.13)

Some comments on this analysis:

- I have suppressed flavor labels. For each twist and helicity structure there are independent distributions for each flavor of quark and antiquark. The flavor singlet quark distributions mix under $q^2$-evolution with gluon distributions of the same twist and helicity structure. Specifically $f_1$ and $G$, $g_1$ and $\Delta G$, and $g_T$ and $G_T$ mix. $e$, $h_1$ and $h_L$ do not mix with gluon distributions.

- Distributions associated with target helicity flip are in actuality measured by orienting the target spin transverse to the scattering axis-$\hat{e}_3$. The transversely polarized state is a linear superposition of helicity eigenstates (e.g. $|\perp\rangle = \frac{1}{\sqrt{2}}(|1/2\rangle + | -1/2\rangle)$ and $|\top\rangle = \frac{1}{\sqrt{2}}(|1/2\rangle - | -1/2\rangle)$, where $|\perp\rangle$ and $|\top\rangle$ are transverse spin eigenstates). Then the asymmetry obtained by reversing the target’s transverse spin isolates the helicity flip amplitude. Thus $h_1$ and $g_T$ (and $G_T$ and $\Delta$ as well) are important in processes involving transversely polarized targets.

- If we decompose the independent quark field $\psi_+$ into chirality eigenstates, we find that (of the nucleon distribution functions) $h_1$, $h_L$ and $e$ are chirally odd (i.e. the quark fields
in the correlation function of Eq. (2.1) have *opposite chirality* whereas \( f_1, q_1 \) and \( g_T \) are chirally even (the quark fields in Eq. (2.1) have the *same chirality*). This explains why \( h_1, h_L \) and \( e \) do not mix with gluon distributions (all of which are chirally even). Further it explains why \( f_1, g_1 \) and \( g_T \) are well-known and much studied whereas \( h_1, h_L \) and \( e \) are poorly-known and neglected: QED and *perturbative* QCD both conserve chirality (except for small quark mass terms) so \( h_1, h_L \) and \( e \) are suppressed in deep inelastic electron (muon and neutrino) scattering. As we shall see in Section IV, they are not suppressed in hard processes with hadronic initial states such as Drell–Yan production of lepton pairs.

The results of this section (for a spin-1/2 target like the nucleon) are summarized in Table 3, where the twist-2 and twist-3 quark and gluon distributions are classified according to target spin dependence and chirality.

| Spin Dependence | Twist-2 \( \mathcal{O}(1) \) | Twist-3 \( \mathcal{O}(1/Q) \) |
|-----------------|-----------------|------------------|
|                 | Quark Chiral Even | Quark Chiral Odd | Gluon |
| Spin average    | \( f_1(x) \)      | \(-\)            | \( G(x) \) |
| (unpolarized target) | \(-\)              | \(-\)            | \( e(x) \) |
| Helicity asymmetry | \( g_1(x) \)      | \(-\)            | \( \Delta G(x) \) |
| (longitudinal polarization) | \(-\)              | \(-\)            | \( h_L(x) \) |
| Helicity flip   | \(-\)             | \( h_1(x) \)     | \(-\) |
| (transverse polarization) | \(-\)              | \(-\)            | \( g_T(x) \) |
|                 | \(-\)             | \(-\)            | \( G_T(x) \) |

*Table 3: Summary of Nucleon Quark and Gluon Distributions through Twist-3, \( \mathcal{O}(1/Q) \).*
III. THE $g_1$ SAGA:

Brief Remarks on the Longitudinal Spin Asymmetry Measurement and Interpretation

The renewed interest in deep inelastic spin phenomena dates from a single number appearing in a 1987 paper by the European Muon Collaboration.\(^3\) In Ref. [3] the EMC reported a value for the area under $g_1(x, Q^2)$ for a proton target at a nominal value of $Q^2 = 10 \text{ GeV}^2$. In QCD this integral is related to axial charges measured in $\beta$-decay and to the flavor singlet axial charge which is otherwise unmeasured:\(^8,^{17}\)

$$
\int_0^1 dx \, g_1^{ep}(x, Q^2) = \frac{1}{18} \left\{ (3F + D) \left( 1 - \alpha_s(Q^2)/\pi \right) + 2\Sigma(Q^2) \left( 1 + \left( \frac{33 - 8f}{32 - 2f} \right) \frac{\alpha_s(Q^2)}{\pi} \right) \right\} .
$$

(3.1)

$f$ is the number of flavors, $F$ and $D$ are $SU(3)$ invariant matrix elements of hyperon $\beta$-decay and $\Sigma(Q^2)$ is the renormalization scale-dependent\(^1\) flavor singlet axial charge:

$$
\Sigma(Q^2) = \left\langle Ps \sum_{u,d,s} \bar{q}\gamma^\mu\gamma_5 q|Ps\right\rangle |_{Q^2} .
$$

(3.2)

$\Sigma(Q^2)$ is also the fraction of the proton’s spin to be found on the spins of its quarks. Many issues associated with this sum rule and its interpretation are reviewed in Ref. [10].

