Non-Ideal Equation of State, Refraction and Opacities in Very Cool, Helium-Rich White Dwarf Atmospheres

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Abstract.

The atmospheres of cool, helium-rich white dwarfs constitute an exotic and poorly explored physical regime of stellar atmospheres. Under physical conditions where the temperature varies from 1000K to 10000K, the density can reach values as large as 2 g/cm$^3$, and the pressure is as high as 1 Mbar, the atmosphere is no longer an ideal gas and must be treated as a dense fluid. Helium atoms become strongly correlated and refraction effects are present. Opacity sources such as He$^-$ free-free absorption must be calculated with a formalism that has never been applied to astrophysical opacities. These effects have been ignored in previous models of cool white dwarf atmospheres.

1. Introduction

Very cool white dwarfs (WDs) with $T_{\text{eff}} < 5000$K are among the oldest objects in our Galaxy. They are of great importance for cosmochronology and for understanding the formation and evolution of the Milky Way. Our main interest is in the atmospheres of these stars as the link between their physical characteristics and the observables. In this contribution we concentrate on the atmospheres rich in helium. Due to the high transparency of helium (compared to hydrogen), our current models predicts for them densities that are typical of liquids, up to $2 - 3$ g/cm$^3$. Under such extreme conditions, the input physics of atmosphere models needs to be reconsidered.

There are several physical effects that distinguish fluid from gaseous atmospheres. They all arise from the strong interaction between particles. Helium atoms become strongly correlated, the refractive index of the fluid departs from unity, and free electrons strongly interact with the medium, which affects the opacity.

Non-ideal effects in the equation of state are dominated by He – He interactions. For a given density and temperature, they result in increases in the pressure and ionization fraction and a decrease in the adiabatic gradient.

The refractive index inside helium-rich white dwarf atmospheres departs from unity and can be as large as 1.35. We have solved the equation of radiative
transfer for refractive stellar atmospheres for the first time (Kowalski & Saumon 2004). The impact of the total internal reflection on the radiative equilibrium results in an increase of the temperature in the optically thin atmospheric layers. This decreases the abundance of hydrogen molecules and the $\text{H}_2-\text{He}$ CIA opacity in atmospheres of mixed composition. Due to strong refraction near the surface, the limb darkening almost disappears.

Fluid atmospheres require a new description of the opacity that is different from that for diluted gases. The calculation of the $\text{He}^-$ free-free absorption (or inverse bremsstrahlung) in WD atmospheres must be revised. Preliminary results obtained from quantum molecular dynamics simulations (QMD) suggest that fluid, helium rich, cool WD atmospheres are much more opaque than predicted by current models.

All these effects must be considered to properly model cool WD atmospheres. Realistic models of very cool WDs are of fundamental importance for cosmochronology and to understand the spectra and physical properties of the coolest WDs, especially the recently discovered, peculiar ultracool stars like LHS 3250.

2. The Non-Ideal Equation of State

Our equation of state (EOS) for fluid H/He mixtures includes the following species: $\text{H}_2$, $\text{H}$, $\text{H}^+$, $\text{He}^+$, $\text{He}^-$, $\text{He}$, $\text{He}_2^+$, $\text{He}^+$ and $e^-$. Currently, only the He–He interactions are included as they dominate the non-ideal contributions to the EOS. These interactions are described with an effective pair potential (Ross & Young 1986) that is calibrated to high-pressure EOS data (Nellis et al. 1984). The EOS is computed in a manner similar to that of Bergeron, Saumon &

![Figure 1](image_url)

Figure 1. Non-ideal effects in WD atmospheres of $T_{\text{eff}} = 4000\text{K}$, $\log g(\text{cm/s}^2) = 8$ and various He/H compositions. Starting from the bottom $n(\text{He})/n(\text{H}) = 10^2, 10^4, 10^6$ and pure He. Shown is the ratio of the total pressure $P$ over the ideal gas pressure $P_{\text{id}}$. 

Wesemael (1995) but explicitly includes non-ideal terms in the calculation of the chemical equilibrium of the H/He mixture. We are therefore able to compute continuous sequences of models from pure H to pure He composition in a self-consistent manner with the non-ideal EOS. The importance of the non-ideal terms in the EOS is illustrated in Fig. 1. As the He content increases, the overall atmospheric opacity decreases and the pressure rises. Clearly, the non-ideal effects dominate the EOS for \( n(\text{He})/n(\text{H}) \geq 10^2 \). Because of the repulsive He–He interactions, it is energetically favorable for He to break up into ionized species, and the degree of ionization increases.

3. The Refraction

The variation of the refractive index inside cool WD atmosphere models is displayed in Figure 2.

![Figure 2. Refractive index inside WD atmospheres of \( T_{\text{eff}} = 4000\text{K}, \log q(\text{cm/s}^2) = 8 \) and various He/H compositions. Starting from the bottom \( n(\text{He})/n(\text{H}) = 0 \) (pure H), 10, 10^2, 10^3, 10^5 and pure He. In the pure H atmosphere, the refractive index arises in fluid H\textsubscript{2}.]

