Research of the influence of rectangular cutout dimensions on the stress-strain state of shells using an effective finite element model

V N Bakulin1,2
1Department of Aircraft Design and Certification, Moscow Aviation Institute (National Research University), 4 Volokolamskoe shosse, Moscow 4125993, Russia
2Laboratory of Non-Classical Models of Mechanics of Composite Materials and Structures, Institute of Applied Mechanics, Russian Academy of Sciences, 7 Leningradsky Avenue, Moscow 125040, Russia
E-mail: vbak@yandex.ru

Abstract. The effect of the dimensions of a rectangular cut on the stress-strain state of a cylindrical shell using an effective finite element model is investigated. A cylindrical finite element rectangular in the plan is used for thin moment shells built on the Kirchhoff – Love hypotheses. The considered finite element is based on functions of the form received at approximation of the generalized deformations with the subsequent satisfaction to the equations of compatibility of deformations. The functions of the form accurately consider displacements as rigid bodies. A comparison of convergence of results obtained with the help of the considered finite element and other known finite elements of foreign authors is given. The example of a cylindrical shell with notches shows a good correspondence of the solution radiated by the considered finite element with the method of grids, as well as the efficiency of the applied finite element having 20 degrees of freedom as compared to the common finite element having 24 degrees of freedom. The influence of the cutout dimensions on the maximum values of deflections and longitudinal forces for a cylindrical shell is shown.

1. Introduction
When creating aviation and space technology, it is often necessary to break the continuity of structural elements, including shells, of various kinds of cutouts (having structural, technological or other purposes), which lead to a weakening of the carrying capacity of structures. Insufficient knowledge of the true picture of the stress-strain state can lead to destruction or an increase in the weight of the structure. The study of the stress-strain state of shells weakened by cuts is a current scientific and technical problem, despite many works devoted to solving this problem. This explains the continuous improvement of methods for calculating stress-strain state (SSS) of structural elements, especially shells with cutouts.

A number of reviews are related to the analysis and development of methods for the calculation of shells with notches and shells with variable parameters, for example [1, 2]. Thus, paper [1] gives an overview of publications on analytical, numerical and analytical computational approaches, as well as experimental methods for the study of homogeneous and heterogeneous, including orthotropic shells with cutouts of various shapes and sizes.
The review [2] deals with the studies of static and dynamic deformation of isotropic and anisotropic elastic shells with variable parameters using classical and refined theories. To solve two-dimensional boundary value problems, an unconventional discrete-continuum approach is used which is based on a spline approximation of unknown equation functions in partial derivatives with variable coefficients. It allows to reduce an initial problem to system of the one-dimensional problems solved by a method of discrete orthogonalization. The analysis of numerical results was carried out. Emphasis is placed on the accuracy of the results.

The tasks of shell statics with holes were considered in [3, 4]. A numerical and experimental study of a cylindrical shell with a neckline is presented in [5]. The influence of circular holes, their size and location on mechanical behavior and strength of cylindrical shells was studied in [4]. The influence of circular holes on mechanical behavior of a three-layer carbon fiber cylinder with a lattice filler was considered in [6].

The analysis of the given and many other works shows that in the overwhelming majority of them the strained - deformed condition of shells with curvilinear holes and mainly circular holes was studied. The most common in practice are rectangular cutouts in aspect of shape. If for the shells weakened by curvilinear cutouts it is still possible to try to apply analytical methods to calculate the stress-strain state, then for the shells with rectangular cutouts the use of analytical methods faces great, and often impossible to overcome, mathematical difficulties [1]. Therefore, to solve this scientific and technical problem, experimental methods are widely used to study the stress-strain state of shells with cutouts, for example [5]. But the most common and appropriate are numerical calculation methods and especially the finite element method.

The finite element under consideration in this paper is based on form functions obtained by the approximation of generalized deformations with subsequent satisfaction of deformation compatibility equations. The shape functions take exact account of movements as rigid bodies. Efficiency of the presented approach (high speed of convergence of numerical results) that allows to reduce necessary for calculation number of FE and consequently to reduce an order of systems of equilibrium, i.e. to lower dimension of solved problems in comparison with traditionally used approximations is shown. This is especially relevant in the problems of calculation of shells with notches, because of the observed concentration of stresses near the notches it is necessary to thicken the grid of breaks in FE, i.e. increase the number of FE, reducing their size. This leads to cumbersome finite element models (FEM), which significantly complicates and limits the use of such models, have to work with large amounts of information, which greatly complicates the processing of the numerical results obtained. At the large dimension of the tasks the calculation errors, the necessary computer resources and the time of calculation increase. This makes it difficult to research such structures with proper accuracy. In this connection, the problem of decreasing the dimensions of the FEM for calculating envelope elements of structures with cut-out is important.