In the most naive, non-relativistic quark model, $\Sigma_{\text{NQM}} = 1$, because only the quarks carry spin in the nucleon. A model independent analysis assuming only $SU(3)$ symmetry in hyperon $\beta$-decay and no polarized strange quarks in the nucleon gives\(^8,^{10}\) $\Sigma_{EJ} \cong 0.60 \pm 0.12$. The EMC data give

$$
\Sigma_{\text{EMC}}(Q_{\text{EMC}}^2) = 0.120 \pm 0.094 \pm 0.138 ,
$$

(3.3)
far from the expectations of any model and arguably compatible with zero. Many explanations have been offered by theorists. Reviews with a variety of biases can be found in Refs. [19–21]. Rather than give a (most-likely superfluous) review here I will restrict myself to a few comments.

- Data — The EMC result relies heavily on the extrapolation to $x = 0$ guided by the four lowest-$x$ points (see Fig. 3). Confirmation is urgently needed. Several new polarized electron and muon experiments are in the works:

  → SMC – The Spin Muon Collaboration now running at CERN. They expect to cover roughly the same $x$ and $Q^2$ range as EMC with better statistics and control of systematic errors. They will first measure the longitudinal asymmetry from deuterium$^{22}$ in order to test Bjorken’s sum rule,$^{23}$ so they will not immediately check the EMC result. They plan to run on polarized hydrogen in the near future. If one is willing to accept the validity of the Bjorken sum rule (which is an iron-clad prediction of QCD), then the deuteron data will check the old result (modulo problems extracting neutron data from a deuteron target).

  → SLAC E142 and E143 — will study $^3$He (E142) and deuterium and hydrogen (in NH$_3$ and ND$_3$ targets) (E143). These classic End Station A experiments will have limited sensitivity to low-$x$ at large-$Q^2$. On the other hand, they promise to report data quickly.

  → HERMES — A large and ambitious program to place polarized gas targets in the electron beam at HERA, it is awaiting a full approval, contingent upon demonstrated polarization of order 50% in the electron ring. This facility will be able to study
a variety of targets in various polarization configurations (both longitudinal and transverse). They will not have data until the latter half of this decade.

It seems as though we shall have wait some time for clear confirmation of the EMC data from electron or muon scattering. There is, however, another possible source of information about $\Sigma(Q^2)$:

$\rightarrow$ LSND — A group at LAMPF is presently mounting an experiment to measure very low energy elastic neutrino scattering from liquid scintillator. At very low energies this probes the axial “charge” seen by the $Z^0$-boson — namely $\bar{u}\gamma^\mu\gamma_5u - \bar{d}\gamma^\mu\gamma_5d - \bar{s}\gamma^\mu\gamma_5s$. Heavy quarks can be ignored — see Ref. [24].) Since the first two terms are known from neutron $\beta$-decay, the experiment measures the polarized strange quark content of the proton directly. An earlier version of this experiment using higher energy neutrinos at BNL — and therefore subject to further problems of interpretation — provided weak confirmation of the EMC result.\(^{25}\) LSND can confirm the EMC data and eventually check the $SU(3)$ symmetry assumptions which lurk behind Eq. (3.1). LSND has its own subtleties, some associated with the fact that many nucleons in scintillator are bound in carbon nuclei. An interesting variation suggested by Garvey \textit{et al.}\(^{26}\) may circumvent some of these difficulties.

- The Gluon Controversy — The triangle anomaly is closely connected to $\int_0^1 dx \, g_1^p(x, Q^2)$. It generates the $Q^2$-dependence of $\Sigma(Q^2)$.\(^{17,18}\) In 1988 several groups suggested\(^{27}\) that a proper interpretation of the anomaly alters the sum rule, Eq. (3.1). The claim is that $\Sigma(Q^2)$ measured by experiment consists of two pieces: one, the “true” quark model
piece, $\tilde{\Sigma}$, is $Q^2$-independent and of order unity; the other, $-\frac{\alpha_s(Q^2)}{2\pi}\Delta G(Q^2)$, is a gluonic contribution proportional to the integrated gluonic spin asymmetry $\Delta G(Q^2)$:

$$\Sigma(Q^2) = \tilde{\Sigma}(Q^2) - \frac{\alpha_s(Q^2)}{2\pi}\Delta G(Q^2).$$  \hspace{1cm} (3.4)

The proposal is summarized in Fig. 4. The authors of Ref. [27] suggest that $\Delta G$ may be so large that it cancels the true quark model piece, $\tilde{\Sigma}$, and yields the small result found by EMC. Carlitz, Collins and Mueller$^{27}$ suggested ways to measure $\Delta G$ in two jet production by polarized electrons.

