Controlling of the melting through porous medium and magnetic field

Taza Gul1,2, Raja S Gul1, Waqas Noman1, Fawad Hussain3 and Iraj S Amiri4,5

Abstract
The melting procedure with a direct contact of phase change material is taken into account to consider the porous medium in the presence of a uniform and transverse magnetic field. A permeable rotating disk is taken as a heater in the melting progression of solid phase change material. The three-dimensional melting layer takes place due to the accruing of the temperature difference among the porous disk and solid material. Movement is subject to the effect of pressure loading (counting the weight of solid), direct relation with solid and rotation due to centrifugal force. The removal of melting is controlled due to the joint exertions of the porous media, wall permeability and resistive force generated due to the applied magnetic field. The motion of the melting layer is assumed unsteady and governed the nonlinear similarity equations. Furthermore, magnetic field, porosity, external load and wall suction enhance melting and heat transfer rates at the thin melts film thickness. The melting rate, momentum and thermal boundary layers are estimated under the impact of Stefan number, magnetic field, porosity parameter and unsteadiness parameter. The Eckert number enhances the thermal boundary layer, and consequently the larger amount of melting received. The governing PDEs is highly nonlinear; thus for the solution we use analytical method of HAM and BVPh 2.0 package. The important outputs of the thickness of the thin layer during melting process in the presence and absence of wall suction are mainly focused.

Keywords
Direct contact melting, melting control, porous media, magnetic field, viscous dissipation, BVPh 2.0 package

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Introduction
Most of the useful products are produced due to the melting of solid materials, and these melts are used in many engineering, biological and industrial applications. Various procedures were adopted by the researcher for the melting procedure. The use of disk as a heater for the melting procedure in a direct contact with phase change solid material is the reliable and economical technique. This technique is frequently used in the field of science and technology like Jackson1 had discussed the process of the metallurgy and welding. Pfann2 had discussed crystal growth and Bejan3,4 had discussed heat energy storage system and lubrication. Measurement of high temperature in the welding process was studied by Arora et al.5 The main issue is to remove the resultant melt at the time of melting process and to protect a huge heat conduction, which was studied by Moallemi et al.6 Lacroix7 had examined the melting phenomena inside a capsule in contact with phase change material for melting. He noticed that the close contact melting obtained due to the heat surface of the solid phase change material is more effective than the heat shifted by convection, which normally exists in much thicker layers of liquefied substances. In these circumstances, the heat transfer enhances and the melting time reduces. Measurement in high-temperature process for many industrial applications was studied by Graham.8 The fascinating application of contact

1Department of Mathematics, City University of Science and Information Technology, Peshawar, Pakistan
2Higher Education Department, Peshawar, Pakistan
3Department of Mathematics, Abbottabad University of Science and Technology, Abbottabad, Pakistan
4Computational Optics Research Group, Advanced Institute of Materials Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam
5Faculty of Applied Sciences, Ton Duc Thang University, Ho Chi Minh City, Vietnam

Corresponding authors:
Taza Gul, Department of Mathematics, City University of Science and Information Technology, Peshawar 25000, Pakistan.
Email: tazagul@cusit.edu.pk
Iraj S Amiri, Computational Optics Research Group, Advanced Institute of Materials Science, Ton Duc Thang University, Ho Chi Minh City 758307, Vietnam.
Email: irajsdaghamiri@tdtu.edu.vn

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melting phenomena for the unsteady problem was discussed by Myers et al.9 Saito et al.10 examined the unsteady contact melting of a solid material with a hot plate, and they used the temperature gradient in the Stefan conditions. The experimental and analytical investigations to the closed contact melting were done by Moallemi et al.11 Kumano et al.12 and Bareiss and Beer.13 The importance of the convection in the close contact melting and the concept of the inertia were introduced by Groulx and Lacroix.14 Melting process in the existence of porous media was studied by Jourabian et al.15 Taghavi16 had introduced the idea of rotation and external pressure in the processes of contact melting.