The purpose of the research in this article is:
- consideration and description of the approach that allows reducing the dimensionality of the tasks to be solved in comparison with the traditionally used approximations;
- the study of the convergence of the results obtained with the finite element in question by comparison with other known data;
- study of the effect of cutout sizes on the largest values of deflections and longitudinal forces for a cylindrical shell.

2. Description of the final element in question

In this paper, we study the effect of the dimensions of a rectangular cutout in aspect of shape on the stress-strain state of the cylindrical shells of rotation by means of an effective finite element model. A two-dimensional rectangular cylindrical finite element (FE) is considered for calculating thin moment shells whose theory is based on the hypotheses of Kirchhoff-Lava.

The form functions of the considered FE, as well as in [7], are based on approximation of generalized deformations ($\varepsilon_1$, $\varepsilon_2$, $\gamma$, $\varphi_1$, $\varphi_2$, $\chi$) with subsequent satisfaction of the deformation compatibility
equations and include movements as rigid bodies. In contrast to work [7], in which the three deformation compatibility equations are not justified because of mathematical difficulties, the three deformation compatibility equations are replaced by four equations in obtaining functions that approximate generalized deformations.

Such approach, when functions of the finite element form are based on approximation of generalized deformations with the subsequent satisfaction of the equations of deformation compatibility and include displacements as rigid bodies, allows to reduce the number of finite elements necessary for calculation, that is to increase the rate of convergence of numerical results. Such finite elements with a high rate of convergence of numerical results will be called effective FEs.

Efficiency of the considered finite element of shells of rotation of the cylindrical form is explained by efficiency of finite element approximations of functions of the generalized movings (further we will name them as approximating functions of movings).

At first, the approximating functions of the displacements are obtained, which are responsible for the displacements of the considered finite element as a rigid body. These approximating functions of movements as a rigid body are defined by integration of the relations connecting components of a vector of deformations with movements, at zero values of the generalized deformations.

Then we obtain the approximating functions of displacements responsible for deformation of the considered finite element. These finite element approximation functions of displacements responsible for the deformation of cylindrical shells are based on the initial approximation of generalized deformations of these shells. At the same time, functions which approximate deformations and satisfy the equations of deformation compatibility are constructed.

By integrating relations connecting components of a strain vector with displacements, at the received expressions of approximating functions of the generalized deformations, functions, approximating the displacements responsible for deformation of a considered final element, we define finite element approximations of functions of displacements.

The forces and moments are calculated through the components of the deformation vector using physical relations (Hook's Law). Further solution of the problem is carried out using FEM procedures in the variant of the displacement method.

Reliability, convergence, accuracy of the results obtained with the help of the considered model and efficiency of the constructed finite element of the shell of cylindrical form rotation were investigated on a number of well-known and widely spread for shell finite element tests by comparison with analytical solutions and numerical results obtained with the help of known finite elements of foreign authors. One such test was an example of the solution of S.P. Timoshenko's problem [8] for a free circular cylindrical shell loaded with two symmetrical diametrically opposed concentrated radial forces.

3. Results and discussion
Reliability and accuracy of the results obtained with the help of the considered model, and the efficiency of the constructed finite element for the calculation of the tensely deformed state of the shells of rotation of cylindrical shape with rectangular in plan notches was checked by the following example.

For a circular cylindrical shell with four symmetrically located on a circle rectangular in plan cutouts at action of the axial compressive force uniformly distributed on faces of a shell, good conformity of the decision is shown, obtained with the help of the considered finite element of cylindrical shells (diagrams marked by dots), with the method of grids (diagrams marked by triangles), as well as the efficiency of the considered FE, which has 20 degrees of freedom, in comparison with the widely known finite element of circular cylindrical shell G Cantin, R W Glagh [9], having 24 degrees of freedom (graphs shown by lines without points) (figure 1).
Figure 1. Comparison of numerical results obtained using the considered finite element of the circular cylindrical shell and the widely spread finite element G Cantin, R W Glagh [9], and also comparison with a method of grids on an example of calculation of the stress-strain state of a shell of rotation of the cylindrical form with four symmetrically located on a circle rectangular in the plan cutouts.

Parameters of the investigated shell of rotation of the cylindrical form with rectangular in shape cutouts and notches are as follows:

\[ \frac{L}{R} = 2.24; \quad \frac{h}{R} = 0.225 \times 10^{-2}; \quad \ell_1/L = 3/7; \quad \ell_2 = \frac{\pi}{4} \cdot R; \]

where \( L \) – shell length, \( R \) – shell radius, \( h \) – shell thickness, \( \ell_1 \) and \( \ell_2 \) – the cut dimensions in the axial and circumferential directions respectively.

Due to symmetry, 1/16 of the shell was considered, which was divided into finite elements as follows: in the circumferential direction into 12 elements, in the axial direction into 7 elements. The total number of unknowns is \( n = 366 \), the half-width of the tape of the system of equations is \( m = 46 \).