This interesting idea generated much activity but has run into a couple of problems which cloud the original interpretation suggested in Refs. [27]. First, several groups$^{28}$ pointed out that the separation between $\tilde{\Sigma}$ and $\alpha_s\Delta G/2\pi$ is ambiguous. Second, no convincing argument has emerged for interpreting $\tilde{\Sigma}$ as the true quark model spin contribution. In fact, a counterexample can be found in the heavy quark limit$^{10}$ of QCD. Finally, there is no direct way to measure $\tilde{\Sigma}$. If $\tilde{\Sigma}$ need not be of order unity, $\Delta G$ need not be large and
the predictive value of Eq. (3.4) becomes dubious. Whatever its significance, Eq. (3.4) has stimulated discussion of experiments to measure \( \Delta G(x, Q^2) \) which is interesting in its own right.

- **\( Q\bar{Q} \)-Pairs** — There is a prevailing view that the most natural state for the \( Q\bar{Q} \)-sea in the nucleon is unpolarized. A more careful investigation shows that this is hardly obvious and possibly a factor in explaining the small value of \( \Sigma_{EMC} \). In Ref. [29] Lipkin and I studied the spin and angular momentum content of a nucleon containing an extra \( Q\bar{Q} \)-pair in a variety of constituent quark models. Our most important observation was merely that vacuum quantum numbers, \( J^{PC} = 0^{++} \), for a \( Q\bar{Q} \)-pair require \( L = S = 1 \). The \( L = S = 0 \) state has \( J^{PC} = 0^{-+} \) and could only make a nucleon if combined with a less symmetric three quark state, which in turn would have problems with traditional quark model successes like magnetic moments and octet axial charges. Stimulated by this, we constructed a three-component model of an octet baryon consisting of 1) the bare three-quark state \( |b\rangle \); 2) an additional \( 0^{++} \) pair \( |b[Q\bar{Q}]^{0^{++}}\rangle \); 3) an additional \( 1^{++} \) pair \( |b[Q\bar{Q}]^{1^{++}}\rangle \). The last is the only other \( Q\bar{Q} \) state which can combine with the usual symmetric three-quark state to produce a nucleon with the proper spin and parity. We found that it is rather easy to accommodate the EMC result (and all traditional successes of the quark model) in such a model, though components 2) and 3) dominate the nucleon’s wavefunction. Of course this is hardly an *explanation* of the EMC data. It is merely an example of a rather benign sea quark distribution which accommodates the data.

- **Flavor tagging to measure \( \Delta u \), \( \Delta d \) and \( \Delta s \)** — There is great interest in the contributions of individual quark flavors to the spin asymmetry. They are defined by

\[
\Delta q^a (Q^2) s^\mu \equiv \left( P_s \bar{q}^a \gamma_\mu \gamma_5 q^a \right)^\dagger Q^2 P_s,
\]  

(3.5)
The sum rule for $g_1^{ep}$ can be rewritten as a measurement of one linear combination of $\Delta u$, $\Delta d$ and $\Delta s$. The separate flavor contributions cannot be distinguished in electron scattering. Neutrino scattering from polarized targets could partially unravel flavor dependence, but such experiments are out of the questions for the foreseeable future.

Recently, Close and Milner have suggested using leading particle effects in fragmentation to tag the flavor of the struck quark. The basic idea is an old one: a $\pi^+$ observed at large-$x$ and large-$z$ in the current fragmentation region is most likely a fragment of a $u$-quark since $\bar{d}$ quarks are suppressed at large $x$. Similarly leading $\pi^-$'s are correlated with $d$-quarks and leading $K^\pm$ with $\bar{s}$ and $s$ (though this correlation is less pronounced since $s$ and $\bar{s}$ have a soft $x$-distribution). The idea has been studied in connection with the HERMES proposal but could be implemented in any experiment with adequate particle identification.

IV. $h_1(x,Q^2)$, CHIRALITY, TRANSVERSITY AND POLARIZED DRELL–YAN

The twist-2, transverse spin structure function, $h_1(x, q^2)$ has been largely ignored since its discovery by Ralston and Soper in 1979. I’d like to rectify the situation by giving $h_1(x, Q^2)$ a major place in this talk. One benefit of this will be a clear understanding of transverse spin in the parton model, a subject which has been surrounded by confusion for years.

$h_1(x, Q^2)$ is a chiral-odd structure function. It is projected out of a light-cone quark correlation function with the Dirac matrix $\sigma_{\mu\nu}\gamma_5$,

$$
\frac{1}{2} \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle Ps \bar{\psi}(0) \sigma_{\mu\nu} i\gamma_5 \psi(\lambda n) | Ps \rangle
= h_1(x) (s_{\perp\mu} p_\nu - s_{\perp\nu} p_\mu) / M + h_L(x) M (p_\mu n_\nu - p_\nu n_\mu) s \cdot n
+ h_3(x) M (s_{\perp\mu} n_\nu - s_{\perp\nu} n_\mu)
$$

(4.1)
where \( p^\mu = (p, 0, 0, p), \) \( n^\mu = \frac{1}{2p}(1, 0, 0, -1), \) \( s^\mu \equiv (s \cdot n)p^\mu + (s \cdot p)n^\mu + s^\mu \) and \( h_L(x) \) and \( h_3(x) \) are twist-3 and twist-4 distribution functions, respectively. If we decompose \( \psi \) into left- and right-handed components, it is clear that \( h_1(x) \) is chirally-odd, as illustrated in Fig. 5a.