Applying geometric optics we can obtain the radiative transfer equation modified for refraction (Cox & Giuli 1968), which has to be solved along curved, and eventually totally reflected ray-paths. In the atmospheres of cool WDs, the refractive index returns to unity far above the photosphere. Therefore, the main effects on refraction are: 1) a significant weakening of limb darkening (from 0.60 to 0.96 for pure He models of \( T_{\text{eff}} = 4000\text{K} \)), 2) a reduction of the opacity, and 3) an increase of temperature in the radiative zone (Fig. 3). This last phenomenon is a consequence of total internal reflection, which occurs in the upper atmospheric layers (Kowalski & Saumon 2004). In mixed H/He models, the larger temperature results in the dissociation of H\textsubscript{2} and a reduction in the H\textsubscript{2} CIA absorption in the infrared (Fig. 3). The effect is strongest in the \( K \) band where the flux is increased by \( \sim 30\% \) in this particular model.
4. The Opacity of Fluid Helium

The third, and probably the most important density effect in He-rich WD atmospheres is its impact on the opacity. We suspect that the description of opacities presently used in modeling of He-rich WD atmospheres is inadequate because the widely used standard expressions for opacities of H and He species are valid for tenuous gases, not fluids! The absorption cross section of the important He$^-$ free-free process is derived for an isolated atom-electron collision, while in a dense fluid, the absorbing electron interacts strongly with many surrounding helium atoms.

So far, a reduction of the He$^-$ ff absorption and the Rayleigh scattering due to strong, collective interactions in dense helium, has been reported (Iglesias, Rogers & Saumon 2002; hereafter IRS02). This calculation, done in the Born approximation, is not quite justified for strong interactions. However, IRS02 show that these two sources of opacity in dense helium-rich WD atmospheres may be reduced by factors as large as 20.

Due to its complexity, a fully analytical, quantum mechanical description of absorption processes in strongly correlated fluids does not exist. However, we can get valuable insight into the opacities of fluid helium from quantum molecular dynamics simulations (Mazevet et al. 2003). The results are surprising. The opacity obtained for pure helium at $T = 5802 K$ and $\rho = 0.5 \text{ g/cm}^3$ is flatter and much larger than that of IRS02 (Figure 4). Moreover, Kramer's $\nu^{-3}$ behavior of the free-free absorption coefficient is strongly suppressed.

The frequency behavior of the absorption coefficient obtained from simulations is similar to the prediction of the classical model (Jackson 1975). In the classical picture the opacity for small frequencies is asymptotically constant because in the presence of the slowly varying electric field of the electromagnetic wave (photon), the mobility of electrons is determined by the frequency of...
Figure 4. Absorption coefficient $\alpha$ for pure He at $T = 5802$K and $\rho = 0.5$ g/cm$^3$. Lines represent the results from QMD simulations (solid), He$^-$ ff of IRS02 and the EOS from §2 (dashed), and He$^-$ ff of IRS02 with $f_e$ extracted from QMD simulations and the classical model extension for photon energies $\leq \gamma_c = 2.5$ eV $\sim 12.8$ µm (dotted).

electron-atoms collisions. This effect is present also in quantum systems. The average frequency of electron-atom collisions $\gamma_c$ (damping parameter) for the conditions in Figure 4 is $\gamma_c = 2.5$ eV. The ionization fraction $f_e = n(e)/n(He)$, extracted from the simulation, by a fit to the classical model is $f_e = 4.8 \cdot 10^{-7}$, compared to $f_e = 1.8 \cdot 10^{-10}$ from our present EOS (§2). Moreover, extending the He$^-$ ff opacity with the classical model for frequencies smaller than $\gamma_c$ and using the ionization fraction extracted from the simulations, we get an opacity that is in good agreement with the simulations! This apparently successful description of the quantum system with a classical model is preliminary and needs to be better understood. Taken at face value, these results suggest that multiple electron-atom collisions are important, and that the ionization fraction predicted by our EOS is incorrect.

5. Implications

Based on the dense fluid physics described here (non-ideal EOS, refraction, and He$^-$ ff and Rayleigh scattering of IRS02 but not the QMD opacities), we have computed a sequence of cool WD atmosphere models (Fig. 5). The comparison with the observed sequence of cool WDs (Bergeron, Ruiz & Leggett 1997; hereafter BRL97) suggests that the coolest WDs have mixed H/He atmospheres, rather than the pure helium composition assigned to them in BRL97. However, our prediction for their atmospheric abundance $n(He)/n(H) \sim 10^6$ seems too high for disk WDs. Assuming a hydrogen accretion rate of $10^{-17} M_\odot$/year, and a maximal thickness of the convection zone of $10^{-3} M_\odot$ [Hansen 2004], the largest expected abundance of helium is $n(He)/n(H) \sim 10^4$. Since the QMD simulations predict that pure helium is much more opaque than in these models, we anticipate that the fit of the coolest WDs with more complete models will result in a much smaller He/H ratio.

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Figure 5. Color-color diagram for cool WDs. Data points come from the BRL97 sample of H-rich (filled triangles) and He-rich (open squares) disk WDs. Sequences of models with mixed H/He composition and constant $T_{\text{eff}}$ are shown by dotted lines (from top to bottom $T_{\text{eff}} = 4000\text{K}$, $5000\text{K}$, $6000\text{K}$). Dashed lines connect models with the same He/H ratio. All models have $\log g(\text{cm/s}^2) = 8$.

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