The flow through rotating disk has many applications in the field of mechanical engineering and industrial process, especially in the geophysics, drilling devices, marine sciences, storage devices and so on. Karman17 had studied the flow over a rotating disk for the first time and explained that the rotating disk manages the flow pattern and enhances the temperature field. Turkylilmazoglu18 mentioned the formation of the boundary layer flow over the rotating disk. Appelquist et al.19 commenced their arguments on the linear instabilities in the rotating disk during fluid motion. The flow of the micro polar nanofluid was caused by a rotating disk in the existence of magnetic field explored by Ramzan et al.20 The nature of the oscillating magnetic field in the convective flow of a nanomaterial using a rotating stretchable disk was discussed by Hassan et al.21 Reducing the amount of melting and enhancing heat transfer rates with thicker melt layers by increasing the magnetic field parameter were proven in the latest efforts of Malepati.22 The mechanical behavior of micro-rotating disk was presented in the recent work of Bagheri et al.23

The flow through magnetic hydrodynamic (MHD) is electrically conducting, which affects the behavior of flow, basically the apparent viscosity is developed in the fluid by implementation of MHD which realized reduction in the velocity of flow. There are many applications of MHD such as in the field of astrophysical phenomena, where the earth magnetic field protects us from radiations, MHD pumps, MHD generator and used in fluid mechanics and so on. Many researchers analyzed magnetic-effect problems such as water-based magnetite nanofluid discussed by Toghraie et al.24 The effect of the magnetic field of fluid flow in a duct was studied by Mousavi et al.25 The effect of MHD was numerically studied in porous medium by Sheikholeslami.26 The use of a magnetic storage device to balance the quality issues was discussed by Sivaperumal et al.27 Magnetic effect on nanofluid in permeable cavity was discussed by Sheikholeslami.28 Heat transfer in porous vessels in the presence of magnetic field was discussed by Rahbari et al.29

At the initial stage, the flow in porous media had exposed the Darcy30 law related to the linear flow velocity to pressure gradient through the porous medium. Porous medium is also symbolized by its permeability that measured the flow conductivity. The accumulation of the fluid and the porous medium is much attracted by the cause of hitch to flow diffusion appointed by local edges or local viscosity. The porosity is used in industrial and engineering fields like conduction in porous media, heat transfer process, forced and natural convection in porous medium and so on. A lot of research work is present in the literature. The latest advancement in porous media gets to spread precocious models for the Darcy law as in Forchheimer’s equation31 and Brinkman’s32 equation. The effects discussed above are not taken into account in Darcy’s equation. The excessive extension in research about porous media on fluid flows was also discussed by Nield and Bejan33 and Ingham and Pop.34,36 The fluid in porous ribs was discussed by Toghraie et al.37 Multilayered porous media was introduced by Arasteh et al.38 Analytical study of heat transfer with permeable wall was discussed by Fakour et al.39 Heat transfer in porous media was studied by Moradi et al.40 Hybrid nanofluid in double pipe of heat exchanger was studied by Shahsavari et al.41 Heat transfer in porous medium was studied by Sheikholeslami et al.42 Heat transfer in solidification in the porous energy storage system was studied by Sheikholeslami et al.43 Heat effect on the phase change material was introduced by Sheikholeslami et al.44 Unsteady analytical study of a spherical particle was studied by Rahimpetroudi et al.45 Fakour et al.46 discussed the heat transfer over an unsteady stretching sheet. To control the melting on a rotating permeable disk which was considered as a heater, a valid quantity of suction was incorporated without allowing the increase in melt layer film thickness, which is mentioned in the current study. The earlier study considered the steady case with wall suction, while the present work considered the unsteady that includes wall suction, magnetic field and porous media. The preferred work is supreme and effective melt removal, which make sure by preventing the enhancement of melt film thickness in melting process and protect as thin film thickness as possible. Thus, boundary layer approximations are used for modeling of melting phenomena under the effect of heat.

The extension and novelty of this research work are pointed out as follows:

1. We take over the model as unsteady and three dimensional. The three-dimensional work can also be used for cylindrical problems.
2. The magnetic field is imposed vertically to the flow pattern to enhance the thermal boundary layer and to control the melting rate, and this effect is same as in the published work.47–49
3. The viscous dissipation has been considered in the current study to improve the heat transfer augmentations.
4. Porous medium and wall suction mechanism are jointly taken to control the melt film thickness.
5. The important physical quantities of the drag force on the disk surface and melting interface are calculated for the limiting parameters in the published work, whereas this study is extended for the other physical constraints such as Eckert number, magnetic field, porosity parameter and unsteadiness parameter.