Deep inelastic lepton scattering in QCD proceeds via the “handbag” diagram, Fig. 5b, and various decorations which generate \( \log - Q^2 \) dependences, \( \alpha_s(Q^2) \) corrections and higher twist corrections, examples of which are shown in Figs. 5c–f. All these involve only \textit{chirally-even} quark distributions because the quark couplings to the photon and gluon preserve chirality. Only the quark mass insertion, Fig. 5f, flips chirality. So up to corrections of order \( m_q/Q, \) \( h_1(x, Q^2) \) decouples from electron scattering.

Fig. 5

There is no analogous suppression of \( h_1(x, Q^2) \) in deep inelastic processes with hadronic initial states such as Drell–Yan. The argument can be read from the standard parton diagram.
for Drell–Yan (Fig. 6). Although chirality is conserved on each quark line separately, the two quarks’ chiralities are unrelated. It is not surprising, then, that Ralston and Soper found that $h_1(x, Q^2)$ determines the transverse-target, transverse-beam asymmetry in Drell–Yan.

The parton interpretation of $h_1$ can be made transparent by decomposing the quark fields which appear in Eq. (4.1) first with respect to the light-cone projection operator $P_\pm$ and then with respect to various spin projection operators which commute with $P_\pm$. The two cases of interest are first, the chirality projection operators, $P_L$ and $P_R$,

$$P_L \equiv \frac{1}{2} (1 \mp \gamma_5)$$  \hspace{1cm} (4.2)

which satisfy

$$[P_L, P_\pm] = 0$$  \hspace{1cm} (4.3)

and second, the transversity projection operators, $Q_\pm$,

$$Q_\pm \equiv \frac{1}{2} (1 \mp \gamma_5 \gamma^\perp)$$ \hspace{1cm} (4.4)

where $\gamma^\perp$ is either $\gamma^1$ or $\gamma^2$, and $Q_\pm$ satisfies

$$[Q_\pm, P_\pm] = 0$$ \hspace{1cm} (4.5)
As explained in Section II, $h_1$ involves only independent light-cone degrees of freedom. The helicity structure of $h_1$ described in Section II is apparent in a chiral basis because helicity and chirality coincide up to irrelevant mass corrections,

$$h_1(x) = \frac{2}{x} \Re \langle P \hat{e}_1 | L^\dagger(xP) R(xP) | P \hat{e}_1 \rangle$$

(4.6)

called compared with $f_1$ and $g_1$

$$f_1(x) = \frac{1}{x} \langle P | R^\dagger(xP) R(xP) + L^\dagger(xP) L(xP) | P \rangle$$

(4.7)

$$g_1(x) = \frac{1}{x} \langle P \hat{e}_3 | R^\dagger(xP) R(xP) - L^\dagger(xP) L(xP) | P \hat{e}_3 \rangle .$$

(4.8)

In Eqs. (4.6) – (4.8) $R(xP)$ and $L(xP)$ are operators which annihilate independent light cone components of the quark field in eigenstates of $P_R$ and $R_L$, respectively, with momentum $k^+ \equiv xP^+$ and integrated over $\vec{k}_\perp$. According to Eq. (4.7) $f_1(x)$ counts quarks with $k^+ = xP^+$ irrespective of helicity, while from Eq. (4.8) $g_1(x)$ counts quarks with helicity parallel to the target helicity minus those antiparallel. $h_1(x)$ is obscure in this basis. If, instead, we diagonalize transversity, then

$$f_1(x) = \frac{1}{x} \langle P | \alpha^\dagger(xP) \alpha(xP) + \beta^\dagger(xP) \beta(xP) | P \rangle$$

(4.9)

$$h_1(x) = \frac{1}{x} \langle P \hat{e}_1 | \alpha^\dagger(xP) \alpha(xP) - \beta^\dagger(xP) \beta(xP) | P \hat{e}_1 \rangle$$

(4.10)

$$g_1(x) = \frac{2}{x} \Re \langle P \hat{e}_3 | \alpha^\dagger(xP) \beta(xP) | P \hat{e}_3 \rangle$$

(4.11)

where $\alpha(xP)$ and $\beta(xP)$ annihilate independent components of the quark field in eigenstates of $Q_+$ and $Q_-$, respectively. From Eq. (4.10) it is clear that $h_1(x)$ counts quarks with

* The $Q^2$-dependence of $f_1$, $g_1$ and $h_1$ due to QCD radiative corrections can be restored (to leading logarithmic order) by integrating over $\vec{k}_\perp$ only up to $\vec{k}_\perp^2 \lesssim Q^2$. For a complete discussion, see Ref. [12].
\[ k^+ = xP^+ \] signed according to whether their transversity is parallel or antiparallel to the target transversity. In this basis the interpretation of \( g_1(x) \) is obscure.