Figure 1 shows the graphical dependencies of the coefficient \( k_w = \frac{w}{Eh/pR} \) for the points of the considered shell rotation of cylindrical form with rectangular in plan notches for three cross sections perpendicular to the axis of the shell. Points 1-3 of the investigated circular cylindrical shell with rectangular cutouts are located in the cross section perpendicular to the axis of the shell and passing on the curvilinear edge of the cutout, and points 4, 5 are located in the cross section of the shell perpendicular to the axis of the shell and passing through the middle of the rectangular edge of the cutout. The upper two graphs refer to points located in the cross section of the shell, perpendicular to the axis of the shell and passing at an equal distance from the edge of the shell and curved edge of the cutout.

The comparison of numerical results obtained by the mesh method, as well as by means of the considered finite element of the circular cylindrical shell and the widely spread finite element G Cantin, R W Glagh was carried out [9], has shown reliability and accuracy of the model used for construction of a finite element, and also efficiency (higher convergence speed) of applied FE that has allowed to reduce number of finite elements necessary for calculation in comparison with widespread finite
elements on an example of calculation of tensely-deformed state of a shell of rotation of the cylindrical form with four rectangular in the plan of cutouts.

At present, the most accurate models for calculating inhomogeneous shells are models based on layer-by-layer analysis of the stress-strain state. Due to its effectiveness, the considered approach to obtaining the functions of the FE form will be relevant for the construction of models of layer-by-layer analysis of the stress-strain state of non-homogeneous cylindrical irregular shells, especially weakened by cut-outs.

The problem of investigating the influence of rectangular cut sizes on the tensely deformed state was solved for a hinged cylindrical shell rotation with a rectangular cutout under the action of an axial compression load uniformly distributed at the ends of the shell.

The parameters of the investigated shell are as follows: \( \frac{L}{R} = 2.24; \frac{h}{R} = 0.0025; E=19.6 \cdot 10^4 \text{MPa}; \mu = 0.3 \), where \( L \) is shell length, \( R \) is shell radius, \( h \) is shell thickness, Young modulus and Poisson's coefficient respectively.

As an example, we considered a cut with an axial dimension \( l_c \) of 20% of the shell length, and in the circumferential direction, the angle of the cutout was varied within \( 36^\circ \leq \theta \leq 96^\circ \) with a step of \( 12^\circ \).

Due to the symmetry, 1/4 of the shell was considered, which was divided into the same finite elements as follows: in the axial direction of 10 elements, in the circumferential direction - into 30 elements. The kinematic boundary conditions at the ends of the shell correspond to an articulated support and look:

\[
\begin{align*}
v &= 0 \\
w &= 0 \\
\vartheta_x &= 0,
\end{align*}
\]

where \( v \) and \( w \) are moves in radial and tangential directions; \( \vartheta_x \) – rotation angle relative to the axis of the shell.

The influence of the cutout size on the maximum values of deflections and longitudinal forces is shown in figure 2. It shows the graphical dependences \( k_w = \frac{w \cdot E h}{p R} \) at points located in the middle of the curved (maximum values are observed here) and the rectilinear edges of the cutout (graphs 1 and 2, respectively), as well as \( k_N = \frac{N}{p} \) (where \( p \)-axial compressive forces are \( p \)-uniformly distributed over the shell ends) in the vicinity of the cut-out corner point (graph 3) from the cut-out solution angle \( \theta \).

The value of deflections and longitudinal forces increases significantly as the size of the cut increases, and the dependencies of \( k_w \) and \( k_N \) on the angle \( \theta \) are nonlinear. The effect of changing the size of the cutout on \( k_w \) and \( k_N \) in the circumferential direction is stronger than in the axial direction.

The received graphic dependences can be used at designing of designs in the form of a shell of rotation of the cylindrical form with rectangular in the plan cut-outs.

4. Conclusion
The approach and the algorithm of construction of effective finite element model for calculation of the stress-strain state of a shell of rotation of the cylindrical form with rectangular cutouts in the plan is considered. The reliability and accuracy of the results obtained with the help of the considered model is shown, as well as the efficiency of the constructed finite element of the circular cylindrical shell with four symmetrically arranged rectangular notches on the circumference under the action of the axial compression force uniformly distributed at the ends of the shell, by comparison with the method of grids and with the widespread finite element of the circular cylindrical shell G Cantin, R W Glagh. The influence of rectangular cut solution angle on the stress-strain state of circular cylindrical shell is studied. Graphic dependencies of the cutout size influence on the largest values of deflections and longitudinal forces for the rotation shell of cylindrical shape with rectangular cutout are given.
Figure 2. The influence of the angle of the cutout solution on the maximum values of deflections observed in the area of the middle of the curvilinear and straight edge of the cutout (Graphs 1 and 2, respectively) and longitudinal forces in the vicinity of the corner point of the cutout (Graph 3).

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