6. The HAM technique and BVP 2.0 package have been used for the solution of the nonlinear problem.

**Formulation**

The current study deals with the melting process of a solid in a contact with a permeable and rotating disk. The disk is considered to work as a heater and rotate with a constant angular velocity ($\Omega/(1 - \beta ti)$). The phase change material is defined as an exterior force, incorporating the melting solid weight and pressure. The temperature of disk surface and melting solid remains unchanged, mutually $T_m$ and $T_w$ with $T_w - T_m > 0$. To keep this difference in temperature, the melting procedure of solid is being gains accordingly, with a layer of melt thickness above the disk $Z = \delta$. We apply similar assumptions to make simpler as assumed in earlier works. The basic governing equations in the cylindrical coordinates ($r, \Theta, Z$) are reflected as

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial r}$$

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial r}$$

$$\rho \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} - \frac{\mu}{K} = 0$$

$$\rho \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z}$$

$$\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = k$$

The temperature of disk surface and melting interface incorporating the melting solid weight and pressure. The current study deals with the melting process of a solid in a contact with a permeable and rotating disk. The disk is considered to work as a heater and rotate with a constant angular velocity ($\Omega/(1 - \beta ti)$). The phase change material is defined as an exterior force, incorporating the melting solid weight and pressure. The temperature of disk surface and melting solid remains unchanged, mutually $T_m$ and $T_w$ with $T_w - T_m > 0$. To keep this difference in temperature, the melting procedure of solid is being gains accordingly, with a layer of melt thickness above the disk $Z = \delta$. We apply similar assumptions to make simpler as assumed in earlier works.

The transformations are chosen from the available literature. The governing equations after the transformation are altered into the set of nonlinear differential equations as

$$u(r, \theta, z) = \frac{\partial u}{\partial r} F(\eta), \quad v(r, \theta, z) = \frac{\partial v}{\partial r} G(\eta), \quad w(r, \theta, z) = \frac{\partial w}{\partial r} H(\eta), \quad p(r, \theta, z) = \frac{\partial p}{\partial r} P(\eta)$$

The boundary conditions after the transformation are settled as

$$F(0) = 0, \quad G(0) = 1, \quad H(0) = - \Gamma, \quad \Theta(0) = 0$$

The valuable physical constraint of the Stefan number is defined as
The melting film thickness $\alpha$ and Stefan number $St$ in the simplified form are calculated from equation (15), while the unsteadiness parameter $S$, Eckert number $Ec$, Prandtl number $Pr$, porosity parameter $Kr$, magnetic parameter, $M$ and wall loading/removal parameter $\Gamma$ are as follows

$$S = \frac{\beta}{\Omega^2}, \quad \lambda = \frac{\frac{(1-\beta t)^2}{\Omega^2 \delta \rho}}{p'}, \quad Ec = \frac{\frac{\epsilon^2 \mu}{k(T_w-T)}},$$
$$Pr = \frac{\mu C_p}{k}, \quad Kr = \frac{(1-\beta t)\nu}{K_\gamma \Omega},$$
$$St = \frac{c_p}{\omega} \frac{T_w-T_m}{h_s}, \quad \Gamma = W_0 \sqrt{\frac{1-\beta t}{\Omega v}},$$
$$\alpha = \sqrt{\frac{\Omega}{v(1-\beta t)}}, \quad M = \frac{\sigma B_0^2 (1-\beta t)}{\Omega \rho}$$

Heat transfer rates of wall and melting are reduced due to heat fluxes at the melting surface and at the disk surface ($\delta$)

$$N_{\text{tw}} = \frac{\Theta'(0)}{\alpha}, \quad N_{\text{tm}} = \frac{\Theta'(1)}{\alpha}$$

Similarly, the shear stresses at the melting surface and at the disk surface lead to reduced wall heat rates and melting heat rates in the tangential direction

$$-\frac{G'(0)}{\alpha}, \quad -\frac{G'(1)}{\alpha}$$

The melting rate of the solid at the interface can be considered as melt velocity of phase change material by way of

$$\frac{H(1)}{\alpha}$$

The above relation defines the ultimate key and important physical quantities of interest at the disk surface and melting interface $\alpha$, $F'(0)$, $G'(0)$, $\Theta'(0)$, $F'(1)$, $\Theta'(1)$ and $H(1)$ which are calculated by fluctuating the physical constraints $M$, $Pr$, $Ec$, $\Gamma$, $S$ and $St$.