The simple structure of Eqs. (4.6) – (4.8) and (4.9) – (4.11) shows that transverse spin effects and longitudinal spin effects are on a completely equivalent footing in perturbative QCD. Not knowing about \( h_1(x) \), many authors, beginning with Feynman,\(^{36}\) have attempted to interpret \( g_\perp(x) \) as the natural transverse spin distribution function. Since \( g_\perp(x) \) is twist-3 and interaction dependent, this attempt led to the erroneous impression that transverse spin effects were inextricably associated with off-shellness, transverse momentum and/or quark-gluon interactions.\(^{37}\) The resolution contained in the present analysis is summarized in Table 4 where the symmetry between transverse and longitudinal spin effects is apparent. Only ignorance of \( h_1 \) and \( h_L \) prevented the appreciation of this symmetry at a much earlier date.

| Longitudinal Spin | Transverse Spin |
|------------------|-----------------|
| Twist-2 \( g_1(x, Q^2) \) | \( h_1(x, Q^2) \) |
| Twist-3 \( h_L(x, Q^2) \) | \( g_T(x, Q^2) \) |

Table 4: The symmetry of transverse and longitudinal spin distribution functions.

It is useful to summarize some of the known properties of \( h_1(x, Q^2) \) and compare them with analogous properties of \( g_1(x, Q^2) \).

- Inequalities:
  \[
  |g_1(x, Q^2)| < f_1(x, Q^2) \\
  |h_1(x, Q^2)| < f_1(x, Q^2)
  \]

for each flavor of quark and antiquark.
Physical interpretation: $h_1(x, Q^2)$ measures transversity. It is chirally odd and related to a bilocal generalization of the tensor operator, $\bar{q}\sigma_{\mu\nu}i\gamma_5q$. $g_1(x, Q^2)$ measures helicity. It is chirally even and related to a bilocal generalization of the axial charge operator, $\bar{q}\gamma_\mu\gamma_5q$.

Sum rules: If we define a “tensor charge"

\[
2s^i\delta q^a(Q^2) \equiv \langle Ps\big|\bar{q}\sigma^{0i}i\gamma_5\frac{\lambda^a}{2}q\big|Q^2 Ps\rangle ,
\]

where $\lambda^a$ is a flavor matrix and $Q^2$ is a renormalization scale, then $\delta q^a(Q^2)$ is related to an integral over $h^a_1(x, Q^2)$,

\[
\delta q^a(Q^2) = \int_0^1 dx \left(h^a_1(x, Q^2) - \bar{h}^a_1(x, Q^2)\right)
\]

where $h^a_1$ and $\bar{h}^a_1$ receive contributions from quarks and antiquarks, respectively. The analogous expressions for $g_1(x, Q^2)$ involve axial charges,

\[
2s^i\Delta q^a(Q^2) \equiv \langle Ps\big|\bar{q}\gamma^i\gamma_5\frac{\lambda^a}{2}q\big|Q^2 Ps\rangle
\]

\[
\Delta q^a(Q^2) = \int_0^1 dx \left(g^a_1(x, Q^2) + \bar{g}^a_1(x, Q^2)\right)
\]

Note the contrast: $h_1(x, Q^2)$ is not normalized to a piece of the angular momentum tensor, so $h_1$, unlike $g_1$, cannot be interpreted as the fraction of the nucleons’ spin found on the quarks’ spin. Note the sign of the antiquark contributions: $\delta q^a$ is charge-conjugation odd, whereas $\Delta q^a$ is charge conjugation even. All tensor charges $\delta q^a$ have non-vanishing anomalous dimensions, but none mix with gluonic operators under renormalization. In contrast, the flavor non-singlet axial charges, $\Delta q^a, a \neq 0$, have vanishing anomalous
dimension, whereas the singlet axial charge $\Delta q^0 \propto \Sigma$ has an anomalous dimension arising from the triangle anomaly.\textsuperscript{17,18,10}

- Models: $h_1$ and $g_1$ are identical in non-relativistic quark models, but differ in relativistic models like the bag model — see Fig. 7.\textsuperscript{34}

- Role in polarized Drell–Yan: $h_1$, $g_1$ and their twist-3 counterparts $g_T$ and $h_L$ can be measured in lepton-pair production with appropriately polarized beams and targets (Drell–Yan process). If both target and beam are longitudinally polarized,\textsuperscript{39}

\[ A_{LL} = \frac{\sum_a e_a^2 g_1^a(x) g_1^a(y)}{\sum_a e_a^2 f_1^a(x) f_1^a(y)}. \]  

(4.17)

If both target and beam are transversely polarized,\textsuperscript{33}

\[ A_{TT} = \frac{\sin^2 \theta \cos 2\phi}{1 + \cos^2 \theta} \frac{\sum_a e_a^2 h_1^a(x) h_1^a(y)}{\sum_a e_a^2 f_1^a(x) f_1^a(y)}. \]  

(4.18)

and if one is longitudinal and the other transverse,\textsuperscript{34}

\[ A_{LT} = \frac{2 \sin 2\theta \cos \phi}{1 + \cos^2 \theta} \frac{M}{\sqrt{Q^2}} \frac{\sum_a e_a^2 (g_1^a(x) y g_T^a(y) - x h_L^a(x) h_T^a(y))}{\sum_a e_a^2 f_1^a(x) f_1^a(y)}. \]  

(4.19)
In each case, $A_{AB}$ is the appropriate spin asymmetry. [The $Q^2$-dependence of the distributions has been suppressed here.] The appearance of $h_1(x)$ at leading-twist (i.e. scaling) in Eq. (4.18) illustrates its importance in processes in which it is not suppressed by a chirality selection rule. The explicit factor of $m/\sqrt{Q^2}$ in Eq. (4.19) confirms the twist-3 assignment of both $g_T$ and $h_L$.

- Measuring $h_1$ in electroproduction: $^{14}$ $h_1(x)$ does not appear at leading twist in electroproduction because the diagrams of Fig. 5b – 5e preserve the quark chirality. The quark mass insertion in Fig. 5f gives rise to a contribution of order $m/Q$ to the measured $g_2(x,Q^2)$ obtained from electroproduction off a transversely polarized target. $u$ and $d$ quarks are abundant in the nucleon but have very small (current) masses. Heavy quarks have non-negligible masses but are rare in the nucleon. In all, $h_1(x,Q^2)$ makes a negligible impact on inclusive electroproduction. [It is interesting to note, however, that $h_1(x,Q^2)$ was discovered and its anomalous dimensions calculated in Ref. [38] on account of this small contribution to electroproduction.]

This situation can be changed by observing a particle — most easily a single pion — in the current fragmentation region: $ep \rightarrow HX$. The spin and twist properties of this process depend on the quark fragmentation function as well as the distribution function. If no spin variables are measured in the final state, then two fragmentation functions can enter: first $\hat{f}_1(x,Q^2)$ which is the familiar twist-two fragmentation function. It is chirally even, and measures the probability for a quark ($a$) to fragment into a hadron ($H$) with longitudinal momentum fraction $z$. [I have suppressed the labels $a$ and $H$ on $\hat{f}_1$.] $\hat{f}_1(z,Q^2)$ is analogous to the distribution function $f_1(x,Q^2)$. Continuing the analogy, there is a twist-three, chiral-odd fragmentation function, $\hat{e}(z,Q^2)$, analogous to the
distribution function \( e(x, Q^2) \) described in Section II. Polarizing the target transversely to the beam, the distribution functions \( h_1(x, Q^2) \) and \( g_T(z, Q^2) \) can enter. \( h_1(x, Q^2) \) can only contribute if \( \hat{e}(z, Q^2) \) provides a compensating chirality flip. \( g_T(z, Q^2) \) does not flip chirality, and, to leading twist, fragments via \( \hat{f}_1(z, Q^2) \). The resulting asymmetry is the sum of two terms — \([h_1 \otimes \hat{e}] \oplus [g_T \otimes \hat{f}_1]\) — each of which is suppressed by \( \mathcal{O}(1/Q) \) because one twist-three object (\( g_T \) or \( \hat{e} \)) is required in each case:

\[
A_T^H(x, z, Q^2) \propto \frac{\Lambda}{\sqrt{Q^2}} \sum_a e_a^2 \left[ h_1^a(x, Q^2) \hat{e}^{a/H}(z, Q^2) + g_T^a(x, Q^2) \hat{f}_1^{a/H}(z, Q^2) \right] . \tag{4.20}
\]

Since \( \hat{f}_1(z, Q^2) \) can be measured in \( e^+e^- \rightarrow HX \) and \( g_T(x, Q^2) \) can be measured in inclusive, polarized electroproduction, it is possible, at least in principle, to extract both \( h_1(x, Q^2) \) and \( \hat{e}(z, Q^2) \) from a measurement of the \( x- \) and \( z- \)dependence of the transverse spin asymmetry in \( ep \rightarrow eHX \).\(^{14} \)

V. TWIST THREE:

Measuring Quark-Gluon Correlations with \( g_T(x, Q^2) \) and \( h_L(x, Q^2) \)