The Stefan number is calculated and obtained from equation (15)

$$St = -\frac{Pr H(1)}{\theta(1)} \quad (15)$$

$$\alpha = \frac{Pr}{St} \left(\frac{15\alpha^4 - 24\alpha^3 + 40\alpha^2 + 15}{\alpha^2} - \frac{25\alpha^2 (\alpha - \gamma)}{3} + \frac{1}{4} Ec Ma^2\right) \quad (21)$$

### Solution and average square residual error

Liao$^{50,51}$ has introduced the average square residual error technique. The initial guesses are chosen as follows

$$F_0(\eta) = \alpha \eta - \eta^2, \quad G_0(\eta) = -\frac{\eta}{\alpha} + 1, \quad H_0(\eta)$$

The initial guesses satisfy the initial and boundary conditions, while the average and square residual errors are calculated through the following technique

$$\mathbf{e}_k = \frac{1}{N+1} \sum_{j=0}^{N} \left[ N_F \left( \sum_{i=0}^{k} (F_i), \sum_{i=0}^{k} (G_i), \sum_{i=0}^{k} (H_i) \right) \right] \left[ \eta - j \eta \right]^2$$

$$\mathbf{e}_k^G = \frac{1}{N+1} \sum_{j=0}^{N} \left[ N_G \left( \sum_{i=0}^{k} (G_i), \sum_{i=0}^{k} (H_i) \right) \right] \left[ \eta - j \eta \right]^2$$

$$\mathbf{e}_k^H = \frac{1}{N+1} \sum_{j=0}^{N} \left[ N_H \left( \sum_{i=0}^{k} (H_i) \right) \right] \left[ \eta - j \eta \right]^2$$

$$\mathbf{e}_k^\Theta = \frac{1}{N+1} \sum_{j=0}^{N} \left[ N_\Theta \left( \sum_{i=0}^{k} (F_i), \sum_{i=0}^{k} (G_i), \sum_{i=0}^{k} (H_i), \sum_{i=0}^{k} (\Theta_i) \right) \right] \left[ \eta - j \eta \right]^2$$

Liao$^{50,51}$ defined the average squared residual error

$$\epsilon' = e'_k + e'_k^G + e'_k^H + e'_k^\Theta$$

Here, $\epsilon'$ represents the total square residual error.

### Results and discussion

The thin layer of melting through direct contact of the solid material with the rotating and the permeable disk
in the unsteady and three-dimensional space has been inspected. The solution of the nonlinear problem has been tackled through the BVPh 2.0 package. The sum of the total and square residual error has been analyzed. The film thickness in the absence of the wall suction and its impact versus the various constraints has been studied. The drag force coefficient and heat transfer rate at different values of the Stefan number have also been observed and displayed in the tables. The geometry of the problem is shown in Figure 1.

Figure 2 illustrates the effect of the magnetic parameter $M$ on the velocity profile $F(\eta)$. The extension in the value of $M$ declines the velocity field $F(\eta)$ and it happens due to the enhancement in the resistant force (Lorentz force). In fact, magnetic field does not allow the velocity for a cheap and clean way to flow. It is added that magnetic field is not interested in clean, run of the fluid and easy flow, but interested in imported resistance-based environment generation.

The influence of the unsteady parameter $S$ on the velocity profile $F(\eta)$ is revealed in Figure 3. The increasing value of parameter $S$ improves the resistance force to stop the flow and reduce the flow motion. Figure 4 shows the porosity influence on the velocity profile $F(\eta)$. The larger value of the porosity parameter $Kr$ prevents the growth of melting layer and consequently reduces the velocity profile. Figure 5 represents the unsteadiness with the velocity profile $G(\eta)$ (azimuthal direction) by getting increment in the value of unsteadiness parameter $S$, and it shows decrease in the velocity. In Figure 6, appearance of velocity $G(\eta)$ (azimuthal direction) displays the decrease due to porosity parameter $Kr$ by increasing its values. The Darcy law
described here by porosity parameter $Kr$ relates the flow through porous media, and this relation shows decrease in the velocity profile $G(\eta)$ (Figure 7). Figure 8 shows the temperature graph $\Theta(\eta)$ dedicated to the magnetic effect $M$ by increasing its value, and the velocity profile shows an increase due to the effect of Lorentz force. The direct relation between the unsteady and the temperature profile shows increase in $S$, which in turn increases the temperature.

Figure 9 shows increasing and decreasing behaviors of temperature for higher and lower estimation of Eckert number $Ec$. The temperature field is expanding on the high values of the Eckert number and declining at the negative values of the Eckert number based on the melting nature of the thin layer. In fact, the Eckert number is the ratio of specific kinetic energy and the specific enthalpy; due to more enthalpy, the velocity profile showed more increase.

Figure 10 indicates the Prandtl number effect versus the temperature field. It expresses that Prandtl is estimated with higher value and produces a more decrease in temperature. In other words, the temperature is ordered to fall down when the instant Prandtl number $Pr$ increases. Prandtl number $Pr$ is directed toward one of the cooling or heating position.