The twist-three quark distributions \( g_T(x, Q^2) \) and \( h_L(x, Q^2) \) are unique windows into quark gluon correlations in the nucleon. All higher twist effects probe quark-gluon correlations. \( g_T \) and \( h_L \) are unique in that they dominate certain observables, in contrast to generic higher-twist effects which must be extracted as corrections to QCD fits to leading twist. For example, consider electron scattering from a nucleon target polarized at an angle \( \alpha \) to the incident electron beam (Fig. 8). The spin-dependent part of the cross-section is given by\(^{40} \)

\[
\frac{d\Delta \sigma}{dx \, dy \, d\phi} = \frac{e^4}{4\pi^2 Q^2} \left\{ \cos \alpha \left[ \left( 1 - \frac{y}{2} - \frac{y^2}{4} (\kappa - 1) \right) g_1 - \frac{y}{2} (\kappa - 1) g_2 \right] 
- \sin \alpha \cos \phi \sqrt{\kappa - 1} \left( 1 - \frac{y}{2} - \frac{y^2}{4} (\kappa - 1) \right) \left( \frac{y}{2} g_1 + g_2 \right) \right\} . \tag{5.1}
\]
where $x = Q^2 / 2P \cdot q$, $y = P \cdot q / ME$, $\kappa = 1 + 4M^2 x^2 / Q^2$ and $\phi$ is the dihedral angle between the scattering plane and the plane defined by the beam and the target spin. As promised, effects associated with $g_1$ scale, but effects of $g_T (= g_1 + g_2)$ fall at least like $1/Q$. An experimenter can measure $g_T$ by (first measuring $g_1$, then) orienting his target at $90^\circ$ to the electron beam. This should be contrasted with the elaborate theoretical analysis necessary to isolate higher-twist in (say) spin averaged electron scattering. Of course, the experiment is still non-trivial: the asymmetry is suppressed relative to the rate by $\mathcal{O}(1/Q)$ necessitating a high statistics experiment.

Fig. 8

The precision with which the operator product expansion and perturbative QCD allow us to analyze electron scattering makes $g_T$ and $h_L$ very useful tools. Moments of $g_T$ and $h_L$ measure the expectation values of specific, well-defined local operators. One must first separate out a “contamination” of twist-2 operators from $g_T$ and $h_L$, namely

$$g_T(x, Q^2) = \int_0^1 \frac{dy}{y} g_1(y, Q^2) + \bar{g}_2(x, Q^2) \quad (5.2)$$
\[ h_L(x, Q^2) = 2x \int_0^1 \frac{dy}{y^2} h_1(y, Q^2) + \bar{h}_2(x, Q^2) \]  

(5.3)

This leaves \( \bar{g}_2 \) and \( \bar{h}_2 \) which depend explicitly on quark gluon interactions and on the gauge coupling \( g \), schematically,

\[
\bar{g}_2(x) \sim g \left\langle Ps \left| \bar{q}_\gamma \gamma_\mu \bar{G}_{\alpha\beta} q \right| Ps \right\rangle \]  

(5.4)

\[
\bar{h}_2(x) \sim g \left\langle Ps \left| \bar{q}_i \sigma^\lambda \gamma_5 G^\beta_\lambda q \right| Ps \right\rangle . \]  

(5.5)

To be more precise, for example, \( \int_0^1 dx x^2 \bar{g}_1(x, Q^2) \) is directly related to the nucleon matrix element of the operator,\(^{42,40}\)

\[
\theta_{[\sigma, \{\mu_1\} \mu_2]} = \frac{g}{8} \bar{\psi} \left( \bar{G}_{\sigma\mu_1} \gamma_\mu_2 + \bar{G}_{\sigma\mu_2} \gamma_\mu_1 \right) \psi . \]  

(5.6)

Similar sum rules for other moments of \( \bar{g}_2 \) and \( \bar{h}_2 \) can be found in Refs. [42] and [34], respectively. Admittedly, at the present time we cannot compute the right-hand side of the sum rules. However, models and more ambitious programs like lattice QCD would be aided by experimental information on matrix elements such as Eq. (5.6). A bag model calculation of \( \bar{g}_2(x) \) and \( \bar{h}_2(x) \) is shown in Fig. 9 for the sake of a rough estimate.\(^{34}\)
Twist-three distribution functions evolve with $Q^2$ in a more complicated manner than leading twist ones. Typically, at leading twist (and leading order), distributions obey an Altarelli–Parisi equation of the form,

$$\frac{d}{d\ln Q^2} f(x, Q^2) = \frac{\alpha_s(Q^2)}{\pi} \int x dy P\left(\frac{x}{y}\right) f(y, Q^2)$$  \hspace{1cm} (5.7)

with a perturbative “splitting function” $P(x/y)$. If $f(x, Q^2)$ is known at $Q^2 = Q_0^2$ (with $Q_0^2 \gg \Lambda^2$) it can be “evolved” to another $Q^2$ using Eq. (5.7). At the very least, Eq. (5.7) allows experimenters to amalgamate data at a variety of different (large) $Q^2$-values.