The impact of the suction parameters versus the radial, angular and axial velocities is shown in Figures 11–13. The increasing values of the suction parameter produce hurdles in the expansion of melting layer and consequently decrease the radial, angular and axial velocities. The temperature field expanded with the enlarging values of the suction parameters as reflected in Figure 14. Also, the graphical results are compared with the velocity profile and temperature distribution of Turkyilmazoglu.\textsuperscript{61}

In the above discussion on the velocity profile and temperature distribution, it has been shown graphically that the magnetic field, porosity and unsteadiness
parameter decrease the velocity of the flow due to which the melting liquid can easily control through suction. In fact, a large amount of melt can be saved with the joint influence of the magnetic field and porosity. The strengthening in a magnetic field, Eckert number, suction constraint and unsteadiness parameter expands the temperature profile, which will help in the melting process as shown in Figures 1–15.

Figures 16 and 17 indicate the melting rate in axial direction which enhance the rate with the rising value of suction and Stefan constraints. Figures 18–20 illustrate the melting rate at the disk surface and at the melting interface in angular direction and the effect matching of Turkyilmazoglu. Figures 21–23 directed the heat transfer rate with different approaches of suction and Stefan number. Again the influence of the above figures agrees with of Turkyilmazoglu, whereas Figure 24 shows the Stefan constraints to control the melting rate.

The numerical results of the drag force to find the melting rate at the location of rotating disk surface and
at the point of melting interface are shown in Tables 1 and 2. The increasing values of the parameters $M, S, Kr$ increase the coefficients of the drag force surface of the rotating disk and melting interface. Table 3 shows the Nusselt number numerically at the disk surface and at the melting interface. Through Table 3, it is decided that decreasing reforms to be introduced in the $Y(0)$ sector for provision of quick increment to the $Ec$. Table 3 urges to upgrade the $Y(0)$ according to the present needs with the augmentation of the Prandtl number $Pr$.

**Conclusion**

The melting of a solid material past over a permeable disk is studied. The medium is also considered porous in the presence of the transverse magnetic field applied...
to the flow pattern. Furthermore, the three-dimensional heated plate is rotating and unsteady. The previous study considered the steady case with wall suction, while the present work is considered the unsteady that includes wall suction, magnetic field, porous media and viscous dissipation. The BVPh 2.0 package has been used for the solution of the nonlinear problem up to the 20th order approximations. The parameters effect on physical melting flow, such as wall suction parameter, magnetic parameter, porosity effect, unsteadiness effect, skin friction coefficient, Nusselt number and temperature distribution effects, are shown and discussed.

The main findings are pointed out as follows:

- The addition of the porosity is used to preserve a thin melt layer film thickness and also used to control the removed melt.
- In the present study, the insertion of the unsteady parameter accomplishes the flow of melt layer downward more accurately.
- The Stefan number jointly works with magnetic field and Eckert number to provide more specific heat, which facilitates for greater melting rate.
- The wall suction works easily and preserves greater amount of melt at disk surface.

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### ORCID iD

Taza Gul [https://orcid.org/0000-0003-1376-8345](https://orcid.org/0000-0003-1376-8345)

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**Appendix I**

**Notation**

- $B_0$ uniform magnetic field (T)
- $C_p$ specific heat (J kg$^{-1}$ K$^{-1}$)
- $Ec$ Eckert number
- $h_f$ latent heat (J kg$^{-1}$)
- $k$ thermal conductivity (W m$^{-1}$ K$^{-1}$)
- $k_r$ permeability constant (m$^2$)
- $Kr$ porosity parameter
- $Nu$ Nusselt number
- $P$ pressure (Pa)
- $Pr$ Prandtl number
- $(r, \Theta, z)$ cylindrical coordinates in the radial, azimuthal and axial directions
- $R$ radius of the cylinder (m)
- $S$ unsteady parameter
- $St$ Stefan constraint
- $t$ time (s)
- $T$ local temperature (K)
- $T_m$ melting temperature (K)
- $T_w$ temperature at wall (K)
- $(u, v, w)$ velocity components in $(r, \Theta, z)$ directions (m s$^{-1}$)
- $(\alpha, \gamma, \delta, \eta, \lambda, \mu, \rho, \sigma, \omega)$ non-dimensional thickness of the melting wall suction, thickness of the melt film layer, similarity variable, pressure parameter, dynamic viscosity (Kg m s$^{-1}$), density (Kg m$^{-3}$), electrical conductivity (s m$^{-1}$), “s” is Siemens, kinematic viscosity (m$^2$ s$^{-1}$)
- $\Omega$ rotation of disk