The evolution of a twist-3 distribution like $g_T(x, Q^2)$ is more complicated. It does not obey a simple evolution equation. Instead $g_T(x, Q^2)$ is related to a pair of more general distribution functions, $G(x, y, Q^2)$ and $\tilde{G}(x, y, Q^2)$, defined by relations like

$$\int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{i\lambda x} e^{i\mu (y-x)} \langle Ps | \bar{\psi}(0) iD^\alpha (\mu n) \not\! \psi(\lambda n) | Q^2 \rangle^1_{Ps} \rangle = 2i \epsilon^{\alpha \beta \mu \nu} n_\beta s_\mu p_\nu G(x, y, Q^2) + \text{lower twist}$$  \hspace{1cm} (5.8)

and similarly for $\tilde{G}(x, y, Q^2)$. $G$ and $\tilde{G}$ are generic twist-3 parton distributions. $g_T(x, Q^2)$ is a simple projection:

$$g_T(x, Q^2) = \frac{1}{2x} \int_0^1 dy \left[ \tilde{G}(x, y) + \tilde{G}(y, z) + G(x, y) - G(y, x) \right] .$$  \hspace{1cm} (5.9)

So twist-3 is inherently much more complicated than $g_T(x, Q^2)$ suggests. It is a happy accident that only this particular projection of $G$ and $\tilde{G}$ appears in electron scattering. The difficulty with evolving $g_T(x, Q^2)$ is that it is $G(x, y, Q^2)$ and $\tilde{G}(x, y, Q^2)$ which obey Altarelli–Parisi-like integro-differential evolution equations. So a measurement of $g_T(x, Q^2)$ at some $Q^2 = Q_0^2 \gg \Lambda^2$ does not provide enough information to predict $g_T(x, Q^2)$ at some other large $Q^2$. This “impediment to evolution” was recently stressed in Ref. [44].
In a recent paper, Ali, Braun and Hiller\textsuperscript{45} have suggested a way around this problem. They study the anomalous dimension matrices for all the local operators which contribute to $G$ and $\tilde{G}$. In the asymptotic limit of $N_c \to \infty$ ($N_c \equiv$ number of colors) and $x \to 1$ they show that $g_T(x, Q^2)$ is an eigenfunction of the matrix evolution equations. In simpler terms: for $N_c$ large and $x$ near 1, $g_t(x, Q^2)$ evolves approximately according to an Altarelli–Parisi equation like (5.1), although the splitting function is not the naive one which would have been obtained by ignoring the complexity of the problem (as was done, for example, in Ref. [46] in the early days of perturbative QCD). The authors of Ref. [45] argue that for $N_c$ not too large and $x$ not too near 1, their results remain approximately valid. If they are right then $g_T(x, Q^2)$ can be evolved with $Q^2$ more or less like a standard distribution function, making it possible to interpret data taken at a variety of $Q^2$ values in a systematic way.

VI. CONCLUSIONS

Manipulation of the spin, twist and chirality dependence of deep inelastic processes offers us a new sensitivity to the details of nucleon structure. This varied and precise information comes as a consequence of our confidence in the formalism provided by perturbative QCD — yet another example of the adage that yesterday’s novelty is today’s tool (and tomorrow’s background!). A carefully planned sequence of experiments including deep inelastic scattering from a variety of targets (proton, deuteron, $^3$He, nuclei) in a variety of spin states, as well as hadronic processes such as polarized Drell–Yan, can give us much more detailed information on the internal quark and gluon structure of the nucleon (and nuclei) than we presently possess. There is no other program which rivals it in precision or clarity of interpretation within the framework of QCD. The experiments to date only scratch the surface of this rich and challenging subject.
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FIGURE CAPTIONS

Fig. 1: Generalized light-cone distribution function: a) the familiar two-particle case expressed as a probability; b) a generic three-particle distribution.

Fig. 2: Forward scattering of a parton (quark or gluon) of momentum $k$ and helicity $h$ from a target of momentum $P$ and helicity $H$.

Fig. 3: EMC data on $g_1(x)$ and its integral.

Fig. 4: Graphical representation of gluonic contribution to $\Sigma$.

Fig. 5: Chirality in deep inelastic scattering: a) Chirally odd contributions to $h_1(x)$; b)–e) Chirally even contributions to deep inelastic scattering (plus $L \leftrightarrow R$ for electromagnetic currents); f) Chirality flip by mass insertion.

Fig. 6: Chirality in Drell–Yan (plus $L \leftrightarrow R$) production of lepton pairs.

Fig. 7: Bag model estimates of $h_1(x)$ and $g_1(x)$.

Fig. 8: Kinematics for polarized deep inelastic lepton scattering from a spin-1/2 target polarized at an angle $\alpha$ with respect to the beam axis.

Fig. 9: The proton’s twist-3 spin-dependent structure functions $g_2$ and $h_2$ in the Bag model: a) $g_2$ and $\bar{g}_2$; b) $h_2$ and $\bar{h}_2$. 